Three Dimensional Cellular Automata Sand Castle construction Model Based on Step Factor

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Abstract. It seems that everyone has the experience of making sandcastles on the leisure beach. In order to know how to build a beautiful and sturdy beach castle, this article simulates and analyzes the process of sand castles being eroded by water, and obtains some factors that extend the life of sand castles. First, we model the waves. Simplify the waves into a sine wave. Using calculus and fluid mechanics, the water wave impact force on a particle on the sand castle was calculated. Second, we model the sandcastle. Discretizing time, space, and state, we innovatively introduce the concepts of friction and impact into the model, and propose a new Three-dimensional Cellular Automaton model combined with step factors. We consider each gravel or water molecule as a cell, and simultaneously introduce the moving probability and collision probability to describe the moving trend and collision between the gravel cells, respectively. In order to effectively describe the loss of sand castles in the waves, the gravel cells that are in direct contact with the water cells will receive an impact force calculated by the first model. According to the law of large numbers, we use enough cells to simulate the erosion of sandcastles by the waves. It was found that within a certain range, the farther from the ocean and the more the total gravel, the stronger the anti-erosion ability of the sand castle.

1. Introduction
In my childhood, the beach was always a paradise for children. Each grain of sand seemed to carry the laughter and laughter of children, and each sand castle carried the children’s imagination. The boy who dreams of becoming a knight always dreams of building a solid castle for his princess. However, his childhood dream castle always could not resist the day and night washing of the sea. The tide and fall of the night, before the fog disappeared in the morning, turned into a broken wall, washed by the fine waves, and splashed with beautiful white foam. "Based on the sand, how can this 'castle' be strengthened under the wash of the waves?" With a lot of doubts, the children are gradually growing up. The junior high school, high school, and university knowledge have filled their minds. The problem they have also studied is: what kind of sand-water mixing ratio is used to build what kind of 3D model can sand castles be stronger and more durable? At the same time, in recent years, the "tofu dregs project" has frequently appeared. How to make buildings near the water stronger and have a longer service life has always been a concern in the field of construction engineering.

2. Models
2.1. Offshore Wave Impact Module

2.1.1. Model establishment. Because the sea area corresponding to the beach is offshore, and there is a low sea breeze wind speed in the offshore waters, the gentleness of the sea bottom near the beach is considered to be constant. Therefore, in this article we ignore the effect of offshore wind speed on the wave speed and consider only the wave. And we assume that the waves hit the coast at a uniform speed. The following is the assumption of the wave shape of the ocean. Since the shallow water spectrum in the offshore area is mostly a mixed single-peak wave spectrum, the situation is too complicated to be suitable for later modeling. Therefore, this article assumes the waveform of the offshore wave as a sine wave. Based on these assumptions, a force analysis is performed on the offshore coast, and the force per unit area under the impact of a periodic wave is finally obtained. In this way, the problem is simplified to the relationship between pressure and velocity in basic physics.

2.1.2. Model solving. Step 1: the establishment of the basic model diagram of the waves
Based on the above assumptions, we simplified the seawater waveform into a periodic sine wave. During the calculation process, we calculated the wave in a periodic period.

Step 2: Impact of water waves
For a sine wave that simulates an ocean wave, we set its wavelength to $\lambda$, wave velocity to $v$, and amplitude to $A$, and at the same time, the density of the seawater is temporarily set to $\rho$.

The impact force of seawater on a certain point is given by the seawater above the horizontal line at this point. Let the height of this point be $h$. We use the micro-element method to divide the three-dimensional ocean wave into countless thin slices with a thickness of $\Delta x$, and calculate size of the area. Then multiply by the unit thickness $\Delta x$ to get the target seawater volume.

$$ S = \int_{\arcsinh A}^{\pi - \arcsinh A} (\sin x - h) \, dx $$

(1)

The details are shown in the following figure:
From this we can use the integral operation of the function to solve (let \( \text{th} = \sin t \)):

\[
V = (2\cos t + 2t\sin t - \pi \sin t)\Delta x
\]  

(2)

After that, bring in the seawater density \( \rho \) and wave velocity \( v \) to calculate the momentum:

\[
\Delta p = (2\cos t + 2t\sin t - \pi \sin t)\Delta x \rho v
\]  

(3)

Based on the relationship between momentum and force, we can calculate the impact force of the water wave per unit time:

\[
F = \frac{\Delta p \sin \theta}{T}
\]  

(4)

2.2. Collision Module

2.2.1. Issues to consider in grit modelling. We added the concepts of friction and collision forces to the classic cellular automata model, and considered the interaction between gravel and gravel, and between gravel and water molecules, and constructed a new cellular automata model to effectively combining the advantages of continuous and discrete models, the calculation speed and accuracy are improved.

2.2.2. Model establishment. Step 1: Establishment of 3D Cellular Automata.

This paper uses a 3D cellular automaton Moore-type structure to divide the entire space into uniform 3D grids, where each grid (cell) is a basic unit. As shown in the figure 3, there are only three states of any grid: Occupied by gravel, occupied by water molecules, or empty.

![Figure 3. 3D Cellular Automata Structure](image)

The tendency of all gravel to move to 26 adjacent grids around it at each time step (discrete) with the probability shown in the following formula:

\[
P_{ijk} = N_{ijk} e^{w_s S_{ijk} + w_p F_{ijk}(1 - n_{ijk})\sigma_{ijk}(1 - \varphi_{ijk})}
\]

\[
N_{ijk} = \frac{1}{\sum_{ijk} e^{w_s S_{ijk} + w_p F_{ijk}(1 - n_{ijk})\sigma_{ijk}(1 - \varphi_{ijk})}}
\]  

(5)
Among them, \(i, j, k\) are the position coordinates of the cell, \(P_{ijk}\) is the probability that the gravel has a tendency to move; \(n_{ijk}\) is the idle coefficient of the cell. If the cell \((i,j,k)\) is already occupied by the gravel, then \(n_{ijk} = 1\), otherwise \(n_{ijk} = 0\); \(N_{ijk}\) is the normalization parameter, so that the sum of the probability of the gravel in 27 directions is equal to 1;

\(S_{ijk}\) and \(W_s\) represent the attractiveness to the gravels where the grid points \((i,j,k)\) are located and their influence coefficients. We use this to represent the moisture content and compactness between the gravels.

\(F_{ijk}\) and \(w_F\) represent the possibility of the grid point \((i,j,k)\) being impacted by the surrounding gravel and its influence coefficient. We use this to represent the degree of influence of impurities between the sand-water mixture on the gravel.

And \(0 \leq w_s \leq 1\), \(0 \leq w_F \leq 1\). The larger the collision coefficient \(w_F\) and the smaller \(w_s\), the collision between gravels will tend to occur.

Step 2: Specify cell movement trends

Different from 2D cellular automaton Moore-type structure, the \(\varphi_{ijk}\) defined here indicates whether the cell \((i,j,k)\) exists. If the cell \((i,j,k)\) does not exist, \(\varphi_{ijk} = 1\). Otherwise, \(\varphi_{ijk} = 0\). As shown in the figure 4, cell \((1,1,1)\) does not exist, then \(\varphi_{111} = 1\). Different from the two-dimensional cellular automaton Moore-type structure, the \(\varphi_{ijk}\) defined here indicates whether the cell \((i,j,k)\) exists. If the cell \((i,j,k)\) does not exist, \(\varphi_{ijk} = 1\). Otherwise, \(\varphi_{ijk} = 0\). As shown in the figure below, cell \((1,1,1)\) does not exist, then \(\varphi_{ijk} = 1\). And the parameter \(\sigma_{ijk}\) is used to indicate that the gravel is moving to the current layer or the next layer or the previous layer Mobile trend. Under normal circumstances, gravel tends to move to the next layer, we set the movement trend of the gravel to the highest, set to 1.5. The second is at the same level, the least likely is to move to the upper level, which is consistent with objective facts. So set the parameter \(\sigma_{ijk}\) as:

\[
\sigma_{ijk} = \begin{cases} 
1.5, & \text{Next level;} \\
1, & \text{At the same level;} \\
0.5, & \text{Previous level}
\end{cases}
\]

Step 3: Determine the collision probability of gravels A and B:

Within a period of time, the sand particles may be pushed up by the waves, they may be brought down, or they may return to their original position. We can describe this phenomenon with probability, called collision probability.

Suppose that a gravel A is located at the cell \((i,j,k)\), and the collision may come from a cube region with \((i,j,k)\) as the center and a separation distance of 1. Set this area as gravel A. We call Envelope Schematic.

When other gravels B intrude into the envelope, it is considered that there is a possibility of collision between gravels A and B.

Unlike 2D planes, the possibility of collision in 3D space comes from the longitudinal (x), lateral (y), and vertical (z) directions. If the distance in a certain direction is less than a critical value in that direction (set by ourselves), it means that overlap may occur and there is a possibility of collision. We describe this phenomenon through a simple force analysis in figure 3 and figure 4. The force on each gravel cell in a certain direction is expressed as the probability of the movement trend in this direction. The greater the force, the greater the probability.
Figure 4. Force of gravel cells in space

Let the probability that the gravels A and B overlap in the longitudinal, lateral, and vertical directions be $p_{ij}(u = x, y, z)$. Then the product of the probability of the gravels A and B moving is the collision probability:

$$P_u = P_{ijk}(A, u)P_{ijk}(B, u)$$  \hspace{1cm} (7)

Where $p_{ijk}(A, u)$ and $p_{ijk}(B, u)$ are the moving probability components of gravel A and gravel B in direction $u$, respectively.

The gravels A and B can obtain the overall moving probability $p_{ijk}(A)$ and $p_{ijk}(B)$ according to formula 1. $p_{ijk}(A, u)$ and $p_{ijk}(B, u)$ are the component values of the overall movement probability $p_{ijk}(A)$ and $p_{ijk}(B)$ in the direction $u$.

Therefore, the collision probability $F_{ijk}$ of the gravels A and B is equal to the sum of the collision probability in three directions, that is,

$$F_{ijk} = \sum_u F_u = \sum_u P_{ijk}(A, u)P_{ijk}(B, u)$$  \hspace{1cm} (8)

The fourth step:

According to the description of the previous module, we combined a static sea wave impact model and a collision model to build a sandcastle-wave erosion model.

We simplify the wave into a sinusoidal waveform, and calculate the impact force $F$ of the water wave at a certain position per unit time from Equation 4. The corresponding $F$ is assigned to the water cell at this position, that is, the water cell at this position can impact the gravel cell with a force $F$. For each grit cell that is in contact with the water molecule cell, the grit cell is subjected to an impact force of size $F$. This has an impact on the collision probability of the gravel cells in direct contact with the water, in order to simulate the situation of the sand castle being hit by the waves.

Within a unit step, the gravel cells that have collided and come in direct contact with the water molecule cells are removed from the gravel pile. The number of remaining gravel cells at the same time is the size of the gravel pile.

3. Solution

3.1. Solution 1
This question requires us to give the best three-dimensional geometry to use as a sandcastle foundation, which lasts the longest time on the seashore that experiences waves and tides. In other words, who has the strongest impact resistance. We selected cubes, cylinders, elliptical cones, and triangular pyramids as our research objects.

The static sea wave impact model and sand castle collision model were used to simulate the process of sand castle erosion by seawater. To simplify the model, we assume that the gravel separated from the whole will not attach to the sandcastle again, so if the gravel cells that are in contact with the water cells move, we consider that the gravel has left the sandcastle. In addition, we do not consider the collapse of sand castles. In the end, our results reflect the erosion degree of sand castles based on the size of the sand castles with the same foundation area and the size of the foundation. The results are shown in the figure 5. The highest erosion rate of the cube is 97.43%, followed by 78.68% for the cylinder, 30.02% for the triangular pyramid, and 9.80% for the elliptical cone.

Figure 5. Results of question 1

In summary, we can conclude that the 3D geometry with the highest resistance to seawater shock is a cone. According to fluid mechanics, it is not difficult to speculate that shapes similar to "rocket heads" or "fishes" can slow the rate of seawater erosion. Therefore, we changed the three-dimensional geometry into an elliptical cylinder, and ran the sandcastle erosion model again, and we got an erosion rate of 5.49% in the same time. This fully validates our hypothesis.

3.2. Solution 2

According to the sandcastle collision model and the static sea wave impact model we established, we set the coastal wave impact model unchanged and change the sand-water mixing ratio in the sandcastle, that is, $S_{ijk}$ and $W_s$ in the first formula. They express the attraction to the gravels where the grid points $(i,j,k)$ are located and their influence coefficients. We use this to represent the moisture content and compactness between the gravels. In order to simplify the model, as long as the eroded area of the sand castle foundation reaches 98.00%, we believe that the sand castle has been completely eroded and disappeared. Taking the cube as an example, we set the sand-water mixing ratio from 0% to 100% and found that the sand-water mixing ratio does not form a sand castle after 18.28%. So within 0 to 18.28%, we found the function value corresponding to the maximum value. According to the MATLAB program, we calculated that when the sand-water ratio was 8.09%, the sand castle disappeared in the 56th second.

Furthermore, we continued to test the optimal sand-water mixing ratios of cylinders, elliptical cones, and triangular pyramids to be 6.89%, 7.33%, and 9.84%, respectively. Therefore, we can conclude that the optimal sand-water mixing ratio for different shapes is different, and its range is at least 6.89% to 9.84%.
3.3. Solution 3
The title asks us to determine how the best sandcastle identified in Requirement 1 is affected by rain and whether it is still the best 3D shape. For the sandcastle erosion model we created earlier, it is affected by the lateral impact from the seawater, and the rainfall caused the model to receive a vertical downward impact. Considering the influence of wind on the beach and other factors, the impact of raindrops will create an inclined angle with the ground.

We consider improving the previous model to add an oblique downward force on the surface. Let this force be the impact of rain, decompose the force, rebuild the model and calculate the possibility of collision. As shown in figure 6.

![Figure 6. 3D exploded view of raindrop impact](image)

We use the idea of the first question: after comparing the cube, ellipse cone, cylinder, triangular prism, and ellipse cylinder that was added later in the same time (not equal to the time of the first question), we get their scour rates are 86.12%, 7.78%, 54.90%, 38.71%, 18.89%, 14.99%.

We found that the highest scouring rate (weakest anti-interference) is still the cube, and the lowest scouring rate is not the elliptical cylinder in question 1, but the elliptical cone. Furthermore, we found that the scouring rate of the 3D geometry for the cylinder type is generally higher than that for the cone type. This also accords with the relevant laws of fluid mechanics and structural mechanics, which shows that the model established is more reasonable.

3.4. Solution 4
The question asks us what strategies to use to extend the life of the sand castle. We considered two aspects: one is to change the distance between the sand castle and the sea water; the other is the total amount of sand.

The first aspect considers the distance from the seawater: we change the "wave" parameter in the static ocean wave impact model we created earlier. In this model, we use a sine function to simulate the waves. When the distance between the sandcastle and the seawater increases, we reduce the wave speed of the corresponding sine function to achieve the effect we want. When we adjusted the wave velocity \( v \) to be small, it can be seen from the figure 7 that the ratio of the foundation of the remaining gravel to the starting area has increased, which indicates that the sandcastle’s erosion resistance has improved. When the wave speed \( v \) reaches 0.1, the sand castle is not eroded.
Figure 7. Relationship between wave velocity and ground area ratio

The second aspect considers the total amount of sand: In our model, we reflect the total amount of sand by the number of cells. As long as the number of cells is changed, the total amount of sand can be changed. As shown in the figure 8, we changed the change in the number of cells. The remaining gravel’s foundation is in the same relationship with the starting area ratio-the more the number of cells, the larger the ratio. However, we can see that as the number of cells increases, the rate of growth decreases. And tend to a certain value.

Figure 8. Relationship between cell number and ground area ratio

To sum up, in a certain range, as the farther the ocean in the sandcastle is, the smaller the wave speed and the greater the total amount of sand, the stronger the erosion resistance of the sandcastle. As distance and total sand continue to increase, the effect becomes smaller.

4. Conclusions

By establishing an offshore wave model and a collision model, we have solved four problems, and from this we can draw general conclusions: Without rain, to build the longest lasting sandcastle, you need to
design the sand castle into an oval cylinder; in the case of rain, the sand castle needs to be designed as an oval cone. For the best sand-water mixing ratio should be at least 6.89% to 9.84%.

In addition, we also found that the farther the ocean in the sand castle, the smaller the wave speed and the greater the total amount of sand, the stronger the sand castle’s erosion resistance. As distance and total sand continue to increase, the effect becomes smaller.

References
[1] Zhou Rui, Li Ying, Zhu Long. Research on Modeling and Simulation of Offshore Waves [J]. Computer Engineering and Applications, 2016, 52 (13).
[2] Xu Anquan. Study on mechanical characteristics of stacked bodies based on 3D cellular automata [J]. Yunnan Hydropower, 2018, 34 (6).
[3] Hu Jun, You Lei. Cellular Automata Model of Pedestrian Evacuation in 3D Space [J]. Acta Phys. Sin, 2014, (8).
[4] Yang Fulong, Cao Jia, Bai Ye. Three-dimensional simulation of forest fire spread based on cellular automata [J]. Computer Engineering and Applications, 2016, 52 (19).
[5] Chen Jun. Simulation of offshore waves [D]. Hubei: Wuhan University of Technology, 2011.
[6] https://www.quanjing.com/imgbuy/QJ6903991500.html