SPES: A Two-Stage Query Equivalence Verifier

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ABSTRACT

In database-as-a-service platforms, automated verification of query equivalence helps eliminate redundant computation across queries (i.e., overlapping sub-queries). State-of-the-art tools for automated detection of query equivalence adopt two different approaches. The first technique is based on reducing queries to algebraic expressions and proving their equivalence using an algebraic theory. The limitations of this approach are twofold. First, it only proves the equivalence of queries with significant differences in the attributes of their relational operators (e.g., predicates in the filter operator). It does not support certain widely-used SQL features (e.g., NULL values). Its verification procedure is computationally intensive. The second technique is based on deriving the symbolic representation of the queries and proving their equivalence using a satisfiability modulo theory. The limitations of this approach are twofold. It only proves the equivalence of queries under set semantics. It cannot prove the equivalence of queries with significant structural differences in their abstract syntax trees.

In this paper, we present a novel two-stage approach to automated verification of query equivalence that addresses the limitations of these individual techniques. The first stage consists of reducing queries to a novel algebraic representation and then normalizing the resulting algebraic expressions to minimize structural differences. The second stage consists of applying a verification algorithm to convert the normalized algebraic expressions to a novel query pair symbolic representation and proving their equivalence under bag semantics using satisfiability modulo theory. We implement our two-stage approach in SPES. SPES proves the equivalence of a larger set of query pairs (90/232) under bag semantics compared to the state-of-the-art tools based on algebraic (30/232) under bag semantics and symbolic approaches (67/232) under set semantics. Furthermore, the average query equivalence verification time is 83× and 3× shorter than those tools, respectively.

PVLDB Reference Format:
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1. INTRODUCTION

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Proceedings of the VLDB Endowment, Vol. xx, No. yyy
ISSN 2150-8097.
DOI: https://doi.org/10.14778/xxxxxxx.xxxxxx

Database-as-a-service (DBaaS) platforms (e.g., Alibaba’s MaxCompute [1], Microsoft’s Azure Data Lake [9], and Google’s BigQuery [11]) enable users to quickly deploy complex data processing pipelines consisting of SQL queries. In practice, these data processing pipelines exhibit a significant amount of computational overlap (i.e., semantically equivalent sub-queries) [17, 50]. This results in higher resource usage and longer query execution times.

Researchers have developed techniques for minimizing redundant computation by materializing the overlapping sub-queries as views and rewriting the original queries to operate on these materialized views [43, 33]. All of these techniques rely on an effective and efficient algorithm for automatically deciding the equivalence of a pair of SQL queries. Two queries are equivalent if they always return the same output table for any given set of input tables. In general, proving query equivalence (QE) is an undecidable problem [14, 18]. Given this constraint, prior efforts have focused on a subset of SQL queries where this problem is decidable (e.g., SELECT-JOIN queries) [20, 45, 27, 39]. While this line of research has studied the theoretical underpinnings of this problem, these techniques are unable to identify overlap in complex SQL queries.

Prior Efforts: Recently, researchers have formulated two pragmatic approaches for automatically proving QE. These efforts are based on two different representations of a query: (1) algebraic representation, and (2) symbolic representation.

UDP is the state-of-the-art prover based on an algebraic approach [23]. It determines QE using three steps. First, it transforms the queries from an abstract syntax tree (AST) representation to an algebraic representation (AR). Next, it applies a set of rules for rewriting the ARs. Lastly, it attempts to find an isomorphism between the vocabularies of these ARs to determine their equivalence via proof construction. While this algebraic approach works well for proving queries with significant structural differences in their ASTs, it suffers from three limitations. First, UDP cannot prove the equivalence of queries when the attributes in their relational operators exhibit significant differences (e.g., predicates in the filter operator). This is because it uses a set of syntax-driven re-write rules to construct the proof. Second, it does not support certain widely used SQL features (e.g., NULL values). Third, its verification procedure is computationally intensive due to the long sequence of re-writes required for proving equivalence.

EQUITAS circumvents these limitations of the algebraic approach by adopting an alternate approach based on symbolic representation [50]. It determines QE using two steps. First, it transforms the queries from an AST representation to a symbolic representation (SR) (i.e., a set of first-order logic (FOL) formulae). A query’s SR symbolically represents the tuples that it returns. Next, it lever-

We refer to the algebraic representation of a pair of queries as ARs.
ages a general-purpose solver based on satisfiability modulo theory (SMT) to determine the containment relationship between two SRs. If the containment relationship holds in both directions, then the queries are equivalent. While this symbolic approach addresses the drawbacks of the algebraic approach, it suffers from two limitations.

First, it only proves equivalence of queries under set semantics (i.e., output tables must not contain duplicate tuples [41]). In practice, queries rely on bag semantics (i.e., output tables may contain duplicate tuples [16]) proving QE under bag semantics is a strictly harder problem that doing so under set. This is because if two queries are equivalent under bag semantics, then they are also equivalent under set semantics. However, the converse does not hold. Second, it cannot prove the equivalence of queries with significant structural differences in their ASTs (e.g., aggregate and union operators). These limitations constrain the applicability of the SR-based approach.

**OUR APPROACH:** In this paper, we present a two-stage query equivalence prover that addresses the limitations of EQUITAS. To address the first limitation, we introduce a novel SR and verification algorithm that directly proves QE under bag semantics (without determining the query containment relationship). To address the second limitation, we present a novel AR that normalizes the structural differences between queries.

We implemented our two-stage approach in SPES, a tool for automatically verifying the equivalence of SQL queries under bag semantics. We evaluate SPES using a collection of 232 pairs of equivalent SQL queries available in the Apache Calcite framework [3]. Each query pair is constructed by applying various optimization rules on complex SQL queries with diverse features (e.g., arithmetic operations, three-valued logic for supporting NULL, subqueries, grouping, and aggregate functions). Our evaluation shows that SPES proves the semantic equivalence of a larger set of query pairs (90/232) compared to UDP (34/232) and EQUITAS (67/232). Furthermore, SPES is 83× faster than UDP and 3× faster than EQUITAS on this benchmark. In addition to the Calcite benchmark, we evaluate the efficacy of SPES on a cloud-scale workload consisting of 9,486 real-world SQL queries from Ant Financial Services Group [2]. SPES automatically found that 27% of the queries in this workload contain overlapping computation. These queries contain complex relational operators (e.g., aggregate functions).

**CONTRIBUTIONS:** We make the following contributions:

- We illustrate the limitations of AR- and SR-based approaches to determining the equivalence of SQL queries in §2.
- We present a novel AR of queries and a set of rules for normalizing the structural differences between queries in §4.
- We introduce a novel verification algorithm and SR to determine the equivalence of normalized ARs in §5.
- We implement this approach in SPES and evaluate its efficacy. We demonstrate that SPES proves the equivalence of a larger set of query pairs in Calcite benchmark compared to UDP and EQUITAS in §7. More importantly, unlike EQUITAS, SPES demonstrates QE under bag semantics.

## 2. MOTIVATION

We motivate the need for a new approach to automated query equivalence verification by illustrating the limitations of prior approaches. We illustrate the limitations of the algebraic and symbolic approaches using a set of examples based on these two tables:

- **EMP table:** (EMP_ID, SALARY, DEPT_ID, LOCATION)
- **DEPT table:** (DEPT_ID, DEPT_NAME)

### 2.1 Algebraic Approach

UDP is the state-of-the-art tool for automatically determining QE using an algebraic approach [23]. It first converts the given pair of queries to \(U\)-semirings, a family of algebraic structures, to obtain the \(U\)-expression of each query (i.e., their ARs). It then applies a set of semantically equivalent algebraic re-write rules to normalize and simplify each \(U\)-expression. UDP seeks to find isomorphisms between the vocabularies of the two \(U\)-expressions to prove that they are syntactically equivalent. It proves QE by demonstrating that the ARs are equivalent.

The algebraic approach cannot prove the equivalence of queries containing semantically-equivalent, syntactically-different predicates (e.g., predicates with arithmetic operators, predicates based on three-valued logic for handling NULL).

#### EXAMPLE 1. SYNTACTICALLY-DIFFERENT PREDICATES:

**Q1:** SELECT \* FROM (SELECT \* FROM EMP WHERE DEPT_ID = 10) AS T WHERE T.DEPT_ID + 5 > T.EMP_ID;

**Q2:** SELECT \* FROM (SELECT \* FROM EMP WHERE DEPT_ID = 10) AS T WHERE 15 > T.EMP_ID;

Q1 first chooses employee tuples whose department id is 10. It then applies another filter to retrieve the employee tuples where the department id plus five is greater than employee id. Q2 contains the same inner query as that in Q1. The only difference is that Q2 chooses the tuples where the employee id is less than 15. Q1 and Q2 are semantically equivalent since the inner query retrieves tuples whose department id is 10 (and 10 + 5 = 15). UDP converts these queries to the following ARs:

**Q1:** \([t.DEPT_ID = 10] \times [t.DEPT_ID + 5 > t.EMP_ID] \times EMP(t)

**Q2:** \([t.DEPT_ID = 10] \times [15 > t.EMP_ID] \times EMP(t)

Each algebraic expression is a function that returns the cardinality of an arbitrary tuple \(t\) in the output table (i.e., number of times the tuple appears). Each predicate is a boolean function that either returns zero or one depending on whether the tuple satisfies the predicate. For example, \([t.DEPT_ID = 10]\) returns one when the given tuple \(t\) satisfies the condition that the department id equals to 10. Each input table expression is a function that returns the cardinality of an arbitrary tuple \(t\) in the input table. For example, \(EMP(t)\) returns the cardinality of \(t\) in EMP table. \(\times\) represents the arithmetic multiplication operation.

UDP cannot prove the semantic equivalence of Q1 and Q2 since it cannot use its re-write rules to prove that the two predicates \([t.DEPT_ID + 5 > t.EMP_ID]\) and \([15 > t.EMP_ID]\) are logically equivalent when the predicate \([t.DEPT_ID = 10]\) holds. We note that UDP may handle this specific example by adding a re-write rule for constant propagation. However, due to the complexity of predicates in general, it is challenging to formulate a set of re-write rules to normalize all possible predicates. Furthermore, it is challenging to normalize predicates in operators containing additional attributes (e.g., group set in aggregate operator).

### 2.2 Symbolic Approach

EQUITAS is the state-of-the-art tool for automatically determining QE using a symbolic approach [50]. It first converts the given pair of queries to their SR (i.e., a set of FOL formulae). Each SR symbolically represents the output table of that query. It then uses an SMT solver to verify the relational properties between SRs to prove the containment relationship between queries. On all valid input
tables, if any tuple returned by Q2 is also returned by Q1, then Q1 contains Q2. EQUITAS proves QE by proving the containment relationship in both directions (i.e., Q1 contains Q2 and Q2 contains Q1). Using the SRs and the SMT solver, EQUITAS proves the equivalence of queries containing semantically-equivalent, syntactically-different predicates (Listing 2.1).

While it overcomes the limitations of the algebraic approach, it suffers from two limitations. First, EQUITAS cannot prove the equivalence of queries with significant structural differences in terms of aggregation and outer-join operators. Second, it can only prove the equivalence of queries under set semantics if they contain aggregation and outer-join operators.

**Example 2. Structural Differences:**

Q1: **SELECT EMP.DEPT_ID, SUM(EMP.SALARY) FROM EMP, DEPT**

**WHERE** EMP.DEPT_ID = DEPT.DEPT_ID

**AND** EMP.SALARY > 1000

**GROUP BY** EMP.DEPT_ID;

Q2: **SELECT T.DEPART_ID, SUM(T.s) FROM**

(SELECT EMP.DEPT_ID, EMP.LOCATION, SUM(EMP.SALARY) as s)

**FROM** DEPT, EMP

**WHERE** EMP.DEPT_ID = DEPT.DEPT_ID

**AND** EMP.SALARY + 1000 > 2000

**GROUP BY** EMP.DEPT_ID, EMP.LOCATION) as T

**GROUP BY** T.DEPT_ID;

Q1 is an aggregation query that calculates the sum of salaries of employees whose salary is greater than 1000 grouped by their department id. Q2 is a nested query whose inner and outer queries are both aggregation queries. The inner query calculates the sum of salaries of all employees whose salary plus 1000 is greater than 2000, grouped by their department id and location. The outer query then calculates the sum of salaries of those employees grouped by their department id. Since the set of GROUP BY columns in the outer query of Q2 is a subset of the group set of the inner query, Q1 and Q2 are equivalent. EQUITAS reduces these queries to their SRs:

Q1: <COND1, COLS1, ASSIGN1>

**COND1:** \((v_2 > 1000 \text{ and } !n_2)\)

**COLS1:** \((v_3, n_3), (v_4, n_4)\)

Q2: <COND2, COLS2, ASSIGN2>

**COND2:** \((v_2 + 1000 > 2000 \text{ and } !n_2)\)

**COLS2:** \((v_3, n_3), (v_5, n_5)\)

Each SR contains two fields: **COND and COLS.** **COND** is a FOL formula that represents the constraints that a tuple must satisfy to be present in the output table. For example, **COND1** denotes that the value of the salary column must be greater than 1000 and must not be NULL. \((v_2, n_2)\) symbolically represents the salary column in an arbitrary EMP tuple. \(v_2\) represents the value and the boolean symbolic variable \(n_2\) indicates if the value is NULL. **COLS** is a vector of pairs of FOL formula that symbolically represent an arbitrary tuple in the output table. For example, **COLS1** represents the tuple returned by Q1. Here, \((v_3, n_3)\) and \((v_4, n_4)\) represent the department id and sum of salaries, respectively.

EQUITAS cannot prove that Q1 and Q2 are equivalent since it cannot prove that \((v_4, n_4)\) is equivalent to \((v_5, n_5)\) when both conditions **COND1 and COND2** hold. The reasons for this are twofold. First, the result of the summation operation is not constructed from a bounded number of tuples in the input table. So, EQUITAS uses two different set of independent variables (i.e., \((v_4, n_4)\) and \((v_5, n_5)\)) to symbolically represent the results of the summation operations of Q1 and Q2’s outer query, respectively. Second, the input tables for the aggregation operations in Q1 and Q2’s outer query are different. So, EQUITAS cannot establish any relation between \((v_4, n_4)\) and \((v_5, n_5)\).

**Example 3. Set Semantics:**

Q1: **SELECT EMP.DEPT_ID, EMP.LOCATION FROM EMP**

Q2: **SELECT EMP.DEPT_ID, EMP.LOCATION FROM EMP GROUP BY EMP.DEPT_ID, EMP.LOCATION**

Q1 selects the department id and location columns of all employees. Q2 selects the same columns from the employee table grouped by the department id and location columns. Q1 and Q2 are semantically equivalent only under set semantics. They are not equivalent under bag semantics since the output tables would differ if there were two employees in the same department and location. EQUITAS reduces these queries to their SRs:

Q1: <COND1, COLS1, ASSIGN1>

**COND1:** True; **COLS1:** \(((v_1, n_1), (v_4, n_4))\)

Q2: <COND2, COLS2, ASSIGN2>

**COND2:** True; **COLS2:** \(((v_1, n_1), (v_4, n_4))\)

EQUITAS proves that Q1 and Q2 contain each other under set semantics. However, most database systems use bag semantics. So, it is critical to prove that they are not equivalent under bag semantics.

3. **OVERVIEW**

In this section, we first present an overview of the two-stage approach in §3.1. We then use an example to illustrate how this approach proves QE of structurally different queries under bag semantics.

3.1 **Two-Staged Approach**

We decompose the previously monolithic equivalence proving problem to two stages.

- In the first stage, SPES converts the two queries to their ARs. It then normalizes each AR using a set of rewrite rules. These rules differ from those used in UDP in that they only focus on minimizing the structural differences between the relational operators of the queries (e.g., inner join, aggregation, and union). These rules do not transform the attributes of the operators in the queries (e.g., complex predicates and projections). The output of this stage is an AR in **union normal form** (UNF) that represents the semantics of the original query. We defer the description of UNF and normalization rules to §4.2. These normalization rules enable SPES to prove the equivalence of structurally different queries.

- In the second stage, SPES prove the equivalence of the normalized ARs under bag semantics. This stage consists of two steps.

**Cardinal Equivalence:** In the first step, SPES first verifies if the given pair of ARs are **cardinally equivalent** under bag semantics. Two queries are cardinally equivalent if and only if for all valid input tables, their output tables contain the same number of tuples. We defer a formal definition of cardinal equivalence to §5.1. If two queries are cardinally equivalent, then there exists a **bijective map** between the tuples returned by these two queries for all valid inputs,
as illustrated in Figure 1a. In this map, each tuple in first table is mapped to an unique tuple in the second table, and all of the tuples in second table are covered by the map. We note that the contents of the output tables of two cardinally equivalent queries may differ.

SPES constructs a Query Pair Symbolic Representation (QPSR) for two cardinally equivalent ARs to symbolically represent the bijective map between the returned tuples. It proves cardinal equivalence of two ARs by recursively constructing the QPSR of their sub-ARs and using the SMT solver to verify specific properties based on the semantics of different types of ARs. We defer a discussion of how SPES constructs QPSR to prove cardinal equivalence to Sections 5.3 to 5.7.

**FULL EQUIVALENCE:** In the second step, SPES uses the constructed QPSR to verify that the given pair of ARs are *fully equivalent* under bag semantics. Two queries are fully equivalent if and only if for all valid input tables, their output tables contain the same tuples (ignoring the order of the tuples). We defer a formal definition of full equivalence to §5.1. If two queries are fully equivalent, then there exists a bijective, identity map between the tuples returned by these two queries for all valid inputs, as illustrated in Figure 1b. In this map, each tuple in first table is mapped to an unique, identical tuple in the second table. Since the QPSR of two given ARs symbolically represents the bijective map between the returned tuples, SPES proves full equivalence of two ARs by using the SMT solver to show that the bijective map is an identity map.

**SPES vs EQUITAS:** SPES differs from EQUITAS in the following ways:

- SPES constructs a QPSR for ARs of a pair of queries after verifying that the ARs are cardinally equivalent. The QPSR symbolically represents the bijective map between the tuples in their output tables. In contrast, EQUITAS directly constructs a SR for each individual query that represents the tuples in its output table.
- SPES directly proves full equivalence without determining the query containment relationship by showing that the bijective map represented by the QPSR is an identity map. EQUITAS proves QE by showing that the query containment relationship holds in both directions.
- SPES decomposes the problem of proving equivalence of ARs into smaller proofs of equivalence of their sub-ARs. It constructs the bijective map between tuples in the final output tables by recursively constructing the bijective maps between tuples in all of the intermediate output tables.
- SPES uses the SMT solver to verify conditions at each level of the AR tree to verify cardinal equivalence. In contrast, EQUITAS uses the SMT solver to verify the conditions only after constructing the SRs.

These differences allow SPES to prove QE under bag semantics. As shown in Table 1, SPES supports a larger set of SQL features in comparison to UDP and EQUITAS.

**SMT Solver:** SPES leverages a SMT solver to prove cardinal and full equivalence of ARs [30]. An SMT solver determines if a given FOL formula is satisfiable. For example, the solver decides that the following formula can be satisfied: \( x + 5 > 10 \land x > 3 \) when \( x \) is six. Similarly, it determines that the following formula cannot be satisfied: \( x + 5 > 10 \land x < 4 \) since there is no integral value of \( x \) for which this formula holds. A detailed description of an SMT solver is available in [29].

### 3.2 Illustrative Example

**Table 1: Support for SQL Features** – Comparison of the SQL features supported by UDP, EQUITAS and SPES. ✓ denotes that the tool supports this feature. Complex predicates include those using: (1) arithmetic operations, (2) NULL, and (3) CASE.

| Feature                      | UDP | EQUITAS | SPES |
|------------------------------|-----|---------|------|
| Aggregate                    | ✓   | ✓       | ✓    |
| Outer Join                   | ✓   | ✓       | ✓    |
| Complex Predicates           | ✓   | ✓       | ✓    |
| Table Semantics              | set | bag     | bag  |
| DISTINCT                    | ✓   | ✓       | ✓    |
| Union                       | ✓   | ✓       | ✓    |

We use Listing 2.2 (Example 2) to show how SPES proves equivalence of structurally different queries under bag semantics.

**First Stage:** SPES first converts the queries to ARs. Figure 2 shows the ARs of two queries Q1 and Q2.

The AR of Q1 is an aggregate AR that takes a SELECT-PROJECT-JOIN (SPJ) AR as input. An aggregate AR contains three fields: input AR, group set, and a vector of aggregate operations. In this case, the group set contains department id and the aggregate operation is sum of salaries. The input AR is an SPJ AR. An SPJ AR also contains three fields: a vector of input ARs, a filter predicate, and a vector of projection expressions. In this case, the SPJ AR takes two table ARs as input (EMP and DEPT).

The AR of Q2 is an aggregate AR that takes another aggregate AR as input. In this case, the group set contains department id and the aggregate operation is sum of salaries computed by the input aggregate AR. The input aggregate AR takes an SPJ AR as input. Its group set contains department id and location. Its aggregate operation is sum of salaries. The SPJ AR is the same as the SPJ AR in Q1 except that its filter predicate is different (i.e., EMP.SALARY + 1000 > 2000) and the order of input table ARs is reversed.

To prove that Q1 and Q2 are semantically equivalent, SPES first applies a set of re-write rules to normalize these ARs. Specifically, it merges the two aggregate ARs within Q2 into a single one. This normalized AR of Q2 is denoted by Q2’ in Figure 2. The AR of Q1 remains unchanged after normalization. We defer a discussion of how SPES normalizes ARs to §4.

**Second Stage:** In the second stage, SPES first verifies the cardinal equivalence of two aggregate ARs. In order to verify the cardinal equivalence of two aggregate ARs, SPES recursively constructs the QPSR of two SPJ ARs that the aggregate ARs take as inputs. To verify the cardinal equivalence of two SPJ ARs, it constructs the bijective maps between each pair of its inputs and checks if they are cardinally equivalent. If that is the case, then it constructs a QPSR for each pair of table ARs. SPES maps the EMP table AR in Q1 with the EMP table AR in Q2’, and the DEPT table AR in Q2 with the DEPT table AR in Q2’.

![Figure 2: Illustrative Example](image-url)
We present the formal definitions of \( \text{COLS}_v^1 \) and \( \text{COLS}_v^2 \):

\[
\text{COLS}_v^1: \{(v_1,n_1),(v_7,n_7)\} \quad \text{and} \quad \text{COLS}_v^2: \{(v_1,n_1),(v_2,n_2),(v_3,n_3),(v_4,n_4)\}
\]

Here, \( \text{COLS}_1 \) and \( \text{COLS}_2 \) symbolically represent two corresponding tuples returned by the two cardinally equivalent table ARs, respectively. Each symbolic tuple is a vector of pairs of FOL terms. We present the formal definitions of \( \text{COLS}_1 \) and \( \text{COLS}_2 \) in §5.2. This pair of symbolic tuples \( \text{COLS}_1 \) and \( \text{COLS}_2 \) defines the bijective map between the tuples returned by the table ARs.

Since these table ARs refer to same EMP input table, the bijective map is an identity map. \( \{(v_1,n_1),(v_2,n_2),(v_3,n_3),(v_4,n_4)\} \) symbolically represents a tuple returned by the EMP table AR. Each pair of symbolic variables represents a column in an arbitrary EMP tuple. For instance, \( (v_1, n_1) \) denotes EMP_ID in this symbolic tuple. v1 represents the value of EMP_ID, the boolean symbolic variable n1 indicates if the value is NULL. The encoding scheme is the same as the one used by EQUITAS [50]. \( \text{COND} \) is an FOL formula that represents the two predicates in the SPJ AR. It must be satisfied for the tuples to be present in the output table. Since these table ARs return all tuples, \( \text{COND} \) is TRUE.

\[
\text{QPSR-1: SPES constructs a QPSR for the pair of EMP table ARs:} \\
\text{COND: True} \\
\text{COLS}_1: \{(v_1,n_1),(v_7,n_7)\} \quad \text{and} \quad \text{COLS}_2: \{(v_1,n_1),(v_2,n_2),(v_3,n_3),(v_4,n_4)\}
\]

QPSR-2: SPES constructs a QPSR for the pair of DEPT table ARs:

\[
\text{COND: True} \\
\text{COLS}_1: \{(v_5,n_5),(v_6,n_6)\} \\
\text{COLS}_2: \{(v_5,n_5),(v_6,n_6)\}
\]

\( \{(v_5,n_5),(v_6,n_6)\} \) symbolically represents a tuple is returned by the DEPT table AR.

QPSR-3: SPES uses these two QPSRs and leverages the SMT solver to verify that predicates always returns the same boolean results for the corresponding tuples in the join table to verify that the two SPJ ARs are cardinally equivalent. SPES then constructs a QPSR for these two SPJ ARs:

\[
\text{COND: (v_2 + 1000 > 2000 \text{ and } n_2)} \quad \text{and} \quad (v_2 > 1000 \text{ and } n_2) \\
\text{COLS}_1: \{(v_1,n_1),(v_2,n_2),(v_3,n_3),(v_4,n_4)\} \\
\text{COLS}_2: \{(v_1,n_1),(v_2,n_2),(v_3,n_3),(v_4,n_4)\}
\]

\( (v_2 > 1000 \text{ and } n_2) \) preserves the two bijective maps in the two sub-QPSRs between the input tables of two SPJ ARs. This bijective map preserves the two bijective maps in the two sub-QPSRs between the input tables ARs. In other words, if a tuple t1 is mapped to another tuple t2 in QPSR-1, and a tuple t3 is mapped to another tuple t4 in QPSR-2, then the join tuple of t1 and t2 maps to that of t3 and t4 in QPSR-3. In this manner, the mapping in the lower-level QPSRs is preserved in the higher-level QPSR. \( \text{COND} \) is the conjunction of the filter predicates.

QPSR-4: SPES uses QPSR-3 and the SMT solver to verify that the two aggregate ARs are cardinally equivalent. If so, it constructs a QPSR for the aggregate ARs (i.e., Q1 and Q2):

\[
\text{COND: (v_2 + 1000 > 2000 \text{ and } n_2)} \quad \text{and} \quad (v_2 > 1000 \text{ and } n_2) \\
\text{COLS}_1: \{(v_1,n_1),(v_7,n_7)\} \\
\text{COLS}_2: \{(v_1,n_1),(v_7,n_7)\}
\]

Here, \( \text{COLS}_1 \) and \( \text{COLS}_2 \) symbolically represent the bijective map between tuples returned by Q1 and Q2, respectively. \( (v_7,n_7) \) represents the sum of salaries column.

Full Equivalence: After determining cardinal equivalence, SPES proves the full equivalence of Q1 and Q2 by using an SMT solver to verify the following property of QPSR-4: \( \text{COND} \implies \text{COLS}_1 = \text{COLS}_2 \). SPES feeds this formula to the SMT solver:

\[
\text{COND} \land \neg(\text{COLS}_1 = \text{COLS}_2)
\]

The solver determines that it cannot be satisfied, thereby showing that the paired symbolic tuples are always equivalent when \( \text{COND} \) holds. Thus, the bijective map between the tuples returned by the ARs is an identity map. So, Q1 and Q2 are fully equivalent under bag semantics.

Summary: SPES first constructs QPSR-1 for EMP table ARs and QPSR-2 for DEPT table ARs. It then uses these QPSRs to determine the cardinal equivalence of SPJ ARs. Next, it constructs QPSR-3 for the SPJ ARs. SPES then uses QPSR-3 to determine the cardinal equivalence of aggregate ARs and constructs QPSR-4 for the overall queries. Lastly, it uses QPSR-4 to decide the full equivalence of Q1 and Q2. Thus, SPES only establishes cardinal equivalence before constructing the QPSRs. It only checks full equivalence for the top-level QPSR (i.e., QPSR-4).

4. Algebraic Representation

In this section, we first define an AR system that captures the semantics of SQL queries in §4.1. We then introduce the Union Normal Form (UNF) of an AR and how SPES converts each AR to UNF in §4.2. We finally present a minimal set of pre-defined rules for reducing an UNF AR to a simplified, semantically equivalent UNF AR in §4.3.

4.1 Syntax and Semantics

We first present the syntax of the AR. We then describe the semantics of the AR based on the relationships between the input and output tables. The formal definition of the semantics is given in Appendix A. An AR \( \theta \) is defined thus:

\[
\theta ::= \text{TABLE}(n) \mid \text{SPI} (\theta, p, \theta) \mid \text{AGG} (\theta, \theta, \theta) \mid \text{UNION}(\theta)
\]

In SPES, an AR can be: (1) a table AR, (2) an SPJ AR, (3) an aggregate AR, or (4) an union AR. We define the semantics of these four types of AR in terms of the relationship between the input tables and the output table. We consider a table to be a bag (i.e., multi-valued set) of tuples as it best represents real-world databases. SPES supports the DISTINCT keyword for discarding duplicate tuples in a bag. Consequently, it also supports set semantics. This representation is based on the AR presented in [23]. We next describe the semantics of these ARs.

Table AR: \( \text{TABLE}(n) \) represents a table in a database. It contains only one field: the name of the table \( n \). Given valid input tables \( T_s \), this AR returns all the tuples in table \( n \).

SPJ AR: This AR contains three fields: (1) a vector of input ARs \( \theta \), (2) a predicate that determines whether a tuple is selected \( p \), and (3) a vector of projection expressions that transform each selected tuple \( \theta \). Given a set of valid input tables \( T_s \), the SPJ AR first evaluates the vector of input ARs on \( T_s \) to obtain a vector of input tables. For each tuple \( t \) in the cartesian product of the vector of input tables, if \( t \) satisfies the given predicate \( p \), then it applies the vector of expressions \( \theta \) on \( t \) and emits the transformed tuple.

Predicate: A predicate may contain arithmetic operators, logical operators, and functions that check if a term is NULL. SPES supports higher-order predicates (e.g., EXISTS) which are encoded as uninterpreted functions.

Projection Expression: A projection expression may contain columns, constant values, NULL, arithmetic operations, user-defined functions, and the CASE keyword. We present the formal definitions of the syntax of predicate and projection expression in Appendix A.

Aggregate AR: The aggregate AR contains three fields: (1) an input AR \( \theta \), (2) a set of grouping attributes \( \theta \), and (3) a vector of aggregate functions \( \theta \). Given a set of valid input tables \( T_s \), the
aggregate AR first evaluates the input AR on Ts to get an input table \( T_0 \). It then partitions the input table \( T_0 \) into a set of bags of tuples as defined by a set of grouping attributes \( \vec{g} \) (tuples in each bag take the same values for the grouping attributes). Lastly, for each bag of tuples, it applies the vector of aggregate functions and returns one tuple. Each aggregate function generates a column in that tuple.

**UNION AR:** The union AR contains one field: a vector of input ARs (\( \vec{o} \)). Given a set of valid input tables \( T_s \), the union AR first evaluates the vector of input ARs on \( T_s \) to get a vector of input tables. It then returns all the tuples present in the input tables (without discarding duplicate tuples). The union AR captures the semantic of the UNION ALL operator [35].

**Motivation:** The reasons for these definitions are twofold. First, it allows SPES to cover most of the frequently-observed SQL queries. Second, since different types of ARs have different semantics, SPES can leverage AR-specific comparison functions. We treat SPJ queries as a separate category since the problem of determining their equivalence is decidable [15]. We include aggregate and union ARs in our definitions since they are widely used SQL constructs, and their semantics differs from that of SPJ queries.

**Complex SQL Constructs:** SPES reduces certain SQL constructs that do not directly map to these four categories to a combination of these categories. Here are two examples:

- **SPES** expresses the LEFT OUTER JOIN operator as an UNION expression that takes a vector of two SPJ expressions as input. The first SPJ expression represents the INNER JOIN component of the LEFT OUTER JOIN operator. The second SPJ expression represents the OUTER JOIN component of the LEFT OUTER JOIN operator and uses EXISTS in the predicate.
- **SPES** expresses the DISTINCT operator as an aggregate expression where the GROUP BY set contains all columns.

### 4.2 Union Normal Form

The syntax of an UNF AR is defined as follows:

\[
\text{UNF} ::= \text{UNION} (\text{SPJE}) \\
\text{SPJE} ::= \text{SPJ}(E, p, \vec{o}) \\
E ::= \text{TABLE}(n) | \text{AGG}(\text{UNF}, \vec{g}, a\vec{g})
\]

The UNF AR is an AR that takes a vector of normalized SPJ ARs as input (SPJE). Each normalized SPJ AR takes a vector of ARs as input (E). These ARs are either a table AR or a normalized aggregate AR. Each normalized aggregate AR can recursively take an UNF AR as input.

**Normalizing Rules:** An AR can be normalized to UNF by repeatedly applying a set of normalization rules. The number of rule applications is finite and the rules are not applied in a specific order.

1. If an AR is SPJ(\( \vec{o}_0 :: \vec{e}_1, p_1, \vec{o}_1 \)) and \( \vec{o}_0 = \text{SPJ}(\vec{e}_2, p_2, \vec{o}_2) \), then transform the AR to SPJ(\( \vec{e}_1, p_1 \land p_2, \vec{o}_1 \lor \vec{o}_2 \)). Here, \( \lor \) denotes concatenation of two vectors and \( \lor \) represents element-wise composition of two vectors of projection expressions.
2. If an AR is SPJ(\( \vec{o}_0 :: \vec{e}_1, p_1, \vec{o}_1 \)) and \( \vec{o}_0 = \text{UNION}(\vec{e}_2) \), then transform the AR to UNION(\( \vec{e}_1 \)). Each AR in \( \vec{e}_1 \) is SPJ(\( \vec{e}' :: \vec{e}_1, p_1, \vec{o}_1 \)) where \( \vec{e}' \) is an AR in \( \vec{e}_2 \).
3. If an AR is UNION(\( \vec{o}_0 :: \vec{e}_1 \)) and \( \vec{o}_0 = \text{UNION}(\vec{e}_2) \), then transform the AR to UNION(\( \vec{e}_2 :: \vec{e}_1 \)).
4. If an AR is UNION(\( \vec{o}_0 :: \vec{e}_1 \)) and \( \vec{o}_0 \) is aggregate or table AR, then transforms \( \vec{o}_0 \) to SPJ(\( \vec{o}_0, \text{true}, \vec{?} \)). \( \vec{?} \) represents the identity map.

5. If an AR is AGG(\( \vec{e}, \vec{g}, a\vec{g} \)) and \( \vec{e} \) is SPJ or table AR, then transforms \( \vec{e} \) to UNION(\( \vec{e} \)).

The first three rules merge SPJ and Union ARs. The fourth and fifth rules update the input AR of Union and Aggregate to satisfy the UNF.

### 4.3 Pre-defined Rules

We now present a minimal set of pre-defined rules to further simplify the UNF ARs. These rules allow SPES to prove the equivalence of larger set of SQL queries.

**Empty Tables:** For this rule, we first define a special AR called the empty table. For any valid input, this AR always returns an empty table. It is only equivalent to another empty table AR.

For an SPJ AR \( \text{SPJ}(\vec{o}, p, \vec{o}) \), if no tuple satisfies the predicate \( p \), then the rule transforms this SPJ AR to an empty table AR. For an SPJ AR, if there exists an empty table AR in its input, then the rule transforms the SPJ AR to an empty table AR. For an union AR, the rule removes all empty table ARs from its input. If the input of an union AR is an empty vector, then the rule transforms the union AR to an empty table AR. For an aggregate AR, if the input is an empty table AR and the group set is not empty, then the rule transforms the aggregate AR to an empty table AR.

**Predicate Pushdown:** For an SPJ AR \( \text{SPJ}(\vec{o}_0 :: \vec{e}, p_1 \land p_2, \vec{o}_1) \), where \( \vec{o}_0 \) is an aggregate AR AGG(\( \vec{e}, \vec{g}, a\vec{g} \)), if \( p_1 \) only depends on the group set \( \vec{g} \), then the rule updates the SPJ AR to SPJ(\( \vec{o}_0 :: \vec{e}, p_2, \vec{o}_1 \)) and updates the predicates of all SPJ expressions in \( \vec{e} \) by taking the conjunction of their original predicates with \( p_1 \).

**Aggregate Merge:** Given an aggregate AR AGG(\( \vec{e}_1, \vec{g}_1, a\vec{g}_1 \)), if the input union AR only has one SPJ AR as its input, and the SPJ AR only has one aggregate AR AGG(\( \vec{e}_2, \vec{g}_2, a\vec{g}_2 \)) as its input, then SPES checks the following conditions: (1) the predicate of the SPJ AR only depends on the group set \( \vec{g}_2 \), (2) the group set \( \vec{g}_1 \) is a subset of the group set \( \vec{g}_2 \), and (3) for each aggregate operation in \( a\vec{g}_1 \), the operand is either in the group set \( \vec{g}_2 \) or in aggregate operation \( a\vec{g}_2 \) such that they are the same aggregation function, and the aggregation function can only be \( \text{MAX}, \text{MIN}, \text{SUM} \) and \( \text{COUNT} \). If the aggregate AR satisfies all of these conditions, then SPES removes AGG(\( \vec{e}_1, \vec{g}_2, a\vec{g}_2 \)) from the SPJ expression, and adds \( \vec{e}_2 \) to the input vector of the SPJ AR. SPES uses this rule to simplify the AR of Q2 in Figure 2.
$T_2$ are in the map. Thus, it is a bijective (one-to-one) map between tuples in $T_1$ and $T_2$. However, the two mapped tuples may differ in their values, as shown in Figure 1a.

**Definition 2. Full Equivalence:** Given a pair of queries Q1 and Q2, Q1 and Q2 are fully equivalent if and only if, for all valid input tables T, the output tables $T_1$ and $T_2$ of Q1 and Q2 are identical.

If Q1 and Q2 are fully equivalent, for all valid inputs, there exists a bijective map between tuples in $T_1$ and $T_2$, and this bijective map is an identity map. In other words, each tuple in $T_1$ can always be mapped to an unique, identical tuple in $T_2$, and all tuples in $T_2$ are in the map, as shown in Figure 1b.

**Motivation** We first try to prove cardinal equivalence before checking for full equivalence. This is because if Q1 and Q2 are fully equivalent, then they must be cardinal equivalent. To prove full equivalence, we prove that the bijective map between tuples in the output tables is an identity map. In the rest of the paper, equivalent queries without any qualifier refer to fully-equivalent queries.

SPES can prove that ARs are fully equivalent even if their sub-ARs are only cardinally equivalent. Consider the following queries:

Q1: `SELECT EMP, DEPT_ID, SUM(EMP.SALARY) FROM (SELECT DEPT_ID, SALARY FROM EMP) GROUP BY EMP, DEPT_ID;`

Q2: `SELECT EMP, DEPT_ID, SUM(EMP.SALARY) FROM (SELECT DEPT_ID, SALARY, DEPT_ID+1 FROM EMP) GROUP BY EMP, DEPT_ID;`

While these queries are fully equivalent, their sub-queries are not fully equivalent. This is because the second sub-query returns three columns while the first one only returns two columns.

### 5.2 Query Pair Symbolic Representation

We now define the symbolic representation of normalized ARs that SPES uses for proving equivalence. QPSR is an extension of the SR defined in EQUITAS to prove QE under bag semantics [50]. In QPSR, we augment the SR to use a pair of symbolic tuples to track a bijective map between the tuples that are returned by two cardinally equivalent ARs. QPSR of a pair of cardinally equivalent ARs Q1 and Q2 is a tuple of the form:

$$\langle \text{COLS}_1, \text{COLS}_2, \text{COND}, \text{ASSIGN} \rangle$$

$\text{COLS}_1$ is a vector of pairs of FOL terms that represent an arbitrary tuple returned by Q1. Each element of this vector represents a column and is of the form: $(\text{VAL}, \text{Is-Null})$, where VAL represents the value of the column and Is-Null denotes the nullability of the column. $\text{COLS}_2$ is another vector of pairs of FOL terms that represents a tuple returned by Q2. Since Q1 and Q2 must be cardinally equivalent before SPES constructs their QPSR, the two symbolic tuples $\text{COLS}_1$ and $\text{COLS}_2$ define a bijective map between the returned tuples. $\text{COND}$ is a FOL formula that represents the constraints that must be satisfied for the symbolic tuples $\text{COLS}_1$ and $\text{COLS}_2$ to be returned by Q1 and Q2, respectively. They encode the semantics of the predicates in the queries. $\text{ASSIGN}$ is another FOL formula that specifies the relational constraints between symbolic variables used in $\text{COLS}_1$, $\text{COLS}_2$ and $\text{COND}$. This formula is used for supporting complex SQL operators, such as CASE.

**Verifying Full Equivalence:** To prove that two cardinally equivalent ARs Q1 and Q2 are fully equivalent, SPES needs to prove that the bijective map between returned tuples is an identity map. In other words, SPES needs to prove that, for an arbitrary tuple $t$ returned by Q1, the bijective map associates $t$ to an identical tuple returned by Q2 with the same values. SPES verifies this property using the QPSR of Q1 and Q2. When both symbolic tuples satisfy the predicate (i.e., $\text{COND}$), it must verify that $\text{COLS}_1$ is equivalent to $\text{COLS}_2$. This property is formalized as:

$$\text{COND} \land \text{ASSIGN} \implies \text{COLS}_1 = \text{COLS}_2$$

SPES verifies this property using an SMT solver [30]. If the property does not hold, then the following formula is satisfiable:

$$\text{COND} \land \text{ASSIGN} \land \neg(\text{COLS}_1 = \text{COLS}_2)$$

SPES feeds this formula into the SMT solver. If the solver determines that this formula is unsatisfiable, then SPES proves that $\text{COLS}_1$ and $\text{COLS}_2$ are always identical. In this manner, SPES leverages the QPSR to prove full equivalence.

### 5.3 Construction of QPSR

Alg. 1 presents a recursive procedure VeriCard for verifying the cardinal equivalence of two ARs. The VeriCard procedure takes a pair of ARs as inputs (i.e., Q1’s AR and Q2’s AR). It first checks the types of the given ARs. If they are of the same type, then it invokes the appropriate sub-procedure for that particular type. We describe these four sub-procedures in Sections 5.4 to 5.7. If Q1 and Q2 are cardinally equivalent, then VeriCard returns their QPSR. If these ARs are of different types, it returns NULL to indicate that it cannot determine their cardinal equivalence. This is because each type of AR has different semantics (§4.1).

Some sub-procedures recursively invoke VeriCard to verify the cardinal equivalence between their sub-queries. It applies the normalization rules defined in §4 to transform the given two ARs so that they are of the same type (and the sub-queries are also of the same types recursively). This normalization process is incomplete (i.e., SPES may conclude that two ARs are not cardinally equivalent since they cannot be normalized to the same type, even if they are actually cardinally equivalent). We discuss this limitation in §7.4.

Each sub-procedure takes a pair of ARs of the same type as inputs. It first attempts to determine if they are cardinally equivalent. If they are cardinally equivalent, then it constructs the QPSR of Q1 and Q2. Otherwise, it returns NULL to indicate that it cannot determine their cardinal equivalence.

In each of the following sub-sections, we first describe the conditions that are sufficient for proving cardinal equivalence based on the semantics of the AR. We then describe how each sub-procedure verifies these conditions to prove cardinal equivalence. We then discuss how SPES constructs the QPSR if they are cardinally equivalent. Lastly, we describe their soundness and completeness properties.

### 5.4 Table AR

Alg. 2 illustrates the VeriTable procedure for table ARs.

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3 A sub-procedure $P$ is sound if whenever it returns a QPSR, the given ARs are cardinally equivalent and the two symbolic tuples define a bijective map. A sub-procedure $P$ is complete if whenever it returns NULL, the given ARs are not cardinally equivalent.
Algorithm 2: Comparison function for Table ARs

Input : A pair of table ARs
Output: QPSR of the table ARs or NULL

1 Procedure VeriTable(TABLE(n1), TABLE(n2))
2    if n1 = n2 then
3      COLS1 ← InitTuple(T→Schema(n1))
4      COLS2 ← COLS1
5      return (COLS1, COLS2, TRUE, TRUE)
6    else return NULL;

Algorithm 3: Comparison function for SPJ ARs

Input : A pair of SPJ ARs
Output: QPSR of given SPJ ARs or NULL

1 Procedure VeriSPJ(SP1(e1, p1, o1), SP2(e2, p2, o2))
2    if QPSR = VeriVec(e1, e2)
3      foreach QPSR ∈ {QPSR} do
4        (COLS1, COLS2, CONST, ASSIGN) ← Compose(QPSR)
5        (COND1, ASSIGN1) ← ConstPred(p1, COLS1)
6        (COND2, ASSIGN2) ← ConstPred(p2, COLS2)
7        if COND1 ∧ COND2 then
8          (COLS1, ASSIGN1) ← ConstExpr(COLS1, o1)
9          (COLS2, ASSIGN2) ← ConstExpr(COLS2, o2)
10         COND ← COND1 ∧ COND2 ∧ COND1
11         ASSIGN ← ASSIGN1 ∧ ASSIGN2 ∧ ASSIGN3 ∧ ASSIGN4
12         return (COLS1, COLS2, CONST, ASSIGN)
13      end
14    end
15    return NULL

CARDINAL EQUIVALENCE:

Lemma 1. A pair of table ARs TABLE(n1) and TABLE(n2) are cardinally equivalent if and only if their input tables are same. (i.e., n1 = n2).

Since the table AR returns all tuples from the input table, thus if two table ARs’ input tables are same, then they will always have the same number of tuples. So VeriTable compares the names of the two input tables (i.e., n1 and n2). SPES cannot show that tables with differing number of tuples are cardinally equivalent in the presence of integrity constraints.

QPSR: We define the QPSR of the two cardinally equivalent table ARs using an identity map between the returned tuples (e.g., QPSR-1 in Section 3.2). VeriTable first constructs the symbolic tuple COLS1 using a vector of new pairs of variables based on the table schema, and then sets the symbolic tuple COLS2 to be the same as COLS1. These two equivalent tuples COLS1 and COLS2 define a bijective map between returned tuples. VeriTable sets the CONST and ASSIGN fields as TRUE since there are no additional constraints that the tuples in the table must satisfy.

Properties: VeriTable is sound and complete. These two properties directly follow from Lemma 1. We present a formal proof in Appendix C.2.1.

5.5 SPJ AR

Alg. 2 illustrates the VeriSPJ procedure for SPJ ARs. VeriSPJ leverages two procedures from [50]: ConstExpr and ConstPred.

ConstExpr takes a vector of projection expressions and a symbolic tuple as inputs, and returns a new symbolic tuple with additional constraints ASSIGN that models the relation between variables. This new symbolic tuple represents the modified tuple based on the vector of projection expressions. ConstPred takes a predicate and a symbolic tuple as the input and returns a boolean formula CONST with additional constraints ASSIGN. CONST symbolically represents the result of evaluating the predicate on the symbolic tuples. ConstPred supports higher-order predicates, such as EXISTS, by encoding them as an uninterpreted function.

CARDINAL EQUIVALENCE: As covered in §4.1, an SPJ AR first computes the cartesian product of all input ARs as the intermediate table (JOIN). It then selects all tuples in the intermediate table that satisfy the predicate (SELECT), and applies the projection on each selected tuple (PROJECT).

Lemma 2. A pair of SPJ ARs SP1(e1, p1, o1) and SP2(e2, p2, o2) are cardinally equivalent if there is a bijective map m between tuples in intermediate join tables, such that the predicates p1 and p2 always return the same result for the corresponding tuples in m.

To prove that there is a bijective map between the tuples in the two intermediate join tables, VeriSPJ first uses the VeriVec procedure to find a bijective map between sub-ARs such that each pair of sub-ARs are cardinally equivalent. VeriVec exhaustively examines all possible maps and recursively uses VeriCard to verify the cardinal equivalence between two sub-ARs. VeriVec returns all possible candidate maps wherein each pair of sub-ARs are cardinally equivalent (\{QPSR\}).

Each candidate map is represented by a vector of QPSR (QPSR), wherein each QPSR defines a bijective map between tuples returned by a pair of cardinally equivalent sub-ARs. VeriSPJ then uses the Compose procedure to construct two symbolic tuples COLS1 and COLS2 (line 4) that represent a bijective map between the tuples in the two intermediate join tables. These two symbolic tuples are constructed by concatenating symbolic tuples from the QPSRs of sub-ARs based on the order of sub-ARs in the input vectors. Compose also constructs CONST and ASSIGN by taking the conjunction of CONST and ASSIGN from the QPSRs of sub-ARs, respectively.

VeriSPJ then tries to prove that the two predicates always return the same result for the two symbolic tuples. VeriSPJ first leverages the ConstPred procedure to encode predicates p1 and p2 on COLS1 and COLS2, respectively (line 6). VeriSPJ uses an SMT solver to prove this property under sub-conditions CONST and all relational constraints: ASSIGN, ASSIGN1, and ASSIGN2 (line 7). If the property holds, then this formula is unsatisfiable:

CONST ∧ ASSIGN1 ∧ ASSIGN2 ∧ ¬(CONST1 = CONST2)

VeriSPJ feeds this formula to an SMT solver. If the solver determines that this formula is unsatisfiable, then we prove CONST1 and CONST2 are always equivalent when the relational constraints ASSIGN, ASSIGN1, and ASSIGN2 and sub-conditions CONST hold.

Consider the cardinally equivalent SPJ ARs shown in Figure 3. In this case, VeriSPJ first verifies that sub-AR E11 is cardinally equivalent to sub-AR E22, and sub-AR E12 is cardinally equivalent to sub-AR E21. Thus, the two intermediate join tables (i.e., cartesian product of sub-tables) are cardinally equivalent.
constructs two symbolic tuples to represent the bijective map between these intermediate join tables by leveraging the two bijective maps between the underlying tables. We verify that two corresponding tuples in the map either both satisfy the predicate or not satisfy the predicate. Thus, the bijective map between the tuples in the intermediate join tables is the bijective map between the tuples in the output tables before projection.

**QPSR:** Since VeriSPJ verifies that the given pair of SPJ ARs are cardinally equivalent, the two symbolic tuples COŁS₁ and COŁS₂ define a bijective map between tuples in the input tables before projection. Projection does not change the bijective map between tuples as it is applied separately on each tuple. Thus, VeriSPJ leverages ConstExpr to construct new symbolic tuples COŁS₁ and COŁS₂ based on the vector of projection expressions and the given symbolic tuples. The QPSR consists of the derived symbolic tuples COŁS₁, COŁS₂, the conjunction of CONST₁, CONST₂, and CONST, and the conjunction of all the relational constraints.

**Properties:** VeriSPJ is sound. Based on Lemma 2, if VeriSPJ returns the QPSR, then the given SPJ ARs are cardinally equivalent. We present a formal proof in §C.2.2.

In general, VeriSPJ is not complete. The reasons are threefold. First, the SMT solver is only complete for linear operators. If the predicates have non-linear operators (e.g., multiplication between columns), then the solver may return UNSAT when it should return SAT [50]. Second, SPES encodes all user-defined functions, string operations, and higher-order predicates as uninterpreted functions. These encodings do not preserve the semantics of these operations. Third, VeriCard is not complete (§5.3).

VeriSPJ procedure is complete if all input ARs for the given two SPJ ARs are table ARs and the SMT solver can determine the satisfiability of the predicates. This is because the problem of deciding equivalence of two conjunctive (i.e., SPJ) queries is decidable [26]. We present a formal proof in Appendix D.

### 5.6 Aggregate AR

Alg. 4 illustrates the VeriAgg procedure for aggregate ARs.

**Cardinal Equivalence:** An aggregate AR groups the tuples in the input table based on the GROUP BY column, then applies the aggregate function on each group to generate a tuple in the output table.

**Lemma 3.** A given pair of aggregate ARs AGG(e₁, g₁, αg₁₁) and AGG(e₂, g₂, αg₂₂) are cardinally equivalent if these two conditions are satisfied: (1) the two input sub-ARs e₁ and e₂ are cardinally equivalent; (2) for any two pairs of corresponding tuples in a bijective map of the QPSR of e₁ and e₂, two tuples in e₁ belong to the same group as defined by g₁ if and only if their associated tuples in e₂ belong to the same group as defined by g₂.

**Algorithm 4:** Comparison function for aggregate ARs

| Input | Output | QPSR of given aggregate ARs or NULL |
|-------|--------|-------------------------------------|
| Procedure | VeriAgg(AGG(e₁, g₁, αg₁₁), AGG(e₂, g₂, αg₂₂)) | QPSR ← VeriCard(e₁, e₂) |
| if QPSR = NULL then | 10.11.111.1111 | else return NULL |

VeriAgg first recursively invokes the VeriCard procedure to determine the cardinal equivalence of the two input sub-ARs e₁ and e₂ (line 2). If VeriCard returns the QPSR of e₁ and e₂, then VeriAgg has proved the first condition in Lemma 3.

To prove the second condition, VeriAgg collects the symbolic tuples COŁS₁ and COŁS₂ from the QPSR. Since these two symbolic tuples define a bijective map between tuples returned by e₁ and e₂, VeriAgg replaces all variables in COŁS₁ and COŁS₂ by a set of fresh variables to generate a second pair of symbolic tuples COŁS₁ and COŁS₂ that represents the same bijective map with different tuples.

We decompose the proof for the second condition into two stages (line 5). In the first stage, we want to prove that if COŁS₁ and COŁS₁ belong to the same group, then COŁS₂ and COŁS₂ also belong to the same group. To prove this, VeriAgg extracts the GROUP BY column sets g₁, g₁, g₂, and g₂ from COŁS₁, COŁS₁, COŁS₂ and COŁS₂, respectively. It then attempts to prove the property:

\[(\text{COND} \land \text{ASSIGN} \land g₁ = g₁') \implies g₂ = g₂'\]

VeriAgg sends the negation of this property to the solver:

\[(\text{COND} \land \text{ASSIGN} \land g₁ = g₁') \land \neg g₂ = g₂'\]

If the solver decides that this formula is unsatisfiable, then it is impossible to find two tuples returned by e₁ that are assigned to the same group by g₁, such that their corresponding tuples returned by e₂ are assigned to different groups by g₂. In the second stage, we use the same technique in the reverse direction of the implication.

Consider the cardinally equivalent aggregate ARs shown in Figure 4. VeriAgg first verifies that the two input ARs E₁ and E₂ are cardinally equivalent, and then constructs the QPSR to represent the bijective map between their returned tuples. VeriAgg then verifies that if two arbitrary tuples in E₁ belong to same group (e.g., first two tuples), then the two corresponding tuples in E₂ also belong to the same group. It also verifies that if two arbitrary tuples in E₂ belong to different groups (e.g., first and third tuples), then the two corresponding tuples in E₂ also belong to different groups. VeriAgg verifies that the two aggregate ARs are cardinally equivalent by verifying that they emit the same number of groups.

**QPSR:** VeriAgg constructs the QPSR of two given aggregate ARs after proving they are cardinally equivalent. COŁS₁ and COŁS₂ define a bijective map between tuples returned by input ARs, and can also be used to define a bijective map between groups in two aggregate ARs. If two aggregate functions in αg₁₁ and αg₂₂ are same and operate on same values (i.e., input columns of the symbolic tuples are same), then the aggregate values in the output tuples are same, since each group contains the same number of tuples.

VeriAgg invokes the InitAgg procedure on αg₂₂ to construct a vector of pairs of new symbolic variables as the symbolic tuples for aggregate functions. In each pair of symbolic variables, the first variable represents the aggregate value. The second variable indicates if the aggregate value is NULL. VeriAgg concatenates the GROUP BY column set g with the symbolic tuple COŁS₁. VeriAgg then invokes the ConstAgg procedure to construct the symbolic columns for αg₂₂, and then concatenates with the GROUP BY column.
set $\vec{g}_2$. ConstAssign uses the same pairs of symbolic variables for all aggregation operations in $a\vec{g}_1\vec{g}_2$, where the aggregation function type and operand columns are the same in $a\vec{g}_1$. VeriAgg sets COND and ASSIGN to TRUE since all tuples must be returned in case of an aggregation AR and there are no additional constraints, respectively.

**Properties:** VeriAgg is sound. Based on Lemma 3, if VeriAgg returns the QPSR, then the two given ARs are cardinally equivalent. This is because the two symbolic tuples $\vec{C}O\vec{L}_S_1$ and $\vec{C}O\vec{L}_S_2$ are constructed from corresponding groups. Thus, $\vec{C}O\vec{L}_S_1$ and $\vec{C}O\vec{L}_S_2$ define a bijective map between tuples returned by the two aggregate ARs. We present a formal proof in Appendix C.2.3.

VeriAgg is not complete. The sources of incompleteness are threefold: (1) incompleteness of VeriCard, (2) limitations of the SMT solver, and (3) when VeriCard returns the QPSR of two input sub-ARs, the symbolic tuples in the QPSR define only one possible bijective map between tuples in the input tables. If VeriAgg fails to prove the second condition in Lemma 3, it is still possible that there exists another bijective map that satisfies the second condition.

### 5.7 Union AR

Alg. 5 illustrates the VeriUnion procedure for union ARs.

**Cardinal Equivalence:** An union AR returns all tuples in each of the input tables.

**Lemma 4.** A given pair of union ARs $Union(\vec{e}_1)$ and $Union(\vec{e}_2)$ are cardinally equivalent if there exists a bijective map between the two input sub-ARs $\vec{e}_1$ and $\vec{e}_2$, such that each pair of ARs are cardinally equivalent.

The lemma follows from the semantics of the union AR. VeriUnion procedure invokes VeriVec (§5.5) to find a bijective map between $\vec{e}_1$ and $\vec{e}_2$ (line 2), such that each pair of ARs are cardinally equivalent.

**QPSR:** VeriVec finds all candidate bijective maps ($\{QPSR\}$) between two input sub-ARs $\vec{e}_1$ and $\vec{e}_2$, such that each pair of sub-ARs are cardinally equivalent. In each candidate bijective map ($QPSR$), a vector of QPSRs is constructed such that each QPSR defines a bijective map between tuples returned by a pair of sub-ARs. VeriUnion gets an arbitrary $QPSR$ (i.e., one candidate bijective map between the sub-ARs). It seeks to construct a bijective map between tuples returned by two union ARs that preserves all of the bijective maps between tuples returned by sub-ARs in that $QPSR$. It first constructs two fresh symbolic tuples $\vec{C}O\vec{L}_S_1$ and $\vec{C}O\vec{L}_S_2$. It then invokes the ConstAssign procedure to set ASSIGN such that both $\vec{C}O\vec{L}_S_1$ and $\vec{C}O\vec{L}_S_2$ are always equivalent to the symbolic tuples in one sub-QPSR returned by VeriVec. ConstAssign creates a vector of boolean variables to set these constraints. VeriUnion returns these two symbolic tuples, TRUE condition, and ASSIGN as the QPSR of the given union ARs.

### 6. Soundness and Completeness

We now discuss the soundness and completeness of SPES for verifying the equivalence of two queries.

**Soundness:** SPES is sound. Given two queries $Q_1$ and $Q_2$, if SPES constructs the QPSR for two normalized ARs that represent $Q_1$ and $Q_2$, and checks the formula holds for the QPSR: $\text{COND} \land \text{ASSIGN} \implies \text{COL}_S_1 = \text{COL}_S_2$, then $Q_1$ and $Q_2$ are fully equivalent.

**Proof Sketch:** If VeriCard returns the QPSR of $Q_1$ and $Q_2$, then $Q_1$ and $Q_2$ are cardinally equivalent and the symbolic tuples in the QPSR define the bijective map between the tuples returned by $Q_1$ and $Q_2$. If SPES determines that the following formula holds for the QPSR: $\text{COND} \land \text{ASSIGN} \implies \text{COL}_S_1 = \text{COL}_S_2$, then $Q_1$ and $Q_2$ are fully equivalent. We present a formal proof in Appendix C.

**Completeness:** In general, SPES is not complete. We discussed the sources of incompleteness in Sections 5.3 to 5.7. However, SPES is complete for a pair of SPJ queries $Q_1$ and $Q_2$ that do not have predicates or projection expressions whose satisfiability cannot be determined by the SMT solver.

**Proof Sketch:** Since $Q_1$ and $Q_2$ are SPJ queries, after normalization, SPES represents them with SPJ ARs that only take table ARs as inputs. VeriSPJ is complete under these conditions. We present a formal proof in Appendix D. Thus, if $Q_1$ and $Q_2$ are fully equivalent, VeriSPJ returns the QPSR of ARs that represents $Q_1$ and $Q_2$. Since $Q_1$ and $Q_2$ do not have predicates or projection expressions whose satisfiability cannot be determined by the SMT solver, the solver will verify whether the following formula holds for the
7. EVALUATION

In this section, we describe our implementation and evaluation of SPES. We begin with a description of our implementation in §7.1. We next report the results of a comparative analysis of SPES against UDP [23] and EQUITAS [50], the state-of-the-art automated QE verifiers based on AR and SR, respectively. We then present the results of a comparative analysis of SPES against QE verification algorithms used in systems for leveraging materialized views. We next quantify the efficacy of SPES in identifying overlapping queries across production SQL queries in §7.3. We conclude with the limitations of the current implementation of SPES in §7.4.

7.1 Implementation

The architecture of SPES is illustrated in Figure 6. SPES takes a pair of SQL queries (Q1 and Q2) as inputs and returns a boolean decision that indicates whether they are fully equivalent. The QE verification pipeline consists of three components: ❶ The compiler first converts the given queries to logical query execution plans. We use the open-source CALCITE framework [3]. ❷ SPES operates on these logical plans in two stages. First, it converts them to their ARs and normalizes these ARs. Next, it uses the third component to verify the cardinal equivalence of ARs and then constructs their QPSRs. It also uses the third component for verifying the properties of QPSR to determine full equivalence. This component is implemented in Java (2,065 lines of code). ❸ The third component is an SMT solver that SPES leverages for determining the satisfiability of FOL formulae [12]. We will release the source code of SPES after this paper is published.

7.2 Comparative Analysis

BENCHMARK: We use queries in the test suite of Apache CALCITE [3] as our benchmark. This test suite contains 232 semantically equivalent query pairs. The reasons for using this benchmark are twofold. First, the CALCITE optimizer is widely used in data processing engines [4, 5, 6, 7, 8]. So, it covers a wide range of SQL features. Second, since UDP and EQUITAS are both evaluated on this query pair benchmark [23, 50], we can quantitatively and qualitatively compare the efficacy of these tools. We send every query pair with the schemata of their input tables to SPES and ask it to check their QE. We conduct this experiment on a commodity server (Intel Core i7-860 processor and 16 GB RAM).

AUTOMATED SQL QE VERIFIERS: The results of this experiment are shown in Table 2. We compare SPES against EQUITAS.

3 The test cases used in this experiment were obtained from the open-sourced COSETTE repository [10].

in the same environment. We present the results reported in the UDP paper [23]. SPES proves the equivalence of a larger set of query pairs (90/232) compared to UDP (34/232) and EQUITAS (67/232). SPES currently supports 120 out of 232 pairs. The un-supported queries either: (1) contain SQL features that are not yet supported (e.g., CAST), or (2) cannot be compiled by CALCITE due to syntax errors. Among the 120 pairs supported by SPES, it proves that 90 pairs (75%) are equivalent under bag semantics. In contrast, UDP proves the equivalence of 34 pairs under bag semantics. EQUITAS proves the equivalence of 67 pairs, but only under set semantics. We group the proved query pairs into three categories:

- **USPJ**: Queries that are union of SELECT PROJECT JOIN.
- **Aggregate**: Queries containing at least one aggregate.
- **Outer-Join**: Queries containing at least one outer JOIN.

Table 2 reports the number of pairs proved by UDP and EQUITAS in each category. The number of proved pairs containing outer JOIN is not in case of UDP. SPES outperforms the other tools on queries containing aggregate and outer JOIN operators.

We next compare the average time taken by SPES, UDP and EQUITAS to prove the equivalence of a pair of queries in each category. This is an important metric for a cloud-scale tool that must be deployed in a DBaaS platform. We only compute this metric for the pairs that these tools can prove. SPES, UDP, and EQUITAS take 0.05 s, 4.16 s, and 0.15 s on average to prove QE. So, SPES is 83 times faster than UDP and 3 times faster than EQUITAS on this benchmark.

LEVERAGING MATERIALIZED VIEWS: In this experiment, we compare SPES against equivalence verification algorithms used in systems for leveraging materialized views: (1) MINICON [43] and (2) VIEWMATCHER [33].

MINICON only proves containment relationships between conjunctive queries (i.e., SPJ queries). So, it supports 30 pairs of SPJ queries in the CALCITE benchmark. In contrast, SPES proves that 27 of these 30 query pairs are equivalent. It does not support 3 pairs since their equivalence is conditioned on integrity constraints that SPES currently does not support. SPES supports other types of queries in the CALCITE benchmark that MINICON cannot support.

VIEWMATCHER only proves containment relationships between SPJ queries and aggregate queries whose inputs are SPJ queries. It leverages a syntactical comparison scheme to verify the containment relationship between queries. We implemented this comparison scheme and found that it proves 25 pairs of queries are equivalent in CALCITE. Since SPES relies on semantic comparison which subsumes syntactical comparison, it supports all of them.

7.3 Efficacy on Production Queries

In this experiment, we quantify the efficacy of SPES in detecting overlap in production SQL queries. We leverage three sets of real production queries from Ant Financial [2], a financial technology company. These queries are used to detect fraud in business transactions. In each set, we run SPES on each pair of queries that operate on the same set of input tables. If SPES decides that a given pair of queries are not equivalent, then we check any constituent sub-queries that operate on the same input tables. We skip checking queries containing only table scans and those that only differ in the parameters passed on to their predicates. This is because SPES trivially proves their equivalence and the computational resources needed for evaluating such queries are negligible.

3 We were unable to conduct a comparative performance analysis under the same environment since UDP is currently not open-sourced.
Compared Query Pairs refers to number of query pairs that operate on the same set of input tables.

ARs with: (1) union and aggregate [15], (2) join and aggregate [7], AR using a set of pre-defined semantically-equivalent re-write rules

Among the 120 query pairs supported by SPES, it cannot prove the equivalence of two ARs only if it can normalize them into the same type of sub-ARs) in a given query (complex queries will have a larger set of expressions). We found that the average number of algebraic expressions in the Ant Financial workload and the CALCITE benchmark is 45.38 and 5.37, respectively.

7.4 Limitations

In general, the problem of deciding QE is undecidable [15]. Among the 120 query pairs supported by SPES, it cannot prove the QE of 30 pairs. We classify them into three categories: (1) lack of normalization rules [22], (2) support for integrity constraints [7], and (3) support for type casting [1].

NORMALIZATION RULES: SPES can verify the cardinal equivalence of two ARs only if it can normalize them into the same type of AR using a set of pre-defined semantically-equivalent re-write rules (§5.3). We will need to introduce additional normalization rules for ARs with: (1) union and aggregate [15], (2) join and aggregate [7], and (3) multiple aggregates with a complex relationship [2]. Adding these re-write rules in the normalization stage will enable SPES to prove the QE of these 22 pairs. However, that will also increase the average QE verification time. Furthermore, these rules are not required for supporting production queries discussed in §7.3.

INTEGRITY CONSTRAINTS: SPES currently does not support integrity constraints (e.g., distinct values, foreign keys, and primary keys). We will need to encode these integrity constraints in our normalization rules. Updating these rules will enable SPES to prove the QE of 7 pairs. For example, we may normalize an OUTER JOIN operation based on a foreign key to an INNER JOIN operation.

8. RELATED WORK

QUERY EQUIVALENCE: The state-of-the-art QE verification tools are based on either AR [24, 22, 25] or SR [50]. We highlighted the differences between SPES and these tools in §2. Prior efforts have examined the theoretical aspects of equivalence and containment relationships between queries. Since it is an undecidable problem [14, 18], these efforts focused on determining categories of queries for which it is a decidable problem: (1) conjunctive queries [21], (2) conjunctive queries with additional constraints [19, 36, 28], and (3) conjunctive queries under bag semantics [37]. The problem of deciding containment relationship between conjunctive queries can be reduced to a constraint satisfiability problem [40]. Other proposals include decision procedures for proving equivalence of a subset of queries under set [20, 45, 44] and bag semantics [27, 31, 32, 42].

SYMBOLIC EXECUTION IN DBMSs: Researchers have leveraged symbolic execution in DBMSs by reducing the given problem to a FOL satisfiability problem and then using an SMT solver to solve it. These efforts include: (1) automatically generating test cases for database applications [46, 47, 13], (2) verifying the correctness of database applications [48, 38, 34], (3) disproving the equivalence of SQL queries [24], and (4) finding the best application-aware memory layout [49]. SPES differs from these efforts in that we seek to address the limitations of symbolic approaches to QE.

9. CONCLUSION

In this paper, we presented the design and implementation of SPES that takes a two-stage approach to query equivalence. We illustrated how it supports structurally-different queries with complex operators under bag semantics. SPES uses a set of rules for normalizing the differences between complex, structurally-different queries. It then converts these expressions to a QPSR and determines their full equivalence under bag semantics using an SMT solver. Our evaluation shows that SPES proves the equivalence of a larger set of query pairs under bag semantics compared to the state-of-the-art tools based on algebraic and symbolic approaches.
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APPENDIX

A. SEMANTICS OF AR

We now formally define the semantics AR queries, using the following formal notation. $\downarrow$ is the evaluation symbol. The left side of this symbol is an algebraic expression that is evaluated on valid input tables $T_s$. The right side of this symbol is the evaluation result, which is the output table. All output tables are bags (i.e., can contain duplicate tuples). A horizontal line separates the pre- and the post-conditions. The pre-conditions on the top of the line include a set of evaluation relations. The post-conditions on the bottom side of the line is an evaluation relation. If all the relations in the pre-conditions hold, then the relation in the post-condition holds.

$E$-TABLE $\downarrow t\forall t \in n$

$E$-SPJ $\downarrow (\text{SPJ}(\vec{e}, \vec{p}, \vec{a})|T_s)\downarrow (\vec{a}(t)|\forall t \in (T_0 \times \cdots \times T_n), p(t))$

$E$-AGG $\downarrow (\text{AGG}(\vec{e}, \vec{g}, \vec{a}\vec{g}|T_s)\downarrow (\vec{a}\vec{g}(t)|\forall t \in \text{part}(T_0, \vec{g}))$

$E$-UNION $\downarrow (\text{UNION} \vec{e}|T_s)\downarrow t\forall t \in T_0 + \cdots + T_n$

$\vec{e} = e_0, e_1, \ldots, e_n$

$E ::= \text{Column $i$}|\text{Const $v$}|\text{NULL}|\text{Bin} E \text{ OP } E|\text{Fun } N (\vec{e})|\text{CASE}$

CASE $::= \text{Pair } E$

Pair $::= (\text{WHEN } P \text{ E})\text{Pair } e$

OP $::= +|−|×|\div|\text{mod}$

P $::= \text{Bin}E \text{ CP } \text{Bin}L \text{ P LOGIC P}|\text{Not }P|\text{IsNull}$

CP $::= >|<|\leq|\geq$

LOGIC $::= \text{AND} \text{ OR}$

Figure 9: Predicate & Projection Expressions – Types of predicates and projection expressions supported by SPES.

B. PREDICATE & PROJ. EXPRESSION

SPES supports the predicate and project expressions shown in Figure 8. It uses the same encoding scheme as the one employed in EQUITAS (described in Section 3.4 of [50]).

A projection expression $E$ can either be a column that refer to a specific column, a constant value, NULL, a binary expression, an uninterpreted function, or an CASE expression (Eqn. 1). A predicate $P$ can either be a binary comparison between two projection expression, a binary predicate that is composed by two predicates, a not predicate and a predicate decide if a projection expression is NULL (Eqn. 5).

C. SOUNDNESS OF VERIFICATION

In this section, we give the formal proof of the soundness of SPES. The overall proof is structured as follows: we introduce symbolic representations of bijections over tuples in §C.1, prove correctness of the procedure for generating symbolic representations (§C.2), and then prove correctness of the procedure for determining equivalence (§C.3).

C.1 Symbolic bijections between queries

All definitions in this section are given with respect to arbitrary queries $Q_1$ and $Q_2$, whose columns are denoted $\text{COLS}_1$ and $\text{COLS}_2$, respectively.

A cardinality-preserving binary relation between $Q_1$ and $Q_2$ is a relation

$R \subseteq \text{COLS}_1 \times \text{COLS}_2$

such that for each input table set $I$ and all tuples $(t, u) \in R$, it holds that

$|t|_{Q_1(I)} = |u|_{Q_2(I)}$

where $|u|_T$ denotes the number of occurrences of tuple $u$ in table $T$.

Cardinality-preserving binary relations can act as witnesses of full equivalence (see Definition 2) between queries.

Lemma 5. If the identity function is a cardinality-preserving binary relation between $Q_1$ and $Q_2$, then $Q_1$ and $Q_2$ are fully equivalent (denoted $Q_1 \equiv Q_2$).

Proof. Let $I$ be an arbitrary table set and let $t$ be an arbitrary tuple over columns $\text{COLS}$. Then

$|t|_{Q_1(I)} = |t|_{Q_2(I)}$

by the fact that the identity function is cardinality-preserving. Thus $Q_1(I)$ and $Q_2(I)$ are equivalent under bag semantics by the definition of bag semantics. Thus $Q_1 \equiv Q_2$, by definition of equivalence.

A symbolic representation of a binary relation

$R \subseteq \text{COLS}_1 \times \text{COLS}_2$

is an SMT formula over a vocabulary that extends $\text{COLS}_1$ and $\text{COLS}_2$ such that for each $(t, u) \in R$, there is some model $m$ of $\varphi$ such that

$t = m(\text{COLS}_1) \quad u = m(\text{COLS}_2)$

where $m(\text{COLS}_1)$ is the tuple of interpretations of each column name in $\text{COLS}_1$ (and similarly for $m(\text{COLS}_2)$).

Symbolic representations of cardinality-preserving bijections can be viewed as QPSRs (defined in Section 5.2), collapsed into single...
Symbolic cardinality-preserving bijections can be conjoined with equivalent constraints over column fields to form new symbolic cardinality-preserving bijections. In order to formalize this, we will say that for each partial bijection \( b \) between \( \text{COLS}_1 \) and \( \text{COLS}_2 \), each formula \( \varphi_1 \) over vocabulary \( \text{COLS}_1 \), and each formula \( \varphi_2 \) over vocabulary \( \text{COLS}_2 \), \( \varphi_1 \) and \( \varphi_2 \) are equivalent over \( b \) if \( m \) is a model of \( \varphi_1 \) if and only if \( b(m) \) is a model of \( \varphi_2 \).

**Lemma 6.** For each cardinality-preserving bijection \( b \) symbolically represented by \( \varphi \) and all \( \psi_1 \) over \( \text{COLS}_1 \) and \( \psi_2 \) over \( \text{COLS}_2 \) that are equivalent over \( b \),

\[
\varphi \land \psi_1 \land \psi_2
\]

is a symbolic cardinality-preserving bijection.

**Proof.** \( b \mid \varphi_1 \) is the interpretation of \( \varphi \land \psi_1 \land \psi_2 \). It is a cardinality-preserving bijection because it is a restriction of a cardinality-preserving bijection.

Because cardinality-preserving bijections can act as witnesses of equivalence, their symbolic representations naturally can, as well.

**Lemma 7.** If there is some symbolic cardinality-preserving bijection \( \varphi \) between \( Q_1 \) and \( Q_2 \) that entails

\[
\text{COLS}_1 = \text{COLS}_2
\]

then \( Q_1 \equiv Q_2 \).

**Proof.** \( \varphi \) represents the identity function by the assumptions that it represents a total function and that it logically entails a symbolic representation of the identity relation. Thus, \( Q_1 \equiv Q_2 \), by Lemma 5.

### C.2 Synthesizing symbolic bijections

We now prove the soundness of the procedure \( \text{VeriCard} \). The proof is defined using a set of lemmas per form of input query (Appendix C.2.1—Appendix C.2.4), each of which are predicated on assumptions that \( \text{VeriCard} \) is sound on smaller queries. The proof for arbitrary queries combines the lemmas that concern each form of query in a proof by induction on \( \text{VeriCard} \)’s input query (Appendix C.2.5).

#### C.2.1 Symbolic bijections between table queries

We now state and prove the soundness of \( \text{VeriTable} \), which is given in Algorithm 2. For a given pair of table ARs \( \text{TABLE}(n_1) \) and \( \text{TABLE}(n_2) \), \( \text{VeriTable} \) first checks if two table ARs have the same name. If two table ARs have the same names, then \( \text{VeriTable} \) uses procedure \( \text{InitTuple} \) to create a new vector of pair of symbolic variables based on the input table schema, and assign this new vector to \( \text{COLS}_1 \). \( \text{VeriTable} \) then sets \( \text{COLS}_1 \) is equal to \( \text{COLS}_2 \). \( \text{VeriTable} \) returns the QPSR with \( \text{COLS}_1 \) and \( \text{COLS}_2 \), where both \( \text{COND} \) and \( \text{ASSIGN} \) are TRUE. If two table ARs have different names, then \( \text{VeriTable} \) returns NULL.

**Lemma 8.** If \( \text{VeriTable} \), given table ARs \( q_1 = \text{TABLE}(n_1) \) and \( q_2 = \text{TABLE}(n_2) \), returns some QPSR \( \varphi \), then \( \varphi \) is a symbolic cardinality-preserving bijection between \( q_1 \) and \( q_2 \).

**Proof.** \( \text{VeriTable} \) determines that \( n_1 = n_2 \), by the fact that \( \text{VeriTable} \) only returns a QPSR if \( n_1 = n_2 \) and by the assumption that \( \text{VeriTable} \) returns a QPSR. The QPSR returned by \( \text{VeriTable} \) is the symbolic representation of the identity relation, and is thus a symbolic cardinality-preserving bijection.
Q1 is a composite of sub-queries. In particular:  

1. If Q1 is a table name, then the proof follows immediately from Lemma 8.
2. If Q1 is an SPJ query, the proof follows immediately from Lemma 9.
3. If Q1 is a aggregate query of the form \( AGG(Q_1', \vec{q}_1, \vec{a}_1) \), then Q2 is a union query of the form \( AGG(Q_2', \vec{q}_2, \vec{a}_2) \), by the assumption that VeriCard returns some QPSR. The proof follows from applying Lemma 10 to the inductive hypothesis on queries Q1' and Q2'.
4. If Q1 is a union query of the form \( UNION(Q_1) \), then Q2 is a union query of the form \( UNION(Q_2') \), by the assumption that VeriCard returns some QPSR. The proof follows from applying Lemma 11 to the inductive hypothesis on all sub-queries in Q1' and Q2'.

\square

C.3 Soundness of SPES

With the soundness of VeriCard established, we are prepared to state and prove the soundness of SPES. Given a pair of queries Q1 and Q2, SPES uses the procedure normalize to converts each queries to algebraic expressions, and uses a set of semantic preserving rewrite rules to normalize them. These semantic preserving rules are defined in Section 4. Then SPES uses the procedure VeriCard to constructs the QPSR of two normalized queries Q1' and Q2'. If VeriCard returns the QPSR (i.e., the QPSR is not NULL), then SPES returns if the formula is valid. If VeriCard doesn't return the QPSR (i.e., the QPSR is NULL), then SPES returns FALSE.

The soundness of SPES is formalized and proved in the following theorem:

**Theorem 1.** If SPES, given queries Q1 and Q2, returns true, then Q1 \( \equiv \) Q2.

**Proof.** Q1' is equivalent to Q1 and Q2' is equivalent to Q2 because the normalization rules that it applies preserve semantics (Section 4). QPSR is a symbolic cardinality-preserving bijection by the assumption that SPES returns true and thus QPSR is not null, and by Lemma 12. The formula

\[ \varphi \implies \text{COLS}_1 = \text{COLS}_2 \]

is valid by the assumption that SPES returns True. Thus \( \varphi \) entails \( \text{COLS}_1 = \text{COLS}_2 \) by the semantics of SMT. Thus, Q1 \( \equiv \) Q2 by Lemma 7.

D. COMPLETENESS OF SPES

**Lemma 13.** For a pair of SELECT-PROJECT-JOIN queries Q1 and Q2 that do not have predicates and projection expressions whose satisfiability cannot be determined by the SMT solver, then SPES is complete.

**Proof.** Since Q1 and Q2 are SELECT-PROJECT-JOIN queries, after normalization, Q1' and Q2' are SPJ ARs with all inputs sub-ARs are table ARs. Based on Lemma 14, if Q1' and Q2' are fully equivalent, then VeriCard returns an QPSR \( \varphi \), and \( \varphi \) is a symbolic representation of a cardinality-preserving bijection between Q1' and Q2'. Because Q1 and Q2 do not have predicates and projection expressions whose satisfiability cannot be determined by the SMT solver. Using SMT solver to verify the following formula holds for QPSR is complete: \( \varphi \implies \text{COLS}_1 = \text{COLS}_2 \). Thus, if SPES decides this formula does not hold. Based on the model \( M \) SPES generates, we can construct the inputs tables that Q1 and Q2 each return one tuple that satisfy the model \( M \). These two tuples are not identical. Thus, Q1 and Q2 are not fully equivalent. By contradiction, SPES is complete.

**Lemma 14.** If all input ARs for the two SPJ ARs are table ARs and the the SMT solver can determine the satisfiability of the predicates, then VeriSPJ procedure is complete.

**Proof.** We prove this theorem using the method of contraposition. Suppose that VeriSPJ returns NULL for a pair of SPJ ARs. By the definition of VeriSPJ, there are two cases:

**Case 1:** There is no bijective map between \( e_{11} \) and \( e_{12} \), such that each pair of ARs are cardinally equivalent. Since each input ARs are table ARs, there are two possible sub-cases.

For the first sub-cases, \( e_{11} \) has more input table ARs than \( e_{12} \). For this sub-case, we can always construct the input such that the intermediate join table of \( e_{11} \) has more tuples than the intermediate join table of \( e_{12} \). SPES have eliminated the case where the predicates are False (§4.3). Thus, these constructed tuples in intermediate join table all can satisfy the predicate. Thus, the two SPJ ARs return differing number of tuples, and are hence not cardinally equivalent.

For the second sub-cases, \( e_{11} \) have different table ARs than \( e_{12} \). For this sub-case, all input tables in \( e_{11} \) are empty tables. And the different table in \( e_{11} \) has one tuple that satisfy the predicate. Thus, the two SPJ ARs return differing number of tuples, and are hence not cardinally equivalent.

**Case 2:** VeriSPJ cannot verify that the two predicates always return the same result for two corresponding tuples in a bijective map between tuples in the intermediate table. In this case, since both predicates are decidable, the solver will generate a model \( M \) such that one symbolic tuple satisfies the predicate and the other one does not. We then construct inputs such that each intermediate table only contains one tuple that matches the values in \( M \). Then the first SPJ AR returns a table that contains one tuple, while the
other one returns an empty table. Thus, the two SPJ ARs are not
cardinally equivalent. □