Collectivity from interference

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Figure 1: Two-dimensional (2-D) per-trigger-particle associated yield of charged hadrons as a function of $\Delta \eta$ and $\Delta \phi$ with jet peak cutoff for better demonstration of the ridge from high multiplicity ($N \geq 110$) pp collisions at $\sqrt{s} = 7$ TeV, for (a) $2 < p_T^{\text{trig}} < 3$ GeV/c and $1 < p_T^{\text{assoc}} < 2$ GeV/c and (b) $5 < p_T^{\text{trig}} < 6$ GeV/c and $1 < p_T^{\text{assoc}} < 2$ GeV/c.

Figure 5: The 1-D $\Delta \phi$ correlation functions in the multiplicity range $110 \leq N^{\text{offline}}_{\text{trk}} < 150$ of pp collisions at $\sqrt{s} = 7$ TeV after subtracting results from $10 \leq N^{\text{offline}}_{\text{trk}} < 20$, for trigger particles composed of inclusive charged particles (left), $K^{0}_S$ particles (middle), and $\Lambda/\bar{\Lambda}$ particles (right). Selection of a $p_T^{\text{trig}}$ and $p_T^{\text{assoc}}$ range of both 1–3 GeV/c is shown for the long-range regions.
Ridge correlations were first found in A-A collisions at RHIC, and since are considered important sign of creation of Quark-Gluon Plasma. They are successfully described by hydrodynamic model. Jet quenching in AA collisions is another sign of creation of QGP. So both-jet quenching and success of hydrodynamics—signs of large final state interactions/final state rescattering. Recently using kinetic equations it was possible to derive hydrodynamic model from final state interactions (G.Moore, A. Kurkella and their collaborators), taking as a basis bottom-up thermalization scenario (Baier,Mueller,Schiff,Son).

Ridge- 1) long range i.e. rapidity-independent correlations  
2) transverse momentum correlations of created hadrons, described by cumulants, measured by experimentalists for N=2,4,6,..  

However ridge was found also in pp and pA collisions at LHC (along with AA collisions) by all 3 collaborations: CMS(2012), Alice (2013), ATLAS (2013).

Paradox-1) there is no evidence for large final state interactions in pp and probably In pA  
2) jet quenching coefficient in both pp and pA is equal to R=1  
So hydrodynamical interpretation of ridge in pp and pA collisions looks unnatural (A. Mueller In Initial Stages, Lissabon)  

Question: can ridge have different origin on pp than in AA?
Indeed, there were several attempts to explain ridge correlations in pp:
1) Hydrodynamics
2) transport models (all need large final state interactions)
3) High density Color Glass Condensate approach (Altinoliuk, Armesto, Beuf, iancu, Kovner, Lublinsky, Jalalian-Marian, Levin, Gotsman, Tribedi, Venugopalan, …)

However: such an approach means that saturated initial state is needed to describe Underlying Event in pp collisions at LHC. On the other hand we know for quite a long time that UE is well described by MC generators, that work in dilute regime (small relative to CGC gluon densities)
4) The same problem is in applying strongly coupled paradigms i.e. ADS/CFT-radical change of known description of UE by MC generators

Another way-Low density scenario, Collectivity from interference – This Talk
1. No initial density, no initial asymmetry, no final state interactions
2. The cumulants naturally come from quantum-mechanical (QM) interference and color correlations.

Hence: 1) does not imply jet quenching in pp/pA
2) natural extension of MC generators, that contain in UE up to 10-20 MPI events (Sjostrand)
The interference occurs not between gluons radiated by separate partons, but between different Hard processes, i.e. different MPIs, which we call “sources”.

![Diagram](image-url)
Basic ideas about MPI geometry used in this talk:

1. Hadronic cross sections with $N$ hard processes simultaneously are characterized by

$$\sigma_{N \text{MPI}} = \frac{\sigma_1 \cdots \sigma_N}{K_N}.$$  

$N=2$ case is called DPS and characterized by

$$\sigma_{2 \text{MPI}} = \frac{\sigma_1 \sigma_2}{\sigma_{\text{eff}}}$$

$$\frac{1}{K_N} = \int \left( \prod_{i=1}^{N} \frac{d\Delta_i}{(2\pi)^2} \right) \frac{G_N(x_i, \{Q_i^2\}, \{\Delta_i\}) G_N(x_i', \{Q_i^2\}, \{\Delta_i\})}{\prod_{i=1}^{N} f(x_i, Q_i^2) f(x_i', Q_i^2)} \frac{g^2}{\left( \sum_{i=1}^{N} \Delta_i \right)^2}$$

The Generalized parton distributions GPD encode geometry of the process. We work Here in mean-field approximation neglecting 1-2 mechanism (Ladder splitting, probably not so big in UE)

$$G_N(x_i, \{Q_i^2\}, \{\Delta_i\}) = \prod_{i=1}^{N} G_1(x_i, Q_i, \Delta_i) = \prod_{i=1}^{N} f(x_i, Q_i) F_{2g}(\Delta_i).$$

Choosing $F_{2g}^2(\Delta) = \exp(-B\Delta_i^2)$ for simplicity to be of Gaussian form

$$F_N(\Delta_1, \ldots, \Delta_N) = \prod_{i=1}^{N} \exp(-B\Delta_i^2)$$

$$F_N(\Delta_1, \ldots, \Delta_N) = \frac{G_N^2(\Delta_1, \ldots, \Delta_N)}{\prod_{i=1}^{N} f(x_i, Q_i) f(x_i', Q_i)}$$

$$B = 2 \text{ GeV}^{-2} \quad \leftrightarrow \quad \sigma_{\text{eff}} \approx 20 \text{ mb}$$

Number compatible to LHC data
In coordinate space:

\[
\frac{d\sigma_N}{\sigma_N \, dk_1 \cdots dk_m} = \frac{\int \left( \prod_{i=1}^{N} dy_i \right) \int d\mathbf{b} \left| \mathcal{M}^2(k_1, \cdots, k_m; y_1, \cdots, y_N) \right| \rho(y_1 \cdots y_N, \mathbf{b})}{\int \left( \prod_{i=1}^{N} dy_i \right) d\mathbf{b} \rho(y_1 \cdots y_N, \mathbf{b})}.
\]

\[
\rho(\{y_i\}, \mathbf{b}) = \prod_j \frac{1}{(4\pi B)^2} \exp \left[ -\frac{y_j^2}{4B} \right] \exp \left[ -\frac{(y_j - b)^2}{4B} \right]
\]

We work in the case \(b=0\) (central collisions)
Basic ideas of the model:

\[
\frac{d\Sigma}{dk_1...dk_m} = \int \left( \prod_{i=1}^{N} dy_i \right) \rho(\{y_i\}) \sigma(\{k_j, y_i\}) .
\]

Each hadron collision is characterized by a set \( \{y_i, b_i\}, i \in [1, N] \), of \( N \) particle emitting sources distributed at transverse positions \( y_i \) with initial colors \( b_i \) in the adjoint representation.

Gluon emission

\[
\tilde{f}(k) \propto g k / k^2 .
\]

Assume LPHD

Coulombic radiation:
Cumulants:

\[ T_n(k_1, k_2) = \binom{m}{2} \int_0^{2\pi} d\phi_1 d\phi_2 \exp\left[i n(\phi_1 - \phi_2)\right] \left( \int \prod_{b=3}^{m} k_b \, dk_b \, d\phi_b \right) \hat{\sigma} \]

Norm:

\[ \overline{T}(k_1, k_2) = \binom{m}{2} \int_0^{2\pi} d\phi_1 d\phi_2 \left( \int \prod_{b=3}^{m} k_b \, dk_b \, d\phi_b \right) \hat{\sigma} \]

Second cumulant:

\[ \langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle(k_1, k_2) = \frac{T_n(k_1, k_2)}{\overline{T}(k_1, k_2)} \]

\[ v_n^2{2}(k_1, k_2) \equiv \langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle(k_1, k_2) \]

\[ v_n{2}(k) \equiv \sqrt{\langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle(k_1, k_2)} \]

4th cumulant

\[ S_n(k_1, k_2, k_3, k_4) = \binom{m}{4} \int_0^{2\pi} d\phi_1 d\phi_2 d\phi_3 d\phi_4 e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \left( \int \prod_{b=5}^{m} k_b \, dk_b \, d\phi_b \right) \hat{\sigma} , \]

\[ \langle \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle \rangle_c = \langle \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle \rangle - \langle \langle e^{in(\phi_1 - \phi_3)} \rangle \langle \langle e^{in(\phi_2 - \phi_4)} \rangle \rangle - \langle \langle e^{in(\phi_1 - \phi_4)} \rangle \langle \langle e^{in(\phi_2 - \phi_3)} \rangle \rangle , \]

Higher cumulants – in the same way, see Ollitrault and his collaborators.
Dipole interference term
Direct calculation: \( m=2 \) \( N=2 \)

\[
\rho (y_1, y_2) = \frac{1}{(2\pi B)^2} \exp \left[ -\frac{y_1^2}{2B} - \frac{y_2^2}{2B} \right].
\]

\[
\hat{\sigma} \{ k_1, k_2 \}, \{ y_1, y_2 \} \propto N_c^2 \left( N_c^2 - 1 \right)^2 \left| \vec{f}(k_1) \right|^2 \left| \vec{f}(k_2) \right|^2 
\times \left\{ 4 + \frac{1}{(N_c^2 - 1)} \left[ e^{i(k_1+k_2) \cdot (y_1-y_2)} + e^{i(k_1-k_2) \cdot (y_1-y_2)} 
+ e^{i(k_1+k_2) \cdot (y_2-y_1)} + e^{i(k_1-k_2) \cdot (y_2-y_1)} \right] \right\}.
\]

Dipole color suppression

\[
\text{Tr} \left[ T^a T^b \right] \text{Tr} \left[ T^b T^a \right] = N_c^2 \left( N_c^2 - 1 \right).
\]

\[
\text{Tr} \left[ T^a T^b T^c T^d \right] \text{Tr} \left[ 1 \right] = N_c^2 \left( N_c^2 - 1 \right)^2.
\]

Final answer:

\[
\frac{d\Sigma}{dk_1 dk_2} \propto \left| \vec{f}(k_1) \right|^2 \left| \vec{f}(k_2) \right|^2 \left[ 1 + \frac{\left( e^{-B(k_1+k_2)^2} + e^{-B(k_1-k_2)^2} \right)^2}{(N_c^2 - 1)} \right].
\]

For \( B = 1/Q_s^2 \), this QCD dipole radiation agrees with CGC calculations.

Saturated initial state not assumed here.
\[ T^{c_j}T^aT^{c_j} = \frac{1}{2} N_c T^a. \]

\[ \hat{\sigma} \propto (N_c^2 - 1)^N N_c^m \left( \prod_{i=1}^m |\tilde{f}(k_i)|^2 \right) \]

\[ \times \left\{ N^m + F_{\text{corr}}^{(2)}(N, m) \frac{N^{m-2}}{(N_c^2 - 1)} \sum_{(ab)} \sum_{(ij)} 4 \cos (k_a \cdot \Delta y_{ij}) \cos (k_b \cdot \Delta y_{ij}) \right. \]

\[ \left. + O \left( \frac{1}{N (N_c^2 - 1)} \right) + O \left( \frac{1}{(N_c^2 - 1)^2} \right) \right\}. \]

\[ F_{\text{corr}}^{(2)}(N, m) = \frac{1}{N_{\text{incoh}}} \sum_{j=0}^{m-2} N^{m-2-j}(m-1-j) \left( \sum_{l=0}^{j} \binom{j}{l} 2^l (N-2)^{j-l} \frac{1}{2^l} \right) \]

\[ = \frac{2}{m(m-1)} N^{1-m} \left( N(N-1)^m + mN^m - N^{1+m} \right). \]
$v_2^2\{2\}(k_1, k_2) \equiv \langle e^{i2(\phi_1 - \phi_2)} \rangle(k_1, k_2) \equiv \frac{F^{(2)}_{\text{corr}}(N, m) \int_\rho \frac{1}{N^2} \sum_{(ij)} 2 J_2(k_1\Delta y_{ij}) J_2(k_2\Delta y_{ij})}{(N_c^2 - 1) + F^{(2)}_{\text{corr}}(N, m) \int_\rho \frac{1}{N^2} \sum_{(ij)} 2^2 J_0(k_1\Delta y_{ij}) J_0(k_2\Delta y_{ij})} + O\left(\frac{1}{(N_c^2 - 1)^2}\right)$

- Partonic $v_2$, may be modified by hadronization
- Signal persists to multi-GeV region

For fixed average multiplicity per source, $\overline{m} = m/N$

$$\lim_{m \to \infty} F^{(2)}_{\text{corr}}(m/\overline{m}, m) = \frac{2\overline{m} + 2e^{-\overline{m}} - 2}{\overline{m}^2}.$$

Red (green) lines for $\overline{m} = m/N = 3 (5)$
For any multiplicity \( m \), \( v_2^2\{2\} \) is finite in the limit \( N \to \infty \) of a large number of sources.

\( v_2^2\{2\}(k_1, k_2) \) does not factorize except for small transverse momentum.

For any finite number of sources \( N \), \( v_2^2\{2\}(k_1, k_2) \) vanishes in the high-multiplicity limit.
4th cumulant and collectivity.

Full result in large $N$ limit

\[
\hat{\sigma} \propto N_c^m (N_c^2 - 1)^N \left( \prod_{i=1}^{m} |\bar{f}(k_i)|^2 \right) N^{m-4} 
\times \left\{ N^4 + F_{\text{corr}}^{(2)}(N, m) \frac{N^2}{(N_c^2 - 1)^2} \sum_{(ab)} \sum_{(lm)} 2^2 \cos (k_a \cdot \Delta y_{lm}) \cos (k_b \cdot \Delta y_{lm}) \right. 
+ F_{\text{corr}}^{(3i)}(N, m) \frac{N}{(N_c^2 - 1)^2} \sum_{(abc)} \sum_{(lm)(mn)(nl)} 2^3 \cos (k_a \cdot \Delta y_{lm}) 
\times \cos (k_b \cdot \Delta y_{mn}) \cos (k_c \cdot \Delta y_{nl}) 
+ F_{\text{corr}}^{(4i)}(N, m) \frac{1}{(N_c^2 - 1)^2} \sum_{(lm)(no)} \sum_{(ab)(cd)} 2^4 \cos (k_a \cdot \Delta y_{lm}) \cos (k_b \cdot \Delta y_{lm}) 
\times \cos (k_c \cdot \Delta y_{no}) \cos (k_d \cdot \Delta y_{no}) 
+ F_{\text{corr}}^{(4ii)}(N, m) \frac{1}{(N_c^2 - 1)^3} \sum_{(lm)(mn)(no)(ol)} \sum_{(abcd)} 2^4 \cos (k_a \cdot \Delta y_{lm}) \cos (k_b \cdot \Delta y_{mn}) 
\times \cos (k_c \cdot \Delta y_{no}) \cos (k_d \cdot \Delta y_{ol}) 
+ F_{\text{corr}}^{(5)}(N, m) \frac{N^{-1}}{(N_c^2 - 1)^3} \sum_{[(lm)(mn)(nl)](op)} \sum_{(abc)(de)} 2^2 \cos (k_d \cdot \Delta y_{op}) \cos (k_e \cdot \Delta y_{op}) 
\times 2^3 \cos (k_a \cdot \Delta y_{lm}) \cos (k_b \cdot \Delta y_{mn}) \cos (k_c \cdot \Delta y_{nl}) 
+ F_{\text{corr}}^{(6)}(N, m) \frac{N^{-2}}{(N_c^2 - 1)^3} \sum_{(lm)(no)(pq)} \sum_{(ab)(cd)(ef)} 2^2 \cos (k_a \cdot \Delta y_{lm}) \cos (k_b \cdot \Delta y_{lm}) 
\times 2^2 \cos (k_c \cdot \Delta y_{no}) \cos (k_d \cdot \Delta y_{no}) 2^2 \cos (k_e \cdot \Delta y_{pq}) \cos (k_f \cdot \Delta y_{pq}) 
\left. + O \left( \frac{1}{N} \right) + O \left( \frac{1}{(N_c^2 - 1)^4} \right) \right\},
\]

Color correction factors can be determined explicitly as for dipole.
Fourth order cumulant in leading order in $\frac{1}{(N_c^2 - 1)^2}$

$$\langle e^{i2(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle_c = (Bk_1^2) (Bk_2^2) (Bk_3^2) (Bk_4^2)$$

$$\left\{ \frac{1}{(N_c^2 - 1)^2} \left( 2 F^{(4i)}_{\text{corr}} - 2 F^{(2)}_{\text{corr}} F^{(2)}_{\text{corr}} + O(N^{-1}) \right) \right. $$

$$+ \frac{1}{(N_c^2 - 1)^3} \left( 2 F^{(6)}_{\text{corr}} (m - 4)(m - 5) - 4 F^{(2)}_{\text{corr}} F^{(4i)}_{\text{corr}} (m - 2)(m - 3) \right.$$

$$+ 4 F^{(5)}_{\text{corr}} (m - 4) - 4 F^{(2)}_{\text{corr}} F^{(3)}_{\text{corr}} (m - 2) \right.$$

$$+ 2 F^{(4i)}_{\text{corr}} + 8 \left( F^{(2)}_{\text{corr}} \right)^3 - 4 F^{(4i)}_{\text{corr}} F^{(2)}_{\text{corr}} \right) + O(N^{-1}) \right\}$$

$$+ O\left(\frac{1}{(N_c^2 - 1)^4}\right) \ldots$$

However direct calculation shows that term is zero
So first nonzero term is:

\[
\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle_c = (Bk_1^2)(Bk_2^2)(Bk_3^2)(Bk_4^2) \frac{2}{(N_c^2-1)^3} (7 + m - m^2)
+ O \left( \frac{1}{(N_c^2-1)^4} \right).
\]

This term appears only in \( \frac{1}{(N_c^2-1)^3} \) limit.

This term is negative and we have collectivity.

\[
v_n\{4\} \equiv \sqrt[4]{-\langle e^{i4(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle_c}.
\]

\[
v_2\{4\}(k) \simeq \frac{1}{(N_c^2-1)^{3/4}} \sqrt[4]{m B k^2}
\]

CGC result for \( v_2\{4\} \) has same \( N_c \) but different \( m \)- (N?)-dependencies,
Tr \[1\] Tr \[T^c T^b\] Tr \[T^b T^c T^d T^a\] Tr \[T^a T^d\] = \(N_c^4 (N_c^2 - 1)^2\)

Tr \[1\] Tr \[T^b T^c\] Tr \[T^c T^d T^b T^a\] Tr \[T^a T^d\] = \(\frac{1}{2} N_c^4 (N_c^2 - 1)^2\)

Tr \[1\] Tr \[T^b T^c\] Tr \[T^d T^c T^b T^a\] Tr \[T^a T^d\] = \(N_c^4 (N_c^2 - 1)^2\)

Tr \[1\] Tr \[T^c T^b\] Tr \[T^b T^d T^c T^a\] Tr \[T^a T^d\] = \(\frac{1}{2} N_c^4 (N_c^2 - 1)^2\)
Different color factors lead to breaking of a $k\cdot k$ symmetry present in large $N$ limit, and appearance of sinuses:

\[
e^{i k_2 \Delta y_{mn}} \left( e^{i k_3 \Delta y_{mn}} + \frac{1}{2} e^{-i k_3 \Delta y_{mn}} \right) + e^{-i k_2 \Delta y_{mn}} \left( \frac{1}{2} e^{i k_3 \Delta y_{mn}} + e^{-i k_3 \Delta y_{mn}} \right)
\]

\[= 3 \cos (k_2 \Delta y_{mn}) \cos (k_3 \Delta y_{mn}) - \sin (k_2 \Delta y_{mn}) \sin (k_3 \Delta y_{mn}).\]

Arise in the present set-up from a purely non-abelian mechanism.

However: odd harmonics are parametrically suppressed by $1/N$
Conclusions

We calculated QCD interference effects based on MPI and found that
1. We have the correct form for 2 point cumulant
2. Collectivity-4th cumulant is negative
3. The cumulants calculated with MPI parameters have correct order of magnitude
4. Odd harmonics naturally appear as a result of nonabelian nature of the model.

We see that no-interaction baseline including QCD/QM interference effects due to different MPIs make significant if not dominant contribution in ridge in pp and pA.

Our model disentangles in calculation of cumulants QCD interference effects from effects that depend on parton density in initial state. Its relation to other approaches, such as CGC can be further clarified.

Further tasks: get closer to phenomenology, more rigorous derivation of our model from QCD, further understanding of final states in pp collision, effects of 1/N, hadronization/LPHD,.............