Muon conversion to electron in nuclei in Minimal R-symmetric Supersymmetric Standard Model

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Abstract

We analyze the lepton flavor violating process $\mu - e$ conversion in the framework of the minimal R-symmetric supersymmetric standard model. The theoretical predictions are determined by considering the experimental constraint on parameter $\delta_{12}$ from the lepton flavor violating decay $\mu \to e\gamma$. The numerical results show that $\gamma$ penguins and $Z$ penguins dominate the predictions on $\text{CR}(\mu - e, \text{Nucleus})$, and the contributions from Higgs penguins and box diagrams are insignificant. The theoretical predictions on conversion rate $\text{CR}(\mu - e, \text{Nucleus})$ in a Al or Ti target can be enhanced close to the future experimental sensitivities and are very promising to be observed in near future experiment.

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I. INTRODUCTION

Searching for Lepton Flavor Violating (LFV) decays are of great importance in probing New Physics (NP) beyond the Standard Model (SM) in which the theoretical predictions on those LFV decays are suppressed by small masses of neutrinos and far beyond the experimental accessibility. There are many different ways to search LFV such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion in nucleus, $\tau$ decays, hadron decays and so on. However, no LFV signals have been observed in experiment up to now. The $\mu - e$ conversion in nucleus is a process that muons are captured in a target of atomic nucleus and form a muonic atom. Several experiments have been built or planned to built to search for this process. Current limit on the $\mu - e$ conversion rate is $4.6 \times 10^{-12}$ for a Ti target at TRIUMF [1], $4.3 \times 10^{-12}$ for a Ti target and $7 \times 10^{-13}$ for a Au target at SINDRUM-II experiment [2]. In future, this LFV process may be observed by experiments with improved sensitivity. A future prospects of $10^{-13}$ for a C target or $10^{-14}$ for a SiC target at DeeMe [3], $10^{-18}$ for a Ti target at PRISM [4] and $10^{-16} - 10^{-17}$ for a Al target at Mu2e and COMET [5, 6] will be achieved, which improve the current experimental limits by several orders of magnitude.

The $\mu - e$ conversion rate has been calculated in the literature for various extensions of SM. Some seesaw models with right handed neutrinos [7–13], scalar triplets [14–16], fermion singlet [17] and fermion triplets [18], can have $\text{CR}(\mu - e, \text{Nucleus})$ close to the experimental sensitivity. There are a few studies within models of non-SUSY, such as unparticle model [19, 20], littlest Higgs model [21, 22], left-right symmetric models [23], 331 model [24] and so on. There are also a few studies within models of SUSY, such as MSSM [25], R-parity violating SUSY [26], low-scale seesaw models of minimal supergravity [27], BLMSSM [28, 29], the CMSSM-seesaw [30], $\nu$SSM [31] and so on. Some pedagogical introductions on the theoretical motivations for charged LFV and the experimental aspects is provided in Ref. [32–34].

In this paper, we will study the LFV process $\mu - e$ conversion in the Minimal R-symmetric Supersymmetric Standard Model (MRSSM) [35]. The MRSSM has an unbroken global $U(1)_R$ symmetry and provides a new solution to the supersymmetric flavor problem in MSSM. In this model, R-symmetry forbids Majorana gaugino masses, $\mu$ term, $A$ terms and all left-right squark and slepton mass mixings. The $R$-charged Higgs $SU(2)_L$ doublets $\tilde{R}_u$
and $\hat{R}_d$ are introduced in MRSSM to yield the Dirac mass terms of higgsinos. Additional superfields $\hat{S}$, $\hat{T}$ and $\hat{O}$ are introduced to yield Dirac mass terms of gauginos. Studies on phenomenology in MRSSM can be found in literatures [36–54]. Similar to MSSM, the off-diagonal entries $\delta^{ij}$ in slepton mass matrices $m_2^l$ and $m_2^r$ dominate the LFV process $\mu - \epsilon$ conversion. Taking account of the constraints from radiative decays $\mu \rightarrow e\gamma$ on the off-diagonal parameters $\delta^{ij}$, we explore $\mu - \epsilon$ conversion rate as a function of off-diagonal parameter $\delta^{ij}$ and other model parameters.

The paper is organized as follows. In Section II we present the details of the MRSSM. All relevant mass matrices and mixing matrices are provided. Feynman diagrams contributing to $\mu - \epsilon$ conversion in MRSSM are given at one loop level. The $\mu - \epsilon$ conversion rate are computed in effective Lagrangian method, and notations and conventions for effective operators and Wilson coefficients are also listed. The numerical results are presented in Section III and the conclusion is drawn in Section IV.

II. MRSSM

In this section, we firstly provide a simple overview of MRSSM in order to fix the notations we use in this paper. The MRSSM has the same gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ as the SM and MSSM. The spectrum of fields in MRSSM contains the standard MSSM matter, Higgs and gauge superfields augmented by chiral adjoints $\hat{O}$, $\hat{T}$, $\hat{S}$ and two $R$-Higgs iso-doublets. The general form of the superpotential of the MRSSM is given by [36]

$$W_{MRSSM} = \mu_d(\hat{R}_d\hat{H}_d) + \mu_u(\hat{R}_u\hat{H}_u) + \Lambda_d(\hat{R}_d\hat{T})\hat{H}_d + \Lambda_u(\hat{R}_u\hat{T})\hat{H}_u + \lambda_d\hat{S}(\hat{R}_d\hat{H}_d) + \lambda_u\hat{S}(\hat{R}_u\hat{H}_u) - Y_d\hat{d}(\hat{q}\hat{H}_d) - Y_e\hat{e}(\hat{l}\hat{H}_d) + Y_u\hat{u}(\hat{q}\hat{H}_u),$$

(1)

where $\hat{H}_u$ and $\hat{H}_d$ are the MSSM-like Higgs weak iso-doublets, $\hat{R}_u$ and $\hat{R}_d$ are the $R$-charged Higgs $SU(2)_L$ doublets and the corresponding Dirac higgsino mass parameters are denoted as $\mu_u$ and $\mu_d$. Although R-symmetry forbids the $\mu$ terms of the MSSM, the bilinear combinations of the normal Higgs $SU(2)_L$ doublets $\hat{H}_u$ and $\hat{H}_d$ with the Higgs $SU(2)_L$ doublets $\hat{R}_u$ and $\hat{R}_d$ are allowed in Eq. (1). Parameters $\lambda_u$, $\lambda_d$, $\Lambda_u$ and $\Lambda_d$ are Yukawa-like trilinear terms involving the singlet $\hat{S}$ and the triplet $\hat{T}$. For our phenomenological studies we take
the soft-breaking terms involving scalar mass that have been considered in [38]

\[
V_{SB,S} = m_{H_d}^2(|H_d|^2 + |H_d^*|^2) + m_{H_u}^2(|H_u|^2 + |H_u^*|^2) + (B_\mu(H_d^*H_u^* - H_d^0H_u^0) + h.c.)
\]

\[
+ m_{R_u}^2(|R_d|^2 + |R_u|^2) + m_{R_u}^2(|R_d|^2 + |R_u|^2) + m_T^2(|T_0|^2 + |T^-|^2 + |T^+|^2)
\]

\[
+ \tilde{m}_\mu^2 |S|^2 + m_\sigma^2 |O|^2 + \tilde{d}_{L,i}m_i^{q,ij}\tilde{d}_{L,j} + \tilde{d}_{R,i}m_i^{q,ij}\tilde{d}_{R,j} + \tilde{u}_{L,i}m_i^{q,ij}\tilde{u}_{L,j}
\]

\[
+ \tilde{u}_{R,i}m_i^{T,ij}\tilde{u}_{R,j} + \tilde{e}_{L,i}m_i^{S,ij}\tilde{e}_{L,j} + \tilde{e}_{R,i}m_i^{S,ij}\tilde{e}_{R,j} + \tilde{\nu}_{L,i}m_i^{S,ij}\tilde{\nu}_{L,j}
\]

(2)

All trilinear scalar couplings involving Higgs bosons to squarks and sleptons are forbidden in Eq. (2) cause the sfermions have an R-charge and these terms are non R-invariant, and this relaxes the flavor problem of the MSSM [35]. The Dirac nature is a manifest feature of MRSSM fermions and the soft-breaking Dirac mass terms of the singlet \( \hat{S} \), triplet \( \hat{T} \) and octet \( \hat{O} \) take the form as

\[
V_{SB,GG} = M_B^B \tilde{B} \tilde{S} + M_W^W \tilde{W}^a \tilde{T}^a + M_D^D \tilde{g} \tilde{O} + h.c.,
\]

(3)

where \( \tilde{B}, \tilde{W} \) and \( \tilde{g} \) are usually MSSM Weyl fermions. R-Higgs bosons do not develop vacuum expectation values since they carry R-charge 2. After electroweak symmetry breaking the singlet and triplet vacuum expectation values effectively modify the \( \mu_u \) and \( \mu_d \), and the modified \( \mu_i \) parameters are given by

\[
\mu^{eff,+}_d = \frac{1}{2}\Lambda_d v_T + \frac{1}{\sqrt{2}}\lambda_d v_S + \mu_d, \quad \mu^{eff,-}_u = -\frac{1}{2}\Lambda_u v_T + \frac{1}{\sqrt{2}}\lambda_u v_S + \mu_u.
\]

The \( v_T \) and \( v_S \) are vacuum expectation values of \( \tilde{T} \) and \( \tilde{S} \) which carry R-charge zero.

In the weak basis \((\sigma_d, \sigma_u, \sigma_S, \sigma_T)\), the pseudo-scalar Higgs boson mass matrix and the diagonalization procedure are

\[
\mathcal{M}_{A^0} = \begin{pmatrix}
B_{\mu\nu_u} & B_\mu & 0 & 0 \\
B_\mu & B_{\mu\nu_d} & 0 & 0 \\
0 & 0 & m_S^2 + \frac{\lambda_d^2 v_S^2 + \lambda_u^2 v_d^2}{2} & \lambda_d \lambda_d v_S^2 - \lambda_u \lambda_u v_d^2 \\
0 & 0 & \lambda_d \lambda_d v_S^2 - \lambda_u \lambda_u v_d^2 & \frac{2v_T}{2}\frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_S^2}{4} + m_T^2
\end{pmatrix}, \quad Z^A \mathcal{M}_{A^0}(Z^A)^\dagger = \mathcal{M}^0_{A^0}. \quad (4)
\]

In the weak basis \((\phi_d, \phi_u, \phi_S, \phi_T)\), the scalar Higgs boson mass matrix and the diagonalization procedure are

\[
\mathcal{M}_h = \begin{pmatrix}
\mathcal{M}_{11} & \mathcal{M}_{1T}^T \\
\mathcal{M}_{21} & \mathcal{M}_{22}
\end{pmatrix}, \quad Z^h \mathcal{M}_h(Z^h)^\dagger = \mathcal{M}_h^\text{diag}, \quad (5)
\]
where the submatrices \( (c_\beta = \cos \beta, s_\beta = \sin \beta) \) are

\[
M_{11} = \begin{pmatrix}
    m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta \\
    -(m_Z^2 + m_A^2) s_\beta c_\beta & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2
\end{pmatrix},
\]

\[
M_{21} = \begin{pmatrix}
    v_d(\sqrt{2} \lambda_d \mu_d^eff,+) - g_1 M_B^D & v_u(\sqrt{2} \lambda_u \mu_u^eff,- + g_1 M_B^D) \\
    v_d(\Lambda_d \mu_d^eff,+) + g_2 M_W^D & -v_u(\Lambda_u \mu_u^eff,1 + g_2 M_W^D)
\end{pmatrix},
\]

\[
M_{22} = \begin{pmatrix}
    4(M_B^D)^2 + m_2^2 S + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{2} & \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2 \sqrt{2}} \\
    \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2 \sqrt{2}} & 4(M_W^D)^2 + m_2^2 T + \frac{\Lambda_d^2 v_d^2 + \Lambda_u^2 v_u^2}{4}
\end{pmatrix}.
\]

The number of neutralino degrees of freedom in MRSSM is doubled compared to MSSM as the neutralinos are Dirac-type. In the weak basis of four neutral electroweak two-component fermions \( \xi_i = (\tilde{B}, \tilde{W}^0, \tilde{\rho}_d, \tilde{\rho}_u) \) with R-charge 1 and four neutral electroweak two-component fermions \( \varsigma_i = (\tilde{S}, \tilde{T}^0, \tilde{\rho}_d, \tilde{\rho}_u) \) with R-charge -1, the neutralino mass matrix and the diagonalization procedure are

\[
m_{\chi^0} = \begin{pmatrix}
    M_B^D & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u \\
    0 & M_W^D & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u \\
    -\frac{1}{\sqrt{2}} \lambda_d v_d & -\frac{1}{\sqrt{2}} \Lambda_d v_d & -\mu_d^{eff,+} & 0 \\
    \frac{1}{\sqrt{2}} \lambda_u v_u & -\frac{1}{\sqrt{2}} \Lambda_u v_u & 0 & \mu_u^{eff,-}
\end{pmatrix}, (N^j_{1})^{*} m_{\chi^0} (N^j_{2})^{t} = m_{\chi^0}^{diag}. \tag{6}
\]

The mass eigenstates \( \kappa_i \) and \( \varphi_i \), and physical four-component Dirac neutralinos are

\[
\xi_i = \sum_{j=1}^{4} (N_{jj}^1)^{*} \kappa_j, \varsigma_i = \sum_{j=1}^{4} (N_{ij}^2)^{*} \varphi_j, \chi_i^0 = \begin{pmatrix} \kappa_i \\ \varphi_i^t \end{pmatrix}.
\]

The number of chargino degrees of freedom in MRSSM is also doubled compared to MSSM and these charginos can be grouped to two separated chargino sectors according to their R-charge. The \( \chi^\pm \)-charginos sector has R-charge 1 electric charge; the \( \rho \)-charginos sector has R-charge -1 electric charge. In the basis \( \xi_i^\pm = (\tilde{W}^+, \tilde{\rho}_d^+) \) and \( \varsigma_i^- = (\tilde{T}^-, \tilde{\rho}_u^-) \), the \( \chi^\pm \)-charginos mass matrix and the diagonalization procedure are

\[
m_{\chi^\pm} = \begin{pmatrix}
    g_2 v_T + M_B^W & \frac{1}{\sqrt{2}} \Lambda_d v_d \\
    \frac{1}{\sqrt{2}} g_2 v_d & -\frac{1}{2} \Lambda_d v_T + \frac{1}{\sqrt{2}} \lambda_d v_S + \mu_d
\end{pmatrix}, (U_{11})^{*} m_{\chi^\pm} (U_{11})^{t} = m_{\chi^\pm}^{diag}. \tag{7}
\]

The mass eigenstates \( \lambda_i^\pm \) and physical four-component Dirac charginos are

\[
\xi_i^+ = \sum_{j=1}^{2} (V_{ij}^{11})^{*} \lambda_j^+, \varsigma_i^- = \sum_{j=1}^{2} (U_{ij}^{11})^{*} \lambda_j^-, \chi_i^\pm = \begin{pmatrix} \lambda_i^+ \\ \lambda_i^- \end{pmatrix}.
\]
Here, we don’t discuss the ρ-charginos sector in detail since it doesn’t contribute to µ - e conversion. More information about the ρ-charginos can be found in Ref.\[38, 40, 42, 52\].

In MRSSM the LFV decays mainly originate from the potential misalignment in sleptons mass matrices. In the gauge eigenstate basis \( \tilde{\nu}_L \), the sneutrino mass matrix and the diagonalization procedure are

\[
\begin{align*}
    m_{\tilde{\nu}}^2 &= m_{\tilde{\nu}}^2 + \frac{1}{8} (g_1^2 + g_2^2) (v_d^2 - v_u^2) + g_2 v_T M_D^W - g_1 v_S M_D^B, \\
    Z^V m_{\tilde{\nu}}^2 (Z^V)'^T &= m_{\tilde{\nu}}^{2, \text{diag}},
\end{align*}
\]

where the last two terms in mass matrix are newly introduced by MRSSM. The slepton mass matrix and the diagonalization procedure are

\[
\begin{align*}
    m_{\tilde{\ell}}^2 &= \begin{pmatrix}
        (m_{\tilde{\ell}}^2)_{LL} & 0 \\
        0 & (m_{\tilde{\ell}}^2)_{RR}
    \end{pmatrix},
    Z^E m_{\tilde{\ell}}^2 (Z^E)'^T &= m_{\tilde{\ell}}^{2, \text{diag}}, \\
    (m_{\tilde{\ell}}^2)_{LL} &= m_\ell^2 + \frac{1}{2} v_d^2 |Y_e|^2 + \frac{1}{8} (g_1^2 - g_2^2) (v_d^2 - v_u^2) - g_1 v_S M_B^U - g_2 v_T M_B^W, \\
    (m_{\tilde{\ell}}^2)_{RR} &= m_\ell^2 + \frac{1}{2} v_d^2 |Y_e|^2 + \frac{1}{4} g_1^2 (v_u^2 - v_d^2) + 2 g_1 v_S M_B^U.
\end{align*}
\]

The sources of LFV are the off-diagonal entries of the 3 x 3 soft supersymmetry breaking matrices \( m_\ell^2 \) and \( m_{\tilde{\ell}}^2 \) in Eqs.(8, 9). From Eq.(9) we can see that the left-right slepton mass mixing is absent in MRSSM, whereas the A terms are present in MSSM.

The mass matrix for up squarks and down squarks, and the relevant diagonalization procedure are

\[
\begin{align*}
    m_{\tilde{u}}^2 &= \begin{pmatrix}
        (m_{\tilde{u}}^2)_{LL} & 0 \\
        0 & (m_{\tilde{u}}^2)_{RR}
    \end{pmatrix},
    Z^U m_{\tilde{u}}^2 (Z^U)'^T &= m_{\tilde{u}}^{2, \text{diag}}, \\
    m_{\tilde{d}}^2 &= \begin{pmatrix}
        (m_{\tilde{d}}^2)_{LL} & 0 \\
        0 & (m_{\tilde{d}}^2)_{RR}
    \end{pmatrix},
    Z^D m_{\tilde{d}}^2 (Z^D)'^T &= m_{\tilde{d}}^{2, \text{diag}}, \\
    (m_{\tilde{u}}^2)_{LL} &= m_\tilde{u}^2 + \frac{1}{2} v_u^2 |Y_u|^2 + \frac{1}{24} (g_1^2 - 3 g_2^2) (v_u^2 - v_d^2) + \frac{1}{3} g_1 v_S M_B^U + g_2 v_T M_B^W, \\
    (m_{\tilde{u}}^2)_{RR} &= m_\tilde{u}^2 + \frac{1}{2} v_u^2 |Y_u|^2 + \frac{1}{8} g_1^2 (v_d^2 - v_u^2) - \frac{2}{3} g_1 v_S M_B^U, \\
    (m_{\tilde{d}}^2)_{LL} &= m_\tilde{d}^2 + \frac{1}{2} v_d^2 |Y_d|^2 + \frac{1}{24} (g_1^2 + 3 g_2^2) (v_u^2 - v_d^2) + \frac{1}{3} g_1 v_S M_B^U - g_2 v_T M_B^W, \\
    (m_{\tilde{d}}^2)_{RR} &= m_\tilde{d}^2 + \frac{1}{2} v_d^2 |Y_d|^2 + \frac{1}{12} g_1^2 (v_u^2 - v_d^2) + \frac{2}{3} g_1 v_S M_B^U.
\end{align*}
\]

The MRSSM has been implemented in the Mathematica package SARAH \[55, 57\], and we use the Feynman rules generated with SARAH-4.14.3 in our work. In MRSSM, violating of lepton flavor arises at the one loop level. In MRSSM, µ - e conversion is induced by the Feynman diagrams given in FIG.\[1\]. The various contributions to this process can be
FIG. 1: One loop Feynman diagrams contributing to $\mu - e$ conversion in MRSSM.

classified into Higgs penguins, $\gamma$ penguins, $Z$ penguins and box diagrams. In the effective Lagrangian method, one can derive the effective Lagrangian relevant for $\mu - e$ conversion as

\[
\mathcal{L}_{\text{eff}} = e \bar{e} \gamma^\mu \left( K^L_1 P_L + K^R_1 P_R \right) l_\mu A_\mu + \sum_{K=S,V}^{X,Y=L,R} B^K_{XY} \bar{e} \Gamma_K P_X l_\mu d \Gamma_K P_Y d
\]

\[
+ \sum_{K=S,V}^{X,Y=L,R} C^K_{XY} \bar{e} \Gamma_K P_X l_\mu \bar{u} \Gamma_K P_Y u + h.c. \quad (11)
\]
The conversion rate \( CR(\mu - e, Nucleus) \) in nuclei can be calculated by

\[
CR(\mu - e, Nucleus) = \sum_{X=L,R} \frac{p_e E_e m_\mu^3 G_F^2 \alpha^3 Z_{eff} F_p^2}{8\pi^2 Z \Gamma_{capt}} \times \left| (Z + N) \left( g_{XV}^{(0)} + g_{XS}^{(0)} \right) + (Z - N) \left( g_{XV}^{(1)} + g_{XS}^{(1)} \right) \right|^2. \tag{12}
\]

Here \( p_e \) and \( E_e \) (\( \sim m_\mu \) in the numerical evaluation) are the momentum and energy of the electron. \( G_F \) and \( \alpha \) are the Fermi constant and the fine structure constant, respectively. \( Z_{eff} \) is the effective atomic charge. \( Z \) and \( N \) are the number of protons and neutrons in the nucleus. \( F_p \) is the nuclear form factor and \( \Gamma_{capt} \) is the total muon capture rate. The values of \( Z_{eff}, F_p \) and \( \Gamma_{capt} \) that will be used in the phenomenological analysis below are given in Table. III. At quark level, the \( g_{XK}^{(i)} \) factors (with \( i=0,1, X=L,R \) and \( K=S,V \)) can be written as combinations of effective couplings

\[
g_{XK}^{(i)} = \frac{1}{2} \sum_{q=u,d,s} \left( g_{XK(q)} G_K^{(q,p)} + (-1)^i g_{XK(q)} G_K^{(q,n)} \right).
\]

The values of \( G_K \) factors are \( G_S^{(u,p)}=G_s^{(d,n)}=5.1, \ G_S^{(d,p)}=G_s^{(u,n)}=4.3, \ G_S^{(s,p)}=G_s^{(s,n)}=2.5, \ G_V^{(u,p)}=G_V^{(d,n)}=2, \ G_V^{(d,p)}=G_V^{(u,n)}=1 \). The \( g_{XK(q)} \) coefficients can be written as combinations of Wilson coefficients

\[
g_{LV(q)} = \sqrt{2} G_F \left( e^2 Q_q (K_L^1 - K_R^2) - \frac{1}{2} (C_{llqq}^{VLL} + C_{llqq}^{VLR}) \right),
\]

\[
g_{LS(q)} = -\sqrt{2} G_F \left( C_{llqq}^{SLL} + C_{llqq}^{SLR} \right),
\]

where \( Q_q \) are the electric charge of quarks, \( C_{llqq}^{SLL} \) equals \( B_{XY}^K (C_{XY}^K) \) for d-quarks (u-quarks),

\[
g_{RV(q)} = g_{LV(q)} |L \to R \text{ and } g_{RS(q)} = g_{LS(q)} |L \to R.
\]

\[
\begin{array}{cccccccc}
\text{Nucleus} & \frac{4}{2}N & \frac{27}{13}\text{Al} & \frac{48}{22}\text{Ti} & \frac{80}{38}\text{Sb} & \frac{121}{51}\text{Sr} & \frac{197}{79}\text{Au} & \frac{208}{82}\text{Pb} \\
Z_{eff} & 11.5 & 17.6 & 25 & 29 & 33.5 & 34 \\
F_p & 0.64 & 0.54 & 0.39 & 0.32 & 0.16 & 0.15 \\
\Gamma_{capt} \times 10^{18} & 0.464079 & 1.70422 & 4.61842 & 6.71711 & 8.59868 & 8.84868
\end{array}
\]

### III. NUMERICAL ANALYSIS

We now turn to the numerical analysis of the one loop corrections to \( \mu - e \) conversion in nuclei in MRSSM by using the full evaluation within the framework of SARAH-4.14.3 [55–57] and SPheno-4.0.4 [59, 60]. The computation is done in a low scale version of SPheno and
all free parameters are given at the SUSY scale. The experimental values of Higgs mass and \( W \) boson mass can impose stringent and nontrivial constraints on the model parameters. The one loop and leading two loop corrections to the lightest (SM-like) Higgs boson in MRSSM have been computed in Ref.\[38\] and the new fields and couplings can give large contributions to the Higgs mass even for stop masses of order 1 TeV and no stop mixing. Meanwhile, the new fields and couplings can not give too large a contribution to the \( W \) boson mass and muon decay in the same regions of parameter space. A better agreement with the latest experimental value for \( W \) boson mass has been investigated in Ref.\[41\]. It combines all numerically relevant contributions that are know in SM in a consistent way with all MRSSM one loop corrections. A set of updated benchmark point BMP1 is given in Ref.\[41\] and we display them in Eq.(13) where all mass parameters are in GeV or GeV^2.

\[
\begin{align*}
\tan \beta &= 3, B_\mu = 500^2, \lambda_d = 1.0, \lambda_u = -0.8, \Lambda_d = -1.2, \Lambda_u = -1.1, \\
M_B^R &= 550, M_B^W = 600, \mu_d = \mu_u = 500, v_S = 5.9, v_T = -0.33, \\
(m_l^2)_{11} &= (m_l^2)_{22} = (m_l^2)_{33} = (m_r^2)_{11} = (m_r^2)_{22} = (m_r^2)_{33} = 1000^2, \\
(m_q^2)_{11} &= (m_q^2)_{11} = (m_d^2)_{11} = (m_q^2)_{22} = (m_d^2)_{22} = (m_d^2)_{22} = 2500^2, \\
(m_q^2)_{33} &= (m_u^2)_{33} = (m_u^2)_{33} = 1000^2, m_T = 3000, m_S = 2000.
\end{align*}
\] (13)

In following numerical analysis, the values in Eq.(13) will be used for all results. Note that, the off-diagonal entries of squark mass matrices \( m_q^2, m_u^2, m_d^2 \) and slepton mass matrices \( m_l^2, m_r^2 \) in Eq.(13) are zero, i.e., the flavour mixing of squark and slepton is absent.

Similarly to most supersymmetry models, the LFV processes in MRSSM originate from the off-diagonal entries of the soft breaking terms \( m_l^2 \) and \( m_r^2 \), which are parameterized by mass insertion

\[
(m_{1J})_{IJ} = \delta_{1J}^{IJ} \sqrt{(m_l^2)_{II}(m_l^2)_{JJ}}, (m_{rJ})_{IJ} = \delta_{rJ}^{IJ} \sqrt{(m_r^2)_{II}(m_r^2)_{JJ}},
\] (14)

where \( I, J = 1, 2, 3 \). To decrease the number of free parameters involved in our calculation, we assume that the off-diagonal entries of \( m_l^2 \) and \( m_r^2 \) in Eq.(14) are equal, i.e., \( \delta_{1J}^{IJ} = \delta_{rJ}^{IJ} = \delta^{IJ} \). The experimental limits on LFV decays, such as radiative two body decays \( l_2 \to l_1 \gamma \), leptonic three body decays \( l_2 \to 3l_1 \), can give strong constraints on the parameters \( \delta^{IJ} \). In the following, we will use LFV decays \( \mu \to e\gamma \) to constrain the parameters \( \delta^{12} \) which has been discussed in Ref.\[53\]. It is noted that \( \delta^{23} \) and \( \delta^{13} \) have been set zero in following discussion since they have no effect on the predictions of CR(\( \mu - e \),Nucleus). Current limits
of LFV decays $\mu \to e\gamma$ is $\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ from MEG \cite{61} and new sensitivity for this decay channel in the future projects will be $\text{BR}(\mu \to e\gamma) \sim 6 \times 10^{-14}$ from MEG II \cite{62}.

In FIG.2 the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ for Al, Ti, Sr, Sb, Au, and Pb are shown as a function of mass insertion parameter $\delta^{12}$. The prediction for $\text{BR}(\mu \to e\gamma)$ exceeds the future experiment sensitivity at $\delta^{12} \sim 0.001$. In a recent Ref.\cite{53} the analytical computation and discussion of $\text{BR}(\mu \to e\gamma)$ in MRSSM has been performed. The valid region for $\delta^{12}$ in Ref.\cite{53} calculated with the Mathematica package Package-X is compatible with that in this work calculated with SARAH and SPheno. We clearly see that both the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ in nuclei are sensitive to $\delta^{12}$, and they increase along with the increase of $\delta^{12}$ which have a same behavior as those in most SUSY models (e.g. \cite{63}). At $\delta^{12} \sim 0.001$, the prediction on $\text{BR}(\mu \to e\gamma)$ is very close to the current experimental limit, and the predictions on $\text{CR}(\mu - e, \text{Nucleus})$ are around $10^{-15} - 10^{-16}$ which are two orders of magnitude below current experimental limits. The predicted $\text{CR}(\mu - e, \text{Nucleus})$ for Ti is around $10^{-15}$ and this is three orders of magnitude above future experimental sensitivity \cite{4}. The predicted $\text{CR}(\mu - e, \text{Nucleus})$ for Al is around $10^{-16}$ and this is in region of the future experimental sensitivity \cite{5,6}.

Taking $\delta^{12} = 0.001$, the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ for Al, Ti, Sr, Sb, Au, and Pb are shown in FIG.3 as a function of the diagonal entries $m_l$ of the soft breaking term $m_l^2$ and $m_r^2$. Here, $m_l = \sqrt{(m_l^2)_{11}} = \sqrt{(m_l^2)_{22}} = \sqrt{(m_l^2)_{33}} = \sqrt{(m_r^2)_{11}} = \sqrt{(m_r^2)_{22}} = \sqrt{(m_r^2)_{33}}$. We clearly see that both the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ in nuclei are sensitive to $m_l$, and they decrease along with the increase of $m_l$ which have a same behavior as those in Ref.\cite{63}. At $m_l=1$ TeV, the prediction on $\text{BR}(\mu \to e\gamma)$ is below $10^{-13}$. Thus, all points in plot of $\text{BR}(\mu \to e\gamma)$ are compatible with the current

FIG. 2: Dependence of $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ on mass insertion parameter $\delta^{12}$.\n
In FIG.3 the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ for Al, Ti, Sr, Sb, Au, and Pb are shown as a function of the diagonal entries $m_l$ of the soft breaking term $m_l^2$ and $m_r^2$. Here, $m_l = \sqrt{(m_l^2)_{11}} = \sqrt{(m_l^2)_{22}} = \sqrt{(m_l^2)_{33}} = \sqrt{(m_r^2)_{11}} = \sqrt{(m_r^2)_{22}} = \sqrt{(m_r^2)_{33}}$. We clearly see that both the predictions for $\text{BR}(\mu \to e\gamma)$ and $\text{CR}(\mu - e, \text{Nucleus})$ in nuclei are sensitive to $m_l$, and they decrease along with the increase of $m_l$ which have a same behavior as those in Ref.\cite{63}. At $m_l=1$ TeV, the prediction on $\text{BR}(\mu \to e\gamma)$ is below $10^{-13}$. Thus, all points in plot of $\text{BR}(\mu \to e\gamma)$ are compatible with the current
FIG. 3: Dependence of BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) on mass parameter \(m_t\).

experimental limit. At \(m_t = 5\) TeV, the predicted CR(\(\mu - e\),Nucleus) for Ti is around \(10^{-16}\) and this is still two orders of magnitude above future experimental sensitivity \[4\]. At \(m_t = 5\) TeV, the predicted CR(\(\mu - e\),Nucleus) for Al is below \(10^{-16}\) and this is still in region of the future experimental sensitivity \[5, 6\].

FIG. 4: Dependence of BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) on mass parameter \(M_W^{WD}\).

In FIG[4] the predictions for BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) are shown as a function of the wino-triplino mass parameter \(M_W^{WD}\). \(M_W^{WD}\) is a new parameter introduced in MRSSM. We clearly see that both the predictions for BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) in nuclei show a weak dependence on \(M_W^{WD}\), and they decrease slowly along with the increase of \(M_W^{WD}\).

In FIG[5] the predictions for BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) are shown as a function of \(\tan \beta\). We clearly see that both the predictions for BR(\(\mu \to e\gamma\)) and CR(\(\mu - e\),Nucleus) in nuclei are not sensitive to \(\tan \beta\), and they take values along a narrow band. This is a striking difference to MSSM \[42\]. Due to the existence of the transition from \(d\)-Higgsino to \(u\)-Higgsino in MSSM, which is governed by \(\mu\)-term, the well-known \(\tan \beta\)-enhancement is
possible. A well-established way to understand the tan$\beta$-enhancement is provided by mass-insertion diagrams involving insertions of the $\mu$-parameter and Majorana gaugino masses. However, the $\mu$-term and Majorana gaugino masses are forbidden in MRSSM and this leads to the result that BR($\mu \rightarrow e\gamma$) and CR($\mu - e$,Nucleus) are not enhanced by tan$\beta$.

In FIG 6 the predictions for BR($\mu \rightarrow e\gamma$) and CR($\mu - e$,Nucleus) are shown as a function of the squark mass $M_{QUd}$. Here, $M_{QUd} = \sqrt{(m_{\tilde{q}}^2)_{11}} = \sqrt{(m_{\tilde{u}}^2)_{11}} = \sqrt{(m_{\tilde{d}}^2)_{11}} = \sqrt{(m_{\tilde{q}}^2)_{22}} = \sqrt{(m_{\tilde{u}}^2)_{22}} = \sqrt{(m_{\tilde{d}}^2)_{22}} = \sqrt{(m_{\tilde{q}}^2)_{33}} = \sqrt{(m_{\tilde{u}}^2)_{33}} = \sqrt{(m_{\tilde{d}}^2)_{33}}$. We clearly see that the predictions for BR($\mu \rightarrow e\gamma$) is not sensitive to $M_{QUd}$, and it takes values along a narrow band. This is because there is no squark medicated diagram contributing to $\mu \rightarrow e\gamma$. Only the box diagrams contributing to CR($\mu - e$,Nucleus) depend on the squark masses. The predictions for CR($\mu - e$,Nucleus) show a weak dependence on $M_{QUd}$, and they decrease slowly along with the increase of $M_{QUd}$ which have a baseline behaviour as those in Ref.[42].

We are also interesting to the effects from other parameters on the predictions of CR($\mu -
e,Nucleus) in MRSSM. By scanning over these parameters, such as $B_{\mu}$, $M_B^D$, $\lambda_d$, $\lambda_u$, $\Lambda_d$, $\Lambda_u$, $\mu_d$ and $\mu_u$, it shows the predictions for CR($\mu$ - e,Nucleus) take values along a narrow band in valid regions.

![Graphs showing predictions on CR($\mu$ - e,Nucleus) as a function of tan $\beta$ from various parts: (a) $\gamma$ penguins; (b) Z penguins; (c) Higgs penguins; (d) box diagrams.](image)

**FIG. 7:** Predictions on CR($\mu$ - e,Nucleus) as a function of tan $\beta$ from various parts: [a] $\gamma$ penguins; [b] Z penguins; [c] Higgs penguins; [d] box diagrams.

In FIG. 7 we show the predictions on CR($\mu$ - e,Nucleus) as a function of tan $\beta$ with the same parameter setup in FIG. 5 but independently considering the contributions from each diagram, and the values of CR($\mu$ - e,Nucleus) are given by only the listed contribution with all others set to zero. The predictions on BR($\mu$ $\to$ $e\gamma$) are not displayed cause they only receive the contribution from $\gamma$ penguins. We observe that the $\gamma$ penguins dominate the predictions on CR($\mu$ - e,Nucleus) and the contributions from Higgs penguins and box diagrams are negligible. The Z penguins are less dominant in the predictions on CR($\mu$ - e,Nucleus) in a large parameter region. The decrease for total predictions CR($\mu$ - e,Nucleus) at tan $\beta$ < 5 in FIG. 5 can be explained by a cancellation of predictions between $\gamma$ penguins and Z penguins shown in FIG. 7 [a] and [b].
IV. CONCLUSIONS

In this work, taking account of the constraints from $\mu \rightarrow e\gamma$ on the parameter space, we analyze the LFV process $\text{CR}(\mu - e, \text{Nucleus})$ in the framework of the Minimal R-symmetric Supersymmetric Standard Model.

In MRSSM, the theoretical predictions on $\text{CR}(\mu - e, \text{Nucleus})$ mainly depend on the mass insertion $\delta^{12}$. The predictions on $\text{CR}(\mu - e, \text{Nucleus})$ would be zero if $\delta^{12}=0$ is assumed. Taking account of experimental bounds on radiative decays $\mu \rightarrow e\gamma$, the values of $\delta^{12}$ is constrained around 0.001. Assuming $\delta^{12} = 0.001$ and other parameter settings in Eq.(13), the predictions on $\text{CR}(\mu - e, \text{Nucleus})$ are at the level of $\mathcal{O}(10^{-15} - 10^{-16})$, which are two or three orders of magnitude above the future experimental prospects for a Al or Ti target. Thus, the LFV processes $\mu - e$ conversion in Al and Ti are very promising to be observed in near future experiment.

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