Gain of photon-axion conversion in paramagnetics

G N Izmaïlov$^1$ and V S Gorelik$^{2,3}$

1 Moscow Aviation Institute (National Research University), Moscow, Russia
2 Lebedev Physical Institute of the Russian Academy of Sciences, Moscow, Russia
3 Bauman Moscow State Technical University, Moscow, Russia
E-mail: gorelik@sci.lebedev.ru

Abstract. The possibility of realization of a photon-axion and axion-photon conversion processes in transparent dielectric media within the visible frequency region is analyzed. Materials contain paramagnetic ions with large magnetic moment: Tb$^{3+}$, Er$^{3+}$, etc. Therefore an additional strong magnetic field is present in the region of the paramagnetic ion. The reinforced field provides increased probability of the conversion processes. In order to ensure the synchronism condition it is supposed to use laser radiation whose frequency coincides with the frequency of unitary polaritons with a refractive index equals to one. Schemes akin the known scheme “Light Shining through a Wall” can be used to observe the photon-axion conversion process. The estimation of efficiency of conversion processes using real sources of laser radiation and the specific dielectric media with paramagnetic ions is given.

1. Introduction

According to modern concepts [1], pseudoscalar bosons called axions are representative of the Dark Matter (DM) components. This particles are weakly interacting with the visible matter and have very small rest energy ($E_{\text{ia}} \sim 1-100 \ \mu\text{eV}$). Axions were introduced as a consequence of deviations from the Standard Model. It follows, in particular, from the CP violation, due to the absence of the electric dipole moment of the proton. Axions as pseudo scalar particles, preserving CP invariance, remove the problem existing within the framework of the Standard Model [2-4]. Astrophysical and cosmological problems arising in the analysis of the energy balance of the stars radiation, a halo presence around the galaxies [5-6], the cooling patterns of white dwarfs [7] and to reduce the duration of supernova SN 1987A [8-9] - can also be described on the basis of the existence of axions. According to the estimates of [1], the axions density in an environment is about $10^{21} \ \text{m}^{-3}$.

Recently the assumptions [5, 9] that axions included in the DM can be detected not only in the astrophysical processes, but in laboratory have been expressed. Several experimental solutions have been proposed for axion searches in the plausible axion-photon coupling range [10]. For the easy registration of axions, the axion conversion into a photon in an external magnetic field was selected (Primakov’s effect, see Figures 1, 2) [11]. As it was shown previously [12], the probability of $N_a$ pseudoscalar bosons emerging, resulting from the conversion of $N_\gamma$ quanta (photons) of the exciting electromagnetic radiation in the pseudoscalar bosons is given by the expression:

$$P_{\gamma \rightarrow a} = \frac{N_a}{N_\gamma} = \frac{1}{4} \left( \frac{Q}{\pi} \right) \left( \frac{\omega}{k_0} \right)^2 \left( dB_0 l \right)^2 \left( \frac{2}{ql} \sin \frac{ql}{2} \right)^2.$$

(1)
Here \( d = 2.7 \cdot 10^{-11} \ \text{GeV}^{-1} \) is an axion-photon coupling constant, \( q = k_a - k_\gamma \) is the change of axion momentum \( k_a \) during the emission of a photon, \( Q \) is a resonator quality factor. It is assumed that \( \hbar = c = 1 \).

**Figure 1.** Feynman diagrams illustrating the effect of a magnetic field in Primakov effect.

It is possible also to reverse the process of turning photons into axions in the presence of the magnetic field. In the effect of “Light shining through walls” (LSTW) the intense laser light as the photon source should be used [12]. (See Figure 2.).

**Figure 2.** A schematic diagram of the setup for the observation of the effect of conversion of laser radiation photons in axions (\( \gamma \rightarrow a \)) and inverse process - reconversion - (\( a \rightarrow \gamma' \)) using the resonators; 1 - a laser light source; 2 - coils; 3 - an opaque wall; 4 - receivers of secondary radiation; 5 - an input translucent mirror of the resonator; 6 - resonators as Fabry-Perot interferometers

From formula (1) one can conclude that the increase in the probability of the photon-axion conversion in a vacuum can be implemented by an increase in the photons optical path length by the use of optical resonators [13], as well as by an increase of the magnetic field \( B_0 \). At the same time an axion-photon reconversion can be carried out in a dielectric medium. In this case the group velocity of the corresponding electromagnetic wave can be significantly reduced. In magnetic crystals the internal magnetic field can be created due to the presence of the paramagnetic ions with a large magnetic moment.

2. The dispersion relation of electromagnetic waves in a dielectric medium

In the absence of free charges \( \rho \) and currents \( j \) for dielectric nonmagnetic medium the electromagnetic waves are given by equation:
\[(\nabla^2 - \frac{\varepsilon \mu}{c^2} \frac{\partial^2}{\partial t^2}) \vec{E} = 0.\]  

(2)

Solutions of this equation are relations:

\[i \varepsilon \varepsilon_0 \vec{k} \vec{E}_0 \exp i (\vec{k} \vec{r} - \omega t) = 0,\]  

(3)

\[\omega^2 = \varepsilon_0^2 k^2 \frac{\varepsilon}{\varepsilon(\omega)},\]  

(4)

where \(\varepsilon(\omega)\) is the corresponding dispersion dependence of the dielectric constant. In the simplest case of an isotropic dielectric for one polar mode the equations take place:

\[\ddot{u} = -\omega_0^2 u + \frac{e \sqrt{F}}{m} \vec{E}; \quad \dot{u} = u_0 \exp i (\vec{k} \vec{r} - \omega t).\]  

(5)

Here \(m\) is an oscillator mass, \(e\) is charge of electron, \(F\) is the coefficient characterizing the power of an oscillator \((F \sim 1)\). \(u\) is an oscillating particle deviation from the equilibrium position. The amplitude of the oscillating deflection from the equilibrium of a charged particle, in accordance with equations (4) and (5) takes the form:

\[u_0 = \frac{e \sqrt{F}}{m (\omega_0^2 - \omega^2)} \vec{E}_0.\]  

(6)

Respectively for the amplitude of the polarization vector we obtain:

\[\vec{P}_0 = \frac{e \varepsilon F}{m V_0 (\omega_0^2 - \omega^2)} \vec{E}_0.\]  

(7)

The equation of motion for the polarization vector is represented as:

\[\vec{P} = -\omega_0^2 \vec{P} + \frac{e^2 F}{m V_0} \vec{E} + \frac{e \sqrt{F}}{m V_0} \vec{P}_0 \exp i (\vec{k} \vec{r} - \omega t).\]  

(8)

Introducing plasma frequency \(\omega_p\), we have the relation

\[\vec{P} = -\omega_0^2 \vec{P} + \omega_p^2 \vec{E}; \quad \omega_p^2 = \frac{e^2 F}{m V_0}.\]  

(9)

Accordingly for \(\vec{D}_0\) we have:

\[\vec{D}_0 = \varepsilon_0 \vec{E}_0 + \vec{P}_0 = \varepsilon_0 \left[1 + \frac{e^2 F}{m V_0 (\omega_0^2 - \omega^2)}\right] \vec{E}_0 = \varepsilon_0 \varepsilon(\omega) \vec{E}_0\]  

(10)

Thus, the dielectric function can be represented as:

\[
\begin{align*}
\varepsilon(\omega) &= 1 + \frac{e^2 F}{m V_0 (\omega_0^2 - \omega^2)} = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} = \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2}; \\
\omega_p^2 &= \omega_0^2 + \omega_0^2; \\
\omega_p^2 &= \frac{e^2 F}{m V_0}.
\end{align*}
\]  

(11)

Account of an electronic oscillator in the elementary volume \(V_0\) leads to factorization of the dielectric function in the form:
\(\varepsilon(\omega) = \frac{\omega^2 - \omega^2_0}{\varepsilon_\infty - \omega^2} + \varepsilon_\infty = n^2_\infty.\)  \hspace{1cm} (12)

Respectively the dispersion relation for polaritons in the considered view of the dielectric medium with an electronic oscillator takes the form [14-15]:

\[\omega^2 = \frac{c^2 k^2}{\varepsilon(\omega)} = \frac{\varepsilon_\infty k^2}{\varepsilon_\infty - \omega^2_0} = \left(\frac{c^2 k^2}{\omega^2_0 - \omega^2}\right), \quad c^2 = \frac{c^2_0}{\varepsilon_\infty}.\]  \hspace{1cm} (13)

From equation (13) we have the biquadratic equation:

\[\omega^2 - \omega^2_0 \left(\omega^2_0 + c^2 k^2\right) + \omega^2_0 c^2 k^2 = 0.\]  \hspace{1cm} (14)

The exact solution of this equation defines two polariton branches:

\[\omega^2 = \frac{\omega^2_0 + c^2 k^2}{2} \left(1 \pm \sqrt{1 - \frac{4 \omega^2_0 c^2 k^2}{\left(\omega^2_0 + c^2 k^2\right)^2}}\right).\]  \hspace{1cm} (15)

Here indices \(l, 0\) mean longitudinal and transversal modes. Graphically this dependence is shown in Figure 3.

**Figure 3.** The dispersion law for polariton waves in terbium nitrate hydrate in the visible wavelength range. The corresponding wavelengths are A1 - 488 nm, A2 - 444 nm. The corresponding energy transition is \(^5D_4 \rightarrow ^7F_6\) (\(\lambda = 448\) nm)

### 3. Peculiarities of the rare earth ions spectra

The photoluminescence spectra of terbium ion \(\text{Tb}^{3+}\) in the optical range is determined by electron transitions between the levels of unfilled inner 4f-shell, which is shielded from external influences of external electrons (usually \(p\)- and \(s\)-) shells. Such transition is forbidden in the electron dipole approximation [16].

The disturbance of the parity is possible if a terbium ion is surrounded by lattice or impurities (oxygen, carbon, and nitrogen). In such case, under the influence of the disturbance fields the opposite parity to the 4f-states occurs and the transition becomes permitted [17]. Such screening leads to narrow spectral lines and intense fluorescence.

It is known that the atomic configuration of the rare earth elements including terbium has the form \([\text{Xe}]4f^76s^2\) ([Xe] is the configuration corresponding to the electron shell of an xenon atom). In the presence of the crystalline lattice the terbium ion \(\text{Tb}^{3+}\) assumes the configuration \([\text{Xe}]4f^{10}\).

The splitting of the levels is determined by the symmetry of the nearest environment of the ion \(\text{Tb}^{3+}\) in the emitting complex [17]. Since the large number of different types of centers of symmetry in the same sample is realized, the emission spectrum has very complex structure.
As a result of the spin-orbit interaction between the electrons in the f-shell of Tb$^{3+}$ 7F ground state splits into a number of terms $^5$F$_6$, $^5$F$_5$, $^5$F$_4$, $^5$F$_3$, etc. (the left part of Figure 4). Accordingly for excited states term $^5$D splits into $^5$D$_0$, $^5$D$_1$, $^5$D$_2$, etc. (see Figure 4). The crystal field of ligands splits the terbium triple ion levels into the sub-levels, thus leading to resolve of degeneracy (the right part of Figure 4).

As we can see from the Figure 5, the tangents to the polariton wave graphs at points U1 and U2 have a small slope. This corresponds to a small value of the group velocity of photons in the medium.

$$V = \frac{d\omega}{dk} = \frac{c^2}{\varepsilon_\infty} = \frac{c_0}{\varepsilon_\infty}.$$  \hspace{1cm} (16)

Here $c_0$ is the photon velocity in vacuum, $V$ is the group velocity of polariton wave in the crystal.

The slowing down of the group velocity (the value of the permittivity $\varepsilon_\infty \sim 10^3$) determines the increase in the photon-axion interaction time in the medium by a factor of $c_0 / V$. These effects contribute to the increase in the probability of a photon-axion conversion $\sim 10^3$.

4. Experimental setup

For experimental confirmation the splitting of the levels due to the presence of the crystalline lattice of the terbium ion Tb$^{3+}$ we studied reflection spectra in an installation whose block diagram is shown in Figure 6 (a, b) The installation is consisted of four functional blocks: nitrogen laser ($\lambda = 337$ nm)-source of UV-radiation, samples, receivers and signal processing device. The obtained primary...
excitation (see Figure 6a) was fed along the optical fiber (2) to the holder (4), on which the capillary light guide (5) with the sample (6) was placed. We put in reflected radiation through the optical fiber (3) to the spectrometer (7). The obtained spectra were transmitted to the computer (8) via the interface.

The transmission spectra were studied in an installation whose block diagram is shown in Figure 6b. The primary excitation from nitrogen laser was focused into optical fiber (2) and fed to the holder (4).

Figure 6. Schematic diagram of the installation for recording the reflection spectra (a) and transmission (b): 1 – nitrogen laser ($\lambda = 337$ nm); 2, 3 - light guides; 4 – a holder; 5 - quartz plates; 6 – a sample; 7 - spectrometers; 8 – a computer

The capillary light guide (5) contained the sample (6). The transmitted radiation was fed along the optical fiber (3) to the spectrometer (7). The obtained spectra were transmitted by the interface to the computer (8).

5. An increase of conversion efficiency due to use of paramagnetic crystal

According to the previous experimental results, photon-axion conversion in vacuum requires more than 10 T values of magnetic induction with superconductor elements and cavity length more than 1 m [12]. In this paper we analyze the opportunity of photon-axion conversion efficiency gain by using of paramagnetic crystal Tb(NO$_3$)$_3$·6H$_2$O [18].

The unpaired electrons of the lanthanide elements in the 4f shell create a high value of the magnetic moment: $\mu = 11.4 \mu_B$. An estimate of the magnetic moment for the 4f shell electrons is given by dependence $\mu_{eff} = \sqrt{n(n+2)}\mu_B$. Here, $n$ is the number of unpaired electrons on the 4f shell [19, 20]. Crystal Tb(NO$_3$)$_3$·6H$_2$O has a low point symmetry (C$_{point}$ point group) which corresponds to the axial symmetry, resemble to magnetic field symmetry. The presence of a large magnetic moment of the Tb$^{3+}$, located at the site of the crystal structure, gives rise to local strong magnetic field inside the crystalline lattice. The magnitude of the internal magnetic field induction can be estimated from the experimental data on fluorescence spectrum (Figures 7, 8) provided by the experimental setup described above. Assuming that the splitting of the spectral lines $\Delta \lambda$ in the spectrum of the Tb$^{3+}$ ion is due to effective the Zeeman effect, we have

$$\Delta E = \hbar \Delta \omega = -\mu_i B.$$  \hspace{1cm} (17)

Here $\mu = 11.4 \mu_B$ is the magnetic moment of the lanthanide ion, $\mu_B = 0.93 \cdot 10^{-23}$ J/T is Bohr magneton, $B$ is the induction of the magnetic field in the crystal lattice. So we obtain for effective magnetic field:

$$B = \hbar \Delta \omega / \mu_i = \hbar \frac{c}{\mu_i \lambda^2} \Delta \lambda.$$  \hspace{1cm} (18)

Using the data of experimental measurements (Figure 8a) and the known constants, we obtain:
Thus, the local internal strong magnetic field produced by a terbium ion in the low symmetrical crystalline lattice (Cs - symmetry) can increase the probability of axion-photon conversion in the paramagnetic ion region by about fifty times compared with the conversion probability in an external superconductor magnetic field of 10 T in vacuum. For approving of momentum conservation law in elemental conversion processes, we should use exciting radiation with frequency, close to the unitary polaritons value (see Figure 5, points U1, U2, U3)

Additional increase in the efficiency of the photon-axion transformation (1) will occur when Fabry-Perot resonators are used, inside which an active medium is placed

\[ P_{\gamma \to \omega} = \frac{N_{\omega}}{N_{\gamma}} = \frac{1}{4} \left( \frac{Q}{\pi} \right) \left( \frac{\alpha}{k} \right) (gB_\omega)^2 F^2(q). \]  

(20)

Here \( Q \) is the quality factor of the resonator, \( F(q) = \frac{2}{q l} \sin \frac{ql}{2} \) is a factor of the order of unity at a resonance. For \( g = 2.7 \times 10^{-11} \text{ GeV}^{-1} \), \( B = 10 \text{ T} \), \( l = 1 \text{ m} \), and \( Q \sim 10^5 \), the probability of the conversion is \( P_{\gamma \to \omega} \sim 10^{-14} \). Estimates made for laser \( P \sim 10 \text{ W} \) (\( \lambda = 0.5 \mu \text{m} \)) emitting in a quasicontinuous regime
with the number of photons at the output $N_\gamma \sim 10^{20}$ per second determine the number of photons at the output of the second identical resonator (see Figure 2) - $N'_\gamma \sim 10^8$.

Using of a paramagnetic medium placed in a Fabry-Perot resonator, taking into account the local magnetic field $B_{\text{int}} = 71.7$ T and reducing the group photon velocity $c/c_0 \sim 10^{-3}$ within the crystal, will lead to increase in the estimate of the number of photons at the output of the apparatus by a factor of $10^7$.

6. Conclusion

Thus in this paper the gain of axion-photon conversion process is proposed by means of paramagnetic crystalline media using instead of vacuum and outer magnetic field, having used before. Low crystalline symmetry of proposed paramagnetic crystal (Tb(NO$_3$)$_3$·6H$_2$O) proves the strong internal local effective magnetic field, larger than outer superconductor magnetic field, having been used before. For approving of momentum conservation law in elemental conversion processes, it was proposed to use the exciting laser emission with frequency, close to unitary polaritons value (see Figure 5, points U1,U2,U3). In this case the group velocity of exciting light is essentially less with comparing to light velocity in vacuum. Accordingly the efficiency of photon – axion conversion is waited be increased enough for hopeful recording of the secondary emission in Primakoff effect by modern detectors.

References

[1] Sloan J V et al 2016 Physics of the Dark Universe 14 95
[2] Peccci R D and Quinn H R 1977 Phys. Rev. Lett. 38 1440
[3] Weinberg S 1978 Phys. Rev. Lett. 40 223
[4] Wilczek F 1978 Phys. Rev. Lett. 40 279
[5] Sikivie P 1983 Phys. Rev. Lett. 51 1415
[6] Ruffelt G G 1990 Phys.Rept. 198 1
[7] Melendez B, Bertolami M M and Althaus L 2012 Revisiting the Impact of Axions in the Cooling of White Dwarfs (Preprint arXiv: 1210.0263v1)
[8] Okun L V 1982 Sov. Phys. JETP 56 502
[9] Ringwald A 2015 The hunt for axions (Preprint arXiv: 1506.04259v1)
[10] Rosenberg L J 2015 PNAS 112 40 12278
[11] Primakoff H 1951 Phys. Rev. 81 899
[12] Sikivie P, Tanner D and van Bibber K 2007 Phys. Rev. Lett. 98 172002
[13] Gorelik V S and Izmailov G N 2011 Bull. Lebedev Phys. Inst. 38 177
[14] Agranovich V M 1968 Theory of excitons (M., Nauka, 384 pp.)
[15] Rumyantsev V V and Gumennyk K V 2015 J. of Photonic Materials and Techn. 1 11
[16] Bunker P R and Jensen P 2006 Molecular Symmetry and Spectroscopy (NRC Research Press 2nd ed. 748 pp.)
[17] Dieke G H 1968 Spectra and Energy levels of Rare Earth Ions in Crystal (Int. Pub.: A division of John Wiley & Sons, N.Y. 401 pp.)
[18] Hoard R et al 1985 IEEE Transactions on Magnetics 21 2
[19] Audi G, Bersillon O, Blachot J and Wapstra A H 2013 Nuclear Physics A 729 3
[20] Huray P G, Nave S E and Haire R G 1983 Journal of the Less Common Metals 93 2 p 293