Thermodynamic Prescription of Cosmological Constant in Randall Sundrum-II Brane

Tanwi Bandyopadhyay*
Adani Institute of Infrastructure Engineering, Ahmedabad-382421, India.
(Dated: October 10, 2018)

In this work, we apply quantum corrected entropy function derived from the Generalized Uncertainty Principle (GUP) to the Holographic Equipartition Law to study the cosmological scenario in Randall-Sundrum (RS) II brane. An extra driving term has come up in the effective Friedmann equation for a homogeneous, isotropic and spatially flat universe. Further, thermodynamic prescription of the universe constraints this term eventually with order equivalent to that of the cosmological constant.

Keywords: Brane world gravity, Equipartition Law, Generalized Uncertainty Principle, Cosmological Constant.

PACS numbers: 04.70.Dy, 04.50.Kd, 98.80.Es

I. INTRODUCTION

In order to give an explanation of higher dimensional theory, Randall and Sundrum ([1], [2]) proposed an idea of bulk-brane model, where the four dimensional world in which we live is called the 3-brane (a domain wall) that is embedded in a higher dimensional spacetime (bulk). According to the theory, the brane confines all the matter field, only gravity propagates in the bulk. Moreover the extra fifth dimension need not be finite, it can extend to infinity in either side of the brane. The concept of brane world scenarios shows a possibility to resolve the problem of unification of all forces and particles in nature. The main equations governing the cosmological evolutions of the brane differ from the corresponding Friedmann equations in standard cosmology ([3] -[6]). The difference lies in the fact that the energy density of the brane appears to be in a quadratic form whereas in standard cosmology, the energy density appears linearly in the field equations. This model is also consistent with the string theory and may resolve the so called hierarchy problem or the source of dark energy and dark matter ([7], [8]). The later theory is one of the overwhelming theories of the current era. The concept of dark matter had been first proposed ([9], [10]) in the context of studying galaxy clusters. The dark energy, on the other hand, is a completely new component which produces sufficient negative pressure. This drives the cosmic acceleration which has also been substantiated by the observational evidences over the years. The observational data clearly states that the current universe is flat having approximate cosmic content of 21% dark matter, 72% dark energy and rest in the form of visible matter and radiation. All these imply that the standard cosmological models are needed to be modified with the models of dark matter and dark energy. Unfortunately, very less is known about dark energy. Hence there exist many prospective candidates for this cosmic component. Among them, cosmological constant $\Lambda$ is the most popular having an equation of state $p_\Lambda = -\rho_\Lambda$. This model is known as $\Lambda$CDM model (cold dark matter) ([11]-[14]). This theory has a major drawback in terms of order of measurement. The observed value of $\Lambda$ is many order of magnitude smaller than its theoretical value predicted in quantum field theory. This is termed as the cosmological constant problem and to resolve this, one of the many proposed cosmological models is varying cosmological constant ($\Lambda(t)$CDM) model ([15]-[20]).

On the other hand, one of the key features of quantum theory of gravity is called the holographic principle. This states that in a bounded system, the number of degrees of freedom is associated to entropy and scales with the area enclosed ([21]-[23]). Under this principle, gravity is shown to be an entropic force derived from the changes in the Bekenstein-Hawking entropy ([24]-[26]). Further, many studies focussed on derivation and investigation of the Friedmann and acceleration equations in the background of entropic cosmology ([27]-[29]). Various forms of entropy have been applied in these studies ([30]-[37]). In some of them, an extra driving term is derived from entropic forces on the horizon of the universe in order to explain its accelerated expansion. Intrigued by the holographic principle, very recently Padmanabhan ([38]) proposed a different approach saying

* tanwi.bandyopadhyay@aiim.ac.in
that the cosmic space is emergent as the cosmic time progresses. It has been termed as the holographic
equipartition law. According to this, the rate of expansion of the universe is related to the difference between
the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside. Keeping
this in the background, the cosmological equations were derived and examined both in classical and modi-

ified theories of gravity \([39]-[45]\). For most of these studies, Bekenstein-Hawking entropy played the major role.

Very recently, a similar study has been carried out in \([46]\), where a modified Rényi entropy was chosen
instead of Bekenstein-Hawking entropy and a constant like term was obtained in the field equations. Imposing
an analytical constraint, this term showed behavior similar to the varying cosmological constant. Further, the
power-law corrected entropy was also tested in the same mechanism and similar results were found in \([47]\). This
surely necessitates more investigation to the alternative studies of dark energy and cosmological constant in
modified gravity theories. We have followed this novel approach to study the underlying cosmological scenario
in the RS-II brane model considering the quantum corrected form of the entropy function derived from the
Generalized Uncertainty Principle (GUP) \([48]\). A similar study has been carried out in \([44]\) in Einstein’s gravity
but our entropy function in unique in its \(\sqrt{\text{Area}}\) form. The necessity and motivation for choosing this entropy
function was discussed later in details. The GUP corrected entropy was applied to the holographic equipartition
law in a four dimensional universe embedded in a conformally flat five dimensional space-time. Consequently,
an analogous extra driving term is derived in the modified Friedmann equations. Further thermodynamical
investigations showed that this extra term is of order identical to the order of cosmological constant.

The paper is organized in the following way: In section II, we briefly review the \(\Lambda(t)\)CDM model and the
modified field equations in the context of brane world gravity. In section III, the expansion of the cosmic space
is treated as an emergent process and the modified Friedmann equations are retrieved from the Holographic
Equipartition Law in the absence of any dark energy component. Section IV presents a brief review of GUP corrected entropy. In this section, subsection A discusses the results of application of GUP corrected entropy into the holographic equipartition law. In subsection B, the validity of the Generalized Second Law of Thermodynamics (GSLT) is assumed and the behavior of the extra driving term is analyzed. Finally, a brief
discussion on our study is made in section V.

II. MAIN EQUATIONS: \(\Lambda(t)\)CDM MODEL IN BRANE WORLD

A homogeneous, isotropic, spatially flat Friedmann-Robertson-Walker (FRW) universe in the natural unit
system \((G = c = h = k_B = 1)\) is given by

\[
ds^2 = dt^2 - a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

which is considered to be embedded in a conformally flat five dimensional space-time. The form of the energy
momentum tensor for a combination of dark matter and dark energy is

\[
T_{\mu\nu} = (\rho_m + p_m + \rho_\Lambda + p_\Lambda) u_\mu u^\nu - (p_m + p_\Lambda) \delta_\mu^\nu
\]

Generally a barotropic equation of state \(p_m = \omega_m \rho_m\) is chosen for the matter part on the brane having energy
density \(\rho_m\) and pressure \(p_m\) and a variable cosmological constant is chosen as the component of dark energy
having energy density \(\rho_\Lambda\) and pressure \(p_\Lambda = -\rho_\Lambda\). The four velocity \(u_\mu\) in comoving coordinate system takes
the form \(u_\mu = \delta_\mu^t\). Thus the effective Einstein equations on the brane are \([42]\)

\[
\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi}{3} \left[ \rho_T \left( 1 + \frac{\rho_T}{2\lambda} \right) \right]
\]

and

\[
\frac{\ddot{a}}{a} + H^2 = -\frac{4\pi}{3} \left[ \rho_T \left( 1 + \frac{2\rho_T}{\lambda} \right) + 3\rho_\Lambda \left( 1 + \frac{\rho_T}{\lambda} \right) \right]
\]
where $\rho_T = \rho_m + \rho_\Lambda$ is the total energy density, $p_T = p_m + p_\Lambda$ is the total pressure, $\lambda$ is the positive brane tension, the Hubble parameter is given by $H(t) = \frac{\dot{a}}{a}$ and $a(t)$ is the scale factor in flat FRW brane model.

Equations 3 and 4 can be explicitly written as

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} \rho_{m\text{eff}} + \frac{1}{3} \Lambda(t)_{\text{eff}}$$

and

$$\frac{\dot{a}}{a} = -\frac{4\pi}{3} \left[ (\rho_m + 3p_m) + \frac{1}{\lambda}(2\rho_m + 3p_m + 2\rho_m\rho_\Lambda + 3\rho_mp_m) \right] + \frac{1}{3} \Lambda(t)_{\text{eff}}$$

where $\rho_{m\text{eff}} = \rho_m \left( 1 + \frac{\rho_m}{\lambda} \right)$ and $\Lambda(t)_{\text{eff}} = 8\pi \left[ \rho_\Lambda \left( 1 + \frac{\rho_\Lambda}{2\lambda} + \frac{\rho_m}{\lambda} \right) \right]$.

For the present brane model with matter field given by equation 2, the explicit form of the energy momentum conservation relation ($T^\mu_{\nu;\nu} = 0$) is

$$\dot{\rho}_m + 3 \frac{\dot{a}}{a} (\rho_m + p_m) = -\dot{\rho}_\Lambda \simeq -\frac{\Lambda(t)_{\text{eff}}}{8\pi}$$

Instead of a variable $\rho_\Lambda$, if we choose a constant $\rho_\Lambda$, then the field equations together with the continuity equation will be identical to the corresponding equations in the standard $\Lambda$CDM model.

### III. FIELD EQUATIONS DERIVED FROM THE HOLOGRAPHIC EQUIPARTITION LAW

For a pure de Sitter universe with Hubble parameter $H$, the holographic principle can be described by the relation

$$N_{\text{sur}} = N_{\text{bulk}}$$

where $N_{\text{sur}}$ denotes the number of the degrees of freedom on the holographic screen with Hubble radius $r_H = 1/H$

$$N_{\text{sur}} = \frac{4\pi}{H^2} = 4S_H$$

Here $S_H$ is the entropy on the Hubble horizon. The number of degrees of freedom in bulk is said to obey the equipartition law of energy

$$N_{\text{bulk}} = \frac{2|E|}{T}$$

In the context of brane world models, the induced active gravitational mass on the brane $|M| = |E|$ has the form

$$|M| = \frac{4\pi}{3H^3} \left| \left( \rho_T + 3p_T + \frac{3p_T\rho_T}{\lambda} + \frac{2\rho_T^2}{\lambda} \right) \right|$$

$$= -\epsilon \frac{4\pi}{3H^3} \left\{ (\rho_m + 3p_m) + \frac{1}{\lambda}(2\rho_m + 3p_m + 2\rho_m\rho_\Lambda + 3\rho_mp_m) + \frac{1}{4\pi} \Lambda(t)_{\text{eff}} \right\}$$
for the choice of the matter field (2). The parameter $\epsilon$ is defined later. Using the above expression of $|M|$ and the horizon temperature $T = H/2\pi$, we get the expression of $N_{\text{bulk}}$ as

$$N_{\text{bulk}} = -\frac{16\pi^2}{3H^4} \left\{ \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right\} + \frac{1}{4\pi} \Lambda(t)_{\text{eff}} \right\} \tag{12}$$

Since the real world is not purely but asymptotically de Sitter, therefore one may propose that the expansion rate of the cosmic volume is related to the difference of these two degrees of freedom. The analytical form of this is described as \[38\]

$$\dot{V} = l_p^2 \left( N_{\text{sur}} - \epsilon N_{\text{bulk}} \right) \tag{13}$$

Equation (13) is known as the holographic equipartition law. Here $V = \frac{4\pi^3}{3H^3}$ is the cosmic volume and the parameter $\epsilon$ is defined by \[38, 51\]

$$\epsilon \equiv \begin{cases} +1, & \text{when } \left[ \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right] < 0 \\ -1, & \text{when } \left[ \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right] > 0 \end{cases} \tag{14}$$

Here, we have considered that there is no dark energy component in the 3-brane, i.e., $\Lambda(t)_{\text{eff}} \sim \rho_\Lambda = 0$. In this case $\left[ \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right] < 0$ for the acceleration of the universe. Hence from equation (11) and (14), the definition of the parameter $\epsilon$ is well justified.

One can write from equations (9), (12) and (13)

$$-4\pi \frac{\dot{H}}{H^4} = \left\{ 4S_H + \frac{16\pi^2}{3H^4} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right\} \tag{15}$$

or equivalently

$$\dot{H} = -\frac{4\pi}{3} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right\} - \frac{H^4 S_H}{\pi} \tag{16}$$

The acceleration equation is therefore read as

$$\ddot{a} = -\frac{4\pi}{3} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right\} + H^2 \left( 1 - \frac{H^2 S_H}{\pi} \right) \tag{17}$$

Thus we have derived the acceleration equation from the holographic equipartition law and an extra driving term appears in the right side of the equation. This term vanishes when one chooses the Bekenstein-Hawking entropy for $S_H$. The acceleration equation will then be

$$\ddot{a} = -\frac{4\pi}{3} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 3p_m + 2\rho_m + 3\rho_m p_m \right) \right\} \tag{18}$$

which is identical to the equation (6) with $\Lambda(t)_{\text{eff}} \sim \rho_\Lambda = 0$. Hence in this case, the field equation and the corresponding energy conservation equation become

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_m \text{eff} \tag{19}$$

and

$$\dot{\rho}_m + 3\frac{\dot{a}}{a} (\rho_m + p_m) = 0 \tag{20}$$

However, any other form of $S_H$ will not result in the above set of equations and the cosmological implications will definitely be something else.
IV. GUP CORRECTED ENTROPY ON THE HORIZON

In recent years, a number of studies in general relativity and modified gravity theories came to surface due to the discovery of different aspects of black hole solutions. Black holes are thermodynamic objects with well defined entropy. Generally, the Bekenstein Hawking entropy (52, 53, 54)

\[ S_{BH} = \frac{A}{4l_p^2} \] (21)

is chosen for the same. Here \( A \) is the surface area of the sphere with the Hubble horizon \( r_H = \frac{1}{H} \) and \( l_p = \sqrt{\frac{G \hbar}{c}} \approx 10^{-35} \text{m} \) is the Planck length. With \( A = 4\pi r_H^2 \), we can write

\[ S_{BH} = \frac{\pi r_H^2}{l_p^2} \] (22)

Instead of a flat universe, if we choose a non-flat universe, then the apparent horizon \( r_A = \frac{1}{\sqrt{H^2 + k a^2}} \) should be used as the horizon radius instead of the Hubble horizon. Corrections in this entropy formula were needed to accommodate the newly emerging physics from string theory and loop quantum gravity (LQG). Several of these theories predicted quantum corrections to the entropy-area relation (55-64)

\[ S_{QG} = \frac{A}{4l_p^2} + C_0 \ln \left( \frac{A}{4l_p^2} \right) + \sum_{n=1}^{\infty} C_n \left( \frac{A}{4l_p^2} \right)^{-n} \] (23)

where the coefficients \( C_n \) are model dependent parameters. Recent rigorous calculations from LQG has fixed the value of \( C_0 = -1/2 \) [59]. On the other hand, Mead [65] first pointed out that Heisenberg uncertainty principle could be affected by gravity. Later, a considerable amount of effort had been put to the modified commutation relations between position and momenta commonly known as the Generalized Uncertainty Principle (GUP) from different perspectives of quantum aspects of gravity. All these studies eventually led to the GUP corrected entropy form (66-69)

\[ S_{GUP} = \frac{A}{4l_p^2} + \sqrt{\pi \alpha_0} \sqrt{\frac{A}{4l_p^2}} - \frac{\pi \alpha_0^2}{64} \ln \left( \frac{A}{4l_p^2} \right) + O(l_p^3) \] (24)

Here \( \alpha_0 \) is a dimensionless constant prescribed in the deformed commutation relations [70]. The leading contribution of this new entropy function lies in its second term \( \sim \sqrt{\text{Area}} \). This is an extra term to the already existing logarithmic correction to entropy derived from the quantum gravity effects. Due to the difference in the leading order correction term, the underlying nature of such model needs to be investigated in four dimensional Einstein’s gravity as well as in higher dimensional modified theories of gravity. Based on many similarities between the black hole horizon and cosmological horizon and on the assumption that the universe should be described by the quantum language, we employ this newly obtained GUP corrected entropy of the black hole horizon as the entropy of the cosmological horizon in the natural unit system

\[ S_Q = \frac{A}{4} + \sqrt{\frac{\pi \alpha_0}{4}} \sqrt{\frac{A}{4}} - \frac{\pi \alpha_0^2}{64} \ln \left( \frac{A}{4} \right) \] (25)

which on further calculation becomes

\[ S_Q = S_{BH} \left[ 1 + \frac{\alpha_0 H}{4} - \frac{\alpha_0^2 H^2}{64} \ln \left( \frac{\pi}{H^2} \right) \right] \] (26)

Here \( S_{BH} = \frac{\pi}{H^2} \). The novelty of this expression is that, when \( \alpha_0 = 0 \), then \( S_Q \) becomes \( S_{BH} \).
A. Consequences of GUP Corrected Entropy into the Holographic Equipartition Law

Here, we apply the GUP corrected entropy function $S_Q$ into the Holographic Equipartition Law, i.e., we consider that

$$S_H = S_Q = S_{BH} \left[ 1 + \frac{\alpha_0 H}{4} - \frac{\alpha_0^2 H^2}{64} \ln \left( \frac{\pi}{H^2} \right) \right]$$  \hspace{1cm} (27)

Substituting this new form of $S_H$ in (17), we have

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( \rho_m + 2\rho_m + 3\rho_mp_m \right) + \frac{\alpha_0^2 H^4}{64} \ln \left( \frac{\pi}{H^2} \right) - \frac{\alpha_0 H^3}{4}$$  \hspace{1cm} (28)

The extra driving term appearing in the right side of the equation needs to be positive for the current cosmic acceleration.

In the brane world gravity, the field equations together with the continuity equation then become

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} \rho_{\text{eff}} + f_\alpha(H)$$  \hspace{1cm} (29)

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} \left( \rho_m + 3p_m \right) + \frac{1}{\lambda} \left( 2\rho_m + 3p_m + 3\rho_mp_m \right) + f_\alpha(H)$$  \hspace{1cm} (30)

and

$$\rho_m' + 3\frac{\ddot{a}}{a} (\rho_m + p_m) = -\frac{3f_\alpha(H)}{8\pi}$$  \hspace{1cm} (31)

where the extra term $f_\alpha(H)$ is given by

$$f_\alpha(H) = \frac{\alpha_0^2 H^4}{64} \ln \left( \frac{\pi}{H^2} \right) - \frac{\alpha_0 H^3}{4}$$  \hspace{1cm} (32)

Let us now discuss about the evolution of this extra driving terms from the entropy function (27) and the acceleration equation (30). Equation (30) is the final equation incorporating all three corrections. As $S_{BH}$ is positive, hence the following restriction is to be obeyed by the parameters for $S_Q$ to be positive

$$\left[ \frac{\alpha_0 H}{16} \ln \left( \frac{\pi}{H^2} \right) - 1 \right] < \frac{4}{\alpha_0 H}$$  \hspace{1cm} (33)

Again for the current cosmic acceleration

$$0 < f_\alpha(H) = \frac{\alpha_0 H^3}{4} \left[ \frac{\alpha_0 H}{16} \ln \left( \frac{\pi}{H^2} \right) - 1 \right]$$  \hspace{1cm} (34)

Hence it is clear from (33) and (34) that

$$f_\alpha(H) < H^2$$  \hspace{1cm} (35)

A similar constraint can be derived from the study of the Generalized Second Law of Thermodynamics (GSLT) as presented in the following subsection.
B. Generalized Second Law of Thermodynamics (GSLT)

Here we shall discuss the GSLT in the current prescription. Considering $S_T$ as the total entropy of the universe, one can write

$$\dot{S}_T = \dot{S}_Q + \dot{S}_I$$  \hfill (36)

where $S_I$ is the entropy of matter inside the horizon. From (27), we can write

$$\dot{S}_Q = S_{BH} \left[ 1 - \left( \frac{\alpha_0 H^2}{64} - \frac{\alpha_0 H}{8} \right) \right]$$  \hfill (37)

where

$$S_{BH} = \frac{d}{dt} \left( \frac{\pi}{H^2} \right) = -\frac{2\pi \dot{H}}{H^3}$$  \hfill (38)

Since $\dot{S}_{BH} > 0$, to satisfy $\dot{S}_Q > 0$, the following restriction needs to be obeyed

$$\left( \frac{\alpha_0^2 H^2}{64} - \frac{\alpha_0 H}{8} \right) < 1$$  \hfill (39)

In order to obtain the rate of change of entropy of the matter inside the horizon, we consider the Gibbs’ equation ($[71],[72]$)

$$T_I dS_I = dE_I + p_T dV$$  \hfill (40)

where $V$ is the volume inside the horizon and $E_I = \rho_T dV$ stands for the internal energy. The temperature of the matter $T_I$ inside the horizon has been assumed to be equivalent to the horizon temperature $T = \frac{H}{2\pi}$. In absence of any dark energy component, this equation takes the form

$$T_I \dot{S}_I = \left[ \rho_m + 3\dot{H}(\rho_m + p_m) \right] V$$

$$= -\frac{3\rho_m \dot{H} V}{8\pi}$$  \hfill (41)

where we have used the modified continuity equation $[31]$ to obtain the expression of $\dot{S}_I$. Taking time derivative of (32) and using the expression of horizon temperature $T$, one can yield

$$\dot{S}_I = S_{BH} \left[ \frac{\alpha_0^2 H^2}{32} \ln \left( \frac{\pi}{H^2} \right) - \frac{3\alpha_0 H}{8} - \frac{\alpha_0^2 H^2}{64} \right]$$  \hfill (42)

Thus from (37) and (42), the rate of change of total entropy of the universe becomes

$$\dot{S}_T = S_{BH} \left[ 1 - \left\{ \frac{\alpha_0 H}{4} + \frac{\alpha_0^2 H^2}{32} - \frac{\alpha_0^2 H^2}{32} \ln \left( \frac{\pi}{H^2} \right) \right\} \right]$$  \hfill (43)

Again as $\dot{S}_{BH} > 0$, to satisfy $\dot{S}_T > 0$, the following condition must be attained

$$\left[ \frac{\alpha_0 H}{4} + \frac{\alpha_0^2 H^2}{32} - \frac{\alpha_0^2 H^2}{32} \ln \left( \frac{\pi}{H^2} \right) \right] < 1$$  \hfill (44)
From (39) and (44), one can easily derive

\[ f_\alpha(H) > \frac{H^2}{2} \]  

(45)

Thus, we attain a very interesting result from (35) and (45)

\[ \frac{H^2}{2} < f_\alpha(H) < H^2 \]

(46)

Following the arguments of [47] as for the observational constraint \( \dot{H} < 0 \) [73], one can assume \( H_0 \) to be the minimum value for \( H \) and arrive at a stricter constraint

\[ \frac{H_0^2}{2} < f_\alpha(H) < H_0^2 \]

\[ \Rightarrow O(f_\alpha(H)) \lesssim O(H_0^2) \]  

(47)

This result is analogous to the one presented in both [46] and [47], though in the former study, a mathematical condition was imposed to obtain similar restriction while in the later, it evolved through the validity of the GSLT. Further probing into the standard \( \Lambda \)CDM model, we obtain \( \Lambda = 3H_0^2\Omega_\Lambda \). This implies that

\[ O\left(\frac{\Lambda}{3}\right) = O(H_0^2\Omega_\Lambda) \]

(48)

As from Planck (2015) results [14], \( \Omega_\Lambda = 0.692 \), which is of order one. This yields to

\[ O\left(\frac{\Lambda}{3}\right) \simeq O(H_0^2) \]  

(49)

Thus the order of the extra driving term in the acceleration equation becomes equivalent to the order of the cosmological constant term. This result however seems to be model-independent as the positive brane tension did not play any significant role in deriving the analogy.

V. DISCUSSIONS

In the present work, our aim was to study the cosmic evolution in the Brane world gravity with the help of the Holographic Equipartition Law. We have applied quantum corrected form of the entropy function derived from the Generalized Uncertainty Principle in the Holographic Equipartition Law to derive the modified cosmological equations in a homogeneous, isotropic and spatially flat 3-brane embedded in a five dimensional bulk. The novelty of the study lies in \( \sqrt{\text{Area}} \) form of the entropy function. It was noticed that the acceleration equation contains an extra driving term of order consistent with the order of the cosmological constant. A similar constraint was obtained assuming the validity of GSLT. The study remained to be model independent and the positive brane tension did not play any crucial role for the attained result. However, it should be understood that our aim was not to verify the GSLT in the modified gravity theory. Rather we were interested in the evolution of the extra driving term appearing in the acceleration equation due to imposition of the holographic equipartition law for a specific GUP corrected entropy function whose leading order term is different from the existing forms. This may reflect new light to the studies of the cosmological constant problem in modified gravity theories.

Acknowledgement:
The author is thankful to IUCAA, Pune, for their warm hospitality and excellent research facilities where part of the work has been done during a visit under the Associateship Programme.

[1] L. Randall and R. Sundrum, Phys.Rev.Lett. 83, 3370 (1999).
[2] L. Randall and R. Sundrum, Phys.Rev.Lett. 83, 4690 (1999).
[3] P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys.Letts.B 477, 285 (2000).
[4] P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, Nucl.Phys.B 565, 269 (2000).
[5] J. Ponce de Leon, Mod.Phys.Letts.A 17, 2425, (2002).
[6] J. Ponce de Leon, Mod.Phys.Letts.A 16, 2291, (2001).
[7] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, Phys.Rev.D 59, 086001 (1999).
[8] A. Lukas, B. A. Ovrut and D. Waldram, Phys.Rev.D 60, 086001 (2000).
[9] F. Zwiecky, Helv.Phys.Acta 6, 110 (1933).
[10] F. Zwiecky, Phys.Rev. 51, 290 (1937).
[11] S. Perlmutter et al., [Supernova Cosmology Project Collaboration], Nature (London), 391, 51 (1998).
[12] A. G. Riess et al., [Supernova Search Team Collaboration], Astron.J. 116, 1009 (1998).
[13] A. G. Riess et al., Astrophys.J. 659, 98 (2007).
[14] P. A. R. Ade et al., Astron.Astrophys. 594, A13, (2016).
[15] J. L. Shapiro and J. Solà, JHEP 02, 006 (2002).
[16] S. Basilokas, M. Plionis and J. Solà, Phys.Rev.D 80, 083511 (2009).
[17] J. P. Mimoso and D. Pavón, Phys.Rev.D 87, 047302 (2013).
[18] M. H. P. M. Putten, Mon.Not.R.Astron.Soc. 450, L48 (2015).
[19] A. Gómez-Valent, E. Karimkhani and J. Solà, J.Cosmol.Astropart.Phys. 12, 048 (2015).
[20] J. A. S. Lima, S. Basilakos amd J. Solà, Gen.Relativ.Grav. 47, 40 (2015).
[21] G. ’tHooft, Salamfest, 0284 (1993).
[22] L. Susskind, J.Math.Phys., 36, 6377, 1995.
[23] R. Bousso, Rev.Mod.Phys. 74, 825 (2002).
[24] T. Padmanabhan, Mod.Phys.Lett.A 25, 1129 (2010).
[25] T. Padmanabhan, Rept.Prog.Phys. 73, 046901 (2010).
[26] E. Verlinde, JHEP 04, 029 (2011).
[27] A. Sheykhi, Phys.Rev.D 81, 104011 (2010).
[28] A. Sheykhi and S. H. Hendi, Phys.Rev.D 84, 044023 (2011).
[29] S. Mitra, S. Saha and S. Chakraborty, *Mod.Phys.Lett.A*, 30, 1550058 (2015).
[30] D. A. Easson, P. H. Frampton and G. F. Smoot, *Phys.Lett.B* 696, 273 (2011).
[31] D. A. Easson, P. H. Frampton and G. F. Smoot, *Int.J.Mod.Phys.A* 27, 1250066 (2012).
[32] Y. F. Cai, J. Liu and H. Li, *Phys.Lett.B* 690, 213 (2010).
[33] T. S. Koivisto, D. F. Motaa and M. Zumalacárregui, *J.Cosmol.Astropart.Phys.* 02, 027 (2011).
[34] N. Komatsu and S. Kimura, *Phys.Rev.D* 87, 043531 (2013).
[35] N. Komatsu and S. Kimura, *Phys.Rev.D* 89, 123501 (2014).
[36] M. P. Dabrowski and H. Gohar, *Phys.Lett.B* 748, 428 (2015).
[37] C. Tsallis and L. J. L. Cirto, *Eur.Phys.J.C* 73, 2487 (2013).
[38] T. Padmanabhan, arXiv:1206.4916 (hep-th).
[39] R. G. Cai, *JHEP* 1211, 016 (2012).
[40] K. Yang, Y.-X. Liu and Y.-Q. Wang, *Phys.Rev.D* 86, 104013 (2012).
[41] Y. Ling and W.-J. Pan, *Phys.Rev.D* 88, 043518 (2013).
[42] A. F. Ali, *Phys.Lett.B* 732, 335 (2014).
[43] T. Padmanabhan, *Mod.Phys.Lett.A*, 30, 1540007 (2015).
[44] F.-Q. Tu and Y.-X. Chen, *Gen.Relativ.Grav.* 47, 87 (2015).
[45] H. Moradpour and A. Sheykhi, *Int.J Theor.Phys.* 55, 1145 (2016).
[46] N. Komatsu, it *Eur.Phys.J.C* 77, 229 (2017).
[47] N. Komatsu, *Phys.Rev.D* 96, 103507 (2017).
[48] P. Vergueno and E. C. Vagenas, *Phys.Lett.B*, 742, 15 (2015).
[49] T. Bandyopadhyay, A. Baveja and S. Chakraborty, *Mod.Phys.Lett.A* 23, 685 (2008).
[50] Y. Ling and J. P. Wu, *J.Cosmol.Astropart.Phys.* 017, 1008 (2010).
[51] T. Padmanabhan, *Res.Astron.Astrophys.* 12, 891 (2012).
[52] J. D. Bekenstein, *Phys.Rev.D* 7, 2333 (1973).
[53] J. D. Bekenstein, *Phys.Rev.D* 9, 3292 (1974).
[54] J. D. Bekenstein, *Phys.Rev.D* 12, 3077 (1975).
[55] R. K. Kaul and P. Majumdar, *Phys.Rev.Lett.* 84, 5255 (2000).
[56] A. Ghosh and P. Mitra, *Phys.Lett.B* 616, 114 (2005).
[57] A. J. Medved and E. C. Vagenas, *Phys.Rev.D* 70, 124021 (2004).
[58] G. A. Camelia, M. Arzano and Proccaccini, *Phys.Rev.D* 70, 107501 (2004).
[59] K. A. Meissner, *Class.Quant.Grav* 21, 5245 (2004).
[60] S. Das, P. Majumdar and R. K. Bhaduri, *Class. Quant. Grav* **19**, 2355 (2002).

[61] Y. S. Myung, *Phys. Lett. B* **579**, 205 (2004).

[62] M. Domagala and J. Lewandowski, *Class. Quant. Grav* **21**, 5233 (2004).

[63] A. Chatterjee and P. Majumdar, *Phys. Rev. Lett.* **92**, 141301 (2004).

[64] M. M. Akbar and S. Das, *Class. Quant. Grav* **21**, 1383 (2004).

[65] C A. Mead, *Phys. Rev. D* **135**, 849 (1964).

[66] B. Majumdar, *Phys. Lett. B* **703**, 402 (2011).

[67] R. J. Adler, P. Chen and D. I. Santiago, *Gen. Relativ. Grav.* **33**, 2101 (2011).

[68] G. A. Camelia, M. Arzano, Y. Ling and G. Mandanici, *Class. Quant. Grav* **23**, 2585 (2006).

[69] B. Majumdar, *Gen. Relativ. Grav.* **45**, 2403 (2013).

[70] S. Das and E. C. Vagenas, *Phys. Rev. Lett.* **101**, 221301 (2008).

[71] B. Wang, Y.G. Gong, E. Abdalla, *Phys. Rev. D* **74**, 083520 (2006).

[72] G. Izquierdo, D. Pavon, *Phys. Lett. B* **633**, 420 (2006).

[73] Krishna P B and Titus Mathew, *Phys. Rev. D* **96**, 063513 (2017).