A$_4$ model of Magic neutrino mass matrix with broken $\mu - \tau$ symmetry and Leptogenesis

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We investigate baryogenesis via leptogenesis in A$_4$ flavor model within the paradigm of type-I and II seesaw mechanism resulting in magic neutrino mass matrix with broken $\mu - \tau$ symmetry in a minimal scenario with two right-handed neutrinos(2RHN). Additional $Z_2$ cyclic symmetry is employed to constrain the Yukawa structure of model. In general, the type-II seesaw terms play crucial role in generating non-degenerate neutrino masses and non-zero $\theta_{13}$, and contribute in baryogenesis. In particular, after the spontaneous symmetry breaking, the Yukawa coupling $y_{13}$ at dimension-four is responsible for the breaking of $\mu - \tau$ symmetry. We, also, study the implication of the model for successful leptogenesis using approximated solution of the Boltzmann equations. We find that the observed baryon asymmetry of the universe requires the lightest right-handed neutrino mass to be $(2.36 - 2.40) \times 10^{9}$GeV for NH ($(2.15 - 2.17) \times 10^{9}$GeV for IH) which is consistent with the Davidson-Ibarra bound of $10^{9}$GeV.

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I. INTRODUCTION

In last two decades neutrino oscillation experiments have evinced that neutrino change identity as they travel from source to detector. This metamorphosis, in turn, requires that neutrinos are massive which, remarkably, is in contradiction with the prediction of the most celebrated theory in particle physics called “Standard Model(SM)”.

Within SM, neutrinos are massless because (i) there are no right-handed(RH) neutrinos in the SM (ii) SM Lagrangian contains only renormalizable terms (iii) No Higgs triplet of SU(2)$_L$ in SM. In fact, Sudbury Neutrino Observatory(SNO)$^{1,2}$ and KamLAND$^{3,4}$ neutrino oscillation experiments have, decisively, demonstrated that neutrinos have tiny but non-zero mass and flavor eigenstates are different from the mass eigenstates. This observation has, since then, been augmented by other experiments as well$^{5,6}$. Thus, the existence of non-zero neutrino mass is smoking gun signal of physics beyond SM(BSM). The mass-squared differences, mixing angles and Dirac-type CP violation phase(s) can be probed in oscillation experiments, however, they are insensitive to the absolute neutrino mass scale. Consequently to the reasons described above, neutrino masses cannot be generated, the way quarks and charged lepton masses are generated within SM. Now, apart from identifying the underlying symmetry responsible for emerged picture of neutrino oscillation parameters(two mass-squared differences $\Delta m_{12}^2$, $\Delta m_{23}^2$) and three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) in neutrino oscillation experiments, the question is what mechanism is responsible for the origin of neutrino mass? In addition, there still remain experimental unknowns in the neutrino sector, some of which are:

1. Mass hierarchy-normal or inverted?
2. Atmospheric mixing angle $\theta_{23}$- above or below maximality?
3. Whether $CP$ symmetry is violated in neutrino oscillations?
4. What is absolute scale of neutrino mass?
5. Neutrino- Dirac or Majorana particle? etc.

The answers to first three questions are(will be) investigated in present(future) neutrino oscillation experiments like India-Based Neutrino Observatory(INO)$^{10}$, Deep Underground Neutrino Experiment(DUNE)$^{11}$, T2HK$^{12}$ and NOvA$^{13}$ etc. whereas direct search neutrino-mass-experiments$^{14}$ and lepton number violating processes like $0\nu\beta\beta$ decay will probe the answers to remaining two questions. These experiments will, further, help in ameliorating the lepton mixing paradigm. Thanks to the arduous experimental efforts that the dominant structure of the neutrino mixing matrix has been revealed and mass-squared differences, mixing angles are known to an unprecedented accuracy. The emerged picture of neutrino masses and mixing is dissimilar to the one in quark sector. For example, the quark mixing angles are small as compared to in neutrino sector(except $\theta_{13}$). In a more general framework like Grand Unified Theories(GUTs) in which quarks and leptons belong to the same representation of the gauge group and Yukawa couplings are related, it is a formidable task to understand such diasporical hierarchies in fermion masses and mixing.

On the theoretical front, non-zero neutrino mass and the emerged spectrum of neutrino masses and mixing angles have posed outstanding fundamental questions before the theorists. In fact, there exist alternative theoretical mass models to explain neutrino parameters, for a review see Refs.$^{14,15}$. The reason for the possibility of variety of models is the unresolved experimental ambiguities, discussed in the last section, that still exist. We can
broadly classify various model building frameworks as (i) theoretical models based on flavor symmetries to explain observed mixing parameters (ii) GUTs, as an attempt to unify quarks and leptons family structures (iii) models based on extra dimensions. The physics of underlying flavor structure is contained in the Yukawa sector of the theory. The Yukawa couplings are, in general, complex, thus, also, seeds the possible leptonic CP violation. To explain the observed spectrum of neutrino masses and mixing, Yukawa sector needs to be extended through introduction of additional fermionic and scalar fields. One way to accomplish it, is the imposition of non-abelian discrete flavor symmetry group at some high energy scale and subsequent spontaneous breaking of flavor symmetry in charged lepton and neutrino sector at low energy scale. In this approach, appropriate charge assignments to the fermionic and scalar field(s) under the flavor group alongwith vacuum alignment of the Higgs or Higgs like field(s) constrain the Yukawa structure of the flavor theory. In general, the proliferation of the field content leads to deterioration of the predictability of the model. Also, we may have to apply additional symmetry(ies) to avoid the unwanted couplings in the Lagrangian of the theory. So, we need to have a minimal extension of the fermionic and scalar sector to reduce free parameters in the theory.

The observation of large mixing is another unresolved puzzle in the neutrino sector. There exist myriads of mixing paradigms to explain observed mixing spectrum such as tri-bimaximal mixing(TBM), bi-maximal mixing, golden ratio and tri-maximal mixing, to name a few. TBM mixing predicts maximal atmospheric mixing ($\theta_{23} = 45^\circ$) and vanishing reactor angle($\theta_{13} = 0$) and as such requires corrections to generate non-zero value of $\theta_{13}$ consistent with experimental observations at T2K, Daya bay, Double Chooz and RENO. The neutrino mass matrix resulting from TBM is, in general, “magic” and “$\mu$–$\tau$” symmetric which means that sum of elements of each row/column remains same and mass matrix is invariant under exchange of $\mu$–$\tau$ indices, respectively.

In the present work, we propose a neutrino mass model for the amendment of TBM ansatz such that $\mu$–$\tau$ symmetry is broken i.e. $\theta_{13}$ is non-zero, however, the mass matrix still has magic symmetry. In an effective theory layout, dimension-five operator instinctively generates small neutrino mass due to suppression by the cut-off scale($\Lambda$). Furthermore, the large mixing angles in leptonic sector hints toward additional CP violation sources apart from the quark sector. The leptonic CP violation sources may generate the observed baryon asymmetry of the universe(BAU) and triggered ardupus studies based on flavor models which, simultaneously, account for correct neutrino phenomenology as well. The matter-antimatter asymmetry is measured as baryon to photon ratio which through cosmological findings is $\eta_B = (6.12 \pm 0.04) \times 10^{-10}$.

Baryon symmetric universe must satisfy the Sakharov’s three necessary conditions

- baryon number violation
- $C$ and CP violation
- out-of-equilibrium decay

essential to account for observed baryon asymmetry. BSM scenarios with one or more right-handed neutrino, scalar Higgs triplet and/or scalar fermion triplet must account for observed neutrino phenomenology. Also, their decay contributes to CP asymmetry and hence matter-antimatter asymmetry. Additional BSM fields generates lepton asymmetry which is converted to baryon asymmetry through sphaleron transitions. Admittedly, the next ambitious goal is to understand, coe tenously, the mechanism of neutrino mass generation, large leptonic mixing and observed baryon asymmetry of the Universe(BAU).

Motivated by this, we present an attractive model based on $A_4$ flavor symmetry for generation of both neutrino masses and leptogenesis in the framework of type-I and II seesaw mechanism. In this model, type-II seesaw plays a vital role in fulfilling the purpose of breaking $\mu$–$\tau$ symmetry and obtaining non-degenerate neutrino masses. The additional flavor fields introduced in the model drives appropriate breaking of the $A_4$ symmetry. Also, we have studied the leptogenesis in the type-I and II scenario using approximate solutions to the Boltzmann equations. For the successful generation of BAU, there exist a lower bound on the right-handed neutrino mass scale just above the Davidson-Ibarra bound in both normal as well as inverted hierarchies(NH, IH) of the neutrino masses.

In Sec. II we have discussed the $A_4 \times Z_3$ model where $Z_3$ controls the Yukawa structure of the theory. In Sec. III and IV, we have presented the leptogenesis framework and constraining equations used in the numerical analysis, respectively. Finally, we present major conclusions of the work, in Sec. V.

II. THE $A_4$ MODEL

The $A_4$ is a non-Abelian discrete group of even permutations of four objects having geometrical resemblance with tetrahedron. It has four irreducible representations(1Rs), viz.: 1, 1’, 1” and 3. The multiplication rules of the 1Rs are: $1 \otimes 1’ = 1”$, $1” \otimes 1’ = 1’$, $1 \otimes 1’’ = 1$, $3 \otimes 3 = 1 \oplus 1’ \oplus 1’’ \oplus 3_1 \oplus 3_2 \oplus 3_3 \oplus 3_4$ with,

$$
\begin{align*}
(3 \otimes 3)_1 &= a_1 b_1 + a_2 b_3 + a_3 b_2, \\
(3 \otimes 3)_1’ &= a_3 b_3 + a_1 b_2 + a_2 b_1, \\
(3 \otimes 3)_1’’ &= a_2 b_2 + a_3 b_1 + a_1 b_3, \\
(3 \otimes 3)_{3_1} &= \frac{1}{3}(2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, 2a_2 b_2 - a_1 b_3 - a_3 b_1), \\
(3 \otimes 3)_{3_2} &= \frac{1}{2}(a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_3 b_1 - a_1 b_3, )
\end{align*}
$$

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where \((a_1, a_2, a_3), (b_1, b_2, b_3)\) are basis vectors of the two triplets. Here, we present an \(A_4 \otimes Z_3\) model within type-I and II seesaw framework. In this model, we employed one \(SU(2)_L\) Higgs doublet, \(H\), four \(SU(2)_L\) singlet flavon fields \(\phi_1, \phi_2, \chi_1, \chi_2\) and one \(SU(2)_L\) triplet Higgs fields \(\Delta\). The transformation properties of different fields under \(SU(2)_L\), \(A_4\) and \(Z_3\) are given in Table I. The charge assignments under \(SU(2)_L\), \(A_4\) and \(Z_3\) lead to the following Yukawa Lagrangian

\[
\mathcal{L} = \frac{y_1}{\Lambda} \bar{D}_{iL} H \phi_1 e_R + \frac{y_\mu}{\Lambda} \bar{D}_{iL} H \phi_2 \mu_R + \frac{y_\tau}{\Lambda} \bar{D}_{iL} H \phi_3 \tau_R + \frac{y_4}{\Lambda} \bar{D}_{iL} \tilde{H} \phi_4 \nu_1 + \frac{y_5}{\Lambda} \bar{D}_{iL} \tilde{H} \phi_5 \nu_2 + h_{\chi_1 \chi_1} N_1^T C^{-1} N_1 + h_{\chi_2 \chi_2} N_2^T C^{-1} N_2 - \Delta v_{\Delta} \bar{D}_i \Delta \tilde{D}_i \phi_\nu + H.c. \quad (1)
\]

where, \(\tilde{H} = i r H^* (H\) is SM Higgs doublet) and \(y_i (i = e, \mu, \tau, \nu_1, \nu_2, \chi_1, \chi_2, \Delta_1, \Delta_2)\) are Yukawa coupling constants. We consider the vacuum expectation (vev) alignment for flavon fields \(\phi_1, \phi_2\) as \(< \phi_1 >= (v_1, 0, 0)\) and \(< \phi_2 >= (v_\nu, v_\nu, v_\nu)\) and vev for \(SU(2)_L\) Higgs doublet to be \(v_H\). The Lagrangian in Eqn. (1) up to dimension-five terms results in diagonal charged lepton matrix

\[
m_{\ell} = \frac{v_\nu v_H}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad (2)
\]

and Dirac neutrino mass matrix

\[
m_D = \frac{v_\nu v_H}{\Lambda} \begin{pmatrix} y_{\nu_1} & y_{\nu_2} \\ y_{\nu_1} & y_{\nu_2} \\ y_{\nu_1} & y_{\nu_2} \end{pmatrix}, \quad (3)
\]

Also, the right-handed neutrino mass matrix is given by

\[
m_R = \begin{pmatrix} y_{\chi_1} v_{\chi_1} & 0 \\ 0 & y_{\chi_2} v_{\chi_2} \end{pmatrix}, \quad (4)
\]

where \(v_{\chi_1}\) and \(v_{\chi_2}\) are the vacuum expectation values of the flavon fields \(\chi_1\) and \(\chi_2\), respectively. Using type-I seesaw relation, the effective neutrino mass matrix is given by

\[
m_{\nu_1} = -m_D m_R^{-1} m_D^T = \begin{pmatrix} c & c & c \\ c & c & c \\ c & c & c \end{pmatrix} , \quad (5)
\]

where, \(c = \frac{a^2}{m_1^2} + \frac{b^2}{m_2^2}\) and Yukawa couplings are rearranged as

\[
a = \frac{v_{\nu_1}}{\Lambda} v_{\nu H}, \quad b = \frac{v_{\nu_2}}{\Lambda} v_{\nu H}, \quad M_1 = y_{\chi_1} v_{\chi_1}, \quad M_2 = y_{\chi_2} v_{\chi_2}, \quad (6)
\]

where, \(M_1, M_2\) are masses of right-handed neutrinos \(N_1, N_2\), respectively. The mass matrix in Eqn. (5) results in degenerate light neutrino masses and vanishing reactor angle \(\theta_{13} = 0\) which is in contradiction to the neutrino oscillation experiments. In order to have correct low energy phenomenology we next consider type-II seesaw scenario in conjunction with type-I. The conjoining of type-II seesaw terms have interesting implications on neutrino mass and leptonogenesis. Assuming vev for scalar triplet field \(\Delta\) to be \(v_\Delta\), the type-II contribution to effective neutrino mass matrix is given by

\[
m_{\nu_2} = \begin{pmatrix} 2p & d - p & -p \\ d - p & 2p & -p \\ -p & -p & 2p + d \end{pmatrix} + \begin{pmatrix} 0 & d & 0 \\ d & 0 & 0 \\ 0 & 0 & d \end{pmatrix} = \begin{pmatrix} 2p & -p & -p \\ -p & 2p & -p \\ -p & -p & 2p \end{pmatrix}
\]

where, \(d = y_{\Delta_1} v_\Delta\) and \(p = \frac{v_{\Delta_1}}{\Lambda} v_\Delta v_\nu\). Since, charged lepton mass matrix in Eqn. (2) is diagonal, therefore effective neutrino mass matrix is given by

\[
m_{\nu_\nu} = m_{\nu_1} + m_{\nu_2}.
\]

Using Eqns. (5) and (6)

\[
m_{\nu_\nu} = \begin{pmatrix} c + 2p & c + d - p & c - p \\ c + d - p & c + 2p & c - p \\ c - p & c - p & c + 2p + d \end{pmatrix} . \quad (8)
\]

The neutrino mass matrix in Eqn. (5) obey magic symmetry and is \(\mu - \tau\) asymmetric. Also, dimension-four and dimension-five terms in Eqn. (1) involving the scalar triplet, \(\Delta\), provide non-degenerate light neutrino masses. After the spontaneous symmetry breaking, the Yukawa coupling \(y_{\Delta_1}\) (contained in \(d\)) at dimension-four is responsible for the breaking of \(\mu - \tau\) symmetry. To see this explicitly we can write Eqn. (5) as
achieved if the scalar triplet $\Delta$ is assigned to 

The scenario. The retaining magic symmetry in type-I and II seesaw 

of such breaking pattern for 

omenology. Here, we have obtained the symmetry ori-

equilibrium decay of lightest right-handed neutrino (a) and Fig.1(b) can 

The different decay modes at one loop-level correction 

be written as $[36, 37]$

The contribution due to out-of-

level and one loop level decay amplitudes involving right-

will survive.

CP asymmetry due to interference of tree 

asymmetry induced by the decay of right-handed 

asymmetry. This feature of the model is due to the 

m

contains two model parameters $d$ and $p$ but the CP asymmetry $\epsilon_\Delta$ contain $|d|$ only reducing number of model parameters responsible for producing net CP asymmetry. This feature of the model is due to the structure of $m_{\nu_1}^\alpha$ in Eqn. 7 wherein the sum of elements of row/column is vanishing. Also, scalar triplet taking part in type-II seesaw contributes to the CP asymmetry as well as, is, responsible for $\mu - \tau$ symmetry breaking, thus, results in non-zero $\theta_{13}$.

IV. NUMERICAL ANALYSIS AND RESULTS

In order to the find the CP asymmetry parameter $\epsilon$, we need to find the values of model parameters. To attain
FIG. 1: One loop-level diagrams contributing to $CP$ asymmetry.

| Parameters                        | Best-fit±1σ             | 3σ range        |
|-----------------------------------|-------------------------|-----------------|
| $\Delta m_{21}^2[10^{-5}\text{ eV}^2]$ | $7.55^{+0.20}_{-0.16}$ | $7.05 - 8.14$  |
| $\Delta m_{31}^2[10^{-3}\text{ eV}^2](\text{NH})$ | $2.50 \pm 0.03$        | $2.41 - 2.60$  |
| $\Delta m_{31}^2[10^{-3}\text{ eV}^2](\text{IH})$ | $2.42^{+0.04}_{-0.03}$ | $2.31 - 2.51$  |
| $\sin^2\theta_{12}/10^{-1}$      | $3.20^{+0.20}_{-0.16}$ | $2.73 - 3.79$  |
| $\sin^2\theta_{23}/10^{-1}(\text{NH})$ | $5.47^{+0.20}_{-0.30}$ | $4.45 - 5.99$  |
| $\sin^2\theta_{23}/10^{-1}(\text{IH})$ | $5.51^{+0.18}_{-0.39}$ | $4.53 - 5.98$  |
| $\sin^2\theta_{13}/10^{-2}(\text{NH})$ | $2.160^{+0.083}_{-0.069}$ | $1.96 - 2.41$ |
| $\sin^2\theta_{13}/10^{-2}(\text{IH})$ | $2.220^{+0.074}_{-0.076}$ | $1.99 - 2.44$ |

TABLE II: The latest global data on neutrino oscillation parameters used in the numerical analysis$^{[35]}$.

FIG. 2: The correlation between model parameters involving type-II couplings $(d, p)$ in normal hierarchy(NH) and inverted hierarchy(IH).

In general, effective neutrino mass matrix in diagonal charged lepton basis is given by

$$ M_{\nu} = V^* M_{\nu}^{\text{diag}} V^\dagger, $$

where $V = U_P$ and $M_{\nu}^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$. $U$ is the PMNS matrix given by

$$ U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & s_{12}s_{23} + c_{12}s_{13}c_{23}e^{i\delta} \\
    s_{12}c_{13} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
    s_{12}s_{13} & c_{12}s_{23} - s_{12}c_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}, $$

where $s_{kl} = \sin \theta_{kl}$ and $c_{kl} = \cos \theta_{kl}$ and $P$ is the diagonal phase matrix, $P = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$, where $\delta$ is Dirac
The sum of neutrino mass matrix, in general, can be written as
\[ M_\nu = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}, \] (20)
where \( M_{ij} \) (for \( i, j = 1, 2, 3 \)) contains nine parameters viz., three neutrino masses \( m_1, m_2, m_3 \), three mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \), and three \( CP \) violating phases \( \delta, \alpha, \beta \).

In the numerical analysis, the known neutrino oscillation parameters such as mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) and mass-squared differences \( \Delta m^2_{21}, \Delta m^2_{31} \) are randomly generated using Gaussian distribution with the best-fit and errors given in Table III. The unknown parameters viz., the three \( CP \) phases \( \delta, \alpha, \beta \) are randomly generated in their full physical range, \( 0^\circ - 360^\circ \), with uniform distribution. The remaining unknown parameter i.e. absolute mass-scale \( m_{\text{lightest}} (m_{\text{lightest}} = m_1) \), in normal hierarchy and \( m_3 \) in inverted hierarchy), is generated within a conservative range \( 0 - 0.24 \) guided by the cosmological limit on the sum of neutrino masses \( \sum m_i < 0.24 \) eV (at 95 % C.L.) [30]. The sample size for all these parameters constitute of \( 10^7 \) points. The neutrino masses \( m_2 \) and \( m_3 (m_1 \text{ and } m_2) \) are calculated using the relations viz.,
\[ m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{31}} \] for NH,
and
\[ m_1 = \sqrt{m_3^2 - \Delta m^2_{31}}, \quad m_2 = \sqrt{m_3^2 - \Delta m^2_{31} + \Delta m^2_{21}} \] for IH.

The sum of \( i^{th} \) row of neutrino mass matrix \( M_\nu \) can be written as
\[ S_i = M_{i1} + M_{i2} + M_{i3}, \] (21)
where \( i = 1, 2, 3 \) is row index. The magic symmetry of \( M_\nu \) implies \( S_1 = S_2 = S_3 \) = constant(\( \kappa \)). Using the procedure outlined above, we require that \( M_\nu \) is magic symmetric within a tolerance of \( 10^{-3} \) i.e. if
\[ |S_1 - S_2| \leq 10^{-3} \quad \text{and} \quad |S_2 - S_3| \leq 10^{-3}. \]

Furthermore, Eqns. (8) and (20), in conjunction, yield relations amongst the model parameters \( c, p, d \) and the elements of \( M_\nu \) viz.,
\[ c = \frac{2M_{13} + M_{11}}{3}, \]
\[ p = \frac{M_{11} - M_{13}}{3}, \]
and
\[ d = M_{12} - M_{13}. \]

The constraints imposed by the magic symmetry alongwith the tolerance defined above are used to obtain the model-parameters \( c, p, d \). The parameter space, thus, identified is, further, used to obtain the \( CP \) asymmetry parameter \( \epsilon \) for normal hierarchy(NH) as well as inverted hierarchy(IH) and, hence, the baryon asymmetry. In the numerical calculations, we have considered a simple scenario where the value of total phase factor, in Eqns.(10) and (17), is taken to be maximal \( \sin(2(\psi_2 - \psi_1)) = 1, \sin(\psi - 2\psi_1) = 1 \) to avoid convolution of phases. Also, in order to fulfill the condition for out-of-equilibrium decay in Eqn.(13), we find that \( \frac{\langle m_1 \rangle_{\text{mix}}}{M_1} < 4 \times 10^{-3} \text{eV}. \) Hence, it is judicious to consider the approximation, \( \frac{a^2}{M_1} \approx 10^{-4}. \) Using the calculated parameter space of \( c = \frac{a^2}{M_1} + \frac{b^2}{M_1} \) and \( d \) alongwith \( \frac{a^2}{M_1} \approx 10^{-4} \), in Eqns. (14-17) we obtain the correlation of baryon asymmetry with right-handed neutrino mass scale \( M_1 \).
The type-II seesaw plays pivotal role to have correct neutrino phenomenology (non-zero $\theta_{13}$ and non-degenerate neutrino masses) and leptogenesis so we have focussed on parameters $d$ and $p$ involving type-II seesaw couplings. We have shown the correlations between the model parameters $d$ and $p$ in Fig. 4(a), Fig. 5(a) and Fig. 6(a).
FIG. 7: The variation of baryon asymmetry with model parameter $\frac{|b|^2}{M_2}$. The dot-dashed line is the observed baryon asymmetry $|\eta_B| = 6.12 \times 10^{-10}$.

FIG. 8: The variation of baryon asymmetry with model parameter $|d|$. The dot-dashed line is the observed baryon asymmetry $|\eta_B| = 6.12 \times 10^{-10}$.

FIG. 9: The variation of baryon asymmetry with lightest right-handed neutrino mass $M_1$. The dashed line is the observed baryon asymmetry $|\eta_B| = (6.12 \pm 0.04) \times 10^{-10}$. The dotted lines represent the bounds on the right-handed neutrino mass consistent with observed baryon asymmetry $|\eta_B|$.

It is observed that imaginary parts of model parameters $d$ and $p$ are symmetric about zero because the phenomenology of $m_\nu$ will be same if we change $d \rightarrow -d$ and $p \rightarrow -p$ in Eqn. (8). The variation of model parameter $|d|$ and $|p|$
with lightest neutrino mass $m_{\text{lightest}}$ is shown in Fig. (5) and Fig. (6), respectively. The variation of baryon asymmetry with the model parameters $|\delta^c_M|$ and $|d|$ is shown in Fig. (7) and Fig. (8), respectively.

In Fig. (9), we have depicted the variation of baryon asymmetry $|\eta_B|$ with the lightest right-handed neutrino mass $M_1$ for best-fit value of parameters $|d|$ and $|\eta|$. We can see from the figure that the observed baryon asymmetry of the universe requires the lightest right-handed neutrino mass in the range $(2.36 - 2.40) \times 10^9$GeV for NH $((2.15 - 2.17) \times 10^9$GeV for IH) which is consistent with the Davidson-Ibarra bound of $10^9$GeV. It is to be noted that right-handed neutrino mass scale is much below the GUT scale and will have negligible renormalization group evolution effects.

V. CONCLUSIONS

In conclusion, we have presented a model based on $A_4$ flavor symmetry augmented by $Z_3$ cyclic group within the framework of type-I and II seesaw mechanism in a minimal scenario of two right-handed neutrinos (2RHN). The type-II seesaw terms play a crucial role in generating non-degenerate neutrino masses and non-zero $\theta_{13}$ (i.e. breaking $\mu - \tau$ symmetry). The Yukawa coupling involving scalar triplet $\Delta$ is responsible for breaking of $\mu - \tau$ symmetry and contributes to the baryogenesis. The resulted neutrino mass matrix retains magic symmetry and non-degenerate eigenvalues. Using the data on neutrino masses and mixing, we have evaluated the model parameter space using the constraints emanating from the magic symmetry with a defined tolerance of $10^{-3}$. We have also, investigated the leptogenesis through approximated solutions of Boltzmann equations to find a lower bound on lightest right-handed neutrino mass. We have worked in the approximation which is in accordance with the essential condition required for out-of-equilibrium decay of lightest right-handed neutrino. We find that the right-handed neutrino mass scale consistent with the observed baryon asymmetry is in the range $(2.36 - 2.40) \times 10^9$GeV((2.15 - 2.17) $\times 10^9$GeV) for NH (IH) which is just above the Davidson-Ibarra bound.

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