Simultaneous Measurement and Simulation of Angular Position and Angular Speed for Time Grating

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Abstract. A method is presented in order to realize the measurement of angular position and angular speed simultaneously based on the theory of time grating and the measurement of angular position and angular speed is transformed into the measurement of phase difference. Four orthogonal excitation signals are generated by the internal phase-locked loop module of FPGA. The excitation signals are superposed and input to the stator sensor. At this time, the rotation of the motor causes the rotation of the rotor sensor to generate the induction signal. The phase difference of the two signals will be obtained by the input comparison circuit of the induction signal and the excitation signal. The phase difference will be processed in FPGA, and finally the angular position and angular speed values will be obtained. The simulation results show that the method is feasible. The experimental results show that the accuracy error of angular position measurement can reach $\pm 3''$, and the measurement speed stability can reach 3rpm(3‰) when the angular speed is 1000rpm.

1. Introduction

Position and speed sensors have been widely used in the measurement of length and speed in the measurement system, and the measurement of angular position and angular speed in circular motion is also very important. With the continuous development of angular position and angular speed sensors, scholars at home and abroad have carried out extensive research and achieved remarkable results. Liu Wei and his colleagues put forward the idea of transforming synchronization of position and speed into synchronization of position and time in [1]. The correlation time between them has been reduced to microsecond level which greatly improves the real-time performance of position and speed measuring instrument, but the accuracy of position and speed has not been significantly improved. Angular position and angular speed are measured by two sensors, which makes the system more complex. In [2], Zhu Hongbin and his colleagues proposed a birefringent and dual-frequency laser Doppler speed measurement method based on low-frequency phase-locked amplification technology for speed measurement, and proposed a zero-difference laser interferometer position measurement method based on orthogonal demodulation technology for position measurement which is two independent measurement methods.

Foreign scholars have made extensive research on angular position and angular speed sensors. In [3], Johnny Rodriguez-Maldonado only uses position measurements to combine Kalman filter with a model that can simultaneously (synchronously) obtain instantaneous estimates of position, speed and...
acceleration to form a Kalman Differentiator (KD), but it is based on an analysis function that cannot be processed for sudden changes in the process. In [4], Naqui J. and his colleagues proposed a resonator based on asymmetric coplanar waveguide technology to measure angular speed directly. The angular speed is measured by measuring the two peak time intervals of carrier signals. This method realizes the synchronous measurement of angular position and angular speed, but requires two sets of MODEM (modulation-demodulation) circuits, and the circuit is complex.

Through the research and learning of many kinds of grid position and speed sensors, found that the accuracy and resolution ratio of the sensor are mostly related to the scale precision of the grid ruler used. The higher the scale precision is, the higher the precision of the sensor is, and the cost is also greatly increased. With the appearance of time grating position sensor, the idea of space-time coordinate conversion is widely used in measurement system, and the method of space-time coordinate conversion has become the theoretical basis of many kinds of measurement sensors.

2. Methodology

2.1. Principle of Time-grating Position Sensor

In [5], the core of the time grating measurement principle is the time-space measurement datum conversion theory, that is, by constructing a double coordinate system of relative uniform motion, the spatial position is converted into the time difference, and the position measurement of the object with arbitrary variable speed is realized by the method of time measurement space. Next, the principle of time grating measurement is introduced through the "ideal" model experiment. As shown in Fig 1 (a).

![Figure 1. Ideal model of time-grating position sensor](image)

$M$ is an infinitely long train running at a constant speed $V$ and taking the earth as the static reference coordinate system. There are two particles $a$ and $b$ on m, where $a$ is stationary, the relative geodetic reference coordinate system $a$ is moving at a constant velocity $V$; $b$ is moving at a velocity $v$, and its relative geodetic velocity is $V+v$. In the geostationary reference coordinate system, railways with sequential average intervals of $W$ are used as reference points, and the relative position $x$ of $a$ and $b$ is measured.

Ideally, when $t=0$, $a$ and $b$ start to move at the same $O$ position as the starting point. When $b$ passes through the first rail $W$, the recording time is $T_0$, and when $a$ passes through the recording time is $T_i$, the interval between rails $W$ is

$$W = \int_{T_0}^{T_i} Vdt = \int_{T_0}^{T_i} (V + v)dt$$

Transformation Formula (1)

$$\int_{T_0}^{T_i} Vdt = \int_{T_0}^{T_i} (V + v)dt$$

$$\int_{T_0}^{T_i} vdt = V(T_i - T_0)$$

As shown in Fig 1 (b), the relative position of $a$ and $b$ is
$$x = W - \int_{0}^{T} Vdt$$
$$= \int_{0}^{T} (V + v)dt - \int_{0}^{T} Vdt$$
$$= \int_{0}^{T} vdt = V(T - T) = V\Delta t$$

(3)

In summary, the principle of time-grating position sensors can be summarized as follows: in a two-coordinate system, one object moves at any speed, the other object moves at a relatively static coordinate system. Then the position value of the measured object is the product of the time difference between two objects passing through the punctuation point and the speed of the coordinate system, that is, the position value is obtained by measuring the time difference. With the advantage of high precision and high resolution ratio of time, as shown in Fig.1(b), replacing time value with high frequency clock pulse $P$, will greatly improve the measurement accuracy of position.

2.2. Measurement of Angular Position

Fig.1 is completed in an ideal state, an infinite train. This condition is now converted to a disk moving at a constant speed $V$, as shown in Fig. 2.

![Figure 2. Measurement of angular position](image)

In the circular motion with the earth as the reference system, there are two marked objects $a$ and $b$, respectively, with $O$ as the center of the circular motion and $R$ as the stationary rod on the ground. $a$ moves at a constant speed $V$ relative to $R$ and $b$ at any speed $V$ relative to $a$, then $b$ moves at a speed $V+v$ relative to $R$. From the working principle of the time-grating position sensor, it is known that the angular position of $a$ relative to $b$ is $\theta$

$$\theta = V(t_0 - t) = V\Delta t$$

(4)

Where $t_0$ and $t$ are the time when $a$ and $b$ ticule passes through $R$. Formula (4) shows that the angular position value is the product of constant speed $V$ and time difference.

Formulas (3) and (4) show that both position and angular position measurements can be converted to measurements of time values.

2.3. Measurement of Angular Speed

Projecting Fig. 2 as shown in Fig. 3, $W$ is geodetic, considered a stationary coordinate system, $W'$ is a stationary wave source, and $W'$ is a reference coordinate system with uniform speed of $V$. There are two observation points $a$ and $b$ in the system established in the two coordinate systems, where $b$ moves at any speed $v$, then moves at speed $V+v$ relative to $W$ coordinate system, and $a$ is stationary in $W'$ coordinate system, then $a$ moves at uniform speed $V$ relative to $W$ coordinate system. Thus, a relative motion occurs between $a$ and $b$. 
Figure 3. Measurement model of angular speed

Set the frequency of the source to \( f \), and when \( v = 0 \), \( b \) observes that \( a \) has a frequency of \( f \). When \( v \neq 0 \), according to \([6-7]\), \( b \) observed a frequency of

\[
f' = \left(\frac{V + v}{v}\right) f
\]

(5)

Where "±" indicates whether the movement direction of observation point \( b \) is consistent with that of \( W' \). From formula (5), the angular speed of point \( b \) is

\[
v = \pm \left(\frac{f' - f}{f}\right) V = \pm \frac{T - T'}{T'} V = \pm \frac{\Delta T}{T'} V
\]

(6)

Where \( T \) is the period of motion of the wave source and \( T' \) is the period of motion of \( b \). It can be seen from formulas (6) that the value of angular speed can be obtained by measuring the speed and period of the wave source and the period difference of the two waves.

2.4. Simultaneous Measurement of Angular Position and Angular Speed

Through the analysis of angular position and angular speed measurement, in order to realize the simultaneous measurement of angular position and angular speed, we only need to make them in the same motion coordinate system, that is, to achieve the unity of space quantity \( V \). It can be seen from the models in Fig.1 and Fig.2, Fig.3 that \( V \) can be regarded as a series of waveforms of uniform motion in space, it is traveling waves. Therefore, the first condition of simultaneous measurement of angular position and angular speed is to produce a series of traveling waves.

According to \([8]\), a rotating magnetic field will be generated in the stator when the three-phase winding coil with a space difference of 120° passes through the AC excitation with a time difference of 120° and the expression of the rotating magnetic field potential is

\[
f_n = F_n \cos 2\pi \left(\frac{t}{T} + \frac{x}{W}\right)
\]

(7)

formula (7) is a traveling wave equation, whose rotational speed \( V \) is

\[
V = \frac{60f}{p} (r/min) = \frac{2\pi f}{p} (arc/s)
\]

(8)

Where \( p \) is the pole number of the motor, which is a constant; \( f \) is the frequency of AC excitation.

Now, formula (8) is substituted into formula (4) and formula (6) respectively. In addition, using the method of Figure 1 (b), insert the high frequency clock pulse into the phase difference, period and period difference, and obtain the measurement formula of angle and angular velocity as follows

\[
\theta = \frac{360}{p} \cdot \frac{\Delta P}{P_{Te}} (°)
\]

(9)

\[
v = \frac{360}{p} \cdot \pm \left(\frac{P_{Te}}{P_{T}} - 1\right) \frac{1}{P_{Te}} (°/s)
\]

(10)
Where $P_0$ is the periodic pulse of the excitation signal, and $\Delta P$ is the pulse of $t_0$ and $t_i$ time periods. $P_T$ is the periodic pulse of the observation head $b$ relative to the motion of the source, and $P_T'$ is the periodic pulse of the observation head $a$.

To sum up, in a system with uniform $V$ motion, one column of waveforms is stationary relative to the system, and the other column of waveforms moves in the system at any speed $v$. The relative motion angular position and angular speed values of the two columns of waveforms can be calculated simultaneously by calculating the operating speed of the system and the phase difference and period of the two columns of waveforms.

3. Results

3.1. Experimental Device

The experimental device is shown in Fig.4. The test-bed is based on granite, avoiding the error caused by temperature deformation, and uses high-precision optical grating as the reference instrument for comparison. The time grating consists of a rotor and a stator. The stator is fixed on the granite test-bed through tooling. The rotor is coaxially connected with the optical grating through the shafting. The servo motor drives the optical grating and the rotor of the time grating to rotate synchronously through the synchronous pulley. The angular position and angular speed of the time grating are compared with that of the optical grating, and the error of the angular position and stability of angular speed of the time grating is obtained.

![Figure 4. Experimental device](image)

3.2. Measurement Error of Angular Position and Angular of Speed Stability

Fig.5 is the test curve of the angular position error of the time grating. It can be seen that the error of the time grating is $\pm 3''$.

![Figure 5. Error of angular position](image)

Fig.6 and Fig.7 are the stability test curves of the angular speed at 200rpm and 1000rpm respectively.
In Fig.6 and Fig.7, blue is the measured value and red is the theoretical value. It can be seen that the speed at 200 rpm is stable at 1 rpm(5‰) and at 1000 rpm is stable at 3 rpm(3‰).

4. Conclusions
By analyzing the working principle of time-grating position sensor, a mathematical model for synchronous measurement of time-grating angular position and angular speed is derived. Based on the angular position measurement theory of time grating and combined with the Doppler effect generated inside the time grating, the angular position measurement and angular speed measurement are unified on the time datum, and an experimental prototype of time grating is developed. The experimental results show that this method can effectively synchronize the grating angular position and angular speed measurements. The accuracy of the angular position is ±3”, and the stability of the angular speed is less than 3‰ at 1000 rpm.

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