Effect of magnetic field on the charge and thermal transport properties of hot and dense QCD matter

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Abstract

We have studied the effect of strong magnetic field on the charge and thermal transport properties of hot QCD matter at finite chemical potential. For this purpose, we have calculated the electrical ($\sigma_{el}$) and thermal ($\kappa$) conductivities using kinetic theory in the relaxation time approximation, where the interactions are subsumed through the distribution functions within the quasiparticle model at finite temperature, strong magnetic field and finite chemical potential. This study helps to understand the impacts of strong magnetic field and chemical potential on the local equilibrium by the Knudsen number ($\Omega$) through $\kappa$ and on the relative behavior between thermal and electrical conductivities through the Lorenz number ($L$) in the Wiedemann-Franz law. We have observed that, both $\sigma_{el}$ and $\kappa$ get increased in the presence of strong magnetic field, and the additional presence of chemical potential further increases their magnitudes, where $\sigma_{el}$ shows decreasing trend with the temperature, opposite to its increasing behavior in the isotropic medium, whereas $\kappa$ increases slowly with the temperature, contrary to its fast increase in the isotropic medium. The variation in $\kappa$ explains the decrease of the Knudsen number with the increase of temperature. However, in the presence of strong magnetic field and finite chemical potential, $\Omega$ gets enhanced and approaches unity, thus, the system may move little away from the equilibrium state. The Lorenz number ($\kappa/(\sigma_{el}T)$) in the abovementioned regime of strong magnetic field and finite chemical potential shows linear enhancement with the temperature and has smaller magnitude than the isotropic one, thus, it describes the violation of the Wiedemann-Franz law for the hot and dense QCD matter in the presence of strong magnetic field.

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1 Introduction

At high temperatures and/or chemical potentials the system can transit to a state consisting of deconfined quarks and gluons, called as quark-gluon plasma (QGP) and such conditions are evidenced in ultrarelativistic heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) \[1\], Large Hadron Collider (LHC) \[2\], and are expected to be produced in the Compressed Baryonic Matter (CBM) experiment at Facility for Antiproton and Ion Research (FAIR) \[3\]. In addition, the collision for nonzero impact parameter also produces a strong magnetic field, whose magnitude varies from $eB = m_\pi^2 \approx 10^{18}$ Gauss at RHIC to $15 m_\pi^2$ at LHC \[4\]. Although the quark chemical potential is very small in the initial stages of ultrarelativistic heavy ion collisions, it is not zero. Some studies also suggest that at temperature around 160 MeV, the baryon chemical potential is approximately 300 MeV \[5–7\]. Especially in the strong magnetic field regime, the baryon chemical potential is observed to go up from 0.1 GeV to 0.6 GeV \[8\], which also implies an increase in the quark chemical potential. In recent years, a lot of observations have been made to explore the effect of strong magnetic field on the properties of hot QCD matter, such as the thermodynamic and magnetic properties \[9–12\], the chiral magnetic effect \[13, 14\], the dilepton production from QGP \[15, 16\] etc.

It has been observed that, in a model where the hot QCD medium is approximated as a gas of hadron resonances, the increase of the electrical conductivity with increasing magnetic field is significant at finite chemical potential \[8\]. According to the field-theoretical calculation in the presence of magnetic field \[17\], the longitudinal electrical conductivity increases with the increase of quark chemical potential. Thus, it would be interesting to see how different transport coefficients of hot QCD matter behave at finite chemical potential in the presence of strong magnetic field. So, in this work, we have calculated the electrical ($\sigma_{el}$) and thermal ($\kappa$) conductivities in the presence of both strong magnetic field and chemical potential ($\mu$) and compared them with their counterparts in the absence of magnetic field and chemical potential. After understanding these transport properties, we have further studied the effects of strong magnetic field and chemical potential on the local equilibrium by the Knudsen number ($\Omega$) through the thermal conductivity and on the relative behavior between electrical and thermal conductivities by the Lorenz number in Wiedemann-Franz law. In the presence of an external magnetic field, the dispersion relation of a charged particle becomes modified as $\omega_{i,n} = \sqrt{p_L^2 + 2n|q_iB| + m_i^2}$, where the
motion along the longitudinal direction ($p_L$) (with respect to the direction of magnetic field) resembles with that for a free particle and the motion along the transverse direction ($p_T$) is expressed in terms of the Landau levels ($n$). In the strong magnetic field (SMF) limit ($|q_i B| \gg T^2$ and $|q_i B| \gg m_i^2$), the charged particles can not jump to the higher Landau levels due to very high energy gap $\sim O(\sqrt{|q_i B|})$, and only the lowest Landau level (LLL) is populated. Thus, the motion of charged particle along the longitudinal direction is much greater than the motion along the transverse direction, i.e. $p_L \gg p_T$ and this develops an anisotropy in the momentum space.

According to some recent observation, the medium is formed at the similar time scale of the production of strong magnetic field due to faster thermalization. The electrical conductivity of QGP plays an important role in extending the lifetime of such strong magnetic field [18, 19]. Thus the transport properties of the medium are expected to be significantly affected by the strong magnetic field. One of the transport coefficients is the electrical conductivity, because of which, electric current is produced in the early stage of the heavy-ion collision and its value is also important for the strength of chiral magnetic effect [13]. Moreover, the strength of the charge asymmetric flow in mass asymmetric collisions is given by the electrical conductivity [20]. Different models have previously investigated the influence of magnetic field on $\sigma_{el}$, such as the quenched SU(2) lattice gauge theory [21], the dilute instanton-liquid model [22], the nonlinear electromagnetic currents [23, 24], axial Hall current [25], real time formalism using the diagrammatic method [26], effective fugacity approach [27] etc. Another transport coefficient is the thermal conductivity ($\kappa$), that is associated with the heat flow or thermal diffusion in a medium. According to the estimation of effective fugacity model [28], $\kappa$ becomes larger in the presence of magnetic field. In this work, we follow the kinetic theory approach to calculate the electrical and thermal conductivities in the presence of strong magnetic field and finite chemical potential.

Using the thermal conductivity, we intend to observe the effects of strong magnetic field and chemical potential on the local hydrodynamic equilibrium of the medium through the Knudsen number ($\Omega$). The Knudsen number is defined as the ratio of the mean free path ($\lambda$) to the characteristic length ($l$) of the system, where $\lambda$ is defined in terms of $\kappa$ as $\lambda = 3\kappa/(\nu C_V)$, with $\nu$ and $C_V$ represent the relative speed and the specific heat at constant volume, respectively. For the validity of the hydrodynamic equilibrium, $\lambda$ needs to be smaller than $l$. The relative importance of the thermal and electrical conductivities
can be understood through the Wiedemann-Franz law, which states that the ratio of the
electronic contribution of the thermal conductivity to the electrical conductivity ($\kappa/\sigma_{el}$)
is the product of Lorenz number ($L$) and temperature. This law is well satisfied by
metals, because they are good heat and electrical conductors. However, different systems
also report the violation of the Wiedemann-Franz law, such as, the thermally populated
electron-hole plasma in graphene [29], the two-flavor quark matter in the Nambu-Jona-
Lasinio model [30] and the strongly interacting QGP medium [31]. Now it is interesting
to see how the Lorenz number, $L = \kappa/(\sigma_{el}T)$ in the Wiedemann-Franz law becomes
influenced by the strong magnetic field and finite chemical potential.

In this work, we have studied the charge and thermal transport properties and their
applications through the Knudsen number and the Lorenz number in the presence of
strong magnetic field and finite chemical potential and observed how these properties
are different from their respective behaviors in the medium in the absence of magnetic
field and chemical potential. The electrical conductivity and the thermal conductivity
of the hot QCD medium can be calculated using different approaches, viz., the relativis-
tic Boltzmann transport equation [32–35], the Chapman-Enskog approximation [31, 36],
the correlator technique using Green-Kubo formula [22, 37, 38], the lattice simulation
[39–41] etc. However, we have used the kinetic theory approach by solving the relativis-
tic Boltzmann transport equation in the relaxation-time approximation to calculate the
electrical and thermal conductivities, where the interactions among particles are incor-
porated through their effective masses in the quasiparticle model at finite temperature,
strong magnetic field and finite chemical potential.

The rest of this paper is organized as follows. In section 2, we have first revisited
the charge transport properties by calculating the electrical conductivity for an isotropic
thermal medium at finite chemical potential and then calculated the same in an ambience
of strong magnetic field. Similarly in section 3, after revisiting the thermal transport
properties for an isotropic thermal medium at finite chemical potential by determining
the thermal conductivity, we have calculated it in the presence of strong magnetic field.
In the similar environment, we have studied some applications of the charge and thermal
transport properties, viz., the Knudsen number and the Wiedemann-Franz law in section
4. In section 5, we have determined the quasiparticle mass of quark in the presence
of strong magnetic field and finite chemical potential. In section 6, we have discussed
our results regarding electrical conductivity, thermal conductivity, Knudsen number and
Wiedemann-Franz law. Finally, in section 7, we have written the conclusions.

2 Charge transport properties

In this section, we are going to study the charge transport properties. Subsections 2.1 and 2.2 are devoted to the calculations of electrical conductivity for an isotropic dense QCD medium in the absence of magnetic field and for a dense QCD medium in the presence of strong magnetic field, respectively.

2.1 Isotropic dense QCD medium in the absence of magnetic field

When an external electric field disturbs the system infinitesimally, an electric current is induced, which is given by

\[ J_\mu = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} p_\mu [q_i \delta f_i(x, p) + \bar{q}_i \delta \bar{f}_i(x, p)] , \]  

(1)

where 'i' stands for flavor and here we have used \( i = u, d, s \). In the above equation \( g_i, q_i (\bar{q}_i) \) and \( \delta f_i (\delta \bar{f}_i) \) represent the degeneracy factor, electric charge and infinitesimal change in the distribution function for the quark (antiquark) of \( i \)th flavor, respectively. According to the Ohm’s law, the spatial component of the four-current is the product of the electrical conductivity and the external electric field,

\[ J = \sigma_{el} E . \]  

(2)

Thus by comparing equations (1) and (2), one can obtain the electrical conductivity. The infinitesimal disturbance \( \delta f_i \) can be determined from the relativistic Boltzmann transport equation (RBTE), which is written in the relaxation time approximation (RTA) \[42\] as

\[ p_\nu \frac{\partial f_i(x, p)}{\partial x_\nu} + q_i F^{\rho\sigma} p_\sigma \frac{\partial f_i(x, p)}{\partial p_\rho} = -\frac{\rho_\nu u_\nu}{\tau_i} \delta f_i(x, p) , \]  

(3)

where \( f_i = \delta f_i + f_i^{iso} \), \( F^{\rho\sigma} \) is the electromagnetic field strength tensor and the relaxation time for quarks (\( \tau_i \)) in a thermal medium is given \[43\] by

\[ \tau_i = \frac{1}{5.1 T \alpha_s^2 \log (1/\alpha_s) [1 + 0.12(2N_i + 1)]} . \]  

(4)
The isotropic distribution functions for quark and antiquark of $i$th flavor are written as

$$f_{i}^{\text{iso}} = \frac{1}{e^{\beta(\omega_i - \mu)} + 1},$$

$$\bar{f}_{i}^{\text{iso}} = \frac{1}{e^{\beta(\omega_i + \mu)} + 1},$$

respectively, where $\omega_i = \sqrt{p^2 + m_i^2}$ and $\mu$ is the chemical potential. For response of electric field, we set $\rho = i$ and $\sigma = 0$ and vice versa in the calculation, so, $F^{i0} = E$ and $F^{0i} = -E$. Thus, RBTE (3) turns out to be

$$q_i E \cdot \partial f_{i}^{\text{iso}} \frac{\partial}{\partial p_0} + q_i p_0 E \cdot \partial \bar{f}_{i}^{\text{iso}} \frac{\partial}{\partial p} = -p_0 \tau_i \delta f_{i}.$$  

(7)

Now solving we get $\delta f_{i}$ as

$$\delta f_{i} = 2q_i \tau_i \beta \frac{E \cdot p}{\omega_i} f_{i}^{\text{iso}} (1 - f_{i}^{\text{iso}}).$$

(8)

Similarly, $\delta \bar{f}_{i}$ is obtained as

$$\delta \bar{f}_{i} = 2\bar{q}_i \tau_i \beta \frac{E \cdot p}{\omega_i} \bar{f}_{i}^{\text{iso}} (1 - \bar{f}_{i}^{\text{iso}}).$$

(9)

After substituting the values of $\delta f_{i}$ and $\delta \bar{f}_{i}$ in eq. (1), we get the electrical conductivity for an isotropic dense medium in the absence of magnetic field as

$$\sigma_{\text{el}}^{\text{iso}} = \frac{\beta}{3\pi^2} \sum_i g_i q_i^2 \int dp \frac{p^4}{\omega_i^4} \tau_i \left[ f_{i}^{\text{iso}} (1 - f_{i}^{\text{iso}}) + \bar{f}_{i}^{\text{iso}} (1 - \bar{f}_{i}^{\text{iso}}) \right].$$

(10)

### 2.2 Dense QCD medium in the presence of strong magnetic field

When the thermal medium comes under the influence of a strong magnetic field, the quark momentum gets split into the components which are transverse and longitudinal to the direction of magnetic field (say, z or 3-direction). As a result the dispersion relation for the quark of $i$th flavor takes the following form,

$$\omega_{i,n}(p_L) = \sqrt{p_L^2 + m_i^2 + 2n |q_i B|},$$

(11)

where $n = 0, 1, 2, \cdots$ represent the Landau levels. In the SMF limit ($|q_i B| \gg T^2$), the quarks only occupy the lowest Landau level ($n = 0$), because they could not be excited thermally to higher Landau levels due to very large energy gap $\sim O(\sqrt{|q_i B|})$. Therefore $p_T \ll p_L$, which results in a momentum anisotropy and the distribution functions for
quark and antiquark are written as

\[ f_i^B = \frac{1}{e^{\beta(\omega_i - \mu)} + 1}, \]  
(12)

\[ \bar{f}_i^B = \frac{1}{e^{\beta(\omega_i + \mu)} + 1}, \]  
(13)

respectively. Here \( \omega_i = \sqrt{p_3^2 + m_i^2} \) according to the dispersion relation in the strong magnetic field limit, where the quark momentum becomes purely longitudinal [44]. So, in the strong magnetic field regime, when an external electric field disturbs the system infinitesimally, an electromagnetic current is induced along the longitudinal direction (3-direction), which is given by

\[ J_3 = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} p_3 [q_i \delta f_i(\tilde{x}, \tilde{p}) + \bar{q}_i \delta \bar{f}_i(\tilde{x}, \tilde{p})], \]  
(14)

where \( \tilde{x} = (x_0, 0, 0, x_3) \) and \( \tilde{p} = (p_0, 0, 0, p_3) \). The (integration) phase factor also gets modified in terms of magnetic field [45, 46] as \( \int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{i|q_i B|}{2\pi} \int \frac{dp_3}{2\pi} \). In the strong magnetic field limit, the electrical conductivity can be determined from the third component of current in Ohm’s law,

\[ J_3 = \sigma_{el} E_3. \]  
(15)

The infinitesimal disturbance \( \delta f_i \) can be evaluated from the relativistic Boltzmann transport equation in the relaxation time approximation, in conjunction with the strong magnetic field limit,

\[ p_0 \frac{\partial f_i}{\partial x_0} + p_3 \frac{\partial f_i}{\partial x_3} + q_i F^\rho\sigma p_\sigma \frac{\partial f_i}{\partial p_\rho} = -\frac{p_0}{\tau_i^B} \delta f_i, \]  
(16)

where the relaxation time, \( \tau_i^B \) in the strong magnetic field regime is given [47] by

\[ \tau_i^B = \frac{\omega_i \left( e^{\beta \omega_i} - 1 \right)}{\alpha_s C_2 m_i^2 (e^{\beta \omega_i} + 1) \int \frac{dp_3}{\omega_i (e^{\beta \omega_i} + 1)}}, \]  
(17)

where \( C_2 \) denotes the Casimir factor. For the response of electric field in the presence of strong magnetic field, we have set \( \rho = 3 \) and \( \sigma = 0 \) and vice versa, so, \( F^{30} = E_3 \) and \( F^{03} = -E_3 \). Solving the eq. (16), we get \( \delta f_i \) as

\[ \delta f_i = \frac{2\tau_i^B \beta q_i E_3 p_3}{\omega_i} f_i^B (1 - f_i^B). \]  
(18)

Similarly for antiquark, \( \delta \bar{f}_i \) is evaluated as

\[ \delta \bar{f}_i = \frac{2\tau_i^B \beta \bar{q}_i E_3 p_3}{\omega_i} \bar{f}_i^B (1 - \bar{f}_i^B). \]  
(19)
Finally substituting $\delta f_i$ and $\delta \bar{f}_i$ in eq. (14) and then comparing with eq. (15), we have calculated the electrical conductivity in the presence of strong magnetic field at finite chemical potential as

$$
\sigma^{B}_{el} = \frac{\beta}{2\pi^2} \sum_i g_i q_i^2 |q_i B| \int dp_3 \frac{p_3^2}{\omega_i^2} \tau_i^B \left[ f_i^B (1 - f_i^B) + \bar{f}_i^B (1 - \bar{f}_i^B) \right].
$$

(20)

3 Thermal transport properties

In this section, we are going to study the thermal transport properties. In subsections 3.1 and 3.2, we have calculated thermal conductivity for an isotropic dense QCD medium in the absence of magnetic field and for a dense QCD medium in the presence of strong magnetic field, respectively.

3.1 Isotropic dense QCD medium in the absence of magnetic field

In a system, the flow of heat is directly proportional to the temperature gradient and the proportionality factor is known as the thermal conductivity. The flow of heat is not continuous, rather it diffuses, depending on the thermal properties of the medium. Thus, through the study of thermal conductivity in a medium, one can get the information on how the heat flows in that medium and how it affects the hydrodynamic equilibrium of the system with finite chemical potential.

The difference between the energy diffusion and the enthalpy diffusion gives the heat flow four-vector as

$$
Q_\mu = \Delta_{\mu\alpha} T^{\alpha\beta} u_\beta - h \Delta_{\mu\alpha} N^\alpha,
$$

(21)

where $\Delta_{\mu\alpha}$ is the projection operator, $\Delta_{\mu\alpha} = g_{\mu\alpha} - u_\mu u_\alpha$, $T^{\alpha\beta}$ is the energy-momentum tensor, $N^\alpha$ is the particle flow four-vector, $h$ is the enthalpy per particle, $h = (\epsilon + P)/n$ with $\epsilon$, $P$ and $n$ represent the energy density, the pressure and the particle number density, respectively. $N^\alpha$ and $T^{\alpha\beta}$ are also defined in terms of the distribution function as

$$
N^\alpha = \sum_i g_i \int \frac{d^3p}{(2\pi)^3 \omega_i} p^\alpha \left[ f_i(x,p) + \bar{f}_i(x,p) \right],
$$

(22)

$$
T^{\alpha\beta} = \sum_i g_i \int \frac{d^3p}{(2\pi)^3 \omega_i} p^\alpha p^\beta \left[ f_i(x,p) + \bar{f}_i(x,p) \right],
$$

(23)
respectively. From the above equations, the particle number density, energy density and pressure can be obtained as \( n = N^{\alpha}u_\alpha, \varepsilon = u_\alpha T^{\alpha\beta}u_\beta \) and \( P = -\Delta^{\alpha\beta}T^{\alpha\beta}/3 \), respectively. As heat flow four-vector in the rest frame of the heat bath is orthogonal to the fluid four-velocity, i.e. \( Q_\mu u^\mu = 0 \), heat flow is spatial, which under the action of external disturbance can be written in terms of the infinitesimal change in the distribution function as

\[
Q = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} \frac{p}{\omega_i} \left[ (\omega_i - h_i) \delta f_i(x, p) + (\omega_i - \bar{h}_i) \delta \bar{f}_i(x, p) \right].
\]  

The Navier-Stokes equation relates the heat flow with the thermal potential (\( U = \mu/T \)) [48] as

\[
Q_\mu = -\kappa \frac{nT^2}{\varepsilon + P} \nabla_\mu U = \kappa \left[ \nabla_\mu T - \frac{T}{\varepsilon + P} \nabla_\mu P \right],
\]

where \( \kappa \) is the thermal conductivity and \( \nabla_\mu \) is the four-gradient, \( \nabla_\mu = \partial_\mu - u_\mu u^\nu \partial_\nu \). In the local rest frame, the spatial component of the heat flow is written as

\[
Q = -\kappa \left[ \frac{\partial T}{\partial x} - \frac{T}{n\hbar} \frac{\partial P}{\partial x} \right].
\]

One can thus obtain the thermal conductivity (\( \kappa \)) by comparing equations (24) and (26). Expanding the distribution function in terms of the gradients of flow velocity and temperature, the relativistic Boltzmann transport equation (3) can be written as

\[
p^{\mu} \partial_\mu T \frac{\partial f_i}{\partial T} + p^{\mu} \partial_\mu (p^{\nu} u_\nu) \frac{\partial f_i}{\partial p^0} + q_i \left[ F^{0j} p_j \frac{\partial f_i}{\partial p^0} + F^{p0} p_0 \frac{\partial f_i}{\partial p^j} \right] = -\frac{p^\nu u_\nu}{\tau_i} \delta f_i,
\]

where \( p_0 = \omega_i - \mu \) and for very small \( \mu \), it can be approximated as \( p_0 \approx \omega_i \). Using following partial derivatives,

\[
\frac{\partial f_i^{\text{iso}}}{\partial T} = \frac{p_0}{T^2} f_i^{\text{iso}}(1 - f_i^{\text{iso}}),
\]

\[
\frac{\partial f_i^{\text{iso}}}{\partial p^0} = -\frac{1}{T} f_i^{\text{iso}}(1 - f_i^{\text{iso}}),
\]

\[
\frac{\partial f_i^{\text{iso}}}{\partial p^j} = -\frac{p^j}{T p_0} f_i^{\text{iso}}(1 - f_i^{\text{iso}}),
\]

we solve eq. (27) and get the infinitesimal disturbance,

\[
\delta f_i = -\frac{\tau_i f_i^{\text{iso}}(1 - f_i^{\text{iso}})}{T} \left[ \frac{p_0}{T} \partial_0 T + \frac{1}{T} p^\nu \partial_\nu T + T \partial_0 \left( \frac{\mu}{T} \right) + \frac{T}{p_0} p^\nu \partial_\nu \left( \frac{\mu}{T} \right) - p^\nu \partial_\nu u_\nu - \frac{p^j p^\nu}{p_0} \partial_\nu u_\nu - \frac{2q_i}{p_0} \mathbf{E} \cdot \mathbf{p} \right].
\]
Using \( \partial_j \left( \frac{\mu}{T} \partial_j T - \frac{\tau_{n\mu}}{T} \partial_j P \right) \) and \( \partial_0 u_\nu = \nabla_\nu P/(nh) \) from the energy-momentum conservation, we obtain \( \delta f_i \) as

\[
\delta f_i = -\tau_i f_{i,iso} (1 - f_{i,iso}) \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p_0 - h_i}{p_0} \right) \frac{p^j}{T} \left( \partial_j T - \frac{T}{n h_i} \partial_j P \right) + T \partial_0 \left( \frac{\mu}{T} \right) - \frac{p^j p^\nu}{p_0} \partial_j u_\nu - \frac{2 q_i}{p_0} E \cdot p \right].
\] (32)

Similarly \( \delta \tilde{f}_i \) is calculated as

\[
\delta \tilde{f}_i = -\tau_i \tilde{f}_{i,iso} (1 - \tilde{f}_{i,iso}) \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p_0 - \tilde{h}_i}{p_0} \right) \frac{p^j}{T} \left( \partial_j T - \frac{T}{n \tilde{h}_i} \partial_j P \right) - T \partial_0 \left( \frac{\mu}{T} \right) - \frac{p^j p^\nu}{p_0} \partial_j u_\nu - \frac{2 \tilde{q}_i}{p_0} E \cdot p \right].
\] (33)

Then substituting the expressions of \( \delta f_i \) and \( \delta \tilde{f}_i \) in eq. (24) and comparing with eq. (26), the thermal conductivity for the isotropic dense medium in the absence of magnetic field is obtained as

\[
\kappa_{iso} = \frac{\beta^2}{6 \pi^2} \sum_i g_i \int dp \frac{p^4}{\omega^2} \tau_i \left[ (\omega_i - h_i)^2 f_{i,iso} (1 - f_{i,iso}) + (\omega_i - \tilde{h}_i)^2 \tilde{f}_{i,iso} (1 - \tilde{f}_{i,iso}) \right].
\] (34)

### 3.2 Dense QCD medium in the presence of strong magnetic field

The presence of strong magnetic field reduces the dynamics of quarks from three spatial dimensions to one spatial dimension, as a result, they can move only along the direction of magnetic field. Thus, in the strong magnetic field regime, the spatial component of heat flow is written as

\[
Q_3 = \sum_i g_i |q_i B| \int dp_3 \frac{p_3}{\omega_i} \left[ (\omega_i - h_i^B) \delta f_i(\tilde{x}, \tilde{p}) + (\omega_i - \tilde{h}_i^B) \delta \tilde{f}_i(\tilde{x}, \tilde{p}) \right].
\] (35)

In addition, eq. (26) also gets modified into

\[
Q_3 = -\kappa^B \frac{\partial T}{\partial x_3} - \frac{T}{n h_B} \frac{\partial P}{\partial x_3} = \kappa^B \left[ \partial_3 T - \frac{T}{n h_B} \partial_3 P \right].
\] (36)

Here \( h^B = (\varepsilon + P)/n \) denotes the enthalpy per particle in the presence of strong magnetic field. In this regime, the particle number density \( (n) \), the energy density \( (\varepsilon) \) and the pressure \( (P) \) are obtained from the following particle flow four-vector and energy-momentum
tensor,
\[ N^\mu = \sum_i g_i |q_i B| \frac{1}{4\pi^2} \int \frac{dp}{\omega_i} \frac{\partial \tilde{p}}{\partial \tilde{x}^\mu} \left[ f_i(\tilde{x}, \tilde{p}) + \tilde{f}_i(\tilde{x}, \tilde{p}) \right], \quad (37) \]
\[ T^{\mu\nu} = \sum_i g_i |q_i B| \frac{1}{4\pi^2} \int \frac{dp}{\omega_i} \frac{\partial \tilde{p} \tilde{p}^\nu}{\partial \tilde{p}^\mu} \left[ f_i(\tilde{x}, \tilde{p}) + \tilde{f}_i(\tilde{x}, \tilde{p}) \right], \quad (38) \]
respectively. The RBTE (16), in terms of the gradients of flow velocity and temperature is written as
\[
\tilde{p}^\mu \frac{\partial T}{\partial \tilde{x}^\nu} \frac{\partial f_i}{\partial T} + \tilde{p}^\nu \frac{\partial (\tilde{p} \tilde{u}^\nu)}{\partial \tilde{x}^\mu} \frac{\partial f_i}{\partial \tilde{p}^\nu} + q_i \left[ F_{\nu}^{\mu3} \frac{\partial f_i}{\partial \tilde{p}^\nu} + F_{\nu}^{\mu0} \frac{\partial f_i}{\partial \tilde{p}^\nu} \right] = -\tilde{p}^\nu \tilde{\nu} f_i, \quad (39)
\]
where \( \tilde{p}^\mu = (p^0, 0, 0, \tilde{p}^3) \) and \( \tilde{x}^\mu = (x^0, 0, 0, x^3) \) are applicable for the calculation in strong magnetic field limit. Now using the following partial derivatives,
\[
\frac{\partial f_i^B}{\partial T} = \frac{p_0}{T^2} f_i^B (1 - f_i^B), \quad (40)
\frac{\partial f_i^B}{\partial \tilde{p}^\nu} = -\frac{1}{T} f_i^B (1 - f_i^B), \quad (41)
\frac{\partial f_i^B}{\partial \tilde{p}^3} = -\frac{p^3}{T p_0} f_i^B (1 - f_i^B), \quad (42)
\]
we get \( \delta f_i \) from eq. (39) as
\[
\delta f_i = -\frac{\tau_i^B f_i^B (1 - f_i^B)}{T} \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p_0 - \tilde{h}_i^B}{p_0} \right) \frac{p^3}{T} \left( \partial_3 T - \frac{T}{n \tilde{h}_i^B} \partial_3 P \right) + T \partial_0 \left( \frac{\mu}{T} \right) \right] - \frac{\tilde{p}^3 \tilde{\nu}}{p_0} \partial_3 u_\nu - \frac{2 q_i}{p_0} E_3 p_3. \quad (43)
\]
Similarly \( \delta \tilde{f}_i \) is evaluated as
\[
\delta \tilde{f}_i = -\frac{\tau_i^B \tilde{f}_i^B (1 - \tilde{f}_i^B)}{T} \left[ \frac{p_0}{T} \partial_0 T + \left( \frac{p_0 - \tilde{h}_i^B}{p_0} \right) \frac{p^3}{T} \left( \partial_3 T - \frac{T}{n \tilde{h}_i^B} \partial_3 P \right) - T \partial_0 \left( \frac{\mu}{T} \right) \right] - \frac{\tilde{p}^3 \tilde{\nu}}{p_0} \partial_3 u_\nu - \frac{2 \tilde{q}_i}{p_0} E_3 p_3. \quad (44)
\]
Substituting \( \delta f_i \) and \( \delta \tilde{f}_i \) in eq. (35) and then comparing with eq. (36), we get the thermal conductivity in the presence of strong magnetic field at finite chemical potential as
\[
\kappa^B = \frac{\beta^2}{4\pi^2} \sum_i g_i |q_i B| \int \frac{dp}{\omega_i} P_i f_i^B \left[ (\omega_1 - \tilde{h}_i^B)^2 f_i^B (1 - f_i^B) + (\omega_1 - \tilde{h}_i^B)^2 \tilde{f}_i^B (1 - \tilde{f}_i^B) \right]. \quad (45)
\]

4 Applications

This section is devoted to study some applications of the electrical and thermal conductivities. In subsection 4.1, we will observe the local equilibrium property of the medium.
through the Knudsen number in the presence of both strong magnetic field and finite
chemical potential. In subsection 4.2, we will observe the relative behavior between the
electrical conductivity and the thermal conductivity through the Wiedemann-Franz law
for a thermal QCD medium in the aforesaid regime.

4.1 Knudsen number

The local equilibrium of a medium can be understood through the Knudsen number and
it is defined by the ratio of mean free path ($\lambda$) to the characteristic length scale ($l$) of the
medium,

$$\Omega = \frac{\lambda}{l}.$$  \hspace{1cm} (46)

For the equilibrium hydrodynamics to be valid, the Knudsen number needs to be smaller
than 1 or the mean free path requires to be smaller than the characteristic length scale
of the medium. The mean free path is in turn related to the thermal conductivity ($\kappa$) by
the following relation,

$$\lambda = \frac{3\kappa}{vC_V},$$  \hspace{1cm} (47)

where $C_V$ and $v$ are the specific heat at constant volume and the relative speed, respectiv-
ely. So, $\Omega$ is now written as

$$\Omega = \frac{3\kappa}{lvC_V}.$$  \hspace{1cm} (48)

In our calculation, we have set $v \simeq 1$, $l = 4$ fm, and $C_V$ is determined from the energy-
momentum tensor as $C_V = \partial(u_\mu T^{\mu\nu} u_\nu)/\partial T$.

4.2 Wiedemann-Franz law

The Wiedemann-Franz law helps us to understand the relation between the charge trans-
port and the thermal transport in a system. This law states that the ratio of charged
particle contribution of the thermal conductivity ($\kappa$) to the electrical conductivity ($\sigma_{el}$) is
equal to the product of Lorenz number ($L$) and temperature,

$$\frac{\kappa}{\sigma_{el}} = LT.$$  \hspace{1cm} (49)
The metals which are good thermal and electrical conductors, perfectly satisfy this law. So, $\kappa/\sigma_d$ has approximately the same value for different metals at the same temperature, i.e. Lorenz number ($\kappa/(\sigma_d T)$) remains the same. Our observation on the Wiedemann-Franz law for the hot QCD matter in the presence of strong magnetic field and finite chemical potential with the quasiparticle masses has been described in section 6.

5 Quasiparticle model in the presence of strong magnetic field and finite chemical potential

In a thermal medium, the particles generally acquire thermally generated masses, known as quasiparticle masses. This mass arises due to the interaction of the particle with the other particles of the medium. Thus, in the quasiparticle model, QGP is treated as a system containing massive noninteracting quasiparticles. For a pure thermal medium, quasiparticle mass is temperature dependent, whereas for a thermal medium in the presence of strong magnetic field and chemical potential ($\mu$), quasiparticle mass becomes temperature, magnetic field and chemical potential dependent. The quasiparticle mass has been derived previously in different approaches, viz., the Nambu-Jona-Lasinio (NJL) and Polyakov NJL based quasiparticle models [49–51], quasiparticle model with Gribov-Zwanziger quantization [52, 53], thermodynamically consistent quasiparticle model [54] etc. In a pure thermal medium, the effective mass (squared) of quark of $i$th flavor at finite $\mu$ is given [55, 56] by

$$m_{qT}^2 = \frac{g'^2 T^2}{6} \left( 1 + \frac{\mu^2}{\pi^2 T^2} \right),$$

where $g'$ is the running coupling at finite temperature, finite chemical potential and zero magnetic field. In an ambience of strong magnetic field, the effective mass of quark can be determined from the effective quark propagator in the $p_0 = 0, p_z \to 0$ limit. The effective quark propagator is evaluated from the self-consistent Schwinger-Dyson equation in a strong magnetic field,

$$S^{-1}(p_\parallel) = \gamma^\mu p_\parallel \mu - \Sigma(p_\parallel).$$
Thus, one requires to calculate the quark self-energy in the strong magnetic field regime, which is given by

\[ \Sigma(p) = -\frac{4}{3} g^2 i \int \frac{d^4 k}{(2\pi)^4} [\gamma_\mu S(k) \gamma^\mu D(p - k)] , \]  

(52)

where \( g \) is the running coupling in the presence of a strong magnetic field [57–60]. The quark propagator \( S(k) \) in vacuum in the strong magnetic field limit is given [61, 62] by the Schwinger proper-time method in momentum space as

\[ S(k) = ie^{-\frac{i k^2}{4m^2}} \frac{(\gamma^0 k_0 - \gamma^3 k_z + m_i)}{k^2 - m_i^2} (1 - \gamma^0 \gamma^3 \gamma^5) , \]

(53)

where the following representations of the metric tensors and four vectors have been used,

\[ g_{\perp \mu} = \text{diag}(0, -1, -1, 0), \quad g_{|| \mu} = \text{diag}(1, 0, 0, -1), \]

\[ k_{\perp \mu} \equiv (0, k_x, k_y, 0), \quad k_{|| \mu} \equiv (k_0, 0, 0, k_z) . \]

Since gluon is an electrically neutral particle, its propagator in vacuum remains unaffected by the magnetic field and retains the form same as in the absence of magnetic field,

\[ D^{\mu\nu}(p - k) = \frac{ig^{\mu\nu}}{(p - k)^2} . \]

(54)

Substituting the quark and gluon propagators in eq. (52), the quark self-energy is determined using imaginary-time formalism in the presence of strong magnetic field at finite chemical potential, where we have replaced the energy integral (\( \int \frac{dm}{2\pi} \)) by sum over Matsubara frequencies and written the integration over the transverse component of the momentum in terms of \( |q_i B| \). Thus, the self-energy (52) gets simplified into

\[ \Sigma(p_{||}) = \frac{2g^2 |q_i B| T}{3\pi^2} \sum_n dk_z \left[ \frac{(1 + \gamma^0 \gamma^3 \gamma^5) (\gamma^0 k_0 - \gamma^3 k_z) - 2m_i}{[k_0^2 - \omega_k^2][ (p_0 - k_0)^2 - \omega_{pk}^2]} \right] \]

\[ = \frac{2g^2 |q_i B|}{3\pi^2} T \sum_n dk_z \left[ (\gamma^0 + \gamma^3 \gamma^5)W^1 - (\gamma^3 + \gamma^0 \gamma^5)k_z W^2 \right] , \]

(55)

where \( \omega_k^2 = k_z^2 + m_i^2, \omega_{pk}^2 = (p_z - k_z)^2 \) and \( W^1 \) and \( W^2 \) are the two frequency sums, which are given by

\[ W^1 = T \sum_n \frac{k_0}{[k_0^2 - \omega_k^2][ (p_0 - k_0)^2 - \omega_{pk}^2]} , \]

(56)

\[ W^2 = T \sum_n \frac{1}{[k_0^2 - \omega_k^2][ (p_0 - k_0)^2 - \omega_{pk}^2]} . \]

(57)
After calculating the above frequency sums and then substituting, we get the simplified form of the quark self-energy \((55)\) as

\[
\Sigma(p_{||}) = \frac{g^2|q_{B}|}{3\pi^2} \int \frac{dk_z}{\omega_k} \left[ \frac{1}{e^{\omega_k} - 1} + \frac{1}{2} \left\{ \frac{1}{e^{\omega_k + \mu}} - 1 + \frac{1}{e^{\omega_k - \mu}} + 1 \right\} \right] \times \left[ \frac{\gamma^0 p_0 + \gamma^3 p_z}{p^2_{||}} + \frac{\gamma^0 \gamma^5 p_z}{p^2_{||}} + \frac{\gamma^3 \gamma^5 p_0}{p^2_{||}} \right], 
\]

which after integration over \(k_z\) takes the following form,

\[
\Sigma(p_{||}) = \frac{g^2|q_{B}|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] \times \left[ \frac{\gamma^0 p_0}{p^2_{||}} + \frac{\gamma^3 p_z}{p^2_{||}} + \frac{\gamma^0 \gamma^5 p_z}{p^2_{||}} + \frac{\gamma^3 \gamma^5 p_0}{p^2_{||}} \right],
\]

where \(\zeta(3)\) and \(\zeta(5)\) are the Riemann zeta functions.

The general covariant structure of the quark self-energy at finite temperature and magnetic field can be written \([12, 60]\) as

\[
\Sigma(p_{||}) = A\gamma^\mu u_\mu + B\gamma^\mu b_\mu + C\gamma^5\gamma^\mu u_\mu + D\gamma^5\gamma^\mu b_\mu,
\]

where \(A\), \(B\), \(C\) and \(D\) represent the form factors, \(u^\mu (1,0,0,0)\) denotes the preferred direction of heat bath which breaks the Lorentz symmetry and \(b^\mu (0,0,0,-1)\) denotes the preferred direction of magnetic field which breaks the rotational symmetry. The form factors are evaluated as

\[
A = \frac{1}{4} \text{Tr} \left[ \Sigma^{\mu} u_\mu \right] = \frac{g^2|q_{B}|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] \frac{p_0}{p^2_{||}}, \quad (61)
\]

\[
B = -\frac{1}{4} \text{Tr} \left[ \Sigma^{\mu} b_\mu \right] = \frac{g^2|q_{B}|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] \frac{p_z}{p^2_{||}}, \quad (62)
\]

\[
C = \frac{1}{4} \text{Tr} \left[ \gamma^5 \Sigma^{\mu} u_\mu \right] = -\frac{g^2|q_{B}|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] \frac{p_z}{p^2_{||}}, \quad (63)
\]

\[
D = -\frac{1}{4} \text{Tr} \left[ \gamma^5 \Sigma^{\mu} b_\mu \right] = \frac{g^2|q_{B}|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] \frac{p_0}{p^2_{||}}, \quad (64)
\]

where \(C = -B\) and \(D = -A\). In terms of the right-handed \((P_R = (1 + \gamma^5)/2)\) and left-handed \((P_L = (1 - \gamma^5)/2)\) chiral projection operators, the quark self-energy \((60)\) is written as

\[
\Sigma(p_{||}) = P_R \left[ (A + C)\gamma^\mu u_\mu + (B + D)\gamma^\mu b_\mu \right] P_R + P_L \left[ (A - C)\gamma^\mu u_\mu + (B - D)\gamma^\mu b_\mu \right] P_L,
\]

(65)
which after the substitutions $C = -B$ and $D = -A$ gets simplified into
\[ \Sigma(p_\parallel) = P_R [(A - B)\gamma^\mu u_\mu + (B - A)\gamma^\mu b_\mu] P_L + P_L [(A + B)\gamma^\mu u_\mu + (B + A)\gamma^\mu b_\mu] P_R . \]  
(66)

Now with the quark self-energy (66), the self-consistent Schwinger-Dyson equation in the presence of a strong magnetic field is written as
\[ S^{-1}(p_\parallel) = \gamma^\mu p_{\parallel\mu} - \Sigma(p_\parallel) = P_R\gamma^\mu X_\mu P_L + P_L\gamma^\mu Y_\mu P_R , \]  
(67)

where
\[ \gamma^\mu X_\mu = \gamma^\mu p_{\parallel\mu} - (A - B)\gamma^\mu u_\mu - (B - A)\gamma^\mu b_\mu , \]  
(68)
\[ \gamma^\mu Y_\mu = \gamma^\mu p_{\parallel\mu} - (A + B)\gamma^\mu u_\mu - (B + A)\gamma^\mu b_\mu . \]  
(69)

Thus we get the effective propagator as
\[ S(p_\parallel) = \frac{1}{2} \left[ P_R\gamma^\mu Y_\mu P_L + P_L\gamma^\mu X_\mu P_R \right] , \]  
(70)

where
\[ \frac{X^2}{2} = X_1^2 = \frac{1}{2} [p_0 - (A - B)]^2 - \frac{1}{2} [p_z + (B - A)]^2 , \]  
(71)
\[ \frac{Y^2}{2} = Y_1^2 = \frac{1}{2} [p_0 - (A + B)]^2 - \frac{1}{2} [p_z + (B + A)]^2 . \]  
(72)

The thermal mass (squared) at finite temperature and finite chemical potential in the presence of strong magnetic field is finally calculated by taking the \( p_0 = 0, p_z \to 0 \) limit of either \( X_1^2 \) or \( Y_1^2 \) (both of them are equal in this limit) as
\[ m^2_{T,B} = X_1^2 \bigg|_{p_0=0,p_z\to0} = Y_1^2 \bigg|_{p_0=0,p_z\to0} = \frac{g^2|qB|}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4} \right] , \]  
(73)

which depends on temperature, chemical potential and magnetic field.

In the calculation, we have chosen a specific range of temperature and magnetic field in such a way that the condition of strong magnetic field limit (\( eB \gg T^2 \)) is satisfied. Thus, we have set the magnetic field at 15 \( m_n^2 \) and the temperature in the range 0.16 GeV - 0.4 GeV. In addition, we have used chemical potential, \( \mu = 0.06 \) GeV, which is smaller than temperature and magnetic field. In the next section, we are going to discuss the results in quasiparticle model by using the temperature and chemical potential-dependent quasiparticle mass (50) for the isotropic dense thermal medium, and temperature, chemical potential and magnetic field-dependent quasiparticle mass (73) for the dense thermal medium in the presence of strong magnetic field.
6 Results and discussions

From figure 1, we have noticed an overall increase of electrical conductivity in the presence of strong magnetic field, whereas with the rising temperature it shows decreasing trend as opposed to the increasing trend in the isotropic medium in the absence of magnetic field. We have also observed that the finite chemical potential enhances the \( \sigma_{el} \) both in the absence of magnetic field (figure 1a) and in the presence of strong magnetic field (figure 1b). This increase in the value of \( \sigma_{el} \) due to finite \( \mu \) is more noticeable in the latter case. This difference in the behavior of electrical conductivity in the absence and presence of strong magnetic field and chemical potential is mainly due to the differences in the distribution function, the relaxation time and the dispersion relation in the absence and presence of magnetic field and chemical potential.

Figure 2 shows the variation of thermal conductivity with temperature in the absence and in the presence of strong magnetic field and chemical potential. The presence of strong magnetic field makes \( \kappa \) increased as compared to the zero magnetic field case and it shows slow increasing trend with temperature (figure 2b), contrary to its rapid increasing trend in the isotropic medium in the absence of magnetic field (figure 2a). The existence of finite chemical potential rises the value of \( \kappa \) and this rise is more pronounced in the presence of strong magnetic field. This difference in the behavior of thermal conductivity in the
absence and presence of strong magnetic field and chemical potential is attributed to the differences in the distribution function, the relaxation time and the dispersion relation in the absence and presence of magnetic field and chemical potential.

From figure 3 we have found that, in the presence of strong magnetic field, the Knudsen number becomes larger than its value in the isotropic medium with no magnetic field. It is also noticed that, in the additional presence of \( \mu \), the Knudsen number becomes further increased and approaches 1 near \( T_c = 0.16 \) GeV (figure 3b), i.e the mean free path gets closer to the size of the system, unlike the isotropic case in the absence of magnetic field where although \( \Omega \) gets risen at finite \( \mu \), but it remains much lower than one (figure 3a). Thus, the medium moves slightly away from the local equilibrium at finite chemical potential in an ambience of strong magnetic field.

Figure 4 depicts the variations of the Lorenz number (\( L \)), i.e. \( \kappa/(\sigma_{el} T) \) with temperature for isotropic medium in the absence of magnetic field and for the medium in the presence of strong magnetic field. We have noticed a decrease in the magnitude of \( L \) due to the strong magnetic field (figure 4b) as compared to the zero magnetic field case (figure 4a). In both the cases, finite chemical potential further reduces the magnitude of \( L \). In all cases, with the increase of temperature, the Lorenz number does not remain constant, rather it increases. Therefore, it indicates the violation of Wiedemann-Franz law for a
Figure 3: Variations of the Knudsen number with temperature in the (a) absence and (b) presence of strong magnetic field.

Figure 4: Variations of the Lorenz number with temperature in the (a) absence and (b) presence of strong magnetic field.
hot and dense QCD matter in the presence of strong magnetic field.

7 Conclusions

In this work, we have studied how the strong magnetic field affects the charge and thermal transport properties of the hot QCD matter at finite chemical potential and observed the deviations from their isotropic counterparts in the isotropic medium in the absence of magnetic field. In calculating the electrical and thermal conductivities we have followed the kinetic theory approach in the relaxation time approximation, where the interactions are incorporated through the effective masses of particles at finite temperature, chemical potential and strong magnetic field in quasiparticle model. After revisiting the calculations of electrical and thermal conductivities for the isotropic dense medium, we determined these conductivities in the presence of strong magnetic field. We have observed that the values of electrical and thermal conductivities get increased in the presence of strong magnetic field in comparison to those in the isotropic medium at zero magnetic field, and the additional presence of chemical potential further increases their values. In applications of the aforesaid conductivities, we have investigated the local equilibrium property through the Knudsen number and the relative behavior between thermal conductivity and electrical conductivity through the Lorenz number in Wiedemann-Franz law in the presence of strong magnetic field and finite chemical potential. We have observed that the Knudsen number in the presence of strong magnetic field and finite chemical potential gets increased in comparison to the isotropic one at zero magnetic field and approaches 1 around $T_c = 0.16$ GeV, but at higher temperatures, it becomes less than 1. Thus around $T_c$, the mean free path gets closer to the characteristic length scale of the system and the medium may move slightly away from the local equilibrium state, whereas at higher temperatures, the medium returns back to its equilibrium state. In the Wiedemann-Franz law, the Lorenz number $\left(\kappa/(\sigma_{el}T)\right)$ in the absence and also in the presence of strong magnetic field and chemical potential does not remain constant, rather it increases with the rise of temperature. So, this behavior of the Lorenz number indicates the violation of the Wiedemann-Franz law for a hot and dense QCD matter in the presence of strong magnetic field.
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