Effect of varying road profile amplitude on the behavior of a nonlinear quarter car model.

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Abstract
Vehicle suspensions are one of the most important equipment in an automobile which impart comfortable ride to the passengers. As the passive suspensions are most widely used in the automobiles, therefore this paper deals with the investigations of the suspension behavior at the different values of the road profile amplitude. The effects on the frequency plots are studied by using fast Fourier transform functions. The phase plots are also drawn to understand the periodic and quasi periodic transformations at the different values of road profile amplitude. It is seen that the natural frequency and its harmonics in the spectrum dies out with the rise in the values of the road profile amplitude and also the increase in the strength of forcing is observed. The in harmonics obtained is seen to be withdrawn with the increase in the road profile amplitude.

1. Introduction
Providing comfortable ride to passengers is one of the vital tasks handled by the automobile suspensions by eliminating all types of disturbances issuing from the road perturbations and bumps. Suspensions can be active or passive; generally active suspensions are much expensive due to their complexities but are more effective because of their faster and better response to road disturbances. Most commonly used are the passive suspensions as they are less costlier and more reliable due to their simplicity, therefore, a study has been conducted for a passive suspension and also the behavior of the suspension is investigated at different values of the road profile amplitude with the help of phase plots and fast fourier transform functions.

Mahajan et al. [1] in their study determined the performance of the suspension system for sine input with variable frequencies and had shown that the increase in the frequency of the sine wave will result in the decrease in the settling time of sprung mass acceleration which will directly result in the effect the passenger ride discomfort. Fakhraei et al. [2] examined the sinusoidal road input in the form of speed bump on a non-linear system to study the effect of chaotic vibration on the ride comfort of driver and also, the response of amplitude and frequency on the chaotic behavior are determined specifying the elevated intensity of vehicle vibration in the chaotic regions. Their results are helpful in the successful design of the dynamic vehicle suspensions free from chaotic vibrations thereby increasing the ride comfort of passengers and drivers. Fakhraei et al. [3] in their study had shown that the chaotic vibration nature in the suspensions can be downsized by selecting the pertinent values of the suspension parameters. Chatterjee. A. [4] discussed the various methods to study and analyze the behavior of non linear dynamic systems. He showed that the feedback of the non linear system involve the frequencies other than the forcing frequency which are the multiples of this forcing frequency. Their study can be used to prove the non linear behavior of the vehicle suspensions.

Li et. al. [5] examined the vibrations of the quarter car model at the different values of the amplitudes concentrating on the synchronized, quasi periodic and chaotic motion. They founded the critical value of
the amplitude at which the behavior of the system changes to chaotic which can be lowered by decreasing the value of the damping coefficient. Their study can be used to determine the quasi periodic or chaotic regions for the quarter car vibrations which can be used further to improve the ride quality of the suspension systems.

Zhou et al. [6] Analyzed the non linear dynamics of the single degree of freedom (SDOF) quarter car model and examined the effect of excitation amplitude on the dynamic response of the system. They examined the behavior at the different values of the frequency and the higher values of the excitation amplitude. Their study shows that the vibration amplitude and the peak resonance increase substantially with the greater values of the excitation amplitude.

Borowiec et al. [7] Analyzes chaotic vibrations and bifurcations by altering the road profile amplitude and excitation frequency. Their study reveals that with the increase in the excitation amplitude, behavior of the quarter car drifts towards chaotic and quasi periodic regions starts to appear. Their research is used to analyze the regions at which chaos would increase in the quarter car model and the parameters are to be assorted so that lesser chaotic vibrations are obtained which would result in better ride comfort of passengers.

This paper extends this work [5] and [7] and the frequency curves are studied at the lower values of the road profile amplitudes. The behavior of the SDOF quarter car model is analyzed and the values of amplitudes are determined at which the quasi periodic motion transforms to periodic motion resulting in the better design of the vehicle suspension system with the help of the phase plot diagrams.

2. Methodology
The motion of single degree of freedom quarter car model is governed by a second order differential equation as given below.

$$\frac{d^{2} y}{dt^{2}} + \omega^{2} y + B_{1}y^{3} + B_{2} \frac{dy}{dt} + B_{3} \left( \frac{dy}{dt} \right)^{3} = -g + A\Omega^{2} \sin(\Omega t),$$

where $y$ is the relative displacement of the mass from the road, $\omega^{2}$, $B_{1}$, $B_{2}$ and $B_{3}$ are $k_{1}/m$, $k_{2}/m$, $c_{1}/m$ and $c_{2}/m$ respectively. $k_{i}$ and $c_{i}$ $(i=1,2)$ are the linear and nonlinear coefficients of the spring and damping respectively. This system was proposed by Li et al. [5] and further investigated by Litak et al. [10] and Borowiec et al. [7]. The suspension system was excited harmonically with a road profile containing a single frequency, as shown in figure 1. The system may oscillate at a single frequency (similar to a linear system) or it may undergo quasi periodic or chaotic oscillations depending upon the excitation frequency and the parameter values. Sinusoidal analysis is done so that the effect of non linearity in suspension elements can easily be observed. Since sinusoidal excitation is a signal that contains a single frequency, the analysis done using this can be easily conducted as well as revealing as compared to other non linear analysis.

Dynamics of a non linear system may be qualitatively understood by using the phase plots. Displacement of the sprung mass is plotted against velocity. Phase plot gives an idea of the complicacy of the motion of the sprung mass. Using the phase plot, the range of displacement and velocity and also the periodicity can be judged. Quasi periodic motions occur in case of two incommensurate frequencies. While chaotic oscillations are non repeating and are a periodic. Solutions of equation (1) were investigated numerically by using fourth order Runge Kutta method.
3. Results and discussions

Dynamical behavior of a nonlinear system can be effectively understood from the phase plot diagrams and fft of the time series. Figure 2 (a), (b) (c) shows the time series of relative displacement, phase plot diagrams and the corresponding spectra. The values of spring constant $k_1 = 160000 \, \text{N/m}$, $k_2 = 150000\,\text{N/m}$, damping constant $c_1 = -250 \, \text{Ns/m}$ and $c_2 = 25 \, \text{Ns}^3/\text{m}^3$ are used. These values are same as used in [7]. Values of excitation amplitude are varied and the results are plotted as shown below.

In Fig.2 excitation amplitude was selected as 0.07. It is observed that the relative displacement is having various closed profiles. The phase plot in fig. 2(a) is also shows the motion with multiple periodicity. Fig.2(c) is showing a weak sub harmonic and also the sharp peak at the forcing frequency and natural frequency ($\omega$) representing incommensurate frequencies. The complexity of the phase plot is due to these incommensurate frequencies. The inharmonics are also seen at frequency ratio below 1.4 and 2.8 indicating the non linear behavior of the system as in [4] (any departure from the ideal harmonic series is called inharmonicity). In addition second and third harmonic of the natural frequency can also be seen.
Figure 2. (a) shows the relative displacement as a function of time while (b) shows the phase plot and (c) shows the spectra corresponding to figure (a) for excitation amplitude of 0.07m.

In Fig. 3, excitation amplitude was increased to 0.11m, while keeping the system parameters same as before. In fig.3 (a) less disturbance is observed in the relative displacement profiles. The amplitude at forcing frequency and its first inharmonic is enhanced. A somewhat decrease is seen in the inharmonic at frequency ratio below 2.8. The first harmonic of the natural frequency at frequency ratio of 2 is not active as in the previous case. Multiple peaks as obtained imply the intensified non linear behaviour of the system.
Figure 3. (a) shows the relative displacement as a function of time while (b) shows the phase plot and (c) shows the spectra corresponding to figure (a) for excitation amplitude of 0.11 m.

In all Fig.4, excitation amplitude was further increased to 0.17 m. In fig. 4(a) near sinusoidal profiles of relative displacement are obtained representing a single dominant frequency at higher values of amplitude. In phase plot fig. 4(b), periodic motion can be seen with the same periodicity as that of the forcing frequency. In fig.4(c) it can be seen that the forcing frequency becomes even more dominant as compared to the previous two cases with its second and third harmonic present with the inharmonicity absent in this case along with the natural frequency and its harmonics.
4. Conclusion

Behavior of a nonlinear single degree of freedom quarter car model has been investigated by running it over a sinusoidal road profile. The model contains a nonlinear damper with linear and cubic terms. Following conclusions are derived based on the above results.

1. At lower values of the excitation amplitude, the forcing frequency along with the natural frequency mainly contribute to the motion of the sprung mass with the natural frequency dominating. With the increase in amplitude it can be seen that the contribution from the forcing frequency becomes stronger and eventually the natural frequency and its harmonics disappear from the spectrum.

2. This can be understood since at low amplitude of road profile the forcing is weaker and the natural frequency of the oscillator can dominate the output. Upon increasing the road profile amplitude the forcing becomes stronger and finally the output is completely determined by the forcing frequency. Due to low amplitude of road profile the system is not perturbed strongly, therefore the inherent dynamics of the oscillator can come into play and so the natural frequency dominates the output.

3. It should also be noted that the in harmonics are present only in the first two cases when both the forcing frequency and the natural frequency is active. It is suspected that these in harmonics may be due to the interference of the inherent dynamics of the system with the forcing.

4. Increase in the values of road profile amplitude as studied in this paper shows that the system becomes more stable at the amplitude of 0.17, hence resulting in the better ride comfort of the passenger and better vehicle suspension performance.

5. The results can be used in the better understanding of the non linear dynamics, ultimately leading to better designed vehicle suspension systems. Passenger ride comfort can be significantly enhanced if the values of amplitude are kept in the mentioned ranges as shown in the research carried out in this paper.

6. In future work the combined effect of frequency and amplitude variation on the quarter car can be carried out for providing better understanding of the non-linearity in suspension systems.

5. References

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