Intersecting Noncommutative M5-branes from Covariant Open Supermembrane

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Abstract

We study intersecting noncommutative (NC) M5-branes from $\kappa$-invariance of an open supermembrane action with constant three-form fluxes. The $\kappa$-invariance gives rise to possible D-brane configurations for which projection operators can be determined. We construct projection operators for two types of 1/4 BPS intersecting NC M5-branes. The one is an intersection of two NC M5-branes: NC $\text{M5}_{\perp} \text{NC M5}$ (3). The other is that of a NC M5-brane and a commutative (C) M5-brane: NC $\text{M5}_{\perp} \text{C M5}$ (1). A NC M5-brane can be viewed as a bound state of M5 and M2, and the configurations $\text{M2}_{\perp} \text{M5}$ (1) and $\text{M2}_{\perp} \text{M2}$ (0) are realized on the intersecting M5-branes. Taking a commutative limit the allowed intersecting M5-branes are surely reproduced: $\text{M5}_{\perp} \text{M5}$ (3) and $\text{M5}_{\perp} \text{M5}$ (1).
1 Introduction

Supermembrane theory in eleven dimensions [1, 2] is closely related to the M-theory formulation [3]. Membranes (M2-branes) are the fundamental objects and open membranes [4, 5] as well as closed ones can be considered. Open membranes can end on a p-dimensional hypersurface (Dirichlet brane) with $p = 1, 5$ and 9 [6, 7] just like an open string can attach to D-branes. The $p = 5$ case corresponds to M5-brane and the $p = 9$ is the end-of-world 9-brane appearing in the Horava-Witten theory [8]. Open M5-branes have been discussed in the recent work [9].

The Dirichlet branes can be investigated from the $\kappa$-symmetry argument [6]. This method is also applicable to string theory [11]. It is a covariant way and a specific gauge-fixing such as light-cone gauge is not necessary. Then it is sufficient to consider a single action of open string or open membrane, rather than each of D-brane actions. It is moreover easy to find what configurations are allowed to exist for rather complicated D-brane setups such as intersecting D-branes or less supersymmetric D-branes, which are difficult to discuss within a brane probe analysis. Finally the method is not restricted to a flat spacetime and can be generalized to curved backgrounds.

In this paper we discuss an application of the $\kappa$-symmetry argument to supersymmetric intersecting M-branes [15–17]. There is, however, an obvious obstacle that an open supermembrane can attach to M5-branes but not M2-branes due to the charge conservation law [4]. Thus it is an easy task to find intersecting M5-branes, but it would be more involved to consider intersecting configurations including M2-branes. A possible way to discuss M2 within the framework of this method is to consider an M5-brane with electric and magnetic fluxes, which is called noncommutative (NC) M5-brane [18, 19]. It can be viewed as a bound state of M5 and M2, and hence one may find a configuration of intersecting M-branes with M2 on the intersecting M5-branes.

By following our previous paper [20], we discuss two types of 1/4 BPS intersecting NC M5-branes: NC $\text{M5} \perp \text{NC M5}$ (3) and NC $\text{M5} \perp \text{C M5}$ (1). The allowed configurations $\text{M2} \perp \text{M2}$ (0) [15, 17] and $\text{M2} \perp \text{M5}$ (1) [15, 17] can be found on the intersecting M5-branes.

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1 M5-branes can be discussed from the superembedding method [10].

2 For Dirichlet branes of open supermembrane in $\text{AdS}_{4/7} \times S^{7/4}$ and pp-wave see [12]. For applications to D-branes in $\text{AdS}_5 \times S^5$ and pp-wave, see [13]. For the results of the brane probe, see [14].
The supergravity solution corresponding to NC M5⊥NC M5 (3) has been constructed [21]. These are the possible configurations of intersecting M-branes (For a review of intersecting D-branes and M-branes without fluxes, see [22]). Taking a commutative limit leads to the possible configuration of intersecting M5-branes: M5⊥M5 (3) [15–17] and M5⊥M5 [17].

This paper is organized as follows. In section 2 we introduce the covariant Green-Schwarz (GS) action of an open supermembrane in flat space with constant three-form and derive the surface terms coming from the κ-variation of the Wess-Zumino term in the action. In section 3 we elaborate NC M5-branes and strong flux limit of them. This section is basically an exposition of [20]. In section 4 two types of intersecting NC M5-brane configurations are constructed. The configurations, which include M2-branes, M2⊥M5 (1) and M2⊥M2 (0) are found on the intersecting M5-branes. The projection operators for them are found by considering a strong flux limit and we can check that the surface terms from the κ-variation surely vanish. Section 5 is devoted to a summary and discussions.

2 Open supermembrane and κ-symmetry

The GS action of a supermembrane in flat spacetime is composed of the Nambu-Goto (NG) part and the Wess-Zumino (WZ) part [1]

\[ S = \int \Sigma d^3 \sigma [\mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZ}}] . \] (2.1)

The NG part is given by

\[ \mathcal{L}_{\text{NG}} = -\sqrt{-g(X, \theta)} , \quad g_{ij} = E_i^A E_j^B \eta_{AB} , \quad E_i^A = \partial_i X^A - i \bar{\theta} \Gamma^A \partial_i \theta . \]

The WZ part is given by

\[
\mathcal{L}_{\text{WZ}} = \epsilon^{ijk} \left[ \frac{1}{6} e_i^A e_j^B e_k^C \mathcal{H}_{ABC} + \frac{i}{2} \bar{\theta} \Gamma_{AB} \partial_i \theta \partial_j X^M e_M^A \partial_k X^N e_N^B + \frac{1}{2} \bar{\theta} \Gamma_{AB} \partial_i \theta \bar{\theta} \Gamma^A \partial_j \theta \partial_k X^M e_M^B - \frac{i}{6} \bar{\theta} \Gamma_{AB} \partial_i \theta \bar{\theta} \Gamma^A \partial_j \theta \bar{\theta} \Gamma^B \partial_k \theta \right].
\]

This part includes a coupling term to a three-form field \( \mathcal{H} = C - db \), where \( C \) and \( b \) are a three-form gauge potential and a two-form gauge potential on the brane, respectively.

The total action (2.1) is invariant under the κ-variation

\[ \delta_\kappa X^M e_M^A = -i \bar{\theta} \Gamma^A \delta_\kappa \theta . \]
This local fermionic symmetry ensures the consistency of the theory. This symmetry is obviously maintained considering a closed supermembrane. However our current interest is an open supermembrane and for the action to be $\kappa$-symmetric we need to impose some appropriate boundary conditions. These conditions should describe Dirichlet branes of open supermembrane.

First of all, let us see boundary conditions for the bosonic coordinates [19]. Suppose a $p$-dimensional hypersurface as a Dirichlet brane of an open supermembrane. When a constant $\mathcal{H}$ is turned on along the Dirichlet $p$-brane worldvolume $\Sigma$, they should satisfy either of the following boundary conditions at a boundary of the world volume $\partial \Sigma$,

$$\begin{align*}
\partial_n X^A + \mathcal{H}^{A}_{BC} \partial_\tau X^B \partial_t X^C &= 0, & \bar{A}_a(a = 0, \cdots, p) \in \text{Neumann} \\
\partial_\tau X^A = \partial_t X^A &= 0, & \bar{A}_a(a = p + 1, \cdots, 10) \in \text{Dirichlet},
\end{align*}$$

where $n$ is a normal direction to $\partial \Sigma$, and $\tau$ and $t$ denote tangential ones. In a large $\mathcal{H}$ limit the Neumann directions for which fluxes are turned on are frozen and replaced by Dirichlet ones.

Then let us discuss boundary conditions for the fermionic variables $\theta$, which is sensitive to how much supersymmetries can be maintained. For the Dirichlet brane to be supersymmetric, “gluing conditions” should be imposed on $\theta$ by constructing projection operators. The main subject is how to construct the projection operators for supersymmetric Dirichlet branes. The projection operators can be determined from the requirement that the surface terms coming from the $\kappa$-variation should vanish. No surface term appears when a membrane is closed. However we are now considering an open membrane and hence surface terms may appear and must be deleted in order to ensure the consistency of the theory.

Since the bulk action admits $\kappa$-symmetry, the $\kappa$-variation of the action $\delta_\kappa S$ leaves only surface terms. The NG part does not give rise to any surface terms. Thus it is sufficient

3In the large $\mathcal{H}$ limit, say $\mathcal{H}_{\bar{A}_0, \bar{A}_1, \bar{A}_2} \to \infty$, the Neumann directions $\{\bar{A}_0, \bar{A}_1, \bar{A}_2\}$ reduce into $\{\bar{A}_0 \in D \cup \bar{A}_1 \in D\} \cap \{\bar{A}_1 \in D \cup \bar{A}_2 \in D\} \cap \{\bar{A}_2 \in D \cup \bar{A}_0 \in D\}$, where $\bar{A}_0 \in D$ denotes that $\bar{A}_0$ is Dirichlet. For example, this is satisfied $\{\bar{A}_1 \in D \cap \bar{A}_2 \in D\}$. Thus taking this limit a few Neumann directions are frozen and the value $p$ reduces to $(p - 2)$ or $(p - 3)$.
to examine the \( \kappa \)-variation of the WZ part,

\[
\delta_\kappa S_{\text{WZ}} = \int_{\partial \Sigma} d^2 \xi \left[ L^{(2)} + L^{(4)} + L^{(6)} \right],
\]

\[
L^{(2)} = -i \left[ \bar{\theta} \Gamma_{\bar{A}\bar{B}} \delta_\kappa \theta + \mathcal{H}_{\bar{A}\bar{B}C} \bar{\theta} \Gamma^C \delta_\kappa \theta \right] \dot{X}^A X'^B,
\]

\[
L^{(4)} = \left[ -\frac{3}{2} \bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_{\bar{A}\bar{B}} + \frac{1}{2} \bar{\theta} \Gamma_{\bar{A}\bar{B}D} \delta_\kappa \theta \bar{\theta} \Gamma^D \right] (\theta' \dot{X}^B - \dot{\theta} X'B),
\]

\[
L^{(6)} = \frac{i}{6} \left[ \bar{\theta} \Gamma_{\bar{A}\bar{B}} \dot{\theta} \Gamma^A \theta' \bar{\theta} \Gamma^B \delta_\kappa \theta - \bar{\theta} \Gamma_{\bar{A}\bar{B}} \theta' \bar{\theta} \Gamma^A \dot{\theta} \bar{\theta} \Gamma^B \delta_\kappa \theta
\]

\[- \frac{2}{3} \bar{\theta} \Gamma_{\bar{A}\bar{B}D} \delta_\kappa \theta \bar{\theta} \Gamma^D \right] \theta'(\dot{X}^B - \dot{\theta} X'B),
\]

where \( \dot{Z} = \partial_\tau Z \) and \( Z' = \partial_t Z \). \( L^{(n)} \) represents the term with the \( n \)-th order of \( \theta \).

It has been shown in [20] that \( L^{(6)} \) in (2.4) vanishes due to the Fierz identity

\[
(CT_{\alpha\beta})(\gamma\delta) = 0.
\]

For the configuration that satisfies that \( L^{(2)} = 0 \), the surface terms in (2.3) are rewritten with (2.5) as

\[
L^{(4)} = -\frac{1}{2} \left[ \bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_{\bar{A}\bar{B}} + \bar{\theta} \Gamma_{\bar{A}\bar{B}D} \delta_\kappa \theta \bar{\theta} \Gamma^D \right] (\theta' \dot{X}^B - \dot{\theta} X'B).
\]

Therefore the problem of finding possible Dirichlet branes is boiled down to constructing the projection operators to make (2.2) and (2.6) vanish.

In the next section we will construct projection operators for NC M5-branes.

### 3 NC M5-brane and strong flux limit

Let us elaborate NC M5-branes as Dirichlet branes of an open supermembrane. This section is an exposition of [20], but more careful derivation will be presented. It would also be helpful to make the manuscript self-contained.

The construction is slightly modified in the presence of the flux. We shall first re-member the case without flux for simplicity. Then we construct projection operators for a NC M5-brane. It is regarded as a bound state of M5 and M2 and in a strong flux limit, infinitely many M2-branes are dissolved on the M5-brane.
3.1 Dirichlet branes without flux

Let us concentrate here on 1/2 BPS Dirichlet branes without flux. By using a gluing matrix $M$, which consists of a product of gamma matrices, the gluing condition is written as

$$\theta = M \theta, \quad M = \ell \Gamma^{\hat{A}_0 \cdots \hat{A}_p}, \quad (\ell^2 (-1)^{\frac{p+1}{2}}) s = 1. \quad (3.1)$$

Here $s = -1$ when $0 \in \{ \hat{A}_0, \hat{A}_1, \cdots, \hat{A}_p \}$ and $s = 1$ otherwise. It can be easily seen that $M$ satisfies the following relations,

$$M^2 = 1, \quad \bar{\theta} = \bar{\theta} M', \quad M' = (-1)^{p+1+\frac{p+1}{2}} M.$$  

First of all, we find a condition under which (2.2) should vanish. It is an easy task to show the following identities,

$$\bar{\theta} \Gamma_{AB} \delta_\kappa \theta = \frac{1}{2} \bar{\theta} (M' \Gamma_{AB} + \Gamma_{AB} M) \delta_\kappa \theta = 0 \quad \text{for } p = 1, 4 \text{ mod } 4,$$

$$\bar{\theta} \Gamma_{\bar{C}} \delta_\kappa \theta = 0 \quad \text{for } p = 3, 4 \text{ mod } 4.$$  

Hence (2.2) vanishes when $p = 1, 4 \text{ mod } 4, \mathcal{H} \equiv 0$. Then let us consider a condition to delete (2.6). Noting that

$$\bar{\theta} \Gamma_{\hat{A}_0 \hat{A}_1 \cdots \hat{A}_5} \delta_\kappa \theta = 0 \quad \text{for } p = 2, 3 \text{ mod } 4,$$

$$\bar{\theta} \Gamma_{\hat{A}_3 \hat{A}_4 \hat{A}_5} \delta_\kappa \theta = 0 \quad \text{for } p = 1, 2 \text{ mod } 4,$$  

one can see that (2.6) vanishes for $p = 1 \text{ mod } 4$. Thus we have reproduced the classification of the 1/2 BPS Dirichlet branes in flat spacetime without flux [6].

3.2 NC M5-branes

Next we shall consider a NC M5-brane by including constant fluxes. Then we need to generalize the ansatz for the gluing matrix as follows:

$$M = h_0 \Gamma^{\hat{A}_0 \hat{A}_1 \cdots \hat{A}_5} + h_1 \Gamma^{\hat{A}_0 \hat{A}_1 \hat{A}_2}. \quad (3.3)$$

The gluing matrix $M = h_0 \Gamma^{\hat{A}_0 \hat{A}_1 \hat{A}_2} + h_1 \Gamma^{\hat{A}_3 \hat{A}_4 \hat{A}_5}$ leads to the same results [20]. This provides the same NC M5-brane with a different parametrization.
It is characteristic of NC branes that $M$ is represented by a sum of the products of gamma matrices in comparison to the case without flux (3.1).

For the gluing matrix $M$ to define a projection operator, the condition $M^2 = 1$ should be satisfied. Then we obtain the following condition,

$$-s_0 h_0^2 - s_1 h_1^2 = 1. \tag{3.4}$$

Here $s_0 = -1$ when $0 \in \{\tilde{A}_0, \tilde{A}_1, \cdots, \tilde{A}_5\}$ and $s_0 = 1$ otherwise, and $s_1 = -1$ when $0 \in \{\tilde{A}_0, \tilde{A}_1, \tilde{A}_2\}$ and $s_1 = 1$ otherwise. It would be helpful to see that the matrix $M$ satisfies

$$\bar{\theta} = \bar{\theta} M', \quad M' = -h_0 \Gamma^{\tilde{A}_0 \tilde{A}_1 \cdots \tilde{A}_5} + h_1 \Gamma^{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2} .$$

Let us first examine (2.2). Since we can easily show the following identities,

$$\bar{\theta} \Gamma_{\tilde{A}_0 \tilde{A}_1} \delta \kappa \theta = \frac{1}{2} \bar{\theta} (M' \Gamma_{\tilde{A}_0 \tilde{A}_1} + \Gamma_{\tilde{A}_0 \tilde{A}_1} M) \delta \kappa \theta = -h_1 \bar{\theta} \Gamma^{\tilde{A}_2} \delta \kappa \theta ,$$

$$\bar{\theta} \Gamma_{\tilde{A}_3 \tilde{A}_4} \delta \kappa \theta = h_1 \bar{\theta} \Gamma^{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2} \delta \kappa \theta , \quad \bar{\theta} \Gamma_{\tilde{A}_2 \tilde{A}_3} \delta \kappa \theta = 0 ,$$

$$\mathcal{H}^{\tilde{A}_3 \tilde{A}_4 \tilde{A}_5} \bar{\theta} \Gamma^{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2} \delta \kappa \theta = -h_0 \mathcal{H}^{\tilde{A}_3 \tilde{A}_4 \tilde{A}_5} \bar{\theta} \Gamma^{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2} \delta \kappa \theta ,$$

we can see that (2.2) may vanish by imposing the conditions

$$h_1 - \mathcal{H}_{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2} = 0 , \quad h_1 - h_0 \mathcal{H}^{\tilde{A}_3 \tilde{A}_4 \tilde{A}_5} = 0 . \tag{3.5}$$

With (3.2), (2.6) also becomes zero. Therefore the gluing matrix (3.3) with the two conditions (3.4) and (3.5) gives a possible M5-brane configuration.

Then let us consider the interpretation of the solution constructed above. For reality of $\mathcal{H}$, it is sufficient to consider $(s_0, s_1) = (-1, \pm 1)$. By substituting (3.5) for (3.4), we obtain the following condition,

$$\frac{1}{(\mathcal{H}^{\tilde{A}_3 \tilde{A}_4 \tilde{A}_5})^2} - \frac{1}{(\mathcal{H}_{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2})^2} = s_1 .$$

This is nothing but the self-dual condition [23] of the gauge field on the M5-brane [18]. That is, we have reproduced the information on the NC M5-brane from the $\kappa$-symmetry argument for the open supermembrane action. Thus we recognize that the projection operator should describe a NC M5-brane $(\tilde{A}_0 \cdots \tilde{A}_5)$ with $\mathcal{H}_{\tilde{A}_0 \tilde{A}_1 \tilde{A}_2}$ and $\mathcal{H}^{\tilde{A}_3 \tilde{A}_4 \tilde{A}_5}$. It is worth noting that the corresponding supergravity solution is found in [21].
Commutative and strong flux limits

Now let us examine a commutative limit and a large $\mathcal{H}$ limit of the NC M5-brane.

We first consider the case with $s_1 = -1$, say NC M5 (012345) with $\mathcal{H}_{012}$ and $\mathcal{H}^{345}$. The conditions (3.4) and (3.5) are solved by using an angle variable $\varphi$ ($0 \leq \varphi \leq \pi/2$),

$$h_0 = \cos \varphi, \quad h_1 = \sin \varphi, \quad \mathcal{H}_{012} = \sin \varphi, \quad \mathcal{H}^{345} = \tan \varphi. \quad (3.6)$$

With (3.6), we can express the gluing matrix $M$ as

$$M = e^{\varphi \Gamma^{345}} \Gamma^{012345}. $$

For a commutative limit $\varphi \to 0$, the NC M5 reduces to commutative M5 (012345), since $\mathcal{H} \to 0$ and $M \to \Gamma^{012345}$.

On the other hand, for $\varphi \to \pi/2$, we see that $\mathcal{H}^{345} \to \infty$ and so the gluing condition reduces to $M \to \Gamma^{012}$ with a critical flux $\mathcal{H}_{012} = 1$. It seems that the resulting projection operator should describe a critical M2-brane (012). Eventually this limit is nothing but the OM limit [24] and it should correspond to one of infinitely many M2-branes dissolved on the M5-brane. This is analogous to the D2-D0 setup where a D2-brane with a flux reduces to a D2-brane with infinitely many D0-brane in a strong magnetic flux limit. We summarize the results in Fig. 1.

Next we examine the case with $s_1 = 1$, say the NC M5 (012345) with $\mathcal{H}_{345}$ and $\mathcal{H}^{012}$. The conditions (3.4) and (3.5) are solved again by using a single variable $\varphi$ ($0 \leq \varphi < \infty$),

$$h_0 = -\cosh \varphi, \quad h_1 = \sinh \varphi, \quad \mathcal{H}_{345} = \sinh \varphi, \quad \mathcal{H}^{012} = -\tanh \varphi. $$

In this case the range of $\varphi$ is not bounded. Then the gluing matrix can be expressed as

$$M = \cosh \varphi \Gamma^{012345} + \sinh \varphi \Gamma^{345} = e^{\varphi \Gamma^{012}} \Gamma^{012345}. $$

In a commutative limit $\varphi \to 0$, the NC M5 reduces to a commutative M5 (012345), since $h_0 = -1, h_1 = 0$ and $\mathcal{H}_{345} = \mathcal{H}^{012} = 0$.

On the other hand, to discuss a strong flux limit $\varphi \to \infty$, we should note that the boundary condition for $\theta$ can be rewritten as

$$2e^{-\varphi} \theta = [(1 + e^{-2\varphi})\Gamma^{012345} + (1 - e^{-2\varphi})\Gamma^{012}\Gamma^{012345}]\theta.$$
Then, after taking this limit, it turns to
\[ 0 = \Gamma^{012345}(1 - \Gamma^{012})\theta \]
and so we obtain that
\[ \theta = \Gamma^{012}\theta . \] (3.7)

The physical interpretation of the resulting M2-brane with a critical flux \( H^{012} = -1 \) is the same as in the case with \( s_1 = -1 \).

\[ \kappa \] -invariance for a critical M2-brane

When a strong flux limit is taken, the gluing condition for a NC M5-brane reduces to that for one of the infinitely many M2-branes. As we have already seen in Sec. 3.1, the \( p = 2 \) case is not allowed as a projection operator in the case without fluxes. Therefore it is a non-trivial problem whether the resulting projection operator for a critical M2 is consistent to the \( \kappa \)-symmetry, or equivalently whether the projection should delete the surface terms coming from the \( \kappa \)-variation.

The \( p = 2 \) case is actually special among other \( p \), and we can find the identities intrinsic to \( p = 2 \),
\[ \mathcal{H}_{\bar{A}_0 A_1 A_2} \bar{\theta} \Gamma A_2 \delta_x \theta = \mathcal{H}_{\bar{A}_0 A_1 A_2} \bar{\theta} \Gamma A_2 \ell \Gamma \bar{A}_0 \bar{A}_1 \bar{A}_2 \delta_x \theta = \mathcal{H}^{\bar{A}_0 \bar{A}_1 \bar{A}_2} \ell \bar{\theta} \Gamma \bar{A}_0 \bar{A}_1 \delta_x \theta \]
and so (2.2) vanishes when

\[ 1 + \ell \mathcal{H}^{\bar{A}_0, \bar{A}_1, \bar{A}_2} = 0. \]  

(3.8)

The flux \( \mathcal{H} \) should be real so that \( \ell \) is real and thus \( s = -1 \). It follows from (3.2) that (2.6) disappears. Thus we have checked that the \( \kappa \)-variation surface terms should vanish for an M2-brane with a critical \( \mathcal{H} \) fixed by (3.8).

Although the \( \kappa \)-symmetry is maintained for the M2-brane, the charge conservation requires the existence of M5-brane behind M2-branes. That is, a NC M5-brane should be regarded as a bound state of M5 and M2.

4 Intersecting NC M5-branes

The main subject of this paper is to construct a projection operators for intersecting NC M5-branes. A single NC M5-brane is characterized by the product of gamma matrices with an exponential factor, as we have already seen. Hence let us consider to describe intersecting NC M5-branes by introducing the two gluing matrices,

\[ M_1 = e^{\varphi_1 \Gamma^{A_0, A_1, A_2}} \Gamma^{A_0 \cdots A_5}, \quad M_2 = e^{\varphi_2 \Gamma^{B_0, B_1, B_2}} \Gamma^{B_0 \cdots B_5}, \quad [M_1, M_2] = 0. \]  

(4.1)

In order to avoid an imaginary \( \mathcal{H} \), the conditions \( 0 \in \{ A_0, \cdots, A_5 \} \) and \( 0 \in \{ B_0, \cdots, B_5 \} \) are assumed.

In the case without flux, i.e., \( \varphi_1 = \varphi_2 = 0 \), the matrices commute each other, if the number, say \( n \), of the common indices contained in \( \Gamma^{A_0 \cdots A_5} \) and \( \Gamma^{B_0 \cdots B_5} \) is even. For \( n = 4 \) it is an intersection of M5-branes along a 3-brane, M5\( \perp \)M5 (3). For \( n = 2 \) it is that of M5-branes along a 1-brane, M5\( \perp \)M5 (1).

We will examine NC versions of these two cases below. In a commutative limit they should be reduced to M5\( \perp \)M5 (3) or M5\( \perp \)M5 (1), and hence we hereafter assume that \( n \) is even.

4.1 Intersecting NC M5\( \perp \)NC M5 (3)

First of all, in order to make our analysis simpler, let us impose the following condition,

\[ \Gamma^{A_0, A_1, A_2} = \Gamma^{B_0, B_1, B_2}. \]  

(4.2)
in addition to the ansatz (4.1). Then the commuting relation \([M_1, M_2] = 0\) requires that

\[
[\Gamma^{A_0 \cdots A_5}, \Gamma^{B_0 \cdots B_5}] = 0, \quad [\Gamma^{A_0 \cdots A_5}, \Gamma^{B_0 B_1 B_2}] = 0. \tag{4.3}
\]

The conditions (4.3) are satisfied if \(\Gamma^{A_0 \cdots A_5}\) and \(\Gamma^{B_0 \cdots B_5}\) share even number of indices, and \(\Gamma^{A_0 \cdots A_5}\) and \(\Gamma^{B_0 \cdots B_2}\) share odd number of indices, respectively. We find two kinds of intersections under the conditions (4.2) and (4.3). After describing some examples of them, we move on to more general solutions.

The first example

The first example is \(\text{M5} (012345) \perp \text{M5} (012367)\) with \(\mathcal{H}^{012}, \mathcal{H}_{345}\) and \(\mathcal{H}_{367}\). To begin with, the vanishing condition of the \(\kappa\)-variation surface term is solved for each of the two M5-branes with two parameters. For the first M5, the gluing matrix is given by

\[
M_1 = e^{\phi_1 \Gamma^{012}} \Gamma^{012345}, \quad \mathcal{H}^{012} = -\tanh \phi_1, \quad \mathcal{H}_{345} = \sinh \phi_1,
\]

and for the second M5 it is

\[
M_2 = e^{\phi_2 \Gamma^{012}} \Gamma^{012367}, \quad \mathcal{H}^{012} = -\tanh \phi_2, \quad \mathcal{H}_{367} = \sinh \phi_2.
\]

From the expression of the electric flux we can read off the condition \(\phi_1 = \phi_2\), and so the gluing matrices and fluxes are given by, respectively,

\[
M_1 = e^{\phi \Gamma^{012}} \Gamma^{012345}, \quad M_2 = e^{\phi \Gamma^{012}} \Gamma^{012367}, \tag{4.4}
\]

\[
\mathcal{H}^{012} = -\tanh \phi, \quad \mathcal{H}_{345} = \sinh \phi, \quad \mathcal{H}_{367} = \sinh \phi. \tag{4.5}
\]

Taking a commutative limit \(\phi \to 0\), the configuration described by (4.4) and (4.5) reduces to an intersecting M5-branes, \(\text{M5} (012345) \perp \text{M5} (012367)\) [15–17]. Taking a strong flux limit \(\phi \to \infty\), both fermionic boundary conditions reduce to

\[
\theta = \Gamma^{012} \theta,
\]

so that it should be regarded as a couple of M2-branes in two sets of infinitely many M2-branes with \(\mathcal{H}^{012} = -1\) on the \(\text{M5} (012345) \perp \text{M5} (012367)\). These limits are depicted in Fig. 2.
The second example

The second example is M5 \((012345) \perp M5 (012367)\) with \(\mathcal{H}^{123}, \mathcal{H}_{045}\) and \(\mathcal{H}_{067}\). The construction of projection operators can be carried out in the similar way and the result is given by

\[
M_1 = e^{i\phi \Gamma^{123}} \Gamma^{012345}, \quad M_2 = e^{i\phi \Gamma^{123}} \Gamma^{012367},
\]

\[
\mathcal{H}^{123} = \tan \varphi, \quad \mathcal{H}_{045} = \mathcal{H}_{067} = \sin \varphi.
\]

By taking a commutative limit \(\varphi \to 0\), this configuration reduces to intersecting M5-branes, M5 \((012345) \perp M5 (012367)\). Taking a strong flux limit \(\varphi \to \pi/2\), the projection operators reduce to M2-branes again. But we turned on two different electric fluxes and so the projection operators are different each other. Hence we find intersections of two sets of infinitely many M2-branes, M2 \((045) \perp M2 (067)\) with \(\mathcal{H}_{045} = \mathcal{H}_{067} = 1\). See Fig. 3. This sub-brane system M2 \(\perp M2 (0)\) is nothing but a possible intersecting configuration of two M2-branes. The corresponding supergravity solution is found in \([15, 17]\).

The third example

Finally, by removing the condition \((4.2)\), we consider the case with \(\Gamma^{A_0 A_1 A_2} \neq \Gamma^{B_0 B_1 B_2}\). In this case, \([M_1, M_2] = 0\) requires that

\[
\begin{align*}
[\Gamma^{A_0 \cdots A_5}, \Gamma^{B_0 \cdots B_5}] &= 0, & [\Gamma^{A_0 \cdots A_5}, \Gamma^{B_3 B_4 B_5}] &= 0, \\
[\Gamma^{B_0 \cdots B_5}, \Gamma^{A_3 A_4 A_5}] &= 0, & [\Gamma^{B_3 B_4 B_5}, \Gamma^{A_3 A_4 A_5}] &= 0.
\end{align*}
\]
By examining the above conditions, we shall find intersecting M5-branes with four fluxes, say, M5 (012345) ⊥ M5 (012367) with $H_{014}$, $H_{235}$, $H_{026}$ and $H_{137}$.

For the first M5-brane, the gluing matrices and fluxes are

\[ M^t_1 = e^{\varphi_1 \Gamma^{012345}} \Gamma^{012345}, \quad H_{014} = \sin \varphi_1, \quad H_{235} = \tan \varphi_1, \]

or

\[ M^h_1 = e^{\phi_1 \Gamma^{012345}} \Gamma^{012345}, \quad H_{235} = \sinh \phi_1, \quad H_{014} = -\tanh \phi_1. \]

For the second M5-brane those are

\[ M^t_2 = e^{\varphi_2 \Gamma^{012367}} \Gamma^{012367}, \quad H_{026} = -\sin \varphi_2, \quad H_{137} = \tan \varphi_2, \]

or

\[ M^h_2 = e^{\phi_2 \Gamma^{012367}} \Gamma^{012367}, \quad H_{137} = -\sinh \phi_2, \quad H_{026} = -\tanh \phi_2, \]

where $0 \leq \varphi_{1,2} \leq \pi/2$ and $0 \leq \phi_{1,2} < \infty$. Therefore there are four possibilities, characterized by boundary conditions with

1) $(M^t_1, M^t_2)$, 2) $(M^t_1, M^h_2)$, 3) $(M^h_1, M^t_2)$, 4) $(M^h_1, M^h_2)$.

All of the configurations reduce to the intersecting M5-branes, M5 (012345) ⊥ M5 (012367) in a commutative limit $\varphi_{1,2} \to 0$ and $\phi_{1,2} \to 0$.

The NC M5-brane described by one of $M^t_{1,2}$ and $M^h_{1,2}$ reduces to infinitely many M2-branes with critical $H$ on the M5-brane for $\varphi_{1,2} \to \pi/2$ and $\phi_{1,2} \to \infty$, respectively. So all of four intersections M5 ⊥ M5 reduce to M2 ⊥ M2 with critical fluxes in the large $H$ limit

\[ \theta = \Gamma^{014} \theta \quad \text{and} \quad \theta = -\Gamma^{026} \theta. \]
An intersecting configuration $M_2 \perp M_5 \ (1)$ [15, 17] can also be realized from the NC $M_5 \perp M_5$ obtained above by taking a strong flux limit. For example, let us consider the NC $M_5 \perp M_5$ characterized by $(M_1^t, M_2^t)$. A NC $M_5$-brane can be regarded as a bound state of $M_5$ and $M_2$, hence we can see $M_2 \perp M_5 \ (1)$ as a sub-brane system of NC $M_5 \perp NC M_5$. By taking the limit $\varphi_2 \to \pi/2$ of the NC $M_5 \perp M_5$ with $(M_1^t, M_2^t)$, we obtain the gluing matrices for $M_2 \perp M_5 \ (1)$,

$$M_1 = e^{\varphi_1 \Gamma^{235}} \Gamma^{012345} \quad M_2 = -\Gamma^{026}.$$ 

It should be remarked that the two gluing matrices commute each other even at this stage. Further taking $\varphi_1 \to \pi/2$, we obtain the intersecting configuration $M_2 \ (014) \perp M_2 \ (026)$ as a sub-brane system on the $M_5$-branes, again.

On the other hand, by taking the limit $\varphi_1 \to \pi/2$, we can obtain another gluing matrices for $M_2 \perp M_5 \ (1)$.

$$M_1 = \Gamma^{014} \quad M_2 = e^{\varphi_2 \Gamma^{137}} \Gamma^{012367}.$$ 

These also commute each other. Then taking $\varphi_2 \to \pi/2$, it reduces to $M_2 \ (014) \perp M_2 \ (026)$. These sequences of the strong flux limits are depicted in Fig. 4.

4.2 Intersecting NC $M_5 \perp M_5 \ (1)$

Here let us consider configurations of $M_5 \perp M_5 \ (1)$, say $M_5 \ (012345) \perp M_5 \ (056789)$ with fluxes $H_{015}$ and $H_{234}$. There are two kinds of configurations. The first is characterized by

$$M_1 = e^{\varphi \Gamma^{015}} \Gamma^{012345} \quad M_2 = \Gamma^{056789} \quad H^{015} = \tanh \varphi \quad H_{234} = \sinh \varphi,$$

and the other is

$$M_1 = e^{\varphi \Gamma^{234}} \Gamma^{012345} \quad M_2 = \Gamma^{056789} \quad H_{015} = -\sin \varphi \quad H^{234} = \tan \varphi.$$ 

For $\varphi \to 0$, both reduce to commutative $M_5 \perp M_5 \ (1)$. For a strong flux limit, $M_1$ reduces to $\theta = \pm \Gamma^{015} \theta$, and so it describes an intersection of an $M_2 \ (015)$ on the $M_5 \ (012345)$ and a commutative $M_5 \ (056789)$. That is, $M_2 \perp M_5 \ (1)$ has been found again. See Fig 5.
5 Summary and Discussions

We have discussed 1/4 BPS intersecting NC M5-branes from the viewpoint of $\kappa$-symmetry of a covariant open supermembrane action. We constructed projection operators for two types of 1/4 BPS intersecting NC M5-branes. The one is an intersection of two NC M5-branes: NC M5$\perp$NC M5 (3). The other is that of a NC M5-brane and a commutative (C) M5-brane: NC M5$\perp$C M5 (1). A NC M5-brane can be viewed as a bound state of M5 and M2, and the configurations M2$\perp$M5 (1) and M2$\perp$M2 (0) are realized on the intersecting M5-branes. Taking a commutative limit the allowed intersecting M5-branes are surely reproduced: M5$\perp$M5 (3) and M5$\perp$M5 (1). By considering a strong flux limit, we have found projection operators even for M2$\perp$M5 (1) and M2$\perp$M2 (0), which still ensures the $\kappa$-invariance of the membrane action.

As another task we are going to consider AdS M-branes with constant three-form fluxes on AdS$_{4/7} \times S^{7/4}$ and pp-wave by using the $\kappa$-symmetry argument (For NC D-branes in flat...
Fig. 5: Strong flux limit of C $\perp$ NC M5 (1).

space and a pp-wave, see [25]. Intersecting D-branes in pp-waves are discussed in [26]. We hope that it could be reported in the near future [27]. It is also interesting to consider a $\kappa$-symmetry argument for M-branes at angle [28].

There are some interesting issues related to open supermembranes, such as the Poisson structure on the M5-brane [29,30], the area-preserving diffeomorphism [31], S-duality [32], and open membrane field theory [33]. Since the $\kappa$-symmetry of open supermembrane theory should be closely related to the equation of motion of M5-brane, it might be interesting to reveal the connection between those issues and the $\kappa$-symmetry argument as a future direction.

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References

[1] E. Bergshoeff, E. Sezgin and P. K. Townsend, “Supermembranes and Eleven-Dimensional Supergravity,” Phys. Lett. B 189 (1987) 75; “Properties of the Eleven-Dimensional Supermembrane Theory,” Ann. Phys. 185 (1988) 330.

[2] B. de Wit, J. Hoppe and H. Nicolai, “On The Quantum Mechanics Of Supermembranes,” Nucl. Phys. B 305 (1988) 545.

[3] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D 55 (1997) 5112 [arXiv:hep-th/9610043].

[4] A. Strominger, “Open p-branes,” Phys. Lett. B 383 (1996) 44 [arXiv:hep-th/9512059].

[5] P. K. Townsend, “D-branes from M-branes” Phys. Lett. B 373 (1996) 68 [arXiv:hep-th/9512062].

[6] K. Ezawa, Y. Matsuo and K. Murakami, “Matrix regularization of open supermembrane: Towards M-theory five-brane via open supermembrane,” Phys. Rev. D 57 (1998) 5118 [arXiv:hep-th/9707200].

[7] B. de Wit, K. Peeters and J. C. Plefka, “Open and closed supermembranes with winding,” Nucl. Phys. Proc. Suppl. 68 (1998) 206 [arXiv:hep-th/9710215].

[8] P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” Nucl. Phys. B 460 (1996) 506 [arXiv:hep-th/9510209]; “Eleven-Dimensional Supergravity on a Manifold with Boundary,” Nucl. Phys. B 475 (1996) 94 [arXiv:hep-th/9603142].

[9] E. A. Bergshoeff, G. W. Gibbons and P. K. Townsend, “Open M5-branes,” Phys. Rev. Lett. 97 (2006) 231601 [arXiv:hep-th/0607193].

[10] C. S. Chu and E. Sezgin, “M-fivebrane from the open supermembrane,” JHEP 9712 (1997) 001 [arXiv:hep-th/9710223].

[11] N. D. Lambert and P. C. West, “D-branes in the Green-Schwarz formalism,” Phys. Lett. B 459 (1999) 515 [arXiv:hep-th/9905031].
For open supermembranes on pp-wave and AdS, see M. Sakaguchi and K. Yoshida, “Dirichlet branes of the covariant open supermembrane in $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$,” Nucl. Phys. B 681 (2004) 137 [arXiv:hep-th/0310035]; “Open M-branes on $\text{AdS}_{4/7} \times S^{7/4}$ revisited,” Nucl. Phys. B 714 (2005) 51 [arXiv:hep-th/0405109]; “Dirichlet branes of the covariant open supermembrane on a pp-wave background,” Nucl. Phys. B 676 (2004) 311 [arXiv:hep-th/0306213]; K. Sugiyama and K. Yoshida, “Supermembrane on the pp-wave background,” Nucl. Phys. B 644 (2002) 113 [arXiv:hep-th/0206070].

P. Bain, K. Peeters and M. Zamaklar, “D-branes in a plane wave from covariant open strings,” Phys. Rev. D 67 (2003) 066001 [arXiv:hep-th/0208038]. S. Hyun, J. Park and H. Shin, “Covariant description of D-branes in IIA plane-wave background,” Phys. Lett. B 559 (2003) 80 [arXiv:hep-th/0212343]. M. Sakaguchi and K. Yoshida, “D-branes of covariant AdS superstrings,” Nucl. Phys. B 684 (2004) 100 [arXiv:hep-th/0310228]. “Notes on D-branes of type IIB string on $\text{AdS}_5 \times S^5$,” Phys. Lett. B 591 (2004) 318 [arXiv:hep-th/0403243]. “D-branes of covariant AdS superstrings: An overview,” arXiv:hep-th/0408208. “Noncommutative D-brane from covariant AdS superstring,” arXiv:hep-th/0604039.

K. Skenderis and M. Taylor, “Branes in AdS and pp-wave spacetimes,” JHEP 0206 (2002) 025. N. Kim and J. T. Yee, “Supersymmetry and branes in M-theory plane-waves,” Phys. Rev. D 67 (2003) 046004 [arXiv:hep-th/0211029].

G. Papadopoulos and P. K. Townsend, “Intersecting M-branes,” Phys. Lett. B 380 (1996) 273 [arXiv:hep-th/9603087].

A. A. Tseytlin, “Harmonic superpositions of M-branes,” Nucl. Phys. B 475 (1996) 149 [arXiv:hep-th/9604035].

J. P. Gauntlett, D. A. Kastor and J. H. Traschen, “Overlapping Branes in M-Theory,” Nucl. Phys. B 478 (1996) 544 [arXiv:hep-th/9604179].

N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909 (1999) 032 [arXiv:hep-th/9908142].
[19] E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, “A noncommutative M-theory five-brane,” Nucl. Phys. B 590 (2000) 173 [arXiv:hep-th/0005026].

[20] M. Sakaguchi and K. Yoshida, “Noncommutative M-branes from covariant open supermembranes,” Phys. Lett. B 642 (2006) 400 [arXiv:hep-th/0608099].

[21] N. Kim, “Intersecting noncommutative D-branes and baryons in magnetic fields,” Phys. Rev. D 62 (2000) 066002 [arXiv:hep-th/0002086].

[22] J. P. Gauntlett, “Intersecting branes,” arXiv:hep-th/9705011.

[23] P. S. Howe, E. Sezgin and P. C. West, “The six-dimensional self-dual tensor,” Phys. Lett. B 400 (1997) 255 [arXiv:hep-th/9702111]; “Covariant field equations of the M-theory five-brane,” Phys. Lett. B 399 (1997) 49 [arXiv:hep-th/9702008].

[24] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, “OM theory in diverse dimensions,” JHEP 0008 (2000) 008 [arXiv:hep-th/0006062].

[25] C. S. Chu and P. M. Ho, “Noncommutative open string and D-brane,” Nucl. Phys. B 550 (1999) 151 [arXiv:hep-th/9812219]; “Constrained quantization of open string in background B field and noncommutative D-brane,” Nucl. Phys. B 568 (2000) 447 [arXiv:hep-th/9906192]; “Noncommutative D-brane and open string in pp-wave background with B-field,” Nucl. Phys. B 636 (2002) 141 [arXiv:hep-th/0203186].

[26] N. Ohta, K. L. Panigrahi and S. Siwach, “Intersecting branes in pp-wave spacetime,” Nucl. Phys. B 674 (2003) 306 [Erratum-ibid. B 748 (2006) 309] [arXiv:hep-th/0306186].

[27] M. Sakaguchi and K. Yoshida, “M-branes with world-volume fluxes in AdS$_{4/7} \times$S$^{7/4}$ and pp-wave,” in preparation.

[28] N. Ohta and P. K. Townsend, “Supersymmetry of M-branes at angles,” Phys. Lett. B 418 (1998) 77 [arXiv:hep-th/9710129].

[29] S. Kawamoto and N. Sasakura, “Open membranes in a constant C-field background and noncommutative boundary strings,” JHEP 0007 (2000) 014 [arXiv:hep-th/0005123].
[30] I. Rudychev, “From noncommutative string/membrane to ordinary ones,” JHEP 0104 (2001) 015 [arXiv:hep-th/0101039].

[31] Y. Matsuo and Y. Shibusa, “Volume preserving diffeomorphism and noncommutative branes,” JHEP 0102 (2001) 006 [arXiv:hep-th/0010040].

[32] T. Kawano and S. Terashima, “S-duality from OM-theory,” Phys. Lett. B 495 (2000) 207 [arXiv:hep-th/0006225].

[33] P. M. Ho and Y. Matsuo, “A toy model of open membrane field theory in constant 3-form flux,” arXiv:hep-th/0701130.