Radiative electroweak effects in deep inelastic scattering of polarized leptons by polarized nucleons

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Abstract. The one-loop electroweak radiative correction of the lowest order to lepton current for deep inelastic scattering (DIS) of longitudinally polarized leptons by polarized nucleons is obtained in model independent way. The detailed numerical analysis within kinematical requirements of future polarization collider experiments is performed. The possibility to reduce radiative effects by detection of hard photons is studied.

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1. Introduction

Deep inelastic lepton-hadron scattering, starting with the discovery of Bjorken scaling in the end of nineteen-sixties has played a crucial role in the development of our present understanding of the nature of particle interaction. The appearance of the first data on the polarization DIS opened a new field of experimental and theoretical investigations and, as a result it led to come the disturbing data of EMC in 1988 [1].

It is natural, that data processing of the modern experiments on DIS of the polarized lepton on the polarized nuclear target requires correct account of the radiative corrections (RC). Up to now there is a series of works where RC are taken into account in the frame of QED-theory for polarization experiments (see f.e. [2, 3, 4, 5]). Those results are used for experiments on a fixed target. However the polarization DIS experiments at collider will be possible in future [6].

For the calculation of RC for such kinds of experiments we cannot restrict our consideration to $\gamma$-exchange graphs only, because in this case the squared transfer momentum $Q^2$ is so high that weak effects begin to play an essential role in the total cross section and spin asymmetries. At the same time the ratio $m^2/Q^2$ (where $m$ is the mass of a scattering lepton) becomes so small that it could be restricted to non-vanishing terms for $m \to 0$. The similar work was already done by us [7], but there we used only the naive parton model. In this article we present the explicit expressions for model independent part of the one-loop lowest-order RC (figure 1) to polarized lepton-nucleon scattering which is described in the terms of the electroweak structure functions (SF). Moreover, the target has an arbitrary polarization and the lepton has a longitudinal one. The contributions appearing from additional virtual particles (V-contribution) in the on-mass renormalization scheme and t’Hooft-Feynman gauge are presented. The results of calculation for the unitary gauge can be found in ref. [8]. We note also that the separation of variables in accordance with [3] allows to write all formulae in a compact form that provided more clearness of the results than it was done in [8].

The present article is organized as follows. In the section 2 the Born contribution and all necessary kinematic invariants are presented. The section 3 is devoted to the electroweak one-loop correction. Numerical analysis for kinematical conditions of the collider experiments is presented in the section 4. Conclusion is given in the last section.

2. Born contribution

Here we consider the deep inelastic polarized lepton-hadron scattering

$$\ell(k_1, \xi) + N(p, \eta) \rightarrow \ell(k_2) + X$$ (1)

in the frame of electroweak standard theory. The quantities in brackets $k_1(k_2), p$ define the momenta of an initial (final) lepton and proton respectively ($k_1^2 = k_2^2 = m^2, \ p^2 = M^2$), $\xi$ and $\eta$ are the polarization vectors of the scattering particles.
Since the lepton is considered to be longitudinally polarized, its polarization vector has the form:\[\xi = \frac{S}{m\sqrt{\lambda_s}}k_1 - \frac{2m}{\sqrt{\lambda_s}}p = \xi_0 + \xi'.\] (2)
The kinematical invariants are defined in a standard way:\[S = 2k_1p, \quad X = 2k_2p = (1 - y)S, \quad Q^2 = -q^2 = -(k_1 - k_2)^2 = xyS, \quad S_x = S - X, \quad S_p = S + X, \quad \lambda_s = S^2 - 4m^2M^2,\] (3)
where \(x\) and \(y\) are usual scaling variables.

The double-differential cross section of lepton-nucleon scattering on the Born level \((d\sigma^B/dxdy \equiv \sigma^B)\) in the frame of electroweak theory reads:\[\sigma^B = \frac{4\pi\alpha^2SxS}{\lambda_sQ^4}\left[L_{\mu\nu}^{\gamma\gamma}W_{\mu\nu}^{\gamma\gamma}(p, q) + \frac{1}{2}(L_{\mu\nu}^{\gamma Z} + L_{\mu\nu}^{Z\gamma})W_{\mu\nu}^{\gamma Z}(p, q)\chi + L_{\mu\nu}^{ZZ}W_{\mu\nu}^{ZZ}(p, q)\chi^2\right],\] (4)
where \(\chi = Q^2/(Q^2 + M_Z^2)\) (\(M_Z\) is the Z-boson mass). The lepton tensors \(L_{\mu\nu}^{mn}\) can be given as:\[L_{\mu\nu}^{mn} = \frac{1}{4}Sp \gamma(v^m - a^m\gamma_5)(\hat{k}_1 + m)(1 - P_L\gamma_5)\gamma_\nu(v^n - a^n\gamma_5)(\hat{k}_2 + m),\] (5)
where:\[v^\gamma = 1, \quad v^Z = (-1 + 4s_w^2)/4s_wc_w, \quad a^\gamma = 0, \quad a^Z = -1/4s_wc_w\] (6)
are the standard electroweak coupling constants, \(c_w\) and \(s_w\) are cosin and sine of Weinberg’s angle respectively and \(P_L\) is degree of lepton polarization.

All of the hadronic tensors \(W_{\mu\nu}^{\gamma\gamma,\gamma Z, Z Z}(p, q)\) can be expressed in terms of eight electroweak SF \([9, 10]\):\[W_{\mu\nu}^I(p, q) = \sum_{i=1}^{8} w_{\mu\nu}^I F_i^I = -\tilde{g}_{\mu\nu}F_1^I + \frac{1}{M^2}\tilde{p}_\mu\tilde{p}_\nu F_2^I + i\epsilon_{\mu\nu\lambda\sigma}\left[p^\lambda q^\sigma F_3^I + q^\lambda\eta^\sigma\frac{1}{M^2}F_4^I - q^\lambda p^\sigma\eta^\mu F_5^I\right] - \frac{1}{2M}[\tilde{p}_\mu\tilde{q}_\nu + \tilde{p}_\nu\tilde{q}_\mu]F_6^I + \frac{\eta q}{M^3}\tilde{p}_\mu\tilde{p}_\nu F_7^I - \frac{\eta q}{M^3}\tilde{q}_\mu\tilde{p}_\nu F_8^I.\] (7)
where $I = \gamma, \gamma Z, Z, \bar{F}$ are defined as ($\epsilon = M^2/pq$)
\[
\tilde{F}_1^I = F_1^I, \quad \tilde{F}_{2,3}^I = \epsilon F_{2,3}^I, \quad \tilde{F}_4^I = \epsilon P_N(g_1^I + g_2^I), \quad \tilde{F}_{5,7}^I = \epsilon^2 P_N g_{2,4}^I, \quad \tilde{F}_{6,8}^I = \epsilon P_N g_{3,5}^I
\]
and
\[
\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}, \quad \tilde{p}_\nu = p_\nu + \frac{pq}{Q^2} q_\nu, \quad \tilde{\eta}_\nu = \eta_\nu + \frac{\eta q}{Q^2} q_\nu.
\]
Hadronic tensors in the form (7-9) correspond to the definition of electroweak SF given in (2.2.7) of ref. [9]. For contraction of leptonic tensor (5) with Hadronic tensors in the form (7-9) correspond to the definition of electroweak SF given in (2.2.7) of ref. [9]. For contraction of leptonic tensor (3) with $w^i_{\mu\nu}$, we have
\[
L_{mn}^m w^i_{\mu\nu} = \theta^B_i R_{mn}^m (i = 1, 2, 6 - 8),
\]
\[
L_{mn}^m w^i_{\mu\nu} = \theta^B_i R_{mn}^m (i = 3 - 5).
\]
The quadratic combinations of the electroweak coupling constants are defined as:
\[
R_{V}^m = (v^m v^n + a^m a^n) - P_L(v^m v^n + a^m a^n),
\]
\[
R_{A}^m = (v^m a^n + a^m v^n) - P_L(v^m v^n + a^m a^n).
\]
The quantities $\theta^B_i$ depend only on the target polarization vector and kinematical invariants:
\[
\theta_1^B = Q^2, \quad \theta_5^B = \eta_q Q^2 S_p / 2 M^3,
\]
\[
\theta_2^B = (X S - M^2 Q^2) / 2 M^2, \quad \theta_6^B = - (X \eta k_1 + S \eta k_2) / 2 M,
\]
\[
\theta_3^B = Q^2 S_p / 4 M^2, \quad \theta_7^B = \eta_q (X S - M^2 Q^2) / 2 M^3,
\]
\[
\theta_4^B = - Q^2 \eta (k_1 + k_2) / M, \quad \theta_8^B = - \eta Q^2 / M.
\]

Covariant representation for the proton polarization vector both for longitudinally and transversely polarized nucleons $\eta$ can be found in Appendix A of ref. [3]. It does not contain a nucleon polarization degree $P_N$ which is included in SF definition (8).

We note, that only leading part of $\xi$ ($\xi_0$) contributes to $\theta^B$ in the ultrarelativistic approximation. The second term ($\xi'$) gives non-vanishing correction to the cross section of radiated process which is considered below.

Using the hadronic tensor (7) and the results for contractions (10) the cross section (4) can be rewritten in a simple form
\[
\sigma^B = \frac{4 \pi \alpha^2 y}{Q^2} \sum_{i=1}^{8} \theta_i^B \mathcal{F}_i.
\]

Both for the Born cross section and for RC considered in the next section the electroweak SF are gathered in eight combinations. So it is convenient to define the generalized SF:
\[
\mathcal{F}_i = R_{V}^i \tilde{F}_i^\gamma + \chi R_{V}^i \tilde{F}_i^\gamma Z + \chi^2 R_{V}^i \tilde{F}_i^{\gamma Z} (i = 1, 2, 6 - 8),
\]
\[
\mathcal{F}_i = R_{A}^i \tilde{F}_i^\gamma + \chi R_{A}^i \tilde{F}_i^\gamma Z + \chi^2 R_{A}^i \tilde{F}_i^{\gamma Z} (i = 3 - 5).
\]
3. The Lowest Order Radiative Correction

The RC of the lowest order appears as the result of one-loop effects and the process with a real photon radiation:

\[ \ell(k_1, \xi) + N(p, \eta) \rightarrow \ell(k_2) + \gamma(k) + X. \]  

The cross section of the radiated process \( (d\sigma^R/dx dy \equiv \sigma^R) \) can be presented as the sum of two parts:

\[ \sigma^R = \bar{\sigma}^R + \hat{\sigma}^R. \]  

The first one \( (\bar{\sigma}^R) \) includes a part of the cross section independent of the leptonic polarization vector \( \xi \) and contribution of its leading term \( \xi_0 \). The second term \( (\hat{\sigma}^R) \) comes from \( \xi' \) only. We can write them again in the terms of leptonic and hadronic tensors:

\[
\bar{\sigma}^R = -\frac{\alpha^3S_2S}{\pi\lambda_s} \int \frac{d^3k}{k_0} \frac{1}{Q_h} \left[ \mathcal{L}^{\gamma\gamma W} \Gamma_{\mu}^\mu(p, q_h) + \frac{1}{2}(\mathcal{L}^{\gamma\mu Z} + \mathcal{L}^{Z\gamma\mu}) W_{\mu}^{\gamma\mu}(p, q_h) \chi_h 
+ \mathcal{L}^{Z\mu\nu} W_{\mu\nu}^Z (p, q_h) \chi_h^2 \right],
\]

\[
\hat{\sigma}^R = -\frac{\alpha^3S_2S}{\pi\lambda_s} \int \frac{d^3k}{k_0} \frac{1}{Q_h} \left[ \hat{\mathcal{L}}^{\gamma\gamma W} \Gamma_{\mu}^\mu(p, q_h) + \frac{1}{2}(\hat{\mathcal{L}}^{\gamma\mu Z} + \hat{\mathcal{L}}^{Z\gamma\mu}) W_{\mu}^{\gamma\mu}(p, q_h) \chi_h 
+ \hat{\mathcal{L}}^{Z\mu\nu} W_{\mu\nu}^Z (p, q_h) \chi_h^2 \right].
\]

Here \( \chi_h = Q^2_h/(Q^2_h + M^2_Z) \) and \( Q^2_h = -q^2_h = -(k_1 - k - k_2)^2 \) are the variables dependent of the momentum of a real photon \( k \).

The hadronic tensors \( W^{\gamma\gamma, \gamma Z, Z, Z}_{\mu\nu} (p, q_h) \) are defined by \( (7) \). The leptonic tensors in \( (17) \) include spin averaged and leading spin dependent parts

\[ \bar{\mathcal{L}}^{mn}_{\mu\nu} = \frac{1}{4} Sp \Gamma^{m}_{\mu} (p_1 + m)(1 - P_L \gamma_5 \hat{\xi}) \Gamma^{n}_{\alpha} (\hat{k}_2 + m), \]

and \( \hat{\sigma}^R \) \( (18) \) comes from the contribution of \( \xi' \) only:

\[ \hat{\mathcal{L}}^{mn}_{\mu\nu} = P_L \frac{1}{4} Sp \Gamma^{m}_{\mu} (p_1 + m) \gamma_5 \hat{\xi} \Gamma^{n}_{\alpha} (\hat{k}_2 + m), \]

where

\[ \Gamma^{m}_{\mu\alpha} = \left[ \left( \frac{k_1}{kk_1} - \frac{k_2}{kk_2} \right) \gamma_{\mu \alpha} - \frac{\gamma_{\mu} k_{\alpha} \gamma_{\mu}}{2kk_1} + \frac{\gamma_{\alpha} k_{\mu} \gamma_{\mu}}{2kk_2} \right] (v^m - a^m \gamma_5), \]

\[ \bar{\Gamma}^{n}_{\alpha\mu} = \left[ \left( \frac{k_1}{kk_1} - \frac{k_2}{kk_2} \right) \gamma_{\alpha \mu} - \frac{\gamma_{\alpha} k_{\mu} \gamma_{\mu}}{2kk_1} + \frac{\gamma_{\mu} k_{\alpha} \gamma_{\mu}}{2kk_2} \right] (v^n - a^n \gamma_5). \]

The results for the contraction of these tensors with \( w_{\mu\nu}^i \) (see \( (6) \)) can be presented using notations of ref.\( \cite{3, 11} \):

\[
\frac{1}{\pi} \int \frac{d^3k}{k_0} \bar{\mathcal{L}}^{mn}_{\mu\nu} w_{\mu\nu}^i = R^m \int dR dT \sum_{j=1}^{k_1} R^{j - 2} \theta_{ij}(T) \quad (i = 1, 2, 6 - 8),
\]

\[
\frac{1}{\pi} \int \frac{d^3k}{k_0} \hat{\mathcal{L}}^{mn}_{\mu\nu} w_{\mu\nu}^i = R^n \int dR dT \sum_{j=1}^{k_1} R^{j - 2} \theta_{ij}(T) \quad (i = 3 - 5),
\]
where \( j \) runs from 1 to \( k_i = (3, 3, 4, 4, 5, 3, 4, 4) \) and the quantities \( \theta_{ij}(\tau) \) are independent of \( R \). Arguments of \( SF \) \( x \) and \( Q^2 \) in the case of radiated process acquire dependence on two photonic variables \( R = 2k_p \) and \( \tau = 2k_{q_h}/R \):

\[
Q^2 \rightarrow Q^2 + R\tau, \quad \tau \rightarrow \frac{Q^2 + R\tau}{S_x - R}.
\]

Integration over third photonic variable is performed analytically. Then the cross section \( \hat{\sigma}^R \) can be obtained in the form

\[
\hat{\sigma}^R = -\alpha'^3 y \int_{\tau_{\min}}^{\tau_{\max}} \int_{i=1}^{8} \int_{j=1}^{8} \theta_{ij}(\tau) \int_0^{R_{\max}} dR \frac{R_{\tau}^{j-2}}{(Q^2 + R\tau)^2} \mathcal{F}_i(R, \tau),
\]

where integration limits are:

\[
\tau_{\max,\min} = \frac{S_x \pm \sqrt{S_x^2 + 4M^2Q^2}}{2M^2}, \quad R_{\max} = \frac{W^2 - (M + m_\pi)^2}{1 + \tau}.
\]

Here \( m_\pi \) is the pion mass and \( W^2 = S_x - Q^2 + M^2 \) is the squared mass of final hadrons. Summing up over \( i = 1, ..., 8 \) corresponds to the contribution of the generalized SF \( \mathcal{F}_i(R, \tau) \) defined by (14) with the replacement of arguments (23). The infrared divergence occurs in the integral for \( R \rightarrow 0 \) in the term where \( j = 1 \) (and only in it). Explicit expressions for \( \theta_{ij}(\tau) \) are given in Appendix.

The cross section \( \hat{\sigma}^R \) can be found in the following way. For the contraction we have

\[
\frac{1}{\pi} \int d^3k k_0 \hat{\epsilon}^{mn}_{\mu\nu} w^{\mu}_{\nu} = P_L(v^a a^m + a^n v^m) \int dR d\tau R \hat{G}(R, \tau) \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \quad (i = 1, 2, 6 - 8),
\]

\[
\frac{1}{\pi} \int d^3k k_0 \hat{\epsilon}^{mn}_{\mu\nu} w^{\mu}_{\nu} = P_L(v^a v^m + a^n a^m) \int dR d\tau R \hat{G}(R, \tau) \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \quad (i = 3 - 5),
\]

where \( C_1(\tau) \) and \( B_1(\tau) \) are given in Appendix (19). Integrand of (26) has a peak coming from the region \( \tau \sim \tau_s \equiv -Q^2/S \), where \( C_1(\tau) \sim m^2 \). In the ultrarelativistic approximation the integration over \( \tau \) can be carried out analytically

\[
\int_{\tau_{\min}}^{\tau_{\max}} d\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \mathcal{G}(\tau) = \mathcal{G}(\tau_o) + \int_{\tau_{\min}}^{\tau_{\max}} d\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} [\mathcal{G}(\tau) - \mathcal{G}(\tau_o)],
\]

where

\[
\int_{\tau_{\min}}^{\tau_{\max}} d\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} = 1
\]

was used. The quantity

\[
\mathcal{G}(\tau) = \int_0^{R_{\max}} \frac{RdR}{(Q^2 + R\tau)^2} \sum_{i=1}^{8} \hat{\theta}_i(R, \tau) \mathcal{F}_i^{pl}(R, \tau)
\]

is a function over \( \tau \) regular in the ultrarelativistic approximation. The second term in (27) \( \sim m^2 \) and has to be dropped in the approximation considered. The SF \( \mathcal{F}_i^{pl}(R, \tau) \)
are parts of the generalized SF containing $P_L$. The quantities $\tilde{\theta}_i(R, \tau_s)$ can be obtained from Born ones \[12\] by the following replacements:

$$\tilde{\theta}_i(R, \tau_s) = \frac{4}{S(S-R)} \theta_i^B \left( k_1 \rightarrow \left( 1 - \frac{R}{S} \right) k_1 \right)$$

(30)

As a result the contribution $\hat{\delta} R$ can be expressed in terms of the Born cross section.

Below we give an explicit result for the total one-loop lowest order correction which includes the contribution from the radiation of a real photon ($\sigma_R$, see figure \[1\] (a, b)) and from additional virtual particles ($\sigma_V$, see figure \[1\] (c - e)) and can be presented as the sum of four infrared free terms:

$$\sigma_V + \sigma_R = \frac{\alpha}{\pi} \delta_{VR} \sigma^B + \sigma^r_V + \sigma^F + \hat{\sigma}_R$$

(31)

The factor

$$\delta_{VR} = (\ln \frac{Q^2}{m^2} - 1) \ln \frac{(W^2 - (M + m^2))^2}{(X + Q^2)(S - Q^2)}$$

$$+ \frac{3}{2} \ln \frac{Q^2}{m^2} - 2 - \frac{1}{2} \ln^2 \frac{X + Q^2}{S - Q^2} + \ln_2 \frac{S X - Q^2 M^2}{(X + Q^2)(S - Q^2)} - \frac{\pi^2}{6}$$

(32)

appears in front of the Born cross section after cancellation of infrared divergence by summing of an infrared part separated from $\sigma_R$ and so-called 'QED-part' of $V$-contribution which arises from the lepton vertex graphs including an additional virtual photon.

The contribution from electroweak loops with the exception of 'QED-part' can be written in terms of the Born cross section with the following replacement:

$$\sigma_V = \sigma^B \left( R_{V,A}^{mn} \rightarrow \delta R_{V,A}^{mn} \right),$$

(33)

where

$$\delta R_{V,A}^{\gamma\gamma} = -2 \Pi^\gamma R_{V,A}^{\gamma\gamma} - 2 \Pi^Z \chi R_{V,A}^{\gamma\gamma}$$

$$+ \frac{\alpha}{4\pi} \left[ 2 R_{V,A}^{\gamma\gamma} A_2(-Q^2, M_Z) + (1 - P_L) \frac{3}{2s_w^2} \Lambda_3(-Q^2, M_W) \right],$$

$$\delta R_{V,A}^{\gamma Z} = -2 (\Pi^\gamma + \Pi^Z) R_{V,A}^{\gamma Z} - 2 \Pi^Z (R_{V,A}^{\gamma\gamma} + \chi R_{V,A}^{Z\gamma})$$

$$+ \frac{\alpha}{4\pi} \left[ 2 (v^Z R_{V,A}^{\gamma Z} + a^Z R_{A,V}^{\gamma Z}) A_2(-Q^2, M_Z)$$

$$+ (1 - P_L) \left\{ \frac{1}{8s_w^2 c_w} \Lambda_2(-Q^2, M_W) + \frac{3}{4s_w^2} (v^Z + a^Z - \frac{c_w}{s_w}) \Lambda_3(-Q^2, M_W) \right\} \right],$$

$$\delta R_{V,A}^{Z\gamma} = -2 \Pi^Z R_{V,A}^{Z\gamma} - 2 \Pi^Z \chi R_{V,A}^{\gamma\gamma}$$

$$+ \frac{\alpha}{4\pi} \left[ 2 \left( (v^Z)^2 + (a_Z)^2 \right) R_{V,A}^{Z\gamma} + 4 v^Z a^Z R_{A,V}^{Z\gamma} \right] A_2(-Q^2, M_Z)$$

$$+ (1 - P_L) (v^Z + a^Z) \left\{ \frac{1}{4s_w^3 c_w} \Lambda_2(-Q^2, M_W) - \frac{3c_w}{2s_w^2} \Lambda_3(-Q^2, M_W) \right\} \right].$$

(34)

Here $M_{Z,W}$ are the masses of $Z$ and $W$ bosons and

$$\Pi^\gamma = -\frac{\hat{\Sigma}^\gamma(-Q^2)}{Q^2}, \quad \Pi^Z = -\frac{\hat{\Sigma}^Z(-Q^2)}{Q^2 + M_Z^2}, \quad \Pi^{\gamma Z} = -\frac{\hat{\Sigma}^{\gamma Z}(-Q^2)}{Q^2}.$$

(35)
Quantities $\hat{\Sigma}^{\gamma,\gamma Z,Z}$ are defined by the formulae (A.2,3,17,B.2-5) of [12] and $\Lambda_{2,3}$ by (B.4,B.6) of [13].

The infrared free part of the cross section of the process $\Box$ has the form

$$\sigma^F_R = -\alpha^3 y \frac{R_{\max}}{\tau_{\min}} \sum_{i=1}^8 \theta_{i1}(\tau) \int_0^{R_{\max}} \frac{dR}{R} \left[ \frac{\mathcal{F}_i(R,\tau)}{(Q^2 + R\tau)^2} - \frac{\mathcal{F}_i(0,0)}{Q^4} \right]$$

$$+ \sum_{j=2}^{k_1} \theta_{ij}(\tau) \int_0^{R_{\max}} dR \frac{R^{j-2}}{(Q^2 + R\tau)^2} \mathcal{F}_i(R,\tau).$$

As it was shown in (26-30) the last term of (31) is obtained in terms of the Born cross section

$$\hat{\sigma}_R = \frac{\alpha y}{\pi S} \int_0^{R_{\max}} \frac{RdR}{(S_x - R)} \hat{\sigma}_{pl}^B,$$

where the upper limit $R_{\max} = S(W^2 - (M + m)^2)/(S - Q^2)$, and $\hat{\sigma}_{pl}^B$ is lepton polarization part of the Born cross section with the following replacement of kinematical variables: $S \to S - R$, $Q^2 \to Q^2(1 - R/S)$ and $k_1 \eta \to k_1 \eta(1 - R/S)$.

4. Numerical Analysis

In this section the RC to different observable quantities in deep inelastic electron-proton scattering at collider are studied numerically.

The double differential cross section as a function of the polarization characteristics of the scattering particles can be presented as the sum of four terms:

$$\sigma = \sigma^u + P_L \sigma^\xi + P_N \sigma^\eta + P_N P_L \sigma^{\xi\eta},$$

the first of them is an unpolarized cross section and three others characterize the polarized contributions independent on polarization degrees. There are no problems with the luminosity measurement in the current collider experiments, so apart from the usual measurement of polarized asymmetries the absolute measurement of cross sections with different polarization configurations of beam and target will be probably possible in future polarization experiments at collider. Besides, now the new methods of data processing, when experimental information of spin observables is extracted directly from the polarized part of the cross section [15, 16] are actively developed. In [16] it is shown how to separate completely unpolarized and polarized cross sections from a sample of experimental data using a special likelihood procedure and a binning on polarization degrees. All above mentioned as well as the fact that RC to asymmetry is always constructed from RC to parts of the cross section [15, 16] allow to restrict our consideration to numerical studying of RC to all of the cross sections in the equation (38) and their combinations.

Radiative correction to these cross sections defined as a ratio of the cross section including one-loop RC only to the Born one

$$\delta^a = \frac{\sigma^{a}_{RC}}{\sigma^a_B} = \frac{\sigma^{a}_{tot}}{\sigma^a_B} - 1, \quad (a = u, \xi, \eta, \xi\eta)$$
Figure 2. Radiative correction to $\sigma^n$, $\sigma^\xi$, $\sigma^\eta$ and $\sigma^\xi\eta$ defined in (38) with (dashed curves) and without cut (full curves). The down indexes $\perp$ and $\parallel$ correspond to longitudinally and transversely polarized proton beam respectively. $\delta^\eta|$ and $\delta^\eta\perp$ are singular for $x = 0.001$ due to the Born cross section $\sigma^n$ crossing zero, so the correspondent curves $\delta^n|$ are rejected.
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Figure 3. Radiative correction to $\Delta \sigma$ defined in (40) with (dashed curves) and without cut (full curves). The down indexes $\perp$ and $\parallel$ correspond to longitudinally and transversely polarized proton beam respectively.

is presented on figure 2 as a function of scaling variables $x$ and $y$. The kinematical region corresponds to the present unpolarization collider experiment at HERA [14]. The results were obtained using GRV-parton model [17, 18] for the electroweak SF $F_i^{\gamma \gamma Z Z}$ and $g_i^{\gamma \gamma Z Z}$ which are defined by the formulae (23-25) of ref.[19] (see also [9]).

In experiments at collider a detection of a hard photon in calorimeter is used to reduce radiative effects. The dashed lines on figure 2 demonstrate the influence of an experimental cut on the RC. One of the simplest variant of the cut when events having a radiative photon energy $E_\gamma > 10\text{GeV}$ are rejected from analysis, is considered only.

From these plots one can see that the relative RC for polarized part of the cross section has the same behavior as the unpolarized one: it goes up when $y$ tends to kinematical boards ($y \to 0, y \to 1$) and when $x$ goes down. At the same time the correction to polarized parts can exceed the correction to unpolarized ones several times more. Usage of the cut on photon energy does not influence RC in the region of small $y$ and suppresses RC in the rest of the region. More detailed discussion of RC at collider with and without experimental cuts can be found in ref.[20].

The simplest case for absolute measurements in polarization experiments is the observation of difference of cross sections with the opposite configuration of the proton spin

$$\Delta \sigma_\parallel = \sigma^{\uparrow \uparrow} - \sigma^{\uparrow \downarrow}, \quad \Delta \sigma_\perp = \sigma^{\uparrow \downarrow} - \sigma^{\downarrow \uparrow}. \quad (40)$$

The first and the second arrows correspond to the lepton and proton polarization degrees equal to $\pm 1$. Main contribution to $\Delta \sigma$ comes from the electromagnetic structure functions $g_1$ and $g_2$. The values of RC to $\Delta \sigma$ defined the same as (39) are presented
Radiative electroweak effects in polarized DIS

Figure 4. The quantity $\delta_{WW}$ on the Born level (dashed curves) and on the level of radiative correction (full curves).

by figure 3 for the longitudinally and transversely polarized protons. This result was obtained using usual approximation $g_2^\gamma = g_{WW}^\gamma = g_{WW}^\gamma = 0$. At the same time there is no reason to be restricted to such a case, especially when we deal with transversal polarized protons. One of the alternative models is a well-known Wandzura and Wilcheck’s formula [21]

$$g_{WW}^\gamma (x, Q^2) = -g_1(x, Q^2) + \frac{1}{x} g_1(\xi, Q^2) \frac{d\xi}{\xi}. \quad (41)$$

The quantity characterizing the influence of a model on the cross section (40) can be defined as

$$\delta_{WW} = \frac{\Delta \sigma_\perp (g_2^\gamma = g_{WW}^\gamma) - \Delta \sigma_\perp (g_2^\gamma = 0)}{\Delta \sigma_\perp (g_2^\gamma = 0)}. \quad (42)$$

Figure 4 shows that the influence is important and the cross sections (40) calculated with $g_2^\gamma = g_{WW}^\gamma$ and with $g_2^\gamma = 0$ differ in some dozen times in the region of small $x$ and $y$.

Experimental information on electroweak structure functions can be obtained using the data of electron and positron scattering with different spin configurations both of leptons and protons. Correspondent combinations of the cross sections were offered in ref. [19] and discussed in review [9]. Here we consider two of them, which allow to extract SF $g_{1Z}^\gamma Z$ and $g_{5Z}^\gamma Z$. On the Born level they read

$$\sigma_{\perp_{\gamma}}^{\uparrow \downarrow} - \sigma_{\perp_{\gamma}}^{\uparrow \uparrow} + \sigma_{\perp_{\gamma}}^{\downarrow \downarrow} - \sigma_{\perp_{\gamma}}^{\downarrow \uparrow} + \sigma_{\perp_{\gamma}}^{\uparrow \downarrow} - \sigma_{\perp_{\gamma}}^{\downarrow \uparrow} + \sigma_{\perp_{\gamma}}^{\downarrow \downarrow} - \sigma_{\perp_{\gamma}}^{\uparrow \uparrow} =$$

$$= \frac{32\pi \alpha^2 S}{Q^4} x(2 - 2y - y^2)[v^\gamma Z g_5^\gamma Z + ((v^\gamma Z)^2 + (v^Z)^2) \chi^2 g_5^Z] \quad (43)$$

and

$$\sigma_{\perp_{\gamma}}^{\uparrow \downarrow} - \sigma_{\perp_{\gamma}}^{\uparrow \uparrow} - \sigma_{\perp_{\gamma}}^{\downarrow \downarrow} - \sigma_{\perp_{\gamma}}^{\downarrow \uparrow} + \sigma_{\perp_{\gamma}}^{\uparrow \downarrow} - \sigma_{\perp_{\gamma}}^{\downarrow \uparrow} - \sigma_{\perp_{\gamma}}^{\downarrow \downarrow} - \sigma_{\perp_{\gamma}}^{\uparrow \uparrow} =$$

$$= \frac{32\pi \alpha^2 S}{Q^4} xy(2 - y)[a^Z \chi g_1^\gamma Z + 2v^Z a^Z \chi^2 g_1^Z]. \quad (44)$$
where the lower index -(+) corresponds to the electron(positron)-proton scattering. These expressions were obtained using model \[9, 19\] where \( g_{\gamma Z}^2 = g_{\gamma Z}^4 = g_{\gamma}^Z = 0 \) and \( g_{\gamma Z,Z}^3 = 2xg_{\gamma}^Z,Z \).

From figure 5 one can see that in the region of small \( x \) the radiative correction cross section \([43]\) exceeds the Born one in several times. The main effect comes from \( \sigma^F_R \) \([33]\), integrand of which over \( R \) for the sum of the cross section \([43]\) is sketched on figure 6.

It can be understood from the analysis of this figure that using the experimental cut discussed above allows essentially to reduce radiative effects for high \( y \) and does not influence the magnitude of RC for \( y < 0.3 \).
5. Conclusion

Thus we obtained the compact and transparent formulae for the lowest order model independent electroweak radiative correction within t’Hooft-Feynman gauge. These formulae can be applied to data processing of experiments at collider. Numerical analysis of the obtained formulae for kinematics of collider experiments shows that radiative correction to the unpolarized cross section and polarized parts of cross section have the same behavior, however polarized correction can exceed the unpolarized one in several times. The detection of a hard photon in calorimeter allows to reduce the radiative effects in the region $y > 0.3$.

Appendix

In this appendix the explicit expressions for quantities $\theta_{ij}(\tau)$ are represented. Due to the factorization of infrared terms all $\theta_{i1}$ are proportional to Born contributions:

$$ \theta_{i1}(\tau) = 4F_{IR}\theta_i^B. $$

The others $\theta_{ij}(\tau)$ functions are given

$$ \begin{align*}
\theta_{12} &= 4\tau F_{IR} \\
\theta_{13} &= -2(2F + F_d\tau^2) \\
\theta_{22} &= (F_{1+}S_xS_p - F_dS_p^2\tau + 2m^2F_{2-}S_p - 2(2M^2\tau - S_x)F_{IR})/2M^2 \\
\theta_{23} &= (4FM^2 + 2F_dM^2\tau^2 - F_dS_x\tau - F_{1+}S_p)/2M^2 \\
\theta_{32} &= (F_{1+}S_xQ^2 + 2m^2F_{2-}Q^2 - m^2F_{2+}S_p\tau + 3F_{IR}S_p\tau)/2M^2 \\
\theta_{33} &= (2m^2F_{2-}\tau - F_dS_p\tau^2 - 2F_{1+}Q^2)/2M^2 \\
\theta_{34} &= -\tau F_{1+}/2M^2 \\
\theta_{42} &= 2(\eta\mathcal{K}\tau(m^2F_{2+} - 3F_{IR}) - \eta qF_{1+}Q^2 - 2m^2F_{2-}Q^2)/M \\
\theta_{43} &= 2(\eta\mathcal{K}F_d\tau^2 + 2F_{1+}^nQ^2 - 2m^2F_{2-}^n\tau)/M \\
\theta_{44} &= 2\tau F_{1+}/M \\
\theta_{62} &= (\eta\mathcal{K}(F_{1+}S_x - 2F_dS_p\tau + 2m^2F_{2-}) \\
&\quad + \eta q(F_{1+}S_p + 2F_{1R}) + 2F_{1R}S_x + 2m^2F_{2-}^nS_p)/M \\
\theta_{63} &= -(\eta\mathcal{K}F_{1+} + \eta qF_{2-}\tau + F_{1+}^nS_x\tau + F_{1+}^nS_p)/M,
\end{align*} $$

and for $i = 5, 7, 8$:

$$ \begin{align*}
\theta_{51} &= 2\eta q\theta_{31}/M, \quad \theta_{71} = \eta q\theta_{21}/M, \quad \theta_{81} = -\eta q\theta_{11}/M, \\
\theta_{52} &= 2(\eta q\theta_{32} - \theta_{33}^n)/M, \quad \theta_{72} = (\eta q\theta_{22} - \theta_{23}^n)/M, \quad \theta_{82} = -(\eta q\theta_{12} - \theta_{13}^n)/M, \\
\theta_{53} &= 2(\eta q\theta_{33} - \theta_{33}^n)/M, \quad \theta_{73} = (\eta q\theta_{23} - \theta_{23}^n)/M, \quad \theta_{83} = -(\eta q\theta_{13} - \theta_{13}^n)/M, \\
\theta_{54} &= 2(\eta q\theta_{34} - \theta_{34}^n)/M, \quad \theta_{74} = -\theta_{23}^n/M, \quad \theta_{84} = \theta_{13}^n/M, \\
\theta_{55} &= -2\theta_{34}^n/M.
\end{align*} $$
where $K = k_1 + k_2$. The following equalities define the functions $F$:

\[
F = \lambda_Q^{-1/2}, \\
F_{1R} = m^2 F_{2+} - Q^2 F_d, \\
F_d = \tau^{-1}(C_2^{-1/2}(\tau) - C_1^{-1/2}(\tau)), \\
F_{1+} = C_2^{-1/2}(\tau) + C_1^{-1/2}(\tau), \\
F_{2\pm} = B_2(\tau)C_2^{-3/2}(\tau) \mp B_1(\tau)C_1^{-3/2}(\tau), \\
F_i = -\lambda_Q^{-3/2}B_1(\tau).
\]

(48)

Here $\lambda_Q = S^2_x + 4M^2 Q^2$ and

\[
B_{1,2}(\tau) = -\frac{1}{2} \left( \lambda_Q \tau \pm S_p(S_x\tau + 2Q^2) \right), \\
C_1(\tau) = (S\tau + Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2), \\
C_2(\tau) = (X\tau - Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2).
\]

(49)

For all $i, j \theta_{ij}^{\eta}$ in (47) are defined as:

\[
\theta_{ij}^{\eta} = \theta_{ij}(F_{\text{All}} \rightarrow F_{\text{All}}^{\eta}),
\]

(50)

where

\[
2F_{\eta} = F(r_{\eta} - \tau s_{\eta}) + 2F_{1}s_{\eta}, \\
2F_{2+}^{\eta} = (2F_{1+} + \tau F_{2-})s_{\eta} + F_{2+}r_{\eta}, \\
2F_{2-}^{\eta} = (2F_d + F_{2+})\tau s_{\eta} + F_{2-}r_{\eta}, \\
2F_{d}^{\eta} = F_{1+}s_{\eta} + F_{d}r_{\eta}, \\
2F_{1+}^{\eta} = (4F + \tau^2 F_d)s_{\eta} + F_{1+}r_{\eta}.
\]

(51)

The quantities

\[
s_{\eta} = a_{\eta} + b_{\eta}, \quad r_{\eta} = \tau(a_{\eta} - b_{\eta}) + 2c_{\eta}
\]

are the combination of coefficients of the polarization vector $\eta$ expansion over the basis (see also Appendix A of ref. [3])

\[
\eta = 2(a_{\eta}k_1 + b_{\eta}k_2 + c_{\eta}p).
\]

(53)

For example in the case of the longitudinally polarized beam they read:

\[
a_{\eta} = \frac{M}{\sqrt{\lambda_s}}, \quad b_{\eta} = 0, \quad c_{\eta} = -\frac{S}{2M \sqrt{\lambda_s}}.
\]

(54)
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