RESPONSE TO “CONCERNING THERMAL TIDES ON HOT JUPITERS” ( GOODMAN 2009; ASTRO-PH 0901.3279)

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ABSTRACT

Motivated by the comments of [Goodman] on our paper concerning thermal tides ([Arras & Socrates 2009a]), we have studied an idealized problem to understand the global response of a completely fluid gas giant planet to thermal forcing at the surface ([Arras & Socrates 2009b]). Our findings disagree with the main claims in [Goodman 2009]. We find that significant quadrupole moments can indeed be induced as a result of thermal forcing. Furthermore, we find that it is possible for the orientation of the quadrupoles to be such that the planet is torqued away from synchronous rotation. Given these results, we believe our proposed thermal tide mechanism ([Arras & Socrates 2009a]) provides a viable scenario for generating steady-state asynchronous rotation, inflated radii and possibly eccentric orbits of the hot Jupiters.

Subject headings: planets

1. THE ISSUE

Gold & Soter (1969; GS from here on) originally developed the idea of thermal tide torques to explain Venus’ asynchronous spin rate. Drawing on their work, [Arras & Socrates 2009a] assessed the importance of thermal tides for the hot Jupiters. Using the simple GS prescription for the quadrupole moment, they found that thermal tides could induce large asynchronous spin, and generate tidal heating rates more than sufficient to power the observed radii. [Goodman 2009] correctly pointed out that the GS ansatz does not faithfully represent the fluid motion induced by time-dependent heating in a completely fluid atmosphere. He argued that the induced quadrupole moment would be many orders of magnitude smaller than the GS value, and with an orientation that would act to synchronize the spin, opposite the GS result.

Motivated by the criticism of [Goodman 2009], we attempted to carefully analyze a simplified problem which captures the basic physics of thermal tide excitation in fluid planets. Our results are presented in [Arras & Socrates 2009b]. From here on, and unless stated otherwise, all references to equations, figures and paper sections are to [Arras & Socrates 2009b].

In this note we compare our solutions to the fluid equations ([Arras & Socrates 2009b]) to the arguments presented in [Goodman 2009]. As [Goodman 2009] is unpublished, we quote the text from his paper posted on the Cornell University astro-ph archive (http://arxiv.org/). We then comment on the accuracy of the GS formula for the quadrupole moment. Contemporaneous with the posting of [Arras & Socrates 2009a], [Gu & Ogilvie 2003] published results on a related problem concerning thermal forcing of hot Jupiters. We briefly comment on the assumptions and results in these two papers.

1.1. [Goodman 2009]

The heart of Goodman’s argument is contained in the fourth paragraph on his page 1: “A jovian planet, being gaseous, lacks elastic strength. The excess column density of the colder parts of the atmosphere is counter-balanced to the degree that hydrostatic equilibrium holds by an indentation of the convective boundary and a redistribution of the cores mass toward the hotter longitudes. Insofar as the radial range over which mass redistribution occurs is small compared to the planetary radius, the thermal tide therefore bears no net mass quadrupole. The torque on the atmosphere is opposed by a torque on the upper parts of the convection zone.”

Through these intuitive arguments, Goodman realized that the Gold and Soter approximation ignores the following fact: though there is a flow from hot to cold at high altitudes, there is also a return flow at lower altitudes. In §5.1, we explicitly calculate the pattern of such a flow, by directly solving the fluid equations in the limit that inertia is ignored. The fact that this basic flow pattern is not contained in the derivation of the GS formula is a serious conceptual shortcoming.

The analysis in §5.1 examines the limit of zero forcing frequency, and departs from Goodman’s aforementioned paragraph in two respects. First, in the limit of zero forcing frequency, figure 3 shows that the return flow need not extend as deep as the convection zone. The bulk of the fluid motion is confined in the radiative zone, near the photosphere of the starlight. Second, in §5.1 we do not find that density perturbations at high altitude are compensated by density perturbations of the opposite sign at lower altitude. Rather, we find that the density perturbation, and hence torque, is identically zero when fluid inertia is ignored.

Given that density perturbations are zero in the limit of zero frequency, our next step was to derive finite frequency corrections. Eq. 45 in §5.2 shows that finite forcing frequency corrections give rise to a nonzero quadrupole moment. Have we violated hydrostatic balance, assumed by Goodman, by including finite frequency? The answer rests on a technical detail, which may be important in future investigations. The equation
of hydrostatic balance \( \frac{dP}{dz} = -\rho g \) is obtained from the radial momentum equation by throwing away the inertial terms. Even if we were to throw away the inertial terms in the radial momentum equation, but kept them in the horizontal momentum equations, we would still find a nonzero quadrupole moment of the correct sign, although its magnitude would be slightly different (throwing away the \(-1\) in the parenthesis in eq. 45 changes the prefactor from \(4\) to \(5\)).

By what factor should the GS quadrupole moment be reduced due to the “isostatic compensation” from the return flow? Goodman (2009) argued above that the reduction factor should be a power of \( \frac{H}{R} \). By contrast, solution of the fluid equations in the limit of small forcing frequency, ignoring gravity waves, finds a frequency dependent reduction factor \( \sim 4(N/\sigma)^2 \) (see eq. 40). Allowing for gravity waves, the calculations in figures 4 and 5 show that the response is larger than eq. 45 by 1-3 orders of magnitude in the relevant period range 1 day - 1 month, due to the excitation of gravity waves.

Goodman (2009) clarifies the range of forcing period over which the quadrupole moment should be isostatically compensated on his page 2: “Whereas terrestrial isostasy operates on such long timescales that rock behaves as fluid, the corresponding timescale for gaseous planets is dynamical, hence less than the tidal period.”

One of the key results of Arras & Socrates (2009b) is that low radial order gravity waves dominate the overlap with the thermal tide forcing. Hence the low frequency limit in eq. 45 does not apply until forcing periods \( \sim 1 \) month, comparable to or longer than the forcing periods of interest (Arras & Socrates 2009a). Note that this surprising result is completely different from the case of gravitational forcing of incompressible fluid bodies, where the low frequency limit applies below the characteristic dynamical frequency \( \sim \frac{GM_p}{R_p^3} \sim (\text{hours})^{-1} \) for a gas giant planet.

Lastly, Goodman (2009) discusses the orientation of the induced quadrupole on his page 2: “Thus, the tidal torque claimed by Arras & Socrates (2009) vanishes to first order in the density variations of the thermal tide. To the next order, the quadrupole moment of the thermal tide aligns with the hottest and most distended parts of the atmosphere, because mass elements are weighted by the squares of their distances from the center. This will lead to a torque of the opposite sign to that of \( \Delta \Omega \), hence driving the planet toward synchronous rotation. Similarly, the phase lag of the thermal tide associated with an orbital eccentricity will affect the orbit only to second order, and will tend to circularize the orbit.”

The analytic solution to the fluid equations in the low frequency non-resonant limit (eq. 45) has the correct sign to drive asynchronous rotation, contrary to Goodman’s claim. Including the effect of gravity waves, figures 4 and 5 show that the sign of the quadrupole can alternate with forcing frequency. These sign changes are due to both the Lorentzian factors in eq. 60, as well as the signs of the quadrupole moments for individual modes (figure 6). Even in this more complicated case, frequency ranges still exist where the thermal and gravitational tide torques may oppose each other, leading to an equilibrium spin state.

In summary, Goodman (2009) correctly points out the deficiencies in the Gold and Soter approximation employed by Arras & Socrates (2009a). However, the solutions to the fluid equations presented in Arras & Socrates (2009b) differ both qualitatively and quantitatively from the basic picture outlined in his work. Consequently, we disagree with Goodman’s criticism of Arras & Socrates (2009a) i.e., that thermal tides cannot lead to asynchronous spin and eccentric orbits.

1.2. The Gold & Soter approximation, and the calculations of Arras & Socrates (2009a)

Gold and Soter’s ansatz involves a major assumption: that the fluid elements remain at roughly constant pressure, so that density perturbations are related to temperature perturbations by \( \delta \rho / \rho = -\delta T / T = -\Delta s / c_p \). From eq. 29, we see this is indeed true if the \( \delta \rho \) and \( \xi \) terms can be ignored. For low frequency forcing, ignoring the \( \delta \rho \) term may be a good approximation, but for fluid atmospheres we have seen it is not a good approximation to ignore the \( \xi \) term. If there is a solid surface, and the boundary condition at this surface is \( \xi = 0 \), then the Gold and Soter ansatz may, in fact, hold. This might be realized if the heat was all deposited at or near the solid surface, rather than well above the surface.

On a more practical level, for fluid planets with surface radiative layer, we have found the Gold and Soter approximation overestimates the quadrupole moment and torque by more than an order of magnitude for the calculations in this paper. Since the steady state planetary radii powered by tidal heating found by Arras & Socrates (2009a) were rather large compared to observed planets, the reduction in torque found in this paper may bring their theory into better agreement with observations.

To summarize, we have found that the Gold and Soter ansatz for the quadrupole moment is qualitatively, but not quantitatively correct. It may be viewed as a convenient order of magnitude estimate.

1.3. Gu & Ogilvie (2009)

Gu & Ogilvie (2009) studied perturbations to a hot Jupiter atmosphere induced by time-dependent radiative heating due to asynchronous rotation. Their primary result is that waves excited by the thermal forcing can radiatively damp and transfer angular momentum vertically in the atmosphere.

Gu & Ogilvie (2009) did not study if net quadrupoles could be induced in the atmosphere. By working in plane parallel geometry and requiring the pressure perturbation to vanish at the base of the grid, they set the quadrupole to zero by hand. As the goal of their paper was to study differential rotation induced by the damping of downward propagating gravity waves, it is likely a good approximation to work in plane parallel geometry, ignoring the possible existence of net quadrupoles. Note that Arras & Socrates (2009b) focus on the complementary issue of net quadrupoles, ignoring possible driving of differential rotation.

2. Summary

In summary, the results in this paper confirm that quadrupole moments of the correct sign and approximately correct magnitude may be induced by time-dependent insolation, confirming the basic assumptions.
of Arras & Socrates (2009a). A future study will use the results from this paper in concert with a thermal evolution code for the hot Jupiters (Arras & Bildsten 2006; Arras & Socrates 2009a) to construct more detailed steady state solutions for the planetary rotation and radius, and the orbital eccentricity.

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