Cooperative backscatter NOMA with imperfect SIC: Towards energy efficient sum rate maximization in sustainable 6G networks

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A B S T R A C T

The combination of backscatter communication with non-orthogonal multiple access (NOMA) has the potential to support low-powered massive connections in upcoming sixth-generation (6G) wireless networks. More specifically, backscatter communication can harvest and use the existing RF signals in the atmosphere for communication, while NOMA provides communication to multiple wireless devices over the same frequency and time resources. This paper has proposed a new resource management framework for backscatter-aided cooperative NOMA communication in upcoming 6G networks. In particular, the proposed work has simultaneously optimized the base station’s transmit power, relaying node, the reflection coefficient of the backscatter tag, and time allocation under imperfect successive interference cancellation to maximize the sum rate of the system. To obtain an efficient solution for the resource management framework, we have proposed a combination of the bisection method and dual theory, where the sub-gradient method is adopted to optimize the Lagrangian multipliers. Numerical results have shown that the proposed solution provides excellent performance. When the performance of the proposed technique is compared to a brute-forcing search technique that guarantees optimal solution however, is very time-consuming, it was seen that the gap in performance is actually 0%. Hence, the proposed framework has provided performance equal to a cumbersome brute-force search technique while offering much less complexity.

1. Introduction

The upcoming sixth-generation (6G) systems are expected to connect billions of communication devices all over the world (Giordani et al., 2020; Ali, 2021). Most promising 6G technologies are artificial intelligence/machine learning (Shome et al., 2021; Ali et al., 2021), reconfigurable intelligent surfaces (Basar, 2020), backscatter communication (Van Huynh et al., 2018), non-orthogonal multiple access (NOMA) (Khan et al., 2020), blockchain (Sekaran et al., 2020; Jameel et al., 2020), Tera-hertz communication (Oleiwi and Al-Raweshidy, 2022), and simultaneous wireless information and power transfer (Oleiwi et al., 2022). These technologies will integrate to the current communication networks.
such as unmanned aerial vehicles (Haider et al., 2021), intelligent transportation systems (Jameel et al., 2020), cognitive radio networks (Tanveer et al., 2021), Internet of Things (Ali et al., 2021), device to device communication (Yu et al., 2021), and physical layer security (Jameel et al., 2019). However, the main challenges would be the spectrum scarcity and limited energy reservoirs, especially for those systems using conventional orthogonal multiple access (OMA) protocol (Khan et al., 2019). In this regard, researchers in academia and industry are studying the above new technologies.

Backscatter communication and NOMA are two examples of emerging technologies that enhance spectrum and energy efficiency of 6G systems (Jameel et al., 2020). Further, NOMA has been shown to outperform OMA protocol (Khan et al., 2020). With the help of the ambient energy harvesting approach, backscatter communication allows sensor devices to transmit data towards surrounding users by reflecting and modulating radio frequency (RF) signal (Li et al., 2021). The basic architecture of backscatter sensor device can be seen in Fig. 1. One the other hand, NOMA enables the transmission of multiple users over the same spectrum/time resources using the superposition coding, and successive interference cancellation (SIC) techniques (Liu et al., 2017; Khan et al., 2021). The performance of backscatter communication has been previously studied in OMA networks (F. Jameel et al., 2021; Khan et al., 2021; Jameel et al., 2019). The integration of NOMA with backscatter communication is a hot topic and some works in literature have investigated different problems related to backscatter communication in NOMA wireless networks.

### 1.1. Recent Advances in NOMA Backscatter Communication Networks

Cooperative communication has been shown to improve the performance of communication systems significantly (Jameel et al., 2019). In cooperative communication, either a dedicated device (called relay) is used to forward the data of a specific user (Ali et al., 2021) or a communicating user cooperates by relaying the data to other users (Jiang et al., 2021). The works in Ali et al. (2021) and Jiang et al. (2021) optimized power allocation in cooperative NOMA systems to maximize the sum rate of the system and to achieve fairness, respectively. For sum rate maximization in cooperative NOMA based device-to-device communication, Jiang et al. (2018) proposed an optimization framework. Kim et al. (2018) proposed a power optimization algorithm to achieve the maximum capacity scaling in a cooperative NOMA scenario. Further, Reference (Khan, 2019) explored an optimization problem to enhance the secrecy rate of NOMA cooperative communication. Of late, the work of authors in Oleiwi and Al-Raweshidy (2022) have proposed a cooperative simultaneous wireless information and power transfer in NOMA-enabled terahertz communications to improve energy and spectral efficiency of the system.

Recently, researchers have studied the integration of NOMA with backscatter communication in next generation wireless networks (Guo et al., 2018). For instance, Zhang et al. (2019) provided the closed-form expressions for the outage probability and ergodic capacity in backscatter-aided NOMA symbiotic system. Khan et al. (2021) proposed backscatter-aided vehicular-to-everything network and jointly optimized the transmit power of base station (BS) and roadside units to maximize the sum capacity of the NOMA system. The work in Yang et al. (2020) jointly optimized the time allocation and reflection coefficient of the backscatter tag to maximize the minimum throughput of the NOMA Internet of Things network. To maximize the energy efficiency, Xu et al. (2021) explored a joint optimization framework of transmit power and reflection coefficient in backscatter-aided NOMA network. The authors of Nazar et al. (2021) derived a closed-form expression for bit error rate in backscatter-aided NOMA network under imperfect SIC. Besides, the authors of Khan et al. (2021) investigated the optimization problem of transmit power and reflection coefficient in backscatter-aided NOMA network under imperfect SIC. Reference (Li et al., 2021) calculated the closed-form expressions of intercept and outage probability for backscatter-aided NOMA system under residual hardware impairments and imperfect channel state information (CSI) and SIC. To improve the energy efficiency of the system, the work in Khan et al. (2021) investigated a new optimization approach for efficient power allocation and reflection coefficient under imperfect SIC. Ihsan et al. (2021) proposed an uplink optimization framework for NOMA backscatter sensor communication under channel estimation errors in intelligent transportation systems. The research work in Khan et al. (2021) has maximized the spectral efficiency of NOMA backscatter communication networks. Ahmed et al. (2021) also maximized energy efficiency of multi-cell NOMA backscatter sensor networks under imperfect SIC. Of late, the authors of Khan et al. (2021) also considered imperfect SIC in multi-cell NOMA backscatter communication to maximize the spectral efficiency of the network.

### 1.2. Motivation and Contributions

Most of the above literature (Zhang et al., 2019; Khan et al., 2021; Yang et al., 2020; Xu et al., 2021; Ihsan et al., 2021; Khan et al., 2021; Ali et al., 2021; Jiang et al., 2021; Jiang et al., 2018; Kim et al., 2018) assumes perfect SIC in their systems which is impractical. The works in (Nazar et al., 2021; Khan et al., 2021; Li et al., 2021; Khan et al., 2021; Ahmed et al., 2021; Khan et al., 2021) consider imperfect SIC, however, cooperation among the communicating users was not considered. Further, the authors in (Ali et al., 2021; Jiang et al., 2021; Jiang et al., 2018; Kim et al., 2018; Khan, 2019) just optimized the power allocation while considering equal time allocation on both hops. To the best of our knowledge, the problem of resource management that simultaneously optimizes the transmit power of BS and relaying node, the reflection coefficient of backscatter tag, and time allocation in cooperative NOMA network under imperfect SIC has not yet been investigated. To fill this bridge, we aim to provide a resource management framework to maximize the sum rate of backscatter-aided cooperative NOMA network under imperfect
SIC. Closed-form solutions are derived by dual theory and KKT conditions, where numerical results demonstrate the superiority of joint optimization with backscattering enabled system over the conventional fix time cooperation and communication without any backscattering. The main contributions of our work can also be summarized as:

1. This paper considers a new optimization framework for a backscatter-aided NOMA cooperative communication, where a BS transmits superimposed data to two NOMA users. This work also considers that the near user performs cooperation by relaying data to a far user. Meanwhile, a backscatter tag also receives superimposed signal from BS and cooperative user. The backscatter tag modulates its information and then reflect it towards both users. Thus, users also act as readers. The communication process takes two-time slots. In the first time slot, BS transmits to both users, and the backscatter tag reflects the received signal of BS towards both users. In the second time slot, near user cooperate by relaying data to a far user, and the backscatter tag reflects the relaying signal to both users by adding useful information.

2. We formulate a new optimization problem to maximize the sum rate of backscatter-aided NOMA cooperative communication under imperfect SIC decoding while satisfying various practical constraints. In particular, we jointly optimize the transmit power of BS, cooperative power of near user, the reflection coefficient of backscatter tag, and time allocation of both time slots while ensuring the minimum data rate of both users. The formulated problem is non convex optimization, and joint optimization cannot be designed to obtain the solution. Thus, we adopt the bisection method and dual theory to obtain an efficient solution, where the values of dual variables are iteratively updated.

3. To see the benefits of backscatter communication, the proposed work also provides the optimization of conventional cooperative NOMA communication without backscattering and the Brute Force Search technique for comparison. Numerical results are plotted using Monte Carlo simulations. Results demonstrate that the proposed optimization approach obtains a higher sum rate than the other benchmark scheme and converges within a reasonable number of iterations.

The remaining of our work can be organized as follows. Section II will provide the system model, various assumptions, and optimization problem. Section III will discuss different steps of proposed optimization solution to enhance the sum rate of the system. Section IV will present and discuss the numerical results based on Monte Carlo simulations while Section V will conclude this paper.

| Parameter | Description |
|-----------|-------------|
| $\rho$    | Available power at the BS |
| $\rho_r$  | Available power at $U_1$ for relaying |
| $\Lambda$ | Fraction of $\rho$ allocated for transmission of $U_1$ |
| $(1-\Lambda)$ | Fraction of $\rho$ allocated for transmission of $U_2$ |
| $T$ | Time allocated for direct transmission |
| $(1-T)$ | Time allocated for cooperation |
| $\phi_1$ | Reflection coefficient in first time slot $(T)$ |
| $\phi_2$ | Reflection coefficient in second time slot $(1-T)$ |
| $g_1$ | Channel gain from BS to $U_1$ |
| $g_2$ | Channel gain from BS to $U_2$ |
| $g_{tt}$ | Channel gain from BS to Backscatter Tag |
| $h_1$ | Channel gain from $U_1$ to $U_2$ |
| $h_2$ | Channel gain from $U_2$ to Backscatter Tag |
| $f_1$ | Channel from Backscatter Tag to $U_1$ |
| $f_2$ | Channel from Backscatter Tag to $U_2$ |
| $\sigma^2$ | Variance of additive white Gaussian noise |

**Table 1** Description of important parameters.

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**2. System Model and Problem Formulation**

We consider a backscatter-aided downlink cooperative NOMA communication as shown in Fig. 2, where a BS serves data to two users (end users) in the cell. The coverage area also contains a backscatter tag that sends data to both the receivers using the existing radio signals. More specifically, the backscatter tag also receives the superimposed signal of BS and harvests some energy from the signal to operate the circuit. Then modulate useful data and reflect it towards users using the reflection coefficient. This work considers that all the nodes are equipped with a single antenna and that the perfect channel state information is available at the BS. The channels between different links are independent and identical and undergo Rayleigh fading. The user closer to the BS has better channel conditions is represented as $U_1$, and the far user with a comparatively much lower value of channel gain is denoted as $U_2$. We have assumed that $U_1$ is also closer to the backscatter tag than $U_2$. The available transmission time is divided into two slots. In the first time slot $(T)$, the BS transmits data to both users; the signal from the BS is also received and reflected by the backscatter tag. The $U_1$ employs SIC to decode its own signal; during the process of SIC, the $U_1$ first decodes the signal of $U_2$ and then subtracts this decoded data from the received signal to decode its own signal. In the remaining time $(1-T)$, the $U_1$ cooperates by relaying the decoded data to $U_2$. This relayed data is also reflected by the backscatter tag. Based on the above discussion and consideration, the signal received at $U_1$ and $U_2$ in the first time slot is given as:

$$
y_{U1,1} = \sqrt{\beta_1} \left( \sqrt{P} x_{U1} + \sqrt{P(1-\Lambda)} x_{U2} \right) + \sqrt{\phi_1 \beta_1} \left( \sqrt{P} x_{U1} + \sqrt{P(1-\Lambda)} x_{U2} \right) z(t).
$$

(1)
\[ y_{U2,1} = \sqrt{P}h_1x_{U2} + \sqrt{P}f_1h_2x_{U2}z(t), \]

where \( z(t) \) is the signal added by the backscattering tag, with 
\[ E \left[ |z(t)|^2 \right] = 1 \] and \( P \) denotes the total available power at the BS. 
\( x_{U1} \) is the data symbol of \( U_1 \) and \( x_{U2} \) represents the data symbol of \( U_2 \), respectively. \( \lambda \) and \( (1-\lambda) \) are the fractions of \( P \) allocated for the signal of \( U_1 \) and \( U_2 \) by the BS. \( g_1, g_2, h_1, \) and \( h_2 \) are the channel gains from BS to \( U_1, U_2 \), from BS to backscatter tag from \( U_1 \) to \( U_2 \) and from \( U_2 \) to the backscatter tag, respectively. Similarly, \( f_1 \) and \( f_2 \) are the channel gains from backscatter tag to \( U_1 \) and \( U_2 \). \( \phi_i \) is the reflection coefficients in the first time slot. Then, in the second time slot, the signal received by \( U_2 \) is given as:

\[ y_{U2,2} = \sqrt{P}h_1x_{U2} + \sqrt{P}f_1h_2x_{U2}z(t), \]

where \( P \) is the power invested by \( U_1 \) for relaying data to \( U_2 \). Note that we have considered that on receiving the signal from the backscattering tag, \( U_1 \) successfully perform SIC to decode its symbol without any interference. However, as the BS is much far compared to the backscattering tag, hence the SIC is imperfect and the \( U_1 \) faces some interference while decoding this symbol. Then, the achievable rates at the \( U_1 \) and \( U_2 \) in the first time slot are given as \( R_1 \) and \( R_2 \). The rate due to relaying at the \( U_2 \) is \( (1-T)R_3 \), and the rate of decoding of the \( U_2 \) symbol at the \( U_1 \) in the first time slot is \( TR_3 \). The values of \( R_1, R_2, R_3 \) and \( TR_3 \) are computed as:

\[ R_1 = \log_2 \left( 1 + \frac{P(1-\lambda)(g_1 + \phi_f h_2 g_3)}{P(1-\lambda)g_1\beta + \sigma^2} \right), \]

\[ R_2 = \log_2 \left( 1 + \frac{P(1-\lambda)(g_2 + \phi_f h_2 g_3)}{P(1-\lambda)g_2\beta + \sigma^2} \right), \]

\[ R_3 = \log_2 \left( 1 + \frac{P(1-\lambda)(g_1 + \phi_f h_2 g_3)}{P(1-\lambda)g_1\beta + \sigma^2} \right), \]

\[ TR_3 = \log_2 \left( 1 + \frac{P(1-\lambda)(g_1 + \phi_f h_2 g_3)}{P(1-\lambda)g_1\beta + \sigma^2} \right), \]

where \( \sigma^2 \) is the variance of additive white Gaussian noise. As this work considers the practical scenario of imperfect SIC, \( \beta \) signifies the fraction of interference faced by the \( U_1 \) while decoding its own data. Then, the problem of maximizing the sum rate is written as:

\[ \mathscr{P} : \max_{(\lambda, \phi_f, h_1, h_2)} \sum R_1 + R_2 + (1-T)R_3 \]

subject to:

\[ \begin{cases} c_1 : TR_1 \geq R_{\text{min}}, \\ c_2 : TR_2 + (1-T)R_1 \geq R_{\text{min}}, \\ c_3 : TR_3 \geq R_{\text{min}} + (1-T)R_3, \\ c_4 : P_r \leq P_{\text{max}}, \forall i, j, \\ c_5 : 0 \leq T \leq 1, 0 \leq \phi_1 \leq 1, 1 \leq \phi_2 \leq 1, 0 \leq \lambda \leq 1, \end{cases} \]

where the objective in (8) is to maximize the sum rate of the system. The first two constraints \( c_1 \) and \( c_2 \) ensure that the minimum rate requirement of the users is satisfied, where \( R_{\text{min}} \) denotes the rate required by the users. Similar to Zhao et al. (2010), the third constraint \( c_3 \) guarantees that the cooperation required is fulfilled. Then, \( c_4 \) makes sure that power allocation at \( U_1 \) will follow the power budget, where \( P_{\text{max}} \) is the battery capacity of the user. Finally, constraint \( c_5 \) ensures that the values of time and reflection coefficients will remain within the practical range.

### 3. Proposed Solution

The considered problem \( \mathscr{P} \) is a multi-variable complex optimization problem as the objective function is not concave in all variables. Thus, a joint optimization framework cannot be designed to obtain the solution. For optimization, we take into consideration the independent impact of all parameters on the objective function and propose solutions subject to the nature of the objective function with respect to the specific parameters.

First we investigate the efficient value of reflection coefficient at backscatter tag for both time slots. As the objective function is concave with respect to \( \phi_1, \phi_2 \) and \( P_r \). We employ duality theory to find the solution for these variables, where the Lagrangian of the problem is written as:

\[ L = TR_1 + TR_2 + (1-T)R_3 + \lambda_1(TR_1 - R_{\text{min}}) \]

\[ + \lambda_2(TR_2 + (1-T)R_3 - R_{\text{min}}) + \mu(TR_3 - TR_2 - (1-T)R_3) \]

\[ + \eta(P_{\text{max}} - P_r) + \zeta_1(1-\phi_1) + \zeta_2(1-\phi_2), \]

where \( \lambda_1, \lambda_2, \mu, \eta, \zeta_1 \) and \( \zeta_2 \) are the Lagrangian multipliers. Then applying Karush–Kuhn–Tucker (KKT) conditions and differentiating with respect to \( \phi \) results in:

\[ \phi_1^* = \max(0, \eta), \]

\[ \phi_2^* = \max(0, \omega), \]

\[ \omega = \frac{-f_2g_3(1+\lambda_2-\mu)P_r(-1+T)-h_1P_r+\sigma^2}{f_2g_3P_{\text{max}}}, \]

Next we compute the efficient value of relayed power at user. For the solution of \( P_r \), we take advantage of the fact that the first two terms in \( \mathscr{E}_3 \) of problem (8) are constant with respect to \( P_r \). Thus, we write the constraint in \( \mathscr{E}_3 \) of problem (8) as:

\[ \mathscr{E}_3 : \left( 2^\gamma - 1 \right) \sigma^2 \geq P_r(h_1 + \phi_f h_2). \]

where \( \gamma = (TR_3 - TR_2)/(1-T) \). After this transformation, next applying KKT conditions and differentiating the Lagrangian with respect to \( P_r \), gives us:

\[ P_r^* = \max(0, \Psi), \]

\[ \Psi = \frac{h_1(1+\lambda_2)(1-T)+f_2g_3(1+\lambda_2)\phi_2(1-T)-\eta(\mu+\sigma^2)}{(\eta+\mu)h_1+f_2g_3P_{\text{max}}}. \]

Now, we calculate the power allocation coefficient at BS. For perfect SIC, i.e. \( \beta = 0 \), the objective function in problem (8) is concave with respect to \( \Lambda \). Thus the solution can be found using techniques proposed in Reference (Khan et al., 2020). However, for the considered imperfect SIC case, the eigenvalue function of the objective with respect to \( \Lambda \) is given as \( EV = \frac{1}{\Lambda} \), where the values of \( \kappa \) and \( \chi \) are:
\[ \kappa = a^2 \left( c^4 - (1 + \Lambda)^4 + \sigma^4 (2b^2 + \sigma^2) \right) \\
+ 2c^2 \sigma (b(3 - 2\Lambda)\Lambda - 2(1 + \Lambda)^2) \sigma^2 \\
- 2c^3 (1 + \Lambda) (b(1 - \Lambda + \Lambda^2) + 2(-1 + \Lambda)^2 \sigma^2) \]
+ c^7 (b^2 \Lambda^4 + 2b \Lambda^2 (3 - 4an + 2\Lambda)^2) + 6(1 + \Lambda)^2 \sigma^4) \]
\[ - 2ab\Lambda^2 (c + \sigma^2) (2c(1 + \Lambda - \sigma^2) + b(c - 2a - \sigma^2)) = 0, \quad \chi = (a + \sigma^2)^2 (c + b - \Lambda + \sigma + \sigma^2)^2 \]
where \( a = \rho(g_2 + \phi f_2 g_3), b = \rho(g_1 + \phi f_1 g_3) \) and \( c = \rho g_1 \beta \). The eigenvalue is positive for \( 0 < \alpha < 1 \) and \( \alpha > \sigma^2 \). Hence, the objective of the problem is convex with respect to \( \Lambda \). For \( \omega(\rho) \), denoting the value of the objective at \( \Lambda = \mu \), convexity implies that we have \( \omega(\mu) + (1 - \mu) \omega(1) \leq \omega(\mu), \) for \( \mu \in [0, 1) \). This shows that the maximum value of the objective function lies on either of the two extremes of \( \Lambda \). Thus, \( U_1 \) is closer to the BOS compared to \( U_2 \), thus, the lower bound can be sorted as \( g_1 > g_3 \). This shows that \( g_1 \) in problem (8) is always satisfied for any value of \( \Lambda \) and the feasibility of the constraint depends only on the value of \( P_P \). Thus the value of \( \Lambda \) is bounded by the rate requirements of both users. The lower bound due to the rate requirement of \( U_1 \) is given by:
\[ \chi_{U_1} = \frac{2^{R_{\text{sec}}(1) - 1}}{P(g_1 - \rho g_1 + \beta 2^{R_{\text{sec}}(1) - 1} g_2 + f_1 g_3 \phi_1)}, \]
where \( R_k = (1 - T) \log_2 \left( 1 + \frac{P}{h(1+\omega)\sigma^2} \right) \). Then, the lower bound and upper bounds are given as \( x_{L} = \min(x_{U_1}, 1) \) and \( x_{U} = \max(x_{U_1}, 0), \) respectively. After this, calculating \( \Lambda^* \) is straightforward, if \( \omega(x_{U}) > \omega(x_{L}) \), then \( \Lambda^* = x_{L} \), otherwise, \( \Lambda^* = x_{U} \). Here, \( \omega(\rho) \) represents the value of the objective function at \( \Lambda = \rho \). For finding the solution of Lagrangian multiplier, we employ subgradient method where in each iteration, the values of the dual variables are updated as:
\[ x_{L}^{i+1} = x_{L} - \delta(T_k - R_{\text{min}}), \]
\[ x_{U}^{i+1} = x_{U} - \delta(T_k - R_k), \]
\[ \mu^{i+1} = \mu^{i} + \delta(T_k - T_k), \]
\[ \eta^{i+1} = \eta^{i} + \delta(P_{\text{max}} - P_k), \]
\[ \xi_{L}^{i+1} = \xi_{L} - \delta(1 - \eta), \]
\[ \xi_{U}^{i+1} = \xi_{U} - \delta(1 - \phi), \]
\[ \theta = f_1 g_3 P(g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2) \]
\[ \times \left( g_2^2 \Lambda^2 P^2 + g_1 \Lambda^2 \Gamma + \Gamma (\beta T - \beta (1 + \beta) \Lambda P)^2 \right) + (\Lambda + \Lambda^L + \mu - \mu^P)^2 \]
\[ + (g_2^2 \Gamma^2 - g_2 \Lambda^P - \sigma^2)(g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2) \]
\[ + (g_2^2 \Gamma^2 - g_2 \Lambda^P - \sigma^2)(g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2) \]
\[ \times (g_2 g_1 \Gamma (1 + \Lambda - \sigma^2) + (g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2)), \]
\[ \theta_1 = g_1 P \left( f_1 g_3 \Lambda^P (g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2) \right) \\
(2g_1 \Lambda^P + (\Gamma + \Lambda^L + \mu - \mu^P)^2 \sigma^2) \\
+ f_2 (\beta g_2 \Gamma - g_2 P - \Lambda^P + \sigma^2) (g_2 P + \sigma^2) \]
\[ (g_2 \Lambda^P + \sigma^2) \]
\[ + f_1 f_2 g_2 \left( g_2 \Lambda^P + g_1 \Lambda^P (\Gamma + \Lambda^L + \mu - \mu^P) + g_1 4 + \Lambda + 3 \Lambda - 3 \mu \]
\[ + (\Lambda + 1 + \mu + \Lambda) g_3 (g_2 P + \sigma^2) (g_2 \Lambda^P + \sigma^2) \]
\[ + (g_2 P + \sigma^2) \sigma^2 (g_2 \Lambda^P + \sigma^2) \]
Algorithm 1 Solution for $T^*$ using Bisection method

(1) **Initialize**: system parameters and variables.
(2) Calculate $\phi_1^*, \phi_2^*, \Lambda^*$ and $P^*$ for $T = 0.5 - \Delta$
(3) Set $t_1 = 0$ and $t_2 = 1$, $R_{\text{best}} = \omega(\phi_1^*, \phi_2^*, \Lambda^*, P^*, T)$
(4) while $|t_1 - t_2| > \varepsilon$
(5) Set $\tau = \frac{t_1 + t_2}{2}$
(6) Calculate $\phi_1^*, \phi_2^*, \Lambda^*, P^*$ for $T = \tau$
(7) if $\omega(\phi_1^*, \phi_2^*, \Lambda^*, P^*, \tau) > R_{\text{best}}$
(8) set $R_{\text{best}} = \omega(\phi_1^*, \phi_2^*, \Lambda^*, P^*, \tau), t_1 = \tau, T^* = \tau$
(9) else
(10) set $t_2 = \tau$
(11) end while
(12) Return $T^*$

In the bisection method, first all the system parameters are initialized. In the second step, for the given T, where $T = 0.5 - \Delta$ (for $\Delta$ be a small positive number close to zero), the values of $\phi_1^*, \phi_2^*, \Lambda^*$ and $P^*$ are calculated. In third step, the bounds of the bisection method are initialized, where the lower bound $(t_1)$ and the upper bound are set equal to 0 and 1, respectively. Then, we calculate the value of objective function for the given values of $\phi_1^*, \phi_2^*, \Lambda^*, P^*$, and $T$. The function $\omega(\phi_1^*, \phi_2^*, \Lambda^*, P^*, T)$ signifies the value of objective function for the given parameters. We set Rbest equal to this rate, where Rbest denotes the maximum value of sum rate achievable till now. In step 4, if the difference between $t_1$ and $t_2$ is greater than the permitted error $\varepsilon$, the expected solution of $T$ represented as $\tau$ in step 5 is calculated. Next in step 6, the values of $\phi_1^*, \phi_2^*, \Lambda^*$ and $P^*$ for $T = \tau$ are calculated. If the value of $\omega(\tau)$ is greater than Rbest for these parameters, then we update Rbest, $T^*$ and set the lower bound $t_1$ equal to $\tau$. Otherwise, the upper bound is set equal to $\tau$ in step 10. Steps 5 to 10 are repeated until the difference between $t_1$ and $t_2$ falls below the permitted error.

The computational complexity of the proposed scheme can be given as $O(BIC)$, where $I$ denotes the number of iterations required by the Duality based method to provide the solution, $B$ represents the steps taken by the bisection method to reach the best solution of time allocation, and $C$ is the computational complexity of computing Eqs. (11), (14), (18)–(25). Note that in the case where time allocation is not optimized, the computational complexity of the framework will be $O(I)$. In the case of multi-cell system, the complexity will remain unchanged, because the optimization framework will run in each cell independently of all the other cells in the network.

### 4. Numerical Results and Discussion

In this section, we present and discuss the simulation results. For the simulations, we have taken $\sigma^2 = 0.001, R_{\text{min}} = 0.1, P = 40$ dBm, $P_{\text{max}} = 20$ dBm, $\beta = 0.1, \epsilon = 0.001$ and $\Delta = 0.01$, until specified otherwise. We use Monte Carlo simulation to obtain the average results. The detailed simulation parameters are also provided in Table 2. In this work, we consider 1 Hertz bandwidth over each link. More specifically, we compute the sum rate per Hertz. Moreover, our optimization framework is independent to the effect of bandwidth/frequency, any bandwidth can be efficiently used to obtain the simulation results. We provide the comparison of four systems Opt, NBS, ET and NBS-ET, respectively. More specifically, Opt refers to the proposed backscattering aided optimization framework. Then, NBS is the system without backscattering tag and we use the same proposed optimization technique to optimize all the system parameters. In ET scheme, all the parameters are optimized for the fixed value of time allocation i.e. $T = 0.5$.

**Table 2** Simulation parameters and values.

| Parameter                | Value       |
|--------------------------|-------------|
| Power budget of BS, $P$  | 40 dBm      |
| Reflection coefficient of backscatter, $\beta$ | $0 < \beta < 1$ |
| Channel type             | i.i.d Raleigh fading |
| Cooperation power, $P_{\text{max}}$ | 30 dBm |
| Imperfect SIC             | 0.1–0.5     |
| Antenna type             | Omni-directional |
| Channel realization      | $10^3$      |
| Minimum data rate $R_{\text{min}}$ | $0.1–1.0$ b/s/Hz |
| Pathloss exponent        | 3           |
| Bandwidth                | 1 Hertz     |
| Noise power density, $\sigma^2$ | 0.001 |
| Permitted error value, $\epsilon$ | 0.001 |
| Circuit power            | 5 dBm       |

**NBS-ET** scheme signifies a system with equal time allocation with no backscattering tag in the system. Since the considered problem has never been solved in the literature before. Thus, to evaluate the performance of the proposed frameworks, we compare the performance with a brute force search BFS technique, where the value of the objective function is checked for each possible value of the optimization variables $(\phi_1, \phi_2, \Lambda, P_t, T)$. This is a very slow technique, thus, it can not be employed in practical systems. However, this technique provides an optimal solution which can be used to evaluate the performance of the proposed optimization frameworks. The effect of increasing $P$ on the sum rate is shown in Fig. 3. It is clear from the figure that the Opt provides the same results as BFS technique. This proves the optimality of the proposed solution technique. It can be seen that an increase in the value of $P$ results in increasing the sum rate of the system as more power becomes available for the transmission. This is because the objective function is a concave monotonically increasing function of $P$. Further, for fixed $R_{\text{min}}$ when the value of available power is increased, the difference in the rates of equal time schemes and optimal time schemes also increases. The reason behind this is that, at a small value of $P$ if we reduce the time allocated for the transmission of a user, then the rate requirement might not be satisfied. However, when $P$ is increased, more power is allocated for the transmission and so the parameter $T$ becomes more flexible. Thus, optimizing $T$ gives us much better rate compared to equal $T$ cases (ET and NBS-ET).

The impact of increasing the minimum rate requirement of each user on the sum rate of the system is shown in Fig. 4. It can be seen that for each case, an increase in $R_{\text{min}}$ results in decreasing the overall rate of the system. This is because when $R_{\text{min}}$ is increased, more
resources are required to meet the rate requirement of all users, so the optimization becomes more tightly bounded. Thus, the optimization is performed for comparatively less resources and the sum rate decreases. The figure shows that the best performance is provided by the proposed Opt scheme. This is because in Opt, all the resources are being optimized and the users benefit from the additional gain due to backscattering tag. In NBS, as the system has no backscattering tag, the SINR of the users is less compared to the Opt case. Hence, the sum rate of the system is less compared to Opt. Similarly, the result shows that optimizing $T$ has a significant impact on the performance, as the sum rates offered by ET and NBS-ET are far less compared to NBS and Opt.

The Fig. 5 shows that larger value of imperfect SIC $\beta$ results in smaller sum rate of the system. With an increase in the interference faced by the $U_i$, the amount of available resources required by the user to meet the rate requirement also increase. Thus, the sum rate of the system decreases. All the schemes provide better performance when the value of $\beta$ is small. Another point worth mentioning here is that the gap in the rates provided by the backscattering system and networks with no backscattering increases if we optimize $T$. As for the same amount of transmission power the backscattering increases the SINR of the users as compared to the SINR in no backscattering case. Hence, the users in the backscattering systems can achieve the minimum required rate at comparatively smaller values of allocated power and time. Moreover, for backscattering system, the benefit of optimizing $T$ also increases. This behavior is also consistent in all the simulation results.

The convergence behavior of the proposed Opt framework is shown in Fig. 6. In the Opt framework, when the dual variables converge, the solution is returned to the bisection method as shown in Algorithm 1. After this, the bisection method provides the updated value of $\tau$. This updated $\tau$ is again used to calculate optimal $\phi_1^*, \phi_2^*, \Lambda^*, P_r^*$, where the dual variables are again updated by using subgradient method. Due to this alternate optimization, the dual variables are updated several times till the optimal value of $\tau$ is reached by the bisection method. This behavior is clear from the Fig. 6, e.g. at iterations $t = 4000$ once the dual variables converge, the value of $\tau$ is updated by the bisection method, and the process of dual variable update starts again. However, it is clear from the figure that after a certain number of iterations, the optimal values of all the variables are reached and so the dual variables converge for iterations $t > 6000$. In addition, the convergence of variable $T$ is shown in Fig. 7. It can be seen that the bisection method provides much faster convergence compared to the dual variables in Fig. 6.

5. Conclusion

Backscatter communication and NOMA are two promising technologies for upcoming 6G networks due to high energy and
spectral efficiency. This paper has provided the resource management framework for backscatter-aided cooperative NOMA network under imperfect SIC decoding. In particular, time allocation, power loading at BS and cooperative user, and reflection coefficient of the backscatter tag have been simultaneously optimized to maximize the sum rate of cooperative NOMA system. Closed-form solutions have been calculated by dual theory and KKT conditions. The numerical results show the efficiency of the proposed framework. Further, the results make it clear that optimizing time allocation along with power loading is very important because it significantly enhances the performance of the system, however, joint optimization of time with other optimization parameters are usually ignored in literature because of the increased complexity.

Our proposed framework can be extended in many ways. For example, it can be extended to multi-cell NOMA cooperative communication. In that case, interference due to neighboring BSs and backscatter tags will be taken into account. This will make the problem more interesting and hard. Besides, multiple backscatter tags can also be considered in one cell to maximize spectral and energy efficiency. Further, reconfigurable intelligent surfaces is emerging 6G technology and can be used in the existing model to improve the received signal strength of the far user and replace unreliable near user cooperation. These important yet solved problems will be investigated in the future.

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Appendix A
This section provides the values of $\theta_1, \theta_2, \theta_3$ and $\theta_4$ as (26)–(29); where in (28)–(29), the values of $\Gamma = 1 + \Lambda, \Upsilon = 1 - \Lambda$, and $v = 1 + \lambda$, respectively.

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