Vibration power flow analysis of pipe perpendicular to plate structures

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Abstract. Pipe perpendicular to plate structures are commonly used in ships. By using the mobility approach, the characteristics of vibration power flow propagation are studied. The dynamic motions of the structure are derived by Euler-Bernoulli beam theory and Helmholtz equation. By applying the lateral and axial excitations, harmonic responses of the displacements of pipe and radiated acoustic pressure in the air are analyzed. It is found that characteristics of the vibration power flow vary with frequency and the direction of exciting forces. Additionally, parametric studies are carried out to investigate the effects of vibration absorber to control the propagation path of vibration power flow.

1. Introduction
Vibration power flow method studies structural vibration and noise response from the standpoint of energy. Compared with traditional vibration analysis methods, it can not only give the absolute measurement of vibration energy, but also give the information of vibration energy transfer path. Numerous previous work have lead to the numerical of typical structures. The study of the vibration power of these structures is of considerable importance in many engineering application, especially in the field of ship industry. The vibration power of pipe perpendicular plate structure has been extensively investigated in the literature.

In past decades there have been numerous works on vibration characteristics. Goyder et al. first introduced the vibration power flow analysis techniques, and studied the near and far field power flow of beam and plates under force and torque excitation [1-3]. J Yan et al. studied the vibration power flow propagation in an infinite periodic ring-stiffened cylindrical shell immersed in water by using space harmonic analysis method [4]. MB Xu et al. studied the effect of fluid on the vibration power propagation of an infinite elastic circular cylindrical shell excited by a line circumferential cosine harmonic force [5-6]. On the basis of this study, Jingxi LIU et al. studied the characteristics of vibration power flow in an infinite laminated composite cylindrical shell filled with fluid [7]. Until now, most vibration power research has concentrated on a single structure, such as rods, beams, thin plates and orthotropic thin plates, J H Song et al. used the wave transmission approach to study the transmission of vibration energy flow though beam-plate junction structure [8]. Seong et al. predicted
the vibration response of reinforced beam-plate coupled structures in frequencies ranging from medium to high by using power flow analysis [9].

Pipe perpendicular plate is one of the most commonly used structures in ship structures. However, little attention has been focused on the vibro-acoustic characteristics of such structure, and exactly how vibration absorber influences the transfer path of vibration power flow remains to be elucidated.

The present work established a vibration power analysis model, aimed at carrying out a complete vibro-acoustic characteristics of the pipe-plate configuration. The effects of different excitation and vibration absorbers are investigated in the following section. The proposed approach helps to give some physical insight into the configuration and offers some proposals in the design process.

2. The vibration power flow of pipe-to-plate

The transmission of vibration in structure is essentially the transmission of vibration energy, and the magnitude of vibration energy can be described by the magnitude of power flow. Power flow is defined as the vibration energy per unit area of time flowing perpendicular to the direction of wave propagation. It takes into account not only the magnitude of force and velocity, but also the phase relationship between them. Obviously, when the structure is applied on a harmonic force $F e^{i\omega t}$ and the velocity is $V e^{i\omega t}$, the time averaged power flow input at this point is written as:

$$P_\text{in} = \frac{1}{2} \text{Re}\{F \cdot V^*\} = \frac{1}{2} \text{Re}\{F \cdot (V^*)^*\} = \frac{1}{2} |F|^2 \text{Re}\{D\}$$

(1)

where "**" represents the conjugate and $D$ is the origin mobility at the location of the structure.

![Fig. 1 Pipe-to-plate structure](image)

Considering the pipeline-plate coupling structure shown in the figure above, the model can be decomposed into two sub-structures separately, pipeline and plate, as shown in Figure 1b. Set the concentrated load force $F$ at the end of the pipeline at point1. Note that the points on the pipeline at the pipeline-plate junction are point2, and the points on the plate are point3. When considering the transverse vibration of pipeline, the influence of shear force on the vibration of pipeline is neglected, so the whole structure can be regarded as a combination of two substructures coupled by bending moment and angular displacement. When considering the longitudinal excitation vibration of pipeline, the bending moment of pipeline at the pipeline-plate junction can be neglected. At this time, the whole structure can be regarded as two substructures coupled by shear force and vertical displacement. The plate has its own boundary conditions, and the boundary condition of the beam is simply supported free. The input power flow exciting by external force is defined as $P_\text{in}$, the power flow from pipe to plate is defined as $P_\text{out}$:

$$P_\text{in} = \frac{1}{2} \text{Re}\{F \cdot V^*\} \quad P_\text{out} = \frac{1}{2} \text{Re}\{M_v \cdot \theta^*\}$$

(2)

Taking the lateral load as an example, the dynamic response can be obtained by the method of principle of linear superposition:
\[ V_1 = V_{1f} + V_{1M_1} \quad \theta_2 = \theta_{2f} + \theta_{2M_1} \quad (3) \]

Because of the fixed connection between the pipeline and the plate, the boundary conditions of bending moment and angular displacement at the connection between the pipeline and the plate are continuous.

\[ M_1 = -M_2 \quad \theta_2 = \theta_3 \quad (4) \]

Define the origin mobility and transfer mobility of the pipeline as:

\[ D_{11} = \frac{V_{1f}}{F}, \quad D_{21} = \frac{\theta_{2f}}{F}, \quad D_{12} = \frac{V_{1M_1}}{M_1}, \quad D_{22} = \frac{\theta_{2M_1}}{M_1} \quad (5) \]

And the origin mobility of the plate is:

\[ D_{33} = \frac{\theta_3}{M_3} \quad (6) \]

The substitution of Eq.(5) in Eq. (3) leads to the expressions in terms of the unknown sets \( V_1 \), \( M_3 \), \( \theta_3 \), substitute them in Eq. (2), the input power and out power can be obtained:

\[ P_{in} = \frac{1}{2} |F|^2 \text{Re} \left\{ D_{11} \frac{D_{21}}{D_{22} + D_{33}} \right\} \quad (7) \]

\[ P_{out} = \frac{1}{2} |F|^2 \left| \frac{D_{21}}{D_{22} + D_{33}} \right|^2 \text{Re} \left\{ D_{33} \right\} \quad (8) \]

The Euler beam model is used to solve the pipeline vibration in this paper. when an harmonic concentrated force \( F = F \text{e}^{i\omega t} \) is applied at the free end of pipe, the dynamic response of pipe is:

\[ Y_r(x,t) = \sum_{j=1}^{\infty} \frac{Y_r(0) Y_j(x) F \text{e}^{i\omega t}}{\omega_j^2 (1 + i\eta) - \omega^2} \quad (9) \]

when an harmonic concentrated moment \( M = M_0 \text{e}^{i\omega t} \) is applied at the support end of pipe (the connection between pipe and plate), the dynamic response of pipe is:

\[ Y_M(x,t) = \sum_{j=1}^{\infty} \frac{Y_r(0) Y_j(x) M_0 \text{e}^{i\omega t}}{\omega_j^2 (1 + i\eta) - \omega^2} \quad (10) \]

where \( Y_r(x) \) is the canonical mode function of pipeline, \( \omega \) is the natural frequency, \( \eta \) is the loss factor. Therefore, the expression of the mobility of the pipeline can be obtained.
Similarly, the mobility at the origin of the plate can be obtained by using the vibration response of the thin plate subjected to concentrated bending moments in the Y direction at the point.

\[
D_{33} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \frac{\partial W_{nm}(x,y)}{\partial y} \right)_{x-x_0, y-y_0}^2 \omega \frac{1}{\omega^2 + \omega_0^2 (1 + i\eta) - \omega^2} \tag{12}
\]

Therefore, the vibration power flow of the pipeline input plate can be obtained by substituting Eq. (11) and Eq. 12 into the Eq. (7) and Eq. (8).

3. Effective of excitation forms

The calculation model: same as the data in the paper1, size of pipe is \( \Phi30 \times 3 \text{mm} \), size of foundation plate is: \( 1000\text{mm} \times 500\text{mm} \times 6\text{mm} \), both of them have the same materials of steel. The position coordinates of the pipe on plate are \((200, 100) \text{mm} \). The boundary condition of the plate is simply supported by four sides.

3.1. Input power flow under two excitation

Since longitudinal and flexural waves are all important of pipe structures, both two types of waves are considered here. Longitudinal waves obey the second order equation, while flexural waves cause two internal forces to act in a pipe. As will be shown these two forces(one associated with bending, the other with shear) are both important since they carry equal amounts of power in the far filed.

Applying harmonic axial unit force and lateral unit force at the free end of the pipe, we can get the dynamic response of the structure. According to Eq. (2), the input vibration power flow under two excitation load can be carry out. As can be seen from Fig.2 :The power flow under axial load is greater than the lateral excitation load, especially in the 200-600Hz frequency range. Affected by the natural frequency of the structure itself, the peak position of the two is the same in the high frequency band, but the magnitude of the amplitude is different, and the input power flow value of the axial load is larger.
The kinetic energy of the plate is defined as the integration of all points kinetic energy on the plate, which can evaluate the vibration energy of plates at frequencies. The vibration energy of the flat plate under the two excitation modes are shown in Figure 2.

3.2. The far field radiated sound power

The effect of vibration control of plate is not only reflected in the vibration control of the plate itself, but also in the sound radiation from the plate to the surrounding fluid. The vibration is not consistent with the sound radiation, even there will be a decrease in acoustic radiation when vibration increases. The sound radiation is closely related to the spatial distribution and frequency of the vibration. The relationship between them can be expressed by radiation efficiency.

The calculation of sound radiation can use the Rayleigh integral in space, or the integral in wave number space. Wallace used the Rayleigh integral method to calculate the far field acoustic radiation problem of simply supported rectangular plates.

\[
\rho(r, \theta, \phi) = -ik_0\rho_0c_0 \frac{e^{ik_0r}}{2\pi r} \sum_{m,n=1}^\infty \hat{w}_{mn} \frac{l_i / l_y}{m\pi} \times \left[ \frac{-1}{(\alpha / m\pi)^n} - 1 \right] \left[ \frac{-1}{(\beta / m\pi)^n} - 1 \right] \] (13)

The radiated sound power flow is:

\[
l = \frac{1}{2\rho c_0} \iint |\rho(r, \theta, \phi)|^2 r^2 \sin\theta d\theta d\phi \] (14)

here $\alpha = k_0 l_x \sin\theta \cos\phi$, $\beta = k_0 l_y \sin\theta \cos\phi$, $\rho_0$, $\rho$, $c_0$ are respective outside fluid filed wave number, density and sound velocity. $r$ is the distance between far field points and rectangular plate. $\theta$, $\phi$ are respective the polar angle and azimuth angle of far field points.

The radiated sound power is related to the input power flow of the structure and the vibration energy of the plate. In the case of axial excitation vibration, the three results are compared together in Fig. 3. we can learn from the picture that, the frequency of input power flow determines the vibration energy and far-field acoustic radiation power of the plate.
Comparing the radiated sound power and radiated efficiency under the two kinds of loads, the radiated sound power caused by axial excitation is greater than that caused by lateral excitation in the low frequency band, which is consistent with the trend of vibration energy of the plate shown in Fig. 3. The acoustic radiation efficiency under axial excitation is much higher than that under transverse excitation, and the two forms of radiation efficiency are almost same after 200 Hz.

3.3. The vibration power flow vector and streamline

Vibration power flow contains the information of force and velocity. It includes not only the magnitude of energy, but also the direction of energy flow. It can also reflect the phase information between force and velocity when using power flow to study vibration. Taking 73Hz and 242Hz as an example, the power flow vector and streamline diagram of the two types of loads are compared. The location of the connection between the pipe and the plate can be clearly seen from the figure. That is the same place where the vibration energy out flow origin. The characteristics of power flow at the same frequency are different under the two kinds of excitation. At 242 Hz, eight symmetrical energy vortices are distributed in the plate under lateral excitation, while only seven energy vortices are distributed under axial excitation. As can be seen from the 73 Hz diagram, the power flow does not simply flow to the boundary due to the existence of boundary conditions, but flows along the boundary to form a certain energy vortex.
Comparing the power flow at two frequencies, it is found that the power flow is strongly frequency dependent. The change of excitation frequency may also lead to a thorough change in the distribution of power flow when other conditions remain unchanged.

4. Effective of vibration absorber on power flow

In order to control the vibration power flow stream line path of plate at the frequency=242Hz under the axial excitation, two passive dynamic vibration absorbers are set up. They are symmetrically distributed at the position of the maximum power flow in the vector diagram. Their position coordinates on the flat plate are (100 \text{mm}, 50 \text{mm}) and (300 \text{mm}, 50 \text{mm}). The parameters of the absorber are $K = 231200 \text{Nm}^{-1}$, $m = 0.1 \text{kg}$.

From the vibration energy diagram of the plate, it can be found that the power flow peak value of 242Hz is effectively reduced, but the peak value of other frequencies is introduced before and after 242Hz. This is the result that is not expected to be seen in vibration control. This problem is not discussed in this paper. The purpose of this paper is to study the effective of vibration absorber on propagation path of vibration power flow in the structure. From the vector graph and streamline diagram, it can be seen that the power flow in the position of the absorber is effectively suppressed, and the power flow that originally propagated mainly to the upper left and the upper right is flowed downward in this direction, so the energy vortex in the structure has also undergone new changes.
5. Conclusions:
This paper derived the vibration power flow expression of pipe-to-plate structure by using the method of mobility. The input power flow of pipeline, vibration energy of plate, radiated sound power and radiated efficiency of plate under transverse and axial loads are studied. The research results show that the power flow is greatly affected by the frequency. In the low frequency band, the power flow of the axial load input structure is greater, and the frequency of radiated sound power and the peak frequency of vibration energy of the plate are consistent with the input power flow frequency. Aiming at the power flow transmission path under 242Hz, vibration control of vibration absorber is carried out. The calculation results show that the absorber can effectively change the power flow path in the plate. The results of this paper can provide a new idea for controlling the vibration and acoustic radiation of pipe-to-plate structures.

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