Simulation of femtosecond pulse in a Kerr-lens mode locked Ti: sapphire laser

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Abstract. The Kerr-lens mode-locking (KLM) is known as a suitable method for generation of femtosecond pulses and mode-locked Ti:sapphire laser is now widely used sources of stable, energetic femtosecond pulses. We will present the simulation of KLM in Ti:sapphire laser cavities with a folded-cavity four-mirror by applying the \textit{ABCD} ray-tracing technique for a Gaussian beam. Simulations will be performed for an asymmetric resonator design. Based on the numerical analysis, we will find the optimum design parameters (slit position, gain cavity spacing, gain medium position) for KLM.

Introduction

The Kerr-lens mode-locking (KLM) based on the lens-effect induced in suitable material by a Kerr nonlinearity is straight forward method to generate pulses as short as few femtosecond laser pulses [1]. When a beam propagates through a medium which a refractive index linearly varies with the intensity (Kerr nonlinearity), the well-known optical phenomenon that a self-focusing effect is observed [2]. The self-focusing effect of an intense pulses is successfully exploited in the mode-locking [3]. The optical Kerr effect is responsible for the intensity dependent refractive index change which causes the lensing in the gain medium. When a suitable aperture is located inside the cavity where the intense pulses beam profile is narrowed by the Kerr-lens effect, the continuous-wave (CW) radiation propagation losses are higher than the intense pulse propagation losses [4]. The combination of the Kerr-lens and aperture acts as a fast saturable absorber, then a fast passive gain modulation is obtained [5]. The intensity fluctuations in laser start-up are not intense enough to induces Kerr lensing [6,7], thus acousto-optic modulation, additive pulse mode-locking, impulsive starting i.e [8-17] are required before the KLM. The self-starting KLM is achieved by using a highly nonlinear Kerr medium in the folded cavity.

In this paper we simulate the influence of the Kerr lensing on the beam propagation in a folded-cavity four-mirror resonator by applying the \textit{ABCD} ray-tracing technique for a Gaussian beam. With a split-step method that dividing the Ti:sapphire crystal to many thin lens-like slices [18] the intensity dependent Kerr-lens effect of the Brewster-plate shaped active medium is described. We introduce the optimum design parameters and discuss the modifications of the beam spot size and radius of curvature in the nonlinear Kerr medium, where the refractive index increases with intensity.

Beam propagation in the resonator

The folded-cavity four-mirror resonator, shown in Fig.1 is used for Ti:sapphire lasers. Parameters \(l_{12}, l_{23}, l_{34}\) are lengths of the three arms and \(R_2, R_4\) are curvature radii of \(M_2\) and \(M_4\) mirrors. The beam propagation inside the cavity is expressed by the \textit{ABCD} round-trip matrices. Due to the astigmatic effect the beam propagation behaviour is different in the tangential \(xz\) plane and in the sagittal \(yz\) plane [19]. The tangential and sagittal rays focus at different positions (astigmatism). This cavity astigmatism is compensated by the Brewster plate at a certain tilting angle. Without the Kerr lensing the Brewster plate behaves linearly, i.e. \(n_z=0\) [3]. In the case with Kerr lensing the intensity
dependence of the refractive index is taken into account in the beam propagation through the gain medium.

Figure 1. The folded-cavity four-mirror resonator with the Brewster-plate shaped active medium.

For one round trip through the resonator the total ABCD matrix which is the multiplication of all the matrices of the elements is given by

\[
M_\text{tot} = \left( \begin{array}{cc} A_i & B_i \\ C_i & D_i \end{array} \right) = \prod M_i = \prod \left( \begin{array}{cc} A_i & B_i \\ C_i & D_i \end{array} \right)
\]

\[
= M_{12}M_{24}M_{43}M_{34}M_{45}M_{52}M_{21}M_{12}
\]

The optical elements in the resonator are listed in Table I. To be a stable resonator, a stability condition, \(|1/C| \leq 2 \) [20] is required which gives the allowed regions of mirror separation for stable laser operation. In this stability region the resonator misalignment sensitivity parameter is determined by \(|1/C|\) where small value of \(|1/C|\) implies lower sensitivity of the resonator [5].

Using the ABCD matrices one can calculate the fundamental Gaussian \( \text{TEM}_{00} \) mode of the cavity for CW operation inside the stability zones of the resonator. The Gaussian mode is represented by the complex beam parameter as

\[
\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda_e}{\pi w^2(z)}
\]

(1)

where \( R(z) \) and \( w(z) \) are the wavefront curvature and the spot size at position \( z \), \( \lambda_e \) is the wavelength.

The beam diameter and spot size are related by \( \Delta d = [2 \ln 2]^{1/2} w \). The beam parameter \( q(z) \) for the propagation direction \( z \) from \( z_1 \) to \( z_2 \) is given by

\[
q(z_2) = \frac{A_{12}q(z_1) + B_{12}}{C_{12}q(z_1) + D_{12}}
\]

(2)

The incident beam has the same curvature as mirrors. In our resonator the input mirror \( M_1 \) at \( z = 0 \) is plane, with the radius \( R_1(0) = \infty \). Then, the beam parameter for one round trip on this mirror is found as

\[
q(0) = i \pi w^2(0) / \lambda_e = [A_0 q(0) + B_0] / [C_0 q(0) + D_0].
\]

Without the Kerr lensing the ABCD matrix components in the equation are constant, thus this quadratic equation can be solved for \( q(0) \) and \( w(0) \). Considering the Kerr-lens effect, the ABCD matrix components depend on the beam spot size \( w \) [3].

To start the ray tracing in the resonator an arbitrary beam parameter is chosen and the calculation is repeated until the real part of the beam parameter \( q_{m+1}(0) \) and \( q_m(0) \) at the \( M_1 \) mirror equals to zero for one round trip.

\[
q_{m+1}(0) = \frac{i \pi w^2_{m+1}(0)}{\lambda_e} = \frac{[A_0 q_m(0) + B_0]}{[C_0 q_m(0) + D_0]}
\]

(3)

The intensity changes inside the Kerr lens medium is achieved by dividing the crystal to many thin lens-like slices \( (l_B \to 0) \) and the beam propagation through each slices is calculated by using the Kerr-
lens ABCD matrices given in Table I with a constant intensity $I_0L$. The solution depends on the crystal position [21].

**Table I. ABCD matrices**

| Optical element                  | Matrix |
|----------------------------------|--------|
| Mirror                           | $M_1, M_4$ |
| Normal incidence                 | $egin{pmatrix} 1 & 0 \\ -2/R & 1 \\ \\ 1 & 0 \\ -2/(R \cos \vartheta) & 1 \end{pmatrix}$ |
| Tilted, sagittal plane           | $M_2, M_3$ |
| Tangential plane                 | $M_2, M_3$ |
| Propagation in space             | $M_{12}, M_{24}, M_{41}, M_{43}$ |

**Table II. Resonator parameters ($\lambda_L=800\text{nm}$)**

| Parameter                              | Value |
|----------------------------------------|-------|
| Mirror curvatures (mm)                 | $R_1 \infty$ |
|                                       | $R_2 50$ |
|                                       | $R_3 50$ |
|                                       | $R_4 \infty$ |
| Mirror tilting angles (deg)            | $\theta_1 0$ |
|                                       | $\theta_2 15.22$ |
|                                       | $\theta_3 15.22$ |
|                                       | $\theta_4 0$ |
| Total resonator length (m)             | $l_t 1.05$ |
| Mirror separators (cm)                 | $l_{12}$ |
|                                       | $l_{23}=l_{2k}+l_B+l_{3k}$ varied around 50 |
| Intracavity plate (Brewster-cut Ti:sapphire crystal) | $n_L 1.76$ |
|                                       | $n_2 1.6 \times 10^{-22} \text{ m}^2 \text{V}^{-2}$ [22] |
| Intracavity plate (Brewster-cut Ti:sapphire crystal) | $l_B 2 \text{mm}$ |
|                                       | $l_{2k}$ varied around $l_{2k}/2$ |

Changing the position of the crystal along the beam affects the mode intensity in the crystal. The optical Kerr effect is a third-order nonlinear process in which the refractive index of the material is intensity dependent [23]. At sufficiently high intensity, the refractive index of the medium will be influenced to a readily observable extent by the field intensity. The Kerr medium is described by the intensity dependent refractive index $n$ that is given [1] by

$$n = n_k + \frac{1}{2} n_z E_{0L}^2 = n_L + \gamma_L I_L = n_k + \frac{n_z}{n_k c_0 \epsilon_0} I_L$$

(4)

where $n_z$ and $\gamma_L$ are the nonlinear refractive index and nonlinear coefficient of medium. $E_{0L}^2$ is the electric field amplitude, $I_L = (n_k c_0 \epsilon_0 / 2)E_{0L}^2$ is the light intensity $c_0$ being the light velocity in vacuum and $\epsilon_0$ the electric permittivity of vacuum).

The Gaussian beam intensity is approximated by a Taylor expansion as
\[ I_L = I_{0L} \exp[-2(r/w)^2] \approx I_{0L}[1-2(r/w)^2] \]  

(5)

then, the (4) equation becomes parabolic [24].

\[ n = n_L + \frac{n_L I_{0L}}{n_L c_0 e_0}[1-2(r/w)^2] \]

\[ = (n_L + \frac{n_L I_{0L}}{n_L c_0 e_0}(1 - \frac{2n_L I_{0L}}{n_L c_0 e_0(n_L + n_L I_{0L})})w^2) \]

(6)

\[ \tilde{n} = 1 - \frac{1}{2}\gamma^2 r^2 \]

where \( \tilde{n} = n_L + \frac{n_L I_{0L}}{n_L c_0 e_0} \) and \( \gamma = (\frac{4n_L I_{0L}}{n_L c_0 e_0 \tilde{n}})^{1/2} \frac{1}{w} = (\frac{8n_L P}{n_L c_0 e_0 \tilde{n} \pi})^{1/2} \frac{1}{w^2} \)

The laser power is \( P = \pi w^2 I_{0L} / 2 \). Since the Kerr lensing for the Brewster plate in the sagittal and tangential plane is different, the \( w \) and \( I_{0L} \) in (6) equation are replaced by \( w' = n_L w \), \( w \rightarrow w' = w'_s \) and \( I_{0L} \rightarrow I_{0L} w'^2 \). The equations for the sagittal plane of a Brewster plate are given by

\[ n_s = \tilde{n}_B (1 - \frac{1}{2}\gamma^2 r^2) \]

where \( \tilde{n}_B = n_L + \frac{n_L I_{0L}}{n_L c_0 e_0} \) and \( \gamma_s = (\frac{4n_L I_{0L}}{n_L c_0 e_0 \tilde{n}_B})^{1/2} \frac{1}{w} = (\frac{8n_L P}{n_L c_0 e_0 \tilde{n}_B \pi})^{1/2} \frac{1}{w^2} \)  

(7)

For the tangential plane of a Brewster plate the equation are written as

\[ n_t = \tilde{n}_B (1 - \frac{1}{2}\gamma_t^2 r^2) \]

with \( \gamma_t = (\frac{4n_L I_{0L}}{n_L c_0 e_0 \tilde{n}_B})^{1/2} \frac{1}{w} = (\frac{8n_L P}{n_L c_0 e_0 \tilde{n}_B \pi})^{1/2} \frac{1}{w^2} \)

(8)

The Kerr-lens ABCD matrices in the tangential and sagittal plane are given in Table I.

**Figure 2.** The misalignment sensitivity parameter of the folded-cavity four-mirror resonator without the Kerr lensing (P=0). a) Sagittal plane, b) tangential plane

**Simulations**

There are two stability regions observed in the tangential and sagittal plane of the four-mirror cavity with the astigmatism compensated. Due to the folded mirror separation \( l_{23} \) it is divided into two stability zones I and II as shown in the misalignment sensitivity parameter plot (Fig.2) without the Kerr lensing.
Figure 3. The influence of the Kerr lens position \( l_{2k} \) position on beam parameters. Laser power, \( P = 2 \times 10^5 \text{W} \). Parameters listed in Table II are applied. a.) sagittal plane, b.) tangential plane.

In the case with the Kerr lensing the parameter \( l_{23} \) is fixed and the Kerr lens position \( l_{2k} \) only in the folded cavity is varied. In Fig.3 the beam parameters modifications caused by the Kerr lensing are compared in the tangential and sagittal plane, respectively. The applied resonator parameters are listed in Table II. The plots, shown in Fig.3 corresponds to the beam diameters at the mirrors, the minimum beam diameters in the gain cavity and the position of the beam waist \( Z_{2,\text{min}} \) measured from the mirror \( M_2 \) against the Kerr lens position \( l_{2k} \). The laser power inside the resonator is set to \( P = 2 \times 10^5 \text{W} \).

From the beam waist position plots, one can see that by increasing the distance from the \( M_2 \) to the crystal (for \( l_{2k} > z_{2,\text{min}} \)), the beam waist exists outside of the crystal, then its position remains constant. It is same for \( l_{2k} < z_{2,\text{min}} - l_B \) and it shows the lensing effect getting small. When the beam waist exists inside the crystal (\( l_{2k} < z_{2,\text{min}} < l_{2k} + l_g \)), its position decreases until the end of the crystal because of the increase of the beam intensity inside the crystal. Since the beam intensity is high at the face of the crystal, around \( l_{2k} = z_{2,\text{min}} - l_B \) and \( l_{2k} = z_{2,\text{min}} \) the Kerr-lens effect is largest.

The beam size is changed in the Brewster plate, causing beam narrowing at some mirrors and beam broadening at others as shown in the plots with the beam diameters \( d_{Wj} \) (Fig.3). An aperture setting in regions of beam narrowing leads to Kerr-lens mode locking.

Conclusion
We introduced the beam parameters modifications caused by the Kerr lensing in the four-mirror folded-cavity resonator. In the simulation the real parameters of a femtosecond Ti:sapphire laser are used. The astigmatism due to the beam propagation in the sagittal and tangential plane of the folded-cavity is compensated.

The optimum resonator design parameters (slit positioning, gain cavity spacing, gain medium position) are obtained for maximum Kerr-lens mode-locking effect.
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