Finite Element Analysis and Theoretical Verification of Natural Vibration of Novel Simply-supported Beams

Rongqiang Zhong¹, Yi Zhao¹, Lirong Yao¹, Jining Liu¹, Zhiming Li¹ and jing Guo*¹

¹Sinopec Northwest Oil Field Company, Wulumuqi, Xinjiang Uygur Autonomous Region, 830000, China

* Corresponding author: jijing@nepu.edu.cn

Abstract. In order to investigate the dynamic characteristics of I-shaped novel simply-supported beams with the upper flange of rectangular high-strength concrete filled steel tube (RHCFST), the modal analysis of 16 simply-supported beams is conducted by ANSYS finite element software. The first order frequencies of the specimen beams are obtained in the Y direction. Based on the concentrated mass method, the frequencies of the specimens are theoretically calculated. By comparison the maximum error of the first-order frequency is 3.80%, which is in good agreement and can meet the precise requirements of practical engineering. The result shows that the first three order vibration modes of the specimen are consistent with the theoretical vibration modes. For the first-order vibration mode, the proportion of the effective mass to the total mass of the specimen is 79.2 % in the Y direction. Therefore, the first-order vibration mode in Y direction should be mainly taken into consideration for this type of beams during the structural design.

1. Introduction
Concrete-filled steel tubular (CFST) flange structure appeared in the 1980s. Since Hollow Flange Beam was proposed by Australian scholar Dempsey [1], CFST flange beams have been widely utilized and studied, and various cross-sectional forms of CFST flange structure have been proposed [2-5]. With the rapid development of high-performance materials in recent years, the interaction between steel tube and high-performance concrete has become a hot issue [6-10]. Owing that the CFST flange beams combine the characteristics of I-shaped steel beams and concrete-filled steel tubes, this type of beams displays the advantages of high bearing capacity, good toughness and good bending performance under earthquake excitation [11-13]. In order to better investigate the dynamic characteristics of the I-shaped novel simply-supported beam with RHCFST, ANSYS finite element software is adopted to analyse the modal of 16 simply-supported beams. Based on the concentrated mass method [14], the natural vibration frequencies in the Y direction of the specimens are calculated theoretically. Through comparing the frequencies of finite element simulation and theoretical calculation, the rationality of finite element modelling is verified. Finally, the main vibration modes in Y direction of the I-shaped novel simply-supported beams with RHCFST are evaluated.
2. Establishment of finite element model

2.1. Specimen design

In this paper, a total of 16 I-shaped novel simply-supported beams with RHCFST are designed with different height-thickness ratios of web. The cross-section shape is shown in Figure 1. The specific parameters of specimens are shown in Table 1, and the specific performance parameters of the material are shown in Table 2.

Table 1. The specific parameters of 16 specimens

| Specimens | \( b_1 \times h_1 \times t_1 \) (mm³) | \( h_2 \times t_w \) (mm²) | \( b_2 \times t_2 \) (mm²) |
|-----------|----------------------------------|--------------------------|--------------------------|
| B-1       | 55×35×3                          | 246×6                    | 60×6                     |
| B-2       | 55×35×3                          | 256×6                    | 60×6                     |
| B-3       | 55×35×3                          | 266×6                    | 60×6                     |
| B-4       | 55×35×3                          | 276×6                    | 60×6                     |
| B-5       | 55×35×3                          | 246×7                    | 60×6                     |
| B-6       | 55×35×3                          | 256×7                    | 60×6                     |
| B-7       | 55×35×3                          | 266×7                    | 60×6                     |
| B-8       | 55×35×3                          | 276×7                    | 60×6                     |
| B-9       | 55×35×3                          | 246×8                    | 60×6                     |
| B-10      | 55×35×3                          | 256×8                    | 60×6                     |
| B-11      | 55×35×3                          | 266×8                    | 60×6                     |
| B-12      | 55×35×3                          | 276×8                    | 60×6                     |
| B-13      | 55×35×3                          | 246×9                    | 60×6                     |
| B-14      | 55×35×3                          | 256×9                    | 60×6                     |
| B-15      | 55×35×3                          | 266×9                    | 60×6                     |
| B-16      | 55×35×3                          | 276×9                    | 60×6                     |

Table 2. The specific parameters of material

| Material | Grade of material | \( E_s \) /MPa | \( \mu \)  | \( \rho \) /kg·m⁻³ |
|----------|-------------------|----------------|-------------|-------------------|
| Concrete | C70               | 3.70×10⁴       | 0.2         | 2450              |
| Steel    | Q235              | 2.06×10⁵       | 0.3         | 7850              |

The length \( (L) \) of the I-shaped novel simply-supported beams with RHCFST is 4 m. The end plates are set at both ends of the specimen beams with a thickness of 20 mm. In addition, the stiffeners are arranged at the mid-span position of the beams. The shape of the specimen is shown in Figure 2.

Figure 1. Cross-sectional shape of specimens
2.2. Finite element model

2.2.1. Element type
Based on ANSYS finite element software, the models of the I-shaped novel simply-supported beams with RHCFST are established. Solid 65 element is adopted for concrete and shell 181 element is used for steel in this paper, and the material performance parameters are selected as shown in Table 2. During the process of modal analysis, the materials of concrete and steel are both in the elastic stage. Therefore, it is assumed that the steel tube and concrete fail to slip and the concrete cannot crack.

2.2.2. Boundary condition and meshing
During the process of finite element modelling, the method of mesh division has a great influence on the accuracy of the analysis results and the speed of calculation. In order to guarantee the mesh quality and calculation accuracy, the initial analysis for each part of the specimen is carried out by using less mesh elements, subsequently, the second analysis is carried out by twice the number of mesh elements as that of the former. When the error between above results is less than 1%, the number of meshes can meet the requirements, otherwise the mesh should continue to be subdivided until the requirements are met [15]. According to above method to meshing, mesh number of the specimens along the length direction ($N_x$) is 100, that of the specimen along the web height direction ($N_{\text{web}}$) is 10, that of the specimen along the flange width direction ($N_{\text{wing width}}$) is 4, and that of specimen along the flange height direction ($N_{\text{wing height}}$) is 2.

When setting the boundary conditions, the two reference points ($RP1$, $RP2$) are established and coupled at the left and right ends, respectively, in order to prevent the phenomenon of stress concentration in the loading process. $RP1$ is constrained by $U_x$, $U_y$, $U_z$ and Rotate $x$, and $RP2$ is constrained by $U_y$, $U_z$ and Rotate $x$. Boundary conditions of the model is shown in Figure 3.

Figure 2. The shape of novel simply-supported beams

Figure 3. Boundary condition and meshing of the specimen
3. Theoretical calculation on the natural vibration of simply-supported beams

3.1. Geometry model of simply-supported beams

To convenient the calculation and research, a simplified model of I-shaped novel simply-supported beam with RHCFST is established in this paper. According to the concentrated mass method proposed in advanced structural dynamics, the structure is divided into two parts, and the mass of each part is concentrated on both ends. The multi-degree-of-freedom system with continuous mass can be simplified into a simple single-degree-of-freedom system with discrete mass, as shown in Figure 4.

The equivalent stiffness method [16] is adopted to further simplify the section of specimens, as shown in Formula (1). Concrete with an area of \( a \times b \) in the upper flange steel tube is equivalent to steel with an area of \( a' \times b' \), as shown in Figure 5.

\[
a' \times b' = \frac{E_c}{E_s} \times a \times b
\]  

(1)

Figure 4. Simplified model

Figure 5. Equivalent stiffness

3.2. The calculation of natural frequency

The motion equation for structure system of the simplified model is shown in formula (2).

\[
m^* \ddot{x} + c^* \dot{x} + k^* x = p(t)
\]  

(2)

Where \( m^* \) is the generalized mass, \( c^* \) is the generalized damping, \( k^* \) is the generalized stiffness, \( p(t) \) is the external load of the system, \( \ddot{x}, \dot{x}, x \) is the acceleration, velocity and displacement of the mass of beam, respectively.

Due that the modal analysis of specimens is studied without considering the damping, the values of both the external load and damping in the structural system are zero. Further, the motion equation in the Y direction of the simplified model is shown in Formula (3).

\[
m^* \ddot{x} + k^* x = 0
\]  

(3)

The circular frequency and the structural natural frequency of the first order modes of the simplified model can be obtained by Formulas (4) and (5). The generalized mass and stiffness can be obtained by Formulas (6) and (7), respectively [17].

\[
\omega_1 = \sqrt{\frac{k^*}{m^*}}
\]  

(4)

\[
f_t = \frac{\omega_1}{2\pi}
\]  

(5)
5. Analysis and results of the modal

4.1. Verification of finite element model

To verify the rationality of the modelling method adopted in this paper, the frequencies of vibration mode of 16 specimens are obtained based on the method of above theoretical calculation, which can be compared with the first order frequencies of the finite element simulation in the Y direction. The specific comparison results are shown in Table 3 and Figure 6.

It can be found that $f_s$ is in good agreement with $f_t$, and the maximum error is 3.80%, which can meet the precise requirement of practical engineering. The result shows that the simplified model can effectively simulate the first-order vibration mode for this kind of beams in the Y direction.

| Specimens | $f_s$/Hz | $f_t$/Hz | $|f_s-f_t|/f_t$% |
|-----------|----------|----------|-----------------|
| B-1       | 48.05    | 49.95    | 3.80            |
| B-2       | 49.59    | 51.55    | 3.79            |
| B-3       | 51.12    | 53.14    | 3.79            |
| B-4       | 52.65    | 54.73    | 3.80            |
| B-5       | 47.24    | 48.95    | 3.49            |
| B-6       | 48.76    | 50.52    | 3.48            |
| B-7       | 50.26    | 52.08    | 3.49            |
| B-8       | 51.76    | 53.64    | 3.50            |
| B-9       | 46.53    | 48.08    | 3.24            |
| B-10      | 48.02    | 49.63    | 3.23            |
| B-11      | 49.50    | 51.16    | 3.24            |
| B-12      | 50.98    | 52.70    | 3.26            |
| B-13      | 45.89    | 47.32    | 3.03            |
| B-14      | 47.36    | 48.85    | 3.03            |
| B-15      | 48.83    | 50.37    | 3.04            |
| B-16      | 50.29    | 51.88    | 3.07            |

Figure 6. The maximum error between finite element and theoretical values
4.2. Mode vibration of specimen beams
The BLOCK LANCZOS method in ANSYS is used to extract the modes of the specimens. Taking B-1 specimen as an example, the first three modes of the specimen are shown in Figure 7. It can be found from the vibration modes that the shape of the first three order vibration modes of the specimen is consistent with that of the theoretical vibration mode. As for the first-order vibration mode, the proportion of the effective mass to the total mass of the specimen in the Y direction is 79.2%. Therefore, the first-order vibration mode in Y direction should be mainly taken into consideration for this type of beams in the structural design.

(a) The first-order vibration mode

(b) The second-order vibration mode

(c) The third-order vibration mode

Figure 7. The first three order modes of B-1 specimen in Y direction

5. Conclusions
Based on above modelling method of the finite element, a total of 16 I-shaped novel simply-supported beams with RHCFST were designed in this paper, and the modal analysis was carried out by ANSYS finite element software. According to the concentrated mass method, the first order natural frequencies in the Y direction of the specimens were obtained. The conclusions can be drawn as follows:

(1) Compared with the theoretical calculation results of the simplified model, the maximum error of the first-order frequency is 3.80%. The result shows that the first-order frequency is in good agreement when only considering the vibration mode in Y direction of the simply-supported beams.

(2) As for the first order vibration mode, the proportion of the effective mass to the total mass of the specimen in the Y direction is 79.2%. Therefore, in order to avoid the sympathetic vibration of the structure under external excitation, the first-order vibration mode frequency should be focused in the Y direction in the structural design.

References
[1] Avery, P., Mahendran, M., Nasir, A. (2000) Flexural capacity of hollow flange beams. J. Constr. Steel Res., 53:201-223.
[2] Zhang, T, Ren, W. D., Wu, Z.Y. (2005) Stability analysis of rectangular hollow flange beam. Low Temp. Archit. Tech., 04:45-47.
[3] Liu, R. F. (2006) Static behavior of a new type of I-beam with a rectangular tube top flange. Harbin Institute of Technology, Harbin.
[4] Ji, J., Xu, Z. C., Jiang, L. Q., Zhang, W. F., Zhou, L. J., Teng, Z. C., Chao, Y. C., Lu, Z. H. (2018) Nonlinear buckling analysis of H-type honeycombed composite column with rectangular concrete-filled steel tube flanges. Int. J. Steel Struct., 18: 1153-1166.

[5] Cheng, F. (2009) Experimental research on static behavior of I-Beam with a rectangular concrete-filled tube top flange. Harbin Institute of Technology, Harbin.

[6] Ji, J., Wang, Y., Chen, X. K., Jiang, L. Q., Zhang, W. F., Zhang, Y. F., Yuan, C. Q., Liu, Y. C. (2017) Analysis of axial compression behavior of strength-gradient composite columns with built-in high-strength concrete filled steel tube. J. Northeast Petroleum Univ., 41: 107-116.

[7] Ji, J., Yu, D. Y., Jiang, L. Q., Liu, Y. C., Yang, M. M., Song, H. Y., Jiang, L. (2020) Research on axial compression bearing capacity of different-strength concrete filled double steel tube short columns. Build. Struct., 50: 120-129.

[8] Ji, J., Yu, D. Y., Jiang, L. Q., Xu, Z. C., Liu, Y. C. (2019) Effect of post-fire curing on the compressive properties of fire-damaged ultra-high toughness cementitious composites. J. Test. Eval., 49: 140-152.

[9] Ji, J., Song, H. Y., Jiang, L. Q., Ren, H. G., Zhang, Y. F., Liu, Y. C. (2021) Tensile performance of high ductility cementitious composites with recycled powder from C&D waste. Front. Mater., 08:1-8.

[10] Ji, J., Xu, Z. C., Jiang, L. Q., Liu, Y. C., Yu, D. Y., Yang, M. M. (2019) Experimental study on compression behaviour of H-shaped composite short column with rectangular CFST flanges and honeycombed steel web subjected to axial load. J. Build. Struct., 40: 63-73.

[11] Ji, J., Zhang, S. L., Jiang, L. Q., Zhou, L. J., Xu, Z. C., Liu, Y. C., Yu, D. Y. (2018) Feasibility of developing engineered cementitious composite with high volumes of fly ash using cost-effective PVA fiber. J. Test. Eval., 48:1-8.

[12] Ji, J., Yang, M. M., Jiang, L. Q., He, J., Teng, Z. C, Liu, Y. C., Song, H. Y. (2019) Output-only parameters identification of earthquake-excited building structure with least squares and input modification process. Appl. Sci., 09: 1-8.

[13] Ji, J., Zhang, R. B., Yu, C. Y., He, L. J., Ren, H. G., Jiang, L. Q. (2021) Flexural Behavior of simply supported beams consisting of gradient concrete and GFRP bars. Front. Mater.,08: 1-14.

[14] Ray, W. C., Joseph, P. (2006) Dynamics of Structures. Higher Education Press, Beijing pp.9-28.

[15] Ji, J., Zhang, W. F., Yuan, C. Q., Liu, Y. C., Xu, Z. C., Wang, Y., Chen, X. K. (2019) Finite element model based test and analysis on ACHC short columns and hoop coefficient. Cluster Comput., 22: 5333-5345.

[16] GB 50017-2017 (2017) Standards for steel structure design. China Construction Industry Press, Beijing.

[17] Shi, X. Q, Cao, X. F. (2021) A modal analysis of planar simply supported beam based on ANSYS. Jiangxi Build. Mater., 03:73-74.