The Production and Decay of Heavy Dimesons

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Abstract

We show that it is necessary to go beyond a single hadron (beyond the quark-antiquark or three-quark systems) in order to distinguish the colour structure of the effective quark-quark interaction and the relevance of 3-body forces. We critically discuss the proposed models which suggest the dimeson $b\bar{b}u\bar{d}$ to be bound by $\sim 100$ MeV and the $c\bar{c}u\bar{d}$ dimeson to be unbound. Only experiment can judge. We estimate the probability of producing $b\bar{b}u\bar{d}$ at LHC by double gluon-gluon fusion and search for a characteristic decay.
I. MOTIVATION

There is a strong motivation to understand the effective interaction between heavy quarks (and antiquarks) since it is expected to be “cleaner” than between light quarks. For heavy particles the nonrelativistic constituent quark model is more acceptable, the perturbative QCD contributions (such as one-gluon-exchange) is more adequate and chiral fields are less important.

While the effective interaction between a heavy quark and a heavy antiquark has been reasonably well studied and fitted by the charmonium and bottomium spectra, there is no free diquark to study the effective interaction between two heavy quarks. One has to dress the diquark in order to obtain a colour singlet object. Therefore, the double-heavy baryons \( ccq, bcq, \) and \( bbq \) (\( q=u,d, \) or \( s \)), as well as the double-heavy dimesons \( cc\bar{q}, bc\bar{q}, \) and \( bb\bar{q} \) (also called tetraquarks) can be considered as a laboratory to study the properties of the diquark. They have not yet been seen experimentally (except some tentative signals \(^1\)). The purpose of our study is to demonstrate, how important information for quark models they would offer, and to stimulate the experimentalists to invest due efforts to discover them in near future.

It is straightforward to extrapolate the one-gluon-exchange (OGE) interaction from \( \bar{Q}Q \) to \( QQ \) (\( Q=\) any quark). The charge conjugation changes the \( \bar{Q} \) antitriplet to \( Q \) triplet. Then the colour factor \( \lambda \cdot \lambda/4 = -4/3 \) for the \( \bar{Q}Q \) singlet changes to \(-2/3\) for the \( QQ \) antitriplet (the “\( V_{QQ} = \frac{1}{2} V_{\bar{Q}Q} \) rule”).

On the other hand, it is questionable whether the (linear) confining potential should also possess such a colour factor and obey the \( V_{QQ} = \frac{1}{2} V_{\bar{Q}Q} \) rule. The fact that the ground state energies and some excited states of light and heavy baryons are reasonably well reproduced with such a “universal” OGE + confining effective interaction is encouraging \(^2\) but not conclusive. There may be other mechanisms for the \( V_{QQ} = \frac{1}{2} V_{\bar{Q}Q} \) rule. For example, the flux tubes in a “Mercedes” configuration can be mimicked by twice weaker two-body flux lines since the length of the arms of the “Mercedes” is approximately half the length of the circumference of the triangle. The colour singlet 3-quark system is insensitive to the features of the \( \text{colour} \cdot \text{colour} \) operator since it is just a constant in the 3-body singlet representation. To explore the colour structure of the effective interaction one has to go beyond mesons and baryons to dimesons and other exotics.

\(^1\)\(^2\)
Moreover, there may be other contributions to the effective interaction. Regarding the short-range part, the important effect is the strong spin-spin splitting which can be due to OGE, pion exchange or some instanton effects, the relative importance of which is not yet clear. Light baryon spectra are better reproduced by the meson exchange interaction than by the OGE interaction but the extension to the heavy baryons and to mesons is still uncertain.

The study of double-heavy baryons and of double-heavy dimesions are complementary. They both refer to the colour triplet heavy diquark and try to determine the strength of the interaction and test the “$V_{QQ} = \frac{1}{2} V_{Q\bar{Q}}$ rule”. The dimesion, however, possesses also a $(6, \bar{6})$ configuration with a sextet heavy diquark and antisextet cloud of two light antiquarks; configuration mixing offers the opportunity to test the colour structure of the interaction.

Furthermore, there are theoretical reasons for three-body and four-body forces. Three-body forces have been shown to be welcome to improve the absolute position of baryons with respect to mesons. The dimeson can give a better hint about the relative strength of the three-body interaction since there are 4 three-body contributions compared to just one in a baryon.

In next Section we give arguments why we expect the bbudos dimeson to be strongly bound while the ccudos, bcudos and others are most likely unbound. In the third Section we estimate the chances to produce the bbudos dimeson in LHC, and in the final Section we call for new ideas how to detect it.

II. BINDING ENERGIES

A. With Two-Body Forces

We estimate the binding energy of the bbudos dimeson assuming the $V_{QQ} = \frac{1}{2} V_{Q\bar{Q}}$ rule and no three-body forces. If this prediction is falsified by the experiment a revision of the two-body QQ interaction and/or introduction of many-quark forces will be needed.

For the sake of clarity we present here a simplified derivation which makes three further assumptions: (i) that the spin-dependent interactions between heavy quarks are not important, (ii) that the configuration with both b quarks bound in a compact diquark with spin 1 and antitriplet colour dominates, and (iii) that the two light antiquarks in the dimeson
behave equally as in a $\Lambda_b$ baryon. We have, however, performed also a careful four-body calculation without such unnecessary assumptions \[4\] and the results were practically the same.

We want the result to be as little model dependent as possible. Therefore we consider any model which reproduces exactly the masses of $\Lambda_b$ baryon and several mesons. Then we can consider Nature as an "analogue computer" and express the mass of the dimeson in terms of those experimental masses. Now we compare the following hadrons

\[
\begin{align*}
M_{bb\bar{u}\bar{d}} &= 2M_b + M_u + M_d + E_{bb} + E_{ud[bb]} \\
M_{Y} &= 2M_b + E_{bb} \\
M_{\Lambda_b} &= M_b + M_u + M_d + E_{ud\bar{b}}
\end{align*}
\]

where $E_{ud[bb]} \approx E_{ud\bar{b}}$ is the potential plus kinetic energy contribution of the two light quarks in the field of a heavy diquark or quark, respectively, and it cancels in the difference. This would be exactly true in the limit where the mass of the $b$ quark goes to infinity and the heavy diquark is point-like so that we can neglect the size of the heavy diquark in the dimeson.

We can estimate the diquark binding energy using the following trick. The binding energy $E_{\text{meson}}$ of a quark and antiquark in the meson is a function of the reduced mass only (neglecting spin forces):

\[
\left[ \frac{p^2}{M_Q} + V_{Q\bar{Q}} \right] \psi = E_{\text{meson}}(M_Q)\psi
\]

For a diquark the Schrödinger equation is similar as for a meson, but with twice weaker interaction. To get the similarity, we mimic the kinetic energy with twice smaller reduced mass.

\[
\left[ \frac{p^2}{M_Q} + V_{Q\bar{Q}} \right] \psi = \frac{1}{2} \left[ \frac{p^2}{M_Q/2} + V_{Q\bar{Q}} \right] \psi = \frac{1}{2} E_{\text{meson}}(M_Q/2)\psi.
\]

We obtain $E_{\text{meson}}(M_b/2)$ by plotting the binding energies of mesons as a function of the reduced mass \[4\]. We do that for different choices of constituent quark masses in order to estimate the uncertainty due to quark masses. The plot is very smooth and we estimate the binding energy of the heavy $bb$ diquark $E_{\text{diquark}}(M_b) = \frac{1}{2}E_{\text{meson}}(M_b/2) = -390 \pm 15$ MeV, whereas for bottomium $\frac{1}{2}E_{\text{meson}}(M_b) = -560 \pm 15$ MeV.

This gives us the phenomenological estimate for the binding energy of the dimeson with respect to the $BB^*$ threshold.
\[
\Delta E_{\text{bbud}} = M_{\Lambda_b} + [M_{\Upsilon} - E_{\text{meson}}(M_{b}) + E_{\text{meson}}(M_{b}/2)]/2 - M_{B} - M_{B^*} = -130 \pm 20 \text{ MeV}.
\]

This agrees well with our detailed calculation \[4\] and with some previous four body calculations \[2\] \[5\] in the constituent quark model.

An analogous calculation using \(\Lambda_c\) and \(J/\psi\) instead of \(\Lambda_b\) and \(\Upsilon\) gives for the mass difference between the \(cc\bar{u}\bar{d}\) dimeson and the \(DD^*\) threshold a value of \(+97\) MeV which means a prediction, that this dimeson is definitely unbound.

**B. With Two- and Three-Body Forces**

Nothing is definite. If a future experiment finds the \(bb\) dimeson unbound or the \(cc\) dimeson bound we shall have to revise our general ideas about the effective quark-quark interaction, and/or introduce many-quark forces.

The recent CDF experiment on double-heavy baryons in Fermilab \[1\] has caused some confusion. The \(cc\bar{u}\bar{d}\) candidate at \(3520\) MeV is somewhat low but still manageable with two-body interactions. One would need smaller constituent quark masses than \[2\]. Why? Smaller quark masses need less negative (kinetic + potential) binding energies to fit the heavy mesons. Therefore, going from \(QQ\) to the \(QQ\) diquark using the \(V_{QQ} = \frac{1}{2}V_{QQ}\) rule one loses less binding and the \(cc\bar{u}\bar{d}\) baryon becomes better bound. (Note that a phenomenological estimate similar to the one for the dimesons would give \(3537\) to \(3560\) MeV). The \(cc\bar{u}\bar{d}\) candidate at \(3783\) seems, however, too light for an excited state and too heavy for the ground state.

On the other hand, the \(cc\bar{u}\) baryon candidate at \(3460\) MeV is not believable due to its \(60\) MeV isospin splitting. If, however, correct, it would require help from many-body forces or some other mechanisms which would lead to an almost bound \(cc\) dimeson (our phenomenological estimate can then be written as \(\Delta E_{cc\bar{u}\bar{d}} = M_{\Xi_{cc}} + M_{\Lambda_c} - M_{D} - M_{D^*} = +3\) MeV).

Some weak three-body terms have been introduced in ref. \[2\] in order to improve (lower) the absolute position of the baryon spectrum if the meson spectrum is fitted. We are exploring which colour structures could be used and whether the parametrisation could be
stretched so as to bind cc dimeson or unbind the bb dimeson without spoiling the fit to mesons and baryons.

III. A MODEL FOR THE SYNTHESIS OF THE HEAVY DIMESON

The bb⃗ u⃗ d dimesons have not been seen in the present machines. Anyway, estimates of the production and detection rate are very pessimistic and have for this reason not been published. Therefore we encourage to look for them at LHC. For the synthesis we propose a three-step model.

(i) First, two b-quarks are formed in the proton-proton collision by a double gluon-gluon fusion: 
\((g + g) + (g + g) \rightarrow (b + \bar{b}) + (b + \bar{b})\). This was shown to be the dominant production mechanism. One might wonder why we need a TeV machine to produce GeV particles. The answer is simple. The two colliding protons can be considered as two packages of virtual gluons whose number is huge for low Bjorken-\(x\). Only the number of gluons with \(x < 0.001\) might be sufficient to make dimesons detectable.

The forward detector LHCb will cover the pseudorapidity region \(1.8 < \eta < 4.9\) and will detect the B and \(\bar{B}\) hadrons in the low \(p_T\) region. We are interested in double-b production in which the two b-quarks are close enough in phase space to synthesize a diquark. By requiring that the two b are produced with \(|p_1(j) - p_2(j)| < \Delta, j = x, y, z,\) we get the cross section \(\sigma \approx 0.4(\Delta/\text{GeV})^3\) nb which is approximately proportional to the momentum volume up to 2 GeV: \(d\sigma/d^3p \approx 0.4\) nb/GeV\(^3\). At the expected luminosity \(L=0.1\) events/(second nb) this corresponds to 144 interesting bb pairs per hour per GeV\(^3\).

(ii) In the second step, the two b quarks join into a diquark. We assume simultaneous production of two independent b quarks with momenta \(\vec{p}_1, \vec{p}_2\). Since they appear wherever within the nucleon volume, we modulate their wavefunctions with a Gaussian profile with the “oscillator parameter” \(B = \sqrt{2/3} \sqrt{<r^2>} = 0.69\) fm corresponding to the nucleon rms radius

\[
\mathcal{N}_B \exp\left(-\vec{r}_1^2/2B^2 + i \vec{p}_1 \vec{r}_1\right)\mathcal{N}_B \exp\left(-\vec{r}_2^2/2B^2 + i \vec{p}_2 \vec{r}_2\right)
\]  
\[
\equiv \mathcal{N}_{B_\sqrt{2}} \exp\left(-\vec{R}^2/(2B/\sqrt{2})^2 + i \vec{P} \vec{R}\right)\mathcal{N}_{B_\sqrt{2}} \exp\left(-\vec{r}^2/2(B/\sqrt{2})^2 + i \vec{p} \vec{r}\right)
\]

where the normalization factor \(\mathcal{N}_\beta = \pi^{-3/4} \beta^{-3/2}\).

We make an impulse approximation that this two-quark state is instantaneously trans-
formed in any of the eigenstates of the two-quark Hamiltonian. Then the amplitude of the diquark formation $M$ is equal to the overlap between the two free quarks and the diquark with the same centre-of-mass motion. By approximating the diquark wavefunction with a Gaussian with the oscillator parameter $\beta = 0.23$ fm we get

$$M(p) = \int d^3r N_B \sqrt{2} \exp(-r^2 / 2B^2) \exp(-\vec{p}\vec{r} - i \vec{p}\vec{r}) = \sqrt{2B \beta^2} \exp((-p^2 / 2)(2B^2\beta^2 / (2B^2 + \beta^2)))$$

For the production cross section we take into account that $\beta << B$ and that $d\sigma / d^3p$ is practically constant and can be taken out of the integral

$$\sigma = \int d^3p \frac{d\sigma}{d^3p} M^2(p) \approx \frac{d\sigma}{d^3p} \left( \frac{\sqrt{2\pi} \hbar}{B} \right)^3 = 0.15 \text{nb}$$

which corresponds to $L\sigma = 54$ diquarks/hour.

(iii) In the third step, the diquark gets dressed. It either acquires one light quark to become the doubly-heavy baryon bbu, bbd or bbs, or two light antiquarks to become a dimeson.

We estimate the probabilities of dressing $f_{\text{dress}}$ using the analogy with dressing a single quark ("fragmentation of a quark into hadrons"). We make use of experimental data obtained at Fermilab and at LEP experiments [7]:

$$\bar{b} \to \bar{b}u, \bar{b}d, \bar{b}s, \bar{b}\bar{u}\bar{d} = 0.37 \pm 0.02, 0.37 \pm 0.02, 0.16 \pm 0.03, 0.09 \pm 0.03.$$

Since a heavy diquark acts similarly as a heavy quark, we expect similar branching ratios:

$$bb \to bbu, bbd, bbs, bb\bar{u}\bar{d} \approx 0.37, 0.37, 0.16, 0.09.$$

This yields the production rate of the dimeson $L\sigma f_{\text{dress}} \sim 5 - 6$ events/hour.

IV. DISCUSSION ABOUT THE DETECTION OF DIMESONS

We expect that the bb\bar{u}\bar{d} dimeson will be stable against strong and electromagnetic decay and will decay only weakly, with a lifetime of about 1 ps (corresponding to the width of 1 meV). The main channel would be the independent decay of each b quark into c quark. This essentially means the independent decay of each B meson in the dimeson, for example.
B \to D + \text{anything. However, such a channel will be difficult to distinguish from the decay of two unbound B mesons which will be the main background contribution. There are no good two-body decay channels of B mesons to allow the reconstruction of the total energy of the dimeson; moreover, each separate exclusive decay channel has a low branching ratio of up to a few percent.}

Much more characteristic would be the simple two-body decay channel $bb\bar{u}\bar{d} \to \Upsilon + \pi$ with the kinetic energy 876 MeV of both mesons together (in the c.m. system). Of course there would be a crowd of other $\Upsilon$ mesons, but few at this energy. The inspiration comes from the $B^0 \to \bar{B}^0$ oscillation which unfortunately is not feasible for bound $B$ mesons because the $BB$ and $B\bar{B}$ states are not degenerate. The weak transition $b\bar{u} \to u\bar{b}$ is negligible because of the low CKM amplitudes.

Some hope is offered by the angular correlation of the $b$ and $c$ quarks after the decay of the first $b \to c$, from which the original $b$-$b$ correlation could be deduced. If a dimeson were formed, the correlation should be isotropic in the c.m. system of the dimeson since the two heavy quarks are expected to be in the relative $1s$ state. On the other hand, two independent $b$-quarks would tend to move more in the same direction.

We call for new ideas for the detection of doubly $b$ dimesons!
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