A Unified Performance Analysis of the Effective Capacity of Dispersed Spectrum Cognitive Radio Systems over Generalized Fading Channels

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Abstract—The effective capacity (EC) has been recently established as a rigorous alternative to the classical Shannon's ergodic capacity since it accounts for the delay constraints imposed by future wireless applications and their impact on the overall system performance. This paper develops a novel unified approach for the EC analysis of dispersed spectrum cognitive radio (CR) with equal gain combining (EGC) and maximal ratio combining (MRC) diversity receivers over generalized fading channels under a maximum delay constraint. The mathematical formalism is validated with selected numerical and equivalent simulation performance evaluation results thus confirming the correctness of the proposed unified approach.

Index Terms—Cognitive radio, dispersed spectrum, delay constraints, effective capacity, equal gain combining, fading channels, maximal ratio combining.

I. INTRODUCTION

R
eal-time emerging applications such as voice over internet protocol (IP), interactive and multimedia streaming, interactive gaming, mobile TV and computing are largely delay-sensitive, which implies that the data will expire if it is not successfully delivered within a certain time frame. As such, an alternative quality of service (QoS) metric that is able to capture the end-to-end communication delay is required. Unfortunately, the conventional notion of Shannon or outage capacity cannot account for the delay aspect. The effective capacity (EC) was introduced in [1] as an alternative metric that quantifies the system performance under QoS limitation.

EC analysis of MISO systems is important research topic because of its inherent operational ability to improve spectral efficiency of next generation wireless communication systems. Dispersed spectrum CR systems have recently attracted the attention of the research community as they can provide full frequency multiplexing and diversity [2]. Because of the fact that modern CR technologies take into account the dispersed nature of the heterogeneous spectrum bands, performance analysis of dispersed spectrum CR system operating is very important, since it can provide useful insights regarding the fundamental limits of wireless communication systems enhanced with CR capabilities [3].

Recently, the concept of EC has attracted the attention of the wireless research community for the performance analysis of single- and multiple-antenna communication systems [4]-[8]. The EC of such systems has been dealt with in several research works, assuming either independent [4]-[6] or correlated [7], [8] fading channels. Specifically, in [4], the EC of multiple-input single-output (MISO) systems assuming independent Rice- and Nakagami-m channels was addressed. In [5] and [6], the EC of MISO systems assuming independent κ−µ and η−µ fading channels, respectively was investigated. In [8], the EC of multiple-antenna systems operating in fading channels with correlation and keyholes has been considered. Finally, in [7], infinite series representations of the EC of correlated Rice and Nakagami-m MISO fading channels have been derived.

It should be noted that the above mentioned approaches presented in [4]-[8] employ the so-called probability density function (PDF)-based approach, which requires knowledge of the PDF of the signal-to-noise ratio (SNR) at the receiver output. Recent wireless applications, however, have become increasingly sophisticated thus using more realistic channel models for performance evaluation purposes [9]. For fading scenarios modeled by generalized fading distributions (e.g., correlated generalized Rice/Nakagami-m [10], generalized gamma (GG) [11], α−κ−µ and α−η−µ [12]), the PDF of the SNR at the receiver end is usually not available in a simple form. In such cases, the evaluation of the EC of wireless systems with L diversity branches using a PDF-based approach requires the computation of an L-fold integral. In general, this integration is tedious and computationally cumbersome even if the diversity branches are assumed to be mutually independent.

On the other hand, equal gain combining (EGC) is regarded as practical alternative to maximal ratio combining (MRC) as its performance is comparable to that of MRC but with lower implementation complexity. Nevertheless, to the best of our knowledge the EC of EGC receivers operating over generalize fading channels has not been considered in the open technical literature yet. This is mainly because of the inherent difficulty in obtaining simple, mathematically tractable expressions for the PDF of the sum of fading envelopes [13]. In addition, the results presented in previously published works, i.e. [4]-[8] are valid for only MRC diversity receivers and cannot be applied to the evaluation of the EC of EGC diversity receivers in a straightforward manner.

Motivated by the above, in this paper we propose a new
moment generating function (MGF)-based approach for obtaining, in a unified way, the exact analysis of the EC of dispersed spectrum CR systems over generalized fading channels, with either MRC or EGC diversity reception. Although our analysis is general enough to accommodate all the well-known fading distributions available in the open technical literature, for performance evaluation purposes, we consider four generic channel fading models, namely arbitrary correlated generalized Rice/Nakagami-\(m\), GG, \(\alpha-k-\mu\) and \(\alpha-\eta-\mu\). Note that the \(\alpha-k-\mu\) and \(\alpha-\eta-\mu\) fading models have not been sufficiently considered for performance evaluation purposes yet, because of the rather complicated mathematical expression of their PDF. Therefore, very few works on performance analysis of wireless communication systems operating over such fading channels are available in the open technical literature. Two representative examples can be found in [14], [15].

In addition to the above mentioned unified approach, the specific novel contributions of the paper are as follows:

- A novel, single integral expression for the EC of dispersed spectrum CR systems, with either MRC or EGC diversity reception, is deduced. This expression can be evaluated for a variety of fading distributions by means of standard numerical integration techniques;
- Novel, computationally efficient, expressions for the MGF of GG, \(\alpha-k-\mu\) and \(\alpha-\eta-\mu\) distributions are presented and the EC of dispersed spectrum CR system is evaluated for different values of their parameters. Such expressions are given as finite sums of exponentials or Bessel functions, avoid the use of Meijer’s G-functions and allow for efficient performance evaluation of the system under consideration;
- In order to get further insight into the effect of system parameters, such as delay constraints, fading, parameters and number of antennas, simple expressions for the EC of dispersed spectrum CR systems are deduced that become tight at low- and high-signal-to-noise ratios (SNR) regimes.

The accuracy of the proposed analysis is substantiated with numerical results, accompanied with equivalent performance evaluation results obtained by means of Monte-Carlo simulations.

The rest of the paper is organized as follows. In Section II the system and channel model is introduced along with a generic expression for the signal-to-noise (SNR) ratio and the EC. In Section III an exact analytical expression for the EC using a unified MGF-based approach is developed, while Section IV presents generic asymptotic expressions for EC in high- and low-SNR regimes. Numerical results are provided in Section V validating the proposed analysis. Finally, Section VI summarizes the paper and the main findings.

**Notations:** \(E(\cdot)\) denotes expectation, \(f_x(\cdot)\) denotes the PDF of the random variable \(X\). \(M_X(\cdot)\) denotes the MGF of the random variable \(X\), \(\Delta(k,a) = \{ \frac{a+1}{k}, \ldots, \frac{a+k-1}{k} \}\), \(I, \det(\cdot)\) and \((\cdot)^{-1}\) denote the \(L \times L\) identity matrix, matrix determinant, and matrix inversion, respectively, \(\| \cdot \|_2\) denotes the squared Frobenius norm, \(L^{-1}\{F(s); s; t\}\) denotes the inverse Laplace transform of \(F(s)\), \(J_a(\cdot)\) is the Bessel function of the first kind and order \(a\) [16 Eq. (8.402)], \(I_a(\cdot)\) is the modified Bessel function of the first kind and order \(a\) [16 Eq. (8.310/1)], and \(G_{p,q}^{m,n}[\cdot]\) is the Meijer’s G-function [16 Eq. (9.301)].

## II. System Model

Let us consider a dispersed spectrum CR in which secondary users (SUs) perform spectrum sensing to identify the available bands. Hereafter, opportunistic spectrum access is considered, where spectrum sensing and allocation are performed in order to determine the available bands and the bands that will be allocated to each user, respectively [2], [17]. The considered system comprises \(L\) available frequency diversity bands with identical bandwidths. During the transmission phase, the transmitted signal is replicated \(L\) times in order to create frequency diversity. Each signal is transmitted over each fading channel and corrupted by additive white Gaussian noise (AWGN). At the receiver side, all the signals received from different channels are first co-phased and then weighted equally or weighted with the fading envelopes when EGC or MRC is employed, respectively. The weighted signals are then being summed to form the combined output.

The instantaneous signal-to-noise ratio (SNR), \(\gamma_{\text{end}}\), at the output of the diversity receiver can be generically expressed as [13 Eq. (2)]

\[
\gamma_{\text{end}} = \frac{E_s}{N_0 \sqrt{L} 1-p+q} \left( \sum_{\ell=1}^{L} R_{\ell} \right)^q \tag{1}
\]

where \(E_s/N_0\) is the SNR per symbol and \(R_{\ell}\) is the fading envelope at the \(\ell\)-th branch, \(\forall \ell \in \{1, 2, \ldots, L\}\). Moreover, \((p,q) = (1,2)\) for EGC whereas \((p,q) = (2,1)\) for MRC. As far as the queuing model is concerned, a simple first-input first-output (FIFO) buffer with constant arrival rate (source data rate) at the transmitter data link layer is considered. By considering ideal modulation and coding at the source physical layer, the service rate of the buffer will be equal to the instantaneous channel capacity. Therefore, using [3 Eq. (4)] and assuming that the transmitter sends uncorrelated circularly symmetric zero-mean complex Gaussian signals, the EC can be expressed as

\[
R(\theta) = -\frac{1}{\theta T B} \ln \left[ E\left(1 + \gamma_{\text{end}}\right)^{-A}\right] \tag{2}
\]

where \(A \triangleq B T B/\ln 2\) represents a metric of delay constraint, with \(B\) denoting the bandwidth of the system, \(T\) the fading block length and \(\theta\) the asymptotic decay-rate of the buffer occupancy. It is noted that a small value for \(\theta\) corresponds to a slow decaying rate thus having less stringent QoS requirements, while a larger one refers to faster decaying rate and thus more stringent QoS requirements.

By substituting (1) into (2), it becomes evident that the evaluation of the EC assuming arbitrarily distributed \(R_{\ell}\) with generalized correlation, involves the numerical evaluation of the following \(L\)-fold integral,

\[
I = \int_{\mathbb{R}} \left[ 1 + \frac{E_s}{N_0 \sqrt{L} 1-p+q} \left( \sum_{\ell=1}^{L} R_{\ell} \right)^q \right]^{-A} f_{\overrightarrow{R}}(\overrightarrow{R}) d\overrightarrow{R} \tag{3}
\]
where \( f_R(\tilde{R}) \) is the joint PDF of the random vector \( \tilde{R} \triangleq \{R_1, R_2, \ldots, R_L\} \). Clearly, this expression becomes prohibitively complex even for small values of \( L \). In fact, even beyond only three branches such an approach becomes computationally intractable and numerical results may not even converge. In order to solve this cumbersome statistical problem, a unified MGF-based approach for the numerical evaluation of \( R(\theta) \) will be presented next that provides a generic single integral expression for the EC of EGC and MRC diversity combiners over arbitrarily correlated generalized fading channels.

### III. THE PROPOSED MGF-BASED APPROACH

#### A. Main Result

In order to obtain a unified MGF-based approach for the evaluation of the EC, the following generic useful lemma is proved first.

**Lemma.** Let \( X \) be a positive RV with PDF \( f_X(x) \) and MGF \( M_X(u) \). Let also \( g(X) \) be a function of \( X \) for which it is assumed that the following inverse Laplace transform \( h(u) = \mathbb{L}^{-1}\{g(x); x; u\} \) exists. Then, the expectation \( \mathbb{E}(g(X)) \) can be expressed in terms of \( M_X(u) \) as

\[
\mathbb{E}(g(X)) = \int_0^\infty h(u)M_X(u)du. \tag{4}
\]

**Proof.** By exploiting the definition of the MGF, i.e., \( M_X(u) \triangleq \int_0^\infty \exp(-ux)f_X(x)dx \), the expectation \( \mathbb{E}(g(X)) \) can be written as

\[
\mathbb{E}(g(X)) = \int_0^\infty f_X(x)g(x)dx = \int_0^\infty f_X(x)\left[\int_0^\infty \exp(-xu)h(u)du\right]dx = \int_0^\infty h(u)\left[\int_0^\infty \exp(-xu)f_X(x)dx\right]du \tag{5}
\]

thus completing the proof.

Based on the above lemma, a unified MGF-based approach for the evaluation of the EC of dispersed spectrum CR systems can be deduced as follows.

**Proposition.** The EC of a dispersed spectrum CR with \( L \) available frequency diversity bands over arbitrary not necessarily independent nor identically distributed fading channels can be expressed in terms of a single integral as

\[
R(\theta) = -\frac{1}{\theta TB} \ln \left[ \frac{1}{\Gamma(A)} \int_0^\infty C_q(u)M_{\tilde{R}_p}(K_{p,q}u)du \right] \tag{6}
\]

where \( M_{\tilde{R}_p}(u) \triangleq \mathbb{E}(\exp(-u\sum_{\ell=1}^L R_{p,\ell}^p)) \) is the joint MGF of the \( p \)-th exponent of the random vector \( \tilde{R} \), \( K_{p,q} = \sqrt{E_s/N_0}L^{(p-q-1)/2} \) and the function \( C_q(u) \) is given by

\[
C_q(u) = (2\pi)^{q+1/2}u^{1/2}G_{q+1,1}^{1,1} \left[ \frac{1}{u^q} |1-A\Delta(q,0)|_0^\infty \right]. \tag{7}
\]

**Proof.** Let us define the RV \( X \triangleq \sum_{\ell=1}^L R_p^p \) and the auxiliary function \( C_q(u) = \mathbb{L}^{-1}\{(1+x^q)^{-A}; x; u\} \). Then, by employing the previous lemma, the expectation of \((1+X^q)^{-A}\) can be expressed as

\[
\mathbb{E}\left((1+X^q)^{-A}\right) = \int_0^\infty C_q(u)M_X(u)du.
\]

By considering the identity \( \left[19, \text{Eq. (8.4.2.5)}\right] \)

\[
(1+x^q)^{-A} = \frac{1}{\Gamma(A)} G_{1,1}^{1,1} \left[ x^q |1^0_0 \right] \tag{8}
\]

and by employing \( \left[20, \text{Eq. (3.38.1)}\right] \) \( C_q(u) \) can be deduced as \( \left[7\right] \) yielding \( \left[6\right] \), which completes the proof.

It is noted that \( \left[6\right] \) is valid for arbitrary and correlated fading channels, as long as the MGF of \( \gamma_{\text{end}} \) exists. For the special case of uncorrelated \( R_\ell \), the EC of the considered system can be readily evaluated by employing the following corollary.

**Corollary.** The EC of a dispersed spectrum CR with \( L \) independent diversity branches is deduced as

\[
R(\theta) = -\frac{1}{\theta TB} \ln \left[ \frac{1}{\Gamma(A)} \int_0^\infty C_q(u) \prod_{\ell=1}^L M_{R_p}(K_{p,q}u)du \right] \tag{9}
\]

**Proof.** When independent branches are considered, \( M_{\tilde{R}_p}(K_{p,q}u) \) can be expressed as the product of the MGFs of \( R_p \), i.e., \( M_{R_p}(K_{p,q}u) = \prod_{\ell=1}^L M_{R_p}(u) \). Then, \( \left[9\right] \) is readily obtained from \( \left[6\right] \).

For the special cases of MRC and EGC diversity receivers, i.e. for \( q = 1 \) and \( 2 \), respectively, it can be shown that \( C_q(u) \) can be expressed in terms of more familiar exponential and Bessel functions. Specifically, for \( q = 1 \), by employing \( \left[19, \text{Eq. (8.4.3.1)}\right], \left[19, \text{Eq. (8.2.2.8)}\right] \) and \( \left[19, \text{Eq. (8.2.2.16)}\right] \), \( C_q(u) \) can be deduced as

\[
C^{\text{MRC}}_q(u) = u^{A-1} \exp(-u), \tag{10}
\]

which is in perfect agreement with \( \left[21, \text{Eq. (5)}\right] \). For \( q = 2 \), by employing \( \left[19, \text{Eq. (8.4.19.1)}\right], \left[19, \text{Eq. (8.2.2.8)}\right] \) and \( \left[19, \text{Eq. (8.2.2.16)}\right] \), \( C_q(u) \) can be deduced as

\[
C^{\text{EGC}}_q(u) = \sqrt{\pi} \left(\frac{u}{2}\right)^{A-1/2} J_{A-1/2}(u). \tag{11}
\]

Note that although in the context of this work \( C^{\text{MRC}}_q(u) \) was derived assuming MRC diversity receivers, it can be used for virtually any diversity scheme, such as selection diversity (SD) and generalized selection combining (GSC), as long as the MGF of \( \gamma_{\text{end}} \) is readily available. For example, for selection diversity (SD), \( \gamma_{\text{end}} \) can be expressed as \( \gamma_{\text{end}} = \max\{R_1^2, R_2^2, \ldots, R_L^2\} \). For generalized selection combining (GSC), the \( K \) strongest branches out of \( L \) available are combined as \( \gamma_{\text{end}} = \sum_{k=1}^K \gamma_{(k)}^2 \) where \( \gamma_{(1)}^2 > \gamma_{(2)}^2 > \ldots > \gamma_{(L)}^2 \). For these two diversity schemes, EC can be computed by employing \( \left[6\right] \) with \( C_q(u) \) given by \( \left[10\right] \).

B. Computational Issues

In the case of MRC, the numerical evaluation of EC is rather straightforward because \( C_q^{MRC}(u) \) is a smooth and well-behaved function. Specifically, EC can be evaluated numerically by employing a \( N \)-point Gauss-Chebyshev quadrature technique or standard built-in functions for numerical integration available in the most popular mathematical software packages, such as Matlab, Maple or Mathematica. In the case of EGC, however, the numerical evaluation of a Hankel transform is required. Such a problem can be considerably more difficult, since \( C_q^{EGC}(u) \) is oscillatory. Efficient methods for the numerical evaluation of the Hankel transform are available in [22]. Thus, the integrals in (6) are first expressed as

\[
\int_0^\infty C_q^{EGC}(u)M_{\bar{R}}(K_{q,p,u})du = \sum_{k=0}^{\infty} \int_{u_k}^{u_{k+1}} C_q^{EGC}(u)M_{\bar{R}}(K_{q,p,u})du \tag{12}
\]

where \( u_0 = 0 \) and \( u_k \) is the \( k \)-th zero of \( J_{A-1/2}(u) \), for \( k \geq 1 \). The series in (12), however, is alternating so that a large number of terms may be required to achieve a sufficient numerical accuracy. Fortunately, by invoking a convergence acceleration algorithm this series can be transformed into another that converges faster. Such an algorithm is the so-called Epsilon algorithm [23]. The algorithm generates a two-dimensional array called the \( \varepsilon \)-table with entries \( \varepsilon^{(k)}_r \) where \( r \) is the column index and \( k \) the location down the column. At the initialization phase, the first column is set to zero as \( \varepsilon^{(k)}_{r-1} = 0 \), \( \forall k > 0 \) and the second column is set to the given partial sums \( s_k \) of (12), i.e. \( \varepsilon^{(k)}_0 = s_k \), \( k = 0, 1, \ldots, N - 1 \), where \( N \) is the number of truncation terms. The remaining elements of the \( \varepsilon \)-table are deduced as

\[
\varepsilon^{(k)}_{r+1} = \varepsilon^{(k)}_{r-1} + \frac{1}{\varepsilon^{(k)}_{r+1} - \varepsilon^{(k)}_r}, \quad r = 1, 2, \ldots. \tag{13}
\]

The even columns of the \( \varepsilon \)-table contain increasingly accurate estimates of the infinite series.

IV. ASYMPTOTIC ANALYSIS

In this section, the asymptotic EC performance of dispersed spectrum CR systems with MRC and EGC in the high- and low-SNR regimes is assessed. More specifically, a novel MGF-based approach for the evaluation of the EC in the high-SNR regime is first presented. The proposed approach provides useful insights as to the factors affecting EC performance in terms of the so-called high-SNR slope and high-SNR power offset [24]. Moreover, the low-SNR performance of the considered system is assessed in terms of the minimum normalized energy per information bit to reliably convey any positive rate and the wide-band slope [25]. It is underlined that the proposed asymptotic performance analysis in both the high- and low-SNR regimes is valid for all well-known fading distributions as long as MGF of \( \bar{R}^p \), \( M_{\bar{R}}(u) \), exists.

A. High-SNR Regime

Following a similar line of arguments as in [26], it is first assumed that \( M_{\bar{R}}(u) \) can be approximated for \( u \to \infty \) as

\[
M_{\bar{R}}(u) = Cu^{-d} + o(u^{-d}). \tag{14}
\]

By substituting (14) into (6), and employing (10) and (11) along with [16] Eq. (6.561/14) and the definition of the gamma function, asymptotic expressions for the EC of MRC and EGC diversity receivers, respectively, can be deduced as

\[
R^{MRC}(\theta) = d \frac{\theta}{TB} \left[ \ln(\frac{E_s}{N_0}) - \frac{1}{d} \ln \left( \frac{\Gamma(A - d)}{\Gamma(A)} \right) \right], \quad A > d \tag{15}
\]

and

\[
R^{EGC}(\theta) = d \frac{\theta}{2TB} \left[ \ln(\frac{E_s}{N_0}) - \ln L - 2 \left( \frac{2 - d}{d} \right) \sqrt{\pi} \Gamma(A - d/2) \right] \frac{\Gamma(0.5 + d/2)}{\Gamma(A)}, \quad d/2 < A < d + 1. \tag{16}
\]

As it can be observed, both asymptotic expressions are of the form

\[
R(\theta) = S_\infty (\log_2(\frac{E_s}{N_0}) - L_\infty) \tag{17}
\]

where \( S_\infty \) is the high-SNR slope, defined as

\[
S_\infty \triangleq \lim_{E_s/N_0 \to 0} \frac{R(\theta)}{E_s/N_0} \tag{18}
\]

and \( L_\infty \) the high-SNR power offset defined as

\[
L_\infty \triangleq \lim_{E_s/N_0 \to 0} \left( \log_2(\frac{E_s}{N_0}) - \frac{R(\theta)}{S_\infty} \right) \tag{19}
\]

Using (14), \( S_\infty \) and \( L_\infty \) can be straightforwardly obtained for any fading model in terms of \( C \) and \( d \).

B. Low-SNR Regime

The low-SNR performance of the considered system can be assessed by using a second-order expansion of the EC around \( E_s/N_0 \to 0^+ \) as [22]

\[
R(\theta) = \hat{R}(0) \frac{E_s}{N_0} + \frac{\hat{R}(0)}{2} \left( \frac{E_s}{N_0} \right)^2 + o \left( \frac{E_s}{N_0} \right)^2 \tag{20}
\]

where \( \hat{R}(0) \) and \( \hat{R}(0) \) denote the first and second order derivatives of the EC with respect to the SNR \( E_s/N_0 \). As it was pointed out in [4], \( \hat{R}(0) \) and \( \hat{R}(0) \) are related to the minimum normalized energy per information bit to reliably convey any positive rate and the wide-band slope respectively [24]. For the case of QoS constraints, these metrics are defined respectively as [4] Eq. (19)]

\[
\frac{E_b}{N_0}_{\text{min}} \triangleq \frac{1}{\hat{R}(0)}, \quad S \triangleq - \frac{2 \ln 2 \left[ \hat{R}(0) \right]^2}{\hat{R}(0)}. \tag{21}
\]

By employing [4] Eq. (22)], \( \hat{R}(0) \) and \( \hat{R}(0) \) can be respectively expressed as

\[
\hat{R}(0) = \frac{1}{\sqrt{L^{1-p+\bar{q}}}} E(\bar{R}^p) \tag{22}
\]
\[
\bar{R}(0) = -\frac{A + 1}{L^{1-p+q} \ln 2} E(\mathbf{R}^{2p}) + \frac{A}{L^{1-p+q} \ln 2} \left( E(\mathbf{R}^{p}) \right)^2 . 
\]

(23)

The expectations in (22) can be readily expressed in terms of \( \mathcal{M}_{\mathbf{R}^{p}}(u) \) as [28]

\[
E(\mathbf{R}^{np}) = (-1)^n \left. \frac{\partial^n \mathcal{M}_{\mathbf{R}^{p}}(u)}{\partial u^n} \right|_{u=0} .
\]

(24)

V. PERFORMANCE ANALYSIS AND DISCUSSION

In this section, the proposed analytical approach presented in the previous sections is employed to determine the EC of dispersed spectrum CR systems. More specifically, the following case studies are considered. i) CR systems operating over arbitrary correlated generalized fading, ii) CR systems operating over (i.i.d) generalized gamma (GG), iii) \( \alpha - \kappa - \mu \) and iv) \( \alpha - \eta - \mu \) fading. For all considered cases, in addition to the exact analysis, asymptotic performance evaluation results are also presented.

A. MRC over Arbitrarily Correlated Generalized Fading Channels

Considering generalized fading channels with arbitrary correlation, the instantaneous SNR at the output of the L-branch receiver is given as \( \gamma_{\text{out}} = E_s/N_0 \sum_{l=1}^{L} R_l^2 = \mathbf{h}^H E_s/N_0 \) where \( \mathbf{h} \) is a \( L \times 1 \) complex Gaussian random vector, having mean value \( \eta \) and covariance matrix \( \mathbf{C} = E((\mathbf{h} - \eta)(\mathbf{h} - \eta)^H) \).

1) Exact Analysis: By employing [10] Eq. (18)], the MGF of \( \| \mathbf{h} \|^2_F \) can be deduced as

\[
\mathcal{M}_{\| \mathbf{h} \|^2_F}(s) = \frac{\exp \left[ -s \mu \eta^H (\mathbf{I} + s \mathbf{C})^{-1} \eta \right]}{\det (\mathbf{I} + s \mathbf{C})^\mu} .
\]

(25)

where the parameter \( \mu \) denotes the diversity order of the signal at each branch. Note that (25) is quite versatile, as it includes as special cases the MGF of \( \| \mathbf{h} \|^2_F \) over correlated Nakagami-\( m \) (\( \eta = 0 \)) [13] Eq. (9.219)] and correlated Rice (\( \mu = 1 \)) fading channels [29] Eq. (5)].

As an example, in Fig. 1 the analytical EC performance of a dispersed spectrum CR system with \( L = 3 \) frequency bands in arbitrarily correlated generalized fading environments is illustrated for various values of \( \mu \) and \( A \). These results have been obtained using \( \eta = [0.25 \exp(i\pi/4) 0.5 \exp(i\pi/6) \exp(i\pi/8)]^T \) and covariance matrix given by [10]:

\[
\mathbf{C} = \begin{pmatrix}
\frac{1}{4} & 0.5 e^{\frac{\pi i}{4}} & 0.25 e^{\frac{3\pi i}{4}} \\
0.5 e^{-\frac{\pi i}{4}} & 2 & 0.125 e^{\frac{3\pi i}{4}} \\
0.25 e^{-\frac{3\pi i}{4}} & 0.125 e^{-\frac{3\pi i}{4}} & 3
\end{pmatrix} .
\]

(26)

It is noted that the EC increases as \( \mu \) increases or \( A \) decreases. In the same figure, equivalent Monte-Carlo computer simulated performance evaluation results have been obtained which perfectly match the analytical ones.

2) High-SNR Analysis: The MGF of \( \| \mathbf{h} \|^2_F \) in (25) when \( s \rightarrow \infty \) can be approximated as

\[
\mathcal{M}_{\| \mathbf{h} \|^2_F}(s) \approx \frac{\exp \left[ \mu \eta^H \eta \right]}{\det (\mathbf{C})^\mu} s^{-\mu} ,
\]

(27)

which is of the form presented in Section IV-A. Therefore using (15) we can obtain the high-SNR asymptotic EC under generalized fading with MRC diversity receiver.

In Fig. 2 the exact analytical EC and high-SNR approximation of a dispersed spectrum CR system with \( L = 3 \) frequency bands and MRC in correlated generalized fading (\( \mu = 2 \))
B. MRC or EGC over Uncorrelated Generalized Gamma Fading Channels

In this case, the PDF or $R_\ell$ is given as \[11\]

$$f_{R_\ell}(r) = \frac{\beta(b/\Omega)^{m\beta/2}r^{\beta m-1} \exp \left[ -\frac{(b/\Omega)^{\beta/2} r^{\beta}}{\Gamma(m)} \right]}{\Gamma(m)}.$$  \hspace{1cm} (28)

where $\beta > 0$ and $m > 1/2$ are two parameters related to the average fading severity and $\Omega$ is related to the average fading power as $E(R_\ell^2) = \left(\Omega/m \right)^{2\beta} \Gamma(m + 2/\beta)/\Gamma(m)$.

1) Exact Analysis: For rational values of $\beta$, the MGF of $R_\ell^p$ can be obtained in terms of Meijer’s G-function by using a similar line of arguments as in [30]. For arbitrary values of $\beta$, the MGF can be expressed in terms of the Fox’s H-function by employing the approach presented in [31]. In what follows, a simple, computational efficient expression for the MGF of $R_\ell^p$, valid for arbitrary values of $\beta$ will be derived. By employing the definition of the MGF and performing the change of variables $(b/\Omega)^{\beta/2} r^{\beta} = t^2$, $\mathcal{M}_{R_\ell^p}(u)$ can be deduced as

$$\mathcal{M}_{R_\ell^p}(u) = \frac{2}{\Gamma(m)} \int_0^\infty t^{2m-1} \exp \left[ -t^2 - ut (b/\Omega)^{\beta/2} \right] dt$$  \hspace{1cm} (29)

The integral in (29) can be solved by employing a Gauss-Chebyshev quadrature technique as

$$\mathcal{M}_{R_\ell^p}(u) = \frac{2}{\Gamma(m)} \sum_{k=1}^{15} w_k t_k^{2m-1} \exp \left[ -u t_k (b/\Omega)^{\beta/2} \right],$$  \hspace{1cm} (30)

where $w_k$ and $t_k$ are the weights and abscissae given in [32].

Figs. 3 and 4 depict the EC performance of dispersed spectrum CR systems with $L = 3$ frequency bands when either MRC or EGC is employed, respectively, for various values of $A$ and fading parameters $\beta$ and $\mu$. It can be observed that there is a fine agreement between analytical and simulated results. Moreover, the EC performance improves as $A$ decreases and $\beta$ or $\mu$ increases.

2) High-SNR Analysis: Hereafter we consider the EGC case only, i.e. $q = 2$. By employing a Taylor series expansion of (28) at $r = 0$ one obtains

$$f_{R_\ell}(r) \approx \frac{\beta(b/\Omega)^{m\beta/2}r^{\beta m-1}}{\Gamma(m)}.$$  \hspace{1cm} (31)

By invoking the definitions of the MGF and the gamma function, the MGF of $R_\ell$ when $u \to \infty$ can be approximated as

$$\mathcal{M}_{R_\ell}(u) \approx \frac{\beta(b/\Omega)^{m\beta/2} \Gamma(\beta m)}{\Gamma(m)} u^{-\beta m}.$$  \hspace{1cm} (32)

Then by employing (15) and (16) the high-SNR asymptotic EC performance under GG fading with EGC diversity can be readily deduced. In Fig. 5 the exact analytical EC and high-SNR approximation of a dispersed spectrum CR system with $L = 3$ frequency bands and EGC over i.i.d. GG fading channels are depicted for various values of $A$. It is evident that the high-SNR approximation provides very tight results for high SNR values.

3) Low-SNR Analysis: We consider hereafter the EGC case only. Then by combining (20) and (22)-(24) the low-SNR asymptotic EC under GG fading with EGC diversity can be readily deduced. In Fig. 6 the exact analytical EC and low-SNR approximation of a dispersed spectrum CR system with $L = 3$ frequency bands and EGC over i.i.d. GG fading channels are depicted for various values of $A$. It is evident that the low-SNR approximation provides very tight results for low SNR values.

C. MRC or EGC over Uncorrelated $\alpha - \kappa - \mu$ Fading Channels

The $\alpha - \kappa - \mu$ is a very general fading model that includes as special cases several well-known distributions, namely the
distribution. In this case, the PDF of $\mathcal{R}_\ell$ is given by \cite{12}

$$f_{\mathcal{R}_\ell}(r) = \frac{\alpha \kappa^{\frac{1+\mu}{\alpha}}}{\exp\left[\frac{\mu r^\alpha}{\mu(1+\kappa)}\right]} \left(2 \sqrt{\kappa(1+\kappa)}I_{\mu-1}\left(2 \sqrt{\kappa(1+\kappa)}\right)\right),$$

where $\alpha, \mu$ and $\kappa \geq 0$ are the distribution parameters.

1) Exact Analysis: A novel computationally efficient expression for $\mathcal{M}_{\mathcal{R}_\ell}^2(u)$ can be obtained by following a similar line of arguments as in the generalized gamma case, as

$$\mathcal{M}_{\mathcal{R}_\ell}^2(u) = 2 \exp[-\mu \kappa](\mu(1-\mu)^{1/2}$$

$$\times \sum_{k=1}^{15} w_k t_k^\mu I_{\mu-1}\left(2 \sqrt{\kappa(1+\kappa)} t_k\right) \exp\left[-u \left(\frac{t_k^2}{\mu(1+\kappa)}\right)^{p/a}\right].$$

where the weights $w_k$ and abscissae $t_k$ are given in \cite{32}.

Figs. 7 and 8 depict the EC of a dispersed spectrum CR system with $L = 3$ frequency bands and MRC or EGC over i.i.d. $\alpha - \kappa - \mu$ fading. As it can be observed, analytical and simulated results are in perfect agreement. Moreover, as expected, EC improves as $A$ decreases and $\alpha, \kappa$ or $\mu$ increases.
2) High-SNR Analysis: Hereafter, the EGC case is considered, i.e. \( q = 2 \). By following a similar line of arguments as in the generalized gamma case, \( M_{R_i}(u) \) when \( u \to \infty \) can be approximated as

\[
M_{R_i}(u) \approx \alpha(1 + \kappa)\mu^u \exp(-\mu u)\Gamma(\alpha\mu)u^{-\alpha\mu}. \tag{35}
\]

Therefore using (15)–(16) asymptotically tight high-SNR expressions for the EC of the considered system can be readily deduced.

D. MRC or EGC over Uncorrelated \( \alpha-\eta-\mu \) Fading Channels

The \( \alpha-\eta-\mu \) distribution is a very general fading distribution that includes as special cases the generalized gamma, the \( \eta-\mu \), the Hoyt and the Nakagami-\( m \) distributions [12]. In this case, the PDF of \( R_i \) is given by [12]

\[
f_{R_i}(r) = \frac{\alpha(\eta - 1)\frac{2}{\sqrt{\eta}}(\eta + 1)^{\frac{\alpha}{\mu}}r^{\alpha\mu - 1}}{\exp\left[-(\eta + 1)^{\frac{1}{\mu}}\right] \sqrt{\eta} \Gamma(\mu)} \prod\mu^{\frac{2}{1 + \mu}}I_{1 - \frac{\alpha}{\mu}}\left(\frac{r^{2\alpha\mu}}{2\eta}\right), \tag{36}
\]

where \( \eta \geq 0 \).

1) Exact Analysis: Following a similar methodology as in the previous test cases, a novel expression for the MGF of \( R_i^p \) is deduced as

\[
M_{R_i^p}(u) = \frac{2\alpha\sqrt{\eta}(\eta - 1)\frac{2}{\sqrt{\eta}}(\eta + 1)^{\frac{\alpha}{\mu}}r^{\alpha\mu - 1}}{\prod\mu^{\frac{2}{1 + \mu}}I_{1 - \frac{\alpha}{\mu}}\left(\frac{r^{2\alpha\mu}}{2\eta}\right)} \times u^{-\alpha(1+2\mu)/(2p)}.
\]

Figs. 9 and 10 depict the EC of a dispersed spectrum CR system with \( L = 3 \) frequency bands and MRC or EGC, respectively, under \( \alpha-\eta-\mu \) fading. Again, numerically evaluated and computer simulation results are in perfect agreement.

2) High-SNR Analysis: Following a similar reasoning as in the previous cases and assuming EGC diversity reception, \( M_{R_i}(u) \) can be approximated as

\[
M_{R_i}(u) \approx \frac{\alpha(\eta - 1)\frac{2}{\sqrt{\eta}}(\eta + 1)^{\frac{\alpha}{\mu}}r^{\alpha\mu - 1}}{\prod\mu^{\frac{2}{1 + \mu}}I_{1 - \frac{\alpha}{\mu}}\left(\frac{r^{2\alpha\mu}}{2\eta}\right)} \times \left[\frac{\eta^2 - 1}{4\eta}\right]^{\frac{\alpha}{\mu} - 1}u^{-2\alpha\mu}. \tag{38}
\]

In Fig. 11 the exact EC and high-SNR approximation of a dispersed spectrum CR system with \( L = 3 \) frequency bands and EGC i.i.d. \( \alpha-\eta-\mu \) fading channels are depicted for various values of \( A \). It is evident that the high-SNR approximation provides exact results, even at moderate SNR values, and can thus accurately predict the respective effective rate.

VI. CONCLUSIONS

Real-time applications are quite delay-sensitive, requiring an alternative performance metric rather than the conventional Shannon or outage capacity. Lately, the effective capacity has attracted attention as a suitable metric quantifying end-to-end system performance under QoS limitations. In this paper a new moment generating function (MGF)-based approach for obtaining, in a unified way, the exact analysis of EC of a dispersed spectrum CR system with either MRC or EGC diversity reception was proposed. A unified MGF-based approach for the asymptotic analysis of EC at low- and high-SNR regions was also proposed thus providing useful insights regarding the factors affecting system performance. The validity of the proposed analytical approach was assessed by considered very generic channel fading models that describe wireless propagation in a more realistic manner than the conventional fading models. The accuracy of the proposed analysis was substantiated with numerical results, accompanied with equivalent performance evaluation results obtained by means of Monte-Carlo simulations.
Fig. 11. Exact analytical EC and high-SNR approximation of a dispersed spectrum CR system with $L = 3$ frequency bands and EGC in i.i.d $\alpha - \eta - \mu$ fading ($\alpha = 2.4, \eta = 64.3, \mu = 1.2$).

[2] K. A. Qaraqe, H. Celebi, M. Mohammad, and A. Ekin, “Performance analysis of ad hoc dispersed spectrum cognitive radio networks over fading channels,” *EURASIP J. Wir. Comm. Networking*, Springer, Jan. 2011.

[3] T. Tsiftsis, F. Foukalas, G. Karagiannidis, and T. Khattab, “On the higher-order statistics of the channel capacity in dispersed spectrum cognitive radio systems over generalized fading channels,” *IEEE Trans. Veh. Technol.*, 2016.

[4] M. Matthaiou, G. Alexandropoulos, H. Ngo, and E. Larsson, “Analytic framework for the effective rate of MISO fading channels,” *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1741–1751, 2012.

[5] J. Zhang, Z. Tan, H. Wang, Q. Huang, and L. Hanzo, “The effective throughput of MISO systems over $\kappa$-$\mu$ fading channels,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 2, pp. 943–947, 2014.

[6] J. Zhang, M. Matthaiou, Z. Tan, and H. Wang, “Effective rate analysis of MISO $\eta - \mu$ fading channels,” in *IEEE Int. Conf. Commun. (ICC)*, 2013, pp. 5840–5844.

[7] X.-B. Guo, L. Dong, and H. Yang, “Performance analysis for effective rate of correlated MISO fading channels,” *Electron. Lett.*, vol. 48, no. 24, pp. 1564–1565, 2012.

[8] C. Zhong, T. Ratajarah, K.-K. Wong, and M.-S. Alouini, “Effective capacity of multiple antenna channels: Correlation and keyhole,” *IET Commun.*, vol. 6, no. 12, pp. 1757–1768, 2012.

[9] M. Dohler, R. W. Heath, A. Lozano, C. Papadias, and R. Valenzuela, “Is the phy layer dead?” *IEEE Commun. Mag.*, vol. 49, no. 4, pp. 159–165, 2011.

[10] F. Yilmaz and M.-S. Alouini, “On the computation of the higher-order statistics of the channel capacity over generalized fading channels,” *IEEE Wireless Comm. Lett.*, vol. 1, no. 1, pp. 573–576, 2012.

[11] E. Stacy, “A generalization of the gamma distribution,” *The Annals of Mathematical Statistics*, vol. 3, no. 33, pp. 1187–1192, Sep. 1962.

[12] G. Fraidenraich and M. D. Yacoub, “The $\alpha - \eta - \mu$ and $\alpha - \kappa - \mu$ fading distributions,” in *2006 IEEE Ninth International Symposium on Spread Spectrum Techniques and Applications*, Aug 2006, pp. 16–20.

[13] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[14] A. K. Papazafeiropoulos and S. A. Kotsopoulos, “Statistical properties for the envelope and phase of the $\alpha$-$\eta$-$\mu$ generalized fading channels,” *Wireless Personal Communications*, vol. 66, no. 4, pp. 651–666, Oct. 2012.

[15] ——, “Second-order statistics for the envelope of $\alpha$-$\kappa$-$\mu$ fading channels,” *IEEE Commun. Lett.*, vol. 14, no. 4, pp. 291–293, Apr. 2010.

REFERENCES

[1] D. Wu and R. Negi, “Effective capacity: A wireless link model for support of quality of service,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 630–643, Jul. 2003.

[16] I. Gradhsheyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[17] S. Gezici, H. Celebi, H. V. Poor, and H. Arslan, “Fundamental limits on time delay estimation in dispersed spectrum cognitive radio systems,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 78–83, Jan. 2009.

[18] F. Yilmaz and M. S. Alouini, “A unified MGF-based capacity analysis of diversity combiners over generalized fading channels,” *IEEE Trans. Commun.*, vol. 60, no. 3, pp. 862–875, 2010.

[19] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series Volume 3: More Special Functions*, 1st ed. Gordon and Breach Science Publishers, 1986.

[20] ——, *Integrals and Series Volume 5: Inverse Laplace Transforms*, 1st ed. CRC, 1992.

[21] K. P. Peppas, P. T. Mathiopoulos, and J. Yang, “On the effective capacity of amplify-and-forward multihop transmission over arbitrary and correlated fading channels,” *IEEE Wireless Comm. Lett.*, vol. 5, no. 3, pp. 248–251, Jun. 2016.

[22] M. J. Cree and P. J. Bones, “Algorithms to numerically evaluate the Hankel transform,” *Computers Math. Appl.*, vol. 26, no. 1, pp. 1–12, 1993.

[23] D. Shanks, “Non-linear transformations of divergent and slowly convergent sequences,” *J. Math. Phys.*, vol. 34, pp. 1–42, 1955.

[24] S. Shamai and S. Verd, “The impact of frequency-flat fading on the spectral efficiency of CDMA,” *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1302–1327, May 2001.

[25] S. Verd, “Spectral efficiency in the wideband regime,” *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, Jun. 2002.

[26] Z. Wang and G. Giannakis, “A simple and general parametrization quantifying performance in fading channels,” *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1390–1398, Aug. 2003.

[27] M. C. Gursoy, “MIMO wireless communications under statistical queueing constraints,” *IEEE Trans. Inf. Theory*, vol. 57, no. 9, pp. 5897–5917, Sep. 2011.

[28] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 4th ed. New York: McGraw Hill, 2002.

[29] K. Hamdi, “Capacity of MRC on correlated Rician fading channels,” *Int’l J. Electron. and Commun.*, vol. 56, no. 5, pp. 708–7011, 2008.

[30] N. C. Sagias and P. T. Mathiopoulos, “Switched diversity receivers over generalized gamma fading channels,” *IEEE Commun. Lett.*, vol. 9, no. 10, pp. 787–783, 2005.

[31] F. Yilmaz and M.-S. Alouini, “Product of the powers of generalized Nakagami-m variates and performance of cascaded fading channels,” in *IEEE Global Telecommunications Conf.*, 2009, pp. 1–8.

[32] N. M. Steen, G. D. Byrne, and E. M. Gelbard, “Gaussian quadratures for the integrals $\int_0^\infty e^{-x^2} f(x)dx$ and $\int_0^\infty e^{-\mu^2} f(x)dx$,” *Mathematics of Computation*, vol. 23, no. 107, pp. 661–671, 1969.