tsmp: An R Package for Time Series with Matrix Profile

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Abstract. This article describes tsmp, an R package that implements the matrix profile concept for time series. The tsmp package is a toolkit that allows all-pairs similarity joins, motif, discords and chains discovery, semantic segmentation, etc. Here we describe how the tsmp package may be used by showing some of the use-cases from the original articles and evaluate the algorithm speed in the R environment. This package can be downloaded at https://CRAN.R-project.org/package=tsmp.

Introduction: time series data mining

A time series (TS) is a sequence of real-valued numbers indexed in time order. Usually, this sequence is taken in a regular period of time, which will be assumed to be true in this context. The interests in TS data mining have been growing along with the increase in available computational power. This kind of data is easily obtained from sensors (e.g., electrocardiogram (ECG)), (ir)regular registered data (e.g., weekly sales, stock prices, brachial blood pressure). Even other kinds of data can be converted to TS format, such as shapes (Wei et al., 2006) and DNA sequences (Shieh and Keogh, 2008). TS are generally large, high dimensional and continuously updated which requires algorithms fast enough in order to be meaningful. Besides, unlike other kinds of data, which usually have exact answers, TS are usually analysed in an approximated fashion.

These characteristics have been challenging researchers to find faster and more accurate methods to retrieve meaningful information from TS. This required one or more of these methods: dimensionality reduction, constraints, domain knowledge, parameter tweaks. Only afterwards could the data mining tasks be applied in feasible time. Typical tasks include motif and discord discovery, subsequence matching, semantic segmentation, rule discovery, similarity search, anomaly detection, clustering, classification, indexing, etc. (Fu, 2011).

This paper describes the tsmp package (Bischoff, 2018) which uses a novel approach to TS data mining: the matrix profile (MP) Yeh et al. (2017b), which is based on the all-pairs similarity search (APSS) (also known as similarity join). The APSS’ task is to, given a collection of data objects, retrieve the nearest neighbour for each object. The remaining part of this paper is organised as follows: In Section 2.2 we describe the reasoning behind the MP, in Section 2.3 we present the tsmp package with examples, in Section 2.4 we compare the performance of the R implementation, and in Section 2.5 we conclude with a brief discussion.

The matrix profile

The reader may be aware of what a distance matrix (DM) is. It is widely used in TS for clustering, classification, motif search, etc. But, even for modestly sized datasets, the algorithms can take months to compute even with speed-up techniques such as indexing (Shieh and Keogh, 2008; Fu et al., 2008), lower-bounding (Keogh and Ratanamahatana, 2005), data discretization (Lin et al., 2003) and early abandoning (Faloutsos et al., 1994). At best, they can be one or two orders of magnitude faster.

The MP is an ordered vector that stores the Euclidean distance between each pair within a similarity join set. One (inefficient) way would be to use the full DM of every iteration of a sliding window join and retrieve just the smallest (non-diagonal) value of each row. The MP also has a companion vector called profile index (PI), that gives us the position of the nearest neighbour of each subsequence.

This method has a host of interesting and exploitable properties. For example, the highest point on the MP corresponds to the TS discord, the (tied) lowest points correspond to the locations of the best TS motif pair, and the variance can be seen as a measure of the TS complexity. Moreover, the histogram of the values in the MP is the exact answer to the TS density estimation. Particularly, it has implications for TS motif discovery, TS joins, shapelet discovery (classification), density estimation, semantic segmentation, visualisation, rule discovery, clustering, etc. (Yeh et al., 2017b).

Some of the advantages/features of this method:

- It is exact, providing no false positives or false dismissals.
• It is \textit{simple} and parameter-free. In contrast, the more general metric space APSS algorithms require building and tuning spatial access methods and/or hash functions.
• It requires an inconsequential space overhead, just $O(n)$ with a small constant factor.
• It is extremely \textit{scalable}, and for \textit{massive} datasets, we can compute the results in an anytime fashion, allowing ultra-fast \textit{approximate} solutions.
• Having computed the similarity join for a dataset, we can incrementally update it very efficiently. In many domains, this means we can effectively maintain exact joins on \textit{streaming} data forever.
• It provides \textit{full joins}, eliminating the need to specify a similarity threshold, which is a near-impossible task in this domain.
• It is \textit{parallelizable}, both on multicore processors and in distributed systems (Zhu et al., 2016).

The \textit{tsmp} package

The \textit{tsmp} package provides several functions that allow for an easy workflow using the matrix profile (MP) concept for TS mining. The package is available from the Comprehensive R Archive Network (CRAN) at \url{https://CRAN.R-project.org/package=tsmp}. In Section 2.3.1 we explain how to install this package. In Section 2.3.2 we describe the syntax for the main functions in \textit{tsmp}, giving an example of a particular model. In Section 2.3.3 we will further explain the available algorithms for MP computation and its current use. In Section 2.3.4 we show some examples of MP application for data mining.

Installation

The \textit{tsmp} package can be installed in two ways:

The release version from CRAN:

\begin{verbatim}
install.packages("tsmp")
\end{verbatim}

or the development version from GitHub:

\begin{verbatim}
# install.packages("devtools")
devtools::install_github("franzbischoff/tsmp")
\end{verbatim}

Input arguments and example

The \textit{tsmp} has a simple and intuitive workflow. First, you must compute the MP of the desired TS. Depending on the task, the user might want to follow one of three paths: univariate self-join, AB-join or multivariate self-join. One exception is the SiMPle algorithm that is a multivariate AB-join and will be explained in Section 2.3.3.

The main function is \texttt{tsmp()}, which has the following usage:

\begin{verbatim}
tsmp(..., window_size, exclusion_zone = 1/2, 
    mode = c("stomp", "stamp", "simple", "mstomp", "scrimp"),
    verbose = 2, a_size = Inf, must_dim = NULL, exc_dim = NULL, 
    n_workers = 1, .keep_data = TRUE)
\end{verbatim}

The first argument \texttt{ellipses} (the three dots) receives one or two TS. For self-joins, the user must input just one TS; two for AB-joins. Multivariate TS may be input as a matrix where each column represents one dimension. Alternatively, the user may input the Multivariate TS as a list of vectors. The second argument \texttt{window_size} is the size of the sliding window. These are the most basic parameters you need to set.

Further parameters are:

• \texttt{exclusion_zone}, is an important parameter for self-joins. This is used to avoid trivial matches and is a modifier of the \texttt{window_size}, i.e., for an \texttt{exclusion_zone} of 1/2, and \texttt{window_size} of 50, internally the result will be 25.

• \texttt{mode}, here the user may choose the algorithm used for the MP calculation. \texttt{stomp}, \texttt{stamp} and \texttt{scrimp} return equal results, although differing in some practical attributes, and they will be further explained in Section 2.3.3. \texttt{mstomp} is designed for Multivariate TS self-join only. \texttt{simple} is designed for Multivariate TS for self-join and AB-join, which will also be further explained in Section 2.3.3.
• **verbose**, controls the verbosity of the function. 0 means no feedback, 1 means text messages only, 2 (the default) means text messages and progress bar, and 3 also plays a sound when finished.

• **s_size**, controls the anytime algorithms. This is just a way to end the algorithm in a controlled manner because the anytime algorithms can be stopped anytime and the result will be returned.

• **must_dim**, is an optional argument for the **mstomp** algorithm. See next item.

• **exc_dim**, as **must_dim**, is an optional argument for the **mstomp** algorithm. These arguments control which dimensions must be included and which must be excluded from the multidimensional MP.

• **n_workers**, controls how many threads will be used for the **stamp**, **stomp**, and **mstomp**. Note that for small datasets, multiple threads add an overhead that makes it slower than just one thread.

• **keep_data**, TRUE by default, keeps the input data inside the output object. This is useful for chained commands.

**Example data**

We think that the best and simple example to demonstrate the **tsmp** package is the motif search.

The **tsmp** package imports the `%>%` (pipe) operator from the **magrittr** package that makes the **tsmp** workflow easier.

The following code snippet shows an example of the workflow for motif search:

```r
R> data <- mp_fluss_data$walkjogrun$data
R> motifs <- tsmp(data, window_size = 80, exclusion_zone = 1/2) %>% find_motif(n_motifs = 3, radius = 10, exclusion_zone = 20) %>% plot()
```

The **find_motif()** function is an S3 class that can receive as the first argument the output of **tsmp()** function as a univariate or multivariate MP. This allows us to use the pipe operator easily. The **plot()** function is also an S3 class extension for plotting objects from the **tsmp** package and works seamlessly.

**Computational methods**

There are several methods to compute the matrix profile (MP). The reason for that is the unquenchable need for speed of the UCR’s researchers. Before starting, let’s clarify that the time complexity of a brute force algorithm has a time complexity of $O(n^2m)$, for $n$ being the length of the reference TS and $m$ the length of the sliding window (query) that is domain dependent.

**Scalable time series anytime matrix profile (STAMP)**

This was the first algorithm used to compute the MP. It uses the Mueen’s algorithm for similarity search (MASS) (Mueen et al., 2015) as the core algorithm for calculating the similarity between the query and the reference TS, called the distance profile (DP). The ultimate MP comes from merging the element-wise minimum from all possible DPs. This algorithm has the time complexity of $O(n^2 \log n)$ and space complexity of $O(n)$ (Yeh et al., 2017b). The anytime property is achieved using a random approach where the best-so-far MP is computed using the DPs that have been already calculated.

**Scalable time series ordered-search matrix profile (STOMP)**

This was the second algorithm used to compute the MP. It also uses the MASS to calculate the DP but only for the first iteration of each batch. The researchers noticed that they could reuse the values calculated of the first DP to make a faster calculation in the next iterations. This results on a time complexity of $O(n^2)$, keeping the same space complexity of $O(n)$. This algorithm is also suitable for a graphics processor unit (GPU) framework (although this was not yet implemented in **tsmp** package) (Zhu et al., 2016). The main drawback of STOMP compared with STAMP is the lack of the anytime property. In scenarios where a fast convergence is needed (e.g., finding the top-k motifs) it may be required only 5% of the MP computation to provide a very accurate approximation of the final result.
Scalable column independent matrix profile (SCRIMP)

The SCRIMP algorithm is still experimental at the time of this article. It combines the best features of STOMP and STAMP, having a time complexity of $O(n^2)$ and the anytime property (UCR, 2016).

Similarity matrix profile (SiMPle)

The SiMPle algorithm is a variation designed for music analysis and exploration (Silva et al., 2018). Internally it uses STOMP for MP computation and allows multidimensional self-joins and AB-joins. The resulting MP is computed using all dimensions. One major difference is that it doesn’t apply any z-normalization on the data, since for music domain this would result in spurious similarities.

Multivariate STOMP (mSTOMP)

The mSTOMP algorithm was designed to motif search in multidimensional data (Yeh et al., 2017a). Performing motif search on all dimensions is almost guaranteed to produce meaningless results, so this algorithm, differently from SiMPle, doesn’t compute the MP using all dimensions naively, but the $d$-dimensional MPs for every possible setting of $d$, simultaneously, in $O(dn^2 \log d)$ time and $O(dn)$ space. The resulting MPs allow motif search in multiple dimensions and also to identify which dimensions are relevant for the motifs founded.

Data mining tasks

Motif search

In Section 2.3.2 we have shown a basic example of the workflow for motif search. Let’s take a look at the result of that code:

```r
R> motifs
Matrix Profile
--------------
Profile size = 9922
Window size = 80
Exclusion zone = 40
Contains 1 set of data with 10001 observations and 1 dimension

Motif
-----
Motif pairs founded = 3
Motif pairs indexes = [584, 741] [3917, 5088] [8543, 8843]
Motif pairs neighbors = [2948] [6719] [6849, 4944]
```

As we can see, this is a summary that tsmp package automatically generates from the resulting object. One nice property is that the object always holds the original MP and by default also holds the input data so that you can keep mining information from it. If the dataset is too big or you are concerned about privacy, you may set the argument `keep_data = FALSE`.

In addition to this summary, you can see the results using `plot()` in Figure 1:
```r
R> plot(motifs, type = "matrix")
```

![Matrix Profile (w = 80; ez = 0.5)](image)

**Figure 1:** The upper graphic shows the computed MP with each motif pair as a coloured vertical bar. The lower graphics show each motif in colour and the founded neighbours in grey.

This dataset is the *WalkJogRun* PAMAP's dataset (Reiss and Stricker, 2012). It contains the recording of human movements in three states, walking, jogging and running. As we can see, the plot shows the motifs of each state. Experienced readers might say that this is not the purpose of motif search, and we agree. The result shown here was achieved using a large `radius` and `exclusion_zone` to force the algorithm to look for distant motifs. Semantic segmentation is the proper algorithm for this task, and we will show this in the next section.

**Semantic segmentation**

As previously explained, the resulting object holds the original data and MP. So let's save some time and use the resulting object from the last section to try to find where the human subject started to jog and to run:

```r
R> segments <- motifs %>% fluss(num_segments = 2)
R> segments
```

```
Matrix Profile
-------------
Profile size = 9922
Window size = 80
Exclusion zone = 40
Contains 1 set of data with 10001 observations and 1 dimension

Arc Count
---------
Profile size = 9922
Minimum normalized count = 0.063 at index 3448

Fluss
-----
Segments = 2
Segmentation indexes = 3448 6687
```

We can see that this object now holds information of the fast low-cost unipotent semantic segmentation (FLUSS) algorithm (Gharabbi et al., 2017), but the motif information is still there and can be retrieved using `as.motif()`. In Figure 2 we can see the graphic result of the segmentation.
**Figure 2:** Semantic segmentation using MP. The upper graphic shows the arc plot of predicted semantic changes (ground truth is 3800 and 6800). The middle graphic shows the data. The lower graphic shows the normalised arc counts with correction for the "edge-effect" (Gharghabi et al., 2017).

**Time series chains**

As a final example of practical application, let’s search for a new kind of primitive: time series chains (Zhu et al., 2018a). This algorithm looks for patterns that are not just similar but evolve through time. The dataset used in this example is a record of the Y-axis of a mobile phone accelerometer while placing it on a walking subject’s pocket (Hoang et al., 2015). The authors of this dataset wanted to analyse the stability of the mobile phone as it slowly settles in the pocket. This is a good example of a pattern that changes through time. Let’s start with the workflow for this example:

```r
R> chains <- mp_gait_data %>% tsmp(window_size = 50, exclusion_zone = 1/4,
+   verbose = 0) %>% find_chains()
R> chains
```

**Matrix Profile**

--------------
Profile size = 855
Window size = 50
Exclusion zone = 13
Contains 1 set of data with 904 observations and 1 dimension

**Chain**

-----
Chains founded = 58
Best Chain size = 6
Best Chain indexes = 148 380 614 746 778 811

Here we see that the algorithm found 58 chains. *Id est*, it found 58 evolving patterns with at least three elements, and the best one is presented in the last line, a chain with six elements. Figure 3 shows the patterns discovered.
$R> plot(chains, ylab = \"\"")$

**Figure 3:** Finding evolving patterns using MP. The upper graphic shows the arc plot of the discovered patterns. The middle graphic shows the data and the position of every pattern as a vertical coloured line. The lower graphic shows the patterns for comparison. They are y-shifted for visualisation only.

### Speed

While this new method for TS data mining is extremelly fast, we have to take into consideration that the R environment is not as fast as a low-level implementation such as C/C++. In Table 1 we present the comparison to the MATLAB version that is available at the University of California Riverside (UCR). Yeh et al. (2017b) shows that the slowest algorithm (STAMP) can be hundreds of times faster than the MK algorithm (the fastest known exact algorithm for computing TS motifs) (Yoon et al., 2015), while the R implementation is just 1.65 to 8.04 times slower than MATLAB’s, which is not a problem for an R researcher.

$R> set.seed(2018)$

$R> data <- cumsum(sample(c(-1, 1), 40000, TRUE))$

| Algorithm | R Time* | MATLAB Time* | Threads |
|-----------|---------|--------------|---------|
| scrimp    | 45.30   | 27.49        | 1       |
| stomp     | 52.72   | 10.27        | 8       |
| stomp     | 136.01  | 16.91        | 1       |
| stamp     | 140.25  | 55.57        | 8       |
| stamp     | 262.03  | 113.18       | 1       |

**Table 1:** Performances of R and MATLAB implementations on an Intel(R) Core(TM) i7-7700 CPU @ 3.60GHz using a random walk dataset. *Median of 5 trials, in seconds.
Conclusion

The examples in Section 2.3.4 show how straightforward the usage of tsmp package is. Regardless, these examples are just a glimpse of the potential of the MP. Several new algorithms based on MP are being developed and will be gradually implemented in the tsmp package (Linardi et al., 2018; Zhu et al., 2018b; Gharghabi et al., 2018; Imani et al., 2018). Yeh et al. (2017a) for example, have developed an algorithm to allow multidimensional space (MDS) visualisation of motifs. Gharghabi et al. (2018) have developed a new distance measure that better suits repetitive patterns (Imani et al., 2018).

The MP has the potential to revolutionise the TS data mining due to its generality, versatility, simplicity and scalability (UCR, 2016). All existing algorithms for MP have been proven to be flexible to be used in several domains using very few parameters and they are also robust, showing good performance with dimensionality reduced data and noisy data. In addition, a yet to be published article shows a fantastic score of \( > 10^{18} \) pairwise comparisons a day using GPU for motif discovery (Zimmerman et al., 2018).

The tsmp package is the first known MP toolkit available on any statistical language, and we hope it can help researchers to better mining TS and also to develop new methods based on MP.

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## Acronyms

| Acronym | Description |
|---------|-------------|
| APSS    | all-pairs similarity search |
| CRAN    | Comprehensive R Archive Network |
| DM      | distance matrix |
| DP      | distance profile |
| ECG     | electrocardiogram |
| FLUSS   | fast low-cost unipotent semantic segmentation |
| GPU     | graphics processor unit |
| MASS    | Mueen's algorithm for similarity search |
| MDS     | multidimensional space |
| MP      | matrix profile |
| mSTOMP  | multivariate scalable time series ordered-search matrix profile |
| PI      | profile index |
| SCRIMP  | scalable column independent matrix profile |
| SiMPle  | similarity matrix profile |
| STAMP   | scalable time series anytime matrix profile |
| STOMP   | scalable time series ordered-search matrix profile |
| TS      | time series |
| UCR     | University of California Riverside |
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