Detecting Physics At The Post-GUT And String Scales
By Linear Colliders

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Abstract
The ability of linear colliders to test physics at the post-GUT scale is investigated. Using current estimates of measurements available at such accelerators, it is seen that soft breaking masses can be measured with errors of about (1-20)%. Three classes of models in the post-GUT region are examined: models with universal soft breaking masses at the string scale, models with horizontal symmetry, and string models with Calabi-Yau compactifications. In each case, linear colliders would be able to test directly theoretical assumptions made at energies beyond the GUT scale to a good accuracy, distinguish between different models, and measure parameters that are expected to be predictions of string models.

1. Introduction
Much of current high energy theory considerations have centered around the possibility that the aspects of the Standard Model (SM) that are not presently understood (e.g. Yukawa couplings, CKM parameters etc.) are
consequences of new physical principles arising at or near the Planck scale
\( M_{Pl} = (\hbar c/8\pi G_N)^{1/2} \approx 2.4 \times 10^{18} \text{ GeV} \). It is thus important to ask whether hypotheses made at such high energies can be experimentally verified.

One normally thinks that a high energy accelerator allows one to learn about physics below its energy reach, but physics above this energy is unprobed. Thus, for example, LEP1 has determined the lower bound on the Higgs mass to be 65 GeV, and one will have to wait until LEP2 to learn more. However, at least two theoretical results have moderated this viewpoint. First the renormalization group equations (RGE) allow one to take data at one energy and extrapolate it to a higher energy to test theoretical ideas at this higher energy. A second, related, result is supersymmetric grand unification. It implies that this upward extrapolation can be made over an enormous energy domain, i.e. of over \( 10^{14} \) GeV.

The fact that the three coupling constants \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) unify for supersymmetric (SUSY) models to a GUT value \( \alpha_G \approx 1/24 \) at a scale \( M_G \) of about \( 10^{16} \) GeV was seen in the 1990 LEP precision data [1], appears quite firm. Thus there have been refinements in the data and refinements in the theoretical treatment (i.e. inclusion of SUSY threshold effects at \( M_S \approx 100 \) GeV - 1 TeV [2], GUT scale threshold effects [3,4], and small Planck scale effects [4]). It is possible that the unification of the three coupling constants at \( M_G \approx 2 \times 10^{16} \) GeV is purely a numerical accident, particularly since there is an adjustable parameter, the SUSY mass scale, \( M_S \). However, such an accident is not too easy to achieve for several reasons: there is only a narrow window of values for \( M_G \) between about \( 5 \times 10^{15} \) GeV (below which in most models a too rapid proton decay \( p \to e^+\pi^0 \) would occur) and the “string” scale about \( 5 \times 10^{17} \) GeV, (above which gravitational effects become strong invalidating the analysis.) Further, unification does not occur for the Standard Model, and satisfactory SUSY unification occurs only for two Higgs doublets (the minimal number) and for no more than four families. Finally, naturalness requires that \( M_S \approx 1 \) TeV, which indeed turns out to be the case.

In order to see whether grand unification is an accident or has deeper physical significance, it is necessary to have other measurements to experimentally test the idea. We will see below that linear colliders (LC) can indeed do this, and can not only probe GUT scale physics, but also physics that may be occurring above the GUT scale, and do this with a high degree of precision.

To analyse grand unification we use here the supergravity models [5] where supersymmetry is broken in a hidden sector at a scale \( \tilde{M}_G \). These models have a number of positive attributes including the following: (1) They account for the unification of the coupling constants. (2) They allow for spontaneous
breaking of supersymmetry at the GUT (or Planck) scale. (This is a crucial feature for without SUSY breaking it is not possible to confront a theoretical model with experiment.) (3) Using the RGE to go to low energies, one finds that the spontaneous breaking of supersymmetry at energy $\sim M_G$ triggers the spontaneous breaking of $SU(2) \times U(1)$ at the electroweak scale $M_{EW} = O(M_Z)$ [6]. Thus supergravity models give an explanation of the Higgs phenomena (and predicted that the top quark would be heavy i.e. $90 \text{ GeV} \lesssim m_t \lesssim 200 \text{ GeV}$) a decade before its discovery. The above three items together imply a very predictive theory, and most of the phenomenological SUSY analyses make use of some or all of the constraints implied by supergravity.

In order to understand the nature of quantities in GUT models that might be measured, we briefly summarize some of the structure of the supergravity models. These models depend on three functions of the scalar fields $\phi_i(x)$ (representing sleptons, squarks, etc.): the gauge kinetic function $f_{\alpha\beta}(\phi_i)$ (which enters in the Lagrangian as $f_{\alpha\beta} F_{\mu\nu}^\alpha F^{\mu\nu\beta}$ with $\alpha, \beta = $ gauge indices), the Kahler potential $K(\phi_i, \phi_i^\dagger)$ (which appears in the scalar kinetic energy as $\kappa^2 \partial^\mu \phi_i \partial^\mu \phi_i^\dagger$, $K_j \equiv \partial^2 K/\partial \phi_i \partial \phi_j^\dagger$, and elsewhere) and the superpotential $W(\phi_i)$. The latter two enter only in the combination

$$G(\phi_i, \phi_i^\dagger) = \kappa^2 K(\phi_i, \phi_i^\dagger) + \ell n \left[ \kappa^6 |W(\phi_i)|^2 \right]$$

(1)

where $\kappa = 1/M_{Pl}$. Writing $\{\phi_i\} = \{\phi_a, z\}$ where $\{\phi_a\}$ are the physical sector fields (quarks, leptons, Higgs) and $z$ are the superHiggs fields whose VEVs, $\langle z \rangle = O(M_{Pl})$, break supersymmetry, the superpotential decomposes into a physical and a hidden part

$$W(\phi_i) = W_{phys}(\phi_a) + W_{hid}(z)$$

(2)

with $\kappa^2 \langle W_{hid} \rangle = O(M_S)$.

The quantities $f_{\alpha\beta}$, $K$ and $W$, at the level of supergravity theory, and are determined by a new physical principle that operates at the Planck scale. However, if one expands these functions in a polynomial in $\phi_a$, those terms carrying the mass dimensions of $f_{\alpha\beta}$, K and W are accessible to low energy discovery (their coupling constants are dimensionless) while higher terms, representing Planck physics corrections, are scaled by $\kappa$. Thus

$$f_{\alpha\beta}(\phi_i) = c_{\alpha\beta}(x) + \kappa^2 c_{\alpha\beta}^{ab}(x)\phi_a \phi_b + \cdots$$

(3)

$$K(\phi_i, \phi_i^\dagger) = \kappa^{-2} c(x, y) + c_b^a(x, y) \phi_a \phi_b^\dagger$$

3
\[ + (c^{ab}(x,y) \phi_a \phi_b + h.c.) \]
\[ + (c^{ab}_{bc}(x,y) \phi_a \phi_b \phi_c^\dagger + h.c.) + \cdots \quad (4) \]

\[ W_{\text{phys}}(\phi_i) = \frac{1}{6} \lambda^{abc}(x) \phi_a \phi_b \phi_c + \frac{1}{24} \kappa \lambda^{abcd}(x) \phi_a \phi_b \phi_c \phi_d + \cdots \quad (5) \]

where \( x \equiv \kappa z, y \equiv \kappa z^\dagger \), and the condition \( K^\dagger = K \) implies

\[ c^a_b(x,y) = c^b_a(y,x)^\dagger; c(x,y) = c(y,x)^\dagger \quad (6) \]

The coefficients in the above expansions, \( c_{\alpha \beta}(x) \), \( c^a_b(x,y) \), etc. have been scaled so that when super Higgs VEVs are taken (\( \langle x \rangle, \langle y \rangle = O(1) \)) they are of \( O(1) \). Also, as is well known [7], one may always transfer the holomorphic \( c^{ab} \) terms of (4) into the superpotential by a Kahler transformation

\[ W \to W' = W \exp[\kappa^2 c^{ab} \phi_a \phi_b] = W + \kappa^2 W c^{ab} \phi_a \phi_b + \cdots \quad (7) \]

which gives rise to an effective \( \mu \) term of the correct electroweak size

\[ \mu^{ab}(x) \phi_a \phi_b; \quad \mu^{ab}(x) = \kappa^2 W_{\text{hid}} c^{ab}(x) \quad (8) \]

Thus \( \mu^{ab} \equiv \langle \mu^{ab}(x) \rangle = O(M_S) \), and the \( \mu \) term arises naturally. One may rescale the gauge and chiral fields so that their kinetic energies have canonical form, after which (for a simple gauge group)

\[ \langle c_{\alpha \beta}(x) \rangle = \delta_{\alpha \beta}; \quad \langle c^a_b(x,y) \rangle = \delta^a_b; \quad \langle c_{xy} \rangle = 1 \quad (9) \]

(where \( c_{xy} = \partial^2 c/\partial x \partial y \).

The higher terms in Eqs. (3-5) scaled by \( \kappa = 1/M_{Pl} \), give rise to non-renormalizable operators (NROs), emphasizing the fact that supergravity models are effective field theories below the Planck scale. Indeed, it would be surprising if such terms did not exist (e.g. in string theory one expects such NROs to arise upon integrating out the tower of Planck mass states). The non-zero gaugino masses at \( M_G \)

\[ (m_{1/2})_{\alpha \beta} = \frac{1}{4} \kappa^{-3} \langle e^{G/2} G^i (K^{-1})^i_{\alpha \beta} \rangle \quad (10) \]

imply such structures occur in the expansion of \( c_{\alpha \beta}(x) \), i.e. \( c_{\alpha \beta}(x) = \delta_{\alpha \beta} + \kappa c^{(1)}_{\alpha \beta} z + \cdots \). A non-zero \( c^{(1)}_{\alpha \beta} \) is required if \( m_{1/2} \) is to be of \( O(M_S) \). Thus it would not be surprising if the corresponding term in the physical sector, \( \kappa c^{(1)}_{\alpha \beta} \phi_a \) were also present. Such a term would not be negligible if \( \phi_a \) were the field that breaks the GUT group to the SM group (e.g. the 24 of SU(5) [8]) for then
\( \langle \phi_a \rangle = O(M_G) \) and this term is \( O(M_G/M_P \ell) \) i.e. a (1-10)\% correction to the leading term. Indeed, the current value, \( \alpha_3 (M_Z) = 0.118 \pm 0.003 \) [9], suggest the existence of a few percent correction of this type [4]. Thus the effects of Planck scale physics on the low energy domain may have already been seen.

In this paper, we consider the possibility of using linear colliders to investigate the post GUT regime. In Sec. 2 we first review what may be determined about GUT scale physics at colliders. We then consider three possible scenarios of the nature of post GUT physics. In Sec. 3, we examine the possibility that the soft breaking masses at the string scale are universal (the RGE producing non-universal effects at \( M_G \)). It is shown there that a LC can distinguish between different gauge groups and even determine the value of \( M_{str} \). In Sec. 4 we examine a simple horizontal group which determines the nature of non-universal soft breaking. In Sec. 5 we examine the Kahler potential arising in a class of Calabi-Yau string models. Sec. 6 contains conclusions.

### 2. GUT Scale Physics

Aside from small GUT scale threshold corrections, most physics below \( M_G \) is insensitive to the nature of the physical sector GUT group \( G \), if \( G \) breaks to the SM group at \( M_G \). Thus relatively model independent tests of some GUT scale physics can be done.

The gaugino soft breaking mass of Eq. (10) is

\[
(m_{1/2})_{\alpha \beta} = \frac{1}{4} \kappa^{-1} \langle G^i (K^{-1})^j_i f^\dagger_{\alpha \beta j} \rangle m_{3/2}
\]  

where \( m_{3/2} = \kappa^{-1} \langle \exp [G/2] \rangle \) is the gravitino mass. In all models where the physical gauge group is a simple group and the hidden sector fields are \( G \) singlets, the gaugino masses will be universal at \( M_G \) to a very good approximation. For this situation the leading term occurs when \( i, j \) are both in the hidden sector yielding \( (m_{1/2})_{\alpha \beta} = m_{1/2} \delta_{\alpha \beta} \) with

\[
m_{1/2} = \frac{1}{4} \langle [c_x(x) + \kappa^{-1} \partial_x W_{hid}/W_{hid}] f x \rangle m_{3/2}
\]  

where \( c_{\alpha \beta}(x) \equiv \delta_{\alpha \beta} f (x) \). Eq. (12) implies \( m_{1/2} = O(M_S) \), though \( m_{1/2} \) could deviate considerably from \( m_{3/2} \). Leading corrections to Eq. (12) arise when \( j = a \) and \( i \) is in the hidden sector, giving from Eqs. (3,4) terms proportional to \( \kappa c_{a b}^\dagger \phi_{\alpha \beta}^a \phi_b \). If \( \phi_b \) is a physical sector field whose VEV breaks \( G \) and hence is of \( O(M_G) \), then one might expect a non-universal correction of order \( M_G/M_P \ell \approx (1-10)\% \). Note that this type of Planck scale term is precisely the one entering the grand unification discussion above which can lead to a reduction in
predicted value of $\alpha_3(M_Z)$ [4].

Universality at $M_G$ leads to the well known relations for the U(1), SU(2) and SU(3) gaugino masses at the electroweak scale [10]

$$\tilde{m}_i = (\alpha_i/\alpha_G) m_{1/2}; \quad i = 1, 2, 3$$

(13)

with the gluino mass $m_{3g} \cong \tilde{m}_3$ [11]. The electroweak sector of this prediction can be well tested at the NLC, i.e. to about 5% [12]. Current analysis shows that $m_{3g}$ can be measured to within about (1-10)% at the LHC (depending on the parameter point) [13], allowing a good test of the third relation $\tilde{m}_2/\tilde{m}_3 = \alpha_2/\alpha_3$ as well, and an experimental determination of the GUT scale parameter $m_{1/2}$. With improved precision one may even be able to detect the small deviations of $\tilde{m}_i$ from universality and correlate them with their effects on the grand unification predictions of $\alpha_3(M_Z)$ mentioned above.

A second important prediction of these supergravity models is the determination of $\mu^2$ at the electroweak scale from the RGE's:

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2$$

(14)

where $m_{H_{1,2}}$ are the running Higgs masses at the electroweak scale (including loop corrections) and $\tan \beta = \langle H_2 \rangle/\langle H_1 \rangle$. The $m_{H_{1,2}}^2(M_Z)$ are scaled by their values $m_{H_{1,2}}(0)$ at $M_G$ and by $m_{3g}$. We restrict these to be within the ranges $0 \leq m_{H_{1,2}}(0) \leq 1$ TeV, 180 GeV < $m_{3g}$ < 1 TeV, where the upper bound is a naturalness requirement, and the lower bound on $m_{3g}$ is the current experimental limit [12]. Hence over most of the parameter space $\mu$ is large and $\mu^2/M_Z^2 >> 1$ (i.e. $\mu \sim 3M_Z$). In this domain, a set of scaling laws hold for the neutralinos ($\tilde{\chi}_i^0$, $i = 1 \cdots 4$), charginos ($\tilde{\chi}_i^\pm$, $i = 1, 2$) and Higgs bosons (h, $H^0$, $H^\pm$, A) [14]:

$$2m_{\tilde{\chi}_1^0} \cong m_{\tilde{\chi}_2^0} \cong m_{\tilde{\chi}_3^\pm} \cong \left(\frac{1}{3} - \frac{1}{4}\right) m_{3g}$$

(15)

$$m_{\tilde{\chi}_3} \cong m_{\tilde{\chi}_4} \cong m_{\tilde{\chi}_2^\pm} >> m_{\tilde{\chi}_1^0}$$

(16)

$$m_{H^0} \cong m_{H^\pm} \cong m_A >> m_h$$

(17)

At a linear collider (LC) the $\tilde{\chi}_i^0$, $\tilde{\chi}_i^\pm$ masses can be measured to $\sim 1\%$ [12,15] and probably good measurements of the Higgs masses will also be available. Thus if these relations were satisfied, it would be good circumstantial evidence for the minimal supergravity unification (though Eqs. (15-17) are not absolutely required by such models as Eq. (14) also has solutions where $\mu =$
It should be noted that many of the particles in Eqs. (15-17) may lie beyond the reach of the NLC, and so higher energy linear colliders will play an important role in testing such relations.

3. Testing String Scale Universality

At scales beyond $M_G$, the theory becomes sensitive to both the nature of the GUT group and the particle spectrum above $M_G$ (which may no longer be just that of the MSSM, i.e. only what was needed to achieve grand unification at $M_G$). Linear colliders will be sensitive to both these types of model dependences, and hence help distinguish between different possibilities. While universality of the gaugino masses is an expected consequence of supergravity grand unification with supersymmetry breaking at the Planck scale, the same is not true for the other soft breaking masses. These arise from the effective potential which has the form [5]

$$V = e^{\kappa K}[(K^{-1})^j_i (W^i + \kappa^2 K^i W)] (W^j + \kappa^2 K^j W)^\dagger - 3\kappa^2 |W|^2 + V_D$$  \hspace{1cm} (18)

where

$$V_D = \frac{1}{2} g^\alpha g^\beta (Re f^{-1})_{\alpha \beta} (K^i (T^\alpha)_{ij} \phi_j) (K^k (T^\beta)_{kl} \phi_l)^\dagger$$  \hspace{1cm} (19)

where $W^i \equiv \partial W/\partial \phi_i$, $T^\alpha$ are the group generators, $K^i = \partial K/\partial \phi_i$, $(K^{-1})^j_i$ is the inverse of the Kahler metric, and $g^\alpha$ are coupling constants. The scalar soft breaking masses $m_0$ and the cubic $A_0$ and quadratic $B_0$ soft breaking masses arise from two types of terms: the superpotential in Eq. (18) which indeed yield a universal contribution, and the Kahler potential which may or may not give a universal contribution [5,7,16]. In terms of the expansions of Eqs. (3-5), one finds [5,7,16]

$$(m^2_0)^a_{\alpha b} = \langle \delta^a_{\alpha b} |c|^2 - 2 + (c_x + c_z) (\kappa^{-1} W_{hid}^x/W_{hid}) + |\kappa^{-1} W_{hid}^x/W_{hid}|^2 \rangle$$

$$+ [c_{\alpha c} c_{\alpha b} - c_{\alpha c}^\dagger] [||c_x|^2 + (c_x + c_z^\dagger) (\kappa^{-1} W_{hid}^x/W_{hid}) +$$

$$+ |\kappa^{-1} W_{hid}^x/W_{hid}|^2] \rangle m_{3/2}^2$$  \hspace{1cm} (20)

where $W_{hid}^x = \partial W_{hid}/\partial x$, and for simplicity we have assumed only one field $z$ grows a Planck mass VEV to break supersymmetry: $<x> = \kappa^{-1} <z>$. (One may easily generalize this.) Imposing the condition that the cosmological constant vanish, i.e. $\langle V \rangle = 0$, reduces $m_0^2$ to the simpler form

$$(m^2_0)^a_{\alpha b} = [\delta^a_{\alpha b} + 3\langle c_{\alpha c} c_{\alpha b} - c_{\alpha c}^\dagger \rangle] m_{3/2}^2$$  \hspace{1cm} (21)
We see that $m_0$ is scaled by $m_{3/2}$ but can differ considerably from it.

Non-universal scalar masses can arise from $c_{cr}^a$ etc., i.e. from derivatives of the Kahler metric $K^a_b$ with respect to the super Higgs fields $z \equiv x\kappa$ and $z^\dagger \equiv y\kappa$ [16]. These terms will be universal only if $K^a_b = \delta^a_b K (x,y)$ i.e. the super Higgs couples universally in K to the physical particles. One possibility is that the symmetry of the Kahler potential, which controls the amount of universality, originates at the higher mass scale where supersymmetry is broken. The highest scale that one can still treat this phenomena field theoretically is the string scale, $M_{str} \simeq 5 \times 10^{17} \text{ GeV}$. In this section we consider then the suggestion that $K^a_b = \delta^a_b K$ at $M_{str}$ and the scalar masses are universal at $\mu = M_{str}$ [17] (a possibility that can actually occur in Calabi-Yau compactification of four dimensional superstrings [18]). The RGE from $M_{str}$ to $M_G$ would then lead to non-universal soft breaking contributions, even if universality held at $M_{str}$. Such phenomena can then be tested at a LC since they produce effects at low energy. It is thus possible to explore experimentally the physics between $M_G$ and $M_{str}$. To illustrate this, we consider several examples of GUT theories.

(i) SU(5) GUT

We assume here for simplicity the minimal particle content above $M_G$, i.e. that matter exists in three generations of $10 = M_{iXY}$ and $5 \equiv M_{ix}$ representations ($i = 1,2,3$), and there is a $5 = H_{1X}$ and $5 = H_2^X$ of Higgs (which contain the two light Higgs doublets coupling to matter) and a $24 = \sum^X_Y$ to break SU(5) to the SM. ($X,Y = 1 \cdots 5$ are SU(5) indices.) The superpotential has the form (retaining only the large third generation Yukawas),

$$ W = \left[ \frac{1}{4} h_t \epsilon_{XYZWU} M^{XY} M^{ZW} H_2 + h_b M^{XY} M_X H_{1Y} \right] + \left[ M_{tr} \Sigma^2 + \frac{1}{6} \lambda_1 tr \Sigma^3 + \lambda_2 H_1 \Sigma H_2 + \mu H_1 H_2 \right] $$

(22)

If one were to assume that the soft breaking masses were universal at $M_{str}$, then SU(5) invariance implies that there would be four soft breaking masses at $M_G$ (which would then modify low energy phenomena). These are $m_{10}$ (which contains $q \equiv (\tilde{u}_L, \tilde{d}_L)$, $u \equiv \tilde{u}_R$, $e \equiv \tilde{e}_R$), $m_5$ (which contains $\ell \equiv (\tilde{u}_L, \tilde{e}_L)$, $d \equiv \tilde{d}_R$) and $m_{H_{1,2}} = m_{H_{1,2}}$ where $H_{1,2}$ are the two light Higgs doublets. One may choose one of the soft breaking masses as the reference mass, $\tilde{m}_0$, and consider deviations of the other masses from $\tilde{m}_0$. A convenient choice is $m_{10} \equiv \tilde{m}_0$ and we write

$$ m_5^2 = \tilde{m}_0^2 (1 + \delta_5); \quad m_{H_{1,2}}^2 = \tilde{m}_0^2 (1 + \delta_{1,2}) $$

(23)
Using Eq. (22), one may calculate the expected deviations from universality that result at $M_G$. These are in general significant ($\approx 50\%$) though not enormous, and are sensitive to the parameters of the model.

One can determine the values of $\tilde{m}_0$ and $\delta_5$ at a linear collider and test the breakdown of universality experimentally that occurs in the post-GUT regime. Thus using the RGE to take the masses down to the electroweak scale, one finds [19]

$$\tilde{m}_0^2 = m_{\tilde{e}_R}^2 - 0.151 m_{1/2}^2 + \sin^2 \theta_W M_Z^2 \cos 2\beta$$  \hspace{1cm} (24)$$

$$\tilde{m}_0^2 \delta_5 = m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2 - 0.377 m_{1/2}^2 + \left( \frac{1}{2} - \sin^2 \theta_W \right) M_Z^2 \cos 2\beta$$  \hspace{1cm} (25)$$

with $m_{1/2} = (\alpha_G/\alpha_2) \tilde{m}_2$. At the NLC one expects to be able to measure $m_{\tilde{e}_R}$, $m_{\tilde{e}_L}$ to about 1%, $m_2$ to about 3%, $\tan \beta$ to 10% [12,15,20] and $\alpha_G$ to perhaps 3%. As an example, for the case $m_{\tilde{e}_L} = 240$ GeV, $m_{\tilde{e}_R} = 200$ GeV, $m_2 = 120$ GeV and $\tan \beta = 5$, one finds from Eqs. (24, 25) and the estimated errors that

$$\tilde{m}_0 \cong (187 \pm 3) \text{ GeV}; \quad \delta_5 \cong 0.206 \pm 0.031$$  \hspace{1cm} (26)$$

Eq. (26) gives an indication of the remarkable level of accuracy obtainable at a LC for the post-GUT parameters. Further, there are many other relations that can be used to determine $\tilde{m}_0$ and $\delta_5$. For example, one can use squarks instead of sleptons since their masses can also be measured at the NLC to 1% provided $\tilde{m}_q < m_{\tilde{g}}$ [21] and $m_{\tilde{q}}$ is within the reach of the NLC. The difference $m_{\tilde{e}_L}^2 - m_{\tilde{d}_L}^2$ determines $\delta_5$ to a similar accuracy. The many different ways to determining $\tilde{m}_0$ and $\delta_5$ would cross check the validity of the SU(5) model.

The parameters $\delta_1$ and $\delta_2$ enter sensitively into $\mu$ and $m_A$. In the scaling region of Eqs. (7-9), $\mu$ can be accurately determined from the $\tilde{\chi}_2^\pm$ and $\tilde{\chi}_{3,4}^0$ masses provided the LC has high enough energy to reach these thresholds [21]. Measurement of $m_A$ requires that the A be pair produced at the LC [22]. The relations one could use to determine $\delta_1$ and $\delta_2$ are

$$\mu^2 (t^2 - 1) = \left[ \delta_1 - \frac{1}{2} t^2 (1 + D_0) \delta_2 \right] \tilde{m}_0^2 + \left[ 1 - \frac{1}{2} t^2 (3 D_0 - 1) \right] \tilde{m}_0^2$$

$$+ \left[ 0.528 + t^2 (3.22 - 3.80 D_0 + 0.060 D_0^2) \right] m_{1/2}^2$$

$$+ \left[ \frac{1}{2} t^2 (1 - D_0) A_F^2 D_0 - \frac{1}{2} M_Z^2 (t^2 - 1) \right]$$  \hspace{1cm} (27)$$

$$m_A^2 \left( \frac{t^2 - 1}{t^2 + 1} \right) = \left[ \delta_1 - \frac{1}{2} (1 + D_0) \delta_2 + \frac{3}{2} (1 - D_0) \right] \tilde{m}_0^2$$

$$+ \left[ 3.22 - 3.80 D_0 + 0.060 D_0^2 \right] m_{1/2}^2 + \frac{1}{2} (1 - D_0) A_F^2 D_0$$
where \( t \equiv \tan \beta \), \( D_0 \) is the Landau pole denominator, \( D_0 = 1 - m_t^2/m_\ell^2 \), \( m_\ell \cong 200 \sin \beta \) GeV, and \( A_R \) is the residue at the pole, \( A_R = -A_t - 1.74 \ m_{1/2} \).

(The numerical coefficients come from running the RGE from \( M_G \) to the electroweak scale.) Thus in order to determine \( \delta_1 \) and \( \delta_2 \), one needs to know \( A_t \), and this could be determined from the light stop (\( \tilde{t}_1 \)) production cross section [13,23]. As an example we consider the parameters \( m_0 = 200 \) GeV, \( \tilde{m}_2 = 120 \) GeV (\( \tilde{m}_2 \equiv (\alpha_2/\alpha_G) m_{1/2} \)), \( \mu = 325 \) GeV, \( m_A = 400 \) GeV, \( A_t = -0.5 \ m_0 \), \( \tan \beta = 5 \) and \( m_t = 175 \) GeV. We assume \( m_0, m_A \) and \( \mu \) are determined with \( \pm 2\% \) error, \( \tilde{m}_2 \) with \( \pm 3\% \) error, \( A_R \) with \( \pm 5\% \) error and \( \tan \beta \) with \( \pm 10\% \) error. One finds

\[
m_{H_1}(0) = (256 \pm 15) GeV; \quad m_{H_2}(0) = (144 \pm 35) GeV
\]  

(29) which corresponds to \( \delta_1 = 0.634 \pm 0.220 \) and \( \delta_2 = 0.485 \pm 0.178 \). Thus deviations from universality would be clearly observable for this situation.

There are numerous other experimental tests one can put this SU(5) model to: (i) There are three mass differences where non-universal effects cancel out:

\[
m_{\tilde{u}_L}^2 - m_{\tilde{u}_R}^2, \quad m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2, \quad m_{\tilde{d}_L}^2 - m_{\tilde{d}_R}^2 .
\]  

(30) These quantities depend only on \( \tilde{m}_0^2 \) and the known RGE form factors in going from \( M_G \) to the electroweak scale. (ii) One can examine generational dependences in these relations and others discussed above. (We have suppressed generational indices in the above equations.) In this way one can check on the symmetry of the Kahler potential. Thus there are many ways of testing such an SU(5) model in the post-GUT regime.

We discuss now how one may determine \( M_{str} \). Assuming that the particle spectrum is that of Eq. (22), one may use the RGE to run the soft SUSY breaking masses to higher scales, and they would unify at \( \mu = M_{str} \). It can then determine experimentally the value of \( M_{str} \). The value of \( M_{str} \) is one of the fundamental parameters of string theory, and it is truly remarkable that it is accessible to experimental test at linear colliders.

(ii) SO(10) GUT

In SO(10) models, each family is put into an SO(10) 16-plet representation, which decomposes into its SU(5) content as 16 = \( 10 + 5 + 1 \), (the SU(5) singlet being \( \nu_R \)). There are a number of ways in which SO(10) can break to the SM
group. We consider here the simplest variant where SO(10) breaks directly to SU(3) x SU(2) x U(1) at $M_G$. One might have expected that SO(10) symmetry implies that $m_5 = m_{10}$ since the 10 and $\bar{5}$ are part of the same 16 representation. However, SO(10) is a rank 5 group and the SM group is rank 4. When one breaks a higher rank group to a lower one, additional D terms can effect the mass relations [24, 25]. Thus at $M_G$, one has $m_{10} \neq m_5$. We will assume here again the simplest possibility that the 5 and $\bar{5}$ Higgs of SU(5) lie in the same 10 of SO(10) (i.e. 10 = 5 + $\bar{5}$). Then at $M_G$ one has [25]

$$m_{10}^2 = m_{16}^2 + \frac{1}{4} (m_{H_1}^2 - m_{H_2}^2)$$

(31)

$$m_5^2 = m_{16}^2 - \frac{3}{4} (m_{H_1}^2 - H_2^2)$$

(32)

where, as in SU(5), $m_{10} = m_q = m_u = m_e$ and $m_5 = m_\ell = m_d$. It is again convenient to chose our reference mass as $\tilde{m}_0 \equiv m_{10}$ with non-universal deviations parameterized as $m_{16}^2 = \tilde{m}_0^2 (1 + \delta_5)$, $m_{H_1,2}^2 = \tilde{m}_0^2 (1 + \delta_{1,2})$. One has then that $m_{16}^2 = \tilde{m}_0^2 [1 - \frac{1}{4} (\delta_2 - \delta_1)]$ and

$$\delta_5 = \delta_2 - \delta_1$$

(33)

In addition, the other SU(5) relations discussed above still hold. Eq. (33) represents the one additional constraint in SO(10) for this pattern of symmetry breaking. It would be testable to about 20% at a LC. For example, using the numbers calculated in Eqs. (26, 29) one finds $\delta_5/(\delta_2 - \delta_1) = 0.160 \pm 25\%$, showing for that case that the SO(10) relation would be significantly violated.

(iii) SU(3) x SU(2) x U(1)

Some string models assume that the SM gauge group holds all the way up to the string scale, after which unification will occur at $M_{str}$ [26]. The RGE will then split the soft breaking masses at $M_G$. Choosing here $m_q \equiv \tilde{m}_0$ as the reference mass one has

$$m_u^2 = \tilde{m}_0^2 (1 + \delta_u); \quad m_\ell^2 = \tilde{m}_0^2 (1 + \delta_2)$$

$$m_d^2 = \tilde{m}_0^2 (1 + \delta_d); \quad m_\ell^2 = \tilde{m}_0^2 (1 + \delta_\ell)$$

$$m_{H_1,2}^2 = \tilde{m}_0^2 (1 + \delta_{1,2})$$

(34)

with the notation $u = \tilde{u}_R$, $e = \tilde{e}_R$ etc. as in SU(5). There are a priori no relations between the different $\delta$'s and hence mass differences that were universal
for SU(5) or SO(10), i.e. Eq. (30), will no longer in general be universal. The different $\delta$’s can, of course, be measured at a LC. Thus $m_{\tilde{u}_L}^2 - m_{\tilde{u}_R}^2$ will determine $\delta_u$, $m_{\tilde{e}_R}^2$ determines $\delta_e$, etc.

(iv) Distinguishing Post-Gut Groups

The LHC can give information about the nature of physics beyond the GUT scale, and in fact distinguish between different gauge groups that may hold beyond $M_G$. In the examples discussed above, one would conclude that if $\delta_{u,e} \neq 0$ or $\delta_d \neq \delta_{\ell}$, then SU(5) and SO(10) would not be valid GUT groups, but the SM group could still hold above $M_G$. But if, $\delta_u = 0 = \delta_e$, $\delta_d = \delta_{\ell}$, then both SU(5) and SO(10) would be consistent with this result. But if, $\delta_5 \neq \delta_2 - \delta_1$, then the specific SO(10) model considered would be eliminated. Similar considerations can be carried out for other gauge groups and other symmetry breaking patterns.

4. Horizontal Symmetry

As discussed in Sec. 2, non-universality of soft breaking masses is controlled by the structure of the Kahler potential. We consider here a model where K possesses a horizontal SU(2)$_H$ symmetry [27] and is based on the total gauge group

$$SU(5) \times SU(2)_H \quad (35)$$

While this model is not completely satisfactory phenomenologically, it does illustrate which aspects of such ideas would be accessible to a LC.

We assume that the first two generations form an SU(2)$_H$ doublet, and the third generation is a singlet. The matter is in the usual SU(5) 10 and 5 representations, while the Higgs are SU(2)$_H$ singlets in 5+\bar{5} and 24 representations (where the 24 breaks SU(5) to the SM at $M_G$). In addition it is assumed that there are three SU(5) singlet, SU(2)$_H$ doublet Higgs, $\phi_i^{(r)}$, $r = 1,2,3$, whose VEV’s have the form $(\langle \phi_{(1)} \rangle, 0)$ and $(0, \langle \phi_{(2)} \rangle)$ to break the SU(2)$_H$ [27]. The superpotential then is

$$W = \left[ \lambda_{ab} M_a^{XY} \bar{M}_{ba} \bar{H}_Y + \lambda_{ab} \epsilon_{XYZW} M_a^{XY} M_b^{ZW} H^U \right]$$

$$+ \left[ \lambda_{e} \phi_{(r)}^i M_i^{XY} \bar{M}_X \bar{H}_Y + \lambda_{e} \epsilon_{XYZWU} \phi_{(r)}^i M_i^{XY} M^{ZW} H^U + \cdots \right] \quad (36)$$
where $a, b = 1, 2, 3$, $i, j = 1, 2$ are SU(2)$_H$ doublet generation indices, and matter fields without generation subscripts are third generation SU(2)$_H$ singlets. One may make a bi-unitary transformation to diagonalize $\lambda^a_{ab}$ to the form diag $\lambda^a_{ab} = (\lambda^d, \lambda^d, \lambda^b)$ while the anti-symmetry of the $\epsilon$-symbol implies diag $\lambda^a_{ab} = (0, 0, 0)$.

The second bracket in Eq. (36) gives rise to mixing between the third and first two generations in the quark mass matrix of size

$$\epsilon = O(\kappa\langle\phi(r)\rangle)$$

and thus second generation masses of size $O(\epsilon^2)$. Hence reasonable first and second generation quark masses require [27] $\lambda^d << 1$ and

$$\epsilon \approx 1/10$$

Thus the picture this model presents is that supersymmetry breaks (in the hidden sector) at the Planck scale ($z \approx M_P$), SU(2)$_H$ breaks at the string scale ($\langle\phi(r)\rangle \approx 1/10 M_P$) and SU(5) at the GUT scale ($\langle\Sigma\rangle \approx M_G \approx 1/100 M_P$), and we will assume all this in the following.

The SU(2)$_H$ Higgs fields can produce corrections to the SU(5) gauge function, but since the $\phi(r)$ are SU(5) singlets, they have the form

$$f_{\alpha\beta}(\phi(r)) = \delta_{\alpha\beta} \kappa^2 c_{rs}(x)\phi^i_{(r)}\epsilon_{ij}\phi^j_{(3)}$$

These corrections are small, i.e. $O(\epsilon^2)$, and maintain the universality of the gaugino masses at the string scale, and down to the GUT scale. Thus they can be neglected. The Kahler potential has the expansion

$$K = \kappa^{-2} c_0^0 (x, y) + \left[ c^{10}_s M^{XY} M^{XY\dagger} + c^{10}_d M^{XY} M^{XY\dagger} + c^5_s M^1_X \bar{M}^1_X \right. + \left. c^5_d \bar{M}^1_X M^1_X \right] + [c^h H^{XY} H^{XY\dagger} + c^{\bar{H}} \bar{H} X \bar{H} X] + c_5 \Sigma^X \Sigma^X \Sigma^X] + \left[ c^{10}_{(r)} \kappa \phi^i_{(r)} M^{XY} M^{XY\dagger} + c^5_{(r)} \kappa \phi^i_{(r)} \bar{M}^1_X \bar{M}^1_X + h.c. \right] + \frac{1}{2} \left[ c^{10}_{[rs]} \kappa \phi^i_{(r)} M^{XY} \kappa \phi^j_{(s)} M^{XY\dagger} + c^{10}_{[rs]} \kappa^2 \phi^i_{(r)} \phi^j_{(s)} M^{XY} M^{XY\dagger} \right] + \ldots$$

where the subscripts (s,d) stand for SU(2)$_H$ (singlet, doublet).

The first two brackets in Eq. (40) are the SU(2)$_H$ invariant terms which give rise to universal soft breaking masses for the first two generations in the 10 and 5 representations, but are, however, split from the singlet third generation and from the Higgs soft breaking masses. Thus in a notation analogous to Eq. (23) they give rise to soft breaking masses.
\[(m_5^i)^2 = \tilde{m}_0^2(1 + \delta_5^i); \quad (m_5)^2 = \tilde{m}_0^2 (1 + \delta_5^i); \quad m_{10}^2 = \tilde{m}_0^2 (1 + \delta_{10}^s); \quad m_{H_{1,2}}^2 = \tilde{m}_0^2(1 + \delta_{1,2}) \quad (41)\]

where the reference mass \(\tilde{m}_0\) is now chosen to be the common mass of the doublet of squarks in the 10 representation. The third and fourth brackets of Eq. (40) give rise to \(\epsilon\) and \(\epsilon^2\) breakings of SU(2)\(_H\) at \(M_{str}\), the former arising only in the mixing of the third (SU(2)\(_H\) singlet) generation with the doublets. Using Eq. (21), one has that the mass matrix for example, for the \(d_L\) squarks at \(M_{str}\) is of the form

\[
\begin{align*}
\tilde{m}_{d_L}^2 &= \begin{pmatrix}
\tilde{m}_d^2 + \epsilon^2 m_{11}^2 & \epsilon^2 m_{12}^2 & \epsilon m_{13}^2 \\
\epsilon^2 m_{12}^2 & \tilde{m}_d^2 + \epsilon^2 m_{22}^2 & \epsilon m_{23}^2 \\
\epsilon m_{13}^2 & \epsilon m_{23}^2 & \tilde{m}_b^2 + \epsilon^2 m_{33}^2
\end{pmatrix}
\end{align*}
(42)
\]

where \(\tilde{m}_d^2 = \tilde{m}_0^2 (1 + \delta_d^i)\), \(\tilde{m}_b^2 = \tilde{m}_0^2(1 + \delta_s^i)\) and \(m_{ij} = O(m_3/2)\). Eq. (42) implies that the first two generation masses at \(M_{str}\) are split by only \(O(\epsilon^2)\) from their values of Eq. (41). The smallness of the splitting, which is natural for these models, is necessary to suppress FCNC [27].

We now discuss what parts of the post GUT hypotheses of these models are directly accessible to experimental test. As seen in Eq. (26), \(\tilde{m}_0\) is determinable to perhaps 2%, and so the \(O(\epsilon^2) \approx 10^{-2}\) splitting of the SU(2)\(_H\) doublets would not be observable without a significant improvement of measurement technique. However, it should be possible to distinguish this class of models from those of Sec. 3. While the non-universal effects in the (mass)\(^2\) differences of Eq. (30) will still cancel out if both masses are in the doublet (first two generation) or singlet (third generation) SU(2)\(_H\) representations, they will not cancel if one is a doublet and the other is a singlet. Thus, the first two differences of Eq. (30) for this “doublet-singlet” type difference determine \(\delta_{10}^s\), and if the model is correct, the value obtained should be the same to \(O(\epsilon^2)\) for each such difference. (There are eight independent measurements of \(\delta_{10}^s\) that should produce the same value of \(\delta_{10}^s\).) The last difference, when one sfermion is in the doublet and one in the singlet determines \(\delta_d^5 - \delta_s^5\) which one expects to be non-zero (and there are four independent measurements which should give the same value for this quantity). As can be seen from Eq. (26), one expects the values of \(\delta_{10}^s\), \(\delta_d^5\), \(\delta_s^5\) to be determined to about 15 % accuracy,
which should allow good tests of the model. Also, unlike the models of Sec. 3, one does not expect the soft SUSY breaking masses to become equal as one extrapolates upwards towards $M_{str}$. Thus while the very small ($\approx 1\%$) effects of the breaking of $SU(2)_H$ are difficult to directly measure, the general constraints of the $SU(2)_H$ symmetry of the Kahler potential should be testable to a reasonable accuracy. Physical assumptions made in theories of this type at energies above $M_G$ (e.g. at $M_{str}$) can be explicitly checked at a LC, and such models distinguished from other models.

5. Superstring Models

The mechanism of supersymmetry breaking in superstring theory is not yet understood, and as a consequence it is not possible to make phenomenological predictions in string theory from first principles. However, it has been suggested that supersymmetry breaking may arise from dilaton(S) and moduli $(T_i, U_i)$ VEV formation. With this assumption, it is possible to calculate soft breaking parameters at the string scale in terms of these unknown VEVs, and this has led to a large amount of analysis in the literature. (See e.g. [28,18].) We consider in this section models of this type arising in Calabi-Yau compactifications with $(2,2)$ vacua based on the gauge group $E_6 \times E_8$, matter then being in $27$ and $\bar{27}$ representations. While the models considered here do not lead to phenomenologically realistic predictions, they will allow us to examine how accurately these string assumptions can be verified experimentally by a LC.

For the case where there is only a single modulus $T$ [29], or when there are many moduli with equal $|F^{T_i}|$ terms [18], the soft breaking masses are universal at the string scale and have the general form (for vanishing U moduli F-terms) [18]:

$$m_{1/2} = \sqrt{3}\sin\theta e^{-i\gamma_S} m_{3/2}$$  \hspace{1cm} (43)

$$m_0^2 = [\sin^2\theta + (\cos^2\theta) \Delta(T, T^*)] m_{3/2}^2$$  \hspace{1cm} (44)

$$A_0 = -\sqrt{3}[\sin\theta e^{-i\gamma_S} + \cos\theta e^{-i\gamma_T} \omega(T, T^*)] m_{3/2}$$  \hspace{1cm} (45)

In Eqs. (43-45), the angle $\theta$ parameterizes the direction between the Goldstino and the dilaton, $\Delta$ and $\omega$ include the $\sigma$-model contribution and instanton correction to the Kahler potential, and $\gamma_S, \gamma_T$ are possible CP violating phases. In the following we will for simplicity set, $\gamma_{S,T}$ to zero. The quantities $\theta, \Delta$ and $\omega$ are model dependent, and we will leave them arbitrary for the moment.

The models considered here are examples of those of Sec. 3 with specific string theory constraints. As discussed in Sec. 2, $m_{1/2}$ can be determined at
a LC with error of about 5%, and from Eq. (26), \( m_0 \) can be determined with error of about 2%. Eqs. (43,44) imply

\[
\frac{m_0^2}{m_{1/2}^2} = \frac{1}{3} \left[ 1 + \Delta \cot^2 \theta \right]
\]  

(46)

and using the parameters of Sec. 2(ii) \( (\bar{m}_2 = 120 \text{ GeV}, m_0 = 187 \text{ GeV}) \) one finds

\[
\Delta \cot^2 \theta = 3.73 \pm 0.25
\]  

(47)

Eqs. (43,45) give

\[
\frac{A_0}{m_{1/2}} = -1 - \omega \cot \theta
\]  

(48)

One may relate \( A_0 \) to \( A_t \) by the RGE:

\[
A_0 = \frac{A_R}{D_0} - 2.20 m_{1/2}
\]  

(49)

where \( A_R = -A_t - 1.74 m_{1/2} \) is the residue at the Landau pole, and \( D_0 = 1 - m_t^2/m_f^2 \) where \( m_f \approx 200 \sin \beta \text{ GeV} \) [30]. For the parameter choice \( A_t = -285 \text{ GeV} \) and \( \tan \beta = 5 \) (with errors of 5% for \( A_R \) and 10% for \( \tan \beta \) as in Sec. 3) one finds

\[
\frac{A_0}{m_{1/2}} = -1.539 \pm 0.047
\]  

(50)

and hence from (48) one has

\[
\omega \cot \theta = 0.539 \pm 0.047
\]  

(51)

Specific Calabi-Yau compactifications determine the values of \( \Delta \) and \( \omega \). Eqs. (47) and (51) then allow for two independent experimental determinations of \( \cot \theta \) to check the validity of a given model. We consider two examples of models discussed in [18].

(i) One-modulus Models

The values of \( \Delta \) and \( \omega \) for the four one-modulus models of [29] can be calculated in the large Calabi-Yau radius limit [31]. In this limit, the instanton contributions are negligible, and for \( \text{Re}T = 5 \), \( \Delta \) and \( \omega \) have average values of \( [18] \Delta \approx 0.40, \omega \approx 0.17 \). Eqs. (47) and (51) then give respectively

\[
| \cot \theta | = 3.05 \pm 0.14
\]  

(52)
\[ ctn\theta = 3.17 \pm 0.28 \] (53)

We see that for these parameters, the value of \( ctn\theta \) is well determined at a LC, and the two values are consistent with each other (with the choice \( ctn\theta > 0 \)). One may now return to (43) and evaluate \( m_{3/2} \). Thus Eq. (52) yields

\[ sin\theta = 0.311 \pm 0.013 \] (54)

and hence

\[ m_{3/2} = (276 \pm 18) GeV \] (55)

Of course, other tests of the validity of these models can be made, such as those discussed in Sec. 3 (where now the gauge group is \( E_6 \)) and elsewhere [28]. Both \( \sin\theta \) and \( m_{3/2} \) are aspects of supersymmetry breaking. When a string understanding this phenomena becomes known, these quantities would presumably be predicted by the model. Thus Eqs. (54) and (55) would then represent precision tests of the string picture of SUSY breaking.

(ii) Maximum \( \Delta \) Model

A model which maximizes the value of \( \Delta \) occurs when \( \text{Im}T = 1/4 \). Then for \( \text{Re}T = 5 \), Ref. [18] finds \( \Delta = 1.62 \) and \( |\omega| = 0.64 \). Eqs. (47) and (51) would now yield respectively

\[ |ctn\theta| = 1.516 \pm 0.071; \quad |ctn\theta| = 0.842 \pm 0.073 \] (56)

In this case the two determinations of \( ctn\theta \) are inconsistent, which would imply that this model is experimentally ruled out for the given choice of low energy parameters.

We see from the above discussion that a LC is capable of testing the validity of different string compactifications as well as being able to distinguish among different compactifications. Further, these determinations can be made with very good accuracy. In particular, one can test those assumptions that are specifically string related.

6. Conclusions

It is generally expected that the LHC and NLC will be able to unravel the physics that lies above the Standard Model. Thus if supersymmetry is correct, these machines should be able to observe much of the SUSY mass spectrum, as well as test GUT scale assumptions. However, supergravity grand unification is an incomplete theory, and many of the hypotheses used there presumably
reside in a more fundamental theory that exists above $M_G$. A remarkable feature of linear colliders is that they will be able to test theoretical assumptions in this post-GUT domain.

In this paper, we have examined three classes of such post-GUT models: supergravity models with universal soft breaking at the string scale $M_{str}$, models with $SU(2)_H$ horizontal symmetry, and Calabi-Yau string models. In each of these it was seen that theoretical assumptions made at post-GUT scales could be checked with generally very good accuracy by a LC. Thus in the first class of models the predicted loss of universality at $M_G$ could be well measured. Different gauge groups, e.g. $SU(5)$, $SO(10)$ could be distinguished, and the value of $M_{str}$ could be determined. For the $SU(2)_H$ model, the very small splittings resulting from the breaking of $SU(2)_H$ will probably require a reduction by a factor of 5-10 in currently expected errors to be observed at the LC. However, the general $SU(2)_H$ symmetry should be easily observable. Finally, different Calabi-Yau compactifications would be distinguishable, and in the cases considered, accurate direct measurements of such explicitly string quantities as the partition of the goldstino between the dilaton and the moduli, and the value of the gravitino mass are obtainable.

However, the reach of the NLC with $\sqrt{s} = 500$ GeV, while good for studying light neutralinos, charginos and perhaps sleptons is likely to be insufficient for the heavier SUSY particles needed to give a full knowledge of what is happening at energies $\sim M_G$. For example, if LEP 1.9 does not discover the lightest chargino $\tilde{\chi}_1^\pm$, then $m_{\tilde{\chi}_1^\pm} \gtrsim 90$ GeV. In the scaling domain, this would imply $m_{\tilde{g}} \gtrsim 270$ GeV. The $\tilde{u}_L$ mass is given by $m_{\tilde{u}_L}^2 \approx \tilde{m}_0^2 + 0.893 m_{\tilde{g}}^2 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta$ which implies $m_{\tilde{u}_L} > 250$ GeV, with similar results for other squarks. Thus in order to sample the full SUSY spectrum, one needs colliders with $\sqrt{s} > 1$ TeV (preferably up to $\sqrt{s} = 2$ TeV).

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30. In Eq. (49) we have neglected the contribution in running the RGE from $M_G$ to $M_{str}$. This contribution depends upon the particle content above $M_G$ and on the Yukawa couplings, and hence requires fixing the Calabi-Yau compactification to calculate it. Thus our analysis here is meant to illustrate what a LC could determine rather than being a detailed calculation for a given model. (We also note that the choice of large $\text{Re} T$ made below moves $M_{str}$ closer to $M_G$ reducing this contribution).

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