Diffusion Processes in Turbulent Magnetic Fields

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Abstract. We study of the effect of turbulence on diffusion processes within magnetized medium. While we exemplify our treatment with heat transfer processes, our results are quite general and are applicable to different processes, e.g. diffusion of heavy elements. Our treatment is also applicable to describing the diffusion of cosmic rays arising from magnetic field wandering. In particular, we find that when the energy injection velocity is smaller than the Alfven speed the heat transfer is partially suppressed, while in the opposite regime the effects of turbulence depend on the intensity of driving. In fact, the scale \( l_A \) at which the turbulent velocity is equal the Alfven velocity is a new important parameter. When the electron mean free path \( \lambda \) is larger than \( l_A \), the stronger the the turbulence, the lower thermal conductivity by electrons is. The turbulent motions, however, induces their own advective transport, that can provide effective diffusivity. For clusters of galaxies, we find that the turbulence is the most important agent for heat transfer. We also show that the domain of applicability of the subdiffusion concept is rather limited.

Keywords: Turbulence, MHD, Interstellar Medium, Plasma

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ASTROPHYSICAL TURBULENCE

The magnetohydrodynamic (MHD) approximation is widely used to describe the actual magnetized plasma turbulence over scales that are much larger than both the mean free path of the particles and their Larmor radius (see Kulsrud 2004). The theory of MHD turbulence has become testable recently due to numerical simulations (see Biskamp 2003) which confirm (see Cho & Lazarian 2005 and ref. therein) the prediction of magnetized Alfvenic eddies being elongated along magnetic field (see Shebalin, Matthaeus & Montgomery 1983, Higdon 1984) and provided results consistent with the quantitative relations for the degree of eddy elongation obtained in Goldreich & Sridhar (1995, henceforth GS95).

GS95 model assumes the isotropic injection of energy at scale \( L \) and the injection velocity equal to the Alfven velocity in the fluid \( V_A \), i.e. the Alfven Mach number \( M_A \equiv (\delta V/V_A) = 1 \). This model can be easily generalized for both \( M_A > 1 \) and \( M_A < 1 \) at the injection. Indeed, if \( M_A > 1 \), instead of the driving scale \( L \) for one can use another scale, namely \( l_A \), which is the scale at which the turbulent velocity gets equal to \( V_A \). For \( M_A \gg 1 \) magnetic fields are not dynamically important at large scales and the turbulence follows the incompressible Kolmogorov cascade \( V_l \sim l^{1/3} \) over the range of scales \([L, l_A] \). This provides \( l_A \sim L D M_A^{-3} \). If \( M_A < 1 \), the turbulence obeys GS95 scaling (also called “strong” MHD turbulence) not from the scale \( L \), but from a smaller scale \( l_{trans} \sim L M_A^2 \) (Lazarian & Vishniac 1999), while in the range \([L, l_{trans}] \) the turbulence is “weak”.
BASICS OF DIFFUSION IN MAGNETIZED PLASMAS

The issue of diffusion in magnetized plasma has been mostly dealt with in the context of heat transfer. Heat transfer in turbulent magnetized plasma is an important astrophysical problem which is relevant to the wide variety of circumstances from mixing layers in the Local Bubble (see Smith & Cox 2001) and Milky way (Begelman & Fabian 1990) to cooling flows in intracluster medium (ICM) (Fabian 1994). The latter problem has been subjected to particular scrutiny as observations do not support the evidence for the cool gas (see Fabian et al. 2001). This is suggestive of the existence of heating that replenishes the energy lost via X-ray emission. Heat transfer from hot outer regions is an important process to consider in this context.

It is well known that magnetic fields can suppress thermal conduction perpendicular to their direction. The issue of heat transfer in realistic turbulent magnetic fields has been long debated (see Bakunin 2005 and references therein). An influential paper by Narayan & Medvedev (2001, henceforth NM01) obtained estimates of thermal conductivity by electrons using the GS95 model of MHD turbulence with the velocity \( V_L \) at the energy injection scale \( L \) that is equal to the Alfven velocity \( V_A \), i.e. the turbulence with the Alfven Mach number \( M_A \equiv (V_L/V_A) = 1 \). This is rather restrictive, as in the ICM \( M_A > 1 \), while in other astrophysical situations \( M_A < 1 \). Below we discuss turbulence for both \( M_A > 1 \) and \( M_A < 1 \) and compare the particle diffusion to that by turbulent fluid motions. For heat transfer, we refer to our results in Lazarian (2006, 2007).

Let us initially disregard the dynamics of fluid motions on diffusion, i.e. consider diffusion induced by particles moving along static magnetic fields. Magnetized turbulence in the GS95 model is anisotropic with eddies elongated along (henceforth denoted by \( || \) ) the direction of local magnetic field. Consider isotropic injection of energy at the outer scale \( L \) and dissipation at the scale \( l_{\perp,\min} \), where \( \perp \) denotes the direction of perpendicular to the local magnetic field. NM01 observed that the separations of magnetic field lines for \( r_0 < l_{\perp,\min} \) are mostly influenced by the motions at the scale \( l_{\perp,\min} \), which results in Lyapunov-type growth: \( \sim r_0 \exp(l/l_{\perp,\min}) \). This growth is similar to that obtained in earlier models with a single scale of turbulent motions (Rechester & Rosenbluth 1978, Chandran & Cowley 1998). This is not surprising as the largest shear that causes field line divergence is provided by the marginally damped motions at the scale around \( l_{\perp,\min} \).

In NM01 \( r_0 \) is associated with the size of the cloud of electrons of the electron Larmor radius \( r_{\text{Lar,particle}} \). They find that the electrons should travel over the distance

\[
L_{RR} \sim l_{\perp,\min} \ln(l_{\perp,\min}/r_{\text{Lar,particle}}) \tag{1}
\]

to get separated by \( l_{\perp,\min} \).

Within the single-scale model which formally corresponds to \( L = l_{||,\min} = l_{\perp,\min} \) the scale \( L_{RR} \) is called Rechester-Rosenbluth distance. For the ICM parameters the logarithmic factor in Eq. (1) is of the order of 30, and this causes 30 times decrease of thermal conductivity for the single-scale model\(^1\). In the multi-scale models with a limited (e.g. a few decades) inertial range the logarithmic factor stays of the same order.

\(^1\) For the single-scale model \( L_{RR} \sim 30L \) and the diffusion over distance \( \Delta \) takes \( L_{RR}/L \) steps, i.e. \( \Delta^2 \sim L_{RR}L \), which decreases the corresponding diffusion coefficient \( \kappa_{\text{particle, single}} \sim \Delta^2/6t \) by the factor of 30.
but it does not affect the thermal conductivity, provided that $L \gg l_{\parallel,\text{min}}$. Indeed, for the electrons to diffuse isotropically they should spread from $r_{\text{Lar,particle}}$ to $L$. The GS95 model of turbulence operates with field lines that are sufficiently stiff, i.e. the deviation of the field lines from their original direction is of the order unity at scale $L$ and less for smaller scales. Therefore to get separated from the initial distance of $l_{\perp,\text{min}}$ to a distance $L$ (see Eq. (5) with $M_A = 1$), at which the motions get uncorrelated the electron should diffuse the distance slightly larger (as field lines are not straight) than $\sqrt{2}L$, which is much larger than the extra travel distance $\sim 30l_{\parallel,\text{min}}$. Explicit calculations in NM01 support this intuitive picture.

### Diffusion for $M_A > 1$

Turbulence with $M_A > 1$ evolves along hydrodynamic isotropic Kolmogorov cascade, i.e. $V_l \sim V_L(l/L)^{1/3}$ over the range of scales $[L, l_A]$, where

$$l_A \approx L(V_A/V_L)^3 \equiv LM_A^{-3}, \quad (2)$$

is the scale at which the magnetic field gets dynamically important, i.e. $V_l = V_A$. This scale plays the role of the injection scale for the GS95 turbulence, i.e. $V_l \sim V_A(l_{\perp}/l_A)^{1/3}$, with eddies at scales less than $l_A$ getting elongated in the direction of the local magnetic field. The corresponding anisotropy can be characterized by the relation between the semi-major axes of the eddies

$$l_{\parallel} \sim L(l_{\perp}/L)^{2/3}M_A^{-1}, \quad M_A > 1,$$

where $\parallel$ and $\perp$ are related to the direction of the local magnetic field. In other words, for $M_A > 1$, the turbulence is still isotropic at the scales larger to $l_A$, but develops $(l_{\perp}/l_A)^{1/3}$ anisotropy for $l < l_A$.

If particles (e.g. electrons) mean free path $\lambda \gg l_A$, they stream freely over the distance of $l_A$. For particles initially at distance $l_{\perp,\text{min}}$ to get separated by $L$ the required travel is the random walk with the step $l_A$, i.e. the mean-squared displacement of a particle till it enters an independent large-scale eddy $\Delta^2 \sim l_A^2(L/l_A)$, where $L/l_A$ is the number of steps. These steps require time $\delta t \sim (L/l_A)l_A/C_1v_{\text{particle}}$, where $v_{\text{particle}}$ is electron thermal velocity and the coefficient $C_1 = 1/3$ accounts for 1D character of motion along magnetic field lines. Thus the electron diffusion coefficient is

$$\kappa_{\text{particle}} \equiv \Delta^2/\delta t \approx (1/3)l_Av_{\text{particle}}, \quad l_A < \lambda,$$

which for $l_A \ll \lambda$ constitutes a substantial reduction of diffusivity compared to its unmagnetized value $\kappa_{\text{unmagn}} = \lambda v_{\text{particle}}$. We assumed in Eq. (4) that $L \gg 30l_{\parallel,\text{min}}$ (see §2.1).

For $\lambda \ll l_A \ll L$, $\kappa_{\text{particle}} \approx 1/3\kappa_{\text{unmagn}}$ as both the $L_{RR}$ and the additional distance for electron to diffuse because of magnetic field being stiff at scales less than $l_A$ are negligible compared to $L$. For $l_A \to L$, when magnetic field has rigidity up to the scale $L$, it gets around $1/5$ of the value in unmagnetized medium, according to NM01.
Note, that even dynamically unimportant magnetic fields do influence heat conductivity over short time intervals. For instance, over time interval less than $t_\parallel^2/C_1\kappa_{\text{unmagn}}$ the diffusion happens along stiff magnetic field lines and the difference between parallel and perpendicular diffusivities is large. This allows the transient existence of sharp small-scale temperature gradients.

**DIFFUSION FOR $M_A < 1$**

It is intuitively clear that for $M_A < 1$ turbulence should be anisotropic from the injection scale $L$. In fact, at large scales the turbulence is expected to be weak (see Lazarian & Vishniac 1999, henceforth LV99). Weak turbulence is characterized by wavepackets that do not change their $l_\parallel$, but develop structures perpendicular to magnetic field, i.e. decrease $l_\perp$. This cannot proceed indefinitely, however. At some small scale the GS95 condition of critical balance, i.e. $l_\parallel/V_A \approx l_\perp/V_l$, becomes satisfied. This perpendicular scale $l_{\text{trans}}$ can be obtained substituting the scaling of weak turbulence (see LV99) $V_l \sim V_L(l_\perp/L)^{1/2}$ into the critical balance condition. This provides $l_{\text{trans}} \sim L M_A^2$ and the corresponding velocity $V_{\text{trans}} \sim V_L M_A$. For scales less than $l_{\text{trans}}$ the turbulence is strong and it follows the scalings of the GS95-type, i.e. $V_l \sim V_L(L/l_\perp)^{-1/3}M_A^{1/3}$ and

$$l_\parallel \sim L(l_\perp/L)^{2/3}M_A^{-4/3}, \quad M_A < 1.$$  

For $M_A < 1$, magnetic field wandering in the direction perpendicular to the mean magnetic field (along y-axis) can be described by $d\langle y^2 \rangle/dx \sim \langle y^2 \rangle/l_\parallel$ (LV99), where $l_\parallel$ is expressed by Eq. (5) and one can associate $l_\perp$ with $2\langle y^2 \rangle$.

$$\langle y^2 \rangle^{1/2} \sim \frac{x^{3/2}}{3^{3/2}L^{1/2}}M_A^2, \quad l_\perp < l_{\text{trans}}$$  

For weak turbulence $d\langle y^2 \rangle/dx \sim L M_A^4$ (LV99) and thus

$$\langle y^2 \rangle^{1/2} \sim L^{1/2}x^{1/2}M_A^2, \quad l_\perp > l_{\text{trans}}.$$  

Fig. 1 confirms the correctness of the above scaling numerically.

Eq. (6) differs by the factor $M_A^3$ from that in NM01, which reflects the gradual suppression of thermal conductivity perpendicular to the mean magnetic field as the magnetic field gets stronger. Physically this means that for $M_A < 1$ the magnetic field fluctuates around the well-defined mean direction. Therefore the diffusivity gets anisotropic with

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2 The relation between the mean squared displacements perpendicular to magnetic field $\langle y^2 \rangle$ and the displacements $x$ along magnetic field for $x < l_\parallel$ can be obtained through the diffusion equation approach in §2.3 and Eq. (3). This gives $\langle y^2 \rangle^{1/2} \sim \frac{x^{3/2}}{3^{3/2}L^{1/2}}M_A^2$.

3 The terms “weak” and “strong” turbulence are accepted in the literature, but can be confusing. As we discuss later at smaller scales at which the turbulent velocities decrease the turbulence becomes strong. The formal theory of weak turbulence is given in Galtier et al. (2000).

4 The fact that one gets $l_\parallel,\min$ in Eq. (1) is related to the presence of this scale in this diffusion equation.
the diffusion coefficient parallel to the mean field $\kappa_{\parallel,\text{particle}} \approx 1/3 \kappa_{\text{magn}}$ being larger than coefficient for diffusion perpendicular to magnetic field $\kappa_{\perp,\text{particle}}$.

Consider the coefficient $\kappa_{\perp,\text{particle}}$ for $M_A \ll 1$. As NM01 showed, particles become uncorrelated if they are displaced over the distance $L$ in the direction perpendicular to magnetic field. To do this, a particle has first to travel $L_{RR}$ (see Eq. (11)), where Eq. (5) relates $l_{\parallel,\min}$ and $l_{\perp,\min}$. Similar to the case in §2.1, for $L \gg 30l_{\parallel,\min}$, the additional travel arising from the logarithmic factor is negligible compared to the overall diffusion distance $L$. At larger scales electron has to diffuse $\sim L$ in the direction parallel to magnetic field to cover the distance of $LM_A^2$ in the direction perpendicular to magnetic field direction. To diffuse over a distance $R$ with random walk of $LM_A^2$ one requires $R^2/L^2M_A^4$ steps. The time of the individual step is $L^2/\kappa_{\parallel,\text{particle}}$. Therefore the perpendicular diffusion coefficient is

$$\kappa_{\perp,\text{particle}} = R^2/(R^2/[\kappa_{\parallel,\text{particle}}M_A^4]) = \kappa_{\parallel,\text{particle}}M_A^4, \quad M_A < 1,$$

An essential assumption there is that the particles do not trace their way back over the
individual steps along magnetic field lines, i.e. \( L_{RR} \ll L \). Note, that for \( M_A \) of the order of unity this is not accurate and one should account for the actual 3D displacement. This introduces the change by a factor of order unity (see above).

**TURBULENT DIFFUSIVITY**

Turbulent motions themselves can induce advective transport. In Cho et al. (2003) we dealt with the turbulence with \( M_A \sim 1 \) and estimated

\[
\kappa_{\text{dynamic}} \approx C_{\text{dyn}} L V_L, \quad M_A > 1,
\]

where \( C_{\text{dyn}} \sim 0(1) \) is a constant, which for hydro turbulence is around 1/3 (Lesieur 1990). If we deal with heat transport, for fully ionized non-degenerate plasma we assume \( C_{\text{dyn}} \approx 2/3 \) to account for the advective heat transport by both protons and electrons.\(^5\) Thus eq. (9) covers the cases of both \( M_A > 1 \) up to \( M_A \sim 1 \). For \( M_A < 1 \) one can estimate \( \kappa_{\text{dynamic}} \sim d^2 / \omega \), where \( d \) is the random walk of the field line over the wave period \( \sim \omega^{-1} \). As the weak turbulence at scale \( L \) evolves over time \( \tau \sim M_A^{-2} \omega^{-1} \), \( \langle y^2 \rangle \) is the result of the random walk with a step \( d \), i.e. \( \langle y^2 \rangle \sim (\tau \omega) d^2 \). According to eq. (6) and (7), the field line is displaced over time \( \tau \) by \( \langle y^2 \rangle \sim L M_A^3 V_A \tau \). Combining the two one gets \( d^2 \sim L M_A^3 V_L \omega^{-1} \), which provides \( \kappa_{\text{dynamic}} \approx C_{\text{dyn}} L V_L M_A^3 \), which is similar to the diffusivity arising from strong turbulence at scales less than \( l_{\text{trans}} \), i.e. \( \kappa_{\text{dynamic}} \approx C_{\text{dyn}} l_{\text{trans}} V_{\text{trans}} \). The total diffusivity is the sum of the two, i.e. for plasma

\[
\kappa_{\text{dynamic}} \approx (\beta / 3) L V_L M_A^3, \quad M_A < 1,
\]

where \( \beta \approx 4 \).

**EXAMPLE: THERMAL CONDUCTIVITY**

In thermal plasma, electrons are mostly responsible for thermal conductivity. The schematic of the parameter space for \( \kappa_{\text{particle}} < \kappa_{\text{dynamic}} \) is shown in Fig 2, where the Mach number \( M_A \) and the Alfvén Mach number \( M_A \) are the variables. For \( M_A < 1 \), the ratio of diffusivities arising from fluid and particle motions is \( \kappa_{\text{dynamic}} / \kappa_{\text{particle}} \sim \beta \alpha M_s M_A (L / \lambda) \) (see Eqs. (8) and (10)), the square root of the ratio of the electron to proton mass \( \alpha = (m_e / m_p)^{1/2} \), which provides the separation line between the two regions in Fig. 2, \( \beta \alpha M_s \sim (L / L) M_A \). For \( 1 < M_A < (L / L)^{1/3} \) the mean free path is less than \( l_A \) which results in \( \kappa_{\text{particle}} \) being some fraction of \( \kappa_{\text{unmagn}} \), while \( \kappa_{\text{dynamic}} \) is given by Eq. (9). Thus \( \kappa_{\text{dynamic}} / \kappa_{\text{particle}} \sim \beta \alpha M_s (L / \lambda) \), i.e. the ratio does not depend on \( M_A \) (horizontal line in Fig. 2). When \( M_A > (L / L)^{1/3} \) the mean free path of electrons is constrained by \( l_A \). In this case \( \kappa_{\text{dynamic}} / \kappa_{\text{particle}} \sim \beta \alpha M_s M_A^3 \) (see Eqs. (9) and (10)). This results in the separation line \( \beta \alpha M_s \sim M_A^{-3} \) in Fig. 2.

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5 This gets clear if one uses the heat flux equation \( q = -\kappa \nabla T \), where \( \kappa_e = n k_B \kappa_{\text{dynamic/elec}} \), \( n \) is electron number density, and \( k_B \) is the Boltzmann constant, for both electron and advective heat transport.
HEAT TRANSFER IN INTRACLUSTER MEDIUM

It is generally believed that Intracluster Medium (ICM) is turbulent. The considerations below can be used as guidance. In unmagnetized plasma with the ICM temperatures $T \sim 10^8$ K and and density $10^{-3}$ cm$^{-3}$ the kinematic viscosity $\eta_{\text{magn}} \sim v_{\text{ion}} \lambda_{\text{ion}}$, where $v_{\text{ion}}$ and $\lambda_{\text{ion}}$ are the velocity of an ion and its mean free path, respectively, would make the Reynolds number $Re \equiv LV/\eta_{\text{magn}}$ of the order of 30. This is barely enough for the onset of turbulence. For the sake of simplicity we assume that ion mean free path coincides with the proton mean free path and both scale as $\lambda \approx 3T_3^2n_{-3}^{-1}$ kpc, where the temperature $T_3 \equiv kT/3$ keV and $n_{-3} \equiv n/10^{-3}$ cm$^{-3}$. This provides $\lambda$ of the order of 0.8–1 kpc for the ICM (see NM01).

It is accepted, however, that magnetic fields decrease the diffusivity. Somewhat naively assuming the maximal scattering rate of an ion, i.e. scattering every orbit (the so-called Bohm diffusion limit) one gets the viscosity perpendicular to magnetic field $\eta_{\perp} \sim v_{\text{ion}}r_{\text{Larmor}}$, which is much smaller than $\eta_{\text{magn}}$, provided that the ion Larmor radius $r_{\text{Larmor}} \ll \lambda_{\text{ion}}$. For the parameters of the ICM this allows essentially invicid mo-
tions\textsuperscript{6} of magnetic lines parallel to each other, e.g. Alfvén motions.

In spite of the substantial progress in understanding of the ICM (see Enßlin, Vogt & Pfrommer 2005, henceforth EVP05, Enßlin & Vogt 2006, henceforth EV06 and references therein), the basic parameters of ICM turbulence are known within the factor of 3 at best. For instance, the estimates of injection velocity $V_L$ varies in the literature from 300 km/s to $10^3$ km/s, while the injection scale $L$ varies from 20 kpc to 200 kpc, depending whether the injection of energy by galaxy mergers or galaxy wakes is considered. EVP05 considers an illustrative model in which the magnetic field with the 10 $\mu$G fills 10% of the volume, while 90% of the volume is filled with the field of $B \sim 1 \mu$G. Using the latter number and assuming $V_L = 10^3$ km/s, $L = 100$ kpc, and the density of the hot ICM is $10^{-3}$ cm$^{-3}$, one gets $V_A \approx 70$ km/s, i.e. $M_A > 1$. Using the numbers above, one gets $l_A \approx 30$ pc for the 90% of the volume of the hot ICM, which is much less than $\lambda_{\text{ion}}$. The diffusivity of ICM plasma gets $\eta = v_{\text{ion}} l_A$ which for the parameters above provides $Re \sim 2 \times 10^3$, which is enough for driving superAlfvénic turbulence at the outer scale $L$. However, as $l_A$ increases as $\propto B^3$, $Re$ gets around 50 for the field of 4 $\mu$G, which is at the border line of exciting turbulence\textsuperscript{4}. However, the regions with higher magnetic fields (e.g. 10 $\mu$G) can support Alfvénic-type turbulence with the injection scale $l_A$ and the injection velocities resulting from large-scale shear $V_L(l_A/L) \sim V_L M_A^{-3}$.\textsuperscript{7}

For the regions of $B \sim 1 \mu$G the value of $l_A$ is smaller than the mean free path of electrons $\lambda$. According to Eq. (3) the value of $\kappa_{\text{electr}}$ is 100 times smaller than $\kappa_{\text{Spitzer}}$. On the contrary, $\kappa_{\text{dynamic}}$ for the ICM parameters adopted will be $\sim 30 \kappa_{\text{Spitzer}}$, which makes the dynamic diffusivity the dominant process. This agrees well with the observations in Voigt & Fabian (2004). Fig. 2 shows the dominance of advective heat transfer for the parameters of the cool core of Hydra A ($B = 6$ $\mu$G, $n = 0.056$ cm$^{-3}$, $L = 40$ kpc, $T = 2.7$ keV according to EV06), point “F”, and for the illustrative model in EVP05, point “V”, for which $B = 1$ $\mu$G.

Note that our stationary model of MHD turbulence is not directly applicable to transient wakes behind galaxies. The ratio of the damping times of the hydro turbulence and the time of straightening of the magnetic field lines is $\sim M_A^{-1}$. Thus, for $M_A > 1$, the magnetic field at scales larger than $l_A$ will be straightening gradually after the hydro turbulence has faded away over time $L/V_L$. The process can be characterized as injection of turbulence at velocity $V_A$ but at scales that increase linearly with time, i.e. as $l_A + V_A t$. The study of heat transfer in transient turbulence and magnetic field “regularly” stretched by passing galaxies will be provided elsewhere.

\begin{itemize}
\item \textsuperscript{6} A regular magnetic field $B_A \approx (2mkT)^{1/2}c/(e\lambda)$ that makes $r_{\text{Lar,ion}}$ less than $\lambda$ and therefore $\eta_{\perp} < \nu_{\text{vagn}}$ is just $10^{-20}$ G. Turbulent magnetic field with many reversals over $r_{\text{Lar,ion}}$ does not interact efficiently with a proton, however. As the result, the protons are not constrained until $l_A$ gets of the order of $r_{\text{Lar,ion}}$. This happens when the turbulent magnetic field is of the order of $2 \times 10^{-3}(V_L/10^3\text{km/s})$ G. At this point, the step for the random walk is $\sim 2 \times 10^{-8}$ pc and the Reynolds number is $5 \times 10^{10}$.
\item \textsuperscript{7} One can imagine dynamo action in which superAlfvénic turbulence generates magnetic field till $l_A$ gets large enough to shut down the turbulence.
\end{itemize}
FIGURE 3. Energy density of compressive modes and Alfvénic slab-type waves, induced by CRs. The energy is transferred from the mean free path scale to the CR Larmor radius scale. If the mean free path falls below compressive motions cutoff or feedback suppression scale, the spectrum of slab waves becomes steeper. From Lazarian & Beresnyak (2006).

ASPECTS OF COSMIC RAY DIFFUSION

Diffusion of cosmic rays (CR) includes diffusion arising from CR scattering and that of wandering magnetic field lines. For our purposes, we disregard the diffusion perpendicular to magnetic field lines that arises from gyroresonance interactions.

Naturally, the diffusion of magnetic field lines depends on the adopted model of turbulence. For instance, Lazarian & Beresnyak (2006) discuss a mechanism by which slab-type perturbations can be generated on the scale of CR gyroradius (see Fig. 3). These slab-type perturbations are likely to contribute subdominantly to field line wandering. Therefore below we concentrate, as in the rest of the paper, on the field wandering induced by Alfvénic modes of GS95 turbulence.

Generalizing our arguments above in terms of the diffusion parallel and perpendicular to mean magnetic field, one can write that $\kappa_{\perp,cr} = \kappa_{\|,cr}(\delta R)^2/(\delta z)^2$, where $\delta R$ is a random walk step in the direction perpendicular to magnetic field. If the corresponding scale $\delta z$ is less than $L_{RR}$, the motion along the field line is one dimensional diffusion that retraces its steps. In this case, $\delta z = (\kappa_{\|,cr}\delta t)^{1/2}$. If, following earlier authors (see Kota & Jokipii 2000, Web et al. 2006) we introduce a spatial field diffusion coefficient $\kappa_{spat} = (\delta R)^2/(\delta z)$, we easily get $\kappa_{\perp,cr} \approx \kappa_{spat} \kappa_{\|,cr}(\delta t)^{1/2}$, which results in the subdiffusion in
perpendicular direction, i.e. in the distance perpendicular to magnetic field growing as \( \kappa_{\perp,cr} \delta t = \kappa_{\text{spat}} k_{||,cr}^{1/2} (\delta t)^{1/2} \). However, as soon as the distance to diffuse is much larger than \( L_{RR} \) (see Eq. (1)), the subdiffusive effects are negligible.

Although CR velocities are of the order of light speed, for sufficiently small mean free paths the advection of CR by turbulent motions may become important.

**CONCLUDING REMARKS**

In the paper above we attempted to describe the diffusion by particle and turbulent motions for \( M_A < 1 \) and \( M_A > 1 \). Unlike earlier papers, we find that turbulence may both enhance diffusion and suppress it. For instance, when \( \lambda \) gets larger than \( l_A \) the conductivity of the medium \( \sim M_A^{-3} \) and therefore the turbulence inhibits heat transfer, provided that \( \kappa_{\text{particle}} > \kappa_{\text{dynamic}} \). Along with the plasma effects that we mention below, this effect can, indeed, support sharp temperature gradients in hot plasmas with weak magnetic field.

As discussed above, rarefied plasma, e.g. ICM plasma, has large viscosity for motions parallel to magnetic field and marginal viscosity for motions that induce perpendicular mixing. Thus fast dissipation of sound waves in the ICM does not contradict the medium being turbulent. The later may be important for the heating of central regions of clusters caused by the AGN feedback (see Churasov et al. 2001, Nusser, Silk & Babul 2006 and more references in EV06). Note, that models that include both heat transfer from the outer hot regions and an additional heating from the AGN feedback look rather promising (see Ruszkowkski & Begelman 2002, Piffaretti & Kaastra 2006). We predict that the viscosity for 1 \( \mu G \) regions is less than for 10 \( \mu G \) regions and therefore heating by sound waves (see Fabian et al. 2005) could be more efficient for the latter. Note, that the plasma instabilities in collisionless magnetized ICM arising from compressive motions (see Schekochihin & Cowley 2006, Lazarian & Beresnyak 2006) can resonantly scatter particles and decrease \( \lambda \). This decreases further \( \kappa_{\text{particle}} \) compared to \( \kappa_{\text{unmagn}} \) but increases \( Re \). In addition, we disregarded mirror effects that can reflect electrons back (see Malyshev & Kulsrud 2001 and references therein), which can further decrease \( \kappa_{\text{particle}} \).

All in all, we have shown that it is impossible to characterize the diffusion in magnetized plasma by a single fraction of the diffusion coefficient of unmagnetized value. The diffusion depends on sonic and Alfven Mach numbers of turbulence and the corresponding diffusion coefficient may be much higher and much lower than the unmagnetized one. As the result, turbulence can inhibit or enhance diffusivity depending on the plasma magnetization and turbulence driving.

Our study indicates that in many cases related to ICM the advective heat transport by dynamic turbulent eddies dominates thermal conductivity. In addition, “subdiffusivity” is probably subdominant for many astrophysical problems.

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