Neutrinos in the simplest little Higgs scenario and TeV leptogenesis

Asmaa Abada\textsuperscript{1}, Gautam Bhattacharyya\textsuperscript{2}, Marta Losada\textsuperscript{3}

\textsuperscript{1)LPT, Université de Paris-Sud XI, Bâtiment 210, 91405 Orsay Cedex, France}
\textsuperscript{2)Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India}
\textsuperscript{3)Centro de Investigaciones, Universidad Antonio Nariño, Cll. 58A No. 37-94, Santa Fe de Bogotá, Colombia}

Abstract

The little Higgs scenario may provide an interesting framework to accommodate TeV scale leptogenesis because a TeV Majorana mass of the right-handed neutrino that we employ for the latter may find a natural place near the ultraviolet cutoff of the former. In this work we study how a light neutrino spectrum, generated radiatively, and TeV scale leptogenesis can be embedded in the simplest little Higgs framework. Alternatively, we highlight how the neutrino Yukawa textures of the latter are constrained.

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I Introduction

The Standard model (SM) with right-handed (RH) neutrinos provides an elegant mechanism for thermal leptogenesis. These RH neutrinos may also be instrumental in generating masses and mixings for the light neutrinos through the see-saw mechanism. There are two intertwined requirements: first, reproduce the spectrum for light neutrinos in the observed range, and second, generate enough CP asymmetry through the out-of-equilibrium decay of heavy RH neutrinos \((\mathcal{M}_R)\). This asymmetry can be transmitted to the baryonic sector through sphaleron induced processes to explain the baryon asymmetry of the Universe. To achieve these two requirements, the RH neutrinos should have Majorana masses and they should couple to the left-handed (LH) lepton doublets and the SM Higgs via complex Yukawa couplings. For natural choices of such couplings, in theories with a large cut-off scale, the RH Majorana masses turn out to be quite close to the GUT scale \((\sim 10^{16} \text{ GeV})\). An alternative and attractive mechanism would be to consider RH neutrinos at the TeV scale \((5-10)\). This scale is accessible in ongoing and near future colliders. Moreover, interesting new physics (like supersymmetry, extra dimensions, etc) could be revealed around that scale. It is now known that with three RH neutrinos it is difficult to achieve TeV scale leptogenesis and reproduce at the same time the small LH neutrino masses \([10]\), unless one considers quasi-degenerate Majorana neutrinos where the exact
degeneracy is lifted by e.g. renormalisation group effects [5] or small fine-tuning [6]. In the present analysis, we first introduce a novel way of generating a lepton asymmetry with 3 RH neutrinos at the TeV scale and secondly, we adapt the scenario proposed in [10], which is a simple extension of the Fukugita-Yanagida model [1] by introducing a fourth generation in addition to the existing three of the SM plus a RH neutrino for each of the four families.

Little Higgs models, in which the SM Higgs doublet is conceived as a pseudo-Goldstone boson of a larger symmetry group, may provide an interesting framework to accommodate TeV scale leptogenesis because the UV cutoff of such models is also around a few TeV. In this paper we study how a light neutrino spectrum and TeV scale leptogenesis with RH neutrinos can be embedded in the simplest little Higgs framework [11, 12].

II Neutrinos in the simplest little Higgs scenario

In the simplest little Higgs scenario the SM gauge group is enlarged to $SU(3)_W \times U(1)_X$ which entails the fermion doublets to be extended to triplets. The left-handed $SU(3)$ lepton triplet is expressed as

$$\psi_{iL} = (\nu_i, \ell_i, N_i)^T_L,$$

where $i$ corresponds to a generation index. The minimal choice for the RH $SU(3)$ singlet components are:

$$n_{iR} \text{ and } \ell_{iR}.$$ (2)

Two $SU(3)$ scalar triplets $\Phi_1$ and $\Phi_2$ are employed for the spontaneous breaking of the $SU(3)_W$ gauge symmetry to $SU(2)_W$ with the vacuum expectation values (vevs) $f_{1,2}$ around the TeV scale. Now, $\Phi_{1,2}$ can be expressed as

$$\Phi_{1,2} = \exp\left(\pm i\frac{\Theta}{f_{1,2}}\right) \begin{pmatrix} 0 \\ 0 \\ f_{1,2} + \rho_{1,2} \end{pmatrix},$$ (3)

where $\rho_{1,2}$ are radial excitations on which we comment later. The phase $\Theta$ is given by (keeping only the SM Higgs field components)

$$\Theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix},$$ (4)

which contains the SM Higgs doublet $h = (h^+ h^0)^T/\sqrt{2}$. Given the aim of our work, we take $f_1 = f_2 = f$ for simplicity. The little Higgs framework requires $f \sim 4\pi v$, where $v$ is the vev of the SM Higgs doublet.

We arrange that only $\Phi_1$ couples to the lepton triplet through the Yukawa interaction (similar to the approximation used in [13])

$$-\mathcal{L}_Y = n^c_{iR} \lambda_{ij} \psi_{jL} \tilde{\Phi}_1^+ + \text{h.c.},$$ (5)
where $\tilde{\Phi}_1$ is obtained by replacing the Higgs doublet by $\tilde{h} \equiv i\tau_2 h^*$ in Eq. (4), and $(i, j)$ run over families. Since this Lagrangian involves only one scalar triplet, the global axial SU(3) remains unbroken, and hence there is no contribution to the Higgs mass divergence from this Yukawa sector. Expanding the above Lagrangian in terms of the SU(2) doublets $L_i = (\nu_i, \ell_i)^T$ and the SU(2) singlets $N_i$, up to the second order in the field $h$, yields (after a slight redefinition, $\psi_L^T \equiv (-iL, N)$)

$$-L_Y = -\bar{n}_iR\lambda_{ij}L_jLh^* + f \left(1 - \frac{h^2}{2f^2}\right)\bar{n}_iR\lambda_{ij}N_{jL} + \text{h.c.}. \quad (6)$$

### III Neutrino mass matrix and eigenvalues

Taking into account the neutrino fields of the model we build the mass matrix. There are two LH states $\nu_L, N_L$ and one RH state $\nu_R$ for each generation. We assume that the gauge singlet field $n_R(\equiv n_L^c)$ has a Majorana mass $M$ around the TeV scale. The mass matrix (tree level) in the basis $(\nu_L, N_L, \nu_L^c)$ turns out to be

$$M = \begin{pmatrix}
0 & 0 & m_D \\
0 & 0 & M_D \\
m_D^T & M_D^T & M
\end{pmatrix}, \quad (7)$$

where, $m_D = -\lambda v$ and $M_D = \lambda f \left(1 - v^2/2f^2\right)$. Strictly speaking, each entry in the mass matrix should be interpreted as a matrix over the generation indices. But for the ease of illustration we concentrate on a single family. The above mass matrix yields two massive and one massless eigenstates with eigenvalues $\sim M, M_D^2/M, 0$. The eigenstate which is dominantly the SM neutrino ($\nu' = \nu + (v/f)N$) is massless at this stage. But one-loop radiative corrections, obtained by integrating out the RH singlet neutrino field, generates a dimension-5 effective operator (valid up to the scale $M$)

$$L_5 \sim \frac{\lambda^2}{16\pi^2} \frac{(\Phi_2^\dagger \psi_L)(\Phi_1^\dagger \psi_L)}{M}, \quad (8)$$

which is realised through Yukawa interaction in conjunction with the scalar quartic interaction $|\Phi_1^\dagger \Phi_2|^2$ (the latter interaction is generated at one-loop). It is straightforward to see by expanding the triplet fields in the above operator in terms of the fields under SU(2) representation that the zeros of the first two by two block of the mass matrix are now lifted. The modified mass matrix now reads ($c \equiv 1/16\pi^2$; the loop factor containing the Higgs quartic coupling $\lambda_h$ is order one and we do not display it for simplicity)

$$M = \begin{pmatrix}
c \cdot m_D^2/M & -c \cdot M_D M_D/M & m_D \\
-c \cdot m_D M_D/M & c \cdot M_D^2/M & M_D \\
m_D^T & M_D^T & M
\end{pmatrix}. \quad (9)$$

The mass matrix has the following eigenvalues:

$$M_1 \simeq cm_D^2/2M; \quad M_2 \simeq M_D^2/M; \quad M_3 \simeq M. \quad (10)$$

It is not difficult to see why the modified eigenvalues are as in Eq. (10). Radiative corrections, which come with the factor ‘c’ in the first two by two block of Eq. (9), are small enough to appreciably alter

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the previously obtained nonzero eigenvalues $M$ and $M_2^2/M$ obtained from diagonalisation of Eq. (7). Clearly, the nonzero determinant of the matrix (9) immediately points to the smallest eigenvalue $M_1$ in Eq. (10). It is straightforward to see that $M_1$ corresponds to the light SM neutrino mass, $M_2$ is the Majorana mass of the state which is dominantly $N_L$, while the RH singlet weighs around $M_3 \sim \text{TeV}$. Note that even though $N_L$ is an SU(2) singlet, it acquires a Majorana see-saw type mass. This happens because $N_L$ is a component of the SU(3) triplet which experiences the interaction in Eq. (8). It is to be noted that the light active neutrino masses have been generated through radiative corrections and not by the conventional see-saw.

Now we see how light active neutrino data constrain the different parameters. We take $M \in [0.5 - 1] \text{ TeV}$. This range is motivated by the requirements of TeV scale leptogenesis, as we shall see later. A light neutrino eigenvalue $\sim \sqrt{\Delta m_{ats}^2} \simeq 0.05$ eV implies the Yukawa coupling $\lambda \sim 10^{-5}$. The Majorana (Dirac) mass of the SU(2) singlet field turns out to be in the keV (MeV) scale. At this point, it is worth comparing our analysis with that of [13]. The aim of [13] was to generate light neutrino masses with unsuppressed Yukawa couplings, which requires the RH Majorana masses in the keV range. On the contrary, we assume that lepton number is violated at the TeV scale. As a result, our Yukawa couplings for the light neutrinos are suppressed. As we shall see later, we can achieve a successful leptogenesis through the decay of RH TeV scale Majorana neutrinos.

IV TeV scale leptogenesis

Since the intrinsic scale of the little Higgs scenario is around a TeV, one may ask whether all the requirements for a successful TeV scale leptogenesis can be accommodated within such a framework. First, we recall that there are two approaches in the literature to realise TeV scale leptogenesis: (a) consider a quasi-degenerate spectrum of heavy RH neutrinos and enhance CP asymmetry through resonant effects [13]; (b) extend the phase space parameters, either (i) by admitting, for example, extra couplings that allow three body decays of the RH neutrinos leading to an enhancement of CP asymmetry [15], or, (ii) by extending the particle content. As regards the latter possibility, one may adopt among others either of two approaches: (1) consider a supersymmetric framework [16], (2) minimally extend the SM by having a fourth chiral generation and add a heavy RH neutrino for each of the four generations, assuming that the lepton asymmetry is due to the decay of the lightest RH neutrino ($n_{R1}$) in the TeV scale [10].

There are two key points: (a) the CP asymmetry

$$\epsilon_1 = \frac{1}{8\pi [\lambda \lambda^\dagger]_{11}} \sum_{j \neq 1} \text{Im}[\lambda \lambda^\dagger]_{1j} f(M_{n_{Rj}}^2/M_{n_{R1}}^2),$$

where $f$ is the loop factor [2], given by

$$f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \frac{1 + x}{x} + \frac{1}{1 - x} \right],$$

has to be magnified by the presence of a large Yukawa coupling, and (b) the condition of out-of-equilibrium decay of $n_{R1}$ has to be ensured, i.e.,

$$\Gamma_{n_{R1}} = \frac{(\lambda \lambda^\dagger)_{11} M_{n_{R1}}^2}{8\pi} < H(T = M_{n_{R1}}),$$

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where $H$ is the Hubble expansion rate at $T = M_{n_{R_1}}$. These conditions restrict the size of the Yukawa couplings. In the following, we discuss two possible scenarios for TeV scale leptogenesis in the context of the simplest little Higgs model.

IV.1 Scenario I: A 3-generation case

First we consider the case with only 3 generations of fermions. We emphasize that this is a novel approach to realize the marriage between TeV scale leptogenesis and little Higgs. We take the first two generation of RH neutrino masses $M_{n_{R_1}} < M_{n_{R_2}}$ of the order of a TeV, thus the associated light LH neutrino masses arise through Eq. (9). The corresponding Yukawa couplings are therefore suppressed $\sim 10^{-5}$. The third generation of RH neutrino in contrast has a much smaller Majorana mass on the order of $M_{n_{R_3}} \sim \text{keV}$, and the associated light LH active neutrino acquires mass via the mechanism of Ref. [13]. This mechanism relies on the same operator of Eq. (8), but with lepton number violating mass $M \sim \text{keV}$ and importantly with an unsuppressed Yukawa coupling $\lambda_{33} \sim 1$. We note here that $\lambda_{31}$ and $\lambda_{32}$ can also be order one, or a little smaller, e.g. order 0.1 to ensure the validity of the mechanism of [13].

We consider the decay of the (next to lightest) RH neutrino ($n_{R_1}$) which weighs around a TeV. All Yukawa couplings entering the decay rate, i.e. $\lambda_{1i}$, for all $i = 1, 2, 3$, have to be order $10^{-7}$ or smaller to ensure an out-of-equilibrium decay, see Eq. (13). Thus the constraints on $\lambda_{1i}$ are about two orders of magnitude stronger than those obtained from light active neutrino masses. Thus the Yukawa matrix (rows corresponding to the three RH neutrinos, columns to the active LH neutrinos) looks like

$$\lambda = \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \beta & \beta & \beta \\ \alpha & \alpha & \mathcal{O}(1) \end{pmatrix},$$

where $\epsilon \sim 10^{-7}$, $\beta \sim 10^{-5}$, and $\mathcal{O}(0.1) < \alpha < \mathcal{O}(1)$. It is understood that all entries are subject to order one uncertainties. Clearly, all entries of the $(3 \times 3)$ light active neutrino masses, proportional to $\lambda^T \lambda$ assuming the RH mass matrix to be diagonal, receive contributions from both the mechanisms cited above.

Now we consider the CP asymmetry from the decay of $n_{R_1}$ into any leptonic flavour, though the most dominant contribution would come from the $\tau$ direction. Note that although we gain from a large Yukawa coupling $\lambda_{33} \sim 1$, we pay the price of a very small $M_{n_{R_3}} \sim \text{keV}$ in the loop. An order-of-magnitude estimate is

$$\epsilon_1 \simeq \frac{1}{6\pi} |\lambda_{33}|^2 \left( \frac{M_{n_{R_3}}}{M_{n_{R_1}}} \right) \ln \left( \frac{M_{n_{R_3}}}{M_{n_{R_1}}} \right) \delta,$$

where $\delta$ captures the CP violating phases which can be order one. On the other hand, when we have $n_{R_3}$ inside the loop, we lose in the smallness of the Yukawa coupling $\sim 10^{-5}$, but we gain in the loop factor. Still, the contribution from $n_{R_3}$ exchange dominates over the one from $n_{R_2}$ exchange$^1$. It is worth noting that the bound derived by Davidson and Ibarra [4] on CP asymmetry is not applicable because what we are considering is the decay of the next-to-lightest RH neutrino, and not the lightest

$^1$Since $n_{R_3}$ is in the keV range, 3-body decays like $n_{R_1} \rightarrow \ell \bar{\ell} n_{R_3}$ are possible. However, this channel will give a null CP asymmetry.
one. Putting numbers, the CP asymmetry can be $\epsilon_1 \sim 5 \cdot 10^{-9}$, which is still quite interesting. This can be further enhanced by invoking e.g. resonance effects [14].

An important point is that although the Yukawa coupling $\lambda_{33}$ is large, the light active mass generated through the mechanism of Ref. [13] is small (less than an eV), hence the washout effect from $\Delta L = 2$ process via $n_{R3}$-exchange graph would be of little numerical significance. This turns the scenario to be very attractive. The relation between the baryonic asymmetry and the leptonic remains the usual one, given by

$$Y_B = -\left(\frac{8N + 4}{14N + 9}\right) \sum_{\alpha=e,\mu,\tau} Y_{L\alpha},$$

(16)

where the number of generation is $N = 3$.

### IV.2 Scenario II: A 4-generation case

Our second scenario consists of invoking a fourth chiral family, i.e. each entry of the mass matrix (9) is actually a $(4 \times 4)$ matrix. We assume that in this picture all four RH neutrinos weigh in the TeV range. We mention here that in spite of stringent constraints from LEP electroweak measurements, there is still a window left for the fourth family (see [17]). We emphasize that in the context of leptogenesis the rôle of the fourth lepton doublet is not much different from that of a supersymmetric partner, so the leptogenesis consequences of a four generation scenario are indicators of a more generic picture.

We now try to see what are the constraints on the different elements of the Yukawa matrix. Recall, the first index of $\lambda$ corresponds to the RH neutrino and the second one to the LH active neutrino. The requirement that light active neutrino masses corresponding to the first three families is less than $\sim 0.05$ eV constrains all the elements of the first three columns to be less than $\beta \sim 10^{-5}$. If we assume that the generated CP asymmetry is due to the decay of $n_{R1}$, then all entries in the first row should be less than $\epsilon \sim 10^{-7}$. The remaining elements, $\lambda_{i4}$ for $i = 2, 3, 4$, have to satisfy the requirement that the fourth generation active neutrino has to weigh above $M_Z/2$. To ensure this one must have each $\lambda_{i4}$ at least $\alpha \sim 1.4 \times \pi$, assuming the RH masses are around 500 GeV (Note, as mentioned already, $\lambda_{14}$ receives a stronger constraint $< 10^{-7}$, necessary for out-of-equilibrium decay of $n_{R1}$). Recall that perturbation theory is assumed to be valid as long as $\lambda^2/4\pi$ remains approximately within unity. Our requirement for $\lambda_{i4}$ for $i = 2, 3, 4$ barely exceeds that limit. Actually the radiative origin of the LH active mass, even for the fourth generation, is responsible for pushing the Yukawa couplings to such large values. It could have been avoided by adding an extra RH singlet neutrino for the fourth family and switching on the $\Phi_2$ interaction in Eq. (5). However, for the moment we stick to our present scenario. So the Yukawa matrix looks like (again, with all entries subject to order one uncertainties)

$$\lambda = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon \\ \beta & \beta & \beta & \alpha \\ \beta & \beta & \beta & \alpha \\ \beta & \beta & \beta & \alpha \end{pmatrix},$$

(17)

Now we turn our attention to CP asymmetry, which we assume to be generated by the decay of the lightest RH neutrino $\nu_{R1}$. We have seen that we must require some large couplings to satisfy the fourth generation neutrino mass constraints: $\lambda_{24} \sim \lambda_{34} \sim \lambda_{44} \sim \alpha$. These large couplings enhance CP
asymmetry, see Eq. (11). This is precisely the reason why a fourth family has been added. We assume that all RH masses weigh in the range 500 GeV to 1 TeV. This obtains

\[ \epsilon_1 \simeq \frac{9}{16\pi} |\beta||\alpha|f\delta. \]  

(18)

The loop function \( f \) turns out to be order one for our choice of RH masses. All the phases are contained in \( \delta \) which can be pushed towards its maximum value of unity. So we gain both in the Yukawa coupling and in the loop factor, contrary to what happens in scenario I. Putting numbers, we may expect \( \epsilon_1 \) to be a few times \( 10^{-6} \). We have taken note of the fact that the \( \Delta L = 2 \) processes involving the fourth family of leptons in external legs are rapid and hence in thermal equilibrium. It is intuitively easy to see this by considering such a process with fourth generation active LH neutrino (\( \nu_4 \)) in external legs \((\nu_4\phi \rightarrow \nu_4\bar{\phi} \text{ with } n_R \text{ exchange})\). This is equivalent to the see-saw diagram that produces the heavy (> 45 GeV) mass for \( \nu_4 \). Consequently, the lepton asymmetry in the fourth leptonic direction is washed out. This modifies the relationship between baryon and lepton asymmetry, which we obtain as

\[ Y_B = -\left( \frac{8N + 4}{14N + 25} \right) \sum_{\alpha = e, \mu, \tau} Y_{L\alpha}, \]  

(19)

where \( Y_L \) is the produced leptonic asymmetry only for the light active leptonic flavours, but \( N = 4 \) is the total number of generations.

V Observations and Outlook

1. We have explored the liaison between little Higgs mechanism and TeV scale leptogenesis in two ways. The first one relies on 3 generations only but the mass textures for the third generation is different from the first two. The second approach is a generic extension of the particle content by invoking a fourth chiral family, which opens the window for a more general framework like supersymmetry. A detailed description of flavour mixings among the light active neutrinos is beyond the scope of the present work. However, we emphasize that a hierarchical pattern of light active neutrino states is necessary to enhance the CP asymmetry. This can be arranged by adjusting the order one uncertainties in different elements of the Yukawa coupling matrices in Eqns. (14) and (17), ensuring agreement with the oscillation data and WMAP constraints.

2. Although \( N_L \) is an SU(2) singlet, a lepton asymmetry cannot be generated from its decay. Its rôle is to influence the mass matrix in Eq. (19) the way we have shown for individual generations.

3. Does \( n_R \) decay into the SU(2) singlet \( N_L \), thus opening a new channel for leptogenesis? For this we look into the Yukawa interaction involving \( N, n_R \) and the radial excitation \( \rho_1 \) (see Eqs. (6) and (10))

\[ -L_{\rho} = \bar{n}_{iR} \left( 1 - \frac{h^\dagger h}{2f^2} \right) \rho_1^I \lambda_{ij} N_{jL} + \text{h.c.} \]  

(20)

But the decay \( n_R \rightarrow N\rho_1 \) is kinematically either disallowed or suppressed since \( M_{\rho_1} \sim \text{TeV} \).

4. Light active neutrino data provides an access to the Majorana mass of \( N_L \)'s. In scenario II this mass turns out to be in the keV range for the first three generations, while the fourth \( N_L \) would
weigh around 100 TeV $^2$. Its decay is however in equilibrium and will not produce any leptonic asymmetry.

The $N_L$'s can mix with $\nu_L$'s but this mixing ($\sin \theta_m \sim v/f$) can be kept just consistent with the few per mille precise LEP data on $Z$ invisible decay width [19].

5. As has been shown in the context of 331 models [20-21] and for the simplest little Higgs model [22], given the assigned charges for the fermion fields each generation is anomalous. However, an anomaly free model is obtained if the number of generations is a multiple of 3. In general, one may argue that the UV completion will make the model anomaly-free [11]. In this case the four generation model proceeds as discussed above. Alternatively one could add fermionic particles to each generation to ensure it is anomaly free per generation as was done in [23].

6. Besides the light active physical state $\nu' \simeq \nu + (v/f)N$, we also have the orthogonal, dominantly sterile, $N' \simeq N - (v/f)\nu$ state which couples to the gauge bosons. These would lead to additional missing energy signatures at colliders.

7. In the context of the littlest Higgs model [24], a recent paper discussed the production of the neutrino masses [25] without having RH neutrinos. We note that if RH neutrinos are included, then it may be possible to produce a lepton asymmetry with only 3 generations consistent with observed data as the smallness of the neutrino masses arising via the interaction with the triplet field can be due to a small vev for the triplet. This allows some of the Yukawa couplings entering the expression of the CP asymmetry to be unsuppressed [15,26] compared to the usual analysis of TeV scale models of leptogenesis with 3 generations. The details of this scenario will be discussed elsewhere.

To conclude, the treasures of the TeV scale could include both the little Higgs scenario and thermal leptogenesis. In this paper, we have studied the synergy between these two mechanisms. The little Higgs scenario has direct consequences on the structure of neutrino mass matrix. The radiative origin of the light active neutrino masses is an interesting feature of our scenario. We have used the constraints derived from there to embed leptogenesis in the simplest little Higgs framework.

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