The extended uncertainty principle effects on the phase transitions of Reissner-Nordström and Schwarzschild black holes

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Abstract

In this paper, we investigate the phase transitions of Reissner-Nordström (RN) and Schwarzschild black holes for the extended uncertainty principle (EUP) framework. Considering temperature $T$, charge $Q$ and electric potential $\Phi$ as the state parameters, we show the van der Waals (vdW) like phase transition of RN black hole in $Q - \Phi$ diagrams and find the critical points depending on EUP parameter $\alpha$. Furthermore, we find Hawking-Page like phase transition for Schwarzschild black hole. The results imply that the black holes in asymptotically flat space have the similar phase structure with the black holes in anti-de Sitter (AdS) space.

Keywords: extended uncertainty; critical phenomena; Hawking-Page phase transition.

1 Introduction

The black hole thermodynamics is one of the most interesting subject in the theoretical physics. Black holes have been considered as thermodynamic systems since the breakthrough studies of Bekenstein and Hawking [1,2]. The black hole thermodynamics may play a vital role for understanding the quantum gravity theories. It may also provide the fundamental relations between general relativity, thermodynamics and quantum mechanics. Besides, considering black holes with a temperature and an entropy reveals many interesting similarities with the conventional thermodynamics systems. Specifically, similar phase transitions and critical phenomena can be found for the black holes. The studies in this direction were

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pioneered by Davies who considered the phase transition of Kerr-Newmann black holes [7]. Hawking and Page later showed a phase transition between Schwarzschild-AdS black hole and thermal AdS space [8]. Once they paved the way to phase transition of black holes, many papers devoted to this direction have been widely studied in the literature and new phase structures have been discovered [9–38]. In Refs. [9, 10], Chamblin et al. studied the thermodynamics of charged AdS black hole and showed a vdW like phase transition. In Refs. [11–16], the authors proposed the novel Ehrenfest equations by considering the analogies $V \leftrightarrow Q$, $P \leftrightarrow \Phi$ or $V \leftrightarrow J$, $P \leftrightarrow \Omega$. Furthermore, $Q - \Phi$ and $J - \Omega$ criticalities revealed more analogies between the AdS black holes and vdW fluids [17–22].

Moreover, these analogies become more exact in the extended phase space where the cosmological constant and its conjugate quantities are identified as (we use the units $\hbar = G_N = c = k_B = L_{Pl} = 1$ throughout the paper.)

\[ P = -\frac{\Lambda}{8\pi}, \quad (1) \]
\[ V = \left( \frac{\partial M}{\partial P} \right)_{S,Q,J}, \quad (2) \]

pressure and thermodynamic volume, respectively. Based on this fact, Kubiznak and Mann showed the $P - V$ criticality of the charged AdS black holes [23]. They obtained the critical points and showed that the critical exponents of the charged black holes are identical to the exponents of vdW fluids. Then, the same phase transition was studied for different AdS black holes [24–34]. Also the existence of triple points and reentrant phase transitions for AdS black holes were reported in Refs. [35–38] (for a more comprehensive review, please see Ref. [39]).

On the other hand, black hole thermodynamics and phase transitions can be considered beyond the semiclassical approximation. Considering the first order correction to entropy and temperature, authors studied the $\Omega - J$ criticality for Myers-Perry black holes in Ref. [40]. In Ref. [41], Schwarzschild black entropy was considered in Tsallis-Renyi approach. The authors found a Hawking-Page like phase transition for Schwarzschild black hole in asymptotically flat spacetime. vdW phase transition of charged black hole in flat space was studied for Renyi statistics in Ref. [42].

The modifications of Heisenberg uncertainty principle, i.e., the generalized uncertainty principle (GUP) and EUP, also have some interesting results on the black hole thermodynamics and phase transitions. Various GUP and EUP models were proposed in the literature [43–55]. Since GUP plays a remarkable role in the Planck scale physics, the modifications of black hole thermodynamics may not be unavoidable. Therefore, various GUP corrections to black hole thermodynamics were investigated in the literature [56–69]. Furthermore, richer and more complicated phase structures can be considered in the GUP framework. In Refs. [67–68], Ma et al. studied GUP effects on the phase transitions of charged AdS black hole. They found reentrant phase transition similar to higher dimensional Kerr-AdS black hole case. In Ref. [69], we considered the GUP for vdW black hole. We found that $P - V$ criticality is physically acceptable for the GUP case.
On the other hand, EUP may affect the black hole thermodynamics at large scales since it introduces the maximum position uncertainty notion [70–77]. Based on the fact that EUP effects are not negligible at the large distance scale, Bolen and Cavaglia showed that Schwarzschild-(A)dS black hole temperature can be derived from the EUP in Ref. [70]. In Ref. [71], modified dispersion relations and EUP effects were investigated for (A)dS black holes. In Ref. [72], the authors employed the EUP to obtain the Renyi entropy for the black holes. Recently, the black hole thermodynamics has been considered for the various EUP models [73–77].

Since (A)dS black hole thermodynamics may be related to EUP [70, 71], EUP corrected black holes in flat spacetime may have the same phase structures similar to AdS case. Therefore, we would like to investigate the phase transition properties of Schwarzschild and RN black holes for the EUP case.

The paper is organized as follows: In Sect. 2 we obtain the EUP modified temperature, entropy and heat capacity of RN black holes. In Sect. 3 we investigate vdW like phase transition in $Q - \Phi$ diagrams. We also study the behaviours of heat capacity and Gibbs free energy for the phase transition. In Sect. 4 we show a Hawking-Page like phase transition for Schwarzschild black hole. In Sec. 5 we finally discuss our results.

## 2 RN black holes thermodynamics with EUP correction

RN black hole solution in four dimensional spacetime is given by [35]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

(3)

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$  

(4)

From $f(r_h) = 0$, we obtain the black hole mass

$$M = \frac{r_h}{2} \left(1 + \frac{Q^2}{r_h^2}\right),$$

(5)

where $r_h$ is the event horizon of black hole.

In order to obtain the EUP correction, we consider the analysis of Xiang and Wen [63]. Now, let us consider the first law of black hole thermodynamics

$$dM = TdS + \Phi dQ.$$  

(6)

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1At this point, we bring the Ref. [78] to the reader’s attention. The motivation of Ref. [70] was criticized in Ref. [78]. In order to obtain the temperature of Schwarzschild-(A)dS black hole, Bolen and Cavaglia chose the EUP parameter identified with (A)dS radius. In Ref. [78], Scardigli showed that the temperature of (A)dS black holes can heuristically be derived from Heisenberg uncertainty principle without any adjustable numerical constant. There is no need the EUP to compute (A)dS black hole temperature. Therefore, we may assume that there is not a necessary link between (A)dS black hole temperature and EUP.
Since entropy is generally the function of horizon area $A$, the temperature of black hole can be written as

$$T = \left( \frac{\partial M}{\partial S} \right)_Q = \frac{dA}{dS} \times \left( \frac{\partial M}{\partial A} \right)_Q = \frac{dA}{dS} \times \frac{\kappa}{8\pi}, \quad (7)$$

where $\kappa$ is the surface gravity of black hole and is defined by

$$\kappa = \frac{f'(r_h)}{2} = \frac{r_h^2 - Q^2}{2r_h^3}. \quad (8)$$

When a particle is absorbed by a black hole, the smallest increase of area is given by \[2\]

$$\Delta A \sim bm \quad (9)$$

where $b$ and $m$ correspond to particle's size and mass, respectively. In order to define the Eq.(9) in a more familiar form, we consider the limitations on $b$ and $m$. In quantum mechanics, we define a particle by a wave packet. The width of a wave packet is characterized by the size of particle $b$, i.e., $b \sim \Delta x$. On the other hand, the momentum uncertainty is not greater than the particle mass $m$, i.e., $m \leq \Delta p$. Therefore, we can write

$$\Delta A \sim bm \geq \Delta x \Delta p. \quad (10)$$

We consider the simplest EUP [55],

$$\Delta x \Delta p \geq 1 + \alpha \Delta x^2, \quad (11)$$

where the EUP parameter $\alpha$ is defined by $\alpha = \alpha_0/L_*^2$, $\alpha_0$ is the dimensionless positive constant and $L_*$ is the new fundamental length scale. Using the EUP and taking $\Delta x \sim 2r_h$, one can get

$$\Delta A \geq \gamma (1 + 4\alpha r_h^2), \quad (12)$$

where $\gamma$ is a calibration factor which is determined from standard results. The minimum increase of entropy is given by $(\Delta S)_{\text{min}} = \ln 2$. Therefore, the EUP modified entropy-area relation is given by

$$\frac{dA}{dS} \simeq \frac{\Delta A_{\text{min}}}{\Delta S_{\text{min}}} = \frac{\gamma (1 + 4\alpha r_h^2)}{\ln 2}. \quad (13)$$

In the limit $\alpha \to 0$, the Eq. (13) must give $dA/dS = 4$. So we find $\gamma = 4\ln 2$. Using Eqs. (7), (8) and (13), the EUP corrected black hole temperature is obtained as

$$T = \frac{(r_h^2 - Q^2)(1 + 4\alpha r_h^2)}{4\pi r_h^3}. \quad (14)$$

The EUP corrected entropy and heat capacity are given by

$$S = \int \frac{dM}{T} = \frac{\pi}{4\alpha} \ln (1 + 4\alpha r_h^2), \quad (15)$$

$$C_Q = T \left( \frac{\partial S}{\partial T} \right)_Q = \frac{2\pi r_h^2 (r_h^2 - Q^2)}{3Q^2 - r_h^2 + 4\alpha r_h^2 (Q^2 + r_h^2)}, \quad (16)$$

respectively. Finally, the electric potential is given by

$$\Phi = \left( \frac{\partial M}{\partial Q} \right)_S = \frac{Q}{r_h}. \quad (17)$$
Figure 1: $Q - \Phi$ diagram of RN black hole for the EUP case. Blue dotted isotherm corresponds to $T < T_c$. The black solid isotherm is for the critical temperature. Red dashed isotherm corresponds to $T > T_c$. We take $\alpha = 1$.

## 3 $Q - \Phi$ criticality of RN black holes

Using Eqs. (14) and (17), we obtain the equation of state $Q = Q(T, \Phi)$ as

$$Q = \Phi \frac{\pi T - \sqrt{\pi^2 T^2 - \alpha (\Phi^2 - 1)^2}}{2\alpha (1 - \Phi^2)}. \quad (18)$$

The critical points can be obtained from the derivatives of Eq. (18) with respect to electric potential, i.e.,

$$\left(\frac{\partial Q}{\partial \Phi}\right)_T = 0, \quad \left(\frac{\partial^2 Q}{\partial \Phi^2}\right)_T = 0. \quad (19)$$

From Eqs. (18) and (19), we find

$$T_c = \frac{4\sqrt{(2\sqrt{3} - 3)\alpha}}{3\pi}, \quad (20)$$

$$\Phi_c = \sqrt{\frac{2\sqrt{3} - 3}{3}}, \quad (21)$$

$$Q_c = \frac{2 - \sqrt{3}}{2\sqrt{\alpha}}. \quad (22)$$
At this point, we give some comments on the critical points. The critical temperature and charge depend on the EUP parameter $\alpha$, but the critical electric potential does not depend on $\alpha$. A similar case was also found for $(n+1)$-dimensional RN-AdS black holes in Ref. [18] where the authors find the critical temperature and charge depend on $\Lambda$ while the critical electric potential does not depend on $\Lambda$.

In Fig. 1, we present the $Q - \Phi$ diagram for the EUP case. It is clear that the $Q - \Phi$ diagram resembles to the $P - V$ diagram of vdW fluids. The temperatures of curves increase from top to bottom. The upper isotherm ($T = 0.28 < T_c$) shows the ideal gas behaviour. The middle isotherm corresponds to critical temperature. For $T = 0.3 > T_c$, the lower isotherm shows a vdW liquid-gas like phase transition with an unstable region, namely oscillating part corresponds to $(\frac{\partial Q}{\partial \Phi})_T > 0$. This behaviour is similar to AdS black holes and vdW fluids with $(T, P, V)$ as state parameters [23]. In Fig. 2, we also investigate effects of the parameter $\alpha$ on isothermal curves. It can be seen that $\alpha$ does not change the behaviour of the isotherms, but it changes the positions of isotherms.

Moreover, we plot both semiclassical and EUP corrected heat capacities in Fig. 4. EUP corrected heat capacity has a different behaviour for large event horizon values. It diverges near the critical point $r_c = Q_c/\Phi_c \sim 0.34$. In contrast to semiclassical case, EUP corrected heat capacity is positive for the large values of $r_h$ and unstable black holes disappear. So EUP corrected heat capacity shares the same behaviours with heat capacity of RN-AdS black holes near the critical points [17].

Finally, we consider the behaviour of Gibbs free energy. In the non-extended phase space, Gibbs free energy can be defined by [18]

$$G = M - TS - \Phi Q = \frac{(Q^2 - r_h^2) [ -8\alpha r_h^2 + (1 + 4\alpha r_h^2) \ln (1 + 4\alpha r_h^2)]}{16\alpha r_h^3}.$$  \hspace{1cm} (23)

In Fig. 5, we plot the Gibbs free energy as a function of the temperature for the different values of the charge. For $Q < Q_c$, the Gibbs free energy shows the characteristic swallow tail behaviour which implies the first order phase transition.

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2At this point, we would like to denote the differences between $(P, V, T)$ and $(Q, \Phi, T)$ state parameters. The ideal gas and vdW like behaviours of $(Q, \Phi, T)$ systems occur for $T < T_C$ and $T > T_C$, respectively while the ideal gas and vdW phase transition behaviours of $(P, V, T)$ systems occur for $T > T_C$ and $T < T_C$, respectively (please see Figs. 2 and 6 in Ref. [23]). In order to investigate the AdS like phase structure for EUP corrected case, we consider the analogies $P \leftrightarrow Q$, $\Phi \leftrightarrow V$ and $T \leftrightarrow \beta = 1/T$ where $\beta$ is inverse temperature. In fact, we choose inverse temperature $\beta$ as the state parameters rather than temperature. In this case, $Q - \Phi$ criticality is more similar to conventional $P - V$ criticality, i.e., ideal gas case for $\beta > \beta_C$ and vdW like phase transition for $\beta < \beta_C$. 

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Figure 2: Isothermal curves for different values of the EUP parameter. (Top-left) $T > T_c$. (Top-right) $T = T_c$. (Bottom) $T < T_c$.

Figure 3: $Q - \Phi$ diagram of RN black hole for $\alpha = 0$. Red dashed, blue solid and black dotted curves correspond to $T = 0.5, 1$ and $1.5$, respectively.
Figure 4: Semiclassical (black solid line) and EUP modified (red dashed line) heat capacities versus $r_h$. We take $Q = Q_c$.

Figure 5: EUP modified Gibbs free energy versus temperature. We take $\alpha = 1$. 
Another important phase transition is the Hawking-Page phase transition. In Ref. [8], Hawking and Page studied the thermodynamics properties of Schwarzschild-AdS black holes. They also showed that the canonical ensemble exists for Schwarzschild-AdS black holes, unlike the counterparts in flat space. They also found a phase transition between black hole and thermal AdS space.

Now, we show a Hawking-Page like phase transition for the EUP case. For $Q = 0$, EUP corrected Gibbs free energy of Schwarzschild black hole is given by

$$ G = M - TS = r_h - \frac{1 + 4\alpha r_h^2}{16\alpha r_h} \ln \left(1 + 4\alpha r_h^2\right). \tag{24} $$

Using Eq. (24), we plot the Gibbs free energy with respect to temperature in Fig. 6. The figure clearly reveals the Hawking-Page like phase transition for the Schwarzschild black hole in flat spacetime. Just like AdS case, there are two branches. The upper branch (black solid line) corresponds to negative heat capacity and defines the small unstable black holes. The lower branch (dashed black line) defines the large stable black holes with positive heat capacity. Just like AdS case, there is a minimum temperature $T_{\text{min}} = \sqrt{\alpha}/\pi$. Below this temperature, black holes cannot exist. On the other hand, radiation-black hole phase transition occurs at Hawking-Page temperature $T_{HP}$. From the $G = 0$ condition, we find $T_{HP} = 1.24\sqrt{\alpha}/\pi$. 

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5 Conclusions

In this paper, we focused on the phase transitions of RN and Schwarzschild black holes for the EUP case. Taking $T$, $Q$ and $\Phi$ as the state parameters, we studied the $Q - \Phi$ criticality of charged black holes. We obtained the critical points and showed that there is a vdW like phase transition for the charged black hole in the EUP case. We also calculated the heat capacity at constant charge and Gibbs free energy. We plotted Gibbs free energy and found the characteristic swallow tail behaviour which is similar to vdW fluids. Finally, we found Hawking-Page like phase transition for the Schwarzschild black hole. In summary, black holes in flat spacetime have the same phase structures with the black holes in AdS spacetime in the presence of EUP effects.

The black holes in flat spacetime may share similar thermodynamic behaviours with black hole in AdS spacetime due to the non-negligible EUP effects at large distance scales. Just like cosmological constant $\Lambda$, EUP effects play the same role for the thermodynamic stability and the phase transition of black holes in flat space. Moreover, Renyi entropy of a black hole may be obtained from EUP [72]. On the other hand, black holes in flat spacetime has similar thermodynamic behaviours with the AdS black holes for the Tsallis-Renyi approach [41,42]. Therefore, our study also reveals the more similarities between the EUP and Renyi entropy.

It is also possible to find similar phase structures in the flat spacetime by considering the black holes in a cavity [79–81]. The different approaches may play the same role at the large distance scales. Therefore, we may consider that they are related to each other.

CRediT authorship contribution statement

Özgür Ökçü: Conceptualization, Writing - original draft, Writing - review & editing. Ekrem Aydiner: Supervision, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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