Dynamics of Emitting Electrons in Strong Electromagnetic Fields

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We derive a modified non-perturbative Lorentz-Abraham-Dirac equation. It satisfies the proper conservation laws, particularly, it conserves the generalized momentum, the latter property eliminates the symmetry-breaking runaway solution. The equation allows a consistent calculation of the electron current, the radiation effect on the electron momentum, and the radiation itself, for a single electron or plasma electrons in strong electromagnetic fields. The equation is applied to a simulation of a strong laser pulse interaction with a plasma target. Some analytical solutions are also provided.

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I. INTRODUCTION

Lasers now allow us to reach intensities within the focal spot of \( W > 10^{22} \text{ W/cm}^2 \). Electron motion in fields where \( W \gg 10^{18} \text{ W/cm}^2 \), for a typical laser wavelength, \( \lambda \sim 1 \mu m \), is ultra-relativistic:

\[
a = \frac{|e|A}{mc^2} \gg 1, \tag{1}
\]

where \( e \) is the electron charge, \( m \) is its mass, \( c \) is the speed of light and \( A \) is the vector-potential amplitude.

An accelerated electron in a strong laser field emits high-frequency radiation \([2]\). Its back-reaction on the electron motion can not be neglected, if in the frame where the electron is initially at rest, the energy radiated during the interaction time is comparable with \( mc^2 \):

\[
\approx \frac{1}{4\pi} \int E^2 c dt \geq mc^2, \quad \text{where} \quad \sigma_T = \frac{8\pi e^4}{m^2c^2} \quad \text{and} \quad E \text{ is the electric field.}
\]

In the course of a Lorentz transformation, this integral transforms proportionally to a wave frequency, \( \omega_0 \). Indeed, \( \tilde{E} \sim \omega_0 A \). The transversal vector potential, \( A \), as well as the differential of the wave phase, \( \omega_0 dt \), are Lorentz invariant. Since \( \omega_0 E - c^2(k_0 \cdot p) = c^2(k_0 \cdot p) \) is invariant, as is the 4-dot-product of the particle momentum, \( p^4 = (\hat{\mathbf{x}}, \mathbf{p}) \), by the wave number, \( k_0^2 = (\hat{\mathbf{z}}, \mathbf{k}_0) \), the (sufficient) condition for the radiation reaction significance is as follows:

\[
\frac{\sigma_T}{mc^2} \int \frac{E^2 c dt}{4\pi} = \int \frac{W dt}{1.2 \text{ kJ/} (\mu \text{m})^2} \geq \frac{mc^2}{\mathcal{E} - cp_x}. \tag{2}
\]

Here \( \mathcal{E} \) and \( \mathbf{p} \) are the particle energy and momentum and the wave is assumed to propagate along the \( x \)-axis. A high value of the integral in (2) may be reached, in principle, at the cost of higher intensity only, \( W \sim 10^{25} \text{ W/cm}^2 \). In the course of the ELI project (see \([3]\)) a laser is expected to reach focusable pulse energy of 1.5 kJ at \( \lambda \approx 0.8 \mu \text{m} \), so the radiation effects will be dominant: \( \int W dt \approx 2.1 \text{ kJ/} (\mu \text{m})^2 \geq 1.2 \text{ kJ/} (\mu \text{m})^2 \). Another opportunity may be realized while a strong laser pulse interacts with energetic electrons, which move oppositely to the direction of the pulse propagation. In this case \( \mathcal{E} - p_x c \approx 2 \mathcal{E} \gg mc^2 \), facilitating the fulfillment of Eq.(2).

In such a geometry, powerful X-ray radiation is generated in the direction of the electron momentum \([4]\). Ineq.(2) determines the regime, in which the energy is efficiently converted to X-ray or \( \gamma \) bursts.

In the course of a strong laser pulse interacting with a dense plasma the counter-propagating electrons may be accelerated in a backward direction by a charge separation field (see, e.g., \([5]\)). At moderate intensities the generation of short pulses of higher-frequency radiation \([6]\) may be interpreted as the reflection of the laser pulse from these bunches as from a reflecting medium: the frequency of the reflected wave, \( \omega_r \), is upshifted: \( \omega_r \sim \omega_0 \). The reflection occurs for a laser pulse moves with a Lorentz-factor of \( \gamma_r \gg 1 \) towards an incident wave. Here we consider such high laser intensities that emission frequencies are upshifted to the hard X-ray and \( \gamma \) range. At realistic electron density, \( N_e \lesssim 10^{24} \text{ cm}^{-3} \), the averaged field approximation of the reflecting medium is not applicable for emitted photon energies exceeding 10 keV, because \( N_e(c/\omega_r)^3 \ll 1 \). The emission from plasma in this case is taken as an integral of the radiation intensity from separate electrons, rather than as the field of a coherent electric current of a \( \gamma \) range frequency in plasma. Hence, even in a dense plasma the emission from separate electrons is essential to the analysis.

While the physical processes involving significant radiation back-reaction are of growing importance, the Lorentz-Abraham-Dirac (LAD) model \([7]\) which is to account for this effect, is not free of difficulties, such as ‘runaway’ solutions etc. \([8, 9, 10, 11, 12, 13, 14]\). Some flaws of the original Dirac version (see Eq. (9) below), are eliminated in the approximate equation, as derived in
the book by Landau and Lifshits [8], as Eq. (76.3), see also the non-relativistic variant in §75. A slightly different approximation was found by Eliezer [11], and most later versions (see, e.g., [12, 13, 14]) are reducible to those listed above.

New problems arise while considering the transition to even higher field intensities, at which Quantum Electrodynamical (QED) effects come into a power. Delegating the discussion of QED strong fields to a forthcoming publication, we still have to briefly discuss here a more general issue of the LAD model conformity with the QED principles. Particularly, the nature of generalized electron momentum (which is substituted for the \( i\hbar \nabla \) operator in the framework of QED) is highly arguable in the LAD model, but this point has hardly been discussed so far. Also, there is a controversy between the treatment of the radiation in QED as a random process of separate photon emission with some probability, and the description of the back-reaction of the radiation in the LAD model with very smooth functions of time which are allowed to be differentiated many times.

In Section II we describe how we derive the modified LAD equation for electrons and account for radiation from electrons and the electron current in a plasma, in a self-consistent manner and in the way which does not contradict QED fundamentals. As an application, the basic elements of a particle-in-cell (PIC) numerical scheme are discussed, which extends the simulation of plasma-electron interactions to higher intensities \( W \geq 10^{22} \text{ W/cm}^2 \). In Section III we analytically solve for the electron motion in a 1D wave field in vacuum. Results of 3D PIC simulations of laser-plasma interaction at intensities \( W \sim 10^{22} \text{ W/cm}^2 \) are discussed in the concluding section.

II. MODIFIED LAD MODEL

Here we assume an electron moving in an external electromagnetic field and emitting high-frequency radiation. In the case of a plasma electron, the external field is the self-consistent electric field, \( \mathbf{E} \) of the plasma, in a self-consistent manner and in the way which does not contradict QED fundamentals. As an application, the basic elements of a particle-in-cell (PIC) numerical scheme are discussed, which extends the simulation of plasma-electron interactions to higher intensities \( W \geq 10^{22} \text{ W/cm}^2 \). In Section III we analytically solve for the electron motion in a 1D wave field in vacuum. Results of 3D PIC simulations of laser-plasma interaction at intensities \( W \sim 10^{22} \text{ W/cm}^2 \) are discussed in the concluding section.

Illustration: external field of the 1D wave. We start from an example of the field of a 1D harmonic wave with the wave 4-vector, \( k_0^i \). Recall that the energy-momentum exchange of the charged particle with the classical field is governed by the Lorentz force, while the effect of the emitted/absorbed photons should be interpreted in terms of the photon 4-momentum. The case of the 1D wave allows both treatments.

In the course of photon emission with the 4-vector, \( k^i_1 \), the 4-momentum is conserved: \( \delta p^i = \delta p^i_0 + \hbar k^i_1 - \hbar k^i_1 \), where \( \delta p^i_0 \) stand for 4-momenta of the electron before and after the emission. In the classical limit of small recoil, the increment in the electron momentum, \( \delta p^i = p^i_1 - p^i_0 \), should be small. Therefore, the condition \( (p \cdot p) = (E/c)^2 - p^2 = m^2c^2 \) requires that \( \delta p^i_0 = 0 \) and \( \delta p^i = \hbar k^i \frac{(k_1^0 p_0^0)}{k_0^0 p_0^0} - \hbar k_1^i \).

The second term is the 4-momentum transferred to the emission and the first term is the gain in 4-momentum the electron obtained from the field. The latter for the classical external field reduces to the effect of the field tensor, \( F^{ik} \), on the yet unknown current, \( \epsilon \delta (dx^i/\delta t) \):

\[
\int \frac{e}{c} F^{ik} \delta (dx^i/\delta t) d\tau = \hbar k_0 \frac{(k_1^0 p_0^0)}{(k_0^0 p_0^0)}.
\]

Here \( \tau \) is the time in the ‘Momentarily Comoving’ Lorentz Frame (MCLF), such that the spatial components of \( p_0 \) vanish. In strong fields as in Eq. (1), emission characteristics are local functions of the wave field (see [12, §90]). Therefore, the integration in Eq. (3) reduces to a multiplication by \( \Delta \tau \). Expressing \( F^{ik} = (\partial A^i/\partial x_j) - (\partial A^j/\partial x_i), \) in terms of the 4-vector-potential and using the entities, \( (k_0 \cdot A) = 0, (k_0 \cdot k_0) = \omega_0^2/c^2 - k_0^2 = 0 \), one can represent \( k_0 \frac{k_1^0}{(k_0^0 p_0^0)} F^{ik} \frac{p_0^k}{p_0^0 F_0^i} \) and solve Eq. (3):

\[
\delta (dx^i/\delta t) = \frac{-(p_0 \cdot k_1)}{m (f_{L0} \cdot f_{L0})} \frac{dI}{\Delta \tau} - \frac{e I f_{L0}^i}{mc}.
\]

In the MCLF the current has only spatial components and may be expressed in terms of the Lorentz transformed electric field, \( E_{\text{MCLF}} \): \( \delta (dx^i/\delta t) = \hbar c E_{\text{MCLF}} F^{ik} \frac{p_0^k}{p_0^0 F_0^i} \), so that the averaged Eq. (4) reads:

\[
\delta (dx^i/\delta t) = \frac{-(p_0 \cdot k_1)}{m (f_{L0} \cdot f_{L0})} \frac{dI}{\Delta \tau} - \frac{e I f_{L0}^i}{mc}.
\]

Now we average Eqs. (3) over the emitted photon parameters. The averaging (taking a mathematical expectation) is done as a weighted integration over \( d\omega_1 \) with the differential probability of emission per unit of time, \( dW/d\omega_1 \), the result being multiplied by \( \Delta \tau \), to account for the time integration. In the MCLF, averaging \( \hbar \omega_1 \) gives \( I \), where \( I = \int_0^{\infty} \hbar \omega_1 I d\tau \), is the total emission intensity. Below we use its ratio to the dipole emission intensity, \( I_E = \tau_0 \omega_0^2 E_{\text{MCLF}}^2/(mc) \), \( \tau_0 = 2e^2/(3mc^3) \sim 6.2 \times 10^{-24} \text{ s} \). In an arbitrary frame of reference, averaged \( \hbar k_1^i \) is the 4-momentum of emitted radiation, \( (\delta p^i/\delta t)_{\text{rad}} \), expressed in terms of:\n
\[
\frac{(d\alpha)}{d\tau} = \frac{\tau_0 I_E}{f_{L0}} I \frac{f_{L0}^i}{m}.
\]

where the terms on the right hand side are: the Lorentz force, \( f_{L0} \), the 4-momentum of the emitted radiation, \( -(\delta p^i/\delta t)_{\text{rad}} \), and the external field effect, \( F^{ik} f_{L0}^k \), on the current. \( J_{\text{rad}} = e(\delta p^i/\delta t)_{\text{rad}} \). Multiplying Eq. (5) by \( p_i \), we see that \( d(p \cdot p)/d\tau = 0 \), maintaining the entity, \( (p \cdot p) = m^2c^2 \).

General case of an arbitrary external field. Eq. (4) is not specific to the 1D wave case and can be derived for an arbitrary external electromagnetic field.
Seeking the last term in the form of $F^i_kJ^k_{\text{rad}}$, which is mandatory for the 4-momentum exchange with the classical external field, and requiring the conservation of $(p\cdot p)$ we obtain Eq. (5) directly and with no extra assumption.

**Electron current.** Now we re-write Eq. (5) in terms of the total electron current, $e\frac{dx^i}{dt} = e\frac{p^i}{m} + e\left(\frac{dx^i}{dt}\right)_{\text{rad}}$:

$$\frac{dp^i}{d\tau} = \frac{e}{c} F^{ik} \frac{dx_k}{d\tau} - \frac{I p^i}{mc^2},$$

$$\frac{dx^i}{d\tau} = p^i \frac{m}{m} + \frac{\tau_0}{IE} \frac{e F^{ik} p_k}{m^2 c}.$$ 

Integrating by volume the equation for the energy-momentum tensor for the external field, $\partial T^{ik}_{\text{ext}}/\partial x^k = -\frac{1}{\mu} F^{ik} j_k$, and representing the volume integral, $\int dV'$, of the point-wise current density, $j^i = \epsilon c \int \frac{dx}{dx'} \delta^{4}(\vec{r}' - \vec{x}(\tau))d\tau$, ($\vec{x}$ being the coordinate 4-vector in an arbitrary Lorentz frame) in terms of $e\frac{dx^i}{dt}$, we find:

$$\frac{d}{d\tau} \int T^{00}_{\text{ext}}dV = -\frac{e}{c} F^{ik} \frac{dx_k}{d\tau} (\text{cf. } \S 3.3).$$

Hence, Eqs. (6-7) conserve the total energy-momentum: $\frac{dp^i}{d\tau} + \frac{d}{d\tau} \int T^{00}_{\text{ext}}dV + \frac{p^i}{mc^2} = 0$.

The generalized momentum may also be conserved. However, for a radiating electron this conservation takes place, if not only the external field is constant along some direction: $(\mathbf{n} \cdot \nabla) A^i = 0$, but also the projection of the emitted momentum, $(\mathbf{n} \cdot \mathbf{p})_{\text{rad}}$, onto $\mathbf{n}$ vanishes: $I(\mathbf{p} \cdot \mathbf{n}) = 0$. If the latter condition is not fulfilled and $I(\mathbf{p} \cdot \mathbf{n}) \neq 0$, then the change in the generalized momentum, $\mathbf{P}^i = p^i + e A^i/c$, is as follows:

$$\left(\mathbf{n} \cdot \frac{d\mathbf{P}}{d\tau}\right) = - (\mathbf{n} \cdot \mathbf{p}) \frac{I}{mc^2}.$$ 

Eq. (8) also follows from the quantum relationship, $\mathbf{n} \cdot \delta \mathbf{P} = -\hbar \mathbf{n} \cdot \mathbf{q}$, (a conserved generalized momentum corresponds to the constant gradient of the electron wave function phase along $\mathbf{n}$ - see 15).

Discussing possible choices of $I$, we note that the ratio $I/IE$ should be bounded at $IE \rightarrow 0$. Although to take $I = IE$ is physically reasonable, there are other interesting options. Particularly, $I$ can be a random function with its average equal to $IE$ (to trace the quantum theory limit or to include emission with large photon energy). One can apply $I$ expressed in terms of the modified emission probability, to treat the processes (like a gyrosynchrotron emission, see §90 in [18]) in very strong fields, such that the QED effects are not negligible. After all, $I$ can differ from $IE$ by a choice of $\omega_{\text{min}} \neq 0$. The latter approach allows us to separate, if desired, the high-frequency emission from a lower-frequency averaged external field, in simulating laser-plasma interactions.

Eqs. (6-7) and their properties result from the assertion that the electron while emitting moves not strictly along the direction of its momentum. Particularly, in the MCLF the electron while emitting is not at rest and displaces along $e\mathbf{E}_{\text{MCLF}}$. In the MCLF the external electric field produces a work at a moving charge, which entirely balances the emitted energy: $e(\frac{\mathbf{P}}{mc^2} \cdot \mathbf{E}_{\text{MCLF}}) = I$.

The model applicability is limited by the requirement for the current $(dx^i/d\tau)_{\text{rad}}$ to be essentially non-relativistic, which is fulfilled as long as $\tau_0 I^2/IE \ll mc^2$. To neglect QED effects, the field should be weak: $e\mathbf{E}_{\text{MCLF}} \ll mc^2(\mathbf{v}/c)$, and $\tau_0 IE/mc^2 \ll (e^2/hc^2) \ll 1$.

**To compare with the radiation force model** we use MCLF and approximate within a short time interval $p = 0$ and put $I = IE$ in spatial components of Eqs. (6-7): $dp/d\tau \approx eE + \gamma_0 e^2(\mathbf{E} \cdot \mathbf{B})/(mc)$, $dx/d\tau \approx (p + \gamma_0 eE)/m$. Formally, the latter is equivalent to the Newton equation with the approximate radiation force as described in [8, 13, 14]: $md^2x/dt^2 = eE + \gamma_0 (e\mathbf{E}/d\tau + e^2(\mathbf{E} \times \mathbf{B})/(mc))$.

Now we compare Eqs. (6-7) with the LAD equation [7]:

$$\frac{d^2x^i}{d\tau^2} = \frac{eF^{ik} dx_k}{mc} + \gamma_0 \frac{dx^i}{d\tau^3} + \frac{\tau_0}{c^2 d\tau} \left(\frac{d^2x^i}{d\tau^2} + \frac{d^2x^i}{d\tau^2}\right),$$

(cf. [8], Eqs. (76.1-2)). Re-write Eq. (9) introducing $I = -m\gamma_0 (d^2x_k/d\tau^2)(d^2x_k/d\tau^2)$, as in [8], Eqs. (73.4):

$$\frac{dp^i}{d\tau} = \frac{eF^{ik} dx_k}{c} - \frac{dx^i}{d\tau^2} \frac{m}{c^2} = \frac{p^i}{mc^2} - \frac{d^2x^i}{d\tau^2}.$$ 

Comparing this with Eqs. (6-7) we find that both our model and the Dirac theory, as well as the modified version of Eq. (9) as described in [8, 13, 14] (which approximates $d^2x^i/d\tau^2 \approx e\mathbf{E} mc^2 dx_k/d\tau$ in the right hand side of Eq. (9)) differ from each other with small terms $\sim \gamma_0^2$.

The key distinction, however, is the choice of the electron momentum. An interesting survey [13] shows that this choice in the Dirac theory is ambiguous. It is problematic too: for $p^i = m\mathbf{dx}_i/d\tau$ Eq. (9) conserves $(p \cdot p) = m^2 c^2$, but the generalized momentum is not conserved, however symmetric the external field and the radiation may be. Particularly, Eq. (9) allows this electron momentum to change in the absence of the external field (the runaway solution, see [8, 13, 14]), while the conservation of the generalized momentum in the MCLF would enforce $\mathbf{P} = \mathbf{p} = 0$, as long as $\mathbf{n} \cdot \mathbf{A} = 0$ for any $\mathbf{n}$ and $I = 0$. With a different choice of the momentum (say, $p^i_0$ as introduced above) the generalized momentum may conserve, but not $(p \cdot p)$. So, the distinction of our approach from the Dirac model lies in: (1) the incorporation of $\sim (dx^i/d\tau)_{\text{rad}}$ into the relationship between the velocity, $4x^i/c^2$, and momentum, $p^i$, instead of the “self-force” $\sim (dx^i/d\tau)_{\text{rad}}$ into the force equation for $dp^i/d\tau$ and (2) the use of a different relativistic formulation, providing a different set of exact conservation laws.

**Application to the particle-in-cell scheme.** 3-vector formulation of Eqs. (6-7) is as simple as:

$$\frac{dp}{dt} = \mathbf{f}_L + \epsilon \frac{e}{c} \left[\delta \mathbf{u} \times \mathbf{B}\right] - \frac{e^2}{c^2} \frac{\delta (\mathbf{u} \cdot \mathbf{f}_L)}{c^2},$$

$$\frac{dx}{dt} = \mathbf{u} + \delta \mathbf{u},$$

where: $I = IE$, $\mathbf{u} = \frac{p}{\sqrt{m^2 + p^i c^2}}$, $\mathbf{f}_L = eE + \hat{e} (\mathbf{u} \times \mathbf{B})$, and

$$\delta \mathbf{u} = \frac{\tau_0}{m} [\mathbf{f}_L - (\mathbf{u} \cdot \mathbf{f}_L)/(mc^2)].$$
These equations may be applied to plasma electrons in order to simulate laser-plasma interactions at high laser field intensity. More precisely, the equations should be solved for 'particles' consisting of a large number of electrons, radiating independently and incoherently. Their contribution into an averaged electron current is $e(u + \delta u)$. The spectrum of radiation from relativistic electrons is calculated assuming that an angular distribution is peaked in the direction of the electron momentum and can be approximated with an angular function and the frequency spectrum, $F(r) = \frac{\gamma^2}{c^2 r} \int_{\gamma}^{\infty} K_{\gamma-3}(r')dr'$, $r = \omega_1/\omega_c$, $\omega_c = \frac{2}{3} \omega_r \gamma^3$, is momentarily close to that from circular motion with a rotation frequency, $\omega_r = |p \times f_L|/p^2$.

$$\frac{dI}{d\Omega d\omega} \Delta t = \frac{\gamma^2}{c^2} \frac{\omega_c}{\omega_1} F \left( \frac{\omega_1}{\omega_c} \right) \Delta t.$$

The integral of the spectral function is normalized by unity, $\int F(x)dx = 1$. The effect of the radiation on the electron motion, $\frac{dI}{d\Omega d\omega} d\Omega d\omega = \gamma^2 (\delta u \cdot f_L)$, is entirely included into Eq.(10).

### III. ELECTRON IN THE 1D WAVE

In the case of the 1D wave external field, the electron motion can be solved analytically. With the external field being a function of $\xi = (k_0 \cdot x)$, the relation between $\xi$ and $\tau$ is given by a product of Eq.(7) by $k_0$: $d\xi/d\tau = (k_0 \cdot p)/m$. Multiplying Eq.(8) by $k_0$, expressing the derivative over $\tau$ in terms of that over $\xi$ and assuming $I = I_E = a(0) \cdot \rho^2 / c^2 |d\alpha/d\xi|^2 / m$, we obtain:

$$\frac{1}{(k_0 \cdot p)} = \left( \frac{1}{(k_0 \cdot p)} \right)_{\xi=0} + \frac{\gamma_0}{m} \int_0^\xi \frac{|d\alpha/d\xi|^2}{d\xi_1} d\xi_1.$$

The transverse momentums, $p_\perp$, is solved from Eq.(8):

$$p_\perp + \frac{2a}{c} \frac{k_0}{(k_0 \cdot p)} = \left( \frac{p_\perp + \frac{2a}{c} k_0}{(k_0 \cdot p)} \right)_{\xi=0} + c \int_0^\xi \frac{a}{d\xi_1} d\xi_1.$$

To compare with the Dirac solution for a short pulse, consider a single-period symmetric wave: $a = a_0 \sin(\xi)$, $0 < \xi < 2\pi$. In the frame of reference in which the electron was at rest prior to the interaction the transverse components of the electron momentum vanish after the pulse: not only they do not turn to infinity, as they would in the Dirac runaway solution, but the conservation of the generalized momentum in conjunction with the pulse symmetry ($\int_0^{2\pi} a \frac{d\xi}{d\xi_1} d\xi_1 = 0$) entirely eliminates $p_\perp$ at $\xi > 2\pi$. For a pulse of moderate intensity, such that the left hand side of Eq.(2) is much less than unity, the electron gains a small momentum, $p_\perp = \sigma_T \int \frac{E^2 d\tau}{(4\pi)}$, in the direction of the pulse propagation, this momentum is absorbed from the pulse. The energy, $cp_\perp$, is absorbed from the pulse and almost equal energy is emitted. For strong pulses satisfying Ineq.(2), we present an integral spectrum of emission in Fig.1. We see that for a longer pulse duration the spectrum is softened. This is a result of the radiation reaction: without it the spectrum would have the same shape as that of the shorter 5-fs pulse and would only increase proportionally to the pulse duration.

### IV. SIMULATIONS AND DISCUSSION

To demonstrate more realistically the role of the radiation back-reaction in the laser-plasma interaction we perform a 3D PIC simulation for a 10-cycle linearly polarized laser pulse having a step-like profile along the pulse direction including $2\times10^7$ rising and falling edges, and a Gaussian profile in the transverse direction, focal diameter 5$\lambda$, and amplitude $a_0 = 70$. The laser pulse is incident normally on a plasma layer of $10^{-6}$ cm thick and density $n_0 = 3n_{cr}$, where $n_{cr} = m e^2/\lambda^2 c^3$ is the critical density. The simulation is performed in the box $20\lambda \times 20\lambda \times 20\lambda$, with spatial resolution $\lambda/20$ and 8 electrons per cell, requiring in all $6.4 \times 10^{10}$ grid cells and $2.6 \times 10^{18}$ particles. The plasma layer is located after a 5$\lambda$ vacuum layer. Here ions are immobile and the time step is $\Delta t = \lambda/(40c)$.

In this simulation, which corresponds to the intensity $10^{22}$ W/cm$^2$ for $\lambda = 0.8\mu$m, by the instant $t = 20\lambda/c$ the laser pulse loses $\sim 27\%$ of its energy, converting $\sim 0.9\%$ of the incident radiation (or $\sim 3.2\%$ of the lost energy) to the backward scattered high-frequency radiation. The angular distribution of the radiation exceeding 150 keV is shown in Fig.2(a). The emitted radiation has a wider angle along the direction of a transverse electric field. The total energy of the pulse equals $\sim 30$ J, and emitted backward high-frequency radiation accounts to $\sim 0.26$ J, with $0.24$ J of photon energy above 150 keV (see Fig. 2(b)).
FIG. 2: Results of 3D PIC simulation for a linearly polarized laser pulse with amplitude $a_0 = 70$ entering a soft plasma layer ($n_0 = 3n_{cr}$). (a) Angular distribution of the backward scattered radiation with photon energy above 150 keV and (b) total emitted energy as a function of the cut-off frequency. The total radiated energy may be close to the particle energy for some electrons (see [16]). Specifically, this relates to a counter-streaming flow of electrons, with momentum up to $150mc$ propagating in the region of the laser pulse, as we observe in the simulation. These fast electrons are generated in the charge separation field from the flow of cold electrons; due to the loss of their energy in the laser field, combined with an action of the charge separation field of opposite sign, they reverse their motion. However, only a minor fraction of electrons counter-propagate with high energy, and of these, only a fraction moves in the region of the strong laser field, where they can radiate. By this account the overall conversion efficiency is diminished and does not exceed the order of a few percent.

We have also applied the described model within a particle-in-cell code to simulate processes pertinent to fast ignition at laser intensities $W \geq 10^{22} \text{W/cm}^2$ [17]. The processes in higher fields, such that the corrected QED probabilities should be used to simulate the emission, are to be considered in a forthcoming publication. We also plan to use high contrast pulses from the Hercules laser to drive high-density targets with intensities $> 10^{22} \text{W/cm}^2$. We hope that such a study may improve understanding of ultra-intense laser-plasma interactions and may result in short X- or $\gamma$-burst production.

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