Reduced-order modelling of unsteady vortex lattice method

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Abstract. The purpose of this research is to present a new idea of reduced-order modelling of unsteady flows without static correction, which is based on fluid eigenmodes. The new reduced order model is formulated based on a new form of wake vortices without static correction. This technique is expected to perform a reduced-order model similar but faster than the previous techniques. The new method is used to analyse unsteady flows over a three-dimensional wing and comparing with the previous techniques. The performances of the present method are demonstrated with numerical examples. The results show the accuracy of the new reduced-order model of unsteady flow with higher computational efficiency. In conclusion, the new method can be considered as an alternative technique to perform the reduced-order models of unsteady flow.

Keywords: unsteady vortex lattice, reduced-order, flow, three-dimensional wing

1. Introduction

The purpose of this research is to present the new reduced-order modeling (ROM) based on Eigenanalysis of an unsteady vortex lattice method (UVLM) about plate like wing. In the past, this technique was proposed based on an idea, which needed a static correction technique to suppress the quasi-steady term making it obtain acceptable results [1-2]. Later, the technique has been improved to the reduced-order modeling of unsteady flows without static correction requirements [3]. The efficiency of this technique as a result from the quasi-steady or zero eigenvalue is removed by rearranging the UVLM equations to a new form of wake vortex strength.

The difference between these two methods is that one is a conventional method with static correction and another is without static correction. The performance of the latter has been proved by many researches [4-8]. It is found that inefficiency of the technique without static correction as it needs matrix inversion of the new equation form [3]. It is thus interesting to study how to discard the inversion of the matrix from the UVLM equation, which expect to increase the performance of the computational efficiency. In the past, it had two models of the vortex in the wake. The first model considers the unsteady wake vortices shedding in the wake with their strengths being proportional to the time rate of change of circulation about the body wing [1-8].
Another one states that the vortex strength of the wake is shed from the trailing edge remains unchanged. This implies that the vortex strength at a position downstream is equal to the vortex strength forward at the previous time step [9-12]. The formulations may cause an arrangement of the reduced-order modeling (ROM) without static correction technique.

2. Unsteady vortex lattice method

In this study, unsteady aerodynamic analysis of an aircraft wing can be modeled with vortex ring method [13]. The unsteady aerodynamic is based on potential flow, which is called UVLM. In general, the aircraft wing with airfoil can be modeled as a lifting surface. A simple lifting surface without camber is represented with a plate like wing as shown in figure 1.

The model of UVLM consists of vortex rings on the body and wake, while their vortex strengths are denoted by \( \Gamma_i \) and \( \Gamma_{wi,i} \), respectively. All panels have chordwise length \( \Delta x \) and spanwise width \( \Delta y \). Aircraft fly with velocity \( V \) and the time increment is \( \Delta t \) which means \( \Delta x = V \Delta t \). According to the boundary condition that there is no flow normal to the lifting surface and the velocity induced by those vortex rings, the UVLM equation is of the matrix form:

\[
A \Gamma^{n+1} + B \Gamma^n = w^{n+1}
\]  

where \( w \) is the known downwash and A and B are known sparse matrices.

The introduction has revealed two models of vortex wake that lead to an idea of the ROM without static correction as follows.

2.1. First model

The model of tailing edge wake can be considered as unsteady vortex which is shed in the wake. Its strength is proportional to the time rate of change of circulation about the wing. The strength of the first vortex point in the wake at the \( n + 1 \) time level is given by

\[
\Gamma_{W,i+1}^n = - \sum_{j=1}^{M} (\Gamma_j^{n+1} + \Gamma_j^n)
\]

where \( M \) is the number of vortex elements on the lifting surface.

Once the vortex strength has been shed into the wake, it is described numerically by

\[
\Gamma_{wi,i}^{n+1} = \Gamma_{wi,i}^n
\]

Finally, because the vortex sheet is infinitely long, but the computational model has a finite length wake, special treatment is required at the last vortex element. Otherwise, the starting vortex would disappear abruptly when it reaches the end of the computational wake producing a discontinuous change in the induced wash at the airfoil. To alleviate this difficulty, the convective vortices, upon reaching the end of the wake, are allowed to dissipate smoothly using the following relationship:

\[
\Gamma_i^{n+1} = \Gamma_i^N + \alpha \Gamma_i^{n+1}, i = N
\]

where \( \alpha \) is a relaxation factor. For the plate like wing problem, the relaxation factor from [1] is used as \( \alpha = 0.5 \).

From the induced vortex on body and wake equation (2-4), the UVLM equation can be expressed as in equation (1).
2.2. Second model
The second one states that the vortex strength of the wake shed from the trailing edge remains unchanged. This implies that the vortex strength at a position downstream is equal to the vortex strength forward at the previous time step. This means that the strengths of the vortex rings at the trailing edge at time step \( n \) are equal to their corresponding wake vortex ring in the first row at time step \( n + 1 \) and similarly for the strength of the wake vortex rings in following row.

\[
\Gamma_{\text{w},i+1}^{n+1} = \Gamma_i^n \quad \text{(5)}
\]

3. ROM
3.1. ROM without static correction technique
From the equation (1) with the 1st wake model, by using the technique in [3], ROM without static correction can be obtained as follows. Due to matrices \( B_{11} \) and \( B_{12} \) are 0, the UVLM equation can be formulated as

\[
\begin{align*}
[A_{22} - A_{21}A_{11}^{-1}A_{12}]\Gamma_{\text{w}}^{n+1} + [B_{22} - B_{21}A_{11}^{-1}A_{12}]\Gamma_{\text{w}}^{n} &= -A_{21}A_{11}^{-1}W_b^{n+1} - B_{21}A_{11}^{-1}W_b^{n} \\
A_{\text{new}} &= A_{22} - A_{21}A_{11}^{-1}A_{12} \\
B_{\text{new}} &= B_{22} - B_{21}A_{11}^{-1}A_{12} \\
W_{\text{new}} &= -A_{21}A_{11}^{-1}W_b^{n+1} - B_{21}A_{11}^{-1}W_b^{n} \\
A_{\text{new}}\Gamma_{\text{w}}^{n+1} + B_{\text{new}}\Gamma_{\text{w}}^{n} &= W_{\text{new}}^{n+1}
\end{align*}
\quad \text{(6)}
\]

(7), (8), (9), (10)

Equation (10) is based purely on \( \Gamma_{\text{w}} \), in which its corresponding eigenvalues will never be zero. The new eigensystem can lead to accurate ROM without using a static correction technique. The drawback of this equation may cause inefficient computation due to the need of inverse of matrices solved in equation (7-9).
3.2. New ROM without static correction technique

To increase efficiency in computation of ROM without static correction technique, the new form of UVLM is proposed. From the equation (1) with the 2\textsuperscript{nd} wake model, the new UVLM for ROM can be performed by multiplying equation (6) with $A_{11}$ leading to

$$[A_{11}A_{22} - A_{21}A_{12}]\Gamma_{w}^{n+1} + [A_{11}B_{22} - B_{21}A_{12}]\Gamma_{w}^{n} = -A_{21}W_{b}^{n+1} - B_{21}W_{b}^{n}. \tag{11}$$

Due to matrix $A_{21}$ being zero, the new UVLM can be formulated as follows:

$$[A_{11}A_{22}]\Gamma_{w}^{n+1} + [A_{11}B_{22} - B_{21}A_{12}]\Gamma_{w}^{n} = -B_{21}W_{b}^{n} \tag{12}$$

where

$$A_{\text{new}} = A_{11}A_{22},$$

$$B_{\text{new}} = A_{11}B_{22} - B_{21}A_{12},$$

$$w_{\text{new}} = -B_{21}W_{b}^{n}.$$

This method can be calculated faster than the previous technique because there is no matrix inversion required but the wing elements and the wake vortex elements must be equal so that the matrix $A_{11}$ and $B_{22}$ can be operated.

4. Results and Discussion

4.1. Eigenanalysis

To study the performance of the proposed technique, the eigenanalysis of the conventional method, the method based on $\Gamma_{w}$, and the present method are compared. The eigenvalues of the plate like wing for the conventional UVLM with the 1\textsuperscript{st} model of wake are shown in figure 2.

![Figure 2. Eigenvalues of the plate like wing for the conventional UVLM with 1\textsuperscript{st} model of wake.](image-url)
Figure 3. Eigenvalues of vortex lattice model of plate like wing - o conventional method + method based on $\Gamma_w$.

Figure 4. Eigenvalues of the plate like wing for the conventional UVLM with 2\textsuperscript{nd} model of wake.

The eigenvalues of plate like wing using equation (11) and 1\textsuperscript{st} model of wake are shown in figure 3. Note that the nonzero eigenvalues of the conventional method are removed. The eigenvalues of the plate like wing for the conventional UVLM with the 2\textsuperscript{nd} model of wake are shown in figure 4, while the conventional UVLM with 1\textsuperscript{st} model and 2\textsuperscript{nd} model of wake are shown in figure 5. The comparative results show only a few eigenvalues are different between UVLM with 1\textsuperscript{st} and 2\textsuperscript{nd} wake models. The comparison of the UVLM with 2\textsuperscript{nd} model of wake and new ROM is shown in figure 6. The results show the new ROM can remove zero eigenvalues without using static correction technique like the work by
[3], but the new equation requires no matrix inversion, which is expected to reduce the computational time and increase performance in computation. The result of the computational time will be present in the next section.

![Eigenvalues of the plate like wing for the conventional UVLM with 1st model of wake (conventional method) and + 2nd model of wake.](image1)

**Figure 5.** Eigenvalues of the plate like wing for the conventional UVLM with 1st model of wake (conventional method) and + 2nd model of wake.

![Eigenvalues of the plate like wing for the conventional UVLM with 2nd model of wake and the new ROM.](image2)

**Figure 6.** Eigenvalues of the plate like wing for the conventional UVLM with 2nd model of wake and the new ROM.

4.2. **Computational Time Comparison between Conventional and New-ROM**

The comparison of the computational time difference between the method based on $\Gamma_w$ and New-ROM is shown in table 1. The results lead to the conclusion that the new technique can sufficiently reduce computation time and help to increase the performance in computation in cases of the design of aircraft wings with using an optimization technique especially for the method based on evolutionary algorithms (EAs). For example, aerodynamic design of aircraft member using EAs with 100 populations and run
for 500 times, it needs 50,000 times of activating UVLM analyses. It means, when performing with the new ROM, computational time can be reduced for several hours.

Table 1. The Computational Time between Conventional and New-ROM Method.

| Method                  | N-Chord | N-Span | N-Wake | Time (s) |
|-------------------------|---------|--------|--------|----------|
| Method based on $\Gamma_w$ | 35      | 10     | 35     | 3.40554  |
| New-ROM                 | 35      | 10     | 35     | 3.343399 |
| Method based on $\Gamma_w$ | 30      | 30     | 30     | 26.547974|
| New-ROM                 | 30      | 30     | 30     | 25.744878|
| Method based on $\Gamma_w$ | 45      | 20     | 45     | 26.632752|
| New-ROM                 | 45      | 20     | 45     | 25.827317|
| Method based on $\Gamma_w$ | 50      | 25     | 50     | 59.779007|
| New-ROM                 | 50      | 25     | 50     | 58.986076|

5. Conclusions
This research presents a new reduced-order model for unsteady flow without static correction technique. This method can construct the eigensystem in the new form without matrix inversion, which can remove zero eigenvalues from the eigensystem. Furthermore, the new method can reduce computation time when compared with the conventional ROM without static correction. In conclusion, the new ROM can be considered as an alternative technique to perform the reduced-order models of unsteady flow. Future work, the present method is extended to design of a high-lift mechanism, which is combination of aerodynamics design and mechanism motion generation synthesis.

Acknowledgments
The authors are grateful for financial support provided by King Mongkut's Institute of Technology Ladkrabang and the National Research Council Thailand.

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