Emerging Paradigms of Neural Network Pruning

Huan Wang∗, Can Qin, Yulun Zhang and Yun Fu
Northeastern University, Boston, MA, USA
{wang.huan, qin.ca}@northeastern.edu, yulun100@gmail.com, yunfu@ece.neu.edu

Abstract

Over-parameterization of neural networks benefits the optimization and generalization yet brings cost in practice. Pruning is adopted as a post-processing solution to this problem, which aims to remove unnecessary parameters in a neural network with little performance compromised. It has been broadly believed the resulted sparse neural network cannot be trained from scratch to comparable accuracy. However, several recent works (e.g., [Frankle and Carbin, 2019a]) challenge this belief by discovering random sparse networks which can be trained to match the performance with their dense counterpart. This new pruning paradigm later inspires more new methods of pruning at initialization. In spite of the encouraging progress, how to coordinate these new pruning fashions with the traditional pruning has not been explored yet. This survey seeks to bridge the gap by proposing a general pruning framework so that the emerging pruning paradigms can be accommodated well with the traditional one. With it, we systematically reflect the major differences and new insights brought by these new pruning fashions, with representative works discussed at length. Finally, we summarize the open questions as worthy future directions.

1 Introduction

Efficient neural network design has always been non-trivial [LeCun et al., 2015; Schmidhuber, 2015]. The conventional wisdom from learning theory suggests that the best generalization comes from a good trade-off between sample size and model complexity [Kearns et al., 1994; Vapnik, 2013], which implies we should not employ too large neural network models in case of over-fitting. However, it is hard to know in practice which size of neural networks fits best. Therein, we tend to adopt redundant models so as to have enough expressivity for the real-world problems. In the deep neural network era, this principle is much more pronounced not only because we are handling more complex problems, but also because over-parameterized networks are observed practically easier to optimize (with proper regularization) than the compact models [Krizhevsky et al., 2012; Simonyan and Zisserman, 2015; He et al., 2016; Huang et al., 2017]. Besides, several theoretical works [Soltanolkotabi et al., 2018; Allen-Zhu et al., 2019; Zou et al., 2020] also suggest that over-parameterization matters for successful optimization and generalization of deep neural networks.

However, over-parameterization brings cost in either model deployment or training. To palliate the problem, neural network pruning is proposed as a post-processing scheme to remove the unnecessary connections or neurons in a pre-trained neural network without seriously compromising its performance. The resulted sparse neural network can benefit many ways such as offline storage, memory footprint, and computation. This pruning as post-processing paradigm has been practiced for a long time with tons of algorithms having been proposed (an outdated survey [Reed, 1993], recent surveys of pruning alone [Gale et al., 2019; Blalock et al., 2020] or as a sub-topic under the umbrella of model compression and acceleration: [Sze et al., 2017; Cheng et al., 2018a; Cheng et al., 2018b; Deng et al., 2020]). Most of them focus on how to select the unnecessary weights (i.e., the pruning criterion problem; see Sec. 3.3 for details) to strike a better sparsity-performance trade-off, few questioning the authenticity of the paradigm itself. One broadly-accepted reason has been that, training the sparse network from scratch underperforms the “pruning as post-processing” fashion, until recently, lottery ticket hypothesis (LTH) [Frankle and Carbin, 2019a] successfully found sparse networks which can be trained from scratch to an accuracy comparable to their dense counterpart (if proper initialization values used). It opens new doors to training an efficient sparse neural network with possibly less cost. Many follow-up works [Morcos et al., 2019; Zhou et al., 2019; Frankle et al., 2020a; Savarese et al., 2020; Wang et al., 2020; Ramamujan et al., 2020; Evci et al., 2020; Frankle et al., 2021] advance the idea and keep challenging the conventional wisdom on neural network pruning.

Despite these new fashions of pruning, to our best knowledge, there is no survey paper to cover them in depth, and more importantly, unify them under a general framework with the traditional paradigm. The emerging pruning fashions have largely changed the traditional pipeline (Sec. 3) and beliefs about pruning, nevertheless, why they can happen and what they imply are almost uncharted. This paper is
Paradigm | Mask src | Weight value src | Train subnet?
---|---|---|---
TraP | Model $K$ | Model $K$ | Yes
LTH | Model $K$ | Model 0 | Yes
LTH-R | Model $K$ | Model $t$ | Yes
PaI | Model 0 | Model 0 | Depends

Table 1: Major difference comparison of the pruning paradigms investigated in this paper: traditional pruning (TraP), lottery ticket hypothesis (LTH), lottery ticket hypothesis with rewinding (LTH-R), and pruning at initialization (PaI). “src” stands for “source”. See Fig. 1 for a more concrete training process illustration.

pruning. However, several new pruning schemes ask for models at different iterations (e.g., LTH needs the model at iteration 0 [Frankle and Carbin, 2019a] or an early iteration $t$ [Frankle et al., 2020a]). To accommodate these new cases, we allow a pruning algorithm to have access to the whole model sequence (which we can easily cache during the model training) instead of only the last single model checkpoint.

Then, pruning can be formulated as a function that takes the model sequence as input and outputs a pruned model $w'$. We need two pieces of information for any neural network model to specify it: its topology and the corresponding parameter values. In the case of a sparse network, the sparse topology is typically realized by a mask tensor (denoted by $m$, with the same shape as $w$), which can be modeled as a function, namely, $m = F_1(w^{(k_1)}; D)$, where $D$ stands for the training dataset, that is, the function can utilize the data available to help decide the masks. Meanwhile, the weights in the pruned model may be adjusted from their original values to mitigate the incurred damage [Hassibi and Stork, 1993; Wang et al., 2019; Wang et al., 2019c; Wang et al., 2021]. This step can be modeled as another function $F_2(w^{(k_2)}; D)$.

Together, pruning can be formulated as

$$w' = F_1(w^{(k_1)}; D) \odot F_2(w^{(k_2)}; D),$$

where $\odot$ represents the Hadamard (element-wise) product.

For the sake of easy understanding, we use the term “traditional pruning” to refer to the conventional fashion of pruning. However, it is helpful to bear in mind that there is no clear concept or time boundary to define which method is “traditional” or “non-traditional”. The disentanglement of pruning as two sub-problems: obtaining the masks and the weight values. Based on this, it is clear to see how the pruning paradigms in this paper differ from one another: traditional pruning (TraP), lottery ticket hypothesis (LTH), lottery ticket hypothesis with rewinding (LTH-R), and pruning at initialization (PaI). $F_1(\cdot)$ represents the function to find masks and $F_2(\cdot)$ represents the function to find new initial values (see Eq. (2)). The dotted line indicates the line (or weight) is pruned (i.e., set to zero). The $\odot$ mark represents the Hadamard (element-wise) product. Different colors stand for different values (this figure is best viewed in color).

Figure 1: Illustration of the proposed framework of pruning (Sec. 2). We disentangle pruning as two sub-problems: obtaining the masks and the weight values. Based on this, it is clear to see how the pruning paradigms in this paper differ from one another: traditional pruning (TraP), lottery ticket hypothesis (LTH), lottery ticket hypothesis with rewinding (LTH-R), and pruning at initialization (PaI). $F_1(\cdot)$ represents the function to find masks and $F_2(\cdot)$ represents the function to find new initial values (see Eq. (2)). The dotted line indicates the line (or weight) is pruned (i.e., set to zero). The $\odot$ mark represents the Hadamard (element-wise) product. Different colors stand for different values (this figure is best viewed in color).

2 A General Framework of Pruning

The training of neural networks (parameterized by a weight vector $w$) via mini-batch stochastic gradient descent (SGD) [Robbins and Monro, 1951; Bottou, 2010] essentially produces a model sequence that finally converges to a model with desired performance:

$$\{ w^{(0)}, w^{(1)}, \ldots, w^{(k)}, \ldots, w^{(K)} \},$$

where $k$ denotes the $k$-th training iteration and $K$ is the total number of training iterations.

Traditionally, we only need the last model checkpoint ($k = K$) as the base model (i.e., the original unpruned model) for

Note that, the input models for $F_1(\cdot)$ and $F_2(\cdot)$ can be from different iterations, hence we use $k_1$ and $k_2$ to differentiate them. By this definition, the pruning cases discussed in this paper can be specified as follows and in Tab. 1:

- **Traditional pruning (TraP)**: $k_1 = k_2 = K$;
- **Lottery ticket hypothesis (LTH)** [Frankle and Carbin, 2019a]: $k_1 = K$, $k_2 = 0$ and $F_2 = I$ (denoting the identity function); lottery ticket hypothesis with rewinding (LTH-R) [Frankle et al., 2020a]: $k_1 = K$, $k_2 = t$ and $F_2 = I$;
- **Pruning at Initialization (PaI)** [Lee et al., 2019; Wang et al., 2020; Zhou et al., 2019; Ramanujan et al., 2020]: $k_1 = k_2 = 0$.

A figurative illustration for the differences above is shown in Fig. 1. In this illustration, we can see clearly where different pruning paradigms obtain their weight values and masks for the initial sparse model.

**Paradigm shifts and questions of interest.** As we see above, the biggest paradigm shift from the traditional paradigm to the new ones (LTH/LTH-R and PaI) lies in using which base...
model (or models) to obtain the masks and weight values, namely, using which model (or models) as the input for $F_1$ and $F_2$ in the pruning formulation (Eq. (2)):

- Traditional pruning methods remove parameters from a trained model\footnote{Some pruning methods start from a randomly initialized model which is then trained with a sparsity-inducing penalty term (i.e., the regularization-based pruning). This paradigm is deemed not pruning a pre-trained model by some works [Reed, 1993]. We do not take this view since the “weight removing” operation still happens to a trained model rather than the randomly initialized model.}. The common belief behind this is that the remaining parameters possess knowledge learnt by the original redundant model. Inheriting it is better than starting over, which is also empirically justified by many works [Han et al., 2015; Han et al., 2016; Frankle and Carbin, 2019a] and thus becomes a common practice in the pruning community. In stark contrast, LTH/LTH-R and PaI inherit the parameter values from the beginning ($k = 0$) or an early iteration ($k = t$). It is tempting to ask: At such an early phase of neural network training, does the model already possess enough knowledge (to provide base models for subsequent pruning)? Or more fundamentally, is neural network training essentially to learn the knowledge from nothing or to reveal the knowledge the model already has? Conventional wisdom favors the former, while the emerging pruning paradigms (especially [Ramanujan et al., 2020; Malach et al., 2020]) suggest the latter (see Sec. 4 and 5 for more detailed discussions).

- Traditional pruning methods produce the masks and weight values from the same model. In most pruning methods, they are determined by the same algorithm – when the masks are determined, the values for the remaining parameters are also determined. Pal also works this way, while LTH/LTH-R works differently. It obtains the masks from model $k = K$ and they are applied to another model $k = 0$ or $t$. Namely, LTH/LTH-R firstly separates the process to obtain mask from that to obtain parameter values. This combination seems unusual. Why or when it is feasible is especially interesting. Intuitively, it should have something to do with the training dynamics [Jacot et al., 2018] and loss landscapes [Li et al., 2018] of the base model. Extremely, if the base model barely changes during training, model $k = K$ will be very close to model $k = 0$ or $t$, then LTH degrades to TP; but if the model changes dramatically during training, whether LTH still holds true remains open now. Sec. 4 will discuss these questions in depth. Apart from the inputs of $F_1$ and $F_2$, the function $F_1$ and $F_2$ themselves can also change. In this regard, the new pruning methods adopt many techniques the same as (or similar to) those in the traditional pruning approaches. For example, LTH/LTH-R employs iterative magnitude pruning (IMP), which is probably the most prevailing pruning technique since 1980s [Reed, 1993; Han et al., 2015; Li et al., 2017; Gale et al., 2019]; the differential mask technique in PaI [Zhou et al., 2019; Ramanujan et al., 2020] also appears in the traditional pruning context [Huang and Wang, 2018; Junjie et al., 2020; Kusupati et al., 2020; Kang and Han, 2020]. We will also discuss them at length in Sec. 4 and 5.

### 3 Classic Topics in Pruning

**Background:** the traditional three-step pruning pipeline.

Conventionally, a pruning algorithm comprises three steps from scratch: pretraining, pruning, and finetuning.

- The first step is to train a model (which is usually deemed redundant) to its convergence; the second step is to remove the unnecessary connections or neurons in it; the third step is to retrain the pruned model to regain performance since the second step typically degrades the model. Notably, when the new pruning scenarios emerge, all the three steps are changed to some degree. For example, the root idea of LTH [Frankle and Carbin, 2019a] calls into question the necessity of the first step (although finding the winning ticket still requires the first step in [Frankle and Carbin, 2019a]); some PaI methods [Zhou et al., 2019; Ramanujan et al., 2020] completely discard the third step.

Within this pruning pipeline, there are mainly four critical questions we need to ask when pruning a specific model in practice: (1) what to prune, (2) how many (connections or neurons) to prune, (3) which to prune, and (4) how to prune exactly, corresponding to four classic topics in pruning, (1) pruning structure, (2) pruning ratio, (3) pruning criterion, and (4) pruning schedule, respectively. They are discussed in detail as follows.

#### 3.1 Pruning Structure

Weights can be pruned in some pattern. The shape of the pattern, named pruning structure, determines the basic pruning element of a pruning algorithm. The smallest structure, of course, is a single weight element, i.e., no structure at all. Therefore, this kind of pruning is called unstructured pruning. The pruning pattern larger than a single weight element can be called structured pruning in general. Structure is introduced typically out of practical interest (e.g., acceleration).

For unstructured pruning, the locations of pruned weights usually are random, which is hard to leverage for acceleration. Although there are plenty of off-the-shelf software libraries to accelerate sparse matrices, previous works [Wen et al., 2016; Wang et al., 2018] have shown the practical speedup is limited. In contrast, for structured pruning, the zero locations are regular, rendering it much easier to achieve considerable acceleration on even the common hardware. Within the family of structured pruning, there are still many options for granularity (e.g., we can choose to prune a whole filter away or only a channel of a filter). In a narrow context, structured pruning means filter pruning in the literature. We recommend readers to refer to [Mao et al., 2017] for more details. As a rule of thumb, for less storage and communication cost, we should focus on more unstructured pruning; for acceleration and low latency, we should focus on more structured pruning.

In the scope of this paper, for the two emerging pruning paradigms (LTH/LTH-R and PaI), most papers (if not all) explore unstructured pruning by our investigation.
3.2 Pruning Ratio

Pruning ratios indicate how many weights to remove. In general, there are two ways to adjust pruning ratios.

The first is to pre-define them. Namely, we know exactly how many parameters will be pruned before the algorithm actually runs. This scheme can be further specified into two sub-schemes. One is to set a global pruning ratio (i.e., how many weights will be pruned for the whole network), such as [Liu et al., 2017]; the other is to set layer-wise pruning ratios. The latter is more prevailing.

The second is to adjust the pruning ratio by other means. This way mostly appears in the regularization-based pruning methods, which remove weights by firstly pushing them towards zero via penalty terms. A larger regularization factor typically leads to more sparsity, i.e., a larger pruning ratio. However, how to set a proper factor to achieve a specific ratio demands much engineering tuning. Several methods have been proposed to improve this [Wang et al., 2019c; Wang et al., 2021]. The pruning ratios are usually pre-specified. Recent years also see some works automatically search the optimal layer-wise pruning ratio [He et al., 2018], yet no consensus has been reached about which way is better.

In the scope of this work, the new pruning paradigms do not show much new invention in this subject. Most of them simply adopt pre-defined pruning ratios.

3.3 Pruning Criterion

With pruning structure and ratios decided, the next question is to select which weights to prune. The selection hinges on certain pruning criteria, which is (arguably) the most critical problem in pruning. Thus many existing pruning works revolve around this topic. The most simple criterion is weight magnitude, for unstructured pruning [Han et al., 2015; Han et al., 2016]. For structured pruning, it equivalently means the Frobenius norm (typically $L_1$-norm or $L_2$-norm) of a weight group vector [Li et al., 2017]. Because of its simplicity and fair enough performance, it is the most prevailing criterion in pruning now. The topic of the lottery ticket hypothesis in the next section also employs this criterion.

Albeit the abundant exploration in this topic, there are no criteria that prove to be significantly better than the others, as far as we know. Since this topic is already well discussed in the surveys of traditional pruning scenarios (see [Reed, 1993; Gale et al., 2019; Blalock et al., 2020]), we will not cover it in depth here. Yet one point worth mention is that, the most popular idea to propose pruning criteria with a sound theoretical basis (if any) is to select the weights that induce the least loss increase. This idea and its variant have been practiced for a long time in the traditional pruning context [LeCun et al., 1990; Hassibi and Stork, 1993; Wang et al., 2019a; Molchanov et al., 2017; Molchanov et al., 2019; Singh and Alistarh, 2020]. We still see its crystallization in the new fashions, such as Pal methods SNIP [Lee et al., 2019] and GraSP [Wang et al., 2020], which we will discuss later.

3.4 Pruning Schedule

With all three aspects determined, we still need to specify the schedule of pruning. There are three typical choices [Wang et al., 2019b]. (1) One-shot: network sparsity (defined by the ratio of zeroed weights in a network) goes from 0 to a target number in a single step, then finetune. (2) Progressive: network sparsity goes from 0 to a target number gradually, typically along with network training; then finetune. (3) Iterative: network sparsity goes from 0 to an intermediate target number, then finetune; then repeat the process until the target sparsity is achieved. Note, both (2) and (3) are characterized by pruning interleaved with network training. Therefore, there is no fundamental boundary between the two. Some works thus use the two terms interchangeably. One consensus is that, progressive and iterative pruning outperform the one-shot counterpart when pruning the same number of weights (or even exactly the same weights [Wang et al., 2021]) because they allow more time for the network to recover.

In the scope of this paper, LTH [Frankle and Carbin, 2019b] adopts the iterative magnitude pruning (IMP). SNIP [Lee et al., 2019] and GraSP [Wang et al., 2020] is an one-shot pruning method based on an established importance criterion. For other Pal methods that requires training to select the subnet (e.g., [Ramanujan et al., 2020]), the sparsity is kept the same when optimizing the masks.

4 Lottery Ticket Hypothesis (LTH)

With the terminology defined in Sec. 2, here is the pipeline proposed in LTH [Frankle and Carbin, 2019b]: First, a randomly initialized network $w(0)$ is trained to its convergence ($w(K)$); second, apply a certain pruning method to obtain the mask tensor $m$, in which the locations of ones defines the topology of the subnet; third, apply the mask $m$ to the initial network $w(0)$ to obtain a subnet (i.e., $w(0) \odot m$) and train the subnet to convergence. [Frankle and Carbin, 2019b] surprisingly found the subnet can achieve comparable (or even better occasionally) accuracy to the dense network. Note, the size (number of non-zero parameters) of the subnet is non-trivially smaller (1/10 or less [Frankle and Carbin, 2019b]) than the dense network. This paper attracts broad interest because it was believed that a non-trivially sparse network cannot be trained to comparable accuracy to its dense counterpart. [Frankle and Carbin, 2019b] thus propose the lottery ticket hypothesis: Every randomly initialized network has a (non-trivially smaller) subnet which can be trained in isolation to match the accuracy of the original network.

Since its debut, many follow-up works have appeared, focusing on four subjects: (1) The central step in the pipeline of LTH is to obtain the mask, obviously. In [Frankle and Carbin, 2019b], the mask is obtained by pruning the converged dense model ($w(K)$), namely, the dense model has to be trained first. How to save this step and find the mask directly from the initial network ($w(0)$) is of great theoretical and practical interest. (2) Explanation of LTH (e.g., why can it happen on earth?), such as [Zhou et al., 2019]. (3) Large-scale LTH. LTH was originally validated on small scaled datasets (MNIST [LeCun et al., 1998] and CIFAR-10 [Krizhevsky, 2009]). Some follow-ups try to make it scalable to large datasets (especially, ImageNet [Deng et al., 2009]). (4) Applications and extensions of LTH on other domains (such as natural language processing and speech recognition tasks).

This paper mainly focuses on the pruning in new cases.
Thus we focus on the first three aspects here. For the first and second aspects, they directly inspire the following works of pruning at initialization, which will be discussed in depth in the next section (Sec. 5), so here we mainly discuss the third aspect: making LTH scalable to large networks and datasets. The major breakthrough is made by [Frankle et al., 2020a]. They propose a modification to the original LTH pipeline: With the masks, apply them to the model at iteration \( k \) instead of 0 (thus we call it lottery ticket hypothesis with rewinding, shorted as LTH-R). The change is motivated by the observation of linear mode connectivity [Frankle et al., 2020a]. With LTH-R, the selected subnet can be trained to full accuracy on large datasets now. Apparently, the difference happens in the early stage of training. This subject is more about the understanding of neural network training rather than pruning, thus out of the scope of this paper. Interested readers are recommended to see [Frankle et al., 2020b] for detailed discussions.

Discussion. There are multiple perspectives to look at the phenomenon of LTH. One way, suggested by its name, is that a good initialization (it performs so well that we call it a lottery ticket) really matters. Nevertheless, we would like to present another perspective in the language of pruning.

As shown in Fig. 1, LTH borrows the weight values from the model at iter 0 while decides the masks from the model at iter \( K \) or \( k \) in the case of LTH-R. Either way, the stark difference from the traditional way is that the masks and weights values are from different models. At first sight, this combination appears mismatched and seems unreasonable, while empirically verified in many cases. In this sense, LTH/LTH-R shows for the first time that the masks and weight values in pruning can actually be decoupled.

Although there is no theory yet, as far as we know, to explain the feasibility of this mismatched combination, the reason is probably related to the training dynamics [Jacot et al., 2018] and loss landscapes [Li et al., 2018] of neural networks. As an extreme case, if the network is barely updated during the pretraining (Fig. 1), then the model at different iterations will be rather close to one another. Then it probably does not matter which iteration we employ to obtain the masks and weight values. One empirical evidence for this is that, in [Liu et al., 2019], the authors tried to reproduce LTH using different initial learning rate (0.01 vs. 0.1) and found it can only be reproduced with the smaller initial learning rate (0.01). The difference between a large learning rate and the small one lies in how much the network is updated. From this, we can see how training dynamics can affect the validation of LTH. Besides, because of different architectures, the loss landscapes can be starkly different (e.g., BN [Ioffe and Szegedy, 2015] and residual make the loss landscapes flatter in deep networks [Li et al., 2018]). Conceivably, LTH will also hold to different degrees on these networks.

5 Pruning at Initialization (PaI)

There are primarily two motivations behind the rise of PaI. (1) Practically, PaI can save the training cost of the full model, as compared to the traditional pruning methods, which mostly focus on improving inference efficiency. (2) PaI per se is an interesting topic for a better theoretical understanding of either pruning or neural networks – it is interesting to see if pruning can still work (as effectively) if the pretrained model, which is typically demanded in traditional pruning, is absent.

In traditional pruning, the pretrained model provides meaningful weights, based on which certain criteria can pick a subset of them and fix the corresponding sparse topology hence. Differently, in PaI, the sparse topology can also be optimized (while the sparsity level is fixed). In some extreme cases, the topology even is the only object of training, with no weight values updated (such as [Ramanujan et al., 2020]).

SNIP [Lee et al., 2019] is the first PaI work, which proposes a saliency criterion named connectivity sensitivity to select weights based on a straightforward idea of loss preservation, i.e., removing the weights that whose absence leads to the least loss change,

\[
S_j(w_j) = \frac{|g_j(w; D)|}{\sum_{k=1}^m |g_k(w; D)|}, \quad g_j(w; D) = \frac{\partial L(m \odot w; D)}{\partial m_j} \bigg|_{m=1},
\]

where \( D \) denotes the dataset; \( m_j \) stands for the \( j \)-th parameter; \( g_j \) is the derivative of the loss function \( L \) w.r.t. \( m_j \), which is an infinitesimal approximation of the loss change when removing the weight \( w_j \). After initialization, each can be assigned a score with the above saliency criterion, then remove the last-\( p \) fraction (\( p \) is the desired pruning ratio) parameters for subsequent training. The proposed criterion is similar to [Mozer and Smolensky, 1989; Karnin, 1990] albeit the difference that they adopt the absolute sensitivity so as to avoid using the pretrained model.

In spite of the novel idea, the rationale behind Eq. (3) is still elusive. After all, the network is randomly initialized. Preserving a random loss value does not make much sense intuitively. As such, [Wang et al., 2020] argue that it is the training dynamics rather than the loss value itself that matters more at the beginning of training. Based on this motivation, [Wang et al., 2020] propose gradient signal preservation (GraSP) in contrast to the previous loss preservation,

\[
S(\delta) = 2\delta^\top Hg + \mathcal{O}(|\delta|^2),
\]

where \( g \) represents the gradient vector; \( H \) is Hessian; \( \delta \) denotes the weight perturbation vector. The pruning pipeline of GraSP is similar to SNIP, just using a different criterion to calculate the score for each weight. With the scores, select the last-\( p \) fraction (\( p \) is the desired pruning ratio) parameters to remove. GraSP exploits the second-order gradients, which can be approximated with a maintainable cost.

Concurrently to GraSP [Wang et al., 2020], [Lee et al., 2020] seek to explain the feasibility of SNIP through the lens of signal propagation. They empirically found pruning damages the dynamical isometry [Saxe et al., 2014] of neural networks and thus propose an approximated orthogonal initialization method tailored to sparse networks to repair it. The repaired isometry improves signal propagation, leading to a better performance of pruning at initialization.

Sparse Topology Optimization. SNIP and GraSP still need to optimize the values of the subnet after picking it out of...
the dense model. Some other works have found another optimization scheme: instead of optimizing the weight values, optimize the network topology. Namely, when a network is randomly initialized, all the values of each connection are fixed. The goal is to find a subnet from the dense network without further training the subnet. This line of works is firstly pioneered by [Zhou et al., 2019] where they try to understand the mysteries of the lottery ticket hypothesis. They find the subnet picked by LTH can achieve non-trivial accuracies already (without training). This implies that, although the original full network is randomly initialized, the chosen subnet is not really random. The subnet picking process itself serves as a kind of training. Therefore, [Zhou et al., 2019] propose the notion of supermasks (or masking as training) along with a proposed algorithm to optimize the masks in order to find better supermasks.

Despite its effectiveness, the method in [Zhou et al., 2019] is only evaluated on small-scale datasets (MNIST and CIFAR), far from practical use. Later, [Ramanujan et al., 2020] further advance this direction. They introduce a trainable score for each weight and update the score to minimize the loss function. In the whole process, weights are not updated. The method achieves quite strong performance on the ImageNet. For example, they manage to pick a subnet out of a random Wide ResNet50 [He et al., 2016]. The subnet is smaller than ResNet34 while delivers better top-1 accuracy than the trained ResNet34 on ImageNet.

The above are the breakthroughs in terms of specific algorithms. In terms of theoretical progress, [Malach et al., 2020] investigate the theoretical basis for the stronger LTH: A subnet in an over-parameterized neural network can be found with considerable accuracy even without training (namely, a different name for PaI without training weights).

6 Discussions and Open Problems

In this section, we seek to overlook the big picture of pruning and think over why these new fashions emerge, and more importantly, where they may lead in the future.

These new fashions of pruning arise from the reflection of the traditional three-step pruning pipeline, towards cheaper pruning in general. Especially, researchers try to get rid of the pretraining step. Pretraining used to provide the base parameters for the following pruning. For the emerging pruning fashions, both LTH and PaI share the point of not using a pretrained model to get the pruned parameter values. The fundamental difference between a pretrained model and a random model was believed to be that the former possesses knowledge while the latter does not. However, the efficacy of pruning at initialization seems to shake this conventional belief: In [Zhou et al., 2019], they found the subnet picked by the LTH method already shows non-trivial accuracy on MNIST and CIFAR-10. This shows, the random network already possesses knowledge, just covered up by useless weights. Therefore, the training of neural network may actually be a process to reveal the knowledge it already has rather than to learn the knowledge from nothing.

Aside from these breakthroughs at the mindset level, we actually see little progress in terms of specific new pruning techniques. For LTH, the adopted pruning method is simply the common magnitude pruning. For many PaI methods, we can find similar origins for the proposed or adopted techniques. For example, the connection sensitivity formula in SNIP [Lee et al., 2019] is similar to that in [Mozer and Smolensky, 1989; Karlin, 1990]. [Molchanov et al., 2017] also propose a first-order pruning criterion based on the same loss preservation idea. The trainable masks idea in [Ramanujan et al., 2020] is reminiscent of [Kang and Han, 2020]. In [Wortsman et al., 2020], the authors propose a training method to select different masks for different tasks based on a single fixed network, which is also similar to [Mallya et al., 2018] in general idea with differences (the latter focuses on pretrained models while the former uses random models).

As stated in Sec. 3, for the four primary aspects in pruning, the new fashions do not bring much change to the pruning ratio, criterion, and schedule. However, for pruning structure, it does have a significant impact. LTH was proposed for unstructured pruning in ICLR 2019, the same venue where another paper [Liu et al., 2019] reported they cannot validate the hypothesis on structured pruning (or filter pruning). As far as we know, to date, there is still no work that validates the hypothesis with structured pruning. The reason behind this mystery and the possible applications if the problem is solved remain an open problem in the field.

The idea of PaI is intriguing, however, in terms of performance, PaI methods still underperform the conventional pruning methods by an obvious margin. For example, according to the experiments in [Wang et al., 2020], with VGG19 [Simonyan and Zisserman, 2015] and ResNet32 [He et al., 2016] networks on CIFAR-10/100, both SNIP and GraSP are consistently outperformed across different sparsity levels by two traditional pruning methods (OBD [LeCun et al., 1990] and MLPrune [Zeng and Urtasun, 2019]), which are not even close to the state-of-the-art. Therefore, we can see that a pretrained model still lends a considerable advantage to the subsequent pruning, which also agrees with our intuition. In this sense, there is still a long road ahead before we can really eliminate the cost of pretraining in pruning.

7 Conclusion

This paper investigates neural network pruning paradigms within a proposed general framework, where the weight values and masks are treated separately. The general framework accommodates new pruning paradigms well. Two major emerging paradigms are covered in this paper, lottery ticket hypothesis and pruning at initialization. We systematically discuss their origins and the differences they have brought compared to the traditional pruning paradigm. Despite the new inspiring findings from these works, new questions follow in company. We discuss the major ones as worthy future directions. We hope this survey can help the community to understand pruning as well as neural networks better.

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