Hydrophone area-averaging correction factors in nonlinearly generated ultrasonic beams

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Abstract. The nonlinear propagation of an ultrasonic wave can be used to produce a wavefield rich in higher frequency components that is ideally suited to the calibration, or inter-calibration, of hydrophones. These techniques usually use a tone-burst signal, limiting the measurements to harmonics of the fundamental calibration frequency. Alternatively, using a short pulse enables calibration at a continuous spectrum of frequencies. Such a technique is used at PTB in conjunction with an optical measurement technique to calibrate devices. Experimental findings indicate that the area-averaging correction factor for a hydrophone in such a field demonstrates a complex behaviour, most notably varying periodically between frequencies that are harmonics of the centre frequency of the original pulse and frequencies that lie midway between these harmonics. The beam characteristics of such nonlinearly generated fields have been investigated using a finite difference solution to the nonlinear Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation for a focused field. The simulation results are used to calculate the hydrophone area-averaging correction factors for 0.2 mm and 0.5 mm devices. The results clearly demonstrate a number of significant features observed in the experimental investigations, including the variation with frequency, drive level and hydrophone element size. An explanation for these effects is also proposed.

1. Introduction

The propagation of a high amplitude ultrasound wave is a fundamentally nonlinear process that can result in waveform distortion with the resultant generation of harmonics of the frequencies initially present in the wave. For suitable combinations of pressure and propagation distance, this eventually leads to the formation of shocks within the waveform, where the pressure changes very rapidly with position or time. Such frequency-rich waveforms are very useful in ultrasonic metrology.

For example a primary standard for ultrasound in water in the frequency range 1 to 40 MHz can be implemented using a laser interferometer to measure the displacements of a membrane hydrophone in a nonlinearly distorted wave field [1]. In this way the hydrophone output can be related to the pressure amplitude via the particle displacement measured with the interferometer. The use of a nonlinearly distorted wavefield removes the need to use a number of individual transducers.

Calibrations can also be transferred from a standard hydrophone to secondary devices by inter-calibration in a nonlinearly distorted wavefield [2]. If a tone-burst drive is used, and a quasi-CW section of the resultant nonlinearly distorted waveform is analysed, then results are obtained at
harmonics of the source frequency. Calibration is then achieved at these discrete frequencies, with the frequency sampling determined by the fundamental frequency used (typically 1 MHz).

A range of developments are driving the need to perform hydrophone calibrations at higher ultrasonic frequencies (up to 100 MHz). This implies the use of a higher frequency transducer (about 5 MHz) as the source for the nonlinearly distorted wavefield. However, if a 5 MHz tone burst is used then the calibrations will only be obtained with a low frequency resolution due to the harmonic spacing of 5 MHz. An alternative approach, that has been used at Physikalisch-Technische Bundesanstalt (PTB) in Germany, is to use a very short acoustic pulse generated as the result of a pulsed discharge into a transducer (see figure 1) [3]. The initial pulse has a broadband spectrum centred on a peak frequency determined by the resonant frequency of the source. Consequently, the waveform generated by nonlinear propagation has a continuous spectrum extending up to high frequencies. The resulting spectrum does however show maxima at frequencies that are close to harmonics of the centre frequency of the source waveform, with dips in between at ‘intermediate’ frequencies.

A hydrophone will in general give an output that is related to the average pressure across its active area. If the acoustic field varies in amplitude (or phase) across this area then the hydrophone output will be different from that which would have been obtained with a point-like device. The ratio of the spatial average to axial pressure is termed the area-averaging factor. Since spatial averaging has the potential to affect calibration, it is important to know this factor. It will, in general, depend on both the device and the field.

While calibrating hydrophones in the nonlinear pulsed wavefields described, it was noted at PTB that the area-averaging factor varied significantly with frequency (see figure 2). This was calculated by using an optical system to measure the pressure amplitude across the area of the hydrophone element and numerically averaging the resultant values. In particular it was noted that the area-averaging factor varied periodically between the ‘harmonic’ and ‘intermediate’ frequencies.

![Figure 1. Example source waveform used in numerical model; this is based on that used for the experimental measurements performed at PTB in reference [3].](image1)

![Figure 2. Area-averaging factor for planar hydrophones of 0.2 and 0.5 mm diameter calculated from experimental measurements with an optical probe at PTB [3].](image2)
This paper investigates these phenomena by use of numerical modelling. In particular it aims to understand the factors that determine the sensitivity of the area-averaging factor to the pulse characteristics. The modelling was used to identify the causes of the phenomena observed rather than exactly reproduce the experimental situation.

2. Model
A finite difference code, known as the Bergen Code, was used to solve the nonlinear Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation and hence model the propagation of the finite amplitude beam generated by a focusing transducer [4]. The KZK equation is a parabolic approximation to the full nonlinear wave equation that consistently accounts for absorption, nonlinearity and diffraction in sound beams. The parabolic or paraxial approximation assumes that the energy propagates at small angles to the beam axis; this approximation is valid for acoustic sources that are many wavelengths across and not strongly focusing, and for field points that are not too far from the beam axis or too near the source plane [5]. The code used works entirely in the frequency domain, representing the waveform as a Fourier series. For the runs reported an axial step (Δz) of 0.005 mm and radial step (Δr) of 0.031 mm were used.

In order to model the propagation of a short pulse the fundamental frequency in the model was set to be 500 kHz, whereas the centre frequency of the pulse (fc) was 6.5 MHz. The initial pressure pulse was modelled by

\[ p(t) = \begin{cases} 
0, & 0 < t \leq t_s, \\
p_0 \exp\left(-t / \tau\right) \sin\left[2\pi f_c \left(t-t_s\right)\right], & t > t_s, 
\end{cases} \]

where for this work \( t_s = \tau = 0.2 \mu s \). The value of \( p_0 \) was chosen to give peak rarefractional pressures at the source of 300 kPa or 500 kPa for the results presented here.

The field of an ideal circular focusing transducer with radius 6.2 mm and geometrical focal length 56 mm was modelled; results are presented for a transverse plane at an axial distance \( z = 47.5 \) mm from the source where the peak rarefractional pressure was a maximum for the 500 kPa run.

In order to calculate the area-averaging effect the average pressure over the surface of a planar circular hydrophone, assumed perfectly aligned on-axis, was first calculated as a function of frequency, \( f \). This is given by

\[ p_{ave}(z, f) = \frac{1}{\pi b^2} \int_0^b p(z, r, f) 2\pi r dr, \]

where \( b \) is the hydrophone element radius and \( p(r, z, f) \) is the complex pressure distribution as a function of radial position, \( r \). The area-averaging factor (\( \alpha \)) is then given by:

\[ \alpha = \left| \frac{p_{ave}(z, f)}{p(z, 0, f)} \right|. \]

The calculations were performed for hydrophones with a diameter of 0.2 and 0.5 mm. The integral of equation (2) was numerically evaluated by cubic spline interpolation of the amplitude and phase radial profiles, followed by trapezoidal numerical integration.
3. Results

Figure 3(a) shows the calculated area-averaging factor for a relatively low initial pressure at the source of 300 kPa, as a function of frequency for the two hydrophone diameters. This shows that the averaging effect is much less significant for the smaller hydrophone, and only results in a small reduction relative to the on-axis pressure, reaching a minimum value of 0.94 by 100 MHz. In contrast, the 0.5 mm device shows a much more significant effect that increases dramatically with frequency such that \( \alpha \approx 0.47 \) by 100 MHz. This follows from the expected behaviour of nonlinearly propagating beams, with the higher harmonics having narrower beams. Figure 3(a) also shows periodic fluctuations in the area-averaging factor that are more pronounced for the larger hydrophone. Comparison with the pulse spectrum confirms that the minima in \( \alpha \) correspond to frequencies that are harmonics of the centre frequency, that is where \( f \approx Nf_c \) for integer \( N \). The maxima correspond to the intermediate frequencies where \( f \approx (N+1/2)f_c \).

In contrast figure 3(b) shows results for a significantly higher drive level of 500 kPa. For the 0.2 mm device the fluctuations are now much more pronounced, although the minima of the area-averaging factor are not significantly different in amplitude from the lower drive case. However, the peaks are much more pronounced and are visible across the whole of the frequency band. The higher amplitude of the oscillations is such that the area-averaging factor is greater than unity for some frequencies. This indicates that the average pressure over the hydrophone is larger than the pressure on axis, implying that the pressure increases with radial displacement from the axis.

These effects are clearly much more pronounced in Figure 3(b) for the larger hydrophone (0.5 mm diameter), which displays large periodic fluctuations in the area-averaging factor. The factor varies significantly in magnitude, ranging from 0.62 to over 1.24. It should be noted that the maximum values are probably not accurately reproduced due to the limited resolution of the model in the frequency domain (\( \Delta f = 500 \) kHz). It is interesting to note that the minimum \( \alpha \) values are actually larger for the high drive case than for the low drive case. This can be attributed to saturation effects resulting in a flattening of the beam (as has previously been observed during farfield calibration [6]).

![Figure 3](image-url)

**Figure 3.** Area-averaging factor for hydrophones of 0.2 and 0.5 mm diameter calculated from the numerical model at a range of 47.5 mm. Results are calculated for initial peak rarefractional pressures at the source of (a) 300 kPa and (b) 500 kPa.

In order to confirm the origins of the area-averaging factor fluctuations, the radial variation of pressure amplitude is plotted in figure 4 for two particular frequency components. In figure 4(a) the results are shown for the 300 kPa case, while in figure 4(b) the corresponding results for the higher drive level are presented. For the low drive case the pressure amplitude is a maximum on-axis for both a frequency of 39 MHz, the 6\(^{th}\) harmonic of the centre frequency, and 35.5 MHz, an intermediate frequency (half way between the 5\(^{th}\) and 6\(^{th}\) harmonics). Notably the two levels are not that dissimilar on axis and rapidly approach each other off axis. In contrast the corresponding curves for the higher drive case (figure 4(b)) are very different. In particular the curve for the intermediate frequency now...
has a minimum on axis, with the pressure amplitude increasing with radial displacement. This causes the area-averaging factor to be greater than unity. It is also noticeable that the pressure amplitude on axis is much lower than that for the harmonic frequency.

As presented so far, the area averaging factor is usually considered purely an amplitude scaling factor. There is, however, also a phase shift associated with spatial averaging, and this too can be calculated using the numerical model. Such phase changes could be potentially significant if this type of wavefield was used for phase calibration of hydrophones. Figure 5 illustrates an example of the phase change due to area-averaging for the 500 kPa case. This shows that for both the 0.2 mm and 0.5 mm hydrophones, the phase change varies periodically, with rapid changes associated with the intermediate frequencies. These effects can be minimised by making measurements at lower drive levels. In addition, the measurements can be repeated with different matching conditions between the generator and transducer so that the fundamental and harmonic frequencies are shifted and the complete frequency range is covered with high strength signals [7]. Averaging the resulting calibrations reduces the effect of the area-averaging factor oscillations with frequency.

![Figure 4](image1.png)

Figure 4. Radial variation of the pressure amplitude for the 6th harmonic of the centre frequency (39 MHz) and a nearby intermediate frequency (35.5 MHz) at a range of 47.5 mm. Results are presented for initial peak rarefractional pressures at the source of (a) 300 kPa and (b) 500 kPa.

![Figure 5](image2.png)

Figure 5. Phase change due to area-averaging for hydrophones of 0.2 and 0.5 mm diameter, calculated from the numerical model for a peak rarefractional pressure at the source of 500 kPa, at a range of 47.5 mm.

### 4. Discussion and conclusions

The results presented in section 3 have demonstrated all of the phenomena reported in reference [3]. They show that the area-averaging factor is very sensitive to frequency, drive level and hydrophone diameter. The similarity of the response shown in figure 3(b) to that shown in figure 2 is very encouraging and indicates that the modelling technique is capable of reproducing the characteristics of the behaviour.
In order to understand the behaviour presented in section 3 it is informative to look at the distorted waveforms that occur on-axis in the observation plane at a range of 47.5 mm as a result of nonlinear propagation (see figure 6). These waveforms were calculated from the Fourier components in the model and then passed through an FIR filter with a cut off at 200 MHz to avoid oscillations due to Gibbs phenomenon. Comparison of the two graphs shows that for the low drive case only the first cycle is of sufficient amplitude to have significantly distorted and produced a shock front at this range. The resulting rapid rise in pressure results in frequency components across the whole spectrum. In contrast the higher drive case shows significant distortion of both the first two cycles with a shock front present on the second cycle as well as the first. It can be considered that each shock gives rise to a wide spectrum of frequencies. As the second shock occurs at a time \( 1/f_c \) later than the first, frequencies that are multiples of \( f_c \) will add constructively in the overall pulse spectrum. In contrast, the contributions at intermediate frequencies from the two shocks will be out of phase and tend to cancel. Hence the reduction of the level on axis at the intermediate frequencies, seen in figure 4(b), can be attributed to cancellation effects associated with the second cycle going in to shock. As the observation position is moved away from the acoustic axis and the amplitude of the pulse reduces, the second cycle no longer distorts as significantly and the cancellation effect reduces; this results in less cancellation so that the signal amplitude at intermediate frequencies actually increases.

An implication of these observations is that the area-averaging effect will be very sensitive to the exact shape of the initial pulse waveform. For example, inverting the pulse results in a very different response with larger fluctuations (not presented here); the polarity shown here in figure 1 was actually chosen experimentally because of this fact.

A broad conclusion of this work is that if pulsed nonlinear waveforms are to be used in calibration work then the drive level should be adjusted to give rise to just one shock front in the distorted pulse at the observation position. This will ensure that the spectrum is broad but will minimise fluctuations in the spectral level, avoiding rapid fluctuations in the area-averaging factor such as those shown in figure 3(b).

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