An Ultra-long Wavelength Sky Model with Absorption Effect

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ABSTRACT

The radio sky at frequencies below ∼10 MHz is still largely unknown, this remains the last unexplored part of the electromagnetic spectrum in astronomy. The upcoming space experiments aiming at such low frequencies (ultra-long wavelength or ultra-low frequency) would benefit from reasonable expectations of the sky brightness distribution at relevant frequencies. In this work, we develop a radio sky model that is valid down to ∼1 MHz. In addition to the discrete HII objects, we take into account the free-free absorption by thermal electrons in the Milky Way’s warm ionized medium (WIM). This absorption effect becomes obvious at ≲ 10 MHz, and could make the global radio spectrum turn over at ∼ 3 MHz. Our sky map shows unique features at the ultra-long wavelengths, including a darker Galactic plane in contrast to the sky at higher frequencies, and the huge shadows of the spiral arms on the sky map. It would be a useful guidance for designing the future ultra-long wavelength observations. Our Ultralong-wavelength Sky Model with Absorption (ULSA) model could be downloaded at DOI:10.5281/zenodo.4454153.

Keywords: Radio continuum emission; Interstellar emissions; Galaxy structure; HII regions; Extragalactic radio sources

1. INTRODUCTION

The radio sky has been surveyed from ∼10 MHz up to ∼THz. Based on data of surveys at the relevant frequencies, full-sky maps were produced at 408 MHz (Haslam et al. 1982; Remazeilles et al. 2015), 1.42 GHz (Reich 1982; Reich & Reich 1986; Reich et al. 2001), and a number of higher frequencies from 22.8 GHz to 857 GHz by WMAP and Planck satellites, see Hinshaw et al. (2009); Planck Collaboration et al. (2016a). In addition, there are also surveys which cover a large portion but not the full sky. In order to produce sky maps at frequencies in which the full sky have not be surveyed, a number of sky models have been developed, based essentially on interpolation or extrapolation of the available data, and statistical modeling of the sky intensity distribution. Over much of the radio wave band, the sky intensity has a nearly power law spectrum, making the interpolation/extrapolation relatively simple and accurate. These includes the Global Sky Model (GSM) (de Oliveira-Costa et al. 2008), its improvements (Danny C. Price 2016; Zheng et al. 2017; Sathyanarayana Rao et al. 2017; Kim et al. 2018), and the Self-consistent Sky Model (SSM) (Huang et al. 2019). They are very useful in the design of new instruments, study of observation strategies, and testing foreground removal/mitigation methods (e.g., Shaw et al. 2014; Zuo et al. 2019).

However, the extrapolation becomes trickier at frequencies ≲ 10 MHz, where the absorption of the interstellar medium (ISM) becomes significant, and at the same time there is a dearth of observation data. Observations below 10 MHz is severely hampered by the absorption and distortions of the ionosphere (Jester & Falcke 2009). Until now there are only a few ground-based observations performed during ∼1950s – 1960s (George et al. 2015; Cane & Whitham 1977), and a few space-based low-resolution observations performed during ∼1960s – 1970s (Alexander et al. 1969; Brown 1973; Alexander & Novaco 1974; Alexander et al. 1975; Cane 1979). Below we shall call this wave band the ultra-long wavelength or ultra-low frequency band, though these are not the name used by the radio engineers. The
ultra-long wavelength sky is still a largely unexplored regime in the astronomical observations. To open this new window in the electromagnetic spectrum, a number of space missions, e.g. the lunar-orbit array for Discovering the Sky at the Longest wavelength (DSL) (Chen et al. 2019), and the lunar-surface based FARSIDE (Burns et al. 2019) have been proposed, with the aim of obtaining high resolution\(^1\) full-sky maps at ultra-long wavelengths.

However, to design such low frequency experiments and observe the sky at this largely unexplored part of electromagnetic spectrum, one needs a reasonable estimate of the sky brightness at these frequencies, and a fair expectation of the structures, so that the required system gain and dynamic range can be derived. More importantly, at present or in the near future, a lunar-orbit or lunar-surface based array necessarily has a limited number of antenna elements (though the orbital precession could improve the \( uv \omega \) coverage at the price of longer observation time), and the antennas are all-sky sensitive, these make the imaging process quite challenging, especially when the sky brightness itself varies dramatically with frequency and direction, as we will show below. It is therefore crucial to have a reasonable full-sky model for these unexplored frequencies, which could serve as a starting point for the deconvolution process, for both end-to-end simulations in designing such experiments and the upcoming real observations.

At \( \lesssim 10 \) MHz, the existing sky models based on extrapolation are grossly inadequate, because the free-free absorption and synchrotron self-absorption (Orlando & Strong 2013; Ghisellini 2013) become very significant. In the Galactic diffuse ISM the former is much more important (Scheuer & Ryle 1953; Shain 1959; Hurley-Walker et al. 2017). Free-free absorption is the inverse process of bremsstrahlung (free-free) radiation, so it is proportional to the square of free electron density. The absorption level depends not only on the smoothed distribution, but also on the small-scale turbulence.

Since as early as 1960s, it has been discovered that the global radio background (the sum of Galactic and extragalactic radiation) spectrum has a downturn at \( \sim 3 - 5 \) MHz, by ground-based telescope (e.g. Ellis & Hamilton 1966), by radio-astronomy rocket (e.g. Alexander & Stone 1965), and by space satellites (e.g. Ariel II Smith 1965; RAE-1 Alexander et al. 1969; IMP-6 Brown 1973; and RAE-2 Novaco & Brown 1978), and free-free absorption is probably the reason of this downturn in spectrum. However, it is also possible that the intrinsic energy spectrum of the cosmic ray particles becomes softer at low energy (Strong & Moskalenko 1998a). More references about early low frequency observations could be found in Cane (1979).

For dense electron clumps, such as those in discrete HII regions, the free-free absorption could make these objects opaque even for radio signal at higher frequencies (e.g. Odegard 1986; Kassim 1988). This effect can be used to separate the synchrotron emissivity behind and in front of such regions. If there are sufficient number of such HII regions with known distance, the 3D distribution of the synchrotron emissivity could be reconstructed. This has been successfully put into practice, see e.g. Nord et al. (2006); Hindson et al. (2016); Su et al. (2017, 2018); Polderman et al. (2019) and references therein. Since the synchrotron radiation is produced by the cosmic ray electrons in magnetic field, the cosmic ray electron distribution can be inferred if the magnetic field is known by other means (Polderman et al. 2020). The degeneracy between the cosmic ray and the magnetic field could also be broken by combining with gamma-ray observations (Nord et al. 2006), as the Galactic gamma-ray is mainly from the collisions between the cosmic ray particles and ISM particles (Nava et al. 2017), and the ISM density profile could be constructed from 21cm, CO, and dust thermal emission observations (Ackermann et al. 2012).

A sky model for the ultra-long wavelengths has to take into account the free-free absorption so as to give a reasonable prediction for the upcoming observations (Reynolds 1990). In this work, we develop an observation-based sky model that is still valid below \( \sim 10 \) MHz, by including the free-free absorption effect. We start with a description of our methods in Sec. 2, which incorporate a fitting result for the Galactic synchrotron spectral index, a fitted Galactic synchrotron emissivity, and an adopted electron density distribution that is responsible for the free-free absorption. The main results are given in Sec. 3, including the model with constant spectral index, the model with frequency-dependent spectral index and the one with direction-dependent spectral index. At last, we summarize our main conclusions in Sec. 4. A brief description of our software is presented in Appendix A.

\(^1\) At such low frequencies, the resolution is limited by the scattering in the interstellar medium and interplanetary medium. This angular scattering limit is roughly arc-minute level at 1 MHz and roughly scales with \( \propto \nu^{-2} \), see Jester & Falcke (2009).

2. METHODS

The free-free absorption becomes significant at low frequencies. The optical depth can be written as the integration of the absorption coefficient, \( \kappa_\nu \), along the line
of sight,
\[ \tau(\nu, s) = \int_0^s \kappa(\nu, T_e) \, ds', \]
where \( n_e \) and \( T_e \) are the electron density and temperature at \( s' \) respectively. For constant electron temperature, the following approximation holds (Condon & Ransom 2016),
\[ \tau_\nu \approx 3.28 \times 10^{-7} \left( \frac{T_e}{10^4 \text{ K}} \right)^{-1.35} \left( \frac{\nu}{\text{GHz}} \right)^{-2.1} \left( \frac{\text{EM}}{\text{pc cm}^{-6}} \right), \]
where the emission measure (EM) is the integration of electron density squared at all scales,
\[ \text{EM} = \int \left( \frac{n_e}{\text{cm}^{-3}} \right)^2 \left( \frac{ds}{\text{pc}} \right). \]
The ISM inside the Milky Way, the circum-galactic medium (CGM), and the intergalactic medium (IGM) can all absorb low frequency radio waves. We can make some simple estimates to assess the absorption on the various astrophysical scales.

Regarding the CGM, van de Voort et al. (2019) has investigated the distribution of the hydrogen surrounding Milky Way-size galaxies from high resolution zoom-in cosmological hydrodynamic simulations with star formation. They presented the column number density of the hydrogen as a function of \( r \) in their Figure 2. We adopt a similar radial density profile, and assume that the free electron density follows the same distribution, with a clumping factor of \( \sim 3 \) and CGM electron temperature of \( \sim 10^5 \text{ K} \). We find \( \tau_{\text{CGM}} \approx 0.9, 0.09 \) and \( 0.007 \) for 1, 3 and 10 MHz respectively. If, however, there are some cooler dense clumps in the CGM, the absorption would be stronger. Thus, the CGM is nearly transparent above 1 MHz, but becomes opaque below 1 MHz.

The EM of the IGM up to a redshift \( z \) is
\[ \text{EM} = \int_0^z \frac{\kappa(\nu)}{\kappa_e} \int d^5z' \int d^3r' \nu C_{\text{IGM}}(z') \bar{n}_e^2(z') dr'_p, \]
where \( C_{\text{IGM}}(z') \) is the clumping factor at \( z' \), and \( dr'_p = cdz'/[(1 + z')H(z')] \) is the proper distance element at \( z' \). Also \( \bar{n}_e(z) = f_e(z) \bar{n}_H(1 + z)^3 \) is the mean free electrons density at \( z \), with \( \bar{n}_H \approx 1.89 \times 10^{-7} \text{ cm}^{-3} \) the mean hydrogen number density at present. Note that to use the EM as in Eq. (2), we add the factor \( \kappa(\nu)/\kappa_e \) to correct for the redshift-dependence of the absorption for a given observing frequency at \( \nu \). Since \( \kappa(\nu) \propto \nu^{-2.1} \) (Condon & Ransom 2016) and \( \nu' = \nu(1 + z') \), we have \( \kappa(\nu)/\kappa_e = (1 + z')^{-2.1} \), hence
\[ \text{EM} = \int_0^z (1 + z')^{-2.1} C_{\text{IGM}}(z') \bar{n}_e^2(z') \frac{cdz'}{(1 + z')H(z')}. \]

![Figure 1](image_url) The IGM mean optical depth for \( \nu = 1, 3, 10 \text{ MHz} \) to redshift \( z \).

We use the tanh model to describe the reionization history for both HI and HeI, and HeII, i.e.
\[ f_e^{\text{HI,HeI}} = \frac{1 + n_{\text{He}}/n_{\text{H}}}{2} \left[ 1 + \tanh \left( \frac{y_{\text{re}} - y}{\Delta y} \right) \right], \]
\[ f_e^{\text{HeII}} = 0.5 \left[ 1 + \tanh \left( \frac{y_{\text{re}} - y}{\Delta y} \right) \right], \]
and
\[ f_e = f_e^{\text{HI,HeI}} + (n_{\text{He}}/n_{\text{H}}) f_e^{\text{HeII}}. \]
Here \( y(z) = (1 + z)^{3/2}, y_{\text{re}} = y(z_{\text{re}}), \Delta y = 3/2(1 + z_{\text{re}})^{1/2} \Delta z, \) and \( n_{\text{He}}/n_{\text{H}} \approx 0.083 \). For HI and HeI, we assume \( z_{\text{re}} = 7.68 \) and \( \Delta z = 0.5 \), and for HeII, we adopt \( z_{\text{re}} = 3.5 \) and \( \Delta z = 0.5 \) (Planck Collaboration et al. 2020). We use the clumping factor model that fits reionization simulations presented in Iliev et al. (2007), but renormalize it to be 3.0 at \( z = 5 \), i.e.
\[ C_{\text{IGM}}(z) = 6.8345 \exp(-0.1822z + 0.003505z^3). \]
The IGM free-free absorption optical depths at frequencies 1, 3 and 10 MHz are shown in Fig. 1 as a function of \( z \), it is not large even at 1 MHz.

Next we consider the ISM, which is much denser than the IGM and CGM. The observed sky brightness temperature in the direction given by Galactic coordinates \( (l, b) \) is the sum of the Galactic radiation \( T_G(\nu, l, b) \) and the extragalactic background \( T_E(\nu) \),
\[ T(\nu, l, b) = T_G(\nu, l, b) + T_E(\nu) \]
\[ = \int_0^{\Omega_G} c(\nu, R, \phi) e^{-\tau(\nu, s)} ds + T_E^{\text{iso}}(\nu)e^{-\tau(\nu, s_{\text{G}})}, \]
where \( s_G \) is the maximum distance to the edge of the Milky Way along any line of sight, \( \epsilon \) is the three-dimensional radiation emissivity in Galactic-centric cylindrical coordinates \((R, Z, \phi)\), and \( \tau(\nu, s) \) is the free-free absorption optical depth integrated up to a distance of \( s \). Here we set \( s_G = 50 \text{ kpc} \). \( T_E^{iso} \) is the extragalactic radio background for which we assume to be isotropic in the first approximation. With models of an extragalactic radio background \( T_E^{iso} \) at ultra-long wavelengths, a three-dimensional emissivity \( \epsilon(\nu, R, Z, \phi) \), and a three-dimensional distribution of free electrons in the Milky Way \( n_e \), we can derive the sky map at the required frequencies from Eq. (10). In the following subsections, we describe the modeling of \( T_E^{iso}, \epsilon, \) and \( n_e \) respectively.

2.1. The isotropic extragalactic background

The extragalactic background is assumed to be largely isotropic, and a summary of some past results is given in Guzmán et al. 2011. Two methods are commonly used to extract it from the observed sky maps (Kogut et al. 2011; Dowell & Taylor 2018). In the first approach used to extract it from the observed sky maps (Kogut et al. 2011; Guzmán et al. 2011). Two methods are commonly used to extract it from the observed sky maps (Kogut et al. 2011; Dowell & Taylor 2018). In the second approach, the Galaxy is described by a plane-parallel model, so the total radiation is

\[
T(\nu) = T_E^{iso}(\nu) + T_G(\nu) \times \text{csc} |b|,
\]

where both \( T_E^{iso} \) and \( T_G \) depend only on frequency. In the second approach, the Galactic radiation is derived from a tracer, usually the Galactic [C\text{II}] emission, say \( T(\nu) = \sum_{i=1}^{2} T_{E,i}^{iso}(\nu) + a_i(\nu) \times \sqrt{I_{\text{CII}}}, \)

where \( I_{\text{CII}} \) is from the CORE/FIRAS measurements. In practice, all coefficients are fitted from observed radio maps and these two methods give consistent results (Kogut et al. 2011; Dowell & Taylor 2018). However, whether the extracted isotropic background really originates from extragalactic sources is still being debated. Seiffert et al. (2011) extracted a background which is larger than the integrated radio emission of external galaxies (Gervasi et al. 2008). The excess could be from unknown extragalactic source populations, or could be unaccounted Galactic foreground (see also Subrahmanyan & Cowsk 2013).

Here we assume that the isotropic background is from extragalactic sources. We adopt the following simple model of isotropic extragalactic background,

\[
T_E^{iso}(\nu) = 1.2 \left( \frac{\nu}{1 \text{ GHz}} \right)^{-2.58} \text{[K]},
\]

This is the fit obtained by the ARCADE-2 balloon experiment, which observed the absolute brightness temperature of the sky between 3 and 90 GHz (Seiffert et al. 2011), minus the cosmic microwave background (CMB). This is in good agreement with the fit obtained from the Long Wavelength Array (LWA) (Dowell & Taylor 2018) below 100 MHz. Strictly speaking, “background” is the cumulative radiation from unresolved sources, so it depends on the sensitivity and angular resolution of the telescope. For simplicity we directly use the background fit given by the ARCADE-2 experiment, since in our model we do not specify any particular instrument.

In addition to the absorption by our Milky Way, the extragalactic radio background would also be absorbed by the Milky Way along any line of sight. In the ISM in the host galaxies. Moreover, the spectrum of relativistic electrons could become softer at lower energy, resulting in softer extragalactic radio background (Protheroe & Biermann 1996). In a recent estimate for the extragalactic radio background, Nițu et al. (2021) found the extragalactic radio background deviates from power-law below \( \sim 1 \text{ MHz} \). In Sec. 3.3 we investigate the case that the extragalactic radio background is not a power-law form.

2.2. The Galactic free-free emission

The free-free emission also contributes to the low frequency sky brightness and should be subtracted from the observed maps for the purpose of assessing Galactic synchrotron. A free-free template could be constructed by using the H\text{\alpha} emission line as a tracer. After correcting the absorption and scattering, the intrinsic H\text{\alpha} flux could be converted into free-free flux (Dickinson et al. 2003; Lian et al. 2020). Moreover, it can also be constructed from the multi-frequency microwave observations.

In this work, we use the EM and \( T_e \) maps constructed from Planck multi-frequency observations (Planck Collaboration et al. 2016b)\footnote{http://pla.esac.esa.int/pla/aio/product-action?MAP.MAP_ID=COM_CompMap_freefree-commander_0250_R2.00.fits} to derive the free-free map, say \( g_{ff}(\nu) = \ln \left\{ \exp \left[ 5.960 \times \frac{\nu^3}{\pi} \ln \left( \nu_9 T_4^{-1.5} \right) \right] + \exp(1) \right\} \)

\[
\tau_{ff}(\nu) = 0.05468 T_e^{-1.5} \nu_9^{-2} \text{EM}_{ff},
\]

\[
T_{ff}(\nu) = 10^6 T_e[1 - \exp(-\tau_{ff})],
\]

where \( \nu_9 \) is the frequency in units \( 10^9 \) Hz and \( T_4 \) is the electron temperature in units \( 10^4 \) K (Draine 2011). In the top panel of Fig. 2, we show the 408 MHz Galactic free-free emission map constructed from the Planck observations. The bottom panel shows the Galactic free-free spectrum. Clearly, most of the free-free emission is from the Galactic plane.
Comparing the above free-free map with observations at 408 MHz (see next subsection), we find that the free-free emission contributes 4% of the mean observed sky brightness. At $|b| > 10^\circ$, the free-free fraction decreases to 1%. If we smooth both maps with a resolution of $10^\circ$, then the r.m.s. contributed by the free-free emission is 15% of the total, while at $|b| > 10^\circ$ the r.m.s. fraction decreases to 6%. This is consistent with Dickinson et al. (2003).

### 2.3. The absorption-free sky map

We first derive the synchrotron spectral index from the observed sky maps at frequencies where absorption is assumed to be negligible. At low frequencies, there are full-sky maps at 408 MHz (Haslam et al. 1974, 1981, 1982; Remazeilles et al. 2015) and 1.42 GHz (Reich 1982; Reich & Reich 1986; Reich et al. 2001). Here we use the free-free subtracted Haslam 408 MHz map as the base, assuming that in the absence of absorption, the brightness temperature of the Galactic synchrotron radiation follows a power-law form (Platania et al. 1998; Kogut 2012; McKinley et al. 2018),

$$T_G(\nu) = T_G(\nu_*) \left( \frac{\nu}{\nu_*} \right)^{\beta_G},$$

where $\nu_* = 408$ MHz. We determine $\beta_G$ by minimizing

$$\chi^2 = \sum_{\nu_j} \sum_i \frac{1}{T^2_i(\nu_j)} \left[ T_E(\nu_j) + T_{G,i}(\nu_*) \left( \frac{\nu_j}{\nu_*} \right)^{\beta_G} + T_{G,i}(\nu_j) - T_i(\nu_j) \right]^2,$$

where $T_i(\nu_j)$ is the observed brightness temperature of the $i$-th pixel in the sky-map at frequency $\nu_j$, $T_{G,i}(\nu_*)$ is the Galactic radiation for the $i$-th pixel at $\nu_*$. The summation is performed for all available pixels at all observed frequencies of our selected data.

Below several hundred MHz, there are observed sky maps, each of which covers a part of the sky. In determining the spectral index, in addition to the Haslam 408 MHz map, we use a selection of observations that have large sky coverages: the 45 MHz sky map in combination of two observations (Guzmán et al. 2011), and the 35 MHz, 38 MHz, 40 MHz, 50 MHz, 60 MHz, 70 MHz, 74 MHz, and 80 MHz maps observed by the LWA (Dowell et al. 2017). The Guzman 45 MHz map covers 96% of the sky, with resolutions of $4.6^\circ \times 2.4^\circ$ and $3.6^\circ \times 3.6^\circ$ for the southern and northern sky respectively. It combines two surveys performed by devices of similar characteristics. All of the LWA maps cover a total of about 80% of the sky, with resolutions ranging from $4.8^\circ \times 4.5^\circ$ to $2.1^\circ \times 2.0^\circ$. As the different maps have different angular resolutions, we smooth all the maps with a beam with FWHM = 5$^\circ$ when deriving the power-law index. For all the maps we subtract the free-free emission given in Sec. 2.2. Our fiducial model assumes that the spectrum index is a constant, but we will also investigate a model with a frequency-dependent spectral index in Sec. 3.3 and a model with direction dependence in the spectral index in Sec. 3.4.

Using the above method, we derive a spectral index $\beta_G = -2.51$ for the Galactic synchrotron radiation. This is actually quite similar to the spectral index of the extragalactic background, $\beta_E = -2.58$. In Fig. 3 we plot the 1 MHz sky map that is extrapolated from free-free observations.
subtracted Haslam 408 MHz map using $\beta_G = -2.51$ for the Galactic component and $\beta_E = -2.58$ for the extragalactic component. As this is a simple power-law extrapolation, its structure is the same as the Haslam 408 MHz map, except for different temperature scale. This map could be compared with the absorption-included maps presented later in this paper.

2.4. The Galactic emissivity model

We tentatively adopt an axisymmetric form for the Galactic emissivity $\epsilon$. In the cylindrical coordinates, the emissivity can be written as

$$
\epsilon(\nu, R, Z) = A \left( \frac{R + r_1}{R_0} \right)^\alpha e^{-R/R_0} e^{-|Z/Z_0|^{\gamma}} \left( \frac{\nu}{\nu_0} \right)^{\beta_G}.
$$

(17)

Here $r_1$ is a small cut off radius to avoid singularity at the Galactic center, and we take $r_1 = 0.1$ kpc. The five frequency-independent free parameters: $A$, $R_0$, $\alpha$, $Z_0$ and $\gamma$ will be obtained by fitting the free-free subtracted Haslam 408 MHz map. The parameter $\beta_G$ is generally a function of frequency and spatial position. Below we will discuss the cases of a frequency-dependent $\beta_G$ and a direction-dependent $\beta_G$.

Before deriving the emissivity parameters, we mask the Loop I and North Polar Spur (NPS) regions. They are generally believed to be nearby objects (see e.g., Wolleben 2007), which do not suffer from the absorption on the Galaxy scale, hence one should not use these to derive the parameters of the Galactic emission and absorption model. We fit the emissivity parameters from free-free subtracted Haslam 408 MHz map and the fitted parameters are listed in Table 1.

2.5. The Galactic free electron distribution

Most free electrons in the ISM are either in the warm ionized medium (WIM), with typical density of $\sim 0.01 - 0.1$ cm$^{-3}$ and typical temperature of $\sim 10^4$ K (Gaensler et al. 2008; de Avillez et al. 2012), or in the hot ionized medium (HIM), with typical density of $\sim 10^{-3}$ cm$^{-3}$ and typical temperature of $\sim 10^5 - 10^6$ K (Ferrière 2001). The free-free absorption is dominated by the WIM. In addition, there are some dense HII regions with typical density of $\sim 10^2$ cm$^{-3}$ and typical temperature of several thousand K (Hindson et al. 2016). They are almost opaque to the low-frequency radio radiation, and hence good for separating the contributions to the synchrotron radiation from different distances along lines of sight (Su et al. 2017, 2018; Polderman et al. 2019). Moreover, around the dense HII regions, there could be some extended HII region envelopes (EHEs), as inferred from the observations (Anantharamaiah 1986; Kassim 1989). They have smaller density ($\sim 0.5 - 10$ cm$^{-3}$) but larger size ($\sim 0.05 - 0.2$ kpc) than the classical HII regions. These could also potentially contribute more to the absorption.

There are a variety of models for the Galactic electron distribution (e.g. Gaensler et al. 2008; Gómez et al. 2001; Taylor & Cordes 1993; Cordes & Lazio 2002, 2003; Yao et al. 2017 (YMW16); Schnitzeler 2012). In the present work, we mainly focus on the general effect of the absorption, so we adopt the NE2001 model (Cordes & Lazio 2002, 2003) for the electron density, which is one of

| Parameter | Value |
|-----------|-------|
| $A$       | 43.10 K kpc$^{-1}$ |
| $R_0$     | 3.41 kpc |
| $\alpha$  | 0.46 |
| $Z_0$     | 1.12 kpc |
| $\gamma$  | 1.23 |

Table 1. Fitted Galactic emissivity model parameters
the most well-accepted models. In NE2001, the Galactic free electrons have five components, i.e., smooth components including a thick disk, a thin disk and five spiral arms; the Galactic center component; the local ISM; a list of known dense clumps; and a list of voids. The dense clumps are mainly HII regions around massive OB stars or supernova remnants (SNRs). The voids are low density regions found between the Sun and some pulsars. The smooth components contain lots of free electron clouds and their volume filling factor is \( \eta \). The could-by-cloud density fluctuations is described by a parameter

\[
\zeta = \frac{\langle n_e^2 \rangle}{n_e^{\text{smooth}}}, \tag{18}
\]

where \( n_e \) is the free electron density of the cloud, \( \langle \cdot \rangle \) denotes cloud-by-cloud average, \( n_e^{\text{smooth}} \) is the density of the smooth component. Inside each cloud, the free electrons also have small scale structures below a largest scale \( L_0 \). It is assumed that the power spectrum of such electron density fluctuations follows a power-law distribution with the index \(-11/3\) as in the classical Kolmogorov turbulence model (Cordes & Lazio 2002). Inside a cloud the small scale free electron fluctuations is described by the fractional variance

\[
\omega^2 = \frac{\langle \delta n_e^2 \rangle^2}{n_e^2}, \tag{19}
\]

where the bar represents the average inside the cloud for all scales below \( L_0 \). The emission measure is then

\[
\text{EM} = \int \eta^{-1} \zeta \langle \omega^2 \rangle n_e^{2\text{smooth}} \, ds. \tag{20}
\]

\[
= \int L_0^{2/3} F n_e^{2\text{smooth}} \, ds. \tag{21}
\]

where the fluctuation parameter \( F \) is defined as

\[
F = L_0^{-2/3} \omega^2 \zeta \eta^{-1}. \tag{22}
\]

Hence, given the smooth density distribution by NE2001, the fluctuation parameter determines the strength of absorption/emission related to the density squared. The fluctuation parameter can be derived from the observed scattering measure (SM) of Galactic pulsars (Cordes et al. 1985; Cordes & Lazio 1991; Cordes et al. 1991; Taylor & Cordes 1993) and extragalactic AGNs and GRBs (Cordes & Lazio 2003). In NE2001, the fluctuation parameter is given for each of the three smooth components. The default values for the thick disk, thin disk and the spiral arms are \( F_1 = 0.18, F_2 = 120 \) and \( F_\alpha = 5 \) respectively. \( F_1 \) is the most relevant one for our purpose, while \( F_2 \) and \( F_\alpha \) are for components on the Galactic plane.

3. RESULTS

3.1. The cylindrical model

In Fig. 5, we plot the absorption-free sky map at 1 MHz, obtained by integration of our fitted emissivity Eq. (17) without considering absorption. This is a symmetric distribution, as it is derived from a model with cylindrical symmetry. If we mask out the Loop I and NPS regions where the bright nearby feature dominates, the relative difference with the actual map is at the level of \( \sim 16\% \).

With the cylindrical emissivity model and the NE2001 free electron distribution, one can construct maps with free-free absorption. In Fig. 6, we plot the sky map with absorption using the NE2001 fiducial parameters. As one can see in these figures, at the lower frequencies the absorption becomes very significant. In particular, the absorption is strongest along the Galactic plane, and as a result the Galactic disk is darker than high Galactic latitude regions at the lower frequencies. This is in stark contrast with observations at higher frequencies or the model map without absorption (see Fig. 3), where the Galactic disk is the brightest part of the sky.
Figure 6. The ultra-long wavelength sky maps at 10, 3 and 1 MHz respectively (from top to bottom), for the fiducial NE2001 parameters ($F_1 = 0.18$). Note in this cylindrical model the features from nearby sources such as the Loop I and Northern Spur are not included.

Figure 7. The mean sky brightness in our models (curves) and in observations (points). dashed-dotted line is the absorption-free sky brightness, the dashed line is the absorption-included sky brightness with $F_1 = 0.18$ in the NE2001 model, while solid line with $F_1 = 3.0$. The data points are from the IMP-6 (filled circles), the RAE-2 (triangles), and the maps used to derive spectral index (stars). As a comparison we plot the extragalactic radiation Eq. (13) by dotted line.

How do these maps compare with the observation? The data at this low frequency range are scarce, but there are some global spectrum measurements, i.e. the average spectrum for the whole or a large part of the sky. In Fig. 7 we plot the global spectrum derived from averaging the brightness of the maps of our model, together with some observational data points. The data were taken by the IMP-6 satellite (filled circles, Brown 1973.) and the RAE-2 satellite (upward and downward triangles, Cane 1979). Besides instrumental differences, the differences between the measurements may be due partly to the different sky area being averaged over for the two satellites (Keshet et al. 2004). The models are plotted as curves, including the absorption-free case (dot-dashed line), the extragalactic source (dots), the fiducial NE2001 model ($F_1 = 0.18$, cyan dash line), and a model with enhanced fluctuations ($F_1 = 3.0$, orange solid line, see discussions below). Compared with the data, we find that the sky brightness below $\sim 3$ MHz is over-predicted when using the NE2001 fiducial fluctuation parameter for thick disk ($F_1 = 0.18$), as shown by the cyan dashed curve. This has already been noticed by several earlier works (e.g., Peterson & Webber 2002; Webber et al. 2008).

Some works (Gaensler et al. 2008; Sun et al. 2008) show that the scale height of the thick disk could be twice the default value used in NE2001. This would in principle enhance the free-free absorption. However, adopting a larger scale height of 1.8 kpc, as compared to the default 0.97 kpc, results in a decrement of the mean sky brightness by only 11% at 1 MHz, because a large fraction of the sky radiation is from high Galactic latitude regions that are not influenced much by the increase of the disk thickness.

Now there are some large uncertainties in the data itself, but the down-turn of the spectrum below $\sim 3$ MHz seems robust. If we take the data points below 3 MHz at face value, a possible solution of this discrepancy is that there may be small-scale structures not modeled in
the current electron distribution model which enhances the absorption. In order to reach consistent results with the low-frequency observations at $\lesssim 3$ MHz, one can enhance the absorption by adopting larger fluctuation parameters for the Galactic diffuse components, or alternatively, adopt a softer spectrum index at lower frequencies. Since the thick disk has the largest influence on the global mean sky brightness, we test the model in which the thick disk has a larger fluctuation parameter of $F_1 = 3.0$, while keeping other parameters to be the same as the NE2001 fiducial values. Note that the fiducial value of this fluctuation parameter is $F_1 = 0.18$ in NE2001, so the optical depth is enhanced by more than fifteen times. The results is the solid curve in Fig. 7, which now agrees with the observational data below 3 MHz. The corresponding maps are shown in Fig. 8, for frequencies 10, 3 and 1 MHz respectively.

The maps in Fig. 6 and Fig. 8 begin to show difference at 3 MHz, and the two 1 MHz maps are quite different. For the 1 MHz map, in Fig. 8 one can see huge shadows of the spiral arms and Galactic center component in the sky map (the blue-cyan regions at the left most and rightmost, and near the center). Note that the spiral arms do not only have higher WIM density than the gap between arms, but also are more crowded with discrete HII regions (Paladini et al. 2004) and their extended envelopes. They produce especially prominent large-scale shadows even at higher frequencies, as has been recognized much earlier (Bridle 1969; Roger et al. 1999; Guzmán et al. 2011). On the other hand, for regions near the Galactic pole, and at some voids, the absorption is much weaker. Such directions would be recommended for detecting extragalactic sources. There are some dark spots, most of which are near the Galactic plane. They are HII regions around massive stars or SNRs, with enhanced absorption. For example near the Galactic plane at around $l \sim 270^\circ$, there is the Gum Nebula. It is an ancient SNR with a distance between 200 pc and 500 pc (Woermann et al. 2001). So far, we have not taken into account the radiation from these sources themselves, so the brightness at their locations may be underestimated in such models.

The IMP-6 satellite has detected maximum radiation from the direction near Galactic poles at frequencies between 0.13 and 2.6 MHz (Brown 1973), and the modulation index (defined as the ratio between the maximum flux minus minimum flux and maximum flux plus minimum flux) is $\sim 15\%$ at 1 MHz. It has a poor resolution of $\sim 100^\circ$ however. We checked that if we smooth our 1 MHz map by a window with FWHM $\sim 100^\circ$, the modulation index is $\sim 20\%$. Qualitatively, the IMP-6 is in line with our prediction that at ultra-long wavelengths, the Galactic plane becomes darker because of the absorption, leaving brighter regions at high Galactic latitudes. However, our smoothed map shows a darker Galactic plane and the brighter Galactic poles also significantly modulated by the projected spiral arms, so the direction of the maximum flux pole is shifted from the Galactic poles. It is not clear if the anisotropy seen by the IMP-6 also have other reasons.

Because of the free-free absorption, the sky radiation observed at ultra-long wavelength would be dominated by nearby sources. As the frequency increases, more and more contribution would be from further sources. Therefore, by combining observations at various fre-
Figure 9. The half brightness distance for 10, 3 and 1 MHz respectively (from left to right).

Figure 10. The optical depth distribution at 1 MHz as viewed from the Earth, project along the Z(left) and Y(right) axis of the Galactic coordinates respectively. The location of our Sun is marked by a star symbol.

...quencies, the ultra-long wavelength observations provide a 3D tomography tool for measuring the electrons distribution and emissivity distribution in our Milky Way. Moreover, with the synchrotron emissivity distribution, one can further derive the cosmic ray electron spectrum and the magnetic field distribution if the gamma-ray observations are analyzed jointly, see Nord et al. (2006); Polderman et al. (2020). In Fig. 9, we show the maps of the critical distance $d_{50}$, within which about 50% of the radiation we observe is emitted. Here we adopt a thick disk fluctuation parameter of $F_1 = 3.0$, and from left to right, we shown results for 10, 3 and 1 MHz, respectively. For each line-of-sight, by comparing the observed specific intensity at different frequencies, one can derive the contribution from different distances.

We also show the optical depth from the Z-axis and Y-axis perspectives. On the Galactic plane, the shape of the local optical depth distribution is irregular, with obvious anisotropy, as well as ray-like features due to nearby ISM distribution. On larger scales, the electron density increases rapidly toward the inner Galaxy region, forming a fan-like structure. From the side view, we can see there are high transparency cones above and below the Galactic plane. The overall structure is what we would expect: there is more opacity towards the inner region of the Galaxy, but local ISM could have significant impact.

3.2. Non-symmetric variations

The model discussed above is based on an emissivity model with cylindrical symmetry with respect to the Galactic center. For practical usage, it is desirable to have mock sky maps which have non-symmetrical brightness temperature variations. We therefore consider a model which includes such variations, given by

$$\Delta T_G(\nu, l, b) = \int_0^{\phi_G} \Delta \epsilon(\nu, R, Z, \phi, s) e^{-\tau(l, b)} ds. \quad (23)$$

The emissivity fluctuations $\Delta \epsilon(\nu, R, Z, \phi, s)$ is of course very poorly known at present, so we adopt an approximated approach as below. The cylindrical absorption-free sky brightness temperature is

$$T_G^a(\nu, l, b) = \int_0^{\phi_G} \epsilon(\nu, R, Z) ds, \quad (24)$$

and the absorption-included sky brightness temperature is

$$T_G(\nu, l, b) = \int_0^{\phi_G} \epsilon(\nu, R, Z) e^{-\tau(l, b)} ds. \quad (25)$$
A “transmittance” for each line of sight is defined as

\[ R_1(\nu, l, b) = \frac{T_G(\nu, l, b)}{T_G^{af}(\nu, l, b)}. \]  

(26)

We then assume that the non-symmetric temperature variations have the same “transmittance” as \( R_1 \) defined above, therefore, instead of looking for a \( \Delta \) in Eq. (23), we directly derive the non-symmetric variations by

\[ \Delta T_G(\nu, l, b) = R_1 \Delta T_G^{af}(\nu, l, b), \]  

(27)

where \( \Delta T_G^{af}(\nu, l, b) \) is the difference between the extrapolated sky map (using Eq. 15) and the cylindrical \( T_G^{af} \). Note that for some large scale features on the sky, this may be not a good approximation. For example the Loop I is more likely a nearby object (Dickinson 2018), therefore the \( R_1 \) should be higher than the average. We leave this for future work.

We add these fluctuations to the cylindrical symmetry model sky maps and the results are shown in Fig. 11. In Fig. ??, we also show evolution of the Galactic absorption in frequencies from 10 to 0.1 MHz. We compare our generated map at 10 MHz with the observed map by Caswell (1976) at the same frequency. In the common sky region, the relative mean deviation is 18.6%. We also compare our generated map at 22 MHz with the observed one by Roger et al. (1999), the mean relative difference is 19.2%.

3.3. Frequency dependence of the spectral index

So far we have only considered models with constant spectral indices for both extragalactic background and Galactic synchrotron radiation. However, the spectral index may vary with frequency. For example, the energy spectrum of interstellar cosmic ray electrons derived from the combination of PAMELA observations at near-Earth space and Voyager observations at distant space (\( \gtrsim 100 \text{ AU} \)) can be described by a broken power-law form, with broken point at \( \sim 0.1 \text{–} 1 \text{ GeV} \) (Potgieter & Nudanganeni 2013; Bisschoff et al. 2019). Assuming a typical interstellar magnetic field \( \sim 5 \mu \text{G} \), then the broken point corresponds to synchrotron critical frequency \( \sim 1 \text{–} 100 \text{ MHz} \). Inspired by this, in this section we investigate a model that the synchrotron spectral index depends on frequency.

The frequency dependence of the spectral index is not yet confirmed in current observations at \( \gtrsim 10 \sim 22 \text{ MHz} \). To proceed, we tentatively assume the following frequency-dependence for the spectral index, for both Galactic radiation and extragalactic background,

\[ \beta = \beta_0 + \beta_1 \exp \left( -\frac{\nu}{\nu_1} \right), \]  

(28)

where \( \beta_0 = -2.58 \) and \( -2.51 \) is the constant spectral index in our fiducial model for extragalactic background and Galactic radiation respectively. \( \nu_1 \) is a critical frequency, for \( \nu \gg \nu_1 \), \( \beta \approx \beta_0 \); for \( \nu \ll \nu_1 \), \( \beta \approx \beta_0 + \beta_1 \). For simplicity we adopt the same \( \beta_1 = 0.7 \) and \( \nu_1 = 1.0 \text{ MHz} \) for both Galactic radiation and extragalactic background.

The absorption-included sky maps with a frequency-dependent spectral index are shown in Fig. 13. Here we adopt \( F_1 = 0.18 \), as the default of NE2001. Comparing them with the Fig. 8, we find that although the mean sky brightness is similar, the morphology is rather dif-

\[ \frac{\Delta T_G(\nu, l, b)}{T_G^{af}(\nu, l, b)} = R_1 \Delta T_G^{af}(\nu, l, b), \]  

(27)
3.4. Spatial variations of the spectral index

The spectral index could also have spatial variations. However, at low frequency only the Haslam 408 MHz map covers the full sky. To derive the full-sky spectral index map, we first generate a spectral index map by combining the Haslam 408 MHz map with the LWA maps, and another one by combining the 408 MHz map with the Guzman 45 MHz map. These two maps are incomplete around the south and north celestial pole respectively, but for sky region common to both maps, the two maps show similar structure, the difference is about 13.6% standard deviation. We produce a combined map by connecting these two spectral index maps by a smooth function. The spectral index at declination \( \delta \) is

\[
\beta_G(\delta) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{A \delta - \delta_0}{|\delta_0|} \right) \right] \beta_{G1}(\delta)
+ \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{A \delta - \delta_0}{|\delta_0|} \right) \right] \beta_{G2}(\delta),
\]

(29)

where \( \beta_{G1} \) is the spectral index derived by Haslam 408 MHz map and LWA maps, while \( \beta_{G2} \) is the spectral index derived by Haslam 408 MHz map and Guzman 45 MHz map. When \( \delta \gg \delta_0 \), we have \( \beta \approx \beta_{G1} \), and when \( \delta \ll \delta_0 \), \( \beta \approx \beta_{G2} \). We take \( \delta_0 = -15^\circ \) and \( A = 1.7 \).

All maps used here are smoothed by a Gaussian beam with FWHM = 5°. The resulting spectral index map is shown in Fig. 15. At the Galactic plane the spectrum slope is shallower than the higher Galactic latitude, and the southern sky is also shallower than the northern sky. Note here the spectral index is for the frequency range of a few tens MHz to 408 MHz, where the free-free absorption is still small except for dense HII regions, so it reflects the intrinsic emission spectrum.

We then model the emissivity as

\[
\epsilon(\nu, R, Z) = A \left( \frac{R + r_1}{R_0} \right)^{\alpha} e^{-R/R_0} e^{-|Z/Z_0|^{\gamma}} \left( \frac{\nu}{\nu_*} \right)^{\beta_{G}(l,b)}. 
\]

(30)

It is formally same as Eq. (17), except that now \( \beta_G \) is allowed to vary with direction (direction-dependent),
Figure 13. Similar to Fig. 6, however here the synchrotron radiation has frequency-dependent spectrum index as Eq. (28).

and the value is taken from the spectrum index map obtained above. Using the emissivity given above, and adding the free-free absorption and small-scale fluctuations (here we also adopt $F_1 = 3.0$), we finally obtain the map shown in Fig. 16. Compared with Fig. 11, the overall brightness is higher in these maps, because the pixels with steeper spectral index become brighter when extrapolated to the lower frequencies.

The mean sky brightness temperature is also plotted in Fig. 14. In this model, for same absorption strength ($F_1$ parameter) the sky brightness temperature is higher than the constant spectral index model, at frequencies below $\sim 3$ MHz. As we see in Fig. 15, the spectrum slope at higher Galactic latitude is steeper than the lower Galactic latitude, and as a result, the absorption-free brightness at high Galactic latitude is larger than in the constant spectral index model. In addition, the free-free absorption at higher Galactic latitude is weaker. Consequently, the ultra-long wavelength sky is brighter in the direction-dependent spectral index model.

3.5. Absorption from Extended HII Region Envelopes (EHEs)

As noted above, the NE2001 model with its fiducial parameters may predict inadequate absorption to produce the observed downturn in the global radio spectrum. To overcome this difficulty we considered adopting a larger fluctuation parameter, i.e. by assuming there are more
small scale fluctuations in the WIM. We now consider another more physical solution to this problem.

**NE2001** includes 175 known HII objects (SNRs or HII regions) with enhanced electron densities ranging from 0.01 to 40 cm\(^{-3}\). Most of them have density \(\sim 0.5 \text{ cm}^{-3}\) at the center, and are modeled with Gaussian profile of width \(\sim 0.01\) kpc. Of course this is not a complete list, there should be many more SNRs, probably between 1000 – 10000 (Berkhuijsen 1984), and HII regions (the WISE mission has already detected more than 8000, see Anderson et al. 2014). HII regions could be small and dense (compact or classical HII regions), or large and diffuse (diffuse HII regions, e.g. Lockman et al. 1996; Anderson et al. 2018). Around the classical HII regions there could be EHEs, with size extends to \(\sim 0.05 – 0.2\) kpc, density \(\sim 0.5 – 10\) cm\(^{-3}\), and temperature 3000 – 8000 K, as found by the low frequency radio recombination lines (Anantharamaiah 1985a,b, 1986; Roshi & Anantharamaiah 2001) or absorption of the radio continuum towards SNRs (Kassim 1989; Lacey et al. 2001).

We now consider the contribution to free-free absorption if more absorbers are included, to see if they can account for the downturn of the observed global radio spectrum at \(\lesssim 3 – 5\) MHz. Although the classical HII region is opaque for the low frequency radio flux, it is difficult for them to significantly reduce the global spectrum because their volume filling factor is too small, so here we focus only on the EHEs.

We assume the spatial distribution of EHEs in the Milky Way has a form similar to that of SNRs (Strong & Moskalenko 1998b)

\[
q(R, Z) = q_0 \left( \frac{R}{R_\odot} \right)^{\eta_q} \exp \left( -\xi \frac{R - R_\odot}{R_\odot} \right) \exp \left( -\frac{|Z|}{Z_a} \right),
\]

where \(Z_a = 0.2\) kpc, \(\eta_q = 1.25\) and \(\xi = 3.56\) as constrained by the Fermi Galactic gamma-ray observations (Trotta et al. 2011). The normalization factor \(q_0\) is set by

\[
N_{\text{EHE}} = \int dZ \int dR 2\pi R q(R, Z),
\]

where \(N_{\text{EHE}}\) is the total number of EHEs in our Milky Way, and is treated as a free parameter here.

We make random realizations of the EHEs that follow the spatial distribution of Eq. (31), and further assume that their size follows a log-normal distribution, with central value \(\langle r_{\text{EHE}}\rangle = 0.1\) kpc and variance \(\sigma_{\log r_{\text{EHE}}} = 0.5\), and limited in the range 0.05 – 0.2 kpc. For simplicity, we model them with sharp edges, and within the edge the electron density is 1.0 cm\(^{-3}\), to be consistent with the typical EHEs. We adopt the same density for all mock EHEs. Regarding the total number, we adopt \(N_{\text{EHE}} = 20000\). We artificially remove one EHE that occasionally encompasses our Sun, it will not influence the statistics of the sample.

In the top panel of Fig. 17 we plot the free-free absorption optical depth to extragalactic sources, purely by our mock EHEs, at 1 MHz. We then replace the HII clumps list in **NE2001** with these mock EHEs, and then re-calculate the sky map. For all other parameters we still use the fiducial values in **NE2001**, because we want to see if the mock EHEs could account for the downturn on the global radio spectrum without increasing the WIM absorption.
the observed global spectrum is

\[ I(\nu) \approx (1 - f_{\text{sky}})I_0(\nu) + f_{\text{sky}}I_0(\nu)\exp[-\tau(\nu)], \]

where \( I_0 \) is the global spectrum without EHEs absorption, and \( f_{\text{sky}} \) is the sky coverage of the EHEs (not the volume filling factor, so it could be close to 1). Because here \( f_{\text{sky}} \approx 1 \), so \( 1 - f_{\text{sky}} \ll 1 \). When \( \nu \gg 1 \text{ MHz} \), the optical depth is still not that high. As a result the second term of the RHS dominates the observed global spectrum. When \( \nu \approx 1 \text{ MHz} \), the EHEs are opaque enough so that \( I_0(\nu)\exp[-\tau(\nu)] \approx 0 \) and the first term of the RHS dominates. Therefore \( I(\nu) \) shows complex trend with decreasing frequency, as shown by the bottom panel of Fig. 17. The WIM absorption would change the \( I_0(\nu) \), so it also changes the behavior of \( I(\nu) \) a bit, but not significant, see the difference between the curve with and without WIM in the bottom panel of Fig. 17.

We further check that, to produce the downturn in such a model, we must have one low-density EHE encompass the Sun. If our Sun is inside an EHE with a typical density of \( \sim 0.1 \text{ cm}^{-3} \), we can get the global spectrum to turn over at \( \sim 3 - 5 \text{ MHz} \) and be consistent with observations. However, we note that it is known that the Sun resides in a region known as the Local Bubble, which has much lower density than above (Frisch et al. 2011). So if there are many EHEs they could indeed affect the global radio spectrum significantly, but to produce the downturn consistent with observations, while also be consistent with the observed local density around the Sun, is still difficult.

4. CONCLUSIONS

In this work, we have developed an ultra-long wavelength radio sky model that is valid below \( \sim 10 \text{ MHz} \). We first derive a cylindrical emissivity from the observed all-sky map at 408 MHz, extrapolate it to ultra-long wavelength using a power-law form, and then add the free-free absorption by the Galactic diffuse free electrons and some small-scale dense HII regions. The spectral index of the power-law is derived from observed multi-wavelength radio maps and the Galactic free electrons distribution is from the NE2001 model.

We found that if we use a constant spectral index \( \beta_G = -2.51 \) for Galactic synchrotron radiation and adopt the fiducial fluctuation parameter for thick disk \( (\beta_f = 0.18) \) in Galactic electron model NE2001, we would over-predict the mean sky brightness at \( \lesssim 3 \text{ MHz} \) compared with the observations by space satellites. The observed mean sky brightness spectrum (the specific intensity) turns over at \( \sim 3 \text{ MHz} \). To solve the problem, either the Galactic diffuse electrons have much larger small-scale fluctuations than modeled by NE2001, or the

Figure 17. Top: the free-free optical depth of the mock EHEs; Middle: the sky map; Bottom: global radio spectrum.
intrinsic synchrotron radiation spectrum itself has shallower slope at $\lesssim 3$ MHz. We investigated both solutions. The morphology of predicted sky maps in these two cases are quite different, therefore, in the future a high resolution ultra-long wavelength sky survey could potentially distinguish them. Moreover, we also investigated a model in which the spectral index depends on direction.

In particular, we found that, at ultra-long wavelengths, the Galactic plane would be darker than higher Galactic latitude regions. Moreover, one can see the shadows of spiral arms from the sky map, particularly at frequencies as low as $\sim 1$ MHz. From the multi-frequency sky maps one can obtain the 3D information of the Galactic diffuse electrons and emissivity distribution.

Our model would be a useful tool for designing the upcoming ultra-long wavelength experiments. The first generation lunar-orbit and lunar-surface based interferometers would have limited number of antennas, hence a reasonable input sky model would be crucial. Our model can be used to test whether these interferometers can successfully and accurately recover the ultra-long wavelength sky, which has complex brightness distribution and changes dramatically with frequency. Moreover, when the new observational data is available, our model can generate low frequency sky maps, which can be used for imaging deconvolution.

We have made our model ULSA publicly available, including the source code, the maps at different frequencies for different models, and some animated figures. They could be downloaded at DOI:10.5281/zenodo.4454153.

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APPENDIX

A. THE ULTRA-LONG WAVELENGTH SKY MODEL SOFTWARE

A.1. The structure of the code

We developed a Python code package named ULSA (Ultra-Long wavelength Sky model with Absorption) to generate the sky map at very low frequencies, taking into account the free-free absorption effect by free electrons in both discrete HII clumps and the WIM. The code first sets up the model for the Galactic emissivity distribution by fitting available observational data at higher frequencies, and the free electron distribution, then generate the absorbed full-sky map. The code files are organized as follows (all under the ULSA directory), with brief explanation in italics:

- **spectral_index_fitting/** Spectral indices, three models are provided.
  - spectral_index_constant.py
  - spectral_index_frequency_dependent.py
  - spectral_index_direction_dependent.py
- **emissivity_fitting/** Set up emissivity model
  - produce_data_for_fitting.py: Pre-process the data for emissivity fitting, removing nearby structures “NPS” and “Loop I”
  - fit_emissivity_params.py: Fit the emissivity parameters.
- **NE2001/** The Galactic electron model
- **sky_map/** Make the sky map.
  - produce_absorbed_sky_map.py

The computations are done as the following three steps:
1. Spectral Index Parameter Fitting. As described in the text, we have developed three models, with (i) constant spectral index; (ii) frequency-dependent spectral index (iii) direction-dependent spectral index. The user may choose one of these. In this step the code produces the spectral index parameters for use in the next step. Once this is done, the sky map can be extrapolated from 408 MHz to lower frequencies.

2. Galactic Emissivity Model Construction. We then derive the emissivity model parameters using the code in the `emissivity_fitting/` directory. First, we pre-process the data by removing local structures such as the NPS and Loop I from the 408 MHz map, then the emissivity model parameters are obtained by fitting this map.

3. Low Frequency Sky Map Generation. Finally, the sky map is generated with the emissivity model constructed above and free-free absorption effect computed using the NE2001 model.

To run the code, one can simply call the `absorption_JRZ` function, which serves as the drive routine, with appropriate choice of various parameters,

```python
>>> from ULSA.sky_map.produce_absorbed_sky_map import absorption_JRZ
>>> f = absorption_JRZ(v, nside, index_type, distance, using_raw_diffuse, using_default_params, critical_dis, output_absorp_free_skymap).mpi()
```

The parameters in the command are

- `v` (float): frequency in MHz for the output map;
- `nside` (int): the Healpix NSIDE value for the sky map;
- `index_type` (str): spectral index modeling option, one of ‘constant_index’, ‘freq_dependent_index’, ‘direction_dependent_index’;
- `distance` (kpc): maximum integration distance along line of sight, default is set to 50 kpc;
- `using_raw_diffuse` (bool): if False, the data will be smoothed by Gaussian kernel, otherwise use the raw data;
- `v_file_dir` (dict): a dictionary structure used to specify additional input map data. Specify the frequency of the map by the dictionary key, and the relative path of the map data as dictionary value. The input sky map file should be in HDF5 format, such as `{XX:”/dir/xxx.hdf5”}`. if None, the spectral index is calculated with the existing data.
- `using_default_params` (bool): if True, use the default spectral index value in the code, or re-calculate the spectral index value if False.
- `input_spectral_index` (array): one can specify the spectral index value by putting in an array containing the spectral index map in the direction-dependent case, or containing one element for the constant or frequency dependent cases.
- `params_408` (list): the emissivity model parameters \([A, R_0, \alpha, Z_0, \gamma]\) obtained by fitting the Haslam 408 MHz sky map. If this parameter is omitted, the values given in Table 1 will be used as defaults. One can also specify these parameters directly by putting in the values, or force the code to re-fit by setting it to [0.,0.,0.,0.,0.].
- `critical_dis` (bool): if True, calculating the half-brightness distance, otherwise this is not calculated.
- `output_absorp_free_skymap` (bool): if True, produce absorption-free sky map at frequency `v` as well.

The function returns a two dimensional matrix, the first column is the Healpix pixel number, and the second column gives the corresponding sky map. One can then plot this map using `healpy.mollview`. The default coordinate system is set to the Galactic coordinate system, but one can transform it to the equatorial coordinate system by calling the Healpix function `f.change_coord(m,[“G”,“C”])`. See the `healpy` documentation for more information. Additionally, some results are also automatically as an HDF5 file. The map is saved in a file with name of the form `freqMHz_sky_map_with_absorption.hdf5`, where `freq` is the frequency in MHz. This file contains two keys which are respectively named “spectral_index” for spectral index map, and “data” for the sky map at the given frequency.
An absorption-free sky map is computed as intermediate result though not saved automatically, but if the parameter `output_absorb_free_skymap` is set as True, it will be saved in a file named in the format `freqMHz_absor_p_free_skymap.hdf5`. Moreover, the half-brightness distance will be computed and saved as `freqMHz_critical_dist.hdf5` if the parameter `critical_dist` is set as True.

A.2. An example of using the code

In the example shown below, sky maps are produced at 1, 2, ... 10 MHz with the constant spectral index model.

```python
from ULSA.sky_map.produce_absorbed_sky_map import absorption_JRZ
import healpy as hp
import numpy as np
import matplotlib.pyplot as plt

def produce_sky_map():
    sky_map_list = []
    # initial params setting
    # NSIDE in healpix
    nside = 2**6
    # distance in kpc unit
    dist = 50.
    # calculate the skymap from 1 MHz to 10 Mhz
    for v in range(1,10,1):
        f = absorption_JRZ(v = v, nside = nside, index_type = 'constant_index', distance = dist, using_raw_diffuse = False, using_default_params=False, input_spectral_index=None, critical_dis = False, output_absorp_free_skymap = False)
        sky_map_list.append(f.mpi())
    # we got a list of sky_map with frequency from 1 MHz to 10 MHz with step 1 MHz.
    return sky_map_list

# then plot the data using mollview
def plot():
    sky_map_list = produce_sky_map()
    plt.figure(1)
    for sky_map in sky_map_list:
        hp.mollview(np.log10(sky_map),cmap = plt.cm.jet)
        plt.show()
# or plt.savefig('xxx.eps',format='eps')
plot()
```

A.3. Adding new input map data

All the observation maps used in this work are under the directory `obs_sky_data/`, one can also add new map data (e.g. at additional frequency) there. The map file should be in the HDF5 format, with first column HEALPix pixel number, second column sky temperature values in equatorial coordinate. Most observed data in their original form is in this coordinate; if not, one must convert it into equatorial coordinate. To use such new data, first, put the new data under `obs_sky_data/`; then, when calling the our function, register the new data by a dictionary whose key is the frequency of this new input map, and the value is the path relative to `obs_sky_data/`. For example, if there is a new map at 22 MHz, one first creates a file `22MHz_sky_map.hdf5`, putting it under `obs_sky_data/22MHz/22MHz_sky_map.hdf5`; then, when calling the `absorption_JRZ` function, provide `v_file_dir = {22:”/22MHz/22MHz_sky_map.hdf5”}`. The code will then add the new data when calculating the spectral index.

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