The squeezed thermal reservoir as a generalized equilibrium reservoir

Gonzalo Manzano\textsuperscript{1,2}  
\textsuperscript{1}Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126, Pisa, Italy  
\textsuperscript{2}International Center for Theoretical Physics, Strada Costiera 11, Trieste 34151, Italy  
(Dated: June 21, 2018)

We explore the perspective of considering the squeezed thermal reservoir as an equilibrium reservoir in a generalized Gibbs ensemble with two non-commuting conserved quantities. We outline the main properties of such a reservoir in terms of the exchange of energy, both heat and work, and entropy, giving some key examples to clarify its physical interpretation. This new paradigm allows for a correct and insightful interpretation of all thermodynamical features of the squeezed thermal reservoir, as well as other similar non-thermal reservoirs, including the characterization of reversibility and the first and second laws of thermodynamics.

I. INTRODUCTION

Thermodynamics is arguably one of the most robust and successful theories in physics. The source of its strength and wide scope resides on the generality and simplicity of their foundational principles, the laws of thermodynamics. Still nowadays they continue to provide us with new insights in a broad range of physical systems, from black holes down to the microscopic world, and entering the quantum realm \cite{1,2}. In particular, one of the main questions which has recently attracted great attention is the development of a thermodynamic description of quantum effects \cite{3} and the elucidation of its role from an operational point of view \cite{4}.

In this context, ongoing discussions concern the correct interpretation of the energetics and entropy dynamics of systems interacting with quantum non-thermal reservoirs \cite{5–8}. These are reservoirs for which the presence of a quantum property such as coherence \cite{9}, quantum correlations \cite{10} or squeezing \cite{12,13} renders its state nonthermal, that is, which cannot longer be described by a Gibbs state. The use of non-thermal reservoirs in heat engine setups lead to striking situations. In particular, for the case of the squeezed thermal reservoir, these situations include the possibility of work extraction from a single reservoir \cite{6}, the emergence of multiple operational regimes \cite{6,7}, and the surpassing of Carnot’s bound \cite{5,6,11–14}. Some of these predictions has been indeed recently demonstrated in the laboratory in a proof-of-principle experiment \cite{15}.

Nevertheless, even if it is clear that traditional thermodynamic inequalities developed for thermal reservoirs cannot be directly applied to non-thermal ones (e.g. Carnot bound), the laws governing the thermodynamic behavior of such reservoirs still requires clarification. Until now, non-thermal reservoirs such as the squeezed thermal reservoir, has been essentially conceptualized as stationary non-equilibrium reservoirs, constructed e.g. from reservoir engineering techniques \cite{5,12}. Under this view, their role is similar in spirit as the one played by the nonequilibrium environmental conditions in flashing ratchets \cite{18}, molecular motors \cite{19}, or information reservoirs \cite{20–22}. This perspective raised the belief that reversible operations are forbidden when using non-thermal reservoirs, as explicitly manifested in most recent works focusing on the energetics of quantum engines from the point of view of ergotropy \cite{8}.

In this work we show that the squeezed thermal reservoir, contrary to the previous belief, can be regarded as a generalized equilibrium reservoir \cite{23,24}, which exchanges two non-commuting (conserved) quantities with the system to which it is coupled: energy and second order coherence, which we call asymmetry \cite{6}. Squeezing is a paradigmatic quantum effect rooted in Heisenberg’s uncertainty principle. It can be defined as the reduction in the uncertainty of some observable at expenses of the increase in the conjugate one \cite{16}, finding notable applications in quantum metrology, computation, cryptography and imaging \cite{17}. Our proposal allows for a correct and insightful interpretation of all thermodynamical features of the squeezed thermal reservoir, as well as other similar non-thermal reservoirs, including the first and second laws of thermodynamics. Starting with the analysis of entropy changes in the reservoir, we are able to distinguish a work contribution associated to the transfer of squeezing, much in the spirit of chemical work for heat and particle reservoirs, or as in other work producing reservoirs \cite{26}. Finally, we provide a generic reversible protocol for processes in contact with a squeezed thermal reservoir, and apply the framework to the experimentally relevant situation of extracting work from a single reservoir \cite{15}. Our results show the deep link between the extractable mechanical work, coherence, and the work performed by the reservoir.

II. THE SQUEEZED THERMAL RESERVOIR

Let us start by first defining the squeezed thermal reservoir. We consider a set of $N$ non-interacting bosonic modes with Hamiltonian $H_R = \sum_k H^{(k)}_R = \sum_k \hbar \omega_k b_k^\dagger b_k$, with bosonic ladder operators $[b_k, b_l^\dagger] = \delta_{k,l}$. Assume that each of them is in a squeezed thermal state, that is, a canonical Gibbs state at some inverse temperature $\beta_0$...
to which the squeezing operator has been applied

$$
\rho_R^{(k)} = S(\xi) \frac{e^{-\beta_R H_R^{(k)}}}{Z_k} S(\xi),
$$

(1)

where $Z_k = \text{Tr}[e^{-\beta_R H_R^{(k)}}]$ is the partition function, $S(\xi) = \exp[\frac{1}{2} \left( b_k^2 \xi^* - b_k^2 \xi \right)]$ is the unitary squeezing operator and $\xi \equiv |\xi| e^{i\theta}$ is the complex squeezing parameter. Along this paper we will consider without loss of generality $\theta = 0$.

Our main motivation is to interpret the above state as a generalized Gibbs ensemble \[23, 24, 26-30\]. In order to do this, we immediately notice that, since $S(\xi)$ is unitary, and whenever $H_R^{(k)}$ is quadratic, we can rewrite (1) in a more convenient way:

$$
\rho_R = e^{-\beta (H_R - \mu A_R)},
$$

(2)

Here we defined the following inverse temperature and a chemical-like potential of squeezing:

$$
\beta \equiv \beta_0 \cosh(2\xi), \quad \mu \equiv \tanh(2\xi),
$$

(3)

together with the quantity

$$
A_R^{(k)} \equiv \frac{-\hbar \omega_k}{2} \left( b_k^2 + b_k^2 \right),
$$

(4)

with energy units. We will refer to (4) as the second-order moments asymmetry or just the asymmetry for reservoir mode $k$, since it can be rewritten as $A_R^{(k)} = (\hbar \omega_k/2)(p_k^2 - x_k^2)$, where $[x_k, p_i] = i \hbar \delta_{ki}$ are the (dimensionless) position and momentum quadratures of the modes [6].

In the following, we will show that this reservoir behaves in a very similar way to a particle reservoir at thermal equilibrium, but replacing the number of particles by the asymmetry for a bosonic system as the energy change between system and reservoirs \[1, 2\]. While the later identifications is correct when handling with thermal reservoirs, the presence of further conserved quantities requires an approach based on the reservoir entropy.

Eq. (6) allows a physical interpretation of $\mu$ as the change in non-equilibrium free energy needed to increase the asymmetry of the reservoir by a given amount. The nonequilibrium free energy with respect to temperature $T$ is defined as $F_R(\rho_R) \equiv \text{Tr}[H_R \rho_R] - k_B T S(\rho_R)$ [31]. It characterizes the optimal amount of work extractable from a generic system with Hamiltonian $H_R$ in state $\rho_R$ with the help of a thermal reservoir at temperature $T$ [31-33]. Identifying $\beta = 1/k_B T$ in Eq. (3) as the inverse temperature, and using Eq. (6), we have that for any infinitesimal change $\rho_R \rightarrow \rho_R$, the change in nonequilibrium free energy is $\Delta F_R = \Delta E_R - k_B T \Delta S_R = \mu \Delta A_R$. That is, the extractable work from this subset of modes, when access to a thermal reservoir at $\beta$ is possible (e.g. other modes of the present reservoir after dephasing) is directly proportional to the asymmetry of the subset $A_R$. Remarkably, this leads to interpret the quantity $\mu = \Delta F_R/\Delta A_R$, as the work needed to increase the asymmetry by one unit. Therefore $W_R = \mu \Delta A_R$ is the analogous of a chemical work, representing the work needed to increase the asymmetry of the reservoir.

In the following, we will show that this reservoir behaves in a very similar way to a particle reservoir at thermal equilibrium, but replacing the number of particles by the asymmetry for a bosonic system as the energy change between system and reservoirs \[1, 2\]. While the later identifications is correct when handling with thermal reservoirs, the presence of further conserved quantities requires an approach based on the reservoir entropy.

III. REVERSIBLE PROCESSES AND ENTROPY PRODUCTION

Until now we have just elucidated the properties of a squeezed thermal reservoir made up by non-interacting bosonic modes, and characterized by fixed parameters $\beta_0$ and $\xi$ (or equivalently $\beta$ and $\mu$). Now we may move to the description of the interaction between the reservoir and a system of interest. In particular, we consider a driven system, typically another bosonic mode with Hamiltonian $H_S(\lambda) = \hbar \omega_S a_S^\dagger a_S$, where $\omega_S$ and $a_S$ may be time-dependent through the external variation of a control parameter $\lambda(t)$. The system interacts with the resonant modes of the reservoir through an interaction Hamiltonian $H_{\text{int}}(\lambda)$.

In order to model the dynamical evolution we consider a sequence of infinitesimal processes where the system interacts with resonant modes in the reservoir following unitary evolution $U_S = \exp \left[ -i \tau (H_S + H_R + H_{\text{int}})/\hbar \right]$. Here we assume that this interaction occurs in a timescale $\tau$ much faster than the external driving, so that $\lambda$ can be considered constant during the interaction. The much weaker interaction with non-resonant modes in the
reservoir can be neglected [6]. For each interaction system and reservoir start in a generic product state $\rho_S \otimes \rho_R$ with $\rho_R$ in Eq. (2), and change to $\rho'_S = \text{Tr}_R[\rho'_{SR}]$ and $\rho'_R = \text{Tr}_S[\rho_{SR}]$, with $\rho'_{SR} = U_\lambda(\rho_S \otimes \rho_R)U^\dagger_\lambda$. After that, the control parameter changes infinitesimally $\lambda \to \lambda'$, the reservoir mode is replaced by a “fresh” mode in the same state $\rho_R$, and the next interaction takes place (see Fig. 1). This kind of repeated interaction scheme can be modeled by a sequence of completely positive and trace-preserving (CPTP) maps [34–36] leading to the development of Lindblad master equations [6, 34, 37] and quantum jump trajectories [38, 39], whose thermodynamic properties can be addressed even for arbitrary environments [25].

The key assumption of the dynamical evolution will be the presence of conserved quantities. The interaction between system and reservoir $H_{\text{int}}$ should exactly preserve both energy and asymmetry in the global system at any time, that is $[H_S + H_R, H_{\text{int}}] = 0$ and $[A_S + A_R, H_{\text{int}}] = 0$. Here the asymmetry in the system $A_S$ is analogously defined as in Eq. (4) by $A_S(\lambda) \equiv \frac{a^2_\lambda + a_\lambda^2}{2}$ for any value of the external parameter $\lambda$. The later condition for the commutators leads to the conservation of average total energy and asymmetry for any interaction step of the evolution:

$$\Delta E_S + \Delta E_R = 0, \quad \Delta A_S + \Delta A_R = 0,$$

(8)

where $\Delta E_S = \text{Tr}[H_S(\rho'_S - \rho_S)]$, $\Delta A_S = \text{Tr}[A_S(\rho'_S - \rho_S)]$ and analogously for the reservoir. It is worth noticing that this assumption indeed prevents sources of energy or asymmetry from switching on/off interactions considered in [37, 40].

Moreover, taking into account that the system is driven by some external controller, their energy and asymmetry may change after the interaction. Then on a coarse-grained time scale involving both interaction and driving, we have

$$\dot{E}_S + \dot{E}_R = \hat{W}, \quad \dot{A}_S + \dot{A}_R = \hat{A},$$

(9)

where $\hat{W} = \text{Tr}[H_S \rho_S]$ represents the mechanical work performed by the external driver, and analogously $\hat{A} = \text{Tr}[A_S \rho_S]$ is the asymmetry induced by driving. Now combining Eqs. (7) and (9) we state the first law of thermodynamics as

$$\dot{E}_S = W + \dot{W}_\text{eq} + \dot{Q},$$

(10)

where $\dot{Q} \equiv -\dot{Q}_R$ is the heat entering the system and $\dot{W}_\text{eq} \equiv -\dot{W}_R = -\mu \dot{A}_R$ is the chemical-like working performed by the reservoir.

Conservation of energy and asymmetry in Eqs. (8) is verified e.g. by interactions of the form $a_\lambda b_k^\dagger + a_k^\dagger b_{\lambda}$, as the one used in Ref. [6]. For frozen $\lambda$, contact with the reservoir brings any initial state of the system in the long time run to the equilibrium state

$$\rho_{eq}^S(\lambda) = S_\lambda(\xi) \frac{e^{-\beta_0 H_S}}{Z_0} S_\lambda^\dagger(\xi) = \frac{e^{-\beta(H_S - \mu A_S)}}{Z_\lambda}.$$

(11)

Here $S_\lambda(\xi) = \exp[\frac{1}{2}(a_{\lambda}^2 - a_{\lambda}^{2\dagger})]$ is the squeezing operator over the system mode, and $Z_\lambda = \text{Tr}[e^{-\beta(H_S - \mu A_S)}]$. More generally, including the possibility of external driving into the system [Eq. (10)], the entropy production rate of a generic process as described previously can be calculated from the sum of von Neumann entropies of system and reservoir [25]. That results in the following statement of the second law

$$\dot{S}_\text{tot} = \dot{S} + \beta(\hat{W} + \dot{W}_\text{eq} - \hat{E}_S)$$

$$= \dot{S} + \beta(\hat{W} - \mu \hat{A}) - \beta(\hat{E}_S - \mu \hat{A}_S) \geq 0,$$

(12)

where we used Eqs. (7), (8) and (9).

Our objective now is to construct a generic reversible evolution for systems in contact with the squeezed thermal reservoir. We first consider the case of reversible work and asymmetry extraction by transforming an arbitrary initial state $\rho_S$ into the equilibrium state $\rho^S_{eq}$ in Eq. (11). Notice that the transformation $\rho_S \to \rho^S_{eq}$ might be done by simply letting the system relax in contact with the reservoir. However, this process is completely irreversible and prevent us from extracting any mechanical work nor asymmetry from $\rho_S$. On the contrary, a reversible protocol maximizing the extraction of resources can be built by extending the protocol explained in Ref. [31] for standard thermal reservoirs (see also Ref. [41]) to the case of a squeezed thermal reservoir.

This extraction process consists in two steps:

1. An initial quench of the system Hamiltonian from $H_S(\lambda_0)$ to $H_S(\lambda_\star) \equiv H_\star$, where

$$H_\star \equiv -S_\lambda^\dagger(\xi) \ln(\rho_S^\dagger) S_\lambda(\xi)/\beta_0.$$

(13)

This is implemented by a sudden change in the control parameter $\lambda_0 \to \lambda_\star$, which implies as well $A_S(\lambda_0) \to A_S(\lambda_\star) \equiv A_\star$. After the quench we still have $\rho^S_0(\xi) H_\star S_\lambda^\dagger(\xi) = \beta(H_\star - \mu A_\star) + \text{sink}^2(\xi)$, for same reservoir parameters. In this step the system state does not change, while a work $W_\alpha = \text{Tr}[H_\star - H_S] \rho^S_0]$ and asymmetry $A_\star = \text{Tr}[A_\star - A_S] \rho^S_0]$ are required. At the end of the step $\rho_S = \exp(-\beta(H_\star - \mu A_\star))/Z_\star$, i.e. the state $\rho_S$ is now
the (new) equilibrium state of the system for \( H_\ast \) and \( A_\ast \). Notice that this can be always done provided that the initial state \( \rho_S \) is positive definite.

2. A quasi-static driving transforming back the Hamiltonian \( H_\ast \) and \( A_\ast \) to their initial forms. During this transformation the control parameter changes quasi-statically \( \lambda_\ast \rightarrow \lambda_0 \) and the system is maintained in instantaneous equilibrium with the reservoir at any time [Eq. 11)]. This implies zero entropy production during the process, and hence \( \dot{Q} = k_B T \dot{S} \). In App. B we indicate a particular realization of this quasi-static process using a concatenation of CPTP maps.

Using the above protocol a maximal amount of work and asymmetry can be extracted from the initial state \( \rho_S \). We integrate Eq. (12) for the equality case, where \( \beta(\dot{W} - \mu A) = -\dot{Z}_\lambda/Z_\lambda \). Finally we add up the contributions from the quench and quasi-static steps, identifying \( W_{\text{ext}} = -W - W_q \) as the total work extracted and \( A_{\text{ext}} = -A_q - \int \text{Tr}[A_\lambda \rho_{S_\lambda}^\ast]d\lambda \) as the total asymmetry extracted, we obtain

\[
W_{\text{ext}} - \mu A_{\text{ext}} = \Omega(\rho_S) - \Omega(\rho_S^\ast) = k_B T D(\rho_S||\rho_S^\ast) \geq 0. \tag{14}
\]

Here the quantity \( \Omega(\rho_S) = \text{Tr}[(H_S - \mu A_S)\rho_S] - k_B T S(\rho_S) \) is a potential generalizing the non-equilibrium free energy for non-equilibrium systems interacting with a squeezed thermal reservoir. Moreover, in the last line we introduced the relative entropy \( S(\rho||\sigma) = \text{Tr}[\rho \ln \sigma - \ln \rho] \geq 0 \), which reaches zero if and only if \( \rho = \sigma \). \[42\]

The interpretation of Eq. (14) is now clear, it tell us that an out-of-equilibrium state \( \rho_S \) is a resource providing a positive amount of extractable mechanical work on the top of the extracted asymmetry. Indeed the above equation (14) can be rephrased as

\[
W_{\text{ext}} = k_B T D(\rho_S||\rho_S^\ast) + W_{\text{sq}} - \mu \Delta A_S. \tag{15}
\]

Notice that even when our initial state is in equilibrium, \( \rho_S = \rho_S^\ast \), extraction of mechanical work is not forbidden anymore, but it is allowed by cyclic extraction of asymmetry from the reservoir, \( W_{\text{ext}} = \mu A_{\text{ext}} = W_{\text{sq}} \).

The previous protocol may be applied to more general transformations \( \rho_S \rightarrow \rho_S^\ast \) by including slight modifications. In such case step 1. is applied exactly as before, but in the quasi-static step 2. we replace the final value of the control parameter to \( \lambda_\ast^\prime \) such that \( H^\prime_\ast \equiv -S^\dagger(\xi) \ln(\rho_S^\ast(\xi))/\beta_\ast \). This implies that the system after quasi-static driving is now \( \rho_S^\ast \). Then a final step is included: 3. A final sudden quench \( \lambda_\ast^\prime \rightarrow \lambda \) which turns back the Hamiltonian and asymmetry operator to their original forms, \( H_S \) and \( A_S \). Notice that this extension is indeed equivalent to the combination of two reversible strokes, \( \rho_S \rightarrow \rho_{eq} \) as before, followed by the inversion of \( \rho_S^\ast \rightarrow \rho_{eq} \).

IV. CYCLIC WORK EXTRACTION FROM A SINGLE RESERVOIR

Once reversible protocols have been introduced we may now discuss work extraction from a single squeezed thermal reservoir. The idea is to introduce a thermodynamic cycle on the system which combines unitary processes on the system and interaction with the squeezed thermal reservoir. In Ref. [6], we already introduced a thermodynamic cycle for work extraction. Starting with the system in \( \rho_S^\ast \) in Eq. (11), it combines a unitary step unsqueezing the mode \( U_1 = S^\dagger \) and leaving the system in the state \( \rho_S = e^{-\beta \ast H_S}/Z \), with simple relaxation of the mode by contact with the squeezed thermal reservoir bringing the state of the system back to \( \rho_S^\ast \). In such protocol, work is extracted in the unitary step \( W_1 \), while during the second process the system only exchanges energy with the reservoir [15]. This second step is intrinsically irreversible (isochoric process) and therefore it does not allow for an optimal use of the resources. Our aim here is to replace this second step by a reversible one.

We consider a generic two-step process with an arbitrary initial unitary step \( U_1 \), leaving the system in a rather arbitrary state \( \rho_S \), while extracting work \( W_1 = \text{Tr}[H_S(\rho_S^\ast - \rho_S)] \) in this step asymmetry may also be extracted as \( A_1 = \text{Tr}[A_S(\rho_S^\ast - \rho_S)] \), while the entropy production is zero. Now in the second step we turn back to \( \rho_S^\ast \) in contact with the environment, but also allowing external driving, which eventually leads to extracting an extra amount of work \( W_2 \) during the process, together with asymmetry \( A_2 \). For this second step we may apply Eq. (12). Integrating and using that by construction \( S(\rho_S) = S(\rho_S^\ast) \), \( \Delta E_S = W_1 \), and \( \Delta A_S = A_1 \), a bound for the total work extracted \( W_{\text{ext}} = W_1 + W_2 \) is obtained

\[
W_{\text{ext}} \leq \mu(A_1 + A_2) = -\mu \Delta A_R = W_{\text{sq}}. \tag{16}
\]

That is, in any cyclic process one may extract an amount of work less or equal than the squeezing chemical-like work performed by the reservoir. This indeed provides an extra motivation to consider \( W_{\text{sq}} \) as work and the squeezed thermal reservoir as a work producing reservoir [26].

The key point for reaching the equality in Eq. (16) is nothing but fully avoiding any irreversibility in the second step of the process. This is accomplished by using the reversible process introduced below. From Eq. (14) it follows that \( W_2 = \Omega(\rho_S) - \Omega(\rho_S^\ast) + \mu A_2 = -W_1 + \mu(A_1 + A_2) \), and the optimal amount of work \( W_{\text{ext}} = W_1 + W_2 = \mu(A_1 + A_2) = W_{\text{sq}} \) is extracted. Notice that this extraction protocol do not need any particular form of \( \rho_S \) and \( U_1 \), e.g. we may use \( U_1 = I \) [cf. Eq. (15)]. In any case, the chemical-like squeezing work extracted from the reservoir equals the asymmetry extracted by the external driver. Therefore, this can be regarded as an example of a squeezing into work conversion in the context of a generalized resource framework [26].

Finally, it is worth mentioning that the maximum extractable work from a single reservoir \( W_{\text{sq}} \) is larger in
and the ergotropy $\mathcal{W}$ (dashed curve). In the inset we show their ratio, which becomes more important when increasing the squeezing parameter $\xi$. In both plots the black dashed vertical line indicates the boundary between classical and quantum regimes for $\hbar \omega \sim 10 k_B T_0$, leading to $\xi^* \sim 1.5$.

FIG. 2. (color online) Comparison of the optimal work reversibly extractable from a single squeezed thermal reservoir $W_{eq}$ (solid curve) with the protocol detailed in App. C and the ergotropy $\mathcal{W}$ (dashed curve). In the inset we show their ratio, which becomes more important when increasing the squeezing parameter $\xi$. In both plots the black dashed vertical line indicates the boundary between classical and quantum regimes for $\hbar \omega \sim 10 k_B T_0$, leading to $\xi^* \sim 1.5$.

The variation of the Hamiltonian $\hbar \omega$ indicates the boundary between classical and quantum regimes.

A3

A3

general than the so-called ergotropy $\mathcal{W}$ of the non-passive state $\rho_S^{eq}$ induced by the squeezed thermal reservoir. The later is defined as the maximum work extractable from a state using only unitary operations describing a cyclic variation of the Hamiltonian [43]. It can be straightforwardly seen that the ergotropy of the state $\rho_S^{eq}$ is given by $W_1$ when $U_1 = S^\dagger$ [6]. We notice that this is just one part of the total extractable work, which can be made arbitrarily larger when increasing $\xi$, as can be seen in Fig. 2 (details are given in App. C). Moreover, we see there that larger amounts of work can be obtained in the quantum regime $\xi > \xi^* \equiv \ln \coth (\beta_0 \hbar \omega)/2$, where the uncertainty in the squeezed quadrature falls below shot noise.

V. CONCLUSIONS

Our formulation of the first and second laws of thermodynamics solve the major problem of how to correctly define work and heat in a broad range of non-thermal reservoirs, such as the squeezed thermal reservoir or the displaced thermal reservoir. In particular, it shows how previous approaches based on the naive consideration of any energy exchanged with the reservoir as heat [5, 6, 11, 12], or in the concept of ergotropy [7, 8] fails to provide a complete explanation of the energetics. We expect that the identification of the chemical-like squeezing work provided here together with the generic reversible protocols may have a strong impact in new designs of quantum thermal machines or other devices combining thermal and squeezing effects.

Acknowledgements. I would like to thank Juan M. R. Parrondo for encouraging me to do this work and Rosario Fazio for useful comments and discussions. I acknowledge financial support from the Horizon 2020 EU collaborative project QuProCS (Grant Agreement No. 641277).

Appendix A: Proof of Eq. (5)

In the following we provide a proof of Eq. (5) in the main text, adapting the one given previously in Ref. [25]. Consider a change in the state of the reservoir

$$\rho_R^{(k)(\epsilon)} = \rho_R^{(k)} + \epsilon \Delta \rho_R^{(k)},$$  

(A1)

where $\rho_R^{(k)(\epsilon)}$ is the state of the reservoir after some arbitrary interaction with other system, $\rho_R^{(k)}$ is the original state, $\Delta \rho_R^{(k)}$ is a traceless operator accounting for the change, and $\epsilon \geq 0$ is a positive (small) number. In the following we will calculate the change in the von Neumann entropy associated to this process, that is, $\Delta S_R = S(\rho_R^{(k)(\epsilon)}) - S(\rho_R^{(k)})$.

In order to proceed we need to calculate the eigenvalues and eigenvectors of the state $\rho_R^{(k)(\epsilon)}$, which we call $\{\lambda_n^{(\epsilon)}, |\lambda_n^{(\epsilon)}\rangle\}$. This can be done by using perturbation theory when $\epsilon \ll 1$. Indeed, up to second order in $\epsilon$ we will have:

$$\lambda_n^{(\epsilon)} \simeq \lambda_n + \epsilon \lambda_n^{(1)} + \epsilon^2 \lambda_n^{(2)},$$

$$|\lambda_n^{(\epsilon)}\rangle \simeq |\lambda_n\rangle + \epsilon |\lambda_n^{(1)}\rangle + \epsilon^2 |\lambda_n^{(2)}\rangle,$$  

(A2)

where $\{\lambda_n, |\lambda_n\rangle\}$ are the eigenvalues and eigenvectors of $\rho_R^{(k)}$, and we have also introduced the first and second order contributions. In particular the first order ones read

$$\lambda_n^{(1)} = (\lambda_n |\Delta \rho_R^{(k)}| \lambda_n\rangle),$$

(A3)

$$|\lambda_n^{(1)}\rangle = \sum_{l \neq n} \frac{|\lambda_n| |\rho_R^{(k)}| \lambda_l\rangle}{\lambda_n - \lambda_l} |\lambda_l\rangle.$$  

(A4)

Then we may write the change in entropy as

$$\Delta S_R = S(\rho_R^{(k)(\epsilon)}) - S(\rho_R^{(k)}) = - \sum_n \lambda_n^{(\epsilon)} \ln \lambda_n^{(\epsilon)} + \sum_m \lambda_m \ln \lambda_m$$  

(A5)

and using Eqs. (A2) we obtain up to second order in $\epsilon$:

$$\Delta S_R \simeq - \epsilon \sum_n \lambda_n^{(1)} \ln \lambda_n - \epsilon^2 (\sum_n \lambda_n^{(2)} \ln \lambda_n + \sum_m \lambda_m^{(2)})/2$$  

(A6)

If we now drop the second order term and use Eq. (A3) we get

$$\Delta S_R \simeq - \epsilon \sum_n \lambda_n^{(1)} \ln \lambda_n = - \epsilon \sum_n \langle \lambda_n |\Delta \rho_R^{(k)}| \lambda_n\rangle \ln \lambda_m$$  

(A7)

which proofs Eq. (5) of the main text.
Appendix B: Reversible protocol

Here we provide a particular realization of the protocol for reversible transformations with a squeezed thermal reservoir. It will be based on a concatenation of completely positive and trace preserving (CPTP) maps. As explained in the main text, for a bosonic mode with Hamiltonian $H$ and density operator $\rho$, the protocol consists in a first sudden quench, where the Hamiltonian changes as $H \rightarrow H_\ast \equiv -S \ln \rho S^\dagger / \beta_0$ while leaving the system in state $\rho_0$. Then this is followed by a quasi-static process where $H_\ast$ transforms back to $H$. During this path the system remains in equilibrium with the squeezed thermal reservoir at any time, ending thus in $\rho = S e^{-\beta H S^\dagger} / Z$. Notice that here for the ease of notation we drop the subscript $S$ in all quantities.

In order to give a map describing the quasi-static process, we extend the one developed in Ref. [41], where an isothermal processes for the case of a thermal reservoir were constructed by alternating infinitesimal adiabatic and isochoric steps [45]. Let’s assume the following sequence of CPTP maps $E_1 \circ E_2 \circ \ldots \circ E_N$, with $N \rightarrow \infty$. Each map in the sequence is intended to describe an infinitesimal time step, for which $E_n(\rho_{n-1}) = \rho_n$. The changes in the Hamiltonian in the sequence can be also written as $H_{n-1} \rightarrow H_n$, and we set for consistency $H_N \equiv H$.

Now we decompose every CPTP map $E$ in the sequence in the following two steps

$$E_n(\rho) \equiv G_n \circ U_n(\rho). \quad (B1)$$

We introduced $U_n(\rho_{n-1}) = \rho_{n-1}$ as a unitary sudden quench of the system Hamiltonian, where $H_{n-1} \rightarrow H_n$ instantaneously. The second step is provided by a CPTP map verifying $G_n(Se^{-\beta_0 H_n S^\dagger} / Z_n) = S e^{-\beta_0 H_n S^\dagger} / Z_n$, that is, a generalized Gibbs-preserving map, which describes the interaction with the environment. During the action of $G_n$ the Hamiltonian is assumed to remain constant.

The key condition to ensure a reversible process is that the change in the entropy of the system equals (minus) the change in entropy of the reservoir, here given by the last term in Eq. (13) of the main text:

$$\Delta S_n \equiv S(\rho_n) - S(\rho_{n-1})$$

$$\quad = \beta_0 \text{Tr}[(H_n - \mu A_n)S^\dagger(\rho_n - \rho_{n-1})]$$

$$\quad = \beta_0 \text{Tr}[SH_nS^\dagger(\rho_n - \rho_{n-1})] \equiv -\beta Q_n. \quad (B2)$$

where $A_n = [p_{n/2}^2 - x_{n/2}^2]/2$ is the asymmetry. Here we have only $H_n$ in the expression because the entropy of the system only changes during the second step, $G_n(\rho_{n-1}) = \rho_n$, when it interacts with the reservoir. This requires that the system state is close to the instantaneous equilibrium state for every step, namely $S e^{-\beta H_n S^\dagger} / Z_n$. In order to warranty this, we show that when assuming an infinitesimal change in the drive during any step, that is

$$H_n = H_{n-1} + \epsilon \Delta H_n \quad (B3)$$

with $\epsilon \ll 1$, then the sequence of CPTP maps defined by Eq. (B1) verify Eq. (B2) up to first order in $\epsilon$.

In order to give a proof we proceed as follows. First, we will show that if the state of the systems starts close to $\rho_{n-1}$ before the map, then, after the application of the map $E_n$ it remains close to the equilibrium state $\rho_n$. We can rewrite this condition as

$$\rho_n = G_n \left( S e^{-\beta_0 H_{n-1} S^\dagger} / Z_{n-1} + \epsilon \Delta \rho_{n-1} \right)$$

$$= S e^{-\beta_0 H_n} / Z_n S^\dagger + \epsilon \Delta \rho_n + O(\epsilon^2), \quad (B4)$$

where $\Delta \rho_{n-1}$ and $\Delta \rho_n$ are traceless operators accounting for the deviations from the equilibrium states before and after the map. If the above condition is verified, then our construction is self-consistent. Then, as a second step, we will proof that, since we may always rewrite $\rho_n = \rho_{n-1} + \epsilon \sigma_n$ for a suitable traceless $\sigma_n$ (in general $\Delta \rho_n \neq \sigma_n$), this implies Eq. (B2).

We introduce Eq. (B3) into the left-hand-side of Eq. (B4). Then we use the expansions

$$e^{\epsilon \beta_0 \Delta H_n} = 1 + \epsilon \beta_0 \Delta H_n + O(\epsilon^2), \quad (B5)$$

$$Z_{n-1} = Z_n[1 + \epsilon \beta_0 \text{Tr}[\Delta H_n] + O(\epsilon^2)], \quad (B6)$$

which combined with linearity give us the following result

$$G_n \left( S e^{-\beta_0 H_{n-1} S^\dagger} / Z_{n-1} + \epsilon \Delta \rho_{n-1} \right) = G_n \left( S e^{-\beta_0 H_n} / Z_n \left[ 1 + \epsilon \beta_0 \Delta H_n + O(\epsilon^2) \right] S^\dagger \right) + \epsilon \ G_n(\Delta \rho_{n-1})$$

$$= G_n \left( S e^{-\beta_0 H_n} / Z_n \left[ 1 + \epsilon \beta_0 \Delta H_n - \text{Tr}[\Delta H_n] + O(\epsilon^2) \right] S^\dagger \right) + \epsilon \ G_n(\Delta \rho_{n-1})$$

$$= S e^{-\beta_0 H_n} / Z_n S^\dagger + \epsilon \left[ G_n(\Delta \rho_{n-1}) + \beta_0 G_n(S e^{-\beta_0 H_n} \Delta H_n S^\dagger) - \beta_0 \text{Tr}[\Delta H_n S e^{-\beta_0 H_n} S^\dagger] \right] + \epsilon \Delta \rho_n.$$
If we now make the identification

$$\Delta \rho_n \equiv G_n(\Delta \rho_{n-1}) + \beta_0 G_n(S \frac{e^{-\beta_0 H_n}}{Z_n} \Delta H_n S^\dagger)$$

$$- \beta_0 \text{Tr}[\Delta H_n] \frac{e^{-\beta_0 H_n}}{Z_n} S^\dagger,$$  \hspace{1cm} (B8)

then Eq. (B4) is recovered. The second part of the proof now follows by obtaining the traceless matrix $\sigma_n$. Using Eq. (B8) and the above expansions it reads:

$$\sigma_n \equiv \rho_n - \rho_{n-1} = \epsilon [E_n(\Delta \rho_{n-1}) - \Delta \rho_{n-1}]$$

$$+ \beta_0 \mathcal{E}_n(S \frac{e^{-\beta_0 H_n}}{Z_n} \Delta H_n S^\dagger - S \frac{e^{-\beta_0 H_n}}{Z_n} \Delta H_n S^\dagger),$$  \hspace{1cm} (B9)

which, as expected is of order $\epsilon$. Then, we may express the eigenvalues and eigenvectors of $\rho_n$, the set $\{p^k_n, |\psi_n^k\rangle\}$, in terms of the corresponding ones for $\rho_{n-1}$. This needs to use the relation $\rho_n = \rho_{n-1} + \epsilon \sigma_n$, with $\sigma_n$ in Eq. (B9).

We can always write

$$p^k_n = p^k_{n-1} + \epsilon (\psi^k_{n-1}\sigma_n|\psi^k_{n-1}\rangle + O(\epsilon^2),$$  \hspace{1cm} (B10)

$$|\psi^k_n\rangle = |\psi^k_{n-1}\rangle + \epsilon \sum_{l \neq k} \frac{\langle \psi^k_{n-1}|\sigma_n|\psi^l_{n-1}\rangle}{p^l_{n-1} - p^k_{n-1}} |\psi^l_{n-1}\rangle + O(\epsilon^2).$$  \hspace{1cm} (B11)

In the other hand, we may obtain the same quantities to first order in $\epsilon$ from the equation $\rho_n = S \frac{e^{-\beta_0 H_n}}{Z_n} S^\dagger + \epsilon \Delta \rho_n + O(\epsilon^2)$. This leads to:

$$\rho^k_n = \frac{e^{-\beta_0 E^k_n}}{Z_n} + \epsilon (E^k_n|S^\dagger \Delta \rho_{n-1} S E^k_n\rangle + O(\epsilon^2),$$  \hspace{1cm} (B12)

$$|\psi^k_n\rangle = S |E^k_n\rangle + \epsilon \sum_{l \neq k} \frac{\langle E^k_n|S |\Delta \rho_{n-1} S E^l_n\rangle}{e^{-\beta_0 H_n} - e^{-\beta_0 H_n}} |E^l_n\rangle + O(\epsilon^2),$$  \hspace{1cm} (B13)

where $\{E^k_n, |E^k_n\rangle\}$ are the eigenstates and eigenvectors of the Hamiltonian $H_n$. We can now calculate the change in entropy during the n-th step of the process, described by the map $\mathcal{E}_n$. Using Eqs. (B10) and (B11), we obtain:

$$\Delta S_n = S(\rho_n) - S(\rho_{n-1})$$

$$= - \sum_k p^k_n \ln p^k_n + \sum_k p^k_{n-1} \ln p^k_{n-1} - \epsilon \sum_k \langle \psi^k_{n-1}|\sigma_n|\psi^k_{n-1}\rangle \ln p^k_{n-1} + O(\epsilon^2).$$  \hspace{1cm} (B14)

Finally, by combining Eqs. (B10) and (B12), we notice that $p^k_{n-1} = \frac{e^{-\beta_0 E^k_n}}{\sigma_n} + O(\epsilon)$. Therefore we have:

$$\ln(p^k_{n-1}) = \ln \left(\frac{e^{-\beta_0 E^k_n}}{\sigma_n}\right) + \ln[1+O(\epsilon)] = \ln \left(\frac{e^{-\beta_0 E^k_n}}{\sigma_n}\right) + O(\epsilon).$$  \hspace{1cm} (B15)

This leads us to write

$$\Delta S_n = -\epsilon \sum_k \langle \psi^k_{n-1}|\sigma_n|\psi^k_{n-1}\rangle \ln \left(\frac{e^{-\beta_0 E^k_n}}{\sigma_n}\right) + O(\epsilon^2)$$

$$= \epsilon \beta_0 \sum_k E^k_n \langle \psi^k_{n-1}|\sigma_n|\psi^k_{n-1}\rangle$$

$$= -\epsilon \text{Tr}[\Delta H_n S^\dagger \sigma_n] + O(\epsilon^2),$$  \hspace{1cm} (B16)

where, in the last step, we have used $|p^k_{n-1}| = \frac{p^k_n}{\epsilon}$ and $O(\epsilon) = S |E^k_n\rangle + O(\epsilon)$, which follows from Eqs. (B11) and (B12) and the cyclic property of the trace. Eq. (B16) corresponds to Eq. (B2) up to first order in $\epsilon$, and completes the proof.

Appendix C: Cyclic work extraction from a single reservoir details

In this last section we give some details about the extractable work and ergotropy for a particular instance of the cycle introduced in the main text. As introduced in the main text, the cycle starts with the system with Hamiltonian $H_0 = h\omega a^\dagger a \equiv H_0$ in equilibrium with the squeezed thermal reservoir at parameters $\beta = \beta_0 \cosh(2\zeta)$ and $\mu = \tanh(2\zeta)$. We consider an initial unitary stroke unsqueezing the mode $U_1 = S^\dagger(\xi)$, leading to $\rho_S = U_1 \rho^S_S U_1^\dagger = e^{-\beta_0 H_S}/Z$. During this first stroke an amount of work is extracted

$$W_1 = \text{Tr}[H_0 \rho^S_S] - \text{Tr}[H_0 \rho_S]$$

$$= h\omega \text{sinh}(2\zeta)(2n_{th} + 1) \equiv W$$  \hspace{1cm} (C1)

where $n_{th} = (e^{\beta_0 \omega} - 1)^{-1}$ is the mean number of excitations with energy $h\omega$ in a thermal reservoir at $\beta_0$. Notice that $\rho_S$ is the Gibbs state, and then it is the state of lower energy for fix entropy. This implies that the work $W_1 = W$ is by definition the ergotropy of state $\rho^S_S$. During this stroke the asymmetry extracted reads

$$A_1 = \text{Tr}[A_0 \rho^S_S] - \text{Tr}[A_0 \rho_S] = \text{Tr}[A_0 \rho^S_S]$$

$$= h\omega \text{sinh}(2\zeta)(n_{th} + 1/2).$$  \hspace{1cm} (C2)

The second stroke of the cycle is the reversible transformation of $\rho_S$ into $\rho^S_S$ as we introduced in the main text and in the previous section of this Supplemental Material. Following Eq. (14) in the main text, in the first part of the stroke (sudden quench) the Hamiltonian is modified from $H_0$ to

$$H_\star = h\omega b^\dagger b = S^\dagger(\xi) H_S S(\xi),$$  \hspace{1cm} (C3)

where $b = S^\dagger(\xi) a S(\xi)$, while the state of the system does not change. At this point we can see that effectively after the quench we have $\rho_S = S(\xi)e^{-\beta_0 H_S} S^\dagger(\xi)/Z_0$, which is the equilibrium state for Hamiltonian $H_\star$. In this step the work extracted is

$$W_2^{\text{quench}} = \text{Tr}[\rho_S H_\star] - \text{Tr}[\rho_S H_\star]$$

$$= \text{Tr}[\rho_S H_0] - \text{Tr}[S(\xi) \rho_S S^\dagger(\xi) H_0] = -W,$$  \hspace{1cm} (C4)
where we used Eq. (C3). This means that the ergotropy extracted in the first stroke is wasted in implementing the initial sudden quench of the second stroke. Analogously, for the extracted asymmetry we have \( A_{2}^{\text{quench}} = -A_{1} \), so that until now we did not gained anything at all. Nevertheless, as we will shortly see, a greater amount of work and asymmetry is going to be extracted in the quasi-static step of the stroke. This can be implemented by means of the slow change of a control parameter \( \lambda \) from \( \lambda_{s} \) to \( \lambda_{0} \), driving the Hamiltonian back to \( H_{0} \). We will parametrize this process as an unsqueezing of the Hamiltonian

\[
H_{S}(\lambda) = \hbar \omega b_{\lambda}^\dagger b_{\lambda}, \quad b_{\lambda} = S^\dagger(\lambda)aS(\lambda),
\]

with \( \lambda_{s} = \xi \) and \( \lambda_{0} = 0 \). These operations are usually implemented in quantum optical setups like degenerate parametric down conversion by passing the light from a pumping laser field through a photonic crystal with \( \chi^{(2)} \) nonlinearity (see e.g. [46]). This kind of driving in particular implies that

\[
\dot{H}_{S}(\lambda) = \hbar \omega (b_{\lambda}^\dagger b_{\lambda} + b_{\lambda}^\dagger b_{\lambda}) = -\hbar \lambda \dot{b}_{\lambda}^2 + b_{\lambda}^2)
\]

\[
= 2\lambda A_{S}(\lambda), \quad (C6)
\]

\[
\dot{A}_{S}(\lambda) = \hbar \omega (b_{\lambda}^\dagger b_{\lambda} + b_{\lambda}^\dagger b_{\lambda}) = -\hbar \omega (2b_{\lambda}^\dagger b_{\lambda} + 1)
\]

\[
= \lambda(2H_{S}(\lambda) + \hbar \omega), \quad (C7)
\]

where we used \( \dot{b}_{\lambda} = -\lambda \dot{b}_{\lambda}^\dagger \), following Eq. (C3). In the other hand, as shown in the previous section the quasi-static driving implies the state of the system being at any time \( \rho_{\text{eq}}^{S} = S(\xi)e^{-\beta_{0}H_{S}(\lambda)/\hbar}Z_{0}S^{\dagger}(\xi) \). Therefore the work extracted during the quasi-static stroke reads

\[
W_{2}^{0 \rightarrow s} = -\int_{\xi}^{0} d\lambda \text{Tr}[A_{S}(\lambda)\rho_{\text{eq}}^{S}]
\]

\[
= \hbar \omega \sinh(2\xi)(2n_{\text{th}} + 1) \int_{0}^{\xi} d\lambda = \hbar \omega \xi \sinh(2\xi)(2n_{\text{th}} + 1).
\]

For the extracted asymmetry we obtain

\[
A_{2}^{0 \rightarrow s} = -\int_{\xi}^{0} d\lambda \text{Tr}[(2H_{S}(\lambda) + \hbar \omega)\rho_{\text{eq}}^{S}]
\]

\[
= \hbar \omega \cosh(2\xi)(2n_{\text{th}} + 1) \int_{0}^{\xi} d\lambda = \hbar \omega \xi \cosh(2\xi)(2n_{\text{th}} + 1).
\]

In conclusion, during a single cycle we obtain a total amount of extracted mechanical work and asymmetry

\[
W_{\text{ext}} = \hbar \omega \xi \sinh(2\xi)(2n_{\text{th}} + 1) \quad (C10)
\]

\[
A_{\text{ext}} = \hbar \omega \xi \cosh(2\xi)(2n_{\text{th}} + 1), \quad (C11)
\]

fulfilling \( W_{\text{ext}} = \mu A_{\text{ext}} \). Notice that the squeezing work in the setup is strictly greater than the ergotropy for any value of \( \xi \), that is \( W_{\text{sq}} = \mu A_{\text{ext}} > W \), since \( \xi \sinh(2\xi) > \sinh^2(\xi) \forall \xi > 0 \).

Finally, we point that one way of looking at the quantunness of the squeezing effects is to look at the induced uncertainty in position or momenta quadratures of our bosonic mode. In our case, for a squeezed thermal state, the squared uncertainty in the position quadrature \( x = (1/\sqrt{2})(a + a^\dagger) \) reads

\[
\Delta^{2}x \equiv \langle x^2 \rangle - \langle x \rangle^2 = (2n_{\text{th}} + 1)\frac{e^{2\xi}}{2}, \quad (C12)
\]

and the Heisenberg uncertainty principle reads \( \Delta x \Delta p \leq 1/2 \). A genuine quantum state is then reached when this uncertainty falls below shot noise, i.e. when \( \Delta^{2}x < 1/2 \). This is equivalent to squeezing the quadrature over a certain quantity

\[
\xi > \xi^{*} = \frac{1}{2} \ln(\coth(\beta_{0}\hbar \omega)). \quad (C13)
\]

The quantity \( \xi^{*} \) essentially depends on the relation between the magnitude of thermal fluctuations and the energy of the system. It diverges for high temperatures when \( \beta_{0} \rightarrow 0 \) and \( \xi^{*} \rightarrow 0 \) when \( \beta_{0} \rightarrow \infty \). For a relevant parameter range \( \beta_{0}\hbar \omega \simeq [0.01, 1] \) we obtain vales of \( \xi^{*} \) from 2.65 to 0.39.

[1] J. Goold, M. Huber, A. Riera, L. del Río, P. Skrzypczyk, The role of quantum information in thermodynamics: a topical review, J. Phys. A: Math. Theor. 49, 143001 (2016).
[2] S. Vinjanampathy, and J. Anders, Quantum Thermodynamics, Contemp. Phys. 57, 545-579 (2016).
[3] M. Lostaglio, D. Jennings, T. Rudolph, Quantum coherence, time-translation symmetry, and thermodynamics, Phys. Rev. X 5, 021001 (2015); M. N. Bera, A. Riera, M. Lewenstein, A. Winter, Generalized laws of thermodynamics in the presence of correlations, Nature Commun. 8, 2180 (2017).
[4] L. del Río, J. Aberg, R. Renner, O. Dahlsten, and V. Vedral, The thermodynamic meaning of negative entropy, Nature 474, 61-63 (2011); P. Skrzypczyk, A. J. Short, and S. Popescu, Work extraction and thermodynamics for individual quantum systems, Nature Commun. 5, 4185 (2014); M. Perarnau-Llobet, K. V. Hovhannisyan, M.
Y. Gurnayova, S. Popescu, A. Short, R. Silva, and P. G. Manzano, J. M. Horowitz, and J. M. R. Parrondo, *Information theory and statistical mechanics with information reservoirs*, Phys. Rev. Lett. 5, 041011 (2015).

O. Abah and E. Lutz, *Efficiency of heat engines coupled to nonequilibrium reservoirs*, EPL 106, 20001 (2014).

G. Manzano, F. Galve, R. Zambrini, and J. M. R. Parrondo, *Entropy production and thermodynamic power of the squeezed thermal reservoir*, Phys. Rev. E 93, 052120 (2016).

W. Niedenzu, D. Gellwasser-Klimovsky, A. G. Kofman, and G. Kurizki, *On the operation of power supplies by quantum non-thermal baths*, New. J. Phys. 18, 083012 (2016).

W. Niedenzu, V. Mukherjee, A. Ghosh, A. G. Kofman, and G. Kurizki, *Quantum engine efficiency bound beyond the second law of thermodynamics*, Nat. Commun. 9, 165 (2018).

M.O. Scully, M.S. Zubairy, G.S. Agarwal, and H. Walther, *Extracting work from a single heat bath via vanishing quantum coherence*, Science 299, 862-864 (2003).

R. Dillenschneider and E. Lutz, *Energetics of quantum correlations*, EPL 88, 5003 (2009).

X. L. Huang, T. Wang, and X. X. Yi, *Effects of reservoir squeezing on quantum systems and work extraction*, Phys. Rev. E 86, 051105 (2012).

J. Roßnagel, O. Abah, F. Schmidt-Kaler, K. Singer, and E. Lutz, *Nanoscale heat engine beyond the Carnot limit*, Phys. Rev. Lett. 112, 030602 (2014).

L. A. Correa, J. P. Palao, D. Alonso, and G. Adesso, *Quantum-enhanced absorption refrigerators*, Sci. Rep. 4, 3949 (2014).

B. K. Agarwalla, J.-H. Jiang, and D. Segal, *Quantum efficiency bound for continuous heat engines coupled to noncanonical reservoirs*, Phys. Rev. B 96, 104304 (2017).

J. Klaers, S. Fuelt, A. Imamoglu, and E. Togan, *Squeezed thermal reservoirs as a resource for a nanomechanical engine beyond the Carnot limit*, Phys. Rev. X 7, 031044 (2017).

P. D. Drummond and Z. Ficek (Eds.), *Quantum squeezing* (Springer-Verlag, Berlin Heidelberg, 2004).

E. S. Polzin, *The squeeze goes on*, Nature 453, 45-46 (2008).

J.M.R. Parrondo, B.J. de Cisneros, Energetics of Brownian motors: a review , App. Phys. A 75, 179-191 (2002).

U. Seifert, *Efficiency of autonomous soft nanomachines at maximum power*, Phys. Rev. Lett. 106, 020601 (2011).

D. Mandal and , *Work and information processing in a solvable model of Maxwell’s demon*, Proc. Natl. Acad. Sci. USA 109, 11641 (2012).

S. Deffner, and C. Jarzynski, *Information processing and the second law of thermodynamics: An inclusive, Hamiltonian approach*, Phys. Rev. X 3, 041003 (2013).

A. C. Barato and U. Seifert, *Stochastic thermodynamics with information reservoirs*, Phys. Rev. E 90, 042150 (2014).

E. T. Jaynes, *Information theory and statistical mechanics*, Phys. Rev. 106, 620 (1957); *Information theory and statistical mechanics II*, Phys. Rev. 108, 171 (1957).

Y. Gurnayova, S. Popescu, A. Short, R. Silva, and P. Skrzypczyk, Thermodynamics of quantum systems with multiple conserved quantities, Nat. Commun. 7, 12049 (2016).

G. Manzano, J. M. Horowitz, and J. M. R. Parrondo, *Quantum fluctuation theorems for arbitrary environments: adiabatic and non-adiabatic entropy production*, arXiv:1710.00054 (2017).

J. M. Horowitz and M. Esposito, *Work producing reservoirs: Stochastic thermodynamics with generalized Gibbs ensembles*, Phys. Rev. E 94, 020102(R) (2016).

M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, *Relaxation in a completely integrable many-body quantum system: An ab initio study of the dynamics of the highly excited states of 1D lattice hard-core bosons* Phys. Rev. Lett. 98, 050405 (2007).

T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, and J. Schmiedmayer, *Experimental observation of a generalized Gibbs ensemble*, Science 348, 207-211 (2015).

M. Perarnau-Llobet, A. Riera, R. Gallego, H. Wilming and J. Eisert, *Work and entropy production in generalised Gibbs ensembles*, New J. Phys. 18, 123035 (2016).

N. Y. Halpern, P. Faist, J. Oppenheim, and A. Winter, *Microcanonical and resource-theoretic derivations of the thermal state of a quantum system with noncommuting charges*, Nat. Commun. 7, 12051 (2016).

J. M. R. Parrondo, J. M. Horowitz and T. Sagawa, Nat. Phys. 11, 131-139 (2015).

P. Skrzypczyk, A. J. Short and S. Popescu, *Work extraction and thermodynamics for individual quantum systems*, Nat. Commun. 4, 4185 (2013).

R. Gallego, J. Eisert and H. Wilming, *Thermodynamic work from operational principles*, New J. Phys. 18, 103017 (2016).

V. Scarani, M. Ziman, P. Stelmachovic, N. Gisin, and V. Buzek, *Thermalizing quantum machines: dissipation and entanglement*, Phys. Rev. Lett. 88, 097905 (2002).

G. Manzano, J. M. Horowitz, and J. M. R. Parrondo, *Nonequilibrium potential and fluctuation theorems for quantum maps*, Phys. Rev. E 92, 032129 (2015).

F. Barra, and C. Lledó, *Stochastic thermodynamics of quantum maps with and without equilibrium*, Phys. Rev. E 96, 052114 (2017).

P. Strasberg, G. Schaller, T. Brandes, M. Esposito, Quantum and information thermodynamics: a unifying framework based on repeated interactions, Phys. Rev. X 7, 021003 (2017).

J. M. Horowitz, *Quantum-trajectory approach to the stochastic thermodynamics of a forced harmonic oscillator*, Phys. Rev. E 85, 031110 (2012).

J. M. Horowitz and J. M. R. Parrondo, *Entropy production along nonequilibrium quantum jump trajectories*, New J. Phys. 15, 085028 (2013).

F. Barra, *The thermodynamic cost of driving quantum systems by their boundaries*, Sci. Rep. 5, 14873 (2015).

G. Manzano, F. Plastina, and R. Zambrini, *Optimal work extraction and thermodynamics of quantum measurements and correlations*, arXiv:1805.08184 (2018).

T. Sagawa, *Second law-like inequalities with quantum relative entropy: An introduction*, in Lectures on quantum computing, thermodynamics and statistical physics, Kinki University Series on Quantum Computing, Vol. 8 (World Scientific, New Jersey, 2013).

A. E. Allahverdyan, R. Balian, and Th. M. Nieuwenhuizen, *Maximal work extraction from finite quantum systems*, EPL 67, (2004).

H. T. Quan, Yu-xi Liu, C. P. Sun, and F. Nori, *Quantum thermodynamic cycles and quantum heat engines*, Phys. Rev. E 76, 031105 (2007).
[45] H. T. Quan, S. Yang, and C. P. Sun, *Microscopic work distribution of small systems in quantum isothermal processes and the minimal work principle*, Phys. Rev. E 78, 021116 (2008).

[46] C. C. Gerry and P. L. Knight, *Introductory Quantum Optics* (Cambridge University Press, 2005).