Magnonics and Supermagnonics

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The magnetic community continues to discuss the possibility to observe the magnetic superfluidity, despite the fact that it has been discovered long time ago. It was observed in antiferromagnetic states of superfluid ⁴He in 1984. In this article we reminds the main principles of spin superfluidity and related Bose-Einstein magnon condensation. We discuss applications of this phenomenon in supermagnonic devises.

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SPIN CURRENT AND SPIN SUPERCURRENT

The magnetically ordered systems characterized by an order parameter, which is the result of spontaneous breaking of spin-orbit SU(2) symmetry in the equilibrium magnetic states. These states shows the rigidity with respect to the inhomogeneous spin rotations $\theta_\alpha(r)$ i.e. to the dependence of the energy on the gradients $\omega_\alpha = \nabla_i \theta_\alpha$. This rigidity leads to formation of spin waves (the collective oscillations of magnons) and spin transport in magnetic textures. Particularly the flow of magnetization can circulate in the case of topological defect in magnetically ordered materials. The spatial spin current can be excited also by pumping of magnetization in one side of the sample and its sink in the other side. The experimental observation of this current is discussed for a long time. See [1–3] and references there. The spin current in this case is proportional to the gradient of order parameter. Indeed its nature is very different from an origin of superfluid and supercurrent phenomena. Let as consider the analogy with the cosmology. The ground state of the Universe - the quantum vacuum is the ordered state described by a complex matrix. The gradients of this state leads to a flow of vacuum, particularly in the case of cosmic strings. Similar case take place in magnetically ordered states. The gradient of ground state leads to a flow of magnetization. Can we call this flow a supercurrent? Yes and Not. From one side the gradient of ground state leads to the flow of magnetization. But originally the term superfluidity and superconductivity applied for the flow of excitations and not a flow due to the gradient of ground state - quantum vacuum.

This difference is very clear in magnetically ordered systems. There is a ground state and excitations - magnons. Physicist usually deal with the dynamics of magnetic ground state at a small excitation described by Landau-Lifshits equations. But the situation drastically changes in the case of a big density of magnons. In this case magnons may form a Bose-Einstein condensate, which gradients leads to a spin supercurrent in full analogy with mass superfluidity and superconductivity. This type of flow has all rights to be called supercurrent while the flow of magnetization in the case of magnetic texture of ground state we can call textural current.

Since magnons obey the Bose statistics, they may form Bose-Einstein condensate (BEC) similar to an atomic BEC. Atoms forms a coherent quantum state at the temperature below a critical one for given density of atoms. These conditions was predicted by Einstein [1] and follows from Bose statistics:

$$T_{BEC} \simeq \frac{\hbar^2}{k_B m} \left(\frac{N_C}{3}\right)^{2/3},$$

where $N_C$ is the density of atomic gas and $T_{BEC}$ is the critical temperature, below which atoms condense in a BEC state. The magnon BEC state should forms at about the same ratio between the temperature and density. Magnons condensed to a Bose-Einstein condensate at a higher density.

Indeed, magnons have a finite lifetime and the total number of equilibrium magnons decreases with decreasing of temperature (and reach zero at $T = 0$). The density of magnons at thermodynamic equilibrium is always below the critical density of magnons BEC formation. However, the density of excited non-equilibrium magnons $N_M$ can be drastically increased up to about Avogadro density by a magnetic resonance methods. The excited magnons can form the quasi-equilibrium excited state with a time scale of about few quasiparticles scattering time. The usual 4-magnon scattering conserves the total number of quasiparticles and hold the distribution function with the effective temperature $T$ and effective chemical potential $\mu$. The critical density of magnons BEC formation $N_{BEC}$ can be estimated from equation (1). The initial temperature of magnons correspond to phonon subsystem temperature, which determines the value of magnetization $M$ and density of thermal magnons. We are able to increase the density of excited magnons above the critical one by dynamical magnetization deflection. Magnons should forms a BEC state when $N_M > N_{BEC}$, under certain conditions, which will be discussed below. The critical magnons concentration $N_{BEC}$ for ferro and antiferromagnets was calculated in [3,4]. Particularly for
easy plain antiferromagnets with wave function
\[ \varepsilon_k = \sqrt{\varepsilon_0^2 + \varepsilon_{xx}^2 (ak)^2} \]  
(2) it reads:
\[ N_{BEC} \approx \frac{(k_B T)^2}{2\pi^2} \frac{\varepsilon_0}{a^2 \varepsilon_{xx}} \]  
(3)

The magnetic ordering in a magnon BEC state arises due to spontaneous breaking of $U(1)$ symmetry in the non-equilibrium magnon condensate. In atomic BEC and in helium superfluids such symmetry breaking leads to a non-zero value of the superfluid rigidity - the superfluid density $\rho_s$ which enters the non-dissipative supercurrent of particles and thus to mass supercurrent. The same takes place for magnon BEC. The supercurrent of magnons can be described by traditional equations:
\[ J = \rho_s v_s \quad , \quad v_s = \frac{\hbar}{m_M} \nabla \alpha \quad , \quad \rho_s (T = 0) = N_M m_M \]  
(4)

where $m_M$ is the effective mass of magnons. In translationally invariant systems, where the mass current coincides with density of linear momentum, Eq.(4) can be obtained directly from the definition of linear momentum density in spin systems:
\[ P = (S - S_z) \nabla \alpha = N_M \hbar \nabla \alpha \]  
(5)

where we used the fact that density of magnons $N_M = S - S_z$ and phase of magnetization precession $\alpha$ are canonically conjugated variables. As in conventional superfluids, the superfluid density of the magnon liquid is determined by the magnon density $N_M$ and magnon mass $m_M$. The superfluid mass current (4) carries magnons with mass $m_M$, while in atomic superfluids the superfluid mass current carries atoms. The magnon mass current generated by precessing magnetization in magnon BEC is similar to electric current generated by precessing magnetization in ferromagnets [3].

The best magnetically ordered spin systems for magnon BEC and spin supercurrent investigations are antiferromagnetic states of superfluid $^3$He owing the very small Gilbert damping (about $10^{-9}$), its absolute purity and different types of magnon-magnon interactions. It is very important to know that the dynamic properties of these states are the results of magnetic ordering and does not related with its mass superfluid properties [8].

**SPIN SUPERFLUIDITY**

In 1984 the first magnon BEC state and related spin superfluidity has been discovered in antiferromagnetic superfluid $^3$He. The magnon BEC demonstrate the spontaneously self-organized phase-coherent precession of spins $\hat{\mathbf{S}}$ [10]. This state is radically different from the conventional ordered states in magnets. It is the quasi-equilibrium state, which emerges on the background of the ordered magnetic state, and which can be represented in terms of the Bose condensation of magnetic excitations - magnons [8].

The magnon BEC opened the new class of the systems, the Bose-Einstein condensates of quasiparticles, whose number is not conserved. Representatives of this class in addition to BEC of magnons are the BEC of phonons [11], excitons [12], exciton-polaritons [13], photons [14], rotons [15] and other bosonic quasiparticles.

Owing the coherence the magnon BEC radiates a very Long Living Induction Decay Signal (LLIDS). It may be considered as a time crystals [16] with a very long, but finite lifetime. It may reach minutes in antiferromagnetic superfluid $^3$He-B. Furthermore, the Goldstone modes - the space-time excitations of the time crystal (the analog of second sound in superfluid $^4$He) have been observed in magnon BECs [17–19]. The lifetime of magnon BEC states may be infinite in the case, when the losses (evaporation) of quasiparticles are replenished by an excitation of new quasiparticles.

The formation of a magnon BEC state was first observed in antiferromagnetic superfluid $^3$He-B [9, 20]. Usually, in the linear case, the induction decay signal after a RF pulse ringing the time, inversely proportional to an inhomogeneous broadening of magnetic system $\Delta \omega$. In the experiments, described in [9, 20] the induction signal also lost coherency at the time scale about 1/\(\Delta \omega\) but then spontaneously reappears and ringing a few orders of magnitude longer! The formation of LLIDS manifest itself the condensation of magnons in a common wave function in all the sample with a common phase and frequency of precession. The mechanism of BEC state formation related to a repulsive intersection between magnons. The higher local magnon density - the bigger dynamical frequency shift in the system. The local inhomogeneity of Larmore precession generates the gradient of phase of precession and, consequently, the superfluid transport of magnons. This spin supercurrent redistribute magnons until the dynamic frequency shift compensate the magnetic field inhomogeneity. The so named Homogeneously Precessing Domain (HPD) forms. The of coherently precession magnetization in HPD radiates LLIDS signal.

The LLIDS obey all the requirement for BEC of quasiparticles, which much later was postulated as an requirement of magnon BEC in well known article by Snoke [21]. Magnon BEC has one to one analogy with the experiments of atomic BEC [22]. Owing the slow magnons relaxation, the number of magnons decrease, but the magnons remains in a coherent state. It is important to note that the BEC state is the eigen state of excited magnons. It was shown experimentally, that the small RF pumping on a frequency of magnon BEC $\omega_{BEC}$ can compensate the magnons relaxation by creation of an
additional magnons. In this case the magnons BEC may
maintains permanently for an infinite time [23]. The 35
years of magnons BEC investigations in different antifer-
romagnetic states of superfluid $^3$He well established the
physics of excited magnon BEC and phenomena of spin
superfluidity. The review of this investigations one can
found, for example in [24, 25] and in the book [8].

**EXPERIMENTAL OBSERVATION OF SPIN
SUPERCURRENT**

The excitation of the homogeneously precessing do-
main is an interesting discovery in itself. The next step
in investigations of the magnon BEC was the experimen-
tal studies of spin supercurrent between two independent
HPD states, connected by a channel which was either per-
pendicular [13, 26, 27] or parallel to magnetic field [28].
The idea of these experiments was very straightforward
and based on the analogy of a superconducting bridge
between two massive superconducting electrodes. Here
we can consider two cells filled with HPD as such elec-
trodes. The role of the potential difference between the
electrodes is equivalent to the difference of the HPD pre-
cession frequencies. This difference leads to an increase
in the gradient of phase of precession in the channel and
consequently to the growth of spin supercurrent. If one
keeps the frequencies of HPD’s precession the same, then
the phase gradient in the channel remains constant and a
steady state supercurrent has to pass through the chan-
nel. In the case of superconductivity this current is sup-
plied by the leads of the normal metal, which have some
resistance and consequently there is a voltage difference.
In the case of the spin supercurrent the longitudinal mag-
netization is not conserved in the RF field. Therefore, the
RF field can pump the longitudinal magnetization into
one cell and pump it out in the other cell. The trans-
port of the longitudinal magnetization along the channel
in a magnetic field is accompanied by transport of Zeem-
man energy. This transport has been measured by the in-
crease in one cell of the energy absorbed from the RF
field with increasing phase difference, and its decrease in
the other cell. In this way we were able to measure the
current of longitudinal magnetization flowing out from
one cell and into the other.

Owing to the direct relation between the phase of mag-
netization precession and the phase of the order para-
meter, we were able to control the spin supercurrent through
measurement of the phase gradient of the magnetization
precession in the channel. This method has no analogue
with superconductivity because there is no field that is
sensitive to the phase of the wave function of electron
Cooper pairs.

The first observation of a spin supercurrent in the
channel have been published in [13]. The experimen-
tal set-up consists of two cells in the form of a barrel
with axes parallel or perpendicular to the magnetic field,
connected by a channel perpendicular to field (see Fig. 2).
The cells were surrounded by RF coils, and copper
shielding prevented interaction between the coils. The
channel was surrounded by additional shielding to pre-
vent RF field penetration into the channel. The coils 1
and 2 were used to excite HPD states in both cells and to
control them. The frequency and phase of the precession
of the domain with homogeneous precession in each of the
volumes was determined by the frequency and phase of the
radio-frequency field of the corresponding coil, sup-
plied from separate highly stable generators. The cells
were filled with HPD by sweeping down the magnetic
field. When the domain boundary crossed the inlet to
the channel, the HPD filled the channel. Miniature re-
ceiving radio-frequency coils 3 and 4 were set up in the
channel, and received a signal from the precessing magn-
etization in the channel. A small signal induced by the
exciting coils was compensated by an electronic circuit.
For HPD creation, equal frequency and phase of both RF
generators was chosen, so we can assume that the differ-
ence of phase of precession in the channel is zero. Then
the frequency of one of the rf generators was changed by
$\delta \omega \simeq 0.1$Hz. This causes the difference between phases of
precession $\Delta \alpha$ to grow. A phase difference between two
HPD’s determines the phase gradient along the channel.

If we keep increasing this phase difference, the spins in
the channel will “wind up” to maintain boundary condi-
tions. The spin current in the channel increases, until it
reaches a certain critical point, after which it drops by a
certain, specified amount. Here, the misquestionment
of the spins is too great to warrant transfer, and locally
the HPD is disrupted, and the spins in the channel “un-
wind”. At this spot, the magnetisation will locally be
equationed parallel to the external magnetic field, and
in this way “lose” several times $2\pi$ of twisting. Once the
tenseness has gone out of the system, the spins reequation
with the surrounding precession angles, and the spiral re-
forms with a few windings less. If the precession phase
difference is increased again, they will wind up until the
critical value once more, as is shown by the measurement
in Fig. 3. There is shown the rise of the absorption signal
in cell 1 and its diminishes in cell 2. ( Due to the sym-
metry, the signal from cell 1 at negative $\Delta \alpha$ corresponds
to the signal from cell 2 at positive $\Delta \alpha$). This process
corresponds to a transfer of longitudinal magnetization,
and consequently the Zeeman energy, from one chamber
to the other. All experimental curves correspond to sta-
nionary solutions in the channel. To check this, we made
the frequencies of the HPD’s equal at a certain time.
Then the absorption signals from both HPD and gradi-
ent distribution in the channel did not change any more -
a steady state spin supercurrent continued to flow along
the channel.

With increasing $\Delta \alpha$ one can see that on reaching a
critical phase difference $\Delta \alpha^c$ at point B the absorption
FIG. 1: The experimental cell for studies of spin supercurrent between two HPD states, excited by RF from two independent coils 1 and 2. The channel of 1.4 mm in diameter connecting HPD states. The pick up coils 3 and 4 monitoring the amplitude and phase of the magnetization precession inside the channel. Pt NMR thermometer (6) monitoring the temperature. The screen (5) suppress the RF crossover signal between the coils. The cells connected to a main volume of $^3$He by a channel.

FIG. 2: Illustration of experimental observation of spin supercurrent between two HPD states. The spin supercurrent transports the magnetization from cell 1 to cell 2. The current is proportional to the gradient of phase of precession in the channel.

Jumps to a smaller value (point C), then increases to the critical value again, etc. In this case the jumps occur with period $2n\pi$ in $\Delta\alpha$. The critical phase gradient determines by the inverse value of the Ginzburg-Landau coherence length.

$$\nabla \alpha_c^2 = 1/\xi_{GL}^2 = \omega_L(\omega_{RF} - \omega_L)/\xi_\perp^2. \quad (6)$$

Its increase with the increasing the difference $\omega_{RF} - \omega_L$. The value of phase slippage also increase. Experimentally the phase slip up to $16\pi$ have been observed. Similar jumps can be seen in the phase of precession in the channel. The gradient of the phase of precession in the channel produces a spin supercurrent which, for the channel perpendicular to $H$, reads:

$$J_P = -\chi \gamma (1-\cos \beta) [(1-\cos \beta) c_\parallel^2 + (1+\cos \beta) c_\perp^2] \nabla \alpha. \quad (7)$$

This supercurrent transports the longitudinal magnetization from cell 2 to cell 1. The rise of the magnetization in cell 1 means a decrease of the angle $\beta$. To maintain the resonance condition, the HPD in this cell begins to absorb more RF power (curve AB). The same supercurrent leads to an increase of angle $\beta$ in cell 2. To prevent this
FIG. 3: The record of energy dissipation in both HPD’s as a function of phases of precession in HPD’s and the phase of precession in pick-up coils 3 and 4. The spin supercurrent transports the magnetization from cell 1 to cell 2. The absorption of energy in cell 1 increases to compensate the Zeeman energy, transported out by spin supercurrent. This energy partly dissipates in the channel due to diffusion relaxation and partly arrives to cell 2. Consequently, the energy, absorbed from the RF field in cell 2 decreases. At point B, the phase slippage of 4π appears. This critical spin current in the channel determines by \( \omega - \omega_L \) and thus as a function of the coherence length \( \xi \) in (6). The signals from pick-up coils measured the phase distribution inside the channel.

the NMR absorption must fall in this cell (curve AB'). In other words, the magnetic supercurrent transports some magnetic energy \( J_E = -J_F H \) from cell 1 to cell 2. To compensate this energy flow, the RF absorption rises in one cell by \( \delta W_1 \) and falls in the other one by \( -\delta W_2 \). If the magnetization transported by the supercurrent were conserved, we would have \( \delta W_1 = -\delta W_2 \). However, there are some relaxation processes caused by interaction between the magnetization of the normal and superfluid components. Spin diffusion of the normal component leads to a dissipation of magnetic energy in the channel, that grows with phase gradient. To maintain the resonance conditions for the HPD in the channel, the energy losses should be compensated by additional energy supply by spin supercurrent. So the spin current is greater at the inlet of the channel than at the outlet. The asymmetry of the experimental curve about \( \Delta \alpha = 0 \) is the result of magnetic relaxation within the channel.

But this relaxation is not the result of friction, it can be treated as a relaxation of the eigenstate, which can not be seen in the case of mass superfluidity or superconductivity due to the conservation of mass and charge. By taking this relaxation into account one can recalculate the distribution of \( \nabla \alpha \) along the channel:

\[
\nabla \alpha(x) = \frac{\exp \Lambda \Delta \alpha - 1}{\Lambda [L + (\exp \Lambda \Delta \alpha - 1)x]} \tag{8}
\]

where \( L \) is the length of the channel and \( \Lambda = \frac{64}{145} D \omega_{RF} c_\perp^{-2} \), where \( D = (D_\parallel + D_\perp)/2 \) is the effective spin diffusion coefficient along the channel as defined in [29]. With cooling this relaxation significantly decreases due to decrease of the normal component of liquid. Indeed, we were not able to investigate this phenomena at temperature below 0.4 °C due to instability of HPD state, which was named “Catastrophic relaxation” [30].

PHASE SLIPAGE.

The spin supercurrent in a channel is limited by the instability of current against phase slippage. In this section we shall analyse the nature of phase slip centres for spin supercurrent. From a general point of view the phase slippage of spin supercurrent is analogous to that observed in superconducting wires [31] and mass superflow through a small hole [32]. We have learned from these superfluidity and superconductivity experiments that the superfluid density should be zero at the phase slip centre. As a result the phase of the order parameter is not determined and the phase relation along the channel can have a discontinuity. The formation of phase slip is related to a change in some energy. If this energy is less, than the density of the kinetic (gradient) energy of the supercurrent, the phase slip appears. As a result of phase slippage the phase difference along a channel will be decreas...
on $2n\pi$. Upon decreasing the kinetic (gradient) energy density the phase slip centre becomes unstable and disappears. The main difference between phase slip in superfluidity and superconductivity and the phase slip of spin supercurrent is that in the latter case it is not necessary to destroy the superfluid state to create the phase slip. It is sufficient to destroy the spin supercurrent density which is proportional to $(1 - \cos \beta)$ (see (5.1)) to maintain the spin supercurrent phase slip centre. If $\beta = 0$ in any part of the channel, the phases of precession of the HPD in the cells are no longer connected and the phase difference between the two HPD's can change by a multiple of $2\pi$. The critical current for creation of the phase slip can be estimated by comparing the stiffness of the HPD state in a channel and kinetic (gradient) energy of a current. This corresponds to the phase gradient equal to the inverse value of the Ginzburg-Landau coherence length according to Eq.(6)

As was shown in [33] the local gradient energy is equal to the energy of HPD formation. In reality the situation is more complicated. One should take into account the spectroscopic correction to the gradient energy that leads to the frequency shift of precession $\Omega_\nabla = \partial F_\nabla / \partial P$:

$$\Omega_\nabla = \frac{5c_2^2 - c_1^2}{4\omega} \nabla \alpha^2$$  \hspace{1cm} (9)

The value of the dipole-dipole frequency shift decreases with increasing current in order to compensate this gradient energy frequency shift and to keep the HPD in the channel in resonance. But when $\Omega_\nabla$ surpasses the difference between the HPD frequency and the Larmor frequency in the channel, the HPD can no longer exist and the angle $\beta$ decreases. Therefore the density of $P$, proportional to $(1 - \cos \beta)$, decreases which makes the spin current solution unstable. Interestingly an analogous instability takes place in the case of mass supercurrent in $^3$He-B due to Fermi liquid corrections. As was shown in [34], the superfluid density in $^3$He-B decreases with increasing gradient of phase of the wave function (velocity). Consequently the critical supercurrent corresponds to a maximum value of current as a function of this gradient. By taking into account the circumstances given above, the critical spin supercurrent should correspond to the gradient:

$$\nabla \alpha_c = \sqrt{\frac{4\omega_L (\omega_{RF} - \omega_L)}{5c_2^2 - c_1^2}} \hspace{1cm} (10)$$

In Fig. 4 we show the experimental value of the critical phase difference between two HPD as function of $\omega_{RF} - \omega_L$. In order to compare these results with theory one should take into account the distribution of phase gradient in a channel, given by [8]. There is a good agreement with the theory, particularly if we use the spin diffusion coefficient as a fitting parameter, that is $D = 0.035 \text{ cm}^2/\text{s}$. (solid line in Fig. 4). For the $D_\perp$, measured under the same conditions we have $D_\perp = 0.058 \text{ cm}^2/\text{s}$. This discrepancy is probably caused by spin - diffusion anisotropy [29, 35], demonstrated experimentally for the first time.

**SPIN-CURRENT JOSEPHSON EFFECT**

The Josephson effect is the response of the current to the phase between two weakly connected regions of coherent quantum states. It was described by Josephson [36]...
FIG. 5: For observation of the dc and ac Josephson effects the orifice of diameter about 0.48 mm was installed inside the channel.

FIG. 6: The Josephson effect for magnon BEC demonstrates the interference between two magnon condensates. Spin current as a function of the phase difference across the junction, $\alpha_2 - \alpha_1$, where $\alpha_1$ and $\alpha_2$ are phases of precession in two coherently precessing domains. Different experimental records correspond to a different ratio between the diameter of the orifice and the coherence length $\xi$ of magnon BEC. The pure dc Josephson phenomenon was observed for magnetic coherent length $\xi = 1.3$ mm (a) and the distorted one for $\xi = 0.8$ mm (b). The phase slippage processes were observed for $\xi = 0.7$ mm (c).

for the case of two quantum states, separated by the potential barrier. This phenomenon is usually studied for the case of quantum states connected by a conducting bridge with the dimensions smaller than the coherence length. In this case the coherent state in the bridge cannot be established so there is no phase memory, which determines the direction of the phase gradient. As a result the supercurrent is determined only by the phase difference between the two states. As the dimensions of the conducting bridge increase, the more complex current-phase relation is observed. For bridge dimensions of the order of the coherence length, a transition to a hysteretic scenario with phase slippage appears.

In the case of mass and electronic supercurrents the coherence length is a function of the temperature. In the case of spin supercurrents, however, the Ginzburg-Landau coherence length $\xi$ is not only a function of temperature, but also a function of the difference between the HPD precession frequency and the local Larmor frequency, according to Eq. (6).
able to change the coherence length in the region of the orifice in the channel and observe the change from the canonical current-phase relation to phase slip behavior. This experiment made in Kapitza Institute [27, 37, 38] is schematically presented in Fig. 9. The orifice, of diameter 0.48 mm, was placed in the central part of the channel. The current-phase characteristics, observed in this experiment are represented for different positions of the domain boundary related to the orifice. One can easily see that the current in Fig. (a) corresponds to the canonical current-phase relation, which transforms to the nonlinear relation in Fig. (b) and then to a phase slip phenomenon in Fig. (c).

In the insertion the modification of the channel profile and screen for the observation of Josephson phenomena is shown.

The first attempt to describe theoretically the spin supercurrent Josephson phenomenon was made in [39]. In spite of some difficulties in presenting a simple mathematical model of the spin supercurrent in an orifice, his calculations have a qualitative agreement with the observed phenomena.

**MAGNON BEC STATES**

The formation of magnon BEC states was confirmed by a many observation. First of all it is the observation of Nambu-Goldstone (NG) mode of magnons condensate [17, 18]. This modes are the magnetic analog of second sound in superfluid $^4$He. It is very important achievement to support the BEC state permanently. It allow to perform the steady state experiments with a two BECs connected by a channel. The spin supercurrent [40], phase slippage [41], Josephson effect [37, 38], spin vortex [42, 43] and other supermagnonic quantum phenomena was observed.

There was discovered the other types of magnon BEC states in $^3$He-B; the self-trapped BEC state, named Q-ball [44], the state at a global minimum of dipole-dipole energy, named HPD2 [45] and the state with partial magnetization [46, 47]. The magnon BEC was suggested in superfluid $^3$He-A [48] and it was observed by pulse [49] and CW [50, 51] methods. Recently the magnon BEC and spin superfluidity was observed in a new antiferromagnetic superfluid state - $^3$He-P [52].

There are no any specific properties of antiferromagnetic superfluid states of $^3$He, which give advantage for magnon BEC formation, except the very small Gilbert damping factor of magnons, which can be as low as $10^{-8}$. Indeed it should be possible to found magnon BEC phenomena in other solid magnetic, as was predicted in [53]. Particularly it was very interesting to search the magnon BEC in systems with coupled nuclear-electron precession, which properties are very similar to $^3$HE-A in aerogel [54]. We have successfully found the formation of magnon BEC in antiferromagnets $\text{MnCO}_3$ and $\text{CsMnF}_3$ at $1.5^\circ$K temperature by CW [55] and pulsed [56] NMR. The observation was done on quasinuclear branch of precession, which characterized by Gilbert damping factor of about $10^{-5}$ and the repulsive interaction between magnons. The new techniques of magnon BEC formation was developed: the non-resonance excitation of magnon BEC [57] and the switch off RF field method [58]. The magnon BEC was observed even in very inhomogeneous conditions [59].

A new breakthrough in research has taken place in YIG. The BEC state of magnons with wave vector $k = 0$ was observed in normally magnetized Yttrium Ferrite Garnet thin film at room temperature [60]. This magnon BEC state differed from observed early magnon BEC state in YIG film magnetized tangentially [61], where magnons with non-zero $k$ were condensed.

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