The study of lepton EDM in CP violating BLMSSM

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(Dated: November 18, 2014)

Abstract

In the supersymmetric model with local gauged baryon and lepton numbers(BLMSSM), the CP-violating effects are considered to study the lepton electric dipole moment(EDM). The CP-violating phases in BLMSSM are more than those in the standard model(SM) and can give large contributions. The analysis of the EDMs for the leptons $e, \mu, \tau$ is shown in this work. It is in favour of exploring the source of CP violating and probing the physics beyond SM.

PACS numbers: 13.40.Em, 12.60.-i

Keywords: CP-violating, electric dipole moment, lepton

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I. INTRODUCTION

The theoretical predictions for EDMs of leptons and neutron are very small in SM. In SM, the electron EDM is estimated as $|d_e| \simeq 10^{-38} e.cm$, which is too small to be detected by the current experiments. The ACME Collaboration report the new result of $d_e = (-2.1 \pm 3.7_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-29} e.cm$. The upper bound of electron EDM is $|d_e| < 8.7 \times 10^{-29} e.cm$ at the 90% confidence level. Therefore, if large EDM of electron is probed, one can ensure it is the signal of new physics beyond SM. $|d_\mu| < 1.9 \times 10^{-19} e.cm$ and $|d_\tau| < 10^{-17} e.cm$ are the EDM upper bound of leptons $\mu$ and $\tau$ respectively. The minimal supersymmetric extension of SM (MSSM) is very attractive and physicists have studied it for a long time. In MSSM, there are a lot of CP violating phases and they can give large contributions to the EDMs of leptons and neutron.

When the CP-violating phases are of normal size, and the SUSY particles are at TeV scale, very big EDMs of elementary particles are obtained, which exceeds the current experimental limit. Three approaches are used to resolve this problem. 1. make the CP violating phases small, i.e. $O(10^{-2})$. That is the so called fine tuning. 2. use mass suppression through making SUSY particles heavy(several TeV). 3. there is cancellation mechanism among the different components. For lepton EDM and neutron EDM, the main parts of chargino and the neutralino contributions are cancelled.

BLMSSM is the minimal supersymmetric extension of the SM with local gauged B and L(BLMSSM). Because of the local gauged B and L, it can explain both the asymmetry of matter-antimatter in the universe and the data from neutrino oscillation experiment. So, BLMSSM is a favorite model beyond MSSM. Extending SM, the authors study the model with B and L as spontaneously broken gauge symmetries around TeV scale. The lightest CP-even Higgs mass and the decays $h^0 \rightarrow \gamma\gamma$, $h^0 \rightarrow ZZ(WW)$ are also studied in this model. In our previous work, we study the neutron EDM and $B^0 - \bar{B}^0$ mixing in the CP-violating BLMSSM.

Research the MDMs and EDMs of leptons are the effective ways to probe new physics beyond the SM. In MSSM, the one-loop contributions to lepton MDM and EDM are well studied. The authors investigate some two loop corrections to lepton MDM and EDM in the framework of MSSM. In the two Higgs Doublet models with CP-violation, the one loop and Barr-Zee type two-loop contributions to fermionic EDMs are obtained. In
Ref.\[14\], a model-independent study of $d_e$ in the SM is carried out. They take into account the right handed neutrinos, the neutrino seesaw mechanism and the framework of minimal flavor violation. Their results show that when neutrinos are Majorana particles, $d_e$ can reach its experiment bound.

After this introduction, in section 2 we briefly introduce the main ingredients of the BLMSSM. The one-loop corrections to the lepton EDM are collected in section 3. Section 4 is devoted to the numerical analysis for the dependence of lepton EDM on the BLMSSM parameters. We show our discussion and conclusion in section 5.

II. THE BLMSSM

The local gauge group of BLMSSM\[6\] is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$. In BLMSSM, the exotic leptons ($\hat{L}_4 \sim (1, 2, -1/2, 0, L_4)$, $\hat{E}_4^c \sim (1, 1, 1, 0, -L_4)$, $\hat{N}_5^c \sim (1, 1, 0, 0, -L_4)$, $\hat{L}_5^c \sim (1, 2, 1/2, 0, -(3 + L_4))$, $\hat{E}_5 \sim (1, 1, -1, 0, 3 + L_4)$, $\hat{N}_5 \sim (1, 1, 0, 0, 3 + L_4)$) are introduced to cancel $L$ anomaly. In the same way, they introduce the exotic quarks ($\hat{Q}_4 \sim (3, 2, 1/6, B_4, 0)$, $\hat{U}_4^c \sim (\bar{3}, 1, -2/3, -B_4, 0)$, $\hat{D}_4^c \sim (\bar{3}, 1, 1/3, -B_4, 0)$, $\hat{Q}_5^c \sim (3, 2, -1/6, -(1 + B_4), 0)$, $\hat{U}_5 \sim (3, 1, 2/3, 1 + B_4, 0)$, $\hat{D}_5 \sim (3, 1, -1/3, 1 + B_4, 0)$) to cancel the $B$ anomaly. The Higgs mechanism is of solid foundation, because of the detection of the lightest CP even Higgs $h^0$ at LHC\[15\].

The Higgs superfields $\hat{\Phi}_L, \hat{\varphi}_L$ are used to break lepton number spontaneously, as well as baryon number is broken by the Higgs superfields $\hat{\Phi}_B, \hat{\varphi}_B$. Therefore, these Higgs superfields $\hat{\Phi}_L, \hat{\varphi}_L$ and $\hat{\Phi}_B, \hat{\varphi}_B$ need nonzero vacuum expectation values (VEVs).

The superpotential of BLMSSM is shown as

$$W_{BLMSSM} = W_{MSSM} + W_B + W_L + W_X,$$
$$W_B = \lambda_Q \hat{Q}_4 \hat{Q}_5^c \hat{\Phi}_B + \lambda_U \hat{U}_4^c \hat{U}_5^c \hat{\varphi}_B + \lambda_D \hat{D}_4^c \hat{D}_5 \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B,$$
$$W_L = \lambda_e \hat{L}_4 \hat{H}_u \hat{E}_4^c + \lambda_{N} \hat{N}_4^c \hat{H}_u \hat{E}_4^c + \lambda_{\nu} \hat{N}_5 \hat{N}_5^c \hat{H}_u \hat{E}_5 + \lambda_{\nu} \hat{N}_5 \hat{N}_5^c \hat{H}_u \hat{E}_5,$$
$$W_X = \lambda_1 \hat{Q}_5 \hat{Q}_5^c \hat{X} + \lambda_2 \hat{U}_5^c \hat{U}_5^c \hat{X} + \lambda_3 \hat{D}_5^c \hat{D}_5^c \hat{X} + \mu_X \hat{X} \hat{X}^c.$$  \hfill (1)

Here, $W_{MSSM}$ represents the superpotential of the MSSM. We give out the concrete form
of the BLMSSM soft breaking terms $\mathcal{L}_{soft}$:

$$
\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} - (m^2_{\nu_u})_{ij} \tilde{N}^c_i \tilde{N}^c_j - m^2_{Q_4} \tilde{Q}_i^c \tilde{Q}_4 - m^2_{U_4} \tilde{U}_i^c \tilde{U}_4 - m^2_{D_4} \tilde{D}_i^c \tilde{D}_4 - m^2_{\phi_B} \Phi^*_B \Phi_B
$$

$$
- m^2_{\tilde{e}_4} \tilde{E}_i^c \tilde{E}_4 - m^2_{\tilde{N}_5} \tilde{N}_i^c \tilde{N}_5 - m^2_{\tilde{N}_6} \tilde{N}_i^c \tilde{N}_6 - m^2_{\phi_L} \Phi^*_L \Phi_L - \left(m_B \lambda_B \lambda_B + m_L \lambda_L \lambda_L + h.c.\right)
\left\{ A_{u_i} Y_{u_i} \tilde{Q}_4 H_u \tilde{U}_4 + A_{d_i} Y_{d_i} \tilde{Q}_4 H_d \tilde{D}_4 + A_{u_5} Y_{u_5} \tilde{Q}_5 H_d \tilde{U}_5 + A_{d_5} Y_{d_5} \tilde{Q}_5 H_u \tilde{D}_5
\right.
\left. + A_{BQ} \lambda_Q \tilde{Q}_4 \Phi_B + A_{BU} \lambda_U \tilde{U}_4 \tilde{U}_5 \varphi_B + A_{BD} \lambda_D \tilde{D}_4 \tilde{D}_5 \varphi_B + B_B \mu_B \Phi_B \varphi_B + h.c.\right\}
\left\{ A_{y_4} \tilde{L}_4 H_d \tilde{E}_4 + A_{y_5} \tilde{L}_4 H_u \tilde{N}_4 + A_{y_6} \tilde{L}_5 H_u \tilde{E}_5 + A_{y_5} \tilde{L}_5 H_u \tilde{N}_5
\right.
\left. + A_{y_i} \tilde{L}_4 H_u \tilde{N}_i + A_{\nu_1} \gamma_\nu \tilde{N}_i \tilde{N}_i \varphi_L + B_{\nu_i} \mu_L \Phi_L \varphi_L + h.c.\right\}
\left\{ A_{1_\nu_1} \tilde{Q}_6 X + A_{2_\nu_2} \tilde{U}_5 X' + A_{3_\nu_3} \tilde{D}_5 X' + B_X \mu_X X X' + h.c.\right\},
$$

(2)

In order to break the local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ down to the electromagnetic symmetry $U(1)_e$, the $SU(2)_L$ doublets $H_u$, $H_d$ should obtain nonzero VEVs $v_u$, $v_d$, and the $SU(2)_L$ singlets $\Phi_B$, $\varphi_B$, $\Phi_L$, $\varphi_L$ should obtain nonzero VEVs $v_B$, $\overline{v}_B$, $v_L$, $\overline{v}_L$ respectively. The Higgs fields and the Higgs superfields are defined as

$$
H_u = \left( \begin{array}{c} H_u^+ \\ \frac{1}{\sqrt{2}} (v_u + H_u^0 + iP_u^0) \end{array} \right), \quad H_d = \left( \begin{array}{c} H_d^- \\ \frac{1}{\sqrt{2}} (v_d + H_d^0 + iP_d^0) \end{array} \right),
$$

$$
\Phi_B = \frac{1}{\sqrt{2}} (v_B + \Phi_B^0 + iP_B^0), \quad \varphi_B = \frac{1}{\sqrt{2}} (\overline{v}_B + \varphi_B^0 + i\overline{P}_B^0),
$$

$$
\Phi_L = \frac{1}{\sqrt{2}} (v_L + \Phi_L^0 + iP_L^0), \quad \varphi_L = \frac{1}{\sqrt{2}} (\overline{v}_L + \varphi_L^0 + i\overline{P}_L^0),
$$

(3)

The detailed discussion of Higgs mass matrices can be found in Ref. [8]. There are the super fields $\tilde{N}^c$ in BLMSSM. Therefore, the neutrinos and scalar neutrinos are doubled as those in MSSM. Through the see-saw mechanism, light neutrinos obtain tiny masses.

In BLMSSM, there are 10 neutralinos: 4 MSSM neutralinos, 3 baryon neutralinos and 3 lepton neutralinos. The MSSM neutralinos, baryon neutralinos and lepton neutralinos do not mix with each other. 3 baryon neutralinos are made up of $\lambda_B$ (the superpartner of the new baryon boson) and $\psi_{\Phi_B}, \psi_{\varphi_B}$ (the superpartners of the $SU(2)_L$ singlets $\Phi_B, \varphi_B$). 3 lepton neutralinos are made up of $\lambda_L$ (the superpartner of the new lepton boson) and $\psi_{\Phi_L}, \psi_{\varphi_L}$ (the superpartners of the $SU(2)_L$ singlets $\Phi_L, \varphi_L$). Baryon neutralinos and lepton neutralinos do not contribute to the lepton EDM at one loop level.
At one loop level, scalar neutrinos can give contributions to lepton EDM. Because the super fields $\tilde{N}^c$ are introduced in BLMSSM, the neutrino mass matrix and the scalar neutrino squared mass matrix are different from those in MSSM.

In the left-handed basis $(\nu, N^c)$, we deduce the mass matrix of neutrinos after symmetry breaking

$$-\mathcal{L}^\nu_{\text{mass}} = (\bar{\nu}^I_R, \bar{N}^c_{R}^I) \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} (Y_{\nu})^{IJ} \\ \frac{v_u}{\sqrt{2}} (Y_{\nu})^{IJ} & \frac{v_u}{\sqrt{2}} (\lambda_{N^c})^{IJ} \end{pmatrix} \begin{pmatrix} \nu^I_L \\ N^c_L^I \end{pmatrix} + \text{h.c.} \quad (4)$$

With the unitary transformations

$$\begin{pmatrix} \nu^I_L \\ \nu^L_{2I} \end{pmatrix} = U_{\nu,L}^I \begin{pmatrix} \nu^I_L \\ N^c_L^I \end{pmatrix}, \quad \begin{pmatrix} \nu^I_R \\ \nu^R_{2I} \end{pmatrix} = W_{\nu,R}^I \begin{pmatrix} \nu^I_R \\ \tilde{N}^c_{R}^I \end{pmatrix}, \quad (5)$$

the mass matrix of neutrinos are diagonalized as

$$W_{\nu,L}^I \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} (Y_{\nu})^{IJ} \\ \frac{v_u}{\sqrt{2}} (Y_{\nu})^{IJ} & \frac{v_u}{\sqrt{2}} (\lambda_{N^c})^{IJ} \end{pmatrix} U_{\nu,R}^I = \text{diag}(m_{\nu^I}, m_{\nu^L}). \quad (6)$$

The squared mass matrix of the scalar neutrinos is obtained from the superpotential and the soft breaking terms in BLMSSM Eqs. (1) and (2).

$$-\mathcal{L}^\nu_{\text{mass}} = \bar{\tilde{n}} \cdot \mathcal{M}_{\tilde{n}} \cdot \tilde{n}, \quad (7)$$

with $\tilde{n}^T = (\bar{\nu}^I, \tilde{N}^c_{I}^*)$. The scalar neutrinos are enlarged by the superfields $\tilde{N}^c$ and the squared mass matrix reads as

$$\mathcal{M}_{\tilde{n}}^2(\bar{\nu}^I, \tilde{N}^c_{I}^*) = \frac{g^2}{8} (v_d^2 - v_u^2) \delta_{IJ} + g^2 (\bar{\nu}^I_L - v_u^2) \delta_{IJ} + \frac{v_u^2}{2} (Y^\dagger_{\nu} Y_{\nu})^{IJ} + (M_{\nu,L}^2)_{IJ},$$

$$(\tilde{N}^c_{I}^*)^2 = -g_2^2 (\bar{\nu}^I_L - v_u^2) \delta_{IJ} + \frac{v_u^2}{2} (Y^\dagger_{\nu} Y_{\nu})^{IJ} + 2v_u^2 (\lambda_{N^c})^{IJ} + (M_{\nu,L}^2)_{IJ} + \mu L \frac{v_d}{\sqrt{2}} (\lambda_{N^c})^{IJ} \frac{v_u}{\sqrt{2}} (A_{N})^{IJ},$$

$$(\tilde{N}^c_{I}^*)^2 = \mu L \frac{v_d}{\sqrt{2}} (Y^\dagger_{\nu} Y_{\nu})^{IJ} - v_u \bar{\nu}^I_L (Y^\dagger_{\nu} Y_{\nu})^{IJ} + \frac{v_u}{\sqrt{2}} (A_{N})^{IJ}. \quad (8)$$

The squared mass matrix of the scalar neutrinos are diagonalized through the formula

$$Z^\dagger_{\nu, IJ} \mathcal{M}_{\tilde{n}}^2 Z^\dagger_{\nu, IJ} = \text{diag}(m_{\nu^I}, m_{\nu^L}, m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2, m_{\tilde{\nu}_3}^2).$$

Because of the introduction of the superfields $\tilde{N}^c$ in BLMSSM. The corrected charginolepton-scalar neutrino couplings are adapted as

$$-\mathcal{L}_{\chi^{\pm} L \tilde{\nu}} = - \sum_{I,J = 1}^{3} \sum_{i,j = 1}^{2} \bar{\chi}^{-}_{i,j} (Y^{IJJ}_I Z^{2j\dagger}_I (Z^\dagger_{I})^{i1} P_R$$

$$+ \left[ \frac{e}{s_W} Z^{IJJ}_I (Z^\dagger_{I})^{i1} + Y^{IJJ}_I Z^{2j\dagger}_I (Z^\dagger_{I})^{i2} \right] P_L \right) e^{IJJ}_I. \quad (9)$$
III. FORMULATION

To obtain the lepton EDM, we use the effective Lagrangian method, and the Feynman amplitude can be expressed by these dimension 6 operators.

\[ O_{1}^{\pm} = \frac{1}{(4\pi)^2} \bar{l} (i\mathcal{D})^3 \omega_{\pm} l, \]
\[ O_{2}^{\pm} = \frac{eQ_f}{(4\pi)^2} \frac{\bar{l} (i\mathcal{D} l)}{\omega_{\pm}} F \cdot \sigma \omega_{\pm} l, \]
\[ O_{3}^{\pm} = \frac{eQ_f}{(4\pi)^2} \frac{\bar{l} F}{\omega_{\pm}} \gamma F \cdot \sigma \omega_{\pm} l, \]
\[ O_{4}^{\pm} = \frac{eQ_f}{(4\pi)^2} \frac{\bar{l} (i\mathcal{D} l)}{\omega_{\pm}} F \cdot \sigma \omega_{\pm} l, \]
\[ O_{5}^{\pm} = \frac{me}{(4\pi)^2} \frac{\bar{l} F}{\omega_{\pm}} l, \]
\[ O_{6}^{\pm} = \frac{eQ_f m_l}{(4\pi)^2} \bar{l} F \cdot \sigma \omega_{\pm} l. \] (10)

with \( \mathcal{D}_{\mu} = \partial_{\mu} + ieA_{\mu}, \omega_{\pm} = \frac{1 + \gamma_5}{2}, l \) denoting the lepton fermion, \( m_l \) being the lepton mass, \( F_{\mu\nu} \) being the electromagnetic field strength. Adopting on-shell condition for external leptons, only the \( O_{2,3,6}^{\pm} \) contribute to lepton EDM. Therefore, the Wilson coefficients of the operators \( O_{2,3,6}^{\pm} \) in the effective Lagrangian are of interest.

The lepton EDM can be expressed as

\[ \mathcal{L}_{EDM} = -\frac{i}{2} d_l \bar{l} \sigma^{\mu\nu} \gamma_5 l F_{\mu\nu}. \] (11)

The fermion EDM is a CP-violating amplitude which can not be obtained at tree level in the fundamental interactions. However, in the CP violating electroweak theory, one loop diagrams should contribute nonzero value to fermion EDM. Considering the relations between the Wilson coefficients \( C_{2,3,6}^{\pm} \) of the operators \( O_{2,3,6}^{\pm} \)[13], the lepton EDM is obtained

\[ d_l = -\frac{2eQ_fm_l}{(4\pi)^2} \text{Im}(C_2^+ + C_2^{-*} + C_6^+). \] (12)

The one loop triangle diagrams in BLMSSM are divided into two types according to the virtual particles: 1 the neutralino-scalar lepton diagram; 2 the chargino-scalar neutrino diagram. After the calculation, using the on-shell condition for the external leptons, we obtain the one loop diagrams contribution to lepton EDM.

\[ d_{\mu} = \frac{e}{32\pi^2 A_{NP}} \text{Im} \left\{ (A_{1})_{ij}^f (A_{2})_{ij}^f \sqrt{x_{\chi^0_j}} \left[ \frac{\partial^2}{\partial x_{L_i}^2} \varrho_{2,1}(x_{\chi^0_j}, x_{L_i}) - 2 \frac{\partial}{\partial x_{L_i}} \varrho_{1,1}(x_{\chi^0_j}, x_{L_i}) \right] \right\}. \]
\begin{align}
+ (B_1)_{ij}^{IJ} (B_2)_{ij}^{IJ} \sqrt{x_{\chi_j}^i} \left[ \frac{\partial^2}{\partial x_{\nu_{\ell_i}^j}^i} \varrho_{2,1}(x_{\chi_j}^i, x_{\nu_{\ell_i}^j}) - \frac{2}{\partial x_{\nu_{\ell_i}^j}} \varrho_{1,1}(x_{\chi_j}^i, x_{\nu_{\ell_i}^j}) \right] \left( x_{\chi_j}^i \right) \right)
\end{align}

with \( x_i \) denoting \( \frac{m_i^2}{\Lambda_{NP}} \), \( \Lambda_{NP} \) representing energy scale of the new physics. The concrete form of the function \( \varrho_{i,j}(x, y) \) is shown here:

\[ \varrho_{i,j}(x, y) = \frac{x^i \ln x - y^j \ln y}{x - y}. \]

The couplings \( (A_1)_{ij}^{IJ}, (A_2)_{ij}^{IJ}, (B_1)_{ij}^{IJ} \) and \( (B_2)_{ij}^{IJ} \) read as:

\[ (A_1)_{ij}^{IJ} = \frac{e}{\sqrt{2} s_W c_W} Z_L^{ij*}(Z_N^{ij*} s_W + Z_N^{ij*} c_W) + Y_1^{ij*} Z_L^{(ij+3)*} Z_N^{ij*}, \]
\[ (A_2)_{ij}^{IJ} = -\frac{e}{s_W} Z_L^{ij*} Z_N^{ij*} + Y_1^{ij*} Z_L^{ij*} Z_N^{ij*}, \]
\[ (B_1)_{ij}^{IJ} = \frac{e}{s_W} Z_+^{ij*}(Z_+^{ij*}) + Y_1^{ij*} Z_+^{ij*} Z_+^{ij*}, \]
\[ (B_2)_{ij}^{IJ} = Y_1^{ij*} Z_+^{ij*}(Z_+^{ij*})^{ij*}. \]

The matrices \( Z_L, Z_N \) respectively diagonalize the mass matrices of scalar lepton and neutralino.

**IV. THE NUMERICAL RESULTS**

For the numerical discussion, we take into account of the lightest neutral CP even Higgs mass \( m_{h_0} \approx 125.7 \text{ GeV} \) \[15\] and the neutrino experiment data \[16, 17\]:

\[ \sin^2 2\theta_{13} = 0.090 \pm 0.009, \quad \sin^2 \theta_{12} = 0.306^{+0.018}_{-0.015}, \quad \sin^2 \theta_{23} = 0.42^{+0.08}_{-0.03}; \]
\[ \Delta m_2^2 = 7.58_{-0.26}^{+0.22} \times 10^{-5} \text{eV}^2, \quad |\Delta m_A|^2 = 2.35_{-0.09}^{+0.12} \times 10^{-3} \text{eV}^2. \]
\( \text{Eq. (16)} \)

In our previous work the neutron EDM is studied \[9\], so the constraint from neutron EDM is also considered here.

We give out the used parameters \[18-20\]:

\[ m_e = 0.51 \times 10^{-3} \text{GeV}, \quad m_\mu = 0.105 \text{GeV}, \quad m_\tau = 1.777 \text{GeV}, \]
\[ m_W = 80.385 \text{GeV}, \quad \alpha(m_Z) = 1/128, \quad s_W^2(m_Z) = 0.23, \quad \Lambda = 1000 \text{GeV}, \]
\[ \tan \beta_L = 2, \quad L_4 = \frac{3}{2}, \quad m_{Z_L} = 1 \text{TeV}, \quad v_{L_t} = \sqrt{v_L^2 + \bar{v}_L^2} = 3 \text{TeV}, \]

\[ \text{Eq. (17)} \]
$A_e = -800\text{GeV}$, $A_\mu = -1300\text{GeV}$, $A_\tau = -1300\text{GeV}$, $\lambda_{\nu e} = 1$, 
$A_e' = -53\text{GeV}$, $A'_\mu = 1300\text{GeV}$, $A'_\tau = 1300\text{GeV}$, $\mu_L = 500\text{GeV}$, 
$A_{\nu e} = A_{\nu \mu} = A_{\nu \tau} = A_{\nu e} = A_{\nu \mu} = A_{\nu \tau} = -500\text{GeV}$, $g_L = 1/6$, 
$Y_\nu^{11} = 1.3031 \times 10^{-6}$, $Y_\nu^{12} = 9.0884 \times 10^{-8}$, $Y_\nu^{12} = 6.9408 \times 10^{-8}$, 
$Y_\nu^{22} = 1.6002 \times 10^{-6}$, $Y_\nu^{23} = 3.4872 \times 10^{-7}$, $Y_\nu^{33} = 1.7208 \times 10^{-6}$, 
$m_1 = 1000 \times e^{i\theta_1}\text{GeV}$, $m_2 = 750 \times e^{i\theta_2}\text{GeV}$, $\mu_H = -800 \times e^{i\theta_3}\text{GeV}$, 
$m_{\nu e} = m_{\nu \mu} = m_{\nu \tau} = 1000\text{GeV}$. \hspace{1cm} (17)

Here, $\theta_1, \theta_2, \theta_\mu$ are the CP-violating phases of the parameters $m_1, m_2$, and $\mu_H$. To simplify 
the numerical discussion, we use the following relations.

\begin{equation}
(m_L^2)_{11} = (m_R^2)_{11} = N_e^2, \quad (m_L^2)_{22} = (m_R^2)_{22} = N_\mu^2, \quad (m_L^2)_{33} = (m_R^2)_{33} = N_\tau^2, \hspace{1cm} (18)
\end{equation}

with $N_e, N_\mu, N_\tau$ representing real parameters.

A. the electron EDM

At first, we study electron EDM, because its upper bound is the most strict one. The 
parameters $\theta_1, \theta_2, \theta_\mu$, $N_e$ and $\tan \beta$ have close relationship with electron EDM. To obtain 
significant one loop contribution to muon MDM \cite{19}, we take $\tan \beta = 15$ and $N_\mu = N_\tau = 1000\text{GeV}$. Supposing $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0), (\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$ and $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$, we plot three lines varying with the parameter $N_e$ in the Fig.\hspace{1cm} The dashed line 
corresponds to the result with $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$. From Fig.\hspace{1cm} one can easily find that 
the effect of $N_e$ to the dashed line is strong. $d_e$ turns large quickly with the decreasing $N_e$ as 
$N_e < 800\text{GeV}$. When $N_e$ is around $600\text{GeV}$ and even smaller, $d_e$ can reach $3.0 \times 10^{-28}(e\text{.cm})$. 
With $N_e = 1000\text{GeV}$ and $N_e = 1500\text{GeV}$, $d_e$ is $7.68 \times 10^{-29}(e\text{.cm})$ and $2.63 \times 10^{-29}(e\text{.cm})$. 
The dotted line and the solid line are almost coincident, and they vary slightly with $N_e$. 
With $N_e = 600\text{GeV}$, the solid line and dotted line are both around $1.5 \times 10^{-28}(e\text{.cm})$. When 
$N_e = 1300\text{GeV}$, the solid line and dotted line can reach $8.9 \times 10^{-29}(e\text{.cm})$, and turn small 
slowly with the enlarging $N_e$. One finds that the effect of $\theta_1$ to $d_e$ is stronger than the effects 
of $\theta_2$ and $\theta_\mu$.

To embody the $\tan \beta$ effect to $d_e$, in Figs.\hspace{1cm} we plot the diagram with $\tan \beta$ varying 
from 5 to 40. With $N_e = N_\mu = N_\tau = 1000\text{GeV}$ and $(\theta_1 = 0.1\pi, \theta_2 = \theta_\mu = 0)$, the result
FIG. 1: With $\tan \beta = 15$ and $N_\mu = N_\tau = 1000 \text{GeV}$, the contributions to electron EDM vary with $N_e$. The dashed line represents the result for $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$; the dotted line represents the result for $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$; the solid line represents the result for $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$.

is plotted in the Fig. 2 by the dotted line. The dotted implies that $\tan \beta$ influences $d_e$ strongly. As $\tan \beta$ is in the region near 15, $d_e$ varies in the region $10^{-29} \sim 10^{-28}(e.cm)$. $d_e$ turns large quickly with the decreasing $\tan \beta$ for $\tan \beta < 10$ and it can even reach $6 \times 10^{-27}(e.cm)$. Negative value is obtained for $d_e$, when $\tan \beta$ is larger than 16. $d_e$ varies in the region $-1 \times 10^{-27} \sim -2.3 \times 10^{-27}(e.cm)$ within $\tan \beta$ range from 20 to 40. The results represented by the solid line and dashed line corresponding to $(\theta_2 = 0.25\pi, \theta_\mu = \theta_1 = 0)$ and $(\theta_\mu = 0.25\pi, \theta_1 = \theta_2 = 0)$ respectively are drawn in the Fig. 3. In the whole, the behaviors of the solid line and the dashed line are similar. They both vary from $0.7 \times 10^{-28}(e.cm)$ to $2.0 \times 10^{-28}(e.cm)$ with the changing $\tan \beta$.

FIG. 2: With $N_e = N_\mu = N_\tau = 1000 \text{GeV}$, the contributions to electron EDM vary with $\tan \beta$. The dotted line in the left diagram represents the result for $(\theta_1 = 0.1\pi, \theta_2 = \theta_\mu = 0)$. 
FIG. 3: With $N_e = N_\mu = N_\tau = 1000\text{GeV}$, the contributions to electron EDM vary with $\tan \beta$. The dotted line represents the result for $(\theta_2 = 0.25\pi, \theta_1 = \theta_\mu = 0)$; The solid line represents the result for $(\theta_\mu = 0.25\pi, \theta_1 = \theta_2 = 0)$.

Supposing $N_e = N_\mu = N_\tau = 1000\text{GeV}$ and $\tan \beta = 15$, we study the contributions to electron EDM varying with the CP violating phases $\theta_1, \theta_2, \theta_\mu$ in Figs. 4, 5, 6. With $\theta_2 = \theta_\mu = 0$, we draw the numerical result versus $\theta_\mu$ by the solid line in the Fig. 4. The result is in the range of $-7.7 \times 10^{-29}(\text{e.cm}) \sim 7.7 \times 10^{-29}(\text{e.cm})$. The dotted line in the Fig. 5 represents the relation between $d_e$ and the CP violating phase $\theta_2$ as $\theta_1 = \theta_\mu = 0$. In the Fig. 6, we use the dashed line to describe electron EDM varying with $\theta_\mu$ based on the assumption $\theta_1 = \theta_2 = 0$. The absolute values of the results denoted by the dotted line and dashed line can both reach $1.1 \times 10^{-28}(\text{e.cm})$. The behaviors of these three lines (solid, dotted, dashed) are similar. That is to say they all look like sine function. From the Figs. 1, 2, 3, 4, 5, 6, one can find the upper bound of electron EDM is strict and has rigorous bound on the parameter space.

B. the muon EDM

In the similar way, the muon EDM is numerically studied. Lepton EDM is CP violating which is generated by the CP violating phases such as $\theta_1, \theta_2, \theta_\mu$. With $\tan \beta = 15$ and $N_e = N_\tau = 1000\text{GeV}$, we study muon EDM varying with the parameter $N_\mu$ which is related with the scalar muon mass. We obtain the solid line with $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$ in Fig. 7. It shows that $N_\mu$ strongly influence the solid line, and it turns large quickly with the decreasing $N_\mu$ when $N_\mu < 1000\text{GeV}$. At the point $N_\mu = 600\text{GeV}$, the muon EDM $d_\mu$ can
FIG. 4: With $N_e = N_\mu = N_\tau = 1000\text{GeV}$, $\theta_2 = \theta_\mu = 0$ and $\tan \beta = 15$, the contributions to electron EDM versus the CP violating phase $\theta_1$ are plotted by the solid line.

FIG. 5: With $N_e = N_\mu = N_\tau = 1000\text{GeV}$, $\theta_1 = \theta_\mu = 0$ and $\tan \beta = 15$, the contributions to electron EDM versus the CP violating phase $\theta_2$ are plotted by the dotted line.

reach $1.16 \times 10^{-24}(e.cm)$. In the $N_\mu$ range ($1200 \sim 2000\text{GeV}$), $d_\mu$ is small and varies from $2 \times 10^{-25}(e.cm)$ to $0.5 \times 10^{-25}(e.cm)$. The dashed line and dotted line are almost coincident and their values are much smaller than that of the solid line. In the whole scope of $N_\mu$, they both vary in the range from $0.32 \times 10^{-25}(e.cm)$ to $0.12 \times 10^{-25}(e.cm)$. Though their effects by $N_\mu$ are very weak, one can still find they are both decreasing functions of the enlarging $N_\mu$.

The upper bound of the muon EDM is at the order of $10^{-19}(e.cm)$, and it is much larger than that of electron. Considering this fact, we take $N_e = N_\tau = 1000\text{GeV}$ and $N_\mu = 600\text{GeV}$ to obtain biggish contribution to muon EDM. As $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$, the result varying with $\tan \beta$ is represented by the dotted line in the Fig. 5. The dotted line implies that it is
FIG. 6: With $N_e = N_\mu = N_\tau = 1000\text{GeV}$, $\theta_1 = \theta_2 = 0$ and $\tan \beta = 15$, the contributions to electron EDM versus the CP violating phase $\theta_\mu$ are plotted by the dashed line.

FIG. 7: With $\tan \beta = 15$ and $N_e = N_\tau = 1000\text{GeV}$, the contributions to muon EDM vary with $N_\mu$. The solid line represents the result for $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$; the dotted line represents the result for $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$; the dashed line represents the result for $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$.

the decreasing function of $\tan \beta$, and changes from $13.5 \times 10^{-25}(e.cm)$ to $11.1 \times 10^{-25}(e.cm)$. With $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$ and $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$ we plot the numerical results by the solid line and dashed line respectively in the Fig.9. The effects of $\theta_2$ and $\theta_\mu$ to muon are almost same and they both are increasing functions of $\tan \beta$. In general the solid line and dashed line are at the order of $10^{-26}(e.cm)$ and are much smaller than the dotted line.

From the Fig.7 and Figs.8,9, we find $\theta_1$ is the CP violating phase that gives dominate contribution to muon EDM. Therefore, we study the contribution to the muon EDM varying with the CP violating phase $\theta_1$. Supposing $N_e = N_\tau = 1000\text{GeV}, N_\mu = 600\text{GeV}$ and $\theta_2 = \theta_\mu = 0$, the contributions to muon EDM are represented by the dotted line, solid line
FIG. 8: With $N_e = N_\tau = 1000\text{GeV}$ and $N_\mu = 600\text{GeV}$, the contributions to muon EDM vary with $\tan \beta$. The dotted line represents the result for $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$.

FIG. 9: With $N_e = N_\tau = 1000\text{GeV}$ and $N_\mu = 600\text{GeV}$, the contributions to muon EDM vary with $\tan \beta$. The solid line represents the result for $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$; the dashed line represents the result for $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$.

and dashed line corresponding to $\tan \beta = 5(15, 25)$ respectively. They are shown in Fig. 10 and all can reach $10 \times 10^{-25}(e.cm)$. These three lines all look like sine function. With our used parameters, the big value of the numerical results for muon EDM shown as the Figs. (7, 8, 9, 10) is about at the order of $10^{-24}(e.cm)$, which is almost five order smaller than muon EDM upper bound.
FIG. 10: With $N_e = N_\tau = 1000\text{GeV}$, $N_\mu = 600\text{GeV}$ and $\theta_2 = \theta_\mu = 0$, the contributions to electron EDM vary with the CP violating phase $\theta_1$. With $\tan \beta = 5(15, 25)$, the results are plotted by the dotted line, solid line and dashed line respectively.

C. the tau EDM

Tau is the heaviest lepton, whose EDM upper bound is the largest one and at the order of $10^{-17}\text{e.cm}$. The tau EDM is also of interest and is calculated here with the supposition $N_e = N_\mu = 1000\text{GeV}$. We study the relation between $d_\tau$ and $N_\tau$ relating with the scalar tau masses. The masses of SUSY particles are important parameters that effect the SUSY contributions to lepton EDM. With $\tan \beta = 15$, $d_\tau$ is numerically studied with the varying $N_\tau$. The solid line represents the result for $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$; the dashed line represents the result for $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$; the dotted line represents the result for $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$. 

FIG. 11: With $\tan \beta = 15$ and $N_e = N_\mu = 1000\text{GeV}$, the contributions to tau EDM vary with $N_\tau$. The solid line represents the result for $(\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)$; the dashed line represents the result for $(\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)$; the dotted line represents the result for $(\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)$. 

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represents the result for \((\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)\); the dotted line represents the result for \((\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)\). These three lines are all decreasing functions of \(N_\tau\), and it conforms with the analysis. The solid line is sensitive to \(N_\tau\), whose biggest value is \(12 \times 10^{-25}\text{(e.cm)}\) with \(N_\tau = 600\text{GeV}\). The solid line diminish fast with enlarging \(N_\tau\), and it is about \(0.5 \times 10^{-25}\text{(e.cm)}\) when \(N_\tau > 1800\text{GeV}\). The dotted line and dashed line are very likeness, and they both vary in the range \(5 \times 10^{-25}(e.cm) \sim 2 \times 10^{-25}(e.cm)\).

\(\tan \beta\) is an important parameter shown as the front \(d_e\) and \(d_\mu\), and it influences the lepton EDM strongly. Here, we take the parameter as \(N_e = N_\mu = 1000\text{GeV}\) and \(N_\tau = 600\text{GeV}\).

![Graph](image)

**FIG. 12:** With \(N_e = N_\mu = 1000\text{GeV}\) and \(N_\tau = 600\text{GeV}\), the contributions to tau EDM vary with \(\tan \beta\). The solid line denotes the result for \((\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)\); the dashed line represents the result for \((\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)\); the dotted line represents the result for \((\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)\).

The numerical result for \(d_\tau\) with \((\theta_1 = 0.5\pi, \theta_2 = \theta_\mu = 0)\) is plotted by the solid line. While the dashed line and dotted line represent \(d_\tau\) as \((\theta_2 = 0.5\pi, \theta_1 = \theta_\mu = 0)\) and \((\theta_\mu = 0.5\pi, \theta_1 = \theta_2 = 0)\) respectively. The solid line changes weakly with the varying \(\tan \beta\), and the result is around \(13 \times 10^{-25}(e.cm)\). The solid line is the decreasing function of \(\tan \beta\). The dotted line and the dashed line are almost liner increasing functions of \(\tan \beta\), and they vary from \(1.7 \times 10^{-25}(e.cm)\) to \(13 \times 10^{-25}(e.cm)\) in \(\tan \beta\) range \((5 \sim 40)\). The dotted line is above the dashed line in a small degree.

In the end, we investigate the relations between \(d_\tau\) and the CP violating phases \(\theta_1, \theta_2\) and \(\theta_\mu\). The parameters used in the Figs. (13, 14, 15) are \(N_e = N_\mu = 1000\text{GeV}, N_\tau = 600\text{GeV}\) and \(\tan \beta = 15\). In the Fig. 13, the solid line denotes the \(d_\tau\) varying with \(\theta_1\) for \(\theta_2 = \theta_\mu = 0\). The biggest value represented by the solid line can reach \(12 \times 10^{-25}(e.cm)\). The dashed line in
FIG. 13: With $N_e = N_\mu = 1000\text{GeV}$, $N_\tau = 600\text{GeV}$, $\theta_2 = \theta_\mu = 0$ and $\tan \beta = 15$, the contribution to tau EDM versus the CP violating phase $\theta_1$ is plotted by the solid line.

FIG. 14: With $N_e = N_\mu = 1000\text{GeV}$, $N_\tau = 600\text{GeV}$, $\theta_1 = \theta_\mu = 0$ and $\tan \beta = 15$, the contribution to tau EDM versus the CP violating phase $\theta_2$ is plotted by the dashed line.

the Fig.14 implies the relation between tau EDM and $\theta_2$ when $\theta_1$ and $\theta_\mu$ are supposed as 0. Based on the assumption $\theta_1 = \theta_2 = 0$, we plot $d_\tau$ varying with $\theta_\mu$ by the dotted line in the Fig.15. The numerical values represented by the dotted line and dashed line are adjacent, and they vary in the range $-5 \times 10^{-25}(e.cm) \sim 5 \times 10^{-25}(e.cm)$. The Figs. 11, 12, 13, 14, 15 show that the one loop contributions to tau EDM are at the order of $10^{-24}(e.cm)$ in our used parameter space. These contributions are about seven order smaller than the upper bound of tau EDM.
FIG. 15: With $N_e = N_\mu = 1000\text{GeV}, N_\tau = 600\text{GeV}$, $\theta_1 = \theta_2 = 0$ and $\tan \beta = 15$, the contribution to tau EDM versus the CP violating phase $\theta_\mu$ is plotted by the dotted line.

V. DISCUSSION AND CONCLUSION

In the frame work of CP violating BLMSSM, we study the one loop contributions to the lepton($e, \mu, \tau$) EDMs. The used parameters can satisfy the experiment data of Higgs and neutrino. We study the effects of the CP violating phases $\theta_1, \theta_2$ and $\theta_\mu$ to the lepton EDM. The upper bound of electron EDM is $8.7 \times 10^{-29}(e.cm)$, which gives strict confine on the BLMSSM parameter space. In our used parameter space, the contributions to electron EDM can easily reach it’s upper bound and even exceed the bound. The numerical values obtained for muon EDM and tau EDM are both at the order of $10^{-24}(e.cm)$, and they are several order smaller than their upper bounds. In general, the numerical results of the lepton EDMs are large, and they maybe detected by the experiments in the near future.

Acknowledgements

This work has been supported by the National Natural Science Foundation of China (NNSFC) with Grants No. 11275036, the open project of State Key Laboratory of Mathematics-Mechanization with Grant No. Y3KF311CJ1, the Natural Science Foundation of Hebei province with Grant No. A2013201277, and the Found of Hebei province with the Grant NO. BR2-201 and the Natural Science Fund of Hebei University with Grants No. 2011JQ05 and No. 2012-242.
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