Effective Action and Measure in Matrix Model of IIB Superstrings

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Abstract

We calculate an effective action and measure induced by the integration over the auxiliary field in the matrix model recently proposed to describe IIB superstrings. It is shown that the measure of integration over the auxiliary matrix is uniquely determined by locality and reparametrization invariance of the resulting effective action. The large–$N$ limit of the induced measure for string coordinates is discussed in detail. It is found to be ultralocal and, thus, possibly is irrelevant in the continuum limit. The model of the GKM type is considered in relation to the effective action problem.
1 Introduction

The matrix model formulation of M theory [1] renewed an interest in the problem of a non-perturbative approach to superstrings. M(atrix) model [1] is expected to provide a complete quantum-mechanical description of M theory thus unifying all known string theories. The emergence of perturbative strings in M(atrix) theory is discussed in a series of recent papers [2].

In the other approach [3], the matrix model is directly identified with type IIB superstring. The action of this model is obtained from the ten-dimensional $U(N)$ super Yang–Mills theory by the reduction to a point. The string action follows from the matrix model by substituting the Poisson brackets for the commutators and the integrals over the world sheet for the traces. This is justified at infinite $N$ if the large $N$ limit is identified with the semiclassical one. As a result of this procedure, the Schild action of IIB Green–Schwarz superstring with fixed $\kappa$–symmetry arises:

$$S_{\text{Schild}} = \int d^2\sigma \left( \frac{\alpha}{4\sqrt{g}} \{X^\mu, X^\nu\}_P B - \frac{i}{2} \bar{\psi} \Gamma^\mu \{X^\mu, \psi\}_P B + \beta \sqrt{g} \right),$$  \hspace{1cm} (1.1)

where $\{,\}_P$ are the usual Poisson brackets.

The Schild action (1.1) is equivalent to the Nambu–Goto action, if the auxiliary field $\sqrt{g}$ is excluded via its classical equations of motion. On the quantum level, one should average w.r.t. $\sqrt{g}$ in the functional integral. The arguments have been given that the Polyakov string is reproduced in this manner, provided that the measure of integration is chosen properly and conformal anomaly cancels [4].

In the IKKT approach [3], the integration over $\sqrt{g}$ is modelled by a summation over the matrix size $N$, which, therefore, is a dynamical variable. A modification of the IKKT model was proposed [5] with an additional matrix variable $Y$ introduced, which enters the action of the matrix model the same way as $\sqrt{g}$ enters the string action:

$$S_{NBI} = -\frac{\alpha}{4} \text{tr} \, Y^{-1} [A_\mu, A_\nu]^2 + \beta \text{tr} \, Y - \frac{1}{2} \text{tr} \, \bar{\psi} \Gamma^\mu [A_\mu, \psi].$$  \hspace{1cm} (1.2)

There is no summation over $N$ in this model and the large $N$ limit can be taken straightforwardly, yielding the string action (1.1) in the semiclassical (large $N$) approximation.

The main idea of the matrix model approach is to consider the matrix integral in the infinite $N$ limit as the nonperturbative definition of the string partition function. The crucial point is the choice of an integration measure, which should respect all the symmetries of the underlying string theory. In [3], it was suggested to define the partition function as follows:

$$Z_{NBI} = \int dA_\mu \, d\bar{\psi} \, d\psi \, dY \, (\det Y)^{-\gamma} \, e^{-S_{NBI}}.$$  \hspace{1cm} (1.3)

The integration over $Y$ ranges over positive Hermitean matrices. In this paper, we focus on the result of this integration, which yields an effective action and measure for $A_\mu$ and their superpartners:

$$J(G) \, e^{-S_{\text{eff}}(G) + \frac{1}{2} \text{tr} \, \bar{\psi} \Gamma^\mu [A_\mu, \psi]} = \int dY \, (\det Y)^{-\gamma} \, e^{-S_{NBI}},$$  \hspace{1cm} (1.4)
where
\[ G = -[A_\mu, A_\nu]^2 \] (1.5)
is a Hermitean positively definite matrix. At large \( N \), when the commutator is replaced by the Poisson bracket, \( G \) becomes the determinant of an induced metric on the world sheet:
\[ G \longrightarrow \{X_\mu, X_\nu\}^2_{PB} = 2 \det_{ab} \partial_a X_\mu \partial_b X^\mu. \] (1.6)

Note that \( Y \) does not interact with fermions and the corresponding term in the action remains unchanged.

The separation of the result of integration over an auxiliary field \( Y \) into the effective action and measure is to some extent arbitrary. As a rule, all power-like terms in the exponential are naturally interpreted as an effective action. We also encounter the logarithmic terms which we refer to the measure of integration over \( A_\mu \). Such interpretation is justified in the large–\( N \) limit, as discussed in sec. 4.

It is evident that in the saddle point approximation the effective action does not differ from the matrix counterpart of the Nambu–Goto one:
\[ S_{NG} = \sqrt{\alpha \beta} \text{tr} \sqrt{G} = \sqrt{\alpha \beta} \text{tr} \sqrt{-[A_\mu, A_\nu]^2}, \] (1.7)
If \( A_\mu \) are regarded as a zero dimensional gauge potentials, this expression looks like the strong coupling non-Abelian Born–Infeld (NBI) action. The Nambu–Goto action is reproduced from the NBI matrix model (1.2), (1.3) even quantum–mechanically, up to a measure factor, for the special value of \( \gamma = N - 1/2 \), where the integration over \( Y \) can be performed explicitly (4). Our main goal is to check by the direct calculation, whether or not this result is valid in general, and how does the effective action look for various values of \( \gamma \).

We demonstrate that \( \gamma = N + O(1) \) is singled out by general principles of locality and reparametrization invariance of the string action. It seems, however, interesting to investigate the model (1.3) at arbitrary \( \gamma \), since this may correspond to some soft breaking of the reparametrization invariance in the theory. It is worth mentioning that the value of \( \gamma = N + O(1) \) is distinguished from many points of view. It was found to be critical for the matrix model, obtained from (1.3) by excluding \( A_\mu \) via their classical equations of motion (4). On the other hand, the integral over the Hermitean matrix \( Y \) with \( A_\mu \) treated as the external fields was shown (4) to be equivalent for \( \gamma = N \) to the unitary matrix integral of (8, 9).

In the general case, the integral over \( Y \) can not be calculated exactly and we use the methods of the large \( N \) expansion, systematically dropping the corrections in \( 1/N \). The parameters \( \gamma, \alpha \) and \( \beta \) are assumed to be of order \( N \).

## 2 Matrix model

According to the definition of the effective action (1.4),
\[ Z(G) \equiv J(G) e^{-S_{eff}(G)} = \int dY \ e^{-\frac{4}{\alpha} \text{tr} Y^{-1} G - \beta \text{tr} Y - \gamma \text{tr} \log Y}. \] (2.1)
This integral can be viewed as a one–matrix model with the external field $G$. It is convenient to make the change of the integration variables: $Y = \frac{N}{\beta} X^{-1}$, then $dY \propto (\det X)^{-2N} dX$ and, up to an irrelevant constant, eq. (2.1) can be rewritten in the following form:

$$Z = \int dX \ e^{-N \text{tr} \left[ X \Lambda + X^{-1} + (2\eta + 1) \log X \right]} ,$$

(2.2)

where we have introduced the notations:

$$\Lambda = \alpha \beta 4 N^2 G$$

(2.3)

and

$$\eta = \frac{1}{2} \left( 1 - \frac{\gamma}{N} \right).$$

(2.4)

The constant $\eta$ is normalized to be of order unity and to vanish for the critical value of $\gamma$.

The matrix integral (2.2) belongs to a class of generalized Kontsevich models [10]. Such models with negative powers of the matrix $X$ have been previously discussed in the context of $c = 1$ bosonic string theory [11]. In [9], the $\tau$-function approach to such models was developed. There, the parameter $\eta$ plays the role of the zeroth time in the corresponding integrable hierarchy. Moreover, at the conformal point $\eta = 0$, this model was shown [7] to have the same Schwinger–Dyson equations as the $U(N)$ model solved in [8, 9].

For the models of this type, the large $N$ solution is known explicitly only in some special cases. The models with cubic potential for $X$ [12] and the combination of the logarithmic and quadratic potentials [13, 14] were solved by a method based on Schwinger–Dyson equations, developed first for the unitary matrix models with external field [8, 9]. The same technique, being applied to the integral (2.2), also allows to find its large $N$ asymptotics in the closed form for arbitrary $\eta$.

The Schwinger–Dyson equations for (2.2) follow from the identity

$$\frac{1}{N^3} \frac{\partial}{\partial \Lambda_{jk}} \frac{\partial}{\partial \Lambda_{li}} \int dX \frac{\partial}{\partial X_{ij}} e^{-N \text{tr} \left[ X \Lambda + X^{-1} + (2\eta + 1) \log X \right]} = 0.$$  

(2.5)

This identity gives rise to the differential equation for $Z(\Lambda)$:

$$\left[ -\frac{1}{N^2} \Lambda_{ji} \frac{\partial}{\partial \Lambda_{jk}} \frac{\partial}{\partial \Lambda_{li}} + \frac{1}{N} (2\eta - 1) \frac{\partial}{\partial \Lambda_{ik}} + \delta_{kl} \right] Z(\Lambda) = 0.$$  

(2.6)

Due to the invariance of the integration measure and the action under the unitary transformations, the partition function $Z$ depends only on the eigenvalues $\lambda_i$ of the matrix $\Lambda$ and is symmetric under their permutations. Hence, only $N$ of the $N^2$ Schwinger–Dyson equations (2.6) are linearly independent. Being written in terms of the eigenvalues, these $N$ equations read

$$\left[ -\frac{1}{N^2} \lambda_i \frac{\partial^2}{\partial \lambda_i^2} - \frac{1}{N^2} \sum_{j \neq i} \lambda_j \frac{1}{\lambda_j - \lambda_i} \left( \frac{\partial}{\partial \lambda_j} - \frac{\partial}{\partial \lambda_i} \right) + \frac{1}{N} (2\eta - 1) \frac{\partial}{\partial \lambda_i} + 1 \right] Z(\lambda) = 0.$$  

(2.7)

For $\eta = 0$, these formulas coincide with the corresponding formulas for the $U(N)$ model [8, 9]. Here, we solve this model for the case $\eta \neq 0$.  

3
It is convenient to set
\[ W(\lambda_i) = \frac{1}{N} \frac{\partial}{\partial \lambda_i} \log Z. \] (2.8)
We also introduce the eigenvalue density of the matrix Λ:
\[ \rho(x) = \frac{1}{N} \sum_i \delta(x - \lambda_i). \] (2.9)
The density obeys the normalization condition
\[ \int dx \rho(x) = 1 \] (2.10)
and in the large \( N \) limit becomes a smooth function.
A simple power counting shows that the derivative of \( W(\lambda_i) \) in the first term on the left hand side of equation (2.7) is suppressed by the factor \( 1/N \) and can be omitted at \( N = \infty \).
The remaining terms are rewritten as follows:
\[ -xW^2(x) - \int dy \rho(y) \frac{W(y) - W(x)}{y - x} + (2\eta - 1)W(x) + 1 = 0, \] (2.11)
where \( \lambda_i \) is replaced by \( x \). The equation (2.11) can be simplified by the substitution
\[ \tilde{W}(x) = xW(x) - \eta. \] (2.12)
After some transformations, using the normalization condition (2.10), we obtain
\[ \tilde{W}^2(x) + x \int dy \rho(y) \frac{\tilde{W}(y) - \tilde{W}(x)}{y - x} = x + \eta^2. \] (2.13)
The nonlinear integral equation (2.13) can be solved with the help of the anzatz
\[ \tilde{W}(x) = f(x) + \frac{x}{2} \int dy \rho(y) \frac{\rho(y) - f(y)}{y - x}, \] (2.14)
where \( f(x) \) is an unknown function to be determined by substituting (2.14) into eq. (2.13).
The asymptotic behaviors of \( \tilde{W}(x) \) and \( f(x) \) as \( x \to \infty \) follow from eq. (2.13): \( \tilde{W}(x) \sim \sqrt{x} + 1/2 \), and the analytic solution with minimal set of singularities is simply
\[ f(x) = \sqrt{ax + b}. \] (2.15)
The parameters \( a \) and \( b \) are unambiguously determined from eq. (2.13). We find that \( b = \eta^2 \) and \( a \) is implicitly defined by the equality
\[ 1 + \frac{1}{2} \int dy \frac{\rho(y)}{\rho(y)} = \frac{1}{\sqrt{a}}, \] (2.16)
or, in terms of the eigenvalues,
\[ 1 + \frac{1}{2N} \sum_j \frac{1}{\sqrt{a\lambda_j + \eta^2}} = \frac{1}{\sqrt{a}}. \] (2.17)
For the logarithmic derivative of the partition function, \( W(\lambda_i) \), we have, according to eqs. (2.12) and (2.14):
\[
W(\lambda_i) = \frac{\eta}{\lambda_i} + \frac{\sqrt{a\lambda_i + \eta^2}}{\lambda_i} + \frac{a}{2N} \sum_j \frac{1}{\sqrt{a\lambda_j + \eta^2}} \left( \frac{1}{\sqrt{a\lambda_i + \eta^2}} + \frac{1}{\sqrt{a\lambda_j + \eta^2}} \right).
\]
(2.18)

To calculate the partition function \( Z(\lambda) \) one should integrate eq. (2.8). This integration is complicated by the fact that \( a \) is also the function of \( \lambda_i \). However, the problem can be avoided by the following trick. We integrate eq. (2.18) w.r.t. \( \lambda_i \) as if \( a \) is a constant. Then we can add an arbitrary function of \( a \) to the expression obtained. The proper choice of this function makes the final expression stationary in \( a \). Thus, we obtain:
\[
\log Z = N^2 \left[ \left( \eta^2 + \frac{1}{4} \right) \log a + \frac{4\eta^2}{\sqrt{a}} - \frac{\eta^2}{a} \right]
\]
\[
+ N \sum_i \left[ \frac{2}{\sqrt{a}} \sqrt{a\lambda_i + \eta^2} + \eta \log \left( \lambda_i \sqrt{a\lambda_i + \eta^2} - \eta \right) \right]
\]
\[
- \frac{1}{2} \sum_{ij} \log \left( \sqrt{a\lambda_i + \eta^2} + \sqrt{a\lambda_j + \eta^2} \right).
\]
(2.19)

One can verify directly that \( \frac{\partial}{\partial a} \log Z = 0 \) and \( \frac{1}{N} \frac{\partial}{\partial \lambda_i} \log Z = W(\lambda_i) \), as far as eq. (2.17) holds.

3 Effective action and measure

We refer all logarithmic terms in (2.19) to the induced measure. Then the remaining terms represent the effective action. Returning to the original notations (2.3), (2.4), after some transformations we find from eq. (2.19):
\[
S_{\text{eff}}(G) = \text{tr} \sqrt{\alpha \beta G + F^2} + (N - \gamma)F - \frac{1}{4} F^2,
\]
(3.1)
\[
J(G) = F^{-\frac{1}{4}(N-\gamma)^2} \prod_i \left( \frac{g_i \sqrt{\alpha \beta g_i + F^2} - F}{\sqrt{\alpha \beta g_i + F^2} + F} \right)^{\frac{1}{2}(N-\gamma)} \prod_{i<j} \frac{1}{\sqrt{\alpha \beta g_i + F^2} + \sqrt{\alpha \beta g_j + F^2}}
\]
(3.2)

Here \( g_i \) are the eigenvalues of the matrix \( G \). The parameter \( F \) is related to \( a \) by
\[
F^2 = \frac{4\eta^2 N^2}{a}.
\]
(3.3)

It can be found from eq. (2.17), which is rewritten as follows:
\[
F = N - \gamma + F \text{ tr} \frac{1}{\sqrt{\alpha \beta G + F^2}}.
\]
(3.4)

General expressions (3.1) and (3.2) considerably simplify for \( \gamma = N + O(1) \). In this case, \( F \to 0 \) and (3.1) reduces to the Nambu–Goto action (1.4), as expected from [5]. We also
find that this value of $\gamma$ is critical for the matrix model involved – the measure $J(G)$ is nonanalytic at $\gamma = N$:

$$J(G) = e^{-\frac{1}{2}(N-\gamma)^2 \log(N-\gamma)} \times \text{regular}. \quad (3.5)$$

The critical behavior we have found is typical for all matrix models with logarithmic potentials \[13, 13\] and, probably, it is the same that was discussed in \[6\]. This nonanalyticity of the measure (3.3) is harmless – the limit at $\gamma = N$ exists. Moreover, this singularity seems to be irrelevant for the description of superstrings, since the nonanalytic piece of the measure factorizes out and does not depend on $G$, hence, it cancels in all correlation functions of $A_\mu$ and $\psi$.

At the critical point, we have, dropping an overall constant factor,

$$J(G) = \prod_{i<j} \frac{1}{\sqrt{g_i} + \sqrt{g_j}}. \quad (3.6)$$

In fact, for $N = \gamma - 1/2$ this result is exact, i.e., it is valid beyond the large $N$ approximation \[4\]. It is interesting to compare (3.3) and (3.4) at $F = 0$ with the solution of the unitary matrix model \[8, 9\]. In accord with \[7\], we find the agreement with the $U(N)$ model in the weak coupling (large $G$) phase \[8, 9\]. The strong coupling (small $G$) phase \[9\] of the unitary model is characterized by the partition function $Z(G)$, which is analytic at $G = 0$, and this phase corresponds to the different branch of the solution to eq. (3.4). However, it is never realized for the Hermitean model, since the matrix $G$ is responsible for the convergence of the integral, and one can not expand it in $G$, whatever small it is. In the case of the unitary matrix model, there are no problems with convergence because of the compactness of the integration domain.

## 4 Discussion

In the large $N$ limit the matrix $G$ is replaced by the determinant of the induced metric on the string world sheet according to eq. (1.6). Let us consider the induced measure (3.6) in this limit\[4\]. For this purpose, we rewrite eq. (3.6) in the form

$$J(G) = \left[ \frac{\Delta^2(\sqrt{g})}{\Delta^2(g)} \right]^{1/2} \left( \frac{\det' \left\{ \sqrt{G}, \cdot \right\} }{\det' \left\{ G, \cdot \right\} } \right)^{1/2}, \quad (4.1)$$

where $\Delta$ is the Vandermonde determinant, $\Delta(g) = \prod_{i<j} (g_i - g_j)$, and the prime denotes that zero modes are omitted in the determinants. Replacing the commutators by the Poisson brackets, we obtain:

$$J[G] = \left( \frac{\det' \left\{ \sqrt{G}, \cdot \right\} }{\det' \left\{ G, \cdot \right\} } \right)^{1/2} \left( \frac{\det' \varepsilon^{ab} \partial_a \sqrt{G} \partial_b}{\det' \varepsilon^{ab} \partial_a G \partial_b} \right)^{1/2}. \quad (4.2)$$

\[1\]We are thankful to P. Olesen for the discussion of this point.
This fraction of the determinants can be substantially simplified, since
\[ \varepsilon^{ab} \partial_a \sqrt{G} \partial_b = \frac{1}{2\sqrt{G}} \varepsilon^{ab} \partial_a G \partial_b. \] (4.3)

Therefore,
\[ J[G] = \text{const} \left( \text{Det} \sqrt{G} \right)^{-1/2}. \] (4.4)

This means that the measure of integration over the string coordinates has the form:
\[ DX_\mu = \prod_\sigma dX_\mu(\sigma) G^{-1/4}(\sigma). \] (4.5)

This expression confirms our interpretation of \( J(G) \) as the induced measure factor, rather than the part of the effective action.

Some comments concerning the measure (4.5) are in order. First of all, the measure (4.5) respects the reparametrization invariance, because under the world sheet diffeomorphisms \( \sigma \rightarrow \sigma' \) the measure is multiplied by the (infinite) constant:
\[ DX_\mu \rightarrow \text{const} \, DX_\mu. \] (4.6)

Since this constant does not depend on the fields, it cancels in all correlation functions. Another important property of the induced measure is its ultralocality:
\[ J[G] = \exp \left( -\frac{1}{4} \int d^2 \sigma \, \delta(\sigma - \sigma') \bigg|_{\sigma' \rightarrow \sigma} \ln G(\sigma) \right). \] (4.7)

In some regularizations \( \delta(0) = 0 \). So, probably the extra factor in the measure can be even omitted. It is worth mentioning that the replacement of the string coordinates by the matrices already introduces some kind of momentum cutoff. The number of the Fourier harmonics in the matrix regularization is effectively of order \( N \), consequently \( N \) plays the role of the largest possible momentum. Thus, in the matrix regularization \( \delta(0) = N^2 \).

It seems likely that the matrix model (2.1) deserves the investigation. Its partition function can be developed into the power series in \( 1/N \) similar to the solutions obtained in [14] for the Kontsevich matrix model and in [17] for the Kontsevich–Penner (Hermitian one-matrix) model. It would be interesting to find whether the model (2.1) has a geometrical meaning from the point of view of the moduli spaces (as the two previous models have).

However, from the string theory point of view, the effective action (3.1) looks rather complicated unless \( \gamma = N + O(1) \). After replacing the traces and the commutators by the integrals and the Poisson brackets it should be interpreted as a world sheet action of the string. In general, this action is nonlocal due to the implicit dependence of the parameter \( F \) on \( G \). What is more important, it is not reparametization invariant – while \( \sqrt{G} \) transforms under general diffeomorphisms as a scalar density, \( F \) has no definite transformation law, as it can be seen from its defining equation (3.4). The only way to bypass these difficulties is to set \( F = 0 \), which is only possible for \( \gamma = N + O(1) \). Thus, the general principles of the string theory select the unique, up to the terms subleading in \( 1/N \), value of \( \gamma \).

Qualitatively, these results can be explained, if we consider the scaling transformations:
\[ \sigma \rightarrow \lambda \sigma, \{ , \}_{PB} \rightarrow \lambda^{-2} \{ , \}_{PB}, \sqrt{g} \rightarrow \lambda^{-2} \sqrt{g}. \] Strictly speaking, they have no analogues in
the matrix model, since the measure of integration over the world sheet of the string, $d^2\sigma$, also should transform. In the matrix model this would correspond to the transformation of traces: $\text{tr} \rightarrow \lambda^2 \text{tr}$. However, if the functional measure in the matrix model is invariant under the rescaling of $Y$, which is the counterpart of $\sqrt{g}$, the value of $\gamma$ is immediately fixed to be equal to $N$.

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