Fourth SM Family Manifestations at CLIC

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The latest electroweak precision data allow the existence of additional chiral generations in the standard model. We study prospects of search for the fourth standard model family fermions and quarkonia at $e^+e^-$ and $\gamma\gamma$ options of CLIC. It is shown that CLIC will be a powerful machine for discovery and investigation of both fourth family leptons and quarkonia. Moreover, the formation of the fourth family quarkonia will give a new opportunity to investigate Higgs boson properties.

I. INTRODUCTION

Today, the mass and mixing patterns of the fundamental fermions are the most mysterious aspects of particle physics. Even, the number of fermion generations is not fixed by the Standard Model (SM). In this sense, SM may be deliberated as an effective theory of fundamental interactions rather than fundamental particles. The statement of the flavor democracy (or, in other words, the Democratic Mass Matrix (DMM) approach), which is quite natural in the SM framework, may be considered as the interesting step in the true direction [1–4]. It is intriguing that, flavor democracy favors the existence of the fourth standard model family [5–8]. The lower limits to masses of the fourth family fermions are [9]:

\[ m_{\nu_4} \gtrsim 50 \text{ GeV from LEP 1,} \]
\[ m_{l_4} \gtrsim 100 \text{ GeV from LEP 2,} \]
\[ m_{d_4} > 199 \text{ GeV (neutral current decays, } d_4 \to qZ) \text{ and } m_{d_4} > 128 \text{ GeV (charged current decays, } d_4 \to qW) \text{ from FNAL (Tevatron Run I).} \]

Recently [10] it is shown that a single extra chiral family with a constrained spectrum is consistent with latest precision data without requiring any other new physics source. Then, as quoted from a recent paper [11]: "It is shown that additional chiral generations are not excluded by the latest electroweak precision data if one assumes that there is no mixings with the known three generations. In the case of "heavy extra generations", when all four new particles are heavier than Z boson, quality of the fit for the one new generation is as good as for zero new generations (Standard Model)". It should be noted that, a degenerate fourth SM family is still disfavored by the data. Although the number of families for degenerate case ($T = U = 0$) is $2.97 \pm 0.30$, this restriction can be relaxed by allowing $T \neq 0$ (which means either non-degenerate extra generations or a new physics), for example, $N_{fam} = 3.27 \pm 0.45$ for $T = 0.10 \pm 0.11$ [12,13].

The fourth family quarks will be copiously produced at the LHC [14,15] if their masses are less than 1 TeV. Concerning the Tevatron Run II, its higher luminosity will increase the discovery limits of $d_4$ quarks with respect to Tevatron Run I data approximately 50%. In addition, the Higgs boson "golden mode" ($H \to ZZ \to 4l$, where $l = e, \mu$) will be observable at the upgraded Tevatron for $125 < m_H < 165$ GeV and $175 < m_H < 300$ GeV with more than 3 $\sigma$ significance [16], provided that the fourth SM family exists. For the same reasons, the SM Higgs boson could be seen at the LHC via the "golden mode" even with an integral luminosity of only a few fb$^{-1}$ [17]. Therefore, the presumed fourth family quarks will be discovered at LHC well before getting CLIC into operation. The same statement is valid for pseudoscalar quarkonia ($\eta_4$) formed by the fourth family quarks [15,18]. Observation of the fourth family leptons at LHC is problematic due to large backgrounds from the production of real W and Z bosons, both singly and in pairs, that submerge these heavy lepton signals [15,19]. For these reasons, lepton colliders will be advantageous for investigation of the fourth SM family leptons and vector ($\psi_4$) quarkonia. The potential of muon colliders in this context was analyzed in [20]. With this paper we complete general overview of the subject by considering the potential of CLIC [21].
II. FLAVOR DEMOCRACY AND THE STANDARD MODEL

It is useful to consider three different bases:
- Standard Model basis \( \{ f^0 \} \),
- Mass basis \( \{ f^m \} \),
- Weak basis \( \{ f^w \} \).

According to the three family SM, before the spontaneous symmetry breaking quarks are grouped into following SU(2) × U(1) multiplets:

\[
\begin{pmatrix}
  (u^0_L, d^0_L), (c^0_R, s^0_R), (t^0_R, b^0_R)
\end{pmatrix}
\]

In one family case all bases are equal and, for example, d-quark mass is obtained due to Yukawa interaction

\[
L^{(d)}_Y = a_d \left( \overline{\nu}_L \, \mathbf{1}_L \right) \left( \begin{pmatrix} \varphi^+ \\ \varphi^- \end{pmatrix} \right) d_R + h.c. \Rightarrow L^{(d)}_m = m_d \overline{d} d
\]

where \( m_d = a_d \eta, \quad \eta = \langle \varphi^0 \rangle \cong 249 \text{ GeV} \). In the same manner \( m_u = a_u \eta, \quad m_e = a_e \eta \) and \( m_\nu = a_\nu \eta \) (if neutrino is Dirac particle). In n family case

\[
L^{(d)}_Y = \sum_{i,j=1}^n a^d_{ij} \left( \overline{\nu}_{L_i} \, \mathbf{1}_{L_i} \right) \left( \begin{pmatrix} \varphi^+ \\ \varphi^- \end{pmatrix} \right) d^d_{R_j} + h.c. \Rightarrow L^{(d)}_m = \sum_{i,j=1}^n m^d_{ij} \overline{d}^d_i d^d_j, \quad m^d_{ij} = a^d_{ij} \eta
\]

where \( d^0_i \) denotes \( d^0, \quad s^0_i \) denotes \( s^0 \) etc. The diagonalization of mass matrix of each type of fermions, or in other words transition from SM basis to mass basis, is performed by well-known bi-unitary transformation. Then, the transition from mass basis to weak basis result in CKM matrix

\[
U_{CKM} = (U^d_L)^T \; U^d_L
\]

which contains 3(6) observable mixing angels and 1(3) observable CP-violating phases in the case of three(four) SM families.

Before the spontaneous symmetry breaking all quarks are massless and there are no differences between \( d^0, s^0 \) and \( b^0 \), fermions with the same quantum numbers are indistinguishable. This leads us to the first assumption [1], namely, Yukawa couplings are equal within each type of fermions:

\[
a^d_{ij} \equiv a^d, \quad a^u_{ij} \equiv a^u, \quad a^l_{ij} \equiv a^l, \quad a^\nu_{ij} \equiv a^\nu
\]

The first assumption result in \( n - 1 \) massless particles and one massive particle with \( m = na^F \eta (F = u, d, l, \nu) \) for each type of the SM fermions.

Because there is only one Higgs doublet which gives Dirac masses to all four types of fermions (up quarks, down quarks, charged leptons and neutrinos), it seems natural to make the second assumption [5,6], namely, Yukawa constants for different types of fermions should be nearly equal:

\[
a^d \approx a^u \approx a^l \approx a^\nu \approx a
\]

Taking into account the mass values for the third generation, the second assumption leads to the statement that according to the flavor democracy the fourth SM family should exist.

In terms of the mass matrix above arguments mean

\[
M^0 = a \eta \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix} \Rightarrow M^m = 4a \eta \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Now, let us make the third assumption, namely, \( a \) is between \( e = g_u \sin \theta_W \) and \( g_w / \cos \theta_W \). Therefore, the fourth family fermions are almost degenerate, in good agreement with experimental value \( \rho = 0.9998 \pm 0.0008 \) [12], and their common mass lies between 320 GeV and 730 GeV. The last value is close to upper limit on heavy quark masses, \( m_Q \leq 700 \text{ GeV} \), which follows from partial-wave unitarity at high energies [22]. It is interest that with preferable value \( a \approx g_w \) flavor democracy predicts \( m_4 \approx 8 m_{t_w} \approx 640 \text{ GeV} \).

The masses of the first three family fermions, as well as an observable interfamily mixings, are generated due to the small deviations from the full flavor democracy [5,7].
III. PAIR PRODUCTION

As a result of the DMM approach, the SM is extended to include a fourth generation of fundamental fermions, with masses typically in the range from 300 GeV to 700 GeV. Therefore, CLIC with 1-3 TeV center of mass energy will give opportunity to search the alleged fourth SM family fermions and quarkonia in details.

A. $e^+e^-$ option

The annihilation of $e^+e^-$ is a classic channel to produce and study new heavy fermions, because the cross sections are relatively large compared with backgrounds [19]. The cross section for the process $e^+e^- \to f \bar{f}$ has the well-known form

$$\sigma = \frac{2\pi\alpha^2}{3s} \xi \beta \left\{ Q_f (Q_f - 2\chi_1 v_{f\bar{f}}) (3 - \beta^2) + \chi_2 (1 + v^2) \left[ v_f^2 (3 - \beta^2) + 2\beta^2 a_f^2 \right] \right\}$$  \hspace{1cm} (8)

where

$$\chi_1 = \frac{1}{16\sin^2\theta_W \cos^2\theta_W (s-M_f^2)^2 + \Gamma_f^2 M_f^2}$$

$$\chi_2 = \frac{1}{256\sin^2\theta_W \cos^2\theta_W (s-M_f^2)^2 + \Gamma_f^2 M_f^2}$$

$$v = -1 + 4\sin^2\theta_W$$

$$a_f = 2T_3 f - 4Q_f \sin^2\theta_W$$

$$\beta = \sqrt{1 - 4m_f^2/s}.$$  

$T_3 = \frac{1}{2}$ for $\nu_4$ and $u_4$, $T_3 = -\frac{1}{2}$ for $l_4$ and $d_4$

$\xi = 1$ for leptons, $\xi = 3$ for quarks.

Obtained cross section values for pair production of the fourth SM family fermions with $m_4 = 320$ (640) GeV and corresponding number of events per working year ($10^7$ s) are given in Table I (Table II). Event signatures are defined by the mass pattern of the fourth family and 4 x 4 Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. According to scenario given in [7], dominant decay modes are $u_4 \to b W^-$, $d_4 \to t W^+$, $l_4 \to \nu_rW^-$ and $\nu_4 \to \tau^- W^+$. We have mentioned in the introduction that the fourth family quarks with $m_4 < 1$ TeV will be discovered at LHC. However, CLIC will raise an opportunity to investigate their properties in details due to sufficiently large event numbers and clean environment. The observation of the fourth family leptons at hadron colliders is not very promising because of low statistics and large background. The charged $l_4$ lepton will have clear signature at CLIC. For example, if produced $W^\pm$ bosons decay leptonically, one deals with two acoplanary opposite charge leptons and large missing energy. Pair production of the neutral $\nu_4$ leptons will lead to more complicated event topology. In this case, $\tau$ tagging will be helpful in identification of events. Indeed, produced $\tau$ leptons will decay at the distance 1-2 cm from interaction point, which can be easily measured by vertex detector.

In addition, polarization of electron and positron beams should help to experimentally determine axial and vector neutral current coupling constants of the fourth family fermions. Note that hadron colliders can not provide sufficient information about them. This subject is under study.

B. $\gamma\gamma$ option

It is well known that linear $e^+e^-$ colliders will give opportunity to construct TeV energy $\gamma\gamma$ colliders on their basis [23–25]. The fourth SM family quarks and charged leptons will be copiously produced at $\gamma\gamma$ machines. The cross section for $\gamma\gamma \to f\bar{f}$ at fixed $s$ has the form

$$\bar{\sigma} = \frac{2\xi\alpha^2_{em} Q_f^2}{s(1 + \beta^2)} \left[ 2\beta(\beta^4 - \beta^2 - 2) + (\beta^6 + \beta^4 - 3\beta^2 - 3) \ln \left( \frac{1 - \beta}{1 + \beta} \right) \right]$$  \hspace{1cm} (9)

where $\beta = \sqrt{1 - 4m_f^2/s}$. Since Compton backscattered photons are not monochromatic, one should perform further integrations over the photon spectrum to obtain the outcome cross section.
\[ \sigma = \int_{\tau_{\text{min}}}^{(0.83)^2} d\tau \int_{\tau/0.83} dx \frac{f_{\gamma}(x)}{x} f_{\gamma}(x) \tilde{\sigma}(\tau s) \]  

(10)

where \( \tau_{\text{min}} = 4m^2/s \) and \( \tilde{\sigma} = \tau s \). The energy spectrum of the high energy photons obtained through Compton backscattering of laser photons on the high energy electron beam has the form

\[ f_{\gamma}(y) = \frac{1}{1.84} \left[ 1 - y + \frac{1 - y}{1 - y} - \frac{4y}{\zeta(1 - y)} + \frac{4y^2}{\zeta^2(1 - y)^2} \right] \]  

(11)

with \( \zeta = 4.8 \). For \( \sqrt{s_{ee}} = 1 \text{ TeV} (3 \text{ TeV}) \), which corresponds to \( \sqrt{s_{\gamma\gamma}} \text{max} = 0.83\sqrt{s_{ee}} = 0.83 \text{ TeV} (2.5 \text{ TeV}) \), the obtained values of cross section and event number per year are presented in Tables I (Table II).

### IV. QUARKONIUM PRODUCTION

The condition for forming \((Q\bar{Q})\) quarkonia states with new heavy quarks is [26]

\[ m_Q \leq (125 \text{ GeV} \ |V_{Qq}|)^{-2/3} \]  

(12)

where \( q = d, s, b \) for \( Q = u_4 \) and \( q = u, c, t \) for \( Q = d_4 \). Differing from t quark, fourth family quarks will form quarkonia because \( u_4 \) and \( d_4 \) are almost degenerate and their decays are suppressed by small CKM mixings [6–8]. Below, we consider resonance productions of \( \psi_4 \) quarkonia at \( e^+e^- \) and \( \eta_4 \) quarkonia at \( \gamma\gamma \) options of CLIC.

#### A. \( e^+e^- \) option

The cross section for the formation of the fourth family quarkonium is given with the well-known relativistic Breit-Wigner equation

\[ \sigma \left( e^+e^- \to (Q\bar{Q}) \right) = \frac{12\pi \left(s/M^2\right) \Gamma_{ee}\Gamma}{(s - M^2)^2 + M^2\Gamma^2} \]  

(13)

where \( M \) is the mass, \( \Gamma_{ee} \) is the partial decay width to \( e^+e^- \) and \( \Gamma \) is the total decay width of the fourth family quarkonium. Using corresponding formulas from [27] in the framework of Coulomb potential model, we obtain decay widths for the main decay modes of \( \psi_4(u_4\bar{u}_4) \) and \( \psi_4(d_4\bar{d}_4) \), which are given in Table III. One can see that dominant decay mode for both \( \psi_4(u_4\bar{u}_4) \) and \( \psi_4(d_4\bar{d}_4) \) quarkonia is \( \psi_4 \to W^+W^- \). Next important decay modes are \( \psi_4 \to \gamma Z \) and \( \psi_4 \to \gamma H \).

In order to estimate the number of produced quarkonium states, one should take into account the luminosity distribution at CLIC which is influenced from energy spread of electron and positron beams and beamstrahlung. In our calculations we use GUINEA-PIG simulation code [28]. For illustration, we assume that \( m_{\psi_4} \approx 1 \text{ TeV} \). The estimated event numbers per year for \( \psi_4 \) production, as well as, \( \psi_4 \to \gamma H \) and \( \psi_4 \to ZH \) decay channels are presented in Table IV. In our opinion, \( \gamma H \) decay mode of \( \psi_4(u_4\bar{u}_4) \) quarkonium is promising for investigation of Higgs boson properties, especially if energy spread of electron and positron beams about \( 10^{-3} \) could be managed succesfully.

Since quarkonia are produced at resonance \((\sqrt{s} \approx m_{\psi_4})\) and the mass of the fourth family quarks are close to \( m_{\psi_4}/2 \), the uncertainty in measurement of quarks’ masses is determined by energy spread of the colliding beams. Therefore, if \( m_4 = 500 \text{ GeV} \), \( \Delta m = 5 \text{ GeV} \) for \( \Delta E/E = 10^{-2} \) and \( \Delta m = 0.5 \text{ GeV} \) for \( \Delta E/E = 10^{-3} \). For comparison at LHC \( \Delta m = 22 \text{ GeV} \) for \( m_4 = 320 \text{ GeV} \) and \( \Delta m = 36 \text{ GeV} \) for \( m_4 = 640 \text{ GeV} \) [14,15].

#### B. \( \gamma\gamma \) option

Pseudoscalar \( \eta_4 \) quarkonia formed by the fourth SM family quarks will be copiously produced at LHC [15,18]. Decay widths for main decay modes of \( \eta_4(u_4\bar{u}_4) \) and \( \eta_4(d_4\bar{d}_4) \) are given in Table V. As seen from the table, the dominant decay mode is \( \eta_4 \to ZH \). One can estimate \( \gamma\gamma \to \eta_4 \) production cross section approximately by using following relation [29]:

4
\[ \sigma \approx 50\text{fb}(1 + \lambda_1 \lambda_2) \left( \frac{\text{Br}_{\gamma\gamma}}{4 \times 10^{-3}} \right) \left( \frac{\Gamma_{\text{tot}}}{1 \text{MeV}} \right) \left( \frac{200 \text{GeV}}{M_\eta} \right)^3 \] (14)

where \( \text{Br}_{\gamma\gamma} \) is the branching ratio of \( \eta_4 \to \gamma\gamma \) decay mode, \( \Gamma_{\text{tot}} \) is the quarkonium total decay width and \( \lambda_{1,2} \) are helicities of initial photons. Assuming \( \lambda_1 \lambda_2 = 1 \), we obtain total event numbers of \( \eta_4 \) production, as well as numbers of \( \eta_4 \to ZH \) events, which are given in Table VI. The advantage of \( \eta_4(u_4\bar{u}_4) \) with respect to \( \eta_4(d_4\bar{d}_4) \) is obvious.

**V. CONCLUSION**

We have shown that both fourth family fermions and quarkonia will be copiously produced at CLIC. If the fourth SM family exists, CLIC will be excellent place for investigation of fourth family quarkonia and leptons. Formation of the fourth SM family quarkonia will give a new opportunity to investigate Higgs bosons properties especially due to \( e^+e^- \to \psi_4 \to \gamma H \) channel. In addition, masses of \( u_4 \) and \( d_4 \) quarks will be measured with high accuracy.

It is possible that other lepton colliders such as NLC, TESLA or JLC can get into operation before CLIC. Of course, the fourth family fermions can be produced at these machines of kinematically allowed. The first stage of all of them (including CLIC) is designed with \( \sqrt{s} = 0.5 \text{ TeV} \). The second stage is thought of \( \sqrt{s} = 1 \text{ TeV} \) with the exception of TESLA (with \( \sqrt{s} = 0.8 \text{ TeV} \). Our result presented in Tables I, IV and VI are applicable for second stage NLC and JLC with a suitable scaling factors coming from luminosity and energy spread. Therefore, all linear electron-positron colliders are almost equal as long as kinematics allows. Finally, obvious advantage of CLIC is 3 TeV option.

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TABLE I. Cross sections and event numbers per year for pair production of the fourth standard model family fermions with mass 320 GeV at CLIC ($\sqrt{s_{ee}} = 1$ TeV, $L_{ee} = 2.7 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ and $L_{\gamma\gamma} = 10^{34}$ cm$^{-2}$ s$^{-1}$)

| $e^+e^-$ option | $\sigma$ (fb) | $N_{ev}$/year |
|------------------|---------------|---------------|
| $u_4\bar{u}_4$   | 130           | 35000         |
| $d_4\bar{d}_4$   | 60            | 16000         |
| $l_4\bar{l}_4$   | 86            | 23000         |
| $\nu_4\bar{\nu}_4$ | 15          | 4100          |

| $\gamma\gamma$ option | $\sigma$ (fb) | $N_{ev}$/year |
|------------------------|---------------|---------------|
|                         | 34            | 3400          |
|                         | 2             | 200           |
|                         | 58            | 5700          |

TABLE II. Cross sections and event numbers per year for pair production of the fourth standard model family fermions with mass 640 GeV at CLIC ($\sqrt{s_{ee}} = 3$ TeV, $L_{ee} = 1 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ and $L_{\gamma\gamma} = 3 \times 10^{34}$ cm$^{-2}$ s$^{-1}$)

| $e^+e^-$ option | $\sigma$ (fb) | $N_{ev}$/year |
|------------------|---------------|---------------|
| $u_4\bar{u}_4$   | 16            | 16000         |
| $d_4\bar{d}_4$   | 8             | 8000          |
| $l_4\bar{l}_4$   | 10            | 10000         |
| $\nu_4\bar{\nu}_4$ | 2            | 2000          |

| $\gamma\gamma$ option | $\sigma$ (fb) | $N_{ev}$/year |
|------------------------|---------------|---------------|
|                         | 27            | 8100          |
|                         | 2             | 600           |
|                         | 46            | 14000         |

TABLE III. Decay widths for main decay modes of $\psi_4$ for $m_H = 150$ (300) GeV with $m_{\psi_4} \simeq 1$ TeV

| ($u_4\pi^0$) | ($d_4\bar{d}_4$) | ($u_4\pi^\pm$) | ($d_4\bar{u}_4$) | ($d_4\bar{d}_4$) | ($u_4\pi^\mp$) | ($d_4\bar{d}_4$) |
|--------------|------------------|----------------|------------------|------------------|----------------|------------------|
| $\Gamma(\psi_4 \rightarrow \ell^+\ell^-)$, $10^{-6}$ MeV | 18.9 | 18.9 | 7.3 | 7.3 |
| $\Gamma(\psi_4 \rightarrow u\bar{u})$, $10^{-2}$ MeV | 3.2 | 3.2 | 1.9 | 1.9 |
| $\Gamma(\psi_4 \rightarrow d\bar{d})$, $10^{-2}$ MeV | 1.4 | 1.4 | 1.7 | 1.7 |
| $\Gamma(\psi_4 \rightarrow Z\gamma)$, $10^{-1}$ MeV | 15 | 15 | 3.7 | 3.7 |
| $\Gamma(\psi_4 \rightarrow ZZ)$, $10^{-1}$ MeV | 1.7 | 1.7 | 5.4 | 5.4 |
| $\Gamma(\psi_4 \rightarrow ZH)$, $10^{-1}$ MeV | 1.7 | 1.6 | 5.5 | 5.2 |
| $\Gamma(\psi_4 \rightarrow \gamma H)$, $10^{-1}$ MeV | 14.4 | 13.4 | 3.6 | 3.4 |
| $\Gamma(\psi_4 \rightarrow W^+W^-)$, MeV | 70.8 | 70.8 | 71.2 | 71.2 |
TABLE IV. The production event numbers per year for the fourth SM family $\psi_4$ quarkonia at CLIC 1 TeV option with $m_{\psi_4} \simeq 1$ TeV

| Process | $\Delta E/E$ | Events per year |
|---------|--------------|-----------------|
| $e^+e^- \to \psi_4 (u\bar{u})$ | $10^{-2}$ | 3500 |
| | $10^{-3}$ | 26600 |
| $e^+e^- \to \psi_4 \to \gamma H (d\bar{d})$ | $10^{-2}$ | 1400 |
| | $10^{-3}$ | 10400 |

TABLE V. Decay widths for main decay modes of $\eta_4$ for $m_H = 150$ (300) GeV with $m_{\eta_4} = 0.75$ TeV

| Decay Mode | $m_H = 150$ GeV | $m_H = 300$ GeV |
|------------|-----------------|-----------------|
| $\Gamma(\eta_4 \to \gamma \gamma)$, 10$^{-3}$ MeV | 19.5 | 19.5 |
| $\Gamma(\eta_4 \to Z \gamma)$, 10$^{-3}$ MeV | 4.6 | 4.6 |
| $\Gamma(\eta_4 \to ZZ)$, 10$^{-1}$ MeV | 2.2 | 2.2 |
| $\Gamma(\eta_4 \to gg)$, MeV | 5.1 | 5.1 |
| $\Gamma(\eta_4 \to ZH)$, MeV | 47.3 | 47.3 |
| $\Gamma(\eta_4 \to W^+W^-)$, 10$^{-2}$ MeV | 5.7 | 5.7 |
| $\Gamma(\eta_4 \to b\bar{b})$, MeV | 16.4 | 16.4 |
| $\Gamma(\eta_4 \to b\bar{b})$, 10$^{-2}$ MeV | 1.0 | 1.0 |

TABLE VI. The production event numbers per year for the fourth SM family $\eta_4$ quarkonia at $\gamma\gamma$ option with $m_{\eta_4} = 0.75$ TeV

| Process | $m_H = 150$ GeV | $m_H = 300$ GeV |
|---------|-----------------|-----------------|
| $\gamma\gamma \to \eta_4 (u\bar{u})$ | 900 | 900 |
| | $d\bar{d}$ | 56 | 56 |
| $\gamma\gamma \to \eta_4 \to ZH (u\bar{u})$ | 610 | 520 |
| | $d\bar{d}$ | 38 | 33 |