Trapping of Projectiles in Fixed Scatterer Calculations*

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Abstract

We study multiple scattering off nuclei in the closure approximation. Instead of reducing the dynamics to one particle potential scattering, the scattering amplitude for fixed target configurations is averaged over the target groundstate density via stochastic integration. At low energies a strong coupling limit is found which can not be obtained in a first order

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optical potential approximation. As its physical explanation, we propose it to be caused by trapping of the projectile. We analyse this phenomenon in mean field and random potential approximations. (PACS: 24.10.-i)

1 Introduction

Multiple scattering off composite targets has been a field of interest for the last decades in several branches of physics. Whenever the interaction between target and projectile is strong or the target system is dense, calculations which take into account multiple scattering by truncating the Born series fail. Unless very specific reactions are considered, physics in this regime will be dominated by genuine multiple scattering effects which is particularly the case for elastic scattering. Although for elastic scattering it may be proven that the many body problem is equivalent to a one body problem with a nonlocal potential [1], the explicit construction of this optical potential requires solving a many body problem involving the target degrees of freedom. Approximate solutions are obtained by assuming that only a small number of target states dominate the dynamics. In the first order optical potential approximation target propagation is restricted to its groundstate, in which case the optical potential for s–wave projectile–nucleon interaction becomes local with complex strength. Despite its simplicity, the first order optical potential has been applied successfully to many reactions in nuclear physics. More recently it has been used in the analysis of low energy scattering of $\bar{p}$ and $K^-$–mesons off nuclei and the corresponding atomic systems (for a review and references see [2]). In these cases, however, it is found that a description of the experimental data is not possible using the free space $t$–matrices extracted from projectile nucleon scattering data. Due to the large $\bar{p}p$ and $K^-p$ scattering amplitudes the projec-
tile wavefunction calculated in the first order optical model is suppressed in the nuclear interior, thus giving rise to scattering phase shifts close to hard sphere scattering. Phenomenological description of the data requires a change in the real part of the potential from repulsive to attractive which has been attributed to missing dynamical properties in a potential model [3].

In this work we do not investigate a specific reaction process. Rather we want to demonstrate that, within a theoretical model study, there exists a so far unobserved multiple scattering phenomenon which is not found when reducing the complex scattering dynamics to potential scattering. To this end we contrast the first order optical potential with another approach, in which all excited nuclear states contribute as intermediate states. The underlying approximation which is the so-called fixed scatterer or frozen nucleus approximation leads to a dynamically richer model in which projectile scattering off the nucleus can even for s-wave interaction not be reduced to a scattering problem with a local potential. The scattering amplitude in this approach is calculated for a fixed spatial configuration of nucleons first and then averaged over the nuclear groundstate density (for an application of this method to $\pi$-scattering see [4, 5]). For large projectile nucleon scattering amplitude no hard sphere phaseshifts like for the first order optical potential are found. The projectile nucleus scattering amplitude becomes independent of the elementary scattering amplitude and is strongly imaginary. This strong coupling limit is related to the generation of a new lengthsacle by averaging over resonant individual configurations. We propose trapping of the projectile in the target system as an explanation.

The paper is organized as follows: In section 2 we briefly review the formal multiple scattering theory and the derivation of our model. We concentrate on zero energy scattering in section 3. Section 4 presents a study of inelastic cross...
sections in which the physical interpretation of the strong coupling limit becomes more transparent. Section 5 is devoted to the discussion of random potential models which will be found to be insufficient for reproducing the strong coupling limit. We end up with a summary and concluding remarks.

2 Derivation of the models

As the formal theory of multiple scattering may be found in many textbooks [6, 7], we discuss this topic only briefly in order to be able to state the approximations underlying the fixed scatterer and the first order optical potential model. We start from a Hamiltonian of the form

$$H = H_0 + K + \sum_{i=1}^{A} v_i ,$$  \hspace{1cm} (1)

describing a system of $A$ target particles whose motion is governed by $H_0$ and a projectile with kinetic energy $K$ interacting with the $i$-th target particle via a twobody potential $v_i$. The usual way of formally solving this problem is to introduce projectile-nucleon transition operators $\tau_i$

$$\tau_i = v_i + v_i G \tau_i ,$$ \hspace{1cm} (2)

where $G$ is the Green’s function for the noninteracting projectile target system

$$G = [E - H_0 - K + i\epsilon]^{-1} = \sum_n |n\rangle \langle n| [E - E_n - K + i\epsilon]^{-1}$$ \hspace{1cm} (3)

with outgoing wave boundary conditions. $|n\rangle$ is a complete set of nuclear energy eigenstates of $H_0$ with energies $E_n$. The transition matrix describing the scattering of the projectile by the many body target is found to be the solution of the following set of linear operator equations [4]

$$T = \sum_i T_i$$ \hspace{1cm} (4)
\[ T_i = \tau_i + \tau_i G \sum_{i \neq j} T_j. \]  

The transition operators \( \tau_i \) are the solution of the scattering problem if only one potential \( v_i \) is different from zero. As their calculation involves \( G \) they still contain all the target dynamics and are in general many body operators. Therefore a solution may only be obtained after approximating this set of operator equations. One approximation has to be made on the level of the projectile nucleon interaction. We assume that \( \tau_i \) may be replaced by the free space projectile-nucleon transition matrix \( t_i \). This so called impulse approximation is valid if the projectile-target interaction time is much shorter than the typical time scale of the target. The impulse approximation is common to the scatterer and first order optical potential model. To obtain a solvable model, an additional approximation on the intermediate states between two scattering events is needed. The fixed scatterer approximation consist of replacing the Green’s function in eq.(5) by the so called closure Green’s function

\[ G_0 \approx G_{\text{closure}} = [E - E_0 - K + i\epsilon]^{-1}, \]  

which means that we neglect the motion of the target particles while the projectile travels inside the target. Note that both these assumptions become justified if the mass difference between target particles and projectile is very large. The consequence of the fixed scatterer approximation is that the projectile cannot loose energy while propagating inside the target system. Although the target particles remain fixed in a scattering event, the many body aspect is still present since quantum mechanical scattering amplitudes are obtained only after averaging over different sets of configurations of the target particles. For an excellent discussion of this approximation see [1]. We shall see that even after reducing the target dynamics to a mere averaging process the model is still capable of producing
nontrivial phenomena.

If the target particles do not overlap and the nucleon projectile interaction is s-wave dominated the scattering amplitude of a configuration of target centers at positions \( \{ \vec{r}_i \} \) is given by [1, 8]

\[
F_{\vec{k}', \vec{k}}(\vec{r}_1, \ldots, \vec{r}_A) = f_0 \sum_i e^{-i\vec{k}' \cdot \vec{r}_i} \psi_i
\]

\[
\psi_i = e^{i\vec{k} \cdot \vec{r}_i} + f_0 \sum_{j \neq i} e^{i|\vec{r}_j - \vec{r}_i|} \psi_j,
\]

where \( f_0 \) is the elementary projectile-nucleon s-wave scattering amplitude taken from \( \langle \vec{k}'|t_i|k \rangle \) which is assumed to be equal for all target particles. The full elastic scattering amplitude is obtained by averaging the individual amplitudes with the weight given by the target ground state density \( \rho_0 = |\langle \vec{r}_1, \ldots, \vec{r}_A|0 \rangle|^2 \):

\[
F(\vec{k}', \vec{k}) = \langle F_{\vec{k}', \vec{k}}(\vec{r}_1, \ldots, \vec{r}_A) \rangle := \int d^3r_1 \ldots d^3r_A \rho_0(\vec{r}_1, \ldots, \vec{r}_A) F_{\vec{k}', \vec{k}}(\vec{r}_1, \ldots, \vec{r}_A).
\]

(9)

Due to its high-dimensional nature we solve the integral [9] by stochastic integration (for details of the method see [9, 10]). \( N \) configurations of nucleon positions \( \{ \vec{r}_i^{(n)} \} \) are sampled from the ground state density and the integral is then approximated by

\[
\langle F_{\vec{k}', \vec{k}} \rangle \approx \langle F_{\vec{k}', \vec{k}} \rangle_N := \frac{1}{N} \sum_{n=1}^N F_{\vec{k}', \vec{k}}(\vec{r}_1^{(n)}, \ldots, \vec{r}_A^{(n)}).
\]

(10)

As in the limit \( N \to \infty \) the sum \( \langle F_{\vec{k}', \vec{k}} \rangle_N \) approaches the exact value of the integral, the elastic scattering amplitude can be calculated in principle to arbitrary precision.

The first order optical potential which will be used for comparing the results of our model is constructed from eq.(5) by approximating the Green’s function assuming that the nucleus will stay in the ground state between successive scatterings.
with the projectile
\[ G_{\text{opt.}} \approx |0\rangle \langle 0| \left[ E - E_0 - K + i\epsilon \right]^{-1}. \]

If one further drops the summation restriction \((i \neq j)\) in eq.(5), the elastic transition matrix for the projectile \( t = \langle 0|T|0 \rangle \) can be calculated from a one-body equation.

\[ t = U + U \left[ E - E_0 - K + i\epsilon \right]^{-1} t \]

with the first order optical potential
\[ U = \sum_i \langle 0|t_i|0 \rangle. \]

The averaging over the nucleon positions is performed in this model already on the level of the projectile-nucleon scattering operators. This is in strong contrast to the fixed scatterer model discussed before, where this averaging is done at the latest stage after multiple scattering has been calculated to infinite order. The consequence of the different treatment of intermediate states can be clearly seen from an expansion of the Born series of both models. In the fixed scatterer model diagrams are found in which the projectile returns to a scattering center after visiting another. These diagrams are are absent in the first order optical potential [1, 5].

The importance of backscattering correlations is well known e.g. in the tight binding model description of disordered lattices [11, 12, 13]. Anderson localization is found only if certain classes of backscattering diagrams are taken into account to infinite order. Coherent potential approximations which are the tight binding model analog of the first order optical potential approximation are not able to describe the metal-insulator transition the Anderson model predicts (see [14] for a recent review). The analysis of the tight binding model in general makes use of the fact that a lattice site is only coupled to a small number of neighbouring sites.
Formally this corresponds to the situation where the sum in equation (8) extends only over few centers. Under such conditions it was shown [11] that iteration of a small number of elementary diagrams is sufficient to understand the physics contained in backscattering correlations.

Similar approaches have been used in nuclear physics in order to construct improved optical potentials [15]. The results to be discussed below, show that at low energy and for large elementary scattering amplitude there exists a strong coupling limit, which is related to backscattering correlations of at least four nucleons. Therefore improved optical potential may in this limit still miss important physics. This seems to be plausible because of the formal differences between the multiple scattering equation (8) and the corresponding equation in the tight binding model. In the latter the assumption of only nearest neighbour interaction is frequently used, in contrast to equation (8), where the coupling between two centers decreases only like the inverse of their distance. Therefore an expansion of (8) in terms which only involve the positions of part of the scattering centers will fail as soon as $f_0$ is larger than the typical internucleon distance. In this limit configurations of nucleons which trap the projectile by multiple scattering cause a completely different result as compared to the first order optical potential.

The qualitatively different behavior of the results in the first order optical potential model and the fixed scatterer model can be most clearly seen in the zero energy limit where an understanding can be gained with a minimum of formal tools. Therefore we first concentrate on this case.
3 Zero energy scattering

In this section we demonstrate the existence of a strong coupling limit in the fixed scatterer model, we study the target particle number dependence and we compare the results of the fixed scatterer model with first order optical potential calculations. First the results for scattering off $^4$He and $^{16}$O are discussed. We used harmonic oscillator densities with size parameter $b = 1.41\text{ fm}$ for $^4$He and $b = 1.71\text{ fm}$ for $^{16}$O. These values were chosen to fit the nuclear rms–radius $^9,^{16}$. Figure 1 shows real and imaginary part of the forward scattering amplitude $F(\theta = 0)$ for $^4$He as function of $|f_0|$ for three different arguments of $f_0$. For all calculations $N = 2 \times 10^4$ configuration were used. We clearly observe that in the region where $|f_0|$ is comparable with the typical internucleon distance the results still depend strongly on the phase of $f_0$. However, for larger $|f_0|$ a saturation value $F(\theta = 0) \approx -2.2\text{ fm} + i \times 0.6\text{ fm}$ is reached. A comparison between the optical potential and the fixed scatterer calculation is shown in figure 2 where the ratio of imaginary to real part of the scattering amplitude off $^{16}$O is plotted. The stochastic integration was performed with $N = 10^4$ configurations. As we can see, first order optical potential and stochastic calculation are still in qualitative agreement for not too large elementary scattering amplitude. In the limit of large elementary amplitude the stochastic result reaches a phase independent limit of $\text{Im}F/\text{Re}F \approx -0.55$, whereas the optical potential calculation yields a strongly phase dependent and decreasing result which is easily understood from the fact that the wavefunction vanishes inside potentials with large negative imaginary part. Therefore the optical potential predicts that the projectile can not penetrate deeply into the nucleus but is scattered in a surface region. The phase dependence is due to to the change of the real part of the potential from attractive to repulsive.
For a repulsive real part the surface region becomes smaller than it is for an attractive one, which leads to a smaller ratio of imaginary to real part of the scattering amplitude.

The universal limit in the fixed scatterer calculation is reached at values which are much larger than scattering lengths which are found experimentally in strongly interacting systems as \( \bar{\text{pp}} \). Nevertheless from figure 2 it may be deduced that precursors of this limit appear much earlier. To find the reason for the different behavior we study in the following our model in the limit \( |f_0| \to \infty \).

In order to find the origins for the constancy of the results in the fixed scatterer model we formulate a simple mean field model. We replace the \( k = 0 \) propagator in eq.8 by its average

\[
\frac{1}{|\vec{r}_i - \vec{r}_j|} \approx \frac{1}{R_0} = \langle \frac{1}{|\vec{r}_i - \vec{r}_j|} \rangle,
\]

which means that we drop all the fluctuations in the configuration ensemble. With this approximation the system of equations is trivial and the scattering amplitude becomes

\[
\langle F \rangle_{mf} = \frac{Af_0}{1 - (A - 1)f_0/R_0} \xrightarrow{f_0 \to \infty} - \frac{A}{A - 1} R_0.
\]

(11)

For not too large elementary scattering amplitude the mean field model describes real and imaginary part of the scattering amplitude rather well but in the limit \( |f_0| \to \infty \) it predicts a zero imaginary part which will be discussed later. To test the \( A \) dependence of the scattering amplitude in this approximation we placed \( A \) centers in a homogenous density of the form

\[
\rho^{(A)}(\vec{r}_1,..,\vec{r}_A) = \rho_0 \prod_{i=1}^{A} \Theta(R - |\vec{r}_i|)
\]

with \( R = 2.6 \text{ fm} \), leading to a value \( R_0 \approx 2.2 \text{ fm} \). Figure 3 shows a comparison of the mean field model and the exact fixed scatterer results for \( f_0 = 250.0 \text{ fm} \) +
\( i \times 6.3 \text{fm} \). The scattering amplitude was choosen on the one hand to be much larger than \( R_0/A \), which is the relevant scale for the set in of universality. On the other hand the imaginary part was taken small compared to the real part to demonstrate that our result of the previous examples is not caused by large elementary imaginary parts. However, due to the almost real value of \( f_0^{-1} \), \( N = 4 \times 10^6 \) configurations were needed to ensure stochastic convergence.

We observe that for a large enough number of scattering centers the mean field prediction for the real part of the scattering amplitude is in excellent agreement with the stochastic calculation. The imaginary part of the scattering amplitude is almost zero for 2 and 3 centers whereas for 4 and more centers it reaches a nonzero value which is only weakly dependent on \( A \) from \( A = 8 \) on. To understand the origin of the imaginary part of the scattering amplitude we rewrite eq. (8) in the following way

\[
F = \sum_{ij} \left( \frac{1}{f_0 - G} \right)_{ij}, \quad G_{ij} = \frac{1}{|\vec{r}_i - \vec{r}_j|} (1 - \delta_{ij}).
\]  

(12)

If one inserts a complete set of eigenvectors of \( G \) defined by

\[
\sum_j G_{ij} \Phi_j^{(n)} = \lambda_n \Phi_i^{(n)},
\]

the scattering amplitude reads

\[
F = \sum_n \frac{r_n^2}{\text{Re} \frac{1}{f_0} + i \text{Im} \frac{1}{f_0} - \lambda_n}, \quad r_n := \sum_i \Phi_i^{(n)}.
\]

(13)

In the strong coupling limit where \( 1/f_0 \approx 0 \) only configurations with a vanishing eigenvalue of \( G \) have a finite imaginary part.

We see from a simple electrodynamical analog that for 2 and 3 centers \( G \) can not have vanishing eigenvalues. A zero eigenvalue is equivalent to the existence of a nontrivial solution of the equation

\[
V(\vec{r}_j) := \sum_{i \neq j} \frac{1}{|\vec{r}_j - \vec{r}_i|} q_i = 0.
\]
This is the condition for placing $A$ charges $q_i$ such that the electrostatic potential $V$ at the position of every charge vanishes. Obviously there is no solution for less than four charges unless one charge is removed to infinity. If the number of scattering centers exceeds 3 there is a continuous distribution of eigenvalues around zero in the configuration ensemble. Therefore there are configurations in the ensemble which can produce scattering resonances at values of $f_0$ much larger than the geometrical length scale given by the internucleon distance. This loss of the connection between the internal length scale given by the eigenvalues of $G$ and the geometrical length scale introduced by the density is the central result of the fixed scatterer calculation. Note also that the observation of a critical particle number, trapping and the loss of the geometrical length scale is very reminiscent of observations made in the investigation of classical irregular scattering [17].

As we have seen, a nonvanishing eigenvalue distribution around zero is responsible for the formation of a finite imaginary part of the scattering amplitude in the stochastic model. In figure 4 the eigenvalue distribution for $^4\text{He}$ is shown. Its gross features can be understood again in the mean field picture. The mean field matrix

$$\tilde{G}_{ij} = \frac{1}{R_0}(1 - \delta_{ij})$$

has one eigenvalue $\lambda_0 = (A - 1)/R_0$ and a $(A - 1)$-fold degenerate eigenvalue $\lambda_1 = -1/R_0$. Only $\lambda_0$ contributes to the scattering amplitude as its eigenvector $\Phi^{(0)} = (1, 1, ..., 1)$ is the only one which has nonvanishing overlap with the incoming wave $\exp(ik\vec{r}_i)$ at $k = 0$. The right peak in figure 4 corresponds to $\lambda_0$, the left peak to $\lambda_1$. The smearing of the left peak due to fluctuations around the mean field matrix causes zero eigenvalues to appear. Note that due to fluctuations no qualitative change may occur since the existence of an eigenvalue $\lambda_0$ and the properties of the corresponding eigenvector are guaranteed by the Perron–Frobenius theorem [18].
We did not yet succeed finding an analytical model for the small eigenvalues. From the point of view of random matrix theory a theoretical calculation of the eigenvalue distribution seems extremely difficult as the matrix entries are highly correlated. On the other hand these correlations are found to be essential since the independence of the imaginary part of $F$ on $A$ is closely related to correlations of the matrix entries. If the matrix entries of $G$ are sampled independently, a decreasing density of small eigenvalues is found when $A$ is increased whereas the correlated matrices produce a stable value \cite{19}.

4 Inelastic cross sections

We will demonstrate in this section that trapping of the projectile has a strong impact on the behaviour of total and inelastic cross sections. In order to show that our observation of a finite imaginary part of the scattering amplitude is not connected with the limit of zero energy scattering or large imaginary parts of the elementary scattering amplitude let us consider a purely elastic elementary interaction; i.e. annihilation processes like in īp-scattering are absent. In this case all inelasticities stem from transitions to excited states of the nucleus. To achieve this we choose the s–wave amplitude to be unitary and use a simple parametrization of the phase shift $\delta_0$ in terms of a real scattering length $a$

$$f_0 = \frac{e^{2i\delta_0} - 1}{2ik}; \quad \delta_0 = ka \quad . \quad (14)$$

With this choice of $f_0$ the projectile-nucleon cross section $\sigma_0$ becomes constant in the limit $k \rightarrow 0$ and equal to $\sigma_0 = 4\pi a^2$. The total cross section of the projectile-nucleus scattering amplitude is calculated via the optical theorem and the inelastic
cross section by subtracting the elastic cross section.

\[
\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im}\langle F(\theta = 0)\rangle
\]

(15)

\[
\sigma_{\text{inelast}} = \sigma_{\text{tot}} - \sigma_{\text{elast}}.
\]

(16)

For one individual configuration the imaginary part of the scattering amplitude \( F \) vanishes like \( k \) if the elementary amplitude (14) is used, because \( F \) is the solution of a potential scattering problem with a real potential. This statement is independent of the number of scattering centers or the value of \( a \). The inelastic cross section for one configuration is zero, because in potential scattering total and elastic cross section are identical. To see the origin of inelasticities in the fixed scatterer model we rewrite equation (16) in the following way:

\[
\sigma_{\text{inelast}} = \frac{4\pi}{k} \text{Im}\langle F(\theta = 0)\rangle - \int d\Omega |\langle \Omega \rangle|^2
\]

(17)

\[
= \int d\Omega \langle |F(\Omega)|^2 \rangle - \int d\Omega |\langle \Omega \rangle|^2
\]

(18)

The second line was obtained by using that averaging over configurations is a linear and real operation and that the optical theorem for one configuration yields the elastic cross section. Inelastic processes in the fixed scatterer model are thus related to fluctuations of the scattering amplitude for the individual configurations.

We plot the cross sections as functions of \( k \) for a scattering length of \( a = 10 \text{ fm} \) in figures 5 (three scattering centers) and 6 (\(^4\text{He}\)). For both calculations CM corrected harmonic oscillator densities have been used [9]. The width of the three center density was fitted to give the same elastic cross section as \(^4\text{He}\). In the case of \(^4\text{He}\), total and inelastic cross section rise like \( 1/k \) since the imaginary part of the scattering amplitude becomes constant and equal to the value obtained in the strong coupling limit at zero energy. Scattering in this example is dominated by inelastic processes. The results are again found to be independent of the scattering
length $a$ once it is chosen large enough. This is in contrast to the three center case where the total cross section becomes constant at small values of $k$, which means that the imaginary part of the scattering amplitude is proportional to $k$.

Obviously the existence of a strong coupling limit does not depend on the existence of projectile-nucleon inelastic scattering. The rise of the inelastic cross section is entirely due to excitation of the nucleus. Although in the closure approximation all states are degenerate, this fact alone is not sufficient to produce a rising inelastic cross section as the three center calculation shows. The reason is that despite the degeneracy transitions at low projectile momentum are suppressed by the orthogonality of the nuclear wave functions which requires a finite momentum transfer to be overcome. Therefore it may be concluded that it is rather the existence of resonant configurations which cause this rise in the inelastic cross section in spite of decreasing momentum. This can again be understood on the level of one configuration. By an expansion of the solution of (8) for small $k$ one can see that if the real part of the denominator in (13) becomes small, the configuration almost develops a zero energy bound state. This means that at a given small value of the energy the imaginary part of the scattering amplitude can be very large although it ultimately decreases like $k$. The finite imaginary part of the averaged scattering amplitude $\langle F \rangle$ is build up by these resonant configurations.

For scattering lengths which are comparable with the typical internucleon distances a rising inelastic cross section is also observed for less than four centers. The regime of $a$ values where rising inelastic cross sections appear can be interpreted as the typical resonance length scale of the system. The result of the fixed scatterer calculation can therefore be restated in the following way: More than three target particles give rise to resonant behaviour for arbitrarily large values of $a$. Therefore the geometrical length scale given by the internucleon distance is no
longer connected to the resonant length scale.

5 Random potential calculation

In the previous section we identified configurations with almost zero energy bound states as the origin of large inelastic cross sections. Nevertheless, averaging over \( k = 0 \) bound states is not sufficient to produce a strong coupling limit. We demonstrate this by comparing our results to scattering off a random potential in the strong coupling limit. Consider an attractive square well potential

\[
V_R(r) = \alpha^2(\Theta(|r| - R) - 1)
\]

of range \( R \) which we regard as a random variable with normalized distribution \( \rho(R) \). For infinitesimal small \( k \) the scattering amplitude is (see e.g. [20])

\[
F = \frac{1}{\alpha \cotan \alpha R - ik} - R,
\]

and the condition to find a bound state at \( k = 0 \) becomes

\[
\cotan \alpha R = 0 \Rightarrow \alpha R = \frac{\pi}{2}(2n + 1), \; n = 0, 1, \ldots.
\]

For large \( \alpha \) these resonant values of \( R \) lie very dense and averaging over a finite range of \( R \) values always means that many resonances are included. The average imaginary part is then

\[
\langle \text{Im}F \rangle = \pi \int dR \rho(R) \delta(\alpha \cotan \alpha R) = \frac{\pi}{\alpha^2} \sum_{n=0}^{\infty} \rho\left(\frac{\pi}{2\alpha}(2n + 1)\right),
\]

where we used \( \lim_{\epsilon \to 0} \text{Im}(x - i\epsilon)^{-1} = \pi \delta(x) \). In the limit \( \alpha \to \infty \) the sum can be converted to an integral as \( \pi/\alpha \) is small yielding

\[
\langle \text{Im}F \rangle = \frac{1}{\alpha} \int dx \rho(x) = \frac{1}{\alpha}.
\]
The imaginary part of the average scattering amplitude vanishes like $1/\alpha$. This surprising result can be understood by studying the wavefunctions inside the potential. They are rapidly oscillating for large $\alpha$ and therefore the overlap with the incoming wave decreases with increasing $\alpha$. For the fixed scatterer model, the numerical results indicate an entirely different behaviour [19]. The overlap of the incoming wave with the eigenvectors of small eigenvalues of $G$ does not depend strongly on the size of the eigenvalue when weighted with their probability density. Thus we can find resonances at arbitrarily large coupling and with nonvanishing overlap of wavefunction and incoming wave. Obviously a minimal criterion for a potential model to reproduce this feature is that a decoupling between the scattering amplitude $f_0$ and the strength of the potential must occur. So far we have not been able to find a consistent model which would fulfill all requirements.

6 Summary and Conclusion

In this work we investigated in comparison to the first order optical potential a model in which in a simple way the many body aspect of multiple scattering is partly retained. We showed that as a consequence of the correlations occurring in the multiple scattering process a strong coupling limit is found. In terms of physical observables our result of a finite imaginary part of the forward scattering amplitude means that the total cross section increases like $1/k$ for small values of $k$. As the elastic cross section remains finite this implies that inelastic processes dominate low energy scattering for four and more nucleons. Our explanation for this is that, while trapped, the projectile travels for a long time inside the nucleus thus having the chance of exciting arbitrary nuclear states. The trapping mechanism causes a new resonant length scale to emerge which is independent of the geometrical
length scale and is responsible for the non-suppression of the projectile wave in regions of high target density which, however, could not be discussed in this work.

To formulate the fixed scatterer model we had to make approximations which may be regarded as unrealistic in usual nuclear physics situations. Therefore a direct application of our work to nuclear reaction data seems not to be possible. However, the existence of trapping in this model indicates that similar phenomena may be observable under realistic circumstances. For example, the divergence of the total cross section at zero energy is clearly an artefact of the closure approximation since exciting the nucleus costs no energy. Nevertheless a trapping mechanism may increase the probability of inelastic processes, even if energy loss is implemented. This will be subject to further investigations.

Another interesting feature is the particle number dependence of the scattering amplitude. The fact that we find trapping only for four and more centers shows that earlier work where only low order backscattering effects were included in an improved optical potential may still have serious drawbacks. Systematic studies of the solutions, eigenvectors and eigenvalues of the system of equations which determines the scattering solutions for fixed target centers show that always a large number of centers contribute to the trapping process [19]. For this reason we failed to develop analytical models for our numerical results by decomposing one configuration into subclusters which trap the projectile. Perturbative approaches which take into account only a subclass of backscattering processes have little chance to describe the correct physics.

Further investigations are needed for finding models which can reproduce the observed phenomena in a simple way. Once this has been achieved one may be able to find the relation of our results to seemingly similar phenomena which are known as Anderson localization and classical irregular scattering.
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Figure 1: Zero energy scattering amplitude for $^4$He as function of $|f_0|$ for various arguments $\varphi$ of $f_0$. The full line is the real part, dotted line imaginary part.

Figure 2: Ratio of imaginary to real part of the zero energy scattering amplitude for $^{16}$O as function of $|f_0|$ for various arguments $\varphi$ of $f_0$. The full line is the stochastic calculation, dashed line the optical potential.

Figure 3: Zero energy scattering amplitude as function of the number of scattering centers $A$. A homogenous density has been used, $f_0 = 250.0 \text{ fm} + i \times 6.3 \text{ fm}$. The full curve is the prediction of the mean field calculation, the points are the results of the stochastic calculation (squares: real part; triangles: imaginary part).

Figure 4: Eigenvalue distribution of $G$ (eq.12) for $^4$He. The distribution is normalized to 1, therefore it is the probability density of finding a given eigenvalue in a randomly picked configuration. The plot is taken from Dirk Lehmanns diploma thesis[19].

Figure 5: Cross sections for 3 scattering centers as function of projectile momentum $k$. The scattering length is $a = 10 \text{ fm}$. The full curve is the total cross section, dotted curve elastic cross section, dashed curve inelastic cross section.

Figure 6: Cross sections for 4 scattering centers as function of projectile momentum $k$. The scattering length is $a = 10 \text{ fm}$. The full curve is the total cross section, dotted curve elastic cross section, dashed curve inelastic cross section.