Translationally invariant calculations of form factors, nucleon densities and momentum distributions for finite nuclei with short-range correlations included

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Abstract Relying upon our previous treatment of the density matrices for nuclei (in general, nonrelativistic self-bound finite systems) we are studying a combined effect of center-of-mass motion and short-range nucleon-nucleon correlations on the nucleon density and momentum distributions in light nuclei ($^4\text{He}$ and $^{16}\text{O}$). Their intrinsic ground-state wave functions are constructed in the so-called fixed center-of-mass approximation, starting with mean-field Slater determinants modified by some correlator (e.g., after Jastrow or Villars). We develop the formalism based upon the Cartesian or boson representation, in which the coordinate and momentum operators are linear combinations of the creation and annihilation operators for oscillatory quanta in the three different space directions, and get the own "Tassie-Barker" factors for each distribution and point out other model-independent results. After this separation of the center-of-mass motion effects we propose additional analytic means in order to simplify the subsequent calculations (e.g., within the Jastrow approach or the unitary correlation operator method). The charge form factors, densities and momentum distributions of $^4\text{He}$ and $^{16}\text{O}$ evaluated by using the well known cluster expansions are compared with data, our exact (numerical) results and microscopic calculations.

1 Introduction

Many efforts have been made to get a deeper understanding of the nuclear structure at small distances (less than the pion Compton wavelength) with realistic many-body calculations for the nuclear wave function (WF) whose short-range part strongly deviates from a mean-field description. In this respect, as well known (see, e.g., survey\textsuperscript{[1]}, ref. \textsuperscript{[2]} and refs. therein), the nucleon density matrices and their Fourier transforms are of great interest, being related, on the one hand, to the nuclear ground-state (g.s.) properties and, on the other hand, to the cross sections of various medium- and high-energy scattering processes off nuclei. Regarding the second aspect, we mean firstly a comparatively simple relation in the Born approximation to express the elastic electron scattering cross section through the charge form factor (FF) $F_{\text{ch}}(q)$ of the target-nucleus and its charge density $\rho_{\text{ch}}(r)$ being defined by the Fourier transform of $F_{\text{ch}}(q)$. In addition, in the so-called approximation of small interaction times (see \textsuperscript{[3]-\textsuperscript{[5]}}) the double differential $(e, e')$ reaction cross section becomes proportional to an integral of the momentum distribution (MD) $\eta(p)$ over the momentum range that is fixed with certain combination (the $q$-scaling variable) of the momentum transfer $q$ and the energy transfer $\omega$ (cf. \textsuperscript{[6]})

Other links with $\eta(p)$ we find in approximate calculations of the spectral function that determines the exclusive $A(e, e'N)X$ cross sections (see, e.g., review\textsuperscript{[7]}, ref. \textsuperscript{[8]} and earlier papers\textsuperscript{[9], [10]})

Of course, two-body and more complicated reaction mechanisms, in particular, due to meson exchange currents (see, e.g., \textsuperscript{[11]} and \textsuperscript{[12]}) in electromagnetic interactions with nuclei, may obscure such links.

Note also the distorted-wave-impulse-approximation calculations\textsuperscript{[13]} of proton MDs in $^{12}\text{C}$ and $^{16}\text{O}(e,e'p)$ reactions at Saclay kinematics, where the authors have shown a strong enhancement of the reaction cross sections with account for the final-state interaction at recoil momenta $q_R$ greater than $1.5 \text{ fm}^{-1}$. In the range the corresponding distributions of outgoing protons, having a considerably slower fall-off with the $q_R$-increasing compared to the plane-wave-impulse-approximation ones, may imitate some SRC effect. Therefore, the corresponding theoretical approaches are needed in certain refinements to bringing a reliable information on the distributions in question from experimental data. Neglecting these complexities one has to deal \textsuperscript{[3], [14], [15]} with the two structure quantities, viz., the intrinsic density distribution (DD) or simply the intrinsic density $\rho_{\text{intra}}(r)$ and the intrinsic MD $\eta_{\text{intra}}(p)$. They are expectation values in the

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translationally invariant (intrinsic) g.s. WF of appropriate many-body (multiplicative) operators which depend on the respective Jacobi variables. These definitions (see the next section) coincide with those by the Sapporo group \cite{16, 17} in studying the properties of few-body systems, but differ from the ones used by the authors of refs. \cite{2, 18, 19} in their calculations of the densities and momentum distributions in \( s - p \) and \( s - d \)shell nuclei. There we encounter the other (not intrinsic) quantities \( \rho(r) \) and \( n(k) \) introduced as in the case of infinite systems (e.g., the nuclear matter) by means of the expectation value of the one-body "density operator" with a trial Jastrow-type WF. The latter in its schematic form \( \Psi = F \Phi \) incorporates a correlation operator \( F \) which incorporates correlations into the mean-field WF \( \Phi \). It is required that \( F \) be translationally invariant and symmetrical in particle permutations. However, when starting with a Slater determinant (SD) \( \Phi \), e.g., as in \cite{2, 18} the function \( \Psi \) is translationally non-invariant ("bad"), that is, it contains spurious components which result from the CM motion (CMM) in a non-free state. In this connection, let us recall earlier and more recent attempts \cite{20, 21, 22, 23, 24, 25} to remedy such a deficiency of the nuclear WF, namely its lack of translational invariance (TI) wherever shell-model WFs (commonly built up from single-particle (s.p.) orbitals) are used.

In most cases the CM correction has been made to calculate the FF \( F_{ch}(q) \) and, respectively, the density \( \rho_{ch}(r) \) using, as a rule, the Tannou-Barker (TB) prescription (a comparison of the relevant effects can be found in ref. \cite{13}) while the not intrinsic DM \( n(k) \) has been corrected (without any good reasons) via the renormalization \( b \to \sqrt{\frac{A-1}{A}} b \) of the corresponding oscillator parameter \( b \) (see, e.g., \cite{26}), i.e., as in the case of \( \rho_{ch}(r) \). An alternative evaluation \cite{3, 4, 13} of the intrinsic FFs, densities and momentum distributions, put forward in \cite{24} to overcome some obstacles in describing the elastic and inclusive momentum distributions compared to the latter. A careful comparison of the correlated one-body properties of \( s - p \) and \( s - d \) nuclei, evaluated within the Jastrow formalism by truncating the FIY, FAHT (factor analogue of the expansion from \cite{37, 38}) and in the low-order approximation (LOA) from \cite{42} for the one-body density matrix (IDM), has been carried out in \cite{43}. In the three cases the CM correction has been taken into account by the commonplace TB factor when extracting the model parameters (the HO parameter and correlation radius) from the experimental charge FF (we will come back to the point later). Of great interest are also the exact Jastrow calculations of the elastic FF, MD and two-body density of \( ^{4}\text{He} \) performed in \cite{44} without any CMM correction (see our discussion below).

The paper is organized as follows. The underlying formalism with basic definitions is exposed in the following section. Sect. 3 is devoted to constructing the translationally invariant correlated WFs, while sect. 4 is concerned with the formulae obtained with the help of the UCOA decomposition of the similarity transformation \( \hat{C} \hat{O}^{(1)} \hat{C} \) truncated at the two-body terms. Here \( \hat{O}^{(1)} \) is a relevant one-body operator additive by nucleons. Explicit expressions for the DDs and MDs of nucleons in \( ^{4}\text{He} \) and \( ^{16}\text{O} \) are shown together with their FFs separately in subsect. 4.1 and 4.2. Our results are discussed and compared with the data in sect. 5. Some intermediate derivations can be found in Appendices.

2 The intrinsic form factor, density and momentum distributions and their evaluation in the Cartesian representation

By definition, the intrinsic (elastic) FF of a nonrelativistic system with the mass number \( A \) and the total angular momentum equal to zero is

\[
F(q) = F_{\text{int}}(q) = \frac{1}{A} \sum_{\alpha=1}^{A} \langle \psi_{\text{int}} | \exp[i \mathbf{q} \cdot (\mathbf{r}_\alpha - \bar{\mathbf{R}})] | \psi_{\text{int}} \rangle
\]

or

\[
F(q) = \langle \psi_{\text{int}} | \exp[i \mathbf{q} \cdot (\bar{\mathbf{r}}_1 - \bar{\mathbf{R}})] | \psi_{\text{int}} \rangle = \ldots
\]

\footnote{Below, the notation \( \hat{F} = \hat{C} \) is employed as well}
\[= \langle \Psi_{\text{int}} \mid \exp[iq \cdot (\hat{r}_A - \hat{R})] \mid \Psi_{\text{int}} \rangle,\]

where \(\Psi_{\text{int}}\) is the intrinsic WF of the system (nucleus), \(\hat{r}_\alpha\) the coordinate operator for nucleon number \(\alpha\), and \(\hat{R} = A^{-1} \sum_{A} \hat{r}_\alpha\) the CM operator.

Recall that \(\mid \Psi_{\text{int}}\rangle\) enters the eigenvector \(|\Psi P\rangle\) of the total Hamiltonian \(\hat{H}\) of the system, which belongs to the eigenvalue \(P\) of the total momentum operator \(P = \sum P_\alpha\): \n
\[
|\Psi P\rangle = |P\rangle \mid \Psi_{\text{int}}\rangle. \tag{2}
\]

Here \(P_\alpha\) is the momentum operator of the \(\alpha\)-th particle. Henceforth the bracket \(\mid \ )\) is used to represent a vector in the space of the center-of-mass coordinates, so that \(P |P\rangle = P |P\rangle\). A ket (bra) with an index \(\mid \alpha\rangle\) \((\langle \cdot \cdot \cdot |\alpha\rangle)\) will refer to the state of the \(\alpha\)-th particle. The intrinsic WF \(\Psi_{\text{int}}\) depends upon the \(A - 1\) independent intrinsic variables. These may be expressed in terms of the Jacobi coordinates, e.g.,

\[
\xi_\alpha = r_{\alpha + 1} - \frac{1}{\alpha + 1} \sum_{\beta = 1}^{\alpha} r_\beta \quad (\alpha = 1, 2, \ldots, A - 1) \tag{3}
\]
or the corresponding canonically conjugate momenta

\[
\eta_\alpha = \frac{1}{\alpha + 1} (\alpha p_{\alpha + 1} - \sum_{\beta = 1}^{\alpha} p_\beta) \quad (\alpha = 1, 2, \ldots, A - 1). \tag{4}
\]

The WF \(\Psi_P(r_1, r_2, \ldots, r_A)\) in the coordinate representation satisfies the requirement of TI,

\[
\Psi_P(r_1 + a, r_2 + a, \ldots, r_A + a) = \exp[iP \cdot a] \Psi_P(r_1, r_2, \ldots, r_A), \tag{5}
\]

for any arbitrary displacement \(a\).

The intrinsic density \(\rho_{\text{int}}(r)\) is the Fourier transform of the elastic FF, or inversely,

\[
F_{\text{int}}(q) = \frac{1}{A} \int \exp[iq \cdot r] \rho_{\text{int}}(r) d^3r. \tag{6}
\]

From eq. \(6\) it follows that \(\rho_{\text{int}}(r) = A \langle \Psi_{\text{int}} | \hat{\rho}_{\text{int}}(r) | \Psi_{\text{int}} \rangle\), where

\[
\hat{\rho}_{\text{int}}(r) = \delta(r - \hat{r}_A + \hat{R}) = \delta(r - \frac{A - 1}{A} \xi_{A - 1}). \tag{7}
\]

Further, the 1DM may be defined as

\[
\rho_{1\text{DM}}^{[1]}(r, r') = A \langle \Psi_{\text{int}} | \hat{\rho}_{1\text{DM}}^{[1]}(r, r') | \Psi_{\text{int}} \rangle
\]

\[
= A \int d^3 \xi_1 \ldots d^3 \xi_{A - 2} \psi_{\text{int}}^{\dagger}(\xi_1, \ldots, \xi_{A - 2}, r)
\]

\[
\times \psi_{\text{int}}(\xi_1, \ldots, \xi_{A - 2}, r'), \tag{8}
\]

so that the normalization condition \(\int d^3r \rho_{1\text{DM}}^{[1]}(r, r) = A\) is satisfied. We would like to emphasize that this is not an "imposed" definition. It appears naturally when evaluating the dynamical FF \(\tilde{F}_{\text{int}}\) (or its diagonal part, if one uses the terminology adopted in Chapter XI of the monograph \cite{17}), which is related to the intrinsic MD \cite{4}

\[
\eta_{\text{int}}(p) = A \langle \Psi_{\text{int}} | \hat{\eta}_{\text{int}}(p) | \Psi_{\text{int}} \rangle \tag{9}
\]

with

\[
\hat{\eta}_{\text{int}}(p) = \delta(p - \hat{p}_A + \hat{P}/A) = \delta(p - \eta_{A - 1})
\]

\[
= |\eta_{A - 1} = p \rangle \langle \eta_{A - 1} = p |. \tag{10}
\]

The OBMD is the Fourier transform of the 1DM

\[
\rho_{1\text{DM}}^{[1]}(r, r'). \tag{11}
\]

As in \cite{15} we would like to point out that

\[
\rho_{\text{int}}(r) = \left[ \frac{A + 1}{A} \right]^3 \rho_{1\text{DM}}^{[1]}(\frac{A + 1}{A} r, \frac{A + 1}{A} r). \tag{12}
\]

In other words, the intrinsic 1DM does not have the property \(\rho_{1\text{DM}}^{[1]}(r, r')\) which can be justified for infinite systems, although it has often been exploited in approximate treatments of finite systems (cf., however, ref. \cite{15}, where an alternative definition of the 1DM for finite self-bound systems was proposed).

Each of these intrinsic quantities can be written as the expectation value of a product of \(A\) operators acting on the subspaces of the separate \(A\) particles. For example, we have

\[
F_{\text{int}}(q) = \langle \Psi_{\text{int}} | \hat{F}_{\text{int}}(q) | \Psi_{\text{int}} \rangle \tag{13}
\]

with the multiplicative operator

\[
\hat{F}_{\text{int}}(q) = \exp[iq \cdot (\hat{r}_1 - \hat{R})] = \exp[iq \cdot r_1 - i\eta_1 \cdot \hat{r}_1 + \cdots + i\eta_{A - 1} \cdot \hat{r}_{A - 1}], \tag{14}
\]

whereas

\[
\hat{\rho}_{\text{int}}(r) = \delta(\hat{r}_1 - \hat{R} - r) = (2\pi)^{-3} \int e^{-iq \cdot \hat{r}_1} \hat{F}_{\text{int}}(q) d^3q. \tag{15}
\]

Now, we will use the Cartesian representation, in which the coordinate (momentum) operator \(\hat{r}_\alpha\) \((\hat{p}_\alpha)\) of the \(\alpha\)-th particle is the linear combination of the Cartesian creation and annihilation operators \(\hat{a}_\alpha^\dagger\) and \(\hat{a}_\alpha\),

\[
\hat{r} = \frac{r_0}{\sqrt{2}} (\hat{a}_1^\dagger + \hat{a}_1), \quad \hat{p} = \frac{p_0}{\sqrt{2}} (\hat{a}_1^\dagger - \hat{a}_1), \quad r_0 p_0 = 1, \tag{16}
\]

with the Bose commutation rules

\[
[\hat{a}_l, \hat{a}_j^\dagger] = [\hat{a}_l, \hat{a}_j] = [\hat{a}_l, \hat{a}_j^\dagger] = 0, \quad [\hat{a}_l, \hat{a}_j] = \delta_{l,j}. \tag{17}
\]

The indices \(l, j = 1, 2, 3\) label the three Cartesian axes \(x, y, z\).

As the "length parameter" \(r_0\) one can choose the oscillator parameter of a suitable HO basis in which the nuclear WF is expanded. Its basis vectors \(|n_x n_y n_z\rangle\), where the quantum numbers \(n_x, n_y, n_z\) take on the values \(0, 1, \ldots, \) are composed of the s.p. states

\[
|n_x n_y n_z\rangle = |n_x! n_y! n_z!\rangle \frac{1}{n_x! n_y! n_z!} (\hat{a}_1^\dagger)^{n_x} (\hat{a}_2^\dagger)^{n_y} (\hat{a}_3^\dagger)^{n_z} |000\rangle, \tag{18}
\]
which are the eigenstates of the Hamiltonian \( \hat{H}_{\text{osc}} = \omega (\hat{a}^+ \cdot \hat{a} + \frac{1}{2}) \),

\[
\hat{H}_{\text{osc}} | n_x n_y n_z \rangle = (n_x + n_y + n_z + \frac{3}{2}) \omega | n_x n_y n_z \rangle ,
\]

where \( \omega \) is the oscillation frequency along the three axes \( x, y \) and \( z \). We use the system of units with \( \hbar = c = 1 \).

The s.p. WF in coordinate representation is written

\[
\psi_{n}(\mathbf{r} = (x, y, z)) = \psi_{n_{x}}(x)\psi_{n_{y}}(y)\psi_{n_{z}}(z),
\]

where (see, e.g., [49])

\[
\psi_{n}(s) = \left[ \sqrt{2^{n}n!}r_{0} \right]^{-\frac{3}{2}} H_{n}(s/r_{0}) \exp(-s^{2}/2r_{0}^{2})
\]

and \( H_{n}(x) \) is a Hermite polynomial. By definition, the oscillator parameter equals \( r_{0} = [m\omega]^{-\frac{1}{2}} \).

Using eqs. [17] [15], after some algebra one can get

\[
\hat{F}_{\text{int}}(q) = F_{TB}(q) F_{HOM}(q) \times \exp \left[ i\frac{\lambda}{A} \frac{\eta}{\sqrt{2}} \hat{a}_{1} \right] \times \\
\exp \left[ -i\frac{\lambda}{\sqrt{2A}} \hat{a}_{2} \right] \exp \left[ -i\frac{\lambda}{\sqrt{2A}} \hat{a}_{3} \right] \times \\
\exp \left[ -i\frac{\lambda}{\sqrt{2A}} \hat{a}_{4} \right] \exp \left[ -i\frac{\lambda}{\sqrt{2A}} \hat{a}_{5} \right],
\]

with \( F_{TB}(q) = \exp(\frac{1}{2q^{2}/\eta^{2}}) \), \( F_{HOM}(q) = \exp(-\frac{1}{4}q^{2}r_{0}^{2}) \).

Thereat, the TB factor \( F_{TB}(q) \) appears automatically due to a specific structure of the operators involved. In other words, its appearance is independent of any nuclear properties (in general, properties of a finite system). The only mathematical tool that has been used is the Baker-Hausdorff relation:

\[
e^{\hat{A} + \hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-\frac{1}{2}[\hat{A}, \hat{B}]},
\]

that is valid with arbitrary operators \( \hat{A} \) and \( \hat{B} \) for which the commutator \([\hat{A}, \hat{B}]\) commutes with each of them.

### 2.1 Constructing intrinsic wave functions. Inclusion of nucleon-nucleon correlations

A Slater determinant,

\[
| \text{Det} \rangle = \frac{1}{\sqrt{A!}} \sum_{\mathcal{P} \in S_{A}} \epsilon_{\mathcal{P}} \hat{P}^{\dagger} \{ \phi_{p_{1}}(1) \} \cdots \{ \phi_{p_{A}}(A) \},
\]

as the total WF \( \Phi \) for an approximate and convenient description of the nuclear g.s., in the framework of the IPM or the Hartree-Fock(HF) approach exemplifies WF’s which do not possess the property of TI, eq.[3]. Here \( \epsilon_{\mathcal{P}} \) is the parity factor for the permutation \( \mathcal{P} \), \( \phi_{n} \) the occupied orbital with the quantum numbers \( \{a\} \) and the summation runs over all permutations of the symmetric group \( S_{A} \).

There are different ways to restore TI if one starts with such a bad WF as \( | \text{Det} \rangle \) (see [20] [22] [24]).

According to Ernst, Shakin and Thaler (EST) prescription [31] [2] in the fixed-CM approximation the nuclear many-body WF with the total momentum \( \mathbf{P} \) can be written in the form:

\[
| \Psi_{F} \rangle = | \mathbf{P} \rangle | \Psi_{int}^{EST} \rangle.
\]

The intrinsic WF after EST

\[
| \Psi_{int}^{EST} \rangle = \frac{(R = 0 \mid \Phi)}{(\langle \Phi \mid R = 0 \rangle)(R = 0 \mid \Phi)\frac{1}{2}},
\]

is constructed from an arbitrary (in general, translationally non-invariant) WF \( \Phi \), by requiring that the CM coordinate \( \mathbf{R} \) be equal to zero. The corresponding FF is the ratio

\[
F^{EST}(q) = \frac{A(q)}{A(0)},
\]

\[
A(q) = \langle \Phi \mid (2\pi)^{3} \delta(\mathbf{R}) \exp[iq \cdot (\mathbf{R} - \mathbf{R})] \mid \Phi \rangle,
\]

while the intrinsic MD

\[
\eta^{EST}(p) = \frac{\langle \Phi \mid (2\pi)^{3} \delta(\mathbf{R}) \exp[i\mathbf{p} \cdot (\mathbf{R} - \mathbf{R})] \mid \Phi \rangle}{\langle \Phi \mid (2\pi)^{3} \delta(\mathbf{R}) \mid \Phi \rangle},
\]

so that we have the Fourier transform

\[
\eta^{EST}(p) = (2\pi)^{-3} \int \exp(-ipz) N(z)/N(0)dz.
\]

with

\[
N(z) = \langle \Phi \mid (2\pi)^{3} \delta(\mathbf{R}) \exp[i(\mathbf{p} - \mathbf{P}/A)z] \mid \Phi \rangle.
\]

We see the certain resemblance between the structure functions \( N(z) \) and \( A(q) \), viz., both are determined by the expectation values of similar multiplicative operators with one and the same trial WF \( \Phi \). Owing to this with the help of the same algebraic techniques (cf. eq.[17]) we get

\[
A(q) = \exp \left[ -\frac{q^{2}r_{0}^{2}}{4} \right] U(q),
\]

\[
U(q) = \int d\lambda \exp \left[ -\frac{q^{2}\lambda^{2}}{4A} \right] F(v, s),
\]

with

\[
v = \frac{r_{0}}{\sqrt{2A}} (\lambda - q), \quad s = \frac{r_{0}}{\sqrt{2A}} \frac{1}{A} \lambda
\]

and the renormalized "length" parameter

\[
r_{0} = \sqrt{\frac{A - 1}{A} r_{0}}.
\]

\footnote{Other projection recipes can be applied without essential changes, see [15].}
and, in parallel,
\[
N(z) = \exp \left( -\frac{z^2 \tilde{p}_0^2}{4} \right) D(z), \tag{29}
\]
\[
D(z) = \int d\lambda \exp \left( -\frac{\tilde{r}^2 \lambda^2}{4A} \right) F(v', s'), \tag{30}
\]
with
\[
s' = -\frac{\tilde{p}_0}{\sqrt{2}} z, \quad v' = \frac{\tilde{v}_0}{\sqrt{2}} (\lambda - \tilde{v}_0 \tilde{p}_0 z) \tag{31}
\]
and
\[
\tilde{p}_0 = \sqrt{\frac{A - 1}{A}} \tilde{p}_0.
\]
When deriving these relations we have applied again eq.\(18\) in combination with the representation
\[
(2\pi)^3 \delta (\hat{R}) = \int \exp \left( i\lambda \hat{R} \right) d\lambda. \tag{32}
\]
After this we see that the expectations \(A(q)\) and \(N(z)\) are expressed through one and the same function \(F(x, y)\)
\[
F(x, y) = \langle \Phi | \hat{O}_1(x + y) \hat{O}_2(x) \cdots \hat{O}_A(x) | \Phi \rangle, \tag{33}
\]
where
\[
\hat{O}_\gamma(x) = \exp(-x^* a^\dagger_\gamma) \exp(x a_\gamma) \equiv \hat{E}_\gamma^1(-x) \hat{E}_\gamma(x) \tag{34}
\]
\[(\gamma = 1, \ldots, A).\]
In other words, we have constructed the generating function for both. One should stress that this result has been obtained independently of the model WF \(\Phi\).

Following a common practice let us consider a correlated A-body trial WF,
\[
| \Phi \rangle = | \Phi_{\text{corr}} \rangle = \hat{C}(1, 2, \ldots, A) \mid \text{Det} \rangle. \tag{35}
\]
The A-particle operator \(\hat{C} = \hat{C}(\hat{r}_\alpha - \hat{\hat{r}}_\beta, \hat{p}_\alpha - \hat{\hat{p}}_\beta)\) introduces the SRCs and meets all necessary requirements of the translational and Galileo invariance, the permutable and rotational symmetry, etc. However, being translationally invariant itself such a model introduction of correlations does not enable to restore the TI violated with such a shell-model WF as the Slater determinant.

What follows can be used with the Jastrow correlator \(29\)
\[
\hat{C} = \frac{\hat{J}}{\sqrt{C_J}}, \quad \hat{J} = \prod_{\alpha < \beta} \hat{f}(\hat{r}_{\alpha \beta}) \tag{36}
\]
The normalization constant \(C_J = \langle \text{Det} | \hat{J}^\dagger \hat{J} | \text{Det} \rangle\) (in general, a constant \(\langle \text{Det} | C^\dagger C | \text{Det} \rangle\), if any) may be omitted keeping in mind the ratios \(A(q)/A(0)\) and \(N(z)/N(0)\). The function \(f(r_{\alpha \beta})\) of the distance \(r_{\alpha \beta} = |\hat{r}_\alpha - \hat{\hat{r}}_\beta|\) is required to come to zero when particles \(\alpha\) and \(\beta\) are inside a correlation volume of a radius \(r_c\).

Another popular option goes back to the lectures by Villars in \(30\) (see also \(31\)) with a unitary operator
\[
\hat{C} = \exp(-i\hat{G}), \tag{37}
\]
\[
\hat{G} = \sum_{\alpha < \beta} \hat{g}(\alpha, \beta), \tag{38}
\]
where the Hermitian operator \(\hat{g}(\alpha, \beta)\) acts onto the space of the pair \((\alpha, \beta)\). In particular, we could follow the simplest Darmstadt ansatz \(33\):
\[
\hat{g}(\alpha, \beta) = \frac{1}{2} \{s (\hat{r}_{\alpha \beta}) \hat{p}_{\alpha \beta} + \hat{p}_{\alpha \beta} s (\hat{r}_{\alpha \beta})\}, \tag{39}
\]
where \(s\) is a function of the relative coordinate \(\hat{r}_{\alpha \beta} = \hat{r}_\alpha - \hat{\hat{r}}_\beta\). Its canonically conjugate momentum \(\hat{p}_{\alpha \beta} = \frac{1}{2} (\hat{p}_\alpha - \hat{\hat{p}}_\beta)\).

Keeping in mind similar constructions we rewrite expectation \(33\) as
\[
F(x, y) = \langle \Phi(-x) | \hat{E}_1^\dagger(-y) \hat{E}_1(y) | \Phi(x) \rangle, \tag{40}
\]
where
\[
| \Phi(x) \rangle = \hat{E}_1(x) \cdots \hat{E}_A(x) | \Phi \rangle,
\]
since \(\hat{E}_1(x + y) = \hat{E}_1(x) \hat{E}_1(y)\) and \([\hat{E}_\alpha(x), \hat{E}_\beta(y)] = 0\) \((\alpha, \beta = 1, \ldots, A)\) for any vectors \(x\) and \(y\).
Moreover, we find that
\[
\hat{E}(x) \hat{r} \hat{E}^{-1}(x) = \hat{r} + \frac{\tilde{v}_0}{\sqrt{2}} x \tag{41}
\]
and
\[
\hat{E}(x) \hat{p} \hat{E}^{-1}(x) = \hat{\hat{p}} - \frac{\tilde{p}_0}{\sqrt{2}} x. \tag{42}
\]
Remind that \(E^\dagger \neq E^{-1}\). In other words, \(\hat{E}_\alpha(x)\) is the displacement operator in the space of nucleon states with the label \(\alpha\).

Due to this property when handling the similarity transformation
\[
\hat{C}' = \hat{E}_1(x) \cdots \hat{E}_A(x) C(\hat{r}_\alpha - \hat{\hat{r}}_\beta, \hat{p}_\alpha - \hat{\hat{p}}_\beta) \times \hat{E}^{-1}_1(x) \cdots \hat{E}^{-1}_A(x),
\]
we get
\[
\hat{C}' = C(\hat{E}_\alpha(x) \hat{r}_\alpha \hat{E}_{\alpha}^{-1}(x) - \hat{E}_\beta(x) \hat{r}_\beta \hat{E}_{\beta}^{-1}(x)),
\]
\[
\hat{E}_\alpha(x) \hat{p}_\alpha \hat{E}_{\alpha}^{-1}(x) - \hat{E}_\beta(x) \hat{p}_\beta \hat{E}_{\beta}^{-1}(x)) = C(\hat{r}_\alpha - \hat{\hat{r}}_\beta, \hat{p}_\alpha - \hat{\hat{p}}_\beta) = \hat{C}
\]
i.e.,
\[
\hat{C}' = \hat{C}. \tag{43}
\]
Recall that \(C\) is a function of all the relative coordinates and their canonically conjugate momenta. From eqs. \(35\) and \(43\) it follows that
\[
| \Phi_{\text{corr}}(x) \rangle = \hat{E}_1(x) \cdots \hat{E}_A(x) | \Phi_{\text{corr}} \rangle =
Then we have the decomposition
\[
\hat{\phi}_a(\alpha; x) = \hat{E}_a(\omega) \phi_a(\alpha) \quad (\alpha = 1, \ldots, A),
\]
which is a consequence of the renormalization of the orbitals, viz.,
\[
| \text{Det}(x) \rangle = \frac{1}{\sqrt{A!}} \sum_{|p|} \epsilon_p \hat{P} \{ | \phi_{p,1}(1;x) \rangle \cdots | \phi_{p,A}(A;x) \rangle \}
\]
In turn, such orbitals can be evaluated in a concise analytic form as linear combinations of the HOM orbitals (see Appendix A).

Expressions (26) and (29) with expectations \( \langle \vec{v}, \vec{s} \rangle \) and \( \langle \vec{v}', \vec{s}' \rangle \), which are determined by eq. (47), are certain base for our calculations.

### 2.2 Calculations with the Jastrow-type correlator

We have seen how expectations \([22, 23]\) with respect to the correlated WF \([25]\) can be expressed through the generating function
\[
\hat{F}_{\text{corr}}(\vec{x}, \vec{y}) = \frac{1}{A} \langle \text{Det}(-x) | \hat{\phi}_{\text{corr}}(\vec{y}) | \text{Det}(x) \rangle,
\]
Following (40) we arrive to
\[
\hat{F}_{\text{corr}}(\vec{x}, \vec{y}) = \frac{1}{A} \langle \text{Det}(-x) | \hat{\phi}_{\text{corr}}(\vec{y}) | \text{Det}(x) \rangle = \langle \hat{C}_{\text{corr}}(\vec{y}) \rangle,
\]
Since we are going to demonstrate (at least, qualitatively) the CMM effects on the FFs and MDs against the SRCs inclusion \([35]\), let us employ, first of all, the Jastrow ansatz \([36]\),
\[
\hat{C} = \hat{J} = \hat{f}(1, 2)\hat{f}(1, 3)\ldots\hat{f}(1, A) \times \hat{f}(2, 3)\ldots\hat{f}(2, A) \times \hat{f}(A, 1, A).
\]
Then we have the decomposition
\[
\hat{Q}(\vec{y}) \equiv \hat{f}^\dagger \hat{Q}^{[1]}(\vec{y}) \hat{J} = \hat{Q}^{[1]}(\vec{y}) + \hat{Q}^{[2]}(\vec{y}) + \ldots + \hat{Q}^{[A]}(\vec{y}),
\]
where \( \hat{Q}^{[n]}(\vec{y}) \) is an \( n \)-body operator so that
\[
\hat{Q}^{[1]}(\vec{y}) = \sum_{\alpha=1}^{A} \hat{E}_\alpha^\dagger(\vec{y}) \hat{E}_\alpha(\vec{y}),
\]
\[
\hat{Q}^{[2]}(\vec{y}) = \sum_{\alpha < \beta} \hat{Q}_{\alpha\beta}(\vec{y}),
\]
etc.

A systematic way of obtaining separate contributions \( \hat{Q}^{[n]}(n \geq 2) \) is prompted by the UCOA \([33]\), where one can find general analytic expressions for the corresponding correlated operators. In case of commuting operators \( \hat{f}(\alpha, \beta) \) (e.g., for the central correlation factors \( \hat{f}(\alpha, \beta) = 1 + \hat{h}(|\vec{r}_\alpha - \vec{r}_\beta|) \) depending only on the distance between particles) one can write (cf. Appendix A in \([32]\))
\[
\hat{J} = \exp\left(\sum_{\alpha < \beta} \ln(1 + \hat{h}(\alpha, \beta))\right).
\]
After this, applying the UCOM procedure we get
\[
\hat{Q}_{\alpha\beta}(\vec{y}) = \left[1 + \hat{h}(\alpha, \beta)\right] \{ \hat{E}_\alpha^\dagger(\vec{y}) \hat{E}_\alpha(\vec{y}) + \hat{E}_\beta^\dagger(\vec{y}) \hat{E}_\beta(\vec{y}) \} [1 + \hat{h}(\alpha, \beta)]
\]
\[
= \hat{F}_{\text{corr}}(\vec{x}, \vec{y}) - \hat{F}_{\text{corr}}(\vec{x}, \vec{y}) \hat{C}_{\text{corr}}(\vec{y}) \hat{C}_{\text{corr}}(\vec{y})
\]
Along such a guideline we obtain putting in eq. (48) once \( \vec{x} = \vec{v} \) and \( \vec{y} = \vec{s} \) by eq. (28)
\[
\hat{F}_{\text{corr}}(\vec{v}, \vec{s}) = \exp\left(\frac{\hat{q}_v^2 - \hat{q}_s^2}{4}\right) F_C(q, v),
\]
\[
F_C(q, v) = \frac{1}{A} \langle \text{Det}(-v) | \hat{C}^\dagger \sum_{\alpha=1}^{A} e^{q_p \hat{r}_\alpha} \hat{C} | \text{Det}(v) \rangle,
\]
and then \( \vec{x} = \vec{v}' \) and \( \vec{y} = \vec{s}' \) by eq. (31)
\[
\hat{F}_{\text{corr}}(\vec{v}', \vec{s}') = \exp\left(\frac{\hat{q}_v^2 - \hat{q}_s^2}{4}\right) N_C(z, v'),
\]
\[
N_C(z, v') = \frac{1}{A} \langle \text{Det}(-v') | \hat{C}^\dagger \sum_{\alpha=1}^{A} e^{q_p \hat{r}_\alpha} \hat{C} | \text{Det}(v') \rangle,
\]
When deriving these formulae, we have used the relation,
\[
\exp(-\hat{y}^\dagger \hat{a}_\alpha) \exp(-\hat{y} \hat{a}_\alpha) = e^{\hat{y}^\dagger \hat{a}_\alpha} \hat{y} \exp[-\hat{y}^\dagger \hat{a}_\alpha + \hat{y} \hat{a}_\alpha],
\]
this specific realization of formula \([15]\) for any \( c \)-vector \( \vec{y} \).

Our consideration is simplified if \( \text{Det}(x) \) becomes independent of the vector \( \vec{x} \), i.e.,
\[
| \text{Det}(x) \rangle = | \text{Det}(0) \rangle = | SD \rangle,
\]
where \( | SD \rangle \) is an original Slater determinant (see below). Then
\[
F_C(q, v) = F_C(q, 0) = \frac{1}{A} (SD | \hat{C}^\dagger \sum_{\alpha=1}^{A} e^{q_p \hat{r}_\alpha} \hat{C} | SD),
\]
and
\[
N_C(z, v') = N_C(z, 0) = \frac{1}{A} (SD | \hat{C}^\dagger \sum_{\alpha=1}^{A} e^{z \hat{p}_\alpha} \hat{C} | SD).
\]
In accordance with eqs. [22] and [24] the corresponding FF and MD can be written as

\[ F_{\text{EST}}(q) = F_{\text{TB}}(q)F_{\text{C}}(q) \]  \hspace{1cm} (64)

with

\[ F_{\text{C}}(q) = \frac{(SD | \hat{C}^{\dagger} e^{i\vec{p}\cdot\vec{z}} \hat{C} | SD)}{(SD | \hat{C}^{\dagger} \hat{C} | SD)} \]  \hspace{1cm} (65)

and

\[ \eta_{\text{EST}}(p) = \frac{1}{(2\pi)^3} \int e^{-ipz} N_{\text{TB}}(z)N_{\text{C}}(z)dz \]  \hspace{1cm} (66)

with

\[ N_{\text{C}}(z) = \frac{(SD | \hat{C}^{\dagger} e^{i\vec{p}\cdot\vec{z}} \hat{C} | SD)}{(SD | \hat{C}^{\dagger} \hat{C} | SD)} \]  \hspace{1cm} (67)

The canonical TB factor

\[ F_{\text{TB}}(q) = \exp\left(\frac{q^2 \rho_0^2}{4\rho}\right) \]  \hspace{1cm} (68)

has appeared in formula [17] for the intrinsic operator \( \hat{F}_{\text{int}}(q) \), while

\[ N_{\text{TB}}(z) = \exp\left(\frac{z^2 \rho_0^2}{4\rho}\right) \]  \hspace{1cm} (69)

is the own TB factor (see discussion in ref. [15]) for the intrinsic MD. Respectively, the function \( F_{\text{C}}(q) \) and the Fourier transform

\[ \eta_{\text{C}}(p) = \frac{1}{(2\pi)^3} \int e^{-ipz} N_{\text{C}}(z)dz \]  \hspace{1cm} (70)

determine the no CM corrected FF and MD with the correlated g.s. [35] normalized to unity.

To go on our exploration with Jastrow-type correlations, let us write down instead of eqs. [57] and [59] as in eq. [51],

\[ F_j(q, v) = F^{[1]}(q, v) + F^{[2]}(q, v) + \ldots + F^{[A]}(q, v) \]  \hspace{1cm} (71)

and

\[ N_j(z, v') = N^{[1]}(z, v') + N^{[2]}(z, v') + \ldots + N^{[A]}(z, v') \]  \hspace{1cm} (72)

to obtain with the help of the UCOM the following expressions:

\[ F^{[1]}(q, v) = \frac{1}{A} \langle \text{Det}(-v') | \sum_{\alpha=1}^{A} e^{i\vec{p}\cdot\vec{\alpha}} | \text{Det}(v') \rangle, \]  \hspace{1cm} (73)

\[ F^{[2]}(q, v) = \frac{1}{A} \langle \text{Det}(-v') | \sum_{\alpha<\beta}^{A} \left[ f^{2}(\alpha, \beta) - 1 \right] e^{i\vec{q}\cdot\vec{\alpha}} + e^{i\vec{q}\cdot\vec{\beta}} \} | \text{Det}(v') \rangle, \]  \hspace{1cm} (74)

\[ N^{[1]}(z, v') = \frac{1}{A} \langle \text{Det}(-v') | \sum_{\alpha=1}^{A} e^{i\vec{\rho}\cdot\vec{\alpha}} | \text{Det}(v') \rangle, \]  \hspace{1cm} (75)

\[ N^{[2]}(z, v') = \frac{1}{A} \langle \text{Det}(-v') | \sum_{\alpha<\beta}^{A} \left[ f(\alpha, \beta) e^{i\vec{\rho}\cdot\vec{\alpha}} + e^{i\vec{\rho}\cdot\vec{\beta}} \right] f^{2}(\alpha, \beta) - 1 \right] e^{i\vec{q}\cdot\vec{\alpha}} + e^{i\vec{q}\cdot\vec{\beta}} \} | \text{Det}(v') \rangle, \]  \hspace{1cm} (76)

for central correlation factor \( f(\alpha, \beta) = f(\hat{\rho}_\alpha - \hat{\rho}_\beta) \) \((\alpha, \beta = 1, \ldots, A)\).

### 2.3 Application to \(^4\text{He}\)

On the condition [61] the matrix elements [73]—[76] are transformed into the corresponding expectations with respect to the \((SD)\). Such a situation is realized for the pure HOM \((1s)^4\) configuration occupied by the four nucleons in \(^4\text{He}\). Indeed, it is the case, where the orbitals

\[ | \phi_\alpha(\alpha) \rangle = | \varphi_\alpha(x) \rangle \]  \hspace{1cm} (77)

is annulled with the operators \( \hat{a}_\alpha \) \((\alpha = 1, \ldots, 4)\) so the renormalized orbitals [48] coincide with the initial \| \phi_\alpha(\alpha) \rangle \). Here \( \chi_{\sigma \tau} \) is the spin (isospin) part of the orbital \((\sigma \tau = + +, + - , - +, - -)\). In other words, the corresponding determinant [46] does not depend on \( x \), i.e.,

\[ | \text{Det}(x) \rangle = | \text{Det}(0) \rangle = | (1s)^4 \rangle \]  \hspace{1cm} (77)

Taking into account the definitions [65] and [67], the quantities in question can be represented as the ratios,

\[ F_{\text{C}}(q) = \frac{A_j(q)}{A_j(0)} \]  \hspace{1cm} (78)

and

\[ N_{\text{C}}(z) = \frac{B_j(z)}{B_j(0)} \]  \hspace{1cm} (79)

where

\[ A_j(q) = \langle (1s)^4 | \hat{f}^{[1]} e^{i\vec{p}\cdot\vec{J}} | (1s)^4 \rangle \]  \hspace{1cm} (80)

and

\[ B_j(z) = \langle (1s)^4 | \hat{f}^{[1]} e^{i\vec{\rho}\cdot\vec{J}} | (1s)^4 \rangle \]  \hspace{1cm} (81)

so that \( B_j(0) = A_j(0) \).

One should point out that we prefer to deal with finite decompositions [80] and [81] retaining for our approximations only a few first terms of them. Effects of the neglected terms can be estimated (at least, for \(^4\text{He}\) as in [83]).
by means of a direct computation without any decomposition (see sec. 5). Of course, the numerator and denominator in each ratio \( (82) \) and \( (79) \) should be equally truncated to meet the requirements \( F_7(0) = 1 \) and \( N_7(0) = 1 \), which guarantee the correct normalization of DDs and MDs. In the context, we will recall many works based upon the so-called \( \eta \)-expansion (see paper [2] and refs. therein) of the inverse denominator \( A_7^{-1}(0) \) in a series. In our opinion, such a procedure create some problem of convergence even for finite \( A \).

Thus we assume

\[
A_J(q) = A^{[1]}(q) + A^{[2]}(q), \tag{82}
\]

\[
B_J(z) = B^{[1]}(z) + B^{[2]}(z), \tag{83}
\]

with

\[
A^{[1]}(q) = \langle e^{i q \varphi_1} \rangle = \int \varphi_1^2(r) e^{i q r} d r, \tag{84}
\]

\[
B^{[1]}(z) = \langle e^{i z \varphi_1} \rangle = \int \varphi_1^2(p) e^{i z p} d p, \tag{85}
\]

\[
A^{[2]}(q) = \frac{1}{A} \sum_{\alpha < \beta} \hat{A}_{\alpha \beta}(q) = \frac{A - 1}{2} (\hat{A}_{12}(q)) \tag{86}
\]

and

\[
B^{[2]}(z) = \frac{1}{A} \sum_{\alpha < \beta} \hat{B}_{\alpha \beta}(z) = \frac{A - 1}{2} (\hat{B}_{12}(z)). \tag{87}
\]

Here

\[
\hat{A}_{\alpha \beta}(q) = \exp[\frac{i}{2} q (\tilde{r}_\alpha + \tilde{r}_\beta)]
\times \{ \hat{f}(\alpha, \beta) - 1 \} e^{i q (\tilde{r}_\alpha - \tilde{r}_\beta)} + H.c., \tag{88}
\]

\[
\hat{B}_{\alpha \beta}(z) = \exp[\frac{i z}{2} (\tilde{p}_\alpha + \tilde{p}_\beta)]
\times \{ \hat{f}(\alpha, \beta) e^{i z (\tilde{p}_\alpha - \tilde{p}_\beta)} \hat{f}(\alpha, \beta) - e^{i z (\tilde{p}_\alpha - \tilde{p}_\beta)} + H.c. \}, \tag{89}
\]

\( (\alpha, \beta = 1, \ldots, A) \)

since \( \hat{f}(\alpha, \beta), \tilde{r}_\alpha + \tilde{r}_\beta = \hat{f}(\alpha, \beta), \tilde{p}_\alpha + \tilde{p}_\beta = 0 \) and the symbol (…) is used to denote the expectation with respect to the determinant | \( (1s)^4 \) \rangle (generally a | SD \rangle). In eqs. \( (84) \)–\( (85) \), \( \varphi_{1s}(r) \ (\varphi_{1s}(p)) \) is the 1s orbital in coordinate (momentum) representation. For convenience, the general HOM orbitals are given in Appendix A.

Further, calculations by formulae \( (B.3) \)–\( (B.8) \) with the HOM orbital \( \varphi_{1s} \), and the correlation factor \( (B.12) \) are reduced to simple quadratures. In particular, the approximation \( (82) \) results in the FF,

\[
F_J(q) = \frac{A_J(q)}{A_J(0)}, \tag{90}
\]

\[
A_J(q) = \alpha_1 \exp(-\frac{q^2}{4b_1^2}) + \alpha_2 \exp(-\frac{q^2}{4b_2^2}) + \alpha_3 \exp(-\frac{q^2}{4b_3^2}) \tag{91}
\]

with the coefficients

\[
\alpha_1 = 1, \quad \alpha_2 = -\frac{6}{(1 + 2y)^{3/2}}, \quad \alpha_3 = \frac{3}{(1 + 4y)^{3/2}}
\]

and the falloff parameters

\[
b_1 = r_0^{-1} = p_0, \quad b_2 = b_1 \sqrt{\frac{1 + 2y}{1 + y}}, \quad b_3 = b_1 \sqrt{\frac{1 + 4y}{1 + 2y}}.
\]

The DD associated with FF \( (90) \), i.e., its Fourier transform, can be represented as

\[
\rho_J(r) = \frac{\pi^{-3/2} b_1^3}{A_J(0)} \times [d_1 \exp(-b_1^2 r^2) + d_2 \exp(-b_2^2 r^2) + d_3 \exp(-b_3^2 r^2)] \tag{92}
\]

At the same time the approximation \( (83) \) gives rise to the MD (cf. eq. \( (70) \)),

\[
\eta_J(p) \equiv \frac{1}{(2\pi)^3} \int e^{-i p z} N_J(z) dz = \frac{\pi^{-3/2} b_1^{-3}}{A_J(0)} \times [\beta_1 \exp(-\frac{p^2}{\gamma_1 b_1^2}) + \beta_2 \exp(-\frac{p^2}{\gamma_2 b_2^2}) + \beta_3 \exp(-\frac{p^2}{\gamma_3 b_3^2})], \tag{93}
\]

with

\[
\beta_1 = 1, \quad \beta_2 = -\frac{6}{(1 + 3y)^{3/2}}, \quad \beta_3 = \frac{3}{[(1 + 4y)(1 + 2y)]^{3/2}}.
\]

and

\[
\gamma_1 = 1, \quad \gamma_2 = 1 + \frac{3y}{1 + 2y}, \quad \gamma_3 = 1 + 2y.
\]

Henceforth we introduce the dimensionless parameter

\[
y = \left( \frac{r_0}{b_c} \right)^2
\]

The corresponding CM corrected quantities are determined by

\[
F_{J, EST}(q) = F_{TB}(q) F_J(q), \tag{94}
\]

\[
\rho_{J, EST}(r) \equiv \frac{1}{(2\pi)^3} \int e^{-i q r} F_{J, EST}(q) dq, \tag{95}
\]

\[
\eta_{J, EST}(p) \equiv \frac{1}{(2\pi)^3} \int e^{-i p z} N_{TB}(z) N_J(z) dz, \tag{96}
\]

so

\[
\rho_{J, EST}(r) = \frac{\pi^{-3/2} b_1^3}{A_J(0)} \times [d_1 \exp(-b_1^2 r^2) + d_2 \exp(-b_2^2 r^2) + d_3 \exp(-b_3^2 r^2)] \tag{97}
\]
with
\[
ed_1 = \left(\frac{b_1}{b_1}\right)^3 \alpha_1, \quad d_2 = \left(\frac{b_2}{b_1}\right)^3 \alpha_2, \quad d_3 = \left(\frac{b_3}{b_1}\right)^3 \alpha_3,
\]
where
\[
\bar{b}_1 = \frac{b_1}{\sqrt{1-A^2}}, \quad \bar{b}_2 = \frac{b_2}{\sqrt{1-\left(\frac{b_2}{b_1}\right)^2 A^{-1}}},
\]
\[
\bar{b}_3 = \frac{b_3}{\sqrt{1-\left(\frac{b_3}{b_1}\right)^2 A^{-1}}}
\]
and
\[
\eta_{J,EST}(p) = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)} \left[\beta_1 \exp(-\frac{1}{\gamma_1} p^2) \beta_2 \exp(-\frac{1}{\gamma_2} p^2) + \beta_3 \exp(-\frac{1}{\gamma_3} p^2)\right]
\]
whence
\[
F^{[1]}_{(1s)}(q, v) = \langle \phi'_{1s}(v) | e^{iq\bar{R}} | \phi_{1s}(v) \rangle (\phi'_{1s}(v) | \phi_{1s}(v))^3
\]
where \( | \phi'_{1s}(\alpha; v) \rangle = E_{\alpha}(v) | \phi_{1s}(\alpha) \rangle \) (\( \alpha = 1, 2, 3, 4 \)) the renormalized s.p. state and omitting the label \( \alpha \) we denote \( | \phi_{1s}(v) \rangle = E(v) | \phi_{1s} \rangle \) (cf. eq. 48). Analogously, one can get
\[
F^{[2]}_{(1s)}(q, v) = \frac{3}{2} \langle \phi'_{1s}(1; -v) | \langle \phi'_{1s}(2; -v) | \langle \phi'_{1s}(1; -v) | \langle \phi'_{1s}(2; -v) | \phi_{1s}(v) \rangle (\phi'_{1s}(v) | \phi_{1s}(v))^2
\]
Let us stress once more that if the vector \( | \phi_{1s} \rangle \) is a linear combination of the Cartesian states \( | n_x n_y n_z \rangle \) the s.p. matrix elements involved are calculated using purely algebraic means.

In addition, we would like to show some results obtained with the Darmstadt (D) correlator, which is determined by eqs. (57–59). It is the case, where, e.g., instead of the operator \( A_{12}(q) \) in expectation one should write,
\[
\hat{A}_{12}^D(q) = e^{-iqR} \left(e^{ij(1,2)} e^{i\frac{q^2}{2} R^2} (e^{-ij(1,2)} - e^{-i\frac{q^2}{2} R^2} + H.c.)\right)
\]
For brevity, we introduce the CM coordinate \( R = \frac{1}{2}(r_1 + r_2) \) of particles 1 and 2 with their relative coordinate \( r = r_1 - r_2 \) and momentum \( p = \frac{1}{2}(p_1 - p_2) \).

The hermitian generator used in [33] looks as
\[
\hat{g}(1, 2) = \frac{1}{2} \left( \frac{s(r)}{p} \hat{r}^a \bar{p}^\mu s(r) \right)
\]
One expects the unitary operator \( \hat{c} = \exp[-i\hat{q}(1, 2)] \) to shift the relative distance \( r \) between the particles via the position-dependent displacement \( s(r) \). A key point is to find an appropriate function \( s(r) \) such that \( \hat{c}(1, 2) \) could be tractable as a correlator in coordinate space. In the context, the authors of work [33] have shown that
\[
\hat{r}_g \equiv \hat{c}(1, 2) \hat{r} \hat{c}(1, 2) = \frac{R(r)}{p} \hat{r},
\]
where the shift \( R(r) - r \) characterizes some deviation of the transformed distance \( r_g \) from the uncorrelated original \( r \).

The relationship [103] enables us to write
\[
\hat{c}(1, 2) e^{i\frac{q^2}{2} R} \bar{c}(1, 2) = e^{1/2 \left( \frac{R(\hat{r})}{p} - \frac{q^2}{2} \right)}
\]
Substituting [104] into eq. [101], we obtain with the (1s)\(^4\) configuration,
\[
A_{12}^{[2]}(q) = \frac{3}{2} \langle \hat{A}_{12}^{[2]}(q) \rangle = 3 \frac{e^{-i\frac{1}{2} q^2 R^2}}{2\sqrt{2}} C(q),
\]
\[
C(q) = \frac{8\pi}{q} \int_0^\infty r^2 dr e^{-\frac{1}{2} q^2 r^2} \left( \sin \frac{1}{2} q R_+(r) - \frac{1}{2} q R_+(r) \right)
\]
The property $C(0) = 0$ provides the required value $F_D(0) = 1$ of the corresponding FF,
\[ F_D(q) = \frac{A_D(q)}{A_D(0)} = A^{[1]}(q) + A^{[2]}_D(q) \] (106)

Furthermore, one can find the relation,
\[ R_+(r) = r + \Phi(1, 2; s \partial_r) s(r) = r + \int_0^1 du \exp(us \partial_r) s(r) \] (107)

As anticipated, for a smooth shift function $s(r)$ small compared to $r$ from (107), it follows (cf. eq. (62) in [33]),
\[ R_+(r) = r + s(r) + ... \] (108)

One should note that the authors of [33] not indicating any model for $s(r)$ have preferred to work with the correlation function $R_+(r)$ directly. Our calculation with a parameterized (sophisticated) form for $R_+(r)$, taken from [33], will be presented somewhere else.

2.4 Application to $^{16}O$

For another $j$-closed nucleus $^{16}O$ we will start with the fully occupied $(1s)^4(1p)^{12}$ configuration which is built from the corresponding HOM orbitals in the $ls$-coupling scheme (see Appendix A). Now, all we need is to show that the relevant SD [46] has the property [61]. In other words, let us verify the relation
\[ |\text{Det}(v)\rangle = \hat{E}_1(v)...\hat{E}_{16}(v) \mid (1s)^4(1p)^{12} = |\langle 1s | 000 \rangle \rangle \] (109)

for any vector $v$.

Indeed, along with the evident equation
\[ \hat{E}(v) \mid 1s \rangle = e^{v\mathbf{a}} \mid 1s \rangle = | 1s \rangle \mid 000 \rangle \equiv 0 \] we find step by step,
\[ | 1p1 \rangle = -\frac{1}{\sqrt{2}} | 100 \rangle - i \frac{1}{\sqrt{2}} | 010 \rangle = (\frac{1}{\sqrt{2}} \hat{a}_x^+ - i \frac{1}{\sqrt{2}} \hat{a}_y^+ \mid 0 \rangle, \]
\[ | 1p0 \rangle = | 001 \rangle = \hat{a}_x^+ \mid 0 \rangle, \]
\[ | 1p \pm \rangle = \frac{1}{\sqrt{2}} | 100 \rangle - i \frac{1}{\sqrt{2}} | 010 \rangle = (\frac{1}{\sqrt{2}} \hat{a}_x^+ - i \frac{1}{\sqrt{2}} \hat{a}_y^+ \mid 0 \rangle, \]

and
\[ e^{v\mathbf{a}} \mid 1p \rangle \mid 1p \rangle = | v_{+} + 1s \rangle, \]
\[ e^{v\mathbf{a}} \mid 1p \rangle \mid 00 \rangle = | v_{0} + 1s \rangle, \]
\[ e^{v\mathbf{a}} \mid 1p \rangle \mid 0 \rangle = | v_{0} + 1s \rangle, \]

with the cyclic components
\[ v_{\pm} = \mp \frac{1}{\sqrt{2}} (v_{x} \pm iv_{y}), \quad v_{0} = v_{z}, \]

Thus
\[ \hat{E}(v) \mid 1pm \rangle = | 1pm \rangle + v_{m} \mid 1s \rangle, \quad (m = 1, 0, -1) \] (110)

Obviously, the second term in the r.h.s. of eq. (110) does not contribute to the determinant $|D(v)\rangle$ that immediately gives rise to (109).

As before, such an observation essentially simplifies our consideration since the matrix elements in (73), (70), and so on are reduced to the expectations with respect to the customary shell determinant $| (1s)^4(1p)^{12} \rangle$. Owing to this, one can again employ formulae (B.3)–(B.8) to get the FFs, DDs, and MDs without any CMM correction,
\[ F_J(q) = \frac{A_J(q)}{A_J(0)} \] (111)

\[ A_J(q) = \alpha_1(q) \exp\left(\frac{-q^2}{4b_1^2}\right) + \alpha_2(q) \exp\left(-\frac{q^2}{4b_2^2}\right) + \alpha_3(q) \exp\left(-\frac{q^2}{4b_3^2}\right), \] (112)

\[ \rho_J(r) = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)}|d_1(r)\exp(-b_1^2 r^2) \]
\[ + d_2(r)\exp(-b_2^2 r^2) - d_3(r)\exp(-b_3^2 r^2)|, \] (113)

\[ \eta_J(p) = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)}[\beta_1(p) \exp\left(-\frac{1}{b_2^2} \right) + \beta_2(p) \exp\left(-\frac{1}{b_2^2} \right) + \beta_3(p) \exp\left(-\frac{1}{b_3^2} \right)], \] (114)

vs. the CMM corrected ones,
\[ F_{J,\text{est}}(q) = F_{TB}(q)F_J(q), \] (115)

\[ \rho_{J,\text{est}}(r) = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)}[d_1(r)\exp(-b_1^2 r^2) \]
\[ + d_2(r)\exp(-b_2^2 r^2) - d_3(r)\exp(-b_3^2 r^2)], \] (116)

\[ \eta_{J,\text{est}}(p) = \frac{\pi^{-3/2}b_1^{-3}}{A_J(0)}[\beta_1(p) \exp\left(-\frac{p^2}{b_2^2} \right) + \beta_2(p) \exp\left(-\frac{p^2}{b_2^2} \right) + \beta_3(p) \exp\left(-\frac{p^2}{b_3^2} \right)], \] (117)

Of course, here we have the relevant TB factor,
\[ F_{TB}(q) = \exp\left(\frac{q^2r_0^2}{64}\right), \] (118)

Analytic (in general, cumbersome) expressions for the polynomials $\alpha_i(q), d_i(r), \beta_i(p)$ and $\beta_i(p)$ ($i = 1, 2, 3$) can be obtained using formulae of Appendix B that results in (by taking, respectively, $x = q/b_1$ and $z = p/b_1$)
\[ \alpha_1(q) = 1 - \frac{x^2}{8}, \quad \alpha_2(q) = 2 \pi_1(y) + \pi_2(y) + \pi_3(y) + \pi_4(y), \]
\[ \alpha_3(q) = \frac{1}{(1 + 2y)^{3/2}}, \] (119)
\[ \alpha_3(q) = -\frac{\pi_21(y) + \pi_22(y)x^2 + \pi_23(y)x^4}{(1 + 4y)^{3/2}}, \]

where

\[
\begin{align*}
\pi_{11}(y) &= -1 - \frac{50 + 116y + 77y^2}{4(1 + 2y)^2}, \\
\pi_{12}(y) &= \frac{13 + 25y + 22y^2 + 18y^3}{8(1 + 2y)^2}, \\
\pi_{13}(y) &= \frac{y^2(-3 - 2y + 5y^2)}{16(1 + 2y)^4}, \\
\pi_{21}(y) &= \pi_{11}(y)^{1/2} - \pi_{12}(y)^{1/2} (1 + 3y^2) (1 + 3y^2) (1 + 3y^2) (1 + 3y^2)
\end{align*}
\]

so

\[ d_1(r) = 1 - \frac{b_2^2 r^2}{2} \left( \frac{3}{2} - \frac{b_2^2 r^2}{2} \right), \]

\[ d_2(r) = \frac{2}{(1 + y)^{3/2}} \pi_{11}(y) + 2\pi_{12}(y) (3 - 2b_2^2 r^2) \left( \frac{b_2}{b_1} \right)^2 \]

\[ + 4\pi_{13}(y) \left( 15 - 20b_2^2 r^2 + 4b_2^2 r^4 \right) \left( \frac{b_2}{b_1} \right)^4, \]

\[ d_3(r) = \frac{1}{(1 + y)^{3/2}} \pi_{21}(y) = 2\pi_{22}(y) (3 - 2b_2^2 r^2) \left( \frac{b_2}{b_1} \right)^2 \]

\[ + 4\pi_{23}(y) \left( 15 - 20b_2^2 r^2 + 4b_2^2 r^4 \right) \left( \frac{b_2}{b_1} \right)^4. \]

At the same time we find for the MD,

\[ \beta_1(p) = \frac{1}{4} + \frac{z^2}{2}, \]

\[ \beta_2(p) = \frac{1}{(1 + 3y)^{3/2}} \left( \eta_{11}(y) + 4 \left( \frac{3}{2} - \frac{z^2}{2} \right) \eta_{12}(y) / \gamma_{21} \right) \]

\[ + 8 \left( 15 - \frac{10z}{\gamma_2} + \frac{2z^2}{\gamma_2^3} \right) \eta_{12}(y) / \gamma_{22}, \]

\[ \beta_3(p) = \frac{\eta_{21}(y) + 4 \left( \frac{3}{2} - \frac{z^2}{2} \right) \eta_{22}(y) / \gamma_3}{(1 + 4y)^{3/2}(1 + 2y)^{3/2}}. \]

where

\[
\begin{align*}
\eta_{11}(y) &= 3 \frac{31y^2 + 44y + 18}{(1 + 2y)^2}, \\
\eta_{12}(y) &= \frac{13 + 69y + 92y^2 + 42y^3}{4(1 + 2y)^4}, \\
\eta_{13}(y) &= \frac{y}{8} \left( 1 + 4y + 3y^2 \right) / (1 + 2y)^4, \\
\eta_{21}(y) &= \frac{3}{8} \left( 9 + 44y + 62y^2 \right) / (1 + 4y)^2, \\
\eta_{22}(y) &= - \frac{13 + 34y}{8(1 + 4y)}. 
\end{align*}
\]

The cutoffs \( b_1, b_2, b_3, \gamma_1, \gamma_2 \) and \( \gamma_3 \) are determined as in eq. (91) and eq. (93). The analytic expressions for the polynomials \( d_i(r) \) and \( \beta_i(p) \) are obtained by following the recipes:

\[ d_1(r) = d_1(r) |_{b_1 \to b_1}, \]

\[ d_2(r) = \left[ 1 - \frac{1}{A} \frac{1 + 2y}{1 + y} \right]^{-3/2} d_2(r) |_{b_1 \to b_1}, \]

\[ d_3(r) = \left[ 1 - \frac{1}{A} \frac{1 + 4y}{1 + 2y} \right]^{-3/2} d_3(r) |_{b_1 \to b_1}, \]

\[ \beta_1(p) = \beta_1(p) |_{\gamma_1 \to \gamma_1}, \]

\[ \beta_2(p) = \gamma_2^{-3/2} \beta_2(p) |_{\gamma_1 \to \gamma_1}, \]

\[ \beta_3(p) = \gamma_3^{-3/2} \beta_3(p) |_{\gamma_1 \to \gamma_1}, \]

where \( b_1, b_2, b_3, \gamma_1, \gamma_2 \) and \( \gamma_3 \) are determined in the same way as in the case of \( ^4He \).

### 3 Results and discussion

The analytic expressions derived in sect. 4 for density and momentum distributions and their Fourier transforms are sufficiently general to be applied in different translationally invariant treatments with the SRCs included. Our calculations carried out by formulae (90-93) for the \( ^4He \) nucleus and by formulae (111)-(118) for \( ^4O \) nucleus are displayed in figs. 1-5 together with available data. In these figures we distinguish two cases in which along with the model Jastrow correlations the CMM correction is either included or not.

In order to calculate the charge FFs we have used the relation

\[ F_{CH}(q) = F_{TFB}(q) F_{DF}(q) F_{proton}(q) F_{int}(q), \]

where \( F_{DF}(q) = 1 - q^2/2m^2 \) is the Darwin-Foldy correction and \( F_{proton}(q) \) is the finite proton size factor with the parametrization from [83].

The parameters \( r_0 \) and \( r_c \) (or, equivalently, \( y = (r_0/r_c)^2 \)) have been extracted from the data in fig. 1 for each nucleus via a least squares fit to the experimental \( F_{CH}(q) \); their best-fit values are \( r_0 = 1.163 \text{ fm} \) and \( y = 3.620 \text{ (rc = 0.658 fm) for } ^4He \) and \( r_0 = 1.217 \text{ fm} \) and \( y = 2.192 \text{ (rc = 0.261 fm) for } ^4O \). Being fixed in such a way, they remain unchanged for subsequent calculations. Along with the best-fit solid curves we have drawn the corresponding dashed curves to demonstrate the CMM influence (sometimes considerable) on the distributions in question. As seen in fig. 1, the CMM-corrected calculations reproduce the observed \( q \)-dependencies of the FFs, viz., the envelopes of diffraction minima and the positions of diffraction minima.

In order to evaluate validity of the approximation given by eqs. (24) and (25) we have calculated quantities \( A_1(q) \) and \( B_j(z) \) without any truncation of decompositions [80] and [81]. Comparison between the corresponding curves shows...
that some qualitative changes of the $r-$ and $p-$ dependencies $\rho_{J,EST}(r)$ and $\eta_{J,EST}(p)$, which are determined, respectively, by \(k \leq 10^{-1}\) and \(k \leq 2.5\), can be by-products of the approximation. In fact, considerable dips in the solid curves on the left panels of figs. 3 and 5 do not appear for exact calculations. At the point, one should note that the additional depression of $\rho_{J,EST}(r)$ with respect to $\rho_J(r)$ at a moderate $y-$ value (cf. the solid and dashed curves in fig.3 for the alpha-particle in the range $0 \leq r \leq 1$) is obscured in the charge density. The latter, being defined as the Fourier transform of the charge FF by formula (119), is calculated via the convolution of $\rho_{J,EST}(r)$ with a smoothed charge distribution in the proton. Moreover, it turns out that even with the lack (at smaller $y-$values) of the necessary property of $\rho_{J,EST}(r)$ to be positively definite the convolution results in a distribution $\rho_{CH}(r)$ which has much in common with that shown by the solid curve in the left panel of fig.2. Perhaps, in spite of similar observations many authors (see, e.g., Table I in [18], with the parameters $b = r_0 = 1.1732$ $fm$ and $\beta = r_0^{-2} = 2.3127$ $fm^{-2}$ for $^4He$ that is equivalent to $y = 3.183$) show only the charge densities of nuclei. Further, the exact distribution $\rho^{exact}_{J,EST}(r)$ (the thick solid curve in fig. 3) has a plateau in the vicinity of $r = 0$ with a shallow dip. When increasing the $y-$values the $\rho_J(r)$ dependencies (both exact and approximate) become smoothly varying functions of the nucleon coordinate $r$.

In addition, as seen from figs. 3 and 4, the CMM correction diminishes the expected depression of the intrin-
Going on our discussion of the interplay between the CM fixation and the phenomenological introduction of $N-N$ repulsion in the nuclear wave function, we will note a simultaneous shrinking of the OBDD and OBMD (cf. the thick solid curves vs dash-dotted ones in Figs. 3, 4 and 5). Following [15] the term ‘shrinking’ implies that the EST prescription gives rise to increasing each of these densities in their central regions (respectively, $0 \leq r \leq r_0$ and $0 \leq p \leq p_0 = r_0^{-1}$) compared to the nTI quantities. But unlike refs. [15] and [57], where the effect has been confirmed within the HOM and its modification [54], the present observation is related to the exact numerical results obtained beyond such simple models. In the context, note the relations under the strong inequality $r_c \ll r_0$ with

$$\rho_{EST}(r) = \lim_{y \to \infty} \rho_{J,EST}(r) = r_0^{-3} \pi^{-3/2} \exp(-r^2/r_0^2)$$

vs.

$$\rho_{HOM}(r) = r_0^{-3} \pi^{-3/2} \exp(-r^2/r_0^2).$$

**Figure 3.** The point-proton density of the nuclei $^4$He and $^{16}$O: calculated by formulae [92] (dashed curve) and [97] (solid curve) on the left and by formulae [113] (dashed curve) and [116] (solid curve) on the right. Distinctions between other curves are the same as in fig.2. The normalization is $\int \rho_J(r)dr = 1$.

**Figure 4.** The one-body density (on the left) and momentum distribution (on the right) of the alpha particle at different $y$-values and fixed $r_0 = 1.163 \text{fm}$. As in fig. 3, curves on the left panel calculated by formulae [92] and [97] whereas the right panel demonstrates the dependence $\eta_{J,EST}(p)$ (eq. (98): solid curves) vs. $\eta_J(p)$ (eq. (99): dashed curves). Distinctions between the thick solid and dash-dotted curves are the same as in fig.3. The normalization is $\int \eta_J(p)dp = 1$. 
and

\[ \eta_{HOM}(p) = \lim_{y \to \infty} \eta_J(p) = \frac{r_0^3}{6} \pi^{-3/2} \exp(-p^2/p_0^2) \]

vs.

\[ \lim_{y \to \infty} \eta_J(p) = \frac{r_0^3}{6} \pi^{-3/2} \exp(-p^2/p_0^2). \]

Remind that here \( r_0 = \frac{\sqrt{3}}{2} r_0 \) and \( p_0 = \frac{\sqrt{3}}{2} p_0 \neq r_0^{-1} \) so we see one and the same renormalization of the parameters \( r_0 \) and \( p_0 \) in accordance with the conclusion from [15] that the so-called Tassie-Barker factors should be different for different distributions of particles in finite systems.

Now, one can ask to what extent the mean square radii of these DDs are modified due to the CMM corrections and the SRCs effects. The analytical expressions of the FFs obtained here enable us to find an explicit dependence of the corresponding radius on parameters \( r_0 \) and \( y \). In this connection, let us recall that it can be found as coefficient of \(-q^2/6\) in the conventional expression \( F(q) = 1 - \frac{1}{6} q^2 r^2 + \cdots \). In particular, we get

\[ F_J(q) = 1 - \frac{1}{6} \langle r^2 \rangle_J + \cdots \]

with

\[ \langle r^2 \rangle_J = -\frac{6 A_J(0)}{A_J(0)}, \tag{120} \]

where \( A_J(q) \) is given by eq. [91] (eq. [121]) in case of \(^4\text{He} (^{16}\text{O}) \). Here \( A_J(0) = \frac{1}{q^2} A_J(q) \big|_{q=0} \). Doing so, we can evaluate the difference \( \Delta_J = \langle r^2 \rangle_J - \langle r^2 \rangle_{HOM} \), where \( \langle r^2 \rangle_{HOM} = \frac{2}{7} r^2_0 \left( \frac{4}{7} r^2_0 \right) \) for \(^4\text{He} (^{16}\text{O}) \). For example, \( \Delta_J = 0.282 \) \( fm^2 \) at \( r_0 = 1.163 \) \( fm \) and \( y = 3.120 \) in case of \(^4\text{He} \) and \( \Delta_J = 0.195 \) \( fm^2 \) at \( r_0 = 1.710 \) \( fm \) and \( y = 20.192 \) in case of \(^{16}\text{O} \). It means that along with the aforementioned depression the SRCs inclusion results in broadening the OBDD.

In its turn, the CMM correction contributes to

\[ F_{J,EST}(q) = 1 - \frac{1}{6} \langle r^2 \rangle_{J,EST} + \cdots \]

with

\[ \langle r^2 \rangle_{J,EST} = c_T + \langle r^2 \rangle_J, \]

where \( c_T = -\langle r^2 \rangle_{HOM}/A \). These quantities enter the expression

\[ \langle r^2 \rangle_{CH} = c_{DW} + \langle r^2 \rangle_p + \langle r^2 \rangle_{J,EST} \]

that determines the rms charge radii \( \langle r^2 \rangle_{CH} \) to be extracted from

\[ F_{CH}(q) = 1 - \frac{1}{6} \langle r^2 \rangle_{CH} + \cdots \]

Remind their experimental values: 1.676(2.730) \( fm \) for \(^4\text{He} (^{16}\text{O}) \), taken from [55]. One can verify that these values are reproduced by our calculations with \( \langle r^2 \rangle_{J,EST} = 2.000 \) \( (6.644) \) \( fm^2 \) for \(^4\text{He} (^{16}\text{O}) \) \( \langle r^2 \rangle_J \) is equal to 2.31 \( fm^2 \) and 6.77 \( fm^2 \), respectively). Note also that accordingly the prescription [55] \( \langle r^2 \rangle_p = 0.775 \) \( fm^2 \).

Finally, one should note that we do not attach great importance to a fair agreement of our calculations with the data in figs. 1-2 and not too good one in fig. 5. In fact, as mentioned in sect.1, the IA, in which the charge FF is determined by formula [119], is insufficient (see, e.g., [11]) to give an adequate treatment of the elastic electron scattering off nuclei with the q-increasing when MEC effects become more and more important. In addition, one has to account for the higher-order contributions to the decompositions by eqs. [20]-[21]. Once more it illustrates fig.6, where we can see a considerable shift of the first diffraction minimum towards the larger q-values. Of course, the shift may be compensated by modifying the values of the parameters involved.
other works (see, e.g., [18] and [2]), for the DD and MD values of the corresponding intrinsic operators. Each of them is a Gaussian whose behavior in the space of variables is governed by the size parameter \( r_0 \) (or its reciprocal \( p_0 \)) and the particle number \( N \) for a given finite system (nucleus), but it does not depend upon the choice of the g.s. WF. The latter can be a simple Slater determinant, include SRCs or not, be CMM-corrected or not, etc.

The use of the Cartesian or boson representation, in which the Jacobi variables are linear combinations of the creation \( \hat{a} \) and destruction \( \hat{a}^\dagger \) operators for oscillator quanta, has allowed us to simplify the calculations for the closed shell nuclei \(^4\)He and \(^{16}\)O. Certainly, the underlying idea based upon the normal ordering of the operators that meet the Bose commutation rules may be helpful in case of other closed and open shell nuclei. The analytic expressions for the intrinsic densities, form factors and momentum distributions derived in sect.2 with the Jastrow correlators are convenient in getting a deeper understanding of some nuclear properties. In particular, after restoring the TI on the SRCs background we have both in \( \rho_J(r) \) and \( \eta_J(p) \) their shrinking at enough large values of the ratio \( y = \left( \frac{r_0}{r_c} \right)^2 \).

Along with the pronounced flattening of the thick solid curve in fig. 5 in the vicinity of the \( p = 2 \, \text{fm}^{-1} \) it means that every time higher-order correlations effects should be investigated separately (cf. similar results obtained in [45] for \(^4\)He without any CM corrections). In the context, the large difference between the Argonne [61] and our calculations in fig. 5 at the \( p \)-values \( \gtrsim 2 \, \text{fm}^{-1} \) can be explained to great extent by the inclusion of many-nucleon correlations in the former. Their role becomes stronger with increasing the mass number. In addition, being aware of the necessity [31] of introducing noncentral correlations (see also [2]), we note that our method of restoring of the TI may be helpful for such complex numerical calculations as well.

4 Summary

We have shown how the approach developed in [15] when studying the one-body and two-body density matrices of finite nuclei can be realized beyond the independent particle shell model. The appropriate treatment of the CMM is combined with the inclusion of the SRCs in the nuclear WF, e.g., regarding either the Jastrow ansatz or the UCOA. In our translationally invariant calculations the OBDD and OBMD are expectation values of the particle multiplicative operators which are dependent on the relative coordinates and momenta (the Jacobi variables) and sandwiched between intrinsic nuclear ground states.

An algebraic procedure proposed earlier helps us to avoid a cumbersome integration and see certain links between the distributions in question being expressed through one and the same generating function. In the course of the procedure the so-called Tassie-Barker factors stem directly from the intrinsic operators (not the WFs). One can stress that these factors being different, unlike other works (see, e.g., [18] and [2]), for the DD and MD occur by reflecting the translationally invariant structure of the corresponding intrinsic operators. Each of them is a Gaussian whose behavior in the space of variables is governed by the size parameter \( r_0 \) (or its reciprocal \( p_0 \)) and the particle number \( N \) for a given finite system (nucleus), but it does not depend upon the choice of the g.s. WF. The latter can be a simple Slater determinant, include SRCs or not, be CMM-corrected or not, etc.

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Finally, regarding prospects of our approach in describing the interplay between the CMM and the SRC effects we mean, first of all, its application for calculations of the two-body momentum distributions in such reactions as \(^4\)He\(e, e'N\)\(X\) and \(^{16}\)O\(e, e'N\)\(X\) (cf. the corresponding qualitative findings in [15]). Our work in the subfield is in progress.

A A key point of calculations beyond HOM

The algebraic technique, shown in sects. 2 and 3, can be also helpful in calculating the expectations by eqs. (22) and (25) (or something like this) with WF \( \Phi \) that is either a linear superposition of SDs or a SD which is composed of (HF) or other model orbitals expanded in the HOM s.p. states. We find such expansions, e.g., for HF solutions [53] and an effective inclusion [54] of short-range repulsion between nucleons (in both cases in spherical representation).

By definition, the normalized RKB-orbital (for a 1s\(^4\) configuration in \(^4\)He nucleus) is

\[
|\phi_s\rangle = \frac{1}{\sqrt{1 + \beta^2}} \left( |\phi_{1s}\rangle + \beta |\phi_{2s}\rangle \right) \quad (A.1)
\]

with an adjustable parameter \( \beta \). In this connection, let us recall the well-known expressions for the HO orbitals \(|nlm\rangle\) that are specified by the principal (spectroscopic), orbital angular momentum and its projection quantum numbers \( n, l \) and \( m \). One has in coordinate space

\[
|\phi_{nlm}\rangle = \langle r |nlm\rangle = R_{nl}(r)Y_{lm} \left( \frac{r}{r_0} \right) \quad (A.2)
\]

\[
R_{nl}(r) = C_{nl}r_0^{-3/2} \left( \frac{r}{r_0} \right)^l
\]
\[ \times \Phi \left( 1 - n, l + \frac{3}{2}, \frac{r^2}{\rho_0} \right) \exp \left( - \frac{1}{2} \frac{r^2}{\rho_0^2} \right), \]

or taking into account eq. (16),

\[ C_{nl} = \sqrt{\frac{\Gamma}{\Gamma(l + \frac{3}{2})}} \left[ \frac{\Gamma(l + n + \frac{1}{2})}{\Gamma(n)} \right]^{1/2}, \]

while in momentum space,

\[ \tilde{\varphi}_{nlm}(p) = \langle p | nlm \rangle = \tilde{R}_{nl}(p) Y_{lm} \left( \frac{p}{p_0} \right), \quad (A.3) \]

\[ \tilde{R}_{nl}(p) = (-1)^{n-l}(i)^l C_{nl} p_0^{3/2} \left( \frac{p}{p_0} \right)^l \times \Phi \left( 1 - n, l + \frac{3}{2}, \frac{p^2}{2p_0^2} \right), \]

where following \[ \Phi(a, c, x) \]

is the confluent function. By passing, remind also the link with the associated Laguerre polynomials,

\[ L_{n-1}^{l+1/2}(x) = \frac{\Gamma(l + n + \frac{1}{2})}{\Gamma(l + \frac{3}{2})} \Phi \left( 1 - n, l + \frac{3}{2}, x \right), \quad n = 1, 2, \ldots \]

In turn, we find in the Cartesian representation

\[ | \varphi_{2s} \rangle = \sum_{n_x n_y n_z} | n_x n_y n_z \rangle \langle n_x n_y n_z | \varphi_{2s} \rangle \quad (A.4) \]

one can show (cf. [39])

\[ | \varphi_{2s} \rangle = -\frac{1}{\sqrt{6}} \langle \hat{a} \dagger \hat{a} \dagger | 000 \rangle, \quad (A.5) \]

or taking into account eq. (16),

\[ | \varphi_{2s} \rangle = -\frac{1}{\sqrt{6}} \hat{a} \dagger \cdot \hat{a} \dagger | 000 \rangle, \quad (A.6) \]

i.e., for the RKB-orbital,

\[ | \phi_s \rangle = [1 + \beta^2]^{-1/2} \left[ 1 - (\beta / \sqrt{6}) \hat{a} \dagger \hat{a} \dagger - \right] | 0 \rangle. \quad (A.7) \]

Substituting (A.7) into (A.6) (when calculating the ratio \( A^{IPM}(q)/A^{IPM}(0) \), the normalization factor \( [1 + \beta^2]^{-1/2} \]

can be omitted) we find

\[ \exp ( \chi \cdot a ) | \phi_s \rangle = [1 - (\beta / \sqrt{6})(\hat{a} \dagger + \chi)(\hat{a} \dagger + \chi)] | 0 \rangle \quad (A.8) \]

for any complex vector \( \chi \).

Now, after modest effort we obtain

\[ \langle \phi_s | \exp (-\chi^* \cdot \hat{a} \dagger) \exp (\chi \cdot \hat{a}) | \phi_s \rangle = \]

\[ = 1 + \beta^2 - \frac{2}{3} \beta^2 \chi^* \chi - \frac{\beta}{\sqrt{6}} [\chi^* \chi^* + \chi^* \chi] + \frac{\beta^2}{6} (\chi^* \chi^*) (\chi \chi) \quad (A.9) \]

**B Relevant calculations**

The expectations of interest can be expressed in terms of these orbitals (in general, the s.p. orbitals \( | \lambda \rangle \) occupied in the g.s.) in different ways. For example, using the formalism of secondary quantization, one has

\[ A^{(2)}(q) = \frac{1}{2} \hat{S}_{\sigma \tau} \sum_{\lambda_1 \lambda_2 \in F} \langle \lambda_1 \lambda_2 | \hat{A}_{12}(q) \rangle x | \lambda_1 \lambda_2 - \lambda_2 \lambda_1 \rangle, \quad (B.1) \]

and

\[ B^{(2)}(z) = \frac{1}{2} \hat{S}_{\sigma \tau} \sum_{\lambda_1 \lambda_2 \in F} \langle \lambda_1 \lambda_2 | \hat{B}_{12}(z) \rangle x | \lambda_1 \lambda_2 - \lambda_2 \lambda_1 \rangle, \quad (B.2) \]

where \( F \) means the Fermi sea, so

\[ A^{(2)}(q) = A^{(2)}_{dir}(q) - A^{(2)}_{exc}(q), \quad (B.3) \]

\[ A^{(2)}_{dir}(q) = \frac{8}{A} \sum_{\lambda_1 \lambda_2 \in F} \langle \varphi_{\lambda_1} \varphi_{\lambda_2} | \hat{A}_{12}(q) \rangle x | \varphi_{\lambda_1} \varphi_{\lambda_2} \rangle, \quad (B.4) \]

\[ A^{(2)}_{exc}(q) = \frac{2}{A} \sum_{\lambda_1 \lambda_2 \in F} \langle \varphi_{\lambda_1} \varphi_{\lambda_2} | \hat{A}_{12}(q) \rangle x | \varphi_{\lambda_2} \varphi_{\lambda_1} \rangle, \quad (B.5) \]

and analogously

\[ B^{(2)}(z) = B^{(2)}_{dir}(z) - B^{(2)}_{exc}(z), \quad (B.6) \]

\[ B^{(2)}_{dir}(z) = \frac{8}{A} \sum_{\lambda_1 \lambda_2 \in F} \langle \varphi_{\lambda_1} \varphi_{\lambda_2} | \hat{B}_{12}(z) \rangle x | \varphi_{\lambda_1} \varphi_{\lambda_2} \rangle, \quad (B.7) \]

\[ B^{(2)}_{exc}(z) = \frac{2}{A} \sum_{\lambda_1 \lambda_2 \in F} \langle \varphi_{\lambda_1} \varphi_{\lambda_2} | \hat{B}_{12}(z) \rangle x | \varphi_{\lambda_2} \varphi_{\lambda_1} \rangle, \quad (B.8) \]

We take the \( ls \)-coupling scheme with the orbitals

\[ | \lambda \rangle = | \varphi \lambda \rangle | \chi_{\sigma \tau} \rangle. \quad (B.9) \]

Accordingly eqs. (88, 89)

\[ \hat{A}_{12}(q) = \hat{h}^{\dagger}(1, 2) \left[ e^{i\bar{q}\bar{r}_1} + e^{i\bar{q}\bar{r}_2} \right] \hat{h}(1, 2) \]

\[ + \hat{h}^{\dagger}(1, 2) \left[ e^{i\bar{q}\bar{r}_1} + e^{i\bar{q}\bar{r}_2} \right] + \left[ e^{i\bar{q}\bar{r}_1} + e^{i\bar{q}\bar{r}_2} \right] \hat{h}(1, 2), \quad (B.10) \]

\[ \hat{B}_{12}(z) = \hat{h}^{\dagger}(1, 2) \left[ e^{i\bar{z}\bar{r}_1} + e^{i\bar{z}\bar{r}_2} \right] \hat{h}(1, 2) \]

\[ + \hat{h}^{\dagger}(1, 2) \left[ e^{i\bar{z}\bar{r}_1} + e^{i\bar{z}\bar{r}_2} \right] + \left[ e^{i\bar{z}\bar{r}_1} + e^{i\bar{z}\bar{r}_2} \right] \hat{h}(1, 2), \quad (B.11) \]

once \( \hat{f}(\alpha, \beta) = 1 + \hbar(\alpha, \beta) \) (\( \alpha, \beta = 1, \ldots, A \)). In this work calculations have been carried out with the state-independent correlator

\[ \hat{h}(\alpha, \beta) = h \left( | \mathbf{r}_{\alpha} - \mathbf{r}_\beta \rangle = -\exp \left[ - \frac{(\mathbf{r}_{\alpha} - \mathbf{r}_\beta)^2}{r_c^2} \right] \right), \quad (B.12) \]
where $r_c$ is a correlation radius.

Further, putting the relation (cf. eq. 60),

$$\exp[-u \hat{a}^\dagger + u \hat{a}] = e^{-\frac{1}{2} u^* u} \exp[-u \hat{a}^\dagger] \exp(u \hat{a})$$

the vector $u$ equal first to

$$u = i \frac{r_0}{\sqrt{2}} q$$  \hspace{2cm} (B.13)

and second to

$$u = \frac{p_0}{\sqrt{2}} z$$  \hspace{2cm} (B.14)

we split exponents $\exp(iq \hat{r})$ and $\exp(i z \hat{p})$, respectively, in eqs. (B.10) and (B.11) into such a normally ordered form. Then, when evaluating the sums in eqs. (B.4)–(B.5) (B.7)–(B.8), it suffices to consider the matrix elements:

$$M^{(k)}_{\lambda_1 \lambda_2}(u) = (\varphi_{\lambda_1} \varphi_{\lambda_2} | e^{-u \hat{a}^\dagger} H^{(k)}(\hat{r}; u) \times \exp(u \hat{a}) | \varphi_{\lambda_1} \varphi_{\lambda_2})$$

$$M^{(k)}_{\lambda_1 \lambda_2}(u) = (\varphi_{\lambda_1} \varphi_{\lambda_2} | e^{-u \hat{a}^\dagger} H^{(k)}(\hat{r}; u) \times \exp(u \hat{a}) | \varphi_{\lambda_1} \varphi_{\lambda_2})(k =1, 2)$$

we have employed the property (41) and introduced the operators

$$H^{(1)}(\hat{r}; u) = h \left( \hat{r} + \frac{r_0}{\sqrt{2}} u \right)$$

and

$$H^{(2)}(\hat{r}; u) = H^{(1)}(\hat{r}; -u^*) H^{(1)}(\hat{r}; u)$$

dependent on the distance $r = r_1 - r_2$ between the nucleons. Obviously, the superscript $k$ in $H^{(k)}(\hat{r}; u)$ labels the order in the correlations involved.

Using the definition (110) the contributions of interest can be represented as

$$\frac{3}{\pi} \sum_{m} M^{(k)}_{1;1;1}(u) = r_0^{-6} \int dr_1 \int dr_2 \frac{r_1^2 + r_2^2}{r_0^6} \left( \frac{r_1^2 + r_2^2}{r_0^2} - u^* u + \frac{u - u^*}{\sqrt{2}} r_1 - r_2 \right) \equiv I^{(k)}(u)$$

$$\frac{3}{\pi} \sum_{m} M^{(k)}_{1pm;1}(u) = r_0^{-6} \int dr_1 \int dr_2 \frac{r_1^2 + r_2^2}{r_0^6} H^{(k)}(r; u) \equiv I^{(k)}(u)$$

$$\frac{3}{\pi} M_{pp}^{(k)}(u) = r_0^{-6} \int dr_1 \int dr_2 \frac{r_1^2 + r_2^2}{r_0^6} \left( \frac{r_1^2 + r_2^2}{r_0^2} - \frac{r_1^2 + r_2^2}{r_0^2} + \frac{u - u^*}{\sqrt{2}} \right) \equiv P^{(k)}(u)$$

while

$$\frac{3}{\pi} \sum_{m} M^{(k)}_{1;1;1}(u) = 2r_0^{-6} \int dr_1 \int dr_2 \left( \left[ \frac{r_1}{r_0} + \frac{u}{\sqrt{2}} \right] \frac{r_2}{r_0} e^{-\frac{r_1^2 + r_2^2}{r_0^2}} H^{(k)}(r; u) \equiv I^{(k)}(u) \right)$$

$$\frac{3}{\pi} \sum_{m} M^{(k)}_{1pm;1}(u) = 2r_0^{-6} \int dr_1 \int dr_2 \left( \left[ \frac{r_1}{r_0} - \frac{u^*}{\sqrt{2}} \right] \frac{r_2}{r_0} e^{-\frac{r_1^2 + r_2^2}{r_0^2}} H^{(k)}(r; u) \equiv J^{(k)}(u) \right)$$

$$\frac{3}{\pi} M_{pp}^{(k)}(u) = \left( \left[ \frac{r_1}{r_0} - \frac{u^*}{\sqrt{2}} \right] \frac{r_2}{r_0} e^{-\frac{r_1^2 + r_2^2}{r_0^2}} H^{(k)}(r; u) \equiv J^{(k)}(u) \right)$$

Substituting expressions (B.17)–(B.18) into these equations, we find with the correlator (B.12),

$$e^{\frac{1}{2} y \nu^2} I^{(1)}(u) = - \int dr_1 \int dr_2 e^{-r_1^2 - r_2^2} e^{-\sqrt{2} y \nu r} \left( r_1^2 + r_2^2 \right)$$

$$e^{\frac{1}{2} y \nu^2} J^{(1)}(u) = - \int dr_1 \int dr_2 e^{-r_1^2 - r_2^2} e^{-\sqrt{2} y \nu r} \left( r_1^2 + r_2^2 \right)$$

$$e^{\frac{1}{2} y \nu^2} P^{(1)}(u) = - \int dr_1 \int dr_2 e^{-r_1^2 - r_2^2} e^{-\sqrt{2} y \nu r} \left( r_1^2 + r_2^2 \right)$$

and

$$e^{\frac{1}{2} y \nu^2} I^{(1)}(u) = -2 \left[ r_1 + \frac{\nu}{\sqrt{2}} \right] r_2$$

$$e^{\frac{1}{2} y \nu^2} J^{(1)}(u) = -2 \left[ r_1 + \frac{\nu}{\sqrt{2}} \right] r_2$$

$$e^{\frac{1}{2} y \nu^2} P^{(1)}(u) = -4 \left[ r_1 + \frac{\nu}{\sqrt{2}} \right] r_2$$
It is readily seen that the corresponding counterparts of the second order, multiplied by the same factor \(-e^{-y^2 - \sqrt{2}yr} \exp[bkr_1 + bkr_2]\), can be obtained from the integrals \([\text{B.25}] - [\text{B.27}]\) and \([\text{B.28}] - [\text{B.30}]\) by doing in their integrands the two independent changes: \(y \to 2y\) and \(yu \to y(u - u^*)\). In turn, these integrals may be calculated by addressing an auxiliary integral

\[
I(u; a, b_1, b_2) = \int dr_1 \int dr_2 e^{-a(r_1^2 + r_2^2)}
\]

\[
e^{-y^2} e^{-\sqrt{2}yr} \exp[bkr_1 + bkr_2]
\]

in the vicinity of the parameter values: \(a = 1, b_1 = b_2 = 0\). Indeed, we have

\[
I(u; a, b_1, b_2) = \frac{\pi^3}{|a (a + 2y)|^{3/2}}
\]

\[
\times \exp \left[ \frac{B^2}{8a} + \frac{(b - \sqrt{2}yu)^2}{2a + 4y} \right],
\]

(B.31)

where \(B = b_1 + b_2\) and \(b = \frac{1}{2} (b_1 - b_2)\), and after evident differentiating (for instance, using analytic means of Mathematica) we get formulae \((\text{eq.111}) - (\text{eq.118})\).

It yields

\[
\begin{align*}
A^{[2]}_{\text{dir}}(q) &= e^{-\frac{1}{2} u^* u} \left[ M^{(2)}_{\text{dir}}(u) + 2 Re M^{(1)}_{\text{dir}}(u) \right] \\
B^{[2]}_{\text{dir}}(z) &= \frac{1}{4} e^{-\frac{1}{2} u^* u} \left[ M^{(2)}_{\text{exc}}(u) + 2 Re M^{(1)}_{\text{exc}}(u) \right]
\end{align*}
\]

(B.32)

and

\[
\begin{align*}
A^{[2]}_{\text{exc}}(q) &= e^{-\frac{1}{2} u^* u} \left[ M^{(2)}_{\text{exc}}(u) + 2 Re M^{(1)}_{\text{exc}}(u) \right] \\
B^{[2]}_{\text{exc}}(z) &= \frac{1}{4} e^{-\frac{1}{2} u^* u} \left[ M^{(2)}_{\text{exc}}(u) + 2 Re M^{(1)}_{\text{exc}}(u) \right]
\end{align*}
\]

(B.33)

where the argument \(u = \frac{1}{\sqrt{\pi}} q (u = \frac{b}{\sqrt{\pi}} z)\) for \(A(B)\), with

\[
M^{(k)}_{\text{dir}}(u) = \sum_{\chi_1, \chi_2 \in F} M^{(k)}_{\chi_1, \chi_2}(u)
\]

(B.34)

and

\[
M^{(k)}_{\text{exc}}(u) = \sum_{\chi_1, \chi_2 \in F} M^{(k)}_{\chi_1, \chi_2}(u), \quad (k = 1, 2).
\]

(B.35)

Now we will separate out the purely 1s subshell, mixed \(1s - 1p\) and purely \(1p\) subshell contributions assuming

\[
M^{(k)}_{\text{dir}}(u) = M^{(k)}_{ss}(u) + M^{(k)}_{\text{mix}}(u) + M^{(k)}_{pp}(u)
\]

(B.36)

and

\[
M^{(k)}_{\text{exc}}(u) = M^{(k)}_{ss}(u) + M^{(k)}_{\text{mix}}(u) + M^{(k)}_{pp}(u), \quad (k = 1, 2)
\]

(B.37)

with

\[
M^{(k)}_{ss}(u) = M^{(k)}_{ss}(u) = \langle 1s1s | H^{(k)}(\vec{r}; u) | 1s1s \rangle,
\]

(B.38)

\[
M^{(k)}_{\text{mix}}(u) = \sum_m \sum_{\chi_1, \chi_2} M^{(k)}_{\chi_1, \chi_2}(u) + M^{(k)}_{1p1s, 1s}(u),
\]

(B.39)

\[
M^{(k)}_{pp}(u) = \sum_{m_1, m_2} M^{(k)}_{1p1s, 1p1s}(u),
\]

(B.40)

and analogously for the bar quantities.
36. F. Iwamoto, M. Yamada, Progr. Theor. Phys. 17, 543 (1957).
37. J. B. Aviles, Ann. Phys. 5, 251 (1958).
38. C. D. Hartogh, M. A. Tolhoek, Physica 24, 721 (1958).
39. N. G. Van Kampen, Physica 27, 783 (1961).
40. J.W. Clark, J. Westhouse, Math. Phys. 9, 131 (1968).
41. J.W. Clark, M.L. Ristig, IL Nuovo Cimento LXX A, 313 (1970).
42. M. Gaudin, J. Gillespie, G. Ripka, Nucl. Phys. A 176, 237 (1971).
43. Dal Ri, S. Stringari, O. Bohigas, Nucl. Phys. A 376, 81 (1982).
44. C. Ciofi degli Atti, M.E. Grypeos, Lett. Nuovo Cimento 2, 587 (1969).
45. Ch.C. Moustakidis et al., Phys. Rev. C 64, 014314 (2001).
46. A. Shebeko, N. Goncharov, Sov. J. Nucl. Phys. 18, 532 (1974).
47. M.L. Goldberger, K.M. Watson, Collision theory, John Wiley and Sons, 1964.
48. D. Van Neck, M. Waroquier, Phys. Rev. C 58, 3359 (1998).
49. V. Neudachin, Yu. Smirnov, Nucleon clusters in light nuclei. “Nauka”: Moscow, 1964.
50. R. Peierls, J. Yoccoz, Proc. Phys. Soc. A 70, 381 (1957).
51. D.J. Ernst, C.M. Shakin, R.M. Thaler, Phys. Rev. C 7, 925 (1973); ibid, 1340.
52. K.W. Schmid, F. Grümmer, Z. Phys. A 336, 5 (1990); ibid. A 337, 267 (1990).
53. V.Yu.Gonchar, E.V.Inopin, V.I.Kuprikov, Yad. Fiz. 25, 46 (1977).
54. S.Radhakant, S.B.Khadkikar, B.Banerjee, Nucl. Phys. A 142, 81 (1970).
55. H.Chandra, G.Sauer, Phys rev. C 13, 245 (1976).
56. H. de Vries, C.W. de Jager, C. de Vries, At. Data Nucl. Data Tables 36, 495 (1987).
57. A. Shebeko, P. Grigorov, Ukr. J. Phys. 52, 830 (2007).
58. R.F. Froshch et al., Phys. Rev. 160, 874 (1967); R.G. Arnold et al., Phys. Rev. Lett. 40, 1429 (1978).
59. I. Sick, J.S. McCarthy, Nucl. Phys. A150, 631 (1970).
60. C. Ciofi degli Atti, E. Pace, G. Salme, Phys. Rev. C43, 1155 (1991).
61. S.C. Pieper, R.B. Wiringa, V.R. Pandharipande, Phys. Rev. C46, 1741 (1992).
62. H. Batemann and A. Erdélyi, Tables of Integral Transforms. Vol.1 (McGraw-Hill, New York, 1954).