Possible way out of the Hawking paradox: Erasing the information at the horizon

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Abstract

We show that small deviations from spherical symmetry, described by means of exact solutions to Einstein equations, provide a mechanism to “bleach” the information about the collapsing body as it falls through the apparent horizon, thereby resolving the information loss paradox. The resulting picture and its implication related to the Landauer’s principle in the presence of a gravitational field, is discussed.

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1 Introduction

The discovery by Hawking [1], that quantum effects would cause a black hole to radiate and eventually to evaporate, together with the fact that such radiation is completely thermal (i.e. it conveys no information) [2], lead to two related contradictions, which form the well known information loss paradox.

On the one hand it follows that a pure quantum state may evolve into a mixed state (the thermal radiation) in contradiction with the principle of unitary evolution. On the other hand, since the Hawking radiation is thermal, it appears that all information about the collapsing object is lost forever.

So far this problem has been extensively discussed (see for example [3]–[12] and references therein) but no consensus has been reached until now, concerning a satisfactory resolution of this quandary.

In this work we want to call the attention to a possible resolution of the loss information paradox, which is based on the fact that close to the horizon there is a bifurcation between any exact solution representing the external field of a perturbed sphere and the perturbed Schwarzschild space-time (although strictly speaking the term “horizon” refers to the spherically symmetric case, we shall use it when considering the $r = 2m$ surface, in the case of small deviations from sphericity).

In the next section we shall elaborate on this issue, and in section III we shall present the calculation of the area surface of the object for an exact solution to the Einstein equations representing a perturbed sphere. In section IV we shall see that the evolution of the area surface, as the object contracts and approaches the horizon, will provide the clue for the resolution of the information loss paradox that we propose. Finally, some discussion is presented in the last section.

2 The Israel theorem

It is well known [13], that the only static and asymptotically-flat vacuum space-time possessing a regular horizon is the Schwarzschild solution. For all the others Weyl exterior solutions [14], the physical components of the Riemann tensor exhibit singularities at $r = 2m$. This result is usually referred to as the Israel theorem.
On the other hand, we know that all physical systems are submitted to fluctuations and, of course, this also applies to self–gravitating systems. Accordingly we have to assume that in the process of collapse (without angular momentum), the spherical symmetry, is permanently submitted to such fluctuations.

Now, if the field produced by a self–gravitating system is not particularly intense (the boundary of the source is much larger than the horizon) and fluctuations off spherical symmetry are slight, then there is no problem in representing the corresponding deviations from spherical symmetry (both inside and outside the source) as a suitable perturbation of the spherically symmetric exact solution [15].

However, as the object becomes more and more compact, such perturbative scheme will eventually fail near the source. Indeed, as it is well known [16], as the boundary surface of the source approaches the horizon, any finite perturbation of the Schwarzschild spacetime, becomes fundamentally different from any exact solution, even if the latter is characterized by parameters whose values are arbitrarily close to those corresponding to spherical symmetry. In other words, for strong gravitational fields, no matter how small the multipole moments (higher than monopole) of the source are, there is a bifurcation between the perturbed Schwarzschild metric and all the other Weyl metrics. This of course is a consequence of the Israel theorem.

From the above, a fundamental question arises: How should we describe the quasi–spherical space–time resulting from the fluctuations off Schwarzschild?

• By means of a perturbed Schwarzschild metric,

or

• By means of an exact solution to Einstein equations, whose (radiatable) multipole moments are arbitrarily small, though non–vanishing.

Our point of view is that the description of such deviations should be done from an exact solution of Einstein equations (of the Weyl family, if we restrict ourselves to vacuum static axially–symmetric solutions) continuously linked to the Schwarzschild metric through one of its parameters, instead of considering a perturbation of the Schwarzschild space–time.

It could be argued that the black hole has no–hair theorem, according to which, in the process of contraction all (radiatable) multipole moments are radiated away [17], is at variance with that point of view. However this is not so.
Indeed, the fact remains that perturbations of spherical symmetry take place all along the evolution of the object. Thus, even if it is true that close to the horizon any of these perturbations is radiated away, it is likewise true that this is a continuous process. In other words, as soon as a “hair” is radiated away, a new perturbation appears which will be later radiated and so on. Therefore, since “hairs” are radiated away at some finite time scale, then at that time scale there will be always a fluctuation acting on the system (see [18] for a more detailed discussion on this point).

Thus, if one wishes to describe the gravitational field of a quasi-spherical source close to the horizon, one should use an exact solution of Einstein equations, rather than a perturbed Schwarzschild, no matter how small the non-sphericity might be. If, for simplicity, one restricts oneself to the family of axially symmetric non-rotating sources, then one has to choose among the Weyl solutions.

The spacetime to be considered here is the so-called gamma metric (γ-metric) [19], [20]. This metric, which is also known as Zipoy-Vorhees metric [21], belongs to the family of Weyl’s solutions, and is continuously linked to the Schwarzschild space-time through one of its parameters. The motivation for this choice stems from the fact that the γ-metric corresponds to a solution of the Laplace equation (in cylindrical coordinates) with the same singularity structure as the Schwarzschild solution (a line segment [19]). In this sense the γ-metric appears as the “natural” generalization of Schwarzschild space-time to the axisymmetric case. On the other hand it is worth noticing that physically meaningful sources for this metric have been found [22].

3 The γ metric

In cylindrical coordinates, static axisymmetric solutions to Einstein equations are given by the Weyl metric [14]  

\[ ds^2 = e^{2\lambda} dt^2 - e^{-2\lambda} \left[ e^{2\mu} \left( d\rho^2 + dz^2 \right) + \rho^2 d\phi^2 \right], \]  

(1)

with  

\[ \lambda_{,\rho \rho} + \rho^{-1} \lambda_{,\rho} + \lambda_{,zz} = 0 \]  

(2)

and  

\[ \mu_{,\rho} = \rho \left( \lambda_{,\rho}^2 - \lambda_{,z}^2 \right) \quad \mu_{,z} = 2\rho \lambda_{,\rho} \lambda_{,z}. \]  

(3)

Observe that (2) is just the Laplace equation for \( \lambda \) (in the Euclidean space).
The $\gamma$-metric is defined by [19]

$$\lambda = \frac{\gamma}{2} \ln \left[ \frac{R_1 + R_2 - 2m}{R_1 + R_2 + 2m} \right],$$

(4)

$$e^{2\mu} = \left[ \frac{(R_1 + R_2 + 2m)(R_1 + R_2 - 2m)}{4R_1R_2} \right]^\gamma,$$

(5)

where

$$R_1^2 = \rho^2 + (z - m)^2 \quad R_2^2 = \rho^2 + (z + m)^2.$$  

(6)

It is worth noticing that $\lambda$, as given by (4), corresponds to the Newtonian potential of a line segment of mass density $\gamma/2$ and length $2m$, symmetrically distributed along the $z$ axis. The particular case $\gamma = 1$, corresponds to the Schwarzschild metric.

It will be useful to work in Erez-Rosen coordinates, given by

$$\rho^2 = (r^2 - 2mr) \sin^2 \theta \quad z = (r - m) \cos \theta,$$

(7)

which yields the line element as [19]

$$ds^2 = F dt^2 - F^{-1} \left\{ G dr^2 + H d\theta^2 + \left( r^2 - 2mr \right) \sin^2 \theta d\varphi^2 \right\},$$

(8)

where

$$F = \left( 1 - \frac{2m}{r} \right)^\gamma,$$

(9)

$$G = \left( \frac{r^2 - 2mr}{r^2 - 2mr + m^2 \sin^2 \theta} \right)^{\gamma^2 - 1},$$

(10)

and

$$H = \frac{(r^2 - 2mr)^{\gamma^2}}{(r^2 - 2mr + m^2 \sin^2 \theta)^{\gamma^2 - 1}},$$

(11)

Now, it is easy to check that $\gamma = 1$ corresponds to the Schwarzschild metric.

The total mass of the source is [19, 20] $M = \gamma m$, and the quadrupole moment is given by

$$Q = \frac{\gamma}{3} m^3 \left( 1 - \gamma^2 \right).$$

(12)

So that $\gamma > 1$ ($\gamma < 1$) corresponds to an oblate (prolate) spheroid.
Let us now calculate the area of the surface \( r = \text{const}; t = \text{const} \) for the line element (8). We have

\[
A = \int \sqrt{|g_{ij}|} \, d\phi d\theta, \tag{13}
\]

where \(|g_{ij}|\) is the determinant of the spatial metric. From (8–11) and (13) it follows

\[
A = 2\pi r^2 \gamma (r^2 - 2mr)^{\frac{(\gamma - 1)^2}{2}} \int_0^\pi \frac{\sin \theta}{(r^2 - 2mr + m^2 \sin^2 \theta)^{\frac{\gamma - 1}{2}}} \, d\theta, \tag{14}
\]

if \( \gamma = 1 \) we obtain

\[
A_{\text{Sch.}} = 4\pi r^2, \tag{15}
\]

which is the well known result for the area surface in the Schwarzschild spacetime. However, if \( \gamma \neq 1 \) the integral above is a hypergeometric function, producing

\[
A = 4\pi r^2 \gamma (r^2 - 2mr)^{\frac{(\gamma - 1)^2}{2}} (r - m)^{(1 - \gamma)^2} \, _2F_1 \left( \frac{1}{2}, \frac{\gamma^2 - 1}{2}; \frac{3}{2}; \frac{m^2}{(r - m)^2} \right), \tag{16}
\]

a result obtained in [19] (except for a minor misprint in their equation (11)). Now using the relationship [23]

\[
_2F_1 (a, b; c; 1) = \frac{\Gamma(c)\Gamma(c - b - a)}{\Gamma(c - b)\Gamma(c - a)}, \tag{17}
\]

where \( \Gamma \) denotes the Gamma function, we see that for \( r = 2m \), we have that for values of \( \gamma \) sufficiently close to 1, i.e. \( \gamma = 1 + \epsilon \) with \( |\epsilon| \ll 1 \), \( A \to 0 \) as \( r \to 2m \). Thus as the object contracts the area surface diminishes, vanishing for \( r = 2m \). This is true for both possible signs of \( \epsilon \), even though the pattern of evolution of \( A \), is not the same in both cases. Indeed, in the case \( \epsilon < 0 \) the ratio \( \frac{A}{A_{\text{Sch.}}} \) monotonically decreases in the process of contraction, vanishing at the horizon. Instead, in the case \( \epsilon > 0 \), the ratio \( \frac{A}{A_{\text{Sch.}}} \), initially increases until it reaches a maximum, and then start to decrease vanishing at the horizon. These specific differences are irrelevant for our discussion, the important point being the vanishing of \( A \) at the horizon.

Let us now relate this result with the information loss paradox.
4 Bleaching of the information at the horizon

Let us first recall that according to the black–hole thermodynamics [21]–[23], the entropy of the black–hole is related to the area surface of the horizon \( A_H \) by (in Planck’s units \( c = G = l_p = k_B = 1 \))

\[
S = \frac{A_H}{4}.
\]  
(18)

On the other hand this entropy is a measure of the information about the black hole interior [24], implying that as the object with area \( A \) contracts, the maximal information it can hold should satisfy the inequality [26]

\[
I_{\text{max}} < A.
\]  
(19)

Let us now get back to the information loss paradox.

One way to resolve the quandary would be to establish the existence of correlations between the quanta emitted at different times, which would in principle carry all of the information about the quantum state of the collapsing body. However, as stressed by Preskill [3], this would necessarily imply the bleaching of the information at the horizon, unless violation of causality is allowed.

If one assumes that the exterior spacetime is described by the Schwarzschild metric (plus some small perturbations to take into account fluctuations of spherical symmetry), then one should not expect this bleaching to occur, since for any freely falling observer the horizon is not a very special place.

However, the situation is dramatically different if, in order to take into account the effects of fluctuations on the spherical symmetry, we describe the spacetime by means of an exact solution (no matter how close to the spherical symmetry), e.g. the \( \gamma \)– metric with \( \gamma = 1 + \epsilon \) with \( \epsilon \ll 1 \). Now we can see that such a bleaching does in fact occur.

Indeed, in this later case, as the collapse proceeds, the area decreases according to (16), vanishing at the horizon, and so does the information held by the object, as it follows from (19).

It is instructive to take a look on this issue from a different perspective.

According to the Landauer’s principle [28], [29], the erasure of one bit of information stored in a system, requires the dissipation into the environment of a minimal amount of energy, whose lower bound is given by (in Planck units)

\[
\Delta E = T \ln 2,
\]  
(20)
where $T$ denotes the proper temperature. However if the system is located in a (weak) static gravitational field, the Landauer’s principle takes the form

$$\triangle E = T(1 + \phi) \ln 2.$$  \hspace{1cm} (21)

where $\phi$ denotes the (negative) gravitational potential, and $T(1 + \phi)$ (the Tolman’s temperature) is the quantity which is constant in thermodynamic equilibrium \[31\].

In the case of a field of arbitrary strength, Tolman’s temperature becomes $T \sqrt{g_{tt}}$, accordingly (21) generalizes to

$$\triangle E = T \sqrt{g_{tt}} \ln 2,$$ \hspace{1cm} (22)

implying that at the horizon (the $r = 2m$ surface), either the proper temperature becomes singular or the erasure of information can be done without any dissipation of energy. If we exclude the former possibility on physical grounds, then we are left with the fact that $\triangle E$ should vanish at the horizon.

Now, as it follows from the information theory \[29\], this situation (erasure without dissipation) corresponds to the case where all bits are already in one state only. Which is exactly the situation that emerges from the assumption that the radiation state is nearly pure, which in turn implies the bleaching of information at the horizon (see the discussion in pages 4 and 5 in \[3\]). Thus the (modified) Landauer’s principle seems to support our conclusion about the bleaching of the information at the horizon.

## 5 Conclusions

We have seen that if the gravitational field of the collapsing body, including small deviations from spherical symmetry, is described by means of an exact solution to Einstein field equations (instead of perturbations of Schwarzschild spacetime) then it appears that information about the collapsing body is stripped away, as the body falls through the horizon, thereby resolving the information loss paradox.

We have also shown that this picture is fully consistent with the Landauer’s principle in the presence of a gravitational field. Indeed, the fact that the required energy to be dissipated, in order to erase one bit of information at $r = 2m$ vanishes, implies that when the object reach that point, is in a single state.
We are well aware of the fact that describing departures from sphericity due to fluctuations, by means of exact solutions to Einstein equations instead of perturbations off Schwarzschild questions the very nature of black holes and by the same reason its thermodynamics. However the important point here is that whatever the end state of a collapsing body (whose surface boundary approaches $r = 2m$) is, the emerging system will be deprived of any information. This is inferred by the Landauer’s principle and/or assuming the inequality to be valid.

In this work we have restricted ourselves to a specific solution (the $\gamma$–metric) for reasons exposed at the Introduction. However, since there are a large number of different Weyl solutions which could be used to describe perturbations of spherical symmetry, then the question arises: which one among the Weyl solutions is better entitled to describe small deviations from spherical symmetry? It should be obvious that the question above has not a unique answer (there is an infinite number of ways of being non–spherical, so to speak). The important point to retain is that, for any Weyl solution (different from Schwarzschild), the horizon is a very special place. Therefore processes such as the bleaching of information discussed here, could be expected in other Weyl spacetimes. At any rate, the choice of any specific Weyl metric has to be reasoned.

Finally it is worth noticing that we have referred exclusively to non–rotating sources. However we know that a result similar to Israel theorem exists for stationary solutions with respect to the Kerr metric [32]. Accordingly, it should be expected that a mechanism of bleaching of information, related to fluctuations off Kerr (described by means of exact stationary solutions to Einstein equations) would exist, solving the information loss paradox in the general case. To prove that, is of course out of the scope of this work.

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