Effects of quintessence on observations of Type Ia SuperNovae in the clumpy Universe

M. Sereno, E. Piedipalumbo, M.V. Sazhin

1 Dipartimento di Scienze Fisiche, Università degli Studi di Napoli “Federico II”, Comp. Univ. Monte S. Angelo, Ed. G, Via Cintia, 80126, Napoli, Italia.
2 Istituto Nazionale di Fisica Nucleare, sez. Napoli, Comp. Univ. Monte S. Angelo, Ed. G, Via Cintia, 80126, Napoli, Italia.
3 Sternberg Astronomical Institute, Moscow, Russia.
4 Osservatorio Astronomico di Capodimonte, Via Moiariello, 16, 80131 Napoli, Italia.

14 December 2021

ABSTRACT

We discuss the amplification dispersion in the observed luminosity of standard candles, like supernovae (SNe) of type Ia, induced by gravitational lensing in a Universe with dark energy (quintessence). We derive the main features of the magnification probability distribution function (pdf) of SNe in the framework of on average Friedmann-Lemaître-Robertson-Walker (FLRW) models for both lensing by large-scale structures and compact objects. Analytic expressions, in terms of hypergeometric functions, for luminosity distance–redshift relations in a flat Universe with homogeneous dark energy have been corrected for the effects of inhomogeneities in the pressureless dark matter (DM). The magnification pdf is strongly dependent on the equation of state, $w_Q$, of the quintessence. With no regard to the nature of DM (microscopic or macroscopic), the dispersion increases with the redshift of the source and is maximum for dark energy with very large negative pressure; the effects of gravitational lensing on the magnification pdf, i.e. the mode biased towards de-amplified values and the long tail towards large magnifications, are reduced for both microscopic DM and quintessence with an intermediate $w_Q$. Different equations of state of the dark energy can deeply change the dispersion in amplification for the projected observed samples of SNe Ia by future space-born missions. The “noise” in the Hubble diagram due to gravitational lensing strongly affects the determination of the cosmological parameters from SNe data. The errors on the pressureless matter density parameter, $\Omega_M$, and on $w_Q$ are maximum for quintessence with not very negative pressure. The effect of the gravitational lensing is of the same order of the other systematics affecting observations of SNe Ia. Due to the lensing by large-scale structures, in a flat Universe with $\Omega_M = 0.4$, at $z = 1$ a cosmological constant ($w_Q = -1$) can be interpreted as dark energy with $w_Q < -0.84$ (at 2-$\sigma$ confidence limit).

Key words: cosmology: theory – dark matter – distance scale – gravitational lensing – large-scale structure of the Universe – supernovae: general

1 INTRODUCTION

During the last years, two independent groups, the High-$z$ SuperNova Search Team (Schmidt et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1999) have given strong evidences of the acceleration of the Universe’s expansion (Riess et al. 1998; Perlmutter et al. 1999). Several other observational and theoretical evidences, like measurements of the anisotropy of the Cosmic Microwave Background Radiation (de Bernardis et al. 2000) and inflationary theories, strongly support a flat or nearly flat Universe. On the other hand, direct measurements of $\Omega_M$ from dynamical estimates or X-ray and lensing
observations of clusters of galaxies indicates that $\Omega_M$ is significantly less than unity, $\Omega_M \simeq 0.3$ (Turner 2000). To solve this puzzle, a new type of energy component in the Universe, now called dark energy or quintessence, was proposed; dark energy with strongly negative pressure is required to explain acceleration ($w_Q \equiv p_Q/\rho_Q < -1/3$, where $p_Q$ and $\rho_Q$ are, respectively, the pressure and energy density of the dark energy).

Observations of SNe Ia, which at low redshifts are sensitive to the deceleration parameter $q_0 = (\Omega_M + (1 + 3w_Q)\Omega_Q)/2$ where $\Omega_Q$ is the density parameter of the dark energy, rely on several properties of these sources. SNe Ia are very luminous and have a small intrinsic dispersion in their peak absolute magnitude, $\delta M \sim 0.3$ (Filippenko & Riess 2000). These features make them the long expected standard candles for cosmology.

A standard candle is a source with known intrinsic luminosity ($L$) (or absolute magnitude). Measurements of its apparent flux ($F$) allow us to determine the photometric distance $D_L$ to the source via equation

$$D_L \equiv \sqrt{\frac{L}{4\pi F}}.$$  

Using standard candles, it is possible to plot the Hubble diagram (that is, the redshift of an object versus cosmological distance to it or vice versa) with very high precision and estimate the global cosmological parameters.

There are several candidates for the dark energy. The oldest one, initially introduced by Albert Einstein as a new fundamental constant of nature, is the cosmological constant ($w = -1$). After the formulation of inflationary theory, cosmologists found that a Λ term can be introduced dynamically (Dolgov, Sazhin & Zeldovich 1990; Zel’dovich 1992; Sahni & Starobinsky 2000), a dynamical Λ term by a scalar field slowly rolling down its potential ($w_Q \geq -1$) (Peebles & Ratra 1988; Wetternich 1988; Ostriker & Steinhardt 1995; Caldwell et al. 1998; Zlatev, Wang & Steinhardt 1999; de Ritis et al. 2000; Rubakov 2000; Rubano & Scudellaro 2001) can support a static energy component with positive energy density but negative pressure. Other possibilities for the quintessence are represented either by networks of light, non-intercommuting topological defects (Vilenkin 1984; Spergel & Pen 1997) ($w_Q = -m/3$, where $m$ is the dimension of the defect: for a string $w_Q = -1/3$; for a domain wall $w_Q = -2/3$) or by the so called X-matter (Chiba, Sugiyama & Nakamura 1997; Turner & White 1997). Alternatively to quintessence, a Universe filled with Chaplygin gas (Kamenshchik, Moschella & Pasquier 2001) is an additional alternative to obtain a negative pressure. Generally, the equation of state $w_Q$ evolves with the redshift, and the feasibility of reconstructing its time evolution has been investigated (Cooray & Huterer 1999; Chiba, Nakamura 2000; Maor, Brustein & Steinhardt 2001; Sahni et al. 2000; Goliath et al. 2001; Huterer & Turner 2001; Nakamura & Chiba 2001; Pavlov et al. 2002; Wang & Garnavich 2001; Wang & Lovelace 2001; Weller & Albrecht 2001a; Weller & Albrecht 2001b; Yamamoto & Futamase 2001; Gerke & Efstathiou 2002). Since in flat FLRW models the distance depends on $w_Q$ only through a triple integral on the redshift (Maor, Brustein & Steinhardt 2001), $w_Q(z)$ can be determined only provided a prior knowledge of the matter density of the Universe (Goliath et al. 2001). In what follows, we will consider only the case of a constant equation of state.

Astrophysical sources other than SNe have been long investigated to build the Hubble diagram. Two independent luminosity estimators, the first one based on the variability of Gamma-Ray Bursts (GRBs) (Reichart & Lamb 2001; Reichart et al. 2001), and the second one derived from the time lag between peaks in hard and soft energies (Norris, Marani & Bonnell 2000), have been recently proposed to infer the luminosity distance to these sources. On the other hand, standard rods, as compact radio sources (Gurvits et al. 1999) or double radio galaxies (Gurevich & Zybin 2000), have been long studied to evaluate the angular diameter distance to cosmological sources. With no regard to their different physical origins, all these observations are affected by gravitational lensing of the sources. In this paper, we want to quantify the effect of inhomogeneities in the pressureless matter on the determination of the distance. In particular, we will study SNe Ia, whose importance in the determination of the cosmological parameters makes necessary a complete study of all systematics. For light beams propagating in the inhomogeneous Universe, the expression of the luminosity distance in terms of the cosmological parameters, as obtained from Eq. (1), changes with respect to the corresponding FLRW model. Mathematical considerations for non-flat models of Universe are done in Sereno et al. (2001). In this paper, we will discuss the simple case of the flat Universe and the influence of clumpiness on the Hubble diagram. We consider inhomogeneous pressureless matter and smooth dark energy. For the more general case of inhomogeneous quintessence see Linder (1988) and Sereno et al. (2001).

The paper is as follows. In Sect. 2, we introduce the on average FLRW models; we discuss the Dyer-Roeder (DR) equation and its analytical solution in terms of hypergeometric functions. In Sect. 3, we consider the case of the homogeneous Universe. Section 4 contains the discussion on the statistical nature of the lensing dispersion induced either by large-scale structures or compact objects; we study the magnitude pdf induced by gravitational lensing and the connected systematic errors in the estimate of $\Omega_M$ and $w_Q$. Section 5 is devoted to some final considerations.

2 THE LUMINOSITY DISTANCE–REDSHIFT RELATION

The standard Hubble diagram is computed with relations that hold in FLRW models, that is assuming all gravitating pressureless matter homogeneously distributed. Instead, observational data are taken in the inhomogeneous Universe, where
sources most likely appears to be de-magnified relative to the standard Hubble diagram. In fact, light bundles, propagating far from clumps along the line of sight from the source to the observer, experience a matter density less than the average matter density of the Universe. Several attempts have addressed the problem of the redshift dependence of the distance in a clumpy Universe: by relaxing the hypothesis of homogeneity and using the Tolman-Bondi metric instead of the FLRW one (Cérélier 2000); by quantifying the small deviations from the isotropy and homogeneity of the Ricci scalar (Trentham 2001); by considering a local void (Tomita 2001). One of the historically more important and widely used framework is the on-average FLRW Universe (Schneider et al. 1992; Seitz, Schneider & Ehlers 1994), where it is assumed that the relations on a large scale are the same of the corresponding FLRW Universe, while inhomogeneities only affect local phenomena like the propagation of light. A simple way to account for this scenario is to introduce the smoothness parameter \( \alpha_M \), that, in a phenomenological way, represents the magnification effect experienced by the light beam (Tomita 1998; Wang 1999). In general, the smoothness parameter is redshift dependent since the degree of smoothness in pressureless matter evolves with time. When \( \alpha_M = 1 \), we reduce to the FLRW case; when \( \alpha_M < 1 \), there is a defocusing effect; \( \alpha_M = 0 \) represents a totally clumped Universe. This limiting case, sometimes known as "empty-beam approximation", corresponds to the maximum distance to a source for light bundles which have not passed through a caustic (Schneider et al. 1992). Historically, \( \alpha_M \) is defined as the fraction of pressureless matter smoothly distributed. The distance recovered in an average FLRW models, sometimes known as DR distance, has been long studied (Zel’dovich 1964; Kantowski 1969; Dyer & Roeder 1972; Dyer & Roeder 1973; Korolev & Sazhin 1986; Linder 1988; Seitz, Schneider & Ehlers 1994) and now is becoming established as a very useful tool for the interpretation of experimental data (Kantowski 1998; Kantowski & Thomas 2001; Perlmutter et al. 1999; Giovi & Amendola 2001). The DR equation for the angular diameter distance, \( D_A \), in the case of a Universe with inhomogeneous pressureless matter and dark energy with constant equation of state, has already been obtained starting from the optical scalar equations (Linder 1988; Giovi & Amendola 2001) or using the multiple lens-plane theory (Sereno et al. 2001). For flat Universes, \( \Omega_M + \Omega_Q = 1 \), it is
\[
(1+z)^2(1 + \mu (1+z)^{3w_Q}) \frac{d^2D_A}{dz^2} + \frac{(1+z)}{2} \left( 7 + (3w_Q + 7)\mu(1+z)^{3w_Q} \right) \frac{dD_A}{dz} \]
\[
+ \frac{3}{2} \left( \alpha_M + (w_Q +1)\mu(1+z)^{3w_Q} \right) D_A = 0,
\]
where \( \mu \equiv \frac{\Omega_M}{\Omega_M} \) and \( \alpha_M \) is the smoothness parameter. The boundary conditions on Eq. (2) are
\[
D_A(0) = 0, \quad \frac{d}{dz}D_A \bigg|_{z=0} = \frac{c}{H_0}.
\]
It is straightforward to obtain the corresponding equation for the luminosity distance. Using the Etherington principle (Etherington 1933),
\[
D_L = (1+z)^2D_A,
\]
we can substitute in Eq. (3),
\[
(1+z)2(1 + \mu (1+z)^{3w_Q}) \frac{d^2D_L}{dz^2} - \frac{(1+z)}{2} \left( 1 + (1 - 3w_Q)\mu(1+z)^{3w_Q} \right) \frac{dD_L}{dz} \]
\[
+ \left( \frac{3\alpha_M - 2}{2} + \frac{1 - 3w_Q}{2} \mu(1+z)^{3w_Q} \right) D_L = 0;
\]
the boundary conditions are, again,
\[
D_L(0) = 0, \quad \frac{d}{dz}D_L \bigg|_{z=0} = \frac{c}{H_0}.
\]
The solution of Eq. (4), satisfying the boundary conditions in Eq. (6), takes the form
\[
D_L(z) = \frac{c}{H_0} D_1(0)D_2(z) - D_1(z)D_2(0),
\]
where \( D_1(z) \) and \( D_2(z) \) are two linearly independent solutions of Eq. (5) and \( W(z) \equiv D_1(z) \frac{D_2(z)}{dz} - \frac{dD_1(z)}{dz}D_2(z) \) is the Wronskian of the solutions system. In what follows, we will consider the DR equation with a constant smoothness parameter. We introduce the parameter
\[
\beta \equiv \frac{\sqrt{25 - 24\alpha_M}}{4}.
\]
To solve Eq. (4), we perform the transformation of both the independent and dependent variables,
\[
u \equiv -\mu(1+z)^{3w_Q}, \quad D_L(z) \equiv u^{\frac{3+\beta}{12}}R_L(z).
\]
With such a transformation, Eq. (5) reduces to the hypergeometric equation for $R_L$,
\[
\frac{d^2 R_L}{d\tau^2} + \left[ \frac{1 + 2\beta}{3w_Q} \frac{1}{u} - \frac{1}{2(1-u)} \right] \frac{dR_L}{du} - \left( \frac{4\beta - 1}{12w_Q} \right) \left( \frac{4\beta + 1}{12w_Q} + 1/2 \right) \frac{1}{u(1-u)} R_L = 0.
\] (10)

A pair of independent solutions of Eq. (10) is
\[
R_1(u) = {}_2F_1 \left[ \frac{\beta - 1}{12w_Q}, \frac{\beta + 1}{12w_Q}, -\frac{1}{2} - \frac{2\beta}{3w_Q} + 1, u \right],
\] (11)
\[
R_2(u) = u \frac{\beta}{w_Q} {}_2F_1 \left[ -\frac{\beta + 1}{12w_Q}, -\frac{\beta + 1}{12w_Q}, -1/2 + \frac{2\beta}{3w_Q}, 1 - \frac{2\beta}{3w_Q} + 1, u \right],
\]
where $F_1$ is the hypergeometric function of the second type. Inserting the expressions for $R_1$ and $R_2$ in Eq. (10) and substituting in Eq. (6), we have the final expression for the luminosity distance,
\[
D_L(z) = \frac{c}{H_0 \beta \sqrt{\Omega_M}} \times \left\{ (1+z)^{\frac{3}{2}+\beta} {}_2F_1 \left[ \frac{\beta + 1}{12w_Q}, \frac{\beta + 1}{12w_Q}, \frac{1}{2} \right] \frac{1}{\beta} - \frac{1 - \beta}{3w_Q} \frac{1}{\Omega_M - 1} \right\}
\]
\[
\times {}_2F_1 \left[ \frac{\beta - 1}{12w_Q}, \frac{\beta + 1}{12w_Q}, \frac{1}{2} \right] \frac{1}{\beta} - \frac{1 - \beta}{3w_Q} \frac{1}{\Omega_M - 1} \right\}.
\] (12)

For the case of a cosmological constant, $w_Q = -1$, Eq. (12) reduces to equation (16) in Kantowski & Thomas (2001), as we can see by using the property of the hypergeometric functions
\[
_2F_1 \left[ a, b, c, x \right] = \frac{1}{(1-x)^a} \_2F_1 \left[ a, a - b, a - c, \frac{x}{x - 1} \right],
\] (13)
and noting that the clumping parameter $\nu$ in Kantowski & Thomas (2001) corresponds to $(\beta - 1)/2$. The case of the cosmological constant is also studied in Kantowski & Thomas (2001), Demianski et al. (2001).

3 THE LUMINOSITY DISTANCE IN THE HOMOGENEOUS UNIVERSE

For the FLRW case ($\alpha_M = 1$), Eq. (5) is solved by
\[
D_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{1}{\sqrt{\Omega_M(1+z')^3 + (1 - \Omega_M)(1+z')^{3(w_Q+1)}}} \, dz'.
\] (14)

This expression is equivalent to Eq. (12) when $\beta = 1/4$,
\[
D_L(z) = \frac{c}{H_0 \sqrt{\Omega_M}} \left\{ \_2F_1 \left[ \frac{1}{6w_Q}, \frac{1}{2}, \frac{1}{6w_Q}, \frac{\Omega_M - 1}{\Omega_M} \right] \right\}
\] (15)
−\frac{1}{\sqrt{1+z}} zF_1\left[−\frac{1}{6Q}\cdot\frac{1}{2}−\frac{1}{6Q}\cdot\frac{\Omega_0−1}{\Omega_0}(1+z)^{3w_Q}\right].

Our Eq. (15) is equivalent to the expression found by Bloomfield & Waga (1999). In the Einstein-de Sitter case (\Omega_M = 1 or \omega_Q \to 0), Eq. (7) reduces to

\begin{equation}
D_L(z) = \frac{c}{H_0}2(1+z)\left(1−\frac{1}{\sqrt{1+z}}\right),
\end{equation}

as can also be seen directly by solving the integral in Eq. (14). Figure (1) plots the luminosity distance for different equations of state: the distance increases for decreasing \omega_Q.

4 THE MAGNIFICATION PROBABILITY DISTRIBUTION FUNCTION

The amplification of a source at a given redshift has a statistical nature. For narrow light-beams, the effect of gravitational lensing results in the appearance of shear and convergence in images of distant sources according to the different amount and distribution of matter along different lines of sight. So, gravitational lensing increases the level of errors in the Hubble diagram (Kantowski et al. 1993; Kantowski 1998; Frieman 1997; Wambsganss et al. 1997; Holz 1998; Holz & Wald 1998; Metcalf 1999; Barber 2000; Porciani & Madau 2000). In the framework of the on average FLRW Universe, we can account for this effect by considering a direction dependent smoothness parameter \alpha_M. Now, \alpha_M represents the effective fraction of matter density in the beam connecting the observer and the source and depends on the distribution of matter in the beam (Wang 1999), values of \alpha_M greater than 1 account for amplification effects.

There is an unique mapping between the magnification \mu of a standard candle at redshift \zeta and the direction-dependent smoothness parameter at \zeta (Wang 1999). According to Eq. (4), the magnification \mu of the source with respect to the maximum empty-beam case (\alpha_M = 0) is

\begin{equation}
\mu = \left[\frac{D_L(\alpha_M = 0)}{D_L(\alpha_M)}\right]^2.
\end{equation}

Once derived the magnification (that is, once found the distance by integrating the null-geodesic equation or using ray-tracing techniques along a line of sight) of a source at epoch \zeta, the corresponding smoothness parameter is determined in comparison with the DR distance: the solution of the DR equation for that constant value of \alpha_M matches, at redshift \zeta, that given value of the distance (Tomita 1998; Wang 1999).

The shape of the magnification pdf depends on the redshift of the source, on the cosmological parameters and on the nature of the DM. The dark matter can be classified according to its clustering properties (Metcalfe & Silk 1999; Seljak & Holz 1999; Mörtsell et al. 2001): microscopic DM consists of weakly interacting massive particles (WIMPs), such as neutralinos (Gurevich & Zybin 1993; Gurevich, Zybin & Sirota 1997) and clumps on galaxy halo-scales; macroscopic DM consists of compact objects, such as massive compact halo objects (MACHOs) or primordial black holes.

According to N-body simulations of large scale structures in cold dark matter models, galactic halos are expected to contain a large number of substructures besides their overall profile. However, this type of small-scale structure does not act as a compact object and only clumps of galaxy-size contribute appreciably to the lensing (Mörtsell et al. 2001).

In the framework of the on average FLRW models, the \mu-pdf is characterized by some general features with no regard to the nature of the DM. Under the assumption that the area of a sphere at redshift \zeta centred on the observer is not affected by the mass distribution, the photon number conservation implies that the mean apparent magnitude of a source at \zeta is identical to the FLRW value (Weinberg 1976; Schneider et al. 1992),

\begin{equation}
\langle \mu \rangle = \mu_{FLW} \equiv \mu(\alpha_M = 1) > 1.
\end{equation}

Since the matter is clumped, most of the narrow light-beams from distant sources do not intersect any matter along the line of sight resulting in a dimming of the image with respect to the filled-beam case: the mode of the pdf, \mu_{peak}, is biased towards the empty beam value,

\begin{equation}
\mu_{peak} < \langle \mu \rangle.
\end{equation}

The third feature is a tail towards large amplifications which preserves the mean. In terms of the magnification relative to the mean,

\begin{equation}
\delta \mu \equiv \frac{D_L(\alpha_M = 0)}{D_L(\alpha_M)} \left[\frac{D_L(\alpha_M = 0)}{D_L(\alpha_M = 1)}\right]^2,
\end{equation}

the \delta \mu-pdf has the mean at \delta \mu = 0, the peak value at \delta \mu_{peak} < 0 and a long tail for \delta \mu > 0, with no regard to the source redshift and to the cosmological parameters. It follows from these very general considerations that a simple way to characterize
the pdf is to consider the parameter \( \Delta \mu \), defined as the difference in amplification between the mean FLRW value and the magnification in the empty beam case \((\alpha_M = 0)\), \( \Delta \mu \equiv -\partial \mu / \partial \mu \). When \( \Delta \mu \) increases, the mode value moves towards greater de-magnification: to preserve the total probability and the mean value, the pdf must both reduce its maximum and enlarge its high amplification tail. From the properties of the angular diameter distance in a clumpy Universe (Sereno et al. 2001), it follows that \( \Delta \mu \) increases with the redshift of the source and with dark energy with large negative pressure. So, the dispersion in the \( \mu \)-pdf due to gravitational lensing increases with \( z \) and it is maximum for the case of the cosmological constant (see also Bergström et al. (2000)): quintessence with \( w_Q > -1 \) reduces the bias towards large de-amplifications of the peak value of the pdf, partially attenuating the effect of the clumpiness.

### 4.1 Lensing by microscopic dark matter

The gravitational lensing effect by large-scale structure on the apparent luminosity of distant sources in the Universe has been studied either with N-body simulations (Wambsganss et al. 1997; Tomita 1998; Barber 2000; Barber et al. 2000; Jain et al. 2000) or with the integration of the geodesic deviation equation (Holz & Wald 1998; Bergström et al. 2000) in a Universe filled with either isothermal spheres or Navarro-Frenk-White profiles (Navarro, Frenk & White 1995; Navarro, Frenk & White 1999).

The \( \mu \)-pdf for the smoothly distributed DM is characterized by two main trends with increasing redshift: an increase in the dispersion and an increasing gaussianity. As we look back to earlier times, the Universe becomes smoother on average and lines of sight become more filled in with matter: light bundles intersect more independent regions along their paths and the resulting \( \mu \)-pdf approaches a gaussian by the central limit theorem (Seljak & Holz 1999; Wang 1999). The corresponding \( \alpha_M \)-pdf also becomes symmetric but it reduces its dispersion and its mode goes to the filled-beam value (Tomita 1998; Wang 1999; Barber et al. 2000). The trends in dispersions in the \( \mu \)-pdf and \( \alpha_M \)-pdf are opposite since, with increasing redshift, a large variation in the distance corresponds to a small variation in the smoothness parameter (Sereno et al. 2001). Wambsganss et al. (1997) used the ray-tracing method for large-scale simulations in a cold dark matter Universe, normalized to the first year COBE data with \( \Omega_M = 0.4 \), \( \Omega_Q = 0.6 \), \( w_Q = -1 \), with a spatial resolution on small scales of the order of the size of a halo, to derive the \( \mu \)-pdf at different redshifts. Wang (1999) was able to find empirical formulae for the fitting of the \( \mu \)-pdf and of the corresponding \( \alpha_M \)-pdf,

\[
p_\mu(\mu, z) = p_{\alpha_M}(\alpha_M, z) \left| \frac{\partial \alpha_M}{\partial \mu} \right| = p_{\alpha_M}(\alpha_M, z) \frac{D_\alpha(\alpha_M = 0)}{2 \mu^{3/2}} \left| \frac{\partial D_A}{\partial \alpha_M} \right|^{-1}.
\]

As noted by Tomita (1998), the angular diameter distance depends on \( \alpha_M \) linearly for \( 0 \leq z \lesssim 5 \) and, with high precision, we can approximate

\[
\frac{\partial D_A}{\partial \alpha_M} \simeq D_A(\alpha_M = 1) - D_A(\alpha_M = 0).
\]

In Fig. (3), we plot the \( \delta \mu \) corresponding to the mode of the \( \alpha_M \)-pdf (as plotted in figure (2b) in Wang (1999)) as a function of the redshift: while the mode value of the \( \alpha_M \)-pdf goes to the filled-beam value for increasing redshift, the variation in magnification with respect to the FLRW mean increases: that is, the bias increases with \( z \).

To study the role of the quintessence in the magnification dispersion of standard candles, we consider the same matter content, that is the same \( \alpha_M \)-pdf (Wang 1999), for different equations of state. Models with different cosmological parameters produce, in general, different \( \alpha_M \)-pdf, predictable by numerical simulations; but, to consider the influence of the dark energy on the \( \mu \)-pdf, it suffices to use the same matter distribution in Eq. (21). This is equivalent to assume that the dependence on quintessence enters Eq. (21) through the angular diameter distances and that the effect on \( p_{\alpha_M} \) is of the second order. So, for analytical convenience, we can use the same \( p_{\alpha_M} \) derived in Wang (1999) for several cosmological models with the same \( \Omega_M \) but different equations of state. In Fig. (3), the \( \mu \)-pdf is plotted for two source redshifts and for two different equations of state: the \( \mu \)-pdf becomes more and more symmetric with \( z \) and the dark energy reduces both the dispersion and the bias.

The effect of gravitational lensing by large-scale structure affects significantly the determination of the cosmological parameters from observations of standard candles. Observed SNe Ia represent individual sources at each redshift and do not sample evenly the probability distribution: at a fixed redshift, we will observe the mode value of the distribution and not the mean one (Wambsganss et al. 1997; Barber 2000). For \( \Omega_M = 0.4 \), \( w_Q = -1 \) and \( z = 1 \), the mode is \( \mu_{\text{peak}} = 1.14 \) and the magnification values above and below which 97.5% of all the lines of sight fall are \( \mu_{\text{low}} = 1.11 \) and \( \mu_{\text{high}} = 1.28 \). This dispersion induces uncertainties in determining \( \Omega_M \) and the equation of state. Assuming a flat Universe with cosmological constant, a Universe with \( \Omega_M = 0.4 \) will be interpreted as a model with \( \Omega_M = 0.42^{+0.03}_{-0.11} \) only because of the gravitational lensing noise. Here and in what follows, the error bars represent 2-σ limits. With the constraint of \( \Omega_M = 0.4 \), a cosmological constant might be interpreted as dark energy with \( w_Q = -0.84 \).

For a flat Universe with \( \Omega_M = 0.4 \) and \( w_Q = -0.5 \), at \( z = 1 \) it is \( \mu_{\text{peak}} = 1.11 \), \( \mu_{\text{low}} = 1.09 \) and \( \mu_{\text{high}} = 1.23 \). With the constraint \( w_Q = -0.5 \), we should estimate \( \Omega_M = 0.43^{+0.05}_{-0.18} \); assuming \( \Omega_M = 0.4 \), it is \( w_Q = -0.46^{+0.05}_{-0.24} \).
Effects of quintessence on observations of Type Ia Supernovae in the clumpy Universe

Figure 2. The magnification relative to the mean calculated for the peak value of the $\alpha_M$-pdf as found in Wang (1999). It is $\Omega_M = 0.4$, $\Omega_Q = 0.6$ and $w_Q = -1$.

Figure 3. The amplification pdf as a function of $\delta \mu$, the magnification relative to the mean, for microscopic DM. The sharply peaked line are for $z = 0.5$, the smoother ones for $z = 2$. Solid and dashed lines correspond, respectively, to $w_Q = -1$ and $-1/2$. It is $\Omega_M = 0.4$ and $\Omega_Q = 0.6$. Solid and dashed lines have the same matter distribution but different cosmological backgrounds.

Although the lensing dispersion is reduced in a quintessence cosmology, the errors induced on the cosmological parameters increase. The reason is that in this models the luminosity distance is less sensitive to the cosmology (Sereno et al. 2001).

4.2 Lensing by compact objects

The effect of gravitational lensing is maximum when the matter in the Universe consists of point masses (Holz & Wald 1998); as seen above, this case is not included in the small-scale structures in the microscopic DM (Mörsell et al. 2001). The universal fraction of macroscopic DM is still unknown. Direct searches for MACHOs in the Milky Way have been performed by the MACHO and EROS collaborations through microlensing surveys. According to the MACHO group (Alcock et al. 2000), the most likely halo fraction in form of compact objects with a mass in the range $0.1 - 1 \, M_\odot$ is of about 20%; the EROS collaboration (Lasserre et al. 2000) has set a 95% confidence limit that objects less than $1 \, M_\odot$ contribute less than 40% of the dark halo. However, the average cosmological fraction in macroscopic DM could be significantly different from these local estimates.

The properties of the $\mu$-pdf are essentially independent of both the mass spectrum of the lenses (this statement is strictly true for point sources (Schneider & Weiss 1988)) and the clustering properties of the point masses, provided that the clustering is spherically symmetric (Holz & Wald 1998). The dispersion in luminosity of standard candles is non-gaussian, sharply peaked at the empty beam value and has a long tail towards large magnifications falling as $\mu^{-3}$ (Paczyński 1986, Rauch 1991, Holz & Wald 1998), caused by small impact parameter lines of sight near the compact objects; so, its second moment is logarithmically divergent and the law of large numbers fails: if strongly lensed events are removed from the data sample, a bias will be introduced towards smaller apparent luminosities (Holz & Wald 1998).

A comparative analysis of the $\mu$-pdf in the case of either microscopic DM or compact objects has put in evidence two main differences: the high magnification tail is larger for macroscopic DM and the mode of the distribution is nearer the average value in the case of lensing by large-scale structures (Seljak & Holz 1999, Mörsell et al. 2001).

The $\mu$-pdf in a Universe filled with a uniform comoving density of compact objects depends on a single parameter, the
mean magnification \( \langle \mu \rangle \) (Rauch 1991; Seljak & Holz 1999). Based on Monte-Carlo simulations, Rauch (1991) gives the fitting formula

\[
p(\mu) \propto \left[ \frac{1 - e^{b(\mu - 1)}}{\mu^2 - 1} \right]^{3/2},
\]

where the parameter \( b \) is related to the mean magnification by \( b = 247 \exp \left[ -22.3(1 - \langle \mu \rangle^{-1/2}) \right] \). The approximation holds for \( \langle \mu \rangle^{-1/2} \gtrsim 0.8 \), a condition verified up to \( z \sim 2 \) in a Universe with low matter density, with no regard to the equation of state \( w_\text{Q} \).

The \( \alpha_M \)-pdf corresponding to the distribution in Eq. (23) is highly non-gaussian, see Fig. 4. The pdf decreases monotonically from the empty beam value to high values of the smoothness parameter. With increasing redshift, the \( \alpha_M \)-pdf tends to flatten and the probability for the filled-beam case and for high values of \( \alpha_M \) grows.

In Fig. 4, we show the \( \delta \mu \)-pdf for two source redshifts and for two values of \( w_\text{Q} \): quintessence with \( w_\text{Q} > -1 \) reduces the effect of clumpiness. For \( z = 0.5 \), the variation in the distance modulus from the empty-beam case to the filled-beam one is 0.033(0.039) mag for \( w_\text{Q} = -1/2(-1) \); for \( z = 1 \), it is 0.108(0.138) mag for \( w_\text{Q} = -1/2(-1) \); for \( z = 1.5 \), it is 0.205(0.268) mag for \( w_\text{Q} = -1/2(-1) \). For \( z \gtrsim 1 \), the bias towards the empty-beam value can be compared with the dispersion of 0.17 mag in the peak magnitudes of SNe Ia after the application of methods as the “multi-colour light curve” method (Riess, Press & Kirshner 1996). The effect of gravitational lensing is of the same order of magnitude as the other systematic uncertainties that limit the conclusions on the cosmological parameters based on SNe Ia Hubble diagram (Filippenko & Riess 2000). The correlation between host galaxy type and both luminosity and light-curve shape of the source; interstellar extinction occurring in the host galaxy and the Milky Way; selection effects in the comparison of nearby and distant SNe; sample contamination by SNe that are not SNe Ia can produce changes as large as 0.1 mag in the measured luminosities of SNe Ia.

The effect on the estimate of the cosmological parameters of gravitational lensing by a totally clumped model with only macroscopic DM is quite dramatic. For a source redshift of \( z = 1 \), a Universe with \( \Omega_M = 0.3 \) and a cosmological constant can be interpreted as a model with \( \Omega_M = 0.42 \) and \( w_\text{Q} = -1 \) or as one with \( \Omega_M = 0.3 \) and \( w_\text{Q} = -0.71 \). These systematic errors increase in a quintessence cosmology with \( w_\text{Q} > -1 \). For \( z = 1 \), a Universe with \( \Omega_M = 0.3 \) and \( w_\text{Q} = -2/3 \) will be interpreted as a model with \( \Omega_M = 0.45 \) and \( w_\text{Q} = -2/3 \) or one with \( \Omega_M = 0.3 \) and \( w_\text{Q} = -0.46 \).

5 CONCLUSIONS

Observations of SNe Ia are strongly affected by inhomogeneities in the Universe. For redshifts \( z \gtrsim 1 \), the variation in the distance modulus from a standard flat FLRW model to a clumpy Universe with the same content of pressureless matter can be considerably greater than other systematic effects. The effect of amplification dispersion by gravitational lensing must be accurately considered. The prospects of future space-born missions, like the SuperNova Acceleration Probe (SNAP - [Http://snap.lbl.gov]) and the Next Generation Space Telescope, of determining properties of the dark energy have been discussed (Jollith et al. 2001; Weller & Albrecht 2001a; Weller & Albrecht 2001b; Gerke & Efstathiou 2002). According to these studies, SNAP data should only distinguish between a cosmological constant and quintessence with \( w_\text{Q} \) relatively far from \(-1 \). When SNe observations are combined with an independent estimate of \( \Omega_M \), for example from galaxy clustering (Verde et al. 2002), the degeneracies among the quintessence models can be significantly reduced and some constraints on the time evolution of the equation of state can be put (Weller & Albrecht 2001; Gerke & Efstathiou 2002). However, these studies only consider measurement errors and intrinsic dispersion of the sources, neglecting the systematic and redshift dependent
error induced by gravitational lensing. We have shown how, also assuming an exact knowledge of \( \Omega_M \), in the redshift range covered by future missions a cosmological constant can be interpreted as dark energy with \( w_Q > -1 \). For \( \Omega_M = 0.4 \) and \( z = 1 \), a \( \Lambda \) constant may be interpreted as quintessence with \( w_Q < -0.84 \), only due to the lensing by large-scale structure. A fraction of DM in form of compact objects will make the situation even more dramatic. So, also with a prior knowledge of the remaining cosmological parameters, gravitational lensing can make the statements on the properties of dark energy based on SNe data significantly less certain.

The effect of inhomogeneities dominates at high redshifts and should be one of the main systematics in attempting to build the Hubble diagram with GRBs. The physical origin of GRBs is still uncertain, but recent observations suggest that they are related to the violent death of massive stars. Under the hypothesis that GRBs trace the global star formation history of the Universe, their assumed rate is strongly dependent on the expected evolution of the star formation rate with the redshift (Porciani & Madau 2001). While some scenarios prefer a redshift distribution of the GRB comoving rate peaked between \( z = 1 \) and 2, according to other ones the comoving rate remains roughly constant at \( z \approx 2 \) and out to very high redshift (Porciani & Madau 2001). Furthermore, the lack of strong lensing events in the fourth BATSE GRBs catalog (Holz, Miller & Quashnock 1999) suggests that, at the 95\% confidence level, the upper limit to the average redshift of GRBs is \( z \sim 3 \) in a flat, low-matter density Universe with cosmological constant. According to these considerations, the effect of gravitational lensing would be really dominant in the Hubble diagram built with GRBs.

As an example, we consider the GRB redshift distribution derived from a combined analysis of two independent luminosity indicators (Schaefer, Deng & Band 2001). Examining a sample of 112 GRBs from the BATSE catalog, Schaefer et al. (2001) found redshifts varying between 0.25 and 5.9 with a median of 1.5. At \( z = 1.5 \), gravitational lensing by large-scale structures, in a model with \( \Omega_M = 0.4 \) and \( w_Q = -2/3 \), induces a magnification distribution with \( \mu_{peak} = 1.25 \), \( \mu_{low} = 1.20 \) and \( \mu_{high} = 1.46 \). Assuming \( w_Q = -2/3 \), we will estimate \( \Omega_M = 0.43^{+0.05}_{-0.16} \); assuming \( \Omega_M = 0.4 \), we will estimate \( w_Q < -0.51 \).

Although the lensing dispersion on the luminosities of standard candles represents a noise in the determination of the cosmological parameters, it can also be considered as a probe of the clustering properties of the DM. Lensing dispersion has been investigated to search for the presence of compact objects in the Universe (Linder, Schneider & Wagoner 1983; Rauch

---

**Figure 5.** The magnification pdf for macroscopic dark matter as a function of \( \delta \mu \), the magnification relative to the mean. Solid and dashed lines correspond, respectively, to \( w_Q = -1 \) and \( -2/3 \). It is \( \Omega_M = 0.3 \), \( \Omega_Q = 0.7 \). Left panel: the source redshift is \( z = 1 \); right panel: it is \( z = 1.5 \).

**Figure 6.** Amplification dispersion relative to the mean due to gravitational lensing by macroscopic DM for the projected 1-year SNAP sample. Thick and thin lines correspond, respectively, to \( w_Q = -1 \) and \( -1/2 \). It is \( \Omega_M = 0.3 \), \( \Omega_Q = 0.7 \). Intrinsic dispersion of SN luminosities is not considered.
The possibility of determining the fraction of macroscopic DM using future samples of SNe Ia has also been explored (Mörtssell et al. 2001). SNAP should intensively observe SNe up to $z \sim 1.7$. In one year of study, this space-borne mission should be able to discover $\sim 2350$ SNe, most of which in the region $0.5 < z < 1.2$. The discrimination of models of Universe with different fractions of compact objects is mainly based on the shift in the peak of the lensing dispersion (Seljak & Holz 1999; Mörtssell et al. 2001): a shift of $\sim 0.01$ mag in the peak of the lensing dispersion in the projected SNAP sample towards lower amplifications corresponds to a growth of $20\%$ in the fraction of macroscopic DM in a flat Universe with $\Omega_M = 0.3$ and a cosmological constant (see figure (4) in Mörtssell et al. (2001)). In Fig. 4, we plot the dispersion in amplification, for the projected redshift distribution of SNe according to the SNAP proposal, in a Universe with $\Omega_M = 0.3$ filled in with macroscopic DM. High de-amplification are preferred in the case of a cosmological constant, when the maximum of the distribution is depleted and the mode is shifted away from the mean with respect to dark energy with $w_Q > -1$. Changing from $w_Q = -1$ to $w_Q = -1/2$, the peak of the distribution moves for $\sim 0.015$ mag towards higher amplifications. So, a significant reduction in the fraction of compact object can be mimed by quintessence with $w_Q > -1$. Since quintessence reduces the dispersion of gravitational lensing, it also reduces the ability to distinguish between macroscopic and macroscopic DM from the shape of the amplification dispersion. Both quintessence and microscopic DM reduce the bias towards the empty beam value and the high magnification tail and their effect is of the same order. A Universe with an high fraction of macroscopic objects can be misleadingly interpreted as one with dark energy with large negative pressure.

ACKNOWLEDGMENTS

Authors are indebted to M. Capaccioli, A.A. Marino, C. Rubano and P. Scudellaro for helpful discussions. They also thank an anonymous referee for the stimulating reports. MVS also would like to acknowledge the hospitality of the Università degli Studi di Napoli “Federico II” and the Osservatorio Astronomico di Capodimonte during his visit in Napoli. This paper was partially carried out with the support of “Cosmion” center, the Russian Fund for Basic Research (grant N 00-02-16350).

REFERENCES

Alcock, C., et al., ApJ, 542, 281
Barber, A.J., 2000, MNRAS, 318, 195
Barber, A.J., Thomas, P.A., Couchman, H.M.P., Fluke, C.J., 2000, MNRAS, 319, 267
Bergström, L., Goliath, M., Goobar, A., Mörtssell, E., 2000, A&A, 385, 13
Bloomfield Torres, L.F., Waga, I., 1996, MNRAS, 279, 712
Caldwell, R.R., Dave, R., Steinhardt, P.J., 1998, Phys. Rev. Lett., 80, 1582
Célerier, M.N., 2000, A&A, 353, 63
Chiba, T., & Nakamura, T., 2000, Phys. Rev. D, 62, 121301
Chiba, T., Sugiyama, N., Nakamura, T., 1997, MNRAS, 289, L5
Cooray, A.R., & Huterer, D., 1999, ApJ, 513, L96
de Bernardis, P., Ade, P.A.R., Bock, J.J., Bond, J. R., Borrill, J., Boscaleri, A., Coble, K., Crill, B.P., 2000, et al., Nature, 404, 955
Demianski, M., de Ritis, R., Marino, A.A., & Piedipalumbo, E., 2000, astro-ph/0004374
de Ritis, R., Marino, A.A., Rubano, C., Scudellaro, P., 2000, Phys. Rev. D, 62, 043506
Dolgov, A., Sazhin, M., Zeldovich, Ya.B., 1999, Basics of modern cosmology, Gif-sur-Yvette, Editions Frontiere
Dyer, C.C., Roeder, R.C., 1972, ApJ, 174, L115
Dyer, C.C., Roeder, R.C., 1973, ApJ, 180, L31
Etherington, I.M.H., 1933, Phil. Mag., 15, 761
Filippenko, A.V., Riess, A.G., 2000, in “Type Ia Supernovae: Theory and Cosmology”, edited by J.C. Niemeyer and J.W. Truran, Cambridge University Press, Cambridge
Frieman, J.A., 1997, Comments Astrophys., 18, 323
Gerke, B.P., Efstatiiou, G., 2002, astro-ph/0201336
Gioci, F., Amendola, L., 2001, MNRAS, 325, 1097
Goliath, M., Amanullah R., Astier, P., Goobar, A., Pain, R., 2001, A&A, 380, 6
Guerra, E.J., Dalrymple, R., Wan, L., 2000, ApJ, 544, 659
Gurevich, A.V., Zybin, K.P., 1995, Phis. Lett. A, 208, 276
Gurevich, A.V., Zybin, K.P., & Sirota, V.A., 1997, Sov. Phys. Usp. 167, 913
Gurvits L.I., Keller, K.E., Frey S., 1999, A&A 342, 378
Holz, D.E., 1998, ApJ, 506, L1
Holz, D.E., & Wald, R.M., 1998, Phys. Rev. D, 58, 063501
Holz, D.E., Miller, M.C., Quashnock, J.M., 1999, ApJ, 510, 64
Huterer, D., & Turner, M.S., 2001, astro-ph/0103177
Jain, B., Seljak, U., White, S., 2000, ApJ, 530, 547
Kamenshchik, A., Moschella, L., Pasquiera, V., 2001, Phys.Lett. B, 511, 265
Kantowski, R., 1969, ApJ, 155, 89
Kantowski, R., Vaughan, T., Branch, D., 1995, ApJ, 447, 35
Kantowski, R., 1998, ApJ, 507, 483

© 0000 RAS, MNRAS 000, 000–000
Effects of quintessence on observations of Type Ia SuperNovae in the clumpy Universe

Kantowski, R., Thomas, R.C., 2001, ApJ, 561, 491
Kantowski, R., Kao, J.K., Thomas, R.C., 2000, ApJ, 545, 549
Korolev, V.A., Sazhin, M.V., 1986, Preprint ISR, N 1060.
Lasserre, T., et al., 2000, A&A, 355, L39
Linder, E.V., 1988, A&A, 206, 190
Linder, E.V., Schneider, P., Wagener, R.V., 1988, ApJ, 324, 786
Maor, I., Brustein, R., & Steinhardt, P.J., 2001, Phys. Rev. Lett., 86, 6.
Metcalf, R.B., 1999, MNRAS, 305, 746
Metcalf, R.B., Silk, J., 1999, ApJ, 519, L1
Mörsell, E., Goobar, A., Bergström, L., 2001, ApJ, 559, 53
Nakamura, T., & Chiba, T., 2001, ApJ, 550, 1
Navarro, J.F., Frenk, C.S., White, S.D.M., 1995, MNRAS, 275, 56
Navarro, J.F., Frenk, C.S., White, S.D.M., 1996, ApJ, 462, 563
Norris, J.P., Marani, G., & Bonnell, J., 2000, ApJ, 534, 248
Ostriker, J.P., Steinhardt, P.J., 1995, Nature, 377, 600
Paczyński, B., 1986, ApJ, 304, 1
Pavlov, M., Rubano, C., Sazhin, M.V., Scudellaro, P., 2002, ApJ, 566, 619
Peebles, P.J.E., Ratra, B., 1988, ApJ, 325, L17
Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A., Nugent, P., Castro, P.G., Deustua, S., Fabbro, S., et al., 1999, ApJ, 517, 565
Porciani, C., & Madau, P., 2000, ApJ, 532, 679
Porciani, C., & Madau, P., 2001, ApJ, 548, 522
Rauch, K.P., 1991, ApJ, 374, 83
Reichart, D.E., Lamb, D.Q., 2001, astro-ph/0103255
Reichart, D.E., Lamb, D.Q., Fenimore, E.E., Ramirez-Ruiz, E., Cline, T.L., Hurley, K., 2001, ApJ, 552, 57
Riess, A.G., Press, W.H., Kirshner, R.P., 1996, ApJ, 473, 88
Riess, A.G., Filippenko, A.V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P.M., Gilliland, R.L., Hogan, C.J., et al., 1998, AJ, 116, 1009
Rubakov, V.A., 2000, Phys. Rev. D, 61, 061501
Rubano, C., Scudellaro, P., 2001, Gen. Rel. Grav., in press; astro-ph/0103335
Sahni, V., Starobinski, A.A., 2000, Int. J. M. Ph., 9, 4
Saini, T.D., Raychaudhury, S., Sahni, V., & Starobinsky, A.A., 2000, Phys. Rev. Lett., 85, 1162
Schaefer, B.E., Deng, M., Band, D.L., 2001, ApJ, 563, L123
Schmidt, B.P., et al., 1998, ApJ, 507, 46
Schneider, P., Weiss, A., 1988, ApJ, 327, 526
Schneider, P., Ehlers, J., Falco, E.E., 1992, Gravitational Lenses, Springer-Verlag, Berlin
Seitz, S., Schneider, P., Ehlers, J., 1994, Class. Quant. Grav., 11, 2345
Seljak, U., Holz, D.E., 1999, A&A, 351, L10
Sereni, M., Covone, G., Pietripalumbo, E., de Ritis, R., 2001, MNRAS, 327, 517
Silveira, V., Waga, I., 1997, Phys. Rev. D, 56, 4625
Spergel, D., Pen, U., 1997, ApJ, 491, L67
Tomita, K., 1998, Prog. Theor. Phys., 100, 79
Tomita, K., 2001, MNRAS, 326, 287
Trentam, N., 2001, MNRAS, 326, 1328
Turner, M.S., 2000, Physica Scripta, 85, 210
Turner, M.S., White, M., 1997, Phys. Rev. D, 56, 4439
Verde, L., et al., 2002, MNRAS, in press; astro-ph/0112161
Vilenkin, A., 1984, Phys. Rev. Lett., 53, 1016
Wambsganss, J., Cen, R., Xu, G., Ostriker, J.P., 1997, ApJ, 475, L81
Wang, Y., 1999, ApJ, 525, 651
Wang, Y., & Garnavich, P.M., 2001, ApJ, 552, 445
Wang, Y., & Lovelace, G., 2001, ApJ, 562, L115
Weinberg, S., 1976, ApJ, 208, L1
Weller, J., Albrecht, A., 2001a, Phys. Rev. Lett., 86, 1939
Weller, J., Albrecht, A., 2001b, astro-ph/0106079
Wetterich, C., 1988, Nuclear Phys. B, 302, 668
Yamamoto, K., & Futamase, T., 2001, astro-ph/0104253
Zlatev, I., Wang, L., Steinhardt, P.J., 1999, Phys. Rev. Lett., 83, 670
Zel’dovich, Ya.B., 1964, Sov. Astr., 8, 13
Zel’dovich, Ya.B., 1992, My Universe: selected reviews, Gordon and Breach.