Phase-merging enhanced harmonic generation free-electron laser

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Abstract
Together with one of its variants, the recently proposed phase-merging enhanced harmonic generation (PEHG) free-electron laser (FEL) is systematically studied in this paper. Different from a standard high-gain harmonic generation scheme, a transverse gradient undulator is employed to introduce a phase-merging effect into the transversely dispersed electron beam in PEHG. The analytical theory of the phase-merging effect and the physical mechanism behind the phenomenon are presented. Using a representative set of beam parameters, intensive start-to-end simulations for soft x-ray FEL generation are given to illustrate the performance of PEHG. Moreover, some practical issues that may affect the performance of PEHG are also discussed.

Keywords: seeded FEL, PEHG, phase-merging effect, transverse gradient

1. Introduction

The recent success of self-amplified spontaneous emission (SASE)-based x-ray free-electron laser (FEL) facilities [1, 2] is enabling cutting-edge science in various areas. While the radiation from a SASE FEL has excellent transverse coherence, it typically has rather limited temporal coherence as the initial radiation comes from the electron beam shot noise. To overcome this problem, several SASE-based techniques have been developed, mainly including self-seeding [3–5], purified SASE [6], improved SASE [7] and HB SASE [8].
An alternative way to significantly improve the temporal coherence of high-gain FELs is to use frequency up-conversion schemes, which generally rely on the techniques of optical-scale manipulation of the electron beam phase space with external coherent laser sources. In the high-gain harmonic generation (HGHG) scheme [9], typically a seed laser pulse is used to interact with electrons in a short undulator, called a modulator, to generate a sinusoidal energy modulation in the electron beam on the seed laser wavelength scale. This energy modulation is then transformed into an associated density modulation by a dispersive magnetic chicane, called the dispersion section (DS). Taking advantage of the fact that the density modulation shows Fourier components at high harmonics of the seed frequency, intense radiation at shorter wavelengths can be generated. The property of HGHG output is a direct map of the seed laser’s attributes, which ensures a high degree of temporal coherence and small pulse energy fluctuations with respect to SASE. These theoretical predictions have been demonstrated in experiments with the HGHG [10–13]. However, significant bunching at higher harmonics is usually needed to strengthen the energy modulation in HGHG, which will result in a degradation of the amplification process in the radiator. Thus the requirement of FEL amplification on the beam energy spread prevents the possibility of reaching a short wavelength in a single-stage HGHG. In order to improve the frequency multiplication efficiency in a single stage, more complicated phase space manipulation techniques have been developed, e.g. the echo-enabled harmonic generation [14, 15] technique employs two modulators and two dispersion sections, which can be used to introduce an echo effect into the electron beam phase space for enhancing the frequency multiplication efficiency with a relatively small energy modulation.

The idea of using a transverse gradient undulator (TGU) to mitigate the effects of electron beam energy spread in FEL oscillators has been described in [16]. Recently, this idea has been applied to laser-plasma accelerator driven high-gain FELs [17]. Inspired by these early works, a novel phase space manipulation technique, originally termed cooled-HGHG, has been proposed for significantly improving the frequency up-conversion efficiency of harmonic generation FELs [18]. This technique benefits from the transverse-longitudinal phase space coupling, while other harmonic generation schemes only manipulate the longitudinal phase space of the electron beam. In comparison with the general idea of using TGU to compensate for the effects of beam energy spread by making every electron satisfy the resonant condition in the undulator, this novel scheme utilizes a different operation regime of TGU: when the transversely dispersed electrons pass through the TGU modulator, around the zero-crossing of the seed laser, the electrons with the same energy will merge into the same longitudinal phase, which holds great promise for generating fully coherent short-wavelength radiation at very high harmonics of the seed laser.

At first glance, this phase-merging phenomenon is very similar to electron beam energy spread cooling. However, the beam energy spread within the range less than the seed laser wavelength is reduced, while the global beam energy spread does not change in such a process. Therefore, in order to clearly and unambiguously illustrate the physics behind it, we rename such a scheme as phase-merging enhanced harmonic generation (PEHG), although further studies demonstrate that this novel technique can be utilized for real electron beam energy spread cooling in an x-ray FEL linear accelerator [19].

In this paper, a systematic study on the PEHG is presented. The principle of the PEHG is introduced in section 2. Analytical estimates and one-dimensional simulation results are given in section 3 to present the physical mechanism of the phase-merging effect and the possibility of
imprinting ultra-high harmonic microbunching into the electron beam with a relatively small energy spread using this technique. Section 4 gives an optimized design for a soft x-ray FEL with realistic parameters based on the PEHG. Some practical constraints that may deteriorate the performance of PEHG are studied in section 5. Finally, we conclude in section 6.

2. Principles of PEHG

The initially proposed PEHG consists of a dogleg followed by a HGHG configuration with a TGU modulator, as shown in figure 1. Rectangular coordinates are given in scheme 1, where x and y represent the horizontal and vertical directions, respectively. The dogleg with dispersion \( \eta \) is used to transversely disperse the electron beam, while the TGU modulator is used for the beam energy modulation and to precisely manipulate the electrons in the horizontal dimension. It is found that these two functions of the TGU modulator can be separately performed by employing a modified design, as shown in scheme 2 of figure 1. Here, a normal modulator is used for the energy modulation, and the TGU is responsible only for transverse manipulation of the electrons, a design that will be much more flexible for practical operation. In principle, the TGU in scheme 2 can be replaced by other kinds of devices with a transverse gradient magnet field—e.g. specifically designed wigglers or small chicanes. For convenience of theoretical analysis, we consider scheme 2 first, and then extend the conclusions to scheme 1.

Following the notation of [15], we also assume an initial Gaussian beam energy distribution with an average energy \( \gamma_0 mc^2 \) and use the variable \( p = (\gamma - \gamma_0)/\sigma_\gamma \) for the dimensionless energy deviation of a particle, where \( \sigma_\gamma \) is the rms energy spread. Then the initial longitudinal phase space distribution should be \( f_0(p) = N_0 \exp(-p^2/2)/\sqrt{2\pi} \). We use \( \chi = (x - x_0)/\sigma_x \) as the dimensionless horizontal position of a particle, where \( x_0 \) is the central beam position in the horizontal plane and \( \sigma_x \) is the initial horizontal rms beam size. Then the horizontal distribution of the electron beam can be written as \( g_0(\chi) = N_0 \exp(-\chi^2/2)/\sqrt{2\pi} \).

After the dogleg, \( \chi \) is changed to

\[
\chi' = \chi + Dp,
\]

where \( D = \eta\sigma_x/\sigma_\gamma \) is the dimensionless transverse dispersion of the dogleg, and the horizontal distribution becomes
where $\chi$ now refers to the value at the entrance of the modulator. After passing through the modulator, the electron beam is modulated with an amplitude $A = \Delta \gamma / \sigma$, where $\Delta \gamma$ is the energy modulation depth induced by the seed laser, and the dimensionless energy deviation of the electron beam becomes $p' = p + A \sin (kz)$, where $k$ is the wave number of the seed laser. The two-dimensional distribution after the seed laser modulation can be written as

$$h_1(\xi, p, \chi) = \frac{N_0}{2\pi} \exp \left[ -\frac{1}{2} (p - A \sin \xi)^2 \right] \exp \left\{ -\frac{1}{2} \left[ \chi - D (p - A \sin \xi) \right]^2 \right\},$$

where $p$ refers to the value after the modulator and $\xi = kz$ is the phase of the electron beam. When the transversely dispersed electron beam is further sent through a TGU with period length $\lambda_u$, period number $N_u$, transverse gradient $\alpha$, and central dimensionless parameter $K_0$, electrons at different horizontal positions will see different undulator $K$ values, where $K(x) = K_0 (1 + \alpha x)$ \cite{17}. According to the FEL resonant equation, different $K$ values will result in different path lengths for a given beam energy, and this converts the longitudinal coordinate $z$ of electrons with different horizontal positions into

$$z' = z + \frac{N_0 \lambda_u}{2\gamma^2} \left[ \frac{K(x)^2 - K_0^2}{2} \right] = z + \frac{L_u K_0^2}{2\gamma^2} \left[ \alpha \chi \sigma + \frac{1}{2} (\alpha \chi \sigma)^2 \right],$$

where $L_u = N_0 \lambda_u$ is the length of the TGU. Considering that the horizontal beam size is usually quite small in FEL, equation (4) can be re-written as

$$z' = z + \frac{L_u K_0^2}{2\gamma^2} \alpha \chi \sigma,$$

and then the electron beam distribution after TGU becomes:

$$h_2(\xi, p, \chi) = \frac{N_0}{2\pi} \exp \left\{ -\frac{1}{2} \left[ p - A \sin (\xi - T \chi) \right]^2 \right\} \exp \left\{ -\frac{1}{2} \left[ \chi - D (p - A \sin (\xi - T \chi)) \right]^2 \right\},$$

where $\xi$ now refers to the new phase of the electron after passage through the TGU and

$$T = \frac{k L_u K_0^2 \alpha \sigma}{2\gamma^2}$$

is the dimensionless gradient parameter of the TGU. Finally, after passing through the DS with the dispersive strength of $R_{56}$, the longitudinal beam distribution evolves to

$$h_{PEHG}(\xi, p, \chi) = \frac{N_0}{2\pi} \exp \left\{ -\frac{1}{2} \left[ p - A \sin (\xi - T \chi - B p) \right]^2 \right\} \exp \left\{ -\frac{1}{2} \left[ \chi - D (p - A \sin (\xi - T \chi - B p)) \right]^2 \right\},$$

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where $\zeta$ is the new phase of the electron after passage through the DS and $B = R_{55}k_0\sigma_y/\gamma$ is the dimensionless strength of the DS. Integration of equation (8) over $p$ and $\chi$ gives the beam density $N$ as a function of $\zeta$, $N(\zeta) = \int_{-\infty}^{\infty} d\chi \int_{-\infty}^{\infty} dp h_{\text{PEHG}}(\zeta, p, \chi)$, and the bunching factor at the $n$th harmonic can be written as

$$b_n = \frac{1}{N_0} \int_{-\infty}^{\infty} dp e^{-ip(TD-B)-inT} f_0(p) g_0(\chi) \left\{ e^{-in(\zeta+AB\sin\zeta)} \right\} = J_n[nAB] e^{-(1/2)[n(TD+B)]^2} e^{-(1/2)[nT]^2}. \quad (9)$$

For the case without TGU (i.e. $T = 0$), equation (9) reduces to the well-known formula for the bunching factor in a standard HGHG FEL.

For the harmonics of $n > 4$, the maximum of the Bessel function in equation (9) is about $0.67/n^{1/3}$ and is achieved when its argument is equal to $n + 0.81n^{1/3}$. Thus for a given energy modulation amplitude $A$, the optimal strength of the DS should be

$$B = \left( n + 0.81n^{1/3} \right)/nA. \quad (10)$$

The maximum of equation (9) will be achieved when $TD = -B$, which gives an optimized relation of $\alpha$ and $\eta$

$$\alpha \eta = -\frac{2\gamma^3 \left(n + 0.81n^{1/3}\right)}{nAk_LnK_0^2\sigma_y}. \quad (11)$$

When we adopt a large $A$ and $\eta$, or a small horizontal size $\sigma_y$, the third term in the right-hand side of equation (9) can be quite close to unity, and the maximal bunching factor of the $n$th harmonic for PEHG will approach

$$b_n \approx 0.67/n^{1/3}, \quad (12)$$

which is much larger than that of a standard HGHG.

For scheme 1 shown in figure 1, the energy modulation process and the phase-merging process are accomplished simultaneously when the electron beam passes through the TGU modulator. The relative phase advance of the electron caused by the gradient of the TGU is the same in both schemes. However, a factor of 1/2 should be introduced in the right-hand side of equation (7) for scheme 1, because the energy modulation increases approximately linearly with the modulator period number $N_m$, and the phase advance obtained by integration over the modulator length contributes a factor of 1/2. Thus for scheme 1, the optimal dimensionless gradient of the TGU should become $TD = -2B$, and then the relation between $\alpha$ and $\eta$ becomes:

$$\alpha \eta = -\frac{4\gamma^3 \left(n + 0.81n^{1/3}\right)}{nAk_LnK_0^2\sigma_y} \quad (13)$$

This result is precisely consistent with the earlier results given in [18].

3. Physical mechanism of PEHG

The physical mechanism behind PEHG is transverse-longitudinal phase space coupling. The evolution of the beam longitudinal phase space for scheme 2 is illustrated in figure 2. For
simplicity, here we assume the horizontal beam size $\sigma = 0$ and only show the phase space within one seed laser wavelength region. The energy modulation amplitude is chosen to be $A = 3$ here, and the optimized condition for the 50th harmonic bunching is $B = -TD \approx 0.35$ according to equation (10).

The initial longitudinal phase space after passing through the dogleg is shown in figure 2(a), where different colors represent different regions of beam energy and also the different horizontal positions of the electrons with respect to the reference electrons with central beam energy. After interaction with the seed laser in the conventional modulator, the longitudinal phase space of the beam evolves to that shown in figure 2(b). The strong optical field induces a rapid correlated growth of the electron beam energy spread. When the beam travels through the TGU, electrons of different colors (different transverse positions) will undergo different undulator $K$ values, thus resulting in the different travel path lengths in the TGU. By properly choosing the gradient of the TGU according to equation (11), the phase space will evaluate to that in figure 2(c). During this process, the electron energy is unchanged. However, around the zero-crossing of the seed laser, electrons with the same energy will merge into the same longitudinal phase due to the relative phase shift of the electrons in the TGU. This phenomenon is known as the ‘phase-merging effect’. After the TGU, the electrons enter the dispersion section where the beam phase space is rotated and the bunching at the desired harmonic is optimized, as shown in figure 2(d). It has been found that most of the electrons are
compressed into a small region around the zero-phase, which indicates that the density modulation has been significantly enhanced for high harmonics.

It can be deduced from equation (9) that the maximal bunching factor of PEHG is mainly determined by the Bessel function term, and that it weakly depends on the absolute value of $A$ when $\sigma_x$ is small or $\eta$ is quite large. Figure 3 shows the simulation results of the maximal bunching factors of PEHG for different energy modulation amplitudes under the condition of $\sigma_x = 0$. For comparison, the optimized bunching factors for the standard HGHG with the same energy modulation amplitudes are also shown. One can clearly see that the bunching factor exponentially decreases as the harmonic number increases for standard HGHG. However, for PEHG, the bunching factor decreases as $n^{-1/3}$ and its maximum fits quite well with the theoretical prediction for $n > 4$.

For a realistic electron beam, the intrinsic horizontal beam size $\sigma_x$ cannot be neglected. It will induce an effective energy spread into the electron beam because of the transverse field gradient of the TGU. The effective energy spread can be written as [17]

$$\sigma_{\text{eff}} = \frac{\sigma_x}{\eta}. \quad (14)$$

Using the optimized condition of PEHG and plugging equation (14) into equation (9), we arrive at

$$b_n = J_n \left[ nA\beta \right] e^{-\left(\frac{nA\beta}{\sqrt{2\pi}\sigma_{\text{eff}}} \right)^2}. \quad (15)$$

One may find that the bunching factor in equation (15) reduces to the form of a standard HGHG. The only difference is that the initial beam energy spread has been replaced by $\sigma_{\text{eff}}$. Here, we define an energy spread compression factor $C = \eta\sigma_x/\gamma\sigma_x$, which can be used to quantitatively measure the phase-merging effect. The intrinsic beam size is determined by the normalized horizontal emittance $\varepsilon_x$ and the $\beta$ function. For a relatively short modulator with length $L_m$, it is reasonable to take $\beta \approx L_m$, and hence $\sigma_x = \sqrt{\varepsilon_x L_m\gamma}$. By using the realistic parameters of the Shanghai soft x-ray FEL (SXFEL) project [20], figure 4 shows the 30th harmonic bunching factor as a function of the initial horizontal emittance. The beam energy is
840 MeV with an energy spread of about 100 keV, the dispersion of the dogleg is $\eta = 1$ m, and the average beta function in the short modulator is $\beta = 2 \, m$. The energy modulation amplitude has been changed from 1 to 10, and the strength of the DS and the gradient of the TGU have been turned simultaneously to optimize the 30th harmonic bunching factor according to equations (10) and (11). The wavelength of the seed laser is 264 nm. One can see from figure 4 that the bunching factor decreases quickly as the horizontal emittance increases when $A$ is smaller than 3. However, the bunching factor is still acceptable for $\varepsilon_x = 1 \, \mu m \, rad$ when $A$ is larger than 6. For the case of $\varepsilon_x = 1 \, \mu m \, rad$ and $A = 6$, the optimized bunching factor comparison between PEHG and HGHG is shown in figure 5. The energy spread compression factor is calculated to be $C \approx 5.74$ for this case, which increases the harmonic up-conversion efficiency approximately six times for PEHG.

It should be pointed out that the bunching factors are nearly the same for the two schemes in figure 1 when $A$ is much larger than 1. However, for a relatively small $A$, the final longitudinal phase space and the bunching factor will be quite different for these two schemes. Figure 6 shows simulation results for these two schemes when $A = 0.1$ and $\varepsilon_x = 0$. Due to the
non-linear effect during the modulation process for scheme 1, the bunching factor decreases for all harmonics. However, for scheme 2, the optimized bunching factor for $A = 0.1$ is nearly the same as for the $A = 3$ case, which demonstrates the theoretical predictions.

4. Generation of soft x-ray radiation

To illustrate a possible application with realistic parameters and show the parameter optimization method of PEHG, we take the nominal parameters of the SXFEL. The SXFEL aims to generate an 8.8 nm FEL from a 264 nm conventional seed laser through a two-stage cascaded HGHG. The electron beam energy is 840 MeV, with a slice energy spread of about 100 keV. The beam peak current is over 600 A. As mentioned above, the bunching factor of PEHG is quite sensitive to the beam emittance. The optimized 30th harmonic bunching factor and three-dimensional gain length of the 8.8 nm radiation as a function of the initial horizontal emittance are shown in figure 7. From figure 7(a), one can find that the bunching factor decreases quickly as the emittance increases when $\eta$ is smaller than 0.5 m, and that the bunching factor can be well maintained for $\eta > 1$ m. However, when the dispersion $\eta$ is too large, it will contribute to a FEL gain reduction due to the increased beam size. In order to ensure adequate gain in the radiator, the dispersion induced beam size is required to be no larger than the
intrinsic horizontal beam size contributed by the radiator beta function. For SXFEL, the beam size in the radiator is about the 100 μm level, considering the beam energy spread of 100 keV, and the maximum dispersion permitted is about 1 m. The three-dimensional FEL gain length as a function of horizontal emittance with different dispersions is calculated and shown in figure 7(b). The gain length can be well controlled under 2 m for the 1 μm rad emittance and 1 m dispersion case, which is reasonable for a seeded soft x-ray FEL. Because the transverse beam size should be calculated by \( \sigma = \sqrt{\sigma_x^2 + \sigma_y^2} \), where the \( \sigma_y^2 \) is unchanged for different \( \eta \), the three-dimensional gain length will not change too much (from 1.5 m to 1.8 m) when \( \eta \) is smaller than 1 m for a 1 μm rad emittance.

With the above parameters, start-to-end tracking of the electron beam, including all the components of SXFEL, has been carried out. The electron beam dynamics in the photo-injector was simulated with ASTRA [21] to take into account space-charge effects. ELEGANT [22] was then used for the simulation in the remainder of the linac. The slice parameters at the exit of the linac are summarized in figure 8. The beam energy in the central part of the electron beam is around 840 MeV, and the peak current is about 600 A. A constant profile is maintained in an approximately 600 fs wide and over 500 A region. A normalized emittance of approximately
0.65 μm-rad and a slice energy spread about 100 keV are observed in figure 8(b). Figure 9 shows the transverse beam center and beam size changes after passing through the dogleg. The average value of the horizontal beam size \( \sigma_x \) is increased from about 60 μm to about 70 μm, which will not significantly affect FEL performance. However, as shown in figure 9(b), the horizontal beam position changes a lot due to the large energy chirp in the electron beam.

The FEL performance of PEHG was simulated by the upgraded three-dimensional FEL code GENESIS [23] based on the output of ELEGANT. A 264 nm seed pulse with the longitudinal pulse length much longer than the bunch length is used in the simulation. The length of the TGU modulator is about 1 m, with a period length of 80 mm and K value of around 5.8. To maximize the bunching factor at the 30th harmonic of the seed laser, the optimized parameters are set to be \( A = 6, B = 0.18, \alpha = 20 m^{-1}, \eta = 0.5 \) m. The maximal bunching factor along the electron beam at the entrance of the radiator exceeds 5%, which fits quite well with the theoretical prediction of figure 7(a). The period length of the radiator is 25 mm with a K value of about 1.3. The evolution of the radiation peak power is shown in figure 10. The large bunching factor at the entrance to the radiator offered by the PEHG scheme is responsible for the initially steep quadratic growth in power. The significant enhancement in performance using the PEHG is clearly seen in figure 10(a), where the peak power of the 30th harmonic radiation exceeds 400 MW, which is quite close to the output peak power of the original design of SXFEL with two-stage HGHG. Moreover, the 8.8 nm radiation saturates...
within the 15 m long undulator, which is in the range of the original design of SXFEL. As mentioned above, the optimized condition for the bunching factor of PEHG is $TD = -B$, which indicates that the beam energy chirp effect induced by the dispersions of the TGU and the DS will counteract each other. Thus, the output wavelength of PEHG will be immune to the large residual beam energy chirp in figure 8(a). The single-shot radiation spectrum at saturation is shown in figure 10(b), from which one can find that the bandwidth of the radiation at saturation is quite close to Fourier-transform-limited.

5. Some practical issues

The unique feature of PEHG is the utilization of an undulator with a transverse field gradient of $\alpha$. We will discuss some practical issues that may affect the performance of PEHG in this section.

Unlike a conventional planar modulator undulator in standard HGHG, the TGU in PEHG will introduce an additional kick in the transverse dimension due to the gradient field, which will result in a deviation of the electron trajectory in the horizontal plane. For a TGU, the magnetic field distribution can be written as

$$B_y(x, z) = B_0(1 + \alpha x) \sin k_{\alpha} z,$$

where $B_0$ is the reference peak magnetic field. The electric field of the seed laser can be simply represented as

$$E_x(z) = E_0 \sin(k_{\alpha} z + \varphi_0),$$

where $E_0$ is the peak electric field and $\varphi_0$ is the carrier envelop phase of the seed laser.

According to Maxwell’s equations, the trajectory equations of the electron with initial horizontal position $x_0$ in the modulator can be written as

$$\frac{dx}{dz} = \frac{dx}{c dt},$$

$$\frac{d^2x}{dz^2} = \frac{1}{c^2} \cdot \frac{d^2x}{dt^2} = -\frac{e}{\gamma mc^2} \cdot \frac{dx}{dt} \cdot B_0(1 + \alpha x) \sin k_{\alpha} z. \quad (19)$$

Considering that the electron wiggles in the undulator, the horizontal position of the electron beam in the modulator can be written as

$$x(z) = x_0 + \frac{K_0}{k_{\alpha \gamma}} \sin k_{\alpha} z. \quad (20)$$

Then equation (19) is changed to

$$\frac{d^2x}{dz^2} = -\frac{e}{\gamma mc} \cdot \frac{dx}{dz} \cdot B_0 \left[ 1 + \alpha \left( x_0 + \frac{K_0}{k_{\alpha \gamma}} \sin k_{\alpha} z \right) \right] \sin k_{\alpha} z \quad (21)$$

$$= -\frac{e}{\gamma mc} \cdot \frac{dx}{dz} \cdot B_0 \left[ (1 + \alpha x_0) \sin k_{\alpha} z + \frac{\alpha K_0}{k_{\alpha \gamma}^2} \sin^2 k_{\alpha} z \right].$$
Integration of this formula over $z$ gives the total x-deviation of the electron after passage through the TGU modulator

$$\Delta x = \frac{\alpha K_0^2}{4\gamma^2} N_{\mu^2 N_{\mu^2}}^2.$$  \hfill (22)

According to the parameters used in section 5, $\Delta x$ is calculated to be about 96 $\mu$m, which will not significantly affect the performance of PEHG. Moreover, it can be derived from equation (21) that the x-deviation can be compensated by introducing an external magnetic field:

$$B_{\text{external}} = \frac{\alpha K_0}{2k_B^*} B_0.$$  \hfill (23)

To illustrate the particle trajectory in the TGU and check the simulation results of GENESIS, we developed a three-dimensional algorithm based on the fundamentals of electrodynamics when considering the appearance of a gradient undulator magnetic field and laser electric field in the time domain [24]. The simulation results are shown in figure 11. It can
be found from figure 11(a) that the deviation of the electron trajectory in the horizontal dimension at the exit of the modulator is about 95 μm, which fits quite well with the theoretical calculation. As the seed laser size is much larger than the beam size—i.e. about 1000 μm (rms) in the simulation—the horizontal deviation will not significantly affect the modulation process. For the case shown in figure 11(a), the maximal bunching factor decreases from 5.6% to about 5.4%, and this deviation can be easily compensated by introducing an external magnetic field of about 5.6 Gs, as shown in figure 11(b).

The sensitivity of the bunching factor to the shot-to-shot fluctuations of the laser power has also been studied by introducing random fluctuations of the laser power in the modulator. The resulting 1000 shots of the fluctuations of the 30th harmonic bunching factor are shown in figure 12. One can find that, with ±5% tolerance on the seed laser peak power, the bunching factor of PEHG can be well maintained over 5%.

6. Conclusion

In summary, intensive analytical and numerical investigations on PEHG schemes have been performed. The results demonstrate the potential of generating ultra-high harmonic radiation with a relatively small energy modulation by a single-stage PEHG. It is found that the optimized nth harmonic bunching factor of PEHG is almost determined by the maximal value of the nth-order Bessel function, which decreases as $n^{-1/3}$. The transverse dispersion-induced increase in beam size will not degrade the FEL performance when the system parameters are properly set. For a PEHG FEL operated at 8.8 nm directly from a 264 nm conventional seed laser, the numerical example demonstrates an output peak power exceeding 400 MW, which is comparable with that of the original two-stage HGHG design. Considering the ability to exploit the full electron bunch in the PEHG, the output bandwidth and the pulse energy will be significantly improved, thus leading to a FEL average brightness two orders of magnitude higher than the two-stage HGHG baseline.

In addition to the generation of fully coherent radiation, the concept of the phase-merging effect also offers a novel method for flexible beam control, which may be useful for cooling the electron beam energy spread, and for ultra-intense and ultra-short FEL pulse generation.

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References

[1] Emma P et al 2010 Nat. Photon. 4 641
[2] Ishikawa T et al 2012 Nat. Photon. 6 540
[3] Feldhaus J et al 1997 Opt. Commun. 140 341
[4] Geloni G, Kocharyan V and Saldin E 2011 J. Mod. Opt. 58 1391
[5] Amann J et al 2012 Nat. Photon. 6 693
[6] Xiang D, Ding Y, Huang Z and Deng H 2013 Phys. Rev. ST Accel. Beams 16 010703
[7] Wu J et al 2013 Proc. IPAC13 (Shanghai, China) p 2068
[8] McNeil B W J, Thompson N R and Dunning D J 2013 Phys. Rev. Lett. 110 134802
[9] Yu L H 1991 Phys. Rev. A 44 5178
[10] Yu L H et al 2000 Science 289 932
[11] Yu L H et al 2003 Phys. Rev. Lett. 91 074801
[12] Liu B et al 2013 Phys. Rev. ST Accel. Beams 16 020704
[13] Allaria E et al 2012 Nat. Photon. 6 699
[14] Stupakov G 2009 Phys. Rev. Lett. 102 074801
[15] Xiang D and Stupakov G Phys. Rev. ST Accel. Beams 12 030702
[16] Smith T, Madey J M J, Elias L R and Deacon D A G 1979 J. Appl. Phys. 50 4580
[17] Huang Z, Ding Y and Schroeder C B 2013 Phys. Rev. Lett. 109 204801
[18] Deng H and Feng C 2013 Phys. Rev. Lett. 111 084801
[19] Feng C, Huang D, Deng H et al 2013 Flexible control of the electron beam slice energy spread by using a transverse gradient magnet, unpublished
[20] Zhao Z T, Chen S, Yu L H, Tang C, Yin L, Wang D and Gu Q 2011 Proc. IPAC11 (San Sebastián, Spain) p 3011
[21] Floettmann K 2011 A space charge tracking algorithm ASTRA User’s Manual (version 3)
[22] Borland M 2000 Argonne National Laboratory Advanced Photon Source Report No. LS–287
[23] Reiche S 1999 Nucl. Instrum. Meth. A 429 243
[24] Deng H, Lin T, Yan J, Wang D and Dai Z 2011 Chin. Phys. C 35 308