Development of a Computerized Multifunctional Form and Position Measurement Instrument

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Abstract. A model machine of multifunctional form and position measurement instrument controlled by a personal computer has been successfully developed. The instrument is designed in rotary table type with a high precision air bearing and the radial rotation error of the rotary table is 0.08 μm. Since a high precision vertical sliding carriage supported by an air bearing is used for the instrument, the straightaway motion error of the carriage is 0.3 μm/200 mm and the parallelism error of the motion of the carriage relative to the rotation axis of the rotary table is 0.4 μm/200 mm. The mathematical models have been established for assessing planar and spatial straightness, flatness, roundness, cylindricity, and coaxality errors. By radial deviation measurement, the instrument can accurately measure form and position errors of such workpieces as shafts, round plates and sleeves of medium or small dimensions with the tolerance grades mostly used in industry.

1. Introduction
Form and position errors of machine components have significant influence on the quality and function of machinery products. Researchers have always paid close attention to the theoretical study on measuring and evaluating form and position errors and to the development of new form and position measurement instruments. Now, many types of advanced multifunctional form and position measurement instruments are being produced in industrialized countries, such as Talyrond 200 series made in Britain, Form measuring system FMS, MGB and Formmeter F2P made in Germany, and Formcorder EC series made in Japan, etc. But, in China, although some types of roundness measuring instruments are being produced, there seems no advanced multifunctional form and position measuring instrument that has been developed and commercialized.

On the basis of the theoretical research on the mathematical models for evaluating form and position errors[1–11], the authors of this article, have successfully developed a model machine of computerized multi-functional form and position measurement instrument. By the radial deviation measurement, the instrument can accurately measure roundness, cylindricity, planar and spatial straightness, and coaxality errors of such workpieces as shafts, round plates and sleeves of medium or small dimensions with the tolerance grades mostly used in industry.

2. Principles of structure and operation
Figure 1 shows the schematic layout of the instrument. The instrument has an air-bearing supported rotary table and a vertical sliding carriage supported by an air bearing. Driven by a synchronous motor,
the rotary table rotates at a speed of 2.5 rpm. Beneath the rotary table mounted a photoelectric encoder which rotates with the rotary table and generates a synchronous pulse and 256 sampling pulses per revolution. By means of the sampling pulses from the encoder, the number of 128 or 256 equi-spaced sampled points can be chosen, in a sampled cross-section, to the actual profile of the workpiece to be measured. Driven by a stepping motor through a pair of gears and a set of precise leading screw and nut, the carriage moves up and down along the guide of the vertical column. A commercial electro-inductive gauge, which is combined with an electro-inductive probe, is used for the instrument. The output signals from the electro-inductive gauge are input a personal computer through the signal processing circuits and an A/D converter. The signal processing circuits include mainly a precise instrumentation amplifier and a low-pass filter. The specially designed low-pass filter is a second-order Butterworth one which has five pass-band ranges, namely 0–15, 0–50, 0–250, 0–500 undulation/rev, and all-pass. The A/D converter is a 12-bit successive approximation one with a conversion error of ±1 LSB on condition that the environmental temperature is 25°C and the voltage values of the power supply are standard. Based on the mathematical models for evaluating form and position errors and the corresponding software, the personal computer can automatically process the sampled data and output the form or position errors according to the assessment methods selected.

Figure 1. The schematic layout of the instrument.

3. Mathematical models
The mathematical models have been established for assessing planar and spatial straightness, flatness, roundness, cylindricity, and coaxiality errors. Due to limited space, only the mathematical models for evaluating coaxiality errors are introduced here.

The least squares and the positioned minimum zone evaluation methods are used to assess coaxiality errors. A workpiece to be measured is set in an instrument coordinate system OXYZ whose Z-axis is coincident with the rotation axis of the rotary table, as shown in figure 2. The datum feature and the feature to be measured of the workpiece are respectively divided, in the direction of Z-axis, into a number of sampled cross-sections perpendicular to Z-axis. Then the discrete radial deviations are successively measured at a number of equi-spaced sampled points in each of the sampled cross-
sections. Assume that the positional vectors of the sampled points are denoted by \( P_i = \{ \Delta r_{ij}, \theta_{ij}, z_j \} \) for the datum feature and those for the feature to be measured \( P_I = \{ \Delta r_{IJ}, \theta_{IJ}, z_J \} \), where \( \Delta r_{ij} \) and \( \Delta r_{IJ} \) are the radial deviations measured at the sampled points, \( \theta_{ij} \) and \( \theta_{IJ} \) are the polar angles of the sampled points, and \( z_j \) and \( z_J \) are the Z-coordinates of the sampled cross-sections.

![Figure 2. The instrument coordinate system and a workpiece to be measured.](image)

(1) Least squares assessment method

Let the positional vectors of the least squares circle centers of the actual contours in the cross-sections be \( O_j = \{ a_j, b_j, z_j \} \) and \( O_J = \{ a_J, b_J, z_J \} \), respectively, for the datum feature and the feature to be measured. Then it is well known that [2]

\[
\begin{align*}
a_j &= 2 \sum_{i=1}^{n} \Delta r_{ij} \cos \theta_{ij} \\
b_j &= 2 \sum_{i=1}^{n} \Delta r_{ij} \sin \theta_{ij} \\
a_J &= 2 \sum_{I=1}^{N} \Delta r_{IJ} \cos \theta_{IJ} \\
b_J &= 2 \sum_{I=1}^{N} \Delta r_{IJ} \sin \theta_{IJ}
\end{align*}
\]

The actual datum axis is represented by the discrete points \( O_j (j = 1, 2, \ldots, m) \). Let the true reference axis \( L_1 \) be the least squares line of \( O_j (j = 1, 2, \ldots, m) \), its directional vector be \( S_1 = \{ p_1, q_1, 1 \} \), and the positional vector of the intersection point between \( L_1 \) and the coordinate plane \( XOY \) be \( A_0 = \{ \xi_0, \eta_0, 0 \} \). Then the equation of \( L_1 \) can be expressed as

\[
\begin{align*}
x &= \xi_0 + p_1 z \\
y &= \eta_0 + q_1 z.
\end{align*}
\]

In the \( j \)-th cross-section, the distance from \( O_j(a_j, b_j, z_j) \) to \( L_1 \) is

\[
e_j = [(a_j - \xi_0 - p_1 z_j)^2 + (b_j - \eta_0 - q_1 z_j)^2]^{1/2}.
\]

According to the least squares principle, it should be met that

\[
Q(\xi_0, \eta_0, p_1, q_1) = \sum_{j=1}^{m} e_j^2 \text{ minimum.}
\]

According to the least squares principle, the values of \( \xi_0, \eta_0, p_1, \) and \( q_1 \) can be determined [4].

The distance from \( O_J(a_J, b_J, z_J) \) to \( L_1 \) is given by

\[
d_J = \| (O_J - A_0) \times S_1 \| / ||S_1|| \quad J = 1, 2, \ldots, M.
\]

Then the coaxality error by the least squares evaluation is
\[ f_{LS} = 2 \times \max_j \{ d_j \} . \] (6)

(2) Positioned minimum zone assessment method

Suppose that a true datum axis \( L \) passes through the point \( A(\xi, \eta, 0) \) in the coordinate plane \( XOY \) and the directional vector of \( L \) is \( S = \{ p, q, 1 \} \). Then the equation of \( L \) is
\[
\begin{align*}
  x &= \xi + p z \\
  y &= \eta + q z.
\end{align*}
\]

The distance from \( O_j(a_j, b_j, z_j) \) to \( L \) in the \( j \)-th cross-section is
\[\varepsilon_j = \sqrt{((a_j - \xi - pz_j)^2 + (b_j - \eta - qz_j)^2)/2} .\]

Let \( F \) be the maximum value of \( \varepsilon(j=1,2,\ldots,m) \), denoted by
\[ F = \max_j \{ \varepsilon_j \} . \] (7)

It is evident that \( F \) is a function of the variables \( \xi, \eta, p \) and \( q \). In order to make \( L \) be accordant with the minimum condition, \( F \) must attain its minimal value. Hence, the following unconstrained optimization model is established
\[ \min F(\xi, \eta, p, q) \] (8)

Let the minimal point of \( F \) be \( \{ \xi^*, \eta^*, p^*, q^* \} \). Then the distance from \( O_j(a_j, b_j, z_j) \) to \( L \) in accord with the minimum condition is given by
\[ d_j^* = \| (O_j - A^*) \times S^* \| / \| S^* \| \quad J = 1,2,\ldots,M, \] (9)

where \( O_j = \{ a_j, b_j, z_j \} \), \( A^* = \{ \xi^*, \eta^*, 0 \} \), and \( S^* = \{ p^*, q^*, 1 \} \). Hence the coaxality error by the positioned minimum zone assessment is
\[ \hat{f}_{MZ} = 2 \times \max_j \{ d_j^* \} . \] (10)

It can be proved that the objective function \( F \) is a continuous, non-differentiable and convex one defined on the four-dimensional Euclidean space \( R^4 \), its minimal value is unique and any of its minimal point must be its global minimal point. The similar conclusions are true to the other mathematical models established. For the instrument, the pattern search algorithm, proposed by Hooke and Jeeves [12], is used to assess the minimum zone form and position errors.

4. Accuracy inspection and experiment results

The accuracy inspection of the instrument itself was performed by the National Metrology and Test Center of Liaoning. By inspection, the radial rotation error of the rotary table is 0.08 \( \mu m \), the straightaway motion error of the carriage is 0.3 \( \mu m \)/200 mm, and the parallelism error of the carriage’s motion with respect to the rotation axis of the rotary table is 0.4 \( \mu m \)/200 mm. By testing, the linearity error of the signal processing circuits is smaller than \( \pm 0.01\% \) of the full scale range.

Many experiments were performed for checking the measurement accuracy of the instrument. Parts of the experiment results are given here. Table 1 shows five groups of measurement results and the corresponding statistical results for roundness, cylindricity, generating line and axis straightness, and coaxality errors. Each group of results was obtained by measuring a workpiece, under one time of its setting on the rotary table, ten times repeatedly. Each of the statistical results is denoted in the form of \( X \pm 2S \), where \( X \) and \( S \) are, respectively, the average value and the standard deviation of the relevant ten measurement results. The roundness errors were measured from a standard test component with a roundness error of 12.10 \( \mu m \) as an accessory of a Talyrond 73 roundness instrument, the generating line and axis straightness errors from a shaft machined by grinding, and the cylindricity and coaxality errors from a stepped shaft machined by turning. All the experiments were made in a
laboratory without special thermostatic and vibration isolation facilities. It can be seen that the values of uncertainty are different for different items of form or position errors. Because of the effects of the straightaway motion error of the carriage and the parallelism error of the carriage’s motion with respect to the rotation axis of the rotary table, the larger values of uncertainty appeared in the measurement of straightness and cylindricity errors. It can be proved that only the radial rotation error of the rotary table affects the positions of the centers of least squares circles. Hence, the values of uncertainty are relatively smaller in the measurement of axis straightness and coaxality errors. In general, the measurement accuracy of the instrument is high enough to measure the workpieces with the tolerance grades mostly used in industry.

5. Conclusion
Since the instrument can accurately measure more items of form and position errors than the traditional roundness or cylindricity measurement instruments, it is more valuable for application.

The measurement accuracy of the instrument is high enough to measure the workpieces with the tolerance grades mostly used in industry.

The objective functions in the mathematical models, used for the instrument for assessing form and position errors by the minimum zone assessment methods, are continuous, non-differentiable and convex so that they can guarantee to provide reliable minimum zone values.

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Table 1. Parts of the experiment results to verify the instrument’s measurement accuracy.

| Sequence number | Roundness (μm) | Cylindricity (μm) | Straightness (μm) | Axis straightness (μm) | Coaxality (μm) |
|-----------------|----------------|-------------------|-------------------|-----------------------|----------------|
| 1               | 12.09          | 10.21             | 1.57              | 1.47                  | 0.80           |
| 2               | 12.13          | 10.22             | 1.52              | 1.48                  | 0.89           |
| 3               | 12.10          | 10.16             | 1.45              | 1.43                  | 0.85           |
| 4               | 12.08          | 10.24             | 1.49              | 1.42                  | 0.85           |
| 5               | 12.12          | 10.14             | 1.52              | 1.42                  | 0.86           |
| 6               | 12.06          | 10.19             | 1.56              | 1.41                  | 0.83           |
| 7               | 12.11          | 10.16             | 1.52              | 1.40                  | 0.81           |
| 8               | 12.16          | 10.19             | 1.56              | 1.47                  | 0.84           |
| 9               | 12.06          | 10.17             | 1.58              | 1.44                  | 0.85           |
| 10              | 12.05          | 10.20             | 1.49              | 1.42                  | 0.84           |

Statistical results
12.10±0.071       10.19±0.142        1.52±0.169             1.44±0.018            0.84±0.016