Edge Enhancement for Remote Sensing Image Using Norm Algorithm in $L_2$ Vector Space

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Abstract  The Householder transformation-norm structure function in $L_2$ vector space of linear algebra is introduced, and the edge enhancement for remote sensing images is realized. The experiment result is compared with traditional Laplacian and Sobel edge enhancements and it shows that the effect of the new method is better than that of the traditional algorithms.

Keywords  image edge enhancement; Householder-norm transformation; $L_2$ vector space

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Introduction

Matrix decomposition of linear algebra is applied widely in signal processing[1]. In recent years, orthogonal decomposition in $L_2$ vector space of linear algebra such as Gram-Schmidt, Givens algorithms are introduced into remote sensing data feature extraction according to the specialties of remote sensing spectral information and a fair result is derived[2,3]. In this paper, the Householder triangle decomposition algorithm is applied to the analysis of remote sensing data and keeps the norm the same.

1 Theory of Householder-norm method in $L_2$ vector space

1.1 Remote sensing data in $L_2$ vector space

The gray value recorded in the remote sensing data can be regarded as a vector. For a pixel, the dimension of the vector is equal to the number of the bands; while for a band, the dimension of the data is the number of pixels. Pixel vector can be expressed as[4]:

$$DN_y^{(i)} = [DN_{y_1}, DN_{y_2}, \ldots, DN_{y_k}]^T$$  \hspace{1cm} (1)

where $i$ and $j$ are subscripts of the pixel location; and $k$ represents dimension, which is the number of the bands. Similarly, data of the whole band can be treated as one vector:

$$DN_{p}^{(k)} = [DN_{p_1}, DN_{p_2}, \ldots, DN_{p_p}]^T$$  \hspace{1cm} (2)

where $p$ is the pixels of the full image. For example, if the size of the image is $M \times N$, so, $p = M \times N$, $k$ represents the number of the bands. The base vectors are not the same in different denotement systems. For a vector with $k$ dimension, one base vector is a unit matrix whose dimension is $k$; and if a full image is regarded as one vector, although the format of the base vector is the same, the dimension equals the size of the total pixels of the image.
1.2 Mathematics of H-N algorithm in $L_2$ vector space

Given $x = [x_1, ..., x_n]^T \in C^n$, the $L_2$ norm of vector $x$, $\|x\|_2$, is defined as [5]:

$$\|x\|_2 = (|x_1|^2 + ... + |x_n|^2)^{1/2}$$  \hspace{1cm} (3)

Householder transformation is a kind of linear transformation, put forward in the end of the 1950s by Householder, which is an efficient method for computing orthogonal matrix [6].

For vectors $a$ and $b$, reflect $a$ to $b$, getting $P_2(a)$, and then reflect $a$ to hyperplane $b^\perp$ that is perpendicular to $b$, getting another projection $P_2^+(a)$. Projection $P_2^+(a)$ is called the orthogonal projection from vector $a$ to vector $b$. Construct the subtraction of $P_2^+(a)$ and $P_2(a)$, deriving a new vector: $Q_2(a) = P_2^+(a) - P_2(a)$, and this new vector is called the Householder transformation of $a$ corresponding to vector $b$. The difference of $Q_2(a)$ and $a$ lies in the direction of their projections on vector $b$.

Householder transformation belongs to data self-adaptation transformations. The minimum norm solution is used as a restriction in the course of the transformation and this transformation is a norm preservation operator when reflecting a vector. In mathematical expressions, this is called the theory of norm invariability of the Householder transformation. Given three random vectors: $x$, $y$ and $b$, the relational expression is always satisfied:

$$\langle Q_2(x), Q_2(y) \rangle = \langle x, y \rangle$$  \hspace{1cm} (4)

Since the covariance is composed of the sum of the results, Eq.(4) actually develops the covariance invariability of $Q_2$, which is the norm invariability of the Householder transformation.

2 Processing procedure and result of remote sensing data H-N transformation

For $v \in R^n$, a nonzero vector, the $n \times n$ matrix $H = I - \frac{2vv^T}{v^Tv}$ is called the Householder matrix (or the Householder reflection), vector $v$ is called the Householder vector.

According to the definition of $H$, we can get:

$$Ha = a - 2v\langle v, a \rangle / v^Tv = a^*$$  \hspace{1cm} (5)

$$HH^T = (I - 2vv^T) \cdot (I - 2vv^T)^T = I - 4vv^T + 4vv^Tv^Tv = I$$  \hspace{1cm} (6)

From the above definition, we can see that $H$ is determined by the unit vector $v$. In $n$ dimension linear space, for any vector $a$, select a unit vector $I$, let $a^* = sI$, the corresponding mirror unit vector is:

$$v = \frac{1}{d}(a - a^*) = \frac{1}{d}(a - sI)$$  \hspace{1cm} (7)

where $d = -\text{sgn}(a^\perp \cdot I) \cdot \sqrt{a^\perp \cdot a}$

$$d = \sqrt{(a \cdot sI)^T \cdot (a - sI)} = \sqrt{2(s^2 + \|a^\perp \cdot I\|^2)}$$  \hspace{1cm} (8)

The key of the Householder transformation is to find a $m \times m$ mirror matrix $H$ for a $m \times n$ matrix, which satisfies the equation:

$$H \cdot A = \begin{bmatrix} R_{mn} \\ 0 \end{bmatrix} \text{, \hspace{1cm} (9)}$$

According to the mirror transformation principle, switch $A$ column by column. For

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$  \hspace{1cm} (10)

Given $a^{(0)} = (0, 0, \cdots, a_{i_1}^{(0)}, a_{i_2}^{(0)}, \cdots, a_{m_i}^{(0)})^T$ whose length is $s_i = \|a^{(0)}\|$. Suppose the mirror of $a^{(0)}$ is $a^{(0)r}$, and the corresponding unit vector is $I^{(i)}$, which $I^{(i)} = (0, 0, \cdots, 1, 0, \cdots, 0)^T$, then:

$$v^{(i)} = \frac{1}{d_i} (a^{(0)} - a^{(0)r}) = \frac{1}{d_i} (a^{(0)} - s_iI^{(i)})$$  \hspace{1cm} (11)

where $d_i = \|a^{(0)} - s_iI^{(i)}\|$. The corresponding mirror mapping matrix is $H^{(i)} = I - 2v^{(i)}(v^{(i)})^T$.

Left multiple $A^{(i)}$, getting:

$$H^{(i)} \cdot A^{(i)} = \begin{bmatrix} s_1 & \cdots & a_{i_1}^{(i)} \\ 0 & s_2 & \cdots & a_{i_2}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_i & \cdots & a_{i_m}^{(i)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & a_{m_i}^{(i)} \end{bmatrix} = A^{(i+1)}$$  \hspace{1cm} (12)

When column $n$ of $A$ matrix completed the mirror transformation, we have $H = H^{(n)} \cdot H^{(n-1)} \cdot \ldots \cdot H^{(1)}$.

Landsat TM image in the northern part of Beijing
City acquired in May 1, 2003 was selected as an example in this research. Fig.1(a) and 1(b) are the original image and H-N transformed image, respectively (only the fifth band was selected here). From the image, we can see that the edge information of the original image was greatly emphasized by the Householder transformation.

![Fig.1 Original image of Landsat ETM in May 1, 2003 and H-N transformed image](image)

In order to illustrate the effect of the Householder algorithm, the traditional Laplacian edge enhancement and Sobel filter were also carried out and the results are in Fig.2(a) and 2(b).

The templates of Laplacian edge enhancement was a $3 \times 3$ kernel:

$$L = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

(13)

![Fig.2 Laplacian and Sobel transformed results of original image](image)

From Fig.2, the Householder transformation result obtains delicate edge information. Laplacian and Sobel operation results are not obvious in the byroad in the southern part of the image while the Householder transformation result is apparent. As a whole, the Householder transformation can extract tiny edge information. Fig.3 is the profile of the original image, the Householder transformed result, and Laplacian and Sobel edge enhancement results, and it is located at the $A-B$ in Fig.1. The abscissa represents pixel numbers, and 400 pixels were selected; the ordinate represents DN value of the image. From Fig.3, the Householder transformed result, the original image and Laplacian edge enhancement result are symmetrically distributed around 150. In the neighborhood of 100th pixel and the 290th pixel, the change is subtle in the original image, but the edge enhancement results are not the same. In the neighborhood of the 100th pixel, the results of the Householder transformation and Sobel are better than that of the Laplacian, but in the neighborhood of the 290th pixel, the results of the Householder transformation is better than that of both the Laplacian and Sobel.

![Fig.3 Profile of original image and transformed results](image)

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