Five Methods of Exoplanet Detection

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Abstract. The study of exoplanetary systems can help us understand the formation and evolution of the solar system itself and search for terrestrial planets in the habitable and extrasolar lives in exoplanetary systems. Exoplanets have become an important area of astrophysics in the last two decades. This paper reviews five different methods to detect exoplanets, including direct imaging, astrometry, radial velocity, transit event observation, and microlensing. These approaches could expand the sample size of exoplanets and further our understanding of the types, formation and evolution of exoplanets.

1. Introduction

Whether there are other Earth-like planets with life in the universe or whether there are exoplanets has always been attracting the attention of researchers. It was not until 1995 that a paper published by Mayor & Queloz in Nature changed people’s imagination of exoplanets, and researchers discovered a planet with a minimum mass of 51% of the mass of Jupiter through long-term measurement of the radial velocity of the sun-like star 51 Peg [1]. This discovery has started the prelude to the study of extrasolar planets. In addition, as early as 1992, Wolszczan and Frail detected three planetary companion objects orbiting the pulsar PSRB1257+12 [2]. However, due to the extremely strong magnetic field of the host star, the surrounding planets are not suitable for reproducing life like the Earth. On the other hand, these planets may be formed by the remains of the precursor stars that formed pulsars after the explosion, rather than formed together with the precursor stars or after the precursor stars entered the main sequence stage.

After just a few decades, by March 21, 2021, 4699 exoplanets have been confirmed (www.exoplanet.eu). Before 2009, the launch of the Kepler Space Telescope by the National Aeronautics and Space Administration (NASA), the exoplanets were detected dominantly by the radial velocity method, while after the release of the data from the Kepler space survey, the detection was immediately dominated by the predation event observation method. All this is attributed to the superb design of the Kepler Space Telescope, which has unprecedented pointing accuracy and photometric accuracy at the scale of one millionth (ppm), thus ensuring the ability to detect terrestrial planets and even smaller exoplanets. The Transiting Exoplanet Survey Satellite (TESS), which was launched aboard a SpaceX Falcon 9 rocket on April 18, 2018, is the next step in searching for planets beyond our solar
system, including the search for bodies that could support life. TESS aims to survey 200,000 of the brightest stars near the sun to search for exoplanets by catching transits. At present, the extrasolar planet detection technology covers not only the pulsar timing method and the radial velocity method that detected the first exoplanets. It also covers the newly developed gravitational microlensing method, transit event observation method, direct imaging method, and astrometry. These detection methods have been widely used and improved in the ongoing ground-based and space-based exoplanet search projects [3, 4]. In the following sections, we will describe each of these detection methods.

2. Direct Imaging Method
Exoplanet imaging usually means the point source image formed by the light of the host star reflected in target source detection and is different from the resolution of the planet surface. When the radius of the planet is large and the orbit radius around the host star is relatively large, the method of adaptive optics and coronagraph direct imaging can be used to detect exoplanets. This is because, in this case, the planet is bright enough to be detected, and the host star is far enough away from the planet for the telescope to be able to distinguish it. In 2004, Chauvin et al. detected the exoplanet 2M1207b discovered with the direct imaging method for the first time by using Chile's Paranal Observatory's Very Large Telescope [5] (as shown in Figure 1). The mass and effectiveness and temperature are predicted as $5 \pm 2 \, M_{\text{Jup}}$ and $1250 \pm 200 \, \text{K}$. The brightness ratio between the planet and the host star depends on the planet's size, the distance between the planet and the star, and the scattering characteristics of the planet's surface. The brightness ratio between the planet and the host star at wavelength $\lambda$ is [6]:

$$\frac{f_\alpha(\alpha, \lambda)}{f_\lambda(\lambda)} = p(\lambda) \left(\frac{R_p}{a}\right) g(\alpha),$$

where $p(\lambda)$ is the geometric albedo, and $g(\alpha)$ is the phase function of the planet. The brightness ratio between the planet and the host star is usually very small. For example, the brightness ratio between a Jupiter-like planet and a sun-like star in a system with a distance of 11 pc and the angular distance of 0.52 arcsec from us is $10^{-8}$ to $10^{-9}$.

Figure 1. The CCD frame of 2M1207b [5].
3. Astrometry Method

Gatewood expounded the idea of using astrometry to detect exoplanets [7]. As shown in Figure 2, until 2010, Muterspaugh et al. detected the exoplanet HD 176051b by astrometry for the first time [8]. Astrometry is to detect planets by directly detecting the position of the host star perturbed by the planet’s gravitational force, so it can get the planet’s mass and orbital inclination [3]. The host star’s orbit around the center of mass of the planet and the star is projected onto the sky plane usually as an ellipse, and the angular semi-major axis $\alpha$ of the ellipse is [6]:

$$\alpha = \frac{M_p}{M_s + M_p} \cdot \left( \frac{a}{1\text{AU}} \right) \left( \frac{d}{1\text{pc}} \right)^{-1} \text{arcsec}, \quad (2)$$

where $a$ is the semi-major axis of the planetary orbit, and $d$ is the distance from the system to the observer. The observable $\alpha$ is directly proportional to the mass of the planet and the semi-major axis of the orbit and inversely proportional to the distance. From the above derivation, the observation of at least one complete orbit is the key to the astrometric detection of exoplanets. More periodic observations can improve the confidence of the method.

![Figure 2. The perturbation of the position of primary star by the exoplanet HD 176051b detected by astrometry for the first time [8].](image)

4. Radial Velocity Method

So far, about a fifth of all known exoplanets has been discovered by the radial velocity method. For planets with large masses and short periods, the radial velocity method is applicable. The essence of this method is to detect exoplanets by measuring the periodic variation of the star relative to the center of mass of the system. Figure 3 shows the radial velocity of the first solar-like exoplanet, 51 Peg [1]. Currently, the detection accuracy of the radial velocity method has exceeded 1 m/s, which can detect short period terrestrial planets surrounding M-type dwarfs [9,10].

In general, the orbits of the planets are assumed to be elliptical. The reasons for this assumption can be based on the orbits of the planets in our solar system and the current study of other exoplanets. One can use the center of mass of the star and the planet as the origin and the long axis of the planet’s orbit as the x-axis to establish the Cartesian coordinate system [6]. As shown in Figure 4, the position of the planet is...
\[ x = r \cos \nu \]  \hspace{1cm} (3)
\[ y = r \sin \nu \]  \hspace{1cm} (4)
\[ z = 0 \]  \hspace{1cm} (5)

where \( r \) is the distance from the planet to the center of mass. Based on the elliptic equation, \( r \) can be replaced by:
\[ r = a \frac{1 - e^2}{1 + e \cos \nu} \]  \hspace{1cm} (6)

where \( \nu \) represents the angle between the line between the planet and the star and the line between the orbit pericentre and the center of mass, \( a \) is the semi-major axis of the planet’s orbit while \( e \) is the eccentricity of the planet’s orbit. In addition, \( \nu \) can be transformed into a function of time \( t \) with the following equation:
\[ \tan \frac{\nu}{2} = \sqrt{\frac{1 + e}{1 - e}} \tan \frac{E}{2} \]  \hspace{1cm} (7)
\[ M = E - e \sin E \]  \hspace{1cm} (8)
\[ M = \frac{2\pi}{P} (t - t_p) \]  \hspace{1cm} (9)

\( t_p \) as a reference time, which indicates the time when the planet passes through the pericentre while \( P \) is the orbital period of the planet.

Figure 3. Curve of the radial velocity of 51 Peg measured by Mayor & Queloz [1].
In actual observations, the orbit of a planet one observes is a projection of the planet's orbit in the celestial coordinate system. As shown in Figure 5, the line of sight of the observer is parallel to the z-axis, and a Cartesian coordinate system is built with the center of mass as the origin, so the position of the planet can be characterized as [6]:

\[
\bar{x} = r [\cos \Omega \cos (v + \omega) - \sin \Omega \sin (v + \omega) \cos i], \quad (10)
\]

\[
\bar{y} = r [\sin \Omega \cos (v + \omega) + \cos \Omega \sin (v + \omega) \cos i], \quad (11)
\]

\[
\bar{z} = rsin(v + \omega)sin i. \quad (12)
\]

Take the derivative of time over \( z \) to get the radial velocity of the planet, multiplied by the mass ratio of the planet to the star \( \frac{M_p}{M_p + M_*} \) that can get the radial velocity \( V_r \) of the star around the barycentre of the system is obtained:

\[
V_r = V_0 + K[\cos(v + \omega) + e \cos \omega], \quad (13)
\]

\[
K = \left( \frac{2\pi G}{P} \right)^{1/3} \frac{M_p}{(M_p + M_*)^{2/3}} \frac{\sin i}{\sqrt{1-e^2}}, \quad (14)
\]

where \( V_0 \) is the radial velocity of the barycentre, \( G \) is a constant of the gravitation. Because \( M_p \ll M_* \), approximating the expression to the radial velocity, the minimum mass of a planet can be expressed (in terms of the mass of the earth):

Figure 4 Schematic diagram of tracks of exoplanet and its corresponding parameters[11].
6. Transit event observation method

More than 3,000 exoplanets have been discovered by detecting signals from periodic planetary transits. In this method, the observer, the planet, and the host star are close to collinear. When the planet is between the observer and the host star, the planet will pass through the front of the host star, causing part of the star’s light to be blocked by the planet. This blocking will be shown in the photometric measurement of the star. This is called a transit event. Figure 6 shows the eclipse light curve of the first exoplanet observed by Charbonneau et al. using the eclipse event observation method [12]. By analyzing the solar eclipse curve of the planet, not only the radius of the planet but also the inclination of the plane of the orbit and the long half axis of the orbit can be obtained.

In the above part, we obtained the line of sight velocity dependency on the orbit of the parameters and the planet’s mass. Based on the above relation, we can easily obtain the partial relation of the eclipse light curves. The plane formed by- xoy is the sky plane, and the distance from the trajectory of the planet in this plane to the origin multiply by a factor of $1+M_p/M^*$ is the distance $r_{sky}$ from the center of the planetary disk to the center of the stellar disk [6]:

$$r_{sky} = \left(1 + \frac{M_p}{M^*}\right) r \sqrt{1-e^2} \sin\omega \sin^2i$$

(16)

If the planet transit event occur, the required $r_{sky}$ minimum value $r_{sky\ min}$ satisfies $r_{sky\ min} < R_p+R_*$. Taking $r_{sky}$ derivative $v$ and making it zero(0), then we can obtain $r_{sky}$, and take $r_{sky}$ satisfactory needed condition:

$$\frac{a^2(1-e^2)^2}{(1+e \cos\omega)} \left\{(1+e \cos\omega)\sin(2\gamma+2\omega)\sin^2i-2e \sin\psi \sin^2(\nu+\omega)\right\}=0.$$  

(17)
Kipping obtained $r_{sky \, min}$ corresponding true anomaly $v_{min}$ as [6,13]:

$$v_{\min} = \frac{n}{2} - \omega + \sum_{k=1}^{n} \eta_k,$$

(18)

$$\eta_1 = \left( \frac{e \cos \omega}{1 + e \sin \omega} \right) \left( \cos^2 i \right)^{\frac{1}{3}},$$

(19)

$$\eta_2 = \left( \frac{e \cos \omega}{1 + e \sin \omega} \right) \left( \frac{1}{1 + e \sin \omega} \right) \left( \cos^2 i \right)^{\frac{2}{3}},$$

(20)

$$\eta_3 = \left( \frac{e \cos \omega}{1 + e \sin \omega} \right) \left( \frac{-6(1 + e \sin \omega) + e^2 \cos^2 \omega (-1 + 2 e \sin \omega)}{6(1 + e \sin \omega)^3} \right) \left( \cos^2 i \right)^{\frac{3}{3}}.$$  

(21)

For the transit eclipse system, the planet’s orbital plane is straightly close to the line direction, thus the last term of sum in Equation 18 is quite a small quantity. In the actual transit eclipse curve dealing process, we can ignore it. Take the approximate result of Equation 18 to Equation 16 to obtain $r_{sky \, min}$:

$$r_{sky \, min} = \left( 1 + \frac{M_p}{M_*} \right) a \left( 1 - e^2 \right) \cos i,$$

(22)

In the field of exoplanet research, the collision parameter $b$ is usually used to characterize the planet’s eclipse trajectory distance and the center of the star’s disk as shown in Figure 7, and $b$ is defined as:

$$b = \frac{r_{sky \, min}}{R_*},$$

(23)

Due to the duration of the eclipse $T_1 \leq p$, therefore we can reasonably assume that the planet is moving at a customary speed when the eclipse occurs, so the distance between the planet and star remains unchanged. The distance between the planet and star $\bar{r}$ is well approximated:

$$\bar{r} = a \frac{1 - e^2}{1 + e \sin \omega}.$$  

(24)

Then, combined with Kepler’s Second law,
For Equation 25, the light becomes extremely small (corresponds to $r_{sky\ min}$) $t = T_{\ min}$ integral to the fourth touch of the planets and stars $t = T_4$. One can obtain:

$$a = \frac{(1+e \sin \omega)^2}{(1-e^2)^2}.$$  \hspace{1cm} (26)

$$v_4 - v_{\ min} = \frac{2\pi a}{\alpha T_4 (T_4 - T_{\ min})}.$$  \hspace{1cm} (27)

Among them, $v_4$ shows the true apse angle of the planet at the fourth contact of the planet and the star. The fourth contact and the time the light becomes extremely small, the difference is exactly half of the duration $T_{14}$. This time, the distance between the planet and astrolabe is $R_p + R_*$. Therefore, the following relationship exists,

$$R_p + R_* = \left(1 + \frac{M_p}{M_*}\right) \frac{a(1-e^2)}{1+e \sin (\omega+\omega_0)} \sqrt{1-\cos^2 \left(\frac{\omega}{\pi} T_{14}\right)} \sin^2 i.$$  \hspace{1cm} (28)

For a system composed of a Jupiter-like planet and a sun-like star, it is approximately 0.001. For a system composed of a red short star and a Jupiter-like planet, $M_p/M_*$ is approximately 0.01. So in the later calculations, $M_p/M_*$ will take two for approximation, then the physical parameters and model parameters have the following relationship [6]:

Planet to star radius ratio

$$\frac{R_p}{R_*} = \sqrt{\Delta F}.$$  \hspace{1cm} (29)

The inclination of the planet’s orbit

$$\sin i = \sqrt{\frac{b^2 \sin^2 \left(\frac{\omega}{\pi} T_{14}\right)}{\sqrt{b^2 \cos^2 \left(\frac{\omega}{\pi} T_{14}\right) - \Delta F}^2}}.$$  \hspace{1cm} (30)

The semi major axis of the planet’s orbit in units of the star radius
\[ a = \frac{1 + e \sin \omega}{1 - e^2} \sqrt{\frac{\left(1 + \sqrt{\Delta F} \right)^2 - b^2 \cos^2 \left(\frac{\pi}{P_{T14}} T_{14}\right)}{\sin^2 \left(\frac{\pi}{P_{T14}} T_{14}\right)}}. \quad (31) \]

The average density of stars can be derived as,

\[ p* = \frac{3\pi}{GP^2} \left(\frac{a}{R*}\right)^3 = \frac{3\pi}{GP^2} \left(\frac{1 + e \sin \omega}{1 - e^2}\right)^3 \left[\frac{\left(1 + \sqrt{\Delta F} \right)^2 - b^2 \cos^2 \left(\frac{\pi}{P_{T14}} T_{14}\right)}{\sin^2 \left(\frac{\pi}{P_{T14}} T_{14}\right)}\right]. \quad (32) \]

6. Gravity Microlensing Effect

Gravity lensing effect refers to that in accordance with the order of observer, lens celestial body and background light source, when the three are close to collinear, the light of background light source will be deflected in the process of reaching the observer due to the gravitational action of lens celestial body, thus producing an effect similar to the light gathering effect of the lens. If the lenticular body is the host star of an exoplanet, the planet will have a similar lensing effect on the light from the background source. The gravitational lensing effect produced by planets is weaker than the gravitational lensing effect produced by other massive bodies, called the microgravitational lensing effect. Figure 8 shows the gravitational lensing event of the exoplanet system OGLE-2005-BLG-390. This study demonstrates the feasibility of microlensing for exoplanet detection [15]. This detection method can be used to detect planets with earth-like mass. A disadvantage of gravitational lensing is that it usually lasts for a short time, usually only a few days to a few weeks, and is rarely observed again, so it cannot be cross-verified with other detection methods.

Figure 8. The light curve of gravity microlensing effect of OGLE-2005-BLG-390 [15].

7. Conclusion

In this review, we review the current methods of exoplanet detection, including direct imaging, astrometry, radial velocity, transit event observation, and microlensing (we do not review pulsar timing). Compared with the other methods, we give a more detailed mathematical and physical description of methods radial velocity, transit event observation, which also found the vast majority of exoplanets. With these methods, researchers have already found thousands of exoplanets, paving the way for further efforts to find habitable exoplanets.
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