Lambda baryons from lattice QCD - from strange to charm -

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Our target is $\Lambda$

**Negative parity $\Lambda$ (1405)**

- difficulty in mass reproduction in quark models
- possibility of meson-baryon ($NK, \pi\Sigma$) molecule
- NK strong attraction
- Has been problematic in Lattice QCD analyses
- Recently $\Lambda(1405)$ was identified in lattice QCD calculation PRL 108, 112001 (2012)

**Spin-orbit partner of $\Lambda$**

- Properties of spin 3/2 $\Lambda$, which is the $\Lambda(1405)$'s partner

**$\Lambda(1405)$'s structures**

- Flavor structures
  - Internal wavefunctions

**$\Lambda c$'s properties**

- properties of $\Lambda c$, where the strange is replaced with the charm
- What does the HQ symmetry cause?
Recent lattice QCD study
Recent Lattice study of $\Lambda$

Isolating the $\Lambda(1405)$ in Lattice QCD, Adelaide group; PRL 108, 112001 (2012)

2+1 conf. by PACS-CS
Pion mass $\rightarrow 156 \sim 702$ MeV

**Brief summary**

- Correlation matrix analyses with many types of interpolating fields
- lowest state lies between $KN$, $n\Sigma$ thresholds
  $\rightarrow$ Same order as the real world
  $\rightarrow$ Claim to have identified $\Lambda(1405)$ signal

Still too heavy

- Adjusted the valence s-quark mass so as to correctly reproduce the K-meson mass (partially quenching)

Gets experimental value

- lowest $\rightarrow$ singlet dominant
  2$^{nd}$ lowest $\rightarrow$ octet dominant
Recent Lattice study of $\Lambda$

Lattice QCD Evidence that the $\Lambda(1405)$ Resonance is an Antikaon-Nucleon Molecule
Adelaide group; Phys.Rev.Lett. 114 (2015) 13, 132002

2+1 conf. by PACS-CS
Pion mass $\rightarrow$ 156~702 MeV

Electromagnetic properties

- Strange-quark magnetic moments in $\Lambda(1405)$ vanishes.

$\rightarrow$ The strange quark is confined in the spin-0 Kaon in $\Lambda(1405)$

There are several issues to be clarified, but the signal can be the $\Lambda(1405)$. $\hspace{1cm}$

Physical point
Why $\Lambda c$?
Our target is $\Lambda$

**udS** – $\Lambda$

$m_u = m_d < m_s$

SU(3)F symmetry

**udC** – $\Lambda$

$m_u = m_d \ll m_c$

Heavy quark spin symmetry appears
Spin-spin interactions become weak
Heavy quark spin decouples and is irrelevant
Largely broken SU(3)F symmetry
Two excitation modes may decouple
Hadron spectra would be much simpler

Manifestation of Diquark DOF?

Replace S with C
Diquarks

Two diquark motion (\(\rho\) and \(\lambda\) modes) may decouple in heavy baryons

\[ \text{ρ-mode excitation Diquark's relative motion} \]

\[ \text{λ-mode excitation Diquark's CM motion} \]

Then, excited \(\Lambda c\) spectra can be simply explained in terms of diquarks (?)

\[ \text{Λc}(2595) \]

\[ \text{Λc}(2286) \]
Diquarks

\[ m_u = m_d < m_s \]

SU(3)F symmetry

Replace S with C

\[ m_u = m_d \ll m_c \]

HQ symmetry

\( \Lambda(1670) \)

Octet? \( \Lambda(1405) \)

Singlet? \( \Lambda(1116) \)

Octet

Then, what is the relationship between uds-\( \Lambda \) and udc-\( \Lambda \) ?

→ Internal structure change
→ Manifestation of diquark’s degrees of freedom

\( \Lambda_c(2595) \)

ρ-excitation?

\( \Lambda_c(2286) \)

λ-excitation?
Strategies
Our strategy

We investigate **masses and flavor structures** of udQ-Λ baryons

We investigate the Q-dependences of

1. Λ mass spectra
2. Internal flavor structures

Interpolation using lattice QCD by changing Q-quark mass
Our strategy

-- Lattice QCD setups --

2+1 gauge configuration by PACS-CS
Iwasaki gauge action, Wilson quark action
$32^3 \times 64$, $a \approx 0.1$ fm, cut off $\sim 2.2$ GeV

→ Well reproduces light hadron mass spectra

We do not employ Relativistic Heavy Quark actions

→ cut off of 2.2 GeV may be insufficient for $m_Q > 1$ GeV
→ not good, but use a common quark action for $u, d, Q$ quarks
  in order to see the internal structure changes
Our strategy

-- Hadronic operators --

We employ operators classified in terms of Cubic group irreducible rep. → 4x4 correlation matrix (We can extract ground ~ 3rd excited states.)

u, d, Q quark operator sizes are the same.
→ Operator mixing vanishes in the flavor-SU(3) limit (\(\mu = m_d = m_Q\))
→ Easy to see the flavor structures of \(\Lambda\) particles

PRD72 (2005) 074501
LHPC group

Operator examples

| \(\Psi_{\tilde{u}, \tilde{d}, \tilde{q}}\) | \(\Psi_{\tilde{u}, \tilde{d}, \tilde{q}}\) |
|-----------------|-----------------|
| \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) | \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) |
| \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) | \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) |
| \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) | \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) |
| \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) | \(\Lambda_{1/2} + \Lambda_{1/2} + \Lambda_{3/2} + \Lambda_{3/2}\) |

\(1/2\) octet (octet-1)
\(1/2\) octet (octet-2)
\(1/2\) octet (octet-3)
\(1/2\) singlet
3/2 octet
Correlation matrix analyses

-- Mass and flavor structures --

2pt cross correlator

\[ \langle \Lambda^i(T)\Lambda^j(0) \rangle = \langle 0 | \Lambda^i e^{-HT} \Lambda^j | 0 \rangle = \langle 0 | \Lambda^i | k \rangle \exp(-E_k T) \langle k | \Lambda^j | 0 \rangle \]

\[ = c^T \Lambda(E_k) c \]

\[ \Lambda(E_k) = \text{diag}(-E_1 T, -E_2 T, \ldots) \]

Eigenvector of correlation matrix \( \rightarrow \) operator overlaps with the state

Eigenvalue of correlation matrix \( \rightarrow \) mass of the state

\[ \langle 0 | \Lambda^i | \text{target state} \rangle \]

We can investigate internal structures.
Mass spectra
Mass spectra

UDS

½ negative $\Lambda$ channel

UDC

½ negative $\Lambda_c$ channel

Spectra are similar in $\Lambda$ and $\Lambda_c$ channel

Still heavier than 1405 MeV
→ But very close

2$^{\text{nd}}$ state’s behavior is a bit different from the $\Lambda_s$ channel
→ Difference in internal structures
Mass spectra

Hadron mass dependences on Q-quark mass

Λc(∼2950)
Λc(∼2900)
Λc(2595)

Λ(1800)
Λ(1670)
Λ(1405)
Flavor structures
We consider “flavor symmetry” for $u$, $d$, $Q$ quarks. (even if $Q=\text{charm}$, we classify states still in terms of $\text{SU}(3)$)

**Singlet component**

$C_s = \langle \Lambda | O_s | \text{vac} \rangle$

$O_s = \frac{1}{\sqrt{3}} (u(dQ) + d(Qu) + Q(ud))$

**Octet component**

$C_o = \langle \Lambda | O_o | \text{vac} \rangle$

$O_o = \frac{1}{\sqrt{6}} (u(dQ) + d(Qu) - 2Q(ud))$
Flavor structures

Lowest 3 states in ½-channel

$\Lambda(1405)$
$\Lambda(2595)$
$\Lambda(1670)$
$\Lambda(1800)$
$\Lambda(\sim 2900)$

$\Lambda(\sim 2950)$

Preliminary
Flavor structures

Lowest 3 states in $\frac{1}{2}$- channel

Singlet $\rightarrow$ equally mixed

Octet dominant

Singlet amplitude for the 0th - ■
Octet amplitude for the 0th - ○

Singlet amplitude for the 1st - ■
Octet amplitude for the 1st - ○

Singlet amplitude for the 2nd - ■
Octet amplitude for the 2nd - ○

uds $\Lambda(1405)$ \hspace{1cm} udc $\Lambda(2595)$
uds $\Lambda(1670)$ \hspace{1cm} udc $\Lambda(\sim 2900)$
uds $\Lambda(1800)$ \hspace{1cm} udc $\Lambda(\sim 2950)$

The ratio $1/\sqrt{2} : 1/\sqrt{2}$ implies 50:50 mixture

preliminary
Flavor structures to Diquark picture

SU(3) wave functions can be expressed in terms of diquark wave functions

\[ R_\rho \rightarrow \rho \text{ mode (diquark p-wave excitation)} \]
\[ R_\lambda \rightarrow \lambda \text{ mode (diquark's CM p-wave excitation)} \]

**ORBITAL**

**SPIN**
\[ \chi_\rho \rightarrow \text{Diquark has spin 1 (total 1/2)} \]
\[ \chi_\lambda \rightarrow \text{Diquark has spin 0 (total 1/2)} \]
\[ \chi_s \rightarrow \text{Diquark has spin 1 (total 3/2)} \]

**singlet**
\[ |\Lambda ; 1 \rangle = \frac{1}{\sqrt{2}}(R_\lambda \chi_\rho - R_\rho \chi_\lambda) \]
\[ \lambda\text{-mode} - \rho\text{-mode} \]

**octet**
\[ |\Lambda ; 8 \rangle = \frac{1}{\sqrt{2}}(R_\lambda \chi_\rho + R_\rho \chi_\lambda) \]
\[ \lambda\text{-mode} + \rho\text{-mode} \]
\[ |\Lambda ; 8 \rangle = \frac{1}{\sqrt{2}}(R_\rho \chi_s) \]
\[ \rho\text{-mode} \]
Flavor structures to Diquark picture

Spin $\frac{1}{2}$ negative parity channel

$\Lambda(1405)$
Singlet dominant

$\Lambda(1670)$
Octet dominant

$\Lambda(1800)$
Octet dominant

$\Lambda_c(\sim 2900)$
$\rho$-mode dominant (spin total 3/2)

$\Lambda(2595)$
$\lambda$-mode dominant

$\Lambda_c(\sim 2950)$
$\rho$-mode dominant

$d$S $\rightarrow$ $\Lambda$
$m_u = m_d < m_s$
SU(3)F symmetry

Replace S with C
udC $\rightarrow$ $\Lambda$
$\Lambda_c(\sim 2950)$
$\rho$-mode dominant

Broken SU(3)F

Diquark DOF seem to appear around $m_Q \sim 700$ MeV
Flavor structures to Diquark picture

Spin $\frac{1}{2}$ negative parity channel

Replace S with C

$\Lambda(1405)$
- Singlet dominant

$\Lambda(1670)$
- Octet dominant

$\Lambda(1800)$
- Octet dominant

$\Lambda_c(\sim 2595)$
- $\rho$-mode dominant

$\Lambda_c(\sim 2950)$
- $\rho$-mode dominant (spin total 3/2)

$\Lambda_c(\sim 2900)$
- $\lambda$-mode dominant

$\Lambda$ cannot see

$m_u = m_d < m_s$
- SU(3)$F$ symmetry

$m_u = m_d \ll m_c$
- Broken SU(3)$F$

 SU(3)$F$ symmetry

$m_u = m_d < m_s$

SU(3)$F$ symmetry

Broken SU(3)$F$

$O:S=1:1$

$O:S=1:0$

$O:S=1:-1$

Possibility of level crossing (or misidentification?)

Replace S with C
Flavor structures to Diquark picture

Level crossing occurs?
We investigated the flavor structure of the low-lying Λ’s in lattice QCD.

Mass

The mass spectra of Λc and Λs are similar though their flavor structures are completely different.

Flavor structure

Ground state: (UDS) singlet dominant state $\rightarrow$ (UDC) $\lambda$-mode excitation

Excited state: (UDS) octet dominant state $\rightarrow$ (UDC) $\rho$- and $\lambda$-mode excitations

Well classified in terms of diquark excitations above (current) $m_Q > 700$ MeV

Future work?

Other channels?
More sophisticated analysis on the internal structures of Λ baryons