Information Mechanics: Theory, Hypothesis, Evidence & Insight

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Abstract

For many years, major theoretical efforts have been underway to unify gravity and the other forces. Today I suggest a different direction for unification; that of the unification of action and entropy. Exploring a few different derivations of this governing thought, we unveil new physics in that the variance of a particle is dependent on its absolute velocity. While this theory has multiple simple arguments that point in the same direction, it might seem speculative. Thus, a testable hypothesis is put forward that “particles move via the discrete Bernoulli process” which is a core assumption of these derivations. Still anticipating resistance to exploring this line of thinking, an experiment to measure the variance of jitter was undertaken. The experiment has collected 2 years of continuous jitter on 2 clocks and calculated the Fourier Transform of its magnitude. Three spikes in the data, which combine to 19+ sigma away from current theory, are not only predicted by our hypothesis, but the best fit parameters of the hypothesis suggest the Earth is moving with the same velocity as the Planck collaboration measured Earth is moving in the Cosmic Microwave Reference Frame. Lastly, this line of thinking, where one explores the variance instead of the mean value of a particle’s location, is worthwhile as it gives insight into the original desire to unify gravity with quantum mechanics.

1. Summary

It is quite humbling to study the latest literature that outlines current pursuits to merge quantum mechanics with relativity (both special and general) [1,2,3,4]. However, the purpose of this article is not mathematical physics, nor a historical accounting of scientific advancement; rather we suggest taking a step back, examining how we got here, and seeing what is possible with a renewed perspective. While this article challenges a few sacred concepts, like breaking from unitarity & relativity, I ask the reader to suspend their disbelief for the moment, as new avenues are opened, new insights are made, and most importantly as the concepts are tested and verified with data [5].

Let us begin by looking, not at the unification of quantum mechanics and gravity, but rather a different type of unification, that of action and entropy. Action and entropy are two major concepts in physics which if unified could bring unknown and unexpected insights. A couple examples shown below are 1) the Gaussian wavefunction carries one quantum of entropy exactly equal to one natural unit, $\ln(e)$, 2) an explanation of the two-slit experiment, and 3) an independent derivation of the entropy of a black hole.

As Carl Sagan popularized the phrase, “Extraordinary claims require extraordinary evidence,” it is prudent to seek experimental evidence of this governing thought. That is why we propose a hypothesis that “particles move via the discrete Bernoulli process”, which is an assumption of the unification. This hypothesis uncovers unexpected new physics, which deviates from relativity, in that the variance of a particle’s location is dependent on its absolute velocity and that a measurement of this velocity is possible without looking outside.

Today we share the outcome of a 19+ sigma experiment which is explained by our hypothesis. While our hypothesis is just one possible explanation, we are able to calculate the speed and direction of Earth in the preferred frame assuming the hypothesis is true. All three of the minimum RMSE parameters of the hypothesis, found using the phase and magnitude of the experiment, suggest the Earth’s reference frame is the same as the Cosmic Microwave Reference frame.

To finish off, we use the variance to show how the solutions to quantum diffusion are the same as the solutions to Friedmann’s equations. What results is a new particle, dubbed “dark particle”, which is a black hole with the reduced Planck mass and arbitrary temperature. These finding should provide useful insights into the vast ongoing research to unify gravity with quantum mechanics.

1.1. Setting the Landscape

To set the context for our idea, let us go back almost 4 centuries ago, when Galileo introduced the concept of relativity and set the framework for motion. A century after that, Newton came forward with his Laws of Motion and stated that an inertial frame is one where his 1st Law was valid. At the same time, Newton proposed an absolute reference frame determined by the fixed stars.

Yet just over a century ago, Michelson & Morley’s experiment to measure the speed of light and Einstein’s theory of special relativity helped put a nail on the coffin of Newton’s absolute reference frame; and absolutism gave way to relativism. Soon after, in almost parallel tracks, the general theory of relativity and quantum mechanics were developed and accepted independently.

However, general relativity is continuous and does not know how to deal with a singularity, and quantum mechanics is quantized, making those realities difficult to unify. Still many attempts have been made [1,2,3,4], with string theory, loop quantum gravity and other efforts to do such. While elegant, these theories have yet to coalesce into agreement or provide experimental evidence of testable hypotheses.

One common thread between the theories is they consider the mean value of the location of the particle, not the variance. Yet from quantum mechanics we know that the location of the particle is fuzzy, and one needs to consider the variance as well. This paradigm, to consider variance alongside the mean value, is commonplace today in many other disciplines; say take for example a stock trader who knows that risk is just as important as return. Still, this author is unaware of any attempt to unify the variance of the quantum mechanical solution with general relativity.

As we will see in the section 5, equivalence between the solutions to quantum diffusion and those Friedmann’s equations of general relativity are possible when the length scale is the standard deviation, not the mean value. And when considering all relevant equations of state for this new named “dark particle”, its solution to both quantum mechanics and general relativity is a modified Langevin equation with a stationary width, consistent with the idea of a quantized black hole, that at its heart is a harmonic oscillator.

Arguably even more profound, yet more readily testable (as we will see), is that this variance is dependent on its absolute velocity and can be measured in a lab, thus bringing back Newton’s original idea of absolutism and challenging relativity where it to date has yet been challenged. Let us begin with this new type of unification...
2. Unification of Action and Entropy

2.1. Simple argument: information equals energy times time

For over a century we have known that the information content of a signal is its bandwidth times its duration. Work by Nyquist and Hartley after the turn of the 20th century [6,7] tells us the bandwidth of a signal is in direct proportion to its width in the frequency domain.

Gabor was even closer on track in the middle of 20th century when he titled the time-frequency plane quantumized with quantized “logons” on information [8] - see Figure 1. Each logon was one degree of freedom and is represented by a shifted and modulated Gaussian wave packet. These wave packets were used as a basis to represent a signal.

They concluded the rate of information that can be encoded into a signal is linear in the bandwidth (which has a maximum value equal to a carrier frequency, f). With Planck’s work on black body radiation and Einstein’s equation for the photo electric effect, where \( \epsilon = hf \) [9], this proportion reduces to below where \( df/dt \) is the information rate of the signal, f is the frequency of the particle, \( \epsilon \) is Planck’s constant times f and t is how long we track it for:

\[
\frac{df}{dt} \propto f \propto \epsilon
\]

Now if we measure the energy and the time using their standard deviations and using Heisenberg’s Uncertainty Principle [10] we can simply conclude that information is the integral of energy over time divided by the quantum of action.

\[
dl = \frac{edt}{\Delta \epsilon \Delta t} = 2\epsilon dt / \hbar
\]

and when \( \epsilon \) is constant we have

\[
I = 2\epsilon t / \hbar
\]

We can now see how Bohr’s correspondence principle [11] reduces the quantum world to the classical when \( \hbar \) goes to zero, as this would imply there is infinite information available to describe the system.

As we will see later, a split Gaussian wavefunction, which meets the Heisenberg Uncertainty Principle, gives us \( f = 1 \). Furthermore, when you calculate the information theoretic entropy, or the negative expected log probability, of each dual domain and add them together (the Hirshman sum [12]), you get \( \log(e) \).

2.2. Top Down

Moving to a more thermodynamical and mechanical approach we start with entropic forces, of which there is much talk today [13,14]. Using this idea, we are able to give a top down derivation of the unification of action and entropy using only: the definition of entropic forces [13], the Euler-Lagrange equations [15] and Newton’s Laws of motion [16].

Let’s start with Newton’s 2nd Law of Motion and include terms for conservative forces and entropic forces,

\[
F_{\text{entropic}} + F_{\text{conservative}} = ma_{\text{particle}}
\]

Now if we consider the vacuum-particle system and consider the acceleration of the vacuum, \( a \), from Newton’s 3rd Law of Motion, we have

\[
m a_{\text{particle}} = F_{\text{particle}} = -F_{\text{vacuum}} = -ma
\]

We can thus re-write the above equation as a statement of Newton’s 1st Law with the acceleration from the perspective of the vacuum,

\[
F_{\text{entropic}} + F_{\text{conservative}} + F_{\text{vacuum}} = 0
\]

Using the Euler-Lagrange equations as derived from Hamilton’s principle we have [15]

\[
F_{\text{vacuum}} = ma = \frac{d}{dt}(mv) = \nabla \left( \frac{mv^2}{2} \right)
\]

We also know from mechanics that [9]

\[
F_{\text{conservative}} = -\nabla V(r)
\]

And with the definition of entropic forces we have [13]

\[
F_{\text{entropic}} = \nabla V(r) = k_b \nabla V \log(\Omega)
\]

Putting this together we have

\[
k_b \nabla V \log(\Omega) - \nabla V(r) + \nabla \left( \frac{mv^2}{2} \right) = 0
\]

Integrating over the gradient and shifting the Lagrangian to the other side of the equation, we have

\[
k_b T \cdot \log(\Omega) = V - \frac{mv^2}{2} + E_o = E_o - L
\]

Where \( E_o \) is the constant of the integration. Now by introducing the relaxation time, \( \tau = \hbar / 2k_b T \), we have

\[
\log(\Omega) = \left( \frac{2}{k_b T} \right) (E_o - L) \tau
\]

Other than the constant of integration, the negative Lagrangian time is “action” and the log of the number of states is “entropy”. While this is the governing thought of the paper, we will re-derive this relationship a few different ways while also enhancing it.

2.3. Additional derivations

Showing how this unification is consistent with quantum mechanics, we use both the Schrödinger and the Heisenberg formulation and show that both result in information equals energy times time.

Figure 1 – Time frequency plane quantized to individual degrees of freedom, each containing one natural unit of information.
2.3.1. Schrödinger equation

The Schrödinger equation, found during the advent of quantum mechanics, dictates how a wave function and its phase evolve through time. The Hamiltonian or energy operator, $H$, of a system is equal to hbar times the imaginary derivative with respect to time; with the operator's eigenvalue, the energy, $\epsilon$, of the system [9].

$$ H\psi = \hbar \frac{d}{dt} \psi = \epsilon \psi $$

The solution to this equation is the complex exponential; and the incremental evolution over $dt$ is

$$ \psi(dt) = \psi(0) e^{-\i \epsilon dt / \hbar} $$

One can calculate the probability associated with this wave function via its magnitude squared [9].

$$ p(dt) = |\psi(dt)|^2 = |\psi(0)|^2 = p(0) $$

Notice the phase information is lost but that unitarity is preserved. Calculating the information without considering the phase information one would conclude that the information is constant and a function only of its initial state, $\psi(0)$, which we will set to 1. However, performing a Minkowski transformation prior to calculating the probability distribution, results in a different answer.

The Minkowski transformation takes imaginary time and makes it real. We see this transformation appear in relativity and analytic continuation [10, 17]. After applying the Minkowski transformation $\psi(dt) = e^{-\i \epsilon dt / \hbar}$ such that,

$$ p(dt) = |\psi(dt)|^2 = e^{-2\i \epsilon dt / \hbar} $$

We have destroyed the unitarity of the equation; however, let us not be turned away from pursuing this line of investigation.

2.3.1.1. Asymptotic Equipartition Property

Without going into the details of the AEP [18], we consider a series of $N$ steps of the evolution. In this case

$$ \psi(t) = \prod_{n=1}^{N} \psi(dt) = e^{-\i \epsilon N dt / \hbar} $$

And,

$$ p(t) = |\psi(t)|^2 = (|\psi(dt)|^2)^N $$

The AEP and the weak law of large numbers [18] can be used to show the negative log probability approaches the incremental entropy. Calling this the differential information, $dl$, I have

$$ - \frac{\log(p(t))}{N} = - \log(p(dt)) \to dl \ as \ N \to \infty $$

$$ dl = 2\i \epsilon dt / \hbar $$

Or,

$$ I = \int 2\i \epsilon dt / \hbar $$

Here we see again that information equals energy times time divided by the quantum of action (in integral form which is important if the energy changes over time).

2.3.2. Heisenberg formulation

It is possible to remove the dependency on the Minkowski transformation and arrive at the same result by replacing the energy eigenvalue $\epsilon$, with the energy operator. When this is the case, the complex conjugate operation also requires the transpose of the operators since they do not commute. Using the power rule to expand the exponent and the logarithm to first order, we now have, with $H$ the energy operator,

$$ - \log(p(t)) = \frac{-i H t}{\hbar} $$

With this approach the negative log probability is now

$$ - \log(p(t)) = \frac{i H t - i H t}{\hbar} $$

Since the commutator $[H, t] = i\hbar$ [9], one has,

$$ - \log(p(t)) = i(-i) = 1 $$

Since the commutator can be interpreted as the area of the time-energy plane enclosed by the difference in path between $H t$ and $i H t$, we see an analogy with Gabor’s tiling of the time-frequency plane [8]. If this is true of every step in the process with the step of size $\delta t = \hbar / 2\i \epsilon$. For a large number of $K$ independent steps, where the A.E.P. [18] kicks in, (and with, $K = t / \delta t$), the negative log probability approaches the information, $-K \log(p(t)) = K \to l$, or

$$ l = K = 2\i \epsilon t / \hbar $$

2.4. Examples

2.4.1. The Gaussian

I argue that when one considers the Hirshman entropy of the Gaussian and adds it to the binary entropy of positive and negative energy solutions, the result is one natural unit of information.

Hirshman [12] proposed that to properly measure the information contained in a pair of distributions linked through the Fourier Transform (FT) one must add the differential entropy of the probability distribution in the time domain to the differential entropy of the probability distribution in the standard frequency domain. Hirshman found that any FT pair contained at least $\log(e/2)$ of information and that the Gaussian has exactly $\log(e/2)$.

To see how an addition $\log(2)$ from the positive and negative energy states can be added to the wave function, let’s first look at the time-frequency dual domain and break the frequency into a positive state and a negative state (as we know to be the case from Dirac [19]). Any
of the dual domains can be used and the analysis to separate the
states into a positive and negative state is symmetric between the two
dual domains. If we have

$$\Phi_+(t) = \frac{1}{\sqrt{2\pi\Delta t^2}} e^{-\frac{(-t)^2}{4\Delta t}}$$

And for the negative eigenvalue,

$$\Phi_-(t) = \frac{1}{\sqrt{2\pi\Delta t^2}} e^{-\frac{(-t)^2}{4\Delta t}}$$

If $\Delta f \ll \bar{f}$ the two functions don’t overlap, they don’t interfere and thus
according to Feynman [9] it’s their probability distributions that add
not the probability amplitudes (or wave functions). The resulting
probability distribution for the frequency domain $P_\Phi(\Delta f)$ is,

$$P_\Phi(\Delta f) = \frac{1}{2}\Phi_+(\Delta f) + \frac{1}{2}\Phi_-(\Delta f)$$

Taking the inverse FT we have

$$\Phi_+(t) = \int \Phi_+(\Delta f) e^{2\pi i \Delta f t} d\Delta f$$

$$\Phi_-(t) = \int \Phi_-(\Delta f) e^{2\pi i \Delta f t} d\Delta f$$

The resulting probability distribution for the time domain, $p_\Phi(t)$ is,

$$p_\Phi(t) = \frac{1}{2}|\Phi_+(t)|^2 + \frac{1}{2}|\Phi_-(t)|^2$$

Given the modulation properties of the FT, $|\Phi_+(t)|^2 = |\Phi_-(t)|^2$ and
with $4\pi\Delta t\Delta f = 1$, $p_\Phi(t)$ reduces to

$$p_\Phi(t) = \frac{1}{\sqrt{2\pi\Delta t^2}} e^{-\frac{t^2}{4\Delta t}}$$

Now using Hirshman’s sum the result is, $\log(e)$

$$I = h(p_\Phi(t)) + h(p_\Phi(t))$$

$$= \frac{1}{2}\log(8\pi\Delta t^2) + \frac{1}{2}\log(2\pi\Delta t^2) = \log(e) = 1$$

2.4.2. Two slit experiment

While the two slit experiment looks like a one-bit measurement
(upper slit or lower slit with equal probability), it is not. We know
this because of the governing hypothesis and the Heisenberg
uncertainty relation that when put together says.

$$I \geq 1$$

natural unit

To see this in detail, assume an experimental setup like in Figure 3,
with "a" approaching zero and "b" a half integer wavelength of
the incoming wave. If a measuring device is asked to locate the particle
at one slit, then the probability distribution on a circular
measurement screen away from the partition in the two-slit
experiment is uniform [20].

If a measurement of the particle’s location at the slits is not
attempted, the particle emerges from both slits with equal amplitude,
and produces the cosine squared distribution on the circular
measurement screen away from the partition [20]. See Figure 4.

$$u(x) = 1/(2\pi)$$ for $x \in [-r, r]$

$$c(x) = (1/r)\cos^2(\pi(b/\lambda)(x/r))$$

For $x \in [-r, r]; \; b/(2\lambda) \in \mathbb{Z}$

If one has $N$ measurement locations on the measurement screen one
can collect $\log(N)$ minus the difference in differential entropy
between the uniform distribution and the actual distribution [18].

Figure 3- Experimental setup for two slit experiment

Figure 4- Probability distributions on measurement screen
depending on if a measurement occurred at the slits.

A fact about the sinusoidal distribution is that it has $\log(e/2)$ less
differential entropy than the uniform distribution. Since we are
taking the difference between two differential entropies the scale
property of the differential entropy does not come into play.

It is worthwhile to mention Gabor’s original analysis [8], to tile of the
time-frequency plane with "logons" of information. Gabor suggested
that each tile, with area proportional to the Heisenberg uncertainty
principle, $A = 4\pi\Delta t\Delta f$, represents a real number with a quantized
piece of information. I suggest the magnitude of this "logon" is one
natural unit consistent with section 2.3.2.

Figure 2 – Probability for Gaussian with non-overlapping positive and
negative states.

Figure 2 - Probability for Gaussian with non-overlapping positive and
negative states.
\[ h(u) - h(c) = -\int_{x}^{r} u(x) \log(u(x)) dx - \int_{x}^{r} c(x) \log(c(x)) dx \]

This integral can be solved analytically for any rand \( b/f(21) \in \mathbb{Z}, \) and is equal to the \( \log(e/2) \).

\[ h(u) - h(c) = \log\left(\frac{e}{2}\right) \]

If a measurement were to occur at the partition, one bit of information, \( \log(2) \), could be extracted by knowing which slit the particle went through plus \( \log(N) \) from the measurement screen. If a measurement at the partition doesn’t happen the total entropy would be \( \log(N) - \log(e/2) \) because in this case the resulting distribution is the cosme distribution. The difference is \( \log(e) = 1 \) nat.

We can see that a simple measurement of a Boolean choice spills over to where the distribution is disturbed, resulting in an additional \( \log(e/2) \) which can be extracted.

### 2.4.3. Black Hole Entropy

One last example of the governing thought is the black hole. Here we see what happens when we cannot consider either \( t \) or \( E \) to be constant, but rather a function of each other. Because of this we see a factor of 2 disappear when we make the integration from \( dt \) to \( l \) as we consider the distance in time as the circumference of the black hole Schwarzschild radius, \( t = 2\pi R/c \). With

\[ E = Mc^2 = Rc^4/2G \]

we integrate to arrive at the well-known entropy of a black hole. [21]

\[ l = \int \frac{2Edt}{h} = \int \frac{2\pi R \kappa c^4 (2\pi R/c)}{2Gh} = \pi (Rc)^2 c^3 \frac{c^2}{Gh} = \frac{Ac^3}{4Gh} \frac{Et}{h} \]

### 2.5. Bottom up Unification

Seeing a few different derivations of “information equals energy times time” we now turn to our capstone bottom up derivation. Here we use an assumption of the Bernoulli Process and information theory to derive the most explicit form of our governing thought.

\[ \log(\Omega) = \left(\frac{2}{h}\right) \int (mc^2 - L)dt \]

### 2.5.1. Bernoulli Process

The Bernoulli process shows how particles step to the left or right with certain probability. Each subsequent step is independent & identically distributed (i.i.d.) and is known as a discrete random walk. Chandrasakhar and Reif [22,23] calculate the average displacement, where \( \beta \) is the probability of stepping to the right, \( N \) is the number of steps and \( \delta x \) is the step size. We have,

\[ \frac{\delta x(N)}{N} = (2\beta - 1)\delta x/\delta N \]

With \( \delta x = c\beta dt \) and \( \delta N = t \) we have

\[ \frac{\delta x(N)}{N} = (2\beta - 1)c \]

Or,

\[ (2\beta - 1) = v/c \]

Notice this equation is mathematically nice since \( \beta \in [0,1] \) and \( v \in [-c,c] \). The variance of position is,

\[ (\Delta x(N))_p^2 = 4\beta(1 - \beta)(\delta x)^2 N \]

Plugging in the equation above between \( v \) and \( \beta \)

\[ (\Delta x(N))_p^2 = \left(1 - \left(\frac{v}{c}\right)^2\right)(\delta x)^2 N \]

We also need to consider the momentum space

\[ \beta = (2\beta - 1)mc \]

Which leads to

\[ (\Delta p(N))_p^2 = 4\beta(1 - \beta)(mc)^2 = \frac{1 - (\frac{v}{c})^2}{N} (mc)^2 \]

A calculation of the variance is,

\[ \text{var} = \left|\frac{(\Delta x(N))_p^2}{m} + \frac{(\Delta p(N))_p^2}{m}\right|^2 \]

\[ \text{var} = \left|\frac{(\Delta x(N))_p^2}{m} + \frac{(\Delta p(N))_p^2}{m} + 2 \cos(\theta_p - \theta_p) \Delta x(N)_p \frac{(\Delta p(N))_p^2}{m} \right| \]

\[ = \left|1 - \left(\frac{v}{c}\right)^2\right| \frac{m}{4} \left(1 + \cos(\theta_p - \theta_p)\right) \]

Where \( \cos(\theta_p - \theta_p) = r \cdot \bar{r} \) is the inner product of the radius unit vector and the momentum unit vector, which is important for section 3 for right now we will assume the special case of a particle moving in an inertial frame or a straight line. In this case the particle will have an infinite radius of curvature and the radius unit vector will be perpendicular to the momentum unit vector, in which case,

\[ \text{var}_{\text{inertial-frame}} = \left(1 - \left(\frac{v}{c}\right)^2\right) \frac{m}{4} \left(1 - \left(\frac{v}{c}\right)^2\right) 2(\delta x)^2 K \]

### 2.5.2. Mutual information: particle and vacuum

Let’s consider the mutual information between the vacuum and the particle. Using the Gaussian channel [18] where the signal is a step in the vacuum Bernoulli process and the noise is the diffusion of the particle over the relaxation time, \( \tau = h/2kBT \), the incremental information is \( dC \) [18]

\[ dC = \frac{1}{2} \log \left(1 + \frac{S}{N_0}\right) \]

Where from section 2.5.1,

\[ S = (\Delta x)^2_{\text{vacuum}} = \left(1 - \left(\frac{v}{c}\right)^2\right) 2(\delta x)^2 K \]

And using Einstein’s kinetic relation for \( D_p \) [24]

\[ N_0 = 2D_p \tau = 2\mu k_BT \tau = \frac{v^2}{c^2} \]

Since \( S \ll N_0 \) we can Taylor expand the logarithm. We also account for both the positive and negative energy states [19], which give a factor of 2 when calculating the differential mutual information, \( dI_M \),

\[ dI_M = 2dC = 2 \left(1 - \left(\frac{v}{c}\right)^2\right) Fc^4(\delta t)^2 \frac{v^2/h}{\nu h} \]

We now make the interpretation that \( \delta t \), which for the dark particle is the reduced Planck length, is the time differential \( dt \) and \( v = dx/dt \). We make these two changes and divide by \( dt \).
\[
\frac{dl_{\mu}}{dt} = \frac{2Fe^2}{\hbar dx} - \frac{2F\text{dt}}{\hbar}
\]

Replacing \( F = m dx/(dt)^2 \) and \( \text{d}t = dx \); we next consider conservative forces so we can replace the incremental work, \( F dx \), with the negative of the potential energy, \( V \), minus the reference potential, \( V_0 \).

\[
\frac{dl_{\mu}}{dt} = \frac{2}{\hbar} (mc^2 + V - V_0)
\]

To solve for \( V_0 \), we postulate that the particle and the vacuum are independent when the potential is zero, \( V = 0 \). When the two are independent, the mutual information rate is zero [18].

\[
\frac{2}{\hbar} (mc^2 - V_0) = 0
\]

And thus

\[
\frac{dl_{\mu}}{dt} = \frac{2}{\hbar} V
\]

2.5.3. Conditional entropy of a kinetic particle

We saw in the section 2.4.1 how the Gaussian distribution can be separated between a positive and negative state to produce one natural unit of information; and through the Bernoulli process this entropy rate happens at each time step, \( \delta t = h/2mc^2 \). For purposes here let’s have the particle drift in one direction, \( \beta \neq 1/2 \). We take the Gaussian probability distribution from section 2.4.1, turn it into the space domain and weigh one of the sides disproportionately as below. With

\[
\Phi(x) = \frac{1}{\sqrt{2\pi(\Delta x)^2}} e^{-\frac{(x)^2}{2(\Delta x)^2}}
\]

We have,

\[
p_\beta(x) = \beta|\Phi(x - \Delta x)|^2 + (1 - \beta)|\Phi(x + \Delta x)|^2
\]

Now the information in one sample of this distribution is the log\((e/2)\) from the Hirshman sum plus \( H_2(\beta) \), where

\[
H_2(\beta) = -\beta \log(\beta) - (1 - \beta) \log(1 - \beta)
\]

With \( (2\beta - 1) = v/c \ll 1 \), we can Taylor expand the logarithm to get

\[
H_2(\beta) = \log(2) - \frac{(v^2)}{2}
\]

which when added to the log\((e/2)\) from the base Gaussian distribution gives a total entropy of the particle

\[
H_2(\beta) = H_2(\beta) + \log\left(\frac{e}{2}\right) = 1 - \frac{(v^2)}{2}
\]

in natural units per \( \Delta t = h/2mc^2 \). Thus, the conditional entropy, \( H_c \), rate is

\[
\frac{dH_c}{dt} = \frac{2}{\hbar} \left( mc^2 - \frac{mv^2}{2} \right) = \frac{2}{\hbar} \left( mc^2 - \xi \right)
\]

Because the particle has interacted with the vacuum potential to make it kinetic, we call \( H_c \) the conditional entropy, \( H(\text{particle|vacuum potential}) \). The resulting biased diffusion is a function of the state of the particle after its interaction with the vacuum.

2.5.4. “Entropy” and “Action”

From information theory, one can prove [18] that the total self-information, \( I_\mu \), is equal to the conditional entropy plus the mutual information. Thus, we have,

\[
I_\mu = H_c + I_{\mu}
\]

Or,

\[
H = H_c + I_{\mu}
\]

We now see that the self-information of the particle is equal to the time integral of the mass energy minus the Lagrangian with a proportionality being the quantum of “action”.

\[
H = -E[\log(p)] = \frac{2}{\hbar} \int (mc^2 - L)dt
\]

Furthermore, with an assumption of the canonical distribution we can show the negative expected log probability is also the thermodynamic entropy over Boltzmann’s constant [23].

\[
\log(\Omega) = S/k_B = -E[\log(p)]
\]

Thus,

\[
\log(\Omega) = \frac{2}{\hbar} \int (mc^2 - L)dt
\]

As we can see, we were able to re-derive the governing thought, this time from information theoretic reasons from the bottom up. We can also see it in its integral form and that the correct constant. Of integration for \( E_{\mu} \) is \( mc^2 \) which comes from the assumption that the mutual information between the particle and the vacuum is zero when the potential is zero.

3. Hypothesis

3.1. Relativity

Centuries ago, Galileo argued that one cannot determine the speed of an inertial frame without looking outside that inertial frame. He used the example of a large ship on the ocean, which when sailing smoothly, could not be distinguished from a laboratory on solid ground [25]. He then extended this to suggest that the Earth is a large ship moving around the Sun. While many experiments have provided evidence of relativity; much has changed since that time and it might behoove one to re-consider that a deeper level of absolute space-time exists beyond what has been measured.

Getting specific here, let’s show how this framework plays out to give the same results as does special relativity for the questions we have been asking. We will use the Lorenz transformation on the mean value of the location of a particle drifting in an inertial frame.
And time. It is also instructive to look at the variance of position relativity, not the instantaneous value. However it is the mean value or expected value that satisfies special relativity. Thus, multiply the triangle on the left by

\[ \frac{\delta x}{\delta t} = \frac{v}{c} \]

We can find the proper time \( s \) and its probability \( \beta_s \), when we multiply the triangle on the left by \( t = N \delta t = N \delta s \)

\[ (2 \beta_s - 1) = \pm \sqrt{1 - (v/c)^2} \]

Thus,

\[ \delta s = \pm t \sqrt{1 - (v/c)^2} \]

and when considering the positive solution,

\[ \delta s = v \delta s \sqrt{1 - (v/c)^2} \]

as it should be when one applies the Lorentz transformation. However it is the mean value or expected value that satisfies special relativity, not the instantaneous value.

FIGURE 5 – The geometry of the Lorentz Transformation

The Lorentz transformation, at its heart, separates one dimension into two when a particle moves at \( v \in (0,1) \). When \( c = 1 \), we have the relationships in Figure 5.

While these three triangles are similar, they are not the same. Let us define the parameters of the Bernoulli process such that these triangles hold.

First define the process for the time. This one is easy, set the probability that time increments by a time step, \( \delta t = h/2m_0c^2 \) equal to one. Thus \( \bar{t} = N \delta t \), where \( N \) is the number of steps. Since this has no variance (as its probability parameter is one) we can set \( \bar{t} = t \).

Next for \( x \), as we saw in section 2.5.1, \( \bar{x} = (2 \beta_x - 1) N \delta x = vt \) when \( \delta x = c \delta t \). However in this case since the probability parameter is neither zero nor one, \( x \) will have a variance and thus we cannot rewrite \( \bar{x} \) as \( x \).

Here we see the variance of position is equal to the speed of light (as Dirac found [19]) but that the mean displacement of the particle is equal to the average velocity times time.

We can find the proper time \( s \) and its probability \( \beta_s \), when we multiply the triangle on the left by \( t = N \delta t = N \delta s \),

\[ (2 \beta_s - 1) = \pm \sqrt{1 - (v/c)^2} \]

Thus,

\[ \delta s = \pm t \sqrt{1 - (v/c)^2} \]

and when considering the positive solution,

\[ \delta s = v \delta s \sqrt{1 - (v/c)^2} \]

as it should be when one applies the Lorentz transformation. However it is the mean value or expected value that satisfies special relativity, not the instantaneous value.

FIGURE 5 – The geometry of the Lorentz Transformation

The geometry of the Lorentz Transformation

\[ \beta_x \]

3.2 Testable Hypothesis

We put forward the hypothesis that: “particles move via the discrete Bernoulli process”.

This hypothesis is testable in a variety of environments (See section 4.4) but for the purposes here, as jitter on a clock. At the most microscopic level, a particle moves in each of the 3+1 direction the length \( \delta x = c \delta t \) with a ratio of \( \delta x \) to \( \delta t \) being the speed of light. This is consistent with the idea of zitterbewegung [26].

3.3 Experimental Theory

As the Earth spins (Figure 6), the velocity of its center of mass plus its rotational velocity lead to a speed on the equator which lead to \( \bar{v}_{laboratory} \); with: \( f_{SN} = 1/365.25 \) day, and \( v_{zL} \) the absolute velocity of the Sun in the plane perpendicular to the Earth’s axis of rotation we have, Figure 6.

\[ R_E = R_e \cos(-2\pi f_{year} t + \varphi) \hat{x} + R_e \cos(-2\pi f_{year} t + \varphi) \hat{y} \]

\[ R_e = R_e \cos(-2\pi f_{SN} t + \theta) \hat{x} + R_e \cos(-2\pi f_{SN} t + \theta) \hat{y} \]

\[ R_y = v_{zL} t \hat{y} \]

FIGURE 6 – A diagram of the Sun/Earth motion

Differentiating \( \bar{v}_{laboratory} \) one can calculate, \( \bar{v}_{laboratory} \)

\[ |\bar{v}_{laboratory}|^2 = |v_{zL}|^2 + |v_e|^2 + |v_{1L}|^2 + 2 |v_{zL}| |v_e| \cos(-2\pi f_{year} t + \varphi) + 2 |v_{1L}| |v_{1L}| \cos(-2\pi f_{SN} t + \theta) + 2 |v_e| |v_{1L}| \cos(-2\pi(f_{SN} - f_{year}) t + \theta - \varphi) \]

Putting this together the jitter will go like:

\[ \left( \frac{\bar{v}_{laboratory}}{c} \right)^2 \frac{\hbar}{m} \left( 1 + \cos(\theta_p - \theta_e) \right) \]

In this case, \( \cos(\theta_p - \theta_e) \) is not 0 since the dominating momentum (which determines \( \beta \)) is \( 2\pi \cdot \text{mass} \cdot R_{eff/year} \) in the direction of Earths
revolution, with the dominating centrifugal acceleration (which determines \( F \)) pointing out from the center of the Earth. Thus,

\[
\cos(\theta_p - \theta_q) = \cos \left( 2\pi (f_{o} - f_{year}) t + \varphi - \frac{\pi}{2} \right)
\]

\[
(\Delta S)^{2}_{\text{varying}} \approx \left( |v_{x1}|^2 + 2 |v_{x2}| |v_{y}| \cdot \cos(2\pi f_{year} t - \varphi) \right) 
\cdot \cos \left( 2\pi (f_{o} - f_{year}) t + \varphi - \frac{\pi}{2} \right) \frac{ht}{m}
\]

4. Experiment

The experiment is very straightforward: take two atomic clocks\(^3\) and measure the timestamps of zero crossings of each clock; take the derivative of each of the time stamps to get the periods, subtract the two channels, take another derivative of each (to remove drift) and lastly take the absolute value\(^2\). This time series is hypothesized to vary with frequencies dominated by, \( f_{day} \), \( f_{sidereal} \), and \( f_{sidereal} - 2f_{year} \).

\[\text{Figure 7 – A diagram of the experimental apparatus}\]

\[\text{Figure 8 – A diagram of the experimental analysis}\]

The specific equipment includes: 2 LPFRS Rb atomic clocks from Spectratime, 2 LTC6957-1 square wave converters, a gt668 jitter analyzer from GuideTech, an i5 Dell Inspireon to host the analyzer and DB, and 2 i7 Dell Inspireons to process the Fourier Transform. All the code, data and further analysis can be found on Drive [27].

4.1. Experimental Result

The FT of the absolute value of the 2nd derivative of time stamps is computed for a select set of 161 frequencies around one cycle per day with a precision of \( \sim 1/(2.0\text{years}) \). Since we are tallying the results in real time (due to the experiment producing 1TB of data daily) the output of the Fourier Transform is similar to a spectrogram with a complex value stored for each hour for each frequency.

For each frequency, the only processing is to subtract a scaled and smoothed FT of the unit function to remove the phase noise (see Figure 9 and 10), then sum the spectrogram coefficients over time. Lastly, we normalize the frequency coefficients which result in Figure 11.

\[\text{Figure 9 – Flow diagram of post processing analysis}\]

\[\text{Figure 10 – Illustration of phase noise cancelation – during the rare times of bad or missing data we preserves the slight aggregate phase impact of the signal over the cycle. When data is not missing, we subtract a constant full cycle which is zero.}\]

\[\text{Figure 11 – Magnitude of FT of magnitude of jitter from an actual experiment over the past two years with 2 clocks. Three peaks are noticeable at one cycle per: anti-sidereal day, day and sidereal day.}\]

The peaks of the graph are at:

\[\text{While the theory does not suggest this is needed, the atomic clocks were orientated 90° to each other. In any case one clock alone should show the signal.}\]

\[\text{It is essential to take the absolute value, just like when you rectify an amplitude-modulated signal before low pass filtering to get the encoded envelope.}\]
Since current theory does not predict any variation of $\Delta\tau$ with these periodicities, $\sigma_{\text{theory}} = 0$, and thus the peaks of this graph are a measure of sigma.

By analyzing the phase and magnitude of the signal we can determine the speed and direction of the Sun/Earth in the preferred frame. In section 2.5.1 we outline the noise in time due to the Bernoulli process. Looking at the oscillating noise, the resulting SNR of $K$ samples, will go like,

$$SNR_{1/day} \approx \frac{1}{2} \sqrt{\frac{6 \cdot 2 \cdot 1.1 |v_{x1}| (\frac{\Delta \tau}{m c^2})}{\eta^2} \cdot K}$$

$$SNR_{1/day+1/year} \approx \frac{1}{2} \sqrt{\frac{6 \cdot 2 \cdot 1.1 |2v_{x1}| v_{z1} (\frac{\Delta \tau}{m c^2})}{\eta^2} \cdot K}$$

Where, $K$ is the number of samples, $\Delta\tau$ is the period of an individual sample (in this case 1/1.25MHz), the factor of 6 is due to 2nd derivative of the time stamps amplifying the signal, the factor of 2 is because there are two clocks each with a signal, the factors of 1.1 inside the square root are due to the cosine splitting the power between positive and negative frequencies, the factor of 1.1 outside the square root is because we measure the absolute value of the jitter not the square of the jitter, and $\eta$ is the standard deviation of the resulting jitter samples.

The mass $m$ is a little harder to determine\(^3\), but if we look at the Rubidium atom as $87$ correlated nucleons then the variance will be $87^2$ more than the variance of one nucleon with a mass of $87$AMU. Thus, the effective mass is one AMU over 87. Since we know, $\eta = 6.2e - 12sec$, $K = 6.81 e13$, and $m = 1.92e - 29 kg$, we can determine the expected $|v_{x1}|$.

4.2. Applying the data to the hypothesis

All of the data and code that can replicate this analysis can be found on Google drive [27]

4.2.1. Calculation of Earth’s Preferred Reference Frame

Since we measure the signal to noise ratio for the three frequencies in question, we can back out $v_{x1}$. By minimizing the mean absolute error of the three observations if our hypothesis is true, we conclude

$$\min \text{RMSE: } v_{x1} = .001c$$

We can also determine the direction from the three phases. We have 2 degrees of freedom $\varphi$ and $\theta$, and three data points (the phases of the three frequencies). Again, by minimizing the RMSE we find the best fit values for $\varphi$ and $\theta$ are:

$$\varphi = 0.41 \text{ radians and } \theta = 2.62 \text{ radians}$$

To find the direction of the velocity of the Sun in the preferred frame, we start with the equation $\varphi = 2\pi f_{\text{year}}t_{\varphi}$ to find $t_{\varphi} \approx 205000$; On that date our velocity is pointed toward the fixed stars on the Eastern Horizon at solar midnight local time in St George, UT (the location of the laboratory). Similarly, we can confirm this direction by, using $\theta$ and its associated unix time $t_{\theta} = 360000$, when it so happens the same constellation is again on the Easter Horizon in St George, UT on Jan 1 1970. That constellation is Virgo. [28,29]

4.2.2. Cosmic Microwave Background Reference Frame

From analysis on the Planck satellite data [30] we see that in the Cosmic Microwave Background frame, the Sun is moving at a speed of $\approx 0.001c$ towards the boundary of Leo and Crater. 1.5 hours away from Virgo. Since our experiment determined the similar speed of the Sun/Earth as the CMB and confirming phases to within 7% of $2\pi$, we conclude that if our original hypothesis is correct, then the CMB reference frame is the preferred absolute frame for the Sun/Earth.

4.3. Affirming Hypotheses

Before we suggest an affirmation of the original hypotheses that particles move via the Bernoulli process on top of an absolute preferred frame, we can run a short calculation on the noise term and show it is below the noise floor of our rubidium atomic clock. When the averaging time $100sec$ we have

$$\Delta f \approx \frac{\Delta t}{t} = \frac{\hbar (v_{x1})^2}{tm_{\text{effective}}c^2} \approx 8x10^{-16} < u_{v_{x1}}$$

A similar calculation on state-of-the-art optical clocks shows $u_{v_{x1}}$ is similar to this noise term when one accounts for a different effective mass and averaging times. But certainly, a thorough investigation is needed if one is to affirm the hypothesis here.

While, more analysis is needed to secure the mathematics, detail the noise, and verify the interpretations, it appears at first pass there is evidence that we can suggest the following:

- The space time motion of an rubidium atom in a locally stationary laboratory on Earth can be modeled using the Bernoulli Process with an artifact in the jitter varying to first and second order with frequencies:

\(^3\) A deeper analysis is needed to consider multiple particles, acting correlated or uncorrelated, which might impact the calculation.
With the insight of how the variance of a particle uncovers new physics, we are emboldened to see how else variance can be used to make progress in other areas.

The concept of quantum diffusion and the length scale of width of the diffusive wave packet allow us to see parallels with Friedmann’s equation. While this does not solve the unification of gravity and quantum mechanics it does provide new insight.

5. Dark Particles

After reviewing the evidence of variance that is a function of velocity, we explore other areas where variance is valuable.

We begin with the theoretical justification for the idea that the energy scale of the vacuum is precisely the reduced Planck mass and expose a new particle dubbed a "dark particle".

5.1. Foundation

We begin by setting the context on a particle of mass \( m \) in equilibrium with a heat bath at temperature \( T \). We assume a particle is in the dual Gaussian ground state.

\[
X \sim p(x)dx = \frac{1}{\sqrt{2\pi (\Delta x)^2}} e^{-x^2/(2(\Delta x)^2)}dx
\]

\[
p_x \sim p(p_x)dp_x = \frac{1}{\sqrt{2\pi (\Delta p_x)^2}} e^{-p_x^2/(2(\Delta p_x)^2)}dp_x
\]

Using the equipartition theorem on the kinetic energy \([9]\), one has

\[
(\Delta p_x)^2 = mk_BT
\]

And using Heisenberg’s Uncertainty equation \([1]\),

\[
(\Delta x)^2 = \frac{\hbar}{2(\Delta p_x)^2} = \frac{\hbar^2}{4mk_BT}
\]

Note the equipartition theorem implies that the particle is coupled to an ensemble or heat bath \([9]\).

5.1.2. Tiny Black Holes

We will now apply these lengths to our understanding of black holes, specifically holes with a mass equal to the reduced Planck mass.

\[
m_p = \sqrt{\frac{\hbar c}{8\pi G}}
\]

A number of special conditions arise at this value of mass. First, the quantum limit, \( \delta x = \hbar/2mc \) is equal to a circle’s circumference with the Schwarzschild radius, Figure 1.

\[
\delta x_p = \frac{\hbar}{2m_pc} = 2\pi R_s = \frac{4\pi Gm_p}{c^2}
\]

Indeed, this is a small cross-sectional area for the black hole.

Second, the Hawking temperature is equal to the mass of the black hole.

\[
k_B T_{\text{Hawking}} = \frac{\hbar c^3}{8\pi G m_p} = m_p c^2 = (4.3 \times 10^{-26} \text{ grams}) c^2
\]

Third, it is not clear that the Hawking temperature is valid at this value of the mass. Specifically Hawking stated in his seminal paper from 1975 \([21]\), “Eventually, when the mass of the black hole is
reduced to $10^{-5}g$, the quasi-stationary approximation will break down. At this point, one cannot continue to use the concept of a classical metric."

Even more recent derivations of Hawking's work still break down at this mass [31]. I will argue that when a black hole has the reduced Planck mass, the Hawking temperature breaks down because a secondary quantum boundary is greater than the Schwarzschild radius and it is this boundary that defines the near horizon's surface gravity. The length of the boundary is such that its surface gravity/temperature is arbitrary.

5.1.3. Quantum Boundary

As the event horizon is defined by the quantum limit, $\delta x$, the outer quantum boundary is defined by the square root of the position's variance ($\Delta x_0$). If $\Delta x_0$ defines the circumference of the boundary (as $\delta x$ defines the circumference of the event horizon), the radius of the outer boundary will be $R_{QB}$, Figure 12.

![Figure 12 - Event horizon (solid line) and quantum boundary (dotted line) of dark particle](image)

$$R_{QB} = \frac{\Delta x_0}{2\pi} = \frac{\hbar}{4\pi\sqrt{m_p k_B T}}$$

The surface gravity at radius $r$ is [32],

$$\kappa(r) = -\frac{1}{2} \frac{d}{dt} \left( -1 + \frac{2Gm_p}{c^2 r} \right) = \frac{Gm_p}{c^2 r^2}$$

The effective temperature [32] for surface gravity at radius $R_{QB}$ will thus be,

$$T_{\text{Dark Particle}} = \frac{\kappa(R_{QB}) \hbar}{2\pi k_B c} = \frac{8\pi Gm^2 T}{\hbar c} = T$$

The width of the black hole's wave packet (which is set by the temperature of the heat bath) that defines the outer quantum boundary is just the right size to define a surface gravity such that the temperature is arbitrary and not a function of mass or other defining feature of the black hole. The temperature is its own independent parameter of the black hole. A black hole with the reduced Planck mass and arbitrary temperature is called a dark particle.

5.2. Quantum Solution

In this section we will review the equations of quantum diffusion as well as modify the Langevin equation to consider a stationary solution which is more consistent with dark particles.

5.2.1. Free Particle Diffusion

We begin with the quantum diffusion of a free particle, which can be derived from the equations of motion [24]. With zero force and $m\dot{x} = p$ one can deduce,

$$x(t) = x_0 + \frac{p}{m} t$$

Calculating the variance and using $x(x)(t = 0) = h/2m$ we have,

$$(\Delta x)^2(t) = (\Delta x_0)^2 + \frac{h}{m} t + \frac{(\Delta p_0)^2}{m^2} t^2$$

This solution has three parts. The linear term is from classical diffusion and Einstein’s kinetic theory [24].

$$\frac{d}{dt} f = \frac{h}{2m} \frac{d^2}{dx^2} f$$

$$D = \mu k_B T = \frac{\tau}{m} k_B T$$

$$(\Delta x)^2(t)_{\text{linear}} = \frac{h}{m} t$$

The constant and quadratic parts are from quantum diffusion which is solved (typically by) Fourier Analysis [33] on the kinetic energy Hamiltonian.

$$H = \frac{p^2}{2m}$$

$$\frac{d}{dt} \psi = \frac{i\hbar}{2m} \frac{d^2}{dx^2} \psi$$

$$(\Delta x)^2(t)_{\text{constant & quadratic}} = (\Delta x_0)^2 + \frac{(\Delta p_0)^2}{m^2} t^2$$

5.2.2. Resistive Force

Here we will derive a resistive force from kinematic arguments. If we look at classical diffusion term and consider the value at $t = \tau$

$$2D \tau = \frac{h}{m} \tau = \frac{\hbar^2}{2mk_B T}$$

Next rearrange the diffusion constant

$$2D \tau = 2\mu k_B T = \frac{-2\nu F}{F} k_B T$$

Replacing $x = vt$ and equating $2D \tau$ to $2D t$ we have,

$$F = \frac{-mx}{\tau^2}$$

5.2.3. Modified Langevin Equation

With a particle no longer free we must re-solve for the variance using the Langevin equation. However contrary to the ordinary Langevin equation [24,33] we will change the assumption that the noisy driving force is uncorrelated with the particle’s location. As we just derived, the force is anti-correlated with the position $F = -\mu r/t^2$. The $1 - D$ equations of motion become,

$$m \ddot{x} = \frac{m}{\tau} (\dot{x}) - \frac{m}{\tau^2} x$$
This equation can be used to solve $\bar{x}^2$ if one assumes the virial theorem [9] where the average quadratic potential energy is equal to the average kinetic energy. The initial condition

$$\bar{x}(0) = \frac{h}{2m}$$

is also assumed ensuring the equation’s boundary conditions obey Heisenberg’s Uncertainty. With calculus and the chain rule, one has,

$$\bar{x}^2 = \frac{h^2}{2m k_B T} (1 - e^{-2a T \tau / h}) = 2D \tau (1 - e^{-1/\tau})$$

This version of the Langevin equation has the familiar $2D \tau$ term; however, it represents a stationary process where the ordinary Langevin equation is non-stationary.

The quantum solution from the modified Langevin presented here is very interesting, as it has a finite asymptotic value, which is what we would expect for a quantum solution to a black hole. We would expect that a black hole has a finite width to it and the outward diffusive pressure is balanced by an inward gravitational pressure.

5.3. Friedmann’s Equations Solutions

We now show that by combining the energy density with three different equations of state, $w = -1, -1/3, & 1/3$ we arrive at the same solution as what was derived in both the continuous and discrete quantum solutions. The solutions to Friedmann’s equation with the densities standing alone correspond to the solutions to the linear and quadratic time terms of the variance when the particle is free. We need to assume the particles come as pairs such that we can define a general relativistic length scale $L$ and a quantum mechanical length scale $\ell$.

5.3.1. Length Scales

We define $L$ as twice the light time $\tau$, the maximum distance two particles can traverse in time $\tau$.

$$L \equiv 2 \tau c = \frac{h c}{k_B T}$$

We define $\ell$ as the variance between the two particles. If the two particles have independent wave functions, we have

$$\ell \equiv \sqrt{2} \cdot \Delta x$$

Using these two definitions we will show that under three different equations of state $L$ (the solution to Friedmann’s equation) will be equal to $\ell$ (the variance of quantum diffusion).

5.3.2. Equation of State, $w = 1/3$

First for the equation of state $w = 1/3$, we have the energy in the 3-D oscillator

$$\hbar \omega = \frac{m}{2 \pi} \left( \alpha^2 + \alpha^2 + \alpha^2 \right) + \frac{1}{2m} \left( p_x^2 + p_y^2 + p_z^2 \right)$$

The average energy of this distribution is the three-dimensional ground state energy of the harmonic oscillator, $3k_B T$.

If we consider a volume $V = L^3$ the energy density is

$$\rho_{w=1/3}(L) = \frac{3k_B T}{c^2 L^3} \approx \frac{3h}{c L}$$

The Friedmann equation when the density is dominated by this equation of state, $w = 1/3$ becomes,

$$\left( \frac{L}{L_0} \right)^2 = \frac{8 \pi G}{3} \rho_{w=1/3} - \frac{8 \pi G m}{c L^2}$$

Solving for $L(t)$ we see it is equal to linear diffusive term from section 3.1.

$$L_{w=1/3}^2 = \left( \frac{32 \pi G h}{c^2} \right) \frac{1}{2} t = \frac{2 h}{m} t = \ell_{linear}^2$$

5.3.3. Equation of State, $w = -1/3$

In deriving the density and solution for this equation of state we turn to a derivation of Friedmann’s equation [34]. We will start by deriving the gravitational explanation of the resistive spring force we introduced earlier. Equating the average gravitational potential energy to $3k_B T/2$ for 3 dimensions gives,

$$\frac{P_{E_{gravity}}}{m} = -\frac{G M m}{r} = \frac{3k_B T}{2}$$

When $M = 4 \pi r^2 p/3$

$$\frac{-G M m}{r} = \frac{-4 \pi G m r^2}{3}$$

Due to symmetry we can re-write $r^2$ as $3(\Delta x)^2 = 3h^2/4m k_B T$ [9] to arrive at,

$$\frac{-8 \pi G m r^2}{3} = \frac{2 k_B T}{h^2}$$

Plugging this back into the relationship between potential energy and force [9] and with time constant $\tau = h/2k_B T$ we have,

$$F_r = \frac{-d}{dr} P_E = \frac{-d}{dr} \frac{G M m}{r} = \frac{-d}{dr} \frac{-4 \pi G m r^2}{3} = \frac{8 \pi G m r}{3}$$

$$\frac{-4 \pi G m r^2}{3} \frac{-m r}{r^2} = \frac{3k_B T_0}{h c}$$

When a particle moves within the space curved by the black hole, a resistive spring force is in play. Here we see a gravitational explanation for the spring force that was also derived earlier.

Going back to solve for the density we have,

$$\frac{-4 \pi G m r^2}{3} \frac{-m r}{r^2} = \frac{3k_B T_0}{h c}$$

Where $T_0$ is constant. With $r^2 = 3(\Delta x)^2 = 3\ell^2/2$ and $m$ the reduced Planck mass, the density is

$$\rho_{w=-1/3}(\ell) = \frac{-6 m k_B T_0}{h c (\ell^2)}$$

Friedmann’s equation and its solution show $L_{w=-1/3}$ is equal to the quadratic diffusive term from section 3.1.

$$\left( \frac{L}{L_0} \right)^2 = \frac{8 \pi G}{3} \rho_{w=-1/3} - \frac{2 k_B T_0}{m (L^2)}$$

$$L_{w=-1/3}^2 = \left( \frac{-2 k_B T}{m} \right) \ell^2 = \ell_{2D}^2$$

Notice the solution of $L_{w=-1/3}$ is imaginary because of the positive curvature associated with this equation of state [34]. Yet we still have $|L| = |\ell|$. In the next two paragraphs we derive the holistic energy density, $\rho_{dark\ particle}$ which is always positive and thus $L_{dark\ particle}$ is real.
The last term we need is a constant energy density, $w = -1$. To solve for the constant density, we insert $L_0 = h/\sqrt{m_0 k_0 T_0}$ (which we show is the asymptotic value of the solution) into the density of the oscillator.

$$
ρ_{w=-1} = ρ_{w=1/3}(L = L_0) = \frac{3h^2}{c \ell_0^4} \left( \frac{3m^2(k_0 T_0)^2}{c^3} \right)
$$

The solution to Friedmann's equation with this density is exponentially increasing, however if we add this density term to our other two densities, we see the solution is equal to the solution of the Langevin equation.

$$
ρ_{\text{dark particle}} = ρ_{w=-1} + ρ_{w=-1/3} + ρ_{w=1/3}
$$

$$
ρ_{\text{DF}}(L) = \frac{3}{h^2} \left( \frac{h^2}{(L(L)4 - \frac{2m k_0 T_0}{L(L)^2}} + \frac{m^2(k_0 T_0)^2}{h^4} \right)
$$

By applying calculus, the solution to Friedmann's equation with this density is

$$
\left( \frac{L}{L_0} \right)^2 = \frac{8πG}{3} ρ_{\text{DF}}(L) = \left( \frac{h}{m L^2} - \frac{κ_0 T_0}{h} \right)^2
$$

$$
L_{\text{DF}}^2 = \frac{h^2}{m k_0 T_0} \left( 1 - e^{−2κ_0 T_0 / h} \right)
$$

With $κ_0 T_0 = h/(2κ_0)$ and the $D = h/(2m)$, this is re-written, showing $L_{\text{DF}}$ equal to the stationary Langevin equation from section 5.2.3.

$$
L_{\text{DF}}^2 = 4Dτ_0 \left( 1 - e^{−1/κ_0} \right) = \ell_{\text{Langevin}}^2
$$

We see the solutions to Friedmann's equation and the equations of quantum diffusion, behave in the same way. It is interesting to note that the density vanishes at the asymptotic value $ρ_{\text{DF}}(L_0) = 0$ so we don't have to worry about this fermionic density contributing to the cosmological constant.

6. Conclusion

We have proposed a new way to look at unification. By seeing the relationship between the entropy and the action we uncover new physics. Suggesting a hypothesis that can be tested, we see that by looking at the variance of a particle's location instead of just its expected or mean value, a preferred reference frame is implied.

By conducting an experiment to measure the magnitude of jitter on a clock, we discover the hypothesized artifact. While the experiment produces a combined 19+ sigma, what is even more encouraging is that all three parameters of the hypothesis point in the same direction, that the reference frame of Earth is the Cosmic Microwave reference frame.

Optimistically we move onto seeing what else we can explore with this new tool of focusing on the variance. We see that when we look to the reduced Planck level, a new dark particle that has a quantum width (square root of its variance) that is equal to the solution to Friedmann's equation.

While this is just one theory, one hypothesis, and one experiment, many other insights are waiting for us. We just need to be willing to consider breaking a few rules.

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Soli Deo Gloria

References

1) A. Zee, Quantum field theory in a nutshell, 2nd edition, Princeton University Press, Princeton, N.J., 2010
2) H. W. Hamber, Quantum Gravitation: The Feynman Path Integral Approach, Springer-Verlag Berlin Heidelberg, 2009
3) M. B. Green, J. H. Schwarz, E. Witten, Superstring theory, vol. 1 & 2, Cambridge University Press, Cambridge, 2012
4) L. Smolin, Einstein’s Unfinished Revolution: The Search for What Lies Beyond the Quantum, Penguin Press, USA 2019
5) M. R. Francis, “Falsifiability and physics”, Symmetry Magazine, https://www.symmetrymagazine.org/article/falsifiability-and-physics, pulled September 22, 2021
6) H. Nyquist, “Certain factors affecting Telegraph speed,” Bell Systems Tech. J., 1924, 3, pp. 324.
7) R. V. L. Hartley, “Transmission of Information,” Bell Systems Tech. J., 1928, 7, pp. 535.
8) D. Gabor, “Theory of Communication,” Journal of Institution of Electrical Engineers, 1946, 93, III p.429
9) R. Feynman, Lectures on Physics, Addison-Wesley Publishing, Reading, 1965
10) R. Shankar, Principles of Quantum Mechanics, 2nd Edition, Plenum Press, New York, NY 1994
11) N. Bohr, ibid
12) Hirsman, I. L, “A Note on Entropy,” American J. of Mathematics, January 1957, 79, No1, p. 152
13) N. Roos, “Entropic forces in Brownian motion,” American Journal of Physics 82, 1161 (2014)
14) E.P. Verlinde, “On the Origin of Gravity and the Laws of Newton”, JHEP, 2011 (4): 29
15) M. Galuzzi, Analytical Mechanics: Translated from the “Mecanique analytique,” nouvelle edition of 1811, J. L. Lagrange, editors Auguste Boissonade & Victor N. Vagliente, London, Kluwer Academic Press, 1997
16) I. Newton, The Principia, translated by A. Motte, Prometheus Books, Amherst, New York, 1995
