Statistical theory of self-similar time series
as a nonextensive thermodynamic system

Alexander I. Olemskoi

Max-Planck-Institute für Physik komplexer System, Nöthnitzer Strasse 38,
D-01187 Dresden, Germany

Abstract

Within Tsallis’ nonextensive statistics, a model is elaborated to address self-similar
time series as a thermodynamic system. Thermodynamic-type characteristics relevant to
temperature, pressure, entropy, internal and free energies are introduced and tested. Stability conditions of time series analysis are discussed in details on
the basis of Van der Waals model.

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1 Introduction

Time series analysis allows to elaborate and verify macroscopic models of complex systems
behaviour on the basis of data analysis. The latter is known to be focused on numerical
calculations of correlation sum for delay vectors that allow to find relevant magnitudes of
fractal dimension and entropy [1]. Being traditionally a branch of the theory of statistics, time
series analysis is based on the class of models of harmonic oscillator, which are relevant to the
simplest case of the Gaussian random process [2]. But it is well known that real time series is
rather the Lévy stable processes related to self-similar system, than the Gaussian ones being
very special case [3]. Therefore, it is important to study self-similar time series. In this Letter
we present a method based on Tsallis’ statistics [4], which is of relevance for self-similar system
analysis [5] – [7].

The paper is organized as follows. Section 2 is devoted to elaboration of a model, which
permits to address a self-similar time series as a nonextensive thermodynamic system. Section
3 is based on calculations of both entropy and internal energy of the time series. As a result,
thermodynamic-type characteristics of the time series such as temperature and entropy, volume
and pressure, internal and free energies are introduced. Their testing for the model of ideal gas
is shown to be basis for statistics of self-similar time series. Section 4 is devoted to discussion of
the physical meaning of the results obtained. Stability conditions of time series mimicked as a

1 Corresponding address: Sumy State University, Rimskii-Korsakov St. 2, 40007 Sumy, Ukraine.
E-mail: olemskoi@ssu.sumy.ua
non-ideal gas are discussed in details when external field and particle interaction are switched on. Several equalities needed in quoting are placed in Appendix.

2 Presenting time series as nonextensive thermodynamic system

Let us consider $D$-dimensional time series $\mathbf{x}(t_n)$ related to the set $\{x_n\}$ of consequent values $x_n \equiv x(t_n)$ of principle variable $x(t)$ taken at discrete time instants $t_n \equiv n \tau$ that we obtain as result of dividing a whole time series length $T \equiv N \tau$ by $N$ equal intervals $\tau$. It is obviously to be relevant to the time series $\mathbf{x}(t_n)$, the set $\{x_n\}$ should be supplemented by conjugated set $\{v_n\}$ of velocities, which show rates of $x_n$-variation with the time jumping. In the simplest case of Markovian consequence, one has $v_n \equiv (x_n - x_{n-1})/\tau$. For more complicated series with $m$-step memory, the velocity magnitude is defined as follows:

$$v_n \equiv \left[ \frac{1}{m} \sum_{i=1}^{m} \delta_i^2(m, n) \right]^{\frac{1}{2}}, \quad \delta_i(m, n) \equiv \frac{x(n-m+i) - x(n-m)+(i-1)}{\tau}. \quad (1)$$

The paradigm of our approach is to address the time series as a physical system defined by an effective Hamiltonian $H = H\{x_n, v_n\}$, on whose basis statistical characteristics of this series could be found. If one proposes that series terms $x_n$ related to different $n$ are not connected, the effective Hamiltonian is additive:

$$H = \sum_{n=1}^{N} \varepsilon_n, \quad \varepsilon_n \equiv (x_n, v_n). \quad (2)$$

Physically, this means that the series under consideration is relevant to an ideal gas comprising of $N$ identical particles with energies $\varepsilon_n$. Further, we suppose different terms of time series to be statistically identical, so that effective particle energy does not depend on coordinate $x_n$: $\varepsilon(x_n, v_n) \Rightarrow \varepsilon(v_n)$. Moreover, since this energy does not vary with inversion of the coordinate jumps $x_n - x_{n-1}$, the function $\varepsilon(v_n)$ should be even. We use the simplest square form

$$\varepsilon_n = \frac{1}{2} v_n^2, \quad (3)$$

which is reduced to the usual kinetic energy for a particle with mass 1. With switching on an external force $\mathbf{F} = \text{const}$, particle energy (3) becomes as follows:

$$\varepsilon_n = \frac{1}{2} v_n^2 - Fx_n. \quad (4)$$

Finally, when time series has a microscopic memory, dimension of the delay vectors $\delta_i(m, n)$ in the definition (1) needs taking $m > 1$. Moreover, if time series terms $x_m, x_n$ with $m \neq n$ are clustered, the Hamiltonian becomes relevant to a non-ideal gas with interaction $w_{mn}, \ m \neq n$:

$$H = \frac{1}{2} \sum_{n=1}^{N} v_n^2 + \frac{1}{2} \sum_{m \neq n} w_{mn}. \quad (5)$$
\begin{equation}
\langle O \rangle_q = \sum_{\{x_n, v_n\}} O\{x_n, v_n\} P_q\{x_n, v_n\}
\end{equation}

defined by relevant functional $O\{x_n, v_n\}$. Hereafter, the principle role is devoted to the sum over states, which reads:

\begin{equation}
\sum_{\{x_n, v_n\}} \Rightarrow \frac{1}{N!} \int \prod_{n=1}^{N} \frac{dx_n dv_n}{\Delta} = \frac{1}{N!} \left( \frac{X^2}{\tau \Delta} \right)^{DN} \prod_{n=1}^{N} \int dy_n \int du_n \Rightarrow N^{-1} \prod_{n=1}^{N} \int dy_n \int du_n; (7)
\end{equation}

\[N \equiv N! \left( \frac{X^2}{\tau \Delta} \right)^{-DN} \simeq \left[ \frac{e^{X^{2D}}}{N(\tau \Delta)^D} \right]^{-N} \quad \text{for } y_n \equiv \frac{x_n}{X}, \; u_n \equiv \frac{\tau v_n}{X}, \quad \int dy_n = \int du_n = 1. (8)\]

Here, $\Delta$ is effective Planck constant that determines a minimal volume of phase space per a particle related to a term of the time series, the factorial takes into account statistical identity of these terms, the factor $(X^2/\tau)^{DN}$ is caused by change of initial variables $x_n, v_n$ into $y_n, u_n$ rescaled with respect to macroscopic length $X$.

At a choice of the probability distribution $P_q\{x_n, v_n\} \Rightarrow P_q\{y_n, u_n\}$, we are stated on a self-similarity of the time series meaning a power-law form of the functional $P_q\{y_n, u_n\}$ instead of usual exponential one type of Gaussian. We adopt known Tsallis' escort distribution [4]

\[P_q\{y_n, u_n\} = \begin{cases} 0 & \text{at } (1 - q) \frac{H\{y_n, u_n\}}{\langle 1 \rangle_q T_0} - E > 1, \\ \frac{1}{Z} \left[ 1 - (1 - q) \frac{H\{y_n, u_n\}}{\langle 1 \rangle_q T_0} - E \right]^{\frac{1}{1-q}} & \text{otherwise}. \end{cases} (9)\]

Here partition function is defined by condition

\[Z \equiv N^{-1} \prod_{n=1}^{N} \int \left[ 1 - (1 - q) \frac{H\{y_n, u_n\}}{\langle 1 \rangle_q T_0} - E \right]^{\frac{1}{1-q}} dy_n du_n, (10)\]

where $0 < q < 1$ is a parameter of nonextensivity, $T_0$ is energy scale, internal energy $E$ and normalization parameter $\langle 1 \rangle_q$ are determined by equalities

\[E \equiv N^{-1} \prod_{n=1}^{N} \int H\{y_n, u_n\} P_q\{y_n, u_n\} dy_n du_n, \]

\[\langle 1 \rangle_q \equiv N^q \prod_{n=1}^{N} \left( \langle P_q\{y_n, u_n\} \rangle \right)^{\frac{1}{2}} dy_n du_n \]

To check the statistical scheme proposed let us address firstly trivial case of time series $x_n = \text{const}$. Here, the particle energy $\varepsilon$ is a constant as well, so that the Hamiltonian is $H = N \varepsilon$. The partition function $Z = N^{-1}$ and the normalization parameter $\langle 1 \rangle_q = N^{-(1-q)}$ are given by inverted normalization factor (8), whereas the internal energy $E = N \varepsilon$ is reduced to the Hamiltonian. Then, the entropy $H = -\ln N$ obtained according to definition (59) given in
Appendix is reduced to zero if only the normalization factor takes value $N = 1$. As a result, we find effective Planck constant:

$$\Delta = \left( \frac{e}{N} \right)^{\frac{1}{N}} \frac{X^2}{\tau}. \quad (12)$$

If $D$-dimensional domain of the coordinate $x_n$-variation supposes to be governed by Lévy-type law

$$X^D = x^D N^{\frac{1}{z}} \quad (13)$$

with microscopic constant $x$ and dynamic exponent $z$, then one obtains the following scaling relation for the phase space volume per a term of the time series:

$$\Delta^D = e \left( \frac{x^2}{\tau} \right)^D N^{\frac{z}{2} - 1}. \quad (14)$$

In the case of Gaussian scattering, when $z = 2$, this volume does not depend on number $N$ of the time series terms, i.e., one obtains a constant.

3 Non-extensive thermodynamics of time series as an ideal gas

Stating on the basis of known results for thermodynamic quantities of nonextensive ideal gas (see Appendix) we are in position to examine pseudo-thermodynamic properties of time series. The quoted equalities (49) – (52) derive to the starting expression for the normalization quantity

$$\langle 1 \rangle_q = \frac{X^{DN} \gamma(q)}{N!} \left[ \frac{\theta(1 + a)^{\frac{2}{1-q}+1}}{1-q} \right]^{\frac{DN}{2}} \frac{1+a}{1-q}. \quad (15)$$

Here the notations are introduced (cf. Eqs. (51))

$$\theta \equiv \frac{2 \pi T_0}{\Delta^2}, \quad \gamma(q) \equiv \frac{\Gamma \left( \frac{1}{1-q} \right)}{\Gamma \left( \frac{1}{1-q} + \frac{DN}{2} \right)}, \quad a \equiv (1 - q) \frac{DN}{2}, \quad (16)$$

where we use $\Gamma$-function. In the limits $N \gg 1$, $(1 - q) \frac{DN}{2} \ll 1$, when

$$\gamma(q) \simeq [e(1 - q)]^{\frac{DN}{2}} (1 + a)^{-\frac{1+a}{1-q}}, \quad (17)$$

one obtains for the entropy (59):

$$H \simeq \frac{Na}{2(1-a)} \ln \left[ e^{2+D} \theta^D \left( \frac{X^D}{N} \right)^2 \right]. \quad (18)$$
With accounting scaling relation (13), this expression takes the usual form
\[ H = N \frac{z-1}{z} \ln \left( \frac{G}{N} \right), \quad G \equiv (2\pi eT_0)^{\frac{Dz}{2}} \left( \frac{x}{\tau} \right)^{-Dz}, \tag{19} \]
if the dynamic exponent is determined as
\[ z = \frac{1}{1 - a}. \tag{20} \]

Respectively, the internal energy (50) and the normalization parameter (15) read:
\[ E = DN \frac{2}{G} \left( \frac{G}{N} \right)^{\frac{2}{z}(1-\frac{1}{z})} T_0, \quad \langle 1 \rangle_q = \left( \frac{G}{N} \right)^{\frac{2}{z}(1-\frac{1}{z})}. \tag{21} \]

Following [8], let us introduce the temperature
\[ T \equiv \langle 1 \rangle_q T_0 = \left( \frac{G}{N} \right)^{\frac{2}{z}(1-\frac{1}{z})} T_0, \quad a \equiv \frac{z-1}{z}, \tag{22} \]
where the second equality takes into account the last relation (21). This definition guarantees the equipartition law
\[ E = CT, \quad C \equiv cN, \quad c \equiv \frac{D}{2}, \tag{23} \]
where the quantity
\[ C = \frac{\partial E}{\partial T} \tag{24} \]
is the specific heat. It is easily to convince that equations (19) – (22) arrive at standard thermodynamic relation
\[ \frac{\partial H}{\partial E} \equiv \frac{1}{T}. \tag{25} \]

Above used treatment is relevant to the method [9] addressed to a fixed value of the internal energy \( E \). In alternative case when the principle state parameter is the temperature \( T \), we should pass to the conjugate formalism [10]. Here, standard definition
\[ F \equiv E - TH \tag{26} \]
of the free energy arrives at the dependence
\[ F = -CT \ln \left( \frac{T}{T_0} \right). \]  

(27)

Then, the thermodynamic identity

\[ \frac{\partial F}{\partial T} \equiv -H \]  

(28)

yields the relation

\[ H = C \ln \left( \frac{T}{T_0} \right) \]  

(29)

that plays a role of a heat equation of state. It arrives at the usual definition of the specific heat (cf. Eq. (24))

\[ C = T \frac{\partial H}{\partial T}. \]  

(30)

Let us introduce now specific entropy per unit time

\[ h \equiv (D\tau)^{-1} \frac{dH}{dN} = \tau^{-1} H_1 - r \]  

(31)

to be determined by a minimal entropy

\[ H_1 = z \ln \sqrt{2\pi e T_0} - z \ln \left( \frac{x}{\tau} \right) \]  

(32)

and a redundancy

\[ r = (D\tau)^{-1} \ln(eN). \]  

(33)

Dependencies on the scale \( x \)

\[ h(x) = \text{const} - \frac{z}{\tau} \ln \left( \frac{x}{\tau} \right), \quad r(x) = \text{const} \]  

(34)

notice that the system behaves in a stochastic manner [11].

Let an effective pressure be defined as

\[ p \equiv -\tau \frac{\partial h}{\partial x} \]  

(35)

to measure specific entropy variation with respect to the time series scale. Then, we arrive at a mechanic equation of state.
being additional to the relation (29) of entropy to temperature. According to Eq. (36), definition of the compressibility

$$\kappa \equiv \left( \frac{\partial x^{-1}}{\partial p} \right)^{-1}$$

shows that it is reduced to the dynamic exponent (20). Respectively, stability conditions

$$C > 0, \quad \kappa > 0$$

reduces to natural restriction

$$z > 1.$$  

It is principally important that the "pressure" (35) is introduced as derivative of the specific entropy $h$ with respect to the microscopic scale $x$. This means a microscopic nature of so defined "pressure", which related to a susceptibility (more exactly, the compressibility (37)) of the time series with respect to a choice of the microscopic scale $x$.

4 Discussion

We have addressed above the simplest model of the ideal gas, which have allowed us to examine analytically a self-similar time series in standard statistical manner. Characteristic peculiarity of related equalities (12) – (14), (18) – (21), (27), (29), (32) – (34), (36), (38), (39) is a scale invariance with respect to variation of the nonextensivity parameter $1 - q$, which is contained everywhere as combination (see Eq. (16))

$$a = (1 - q)C \quad C \equiv \frac{DN}{2}$$

with free magnitude of parameter $a$ related to the dynamic exponent $z$ (see Eq. (20)). This invariance is clear to be caused by self-similarity of the system under consideration. For a given time series, the value of exponent $z$ is fixed if it is addressed to a mono fractal manifold and takes a closed set of magnitudes $z$ in the case of self-similar system relevant to a multifractal. In real time series this invariance can be broken, so that a dependence on the parameter of nonextensivity could appear. However, above introduced set of pseudo-thermodynamic characteristics of time series is kept as applicable and accustomed thermodynamic relations (23), (24), (28), (30), (35), (37) can be applied to analysis of arbitrary time series.

Let us focus now on effect of external field and particle interaction, which switching on is expressed by equalities (4), (5). Related entropy additions (54), (56) arrive at total value of the specific heat (30) in the following form:

$$C_{tot} = C - \frac{C}{T} \left( FX - \frac{z - 1}{Dz} \frac{N}{V} w \right),$$
where one supposes $F V \ll T$ for simplicity. Thus, at temperature less than critical magnitude

$$T_c = F x N^{\frac{1}{Dz}} - \frac{z-1}{Dz} w x^{-D} N^{1-\frac{1}{z}}.$$  \hfill (42)

deﬁnition, the system becomes nonstable due to external force $F$ and interparticle attraction $w < 0$. If the number $N$ is much less than critical value $N_c$ defined as

$$N_c^{(1 + \frac{1}{D}) \frac{1}{z} - 1} \equiv \frac{z-1}{Dz} \frac{|w|}{F} x^{-(D+1)}.$$  \hfill (43)

main contribution gets interparticle attraction $w < 0$, in opposite case $N \gg N_c$ – external force $F$. With accounting temperature definition (22) and notation (19), this instability means that a microscopic time interval $\tau$ should be more than a critical magnitude $\tau_c$. In the case $N \ll N_c$, we find

$$\tau_c = \left( \frac{z-1}{Dz} \frac{|w|}{x^{(D+1)}} \right)^{\frac{1}{2}} x^{\frac{1}{2}\tau^{(1+\frac{1}{D})}}.$$  \hfill (44)

In opposite case $N \gg N_c$, one obtains

$$\tau_c = \frac{F \tau^{(1+\frac{1}{D})}}{\sqrt{2\pi e T_0}} x^{\tau^{\frac{1}{2}(\tau + 1)}} N^{\tau^{2+\frac{1}{D}}(\tau + 1)}.$$  \hfill (45)

On the other hand, the entropy additions (54), (56) accompanied by Eqs. (31), (35), (36) arrive at the compressibility (37) in the form

$$\kappa^{-1} = z - (D + 1) \frac{z-1}{z} \frac{N}{X^D} \left( v + \frac{w}{T} \right).$$  \hfill (46)

Hence, the system becomes nonstable if microscopic scale $x$ is less than a critical magnitude determined as

$$x_c^D = (D + 1) \frac{z-1}{z^2} \left( v + \frac{w}{T} \right) N^{1-\frac{1}{z}}.$$  \hfill (47)

In the limit of very large time intervals $\tau$, when the temperature is $T \gg w/v$, one finds the simple estimation

$$x_c \sim \epsilon N^{\frac{1}{z}(1-\frac{1}{D})}.$$  \hfill (48)

Thus, at $z = 1$ critical value $x_c$ is reduced to a scale of the time series resolution $\epsilon$, being a particle radius within the gas model.

Above studied instabilities break the conditions (38), which guarantee predictability of the time series analysis. Our main result is that the predictability becomes impossible if microscopic
scales \( \tau_x \) take values less than critical magnitudes \( \tau_{xc} \), increasing with growth of the number \( N \) of time series terms.

To conclude, we point out that main advantage of the approach proposed is a possibility of its application to numerical analysis of real time series. The basis of this calculations is stated on expressions (1) – (6), (9) – (11), (22), (23), (24), (26), (28), (30), (31), (35), (37), (59). This work is in progress.

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Appendix: Non-extensive thermodynamics of thermodynamic system

Calculation of main thermodynamic quantities of nonextensive ideal gas arrives at the Euler \( \Gamma \)-function to be reduced to the following expressions for partition function and internal energy [8] – [10]

\[
Z = \frac{V^N \gamma(q)}{N!} \left[ \frac{\theta \langle 1 \rangle_q}{1 - q} \right]^{DN} \left[ 1 + (1 - q) \frac{DN}{2} \right]^{-1} \left[ 1 + (1 - q) \frac{E}{\langle 1 \rangle_q T_0} \right]^{\frac{1}{1 - q} + \frac{DN}{2}},
\]

\[
E = \frac{DN V^N \gamma(q) T_0}{2} \left[ \frac{\theta \langle 1 \rangle_q}{1 - q} \right]^{DN} \left[ 1 + (1 - q) \frac{DN}{2} \right]^{-1} \left[ 1 + (1 - q) \frac{E}{\langle 1 \rangle_q T_0} \right]^{\frac{1}{1 - q} + \frac{DN}{2}} Z^{-q}.
\]

Here, \( D \)-dimensional gas in volume \( V \equiv X^D \) is addressed and the notations are introduced

\[
\theta \equiv \frac{2\pi mT_0}{\hbar^2}, \quad \gamma(q) \equiv \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} + \frac{DN}{2}\right)},
\]

where \( m \) is particle mass, \( \hbar \) is Planck constant and \( T_0 \) is energy scale. Normalization quantity

\[
\langle 1 \rangle_q = Z^{1-q}
\]

follows from the normalization condition of the Tsallis’ distribution function [4].

Switching on an external force \( F = \text{const} \) causes the second term in Hamiltonian (4) to arrive at the following factor in Eq. (49):

\[
Z_{ext} = \exp \left[ DN \left( \frac{FX}{2T} \right) \right] \left( \frac{\sinh \left( \frac{FX}{2T} \right)}{\left( \frac{FX}{2T} \right)} \right)^{DN}.
\]
According to the definition (59), this yields the entropy addition

\[ H_{\text{ext}} = DN \left( \frac{FX}{2T} \right) + DN \ln \left( \sinh \left( \frac{FX}{2T} \right) \right) \].

(54)

Within a supposition \( F > 0 \), a homogeneous external field arrives at monotonous decrease of the entropy \( H_{\text{ext}}(T) \) with the temperature growth. At \( T \gg FX \) contribution of the second term is neglecting, in the limit \( T \to 0 \) it tends to the first one.

According to recent work [12], cluster expansion of the particle interaction in Hamiltonian (5) results in additional factor in partition function (49):

\[ Z_{\text{int}} = 1 - \frac{N^2}{2V} \left( v + \frac{w}{T} \right) ; \quad w \equiv S_D \int_{\epsilon}^{\infty} w(x) x^{D-1} dx, \quad S_D \equiv \frac{2\pi^{D/2}}{\Gamma(D/2)}; \quad v \equiv \frac{S_D}{D} \epsilon^D, \]

(55)

where \( \epsilon \) is effective radius of the particle core. Relevant entropy addition

\[ H_{\text{int}} = -\frac{aN^2}{2V} \left( v + \frac{w}{T} \right) \]

(56)

monotonously decreases with growth of the particle volume \( v \). Growth of the temperature \( T \) causes the entropy decrease for attractive interaction \( w < 0 \) and its increase in the case of repelling one \( (w > 0) \).

Finally, we arrive at useful relations between physically defined entropy \( H \) and related Tsallis’ magnitude \( H_q \). One has symbolic equation

\[ H = a \ln[\exp_q(H)] , \quad H_q = \ln_q[\exp(H/a)] ; \quad a \equiv (1 - q) \frac{DN}{2} , \]

(57)

that are alternations of the usual functions logarithm \( \ln(x) \) and exponential \( \exp(x) \) with corresponding Tsallis’ generalizations [4]

\[ \ln_q(x) \equiv \frac{x^{1-q} - 1}{1 - q} , \quad \exp_q(x) \equiv [1 + (1 - q)x]^{\frac{1}{1-q}} . \]

(58)

In explicit form, relations (57) are appeared as

\[ H \equiv a \ln Z , \quad H_q \equiv \frac{\langle 1 \rangle_q - 1}{1 - q} . \]

(59)

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