On the $D^+ \to \omega\pi^+$ decay

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Abstract

The $D^+ \to \omega\pi^+$ decay is studied by decomposing its amplitude into a sum of factorizable and non-factorizable ones. Its rate is predicted as a function of mass and width of a hypothetical hybrid meson.

PACS number(s): 13.25.Ft, 11.30.Hv, 11.40.Ha, 12.39.Mk
(Quasi) two body decays of charm mesons can be described in terms of three different quark-line diagrams, i.e., spectator, color mismatched spectator and annihilation diagrams in the weak boson mass $m_W \to \infty$ limit. As is well known, the naively factorized amplitudes given by the last two diagrams are suppressed (color suppressed and helicity suppressed, respectively) in the BSW scheme while the measured decay rates for color suppressed $D^0 \to \bar{K}^0\pi^0$, $K^*0\pi^0$, etc. are not very small against the expectation. Therefore, the factorization has been implemented by taking into account final state interactions. However, amplitudes with final state interactions are given by non-leading terms in the large $N_c$ expansion which are not factorizable. It implies that the non-factorizable contributions play an essential role, at least, in hadronic weak decays of charm mesons.

In this short note, we study the $D^+ \to \omega\pi^+$ decay decomposing its amplitude into a sum of factorizable and non-factorizable ones as in two body decays. Therefore, our starting point is to decompose the effective weak Hamiltonian into a sum of the BSW Hamiltonian, $H_{BSW}^{SW}$, and an extra term, $\bar{H}_w$, i.e., $H_w = H_{BSW}^{SW} + \bar{H}_w$, where $H_{BSW}^{SW}$ and $\bar{H}_w$ are responsible for factorizable and non-factorizable amplitudes, respectively. For more details, see Refs. [2, 3, 4].

The amplitude for the $D^+ \to \omega\pi^+$ decay includes all the three types of amplitudes mentioned above. Using the naive factorization in the BSW scheme, we calculate factorizable amplitudes $M(D \to VP)_{FA}$ for $D \to VP$ decays, where $D$, $V$ and $P$ denote a charm, a vector and a pseudo-scalar (PS) meson, respectively. A typical result is given in Table I, in which $A$ is defined by, for example, $M(D^+ \to \omega\pi^+)_{FA} = -i(G_F/\sqrt{2})V_{ud}V_{ut}f \pi A(D^+ \to \omega\pi^+)_{FA}$, etc. The CKM matrix elements [6] are taken to be real in this note. In the above, matrix elements [7] are taken to be zero in this note. In the above, matrix elements of currents have been taken in the same form as in Ref. [4]. In both of the annihilation and the color suppressed decays, the factorized amplitudes are actually suppressed.

Next, we study non-factorizable amplitudes. As discussed in Refs. [3] and [4], they are dominated by dynamical contributions of various hadron states and can be estimated by using a hard PS-meson approximation in the infinite momentum frame (IMF). In this approximation, the non-factorizable amplitude for $D \to VP$ is given by $M \simeq M_{ETC} + M_S$, where $M_{ETC}$ is written in the form, $M_{ETC}(D \to VP) = (i/f_P)\langle V||[V_P,\bar{H}_w]|D\rangle$, by using $[V_P + A_P,\bar{H}_w] = 0$. $V_P$ is an $SU_f(3)$ charge and $A_P$ is its axial counterpart. $M_S$ is given by a sum of all possible pole amplitudes, $M_S = \sum_{n} M^{(n)} + \sum_{l} M^{(l)}$, where

\[
M^{(n)}(D \to VP) = -\frac{i}{f_P} \frac{m_v^2 - m_D^2}{m_n^2 - m_D^2} \langle V|A_P|n\rangle \langle n|\bar{H}_w|D\rangle,
\]

\[
M^{(l)}(D \to VP) = -\frac{i}{f_P} \frac{m_v^2 - m_D^2}{m_l^2 - m_V^2} \langle V|\bar{H}_w|l\rangle \langle l|A_P|D\rangle.
\]

The intermediate $n$ and $l$ run over all possible single meson states but the states $\langle n|\omega\rangle$ and $\langle l|A_P\rangle$, respectively, should conserve spins [4]. However, it has been known that the factorized amplitude dominates the one for the $D^+_s \to \phi\pi^+$, which

\[
\begin{aligned}
\langle \text{Pr}angle & \equiv \frac{\langle n|A_P|D\rangle}{\langle n|\bar{H}_w|D\rangle}, \\
\langle \text{NC}angle & \equiv \frac{\langle l|A_P|D\rangle}{\langle l|\bar{H}_w|D\rangle},
\end{aligned}
\]

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Table I. Factorized and non-factorizable amplitudes for typical quasi two-body decays, where \( h_D = -\sqrt{2}(D^0|^A_\pi|^D^+) \) and \( \hat{g}_H = \langle \omega |A_\pi|\bar{n}\rangle / \sqrt{2} \). The amplitudes \( A_{FA} \) and \( M_{NF} \) are defined in the text.

| Decay            | \( A_{FA} \)                      | \( M_{NF} \)                      |
|------------------|----------------------------------|----------------------------------|
| \( D_s^+ \rightarrow \phi\pi^+ \) | \( A_0^{(\phi\pi^+)}(m_{\pi^+}^2 2m_\phi[\epsilon^{(\phi^+)}(p') \cdot p] \) | neglected                      |
| \( D_s^+ \rightarrow \rho^0\pi^+ \) | 0                                | 0                                |
| \( D_s^+ \rightarrow \omega\pi^+ \) | small                            | \( -\langle \hat{\pi}_H^+ |\hat{H}_u|D_s^+ \rangle \left( \frac{m_{D_s}^2 - m_\omega^2}{m_{D_s}^2 - m_{\hat{\pi}_H}^2} \right) \sqrt{2}\hat{g}_H \) |
| \( D^+ \rightarrow \phi\pi^+ \) | small                            | \( -\langle \phi |\hat{H}_u|D^0 \rangle \left( \frac{m_D^2 - m_\phi^2}{m_D^2 - m_\phi^2} \right) \sqrt{1/2}h_D \) |
| \( D^+ \rightarrow \omega\pi^+ \) | \( \sqrt{1/2}A_0^{(\omega\pi)}(m_{\pi^+}^2 2m_\omega[\epsilon^{(\omega^+)}(p') \cdot p] \) | \( -\langle \omega |\hat{H}_u|D^0 \rangle \left( \frac{m_D^2 - m_\omega^2}{m_D^2 - m_\omega^2} \right) \sqrt{1/2}h_D \) |

is an approximate spectator decay, although four-quark states can contribute to its s-channel intermediate states. Since the spin and parity of the intermediate four quark state in this decay should be \( J^P = 0^- \) and such a four-quark meson state includes some orbital excitation in the system, its mass and width will be much higher and broader than the corresponding \( \{q\bar{q}\bar{q}\} \) mesons with positive parity which have played an important role in two body decays of charm mesons [5]. Therefore, pole contributions of four-quark mesons to the \( D_s^+ \rightarrow \phi\pi^+ \) are not very important and the factorized amplitude for the decay can be estimated by using its measured rate. For the same reason, we neglect possible four-quark meson contributions to a color mismatched decay, \( D^+ \rightarrow \phi\pi^+ \), and will estimate its \( D^* \) pole amplitude in the \( u \)-channel using its measured rate. For contributions of excited meson poles in the \( u \)-channel, however, we expect that they are small as discussed in Ref. [3]. In the \( D^+_s \rightarrow \omega\pi^+ \), which is an approximate annihilation decay, its amplitude will be dominated by an s-channel pole contribution of hybrid meson \( (\hat{\pi}_H) \) with \( I^GJ^{P(C)} = 1^+0^-(\pm) \).

As seen in Eq. (1), the non-factorizable amplitudes are described in terms of asymptotic matrix elements (matrix elements taken between single hadron states with infinite momentum) of \( A_P \) and \( \hat{H}_u \) since \( M_{ETC} \) has been discarded. Asymptotic matrix elements of \( A_P \) are parameterized as \( \sqrt{2}\langle D^0|A_\pi|^D^+ \rangle = -h_D \) and \( \langle \omega |A_\pi|\bar{n}\rangle = \sqrt{2}\hat{g}_H \). However, our result on the \( D^+ \rightarrow \omega\pi^+ \) in this short note does not explicitly depend on these matrix elements.

Constraints on (i.e., selection rules of) asymptotic matrix elements of \( \hat{H}_u \) can be obtained by using an intuitive quark counting [3] which is an analogue to the Miura-Minamikawa-Pati-Woo theorem in hyperon decays [10]. Noting that the wave function of the ground-state \( \{q\bar{q}\}_0 \) meson is anti-symmetric under the exchange of its constituent quark and antiquark [11], we obtain [3] \( \langle \{q\bar{q}\}_0 |\hat{O}_+|\{q\bar{q}\}_0 \rangle = 0 \), which leads to
\begin{align}
\langle \pi^+ | \bar{H}_w | D_s^+ \rangle &= - \langle K^0 | \bar{H}_w | D^0 \rangle, \\
\langle \rho^+ | \bar{H}_w | D_s^+ \rangle &= - \langle K^0 | \bar{H}_w | D^0 \rangle,
\end{align}

where approximate equalities \( V_{ud} = V_{cs} \) and \( V_{us} = - V_{cd} \) have been used. Asymptotic flavor \( SU_f(3) \) symmetry \([12]\) relates the asymptotic matrix elements of the CKM-angle favored weak Hamiltonian to those of the CKM-angle suppressed one \([5]\), for example,

\[ \langle \pi^+ | \bar{H}_w | D^+ \rangle = \frac{V_{cd}}{V_{cs}} \langle \pi^+ | \bar{H}_w | D_s^+ \rangle, \text{ etc.} \]  

(3)

Inserting the above parameterizations of asymptotic matrix elements of \( \bar{H}_w \) and \( A_P \)'s into the non-factorizable hard PS meson amplitudes, we obtain amplitudes for the annihilation and color mismatched types of \( D \to VP \) decays whose surface terms are dominated by pole contributions of the ground-state \( \{q\bar{q}\}_0 \) and hybrid \( \{qg\}_0 \) mesons. In the approximation in which pole contributions of \( \{q\bar{q}\}_0 \) and hybrid \( J^P(C) = 0^{-}(\pm) \) mesons are taken into account, the non-factorizable amplitudes for typical quasi two body decays are given in the third column of Table I, where the useless imaginary unit has been factored out, i.e., \( M_{\text{NF}} = i M_{\text{NF}} \). The amplitudes for the \( D_s^+ \to \rho^0 \pi^+ \) and \( D_s^+ \to \omega \pi^+ \) are described by two annihilation diagrams and they cancel each other in the former while they interfere constructively with each other in the latter \([3]\).

To estimate the rate for the \( D^+ \to \omega \pi^+ \) decay, we take the central values of the measured ones \([14]\) of the CKM matrix elements \( [V_{ud}, V_{us}, V_{cd}, V_{cs}] \), decay constants \( [f_\pi, f_K] \), and lifetimes of charm mesons \( [\tau(D^+), \tau(D^0), \tau(D_s^0)] \). As seen in Table I, \( M(D^+ \to \phi \pi^+) \simeq M(D^+) \langle D^+ \to \phi \pi^+ \rangle \). Eliminating useless imaginary unit, we obtain

\[ \mathcal{M}(D^+) \langle D^+ \to \phi \pi^+ \rangle \simeq 1.03 \left[ \frac{\langle \omega | \bar{H}_w | D^0 \rangle}{\langle \phi | \bar{H}_w | D^0 \rangle} \right] \mathcal{M}(D^+ \to \phi \pi^+) \simeq \{-5.3\} \times 10^{-7} \text{ GeV} \]  

(4)

using Eq. \([2]\) and \( |\mathcal{M}(D^+ \to \phi \pi^+)| \simeq 7.4 \times 10^{-7} \text{ GeV} \) from \( \Gamma(D^+ \to \phi \pi^+)_{\exp} = (4.0 \pm 0.4) \times 10^{-10} \text{ GeV} \) \([14]\), where the sign of \( \langle \rho^+ | \bar{H}_w | D^0 \rangle \) [and hence \( \langle \phi | \bar{H}_w | D^0 \rangle \) because of Eq. \([2]\)] have been taken to be the same as that of \( \langle \pi^+ | \bar{H}_w | D^+ \rangle \) in Ref. \([3]\) as was in weak interactions of \( K \) mesons \([3]\) i.e., \( \langle \rho^0 | \bar{H}_w | K^0 \rangle / \langle \pi^0 | \bar{H}_w | K^0 \rangle > 0 \).

The amplitude for the \( D_s^+ \to \omega \pi^+ \) decay is dominated by a pole contribution of \( \hat{\pi}_H \), i.e., \( \mathcal{M}(D_s^+ \to \omega \pi^+) \simeq \mathcal{M}(\hat{\pi}_H)(D_s^+ \to \omega \pi^+) \). From the measured rate \([14]\), \( \Gamma(D_s^+ \to \omega \pi^+)_{\exp} = (3.9 \pm 1.5) \times 10^{-15} \text{ GeV} \), we obtain \( |\mathcal{M}(\hat{\pi}_H)(D_s^+ \to \omega \pi^+)\rangle \simeq 6.8 \times 10^{-7} \text{ GeV} \). Using the asymptotic \( SU_f(3) \) relation in Eq.\( (3) \), we obtain

\[ \mathcal{M}(\hat{\pi}_H)(D^+ \to \omega \pi^+) \simeq -1.3 \times \left( \frac{m_{D_s^0}^2 - m_{\pi_H^0}^2}{m_D^2 - 2m_{\pi_H^0}^2} \right) \hat{\alpha}_H \times 10^{-7} \text{ GeV} \]  

(5)

where \( \hat{\alpha}_H \) is a parameter providing the sign of \( \mathcal{M}(D_s^+ \to \omega \pi^+) \).

The factorized amplitude for the \( D^+ \to \omega \pi^+ \) includes a form factor \( A_0(\omega D)(m_{\pi}^2) \). To estimate it, we put \( A_0(\omega D)(m_{\pi}^2) \simeq A_0(\phi D_s)(m_{\pi}^2) \) which can be obtained by applying (asymptotic)
and \( \cong \) where the first, second and third terms on the right-hand-side of Eq.(6) are from the upper bound \( M \) and \( \Gamma \) \( \hat{\Gamma} \text{GeV} \), \( \Gamma_B \) sum of \( \pi \). It is reasonable since it is not very far from two predicted values \([15]\), \( (\pi \rightarrow + 76) \). In summary, the \( \phi \pi \rightarrow 0\) is around the level of \( m_H \text{GeV} \). When we take account for them, however, we predict \( B(D^+ \rightarrow \omega \pi^+) \) as a function of mass \( m_{\hat{\pi}_H} \) and width \( \Gamma_{\hat{\pi}_H} \) of \( \hat{\pi}_H \). It depends on the relative sign between \( M_{\text{FA}} \) and \( M^{(\hat{\pi}_H)} \). As seen in Fig. I, however, gross features of the branching ratios, \( B_- \) for \( \hat{\alpha}_H = -1 \) and \( B_+ \) for \( \hat{\alpha}_H = +1 \), are not very different from each other since \( M^{(D^+)} \) cancel almost \( M_{\text{FA}} \) in the present case. Except for the neighborhood of the singular point, \( (m_{\hat{\pi}_H} = m_D, \Gamma_{\hat{\pi}_H} = 0) \), both of \( B_- \) and \( B_+ \) are always lower than the experimental upper bound \( [B(D^+ \rightarrow \omega \pi^+)]_{\text{exp}} < 7 \times 10^{-3} \) and are very sensitive to the values of \( m_{\hat{\pi}_H} \) and \( \Gamma_{\hat{\pi}_H} \) in the region, \( (1.8 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.10 \text{ GeV}) \), and \( (1.75 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.15 \text{ GeV}) \), respectively. Outside of the above regions, however, they are very small, i.e., \( B_+ \) is around the level of \( \sim 1.5 \times 10^{-3} \) for \( m_{\hat{\pi}_H} \cong 1.8 \text{ GeV} \) and much smaller for \( m_{\hat{\pi}_H} \cong 1.9 \text{ GeV} \) while \( B_- \cong 0.5 \times 10^{-3} \) outside the region, \( (1.8 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.10 \text{ GeV}) \).

In summary, the \( D^+ \rightarrow \omega \pi^+ \) decay has been studied by decomposing the amplitude into a sum of factorizable and non-factorizable ones. The former has been estimated by using the naive factorization in the BSW scheme while the latter has been calculated by using a hard pion approximation. As the result, its amplitude has been approximately given by a sum of \( M_{\text{FA}}, M^{(D^+)} \) and \( M^{(\hat{\pi}_H)} \). The first two have been estimated by using the measured

\[
\mathcal{M}_{\text{tot}}(D^+ \rightarrow \omega \pi^+) \cong \left\{ 3.6a_1 - 5.3 - 1.3\hat{\alpha}_H \left( \frac{m_{D_k}^2 - m_{\hat{\pi}_H}^2}{m_{D_k}^2 - m_{\hat{\pi}_H}^2} \right) \right\} \times 10^{-7} \text{ GeV},
\]

where the first, second and third terms on the right-hand-side of Eq.(6) are from \( M_{\text{FA}}, \mathcal{M}^{(D^+)} \) and \( \mathcal{M}^{(\hat{\pi}_H)} \), respectively. Taking \( a_1 = 1.09 \) \([14]\), we will study the branching ratio for the \( D^+ \rightarrow \omega \pi^+ \). If we neglected the non-factorizable contributions, we would obtain \( B(D^+ \rightarrow \omega \pi^+)_{\text{FA}} \cong 2.1 \times 10^{-3} \). When we take account for them, however, we predict \( B(D^+ \rightarrow \omega \pi^+) \) as a function of mass \( m_{\hat{\pi}_H} \) and width \( \Gamma_{\hat{\pi}_H} \) of \( \hat{\pi}_H \). It depends on the relative sign between \( M_{\text{FA}} \) and \( M^{(\hat{\pi}_H)} \). As seen in Fig. I, however, gross features of the branching ratios, \( B_- \) for \( \hat{\alpha}_H = -1 \) and \( B_+ \) for \( \hat{\alpha}_H = +1 \), are not very different from each other since \( M^{(D^+)} \) cancel almost \( M_{\text{FA}} \) in the present case. Except for the neighborhood of the singular point, \( (m_{\hat{\pi}_H} = m_D, \Gamma_{\hat{\pi}_H} = 0) \), both of \( B_- \) and \( B_+ \) are always lower than the experimental upper bound \( [B(D^+ \rightarrow \omega \pi^+)]_{\text{exp}} < 7 \times 10^{-3} \) and are very sensitive to the values of \( m_{\hat{\pi}_H} \) and \( \Gamma_{\hat{\pi}_H} \) in the region, \( (1.8 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.10 \text{ GeV}) \), and \( (1.75 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.15 \text{ GeV}) \), respectively. Outside of the above regions, however, they are very small, i.e., \( B_+ \) is around the level of \( \sim 1.5 \times 10^{-3} \) for \( m_{\hat{\pi}_H} \cong 1.8 \text{ GeV} \) and much smaller for \( m_{\hat{\pi}_H} \cong 1.9 \text{ GeV} \) while \( B_- \cong 0.5 \times 10^{-3} \) outside the region, \( (1.8 \cong m_{\hat{\pi}_H} \cong 1.9 \text{ GeV}, \Gamma_{\hat{\pi}_H} \cong 0.10 \text{ GeV}) \).

In summary, the \( D^+ \rightarrow \omega \pi^+ \) decay has been studied by decomposing the amplitude into a sum of factorizable and non-factorizable ones. The former has been estimated by using the naive factorization in the BSW scheme while the latter has been calculated by using a hard pion approximation. As the result, its amplitude has been approximately given by a sum of \( M_{\text{FA}}, M^{(D^+)} \) and \( M^{(\hat{\pi}_H)} \). The first two have been estimated by using the measured

\[
M_{\text{tot}}(D^+ \rightarrow \omega \pi^+) \cong \left\{ 3.6a_1 - 5.3 - 1.3\hat{\alpha}_H \left( \frac{m_{D_k}^2 - m_{\hat{\pi}_H}^2}{m_{D_k}^2 - m_{\hat{\pi}_H}^2} \right) \right\} \times 10^{-7} \text{ GeV},
\]

Fig. I. \( B(D^+ \rightarrow \omega \pi^+) \times 10^3 \) vs. \( (m = m_{\hat{\pi}_H}, \Gamma = \Gamma_{\hat{\pi}_H}) \). (a) \( B_- \) for \( \hat{\alpha}_H = -1 \) and (b) \( B_+ \) for \( \hat{\alpha}_H = +1 \).
rates for the $D^+_s \to \phi \pi^+$ and $D^+ \to \phi \pi^+$. We have predicted $B(D^+ \to \omega \pi^+)$ as a function of mass and width of a hypothetical hybrid meson $\pi_H$ with $I^GJ^P(C) = 1^+0^-(^-)$. If $m_{\pi_H}$ is close to $m_D$ and $\Gamma_{\pi_H}$ is small, it will be strongly enhanced. If not, however, it will be $\sim 1.5 \times 10^{-3}$ or much smaller. The former is not very far from the result by the naive factorization.

The author would like to thank Prof. T. E. Browder for attracting his attention to the $D^+ \to \omega \pi^+$ decay. He also would like to appreciate Prof. H. Yamamoto for discussions and comments.
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