Multi-quality Index Optimization of Injection Compression Process Parameters Based on Grey Robustness

Junjie Zhu, Wenhan Huang, Zhiwen Qiu
School of Electrical and Mechanical Engineering, Heyuan Ploytechni, Heyuan 517200, China
gzb@gdut.edu.cn

Abstract. Aiming at the balance optimization of multiple mass indexes, such as sink marks, warpage and residual stress in the injection compression molding process of optical products, from the perspective of robust design, the experiment of injection compression processing of wedge-shaped light guide plate with five factors and four levels was designed by orthogonal experiment method. The optimal combination of process parameters, the order of influence and the trend of change were obtained by S/N analysis. Finally, the grey relational theory was used to transform the multi-quality index optimization problem into the single objective optimization of grey relational degree, and the optimal process parameter combination considering multiple quality indexes were obtained. Compared with before optimization, each quality index decreased by 23.89%, 8.87% and 24.68% respectively. The research shows that the optimization technology based on grey robustness can effectively solve the problem of balance optimization among multiple mass indexes, and can realize the overall optimization of injection compression process parameters.

1. Introduction
With the continuous progress of injection molding technology, optical products are developing in the direction of high light, high precision and light weight. In molding optical products, injection compression molding technology has more unique advantages than traditional injection molding technology. Injection compression molding is the addition of compression steps to the filling process of conventional injection molding. In the compression stage, the whole moving mold pressure is applied to maintain the pressure, the shrinkage of the plastic melt is better compensated, which is conducive to improve the molding quality of the product.

In order to improve the molding quality of products, many scholars have conducted a large number of studies on the quality of injection compression molding products [1-3]. Taking PMMA lens as an example, Li [4] and others studied the influence of injection compression molding process parameters on volume shrinkage. Based on the finite element analysis theory and viscoelastic constitutive equation, Xie [5] and others used Moldex3D software to carry out three-dimensional simulation analysis of the forming of variable-thickness optical lens. Qiu [6] and others used MPI software to simulate the thermal residual stress of injection compression molding of thin-walled plastic parts.

In this paper, based on CAE finite element technology, the influences of injection compression process parameters on the sink marks, warpage and residual stress of PMMA wedge light guide plate were studied by orthogonal test and SNR analysis. The grey system theory is applied to the robust design of injection molding of light guide plate, and the problem of multi-quality index optimization is transformed into the single objective optimization of grey correlation degree. Finally, the optimal
process parameter combination considering multiple quality indexes is obtained. The problem of balance optimization among multiple quality indexes is solved effectively, and the overall optimization of injection compression process parameters is realized.

2. Grey relational optimization theory

The theory of grey correlation optimization was proposed by Professor Deng Julong in 1982. As an important branch of grey system theory, it is used to analyze the degree of correlation among various factors in the system. It is a method to judge the strength of the correlation based on the degree of correlation among factors [7].

In the grey correlation analysis method, set \( x_0=\{x_0(1), x_0(2), \cdots, x_0(n)\} \) as the reference sequence in the correlation analysis, \( x_i=\{x_i(1), x_i(2), \cdots, x_i(n)\} \) as the sequence being compared, \( i=1, 2, \cdots, m \). For a reference series \( x_0 \), comparison series \( x_i \) can be expressed as the difference between each comparison curve and the reference curve at each point in the following relation [8].

\[
\gamma_i(k) = \frac{\min_i \min_k \left| x_0(k) - x_i(k) \right| + \xi \max_i \max_k \left| x_0(k) - x_i(k) \right|}{\max_i \max_k \left| x_0(k) - x_i(k) \right|}
\]

In this formula, \( \gamma_i(k) \) represents the relative difference between the comparison curve \( x_i \) and the reference curve \( x_0 \) at time \( k \). The relative difference of this form is called the coefficient of correlation of \( x_i \) with respect to \( x_0 \) at time \( k \). \( \xi \) denotes resolution coefficient, \( \xi \in [0,1] \).

Finally, the grey relational degree of multiple optimization targets is calculated, it is the average value of all grey relational degrees.

\[
\lambda_i = \frac{1}{m} \sum_{i=1}^{n} \delta_i \quad (i = 1, 2, 3 \cdots n)
\]

3. Robust design based on grey relational degree

Robustness refers to the insensitivity of the dependent variable to the deterioration of the factor. Robust design is based on the comprehensive consideration of product performance, quality and cost to select the best design, so as to maximize the benefit. The method of robust design is used to minimize the sensitivity of product quality due to the change of error factors, which is suitable for solving the problem of multi-index optimization of injection molding process parameters.

The two main tools of robust design are orthogonal table and signal to noise ratio. By using orthogonal table to arrange the test, the optimal combination of parameters and test factors can be found through fewer tests. Due to the existence of unstable factors, the results will produce deviations. Therefore, the signal to noise (\( \eta \)) ratio is used as the index to measure the robustness of output features. The greater the \( \eta \), the more stable the output features are. The signal to noise ratio is the ratio of noise (S) and signal (N), which can be divided into three types, large sight feature, small sight feature and sight feature [9-10].

If the quality index has the feature of maximization, the SNR calculation formula is shown in equation (3).

\[
\eta=S/N=-10\log\left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{\gamma_i} \right)
\]

(3)

If the quality index has the feature of minimization, the SNR calculation formula is shown in Equation (4).

\[
\eta=S/N=-10\log\left(\frac{1}{n} \sum_{i=1}^{n} \gamma_i \right)
\]

(4)

If the quality index has the visual property, the SNR calculation formula is shown in Equation (5).
\[ \eta = \frac{S/N}{10 \log_{10} \frac{\mu}{\sigma}}, \quad \mu = \frac{1}{n} \sum_{i=1}^{n} y_i, \quad \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \mu)^2 \]

\( \eta \) denotes signal to noise ratio, dB; \( y_i \) represents the result of the test; \( n \) is the number of repeated tests for each test.

Robust design method combined with grey system theory, established the grey relationship between the quality index and the reference target, and calculated the correlation coefficient between each quality index and the reference target. The multi-quality index optimization problem is transformed into a single objective optimization problem with grey correlation degree as the objective, and the correlation degree is analyzed by range and mean value. The optimal and balanced combination of process parameters was obtained to improve the forming quality of the products.

Steps of robust design process based on grey relational degree are as follows: (1) Determination of quality index and constraint conditions. (2) Orthogonal experimental design. (3) CAE finite element analysis. (4) Data processing and analysis. (5) Grey correlation degree calculation (6) Optimize the horizontal combination of factors and verify it.

4. Case study
A wedge-shaped light guide plate in an optical element is taken as an example. The grey robust theory is applied to the optimization of injection compression molding process parameters of light guide plates. The length and width of the light guide plate are 100 mm and 80 mm respectively, and the average thickness is 1.65 mm. The material is Sumipex MH PMMA, produced by Sumitomo Chemical Company. PVT curves of polymer materials are shown in figure 1.

![Figure 1. PVT curve of PMMA](image-url)

The light guide plate is wedge-shaped, and the thickness decreases linearly from 2.5 mm to 0.8 mm. The number of finite element meshes is 3268, the number of nodes is 1715, the maximum aspect ratio is 2.8, the average aspect ratio is 1.44. The mold is formed with one mold and one cavity. In order to minimize the internal stress of the products and make the melt form a smooth material flow before entering the cavity, the glue is fed into the fan gate. The flat gate facilitates better melt entry into the cavity without point injection, thus reducing shear and internal stress and improving warping of the product. The CAE finite element model is shown in Figure 2.
4.1 Parameter design and results of orthogonal test

According to the quality requirements of forming optical products, sink marks ($S$), warpage ($W$) and residual stress ($R$) of the light guide plate were selected as quality evaluation indexes. Five injection compression process parameters, including mold temperature ($A$), melt temperature ($B$), compression delay time ($C$), compression time ($D$) and compression speed ($E$), were selected as test factors. According to the polymer material characteristics, product molding process requirements and injection machine parameters, Four levels were uniformly selected within the range of each process parameter, and 16 tests were carried out using $L_{16}(4^{5})$ orthogonal table. The injection compression process parameters and their horizontal design are shown in table 1, and the test results are shown in table 2.

| Table 1. Injection compression process parameters and levels. |
|-----------------------------------------------|
| Levels | Injection compression process parameters |
| A/℃ | B/℃ | C/s | D/s | E/cm/s |
| 1 | 50 | 225 | 0.8 | 1 | 0.1 |
| 2 | 60 | 240 | 1.3 | 2 | 0.3 |
| 3 | 70 | 255 | 1.8 | 3 | 0.5 |
| 4 | 80 | 270 | 2.3 | 4 | 0.7 |

| Table 2. Orthogonal test results. |
|-----------------------------------|
| Test | A | B | C | D | E | $S$/% | $W$/mm | $R$/MPa |
| 1 | 1 | 1 | 1 | 1 | 1 | 3.224 | 0.3596 | 19.24 |
| 2 | 1 | 2 | 2 | 2 | 2 | 2.790 | 0.3461 | 18.48 |
| 3 | 1 | 3 | 3 | 3 | 3 | 2.875 | 0.3484 | 18.84 |
| 4 | 1 | 4 | 4 | 4 | 4 | 2.986 | 0.3529 | 17.38 |
| 5 | 2 | 1 | 2 | 3 | 4 | 2.020 | 0.3416 | 18.88 |
| 6 | 2 | 2 | 1 | 4 | 3 | 3.244 | 0.3869 | 18.71 |
| 7 | 2 | 3 | 4 | 1 | 2 | 4.038 | 0.3622 | 14.86 |
| 8 | 2 | 4 | 3 | 2 | 1 | 4.548 | 0.3765 | 11.31 |
| 9 | 3 | 1 | 3 | 4 | 2 | 1.762 | 0.3409 | 18.17 |
| 10 | 3 | 2 | 4 | 3 | 1 | 2.034 | 0.3392 | 16.06 |
| 11 | 3 | 3 | 1 | 2 | 4 | 4.589 | 0.4042 | 15.02 |
| 12 | 3 | 4 | 2 | 1 | 3 | 6.010 | 0.4310 | 11.63 |
| 13 | 4 | 1 | 4 | 2 | 3 | 1.561 | 0.3423 | 21.88 |
| 14 | 4 | 2 | 3 | 1 | 4 | 3.856 | 0.3936 | 12.93 |
| 15 | 4 | 3 | 2 | 4 | 1 | 3.692 | 0.4050 | 13.79 |
| 16 | 4 | 4 | 1 | 3 | 2 | 4.840 | 0.4231 | 12.86 |
4.2 SNR analysis of test results

In actual production, considering the small characteristics of sink marks, warpage and residual stress. The smaller the test result is, the larger the SNR is. Therefore, formula (4) is used to process the data in table 2. In order to more objectively evaluate the influence trend of various test factors on quality indicators at different levels, the three quality indicators ($S$, $W$, $R$) in table 2 were converted into SNR. The mean value (MV) and range (RA) were analyzed, as shown in table 3. According to the mean analysis results in Table 3, it can be seen that when A, B, C, D and E are at levels 1, 1, 4, 3 and 3 respectively, the sink marks of the light guide plate is the smallest. The results show that the best combination of parameters to obtain the minimum sink marks is as follows: mold temperature 50 ℃, melt temperature 225 ℃, compression delay time 2.3 s, compression time 3 s, compression speed 0.5 cm/s. In addition, according to the range ranking in table 3, the primary and secondary order of process parameters affecting the sink marks of the light guide plate is: melt temperature > compression delay time > compression time > mold temperature > compression speed.

Similarly, the optimal parameter combination to obtain the minimum warpage is as follows: mold temperature 50 ℃, melt temperature 225 ℃, compression delay time 2.3 s, compression time 3 s, and compression speed 0.3 cm/s. The primary and secondary order of the process parameters affecting the warpage of the light guide plate is: melt temperature > compression delay time > compression time > mold temperature > compression speed. The optimal parameter combination of the minimum residual stress is as follows: mold temperature 80 ℃, melt temperature 270 ℃, compression delay time 1.8 s, compression time 1 s, compression speed 0.1 cm/s. The primary and secondary order of process parameters affecting the residual stress of the light guide plate is: melt temperature > mold temperature > compression time > time > compression delay time > compression speed.

According to the SNR analysis data in table 3, the average analysis chart of each quality index can be drawn. It can more intuitively reflect the change and trend of the influence of each process parameter level on each index.

| Quality index | Levels | A     | B     | C     | D     | E     |
|---------------|--------|-------|-------|-------|-------|-------|
| Sink marks (%)| MV1    | -9.439| -6.266| -11.830| -12.398| -10.209|
|               | MV2    | -10.402| -9.256| -10.485| -9.793| -9.913|
|               | MV3    | -9.975| -11.469| -9.743| -8.786| -9.710|
|               | MV4    | -10.158| -12.983| -7.915| -8.997| -10.142|
|               | RA     | 0.719| 6.717| 3.915| 3.612| 0.499|
| Warpage (mm)  | MV1    | 9.076| 9.218| 8.118| 8.278| 8.653|
|               | MV2    | 8.721| 8.738| 8.427| 8.720| 8.714|
|               | MV3    | 8.479| 8.425| 8.772| 8.838| 8.507|
|               | MV4    | 8.183| 8.078| 9.143| 8.623| 8.586|
|               | RA     | 0.893| 1.14| 1.025| 0.56| 0.207|
| Residual stress (MPa) | MV1 | -25.286| -25.799| -24.211| -23.167| -23.415|
|               | MV2    | -23.868| -24.281| -23.739| -24.184| -24.037|
|               | MV3    | -23.537| -23.772| -23.453| -24.286| -24.720|
|               | MV4    | -23.502| -22.342| -24.789| -24.555| -24.022|
|               | RA     | 1.784| 3.457| 1.336| 1.388| 1.305|
4.3 Optimization of process parameters by grey correlation theory

According to the above SNR analysis, the optimal process parameter combination of each quality index is not consistent. In order to further balance and optimize the process parameters, the grey relation theory is adopted to construct the grey relation between the quality index and the reference target. With the grey correlation degree as the link, the multiple quality indexes were transformed into a single objective, so as to find a better and more balanced combination of process parameters.

The data of 16 groups of test results in table 2 are taken as the basis. The minimum sink marks, minimum warpage and minimum residual stress of the light guide plate were selected as the reference sequence, which were denoized as \{1.561, 0.3392, 11.31\}. The reference sequence is marked as \{0.4842, 0.9466, 1.006\} after processing. The original data sequence is dimensionless processed by the initial value method. The correlation degree of sink marks (δS), the correlation degree of warpage (δW), the correlation degree of residual stress (δR) and the weighted average correlation degree (δSWR) of three quality indexes of each test were calculated. The calculated results are shown in table 4.

According to the grey system theory, the degree of correlation between factors and systems mainly depends on the order of the degree of correlation. The larger the correlation value of a process parameter is, the closer the corresponding parameter is to the optimal parameter [11-12]. The average correlation degree of each factor and each level was obtained, and the range analysis was carried out. The results are shown in table 5.

**Table 4. Grey correlation degree of each index.**

| Test | δS  | δW  | δR  | δSWR | Test | δS  | δW  | δR  | δSWR |
|------|-----|-----|-----|------|------|-----|-----|-----|------|
| 1    | 0.572 | 0.692 | 0.399 | 0.554 | 9    | 0.917 | 0.964 | 0.435 | 0.772 |
| 2    | 0.644 | 0.869 | 0.424 | 0.645 | 10   | 0.824 | 1   | 0.526 | 0.783 |
| 3    | 0.628 | 0.833 | 0.425 | 0.628 | 11   | 0.423 | 0.413 | 0.587 | 0.475 |
| 4    | 0.609 | 0.770 | 0.465 | 0.615 | 12   | 0.333 | 0.333 | 0.942 | 0.536 |
| 5    | 0.829 | 0.950 | 0.411 | 0.730 | 13   | 1   | 0.936 | 0.333 | 0.756 |
| 6    | 0.569 | 0.490 | 0.416 | 0.492 | 14   | 0.492 | 0.457 | 0.765 | 0.571 |
| 7    | 0.473 | 0.666 | 0.598 | 0.579 | 15   | 0.510 | 0.410 | 0.680 | 0.534 |
| 8    | 0.426 | 0.551 | 0.659 | 16   | 0.404 | 0.353 | 0.773 | 0.510 |

**Table 5. Average correlation degree of each factor at each level.**

| Levels | A/°C | B/°C | C/s | D/s | E/cm/s |
|--------|------|------|-----|-----|--------|
| 1      | 0.6112 | 0.7035 | 0.5081 | 0.5606 | 0.6331 |
| 2      | 0.6152 | 0.6234 | 0.6117 | 0.6343 | 0.6269 |
| 3      | 0.6419 | 0.5543 | 0.6581 | 0.6633 | 0.6036 |
| 4      | 0.5932 | 0.5803 | 0.6837 | 0.6034 | 0.5980 |
| Range  | 0.0487 | 0.1492 | 0.1756 | 0.1027 | 0.0351 |
Combined with the data in table 5, the average correlation degree curve of the comprehensive quality index of the light guide plate under various factors and levels is plotted, as shown in figure 4. According to grey correlation theory, the mean value of correlation degree is larger. This indicates that the greater the influence of this level of the process parameter on the target response, this level is the optimal level of this factor. Therefore, based on the various quality indicators of the light guide plate, the optimal combination of process parameters is: mold temperature 70℃, melt temperature 225℃, compression delay time 2.3 s, compression time 3 s, compression speed 0.1 cm/s. The primary and secondary order of the process parameters that affect the forming quality of the light guide plate is: compression delay time > melt temperature > compression time > mold temperature > compression speed.

Figure 4. Average correlation degree curve of comprehensive quality of light guide plate at various factors and levels

4.4 Validation
The process parameter combination obtained by the grey robust optimization technology was used to carry out the simulation experiment. The results are shown in figure 5.

5. Conclusion
(1) Through the SNR analysis of the robust design, the optimal process parameters of the wedge-shaped light guide plate, such as sink marks, warpage and residual stress were obtained.
(2) The optimal combination of process parameters to meet the three quality indicators of the light guide plate is as follows: mold temperature 70 ℃, melt temperature 225 ℃, compression delay time 2.3 s, compression time 3 s, and compression speed 0.1 cm/s. The primary and secondary order of the process parameters that affect the forming quality of the light guide plate is: compression delay time > melt temperature > compression time > mold temperature > compression speed.

(3) The grey robust technique is simple, practical and reliable for the optimization of multiple quality indexes in injection compression processing. This optimization method can effectively solve the balance problem among multiple indexes and realize the overall optimization of injection compression process parameters.

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