Low–high voltage duality in tunneling spectroscopy of the Sachdev-Ye-Kitaev model

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The Sachdev-Ye-Kitaev (SYK) model describes a strongly correlated metal with all-to-all random interactions (average strength $J$) between $N$ fermions (complex Dirac fermions or real Majorana fermions). In the large-$N$ limit a conformal symmetry emerges that renders the model exactly soluble. Here we study how the non-Fermi liquid behavior of the closed system in equilibrium manifests itself in an open system out of equilibrium. We calculate the current-voltage characteristic of a quantum dot, described by the complex-valued SYK model, coupled to a voltage source via a single-channel metallic lead (coupling strength $\Gamma$). A one-parameter scaling law appears in the large-$N$ conformal regime, where the differential conductance $G = dI/dV$ depends on the applied voltage only through the dimensionless combination $\xi = eVJ/\Gamma^2$. Low and high voltages are related by the duality $G(\xi) = G(\pi/\xi)$. This provides for an unambiguous signature of the conformal symmetry in tunneling spectroscopy.

Introduction — The Sachdev-Ye-Kitaev model, a fermionic version of a disordered quantum Heisenberg magnet, describes how $N$ fermionic zero-energy modes are broadened into a band of width $J$ by random infinite-range interactions. The phase diagram of the SYK Hamiltonian can be solved exactly in the large-$N$ limit when a conformal symmetry emerges at low energies that forms a holographic description of the horizon of an extremal black hole in a (1+1)-dimensional anti-de Sitter space.

To be able to probe this holographic behaviour in the laboratory, it is of interest to create a “black hole on a chip” that is, to realize the SYK model in the solid state. Ref. 8 proposed to use a quantum dot formed by an opening in a superconducting sheet on the surface of a topological insulator. In a perpendicular magnetic field the quantum dot can trap vortices, each of which contains a Majorana zero-mode. Chiral symmetry ensures that the band only broadens as a result of four-Majorana-fermion terms in the Hamiltonian, a prerequisite for the real-valued SYK model. A similar construction uses an array of Majorana nanowires coupled to a quantum dot. Since it might be easier to start from conventional electrons rather than Majorana fermions, Ref. 10 suggested to work with the complex-valued SYK model of interacting Dirac fermions in the zeroth Landau level of a graphene quantum dot. Chiral symmetry at the charge-neutrality point again suppresses broadening of the band by two-fermion terms.

The natural way to study a quantum dot is via transport properties. Electrical conduction through chains of SYK quantum dots has been studied in Refs. 12–19. For a single quantum dot coupled to a tunnel contact, as in Fig. 1, Refs. 8, 10 studied the limit of negligibly small coupling strength $\Gamma$, in which the differential conductance $G = dI/dV$ equals the density of states of the quantum dot. Conformal symmetry in the large-$N$ limit gives a low-voltage divergence $\propto 1/\sqrt{V}$, until $eV$ drops below the single-particle level spacing $\delta \simeq J/N$.

Here we investigate how a finite $\Gamma$ affects the tunneling spectroscopy. We focus on the complex-valued SYK model for Dirac fermions, as in the graphene quantum dot of Ref. 10. Our key result is that in the large-$N$ conformal regime regime $\xi \ll eV \ll J$ the zero-temperature differential conductance of the quantum dot depends on $\Gamma$, and $V$ only via the dimensionless combination $\xi = eVJ/\Gamma^2$. Low and high voltages are related by the duality $G(\xi) = G(\pi/\xi)$, providing an experimental signature of the conformal symmetry.

Tunneling Hamiltonian — We describe the geometry of Fig. 1 by the Hamiltonian

$$H = H_{\text{SYK}} + \sum_p \varepsilon_p \psi_p^\dagger \psi_p + \sum_{i,p} (\lambda_i c_i^\dagger \psi_p + \lambda_i^* \psi_p^\dagger c_i),$$

$$H_{\text{SYK}} = (2N)^{-3/2} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j, \quad J_{ijkl} = J_{ij:kl} = -J_{ijkl} = -J_{ij:kl}.$$  \hfill (1)

The annihilation operators $c_i$, $i = 1, 2, \ldots$ represent the $N = h\Phi/e$ interacting Dirac fermions in the spin-polarized zeroth Landau level of the graphene quantum dot (enclosing a flux $\Phi$). Two-fermion terms $c_i^\dagger c_j$ are suppressed by chiral symmetry when the Fermi level $\mu = 0$ is
at the charge-neutrality point (Dirac point)\textsuperscript{10} The operators $\psi_p$ represent electrons at momentum $p$ in the single-channel lead (dispersion $\epsilon_p = p^2/2m$, linearized near the Fermi level), coupled to mode $i$ in the quantum dot with complex amplitude $\lambda_i$. The tunneling current depends only on the sum of $|\lambda_i|^2$, via the coupling strength

$$\Gamma = \pi \rho_{\text{lead}} \sum_i |\lambda_i|^2, \quad \rho_{\text{lead}} = (2\pi \hbar v_F)^{-1}. \quad (2)$$

If $T \in (0, 1)$ is the transmission probability into the quantum dot, one has $\Gamma \simeq \mathcal{T} N \delta \simeq \mathcal{T} J$. The Hamiltonian $H_{\text{SYK}}$ is the complex-valued SYK model\textsuperscript{22} if we take random couplings $J_{ij;kl}$ that are independently distributed Gaussians with zero mean $\langle J_{ij;kl} \rangle = 0$ and variance $\langle |J_{ij;kl}|^2 \rangle = J^2$. The zeroth Landau level then broadens into a band of width $J$, corresponding to a single-particle level spacing $\delta \simeq J/N$ (more precisely, $\delta \simeq J/N \ln N$)\textsuperscript{22} In the energy range $\delta \ll \varepsilon \ll J$ the retarded Green’s functions can be evaluated in saddle-point approximation\textsuperscript{22}

$$G^R(\varepsilon) = \frac{-i \pi^{1/4}}{2\pi J} \frac{\Gamma(1/4 - i\beta \varepsilon/2\pi)}{\Gamma(3/4 - i\beta \varepsilon/2\pi)}, \quad (3)$$

where $\beta = 1/k_B T$ and $\Gamma(x)$ is the Gamma function. At zero temperature this simplifies to

$$G^R(\varepsilon) = \frac{-i \pi^{1/4}}{2\pi J} \exp \left( \frac{1}{4} \pi \text{sgn}(\varepsilon) \right) |J\varepsilon|^{1/2}. \quad (4)$$

Quantum fluctuations around the saddle point cut off the low-$\varepsilon$ divergence for $|\varepsilon| < \delta$\textsuperscript{20,22}

**Tunneling current** — The quantum dot is strongly coupled to a grounded substrate\textsuperscript{23} so the current is entirely determined by the transmission of electrons through the point contact. The current operator $I$ is given by the commutator

$$I = \frac{ie}{\hbar} \left[ H, \sum_p \psi_p^\dagger \psi_p \right] = \frac{e}{\hbar} \sum_{n,p} \left( \lambda_n^c \psi_n^c \psi_p^\dagger - \lambda_n^p \psi_n^\dagger \psi_p \right). \quad (5)$$

We calculate the time-averaged expectation value of $I$ using the Keldysh path integral technique\textsuperscript{23,24} which has previously been applied to the SYK model in Refs.\textsuperscript{12,13,15,28} The expectation value $I$ of the tunneling current is given by the first derivative of cumulant generating function\textsuperscript{28}

$$I = -i \lim_{\chi \to 0} \frac{\partial}{\partial \chi} \ln Z(\chi), \quad (6)$$

$$Z(\chi) = \langle \mathcal{T}_C \exp \left( -i \int_C dt \left[ H + \frac{1}{2} \chi(t) I \right] \right) \rangle. \quad (7)$$

Here $T_C$ indicates time-ordering along the Keldysh contour\textsuperscript{24} of the counting field $\chi(t)$, equal to $+\chi$ on the forward branch of the contour (from $t = 0$ to $t = \infty$) and equal to $-\chi$ on the backward branch (from $t = \infty$ to $t = 0$). The calculation is worked out in the Appendix.

The result for the differential conductance is

$$G = \frac{dI}{dV} = \frac{e^2}{\hbar} \int_{-\infty}^{+\infty} d\varepsilon f'(\varepsilon - eV) \frac{4\Gamma \text{Im} G^R(\varepsilon)}{|1 + i\Gamma G^R(\varepsilon)|^2}, \quad (8)$$

where $f(\varepsilon) = (1 + e^{\beta \varepsilon})^{-1}$ is the Fermi function. Substitution of the conformal Green’s function\textsuperscript{23} gives upon integration the finite temperature curves plotted in Fig.\textsuperscript{2}

At zero temperature $f'(\varepsilon - eV) \to -\delta(\varepsilon - eV)$ and substitution of Eq.\textsuperscript{4} produces a single-parameter function of $\xi = eV J/\Gamma^2$,

$$G(\xi) = \frac{e^2}{\hbar} \frac{2\sqrt{2}}{\sqrt{2 + \pi^{1/4} \xi^{-1/2} + \pi^{-1/4} \xi^{1/2}}}. \quad (9)$$

**Low-high voltage duality** — The $T = 0$ differential conductance\textsuperscript{1} in the conformal regime $J/N \ll eV \ll J$ satisfies the duality relation

$$G(\xi) = G(\pi/\xi), \quad \text{if } N^{-1}(J/\Gamma)^2 \ll \xi, \pi/\xi \ll (J/\Gamma)^2. \quad (10)$$

The $V$-to-$1/V$ duality is visible in the semi-logarithmic scale of Fig.\textsuperscript{2} by a reflection symmetry of the differential conductance along the $\xi = \sqrt{\pi}$ axis. The symmetry is precise at $T = 0$, and is broken in the tails with increasing temperature.

The voltage range in which $V$ and $1/V$ are related by Eq.\textsuperscript{10} covers the full conformal regime for $N \simeq (J/\Gamma)^4$. In this voltage range the $1/\sqrt{V}$ tail at high voltages crosses over to a $\sqrt{V}$ decay at low voltages. The high-voltage tail reproduces the $1/\sqrt{V}$ differential conductance that follows\textsuperscript{8,10} from the density of states in the limit $\Gamma \to 0$ (since $\xi \to \infty$ for $\Gamma \to 0$). The density of states gives\textsuperscript{8,22} a crossover to a $\sqrt{V}$ decay when $eV$ drops below the single-particle level spacing $\delta \simeq J/N$. Our finite-$\Gamma$ result\textsuperscript{11} implies that this crossover already sets in at larger voltages $eV \simeq \Gamma^2/J$, well above $\delta$ for $N \gg (J/\Gamma)^2$.

The symmetrically peaked profile of Fig.\textsuperscript{2} is a signature of conformal symmetry in as much as this produces a power-law singularity in the retarded propagator at low $\varepsilon$. 

**FIG. 2:** Differential conductance $G = dI/dV$ calculated from Eq. (9) as a function of dimensionless voltage $\xi = eV J/\Gamma^2$ for three different temperatures. On the semi-logarithmic scale the duality between low and high voltages shows up as a reflection symmetry along the dotted line (where $\xi = \sqrt{\pi}$).
energies. It is not specific for the square-root singularity\cite{[54x659]}, other exponents would give a qualitatively similar low-high voltage duality. For example, the generalized SYK$_p$ model with $2p \geq 4$ interacting Majorana fermion terms has a $e^{(1-p)/p}$ singularity\cite{[54x659,13]} corresponding to the duality $G(\xi_p) = G(C_p/\xi_p)$ with $C_p$ a numerical coefficient and $\xi_p = (eV)^{(p-1)/p}J^2/J^2 \Gamma^{-2}$. In contrast, a disordered Fermi liquid such as the non-interacting SYK model, with Hamiltonian $H = \sum_{ij} J_{ij} c_i^\dagger c_j$, has a constant propagator at low energies and hence a constant $dI/dV$ in the range $J/N \ll eV \ll J$.

**Conclusion** — We have shown that tunneling spectroscopy can reveal a low-high voltage duality in the conformal regime of the Sachdev-Ye-Kitaev model of $N$ interacting Dirac fermions. A physical system in which one might search for this duality is the graphene quantum dot in the lowest Landau level, proposed by Chen, Ilan, De Juan, Pikulin, and Franz\cite{[54x659]}. As argued by those authors, one should be able to reach $N$ of order $10^2$ for laboratory magnetic field strengths in a sub-micrometer-size quantum dot. This leaves two decades in the conformal regime $J/N \ll eV \ll J$. If we tune the tunnel coupling strength near the ballistic limit $\Gamma \ll J$, it should be possible even for these moderately large values of $N$ to achieve $N \simeq (J/\Gamma)^4$ and access the duality over two decades of voltage variation. For such large $N$ the condition on temperature, $k_B T \ll \Gamma^2/J$ would then also be within experimental reach ($J \simeq 34$ meV from Ref.\cite{[54x659]} and $\Gamma \simeq 10$ meV has $k_B T = 10^{-2} \Gamma^2/J$ at $T = 300$ mK.

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**Appendix A: Outline of the calculation**

We describe the calculation leading to Eq.\cite{[54x707]} for the current-voltage characteristics, generalizing it to nonzero chemical potential $\mu$ and including also the shot noise power. We set $\hbar$ and $e$ to unity, except for the final formulas.

1. **Generating function of counting statistics**

Arbitrary cumulants of the current operator\cite{[54x659]} can be obtained from the generating function\cite{[54x659]}. A gauge transformation allows us to write equivalently

$$Z(\chi) = \langle T_C \exp\left( -i C H(t) \, dt \right) \rangle, \quad (A1)$$

$$H(t) = H_{\text{SYK}} + \sum_{n,p} \varepsilon_p \psi_p^\dagger \psi_p - \mu \sum_n c_n^\dagger c_n + \sum_n \left( e^{i \chi(t)/\beta} \sum_{\nu} \lambda^\dagger_{\nu} c^\dagger_{\nu} \psi_{\nu} + e^{-i \chi(t)/\beta} \sum_{\nu} \lambda_{\nu} c_{\nu}^\dagger \psi_{\nu} \right). \quad (A2)$$

For generality we have added a chemical potential term $\propto \mu$. (In the main text we take $\mu = 0$, corresponding to a quantum dot at charge neutrality.)

We need the advanced and retarded Green’s functions $G^A(\varepsilon) = (G^R(\varepsilon))^*$ and the Keldysh Green’s function

$$G^K(\varepsilon) = F(\varepsilon) \left( G^R(\varepsilon) - G^A(\varepsilon) \right), \quad F(\varepsilon) = \tanh(\beta\varepsilon/2). \quad (A3)$$

These are collected in the matrix Green’s function $G$, which on the Keldysh contour has the representation\cite{[20,21,22]}

$$G = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix} = L \sigma_3 \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} L^\dagger, \quad (A4)$$

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (A5)$$

in terms of the Green’s functions on the forward and backward branches of the contour:

$$G^{++}(t,t') = -i N^{-1} \sum_n \langle T c_n(t) c_n^\dagger(t') \rangle, \quad (A6a)$$

$$G^{+-}(t,t') = i N^{-1} \sum_n \langle c_n^\dagger(t) c_n(t') \rangle, \quad (A6b)$$

$$G^{-+}(t,t') = -i N^{-1} \sum_n \langle c_n(t) c_n^\dagger(t') \rangle, \quad (A6c)$$

$$G^{--}(t,t') = -i N^{-1} \sum_n \langle T^{-1} c_n(t) c_n^\dagger(t') \rangle. \quad (A6d)$$

The operators $T$ and $T^{-1}$ order the times in increasing and decreasing order, respectively.

2. **Saddle point solution**

In the regime $J/N \ll \varepsilon \ll J$ the Green’s function of the SYK model is given by the saddle point solution\cite{[23]}

$$G^R(\varepsilon) = -i C e^{-i\theta} \sqrt{\frac{\beta}{2\pi J}} \left( \frac{1}{4} - i \frac{\beta \varepsilon}{2\pi} + i \frac{\varepsilon}{\pi} \right)^{1/4}, \quad (A7)$$

with the definitions

$$e^{2\pi \varepsilon} = \frac{\sin\left( \frac{\pi}{4} + \theta \right)}{\sin\left( \frac{\pi}{4} - \theta \right)}, \quad C = \left( \pi / \cos 2\theta \right)^{1/4}. \quad (A8)$$

The angle $\theta \in \left(-\pi/4, \pi/4\right)$ is a spectral asymmetry angle\cite{[23]} determined by the charge per site $Q \in (-1/2, 1/2)$ on the quantum dot according to\cite{[23]}

$$Q = N^{-1} \sum_i \langle c_i^\dagger c_i \rangle - \frac{1}{2} = -\theta - \pi - \frac{1}{4} \sin 2\theta. \quad (A9)$$

For $\mu = 0$, when $Q = 0$, one has $\theta = 0$, $C = \pi^{1/4}$. In good approximation (accurate within 15%),

$$\theta \approx -\frac{1}{2} \pi Q \Rightarrow C \approx \left( \pi / \cos \pi Q \right)^{1/4}. \quad (A10)$$

In the mean-field approach the quartic SYK interaction $\frac{1}{2} \sum_{ij} J_{ij}(\phi) \phi^2$ is replaced by a quadratic one with the kernel $G^{-1}$ from Eq.\cite{[A4]}. A Gaussian integration over the Grassmann fields gives the generating function

$$G^K(\varepsilon) = F(\varepsilon) \left( G^R(\varepsilon) - G^A(\varepsilon) \right), \quad F(\varepsilon) = \tanh(\beta\varepsilon/2). \quad (A3)$$

These are collected in the matrix Green’s function $G$, which on the Keldysh contour has the representation\cite{[20,21,22]}

$$G = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix} = L \sigma_3 \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} L^\dagger, \quad (A4)$$

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (A5)$$

in terms of the Green’s functions on the forward and backward branches of the contour:

$$G^{++}(t,t') = -i N^{-1} \sum_n \langle T c_n(t) c_n^\dagger(t') \rangle, \quad (A6a)$$

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$$G^{-+}(t,t') = -i N^{-1} \sum_n \langle c_n(t) c_n^\dagger(t') \rangle, \quad (A6c)$$

$$G^{--}(t,t') = -i N^{-1} \sum_n \langle T^{-1} c_n(t) c_n^\dagger(t') \rangle. \quad (A6d)$$

The operators $T$ and $T^{-1}$ order the times in increasing and decreasing order, respectively.
\[
\ln Z = \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \ln \left( \frac{\det [1 - \Gamma \Xi(\varepsilon) \Lambda \xi G(\varepsilon) \Lambda]}{\det [1 - \Gamma \Xi G(\varepsilon)]} \right) , \quad \Lambda = \begin{pmatrix} \cos(\chi/2) & i\sin(\chi/2) \\ i\sin(\chi/2) & \cos(\chi/2) \end{pmatrix} , \quad \Xi(\varepsilon) = -i \begin{pmatrix} 1 & 2F(\varepsilon - V) \\ 0 & -1 \end{pmatrix} . \quad (A11)
\]

The matrix \( \Xi(\varepsilon) \) is the Keldysh Green’s function of the lead, integrated over the momenta. This evaluates further to
\[
\ln Z = \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \ln \left[ 1 + \frac{i\Gamma (G^R - G^A)}{(1 + i\Gamma G^R) (1 - i\Gamma G^A)} \left( [1 - F(\varepsilon)F(\varepsilon - V)] (\cos \chi - 1) + i [F(\varepsilon) - F(\varepsilon - V)] \sin \chi \right) \right] . \quad (A12)
\]

At zero temperature the distribution function simplifies to \( F(\varepsilon) \rightarrow \text{sgn}(\varepsilon) \), hence
\[
\ln Z = \int_0^V \frac{d\varepsilon}{2\pi} \ln \left[ 1 + \frac{2i\Gamma (G^R - G^A) (e^\chi - 1)}{(1 + i\Gamma G^R) (1 - i\Gamma G^A)} \right] . \quad (A13)
\]

3. Average current and shot noise power

A \( p \)-fold differentiation of \( Z(\chi) \) with respect to \( \chi \) gives the \( p \)-th cumulant of the current. In this way the full counting statistics of the charge transmitted through the quantum dot can be calculated. The first cumulant, the time-averaged current \( I \) from Eq. (6), is given by
\[
I = \frac{e}{h} \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi} \frac{i\Gamma [F(\varepsilon) - F(\varepsilon - V)] (G^R - G^A)}{(1 + i\Gamma G^R) (1 - i\Gamma G^A)} , \quad (A14)
\]
which is Eq. (8) from the main text.

At zero temperature the differential conductance \( G = dI/dV \) is
\[
G(\xi) = \frac{2e^2}{h} \left[ 1 + \frac{1}{2\sin(\pi/4 + \theta)} \left( \frac{\sqrt{\xi}}{C} + \frac{C}{\sqrt{\xi}} \right) \right]^{-1} . \quad (A15)
\]

with \( \xi = eV J/T^2 \). The duality relation
\[
G(\xi) = G(C^2/\xi) \quad (A16)
\]
reduces to the one from the main text, \( G(\xi) = G(\pi/\xi) \), when we set \( \mu = 0 \Rightarrow \theta = 0 \Rightarrow C = \pi^{1/4} \).

The second cumulant, the shot noise power \( P \), follows similarly from
\[
P = -\lim_{\chi \to 0} \frac{\partial^2}{\partial \chi^2} \ln Z(\chi) . \quad (A17)
\]
It has the same one-parameter scaling and duality as \( G \). The fact that higher order cumulants of the current have the same scaling as the differential conductance is a consequence of the single-point-contact geometry, with a single counting field \( \chi(t) \). This does not carry over to a two-point-contact geometry.

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