1/4 Partial Breaking of Global Supersymmetry and New Superparticle Actions

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Abstract

We construct the worldline superfield massive superparticle actions which preserve 1/4 portion of the underlying higher-dimensional supersymmetry. The rest of supersymmetry is spontaneously broken and realized by nonlinear transformations. We consider the cases of $N = 4 \rightarrow N = 1$ and $N = 8 \rightarrow N = 2$ partial breaking. In the first case we present the corresponding Green-Schwarz type target superspace action with one $\kappa$-supersymmetry. It is related to the superfield action via a field redefinition. In the second case we find out two possible models, one of which is a direct generalization of the $N = 4 \rightarrow N = 1$ case, while another is essentially different. For the first model we formulate Green-Schwarz type action with two $\kappa$-supersymmetries. We elaborate on the bosonic part of the superfield action for the second model and find that only in two special limits it takes the standard Nambu-Goto form. In the general case it is determined by a fourth-order algebraic equation. The characteristic common feature of these new superparticle models is that the algebras of their spontaneously broken supersymmetries are non-trivial truncations of the general extensions of $N = 1$ and $N = 2$ Poincaré $D = 4$ superalgebras by tensorial central charges.

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1 Introduction

The most attractive feature of the description of superbranes based upon the concept of partial spontaneous breaking of global supersymmetry (PBGS) [1] - [4] is the manifest off-shell realization of the corresponding worldvolume supersymmetry. In this approach, the physical worldvolume multiplets of the given superbrane are represented by superfields given on the proper worldvolume superspace. They are interpreted as Goldstone superfields realizing spontaneous breaking of the full brane supersymmetry group down to its unbroken worldvolume subgroup. The spontaneously broken supersymmetry is realized on the Goldstone superfields by inhomogeneous and nonlinear transformations. The invariant Goldstone superfields actions, after passing to components and, in general, eliminating auxiliary fields by their equations of motion, coincide with gauge-fixed forms of the relevant Green-Schwarz type actions.

Until now, only the examples of 1/2 breaking of supersymmetry corresponding to the standard BPS p-branes and D-branes were treated in the literature on the PBGS. On the other hand, recently there was some interest in the 1/4 and other unusual fractional PBGS options, mainly caused by the existence of the $D = 11$ supergravity solutions which preserve various fractions of the underlying 32 supersymmetries and admit a nice interpretation in terms of intersecting branes (see, e.g., [15]-[20] and references therein).

In view of this, it seems interesting to extend the manifestly worldvolume supersymmetric PBGS description of branes to the 1/4 breaking and other fractional patterns. In the present paper, as first steps toward this goal, we consider several examples of the 1/4 PBGS superfield actions in the simplest case of massive superparticle.

The construction of the Goldstone superfield action is the most difficult part of the PBGS approach. The generic methods of nonlinear realizations [21]-[23] which nicely work in the case of standard internal symmetry and space-time groups prove to be not too helpful when trying to employ them for constructing PBGS invariants. All the known Goldstone superfield Lagrangians are of the Chern-Simons or WZNW type, in the sense that they are not tensors with respect to the hidden supersymmetry transformations. The latter shift them by a full derivative, thus leaving the action invariant up to surface terms. As the result, one cannot directly apply the powerful method of covariant Cartan forms for constructing invariant actions.

A way around this difficulty was proposed in [8, 9] and used to construct the $N = 1$, $D = 4$ superfield actions providing the PBGS description of $N = 2$ D3-superbrane and super 3-brane. It is based on the idea of embedding the basic Goldstone superfield into some linear multiplet of the underlying supersymmetry group. Initially this multiplet comprises a set of independent worldvolume superfields. After imposing appropriate covariant constraints one succeeds in expressing all these superfields in terms of the basic Goldstone ones. One of the superfields of the initial linear representation is shifted by a full derivative under the broken supersymmetry transformations and so can be chosen as the Goldstone superfields Lagrangian.

This procedure was further elaborated and applied to other cases of PBGS in refs. [11], [12], [14]. However, its generality and usefulness are obscured by the fact that choosing the linear representation to start with and picking up appropriate constraints are a sort of guess-work. This guess-work becomes rather intricate in more complicated cases like the 1/4 PBGS patterns we are interested in here.

In order to construct the Goldstone superfield actions for this case we propose a more
systematic version of the above procedure, following the general method of setting up linear representations of supersymmetry in terms of the appropriate nonlinear realizations [24, 25]. Though this method was worked out originally for the case of total breaking of supersymmetry, it can be straightforwardly extended to the PBGS case. Its main advantage consists in that it allows one to avoid seeking for the constraints on the original linearly transforming superfields. They are expressed through the Goldstone superfields by making use of some algorithmic algebraic computation.

In Sect. 2 we explain this method on the toy examples of the $N = 2 \rightarrow N = 1$ and $N = 4 \rightarrow N = 2$ PBGS in $d = 1$ which were studied earlier from a different point of view in [3, 4, 5, 6]. Then in Sect. 3 we apply it to construct a new model of massive superparticle with the 1/4 PBGS pattern $N = 4 \rightarrow N = 1$. We find that the corresponding $N = 4, d = 1$ superalgebra does not follow by a dimensional reduction from the standard $N = 1$ Poincaré superalgebra in $D = 4$. It is an extension of the latter by the Lorentz tensorial central charges (see [21, 27, 20, 19] and references therein) which yields this $N = 4, d = 1$ superalgebra via dimensional reduction. In Sect. 4 we give the corresponding Green-Schwarz type action revealing just one local $\kappa$-symmetry and establish the precise correspondence between the PBGS and Green-Schwarz actions. In Sect. 5 we construct, using analogous methods, the superparticle model realizing the PBGS $N = 8 \rightarrow N = 2$. We find that there exist two versions of such model. They differ in the superfield contents and in the structure of the physical bosons action. Only in the first case this action has the standard static gauge Nambu-Goto form of the massive particle action. We present the Green-Schwarz type action for this case, with two independent $\kappa$ symmetries.

2 Two examples of 1/2 PBGS in d=1

2.1 N=2 → N=1

To explain the key features of our approach we start from a simplest example of $N = 2 \rightarrow N = 1$ partial breaking of global supersymmetry in $d = 1$.

The anticommutation relation of $N = 1, d = 1$ Poincaré superalgebra reads

$$\{Q, Q\} = 2P.$$ (2.1)

We wish to construct a $N = 1$ superfield action which is also invariant with respect to one extra spontaneously broken $d = 1$ supersymmetry. Thus our basic objects will be superfields given on $N = 1, d = 1$ superspace with the coordinates \{t, \theta\}, (\bar{t} = -t, \bar{\theta} = \theta) \]

Let us define the fermionic and bosonic superfields $\psi(t, \theta)$ and $v(t, \theta)$ related as

$$\psi = \frac{1}{2} Dv, \quad (\bar{\psi} = \psi, \bar{v} = -v),$$ (2.2)

where $D$ is the spinor covariant derivative

$$D = \frac{\partial}{\partial \theta} + \theta \partial_t, \quad \{D, D\} = 2\partial_t.$$ (2.3)

\footnote{Henceforth, we use the following convention for the evolution parameter and action: $t = it'$ and $S = iS'$, where $t'$ and $S'$ are the standard real time and action.}
If we now introduce an additional, arbitrary for the moment, real spinor superfield \( \Upsilon(t, \theta) \) then it becomes possible to realize an extra \( N = 1, d = 1 \) supersymmetry on the spinors \( \psi \) and \( \Upsilon \). Assuming the second supersymmetry to be spontaneously broken, the linear transformation laws can be written as
\[
\delta \psi = \epsilon (1 - D \Upsilon) , \quad \delta \Upsilon = \epsilon D \psi .
\] (2.4)
The presence of the constant shift in the transformations (2.4) just means the spontaneous breaking of this supersymmetry and suggests the interpretation of \( \psi \) as the Goldstone fermionic superfield accompanying the linear realization of this breaking. One can check that the generator \( S \) of the transformations (2.4) forms the \( N = 1, d = 1 \) superalgebra
\[
\{ S, S \} = 2 \tag{2.5}
\]
and anticommutates with the generator \( Q \) of the manifest \( N = 1 \) supersymmetry (2.1) in the given realization. However, from (2.4) one can also extract the transformation law of the scalar superfield \( v \):
\[
\delta v = -2 \epsilon (\theta - \Upsilon) , \quad \delta \Upsilon = \frac{1}{2} \epsilon \partial_t v .
\] (2.6)
Now, the brackets of the manifest and second \( N = 1 \) supersymmetries yield a constant shift of the superfield \( v \), which means that a central charge \( Z \) appears in the anticommutators of these two \( N = 1 \) supersymmetries and \( v \) is the bosonic Goldstone superfield associated with the broken central charge transformations
\[
\{ Q, S \} = 2Z .
\] (2.7)

Thus, as long as we limit our attention to the Goldstone fermionic superfield only, the full supersymmetry algebra is given by (2.1), (2.5) with no need for central charge. Imposing the relation (2.2) introduces the active central charge as in (2.7) with \( v \) as the corresponding Goldstone superfield. As we shall see soon, these options lead to PBGS theories with different physical fields contents.

Let us note that the field \( \Upsilon \) is a good candidate for the Lagrangian density. Indeed, the integral
\[
I = \int dt d\theta \ \Upsilon \tag{2.8}
\]
is manifestly invariant with respect to the first \( N = 1 \) supersymmetry. It is also invariant with respect to the second supersymmetry because the integrand is shifted by a spinor derivative under the variation (2.4).

\[\text{We should stress that hereafter the superalgebras we deal with are those of the superfield variations. It is sufficient to consider them for our aim of constructing invariant actions. The superalgebras of charges and supercharges constructed from the PBGS actions by Noether procedure are different: they inevitably include some constant central charges which are crucial for evading [1, 2] the famous Witten’s no-go theorem [28]. In particular, such a central charge will appear as a shift of \( P \) in the r.h.s. of the anticommutator (2.7) of generators of the spontaneously broken \( S \) supersymmetry. These central charges do not produce any transformation on the fields, as opposed to another, “active” sort of central charges which generate actual symmetries and are of heavy use throughout this paper.}\]
If the superfield $\Upsilon$ is regarded as independent, the “action” (2.8) is of course meaningless. It becomes meaningful after expressing $\Upsilon$ in terms of the Goldstone superfield $\psi$. This can be performed in a covariant way, resorting to the method of [24, 25].

As the first step one finds the finite transformations of the second supersymmetry for the basic superfields $\psi, \Upsilon$ which for the case at hand just coincide with (2.4).

Next, one introduces the Goldstone fermion of nonlinear realization $\eta$ having the following transformation law under the second supersymmetry:

$$\delta \eta = \epsilon + \epsilon \eta \partial_t \eta.$$  (2.9)

This object, together with $N = 1$ superspace coordinates, parametrize the supersymmetry group manifold. The infinitesimal transformation law (2.9) correspond to the standard group composition law for the exponential parametrization of the supergroup element.

The crucial step is to substitute $(-\eta)$ for the parameter $\epsilon$ in the r.h.s. of (2.4) and to define new superfields

$$\tilde{\psi} = \psi - \eta (1 - D\Upsilon) , \quad \tilde{\Upsilon} = \Upsilon - \eta D\psi .$$  (2.10)

One can check that the newly defined superfields $\tilde{\psi}, \tilde{\Upsilon}$ transform homogeneously and independently of each other under the second spontaneously broken supersymmetry [24]:

$$\delta \tilde{\psi} = \epsilon \eta \partial_t \tilde{\psi} , \quad \delta \tilde{\Upsilon} = \epsilon \eta \partial_t \tilde{\Upsilon} .$$  (2.11)

Thus it is the covariant operation to put these superfields equal to zero

$$\tilde{\psi} = 0 , \quad \tilde{\Upsilon} = 0 .$$  (2.12)

These constraints imply the following set of equations for the original superfields $\psi, \eta, \Upsilon$

$$\psi - \eta (1 - D\Upsilon) = 0 , \quad \Upsilon - \eta D\psi = 0 ,$$  (2.13)

providing the manifestly covariant way of expressing them in terms of a few basic ones. As such one can take either the Goldstone fermion of the nonlinear realization, or that of the linear realization. The solution to eqs. (2.13) is given by

$$\Upsilon = \frac{2\psi D\psi}{1 + \sqrt{1 - 4D\psi D\psi}} = \frac{\eta D\eta}{1 + \sqrt{1 - 4\eta D\eta}} ,$$  (2.14)

$$\eta = \frac{2\psi}{1 + \sqrt{1 - 4D\psi D\psi}} .$$  (2.15)

We see that eq. (2.14) gives the expression of $\Upsilon$ through the Goldstone superfields, while eq. (2.13) is just the equivalency relation between the two types of the Goldstone fermionic superfields.

We should stress that the transformation properties of the Goldstone fermion $\eta$ (2.9) actually follow from their expressions (2.13) and the known transformation properties of $\psi, \Upsilon$ (2.4). Therefore we do not need to know them beforehand.

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3In principle, one could choose an arbitrary constant as the r.h.s. of the second constraint. This does not influence our further consideration.
Finally, after the substitution of the expression (2.14) and using (2.2) the integral (2.8) yields the sought Goldstone superfield action

\[ S_v = \int dt d\theta \Psi \equiv \frac{1}{2} \int dt d\theta \frac{\partial_t v Dv}{1 + \sqrt{1 - (\partial_t v)^2}}. \] (2.16)

For the physical bosonic component \( v|_{\theta=0} \) one obtains just the \( D = 2 \) massive particle action in the static gauge:

\[ S_v^{bos} = \frac{1}{2} \int dt \left( 1 - \sqrt{1 - (\partial_t v)^2} \right). \] (2.17)

Thus, the action (2.16) can be naturally interpreted as a manifestly \( N = 1, d = 1 \) supersymmetric worldline action of the superparticle in \( D = 2 \).

Note that the action (2.16) is related by a simple field redefinition to the \( D = 2 \) superparticle action which was constructed in [4] proceeding from the standard nonlinear realizations approach. It was also obtained in [5] as the low-energy collective coordinates action in some two-dimensional solitonic models. We used this system just as a simple illustration of our approach to the construction of the PBGS Goldstone superfields actions.

Finally, we would like to mention that there exists one more possibility for the Goldstone superfield action. Namely, one can consider \( \psi \) as an independent superfield, without assuming the “prepotential” representation (2.2). This choice corresponds to the \( N = 2 \) supersymmetry algebra with \( Z = 0 \) in (2.7). In this case the off-shell worldline Goldstone multiplet consists of the physical fermion \( \psi|_{\theta=0} \) and an auxiliary bosonic field \( D\psi|_{\theta=0} \), i.e. it includes no physical boson at all. Such an exotic structure is of course an artifact of the one-dimensional case where supersymmetry in general does not require matching of the on-shell fermionic and bosonic degrees of freedom. The corresponding Goldstone action is still given by (2.16), with \( \psi \) standing for \( \frac{1}{2}Dv \). The auxiliary field \( D\psi \) vanishes by its equations of motion like in the free case (with \( \psi D\psi \) as the Lagrangian), and the action of the physical fermion turns out to be just the \( N = 1, d = 1 \) Volkov-Akulov action [29]. It corresponds to the total breaking of \( N = 1, d = 1 \) \( S \) supersymmetry. Any trace of unbroken \( Q \) supersymmetry (2.18) disappears on shell. Moreover, this \( d = 1 \) Volkov-Akulov action coincides with the free one.

### 2.2 \( N=4 \to N=2 \)

For completeness we briefly consider one more simple example of PBGS in \( d = 1 \), namely the \( N = 4 \to N = 2 \) one.

The \( N = 2, d = 1 \) Poncaré superalgebra without central charges reads

\[ \{Q, Q\} = \{Q, \bar{Q}\} = 0, \quad \{Q, \bar{Q}\} = P. \] (2.18)

Our basic objects will be the fermionic and bosonic \( N = 2, d = 1 \) superfields \( \psi(t, \theta, \bar{\theta}), \bar{\psi}(t, \theta, \bar{\theta}) \) and \( \rho(t, \theta, \bar{\theta}) \) where \( \{t, \theta, \bar{\theta}\} \) are \( N = 2, d = 1 \) superspace coordinates. We assume that \( \rho \) is a real superfield while \( \psi, \bar{\psi} \) are related to it by

\[ \psi = -\frac{1}{2} \bar{D}\rho, \quad \bar{\psi} = \frac{1}{2} D\rho. \] (2.19)
where the spinor covariant derivatives are defined by
\[
D = \frac{\partial}{\partial \theta} + \frac{1}{2} \bar{\theta} \partial_t, \quad \bar{D} = \frac{\partial}{\partial \bar{\theta}} + \frac{1}{2} \theta \partial_t, \quad \{ D, \bar{D} \} = \partial_t, \quad D^2 = \bar{D}^2 = 0 .
\] (2.20)

By construction, the fermionic superfields are chiral (antichiral)
\[
\bar{D} \psi = 0, \quad D \bar{\psi} = 0 .
\] (2.21)

After introducing an additional scalar superfield \( \Phi(t, \theta, \bar{\theta}) \) one can realize an extra \( N = 2, d = 1 \) supersymmetry on the spinors \( \psi, \bar{\psi} \) and scalar \( \Phi \) (the second supersymmetry with the generators \( S, \bar{S} \) is supposed to be spontaneously broken):
\[
\delta \psi = \epsilon \left( 1 - \bar{D} D \Phi \right), \quad \delta \bar{\psi} = \bar{\epsilon} \left( 1 + D \bar{D} \Phi \right), \quad \delta \Phi = \epsilon \bar{\psi} - \bar{\epsilon} \psi .
\] (2.22)

The transformation law of the scalar superfield \( \rho \) reads:
\[
\delta \rho = 2 \left( \theta \bar{\epsilon} - \bar{\theta} \epsilon + D \Phi \epsilon + \bar{D} \Phi \bar{\epsilon} \right) .
\] (2.23)

Once again, due to the explicit presence of \( \theta, \bar{\theta} \) in (2.23), the brackets of the manifest and second \( N = 2 \) supersymmetries yield a constant shift of the superfield \( \rho \), and therefore a central charge \( Z \) appears in the anticommutators of these two \( N = 2 \) supersymmetries
\[
\{ S, \bar{S} \} = P , \quad \{ Q, \bar{S} \} = 2Z , \quad \{ \bar{Q}, S \} = 2Z .
\] (2.24)

The field \( \Phi \) can be treated as the Lagrangian density and the action
\[
S = \int dt d^2 \theta \Phi
\] (2.25)
is manifestly invariant with respect to both \( N = 2 \) supersymmetries. To express \( \Phi \) in terms of the Goldstone superfields \( \psi, \bar{\psi} \) we use the same method \[24, 25\] as in the previous subsection.

The finite transformations of the second supersymmetry for the basic superfields \( \psi, \bar{\psi}, \Phi \) can be easily computed
\[
\Delta \psi = \epsilon \left( 1 - \bar{D} D \Phi \right) + \frac{1}{2} \epsilon \bar{\epsilon} \partial_t \psi, \quad \Delta \bar{\psi} = \bar{\epsilon} \left( 1 + D \bar{D} \Phi \right) - \frac{1}{2} \epsilon \bar{\epsilon} \partial_t \bar{\psi}, \\
\Delta \Phi = \epsilon \bar{\psi} - \bar{\epsilon} \psi + \epsilon \bar{\epsilon} \left( 1 + \frac{1}{2} [D, \bar{D}] \Phi \right) .
\] (2.26)

Then one introduces the Goldstone fermions of nonlinear realization \( \eta, \bar{\eta} \) with the following transformation laws under the second supersymmetry:
\[
\delta \eta = \epsilon + \frac{1}{2} (\epsilon \bar{\eta} + \bar{\epsilon} \eta) \partial_t \eta, \quad \delta \bar{\eta} = \bar{\epsilon} + \frac{1}{2} (\epsilon \bar{\eta} + \bar{\epsilon} \eta) \partial_t \bar{\eta} .
\] (2.27)

\footnote{This multiplet is a \( d = 1 \) reduction of chiral \( N = 1, D = 4 \) multiplet.}
and substitutes \((-\eta, -\bar{\eta})\) for the parameters \((\epsilon, \bar{\epsilon})\) in the r.h.s. of (2.26) to define the new superfields

\[
\tilde{\psi} = \psi - \eta \left(1 - \bar{D}D\Phi\right) + \frac{1}{2}\eta\bar{\eta}\partial_t \psi, \quad \tilde{\bar{\psi}} = \bar{\psi} - \bar{\eta} \left(1 + D\bar{D}\Phi\right) - \frac{1}{2}\eta\bar{\eta}\partial_t \bar{\psi},
\]

\[
\tilde{\Phi} = \Phi - \eta\bar{\psi} + \bar{\eta}\psi + \eta\bar{\eta} \left(1 + \frac{1}{2} [D, \bar{D}] \Phi\right).
\] (2.28)

Like in the previous example they transform independently of each another and homogeneously under the spontaneously broken supersymmetry.

As the last step one puts these superfields equal to zero

\[
\tilde{\psi} = 0, \quad \tilde{\bar{\psi}} = 0, \quad \tilde{\Phi} = 0.
\] (2.29)

The solution to eqs. (2.29) is given by

\[
\Phi = \frac{2\psi\bar{\psi}}{1 + \sqrt{1 + 4D\psi\bar{D}\psi}} = \frac{\eta\bar{\eta}}{1 + DD(\eta\bar{\eta})},
\]

\[
\eta = \frac{\psi}{1 - \bar{D}D\Phi} + \frac{1}{2}\psi\bar{\psi}\partial_t \psi \quad \bar{\eta} = \frac{\bar{\psi}}{1 + D\bar{D}\Phi} - \frac{1}{2}\psi\bar{\psi}\partial_t \bar{\psi} \quad \frac{1}{1 + \bar{D}D(\bar{\eta}\eta)}.
\] (2.30)

Note that the Goldstone fermions \(\eta, \bar{\eta}\) obey the covariant chirality conditions:

\[
\bar{D}\eta + \frac{1}{2}\bar{\eta}\bar{D}\eta\partial_t \eta = 0, \quad D\bar{\eta} + \frac{1}{2}\bar{\eta}D\bar{\eta}\partial_t \bar{\eta} = 0
\] (2.32)

which are equivalent to the ordinary chirality conditions (2.21) for \(\psi, \bar{\psi}\).

After the substitution of the expression (2.30) and using (2.19) the action (2.25) takes the form

\[
S_{\rho} = \int dt d^2\theta \frac{\bar{D}\rho D\rho}{1 + \sqrt{1 + (\partial_t \rho)^2} (\bar{D}D\rho)}.
\] (2.33)

For the physical bosonic component \(\rho|_{\theta=0}\) one obtains just the \(D = 2\) massive particle action in the static gauge:\footnote{We use the convention \(\int d^2\theta \equiv \int \bar{D}D\).}

\[
S_{\rho}^{\text{bos}} = \frac{1}{2} \int dt \left(1 - \sqrt{1 + (\partial_t \rho)^2}\right).
\] (2.34)

Thus, the action (2.33) can be naturally interpreted as a manifestly \(N = 2, d = 1\) supersymmetric worldline action of the superparticle in \(D = 2\). Note that the auxiliary field in \(\rho\) is vanishing on shell.

Like in other PBGS theories, in our case the Goldstone fermions can be placed into different multiplets of unbroken \(N = 2, d = 1\) supersymmetry. Instead of the real scalar superfield we can choose chiral-anti-chiral bosonic superfields \(\phi, \bar{\phi}\) as the basic Goldstone ones:

\[
\psi = -\frac{1}{2}D\phi, \quad \bar{\psi} = \frac{1}{2}D\bar{\phi}, \quad D\phi = D\bar{\phi} = 0.
\] (2.35)
The transformation properties of \( \phi, \bar{\phi} \) with respect to the spontaneously broken supersymmetry are given by

\[
\delta \phi = 2\epsilon \left( \bar{\theta} - D\Phi \right), \quad \delta \bar{\phi} = -2\bar{\epsilon} \left( \theta + \bar{D}\Phi \right), \quad \delta \Phi = \frac{1}{2} \left( \epsilon D\bar{\phi} + \bar{\epsilon} D\phi \right).
\] (2.36)

From (2.36) one finds that the brackets of manifest and spontaneously broken supersymmetry contain a complex central charge \( Z \):

\[
\{ Q, \bar{S} \} = 2Z , \quad \{ \bar{Q}, S \} = 2\bar{Z} ,
\] (2.37)

which is realized as shifts of \( \phi, \bar{\phi} \).

Substituting the expressions for \( \psi, \bar{\psi} \) (2.35) into (2.30), one finds the action

\[
S_\phi = -\frac{1}{2} \int dt d^2\theta \frac{D\phi D\bar{\phi}}{1 + \sqrt{1 + \partial_t \phi \partial_t \bar{\phi}}}.
\] (2.38)

Its bosonic core is the \( D = 3 \) particle action in the static gauge:

\[
S^\text{bos}_\phi = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t \varphi \partial_t \bar{\varphi}} \right),
\] (2.39)

where \( \varphi = \phi|_{\theta=0}, \bar{\varphi} = \bar{\phi}|_{\theta=0} \). Therefore, (2.38) is a manifestly \( N = 2, d = 1 \) supersymmetric worldline action of the massive \( N = 2 \) superparticle in \( D = 3 \) \(^\text{6} \).

Note that this action was deduced earlier in \([6]\) using a different approach. It was demonstrated there that the corresponding component action is related through a field redefinition to a gauge-fixed form of the relevant Green-Schwarz type action (in the static gauge and with \( \kappa \) symmetry fully used to gauge away two out of four target spinor coordinates).

In the considered case, like in the previous one, there exists one more possibility for the Goldstone superfield action, with \( \psi, \bar{\psi} \) being basic chiral superfields unrelated to any additional bosonic ones. In this case we once again end up with a \( d = 1 \) Volkov-Akulov action for the complex Goldstone fermionic field, such that it is reduced to the free action via the suitable field redefinition.

\section{N=4 \( \rightarrow \) N=1 PBGS}

The conventional way of dealing with the PBGS phenomenon is to start from some supersymmetry algebra, to construct its appropriate nonlinear realization and then to look for the invariant action. As was already mentioned in Introduction, the nonlinear realization formalism is not too helpful in what concerns the last problem. Also, there are no clear principles as to which version of the given Poincaré supersymmetry should be actually chosen as the point of departure: with or without central charges, how many such charges should be taken into account, etc. For instance, an attempt to describe 1/4 breaking of \( N = 1 \) Poincaré supersymmetry in \( d = 11 \) along the standard lines \([13]\) leads to very strong constraints on some

\( ^6 \)To avoid a possible confusion, recall that \( N = 2, D = 3 \) Poincaré supersymmetry has 4 fermionic generators and it is just \( N = 4 \) superalgebra with two central charges from the \( d = 1 \) perspective.
of the involved Goldstone superfields. Similar difficulties come out when trying to apply the
standard formalism for describing the $1/4$ PBGS pattern $N = 2, d = 4 \to N = 1, d = 3$ \cite{2}. In this and next Sections we adhere to a somewhat different point of view already accounted
for in the previous Section. Namely, we start with some “probe” linear representation of the
spontaneously broken supersymmetry realized on a set of world-line superfields involving
the needed number of Goldstone fermions. Then we restore the full structure of the supersymmetry
algebra by studying the closure of supersymmetry transformations. After this we express
the linearly transforming superfields in terms of the Goldstone ones, applying the procedure
explained in the previous Section (or its modifications). Finally, we construct the invariant off-shell Goldstone superfield action like in the previous cases as a world-line superspace integral
of the appropriate $d = 1$ superfield component of the original linear representation.

We firstly apply this general strategy to the case of $N = 4 \to N = 1$ PBGS. Our goal is
to construct a $N = 1, d = 1$ superfield action which would respect three extra spontaneously
broken supersymmetries. Thus, the minimal multiplet should include at least three $N = 1$
fermionic Goldstone superfields $\psi^i(t, \theta)(i = 1, 2, 3)$. Without loss of generality these superfields
can be chosen to form a triplet with respect to some $SO(3)$ automorphism group. As their basic
property, they should have inhomogeneous transformation laws with respect to the broken
supersymmetries, i.e. their transformations should start with the corresponding Grassmann
parameters. One can check that this requirement is met in a minimal way and the algebra of
transformations gets closed at cost of adding one additional fermionic $N = 1$ superfield $\Upsilon(t, \theta)$. The transformations under the broken supersymmetries read

$$\delta \psi^i = \epsilon^i (1 - D \Upsilon) - \epsilon^{ijk} \epsilon^j D \psi^k , \quad \delta \Upsilon = \epsilon^i D \psi^i . \quad (3.1)$$

They form the following algebra

$$\{Q, Q\} = 2P , \quad \{S^i, S^j\} = 2\delta^{ij}P , \quad \{Q, S^i\} = 0 . \quad (3.2)$$

The fermionic $N = 1, d = 1$ superfields contain no bosonic degrees of freedom of physical
dimension. Since we wish to have a superparticle model, with the world-line scalar $N = 1$
multiplets containing such bosonic fields, we are led to introduce bosonic superfields $v^i$

$$\psi^i = \frac{1}{2} D v^i , \quad (\bar{\psi}^i = \bar{\psi}^i , \bar{v}^i = -v^i) . \quad (3.3)$$

Their transformation properties can be extracted from (3.1):

$$\delta v^i = -2 \epsilon^i (\theta - \Upsilon) + \epsilon^{ijk} \epsilon^j D v^k , \quad \delta \Upsilon = \frac{1}{2} \epsilon^i \partial_t v^i . \quad (3.4)$$

Like in the $N = 4 \to N = 2$ case, due to the explicit presence of $\theta$ in the transformations (3.4),
the anticommutators of the manifest $Q$ and spontaneously broken $S^i$ supersymmetry generators
acquire an active central charges $Z^i$ in the right-hand side. Finally, the basic anticommutation
relations extracted from the above superfield variations read

$$\{Q, Q\} = 2P , \quad \{S^i, S^j\} = 2\delta^{ij}P , \quad \{Q, S^i\} = 2Z^i . \quad (3.5)$$
The central charge generators act as pure shifts of $v^i$, suggesting the interpretation of $v^i$ as Goldstone superfields parametrizing transverse directions in a four-dimensional space where $Z^i, P$ act as the translation operators.

Surprisingly, the superalgebra (3.3) cannot be interpreted as a dimensional reduction of the standard $N = 1$ Poincaré superalgebra in $d = 4$, with $Z^i, P$ being the components of full 4-momentum. Moreover, it also cannot be recovered from any one- or two-central charges extension of the $D = 3$ or $D = 2$ Poincaré superalgebras with the relevant Lorentz groups ($SO(1, 2) \sim SL(2, R)$ and $SO(1, 1)$) as the automorphism ones. Indeed, the only automorphism group of (3.3) is $SO(3)$ with respect to which both odd and even generators are split into a singlet and triplet.

Nevertheless, it is still possible to interpret (3.3) in the dimensional reduction language. For this one should proceed not from the standard $N = 1, D = 4$ super Poincaré algebra, but from its extension by tensorial central charges [21, 27, 19]. The generators $Z^i$ turn out to partly come from these central charges and partly from the extra components of 4-momentum. The precise correspondence is given in Appendix.

Let us turn to the issue of constructing invariant action for the system under consideration. Like in the previous case (Sect. 2), we can use $\Upsilon(v)$ as a Lagrangian density,

$$S_v = \int dtd\theta \Upsilon(v), \quad (3.6)$$

in view of its transformation property (3.1). Then the main question is how to covariantly express $\Upsilon$ in terms of $\psi^i$ and, further, $v^i$. One could use just the method of the previous Section. However, it turns out that in the present case it is easier to perform a direct construction of $\Upsilon$.

The idea of this construction is rather simple. In the case at hand there is only one non-nilpotent bosonic dimensionless object $X = D\psi^iD\psi^j$. Therefore the general ansatz for the superfield $\Upsilon$ will contain arbitrary functions of $X$ only. Moreover, the unique objects having positive (one half) dimension (in the length units) are the spinor superfields $\psi^i$. This allows one to write the general ansatz for $\Upsilon$ as

$$\Upsilon = \psi^iD\psi^j A + \psi^2 \psi^i B + \psi^2 D\psi^i \psi^j D\psi^j C + \psi^3 \psi^i D\psi^j E + \psi^3 \psi^2 \psi^j F + \psi^3 \psi^2 D\psi^j G, \quad (3.7)$$

where $A, B, \ldots, G$ are as yet undetermined functions of $X$, and we use the following notations

$$\psi^i_t = \partial_t \psi^i, \quad \psi^2 = \varepsilon^{ijk} \psi^j \psi^k, \quad \psi^3 = \varepsilon^{ijk} \psi^i \psi^j \psi^k. \quad (3.8)$$

Now, using (3.1), (3.7) we can write $\delta \psi^i$ in terms of $\psi^i$ only. Then we can explicitly evaluate $\delta \Upsilon$ and then require it to be equal to $\varepsilon^i D\psi^i$ in accordance with the transformation law (3.1). After rather lengthy calculations we get the system of algebraic equations for the functions $A, \ldots, G$

$$(1 - X A) A = 1 \Rightarrow A = \frac{2}{1 + \sqrt{1 - 4D\psi^i D\psi^i}},$$

$$B = \frac{A^2}{2(A - 2)}, \quad C = -\frac{A^4}{2(A - 2)}, \quad E = \frac{A^3}{A - 2}, \quad F = \frac{A^3(A - 4)}{6(A - 2)^2}, \quad G = -\frac{A^5(A - 4)}{6(A - 2)^2}. \quad (3.9)$$

Thus the integral (3.6) with $\Upsilon$ defined by (3.7), (3.9) provides us with the action for the system realizing the $N = 4 \rightarrow N = 1$ PBGS pattern.
In fact we can greatly simplify this action. First, let us note that the $B$ and $C$ terms in \((3.7)\) can be absorbed (up to full $t$- and $D$- derivatives) into the $F$ term. As the second step, all the remaining $E$, $F$, $G$ terms can be reduced to the single $A$ term, redefining the superfields $v^i$ as follows

$$v^i \rightarrow \phi^i = v^i + \psi^j \varepsilon^{ijk} \psi^k H_1 + \psi^3 \psi^i H_2 + \varepsilon^{ijk} \psi^j \psi^k H_3,$$  \hspace{1cm} (3.10)

where $H_1$, $H_2$, $H_3$ are some functions of $X$. These functions can be given explicitly, but to know the precise expressions is in fact needless for our purpose. The main point is that the action in terms of the redefined bosonic superfield $\phi^i$ takes the very simple form

$$S_\phi = \int dt d\theta \frac{2\xi^i D\xi^i}{1 + \sqrt{1 - 4 D\xi^i D\xi^i}}, \quad \xi^i \equiv \frac{1}{2} D\phi^i.$$  \hspace{1cm} (3.11)

Of course, the transformation properties of the new superfields $\xi^i$, $\phi^i$ essentially differ from \((3.1)\), \((3.4)\), but the action is guaranteed to be invariant by the above construction.

Thus we have found the correct Goldstone superfields action describing the PBGS pattern $N = 4 \rightarrow N = 1$.

Let us end this section by noting that the bosonic core of the action \((3.11)\)

$$S_{bos} = \frac{1}{2} \int dt \left( 1 - \sqrt{1 - \partial_t \phi^i \partial_t \phi^i} \right)$$  \hspace{1cm} (3.12)

is just the standard massive $D = 4$ particle action in the static gauge. It is known to exhibit the hidden nonlinearily realized $D = 4$ Lorentz symmetry $SO(1,3)$. One can ask why this symmetry is present in this action whereas the supersymmetry algebra \((3.3)\) from which we have started respects no such an automorphism. The answer is that the explicit breaking of this Lorentz symmetry occurs just in the fermionic terms of the component action. This becomes transparent in the Green-Schwarz formulation of the same system. Now we turn to describing such a formulation.

### 4 Target space action with one $\kappa$-supersymmetry

To clarify the situation with $N = 4 \rightarrow N = 1$ PBGS, in this section we construct the target space action which possesses only one $\kappa$-supersymmetry and reduces to the action \((3.11)\) in a fixed gauge.

We shall deal with the $N = 4$ superalgebra \((3.3)\). In accordance with the standard strategy of constructing Green-Schwarz type actions (see \[30], \[3], \[31], \[3] for the case of massive superparticles) let us introduce bosonic $X^0(t), Y^i(t)$ and fermionic $\Theta(t), \Psi^i(t)$ $d = 1$ fields, the coordinates of a target $N = 4$ superspace. They have the standard transformation properties under $N = 4$ supersymmetry \((3.3)\)

$$\delta X^0 = -\epsilon \Theta - \epsilon^i \Psi^i, \quad \delta Y^i = -\epsilon^i \Theta - \epsilon \Psi^i, \quad \delta \Theta = \epsilon, \quad \delta \Psi^i = \epsilon^i.$$  \hspace{1cm} (4.1)

Next we construct the supersymmetric invariants $\Pi^0, \Pi^i$

$$\Pi^0 = \partial_t X^0 + \Theta \partial_t \Theta + \Psi^i \partial_t \Psi^i, \quad \Pi^i = \partial_t Y^i - \partial_t \Theta \Psi^i + \Theta \partial_t \Psi^i.$$  \hspace{1cm} (4.2)
After some guess-work, the target sigma-model action invariant under the global target space supersymmetry (4.1), local $t$ reparametrizations and one local fermionic $\kappa$ symmetry was found to have the following almost unique form

$$S_{gs} = -\int dt \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i} - \alpha \int dt \left( \Theta \partial_t \Theta - \Psi^i \partial_t \Psi^i \right),$$  
(4.3)

where $\alpha = \pm 1$. The latter ambiguity in the sign amounts to the fact that the first and second terms in the lagrangian density in (4.3) behave in different way under the reflection $t \to -t$: first one is invariant while the second changes its sign. The cases with $\alpha = \pm 1$ are related by this reflection, so without loss of generality one can choose, e.g., $\alpha = 1$.

The $\kappa$-symmetry transformations are given by

$$\delta \Theta = \kappa , \quad \delta \Psi^i = \kappa \frac{\Pi^i}{\Pi^0 - \alpha \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i}},$$

$$\delta X^0 = -\Theta \delta \Theta - \Psi^i \delta \Psi^i , \quad \delta Y^i = -\Psi^i \delta \Theta - \Theta \delta \Psi^i ,$$  
(4.4)

where $\kappa(t)$ is an arbitrary fermionic gauge parameter. It is straightforward to check the invariance of (4.3) under these transformations.

We claim that the action (4.3) possesses only one $\kappa$-supersymmetry and therefore provides another, “space-time” realization of the same $N = 4 \to N = 1$ PBGS phenomenon. Let us prove that (4.3) indeed possesses no any other local fermionic symmetry apart from $\kappa$-symmetry (4.4).

To this end, we need to study the algebra of the constraints in the Hamiltonian formalism. We first introduce the einbein $e(t)$ and rewrite the action (4.3) (with $\alpha = 1$) as

$$S_{gs} = \int dt L = -\int dt \left[ \frac{1}{2e} \left( \Pi^0 \Pi^0 - \Pi^i \Pi^i \right) + \frac{e}{2} \right] - \int dt \left( \Theta \partial_t \Theta - \Psi^i \partial_t \Psi^i \right).$$  
(4.5)

Then one computes canonically conjugated variables

$$P^0 = \frac{\partial L}{\partial \dot{X}^0} = -\frac{\Pi^0}{e} , \quad P^i = \frac{\partial L}{\partial \dot{Y}^i} = \frac{\Pi^i}{e} , \quad P_e = \frac{\partial L}{\partial \dot{e}} = 0 ,$$

$$\Omega = \frac{\partial L}{\partial \dot{\Theta}} = \left( \frac{\Pi^0}{e} + 1 \right) \Theta - \frac{\Pi^i}{e} \Psi^i , \quad \Omega^i = \frac{\partial L}{\partial \dot{\Psi}^i} = \left( \frac{\Pi^0}{e} - 1 \right) \Psi^i - \frac{\Pi^i}{e} \Theta .$$  
(4.6)

The canonical hamiltonian reads

$$H = P^0 \partial_t X^0 + P^i \partial_t Y^i + \partial_t \Theta \Omega + \partial_t \Psi^i \Omega^i - L = -\frac{e}{2} \left( P^0 P^0 - P^i P^i \right).$$  
(4.7)

There is one primary bosonic constraint, $P_e$, and four fermionic constraints which we denote by

$$r^0 = \Omega + (P^0 - 1) \Theta + P^i \Psi^i , \quad r^i = \Omega^i + (P^0 + 1) \Psi^i + P^i \Theta .$$  
(4.8)

When taking the Poisson bracket of the primary bosonic constraint with the canonical hamiltonian, we obtain the secondary bosonic constraint

$$P^0 P^0 - P^i P^i = 1$$  
(4.9)
We now have to determine which of the fermionic constraints \( \tau^\mu = (\tau^0, \tau^i) \) are first class, and thus generate gauge symmetries, and which are second class. We compute the matrix of the Poisson brackets of the fermionic constraints

\[
\{ \tau^\mu, \tau^\nu \} = C^{\mu\nu}, \quad C = 2 \begin{pmatrix} P^0 - 1 & \vec{P}^t \\ \vec{P} & (P^0 + 1)\mathbf{1} \end{pmatrix},
\]

where \( \mathbf{1} \) is the three dimensional unit matrix. The eigenvalues of the matrix \( C \) are easily computed to be \( P^0 + 1, P^0 + 1, P^0 + \sqrt{\vec{P}^2 + 1}, P^0 - \sqrt{\vec{P}^2 + 1} \). On the constraint surface, the last of these eigenvalues vanishes, and the other three remain non zero. Thus, there is one and only one first class constraint which may be chosen to be

\[
\kappa = \tau^0 - \frac{1}{P^0 + 1} \vec{P} \vec{\tau}.
\]

Its Poisson brackets with the constraints read

\[
\{ \kappa, \tau^0 \} = \frac{2(P^0 P^0 - P^i P^i - 1)}{P^0 + 1}, \quad \{ \kappa, \tau^i \} = 0.
\]

The constraint \( \kappa \) generates the unique local fermionic symmetry (4.4) through the Poisson bracket.

It is natural to expect, by analogy with the 1/2 PBGS examples [3, 4], that the static gauge \((X^0 = t, \Theta = 0)\) form of (4.3) coincides, modulo field redefinitions, with the component action following from the world-line superfield one (3.11). Now we shall show that this is indeed the case.

In the static gauge the action (4.3) (with \( \alpha = 1 \)) reads

\[
S_{gs} = -\int dt \left[ \sqrt{1 + \Phi^i \partial_t \Phi^i} - \partial_t \Phi^i \partial_t \Phi^i - \Psi^i \partial_t \Phi^i \right].
\]

One can reduce it to the form

\[
S_{gs} = -\int dt \left[ \sqrt{1 - \partial_t Y^i \partial_t Y^i - \tilde{\Psi}^i \partial_t \tilde{\Phi}^i} \right],
\]

where

\[
\tilde{\Psi}^i = a_1 \psi^i \left[ 1 + a_2 (\Psi \partial_t \Psi) + a_3 (\Psi \partial_t \Psi)^2 \right], \quad a_1 = 1 + \frac{1}{\sqrt{1 - \partial_t Y^i \partial_t Y^i}},
\]

\[
a_2 = \frac{\partial_t Y^i \partial_t Y^i}{4a_1^2 \left( 1 - \partial_t Y^i \partial_t Y^i \right)^{3/2}}, \quad a_3 = \frac{\partial_t Y^i \partial_t Y^i}{4a_1^2 \left( 1 - \partial_t Y^i \partial_t Y^i \right)^{5/2}} - \frac{a_2}{2}.
\]

On the other hand, the action (3.11) after integration over \( \theta \) and properly rescaling fermions can be rewritten as follows:

\[
S_\phi = \int dt \left[ \frac{1}{1 + \sqrt{1 - \partial_t \phi^i \partial_t \phi^i}} \xi^i \partial_t \xi^i - \frac{\xi^i \partial_t \phi^i \partial_t \xi^i}{(1 + \sqrt{1 - \partial_t \phi^i \partial_t \phi^i}) \sqrt{1 - \partial_t \phi^i \partial_t \phi^i}} \right].
\]
By passing to the new variables $\phi^i, \tilde{\xi}^i$
\[ \xi^i = \tilde{\xi}^i \left( 1 + A \partial_t \phi^j \tilde{\xi}^j \partial_t (\partial_t \phi^k \partial_t \tilde{\xi}^k) \right) \] (4.17)
the action (4.16) can be put in the form
\[ S_\phi = \int dt \left[ \frac{1}{2} - \frac{1}{2} \sqrt{1 - \partial_t \phi^i \partial_t \phi^j + \psi^i \partial_t \psi^j} \right]. \] (4.18)
Here
\[ \phi^i = \phi^i + b_1 \tilde{\xi}^i \partial_t \phi^j \tilde{\xi}^j, \quad \psi^i = \tilde{\xi}^i + b_2 \partial_t \phi^i \partial_t \phi^j \tilde{\xi}^j, \] (4.19)
and $A, b_1, b_2$ are defined by simple algebraic equations
\[ b_2^2 (\partial_t \phi)^2 = \frac{2b_1}{\sqrt{1 - (\partial_t \phi)^2(1 - \sqrt{1 - (\partial_t \phi)^2})}}, \quad A = -\frac{b_1^2}{\sqrt{1 - (\partial_t \phi)^2(1 - \sqrt{1 - (\partial_t \phi)^2})}}, \]
\[ 2b_2 + b_2^2 (\partial_t \phi)^2 = \frac{4}{\sqrt{1 - (\partial_t \phi)^2(1 - \sqrt{1 - (\partial_t \phi)^2})}}. \] (4.20)
It can be easily seen that after rescaling of $\tilde{\psi}^i$ by $1/\sqrt{2}$ the action (4.18) reduces to (4.14) up to an overall constant.

Finally, we note that in terms of the invariants (4.12) the Green-Schwarz action (4.13) looks as if it possess $D = 4$ Lorentz invariance. Indeed, in the limit of vanishing fermions $X^0, Y^i$ and, hence, $\Pi^0, \Pi^i$ can be combined into a $D = 4$ vector. However, the fermionic terms in (4.2) break this “would-be” Lorentz symmetry down to $SO(3)$.

5 N=8 → N=2 PBGS

In this section we will consider two examples of $N = 8 \rightarrow N = 2$ PBGS.

In the previous sections we described the procedure which helps to define the action for the given PBGS pattern if the proper realization of broken supersymmetries is known. To construct a superparticle model which would exhibit $N = 8 \rightarrow N = 2$ PBGS we should, before all, examine how 6 broken supersymmetries could be realized on a set of $N = 2, d = 1$ superfields. We succeeded in finding out two such realizations.

5.1 Case I

The first realization is a more or less straightforward generalization of the $N = 4 \rightarrow N = 1$ case. The basic set of $N = 2, d = 1$ superfields includes seven bosonic superfields: a general real superfield $\Phi$ and two conjugated triplets of chiral-anti-chiral superfields $\bar{v}^i, v^i$:
\[ Dv^i = \bar{D}v^i = 0, \quad i = 1, 2, 3. \]

The broken supersymmetry transformations of these superfields read
\[ \delta v^i = -2 \left( \bar{\theta} - D\Phi \right) \bar{\epsilon}^i + \epsilon^{ijk} \bar{\epsilon}^j Dv^k, \quad \delta \bar{v}^i = 2 \left( \theta + \bar{D}\Phi \right) \bar{\epsilon}^i + \epsilon^{ijk} \epsilon^j D\bar{v}^k, \]
\[ \delta \Phi = \frac{1}{2} \left( \epsilon^i D\bar{v}^i + \bar{\epsilon}^i \bar{D}v^i \right). \] (5.1)
Together with the manifest supersymmetry, they form the algebra with six central charges \( Z^i, \bar{Z}^i \)

\[
\{Q, \bar{Q}\} = P, \quad \{S^i, \bar{S}^i\} = \delta^{ij} P, \quad \{Q, S^i\} = 2Z^i, \quad \{\bar{Q}, \bar{S}^i\} = 2\bar{Z}^i. \tag{5.2}
\]

The fermionic chiral superfields defined by

\[
\psi^i = -\frac{1}{2} \bar{D}v^i, \quad \bar{\psi}^i = \frac{1}{2} D\bar{v}^i
\]

are transformed under (5.1) as

\[
\delta \psi^i = (1 - \bar{D}D\Phi) \epsilon^i + \bar{\epsilon}^{jk} \bar{\epsilon}^i \bar{D} \bar{\psi}^k, \quad \delta \bar{\psi}^i = (1 + D\bar{D}\Phi) \bar{\epsilon}^i + \bar{\epsilon}^{jk} \epsilon^i D\psi^k, \quad \delta \Phi = \epsilon^i \bar{\psi}^i - \bar{\epsilon}^i \psi^i. \tag{5.3}
\]

So they are Goldstone superfields corresponding to the linear realization of six spontaneously broken supersymmetries with the parameters \( \epsilon^i, \bar{\epsilon}^i \). The bosonic superfields \( \bar{v}^i, v^i \) are the Goldstone ones associated with the spontaneously broken central charges transformations.

An interesting feature of this realization is the strange charges of the Goldstone superfields with respect to the \( U(1) \) automorphism group of the manifest supersymmetry algebra. Indeed, from (5.3) one finds the relation between the charges of \( \psi^i \) and \( \theta \)

\[
q_{\psi} = \frac{1}{3} q_{\theta}. \tag{5.4}
\]

Correspondingly, for the charge of \( v^i \) we have

\[
q_{v} = -\frac{2}{3} q_{\theta}.
\]

One should ascribe similar fractional charges, of course, to the spontaneously broken supersymmetry and central charge generators in (5.2).

Once again, the superfield \( \Phi \), in accord with its transformation properties, can be chosen as the Lagrangian density describing this PBGS pattern. The straightforward application of the method [24, 25] for expressing \( \Phi \) in terms of the Goldstone superfields \( \psi^i, \bar{\psi}^i \) yields a rather complicated system of equations. It can be rather easily solved in the limit of vanishing fermions, yielding the static gauge action for a massive particle in a seven-dimensional space-time as the bosonic part of the full superfield action

\[
S_{v}^{bos} = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t v^i \partial_t \bar{v}^i} \right). \tag{5.5}
\]

We have found the full action in terms of \( N = 2 \) superfields as well. It does not look very illuminating, so we do not give it here.

Surprisingly, the target space GS formulation for this case is very similar to the case of \( N = 4 \rightarrow N = 1 \) PBGS. The full action for the physical worldline multiplet can be immediately extracted from this formulation.

As a first step, we define the standard realization of \( N=8 \) superalgebra (5.2) in the superspace with seven bosonic \( X^0, Y^i, \bar{Y}^i \) and eight fermionic \( \Theta, \bar{\Theta}, \Psi^i, \bar{\Psi}^i \) coordinates:

\[
\delta \Theta = \epsilon, \quad \delta \Psi^i = \bar{\epsilon}^i, \quad \delta Y^i = -2e^i \Theta, \quad \delta \bar{\Theta} = \bar{\epsilon}, \quad \delta \bar{\Psi}^i = e^i, \quad \delta \bar{Y}^i = 2\bar{e}^i \bar{\Theta},
\]

\[
\delta X^0 = -\frac{1}{2} \left( \bar{e} \bar{\Theta} + \bar{\epsilon} \Theta + e^i \bar{\Psi}^i + \bar{e}^i \Psi^i \right). \tag{5.6}
\]
Using the supersymmetric invariants
\[
\begin{align*}
\Pi^0 &= \partial_t X^0 + \frac{1}{2} \left( \Theta \partial_t \Theta + \bar{\Theta} \partial_t \bar{\Theta} + \Psi^i \partial_t \bar{\Psi}^i + \bar{\Psi}^i \partial_t \Psi^i \right), \\
\Pi^i &= \partial_t Y^i + 2 \Psi^i \partial_t \Theta , \quad \bar{\Pi}^i \equiv (\bar{\Pi}^i) = -\partial_t \bar{Y}^i + 2 \bar{\Psi}^i \partial_t \bar{\Theta}.
\end{align*}
\]
we can construct the unique action
\[
S_{gs} = -\int dt \sqrt{\Pi^0 \bar{\Pi}^0 - \Pi^i \bar{\Pi}^i} + \int dt \left( \Theta \partial_t \bar{\Theta} - \Psi^i \partial_t \bar{\Psi}^i \right),
\]
with two \(\kappa\)-supersymmetries:
\[
\begin{align*}
\delta X^0 &= -\frac{1}{2} \left( \Theta \delta \Theta + \Theta \delta \bar{\Theta} + \Psi^i \delta \bar{\Psi}^i + \bar{\Psi}^i \delta \Psi^i \right), \\
\delta Y^i &= -2 \Psi^i \delta \Theta , \quad \delta \bar{Y}^i = 2 \bar{\Psi}^i \delta \bar{\Theta} , \\
\delta \Psi^i &= \frac{\Pi^i \delta \bar{\Theta}}{\Pi^0 + \sqrt{\Pi^0 \bar{\Pi}^0 - \Pi^i \bar{\Pi}^i}} , \quad \delta \bar{\Psi}^i = \frac{\bar{\Pi}^i \delta \Theta}{\Pi^0 + \sqrt{\Pi^0 \bar{\Pi}^0 - \Pi^i \bar{\Pi}^i}}.
\end{align*}
\]
The Hamiltonian analysis, which repeats the basic steps of the analysis in the \(N = 4\rightarrow N = 1\) case, shows that there are no further gauge fermionic symmetries in the action (5.8).

In the static gauge, \(X^0 = t, \Theta = 0\), the action (5.8) takes the very simple form
\[
S_{gs} = -\int dt \left[ \sqrt{1 + \frac{1}{2} \partial_t \Psi^i \partial_t \bar{\Psi}^i + \frac{1}{2} \partial_t \bar{\Psi}^i \partial_t \Psi^i} \right]^2 + \partial_t Y^i \partial_t \bar{Y}^i + \Psi^i \partial_t \bar{\Psi}^i.
\]
We are still not aware of the equivalency transformation from this action to the PBGS action. We expect such a transformation to exist like in the other known PBGS cases, though its precise form can be rather complicated in view of complexity of the full PBGS action.

### 5.2 Case II

The second realization of \(N = 8, d = 1\) supersymmetry with six spontaneously broken supersymmetries can be constructed in terms of general bosonic \(N = 2\) superfield \(\Phi\) and six chiral and anti-chiral Goldstone fermions \(\{\psi_\alpha, \bar{\psi}_\alpha, \xi, \bar{\xi}\}, \alpha = 1, 2\)
\[
\bar{D}\psi_\alpha = D\xi = 0 , \quad D\bar{\psi}_\alpha = D\bar{\xi} = 0 ,
\]
which form two doublets and two singlets with respect to \(SO(2)\) automorphism group. The appropriate closed set of the broken supersymmetry transformations reads
\[
\begin{align*}
\delta \xi &= \left( 1 + \bar{D}D\Phi \right) \nu + \varepsilon_{\alpha\beta} \bar{\mu}_\alpha \bar{D}\bar{\psi}_\beta , \quad \delta \bar{\xi} = \left( 1 - D\bar{D}\Phi \right) \bar{\nu} + \varepsilon_{\alpha\beta} \mu_\alpha D\psi_\beta , \\
\delta \psi_\alpha &= \varepsilon_{\alpha\beta} \left( \bar{\mu}_\beta \bar{D}\bar{\psi}_\beta + \nu \bar{D}\bar{\psi}_\beta \right) + \left( 1 - \bar{D}D\Phi \right) \mu_\alpha , \\
\delta \bar{\psi}_\alpha &= \varepsilon_{\alpha\beta} \left( \nu D\psi_\beta + \bar{\mu}_\beta D\psi_\beta \right) + \left( 1 + D\bar{D}\Phi \right) \bar{\mu}_\alpha , \\
\delta \Phi &= \bar{\nu} \xi - \nu \bar{\xi} - \bar{\mu}_\alpha \psi_\alpha + \mu_\alpha \bar{\psi}_\alpha .
\end{align*}
\]
To reveal the underlying central-charges extended supersymmetry algebra and to gain physical bosonic fields, we need to pass as before to bosonic superfields. The minimal realization amounts to expressing \( \psi_\alpha \) through two real scalar superfields \( u_\alpha \):\[
\psi_\alpha = -\frac{1}{2} D u_\alpha , \quad \bar{\psi}_\alpha = \frac{1}{2} D u_\alpha . \tag{5.13}
\]

To learn what kind of “prepotential” one should introduce for the remaining Goldstone superfield \( \xi \), let us examine the relation between \( U(1) \) charges of spinor superfields which follows from (5.12) \[
q_\xi = -2q_\psi - q_D . \tag{5.14}
\]

Here \( q_D \) is the \( U(1) \) charge of the covariant derivative \( D \) (\( q_D = -1 \) if one ascribes the charge +1 to \( \theta \)). From this relation and (5.13) it follows that the \( U(1) \) charges of \( \psi \) and \( \xi \) are equal to \(-q_D \) and \( q_D \), respectively. Recall, however, that \( \psi \) and \( \xi \) have same chiralities (see (5.11)). Bearing this in mind, the only way to introduce the bosonic superfield \( v \) for \( \xi \) is to choose it complex and having the \( U(1) \) charge equal to \(-2q_D \)
\[
\xi = -\frac{1}{2} \bar{D} v , \quad \bar{\xi} = \frac{1}{2} \bar{D} \bar{v} , \quad Dv = \bar{D} \bar{v} = 0 . \tag{5.15}
\]

In terms of the newly introduced bosonic superfields the supersymmetry transformations take the form:
\[
\delta v = -2 \left( \bar{\theta} + D\Phi \right) \nu + \varepsilon_{\alpha\beta} \bar{\mu}_\alpha Du_\beta , \quad \delta \bar{v} = 2 \left( \theta - D\Phi \right) \bar{\nu} + \varepsilon_{\alpha\beta} \mu_\alpha Du_\beta ,
\]
\[
\delta u_\alpha = \varepsilon_{\alpha\beta} \left( \bar{v} D u_\beta + v \bar{D} u_\beta + \bar{\mu}_\beta \bar{D} v + \mu_\beta Dv \right) + 2 \left( \theta + D\Phi \right) \bar{\mu}_\alpha - 2 \left( \bar{\theta} - D\Phi \right) \mu_\alpha ,
\]
\[
\delta \Phi = -\frac{1}{2} \left( \nu D \bar{v} + \bar{v} \bar{D} v - \bar{\mu}_\alpha \bar{D} u_\alpha - \mu_\alpha Du_\alpha \right) . \tag{5.16}
\]

Denoting the generators of the broken supersymmetry by \( S_\alpha, \bar{S}_\alpha \) and \( S, \bar{S} \), and the generators of the manifest \( N = 2 \) supersymmetry by \( Q, \bar{Q} \), one can write the full supersymmetry algebra pertinent to this case as
\[
\{ Q, \bar{Q} \} = \{ S, \bar{S} \} = P , \quad \{ S_\alpha, \bar{S}_\beta \} = \delta_{\alpha\beta} P , \quad \{ Q, \bar{S} \} = 2 \bar{Z} , \quad \{ \bar{Q}, S \} = 2 Z ,
\]
\[
\{ Q, S_\alpha \} = 2 Z_\alpha , \quad \{ \bar{Q}, S_\alpha \} = 2 \bar{Z}_\alpha , \quad \{ S, \bar{S}_\alpha \} = 2 \varepsilon_{\alpha\beta} Z_\beta , \quad \{ S, S_\alpha \} = 2 \varepsilon_{\alpha\beta} \bar{Z}_\beta . \tag{5.17}
\]

Once again, we can take the superfield \( \Phi \) as the Lagrangian density and the main problem is to covariantly express \( \Phi \) in terms of the Goldstone fermions or Goldstone bosons. The method we used in the previous cases works here as well, but it gives rather complicated equations which we for the time being were unable to solve explicitly. Nevertheless, we can try to find the bosonic part of the action. To this end, we should keep in the superfields with tilde (which are obtained by the finite broken supersymmetry transformations of our basic superfields) only a few terms, namely, the terms linear in the fermionic superfields and quadratic terms in \( \Phi \):
\[
\begin{pmatrix}
\xi \\
\bar{\xi} \\
\psi_\alpha \\
\bar{\psi}_\alpha
\end{pmatrix} = \begin{pmatrix}
\xi \\
\bar{\xi} \\
\psi_\alpha \\
\bar{\psi}_\alpha
\end{pmatrix} + \begin{pmatrix}
1 + X & 0 & 0 & \varepsilon_{\beta\gamma} \bar{D} \psi_\gamma \\
0 & 1 + X & \varepsilon_{\beta\gamma} \bar{D} \psi_\gamma & 0 \\
0 & \varepsilon_{\alpha\gamma} \bar{D} \psi_\gamma & (1 - X) \delta_{\alpha\beta} & \varepsilon_{\alpha\beta} \bar{D} \xi \\
\varepsilon_{\alpha\gamma} D \psi_\gamma & 0 & \varepsilon_{\alpha\beta} D \xi & (1 - X) \delta_{\alpha\beta}
\end{pmatrix} \begin{pmatrix}
\nu \\
\bar{\nu} \\
\mu_\beta \\
\bar{\mu}_\beta
\end{pmatrix} = 0 , \tag{5.18}
\]
\[ \ddot{\Phi} = \Phi + \frac{1}{2} \left( \nu \xi - \nu \bar{\xi} + \mu_\alpha \bar{\psi}_\alpha - \bar{\mu}_\alpha \psi_\alpha \right) = 0 , \]  

(5.19)

where \( X = \bar{D}D\Phi \).

Now, substituting \((\nu, \bar{\nu}, \mu_\alpha, \bar{\mu}_\alpha)\) from (5.18) in (5.19), hitting both sides of (5.19) by \( \bar{D}D \) and omitting the terms with fermions we get the following equation for the bosonic part of the action which we denote by \( X \):

\[ (X^2 - X + a)(X^2 + a - 1) + 2D\xi \bar{D}\bar{\xi} = 0 , \]

(5.20)

or, equivalently,

\[ (X^2 - 2X + 1 + a)(X^2 + X + a) - 2\bar{D}\psi_\alpha D\bar{\psi}_\alpha = 0 . \]

(5.21)

Here,

\[ a = D\xi \bar{D}\bar{\xi} + \bar{D}\psi_\alpha D\bar{\psi}_\alpha . \]

The general solution of these equations exists (we are interested in that one which goes to zero when all fields are put equal to zero), but it looks too complicated to explicitly present it here. In the two limits, \( \psi_\alpha = 0 \) or \( \xi = 0 \), it takes the familiar form of the static gauge actions of massive particles moving on some three-dimensional target manifolds

\[ S^\text{bos}_v = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t v \partial_t \bar{v}} \right) , \quad S^\text{bos}_u = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t u_\alpha \partial_t \bar{u}_\alpha} \right) . \]

(5.22)

In the generic case there is a non-trivial cross-interaction between the bosonic fields appearing in (5.22). It can hopefully be interpreted in the geometric language of intersection of the trajectories of two different superparticles, with the physical worldline scalar multiplets represented by the superfields \( u_\alpha \) and \( v, \bar{v} \), respectively.

The fact that the bosonic part of the action cannot be written in the standard form strongly obscures the construction of the GS formulation for this case. Up to now we have not succeeded to find a manageable form even for the bosonic part of the hypothetical target superspace GS action in the case at hand (though the WZ term seems to be a direct generalization of those for the previously studied cases). We believe that the better understanding of this case would be helpful for studying the \( 1/4 \) PBGS systems with higher-dimensional worldvolumes. Such systems hopefully correspond to the new types of superbranes.

6 Conclusions

In this paper we presented, for the first time, the manifestly worldline supersymmetric superparticle actions exhibiting hidden spontaneously broken supersymmetries the number of which is four times the number of the linearly realized manifest ones. We treated in detail the case of \( N = 4 \rightarrow N = 1 \) partial breaking and discussed some basic features of the more complicated \( N = 8 \rightarrow N = 2 \) case. We proposed a general systematic method of constructing actions for these and other PBGS systems. For the first case and for one of the two versions of the second case we found the appropriate GS-type target superspace actions with one and two \( \kappa \)-symmetries. The common unusual feature of the superparticle systems considered is that their space-time interpretation is possible only within the superspaces corresponding to

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higher-dimensional supersymmetries with tensorial central charge generators. The target space coordinates associated with some combinations of these generators and extra components of the translation operator parametrize the transverse spatial bosonic directions in these models. It would be of interest to understand whether this is the general property of systems with fractional PBGS. Another interesting problem is to see whether the $1/4$ PBGS patterns studied here can be related to the existence of the appropriate BPS solutions of higher-dimensional supergravities along the line of refs. [16, 17, 20].

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Appendix: $N=4$, $d=1$ from $N=1$, $D=4$

Here we demonstrate how $N = 4$, $d = 1$ supersymmetry algebra (3.5) can be recovered from the $N = 1$, $D = 4$ algebra extended by tensorial central charges. This extended superalgebra is defined by the following anticommutation relations [27, 19, 20]

\[
\{Q_\alpha, \bar{Q}_\beta\} = 2 (\sigma^\mu)_{\alpha\beta} P_\mu + 2 (\sigma^m)_{\alpha\beta} P_m ,
\]

\[
\{Q_\alpha, Q_\beta\} = 2 T_{(\alpha\beta)} , \quad \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 2 T_{(\dot{\alpha}\dot{\beta})}.
\]

Let us deviate from this standard manifestly $SL(2, C)$ covariant notation by keeping manifest only $SU(2) \in SL(2, C)$. In such a notation the dotted indices are simply upper case doublet $SU(2)$ indices. E.g., the relation (A.1) is rewritten as

\[
\{Q_\alpha, \bar{Q}_\beta\} = 2 \delta^\beta_\alpha P_0 + 2 (\sigma^m)_{\alpha\beta} P_m .
\]

Next, we combine $Q$ and $\bar{Q}$ into doublets of some extra $SU(2)$ and pass to the quartet notation

\[
Q_{\alpha i} = (Q_\alpha, -\bar{Q}_\beta), \quad \bar{Q}_{\alpha i} = \epsilon^{\alpha\beta} \epsilon^{ik} Q_{\beta k} , \quad (\epsilon_{12} = -\epsilon^{12} = 1) .
\]

In terms of these generators the original algebra (A.1) is rewritten as

\[
\{Q_{\alpha i}, Q_{\beta k}\} = 2 F_{(\alpha\beta)(ik)} + 2 \epsilon_{\alpha\beta} \epsilon_{ik} P_0 ,
\]

where

\[
F_{(\alpha\beta)(11)} = T_{(\alpha\beta)} , \quad F_{(\alpha\beta)(22)} = T_{(\alpha\beta)} , \quad F_{(\alpha\beta)(12)} = -(\sigma^m)_{(\alpha\beta)} P_m .
\]

Note that this form of the algebra (A.1), (A.2) reveals that the full automorphism group of the latter is the 16-parametric non-compact general linear group $GL(4, \mathbb{R})$. It contains, in parallel
with the $SO(4) \sim SU(2) \times SU(2)$ subgroup which is manifest in the notation (A.4), also the original Lorentz group, of course. The generators belonging to the coset of the latter over the left manifest $SU(2)$ are constructed as direct products of the generators of this $SU(2)$ by $i(\sigma^3)^k_i$ and so act also on the index $i$ of $Q_{\alpha i}$.

In order to reproduce the algebra (3.3), we need two more steps.

First, we should pass to the notation in which only the diagonal $SU(2)$ in the product $SU(2) \times SU(2)$ realized on $Q_{\alpha i}$ is manifest. Then we split $Q_{\alpha i}$ into a singlet and triplet with respect to this diagonal $SU(2)$

$$Q_{\alpha i} = \epsilon_{\alpha i} Q + i(\sigma^m)_{(\alpha i)} S^m, \quad Q = \bar{Q}, \quad S^m = (S^m).$$

Similarly, the tensorial generator $F_{(\alpha\beta)(ik)}$ in (A.4) is split into a real totally symmetric 4-index tensor $F_{(\alpha\beta ik)}$ (5 independent components), a pseudo-real triplet

$$F_{(\alpha i)} = \frac{1}{2} \{ \epsilon^{\beta k} F_{(\alpha\beta)(ik)} + (\alpha \leftrightarrow i) \} \equiv -2i(\sigma^m)_{(\alpha i)} Z^m, \quad Z^m = \bar{Z^m},$$

and a real singlet

$$F = \frac{1}{2} \epsilon^{\alpha i} \epsilon^{\beta k} F_{(\alpha\beta)(ik)}.$$

The algebra (A.4) in this new basis can be rewritten as follows

$$\{Q, Q\} = P_0 + F, \quad \{Q, S^m\} = 2Z^m, \quad \{S^m, S^n\} = 2Z^{mn} + \delta^{mn} (P_0 - \frac{1}{3} F),$$

where

$$Z^{mn} = -\frac{1}{4} (\sigma^m)_{(\alpha i)} (\sigma^n)^{(\beta k)} F_{(\alpha\beta ik)}, \quad \text{Tr} Z = 0.$$

We observe that (A.9) coincide with (3.5) (up to the rescaling $P = 1/2 P_0$) provided that

$$Z^{mn} = 0, \quad F = 0.$$

These constraints are covariant under the manifest diagonal $SU(2) \sim SO(3)$ which is thus identified with the $SO(3)$ automorphisms of (3.3). Eqs. (A.10) can be shown to fully break the rest of the automorphism group $GL(4, R)$ of the algebra (A.1), (A.4), which explains the absence of any extra automorphisms in (3.5).

Eqs. (A.10) amount to the following constraints on the original bosonic generators

$$T_{12} = P_1 - i P_2, \quad \overline{T}_{12} = -(P_1 + i P_2), \quad \overline{T}_{11} + T_{22} = 0, \quad T_{11} = \overline{T}_{22} = 0, \quad P_3 = 0,$$

leaving us just with four independent generators (together with $P$). Taking account of these relations, the central charges $Z^i$ can be identified with the following combinations

$$Z^1 = P_2, \quad Z^2 = -P_1, \quad Z^3 = \frac{i}{4} (T_{11} - T_{22}).$$

Note that the superalgebra of charges obtained via Nöther’s prescription from the PBGS action (3.11) differs from (3.3) by the presence of non-zero constant generator $F$. It ensures the relative
shift of the translation generators in the $QQ$ and $SS$ anticommutators in accord with the general reasoning of ref. [1, 2].

For the underlying algebras of other $1/4$ PBGS examples considered in this paper one can also obtain a similar identification proceeding from the most general extension of $N = 2$, $D = 4$ superalgebra by the appropriate tensorial central charges (such extensions for the generic $N$ are presented, e.g., in [27, 28]).

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