Non-linear theory for multiple M2 branes

Roberto Iengo\textsuperscript{a} and Jorge G. Russo\textsuperscript{b,c}

\textsuperscript{a}International School for Advanced Studies (SISSA)  
Via Beirut 2-4, I-34013 Trieste, Italy  
INFN, Sezione di Trieste

\textsuperscript{b}Institució Catalana de Recerca i Estudis Avançats (ICREA)

\textsuperscript{c}Departament ECM and Institut de Ciencies del Cosmos,  
Facultat de Física, Universitat de Barcelona,  
Diagonal 647, 08028 Barcelona, Spain

Abstract

We present a manifestly $SO(8)$ invariant non-linear Lagrangian for describing the non-abelian dynamics of the bosonic degrees of freedom of $N$ coinciding M2 branes in flat spacetime. The theory exhibits a gauge symmetry structure of the $BF$ type (semidirect product of $SU(N)$ and translations) and at low energies it reduces exactly to the bosonic part of the Lorentzian Bagger-Lambert Lagrangian for group $SU(N)$. There are eight scalar fields satisfying a free-scalar equation. When one of them takes a large expectation value, the non-linear Lagrangian gets simplified and the theory becomes equivalent to the Myers non-abelian Lagrangian describing the dynamics of $N$ coinciding D2 branes. As an application, we show that the BPS fuzzy funnel solution describing M2 branes ending into a single M5 brane is an exact solution of the non-linear system.
1 Introduction

Understanding the dynamics of multiple M2 branes may reveal important aspects of the microscopic structure of M-theory. Recently several models for M2 brane dynamics with explicit Lagrangian description have appeared in the literature. In [1] Bagger and Lambert found a Lagrangian with maximal superconformal invariance containing the expected degrees of freedom of M2 branes (see also [2, 3]). The construction uses an algebraic structure called Lie 3-algebra, parametrized by structure constants \( f^{abc,d} \) and a bi-invariant metric \( h^{ab} \). The structure constants must satisfy a quadratic condition which turns out to be quite restrictive. It was shown in [4, 5] that for a positive quadratic condition which turns out to be quite restrictive. It was shown in [4, 5] that for a positive large expectation value) the model reduces to the maximally supersymmetric Yang-Mills Lagrangian describing the low-energy dynamics of M2 branes in flat spacetime. By giving an expectation value to one of the scalar fields through the procedure found in [6, 7], one can indeed show [8, 10] that in the IR regime (corresponding to a large expectation value) the model reduces to the maximally supersymmetric Yang-Mills Lagrangian describing the low-energy dynamics of M2 branes. The bosonic part of the Lagrangian is given by

\[
L = \text{Tr} \left( \frac{1}{2} M^{IJK} \partial_\mu X_I^J X_K^\mu + \frac{1}{12} M^{IJK} M^{IJK} \right) + \left( \partial_\mu X_I^J - \text{Tr}[B_\mu, X_I^J] \right) \partial^\mu X_I^J ,
\]

where the trace is over \( su(N) \) matrices of the adjoint representation, generated by \( T^a \), \( a = 1, ..., N^2-1 \) and \( A_\mu = A_\mu^a T^a \), \( B_\mu = B_\mu^a T^a \), \( X_I = X_I^a T^a \), whereas \( X_\pm \) are \( su(N) \) singlets. We have also introduced the notation

\[
M^{IJK} = X_I^J [X^K, X_J^I] + X_I^K [X^J, X_J^I] + X_I^J [X^K, X_J^I] ,
\]

\[
\hat{D}_\mu X_I = D_\mu X_I - X_I^J \partial_\mu A_\mu^J \equiv \partial_\mu X_I + [A_\mu, X_I] .
\]

As a consequence of the Lorentzian signature of \( h^{ab} \), there is a field \( X_+ \equiv X_+^I + X_-^I \) with the wrong sign in the kinetic term, which may lead to violation of unitarity. Different arguments have been given in [8, 9, 10] (see also [12, 13, 14]) that the model may nevertheless be unitary due to the particular form of the interactions, which ensure that \( X_+ \) and \( X_- \) can be integrated out by its equations of motion; they also imply that the ghost-like fields do not run in loops of Feynman diagrams. The role of the \( X_+^I, X_-^I \) fields is to provide a special kind of dressing that leads to the conformal invariance of the model.

A different strategy studied in [15, 16] is to gauging the global translational symmetry \( X_I^J \to X_I^J + c^I \) by means of the introduction of a gauge field \( C_\mu^I \) in a new term in the Lagrangian \( -C_\mu^I \partial_\mu X_+^I \). The equation of motion of \( C_\mu^I \) then freezes out the mode \( X_+^I \) to a constant value. The resulting model seems to be essentially equivalent the maximally supersymmetric Yang-Mills Lagrangian describing the low energy dynamics of D2 branes, though this has not yet been shown in a complete treatment including calculation of observables (see also [17]).

In addition to the \( SU(N) \) gauge symmetry, the above Lagrangian is invariant under the (non-compact) gauge symmetry transformations associated with the \( B_\mu \) gauge field,

\[
\delta X_I^J = X_+^I \Lambda , \quad \delta B_\mu = D_\mu \Lambda , \quad \delta X_+^I = 0 , \quad \delta X_-^I = \text{Tr}(X_I^J \Lambda) .
\]

The symmetry algebra underlying the model is generated by \( J^a, P^a \) satisfying the BF algebra

\[
[J^a, J^b] = C^{ab} c^c , \quad [P^a, P^b] = e^{ab} c c^c , \quad [P^a, J^b] = 0 .
\]

The Lagrangian (1.1) is a candidate to describe M2 brane dynamics in the low-energy approximation. The full M2 brane dynamics is expected to be described by a non-linear theory which at low
energies reduces to (1.1) and in some limit (discussed below) reduces to the non-linear dynamics of $N$ D2 branes. The non-linear Lagrangian describing the dynamics of D branes is not fully understood in the non-abelian case. However, there is a concrete Lagrangian for the bosonic degrees of freedom found by Myers [18] which works quite well up to high orders in $\alpha'$. For flat backgrounds, the action of [18] for D2 branes reduces to

$$L = -T \text{Str} \sqrt{-\det (\eta_{\mu\nu} + \lambda^2 D_\mu \Phi^i Q^{-1}_{ij} D_\nu \Phi^j + \lambda F_{\mu\nu})} \det Q \, .$$

(1.6)

where Str means symmetrized trace [19] and

$$Q^{ij} = \delta^{ij} + i\lambda[\Phi^i, \Phi^j] \, .$$

(1.7)

As usual, $\Phi^i$ represents the transverse displacements, $\Delta x^i = \lambda \Phi^i$, $\lambda = 2\pi l_s^2$. For further details we refer to [18]. The tension is

$$T = \frac{1}{(2\pi)^2 l_s^2 g_s} = \frac{1}{\lambda^2 g_{YM}^2} \, , \quad g_{YM}^2 = \frac{g_s}{l_s} \, .$$

(1.8)

For a single M2 brane, the classical non-linear dynamics is governed by the supermembrane action [20]. For multiple M2 branes, the non-linear action analogous to the Myers D brane Lagrangian is not known. The aim of this paper is to find a non-linear $SO(8)$ invariant Lagrangian for the bosonic degrees of freedom of the M2 branes that reduces to the Myers Lagrangian at large $g_{YM}$ coupling and to the BF membrane Lagrangian (1.1) at low energies.

Another proposal for M2 branes in flat spacetime was presented in [21], called ABJM models, in terms of a Lagrangian that realizes six supersymmetries (see also [22, 23]). We have not found a natural ansatz for the non-linear generalization of the ABJM models, so we will not discuss them in this paper. Some studies of non-linear Lagrangians for M2 branes, which do not overlap with this paper, are in [24, 25]. It would also be interesting to understand the non-linear theory for the Bagger-Lambert construction based on the Nambu-bracket [26, 27, 28, 29].

This paper is organized as follows. In section 2 we start with the abelian case. Here one can write two alternative proposals, but only one of them survives in the non-abelian case. In section 3 we consider the non-abelian case and propose a non-linear M2 brane Lagrangian with the desired symmetry structure, which turns out to be equivalent to the Myers lagrangian when one of the scalar fields is set to a constant value. In section 4 we check that the supersymmetric funnel of eleven dimensions – representing a fuzzy M2-M5 brane intersection – is an exact solution of our proposal, and that is not modified by the non-linearities, just as it happens in the D1-D3 brane case [30].

2 From D2 branes to M2 branes in the abelian case

The connection between the single D2 brane and the single M2 brane action was derived in [31]. Here we will review part of this connection, following [31], and in addition connect with the recently found BF membrane (or “Lorentzian Bagger-Lambert”) theory (1.1) based on the Bagger-Lambert construction. We will only consider the part containing the bosonic fields. The BI Lagrangian for a D2 brane in the static gauge is given by

$$L = -T \sqrt{- \det (g_{\mu\nu} + \lambda F_{\mu\nu})} \, ,$$

(2.1)

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda^2 \partial_\mu \Phi^j \partial_\nu \Phi^j \, , \quad i = 1, \ldots, 7 \, .$$

(2.2)

By introducing a Lagrange multiplier $p$, this can be written as

$$L = \frac{1}{2p} T^2 \det (g_{\mu\nu}) - \frac{1}{2p} \left( 1 + \frac{1}{2} \lambda^2 |F|^2 \right) \, .$$

(2.3)
where we used the identity for $3 \times 3$ matrices
\[
\det (g_{\mu\nu} + \lambda F_{\mu\nu}) = \det (g_{\mu\nu}) \left(1 + \frac{1}{2}\lambda^2 |F|^2\right), \quad |F|^2 = g^{\mu\nu} g^{\sigma\tau} F_{\mu\nu} F_{\sigma\tau},
\]
which applies for any antisymmetric $F_{\mu\nu}$. Introducing an auxiliary field $B_\mu$, we can write the Lagrangian as
\[
L = \frac{1}{2p} T^2 \det (g_{\mu\nu}) \left(1 + \lambda^2 g_{YM} B_\mu B_\nu g^{\mu\nu}\right) + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} - \frac{1}{2} p.
\]
This is the standard duality connecting Chern-Simons and Yang-Mills theory in three dimensions. Solving the equation for $B_\mu$, substituting in (2.5) and using
\[
|F|^2 = \frac{1}{2} (\det g)^{-1} g_{\mu\nu\rho} \epsilon^{\mu\nu\rho\sigma} F_{\lambda\sigma} F_{\nu\lambda},
\]
one can verify that the Lagrangian (2.1) is reproduced.

Next, using the identity for $3 \times 3$ matrices
\[
\det (g_{\mu\nu} + K_\mu K_\nu) = \det g_{\mu\nu} \left(1 + K_\mu K_\nu g^{\mu\nu}\right),
\]
we get
\[
L = \frac{1}{2p} T^2 \det (g_{\mu\nu} + \lambda^2 g_{YM} B_\mu B_\nu) + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} - \frac{1}{2} p.
\]
Solving the equation of motion for $p$ we find
\[
L = -T \sqrt{- \det (g_{\mu\nu} + \lambda^2 g_{YM} B_\mu B_\nu)} + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}.
\]
Now the equation for $A_\mu$ is solved by $B_\mu = \partial_\mu \phi$. This introduces the eight-th scalar field $\Phi^8$ in the Lagrangian, $\Phi^8 \equiv g_{YM}^2 \Phi$. In order to compare with the Lagrangian (1.1) (and to have canonically normalized scalar fields), we introduce new variables $X^I$ by
\[
\Phi^I = g_{YM} X^I, \quad I = 1, \ldots, 8.
\]
Note that $|X^I| = \mu^{1/2}$ carries the standard dimensionality of a bosonic field in $D = 2 + 1$ and that also $[g_{YM}] = \mu^{1/2}$. Therefore, we finally get
\[
L = -T \sqrt{- \det \left(\eta_{\mu\nu} + \frac{1}{T} \partial_\mu X^I \partial_\nu X^I\right)},
\]
where we used $T^{-1} = \lambda^2 g_{YM}^2$ (see eq. (1.8)). The eleven-dimensional Planck length scale $l_p$ is related to $T$ by $T^{-1} = (2\pi)^2 l_p^2$ (we used $l_p^2 = l_s^2 g_s$). Thus we find the Lagrangian for a membrane in the static gauge with the expected $SO(8)$ symmetry.

Now we will show that the Lagrangian (2.5) arises from either one of the following non-linear generalizations of the (abelian) BF membrane Lagrangian:
\[
L_1 = -T \sqrt{- \det \left(\eta_{\mu\nu} + \frac{1}{T} \left(\hat{D}_\mu X^I \hat{D}_\nu X^I - 2[\partial_\mu X^I] \partial_\nu X^I\right)\right)} + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho},
\]
\[
L_2 = -T \sqrt{- \det \left(\eta_{\mu\nu} + \frac{1}{T} \hat{D}_\mu X^I \hat{D}_\nu X^I\right)} + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}
\] \[
+ (\partial_\mu X_+ - X^I \partial_\mu X_+^I) \partial^\mu X_+^I - \frac{X_+ \cdot X_+}{X_+^2} \hat{D}_\mu X^I \partial^\mu X^I_+ + \frac{1}{2} (X_+ \cdot X_+^2)^2 \partial_\mu X_+^I \partial^\mu X_+^I,
\] \[
\text{(2.12)}
\]

\[
\text{(2.13)}
\]

\[\text{Here and in what follows we ignore gauge fixing terms and corresponding ghost contributions. The discussion will be purely classical.}\]
where, as usual, $A_{(\mu}B_{\nu)} \equiv \frac{1}{2}(A_{\mu}B_{\nu} + A_{\nu}B_{\mu})$, and

$$\hat{D}_\mu X^I = \hat{D}_\mu X^I - \frac{X_+ \cdot X}{X_+^2} \partial_\mu X_+^I, \quad \hat{D}_\mu X^I = \partial_\mu X^I - X_+^I B_\mu. \quad (2.14)$$

The Lagrangians $L_1$, $L_2$ are invariant under the non-compact gauge symmetry transformations

$$\delta B_\mu = \partial_\mu \Lambda, \quad \delta X^I = X_+^I \Lambda, \quad \delta X_+^I = \Lambda X_I^I, \quad \delta X_+^I = 0. \quad (2.15)$$

Note that $\delta(\hat{D}_\mu X^I) = 0$ and that $\delta(\partial_\mu X^I - X^I B_\mu) = \Lambda \hat{D}_\mu X^I$ while $\delta(\hat{D}_\mu X^I) = \Lambda \partial_\mu X^I$. Therefore, the last terms of eq. (2.13) are also gauge invariant since they can be written as

$$(\partial_\mu X^I - X^I B_\mu) \partial^\mu X^I = \frac{1}{2}(X_+ \cdot X)^2 \partial_\mu X_+^I \partial^\mu X_+^I =$$

$$= (\partial_\mu X^I - X^I B_\mu) \partial^\mu X^I - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I + \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I, \quad (2.16)$$

i.e. they are given by the same gauge-invariant combination appearing in the low energy lagrangian \[1\] plus the gauge-invariant term $\hat{D}X \hat{D}X$. The full expression \[2.10\] vanishes for constant $X_+^I$.

The basic difference between the two non-linear Lagrangians $L_1$ and $L_2$ is that in the second case the kinetic term $\partial_\mu X_+^I \partial^\mu X^I$ is outside the square root. The remaining terms have to be added to preserve gauge invariance and to preserve the connection with \[1.1\] at low energies. As we will see, in the non-abelian case, the second Lagrangian $L_2$ can be constructed, because $X_+^I, X^I$ are $SU(N)$ singlets and cannot be put inside the trace in a way preserving both $SU(N)$ and $B_\mu$ gauge invariance.

Following the method of \[11\], we assume that $X_+^I$ takes an expectation value, so that $X_+^I$ is equal to constant vector $v^I$ plus a small fluctuation. Then the Lagrangians $L_1$ and $L_2$ become

$$L \equiv L_1 = L_2 = -T \sqrt{\det(\eta_{\mu\nu} + \frac{1}{T} (\partial_\mu X^I - v^I B_\mu)(\partial_\nu X^I - v^I B_\nu)) + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}}, \quad (2.17)$$

where we ignore terms with fluctuations which are suppressed at large $v^I$.

We can use the global $SO(8)$ symmetry to fix $v^I = vg_{8S}$. We get

$$L = -T \sqrt{\det(\eta_{\mu\nu} + \frac{1}{T} \partial_\mu X^I \partial_\nu X^I + \frac{1}{T} (\partial_\mu X^8 - v B_\mu)(\partial_\nu X^8 - v B_\nu)) + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}} \quad (2.18)$$

By choosing the gauge $X^8 = 0$ for the symmetry \[2.15\], and taking $v = g_{YM}$, we finally obtain

$$L = -T \sqrt{\det(\eta_{\mu\nu} + \frac{1}{T} \partial_\mu X^I \partial_\nu X^I + \frac{1}{T} g_{YM}^2 B_\mu B_\nu) + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}}. \quad (2.19)$$

This is precisely the previous Lagrangian \[2.9\].

### 3 Born-Infeld Lagrangian for Non-Abelian BF membrane

Our starting point is the Myers Lagrangian \[1.9\] describing the dynamics of $N$ coinciding D2 branes. Writing as before $\Phi^i = g_{YM} X^i$, the D2 brane Lagrangian is:

$$L = -\frac{1}{X^2 g_{YM}^2} \text{STr} \sqrt{-\det(\eta_{\mu\nu} + \lambda^2 g_{YM}^2 D_\mu X^i Q^{-1} Q_{ij} D_\nu X^j + \lambda F_{\mu\nu}) \det Q}, \quad (3.1)$$

where

$$Q^{ij} = \delta^{ij} + i \lambda g_{YM}^2 [X^i, X^j], \quad i, j = 1, \cdots, 7. \quad (3.2)$$
Due to the symmetrized trace prescription, only the symmetric part of \( Q_{ij}^{-1} \) contributes, that is
\[
\text{STr} \sqrt{\cdots D_\mu X^i Q^{-1}_{ij} D_\nu X^j \cdots} = \text{STr} \sqrt{\cdots D_\mu X^i \frac{Q^{-1}_{ij} + Q^{-1}_{ji}}{2} D_\nu X^j \cdots}
\] (3.3)

Therefore, by defining \( g_{\mu\nu} \equiv \eta_{\mu\nu} + D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j \), where \( (ij) \) denotes symmetrization, we have that, inside the STr prescription, \( g_{\mu\nu} = g_{\nu\mu} \) and we can treat \( g_{\mu\nu} \) as a metric.\(^2\)

We begin by showing that the D2 brane Lagrangian (3.1) has the equivalent form
\[
\mathcal{L} = -T \text{STr} \sqrt{- \det (\eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j + \frac{1}{T^2} B_\mu B_\nu \frac{\det Q}{\det Q}) \det Q + \text{Tr} \left( \frac{1}{2} \epsilon^{\mu\nu\rho} B_{\mu\nu} F_{\nu\rho} \right)},
\] (3.4)
with
\[
v = g_{\text{YM}}.
\] (3.5)

First, we use the relation (2.7) for \( 3 \times 3 \) matrices, with \( g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j \), and \( K_\mu = v B_\mu / \sqrt{T \det Q} \), and write, introducing a Lagrange multiplier \( u \),
\[
- T \text{STr} \sqrt{- \det (\eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j + \frac{1}{T^2} v^2 B_\mu B_\nu \frac{\det Q}{\det Q}) \det Q}
\[
= \frac{1}{2u} T^2 \det Q \det g + \frac{1}{2u} \det g \, T v^2 B_\mu B_\nu g^{\mu\nu} - \frac{u}{2}.
\] (3.6)

Every term in the above expression is a (gauge-group) matrix. In the following manipulations we treat them as c-numbers, assuming that it is justified by the STr prescription.

The equation of motion for \( B_\mu \) gives
\[
g^{\mu\nu} B_\nu = - \frac{u}{2T v^2} \det g \epsilon^{\mu\nu\rho} F_{\nu\rho}.
\] (3.7)

Substituting back we get
\[
- T \sqrt{- \det (\eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j + \frac{1}{T^2} v^2 B_\mu B_\nu \frac{\det Q}{\det Q}) \det Q} + \frac{1}{2} \epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} =
\[
= \frac{1}{2u} T^2 \det Q \det g - \frac{u}{2} (1 + |F|^2) \left( \frac{1}{2T v^2} \right),
\] (3.8)
where \( |F|^2 \equiv g^{\mu\nu} \epsilon^{\mu\nu\rho} F_{\mu\nu} F_{\rho\'\rho} \) and we have made use of \( g_{\mu\nu} \epsilon^{\mu\nu\rho} F_{\mu\nu} F_{\rho\'\rho} = 2 \det g |F|^2 \).

Solving for \( u \), setting \( v = g_{\text{YM}} \) and using eqs. (1.3), (2.3), we finally obtain the D2 brane Lagrangian (3.1).

Just as in the abelian case, the above Lagrangian (3.4) originates from an M2 brane Lagrangian, where the term \( B_\mu B_\nu \) arises from a term \( D_\mu X^J D_\nu X^J \). The \( \text{SO}(8) \) invariant starting point must be of the form \( D_\mu X^I \tilde{Q}_{IJ} D_\nu X^J \), \( I, J = 1, \ldots, 8 \), where \( \tilde{Q}_{IJ} \) and the covariant derivative \( \tilde{D}_\mu \) are to be determined. The connection with the D2 brane Lagrangian (3.1) requires that, upon setting \( X^I_+ = v \delta_8 \), with \( v = g_{\text{YM}} \), one gets
\[
\tilde{D}_\mu X^I \tilde{Q}_{IJ} \tilde{D}_\nu X^J \rightarrow D_\mu X^I Q^{-1}_{IJ} D_\nu X^J + v^2 B_\mu B_\nu \frac{\det Q}{\det Q}.
\] (3.9)

\(^2\)Note that \( (Q^{-1})_{(ij)} \) is different from \( (Q_{(ij)})^{-1} = \delta_{ij} \).
Therefore,\[ X^I_+ = v \delta_{I8} \rightarrow \tilde{Q}^{ij} = Q^{ij}, \quad \tilde{Q}^{I8} = \tilde{Q}^{8j} = 0, \quad \tilde{Q}^{88} = \text{det} \, Q. \] (3.10)

Hence\[ \text{det} \, \tilde{Q} = (\text{det} \, Q)^2. \] (3.11)

Since only the symmetric part of $Q^{-1}_{ij}$ contributes to the symmetrized trace $\text{STr}$, one could in principle relax the condition $\tilde{Q}^{ij} = Q^{ij}$ in (3.10) and impose the weaker condition $(\tilde{Q}^{-1})_{(ij)} = (Q^{-1})_{(ij)}$. However, it turns out that the simplest ansatz for $\tilde{Q}^{Ij}$ naturally gives $\tilde{Q}^{ij} = Q^{ij}$.

Invariance under the non-abelian $B_\mu$ gauge transformations (1.4) is achieved by defining, just like in the abelian case,\[ \tilde{D}_\mu X^I_+ = \tilde{D}_\mu X^I_+ - \frac{X^I_+ \cdot X^I_+}{X^I_+} \partial_\mu X^I_+, \] (3.12)

where $\tilde{D}_\mu X^I_+ = D_\mu X^I_+ - X^I_+ B_\mu$ is the covariant derivative (1.3) appearing in the low energy Lagrangian (now $D_\mu X^I_+ = \partial_\mu X^I_+ + [A_\mu, X^I_+]$). It follows that $\delta (\tilde{D}_\mu X^I_+) = 0$ under (1.4). Recall that $X^I_+$ are $SU(N)$ singlets.

Let us now return to the general form of $\tilde{Q}^{IJ}$. This must be given in terms of $X^I_+$ and $X^I$ in a combination invariant under the $B_\nu$-gauge transformations (1.4). It should not depend on $X^I_+$ in order to maintain the important property of the low energy BF membrane Lagrangian (1.1) that in a combination invariant under the $B_\nu$-gauge transformations (1.4) is achieved by defining, just like in the abelian case,\[ \tilde{D}_\mu X^I = \tilde{D}_\mu X^I - \frac{X^I \cdot X^I}{X^I} \partial_\mu X^I, \]

where $\tilde{D}_\mu X^I = D_\mu X^I - X^I B_\mu$ is the covariant derivative (1.3) appearing in the low energy Lagrangian (now $D_\mu X^I = \partial_\mu X^I + [A_\mu, X^I]$). It follows that $\delta (\tilde{D}_\mu X^I) = 0$ under (1.4). Recall that $X^I_+$ are $SU(N)$ singlets.

Let us now return to the general form of $\tilde{Q}^{IJ}$. This must be given in terms of $X^I_+$ and $X^I$ in a combination invariant under the $B_\nu$-gauge transformations (1.4). It should not depend on $X^I_+$ in order to maintain the important property of the low energy BF membrane Lagrangian (1.1) that interactions do not involve $X^I$ (this ensures, in particular, that $X^I_+, X^I_-$ do not propagate in loops [8]). Some simple gauge-invariant $SO(8)$ tensors are $\delta^{Ij}$, $X^I_+ X^I_-$, $X^K M^{IJK}$, where $M^{IJK}$ was defined in eq. (1.2). More general gauge-invariant operators involving $X^I_+$ and $X^I_-$ can be constructed by forming products $O_n \equiv X_{[I_1} X_{J_2} \ldots X_{J_n]}$, where $[\ldots]$ denotes complete antisymmetrization in all indices [3]. Then one can define $SO(8)$ tensors $P^{I,J} = (O_n \cdot O_{n-2})^{I,J}$ or $R^{I,J} = (O_n \cdot O_n)^{I,J}$ (in a short-hand notation, meaning that all indices are contracted except two indices $I,J$). The simplest gauge-invariant $SO(8)$ tensor $\tilde{Q}^{IJ}$ satisfying the “boundary” conditions (3.11) is in fact of the form\[ \tilde{Q}^{IJ} = a(X, X_+) \delta^{IJ} + b(X, X_+) X^I_+ X^J_+ + c(X, X_+) X^K_+ M^{IJK}, \] (3.13)

where $a, b, c$ are gauge-invariant (and $SO(8)$ invariant) functions of $X^I_+, X^I$.\[ \text{Imposing the condition (3.10) for } X^I_+ = v \delta_{I8}, \text{ with } v = g_\text{YM} \text{ (noting that } T^{-1/2} v = \lambda g_\text{YM}^2 \text{ and } M^{8ij} = v[X^i, X^j] \text{), then } \tilde{Q}^{IJ} \text{ is uniquely determined:} \]

\[ \tilde{Q}^{IJ} \equiv S^{IJ} + \frac{X^I_+ X^J_+}{X^I_+} (\text{det}(S) - 1) = (\delta^{IJ} - \frac{X^I_+ X^J_+}{X^I_+^2} + \frac{i}{\sqrt{T}} \frac{X^K_+ M^{IJK}}{\sqrt{X^I_+^2}}) + \frac{X^I_+ X^J_+}{X^I_+^2} \text{det}(S), \] (3.14)

where\[ S^{IJ} = \delta^{IJ} + \frac{i}{\sqrt{T}} \frac{X^K_+ M^{IJK}}{\sqrt{X^I_+^2}}, \quad X^2_+ = X^I_+ X^I_+. \] (3.15)

In the above formulas, it is understood that $\delta^{IJ}$ and $X^I_+ X^J_+$ are multiplied by the identity matrix $I_{N \times N}$.

One can check that $\tilde{Q}^{IJ}$ is indeed invariant under $B_\mu$-gauge transformations (1.4). Note that the expression (3.14) involves a decomposition in a first term orthogonal to $X^I_+$ (since $X^I_+ M^{IJK} X^K_+ = 0$ by virtue of the fact that $M^{IJK}$ is completely antisymmetric), and a second term proportional to $X^I_+ X^J_+$ (hence $X^I_+ X^J_+ \tilde{Q}^{IJ} = X^2_+ \text{det}(S)$).

\[ ^3 \text{This observation is due to M. Van Raamsdonk.} \]

\[ ^4 \text{The condition } \tilde{Q}^{IJ} = Q^{ij} \text{ seems to leave (3.13) as the unique solution, since } X^K_+ M^{IJK} = R^{I,J}_+ - R^{J,I}_+, \text{ with } R^{I,J}_+ = Q^{I,J}_+ O^{K,J}_+, \text{ is the only gauge-invariant operator which is antisymmetric in } IJ \text{ and quadratic in } X^I. \text{ On the other hand, we have not found any simpler } \tilde{Q}^{IJ} \text{ from the weaker condition } (\tilde{Q}^{-1})_{(ij)} = (Q^{-1})_{(ij)}. \]
One can check that
\[
\frac{X_+^i M^{LJK} X_j^L M^{JK}}{\sqrt{X_+^2}} = -\frac{1}{3} M^{IKJ} M^{LJK} \tag{3.16}
\]
and
\[
\text{Tr} \left( \frac{i}{\sqrt{T}} \frac{X_+^i M^{LJK}}{\sqrt{X_+^2}} + \frac{X_+^i X_j^L}{X_+^2} (\det(S) - 1) \right)^n = \text{Tr} \left( \frac{i}{\sqrt{T}} \frac{X_+^i M^{LJK}}{\sqrt{X_+^2}} \right)^n + (\det(S) - 1)^n \tag{3.17}
\]
\[
\rightarrow \text{Tr} \log(\tilde{Q}^{IJ}) = \text{Tr} \log(S^{IJ}) + \log(\det(S)) \rightarrow \det \tilde{Q} = (\det(S))^2.
\]
Thus we are led to the following nonlinear Lagrangian for multiple M2 branes:
\[
\mathcal{L} = -T \text{Str} \left( \sqrt{-\det \left( \eta_{\mu\nu} + \frac{1}{T} \tilde{D}_{\mu} X^I \tilde{Q}^{-1}_{IJ} \tilde{D}_{\nu} X^J \right)} (\det \tilde{Q})^{1/4} + \text{Tr} \left( \frac{1}{2} \epsilon^{\mu\nu\rho} B_{\mu} F_{\nu\rho} \right) \right.
\]
\[
+ (\partial_{\mu} X^I_+ - \text{Tr}(X^I B_{\mu})) \partial^\mu X^I_+ - \text{Tr} \left( \frac{X_+ X^I_+}{X^2_+} \tilde{D}_{\mu} X^J \partial^\mu X^J_+ - \frac{1}{2} (\frac{X_+ X^I_+}{X^2_+})^2 \partial_{\mu} X^I_+ \partial^\mu X^I_+ \right) \tag{3.18}
\]
The connection with the D2 brane Lagrangian is now easily seen. For $X^I_+ = v_0 s$ we get $S^{ij} = Q^{ij}$, $S^{si} = S^{s8} = 0$, $S^{ss} = 1$, hence $\det S = \det Q$, $\det \tilde{Q} = (\det Q)^2$ and $Q^{-1} = Q^{-1}$, $(\tilde{Q}^{-1})_{ss} = 1/\det \tilde{Q}$. Then, by choosing the gauge $X^8 = 0$ we recover (3.3), which was shown to be equivalent to the D2 brane Lagrangian (3.1).

As in the abelian case, the last term is added in order to match the low-energy Lagrangian. Note that it vanishes for constant $X^I_+$. Its origin is the non-abelian version of the gauge-invariant combination eq. (2.16).

At the linearized approximation
\[
\det \tilde{Q} = (\det(S))^2 \cong 1 + \frac{1}{T} \frac{X^L_+ M^{LJK} X^J_+ M^{IKJ}}{\sqrt{X^2_+}} = 1 - \frac{1}{3T} M^{IKJ} M^{LJK}. \tag{3.19}
\]
Therefore, from (3.18),
\[
\mathcal{L} = -NT + \text{Tr} \left[ \frac{1}{2} \epsilon^{\mu\nu\rho} B_{\mu} F_{\nu\rho} - \frac{1}{2} \tilde{D}_{\mu} X^I \tilde{D}_{\nu} X^I - \frac{1}{12} M^{IKJ} M^{LJK} \right] + (\partial_{\mu} X^I_+ - \text{Tr}(X^I B_{\mu})) \partial^\mu X^I_+ + O(l_p^3) \tag{3.20}
\]
that is, we get the Lagrangian (1.1).

## 4 Fuzzy Funnel for M2-M5 brane Intersection

In this section we compare a BPS solution of the low energy Lagrangian (1.1) with an exact solution of the non-linear system (3.18). The solution generalizes the fuzzy funnel solution of [30] describing $N$ D1 branes ending in a D3 brane to eleven dimensions. Studies of BPS solutions in the Bagger-Lambert system can be found in [2] [33] [35] [36] [37].

\(^5\) The appearance of factors $X_+^2 = X_+^L X_+^J$ in the Lagrangian (3.18), and the fact that the Yang-Mills coupling is $g_{YM}^2 = \langle X_+^L X_+^J \rangle$, may suggest an interpretation of $X_+^2$ as a radial coordinate representing the center of mass position of the M2 branes [33]. However, this does not seem to be the precise role of $X_+^2$ in the Lagrangian (3.18).
4.1 BPS solution in BF membrane model

The BPS equations corresponding to the system (4.1) are given by [8, 9, 10]

\[\begin{align*}
\delta \Psi_+ &= \partial_\mu X^I_+ \Gamma^I \mu \varepsilon = 0 , \\
\delta \Psi_- &= (\partial_\mu X^I_- - \text{Tr}[B_\mu X^I]) \Gamma^I \mu \varepsilon - \frac{1}{3} \text{Tr}[X^I X^J X^K] \Gamma^{IJK} \varepsilon = 0 , \\
\delta \Psi &= (\partial_\mu X^I - B_\mu X^I + [A_\mu, X^I]) \Gamma^I \mu \varepsilon - X^I X^J X^K \Gamma^{IJK} \varepsilon = 0 .
\end{align*}\]  

(4.1)

The world-volume directions are \( N_0 = N_F \) or an irreducible representation, and they are identified with 0, 9, 10 (so that \( \Gamma^0 = \Gamma^9 \), \( \Gamma^1 = \Gamma^9 \), \( \Gamma^2 = \Gamma^{10} \)). Here \( \varepsilon \) is an eleven-dimensional Majorana spinor satisfying the condition \( \Gamma_{012} \varepsilon = \varepsilon \).

To solve the first equation, we set \( X^I_+ = v \delta I_8 \). We then look for solutions with \( B_\mu = A_\mu = 0 \) and set \( X = X^I T^a \), \( \Psi = \Psi_a T^a \), with \( \text{Tr}[T_a T_b] = K \delta_{ab} \). The remaining equations reduce to

\[\begin{align*}
\delta \Psi_+ &= \partial_\mu X^I_+ \Gamma^I \mu \varepsilon - \frac{1}{6} K C^{jcd} X^I_d X^J_e X^K_c \Gamma^{IJK} \varepsilon = 0 , \\
\delta \Psi_a &= \partial_\mu X^I_+ \Gamma^I \mu \varepsilon - \frac{v}{2} C^{jcd} X^I_d X^J c \Gamma^{IJS} \varepsilon = 0 .
\end{align*}\]  

(4.2)

The system admits a solution with \( SU(2) \) symmetry. We set \( T^i = \alpha^i \), \( i = 1, 2, 3 \) to be \( SU(2) \) generators in some \( N \times N \) representation, so that \( C^{ijk} = 2 \varepsilon^{ijk} \). We then consider the ansatz

\[X^I_a = f(\sigma) \delta a_I, \quad a, I = 1, \ldots, 3, \quad X^I_+ = p(\sigma) \delta I_8 ,
\]

where \( \sigma \equiv \sigma_1 \). This gives the equations

\[p'(\sigma) = \mp 2K f(\sigma)^3 , \quad f'(\sigma) = \pm 2vf(\sigma)^2 ,
\]

and the conditions on the spinor

\[\Gamma^{12389} \varepsilon = \mp \varepsilon .
\]

(4.5)

The equation \( f'(\sigma) = \pm 2vf(\sigma)^2 \) is exactly the same equation that arises for the fuzzy funnel in the D1-D3 brane system (taking into account the normalization \( (2.10) \)). The solution is given by

\[vf(\sigma) = \mp \frac{1}{2(\sigma_1 - \sigma_\infty)} ,
\]

(4.6)

where \( \sigma_\infty \) is an integration constant representing the position of the D3 brane. Integrating the equation for \( p \), we get

\[p(\sigma) = \mp \frac{K}{8v^3(\sigma_1 - \sigma_\infty)^2} .
\]

(4.7)

For an irreducible \( N \times N \) \( SU(2) \) representation \( K = \frac{1}{3} N(N^2 - 1) \).

4.2 Funnel in non-linear M2 brane theory

Here we discuss the funnel solution starting from the non-linear M2 brane Lagrangian [3, 18]. The ansatz is:

\[\begin{align*}
X^i &= f(\sigma) \alpha^i , \quad i = 1, 2, 3 , \quad X^I = 0 \text{ for } I > 3 , \\
X^I_+ &= v(\sigma) \delta I_8 , \quad X^I_- = p(\sigma) \delta I_8 \\
B_\mu &= 0 , \quad F_{\mu \nu} = 0 ,
\end{align*}\]  

(4.8)

where, as before, \( \alpha^i \) are the \( SU(2) \) generators in some \( N \times N \) representation, and \( \sigma \equiv \sigma_1 \) is a world-volume space coordinate.
With this ansatz the Lagrangian (3.18) becomes:

$$L = -T \text{Str} \left( \sqrt{\left( I + \frac{1}{T} f^2 \alpha^i Q^{-1}_{ij} \alpha^j \right) \det Q} \right) + p' v'$$  \hspace{1cm} (4.9)

where $I_{N \times N}$ is the identity matrix, and $Q^{ij} = \delta^{ij} + \frac{i}{\sqrt{T}} a^2 f^2 [\alpha^i, \alpha^j]$. In the large $N$ limit, assuming the symmetrized trace prescription, we obtain

$$\alpha^i Q^{-1}_{ij} \alpha^j = C_2 I, \quad \det Q = I + 4 T^{-1} f^4 v^2 C_2 I,$$  \hspace{1cm} (4.10)

where $C_2$ is the quadratic Casimir of the $SU(2) N \times N$ representation. Therefore

$$L = -TN \sqrt{(1 + \frac{1}{T} f^2 C_2)(1 + 4 \frac{1}{T} f^4 v^2 C_2)} + p' v'. \hspace{1cm} (4.11)$$

The variation with respect to $p$ gives $v = \text{const}$. The variation with respect to $v$ gives the equation:

$$p'' + 4 f^4 N C_2 \sqrt{\frac{1 + \frac{1}{T} f^2 C_2}{1 + 4 \frac{1}{T} f^4 v^2 C_2}} = 0.$$  \hspace{1cm} (4.12)

One can substitute the second-order equation for $f$ by the condition

$$f' \delta L \frac{\delta L}{\delta f} + p' \delta L \frac{\delta L}{\delta p} + v' \delta L \frac{\delta L}{\delta v} - L = \text{const} \rightarrow \sqrt{\frac{1 + \frac{1}{T} f^2 C_2}{1 + 4 \frac{1}{T} f^4 v^2 C_2}} = \text{const}.$$  \hspace{1cm} (4.13)

The last equation is solved by the solution of the first order equation:

$$f' = \pm 2 f^2 v, \hspace{1cm} (4.14)$$

whereby it follows that the equation (4.12) for $p$ is equivalent to

$$p' = \mp 2 N C_2 f^3 = \mp 2 K f^3.$$  \hspace{1cm} (4.15)

The above system of two first order equations (4.14), (4.15) is the same as eq. (4.4), obtained by looking for a supersymmetric solution of the linearized Lagrangian. Thus the BPS solution of the leading order theory (1.1) is also a solution of the full nonabelian M2 brane non-linear Lagrangian (3.18).

5 Discussion

Summarizing, we found the following non-linear Lagrangian

$$L = -T \text{Str} \left( \sqrt{\left( \eta_{\mu \nu} + \frac{1}{T} \hat{D}_\mu X^I \hat{Q}_{IJ}^{-1} \hat{D}_\nu X^J \right) \det \hat{Q}^{1/4}} \right) + \text{Tr} \left( \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} B_{\mu \nu} F_{\rho \sigma} \right) \hspace{1cm} (5.1)$$

where

$$\hat{D}_\mu X^I = \tilde{D}_\mu X^I - \frac{X^+ \cdot X}{X_+^2} \partial_\mu X_+^I, \hspace{1cm} (5.2)$$

$$\tilde{D}_\mu X^I = D_\mu X^I - X^+ X^- \partial_\mu X_+^I + [A_\mu, X^I]. \hspace{1cm} (5.3)$$
and ˜Q^{IJ} is defined in eqs. 3.14, 1.2. It is invariant under SU(N) gauge transformations and under the non-compact B_µ gauge transformations (1.4). The equation of motion for X^I_+ gives
\[ \partial_\mu \partial^\nu X^I_+ = 0. \] (5.4)

The second line of eq. (5.1) – which is gauge invariant by itself and vanishes for constant X^I_+ – ensures the match with the low-energy theory. As pointed out in section 2, the kinetic term for X^I_+, X^I_- appearing in the low-energy Lagrangian (1.1) cannot be put inside the square root because X^I_+, X^I_- are SU(N) singlets. Since ˜D_\mu X^I \tilde{D}_\nu X^J alone is not gauge invariant under B_µ transformations (1.4), one is led to introduce the covariant derivative ˜D_\mu X^I to render the square-root term invariant. On the other hand, the factor (det ˜Q)^{1/4} ensures that, after setting X^I_+ = v\delta^I_8, the correct D2 Lagrangian (3.1) is reproduced (modulo terms involving fluctuations of X^I_+ which are suppressed at large v). Note that if the shift symmetry X^I_+ \rightarrow X^I_+ + c^I is gauged as in [15, 16, 17] by adding the term \(-C^I_\mu \partial^\mu X^I_+\), then the theory (5.1) would be formally equivalent to the D2 brane Myers theory. Indeed, the equation of motion of C^I_\mu is \(\partial_\mu X^I_+ = 0\), which sets X^I_+ to a constant value v^I, hence leading to the D2 brane Lagrangian (3.1) by following the same steps of section 3.

In conclusion, the Lagrangian (5.1) satisfies the following properties:

- **SO(8) invariance.**
- Invariance under the local gauge symmetries of the BF theory with algebra (1.5) (i.e. SU(N) gauge invariance and B_µ-gauge transformations (1.4)).
- It contains just one dimensionful parameter \(l^3_p\) (or \(T = 1/(4\pi^2 l^3_p)\)), which disappears in the low energy approximation.
- At low energies the Lagrangian (5.1) reduces to the bosonic part of the BF membrane Lagrangian (3.1).
- When X^I_+ takes a large expectation value the Lagrangian (5.1) reduces to the Myers non-abelian D2 brane Lagrangian (1.6).
- The supersymmetric fuzzy funnel is a solution of the non-linear Lagrangian (5.1) to all orders. It does not receive any correction, just as it is the case for the D brane fuzzy funnel system describing the intersection of a D1 and a D3 brane [30].

**Acknowledgments**

R.I. acknowledges the hospitality at the Department ECM of the University of Barcelona where this work was initiated. This work is also supported by the European EC-RTN network MRTN-CT-2004-005104. J.R. also acknowledges support by MCYT FPA 2007-66665 and CIRIT GC 2005SGR-00564.
References

[1] J. Bagger and N. Lambert, “Gauge Symmetry and Supersymmetry of Multiple M2-Branes,” Phys. Rev. D 77, 065008 (2008) [arXiv:0711.0955 [hep-th]].

[2] A. Basu and J. A. Harvey, “The M2-M5 brane system and a generalized Nahm’s equation,” Nucl. Phys. B 713, 136 (2005) [arXiv:hep-th/0412310].

[3] A. Gustavsson, “Selfdual strings and loop space Nahm equations,” JHEP 0804, 083 (2008) [arXiv:0802.3456 [hep-th]].

[4] G. Papadopoulos, “M2-branes, 3-Lie Algebras and Plucker relations,” JHEP 0805, 054 (2008) [arXiv:0804.2662 [hep-th]].

[5] J. P. Gauntlett and J. B. Gutowski, “Constraining Maximal Supersymmetric Membrane Actions,” [arXiv:0804.3078 [hep-th]].

[6] N. Lambert and D. Tong, “Membranes on an Orbifold,” [arXiv:0804.1114 [hep-th]].

[7] J. Distler, S. Mukhi, C. Papageorgakis and M. Van Raamsdonk, “M2-branes on M-folds,” JHEP 0805, 038 (2008) [arXiv:0804.1250 [hep-th]].

[8] J. Gomis, G. Milanesi and J. G. Russo, “Bagger-Lambert Theory for General Lie Algebras,” JHEP 0806, 075 (2008) [arXiv:0805.1012 [hep-th]].

[9] S. Benvenuti, D. Rodriguez-Gomez, E. Tomni and H. Verlinde, “N=8 superconformal gauge theories and M2 branes,” [arXiv:0805.1087 [hep-th]].

[10] P. M. Ho, Y. Imamura and Y. Matsuo, “M2 to D2 revisited,” JHEP 0807, 003 (2008) [arXiv:0805.1202 [hep-th]].

[11] S. Mukhi and C. Papageorgakis, “M2 to D2,” JHEP 0805, 085 (2008) [arXiv:0803.3218 [hep-th]].

[12] S. Cecotti and A. Sen, “Coulomb Branch of the Lorentzian Three Algebra Theory,” [arXiv:0806.1990 [hep-th]].

[13] P. de Medeiros, J. M. Figueroa-O’Farrill and E. Mendez-Escobar, “Metric Lie 3-algebras in Bagger-Lambert theory,” [arXiv:0806.3242 [hep-th]].

[14] H. Verlinde, “D2 or M2? A Note on Membrane Scattering,” [arXiv:0807.2121 [hep-th]].

[15] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “Ghost-Free Superconformal Action for Multiple M2-Branes,” [arXiv:0806.0054 [hep-th]].

[16] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, “Supersymmetric Yang-Mills Theory From Lorentzian Three-Algebras,” [arXiv:0806.0738 [hep-th]].

[17] B. Ezhuthachan, S. Mukhi and C. Papageorgakis, “D2 to D2,” JHEP 0807, 041 (2008) [arXiv:0806.1639 [hep-th]].

[18] R. C. Myers, “Dielectric-branes,” JHEP 9912, 022 (1999) [arXiv:hep-th/9910053].

[19] A. A. Tseytlin, “On non-abelian generalisation of the Born-Infeld action in string theory,” Nucl. Phys. B 501, 41 (1997) [arXiv:hep-th/9701123].
[20] E. Bergshoeff, E. Sezgin and P. K. Townsend, “Supermembranes and eleven-dimensional supergravity,” Phys. Lett. B 189, 75 (1987).

[21] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” arXiv:0806.1218 [hep-th].

[22] M. Benna, I. Klebanov, T. Klose and M. Smedback, “Superconformal Chern-Simons Theories and AdS4/CFT3 Correspondence,” arXiv:0806.1519 [hep-th].

[23] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “Studies of the ABJM Theory in a Formulation with Manifest SU(4) R-Symmetry,” arXiv:0807.0880 [hep-th].

[24] T. Li, Y. Liu and D. Xie, “Multiple D2-Brane Action from M2-Branes,” arXiv:0807.1183 [hep-th].

[25] J. Kluson, “D2 to M2 Procedure for D2-Brane DBI Effective Action,” arXiv:0807.4054 [hep-th].

[26] P. M. Ho and Y. Matsuo, “M5 from M2,” JHEP 0806, 105 (2008) arXiv:0804.3629 [hep-th].

[27] P. M. Ho, Y. Imamura, Y. Matsuo and S. Shiba, “M5-brane in three-form flux and multiple M2-branes,” JHEP 0808, 014 (2008) arXiv:0805.2898 [hep-th].

[28] I. A. Bandos and P. K. Townsend, “Light-cone M5 and multiple M2-branes,” arXiv:0806.4777 [hep-th].

[29] I. A. Bandos and P. K. Townsend, “SDiff Gauge Theory and the M2 Condensate,” arXiv:0808.1583 [hep-th].

[30] N. R. Constable, R. C. Myers and O. Tafjord, “The noncommutative bion core,” Phys. Rev. D 61, 106009 (2000) arXiv:hep-th/9911136.

[31] E. Bergshoeff and P. K. Townsend, “Super D-branes,” Nucl. Phys. B 490, 145 (1997) arXiv:hep-th/9611173.

[32] H. Nicolai and H. Samtleben, “Chern-Simons vs. Yang-Mills gaugings in three dimensions,” Nucl. Phys. B 668, 167 (2003) arXiv:hep-th/0303213; B. de Wit, H. Nicolai and H. Samtleben, “Gauged supergravities in three dimensions: A panoramic overview,” arXiv:hep-th/0403014.

[33] S. Banerjee and A. Sen, “Interpreting the M2-brane Action,” arXiv:0805.3930 [hep-th].

[34] J. Bagger and N. Lambert, “Comments On Multiple M2-branes,” JHEP 0802, 105 (2008) arXiv:0712.3738 [hep-th].

[35] K. Hosomichi, K. M. Lee and S. Lee, “Mass-Deformed Bagger-Lambert Theory and its BPS Objects,” arXiv:0804.2519 [hep-th].

[36] I. Jeon, J. Kim, N. Kim, S. W. Kim and J. H. Park, “Classification of the BPS states in Bagger-Lambert Theory,” JHEP 0807, 056 (2008) arXiv:0805.3230 [hep-th].

[37] F. Passerini, “M2-Brane Superalgebra from Bagger-Lambert Theory,” arXiv:0806.0363 [hep-th].