Canonical aspects of strangeness enhancement

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Strangeness enhancement (SE) in heavy ion collisions can be understood in the statistical model on the basis of canonical suppression. In this formulation, SE is a consequence of the transition from canonical to the asymptotic grand canonical limit and is predicted to be a decreasing function of collision energy. This model predictions are consistent with the recent NA49 data on Λ enhancement at $p_{lab} = 40, 80, 158$ GeV.

1. INTRODUCTION

The enhanced production of strange particles in nucleus-nucleus (AA) relatively to proton-proton (pp) or proton-nucleus (pA) collisions, was since long, argued to be a signal of a Quark-Gluon Plasma (QGP) formation. This enhancement is seen in all experiments at all energies from AGS to SPS up to RHIC. Furthermore, enhancement of strange baryons and anti-baryons was predicted to depend on their strangeness content and to appear in a typical hierarchy, $E_\Lambda < E_\Xi < E_\Omega$, which was observed in particular by the WA97, NA57 and NA49 Collaborations. These enhancement features and hierarchy, are argued to be expected at all energies as a consequence of a canonical thermal phase-space suppression.

2. CANONICAL STRANGENESS SUPPRESSION

When describing a statistical system there are at least two ways of implementing the conservation laws of quantum numbers. In the Grand Canonical (GC) formulation conservation of a quantum number is ensured on average by use of the corresponding chemical potential. This description is only valid when particles, with a given conserved charge, are produced with a large multiplicity (system of a large volume and/or high temperature). In the Canonical formulation (C) conservation of a quantum number is implemented exactly on an event-by-event basis. This approach is relevant when particles carrying a given
conserved charge are produced with a small multiplicity (system of small volume and/or low temperature). In the usual definition of SE one compares the yield per participant of a given particle $i$ with strangeness $s$ in the large AA system to the yield per participant, of the same particle, in the small pp or pA system. Assuming that the volume parameter scales with the number of participants, $V \sim N_{\text{part}}$, the enhancement of this particle is given by

$$E_s^i = \left( \frac{n_s^i}{n_s^{i,\text{pp}}} \right)^{AA}_{pp}$$

where $n_s^i$ is the number density of particle $i$. From the above considerations, the pp (or pA) system has to be described in the canonical formulation. In principle, all conserved charges (baryon number $B$, electric charge $Q$, and strangeness $S$) have to be treated canonically. However, with a good approximation, $B$ and $Q$ can be treated grand canonically even in high energy pp collisions. This results in a slight overestimate of the canonical effect [5].

The canonical partition function of a gas of particles and resonances having strangeness $s = 0, \pm 1, \pm 2, \pm 3$, and with total strangeness $S = 0$ can be written as [6]

$$Z_{S=0}^C(T, V) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp \left( \sum_{n=-3}^{3} S_n e^{in\phi} \right)$$

where $S_n = V \sum_k Z_k^1$, is the sum over all particles and resonances carrying strangeness $n$ and where $Z_k^1 \equiv (g_k/2\pi^2)m_k^2TK_2(m_k/T)\exp{(B_k\mu_B/T + Q_k\mu_Q/T)}$ is the one-particle partition function of particle $k$ with multiplicity $g_k$, mass $m_k$, baryon number $B_k$, electric charge $Q_k$ and the corresponding chemical potentials $\mu_B$ and $\mu_Q$. From Eq.(4) one can derive the canonical density $(n_s^i)^C$ and show that

$$(n_s^i)^C = (n_s^{i,\text{GC}})F_s(T, V)$$

where $(n_s^{i,\text{GC}})$ is the grand canonical density and $F_s(T, V)$ is the suppression factor that measures a deviation of $(n_s^{i})^C$ from its asymptotic, grand canonical value. For large $V$ and/or $T$, $F_s(T, V) \to 1$, and $(n_s^{i})^C$ reaches its grand canonical value. From Eq.(1), SE for a given $V \sim N_{\text{part}}$ relative to pp is determined from

$$E_s^i = \frac{F_s(T, V)}{F_s(T, V_{pp})} \equiv \frac{F_s(T, N_{\text{part}})}{F_s(T, 2)}$$

Figure 1 displays an example of the suppression factor at $\sqrt{s} = 8.73$ GeV. The enhancement hierarchy at corresponding energy, as seen in Figure 2, is a direct consequence of the behavior of the suppression factor with the strangeness content of the particle [8]. Thus, SE can indeed appear as a canonical suppression effect.

3. ENERGY DEPENDENCE OF STRANGENESS ENHANCEMENT

We have studied the energy dependence of SE at four energies: $\sqrt{s} = 8.73, 12.3, 17.3$ GeV (NA49, NA57, WA97, SPS) and $\sqrt{s} = 130$ GeV (RHIC). At 17.3 and 130 GeV the values of $T$ and $\mu_B$ were taken from the thermal fit from [10,11]. At $\sqrt{s} = 8.73$ and 12.3
Figure 1. Canonical suppression factor for different of particle strangeness: $s = 1, 2, 3$ at energy $\sqrt{s} = 8.73$ GeV.

Figure 2. Centrality dependence of the relative enhancement of particles in central Pb-Pb at energy $\sqrt{s} = 8.73$ GeV.

GeV the values of these parameters were determined following unified freeze-out curve [12] and by using the measured $< \pi > / < N_{part} >$ ratio at the relevant energies. Moreover, as a first approximation, the assumption have been made that the thermal parameters are independent of centrality. We have shown that SE is a decreasing function of collision energy [9]. The result for $\Lambda$ is given in Figure 3. In Figure 4 the model predictions are compared to the recent experimental data of NA49 Collaboration [13]. The experimental points are deduced from the $(< \Lambda > / < \pi >)_{AA}/(< \Lambda > / < \pi >)_{pp}$ ratio [13] multiplied by $< \pi > / < N_{part} >$ in AA and in pp collisions [14] at the same energy. The solid line in Figure 4 is the prediction of the canonical statistical model.

Figure 3. Centrality dependence of the relative enhancement of $\Lambda$ in central Pb-Pb at different collision energies.

Figure 4. Energy dependence of the relative enhancement of $\Lambda$ in central Pb-Pb collisions.

4. CONCLUDING REMARKS

Canonical suppression plays an important role in SE. The present canonical statistical model naturally describes [14] the bulk properties of the WA97 data for $\Lambda, \Xi, \Omega$, that show SE hierarchy, and saturation of the yields per participant for large $N_{part}$. However, the
quantitative understanding of the recent data of the NA57 Collaboration \cite{16}, showing a 
smooth increase of that yields with $N_{\text{part}} > 100$ and a significant change in the behavior 
of $\Xi$ (the yield rises from $N_{\text{part}} = 62$ to $N_{\text{part}} = 121$ by a factor 2.6) would require 
a further study on the behavior of thermal parameters with centrality \cite{17}. These new 
features of NA57 data may be understood admitting a non-linear dependence of the 
canonical correlation volume or of the off-equilibrium strangeness fugacity with centrality \cite{18}. 
Nevertheless, the enhancement hierarchy and the decrease of enhancement with 
increasing collision energy are a generic features of the canonical statistical thermal model 
that are independent on the particular choice of the parameters. The most recent NA49 
data on $\Lambda$ yields seem to support the model prediction that $\Sigma E$ is a decreasing function 
of collisions energy. However, experimental confirmation of these behavior by even more 
spectacular variation of $\Xi$ and $\Omega$ enhancement with $\sqrt{s}$ would be required. We note that 
UrQMD \cite{19} and DPM \cite{20} predicts a larger $\Sigma E$ at RHIC than at SPS energy, which is 
in contrast to present statistical model results.

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