Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model

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In the present work, the mass spectra of doubly heavy tetraquarks $T_{QQ'}$ are systematically investigated in a relativized quark model. The four-body systems including the Coulomb potential, confining potential, spin-spin interactions, and relativistic corrections are solved within the variational method. Our results suggest that the $1^+_{1S}$ $bbūd$ state is 54 MeV below the relevant $BB$ and $BB^*$ thresholds, which indicates that both strong and electromagnetic decays are forbidden, and thus this state can be a stable one. Its large hidden color component and small root mean square radius demonstrate that it is a compact tetraquark rather than a loosely bound molecule or point-like diquark-antidiquark structure. Our predictions of the doubly heavy tetraquarks may provide valuable information for future experimental searches.

I. INTRODUCTION

In the past two decades, plenty of new resonances have been observed in the hadronic physics, and some of them can be hardly classified into the conventional hadron sectors, i.e., mesons and baryons [1]. These exotic structures have attracted extensive theoretical and experimental interests due to their enigmatic properties [2–12]. To describe their inner structures, new effective degree of freedom are introduced to go beyond the traditional quark-antiquark and three-quark configurations. The experimental observations of charged quarkonium-like states $Z_{cc}$ [13–17] and pentaquark states $P_{c}[18, 19]$ provide strong evidences for the existence of the exotic hadrons in QCD. Besides these hidden charm and bottom ones, it is also expected that the open flavor exotic states should exist. However, the experimental searches for these flavored exotic hadrons were beset with difficulties and obstacles, and the experiences of failures, such as $Θ^+(1540)$ [20] and $X(5568)$ [21, 22], have cast a shadow over this research area.

The situation began to change in 2017, when a doubly heavy baryon $Ξ_{cc}^{*+}$ was observed by the LHCb Collaboration [23]. Although the $Ξ_{cc}^{*+}$ is regarded as a $S$-wave conventional baryon, it provides an excellent opportunity to examine the interactions between two heavy quarks and search for more doubly heavy quark systems. Indeed, based on the mass of $Ξ_{cc}^{*+}$, the mass spectra of doubly heavy tetraquark states $T_{QQ'}$ were studied subsequently, which indicate that there should exist at least one stable flavored exotic tetraquark $bbūd$ [24, 25].

Actually, the doubly heavy tetraquarks $T_{QQ'}$ have been discussed for a long time. Before the observation of $Ξ_{cc}^{*+}$, there have been a number of theoretical works on the doubly heavy tetraquarks. Various approaches, involving quark models [24–42], QCD sum rules [43–46], and lattice QCD [47–50], were adopted to estimate their mass spectra. Due to the lack of experimental information on the doubly heavy systems, it is difficult to distinguish those numerous results. Also, several works have investigated their production mechanism, which should be helpful for experimental searches [51–55]. Lately, stimulated by the observation of $Ξ_{cc}^{*+}$, the studies on doubly heavy systems were revived and have interested plenty of theorists and experimenters. In particular, the properties for the doubly heavy tetraquarks, such as their masses, decays, and production rates, have been extensively discussed in the past years [56–83]. Within different frameworks, these studies present distinctive results and conclusions. However, almost all the works agree that the isoscalar $T_{bb}$ state should be stable against its strong and electromagnetic decays. The binding energy relative to the $BB^*$ threshold is predicted to be more than 100 MeV by most studies, which is deeply bound and leads to a compact configuration.

Within the framework of quark models, the previous studies were mainly based on the nonrelativistic quark potential models or simple quark models. Since the doubly heavy tetraquarks also include two light antiquarks, the relativistic corrections for the mass spectra may be significant. For instance, the masses of doubly heavy tetraquarks are calculated within a relativistic quark model under the diquark approximation [30]. However, the four-body calculations together with relativistic effects have not been done in the literature. Therefore, before making a final conclusion on the isoscalar $T_{bb}$ state, it is essential to perform a calculation in a relativized quark model with few-body method for the doubly heavy tetraquark spectra.

In this issue, we investigate the mass spectra of doubly heavy tetraquarks $T_{QQ'}$ in the relativized quark model proposed by Godfrey, Capstick, and Isgur [84, 85]. This model has been extensively adopted to study the properties of conventional hadrons and it may give a unified description of different flavor sectors. Also, under the diquark approximation, the authors have employed the relativized quark model.
to deal with the tetraquark states and achieved satisfactory results [86–92]. Thus, the relativized quark model is suitable for us to deal with the doubly heavy tetraquarks, where all the heavy-heavy, heavy-light, and light-light quark interactions are involved. For the first time, we extend the relativized quark model to investigate the double heavy tetraquark spectra by solving a four-body Schrödinger-type equation. With the present extension, the tetraquark, as well as the conventional hadrons can be described in a uniform frame.

This paper is organized as follows. The framework of relativized quark model and few-body method are introduced in Sec. II. The results and discussions of doubly heavy tetraquark spectra are given in Sec. III. A summary is presented in the last section.

II. MODEL

A. Hamiltonian

To calculate the mass spectra of doubly heavy tetraquarks $T_{QQ'} \equiv QQ' \bar{q}q'$, the relativized Hamiltonian should be constructed. Similar to the procedures of the conventional mesons and baryons [84, 85], the relativized Hamiltonian for a $QQ' \bar{q}q'$ tetraquark state can be written as

$$H = H_0 + \sum_{i<j} V^{\text{oge}}_{ij} + \sum_{i<j} V^{\text{conf}}_{ij},$$

where $H_0$ is a relativistic kinetic energy term

$$H_0 = \sum_{i=1}^{4} \left\{ \frac{p_i^2}{2m_i} + m_i^2 \right\}^{1/2}.$$  \hspace{1cm} (2)

The $V^{\text{oge}}_{ij}$ is the one gluon exchange pairwise potential, and $V^{\text{conf}}_{ij}$ corresponds to the confining part. The kinematic energy of the center-of-mass system can be eliminated by the constraint $\sum_{i=1}^{4} p_i = 0$.

In present work, we only concentrate on the $S$-wave ground states and do not include the spin-orbit and tensor interactions. Then, the potential $V^{\text{oge}}_{ij}$ can be expressed as

$$V^{\text{oge}}_{ij} = \beta_{ij}^{1/2} \tilde{G}(r_{ij}) \beta_{ij}^{1/2} + \sigma_{ij}^{1/2+\epsilon} \frac{2S_i \cdot S_j}{3m_im_j} \nabla^2 \tilde{G}(r_{ij}) \sigma_{ij}^{1/2+\epsilon},$$

with

$$\beta_{ij} = 1 + \frac{p_{ij}^2}{(p_{ij}^2 + m_i^2)^{1/2}}$$

and

$$\sigma_{ij} = \frac{m_im_j}{(p_{ij}^2 + m_i^2)^{3/2}(p_{ij}^2 + m_j^2)^{1/2}}.$$

The $p_{ij}$ is the magnitude of the momentum of either of the quarks in the center-of-mass frame of $ij$ quark subsystem, and

the $\epsilon_i$ is a free parameter reflecting the momentum dependence. The smeared Coulomb potential $\tilde{G}(r_{ij})$ is

$$\tilde{G}(r_{ij}) = F_i \cdot F_j \sum_{k=1}^{3} \frac{\alpha_k}{r_{ij}} \text{erf}(\tau_{kij} r_{ij}),$$

with

$$\frac{1}{\tau_{kij}} = \frac{1}{\gamma_k} + \frac{1}{\sigma_{ij}},$$

and

$$\sigma_{ij}^2 = \sigma_0^2 \left\{ \frac{1}{2} + \frac{1}{2} \left( \frac{4m_i m_j}{(m_i + m_j)^2} \right)^4 + s^2 \left( \frac{2m_i m_j}{m_i + m_j} \right)^2 \right\}.$$  \hspace{1cm} (8)

The $F_i \cdot F_j$ stands for the color matrix and reads

$$F_i = \begin{cases} \frac{\lambda_i}{2} & \text{for quarks}, \\ -\frac{\lambda_i}{2} & \text{for antiquarks}. \end{cases}$$

Similarly, the confining interaction $V^{\text{conf}}_{ij}$ can be expressed as

$$V^{\text{conf}}_{ij} = -\frac{3}{4} F_i \cdot F_j \left\{ b \left( e^{-\sigma_{ij} r_{ij}} + \frac{1}{1 + \frac{1}{2\sigma_i^2 r_{ij}^2}} \text{erf}(\sigma_i r_{ij}) \right) + c \right\}.$$  \hspace{1cm} (10)

All the parameters used here are taken from the original reference [84] and collected in Table I for convenience. The details of the relativized procedure can be found in Refs. [84, 85].

| TABLE I: Relevant parameters of the relativized quark model [84]. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| $m_c/m_Q$(MeV)                  | $m_Q$(MeV)      | $m_b$(MeV)      | $m_b$(MeV)      | $\alpha_1$     |
|                                | 220             | 419             | 1628            | 4977            | 0.25            |
| $\alpha_2$                     | 0.15            | 0.20            | 0.20            | 0.15            | 0.20            |
| $\gamma_1$(GeV)                | 1628            | 4977            | 0.25            | 1628            | 4977            |
| $\gamma_2$(GeV)                | 0.15            | 0.20            | 0.20            | 0.15            | 0.20            |
| $\gamma_3$(GeV)                | 0.15            | 0.20            | 0.20            | 0.15            | 0.20            |
| $b$(GeV$^{-1}$)                 | 1000            | 1000            | 1000            | 1000            | 1000            |
| $c$(MeV)                       | 0.18            | -253            | 1.80            | 1.55            | -0.168          |

B. Matrix elements of color, flavor, and spin parts

The wave function of a $Q_1 Q_2' q_3 q_4'$ state can be divided into color, flavor, spin, and spatial parts. In the color space, one has two kinds of colorless states with well defined permutation properties,

$$|33\rangle = |(Q_1 Q_2' )^3(q_3 q_4')\rangle,$$

$$|66\rangle = |(Q_1 Q_2' )^6(q_3 q_4')\rangle,$$

where the $|33\rangle$ is antisymmetric under the exchange of both quarks and antiquarks, and the $|66\rangle$ is the symmetric one. One can evaluate the color matrix elements $(F_i \cdot F_j)$ with the help of
explicit color wave functions or the SU(3) Casimir operator. The results are collected in Table II.

For the flavor part, the combination between quarks $\bar{u}$ and $d$ can be symmetric with $I = 1$ or antisymmetric with $I = 0$, while the combinations of $\bar{s}s$, $cc$, and $bb$ are always symmetric. For combinations $\bar{u}u$ and $\bar{d}d$, one can also construct the symmetric and antisymmetric flavor wave functions under the flavor SU(3) symmetry. The $c$ and $b$ are treated as different particles and no symmetry constraint should be obeyed. For convenience, the notation $\bar{u}d$ represents the combinations $\bar{u}u$, $\bar{d}d$, $(\bar{u}u+\bar{d}d)/\sqrt{2}$, and $(\bar{u}u-\bar{d}d)/\sqrt{2}$, and notation $\bar{s}s$ stands for the combinations $(\bar{s}s+\bar{s}s)/\sqrt{2}$, $(\bar{s}s-\bar{s}s)/\sqrt{2}$, $(\bar{s}s+\bar{s}s)/\sqrt{2}$, and $(\bar{s}s-\bar{s}s)/\sqrt{2}$ in the present work.

In the spin space, one can construct six spin states,

\begin{align}
\chi^{00}_0 &= \langle \mathcal{Q}_1 \mathcal{Q}_2 | 0 \rangle \langle \bar{q}_3 \bar{q}_4 | 0 \rangle_0, \\
\chi^{01}_0 &= \langle \mathcal{Q}_1 \mathcal{Q}_2 | 0 \rangle \langle \bar{q}_3 \bar{q}_4 | 1 \rangle_0, \\
\chi^{10}_1 &= \langle \mathcal{Q}_1 \mathcal{Q}_2 | 1 \rangle \langle \bar{q}_3 \bar{q}_4 | 0 \rangle_1, \\
\chi^{11}_1 &= \langle \mathcal{Q}_1 \mathcal{Q}_2 | 1 \rangle \langle \bar{q}_3 \bar{q}_4 | 1 \rangle_1, \\
\chi^{11}_2 &= \langle \mathcal{Q}_1 \mathcal{Q}_2 | 1 \rangle \langle \bar{q}_3 \bar{q}_4 | 2 \rangle_2,
\end{align}

where $(\mathcal{Q}_1 \mathcal{Q}_2)_0$ and $(\bar{q}_3 \bar{q}_4)_0$ are antisymmetric for the two fermions under permutations, and the $(\mathcal{Q}_1 \mathcal{Q}_2)_1$ and $(\bar{q}_3 \bar{q}_4)_1$ are symmetric. For the notation $\chi^{S1;S3}_S$, the $S_{12}$, $S_{34}$, and $S$ are the spin of two heavy quarks, spin of two light antiquarks, and total spin, respectively. The relevant spin matrix elements can be evaluated with the standard angular momentum algebra, and the results are listed in Table III.

For a $S-$wave $T_{QQ'}$ state, the spatial part is always symmetric, and then the color-spin-wave function should be antisymmetric for the identical quarks and antiquarks according to the Pauli exclusion principle. From the above discussions, we perform all possible configurations for the $QQ'\bar{q}\bar{q}'$ systems in Table IV. It should be noted that for a given system different configurations with same isospin-spin can mix with each other.

### C. Matrix elements of spatial part

For a $Q_1Q'_2\bar{q}_3\bar{q}'_4$ state, the Jacobi coordinates are shown in Figure 1. In these coordinates, one can define

\[ r_{12} = r_1 - r_2, \]

where \( r_{12} = r_1 - r_2 \)

| \( (O) \) | \( (F_1 \cdot F_2) \) | \( (F_1 \cdot F_3) \) | \( (F_1 \cdot F_4) \) | \( (F_2 \cdot F_3) \) | \( (F_2 \cdot F_4) \) | \( (F_3 \cdot F_4) \) |
|---|---|---|---|---|---|---|
| (33)(33) | -2/3 | -2/3 | -1/3 | -1/3 | -1/3 | -1/3 |
| (66)(66) | 1/3 | 1/3 | -5/6 | -5/6 | -5/6 | -5/6 |
| (33)(66) | 0 | 0 | -1/\sqrt{2} | -1/\sqrt{2} | 1/\sqrt{2} | 1/\sqrt{2} |

### TABLE II: Color matrix elements.

| \( (O) \) | \( (S_1 \cdot S_2) \) | \( (S_1 \cdot S_3) \) | \( (S_1 \cdot S_4) \) | \( (S_2 \cdot S_3) \) | \( (S_2 \cdot S_4) \) |
|---|---|---|---|---|---|
| (\chi^{ij}_1)_{O}^{|} | -3/4 | -3/4 | 0 | 0 | 0 |
| (\chi^{ij}_1)_{O}^{||} | 1/4 | 1/4 | 1/4 | 1/4 | 1/4 |
| (\chi^{ij}_1)_{O}^{\perp} | 0 | 0 | -\sqrt{3}/4 | -\sqrt{3}/4 | \sqrt{3}/4 |
| (\chi^{ij}_1)_{O}^{\|} | -3/4 | 1/4 | 0 | 0 | 0 |
| (\chi^{ij}_1)_{O}^{\perp} | 1/4 | 1/4 | 1/4 | 1/4 | 1/4 |
| (\chi^{ij}_1)_{O}^{\|} | 0 | 0 | -1/4 | -1/4 | -1/4 |
| (\chi^{ij}_1)_{O}^{\perp} | 0 | 0 | -\sqrt{3}/4 | \sqrt{3}/4 | \sqrt{3}/4 |
| (\chi^{ij}_1)_{O}^{\|} | 0 | 0 | -\sqrt{3}/4 | \sqrt{3}/4 | \sqrt{3}/4 |
| (\chi^{ij}_1)_{O}^{\perp} | 1/4 | 1/4 | 1/4 | 1/4 | 1/4 |

### TABLE III: Spin matrix elements.

\[ r_{34} = r_3 - r_4, \]

\[ r = \frac{m_1r_1 + m_2r_2}{m_1 + m_2} - \frac{m_3r_3 + m_4r_4}{m_3 + m_4}, \]

and

\[ R = \frac{m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4}{m_1 + m_2 + m_3 + m_4}. \]

Then, other relevant coordinates of this system can be expressed in terms of $r_{12}$, $r_{34}$, and $r$ as follows

\[ r_{13} = r_1 - r_3 = \frac{m_2}{m_1 + m_2}r_{12} - \frac{m_4}{m_3 + m_4}r_{34} + r, \]

\[ r_{24} = r_2 - r_4 = -\frac{m_3}{m_1 + m_2}r_{12} + \frac{m_3}{m_3 + m_4}r_{34} + r, \]

\[ r_{14} = r_1 - r_4 = \frac{m_2}{m_1 + m_2}r_{12} + \frac{m_3}{m_3 + m_4}r_{34} + r, \]

\[ r_{23} = r_2 - r_3 = -\frac{m_3}{m_1 + m_2}r_{12} - \frac{m_4}{m_3 + m_4}r_{34} + r, \]

\[ r' = \frac{m_1r_1 + m_3r_3}{m_1 + m_3} - \frac{m_2r_2 + m_4r_4}{m_2 + m_4} = \frac{m_1m_2(m_1 + m_2 + m_3 + m_4)}{(m_1 + m_2)(m_1 + m_3)(m_2 + m_4)}r_{12} + \frac{m_3m_4(m_2 + m_3 + m_4)}{(m_3 + m_4)(m_1 + m_3)(m_2 + m_4)}r_{34} + \frac{m_1m_2m_3m_4}{(m_1 + m_3)(m_2 + m_4)}r', \]

and

\[ r'' = \frac{m_1r_1 + m_4r_4}{m_1 + m_4} - \frac{m_2r_2 + m_3r_3}{m_2 + m_3} = \frac{m_1m_2(m_1 + m_2 + m_3 + m_4)}{(m_1 + m_2)(m_1 + m_4)(m_2 + m_3)}r_{12} - \frac{m_3m_4(m_1 + m_3 + m_4)}{(m_3 + m_4)(m_1 + m_4)(m_2 + m_3)}r_{34} + \frac{m_1m_2m_3m_4}{(m_1 + m_3)(m_2 + m_4)}r''. \]
TABLE IV: All possible configurations for the $Q\bar{Q}'\bar{q}'\bar{q}'$ systems. The subscripts and superscripts are the spin quantum numbers and color types, respectively. The braces {}, brackets [] strand for the symmetric, antisymmetric flavor wave functions, respectively. The parentheses () are used for the subsystems without permutation symmetries.

| System | $J^P$ | Configuration |
|--------|-------|---------------|
| $(cc)[\bar{u}\bar{d}]$ | 01* | $|\{cc\}_1^*|\bar{u}\bar{d}\rangle_{10}$ | $|\{cc\}_0^*|\bar{u}\bar{d}\rangle_{01}$ |
| $(cc)[\bar{u}\bar{d}]$ | 10* | $|\{cc\}_1^*|\bar{u}\bar{d}\rangle_{10}$ | $|\{cc\}_0^*|\bar{u}\bar{d}\rangle_{00}$ |
| $(bb)[\bar{u}\bar{d}]$ | 01* | $|\{bb\}_1^*|\bar{u}\bar{d}\rangle_{11}$ | $|\{bb\}_0^*|\bar{u}\bar{d}\rangle_{01}$ |
| $(bb)[\bar{u}\bar{d}]$ | 10* | $|\{bb\}_1^*|\bar{u}\bar{d}\rangle_{11}$ | $|\{bb\}_0^*|\bar{u}\bar{d}\rangle_{00}$ |
| $(cb)[\bar{u}\bar{d}]$ | 00* | $|\{cb\}_0^*|\bar{u}\bar{d}\rangle_{00}$ | $|\{cb\}_1^*|\bar{u}\bar{d}\rangle_{01}$ |
| $(cb)[\bar{u}\bar{d}]$ | 12* | $|\{cb\}_1^*|\bar{u}\bar{d}\rangle_{12}$ | $|\{cb\}_0^*|\bar{u}\bar{d}\rangle_{02}$ |
| $(cb)[\bar{u}\bar{d}]$ | 01* | $|\{cb\}_1^*|\bar{u}\bar{d}\rangle_{11}$ | $|\{cb\}_0^*|\bar{u}\bar{d}\rangle_{01}$ |
| $(cb)[\bar{u}\bar{d}]$ | 10* | $|\{cb\}_1^*|\bar{u}\bar{d}\rangle_{11}$ | $|\{cb\}_0^*|\bar{u}\bar{d}\rangle_{00}$ |
| $(cc)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{1}^*$ | $|\{cc\}_1^*|\bar{a}\bar{s}\rangle_{11}$ | $|\{cc\}_0^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(cc)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{0}^*$ | $|\{cc\}_1^*|\bar{a}\bar{s}\rangle_{10}$ | $|\{cc\}_0^*|\bar{a}\bar{s}\rangle_{00}$ |
| $(bb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{1}^*$ | $|\{bb\}_1^*|\bar{a}\bar{s}\rangle_{11}$ | $|\{bb\}_0^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(bb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{0}^*$ | $|\{bb\}_1^*|\bar{a}\bar{s}\rangle_{10}$ | $|\{bb\}_0^*|\bar{a}\bar{s}\rangle_{00}$ |
| $(cb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{0}^*$ | $|\{cb\}_0^*|\bar{a}\bar{s}\rangle_{00}$ | $|\{cb\}_1^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(cb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{1}^*$ | $|\{cb\}_1^*|\bar{a}\bar{s}\rangle_{11}$ | $|\{cb\}_0^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(cb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{2}^*$ | $|\{cb\}_2^*|\bar{a}\bar{s}\rangle_{22}$ | $|\{cb\}_1^*|\bar{a}\bar{s}\rangle_{12}$ |
| $(cb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{0}^*$ | $|\{cb\}_0^*|\bar{a}\bar{s}\rangle_{00}$ | $|\{cb\}_1^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(cb)[\bar{a}\bar{s}]$ | $\frac{1}{2}\bar{1}^*$ | $|\{cb\}_1^*|\bar{a}\bar{s}\rangle_{11}$ | $|\{cb\}_0^*|\bar{a}\bar{s}\rangle_{01}$ |
| $(bb)[\bar{3}\bar{3}]$ | 00* | $|\{bb\}_0^*|\bar{3}\bar{3}\rangle_{00}$ | $|\{bb\}_1^*|\bar{3}\bar{3}\rangle_{10}$ |
| $(bb)[\bar{3}\bar{3}]$ | 01* | $|\{bb\}_1^*|\bar{3}\bar{3}\rangle_{11}$ | $|\{bb\}_0^*|\bar{3}\bar{3}\rangle_{01}$ |
| $(bb)[\bar{3}\bar{3}]$ | 02* | $|\{bb\}_2^*|\bar{3}\bar{3}\rangle_{22}$ | $|\{bb\}_1^*|\bar{3}\bar{3}\rangle_{12}$ |
| $(cb)[\bar{3}\bar{3}]$ | 00* | $|\{cb\}_0^*|\bar{3}\bar{3}\rangle_{00}$ | $|\{cb\}_1^*|\bar{3}\bar{3}\rangle_{01}$ |
| $(cb)[\bar{3}\bar{3}]$ | 01* | $|\{cb\}_1^*|\bar{3}\bar{3}\rangle_{11}$ | $|\{cb\}_0^*|\bar{3}\bar{3}\rangle_{01}$ |
| $(cb)[\bar{3}\bar{3}]$ | 02* | $|\{cb\}_2^*|\bar{3}\bar{3}\rangle_{22}$ | $|\{cb\}_1^*|\bar{3}\bar{3}\rangle_{12}$ |

Fig. 1: The $Q_1Q_2\bar{q}_3\bar{q}_4'$ tetraquark state in Jacobi coordinates.
In our numerical calculation, the spatial wave function of a few-body system can be expanded in terms of a set of Gaussian basis functions, which forms an approximate complete set in a finite coordinate space [93]. For a $S$-wave $Q_1 Q_2 \bar q \bar q'$ tetraquark, the expanded basis should satisfy the relation $I_{12} + I_{34} + l = 0$, where the $I_{12}$, $I_{34}$, and $l$ are the relative angular momenta of the $Q_1 Q_2$, $\bar q \bar q'$, and $(Q_1 Q_2')\bar q' \bar q$, respectively. The contributions of higher orbital excitations to the ground states arise from the slight mixing via the spin-orbit or tensor interactions, which have been neglected in present calculations. Then, only the $I_{12} = I_{34} = l = 0$ case should be considered, and the spatial wave function for a certain tetraquark configuration can be expressed as

$$\Psi(r_{12}, r_{34}, r) = \sum_{n_0,n_\alpha,n} C_{n_0,n_\alpha,n} \psi_{n_0}(r_{12}) \psi_{n_\alpha}(r_{34}) \psi_n(r),$$

where $C_{n_0,n_\alpha,n}$ are the expansion coefficients. The $\psi_{n_0}(r_{12}) \psi_{n_\alpha}(r_{34}) \psi_n(r)$ stands for the position representation of the basis $|n\rangle = |n_{Q_1 Q_2}\rangle$, where

$$\psi_n(r) = \frac{2\sqrt{3/4}}{\pi^{1/4}} \frac{3/4}{e^{r^2/4n\nu}} Y_{00}(\hat{r}) \left( \frac{2\nu_0}{\pi} \right)^{3/4} e^{-\nu_0 r^2},$$

$$\nu_n = \frac{1}{r^2(2n-1)}, \quad (n = 1 - N_{\text{max}}).$$

The three parameters $r_1$, $a$, and $N_{\text{max}}$ are the Gaussian size parameters in geometric progression for numerical calculations, and the final results are stable and independent with these parameters within an approximate complete set in a sufficiently large space [93]. Besides the position representation $\psi_n(r)$, it is also convenient for the numerical calculations to present the momentum representation $\phi_n(p)$,

$$\phi_n(p) = \frac{2^{3/4}}{\pi^{1/4} \nu_n^{3/4}} e^{-p^2/(4n\nu)} Y_{00}(\hat{p}) \left( \frac{1}{2\pi \nu_n} \right)^{3/4} e^{-p^2/(4\nu_n)}.$$

Similarly, the formulas of $\psi_{n_0}(r_{12})$, $\psi_{n_\alpha}(r_{34})$, and $\phi_n(p)$ can be obtained by replacing the $n$, $r$, and $p$ of the $\psi_n(r)$ and $\phi_n(p)$.

To calculate the spatial matrix elements, we encounter the momentum-dependent factors combined with the position-dependent potentials in the relativized Hamiltonian. This difficulty can be overcome by inserting complete sets of Gaussian functions between the two types of operators. Take the first term of $\hat{V}_{ij}^{\text{pro}}$ for example, the matrix elements between two bases $|\alpha\rangle$ and $|\beta\rangle$ can be written as

$$\langle \alpha | \hat{G}(r_{ij}) | \beta \rangle = \sum_{\gamma, \delta, \rho} \langle \alpha | \hat{G}(r_{ij}) | \gamma \rangle \langle \gamma | N^{-1} \rho \delta | \beta \rangle \times \langle \rho, \delta | \hat{G}(r_{ij}) | \alpha \rangle \times \langle N^{-1} \rho, \delta | \beta \rangle.$$  

The $N$ is the overlap matrix of the Gaussian functions with matrix elements $N_{ij} = \langle ij | j \rangle$, which arises form the nonorthogonality of the bases. Together with the explicit forms of the basis in two representations, one can evaluate the expectations of momentum-dependent parts and position-dependent parts in the momentum representation and position representation, respectively.

D. Generalized eigenvalue problem

When all the matrix elements have been worked out, the mass spectra can be obtained by solving the generalized eigenvalue problem. For a given configuration without mixing, the homogeneous equation set can be expressed as

$$\sum_{j=1}^{N_{\text{max}}} (H_{ij} - E N_{ij}) C_j = 0, \quad (i = 1 - N_{\text{max}}).$$

Where, the $H_{ij}$ are the matrix elements in the total color-flavor-spin-spatial bases, $E$ stands for the eigenvalue, and $C_j$ are the relevant eigenvector. The lowest eigenvalue represents for the mass of this configuration, and the eigenvector corresponds to the expansion coefficients $C_{n_0,n_\alpha,n}$ in the spatial wave function.

From Table IV, a given system may include several different configurations with same $IJ^P$, which can mix with each other. In present calculation, we first solve the generalized eigenvalue problem to get the masses of pure configurations, and then calculate the off-diagonal effects between different configurations. The final mass spectra can be obtained by diagonalizing the mass matrix of these configurations.

III. RESULTS AND DISCUSSIONS

A. Numerical stability

Before discussing the properties of predicted tetraquarks, it is important to concentrate on the stabilities of the numerical procedures. In the nonrelativistic quark model, one can calculate the expectations of Hamiltonian in the trial wave functions, and always obtain the upper limit of the masses. When the number of bases increases, the numerical results decrease and approximate closely to the actual values. Empirically, stable results for $S$-wave states can be achieved within small numbers of bases.

In the relativized quark model, to calculate the matrix elements of Hamiltonian, complete sets of Gaussian functions should be inserted twice for the $V_{ij}^{\text{pro}}$, while the $V_{ij}^{\text{conf}}$ and relativistic kinetic energy term can be evaluated straightforward. The number of basis should be large enough to guarantee approximate completeness, otherwise the matrix elements of $V_{ij}^{\text{pro}}$ terms will be meaningless. For the meson spectra, a dozen bases are adequate, while about one hundred bases are needed for the baryon spectra [84, 85]. One can expect that several hundred or one thousand Gaussian functions are proper for calculating the tetraquark spectra.

Take the six pure configurations of $b\bar{b}t\bar{d}$ system for example, we investigate the dependence of results on the number of bases. The basis number varies from $N_{\text{max}}^3 = 6^3$ to $10^3$, and the dependence is presented in Figure 2. It is found that the eigenvalues are stable when the $N_{\text{max}}^3$ becomes larger. With $N_{\text{max}}^3 = 10^3$ bases, the numerical uncertainties are rather small, which are enough for the quark model calculations. Thus, we
adopt $10^3$ Gaussian bases to study the $S$–wave $T_{00}$ spectra in present work.

![Mass (MeV) vs. $N_{max}$](image)

**FIG. 2:** Numerical stabilities for six pure configurations of $bb\bar{u}\bar{d}$ system. The blue points, red squares, green diamonds, purple triangles, brown inverted triangles, and orange circles stand for the $|\{bb\bar{u}\bar{d}\}_{00}^0, \{bb\bar{u}\bar{d}\}_{11}^1, \{bb\bar{u}\bar{d}\}_{00}^0, \{bb\bar{u}\bar{d}\}_{11}^1, \{bb\bar{u}\bar{d}\}_{00}^0, \{bb\bar{u}\bar{d}\}_{11}^1 \rangle$, and $|\{bb\bar{u}\bar{d}\}_{00}^0, \{bb\bar{u}\bar{d}\}_{11}^1 \rangle$, configurations.

**B. Non-strange systems**

The predicted masses of $cc\bar{u}\bar{d}$, $bb\bar{u}\bar{d}$, and $cb\bar{u}\bar{d}$ systems are presented in Table V and Figure 3. For the $cc\bar{u}\bar{d}$ system, the lowest state is the 1$^{+}$ with 4041 MeV, which is a mixing state of the $|\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1|, |\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1|, |\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1 \rangle$, and $|\{cc\bar{u}\bar{d}\}_{00}^0, \{cc\bar{u}\bar{d}\}_{11}^1 \rangle$, configurations. This mixing is relatively small, and the $|\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1|, |\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1|, |\{cc\bar{u}\bar{d}\}_{00}^0|, |\{cc\bar{u}\bar{d}\}_{11}^1 \rangle$, and $|\{cc\bar{u}\bar{d}\}_{00}^0, \{cc\bar{u}\bar{d}\}_{11}^1 \rangle$, components are predominant. Due to the quantum conservation, the 0$^+$ and 2$^+$ states might decay into a pair of pseudoscalar mesons, while the allowed decay mode of a 1$^+$ state should be a vector meson plus a pseudoscalar one. From Figure 3, it can be seen that the lowest $cc\bar{u}\bar{d}$ state is 165 MeV higher than the $DD^*$ threshold, which can easily decay via falling apart mechanism.

For the $bb\bar{u}\bar{d}$ system, the mixing between different configurations are rather small and can be neglected. The predicted mass of the lowest state is 10550 MeV, which is almost a pure $|\{bb\bar{u}\bar{d}\}_{00}^0|, |\{bb\bar{u}\bar{d}\}_{11}^1 \rangle$ state. From our calculation, its mass is lower than the $B\bar{B}$ and $B\bar{B}^*$ thresholds, which indicates that both strong and electromagnetic decays are forbidden. Compared with $B\bar{B}^*$ channel, the binding energy is 54 MeV and the decay width should be tiny enough. Although the binding energy is smaller than that of the nonrelativistic quark models [31, 32, 37–39, 63, 67, 71, 73, 78, 79], we obtain the same conclusion about the stability of this state. The differences may arise from the relativized Hamiltonian, where the smearing potentials and relativistic corrections are included. This narrow structure can be searched via final states of weak decays, such as $B\bar{D}n^-$ and $B\bar{D}F$ $\nu_1$, in future LHC experiments [94, 95].

For the $cb\bar{u}\bar{d}$ system, there are two lower states around 7.3 GeV. With small mixing, these two states mainly consist of $|\{cb\bar{u}\bar{d}\}_{00}^0|, |\{cb\bar{u}\bar{d}\}_{11}^1 \rangle$, and $|\{cb\bar{u}\bar{d}\}_{00}^0|, |\{cb\bar{u}\bar{d}\}_{11}^1 \rangle$, configurations, respectively. The predicted masses of $cb\bar{u}\bar{d}$ tetrquarks are much higher than the $B\bar{D}$ and $B\bar{D}^*$ thresholds, and they can decay via quark rearrangement. Our calculation suggests that no stable $cb\bar{u}\bar{d}$ state does exist.

Together with the mass spectra, the wave functions are also obtained by solving the generalized eigenvalue problem of Hamiltonian. With these wave functions, we can calculate the proportions of hidden color components and the root mean square radii. Besides the [33] and [66] classifications, one can also define other sets of color representations,

$$|11\rangle = |Q_1\bar{q}_3\rangle |Q_2\bar{q}_3\rangle,$$  \hspace{1cm} (35)

$$|88\rangle = |Q_1\bar{q}_3\rangle ^8 |Q_2\bar{q}_3\rangle ,$$  \hspace{1cm} (36)

and

$$|1'1\rangle = |Q_1\bar{q}_3\rangle |Q_2\bar{q}_3\rangle ,$$  \hspace{1cm} (37)

$$|8'8\rangle = |Q_1\bar{q}_3\rangle |Q_2\bar{q}_3\rangle ^8 .$$  \hspace{1cm} (38)

Then, the three sets of color representations can be related as follows,

$$|11\rangle = \sqrt{\frac{1}{3}}|33\rangle + \sqrt{\frac{2}{3}}|66\rangle ,$$  \hspace{1cm} (39)

$$|88\rangle = -\sqrt{\frac{2}{3}}|33\rangle + \sqrt{\frac{1}{3}}|66\rangle ,$$  \hspace{1cm} (40)

and

$$|1'1\rangle = -\sqrt{\frac{1}{3}}|33\rangle + \sqrt{\frac{2}{3}}|66\rangle ,$$  \hspace{1cm} (41)

$$|8'8\rangle = \sqrt{\frac{2}{3}}|33\rangle + \sqrt{\frac{1}{3}}|66\rangle .$$  \hspace{1cm} (42)

Here, we adopt the [11] and [88] representations to stand for the neutral color and hidden color components, respectively.

The color proportions and root mean square radii of the three lowest $cc\bar{u}\bar{d}$, $bb\bar{u}\bar{d}$, and $cb\bar{u}\bar{d}$ states are presented in Table VI. The large hidden color component and small root mean square radius indicate that the $I^F = 0^{+}$ $bb\bar{u}\bar{d}$ state is a compact tetrquark rather than a loosely bound molecule. Also, the $0.285 \sim 0.484$ fm radii differentiate it from a point-like diquark-antidiquark structure. The sketch of this stable $T_{00}$ state is presented in Figure 4. It can be seen that the two heavy quarks stay close to each other like a static color source, while the light antiquark pair circles around this source and is shared by two heavy quarks.
### TABLE V: Predicted mass spectra for the $cc\bar{u}d$, $bb\bar{u}d$, and $c\bar{b}u\bar{d}$ systems.

| $I\bar{J}$ | Configuration | $\langle H \rangle$ (MeV) | Mass (MeV) | Eigenvector |
|------------|---------------|----------------|-----------|-------------|
| 01$^-$    | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (4053 - 55) | 4041 | $(-0.597, -0.205)$ |
|           | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (-55 4302) | 4313 | $(0.205, -0.979)$ |
| 10$^+$    | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (4241 - 89) | 4195 | $(-0.890, -0.455)$ |
|           | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (-89 4369) | 4414 | $(0.455, -0.890)$ |
| 11$^+$    | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | 4268 | 4268 | 1 |
| 12$^+$    | $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |_2$ | 4318 | 4318 | 1 |

| $I\bar{J}$ | Configuration | $\langle H \rangle$ (MeV) | Mass (MeV) | Eigenvector |
|------------|---------------|----------------|-----------|-------------|
| 01$^-$    | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (10551 20) | 10550 | $(-0.999, 0.050)$ |
|           | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (20 10950) | 10951 | $(-0.050, -0.999)$ |
| 10$^+$    | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (10769 31) | 10765 | $(-0.993, 0.122)$ |
|           | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (31 11015) | 11019 | $(-0.122, -0.993)$ |
| 11$^+$    | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | 10779 | 10779 | 1 |
| 12$^+$    | $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |_2$ | 10799 | 10799 | 1 |

| $I\bar{J}$ | Configuration | $\langle H \rangle$ (MeV) | Mass (MeV) | Eigenvector |
|------------|---------------|----------------|-----------|-------------|
| 00$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (7314 - 67) | 7297 | $(-0.970, -0.245)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (-67 7563) | 7580 | $(0.245, -0.970)$ |
| 01$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (7330 - 35 17) | 7325 | $(-0.992, -0.109, 0.067)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (-35 7658 18) | 7607 | $(0.095, -0.274, 0.957)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (17 18 7611) | 7666 | $(-0.086, 0.956, 0.282)$ |
| 02$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_2$ | 7697 | 7697 | 1 |
| 10$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (7535 - 56) | 7519 | $(-0.964, -0.265)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_0$ | (-56 7724) | 7740 | $(0.265, -0.964)$ |
| 11$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (7553 10 32) | 7537 | $(-0.740, 0.648, 0.183)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (10 7552 -16) | 7561 | $(-0.650, -0.758, 0.054)$ |
|           | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_1$ | (32 -16 7722) | 7729 | $(-0.174, 0.079, -0.982)$ |
| 12$^+$    | $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |_2$ | 7586 | 7586 | 1 |

![Graph](image_url)  
**FIG. 3:** The predicted masses of $cc\bar{u}d$, $bb\bar{u}d$, and $c\bar{b}u\bar{d}$ systems together with relevant thresholds. The blue lines stand for the tetraquarks including antisymmetric light subsystem $[\bar{u}\bar{d}]$, and the red lines correspond to the ones with symmetric light subsystem $[\bar{u}d]$.  

### TABLE VI: The color proportions and the root mean square radii of the three lowest $cc\bar{u}d$, $bb\bar{u}d$, and $c\bar{b}u\bar{d}$ states. The expectations $\langle r_{1d}^2 \rangle^{1/2}$, $\langle r_{2d}^2 \rangle^{1/2}$, and $\langle r_{n2}^2 \rangle^{1/2}$ equal to the values of $\langle r_{1d}^2 \rangle^{1/2}$, $\langle r_{2d}^2 \rangle^{1/2}$, and $\langle r_{n2}^2 \rangle^{1/2}$, respectively, which are omitted for simplicity. The units of masses and root mean square radii are in MeV and fm, respectively.

| System   | Mass | $|33\rangle$ | $|66\rangle$ | $|11\rangle$ | $|88\rangle$ | $\langle r_{1d}^2 \rangle^{1/2}$ | $\langle r_{2d}^2 \rangle^{1/2}$ | $\langle r_{n2}^2 \rangle^{1/2}$ | $\langle r_{1d}^2 \rangle^{1/2}$ | $\langle r_{2d}^2 \rangle^{1/2}$ | $\langle r_{n2}^2 \rangle^{1/2}$ |
|----------|------|-------------|-------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $|cc\rangle |\bar{u}\rangle | \bar{d}\rangle |$ | 4041 | 95.8% | 4.2% | 34.7% | 65.3% | 0.449 | 0.597 | 0.386 | 0.537 | 0.537 | 0.402 |
| $|bb\rangle |\bar{u}\rangle | \bar{d}\rangle |$ | 10550 | 99.8% | 0.2% | 33.4% | 66.6% | 0.285 | 0.484 | 0.370 | 0.465 | 0.465 | 0.274 |
| $|cb\rangle |\bar{u}\rangle | \bar{d}\rangle |$ | 7297 | 94.0% | 6.0% | 35.3% | 64.7% | 0.357 | 0.489 | 0.373 | 0.521 | 0.455 | 0.324 |
C. Strange systems

In present work, we treat the antisymmetric \([\bar{u}\bar{s}]\) and symmetric \([\bar{s}\bar{s}]\) as different flavor parts and do not consider the admixture between them. This situation is similar as the conventional \(\Xi_{c(b)}\) and \(\Xi’_{c(b)}\) baryons, which are usually regarded as two independent families. The mass spectra for the \(cc\bar{u}\bar{s}\), \(bb\bar{u}\bar{s}\), and \(cb\bar{s}\bar{s}\) systems are shown in Table VII and Figure 5. All of the tetraquarks locate above the corresponding thresholds, and the three lowest ones for these systems are 4232, 10734, and 7483 MeV, respectively. Analogously, the \(0^+\) and \(2^+\) states can decay into a pair of pseudoscalar mesons, and the \(1^+\) states can fall apart into a vector meson plus a pseudoscalar one.

It should be mentioned that in the literature some results supported a stable \(\{bb\}[\bar{u}\bar{s}]\) state with \(IJ^P = \frac{1}{2}^- 1^+\) [25, 35, 46, 49, 66, 67], and others predicted a state near the open bottom thresholds [30, 63]. Our results show that the lowest \(\{bb\}[\bar{u}\bar{s}]\) state is about 40 MeV above the \(\bar{B}\bar{B}\) and \(\bar{B}'\bar{B}\) thresholds. Considering the uncertainties of relativized quark model, we conclude that a resonance-like \(\{bb\}[\bar{u}\bar{s}]\) structure may exist. The results of color proportions and root mean square radii of the three lowest \(cc\bar{s}\bar{s}\), \(bb\bar{u}\bar{s}\), and \(cb\bar{s}\bar{s}\) states are also listed in Table VIII for reference. More experimental searches are expected to resolve this problem in the future.

For the \(cc\bar{s}\bar{s}\), \(bb\bar{s}\bar{s}\), and \(cb\bar{s}\bar{s}\) systems, the strange quark pair must be symmetric in flavor part and therefore, less states are predicted. From Table IX and Figure 6, it can been seen that all of them lie much higher than the corresponding thresholds and can easily fall apart into the charmed strange or bottom strange final states. Our results are consistent with other theoretical works [30, 40], and we believe that no stable structure exists in \(cc\bar{s}\bar{s}\), \(bb\bar{s}\bar{s}\), and \(cb\bar{s}\bar{s}\) systems.

D. Mass ratios

With the mass spectra of the doubly heavy tetraquarks \(T_{QQ}\), one can discuss the mass differences between tetraquark states and the corresponding thresholds. For instance, the mass differences between lower \(J^P = 1^+\) tetraquarks and thresholds versus the different systems are plotted in Figure 7. With the fixed light antiquark subsystem, the mass differences decrease when the heavy quarks vary from \(cc\) to \(bb\). Similarly, for a certain heavy quark subsystem, the mass differences show upward trends when the light antiquarks change from \(\bar{u}\bar{d}\) to \(\bar{s}\bar{s}\). The \(IJ^P = 01^+ \{bb\}[\bar{u}\bar{d}]\) state has the largest mass ratio between heavy quarks and light antiquarks, which forms a binding compact tetraquark. With the mass ratios between two subsystems decreasing, we cannot obtain stable doubly heavy tetraquarks.

In Refs. [41, 78], the authors also discussed the dependence of mass ratios between the heavy and light subsystems within nonrelativistic quark model, and showed the same behaviors with our relativized calculations. If one keeps reducing the mass ratios, the doubly heavy tetraquarks will become fully heavy tetraquarks. We can speculate that there is no stable state for the fully heavy tetraquarks since the mass ratios between the two subsystems are sufficiently small. This conjecture is supported by the experimental observations [96, 97] and nonrelativistic quark model works with proper potentials [98–103]. Certainly, the classifications of fully heavy tetraquarks are different with doubly heavy tetraquarks, precise calculations within the relativized quark model are needed before coming to any conclusion.

IV. SUMMARY

In this work, we systematically investigate the mass spectra of doubly heavy tetraquarks \(T_{QQ}\) in a relativized quark model. The four-body systems including the Coulomb potential, confining potential, spin-spin interactions, and relativistic corrections are solved within the variational method. With the present extension, the tetraquark, as well as the conventional hadrons can be described in a uniform frame. Our results suggest that the \(IJ^P = 01^+ bb\bar{u}\bar{d}\) state is 54 MeV below the relevant \(BB\) and \(BB^*\) thresholds, which indicates that both strong and electromagnetic decays are forbidden, and thus this state can be a stable one. The large hidden color component and small root mean square radius demonstrate that it is a compact tetraquark rather than a loosely bound molecule or point-like diquark-antidiquark structure. Compared with the results of nonrelativistic quark models, our calculations present a lower binding energy of this promising isoscalar \(T_{bb}\) state, but the decay behaviors agree with each other. We believe our calculations and predictions of the doubly heavy tetraquarks may provide valuable information for future experimental searches.

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TABLE VIII: The color proportions and root mean square radii of the three lowest including antisymmetric light subsystem $[\bar{u}\bar{s}]$, and the red lines correspond to the ones with symmetric light subsystem $[\bar{s}u]$.

| System | Mass (MeV) | $|\bar{s}\bar{s}|$ | $|\bar{u}\bar{s}|$ | $|\bar{s}u|$ | $|\bar{u}\bar{u}|$ | $|\bar{u}\bar{s}|^{\dagger}$ | $|\bar{s}u|^\dagger$ | $r_{v}^{2}$ | $r_{v}^{3}$ | $r_{v}^{4}$ |
|--------|------------|-----------------|-----------------|--------------|--------------|----------------|----------------|----------|--------|--------|
| $(cc)|[\bar{s}\bar{s}]|$ | 4232 | 93.1% | 6.9% | 35.6% | 64.4% | 0.423 | 0.491 | 0.384 | 0.544 | 0.470 | 0.363 |
| $(bb)|[\bar{s}\bar{s}]|$ | 10734 | 99.6% | 0.4% | 33.5% | 66.5% | 0.284 | 0.484 | 0.364 | 0.503 | 0.425 | 0.269 |
| $(cb)|[\bar{s}\bar{s}]|$ | 7483 | 90.6% | 9.4% | 36.5% | 63.5% | 0.358 | 0.493 | 0.365 | 0.557 | 0.412 | 0.324 |

TABLE VII: Predicted mass spectra for the $cc\bar{s}$, $bb\bar{s}$, and $cb\bar{s}$ systems.

| $IJ^+$ | Configuration | $(H)$ (MeV) | Mass (MeV) | Eigenvector |
|--------|--------------|-------------|------------|-------------|
| $\frac{1}{2}^+$ | $|cc\rangle [\bar{s}\bar{s}]_0^0$ | $\begin{pmatrix} 4246 \\ -50 \end{pmatrix}$ | 4232 | $\begin{pmatrix} -0.965 \\ -0.263 \end{pmatrix}$ |
| $\frac{1}{2}^+$ | $|cc\rangle [\bar{s}\bar{s}]_1^1$ | $\begin{pmatrix} 4370 \\ -82 \end{pmatrix}$ | 4323 | $\begin{pmatrix} -0.865 \\ -0.501 \end{pmatrix}$ |
| $\frac{1}{2}^+$ | $|cc\rangle [\bar{s}\bar{s}]_2^2$ | $\begin{pmatrix} 4394 \\ -82 \end{pmatrix}$ | 4512 | $\begin{pmatrix} 0.501 \\ 0.865 \end{pmatrix}$ |
| $\frac{1}{2}^+$ | $|cc\rangle [\bar{s}\bar{s}]_3^3$ | $\begin{pmatrix} 4400 \\ -50 \end{pmatrix}$ | 4727 | $\begin{pmatrix} 0.263 \\ -0.965 \end{pmatrix}$ |

FIG. 5: The predicted masses of $cc\bar{s}$, $bb\bar{s}$, and $cb\bar{s}$ systems together with relevant thresholds. The blue lines stand for the tetraquarks including antisymmetric light subsystem $[\bar{s}\bar{s}]$, and the red lines correspond to the ones with symmetric light subsystem $[\bar{s}u]$. 

TABLE VIII: The color proportions and root mean square radii of the three lowest $cc\bar{s}$, $bb\bar{s}$, and $cb\bar{s}$ states. The units of masses and root mean square radii are in MeV and fm, respectively.
FIG. 6: The predicted masses of the $cc\bar{s}$, $bb\bar{s}$, and $cb\bar{s}$ systems together with relevant thresholds.

| $IJ^*$ | Configuration | $(H)$ (MeV) | Mass (MeV) | Eigenvector |
|--------|---------------|-------------|------------|-------------|
| $00^*$ | $[cc]^4[\bar{s}\bar{s}]=0_0$ | $[4469, 79]$ | $4417$ | $(-0.832, 0.555)$ |
|        | $[cc]^4[\bar{s}\bar{s}]=0_0$ | $79\ 4535$ | $4587$ | $(-0.555, -0.832)$ |
| $01^*$ | $[cc]^4[\bar{s}\bar{s}]=1_1$ | $4493$ | $4493$ | $1$ |
| $02^*$ | $[cc]^4[\bar{s}\bar{s}]=1_2$ | $4536$ | $4536$ | $1$ |
| $00^*$ | $[bb]^4[\bar{s}\bar{s}]=0_0$ | $[10977, -29]$ | $10972$ | $(-0.987, -0.159)$ |
|        | $[bb]^4[\bar{s}\bar{s}]=0_0$ | $-29\ 11151$ | $11155$ | $(0.159, -0.987)$ |
| $01^*$ | $[bb]^4[\bar{s}\bar{s}]=1_1$ | $10986$ | $10986$ | $1$ |
| $02^*$ | $[bb]^4[\bar{s}\bar{s}]=1_2$ | $11004$ | $11004$ | $1$ |
| $00^*$ | $[cb]^4[\bar{s}\bar{s}]=0_0$ | $[7753, -50]$ | $7735$ | $(-0.941, -0.337)$ |
|        | $[cb]^4[\bar{s}\bar{s}]=0_0$ | $-50\ 7876$ | $7894$ | $(0.337, -0.941)$ |
| $01^*$ | $[cb]^4[\bar{s}\bar{s}]=1_1$ | $[7767, -8\ 29]$ | $7752$ | $(0.784, 0.570, -0.248)$ |
|        | $[cb]^4[\bar{s}\bar{s}]=1_1$ | $-8\ 7769\ 13$ | $7775$ | $(-0.576, 0.816, 0.056)$ |
|        | $[cb]^4[\bar{s}\bar{s}]=1_1$ | $29\ 13\ 7873$ | $7881$ | $(-0.234, -0.099, -0.967)$ |
| $02^*$ | $[cb]^4[\bar{s}\bar{s}]=1_2$ | $7798$ | $7798$ | $1$ |
FIG. 7: Mass differences between lower $J^P = 1^+$ tetraquarks and thresholds versus the different systems. The blue points stand for the tetraquarks including antisymmetric light subsystems, and the red squares correspond to the ones with symmetric light subsystems.

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