Mathematical Model of the Stick-Slip Effect for Describing the "Drumbeat" Seismic Regime During the Eruption of the Kizimen Volcano in Kamchatka

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Abstract. During the eruption of the Kizimen volcano in 2010-2013, there was a uniform squeezing of the viscous lava flow. Simultaneously with its movement, earthquakes with an unusual quasi-periodicity were recorded, the "drumbeat" mode. In this work, we show that these earthquakes were generated by the movement of the flow front, which was observed for the first time in the practice of volcanological research. We represent the movement of the flow as an intermittent slip with the inclusion of the "stick-slip" mechanism with the initiation of a self-oscillating process. The plausibility of the phenomenological model at the qualitative level is confirmed by the mathematical model of a fractional nonlinear oscillator.

INTRODUCTION

As a rule, the "drumbeats" the seismic regime, accompanies the squeezing out of extrusive domes on andesite and dacite volcanoes of the world [1]-[4]. A feature of this regime is a well-pronounced quasi-period of the occurrence of earthquakes from seconds to several tens of minutes with the formation of multiplets lasting from several hours to tens of days.

To simulate the "drumbeats" mode recorded during the eruption extrusion squeeze in. Soufriere Hills in 1997, considered the simplest model of the process observed in polymer technology [5]. The model assumes that the magma is a Newtonian fluid with a constant viscosity independent of the shear rate. When the melt is squeezed out of the reservoir at a constant speed into a channel with a small diameter, then the movement takes on an intermittent character only when the material flow rate is within a certain range. Physical modeling showed that the oscillations are caused by the boundary conditions of the contact of the melt with the wall. For oscillations to occur, a process is needed when energy can be alternately accumulated and released. In this case, the magma will alternately stick and slide along the channel wall, which corresponds to the "stick-slip" mechanism. The simulation results showed that an increase in the oscillation period is due to a change in the volume or length of the channel, as well as an increase in the viscosity of magma. Whereas, an increase in the bulk modulus or channel radius led to a decrease in the cycle period.

During the eruption of the Tungurahua volcano (Andes, Ecuador) in 2015, the "drumbeats" regime was recorded before the extrusion dome appeared [6]. In this case, two mechanisms have been proposed for the occurrence of the drumbeats mode: (1) the process of degassing followed by migration to the surface of the flow of gases; (2) the process of destruction of the geomedium due to the movement of magma. However, the preference is given to the mechanism of the "drumbeats" regime due to the movement of the fluid/gas flow along the network of cracks that appear ahead of the gradually rising magma front. However, this model does not provide an explanation for the quasiregularity of the process.

From the above, we can conclude that the occurrence of multiplets of earthquakes in the "drumbeats" mode is a unique natural process, the mechanism of which has not yet been fully understood. Understanding the generation

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process of quasiregular earthquakes requires a search for suitable mathematical models. In this paper, we consider the relationship of the seismic regime “drumbeats” with the dynamics of lava flow and propose a mathematical model of its occurrence.

**STICK-SLIP MATHEMATICAL MODEL**

At the front of the lava flow, separate lava blocks are formed (we take them in the form of a parallelepiped), which move under the action of an active core moving at a constant velocity $v$. In this case, the block $m$ can be considered as the right element of the system, moving due to the forces of noncumulative friction $F_{nec}$ with a falling characteristic depending on the speed (Fig. 1). Naturally, elements with a stiffness coefficient $k$ and a viscous resistance coefficient $b$ should be included between the active core and individual blocks.

![Figure 1](image)

**FIGURE 1.** Mechanical model of the generation of “drumbeats” seismic regime, which occurs when a lava flow

The mechanical model presented in Figure 1 describes the stick-slip effect. The stick-slip effect arises in problems of the movement of a load on a spring on a solid surface, taking into account friction and adhesion, which leads to adhesion of the load to the surface at certain times [7]. When the load moves along the surface, jumps of the load are observed, which are due to adhesion and separation from the surface, quasi-periodic sliding occurs. The magnitude of these jumps depends on the strength of adhesion and the stiffness of the spring.

Also stick-slip effect may be laid in the foundation of the mechanical model of the earthquake in the subduction zone of lithospheric plates [8]. Consider a mathematical model of the stick-slip effect.

Consider the following Cauchy problem [9]:

$$
\frac{d^2 x(t)}{dt^2} + b \frac{dx(t)}{dt} + \omega^2 x(t) = ut + c \sum_{n=1}^{\infty} a_n \sin(n\pi x(t)), x(0) = x_0, \dot{x}(0) = y_0,
$$

(1)

where $x(t)$ is the displacement function, $t \in [0, T]$ – time $T$ is the simulation time, $v = \omega^2$, $v$ is the displacement speed $\omega$ is the natural frequency, $b$ – coefficient of friction, $c = \frac{U_p}{x_p} = \text{surface adhesion energy}$, $m = k/\omega^2$ – effective mass of the oscillator, $k$ – stiffness coefficient, $U_p$ and $x_p$ – the depth and width of the potential well, $a_n = 2n \int_0^\frac{1}{2} \frac{\cos(n\pi t)dt}{\cosh^2(\pi t)}$ are the coefficients of the expansion of the Fourier series, $x_0$ and $y_0$ are given constants that determine the initial condition of the oscillatory system (1).

The mathematical model of the stick-slip effect is a nonlinear oscillator with in the approximation of equally alternating potential wells. Falling into a potential well means that the displacement rate is close to zero.

Let’s show on a qualitative level that this mathematical model can be applied to the description of the seismic regime “drumbeats”. The solution to problem (1) was obtained by the numerical Runge-Kutta method of the 4th order in the environment of symbolic computer mathematics Maple 2021.

**DATA AND ANALYSIS**

The analysis of the drumbeats seismic regime was carried out on the basis of seismic data from KZV station (Fig. 2, inset) of the Kamchatka Branch of the Geophysical Survey of the Russian Academy of Sciences (KBGS RAS) [10], [11] on the vertical SHZ component. KZV was installed in 2009, 2.5 km from the top of the Kizimen volcano, and
in the period 2011-2012 it worked almost without failures and malfunctions, which is of particular importance for us to study the quasi-periodicity of the earthquake occurrence process. The work with seismic records was carried out in the DIMAS seismic record processing program [12].

Let us consider the change in the nature of the generation of earthquakes in the "drumbeats" mode in the periods of March and October 2011, for which mathematical modeling was carried out. In March (Fig. 3c), a lava flow began to form with a maximum thickness of $h = 70$ m and a length of 1 km [13].

During this period, KZV recorded a multiplet of earthquakes lasting 12 days (March 11-22). On March 20, the average period of earthquake registration was $T = 14$ s (the average frequency of earthquakes was $\bar{f} = 3.7$ min$^{-1}$). The average amplitude of earthquakes in the selected fragment of the seismic record in Fig. 3a was equal to $\bar{A} = 4.5$ $\mu$m/s (Fig. 4a).
After analyzing and comparing the structures of the seismic mode multiplets "drumbeats" in March and October 2011 and their dynamic parameters with the dynamics of the lava flow, it was concluded that the more powerful tongue of the lava flow generated earthquakes with a lower frequency of registration, but a higher amplitude. Conversely, a thin lava flow, which was characterized by a high velocity of movement, generated more frequent earthquakes with a lower amplitude. The results obtained give grounds to propose the following mathematical model (1) for the generation of the "drumbeats" seismic regime.

Consider the results of modeling using the parameters of the lava flow in March 2011 (Fig.4). The values of the parameters of the model (1) are chosen as follows:

\[ b = 0.6, \quad v = 16.72, \quad \omega = 409, \quad m = 10^5, \quad v = 10^{-4}, \quad x_0 = y_0 = 0. \]

For numerical analysis, the number of nodes of the computational grid coincides with the amount of experimental data \( N = 18235 \), the decoding step is \( \tau = 1/128 \).

![FIGURE 4. Seismic mode “drumbeats”, registered on 03/20/2011 at KZV s/st, SHZ channel with time countdown 08:23:08 (a) and 10/05/2011 at KZV s/st, SHZ channel with 14:48 time countdown: 37 (b)](image)

![FIGURE 5. Seismic mode “drumbeats” recorded on 03/20/2011 with time reference 08:23:53 (a) and block displacement velocity function curve (b) based on lava flow parameters (1)](image)
Consider the results of modeling using the parameters of the lava flow in October 2011 (Fig. 5). The values of the parameters of the model (1) are chosen as follows:

\[ b = 0.1, = 572.48, \nu = 5.93, \omega = 243.7, m = 10^5, \nu = 10^{-4}, x_0 = y_0 = 0, N = 112504, \tau = 1/128. \]

Interpreting the results obtained, we can say that this model in the first approximation describes the process of generation of quasiperiodic earthquakes of the “drumbeats” mode. As can be seen from Fig. 4 and 5, the freezing coefficient of the block \( c \), friction \( b \) and the speed of movement of the block \( \nu \) have the greatest influence on the oscillations. With an increase in \( c \), an increase in the energy of earthquakes occurs, since it is necessary to apply a large force in order to overcome the “freezing” of the block at the front of the lava flow.

**CONCLUSION**

The results of mathematical modeling showed that with an increase in the thickness of the front of the Kizimen volcano lava flow, the friction at the contact between the base of the flow and the underlying rock increased. The pressure of the squeezing lava led to the “breakdown” of individual blocks of the lava flow front, their slipping and stopping, thus the model of generation of quasiperiodic earthquakes is consistent with the “stick-slip” model. More drumbeats in multiplets with lower \( A \) were the result of a decrease in sticking, while fewer earthquakes with higher \( A \) were characterized by an increase in sticking, which increased with increasing lava flow front strength.

Estimated calculations performed for two examples showed that the model of a fractional nonlinear oscillator provides self-oscillatory motion of the lava flow front block with a frequency close to the observed one. This is confirmed by the coincidence of the results of mathematical modeling with a fractional nonlinear oscillator on a qualitative level with experimental data.

Based on the simulation results using the known physical parameters of the lava flow, it can be said that the freezing coefficient of the block \( c \) had the greatest influence on the oscillations. To overcome the "freezing" of the block at the front of the lava flow, it was necessary to apply force. The more freezing we observed, the more force needed to be applied. Consequently, with an increase, there was an increase in the energy of earthquakes.

As a continuation of the work, it makes sense to consider a generalization of the mathematical model (1) to the case of taking into account the memory effect [15] and; however, in this case, the computational complexity of the problem that must be solved by supercomputers will sharply increase.
It is also of practical interest to study the amplitude-frequency and phase-frequency characteristics of the stick-slip effect by analogy with [16].

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