Simulating Posterior Distributions for Zero–Inflated Automobile Insurance Data

J.M. Pérez Sánchez\textsuperscript{a} and E. Gómez–Déniz\textsuperscript{b}

\textsuperscript{a} Department of Quantitative Methods, University of Granada, Spain
\textsuperscript{b} Department of Quantitative Methods in Economics and TiDES Institute, University of Las Palmas de Gran Canaria, Spain.

Abstract

Generalized linear models (GLMs) using a regression procedure to fit relationships between predictor and target variables are widely used in automobile insurance data. Here, in the process of ratemaking and in order to compute the premiums to be charged to the policy–holders it is crucial to detect the relevant variables which affect to the value of the premium since in this case the insurer could eventually fix more precisely the premiums. We propose here a methodology with a different perspective. Instead of the exponential family we pay attention to the Power Series Distributions and develop a Bayesian methodology using sampling–based methods in order to detect relevant variables in automobile insurance data set. This model, as the GLMs, allows to incorporate the presence of an excessive number of zero counts and overdispersion phenomena (variance larger than the mean). Following this spirit, in this paper we present a novel and flexible zero–inflated Bayesian regression model. This model includes other familiar models such as the zero–inflated Poisson and zero–inflated geometric models, as special cases. A Bayesian estimation method is developed as an alternative to traditionally used maximum likelihood based methods to analyze such data. For a real data collected from 2004 to 2005 in an Australian insurance company an example is provided by using Markov Chain Monte Carlo method which is developed in WinBUGS package. The results show that the new Bayesian method performs the previous models.

Keywords: Automobile Insurance, Claim, Markov Chain Monte Carlo, Portfolio, Power Series Distribution, Zero–Inflated.

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Generalized linear models (GLMs) using a regression procedure to fit relationships between predictor and target variables are widely used in automobile insurance data. Here, in the process of ratemaking and in order to compute the premiums to be charged to the policy–holders it is crucial to detect the relevant variables which affect to the value of the premium since in this case the insurer could eventually fix more precisely the premiums. We propose here a methodology with a different perspective. Instead of the exponential family we pay attention to the Power Series Distributions and develop a Bayesian methodology using sampling–based methods in order to detect relevant variables in automobile insurance data set. This model, as the GLMs, allows to incorporate the presence of an excessive number of zero counts and overdispersion phenomena (variance larger than the mean). Following this spirit, in this paper we present a novel and flexible zero–inflated Bayesian regression model. This model includes other familiar models such as the zero–inflated Poisson and zero–inflated geometric models, as special cases. A Bayesian estimation method is developed as an alternative to traditionally used maximum likelihood based methods to analyze such data. For a real data collected from 2004 to 2005 in an Australian insurance company an example is provided by using Markov Chain Monte Carlo method which is developed in WinBUGS package. The results show that the new Bayesian method performs the previous models.

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1 Introduction and motivation

There are a lot of applications involving discrete data for which observed data show frequency of an observation zero significantly higher than the one predicted by the assumed model. The problem of high proportion of zeros has been an interest in data analysis and modelling in many situations such as in the medical field, engineering applications, manufacturing, economics,
public health, road safety, epidemiology and, in particular, actuarial data. Models having more number of zeros significantly are known as zero-inflated models. Poisson regression models provide a standard framework for the analysis of count data. However, count data are often overdispersed relative to the Poisson distribution. One frequent factor of overdispersion is that the incidence of zero counts is greater than expected for the Poisson distribution and this is of interest because zero counts frequently have special issue. For example, in counting claims from policyholders, a policyholder may have no claims either because he/she is a good driver, or simply because no risk factors have happened “near” his/her driving. This is the distinction between structural zeros, which are (almost) inevitable, and sampling zeros, which occur by chance. In the other hand, as it is pointed out by Denuit et al. (2009), overdispersion leads to underestimates of standard errors and overestimates of Chi-square statistics and this could derive in serious consequences. For example, some explanatory variables may become not significant after overdispersion has been accounted for.

In the last decades there has been considerable interest in models for count data that allow for excess zeros, particularly in the econometric literature. Mullahy (1986) explores the specification and testing of some modified count data models. Lambert (1992) provides a manufacturing defects application of these models and discusses the case of zero-inflated Poisson (ZIP) models. Gupta et al. (1996) provide a general analysis of zero-inflated models. Gurmu (1997) develops a semi-parametric estimation method for hurdle (two-part) count regression models. Ridout et al. (1998) consider the problem of modelling count data with excess zeros and review some possible models. Hall (2000) adapts Lambert’s methodology to an upper bounded count situation, thereby obtaining a zero-inflated binomial (ZIB) model. Ghosh et al. (2006) introduce a flexible class of zero inflated models which includes other familiar models such as the Zero Inflated Poisson (ZIP) models, as special cases by using a Bayesian estimation method. An overview of count data in econometrics including zero inflated models is provided in Cameron and Trivedi (1998). In insurance, Yip and Yau (2005) provide a better fit to their insurance data by using zero-inflated count models. Boucher et al. (2007) revise zero-inflated and hurdle models with application to a Spanish insurance company and more recently, Mouatassim and Ezzahid (2012) analyze the zero-inflated models with an application to private health insurance data.

GLMs using a regression procedure to fit relationships between predictor
and target variables are widely used in automobile insurance data. Here, in order to compute the premiums to be charged to the policy-holders it is crucial to detect the relevant variables which affect to the value of the premium since in this case the insurer could eventually fix more precisely their premiums. We propose here a methodology with a different perspective. Instead of the exponential family we pay attention to the Power Series Distributions and develop a Bayesian methodology using sampling-based methods in order to model an automobile insurance data set. This model, as the GLMs, led us to incorporate the presence of an excessive number of zero counts and overdispersion phenomena. Following this spirit, in this paper we present a novel and flexible zero-inflated Bayesian regression model. We compare several inflated and standard models focused on applications in automobile insurance. The Bayesian model proposed here includes other familiar models such as the zero-inflated Poisson and zero-inflated geometric models, as special cases. A Bayesian estimation method is developed as an alternative to traditionally used maximum likelihood based methods to analyze such data. For a real data collected from 2004 to 2005 in an Australian insurance company an example is provided by using WinBUGS. The results show that the new Bayesian method performs the previous models.

The structured of the paper is as follows. Section 2 provides the zero-inflated power series distributions and the new Bayesian model proposed here. Sections 3 looks at automobile insurance application and 4 briefly concludes.

2 Modelling zero-inflated data

It is known that power series distributions form a useful subclass of one-parameter discrete exponential families suitable for modelling count data. From the original works of Kosambi (1949) and Noack (1950) the power series distribution has been very popular in the statistical literature dealing with discrete distributions which belong to this simple class. Two references concerning these features are Patil (1962b) and Patil (1962a). A revision of the power series distribution can be viewed in the chapter two in Johnson et al. (2005).

The probability function of the power series distribution becomes

\[
\Pr(X = x) = \frac{b(x)\theta^x}{f(\theta)}, \quad x = 0, 1, \ldots,
\]  

where
where $b(x)\theta^x \geq 0$, $b(x)$ is a function of $x$ or constant, $f(\theta) = \sum_{x=0}^{\infty} b(x)\theta^x$ is convergent and $\theta > 0$ is refereed as the power parameter of the distribution. The family of discrete distributions defined in (1) includes a broad class of known distributions, as the Poisson, binomial, negative binomial, logarithmic series and the Conway–Maxwell–Poisson distributions, among others. After computing the probability generating function, given by $G_X(z) = f(z\theta)/f(\theta)$, $|z| \leq 1$, it is simple to see that the mean and variance of the power series distribution result

$$E(X) = \mu = \frac{\theta f'(\theta)}{f(\theta)},$$

$$\text{var}(X) = \sigma^2 = \frac{\theta^2 f''(\theta)}{f(\theta)} + \mu(1 - \mu).$$

Thus the index of dispersion

$$ID = \frac{\sigma^2}{\mu} = 1 + \frac{\theta f''(\theta)}{f'(\theta)} - \mu$$

accommodates for overdispersion when $\frac{\theta f''(\theta)}{f'(\theta)} - \mu > 0$. For example, when the Poisson distribution is considered we have that $f(\theta) = \exp(\theta)$ and $ID = 0$, i.e. we get equidispersion. If $f(\theta) = (1 - \theta)^r$, $r > 0$, the distribution in (1) reduces to the negative binomial distribution and from (3) we get that $ID = 1 + \theta/(1 - \theta) > 1$ and overdispersion phenomena is obtained. Observe that for the binomial and negative binomial cases, the corresponding additional integer parameters, usually called $n > 0$ and $r > 0$, are considered as nuisance parameters.

Starting with a distribution belonging to the Power series, a more flexible model can be considered by adding a parameter which led us to inflate the zero value of the empirical data when there exists inflation of this. Thus, zero–inflated power series distribution contains two parameters. The first parameter $\omega$ indicates inflation of zeros and the other parameter $\theta$ is that of power series distribution. A zero–inflated power series distribution is a mixture of a power series distribution and a degenerate distribution at zero, with a mixing probability $\omega$ for the degenerate distribution. As Johnson et al. (2005) point out, a very simple alternative for modelling this setting is to add an arbitrary proportion of zeros, decreasing the remaining frequencies in an appropriate manner. In conclusion, zero–inflated models deal with the problem that the data display a higher fraction of zeros (non–claims...
in our case) and therefore appropriate for modelling counts that encounter disproportionally large frequencies of zeros.

If we start with a discrete distribution \( \Pr(Y = y) \), we can build a zero–inflated distribution in a simple form (see Cohen (1966)), by assuming

\[
\Pr(Y = y; \omega) = \begin{cases} 
\omega + (1 - \omega) \Pr(Y = 0), & y = 0, \\
(1 - \omega) \Pr(Y = y), & y \neq 0,
\end{cases}
\]

where \( \Pr(Y = y), x = 0, 1, \ldots \), is the parent distribution and

\[
- \frac{\Pr(Y = 0)}{1 - \Pr(Y = 0)} < \omega < 1.
\]

This last inequality allows the distribution to be well defined for certain negative values of \( \omega \). The counterpart of this representation of the support of \( \omega \) instead of the usual \( 0 \leq \omega \leq 1 \) is that the mixing interpretation is lost but, in practice the \( \omega \) parameter can take negative values into the support given in (5) and therefore (4) is genuine. See for example Bhattacharya et al. (2008). Later we will see that this is the case for the data which it will considered here.

So, the probability mass function of the zero–inflated Power series distribution, \( ZIPS(\omega, \theta) \), results

\[
Pr(Y_i = y_i; \omega) = \begin{cases} 
\omega + (1 - \omega) \frac{b(0)}{f(\theta)}, & y_i = 0, \\
(1 - \omega) \frac{b(k)\theta^k}{f(\theta)}, & y_i = k \neq 0,
\end{cases}
\]

where \( f(\theta) = \sum_{k=0}^{\infty} b(k)\theta^k \), \( 0 \leq \omega < 1 \) and \( \theta > 0 \). The mean and the variance are

\[
E(y_i; \omega) = (1 - \omega)\mu, \quad \text{var}(y_i; \omega) = (1 - \omega)(\sigma^2 + \omega\mu^2),
\]

where \( \mu \) denotes the mean of the power series distribution given in (2).

Now, zero–inflated forms assuming different count distributions belonging to the power series distribution, can be defined easily. Gupta et al. (1996) and Ghosh et al. (2006), for example, investigated the zero–inflated form of the generalized Poisson distribution.
Maximum likelihood estimators of $\omega$ and $\theta$ can be obtained by maximizing $\log \ell(\omega, \theta; y_i)$, $y = 1, \ldots, n$, with respect to $\omega$ and $\theta$, where

$$
\ell(\omega, \theta; y_i) = \prod_{i=1}^{n} \left[ \omega + (1 - \omega) \frac{b(0)}{f(\theta)} \right]^{ny_i} \left[ (1 - \omega) \frac{b(y_i) \theta^ny_i}{f(\theta)} \right]^{n-ny_i}.
$$

(9)

Here, $n$ is the sample size and $n_0$ is the number of zeros counts in the sample. Observe that by using binomial expansion the likelihood function in (9) can be written as

$$
\ell(\omega, \theta; y_i) \propto \theta^{n\bar{x}} \sum_{j=0}^{n_0} \binom{n_0}{j} \omega^j (1 - \omega)^{n-j} \left[ \frac{b(0)}{f(\theta)} \right]^{n-j}.
$$

(10)

After obtaining the normal equations we have to solve the equation

$$
\bar{x} - \frac{\theta f'(\theta)}{f(\theta) - b(0)} = 0
$$

to get the maximum likelihood estimate of $\theta$ and where $\bar{x}$ is the sample mean. Once obtained $\theta$ the parameter $\omega$ is obtained from

$$
\omega = \frac{(n-n_0)f(\theta)}{n[f(\theta) - b(0)]}.
$$

Therefore, the maximum likelihood estimation of the parameters under the power series distribution is simple and, in a similar manner, the regression coefficients when covariates are implemented into the model can also be obtained in a simple way.

2.1 Including covariates

In practice, the practitioner usually uses data set with commonly available exogenous covariates in order to explain the variable $Y_i$, known in this case as the endogenous variate. That is, suppose that for the $i$th observation, covariates $x$ and $z$ are available. In order to adapt the model to this framework we need to relate these covariates with endogenous variable via the parameters $\theta$ and $\omega$. This can be made through the following links,

$$
\theta_i = \exp(x_i^\top \beta),
$$

$$
\log \left( \frac{\omega_i}{1 - \omega_i} \right) = z_i^\top \gamma.
$$
with $\beta^T = (\beta_1, \ldots, \beta_k)$ and $\gamma^T = (\gamma_1, \ldots, \gamma_k)$ vectors of unknown regression parameters associated with covariates. Of course that in practice is common to suppose that the design matrix $X$ and $Z$ are the same.

A nice reformulation of the zero–inflated model above was proposed recently by Ghosh et al. (2006) which considered that the zero–inflated model can be represented as $Y = V(1 - B)$, where $B$ is a Bernoulli, Bernoulli($p$), random variable and $V$ independently to $B$ has a discrete distribution on the power series, PS($\theta$). Under this representation, the mean, $E(Y)$, and variance, $var(Y)$, can be rewritten as

$$E(y) = (1 - \omega)E(V), \quad (11)$$

$$var(y) = \frac{\omega}{1-\omega}E(y)^2 + \delta E(y), \quad (12)$$

where $\delta = var(V)/E(V)$ denotes the coefficient of dispersion of the latent random variable $V$. If the latent variable, $V$ does not have an underdispersed distribution (i.e., $\delta \geq 1$), then the distribution of $Y$ is overdispersed. On the other hand, if $V$ has a underdispersed distribution (i.e., $\delta < 1$), then $Y$ has a underdispersed distribution if and only if $E(V) < (1 - \delta)/\omega$. In their paper, they suppose that $V$ follows the power series distribution. In advance, this model will be called BZIPS (Bayesian Zero–Inflated Power series model).

In this paper two discrete distribution belonging to the Power series distributions will be considered. They are the Poisson distribution with parameter $\theta > 0$ and the geometric distribution with parameter $\frac{1}{(1 + \theta)}$, $\theta > 0$.

### 2.2 The Bayesian model

In this section a Bayesian methodology is carried out which allows for estimating of the model above in a simple way facilitating the process to incorporate covariates and providing exact posterior inference up to a Monte Carlo error. This model can easily accommodate multiple continuous and categorical predictors.

From a Bayesian point of view, prior distributions for $\omega$ and $\theta$ will be required. In this sense and looking to the log–likelihood in (10) it is adequate to assuming a Beta prior distribution for $\omega$ and the natural conjugate prior (Ghosh et al. (2006)) for the power series distribution in the following way,

$$\omega \sim Be(b_1, b_2),$$

$$\theta \sim \pi(\theta) = \frac{\theta^{a_1}}{[f(\theta)]^{a_2}}.$$
Both assumptions establish a congruent model, present important computational advantages and, in addition, have a tradition in Bayesian statistical literature. However, in this article, the covariates that affect $\omega$ and $\mu$ are fixed. So, we specify independent prior distributions for the parameters of the regression models, i.e. $\beta$ and $\gamma$, in the next way,

$$
\beta \sim U(a_\beta, b_\beta), \\
\gamma \sim U(a_\gamma, b_\gamma),
$$

where the constants $a_\beta$, $b_\beta$, $a_\gamma$ and $b_\gamma$ are assumed known. In particular, $a_\beta = a_\gamma = -10^5$ and $b_\beta = b_\gamma = 10^5$ which expresses lack of knowledge about the regression parameters. These noninformative uniform distributions are appropriate if no prior knowledge is available about the likely range of values of the parameters (Lempers (1971); Mitchell and Beauchamp (1988); among others).

Given that the prior distributions for parameters have been assessed, the next procedure is combine the likelihood function in (10) with priors to make a Bayesian inference. Since no closed forms are available for marginal posterior distributions, numerical approaches have to be used for generating them. The numerical approaches used are based on simulating from the posterior distributions which are proper since we consider proper priors although with somewhat large variances. The simulation approach used is Markov Chain Monte Carlo, which can be done using the WinBUGS software. Markov Chain Monte Carlo (MCMC) is in this setting a powerful tool which allows to get the estimates of the parameters involved. As it is well-known, MCMC is a method of posterior simulation and led us to compute the posterior density function for arbitrary points in the parameter space. With MCMC, it is possible to generate samples from an arbitrary posterior density and to use these samples to approximate expectations of quantities of interest such as the mean or second order moment. Several other aspects of the Markov chain method also contributed to its success. The methodology is very simple and consist of generating simulated samples from that posterior density, even though the density corresponds to unknown distributions. In this context, Gibbs sampling is a natural estimation method. A reasonable choice for starting values of $\beta$ and $\gamma$ for the MCMC simulation can be obtained by standard Poisson and Negative Binomial regression models using any statistical software package such as STATA. In this work, all simulations were done using WinBUGS (Spiegelhalter et al. (1999)). We run three parallel chains.
and a single long chain for diagnostic assessment (checked using CODA soft-
ware). A total of 100,000 iterations were carried out (after a 100,000 burn-in
period). A complete Gibbs sampling algorithm is outlined in [Ghosh et al.
(2006)].

3 Experiments with insurance data

In this section an application to the different models considered in the previ-
ous sections is developed in order to see how the proposed Bayesian method
works. The data set considered was taken from the web page of Mac-
Quarie University in Sydney (Australia) in which the different data taken
by Jong and Heller (2008) are available. This page contains numerous data
to be used in actuarial setting.

3.1 The data

In particular, the data studied contain information from policyholders of
several Australian insurance companies in 2004 and 2005, describing certain
characteristics related to the vehicles and the policyholders. It contains 67856
policies, of which 63232 (93.18%) have no claims. Table 1 shows a descriptive
of the dependent and independent variables.

Table 1: Descriptive summary of variables.

| Variables      | Mean     | Variance | Minimum | Maximum |
|----------------|----------|----------|---------|---------|
| Number of claims | 0.07275  | 0.07739  | 0       | 4       |
| Vehicle value   | 1.77702  | 1.45258  | 0       | 34.56   |
| Gender          | 0.43110  | 0.24525  | 0       | 1       |
| Young age       | 0.27436  | 0.19908  | 0       | 1       |
| Medium age      | 0.57492  | 0.24439  | 0       | 1       |
| Old age         | 0.46081  | 0.24846  | 0       | 1       |
| Vehicle age     | 0.57492  | 0.24439  | 0       | 1       |

Table 2 shows some measures which led us to consider departures from
the Poisson distribution. These measures are the proportion of zeros, $p_0$,
the cumulant $\kappa_3$, the zero–inflation index, $z_i = 1 + \log(p_0)/\mu$, and the third
central moment inflation index, $\kappa = \kappa_3/\mu - 1$, (see Puig and Valero (2006) for details). In this Table we can see the sample values of them and its corresponding estimates values obtained by using the Poisson distribution, the geometric distribution, the zero–inflated Poisson and the zero–inflated geometric distributions after estimating the corresponding parameters by maximum likelihood method. We can see that the geometric distribution (in its versions inflated and non–inflated at zero) outperforms the Poisson one.

Table 2: Some measures of inflation

|        | Sample | Poisson | Geometric | ZIP    | ZIG    |
|--------|--------|---------|-----------|--------|--------|
| $p_0$  | 0.93180| 0.92983 | 0.93218   | 0.93180| 0.93186|
| $\kappa_3$ | 0.08757| 0.07276 | 0.08941   | 0.08573| 0.08705|
| $z_i$  | 0.03000| 0.00000 | 0.03470   | 0.03000| 0.03000|
| $\kappa$ | 0.20367| 0.00000 | 0.22886   | 0.17832| 0.19640|

For each policy, the initial information for the period considered and the existence or otherwise of at least a claim were reported within this yearly period. In total, four explanatory variables are considered, together with a dependent variable representing the number of claims. Vehicle value represents its value in 10,000 Australian dollars. Vehicle age is equal to 1 if the vehicle is relatively young (7 years or less). Gender is equal to 1 if the policyholder is a man. This variable is included in the model for didactic purposes but, as expected, is not relevant in all the models considered. Finally, a categorical variable was considered to represent the age of the policyholder by dividing this feature in three dummies: young, medium and old ages. In this sense, we try to identify if there is/are age sets with more propensity to make claims. Several authors have used previously age variable in a dichotomous way. Some of them are Boucher et al. (2007), Bermúdez (2009) and Pérez et al. (2014), among others.

3.2 Results, diagnostic and comparisons

Table 3 shows results under the Bayesian model proposed by Ghosh et al. (2006) (BZIPS). As we can see, vehicle value and older policyholders (in relation to medium age) are relevant for the chance of being in zero–state. The positive value of the first coefficient indicates that this chance increases
with the value of the vehicle (with relevance at 1%) and the negative scope of old age indicates that the change of being in zero–states decreases for older policyholders in relation to the medium age policyholders, at 5% of relevance.

In the other hand, a negative intercept of $-1.883$ (with relevance at 1%) indicates that the average number of claims is less for the medium age policyholders. Furthermore, the more vehicle value, less average number of claims is expected, again at 1% of relevance. These results are consistent with the previous zero–state coefficients.

Table 3: Zero inflated Power Series fitted model (BZIPS).

|                | ZIPS ($\alpha_i$) | ZIPS ($\beta_i$) |
|----------------|-------------------|-------------------|
| Intercept      | $-0.482$          | $-1.883$ ***      |
|                | $(0.295)$         | $(0.122)$         |
| Vehicle value  | $0.543$ ***       | $-0.102$ ***      |
|                | $(0.096)$         | $(0.026)$         |
| Gender         | $0.005$           | $-0.028$          |
|                | $(0.181)$         | $(0.077)$         |
| Young age      | $-0.0008$         | $0.095$           |
|                | $(0.237)$         | $(0.093)$         |
| Old age        | $-0.497$ **       | $0.018$           |
|                | $(0.212)$         | $(0.101)$         |
| Vehicle age    | $0.070$           | $-0.035$          |
|                | $(0.236)$         | $(0.097)$         |

Table 4 shows results under the Bayesian model proposed in this paper (BZIGPS). Now, the fact of being in the medium age class increases the chance of zero–state (the $\alpha$–intercept is relevant at 10%). The average number of claims increases if the vehicle value is high (at 1% of relevance) and decreases in the medium and old age classes (more for older policyholders). For the youngest people, the average number of claims is expected to be greater than for the other two groups of policyholders (with 1% of relevance). Finally, the more vehicle age decreases the average number of claims at 10% of relevance.

Although there are a variety of methodologies to compare several models for a given data set, the deviance information criterion DIC, which is a generalization of the AIC (Akaike information criterion) and BIC (Bayesian
Table 4: Zero inflated Geometric–Power Series fitted model (BZIGPS).

|         | BZIGPS ($\alpha_i$) | BZIGPS ($\beta_i$) |
|---------|---------------------|---------------------|
| Intercept | 5.142 *             | -2.637 ***          |
|          | (3.199)             | (0.051)             |
| Vehicle value | 3.097               | 0.046 ***           |
|          | (3.328)             | (0.022)             |
| Gender   | 2.055               | -0.020              |
|          | (3.962)             | (0.032)             |
| Young age| 0.311               | 0.096 ***           |
|          | (4.628)             | (0.040)             |
| Old age  | 2.421               | -0.211 ***          |
|          | (4.705)             | (0.043)             |
| Vehicle age | 2.672                | -0.054              |
|          | (4.441)             | (0.037) *           |

information criterion), is useful in Bayesian model selection problems where the posterior distributions of the models have been obtained by MCMC simulation. Recall that it is only valid when the posterior distribution is approximately multivariate normal, which is the case considered here. In all the criterions, the model that better fits is a data set is the model with the smallest value.

Finally, it is interesting to compare these values with the frequentist point estimates and standard errors. Table 5 shows the results under two standard Poisson and negative binomial distributions and their zero–inflated versions. We can observe that they detect the same relevant factors than BZIGPS model but, as we can see in Table 6, these models fits worse than the BZIPS and BZIGPS models. The deviance of BZIPS is the smallest, but the confidence interval with a 5% significance is [9336, 9889], which includes the deviance of the BZIGPS model, so there is no significative difference in terms of fitting between these two models. By comparing Tables 4 and 5 we can observe that the estimated coefficients differ considerably, although the signs and the relevant factors remain the same. So, we can say that results from BZIGPS model are more consistent (than BZIPS) with respect to the results from classical models in Table 5 providing much better fitting.
### Table 5: Poisson and negative binomial inflated and non–inflated models

|                  | Poisson     | Negative Binomial | ZIP          | ZIBN         |
|------------------|-------------|-------------------|--------------|--------------|
| Intercept        | -2.6282 *** | -2.6305 ***       | -2.0540 ***  | -2.6306 ***  |
|                  | (0.0407)    | (0.0412)          | (0.0696)     | (0.0412)     |
| Vehicle value    | 0.0381 ***  | 0.0392 ***        | 0.0392 ***   | 0.0393 ***   |
|                  | (0.0114)    | (0.0117)          | (0.0118)     | (0.0117)     |
| Gender           | -0.0159     | -0.0161           | -0.0164      | -0.0162      |
|                  | (0.0300)    | (0.0300)          | (0.0301)     | (0.0301)     |
| Young age        | 0.0958 ***  | 0.0955 ***        | 0.0949 ***   | 0.0956 ***   |
|                  | (0.0337)    | (0.0337)          | (0.0337)     | (0.0337)     |
| Old age          | -0.2081 *** | -0.2083 ***       | -0.2082 ***  | -0.2083 ***  |
|                  | (0.0385)    | (0.0385)          | (0.0385)     | (0.0385)     |
| Vehicle age      | -0.0617 *   | -0.0606 *         | -0.0604     * | -0.0606 *    |
|                  | (0.0334)    | (0.0335)          | (0.0335)     | (0.0335)     |
| Inflation constant|            |                   | -0.2489 *   | -14.1262 *** |
|                  |              |                   | (0.1283)     | (0.7096)     |

### Table 6: Summaries of the fitting values for the models considered: DIC, AIC and BIC

| Model        | DIC      | AIC      | BIC      |
|--------------|----------|----------|----------|
| Poisson      | 36118.353| 36130.353| 36185.105|
| Negative Binomial | 36019.507| 36033.507| 36097.383|
| ZIP          | 36026.881| 36040.881| 36104.757|
| ZINB         | 36019.507| 36035.507| 36108.508|
| BZIGOPS      | 9871.000 | 9893.000 | 10004.501|
| BZIPS        | 9611.000 | 9623.000 | 9744.502 |
4 Final remarks

Insurance literature have paid a lot of attention on the considerable potential of generalized linear models as a comprehensive modelling tool for the study of the claims process in the presence of covariates in automobile insurance. Nevertheless, less attention have been paid to the well-known family of Power Series Distribution.

In this paper we develop a Bayesian methodology using sampling-based methods in order to model an automobile insurance data set by using discrete distributions belonging to the Power Series Distributions. As a consequence, we get a new and flexible model when overdispersion and inflation of zeros is presented in the data set. This is a topic which frequently appears in data related with automobile insurance.

For a real data collected from 2004 to 2005 in an Australian insurance company an example is provided by using Markov Chain Monte Carlo method which is developed in WinBUGS package. Comparisons with standard and Bayesian ZIP models in terms of parameter estimations and information criteria such as DIC, AIC and BIC are carried out. Bayesian ZIP models suggest a much improved fit over the classic ZIP models. Therefore, the results obtained here show that the new Bayesian method performs the previous GLMs models.

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