Echoes in a Single
Quantum Kerr-nonlinear Oscillator

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Abstract: We theoretically study the echo phenomenon in a single impulsively excited ("kicked") Kerr-nonlinear oscillator. These echoes may be useful for studying decoherence processes in a number of systems related to quantum information processing. © 2021 The Author(s)

1. Introduction

Perhaps the most famous example of echoes in physics are spin echoes conceived by Erwin Hahn in 1950 [2]. Spin echo can be induced by irradiating a collection of spins by two delayed magnetic field pulses. Then, a spontaneous pulsed magnetization response emerges at twice the delay between the excitation pulses. Over the years, echoes have been discovered in various physical systems, including systems consisting of many interacting/non-interacting particles, as well as in single quantum systems [3–5].

In this paper, we study echoes in a single impulsively excited quantum Kerr-nonlinear oscillator described by the Hamiltonian, \( H_0 = \hbar \omega_0 \hat{a}^\dagger \hat{a} + \hbar \chi (\hat{a}^\dagger)^2 \hat{a}^2 \), where \( \hat{a} \) and \( \hat{a}^\dagger \) are the canonical creation and annihilation operators, \( \omega_0 \) is the fundamental frequency of the oscillator, and \( \chi \) is the anharmonicity parameter [6]. Echoes studied here occur in individual quantum oscillators, like in [3–5], and rely on their intrinsic unitary dynamics.

2. The model

Quantized Kerr-nonlinear oscillator “kicked” by a pulsed coherent field can be modeled by a Hamiltonian (under rotating wave approximation)

\[
\hat{H} = \Delta \hat{a}^\dagger \hat{a} + (\hat{a}^\dagger)^2 \hat{a}^2 + E_0 f(t) (\hat{a}^\dagger + \hat{a}),
\]

where \( \Delta = (\omega_0 - \omega) / \chi \), \( \omega_0 \), \( E_0 \) are the dimensionless detuning, carrier frequency of the field, and field amplitude. Function \( f(t) \) defines the envelope of the field. Energy (time) is measured in units of \( \hbar \chi (\chi^{-1}) \).

Classical Hamiltonian, formally corresponding to the quantum one in Eq. (1) can be defined as [7] \( \hat{H} = (\Delta/2)(q^2 + p^2) + (1/4)(q^2 + p^2)^2 + \sqrt{2} E_0 f(t)q \), where \( q \) and \( p \) are the position and the momentum of the oscillator. For details, see the extended paper [1].

3. Results

An oscillator being initially in a coherent state \( \vert \alpha_0 \rangle \), is impulsive excited ("kicked") after a delay \( \tau \) (counted from the beginning of the evolution). Figure 1(a) shows the expectation value \( \langle \hat{q} \rangle (t) = \langle \hat{a}^\dagger + \hat{a} \rangle / \sqrt{2} \) calculated numerically in two different ways: (i) by solving the time-dependent Shrödinger equation with \( \hat{H} \) in Eq. (1), and (ii) by simulating the behavior of a classical ensemble corresponding to \( \vert \alpha_0 \rangle \). The curve describing the free propagation is added for comparison. Both quantum and classical simulations predict the expected immediate response to the applied excitation at \( t = \tau = 0.5 \). Later on, at twice the kick delay (\( t = 2\tau = 1.0 \)), coherent oscillations appear again without any additional kicks. On the long time scale, several additional pulsed responses having remarkably large amplitudes emerge at \( t = T_{rev} - \tau = 2.64 \), and \( t = T_{rev} - 2\tau = 2.14 \). Here, \( T_{rev} = \pi \) is the revival time. The described pulsed responses are similar to echo signals known in many other physical system. The echoes can be classified as “classical” and “quantum” ones based on their timing and mechanism of formation. The details can be found in the extended article [1].

Next, we consider the sensitivity of the echoes to damping effects caused by interaction of the oscillator with a reservoir at finite temperature \( T \). Oscillator dynamics is described by the reduced density matrix \( \hat{S}(t) \), defined by the differential equation [6]

\[
\frac{d\hat{S}}{dt} = -i[\hat{H}, \hat{S}] + \gamma \hbar [\hat{a}, \hat{S}, \hat{a}^\dagger] + \frac{\gamma}{2}(2\hat{a}\hat{S}\hat{a}\dagger - \hat{a}\dagger\hat{a}\hat{S} - \hat{S}\hat{a}\dagger - \hat{a}\hat{S}) \tag{2}
\]

*For the extended paper, see [1].
where $\hat{\mathcal{H}}$ is defined in Eq. (1), $\hat{\mathcal{L}} = \hat{\mathcal{A}}\hat{\mathcal{B}} - \hat{\mathcal{B}}\hat{\mathcal{A}}$, $\gamma$ is the dimensionless damping constant, $\bar{n} = [\exp(\epsilon) - 1]^{-1}$ [where $\epsilon = \hbar\omega/(k_B T)$] is the mean number of bosonic excitations in the reservoir’s mode having frequency $\omega$ (the fundamental frequency of the quantized Kerr-nonlinear oscillator), and $k_B$ is the Boltzmann constant.

Figures 1(b,c) show two examples of the damped dynamics of $\langle \hat{q}\rangle$ for the case of a field being initially in a coherent state $|\alpha_0\rangle$ [i.e. $\hat{S}(t = 0) = |\alpha_0\rangle \langle \alpha_0|$. For weak damping, $[\gamma = 10^{-3}$, see Fig. 1(b)], the amplitude of all oscillations (revivals and echoes) gradually diminish with each revival cycle. In the case of stronger damping, $[\gamma = 0.1$, see Fig. 1(c)], even the first revival at $t = T_{rev} = \pi$ is not visible. In contrast, the classical echo appearing on the short time scale is clearly visible. Since the quantum echo appears on the longer time scale (just before the revival), its amplitude is negligible as compared to the amplitude of the classical echo.

4. Conclusions

Two types of echoes, quantum and classical ones, are predicted in the kicked Kerr-nonlinear oscillator. The classical echoes exist on the short time-scale in the semi-classical limit, while the quantum ones show themselves near the quantum revivals of various order. Kerr-nonlinear oscillator model is relevant to several quantum-information processing systems [8], and the discussed echoes may be useful for studying decoherence processes in such systems.

References

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