Simply Modeling $\bar{B} \rightarrow K^* \gamma$

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Abstract

A simple relativistic model of heavy-light mesons is applied to the rare decay $\bar{B} \rightarrow K^* \gamma$ in the standard model. We find $\Gamma(\bar{B} \rightarrow K^* \gamma)/\Gamma(b \rightarrow s \gamma) = (17 \pm 4\%)$ and $\text{BR}(\bar{B} \rightarrow K^* \gamma) = (4.8 \pm 1.9) \times 10^{-5} (|V_{cb}|/0.04)^2$. These numbers are reduced by only 20% in the heavy-quark limit.

We have recently developed a relativistic model of mesons containing a heavy quark $Q$ and a light antiquark $\bar{q}$ [1, 2]. Matrix elements for $Q_1 \bar{q} \rightarrow Q_2 \bar{q}$ meson transitions are represented by quark loop graphs with $Q_2 Q_1$-type operator insertions on the heavy-quark line. The external mesons are joined to the loop with vertices of the form $Z^2/(-k^2 + \Lambda^2)$, where $k$ is the light quark momentum. These vertices suppress large momentum flow into the light quark, which is the essential physical effect of the light quark wave function. The $Z$’s and $\Lambda$’s are different for each meson flavor and spin, but are not arbitrary parameters. They are fixed in terms of the heavy and light masses appearing in the standard quark propagators by requiring the meson self-energies to vanish and have unit slope at the physically-measured meson masses.

Central to the application of a free quark model is the assumption that QCD confinement is characterized by somewhat smaller momentum scales than typical light quark momenta in a heavy-light meson. This situation is realized by our model and it thus suggests the possibility that confinement may not play an essential role. Indeed, our model results are obtained by simply dropping the imaginary parts arising from the free quark loop diagrams. The success or failure of such a model will shed

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light on the role played by confinement. Thus far we have found that the model yields a differential decay spectrum for $B \rightarrow D^* \ell \nu$ whose shape compares well with the data when $m_b = 4.80 \text{ GeV}$, $m_c = 1.44 \text{ GeV}$ and $m_q = 250 \text{ MeV}$ \cite{3}. The model may also be expanded in inverse powers of heavy-quark masses and the vector- and axial-vector current form factors have been shown in \cite{1} to be consistent with all heavy-quark symmetry constraints through order $1/m_Q$ \cite{4,5}.

The purpose of this paper is to apply our model to the rare decay $B \rightarrow K^* \gamma$ \cite{6} within the context of the standard model. We will be mainly interested in the results of our full, unexpanded model, but we will also compare these results to those obtained in the heavy-quark limit. Because the strange quark is not particularly heavy, there is no $a$ priori reason to expect there to be any resemblance. We will see, however, that certain quantities have surprisingly small net corrections.

The relevant $\bar{s}b$-type operators are \cite{7}

$$O_\mu = \bar{s}i\sigma_{\mu\nu} q^\nu b \quad \text{and} \quad O_{5\mu} = \bar{s}i\sigma_{\mu\nu} q^\nu \gamma_5 b. \quad (1)$$

Form factors for $B(M,V) \rightarrow \overline{K}^*(m,v,\varepsilon)$ may be defined by

$$\langle \overline{K}^* | O_\mu | B \rangle = \sqrt{Mm(M + m)} h(\omega) \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu} V^\rho V^\sigma \quad (2)$$

$$\langle \overline{K}^* | O_{5\mu} | B \rangle = - i \sqrt{Mm} \{ [(M - m)(\omega + 1)g_{\mu\nu} - M(V + v)_\mu V_\nu] h_5(\omega) + [(M + m)(\omega - 1)g_{\mu\nu} + M(V - v)_\mu V_\nu] h'_5(\omega) \} \varepsilon^{\nu} \quad (3)$$

where $\omega = V \cdot v$ is the product of the mesons’ four-velocities. The three form factors $h$, $h_5$, and $h'_5$ are not independent at the physical recoil point $\omega_o = (M^2 + m^2)/2Mm = 3.04$, where they satisfy $h(\omega_o) = h_5(\omega_o) + (M - m)(M + m)^{-1} h'_5(\omega_o)$. In the heavy-quark limit, we find $h(\omega) = h_5(\omega) = \xi(\omega)$ and $h'_5(\omega) = 0$, where $\xi$ is the same Isgur-Wise function appearing in meson semileptonic decays.

As described below we will fix the $m_b$ and $m_s$ masses by going to a point of minimal sensitivity in the $m_b$-$m_s$ plane. It is reassuring to find that this point occurs at the physically reasonable values $m_b = 4.83 \text{ GeV}$ and $m_s = .40 \text{ GeV}$. As in our earlier work, we choose $m_q = 250 \text{ MeV}$ as a reasonable light quark constituent mass \cite{2}. This is somewhat smaller than the usual 330 MeV and it models the fact that the actual momentum-dependent light quark mass has fallen somewhat at typical light quark momenta in the loop. We will find little sensitivity to the choice of $m_q$. The vertices are given explicitly by

$$\gamma_5 \frac{Z^2_B}{-k^2 + \Lambda^2_B} \quad \text{and} \quad -i\gamma_\nu \frac{Z^2_{K^*}}{-k^2 + \Lambda^2_{K^*}}, \quad (4)$$
and the various constants are determined by the physical masses $M = 5.279$ GeV and $m = .892$ GeV to be $\Lambda_B = .577$ GeV, $Z_B = 1.013$ GeV, $\Lambda_{K^*} = .617$ GeV and $Z_{K^*} = .871$ GeV. These values of $\Lambda$ characterize the typical light quark momenta.

The form factors computed using the full vertices of Eq. (4) are shown along with $\xi$ in Fig. 1. Numerical results are shown in (5) at the zero recoil point $\omega = 1$ and at the physical recoil point $\omega_o$.

| $\omega$ | $h$ | $h_5$ | $h'_5$ | $\xi$ |
|----------|-----|-------|--------|-------|
| $\omega_o$ | .262 | .205 | .080 | .235 |

We see that the net deviations from the heavy-quark limit are in general not as large as one might have expected. At the physical recoil point $h$ and $h_5$ differ from $\xi$ by less than 15%. But this does not mean that the $1/m_s$ expansion makes any sense; indeed if we write

$$h(\omega) = \xi(\omega \{1 + A/m_s + B/m_b\}$$

we find numerically that $A$ and $B$ are of order $-150$ MeV. This gives a correction going in the opposite direction from the full result, and thus the higher order terms must be significant.

We also find that the leading order corrections to $h$ and $h_5$ vanish at zero recoil in the model; this is analogous to Luke’s theorem [4]. In particular, $h(1) - 1 = .27$ is entirely due to effects at order $1/m_s^2$ and beyond.

The $b \rightarrow s\gamma$ vertex in the effective theory obtained by integrating out the $W$ boson and the top quark in the standard model is [7]

$$\Gamma_\mu = \kappa[\{(1 + r)\mathcal{O}_\mu + (1 - r)\mathcal{O}_5\mu\}]$$

where $\kappa = eG_F V_{ts} V_{tb} \overline{F}_2 m_b/8\sqrt{2}\pi^2$ and $r = m_s/m_b$. The coefficient $\overline{F}_2$ depends on the mass $m_t$ of the top quark and contains the effects of QCD scaling from $\mu = M_W$ down to $\mu = m_b$. In the leading logarithmic approximation it is given by [8]

$$\overline{F}_2 \approx \eta^{-16/23} \left\{ F_2(m_t^2/M_W^2) + 116(\eta^{10/23} - 1)/135 + 58(\eta^{28/23} - 1)/189 \right\},$$

where $\eta = \alpha_s(m_b)/\alpha_s(M_W)$ and [8]

$$F_2(x) = \frac{8x^3 + 5x^2 - 7x}{12(x - 1)^3} - \frac{3x^3 - 2x^2}{2(x - 1)^4} \ln x.$$  

(We note that results in the next-to-leading-logarithmic approximation have been computed and are not drastically different [4].)

Using $\eta = \ln(M_W/\Lambda_{QCD})/\ln(m_b/\Lambda_{QCD}) = 1.96$ with $\Lambda_{QCD} = 250$ MeV, we find $\overline{F}_2/F_2 = 1.92$ for $m_t = 135$ GeV. This is a well-known large enhancement factor from
short-distance QCD \[8, 10\]. We note in passing that attempts have been made to estimate the contribution of internal $\psi - \gamma$ conversion to $B \rightarrow K^* \gamma$ via vector meson dominance \[11\]. However, the Wilson coefficient of the relevant four-quark operator at $\mu = m_b$ is suppressed by roughly a factor of three compared with its value at $\mu = M_W$ \[10\]. We therefore neglect this contribution compared with the QCD-enhanced short-distance one.

The width for $B \rightarrow K^* \gamma$ is given by

$$\Gamma(B \rightarrow K^* \gamma) = M^3|\kappa|^2(16\pi R)^{-1}(1 - R^2)^3(1 + R)^2h(\omega_o)^2,$$  \tag{10}

where $R = m/M$. A fundamental quantity is the ratio of the exclusive $B \rightarrow K^* \gamma$ decay width to that of the inclusive $B \rightarrow X_s \gamma$ decay. This may be taken to be equal to the quark-level $b \rightarrow s \gamma$ width, given by

$$\Gamma(b \rightarrow s \gamma) = m_b^3|\kappa|^2(4\pi)^{-1}(1 - r^2)^3(1 + r^2),$$  \tag{11}

where $r = m_s/m_b$. (The corrections to this relation were shown in \[12\] to be of order $1/m_b^2$ and may be neglected here.) The Kobayashi-Maskawa elements, top quark mass and QCD scaling effects cancel in the ratio, and we find

$$\Gamma(B \rightarrow K^* \gamma)/\Gamma(b \rightarrow s \gamma) = 17\%.$$  \tag{12}

Because $V_{tb}^* V_{tb}$ is not directly measured, it is convenient to use unitarity and the smallness of $V_{ub}$ to write $V_{ts}^* V_{tb} \approx -V_{cs}^* V_{cb}$ and express the branching ratio in terms of $V_{cb}$. With $|V_{cs}| = .974$, $\tau_B = 1.5 \times 10^{-12}$ s and $m_t = 135$ GeV, we find

$$\text{BR}(B \rightarrow K^* \gamma) = 4.8 \times 10^{-5}(|V_{cb}|/.04)^2.$$  \tag{13}

The uncertainties due to a $\pm 100$ MeV shift in $\Lambda_{QCD}$ and a $\pm 25$ GeV shift in $m_t$ are $\pm 10\%$ each. The rate for $B \rightarrow K^* \gamma$ is found to decrease by 20\% if the form factor $h(\omega_o)$ in \(10\) is replaced by its value in the heavy-quark limit.

The CLEO collaboration \[13\] has recently reported a branching ratio of $(4.5\pm1.5\pm0.9) \times 10^{-5}$. We show in Fig. 2 how $|V_{cb}|$ is constrained by the data as a function of $m_t$.

We now discuss the sensitivity of these results to the quark masses. When $h(\omega_o)$ is plotted as a function of $m_b$ and $m_s$ with $m_q$ held fixed at 250 MeV, there is a saddle point at $m_b = 4.83$ GeV and $m_s = .40$ GeV. This point represents the point of minimal sensitivity to the choice of quark masses and we adopt it as our standard reference point. In the circular region of radius 40 MeV centred at this point, we find that $h(\omega_o)$ varies by less than 7.5\% from its value at the saddle point. We thus estimate a 15\% uncertainty in the branching ratio due to the $b$ and $s$ quark masses. The heightened sensitivity to the quark masses which occurs farther away from the saddle point has
been discussed in \cite{3} in the context of $B \rightarrow D^{(*)}$ semileptonic decays. There, it was stressed that the sensitivity is the expected result of constraining the meson masses to their physical values, and not a breakdown of the heavy-quark expansion in the model. We also find that varying the light quark mass $m_q$ by 40 MeV, with the $b$ and $s$ masses fixed, changes the branching ratio by less than 5%. It will be possible in the future to more accurately determine the quark masses appropriate to the model from heavy-meson and -baryon semileptonic decays.

In conclusion, we find in our model $\Gamma(B \rightarrow K^* \gamma)/\Gamma(b \rightarrow s\gamma) = (17 \pm 4)\%$ and $\text{BR}(B \rightarrow K^* \gamma) = (4.8 \pm 1.9) \times 10^{-5}(|V_{cb}|/0.04)^2$. These compare well with values of $(20 \pm 6)\%$ and $(6.8 \pm 2.4) \times 10^{-5}(|V_{cb}|/0.035)^2$ obtained in one recent QCD sum rule approach \cite{4}, and $(17 \pm 5)\%$ and $(4 \pm 1) \times 10^{-5}$ in another \cite{5}.

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Figure Captions

Figure 1: Model results for form factors $h$, $h_5$ and $h'_5$ and Isgur-Wise function $\xi$. The physical recoil point is $\omega = \omega_o$.

Figure 2: Region in $|V_{cb}|$-$m_t$ plane (between dotted lines) allowed by data. The solid line corresponds to the central value of the branching ratio. The figure is plotted for $m_b = 4.83$ GeV and $m_s = 400$ MeV.
Figure 1
Figure 2