The nuclear force problem: Are we seeing the end of the tunnel?

R. Machleidt* and D. R. Entemab†

aDepartment of Physics, University of Idaho, Moscow, Idaho 83844, USA
bNuclear Physics Group, University of Salamanca, E-37008 Salamanca, Spain

Embedded in the historical context, we review recent progress in the development of nucleon-nucleon (NN) potentials based upon chiral effective field theory. A major breakthrough is the construction of the first NN potential at next-to-next-to-next-to-leading order (fourth order) of chiral perturbation theory (χPT). The accuracy of this potential concerning the reproduction of the NN data below 290 MeV lab. energy is comparable to the one of phenomenological high-precision potentials. Since NN potentials of order three or less of χPT are known to be deficient in quantitative terms, the recent advances show that the fourth order of χPT is necessary and sufficient for a reliable NN potential derived from chiral effective Lagrangians. This recent substantial progress raises hopes that we might be getting closer to a solution of the nuclear force problem at low energies that has plagued the community for more than half a century.

1. Introduction

The theory of nuclear forces has a long history. Based upon the Yukawa idea, first field-theoretic attempts to derive the nucleon-nucleon (NN) interaction were focused on pion-exchange, resulting in the NN potentials by Gartenhaus and by Signell and Marshak. Even qualitatively, these potentials were barely in agreement with empirical information on the nuclear force. These “pion theories” of the 1950s, are generally judged as failures—for reasons we understand today: pion dynamics is constrained by chiral symmetry, a crucial point that was unknown in the 1950s.

Historically, the experimental discovery of heavy mesons in the early 1960s saved the situation. The one-boson-exchange (OBE) model emerged and, finally, highly sophisticated meson-theoretic potentials, like the Paris and the Bonn potentials, were developed.

The nuclear force problem appeared to be solved, however, with the discovery of Quantum Chromo-Dynamics (QCD), all “meson theories” had to be relegated to models and the attempts to derive the nuclear force started all over again.

The problem with a derivation from QCD is that this theory is non-perturbative in the low-energy regime characteristic for nuclear physics and direct solutions are impossible. Therefore, during the first round of new attempts, QCD inspired quark models became popular. These models were able to reproduce qualitatively some of the gross features of

*Electronic address: machleid@uidaho.edu
†Electronic addresses: dentem@uidaho.edu, entem@usal.es
the nuclear force. However, on a critical note, it has been pointed out that these quark-based approaches are nothing but another set of models and, thus, do not represent any fundamental progress. Equally well, one may then stay with the simpler and much more quantitative meson models.

The major breakthrough occurred when the concept of an effective field theory (EFT) was introduced and applied to low-energy QCD. As outlined by Weinberg in a seminal paper, one has to write down the most general Lagrangian consistent with the assumed symmetry principles, particularly, the (broken) chiral symmetry of QCD. At low energy, the effective degrees of freedom are pions and nucleons rather than quarks and gluons; heavy mesons and nucleon resonances are ‘integrated out’. So, in a certain sense we are back to the 1950s, except that we are smarter by 40 years of experience: broken chiral symmetry is a crucial constraint that generates and controls the dynamics and establishes a clear connection with the underlying theory, QCD.

The chiral effective Lagrangian is given by an infinite series of terms with increasing number of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Applying this Lagrangian to $NN$ scattering, an unlimited number of Feynman diagrams is generated which may suggest again an untractable problem. However, Weinberg showed that a systematic expansion of the nuclear amplitude exists in terms of $(Q/\Lambda_\chi)^\nu$, where $Q$ denotes a momentum or pion mass, $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale, and $\nu \geq 0$. For a given order $\nu$, the number of contributing terms is finite and calculable; these terms are uniquely defined and the prediction at each order is model-independent. By going to higher orders, the amplitude can be calculated to any desired accuracy. The scheme just outlined has become known as chiral perturbation theory ($\chi$PT).

Following the first initiative by Weinberg, pioneering work was performed by Ordóñez, Ray, and van Kolck who constructed a $NN$ potential in coordinate space based upon $\chi$PT at next-to-next-to-leading order (NNLO; $\nu = 3$). The results were encouraging and many researcher became attracted to the new field. Kaiser et al. presented the first model-independent prediction for the $NN$ amplitudes of peripheral partial waves at NNLO. Epelbaum et al. developed the first momentum-space $NN$ potential at NNLO.

In the 1990s, unrelated, parallel research showed that, for conclusive few-body calculations and meaningful microscopic nuclear structure predictions, the input $NN$ potential must be of the highest precision; i.e., it must reproduce the $NN$ data below about 300 MeV lab. energy with a $\chi^2$/datum $\approx 1$. The family of high-precision $NN$ potentials was developed which fulfills this requirement. Due to the outstanding accuracy of these $NN$ potentials, it was possible to pin down cases of few-body scattering and of nuclear structure that clearly require three-nucleon forces (3NF) for their microscopic explanation. Famous examples are the $A_y$ puzzle of $N-d$ scattering and the ground state of $^{19}$B.

One important advantage of $\chi$PT is that it makes specific predictions for many-body forces. For a given order of $\chi$PT, both 2N and 3N forces are generated on the same footing. At next-to-leading order (NLO), all 3NF cancel; but at NNLO and higher orders, well-defined, nonvanishing 3NF terms occur. However, since 3NF effects are in general very subtle, it is only possible to demonstrate their necessity and relevance when the 2NF is of high precision.

$NN$ potentials based upon $\chi$PT at NNLO are poor in quantitative terms; they repro-
duce the $NN$ data below 290 MeV lab. energy with a $\chi^2$/datum of more than 20 (cf. Table 1 and 2, below) which is totally unacceptable. Clearly, there is a strong need for more precision, implying, one has to go to higher order.

Therefore, a crucial recent progress is the successful construction—by the authors [1]—of the first $NN$ potential that is based consistently on $\chi$PT at next-to-next-to-next-to-leading order ($N^3\text{LO}$; fourth order). It turns out that, at this order, the accuracy is comparable to the one of the high-precision phenomenological potentials (cf. Table 1 and 2, below). Thus, the $NN$ potential at $N^3\text{LO}$ is the first to meet the requirements for a reliable input-potential for exact few-body and microscopic nuclear structure calculations (including chiral 3NF consistent with the chiral 2NF).

2. The $NN$ potential at $N^3\text{LO}$

In $\chi$PT, the $NN$ amplitude is uniquely determined by two classes of contributions: contact terms and pion-exchange diagrams. At $N^3\text{LO}$, there are two contacts of order $Q^0$ [$O(Q^0)$], seven of $O(Q^2)$, and 15 of $O(Q^4)$, resulting in a total of 24 contact terms, which generate 24 parameters that are crucial for the fit of the partial waves with orbital angular momentum $L \leq 2$.

Now, turning to the pion contributions: At leading order [LO, $O(Q^0)$, $\nu = 0$], there is only the well-known static one-pion exchange (OPE). Two-pion exchange (TPE) starts at next-to-leading order (NLO, $\nu = 2$), and there are further TPE contributions in any higher order. While TPE at NNLO was known for a while, TPE at $N^3\text{LO}$ has been calculated only recently by Kaiser [2]. All $2\pi$ exchange contributions up to $N^3\text{LO}$ are summarized in a pedagogical and systematic fashion in Ref. [3] where the model-independent results for $NN$ scattering in peripheral partial waves are also shown. Finally, there is also three-pion exchange, which shows up for the first time at $N^3\text{LO}$ (two loops). In Ref. [4], it was demonstrated that the $3\pi$ contributions at this order are negligible, which is why they can be omitted.

For an accurate fit of the low-energy $pp$ and $np$ data, charge-dependence is important. We include charge-dependence up to next-to-leading order of the isospin-violation scheme (NLO). Thus, we include the pion mass difference in OPE and the Coulomb potential in $pp$ scattering, which takes care of the LO contributions. At order NLO we have pion mass difference in the NLO part of TPE, $\pi\gamma$ exchange, and two charge-dependent contact interactions of order $Q^0$ which make possible an accurate fit of the three different $^1S_0$ scattering lengths, $a_{pp}$, $a_{nn}$, and $a_{np}$.

Chiral perturbation theory is a low-momentum expansion. It is valid only for momenta $Q \ll \Lambda_\chi \approx 1$ GeV. To enforce this, we multiply all expressions (contacts and irreducible pion exchanges) with a regulator function,

$$\exp \left[-\left(\frac{p}{\Lambda}\right)^{2n} - \left(\frac{p'}{\Lambda}\right)^{2n}\right], \quad (1)$$

where $p$ and $p'$ denote, respectively, the magnitudes of the initial and final nucleon momenta in the center-of-mass frame. We use $\Lambda = 0.5$ GeV throughout. The exponent $2n$ is chosen to be sufficiently large so that the regulator generates powers which are beyond
Table 1
$\chi^2$/datum for the reproduction of the 1999 $np$ database [5] below 290 MeV by various $np$ potentials.

| Bin (MeV) | # of data | $N^3$LO$^a$ | NNLO$^b$ | NLO$^b$ | AV18$^c$ |
|-----------|-----------|-------------|-----------|---------|---------|
| 0–100     | 1058      | 1.06        | 1.71      | 5.20    | 0.95    |
| 100–190   | 501       | 1.08        | 12.9      | 49.3    | 1.10    |
| 190–290   | 843       | 1.15        | 19.2      | 68.3    | 1.11    |
| 0–290     | 2402      | 1.10        | 10.1      | 36.2    | 1.04    |

$^a$Ref. [1].  
$^b$Ref. [6].  
$^c$Ref. [7].

The contact terms plus irreducible pion-exchange expressions at $N^3$LO, multiplied by the above regulator, define the $NN$ potential at $N^3$LO. This potential is applied in a Lippmann-Schwinger equation to obtain the $T$-matrix from which phase shifts and $NN$ observables are calculated. The corresponding homogeneous equation determines the properties of the two-nucleon bound state (deuteron).

3. Results

The peripheral partial waves of $NN$ scattering with $L \geq 3$ are exclusively determined by OPE and TPE because the $N^3$LO contacts contribute to $L \leq 2$ only. OPE and TPE at $N^3$LO depend on the axial-vector coupling constant, $g_A$ (we use $g_A = 1.29$), the pion decay constant, $f_\pi = 92.4$ MeV, and eight low-energy constants (LEC) that appear in the dimension-two and dimension-three $\pi N$ Lagrangians (cf. Ref. [3]). In the optimization process, we varied three of them, namely, $c_2$, $c_3$, and $c_4$. We found that the other LEC are not very effective in the $NN$ system and, therefore, we kept them at the values determined from $\pi N$. The most influential constant is $c_3$, which has to be chosen on the low side (slightly more than one standard deviation below its $\pi N$ determination) for an optimal fit of the $NN$ data.

The most important set of fit parameters are the ones associated with the 24 contact terms that rule the partial waves with $L \leq 2$. In addition, we have two charge-dependent

Table 2
$\chi^2$/datum for the reproduction of the 1999 $pp$ database [5] below 290 MeV by various $pp$ potentials.

| Bin (MeV) | # of data | $N^3$LO$^a$ | NNLO$^b$ | NLO$^b$ | AV18$^c$ |
|-----------|-----------|-------------|-----------|---------|---------|
| 0–100     | 795       | 1.05        | 6.66      | 57.8    | 0.96    |
| 100–190   | 411       | 1.50        | 28.3      | 62.0    | 1.31    |
| 190–290   | 851       | 1.93        | 66.8      | 111.6   | 1.82    |
| 0–290     | 2057      | 1.50        | 35.4      | 80.1    | 1.38    |

$^a$Ref. [1].  
$^b$See footnote [8].  
$^c$Ref. [7].
contacts, which brings the number of contact parameters to 26. Since we treated three LEC as semi-free, the total number of parameters of the N$^3$LO potential is 29.

In the optimization procedure, we fit first phase shifts, and then we refine the fit by minimizing the $\chi^2$ obtained from a direct comparison with the data. The $\chi^2$/datum for the fit of the np data below 290 MeV is shown in Table 1, and the corresponding one for pp is given in Table 2. The $\chi^2$ tables demonstrate a dramatic improvement of the NN interaction order by order. It is clearly revealed that, at NLO and NNLO, the reproduction of the NN data is of unacceptably poor quality. However, at N$^3$LO, the quantitative character is comparable to the phenomenological high-precision Argonne V$_{18}$ potential [7].

4. Summary and outlook

In summary, the first NN potential at fourth order of $\chi$PT has been developed by the authors [1]. This potential is as quantitative as some so-called high-precision phenomenological potentials. Due to its basis in $\chi$PT, the many-body forces associated with this two-body force are well-defined. Thus, we have a promising starting point for exact few-body calculations and microscopic nuclear structure theory.

So again, we have reached a point in the history of nuclear forces where one could believe that the nuclear force problem is, essentially, solved. However, based upon the historical experience outlined in the Introduction, caution is in place to avoid another embarrassment. We may be seeing the end of one tunnel; but, is it the last one?

Acknowledgements

This work was supported by the U.S. National Science Foundation under Grant No. PHY-0099444 and by three Spanish foundations: the Ministerio de Ciencia y Tecnología under Contract No. BFM2001-3563, the Junta de Castilla y León under Contract No. SA-109/01, and the Ramón Areces Foundation.

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