Numerical investigation of the Taylor-Couette and Batchelor flows with heat transfer: physics and numerical modelling

K Kiełczewski\textsuperscript{1}, E Tuliszka-Sznitko\textsuperscript{1} and P Bontoux\textsuperscript{2}

\textsuperscript{1}Institute of Thermal Engineering, Poznań University of Technology, ul. Piotrowo 3, 60-965 Poznań, Poland
\textsuperscript{2}M2P2 Laboratory, UMR 7340 CNRS, Aix-Marseille Université, Ecole Centrale Marseille, 13451 Marseille Cedex 20, France

E-mail: ewa.tuliszka-sznitko@put.poznan.pl

Abstract. In the paper the authors present the results obtained during a numerical investigation (Direct Numerical Simulation/Spectral Vanishing Viscosity method - DNS/SVV) of a flow with heat transfer in rotating cavities (i.e. the flow between two concentric disks and two concentric cylinders). These model flows are useful from numerical and experimental point of view among others because of the simplicity of their geometry. Simultaneously, the flows in rotating cavities appear in numerous industrial installations and machines in the field of mechanics and chemistry, e.g., in ventilation installations, desalination tanks and waste water tanks, in cooling system, in gas turbines and axial compressors. In the paper attention is focused on the laminar-turbulent region in the configuration of the large aspect ratio \( \Gamma \) (0.04 is also presented for comparison). The main purpose of computations is to investigate the influence of different parameters (the aspect ratio, the end-wall boundary conditions and temperature gradient) on the flow structure and flow characteristics. For the non-isothermal flow cases the Nusselt number distributions along cylinders are presented and are correlated with the flow structures. The \( \lambda_2 \) method has been used for visualization.

1. Introduction
The flows in rotating cavities, i.e. the flows between two concentric cylinders and two concentric disks, are the simplest possible wall flows which show most of the phenomena the understanding of which enables us to reconstruct the 3D flows in more complex industrial configurations, [1]. These flows are particularly suitable for investigating the phenomena which occur in gas turbines, ventilations, chemical mixing equipment in geophysics and astrophysics. The flow in rotating cavity is characterized by four main parameters: aspect ratio \( \Gamma = \frac{2h}{(R_2 - R_1)} \), curvature parameter \( \eta = \frac{(Rm - 1)}{(Rm + 1)} = \frac{R_1}{R_2} \), Reynolds number \( Re = \frac{(R_2 - R_1) \Omega \nu}{\nu} \) and thermal Rossby number \( B = \beta \Delta T \). The aspect ratio, curvature parameter, Reynolds number and thermal Rossby number create a large parameter space very rich in physical phenomena. For configurations of the small aspect ratio \( \Gamma \) attention was focused on the flows between rotating and stationary disks, in which the flow is pumped radially outward along rotor and recirculates along stator. When two boundary layers are merged, i.e. when the gap between disks is very thin (\( \Gamma < 0.02 \), so called torsional Couette flow) different types of finite-size turbulent structures
(turbulent spots, turbulent spirals) appear with the gradually increasing Reynolds number, which leads finally to turbulent flow. In this flow case the influence of the inner and outer cylinders boundary conditions on the flow structure is small. When the gap between disks is more than the thickness of two disks' boundary layers (the boundary layers are separated by the inviscid rotating core, \(1 > \Gamma > 0.02\)) the flow is named Batchelor. In the Batchelor flow the stationary disk boundary layer is more unstable than the rotating one; the transition process stars with the occurance of circular waves in the stator boundary layer that propagate inward [2]. At higher values of the Reynolds number, another unstable mode appears [2], [3], [4], which creates a spiral pattern (spirals co-exists with the circular waves for a range of Reynolds numbers). The heat transfer in the transitional and turbulent rotor/stator cavity was also discussed in many papers [5], [6], [7], [8].

With the increasing aspect ratio the impact of the inner and outer cylinder boundary conditions on the flow structure gradually increases. The flow between two concentric disks and two concentric cylinders of aspect ratio \(\Gamma \geq 1\) is named Taylor-Couette flow. In its simplest form, the Taylor-Couette flow arises from the shear flow between a rotating inner cylinder and a stationary outer cylinder (the stable, laminar flow for this configuration is known as cylindrical Couette flow). Beyond a certain critical Reynolds number, the Couette flow becomes unstable, which results in the appearance of pairs of counter-rotating, axisymmetric vortices (TVF Taylor vortex flow) that fill in the annulus (first bifurcation, figure 1a). Each pair of vortices has an axial wavelength, figure 1a. For higher Reynolds numbers a supercritical Hopf bifurcation leads to wavy vortex flow, referred here as WVF, the movement of which consists of azimuthally traveling along the Taylor-Couette vortices waves. However, the flow state for wavy vortex flow depends strongly on how the final rotational speed of the inner cylinder is achieved: impulsively or quasi-statically. With further increase of Reynolds number the modulated waves state (MWV), the chaotic and turbulent Taylor vortex flows appear. It was shown in [9] that the spatial and temporal behavior of modulated waves are directly connected to the outflow jet of the Taylor-Couette vortices.

Experimental measurement and flow visualization have traditionally been the main source of information about transitional and turbulent Taylor–Couette flows, [10], [11]. However, recently the Taylor-Couette flow is also intensively investigated by DNS and LES methods, [12], [13], [14]. In many numerical investigations the assumption of infinitely long annuli, with the periodicity condition in axial direction, has been undertaken. But, the presence of end-walls (even in the case of large height of cylinders) destroys the translation invariance in z direction, which exists in the theoretical infinitely long annuli. In experimental investigations, the presence of the end-walls strongly influences the flow structures by constraining the axial motion near the end-walls and by causing Ekman pumping. For the flow cases with the rotating inner cylinder, the outer cylinder fixed and the end-walls rotating with the inner cylinder, centrifugal viscous pumping causes an outflow movement of the fluid at the end-walls. For the end-walls attached to the stationary outer cylinder, the inflow movement at the end-walls occurs. Asymmetric boundary conditions at the end-walls (stationary top and rotating bottom disks, for example) were also investigated [15], but these flow cases are not fully documented. The importance of the top and bottom disks boundary conditions on selecting the flow pattern (including problem of anomalous modes) was investigated among others in [15], [16].

The heat transfer in Taylor-Couette flow has also been an object of great interest for scientists and engineers, [11], [17] and [18]. Investigations have shown, among others, that above the critical threshold, the transport of fluid from one cylinder to the other, enhances the heat transfer. In [19] the authors have found the second, less pronounced, transition altering the evolution of heat transfer. This thermal transition coincides with the shortest wavelength in axial direction, [20]. The review of papers on the Taylor-Couette flow with heat transfer was done recently in [21]. In [21] the authors stated that, although the problem of the heat transfer in the Taylor-Couette flow is well documented, there are still areas which need further research, for example, fluid flows in configurations of large cylindrical gaps.

In the present paper the authors show transitional and turbulent results obtained numerically (using the DNS/SVV method) in the rotating cavity of the aspect ratios from the range \(\Gamma = 0.04 \sim 20.0\). In the paper, the authors propose to research the influence of the presence of the end-walls (symmetric and...
asymmetric boundary conditions) together with the investigation of the influence of temperature variation on the Taylor-Couette flow. The authors propose to study the phenomena in Taylor-Couette flow for multy-parameter space which should help to answer the fundamental question concerning transitional structures, their origins, their role in creating stress and transporting energy, and the relationship between momentum and heat transport. This knowledge can help to control the boundary layer flow in more complex configurations. The results presented in the paper are the authors’ first step towards such formulated scientific purpose. The presented Taylor-Couette calculations are obtained for relatively low Reynolds numbers, however, computations are still being continued in order to obtain a higher Re. In order to perform calculations in conditions that would be similar to the experimental ones the Reynolds number is gradually increased by $\Delta Re = 20.0$ and computations are performed for large global time units up to 2000.0, i.e. in terms of $\Omega^{-1}$. The authors are going to confirm experimentally at least a part of obtained results in the frame of EUHit project.

The paper is organized in the following way: The mathematical and numerical approach used in investigations are described in section 2. The selected results obtained for the Batchelor flows (discussed in detail in [8]) are shortly presented in section 3.1. Different stages of the Taylor-Couette laminar-turbulent transition flow obtained for $\Gamma = 11.75$ are presented in section 3.2. In section 3.3 the exemplary Taylor-Couette flow structures obtained for the symmetric and asymmetric end-wall boundary conditions are presented. For the flow case of $\Gamma = 20.0$ the results with heat transfer are shown and distributions of the local Nusselt number along the inner and outer cylinders are analyzed, section 3.4. Conclusion remarks are given in section 4.

Figure 1. Schematic picture of the rotating cavity, a) for Taylor-Couette flow, b) for Batchelor flow.

2. Mathematical and numerical approach

In the paper the authors investigate the isothermal and non-isothermal fluid flows in the rotating cavity of the inner and outer cylinder radius $R_1$ and $R_2$, respectively (figure 1a and 1b). The inter-disks spacing is denoted by $2h$. The rotor (the inner cylinder and in some flow cases also the bottom disk) rotates at uniform angular velocity $\Omega = e_z \omega$, where $e_z$ is the unit vector on the $z$ axis. The flow is described by the Navier-Stokes, continuity and energy equations written in a cylindrical coordinate system $(R, \varphi, Z)$. The equations are written with respect to rotating frame of reference. The Boussinesq approximation is used to take into account the buoyancy effects induced by the involved body forces. The dimensionless axial and radial coordinates are: $z = Z / h$, $z \in [-1, 1]$, $r = (2R_(R_1 + R_2))/(R_2 - R_1)$, $r \in [-1, 1]$. The velocity components and time are normalized by $\Omega R_2$ and $\Omega^{-1}$, respectively. The dimensionless components of the velocity vector in radial, azimuthal and axial directions are denoted by $u = U / \Omega R_2$, $v = V / \Omega R_2$, $w = W / \Omega R_2$. In the non-isothermal flow cases
the dimensionless temperature is defined in the following way: \( \Theta = (T - T_1)/(T_2 - T_1) \), where \( T_1 \) is the temperature of the cooled surface (the inner cylinder and the top disk, for example), and \( T_2 \) is the temperature of the warmed surface (the bottom or both disks and the outer cylinder). The Prandtl number \( Pr = 0.71 \) and the thermal Rossby number \( B = \beta(T_2 - T_1) \) are constant (\( \beta \) is thermal expansion coefficient). The no-slip boundary conditions are applied to all rigid walls (\( u = w = 0 \)). For the azimuthal velocity component the boundary conditions are: \( v = 0 \) on the rotating inner cylinder and on the rotating disk, and \( v = -(Rm + r)/(Rm + 1) \) on the stationary outer cylinder and on the stationary disks. The time scheme is second-order semi-implicit, which combines an implicit treatment of the diffusive term and an explicit Adams–Bashforth scheme for the non-linear convective terms. The spatial scheme is based on a pseudo-spectral Chebyshev-Fourier-Galerkin collocation approximation. All presented results are obtained with the use of DNS/SVV method (using meshes of 2-30 million collocation points), [7], [8], [23].

3. Results

3.1 Batchelor flows
The authors have performed broad numerical investigations of the flow between rotor and stator with and without heat transfer for different aspect ratios \( \Gamma = 0.04 - 0.5 \) (Batchelor flows), different curvature parameters, and for different Reynolds numbers (including fully turbulent flows). The results were published, among others, in [8]. These investigations delivered many correlational formulas, which can be useful for scientists and engineers. The exemplary temperature field obtained for \( Re=163257, \; \Gamma = 0.04, \; Rm=1.8, \; B=0.1 \) is presented in figure 2. The flow is pumped radially outward along the rotating bottom disk and recirculates along the stationary top disk (the inner cylinder is attached to the rotor and the outer one to the stator, the rotor and the outer cylinder are heated). In the considered flow case the stator boundary layer is fully turbulent, whereas the rotor one is turbulent in the area near the outer cylinder (\( r = 0.0 - 1.0 \)). The azimuthal distributions of the main Reynolds stress tensor components normalized with the wall units (obtained for radius \( r = 0.0 \) and different \( Re \), stator boundary layer) as a function of \( z^+ \) are presented in figure 3 together with the wall asymptotes and [22] numerical results. The definition of Reynolds number in figure 3 is based on \( R_\gamma, \; Re = R_\gamma^2 \Omega / \nu \) and \( L = 1 / \Gamma \). The influence of the cylinders boundary conditions on the flow structure for aspect ratio 0.04 is small, however, the computations performed by the authors for higher \( \Gamma \) have shown that with an increasing aspect ratio this influence increases significantly.

![Figure 2. The meridian temperature field obtained for \( Re=163257, \; L=0.04, \; Rm=1.8, \; B=0.1 \).](image1)

![Figure 3. The axial profiles of the three main Reynolds stress tensor components normalized with wall units versus the wall coordinate obtained for different \( Re \) and \( L = 1 / \Gamma \). Definition of \( Re \) is based on \( R_\gamma, \; Re = R_\gamma^2 \Omega / \nu \). \( Rm=1.8, \; B=0.1 \).](image2)
3.2 Different stages of Taylor-Couette vortices

Figure 4 shows the exemplary 3D flow structures obtained for higher aspect ratio equal \( \Gamma = 11.75 \) and for the radius ratio 0.9 (\( Rm=19 \), isothermal flow case); both end-walls are attached to the outer stationary cylinder. In figure 4 the iso-surfaces of instantaneous \( \lambda_2 \) are presented. The comparison of the results obtained for different Reynolds numbers shows that for Re smaller than 80 only Ekman vortices at the end-walls are present in the cavity (flow is pumped radially inward along end-walls). At higher Re the Taylor vortices gradually appear and the Ekman vortices are squeezed. The vortex flows obtained for Re = 139, 248, 342 and 540 are presented in figure 4a, 4b, 4c and 4d, respectively. For Re=139 the stationary Taylor vortex flow is observed (figure 4a) but for higher Re the modulated waves state occurs (figure 4b, 4c and 4d). The flow is dominated by the outflow jet, which is driven directly by the centrifugal force and throws fluid outward from the rotating inner cylinder. The inflow jet is broader than the outflow jet and at the same time is weaker than the outflow jet (the outflow jet narrows with the increasing Re). Figure 5 presents time histories of the azimuthal velocity component obtained at the monitoring points \((r=0.871, z=0.718, \phi=0,0)\). The instability characteristic obtained for Re=199.8 (figure 5a) shows that the rotation change initiates a disturbance characterized by a wave packet which is rapidly damped (reaching a steady state) and then \((t > 350)\) grows exponentially. At Reynolds number Re=248, the disturbances are oscillatory from the beginning of the time history with constant amplitude.

![Figure 4](image)

**Figure 4.** The iso-surfaces of instantaneous \( \lambda_2 \) obtained for Reynolds numbers: a) Re=139, b) 248, c) 342, d) 540, \( \Gamma = 11.75 \), \( Rm=19 \).

| \( \Gamma \) | \( Rm \) | \( \eta = R_1 / R_2 \) |
|---|---|---|
| 1 | 20 | 0.5 |
| 2 | 12 | 0.9 |
| 3 | 11.75 | 0.9 |
| 4 | 4 | 0.615 |
| 5 | 2.35 | 0.5 |
| 6 | 1.88 | 0.375 |
| 7 | 0.68 | 0.375 |
Figure 5. Instability characteristics obtained at the point \((r=-0.871, z=0.718, \varphi=0.0)\), the azimuthal velocity component as a function of time, a) \(Re=199.8\), b) \(Re=248.4\).

3.3 Influence of the end-walls

The authors consider two types of boundary conditions at the disks: 1) configurations with both end-walls attached to the stationary outer cylinder, and 2) with the top disk attached to the outer stationary cylinder and the bottom one attached to the inner rotating cylinder. These two types of the boundary conditions have been applied to all flow cases listed in table 1. The influence of the end-wall boundary conditions on the Taylor-Couette flow structure is shown in figure 6, where the meridian flow and the iso-lines of the azimuthal velocity component obtained with the symmetric and asymmetric boundary conditions are presented. In figure 6 two flow cases are analyzed: 1) \(\Gamma=4, Re=530\), 2) \(\Gamma=1.88, Re=633\). In the flow cases with the symmetric boundary conditions the onset of instability process gives rise to a primary mode with an even number of cells. The direction of the rotation at both disks is consistent with the Ekman vortex, so there is an inward flow towards the rotating inner cylinder. The symmetry of the flow in the midplane is fully preserved. When the bottom disk is attached to the rotating cylinder and the top one to the stationary outer cylinder there is an inward flow towards the rotating inner cylinder at the stationary top disk and an outward flow towards the stationary outer cylinder. In these flow cases there is an odd number of vortices. The cavity with the asymmetric boundary conditions was investigated experimentally in [24] where the authors observed interaction between three and five cells, and in [15] where interaction between one and three cells was observed. The steady states with the odd number of cells and so called anomalous modes are discussed in [15], [16], [25].

Figure 6. The instantaneous velocity field in meridian section and the iso-lines of the azimuthal velocity component obtained for: a and b) \(\Gamma=4, Re=530\), asymmetric boundary conditions at the disks, c and d) \(\Gamma=4, Re=530\), symmetric boundary conditions at the disks, e and f)
\( \Gamma = 1.88, \text{Re}=633, \) asymmetric boundary conditions, g and h) \( \Gamma = 1.88, \text{Re}=633, \) symmetric boundary conditions.

3.4 Taylor-Couette flow with heat transfer

Figure 7 presents the results obtained by the authors for non-isothermal fluid flow in the cavity of aspect ratio \( \Gamma = 20.0, \) radius ratio \( \eta = 0.5 \) and Reynolds numbers \( \text{Re}=500.0 \) and 300.0 (the outer stationary cylinder and two stationary end-walls are heated). The iso-lines of temperature in figure 7a show the presence of 24 vortices (shorter wavelength in axial direction in comparison to the isothermal flow case). The distributions of the local Nusselt number obtained for \( \text{Re}=500.0 \) and 300.0 along the outer heated cylinder and inner one are presented in figure 7b and 7c, respectively. The maximum values of local Nusselt numbers along the outer cylinder are observed in the areas of outflow jets (figure 7c), whereas the maximum values along the inner cylinder are observed in the areas of inflow jets. The peaks of Nu along the outer cylinder are sharper than the peaks along the inner cylinder. This results from the fact that the outflow jets are thinner than inflow jets. The similar analysis was done in [18].

![Figure 7](image)

**Figure 7.** Numerical results obtained for \( \Gamma = 20.0, \) radius ratio \( \eta = 0.5 \) and \( \text{Re}=500.0 \) (black line) and 300.0 (red line), a) the temperature field in meridian section, b) the distributions of the local Nusselt number along the outer stationary cylinder, c) the distributions of the local Nusselt number along the inner rotating cylinder.

4. Conclusions

In the paper the authors presented the results obtained numerically (DNS/SVV) during the investigations of the flow with the heat transfer in rotating cavity of the aspect ratio from the range 0.04-20.0, for different curvature parameters \( R_m \), different Reynolds numbers, for symmetric and asymmetric boundary conditions at the end-walls and for different thermal conditions. Attention was focused on the transitional results obtained for the Taylor-Couette flow. The computations performed for symmetric and asymmetric boundary conditions at the disks (\( \Gamma = 12, 11.75, 4.0, 2.35, 1.88 \)) showed that the asymmetric boundary conditions break the mirror-plane flow symmetry, which is preserved in symmetric boundary conditions cases. For configuration of \( \Gamma = 4.0 \) the authors observed
4 Taylor-Couette vortices for the symmetric and 3 vortices for asymmetric end-walls conditions. For configurations of \( \Gamma = 1.88 \) the authors obtained 2 and 1 vortices respectively for the symmetric and asymmetric conditions. For non-isothermal flow case (\( \Gamma = 2.00 \), the symmetric boundary conditions at the disks) the authors observed shorter wavelength in axial direction in comparison to the isothermal flow case. The local Nusselt number distributions along the inner and outer cylinders were presented and discussed in the light of the flow structure. The computations will be continued for higher Reynolds numbers and also for modulated in time boundary conditions.

Acknowledgment

Computations were performed in the Poznań Supercomputing and Networking Center, which is gratefully acknowledged.

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