An Integer Programming Model for Flow Shop Scheduling Under TOU and Tiered Electricity Price

Lining Peng*, Mingshun Yang and Renhao Xiao

School of Mechanical and Precision Instrument Engineering, Xi’an University of Technology, Shaanxi, China

*Corresponding author: 2190221166@stu.xaut.edu.cn

Abstract. Reasonable scheduling can achieve energy saving in the production process. Considering the TOU and tiered electricity price policy, an integer linear programming model for flow shop with the goal of minimizing the power cost is constructed, and a particle swarm optimization algorithm (PSO) with a three-layer coding method based on the workpiece, start time and machining state and the iterative process is designed. Finally, the correctness of the mathematical model and the effectiveness of the algorithm are verified by an example.

Keywords: Flow shop, TOU and tiered electricity price, PSO, production scheduling.

1. Introduction

According to the National Bureau of Statistics, the industrial sector accounts for 70 percent of the total energy consumption, 83.5 percent of which comes from manufacturing. As a direct secondary energy, electric energy is widely used in the manufacturing industry. Demand Side Management (DSM) is to guide power users to change their electricity consumption mode by adopting incentive measures (such as time-of-use price policy), so as to truly realize the conservation and efficient use of energy [1].

Wu [2] studied the same time parallel machine scheduling rate and a continuous-time mixed integer linear programming (MILP) model was established to minimize the total power consumption. Ding [3] studied hybrid flow shop scheduling problems under time rate (HFSP), the purpose was to minimize the total electricity in order to ensure the completion of tasks in the production process during the program. Zhang et al. [4] studied the scheduling problem under time-of-use electricity price to maximize the completion time and electricity. For the mixed flow shop scheduling problem, Wang et al. [5] proposed a mixed integer programming model to minimize the total energy consumption and completion time.

Time-of-use electricity price is divided into several periods on the basis of unified electricity price. Different electricity prices are adopted for different periods, generally divided into peak, flat and trough periods. A single time-of-use electricity price may cause the shift of peak and valley periods, and a single step price cannot achieve the purpose of grid stability. Therefore, the TOU price is proposed [6] which combines the TOU price and the step price, to achieve the purpose of energy saving and emission reduction. This paper adopts discrete-time modeling under the consideration of TOU and tiered electricity price, takes the minimum electricity cost as the goal, establishes a
mathematical model for the production scheduling problem, and proposes a particle swarm algorithm to solve the model. Finally, an example is given to verify the established model and algorithm.

2. Problem description

The flow shop scheduling problem considering TOU and electricity price stratification is described as follows. Each workpiece must be processed by multiple machines with the same processing path, one machine can only execute one process. TOU and hierarchical electricity prices are based on the user's electricity demand curve, which divides the processing period into three types: peak period, flat peak period and valley period. A multi-level price for each segment is predetermined.

Given the processing time and power requirements of each workpiece on the corresponding processing machine, considering the time-sharing step price, the processing sequence and start processing time of each workpiece are required to ensure that the processing task is completed within the specified time and minimize processing Capacity cost.

Following assumptions are considered: ① Each machine can only handle one process at a certain time. ② Processing time is known and predetermined. ③ Once each process starts, the process cannot be interrupted. ④ Processing time of each process includes the preparation time. ⑤ Each process of the workpiece must be completed before subsequent processes.

3. Model building

In order to establish a scheduling model, the mathematical symbols are defined as follows:

- \( M \): Manufacturing machine, \( i = 1, 2, 3, ..., M \);
- \( N \): Production target, \( j = 1, 2, 3, ..., N \);
- \( T \): Specified duration, \( t = 1, 2, 3, ..., T \);
- \( PT_{ij} \): The processing time of workpiece \( j \) on the machine \( i \);
- \( q_{ij} \): Machine \( i \) power requirements for workpiece \( j \);
- \( N_{it} \): The total number of products completed on machine \( i \) at time \( t \);
- \( x_{ijt} \): Machine \( i \) processes workpiece \( j \) at time \( t \), then \( x_{ijt} = 1 \), otherwise \( x_{ijt} = 0 \);
- \( y_{ijt} \): Machine \( i \) processes workpiece \( j \) at time \( t \), then \( y_{ijt} = 1 \), otherwise \( y_{ijt} = 0 \);
- \( S \): The number of sub-periods, \( s = 1, 2, 3, ..., S \);
- \( V \): The number of electricity price steps, \( v = 1, 2, 3, ..., V \);
- \( R_{sv} \): The electricity consumption of period \( s \) is included in the range of the \( v \)-th gear price, then \( R_{sv} = 1 \), otherwise \( R_{sv} = 0 \);
- \( PCS_{sv} \): The dividing point of the \( v \)-th step power consumption in time period \( s \);
- \( P_{sv} \): The \( v \)-th price of period \( s \), Electricity consumption in period \( s \);
- \( TEC \): Total power cost.

Without considering electricity consumption such as lighting, only considering power consumption of production machines, the mathematical model is established as follows:

\[
\min \{TEC\} 
\]

Among them:

\[
TEC = \sum_{s=1}^{S} \left[ P_{s1}E_s + \sum_{v=2}^{V} \left( R_{sv} \left( E_s - PCS_{sv} \right) \left( P_{sv} - P_{s,v-1} \right) \right) \right] 
\]

\[
E_s = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{t=1}^{T} x_{ijt} q_{ij} 
\]

In view of the research problem, consider the following constraints:

\[
N_y = \sum_{j=1}^{N} \sum_{t=1}^{T} y_{ijt}, \quad N_{it} = N 
\]
Among them: Formula (1) represents the objective of minimizing the total power cost; Formula (2)
represents the total power cost in the processing; Formula (3) represents the electricity consumption
of each period; the constraints (4) ensures that the processing task is completed within the specified
time, and each workpiece completes the specified number of procedures; the condition (5) guarantees
that the machining process is not interrupted, and once the process of the workpiece starts processing,
it cannot be interrupted; the constraint condition (6) is to ensure that a machine can only process one
workpiece at the same time, to meet the machine constraints; constraint (7) ensure that a workpiece
can only be processed by one machine at the same time to satisfy the workpiece constraints.

4. Algorithm design
The flow shop scheduling that considers TOU and tiered electricity price increases the complexity of
the problem further. PSO was proposed in 1995, and more and more scholars are improving it to solve
the NP-Hard problem. In this paper, the particle swarm algorithm is used to solve the problem, and
according to the complexity of the problem, redesign the key part of particle swarm.

4.1. Coding method
The flow shop scheduling problem under the TOU and tiered electricity price requires comprehensive
consideration of the processing sequence and processing time of the workpiece. In order to facilitate
the determination of the start processing time of the workpiece and the time period of machine
processing, three layers of coding are adopted: workpiece code + starting time code + processing
status code, the first layer is the workpiece code, which is encoded based on the processing sequence
of the workpiece. The second layer is the starting time code, which is based on the starting processing
time of each machine after discretization of the specified time. The third layer is the machine
processing state code, which is based on the processing state of the machine.

4.2. Location updates operation
For the workpiece code, the precedence preserving order-based crossover (POX) is used. For the start
time code and the processing status code, the position update is also carried out according to the
probability. Two points are used to cross, and then two points are generated. The part between two
points is exchanged, and the illegal solution will be generated after the position update. A correction
method is used to ensure the feasibility of the solution.

4.3. Position change operation
For each individual, the position transformation in probability for operation, using two exchange ways,
the corresponding coding layer randomly generated two locations, is located in the two positions
coding swapping. For the workpiece code, this position transformation operation won't produce illegal
solution for the processing of time code and status code, after the illegal solutions are generated, a
later correction method is used to ensure the generation of feasible solutions.

4.4. Algorithm flowchart
The algorithm is divided into two parts, the flowchart is shown in Figure 1. The first part is to perform
position update and position conversion operations on the workpiece code. When the workpiece can...
complete the processing task within the specified time, the second part is executed, and the time code and the status code perform position update and position transformation operations. The second part has a larger search range, so the second part performs a search every time. When the number of iterations of the entire population meets times, the optimal solution in the iteration process is output.

5. Case analysis

For the flow production workshop of a machine tool manufacturing enterprise considering the TOU and tiered electricity price, the shaft parts are processed. There are 4 machines in the workshop, and there are 2 different shaft workpieces, 5 pieces of each type, and a total of 10 pieces of artifacts need to processing, it is required to complete the processing task within 6 hours. The processing time and power requirements of each type of workpiece are shown in Table 1.

Table 1. Processing Time and Power of Each Machine

| Workpiece | (process time (min), power demand (KW)) |
|-----------|----------------------------------------|
| Category  | Machine1 | Machine2 | Machine3 | Machine4 |
| 1         | (25,15)  | (20,20)  | (30,25)  | (25,20)  |
| 2         | (20,20)  | (25,20)  | (20,25)  | (25,20)  |

Table 2. The TOU and Tiered Electricity Price Period and Price List

|             | Peak period | Flat peak period | Trough period |
|-------------|-------------|------------------|---------------|
| First gear  | 0.9831      | 0.6712           | 0.3594        |
| Second gear | 1.3131      | 1.0012           | 0.6894        |

With reference to the current time-of-use electricity price and tiered electricity price, set two levels of electricity prices for each time period, and construct the TOU and tiered electricity price model of this article with a specified time of 6 hours. In order to reduce the complexity of the problem, we take 12 per unit times as an electricity price time. There are 6 electricity price periods within the specified time period, including 2 trough periods, 3 flat peak periods and 1 peak period. Assuming that the
electricity price per period is shown in Table 2. The cutoff point of the two-tier price of the TOU and tiered electricity price is 45 KWH/h.

According to the model established above, the algorithm was written in the m language, and the program was run in MATLAB R2014b. The population size is 50, $\alpha=0.5$, $Pm=0.1$, $D=500$, $DT=800$. After running 10 times, the optimal value was selected, and the scheduling Gantt chart was obtained, as shown in Figure 2.

Different from the above-mentioned goal of minimizing power cost, the comparison group aims to minimize the maximum completion time, and the program runs in the same environment. After 10 times of execution, the optimal value is obtained, and the scheduling Gantt chart is shown in Figure 3.

It can be seen from Figure 2 that the scheduling considering the TOU and tiered electricity price effectively completed the processing task within the specified time, and the starting time of the workpiece was reasonably arranged within the specified time. From the comparison of Figure 2 and Figure 3, the electricity cost comparison of the two groups can be obtained. It can be seen that the dispatch results obtained in this paper considering the TOU and tiered electricity price have effectively reduced the electricity cost by 11%.

![Figure 2. Scheduling Gantt diagram](image2.png)

![Figure 3. Gantt diagram of comparison group scheduling](image3.png)
6. Conclusions

In this paper, a mathematical model of the flow shop scheduling problem considering the hierarchical electricity price is established and the corresponding PSO algorithm is designed. Finally, an example is given to verify the correctness of the model and the effectiveness of the algorithm. The results show that scheduling considering time-of-use electricity price and electricity price stratification can reduce the production cost of enterprises to a certain extent and has certain guiding significance for production. This article does not consider the waiting cost of the production process, such as the standby energy consumption of the machine.

Acknowledgments

This work is supported by National Natural Science Foundation of China (Grant No. 52005404), China Postdoctoral Science Foundation (Grant No: 2020M673612XB) and Doctoral innovation fund of Xi’an University of Technology (Grant No: 310-252072013).

References

[1] Gahm C, Denz F, Dirr M, et al. Energy-efficient scheduling in manufacturing companies: A review and research framework [J]. European Journal of Operational Research, 2016, 248 (3): 744–757.
[2] Wu Y J. Hybrid Tabu Search Algorithm for Parallel-Machine Scheduling Optimization under Time-of-Use Tariffs [J]. Modern Computer, 2019 (13): 26–31
[3] Ding J Y, Song S J, Zhang R, et al. Parallel Machine Scheduling Under Time-of-Use Electricity Prices: New Models and Optimization Approaches[J]. IEEE Transactions on Automation Science and Engineering, 2016, 13 (2): 1138–1154.
[4] Zhang M Y, Yan J H, Zhang Y L, et al. Optimization for energy-efficient flexible flow shop scheduling under time of use electricity tariffs [J]. Procedia CIRP, 2019, 80: 251–256.
[5] Wang S J, Wang X D, Chu F, et al. An energy-efficient two-stage hybrid flow shop scheduling problem in a glass production [J]. International Journal of Production Research, 2020, 58 (8): 2283-2314.
[6] Liu B, Jiang B L, Hao N, et al. Coordinated optimization of TOU & tiered pricing and optimal scheduling model combining microgrid under uncertain bilevel programming [J]. Power System Protection and Control, 2019, 47 (07): 75–83.