Signatures of TeV gravity from the evaporation of cosmogenic black holes

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Abstract

TeV gravity models provide a scenario for black hole formation at energies much smaller than $G_N^{-1/2} \sim 10^{19}$ GeV. In particular, the collision of a ultrahigh energy cosmic ray with a dark matter particle in our galactic halo or with another cosmic ray could result into a black hole of mass between $10^4$ and $10^{11}$ GeV. Once produced, such object would evaporate into elementary particles via Hawking radiation. We show that the interactions among the particles exiting the black hole are not able to produce a photosphere nor a chromosphere. We then evaluate how these particles evolve using the jet-code HERWIG, and obtain a final diffuse flux of stable 4-dimensional particles peaked at 0.2 GeV. This flux consists of an approximate 43\% of neutrinos, a 28\% of electrons, a 16\% of photons and a 13\% of protons. Emission into the bulk would range from a 1.4\% of the total energy for $n = 2$ to a 16\% for $n = 6$.

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1 Introduction

Models with extra dimensions [1] provide one of the most promising solutions to the hierarchy problem, namely, the huge difference between the scale of gravity $M_P = G_N^{-1} \sim 10^{19}$ GeV and the electroweak (EW) scale $M_{EW} \sim 100$ GeV. In these models $M_P$ appears as an effective scale related with the fundamental one, $M_D \sim 1–10$ TeV, by the volume of the compact space or by an exponential warp factor. The difference between $M_{EW}$ and $M_D$ would then just define a little hierarchy problem that should be easier to solve consistently with all collider data. The phenomenological consequences of this framework are quite intriguing: the fundamental scale would be at accessible energies, and processes with $\sqrt{s} \gg M_D$ would probe a transplanckian regime where gravity is expected to dominate over the other interactions [2]. The spin two of the graviton implies then gravitational cross sections that grow fast with $\sqrt{s}$ and become long distance interactions. As a consequence, quantum gravity or other short distance effects become irrelevant as they are screened by black hole (BH) horizons [3].

One of the scenarios in which TeV gravity effects could play a significant role is provided by cosmic rays physics. The Earth is constantly hit by a flux of protons with energy of up to $10^{11}$ GeV and, associated to that flux, it is also expected a flux of cosmogenic neutrinos (still unobserved) with a typical energy peaked around $10^{10} \text{ GeV}$ [4]. These are energies much larger than the ones to be explored at the LHC, where there would be no evidence for gravitational interactions if the scale $M_D$ is above a few TeV. In addition, notice that the new physics should be more relevant in collisions of particles with a small SM cross section, as it is expected for the interaction of a proton with a dark matter particle $\chi$ if it is taken to be a weakly interacting massive particle (WIMP). We will discuss here the interaction of ultra high energy cosmic rays (UHECR) with dark matter particles $\chi$ in our galactic halo. No detail about the nature of $\chi$ other than its mass, which defines the center-of-mass energy $\sqrt{s} = \sqrt{2m_\chi E}$ in the collision, is going to be significant to the present analysis. We also consider collisions of UHECR with other cosmic rays. These are arguably the most energetic elementary processes that we know that occur in nature at the present time, and would produce mini BHs significantly colder and longer-lived than the ones usually considered in the literature. We will focus just on BH production and evaporation, being this analysis a necessary first step in order to understand the full effects of TeV gravity on UHECR phenomenology.

2 Cosmogenic black hole production

BH production processes are the most widely and detailfully discussed aspect of TeV-gravity phenomenology [5], and they have been considered both in the LHC [6] and in the UHECR context [7]. Here we will assume a scenario with $n$ flat extra dimensions of common length where gravity is free to propagate, while matter fields are trapped on a (non-compact) four-dimensional brane. We will use the basic estimate that the collision
of two pointlike particles at impact parameters smaller than the Schwarzschild radius $r_H$ of the system leads to the production of a BH whose mass is given by $M = \sqrt{s}$. The BHs that we are considering ($M < 10^{11}$ GeV) will be described by a $(4+n)$-dimensional metric (they are smaller than the volume of the compact space), being their radius

$$r_H = \left( \frac{2^n \pi^{\frac{n-3}{2}} \Gamma\left(\frac{n+3}{2}\right)}{n+2} \right)^{\frac{1}{n+1}} \left( \frac{M}{M_D} \right)^{\frac{1}{n+1}} \frac{1}{M_D}.$$  

For two pointlike particles, the cross section $\sigma(s) = \sigma_{\nu\nu} = \sigma_{\nu\chi}$ to produce a BH is then written as

$$\sigma = \pi r_H^2.$$  

If the collision involves non-elementary (at the scale $\mu = 1/r_H$) protons, then its partonic structure has to be included in order to find the total cross section, as it is usually done for analyses of BH production at LHC [6]. The $p-\chi$ (or $p-\nu$) cross section may therefore be written as

$$\sigma_{p\nu}(s) = \int_{M_D^2/s}^1 dx \left( \sum_i f_i(x, \mu) \right) \hat{\sigma}(xs).$$

This formula expresses the cross section as the sum of partial contributions $\hat{\sigma}(xs)$ to produce a BH of mass $M = \sqrt{xs}$ resulting from the collision of a parton $i$ that carries a fraction $x$ of momentum with a pointlike target. It is crucial to notice that the scale $\mu$ in the collision is fixed by the inverse Schwarzschild radius, rather than by the BH mass [3] [8], since the scattering is probing a length scale that grows (not decreases!) with $s$. Actually, we expect that for large enough $s$ the scale that we are exploring goes above its radius and a pointlike behaviour for the proton will emerge. In contrast with a QED scattering, here at lower energies ($\approx 10^3$ GeV) we can see the composite structure of the proton, while at higher energies ($\approx 10^9$ GeV) the proton will scatter coherently as a whole. Since Eq. 3 does not reproduce this behaviour, it is necessary to include matching corrections between the two energy regions. The cross section in Eq. 3 describes the low-energy regime, and it is dominated by the large number of partons of low $x$ that may produce a BH of mass near the threshold $M_D$. This scheme explains why $\sigma_{p\nu} > \sigma_{\nu\nu}$. When the cross section $\sigma_{p\chi}$ approaches the proton size ($\approx 20$ mbarn), then the density of partons with enough energy to produce a BH is so large that the parton cross sections overlap, and the BHs produced are big enough to trap other spectator partons. This overlapping reduces the total cross section and increases the average mass of the produced BH. In this regime $\sigma_{p\nu}$ is basically constant with $s$ until it matches the pointlike behaviour in $\sigma_{\nu\nu}$. A similar behavior is also expected in $p-p$ collisions, where the partonic enhancement of the cross section is even more important at lower energies (in this regime $\sigma_{pp} > \sigma_{p\nu} > \sigma_{\nu\nu}$) and the intermediate regime of constant total cross section is reached at lower energies. The smooth transition from these regimes can be modelled numerically discounting the contributions from spectator
We will analyze two processes that can lead to BH production (see [10] for the fluxes of proton, cosmogenic neutrinos and for the dark matter density).

(i) A cosmic ray of energy $E$ colliding with a dark matter particle $\chi$ at rest in the frame of reference of our galaxy. The average number of BHs produced per unit time and volume depends on the density $\rho_\chi$, the cross section $\sigma_{i\chi}$ and the differential flux of cosmic rays $\frac{d\phi_i}{dE}$ (with $i = p, \nu$):

$$\frac{d^2N}{dt\,dV} = 4\pi \int dE \sigma_{i\chi}(s) \frac{d\phi_i}{dE} \rho_\chi. \quad (4)$$

Here the center of mass energy $\sqrt{s} = \sqrt{2m_\chi E}$ can run from $M_D$ to $10^{12}$ GeV.

(ii) A cosmic ray of energy $E_1$ colliding with a cosmic ray of energy $E_2$. In this case the center of mass energy depends upon the relative angle $\theta$, and results into $\sqrt{s} = \sqrt{2E_1E_2(1-\cos\theta)}$. The interaction rate per unit time and volume is expressed by:

$$\frac{d^2N}{dt\,dV} = 16\pi^2 \int dE_1\,dE_2\,d\cos\theta\,\sigma_{ij}(s)\,\sin\theta/2\,\frac{d\phi_i}{dE_1}\,\frac{d\phi_j}{dE_2}. \quad (5)$$

These processes generate BH masses $M = \sqrt{s}$ that can reach $\sim 10^{12}$ GeV.

In Fig. 2 we plot the production rate of BHs from both types of collisions.

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2We assumed a CTEQ6M set of PDF [9]
3 Black hole evaporation

To understand what kind of signal one could observe from such an event, it is necessary to estimate how the BH evolves after its production. It is expected that initially the BH undergoes a quick balding phase, in which it loses its gauge hair and asymmetries. Then it experiences a spin down phase, where its angular momentum is radiated while losing just a small fraction of its mass [11]. Finally, during most of its life the BH is in a Schwarzschild phase, losing mass through spherically symmetric Hawking radiation [12]. The spectrum is, in a first approximation, that of a black body of temperature [13]

\[
T = \frac{n + 1}{4\pi r_H}.
\]  

This means that the scale of emission is fixed by the inverse Schwarzschild radius. This formula has important corrections arising from the gravitational barrier that the particles have to cross once emitted. These corrections are usually expressed in terms of the so called greybody factors, effective emission areas \(\sigma_n^{(i)}(\omega)\) that depend on the dimensionality \((4 + n)\) of the space-time, the spin of the particle emitted, and its energy \(\omega\) [14]. These factors give corrections of order 1 to the black-body emission rates for all particles species except for the graviton, which can have a stronger correction depending upon the number of extra dimensions. We will assume here the numerical greybody factors given in [15].
The number of particles of the species $i$ emitted with $(4 + n)$-dimensional momenta between $k$ and $k + dk$ in a time interval $dt$ can be written as
\[
d N_i(\omega) = g_i \sigma_n^{(i)}(\omega) \left( \frac{1}{\exp(\omega/T_{BH}) + 1} \right) \frac{d^{n+3}k}{(2\pi)^{n+3}} dt,
\] (7)
while the radiated energy is given by
\[
d E_i(\omega) = g_i \sigma_n^{(i)}(\omega) \left( \frac{\omega}{\exp(\omega/T_{BH}) + 1} \right) \frac{d^{n+3}k}{(2\pi)^{n+3}} dt.
\] (8)

Some remarks are here in order.

(i) On dimensional grounds $\dot{E} \sim A_2^{n+1} T^{4+n} \sim 1/r_H^2 \sim T^2$ and $\dot{N} \sim T$, so each degree of freedom should contribute equally (up to order one geometric and greybody factors) to the total emission independently from its bulk or brane localization [16].

(ii) We are considering BH temperatures above $\Lambda_{QCD}$ ($M \lesssim 10^{11}$ GeV leads to $T \gtrsim 1$ GeV), so QCD degrees of freedom (quarks and gluons) are also radiated and dominate the total emission.

Once the instant spectrum is known, we integrate it over time to get the BH lifetime. On dimensional grounds $\tau \sim M^{-1} (M/M_D)^{n+3} n+1$, although the dependence upon the number of the radiated degrees of freedom at different temperatures may be significant. In Fig. 3 we plot the correlation between lifetime, mass and initial temperatures for BHs of mass ranging from $10 \text{ TeV}$ to $10^{11} \text{ GeV}$, $n = 2, 6$ and $M_D = 1 \text{ TeV}$; it is there shown that lifetimes go from a maximum of $10^{-14}$ s for the most heavy BHs to a minimum around $10^{-26}$ s for LHC-like BHs.

4 Thermal properties of the radiation

An important issue about the evolution of the radiation from a BH is the debated question relative to its thermalization. It has been argued [17] that the emitted particles should produce a thick shell of almost-thermal plasma of QED (QCD) particles usually called photosphere (chromosphere). This would occur for BHs above a critical temperature $T_{QED}$ ($T_{QCD}$), and would change the average energy of the emitted particles from $E_{av} \sim T$ to $E_{av} \approx m_e$ (or $E_{av} \approx \Lambda_{QCD}$). The argument leading to these shells is based on the average number $\Gamma$ of interactions of the particles exiting the BH, so $\Gamma \gg 1$ should suffice to confirm the presence of the plasma shell. Initial estimates [17] used the expression
\[
\Gamma = \langle \sigma v \rho \rangle
\] (9)
which describes the case of particles scattering against a fixed target. Recently, however, it has been noticed that the kinematic differences between that case and the case of particles exiting radially from a BH are so significant that lead to a complete suppression of the
interaction rate \[15\]. We will show, following the approach of Carr, McGibbon and Page, that their arguments (formulated for ordinary BHs) hold also for BHs in TeV gravity models.

The first kinematic effect is due to causality, and depends on the fact that the scattered particles do not come from infinity (as in a regular collision), they are created in definite points of space-time. This introduces a minimal separation between particles successively emitted, both in time and length, and induces via Heisenberg’s indetermination principle an UV cutoff in the scale of the exchanged momenta. The scattering cross section is reduced because not all the energies can be interchanged. In particular, in QED (QCD) Bremsstrahlung and pair production the momenta dominating the collision are of order \(Q^2 \sim m_c^2\) (or right above \(\sim \Lambda_{QCD}^2\)). If the particle wave functions do not overlap, and their minimum distance \(\nu^{-1}\) (in units of their Compton wavelength) is larger than the dominant inverse momenta, then the process will be suppressed. Checking the parameter \(\nu\) is sufficient to decide about the effective connection between emitted particles, and eventually exclude thermal interactions. In [10] We have shown that this argument excludes the presence of a photosphere for any number of extra dimensions, but not of a chromosphere when \(n > 2\).

The second suppression effect is based on the fact that the interaction between two particles is not instantaneous, it takes a finite time to complete. It is easy to see that when

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3These arguments are supported by the numerical analysis in [19].
this occurs the particles are already far away from each other, so that they can not interact again. In particular, after completing a QCD interaction partons will be at distances larger than $\Lambda_{QCD}^{-1}$, where QCD is already ineffective. To fully understand this point, one first has to notice that the interaction between particles moving radially in the same direction (within the exclusion cone) is negligible, as the density in such a region is low. Also, that particles moving radially keep on moving radially, as the average angular deviation due to Bremsstrahlung-like processes is small. This implies that the distance of a particle to the particles out of the exclusion cone will always increase (they never approach to each other), and when it reaches a radius $r_{brem}$ this distance will be bigger than $m_e^{-1}$ (or $\Lambda_{QCD}^{-1}$) and the particle is no longer able to interact. If the BH temperature is above $T \sim \Lambda_{QCD}$, as it is the case for the BHs under study here, it is easy to see that after the particle has completed one interaction it will have already crossed $r_{brem}$.

5 Stable particle spectrum

Once the greybody spectrum of emission has been established, it is necessary to study how it evolves at astrophysical distances: unstable particles will decay, and colored particles (which dominate the spectrum) will fragment into hadrons and then shower into stable species. We present our results following the approach of [20], who first studied this issue for primordial BHs. The main difference with their analysis is that while the authors in [20] compute the stationary spectrum at a given $T$ (which only changes on astrophysical time scales), we need here to evaluate the spectrum integrated over the whole (very short) BH lifetime. In any case, our results will be analogous, since the temperature of a BH varies little for most of its lifetime. Of course, our framework also deals with a different scale of gravity $M_D \ll M_P$ and extra dimensions where gravitons propagate. This implies emission into the bulk and different greybody factors for all the species. Notice finally that the spectrum that we are discussing is in the BH rest frame, it is not the one to be observed at the Earth as the BHs produced in cosmic ray collisions will be highly boosted.

We will assume that the evolution of the species $i$ emitted by a BH at rest coincides with the one in $e^+e^- \rightarrow i\bar{i}$ in the center of mass frame, so we will use the MonteCarlo jet code HERWIG6 [21] to evolve the greybody spectrum described before. Namely, we compute the convolution

$$\frac{dN_j}{dtd\omega} = \sum_i \int d\omega' \left( \frac{dN_i}{dtd\omega'}(\omega') \right) \left( \frac{dg_{ji}(\omega, \omega')}{d\omega}(\omega', \omega') \right),$$

(10)

to obtain the number $dN_j$ of stable particles of species $j$ with energy between $\omega$ and $\omega + d\omega$ emitted in a time $dt$. The first term in parenthesis stands for the greybody spectrum of emission for particle species $i$, while the second encodes the probability for the species $i$ of energy $\omega'$ to give a $j$ of energy $\omega$. For a given $T$, this has been implemented via MonteCarlo including all particles of mass $m_i < T$ (leptons, quarks and gauge bosons,
Figure 4: Instant spectrum of stable particles and bulk gravitons (dashed) emitted by a BH of temperature $T = 10$ GeV for $M_D = 1$ TeV and $n = 2$ (left) and $n = 6$ (right).

neglecting the Higgs or the dark matter particle) and has resulted in a final spectrum of neutrinos, electrons, photons and protons. The spectrum includes the same number of particles and antiparticles (they are generated at the same rate), and the three families of neutrinos (their flavor oscillates at astrophysical length scales). In Fig. 4 we plot the spectrum at fixed temperature $T = 10$ GeV, whereas in Fig. 5 we give the complete spectrum for initial masses of $M = 10^4$ GeV and $10^{10}$ and $n = 2$. The results can be summarized as follows.

(i) The main product of the emission is constituted by particles resulting from the showering of QCD species; this explains the primary peak at $\approx 0.2$ GeV observed in the spectrum. It is also possible to detect at $E \sim T$ the direct greybody emission as a secondary peak. Gravitons decouple, since they are not produced by decay of unstable species.

(ii) The relative emissivities of Standard Model particles are an approximate 43% of neutrinos, a 28% of electrons, a 16% of photons and a 13% of protons. This is only mildly sensitive to the BH mass or $M_D$, as it is determined by the showering of colored particles.

(iii) Emission into the bulk goes from the 0.4% of the total number of particles (16% of the total energy) emitted if $n = 6$ and $T = 1.2$ GeV to the 0.02% of the particles (1.4% of the energy) emitted for $n = 2$ and $T = 120$ GeV.
Figure 5: Total spectrum of stable particles and bulk gravitons (dashed) produced by a BH of mass $M = 10$ TeV (left) and $M = 10^{10}$ GeV (right) for $M_D = 1$ TeV and $n = 2$.

6 Summary and outlook

The head to head collision of two cosmic rays provides center-of-mass energies of up to $10^{11}$ GeV. In models with extra dimensions and a fundamental scale of gravity at the TeV such collision should result in the formation of a mini BH. Its evaporation and showering into stable particles could provide an observable signal.

We have estimated the production rate of these BHs (Fig. 2) via collisions of two cosmic rays or, more frequently, in the collision of a cosmic ray and a dark matter particle. In particular, it seems worth to analyze the possibility that (i) extragalactic cosmic rays crossing the galactic DM halo produce a flux of secondary particles with a characteristic shape and strongly dependent upon galactic latitude; (ii) a fraction of the flux of cosmic rays with energy up to $\sim 10^8$ GeV trapped in our galaxy by $\mu G$ magnetic fields can be processed by TeV interactions into a secondary flux peaked at smaller energies. Notice that the physics proposed in this talk is expected to become relevant just at center of mass energies above $\sqrt{s} \sim \sqrt{2Em_\chi} \sim 1$ TeV, i.e., at cosmic ray energies around the cosmic ray knee. These considerations will be worked out in [22], where the additional effects of gravitational elastic interactions will also be included.

Here we have discussed the properties of BHs with masses between $10^4$ and $10^{11}$ GeV. Such objects have a proper lifetime between $10^{-14}$ and $10^{-26}$ s (Fig. 3), and their desintegration products are mainly determined by the fragmentation of QCD species produced via Hawking radiation. Interactions among emitted particles are not able to produce a thermal
shell of radiation, so the spectrum of fundamental species exit the BH with basically the black body spectrum described by Hawking. The final spectrum of stable particles at large distances, however, is peaked around $\Lambda_{QCD}$, and exhibits features weakly dependent upon number of extra dimensions or the BHs mass. Standard Model modes are constituted by an approximate 43% of neutrinos, a 28% of electrons, a 16% of photons and a 13% of protons. The gravitons produced are a fraction that goes from the 0.4% of the total number of particles (16% of the energy) for $M = 10^{10}$ GeV and $n = 6$ to the 0.02% (1.4% of the energy) for $M = 10$ TeV and $n = 2$.

This work is a preliminary analysis, with results that can be useful for future search for effects of TeV gravity on cosmic ray physics.

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