Sources of Charged Higgs Pair Through Double or Triple Higgs Production in Two Higgs Doublet Model Type II at Linear Colliders

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Abstract

The production of triple Higgs ($H^+H^-H^0$), ($H^+H^-h^0$) and pair wise charged Higgs boson ($H^+H^-$) is studied in the context of future linear colliders within the Two-Higgs Doublet Model (2HDM) type II. The 2HDM may offer a clue to disentangle hints of physics beyond the Standard Model (SM) specially at linear colliders. By computing the three Higgs final state production cross-sections at the leading order allow a first insight into the Higgs potential and found several order of magnitude ($\sim 10^4$) enhancement in 2HDM than Minimal Supersymmetric Model (MSSM). While the traditional double charged Higgs production cross-section lie in the same order ($\sim 20$ fb) of magnitude in both 2HDM and MSSM. Due to the extremely clean environment of linear collider, even though with less cross-section and corresponding less decay rate of triple Higgs compare to double Higgs can be used to identify having final state $\mu^+\mu^- \rightarrow H^-H^+H^0 \rightarrow \tau\tau\nu\nu b\bar{b}$ by applying b-tagging. We also compare triple Higgs and double Higgs final states production cross-section in $\mu^+\mu^-$ collider with the $e^+e^-$ linear collider operating at $\sqrt{s} = 1.5$ TeV and find no sizable difference.

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I. INTRODUCTION

The main undisputed highlight of Run 1 of the Large Hadron Collider (LHC) at CERN\cite{1,2} is the discovery of the Standard Model (SM) Higgs boson and the measured signal strengths are quite agree according to the predictions in its several production and decay channels of the discovered particle\cite{3,4}. The mass of the Higgs signal is found to be equal to 126 GeV, which not only confirm the Higgs mechanism\cite{5–7} as a right approach towards giving masses to the electroweak particles and gauge bosons, but also put a question of possibility of existence of further Higgs bosons, which is still not clear at all wether the Higgs sector is indeed minimal, containing only a single Higgs doublet. One of the straight forward way to address such questions is simply to go beyond the SM by adding second Higgs doublet to the field content of model. A particularly well-motivated possibility along these lines is the Minimal Supersymmetric Standard Model (MSSM)\cite{8}. But another, simpler,one is just the general (unconstrained) Two-Higgs-Doublet Model (2HDM)\cite{9}. The general 2HDM Higgs sector contains two CP-even neutral Higgs bosons, $h^0$, $H^0$, a CP-odd (pseudo-scaler) neutral Higgs boson, $A^0$ and pair of charged Higgs boson, $H^\pm$. The lightest neutral Higgs boson, $h^0$, is the SM-like Higgs boson and is the candidate for the signal observed at LHC. There are many motivations for 2HDMs but the best known motivation are from Supersymmetry\cite{10}, axion models\cite{11} and that the SM is unable\cite{12} to generate a barryon asymmetry of the universe of sufficient size. The Higgs sector of the SM is very predictive, with the Higgs mass being the only free parameter, and had been tested successfully at the LHC over some specific theoretically preferred mass ranges, where as MSSM has two free parameters $m_A,\tan\beta$. In contrast, due to the larger number of free parameters in the 2HDM, it will take much longer to probe the entire parameter space of the various models. So it can be easily pre-assumed the importance of 2HDM to use for building the structure of such a model that is more complicated than SM.

The purpose of this paper is to take into account all current constraints on the type-II CP-conserving 2HDM parameter space to determine the allowed ranges of the triple and double Higgs couplings to estimate its corresponding cross-sections in order to prepare the ground for collider studies. Both of these processes are also a major source of charged Higgs boson. The charged Higgs bosons provide a unique signature of a theory beyond the SM due to their property of being electric charged which makes them different from the neutral SM Higgs boson in terms of their production, interaction and decay properties. Therefore they have been extensively searched during the last years at Tevatron (Fermilab), Large Electron-Positron Collider (LEP) and currently at Large Hadron Collider (LHC). The direct searches in LEPII set the lower limit on charged Higgs mass as...
$m_{H^\pm} > 89 \text{ GeV}$ for all $\tan\beta$ values \cite{13}, whereas the CDF \cite{14,17} and D0 \cite{18,21} experiments of Tevatron restrict the $m_{H^\pm}$ to be in the range $m_{H^\pm} > 80 \text{ GeV}$ for $2 < \tan\beta < 30$. The recent results from LHC exclude a large parameter space of the light charged Higgs ($m_{H^\pm} < 160 \text{ GeV}$) and heavy charged Higgs at $\tan\beta > 50$ \cite{22,23}. Therefore the constraints allow to chose $m_{H^\pm}$ in the range $160 < m_{H^\pm} < 600 \text{ GeV}$ or even beyond, to carry this analysis at $\tan\beta = 10$. Also we have to keep the theory in the perturbative regime, which entails that only values of $\tan\beta$ in the approximate range $0.1 < \tan\beta < 60$ are allowed.

Furthermore, a remarkable restrictions over charged Higgs masses comes from Flavour Changing Neutral Currents (FCNC) radiative B-meson decays, whose branching ratio $BR(b \to s\gamma) \approx 3 \times 10^{-4}$ \cite{24} is measured with sufficient precision that becomes sensitive to new physics. The charged Higgs contribution in above branching ratio increases with decrease in $m_{H^\pm}$. This channel has been studied by BaBar and Belle collaborations \cite{25,26} in detail and the recent limit excludes charged Higgs lighter than $327 \text{ GeV}$ at 95% C.L.

Similarly the present measured accuracy of $BR(B_s \to \mu^+\mu^-)$ indicates that the results from LHCb for SUSY and Higgs, a large part of SUSY parameter space is left unconstrained \cite{27}. Therefore the chosen charged Higgs mass points in this paper are not sensitive to $B_s \to \mu^+\mu^-$ constraints.

To observe the impact of CP-odd Higgs boson on the triple and double Higgs production cross sections, the mass point is chosen to be in the range $400 < m_A < 600 \text{ GeV}$ and also check explicitly that the these values respect the $\Delta \rho$ constraint at $m_{H^0} = 300 \text{ GeV}$ and $\tan\beta = 10$ \cite{28}, where $\Delta \rho$ represents the deviations from 1 induced by pure quantum corrections. we must enforce that the additional quantum effects coming from 2HDM ought to satisfy the global fit to electroweak measurements requires $\Delta \rho$ to be $O(10^{-3})$ \cite{29}. It is thus important to stay in a region of parameter space where this bound is respected. Therefore, the set of chosen mass points of CP-odd Higgs are $m_A^0 = 400, 500, 600 \text{ GeV}$ and are consistent with Electroweak precision measurements. These $m_A$ values are also constrained to be relatively heavy due to limits on $\Delta \rho$ \cite{30,31} and are used in the whole paper.

Similarly regarding the neutral Higgs mass $m_{H^0}^0$, recently CMS and ATLAS experiment exclude a wide range of $m_{H^0}$ of MSSM via $H \to \tau\tau$ channel at $\sqrt{s} = 8 \text{ TeV}$ searches \cite{32,33}. It is found that the upper limit of $\tan\beta$ at $m_{H^0} = 300 \text{ GeV}$ is set at $\tan\beta = 10$. Therefore $m_{H^0} = 300 \text{ GeV}$ at $\tan\beta = 10$ is assumed in this paper, as this is the highest $\tan\beta$ achievable at $m_{H^0} = 300 \text{ GeV}$.

The above mentioned study is intended to cover the unexplored parameter space in the 2HDM type II and is observed in the contest of future Muon collider operating at $\sqrt{s} = 1.5 \text{ TeV}$. The muon collider is expecting to get the integrated luminosity around $125 \text{ fb}^{-1}$ at $\sqrt{s} = 1.5 \text{ TeV}$ and
440 $fb^{-1}$ at $\sqrt{s} = 3$ TeV \[34\]. This is a unique machine, to be designed to provide high luminosity, very small energy spread, excellent stability, and good shielding of muon beam decay backgrounds.

II. THEORETICAL FRAMEWORK

The theoretical basis is a two Higgs doublet model with the general potential as follows \[35, 36\].

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c. \right]$$

$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right](\Phi_1^\dagger \Phi_2) + h.c. \right\}$$

(1)

The free parameters of such a model are

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, m_{12}^2, \tan \beta$$

(2)

in the general basis. The CP violation or Flavor Changing Neutral Currents (FCNC) are not assumed as they are naturally suppressed via the Natural Flavor Conservation (NFC) mechanism by imposing a $Z_2$ symmetry on the Lagrangian \[37, 38\] which leads to the following requirement,

$$\lambda_6 = \lambda_7 = 0.$$  

(3)

In order to work in the allowed region of parameter space which is not yet excluded by current experiments, a small value of $\tan \beta = 10$ is chosen for the whole paper. Furthermore we choose $\sin(\beta - \alpha) = 1$ which takes the region of study very close to MSSM parameter space. With the above setting, i.e., $\tan \beta = 10$ and $\sin(\beta - \alpha) = 1$, free parameters can alternatively be taken as:

$$m_h, m_H, m_A, m_{H^\pm}, m_{12}^2, \tan \beta$$

(4)

because there is a correspondence between Higgs boson masses and $\lambda$ values. We use 2hdmc package \[39\] to ensure that chosen parameters are consistent with current experimental limits and respect also the potential unitarity, perturbativity and stability. A point is chosen if it satisfies all above requirements. Figures 1 and 2 show Higgs boson masses used in the analysis and their corresponding $\lambda$ values extracted from 2hdmc.

III. CHARGED HIGGS PAIR PRODUCTION

Double Higgs or pair production of charged Higgs bosons in linear collider, more specifically at $e^+e^-$ collider has been investigated extensively in the literature mainly in MSSM \[40-42\] and
FIG. 1: Higgs boson masses and the value of $m_{12}$ which respect physical requirements on model potential as well as experimental limits and observations on Higgs boson masses.

FIG. 2: Values of $\lambda$ parameters which result in Higgs boson masses in Fig. 1.

surprisingly is paid less attention in 2HDM at $\mu^+\mu^-$ collider. According to our knowledge, with accommodating all the recent constraints on masses of Higgs bosons both experimentally and phenomenologically have not been carried out at the linear colliders e.g., ILC, CLIC and confirmly not done at future muon collider. These kind of processes are strictly prohibited at the leading order in the the SM, so it can not proceed at tree level in SM. However at higher order it might produce but with extremely small cross-section ($10^{-9}$ pb) and corresponding small production rate. A rich literature exist on the one-loop calculation of cross-section for two Higgs final states including
charged Higgs pair essentially in the MSSM \[43\] at \(e^+e^-\). There has been also a number of studies performed in charged Higgs production at \(e^+e^-\) colliders \[44\]–\[54\]. However, none of the production processes lead to a more promising result than the charged Higgs pair production followed by a decay to \(\bar{t}b\) or \(\tau\nu\).

It is important to note that other two Higgs identical neutral final states (\(A^0 A^0\), \(H^0 H^0\), \(h^0 h^0\)) can not proceed at tree level neither in the SM nor in the MSSM due to CP-conservation, even after loop correction very tiny cross sections of the order \(10^{-5}\) pb are obtained \[55\] which means couple of events per 100 \(fb^{-1}\).

In Figure 3 we plot the total production cross-section \(\sigma(\mu\mu \rightarrow H^+H^-)\) (in pb) as a function of

![Graph showing total cross-section as a function of \(\sqrt{s}\) for the tree-level Higgs boson pair production channel at 2HDM](image)

FIG. 3: Total cross-section \(\sigma(H^+H^-)\) (in pb) as a function of \(\sqrt{s}\) for the tree-level Higgs boson pair production channel at 2HDM

center-of-mass energy \(\sqrt{s}\) (in GeV) at fixed \(m_A = 400\) GeV at various \(m_{H^\pm}\) values restricting to 2HDM. It is checked explicitly that the charged Higgs pair production cross section is independent of \(\tan\beta\) and \(m_A\) values. We can see in the Figure 3 the corresponding production rates are quite sizable, attaining in all cases a several thousands events per 500 \(fb^{-1}\) of integrated luminosity, with maximum values reaching \(\sigma(\mu^+\mu \rightarrow H^+H) \approx 20\) fb. On contrary, it is quite interesting to compare with the predicted contributions in the MSSM. After careful calculation and by using simulation tools the results indicate that the predicted \(\sigma(\mu^+\mu \rightarrow H^+H)\) having dependence on \(m_A\) at \(\tan\beta = 10\) in MSSM have identical cross-sections as compared to the cross-section in 2HDM. It is more simple and easier to work with MSSM Higgs sector which is fully determined at the tree level by just of pair of free parameters namely \(\tan\beta\) and \(M_A\) as discussed earlier. So by taking advantage
of simple structure of the parameter space, we search for the maximum cross-section at fixed $\sqrt{s}$ value simultaneously fulfilling all the phenomenological constraints on the SUSY Higgs masses. We know that the charged Higgs boson mass is not so severely restricted as in the case of 2HDM type II models because of the contributions from squark and chargino in the $B(b \to s\gamma)$ which can compensate for the charged Higgs effects [56]. The choice of the default set of MSSM parameters used for double Higgs and triple Higgs final states production are $M_{SUSY} = 1000$ GeV, $\mu = 200$ GeV, $A_t = 1000$ GeV, $A_b = 1000$ GeV and $A_\tau = 1000$ GeV reflect the $m_{h}^{max}$ scenario. Here we want to make an interesting benchmark statement that by choosing the similar values of $m_{H^\pm}$ both in MSSM (corresponding $m_A$) and 2HDM, the $\sigma_{max}(H^+H^-)$ are comparable of the order of $10^{-2}$ ($\sim 0.008$) pb. Consequently, a sizeable production rate of non-SM Higgs boson can be achieved at linear collider. In this sense the two models will be difficult to distinguish using double Higgs ($H^+H^-$) channel. So in order to make a clear and more reliable discrimination between the two models, a detailed study of radiative corrections are performed both in the MSSM [57–59] and the 2HDM [60]. It might be possible that the triple Higgs boson self-interactions may play a crucial role in this endeavor because, in favorable circumstances, they may easily distinguish between the MSSM and the general 2HDM Higgs sectors in the clean environment of linear colliders.

IV. TRIPLE HIGGS PRODUCTION

The triple Higgs production has been studied in different papers in the context of linear colliders [61], where radiative corrections to the triple Higgs coupling have been studied. The ratio of triple Higgs coupling in 2HDM to that in SM has been studied in detail in [62], taking into account the perturbativity requirements on $\lambda_i$, vacuum stability and Higgs boson mass limits from direct and indirect searches. The production cross section of triple Higgs production at $e^+e^-$ collisions has been studied in [63]. A similar study has also been performed in MSSM in [64]. The effect of triple Higgs coupling in the production of Higgs pairs in 2HDM has been discussed in [64] for different set of center of mass energies and integrated luminosities of a linear $e^+e^-$ collider. The effect of quantum corrections and triple Higgs self-interactions in the neutral Higgs pair production in 2HDM as a function of $\tan\beta$ and $\lambda_5$ is reported in [65].

The triple Higgs boson production processes under consideration are $H^+H^-H^0$ and $H^+H^-h^0$ in $\mu^+\mu^-$ annihilations within the 2HDM type II with their corresponding trilinear Higgs bosons couplings given in the Eqns. 5 and 6 (Ref. 30).
\[ H^\pm H^\pm H^0(2HDM) : \frac{-ie}{m_W s_W s_2\beta} \left[ (m_{H^\pm}^2 - m_A^2 + \frac{1}{2} m_H^2) s_2\beta c_{\beta - \alpha} - (m_{H^\pm}^2 - m_A^2) c_2\beta s_{\beta - \alpha} \right] \] (5)

\[ H^\pm H^\pm h^0(2HDM) : \frac{-ie}{m_W s_W s_2\beta} \left[ (m_{H^\pm}^2 - m_A^2 + \frac{1}{2} m_h^2) s_2\beta c_{\beta - \alpha} + (m_h - m_A^2) c_2\beta s_{\beta - \alpha} \right] \] (6)

In case when there is a very large mass difference between the charged Higgs and CP-odd neutral Higgs, the \( H^+ H^- H^0 \) coupling is larger than \( H^+ H^- h^0 \), which is also one way of cross-section enhancement by mass splitting between them. The other way to illustrate the enhancement is for example focusing the coupling of \( H^+ H^- H^0 \), where the coupling effectively grows as \( \tan \beta \) or \( \cot \beta \) for \( \tan \beta \gg 1 \) or \( \tan \beta \ll 1 \) respectively. The corresponding cross-section can vary either by \( \tan^2 \beta \) at larger \( \tan \beta \) values or by \( \cot^2 \beta \) at small \( \tan \beta \) values respectively. In contrast to the situation in MSSM, where the triple Higgs coupling undergo radiative corrections [66], do not have any possible source of enhancement and this can be seen from Eqn. 7 showing tree level coupling in MSSM. The Eqn. 7 clearly indicate that the couplings are naturally gauge like and hence the expected cross-section remains rather small [41]. In our case, since the mass points chosen in this study of both types of Higgs bosons are not so much different in some of the cases, so it would be interesting to study both of these processes logically. We wish to compute the cross-sections and want to compare with the corresponding MSSM values with same parameters defined in the previous section. As mentioned earlier that due to the low energy \( b \to s\gamma \) constraint on the charged Higgs boson mass in type II models, we cannot keep the CP-odd mass \( M_{A^0} \) relatively light for these models. Since we left a very limited parameter space in 2HDM after recent excluded parameter space by LHC experiments so we can not simply take higher \( \tan \beta \) values for the cross-section enhancement as has been a usual practice and more natural.

\[ H^\pm H^\pm H^0(MSSM) = \frac{-ie m_W}{\sin \theta_W} \left[ \cos(\beta - \alpha) - \frac{\cos 2\beta \cos(\alpha + \beta)}{2 \cos^2 \theta_W} \right] \] (7)

In Figs. 4 and 5 we have plotted triple Higgs cross-section \( \sigma(H^+ H^- H^0) \) for 2HDM type 2 as a function of \( \sqrt{s} \) at \( m_A = 400 \text{ GeV} \), 500 GeV and at \( m_A = 600 \text{ GeV} \), \( m_{H^\pm} = 250 \text{ GeV} \) respectively to observe the effect of CP-odd Higgs and charged Higgs on \( \sigma \) in each plot. Whereas, Fig. ?? shows the impact of \( m_A \) on \( \sigma(H^+ H^- H^0) \) at a particular value of charged Higgs mass \( m_{H^\pm} = 250 \text{ GeV} \). Just for completeness the corresponding values of the parameter \( \lambda_5 \) are used as \( \lambda_5 = -1.18978 \) at \( m_A = 400 \text{ GeV} \), \( \lambda_5 = -2.67433 \) at \( m_A = 500 \text{ GeV} \) and \( \lambda_5 = -4.48877 \) at \( m_A = 600 \text{ GeV} \). In order to compare the 2HDM results with the corresponding MSSM values, we have computed cross-sections...
FIG. 4: (Left): Triple Higgs production within 2HDM at $m_A = 400$ GeV for various $m_{H^\pm}$ values. (Right): Triple Higgs production within 2HDM at $m_A = 500$ GeV for various $m_{H^\pm}$ values

FIG. 5: (Left): Triple Higgs production within 2HDM at $m_A = 600$ GeV for various $m_{H^\pm}$ values. (Right): Effect of $m_A$ is observed on production cross-section at $m_{H^\pm} = 250$ GeV as a function of $\sqrt{s}$

of two channels induce the triple Higgs $\sigma(H^+H^-H^0)$, $\sigma(H^+H^-h^0)$ and one channel induce double Higgs final states $\sigma(H^+H^-)$ within the MSSM framework. The selected channels are found to have dominant cross-sections compare to the rest of the possible allowed processes not discussed here. So we searched for the MSSM and 2HDM most optimal parameter space where the maximum allowed cross-section can be achieved. In Table I we provide the $\sigma_{\text{max}}(H^+H^-H^0)$, $\sigma_{\text{max}}(H^+H^-h^0)$ and $\sigma(H^+H^-)$ for each process at two different center of mass energy values ($\sqrt{s} = 1$ TeV and $\sqrt{s} = 1.5$ TeV) after scanning MSSM parameter space e.g., $(m_A, \tan\beta)$ at fixed values and concerning 2HDM in addition to MSSM parameters the $m_{H^\pm}$ dependence is also checked explicitly with respecting all the current experimental and phenomenological constraints. The abbreviated name "NP" stands for "Not Possible", which means this kind of process at these parameters is not possible to produce
at this center of mass energy, as the energy is not enough to meet the production threshold.

The MSSM cross-sections for both triple Higgs cases at most for leading processes are extremely small reaching the largest value of $\sigma_{max} (H^+H^-H^0) \approx 10^{-10}$ pb and $\sigma_{max} (H^+H^-h^0) \approx 10^{-8}$ pb around 2 order larger cross-section than former. This implies that the maximum 2HDM cross-section for $H^+H^-H^0$ at a particular $m_A$ value which maximize cross-section, is typically $10^4$ times larger than the corresponding maximum MSSM values. Similarly the largest cross-section for $H^+H^-h^0$ in 2HDM is typically $10^5$ times larger than the maximum cross-section within MSSM framework. In Figure 6 total cross-section of $\mu\mu \to H^+H^-h$ process in 2HDM type II as a function of $\sqrt{s}$ is plotted for different values of $m_{H^\pm}$ at $m_A = 400$ GeV (left) and at $m_A = 500$ GeV (right) with scanning various charged Higgs mass values. Figure 7 shows the similar plot (right) but with $m_A = 600$ GeV while (right) plot demonstrates the curves showing maximum cross section (at $m_{H^\pm}$ = 250 GeV) taken from previous plots at three $m_A$ values. The plots shows that the largest cross section $\sim 5$ fb at higher $m_A$ can be achieved. On similar pattern, the largest $\sigma(H^+H^-)$ in MSSM is one order of magnitude lesser than the maximum cross-section in 2HDM. Now on the basis of these results this is easy to conclude that the triple Higgs boson channels are generally much more promising in 2HDM than working within MSSM framework, while double Higgs production cross-section and corresponding decay rates are substantially larger than triple Higgs channels during

| Process               | $m_A$ (GeV) | $m_{H^\pm}$ (GeV) | $\sigma_{max}$ (1 TeV) | $\sigma_{max}$ (1.5 TeV) |
|-----------------------|-------------|-------------------|------------------------|-------------------------|
| $\mu^+\mu^- \to H^+H^-$ | 400         | 392               | $5.7347 \times 10^{-3}$ | $7.7463 \times 10^{-3}$ |
| $\mu^+\mu^- \to H^+H^-H^0$ | 400         | 250               | $3.7053 \times 10^{-10}$ | $3.711 \times 10^{-10}$ |
| $\mu^+\mu^- \to H^+H^-h^0$ | 400         | 250               | $1.6963 \times 10^{-9}$  | $4.4221 \times 10^{-8}$  |
| $\mu^+\mu^- \to H^+H^-H^0$ | 500         | 493               | $5.059 \times 10^{-3}$  | $5.2652 \times 10^{-3}$  |
| $\mu^+\mu^- \to H^+H^-H^0$ | 500         | 250               | NP                      | NP                      |
| $\mu^+\mu^- \to H^+H^-h^0$ | 500         | 250               | $1.4453 \times 10^{-8}$  | $1.4026 \times 10^{-8}$  |
| $\mu^+\mu^- \to H^+H^-H^0$ | 600         | 595               | NP                      | $2.6976 \times 10^{-3}$  |
| $\mu^+\mu^- \to H^+H^-H^0$ | 600         | 250               | NP                      | $4.2614 \times 10^{-7}$  |
| $\mu^+\mu^- \to H^+H^-h^0$ | 600         | 250               | $2.8064 \times 10^{-9}$  | $2.4993 \times 10^{-9}$  |

TABLE I: The maximum cross-section (in pb) for the leading order double Higgs and triple Higgs processes within MSSM and 2HDM are given at two different center of mass energy values at $\sqrt{s} = 1$ TeV and $\sqrt{s} = 1.5$ TeV to perform comparison among two models. The values extracted are given at particular $m_A$ value with that $m_{H^\pm}$ value which provide maximum cross-section (lowest $m_{H^\pm}$ e.g., 250 GeV in our case).
working either in MSSM or 2HDM, because of having identical cross-sections as have been seen during plotting the Figure 3.

FIG. 6: (The total $\sigma(H^+H^-h)$ as a function of $\sqrt{s}$ of muon collider at various $m_{H^\pm}$ values by fixing $m_A = 400$ GeV (left) and at $m_A = 500$ GeV (right) with corresponding decay rates at 500 $fb^{-1}$.

FIG. 7: (left) the total $\sigma(H^+H^-h)$ as a function of $\sqrt{s}$ at various $m_{H^\pm}$ points for a fixed $m_A = 600$ GeV value and the $\sigma(H^+H^-h)$ (right) at $m_A = 400$ GeV, 500 GeV, 600 GeV having specific $m_{H^\pm} = 250$ GeV value with decay rates at 500 $fb^{-1}$.

V. CROSS SECTION AT $e^+e^-$ AND $\mu^+\mu^-$ COLLIDERS

Before proceeding to build any collider it is essential to perform the cross section comparison study among available and expected colliders, based on such analysis the correct choice of getting
largest cross-sections are favoured. Here in this section we construct three tables to deeply analyze the matching of cross sections regarding the three selected channels at two possible choices of colliders e.g., $e^+e^-$, $\mu^+\mu^-$. The Table II demonstrates the dominance of $H^+H^-$ cross section over triple Higgs and shows no difference in cross section between $e^+e^-$ and $\mu^+\mu^-$ even though by introducing the non zero masses of electron and muon and their corresponding couplings in the Lagrangian. As shown in Figure 8 the charged Higgs pair production may proceed through three types of diagrams. The left diagram includes s-channel electroweak propagators while middle one is an s-channel consist of neutral Higgs boson as propagator. The CP odd neutral Higgs boson does not contribute in the production cross section due to having zero coupling with charged Higgs pair. Therefore we only take into account the electroweak and CP-even neutral Higgs bosons contribution. We separately calculate the amount of cross section even it is a very tiny which may be considerably become important probably after a long run of any one of these two colliders in future. Whereas, the right diagram depicts a t-channel process which has negligibly small contribution at both $e^+e^-$ and $\mu^+\mu^-$ colliders due to the small Yukawa coupling between the charged Higgs and the leptons. The neutral Higgs contribution in the total cross section is very limited due to the fact that they are not considered to be so heavy in MSSM to be produced as a resonance while on contrary in 2HDM where neutral Higgs masses are free parameters specifically in general 2HDM, sizable contribution to the total cross section might be achieved depending on the neutral Higgs masses. Whereas in 2HDM type II, there are restriction on Higgs masses and contribution is unlike MSSM might be different. Furthermore, the triple Higgs coupling shown in Figure 9 of $H^+H^-H^0$ and $H^+H^-h^0$ shown above varies proportional to $(m_H^2 - m_A^2)$ and $(m_H^2 - m_A^2)$ respectively, make the contribution of $h^0/H^0$ negligibly small in the total cross section of both triple Higgs production which is convincingly shown in Table III for the comparison of MSSM and 2HDM in the context of both colliders.

In summarizing the results we can conclude that for the charged Higgs searches the most suitable channel from cross section point of view is charged Higgs pair production having largest cross-section of the order of $10^{-2}$ pb in 2HDM and one order smaller $\sim 10^{-3}$ in MSSM, while the selected triple Higgs processes $(H^+H^-H^0, H^+H^-h^0)$ have cross-section ($10^{-10}$ pb, $10^{-8}$ pb) in MSSM respectively. Interestingly we found around four or der of magnitude large cross-section of triple Higgs in 2HDM comparing to MSSM in both channels, while lower values of $m_A$ and $m_{H\pm}$ are found to be most favourable for larger cross-section in triple Higgs rather we found same cross-section values for charged Higgs pair production in MSSM and 2HDM when proper settings of Higgs masses are performed. In addition a negligibly small discrepancy in the cross-section
calculation have been seen during comparing $e^+e^-$ and $\mu^+\mu^-$ colliders in those Feynman diagrams where $h^0/H^0$ are mediated as propagators and could not see any difference in the total cross-section of all three channels.

FIG. 8: Double charged Higgs production at lepton colliders. The schannel (left and middle) and tchannel (right) diagrams involved in the signal process

FIG. 9: The possible Feynman diagrams for triple Higgs production at lepton colliders.

VI. SUMMARY

We have studied the most promising channels in double Higgs (charged) pair production and triple Higgs boson production at future linear colliders specifically $\mu^+\mu^-$ collider in the framework of CP-conserving Two Higgs Doublet Model type II and Minimal Supersymmetric Standard Model. We have computed and quantified the sizes of double and triple Higgs production cross-sections
| Collider type | $Z/\gamma$ | $h^0/H^0$ | Total | $Z/\gamma$ | $h^0/H^0$ | Total |
|---------------|------------|------------|--------|------------|------------|--------|
| $\mu^+\mu^-$ | $7.74 \times 10^{-3}$ | $1.07 \times 10^{-10}$ | $7.74 \times 10^{-3}$ | $1.09 \times 10^{-2}$ | $5.63 \times 10^{-9}$ | $1.09 \times 10^{-2}$ |
| $e^+e^-$     | $7.74 \times 10^{-3}$ | $2.0 \times 10^{-15}$ | $7.74 \times 10^{-3}$ | $1.09 \times 10^{-2}$ | $3.04 \times 10^{-18}$ | $1.09 \times 10^{-2}$ |

TABLE II: Total cross sections (in pb) of double charged Higgs are obtained to compare between $e^+e^-$ and $\mu^+\mu^-$ at $\sqrt{s} = 1.5$ TeV at $m_A = 400$ GeV in MSSM and $m_{H^\pm} = 250$ GeV in addition in 2HDM.

| Collider type | $H^+H^-H^0$ | $H^+H^0$ |
|---------------|--------------|------------|
| $\mu^+\mu^-$ | $2.16 \times 10^{-10}$ | $4.42 \times 10^{-8}$ |
| $e^+e^-$     | $1.77 \times 10^{-11}$ | $4.42 \times 10^{-8}$ |

TABLE III: Total cross-section (in pb) comparison between electron-positron and muon-muon collider within MSSM at $m_A = 400$ GeV and $\sqrt{s} = 1.5$ TeV. It is found that the triple Higgs process is not possible to produce at $\sqrt{s} = 1$ TeV, as well as at higher $m_A$ values selected in this study.

both in MSSM and in 2HDM by respecting all the current experimental and phenomenological constraints on Higgs masses, requirement of perturbativity and vacuum stability condition. We have obtained the maximum cross-sections at two different center of mass energies to compare these processes between MSSM and 2HDM where we found several order discrepancies mentioned above. In addition the same channels are reproduced at $e^+e^-$ and $\mu^+\mu^-$ colliders separately to compare the total cross-section and mainly contributing Feynman diagrams in total cross-section in both models. We confirm that no sizable difference in cross sections found at both colliders.

The charged Higgs pair production will be treated as one of the golden channels for the discovery of charged Higgs if the Large Hadron Collider missed it. However, if a charged Higgs is observed at LHC in the coming years, this process would be not only the best candidate for the confirmation of LHC results at a linear collider but also will be an excellent tool to measure its properties more precisely. Since the charged Higgs pair production is observable with a cross section of more than 20 $fb$, such a large cross section would provide observable signals earlier than expected. More over this process is sensitive to the mass of neutral Higgs bosons which are involved in the s-channel Feynman diagrams, therefore it could be reflected as a hint for neutral heavy Higgs boson.

Whereas the triple Higgs ($H^+H^-H^0$) production at linear colliders followed by the charged Higgs decay to pair of $\tau\nu$ and neutral Higgs decay to $b\bar{b}$ with the final state $\tau^+\tau^-b\bar{b}E_T^{miss}$ is considered to be the most suitable choice for electroweak background rejection using b-tagging tools, though
TABLE IV: Total cross-section (in pb) obtained corresponding to triple Higgs and double Higgs production.

| Collider type | $H^+H^-H^0$ | $H^+H^-h^0$ |
|---------------|-------------|-------------|
|               | $Z/\gamma$ | $h^0/H^0$  | Total | $Z/\gamma$ | $h^0/H^0$  | Total |
| $\mu^+\mu^-$ | 1.07 $\times 10^{-6}$ | 3.76 $\times 10^{-7}$ | 1.45 $\times 10^{-6}$ | 4.08 $\times 10^{-4}$ | 1.72 $\times 10^{-10}$ | 4.08 $\times 10^{-4}$ |
| $e^+e^-$     | 1.075 $\times 10^{-6}$ | 8.75 $\times 10^{-12}$ | 1.075 $\times 10^{-6}$ | 4.08 $\times 10^{-4}$ | 9.3 $\times 10^{-20}$ | 4.08 $\times 10^{-4}$ |

The signal cross-section is small i.e., $\sigma(H^+H^-h^0) \approx 0.4 - 4.0$ fb, $\sigma(H^+H^-H^0) \approx 10^{-3}$ fb, with a reasonable background suppression, high signal significance values can be achieved [28] at higher integrated luminosity depending on the charged Higgs mass, $\tan \beta$ and CP-odd neutral Higgs boson mass.

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