Intrinsic noise of a micro-mechanical displacement detector based on the radio-frequency single-electron transistor

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Abstract

We investigate the intrinsic noise of a micro-mechanical displacement detector based on the radio-frequency single-electron transistor (rf-SET). Using the noise analysis of a SET by Korotkov as our starting point, we determine the spectral density of the displacement noise due to the tunneling current shot noise. The resulting mechanical displacement noise decreases in inverse proportion to the increasing gate voltage. In contrast, the displacement noise due to the fluctuating SET island charge increases approximately linearly with increasing gate voltage. Taking into account both of these noise sources results in an optimum gate voltage value for the lowest displacement noise and hence best sensitivity. We show that a displacement sensitivity of about $10^{-4}$ Å and a force sensitivity of about $10^{-16}$ N are predicted for a micron-sized cantilever with a realizable resonant frequency 100 MHz and quality factor $Q \sim 10^4$. Such sensitivities would allow the detection of quantum squeezing in the mechanical motion of the micro-mechanical cantilever and the detection of single-spin magnetic resonance in magnetic resonance force microscopy (MRFM).

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I. INTRODUCTION

Fast and ultra-sensitive displacement and force detection is of great interest for a broad range of fundamental applications. It is an essential requirement for the proposed detection of quantum superposition states\(^1\), quantum squeezed states\(^2\), and single-spin magnetic resonance in MRFM\(^3\) which has also been proposed for the read-out stage of a solid-state NMR quantum computer.\(^4\) The common requirement of these examples is the need to perform the measurement using a micro-to-nanoscale-mechanical detector at, or even below, the “standard quantum limit” (SQL) \(\Delta x_{\text{SQL}} = (\hbar/2m\omega_0)^{1/2}\), and in the radio-frequency range. Considering, for instance, a micron-sized cantilever with a resonant frequency of 100 MHz and quality factor \(Q \sim 10^4\), the detection of quantum squeezed states requires a displacement sensitivity of about \(10^{-4} \text{Å}\).\(^2\) As another example, in detecting single-spin magnetic resonance, the interaction force between a nano-magnet with a targeted electron spin is typically on the order of about \(10^{-16} \text{N}(100 \text{aN})\).\(^3\) If a micron-sized cantilever on which the nano-magnet is mounted is directly coupled at the spin precession frequency (100 MHz or higher), the magnetic force will cause the cantilever to oscillate at an amplitude of about \(10^{-4} \text{Å}\) for a single spin.

Following the development of the rf-SET\(^1\)–\(^7\) it has been proposed to use an rf-SET-based displacement detector for measuring sub-angstrom motion of micron-scale and smaller mechanical oscillators.\(^8\) In the rf-SET, \(1/f\) noise which limits the applications of the conventional dc-SET is considerably improved. A shot noise limit of \(10^{-6} \text{Å}/\sqrt{\text{Hz}}\) may be achievable by using an rf-SET displacement detector,\(^8\) provided the coupling (gate voltage) between the SET and micro-mechanical oscillator is strong enough. However, the back-action of the SET onto the mechanical oscillator must also be taken into account: the fluctuating force acting on the oscillator due to the fluctuating island charge produces another contribution to the displacement noise. The total displacement noise will be the sum of contributions from two sources - the shot noise in the tunneling current and the force noise due to the fluctuating island charge.
In this paper, we give a calculation of the displacement noise of the rf-SET displacement detector. By using the intrinsic noise analysis method of Korotkov, the spectral density of the shot noise $S_I$ in the tunneling current and the spectral density of the force noise $S_F$ are determined, and the total spectral density of the displacement noise $S_X$ is then deduced. We also present two simple expressions from which good estimates of the displacement noise due to the shot noise and the fluctuating island charge can be obtained. Results for the ultimate intrinsic noise limit of the detector for different parameter values are presented and discussed. The displacement and force sensitivities are estimated for an optimized rf-SET displacement detector and we find that they meet the requirement for detecting quantum squeezing and single-spin magnetic resonance.

II. INTRINSIC NOISE OF THE RF-SET DISPLACEMENT DETECTOR

The scheme of the rf-SET displacement detector is shown in Fig. 1. The basic principle of the device involves locating one of the gate capacitor plates of the SET on the cantilever so that, for fixed gate voltage bias, a mechanical displacement is converted into a polarization charge fluctuation. The stray capacitance $C_s$ of the leads contacting the SET and an inductor $L$ form a tank circuit with resonant frequency $\omega_T = (LC_s)^{-1/2}$, loaded by the SET. A monochromatic carrier wave is sent down the cable. At the resonant frequency the circuit impedance is small and the reflected power provides a measure of the SET’s differential resistance $R_d$. When the gate capacitor is biased, mechanical motion of the cantilever is converted into differential resistance changes, hence modulating the reflected signal power.

To simplify the algebra and make the final results concise, we shall in fact consider a symmetric dc-SET in the calculations with junction resistances $R_1 = R_2 = R_j$ and junction capacitances $C_1 = C_2 = C_j$. The differences between the rf- and dc-SET noise formulae are inessential, with appropriate time-averages required in the former resulting in only small quantitative differences in the noise values.

Referring to Fig. 1, when the drain-source voltage $V_{ds}$ is small compared to the voltage
\[ e/C_\Sigma, \text{ where } C_\Sigma = 2C_j + C_g \text{ is the total capacitance of the SET, and also the thermal energy } k_B T \ll eV_{ds}, \text{ the peaks of the tunneling current } I_{ds} \text{ will be well separated in gate voltage } V_g. \] To derive an expression for \( I_{ds} \) in the tunneling region, we consider only the two most probable island electron numbers \( n \) and \( n + 1 \), resulting in the following approximation:

\[ I_{ds} = e[b_1(n) - t_1(n)]\sigma(n) + e[b_1(n + 1) - t_1(n + 1)]\sigma(n + 1). \]  

(1)

The probabilities \( \sigma(n) \) and \( \sigma(n + 1) \) are also given there and the tunneling rates \( b_i(t_i) \) from the bottom (top) across the \( i \)th junction of the rf-SET take the usual form [see expression (A3) in the appendix].

In the previous calculation of the sensitivity of the rf-SET based detector, the shot noise formula \( S_I = 2eI_{ds} \) was used. This expression, which gives the maximum current noise, is only approximately valid when \( V_{ds} \) is close to the tunneling threshold. Taking as our tool the intrinsic noise analysis of the SET by Korotkov, we obtain the following more accurate expression for the spectral density of the current noise \( S_I(\omega) \) [see appendix A for an outline of the deviation]:

\[ S_I(\omega) = \frac{e^2a(n)b(n + 1)}{a(n) + b(n + 1)} + \frac{e^2[f(n) - g(n + 1)]a^2(n)g(n + 1) - b^2(n + 1)f(n)}{(a(n) + b(n + 1))^2 + \omega^2}, \]  

(2)

where

\[
\begin{align*}
    a(m) &= b_1(m) + t_2(m), \\
    b(m) &= b_2(m) + t_1(m), \\
    f(m) &= b_1(m) - t_2(m), \\
    g(m) &= b_2(m) - t_1(m).
\end{align*}
\]  

(3)

The spectral density of the displacement noise due to the shot noise in the tunneling current is:

\[ S_\chi^L(\omega) = S_I(\omega)/(dI_{ds}/dx)^2. \]  

(4)

The dependence on the displacement \( x \) enters through the gate capacitance which is approximately \( C_g \approx C_g^0(1 - x/d) \) for \( |x| \ll d \) where \( d \) is the cantilever electrode-island electrode gap.
The fluctuation of the island charge will produce a fluctuating force on the cantilever, therefore leading to another contribution to the displacement noise. The ultimate force noise is only determined by the stochastic character of the tunneling process. By using the same method of intrinsic noise analysis, the following expressions are obtained for the spectral density of the force noise $S_F(\omega)$ [see appendix B for an outline of the deviation]:

$$S_F(\omega) = \frac{4A^2e^4a(n)b(n+1)}{a(n) + b(n+1)} \times \frac{(2C_j/C_g)^2(2n + 1 + 2\Delta n)^2 + (4C_j/C_g)(2n + 1)(2n + 1 + 2\Delta n) + (2n + 1)^2}{[a(n) + b(n+1)]^2 + \omega^2},$$  \hspace{1cm} (5)

where

$$A = C_g(2C_j - C_g)/2dC^3$$ \hspace{1cm} (6)

and

$$\Delta n = \frac{C_gV_g}{e} - \frac{C_gV_{ds}}{2e} - n - \frac{1}{2}.$$ \hspace{1cm} (7)

We model the cantilever as a simple harmonic oscillator, so that the spectral density of the displacement noise due to the fluctuating force is:

$$S_X^F(\omega) = \frac{S_F(\omega)/m^2_{\text{eff}}}{(\omega^2 - \omega^2_0)^2 + \omega^2\omega^2_0/Q^2},$$ \hspace{1cm} (8)

where $m_{\text{eff}}$ is the motional mass of the cantilever, $\omega_0$ is its resonant frequency, and $Q$ is the quality factor. For a cantilever with geometry $l \times w \times t$ (length \times width \times thickness) and made from material of density $\rho$ and Young’s modulus $E$, the resonant frequency of the lowest-order flexural mode is:

$$\omega_0 = 3.516 \frac{t}{l^2} \sqrt{\frac{E}{12\rho}}.$$ \hspace{1cm} (9)

The motional mass of the cantilever is given by $m_{\text{eff}} = \rho l w t / 4$ and the effective spring constant $k_{\text{eff}}$ at the cantilever’s tip is $k_{\text{eff}} = m_{\text{eff}}\omega_0^2$. The total displacement noise will be given as the sum of the above two contributions (4) and (8):

$$S_X(\omega) = S_X^I(\omega) + S_X^F(\omega).$$ \hspace{1cm} (10)
In addition, the following expressions are given here only for simple estimates of the displacement noise at the resonant frequency of the cantilever [see appendix C for an outline of their derivation]:

\[S^I_X = K \frac{2R_j}{eV_{ds}} \left[ \frac{2.5k_B T dC_{\Sigma}}{C_g(V_g - V_{ds}/2)} \right]^2, \tag{11}\]

\[S^F_X = \frac{Q^2 e^3 R_j}{k_{\text{eff}} V_{ds}} \left[ \frac{C_g(2C_j - C_g)(V_g - V_{ds}/2)}{dC_{\Sigma}^2} \right]^2, \tag{12}\]

where \(K \approx 40\) is a constant.

### III. RESULTS AND DISCUSSION

In this paper, we consider crystalline Si cantilevers with mass density \(\rho = 2.33 \times 10^3\) kg/m\(^3\) and Young’s modulus \(E_{100} = 1.33 \times 10^{11}\) N/m\(^2\). To make the picture clear, we compare the displacement noise for different cantilevers. Table I lists the resonant frequencies \(f_0\) and masses \(m\) (the effective mass of a cantilever is \(m_{\text{eff}} = m/4\)) of cantilevers with the same width and thickness but different lengths. The resonant frequencies \(f_0 = \omega_0/2\pi\) are obtained from expression (3). We mainly focus our attention on the detector using the cantilever with geometry \(1.25 \times 0.6 \times 0.15\) in \(\mu\)m, quality factor \(Q = 10^4\), resonant frequency \(f_0 = 117\) MHz, and effective spring constant \(k_{\text{eff}} = 3.6 \times 10^9\) aN/Å. The standard quantum limit of the fundamental flexural mode of this cantilever is \(\Delta x_{\text{SQL}} = 3.3 \times 10^{-4}\) Å, so that the minimum detectable force at the resonant frequency is \(\Delta F = k_{\text{eff}} \Delta x_{\text{SQL}} / Q = 120\) aN.

Fig. 2 shows the dependence of the displacement noise \(\sqrt{S^I_X}\) on the gate voltage \(V_g\) at the resonant frequency \(f_0 = 117\) MHz of cantilever. The result assumes a symmetric SET with junction capacitance \(C_j = 100\) aF, junction resistance \(R_j = 50\) kΩ, gate capacitance \(C_g = 50\) aF, gate capacitor gap \(d = 0.1\) \(\mu\)m, source-drain voltage \(V_{ds} = 0.38\) mV(0.6\(e/C_{\Sigma}\)), and temperature \(T = 74\) mK(0.01\(e^2/C_{\Sigma}\)), which are all routinely achievable. The solid line is obtained by using expressions (1), (3), and (4) and the dashed line using (11). The minimum displacement noise occurs near the edges of the tunneling current \(I_{ds}\), and decreases
approximately in inverse proportion to increasing gate voltage. The displacement noise diverges in the center of tunneling current peaks and in the Coulomb blockade region where the tunneling current is suppressed. The result here is similar to Fig. 2 in Ref. 8.

The dependence of the displacement noise $\sqrt{S_X}$ on the gate voltage $V_g$ at the resonant frequency $f_0 = 117$ MHz is illustrated in Fig. 3. The result assumes the same parameters as above. The solid line is obtained by using expressions (1) and (8), and the dashed line using (12). The maximum displacement noise increases linearly with the increasing gate voltage.

Fig. 4 shows the total displacement noise $\sqrt{S_X}$ as a function of gate voltage. The tunneling current is shown as the dashed lines. Considering the opposite dependences of $S_X^I$ and $S_X^F$ on gate voltage, we will expect an optimum gate voltage value for the lowest displacement noise and best sensitivity. But there are two problems involved in the determination. The first problem is that near the edges the tunneling current $I_{ds}$ may be too small to be measured accurately. The second is that because $S_X^I$ and $S_X^F$ change in different ways with increasing gate voltage $V_g$, it is rather involved finding the most optimum value of $V_g$ at which the total noise is minimum; the positions of the minima in the displacement noise doublets will change relatively to the tunneling current peaks with increasing gate voltage. An alternative, simpler strategy is to investigate the displacement noise at the gate voltage for which the tunneling current is fixed and measurable, say between 30% and 60% of its maximum value $I_{ds,max}$. Fig. 5 shows the total displacement noise $\sqrt{S_X}$ for our rf-SET displacement detector using different cantilever parameter values listed in Table I. The above method is utilized with the displacement noise determined where the tunneling current is half its maximum value. For the detector using a cantilever with resonant frequency $f_0 = 117$ MHz, the minimum displacement noise is $4.3 \times 10^{-6}$ Å/√Hz, corresponding to an absolute displacement sensitivity of $5 \times 10^{-4}$ Å and force sensitivity 180 aN. Note that the minimum displacement noise increases and shifts to lower gate voltage for increasing cantilever length. As is evident from (12), this is a direct consequence of the effective spring constant dependence: obviously a stiffer cantilever will be displaced less by the same applied
force.

Fig. 6 shows the detector displacement noise for different junction capacitance values. Both $S_X^I$ and $S_X^F$ decrease with decreasing $C_j$. Because small displacements are detected by monitoring changes in tunneling current, taking a smaller junction capacitance serves another advantage for the experiment: with $C_\Sigma V_{ds}/e$ kept fixed, a smaller junction capacitance corresponds to a larger source-drain voltage, so that the maximum tunneling current $I_{ds,\text{max}}$ increases according to expression (C4).

Fig. 7 shows the displacement noise when the tunneling current is biased at various values between 30% and 60% of $I_{ds,\text{max}}$. We see that performing detection at a smaller percentage of the maximum tunneling current becomes another effective way to further decrease the displacement noise. The lowest displacement noise we obtain is $\sqrt{S_X} = 3.2 \times 10^{-6} \, \text{Å/√Hz}$, assuming $C_j = 100 \, \text{aF}$, $C_g = 50 \, \text{aF}$, and $I_{ds} = 0.57 \, \text{nA}$, biased at 30% of its maximum value. The optimum gate voltage is 1.7 V in this case.

In conclusion, approaching the SQL will become possible for a carefully engineered rf-SET displacement detector. A sensitivity at, or even beyond the SQL may be achievable by using the same parameters as in the above analysis but with the smaller junction capacitance $C_j = 75 \, \text{aF}$. Such a sensitivity would enable us to observe quantum squeezed states in micro-mechanical systems. The proposed displacement detector also meets the crucial requirement for advances in MRFM. The radio-frequency micro-mechanical cantilever used in our model allows direct coupling at the spin precession frequency, therefore allowing fast read-out, while the predicted force sensitivity would allow the measurement of single-spin resonance.

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**APPENDIX A: TUNNELING CURRENT SHOT NOISE**

The tunneling current shot noise is given by expression (30) in Ref. [10]. In our notation, it is:

\[ S_I(\omega) = 2U + e^2 \sum_{m,m'} [t_2(m') - b_2(m') + t_1(m') - b_1(m')] B_{m',m} \]
\[ \times \{[t_2(m - 1) - b_1(m - 1)]\sigma(m - 1) + [t_1(m + 1) - b_2(m + 1)]\sigma(m + 1)\}, \] (A1)

where

\[ 2U = (e^2/2) \sum_m \sigma(m) [t_2(m) + b_2(m) + t_1(m) + b_1(m)] \] (A2)

and \( B_{m',m} \) is the real part of the inverse of the tridiagonal matrix \((i\omega I - \Gamma)\), where \( I \) is the unit matrix and

\[ \Gamma_{nm} = \delta_{n-1,m}a(m) + \delta_{n+1,m}b(m) - \delta_{n,m}c(m), \] (A3)

with

\[ a(m) = b_1(m) + t_2(m), \]
\[ b(m) = t_1(m) + b_2(m), \]
\[ c(m) = b_1(m) + t_2(m) + t_1(m) + b_2(m). \] (A4)

The tunneling rates are

\[ b_i(n) = \frac{\Delta E_i^<(n)/e^2R_j}{[1 - e^{-\Delta E_i^<(n)/k_BT}]}, \]
\[ t_i(n) = \frac{\Delta E_i^+(n)/e^2R_j}{[1 - e^{-\Delta E_i^+(n)/k_BT}]}, \] (A5)

where

\[ \Delta E_i^±(n) = [-e/2 \pm (en - C_jV_{ds} - C_gV_g)]e/C_\Sigma, \]
\[ \Delta E^\pm_2(n) = \left[ -\frac{e}{2} \mp (en + (C_j + C_g)V_{ds} - C_gV_g) \right] e/C_{\Sigma}. \]  

(A6)

A convenient form for the tunneling rates is

\begin{align*}
 b_1(m) &= \frac{1}{R_j C_{\Sigma}} \frac{n - m + \Delta n + \tilde{V}_{ds}}{1 - \exp \left[ -(n - m + \Delta n + \tilde{V}_{ds})/T \right]}, \\
 b_2(m) &= \frac{1}{R_j C_{\Sigma}} \frac{m - n - 1 - \Delta n + \tilde{V}_{ds}}{1 - \exp \left[ -(m - n - 1 - \Delta n + \tilde{V}_{ds})/\tilde{T} \right]}, \\
 t_1(m) &= \frac{1}{R_j C_{\Sigma}} \frac{m - n - 1 - \Delta n - \tilde{V}_{ds}}{1 - \exp \left[ -(m - n - 1 - \Delta n - \tilde{V}_{ds})/\tilde{T} \right]}, \\
 t_2(m) &= \frac{1}{R_j C_{\Sigma}} \frac{n - m + \Delta n - \tilde{V}_{ds}}{1 - \exp \left[ -(n - m + \Delta n - \tilde{V}_{ds})/\tilde{T} \right]},
\end{align*}

(A7)

where

\begin{align*}
 \Delta n &= \frac{C_g V_g}{e} - \frac{C_g V_{ds}}{2e} - n - \frac{1}{2}, \\
 \tilde{V}_{ds} &= \frac{C_{\Sigma} V_{ds}}{2e}, \\
 \tilde{T} &= \frac{C_{\Sigma} k_B T}{e^2},
\end{align*}

(A8)

and \( n, n + 1 \) are the most probable island electron numbers, determined by the condition

\[ n < \frac{C_g V_g}{e} - \frac{C_g V_{ds}}{2e} < n + 1, \]

which is equivalent to \(-0.5 < \Delta n < 0.5\).

Under the condition \( k_B T \ll eV_{ds} \) (equivalently \( \tilde{T} \ll \tilde{V}_{ds} \)), we have that \( t_1(n), b_2(n), b_1(n + 1), t_2(n + 1) \approx 0 \) for small drain-source voltage, and furthermore \( b(m) \approx 0 \) for all \( m \leq n \) and \( a(m) \approx 0 \) for all \( m \geq n + 1 \). Because the probabilities \( \sigma(m) \approx 0 \) for all \( m \neq n, n + 1 \), only the terms \( m = n \) and \( m = n + 1 \) need be considered in the summation over \( m \) in eqs. (A1) and (A2). From the form of the matrix (A3) and the above approximate conditions on the probabilities and tunneling rates, we then also have that only \( m' = n \) and \( m' = n + 1 \) need be considered in the sum over \( m' \). The matrix elements \( B_{n,n}, B_{n,n+1}, B_{n+1,n}, \) and \( B_{n+1,n+1} \) contain only the tunneling rates \( a(n), b(n), c(n), \) and \( c(n + 1) \) and it is valid to just consider \( 2 \times 2 \) matrices for \( \Gamma \) and \( B \):
\[ B = \text{Re} \left( \begin{pmatrix} i\omega - c(n) & -b(n) \\ -a(n) & i\omega - c(n + 1) \end{pmatrix} \right)^{-1}. \]  

(A9)

The probabilities \( \sigma(n) \) and \( \sigma(n + 1) \) are given in Ref. 8 as

\[
\sigma(n) = b(n + 1)/[a(n) + b(n + 1)], \\
\sigma(n + 1) = a(n)/[a(n) + b(n + 1)]. \]  

(A10)

Substituting (A9) and (A10) into (A1) and (A2) and neglecting all small terms, we get

\[
S_I(\omega) = e^2 a(n)b(n + 1)/[a(n) + b(n + 1)]^2 + e^2 / [a(n) + b(n + 1)]^2 \omega^2. \]  

(A11)

**APPENDIX B: FORCE NOISE DUE TO THE FLUCTUATING ISLAND CHARGE**

The electrostatic interaction between the fixed island and cantilever electrodes of the gate capacitor is

\[ F = A[C_j(V_{ds} - 2V_g) - ne + Q_p]^2, \]  

(B1)

where \( Q_p \) is the background charge and

\[ A = C_g(2C_j - C_g)/\Sigma C^3. \]  

(B2)

Define

\[ \tilde{Q} = C_j(V_{ds} - 2V_g) = -(2n + 1 + 2\Delta n)eC_j/C_g, \]  

(B3)

where the second equality follows from (A8). Neglecting \( Q_p \) in (B1), the force noise is given as

\[
S_F(\omega) = A^2 \left\{ 4\tilde{Q}^2 e^2 S_{nn}(\omega) + e^4 S_{n^2n^2}(\omega) - 2\tilde{Q} e^3 [S_{n^2n}(\omega) + S_{nn^2}(\omega)] \right\}. \]  

(B4)

Similarly to expression (27) in Ref. 10, we have
\[ S_{n\alpha n\beta}(\omega) = 4 \sum_{m,m'} m'^\alpha B_{m'm} m^\beta \sigma(m). \]  

(B5)

Using expressions (A9), (A10), and (B5) to obtain \( S_{nn}(\omega) \), \( S_{n2n}(\omega) \), and \( [S_{n2n}(\omega) + S_{nn}(\omega)] \), the final result for the force noise is

\[ S_F(\omega) = \frac{4A^2 e^4 a(n)b(n+1)}{a(n) + b(n+1)} \times \frac{(2C_j/C_g)^2(2n+1 + 2\Delta n)^2 + (4C_j/C_g)(2n+1)(2n+1 + 2\Delta n) + (2n+1)^2}{[a(n) + b(n+1)]^2 + \omega^2}. \]  

(B6)

Note that both \( S_I(\omega) \) and \( S_F(\omega) \) have a Lorenzian spectrum.

**APPENDIX C: SIMPLE ESTIMATE FOR THE DISPLACEMENT NOISE**

A simple, approximate formula for the tunneling current is

\[ I_{ds} = I_{ds,max} / \cosh^2 \left[ eC_g(V_{g,\text{res}} - V_g)/2.5k_BT \Sigma \right]. \]  

(C1)

where \( V_{g,\text{res}} \) is the value of the gate voltage at which the tunneling current is maximal:

\[ V_{g,\text{res}} = \frac{V_{ds}}{2} + (n + \frac{1}{2}) \frac{e}{C_g}. \]  

(C2)

Subsituting (C1) and the shot noise limit formula \( S_I = 2eI_{ds} \) into (4), we obtain

\[ S^I_X = \frac{1}{2eI_{ds,max}} \left[ \frac{2.5k_BTdC_{\Sigma}}{C_g(V_{g} - V_{ds}/2)} \right]^2 \frac{\cosh^4 \left[ eC_g(V_{g,\text{res}} - V_g)/2.5k_BT \Sigma \right]}{\sinh^2 \left[ eC_g(V_{g,\text{res}} - V_g)/2.5k_BT \Sigma \right]} \]  

(C3)

Using expression (4) for the tunneling current with \( \sigma(n) = \sigma(n+1) = 0.5 \) and expressions (A7) for the tunneling rates with \( \Delta n = 0 \) gives for the maximum tunneling current:

\[ I_{ds,max} = V_{ds}/4R_j. \]  

(C4)

To make a simpler estimate of the displacement noise, we might replace the dimensionless, parameter-independent function \( \cosh^4[x]/\sinh^2[x] \) by a constant \( K \), say its minimum value which is 4. However, this function poorly approximates the correct doublet shape and it is found numerically that the \( K \approx 40 \) provides a good fit. Thus we obtain the final result for the displacement noise due to the shot noise in the tunneling current:
\[ S_X^I = K \frac{2R_j}{eV_{ds}} \left[ \frac{2.5k_BTdC_\Sigma}{C_g(V_g - V_{ds}/2)} \right]^2. \]  

(C5)

A simple approximation to the maximum force noise is obtained by setting \( \Delta n = 0 \) in (B6) and neglecting small terms:

\[ S_F(0) = \left( A^2 e^5 R_j/V_{ds} \right) \left[ C_\Sigma(2n + 1)/C_g \right]^2. \]  

(C6)

Substituting in the definition (B2) for \( A \) and using \( 2n + 1 = C_g(2V_g - V_{ds})/e \) at maximum tunneling current, we obtain:

\[ S_X^F = \frac{Q^2 e^3 R_j}{k_{\text{eff}} V_{ds}} \left[ \frac{C_g(2C_j - C_g)(V_g - V_{ds}/2)}{dC_\Sigma^2} \right]^2, \]  

(C7)

at the resonant frequency \( \omega_0 \) of the cantilever, which is assumed to be much smaller than the tunneling rates in magnitude.
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TABLE I. The resonant frequency $f_0$ and mass $m$ of crystalline Si cantilevers with different lengths ($l$) and fixed width ($w = 0.6 \, \mu m$) and thickness ($t = 0.15 \, \mu m$).

| $l$ in $\mu m$ | 1.25 | 1.5 | 2 | 4 |
|---------------|------|-----|---|---|
| $f_0$ in MHz  | 117.2| 81.36| 45.77| 11.44 |
| $m$ in pg     | 0.262| 0.315| 0.419| 0.839 |
FIGURES

FIG. 1. Scheme of the rf-SET displacement detector.

FIG. 2. Displacement noise due to the tunneling current shot noise as a function of gate voltage (upper, doublet curves). The estimate of the lower limit on the displacement noise vs. gate voltage is also shown (lower, dashed line). The noise analysis assumes a symmetric SET at $T = 74 \text{ mK}(0.01e^2/C_{\Sigma})$ with junction capacitance $C_j = 100 \text{ aF}$, junction resistance $R_j = 50 \text{ k}\Omega$, and static gate capacitance $C_g = 50 \text{ aF}$ (the gate capacitor gap is $d = 0.1 \mu\text{m}$). The drain-source voltage is $V_{ds} = 0.38 \text{ mV}(0.6e/C_{\Sigma})$. The displacement noise is determined at 117 MHz-the resonant frequency of the cantilever.

FIG. 3. Displacement noise due to the force noise from the fluctuating island charge as a function of gate voltage (lower curves). The estimate of the upper limit on the displacement noise is also shown (upper, dashed line). The noise analysis assumes the same parameters as in Fig. 2.
FIG. 4. The total displacement noise as a function of gate voltage (upper, doublet curves). For illustrative purposes, the sample voltage range is chosen such that the force and current displacement noise contributions are comparable. The tunneling current is given by the lower, dashed line.

FIG. 5. The total displacement noise as a function of gate voltage for the different cantilever lengths listed in Table I. The same parameter values as in Fig. 2 are assumed. The displacement noise at the resonant frequency of the cantilever is determined at the gate voltage for which the tunneling current is fixed at half its maximum value.

FIG. 6. Total displacement noise for different junction capacitances. Except for the junction capacitances, the same parameter values as in Fig. 2 are assumed and the displacement noise is determined at the tunneling current half-maximum.

FIG. 7. The displacement noise for different percentages of the maximum tunneling current. The same parameters as in Fig. 2 are used.
Fig. 1 Zhang
Fig. 2 Zhang
Fig. 3  Zhang
Fig. 4  Zhang
Fig. 5 Zhang
Fig. 6 Zhang
Displacement Noise (angstrom/Hz^{0.5})

Gate Voltage (V)

Fig. 7  Zhang