Negative differential resistance in normal narrow bands - superconducting junctions with Andreev reflection

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We have calculated current-voltage characteristic curves for normal-superconducting junctions with Andreev reflections and different types of electronic bands. We found that when the normal band is narrow, of the order of the superconducting energy gap, a negative differential resistance appears at a voltage of the order of the band width plus the gap value. In case two bands contribute to the total current the conductance can be smaller than unity at voltages above the gap value. Our simulations may provide an answer to different experimental data of the literature that were not yet understood.

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The measurement of the current-voltage ($I - U$) characteristic curves of junctions made from similar or different materials separated by an insulating layer is one of the usual transport methods to obtain information on the electronic band structures near the Fermi level $E_F$. If one of the materials of the junctions is a superconductor, the $I - U$ curve provides a direct measurement of the energy gap $\Delta$, for example. In this case the voltage region smaller than $\Delta/e$ ($e$ is the electronic charge) the conductance $G$ is in general much smaller than $G_N$, the value obtained at $U \gg \Delta/e$. In case one has a plain normal-superconducting junction without any insulating intermediate layer, the conductance $G(U < \Delta/e) \sim 2G(U \gg \Delta/e) = 2G_N$, a rather surprising phenomenon taking into account that for a normal-normal metals junctions the normalized conductance $G/G_N = 1$ at all voltages.

Andreev studied theoretically the heat transport through a normal-superconducting plain junction and obtained the factors 2 and 1 for the normalized conductance in these two voltage regions \cite{1}. Blonder, Tinkham and Klapwijk (BTK) \cite{2} studied the reflection of an electron at this kind of junctions and obtained a similar solution for the conductance, i.e. it decreases from a factor 2 to 1 when the voltage grows from zero to values above the energy gap of the superconducting part of the junction. The physical explanation for the value of 2 at $U < \Delta/e$ is related to the reflection and transmission of the electron of the normal part into the superconducting one. The electron current from the normal part transforms in a Cooper pair current in the superconductor and a hole reflects back into the normal part. Using a different approach Garcia, Flores and Guinea \cite{3} studied a similar phenomenon observed in scanning tunneling microscopy and obtained the same equation as in Ref. \cite{2}.

All the $I - U$ curves discussed above were obtained for the case of ballistic transmittance between a normal metal and a superconductor. This approach has been extended for the case of ferromagnets with a certain spin polarization and for a diffusive regime in a recent publication \cite{4}. In the last years there have been several reports \cite{5} showing experimental data with conductance values less than its normal state value $G_N$ at $U > \Delta/e$. Although some modeling for this behavior on the basis of non-ideal interfaces, proximity effects and energy losses were proposed, the generality of this behavior observed in different kind of samples suggests that other explanation may be needed.

Assuming two different main cases for the normal-superconducting junctions, see Fig. 1 we calculate the $I - U$ characteristic curves and conductance as a function of bias voltage $U$. Our aim is to show in which cases the conductance gets smaller than unity (in reduced scale, i.e. $G/G_N$) at voltage values above the gap implying the contribution of a negative differential resistance to the total current. We show below that this can be observed when the band width $W$ of the normal material is of the order of the energy gap of the superconducting material, what we call a normal narrow band. The conductance exhibits a negative differential resistance with a maximum absolute value at $\Delta < eU \lesssim \Delta + W$, specially in the two dimensional case and it tends to zero for voltages $eU > W + \Delta$.

We assume that the junction is composed by two different materials one of them a superconductor with a energy gap $\Delta$. Following Refs. \cite{2,3} the Andreev conductance at zero temperature (in units of quantum of conductance) $G(U)$ is given by

$$G(U) = 1 - \frac{((1 - a^2)^2(1 - |T|))}{(1 - a^2(1 - |T|))} = \frac{a^2|T|^2}{(1 - a^2(1 - |T|))^2},$$

which depends on the transmittivity $T$ in the normal state; $a = (U/\Delta) - (|U/\Delta|)^2 - 1)^{0.5}$. The current $I$ as a function of the voltage $U$ at the junction can be calculated taking into account that it is
controlled by Cooper pairs Andreev currents if $eU < \Delta$ \cite{1}. However at higher $U$ the current is controlled by quasiparticles and for $U > 7\Delta/e$ we have practically conduction between normal materials \cite{123}. The solution for the current $I$ is

$$I(U) = B \cdot A \int_{0}^{U} G(U')dU', \tag{2}$$

where $A$ is a constant that depends on the junction geometry and on the integration average on $\alpha$, the angle that the incoming particles form to the interface and $B = N(0)ev_{F}$, with $N(0)$ the density of states at $E_F$ and $v_F$ the Fermi velocity. This integral corresponds to ballistic 3D case for a system in which the occupied normal band is much broader than $\Delta$, i.e. $I_{W>\Delta}$. The transmittivity can be expressed as $T = 1/(1 + Z_0^2)$, where $Z_0$ was defined in \cite{2}. Notice that $Z_0 = 0$ and $\infty$ means $T = 1$ and 0.

For the case of Fig.1(a) the conductance (calculated assuming a transmittivity $T = 1$ , see Eq. \ref{conductance}) has the known value of 2 at $U \leq \Delta$ and reduces gradually to 1 at voltages above $\Delta$, see Fig.2(a). The total current $I$ through the junction is calculated following Eq. \ref{current} and is depicted in Fig.2(c). In case we have $T = 0.5$ we get the results shown in Fig.2(b,d) for the conductance and current. With exception of the curves in Fig.6, all others are normalized to their normal-state values.

In the case we have a normal metal with a narrow band width $W \sim \Delta$, then the situation changes and the equation for the current is given by

$$I_{W<\Delta} = Am^* \int_{0}^{U} (W-eU'^{b})G(U')dU', \tag{3}$$

where the parameter $b = 0.5, 0, -0.5$ corresponds to the 3D, 2D and 1D case and the parameter $A$ has the same meaning as in Eq. \ref{current} and $m^*$ is the effective mass of the carriers. The correction term $(E_F + eU'^{b})$ can be in general neglected if $E_F \gg eU$. The theoretical results below are calculated at zero temperature.

If we have a narrow band in energy with its minima at $U = 0$ near the superconducting gap, the integral to estimate the current is like in Eq. \ref{current} but with the bottom of the band that depends on the assumed band width.
corresponding conductances $dI/dU$.

The same as in (a) but for $T = 0.5$. (c) and (d) show the corresponding conductances $dI/dU$. In (d) the conductance is normalized by its value at $eU \gg \Delta$.

FIG. 4: (a) Total current $I$ vs. normalized voltage $eU/\Delta$ for $T = 1$ for the case of a normal part having two materials touching in parallel the superconducting part, one with $W \gg \Delta$ and a second with $W = 1.5 \Delta$, calculated using $I = I_{W=\Delta} + 0.5 I_{W=\Delta}$ (Eq. 2 and Eq. 9) for the 3D case. (b) The same as in (a) but for $T = 0.5$. (c) and (d) show the corresponding conductances $dI/dU$. In (d) the conductance is normalized by its value at $eU \gg \Delta$.

2$W$ and therefore of the order of $\Delta$. Note that in this case the current will not be symmetric for positive and negative voltages. For positive voltages Fig. 4 shows the current for the case of a conductor with a narrow band (see Fig. 1(b)) filled up to $W = 2\Delta$ in the 3D and 2D cases for two different transmittivities. Note that in both 3D and 2D cases there appears a negative differential resistance region at $2 \lesssim eU/\Delta \lesssim 3$ for $T = 1$ and at slightly higher voltage values for $T = 0.5$, see Fig. 3. The behavior of the conductance $dI/dU$ depends on the transmittivity value as well as the dimensionality of the normal band, see dashed lines in Fig. 3. In case the voltage is negative we will obtain the same curve if the Fermi energy was at the middle of the band. Otherwise we need to do the calculations for the specific case.

One could also treat the case in which the narrow band material is present in parallel to the larger band one, both contributing in parallel to the current through the junction. This is an interesting case that may occur at the interfaces of the contacts. When the two materials, the normal and superconducting one, are put together narrow bands at the Fermi energy may appear upon materials used, junction geometry and quality. The current and the conductance are presented in Figures 4 to 6 for different cases.

In the particular cases treated below we have reduced the transmittivity of the narrow band respect to the large one by a factor of two. This assumption means that the effective mass of the carriers in the smaller band is half of that in the larger one. Nevertheless, we will see that the negative differential resistance contribution from the narrow band influences the total conductance. We note that qualitatively similar characteristic curves as we describe below were observed in many experiments but in general their meaning was not discussed. The cases we discuss below can really happen because these narrow bands can be formed at the contacts between the normal and superconducting parts. Figure 4 shows the calculated total current $I$ given by the sum of two normal conductors in parallel, one with a large band width $W \gg \Delta$ and the other with $W = 1.5\Delta$, for $T = 1$ (a) and $T = 0.5$ (b). The figures (c) and (d) show the calculated conductances $dI/dU$. Figure 5 shows the results for the 2D case and for $T = \exp(-1)$ or $T = \exp(-0.5)$.

We describe now the last case when the small and large bands contribute to the current but their carriers have to overcome a barrier of height $\phi$ existing between them and the superconductor. If $\phi$ is not very large then it will allow more current as the potential $U$ increases. An example can be seen in Ref. 12. Figure 6 shows the results of the calculations for two bands using $T = \exp(-(2 - 0.4U)^{0.5})$. It can be seen that at low voltages the conductance is smaller than at high voltages with an intermediate region where it decreases due to the negative resistance contribution from the narrow band. This case is different from the previous cases, see for example Figs. 4 and 5 because there the bands just
FIG. 6: Total current vs. normalized voltage for a normal conducting part with two parallel contributions with $W \gg \Delta$ and $W = 1.5\Delta$ with a transmissivity $T = \exp(-2r_{4U}^{D})$ for the 3D case (a) and 2D case (b). The dashed line in (a) is the conductance $G$ following Eq. (1). The figures in (c) and (d) show the corresponding conductances $dI/dU$.

Contribute with different transmittivities but these do not depend on $U$, i.e. there is no tunnel barrier that change the conductivity appreciably. The increase of the conductance at large $U$ values, as is the case of Fig. 4 may occur because when a contact is formed it could have some oxide that acts as a potential barrier.

In conclusion, in this paper we have discussed the contribution of different types of bands of normal–superconducting junctions with Andreev reflections. The results show: (1) For a single normal wide band $W \gg \Delta$ the conductance $G$ and the current vs. applied voltage behave as expected, i.e. $G = 2$ for $eU \ll \Delta$ and tends to 1 for $eU > \Delta$. (2) For a junction with a normal narrow-band width $2W \sim \Delta$ and at $eU > W + \Delta$ the current tends to a constant and the conductance to zero. At intermediate voltages we find a region of negative differential resistance (current decreases with voltage) as shown in Fig. 3. (3) As two bands, one with a large and the other with a narrow band width, contribute in parallel, a variety of cases appear. In any of those discussed here the negative differential resistance part coming from the narrow band has a clear influence on the expected $I - U$ characteristics. (4) Finally, if we assume a not very high potential barrier between the normal and superconducting parts, the conductance does not saturate but steadily increases at high voltages $eU > W + \Delta$. These results indicate that two electronic bands at the Fermi level can complicate the form of the characteristic $I - U$ curves in real junctions. A comparison of our results with the available experimental data [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] indicates that our model should be useful to understand experimental results.

For wide band ferromagnetic materials and ballistic transport (which produces minima for bias voltages smaller than the energy gap) further extensions of the BTK model were reported in Ref. [4, 13], approaches that can be further developed including the contributions of narrow bands. A narrow band effect can be also treated taking into account a proximity effect [10]. However, if the conductance minima are at higher voltages than the equivalent to the energy gap (as e.g. the case of Fe/Ta [5]), the proximity effect can unlikely explain it, but it is a clear sign for a narrow band effect. These possibilities have not been yet discussed in any of the existing experimental papers and stress the possible existence of interesting physics at contacts and their interfaces.

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[1] A. F. Andreev, Sov. Phys. JETP 19, 1228 (1964).
[2] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B 25, 4515 (1982).
[3] N. García, F. Flores, and F. Guinea, J. Vac. Sci. Technol. A 6, 323 (1988).
[4] I. I. Mazin, A. A. Golubov, and B. Nadgorny, J. Appl. Phys. 89, 7576 (2001).
[5] R. J. Soulen, M. S. Osofsky, B. Nadgorny, T. Ambrose, S. F. Cheng, P. R. Broussard, C. T. Tanaka, J. Nowak, J. S. Moodera, et al., Science 282, 85 (1998).
[6] J. Y. T. Wei, N.-C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. 81, 2542 (1998).
[7] R. J. Soulen, M. S. Osofsky, B. Nadgorny, T. Ambrose, P. Broussard, J. Byers, C. T. Tanaka, J. Nowak, J. S. Moodera, G. Laprade, et al., J. Appl. Phys. 85, 4589 (1999).
[8] P. M. C. Rourke, M. A. Tanatar, C. S. Turel, J. Berdeles, C. Petrovic, and J. Y. T. Wei, Phys. Rev. Lett. 94, 107005 (2005), see also the comments from G. Sheet and P. Raychaudhuri, Phys. Rev. Lett. 96, 259701 (2006), and W. K. Park and L. H. Greene, Phys. Rev. Lett. 96, 259702 (2006), and the reply from the authors in Phys. Rev. Lett. 96, 259703 (2006).
[9] R. P. Panguluri, C. Zeng, H. H. Weitering, J. M. Sullivan, S. C. Erwin, and B. Nadgorny, phys. stat. sol. (b) 242, R67 (2005).
[10] G. J. Strijkers, Y. Ji, F. Y. Yang, C. L. Chien, and J. M. Byers, Phys. Rev. B 63, 104510 (2001).
[11] R. P. Panguluri, K. C. Ku, T. Wojtowicz, X. Liu, J. K. Furdyna, Y. B. Lyanda-Geller, N. Samarth, and B. Nadgorny, Phys. Rev. B 72, 054510 (2005).
[12] S. H. Yun, N. Anderson, B. Liang, R. L. Greene, and A. Biswas, arXiv:0712.1614.

[13] K. Xia, P. J. Kelly, G. E. W. Bauer, and I. Turek, Phys. Rev. Lett. 89, 166603 (2002).