Modeling of burden at automated strength calculation of mobile vehicles supporting structures

M Tokareva, T Zubkova

Faculty of mathematics and information technology, Orenburg State University, 13 Pobeda ave., Orenburg, 460018, Russian Federation

E-mail: tokareva@mail.osu.ru

Abstract. A method, its algorithmic and software implementation, based on the use of the Ritz variational principle, which allows to calculate the values of stresses, displacements, reactions at the points of interest to the researcher, has been developed to simulate static loading when calculating the strength of structures consisting of flat elements. This provides an opportunity to reduce the complexity and timeline of final testing of the supporting structures of mobile vehicles.

1. Introduction

Meeting the needs of society in the effective functioning of vehicles requires the solution of a complex of scientific and technical problems, in particular, the use of high-tech information technologies in the production and restoration of the supporting structures of vehicles. A promising direction of increasing their reliability is the automation of calculation using the methods of the theory of elasticity and structural mechanics of vehicles and structures, which allows to choose designs that are rational in terms of strength and manufacturability at the design stage, which significantly reduces development time and developmental tests [2,3,4,5].

Depending on the objectives, the following objects may be subjected to testing: the supporting structures in the vehicle; separate complete support systems (frames, dump frames, the foundations of cargo platforms or bodies, etc.); separate closed contours of the supporting structures; open nodal connections and individual elements (spars, cross-pieces, racks, etc.) or their sections.

Testing of objects from the first three groups is most often carried out with the purpose of comparative assessment of the design options durability or individual assessment of the experimental structures durability. Objects that are preferable for conducting final testing are nodal joints or separate parts of structural elements, depending on which local area limits the durability of the entire supporting structure [6].

When repairing the supporting structures, it is necessary to take into account their condition at the time of repair, the possibility of replacing the elements, taking into account the variation of parameters such as geometric shapes and sizes, material properties. With the possible change of these parameters, the construction that has been repaired should have sufficient, as a rule, regulated, reliability and durability with a minimum amount of various types of tests of modified or modernized variants.

The authors have developed a technique enabling rapid assessment of stationary loads, which allows you to choose from a variety of design options for supporting structures assemblies an
acceptable design and technological option that meets the requirements for strength and material intensity. The technique uses a modification of the well-known Ritz method, and due to it, it is possible to avoid the numerical integration used in the finite element method [1], which makes it possible to reduce the design calculation time without loss of accuracy.

2. A description of the methodology

Spatial structures consisting of flat elements can be divided into their constituent parts with the introduction of unknown force bonds. Depending on the location of forces relative to the flat element, the latter may be of the following types:

A - with forces acting in the plane of the element;
B - with forces acting perpendicular to the plane of the element;
C - with forces acting both in the plane of the element and perpendicular to it.

In solving the problem, the finite-difference scheme obtained from the Ritz variational principle is used. To this end, the elements are covered with rectangular grids so that they are combined at the joints. Forces can only be applied at grid points. If it is inconvenient to combine an arbitrary force with a node, it should be decomposed into two components applied at the nearest nodes.

The elements of type A with the forces acting along the middle surface of the element can be applied to the ratio of the plane problem of the theory of elasticity.

In particular, the Ritz method and the formula for the specific potential energy, which has the following form for bulk particles, can be applied:

\[
W = G(\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2 + \frac{\nu \Delta}{1-2\nu} + \frac{1}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)),
\]

where \( \Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z; \nu \) is Poisson ratio.

In the case of a plane-stressed problem, the condition \( \sigma_z = 0 \) in the formulas of the generalized Hooke’s law gives

\[
\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y).
\]

Substituting the values into the formula for the specific potential energy and assuming that \( \gamma_{yx} = \gamma_{zx} = 0 \) we get:

\[
W = G(\varepsilon_x^2 + \varepsilon_y^2 + \nu(\varepsilon_x + \varepsilon_y)^2 + \frac{1}{2} \gamma_{xy}^2)
\]

It is obvious that the total potential energy in a certain volume \( V \) with a constant value of stresses \( \sigma_x, \sigma_y, \tau_{xy} \) it is equal to \( W_n = WV \). In the case of a plane-deformed state, the denominator of the third term instead of \( 1-\nu \) should be \( 1-2\nu \).

Talking the coordinate system, as shown in Figure 1, we write the relative deformation:

- for section KI \( \varepsilon_x = \frac{\partial U}{\partial x} = \frac{U_x - U_k}{\lambda_i}; \)
- for section QK \( \varepsilon_y = \frac{\partial V}{\partial y} = \frac{V_y - V_k}{s_i}; \)
- for section QKI \( \gamma_{xy} = \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} = \frac{U_q - U_k}{s_i} + \frac{V_q - V_k}{\lambda_i}. \)

The total potential energy of the system \( W_n \), divided by a rectangular grid into a series of elements and containing \( n \) points can be written as a function of \( 2n \) unknown displacements:

\[
W_n = f(x_1, x_2, x_3, ..., x_{2n-1}, x_{2n}).
\]
The task is to determine the values of the unknowns \( x_1, \ldots, x_{2n} \) with a sufficient approximation. The Ritz method gives for this condition the minimum potential energy:

\[
\frac{\partial W_n}{\partial x_1} = \frac{\partial W_n}{\partial x_2} = \ldots = \frac{\partial W_n}{\partial x_i} = \ldots = \frac{\partial W_n}{\partial x_{2n}} = 0.
\]

You can make sure that for the derivatives \( \frac{\partial W_i}{\partial x_1} \) and \( \frac{\partial W_i}{\partial x_{2n}} \) there are only unknowns that are located not more than one step away from point \( i \), that is, only the unknown displacements of the points indicated in Figure 1. Therefore, we write down the potential values energy for only four plots containing point \( i \). Figure 2 shows a diagram of the regions defining the values of the relative deformations \( \varepsilon_x, \varepsilon_y \) and \( \gamma_{xy} \), as well as 12 regions, in each of which the values of \( \varepsilon_x, \varepsilon_y \) do not change.

\[
\begin{align*}
(\varepsilon_x + \varepsilon_y) & \\
(\gamma_{xy}) & \\
\end{align*}
\]

![Diagram of the areas defining the value of relative deformations.](image)

Writing the values of all four terms of the potential energy and taking the derivatives with respect to \( x_i \) and \( x_{i+1} \), we obtain 2 equations from the general system consisting of \( 2n \) equations.

The equations are written as follows:

\[
A_p x_p + A_N x_N + \ldots + A_p x_p + A_q x_q + \ldots + A_p x_p = \frac{P_i}{2G};
\]

\[
B_p x_p + \ldots + B_p x_p + B_q x_q + \ldots + B_p x_p = \frac{P_i}{2G},
\]

where \( P_i \) and \( P_i \) are the horizontal and vertical forces applied at point \( i \), which are positive if directed along the axes \( Ox \) and \( Oy \), respectively; \( G \) is the shear modulus equal to \( E/2(1+\nu) \), where \( E \) is the elastic modulus of the 1st kind, \( \nu \) is the Poisson ratio.
The coefficients \( A \) and \( B \) in equations (3), (4) are determined on the basis of the weighting factors, some of which are recorded in the information files of the task, and some - programmatically determined for areas A and B.

\[
\varepsilon_x = \frac{x_b - x_A + x_B - x_C}{2\lambda_i}; \quad \varepsilon_y = \frac{x_d - x_c + x_b - x_d}{2\lambda_j};
\]
\[
\tau_{xy} = \frac{1}{2} \frac{(x_A - x_C + x_B - x_D) + (x_D - x_B + x_A - x_D)}{s_j};
\]

- tension along the axes \( Ox \) and \( Oy \):
  \[
  \sigma_x = E \frac{\varepsilon_x - \nu \varepsilon_y}{1 - \nu^2}; \quad \sigma_y = E \frac{\varepsilon_x + \nu \varepsilon_y}{1 - \nu^2}; \quad \tau_{xy} = \frac{E}{2(1 - \nu)} \gamma_{xy};
  \]

- main stresses:
  \[
  \sigma_{1,2} = \frac{1}{2} (\sigma_x + \sigma_y) \pm \sqrt{\left(\sigma_x - \sigma_y\right)^2 - 4\tau_{xy}^2},
  \]
where \( \sigma_1 \) is large and, \( \sigma_2 \) is smaller in absolute value.

The tangential stresses are equal
\[
\tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2}.
\]

The given stresses are calculated according to the theory of strength formula:
\[
\sigma_{\text{sp}} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}.
\]

The angle of inclination of the main site \( \alpha \) is obtained from the dependency:
\[
tg 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}.
\]

The positive angle \( \alpha \) is deposited from the positive direction of the axis \( Ox \) clockwise.

For elements of type B, the voltage is determined by the formulas for the plates (Figure 4).

Usually, to solve the problem of stress-strain state of spatial systems, taking into account their plane deformation and bending, the structural components are covered with a grid of a regular structure. However, when solving some problems, it is more convenient to have a grid of irregular structure, since this facilitates the determination of stresses in the places of their concentration. The use of an irregular grid complicates the solution of the problem, therefore, for composite structural elements that carry the load in their plane, an irregular grid is adopted, and for flexural elements - a regular one.

Figure 5 shows one of the structural elements in a plane-stressed state with an irregular grid applied.
Figure 4. To the calculation of stresses in area B.

At the edges, unknown reactions of bonds $R_1$, $R_2$, etc., are applied, which are taken into account in equations (3) and (4) by introducing additional terms like $-R_i/2GB$, where $B$ is the thickness of the element. Missing equations are obtained from the compatibility condition.

Figure 5. Irregular grid for a flat task.

The method of strength calculation is embodied in the “Plosk” software package, which implements:

1. the algorithm to automate the compilation of the system, taking into account the different configuration of the boundaries of the flat element;
2. obtaining the values of stresses, main stresses, as well as the values of tilt angles and reduced stresses in a user-friendly form;
3. the packaging algorithm and the algorithm for solving the system in a packaged form by the Gauss method with the choice of the main element;
4. the algorithm for organizing software checkpoints, which allows you to take breaks in the decision and resume the decision from the place where the stop occurred;
5. the algorithm for taking into account the input of additional equations or the replacement of already formed ones, which include the equations of support or forced displacements and the compatibility equation;
6. the possibility of correction of weighting coefficients, which allows introducing additional terms with new unknowns into the already formed equations, which may occur when inserting bonds at the joints or when entering bearing links;
7. the algorithm for selective calculation of reactions at points of interest to the researcher;
8. the ability to enter up to three groups of free members corresponding to three types of loading of the same design, which allows one to obtain stresses, displacements and reactions for the specified loading options.

The validity and reliability of the proposed methodology is confirmed by the correct use of the analytical apparatus of the study and the adequacy of the calculated results to the experimental data obtained from the bench tests of local models of tractor semi-trailers and typical operating conditions.
3. Conclusion
Using the software package that implements the modification of the Ritz method, the strength of the frame assembly of the heavy-duty tractor dumping semi-trailer OZTP-9554, which has a wide range of operating conditions and is able to adequately represent other transport vehicles, was calculated. The results of stress calculations are on average 8-10% higher than the experimental ones, i.e. sent to the margin of safety.

Using the results of testing large-scale models with a limited range of modeling and their subsequent approximation to full-scale structures will allow the use of universal testing equipment and significantly reduce energy and material costs in tests related to fatigue life testing of large-sized bearing systems of transport, road-building, metallurgical and other machines working under conditions of periodic loading.

It is possible to use the results in the following main areas:
- studies on the static strength of these structures for various types of loading;
- fine-tuning of new and modified structures and the development of technologies for their production and repair;
- the creation of a data bank on the stress-strain state of various versions of standard design nodes.

Using the results obtained when using the “Plosk” software package, the authors propose a technique and its software implementation for building models of displacements and reactions using correlation and factor analyzes.

The proposed statistical methodology and its software implementation was applied by the authors to determine the conditionality of the sizes of the elements of the chain-drive in the PR-15.875 chain. For the given nominal values of dimensions and their deviations, under the condition of normal distribution, a study matrix was constructed, the column parameters of which were the main technical characteristics with normal distributions of their values, the observations — generation lines.

Further work of the authors in this direction is planned in carrying out calculations of the dynamic characteristics of more complex load-bearing structures and checking the adequacy of the models constructed to the experimental data.

References

[1] Finite-Element Method for Three-Dimensional Problems of Elastic Structures Buckling Theory. Dimitrienko Yu.I., Bogdanov I.O. Herald of the Bauman Moscow State Tech. Univ., Nat. Sci. 2016, no. 6, pp. 73–92. DOI: 10.18698/1812-3368-2016-6-73-92

[2] More on the Development of Rigidity Evaluation Techniques of Freight Car Structures and Their Components in the Design and on Trials. Adjustment of Design Loads. Plotkin V.S., Krasnobaev A.M., Konkova T.Y., Krasnobaev O.A. JSC Railway Research Institute, 2015, no. 6, pp.50-57.

[3] Increasing the Reliability the Combined Criteria of the Static Strength of a Material of Complexly Loaded Deformable Structures Zenkov E.V., Tsvik L.B. Materials physics and mechanics. 2018. t. 40. № 1. pp.124-132.

[4] Numerical Methods for Calculating the Strength and Stability of Stiffened Orthotropic Shell Skarpov V.V., Semenov A.A. Materials physics and mechanics. 2017. t. 31. № 1-2. c. 16-19.

[5] Application of Unstructured Approximating meshes in discrete-continual finite element method of structural analysis Akimov P.A., Negrozov O.A. Materials Physics and Mechanics. 2016. T. 26. № 1. pp. 5-8.

[6] On the issue of assessing the stress-strain state of frame metal structures of mobile vehicles. Tokareva M.A, Rassokha V.I., Filippov V.Yu. Control. Diagnostics, 1999. №11.- pp. 7-11.