A resonance problem on the low-lying resonant state in the $^9$Be system

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Abstract. The photo-disintegration cross section of $^9$Be is investigated in a framework of the $\alpha+\alpha+n$ three-body cluster model. Much interest is concentrated on the nature of the $1/2^+$ state of $^9$Be located just above the three-body threshold. The existence of this $1/2^+$ resonance is a long-standing problem and has been a topic of quite interest in relation to the breakup mechanism of $^9$Be into the $^8$Be($0^+$)+n and ($\alpha+\alpha+n$) channels. The newly measured breakup cross sections of the $1/2^+$ state of Ref. [1] is inconsistent with the old experimental values [2, 3]. The purpose of this study is to clarify the properties of the $1/2^+$ state of $^9$Be with the possibility of a genuine three-body ($\alpha+\alpha+n$) resonance or the two-body ($^8$Be($0^+$)+n) virtual state.

1. Introduction
The neutron capture ($\mathrm{n},\gamma$) and its inverse ($\gamma,n$) reactions are the highly interesting topics in current nuclear physics, because these reactions play important roles in nuclear reactor physics and astrophysical physics. So far, the resonant structure [4, 5] and a virtual characteristic state [6] of $^9$Be($1/2^+$) have been studied based on the various theoretical approaches and photodissociation experiments. However, a complete understanding on the nature of $^9$Be($1/2^+$) state has not yet been obtained. It is desired to perform a more comprehensive study based on an $\alpha+\alpha+n$ three-body calculations which can treat the bound ground state and also the unbound resonant states of $^9$Be in an unified framework. In this work, we apply the complex scaling method (CSM) [7] to an $\alpha+\alpha+n$ three-cluster model for understanding the nuclear structure and ($\gamma,n$) reactions for low-lying states in $^9$Be.

For the purpose of this work we treat the unbound nature of the $1/2^+$ state of $^9$Be and investigate the E1 transition between $1/2^+$ and $3/2^-$ states, which contributes to the ($\gamma,n$) reaction dominantly. We examine the resonance formation in the ($\gamma,n$) reaction to see whether a direct three-body breakup or a two-step process through the $^8$Be+n intermediate resonant state is dominant.
2. Three-body model and complex scaling method

The Hamiltonian for the relative motion of the $\alpha + \alpha + n$ three-body system for $^9$Be is given as

$$\hat{H} = \sum_{i=1}^{3} t_i - T_{c.m} + \sum_{i=1}^{2} V_{\alpha n}(\xi_i) + V_{\alpha \alpha} + V_3 + V_{PF}. \quad (1)$$

The Hamiltonian consists of the kinetic energy operators of the first and second terms, the two-body potential terms of subsystems for $\alpha + \alpha$ and $\alpha + n$, the three-body $\alpha + \alpha + n$ potential term and the last term that projects out the Pauli forbidden (PF) state from the relative motion. The details are given in Ref. [8]. The KKNN potential [9] and the folding potential of an effective nuclear and the Coulomb forces are applied to $V_{\alpha n}$ and $V_{\alpha \alpha}$, respectively. Here $\xi_i$ is the relative coordinate between two clusters ($\alpha$ and $n$). The three-body (three-cluster) potential $V_3$ is explicitly given by the following one-range Gaussian form with the strength $v_{3\beta}$:

$$V_3(r_1, r_2) = v_{3\beta} \exp(-\rho r^2), \quad (2)$$

where $\rho$ is the hyper-radius of the $\alpha + \alpha + n$ system.

The lowest threshold of $^9$Be is a three-body breakup into $\alpha + \alpha + n$. Only the ground state of $^9$Be is bound, so $^9$Be is a Borromean system. We investigate the properties of the unbound states of $^9$Be using the CSM. The CSM has been applied to many kinds of nuclear systems, particularly as a useful method to describe multi-body resonance states together with bound states [7, 8, 10]. The complex-scaled Schrödinger equation is expressed using the complex-scaled Hamiltonian $H(\theta)$ as

$$H(\theta)\Psi(\theta) = E\Psi(\theta). \quad (3)$$

The relative coordinates $\vec{r_j}$ of the three-body system $\alpha + \alpha + n$ are transformed as $\vec{r_j} \to \vec{r_j}e^{i\theta}$ ($j = 1, 2$) with a real parameter $\theta$. We can approximate the eigen function $\Psi_j(\theta)$ for the state $\nu$ with the total spin $J$ in Eq. (3) using the following basis expansion:

$$\Psi_j^\nu(\theta) = \sum_{\beta=1}^{N} \psi_{\beta}^j(\theta) \psi_{\beta}^J,$$  \quad (4)

where $\psi_{\beta}^J$ is the basis function for three-body states of $^9$Be and given as

$$\psi_{\beta}^J = \left[\phi^{i\beta}_{\alpha}(\vec{r}_1) \otimes \phi^{j\beta}_{\alpha}(\vec{r}_2) \right]_{L_{\beta}} \otimes \chi_{1/2}^L.$$  \quad (5)

The index $\beta$ represents a set of $\{c, i, l, j, \lambda, L\}$ where $c = 1$ and 2 specifies a channel of $(\alpha + \alpha) + n$ and $(\alpha + n) + \alpha$, respectively. The orbital angular momenta $l$ and $\lambda$ are corresponding to the relative motion for $\vec{r}_1$ and $\vec{r}_2$, respectively, in the channel $c$. Furthermore, $L$ is the total orbital angular momentum. The indices $i$ and $j$ are to distinguish the radial basis functions. We employ the Gaussian basis functions to describe the radial component for each partial wave:

$$\phi^{k}_{\ell}(\vec{r}) = N_{\ell}^{k} r^\ell e^{-\frac{a_k}{2}r^2} Y_{\ell}(\hat{r}), \quad a_k = a_0\eta_{k}^{-1},$$ \quad (6)

where the details are explained in Ref. [11].

The cross section $\sigma_{\nu n}$ of the photo-disintegration

$$^9\text{Be}(3/2^-) + \gamma \to n + \alpha + \alpha. \quad (7)$$

is calculated in terms of the multi-pole response can be expressed as the following form

$$\sigma^{\gamma}_{E,\lambda}(E_{\gamma}) = \frac{(2\pi)^2(\lambda + 1)}{\lambda[(2\lambda + 1)!]^2} \left(\frac{E_{\gamma}}{\hbar c}\right)^{2\lambda - 1} \frac{dB(E\lambda, E_{\gamma})}{dE_{\gamma}}. \quad (8)$$
In this work, we consider the $E1$ transitions from the $3/2^-$ ground state to the unbound $1/2^+$ state in $^9$Be using the complex-scaled Green’s function [7, 8], which contribute to the $^9$Be($\gamma,n$) $^8$Be cross section as

$$\sigma_{E1}^\gamma(E_\gamma) = \frac{16\pi^3}{9\hbar c} E_\gamma \frac{dB(E1,E_\gamma)}{dE_\gamma}.$$  \hfill (9)

3. Results and Discussion
The $3/2^-$ ground state of $^9$Be is calculated at $-2.16$ MeV from the $\alpha+\alpha+n$ threshold with no three-body potential ($v_{3b}=0$) [12]. The two-cluster potentials $V_{\alpha n}$ and $V_{\alpha\alpha}$ are fixed so as to reproduce the experimental data of the corresponding sub-systems. Here, we introduce a repulsive three-body potential ($v_{3b}=6.57$ MeV, $\mu = 0.1$ fm$^{-2}$) to fit the observed binding energy ($-1.574$ MeV) of $^9$Be. The Hamiltonian Eq. (1) reproduces all the threshold energies of $\alpha+\alpha+n$, $^8$Be+$n$ and $^9$He+$\alpha$ in the $^9$Be nucleus. Using this Hamiltonian, we do not obtain the $1/2^+$ resonance of $^9$Be. This result is the same as that obtained in Refs. [6, 12]. To clarify the resonance nature of the $1/2^+$ state, we carry out three-body calculations of the $1/2^+$ state by adding the three-body potential. With a strong attractive three-body potential with a negative value of $v_{3b}$, we can obtain the $1/2^+$ state as a bound state as shown in Fig. 1. From calculations for various values of $v_{3b}$, we find that the $1/2^+$ state becomes bound with the value of $v_{3b}$ below $-26$ MeV.

![Figure 1](image1.png)  
**Figure 1.** Energies of the $1/2^+$ state measured from the $\alpha+\alpha+n$ threshold, as a function of the strength three-body potential $v_{3b}$.

![Figure 2](image2.png)  
**Figure 2.** The eigenvalue trajectory of the $1/2^+$ state in the complex-energy plane.

Increasing the value of $v_{3b}$ from $-26$ MeV gradually, we search for the $1/2^+$ resonance solutions in the CSM, whose eigenvalue trajectory is shown in Fig. 2. We see a jump from the $\alpha+\alpha+n$ threshold energy to a complex value, the real part of which corresponds to a energy just above the $^8$Be+$n$ threshold. This discontinuity of the trajectory may be understood by considering the properties of solutions in the CSM. In the CSM, resonance poles existing below the 2$\theta$-line of continuum states starting from the lowest threshold of $\alpha+\alpha+n$ cannot be obtained as isolated eigenvalues. Because of the analyticity condition for the present complex scaled Hamiltonian, the $\theta$ value must be smaller than $45^\circ$ [7]. For virtual states (whose energy eigenvalue is located at the negative real axis of the second Riemann surface), the angle $\theta$ has to be larger than $90^\circ$. When a virtual state exists in the $^8$Be+$n$ channel, a jump is expected in the resonance trajectory as discussed in Ref. [13]. The present case is not a two-body system.
but the $\alpha + \alpha + n$ three-body one. It is difficult to trace the jump behavior of the pole solutions in the CSM.

Figure 3 displays an example of the eigenvalue distribution of $^9$Be$(1/2^+)$ for $v_{3b}=-17$ MeV with $\theta = 15^\circ$. Besides the triangle corresponding to the resonance, all solutions for continuum states described by open circles lie on two straight lines starting from positions of the $\alpha + \alpha + n$ three- and $^8$Be$(0^+)+n$ two-body thresholds.

Using the solutions of $1/2^+$ and ground states, we calculate the $E1$ transition strength to investigate the effect of the $1/2^+$ resonance of $^9$Be. In Fig. 4, the results are shown as a function of the energy measured from the $\alpha + \alpha + n$ threshold for several values of the three-body strength $v_{3b}$. We applied attractive ($v_{3b} = -15$ and $-17.1$ MeV) or repulsive ($v_{3b} = 0$ and $10$ MeV) three-body potentials. The $E1$ transition strength for $v_{3b} = -17.1$ MeV rises sharply from the $^8$Be$(0^+)+n$ two-body threshold. The strength reaches a maximum of $0.26 e^2\text{fm}^2/\text{MeV}$

Figure 5. The $E1$ transition strength of $^9$Be$(3/2^-\rightarrow 1/2^+)$ calculated for $v_{3b} = -20, -21, -22, -25$ MeV.

Figure 6. The $^9$Be$(\gamma, n)^8$Be cross section $\sigma_{\gamma n}$ as a function of excitation energy $E$. The experimental data are taken from Refs. [1, 2, 3].
at $E = 0.11$ MeV, and decreases to 0.01 $e^2$fm$^2$/MeV at $E = 0.5$ MeV. As $v_{3b}$ increases, the peaks of the $E1$ transition strength decrease gradually and move into higher energies. In the cases of $v_{3b}$=0 and 10 MeV, the peaks of the $E1$ strength are broad and their energy positions are distant from the $^8$Be($0^+$) + n threshold. It is noted that the $E1$ transition strength is very small in the energy region below the $^8$Be($0^+$) + n two-body threshold.

The $E1$ transition strengths calculated for $v_{3b}$ = $-25 \sim -20$ MeV are shown in Fig. 5. They have strengths in the energy region between $\alpha + \alpha + n$ and the $^8$Be($0^+$) + n thresholds. Each $E1$ transition strength seems to be expressed by the Breit-Wigner form. In the case of $v_{3b} < -25$ MeV, where the $1/2^+$ state becomes a bound state, the $E1$ transition strengths have not a characteristic structure around $\alpha + \alpha + n$ and $^8$Be($0^+$) + n threshold energies.

Comparing the calculated photo-disintegration cross section of the $1/2^+$ state with the recent new experiment [1], we find that $v_{3b} = -17.1$ MeV gives a good agreement as shown in Fig. 6. Using this $v_{3b}$, we obtain $1/2^+$ as a three-body resonance. This result indicates that the experimental cross section can be explained in terms of the $1/2^+$ resonance of $^9$Be.

4. Summary
In this work, we investigate the properties of the $1/2^+$ state of $^9$Be varying values of three-body potential. Without the three-body potential, we cannot obtain the $1/2^+$ resonance of $^9$Be, which is consistent to the results in Refs. [6, 12]. Using the attractive three-body potential, we can obtain the $1/2^+$ resonance, which is responsible to explain the $E1$ transition strength of $^9$Be with an enhancement near the $\alpha + \alpha + n$ threshold energy. The calculated photo-disintegration cross section is in good agreement with new experimental data [1] using the resonance solution of $^9$Be(1/2+) with an appropriate strength of the three-body potential.

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