Supersymmetry and electroweak breaking with large and small extra dimensions

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Abstract

We consider the problem of supersymmetry and electroweak breaking in a 5d theory compactified on an $S^1/Z_2$ orbifold, where the extra dimension may be large or small. We consider the case of a supersymmetry breaking 4d brane located at one of the orbifold fixed points with the Standard Model gauge sector, third family and Higgs fields in the 5d bulk, and the first two families on a parallel 4d matter brane located at the other fixed point. We compute the Kaluza-Klein mass spectrum in this theory using a matrix technique which allows us to interpolate between large and small extra dimensions. We also consider the problem of electroweak symmetry breaking in this theory and localize the Yukawa couplings on the 4d matter brane spatially separated from the brane where supersymmetry is broken. We calculate the 1-loop effective potential using a zeta-function regularization technique, and find that the dominant top and stop contributions are separately finite. Using this result we find consistent electroweak symmetry breaking for a compactification scale $1/R \approx 830$ GeV and a lightest Higgs boson mass $m_h \approx 170$ GeV.
1 Introduction

The Standard Model (SM) of electroweak and strong interactions provides a description of the fundamental particles and forces present in Nature. It has been rigorously tested at high-energy colliders with excellent agreement. However there are many theoretical reasons to believe that the SM is not a complete description of Nature. Recently there have also been experimental signals of “new physics” beyond the SM such as neutrino oscillations \[1\] and discrepancies in \(g_{\mu} - 2\) measurements \[4\].

An outstanding candidate for new physics beyond the SM is supersymmetry (SUSY) which solves many theoretical problems in a natural way and has lead to a SUSY extension of the SM called the Minimal Supersymmetric Standard Model (MSSM). SUSY is also attractive since it is a fundamental symmetry in string theories, which provide the only consistent method of combining gravity with strong and electroweak forces in a single unified theory. Recently there has been considerable interest in low-energy superstring-inspired models with heterotic (and now type I) models leading the way towards fully realistic models. An unresolved problem is to understand the mechanism responsible for SUSY breaking, whereby supersymmetric partners acquire a large mass beyond the reach of current accelerators. This has been an active area of research for many years - some of the leading candidates are gravity \[3\], gauge \[4\], anomaly \[5\] and gaugino mediated SUSY breaking \[6\]. Our previous work considered the embedding of gaugino mediation into a type I string model involving intersecting D-branes \[7\].

Regardless of ones opinion about superstring-inspired models involving D-branes, extra-dimensional “brane world” scenarios have become an active area of research in their own right. They provide a novel environment for investigating familiar problems such as electroweak symmetry breaking (EWSB). This problem has been the focus of much recent work in models involving large extra dimensions \(R \sim TeV^{-1}\) or equivalently low string scales \(M_s\) \[8, 9, 10, 11, 12\]. The models \[8, 9, 10, 11, 12\] share similar features such as starting from a 5d theory then compactifying the extra dimension on an \(S^1/Z_2\) orbifold (with the exception of \[10\] that has \(S^1/Z_2 \times Z'_2\) instead). This leads to fixed points invariant under \(Z_2\)-parity, where 4d D-branes can be lo-
SUSY is broken in the bulk by Scherk-Schwarz (SS) compactification of the fifth dimension. This compactification results in an infinite tower of Kaluza-Klein (KK) excitations for bulk fields, but not for fields localized on either 4d brane. The Yukawa interactions are localised at the orbifold fixed points.

The models differ in the type of SUSY breaking, the choice of orbifold and the location of the MSSM fields, and in the methods used to analyse the spectrum and electroweak symmetry breaking. Table 1 illustrates the important differences between these models.

| Model | [9] | [10] | [11] | [12] |
|-------|-----|-----|-----|-----|
| Bulk fields | G,H | G,H,S,D | G,S,D | G,S |
| Brane fields | S,D | S,D | H | H,D |
| SUSY breaking | SS | SS | SS + SUSY brane | SS |
| Higgs mass | $m_h \leq 110$ GeV | $m_h \sim 128$ GeV | $m_h \leq 150$ GeV |
| Compactification scale | $R^{-1} \sim 1$ TeV | $R^{-1} \sim 350$ GeV | $R^{-1} \sim 1$ TeV | $4 \leq R^{-1} \leq 10 - 15$ TeV |

Table 1: Comparison between the various models showing where the gauge (G), Higgs (H), $SU(2)_L$ singlets (S) and $SU(2)_L$ doublet fields (D) live in the extra dimension. The mechanisms that breaks SUSY are either the Scherk-Schwarz (SS) boundary conditions or a SUSY breaking brane. The models also make EWSB predictions for the lightest Higgs boson mass $m_h$ and the extra dimensional compactification scale $1/R$. In model [11], the “Higgs mass correction” at 1-loop and zero external momenta is calculated. However this is not the physical mass since it corresponds to the second derivative of the effective potential at $\langle H \rangle = 0$.

In this paper we consider a 5d theory compactified on an $S^1/Z_2$ orbifold. SUSY is broken on a 4d “source” brane located at one of the fixed points. The first two MSSM families live on another 4d “matter” brane located at the other fixed point, while the third family, MSSM gauge sector and the Higgs fields live in the extra dimensional bulk and therefore acquire non-trivial soft parameters due to their direct coupling to the SUSY breaking brane. This set-up, which differs from all the other models in Table 1, is motivated by the string-inspired model in [7]. Notice that the presence of the third family in the bulk, particularly the top and stop, is phenomenologically desirable for its important contribution to EWSB where the up-like Higgs mass-squared is driven negative by 1-loop radiative corrections which trigger the spontaneous

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1Notice that ref. [11] also consider a scenario where SUSY is broken on a hidden sector brane.
breakdown of $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$ via the Higgs mechanism. Within this set-up we shall calculate the mass spectra of bulk field KK-resonances using two standard methods, then using a matrix method which enables us to interpolate between large and small extra dimensions and compare the results. We will also consider the problem of EWSB in this extra dimensional model where the top/stop Yukawa couplings are localized on the matter brane. We calculate the 1-loop effective potential using dimensional regularization to perform the momentum integral and zeta-function regularization to sum over an infinite tower of KK-modes [17, 18]. We find that the top contribution is separately finite due to a cancellation between the top and its CP-mirror field. The stop contribution is also separately finite due to a cancellation between stop and its CP-mirror fields and after cancellation gives a constant contribution (independent of the Higgs background field). Therefore we find that EWSB is triggered only by the finite 1-loop top contribution alone which, unlike the stop contribution, depends on the Higgs background field. If we neglect the Higgs interaction with the SUSY breaking brane and take $\tan \beta \rightarrow \infty$, minimization of the effective potential allows us to make a prediction for the compactification scale $1/R \approx 830$ GeV and the lightest Higgs boson mass $m_h \approx 170$ GeV which is heavier than for the models [9, 10, 11, 12] in Table 1.

The layout of the remainder of the paper is as follows. In section 2 we introduce our string-inspired 5d model and discuss the $\mathcal{N} = 2$ SUSY formalism and allocation of MSSM fields. In section 3 we calculate the KK-mode mass spectra in the absence of Yukawa couplings, for models with a large or small extra dimension using three different methods. Section 4 considers the localization of the Yukawa couplings on the matter brane and we revisit the third family mass spectra in the presence of Yukawa couplings. Then in section 5 we calculate the effective potential and discuss EWSB in this model. Section 6 concludes the paper.

2 Our model
2.1 Outline

In this section we introduce our string-inspired model and discuss the location of the MSSM fields that arise from our string construction. We will also review the $\mathcal{N} = 2$ formalism that is commonly used to describe supersymmetry in 5d. The setup shown in Figure 1 is a simplification of our previous model, but with a single extra dimension compactified on an $S^1/Z_2$ orbifold.

Figure 1: The model showing the parallel 3-branes spatially separated along the extra dimension $y$. This extra dimension is compactified on the orbifold $S^1/Z_2$ that leads to two fixed points at $y = 0, \pi R$, where the two D3-branes are located. The first two chiral families live on the “matter” brane at $y = 0$, while SUSY is broken by the F-term of a gauge singlet field $S$ on the source brane at $y = \pi R$. Following an explicit type I string construction, the third family, gauge fields, Higgses and Higgsinos live in the extra dimensional bulk. The bulk is required to be $\mathcal{N} = 2$ supersymmetric which requires the inclusion of MSSM “mirror” fields into the spectrum. Yukawa couplings are localized on the matter brane at $y = 0$. The fields present in the model are summarised in Table 2.

From a 4d viewpoint, $\mathcal{N} = 1$ SUSY in 5d is equivalent to $\mathcal{N} = 2$ in 4d, since the Kaluza-Klein (KK) states can combine in pairs to form $\mathcal{N} = 2$ states. MSSM mirror fields also need to be added to the theory to respect the $\mathcal{N} = 2$ SUSY and form hypermultiplets. The $Z_2$-parity of the orbifold provides a classification of bulk (5d) fields into odd and even classes. Odd fields vanish on the 4d branes at the fixed points, while even parity fields do not vanish and can therefore couple to boundary fields. Only even fields have $k = 0$ KK-modes which can be associated with...
MSSM fields. The mirror states are chosen to be odd and therefore do not appear in the MSSM spectra. Therefore an $\mathcal{N} = 1$ supersymmetric theory in 5d is equivalent to an $\mathcal{N} = 2$ theory in 4d, where 5d bulk fields are equivalent to an infinite tower of 4d KK resonances and $\mathcal{N} = 2$ SUSY is required for the KK modes to form Dirac masses in the bulk. The minimal supersymmetric multiplets in 5d are matter hypermultiplets (chiral and Higgs) and vector supermultiplets (gauge fields) constructed from $\mathcal{N} = 1$ superfields and their “mirror” superfields. The definition of a mirror superfield is discussed in appendix A.2.

The 5d vector supermultiplets $V$ contain a five-dimensional gauge field $A_{M=\mu,5}$, a real scalar $\sigma$ and two Weyl fermions $\lambda_{1,2}$ that all transform in the adjoint representation of the gauge group $\text{SO}(10,19)$. The 5d vector supermultiplet $V$ can be decomposed into an $\mathcal{N} = 1$ vector supermultiplet (containing a gauge boson $A_\mu$ and a gaugino $\lambda_1$) and an $\mathcal{N} = 1$ chiral supermultiplet (containing a scalar $\Sigma \sim \sigma + iA_5$ and a fermion $\lambda_2$).

Similarly each 5d matter hypermultiplet can be decomposed into an $\mathcal{N} = 1$ chiral supermultiplet and its CP-mirror chiral supermultiplet. For example, the up-like Higgs hypermultiplet $H_u$ contains the MSSM Higgs superfield $H_u \sim h_u, \tilde{h}_u$; and its CP-mirror $H^{mc}_u \sim h^{mc}_u, \tilde{h}^{mc}_u$. Similarly for the other matter hypermultiplets. The model has twice the particle content of the MSSM since bulk fields and their mirrors are both needed to form $\mathcal{N} = 2$ invariant states. See appendix A.3 for a discussion of constructing fermion 4-component Dirac spinors from MSSM fields and the CP-conjugates of their mirrors. The location of the fields present in our model are shown in Table 2.

There are two types of field present in the model: boundary fields that are localized on either 4d brane, and bulk fields that feel the extra dimension between the parallel 3-branes. The compactification of this dimension on the $S^1/Z_2$ orbifold leads to a classification of the bulk superfields ($\xi$) into odd ($\xi_{\text{odd}}$) and even ($\xi_{\text{even}}$) states, depending on their transformation under the $Z_2$ reflection $y \leftrightarrow -y$. 
Table 2: The location of the states present in our model. Bulk fields are also classified by their transformation with respect to $Z_2$-parity. Notice that the superfields $Q$, $U$, $D$, $L$, $E$, $N$ implicitly include the scalar and fermion components, e.g. $Q_{iL} \sim \tilde{q}_{iL}$, $q_{iL}$.

\[
\begin{array}{|c|c|c|}
\hline
\text{States} & \text{Location} & \text{$Z_2$-parity} \\
\hline
F_{iL} \sim Q_{iL}, L_{iL} & y = 0 & \\
F_{iR} \sim U_{iR}, D_{iR}, E_{iR}, N_{iR} & y = 0 & \\
F_{3L} \sim Q_{3L}, L_{3L} & \text{bulk} & \text{even} \\
F_{3R} \sim U_{3R}, D_{3R}, E_{3R}, N_{3R} & \text{bulk} & \text{even} \\
F_{mc}^{3L} \sim Q_{3L}, L_{3L} & \text{bulk} & \text{odd} \\
F_{mc}^{3R} \sim U_{3R}, D_{3R}, E_{3R}, N_{3R} & \text{bulk} & \text{odd} \\
V \sim A_\mu, \lambda_1 & \text{bulk} & \text{even} \\
\Sigma \sim \sigma + iA_5, \lambda_2 & \text{bulk} & \text{odd} \\
H_u \sim h_u, \tilde{h}_u & H_d \sim h_d, \tilde{h}_d & \text{bulk} & \text{even} \\
H_u^{mc} \sim h_u^{mc}, \tilde{h}_u^{mc} & H_d^{mc} \sim h_d^{mc}, \tilde{h}_d^{mc} & \text{bulk} & \text{odd} \\
S & y = \pi R \\
\hline
\end{array}
\]

The odd fields have KK expansions involving $\sin \left(\frac{k y}{R}\right)$ or $\sin \left(m_k y\right)$ where $k$ is the KK number and $m_k$ is the $k^{th}$ KK-mode mass\[. They vanish at the fixed points, which means that odd fields do not have zero modes which are associated with MSSM fields. Whereas the even fields have $\cos \left(\frac{k y}{R}\right)$ or $\cos \left(m_k y\right)$ expansions and therefore do not vanish at the orbifold fixed points\[. These $Z_2$-parity transformation properties are important when we come to couple bulk fields to boundary fields at either fixed point, for example in section\[ we localize the third family Yukawa couplings at $y = 0$ using a neat method involving an off-shell formulation of supersymmetry in 5d \[.}

\[2\] Usually the KK modes have masses of the form $m_k = k/R$.

\[3\] We can choose that the familiar MSSM fields are even with respect to the $Z_2$-symmetry, and so have massless zero modes before SUSY breaking.
2.2 Lagrangian

The 5d lagrangian can be split into an $\mathcal{N} = 2$ invariant bulk term \[19\] consisting of 5d bulk fields, and 4d $\mathcal{N} = 1$ invariant brane terms localized on either 3-brane. The 4d brane terms are formed from the boundary fields and the 4d even projections of the bulk fields on to the boundary branes. There is a SUSY breaking term localized on the source brane at $y = \pi R$. The off-shell formalism of $\mathcal{N} = 2$ SUSY in 5d is discussed in ref. \[20\].

\[
\mathcal{L} = \mathcal{L}_5 \left[ \xi (x, y) \right] + \sum_j \delta (y - y_j) \mathcal{L}_j \left[ \xi (x, y_j), \eta_j (x) \right]
\]  

(2)

where $j$ runs over the two branes at the orbifold fixed points, $x$ are coordinates for the 4 non-compact dimensions, $y$ is the coordinate for the extra compact spatial dimension, $\xi$ is a bulk field, and $\eta_j$ is a field localized on the $j^{th}$ brane.

The 5d lagrangian for vector $(A_M, \sigma, \lambda_i)$ and matter hypermultiplets $(\Phi^a_i, \Psi_a)$ given below \[13, 19\] includes the standard kinetic energy terms and supersymmetric Yukawa interaction terms:

\[
\mathcal{L}_5 = Tr \frac{1}{g^2} \left\{ -\frac{1}{2} F^2_{MN} + |D_M \sigma|^2 + i \bar{\lambda}_i \gamma^M D_M \lambda^i - \bar{\lambda}_i \left[ \sigma, \lambda^i \right] \right\} + |D_M \Phi^a_i|^2 + i \bar{\Psi}_a \gamma^M D_M \Psi^a - \left( i \sqrt{2} \bar{\Phi}_a^i \bar{\lambda}_i \Psi^a + h.c. \right) - \bar{\Psi}_a \sigma \Psi^a
\]

\[
-\Phi^\dagger_a \sigma^2 \Phi^a - g^2 \sum_{m,\alpha} \left[ \Phi^\dagger_a (\tau^m)_i \right] T^\alpha \Phi^a_j \right]^2
\]

(3)

where $a$ labels the bulk matter fields (including both Higgs doublets and the third family superfields); $i, j = 1, 2$ are $SU(2)_R$ (R-parity) indices and $M, N = 0 - 3, 5$. $D_M$ is a covariant derivative and $\tau^m$ are $SU(2)$ generators where $m=1,2,3$. $\Phi^a_i(\Psi_a)$ are the scalar (Dirac fermion) components of the Higgs and third family superfields.

Supersymmetry is broken by the F-term of a 4d gauge-singlet field $S$ on the source brane at the fixed point $y = \pi R$ and mediated across the extra dimensional bulk by gauginos, third family scalars and Higgs fields as discussed in ref. \[4\]. The source field couples directly to some of the even parity 5d bulk fields - Higgses, gauginos and third family scalars - to form soft SUSY breaking terms localized at the $y = \pi R$ fixed point\[4\]. The presence of powers of the cutoff scale

\[4\] Notice that if different gauge singlets on the source brane couple to different bulk fields then non-universal


\[ M_* \text{ appear due to dimensional analysis and the effective nature of the theory.} \]

We have third family scalar masses and gaugino masses from the following lagrangian:

\[ \delta \mathcal{L}_{\pi R}^{(1)} = \delta (y - \pi R) \left[ - \int d^4 \theta \frac{c_{F_{3L}}}{M_*^3} F_{3L}^\dagger F_{3L} S - \int d^4 \theta \frac{c_{F_{3R}}}{M_*^3} F_{3R}^\dagger F_{3R} S + \int d^2 \theta \frac{c_w}{16g_5^2 M_*^2} S \operatorname{tr} W^\alpha W_\alpha + h.c. \right] \]

where \( F_{3L} \) and \( F_{3R} \) represent the third family superfields \( Q_{3L}, L_{3L}, U_{3R}, D_{3R}, E_{3R}, N_{3R} \); \( c_{F_{3L}} \) and \( c_{F_{3R}} \) are the coupling to the SUSY breaking field \( S \); \( g_5 \) is the 5d gauge coupling; and \( W_\alpha \) is the 5d gauge field-strength superfield that contains the gaugino as its lowest component.

We can also generate soft Higgs masses, \( B\mu \) and \( \mu \)-terms:

\[ \delta \mathcal{L}_{\pi R}^{(2)} = \delta (y - \pi R) \left[ - \int d^4 \theta \frac{1}{M_*^3} \left( c_{H_u} H_u^\dagger H_u + c_{H_d} H_d^\dagger H_d + c_{B\mu} H_u H_d + h.c. \right) S^\dagger S - \int d^2 \theta \frac{c_w}{16g_5^2 M_*^2} H_u H_d S^\dagger + h.c. \right] \]

Notice that terms with even hypermultiplet fields replaced by their mirror pairs are forbidden by \( Z_2 \)-parity as only even fields couple directly to the 3-brane boundaries at the orbifold fixed points. However, the \( y \)-derivative of an odd field is actually even with respect to \( Z_2 \)-parity, so terms like \( \delta (y - \pi R) \int d^4 \theta \frac{1}{M_*^3} \partial_y Q_{3L}^\dagger \partial_y Q_{3L}^\dagger S \) are allowed by the \( Z_2 \) symmetry, but are heavily suppressed by higher powers of the cutoff scale \( M_* \), and can therefore be neglected.

So far we have not specified where the Yukawa couplings arise in our model. We adopt the standard approach of previous models and localize the Yukawas on either 4d brane since bulk Yukawa couplings explicitly break the \( \mathcal{N} = 2 \) invariance\(^5\). We will postpone this discussion until section 4.

### 3 Mass spectra - in the absence of Yukawa couplings

In this section we will calculate the KK-mode mass spectra of some bulk fields for large or small extra dimensions using two different standard methods. First we will review the results

\(^5\)Notice that it is possible to construct higher dimensional operators in the bulk that respect the weaker constraint of \( SU(2)_R \) (R-parity) invariance, but \( \mathcal{N} = 2 \) SUSY is still explicitly broken in the bulk.
for a small extra dimension where the non-zero KK-modes are effectively decoupled from the theory [6, 7]. Then we will use an equation-of-motion method developed in refs. [11, 21, 22] to find KK mass eigenvalues using a KK expansion in terms of a mass eigenstate ansatz - $\cos(m_k y)$. In section 3.3 we will introduce a variation of a matrix method proposed in ref. [23] that we feel is more powerful since it can (in principle) solve for large or small extra dimensions. We will find that the mass eigenvalues $m_k$ satisfy equations in terms of the SUSY breaking parameters. These relations can often be solved iteratively by considering two different limits of strong and weak SUSY breaking. We will show explicitly that the equation-of-motion method is only applicable in either very strong or very weak SUSY breaking limits.

3.1 Mass spectra - small extra dimensions

In the limit of a small extra dimension, we recover some results from the original $\tilde{g}MSB$ model [6] - but with the third family also in the bulk [7]. Physically the compactification scale is very high, of order the ultraviolet cutoff string scale, and so the KK-mode masses are very heavy and effectively decouple from the theory. Hence we only consider the ground state zero-modes (MSSM fields).

From our previous model [7] we have the following zero-mode mass predictions before EWSB. They are expressed in terms of the cutoff scale $M_*$, the SUSY breaking parameter $F_S$ and a coupling parameter $\epsilon$. The first and second family scalar ($\phi_{1,2}$) soft masses are exponentially suppressed due to their displacement from the SUSY breaking sector. Therefore these masses are negligible at the high scale.

$$m_{\phi_{1,2}} = m_{\psi_{1,2}} = 0$$  \hspace{1cm} (6)

The third family scalars ($\phi_3$) couple directly to the source brane to obtain a soft mass.

$$m_{\phi_3}^2 \sim \frac{1}{\epsilon l_4^4 M_*^2}, \quad m_{\psi_3} = 0$$  \hspace{1cm} (7)

\footnote{In particular, this method breaks down when we include Yukawa couplings and attempt to calculate the mass spectra.}

\footnote{See refs. [6, 7] for a discussion of $\epsilon$, but essentially it represents the coupling strength of the theory. $\epsilon \sim 1$ for strong coupling.}
The MSSM gauginos ($\lambda$) are also coupled directly to the SUSY breaking and acquire soft masses, while the gauge bosons ($A_\mu$) do not.

$$m_\lambda \sim \frac{1}{\ell l_4 M} \frac{F_S}{M_s}, \quad m_{A_\mu} = 0$$

(8)

The Higgs scalars ($h_u, h_d$) acquire soft masses, and their coupling also generates a mixing $B\mu$-term.

$$B\mu, m_{h_u}^2, m_{h_d}^2 \sim \frac{1}{\ell l_4 M^2} \frac{F_S^2}{M^2}$$

(9)

The $\mu$-problem is solved by the Giudice-Masiero mechanism [24] to give an effective soft mass to the higgsinos ($\tilde{h}_u, \tilde{h}_d$).

$$\mu \sim \frac{1}{\ell l_4 M} \frac{F_S}{M_s}$$

(10)

These zero-mode predictions are approximate and arise from a naive dimensional analysis.

In ref. [7] we found that FCNC experimental data and the desire for a phenomenologically valid ratio between gaugino and squark masses leads to an allowed region of the coupling strength parameter $0.01 \leq \epsilon \leq 0.1$.

### 3.2 Mass spectra - equation-of-motion method

We use a dynamical method, developed in refs. [11, 21, 22], to find the mass eigenvalue $m_k$ by proposing a mass eigenstate ansatz for the KK-mode expansion. For example, in the case of weak SUSY breaking, we would expect the KK-mode masses to be slightly perturbed away from the usual free-wave expansions - $\sin(ky/R)$ or $\cos(ky/R)$. However in the strong SUSY breaking limit, we expect the KK masses to be different - $\sin(m_k y)$ or $\cos(m_k y)$. We obtain a set of coupled, simultaneous differential equations in terms of the KK mass, that can usually be solved iteratively. This method works for both scalars and fermions - where the requirement for $\mathcal{N} = 2$ SUSY in the bulk couples odd and even parity fermion fields together. This point is discussed in appendix A.3. Explicit details of the gaugino calculation are given in appendix A.4.
3.2.1 Gauginos - $\lambda_{1,2}$

The even ($\lambda_1$) and odd ($\lambda_2$) parity gauginos are combined together to form a 4-component Dirac spinor. Following Eqn.4, we see that the even-parity gaugino $\lambda_1$ couples directly to the SUSY breaking sector at $y = \pi R$ and acquires a localized soft mass. However, the odd-parity $\mathcal{N} = 2$ superpartner $\lambda_2$ is coupled to the even gaugino through the extra dimensional kinetic term as shown in Eqn.3 and appendix A.3.

We can use the equation-of-motion method - discussed explicitly in appendix A.4 for gauginos - to obtain an expression relating the KK mass to the SUSY breaking F-term vev:

$$\tan [m_{\lambda,k} \pi R] = \frac{c_w F_S}{4 M^2_s}$$

which can be solved iteratively by treating $F_S$ as a small (large) parameter for the weak (strong) SUSY breaking limit.

3.2.2 Third family scalars - $\tilde{t}, \tilde{b}$

Eqn.4 shows that the third family scalars ($\tilde{t}, \tilde{b}$) live in the extra dimensional bulk and couple directly to the SUSY breaking brane to generate soft masses localized at the $y = \pi R$ fixed point. The mirror scalar partners are odd under the $Z_2$-parity transformation and therefore do not couple to the SUSY breaking sector to acquire soft masses. Following the method of section 3.2.1 and refs. [11, 22], we find that the mass eigenvalues (in the absence of Yukawa couplings) satisfy the following relation:\footnote{Notice that with large $\tan \beta$, we are only interested in the top/stop sector, particularly their contribution to the effective potential. Throughout the rest of the paper we will ignore the bottom contributions, except in this subsection where $m_{t,k}$ and $m_{b,k}$ coincide.}

$$m_{\tilde{t},k} \tan [m_{\tilde{t},k} \pi R] = \frac{c_i F_S^2}{2 M^3_s}$$

3.3 Mass spectra - matrix method

In this section we will discuss a matrix method that interpolates between large and small extra dimensions. It is a variant of a technique developed in ref. [23]. We will give explicit examples of using the matrix method to find the stop and gaugino KK-masses in the absence of Yukawa...
couplings. The Higgs scalar mass spectrum is complicated by the presence of the $B\mu$ and $\mu$ mixing parameters. However, in the limit of a small $\mu$-term we find a spectrum similar to the stop KK-spectrum. In the absence of Yukawa couplings the top field has the usual KK mass spectra $m_{t,k} = k/R$ where $(k = 0, 1, 2, \ldots)$.

### 3.3.1 Third family scalars - $\tilde{t}(\tilde{b})$

Now we will calculate the stop(sbottom) mass spectra using the matrix method. Using the following 5d KK-mode expansions:

\[
\tilde{t}_L(x, y) = \frac{1}{\sqrt{2\pi R}} \tilde{t}_{L,0}(x) + \sum_{k=1}^{\infty} \frac{1}{\sqrt{\pi R}} \cos \left( \frac{ky}{R} \right) \tilde{t}_{L,k}(x)
\]

\[
\tilde{t}_{L,0}(x) = \sum_{k=1}^{\infty} \frac{1}{\sqrt{\pi R}} \sin \left( \frac{ky}{R} \right) \tilde{t}_{L,k}(x)
\]

and inserting this into Eqs. 3 and 4, we get the lagrangian mass term for the stop after integrating over the extra dimension coordinate $(y)$,

\[
- \mathcal{L}_4^{mass} = \frac{1}{2R^2} \left[ \sum_{k=1}^{\infty} k^2 \left( \tilde{t}^{*}_{L,k} \tilde{t}_{L,k} + \tilde{t}^{mc*}_{L,k} \tilde{t}^{mc}_{L,k} \right) + \sum_{k,l=1}^{\infty} \frac{2\alpha}{\pi^2} (-1)^{k+l} \tilde{t}^{*}_{L,k} \tilde{t}_{L,l} \right. \\
+ \sum_{k=1}^{\infty} \frac{2\alpha}{\sqrt{2\pi^2}} (-1)^k \tilde{t}^{*}_{L,k} \tilde{t}_{L,0} + \frac{\alpha}{\pi^2} \tilde{t}^{*}_{L,0} \tilde{t}_{L,0} + h.c. + (L \leftrightarrow R) \right]
\]

where the dimensionless parameter is

\[
\alpha = c_t \pi \left( \frac{F_S^2}{M_*^4} \right) M_* R
\]

The strong SUSY breaking limit ($\alpha \to \infty$) corresponds to a large extra dimension since the soft mass $m_{soft}^2 \sim 1/R^2$ and realistic phenomenology requires $1/R \approx \mathcal{O}(TeV)$. Similarly in the weak limit ($\alpha \to 0$), the soft mass $m_0^2 \sim \alpha/R^2$ which requires $1/R \approx M_P$ which corresponds to a small extra dimension.

\[
\alpha \to 0 \quad \Rightarrow \quad R \sim M_P^{-1}
\]

\[
\alpha \to \infty \quad \Rightarrow \quad R \sim (TeV)^{-1}
\]

\footnote{As discussed earlier, we are primarily interested in the top/stop sector since they provide the dominant contributions to the 1-loop effective potential due to the large size of the top/stop Yukawa coupling.}
Notice that the mixing between different KK-modes in the lagrangian of Eqn.15 arise from the localization of the SUSY breaking on the brane at \( y = \pi R \), and therefore the inclusion of a delta function in the lagrangian. The mass matrix is symmetric and may be written in the basis \((\tilde{t}_{L,0} \tilde{t}_{L,1} \tilde{t}_{L,2} \ldots)^T\) as,

\[
\mathcal{M}^2 = \frac{1}{2R^2} \begin{pmatrix}
\frac{\alpha}{\pi^2} & -\frac{2\sqrt{2}\alpha}{\pi^2} & \frac{2\alpha}{\pi^2} & -\frac{2\sqrt{2}\alpha}{\pi^2} & \ldots \\
-\frac{2\sqrt{2}\alpha}{\pi^2} & 1^2 + 2\alpha/\pi^2 & -2\alpha/\pi^2 & 2\alpha/\pi^2 & \ldots \\
\frac{2\alpha}{\pi^2} & -2\alpha/\pi^2 & 2^2 + 2\alpha/\pi^2 & -2\alpha/\pi^2 & \ldots \\
-\frac{2\sqrt{2}\alpha}{\pi^2} & 2\alpha/\pi^2 & -2\alpha/\pi^2 & 3^2 + 2\alpha/\pi^2 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\] (19)

Suppose that \( \lambda^2 \) is the eigenvalue associated with the eigenvector \( Q = (Q_0 \ Q_1 \ Q_2 \ldots)^T \). We obtain the following set of eigenvalues equations \( \mathcal{M}^2 Q = (\lambda^2/(2R^2))Q \) which yield

\[
\alpha Q_0 - \frac{2}{\sqrt{2}} \alpha S_o + \frac{2}{\sqrt{2}} \alpha S_e = \pi^2 \lambda^2 Q_0 \quad n = 0
\] (20)

\[
-\frac{2}{\sqrt{2}} \alpha Q_0 + 2\alpha S_o - 2\alpha S_e = \pi^2 (\lambda^2 - n^2) Q_n \quad n \in \text{odd}
\] (21)

\[
\frac{2}{\sqrt{2}} \alpha Q_0 - 2\alpha S_o + 2\alpha S_e = \pi^2 (\lambda^2 - n^2) Q_n \quad n \in \text{even}
\] (22)

where

\[
S_o = \sum_{n \in \text{odd}} Q_n \quad S_e = \sum_{n \in \text{even}} Q_n
\] (23)

It is straightforward to see that

\[
S_e = \frac{2}{\sqrt{2}} \lambda^2 Q_0 \sum_{n \in \text{even}} \frac{1}{\lambda^2 - n^2}
\] (24)

\[
S_o = \frac{\alpha}{\pi^2} \left(-\frac{2}{\sqrt{2}} Q_0 + 2S_o - 2S_e\right) \sum_{n \in \text{odd}} \frac{1}{\lambda^2 - n^2}
\] (25)

Following some algebra and using Eqn.20 we find the relation:

\[
\pi^2 \lambda^2 = \alpha + 2\alpha \lambda^2 \sum_{n=1}^{\infty} \frac{1}{\lambda^2 - n^2}
\] (26)

where the physical mass eigenvalue is \( m_{i,k} = \lambda/R \) and we use the identity

\[
\sum_{n=1}^{\infty} \frac{1}{\lambda^2 - n^2} = -1 + \pi \lambda \cot(\pi \lambda) / 2\lambda^2
\] (27)

\[\text{10}^{10}\text{The presence of the brane at } y = \pi R \text{ explicitly breaks the translational invariance along the extra dimension, and so the fifth dimensional momentum - and therefore KK number - is no longer conserved.}\]
to obtain the transcendental equation for the even stop (and sbottom) KK-mode masses

\[ m_{\tilde{t},k} \tan \left[ m_{\tilde{t},k} \pi R \right] = \frac{\alpha}{\pi R} \]  

(28)

We can solve Eqn.\(^\text{28}\) iteratively by considering the limits of strong(weak) SUSY breaking, where \(\alpha\) is a large(small) parameter \([11, 22]\). In the strong SUSY breaking limit \(\alpha \gg 1\) (or equivalently \(\sqrt{F_s} \sim M_*\)) and the extra dimension is large \((RM_* \gg 1)\). In this case, Eqn.\(^\text{28}\) yields a spectrum where the low-lying mass eigenvalues are approximately

\[ m_{\tilde{t},k} \approx \left( k + \frac{1}{2} \right) \frac{1}{R} \left( 1 - \frac{1}{\alpha} + O\left( \frac{1}{\alpha^2} \right) \right) \quad (k = 0, 1, 2, \ldots) \]  

(29)

Neglecting terms of order \(1/\alpha\) and higher, we see that each KK-mode mass is shifted up by half a unit relative to the usual unperturbed KK mass \(k/R\). The same conclusion applies when \(\sqrt{F_s} \gg M_*\) and \(M_* R \approx 1\). However it seems quite unnatural to have the SUSY breaking \(F\)-term much larger than the cutoff of the theory \(M_*\). Physically, this implies that the SUSY breaking brane acts as an impenetrable wall that makes the masses insensitive to the precise values of \(c_{\tilde{t}}, F_s^2\) and \(M_*\), with the dependence only arising in small higher-order corrections. This limit can only be phenomenologically viable if the compactification scale \(1/R = O(\text{TeV})\) i.e. if the extra dimension is large.

Compare the stop mass eigenvalues found using the equation-of-motion method (section 3.2.2) and the matrix method (section 3.3.1) in the limit of strong SUSY breaking. We see that the resulting relations for the masses in terms of SUSY breaking parameters - shown in Eqns.\(^\text{12}\) and \(^\text{28}\) respectively - differ by a factor of \(1/2\). This difference arises since the equation-of-motion method uses a KK-mode expansion where the \(y\)-dependence of the input wavefunction is \(\tilde{t}(x,y) \sim \sum_k f_k(y)\tilde{t}_k(x)\), where \(f_k(y) \sim \cos(m_{\tilde{t},k} y)\) and \(m_{\tilde{t},k}\) is the solution of Eqn.\(^\text{12}\). However the \(y\)-dependence of the wavefunction input in to the matrix method is \(m_{\tilde{t},k} = k/R\) which explicitly generates a mass matrix which can be diagonalized to find the physical mass eigenvalues. The difference between these wavefunction profiles introduces some

\(^{11}\)Remember that the odd parity mirror fields remain massless since they do not couple to the even stop fields.
corrections into the mass term \[22\]. For example, consider the strong SUSY breaking limit where

\[
\int_0^{\pi R} f_k(y) f_l(y) \sim \frac{R \pi}{2} \left(1 + \frac{2}{\alpha}\right) \delta_{kl} + \mathcal{O}\left(\frac{1}{\alpha^2}\right)
\]  

(30)

After integrating out the fifth dimension, the mass correction is given by

\[
m^2_{t,k} \sim \left(1 + \frac{1}{2}\right)^2 \frac{1}{R^2} \left[1 - \frac{2}{\alpha} + \mathcal{O}\left(\frac{1}{\alpha^2}\right)\right]
\]

(31)

which agrees with the mass corrections found using the more general matrix method (Eqn. 29).

Figure 2: Comparison of dimensionless stop mass parameter \(M_k = m_{t,k} \pi R\) against SUSY breaking parameter \(\alpha\), where \(m_{t,k}\) is the corresponding KK-mode mass. Only the first two KK-mode masses are shown - \(k = 0\) (lower) and \(k = 1\) (upper). The continuous-line is the solution of Eqn. 28 and the dashed-line arises from Eqn. 12.

However for arbitrary values of \(\alpha\) the relationship between the two methods is more complicated. In general, the equation-of-motion method leads to a non-diagonal propagator in 4d. In figure 2 we have shown the numerical difference between the two methods - considering the first two KK-excitations \(k = 0\) (lower) and \(k = 1\) (upper). As expected the two methods converge in the limits \(\alpha \to \infty\) and \(\alpha \to 0\). However the region \(0.1 \leq \alpha \leq 10\) highlights a clear
discrepancy, which becomes more apparent for higher KK-excitations. We can also conclude from $\alpha \sim 1$ that the extra dimension should have a compactification scale $1/R \sim \mathcal{O}(\text{TeV})$ to give phenomenologically reasonable zero-mode soft masses\footnote{Remember that we associate the bulk field zero-modes with the MSSM fields.}. 

In the weak supersymmetry limit ($\alpha \ll 1$) when $F_s \ll M_*$ and the fifth dimension length is small ($RM_* \sim 1$), we can utilize effective field theory techniques to find the mass of the lightest KK-mode ($k = 0$) by decoupling higher(heavier) and lower(lighter) KK-modes. As a first approximation we suppose that interactions between the heavier modes conserve extra dimensional momentum (hence KK-number).

Therefore, the lagrangian of Eqn.\ref{eq:lagrangian4} can be rewritten as\footnote{This approximation is valid since the mixing between non-zero KK-modes do not affect the mass of the zero mode.}: 

$$ - \mathcal{L}^{\text{mass}}_4 = \frac{\alpha}{2\pi^2 R^2} \tilde{t}_{L,0} \tilde{t}_{L,0} + \frac{1}{2R^2} \sum_{k=1}^{\infty} \left[ k^2 \frac{2\alpha}{\pi^2} \tilde{t}_{L,k} \tilde{t}_{L,k} + \frac{2\alpha}{\sqrt{2\pi^2}} (-1)^k \left( \tilde{t}_{L,k} \tilde{t}_{L,0} + \text{h.c.} \right) \right] $$ \hspace{1cm} (32) 

where the dimensionless parameter $\alpha$ is given in Eqn.\ref{eq:alpha}.

Integrating out the $\tilde{t}_{L,k}$ modes we obtain an expression for the zero-mode mass-squared:

$$ m^2_{k=0} = \frac{\alpha}{\pi^2 R^2} \left( 1 - 2\alpha \sum_{k=1}^{\infty} \frac{1}{k^2 \pi^2 + 2\alpha} \right) $$

$$ = \frac{3}{2\pi^2 R^2} \left( 1 - \frac{\sqrt{2\alpha}}{3} \coth(\sqrt{2\alpha}) \right) $$ \hspace{1cm} (33) 

Expanding around $\alpha = 0$ the zero-mode mass takes the compact form:

$$ m^2_{k=0} = \frac{\alpha}{\pi^2 R^2} \left( 1 - \frac{\alpha}{3} + \mathcal{O}(\alpha^2) \right) $$ \hspace{1cm} (34) 

Notice that the linear term in $\alpha$ comes directly from the lagrangian of Eqn.\ref{eq:lagrangian4} as expected. The heavier modes now only contribute to the zero-mode mass at $\mathcal{O}(\alpha^2)$.

The weak SUSY breaking limit is very interesting because SUSY is broken at the high scale and the compactification scale $(1/R)$ is also very large. This leads to a low-energy effective theory where SUSY is broken softly at a much lower scale\footnote{Recently the same result has been observed in a similar model where SUSY is broken through the Scherk-Schwarz mechanism and the $\alpha$ parameter arises from a twisting of the fields due to the requirement of translational invariance of the extra dimension.} $\sim \alpha/R$. 

\begin{table}[h] 
\centering 
\begin{tabular}{|c|c|}
\hline 
Parameter & Value \\
\hline 
$\alpha$ & $\sim 1$ \\
$\mathcal{O}(\text{TeV})$ & $1/R$ \\
\hline 
\end{tabular} 
\caption{Parameter values for the weak SUSY breaking limit.} 
\end{table}
We can also make a connection with the method of section 3.1 if $M_s R \approx 1$ and $F_S \ll M_s$ ($\alpha \ll 1$). We find that the linear term in $\alpha$ in Eqn.34 gives the following zero-mode mass:

$$m_{k=0}^2 = \frac{c_i F_S^2}{\pi M_s^2}$$  \hspace{1cm} (35)

Suppose that we set $c_i/\pi \approx 1/(\epsilon_4)$. We recover the same relation for the zero-mode mass - Eqn.34 - found using the method for small extra dimensions in section 3.1. In the small extra dimension scenario, the theory cutoff is associated with the Planck scale $M_s \approx 1/R = 10^{19} GeV$. This result implies that for a realistic mass spectra ($m_0 = \mathcal{O}(\text{TeV})$), the SUSY breaking $F$-term must be at an intermediate scale, $\sqrt{F_S} = 10^{11} GeV$.

![Graph](image_url)

Figure 3: Comparison of dimensionless stop mass parameter $M_0 = m_{\tilde{t}_0} \pi R$ against SUSY breaking parameter $\alpha$, where $m_{\tilde{t}_0}$ is the corresponding $k = 0$ KK-mode mass. The continuous-line is the exact zero-mode solution of Eqn.28 and the dashed(dotted) lines are approximate solutions from Eqns.33 and 34 respectively. Notice that the dotted line only includes terms linear in $\alpha$ from Eqn.34.

We find the non-zero KK-mode masses by solving Eqn.28 in the limit $\alpha \ll 1$

$$m_{\tilde{t}_k} = \frac{k}{R} \left(1 + \frac{\alpha}{k^2} + \mathcal{O}(\alpha^2)\right) \hspace{1cm} (k = 1, 2, 3, \ldots)$$  \hspace{1cm} (36)
We see that for large KK-mode number, the spectra is approximately \( m_{i,k} \approx k/R \) which is the result we would expect since in this limit the SUSY breaking brane is completely “transparent” to the heavier KK-modes.

In figure 3 we plot the zero-mode masses found numerically from matrix method relations against the SUSY breaking parameter \( \alpha \). Consider the zero-mode solutions of Eqn.28 (continuous-line), the low energy spectra of Eqn.33 (dashed-line) and the mass found by neglecting terms quadratic in \( \alpha \) from Eqn.34 (dotted-line). The deviation between the three zero-mode masses occurs for \( \alpha \sim 1 \), which implies that inside this range the decoupling theorem no longer works. A physical interpretation for \( \alpha \geq 1 \) is that the infinite tower of KK-modes correlate and behave like a single particle and the decoupling of the lighter modes is non-trivial. In this case, it is necessary to consider the interactions between all of the KK-modes.

### 3.3.2 Gauginos - \( \lambda_{1,2} \)

Following the discussion in section 3.2.1 and appendix A.4, we know that the even parity MSSM gaugino \( \lambda_1 \) and its odd \( \mathcal{N} = 2 \) mirror \( \lambda_2 \) are coupled together in the 5d lagrangian. The lagrangian mass terms are

\[
\mathcal{L}_5^{mass} = -\lambda_2 \partial_5 \lambda_1 + \lambda_1 \partial_5 \lambda_2 - \frac{c_w F_S}{2 M_s^2} \delta(y - \pi R) \lambda_1 \lambda_1 + h.c + \ldots
\]  

(37)

Notice that only the even parity gaugino \( \lambda_1 \) couples to the supersymmetry breaking directly.

After integrating out the fifth dimension \( y \) and using the following KK-mode expansions:

\[
\lambda_1(x, y) = \frac{1}{\sqrt{\pi R}} \lambda_0(x) + \sqrt{\frac{2}{\pi R}} \sum_{k=1}^{\infty} \cos \left( \frac{k y}{R} \right) \lambda_{1,k}(x)
\]

(38)

\[
\lambda_2(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{k=1}^{\infty} \sin \left( \frac{k y}{R} \right) \lambda_{2,k}(x)
\]

(39)

we can rewrite the 4d lagrangian associated with the gaugino mass term:

\[
-\mathcal{L}_4^{mass} = \frac{\beta}{\pi R} \lambda_0 \lambda_0 + \frac{1}{R} \sum_{k,n=1}^{\infty} \left[ \left( \sqrt{2}(-1)^k \frac{\beta}{\pi} \lambda_{1,k} \lambda_{1,k} \right) + 2(-1)^{k+n} \frac{\beta}{\pi} \lambda_{1,k} \lambda_{1,n} - 2 k \lambda_{1,k} \lambda_{2,k} \right] + h.c
\]

(40)

where \( \lambda_0 = \lambda_{1,0} \) and \( \beta = c_w F_S/(2 M_s^2) \).
Repeating the method discussed for stops in section [3.3.1], we can find the eigenvalues and eigenvectors of the mass matrix by choosing the basis \((\lambda_0, \lambda^+_1, \lambda^-_1, \ldots, \lambda^+_k, \lambda^-_k, \ldots)\) where
\[
\lambda^+_k = \frac{\lambda_{1,k} + \lambda_{2,k}}{\sqrt{2}} \quad \lambda^-_k = \frac{\lambda_{1,k} - \lambda_{2,k}}{\sqrt{2}} \quad (k \neq 0)
\] (41)

In this basis, the gaugino mass matrix has the form:
\[
\mathcal{M} = \frac{1}{\pi R} \begin{pmatrix}
\beta & -\beta & -\beta & \beta & \beta & \ldots \\
-\beta & \beta - \pi & \beta & -\beta & -\beta & \ldots \\
-\beta & \beta & \beta + \pi & -\beta & -\beta & \ldots \\
\beta & -\beta & -\beta & \beta & -2\pi & \beta & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\] (42)

Suppose that the eigenvectors of \(\mathcal{M}\) are found to be \((\Lambda_0, \Lambda^+_1, \Lambda^-_1, \ldots, \Lambda^+_k, \Lambda^-_k, \ldots)\) with corresponding eigenvalue \(\lambda\). We obtain the following eigenvalue equations for odd and even KK-modes:
\[
\begin{align*}
\beta (\Lambda_0 - S_0 + S_e) &= \lambda \Lambda_0 & n = 0 \\
\beta (-\Lambda_0 + S_0 - S_e) &= (\lambda + n\pi)\Lambda^+_n & n \in \text{odd} \\
\beta (-\Lambda_0 + S_0 - S_e) &= (\lambda - n\pi)\Lambda^-_n & n \in \text{odd} \\
\beta (\Lambda_0 - S_0 + S_e) &= (\lambda + n\pi)\Lambda^+_n & n \in \text{even} \\
\beta (\Lambda_0 - S_0 + S_e) &= (\lambda - n\pi)\Lambda^-_n & n \in \text{even}
\end{align*}
\] (44) - (45)

where
\[
S_0 = \sum_{n \in \text{odd}} \left(\Lambda^+_n + \Lambda^-_n\right) \quad S_e = \sum_{n \in \text{even}} \left(\Lambda^+_n + \Lambda^-_n\right)
\] (46)

After some algebra and using the identities
\[
\sum_{n \in \text{even}} \frac{1}{\lambda^2 - n^2\pi^2} = \frac{-2 + \lambda \cot(\lambda/2)}{4\lambda^2}
\] (47)
\[
\sum_{n \in \text{odd}} \frac{1}{\lambda^2 - n^2\pi^2} = \frac{-\tan(\lambda/2)}{4\lambda}
\] (48)

we find a relation for the mass eigenvalue \(m_{\lambda,k} = \lambda/(\pi R)\) in terms of the SUSY breaking parameter \(\beta\)
\[
\tan(m_{\lambda,k}\pi R) = \beta = \frac{c_w F_S}{2M^2_*}
\] (49)
which has the following solutions

\[ m_{\lambda,k} = \frac{k}{R} + \frac{1}{\pi R} \arctan(\beta) \quad (k = 0, \pm 1 \pm 2, \ldots) \]  \hspace{1cm} (50)

Solutions exist for both positive and negative \( k \) where \( k > 0 \) (\( k < 0 \)) is associated with the eigenvector \( \Lambda_k^+ \) (\( \Lambda_k^- \)), however the absolute value gives the physical mass. Hence, even though only the even gaugino \( \lambda_1 \) couples directly to the SUSY breaking sector, the two gaugino get mass due to mixing in the 5d kinetic term. Notice that this is completely different to the scalar sector where only the even parity fields acquire masses.

Figure 4: Comparison of dimensionless gaugino mass parameter \( M_0 = m_{\lambda,0} \pi R \) against SUSY breaking parameter \( \beta \), where \( m_{\lambda,0} \) is the corresponding \( k = 0 \) KK-mode mass. The full line is the zero-mode solution of Eqn.49 using the matrix method, and the dashed line arises from the equation-of-motion method Eqn.11.

For example, in the case of weak SUSY breaking \((\beta \ll 1)\), the masses are given by

\[ m_{\lambda,k} = \frac{k}{R} + \frac{\beta}{\pi R} - O(\beta^3) \]  \hspace{1cm} (51)

which is the same mass spectrum that was obtained in \[26\] where supersymmetry was broken.
by an F-term defined in the bulk. In the strong SUSY breaking limit \( \beta \gg 1 \), we have

\[
m_{\lambda,k} = \left( k + \frac{1}{2} \right) \frac{1}{R} - \frac{1}{\beta \pi R} + \mathcal{O} \left( \frac{1}{\beta^3} \right)
\]

(52)

Neglecting the \( \beta \) contribution we get the same spectrum as in [10, 11], where the compactification \( S^1/(Z_2 \times Z'_2) \) was used.

The different methods are related in a similar way to the stop analysis in section 3.3.1. We will not repeat the full discussion here. Figure 4 illustrates the differences in the zero-mode mass found using the equation-of-motion method (dashed-line) and the matrix method (continuous-line). Notice that for the range \( 0.01 \leq \beta \leq 10 \) there is a very large discrepancy between methods, which suggests that the equation-of-motion method breaks down since the propagator is non-diagonal in this region.

4 Mass spectra - including Yukawa couplings

In this section we will recalculate the stop mass spectra in the presence of Yukawa couplings using the matrix method of section 3.3. We adopt the conventional view of refs. [9, 10, 11, 12] and localize the Yukawa couplings on the Yukawa brane at \( y = 0 \). We will only consider the third family Yukawa couplings since the the top/stop sector provides the dominant radiative corrections to the 1-loop effective potential. Notice that it is impossible to simultaneously maintain \( \mathcal{N} = 2 \) SUSY in the bulk and have bulk Yukawa couplings since any higher-dimensional bulk operator that could generate bulk Yukawas explicitly breaks \( \mathcal{N} = 2 \) SUSY.

We can immediately write down the 5d lagrangian term that generates top/stop Yukawa couplings on the Yukawa brane at \( y = 0 \):

\[
- \mathcal{L}_{5}^{Yuk} = \delta(y) \left[ \frac{f_t}{M^3/2_s} \int d^2 \theta \; Q_3 \cdot H_u U^c_3 R + h.c. \right]
\]

(53)

where \( f_t = (2 \pi R M_s)^{3/2} y_t \) and \( f_t(y_t) \) is the 5d(4d) Yukawa coupling. Powers of the cutoff \( M_s \) are required for a lagrangian with the correct mass dimension\(^\text{15}\).

\(^{15}\) The lagrangian is required to have a total mass dimension of 5. Chiral superfields have a mass dimension equal to that of the lowest component, the scalar field. In 5d, the mass dimensions are: \( [Q_3] = [U_3] = [H_u] = 3/2 \). The \( \theta \)-integration and delta function have unit mass dimension, so the Yukawa couplings are dimensionless.
4.1 Stop mass spectrum

We begin by calculating the stop mass spectrum in the presence of Yukawa couplings. Following the method developed in ref. [20] for coupling bulk fields to localized brane fields using an off-shell formulation of SUSY, we can combine Eqns. 3 and 53. We find the following expression for the auxiliary F-field \( F_{Q_3} \):

\[
F_{Q_3}^\dagger = \delta(y) \frac{\tilde{f}_t}{M_*^{3/2}} \tilde{t}_R h_u - \partial_5 \tilde{t}_{L}^{\mu c}
\]

which can then be substituted back into the lagrangian Eq. 53 with the other F-terms. However it is important to notice the presence of a delta-function squared \( \delta^2(y) \) that can be re-expressed as \( \delta^2(y) = \delta(0) \delta(y) \). We recast \( \delta(0) \) as

\[
\delta(0) = \frac{1}{\pi R} \sum_{n=-\infty}^{\infty} 1 = \frac{1}{\pi R} + \frac{2}{\pi R} \sum_{n=1}^{\infty} \frac{1}{2} \left( \frac{1}{2} \right) = \frac{2}{\pi R} D
\]

where \( D \) is an infinite quantity.

We can now integrate over the extra dimension to find the 4d lagrangian for the stop Yukawas in terms of KK modes\(^\text{16}\) from Eq. 53 we get:

\[
- L_{Yuk}^4 = 8y_t^2 D \sum_{k,l=1}^{\infty} \tilde{t}_{L,k}^* \tilde{t}_{L,l} h_{u,0}^* h_{u,0} + 4y_t^2 D \tilde{t}_{L,0}^* \tilde{t}_{L,0} h_{u,0}^* h_{u,0} + 4\sqrt{2} y_t^2 D \sum_{k=1}^{\infty} \tilde{t}_{L,k}^* \tilde{t}_{L,0} h_{u,0}^* h_{u,0} - 2y_t \sum_{k,l=1}^{\infty} \frac{k}{R} \tilde{t}_{R,k}^{\mu c} \tilde{t}_{L,l} h_{u,0} + h.c. + (L \leftrightarrow R)
\]

Combine Eqs. 14 and 56 to rewrite the 4d stop mass terms including both soft SUSY breaking masses and Yukawa contributions.

\[
- L_{mass}^4 = \frac{1}{2R^2} \left[ \sum_{k,l=1}^{\infty} k^2 \left( \tilde{t}_{L,k}^* \tilde{t}_{L,k} + \tilde{t}_{R,k}^{\mu c} \tilde{t}_{R,k}^{\mu c} \right) + \sum_{k,l=1}^{\infty} \left( \frac{2\alpha}{\sqrt{2}} (1)^{k+l} + 16m_t^2 R^2 D \right) \tilde{t}_{L,k}^* \tilde{t}_{L,l} \right.
\]

\[
+ \sum_{k=1}^{\infty} \left( \frac{2\alpha}{\sqrt{2}} (1)^k + 8\sqrt{2} m_t^2 R^2 D \right) \tilde{t}_{L,k}^* \tilde{t}_{L,0} + \left( \frac{\alpha}{\pi^2} (1)^k + 8m_t^2 R^2 D \right) \tilde{t}_{L,0}^* \tilde{t}_{L,0} \left. + \sum_{k,l=1}^{\infty} 4km_t R \left( \tilde{t}_{R,k}^{\mu c} \tilde{t}_{L,l}^* + \tilde{t}_{R,k}^{\mu c} \tilde{t}_{R,l} \right) - \sum_{k=1}^{\infty} 2\sqrt{2} km_t R \left( \tilde{t}_{R,k}^{\mu c} \tilde{t}_{L,0}^* + \tilde{t}_{L,k}^{\mu c} \tilde{t}_{R,0} \right) \right] + (L \leftrightarrow R)
\]

\(^\text{16}\)The Higgs field has the standard KK-mode expansion, but we assume that only the zero-mode acquires a non-zero vev that is identified with the corresponding MSSM vev.
where we have replaced the Higgs field zero-mode by its vev \( H_c \) so that \( m_t = y_t H_c \), and the SUSY breaking parameter is defined by \( \alpha = c_t \pi \left( \frac{F_S^2}{M_s^4} \right) M_s R \) as in Eq.10.

We can now diagonalize the infinite mass matrix using the most convenient basis:

\[
\begin{pmatrix} \tilde{t}_{L,0} & \tilde{t}_{L,k} & \tilde{t}_{R,l} \end{pmatrix}^T \quad k, l = 1, \ldots, \infty
\]

and the square mass matrix is:

\[
M^2 = \frac{1}{2R^2} \begin{pmatrix}
\alpha/\pi^2 + 8m_t^2 R^2 \mathcal{D} & \sqrt{2} \alpha \tilde{I}/\pi^2 + 8\sqrt{2}m_t^2 R^2 \mathcal{D} I & -2\sqrt{2}m_t R (I \cdot M) \\
\sqrt{2} \alpha \tilde{I}^T/\pi^2 + 8\sqrt{2}m_t^2 R^2 \mathcal{D} I^T & M^2 + 2\alpha \tilde{J}/\pi^2 + 16m_t^2 R^2 \mathcal{D} J & -4m_t R (J \cdot M) \\
-2\sqrt{2}m_t R (M \cdot I^T) & -4m_t R (M \cdot J) & M^2
\end{pmatrix}
\]

(59)

where \( I = (1, 1, 1, \ldots), \tilde{I} = (-1, 1, -1, 1, \ldots), M_{kl} = k\delta_{kl} \) and

\[
J = I^T I = \begin{pmatrix} 1 & 1 & 1 & \cdots \\
1 & 1 & 1 & \cdots \\
1 & 1 & 1 & \cdots \\
\vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \tilde{J} = \tilde{I}^T \tilde{I} = \begin{pmatrix} 1 & -1 & 1 & \cdots \\
-1 & 1 & -1 & \cdots \\
1 & -1 & 1 & \cdots \\
\vdots & \vdots & \vdots & \ddots \end{pmatrix}
\]

(60)

Following a similar procedure as section 3.3.1 we can derive an equation for the physical field-dependent mass-eigenvalue \( m_k = \lambda/R \) in terms of the SUSY breaking parameters:

\[
\lambda^2 = \mathcal{P}_+ \pi \lambda \cot(\pi \lambda) + \frac{\pi^2}{4} (\mathcal{P}_+^2 + \mathcal{P}_-^2)
\]

(61)

where \( \mathcal{P}_\pm = \pm \alpha/\pi^2 + 4m_t^2 R^2 \pi \lambda \cot(\pi \lambda) \). This gives after some algebra the following transcendental equation \(^{17}\):

\[
m_{\tilde{t},k} R \left[ \tan(\pi m_{\tilde{t},k} R) - \frac{4m_t^2 R^2 \pi^2}{\tan(\pi m_{\tilde{t},k} R)} \right] = \frac{\alpha}{\pi} \left( 1 + 4m_t^2 R^2 \pi^2 \right)
\]

(62)

Once more we can solve this equation in the limit of large or small SUSY breaking.

### 4.1.1 No SUSY breaking \((\alpha = 0)\)

First consider the SUSY conserving limit \((\alpha = 0)\), Eqn 62 can be rewritten as

\[
\tan^2(\pi m_{\tilde{t},k} R) = 4m_t^2 R^2 \pi^2
\]

(63)

which has solutions

\[
m_{\tilde{t},k} = \frac{k}{R} \pm \frac{1}{\pi R} \arctan(2m_t \pi R)
\]

(64)

\(^{17}\)Note that \( \mathcal{D} \) defined in Eqn 55 has cancelled to give a finite eigenvalue in Eqn 61.
4.1.2 Weak SUSY breaking \((\alpha \ll 1)\)

Now consider the weak SUSY breaking limit (small extra dimension) where Eqn.62 can be solved in powers of the SUSY breaking parameter \(\alpha\). The general result is very complicated, so we will only discuss the zero-mode which has a mass eigenvalue with the following expansion:

\[
m_{\tilde{t},k=0}^2 = \frac{\alpha}{\pi^2 R^2} + \frac{1}{\pi^2 R^2} [\arctan(2m_t \pi R)]^2 + \alpha^2 f(2m_t \pi R) + \alpha^3 g(2m_t \pi R) + \ldots
\]

(65)

where \(f\) and \(g\) are some functions of \((2m_t \pi R)\). We can ignore the terms \(\mathcal{O}(\alpha^2)\) and higher when \(\alpha \ll 1\) and we will see in section 4.2 that we recover a mass eigenvalue with the same field-dependence as the top. However there is a discrepancy between the stop and top zero-mode masses when \(\alpha \approx 1\) and the higher-order terms cannot be ignored necessarily.

4.1.3 Strong SUSY breaking \((\alpha \rightarrow \infty)\)

Finally consider the strong SUSY breaking limit (large extra dimension) where \(\alpha \gg m_t\). Eqn.62 has a meaningful solution when \(m_k\) satisfies the following relations:

\[
m_{i,k} R \tan(\pi m_{i,k} R) = \frac{\alpha}{\pi} \rightarrow \infty
\]

(66)

or \[-m_{i,k} R \cot(\pi m_{i,k} R) = \frac{\alpha}{\pi} \rightarrow \infty\]

(67)

which yields the familiar eigenvalue of section 3.2.2 (if we ignore the small SUSY breaking parameter-dependent correction)

\[
m_{i,k} = \left( k + \frac{1}{2} \right) \frac{1}{R} \quad (k = 0, \pm 1, \pm 2, \ldots)
\]

(68)

and \[m_{\tilde{t},k} = (2k + 1) \frac{1}{R} \quad (k = 0, \pm 1, \pm 2, \ldots)\]

(69)

Notice that the eigenvalue is independent of the Higgs vev which implies that in this limit the stop contribution to the effective potential is absorbed by the cosmological constant. This implies that the strong SUSY breaking limit, in combination with a localized Yukawa coupling brane, washes out the field-dependence.
4.2 Top mass spectrum

Now we repeat the analysis for the top mass spectrum in the presence of the Yukawa couplings. Using the 5d lagrangian in appendix A.3 and Eqn.53, we can write down the lagrangian terms that contribute to the top mass:

\[ L_{\text{mass}}^5 = t_L^\dagger \partial_5 t_L^{mc} - i_{L,L}^{mc} \partial_5 t_L + i_{R,L}^{mc} \partial_5 t_R - \frac{f_t H_c}{M^3/2} \delta(y) \left( i_{L,L}^{mc} + i_{R,L}^{mc} \right) \]

where \( f_t = (2\pi R M_s)^{3/2} y_t \) and \( f_t(y_t) \) is the 5d(4d) Yukawa coupling; \( H_c \) is the classical vev of the up-like Higgs field as before. Using the familiar expansion in terms of 4d KK-modes, we obtain

\[ -L_{\text{mass}}^4 = -\sum_{k=1}^{\infty} \frac{k}{R} \left( t_{L,k}^{\dagger} t_{L,k}^{mc} + t_{R,k}^{\dagger} t_{R,k} + h.c. \right) + 2m_t t_{L,0}^{\dagger} t_{R,0} + 2\sqrt{2}m_t \sum_{k=1}^{\infty} \left( t_{L,0}^{\dagger} t_{R,k} + t_{L,k}^{\dagger} t_{R,0} + h.c. \right) + 4m_t \sum_{k,l=1}^{\infty} \left( t_{L,k}^{\dagger} t_{R,k} + h.c. \right) \]

This can be rewritten in block-diagonal matrix form:

\[ -L_{\text{mass}}^4 = \frac{1}{R} \sum_{k,l} \left( t_{L,0}^{\dagger} t_{L,k}^{\dagger} t_{R,k} t_{R,0} \right) \begin{pmatrix} 2m_t R & 2\sqrt{2}m_t R I & 0 \\ 2\sqrt{2}m_t R I^T & 4m_t R (I^T I) & -M \\ 0 & -M & 0 \end{pmatrix} \begin{pmatrix} t_{R,0}^{\dagger} \\ t_{R,l}^{\dagger} \\ t_{L,l}^{mc} \end{pmatrix} + h.c. \]

where \( I = (1, 1, 1, 1, \ldots) \), \( M_{kl} = k\delta_{kl} \) and \( m_t = y_t H_c \).

We can derive a relation for the KK-mode mass eigenvalue in terms of the classical Higgs vev:

\[ \tan(\pi m_{t,k} R) = 2m_t \pi R \]

which yield the mass eigenvalues

\[ m_{t,k} = \frac{k}{R} + \frac{1}{\pi R} \arctan(2m_t \pi R) \quad (k = 0, \pm 1, \pm 2, \ldots) \]

This result is identical to the stop mass of Eqn.64 as expected since SUSY is not broken and it is also consistent with the result in ref. [10]. Note that the zero mode top mass \( m_{t,k=0} \) is different from the usual 4D top mass.
5 Electroweak Symmetry Breaking

In this section, we will use the mass eigenvalues found in section 4 with Yukawa couplings on the brane, to calculate the 1-loop effective potential. We follow a method developed in ref. [17] and consider the top/stop contributions to a summation over the infinite tower of KK-modes. We will minimize the effective potential to determine whether EWSB is possible in the limit of \( \tan \beta \to \infty \). We also obtain a prediction for the lightest scalar Higgs mass.

We are interested in calculating the dominant radiative corrections to the 1-loop effective potential arising from the top/stop sector\(^{18}\). From Eqns.68 and 74 we have the following field-dependent masses for the top and stop fields:

\[
m_{t,k}(H_c) = \left| \pm \frac{k}{R} + \frac{1}{\pi R} \arctan(2y_t H_c \pi R) \right| \tag{75}
\]

\[
m_{t,k}^2(H_c) = \left( \pm k + \frac{1}{2} \right)^2 \frac{1}{R^2} \tag{76}
\]

The 1-loop effective potential is given by the formula

\[
V_{1-loop} = \frac{1}{2} Tr \sum_{k=\pm\infty} \int \frac{d^4p}{(2\pi)^4} \ln \left[ \frac{R^2 \left( p^2 + m_{t,k}^2(H_c) \right)}{R^2 \left( p^2 + m_{\tilde{t},k}^2(H_c) \right)} \right] = V_t(H_c) + V_{\tilde{t}}(H_c) \tag{77}
\]

where the trace is over all degrees of freedom. Each top/stop (and mirror) field has three colours and four degrees of freedom, i.e. a four-component fermion or two complex scalars. The trace gives an overall factor of \( N_c = 12 \).

Interchanging the summation over KK-modes with the integration over momenta, one finds

\[
V_t = N_c \int \frac{d^4p}{(2\pi)^4} \left( \pi Rp + \ln \left( 1 + e^{-2\pi R p} \right) \right) \tag{78}
\]

\[
V_{\tilde{t}} = -\frac{N_c}{2} \int \frac{d^4p}{(2\pi)^4} \left( 2\pi Rp + \ln \left( 1 - re^{-2\pi R p} \right) + \ln \left( 1 - \frac{1}{r} e^{-2\pi R p} \right) \right) \tag{79}
\]

where \( r = \exp(2i \arctan(2y_t H_c \pi R)) \). When we calculate these integrals using a cutoff we see the “infinite contribution” which comes from the first terms \( \pi Rp \) of Eqns.78 and 79 cancels\(^{18}\). The top/stop fields give the largest contribution to the 1-loop effective potential due to the large size of their Yukawa coupling relative to the other fields.
out in the final expression\(^{17}\). The remaining terms give a finite contribution because they are exponentially suppressed with respect to the momentum.

Alternatively we can first perform the momentum integration using standard dimensional regularization, leading to a result which consists of an infinite pole part proportional to \(1/\epsilon\) plus a finite part. Then we can argue that the pole term gives a zero contribution once the KK sums are properly regulated using zeta-function regularization.

There has recently been some criticism regarding how to calculate the effective potential\(^{27}\). The first part of the criticism states that for each KK-mode, the integral in Eqn.\(^ {77}\) is ultraviolet-divergent, and so it is unclear whether it is possible to interchange the summation with the momentum integral. The second part of the criticism is concerned that KK-modes above the theory cutoff can contribute to the effective potential. The zeta-function regularization approach successfully overcomes the first problem by using dimensional regularization to evaluate the momentum integrals first, and then afterwards performing the infinite sum using zeta-function regularization\(^{17}\). Since in this approach we integrate over all 4d momenta, it is natural that we should also sum over all KK number, since the KK number is related to momentum flowing along the direction of the extra dimension, so in this approach the second criticism does not apply either.\(^{19}\)

Performing the integrals in Eq.\(^ {77}\) using dimensional regularization, the infinite part of the effective potential arising from the top contribution can be expressed as\(^{17}\):

\[
V_t^\infty (H_c) = \frac{N_c}{32\pi^2\epsilon} \left[ (m_{t,k=0})^4 + \sum_{k=1}^{\infty} \left( \frac{k}{R} + m_{t,k=0} \right)^4 + \sum_{k=1}^{\infty} \left( -\frac{k}{R} + m_{t,k=0} \right)^4 \right] \tag{80}
\]

Then, using zeta-function regularization\(^ {17, 18}\) to perform the infinite summation, we find that the three terms in Eqn.\(^ {80}\) cancel each other, and \(V_t^\infty (H_c)\) is exactly zero. This cancellation occurs due to the explicit \(\mathcal{N} = 2\) SUSY in the bulk that would otherwise not occur for models where the higher KK-modes are decoupled or the infinite sum is truncated. For a non-supersymmetric model where only the first two terms in Eqn.\(^ {80}\) arise, the cancellation also would not occur.\(^{27}\)

\(^{19}\)In approaches where both a momentum cut-off and an appropriate symmetry-preserving KK cut-off are used, the heavier KK-mode contributions are found to be exponentially suppressed\(^ {28}\).
Note that in this approach the top contribution to the effective potential is therefore finite and the infinite contribution in Eq.79 does not appear, because in dimensional regularization this is defined to be zero. Similarly the stop contribution also gives a finite contribution and the infinite contribution in Eq.78 also does not appear.

The finite top contribution is:

\[ V_t(H_c) = \frac{3N_c}{64\pi^6 R^4} \sum_{n=1}^{\infty} \frac{\cos[2\pi R n m, k=0(H_c)]}{n^5} \]

which coincides with the finite part of Eqn.79. Similarly, the stop contribution is a finite constant:

\[ V_{\tilde{t}}(H_c) = -\frac{3N_c}{64\pi^6 R^4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \]

\[ = \frac{45N_c\zeta(5)}{1024\pi^6 R^4} \]

again this result is the same obtained in the finite part of Eqn.78.

Notice that in the case of small extra dimensions, the non-zero KK-modes are decoupled from the zero-mode in an effective theory. Hence, when we calculate the infinite part of the effective potential \( V_{t,\tilde{t}}^\infty(H_c) \) we only have the first term \( (k = 0) \) in Eqn.80, and therefore recover the familiar MSSM effective potential.

In general, the tree-level effective potential is given by:

\[ V_{tree} = m^2 H_c^2 + \frac{g^2 + g'^2}{8} H^4 + \Lambda \]

where \( g \) and \( g' \) are the gauge couplings of \( SU(2)_L \) and \( U(1)_Y \) respectively; \( \Lambda \) is the cosmological constant; and \( m^2 \) is a soft mass generated by the coupling of the Higgs field to the SUSY breaking brane

\[ m^2 = \frac{c_{H_u}\alpha}{2c_{\tilde{t}}R^2} \]

where \( \alpha \) is given by Eqn.16. \( c_{H_u}, c_{\tilde{t}} \) are the coupling constants from Eqn.16.

In the strong SUSY breaking limit \( (\alpha \gg 1) \) and \( m^2 \gg 0 \). This is a significant problem in order for EWSB to be triggered via radiative corrections. One possible solution is to introduce a hierarchy between \( c_{H_u} \) and \( c_{\tilde{t}} \), or alternatively assume that the Higgses and third family couple
to different gauge-singlets on the SUSY breaking brane (non-universality). However, we will make a simplification and set $c_{H_u} = 0$ in what follows.

The total effective potential is now

$$V_{\text{eff}}(H_c) = \frac{g^2 + g'^2}{8} H_c^4 + \bar{\Lambda} + \frac{3N_c}{64\pi^6 R^4} \sum_{n=1}^{\infty} \frac{\cos [2\pi R n m_{t,k=0}(H_c)]}{n^5}$$

(85)

where we have absorbed the constant stop contribution into a redefinition of the cosmological constant $\bar{\Lambda}$.

Taking the second derivative at the origin gives:

$$\left. \frac{d^2 V_{\text{eff}}}{dH_c^2} \right|_{H_c=0} = \frac{-3N_c g_t^2 \zeta(3)}{4\pi^4 R^2}$$

(86)

Then, EWSB is triggered by radiative corrections at the compactification scale. The last equation is consistent with the result using Feynman diagrams to evaluate the Higgs scalar two-point function at zero external momenta [11]. However this mass is not identified with the physical Higgs boson mass.

Using the $\overline{\text{MS}}$ top mass $m_{t,0} = 166$ GeV, we can calculate the compactification scale by imposing the following minimization conditions around $v = 175$ GeV:

$$\left. \frac{dV_{\text{eff}}}{dH_c} \right|_{H_c=v} = 0$$

(87)

$$\left. \frac{d^2 V_{\text{eff}}}{dH_c^2} \right|_{H_c=v} = m_h^2$$

(88)

Numerically we find:

$$\frac{1}{R} \approx 830 \text{ GeV}$$

(89)

which is approximately 2.5 times larger than the compactification scale calculated in ref. [10].

The second-derivative of the effective potential at the vacuum ($H_c$) yields a lightest Higgs scalar mass

$$m_h \approx 170 \text{ GeV}$$

(90)

We have seen that in the strong SUSY breaking limit, the stop KK mass spectra is independent of the background Higgs field which gives a constant contribution to the effective potential.
Electroweak symmetry breaking is radiatively triggered by the top sector at 1-loop. However these results cannot be generalized to the Standard Model embedded in extra dimensions since $\mathcal{N} = 2$ SUSY is required for a well-defined theory.

These predictions apply for our simplified toy model where we have set $\tan \beta \rightarrow \infty$ and neglected the coupling of the Higgs fields to the SUSY breaking brane which would generate soft masses. It will be interesting to repeat our analysis in a more general two Higgs doublet model with more realistic values of $\tan \beta$.

6 Conclusions

In conclusion we have considered the problem of supersymmetry and electroweak breaking in a 5d theory compactified on an $S^1/Z_2$ orbifold, where the extra dimension may be large or small. In our model there is a supersymmetry breaking 4d brane located at one of the orbifold fixed points with the Standard Model gauge sector, third family and Higgs fields in the 5d bulk, and the first two families on a parallel 4d matter brane located at the other fixed point. This set-up was motivated by a recent string-inspired analysis [7], and as discussed in that model will lead to a characteristic SUSY mass spectrum where the third family sparticles are heavier than the second family sparticles, which only receive masses through radiative corrections, thereby solving the SUSY flavour changing neutral current problem. We have computed the Kaluza-Klein mass spectrum in this theory using a matrix technique which allows us to interpolate between large and small extra dimensions, ranging from the GUT scale down to a TeV. The matrix method was shown to lead to a more reliable estimate of the mass spectrum especially in the parameter regions $\alpha, \beta \sim 0.1 - 10$ which may be relevant for TeV scale extra dimensions.

Using the reliable matrix method we have also calculated the KK mass spectra including the important third family Yukawa couplings on the matter brane. Remarkably, in the strong SUSY breaking limit, background (Higgs) field dependence of the stop mass is washed out and the only remaining field dependence is in the top mass. Using these results we calculated the 1-loop effective potential, including the dominant top and stop corrections. By performing the
momentum integrations first, using standard dimensional regularization, then regulating the KK sums using zeta-function regularization, we obtained a finite result for the 1-loop effective potential which is not subject to the criticisms that have been made concerning previous approaches. The resulting effective potential has a second derivative at the origin which agrees with a recent Feynman diagram calculation, which gives us some confidence in our effective potential. By minimizing our effective potential in the limit of large tan $\beta$, and making the simple approximations that the tree level Higgs mass and $\mu$ parameter are both zero, enables us to predict the compactification scale $1/R \sim 830$ GeV and the lightest Higgs boson mass $m_h \approx 170$ GeV. Our Higgs mass prediction is significantly higher than other models based on SS SUSY breaking. Of course in the limit of small extra dimensions the model reduces to the MSSM with usual radiative electroweak symmetry breaking and the standard Higgs mass bound applies.
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A Appendix

A.1 5d Dirac matrices

In this appendix we will review the Dirac matrices that appear in the fermion terms of the 5d Lagrangian. We will use the notation that the indices $M, N$ run over $0,1,2,3,5$; and $\mu$ runs over $0,1,2,3$ as usual. We use a timelike metric $\eta_{MN} = \text{diag} (1,-1,-1,-1,-1)$, and take the following basis for the 5d Dirac matrices:

$$\gamma^M = \begin{bmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

where $\sigma^\mu = (1, \sigma)$ and $\overline{\sigma}^\mu = (1, -\sigma)$.

A.2 Mirror fields

$\mathcal{N} = 2$ SUSY requires that the mirror partners of $\mathcal{N} = 1$ MSSM superfields need to be included to construct $\mathcal{N} = 2$ hypermultiplets. We will remove any ambiguity by specifying what we mean by a “mirror” partner. Consider the left-handed quark MSSM doublet, $Q_L$, as an explicit example. Under the MSSM “321” gauge symmetry and Lorentz symmetry, $Q_L$ has the following quantum numbers respectively:

$$Q_L : \begin{pmatrix} 3 & 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \end{pmatrix}$$

Now the mirror $Q^m_L$ has the opposite gauge quantum numbers, but still transforms like a left-handed field:

$$Q^m_L : \begin{pmatrix} 3 & 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \end{pmatrix}$$
However, the CP-operation recovers a right-handed “CP-mirror” with the same gauge quantum numbers:

\[
CP \{Q_L^m\} \mapsto Q_L^{mc} : \begin{pmatrix} 3 \ 2 \ 1/6 \\ 0 \ 1/2 \end{pmatrix}
\]

(94)

When we consider the higgsinos and top quark, it will be useful to form 4-component Dirac spinors from the \( \mathcal{N} = 1 \) MSSM fields and the CP-conjugates of their mirror fields. This leads to mixing in the classical equations of motion as discussed in Appendix A.3. Similar for gauginos we associate the usual \( \mathcal{N} = 1 \) gaugino and its \( \mathcal{N} = 2 \) superpartner together in a 4-component spinor. Note that use of the term “mirror” will now include CP-conjugation implicitly.

### A.3 \( \mathcal{N} = 2 \) spinors and 5d kinetic terms

As discussed in section 2, it is convenient to work in terms of \( \mathcal{N} = 2 \) hypermultiplets. These are formed from conventional \( \mathcal{N} = 1 \) supermultiplets by adding the CP-conjugates of their mirror superfields with opposite quantum numbers.

We will consider an explicit example of the third family that lives in the extra-dimensional bulk. The third family scalars and their “mirrors” are uncoupled, and so only the even parity (MSSM) scalars couple directly to the SUSY breaking sector to acquire a soft mass. This is shown in section 3. However, the form of the 5d Dirac matrices causes mixing between fermion fields of even and odd \( Z_2 \)-parity.

Consider the top fields charged with respect to the (unbroken) \( SU(2)_L \) gauge group in the MSSM - the left-handed top is contained within the left-handed quark doublet \( q_{3L} \) along with the left-handed bottom quark. The right-handed top is a singlet with respect to \( SU(2)_L \), and so a Dirac mass term \( \sim m_t \left( t_L t_R^\dagger + t_R^\dagger t_L \right) \) is forbidden by gauge invariance.

In the \( \mathcal{N} = 2 \) generalisation, we must include additional mirror fields to construct the full 5d hypermultiplet. The left-handed top \( t_L \) and the CP-conjugate of its mirror, \( t_L^{mc} \), can be combined into a 4-component Dirac spinor, since the charge-conjugated left-handed mirror is equivalent to a right-handed fermion. Similarly for the right-handed top \( t_R \) and its mirror \( t_R^{mc} \). Notice that

\[\text{A Dirac mass may be formed after the } SU(2)_L \text{ gauge symmetry is broken, and this is what happens in the (MS)SM through the Higgs mechanism and EWSB.}\]
SU(2)_L singlets and doublets appear in different Dirac spinors, and therefore do not break the gauge symmetry. We have two 4-component Dirac spinors for the top sector, where the index labels the handedness of the MSSM fermion:

\[ T_L = \begin{pmatrix} t_L \\ t_{mc}^L \end{pmatrix}, \quad T_R = \begin{pmatrix} t^m_{mc} \\ t_R \end{pmatrix} \tag{95} \]

and similarly \( \bar{T} = T^\dagger \gamma^0 \)

\[ \bar{T}_L = \begin{pmatrix} t_{mc}^L \dagger \\ t_L^\dagger \end{pmatrix}, \quad \bar{T}_R = \begin{pmatrix} t_R^\dagger \\ t^m_{mc} \end{pmatrix} \tag{96} \]

We can now construct the kinetic terms in the 5d lagrangian in the absence of interactions.

\[
L_{KE} = i\bar{T}_L \gamma^M \partial_M T_L + i\bar{T}_R \gamma^M \partial_M T_R \\
= i\bar{t}^L \sigma^\mu \partial_\mu t_L + i\bar{t}^R \sigma^\mu \partial_\mu t_R + \bar{t}_{mc}^L \sigma^\mu \partial_\mu t_{mc}^L + \bar{t}_{mc}^R \sigma^\mu \partial_\mu t_{mc}^R \tag{97} \\
- t_{mc}^L \partial_5 t_L + \bar{t}_{mc}^R \partial_5 t_{mc}^R - \bar{t}_{mc}^R \partial_5 t_{mc}^R
\]

Notice that the 4d kinetic terms do not mix fields, while \( \gamma^5 \) leads to mixing between fields and their mirror states. This leads to non-trivial classical equations of motion.

### A.4 Equation-of-motion method details - gauginos

In this appendix we will show the details of the calculation for the gaugino mass spectrum using the equation-of-motion method (section 3.2.1). We derive Eqn.11 which is a relation for the KK-mode mass in terms of the SUSY breaking parameters. From Eq.4, we see that the even-parity gaugino \( \lambda_1 \) couples directly to the SUSY breaking sector at \( y = \pi R \) leading to a localized soft gaugino mass. However, the odd-parity \( N = 2 \) superpartner \( \lambda_2 \) is coupled to the even gaugino through the kinetic term as shown in Appendix A.3.

The gaugino lagrangian includes kinetic terms and the coupling of the even-parity gaugino to the SUSY breaking sector, where the even and odd gauginos form a Dirac spinor \( \Gamma = (\lambda_1 \lambda_2)^T \)

\[
L_\lambda = i\bar{\Gamma} \gamma^M \partial_M \Gamma + \delta (y - \pi R) \int d^2 \theta \frac{c_w}{16g_5^2 M_*^2} S (tr W^a W_\alpha + h.c.) \tag{98} 
\]
which leads to the classical equations of motion that couple the two gauginos:

\[ i\bar{\sigma}^{\mu}\partial_{\mu}\lambda_{2}(x, y) + \partial_{5}\lambda_{1}(x, y) = 0 \] (99)

\[ i\bar{\sigma}^{\mu}\partial_{\mu}\lambda_{1}(x, y) - \partial_{5}\lambda_{2}(x, y) - \delta(y - \pi R)\frac{c_{W}F_{S}}{2M_{s}^{2}}\lambda_{1}(x, y) = 0 \] (100)

Following refs. [11, 22], we can solve these equation using the following solutions:

\[ \lambda_{i}(x, y) = \sum_{k}\eta_{i,k}(x)g_{i,k}(y) \] (101)

where \( g_{1,k}(g_{2,k}) \) are even (odd) with respect to the \( Z_{2} \)-parity transformation \( y \rightarrow -y \). We integrate Eqn.100 in a region \((\pi R - \epsilon, \pi R + \epsilon)\) where \( \epsilon \rightarrow 0 \) to obtain boundary conditions at \( y = \pi R \) which must be satisfied by each KK mode:

\[ \eta_{2,k}(x) = \frac{c_{W}F_{S}}{4M_{s}^{2}}\frac{g_{1,k}(\pi R)}{g_{2,k}(\pi R - \epsilon)}\eta_{1,k}(x) \] (102)

Using the 2-component Weyl form of the Dirac equation, \( i\bar{\sigma}^{\mu}\partial_{\mu}\eta_{1,k} = m_{\lambda,k}\eta_{1,k} \), we can rewrite Eqn.100 at \( y \neq \pi R \) (so neglecting the delta function)

\[ m_{\lambda,k}\eta_{1,k}(x)g_{1,k}(y) - \frac{c_{W}F_{S}}{4M_{s}^{2}}\frac{g_{1,k}(\pi R)}{g_{2,k}(\pi R)}\eta_{1,k}(x)\partial_{5}g_{2,k}(y) = 0 \] (103)

which give solutions

\[ g_{1,k}(y) \sim \cos[m_{\lambda,k}y], \hspace{1cm} g_{2,k}(y) \sim \sin[m_{\lambda,k}y] \] (104)

The final result gives a relation for the gaugino KK-mode masses \( m_{\lambda,k} \) in terms of the SUSY breaking F-term

\[ \tan[m_{\lambda,k}\pi R] = \frac{c_{W}F_{S}}{4M_{s}^{2}} \] (105)

References

[1] For a review and further references, see for example S.F.King, hep-ph/0105261, J. Phys. G27 (2001) 2149.

\[ \text{Notice that this } y\text{-dependence is consistent with the identification of } \lambda_{1}(\lambda_{2}) \text{ with the even (odd) parity gaugino states, since we only want } \lambda_{1} \text{ to couple to the boundary fields and have a non-vanishing zero mode which we can associate with the MSSM gaugino.} \]
[2] Muon g-2 Collaboration, hep-ex/0102017, *Phys. Rev. Lett.* **86** 2227 (2001).

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