A Novel Method for State of Charge Estimation of Lithium-ion Batteries using Embedded Cubature Kalman Filter

J L Xu, W Xu, F G Huang*, W Xia and B L Liu

School of Mechanical and Electrical Engineering, Wuhan University of Technology of China, Wuhan, Hubei 430070, China
*E-mail: hawkfly@whut.edu.cn

Abstract. The accurate estimation of state of charge (SOC) of Lithium-ion battery is one of the most crucial issues for battery management system (BMS). This paper proposes a novel method for SOC estimation using the embedded cubature Kalman filter (ECKF). ECKF computes the cubature points with a weight depends on an optional parameter, which differs from the standard cubature Kalman filter (CKF). An online model identification method for a second-order RC networks equivalent circuit model using the forgetting factor recursive least squares (FRLS) algorithm has been presented. The Dynamic cycles are used to assess the superiority in improving the accuracy and stability of proposed method compared with the widely used algorithms. Experimental results show that, with 20% initial SOC error, the maximum estimation error is within 1%, which indicates that the proposed ECKF method has higher estimation accuracy and robustness for SOC estimation.

1. Introduction
With the depletion of conventional fossil energy and the concerns for environmental pollution, electric vehicles (EVs) have the best development opportunity in recent years. Due to high energy density, Lithium-ion batteries functioned as the energy system are widely applied in EVs. The performance of Lithium-ion batteries affects the safety, reliability and efficiency of the EVs [1]. Therefore, battery management system (BMS) plays a great role in providing the state of battery, such as the state of charge (SOC) which describes the remaining energy of battery [2]. Accurate SOC estimation can optimize the energy and battery balance management. However, SOC estimation faces a lot of challenges. Not only because SOC cannot be measured directly [3].

Different kinds of approaches to SOC estimation have been proposed, generally, they are categorized into four main types, namely, conventional method, Non-linear observer, learning algorithm and adaptive filter algorithm [4]. Conventional method is applied extensively due to its simplicity and convenience. Coulomb counting and open circuit voltage (OCV) methods are the typical representatives. Non-linear observer methods, including Sliding Mode Observer (SMO) and Proportional-integral Observer (PIO), have been applied to online SOC estimation. With the development of artificial intelligence, various Learning algorithms for SOC estimation have been proposed. The Learning algorithm includes Neural Network (NN) and Support Vector Machine (SVM). Adaptive filter algorithms based on model are being widely applied in practical application due to its close-loop feedback for correcting the estimation, for instance, extended Kalman filter. Nevertheless, the EKF algorithm uses a first-order Taylor polynomial to approximate the nonlinear system, which may reduce the accuracy even lead divergence. After, the unscented Kalman filtering (UKF), cubature Kalman filter (CKF) have been developed for pursuing higher accurate SOC estimation. CKF updates the mean and variance of state-vector by \(2n\) cubature points, which makes a
great achievement. However, in standard CKF, the defect is that CKF collects 2n equal-weight cubature points, which may result in inconvenience of calculation if the cubature points exceeds the defined interval, or the cubature points is plural. It is not conducive to improve accuracy and convergence time [5].

To address this problem and improve the SOC estimation accuracy and robustness, this paper proposes a novel SOC estimation algorithm using embedded cubature Kalman filter (ECKF). ECKF is established based on embedded cubature rules. The weight cubature points are related with a free parameter. It is show that a proper value of the parameter can result a high accuracy.

2. Embedded cubature Kalman filter

2.1. Embedded cubature rules

Firstly, we depict the embedded cubature rules before constructing the ECKF algorithm. Summarizing embedded cubature rules as follows according to prior knowledge [6]:

Considering the infinite integral over in the field of real number \( R \):

\[
I(f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} w(x_1) \cdots w(x_n) f(x_1, \ldots, x_n) dx_1 \cdots dx_n
\]

(1)

Where, \( f(x_i) \) is the integral function, \( \omega(x_i) \geq 0 \) is the cubature weight of the state-variable \( x_i \) (i = 1, \ldots, n). Let a symmetric formula be the integral \( I(f) \) and \( R^{(m,n)} \) be a element of \( R \) of \( 2m+1 \) degree. We get:

\[
R^{(m,n)}(f) = I(f)
\]

(2)

Constructing a set of all distinct \( n \)-partitions \( P \) consisting of a series integers \( 0,1,2,\cdots,m \), and for arbitrary \( i,j, P^{(m,n)} \neq P^{(m,n)} \), so that:

\[
P^{(m,n)} = \{(p_1, \cdots, p_n) \mid 0 \leq p_1 \leq \cdots \leq p_m \leq m, \ p \leq m \}
\]

(3)

Here, \(|p| = \sum_p \). Defines a sequence of non-negative generators: \( u_0, u_1, \ldots, u_m \), and \( u_0 = 0 \). Let \( u = (u_{p_1}, \cdots, u_{p_m}) \), and let \( f(u) \) as a symmetric rule defined by:

\[
f(u) = \sum_{q \in \Pi_p} \sum_{x} f(s_1u_{q_1}, s_2u_{q_2}, \ldots, s_mu_m)
\]

(4)

Where, \( \Pi_p \) represents all distinct permutations of \( P \) and \( s_i = \pm 1 \). Define \( \delta = (\delta, \cdots, \delta) \) as the known numbers, the \( R^{(m,n)} \) can be defined as:

\[
R^{(m,n)}(f) = \sum_{p \in P^{(m,n)}} W^{(m,n)} f(u) + W^{(m,n)} f(|\delta|)
\]

(5)

Here, \( \delta \) is a free parameter, the reason why \( R \) is called an embedded family is that \( R^{(m,n)} \) uses whole function values used by \( R^{(m-1,n)} \).

2.2. Third-degree ECKF

According the embedded cubature rule, the third-degree ECKF can be formulated as [7]:

\[
l(f) = \int_{x} f(x) \delta(x, \theta, I) \delta \left(1 - \frac{1}{2\delta^2} \right) f(\xi) = \sum_{\delta} \frac{1}{2\delta^2} f(\xi) d\delta
\]

(6)

Where, \( \theta \) and \( I \) denote the zero and identity matrices. The free parameter \( \delta \) determines the embedded cubature formulation. The set of embedded cubature points \( \xi \) and weights \( \omega_i \) are:
Generally, Kalman filter consists of state vector and measurement vector defined as follows:

\[
\begin{align*}
    \hat{x}_k & = f(x_{k-1}, u_{k-1}) + \omega_{k-1} \\
    y_k & = h(x_k, u_k) + v_k
\end{align*}
\]

where \(x_k\) is the state vector, \(y_k\) is the measurement vector, \(u_k\) is the input vector, \(\omega_{k-1}\) is the Gaussian process noise with zero mean, and covariance \(Q\), \(v_k\) is the Gaussian measurement noise with zero mean, and covariance \(R\), the error covariance matrix is \(P\).

The ECKF algorithm for SOC estimation can be depicted as Table 1 [8]:

| Step 1: Initialize values. | Set the initial noises value and state vector. |
|----------------------------|------------------------------------------------|
| \(Q_0\), \(R_0\), \(\hat{x}_0 = E(x_0)\), \(P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]\) |

| Step 2: Time update | Compute embedded cubature points: \(X_{e,k-1|k-1} = S_{k-1|k-1} \tilde{\xi}_k + \hat{x}_{k-1|k-1}\) |
|----------------------|------------------------------------------------------------------|
| Where, \(S_{k-1|k-1} = \text{chol}(P_{k-1})\). | Propagate the embedded cubature points: \(x_{e,k|k-1} = f(X_{e,k-1|k-1}, u_{k-1}) \hat{x}_{k|k-1} = \frac{1}{2^n + 1} \sum_{i=1}^{2^n+1} X_{e,i|k-1}\) |
| Compute the propagated state covariance: \(R_k = \frac{1}{2^n + 1} \sum_{i=1}^{2^n+1} (x_{e,i|k-1} - \hat{x}_{k|k-1})(x_{e,i|k-1} - \hat{x}_{k|k-1})^T + Q_{k-1}\) |

| Step 3: Measurement update | Recompute the embedded cubature points: \(X_{e,i|k-1} = S_{k-1|k-1} \tilde{\xi}_k + \hat{x}_{i|k-1}\) |
|---------------------------|------------------------------------------------------------------|
| Where, \(S_{i|k-1} = \text{chol}(P_{i})\). | Propagate embedded cubature points: \(y_{e,i|k-1} = h(X_{e,i|k-1}, u_k) \hat{y}_{i|k-1} = \frac{1}{2^n + 1} \sum_{i=1}^{2^n+1} Y_{e,i|k-1}\) |
| Compute the propagated measurement covariance: \(R'_k = \frac{1}{2^n + 1} \sum_{i=1}^{2^n+1} (y_{e,i|k-1} - \hat{y}_{i|k-1})(y_{e,i|k-1} - \hat{y}_{i|k-1})^T + R_{k-1}\) |
| Calculate the filter covariance matrix: \(P_k = \frac{1}{2^n + 1} \sum_{i=1}^{2^n+1} (x_{e,i|k-1} - \hat{x}_{k|k-1})(y_{e,i|k-1} - \hat{y}_{i|k-1})^T + R_{k-1}\) |
| Obtain the gain matrix: \(K_k = P_k^y (P_k^y)^{-1}\) |
| Update the state value: \(\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_{k|k-1})\) |
| Update the error covariance matrix: \(P_k = P_k - K_k P_k^y K_k^T\) |
3. Battery model and experiment

3.1. Battery model
Comparing with the electrochemical and impedance models of lithium-ion batteries, the equivalent circuit model (ECM) has been widely applied in SOC estimation due to its easiness to implement and enough accuracy. In this paper, the seconds-order RC networks equivalent circuit model is proposed. The schematic of the model is presented in figure 1. The model consists of an internal resistance $R_0$, second RC networks $R_aC_a$, $R_pC_p$ and a power source $U_{OC}$ (which is the open circuit voltage (OCV)). The dynamic behavior of the model is depicted by equation (10).

\[
\begin{align*}
U_t &= U_{oc} - I_0R_0 + U_p \\
U_p &= I_a/C_p - U_p/R_pC_p \\
U_a &= I_a/C_a - U_p/R_pC_a
\end{align*}
\]

Where, $U_t$ denotes the terminal voltage. Let equation (10) transform into a discrete form:

\[
\begin{align*}
U_{t,k} &= U_{oc,k} + I_{0,k}R_{0,k} + U_{p,k} \\
U_{p,k} &= I_{a,k}/C_p - U_{p,k}/R_pC_p \\
U_{a,k} &= I_{a,k}/C_a - U_{p,k}/R_pC_a
\end{align*}
\]

Where, $\tau_i$ is the time constant, which equals to $\tau_i = RC_i$, $\Delta t$ denotes the sampling time.
The most common way to define the SOC is the ratio of the available capacity to the nominal capacity [9], which is shown in equation (12).

\[
SOC_i = SOC_{i-1} - \int_{t_{i-1}}^{t_i} \frac{I_0\eta dt}{C_n}
\]

Where, $SOC_i$ is the SOC at time $t$, $C_n$ is the nominal capacity, $\eta$ is the columbic efficiency ($\eta = 0.99$). Use $z$ as the abbreviation of SOC, $k$ as the kth sample time and transform equation (12) to discrete form:

\[
z_k = z_{k-1} - I_0\eta / C_n
\]

Now, we choose the $z$ and $U_p$ as the state vectors, $U_t$ as the measurement vector. The current $I_0$ is the input variable. We get:

\[
\begin{bmatrix}
z_k \\
U_{p,k} \\
U_{a,k}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & \exp(-\Delta t / \tau_p) & 0 \\
0 & 0 & \exp(-\Delta t / \tau_a)
\end{bmatrix}
\begin{bmatrix}
z_{k-1} \\
U_{p,k-1} \\
U_{a,k-1}
\end{bmatrix}
+ \begin{bmatrix}
1/C_n \\
R_p(1-\exp(-\Delta t / \tau_p)) \\
R_p(1-\exp(-\Delta t / \tau_a))
\end{bmatrix}
I_{0,k-1}
+ \begin{bmatrix}
\omega_{0,k-1} \\
\omega_{2,k-1} \\
\omega_{3,k-1}
\end{bmatrix}
\]

\[
U_{t,k} = U_{oc,k} + U_{a,k} + U_{p,k} + R_0I_{0,k} + \nu_k
\]

Where, $\omega_i (i = 1, 2, 3)$ is the process noise. $\nu$ represents the measurement noise.
3.2. Experiments

To prove the proposed algorithm, a test bench is constructed to obtain experimental data such as current and voltage, OCV. It includes a computer for experimental control, date storage and analysis, a battery tester (NEWARE BTS4008), and lithium-ion cells. The nominal capacity of the lithium-ion cells is 2.4 Ah, nominal voltage is 3.6 V, charge and discharge cut-off voltage are 4.2 V and 2.75 V, respectively. Some experiments of lithium-ion batteries need to be tested for identifying parameters of the battery model and executing the Kalman filter system. Figure 2 shows the test bench and the test procedure.

![Test Bench Diagram](image)

**Figure 2.** Schematic of the battery test bench and procedure: (a) test bench (b) procedure.

![Measuring OCV](image)

**Figure 3.** The measured HPPC voltage and fitting OCV curve.

As shown in Fig. 2 (b), The HPPC test is used to obtain OCV and identify the parameters online [10]. Dynamic stress test (DST) and Federal Urban Driving Schedule (FUDS) are used to estimate SOC for proving the algorithm ability.

3.3. OCV and parameters

In this paper, the polyfit method is employed to fit the function of OCV using the formula \(y=ax^6+bx^5+cx^4+dx^3+ex^2+fx+g\). Table 2 shows the results of those coefficients. Figure 3 describes the fitting curve and measured open circuit voltage values, which indicates that the fitting curve trace of the measured values quite well.

| coefficients | a       | b             | c         | d       | e       | f       | g       |
|--------------|---------|---------------|-----------|---------|---------|---------|---------|
| Values       | 3.95e-12| -7.28e-10     | -1.19e-8  | 1.06e-5 | -7.52e-4| 0.023   | 3.33    |

Table 2. The parameters of the fitting curve.
Table 3. Process of the FRLS algorithm.

| Step 1: Initialize the initial state values, gain vector and covariance values |
| Step 2: Calculate the estimation error: $y(k)$ |
| Step 3: Compute the estimation error: $e(k) = y(k) - \varphi(k)\hat{\theta}(k-1)$ |
| Step 4: Update the gain vector using: $G(k) = \frac{P(k)\varphi(k)}{\lambda + \varphi'_{\infty}(k)P(k-1)\varphi(k)}$ |
| Step 5: Update the covariance matrix: $P(k) = \frac{1}{\lambda}(P(k-1) - G(k)\varphi_{\infty}(k)P(k-1))$ |
| Step 6: Update the estimation value: $\hat{\theta}(k) = \hat{\theta}(k-1) - G(k)e(k)$ |

In this work, the widely used online method—forgetting factor recursive least squares (FRLS) algorithm [11] is used to identify the parameters of the battery model online. Table 3 depicts the online identification process. Where, $e(k)$ is the estimation error of the battery terminal voltage, $G(k)$ is the gain vector, $P(k)$ is the covariance matrix, $\varphi(k)$ is the state variables vector, and $\theta(k)$ is the parameters vector. $y(k)$ is the measured vector. According to the proposed battery model of this paper, they are expressed by followed.

$$\varphi(k) = \left[ (U_{i-1} - U_{i-2} - U_{i-3} + U_{i-4} - U_{i-5}), (I_0(k), I_0(k-1), I_0(k-2)) \right]$$

$$\theta(k) = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]$$

$$y(k) = U_i(k) - U_{i-1}(k)$$

(16)

4. Results and discussion

This section represents two instances to prove the performance of the ECKF algorithm for estimating SOC. One case is the accuracy demonstration under DST cycle. Another case is the robustness reveal with an initial SOC error under FUDS cycle. The proposed algorithm is comparison with EKF, UKF, CKF, ECKF under both two cases.

![Figure 4. SOC estimation results for DST test: (a) SOC estimation; (b) Estimation error.](image)

4.1. Estimation accuracy under DST cycle

Figure 4 (a) presents the results of SOC estimation under DST cycle using those algorithms under the same initial condition. The black line denotes the reference SOC, which is calculated by the ampere hour counting through the tester BTS4008. Figure 4 (b) shows the errors of estimated SOC. It is clear that the ECKF algorithm has better accuracy than others and tracks the true SOC very well in all test time. The results of EKF-based has the larger fluctuation and the results of UKF-based has a tendency to divergence as the test goes on. The CKF-based results has a better accuracy than EKF and UKF-based. In a word, the ECKF algorithm has the best performance for SOC estimation compared with EKF, UKF and CKF algorithms. This indicates that the embedded cubature rule with an optional value $\delta$ can make the standard CKF more precision. The maximum error based on the proposed algorithm is less than 0.7%.
4.2. Robustness demonstration with initial error under FUDS cycle

Robustness is another factor in SOC estimation algorithm. In this work, we evaluate the robustness through the time converged to 5% error with 20% initial SOC error as an example. Figure 5 compares the estimation results with the four algorithms. The results of the fastest to the slowest convergence are ECKF, CKF, UKF and EKF, respectively. It can be seen that the proposed algorithm presents the fastest convergence speed, which is almost less 40% than standard CKF algorithm (where, the ECKF is about 144 seconds, CKF is about 224 seconds). The proposed algorithm based on embedded cubature rule exhibits high effectiveness in both accuracy and robustness from above results.

5. Conclusion

In this work, a novel method for SOC estimation based on the ECKF algorithm has been proposed. The proposed algorithm is constructed using the merit of embedded cubature rule based on standard cubature Kalman filter. In order to reduce the estimated SOC error by offline parameter identification, the FRBS algorithm is selected to update the battery parameters online. SOC estimation methods based on EKF, UKF, CKF, ECKF are established to demonstrate the effectiveness of the proposed algorithm. The DST and FUDS cycles are executed to evaluate the ability for estimated SOC based on ECKF compared with the other methods. It can be concluded from the results of experiments that the ECKF algorithm for SOC estimation has higher precision than others under DST cycle. The FUDS test is used to assess the robustness of SOC estimation with 20% initial SOC error. The time converged to 5% estimated error with the ECKF algorithm is approximately 144s, which is about 1.5 times as quick as the standard CKF method. Therefore, the proposed ECKF based on the embedded cubature rule can improve the accuracy and robustness for SOC estimation.

6. References

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Figure 5. SOC estimation results for FUDS test: (a) SOC estimation; (b) Estimation error.