Electron transport in Coulomb- and tunnel-coupled one-dimensional systems

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Abstract

We develop a linear theory of electron transport for a system of two identical quantum wires in a wide range of the wire length $L$, unifying both the ballistic and diffusive transport regimes. The microscopic model, involving the interaction of electrons with each other and with bulk acoustical phonons allows a reduction of the quantum kinetic equation to a set of coupled equations for the local chemical potentials for forward- and backward-moving electrons in the wires. As an application of the general solution of these equations, we consider different kinds of electrical contacts to the double-wire system and calculate the direct resistance, the transresistance, in the presence of tunneling and Coulomb drag, and the tunneling resistance. If $L$ is smaller than the backscattering length $l_P$, both the tunneling and the drag lead to a negative transresistance, while in the diffusive regime ($L \gg l_P$) the tunneling opposes the drag and leads to a positive transresistance. If $L$ is smaller than the phase-breaking length, the tunneling leads to interference oscillations of the resistances that are damped exponentially with $L$.

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I. INTRODUCTION

One-dimensional (1D) electron systems, such as they occur in semiconductor quantum wires, are in the forefront of research in modern condensed-matter physics. In submicrometer-long quantum wires at low temperatures the electron transport occurs in the ballistic regime and the wire conductance reaches its fundamental value of $G_0 = e^2/\pi \hbar$. On the other hand, in sufficiently long wires the conductance is limited by scattering processes. If quantum-interference effects are neglected, as is the case when the inelastic scattering dominates, the conductance is given by $\sigma/L$, where $L$ is the wire length and $\sigma$ the conductivity described by the Drude expression $\sigma = e^2 n \tau_{tr}/m$, where $n$ is the electron density, $\tau_{tr}$ the transport time, and $m$ the effective mass of the electron. This regime is referred to as the diffusive transport.

Modern technology allows to create various systems comprising two quantum wires put closely to each other so that the tunneling of electrons between the wires and/or interlayer electron-electron interaction is essential. Both these effects give rise to coupling between the electron sub-systems in single wires and in that way modify their electronic properties. This renders the coupled double-wire systems a subject of interest. In the past years, there have been experimental and theoretical studies of 1D-1D tunneling and electron transport along the wires of such systems. Investigations of the transport are mostly devoted to interlayer tunneling in the purely ballistic regime and in connection with the idea of electron-wave coupler. On the other hand, there are theoretical papers describing the momentum transfer between the wires due to interlayer Coulomb interaction and the corresponding interlayer transresistance (Coulomb drag). Calculations of the Coulomb drag have been done both for the diffusive and ballistic transport regimes, as well as for the regime in which the electron sub-systems are described by the Luttinger liquid model.

Despite this progress, there is a substantial lack of description of the electron transport in coupled quantum wires. Even if we accept the concept that the electrons are described by a normal Fermi liquid, two important questions arise. The first one is how to describe
the electrical properties when both tunneling and interactions of the electrons (with each other and with impurities or phonons) are essential. The second one is how to bridge the gap between the ballistic and diffusive transport regimes in such a description.

In this paper we present a linear-response theory of electron transport in coupled quantum wires that gives a reasonable answer to both questions stated above. We consider two parallel, tunnel-coupled 1D systems of degenerate electron gases adiabatically contacted to four equilibrium reservoirs, as shown and labeled in Fig. 1. This general scheme of a four-terminal device may describe both planar\textsuperscript{2,4,5,15} and vertically coupled\textsuperscript{3,9,12,17} double-wire devices. We take into account the interaction of electrons with themselves as well as that with acoustical phonons. We start from the quantum kinetic equation and finally transform it to a set of linear differential equations describing the distributions of the local chemical potentials for the systems of forward- and backward-moving electrons along the wires. The boundary conditions for such equations are dictated by the Landauer-Büttiker-Imry theory. This transformation is justified from microscopic calculations, which also give us expressions for the characteristic times, associated with the interactions involved, that enter the equations for chemical potentials.

As an application of our transport theory, we analyze in detail different kinds of electrical contacts to the double-wire system. First we consider the case when the voltage is applied between the ends of one wire and calculate the “direct” resistance of this wire as it is affected by the presence of the other one, as well as the transresistance, i.e., the resistance associated with the voltage induced at the ends of the uncontacted wire. Details about experimental measurements of the transresistance in such systems can be found in Ref. 23. Next we consider the case corresponding to the tunneling measurements\textsuperscript{2}, when the voltage is applied between the wires, and calculate the tunneling resistance. Several previously obtained theoretical results for such quantities (some of them are for coupled 2D systems) follow from our theory as limiting cases. A brief account of the main results appeared in Ref. 23.

The paper is organized as follows. In Sec. II we present the microscopic model and derive
the equations for the local chemical potentials in the layers. In Sec. III we solve these equations, apply the obtained results to the calculation of the direct resistance, transresistance, and tunneling resistance of the double-wire system, and describe possible transport regimes. Concluding remarks and discussion of the approximations made are given in Sec. IV. The Appendix contains detailed microscopic calculations and expressions of the characteristic times entering the equations for chemical potentials.

II. FROM QUANTUM KINETIC THEORY TO LOCAL DESCRIPTION

Consider two homogeneous 1D quantum layers of length \( L \), labeled left \((l)\) and right \((r)\) along the \( x \) axis, see Fig. 1. The quantum kinetic equation for the density matrix \( \hat{\rho} \) reads

\[
\frac{\partial \hat{\rho}}{\partial t} + \frac{i}{\hbar} [\hat{H}_0 + \hat{H}_C + \hat{H}_{e-ph}, \hat{\rho}] = 0. \tag{1}
\]

Here we assume that electrons interact with each other via the Coulomb field \( \hat{H}_C \) and with acoustical phonons \( \hat{H}_{e-ph} \). Elastic scattering is neglected, i.e., we assume ideal wires. The unperturbed Hamiltonian \( \hat{H}_0 \) includes both the kinetic and potential energy operators. Below we use the basis of the isolated \( l \) and \( r \) layer states \( F_l(y, z) \) and \( F_r(y, z) \) and assume that only the lowest level is occupied in each layer. In this basis the potential energy is the matrix

\[
\hat{h} = (\Delta/2)\hat{\sigma}_z + T\hat{\sigma}_x. \tag{2}
\]

Here \( \hat{\sigma}_i \) are the Pauli matrices, \( \Delta \) is the level-splitting energy, and \( T \) the tunneling matrix element characterizing the strength of the tunnel coupling. Such tight-binding description is often used in application to two-level systems.

The kinetic equation can be written\(^\text{25}\) as one for the Keldysh’s Green’s function \( \hat{G}^{-+} \). Below we consider the case when the characteristic spatial scale of the electronic distribution is large in comparison to the Fermi wavelength \( \pi h/p_F \) and use the Keldysh’s matrix Green’s function in the Wigner representation \( \hat{G}_{\epsilon, f}^{-+}(p, x) \), where \( p \) and \( \epsilon \) are the momentum and
energy and $t$ the time. The time dependence of $\hat{G}^{+-}$ is not essential in the following, since we study a time-averaged, steady-state problem. The linear response theory uses a Green’s function of the form

$$\hat{G}^{\alpha\beta}_\varepsilon (p, x) = \hat{G}^{(0)\alpha\beta}_\varepsilon (p, x),$$

where $\alpha$ and $\beta$ are $+$ or $-$. The unperturbed part $\hat{G}^{(0)\alpha\beta}_\varepsilon (p)$ is given by

$$\hat{G}^{(0)\alpha\beta}_\varepsilon (p, x) = f(\varepsilon)[\hat{G}^A_\varepsilon (p) - \hat{G}^R_\varepsilon (p)], \quad \hat{G}^{(0)+-}_\varepsilon (p) = (f(\varepsilon) - 1)[\hat{G}^A_\varepsilon (p) - \hat{G}^R_\varepsilon (p)],$$

where $f(\varepsilon) = [1 + e^{(\varepsilon - \mu)/k_B T}]$ is the equilibrium Fermi distribution function and $\hat{G}^{RA}_\varepsilon$ are the retarded and advanced Green’s functions which satisfy the equations

$$\left[ \varepsilon - \varepsilon_p - \hat{h} - \hat{\Sigma}^{R,A}_\varepsilon (p) \right] \hat{G}^{R,A}_\varepsilon (p) = 1;$$

here $\hat{\Sigma}^{R,A}_\varepsilon (p)$ are the self-energy functions.

The linearized kinetic equation reads

$$\frac{\hbar}{2} \left\{ \hat{v}_p, \frac{\partial}{\partial x} \delta \hat{G}^{+-}_\varepsilon (p, x) \right\} + i \left[ \hat{h}, \delta \hat{G}^{+-}_\varepsilon (p, x) \right]$$

$$- \frac{\hbar}{2} \left\{ \frac{\partial}{\partial x} \hat{\phi}, \frac{\partial}{\partial p} \hat{G}^{(0)+-}_\varepsilon (p) \right\} + i \left[ \hat{\phi}, \hat{G}^{(0)+-}_\varepsilon (p) \right] = i\delta \hat{I}(\varepsilon, p, x).$$

Here {...} denotes anticommutators, $\hat{v}_p = \hat{P}_l v_{lp} + \hat{P}_r v_{rp}$ is the diagonal matrix of the group velocities, and $\hat{P}_l = (1 + \hat{\sigma}_z)/2$ and $\hat{P}_r = (1 - \hat{\sigma}_z)/2$ are the projection matrices. In this paper we consider the case of equal group velocities in the layers, with $v_{lp} = v_{rp} = v_p = p/m$. If a magnetic field is applied perpendicular to the wire plane though, $v_{lp}$ and $v_{rp}$ become different. Further, $\hat{\phi}$ is the matrix of the self-consistent electrostatic potential arising due to perturbation of the electron density. In the mean-field (Hartree) approximation this matrix is diagonal. Finally, the generalized collision integral $\hat{I}$ is given by

$$\hat{I} = - \left\{ \hat{\Sigma}^{++} \hat{G}^{++} + \hat{\Sigma}^{--} \hat{G}^{--} + \hat{G}^{+-} \hat{\Sigma}^{++} + \hat{G}^{-+} \hat{\Sigma}^{--} \right\},$$
where all Green’s functions $\hat{G}^{\alpha\beta}$ and self-energy functions $\hat{\Sigma}^{\alpha\beta}$ have the same arguments $\varepsilon$, $p$, and $x$. This corresponds to a quasiclassical description of the scattering. However, the matrix structure of $\hat{G}^{\alpha\beta}$ and $\hat{\Sigma}^{\alpha\beta}$ remains important and Eq. (7) is not reduced to a classical Boltzmann equation. Since we consider the interaction of electrons with each other and with acoustical phonons, the corresponding lowest-order contributions to the self-energy are given by the diagrams of Fig. 2. We neglect the exchange part of the Coulomb interaction for the following reasons. The first-order exchange contributions do not influence the imaginary part of the self-energy and are not, therefore, essential for the calculation of the collision integral. The second-order exchange contributions are small as compared to the second-order direct Coulomb contributions, represented by the diagram of Fig. 2 (b), if the momentum transfer $q$ is small in comparison with the Fermi momentum. Finally, there is no exchange contributions to the interlayer Coulomb interaction.

We consider low temperatures and degenerate electrons. We also assume that the Fermi energy is large in comparison with both the tunneling matrix element $T$ and the level splitting $\Delta$, thereby neglecting the difference between the electron densities in the layers. We sum up Eq. (7) over the electron momentum $p$ in the regions of positive (+) (or forward) and negative (−) (or backward) group velocities and introduce the nonequilibrium part $\hat{g}_\varepsilon(x)$ of the energy distribution function in the manner

$$\hat{g}_\varepsilon^\pm(x) = \int_{\pm} \frac{dp}{2\pi i} |v_p| \delta\hat{G}_\varepsilon^\pm(p, x).$$

(9)

Since $\delta\hat{G}^{\pm}$ is essentially nonzero only in narrow intervals of energy and momentum near the equilibrium chemical potential $\mu$ and Fermi momentum $p_F$, we can replace $|v_p|$ in this equation by the Fermi velocity $v_F$, common to both layers. The integration in the + and − regions in Eq. (7) removes the contributions proportional to the potential matrix $\hat{\varphi}(x)$ and we obtain

$$\pm v_F \frac{\partial}{\partial x} \hat{g}_\varepsilon^\pm(x) + \frac{i}{\hbar} \left[ \hat{h}, \hat{g}_\varepsilon^\pm(x) \right] = \delta\hat{I}_\varepsilon^\pm(\varepsilon, x)$$

(10)

where the collision integral $\delta\hat{I}_\varepsilon^\pm(\varepsilon, x) = (2\pi\hbar)^{-1} \int_{\pm} dp |v_p| \delta\hat{I}(\varepsilon, p, x)$ depends on both $\hat{g}^+$ and $\hat{g}^-$, since it accounts for both forward and backward scattering processes. However, when
we integrate Eq. (10) over the energy, the diagonal part of $\delta I_\pm(\varepsilon, x)$ vanishes for forward-scattering contributions, and only the backscattering contributions remain, see below. In contrast, the forward-scattering contributions for the nondiagonal part of the collision integral are not eliminated by the energy integration.

The matrix kinetic equation (10) is equivalent to eight scalar equations for the four components of $\hat{g}^+$ and the four ones of $\hat{g}^-$, corresponding to forward- and backward-propagating electrons, respectively. These equations must be accompanied by boundary conditions connecting the components of $\hat{g}^\pm$ with the quasi-equilibrium distribution functions of the four leads which the quantum wires are contacted to, cf. Fig. 1. The distribution functions of the leads are defined by the four chemical potentials $\mu_{1l}, \mu_{1r}, \mu_{2l},$ and $\mu_{2r}$. If we assume that the potentials in the contact regions are sufficiently smooth in comparison with the Fermi wavelength but abrupt enough as compared to the characteristic scale of the electronic distribution, we can apply Eq. (10) in the contact region as well. It gives us the conditions of continuity for all components of $\hat{g}_\varepsilon^\pm(x)$ across the contact regions and we obtain

$$
\hat{g}_\varepsilon^+(0) = -\frac{\partial f(\varepsilon)}{\partial \varepsilon} [\hat{P}_l \delta \mu_{1l} + \hat{P}_r \delta \mu_{1r}], \quad \hat{g}_\varepsilon^-(L) = -\frac{\partial f(\varepsilon)}{\partial \varepsilon} [\hat{P}_l \delta \mu_{2l} + \hat{P}_r \delta \mu_{2r}],
$$

(11)

with $\delta \mu_{1l} = \mu_{1l} - \mu$, etc. The forward- and backward-propagating states are "connected", respectively, to the leads 1 and 2. The nondiagonal components vanish at the contacts because the tunneling is absent outside the region $x = [0, L]$.

The problem described by the matrix equation (10) and the boundary conditions (11) can be considerably simplified and solved analytically if we assume that both backscattering and the interlayer tunneling occur much less frequently than the scattering of electrons inside the layers and inside the $+$ or $-$ regions. The tunneling can be made weak if, for example, the potential barrier between the wires is thick enough. As concerns the backscattering, this condition is often fulfilled at low temperatures for both the electron-electron and electron-phonon scattering mechanisms. In the first case, the backscattering probability contains a factor $[K_0(2p_F a)]^2$, where $K_0$ is the modified Bessel function and $a$ is the wire width. This factor is exponentially small for $2p_F a > 1$. The acoustic phonon-assisted backscattering
gives a small contribution in comparison with the electron-electron forward-scattering due to the smallness of the electron-phonon coupling constant. In addition, this backscattering is suppressed at very low temperatures \( T_e < 2p_F s \), where \( s \) is the sound velocity. If the stated conditions are fulfilled, the diagonal part of the electron-electron forward-scattering locally satisfies a kinetic equation with the electron-electron forward-scattering and backscattering contributions. It means that the diagonal part of \( \hat{g}_\pm^\varepsilon(x) \) is given by the following equation

\[
[\hat{g}_\pm^\varepsilon(x)]_{jj} = -\frac{\partial f(\varepsilon)}{\partial \varepsilon} \delta \mu_j^\pm(x)
\]  

(12)

where \( \delta \mu_j^\pm(x) = \mu_j^\pm(x) - \mu \ (j = l, r) \) do not depend on the energy. The quantities \( \mu_j^\pm(x) \) have the direct meaning of local chemical potentials for the layers \( l \) and \( r \). It is convenient to introduce also the nondiagonal components of the chemical potentials \( \mu_{l,r}^\pm(x) = \mu_l^\pm(x) - i\phi_l^\pm(x) \) and \( \mu_{r,l}^\pm(x) = \mu_r^\pm(x) + i\phi_r^\pm(x) \), by writing the whole chemical potential matrix as

\[
\delta \hat{\mu}^\pm(x) = \int d\varepsilon \ \hat{g}_\pm^\varepsilon(x).
\]

(13)

Below we drop the symbol ”\( \delta \) in \( \delta \hat{\mu}^\pm(x) \) and the contact potentials \( \delta \mu_{1l,r} \) and \( \delta \mu_{2l,r} \) since all chemical potentials are counted from the same equilibrium value \( \mu \).

Substituting Eq. (12) into Eq. (10) and integrating the latter over the energy, we finally obtain eight coupled, first-order differential equations for the eight components of \( \mu_k^\pm(x) \):

\[
\pm d\mu_k^\pm/dx + (\mu_k^\pm - \mu_{k'}^\mp)(1/l_P + 1/l_D) - (\mu_k^\mp - \mu_{k'}^\pm)/l_D - 2t_F \mu_v^\pm = 0,
\]

(14)

\[
\pm d\mu_{r}^\pm/dx + (\mu_r^\pm - \mu_{r'}^\mp)(1/l_P + 1/l_D) - (\mu_r^\mp - \mu_{r'}^\pm)/l_D + 2t_F \mu_v^\pm = 0,
\]

(15)

\[
\pm d\mu_u^\pm/dx + \delta_F \mu_v^\pm + \mu_u^\pm/l_C = 0,
\]

(16)

\[
\pm d\mu_v^\pm/dx - \delta_F \mu_u^\pm + \mu_v^\pm/l_C + t_F (\mu_u^\pm - \mu_v^\pm) = 0.
\]

(17)

Here \( t_F = T/hv_F \) and \( \delta_F = \Delta/hv_F \). The boundary conditions for all potentials follow from Eqs. (11)-(13) and are \( \mu_1^+(0) = \mu_{u1}, \mu_1^-(L) = \mu_{u2l}, \mu_r^+(0) = \mu_{r1}, \mu_r^-(L) = \mu_{r2l} \) and \( \mu_{u,v}^+(0) = \mu_{u,v}^-(L) = 0 \). The characteristic lengths \( l_P, l_D, \) and \( l_C \) result from the collision integral \( \delta I_\pm(\varepsilon, x) \), evaluated to the lowest order with respect to the tunneling matrix element.
The transport time $\tau_P$ is common to both layers since we assume that the confining potentials for the wires $l$ and $r$ are almost identical. The analytical expressions for the $\tau_P$, $\tau_D$ and $\tau_C$ are given in the Appendix. All characteristic lengths are sensitive to the temperature $T$ and the level splitting $\Delta$. It is essential that $l_C$, which is controlled by electron-electron interaction, is always much smaller than $l_D$ and $l_P$. On the other hand, depending on the temperature and level splitting one can have different relations between $l_D$ and $l_P$: both cases, $l_P \ll l_D$ and $l_P \gg l_D$, are possible.

Equations (14)-(17) with the stated boundary conditions give us a complete description of the electrical properties of double quantum-wire systems in a wide range of regimes starting from the purely ballistic transport regime $L \ll l_C$ to the diffusive transport regime $L \gg l_P, l_D$. The local currents flowing in the layers $j = l, r$ are expressed by

$$J_j(x) = G_0[\mu_j^+(x) - \mu_j^-(x)]/e,$$

and the local tunnel currents are proportional to $T \mu_v^\pm(x)$.

Below we present the general solution of Eqs. (14)-(17) and describe two important cases, that of long systems, with $L \gg l_C$, and that of short systems with $L \sim l_C$. To characterize the effects of drag and tunneling, we then consider different kinds of electrical contact to the double-wire system. First we consider a typical setup for the drag measurements, when the current $J_r = J_r(0) = J_r(L)$ is injected in wire $r$ ("drive wire") while no current is allowed to flow into wire $l$, $J_l(0) = J_l(L) = 0$, and calculate the transresistance $R_{TR}$ defined as $R_{TR} = [\mu_1 - \mu_2]/eJ_r$ as well as the "direct" resistance $R = [\mu_1 - \mu_2]/eJ_r$. Next, we turn to the tunneling measurements, when the voltage is applied between the wires. We consider both the symmetric setup, when all four ends of the wires are connected to external sources, with $\mu_{1l} = \mu_{2l}$ and $\mu_{1r} = \mu_{2r}$, and the non-symmetric one, when the voltage is applied between the ends $1l$ and $2r$ while the remaining ends are not contacted,
\( J_r(0) = 0, \ J_l(L) = 0. \) For each of these cases we calculate the tunneling resistances \( R_{Ts} \) (symmetric) and \( R_{Tn} \) (non-symmetric). Both of them can be defined as \([\mu_{1l} - \mu_{2r}] / e J_T\), where \( J_T \) is the total current injected. \( J_T \) is equal to \( 2J_l(0) \) and \( J_l(0) \) for the symmetric and non-symmetric contacts, respectively.

\[ \] 

**III. Results**

Since Eqs. (14)-(17) are linear, their general solution is easily obtained as

\[
  2\mu^+_{l,r}(x) = (1 + L/l_p)^{-1} \left[ (\mu_{1l} + \mu_{1r})(1 + (L - x)/l_p) + (\mu_{2l} + \mu_{2r})x/l_p \right] \\
  \pm \sum_i \left( A_i^+ e^{\lambda_i x} + B_i^+ e^{-\lambda_i x} \right) \tag{19}
\]

\[
  2\mu^-_{l,r}(x) = (1 + L/l_p)^{-1} \left[ (\mu_{1l} + \mu_{1r})(L - x)/l_p + (\mu_{2l} + \mu_{2r})(1 + x/l_p) \right] \\
  \pm \sum_i \left( A_i^- e^{\lambda_i x} + B_i^- e^{-\lambda_i x} \right) \tag{20}
\]

\[
  \mu^\pm(x) = \sum_i \left( C_i^\pm e^{\lambda_i x} + D_i^\pm e^{-\lambda_i x} \right) \tag{21}
\]

\[
  \mu^\pm(x) = -\sum_i \left( \frac{\delta_F}{\pm \lambda_i + l_C^{-1}} C_i^\pm e^{\lambda_i x} + \frac{\delta_F}{\mp \lambda_i + l_C^{-1}} D_i^\pm e^{-\lambda_i x} \right) \tag{22}
\]

Here \( \lambda_i = \sqrt{y_i} \) and \( y = y_i \) are the solutions of the cubic equation

\[
y^3 - 2y^2[l_C^{-2} - \delta_F^2 - 4t_F^2] + y[l_C^{-4} + (\delta_F^2 + 4t_F^2)^2 - 8(t_F/l_C)^2 + 2(\delta_F/l_C)^2 + 8t_F^2/l_C l_1] \\
-4t_F^2[4(t_F/l_C)^2 + (2/l_C l_1)(l_C^{-2} + \delta_F^2)] = 0, \tag{23}
\]

where \( l_1^{-1} = l_p^{-1} + 2l_D^{-1} \). The coefficients \( A_i^\pm, B_i^\pm, C_i^\pm, \) and \( D_i^\pm \) are to be found from Eqs. (14)-(17) and the relevant boundary conditions. Below we use the property \( l_C \ll l_p, l_D \) and the condition of weak tunnel coupling \( t_F \ll l_C^{-1} \) to simplify this procedure. Then the three roots of Eq. (23) are easily obtained as

\[
  \lambda_1 = \lambda \simeq 2(1/l_T^2 + 1/l_T l_1)^{1/2}, \ \lambda_{2,3} = \lambda \pm \simeq 1/l_C \pm i\delta_F, \tag{24}
\]

where we introduced the tunneling length \( l_T = v_F \tau_T \). The tunneling time \( \tau_T \), which contains a resonance dependence on the level splitting, is defined by
\[ \tau_T^{-1} = \tau_C^{-1} \frac{2T^2}{\Delta^2 + (\hbar/\tau_C)^2}. \] (25)

The root \( \lambda \) describes long-scale variations of the chemical potentials while \( \lambda_\pm \) corresponds to short-scale variations. Accordingly, we consider the regimes that follow.

**A. Long wires, \( L \gg l_C \).**

This length range comprises the region from the "pseudo-ballistic" \( (l_C \ll L \ll l_P, l_D) \) to the diffusive \( (L \gg l_P) \) regimes. All solutions containing \( \lambda_\pm \) exist only in short regions in the vicinity of the contacts. They are evanescent inside the wire region and not essential in the calculation of the currents. Considering only the solutions involving \( \lambda \), we find

\[ 2\mu_{l,r}^+(x) = (1 + L/l_P)^{-1} \left[ (\mu_{1l} + \mu_{1r}) \left( 1 + (L - x)/l_P \right) + (\mu_{2l} + \mu_{2r})x/l_P \right] \]
\[ \pm(\mu_{1l} - \mu_{1r})P(L - x)/P(L) \pm (\mu_{2l} - \mu_{2r}) \sinh \lambda x/P(L) \] (26)

\[ 2\mu_{l,r}^-(x) = (1 + L/l_P)^{-1} \left[ (\mu_{1l} + \mu_{1r})(L - x)/l_P + (\mu_{2l} + \mu_{2r})(1 + x/l_P) \right] \]
\[ \pm(\mu_{1l} - \mu_{1r}) \sinh \lambda(L - x)/P(L) \pm (\mu_{2l} - \mu_{2r})P(x)/P(L), \] (27)

where \( P(x) = (1 + 2l_1/l_T) \sinh \lambda x + \lambda l_1 \cosh \lambda x \). The same expressions can be obtained from the four coupled balance equations compactly presented as

\[ \pm d\mu_{j,r}^+/dx + (\mu_{j,r}^+ - \mu_{j,r}^-)(1/l_P + 1/l_D) \]
\[ -(\mu_{j,r}^+ - \mu_{j,r}^-)/l_D + (\mu_{j,r}^+ - \mu_{j',r}^-)/l_T = 0, \] (28)

where \( j = l, r \) and \( j' \neq j \). These equations follow from Eqs. (14)-(17) in the limit \( L \gg l_C \), when one can neglect the derivatives \( d\mu_{u,v}^+/dx \) in comparison to \( \mu_{u,v}^+/l_C \). With Eqs. (26) and (27) we obtain

\[ R = \frac{\pi \hbar}{2c^2} \left[ 2 + L/l_P + (1 + l_T/l_1)^{1/2} \tanh(\lambda L/2) \right], \] (29)

\[ R_{TR} = \frac{\pi \hbar}{2c^2} \left[ L/l_P - (1 + l_T/l_1)^{1/2} \tanh(\lambda L/2) \right], \] (30)
For $L \ll l_P, l_T$ we have $R \simeq \pi h/e^2 = G_0^{-1}$, while the transresistance is given by $R_{TR} \simeq -(\pi h/e^2)L[1/l_D + 1/l_T]$. As seen, $R_{TR}$ is small, always negative, and proportional to the wire length $L$ multiplied by a sum of drag and tunneling rates. If one neglects tunneling, the resulting expression for $R_{TR}$, with $\tau_D$ given by Eq. (A11), describes the Coulomb drag in the ballistic regime previously investigated $^{21}$ by Gurevich et al.. When $L$ increases and the electron transport becomes diffusive ($L \gg l_P$), we obtain, for $\lambda L/2 \ll 1$, $R \simeq (\pi h/e^2)L[1/l_P + 1/l_D]$. This resistance, if one omits the drag contribution, is expressed in terms of the usual Drude conductivity $\sigma = L/R = e^2l_P/\pi h = e^2n\tau_P/m$. The corresponding transresistance is

$$R_{TR} = -\frac{\pi h}{e^2}(L/l_D)\left[1 - (L/L_0)^2\right], \quad L_0 = (6l_P^2l_T/l_D)^{1/2}. \quad (31)$$

Expressing $l_D$ and $l_T$ through the drag transresistivity $\pi h/e^2l_D$ and the tunneling conductance $G_T = e^2\rho_{1D}/\tau_T = 2e^2/\pi h l_T$, where $\rho_{1D}$ is the 1D density of states at the Fermi level, one can see that Eq. (31) formally coincides with that obtained in Ref. 27, where a competition of drag and tunneling effects was investigated for double quantum-well systems. For $\lambda L \sim 1$, the transresistance is large and comparable to the direct resistance, because a considerable fraction of the current penetrates the $l$ layer due to tunneling. This regime for double quantum wells has been investigated both experimentally $^{28}$ and theoretically $^{29}$. If one neglects the drag and assumes the diffusive regime ($L \gg l_P$) with weak tunneling ($l_P \ll l_T$), Eqs. (29) and (30) describe the results obtained in Ref. 29. For $\lambda L \gg 1$ we have $R_{TR} = R = (\pi h/e^2l_P)(L/2) = L/2\sigma$. This is the case when the current, though injected only in one layer, is equally distributed among the layers due to tunneling.

Figure 3 shows the length dependence of the transresistance calculated for different relative contributions of the Coulomb drag and tunneling. The transresistance is negative for small $L$ but always changes its sign and becomes positive as $L$ increases and the backscattering occurs often [see also Eq. (31)]. This behavior can be explained with the help of the balance equation (28), which shows that the tunneling tends to decrease the difference between $\mu_{l}^{\pm}$ and $\mu_{r}^{\pm}$ while the backscattering tends to decrease the difference between $\mu_{l,r}^{++}$. 

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and $\mu_{l,r}$. Thus, for $\mu_+(0) = \mu_-(0)$ and $\mu_+(L) = \mu_-(L)$ the change of $\mu_+(\mu_-$), with $x$, is opposite to that of $\mu_+(\mu_-)$ at small $L$ and becomes the same as that of $\mu_+(\mu_-)$ as $L$ increases, leading to the change of $R_{TR} = [\mu_+(0) - \mu_-(L)]/eJ_r$ from negative to positive. This transition occurs at smaller $L/l_P$ if the tunneling is stronger (larger $l_P/l_T$) and the drag weaker (smaller $l_P/l_D$).

Although $l_T$ is normally longer than $l_P$, the opposite condition can also be realized. A particularly interesting transport regime, corresponding to long quantum wires without backscattering, occurs in tunnel-coupled magnetic edge states\textsuperscript{30,31}, since an edge state represents a 1D system where the electrons can move only in one direction. Assuming $1/l_P = 1/l_D = 0$ in Eqs. (29) and (30), we obtain the result of Ref. 31 in the form

$$R = \frac{\pi \hbar}{e^2} \left[ 1 + \frac{1}{2} \tanh(L/l_T) \right], \quad R_{TR} = -\frac{\pi \hbar}{2e^2} \tanh(L/l_T). \quad (32)$$

Consider now the behavior of the tunneling resistances. With Eqs. (26) and (27) we obtain

$$R_{Ts} = \frac{\pi \hbar}{2e^2} \left[ 1 + (1 + l_T/l_1)^{1/2} \coth(\lambda L/2) \right] \quad (33)$$

and

$$R_{Tn} = \frac{\pi \hbar}{2e^2} \left[ 2 + L/l_P + (1 + l_T/l_1)^{1/2} \coth(\lambda L/2) \right], \quad (34)$$

for symmetric and non-symmetric contacts, respectively. For conditions $\lambda L/2 \ll 1$ we have $R_{Ts} \approx R_{Tn} \approx (\pi \hbar/2e^2)(L/l_T)$, i.e., the tunneling resistances depend only on the ratio of the tunneling length to the wire length. This is because the regime of $\lambda L/2 \ll 1$ corresponds to weak tunneling and the chemical potentials $\mu_\pm(x)$ and $\mu_\pm(x)$ are close to $\mu_1$ and $\mu_2$, respectively. With the use of the tunneling conductance $G_T$ (see above) one can rewrite the expression for the tunneling resistances in a more transparent way: $R_{Ts} \approx R_{Tn} \approx (G_T L)^{-1}$. For $\lambda L/2 \sim 1$, when the coordinate dependence of the chemical potentials in the layers is essential, $R_{Tn}$ is different from $R_{Ts}$ and both of them depend on the scattering length $l_P$. The drag effect is not so important as for the transresistance: the tunneling resistances depend on $l_D$ only if $l_D$ is comparable to or smaller than both $l_P$ and $l_T$. 
Figure 4 shows the length dependence of the tunneling resistances $R_{Ts}$ and $R_{Tn}$, as given by Eqs. (33) and (34), for several different values of the ratio $l_p/l_T$ describing the strength of the tunneling with respect to the backscattering. The drag effect is neglected, $1/l_D = 0$. As the wire length becomes larger than the backscattering length, the $1/L$ decrease of the tunneling resistance changes to either a $L$-independent behavior (for $R_{Ts}$) or to a linear increase (for $R_{Tn}$). In the first case the dependence on $L$ disappears because all tunneling occurs near the ends. In contrast, for non-symmetric contacts the resistance $R_{Tn}$ is determined by the Ohmic resistance of the wires instead of the tunneling effects, and increases linearly with $L$. A similar effect, in applications to coupled quantum wells, is discussed in Ref. 29.

B. Short wires, $L \ll l_P, l_D$

This length range comprises the region from the purely ballistic ($l \ll l_C$) to the ”pseudo-ballistic” ($l_C \ll L \ll l_P, l_D$) regimes. Since the electrons pass along the wires almost without backscattering, $R$ is close to $\pi \hbar/e^2$, and $R_{TR}$ is small. However, for $L \sim l_C$ an electron tunneling between the layers does not lose its phase memory completely and tunnel coherence effects can manifest themselves on such short lengths giving additional contributions to the transresistance $R_{TR}$ and the tunneling resistances $R_{Ts}$ and $R_{Tn}$; accordingly the expressions for these quantities obtained in the previous subsection for $L \ll l_P, l_D$ should be modified.

A convenient analytical approach to the problem in this regime is to solve Eqs. (14)-(17) by iterations taking $\mu_{l,r}^+(x) = \mu_{1l,r}, \mu_{l,r}^-(x) = \mu_{2l,r}$ and $\mu_{u,v}^\pm(x) = 0$ as an initial approximation. Another way is to use Eqs. (19)-(22) directly. We obtain

$$2\mu_{l,r}^+(x) = (\mu_{1l} + \mu_{1r})(1 - x/l_P) + (\mu_{2l} + \mu_{2r}) x/l_P$$
$$\pm (\mu_{1l} - \mu_{1r}) [1 - x/l_1 - 2x/l_T + 2(l_C/l_T)\Phi(x)] \pm (\mu_{2l} - \mu_{2r}) x/l_1$$

$$2\mu_{l,r}^-(L - x) = (\mu_{2l} + \mu_{2r})(1 - x/l_P) + (\mu_{1l} + \mu_{1r}) x/l_P$$
\[
\pm (\mu_{2l} - \mu_{2r}) [1 - x/l_1 - 2x/l_T + 2(l_C/l_T)\Phi(x)] \pm (\mu_{1l} - \mu_{1r}) x/l_1 
\]  
(36)

where

\[
\Phi(x) = \left[\frac{1}{(l_C^2 + \delta_F^2)} \right] \left[ 2(\delta_F/l_C)e^{-x/l_C} \sin(\delta_F x) + (l_C^{-2} - \delta_F^2) \left( 1 - e^{-x/l_C} \cos(\delta_F x) \right) \right], 
\]  
(37)

is an oscillating function of the coordinate \(x\) and \(\delta_F = \Delta/\hbar v_F\). Now \(R\) and \(R_{TR}\) are given, respectively, by

\[
R = \frac{\hbar \pi}{e^2} \left[ 1 + \frac{L}{l_F} + \frac{L}{l_D} + \frac{L}{2l_T} - \frac{(l_C/2l_T)\Phi(L)}{} \right], 
\]  
(38)

and

\[
R_{TR} = \frac{\hbar \pi}{e^2} \left[ -\frac{L}{l_D} - \frac{L}{2l_T} + \frac{(l_C/2l_T)\Phi(L)}{} \right], 
\]  
(39)

The contribution to \(R_{TR}\) coming from the term proportional to \(\Phi(L)\) is not small for \(L \sim l_C\). It describes oscillations damped due to the factor \(\exp(-L/l_C)\). The periodic behavior can be described as a result of the interference of electron waves of the left and right layers along the length \(L\): due to a finite level splitting \(\Delta\) these waves have different phase velocities.

Similar interference effects occur in the tunneling resistances

\[
R_{Ts} \simeq R_{Ta} \simeq \frac{\hbar \pi}{2e^2} \left( l_T/L \right) \left[ 1 - \frac{(l_C/L)\Phi(L)}{} \right]^{-1}. 
\]  
(40)

From Eq. (37) for \(\Phi(L)\) we see that both the transresistance and the tunneling resistances, being functions of \(\Delta L/\hbar v_F\), oscillate with level separation \(\Delta\). The oscillations are damped when the wire length \(L\) exceeds \(l_C\) so that the tunnel coherence over the wire length is suppressed.

Changing \(\Delta\) by applying a voltage across the wires would lead to oscillations of \(R\), \(R_{TR}\), \(R_{Ts}\), and \(R_{Tn}\). Another way to change \(\Delta\) is to apply a magnetic field \(B\) perpendicular to the plane of the wires\(^{15,16}\). For sufficiently weak \(B\) the results presented so far still hold with the phase \(\delta_F L\) having an additional contribution \(2\pi\phi/\phi_0\), where \(\phi_0 = \hbar/e\) is the magnetic flux quantum and \(\phi = BwL\) the flux enclosed by the area between the wires. Though the double-wire system does not form a closed current loop, this should lead to Aharonov-Bohm-type oscillations in the resistances defined by Eqs. (38)-(40).
In very short wires, with $L \ll l_C, \delta_F^{-1}$, Eqs. (39) and (40) become

$$R_{TR} = -\frac{\pi \hbar}{e^2} \left( \frac{L/l_D + L^2t_F^2}{2} \right), \quad R_{Ts} = R_{Tn} = \frac{\pi \hbar}{2e^2} L^{-2} t_F^{-2}. \quad (41)$$

In this regime only a small fraction of the electronic wave packet is coherently transmitted from one wire to another. The tunneling contribution to $R_{TR}$ follows a $L^2$ dependence, instead of the linear dependence occurring for $l_C \ll L \ll l_P, l_D$, when the tunneling is non-coherent. The length dependence of the tunneling resistance follows a $L^{-2}$ law.

In the investigation of the purely ballistic regime ($L \ll l_C$) we can neglect the collision integral in Eq. (10) and need not make the assumption about the smallness of the tunneling matrix element which was essential for evaluation of the scattering-induced contributions in Eqs. (14)-(17). The electron transport in coupled quantum wires in this regime is pertinent to the problem of electron-wave directional couplers. Theoretical studies of this problem$^4$-$^{14}$, although rather extensive, included only a quantum-mechanical calculation of the electronic transmission. Below we show how the essential results of these studies can be obtained in a simple way from the quantum-kinetic analysis. Integrating Eq. (10), with $\delta \hat{I}_\pm(\varepsilon, x) = 0$, over the energy and taking Eq. (13) into account, we find that the distribution of the chemical potentials is again described by Eqs. (14)-(17) without the terms containing the scattering lengths $l_C, l_P$, and $l_D$. Since there is no backscattering, the solutions for $\mu^{+}_{l,r}$ and $\mu^{-}_{l,r}$ are decoupled

$$\mu^{+}_{l,r}(x) = \mu_{1l,r} \mp (\mu_{1l} - \mu_{1r}) r \sin^2(\Delta_T x/2\hbar v_F), \quad (42)$$
$$\mu^{-}_{l,r}(x) = \mu_{2l,r} \mp (\mu_{2l} - \mu_{2r}) r \sin^2(\Delta_T (L - x)/2\hbar v_F); \quad (43)$$

here $\Delta_T = (\Delta^2 + 4T^2)^{1/2}$ and $r = 4T^2/\Delta_T^2$. Equations (42) and (43) describe oscillations of the electronic wave packets between the layers due to coherent tunneling. A complete transfer of the wave packet can be achieved for $\Delta = 0$. One can calculate the resistance and transresistance as

$$R = \frac{\pi \hbar}{e^2} [1 - (r/2) \sin^2 \psi] /[1 - r \sin^2 \psi], \quad (44)$$
$$R_{TR} = -\frac{\pi \hbar}{2e^2} r \sin^2 \psi/[1 - r \sin^2 \psi] \quad (45)$$

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and the tunneling resistances as

\[ R_{Ts} = R_{Tn} - \frac{\pi \hbar}{2e^2} = \frac{\pi \hbar}{2e^2} r^{-1} \sin^{-2} \psi, \quad (46) \]

Here \( \psi = \Delta_T L / 2 \hbar v_F \). The oscillations of these quantities occur in a way similar to the one described by Eqs. (38)-(40): \( \delta_F L \) coincides with \( 2\psi \) if one replaces \( \Delta \) by \( \Delta_T \). However, since the tunnel coupling is strong, the oscillations described by Eqs. (44)-(46) have large amplitudes. In particular, when \( \Delta \) is small \( (r \approx 1) \), all the quantities given by Eqs. (44)-(46) show giant oscillations with amplitude large in comparison to \( G_0^{-1} \).

IV. CONCLUSIONS

In this paper we carried out a theoretical study of electron transport in parallel 1D layers coupled by tunneling and Coulomb interaction and contacted, at their ends, to quasi-equilibrium reservoirs. A linear-response, steady-state regime has been investigated, and the wires were assumed to be ideal, i.e., without defects and, therefore, the elastic scattering of electrons by them was neglected. As the most important result of our study, we found that a full quantum-kinetic description of the problem is reduced, with physically reasonable assumptions, to a set of linear, first-order differential equations describing the distribution of local chemical potentials for forward- and backward-moving electrons. The boundary conditions for the chemical potentials are determined by the potentials of the reservoirs controlled by applied voltages. The solution of this set was obtained analytically and allowed us to describe the local currents flowing in each layer from the pure ballistic regime, when the electrons do not suffer any scattering along the wires, to the diffusive regime, when the electrons experience many backscattering events during the transport.

In particular, we applied our approach to the description of the resistance \( R \), transresistance \( R_{TR} \), and tunneling resistances \( R_{Ts} \) and \( R_{Tn} \) of double quantum wires. The most important result is that \( R_{TR} \), which is caused by both tunneling and Coulomb drag effects, depends on the wire length \( L \) non-monotonically and always changes its sign as \( L \) increases,
because in shorter wires, when backscattering is rare, the tunneling, as well as the drag, leads to a \textit{negative} $R_{TR}$, while in longer wires, when the transport becomes diffusive, the tunneling leads to a \textit{positive} $R_{TR}$, in a way similar to that for coupled two-dimensional (2D) systems\textsuperscript{28,29}, and overcomes the drag as the length $L$ increases. This sign inversion is qualitatively understood from an analysis of the balance equation (28) and is mathematically described by Eq. (30). In the diffusive limit and for $l_T \gg l_P$, Eqs. (29) and (30) formally coincide with those obtained previously\textsuperscript{27,29} for coupled 2D systems. Besides, some recent studies of transport phenomena in coupled 1D layers, namely transport without backscattering in tunnel-coupled edge states\textsuperscript{30,31} and Coulomb drag between quantum wires in the ballistic regime\textsuperscript{21}, constitute limiting cases of the more general results given by Eqs. (29) and (30).

One should stress the importance of the phase-breaking processes that suppress the tunnel coherence. In our model, i.e., without elastic scattering, these processes are proved to be much more frequent than the backscattering processes. This allowed us to distinguish two transport regimes: the pure ballistic regime, without any scattering, and the pseudo-ballistic one, without backscattering but with essential forward-scattering due to electron-electron interaction, and with suppressed coherence. If one considers just a single wire, there is no difference between these regimes as concerns the electrical properties: the resistance is equal to $G_0^{-1}$ in both cases. However, the electrical properties of a tunnel-coupled double-wire system behave differently as one passes from one regime to another, because the contribution of the tunneling to the electrical properties becomes different. In the ballistic regime, as well as in the transition region between the two regimes ($L \sim l_C$), all calculated resistances oscillate with $L$ and with the level splitting energy $\Delta$ due to interference of the electron waves. One can vary the level splitting energy by applying either a transverse voltage across the wires or a magnetic field perpendicular to the wire plane. In the latter case the oscillations show an Aharonov-Bohm periodicity associated with the magnetic flux penetrating through the area $Lw$ between the wires. The oscillations become exponentially damped as the ratio $L/l_C$ increases. If the tunnel coupling is strong, the oscillations have large amplitudes,
which, from a theoretical point of view, can be much larger than the resistance quantum $G_0^{-1}$. So far the experimentally observed\textsuperscript{9,12} resistance oscillations in tunnel-coupled ballistic quantum wires were of small ($\sim 0.5$ KΩ) amplitude. This is not surprising because there are many factors which compete against the tunnel coherence. Apart from inelastic scattering considered in this paper, there are elastic scattering and long-scale inhomogeneities of the wires which would lead to a coordinate dependence of the level splitting $\Delta$. If these variations of $\Delta$ are larger than the tunneling matrix element, the coherence would be considerably suppressed.

We now discuss the approximations made in this paper. The main approximation is the neglect of elastic scattering. Since this scattering tends to be dominant at low temperatures, the presence of impurities in the 1D channels will considerably modify the transport. The elastic scattering will lead to an increase of backscattering and interference between the forward- and backward-moving electron waves. As a consequence, $R$, $R_{TR}$, $R_{Ts}$ and $R_{Tn}$ will depend on the spatial positions of the impurities in the channels and the regular expressions obtained in this paper will not be valid. A further development of the transport theory for tunnel-coupled wires in the presence of elastic scattering is therefore desirable. On the other hand, advances in the technology of nanostructures, in particular selective doping, can make it possible to achieve structures where the elastic scattering in 1D channels is minimized for wire lengths smaller than a few microns which is the current standard of the impurity mean free path at low temperatures.

Another approximation concerns the transition from the quantum kinetic equation (1) to the semi-classical description given by Eq. (7). It is valid when the spatial scale of the electronic distribution is large in comparison to the Fermi wavelength $\pi\hbar/p_F$. We have seen that this scale is determined either by $\lambda$, given by Eq. (24) for long wires, or by $\lambda_{\pm}$ for short wires. In the case of strong tunnel coupling the characteristic scale is given by $\hbar v_F/\Delta_T$. Therefore, the necessary requirement is fulfilled if the tunneling matrix element $T$, level splitting $\Delta$, and the energy $\hbar/\tau_C$, associated with the smallest scattering time $\tau_C$, are small in comparison to the Fermi energy. These conditions have been assumed throughout the
paper. This also allowed us to neglect the difference between the electron densities in the layers and characterize the electrons in different layers by the same Fermi velocity $|v_p| \simeq v_F$.

The assumption about the adiabatic connection of the wires to the leads, which allowed us to neglect elastic scattering of electrons near the ends of the wires, implies that the Fermi wavelength $\pi \hbar / p_F$ must be small in comparison to the contact lengths, i.e., to the lengths of transition from the leads to the wires. On the other hand, the oscillations associated with the tunnel coherence, cf. Sec. III B, can be seen if the contact lengths are smaller than both $l_C$ and $\hbar v_F / \Delta_T$. In principle, both requirements can be fulfilled.

The next approximation, which allowed us to solve the kinetic equation analytically in the whole range of regimes from ballistic to diffusive, is equivalent to the following statement. In each layer the forward- and backward-moving electrons can be described as weakly coupled sub-systems characterized by their own local chemical potentials. This statement is obvious for the case of pure ballistic or pseudo-ballistic transport, when these potentials are merely dictated by the reservoirs (leads) and do not change with coordinate $x$. When the backscattering becomes essential, this statement is still true if we assume that the forward-scattering events are much more frequent than the backscattering and tunneling events. For example, it is always true for magnetic edge states, where one can completely neglect backscattering, and the introduction of local chemical potentials (see Ref. 31) is well-justified. In our case, a consideration of the electron-electron collision integral allowed us to estimate the characteristic time of the Coulomb-assisted forward scattering, and we find that it is of the order of $\tau_C$, which is small in comparison with both backscattering times $\tau_P$ and $\tau_D$. Thus, the electron-electron interaction provides an effective mechanism for forward scattering and can maintain quasi-equilibrium Fermi distribution functions for forward- and backward-moving electron sub-systems. However, these conditions may be violated when the conducting channels contain impurities with short-range potentials and the elastic backscattering becomes important.

Our evaluation of the characteristic scattering times from the collision integral has employed only the lowest-order essential contributions of the electron-phonon and electron-
electron interactions, given by the diagrams of Fig. 2 and leading to collision integrals with scattering amplitudes in the Born approximation. While it is normally good for electron-phonon interaction due to the weakness of the coupling constant, a rigorous evaluation of the electron-electron part requires also a consideration of higher-order contributions, given by more complex diagrams, because the ratio of the Bohr energy to Fermi energy $\varepsilon_F$, which is the parameter of the perturbation expansion for the Coulomb interaction, is not small. Nevertheless, using the Born approximation in the evaluation of the drag time is still reasonable if the momentum $2p_F$ transferred in backscattering is large and the electron-electron backscattering probability is small. As concerns $\tau_C$, it is determined by forward-scattering processes with small momentum transfer and the Born approximation is not well justified.

On the other hand, our theory leads to a non-divergent expression (A14) for $\tau_C$ and gives, for typical parameters of the electron system, physically reasonable values. We remind that in our theory both $k_B T_e$ and $\Delta$ are much smaller than $\varepsilon_F$. Therefore, one may expect that Eq. (A14) provides a correct order-of-magnitude estimate of the phase-breaking time caused by electron-electron interaction.

Finally, we stress that the results obtained in this paper hold for a normal Fermi-liquid state of the electron system. If the electrons in the wires are in the Luttinger-liquid state, these results have to be reconsidered.

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APPENDIX

Below we give a microscopic calculation of the characteristic times $\tau_P$, $\tau_D$, and $\tau_C$. The coordinate index $x$ in the Green’s functions and self-energies is omitted and $\hbar$ is set equal to 1. The normalization lengths are also set equal to 1. The electron-phonon self-energies given by the diagrams shown in Fig. 2 (a) are explicitly expressed as
\[ \Sigma_{\alpha\beta}(p) = i \sum_{Q} \frac{d\omega}{2\pi} G_{\alpha\beta}^{ij',\varepsilon} - \omega(p - q)D_{\alpha\beta}^{ij}(\omega, Q)M_{\alpha\beta}^{e-\text{ph}}(Q)M_{\alpha\beta}^{e-\text{ph}}(-Q), \]

where the unperturbed Green’s functions of phonons \(D^+\) and \(D^-\) (we do not need \(D^{--}\) and \(D^{++}\) in the following) are given as

\[ iD_{\alpha\beta}^{\varepsilon\pm}(\omega, Q) = 2\pi \left[ NQ\delta(\omega \mp sQ) + (1 + NQ)\delta(\omega \pm sQ) \right] \]

and the matrix elements of the electron-phonon interaction are

\[ M_{\alpha\beta}^{e-\text{ph}}(Q) = i\sqrt{E_1^2Q/2\rho s} J_{\alpha\beta}^{e-\text{ph}}(q_y, q_z), \]

\[ J_{\alpha\beta}^{e-\text{ph}}(q_y, q_z) = \int \int d\omega d\varepsilon (2\pi)^2 G_{\alpha\beta}^{ij',\varepsilon'} - \omega(p - q)G_{\alpha\beta}^{\varepsilon\pm}(p')G_{\alpha\beta}^{\varepsilon'} + \omega(p' + q), \]

where \(l = 0\) for \(\alpha = \beta\) and \(l = 1\) for \(\alpha \neq \beta\), the factor of 2 comes from the spin summation in the ”loop”. Here

\[ M_{\alpha\beta}^{e-\text{ph}}(q) = (2\epsilon^2/\epsilon) \int \int \int dydy'dzdz'K_0(|q||r - r'|)F_2^2(y, z)F_2^2(y', z') \]

are the matrix elements for electron-electron interaction, \(\epsilon\) is the dielectric constant, \(K_0\) is the modified Bessel function, and \(|r - r'| = [(y - y')^2 + (z - z')^2]^{1/2}\).
For the evaluation of the collision integral we use Eqs. (3)-(6) and express the non-equilibrium part of the matrix Green’s functions according to [see also Eq. (9)]

\[ \delta \hat{G}_{\varepsilon}^{ij} (\pm |p|) = \hat{g}_{\varepsilon}^{\pm} \hat{G}_{\varepsilon}^{A}(p) - \hat{G}_{\varepsilon}^{R}(p) \hat{g}_{\varepsilon}^{\pm}. \] (A6)

The collision integrals are evaluated below assuming of weak tunneling, when the non-diagonal contributions of \( \hat{G}_{\varepsilon}^{R,A}(p) \) are neglected. This approximation is valid when the tunneling matrix element \( T \) is small in comparison to the imaginary part of the self-energies and when the level splitting \( |\Delta| \) is small in comparison to the Fermi energy. Both requirements are assumed fulfilled. Then the components \( [\hat{g}_{\varepsilon}^{\pm}]_{jj'} \) enter only in the corresponding parts \( \delta [\hat{I}(\varepsilon)]_{jj'} \) of the collision integral.

Now we calculate the diagonal parts for the electron-phonon scattering contribution to the collision integral. Taking the self-energy given by Eqs. (A1)-(A3) we find

\[ \int d\varepsilon \delta [\hat{I}^{\text{ph}}_{\varepsilon}]_{jj} = \frac{-(\mu_j^+ - \mu_j^-)}{2 \tau_{Pj}} \] where \( j = l, r \). The phonon-assisted transport time is given by

\[ \tau_{Pj}^{-1} = \frac{E_i^2}{\rho s k_BT_e} \sum_{q_y, q_z} |J^{\varepsilon - \text{ph}}_{jj}(q_y, q_z)|^2 \sum_{p, q(p>0, p<0)} v_p Q \int d\varepsilon \frac{|f(\varepsilon) - f(\varepsilon - sQ)|}{4 \sinh^2(sQ/2\varepsilon)} \]

\[ \times \left[ G_{jj,\varepsilon}^c - sQ(p)G_{jj,\varepsilon}^c(p - q) + G_{jj,\varepsilon}^c(p)G_{jj,\varepsilon}^c(p - q) \right], \] (A7)

where we defined \( G^c = G^A - G^R \). For further evaluation of \( \tau_{Pj} \) we use the free-particle (unperturbed) Green’s functions, i.e., \( G_{il,\varepsilon}^{R,A}(p) = [\varepsilon - \Delta/2 - p^2/2m \pm i0]^{-1} \) and \( G_{rr,\varepsilon}^{R,A}(p) = [\varepsilon + \Delta/2 - p^2/2m \pm i0]^{-1} \), and obtain

\[ \tau_{Pj}^{-1} = \frac{E_i^2}{\rho s k_BT_e} \sum_{q_y, q_z} |J^{\varepsilon - \text{ph}}_{jj}(q_y, q_z)|^2 \int_{-\infty}^{\infty} dq_1 \frac{Q[f(\mu + v_F q_1/2 - sQ/2) - f(\mu + v_F q_1/2 + sQ/2)]}{4 \sinh^2(sQ/2k_BT_e)}, \] (A8)

where \( q_1 = q - 2p_F \) is a small variable. In the limit \( k_BT_e \gg ms^2 \) the integral over \( q_1 \) is easily calculated. In addition, if \( k_BT_e \gg s[(\pi/a)^2 + (2p_F)^2]^{1/2} \), where \( a \) is the wire width, the scattering becomes quasi-elastic, and Eq. (A8) is reduced to a known\(^26\) result
\[
\tau_{Pj}^{-1} = \frac{2E_2^2k_BT_e}{\rho s^2v_F} \int \int dydz F_j^4(y, z) .
\]
(A9)

Since we assume that the confining potentials for the layers are almost identical, the difference between \(\tau_{Pl}\) and \(\tau_{Pr}\) is neglected: \(\tau_{Pl} = \tau_{Pr} = \tau_P\). A numerical estimate, using Eq. (A9) and GaAs material parameters, gives \(\tau_P^{-1} \sim 10^{-2} k_BT_e\).

The electron-electron scattering contribution to the diagonal parts of the collision integrals gives the Coulomb drag terms:

\[
\int d\varepsilon \delta (\hat{\varepsilon}_e - \varepsilon) [\hat{I}_{\varepsilon e}(\varepsilon)]_{jj} = - (\mu_j^\pm - \mu_j^\mp) / 2\tau_D + (\mu_j^\pm - \mu_j^\mp) / 2\tau_D,
\]

where \(j' \neq j\). The drag time is given by

\[
\tau_D^{-1} = \frac{4}{k_BT_e} \sum_{p, p', q} |M_{tr} e^{-e}(q)|^2 v_p \int \int \frac{d\varepsilon d\varepsilon' d\omega}{(2\pi)^2} G_{ll, \varepsilon}^c(p) G_{ll, \varepsilon - \omega}(p - q)
\]

\[
\times G_{rr, \varepsilon'}^c(p') G_{rr, \varepsilon' + \omega}(p' + q) \frac{[f(\varepsilon - \omega) - f(\varepsilon)][f(\varepsilon') - f(\varepsilon' + \omega)]}{4 \sinh^2(\omega / 2k_BT_e)}
\]

The sum here must be evaluated for \(p > 0, p - q < 0, p' < 0, \) and \(p' + q > 0\). The evaluation of Eq. (A10) using the free-particle Green’s functions gives a simple result

\[
\tau_D^{-1} = \frac{k_BT_e}{\pi v_F^2} \left( \frac{2|M_{tr} e^{-e}(2pF)|^2}{\sinh^2(\Delta / 2k_BT_e)} \right) \left( \frac{\Delta / 2k_BT_e}{2k_BT_e} \right)^2
\]

(A11)

One can estimate \(M_{tr} e^{-e}(2pF)\) as \((2e^2/\varepsilon)K_0(2pFw)\), where \(w\) is the distance between the centers of the wires. If \(2pFw \ll 1\), which is easily achieved for \(w \sim 30 \text{ nm}\), \(K_0(2pFw)\) is exponentially small.

Finally, we calculate the electron-electron part of the nondiagonal components of the collision integral. Since the main contribution to it comes from the forward-scattering processes \(|q| \ll p_F\), only such processes are considered below. The integral of \([\hat{I}_{\varepsilon e}(\varepsilon)]_{jj'} (j \neq j')\) over the energy \(\varepsilon\) can be reduced to a sum of three terms characterized by three different statistical factors:

\[
\int d\varepsilon \delta (\hat{\varepsilon}_e - \varepsilon) [\hat{I}_{\varepsilon e}(\varepsilon)]_{jj'} = -2 \int d\varepsilon [g_{\varepsilon}^\pm]_{jj'} v_p \int \int \frac{d\varepsilon' d\omega}{(2\pi)^2}
\]

\[
\times \left\{ (\Lambda_{j,j}^{ARR} + \Lambda_{j,j'}^{RRA}) [f(\varepsilon') - f(\varepsilon' + \omega)] + f(\varepsilon - \omega) [f(\varepsilon + \omega) - f(\varepsilon')] \right\}
\]
\[
\left( \Lambda_{jj'}^{ARRA} + \Lambda_{jj'}^{RAAR} \right) f(\epsilon' + \omega)[1 - f(\epsilon')] + \left( \Lambda_{jj'}^{ARRA} + \Lambda_{jj'}^{RAAR} \right) f(\epsilon - \omega)[f(\epsilon' + \omega) - f(\epsilon')]
\] (A12)

In Eq. (A12) we used the shortcuts

\[
\Lambda_{\alpha\beta\gamma\delta}^{ijj'} = G_{\alpha}^{ij\epsilon}(p)G_{\beta}^{ij\epsilon'}(p')G_{\gamma}^{i'i'}G_{\delta}^{i'i'} + \omega(p - q) \sum_{i} G_{i'i'}^{i'i'}G_{i'i'}^{i'i'} + \omega(p' + q) \times \left[ \left( M_{ij}^{\epsilon\epsilon}(q) \right)^2 - M_{ij}^{\epsilon\epsilon}(q)M_{ij}^{\epsilon\epsilon}(q) \right],
\] (A13)

and neglected the terms with \( \alpha = \beta \) and \( \gamma = \delta \) because they vanish after the summations over \( p \) and \( p' \), respectively. Since \([g_{\epsilon}]_{lt} = [g_{\epsilon}]_{l't} = g_{\epsilon}^{\pm} \), one can see that \( \int d\epsilon \delta[I_{\epsilon}^{\epsilon\epsilon}(\epsilon)]_{jj'} = \int d\epsilon \delta[I_{\epsilon}^{\epsilon\epsilon}(\epsilon)]_{jj'} \).

Calculating the integrals in the expression (A12) within the approximation of the free-particle Green’s functions, we find that the third term on the right-hand side of Eq. (A12) vanishes. The first term diverges for \( \Delta = 0 \) but it is close to zero for \( \Delta \neq 0 \) and can be neglected in the following. In contrast, the second term gives a regular contribution, which can be represented, on account of Eq. (13), as \( \int d\epsilon \delta[I_{\epsilon}^{\epsilon\epsilon}(\epsilon)]_{jj'} = -\mu_{jj'}/\tau_C \). The "nondiagonal" relaxation time \( \tau_C \) (we take into account only its real part) is given by

\[
\tau_C^{-1} = \frac{e^4 S^2 \Delta}{2\pi e^2 v_F^2} \coth \frac{\Delta}{4k_B T_e},
\] (A14)

where

\[
S = -\int d\epsilon d\epsilon' d\epsilon d\epsilon' \ln |r - r'| |F^2_{\epsilon}(y, z)| |F^2_{\epsilon'}(y', z') - F^2_{r}(y', z')|.
\] (A15)

In the calculation we took into account \( M_{li}^{\epsilon\epsilon}(q) \simeq M_{ir}^{\epsilon\epsilon}(q) \) and \( qa \ll 1 \). The last property allowed us to use the approximation \( K_0(x) \simeq -[C + \ln(x/2)] \), where \( C \) is Euler’s constant; we found \( M_{li}^{\epsilon\epsilon}(q) - M_{ir}^{\epsilon\epsilon}(q) \simeq (2e^2/\epsilon)S \). The overlap integral \( S \) can be approximated, to a good accuracy, by \( \ln(w/a) \).

If \( \Delta \gg 4k_B T_e \), the relaxation rate \( \tau_C^{-1} \) given by Eq. (A14) is temperature-independent and proportional to \( |\Delta| \). For \( \Delta \ll 4k_B T_e \), \( \tau_C^{-1} \) is proportional to \( T_e \). A comparison of Eq.
(A11) and Eq. (A14) shows that $\tau_C$ is always much smaller than $\tau_D$, since $\tau_C$ is controlled by forward-scattering processes and does not contain the smallness associated with the factor $[K_0(2p_Fw)]^2$. A numerical estimate also shows that $\tau_C \ll \tau_P$, because of the weakness of the electron coupling to acoustical phonons. For this reason we neglected the contribution of electron-phonon scattering to the nondiagonal part of the collision integral.
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FIGURES

FIG. 1. Schematic representation of two coupled quantum wires.

FIG. 2. Feynman diagrams describing the contributions of electron-phonon (a) and electron-electron (b) interaction to the self-energies.

FIG. 3. Transresistance $R_{TR}$ as a function of the wire length $L$. The solid curves correspond to $l_P/l_D = 0.1$ (weaker drag) and the dashed ones to $l_P/l_D = 1$ (stronger drag). Each curve is marked by the value of $l_P/l_T$.

FIG. 4. Tunneling resistances for symmetric (solid) and non-symmetric (dashed) setups as a function of the wire length $L$, at $1/l_D = 0$. Each curve is marked by the value of $l_P/l_T$. The inset shows the currents (arrows) injected in and coming out of the wires (broad lines) for both cases.
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