Simultaneous multiple-users quantum communication across a spin chain channel

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The time evolution of spin chains has been extensively studied for transferring quantum states between different registers of a quantum computer through their natural time evolution. The main advantage of these protocols is their minimal demand for dynamical control and their resilience against disorder and imperfections [4, 5]. The drawback, however, is the dispersive nature of their dynamics which scrambles the information among various degrees of freedom [6, 7]. Many proposals have been put forward to fix this issue. By engineering the nearest neighbor couplings [8, 10] or tuning long range exchange interactions [11], one can change the dispersion relation into a linear function and achieve perfect state transfer. Simpler designs excite the system only in the linear zone of its dispersion relation and achieve pretty good transfer fidelities [12-14]. Dual rail systems [15, 16] and d-level spin chains [17] can asymptotically reach perfect state transfer. Adiabatic attachment and detachment of qubits [18-20] and their faster versions through a short cut to adiabaticity [21, 22], optimal control [23] and machine learning assisted transfer [24] have also been suggested. Routing information between different nodes of two and three dimensional graphs can be achieved by proper combination of ferro and anti-ferromagnetic couplings [25, 26]. Exploiting projective measurements for encoding [27] and counteracting dephasing [28] can enhance quality of transfer and local rotations [29, 30] may yield in better fidelities or enhanced communication rate. In addition, an important class of protocols rely on inducing an effective end-to-end interaction between the sender-receiver sites through either weak boundary couplings [31-37] or large magnetic fields near the ends [38, 39]. Some of the proposals for quantum state transfer have been experimentally implemented in coupled optical fibers [40, 41], nuclear magnetic resonance devices [42], optical lattices [43] and superconducting quantum simulators [44].

In almost all the existing state transfer protocols only one pair of sender-receivers can use the spin chain channel at each time. This significantly reduces the communication rate as users can only access the channel sequentially and not in parallel, a bottleneck that may ultimately limit the speed of big quantum computers. Although multiple qubit communication [37, 39] have been proposed they have no freedom to adjust the choice of the sender and receiver qubits which are predetermined by the symmetry of the system and thus still work as a single sender-receiver protocol with multiple qubits. In classical communication networks (e.g. telecommunication systems), however, the frequency bandwidth of the channel is divided between multiple users who can use the channel simultaneously. This can be achieved by modulating the signal of each pair of sender-receivers with a different carrier signal, which is a periodic function with a distinct frequency, and send it through a common communication channel. Since each sender’s data lies in a different frequency bandwidth, the corresponding receiver can access the relevant information by using a proper frequency filter. Consequently, crosstalks are prevented and the communication rate is significantly enhanced. A key open question is whether one can develop a quantum counterpart of classical communication systems and allow multiple users simultaneously communicating with each other through a common channel.

In this letter, we address this critical problem by proposing a communication scheme which is based on tuning the local parameters at the sender and receiver sites. These local tunings, proposed in three different strategies, excite different sets

FIG. 1: Schematic. The schematic of simultaneous quantum communication between multiple users across a spin chain channel. By optimizing the local parameters at sender and receiver sites multiple pairs can use the channel simultaneously through exciting different sets of eigenenergies and thus keep the crosstalk negligible.
of energy eigenstates for communication of each pair of users and thus result in high transmission fidelities and negligible crosstalk. We have also proposed to implement our protocol with superconducting quantum simulators.

The Model.– We consider $M$ pairs of sender-receivers in a way that pair $\alpha$ ($\alpha = 1, \cdots, M$) communicate between the qubits $S_\alpha$ (sender) and $R_\alpha$ (receiver). All pairs share a common spin chain data-bus between their sender and receiver sites. A schematic of the system is given in Fig. 1. The goal is to establish simultaneous high-fidelity communication between any pair of $(S_\alpha, R_\alpha)$ while suppressing the crosstalk between $(S_\alpha, R_\beta)$ with $\alpha \neq \beta$. The spin chain channel consists of $N$ spin-$1/2$ particles which interact via Hamiltonian

$$H_{ch} = J \sum_{i=1}^{N-1} (\sigma^x_i \sigma^x_{i+1} + \sigma^y_i \sigma^y_{i+1}) + B_0 (\sigma^z_i + \sigma^z_{i+1}),$$

(1)

where $\sigma^x, \sigma^y, \sigma^z$ are the Pauli operators acting on site $i$, $J$ is the spin exchange coupling and $B_0$ is the magnetic field in the $z$ direction acting only on sites 1 and $N$. All senders (receivers) are coupled to the first (last) site of the channel. The interaction between the user qubits and the channel is given by

$$H_I = J_0 \sum_{i=1}^{M} (\sigma^x_{S_\alpha} \sigma^x_i + \sigma^y_{S_\alpha} \sigma^y_i + \sigma^x_{R_\alpha} \sigma^x_N + \sigma^y_{R_\alpha} \sigma^y_N),$$

(2)

where $J_0$ is the coupling between the users and the channel and $B_0$ is the magnetic field acting on the pair user $\alpha$. Without loss of generality, we assume that the sender $\alpha$ initially sets its qubit in an arbitrary, even possibly unknown, pure state

$$|\psi_{S_\alpha}\rangle = \cos(\frac{\theta_\alpha}{2})|0\rangle + e^{i\phi_\alpha} \sin(\frac{\theta_\alpha}{2})|1\rangle, \quad \alpha = (1, \cdots, M)$$

(3)

where $\theta_\alpha$ and $\phi_\alpha$ are the angles determining the quantum state on the surface of the Bloch sphere. The rest of the spins, including all receivers and the channel, are initialized in $|0\rangle$. Therefore, the state of the whole system becomes

$$|\Psi(t)\rangle = |\psi_{S_\alpha}\rangle \otimes \cdots \otimes |\psi_{S_M}\rangle \otimes |0_{ch}\rangle \otimes |0_{n_1}\rangle \otimes \cdots \otimes |0_{n_M}\rangle,$$

(4)

where $|0_{ch}\rangle = |0, \cdots, 0\rangle$ shows the state of the channel. Since this quantum state is not an eigenstate of the total Hamiltonian $H=H_{ch}+H_I$, it evolves as $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$. At any time $t$ the state of the receiver sites is given by $\rho_{n_\beta}(t) = T_{R_\beta}(\langle \Psi(t) | \langle \Psi(t) |)$, where $T_{R_\beta}$ means tracing over all sites except $R_\beta$. To quantify the average fidelity between the sender $\alpha$ and receiver $\beta$ we define a fidelity matrix as $F_{\alpha\beta}(t, \Theta) = \langle \psi_{S_\alpha} | \rho_{n_\beta}(t) | \psi_{S_\alpha} \rangle$ ($\alpha, \beta = 1, \cdots, M$), where $\Theta = \{\theta_1, \cdots, \theta_M, \phi_1, \cdots, \phi_M\}$ accounts for the input parameters of the senders. To get an input-independent quantity one can take the average of these fidelities over all possible initial states on the surface of the Bloch spheres for all $M$ users

$$\bar{F}_{\alpha\beta}(t) = \int F_{\alpha\beta}(t, \Theta) d\Omega_1 \cdots d\Omega_M,$$

(5)

where $d\Omega_\alpha = \frac{1}{4\pi} \sin(\theta_\alpha) d\theta_\alpha d\phi_\alpha$ is the normalized $SU(2)$ Haar measure. For our Hamiltonian $H$ that conserves the total number of excitations, we provide a general form of $\bar{F}_{\alpha\beta}(t)$ in the first section of the Supplementary Materials. The diagonal term $F_{\alpha\alpha}(t)$ quantifies the average fidelity of the transmission between the sender-receiver $\alpha$ and the off diagonal term $\bar{F}_{\alpha\beta}(t)$ with $\alpha \neq \beta$ accounts for the crosstalk between the users $\alpha$ and $\beta$. Our goal is to maximize the transmission fidelities $\bar{F}_{\alpha\beta}$ at a certain time $t=\tau$ while simultaneously keeping the crosstalk fidelities $F_{\alpha\beta}$ around 0.5 (i.e. no crosstalk), through controlling the Hamiltonian parameters $B_0$, $J_0$ and $B_\alpha$’s. Our protocol can be understood in two steps. The first step is to induce an effective end-to-end transmission between the senders and the receivers, namely confining the excitations to the subspace $\{S_1, \cdots, S_M, R_1, \cdots, R_M\}$ and leaving the channel close to $|0_{ch}\rangle$ at all times, by either decreasing $J_0/J$ or increasing $B_0/J$ [28, 31–34, 34] (see the second section of the Supplementary Materials for more details). The second step is to separate the communication between each of the $M$ pairs, by tuning $B_\alpha$’s individually. In the following we, first, restrict ourselves to the case of two pairs, i.e. $M=2$, and consider three different strategies to maximize $\bar{F}_{\alpha\beta}$ with minimum crosstalk. Then, we extend the results to larger $M$.

Strategy 1 ($B_0=0$).– In the first scenario, inspired by Refs. [28, 31, 34] for single user end-to-end communication, we put $B_0=0$ and consider $J_0 \ll J$. This choice of parameters creates an effective direct interaction between the sender sub-
space \{S_1, S_2\} and the receiver ones \{R_1, R_2\}. To suppress the crosstalk and block the flow of information between the two subspaces of \{S_1, R_1\} and \{S_2, R_2\} we apply external fields \(B_1\) and \(B_2\) to make them energetically off-resonant from each other. By sitting at the site \(R_1\) one can see the information arrives from both senders. In Fig. 3(a) we plot the average fidelities \(F_{11}\) and \(F_{21}\) as functions of time in a chain of \(N = 20\) when the parameters are tuned to \(B_1/J = -B_2/J = 1\) and \(J_0/J = 0.04\). The corresponding quantities for the receiver \(R_2\), namely \(F_{22}\) and \(F_{12}\), are plotted in Fig. 3(b). As the figures show, \(F_{11}\) and \(F_{22}\) evolve and at certain times they peak to very high values. It is worth noting that the fast oscillations in the average fidelities are caused by the local magnetic fields \(B_1\) and \(B_2\). In practice, even if the receiver cannot take the state precisely at \(t = \tau\) these fast deterministic rotations can be easily corrected by a local operation. Interestingly, the average fidelities \(F_{12}\) and \(F_{21}\) remain low and oscillate around 0.5 resulting in negligible crosstalk between the two communicating parties. In order to optimize the parameters, one can fix a time window, e.g. we choose \([0, 500)/J\), for the dynamics of the system and then optimize the other parameters to achieve the maximum transmission fidelities and minimum crosstalks. By fixing the time window, one can find optimal values for all the Hamiltonian parameters (namely \(J_{opt}^B\), \(B_1^{opt}\) and \(B_2^{opt}\) as well as the times \(\tau = \tau_1\) at which receivers should take their quantum states. The corresponding transmission fidelities, for optimal parameters, are \(F_{11}^{max} = F_{22}^{max}(\tau_1)\).

In Fig. 3(c) we plot the transmission fidelity \(F_{11}\) as well as the crosstalk fidelity \(F_{21}(\tau_1)\) as functions of \(N\). For channels up to \(N = 32\) sites the fidelity \(F_{11}\) is always above 0.9, even reaching 0.98 in several cases, while the fidelity \(F_{21}(\tau_1)\) remains around 0.5 showing negligible crosstalk. The same situation exists for receiver 2 as depicted in Fig. 3(d) for \(F_{22}\) and \(F_{12}(\tau_2)\). In Fig. 3(e) we plot the optimal times \(\tau_1\) and \(\tau_2\) as functions of \(N\) which show that the two transmissions happen within similar time scales. This is because the separation between the senders and receivers are identical for both parties. For the sake of completeness, we present the optimal values of the coupling \(J_0^{opt}\), obtained for the chosen time window, as a function of \(N\) in Fig. 3(f). The optimal coupling is found to be around \(J_0^{opt}/J \sim 0.05\) for almost all values of \(N\) and weakly depending on the length which is consistent with the results of Ref. [28], for single user communications. Similarly, the optimal values for the local fields are obtained as \(B_1^{opt}/J = -B_2^{opt}/J = 1\) and is almost independent of \(N\).

**Strategy 2** \((J_0=1)\).– Our second strategy is adopted from [38] [39] and is accomplished by applying a strong field \(B_0\) on the ending sites of the channel and instead keep the couplings uniform, i.e. \(J_0 = J\) (see Fig. 1). To see the attainable fidelities for this strategy we plot \(F_{11}\) and \(F_{21}\) as functions of time in Fig. 3(a) and similarly for \(F_{12}\) and \(F_{22}\) in Fig. 3(b) for a chain of \(N = 20\) in which \(B_0/J = 24\) and \(B_1/J = -B_2/J = 1\). As figures show the transmission fidelities \(F_{11}\) and \(F_{22}\) reach very high values, while rapidly oscillating due to the presence of local fields. Remarkably, the crosstalks \(F_{12}\) and \(F_{21}\) fluctuate around 0.5, showing very small crosstalk. Analogous to the previous strategy, one can optimize the time of the evolution as well as the Hamiltonian parameters (namely, \(B_0, B_1\) and \(B_2\)) within a chosen time window, here again \([0, 500)/J\). In Fig. 3(c) we report the maximum transmission fidelity \(F_{11}^{max}\) and the crosstalk fidelity \(F_{21}(\tau_1)\) as functions of \(N\). Likewise, for the receiver site \(R_2\), we plot \(F_{22}^{max}\) and \(F_{12}(\tau_2)\) as functions of \(N\) in Fig. 3(d). Both of these figures show that while the transmission fidelities for both parties achieve above 0.95 the crosstalks between them remain negligible. The optimal times for the transmissions are shown in Fig. 3(e). The reason that the optimal times oscillate with length is because the chosen time window allows for several peaks and their maximum changes as the length vary. Finally, in Fig. 3(f), we present the optimal values of the field \(B_0^{opt}\) for different system sizes which shows that by tuning \(B_0/J \sim 30 – 40\) the system can operate optimally. Interestingly, the optimal values for \(B_1\) and \(B_2\) are very close to the previous strategy, i.e. \(B_1/J = -B_2/J = 1\).

**Strategy 3.**– The last scenario is a hybrid of both outlined strategies and the performance of the channel is investigated when both \(B_0\) and \(J_0\) are optimized. Again we fix the time window to \([0, 500)/J\) and optimize the time and the parameters \(B_0, J_0, B_1, \) and \(B_2\) to maximize the transmission fidelities.
and minimize the crosstalk. In Fig. 4(a) we report the maximum attainable fidelities $F_{11}(\tau_1)$ and $F_{22}(\tau_2)$ as functions of length $N$. These quantities are obtained at optimal times $\tau_1 \in [1, 500]/J$ (b), optimal $B_{\mu\nu}^{opt}/J \in [1, 40]$ (c) optimal $J_0^{opt}/J \in [0.01, 1]$ (d) and optimal magnetic fields $B_1^{opt}/J = -B_2^{opt}/J = 1$.

![Strategy 3](image)

**TABLE I: Comparing different strategies.** Maximal average fidelities $F_{\mu\nu}^{max}$, for three outlined strategies.

| $N$ | $F_{11}^{max}$ (Strategy 1) | $F_{11}^{max}$ (Strategy 2) | $F_{11}^{max}$ (Strategy 3) |
|-----|-----------------------------|-----------------------------|-----------------------------|
| 10  | 0.919                       | 0.959                       | 0.982                       |
| 15  | 0.914                       | 0.972                       | 0.980                       |
| 20  | 0.971                       | 0.966                       | 0.973                       |
| 25  | 0.903                       | 0.971                       | 0.971                       |
| 30  | 0.901                       | 0.951                       | 0.961                       |

**Multiple users.**— The proposed protocol with all the three strategies can be generalized to more than two users. The price that one has to pay is to consider a bit longer time window. No matter how many users we consider, one can always tune the parameters to keep the crosstalks negligible. For instance, in a system of length $N=6$ three users can simultaneously communicate with transmission fidelities $\bar{F}_{\alpha\nu}>0.98$ ($\alpha=1, 2, 3$) while keeping the crosstalks $\bar{F}_{\beta\gamma}<0.53$ within the time scales of $500<Jr<1000$ by tuning $B_0/J=1$, $B_0/J=35$, $B_1/J=1.5$, $B_2/J=-1$ and $B_3/J=0.5$. More details are provided in the last section of the Supplementary Materials.

**Bi-localized eigenstates.**— The main reason behind the achievement of high transmission fidelities and low crosstalk is the emergence of bi-localized eigenstates whose excitations are mainly localized at sender and receiver sites. Since these bi-localized eigenstates are the only ones involving in the dynamics of the system, the channel mostly remains in the state $|0_{1ab}\rangle$. Consequently, an effective end-to-end interaction is generated between the sender and receiver qubits. The emergence of bi-localized qubits is mainly due to the engineering of $J_0$ and $B_0$ and then, to minimize the crosstalk, further localizing the excitations between each pair $(S_\alpha, R_\alpha)$ is achieved by tuning $B_\alpha$'s. A detailed discussion about this is available in the second section of the supplemental materials.

**Experimental proposal.**— The best physical platform to provide the XX Hamiltonian with the required controllability of our protocol is superconducting coupled qubits. Recently, they have been used for simulating non-equilibrium dynamics of many-body systems for single-user perfect state transfer [44], many-body localization [45, 46], spectrometry [47] and quantum random walks [48]. In such devices, the exchange coupling varies between $J \sim 10–50$ MHz, the dephasing time is $\sim 10–20$ $\mu$s and the local energy splitting, equivalent to magnetic fields in our protocol, can be tuned up to 800 MHz (namely $B/J \sim 20$) [44,48]. Adopting our strategy 2, for a system of length $N=8$ and exchange coupling $J=50$ MHz, one can tune the energy splittings to be $B_1 = -B_2 = 50$ MHz (i.e. $B_1/J = -B_2/J = 1$) and $B_0=600$ MHz (i.e. $B_0/J=12$). These parameters result in $\bar{F}_{\alpha\nu} \approx 0.95$ for optimal time of $\tau_\alpha \approx 2.5$ $\mu$s which is within the coherence time of the system.

**Conclusion.**— Spin chains provide fast and high quality data-buses for connecting different registers and processors in a quantum computer. However, in the absence of simultaneous communication between different pairs of sender-receivers the speed of computation will be ultimately limited by the waiting time required for the sequential use of the channel. In this letter, we address this key issue by introducing a protocol for simultaneous quantum communication between multiple users sharing a common spin chain data-bus. Our proposal, presented in three different strategies, is based on creating an effective end-to-end interaction between each pair of sender-receivers and yields very high transmission fidelities. In each proposed strategy, different sets of local parameters are optimized so that each pair of users communicate through a different energy eigenstate of the system. Since the energy of each communication channel is off resonant with the others the crosstalk is negligible. While all the three strategies provide high transmission fidelities, the third strategy which is a hybrid control of both the coupling and the magnetic field outperforms the others. Our protocol can also be realized in current superconducting quantum simulators.

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SUPPLEMENTARY MATERIALS

1. Average Fidelity Matrix for Excitation Conserving Hamiltonians

The key mathematical objects needed to analyze the performance of simultaneous multi-party quantum communication are $F_{ab}(t)$ with $a, b = 1, \ldots, M$ evaluated by integration over the Bloch sphere of all possible pure input states. To obtain a general form of these quantities, let’s start by rewriting the initial state of the total system in the form

$$\Psi_0 = \sum_i a_i(\Theta) |i\rangle \otimes |0_{ch}\rangle \otimes |0_R\rangle \tag{S1}$$

where the vectors $|i\rangle = |i_1, \ldots, i_M\rangle$ ($i_\alpha = 0, 1$), $|0_{ch}\rangle = |0, \ldots, 0\rangle$ and $|0_R\rangle = |0, \ldots, 0\rangle$ denote the state of the senders, channel and receivers, respectively. The coefficient $a_i(\Theta)$ is an abbreviation for $a_i(\Theta) = a_{i_1, \ldots, i_M}(\Theta)$ and contains all the parameters $\Theta = \{\theta_1, \ldots, \theta_M, \phi_1, \ldots, \phi_M\}$ which are inputted by the $M$ senders. Considering the evolved state of the overall system as $\rho(t) = U[|\Psi_0\rangle\langle\Psi_0|]$, the output state of each receiver can be obtained by tracing out the other qubits as

$$\rho_{\alpha}(t) = \sum_{i, j} a_i(\Theta)a_j^*(\Theta)\Gamma_{i,j}(t), \quad \alpha = 1, \ldots, M \tag{S2}$$

where $\Gamma_{i,j}(t) = Tr_R[U||i\rangle\langle j| \otimes |0_{ch}\rangle \otimes |0_R\rangle \langle 0_{ch}||0_R\rangle \langle 0_R|]$. Substituting Eq. (S2) in the fidelity $F_{ab}(t, \Theta) = \langle \Psi_{ab} | \rho_{ab}(t) | \Psi_{ab} \rangle$ and taking the average over all possible initial states on the surface of the Bloch spheres for all users, result in

$$F_{ab}(t) = \int F_{ab}(t, \Theta) d\Omega_1 \cdots d\Omega_M$$

$$= \frac{1}{2} + \frac{1}{3} \sum_{i} \sum_{j} \{ \sum_{i_{\alpha} = 0}^{1} \langle 0 | \Gamma_{i,j}^{\alpha} | 0 \rangle - \sum_{i_{\alpha} = 0}^{1} \langle 0 | \Gamma_{i,j}^{\alpha} | 0 \rangle \} + \sum_{i,j} \sum_{i_{\alpha} \neq i_{\beta}} \langle i_{\alpha} | \Gamma_{i,j}^{\alpha} | i_{\beta} \rangle, \tag{S3}$$

where the first and second summations contain all $|i\rangle$ in which $i_\alpha = 0$ and $i_\alpha = 1$, respectively. While the last summation includes all $|i\rangle$ and $|j\rangle$ that only in $i_{\alpha}$ are different.

For the sake of completeness, we present the form of $F_{ab}(t)$ for a protocol with two users (i.e. $M = 2$). In this case the vector $|i\rangle$ belongs to $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. So, using Eq. (S3) results in

$$F_{1b}(t) = \frac{1}{2} + \frac{1}{12} \{ 0 | \Gamma_{0,0}^{00} + | \Gamma_{0,0}^{01} | 0 \rangle - 0 | \Gamma_{1,0}^{10} + | \Gamma_{1,0}^{11} | 1 \rangle + | \Gamma_{0,1}^{00} + | \Gamma_{0,1}^{01} | 0 \rangle + | \Gamma_{1,1}^{10} + | \Gamma_{1,1}^{11} | 1 \rangle \}, \tag{S4}$$

$$F_{2b}(t) = \frac{1}{2} + \frac{1}{12} \{ 0 | \Gamma_{0,0}^{00} + | \Gamma_{0,0}^{01} | 0 \rangle - 0 | \Gamma_{1,0}^{10} + | \Gamma_{1,0}^{11} | 1 \rangle + | \Gamma_{0,1}^{00} + | \Gamma_{0,1}^{01} | 0 \rangle + | \Gamma_{1,1}^{10} + | \Gamma_{1,1}^{11} | 1 \rangle \}. \tag{S5}$$

FIG. S1: Localization for $J_0 \ll J$. (a): The localization value $\langle n_{i_{\alpha}}^{(1)} \rangle$ of eigenstates with one excitation in different position state $i_{\alpha}$. The inverse participation ratio $IPR^{(1)}(\psi_{\alpha}^{(1)})$ (b) and $IPR^{(2)}(\psi_{\alpha}^{(2)})$ (c) as a function of the number of the eigenstates in one and two-excitation subspaces respectively. These quantities are obtained in chains of length $N = 12$ with the Hamiltonian’s parameters which are considered as $B_1/J = -B_1/J = 1$, $B_0/J = 0$ and $J_0/J = 0.01$.

2. Relevant Eigenstates Localized at Boundaries

Making an effective end-to-end transmission between the senders and the receivers will be possible by either decreasing the coupling between users and the chain, i.e. choosing $J_0/J \ll 1$ or applying strong magnetic field $B_0/J$ on the end sites of the chain. In both cases, the excitations confine to the users sites $\{S_1, \ldots, S_M, R_1, \ldots, R_M\}$ and leave the channel almost unexcited at all times. Besides, by tuning the local magnetic fields $B_{\alpha}$, one can further localize the excitations between each pair $\{S_{\alpha}, R_{\alpha}\}$, to minimize the crosstalk. To investigate these issues, we use the inverse participation ratio $IPR^{(1)}(\psi_{\alpha}^{(1)})$ and $IPR^{(2)}(\psi_{\alpha}^{(2)})$ as a function of the number of the eigenstates in one and two-excitation subspaces respectively. These quantities are obtained in chains of length $N = 12$ with the Hamiltonian’s parameters which are considered as $B_1/J = -B_1/J = 1$, $B_0/J = 0$ and $J_0/J = 0.01$. **Making an effective end-to-end transmission between the senders and the receivers will be possible by either decreasing the coupling between users and the chain, i.e. choosing $J_0/J \ll 1$ or applying strong magnetic field $B_0/J$ on the end sites of the chain. In both cases, the excitations confine to the users sites $\{S_1, \ldots, S_M, R_1, \ldots, R_M\}$ and leave the channel almost unexcited at all times. Besides, by tuning the local magnetic fields $B_{\alpha}$, one can further localize the excitations between each pair $\{S_{\alpha}, R_{\alpha}\}$, to minimize the crosstalk. To investigate these issues, we use the inverse participation ratio $IPR^{(1)}(\psi_{\alpha}^{(1)})$ and $IPR^{(2)}(\psi_{\alpha}^{(2)})$ as a function of the number of the eigenstates in one and two-excitation subspaces respectively. These quantities are obtained in chains of length $N = 12$ with the Hamiltonian’s parameters which are considered as $B_1/J = -B_1/J = 1$, $B_0/J = 0$ and $J_0/J = 0.01$.**
FIG. S2: Localization for $B_0 > J$. (a): The localization value $|\langle n|e^{(1)}_k|n\rangle|^4$ of eigenstates with one excitation in different position state $|n\rangle$. The inverse participation ratio $IPR^{(1)}(|e^{(1)}_k>|)$ (b) and $IPR^{(2)}(|e^{(2)}_k>|)$ (c) as a function of the number of the eigenstates in one- and two-excitation subspaces respectively. These quantities are obtained in chains of length $N = 12$ with the Hamiltonian’s parameters which are considered as $B_1/J = -B_1/J = 1, B_0/J = 30$ and $J_0/J = J$.

(IPR) \[\textbf{38}\], that will be defined below, to quantify the degree of localization of each Hamiltonian’s eigenstate in different sites. Here, without loss of generality, we restrict ourselves to the case of two users and particularly discuss the locality of eigenstates in qubit sites $|S_1,S_2,R_1,R_2\rangle$. Since XX Hamiltonian considered here commutes with the total spin in $z$ direction, and hence, conserves the number of excitations, the dynamic of overall system, in the case of two users, is restricted to evolve within the zero-, one- and two-excitation subspaces. Let $|n\rangle (n \in \{S_1,S_2,1,\ldots,N,R_1,R_2\})$ and $|n_1,n_2\rangle$ with $n_1 < n_2$ ($n_1 \in \{S_1,S_2,1,\ldots,N,R_1\}$ and $n_2 \in \{S_2,1,\ldots,N,R_1,R_2\}$) denote the positions of the excitations in the one- and two-excitation subspaces, respectively. Moreover, consider $|e^{(1)}_k\rangle$ and $|e^{(2)}_k\rangle$ as the sets of the eigenvalues, in increasing order, and the corresponding eigenstates of $H^{(\mu)} (\mu = 1,2)$ which in turn, is the total Hamiltonian within the $\mu$-excitation subspace. Since the type and the number of eigenstates of $H^{(1)}$ and $H^{(2)}$ are different, the IPR should be considered separately in each subspace. In one-excitation subspace, the degree of localization of a given eigenstate $|e^{(1)}_k\rangle$ can be calculated by the $IPR^{(1)}$, defined as

\[
IPR^{(1)}(|e^{(1)}_k\rangle) = \frac{1}{\sum_n |\langle n|e^{(1)}_k|n\rangle|^4}.
\]

When the eigenstate $|e^{(1)}_k\rangle$ is highly localized, i.e. $|\langle n|e^{(1)}_k|n\rangle|$ is nonzero for only one particular position state $|n\rangle$, Eq. \[\textbf{S6}\] gets its minimum value, 1, and when the eigenstate is uniformly distributed on all sites, this quantity attains its maximum value, $N$. Likewise, for the eigenstates of $H^{(2)}$, the $IPR^{(2)}$ is

\[
IPR^{(2)}(|e^{(2)}_k\rangle) = \frac{1}{\sum_{n_1,n_2} |\langle n_1,n_2|e^{(2)}_k|n_1,n_2\rangle|^4}.
\]

Analogues to the previous case the minimum value of $IPR^{(2)}(|e^{(2)}_k\rangle)$ is equal to 1 which indicates that the eigenstate $|e^{(2)}_k\rangle$ is completely localized in a specific position state $|n_1,n_2\rangle$ and it’s maximum value, $O(N^2)$, appears when excitations are distributed on all sites uniformly. In the following we exploit the $IPR^{(1)}$ and $IPR^{(2)}$ to peruse the localization of the Hamiltonian’s eigenstates for the first and second strategies outlined in the main paper.

The first strategy is based on weakly coupling the users to the chain (i.e. $J_0/J \ll 1$ and $B_0 = 0$). The degree of localization for Hamiltonian’s eigenstates in one-excitation subspace, $IPR^{(1)}$, in a chain of length $N = 12$, is reported in Fig. \[\textbf{S1}\](b). Clearly, two couples of degenerate eigenstates $|e^{(1)}_k\rangle$ are highly localized with $IPR^{(1)} = 2$. By considering the numerator of $IPR^{(1)}$, i.e. $|\langle n|e^{(1)}_k|n\rangle|^4$, which is plotted in Fig. \[\textbf{S1}\](a) as a function of $n$ and $k$, one can find that the excitations of these eigenstates are strongly localized on sites $(S_1,R_1)$ and $(S_2,R_2)$. Analogues results can be obtained for eigenstates with two excitations. In Fig. \[\textbf{S1}\](c) the localization’s degree $IPR^{(2)}$ as a function of $k$ is plotted. Strong localization $IPR^{(2)} = 1$ take places for two eigenstates at position states $(S_1,R_1)$ and $(S_2,R_2)$. Besides these two, there are eigenstates with middle energies that show non-negotiable localization, i.e. $IPR^{(2)} < 10$. Our results show that these eigenstates have remarkable overlap only with $(S_1,S_2),(S_1,R_2),(S_2,R_1)$ and $(R_1,R_2)$. Note that in producing Fig. \[\textbf{S1}\] Hamiltonian’s parameters are set as $B_1/J = -B_2/J = 1$ and $J_0/J = J_0 = 0.01$. We remark that the number of high-localized eigenstates is slightly dependent on $N$ and by increasing the chain’s length varies from 2 to 6 in the one-excitation subspace, due to the reduction of the energy gaps between eigenstates.

Excitation confinement to the users’ qubits can be also established by applying magnetic field $B_0$ on the end sites of the chain (corresponding to the second strategy outlined in
Our results show that, while there are three eigenstates with $IPR^1 = 2$ that completely overlap with three states $|S_1, R_1\rangle$ and $|S_2, R_2\rangle$, the others with remarkable localization (i.e. $IPR^2 < 10$) have superposition with the states belong to $(|S_1, 1\rangle, |1, R_1\rangle, |S_1, N\rangle, |N, R_1\rangle, |S_2, 1\rangle, |1, R_2\rangle, |S_2, N\rangle, |N, R_2\rangle, |S_1, S_2\rangle, |S_1, R_2\rangle, |S_2, R_1\rangle, |R_1, R_2\rangle)$. Note that in Fig. S2(c), the localization of $|\phi_2^{(2)}\rangle$ is also considered. While there are three eigenstates with $IPR^1 = 2$, the price that one has to pay is to consider a bit longer time window.

3. Communication with Three Users

For more than two pairs of sender-receivers that share a spin chain channel simultaneously, high quality state transfer can be also achieved by tuning the Hamiltonian parameters. Considering 3 users, in Fig. S3(a) we plot the time evolution of different average fidelities $\bar{F}_{11,21,31}$ in a system of size $N = 6$. In providing this plot the parameters are set as $J_0/J = 0, B_0/J = 35, B_1/J = 1.5, B_2/J = -1$ and $B_3/J = 0.5$. The corresponding quantity for the receivers 2 and 3, namely $\bar{F}_{22,12,32}$ and $\bar{F}_{23,13,33}$ are plotted in Fig. S3(b) and (c) respectively. As the figures show, while $\bar{F}_{\alpha\alpha}$ evolves to get its maximum over than $0.98$ in a particular time $\tau_\alpha$ belonging to the window $[500, 1000]/J$, crosstalks $\bar{F}_{\alpha\beta}$ ($\alpha \neq \beta$) gently fluctuate around 0.5. Such results are independent of the strategy type and can be obtained with all the three strategies even to more than three users. The price that one has to pay is to consider a bit longer time window.