Nonachromaticity and reversals of topological phase as a function of wavelength

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Abstract

Contrary to the property of achromaticity (independence of wavelength) usually associated with topological phases, we describe conditions under which topological phases encountered in optics can show sharp changes and reversals for small changes in wavelength, a phenomenon originating in the occurrence of phase singularities, earlier observed in interference experiments.

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Achromaticity of the topological phase arising from polarization transformations of light [1] has been the basis of several applications [2]. Achromatic retarders based on Pancharatnam’s work are routinely used in Astronomical polarimetry. Using the standard example of the “QHQ retarder”, where Q and H are quarterwave and halfwave retarders, the topological origin of achromaticity was explained in ref.[3] where it was also shown that at wavelengths far removed from the design wavelength $\lambda_0$ of the QHQ retarder it can show the opposite behaviour i.e. sharp changes and reversals of phase retardation originating in phase singularities [4, 5, 6]. Direct interferometric observations of phase singularities were reported in [5, 6]. The new result I wish to report here is that such sharp changes and reversals of phase retardation of a QHQ retarder can in fact be made to occur at wavelengths arbitrarily close to $\lambda_0$. This would happen if the retardation of Q and H were equal to $(2n + 1/2)\pi$ and $(4n + 1)\pi$ respectively, where n is an integer which could be made large.

A simple way to understand the effect is to imagine a standard two-slit interference experiment in which fringes are formed on a screen placed some distance away from the slits. A QHQ retarder is placed in the path of one of the two beams. Let the eigenstates of the retarder at $\lambda_0$ be $|X\rangle$, i.e. linear polarization along the x-direction. For an $|X\rangle$-polarized source illuminating the slits, the intensity on the screen, by Pancharatnam’s prescription, is given by

$$ I = I_1 + I_2 + 2\sqrt{(I_1I_2)} Re(<X | U | X > e^{i\phi}) $$

(1)

where U is the $2\times2$ unitary polarization transformation matrix representing the QHQ device and $\phi$ is a path-dependent phase varying along the screen. U is a function of two parameters, (i) the retardation $2\delta$ of Q (hence $4\delta$ of H) at the wavelength $\lambda$, assuming a dependence $\delta = (\lambda_0/\lambda)(\pi/4)$, (ii) $\rho$, the
orientation of the fast axis of H with respect to the X-direction. The modulus and phase of the complex visibility function $\gamma = \langle X \mid U \mid X \rangle$ determine respectively the fringe contrast and the phase shift caused by the QHQ device. The assumed $\lambda$-dependence of the retardation results when the two refractive indices of Q and H are independent of wavelength. It was shown in ref.[3] that (1) when $2\delta = (2m + 1)(\pi/4)$, which happens when $\lambda = (2\lambda_0)/(2m + 1)$, phase singularities occur at the HWP orientations $\rho = m\pi$; m being an integer. At these values, the modulus of $\gamma$ goes to zero leading to zero contrast of the fringes and its phase undergoes a sharp jump of magnitude $\pi$. In a closed circuit around any one of these singularities, the phase changes by $\pm 2\pi$. Such phase changes have been verified experimentally in interference experiments [5, 6] (2) When $2\delta = 2m\pi - 3\pi/2$, which happens when $\lambda = \lambda_0/(4m - 3)$, the visibility is constant in magnitude and its phase increases linearly with $\rho$, the device acting as a pure phase retarder. (3) When $2\delta = 2m\pi - \pi/2$, which happens when $\lambda = \lambda_0/(4m - 1)$, the visibility is constant in magnitude and its phase decreases linearly with $\rho$, the device acting as a pure phase retarder with the opposite sign.

The key point of this paper is that if the retardation of Q and H were chosen to be $2n\pi + \pi/2$ and $4n\pi + \pi$ respectively (multi-order waveplates), the spacing in $\lambda$ at which singularities and hence the phase reversals occur can be made arbitrarily small. If we assume again a $1/\lambda$ dependence of retardation of Q and H, the condition for phase reversal closest to the design wavelength for such a retarder is given by, $(2n\pi + \pi/2)(\lambda_0/\lambda) = (2n\pi + 3\pi/2)$. This gives, for $n = 1$, $\lambda = .71\lambda_0$; for $n = 2$, $\lambda = .82\lambda_0$; for $n = 3$, $\lambda = .87\lambda_0$ and so on. It can be concluded therefore that while topological phases can be made achromatic.
which can be useful, they are not necessary so.

It may also be pointed out that there are other contexts in interferometry wherein $\pi$ phase jumps accompanying zero crossings of a real visibility function can be seen as manifestations of a phase singularity of a complex visibility function in a higher dimensional space. In such an enlarged description, the $\pi$ phase shift acquires a sign that is measurable. We wish to cite one such example.

In an interesting extension of Pancharatnam’s phase criterion to the physics of interference of mixed states, Sjöqvist et.al. [7] have shown that if a beam of particles with internal degrees of freedom, in a mixed state with a density matrix $\rho_0$, is split in a Mach-Zhender interferometer and a unitary transformation $U_i$ acts on the space of $N$ internal states in one of the two paths, a phase difference given by $\arg \text{Tr}(U_i \rho_0)$ is introduced between the two beams. Here “phase difference” is defined as the shift in the maximum of the interference pattern. This quantity, which is measurable, reduces to the phase shift $\arg(<\psi_0|U_i|\psi_0>)$, defined by Pancharatnam [1] for pure states $|\psi_0>$. We wish to point out that the mixed state phase as defined in ref.[7] becomes indeterminate at points in the parameter space where $|\text{Tr}(U_i \rho_0)|=0$. Eqn.(8) in the paper shows that at such points the interference pattern is uniform, with no fringes. Just as in the pure state case discussed above, such points constitute singularities of phase involving discontinuous phase jumps and, depending upon the context, may occur as singular lines or surfaces. The mixed state phase involves singularities in new kinds of parameter spaces which include variables representing decoherence of quantum states or depolarization in case of polarization states of light. One can therefore have measurable $2n\pi$ topological phases resulting from cycles in parameter spaces consisting of degree of
polarization and parameters of unitary transformation of polarization states. Let us note that in the entire discussion above, we have been concerned with the total phase and not just a geometric part of the phase.

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