Coordinated target tracking of two UAVs based on game theory approach and Lyapunov guidance vector fields

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Abstract. Considering the problems of the lack of tracking the intelligent target, which is "smart" enough to escape from the detection by maximizing the estimation error in the current tracking methods by multi-UAVs, a cooperative control method for tracking intelligent target by two UAVs is proposed based on Lyapunov guidance vector field. The mathematical model of an intelligent target is established. According to the multi-UAV distributed intelligent target state fusion estimation method, the target state information is obtained to control two UAVs to circle the target through the Lyapunov guidance vector method. The simulation results demonstrated that the method enables two UAVs to track the intelligent target stably and improve the intelligent target's positioning accuracy effectively.

1. Introduction
For the target's location and tracking, most of the literature is aimed at the common target [1,2]. However, in the actual situation, the target will have a certain degree of antagonism and have anti-tracking or anti-detection, which increases the estimation and prediction error of its state by the sensor, and leads to the failure of tracking [3]. Reference [4,5] proposed the minimax filter (MF) to track the intelligent target using game theory. However, based on the sensor's fixed position, the tracking accuracy will gradually decrease as the target is far away from the sensor.

Cooperative target tracking based on multi-UAV has become an important direction and has been widely studied. The research shows that the target's optimal estimation can be obtained when the line-of-sight angle between the two UAVs and the target is 90° [6,7]. Another difficult problem of coordinated tracking based on multi-UAVs is maintaining the observation configuration of multi-UAVs to obtain the optimal target estimation. Eric W. Frew et al. [8,9] used the LVFG method to control multi-UAVs to keep standoff tracking of stationary and moving targets. Kim et al. [10,11] obtained the optimal observation and achieved stable tracking by controlling the UAV phase on the circle. Reference [12] realism of multi-UAVs cooperative optimal target tracking based on the receding horizon control method.

At present, most of the researches on cooperative tracking of UAV is aimed at ordinary moving targets, but there are few pieces of research on the intelligent target. To meet the requirements of real-time and optimality, combined with the Lyapunov guidance vector field method, this paper puts forward the optimal control method of the intelligent target by two UAVs, which can realize the stable tracking of the intelligent target and improve the positioning accuracy.

2. Target state fusion estimation based on the MF method
It can perceive the sensor's observation and can carry out anti-tracking through strategies with the reason that the intelligent target has the ability of anti-reconnaissance and anti-resistance, so it is hard to estimate the state of the target accurately by using conventional extended Kalman filter and unscented filter. Therefore, according to the intelligent target's motion characteristics, this section uses the MF method to achieve stable tracking of the target.

2.1. Model of intelligent target and sensor observation

The UAV's normal target and observation are modeled as a discrete-time linear time-invariant system defined by the system equation,

\[
x(k+1) = Ax(k) + Bω(k)
\]

\[
z(k) = Cx(k) + v(k)
\]

where \(x(k)\) represents the target state at \(k\) time, \(ω(k)\) and \(v(k)\) is gaussian process noise with zero mean, the covariance matrix of is \(Q\) and \(R\), respectively. \(A\) and \(B\) are transition matrix of the state, and \(C\) is observation matrix of the sensor.

The intelligent target's modeled as [4]:

\[
x(k+1) = Ax(k) + Bω(k) + d(k)
\]

where \(d(k)\) represents the anti-reactance factor of the target at \(k\) time, which is based on the estimated value of the target state by UAV and increases the deviation between the target motion state and its state estimation, which expression is as follows:

\[
d(k) = L \left[ \hat{x}(k) - x(k) \right]
\]

where \(L\) is the counter coefficient. The anti-noise \(η(k)\) is gaussian process noise with zero mean, the covariance matrix of is \(S\). \(\hat{x}(k)\) represents the estimated state of the target at \(k\) time.

The measurement \(z\) of the sensor can be modeled according to the position of the UAV \((x, y)^T\) and the target position \((x_\text{t}, y_\text{t})^T\) as follows:

\[
z(k) = h(x_\text{t}, y_\text{t}) + v_k
\]

\[
\begin{pmatrix}
\sqrt{(x_\text{t} - x_k)^2 + (y_\text{t} - y_k)^2} \\
\tan^{-1}(y_\text{t} - y_k) / (x_\text{t} - x_k)
\end{pmatrix} + v(k)
\]

The linear observation matrix is obtained by the first-order Taylor expansion of the nonlinear observation function \(h(x_\text{t}, y_\text{t})\):

\[
C(k) = \frac{\partial h(x_\text{t}, y_\text{t})}{\partial x_k} \bigg|_{x(x) = \hat{x}(k)}
\]

\[
= \begin{bmatrix}
\frac{\Delta x_k}{r_k} & \frac{\Delta y_k}{r_k} & 0 \\
\frac{\Delta x_k}{r_k^2} & \frac{\Delta y_k}{r_k^2} & 0 \\
-\frac{1}{r_k} & \frac{1}{r_k} & 0
\end{bmatrix}
\]

where \(r_k = \sqrt{(x_\text{t} - x_k)^2 + (y_\text{t} - y_k)^2}\), \(\Delta x_k = x_k - x_\text{t}\), \(\Delta y_k = y_k - y_\text{t}\).

2.2. Game theory approach of tracking the intelligent target state

The method of minimax filter is proposed to estimate the target for the intelligent target. The minimax filter is an unbiased estimator with a form similar to Kalman filtering.

\[
\hat{x}(k+1) = Ax(k) + K \left[ z(k) - C\hat{x}(k) \right]
\]

\(K\) is the filter gain matrix, the error of target state estimation can be expressed as:

\[
e(k) = x(k) - \hat{x}(k)
\]

The error of the update step is:
\[ e(k+1) = x(k+1) - \hat{x}(k+1) \]
\[ = (A-KC+LC)e(k) + B\omega(k) + L\eta(k) - K\dot{\nu}(k) \]
\[ = e^K(k+1) + e^L(k+1) \]

Let \( F = A-KC+LC \), we can get
\[ e^K(k+1) = Fe^K(k) + B\omega(k) - K\dot{\nu}(k) \]
\[ e^L(k+1) = Fe^L(k) + L\eta(k) \]

The error \( e \) should be minimized to obtain a better target state, and the error \( e^L \) is related to the anti-noise of the target. When considering the maximum antagonism of the target, the error \( e^L \) should be maximized, which is the \( e^K \) should be minimized and the \( e^L \) should be maximized. The parameters \( K \) and \( L \) need to be optimized to reach the equilibrium in the zero-sum game, and the following cost function is defined:
\[ J(K, L) = \text{trace}\left( \sum_{k=0}^{\infty} E\left( \| z^K(k) \|^2 - \| z^L(k) \|^2 \right) \right) \]

where \( h \) represents the time range.

Note that the optimal parameters are expressed as \( K^* \) and \( L^* \), the equilibrium point of the zero-sum game satisfies:
\[ J(K^*, L^*) \leq J(K, L^*) \leq J(K^*, L) \]

Let
\[ J(K, L) = \text{trace}\left( \sum_{k=0}^{\infty} P_k \right) \]

where \( P_k = FP_kF^T + BQB^T + KRK^T - LSL^T \).

According to the process of solving and proving in the reference [4], when \( P_k \geq 0 \), \( CP_kC^T + R \geq 0 \) and \( CP_kC^T \geq 0 \) satisfied, the optimal parameters can be obtained as follows:
\[ K^* = A\sum_i C^T R^{-1} \]
\[ L^* = A\sum_i C^T S^{-1} \]
\[ \Sigma_i^{-1} = P_i^{-1} + C^T \left( R^{-1} - S^{-1} \right) C \]

Finally, the iterative calculation formula of state estimation for tracking of intelligent target is obtained.
\[ \hat{x}(k+1) = A\hat{x}(k) + K^* \left[ z(k) - C\hat{x}(k) \right] \]
\[ \Sigma_i^{-1} = P_i^{-1} + C^T \left( R^{-1} - S^{-1} \right) C \]
\[ P_{k+1} = A\Sigma_i C^T + BQB^T \]
\[ K^* = A\Sigma_i C^T R^{-1} \]

2.3. Multi-UAVs distributed state fusion estimation of the intelligent target using game theory approach

In the distributed model, when multiple UAVs observe the target, each UAV \( i \) will have the observation information \( z_{i,k} \) of the target, predicted observations \( \hat{z}_{i,k} \), observation matrix \( C_{i,k} \), and estimated results \( \hat{x}_{i,k} \) and \( P_{i,k} \) at the time of \( k \). The information is exchanged between the UAV. The information sent by the UAV \( i \) is \( z_{i,k}, \hat{x}_{i,k} \) and \( P_{i,k} \), UAV \( i \) receives the information \( z_{j,k}, \hat{x}_{j,k} \) and \( P_{j,k} \), and the estimation expression of the target state fusion composed of multiple UAV is as follows:
\[ \dot{x}_{i,k} = A \dot{x}_{i,k} + A \sum_{j \neq k} \left( C_{i,k} R_{j}^{-1} z_{j,k} - C_{i,k} R_{j}^{-1} \dot{x}_{i,k} \right) \tag{22} \]

\[ \sum_{i,k} = P_{i,k}^T + C_{i,k}^T \left( R_{i}^{-1} - S_{i}^{-1} \right) C_{i,k} \]

\[ \dot{\theta}_d = 4v_d \frac{R_d r^2}{r^2 + R_d^2} \tag{27} \]

where \( N \) represents the number of UAVs.

3. Cooperative tracking of UAVs based on Lyapunov guidance vector field

For two considerations, the tracking strategy of UAV circling the target at a fixed distance is adopted. On the one hand, the speed of the UAV is fast relative to the target, and hovering observation can maintain the observation from different directions of the target, to obtain more observation information of the target; on the other hand, keeping a certain distance from the target can avoid the threat of the target to the UAV.

3.1. Lyapunov guidance vector field

The Lyapunov guidance vector field is:

\[ g(x,y) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dot{x}_d \\ y_d \end{bmatrix} \]

\[ = \begin{bmatrix} v_d \\ \frac{r}{r^2 + R_d^2} \end{bmatrix} \left[ -x_d (r^2 - R_d^2) - y_d (2 \cdot r \cdot R_d) \right] \]

\[ -y_d (r^2 - R_d^2) + x_d (2 \cdot r \cdot R_d) \]

where \( r = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} = \sqrt{x_d^2 + y_d^2} \), \( R_d \) is the spiral radius of the UAV, and it is also the threat radius of the target, \( v_d \) is the desired speed.

The expected heading angle of the UAV is obtained by the guidance vector field

\[ \theta_d = \arctan \left( \frac{\dot{x}_d}{y_d} \right) \tag{26} \]

Design of proportional Controller based on feedback Control principle.

\[ u = \theta_d - k \left( \theta_d - \theta \right) \]

\[ \dot{\theta}_d = 4v_d \frac{R_d r^2}{r^2 + R_d^2} \]

3.2. Cooperative target tracking by two UAVs

Multi-UAVs cooperative target tracking can improve the performance of target tracking. At the same time, it has better robustness and stability. Taking the cooperative observation of two UAVs as an example, when the angle between two UAVs and the target is 90 degrees, it has the optimal observation configuration, improving the tracking effect of the target to the maximum extent. The schematic diagram of two UAVs loitering around the target is shown in figure 1.

\( \theta_1 \) and \( \theta_2 \) represents the phase angle of UAV1 and UAV2 respectively, which clockwise is positive, counter clockwise is negative.
Supposing the Lyapunov function is described as follows:

$$V_p = (\Delta \theta - \theta_d)^2$$

(28)

where $\Delta \theta = \theta_i - \theta_d$ and $\theta_d$ is the desired phase angle.

According to the principle of Lyapunov stability, when the following formula (29) is satisfied, it will converge to the reference phase $\theta_d$:

$$\frac{d}{dt} V_p = 2(\Delta \theta - \theta_d)(\dot{\theta}_2 - \dot{\theta}_1)$$

(29)

To satisfied the formula (29), the phase changes of the UAV are selected as follows.

$$\theta_1 = k \left( \frac{r_2}{r_1} \right)^2 (\Delta \theta - \theta_d) + u_0 / r_j$$

$$\theta_2 = -k \left( \frac{r_2}{r_2} \right)^2 (\Delta \theta - \theta_d) + u_0 / r_j$$

(30)

By substituting formula (30) into formula (29), we can get:

$$\frac{d}{dt} V_p = -4k \left( \frac{r_2}{r_1} \right)^2 V_p \leq 0$$

(31)

From equation (31), we can know that the system can stably converge to the spiral circle. The phase control of each UAV can be obtained as follows:

$$u_{1,1} = k \left( \frac{r_2}{r_1} \right)^2 (\dot{\theta}_2 - \dot{\theta}_1 - \theta_d) r_j + u_0$$

$$u_{1,2} = -k \left( \frac{r_2}{r_1} \right)^2 (\dot{\theta}_2 - \dot{\theta}_1 - \theta_d) r_j + u_0$$

(32)

4. Simulation analysis

4.1. Analysis results of MF

It is assumed that the target is moving with a constant speed at 5m/s and the heading is $7\pi / 36$, the initial position of the target is at (100m,100m), anti-noise variance of intelligent target is $S = \text{diag}(25, 0.05)$, antagonistic coefficient $L = \{0, 1, 0, 0, 1, 0, 0, 0, 0, 0\}$. The position of the sensor is at (0m,0m), which the variance of observation noise is $R = \text{diag}(20, 0.015)$. The simulation interval is 1s and the simulation last 200s.

Compared with the extended Kalman filter (EKF) and unscented information filter (UIF), the following tracking simulation results are obtained and shown.
UAV1 fluctuation, UIF, 700 same results shows trajectory 100m). Trajectory 1100 100 and 400 4 300 40 900 400 900 the estimated 500 160 targets State range the observation intelligent very 300 different assumed to the filters, 120 160 800 (-100m, track of other 80 15m/s~35m/s, 300 3 filters. 700 is 500 to the the 1000 at the 800 The however, larger, target 140 different is 4.1. intelligent the 700 of 600 EKF 20 target is 400 3 when 20 1000 target use 500 300 Root of 120 1000 target UAV position UIF is the 1000 800 180 becomes target 900 the so error state 300 150m, 100 small UAVs and 800 1000 is 800 120 800 100 target's error 600 the 1000 or 400 500 600 loitering 60 600 in target's error 600 the 1000 40 100 300 900 of MF 120 target kept 600 1000 model of 900 bigger, 800 RMSE/m through 180 400 160 of parameter filter in settings is those 900 at 700 80 700 700 1200 of figure 4 1000 of various filters, and the antagonism of targets is greater through EKF and UIF filters. Figure 3 shows the tracking tracks of the intelligent target by MF, EKF, and UIF, respectively. The MF filter can track the target state better, but with time, the tracking error of EKF and UIF to the target becomes larger, so it is impossible to track the target state continuously. It can also be seen from the RMSE of the target position in figure 4 that the target position error estimated by the MF filter is kept in a small fluctuation range. However, the position error estimated by EKF and UIF filter fluctuates in a wide range.

4.2. Cooperative observation by two UAVs
It is assumed that the initial position of the UAV1 is at (-100m, -100m), the initial position of the UAV2 is at (-100m, 100m). The speed range of UAV is 15m/s~35m/s, the loitering radius is 150m, and the other parameter settings are the same as those in 4.1. The simulation results of the Lyapunov guidance vector field are shown in figure 5 ~ figure 10.

Figure 2. Trajectory of intelligent and normal target through MF, UIF and EKF
Figure 3. State estimate of intelligent target through MF, UIF and EKF.
Figure 4. Root mean square error (RMSE) of intelligent target position through MF, UIF and EKF.

Figure 5. State estimate of target by MF, UIF and EKF.
5. Conclusion
This paper presents a control method for two UAVs to track the intelligent target. According to the intelligent target's motion characteristics, this method uses the MF filter to estimate the motion state of the target, which controls the two UAVs to track the intelligent target and improve the tracking accuracy of the intelligent target. The simulation results show that this method can realize the stable tracking of the intelligent target by two UAVs and has good real-time performance, optimality, and realizability.

6. References
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