Heat transport in turbulent Rayleigh-Bénard convection for $Pr \simeq 0.8$ and $Ra \lesssim 10^{15}$

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Abstract. We report experimental results for the heat transport, as expressed by the Nusselt number $Nu$, by turbulent Rayleigh-Bénard convection in a cylindrical sample of aspect ratio $\Gamma \equiv D/L = 0.50$ ($D = 1.12$ m is the diameter and $L = 2.24$ m the height). The measurements are for the Rayleigh-number range $10^{12} \lesssim Ra \lesssim 10^{15}$ and for a Prandtl number $Pr \simeq 0.86$. At these large $Ra$ the results were exceptionally sensitive to details of the experiment. Near $Ra = 10^{15}$ the Nusselt number could be caused to vary over the range $3500 \lesssim Nu \lesssim 5800$ by minor changes in the apparatus or operating procedure.

1. Introduction

The dependence of the Nusselt number $Nu$ upon the Rayleigh number $Ra$ in the limit of large $Ra$ is of interest for at least two reasons. On the one hand, it is expected theoretically that this dependence may change from an effective power law $Nu = N_0 Ra^{\gamma_{eff}}$ with $\gamma_{eff} \approx 0.31$ to one with $\gamma_{eff} \approx 0.39$; this transition is predicted to be associated with a transition of the viscous boundary layers near the top and bottom plates from a laminar to a turbulent condition (1; 2). On the other hand, an extrapolation of laboratory measurements to astrophysically and geophysically relevant ranges of $Ra$, which often are well above $10^{20}$, requires an understanding of $Nu(Ra)$ well above this transition. Such extrapolations at present are uncertain by orders of magnitude as nominally equivalent laboratory experiments using helium gas at temperatures near 5 K and in the range $Ra \lesssim 10^{15}$ (3) or $10^{17}$ (4) disagree significantly with each other, with a transition in $Nu(Ra)$ apparently present in one (3) but not in the other (4).

2. The High Pressure Convection Facility

The uncertain situation described above motivated us to construct a very large convection sample cell, located in an even larger pressure vessel known as the Uboot of Göttingen at the Max Planck Institute for Dynamics and Self Organization in Göttingen, Germany (5). This sample, known as the High Pressure Convection Facility, was cylindrical and of aspect ratio $\Gamma \equiv D/L = 0.50$ ($D = 1.12$ m is the diameter and $L = 2.24$ m the height). The top and bottom
Figure 1. The reduced Nusselt number $\frac{Nu}{Ra^{0.3}}$ as a function of the Rayleigh number $Ra$ on a logarithmic scale. The dashed, solid red, and solid black lines correspond to $\gamma_{\text{eff}} = 0.308, 0.250,$ and 0.355 respectively in the power law $Nu = N_0 Ra^{\gamma_{\text{eff}}}$. The purple solid line corresponds to $\gamma_{\text{eff}} = 0.318$. (a): Data from the “open” sample (see text). The various symbols and colors are for data taken at various pressures and operating conditions. The data on the upper (lower) branch were obtained at a mean temperature $T_m$ of about 21°C (25°C). (b): The data from the open sample are shown again as black symbols. The blue, green, purple, and red symbols are from the “closed” sample (see text). The green and blue data are for $T_m \approx 21^\circ$C and are close to the upper branch of the open sample; the purple and red data are for $T_m \approx 31^\circ$C. They are much higher than the lower branch of the “open” sample but still below the upper branch. The black plusses are data from Ref. (4). The black stars are from (3) as tabulated in (8).

plates were made of copper, and the sidewall was made of Plexiglas and had a wall thickness of 9.5 mm. The pressure vessel, and with it the sample cell, was filled with a suitable gas, usually sulfur hexafluoride (SF$_6$), at pressures up to 19 bars; this gas served as the convection fluid. As described elsewhere (5), the cell was protected by numerous thermal shields designed to prevent parasitic heat flow from the bottom plate to places other than through the sample.

It is apparent that during the filling process the cell can not be sealed and that at least some small leak between it and the Uboot must exist to permit the gas to enter. To this end a small gap of width approximately one mm (which was considered negligibly small compared to the 1120 mm diameter) was initially permitted to exist between the side wall and both the top and the bottom plate. We shall refer to this sample as the “open sample”.

3. Results from the “open sample”

Measurements with the open sample, which had been reported previously (5; 6; 7), are shown in Fig. 1a. The Nusselt number itself varies by over an order of magnitude over our Ra range and is difficult to display with adequate resolution. Thus we show the reduced Nusselt number $\frac{Nu}{Ra^{0.3}}$, which should be nearly constant if $\gamma_{\text{eff}} \approx 0.3$. The various symbols and colors correspond to various sample pressures and operating conditions. One sees that there are two or more branches. The lower (upper) one was attained when the mean sample temperature $T_m$ was higher (lower) than the Uboot temperature $T_U$. Values of Nu between the two extremes could be attained as well by changing $T_m - T_U$, but the data shown correspond reasonably well to the largest and smallest values that could be reached. It was tentatively concluded that this bistability, or stability range, might be due to a “chimney effect”, where the very small difference in average density of the sample gas and the gas in the Uboot (which exists when $T_m \neq T_U$) caused a flow through the 1 mm gap which would be upward for $T_m > T_U$ and downward for $T_m < T_U$. It is unclear why such a small perturbation should cause such dramatic changes of
the heat transport, by a factor of 1.7 or so at the largest Ra. Apparently the system becomes extremely sensitive to external perturbations at these large values of Ra.

4. Results for the “closed sample”

In order to avoid the “chimney effect” discussed above, the sidewall of the sample was sealed completely to the top and bottom plates. We shall call this the “closed sample”. To permit the filling of the sample with gas, we provided a 2.5 cm diameter tube which passed through the sidewall at mid height. One tube end was accurately flush with the inside of the wall. The tube extended outside the sidewall for several cm through a hole in a side shield before it entered a remotely operable valve. Now the filling of the Uboot and sample cell with gas had to be carried out quite slowly to avoid excessive pressure differentials; it typically took two or more days. Once filled, a desired temperature difference between the top and bottom plates was applied with the valve open, and after equilibration for about 12 hours the valve was closed and all desired measurements were made. Data obtained in this manner are shown in Fig. 1b as colored symbols. Also shown there, for comparison, are the data from Fig. 1a, but these "open cell” data are now given as black symbols. We see that the upper branch, obtained with \( T_m - T_U > 0 \), is similar to that found for the open sample; but now the lower branch no longer exists. Instead the purple and red points, obtained with \( T_m - T_U > 0 \), suggest that the “classical” state with \( T_m > T_U \) continues to exist up to much larger Ra when \( T_m > T_U \). At present we have no experimentally supported explanation for the difference between the upper and the classical branch. The upper branch, both for the open and the closed sample, corresponds to \( \gamma_{eff} \approx 0.36 \) (black solid line), which is significantly larger than the value near 0.31 obtained from numerous measurements at lower Ra but a bit smaller than the value 0.38 often associated with the high-Ra regime predicted by Kraichnan (1). The classical branch, on the other hand, is fit well by a power law with \( \gamma_{eff} = 0.318 \) (purple solid line), which is quite consistent with effective exponents obtained in numerous smaller-Ra measurements.

We note with interest that a possible theoretical explanation for the existence of different branches of Nu(Ra), with values for \( \gamma_{eff} \) in the range from about 0.14 to about 0.38, has been given in a recent paper by Grossmann and Lohse (2).

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