Neutron star’s initial spin period distribution

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\textbf{ABSTRACT}

We analyse different possibilities to explain the wide initial spin period distribution of radio pulsars presented by Noutsos et al. With a population synthesis modelling, we demonstrate that magnetic field decay can be used to interpret the difference between the recent results by Noutsos et al and those by Popov and Turolla, where a much younger population of neutron stars associated with supernova remnants with known ages has been studied. In particular, an exponential field decay with $\tau_{\text{mag}} = 5$ Myr can produce a ‘tail’ in the reconstructed initial spin period distribution up to $P_0 > 1$ s starting with a standard Gaussian with $\langle P_0 \rangle = 0.3$ s and $\sigma_P = 0.15$ s. Another option to explain the difference between initial spin period distributions from Noutsos et al. and Popov and Turolla – the emerging magnetic field – is also briefly discussed.

\textbf{Key words:} stars: neutron – pulsars: general.

\section{1 INTRODUCTION}

Initial parameters of neutron stars (NSs) can neither be observed directly nor calculated from the first principles, yet. Such parameters as the initial magnetic field or initial spin period distributions as well as the initial velocity distribution, etc. are derived from the observational data using different assumptions, sometimes via population synthesis modelling (Popov & Prokhorov 2007).

Recently, Noutsos et al. (2013, hereafter N13) presented a detailed analysis of a sample of radio pulsars (PSRs) with relatively well-determined kinematic ages. Assuming standard magneto-dipole formula, they reconstruct the initial spin period distributions for PSRs and compare it with the previous results. In particular, they provide comparison with the distribution proposed by Popov & Turolla (2012, hereafter PT12) for the population of NSs associated with supernova remnants (SNRs) with known ages.

The sample of pulsars used by N13 has average ages of about a few million years. The NSs used by PT12 are much younger. The two initial spin period distributions do not coincide: the one by N13 is wider; even PSRs with initial spin periods $P_0 \gtrsim 1$ s are present. In PT12, the authors provide for many sources just upper limits on $P_0$, and they do not provide the initial spin period distribution. So, it is difficult to compare directly the two distributions. However, as the data in PT12 are shown to be compatible with a Gaussian with $\langle P_0 \rangle = 0.1$ s and $\sigma_P = 0.1$ s, we compare data from N13 with two Gaussians (see Fig. 1). The Kolmogorov–Smirnov test demonstrates that the probability that the data from N13 are compatible with the Gaussian with $\langle P_0 \rangle = 0.1$ s and $\sigma_P = 0.1$ is $4.5 \times 10^{-9}$. Slightly wider Gaussians are also in correspondence with the data set in PT12, so we also compare the data from N13 with the Gaussian $\langle P_0 \rangle = 0.2$ s and $\sigma_P = 0.2$ s. The probability that they come from the same distribution is 0.0582. We thus conclude that distributions from N13 and PT12 cannot correspond to the same population of sources.

In this paper, we discuss how two initial spin period distributions can be brought in correspondence with each other. We propose two possibilities: field decay or emerging magnetic field buried earlier by the fall-back accretion.

\section{2 DECAYING MAGNETIC FIELD}

In the paper by N13, initial periods are reconstructed from the present ones using the magneto-dipole formula with all parameters (magnetic field, angle between spin and magnetic axes) constant in time. In PT12, the authors used the same assumptions, but objects in their sample are about two orders of magnitude younger. For this young sample of objects in SNRs, evolution of the field or of the angle cannot be very significant, but it can influence the other one studied in N13.

Below, we do not speak separately about the angle evolution. Instead, one can assume that the ‘magnetic field’ in the magneto-dipole formula is actually an ‘effective field’, $B_{\text{eff}} = B \sin \chi$, where $\chi$ is the angle between spin and magnetic axes and $B$ is the magnetic field at the NS equator.

The process of field decay in pulsars can explain the long initial spin periods, inferred under the assumption of a constant magnetic field, as earlier braking was faster than it is now. This would widen the reconstructed (under assumption of constant parameters) $P_0$ distribution. Let us give an example.

Rapid exponential field decay on the time-scale of about a few million years is assumed to be excluded for NSs with standard field...
values (magnetars are not discussed here) basing on the population synthesis of PSRs (Faucher-Giguère & Kaspi 2006). However, longer time-scales closer to $10^7$ yr are possible (Pons, Miralles & Geppert 2009). In particular, we choose for an illustration a characteristic time-scale for the exponential field decay equal to 5 Myr, as it was proposed by Gonthier et al. (2002) (see, however, critics in Faucher-Giguère & Kaspi 2006). More complicated models of field decay can also be used. For example, as the NS crust cools down at ages $>1$ Myr the Hall drift may become important (Pons et al. 2009); then the rate of decay depends on details of the NS parameters, but for illustration purposes we prefer to use a monotonic exponential decay which proceeds with the same rate for all NSs.

Note, also, that the distribution with $(P_0) = 0.1$ s and $\sigma_p = 0.1$ s, for which N13 make closer to 10^7 yr are possible (PT12 just as an illustration, not as the best fit. Variants with larger $(P_0)$ and $\sigma_p$ are also possible, as it was noted in that paper. We take the following initial distributions. Spin periods have a Gaussian distribution with $\sigma_p = 0.15$ and mean value 0.3 s. Initial magnetic field has a Gaussian distribution in log with $\sigma_B = 0.55$ and mean value log $B_0/[G] = 12.65$ (Faucher-Giguère & Kaspi 2006; Popov et al. 2010).

The magneto-rotational evolution of PSRs in the population synthesis model was calculated for exponentially decaying magnetic field

$$B = B_0 \exp \left( -\frac{t}{\tau_{\text{mag}}} \right).$$

(1)

As we want to compare our calculations with results from N13, we need to take relatively young (with respect to the whole population of normal PSRs) and not very faraway objects. To do so, we consider only not-too-old pulsars at distances smaller than 5 kpc from the Sun. The latter condition is related to the potential difficulty in estimating the kinematic ages for distant PSRs. In the sample from N13, the majority of objects with estimated $P_0$ have distances $\lesssim 5$ kpc. In addition, among distant objects dim PSRs can be missed, which potentially may result in a bias in the $P_0$ distribution. To avoid this, we exclude distant NSs in our model limiting the distance to a value close to the maximum in the sample from N13. To select young objects, we take PSRs with true ages below $10^5$ yr. On the other hand, since for very young NSs it is difficult to estimate kinematic ages we neglect objects with $t_{\text{true}} < 10^5$ yr (indeed, such NSs are absent in N13).

Let us briefly describe the population synthesis model we use. Pulsars are born with constant rate (the exact value is not important for our study) in four spiral arms which are parametrized by a logarithmic spiral (Vallée 2005). We follow evolution of each pulsar in the model for $5 \times 10^9$ yr. The motion of the pulsar is numerically integrated in the gravitational potential of the Galaxy. The potential is chosen in the same form as in Faucher-Giguère & Kaspi (2006), i.e. $\phi_0(r, z) = \phi_{\text{dis}}(r, z) + \phi_0(r, z) + \phi_0(r, z)$. Here, $\phi_{\text{dis}}$ is the potential of the disc and halo, $\phi_0(r, z)$ is the potential of the bulge and $\phi_0(r, z)$ is the potential of the Galactic nucleus (for details see Kuijken & Gilmore 1989). The kick velocity is sampled from the double-sided exponential distribution

$$p(v) = \frac{1}{2\langle v \rangle} \exp \left( -\frac{|v|}{\langle v \rangle} \right),$$

(2)

with $\langle v \rangle = 180$ km s^{-1} (Faucher-Giguère & Kaspi 2006) and $|v|$ is the absolute value of $v$. Every component of the kick velocity is randomly generated according to the probability function (equation 2). We neglect the Shklovskii effect as well as changes of the relative position of the Sun and spiral arms.

The dispersion measure is calculated according to the NE2001 electron-density model (Cordes & Lazio 2002). The spin period of a pulsar at the moment of observation is calculated by integrating the standard magneto-dipole formula:

$$P^2(t) = P_0^2 + 2K \int_0^t (t')^2 dt'.$$

(3)

Here, $K = 8\pi^2R^6/(31c^3) = 10^{-39}$ cm s^{-1}. $R$ is the NS radius, $I$ is the moment of inertia and $c$ is the speed of light. $B(t)$ is calculated according to equation (1).

A pulsar is considered to be detected if its luminosity exceeds $S_{\text{min}}$ for the Parkes multibeam (Manchester et al. 2001) or Swinburne survey (Edwards et al. 2001) and if it is located within a 15° (full width) stripe along the Galactic plane. To calculate beaming and radio luminosity we use the same assumptions as in the best model in Faucher-Giguère & Kaspi (2006). A pulsar is assumed to be observable till it crosses the death line (Ruderman & Sutherland 1975; Rawley, Taylor & Davis 1986):

$$\frac{B}{P^2} < 0.12 \times 10^{12} \text{ G s}^{-2}.$$

(4)

After we obtained a synthetic population of the observed PSRs, we reconstructed the initial spin period distribution assuming the magneto-dipole formula with constant parameters, i.e. we follow the procedure used by N13:

$$P_0 = P\sqrt{1 - \frac{t_{\text{true}}}{\tau}}.$$  

(5)

Here, $P$ is the present spin period, $\tau$ is the characteristic (spindown) age and $t_{\text{true}}$ is the true age of a pulsar, as computed from the model, in contrast to the approach by N13 where the most probable estimate based on the kinematic age is used. However, this assumption cannot cause problems because typically for PSRs from N13 the kinematic age, $t_{\text{kin}}$, should be very close to the true age.

We use the classical definition of the characteristic age:

$$\tau = \frac{P}{2P}.$$  

(6)
Results of population synthesis simulations are shown in Fig. 2. A histogram shows the reconstructed distribution of \( P_0 \) for the population synthesis model. The population evolved with exponentially decaying magnetic field, \( \tau_{\text{mag}} = 5 \) Myr. Only PSRs with \( d < 5 \) kpc (at the moment of observation) are taken into account. The solid line in both panels represents the true initial distribution of periods used in our calculations. In the left-hand panel, we show results where PSRs with characteristic ages \( > 10^7 \) yr are neglected. In the right-hand panel, objects with \( t_{\text{true}} < 10^7 \) yr are not taken into account. In both plots, objects with \( t_{\text{true}} < 10^7 \) yr are not used.

It is expected that \( P_0 \) calculated with equation (5) is longer in the case of magnetic field decay. Indeed, we can write

\[
P^2(t) = P_0^2 + 2KB_0^2 \int_0^t f^2(t) \, dt,
\]

where \( B_0 \) is the initial magnetic field and \( f(t) \) is the magnetic-field-decay function. Then

\[
\tau = \frac{\tau_{\text{corr}}}{f^2(t)} + \int_0^t \frac{f^2(t) \, dt}{f^2(t)},
\]

where \( \tau_{\text{corr}} = P_0^2/(2KB_0^2) \) is a correction term, and physically this is the ‘initial spin-down age’.

It is known that for normal pulsars (\( B \sim 10^{12} \) G) with ages larger than \( \sim 10^7 \) yr, \( \tau_{\text{corr}} \) is usually much smaller than \( \tau \) and, therefore, changes in the reconstructed \( P_0 \) in equation (5) are determined mainly by the magnetic field decay. In fact, when decay starts to be important, the second term in equation (8) is always larger than the first one and they grow with approximately the same rate. If the field evolution, contained in \( f(t) \), is the same for all PSRs, then when \( \tau_{\text{corr}} \) starts to be insignificant, the characteristic ages of PSRs with the same \( t_{\text{true}} \) are approximately equal (Igoshev 2012). However, \( P \) is still determined by \( B_0 \), see equation (7). So, the reconstructed initial periods appear to be dependent on \( B_0 \).

Results of population synthesis simulations are shown in Fig. 2. As a histogram, we present the reconstructed distribution of \( P_0 \) (to be compared with fig. 17 in N13, see also Fig. 1). The real initial spin period distribution used in the model is shown with a solid line. We also show a variant of this plot where the condition \( t_{\text{true}} < 10^7 \) yr was replaced by \( \tau < 10^7 \) yr. The motivation is that in N13 all but one PSRs with estimated initial spin periods satisfy this condition. As it can be seen, there is virtually no difference between the two panels.

Obviously, the reconstructed distribution looks much different from the real underlying initial distribution, but very similar to the one derived by N13. This demonstrates that the magnetic field decay can potentially explain the difference between two distributions for realistic parameters. This is the main result of our paper. Of course, some fine tuning is possible, but we think that with the current low statistics this is premature.

3 DISCUSSION

3.1 Re-emerging magnetic field

Another possibility to explain the difference between distributions from N13 and PT12 is related to the idea of emerging magnetic field after it has been buried due to strong fall-back accretion (Muslimov & Page 1995; Ho 2011; Bernal, Page & Lee 2012; Viganò & Pons 2012).\(^1\)

The initial spin period distribution derived by N13 for values below \( \sim 0.3-0.4 \) s is not much in contradiction with PT12. The main problem is due to objects with \( P_1 \approx 1 \) s, which are just a few. One way to explain them is to find a physical reason why they are hidden among young sources (analysed by PT12) and visible among the older population (studied in N13). Fall-back which buries the magnetic field can do the job if the field emerges faster than a few hundred thousand years.

Among young NSs in SNRs there is a group of so-called central compact objects (CCOs) for which spin periods are not measured as no pulsations (in radio or/and in X-rays) are observed. They are assumed to be relatives of the so-called antimagnetars (Gotthelf, Halpern & Alford 2013) which have long \( P_0 \approx 0.1-0.5 \) s. Then, we can suspect that their initial spin periods are also long. One can speculate that they are on average longer than those of CCOs with observed pulsations. For example, it is possible to propose a hypothesis that for longer initial spin periods fall-back is more significant, and so the field is buried deeper, and then we do not see pulsations, which are due to the non-isotropic temperature distribution produced by the magnetic field. For example, Güneydaş & Eksi (2013) provide arguments in favour of a correlation between kick and amount of matter accreted during a fall-back episode. On the other hand, Spruit & Phinney (1998) discuss a correlation between the value of kick velocity and initial spin.\(^2\) Additional kick, in their model, can spin up the star, and so the larger the kick, the faster the spin. Then, NSs with smaller initial kicks can have longer spin

\(^1\) About fall-back see, for example, Colpi, Shapiro & Wasserman 1996 and references to early studies therein.

\(^2\) Note, however, that our analysis using \( P_0 \) data from N13 and PT12, and velocities from the ATNF catalogue does not support any correlation between \( P_0 \) and velocity.
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Figure 3. $P_0 - B$ plot for sources from N13. Initial spin periods are reconstructed from the present day values using the magneto-dipole formula, constant field and kinematic age as the true age. Arrows point to the initial parameters of the pulsars with reconstructed $P_0 > 0.6$ s if the exponential magnetic field decay with $\tau_{\text{mag}}$ was operating. If for a PSR several arrows are plotted, then the solid lines correspond to the most probable estimate of kinematic age, $t_{\text{kin}}$, from N13 and the dashed lines to the lower and upper limits on $t_{\text{kin}}$. If just one solid arrow is plotted, then it corresponds to the lower limit on $t_{\text{kin}}$, because large values do not allow us to derive any estimate for the initial spin period.

3.2 $P_0 - B$ correlation

Here, we want to discuss a possible correlation between present magnetic field (determined as $B \sim \sqrt{\dot{P} P}$) and initial spin period reconstructed with equation (5). We use the data from table 1 in N13 and results are shown in Fig. 3. It seems, mostly due to a group of sources with $P_0 \gtrsim 0.6$ s, that for longer initial spin periods the magnetic field is higher. The plot $P_0 - B$ presented by PT12 is much different. So, again we have to explain the differences. The question is: Is it a real correlation, or this is an artefact due to the assumptions made to derive $P_0$ or is it just a fluctuation?

Statistics is rather poor, and two different conclusions can be made. Either the correlation appears only due to the group of six PSRs with the longest $P_0$ or the correlation is valid for the whole range of $P_0$ (the two outlying PSRs with the largest fields can be due to a fluctuation). Also, of course, it is probable that this feature of the distribution is just due to some unknown selection effects, but we do not discuss this possibility further.

We calculate similar plots with a population synthesis model. Results are shown in Fig. 4. In the left-hand panel, we show the case for the constant magnetic field and in the right-hand panel that of a decaying field with $\tau_{\text{mag}} = 5 \times 10^6$ yr. Initial spin periods are reconstructed using equation (5). We select objects with true ages in the interval $10^5 < t_{\text{true}} < 10^7$ yr. Both plots are not similar to Fig. 3: no obvious correlation is visible in any panel. The main difference between the two distributions is that for decayed fields we have objects with long reconstructed $P_0$, as expected, and the top-left region in the right-hand panel is underpopulated as sources are shifted towards the bottom-right part of the plot. However, if we remove objects with $\tau > 10^7$ yr (which are absent in the sample from N13), then the plot for the decayed field starts to look relatively similar to Fig. 3. If in the original data from N13 the correlation is valid for the whole range, then this can be considered as an argument in favour of the decaying field model. We additionally illustrate it adding to Fig. 3 arrows connecting reconstructed in N13 initial parameters with those obtained in the model with decaying magnetic field. Clearly, PSRs are moved to much shorter initial periods, and so no contradiction with the distribution proposed in PT12 exists.

In the opposite case, if the correlation in Fig. 3 is just due to six objects on the right, then we have a separate population of sources with long initial spin periods, and this, in our opinion, can be an argument in favour of emerging magnetic field. In PT12, the authors did not find any correlation between $P_0$ and $B$, but in their sample of young objects, most probably, there are no (or very few) PSRs with re-emerged fields, and magnetic fields could not change significantly due to decay. Note also that CCOs can have periods and larger amounts of accreted matter. If this is the case, then among young NSs (analysed by PT12) sources with large initial spins should not be visible, but on the time-scale of $10^4 - 10^5$ yr their fields can re-emerge (otherwise, they should be visible among high-mass X-ray binaries, but this is not the case, see Popov & Turolla 2013), and so they contribute to the sample analysed by N13.

Figure 4. $P_0 - B$ plots for synthetic populations. Initial spin periods are reconstructed according to equation (5). Fields represent present day values. Only objects with $10^5 < t_{\text{true}} < 10^7$ yr are plotted. Left: constant magnetic field. Right: decaying field with $\tau_{\text{mag}} = 5$ Myr. The solid line in each plot corresponds to $\tau = 10^7$ yr, the dashed line in the bottom to the death line.
3. One sample (PT12) contains objects with plot with the data from N13.

t distribution. = ∼γ distributions for a large sample of plot for the phenomenological magnetic field evolution C yr in the sample from N13,

τ ∼× = σ ∼yr we have = 12.92, t (9) plane calculated within this yr magnetic fields are assumed to 0.47. For yr. The dashed line = 0.84. Initial distributions are 1.17 and τ ⟨0.2 s, = = t similar to the results by N13.

3.5 10 10 ∼= 0.15 s and τ = 10 yr are plotted. Magnetic field evolution is calculated according 10 τ yr magnetic fields are assumed to have larger magnetic fields (after they fully emerge), as we see in Fig. 3.

To distinguish between the two possibilities more sources with known ages are necessary.

3.3 Phenomenological magnetic-field-decay model

More complicated magnetic-field-decay models exist, and they also can be applied to reproduce the P0−B plot with the data from N13. As an illustration we use a model developed by Igoshev (in preparation) (see also Igoshev 2012) for a different study, but applicable also in this case. Parameters and evolutionary laws in the model were fitted using the data on t distributions for a large sample of known PSRs.

As there are no pulsars with τ > 107 yr in the sample from N13, it may indicate that (despite the field decay) for ages τtrue ∼ 107−109 yr we have τtrue ∼ τ. The phenomenological model of decay suggested by Igoshev (2012) satisfies this condition. The model is parametrized by the following equation:

\[ f_{\text{corr}}(t) = \left( \left[ \frac{t}{t_0} \right]^\gamma + C \right)^{-1}. \] (9)

Parameters of the fine-tuned model have the following values: t0 = 10 000 yr, α = 0.034, γ = 1.17 and C = 0.84. Initial distributions are the following: \( P_0 = 0.2 \text{s}, \sigma_P = 0.15 \text{s} \) and \( \langle \log B_0/|G| \rangle = 12.92, \sigma_B = 0.47. \) For \( t_{\text{true}} > 3.5 \times 10^7 \text{yr} \) magnetic fields are assumed to be constant.

The distribution of pulsars in \( P_0−B \) plane calculated within this model is similar to the one from N13, see Fig. 5. Despite the fact that this distribution is based on a multiparametric, phenomenological model, this exercise illustrates that the field decay can be fine tuned to produce the \( P_0−B \) similar to the results by N13.

4 CONCLUSIONS

In this work, we studied if results on the initial spin period distributions obtained in PT12 and N13 can be brought in correspondence with each other. In both papers, the authors reconstructed initial periods using independent estimates of NSs ages (SNR age in PT12 and kinematic age in N13) and the standard magneto-dipole formula with braking index \( n = 3 \). One sample (PT12) contains objects with ages from a few thousand years up to \( \sim 100 \text{kyr} \). The other sample (N13) contains much older NSs with an average age of about a few million years. We analysed if magnetic field evolution can produce both reconstructed initial period distributions starting with the same true distribution.

We apply the population synthesis technique to scrutinize the effects of magnetic field decay on the reconstructed \( P_0 \) distribution. We found that a simple exponential decay with \( \tau_{\text{mag}} = 5 \text{Myr} \) can reproduce a narrow distribution for younger objects and a much wider one for objects with ages \( \sim \text{few Myr} \), in correspondence with PT12 and N13. Potentially, magnetic field evolution can be fine-tuned to reproduced additional features of the sample presented in N13, but such calculations are beyond the aims of this paper.

In addition, we briefly discussed the possibility that re-emerging magnetic field can be also used to explain the differences between the two initial spin period distributions, as NSs with longer initial spin periods can be hidden among the younger population studied in PT12. However, we do not model this mechanism.

We conclude noticing that the differences between initial spin period distributions obtained by PT12 and N13 are due to evolution of the magnetic field on the time-scale smaller than a few million years. To distinguish between different models of field evolution it is necessary to increase statistics of NSs with known ages.

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REFERENCES

Bernal C. G., Page D., Lee W. H., 2012, preprint (arXiv:1212.0464)
Colpi M., Shapiro S. L., Wasserman I., 1996, ApJ, 470, 1075
Cordes J. M., Lazio T. J. W., 2002, preprint (astro-ph/0207156)
Edwards R. T., Bailes M., van Straten W., Britton M. C., 2001, MNJAS, 326, 358
Faucher-Giguère C.-A., Kaspi V. M., 2006, ApJ, 643, 322
Gonthier P. L., Ouellette M. S., Berrier J., O’Brien S., Harding A. K., 2002, ApJ, 565, 482
Gotthelf E. V., Halpern J. P., Alford J., 2013, ApJ, 765, 58
Guimard E., Kesten Y. K., 2013, MNRAS, 430, L59
Ho W. C. G., 2011, MNRAS, 414, 2567
Igoshev A. P., 2012, preprint (arXiv:1204.3445)
Kuijken K., Gilmore G., 1989, MNRAS, 239, 651
Manchester R. N. et al., 2001, MNRAS, 328, 17
Muslimov A., Page D., 1995, ApJ, 440, L77
Noutsos A., Schnitzeler H. F. M., Keane E. F., Kramer M., Johnston S., 2013, MNRAS, 718 (N13)
Popov S. B., Prokhorov M. E., 2007, Phys.-Usp., 50, 1123
Popov S. B., Turolla R., 2012, Ap&SS, 341, 457 (PT12)
Popov S. B., Turolla R., 2013, in Lewandowski W., Maron O., Kijak J., eds, ASP Conf. Ser. Vol. 466, Initial Parameters of Neutron Stars. Astron. Soc. Pac., San Francisco, p. 191

Figure 5. \( P_0−B \) plot for the phenomenological magnetic field evolution model. Initial spin periods are reconstructed according to equation (5). Present day values of the magnetic field are shown. Only objects with \( t_{\text{true}} < 10^7 \text{yr} \) are plotted. Magnetic field evolution is calculated according to equation (9). The solid line corresponds to \( \tau = 10^7 \text{yr} \). The dashed line in the bottom corresponds to the death line.

Large toroidal fields (Shabaltas & Lai 2012), and dipole fields can be at least correlated with toroidal, then objects with long initial spin periods can have larger magnetic fields (after they fully emerge), as we see in Fig. 3.

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Popov S. B., Pons J. A., Miralles J. A., Boldin P. A., Posselt B., 2010, MNRAS, 401, 2675
Rawley L. A., Taylor J. H., Davis M. M., 1986, Nat., 319, 383
Ruderman M. A., Sutherland P. G., 1975, ApJ, 196, 51
Shabaltas N., Lai D., 2012, ApJ, 748, 148

Spruit H., Phinney E. S., 1998, Nat., 393, 139
Vallée J. P., 2005, AJ, 130, 569
Viganò D., Pons J. A., 2012, MNRAS, 425, 2487

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