Correlation analysis to determine the relationship between the geometric dimensions and the electromagnetic moment of a switched reluctance motor

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Abstract. An electric drive is an indispensable part of any mechanism. Without it, the functioning of any industry is impossible, especially in the railway. It allows you to automate technological processes and increase their efficiency. For efficient railway transport, it is necessary to increase the level of technical development of traction rolling stock, which is possible through the use of promising types of electric machines, one of which is a switched reluctance motor. To create a highly efficient and technological switched reluctance electric drive, it is necessary to have information about the relationship between the geometric dimensions of its magnetic system and the average value of the electromagnetic moment. The purpose of this article is to determine the strength and direction of the relationship between the average value of the electromagnetic moment and the changing geometric values of the tooth-groove zone of the object under study.

The numerical value of the electromagnetic moment was obtained using the stochastic method in the MATLAB application package, which directly interacted with the FEMM version 4.2 program, which is based on the finite element method. The research carried out in the article and the results obtained allow us to conclude that there is or is not an appropriate connection between the considered elements of the magnetic system design and the value of the electromagnetic moment.

1. Introduction
Currently, regulated automated electric drives are used in most areas of energy and industry, including the railway industry. The regulated electric drive is the most widespread at this time and one of the most effective energy-saving and resource-saving technologies known at this time.

The rapid development of power electronics has given humanity a number of important discoveries, without which it is currently impossible to imagine the modern production and operation of regulated electric drives. Powerful controlled power switches – transistors that can switch currents up to 600 A at a voltage of up to 1200 V with a frequency of 30 kHz or more, as well as processor tools that provide the creation of «artificial intelligence» – have become available.

Among all the relatively new electric drives, where the frequency converter and control system are integral parts, we can single out one – switched reliance drive (SRD).

The switched reluctance machine (SRM) currently forms the basis of a modern SRD of industrial production, determines the technical and economic possibilities of wide application of electric traction in railway [1] and automobile transport [2, 3], provides appropriate power functions and operating modes of electric drives of machine equipment, complex robotic systems [4, 5] and other
technological equipment of automated production. Epy afunctional scheme of the switched reliance drive traction rolling stock is shown in Figure 1.

![Functional scheme of the switched reliance drive.](image)

Output parameters such as electromagnetic torque and shaft speed are usually used to match the mechanical characteristics of the SRM and the production facility. These parameters are determined by the configuration of the active part of the SRM, which in turn consists of the yoke and stator teeth, the yoke and rotor teeth, and the stator winding. To create a SRD that is competitive on the world market and has high technical and economic indicators, it is necessary to have reliable, scientifically based information about the relationship and degree of influence of the geometric variables of the magnetic system and the average value of the electromagnetic moment.

Quite often, in the course of research, it is necessary to determine or exclude the existence of a relationship between different values of the process. Variables can be linked by a functional (hard-deterministic) or stochastic (probabilistic, statistical) relationship. As you know, in a functional relationship, each value of one independent attribute corresponds to a strictly defined value of another dependent attribute. In the case of a probabilistic relationship between the observed variables, the value of the factor attribute corresponds to the set of possible values of the effective attribute, i.e. when the value of one value corresponds to a certain distribution of another value. A special case of stochastic dependence is the correlation dependence, which allows you to determine the strength and direction of the relationship between variables. This is the nature of the dependence and have the geometric dimensions of the fragments of the magnetic system and the average value of the electromagnetic moment.

The purpose of this article is to determine the tightness, the direction of communication, and the trend of changes in the average value of the electromagnetic moment under the influence of varying values of the geometric dimensions of the elements of the active part of the SRM. Also, the results obtained make it possible to further determine the degree of influence of various design parameters of the tooth-groove zone of the active part on the average value of the electromagnetic moment.

2. Results and Discussion
The term «correlation» (from lat. correlatio-relation, relation, dependence) was first applied by J. Cuvier in 1806. The first mathematical description of the method was made by O. Brave in 1846. Currently, correlation analysis is a fairly widespread, flexible and effective tool for processing statistical data. This method has found its application in various fields of science and technology [6–8].
In this article, a three-phase SRM with a 12/8 magnetic system configuration (stator teeth – 12, rotor teeth – 8), designed for an electric traction drive of an electric train, with a rated power of 300 kW and a torque equal to 1.99 kN-m, was used as the object under study.

Variations in the numerical values of various design variables of the magnetic system SRM were used as a factor parameter, and the average value of the electromagnetic moment was used as a result parameter. The geometric dimensions, the values of which were considered separately, are shown in figure 2.

![Figure 2](image.png)

**Figure 2.** Fragment of the magnetic system under study.

The study was carried out with the parameters that have the greatest influence on the formation of the numerical value of the moment. This information was taken from earlier dissertation research with the NTI-1200 traction motor, which also has a 12/8 configuration. Thus, the relationship between the average value of the electromagnetic moment and the geometric values of the following parameters of the magnetic system was considered:

- Slope of the side surface of the stator tooth, deg. \( (b_{2s}) \) (figure 1 does not show this element, since \( b_{2s} = 0 \)).
- Radius of the stator in the grooves, mm \( (R_{2s}) \).
- Stator tooth width, mm \( (b_{1s}) \).
- Radius of the rotor along the grooves, mm \( (R_{2r}) \).
- Air gap, mm \( (\delta) \).

Motor dimensions such as the external diameter of the stator, the length of the magnetic core in the axial direction, and the shaft diameter are often specified in the technical specification, since it is not uncommon to integrate a SRM magnetic system into the housing of an asynchronous motor or DC motor. However, if these dimensions are not defined in the terms of reference, it is also possible to include them in the parameters under study as variable variables. The values of the values shown in table 1 were used as restrictions for calculating the numerical values of the above geometric parameters.

| Geometrical parameter                      | Original size, mm | Limitations, mm |
|-------------------------------------------|-------------------|-----------------|
| Slope of the side surface of the stator tooth, deg. \( (b_{2s}) \) | 0                 | 0...10          |
| The radius of the stator in the grooves, \( (R_{2s}) \)   | 301               | 296...306       |
| Stator tooth width, \( (b_{1s}) \)            | 73                | 68...75         |
| Radius of the rotor along the grooves, \( (R_{2r}) \) | 188               | 183...193       |
| Air gap, \( (\delta) \)                       | 2                 | 1...3           |

Table 1. Limitations of the considered geometric parameters.
The data sets required to build this correlation were obtained using an optimization algorithm based on the Monte-Carlo method implemented in a software product designed to automate and optimize the design of the active part of the SRM used in previous studies [9, 10].

The Monte-Carlo method was chosen because it differs from other methods in its simplicity and generality, as well as its guarantee of finding the global extremum. The disadvantage of this method is slow convergence, but this disadvantage can be partially eliminated by modifying the method [11, 12]. The distribution of the random variable was assumed at a uniform probability density. The optimization algorithm and processing of the results of field calculations were carried out using the Matlab application software package, which, in turn, directly interacted with the FEMM 4.2 program for modeling and calculating magnetic fields.

The sample size after calculations for each case consisted on average of 200 values. As a result, scattering diagrams and regression functions were constructed, which are shown in the corresponding figures. The constructed scatter plots allow you to clearly see the nature of the relationship between variables. Each figure shows linear regression equations and their approximation confidence values $R^2$ (coefficient of determination). Some of the figures also present alternative equations that are polynomial in nature, due to the fact that the coefficient of determination of the linear regression function took a lower value for the distributions under consideration. All variants of trend lines available in Excel (exponential, logarithmic, power) were considered, but the values of $R^2$ when using them differed from the linear function by no more than 2 %, so their further consideration and application is impractical.

On all the charts under study, the orange dashed line shows a linear regression line, and the blue dotted line shows a polynomial trend line. Also, the coordinate with the highest value of the moment variable is highlighted in red on each scatterplot.

The first dependence under consideration was a function with variables such as the angle of inclination of the side surface of the stator tooth and the average value of the electromagnetic moment (Figure 3). At the same time, the value of the air gap remained unchanged.

![Graph of the correlation between the electromagnetic moment ($M$) and the angle of inclination of the side surface of the stator tooth ($b2s$).](image)

**Figure 3.** Graph of the correlation between the electromagnetic moment ($M$) and the angle of inclination of the side surface of the stator tooth ($b2s$).

The graphic relationship of the structural element of the magnetic system - the slope of the side surface of the stator tooth and the electromagnetic moment, is shown in figure 3. The maximum value of the moment is obtained at the point highlighted in red with the value $b2s = 8.82$. According to the
initial design of the object under study, the slope of the side surface of the stator tooth was 0 degrees. After the optimization calculation, the value of this structural element was 10 degrees.

The magnitude of the accuracy of the approximation for this nonlinear dependence is maximum for the equation of a curve line polynomial regression. The correlation coefficient for this data set takes a value equal to \( r = 0.96 \) and indicates that this relationship can be characterized as «direct» and close (strong).

The next geometric parameter of the stator, depending on the value of the moment, was considered \( R_{2s} \) (stator radius along the grooves, mm) (Figure 4). In this case, the maximum average value of the moment is equal to the size of the stator radius along the grooves, which practically borders on the accepted numerical restrictions. For this case, the optimal point has moved down from the original size. In the initial version, \( R_{2s} = 301 \) mm, and after the optimization calculation – 296.43 mm.

The approximation confidence value for this dependence takes a rather low value. It shows only 33% of the variations in M values due to the variability of \( R_{2s} \) sizes. The correlation coefficient for this data array takes a value equal to \( r = -0.84 \). The specified value of the coefficient according to the scale of Chedoke indicates a fairly strong negative linear relationship. According to previous studies with SRM 12/8, the \( R_{2s} \) parameter played a significant role in the formation of the moment value, so to establish a more accurate relationship, it is recommended to calculate with a large number of random throws.
The third data set studied was the numerical values of the variables $M$ from the values of the stator tooth width (Figure 5).

The presented correlation has the values of the coefficients of determination for linear and polynomial functions that are almost identical (97.3 and 98.2 %), differing only from each other by 1 %. Since in this case the value of $R^2$ for the linear regression function is quite high, it is highly likely that this dependence can be considered as linear, and the correlation coefficient, which is used only for linear relationships, indicates a «direct», positive and very strong, almost functional relationship ($r = 0.96$). As can be seen from this picture of the distribution of variables, the maximum value of the moment is reached when the stator tooth width is 74.51 mm. This value exceeds the size of the original design (73 mm) by 2 %.

Further, the geometric parameter of the rotor was considered (Figure 6) and the statistical dependence under study was a set of variables $R^2r$ (radius of the rotor along the grooves, mm) and $M$.

![Figure 6. Graph of correlation between the electromagnetic moment ($M$) and the rotor radius along the grooves ($R2r$).](image)

The quality of the regression model for a graphically represented distribution of the observed parameters is almost 30 % better when using a polynomial trend line. The value of the coefficient of determination $R^2 = 0.89$ can be interpreted as follows: 90 % of the variance of the resulting factor ($M$) is explained by this regression equation, and the remaining part, which is 10 %, is explained by other factors of variance. For such a value $R^2$, the proportion of the explained variation $R2r$ is high enough and the constructed regression model with a high degree of probability has good approximating properties. When considering this relationship as linear, we can say that the direction of the correlation is negative ($r = -0.86$), and the tightness of the interaction of variables can be defined as strong, but rather close to moderate. The highest value of the electromagnetic moment is observed when the radius of the rotor along the grooves is equal to 186 mm (less than the original 2 mm), which indicates such a structural change as a narrowing of the rotor yoke.

The last correlation field considered was a distribution consisting of variables of the electromagnetic moment ($M$) and the air gap ($\delta$) (Figure 7).

The maximum value of $R^2$ is 0.9994 if the regression line is linear. A value of 99.9 % indicates that there is a functional relationship, i.e. those 99.9 % of changes in the first variable ($M$) are caused by variations in the second variable ($\delta$). The linear regression equation describes the relationship of these variables with sufficient accuracy. The quality of the model with this value $R^2$ is good and can be used in the future to predict the values of the performance indicator. The value of the correlation coefficient ($r = -0.996$) indicates a linear functional relationship in which all the values of the studied variables are located almost on the same straight line. The relationship between $M$ and $\delta$ can be defined as negative and practically functional (very strong). The maximum torque is achieved when the air gap is 1.01 mm, which is less than the initial 0.99 mm (from $\delta = 2$ mm). The decrease in the value of $\delta$ is natural, since the value of the electromagnetic moment is inversely proportional to the value of the air gap. However, creating a SRM with a very small air gap will lead to technological difficulties and
problems in further operation (for example, an increase in the degree of unevenness of the air gap under different poles).

![Figure 7](image)

**Figure 7.** Graph of the correlation between the electromagnetic moment \( (M) \) and the air gap \( (\delta) \).

Figure 8 shows the magnetic system fragments to illustrate the changes that the geometric fragments under consideration have undergone. The original geometry is shown in black, and the resulting geometry is shown in blue.

![Figure 8](image)

**Figure 8.** Comparison of the initial geometry of the magnetic system and the one obtained after optimization calculation: \( a \) – angle of inclination of the side surface of the stator tooth \( (b2s) \); \( b \) – stator radius along the grooves \( (R2s) \); \( c \) – radius of the stator tooth width \( (b1s) \); \( d \) – rotor radius along the grooves \( (R2r) \).

3. **Conclusion**

To create a SRM with high energy parameters, in particular a high value of the electromagnetic moment, it is necessary to carry out a comprehensive optimization of the geometric parameters of the stator and rotor. In turn, the FEMM program allows you to take into account the features of calculating
electric machines of this type. This program guarantees the consistency of conflicting requirements when the extremum of the target function is reached, for example, the maximum of the average value of the electromagnetic moment.

The use of correlation analysis allowed us to determine the strength and direction of the relationship between the studied structural elements of the active part SRM 12/8 and the average value of the electromagnetic moment. Based on the results of the analysis, it is possible to judge the existence of a corresponding correlation of these parameters.

The results of calculations of the average value of the electromagnetic moment when varying a separate structural size allowed us to see the degree of influence of each parameter of the magnetic system on the formation of the moment value.

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