Phenomenology of scale-dependent space-time dimension.

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Loop-mediated processes characterized by dynamical scale $M$ indirectly measure space-time dimension $d$ at this scale. Assuming the latter to be scale-dependent and taking as examples $B$-oscillations and muon $(g - 2)$ experimental results we address the question about constraints put by this data on $|4 - d(L)|$ at smaller distances, i.e. for $ML < 1$. It is shown that sensitivity is lost for $1/L$ around 300-400 GeV, and any value of $d(L)$ between 2 and 5 at this scale is compatible with the data.

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I. INTRODUCTION

The main goal of modern high energy physics is to find and explore phenomena beyond the Standard Model (SM). There are exciting New Physics (NP) hints such as neutrino oscillations and dark matter. Nevertheless no experiment has shown so far direct and conclusive evidence in favor of any particular NP scenario. Instead currently available data play a role of constraints for different SM extensions. Despite some of these constraints are rather tight, it is fair to say that many reasonable TeV-scale NP scenarios discussed in the literature are still far from being rejected.

In a broad phenomenological prospective almost all NP scenarios can be divided into two large groups. The first one consists of the models which extend particle content of the SM by adding some new particles, according to this or that dynamical principle. The best known example of this kind is supersymmetric extension of the SM. The corresponding phenomenology is well studied, at least in case of MSSM. Dynamical evolution of states in all models of this type takes place on the standard Riemannian (3+1)-dimensional space-time manifold of general relativity, whose internal dynamics is believed to be governed by the genuine Planck scale $L_P = 1.6 	imes 10^{-35}$ m.

The second group of models suggests much more radical extension of the SM. It is assumed that the picture of our familiar space-time as smooth four-dimensional manifold is applicable only at low energies, and becomes inadequate below some distance scale $L$ (which can be much larger than $L_P$ and perhaps of (1-2 TeV)$^{-1}$ range). The so called small extra dimensions and TeV-gravity scenarios [1, 2, 3, 4] are well known examples of the theories of this kind. The most attractive feature of these models is the emergent nature of the Planck scale $L_P$.

Another line of reasoning, having its roots in seminal papers [5] and [6], is to some extent parallel to extra dimensional picture. One can think of space-time geometry becoming of discontinuous or of fractal type at small distances. This idea has been explored from many different points of view: conventional wisdom of Planck-scale quantum gravity fuzziness [5], space-time foam models [7], spin-networks in loop quantum gravity [8], dynamical triangulations [3, 10], fractal space-time structure in asymptotically safe gravity [11], noncommutative geometry phenomenology [12, 13], modified commutation relations [14, 15, 16, 17], modified dispersion relations in the context of Finslerian geometry [18], minimal length phenomenology [19, 20], curved momentum space [21] and other approaches. We refer the interested reader to [22] for bibliographical review devoted to the models of short distance space-time structure. In most of these approaches one associates the corresponding NP length scale $L$ with the Planck length $L_P$, despite nothing prevents to think about $L$ as being different from $L_P$ (e.g., much larger).

The distinctive feature of all these extra approaches is the fact that effective number of space-time dimensions felt by the propagating particle depends on its energy/virtuality. One can think of different signs of this dependence. While in versions of extra dimensions scenarios which allow the SM fields to propagate in extra dimensions they ”open up” with the increase of particle momentum, in fractal scenarios one can imagine low-dimensional dynamics in the ultraviolet (UV) limit, and (3+1)-dimensional infrared description as an emergent phenomenon.

Taking space-time dimension $d$ as a free parameter it is legitimate to ask a question about experimental measurement of this quantity. In this way the quantity $\epsilon = (d - 4)$ is to be constrained by observations. There is extensive list of references on the subject starting from [23, 24]. If $d$ is understood as just a scale-independent constant devoid of any dynamics (as it is done in the cited papers), the constraints come both from celestial physics (Newton gravitational force law, perihelion precession etc) and from atomic (Coulomb law, hydrogen spectra etc. see e.g. [25]) as well as elementary particle physics (muon $g - 2$ etc). It is worth stressing that since any experiment actually deals with finite energy-momentum space-time resolution, one never has access, strictly speaking, to the true UV dimension of a given space-time and in this sense physical dimension always has to be understood as resolution-dependent quantity.
Needless to remind that the idea of $d$ being different from 4 and non-integer is very appealing from field theoretical point of view. As is known since the advent of dimensional regularization [26, 27] and dimensional reduction [28] methods, the loop amplitudes which are divergent at integer values of $d$ (in particular, for $d = 4$) can be analytically continued and self-consistently defined as finite quantities for noninteger values of $d$. Usually understood as a convenient mathematical regularization trick, this property may have a deeper meaning, signaling the preference for interacting quantum field theories to live in non-integer dimensional world. Moreover, despite it is common practice in modern quantum field theory to understand the regularization procedure as being formal, this is not the case for some of the scenarios mentioned above, where space-time dynamics or new particles can play a role of regulators for field-theoretical amplitudes. For example, many of the NP scenarios with NP scale $\Lambda$ can be described as a process-dependent change in loop integration measure

$$\int d^4 p \rightarrow \int d^4 p \, g(p^2, \Lambda^2)$$

such that $g(p^2, \Lambda^2) \approx 1$ for $p^2 \ll \Lambda^2$. On the other hand, at large $p^2$ the function $g(p^2, \Lambda^2)$ is such that the integral becomes convergent. The best known example of this pattern is given by cancelation of divergencies in supersymmetric theories. As is well known minimal length and alike scenarios also provide UV modification of the measure (often process-independent) leading to convergence of loop amplitudes.

In the present paper the question about phenomenological constraints on space-time dimension at short scales is addressed, taking the latter to be scale-dependent in a particular way.

The concept of scale-dependent dimension was thoroughly analyzed in studies of dynamics on multi-fractal sets (see, e.g. [29]). There have been attempts to develop the corresponding formalism in field-theoretical context (see, e.g. [30]). In the present paper we explore some phenomenological consequences of this picture being applied to experimental data on $B$-meson oscillations and muon anomalous magnetic moment. The oscillations phenomena are genuine quantum effects dominated by loop diagrams and therefore they put into test, as any loop process does, the overall integrity of quantum field theory. In particular, $K - \bar{K}$ oscillations constrain extensions of conventional quantum mechanics caused by effects of nontrivial space-time dynamics [31, 32]. Speaking in a broad sense we are exploring the same phenomenon (interplay between short-distance geometry and quantum mechanics of mixing), but our concrete model is very different from the one considered in [31, 32] and subsequent papers.

The organization of the paper is as follows. In the next Section II we present our Ansatz and briefly discuss general features of the corresponding phenomenology. In Section III the $B_s$ mixing rate is analyzed and the exclusion plots of interest are given. We compare the corresponding constraints with the ones provided by muon anomalous magnetic moment data. The conclusions are presented in Section IV. There is an Appendix in the paper, where relevant formulas used in the main text are collected.

Since we are always interested in real parts of the amplitudes we find it convenient to work in Euclidean metric and leave aside an interesting question about Wick rotation in the context of non-integer dimensional theories.

II. THE MODEL

As is well known typical expression for amplitude in perturbative quantum field theory has the following form:

$$\mathcal{A} = \mathcal{A}^{(0)}(q_i, m_i, e_i) + \sum_{j=1}^{\infty} \mathcal{A}^{(j)}(q_i, m_i, e_i)$$

(2)

where $\mathcal{A}^{(0)}(q_i, m_i, e_i)$ is tree amplitude, $q_i, m_i, e_i$ stay for external momenta, masses and couplings of interacting particles, while loop amplitudes are given by

$$\mathcal{A}^{(j)}(q_i, m_i, e_i) = \left\{ \prod_{k=1}^{j} \int d^4 p_k \right\} \mathcal{P}(p_k, q_i, m_i, e_i)$$

(3)

where $p_k$ are loop momenta being integrated over and in the course of renormalization procedure artificial dimensional scale $\mu$ enters individual terms in $\mathcal{P}$. In principle, many different strategies to continue (2) to $d \neq 4$ can be chosen. We adopt the picture of dimensional reduction [28]. The main feature which makes this approach distinct from the standard dimensional regularization is the following: one analytically continues in the number of components $d$ of all loop momenta, but keeps the number of all tensor and spinor field components fixed. Notice that all known problems of dimensional regularization, notably $\gamma_5$-ambiguity have their roots in the fact that nontrivial number of relevant degrees of freedom is - contrary to the number of space-time dimensions - an intrinsically integer quantity.

One immediate consequence of this prescription is that tree amplitude $\mathcal{A}^{(0)}$ never gets modified (notice that this is in contrast with many other approaches to non-integer dimensional theories, e.g. [26, 25]). All incoming and outgoing particles by definition live in $d = 4$, and the only affected element of (3) is the loop integration measure.

From physical point of view there is some analogy with the weak field expansion in gravity where one integrates the factor $\exp(-S) = \exp(-\int d^2 \sqrt{h} L[g_{\alpha\beta}, \phi])$ over small metric fluctuations $h_{\alpha\beta}(x)$ with respect to flat background $g_{\alpha\beta} = g_{\alpha\beta}(x) - h_{\alpha\beta}(x)$. In this case there is no question about dimensionality of the bulk space, which always coincides with $\eta_{\alpha\beta}$. Alternatively
one can think of field theory action defined on a fractal set $K$ (30, see also 22 and references therein): $S^{[K]} = \int_K \mu \mu L(\phi(x))$ and a partition function $Z$ given by quantum average over superposition of subsets $K$ of different and in general non-integer dimensions $d$: $Z = \langle \exp(-S^{[K]}) \rangle_K$.

Then the crucial point is how such theory couples to external currents. One has a choice - to take the coupling factor as $\exp(i \int_K \mu \mu J(x)\phi(x))$ or to keep bulk expression $\exp(i \int_K \mu \mu \mu L(\phi(z))$, where $K \subset \mathbb{R}$. It is important to stress that these options would mediate two different kinds of physics. In the former case there is an interference already of tree level processes mediating by particles propagating over different $K$ one can think of field theory action defined on a fractal set $K$ the parameter $s$

\[ \int \tau \tau \mu \mu \mu L(\phi(z)) \] where the parameter $s$ controls the degree of divergence. In coordinate space this is replaced by corresponding expression for one-loop effective action

\[ \mathcal{A}^{(1)} = \int_0^\infty \frac{d\tau}{\tau} \int d^4x \exp(-\tau p^2) \mathcal{F}(\tau, q_i, m_i, e_i) \] (4)

where the parameter $s$ controls the degree of divergence. In coordinate space this is replaced by corresponding expression for one-loop effective action

\[ \Gamma[\phi] = \int_0^{\infty} \frac{d\tau}{\tau} \int_K d\mu_x \langle x | \exp(-\tau \mathcal{O}[\phi]) \rangle x \] (5)

where the operator $\mathcal{O}[\phi]$ encodes all information about dynamics of the theory and Green’s functions can be extracted from (5) by the standard technique. In case of $K = \mathbb{R}^4$ we have conventional theory in flat four-dimensional Euclidean space with the measure $\int d\mu_x = \int d^4x$. If the space $K$ is curved or has boundaries, one is to apply Schwinger-De Witt technique 33 to get answer in terms of expansion in powers of $\tau$ with the leading term given by

\[ \langle x | \exp(\tau \Delta) \rangle x \sim \frac{1}{(4\pi\tau)^{\frac{d}{2}}} \] (6)

where $d = d(x)$ is Hausdorff dimension of $K$ at the point $x$, $\Delta$ is $d$-dimensional Euclidean Laplace operator, and the intrinsic dimension of random walk on $K$ is taken to be equal to 2, as for the standard Brownian motion (see detailed analysis in 34). The value of $d = d(x)$ is defined by

\[ \lim_{\tau \to 0} \left( \frac{L}{\tau} \right)^d \frac{\Gamma(\frac{d}{2})}{2\pi^{\frac{d}{2}}} \int_{B_c} d\mu_x = L^4 \] (7)

where $B_c \subset K$ is a ball of radius $r$ centered at the point $x$ and $L$ - dimensionful scale, characterizing the set $K$. Notice that (6), (7) are valid even for fractal set $K$ having no differentiable Riemannian structure.

It is very interesting that the integration measure in (6) given for $K = \mathbb{R}^4$ by $\int d\tau / \tau^{d/2-1} \int d^4x$ can be understood 33 as corresponding to curved space $AdS_{d+1}$ with the coordinates $(x, \tau)$. If $h_{\mu\nu}(x)$ is a metric tensor on $K$, the $d + 1$-dimensional metric can be chosen as $ds^2 = \tau^{-2}d\tau^2 + \tau^{-1}h_{\mu\nu}(x, \tau)dx^\mu dx^\nu$ with the condition $h_{\mu\nu}(x, \tau = 0) = h_{\mu\nu}(x)$ (notice that trivial choice $h_{\mu\nu}(x, \tau) = h_{\mu\nu}(x)$ would not be a solution of Einstein’s equations). Thus if one tries to keep the above geometrical interpretation, the properties of the space $K$ become $\tau$-dependent and factorization of the measure is lost, while the Riemannian structure of the manifold is kept intact.

In the present paper we relax the latter condition and investigate a particular deformation of (4) and (5) corresponding to dimension of $K$ being $\tau$-dependent: $d \to d(\tau)$. At the same time the space $K$ is taken as homogeneous in the sense that $d$ does not depend on $x$. We assume the following natural asymptotic conditions: $d(\infty) = 4$, $d(0) = d$ where $d$ is some ”true” ultraviolet space-time dimension. A typical scale $\bar{\tau}$ where the transition takes place is taken as a free parameter of the model $L^2$. The corresponding physics is outlined in the introduction: at small virtuality, corresponding to $\tau$ much larger than $\bar{\tau}$, one has the standard 4-dimensional dynamics, $d(\infty) = 4$. This condition is of prime importance for the approach to preserve unitarity. It guarantees that virtual particles on-shell are indistinguishable from real ones and propagate in the same four-dimensional space-time. The situation is analogous to that in dimensional reduction method (see discussion in 28).

Unfortunately we have no guiding physical principle to fix $d(\tau)$ dependence. One might appeal to numerical simulations from [9, 10] or analytical results from recent papers [17, 36]. However since our attitude here is mostly phenomenological, we find it convenient to choose a particular Ansatz for this function, which allows to proceed with analytical computations. We have chosen the following one:

\[ \int \frac{d^4p}{(2\pi)^4} \exp(-\tau p^2) \to \frac{1}{(4\pi\tau)^{\frac{d}{2}}} \] (8)

where $w = \tau / \bar{\tau}$ and the measure deformation corresponds to $f(w, d) \neq 1$:

\[ f(w, d) = g(w) + (1 - g(w))w^{2-\frac{d}{2}} \] (9)
The function $g(w)$ must obey obvious asymptotic conditions $g(0) = 0$, $g(\infty) = 1$. We have studied two particular choices for $g(w)$ making analytical calculations possible:

$$g_1(w) = \left(1 + \frac{1}{w} + \frac{1}{2w^2}\right) \exp(-1/w)$$  \hspace{1cm} (10)

$$g_2(w) = 1 - \frac{1}{\cosh(w)}$$  \hspace{1cm} (11)

The latter function approaches $w = \infty$ asymptotic exponentially and $w = 0$ polynomially, while the former one does it in the opposite way (the chosen pre-exponential factor for $g_1(w)$ makes large-$w$ convergence faster). Certainly, we expect the results to depend on gross features of the weight functions and not on the details how they approach their asymptotic limits, and indeed this is what has been found.

One can see that the function $f(w, d)$ switches between 4-dimensional dynamics in the infrared (large $w$) and $d$-dimensional dynamics in the ultraviolet (small $w$). One can interpret \cite{37} by saying that at typical virtuality $1/\tau$ particles propagate as if effective dimension of space-time would be

$$d(w) = 4 - \frac{\log f^2(w, d)}{\log w}$$  \hspace{1cm} (12)

The character of this transition is controlled by the choice of $g(w)$. We plot the functions $d_1(\tau)$, $d_2(\tau)$, $r \sim \sqrt{w}$ corresponding to two choices (10), (11) in Fig.1, taking by way of example $d = 2$. We have adjusted scales in such a way that $L = L_2 = 2L_1$, where $L_{1,2}$ correspond to the choices $g_{1,2}(w)$. This allows to have roughly the same transition region for two different functions (10), (11). Notice that for $d = 4$ the function $d(\tau) \equiv 4$ and does not depend on $L^2$, since the scale $\tau = L^2$ has been chosen as a scale where dimensional reduction $4 \rightarrow d$ happens.

At formal level phenomenology the expression \cite{8} leads to is in close correspondence with the one discussed in \cite{37}. We do not need to constrain $d$ to be larger than 4, or to be integer with the increase of momentum scale, while the approach of \cite{37} adopts the extra dimensional logic, where effective dimensionality of space-time always increases at small distances by integer steps. Another, more technical thing is that our approach preserves manifest $O(4)$ symmetry and gauge invariance. Indeed, the general structure of one-loop effective action is such that the integrand for the proper time integral is proportional to the trace of the corresponding Wilson loop and is gauge-invariant by itself, hence gauge invariance cannot be broken by any measure deformation $\int d\tau \rightarrow \int d\tau f(\tau)$.

Since the phenomenology of \cite{8} corresponds to changes of ultraviolet behavior of the Green’s functions, it poses a question about validity of the whole approach in field theoretical framework. This issue will be addressed elsewhere, here we only notice that since deviation of $f(w)$ from unity parameterizes the NP features as small corrections to the SM answers, we are actually never in the regime where the effects caused by $d \neq 4$ become dominant. Speaking differently, the loop integrals for SM observables we look at are always saturated by the values of $\tau$ much larger than $L^2$.

In the next section we apply the Ansatz (8) to the weak ($B$-oscillations) and electromagnetic (muon anomalous magnetic moment) loop processes.

### III. APPLICATIONS

Loop mediated processes have always played important role in exploration of not yet discovered degrees of freedom. We are interested to check the sensitivity of the selected electroweak and electromagnetic observables to the UV measure deformation suggested above. Recent experimental observation of oscillations of neutral $B_s$ mesons complement the well known result for $B_d$ oscillations, and the current number is \cite{38}:

$$\Delta M_s = (17.78 \pm 0.12) \text{ ps}^{-1}$$  \hspace{1cm} (13)

This result is consistent with the SM expectations. For the models we are discussing there is no principal difference between $B_d$ and $B_s$ cases. We consider $\Delta M_s$ in what follows because theoretical uncertainty is slightly smaller for this quantity.

It SM the mass difference between "heavy" and "light" mass eigenstates, determining the oscillation frequency, is given by \cite{39}:

$$\Delta M_s = \frac{G_F^2 M_s^2}{6\pi^2} (V_{ts}V_{tb})^2 M_{B_s} \hat{\eta} B_{B_s} f_{B_s}^2 S_0(x_t)$$  \hspace{1cm} (14)

where the short-distance part (in effective Hamiltonian sense) of this expression is represented by QCD correction $\hat{\eta} = 0.552$ and the function $S_0(x_t)$, $x_t = m_t^2/mL^2$, first computed in \cite{41}, whose exact form can be found in Appendix A. The status of this theoretical prediction and space left for NP is reviewed in \cite{42}. The quantities entering (14) are measured or theoretically computed with uncertainties, whose budget is conservatively summarized in the table below:

| $\delta \Delta M_s/\Delta M_s$ | $\delta (f_{B_s}\sqrt{B_{B_s}})$ | $\delta (S_0(x_t))$ | $\delta m_s$ | $\delta (V_{ts}^*V_{tb})^2$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $0.7\%$ | $8\%-12\%$ | $2.6\%$ | $2.0\%$ |

The uncertainties in $G_F$ and in the mass parameters are negligible. Notice that to get independent prediction for $\Delta M_s$ one has to consider CKM factors $(V_{ts}^*V_{tb})^2$ as input parameters, to be determined by some other observables. The uncertainty indicated above corresponds to unitarity-based determination of $|V_{ts}^*V_{tb}| = \ldots$
and references therein for introduction into the subject. Current experimental value for
\[ a_\mu = \frac{1}{2}(g_\mu - 2) = F_2^{SM}(k^2 = 0) \] (17)
is given by (see [46] and references therein):
\[ a_\mu = 11659208.0(6.3) \times 10^{-10} \] (18)

There is strong interest on this subject since some \( \sim 3\sigma \) discrepancy between theoretical SM prediction for this quantity and the experimental result [13] is seen. The former, however, suffers from hadronic uncertainties at the level about 0.5 ppm, i.e. comparable with experimental accuracy, therefore it is very difficult if not impossible to deduce solid positive statement about the meaning of such discrepancy. But regardless the status of the SM theoretical prediction it is obvious that no NP scenario may bring corrections to the SM which are in contradiction with the experimental result [13]. The unprecedented accuracy of current experimental value \( \delta a_\mu/a_\mu = 0.54 \) ppm - makes this requirement especially challenging. It is clear, that in our case the dominant correction comes from the one-loop term, which is proportional to \( \alpha \). Thus we must require the correction already at the first loop to be smaller than combined experimental uncertainty \( \delta a_\mu = 6.3 \times 10^{-10} \):
\[ w(L, d) = \frac{|F_2(L, d) - F_2^{SM}|}{\delta a_\mu} \lesssim 1 \] (19)

as it should be.

The Figure 2 represents the results as contour plots of \( s(L, d) \) for the choice [11]. Dashed contours correspond to \( \pm 10\% \) deviation of \( S(L, d) \) from the SM result \( S_0 \), thin contours - to \( \pm 5\% \) deviation and thick contours - to \( s(L, d) = 1 \). The results for the choice [11] are depicted at Figure 3. Sensitivity to the scale \( L \) is lost for \( d = 4 \), hence the horizontal line \( s(L, 4) = 1 \) on Figs.2,3. The geography of allowed and excluded regions at the corresponding level of accuracy is clear from the pictures. We see good level of qualitative similarity between the figures, illustrating the expected robustness of the answer with respect to asymptotic properties of chosen transition weights \( g_1(w), g_2(w) \).

It is interesting to see that at the scale \( L \sim 3 \) as small as 300–400 GeV the experimental data on oscillations put almost no any constraints on dimensionality of space-time \( d \) at this scale. It is instructive to compare this result with the \( R^{-1} \sim 600 \) GeV bound on minimal universal extra dimension radius \( R \) from \( B \to X_{s\gamma} \) data recently obtained in [44].

It is interesting to reconcile the above results with the constraints coming from non-flavor physics, namely from QED. Of prime interest in this respect is the anomalous muon magnetic moment. The reader is referred to the reviews [45, 46], recent updates [47, 48, 49].
partly compensates smallness of $m_\mu L$. Anyway we find it remarkable that roughly speaking any number of dimensions between 5 and 2 at a scale as small as 350 GeV is compatible with experimental data. This is to be compared with $10^{-7} - 10^{-9}$ bounds on $\epsilon = |d - 4|$ from [23, 24].

It a sense, this is rather general situation, as can be seen comparing our results with that of [50, 51, 52, 53, 54, 55]. Having just two-dimensional minimal flavor violating NP parameter space is enough to open up possibilities for rather low energy scale of New Physics.

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APPENDIX A: APPENDIX A

To be self-contained we collect here explicit formulas used in the main text to compute the contour plots of interest.

1. The basic integrals are given by

$$I(s, d, y, \bar{\tau}) = \int_0^\infty \frac{d\tau}{\tau^s} f(\tau/\bar{\tau}, d) \exp(-y\tau) \quad (A1)$$

and can be expressed in terms of Bessel functions of the second kind and $\Gamma$-functions in case of $f(\tau/\bar{\tau}, d) = f_1(w, d)$ and it terms of $\Gamma$ and $\zeta$-functions for $f(\tau/\bar{\tau}, d) = f_2(w, d)$. The full expressions are rather cumbersome.

2. The short-distance function $S_0 = S_0^{WW} + S_0^{WH} + S_0^{HH}$ is given by

$$S_0^{WW} = 16\pi^2 m_W^2 \int \frac{d^4p}{(2\pi)^4} \frac{m_t^4}{(p^2 + m_W^2)^2} \frac{1}{(p^2 + m_t^2)^2}$$

$$S_0^{HH} = 4\pi^2 \frac{m_t^4}{m_W^2} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 + m_W^2)^2} \frac{p^2}{(p^2 + m_t^2)^2}$$

$$S_0^{HW} = 32\pi^2 m_t^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 + m_W^2)^2} \frac{1}{(p^2 + m_t^2)^2}$$

with the result

$$S_0 = \frac{4x_t - 11x^2_t + x_t^3}{4(x_t - 1)^2} + \frac{3x_t^3 \log x_t}{2(x_t - 1)^3} \quad (A2)$$

where $x_t = m_t^2/m_W^2$.

3. The SM one-loop contribution to the muon anomalous magnetic moment is given by the following Feynman integral

$$F_2^{SM} = 8e^2 \int_0^1 x^2(1-x) \int \frac{d^4\bar{p}}{(2\pi)^4} \frac{1}{(\bar{p}^2 + x^2)^3} \quad (A3)$$

The function $F_2(L, d)$ corresponds to the replacements [56] in this integral, with the proper account of the loop momentum rescaling $p = m_\mu \bar{p}$.

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FIG. 3: The same as Fig. 2, for the choice (11).

FIG. 4: Contour plot for the function $w(L, d)$ with the choice (10). Contours correspond to $w(L, d) = \pm 1$ (dashed), $w(L, d) = \pm 0.1$ (thin) and $w(L, d) = 0$ (thick).

FIG. 5: The same as Fig. 4, for the choice (11).