Measurement of CP violation of neutral kaon system in J/ψ decay at the Super Tau-Charm Facility

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In this paper, we present a preliminary study of CP violation effect of K°−K̄° system in J/ψ decay. The CP violation parameters η± and η00 as well as their corresponding phase φ+ and φ00 can be determined by the difference of the time-dependent decay rates between K° and K̄° produced from J/ψ → K−π⁺K°+c.c. processes. We investigate the precisions of the measurements of the CP violation effect at the Super Tau-Charm facility(STCF), a e⁺e⁻ collider with a peak luminosity of 0.5 × 10^{35} cm^{-2}s^{-1}. The parameters η± and its phase φ± can be measured at a relative precision of 1 × 10^{-3}, which the statistical accuracy will be several times better than that of the existing PDG average values.

I. INTRODUCTION

The phenomenon of mixing in neutral kaon system has been of special interest for a long time. Neutral kaons have definite quark components and strangeness S when they are produced by strong interaction. The K° and K̄° only differ by strangeness, but strangeness can be changed by flavor changing process(weak interaction), namely K° − K̄° mixing or oscillation. The Feynman diagrams of K° − K̄° mixing are shown in Fig. 1. If we assume CP is symmetric in weak interaction, the CP eigenstates can be defined by K°, K̄° basis:

\[ |K_1⟩ = \frac{1}{\sqrt{2}} [ |K°⟩ + |K̄°⟩] , \text{ with } CP = 1 \]
\[ |K_2⟩ = \frac{1}{\sqrt{2}} [ |K°⟩ - |K̄°⟩] , \text{ with } CP = -1. \quad (1) \]

The decay final states with two pion or three pion have well defined CP eigenvalues: CP |ππ⟩ = + |ππ⟩ and CP |πππ⟩ = − |πππ⟩. The phase space of K_1 → ππ is larger than K_2 → ππ, therefore the lifetime of K_1 is shorter than K_2, so they are called K_S and K_L. The K_S and K_L are kaon states with well defined mass and lifetimes that exist in nature.

However, the CP symmetry is violated in weak interaction. K_L may also decay to ππ final state and K_S can decay to πππ too. It is convenient to express the mass states of two neutral kaons in the |K_1⟩ and |K_2⟩ basis [3]:

\[ |K_S⟩ = \frac{1}{\sqrt{1 + |ɛ_S|^2}} (|K_1⟩ + ɛ_S |K_2⟩) \]
\[ |K_L⟩ = \frac{1}{\sqrt{1 + |ɛ_L|^2}} (|K_2⟩ + ɛ_L |K_1⟩). \quad (2) \]

where ɛ_S and ɛ_L are complex parameters indicating possible CP and CPT violation. The parameters ɛ_S and ɛ_L can be expressed as ɛ_S = ε + δ and ɛ_L = ε − δ without the assumption of CPT conservation; and ɛ_S = ɛ_L = ε, if CPT invariance holds.

There are in general three different types of CP violation effects in neutral kaon decays: (1) indirect CP violation or CP violation in K° – K̄° mixing; (2) direct CP violation in decay amplitude; (3) CP violation in the interference of mixing and decay. We take the neutral kaon decay to ππ as example to explain the direct CP violation. The ππ final states can be decomposed into superposition of isospin eigenstates I = 0 or I = 2. Therefore the amplitudes of a neutral kaon decay to ππ can be described as [4]:

\[ A_I = ⟨\pi\pi; I |H_{wk} |K°⟩ , \quad A_0 = ⟨\pi\pi; I |H_{wk} |K̄°⟩ \]
\[ A_I = (A_I + B_I)e^{iδ_I} , \quad A_0 = (A^*_I - B^*_I)e^{-iδ_I} \quad (3) \]

where I = 0, 2. The amplitudes A_I and B_I are CPT symmetric and antisymmetric, respectively. The factor e^{iδ_I} represents the final state interaction of the pions. Direct CP violation is arised from the phase difference between the A_0 and A_2 amplitudes. This phase difference is produced by the interference of tree diagrams and penguin diagrams for s quark decay, which is shown in Fig. 2.
It is convenient to introduce the \( CP \) violation parameters in the neutral kaon sector:

\[
\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = |\eta_{+-}| e^{i\phi_{+-}} \equiv \epsilon + \epsilon', \\
\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = |\eta_{00}| e^{i\phi_{00}} \equiv \epsilon - 2\epsilon',
\]

where \( \epsilon = |\epsilon| e^{i\phi_{\epsilon}} \) is the mixing parameter and \( \epsilon' = |\epsilon'| e^{i\phi_{\epsilon'}} \) represent the direct \( CP \) violation. At present, the PDG average value of the above \( CP \) violation parameters are \([3]\):

\[
|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}; \quad \phi_{+-} = (43.4 \pm 0.5)^\circ, \\
|\eta_{00}| = (2.220 \pm 0.011) \times 10^{-3}; \quad \phi_{00} = (43.7 \pm 0.6)^\circ, \\
|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}; \quad \phi_{\epsilon} = (43.5 \pm 0.5)^\circ.
\]

In which the value of \( \phi_{+-}, \phi_{00} \) and \( \phi_{\epsilon} \) are taken without assuming \( CPT \) conservation.

**II. ANALYSIS METHOD**

In this paper we first investigate the possibility of measuring the \( CP \) and \( CPT \) violation at a super \( \tau \) – \textit{charm} facility (STCF) \([6, 7]\). The main part of STCF project is a symmetrical electron-positron collider with a peak luminosity of \( 0.5 \times 10^{35} \text{cm}^{-2}\text{s}^{-1} \) or more at center-of-mass energy \( \sqrt{s} = 4.0 \text{ GeV} \). The STCF will operate at the energies \( \sqrt{s} \) from 2.0 to 7.0 GeV, which will provide more than 1 trillion \( J/\psi \) events per year. Much more precise measurements of these \( K^0 - \bar{K}^0 \) mixing and \( CP, CPT \) violation parameters can be achieved at STCF. In experiments, there are two methods to tag the neutral kaon. The first method is \( CP \) tagging method, one can distinguish the \( K_1 \) and \( K_2 \) by reconstruction the decay final \( \pi \pi \) or \( \pi\pi\pi \) states. Another method is flavor tagging method, the \( K^0 \) or \( \bar{K}^0 \) can be distinguished by their quark composition or semileptonical decay final states according to \([8]\).

The method we suggest is using the charged-conjugate particles \( K^0 \) and \( \bar{K}^0 \) produced in \( J/\psi \) decays. Initially-pure \( K^0 \) and \( \bar{K}^0 \) states are produced by decay channels: \( J/\psi \to K^0 K^-\pi^+ / \bar{K}^0 K^+\pi^- \), with the corresponding branching ratio \((5.6 \pm 0.5) \times 10^{-3} \). The strangeness is conserved in the strong interaction, thus the strangeness of the neutral kaon at production can be tagged by the charge sign of the concomitant charged kaon.

The \( CP \) violation parameters \( \eta_{+-} \) and \( \eta_{00} \) can be estimated by measuring the difference of time-dependent decay rates between \( K_0 \) and \( \bar{K}_0 \). With the above definition, for the \( CP \) eigenstates \( f = 2\pi \) or \( 3\pi \), the decay rate \( R_f(\tau) = R[K_{i=0} \to f_{i=\tau}] \) and \( \bar{R}_f(\tau) = R[\bar{K}_{i=0} \to f_{i=\tau}] \) can be written as \([4]\):

\[
R_f(\tau) = \left[ 1 - 2\text{Re}(\epsilon - \delta) \right] \frac{1}{2} \Gamma^f_S \times \left[ e^{-\Gamma^f_S \tau} + |\eta_f|^2 e^{-\Gamma^f_L \tau} + 2 |\eta_f| e^{(1/2)(\Gamma^f_S + \Gamma^f_L)\tau} \cos(\Delta m \tau - \phi_f) \right], \\
\bar{R}_f(\tau) = \left[ 1 + 2\text{Re}(\epsilon - \delta) \right] \frac{1}{2} \Gamma^f_S \times \left[ e^{-\Gamma^f_S \tau} + |\eta_f|^2 e^{-\Gamma^f_L \tau} - 2 |\eta_f| e^{(1/2)(\Gamma^f_S + \Gamma^f_L)\tau} \cos(\Delta m \tau - \phi_f) \right]
\]

where \( \Gamma^f_S \) is the partial decay width of \( K_S \to f \), \( \Gamma_S \) and \( \Gamma_L \) are the total decay width of \( K_S \) and \( K_L \), respectively. The \( \Delta m = m_L - m_S \) is the mass difference between \( K_L \) and \( K_S \). The decay rate asymmetry can be formed by...
the combination of $R_f$ and $\tilde{R}_f$:

$$A_{\text{CP}}(\tau) = \frac{R_f(\tau) - \tilde{R}_f(\tau)}{R_f(\tau) + \tilde{R}_f(\tau)}$$

$$= 2\text{Re}(\epsilon - \delta) - \frac{2|\eta_f|\sigma(1/2)(\Gamma_\eta - \Gamma_L)\cos(\Delta m\tau - \phi_f)}{1 + |\eta_f|^2\sigma(\Gamma_\eta - \Gamma_L)\tau}$$

(6)

$$A_{\text{CP}}(\tau) = \frac{R_f(\tau) - \tilde{R}_f(\tau)}{R_f(\tau) + \tilde{R}_f(\tau)}$$

III. MONTE CARLO SIMULATION

BESIII has collected 10 billion $J/\psi$ events which is largest so far [9], while the statistics are still insufficient to study $CP$ violation in neutral kaon system. Therefore the future STCF will be a promising facility to perform this subject. The STCF will accumulate about $10^{12}$ or even $10^{13}$ $J/\psi$ events per year in monochrome collision mode [10]. Thus there will be more than $5.6 \times 10^9 K^-\pi^+K^0+c.c.$ events before reconstruction. The method of measuring the decay lifetime of neutral kaon at STCF is shown in Fig. 3. The $J/\psi$ particle is produced after the collision of $e^+e^-$ and then rapidly decay into $K^-\pi^+K^0$. The precise measurement of the production and decay vertex of $K^0$ is important for the estimation of $K^0$ proper-time distribution. A vertex constrained fit is performed on the $\pi^+\pi^-$ to get the decay vertex of $K^0$. The two charged pions are required to originate from a common vertex. And then the production vertex of $K^0$ is obtained by performing another vertex constrained fit on $K^-\pi^+$. The position uncertainty of production vertex is taken as $\sigma_x = 13.6 \mu m$, $\sigma_y = 1.4 \mu m$ and $\sigma_z = 50 \mu m$ [6, 7], where the $z$ axis is along the beam direction, and the $x$-$y$ axes are in the plane perpendicular to the beam. The $\sigma_x$ and $\sigma_y$ are taken from the size of $e^+e^-$ beam in the $x-y$ plane, while $\sigma_z$ can be constrained by the vertex constrained fit of $K^-\pi^+$, that is, much smaller than the size of the beam in the $z$-direction. The distance between $K^0$ production vertex and decay vertex is taken as the decay length $L_K$. The detector resolution for $K^0$ momentum $p_K$ is estimated to be $\sigma_p \approx 0.5% \cdot p_K$. The proper-time of $K^0$ can be calculated by decay length $l_K$ and momentum $p_K$:

$$t = \frac{m_{K^0} l_K}{p_K}$$

(7)

where $m_{K^0}$ is the invariant mass of $K^0$.

A Monte Carlo(MC) simulation is performed on the basis of the total number $3.9 \times 10^9 J/\psi \rightarrow K^-\pi^+K^0(K^0 \rightarrow \pi^+\pi^-) + c.c.$ events. The time dependent decay rates of $K^0(K^0 \rightarrow \pi^+\pi^-)$ are simulated based on Eq. 5 with the input parameters fixed to PDG average values [5]. The MC simulation is generated with the considerations of the position uncertainty of $e^+e^-$ interaction point and $\pi^+\pi^-$ vertex reconstruction as well as the uncertainties of momentum reconstruction of $\pi^+\pi^-$. The detector resolution of $K^0$ decay vertex versus decay length as well as momentum are shown in Fig. 4. Figure 5 shows the reconstruction efficiency versus decay length and momentum.

The time dependent number $N(\tau)$ for $K^0 \rightarrow \pi^+\pi^-$ as well as $\bar{N}(\tau)$ for $K^0 \rightarrow \pi^+\pi^-$ are shown in Fig. 6. Considering that this method is not sensitive to the measurement of parameter $\text{Re}(\epsilon - \delta)$, A normalization factor $k = 1 + 4\text{Re}(\epsilon - \delta)$ is used to include the influence of parameter $\text{Re}(\epsilon - \delta)$ on fitting [11]. Then the time dependent decay rate asymmetry $A_{\text{CP}}$ can be expressed as Eq. 6. We use the MINUIT method from ROOT package [12] to determine the fit parameters and the fitting

FIG. 3. A diagram of how to measure the decay lifetime of neutral kaon.

FIG. 4. The detector resolution of $K^0$ decay vertex versus (a) decay length $L_K$ and (b) momentum $p_K$.
FIG. 5. The detection efficiency of $K^0$ decay vertex versus (a) decay length $l_K$ and (b) momentum $p_K$.

FIG. 6. The time-dependent decay rates for $K^0$ (black point) and $\bar{K}^0$ (open circle) separately.

FIG. 7. The time dependent decay rate asymmetry $A_{+−}$ versus the neutral kaon decay time. The points with error bar are from MC simulation and the solid line is the fitting results.

TABLE I. The comparison between PDG average value and fitting results in this work.

| Par. | $|\eta_{+−}|$ (10$^{-3}$) | $\phi_{+−}$ (degree) |
|------|---------------------|---------------------|
| PDG  | 2.232 ± 0.011       | 43.4 ± 0.5          |
| STCF | 2.2320 ± 0.0025 ± 0.0027 | 43.510 ± 0.051 ± 0.059 |

In this fitting the parameters $\Gamma_{S}^{\pi^{+}\pi^{-}}$, $\Gamma_{S}$, $\Gamma_{L}$ and $\Delta m$ are fixed to latest PDG average value [4]. The comparison between PDG average value and fitting results are shown in Table I. The first uncertainty of fitted parameters is statistical uncertainty, and the second is associated with the uncertainties of $e^{+}e^{-}$ interaction point and the reconstruction of $\pi^{+}\pi^{-}$ vertex and momentum. The fitting quality is present by $\chi^{2}$/d.o.f. = 0.7 and the correlation coefficients between $|\eta_{+−}|$, $\phi_{+−}$ and normalization factor $k$ are shown in Table II.

TABLE II. Correlation coefficients of the fitted parameters.

| Par. | $|\eta_{+−}|$ | $\phi_{+−}$ | $k$ |
|------|--------------|-------------|-----|
| $|\eta_{+−}|$ | 1            | -0.04       | 0.59 |
| $\phi_{+−}$ |              | 1           | 0.029 |
| $k$       |              |             | 1    |

IV. CONCLUSIONS

To summarize, this paper present a preliminary study of $CP$ violation effects in neutral kaon produced at a $J/\psi$ facility. By using about 10$^{12}$ $J/\psi$ events collected at STCF, the relative accuracy in measuring the parameters $|\eta_{+−}|$ and $\phi_{+−}$ is estimated to be at the level of 10$^{-3}$, which will be several times better than the exist-
ing measurements. Measurement with such high precision needs a high purity data sample, accurate vertex reconstruction and well-controlled systematic uncertainties, which requiring STCF should have a charged track detector with excellent performance or a vertex detector to effectively reconstruct the neutral kaon far from the collision point. Especially when the proper time of $K^0/\bar{K}^0$ is between 10 and 15 times of the $K_S$ mean lifetime, there will be high sensitivity to measure $|\eta_{+/-}|$ and $\phi_{+/-}$. The corresponding decay length of neutral kaon is between 10 cm and 35 cm. Therefore, the track detector of STCF is required to reconstruct the vertex of $K^0/\bar{K}^0$ efficiently and accurately in this interval. A detector with excellent position resolution and high resolution in the energy loss $dE/dx$ will help to suppress the background of miscombinations and particle misidentification. The values of $\phi_{+/-}$ and $\phi_{00} - \phi_{+/-}$ will be used to set limits on $CPT$ violation [5]. However, the $CPT$ test by comparing the measured $\phi_{+/-}$ and super weak phase $\phi_{sw}$ need to carefully estimate the phase difference from the contribution of the semileptonic decay modes and the $3\pi$ decay modes of neutral kaon [13][17]. This method can also be used to study the time-dependent $CP$ asymmetries in the charmed hadron decays with neutral kaons in the final state at Belle(II) and LHCb [18][21], which not only can help us to study the $CP$ violation of neutral kaon system, but also reveal the direct $CP$ violation in the charm hadron decays after subtracting the contribution from neutral kaon mixing.

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