Accessibility Measure for Eternal Inflation: 
Dynamical Criticality and Higgs Metastability

Justin Khoury

Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania, 
209 South 33rd St, Philadelphia, PA 19104

Abstract
We propose a new measure for eternal inflation, based on search optimization and first-passage statistics. This work builds on the dynamical selection mechanism for vacua based on search optimization proposed recently by the author and Parrikar. The approach is motivated by the possibility that eternal inflation has unfolded for a finite time much shorter than the exponentially long mixing time for the landscape. The proposed accessibility measure assigns greater weight to vacua that are accessed efficiently under time evolution. It is the analogue of the closeness centrality index widely used in network science. The proposed measure enjoys a number of desirable properties. It is simultaneously time-reparametrization invariant, independent of initial conditions, and oblivious to physical vs comoving weighing of pocket universes. Importantly, the proposed measure makes concrete and testable predictions that are largely independent of anthropic reasoning. Firstly, it favors vacua residing in regions of the landscape with funnel-like topography, akin to the energy landscape of naturally-occurring proteins. Secondly, it favors regions of the landscape that are tuned at dynamical criticality, with vacua having an average lifetime of order the de Sitter Page time. Thus the predicted lifetime of our universe is of order its Page time, $\sim 10^{130}$ years, which is compatible with Standard Model estimates for electroweak metastability. Relatedly, the supersymmetry breaking scale should be high, at least $10^{10}$ GeV. The discovery of beyond-the-Standard Model particles at the Large Hadron Collider or future accelerators, including low-scale supersymmetry, would rule out the possibility that our vacuum lies in an optimal region of the landscape. The present framework suggests a correspondence between the near-criticality of our universe and dynamical critical phenomena on the string landscape.

1 Introduction
If the fundamental theory allows eternal inflation [1–3], then a measure is necessary to assign probabilities for different outcomes and therefore make predictions. The task of defining a measure satisfying various physical desirata, such as time-reparametrization invariance and independence on initial conditions, has proven to be formidable. This is the well-known measure problem. (See [4] for a review.) In the 80’s and 90’s the measure problem was viewed by and large as a pesky mathematical subtlety, with the exception of a handful of inflationary cosmologists who studied it seriously. In the early 2000’s, it came to more widely appreciated as a pressing problem with the discovery of an exponentially large number of metastable de Sitter (dS) vacua in string theory [5–8].

Historically two broad classes of measures have been considered:

- **Global measures** define a global foliation of space-time specified by a global time coordinate $t$. One

1Recently, the existence of metastable dS vacua in string theory, at least in parametrically-controlled regimes, has been questioned through the dS swampland conjecture [9–12]. This has sparked a heated debate — see [13] and references therein. In our analysis, we assume for concreteness the existence of a landscape of metastable dS vacua.
counts pocket universes\(^2\) on a late-time cutoff surface \(t = t_c\), and then lets \(t_c \to \infty\) \([14, 21, 23, 25]\). This prescription has the advantage of being independent of initial conditions, consistent with the attractor property of inflation. Its major drawback is that the result depends sensitively on the choice of foliation \([14, 16]\). Another source of ambiguity is whether pocket universes are weighted according to their comoving or physical volume. This can result in exponentially different results, even for fixed foliation.

Faced with these difficulties, recent work on global measures has focused on ruling out certain foliations based on phenomenological input, \(e.g.\) avoiding the youngness paradox \([21]\) or domination by Boltzmann brains \([18, 19]\). This has led to some convergence towards scale-factor time as the most phenomenologically sensible foliation \([18]\). While this interplay between theory and phenomenology is logically reasonable, it would be far more desirable to define \textit{ab initio} a measure based solely on theoretical principles, such as time-reparametrization invariance and independence on initial conditions, and derive phenomenological implications \textit{a posteriori}.

- \textit{Local measures} focus on a space-time region around a time-like observer. Examples include the past light-cone of world-lines (causal diamond measure \([26, 27]\)), a region bounded by the apparent horizon \([28]\), and a space-like region around a world-line (\(e.g.\) the “watcher” measures \([24, 25, 29, 30]\)). Such constructions are manifestly gauge-invariant. However, because a typical geodesic will eventually enter an AdS or Minkowski vacuum, generally assumed to be terminal\(^4\), all but a measure zero of watchers will sample a finite number of bubbles. This regulates the infinities of eternal inflation \([26]\), at the expense of sensitivity to initial conditions\(^5\). In particular, it has been shown that for specific choices of initial conditions certain local measures agree with their global counterpart \([27, 32]\).

Despite the variety of approaches, all proposed measures to date focus on the same statistics: \textit{the stationary distribution of a Markov process describing vacuum dynamics}. Mathematically, the landscape can be modeled by a graph or network whose nodes represent the different vacua, and whose links denote the relevant transitions (Sec. 2). After coarse-graining, the probability \(f_i(t)\) that a time-like observer (“watcher”) occupies vacuum \(i\) at time \(t\) is governed by a linear Markov equation \([23, 24]\). (Equivalently, \(f_i(t)\) is the fraction of comoving volume occupied by vacuum \(i\).)

For generic initial conditions, the solution to the Markov process tends asymptotically to a stationary distribution: \(f(t) \to f^\infty\). This stationary distribution is the zero-mode of the transition matrix, and as such lies entirely within the subspace of terminal vacua. The relative probability to lie in different non-terminal (dS) vacua is therefore determined by the subleading term as \(t \to \infty\), which is in turn set by the slowest-decaying (or “dominant”) eigenvector: \(\delta f \sim v^{(1)}_M e^{\lambda_1 t}\). Here \(\lambda_1 < 0\) is the “dominant” eigenvalue, \(i.e.\), the non-zero eigenvalue of \(M\) with the smallest magnitude, which sets the relaxation time. In turn, \(v^{(1)}_M\) is dominated by the dS vacuum with the \textit{slowest decay rate} anywhere on the landscape. Since this so-called “dominant vacuum” is unlikely to be hospitable, the relative probabilities for different hospitable vacua are

\(^2\)Pocket universes can consist either of thermalized regions, if the evolution on the landscape is dominated by quantum diffusion of scalar fields, or false-vacua dS regions, if the evolution is instead dominated by quantum tunneling and bubble nucleation. To simplify the discussion, in this paper we will focus exclusively on the latter case for concreteness.

\(^3\)Recently the 4-volume cutoff measure has been proposed \([22]\), which improves on certain technical drawbacks of the scale factor measure while otherwise making similar predictions.

\(^4\)An alternative and more speculative possibility is that collapsing AdS regions can sometimes bounce and avoid big crunch singularities \([28, 31]\). In this paper we will treat AdS regions as terminal.

\(^5\)A possible argument \([20, 27]\) is that the question of initial conditions is logically distinct from the measure problem and should be provided by the theory of quantum gravity. While this is a fair point, it would be more satisfactory to have a measure, defined solely within semi-classical gravity, that is both time-reparametrization invariant \textit{and} independent of initial conditions.
determined by the transition rate from the dominant vacuum to each hospitable vacuum. All global and local measures are based on this late-time prescription.

Aside from the technical challenges mentioned earlier, the late-time prescription presents another drawback — its sensitivity to exponentially small terms in the transition matrix. The argument, laid out in the Appendix, can be summarized as follows. Although the detailed nature of the dominant vacuum requires input from string theory, one expects on general grounds that it has very small vacuum energy $V_{\text{dom}}$, and is surrounded by vacua of much higher potential energy. In this configuration, its only allowed Coleman-De Luccia (CDL) transitions involve “up-tunneling”. By detail balance, the rate for such upward transitions is exponentially suppressed by $e^{-48\pi^2 M_{\text{Pl}}^4/V_{\text{dom}}}$ compared to the (already exponentially small) rate for the reverse processes. In particular, transition rates from the dominant vacuum to hospitable vacua, which set the relative probabilities, are sensitive to exponentially small contributions to the transition matrix. Minor tweaks to the landscape can alter these exponentially small corrections, resulting in different predictions. While not logistically inconsistent, we view such sensitivity to minor tweaks to the landscape as undesirable.

1.1 A new measure of centrality

In the language of graph theory the problem of defining a measure for eternal inflation is a question of centrality — which nodes in the network are, in a suitably defined sense, most important? Recently there has been tremendous activity in understanding the properties of real-world networks, such as the world wide web, academic co-authorship, social networks, protein interaction networks, epidemic propagation etc. The question of centrality is of utmost importance in these studies.

Various centrality indices have been proposed, and they can be broadly classified as followed (see [33] for a review). One category of centrality measures is based on spectral properties of the transition matrix. This includes eigenvector centrality, Katz index [34], and Google’s PageRank algorithm [35]. The stationary distribution on which global and local measures are based belongs to this category. Specifically it is an example of left dominant eigenvector centrality.

Another category of centrality measures focuses instead on the shortest paths between nodes. For instance, the closeness measure [36, 37] assigns greater weight to nodes that can be reached on average with the fewest number of steps. Another example is betweenness centrality, which favors nodes that have the most shortest paths between other pairs of nodes passing through them. Intuitively, nodes that are favored by closeness or betweenness centrality are most important in controlling the flow of information in the network.

In this paper we present a new measure for eternal inflation that is analogous to closeness centrality. We call it the accessibility measure. Instead of characterizing the distribution of vacua at equilibrium, the proposed measure pertains to the approach to equilibrium. Indeed, since eternal inflation is past-geodesically incomplete [38], eternal inflation started a finite proper time in our past. The standard approach based on the stationary distribution assumes that we live sufficiently long after the onset of eternal inflation, such that vacuum statistics have reached equilibrium. While logically consistent, this assumption is non-trivial. Globally the landscape features many exponentially long-lived metastable vacua, resulting in glassy dynamics and exponentially long relaxation time [39].

The accessibility measure is motivated instead by the alternative possibility that eternal inflation has unfolded for a time much shorter than the relaxation time. In this case, as first proposed in [40], a vacuum like ours should be likely not because it is typical according to the stationary distribution, but rather because it has the right properties to be accessed early on in the evolution. At times much smaller than the mixing time, most hospitable vacua have been accessed once or perhaps not at all. In such a situation, the occupational probability $f_i(t)$ is not the most reliable statistics. Instead the accessibility measure will be based on first-passage statistics, which are best-suited to study the approach to equilibrium. The accessibility measure will give greater weight to vacua with high first-passage probability, or equivalently short first-passage time. Such vacua are most likely to be accessed quickly, much earlier than the relaxation time.
A natural concern with working at early times is sensitivity to initial conditions. The accessibility measure will be defined by effectively averaging over initial conditions, to give a result that is independent of initial conditions as well as time-reparametrization invariant. Furthermore, because it is based on access time instead of volume fractions, it is oblivious to comoving vs physical volume ambiguity and avoids any youngness bias. The accessibility measure is also insensitive to minor tweaks to the landscape; it can be reliably calculated by neglecting the exponentially small terms in the transition matrix that encode upward-tunneling. Last but not least, the accessibility measure makes testable and falsifiable predictions (summarized below).

This work is a continuation of a recent paper [41], which presented a new dynamical selection mechanism for vacua based on search optimization. Instead of restricting to late-time, stationary distributions for the different vacua, the analysis focused on the approach to equilibrium. It was shown that the mean first-passage time (MFPT) to hospitable vacua is minimized for vacua lying at the bottom of funnel-like regions of the landscape, as sketched in Fig. 1. This is akin to the smooth folding funnels of naturally-occurring proteins [43, 44], where the high-energy unfolded states are connected to the lowest-energy native state by a relatively smooth funnel. Furthermore, it was argued that optimal regions of the landscape achieve a compromise between minimal oversampling and sweeping exploration of the region, resulting in dynamical criticality.

The analysis of [41] stopped short, however, of defining an actual measure for the likelihood of residing in optimal regions of the landscape. The goal of this paper is to fill this gap. We will argue that the accessibility measure favors vacua in optimal regions, characterized by funnel-like topography and dynamical criticality.

1.2 Brief overview of results

Our measure is simplest to define in a toy landscape comprised only of dS vacua (Sec. 4). The key building block is the MFPT \( \langle t_{j \rightarrow i} \rangle \) between a pair of vacua \( i \) and \( j \), defined as the time for a random walker starting from \( j \) to reach \( i \) averaged over all paths connecting the two nodes [43]. We then define a dimensionless partial MFPT (pMFPT) to a given node \( i \) by averaging the MFPT over all initial nodes \( j \)

\[
\mathcal{T}_i \equiv \frac{v_i^{(1)2}}{1 + v_i^{(1)2}} \sum_{j \neq i} v_j^{(1)2} \frac{(t_{j \rightarrow i})}{\Delta t},
\]

Figure 1: An optimal region is characterized by all vacua having at least one allowed downward transition. Its landscape topography is that of a funnel, akin to the free energy landscape of proteins. (Reproduced from [42].)
where $\Delta t$ is the coarse-graining time step, and $v^{(1)}_j$ is the zero-mode of the transition matrix which sets the stationary distribution for the occupational probability\footnote{The partial MFPT \cite{46} is closely related to a quantity introduced by \cite{46}, which was called global MFPT.}. Therefore $T_i$ can be thought of as the MFPT to node $i$, averaged over all initial nodes weighted by the stationary distribution. The overall factor of $\frac{v^{(1)}_j}{1 + v^{(1)}_j}$ is included to simplify some of the results below.

At first sight it may seem strange to weigh initial vacua according to the stationary distribution, since our measure is supposed to capture information about the approach to equilibrium. We offer the following justification. Given a Markov process, one can argue that the stationary distribution is a natural distribution to consider\footnote{Incidentally, it is by similarly averaging over initial conditions using the stationary distribution that certain local measures match the predictions of their global counterpart \cite{27,32}.}. More to the point, however, the stationary distribution offers a conservative estimate of the average time needed to reach a given node. Indeed, as we will see, $v^{(1)}_j$ is dominated by the most stable dS vacuum, which should therefore correspond to the largest MFPT $\langle t_{j \rightarrow i} \rangle$. In other words, the weighted sum in $T_i$ gives greatest weight to the initial node corresponding to the longest average travel time to $i$. From this point of view we expect that any distribution with non-zero support on the most stable dS vacuum (such as a uniform distribution) should give similar results. That said, we cannot claim that the accessibility measure defined below is unique.

The pMFPT can be expressed in alternative ways, each offering different insights. Firstly, we will see that \cite{1} can be cast simply in terms of the eigenvalues and eigenvectors of the transition matrix. This makes the time-reparametrization invariance of $T_i$ manifest. Secondly, it can be written in terms of first-return statistics \cite{46}:

$$T_i = \frac{1}{2} \langle t_{i \rightarrow i}^2 \rangle / \langle t_{i \rightarrow i} \rangle^2,$$

where $\langle t_{i \rightarrow i} \rangle$ is the first-return time, and $\langle t_{i \rightarrow i}^2 \rangle$ is the second moment of the first-return probability. Since by definition first-return statistics for $i$ describe random walks that all start at $i$, this expression makes manifest that $T_i$ is independent of initial conditions. Furthermore, it implies an important lower-bound \cite{46}:

$$T_i \geq \frac{1}{2}.$$

A third, and perhaps most intuitive, interpretation of the pMFPT is in terms of first-passage probability. On a finite landscape every vacuum is guaranteed to be populated eventually, but the required time scale for all vacua to be accessed is of course the relaxation time. At times much shorter than the relaxation time, we will see that maximizing the first-passage probability to a given vacuum is equivalent to minimizing its pMFPT.

Yet another equivalent expression for the pMFPT is in terms of the escape or never-return probability $S_i$ from $i$ in the limit of an infinite network. This represents the probability that a random walker who started from $i$ eventually never returns to $i$. We will find the simple relation $T_i = S_i^{-1}$.

Our proposed accessibility measure for dS-only landscapes is then defined as\footnote{The accessibility measure defined here is related to random walk centrality introduced in \cite{47}, as well as second order centrality studied in \cite{48}.}

$$p_i = \frac{T_i^{-1}}{\sum_k T_k^{-1}},$$

By construction, the measure is both time-reparametrization and independent of initial conditions. Furthermore, it is oblivious to the comoving vs physical volume ambiguity. The measure favors vacua that are easily accessed under time evolution, i.e., those vacua that saturate $\frac{T_i}{T} \sim O(1)$. Thus, unlike standard
measures based on the stationary distribution, the proposed measure does not exponentially favor a single vacuum. There can be many vacua with $T_i \sim \mathcal{O}(1)$, and all will be weighted equally according to (4).

In Sec. 5 we will generalize the accessibility measure to a landscape with terminal vacua. For dS vacua, we have mentioned above four different equivalent expressions for the pMFPT in the case of a dS-only toy landscape: 1. In terms of a weighted average of the MFPT to a given node; 2. In terms of the variance of first-return times to a given node; 3. In terms of the first-passage probability for times shorter than the relaxation time; 4. In terms of the escape probability from a given node.

Once terminals are included, however, these give inequivalent definitions of the pMFPT. It turns out that the last definition in terms of the escape probability offers the most straightforward generalization to the case with terminals. Therefore for dS vacua we define the accessibility measure in terms of the escape probability. For terminal vacua, the natural analogue to an escape probability is a trapping probability. Both escape and trapping probabilities admit simple expressions in terms of eigenvectors and eigenvalues of the transition matrix. The resulting accessibility measure is both time-reparametrization invariant and independent of initial conditions.

### 1.3 Predictions of the accessibility measure

Importantly, the accessibility measure makes concrete and testable predictions (Sec. 6), in particular for new physics (or absence thereof) at the Large Hadron Collider (LHC). These predictions are largely independent of anthropic reasoning. The only input of possible anthropic origin is the vacuum energy, or equivalently the Hubble constant $H_0$. Given the observed value of $H_0$, we derive predictions for other observables, such as the optimal lifetime of our universe, which follow readily from the measure itself.

- **Access time**: The accessibility measure favors vacua that are easily accessed under time evolution. Specifically, it favors vacua that nearly saturate (5), i.e., with order unity pMFPT. In terms of proper time for vacuum $I$, this implies an optimal access time of order its Hubble time:
  \[ \tau_i^{\text{access}} \sim H_i^{-1}. \]
  Given the observed vacuum energy in our universe, this implies that we live approximately $H_0^{-1} \sim 13.8$ billion years after the beginning of eternal inflation. This in turn implies an upper bound on the number of e-folds during the last period of inflation prior to reheating: $N \lesssim H_{\text{inf}}/H_0$.

- **Funnel topography**: To make further predictions we will consider a finite region of the landscape that includes $N_{\text{inf}} \gg 1$ dS vacua. The fiducial region is treated as a closed system for simplicity, though this assumption is relaxed later on. We define in Sec. 6.2 a characteristic time $\langle T \rangle$ for the landscape dynamics of dS vacua in the region, as a suitable average over the pMFPTs. In the downward approximation, where upward transitions are neglected, this average pMFPT reduces to
  \[ \langle T \rangle \simeq \frac{1}{N_{\text{inf}}} \sum_{i=1}^{N_{\text{inf}}} \frac{1}{\kappa_i \Delta t}, \]
  where $\kappa_i$ is the total decay rate of the $i$th vacuum. Thus $\langle T \rangle$ is recognized as a mean residency time. If any vacuum in the region has only upward-tunneling as allowed transitions, then its decay rate will be exponentially small, resulting in an exponentially large $\langle T \rangle$. Such a region is characterized by frustration and glassy dynamics [49]. It should be clear that such frustrated regions are heavily disfavored by the accessibility measure. Instead, the accessibility measure favors regions whose dS vacua all have allowed downward transitions, either to lower-lying dS vacua or to terminals. Such favored regions therefore have the topography of a broad valley or funnel, as shown in Fig. 1. This reaffirms the result obtained in [41], this time motivated by a well-defined measure.
This is a key prediction of the accessibility measure. Unlike stationary measures, according to which our vacuum should be generic among all hospitable vacua on the landscape (the “principle of mediocrity”), the accessibility measure favors vacua residing in special regions of the landscape with funnel-like topography.

• **Dynamical criticality and Page lifetime:** To properly treat regions of the landscape as open systems would require modeling their environment. Following [41], we instead study a proxy requirement that relies solely on the intrinsic dynamics within a given region. Specifically, we demand that random walks in the region are recurrent in the infinite-network limit. Recurrent walks thoroughly explore any region around their starting point, hence recurrence offers a reliable and model-independent benchmark for efficient sampling [41]. We will show in Sec. 6.4 that the joint demands of minimal oversampling, defined by minimal \( \langle T \rangle \), and sweeping exploration, defined by recurrence, selects regions of the landscape that are tuned at dynamical criticality. Vacua in optimal regions have an average lifetime of order the dS Page time:

\[
\tau_{\text{crit}}(H) \sim \frac{M_{\text{Pl}}^2}{H^3} .
\]

Thus the Page time represents an optimal time for vacuum selection, as first realized in [41].

• **Higgs metastability and particle phenomenology:** For our vacuum, (7) implies the lifetime

\[
\tau_{\text{decay}} \sim \frac{M_{\text{Pl}}^2}{H^3} \sim 10^{130} \text{ years}.
\]

Remarkably, this agrees to within \( \gtrsim 2\sigma \) with the Standard Model (SM) estimate for electroweak metastability [50]: \( \tau_{\text{SM}} = 10^{526^{+409}_{-202}} \text{ years} \). In other words, taking \( H_0 \) as input, the optimal lifetime [8] constrains a combination of SM parameters, in particular the Higgs and top quark masses. Closer agreement with the SM lifetime estimate can be achieved if the top quark is slightly heavier, \( m_t \simeq 174.5 \text{ GeV} \), or with new physics at intermediate scales, such as right-handed neutrinos with mass of \( 10^{13} - 10^{14} \text{ GeV} \) [51].

More generally, the accessibility measure offers a dynamical explanation for the near-criticality of our vacuum. It gives a *raison d’être* for the conspiracy underlying Higgs metastability. Therefore, from this point of view the inferred metastability of the electroweak vacuum is sacred. New Beyond-the-SM (BSM) physics discoverable by the LHC, on the other hand, can jeopardize this observable and, barring fine-tunings, will make our vacuum stable. Therefore, *the discovery of BSM particles at the LHC and future colliders, including low-scale SUSY, would rule out the possibility that our vacuum lies in an optimal region of the landscape.* This is a falsifiable prediction of the accessibility measure.

• **Scale of inflation:** If our vacuum lies in an optimal region, then, on the one hand, it was accessed within a Hubble time \( H_0^{-1} \), per [5], and, on the other hand, originated from a parent vacuum whose lifetime was of order the Page time. These two facts together imply a bound on the Hubble scale of the parent vacuum: \( H_{\text{parent}} \gtrsim (M_{\text{Pl}}^2 H_0)^{1/3} \). It is usually assumed that the tunneling event from the parent vacuum is followed by a period of slow-roll inflation, with Hubble scale \( H_{\text{inf}} \). Assuming that \( H_{\text{inf}} \sim H_{\text{parent}} \), then (132) implies a lower bound on the slow-roll inflationary energy scale:

\[
E_{\text{inf}} \sim \sqrt{H_{\text{parent}} M_{\text{Pl}}} \gtrsim 10^8 \text{ GeV} .
\]

Meanwhile, it has been argued that if the inflationary is too high, then Higgs quantum fluctuations during inflation could push the field beyond the potential barrier [52]. Assuming minimal coupling of the Higgs to gravity, for simplicity, Higgs fluctuations will be under control if the inflationary scale satisfies

\[
E_{\text{inf}} \lesssim 10^{14} \text{ GeV} .
\]
(The bound becomes looser with non-minimal coupling of suitable sign \[52\]. It has also been argued that the bound is also sensitive to higher-dimensional operators and deviations from exact de Sitter \[53\].) Equations (9) and (10) together imply an optimal range for the inflationary scale, assuming a minimally-coupled Higgs, of \(10^8 \text{ GeV} \lesssim E_{\text{inf}} \lesssim 10^{14} \text{ GeV}\). Therefore, a detection of primordial gravitational waves, for instance from cosmic microwave background polarization, would either imply that the Higgs must have a non-minimal coupling to gravity, or otherwise disfavor the possibility that our vacuum lies in an optimal region of the landscape.

Our results can also be cast in the language of computational complexity and search optimization. As mentioned earlier, the landscape features many very long-lived false vacua, resulting in frustrated dynamics and exponentially long mixing time \[39\]. Correspondingly, it has been shown \[54\] in the context of simplified landscape models \[5, 55\] that finding a vacuum within a hospitable range of potential energy is an \(\text{NP}\)-hard problem. See also \[56, 57\]. Aside from computational complexity, it has been argued that string compactifications also face issues of undecidability \[58, 59\]. There has been much activity recently in applying deep learning algorithms to the string landscape search problem \[60–71\].

We will show that regions with slow (glassy) transition rates correspond to a mean residency time \(\langle T \rangle\) scaling polynomially in \(N_{\text{inf}}\). Since \(N_{\text{inf}}\) generically scales exponentially with the effective moduli-space dimensionality \(D\), this corresponds to \(\langle T \rangle\) also scaling exponentially in \(D\), which is compatible with the \(\text{NP}\)-hard complexity class of the general problem \[54\]. Optimal regions of the landscape, however, have a mean residency time scaling at logarithmically in \(N_{\text{inf}}\), and hence linearly in \(D\). This does not contradict the \(\text{NP}\)-hard classification — \(\text{NP}\)-hardness is a worst-case assessment which does not preclude the existence of polynomial-time solutions for special instances of the problem. Furthermore, the logarithmic divergence of the mean residency time signals a dynamical phase transition. A similar non-equilibrium phase transition occurs in quenched disordered media, when the probability distribution for waiting times reaches a critical power-law \[72\].

2 Landscape Dynamics as a Random Walk on a Network

The landscape can be modeled as a network (or graph) of nodes representing the different dS, AdS and Minkowski vacua. We assume that AdS and Minkowski vacua are terminal, acting as absorbing nodes. Network links, which define the network topology, represent the relevant transitions between vacua. For concreteness we assume these are governed by Coleman-De Luccia (CDL) instantons \[73–75\].

As shown in the seminal papers of Garriga, Vilenkin and collaborators \[23, 24\], a convenient approach to study landscape dynamics is to follow a time-like geodesic (or “watcher”) in the eternally inflating space-time. See also \[29, 30\]. In time, the watcher passes through a sequence of non-terminal vacua, until it finally hits a terminal vacuum. In the process, the watcher is performing a random walk on the network of vacua.

Let \(N\) denote the total number of vacua in the network, taken to be comprised of \(N_{\text{inf}}\) inflating vacua and \(N_{\text{term}}\) terminal vacua. In what follows, we will use capital indices \(I, J = 1, \ldots, N\) for all vacua; indices \(i, j = 1, \ldots, N_{\text{inf}}\) for dS vacua, and \(a, b = 1, \ldots, N_{\text{term}}\) for terminal (AdS and Minkowski) vacua. Unless otherwise stated, summations over these indices are assumed to run over their respective range.

Let \(f_i(\tau_i)\) denote the probability that the watcher is in vacuum \(I\), as a function of the local proper time \(\tau_i\). Equivalently, \(f_i\) is the fraction of total comoving volume occupied by vacuum \(I\). After coarse-graining over a time interval \(\Delta \tau_i\) longer than transient evolution between periods of vacuum energy domination, the change in occupation probability satisfies the master equation

\[
\Delta f_I = \sum_J \left( \kappa_{IJ}^{\text{proper}} - \delta_{IJ} \sum_K \kappa_{KJ}^{\text{proper}} \right) \Delta \tau_J f_J , \tag{11}
\]
where $\kappa_{IJ}^{\text{proper}}$ is the $J \to I$ proper transition rate. The watcher’s proper time is related to a global time variable $t$ parametrizing the foliation through a lapse function $\mathcal{N}_I$ \cite{23, 25}:

$$\Delta \tau_I = \mathcal{N}_I \Delta t.$$  \hfill (12)

In a discrete setting, we think of $t$ as a uniform discrete counter for transitions. In our analysis we will remain agnostic about the choice of time, as our goal is to define a time-reparametrization invariant measure.\footnote{A restriction on the choice of lapse function is that it should be well-defined in vacua of all types. For instance, scale-factor time, given by $\mathcal{N}_I = H^{-1}_I$, is pathological in Minkowski vacua.} In terms of global time, (11) becomes

$$\Delta f_I = \sum_J \left( \kappa_{IJ} - \delta_{IJ} \sum_K \kappa_{KJ} \right) \Delta t f_J,$$  \hfill (13)

where $\kappa_{IJ} \equiv \kappa_{IJ}^{\text{proper}} \mathcal{N}_J$. Therefore, in the continuum limit (13) becomes

$$\frac{df_I}{dt} = \sum_J M_{IJ} f_J,$$  \hfill (14)

where $M_{IJ}$ is the transition matrix:

$$M_{IJ} \equiv \kappa_{IJ} - \delta_{IJ} \sum_K \kappa_{KJ}.$$  \hfill (15)

The sum over all rows of any column of $M$ vanishes identically, $\sum_I M_{IJ} = 0$, which enforces conservation of probability: $\sum_i f_i = 1$. The solution to (14) is

$$f(t) = e^{Mt} f(0),$$  \hfill (16)

where $f(0)$ is the initial probability vector.

Since $M$ satisfies the sum rule $\sum_I M_{IJ} = 0$ and has positive off-diagonal elements, it follows from Perron-Frobenius’ theorem that it has a single vanishing eigenvalue, while all other eigenvalues have strictly negative real parts \cite{21}. (In fact we will see later that the non-zero eigenvalue with largest real part is also non-degenerate and real.) The zero-eigensubspace is highly degenerate. Indeed, since the rate out of terminal vacua vanishes by assumption, we have

$$\kappa_{Ia} = 0.$$  \hfill (17)

Therefore, any vector lying in the terminal subspace, \textit{i.e.}, of the form

$$f_\infty^a \geq 0 \quad a \in \text{terminals} ;$$

$$f_\infty^i = 0 \quad i \in \text{dS} ,$$  \hfill (18)

is a zero-mode, $M f_\infty = 0$. (This of course assumes there is at least one terminal vacuum. We will consider a toy landscape comprised only of dS vacua in Sec. 2.6.) In particular, the solution \footnote{A restriction on the choice of lapse function is that it should be well-defined in vacua of all types. For instance, scale-factor time, given by $\mathcal{N}_I = H^{-1}_I$, is pathological in Minkowski vacua.} (16) asymptotically tends to $f_\infty^a = f_a(0)$, $f_\infty^i = 0$. Thus the stationary distribution lies entirely in the terminal subspace and is determined by initial conditions.

### 2.1 Occupational probability matrix

It is convenient to express the probability vector in terms of the occupational probability matrix, or Green’s function, $P_{I,J}(t)$. This represents the probability that a random walker starting from $J$ at the initial time is at $I$ at time $t$. The probability vector can be expanded as

$$f_I(t) = \sum_J P_{I,J}(t) f_J(0).$$  \hfill (19)
Substituting into (14), the Green’s function satisfies the master equation
\[
\frac{dP_{IJ}}{dt} = \sum_K \mathcal{M}_{IK} P_{KJ}; \quad P_{IJ}(0) = \delta_{IJ},
\] (20)
with solution
\[
P_{IJ}(t) = (e^{\mathcal{M}t})_{IJ}.
\] (21)

The master equation (20) can be decomposed into rate equations for terminal and non-terminal components. Setting \(J = a\), it is easy to show that the solution is
\[
P_{Ia}(t) = \delta_{Ia}.
\] (22)
This is consistent with the general solution (21), together with \(\sum_K \mathcal{M}_{IK} P_{Ka} = 0\). Not surprisingly, a watcher starting in a terminal vacuum must remain in that terminal forever.

With \(J = j\), the rate equation breaks into
\[
\frac{dP_{ij}}{dt} = \sum_k M_{ik} P_{kj};
\] (23a)
\[
\frac{dP_{aj}}{dt} = \sum_i \kappa_{ai} P_{ij}.
\] (23b)

The matrix \(M\) appearing in (23a) is an \(N_{\inf} \times N_{\inf}\) square matrix defined by
\[
M_{ij} = \kappa_{ij} - \delta_{ij} \kappa_j,
\] (24)
where \(\kappa_j \equiv \sum_I \kappa_{IJ}\) is the total decay rate of vacuum \(j\). The solution to (23a) for \(P_{ij}\) follows immediately:
\[
P_{ij}(t) = (e^{Mt})_{ij}.
\] (25)

### 2.2 CDL transition rates

To proceed we must be more specific about transition rates. For \(\text{dS} \rightarrow \text{dS}\) transitions, the CDL rate is of the form
\[
\kappa_{ij} = \frac{A_{ij}}{w_j}.
\] (26)
Here \(A_{ij} = (\Lambda^4 e^{-S_{\text{bounce}}})_{ij}\) is the adjacency matrix, with \(S_{\text{bounce}}\) denoting the Euclidean action of the bounce solution and \(\Lambda^4\) the fluctuation determinant. The important property for our purposes is that this matrix is symmetric [76]:
\[
A_{ij} = A_{ji}.
\] (27)
The other factor in (26) is the weight \(w_j\) of the parent dS vacuum:
\[
w_j = H_j^3 N_j^{-1} e^{S_j},
\] (28)
where \(S_j = 48\pi^2 M_{\text{Pl}}^4/V_j\) is the dS entropy of the parent vacuum. The factor of \(H_j^3 = \left(V_j/3M_{\text{Pl}}^2\right)^{3/2}\) converts the CDL rate per unit volume to a transition rate, while the factor of \(N_j^{-1}\) converts the rate from unit proper time to global time via (12).

Transitions from inflating to terminal vacua are also assumed to be governed by CDL instantons. In this case the \(\text{dS} \rightarrow \text{AdS}/\text{Minkowski}\) rate is given by
\[
\kappa_{aj} = \frac{(\Lambda^4 e^{-S_{\text{bounce}}})_{aj}}{w_j}.
\] (29)
This is similar to (26), except of course that the numerator is no longer symmetric.
2.3 On time-reparametrization invariance

Since our goal is to define a time-reparametrization invariant measure, it is important to identify the invariant building blocks. First note from (28) that the combination
\[ \omega_j \equiv \frac{w_j}{\Delta t} = \frac{H_j^3 e^{S_j}}{\Delta \tau_j} \]
(30)
is gauge invariant. Similarly, the rates (26) and (29) are both inversely proportional to \( w_j \), hence the dimensionless transition probability
\[ \kappa_{ij} \Delta t = \kappa_{ij}^{\text{proper}} \Delta \tau_j \]
(31)
is also invariant. Thus the transition probability matrix \( M_{ij} \Delta t \) has time-reparametrization invariant elements:
\[ M_{ij} \Delta t = (\kappa_{ij} - \delta_{ij} \kappa_j) \Delta t = (\kappa_{ij}^{\text{proper}} - \delta_{ij} \kappa_j^{\text{proper}}) \Delta \tau_j , \]
(32)
where \( \kappa_j = \sum_i \kappa_{ij} \) is the total decay rate of vacuum \( j \). In particular the eigenvectors and eigenvalues of \( M_{ij} \Delta t \) are invariant.

2.4 Detailed balance and downward approximation

A key feature of the dS-dS transition rate (26) is that it is the ratio of a symmetric matrix and a weight factor proportional to the exponential of the dS entropy. (Our results apply to any rate, CDL or otherwise, of this form.) An immediate consequence is that the rates satisfy detailed balance,
\[ \frac{\kappa_{ji}}{\kappa_{ij}} = \frac{w_j}{w_i} \sim e^{S_j - S_i} . \]
(33)
Thus upward tunneling is suppressed compared to downward tunneling by an exponential of the difference in dS entropy. Importantly, (33) only depends on the false and true vacuum potential energy. It is insensitive to details of the potential barrier and does not rely on the thin-wall approximation.

This allows one to define a “downward” approximation in which upward tunneling is neglected to leading order [77, 78]. By labeling dS vacua in order of increasing potential energy, \( 0 < V_1 \leq \ldots \leq V_{N_{\text{inf}}} \), the transition matrix \( M_{ij} \) for dS vacua defined in (24) takes the form
\[ M = \begin{pmatrix}
-\kappa_1 & \kappa_{12} & \kappa_{13} & \cdots \\
0 & -\kappa_2 & \kappa_{23} & \cdots \\
\vdots & 0 & \ddots & \ddots \\
0 & 0 & 0 & -\kappa_{N_{\text{inf}}}
\end{pmatrix} + M_{\text{up}} . \]
(34)
(Recall that \( \kappa_j = \sum_i \kappa_{ij} \) is the total decay rate of \( j \).) The leading, upper-triangular matrix encodes all downward transitions. The second term, \( M_{\text{up}} \), encodes all (exponentially small) upward transitions. In the “downward” approximation [77, 78], one treats \( M_{\text{up}} \) perturbatively. We will make use of this approximation later on to simplify some of our results.

2.5 Spectral analysis

Although \( M \) is not symmetric, it nevertheless has real eigenvalues, and its eigenvectors form a complete basis of the \( N_{\text{inf}} \)-dimensional subspace of dS vacua. To see this, define the auxiliary matrix
\[ \Sigma = W^{-1/2} M W^{1/2} ; \quad W \equiv \text{diag}(\omega_1, \omega_2, \ldots, \omega_{N_{\text{inf}}}) , \]
(35)
where $\omega_i = w_i / \Delta t$ is the invariant weight defined in [30]. Using [26] for the rate between dS vacua, it is straightforward to see that $\Sigma$ is symmetric and therefore has real eigenvalues. Moreover, since [35] defines a similarity transformation, $\Sigma$ and $M$ have identical spectra. Importantly, per the discussion in Sec. 2.3, the matrix $\Sigma \Delta t \equiv W^{-1/2} M \Delta t W^{1/2}$ is manifestly time-reparametrization invariant, and therefore so are its eigenvalues and eigenvectors.

For simplicity we assume that $M$ is irreducible, i.e., there exists a sequence of transitions connecting any pair of inflating vacua. It then follows from Perron-Frobenius’ theorem that its largest eigenvalue is non-degenerate and negative, $\lambda_1 \leq 0$, while all other eigenvalues are strictly smaller. In other words,

$$0 \geq \lambda_1 > \lambda_2 \geq \ldots \geq \lambda_{N_{\text{inf}}}.$$  \hfill (36)

Furthermore, $\lambda_1 = 0$ if and only if the decay rate into terminals vanishes for all dS vacua. Incidentally, we have already seen that the full transition matrix $M$ has a single vanishing eigenvalue, with $N_{\text{term}}$ degeneracy, while all other $N_{\text{inf}}$ eigenvalues have strictly negative real parts. It is easy to show, given the form of $M$, that the latter set of eigenvalues coincide with [36]. Therefore $M$ has real eigenvalues, and its non-zero eigenvalues coincide with those of $M$. In particular, its largest, non-vanishing eigenvalue is $\lambda_1$, which sets the relaxation time for the Markov process.

The eigenvectors of $\Sigma$, denoted by $v^{(l)}$, $l = 1, \ldots, N_{\text{inf}}$, form a complete, orthonormal and gauge invariant basis of the $N_{\text{inf}}$-dimensional subspace of dS vacua:

$$\sum_{\ell=1}^{N_{\text{inf}}} v^{(l)}_i v^{(l)}_j = \delta_{ij}; \quad \sum_i v^{(l)}_i v^{(l')}_i = \delta_{ll'}.$$ \hfill (37)

We mention in passing a further consequence of Perron-Frobenius’ theorem, namely that the components of the dominant eigenvector can be chosen to all be positive:

$$v^{(1)}_i \geq 0.$$ \hfill (38)

Meanwhile, the eigenvectors of $M$ are simply related to those of $\Sigma$ via

$$v^{(l)}_M = W^{1/2} v^{(l)}.$$ \hfill (39)

Thus the eigenvectors of $M$ also form a complete basis, albeit not orthonormal. Furthermore, they are simply related to those of $M$, with eigenvalues given by [36]. Indeed, it is easy to show, given the form of $M$, that the corresponding eigenvectors are:

$$v^{(l)}_M = v^{(l)}_i; \quad v^{(l)}_M = \frac{1}{\lambda_M} \sum_i \kappa_{ai} v^{(l)}_M.$$ \hfill (40)

Given the form [24] of $M_{ij}$, it is straightforward to show that its dominant eigenvalue is given by

$$\lambda_1 = -\sum_{a=1}^{N_{\text{term}}} \sum_i \kappa_{ai} \sqrt{\omega_i} v^{(1)}_i.$$ \hfill (41)

In the downward approximation [77], in particular, one neglects upward transitions to leading order, and the transition matrix [34] becomes upper-triangular. Hence its eigenvalues are simply given by its diagonal elements. It follows that $\lambda_1$ corresponds to the smallest decay rate [77]:

$$\lambda_1 \simeq -\min \{ \kappa_j \} \quad \text{(downward).}$$ \hfill (42)

Thus the relaxation time is determined by the longest-lived dS vacuum. Similarly, in this approximation the other eigenvalues $\lambda_2, \ldots, \lambda_{N_{\text{inf}}}$ are given by the decay rates of dS vacua in increasing order of instability.
In terms of the eigenvalues and eigenvectors of $\Sigma$, the solution (25) for $P_{ij}$ becomes

$$P_{ij}(t) = \left(W^{1/2}e^{\Sigma t}W^{-1/2}\right)_{ij} = \sqrt{\frac{\omega_i}{\omega_j}} \sum_{\ell=1}^{N_{\text{inf}}} e^{\lambda_{\ell} t} v^{(t)}_i v^{(t)}_j . \quad (43)$$

By orthonormality (37) this satisfies the correct initial condition, $P_{ij}(0) = \sqrt{\frac{\omega_i}{\omega_j}} \sum_{\ell=1}^{N_{\text{inf}}} e^{\lambda_{\ell} t} v^{(t)}_i v^{(t)}_j = \delta_{ij}$. When computing first-passage statistics in Sec. 3 we will make use of the Laplace transform of (43), defined as usual by $\tilde{P}_{ij}(s) = \int_0^\infty dt P_{ij}(t)e^{-st}$. The result is

$$\tilde{P}_{ij}(s) = \sqrt{\frac{\omega_i}{\omega_j}} \sum_{\ell=1}^{N_{\text{inf}}} v^{(t)}_i v^{(t)}_j . \quad (44)$$

Next we can solve for $P_{aj}$ by substituting (43) into the master equation (23b):

$$\frac{dP_{aj}}{dt} = \sum_{\ell} e^{\lambda_{\ell} t} \sum_i \kappa_{ai} \sqrt{\frac{\omega_i}{\omega_j}} v^{(t)}_i v^{(t)}_j . \quad (45)$$

The solution with initial condition $P_{aj}(0) = 0$ is

$$P_{aj}(t) = \sum_{\ell} e^{\lambda_{\ell} t} - \frac{1}{\lambda_{\ell}} \sum_i \kappa_{ai} \sqrt{\frac{\omega_i}{\omega_j}} v^{(t)}_i v^{(t)}_j . \quad (46)$$

Equations (22), (43) and (46) form the solution for the occupational probability matrix $P_{IJ}(t)$.

2.6 dS-only toy landscape

Our measure will be simplest to define in the case of a toy landscape comprised of dS vacua only, i.e., without terminals. This will be the focus of Sec. 4. With this in mind, we collect here a few useful results for dS-only landscapes.

In the absence of terminals, the transition matrix $M_{ij}$ is still given by (24), but with $\kappa_j = \sum_k \kappa_{kj}$. Hence in this case the sum of any column of $M$ vanishes, $\sum_i M_{ij} = 0$. It then follows from Perron-Frobenius’ theorem that the largest eigenvalue of $M$ is non-degenerate and vanishes:

$$\lambda_1 = 0 . \quad (47)$$

The Green’s function (43) therefore reduces to

$$P_{ij}(t) = \frac{v^{(1)}_i v^{(1)}_j}{\sqrt{\frac{\omega_i}{\omega_j}}} + \sum_{\ell \geq 2} e^{\lambda_{\ell} t} v^{(t)}_i v^{(t)}_j , \quad (48)$$

where we have isolated the zero-mode for convenience. Its Laplace transform is

$$\tilde{P}_{ij}(s) = \sqrt{\frac{\omega_i}{\omega_j}} \left( \frac{v^{(1)}_i v^{(1)}_j}{s} + \sum_{\ell \geq 2} \frac{v^{(t)}_i v^{(t)}_j}{s - \lambda_{\ell}} \right) . \quad (49)$$

It is straightforward to derive an explicit expression for $v^{(1)}$ and $v^{(1)}_M$, the zero-modes for $\Sigma$ and $M$ respectively. Note that $v^{(1)}_M$, in particular, sets the stationary distribution: $\frac{df^\infty}{dt} = M f^\infty = 0$. To do so, first write

$$M = ZW^{-1} ; \quad Z_{ij} = A_{ij} - \delta_{ij} \sum_r A_{rj} , \quad (50)$$
where $A_{ij}$ is the symmetric matrix defined in (26). Substituting into (35) gives
\[ \Sigma = W^{-1/2} M W^{1/2} = W^{-1/2} Z W^{-1/2}. \] (51)

Now, notice that the vector with unit entries, $\vec{e} \equiv (1, 1, \ldots, 1)$, is a zero-eigenvector of $Z$. It follows that $\Sigma (W^{1/2} \vec{e}) = 0$, hence $v^{(1)} \sim W^{1/2} \vec{e}$. Normalizing, we obtain
\[ v^{(1)}_i = \sqrt{\frac{\omega_i}{\omega}}; \quad \omega \equiv \sum_i \omega_i. \] (52)

Per (39), the corresponding zero-mode of $M$ is
\[ v^{(1)}_M = \sqrt{\omega} v^{(1)}_i = \frac{\omega_i}{\sqrt{\omega}}. \] (53)

The stationary distribution $f^\infty$ is proportional to $v^{(1)}_M$, and by definition satisfies $\sum_i f^\infty_i = 1$. It follows that
\[ f^\infty_i = \frac{w_i}{w} \sim v^{(1)}_i^2. \] (54)

From this point of view, $v^{(1)}$ can be thought as a gauge invariant generalization of the (dS-only) stationary measure $f^\infty$.

3 First-Passage Statistics

The key building block in defining the accessibility measure is the mean first-passage time (MFPT). This is the average time taken by a random walker starting from a given initial node to reach a given target for the first time. The MFPT has been applied to random walks in various contexts [45] and is a standard measure of search efficiency on networks, e.g., [79]. Notably, in cosmology first-passage statistics have been used in the context of stochastic inflation [80–82], in particular to study tunneling between vacua [83]. The MFPT was applied to landscape dynamics in [41], and some of the concepts covered below also appeared in [41].

3.1 First-passage density

A central quantity in first-passage statistics is the first-passage density, $F_{IJ}(t)$, $I \neq J$. This is the probability density that a random walker who started at node $J$ at $t = 0$ visits node $I$ for the first time at time $t$. All first-passage statistics can be derived from $F$. For instance, the MFPT is given by its first moment:
\[ \langle t_{J \rightarrow I} \rangle = \frac{\int_0^\infty dt t F_{IJ}(t)}{\int_0^\infty dt F_{IJ}(t)} = -\left. \frac{d \ln \tilde{F}_{IJ}(s)}{ds} \right|_{s=0}. \] (55)

Thus $\langle t_{J \rightarrow I} \rangle$ is the average time for a random walker starting from $J$ to reach $I$, averaged over all paths connecting the two nodes.

A well-known equation relating first-passage density and occupational probability is [45]
\[ P_{IJ}(t) = \int_0^t dt' F_{IJ}(t') P_{II}(t - t'); \quad I \neq J. \] (56)

The meaning of this relation is clear: the occupational probability at time $t$ is given by the probability that the walker has reached $I$ at any earlier time $t'$ multiplied by the “loop” probability $P_{II}$ that the walker returned to $I$ in the remaining time $t - t'$. Clearly $F_{IJ}$ is non-vanishing only if the initial node is a dS
vacuum, thus we henceforth set \( J = j \). It remains to specify whether the final node \( I \) is a terminal or non-terminal vacuum.

Consider first the case where the final node is a terminal vacuum (\( I = a \)). Using \( P_{aa}(t - t') = 1 \), which follows from (22), we can differentiate (56) to obtain

\[
F_{aj}(t) = \frac{dP_{aj}}{dt} = \sum_{\ell} e^{\lambda_{\ell} t} \sum_{i} \kappa_{ai} \sqrt{\frac{\omega_{i}}{\omega_{j}}} v_{i}^{(\ell)} v_{j}^{(\ell)} ,
\]

where in the last step we have substituted (45). The Laplace transform of this result, which will be useful later on, is

\[
\hat{F}_{aj}(s) = \sum_{\ell} \frac{1}{s - \lambda_{\ell}} \sum_{i} \kappa_{ai} \sqrt{\frac{\omega_{i}}{\omega_{j}}} v_{i}^{(\ell)} v_{j}^{(\ell)} = \sum_{\ell} \frac{\lambda_{\ell}}{s - \lambda_{\ell}} \frac{v_{j}^{(\ell)}}{\sqrt{\omega_{j}}} v_{j}^{(\ell)} ,
\]

where we have used (40). Differentiating and setting \( s = 0 \) gives the MFPT (55) from dS vacuum \( i \) to terminal \( a \):

\[
\langle t_{j \rightarrow a} \rangle = - \frac{d \ln \hat{F}_{aj}(s)}{ds} \bigg|_{s=0} = \sum_{\ell} |\lambda_{\ell}|^{-1} \frac{v_{j}^{(\ell)}}{v_{i}^{(\ell)}} v_{j}^{(\ell)} v_{j}^{(\ell)} .
\]

Consider next the situation where the final node is a dS vacuum (\( I = i \)). In this case, (56) becomes

\[
P_{ij}(t) = \int_{0}^{t} dt' F_{ij}(t') P_{ii}(t - t') ; \quad i \neq j .
\]

Taking the Laplace transform we obtain

\[
\hat{F}_{ij}(s) = \frac{\hat{P}_{ij}(s)}{\hat{P}_{ii}(s)} .
\]

Substituting (44) gives the desired result

\[
\hat{F}_{ij}(s) = \sqrt{\frac{\omega_{i}}{\omega_{j}}} \frac{v_{i}^{(1)} v_{j}^{(1)}}{v_{i}^{(1)} v_{i}^{(1)}} + (s - \lambda_{1}) \sum_{\ell \geq 2} \frac{1}{s - \lambda_{\ell}} \frac{v_{i}^{(\ell)} v_{j}^{(\ell)}}{v_{i}^{(\ell)} v_{i}^{(\ell)}} .
\]

In particular, let us focus on the special case of a toy landscape without terminals studied in Sec. 2.6. Setting \( \lambda_{1} = 0 \) and using (52) gives

\[
\hat{F}_{ij}(s) = \sqrt{\frac{\omega_{i}}{\omega_{j}}} \frac{v_{i}^{(1)} v_{j}^{(1)}}{v_{i}^{(1)} v_{i}^{(1)}} + s \sum_{\ell \geq 2} \frac{1}{s - \lambda_{\ell}} \frac{v_{i}^{(\ell)} v_{j}^{(\ell)}}{v_{i}^{(\ell)} v_{i}^{(\ell)}} .
\]

Note that for finite \( N_{\text{int}} \) the ever-hitting probability, \( \hat{F}_{ij}(0) = \int_{0}^{\infty} dt F_{ij}(t) \), is unity in this case. (This is in contrast with (62), whose ever-hitting probability is less than unity due to the loss of probability to terminals.) Differentiating and setting \( s = 0 \) gives the MFPT between two dS vacua in a landscape without terminals:

\[
\langle t_{j \rightarrow i} \rangle = \frac{1}{v_{i}^{(1)} v_{i}^{(1)}} \sum_{\ell = 2}^{N_{\text{int}}} \frac{1}{|\lambda_{\ell}|} \left( v_{i}^{(\ell)} v_{i}^{(\ell)} - \frac{v_{i}^{(1)} v_{i}^{(1)} v_{i}^{(\ell)} v_{i}^{(\ell)}}{v_{i}^{(1)} v_{i}^{(1)}} \right) .
\]

We will make use of this result when defining the accessibility measure in Sec. 4.
3.2 First-return density

One can similarly define a first-return density, \( F_{II}(t) \), which represents the probability density that a random walker returns at the initial node at time \( t \). Clearly this is only non-trivial if the node in question is a dS vacuum, hence we set \( I = i \).

We proceed by generalizing (60) to allow for the initial and final nodes to be the same. To do so, it is convenient to discretize time in units of \( \Delta t \), where \( \Delta t \) is the coarse-graining time interval (12). That is, \( t = n \Delta t \), with \( n \) an integer. The generalization of (56) is then

\[
P_{ij}(n) = \delta_{ij} \delta_{n0} + \sum_{m=0}^{n} F_{ij}(m) P_{ii}(n-m) \Delta t ,
\]

where we have dropped corrections of \( \mathcal{O}((\Delta t)^2) \). Taking the discrete Laplace transform, defined as \( \tilde{f}(s) = \sum_{n=0}^{\infty} f(n) e^{-sn} \Delta t \), gives

\[
\tilde{F}_{ij}(s) = \delta_{ij} \Delta t + \tilde{F}_{ij}(s) \tilde{P}_{ii}(s) ,
\]

which agrees with (61) for \( i \neq j \). For the case of interest, \( i = j \), we obtain

\[
\tilde{F}_{ii}(s) = 1 - \frac{\Delta t}{\tilde{P}_{ii}(s)} .
\]

Substituting (44) gives

\[
\tilde{F}_{ii}(s) = 1 - \frac{(s - \lambda_1) \Delta t}{v_i^{(1)} + (s - \lambda_1) \sum_{\ell \geq 2} \frac{1}{s - \lambda_\ell} v_\ell^{(\ell)} }.
\]

In the special case of a toy landscape without terminal vacua, we can set \( \lambda_1 = 0 \) to obtain

\[
\tilde{F}_{ii}(s) = 1 - \frac{s \Delta t}{v_i^{(1)} + s \sum_{\ell \geq 2} \frac{1}{s - \lambda_\ell} v_\ell^{(\ell)} } \quad (\text{dS only}) .
\]

For finite \( N_{\text{inf}} \), this implies that the ever-return probability, \( \tilde{F}_{ii}(0) = \int_0^\infty dt F_{ii}(t) \), is unity. Meanwhile, the first moment of this distribution gives the mean first-return time:

\[
\langle t_{i \rightarrow i} \rangle = -\left. \frac{d\tilde{F}_{ii}(s)}{ds} \right|_{s=0} = \frac{\Delta t}{v_i^{(1)}} \quad (\text{dS only}) .
\]

Thus the mean first-return time is set by the stationary distribution, which is Kac’s celebrated lemma [84].

4 Accessibility Measure Without Terminals

We are now in a position to define the accessibility measure. The measure is conceptually easiest to define in the absence of terminal vacua, i.e., in a toy landscape comprised of dS vacua only. This is the subject of this Section. In Sec. 5 we will generalize the measure to include terminals.

The key building block is a weighted MFPT, defined in Sec. 4.1 called the partial MFPT (pMFPT). The pMFPT can be expressed simply in terms of the eigenvalues and eigenvectors of the transition matrix. Moreover, it admits other equivalent representations, each offering different insights. Firstly, we will show in Sec. 4.2 that the pMFPT can be neatly expressed in terms of first-return statistics. This form makes manifest that the pMFPT is independent of initial conditions, and immediately implies an important lower bound for the pMFPT. Secondly we will show in Sec. 4.3 that minimizing the pMFPT is equivalent to maximizing the first-passage probability at intermediate times. Thirdly, we will see in Sec. 4.4 can be related to the escape or never-return probability. The accessibility measure is then defined in Sec. 4.5 in terms of the reciprocal of the pMFPT.
4.1 pMFPT

We define the dimensionless partial MFPT (pMFPT) to the dS vacuum $j$ by

$$T_i \equiv \frac{v_i^{(1)^2}}{1 + v_i^{(1)^2}} \sum_{j \neq i} v_j^{(1)^2} \frac{(t_{j \rightarrow i})}{\Delta t},$$

(71)

where $(t_{j \rightarrow i})$ is the MFPT (64) between dS vacua $j$ and $i$, and $\Delta t$ is the coarse-graining time interval. Since $v_j^{(1)^2}$ sets the stationary distribution, per (54), $T_i$ can be thought of as the MFPT to node $i$, averaged over all initial nodes weighted by the stationary distribution. The overall factor of $\frac{v_i^{(1)^2}}{1 + v_i^{(1)^2}}$ is included to simplify some of the resulting expressions.

As mentioned in the Introduction, weighing the MFPT's $(t_{j \rightarrow i})$ by the stationary distribution yields a conservative estimate of the characteristic time needed to reach $i$. Indeed, the stationary distribution $v_j^{(1)^2}$ exponentially favors the lowest-lying dS vacuum, as can be seen from (52), which in turn is the most stable vacuum. (To leading order in the downward approximation, this vacuum is absolutely stable.) The corresponding MFPT $(t_{j \rightarrow i})$ should be the largest. Thus weighing with the stationary distribution amounts to giving greatest weight to the initial node with the longest average travel time to $i$. It also stands to reason that any other distribution with non-zero support on the most stable dS vacuum, such as the uniform distribution, should yield similar results. We leave to future work a detailed study of sensitivity to different distributions and proceed for now with (71) as a definition of the pMFPT.

Substituting (64) for $(t_{j \rightarrow i})$, and using the identity

$$\sum_{j \neq i} v_j^{(1)^2} v_j^{(\ell)} = -v_i^{(1)^2} v_i^{(\ell)}$$

(which follows from the orthonormality relation (37)), we obtain

$$T_i = \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)^2}}{|\lambda_i| \Delta t}.$$  

(72)

Thus the pMFPT is simply related to the non-zero eigenvalues and corresponding eigenvectors (‘relaxing modes’) of the transition matrix. The eigenvectors are time-reparametrization invariant, as mentioned earlier. Meanwhile, the $\lambda_i \Delta t$’s are eigenvalues of the gauge invariant probability matrix $\Sigma \Delta t = W^{-1/2} M \Delta t W^{1/2}$. Therefore the pFMPT is manifestly time-reparametrization invariant.

Importantly, the eigenvalues and eigenvectors are to an excellent approximation determined by the leading upper-triangular matrix in (34), encoding downward transitions, up to exponentially small corrections due to $M_{\text{up}}$. As such, unlike stationary measures, the pMFPT is robust against small tweaks to the landscape, of the kind discussed in the Appendix. Finally, we note in passing that summing (72) over $i$ gives a global MFPT, otherwise known as Kemeny’s constant [85]:

$$T_{MFPT} = \sum_i T_i = \sum_{\ell=2}^{N_{\text{inf}}} \frac{1}{|\lambda_i| \Delta t}.$$  

(73)

This matches the MFPT studied in [41] for finite regions of the landscape.

4.2 pMFPT and first-return statistics

The pMFPT can be neatly expressed in terms of first-return statistics [46]. Consider the second moment of the first-return probability,

$$\langle t_{i \rightarrow i}^2 \rangle = \int_0^\infty dt t^2 F_{ii}(t) = \left. \frac{d^2 \tilde{F}_{ii}(s)}{ds^2} \right|_{s=0},$$  

(74)

The partial MFPT is closely related to a quantity first defined in [46], which the authors called the global MFPT. We prefer the term partial MFPT, since, as we will see (73), summing $T_i$ over $i$ gives Kemeny’s constant, $T_{MFPT}$, which to our mind is a more apt measure of global MFPT.
where we have used the fact that the first-return probability for dS-only vacua, given by (69), is normalized: \( \int_0^\infty dt F_{ii}(t) = \bar{F}_{ii}(0) = 1 \). Substituting (69), we obtain
\[
\langle t_i^2 \rangle = \frac{\Delta t}{v_i^{(1)2}} \sum_{\ell=2}^{N_{\text{cd}}} \frac{v_i^{(2)}}{|\lambda|} = 2 \langle t_i \rangle^2 \sum_{\ell=2}^{N_{\text{cd}}} \frac{v_i^{(2)}}{|\lambda|} \Delta t ,
\]
where the last step follows from Kac’s lemma (70). Combining with (72) gives the desired result:
\[
T_i = \frac{1}{2} \langle t_i^2 \rangle .
\]
Thus the pMFPT is simply related to the variance of first-return times. Since first-return statistics by definition consider random walks that start at \( i \), this expression makes clear that \( T_i \) is independent of initial conditions. Furthermore, since \( \langle t_i^2 \rangle \geq \langle t_i \rangle^2 \), this implies a lower bound for the pMFPT:
\[
T_i \geq \frac{1}{2} .
\]
The inequality is saturated if the variance of first-return times vanishes, such that \( \langle t_i^2 \rangle = \langle t_i \rangle^2 \). This bound will play a key role in deriving phenomenological implications of the accessibility measure in Sec. 6.

### 4.3 pMFPT and first-passage probability

Perhaps the most intuitive interpretation of the pMFPT is through its relation to the first-passage probability. As we now show, minimizing \( T_i \) is equivalent to maximizing the first-passage probability to \( i \) at early times compared to the relaxation time.

Let us define the first-passage probability to \( i \) as a weighted average over initial nodes:
\[
H_i(t) = \sum_j v_j^{(1)2} \int_0^t dt' F_{ij}(t') .
\]
On a finite landscape, every vacuum is guaranteed to be accessed eventually, as reflected by the fact that \( H_i \) tends to unity as \( t \to \infty \):
\[
\lim_{t \to \infty} H_i(t) = \lim_{s \to 0} \sum_{j \neq i} v_j^{(1)2} \bar{F}_{ij}(s) = \lim_{s \to 0} \frac{v_i^{(1)2}}{v_i^{(2)}} \left( 1 - s \sum_{\ell \geq 2} \frac{1}{|\lambda|} \frac{v_i^{(2)}}{\lambda} \right) = 1 ,
\]
where we have substituted (63) and used orthonormality (37). Correspondingly, the probability that \( i \) has not yet been visited, \( 1 - H_i(t) \), starts from 1 and tends to 0 as \( t \to \infty \). While each vacuum is guaranteed to be populated eventually, the required time scale for all vacua to be accessed is of course the relaxation time.

Our interest lies instead on the approach to equilibrium, i.e., for \( t \) much smaller than the relaxation time. To quantify the finite-time probability that a site \( i \) has not yet been visited, consider the simple figure of merit:
\[
X_i = \frac{1}{\Delta t} \int_0^\infty dt \left( 1 - H_i(t) \right) .
\]
Nodes with small \( X_i \) have a higher probability of being accessed early, whereas those with high \( X_i \) tend to be populated later. In terms of Laplace transforms, we have
\[
X_i = \frac{1}{\Delta t} \lim_{s \to 0} \left( \frac{1}{s} - \bar{H}_i(s) \right) = \lim_{s \to 0} \left( \frac{1}{s} \left( 1 - \sum_j v_j^{(1)2} \bar{F}_{ij}(s) \right) \right) = \frac{1 + v_i^{(1)2}}{v_i^{(1)2}} \sum_{\ell=2}^{N_{\text{cd}}} \frac{v_i^{(2)}}{|\lambda|} \Delta t ,
\]

18
which implies the simple result

\[ X_i = \frac{1 + v_i^{(1)} \Delta t}{v_i^{(1)} \Delta t} T_i. \]  

(82)

Hence, for fixed stationary distribution, nodes with smaller pMFPT have smaller \( X_i \) and, correspondingly, higher probability of being accessed early on relative to the relaxation time; in contrast, nodes with larger pMFPT are less likely to be accessed early on.

### 4.4 pMFPT and escape probability

Yet another useful expression for the pMFPT is in terms of the escape probability in the limit of an infinite landscape (\( N_{\text{inf}} \to \infty \)). First let us define the pseudo-Green’s function [86]:

\[ \mathcal{R}_i = \frac{1}{\Delta t} \int_0^{\infty} dt \left( P_{ii}(t) - v_i^{(1)} \Delta t \right) \lim_{s \to 0} \left( \tilde{P}_{ii}(s) - \frac{v_i^{(1)} \Delta t}{s} \right). \]  

(83)

Setting \( i = j \) in (49), we have

\[ \tilde{P}_{ii}(s) = v_i^{(1)} \Delta t + \sum_{\ell \geq 2} \frac{v_i^{(\ell)} \Delta t}{s - \lambda_{\ell}}, \]  

(84)

and it follows that

\[ \mathcal{R}_i = \sum_{\ell = 2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} \Delta t}{|\lambda_{\ell}| \Delta t} = T_i. \]  

(85)

Thus \( \mathcal{R}_i \) agrees with the pMFPT.

On the other hand, the pseudo-Green’s function can be related to the escape probability \( S_i \), defined as the probability that the walker never returns to \( i \):

\[ S_i \equiv 1 - \int_0^{\infty} dt F_{ii}(t) = 1 - \lim_{s \to 0} \tilde{F}_{ii}(s), \]  

(86)

where \( \lim_{s \to 0} \tilde{F}_{ii}(s) = \int_0^{\infty} dt F_{ii}(t) \) is the ever-return probability. The escape probability delineates whether random walks are \textit{recurrent} at \( i \) (vanishing escape probability) or \textit{transient} at \( i \) (finite escape probability):

\[ S_i = 0 \iff \text{recurrence at } i \]

\[ S_i = \text{finite} \iff \text{transience at } i. \]  

(87)

Recurrence at \( i \) means that a random walker is certain to return eventually to \( i \), and, because the process is Markovian, will do so infinitely-many times in the future.

Substituting the first-return density (67), we have

\[ S_i = \lim_{s \to 0} \frac{\Delta t}{\tilde{P}_{ii}(s)}, \]  

(88)

with \( \tilde{P}_{ii}(s) \) given by (84). Therefore, whether random walks are recurrent or transient at \( i \) depends on whether \( \lim_{s \to 0} \tilde{P}_{ii}(s) \) is divergent or finite, respectively. For finite \( N_{\text{inf}} \), the first term in (84) diverges as \( s \to 0 \), resulting in a divergent \( \tilde{P}_{ii} \) and hence a vanishing escape probability. Therefore, not surprisingly, random walks on finite networks are always recurrent. In the limit of an infinite network (\( N_{\text{inf}} \to \infty \)), on the other hand, the first term in (84) gives a vanishingly small contribution [87], leaving us with

\[ \lim_{N_{\text{inf}} \to \infty} \frac{\tilde{P}_{ii}(s)}{\Delta t} = \sum_{\ell \geq 2} \frac{v_i^{(\ell)} \Delta t}{|\lambda_{\ell}| \Delta t}. \]  

(89)
The order of limits matters — one must first send $N_{\text{inf}} \to \infty$ before taking the late-time ($s \to 0$) limit. Equation (89) agrees with the pseudo-Green’s function $\mathcal{R}_i$ (and therefore the pMFPT) defined at finite $N_{\text{inf}}$.

Combining (88) and (89) yields the desired relation between pMFPT and escape probability:

$$S_i = \mathcal{R}_i^{-1} = T_i^{-1},$$

(90)

where the large network limit is understood in calculating $S_i$. Importantly, since the escape probability is time-reparametrization invariant, as mentioned earlier, (90) confirms that so is $T_i$.

4.5 Accessibility measure

The accessibility measure is defined as the reciprocal of the pMFPT, suitably normalized:

$$p_i \equiv \frac{T_i^{-1}}{\sum_k T_k^{-1}} = \frac{S_i}{\sum_k S_k}.$$  

(91)

Since the pMFPT is both time-reparametrization invariant and independent of initial conditions, as argued above, so is $p_i$. Furthermore, because the accessibility measure is constructed from first-passage statistics, it is clearly oblivious to any comoving vs physical volume ambiguity. However, we cannot claim that the measure is unique, since the pMFPT involved a choice of distribution to average over initial nodes, as discussed below (71).

The measure favors vacua that are easily accessed under time evolution, i.e., vacua that saturate (77): $T_i \sim O(1)$. As such, it is analogous to closeness centrality [36, 37], a widely-used centrality index in studies of complex networks. The closeness measure assigns greater weight to nodes that can be reached on average with the fewest number of steps.

Unlike standard measures based on the stationary distribution, which exponentially favors a single dominant vacuum, the accessibility measure allows for multiple vacua having comparable weight. Specifically, there can be many vacua that nearly saturate (77), i.e., with $T_i \sim O(1)$, and all will be weighted equally according to (91). Relatedly, unlike stationary measures, the accessibility measure is insensitive to small tweaks to the landscape. It can be reliably calculated to leading order in the downward approximation, where the transition matrix assumes the upper-triangular form (34).

5 Accessibility Measure With Terminals

In the case of a dS-only landscape studied in the previous Section, we saw that the accessibility measure could be expressed in four equivalent ways: 1. In terms of a pMFPT, defined in (71) as a weighted average of the MFPT to a given node; 2. In terms of the variance of first-return times to the starting node; 3. In terms of a first-passage probability to a given node for times shorter than the relaxation time; 4. In terms of the escape probability from a given node.

Once terminals are included, however, these give inequivalent definitions of the pMFPT. It turns out that the last approach in terms of escape probability is most straightforward to generalize in a landscape with terminal vacua. Specifically, in Sec. 5.1 we will define the accessibility measure for dS vacua in terms of the escape probability. In Sec. 5.2 we will instead define the accessibility measure for terminal vacua in terms of a trapping probability. Both escape and trapping probabilities admit simple expressions in terms of eigenvectors and eigenvalues of the transition matrix. Akin to the dS-only case, the resulting measure is both time-reparametrization invariant and independent of initial conditions.
5.1 pMFPT and escape probability for dS vacua

Following the steps in Sec. 4.4, we begin by defining the pseudo-Green’s function $\mathcal{R}_i$. Analogously to (83), the pseudo-Green’s function is obtained by subtracting the dominant eigenvector contribution to the occupational probability:

$$\mathcal{R}_i \equiv \frac{1}{\Delta t} \int_0^\infty dt \ (P_{ii}(t) - e^{\lambda_1 t} v_i^{(1)} t^2) = \frac{1}{\Delta t} \lim_{s \to 0} \left( \tilde{P}_{ii}(s) - \frac{v_i^{(1)} t^2}{s - \lambda_1} \right).$$

(92)

Using (44) with $i = j$,

$$\tilde{P}_{ii}(s) = \frac{v_i^{(1)} t^2}{s - \lambda_1} + \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} t^2}{s - \lambda_\ell},$$

(93)

we obtain

$$\mathcal{R}_i = \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} t^2}{|\lambda_\ell| \Delta t}.$$

(94)

The result is identical in form to the dS-only pseudo-Green’s function (85), though of course the eigenvalues and eigenvectors of the transition matrix are modified by the presence of terminals. In light of this we are led to define the pMFPT for dS vacua as

$$T_i \equiv \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} t^2}{|\lambda_\ell| \Delta t}.$$

(95)

In particular, summing over $i$ gives a measure of the global MFPT or Kemeny’s constant analogous to (78):

$$T^\text{MFPT} = \sum_{i=1}^{N_{\text{inf}}} T_i = \sum_{\ell=2}^{N_{\text{inf}}} \frac{1}{|\lambda_\ell| \Delta t}.$$

(96)

The pseudo-Green’s function is once again related to the escape probability (86),

$$S_i = 1 - \tilde{F}_{ii}(0) = \frac{\Delta t}{P_{ii}(0)}.$$

(97)

As before, whether random walks are recurrent or transient at $i$ depends on whether $\lim_{s \to 0} \tilde{P}_{ii}(s)$ is divergent or finite, respectively. Unlike the dS-only case, where the escape probability vanishes for finite $N_{\text{inf}}$, in this case the escape probability is finite due to the presence of terminals. In any case, in the limit of an infinite landscape ($N_{\text{inf}} \to \infty$), the first term in (93) gives a vanishingly small contribution (87), resulting in

$$S_i = \mathcal{R}_i^{-1} = T_i^{-1}.$$

(98)

Once again, the pseudo-Green’s function coincides with the escape probability in the large-network limit.

Unlike the dS-only case, we have been unable to establish a strict lower bound on $T_i$ akin to (77). It is straightforward, however, to show that\footnote{To prove (99), define the renormalized Green’s function, $P^{R}_{ij}(t) \equiv e^{-\lambda_i t} P_{ij}(t) = \sqrt{\omega_i/\omega_j} \sum_{\ell} e^{(\lambda_\ell - \lambda_i)t} v_i^{(\ell)} v_j^{(1)}$, which remains finite asymptotically: $P^{R}_{ij}(t \to \infty) = \sqrt{\omega_i/\omega_j} v_i^{(1)} v_j^{(1)}$. Thus its behavior is identical to the dS-only result (84), except for the shift: $\lambda_i \to \lambda_\ell - \lambda_i$. Similarly, it follows from (90) that $P^{R}_{ij}(t) = \int_0^\infty dt' P^{R}_{ij}(t') P^{R}_{ii}(t - t')$, where $P^{R}_{ii}(t) = e^{-\lambda_1 t} F_{ii}(t)$ is a renormalized first-passage density. In particular, the moments of the renormalized first-return density $P^{R}_{ii}$ satisfy $\frac{1}{2} \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} t^2}{|\lambda_\ell - \lambda_1| \Delta t}$, which implies (99).}

$$\sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} t^2}{(\lambda_1 - \lambda_\ell) \Delta t} \geq \frac{1}{2}.$$  

(99)
In the limit $|\lambda_1| \ll |\lambda_\ell|$, corresponding to slow decay into terminals relative to dS-dS transitions, this implies $T_i \gtrsim 1/2$. For general $\lambda_1$, we will show in Sec. 6.2 using the downward approximation that the average pMFPT satisfies $\langle T \rangle \gtrsim 1$ — see (117). We have been unable to generalize this to a strict lower bound for individual vacua.

5.2 pMFPT and trapping probability for terminal vacua

For terminal vacua, the natural analogue of the escape probability is the trapping probability. Let us define a weighted trapping probability $S_a$ as the late-time occupational probability at terminal node $a$ averaged over all initial dS nodes $j$:

$$S_a = |\lambda_1| \Delta t \lim_{t \to \infty} \frac{\sum_j P_{aj}(t) \sqrt{\omega_j v_i^{(1)}}}{\sum_i \sqrt{\omega_i v_i^{(1)}}}.$$  (100)

This definition deserves some comments. The weighing factor $\sqrt{\omega_i v_i^{(1)}}$ is recognized as the dominant eigenvector $v_i^{(1)}$ of the transition matrix. Since $v_i^{(1)} \geq 0$, per (38), this weighing factor is well-defined and, as we will see, leads to a simple expression for the trapping probability. The overall factor of $|\lambda_1| \Delta t$ is included for convenience.

Recall from (57) the relation $F_{aj}(t) = dP_{aj}/dt$ between first-passage and occupational probabilities. Integrating this equation implies

$$\lim_{t \to \infty} P_{aj}(t) = \tilde{F}_{aj}(0) = \sum_i \kappa_{ai} \sqrt{\omega_i v_i^{(1)}} v_i^{(1)}.$$  (101)

where in the last step we have used (58). Substituting this into (100) and using the orthonormality of the eigenvectors, we obtain

$$S_a = \frac{\sum_i \kappa_{ai} \Delta t \sqrt{\omega_i v_i^{(1)}}}{\sum_i \sqrt{\omega_i v_i^{(1)}}}.$$  (102)

This expression makes clear that the weighted trapping probability has a number of desirable properties:

1. $S_a$ is manifestly time-reparametrization invariant.

2. Since the trapping probability is defined as a prescribed average over initial nodes, it is clearly independent of initial conditions.

3. As it should, $S_a \to 0$ in the limit $\kappa_{ai} \to 0$, i.e., if all transition rates to $a$ vanish.

4. For node $i$ to even qualify as a vacuum its decay rate should be less than an inverse unit time step: $\kappa_{ai} \Delta t \leq 1$. Therefore the trapping probability satisfies the upper bound:

$$S_a \leq 1.$$  (103)

5. It follows from the explicit expression (44) for $\lambda_1$ that

$$\sum_{a=1}^{N_{\text{term}}} S_a = \sum_{a=1}^{N_{\text{term}}} \kappa_{ai} \Delta t \sqrt{\omega_i v_i^{(1)}} \sum_i \sqrt{\omega_i v_i^{(1)}} = |\lambda_1| \Delta t.$$  (104)

To make this point more explicit, since $\kappa_{ai} \Delta t$ is time-reparametrization invariant, it is convenient to work with proper time: $\kappa_{ai} \Delta t = \kappa_{ai}^{\text{proper}} \Delta \tau_i$. The natural proper unit time step is a Hubble time, $\Delta \tau_i = H_i^{-1}$, hence we obtain $\kappa_{ai} \Delta t = \kappa_{ai}^{\text{proper}} H_i^{-1}$. By definition, a necessary condition for $i$ to be a vacuum is that its proper decay rate is at most its Hubble rate: $\kappa_{ai}^{\text{proper}} \leq H_i$. It follows that $\kappa_{ai} \Delta t \leq 1$. 

22
Analogously to the relation (98) for dS vacua between escape probability and pMFPT, we define a pMFPT for terminals as the reciprocal of the trapping probability:

\[ T_a \equiv S_a^{-1} = \frac{\sum_i \sqrt{\omega_i v_i^{(1)}}}{\sum_i \kappa_{ai} \Delta t \sqrt{\omega_i v_i^{(1)}}}. \]  

(105)

Given (103), \( T_a \) satisfies a lower bound

\[ T_a \geq 1, \]  

(106)

which is akin to (77) in the dS-only case.

5.3 General definition of the accessibility measure

The accessibility measure on a general landscape with non-terminal (dS) and terminal vacua is defined once again as the reciprocal of the pMFPT:

\[ p_I = \frac{T_i^{-1}}{\sum_K T_K^{-1}}, \]  

(107)

where the MFPT is given by (95) and (105) for dS vacua and terminals, respectively:

\[ T_i = \sum_{\ell=2}^{N_{\text{inf}}} \frac{v_i^{(\ell)} 2}{|\lambda_{i\ell}| \Delta t}, \quad i \in \text{dS}; \]

\[ T_a = \frac{\sum_i \sqrt{\omega_i v_i^{(1)}}}{\sum_i \kappa_{ai} \Delta t \sqrt{\omega_i v_i^{(1)}}}, \quad a \in \text{terminals}. \]  

(108)

As argued above, the measure thus defined is gauge invariant and independent of initial conditions. It is well-defined for both non-terminals and terminals alike. Because it is defined in terms of first-passage probabilities, \( p_I \) is oblivious to the comoving vs physical volume ambiguity that afflicts stationary measures. Unlike stationary measures, \( p_I \) can be reliably calculated to leading order in the downward approximation, and therefore is insensitive to small tweaks to the landscape.

Importantly, the accessibility measure makes concrete and potentially testable predictions, as discussed in the next Section.

6 Phenomenological Implications

In this Section we derive various phenomenological implications of the accessibility measure. Importantly, these predictions do not rely on anthropic reasoning, and instead derive from the measure itself. To be precise, in various places below we will use as input the observed value of the cosmological constant, or equivalently, the Hubble constant \( H_0 \). We do not attempt to explain the measured \( H_0 \), and its smallness may ultimately rely on anthropics. However, taking \( H_0 \) as a given we derive predictions for other observables, such as the optimal lifetime of our universe and the absence of new physics at the LHC, which follow readily from the measure.

Some of the predictions derived below, such as the lifetime of our universe, were originally obtained in [41]. The difference in the present analysis is that such predictions are now firmly rooted in a measure. Other predictions, such as the optimal access time, are new.
6.1 Access time

The accessibility measure \( T_I \) favors vacua that are easily accessed under time evolution, specifically vacua with order unity pMFPT \( T_I \sim O(1) \). \( \text{(109)} \)

From \( \text{(108)} \) we see that the pMFPT is made dimensionless by the coarse-graining time step \( \Delta t \). Thus \( \text{(109)} \) implies that optimal vacua are accessed in a physical time of order \( \Delta t \). In terms of proper time for vacuum \( I \), the natural coarse-graining time interval is coarse the Hubble time, \( \Delta t = \Delta \tau_I = H_I^{-1} \). Therefore vacua favored by the measure are reached in a proper time of order their own Hubble time:

\[
\tau_I^{\text{access}} \sim H_I^{-1}. \tag{110}
\]

Given the observed value of the vacuum energy in our universe, \( \text{(110)} \) implies that we live approximately \( H_0^{-1} \sim 13.8 \) billion years after the beginning of eternal inflation. This is not a trivial statement. While eternal inflation is well-known to be geodesically past-incomplete \[38\], the last period of inflation which gave rise to our universe could have occurred an arbitrarily long time after the initial “big bang”, in principle much longer than 13.8 billion years.

In particular, \( \text{(110)} \) implies an upper bound on the duration of the last period of inflation. Denoting the Hubble scale of this last inflationary bout by \( H_{\text{inf}} \), the number of e-folds allowed by the optimal access time is bounded:

\[
N \lesssim \frac{H_{\text{inf}}}{H_0}. \tag{111}
\]

Later on we will combine this with the optimal lifetime of dS vacua to derive a lower bound on \( H_{\text{inf}} \).

6.2 Funnel topography

To proceed, it is helpful to focus on a finite fiducial region of the landscape comprised of \( N \gg 1 \) vacua. For simplicity the region is approximated as a closed system, ignoring the exchange of probability with its surroundings. We will relax this assumption below.

Our task is to define a characteristic time for the landscape dynamics of dS vacua in the region, as a suitable average over the pMFPTs. To start with, recall from \( \text{(96)} \) the global MFPT, or Kemeny’s constant, for dS vacua in the region:

\[
T_{\text{MFPT}} = \sum_{\ell=2}^{N_{\text{inf}}} \frac{1}{|\lambda_\ell| \Delta t}. \tag{112}
\]

This gives a characteristic time for dS vacua to populate each other in the region. Meanwhile, the characteristic time for dS vacua to populate terminals can be estimated as the reciprocal of the total escape probability for terminal vacua:

\[
T_{\text{terms}} = \frac{1}{\sum_a S_a} = \frac{1}{|\lambda_1| \Delta t}. \tag{113}
\]

Equations \( \text{(112)} \) and \( \text{(113)} \) make sense. The time required to populate dS vacua is set by all but the smallest (in magnitude) eigenvalue of the transition matrix, while the smallest eigenvalue \( \lambda_1 \) sets the characteristic leakage time into terminals.

\(^{13}\) For terminal vacua, the pMFPT satisfies a lower bound \( \text{(106)} \), hence \( \text{(109)} \) is justified. For dS vacua, we have not yet been able to derive a strict lower bound, as discussed around \( \text{(99)} \). Nevertheless, it seems reasonable to assume that \( \text{(109)} \) holds with an order unity factor.
We are therefore led to define an average pMFPT in the region by

\[ \langle T \rangle \equiv \frac{\mathcal{T}_{\text{MFPT}} + \mathcal{T}_{\text{terms}}}{N_{\text{inf}}} = \frac{1}{N_{\text{inf}}} \sum_{\ell=1}^{N_{\text{inf}}} \frac{1}{|\lambda_\ell| \Delta t}. \]

(114)

This gives a characteristic time for populating dS and terminal vacua in the region. Conveniently, it only depends on the eigenvalues of the transition matrix, not on its eigenvectors. Indeed, our ulterior motive for considering a finite region of the landscape and defining an average pMFPT over that region was to achieve this simplification.

In particular, to leading order in the downward approximation, \( M_{ij} \) reduces to an upper-triangular matrix given by the first term in (34), and its eigenvalues can therefore be read off from the diagonal entries:

\[ \langle T \rangle \approx \frac{1}{N_{\text{inf}}} \sum_{i=1}^{N_{\text{inf}}} \frac{1}{\kappa_i \Delta t} \]  \hspace{1cm} \text{(downward)}. \]

(115)

Thus \( \langle T \rangle \) is interpreted as a dimensionless mean residency time. Incidentally, since \( \kappa_i \) is the total decay rate of node \( i \) per unit time \( t \), (115) makes the time-reparametrization invariance of \( \langle T \rangle \) manifest. In what follows it will be convenient to work in terms of proper time \( \kappa_i \Delta t = \kappa_{i}^{\text{proper}} \Delta \tau_i \). The natural proper time step is of course the Hubble time, \( \Delta \tau_i = H_i^{-1} \), thus (115) becomes

\[ \langle T \rangle \approx \frac{1}{N_{\text{inf}}} \sum_{i=1}^{N_{\text{inf}}} H_i \tau_{i}^{\text{decay}}, \]

(116)

where \( \tau_{i}^{\text{decay}} \equiv 1/\kappa_{i}^{\text{proper}} \) is the proper lifetime of dS vacuum \( i \). And since each vacuum must have by definition a proper lifetime longer than its Hubble time, \( \tau_{i}^{\text{decay}} \gtrsim H_i^{-1} \), this implies

\[ \langle T \rangle \gtrsim 1. \]

(117)

Now we arrive at a key point. In the downward approximation it is possible for multiple dS vacua to become absolutely stable, \( \kappa_i = 0 \). This will occur whenever such vacua have only up-tunneling as allowed transitions. In this case (115) implies that \( \langle T \rangle \) will diverge to leading order in the downward approximation, meaning that at sub-leading order \( \langle T \rangle \) will be exponentially large. A region that includes such vacua exhibits frustration and glassy dynamics [49]. It should be clear that such frustrated regions are heavily disfavored by the accessibility measure.

Instead, the accessibility measure favors regions whose dS vacua all have allowed downward transitions, either to lower-lying dS vacua or to terminals. Such favored regions therefore have the topography of a broad valley or funnel, as sketched in Fig. 1. This is akin to the principle of minimal frustration of protein energy landscapes [43, 44], where the high-energy unfolded states are connected to the lowest-energy native state by a relatively smooth funnel. This is a key prediction of the accessibility measure. Unlike stationary measures, which rest on the idea that our vacuum should be run-of-the-mill among all hospitable vacua on the landscape (the “principle of mediocrity”), the accessibility measure favors vacua residing in special, funnel-like regions of the landscape. This may have important implications for string phenomenology and model-building.

### 6.3 Average lifetime of vacua and computational complexity

To make further predictions, we follow [41] and take the continuum limit of (115), valid for \( N_{\text{inf}} \gg 1 \). Given the form of CDL transitions, the lifetime of a given vacuum in general depends both on its potential energy \( V \) as well as the various “bounce” parameters \( \theta \) characterizing the shape of the potential barrier:

\[ \tau_{i}^{\text{decay}} = \tau_{i}^{\text{decay}}(V_i, \theta_i) \]

(118)
Let $\mathcal{P}(V, \theta)$ denote the underlying joint probability distribution that a given vacuum has potential energy $V$ and bounce parameters $\theta$. For simplicity we will assume that on the string landscape the absolute height of a vacuum and the shape of the surrounding potential barriers are uncorrelated: $\mathcal{P}(V, \theta) \equiv \mathcal{P}(V) \mathcal{P}(\theta)$. This allows us to marginalize over bounce parameters and define an average lifetime $\tau_{\text{decay}}(V)$ for vacua of given potential energy:

$$\tau_{\text{decay}}(V) \equiv \int d\theta \tau_{\text{decay}}(V, \theta) \mathcal{P}(\theta).$$

Therefore, using the Friedmann relation $H \sim \sqrt{V/M_{\text{Pl}}}$, the mean residency time (116) becomes, in the continuum limit,

$$\langle T \rangle = \int_{V_{\text{min}}}^{V_{\text{max}}} dV \frac{\sqrt{V}}{M_{\text{Pl}}} \tau_{\text{decay}}(V) \mathcal{P}(V),$$

where $V_{\text{min}}$ and $V_{\text{max}}$ are respectively the smallest and largest vacuum energy achieved in the region.

Provided that $\mathcal{P}(V)$ falls off sufficiently fast at large $V$, the result for $\langle T \rangle$ is controlled by the behavior of $\tau_{\text{decay}}(V)$ for small $V$. Assuming as usual that $\mathcal{P}(V)$ is nearly uniform for $V$ much smaller than the fundamental scale [88], (120) can be approximated by

$$\langle T \rangle \sim \int_{V_{\text{min}}}^{V_{\text{max}}} dV \frac{\sqrt{V}}{M_{\text{Pl}}} \tau_{\text{decay}}(V),$$

where, for a uniform distribution, the smallest potential energy is on average inversely proportional to the number of vacua:

$$V_{\text{min}} \sim \frac{M_{\text{Pl}}^4}{N_{\text{inf}}}. \quad (122)$$

Clearly the integral (121) will converge or diverge as $V_{\text{min}} \rightarrow 0$ depending on whether $\tau_{\text{decay}}(V)$ diverges slower or faster than $V^{-3/2}$, with the critical case $\tau_{\text{decay}}(V) \sim V^{-3/2}$ corresponding to a logarithmic divergence.

The accessibility measure favors regions of the landscape where the mean residency time $\langle T \rangle$ nearly saturates (117). This requires the integral (121) to converge or, at worst, depend logarithmically on $V_{\text{min}}$. Using the Planck mass $M_{\text{Pl}}$ to fix dimensions, as it is the natural scale in the problem, the measure favors landscape regions where vacua have an average lifetime in the range

$$\frac{M_{\text{Pl}}}{\sqrt{V}} < \tau_{\text{decay}}(V) \lesssim \frac{M_{\text{Pl}}^5}{N_{\text{inf}}^{1/2}} \quad \text{as } V \rightarrow 0. \quad (123)$$

The shortest allowed lifetime is of course the Hubble time. The longest allowed lifetime is recognized as the Page time [89] for dS space [90] [93]:

$$\tau_{\text{Page}} \sim \frac{M_{\text{Pl}}^5}{V^{3/2}} \sim \frac{M_{\text{Pl}}^2}{H^3}. \quad (124)$$

In slow-roll inflation, the Page time marks the phase transition to slow-roll eternal inflation [94] and has been used to place a bound on the maximum number of e-folds that can be described semi-classically [95]. The appearance of the Page time in the present context of false-vacuum eternal inflation, first noticed in [41], is surprising.

From a computational complexity perspective, the Page time represents a transition in the scaling of $\langle T \rangle$ with the number of vacua. To see this, let us assume for simplicity that $\tau_{\text{decay}}(V)$ is a power-law for small $V$,

$$\tau_{\text{decay}}(V) \sim V^{-\alpha} \quad \text{as } V \rightarrow 0. \quad (125)$$
Then the mean residency time \( \langle T \rangle \) gives
\[
\langle T \rangle \sim \begin{cases} 
\text{constant} & \text{for } \alpha < 3/2 \\
\log v_{\text{min}} \sim \log N_{\text{inf}} & \text{for } \alpha = 3/2 \\
v_{\text{min}}^{3/2-\alpha} \sim N_{\text{inf}}^{\alpha-3/2} & \text{for } \alpha > 3/2.
\end{cases}
\tag{126}
\]

In turn, the number of vacua \( N_{\text{inf}} \) generically scales exponentially with the effective moduli-space dimensionality \( D \) of the landscape region, that is, \( N_{\text{inf}} \sim e^D \). We learn from (126) that regions with slow transition rates, \( \alpha > 3/2 \), correspond to a mean residency time scaling polynomially in \( N_{\text{inf}} \), hence exponentially in \( D \). This is compatible with the NP-hard complexity class of finding vacua within a suitable range of potential energy [54]. Regions with fast enough transition rates, \( \alpha \leq 3/2 \), on the other hand, have a mean residency time scaling at worst logarithmically in \( N_{\text{inf}} \), hence linearly in \( D \). Note that this does not contradict the NP-hard complexity classification, as NP-hardness, being a worst-case assessment, does not preclude the existence of polynomial-time solutions for special instances of the problem.

The case \( \alpha = 3/2 \), where the average lifetime is order the Page time, marks a critical boundary between the other two phases. The mean residency time diverges as \( \log N_{\text{inf}} \), signaling a dynamical phase transition. A similar non-equilibrium phase transition occurs in quenched disordered media, whenever the probability distribution for waiting times reaches a critical power-law [72]. It also describes a computational phase transition [96, 97]. A famous example is the phase transition in heuristic decision-tree pruning from polynomial to exponential search time at a critical value of the effective branching ratio [98].

6.4 Recurrence and dynamical criticality

The lifetime range (123) preferred by the accessibility measure was derived by approximating the finite landscape region of interest as a closed system. With this assumption, minimizing \( \langle T \rangle \) requires that downward transitions are as fast as possible. More realistically, however, one should treat regions as open systems, allowing for the possibility that a random walker escapes a given region before accessing a target vacuum. It stands to reason that vacua residing in regions where the likelihood of escape is high should be disfavored by the measure. We expect that the accessibility measure favors regions where random walks efficiently explore vacua, thereby minimizing the mean residency time, while at the same time minimizing the likelihood of escape before finding viable vacua.

Treating landscape regions as open systems would require modeling the probability leaking into environment, which may introduce unwanted model-dependence in our analysis. Following [41], we instead propose to study a proxy requirement that relies solely on the intrinsic dynamics within a given region. Specifically, we demand that random walks in the region are recurrent in the infinite-network limit, \( N_{\text{inf}} \rightarrow \infty \). As discussed in Sec. 4.4, in recurrent walks every site in the region will be visited with probability one. Recurrent walks thoroughly explore any region around their starting point, whereas transient walks tend to escape to infinity. Although not formally equivalent to modeling regions as open systems, recurrence offers a reliable and model-independent benchmark for efficient sampling [41].

Per (87) random walks will be recurrent at \( i \) when \( N_{\text{inf}} \rightarrow \infty \) if the escape probability \( S_i \) vanishes in this limit. In turn, from (98) this requires that the pMFPT \( T_i \) diverges in the limit. Therefore, random walks in a given region will be recurrent if the mean residency time diverges as \( N_{\text{inf}} \rightarrow \infty \):
\[
\langle T \rangle \rightarrow \infty \quad \text{as} \quad N_{\text{inf}} \rightarrow \infty.
\tag{127}
\]

We therefore have two competing requirements: minimal mean residency time, which requires that vacua have relatively short lifetimes, per (123); and recurrence, which requires that the mean residency time diverge as \( N_{\text{inf}} \rightarrow \infty \). Optimal regions reach a compromise by achieving the shortest \( \langle T \rangle \) compatible with recurrence, i.e., the least-divergent integral (121). Per (126), vacua in optimal regions have an average lifetime of order
the Page time \( \tau_{\text{crit}} \)

\[
\tau_{\text{crit}}(H) \sim \frac{M_{\text{Pl}}^2}{H^3}.
\] (128)

That the Page time represents an optimal time for vacuum selection on the landscape was first realized in [41]. In the present analysis this is now justified by considerations of a well-defined measure.

As discussed earlier, the average lifetime of order the Page time corresponds to a dynamical phase transition. Thus the joint demands of minimal oversampling, defined by minimal mean residency time, and sweeping exploration, defined by recurrence, selects regions of the landscape that are tuned at criticality. Therefore, we are led to conjecture that the accessibility measure favors dynamically critical regions of the landscape. We cannot yet claim a rigorous proof of this statement, because recurrence is only a proxy for minimizing the escape probability, but it is reasonable to expect that the accessibility measure, being rooted in search optimization, is peaked at criticality.

Complex self-organized systems poised at criticality are ubiquitous in the natural world [99]. Examples include brain activity, where the probability distribution for neuronal avalanches of different size is scale invariant [100–103]; and the flocking behavior of starlings, whose velocity correlations are scale invariant [104, 105]. It has been conjectured that dynamical criticality is evolutionarily favored because it offers an ideal compromise between robust response to external stimuli and flexibility of adaptation to a changing environment.

Furthermore, it has been argued that computational capabilities are maximized at the phase transition between stable and unstable dynamical behavior — the so-called “edge of chaos” [106]. This idea goes back to random boolean networks [107] and cellular automata [108–113]. In machine learning, certain recurrent neural networks [114, 115] achieve maximal computational power for vanishing Lyapunov exponent [116]. Meanwhile, the connectivity matrix of well-trained, state-of-the-art deep neural networks has been shown recently to have a power-law spectral density [117], well-described by heavy-tailed random matrix theory [118].

Similarly, our mechanism selects regions of the landscape that are dynamically critical, in the sense of the recurrence/transience dynamical phase transition, and computationally critical, in the sense that \( \langle T \rangle \) lies at the transition between polynomial and non-polynomial search time. Tantalizingly, this suggests a connection between non-equilibrium critical phenomena on the landscape and the near-criticality of our universe. We illustrate this below with Higgs metastability.

6.5 Higgs metastability and particle phenomenology

If our vacuum is part of an optimal region of the landscape, characterized by vacua with critical Page lifetimes [128], then we predict that the lifetime of our universe is

\[
\tau \sim \frac{M_{\text{Pl}}^2}{H_0^3} \sim 10^{130} \text{ years}.
\] (129)

This explains the metastability of the electroweak vacuum. Remarkably, the predicted lifetime agrees to within \( \gtrsim 2\sigma \) with the SM prediction [50]: \( \tau_{\text{SM}} = 10^{526.4\pm409} \) years. To be clear, we of course do not claim to explain the smallness of the cosmological constant. But taking the observed vacuum energy \( \sim M_{\text{Pl}}^2H_0^2 \) as given, the optimal lifetime [129] constrains a combination of SM parameters, including the Higgs and top quark masses. Indeed, what makes Higgs metastability particularly interesting is the relation it entails between the cosmological constant and weak hierarchy problems.

Closer agreement with the SM lifetime estimate can be achieved if the top quark is slightly heavier, \( m_t \approx 174.5 \text{ GeV} \). This can be viewed as a prediction, assuming of course that the SM is valid all the way to the Planck scale. New physics at intermediate scales can reduce the tension. For instance, adding a gauge-invariant, higher-dimensional operator \( h^6/\Lambda_{\text{NP}}^2 \) will affect the predicted lifetime if \( \Lambda_{\text{NP}} \lesssim 10^{13} \text{ GeV} \), assuming the central value \( m_t = 173.5 \text{ GeV} \) [50].

28
More generally, our mechanism offers a dynamical explanation for why our universe is poised at criticality. It gives a raison d'être for the conspiracy underlying Higgs metastability. In other words, from the point of view of the accessibility measure the inferred metastability of the electroweak vacuum is sacred. New physics below the SM instability scale, $\sim 10^{10}$ GeV, on the other hand, can jeopardize this observable. Here are some of the implications for a few BSM candidates already outlined in [41]:

- **Low-scale SUSY**: If the SUSY breaking scale is $\lesssim 10^{10}$ GeV, this will directly impact the stability of our vacuum. There are three obvious possibilities: 1) SUSY makes our vacuum unstable (e.g., via decay to charge/color breaking vacua [119]), which by itself is inconsistent and therefore requires additional new physics; 2) SUSY makes our vacuum stable, which is disfavored by our mechanism; 3) SUSY maintains our vacuum within the metastability region. The latter possibility, while logically consistent with our mechanism, would require further numerical conspiracy, above and beyond that already achieved in the SM. Barring fine-tunings, the natural implication of SUSY below $10^{10}$ GeV is to make our vacuum stable, which is disfavored by the measure. Therefore the accessibility measure favors optimal regions of the landscape characterized by very high-scale SUSY breaking. More generally, the above argument applies to any new physics at the LHC. It follows that the discovery of BSM particles at the LHC, including low-scale SUSY, would rule out the possibility that our vacuum lies in an optimal region of the landscape. This is a falsifiable prediction of the accessibility measure.

- **Sterile neutrinos**: Massive right-handed neutrinos, like the top quark, tend to make the vacuum less stable. Assuming three right-handed neutrinos of comparable mass, for simplicity, the impact on Higgs metastability is negligible if their mass is $\lesssim 10^{13}$ GeV [51]. On the other hand, if their mass is around $10^{13} - 10^{14}$ GeV, then the expected lifetime for our vacuum will be in closer agreement with the optimal lifetime [129].

- **QCD axion**: Consider the QCD axion as a solution to the strong CP problem. The radial part of the $U(1)$ complex scalar is a boson and hence makes the electroweak vacuum more stable. To preserve the desired metastability, the Peccei-Quinn scale must be sufficiently high, $f_a \gtrsim 10^{10}$ GeV [121].

6.6 Scale of inflation and amplitude of gravitational waves

Vacua in optimal regions of the landscape are, on the one hand, accessed in a Hubble time, per (110), and, on the other, have a lifetime of order the Page time (128). This implies a bound on the scale of the last period of inflation for our universe.

Assuming that our vacuum is part of an optimal region of the landscape, then the parent dS vacuum which tunneled to our vacuum had a proper lifetime of order its Page time $\tau_{\text{parent}} \sim M_{\text{Pl}}^2 / H_{\text{parent}}^3$, corresponding to a number of e-folds of

$$N_{\text{parent}} \sim \frac{M_{\text{Pl}}^2}{H_{\text{parent}}^2}. \quad (130)$$

But since our vacuum was accessed within a time $H_0^{-1}$, the number of e-folds is bounded by (111):

$$N_{\text{parent}} \lesssim \frac{H_{\text{parent}}}{H_0}. \quad (131)$$

It follows that

$$H_{\text{parent}} \gtrsim \left( M_{\text{Pl}}^2 H_0 \right)^{1/3} \approx 20 \text{ MeV}. \quad (132)$$

14 This expectation is borne out by an explicit calculation of [120], which showed that if all SUSY partners have masses at the SUSY breaking scale, then the metastability of our vacuum requires a SUSY breaking scale of $\gtrsim 10^{10}$ GeV.
It is usually assumed that the tunneling event from the parent vacuum is followed by a period of slow-roll inflation, with Hubble scale $H_{\text{inf}}$. Assuming $H_{\text{inf}} \sim H_{\text{parent}}$ for concreteness, then (132) implies a lower bound on the slow-roll inflationary energy scale:

$$E_{\text{inf}} \sim \sqrt{H_{\text{parent}} M_{\text{Pl}}} \gtrsim 10^8 \text{ GeV}. \quad (133)$$

Therefore the accessibility measure disfavors low-scale inflationary scenarios.

On the other hand, it has been argued that if the inflationary is too high, then Higgs quantum fluctuations during inflation can push the field beyond the potential barrier [52]. Assuming minimal coupling of the Higgs to gravity, for simplicity, the inflationary scale must satisfy $H_{\text{inf}} \lesssim 10^9$ GeV, or $E_{\text{inf}} \lesssim 10^{14}$ GeV, to keep Higgs fluctuations under control. (The bound becomes looser with non-minimal coupling of suitable sign [52] and can be affected by higher-dimensional operators and deviations from exact de Sitter [53].) Combined with (133), we conclude that the optimal range for the inflationary scale, assuming a minimally-coupled Higgs, is

$$10^8 \text{ GeV} \lesssim E_{\text{inf}} \lesssim 10^{14} \text{ GeV}. \quad (134)$$

A detection of primordial gravitational waves, for instance from cosmic microwave background polarization observations, would imply that the Higgs must have a non-minimal coupling to gravity, or otherwise disfavor the possibility that our vacuum lies in an optimal region of the landscape.

7 Conclusions

The measure problem is arguably the most pressing and formidable challenge in cosmology. If the fundamental theory allows eternal inflation, then our universe is but a small region of a vast multiverse containing infinitely-many other pocket universes with different physical properties. A measure is therefore necessary to make any predictions about physical observables in our universe.

Two broad classes of measures have been proposed: i) global measures, that count pocket universes on a global foliation of the eternally-inflating space-time; ii) local measures, that count pocket universes in a finite region of space-time defined by a single observer. Each approach has pros and cons. Global measures are independent of initial conditions but depend sensitively on the choice of foliation as well as whether bubbles are weighted according to comoving or physical volume. Local measures are manifestly gauge-invariant but are sensitive to initial conditions.

A drawback afflicting both global and local measures is their sensitivity to minor tweaks of the landscape. Global and local measures are based on the nearly-stationary distribution of the Markov process describing vacuum dynamics. The stationary distribution exponentially favors a single dS vacuum — the dominant vacuum, which has the slowest decay rate anywhere on the landscape. This is the robust prediction of global/local measures. But since this so-called dominant vacuum is unlikely to be hospitable, one is forced to keep track of subleading terms in the transition matrix, which encode upward transitions from the dominant vacuum to hospitable vacua. As shown in the Appendix, exponentially small tweaks in the transition rates can result in exponential differences in the relative probabilities for hospitable vacua.

Thus, despite more than three decades of effort, the measure problem remains unsolved. However, two aspects of the problem give us hope that a solution is within reach. Firstly, after suitable coarse-graining the rate equation governing vacuum dynamics reduces to a simple, linear Markov process, free of the conceptual pitfalls of eternal inflation. Secondly, despite the variety of approaches to the measure problem, all proposed measures to date have focused on a single statistics: the stationary distribution of the Markov process. Granted, this is the most natural and simplest statistics to consider. But, as the large body of recent work on complex networks has shown, the stationary distribution offers only a narrow viewpoint of the importance of different nodes. Specifically, it is predominantly sensitive to the local properties of the network. This can
be seen most emphatically in the dS-only case, where \( f_i^\infty = \frac{w_i}{w} \) is determined solely by the vacuum energy of each node and conveys no information about network topology. It is oblivious, in particular, to whether a given node is well-connected or isolated from other vacua.

The purpose of this work is to offer a fresh approach to the measure problem, drawing on recent results in network science. Indeed, from the broader perspective of random walks on graphs, the measure problem translates to a question of network centrality. Various centrality measures have been proposed in the literature, each offering different perspectives on the importance of nodes in a network. Some measures, such as degree centrality, are determined by local properties of the network, akin to the stationary measure on the landscape. Other centrality indices offer insights on dynamical aspects of the network, by identifying nodes that control information flow.

The accessibility measure presented in this paper belongs to this latter category. Instead of characterizing the distribution of vacua near equilibrium, the proposed measure pertains to the approach to equilibrium. It favors vacua that are easily accessed and populated early on in the evolution. Specifically, such vacua are populated within a time of order their Hubble time, much earlier than the exponentially long mixing time for the landscape. This is motivated physically by the possibility that the eternal inflation has been unfolding for a time much shorter than the relaxation time.

As such the accessibility measure naturally connects to issues of computational complexity and search optimization on the landscape. Generic regions of the landscape are characterized by frustrated dynamics, resulting in exponentially long search times compatible with the \( \text{NP} \)-hard complexity class of the general problem. In contrast, the accessibility measure favors regions of the landscape where the search algorithm is efficient. Such optimal regions can be thought of as special, polynomial-time instances of the general problem.

The proposed measure enjoys a number of desirable features. It is simultaneously time-reparametrization invariant, independent of initial conditions, and oblivious to whether pocket universes are weighted according to their comoving or physical volume. It does not suffer from youngness bias or Boltzmann brains. Unlike stationary measures, it is robust against minor tweaks to the landscape. It can be reliably calculated to leading order in the downward approximation, i.e., neglecting the exponentially small terms in the transition matrix that encode upward transitions.

Importantly, the accessibility measure makes concrete, falsifiable predictions that are largely independent of anthropic reasoning. The most enticing prediction is that our vacuum should have a lifetime of order its Page time, \( \sim 10^{130} \) years. Remarkably, this agrees to within \( \sim 2\sigma \) with the SM result for electroweak metastability. To be precise, the predicted lifetime takes \( H_0 \) as an input (perhaps anthropically determined) and then constrains a combination of the SM parameters that are most important in determining vacuum stability, namely the Higgs mass, top quark mass, and gauge couplings. Therefore, given a (possibly anthropic) solution to the cosmological constant problem, the accessibility measure offers a non-anthropic solution to the weak hierarchy problem. The appearance of the Page time as a critical time for landscape dynamics is somewhat mysterious, and it will be interesting to seek a deeper understanding for its origin in this context.

Thus search optimization on the landscape offers a compelling explanation for the delicate numerical conspiracy underlying Higgs metastability. From this point of view the inferred metastability of the electroweak vacuum is no accident — it is a sacred observable. Any new BSM physics discovered at the LHC, including low-scale SUSY, would have to conspire to maintain the metastability bound, which would require further fine-tuning above and beyond that already achieved in the SM. Therefore, barring fine tuning, it stands to reason that the discovery of BSM particles at the LHC or future colliders, including low-scale SUSY, would rule out the possibility that our vacuum lies in an optimal region of the landscape. This still allows the possibility of discovering light particles, such as the axion, as these would have negligible impact on vacuum stability. These are falsifiable predictions of the accessibility measure.

Another key prediction of the accessibility measure is that our vacuum lies a special region of the landscape
with funnel-like topography. This is in stark contrast with the “principle of mediocrity” underlying stationary measures, whereby our vacuum should be run-of-the-mill among all hospitable vacua on the landscape. The most interesting implications of funnel-like regions may be for string phenomenology and model-building. String vacua with realistic particle physics are usually considered in isolation, without much consideration for their accessibility and the topography of the surrounding landscape region. It will be very interesting to study the implications of a funnel topography on particle physics, in particular whether SM-like particle spectra are more prevalent in such regions. Relatedly, as already speculated in [41], it would be interesting to see whether funnel-like regions of the landscape correspond to a low effective moduli-space dimensionality, particularly in the vicinity of the lowest-energy vacuum, as this could offer a dynamical explanation for why our universe does not have more than three spatial dimensions (though anthropic reasoning may be necessary to explain why it does not have fewer than three).

A third key prediction of the accessibility measure is that it favors regions of the landscape tuned at dynamical criticality. This suggests a connection between criticality of vacuum dynamics on the landscape and the near-criticality of our universe, as the vacuum metastability prediction already illustrates. Indeed, it is striking that most fine-tuning problems in fundamental physics, such as the weak hierarchy problem and the cosmological constant problem, can also be understood as problems of criticality. It is tempting to speculate that search optimization might shed new, non-anthropic light on the smallness of the cosmological constant. Last but not least is slow-roll inflation, which itself represents a phenomenon of near-criticality as the inflaton interpolates between an approximately conformally invariant de Sitter phase and standard big bang cosmology. It will be very interesting to see whether slow-roll potentials are more ubiquitous in funnel-like regions of the landscape than in random places in moduli space.

Acknowledgements: We thank Cliff Burgess, Jaume Garriga, Alan Guth for helpful discussions, and are particularly grateful to James Halverson, Cody Long, Onkar Parrikar and Alex Vilenkin for their insights. We thank the Institut de Ciències del Cosmos at the University of Barcelona (ICCUB) for their hospitality while part of this work was completed. This work is supported by the US Department of Energy (HEP) Award DE-SC0013528, NASA ATP grant 80NSSC18K0694, and by the Simons Foundation Origins of the Universe Initiative.

Appendix: Sensitivity of Stationary Measures

In this Appendix we point out a drawback of local and global measures based on the nearly-stationary distribution of the Markov process — their sensitivity to exponentially small terms in the transition matrix. The root of the problem is that the stationary distribution overwhelmingly favors a single dS vacuum — the one with the slowest decay rate. Because this so-called dominant vacuum is unlikely to be hospitable, the relative probabilities for different hospitable vacua are determined by upward transitions from the dominant vacuum, which in turn are sensitive to exponentially small terms in the transition matrix. Therefore, exponentially small tweaks in the transition rates can result in exponential differences in the relative probabilities for hospitable vacua. While not logistically inconsistent, we view such sensitivity to minor tweaks to the landscape as undesirable. After giving the general argument in more detail below, we will illustrate this with a toy mini-landscape comprised of a three dS vacua.

As we have seen in (18), whenever the landscape includes terminal vacua the stationary solution $f^\infty$ lies entirely in the terminal subspace. The relative probability to lie in different dS vacua is then determined by the subleading term as $t \to \infty$, which in turn is set by the eigenvalue of $M$ with the largest (non-vanishing) real part. Per our discussion below (36), this “dominant” eigenvalue is just $\lambda_1$, and the corresponding eigenvector is $v^{(1)}_1$. Therefore at late times we have

$$f(t) \simeq f^\infty + \beta v^{(1)}_1 e^{\lambda_1 t},$$

(135)
with $\beta$ a constant. Thus, since $v^{(\ell)}_{\nu i} = v^{(\ell)}_{Mi}$ per (40), the relative probability for different dS vacua is set by the leading eigenvector of $M$.

In the downward approximation, recall from (42) that $\lambda_1$ is set by the smallest decay rate. Correspondingly, the dominant eigenvector $v^{(1)}_{Mi}$ is set by the \textit{longest-lived dS vacuum} — the so-called dominant vacuum. While the detailed nature of this vacuum requires input from string theory, on general grounds one expects that it has very small vacuum energy and is surrounded by vacua of much higher potential energy. In this configuration, its only allowed CDL transitions involve \textit{“upward”} tunneling, such that its decay rate is exponentially suppressed (per (33)).

\textit{A priori} there is no reason to expect that the dominant vacuum is hospitable. Thus the relative probabilities of different hospitable dS vacua is set by their relative transition rates from the dominant vacuum. And because such transitions necessarily involve up-tunneling, as argued above, the relative probabilities are sensitive to the exponentially small contributions to the transition matrix encoded in $M_{\text{up}}$ in (34). It follows that \textit{expONENTIALLY small changes to the transition matrix can result in dramatically different predictions for the relative abundance of hospitable vacua}. As we will see in the toy example below, small tweaks to the landscape can have a major impact on $M_{\text{up}}$, and consequently on the predictions of the stationary measure. Furthermore, these tweaks can be done in such a way as to preserve all downward transition rates, as well as the hierarchy of potential energies between different dS vacua.

To be clear, the stationary measure \textit{does} make a robust prediction. Among all dS vacua, it overwhelmingly favors a single one — the longest-lived dS vacuum. Indeed, the dominant eigenvector appearing in (135) can be accurately determined in the downward approximation, as argued above, and as such is insensitive to exponentially small corrections encoded in $M_{\text{up}}$. The problem, of course, is that, unless we are exceedingly lucky, the dominant vacuum will be inhospitable. In this case the relative probabilities for hospitable vacua are controlled by exponentially small upward transition rates, hence the sensitivity to small tweaks to the landscape.

To illustrate these general points, consider a simple mini-landscape comprised of three dS vacua (Fig. 2). The mini-landscape also includes terminals, not shown in the Figure. Vacuum 1, the dominant vacuum, is connected to only two other dS vacua labeled by 2 and 3, both assumed hospitable. The assumed hierarchy of potential energies is

$$0 < V_1 \ll V_2, V_3,$$

while we remain agnostic for the moment about the relative magnitude of $V_2$ and $V_3$.

We are therefore interested in determining the relative probability $p_2/p_3$ to occupy vacuum 2 or 3. In \textit{global} measures where vacua are weighted according to volume, $p_2/p_3$ is given directly by $f_2/f_3$. In local measures, such as the watcher measure, the relative probability is instead given by the relative number of bubbles, $N_2/N_3$, encountered along the watcher’s time-like geodesic [24]. In either case, the result is set by
the transition rate from the dominant vacuum to the hospitable ones:

\[
\frac{p_2}{p_3} \sim \frac{\kappa_{21}}{\kappa_{31}} = \frac{w_2}{w_3} \frac{\kappa_{12}}{\kappa_{13}},
\]

(137)

where the last step follows from detailed balance [33]. The upshot is that the ratio \( p_2/p_3 \) is sensitive to the exponentially small upward transition rates \( \kappa_{21} \) and \( \kappa_{31} \).

In particular, it is possible to change the ratio \( p_2/p_3 \) while keeping all downward rates fixed. Indeed, imagine varying \( V_3 \) while keeping \( V_2 \), as well as the downward rates \( \kappa_{12} \) and \( \kappa_{13} \), fixed. (Since the latter is given \( \kappa_{13} = A_{13}/w_3 \), this entails adjusting the shape of the potential such that \( A_{13} \) and \( w_3 \) vary in tandem.) In the process of letting \( V_3 \to \hat{V}_3 \) the relative fraction (137) changes to a new value \( \hat{p}_2/p_3 \) given by

\[
\frac{\hat{p}_2/p_3}{p_2/p_3} = \frac{w_3}{\hat{w}_3} \sim e^{48\pi^2 M_{Pl}^4 \left( \frac{1}{V_3} - \frac{1}{\hat{V}_3} \right)}.
\]

(138)

Clearly this ratio can be \( \ll 1 \) or \( \gg 1 \) depending on whether \( V_3 \) is larger or smaller than \( \hat{V}_3 \). For instance, the relative fraction can change from \( p_2/p_3 \ll 1 \) (vacuum 3 heavily disfavored) to \( \hat{p}_2/p_3 \gg 1 \) (vacuum 3 heavily favored), thereby inverting the predicted volume fraction of type 2 vs type 3 regions. Note that this does not require changing the hierarchy between \( V_2 \) and \( V_3 \). For instance, suppose that \( V_2 \ll V_3 \), then with the above procedure it is possible, for suitable downward rates, to dramatically change the relative probability of vacuum 2 and 3 while maintaining the hierarchy of potential energies.

It is straightforward to concoct more elaborate examples, but the basic point remains the same — the predictions of stationary measures for hospitable vacua are sensitive to exponentially small terms in the transition matrix. Minor tweaks to the landscape can alter these exponentially small corrections, resulting in dramatically different predictions.

In contrast, the accessibility measure presented in the main text is based on first-passage statistics, which in turn are determined by the eigenspectrum of the transition matrix. The eigenvalues and eigenvectors of the matrix (34) are to an excellent approximation determined by the leading upper-triangular matrix, encoding downward transitions, up to exponentially small corrections due to \( M_{up} \). Small tweaks to the landscape, as in the toy example above, have negligible impact on the accessibility measure. In this sense measures based on first-passage statistics are robust against small changes to the landscape.

References

[1] A. Vilenkin, “The Birth of Inflationary Universes,” Phys. Rev. D 27, 2848 (1983).
[2] A. D. Linde, “Eternal Chaotic Inflation,” Mod. Phys. Lett. A 1, 81 (1986).
[3] A. D. Linde, “Eternally Existing Selfreproducing Chaotic Inflationary Universe,” Phys. Lett. B 175, 395 (1986).
[4] B. Freivogel, “Making predictions in the multiverse,” Class. Quant. Grav. 28, 204007 (2011) [arXiv:1105.0244 [hep-th]].
[5] R. Bousso and J. Polchinski, “Quantization of four form fluxes and dynamical neutralization of the cosmological constant,” JHEP 0006, 006 (2000) [hep-th/0004134].
[6] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” Phys. Rev. D 68, 046005 (2003) [hep-th/0301240].
[7] L. Susskind, “The Anthropic landscape of string theory,” In *Carr, Bernard (ed.): Universe or multiverse?* 247-266 [hep-th/0302219].

34
[8] M. R. Douglas, “The Statistics of string / M theory vacua,” JHEP 0305, 046 (2003) [hep-th/0303194].
[9] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, “De Sitter Space and the Swampland,” arXiv:1806.08362 [hep-th].
[10] P. Agrawal, G. Obied, P. J. Steinhardt and C. Vafa, “On the Cosmological Implications of the String Swampland,” Phys. Lett. B 784, 271 (2018) [arXiv:1806.09718 [hep-th]].
[11] S. K. Garg and C. Krishnan, “Bounds on Slow Roll and the de Sitter Swampland,” JHEP 1911, 075 (2019) [arXiv:1807.05193 [hep-th]].
[12] H. Ooguri, E. Palti, G. Shiu and C. Vafa, “Distance and de Sitter Conjectures on the Swampland,” Phys. Lett. B 788, 180 (2019) [arXiv:1810.05506 [hep-th]].
[13] E. Palti, “The Swampland: Introduction and Review,” Fortsch. Phys. 67, no. 6, 1900037 (2019) [arXiv:1903.06239 [hep-th]].
[14] A. D. Linde and A. Mezhlumian, “Stationary universe,” Phys. Lett. B 307, 25 (1993) [gr-qc/9304015].
[15] A. D. Linde, D. A. Linde and A. Mezhlumian, “From the Big Bang theory to the theory of a stationary universe,” Phys. Rev. D 49, 1783 (1994) [gr-qc/9306035].
[16] J. Garcia-Bellido, A. D. Linde and D. A. Linde, “Fluctuations of the gravitational constant in the inflationary Brans-Dicke cosmology,” Phys. Rev. D 50, 730 (1994) [astro-ph/9312039].
[17] A. Vilenkin, “Predictions from quantum cosmology,” Phys. Rev. Lett. 74, 846 (1995) [gr-qc/9406010].
[18] A. De Simone, A. H. Guth, A. D. Linde, M. Noorbala, M. P. Salem and A. Vilenkin, “Boltzmann brains and the scale-factor cutoff measure of the multiverse,” Phys. Rev. D 82, 063520 (2010) [arXiv:0808.3778 [hep-th]].
[19] R. Bousso, B. Freivogel and I. S. Yang, “Properties of the scale factor measure,” Phys. Rev. D 79, 063513 (2009) [arXiv:0808.3770 [hep-th]].
[20] A. De Simone, A. H. Guth, M. P. Salem and A. Vilenkin, “Predicting the cosmological constant with the scale-factor cutoff measure,” Phys. Rev. D 78, 063520 (2008) [arXiv:0805.2173 [hep-th]].
[21] A. H. Guth, “Eternal inflation and its implications,” J. Phys. A 40, 6811 (2007) [hep-th/0702178 [HEP-TH]].
[22] A. Vilenkin and M. Yamada, “Four-volume cutoff measure of the multiverse,” Phys. Rev. D 101, no. 4, 043520 (2020) [arXiv:1912.02187 [hep-th]].
[23] J. Garriga and A. Vilenkin, “Recycling universe,” Phys. Rev. D 57, 2230 (1998) [astro-ph/9707292].
[24] J. Garriga, D. Schwartz-Perlov, A. Vilenkin and S. Winitzki, “Probabilities in the inflationary multiverse,” JCAP 0601, 017 (2006) [hep-th/0509184].
[25] V. Vanchurin and A. Vilenkin, “Eternal observers and bubble abundances in the landscape,” Phys. Rev. D 74, 043520 (2006) [hep-th/0605015].
[26] R. Bousso, “Holographic probabilities in eternal inflation,” Phys. Rev. Lett. 97, 191302 (2006) [hep-th/0605263].
[27] R. Bousso, “Complementarity in the Multiverse,” Phys. Rev. D 79, 123524 (2009) [arXiv:0901.4806 [hep-th]].
[28] R. Bousso, B. Freivogel, S. Leichenauer and V. Rosenhaus, “A geometric solution to the coincidence problem, and the size of the landscape as the origin of hierarchy,” Phys. Rev. Lett. 106, 101301 (2011) [arXiv:1011.0714 [hep-th]].

[29] J. Garriga and A. Vilenkin, “Watchers of the multiverse,” JCAP 1305, 037 (2013) [arXiv:1210.7540 [hep-th]].

[30] Y. Nomura, “Physical Theories, Eternal Inflation, and Quantum Universe,” JHEP 1111, 063 (2011) [arXiv:1104.2324 [hep-th]].

[31] J. Garriga, A. Vilenkin and J. Zhang, “Non-singular bounce transitions in the multiverse,” JCAP 1311, 055 (2013) [arXiv:1309.2847 [hep-th]].

[32] R. Bousso and I. S. Yang, “Global-Local Duality in Eternal Inflation,” Phys. Rev. D 80, 124024 (2009) doi:10.1103/PhysRevD.80.124024 [arXiv:0904.2386 [hep-th]].

[33] L. Da F. Costa, F. A. Rodrigues, G. Travieso and P. R. Villas Boas, “Characterization of complex networks: A survey of measurements,” Advances in Phys. 56, 167 (2007) [arXiv:cond-mat/0505185].

[34] L. Katz, “A new status index derived from sociometric analysis,” Psychometrika 18, 39 (1953).

[35] S. Brin and L. Page, “The anatomy of a large-scale hypertextual Web search engine,” Computer Networks and ISDN Systems 30, 107 (1998).

[36] A. Bavelas, “Communication patterns in task-oriented groups,” J. Acoust. Soc. Am, 22, 725 (1950).

[37] G. Sabidussi, “The centrality index of a graph,” Psychometrika 31, 581 (1966).

[38] A. Borde, A. H. Guth and A. Vilenkin, “Inflationary space-times are incomplete in past directions,” Phys. Rev. Lett. 90, 151301 (2003) [gr-qc/0110012].

[39] F. Denef, “TASI lectures on complex structures,” arXiv:1104.0254 [hep-th].

[40] F. Denef, M. R. Douglas, B. Greene and C. Zukowski, “Computational complexity of the landscape II—Cosmological considerations,” Annals Phys. 392, 93 (2018) [arXiv:1706.06430 [hep-th]].

[41] J. Khoury and O. Parrikar, “Search Optimization, Funnel Topography, and Dynamical Criticality on the String Landscape,” JCAP 1912, 014 (2019) [arXiv:1907.07693 [hep-th]].

[42] N. Gö, “Theoretical Studies of Protein Folding,” Ann. Rev. Biophys. Bioeng. 12, 183 (1983).

[43] S. Redner, “A Guide to First-Passage Processes,” Cambridge University Press, 328 p. (2001).

[44] V. Tejedor, O. Bénichou and R. Voituriez, “Global mean first-passage times of random walks on complex networks,” Phys. Rev. E 80, 065104(R) (2009) [arXiv:0909.0657 [cond-mat.stat-mech]].

[45] J. D. Noh and H. Rieger, “Random Walks on Complex Networks,” Phys. Rev. Lett. 92, 118701 (2004).

[46] A.-M. Kermarrec, E. Le Merrer, B. Sericola and G. Trédan, “Second order centrality: Distributed assessment of nodes criticity in complex networks,” Computer Comm. 34, 619 (2011).
[49] J. C. Mauro and M. M. Smedskjaer, “Statistical mechanics of glass,” Journal of Non-Crystalline Solids 396, 41 (2014).
[50] A. Andreassen, W. Frost and M. D. Schwartz, “Scale Invariant Instantons and the Complete Lifetime of the Standard Model,” Phys. Rev. D 97, no. 5, 056006 (2018) [arXiv:1707.08124 [hep-ph]].
[51] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto and A. Strumia, “Higgs mass implications on the stability of the electroweak vacuum,” Phys. Lett. B 709, 222 (2012) [arXiv:1112.3022 [hep-ph]].
[52] J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia and N. Tetradis, “The cosmological Higgsstory of the vacuum instability,” JHEP 1509, 174 (2015) [arXiv:1505.04825 [hep-ph]].
[53] J. Fumagalli, S. Renaux-Petel and J. W. Ronayne, “Higgs vacuum (in)stability during inflation: the dangerous relevance of de Sitter departure and Planck-suppressed operators,” arXiv:1910.13430 [hep-ph].
[54] F. Denef and M. R. Douglas, “Computational complexity of the landscape. I.,” Annals Phys. 322, 1096 (2007) [hep-th/0602072].
[55] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, “Predictive landscapes and new physics at a TeV,” hep-th/0501082.
[56] N. Bao, R. Bousso, S. Jordan and B. Lackey, “Fast optimization algorithms and the cosmological constant,” Phys. Rev. D 96, no. 10, 103512 (2017) [arXiv:1706.08503 [hep-th]].
[57] J. Halverson and F. Ruehle, “Computational Complexity of Vacua and Near-Vacua in Field and String Theory,” Phys. Rev. D 99, no. 4, 046015 (2019) [arXiv:1809.08279 [hep-th]].
[58] M. Cvetic, I. Garcia-Etxebarria and J. Halverson, “On the computation of non-perturbative effective potentials in the string theory landscape: IIB/F-theory perspective,” Fortsch. Phys. 59, 243 (2011) [arXiv:1009.5386 [hep-th]].
[59] J. Halverson, M. Plesser, F. Ruehle and J. Tian, “Kahler Moduli Stabilization and the Propagation of Decidability,” arXiv:1911.07835 [hep-th].
[60] J. Carifio, W. J. Cunningham, J. Halverson, D. Krioukov, C. Long and B. D. Nelson, “Vacuum Selection from Cosmology on Networks of String Geometries,” Phys. Rev. Lett. 121, no. 10, 101602 (2018) [arXiv:1711.06685 [hep-th]].
[61] Y. H. He, “Deep-Learning the Landscape,” arXiv:1706.02714 [hep-th].
[62] D. Krefl and R. K. Seong, “Machine Learning of Calabi-Yau Volumes,” Phys. Rev. D 96, no. 6, 066014 (2017) [arXiv:1706.03346 [hep-th]].
[63] F. Ruehle, “Evolving neural networks with genetic algorithms to study the String Landscape,” JHEP 1708, 038 (2017) [arXiv:1706.07024 [hep-th]].
[64] J. Carifio, J. Halverson, D. Krioukov and B. D. Nelson, “Machine Learning in the String Landscape,” JHEP 1709, 157 (2017) [arXiv:1707.00655 [hep-th]].
[65] Y. N. Wang and Z. Zhang, “Learning non-Higgsable gauge groups in 4D F-theory,” JHEP 1808, 009 (2018) [arXiv:1804.07296 [hep-th]].
[66] D. Klaewer and L. Schlechter, “Machine Learning Line Bundle Cohomologies of Hypersurfaces in Toric Varieties,” Phys. Lett. B 789, 438 (2019) [arXiv:1809.02547 [hep-th]].
A. Matter, E. Parr and P. K. S. Vaudrevange, “Deep learning in the heterotic orbifold landscape,” Nucl. Phys. B 940, 113 (2019) doi:10.1016/j.nuclphysb.2019.01.013 [arXiv:1811.05993 [hep-th]].

A. Cole and G. Shiu, “Topological Data Analysis for the String Landscape,” JHEP 1903, 054 (2019) [arXiv:1812.06960 [hep-th]].

J. Halverson, B. Nelson and F. Ruehle, “Branes with Brains: Exploring String Vacua with Deep Reinforcement Learning,” JHEP 1906, 003 (2019) [arXiv:1903.11616 [hep-th]].

Y. H. He and S. J. Lee, “Distinguishing elliptic fibrations with AI,” Phys. Lett. B 798, 134889 (2019) [arXiv:1904.08530 [hep-th]].

A. Cole, A. Schachner and G. Shiu, “Searching the Landscape of Flux Vacua with Genetic Algorithms,” JHEP 1911, 045 (2019) [arXiv:1907.10072 [hep-th]].

S. Havlin and D. Ben-Avraham, “Diffusion in disordered media,” Advances in Physics 36, 695 (1987).

S. R. Coleman, “The Fate of the False Vacuum. 1. Semiclassical Theory,” Phys. Rev. D 15, 2929 (1977) Erratum: [Phys. Rev. D 16, 1248 (1977)].

C. G. Callan, Jr. and S. R. Coleman, “The Fate of the False Vacuum. 2. First Quantum Corrections,” Phys. Rev. D 16, 1762 (1977).

S. R. Coleman and F. De Luccia, “Gravitational Effects on and of Vacuum Decay,” Phys. Rev. D 21, 3305 (1980).

K. M. Lee and E. J. Weinberg, “Decay of the True Vacuum in Curved Space-time,” Phys. Rev. D 36, 1088 (1987).

D. Schwartz-Perlov and A. Vilenkin, “Probabilities in the Bousso-Polchinski multiverse,” JCAP 0606, 010 (2006) [hep-th/0601162].

K. D. Olum and D. Schwartz-Perlov, “Anthropic prediction in a large toy landscape,” JCAP 0710, 010 (2007) [arXiv:0705.2562 [hep-th]].

V. Tejedor, O. Bénichou, and R. Voituriez, “Global mean first-passage times of random walks on complex networks,” Phys. Rev. E 80, 065104(R) (2009) [arXiv:0909.0657 [cond-mat.stat-mech]].

V. Vennin and A. A. Starobinsky, “Correlation Functions in Stochastic Inflation,” Eur. Phys. J. C 75, 413 (2015) [arXiv:1506.04732 [hep-th]].

H. Assadullahi, H. Firouzjahi, M. Noorbala, V. Vennin and D. Wands, “Multiple Fields in Stochastic Inflation,” JCAP 1606, 043 (2016) [arXiv:1604.04502 [hep-th]].

V. Vennin, H. Assadullahi, H. Firouzjahi, M. Noorbala and D. Wands, “Critical Number of Fields in Stochastic Inflation,” Phys. Rev. Lett. 118, no. 3, 031301 (2017) [arXiv:1604.06017 [astro-ph.CO]].

M. Noorbala, V. Vennin, H. Assadullahi, H. Firouzjahi and D. Wands, “Tunneling in Stochastic Inflation,” JCAP 1809, 032 (2018) [arXiv:1806.09634 [hep-th]].

M. Kac, “On the notion of recurrence in discrete stochastic processes,” Bull. Amer. Math. Soc. 53, 1002 (1947).

J. G. Kemeny and J. L. Snell, “Finite Markov Chains,” Van Nostrand Comp. Int., New York, 1960.

S. Condamin et al., “First-passage times in complex scale-invariant media,” Nature 450, 77 (2007).
[87] T. M. Michelitsch et al., “Recurrence of random walks with long-range steps generated by fractional Laplacian matrices on regular networks and simple cubic lattices,” Journal of Phys. A: Math. and Theo. 50, 505004 (2017) [arXiv:1707.05843 [cond-mat.stat-mech]].

[88] S. Weinberg, “A Priori probability distribution of the cosmological constant,” Phys. Rev. D 61, 103505 (2000) [astro-ph/0002387].

[89] D. N. Page, “Information in black hole radiation,” Phys. Rev. Lett. 71, 3743 (1993) [hep-th/9306083].

[90] U. H. Danielsson, D. Domert and M. E. Olsson, “Miracles and complementarity in de Sitter space,” Phys. Rev. D 68, 083508 (2003) [hep-th/0210198].

[91] U. H. Danielsson and M. E. Olsson, “On thermalization in de Sitter space,” JHEP 0403, 036 (2004) [hep-th/0309163].

[92] R. Z. Ferreira, M. Sandora and M. S. Sloth, “Asymptotic Symmetries in de Sitter and Inflationary Spacetimes,” JCAP 1704, no. 04, 033 (2017) [arXiv:1609.06318 [hep-th]].

[93] R. Z. Ferreira, M. Sandora and M. S. Sloth, “Patient Observers and Non-perturbative Infrared Dynamics in Inflation,” JCAP 1802, no. 02, 055 (2018) [arXiv:1703.10162 [hep-th]].

[94] P. Creminelli, S. Dubovsky, A. Nicolis, L. Senatore and M. Zaldarriaga, “The Phase Transition to Slow-roll Eternal Inflation,” JHEP 0809, 036 (2008) [arXiv:0802.1067 [hep-th]].

[95] N. Arkani-Hamed, S. Dubovsky, A. Nicolis, E. Trincherini and G. Villadoro, “A Measure of de Sitter entropy and eternal inflation,” JHEP 0705, 055 (2007) [arXiv:0704.1814 [hep-th]].

[96] R. Monasson et al., “Determining computational complexity from characteristic ‘phase transitions’,” Nature 400, 133 (1999).

[97] T. Hogg, B. A. Huberman and C. P. Williams, “Phase transitions and the search problem,” Artificial Intelligence 81, 1 (1996).

[98] B. A. Huberman and T. Hogg, “Phase transitions in artificial intelligence systems,” Artificial Intelligence 33, 155 (1987).

[99] T. Mora and W. Bialek, “Are biological systems poised at criticality?” J. Stat. Phys. 144, 268 (2011) [arXiv:1012.2242 [q-bio.QM]].

[100] M. Usher, M. Stemmler and Z. Olami, “Dynamic pattern formation leads to 1/f noise in neural populations,” Phys Rev Lett 74, 326 (1995).

[101] O. Kinouchi and M. Copelli, “Optimal Dynamical Range of Excitable Networks at Criticality,” Nature Phys. 2, 348 (2006) [arXiv:q-bio/0601037 [q-bio.NC]].

[102] J. M. Beggs and D. Plenz, “Neuronal avalanches in neocortical circuits,” J. Neurosci 23, 11167 (2003).

[103] D. R. Chialvo, “Emergent complex neural dynamics,” Nature Phys. 6, 744 (2010) [arXiv:1010.2530 [q-bio.NC]].

[104] A. Cavagna et al., “Scale-free correlations in starling flocks,” Proc. Natl Acad. Sci USA 107, 11865 (2010).

[105] W. Bialek et al., “Statistical mechanics for natural flocks of birds,” Proc. Natl Acad. Sci USA 109, 4786 (2012) [arXiv:1107.0604 [physics.bio-ph]].
[106] A. Roli, M. Villani, A. Filisetti and R. Serra, “Dynamical criticality: overview and open questions,” J. of Systems Sci and Complexity, 31, 647 (2018).

[107] S. A. Kauffman, “The Origins of Order: Self-Organization and Selection in Evolution,” Oxford University Press, Oxford (1993).

[108] S. Wolfram, “Universality and complexity in cellular automata,” Physica D 10, 1 (1984).

[109] N. H. Packard, “Adaptation toward the edge of chaos,” Dynamic Patterns in Complex Systems 212, 293 (1988).

[110] C. G. Langton, “Computation at the edge of chaos: Phase transitions and emergent computation,” Physica D 42, 12 (1990).

[111] J. P. Crutchfield and K. Young, “Inferring statistical complexity,” Phys. Rev. Lett. 63, 105 (1989).

[112] J. P. Crutchfield and K. Young, “Computation at the onset of chaos,” In W. H. Zurek, editor, Complexity, Entropy, and the Physics of Information, p. 223269, Addison-Wesley (1990).

[113] M. Mitchell, P. Hraber and J. P. Crutchfield, “Revisiting the Edge of Chaos: Evolving Cellular Automata to Perform Computations,” Complex Systems 7, 89 (1993) [arXiv:adap-org/9303003].

[114] N. Bertschinger and T. Natschläger, “Real-time computation at the edge of chaos in recurrent neural networks,” Neural Computation 16, 1413 (2004).

[115] J. Boedecker, O. Obst O, J. T. Lizier, N. M. Mayer and M. Asada, “Information processing in echo state networks at the edge of chaos,” Theory in Biosciences 131, 205 (2012).

[116] F. Matzner, “Neuroevolution on the Edge of Chaos,” Proc. of the Genetic and Evolutionary Computation Conference, 465 (2017) [arXiv:1706.01330 [cs.NE]].

[117] C. H. Martin and M. W. Mahoney, “Implicit Self-Regularization in Deep Neural Networks: Evidence from Random Matrix Theory and Implications for Learning,” arXiv:1810.01075 [cs.LG].

[118] D. Sornette, “Critical phenomena in natural sciences: chaos, fractals, self-organization and disorder: concepts and tools,” Springer-Verlag, Berlin, 2006.

[119] J. F. Gunion, H. E. Haber and M. Sher, “Charge / Color Breaking Minima and a-Parameter Bounds in Supersymmetric Models,” Nucl. Phys. B 306, 1 (1988).

[120] G. F. Giudice and A. Strumia, “Probing High-Scale and Split Supersymmetry with Higgs Mass Measurements,” Nucl. Phys. B 858, 63 (2012) [arXiv:1108.6077 [hep-ph]].

[121] M. P. Hertzberg, “A Correlation Between the Higgs Mass and Dark Matter,” Adv. High Energy Phys. 2017, 6295927 (2017) [arXiv:1210.3624 [hep-ph]].