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Unpredictable nature of nanofluid flow: A study on effects of uncertainties in effective viscosity

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Abstract

A numerical analysis of steady, laminar and 2-D nanofluid flow around a circular cylinder has been carried out to showcase the effects of variable viscosity on flow characteristics. The governing equations of flow are solved using a finite-volume method based on SIMPLE algorithm. Three cases of simulations in which the effective viscosity of the nanofluid is calculated using (a) Classical Brinkman model, (b) Recent correlation from literature and (c) Experimental data from literature are performed. A comparative analysis of the three cases shows that the flow characteristics of nanofluids become unpredictable due to the uncertainties in effective viscosity. In the first case, nanofluids show an accelerated flow with earlier flow separation and longer wake bubbles. Whereas, in other two cases, a decelerated flow with delayed flow separation is noted. This is the first time; a decelerated flow of nanofluids has been reported in literature. It is understood that, flow characteristics of nanofluid vary both qualitatively and quantitatively due to the variations in effective viscosity. This work showcases the importance of precise effective viscosity models to clearly understand the flow features of nanofluids.

Keywords: Nanofluids; Circular cylinder; Steady laminar flow; Effective viscosity; Uncertainties.

1. Introduction

In recent years, flow and convective heat transfer around cylinders using nanofluids has been an interesting area of research; owing to the enhanced thermo-physical properties of nanofluids, interesting flow phenomena and
applications of flow around cylinders. Understanding the flow characteristics of nanofluids is very vital for estimating their real potential as coolants in heat transfer systems. In order to analyze the flow characteristics, a precise prediction of effective viscosity of nanofluids is compulsory. But, numerous viscosity models and correlations are available in literature, which show different levels of enhancement in viscosity of nanofluids. No single model is capable of comprehensively explaining all the available experimental data. Thus, there is an uncertainty prevailing in the prediction of effective viscosity of nanofluids. This dilemma in calculation of effective viscosity is a major problem in the real time application of nanofluids.

A considerable volume of works on nanofluid flow around cylinders is available in literature. But, most of the available works have modeled the flow using conventional theoretical models [1-8]. In spite of the voluminous literature available on nanofluid flow around cylinders, a study on effects of uncertainty in effective viscosity of nanofluids on the flow characteristics is still lacking. Abu-Nada et al. [9] made an initiating study, but their work is limited only to the effects on heat transfer characteristics. Hence in this study, effects of uncertainties in effective viscosity are studied.
viscosity of nanofluids on flow characteristics are highlighted during a flow around an unconfined circular cylinder. A laminar, steady and two-dimensional flow is considered with the Reynolds number varying from 1 to 40. TiO₂-Water nanofluids with particle volume fraction ranging from 0% to 2% have been used for analysis. Three cases of simulations are performed in which the effective viscosity of nanofluids is determined from two different theoretical models [10-11] in the first two cases and experimental viscosity data from literature [12] is used in the third case.

2. Flow configuration and mathematical formulation

An infinitely long cylinder is exposed to nanofluid flowing with a uniform free stream velocity, \( U_∞ \) and directed towards the positive \( x^- \) direction. The center of the cylinder is at the origin of the cylindrical co-ordinate system, \((r, \theta, z)\). The flow is two-dimensional as the cylinder is infinitely long. The outer boundary is placed far-away from the cylinder surface to facilitate the computation. The computational domain used in this work is graphically represented in Fig. 1(a). Owing to the small particle size, the nanofluids are assumed to be homogeneous and the particles follow the trajectory of the fluid with no velocity slip. Based on these assumptions, nanofluids are treated as pure Newtonian fluids with effective viscosity. Thus, the continuity and momentum equations for Newtonian fluid assuming negligible viscous dissipation are presented in non-dimensional form as follows:

**Continuity equation:**

\[
\frac{1}{R} \frac{\partial}{\partial R} (RV) + \frac{1}{R} \frac{\partial}{\partial \theta} (U) = 0
\]

**Momentum equations:**

\[
\frac{\partial U}{\partial \tau} + \left( \frac{U \frac{\partial U}{\partial R}}{R} + V \frac{\partial U}{\partial R} + \frac{U V}{R} \right) = - \frac{\rho_f}{\rho_{nf}} \frac{1}{R} \frac{\partial p}{\partial \theta} + \frac{2 \mu_{eff}}{\nu_f \rho_{nf} \rho_f} \left( \frac{1}{R} \frac{\partial U}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( \frac{1}{R} \frac{\partial U}{\partial \theta} \right) + \frac{2}{R^2} \frac{\partial V}{\partial \theta^2} - \frac{U}{R^2}
\]

\[
\frac{\partial V}{\partial \tau} + \left( \frac{U \frac{\partial V}{\partial R}}{R} + V \frac{\partial V}{\partial R} + \frac{U^2}{R} \right) = - \frac{\rho_f}{\rho_{nf}} \frac{1}{R} \frac{\partial p}{\partial \theta} + \frac{2 \mu_{eff}}{\nu_f \rho_{nf} \rho_f} \left( \frac{1}{R} \frac{\partial V}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( \frac{1}{R} \frac{\partial V}{\partial \theta} \right) + \frac{2}{R^2} \frac{\partial U}{\partial \theta^2} - \frac{U}{R^2}
\]

The subscripts ‘nf’ and ‘f’ correspond to nanofluids and pure fluid, respectively. In the above equation, the Reynolds number is defined as \( Re = 2a U_∞/\nu_f \). Following parameters were used to non-dimensionalize the governing equations:

\[
U = \frac{u}{U_∞}, \quad V = \frac{v}{U_∞}, \quad \theta = \frac{\theta^*}{R}, \quad R = \frac{r}{a}, \quad P = \frac{p}{\rho_f \nu_f^2}
\]

The boundary conditions applied on the inlet, outlet and the cylinder surface are presented below.

**Inlet:** \( U = U_∞ \sin \theta \) and \( V = U_∞ \cos \theta \)

**Outlet:** \( \frac{\partial U}{\partial x} = 0 \) and \( \frac{\partial V}{\partial x} = 0 \)

**Cylinder wall:** \( U = 0 \) and \( V = 0 \)

Table 1. Thermo-physical properties of TiO₂ nanoparticle and water (basefluid).

| S.No | Material | k (W/mK) | \( \mu_f \) (mPaS) | \( \rho \) (kg/m³) | \( c_p \) (J/kgK) |
|------|----------|----------|-------------------|-----------------|-----------------|
| 1    | TiO₂     | 11.700   | -                 | 4230.000        | 711.7556        |
| 2    | Water    | 0.5800   | 0.000891          | 9997.047        | 4181.000        |
2.1. Thermo-physical properties of nanofluids

The thermo-physical properties of nanoparticle and the basefluid are listed in Table 1. In this study, a single phase approach in which the nanofluid is assumed to be a homogeneous liquid with effective physical properties is utilized.

2.1.1 Effective viscosity

Three cases of simulations are carried out in which the effective viscosity of nanofluid is calculated as follows:

**Case 1:** The effective viscosity of nanofluids is calculated by Brinkman model [10] as

\[ \mu_{eff} = \frac{ \mu_f }{(1 - \phi)^{2.5}} \]

**Case 2:** In this case the effective viscosity of nanofluids is calculated using a correlation by Chen et al. [11] which is given by

\[ \mu_{eff} = \mu_f (1 + 10.6\phi + (10.6\phi)^2) \]

**Case 3:** Finally in this case, experimental values of effective viscosity of nanofluids [12] which contain ultra-fine Titania (TiO2) nanoparticles suspended in water as basefluid are taken into account. A comparison between a basic model, a new model which considers the aggregation mechanism and experimental data for effective viscosity is apt for quantifying the uncertainties in flow characteristics caused due to models.

2.1.2 Effective density

Effective density of nanofluid of the nanofluid is given as follows [1-2]

\[ \rho_{eff} = \phi \rho_p + (1 - \phi) \rho_f \]

| S.No | m x n     | CD   | % Difference |
|------|-----------|------|--------------|
| 1    | 199 X 745*| 1.5060 | 0.26         |
| 2    | 401 X 745 | 1.5092 | 0.07         |
| 3    | 601 X 745 | 1.5103 | -            |

* Grid used in this study

3. Numerical methodology, code validation and grid independence study

Governing equations are solved using finite volume method based on SIMPLE algorithm. Time derivatives and convective terms are discretized using first order implicit scheme and third order accurate QUICK scheme, respectively. Central difference scheme is used to discretize diffusion terms. The grid is more refined around the cylinder and slightly coarse away from the cylinder in far field (see Fig. 1(b)). A systematic set of test runs have been conducted with different grid sizes \((m \times n)\) to ensure the grid independency of the solution. The results of grid independence study are shown in Table 3. The CFD code used in this study is validated by comparing the coefficient of drag obtained using present code with data from literature [13] for pure fluid flow around a circular cylinder. The values of the present study are found to match well with the data from [13].
Fig. 2. Comparison of streamline patterns obtained in three cases at Re = 1, 10 and 40.

4. Results and discussion

Flow characteristics of nanofluid flowing around a circular cylinder obtained in three cases are compared with each other. A detailed comparative analysis is presented below.
4.1. Streamlines

The flow field of nanofluids around the circular cylinder is visualized using streamline patterns in Fig. 2. Streamline patterns at 0.2% and 2% particle volume fractions and at Reynolds numbers, \( Re = 1, 10 \) and 40 are alone presented for the purpose of brevity. At \( Re = 1 \), due to low flow velocity; the flow is completely attached to the cylinder surface. At \( Re = 10 \), the flow is unable to adhere in the downstream region and it separates from the cylinder surface forming a recirculation/wake bubble in the trailing edge of cylinder. As the Reynolds number is further increased, length of the recirculating wake bubble gradually increases and it is maximum at \( Re = 40 \). In addition to \( Re \), effects of particle volume fraction in each case of simulation are also presented. In case 1, increase in particle volume fraction results in an accelerated flow causing an increase in wake length with increase in \( \phi \). Whereas in case 2 and case 3, wake length decreases with addition of nanoparticles indicating a deceleration in flow. This is completely different from case 1. Even though case 2 and case 3 show a decelerated flow, length of the wake bubbles in case 2 and case 3 vary from each other. This can be understood from the recirculation lengths shown using vertical lines for easier comparison.

![Fig. 3. Variation of coefficient of drag with nanoparticle volume fraction in three cases at (a) \( Re = 1 \) and (b) \( Re = 10 \).](image)

![Fig. 4. Variation of (a) Wake length and (b) Separation angle in three cases at \( Re = 10 \) to 40.](image)
4.2. Coefficient of drag

Variation of coefficients of drag with particle volume fraction for $1 \leq Re \leq 40$ is presented in Fig. 3 (a) and (b). At $Re = 1$, coefficients of drag of the cylinder are quite high. A sudden drop in $C_D$ is noted when the Reynolds number is increased to 10. As $Re$ is increased further, corresponding decrease in $C_D$ is noted and the lowest value is seen at $Re = 40$. Irrespective of the Reynolds number, coefficients of drag increase with increase in particle volume fraction due to increase in viscosity of nanofluids with the addition of nanoparticles. The basic Brinkman model in case 1 show lower values of $C_D$ and the highest values of $C_D$ are seen in case 2. Even though, case 3 lies between case 1 and 2, it is closer to case 2. The uncertainty in viscosity prediction by the models is more pronounced at lower Reynolds numbers and higher volume fractions. At $Re = 1$ and $\phi = 2\%$, there is 5.5% variation between case 1 and case 3, 4.8% variation between case 2 and case 3 and 10.9% variation between case 1 and 2.

4.3. Wake length

At $Re = 10$, flow separates from the cylinder surface and forms a recirculating wake bubble in the downstream side of cylinder (see Fig. 2). As Reynolds number is increased, the length of the wake bubble increases gradually as shown in Fig. 4 (a). In case 1, a marginal increase in wake length is observed when the volume fraction of nanoparticles is increased, indicating the promotion of flow separation. In case 2 and case 3, wake length decreases remarkably with addition of nanoparticles. This is because of the higher increase in viscosity with increase in particle volume fraction leading to delayed flow separation and shorter wake lengths. Thus, a completely contradicting behavior is exhibited in case 1 when compared to the other two cases. The variations between three cases are very high at higher Reynolds numbers and volume fractions. At $Re = 40$ and $\phi = 2\%$, there is a 9.3% variation between case 1 and case 3, 7.8% variation between case 2 and case 3, while case 1 and case 2 vary by 15.9%.

4.4. Separation angle

At $Re \geq 10$, the flow is separated from the surface of the cylinder on the downstream side as observed in the streamlines shown in Fig. 2. The angle at which the flow separation takes place is known as the separation angle and it varies with $Re$ and $\phi$ as shown in Fig 4 (b). At $Re = 10$ and $\phi = 0\%$, the flow separation occurs at approximately 150º. Flow separation is advanced by increasing the Reynolds number. Hence, the flow separates at smaller angles along the cylinder surface at higher Reynolds numbers. For instance, at $Re = 40$ and $\phi = 0\%$, flow separation is noticed at 126º. Flow separation is also influenced by particle volume fraction. In case 1, the separation angle decreases with by addition of nanoparticles indicating the advancement of flow separation. A completely new phenomenon is seen in case 2 and case 3, where the separation angle increases with increase in volume fraction. This shows that the flow separation is delayed by the addition of nanoparticles. This is due to the sharp increase in viscosity in case 2 and case 3, which prevents the flow separation. At $Re = 10$ and $\phi = 2\%$, there is 1.4% variation between case 1 and case 3, 1.5% variation between case 2 and case 3 and 2.9% variation between case 1 and case 2.

5. Conclusions and future work

A 2-D, steady and laminar nanofluid flow around a circular cylinder, with Reynolds number varying from 1 to 40 has been numerically investigated. Variation of flow characteristics with particle volume fraction and Reynolds number is analyzed. Notable characteristic of this work is that, in addition to analyzing the effects of particle volume fraction and Reynolds number, a comparative study in which different models and experimental data for effective viscosity, available in literature are used has been presented. The results of comparative study of three cases can be concluded as follows:

- The basic model by Brinkman [10] used in case 1 shows an accelerated flow with increase in particle volume fraction. But, a completely new and unique phenomenon is seen in case 2. The recent correlation for effective viscosity of nanofluids by Chen et al. [11] used in case 2, exhibits an decelerated flow
This is because the correlation in case 2 captures the increase in viscosity due to increase in particle volume fraction quite well than the Brinkman model in case 1. Hence, in case 2 an increase in kinematic viscosity is noted, which leads to a decelerated flow.

Even though case 2 and 3 show similar trends of flow characteristics, they vary quantitatively.

From the comparative study it is evident that, the choice of effective property models has a significant effect on the prediction of flow characteristics. These variations observed between the three cases clearly indicate that predicting the real potential of nanofluids is still a question. In future, a much elaborate study including the heat transfer characteristics will be carried out. Multi-phase modeling of nanofluids will also be carried out to give a better prediction of flow and heat transfer characteristics of nanofluids.

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