High Energy Hadron Production, Self-Organized Criticality and Absorbing State Phase Transition

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Abstract

In high energy nuclear collisions, production rates of light nuclei agree with the predictions of an ideal gas at a temperature $T = 155 \pm 10$ MeV. In an equilibrium hadronic medium of this temperature, light nuclei cannot survive. In this contribution, we suggest that the observed behavior is due to an evolution in global non-equilibrium, leading to self-organized criticality and to hadron formation as an absorbing state phase transition for color degrees of freedom. At the confinement point, the initial quark-gluon medium becomes quenched by the vacuum, breaking up into all allowed free hadronic and nuclear mass states, without (or with a very short-live) subsequent formation of thermal hadronic medium.

1 Introduction

The yields for deuteron, $^3$He, hyper-triton, $^4$He and their antiparticles have recently been measured in $Pb - Pb$ collisions by the ALICE collaboration \cite{1,2,3} and are in very good agreement \cite{4} with the statistical hadronization model (SHM) \cite{5}, with a formation temperature of $T \simeq 155$ MeV, corresponding at the (pseudo)critical confinement temperature $T_c = 155 \pm 10$ MeV \cite{6}.

The curious feature is that the all hadron abundances are already specified once and for all at $T_c$ and are not subsequently modified in the evolution of the hadron gas and this enigma is further enhanced by the yields for light nuclei. Indeed, these states have binding energies of a few MeV and are generally much larger than hadronic size, and therefore their survival in the assumed hot hadron gas poses an even more striking puzzle \cite{7}. For example, the hyper-triton root-mean-square size is close to 10 fm, about the same size of the whole fireball formed in Pb-Pb collision at $\sqrt{s} = 2.76$ TeV, and the energy needed to remove the $\Lambda$ from it is $130 \pm 30$ KeV.

Why are the yields for the production of light nuclei determined by the rates as specified at the critical hadronization temperature, although in hot hadron gas they would immediately be destroyed?

In this contribution, following ref. \cite{8}, we want to discuss a solution to this puzzle obtained by abandoning the idea of a thermal hadron medium existing below the confinement point. We propose that the hot quark-gluon system, when it cools down to the hadronization temperature, is effectively quenched by the cold physical vacuum.

The relevant basic mechanism for this is self-organized criticality, leading to universal scale-free behavior, based on an absorbing state phase transition for color degrees of freedom.
2 Self-organized criticality and absorbing state phase transition

The core hypothesis of Self-Organized Criticality (SOC) \cite{9,10} is that systems consisting of many interacting components will, under certain conditions, spontaneously organize into a state with properties akin to that ones observed in an equilibrium thermodynamic system, as the scale-free behavior.

The self-organized evolution indicates that the complex behavior arises spontaneously without the need for the external tuning of a control parameter (the temperature for example). In SOC the dynamics of the order parameter drives the control parameter to the critical value: natural dynamics drives the system towards and maintains it at the edge of stability.

For non-equilibrium steady states it is becoming increasingly evident that SOC is related to conventional critical behavior by the concept of absorbing-state phase transition \cite{11,12}. An absorbing state is a configuration that can be reached by the dynamics but cannot be left and absorbing state phase transitions are among the simplest non-equilibrium phenomena displaying critical behavior and universality \cite{11,12}.

A clear example is given by models describing the growth of bacterial colonies or the spreading of an infectious disease among a population: once an absorbing state, e.g., a state in which all the bacteria are dead, is reached, the system cannot escape from it.

Let us now consider the hadronization dynamics where, for sake of simplicity, an initial $e^+e^-$ annihilation produces a $\bar{q}q$ pair which evolves according to QCD dynamics. The short distances dynamics is due to local interacting color charges, with the QCD processes of parton (quarks and gluons) annihilations and creation.

The dynamics of color degrees of freedom (d.o.f.) ends up with the hadronic production, i.e. with the production of colorless clusters. The final state has no color and the evolution of the system cannot produce colored partons in the final state.

From this point of view, hadron production is a phase transition to an absorbing state for color degrees of freedom. Moreover this phase transition is a non-equilibrium one, since, by definition, the rate out of an absorbing state is zero and an absorbing state can not obey the detailed balance.

A toy model which shows how the competition between hadron (h) formation, i.e. color neutralization, and production and/or annihilation of color charges (partons P) leads to an absorbing state is easily obtained by considering a normalized quantity $\rho(t)$, proportional to color charge., as a function of time $t$ and the processes: $P + P \rightarrow P$ with rate $\lambda$ (parton annihilation); $P \rightarrow P + P$ with rate $\sigma$ (parton production); $P \rightarrow h$ with rate $k$ (color neutralization).

The mean field evolution equation is given by \cite{11,12}

$$\frac{d\rho}{dt} = (\sigma - k)\rho - \lambda \rho^2 = \rho(\sigma - k - \lambda \rho) . \tag{1}$$

If $\sigma < k$ the steady state is $\rho_s = 0$ and is an absorbing state. If $\sigma > k$ the steady state is $\rho_s = (\sigma - k)/\lambda$, the critical value is $\sigma_c = k$ and, as in thermal equilibrium, the critical point is governed by a power law behavior $\rho_s \simeq (\sigma - \sigma_c)^\beta$ with $\beta = 1$.

Absorbing states characterize first order phase transitions also \cite{16} and, indeed, for pure $SU(N)$ gauge theories, where the Polyakov loop, $l$, is an order parameter, one can show...
that the dynamical evolution of the system [17] has a steady state with $l = 0$, which is an absorbing state.

According to previous discussion: 1) The Hadronization mechanism is a non equilibrium phase transition to an absorbing state for color d.o.f.; 2) The dynamical evolution is driven by color d.o.f. up to the hadronization time/temperature; 3) a natural assumption, due to the absorbing state phase transition, is that the system is essentially frozen at the values of the parameters at the transition.

Let us discuss the consequences of this point of view for the hadron production.

3 SOC in hadron formation

3.1 Self-organization and hadronic spectrum

The typical illustration of SOC, proposed in the pioneering work [9], is the avalanches dynamics of sandpiles, where the number $N(s)$ of avalanches of size $s$ observed over a long period is found to vary as a power of $s$, $N(s) = \alpha s^{-p}$, which means that the phenomenon is scale-free.

Another useful example of self-organized criticality provided by partitioning integers [8]. Consider the ordered partitioning of an integer $n$ into integers. The number $q(n)$ of such partitionings is for $n = 3$ equal to four: 3, 2+1, 1+2, 1+1+1, i.e., $q(3) = 4$. It is easily shown [14] that in general $q(n) = 2^{n-1}$, i.e. the number of partitions increases exponentially with the size of the integer.

Given an initial integer $n$, we would now like to know the number $N(k, n)$ specifying how often a given integer $k$ occurs in the set of all partitionings of $n$. To illustrate, in the above case of $n = 3$, we have $N(3, 3) = 1$, $N(3, 2) = 2$ and $N(3, 1) = 5$. To apply the formalism of self-organized criticality, we have to attribute a strength $s(k)$ to each integer. It seems natural use the number of partitions for this, i.e., set $s(k) = q(k)$ and the desired number $N(k, n)$ in a scale-free scenario is then given by

$$N(k, n) = \alpha(n)[s(k)]^{-p}. \quad (2)$$

For small values of $n$, $N(k, n)$ is readily obtained explicitly and one finds that the critical exponent becomes $p \approx 1.26$.

The previous example is immediately reminiscent of the statistical bootstrap model of Hagedorn [13], who had “fireballs composed of fireballs, which in turn are composed of fireballs, and so on”. Indeed, its general pattern has been shown to be due to an underlying structure analogous to the partitioning of an integer into integers [14].

More precisely, Hagedorn’s bootstrap approach [13] proposes that a hadronic colorless state of overall mass $m$ can be partitioned into structurally similar colorless states, and so on. If these states were at rest, the situation would be identical to the above partitioning problem. Since the constituent fireballs have an intrinsic motion, the number of states $\rho(m)$ corresponding to a given mass $m$ is determined by the bootstrap equation which can be asymptotically solved [15], giving $\rho(m) \sim m^{-a} \exp(m/T_H)$ and $T_H$ as solution of

$$\left(\frac{2}{3\pi}\right) \left(\frac{T_H}{m_0}\right) K_2(m_0/T_H) = 2\ln 2 - 1. \quad (3)$$
with \( m_0 \) denoting the lowest possible mass and \( K_2(x) \) is a Hankel function of pure imaginary argument. For \( m_0 = m_\pi \simeq 130 \text{ MeV} \), this leads to the Hagedorn temperature \( T_H \simeq 150 \text{ MeV} \), i.e., to approximately the critical hadronization temperature found in statistical QCD. The cited solution gave \( a = 3 \), but other exponents could also be considered.

The previous form is an asymptotic solution of the bootstrap equation which diverges for \( m \to 0 \) and must be modified for small masses. Using a similar result for \( \rho(m) \) obtained in the dual resonance model \[18\], Hagedorn proposed

\[
\rho(m) = \text{const.} \cdot (1 + (m/\mu_0))^{-a} \exp(m/T_H) \tag{4}
\]

where \( \mu_0 \simeq 1 - 2 \text{ GeV} \) is a normalization constant.

At this point we should emphasize that the form of \( \rho(m) \) is entirely due to the self-organized nature of the components and they are in no way a result of thermal behavior. We have expressed the slope coefficient of \( m \) in terms of the Hagedorn “temperature” only in reference to subsequent applications. In itself, it is totally of combinatorical origin.

### 3.2 Comparison with ALICE data

We now apply the formalism of self-organized criticality to strong interaction physics. Our picture assumes a sudden quench of the partonic medium produced in the collision. The initial hot system of deconfined quarks and gluons rapidly expands and cools; while this system is presumably in local thermal equilibrium, the difference between transverse and longitudinal motion implies a global non-equilibrium behavior. The longitudinal expansion quickly drives the system to the hadronisation point, and it is now suddenly thrown into the cold physical vacuum. The process is not unlike that of a molten metal being dumped into cold water. In this quenching process, the system freezes out into the degrees of freedom presented by the system at the transition point and subsequently remains as such, due to the absorbing state nature of the transition, apart from possible hadron or resonance decays. There never is an evolving warm metal. In other words, in our case there is no hot (or a very short-live) interacting hadron gas. Whatever thermal features are observed, such as radial or elliptic hydrodynamic flow, must then have originated from local equilibrium in the earlier deconfined stage \[19\]. The mechanism driving the system rapidly to the critical point is the global non-equilibrium due to the longitudinal motion provided by the collision.

In such a scenario, high energy nuclear collisions lead to a system which at the critical point, i.e. the color absorbing state, breaks up into components of different masses \( m \), subject to self-similar composition and hence of a strength \( \rho(m) \) as given by the above Eq. (4). In the self-organized criticality formalism, this implies that the interaction will produce

\[
N(m) = \alpha [\rho(m)]^{-p} \tag{5}
\]

hadrons of mass \( m \). With \( \rho(m) \) given by Eq. (4), the resulting powerlaw form

\[
\log N(m) = -m \left( \frac{p \log e}{T_H} \right) \left[ 1 - \left( \frac{a T_H}{m} \right) \log(1 + \frac{m}{\mu_0}) \right] + \text{const.} \tag{6}
\]

is found to show a behavior similar to that obtained from an ideal resonance gas in equilibrium. We emphasize that it is here obtained assuming only scale-free behavior
(self-organized criticality) and a mass weight determined by the number of partitions. No
equilibrium thermal system of any kind is assumed.
We now consider the mentioned ALICE data \[1,3\]. In Fig. 1 the production yields for the
different mass states in central \(Pb - Pb\) collisions at \(\sqrt{s} = 2.76\) GeV are shown; in each
case, the yield is divided by the relevant spin degeneracy. We see that the yields show
essentially powerlike behavior, and the light nuclei follow the same law as the elementary
hadrons. The solid line in Fig. 1 shows the behavior obtained from eqs. (6), ignoring for
the moment the second term in the square brackets,
\[
\log\left(\frac{dN}{dy}\right) / (2s + 1) \simeq -m \left( \frac{0.43 p}{T_H} \right) + A, \tag{7}
\]
with \(T_H = 0.155 MeV\) and fit values \(p = 0.9, A = 3.4\). The form is evidently in good
agreement with the data.
Including the correction term to linear behavior that we had omitted above, we have
\[
\log\left(\frac{dN}{dy}\right) / (2s + 1) \simeq -m \left( \frac{0.43 p}{T_H} \right) + p a \log[1 + \left( \frac{m}{\mu} \right)] + A. \tag{8}
\]
The additional term is, as indicated, rather model dependent. It will effectively turn
the yield curve down for decreasing masses. This is in fact necessary, since the decay of
heavier resonances will enhance the direct low mass meson yields. To illustrate the effect
of the term, we choose \(a = 3\), corresponding to the mentioned solution of the bootstrap
equation \[15\], and \(\mu = 2\) GeV for the normalization. The result is included in Fig. 1.

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Figure 1: Yield rates of species at central rapidity vs. their mass \(m\) \[1,3\]. The solid line
corresponds to Eq. (7), the dashed line to Eq. (8).
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