Regularization of the Nambu-Jona Lasinio model under a uniform magnetic field and the role of the anomalous magnetic moments

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Abstract

The vacuum contribution to quark matter under a uniform magnetic field within the SU(3) version of the Nambu and Jona-Lasinio model is studied. The standard regularization procedure is examined and a new prescription is proposed. For this purpose analytic regularization and a subtraction scheme are used to deal with divergencies depending on the magnetic intensity. This scheme is combined with the standard three momentum cutoff recipe, and reduces to it for vanishing magnetic intensity. Furthermore, the effects of a direct coupling between the anomalous magnetic moments of the quarks and the magnetic field is considered. Single particle properties as well as bulk thermodynamical quantities are studied for a configuration of matter found in neutron stars. A wide range of baryonic densities and magnetic intensities are examined at zero temperature.

1 Introduction

The study of dense matter under strong interaction is usually carried out by employing effective models, due to the intricacies of the fundamental theory . Within this approach, the Nambu-Jona Lasinio (NJL) model has shown to be a useful conceptual tool to tackle different problems. In particular, it has been widely used to describe quarks interacting with magnetic fields [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Within this versatile description a variety of issues have been analyzed such as magnetic catalysis, magnetic oscillations [5], color superconductivity [6, 7, 9, 12], chiral density waves [8], vector [10] and tensor [11] additional couplings, and quark stars [13]. Multiple efforts have been made to extract physical content from the vacuum of the strong interaction affected by a magnetic field [14, 15, 16, 17]. Ref. [14]
provides general expressions for the effective action of a Dirac field interacting with a magnetic field for intensities greater than the mass scale. A description based on the chiral sigma lagrangian has been made in [15], [18] uses the quark-meson model, whereas in [16] the vacuum contribution to the magnetization is evaluated in a one-loop approach to QCD for very intense magnetic fields. Using the chiral quark model and a Ginzburg-Landau expansion Ref. [17] has found that different treatments of the divergences could yield important modifications of the phase diagram.

The investigation of this issue has been particularly active within the NJL model [3, 4, 5, 19, 20, 21, 22, 23, 24, 25, 26], since the contribution coming from the Dirac sea of quarks is responsible for the dynamical breaking of the chiral symmetry and it is a crucial point of the NJL model. There exist several prescriptions to deal with divergent contributions in the NJL at zero magnetic field, all of them yield compatible predictions. But it seems that it is not the case in the presence of an external magnetic field, as was recently pointed out in [23, 24]. In these references it is mentioned that the use of smooth form factors instead of a steep cutoff could change drastically the physical predictions. Following the analytic regularization in terms of the Hurwitz zeta function [14], a residue depending on the squared magnetic intensity is found in [5]. To dispose of this singularity, the authors propose a wavefunction renormalization by associating it to the pure magnetic contribution to the energy density. To deal with the divergencies in the thermodynamic potential, in [19, 20] the pressure at zero baryonic density and finite \( B \) is subtracted and added, in the former case still exhibiting the undesirable divergency and in the latter one it has been regularized in the 3-momentum cutoff scheme. While in the calculations of [25] a softening regulator is used to analyze the effects of the AMM on the quark matter phase diagram, in [26] a step function in momentum space is used with the same purpose.

An interesting aspect to be taken into account for Dirac particles in a magnetic field is the discrepancy of the gyromagnetic factor from the ideal value 2. This can happen in a quasiparticle scheme where the effects of some interactions give rise to the anomalous magnetic moments (AMM) [27, 28, 11]. As a reference one can take the prediction of the non relativistic constituent quark model for the magnetic moments of the light quarks. In order to adjust the experimental values of the proton and neutron magnetic moments, the gyromagnetic ratios \( \tilde{g}_u = 2\mu_u/\mu_N = 3.7 \), \( \tilde{g}_d = 2\mu_d/\mu_N = -1.94 \) are obtained within this approach. The appearance of AMM is closely related to the breakdown of the chiral symmetry. For this reason the NJL model with zero current quark mass have been used to study the origin of the AMM [11, 27, 28]. To analyze the feasibility of the dynamical generation of the AMM, in [27] a one loop correction to the electromagnetic vertex is evaluated within the one flavor NJL model, obtaining

\[
\tilde{g} \approx \frac{2M_p}{M} \frac{e}{M_e} \left[ 1 - \frac{(M/\Lambda)^2 \log(\Lambda/M)}{M^2} \right].
\]

In a more sophisticated treatment they obtain \( \tilde{g}_u \approx 3.72 \), \( \tilde{g}_d \approx -1.86 \), by choosing adequately the constituent quark masses.
In the approach of [28] the AMM is extracted from the low momentum electromagnetic current written in terms of the kernel of the Ward identities. Assuming a four momentum cutoff, they find zero AMM in a one flavor NJL model. However, by using the two flavor version, the authors obtain $\tilde{g}_u \approx 3.813$, $\tilde{g}_d \approx -1.929$, which differ from the phenomenological expectations by less than 1%. Furthermore, in the same work an schematic confining potential for only one flavor is considered. By taking a constituent quark mass $M = 330$ MeV, typical of the NJL model, the magnitude of the AMM predicted is as large as 0.15.

Another point of view is developed in [11], where the one flavor NJL model is supplemented with a four fermion tensor interaction, which induces a condensate in the $\gamma^1 \gamma^2$ channel. In this case the intrinsic relation between the constituent quark mass and its AMM is explicitly exposed, since the vacuum condensate which breaks the chiral symmetry is also responsible for the rising of the AMM. As a consequence, the AMM has a non-perturbative dependence on the magnetic intensity.

The necessity of the AMM of the quarks has been emphasized in [29] in the context of the Karl-Sehgal formula, which relates baryonic properties with the spin configuration of the quarks composing them. By stating the dynamical independency of the axial and tensorial quark contributions to the baryonic intrinsic magnetism, the AMM of the quarks are proposed as the parameters that distinguish between them. Resorting to sound arguments, the author propose $a_u = a_d \approx 0.38$, $a_s \approx 0.2 - 0.38$ as significative values for the AMM for the lightest flavors.

Other investigations have focused on the consequences of a linear coupling between the AMM of the quarks and an external magnetic field [25, 26, 30]. For instance, [25] analyze the phase diagram of the NJL at finite temperature, with special emphasis on a possible chiral restoration due to the non-zero AMM. Furthermore, the possibility of a non linear coupling of the AMM of the quarks is considered. In this model the AMM is related to the quantum correction to the electrodynamic vertex, and a non-perturbative dependence on the magnetic field is introduced through the effective constituent quark mass. The influence of the AMM on the structure of the lightest scalar mesons is analyzed in [26], while [30] is devoted to study their effects on neutral and beta stable quark matter within a bag model.

The aim of the present work is to study the effects of a uniform magnetic field on the properties of matter of quarks that have acquired AMM. In particular we focus on the vacuum effects and we perform an analytical regularization of the NJL that matches the standard three-momentum cutoff scheme for vanishing magnetic intensity. For this purpose, a fermion propagator is used which includes the anomalous magnetic moments and the full interaction with the external magnetic field [32, 33]. This propagator has been used to evaluate meson properties [31, 33], and the effect of the AMM within the NJL model [26]. Previous investigations have considered quarks with AMM within this framework [25, 26], but the divergent one-dimensional integrals were treated with a momentum cutoff which depends on the magnetic intensity. In the case of [25] the cutoff parameter $\Lambda$ is inspired by a covariant 4-momentum scheme.
$E_{n,s}(p, B) < \Lambda$, where $E_{n,s}(p, B)$ represents the energy of the $n$-th Landau level with spin projection $s$ along the direction of the uniform magnetic field. Ref. [26], instead, uses a 3-momentum cutoff $p < \sqrt{\Lambda^2 + M^2 - E_{n,s}(0, B)^2}$.

In the present work a 3-flavor version of the NJL is used. Most of the references just cited, in particular those corresponding to the study of AMM, use the two-flavor formulation. Thus we provide here an insight on the dynamics of the strange degree of freedom. This is particularly useful for applications to astrophysical studies, as for instance the final stage of neutron stars, where quark matter is electrically neutral and it is in equilibrium against weak decay.

This work is organized as follows. In the next section a summary of the NJL model is presented and a set of prescriptions to deal with the divergent contributions of the Dirac sea of quarks with AMM is proposed. Some numerical results are discussed in Sec. III, and the last section is devoted to drawing the conclusions.

2 Effects of the AMM on the vacuum properties in the NJL model

The SU(3) NJL model extended with an AMM term has the Lagrangian density

$$\mathcal{L}_{NJL} = \bar{\psi}(i\slashed{D} - M_0 - \kappa \sigma_{\mu \nu} F^{\mu \nu}) \psi + G \left[ (\bar{\psi} \lambda_a \psi)^2 + (\bar{\psi} \gamma^5 \lambda_a \psi)^2 \right] - K \left[ \det \bar{\psi}(1 + \gamma^5) \psi + \det \bar{\psi}(1 - \gamma^5) \psi \right],$$

(1)

where a summation over color and flavor is implicit and the current mass matrix $M_0 = \text{diag}(M_{0u}, M_{0d}, M_{0s})$ breaks explicitly the chiral symmetry and the covariant derivative $D^{\mu} = \partial^{\mu} - i Q e A^{\mu} / 3$ takes account of the uniform magnetic field, with $Q = \text{diag}(2, -1, -1)$. The AMM are displayed in the matrix $\kappa = \text{diag}(\kappa_u, \kappa_d, \kappa_s)$.

Due to the presence of a vacuum condensate the quark field acquires an enlarged constituent mass, a process that in the usual Hartree approach is described by

$$M_i = M_{0i} - 4G \langle \bar{\psi}_i \psi_i \rangle + 2K \langle \bar{\psi}_j \psi_j \rangle \langle \bar{\psi}_k \psi_k \rangle,$$

for $i \neq j \neq k$.

By using standard techniques one finds the grand partition function per unit volume $\Omega$,

$$\Omega = N_c \sum_f \left[ (\langle \bar{\psi}_f \gamma^0 \partial_0 \psi_f \rangle + 2G \langle \bar{\psi}_f \psi_f \rangle^2 - \mu_f n_f \right] - 4K \langle \bar{\psi}_u \psi_u \rangle \langle \bar{\psi}_d \psi_d \rangle \langle \bar{\psi}_s \psi_s \rangle.$$

(2)

The first term between square brackets stands for the kinetic contribution, and the last one corresponds to the conserved charges, i.e. electric charge and baryonic number.

Both quantities $\langle \bar{\psi}_k \psi_k \rangle$ and $\langle \bar{\psi}_k \gamma^0 \partial_0 \psi_k \rangle$ are ultraviolet divergent and need to
be interpreted adequately. There are several standard recipes within the NJL model, such as the non-covariant 3-momentum cutoff and the Lorentz invariant procedures of Pauli-Villars and 4-momentum cutoff. In the present work a regularization procedure is used which reduces to the 3-momentum cutoff at vanishing magnetic field. For this purpose a fermionic propagator is used corresponding to an effective quark with constituent mass and interacting with an uniform magnetic field through the electric charge and the AMM. It is given by

\[ G_f(x', x) = e^{i\Phi} \int \frac{d^4p}{(2\pi)^4} e^{-ip^\nu(x'_\nu - x_\nu)} \left[ G_{f0}(p) + e^{-p^2_1/\beta^2} \sum_n (-1)^n G_{fns}(p) \right] \]

(3)

where

\[ G_{f0}(p) = e^{-p^2_1/\beta^2} (\not\gamma + M_f - K_f) (1 + i\gamma^1\gamma^2) \Xi, \]

(4)

\[ G_{fns}(p) = \frac{\Delta_n + sM_f}{2\Delta_n} \left\{ (\not\gamma - K_f + s\Delta_n) (1 + i\gamma^1\gamma^2) L_n(2p^2_\perp/\beta_f) - (\not\gamma + K_f - s\Delta_n) \right. \]
\[ \times \left. (1 - i\gamma^1\gamma^2) \frac{s\Delta_n - M_f}{s\Delta_n + M_f} L_{n-1}(2p^2_\perp/\beta_f) + (\not\gamma i\gamma^1\gamma^2 + s\Delta_n - K_f) \not\gamma \frac{s\Delta_n - M_f}{p^2_\perp} \right. \]
\[ \times \left. [L_n(2p^2_\perp/\beta_f) - L_{n-1}(2p^2_\perp/\beta_f)] \right\} \Xi_{ns}, \]

(5)

\[ \Xi_{ns} = \frac{1}{p^2_0 - E_{fns}^2 + i\epsilon} + 2\pi i n_F(p_0) \delta(p^2_0 - E_{fns}^2). \]

(6)

In these expressions the index \( s = \pm 1 \) corresponds to the projection of the spin on the direction of the uniform magnetic field. Eq. \( \Phi \) propagates the lowest Landau level with the unique spin projection \( s = 1 \). The sum over the index \( n \geq 1 \) takes account of the higher Landau levels, and the following notation is used \( \beta_f = eQ_fB/3, K_f = x_fB, \not\gamma = p_0\gamma^0 - p_\perp\gamma^3, \not\gamma^2 = -p_\perp\gamma^1 - p_y\gamma^2, p^2_\perp = p^2_\perp + p_y^2, L_n \) stands for the Laguerre polynomial of order \( m \), and

\[ E_{fns} = \sqrt{p^2_0 + (\Delta_n - sK_f)^2} \]
\[ \Delta_n = \sqrt{M_f^2 + 2n\beta_f} \]

Finally, the phase factor \( \Phi = \beta_f(x + x') (y' - y)/2 \) embodies the gauge fixing.

Using the propagator of Eq. \( \Xi \) the quark condensates and the kinetic contributions are evaluated as

\[ \phi_f = \langle \bar{\psi}_f \psi_f \rangle = -i \lim_{t' \to t^+} \text{Tr}\{G_f(t, \vec{r}, t', \vec{r})\}, \]

(7)

\[ \langle \bar{\psi}_f i\gamma^0 \partial_0 \psi_f \rangle = -i \lim_{t' \to t^+} \text{Tr}\left\{ i\gamma^0 \frac{\partial}{\partial t} G_b(t, \vec{r}, t', \vec{r}) \right\}, \]

(8)

together with

\[ \langle \bar{\psi}_f \gamma^\nu \psi_f \rangle = -i \lim_{t' \to t^+} \text{Tr}\{\gamma^\nu G_f(t, \vec{r}, t', \vec{r})\}. \]

(9)
However, to ensure thermodynamical consistency, the following condition is imposed \[37\]

\[
\langle \bar{\psi}_f \psi_f \rangle = \left( \frac{\partial \Omega}{\partial M_f} \right)_{\phi,\mu,B} \tag{10}
\]

As already mentioned, these quantities have divergent vacuum contributions. In the Appendix a regularization procedure is applied that ensures null vacuum contributions at zero magnetic intensity. This requisite is used to match the 3-momentum cutoff procedure, by simply adding the standard expressions in terms of the cutoff parameter $\Lambda$. Thus at the regularization point these quantities reduce to the commonly used vacuum values. But for any other conditions, finite contributions depending on the density and the magnetic intensity are extracted from the vacuum.

Eq. (11) represents the density of baryonic current, which for infinite homogeneous matter has zero vacuum value.

In the Appendix the derivation of the regularized Dirac sea terms of Eq. (8) is shown. That expression reduces to

\[
\langle \bar{\psi}_f i\gamma^0 \partial_0 \psi_f \rangle^D - \langle \bar{\psi}_f i\gamma^0 \partial_0 \psi_f \rangle^N_{JL} \to \frac{-N_c \beta_f^2}{4\pi^2} \left[ \zeta'(-1, x) - \frac{1}{2} \left( x^2 - x + \frac{1}{6} \right) \ln(x) + \frac{x^2}{4} \right] + \frac{N_c \beta_f^2 N_f}{4\pi^2} \left( x^2 + x + \frac{1}{6} \right) \ln \left( \frac{\nu}{M_f^2} \right) - \frac{N_c \beta_f^2}{4\pi^2} \left( 3 \tilde{x}^2 - \tilde{x} + \frac{1}{6} \right) \ln \left( \frac{\tilde{\nu}}{\tilde{M}_f^2} \right) \tag{11}
\]

for $\kappa_f \to 0$. The notation $x = M_f^2/(2\beta_f)$ and $\tilde{x} = \tilde{M}_f^2/(2\beta_f)$ has been used, where $\tilde{M}_f$ stands for the constituent mass at finite baryonic density and $B = 0$. In this form, it can be compared with previous results, as for instance [9]

\[
-\frac{N_c \beta_f^2}{2\pi^2} \left[ \zeta'(-1, x) - \frac{1}{2} \left( x^2 - x \right) \ln(x) + \frac{x^2}{4} \right]
\]

The first term of Eq. (11) resembles the last equation. However, they differ in two points. First, the polynomial multiplying the logarithm has an extra term, which comes from the definition of $\zeta(-1, x)$. Furthermore in the last term between square brackets the quantities $\tilde{M}$ and $M$ are taken as identical. The remaining terms of Eq. (11) are missing in the mentioned approach. The difference can be minimized by choosing $\nu = M_f^2$. In such case, the third term of Eq. (11) becomes null, and the last one would also be zero if one identify $\tilde{M} = M$. For this reason we adopt in the following $\nu = M_f^2$, but the distinction between $M_f$ and $\tilde{M}_f$ will be kept.

Furthermore, Eqs. (12) and (13) receives finite contributions from the Fermi sea

\[
\langle \bar{\psi}_f \psi_f \rangle^F = \frac{N_c}{2\pi^2} \beta_f M_f \sum_{\Delta_n} \Delta_n - s K_f \ln \left( \frac{\mu_f + p_{fns}}{\Delta_n - s K_f} \right), \tag{12}
\]

\[
\langle \bar{\psi}_f i\gamma^0 \partial_0 \psi_f \rangle^F = \frac{N_c}{2\pi^2} \sum_{\Delta_n, s} \left[ \mu_f p_{fns} + (\Delta_n - s K_f)^2 \ln \left( \frac{\mu_f + p_{fns}}{\Delta_n - s K_f} \right) \right] \tag{13}
\]
\[ n'_f = \langle \bar{\psi}_f \gamma^\nu \psi_f \rangle^F = \delta_{\nu 0} \frac{N_c}{6\pi^2} \beta_f' \sum'_{n,s} p_{fns}, \quad (14) \]

where the primed sum indicates that for \( n = 0 \) only one spin projection must be considered as explained previously. The highest occupied Landau level \( N \) is defined by the condition \( \mu_f^2 - (\Delta_N - sK_f)^2 \geq 0 \). The Lagrange multipliers \( \mu_f \) are determined by the conserved charges, and \( p_{fns} = \sqrt{\mu_f^2 - (\Delta_N - sK_f)^2} \).

The magnetization per unit volume is given by the equation \( \mathcal{M} = -\partial \Omega / \partial B \), which can be simplified by using the stationary point conditions \([35] [22]\),

\[ \mathcal{M} = \frac{N_c}{8\pi^2 B} \sum_f \left( D_f + 2\beta_f \sum'_{n,s} F_{fns} \right), \]

where

\[
D_f = 8\beta_f^2 \zeta'(-1, \omega_f) - 2\beta_f [M_f^2 - K_f^2] \ln \left( \frac{\Gamma(\omega_f)}{\sqrt{2\pi}} \right) - \left[ 2K_f^2 (K_f^2 + M_f^2 - \beta_f) - \beta_f \left( M_f^2 + K_f^2 - \frac{\beta_f}{3} \right) \right] \\
+ \ln \left( \frac{M_f}{2\beta_f} \right) + 2\beta_f (M_f + K_f) (M_f + 3K_f) \ln \left( 1 + \frac{K_f}{M_f} \right) - 4K_f^2 (K_f^2 + \tilde{M}_f^2) \\
+ \beta_f \left( 2\tilde{M}_f K_f + \frac{\beta_f}{3} \right) \ln \left( \frac{\tilde{M}_f}{2\beta_f} \right) - \frac{1}{2} (M_f^2 - K_f^2)^2 - \frac{\beta_f^2}{3} - 2\beta_f K_f (2\tilde{M}_f - M_f), \quad (15) \\
\]

\[ F_{fns} = \mu_f p_{fns} - (s\Delta_n - K_f) \left( 2s\Delta_n - 3K_f - s\frac{M_f}{\Delta_n} \right) \ln \left( \frac{\mu_f + p_{fns}}{s\Delta_n - K_f} \right). \]

The pressure and energy density are given by the canonical results, \( P = -\Omega, \quad E = \sum_f \mu_f n_f - P \) and the transversal component of the stress tensor is defined as \( P_L = P - MB \).

Following a common practice, the quantum corrections to the leptonic properties are neglected, as well as the effects of their AMM, so that they contribute with

\[ n_l = \frac{\beta_l}{2\pi^2} \sum'_{n,s} p_{lns}, \quad (16) \]

\[ P_l = \frac{\beta_l}{4\pi^2} \sum'_{n,s} \left[ \mu_l p_{lns} - (m_l^2 + 2n\beta_l) \ln \left( \frac{\mu_l + p_{lns}}{\sqrt{m_l^2 + 2n\beta_l}} \right) \right], \quad (17) \]

\[ \mathcal{M}_l = \frac{\beta_l}{4\pi^2} \sum'_{n,s} \left[ (m_l^2 + 4\beta_l n) \ln \left( \frac{\mu_l + p_{lns}}{\sqrt{m_l^2 + 2n\beta_l}} \right) - \mu_l p_{lns} \right], \quad (18) \]

to the particle number density, pressure and magnetization, respectively. The definition \( p_{lns} = \sqrt{\mu_l^2 - m_l^2 - 2n\beta_l} \) is used.
3 Results and discussion

In this section the effects of the AMM of the quarks are studied for the case of electrically neutral matter and in equilibrium against weak decay, so it is necessary to include leptons in this description. The leptons get a chemical potential $\mu_l$ associated with the local conservation of the electric charge.

As it is usual, the conditions for the conservation of the baryonic charge $n_B = (n_u + n_d + n_s)/3$ and electric neutrality $2n_u - n_d - n_s - 3n_l = 0$ are imposed. The baryonic density of quarks is given by Eq. (14).

In the present calculations the following NJL parameters are used: $M_{u0} = M_{d0} = 5.5$ MeV, $M_{s0} = 135.7$ MeV, $\Lambda = 631.4$ MeV, $G = 1.835/\Lambda^2$, $K = 9.29/\Lambda^5$ [34]. For the total magnetic moments the prescription $\mu_u = (4 \mu_p + \mu_n)/5$, $\mu_d = (\mu_p + \mu_n)/5$, $\mu_s = \mu_\Lambda$ of the constituent quark model is adopted. Taking the experimental values of the baryonic magnetic moments together with constituent masses estimated within the same framework $M_u = M_d = 363$ MeV and $M_s = 538$ MeV the following AMM are obtained $\kappa_u = 0.074$, $\kappa_d = 0.127$, $\kappa_s = 0.053$ in units of the nuclear magneton, this set will be denoted in the following as AMM1. The values so obtained are small in comparison with other predictions [28, 29], therefore the alternative set $\kappa_u = \kappa_d = 0.38$, $\kappa_s = 0.25$ is also considered. It is compatible with the results of [29], and will be recognized as set AMM2.

The range of magnetic intensities studied $10^{15} \leq B \leq 10^{19}$ G greatly exceeds the phenomenology of strongly magnetized compact stars.

As a first step, different prescriptions for the regularization of the NJL immersed in a uniform magnetic field are considered. A comparison between the present approach and the commonly used procedure as described for instance in [5], is made here. In the following the last approach is referred as case C, while the label AMM0 is used for the results of this work when the AMM are zero. In Fig. 1 the constituent quark masses as a function of the magnetic intensity at zero baryonic density are shown for a wide range of magnetic intensities. In order to appreciate the low intensity behavior a small figure is inserted in the upper panel, restricted to $B < 10^{19}$ G. It can be appreciated that the cases AMM1 and AMM0 are similar for the full range of intensities. The higher AMM of the set AMM2 yield enhanced masses for all the flavors. The increasing magnitude of the AMM seems to cause a higher slope $\partial M/\partial B$. The results corresponding to the approach C differ considerably from the other ones, even for relatively low intensities, although qualitative agreement is obtained for $B < 2 \times 10^{18}$ G. In coincidence all the approaches predict increasing masses but the rate of growth is higher for the case C.

Calculations of the light quark masses at finite temperature including AMM have been presented in [26]. A comparison with these results is risky because they have been obtained in different conditions, i.e. equal particle number of $u$ and $d$ flavors and finite temperature. However, the curves for temperatures below $T = 100$ MeV seems to behave similarly. In Fig. 4 of this reference the quark mass as a function of the magnetic intensity shows quick oscillations around a decreasing mean value when AMM are included and slightly decreasing.
for zero AMM.

To take a view of the influence of the regularization scheme on the density dependence, a model of the variation of the magnetic field in the interior of a magnetar \[\text{[39]}\] is considered. It is given in terms of the ratio \(R = n/n_0\) by the formula

\[
B(n) = B_s + B_0 [1 - \exp(-\beta R^\gamma)],
\]

(19)

where \(n_0 = 0.15 \text{ fm}^{-3}\) is the reference density, \(B_s = 10^{15} \text{ G}\) is the intensity on the star surface, and the remaining parameters have been chosen as \(B_0 = 5 \times 10^{18} \text{ G}\), \(\beta = 0.01\), \(\gamma = 3\). The maximum strength \(5 \times 10^{18} \text{ G}\) corresponds to asymptotic high densities and could not be reached in a realistic description. Due to the facts just discussed one can expect that the effective quark masses do not show significative differences. For this reason the thermodynamical pressure is chosen as an interesting quantity to be examined. It is shown in Fig. 2 as a function of the baryonic density at zero temperature for a range which covers from the surface to medium depths of a typical neutron star. All the results coincide in the zero density limit and for \(R > 2.3\), where an almost linear increase of the pressure is achieved. In between, the case C predicts a thermodynamical instability while the remaining ones are almost coincident and they show a change of concavity near \(R = 1.2\). Thus, in these descriptions there is a region where the isothermal incompressibility becomes zero. Minor discrepancies, not shown in the figure, arise for \(R > 5\).

In conclusion, one can say that there is a qualitative agreement between these procedures in the low magnetic intensity regime, but the differences become important for \(B > 10^{19} \text{ G}\). The agreement also exists for the range of densities of interest in the description of a magnetar, although some features as that discussed for the pressure, can be found.

Next a discussion of the effects of the AMM on the dynamics of quarks is analyzed. In Fig. 1 it can be appreciated that the AMM are the cause of the enhancement of the increasing trend of the quark masses with \(B\). For instance, a comparison with the outcome AMM0 shows a difference of about 50 keV for the light quarks and about 20 keV for the strange one at \(B = 5 \times 10^{18} \text{ G}\) and using the set AMM1. The discrepancy grows to 200 keV for the \(u\) and \(d\) flavors and to 70 keV for the strange one, when using the AMM2 parametrization.

In Fig. 3 the density dependence is shown at fixed intensity \(B = 10^{19} \text{ G}\). The figure extends to \(R = 7\), a density which is feasible in the core of magnetars. A monotonously decreasing behavior is obtained for all the flavors. In the case of \(M_u\) there is an almost linear trend at low density till the point \(R \approx 1.3\) where the first excited Landau level starts to be occupied. Here a noticeable change of slope takes place. The curve for \(M_s\) shows a shoulder shape, after a plateau for \(3 < R < 4.5\) a change of slope together with an inflexion point occurs around \(R = 4.2\). At this density the strange quark comes out to the Fermi sea. The effect of the AMM is almost indistinguishable at the scale shown, but the numerical increments are of 2-5 MeV for the light flavors and around 0.1 MeV for the \(s\) quark.

The influence of the AMM on the masses of the quarks decreases quickly with
the magnetic intensity, so that for $B < 5 \times 10^{18}$ G all the corrections diminish about 30%.

The abundance of particles relative to the total number of quarks is displayed in Fig. 4 as a function of the baryonic density for the fixed intensity $B = 5 \times 10^{18}$ G. There are no appreciable discrepancies between the different treatments. Interestingly, all of them predict a depletion of the electron population for $R \approx 0.5$. Since at this point the constituent masses of the $u$ and $d$ flavors are of the order 300 MeV, this fact could have important consequences in the thermal and electric conductivity of the quark star matter. Furthermore, the inclusion of the AMM produces a slight shift to lower density of the rise of the strange quark population.

The next figure, Fig. 5, is devoted to the energy per particle as a function of the density for a constant magnetic field $B = 5 \times 10^{18}$ G. A comparison shows that due to the AMM the quark interaction in a dense medium becomes slightly less repulsive. The greatest energy difference is about 3 MeV and it takes place in the low density region. For $R > 3$ an asymptotic almost linear regime is reached. In such a situation the difference reduces to a few tenths of MeV. The model for the variation of the magnetic field in the interior of a magnetar, given by Eq. (19), shows the same trend. It has the greatest energy for low densities while asymptotically, as $B$ grows, coincides with the other cases.

The magnetization is a measure of the response of the system to the magnetic excitation, it is shown in Fig. 6 as a function of the density. Since this is a very small quantity, the results are scaled with the proton charge $e = 1.6 \times 10^{-19}$ C, which is appropriate for the range of intensities examined here. The curves correspond to constant intensity for $B = 10^{17}$ G and $B = 5 \times 10^{18}$ G. The bottom of this figure is occupied by the three curves corresponding to the different parametrizations and the lowest intensity. Because of their similar behavior, the high frequency and the small amplitude of their oscillations, they form a band. In the low density regime the mean value remains almost constant up to $R \approx 1.5$ where it increases quickly, the amplitude of the oscillations becomes greater also. In contrast, for $B = 5 \times 10^{18}$ G the quasi-periodicity disappears and the magnitude of the oscillation is multiplied by a factor 10. This is a manifestation of a coherent dynamics, more favorable at higher intensity due to the smaller number of accessible Landau levels. In this figure the result of the density dependent magnetic field is also included. It has an intermediate behavior between the two cases just discussed.

In Fig. 1 the effects of the magnetic intensity on the quark masses in vacuum has been shown. In order to study how the density influence the magnetic dependence, a detail of the results obtained for these masses for non-zero baryonic number is presented in Fig 7. The values chosen for the density, $R = 4$ and $R = 7$, correspond to situations where quark matter is stable and the $s$ quark is only virtual or is able to exist in the Fermi shell of the star matter, respectively. The inclusion of the AMM for the light quarks yields increasing masses as $B$ grows, with a slope more pronounced for the greater AMM of the set AMM2. In contrast, in the AMM0 approach a slightly decreasing or almost constant trend is exhibited for the two baryonic densities examined here. The
mass of the strange quark does not show considerable variations. At medium densities $R = 4$ it varies monotonously, while for the higher density $R = 7$ it exhibits fluctuations whose amplitude increases with $B$. The AMM has a minor effect on this flavor.

Finally, in Fig. 8 the magnetization as a function of the magnetic intensity is shown for the fixed baryonic densities $R = 4$ and $R = 7$. The typical oscillatory behavior is obtained, whose amplitude increases with $B$. The peak values are slightly increased when the AMM are considered. It is interesting to note that the quark matter is always in a paramagnetic regime.

4 Summary and Conclusions

In this work a procedure to remove divergences in the $1/N_c$ approach to the SU(3) NJL model under a uniform magnetic field $B$ has been proposed. The calculations have been made by using a covariant propagator for the quarks with constituent mass, which takes account of the full effect of the magnetic field as well as the effect of the anomalous magnetic moment. There are divergencies which depend on the magnetic intensity. Since the interaction used is an effective model of the strong interaction, a full renormalization is meaningless. Therefore the divergent terms are not ascribed to the renormalization of the external magnetic field since, within the model used, it is not a dynamical variable. To obtain physically meaningful results from the divergent contributions an analytical regularization has been proposed which recovers the standard three momentum cutoff scheme at $B = 0$ and arbitrary baryonic density. For this purpose a subtraction of fourth order in the vertices $q_f B$ and $\kappa_f B$ is performed. Since the regularization point is chosen at $B = 0$ and fixed baryonic density, the regularized quantities depend on the quark masses $\tilde{M}_q$ evaluated in such conditions. The present approach complements previous work, as for instance [5], since it includes $B$ dependent terms not considered before as well as the additional coupling of the AMM of the quarks. The regularization scale parameter, typical of the dimensional regularization, has been chosen so as to maximize the agreement with previous studies.

The regularized model has been used to study quark matter in equilibrium against weak decay and electrically neutral, as can be found in the composition of magnetars. A range of magnetic intensities $10^{15} G \leq B \leq 10^{19} G$ and baryonic densities $n \leq 1 \text{ fm}^{-3}$ have been analyzed, which are adequate to describe such situation.

The results at zero baryonic density have been compared with those obtained with the commonly used prescription of [5]. A model for the magnetic intensity in the interior of a magnetar [39] has been considered to test the results of the different approaches. For this model the intensity $B$ is parameterized in terms of the baryonic density and ranges between $10^{15} G \leq B \leq 5 \times 10^{18} G$. Only minor numerical differences have been obtained for the two schemes of regularization, although they can have physical consequences. As an example the thermodynamical instability for $n < 0.3 \text{ fm}^{-3}$, which does not happen in the
approach of the present work, can be mentioned. In general terms a qualitative agreement is obtained for low intensities, but discrepancies become significative for strong magnetic fields \( B > 5 \times 10^{18} \) G. Hence one can conclude that the study of magnetars will probably not evidence completely these differences as in physical situations where the magnetic field manifests with stronger intensity.

A contrast of the results with or without AMM shows that the variation of the light quark masses with the magnetic intensity in very dense quark matter is monotonously increasing when the AMM are considered, otherwise it is slightly decreasing or almost constant. The magnetization of quark matter does not show clear evidence of the presence of AMM.

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A Regularization of the vacuum contribution to the thermodynamical potential

In this section the Dirac sea contribution to Eq. (8) is regularized, i.e. the contribution coming from the first term of Eq. (6).

In momentum coordinates Eq. (8) can be rewritten as

\[
\langle \bar{\psi} \gamma^0 \partial_0 \psi \rangle = -i \lim_{\epsilon \to 0^+} \int \frac{d^4 p}{(2\pi)^4} e^{-ip_0} p_0 \text{Tr} \{ \gamma^0 G_f(p) \} \tag{20}
\]

Keeping only those terms corresponding to the Dirac sea it reduces to

\[
-\frac{2i}{(2\pi)^3} \sum_{n,s}^' \frac{(-1)^n}{\Delta_n} \int d^4 p \frac{p_0^2 e^{-p_0^2/\beta_f}}{u_n^2 - (\Delta_n - s K_f)^2 + i\epsilon} \left[ (\Delta_n + s M_f) L_n - (\Delta_n - s M_f) L_{n-1} \right]
\]

where the argument of the Laguerre functions \( L_k \) is \( 2p_0^2 / \beta_f \) and the primed sum has the same meaning as in the main text.

As usual in analytic regularization, an undetermined scale factor \( \nu \) can be introduced. After representing the denominator in exponential form and passing to the euclidean space in \( p_0 p_z \), coordinates the momentum integrals can be performed leading to

\[
= \frac{\nu \beta_f}{8\pi^2} \int_{0}^{\infty} d\tau \tau^3 \left[ \sum_{n,s}^' e^{-\tau(\Delta_n - s K_f)^2/\nu} - e^{-\tau(M+K_f)^2/\nu} \right]
\]

\[
= \lim_{\epsilon \to 0} \frac{\nu \beta_f}{8\pi^2} \int_{0}^{\infty} d\tau \tau^{\epsilon-2} \left[ \sum_{n,s}^' e^{-\tau(\Delta_n - s K_f)^2/\nu} - e^{-\tau(M+K_f)^2/\nu} \right] \tag{21}
\]
In order to isolate the pole at $\tau = 0$ a vanishing parameter $\epsilon$ has been introduced and through a change of variable, one arrives to

$$
\frac{\nu \beta_f}{8\pi^2} \lim_{\epsilon \to 0} \left\{ \sum_{s,n=0} \left( \frac{(\Delta_n - s K_f)^2}{\nu} \right)^{1-\epsilon} - \left( \frac{(m + K_f)^2}{\nu} \right)^{1-\epsilon} \right\} \int_0^\infty dt \, t^{-2} e^{-t} \tag{22}
$$

The integral can be identified as $\Gamma(\epsilon - 1)$. To put the double summation in a simpler form, and bearing in mind that for $\epsilon = 0$ it reduces to

$$
\sum_{s,n=0} \left( \frac{(\Delta_n - s K_f)^2}{\nu} \right)^{1-\epsilon} \approx \sum_s \left( \frac{2 \beta_f}{\nu} \right)^{1-\epsilon} \zeta(-1, \omega_f) \tag{23}
$$

By making a Lorenz expansion in $\epsilon$ one obtains

$$
-\frac{1}{8\pi^2} \lim_{\epsilon \to 0} \left\{ \left( \frac{1}{\epsilon} + 1 - \gamma \right) \left[ 4 \beta_f^2 \zeta(-1, \omega_f) - \beta_f (M_f + K_f)^2 \right] + 4 \beta_f^2 \frac{\partial}{\partial z} \zeta(z = -1, \omega_f) \right.
$$

$$
\left. -4 \beta_f^2 \zeta(-1, \omega_f) \ln \left( \frac{2 \beta_f}{\nu} \right) + \beta_f (M_f + K_f)^2 \ln \left( \frac{(M_f + K_f)^2}{\nu} \right) + O(\epsilon) \right\} \tag{24}
$$

Here a simple pole is evident, whose residue is a polynomial of fourth order in $B$. For $B \to 0$

$$
m^4 \frac{8\pi^2}{\epsilon + \frac{3}{2} - \gamma} \tag{25}
$$

the typical behavior for a Dirac particle is obtained. The last divergence is usually tackled within this model by the introduction of a constant 3-momentum cutoff $\Lambda$. In this way the following finite contribution is assigned to it

$$
\langle \bar{\psi}_f i \gamma \partial_0 \psi_f \rangle_{N^JL} = \frac{N_c}{8\pi^2} \left[ M_f^4 \ln \left( \frac{\Lambda + E_{\Lambda}}{M_f} \right) - \Lambda E_{\Lambda} \left( \Lambda^2 + E_{\Lambda}^2 \right) \right], \tag{26}
$$

where $E_{\Lambda} = \sqrt{\Lambda^2 + M_f^2}$.

In the following we apply to Eq. (24) a procedure that gets rid of the pole term and simultaneously ensures the convergence to Eq. (26) as $B \to 0$.

In Eq. (24) it can be observed that the magnetic dependence of the residue reduces to order two for $\kappa_f = 0$, hence the AMM is the cause of new divergencies depending on $B$.

It must be pointed out that this residue satisfies

$$
F(M, \beta_f, K) = 4 \beta_f^2 \zeta \left( -1, \frac{\lambda}{2\beta_f} \right) - \beta_f (M + K)^2
$$

$$
= \left( \sum_{k=0}^2 \frac{\beta^{k-j}}{(k-j)!} \frac{K^j}{\beta^{k-j}} \frac{\partial^{k-j} \partial^j}{\partial K^j} + \frac{K^4}{4!} \frac{\partial^4}{\partial K^4} \right) F(M, 0, K) \tag{27}
$$
Based on this feature the following subtraction procedure is proposed

\[
\langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle^D = \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle_{NJL} + \left[ 1 - \sum_{k=0}^{K_f} \frac{\beta_f^{k-j}}{(k-j)!} K_f^j \left( \frac{\partial^{k-j} \partial_0}{\partial \beta_f^{k-j} \partial K_f^j} \right) \right] \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle_{\bar{\psi} f \psi}\]

Where the subindex 0 indicates that the derivatives must be evaluated at \(\nu\). As explained in the main text, if \(\nu = 0, K_f = 0\). For the properties of the derivatives of the Hurwitz zeta function, see for instance [36].

Thus, the final expression is

\[
\langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle^D = \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle_{NJL} - \frac{N_c}{8\pi^2} \left\{ 4\beta_f^2 \frac{\partial}{\partial z} \zeta(z = -1, \omega_f) + 4\beta_f^2 \zeta(-1, \omega_f) \ln \left( \frac{\nu}{2\beta_f} \right) \right. \\
- \left. 4\beta_f^2 \zeta(-1, \omega_f) \ln \left( \frac{\nu}{M_f^2} \right) + \beta_f \left( M_f + K_f \right) \ln \left( \frac{\left( M_f + M_f \right)^2}{\nu} \right) + \frac{\tilde{M}_f^4}{4} - \frac{K_f^4}{2} \right. \\
- \left. \frac{\beta_f^2}{3} - 2\beta_f K_f + \beta_f \left( \tilde{M}_f + K_f \right) \ln \left( \frac{\nu}{M_f^2} \right) \right\} \\
\]

where \(\tilde{M}\) stands for the effective quark mass at the regularization point and \(\omega_f\) is obtained from \(\omega_f\) by replacing \(M_f\) by \(\tilde{M}_f\). In the present investigation this point is taken at a fixed baryonic number and zero magnetic intensity.

As explained in the main text, if \(\nu = M_f^2\) is adopted the last equation becomes

\[
\langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle^D = \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle_{NJL} - \frac{N_c}{8\pi^2} \left\{ 4\beta_f^2 \frac{\partial}{\partial z} \zeta(z = -1, \omega_f) + 4\beta_f^2 \zeta(-1, \omega_f) \ln \left( \frac{M_f^2}{2\beta_f} \right) \right. \\
- \left. 8\beta_f^2 \zeta(-1, \omega_f) \ln \left( \frac{M_f}{M_f} \right) + 2\beta_f \left( M_f + K_f \right) \ln \left( 1 + \frac{K_f^4}{2M_f^2} \right) + \frac{\tilde{M}_f^4}{4} - \frac{K_f^4}{2} - \frac{\beta_f^2}{3} \right. \\
- \left. 2\beta_f K_f + 2\beta_f \left( \tilde{M}_f + K_f \right) \ln \left( \frac{M_f}{M_f} \right) \right\} \\
\]

Using Eqs. (2), (13) and (30) one obtains the regularized thermodynamic potential by taking

\[
\langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle = \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle^D + \langle \bar{\psi}_f i \gamma^0 \partial_0 \psi_f \rangle^F.
\]

In turn, the quark condensate is evaluated using Eq. (11), obtaining

\[
\langle \bar{\psi}_f \psi_f \rangle = \langle \bar{\psi}_f \psi_f \rangle^F + \langle \bar{\psi}_f \psi_f \rangle^D,
\]

14
\[
\langle \bar{\psi} f \psi \rangle^D = \langle \bar{\psi} f \psi \rangle_{NJL} - \frac{N_c}{4\pi^2} \left\{ 2\beta_f M_f \ln \left( \frac{\Gamma(\omega_f)}{\sqrt{2\pi}} \right) - M_f \left( M_f^2 + K_f^2 - \beta_f \right) \ln \left( \frac{M_f^2}{2\beta_f} \right) \right.
\]
\[
+ 2\beta_f (M_f + K_f) \ln \left( 1 + \frac{K_f}{M_f} \right) + \beta_f M_f - \frac{1}{2} \left( 1 - \frac{M_f}{\tilde{M}_f} \right) \right. 
\]
\[
\left. \times \left[ (M_f + \tilde{M}_f) \left( M_f^2 + \tilde{M}_f^2 + 2K_f^2 \right) + 4\beta_f \right] \right\} 
\]
\[
\langle \bar{\psi} f \psi \rangle_{NJL} = \frac{N_c}{2\pi^2} M_f \left[ M_f^2 \ln \left( \frac{\Lambda + E_\Lambda}{M_f} \right) - \Lambda E_\Lambda \right] 
\]

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Figure 1: The thermodynamical pressure as a function of the baryonic density for a model of the variation of the magnetic intensity in the interior of a magnetar. Two different regularization schemes, and results with or without AMM are compared as explained in the main text.
Figure 2: The relative number of particles as a function of the baryonic density for a fixed magnetic intensity $B = 5 \times 10^{18}$ G. Results corresponding to the AMM0, AMM1 and AMM2 parametrizations are represented in the panel (a), (b) and (c) respectively.
Figure 3: The energy per particle as a function of the baryonic density for a fixed magnetic intensity $B = 5 \times 10^{18}$ G. Results corresponding to a variable magnetic intensity are also represented as case D.