On tail trend detection: modeling relative risk

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Abstract The climate change dispute is about changes over time of environmental characteristics (such as rainfall). Some people say that a possible change is not so much in the mean but rather in the extreme phenomena (that is, the average rainfall may not change much but heavy storms may become more or less frequent). The paper studies changes over time in the probability that some high threshold is exceeded. The model is such that the threshold does not need to be specified, the results hold for any high threshold. For simplicity a certain linear trend is studied depending on one real parameter. Estimation and testing procedures (is there a trend?) are developed. Simulation results are presented. The method is applied to trends in heavy rainfall at 18 gauging stations across Germany and The Netherlands. A tentative conclusion is that the trend seems to depend on whether or not a station is close to the sea.

Keywords Extreme value distribution · Regular variation · Extreme rainfall

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1 Introduction

In the climate change dispute some people suggest (Klein Tank and Können 2003; Groisman et al. 2005; Alexander et al. 2006; Zolina et al. 2009) that perhaps there is no or little change in the mean of the probability distribution of daily rainfall over time but there is a change in the tail that is, more extreme events occur more frequently. The present paper – like (Smith 1989; Hall and Tajvidi 2000; Hanel et al. 2009) – considers a trend in extremes from the point of view of extreme value theory.

If one wants to concentrate on a trend connected with extreme events rather than with the central part of the probability distribution function $F$, one should look at a high quantile $F^{-}(p)$ (i.e. the inverse of $F$) for $p$ close to one or at the exceedance probability $1 - F(x)$ at a high level $x$. Hence we consider the limit behavior of $F^{-}(p)$ as $p \uparrow 1$ or $F(x)$ as $x \uparrow x^*$, which is the right end point of the probability distribution ($x^* := \sup\{x : F(x) < 1\}$). Since the limit relation for $F$ is simpler than for $F^{-}$, we concentrate on the behavior of $F(x)$ as $x \uparrow x^*$.

Two models for trends connected with extremes spring forward: a trend in the extreme value index $\gamma$ or a situation with constant extreme value index but with heteroscedastic observations (in the tail). We consider the latter option here. In that case probabilities of extreme events are not assumed asymptotically equal (which would mean tail equivalence cf. Resnick, 1971) but asymptotically comparable as follows.

Consider random variables $X(s)$ where $s \geq 0$ is time. Write $F_s(x) := P\{X(s) \leq x\}$ for $x \in \mathbb{R}$. We assume that for all $s > 0$

$$
\frac{1 - F_s(x)}{1 - F_0(x)}
$$

tends to a positive constant $d(s)$ for all $s > 0$ when $x$ tends to the right endpoint $x^*$ of $F_0$ where $d$ is a positive continuous function. Hence the exceedance probability at time $s$ is systematically a factor times the exceedance probability at time zero. We consider a simple model for relative risk and assume that for some real trend constant $c$ and all $s \geq 0$

$$
\lim_{x \uparrow x^*} \frac{1 - F_s(x)}{1 - F_0(x)} = e^{cs}. \quad (1)
$$

This means that for example (with $s = 1$ and $c = 1$) that the probability of any extreme event taking place at time $s$ is $e$ times the corresponding probability at time zero. For small $s$ the limit function is approximately linear.

For our analysis we shall need the context of extreme value theory that is, we assume that the distribution function $F_0$ is in the domain of attraction of some extreme value distribution $G_\gamma$ i.e., there exist sequences of constants $a_n > 0$ and $b_n$ ($n = 1, 2, \ldots$) such that the normalized maximum of a sample from $F_0$ converges to $G_\gamma$ for some $\gamma \in \mathbb{R}$:

$$
\lim_{n \to \infty} F_0^n(a_nx + b_n) = G_\gamma(x) := \exp\left\{-(1 + \gamma x)^{-1/\gamma}\right\}. \quad (2)
$$

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