Probing sterile neutrino in meson decays with and without sequential neutrino decay

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We present the most systematic approach to discover a sterile neutrino \( N \) or constrain \( |U_{\nu N}|^2 \), the mixing between active neutrino \( \nu_\ell \) (with \( \ell = e, \mu, \tau \)) and the sterile neutrino \( N \), from \( B \to D\ell N \) decays. Our constraint on \( |U_{\nu N}|^2 \) achievable from Belle II data is comparable with that from the much larger data set of upgraded LHCb, even much better for mass of sterile neutrino \( m_N < 2 \text{ GeV} \). We can also probe the Dirac and Majorana nature of \( N \) by observing the sequential decay of \( N \), including suppression factors associated with observation of a displaced vertex and helicity flip, for Majorana \( N \).

Keywords: sterile neutrino; Majorana neutrino; leptonic decays; semileptonic decays; displaced vertex

Introduction.— In many new physics theories, called see-saw mechanisms, there are one or more heavier cousins of the active flavor neutrinos \( \nu_\ell \) (with \( \ell = e, \mu, \tau \)) which do not have any interaction with Standard Model (SM) particles except mixing with the active neutrinos. These heavy neutrinos are named as sterile neutrinos which can be either Dirac or Majorana fermions. Among the varieties of new physics scenarios where sterile neutrinos appear, the original seesaw mechanism [1] predicts their mass to be much larger than 1 TeV. In other seesaw models heavy neutrinos can have mass in the range \( \sim 0.1 \text{ TeV} \sim 1 \text{ TeV} \) [2], or even close to about 1 GeV [3]. The mixing parameters, \( U_{\nu N} \), which describe the strength of mixing between a sterile (heavy) neutrino \( N \) with the SM flavor (light) neutrinos \( \nu_\ell \) are constrained by various experimental data (see Refs. [4, 5] and references therein for more details on the experimental bounds).

Ascertaining the Dirac or Majorana nature of neutrinos is one of the most important issues in neutrino physics. It is well known that Dirac neutrinos can participate only in the lepton number conserving (LNC) processes, while Majorana neutrinos can get involved in both lepton number violating (LNV) and LNC processes. Therefore, to investigate the Majorana nature of neutrinos, many attempts have been made at studying various LNV processes, including neutrinoless double beta decay \((0\beta\beta)\) [6], specific LNV processes at LHC [7–10], LNV \( \tau \) lepton decays [11], and LNV rare meson decays [12–19]: for instance, rare LNV decays of \( K, D_{(s)}, B_{(s)} \) mesons have been studied extensively in literature. In particular, semileptonic decays such as \( B \to D\ell\pi \) and \( B \to \ell\ell\pi \) were explored in Refs. [18, 19] to not only distinguish between Dirac and Majorana signatures, but also constrain \( |U_{\nu N}|^2 \), without the step-by-step consideration of the feasibility of these decays inside detector including the helicity flip for Majorana case.

In this letter, the most significant result is the stringent constraint achievable on \( |U_{\nu N}|^2 \), especially on \( |U_{\mu N}|^2 \), which can be obtained from non-observation of the decays \( B \to D\ell N \) without considering sequential \( N \) decay. This simple strategy has, however, remained unexplored in the currently existing literature. Instead of considering two-body leptonic decays \( B^+ \to \ell^+ N \), such as in the studies utilizing \( \pi^+ \) (or \( K^+ \)) to constrain \( |U_{\nu N}|^2 \) [20], we consider the three-body semileptonic decays \( B \to D\ell N \) which have bigger branching ratios in a larger mass range. The reach of our study to constrain \( |U_{\nu N}|^2 \) and \( |U_{\mu N}|^2 \) is better by an order of magnitude from existing experimental constraints in certain mass ranges of interest. Interestingly, our constraint on \( |U_{\mu N}|^2 \) obtained by considering only \( \sim 4.8 \times 10^6 \) number of fully reconstructed \( B \to D\mu N \) decays at Belle II [21] is comparable with the constraint achievable from \( 4.8 \times 10^{12} \) number of \( B \to D\mu\mu\pi \) decays at upgraded LHCb [5]. Although the missing sterile neutrino search gives the stringent constraint on \( |U_{\nu N}|^2 \), it can not distinguish Dirac and Majorana neutrinos. Therefore, in this letter, we also conduct a systematic study including the sequential decay of \( N \) with a displaced vertex signature for probing its Majorana nature, and consider the very important but otherwise overlooked effect of helicity flip for sterile neutrinos. Despite the suppression coming from observation of displaced vertices as well as the helicity flip, we find that heavier and less energetic neutrinos have a bigger chance of decaying inside a detector with decay length \( \leq 1 \text{ m} \) provided they exist. Finally, we present an estimate of \( |U_{\nu N}|^2 \) in the case of LNC \( B \to D\mu^+\mu^−\pi^0 \) decay gets observed in Belle II and/or LHCb. The LNV mode \( B \to D\mu^+\mu^−\pi^− \) receives additional suppression from helicity flip.

Choosing appropriate production modes.— It is very well known that any candidate for sterile neutrino must always be (1) electrically neutral, (2) spin-1/2 fermion, (3) massive (non-zero mass, could be light or heavy), and (4) long-living. These four characteristics of a sterile neutrino help us to choose the appropriate three-body semileptonic production process. The sequential decay of the sterile neutrino is not required in this case.

(1) A sterile neutrino is always electrically neutral. This condition allows us to consider three-body semileptonic \( B \) and \( D \) decays and replace the final state active neutrino by sterile neutrino, e.g. \( B \to D\mu N, D \to K\mu N \).

(2) A sterile neutrino is always a spin-1/2 fermion. This condition rules out such decays as \( B \to D^*\mu N \) and \( D \to K^*\mu N \) from our consideration. Similarly decays like \( B \to \)}
$KN\bar{N}$ where $N$ could have spin 3/2 and $B \to Khh$ where $h$ could be a scalar, also get ruled out.

(3) A sterile neutrino is always massive. We are interested in the scenario where there is one light sterile neutrino with mass $m_S < 4$ GeV. The criteria of massive sterile neutrino helps us to eliminate background events to our processes. The decay $B \to D\mu N$, for example, can receive background from the decay $B \to D\mu\nu\nu\bar{\nu}$. However, the invariant mass of all the missing momenta in the background process varies from one event to another unlike the fixed mass of $N$. The mass of $N$ in the decay, e.g. $B \to D\mu N$, is to be determined by measuring the 4-momenta of initial $B$ meson, final $D$ meson and $\mu$, without considering the sequential decay of $N$. This method of determining the mass of $N$ is also applicable when $N$ decays instantly. This is possible in experiments such as Belle II or BES III where $B$ and $D$ mesons are pair produced along with $\bar{B}$ and $D$ from the decays $\Upsilon(4S) \to B\bar{B}$ and $\psi(3770) \to D\bar{D}$ respectively, and the 4-momentum of $\bar{B}, D$ can be precisely measured by full hadronic reconstruction.

(4) A light sterile neutrino is always long-living. Thus it may or may not decay inside a detector, depending on its mass, energy and the size of the detector. If it decays inside the detector, with observation of displaced vertex, we can probe the Dirac or Majorana nature of the neutrino when combined with its production mode. The displaced vertex signatures can also be used to veto any background events for the decays under consideration.

Note that the decays $B^+ \to \tau^+ N$ and $B \to D\tau N$, where the 4-momentum of the tau lepton is reconstructed from its further sequential decay, are less promising for our study. This is due to the presence of at least one neutrino (or antineutrino) in the final state of all tau decays. Nevertheless, taking into account that the tau 4-momentum could be measured accurately with a smaller probability, we shall constrain $|U_{\tau N}|^2$ from $B \to D\tau N$ which has a larger branching ratio than $B^+ \to \tau^+ N$.

Constraints on $|U_{\mu N}|^2$ and $|U_{\tau N}|^2$—The branching ratios of all the decay modes under our consideration are directly proportional to the active-sterile mixing parameter $|U_{\mu N}|^2$. We obtain the canonical branching ratio of a decay, e.g. $B \to D\mu N$, by factoring out $|U_{\mu N}|^2$ from the theoretically calculable branching ratio [18]. Given the value of canonical branching ratio $Br(B \to D\mu N)$, the number of such decays observable in the detector ($N_{B\to D\mu N}$) and the total number of fully reconstructed parent particles ($N_{B}$), we can estimate $|U_{\mu N}|^2$ by

$$|U_{\mu N}|^2 = \frac{N_{B\to D\mu N}}{N_{B} \times Br(B \to D\mu N)}.$$ 

For a numerical study we consider the decays $B \to D\mu N$ and $B \to D\tau N$ in context of Belle II experiment, which is poised to detect $10^{11}$ number of $B$ decays [21]. Out of these about 0.61% of charged $B$ events and 0.34% of neutral $B$ events can be fully reconstructed from hadronic tagging [21], so that only about $4.8 \times 10^8$ number of $B$ mesons get fully reconstructed. Considering only these $B$ decays, we are able to estimate the value of $|U_{\mu N}|^2$, as shown in Fig. 1a, from possible observation of less than 50 events or so for $B \to D\mu N$. It is easy to observe that in the mass range $\sim 2 - 3$ GeV our approach can provide stronger constraint, by about one order of magnitude, than the existing experimental upper-limit (exclusion region at $\sim 90\%$ C.L. from various experiments is shown by the shaded region in gray). From Fig. 1a it is also clear that our constraint is comparable with the $95\%$ C.L. upper-limit on $|U_{\mu N}|^2$, shown as a thick dashed line, obtained in Ref. [5] using $4.8 \times 10^{12}$ number of $B$ decays at upgraded LHCb (with the decay $B \to D\mu\pi$). Another important aspect of our analysis is that for $m_N < 2$ GeV (very important for light sterile neutrino searches) our constraint significantly surpasses even the above LHCb upgrade constraint. This is primarily due to the suppression factors affecting the observation of $B \to D\mu\pi$ decays inside a finite-sized detector for smaller values of $m_N$ (see the next section and Fig. 3a). The minimum value of mass $m_N$ that can be probed in our approach is constrained only by the experimental accuracy of measurement of 4-momenta of $B$, $D$ and $\mu$. It is interesting to note that despite larger num-

![FIG. 1.](image-url) Values of $|U_{\mu N}|^2$ and $|U_{\tau N}|^2$ obtained from observed number of events (less than 50 events) of the decays $B \to D\mu N$ and $B \to D\tau N$, respectively, from a sample of $4.8 \times 10^8$ number of $B$ decays with initial 4-momenta fully measured, and assuming $0.1\%$ chance of full reconstruction of $\tau$ from its decays. The shaded regions in gray are currently excluded at $90\%$ C.L. from various experiments (for details and associated references see Ref. [4, 5]). The thick dashed curve in (a) shows the possible $95\%$ C.L. upper limit on $|U_{\mu N}|^2$ from the decays $B \to D\mu\pi$ from LHCb upgrade as proposed in Ref. [5]. The $B \to D\tau N$ decays include both the charged and neutral modes.
ber of $B$ decays at upgraded LHCb, our methodology can not be applied to LHCb due to lack of full reconstruction of the parent $B$ meson without considering the sequential decay of $N$.

Similarly, we can constrain $|U_{eN}|^2$ from number of observed $B \to D\tau N$ decays if the 4-momentum of the final $\tau$ could be accurately measured. In Fig. 1b we show estimated values of $|U_{eN}|^2$ from number of observed $B \to D\tau N$ decays $\lesssim 50$. Here we have assumed that the 4-momenta of only 0.1% of all the tau could be precisely measured (e.g. more than 3-prong decays of $\tau$ [22]). It is clear from Fig. 1b that our constraint on $|U_{eN}|^2$ in the mass range [0.3, 1] GeV is more stringent than the existing studies, sometimes by an order of magnitude. It should be noted that if we consider bigger tau reconstruction efficiency we can further improve our current result. Thus we have systematically shown we can stringently probe the active-sterile mixing parameters by not considering any sequential decay of sterile neutrino provided the 4-momenta of all the other particles are well measured. However, the Dirac and Majorana nature of the sterile neutrino can be probed only when its sequential decay inside detector is considered.

![Meson-level Feynman diagrams contributing to the decays $B^0 \to D^- \mu^+ \mu^- \pi^+$. The sterile neutrino is produced at the first vertex and decays at the second vertex, which is at an observable distance away from the first vertex. The circular blobs connote the contributions from the corresponding hadronic form factors and decay constants. The cross in the Majorana scenario denotes the helicity flip involved in the decay.](image)

**FIG. 2.** Meson-level Feynman diagrams contributing to the decays $B^0 \to D^- \mu^+ \mu^- \pi^+$. The sterile neutrino is produced at the first vertex and decays at the second vertex, which is at an observable distance away from the first vertex. The circular blobs connote the contributions from the corresponding hadronic form factors and decay constants. The cross in the Majorana scenario denotes the helicity flip involved in the decay.

**Probing the Dirac and Majorana nature of $N$**—If the sterile neutrino $N$ decays inside a detector, we can probe lepton number violation in the entire process (which includes both the production of $N$ and its sequential decay) to ascertain its Majorana nature. For example, let us consider the sequential decay, $B^0 \to D^- \mu^+ \mu^- \pi^+ \equiv (B^0 \to D^- \mu^+ N) \otimes (N \to \mu^- \pi^+)$. The meson-level Feynman diagrams for these decays are shown in Fig. 2. It is very clear that observation of the lepton number violating mode $B^0 \to D^- \mu^+ \mu^- \pi^+$ would imply that the sterile neutrino has Majorana nature. In addition to considering the reconstruction of sterile neutrino from the final states $\mu^- \pi^+$ in the detector, our selection of events must also include a significant spatial separation between the point of production and point of decay of the sterile neutrino. Such a displaced vertex signature is essential to eliminate any contamination from background events in the SM. We can also consider the sterile neutrino decay $N \to \tau^+ \pi^-$ if allowed by kinematics. Similar analysis as above can be done for $D^0 \to K^- \mu^+ \mu^- \pi^+$ and related decays as well.

The displaced vertex signature, which is related to the observation of the decay length of the sterile neutrino, could be smaller or longer depending on its lifetime and energy. For a detector of finite size, say $L_D$, the observation of displaced vertex with decay length $L$ necessarily demands that $L < L_D$.

The feasibility of studying the Dirac and Majorana signatures of the sterile neutrino $N$ (of mass $m_N$, energy $E_N$ and total decay rate $\Gamma_N$) in a $D \mu\mu$ decay by using a detector of finite size $L_D$ depends on two important factors, (1) $P_{\text{decay}}(L)$, the probability of decay of $N$ within $L < L_D$, and (2) $P_{\text{flip}}$, the probability of helicity flip required for observation of the LNV decays which characterize the Majorana neutrino, and these are given by

$$P_{\text{decay}}(L) = 1 - \exp\left(-L m_N \Gamma_N / \sqrt{E_N^2 - m_N^2}\right),$$

$$P_{\text{flip}}(E_N) = m_N^2 / \left(E_N + \sqrt{E_N^2 - m_N^2}\right).$$

In the numerical study shown in Fig. 3 we probe both the feasibility of observing the $B \to D \mu\mu$ decays inside a finite sized detector, and the consequences of such a decay if observed. In Fig. 3a we probe the feasibility of observing the LNC decays $B \to D \mu^+ \mu^- \pi$ inside a finite size detector, considering the existing experimental constraints on $|U_{eN}|^2$ for $\ell = e, \mu, \tau$ which can be found in Refs. [4, 5]. The colorbar in Fig. 3a signifies the probability of observing these decays in which the energy of $N$ or the $\mu^- \pi^+$ system is known and we observe a displaced vertex with the decay length measured in the rest frame of the $B$ meson. The variable $e_N$ used along the horizontal axis of Fig. 3a is a dimensionless variable dependent on the energy $E_N$ as well as the maximum and minimum values of energy $(E_N^{\text{max}}, E_N^{\text{min}})$:

$$e_N = \left(E_N - E_N^{\text{min}}\right) / \left(E_N^{\text{max}} - E_N^{\text{min}}\right),$$

such that $e_N = 0$ (or $e_N = 1$) corresponds to the minimum (or maximum) energy of the sterile neutrino $N$. It is clear from Fig. 3a that for lighter $N$ (e.g. with $m_N = 1, 1.5$ GeV) there is extremely low probability ($\lesssim 10^{-5}$) of observing any displaced vertex signature with a decay length $\lesssim 1$ m. Therefore, if the sterile neutrino is light, its decay inside a finite sized detector is very unlikely, in which case our methodology mentioned before can be used to probe the existence of the sterile neutrino as well as study the active-sterile mixing parameter following Fig. 1. When we consider heavier sterile neutrino (e.g. $m_N = 2, 2.5$ GeV) the probability of detecting displaced vertex signature with a decay length $\lesssim 1$ m in the very low energy regime is higher. This is plausible, as only a heavy and non-relativistic neutrino due to its large lifetime would have a better chance of decaying inside a finite size detector such as Belle II and LHCb. In Fig. 3a we have considered a very generous range of decay lengths, from $1$ mm to about $10$ m. The LNV mode $B \to D \mu^+ \mu^- \pi$ receives additional suppression from helicity flip, and therefore, would have even lesser probability of getting observed within decay lengths $\lesssim 1$ m.
Number of observed events

FIG. 3. Numerical study of feasibility of observing purely Dirac (LNC) signal $B \rightarrow D\mu^+\mu^-\pi$ and purely Majorana (LNV) signal $B \rightarrow D\mu^+\mu^\pm\pi$, inside a finite sized detector, especially using the Belle II detector with decay lengths less than 1 m in the case of (b).

Assuming that in the laboratory we could observe such decays, say in Belle II and LHCb for example, we can estimate the value of the active-sterile mixing parameter $|U_{eN}|^2$ as a function of observed number of events. The LHCb scenario is analyzed in Ref. [5]. Here we consider the Belle II experiment for our numerical study. Considering masses $m_N = 0.5, 1, 1.5, 2, 2.5, 3$ GeV we show the estimated values of $|U_{eN}|^2$ for different number of observed events in Fig. 3b. We consider $P_{\text{decay}}(L)$ with $L = 1$ m for Belle II (considering the size of the central drift chamber [21]). We have also considered the additional suppression from helicity flip factor $P_{\text{flip}}$ while considering the LNV mode (which is on the bottom panel of Fig. 3b). For the total decay rate of $N$ which enters $P_{\text{decay}}(L)$ we have used Eqs. (30–32) of Ref. [18] and it depends on $|U_{eN}|^2$, $|U_{\mu N}|^2$ and $|U_{\tau N}|^2$. Since $|U_{eN}|^2$ is already constrained by $0\nu\beta\beta$ experiments to be much smaller than $|U_{\mu N}|^2$ and $|U_{\tau N}|^2$ (see Ref. [4]), we can safely neglect its contribution, as done in Fig. 3b. It can be inferred from Fig. 3 that for $|U_{eN}|^2$ smaller than the experimental upper limit, both Belle II and LHCb can still aspire to observe more than a handful of $B \rightarrow D\mu\mu\mu\pi$ decays with displaced vertex signatures. This is also valid for Majorana case where additional suppression from helicity flip must be considered.

**Conclusion.**—In this letter we have provided the simple, most systematic and efficient strategy to use the semileptonic decays $B \rightarrow D\ell N$ with $\ell = \mu, \tau$ to put much stringent constraint on the active-sterile mixing parameters $|U_{eN}|^2$, stronger by an order of magnitude than existing experimental constraints. The strength of our suggested methodology can be inferred from the fact that using only about $4.8 \times 10^8$ number of fully reconstructed $B$ decays at Belle II we can predict constraint which is comparable with the upper limit achievable from $4.8 \times 10^{12}$ number of $B$ decays at LHCb. This study does not require us to consider the sequential decay of $N$ at all as long as the 4-momentum of the parent $B$ meson as well as those of $D$ and $\ell$ can be fully determined experimentally. We show that by starting from the four fundamental properties of sterile neutrinos, viz. electrically neutral, spin-1/2 fermion, massive and long-living nature, we can probe the existence of sterile neutrino in these decays. Furthermore, the sequential decay of $N$ provides additional information, about lepton number conservation, which can be used to decipher its Dirac or Majorana nature. Our numerical study clearly shows that the LNV signal is suppressed than the LNC one due to helicity flip. Additional suppression, common to both LNV and LNC scenarios, comes from the probability of decay of $N$ inside a finite sized detector. We find that the active-sterile mixing parameters can also be well constrained, improving the existing experimental upper limits, if no sterile neutrino decays with decay lengths smaller than 1 m could be observed inside detector. Detectors that can probe larger decay lengths, as well as search for heavier neutrinos in their low energy regime would indeed prove to be more advantageous for such studies.
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