Optical Nonlinear Correlations in Disordered Media

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Imaging through scattering and random media is an outstanding problem that to date has been tackled by either measuring the medium transmission matrix or exploiting linear correlations in the transmitted speckle patterns. However, transmission matrix techniques require interferometric stability and linear correlations, such as the memory effect, can be exploited only in thin scattering media. Here we uncover an unexpected nonlinear correlation in randomly scattered fields that connects different realisations of the scattering medium and exists in the absence of the speckle memory effect. Besides the novelty of the nonlinear relationship itself, these results provide a route to imaging through dynamic and thick scattering media with applications for deep-tissue imaging or imaging through smoke or fog.

Introduction. The statistics of individual speckle patterns created by coherent illumination of an optically random medium is well understood. It is widely accepted that as the real and imaginary parts of the scattered field are uncorrelated Gaussian random variables, the field amplitude follows a Rayleigh distribution and the light intensity follows a negative exponential distribution [1]. The joint statistics of the scattered light is a more challenging problem. Various types of correlations in space, time or frequency may exist in the scattered fields, depending on the problem geometry and disorder strength [2–5]. A particularly relevant configuration involves imaging through a scattering slab, where the goal is to reconstruct the image from the transmitted scattered light. The strongest and most evident of the field correlations, the memory effect (ME) [6–8], has proven to be useful in such a situation for example allowing to reconstruct the image from a simple autocorrelation calculation of the speckle pattern [5–7]. Recently more subtle long-range mesoscopic correlations emerging in strongly scattering media [8] have been used to retrieve the image of a hidden object [9]. However, it is well known that correlations capture only a fraction of the total statistical dependence between random variables.

Statistical dependencies within strongly scattered light beyond linear correlations are less known due to the complexity of the problem and the absence of a common criterion for identifying non-linear dependencies without any a priori information about their nature. Indeed, multiple scattering usually involves thousands to millions optical modes and therefore the dimensionality of the total joint probability distribution is extremely high. However, in recent work [10], the universal bounds of spatial information preserved in multiple scattering have been estimated within the context of random matrix theory [11], thus neglecting the details of a realistic optical random scattering potential.

Alternatively to a stochastic approach to multiple scattering, a deterministic description based on the transmission matrix (TM) measurement is possible [12, 13]. This complex-valued matrix completely characterizes the scattering medium and once known, can be used to calculate the input field distribution from the output speckle pattern. The TM approach has been successfully applied to both diffuse imaging and imaging through multimode fibres [14, 15]. However, it suffers from the main drawback that it is sensitive to optical wave-length scale changes in the scattering medium. Measuring the transmission matrix typically requires some form of holography that needs to be repeated for a number of input modes and therefore is very challenging to extend to dynamic scattering media such as live tissue or fog. Even apparently static media such as white paint are known to evolve over relatively short timescales [16].

The TMs of typical disordered media can be extremely large, so data-driven approaches seem a reasonable way to tackle the resulting complicated mapping between the input and output patterns. Machine learning and artificial neural networks (ANNs) have therefore been increasingly applied over the past few years to the problem of classifying objects or imaging through scattering media [17–21]. These methods however, tend to suffer from the same drawback as the TM approaches, i.e. minimal changes of the scattering medium deteriorate or render the reconstruction impossible. Recently, a convolutional neural network based on the U-net architecture [22] was shown to be capable of reconstructing the image of a hidden object through optical disorder, while being trained on a similar, but different disorder realizations [23]. These results were demonstrated in a very thin diffuser that will therefore exhibit a marked (i.e. wide-angle) ME.

In this work we demonstrate the presence of a nonlinear statistical correlation in the optical field distribution transmitted through a random medium. We purposely consider a system in which no memory effect or any other linear correlations could be present, which is achieved by stacking two glass diffusers at a distance from each other. We gain physical insight into the inner workings of the random scattering process by using a dimensionality reduction technique, which we apply to the TMs of the scattering medium. The structure of the dimensionally-
randomly chosen transmission matrices reveals a clear nonlinear dependency when the disorder is varied, therefore generalising the concept of statistical relations in scattered fields beyond simple linear correlations. We then verify the ability of a U-Net ANN to image handwritten digits through such a complex scattering system. We find a similar quality image reconstruction in the presence or absence of the memory effect that underlines the dominant role played by these nonlinear correlations in supervised imaging approaches.

**Random scattering beyond the memory effect.**

The scattering configuration we consider is typical of a number of imaging through obstruction experiments, see Fig. 1(a). A collimated laser beam illuminates a spatial light modulator (SLM). In the first part of the work we use a nematic liquid crystal SLM, in the second, a digital micromirror device (DMD). The SLM is imaged by a 4f lens system onto a camera. The random scattering medium is made of one or two ground glass diffusers (220 grit, Thorlabs) placed between the SLM and the first lens of the imaging system. We note that one of the main goals of this work is to investigate the underlying physics of imaging through unknown or dynamic scattering media in the absence of the ME, i.e. in regime in which standard approaches based on the measurement of a single transmission matrix or on the autocorrelation of transmitted speckle patterns would fail. A simple system allowing to exclude any possibility of the ME to influence image reconstruction, is a stack of spatially separated thin diffusers. Even in the case of a far field measurement, the ME range becomes negligibly small, which we verified by projecting a simple 3 dot pattern shown in Fig. 1(b) and calculating the autocorrelation of the scattered light averaged over 600 different diffuser realizations. For this measurement we removed
The imaging system and collected far-field speckle patterns. As we can see in the middle panel of Fig. 1(b), the autocorrelation of the light scattered by a single diffuser shows a clear hexagonal pattern (the autocorrelation of the projected pattern), indicating the presence of linear correlations and ME. The right panel shows the same autocorrelation for two spatially separated diffusers characterised by absence of any structure, indicating no features of any discernible ME.

**Statistical dependencies in scattered light.** Any statistical relations of the randomly scattered fields are mapped to particular features in the TM of the corresponding disordered medium. For example, ME results in a banded TM structure, with the width of the band proportional to the ME range [24]. In order to search for statistical dependencies or correlations of strongly scattered light, we perform a statistical analysis of the random medium transmission matrices. We both numerically simulated and experimentally measured a set of TMs of the 2-diffuser scattering medium while varying the disorder configuration. Numerical modelling and subsequent determination of the TMs for different diffuser configurations is obtained by treating the diffusers as random phase screens and using the Fresnel propagation formula to calculate the fields at the second diffuser and then at the output plane. In the experiments we used a nematic liquid crystal SLM performing phase only modulation. By using a 4-step internal-reference interferometry, we retrieve amplitude and phase of the output speckle pattern, following the method outlined in Ref. [12]. Both in the numerics and in the experiments, we used discrete-cosine transform (DCT) harmonics as the input basis. In order to simplify the analysis we used the first 1024 DCT patterns and calculated/measured the complex output fields in a 32×32 pixel area. These are then reshaped into 1×1024 vectors, then combining them into a 1024×1024 TM. The average size of the output speckle grains was around 4 pixels so the number of independent modes in the output was around 64.

The resulting TMs have a high dimensionality (in our case, 32×32=1024, dictated by the size of our images) and do not reveal any clear features not upon visual inspection, as can be seen in Fig. 2(a). We therefore applied a dimensionality reduction technique to visualize the structure of the data manifold [25].

Dimensionality reduction is a data processing technique utilised for finding a low-dimension representation of a high-dimensional vector set, while preserving its important features. A well-known approach is principal component analysis, and recent progress in machine learning has allowed to exploit more evolved, nonlinear mapping techniques. Loosely speaking, these have the common goal of grouping data points based on their distance in the high dimensional space and finding the optimal, nonlinear mapping function onto a lower dimensional space where the data is grouped into common, well-separated volumes based on common features shared in the high dimensional space. Examples are stochastic neighbourhood embedding (t-SNE) [26] and, a more recent Uniform Manifold Approximation and Projection (UMAP) approach that we employ here in virtue of its ability to better preserve global structure when compared to t-SNE [27] and to uncover intricate relationships within high dimensional datasets [28, 30].

In Fig. 2(a), we show an example of a single TM (i.e. for just one diffuser configuration) and its dimensionally reduced representation obtained by applying UMAP algorithm. As can be seen, the UMAP projects the 1024×1024 TM onto a 2D surface in 3D space. This can be explained by noting that the input DCT harmonics basis can be parametrised by just two independent spatial frequencies. The scattering medium, by making a complex transformation significantly extends this parameter space and introduces a complex dependence between these new parameters. Yet, the (completely unsupervised) UMAP algorithm is able to identify the correct mapping that reduces the number of parameters back to two. Moreover, by applying an inverse transform to the 3D representation, we can compare the original speckle pattern, examples shown in Fig. 2(b), with those reconstructed from the dimensionality-reduced nonlinear mapping, Fig. 2(c). The back-projected patterns show the same overall structure of the original speckle images, albeit with blurred details, i.e. high k-vector components.
appear to be lost whilst low k-vector components are preserved.

We then apply the UMAP projection to the TMs calculated/measured for 30 different disorder realizations, shown in Fig. 2(d) and (e) respectively. We can see distinct structures in the reduced 3D representations. Both numerical and measurement results show that the 2D surfaces form a hyperboloid-like 3D volume characterised by a clear circular pattern in the top view projection and a X-shaped pattern in the side view. Both of these patterns are classical examples showing no linear correlation, but strong statistical dependence. This is the key finding of this work. Without any prior knowledge of the exact shape of the nonlinear correlations, it would be extremely hard to discover these and they have indeed so far escaped attention.

**Imaging through scattering media beyond the memory effect.** These findings lead to the proposal that these statistical dependencies can be picked up by an ANN that is trained to transform output speckle patterns into (unseen) input images of objects placed before the scatterer. Indeed, both ANNs and the UMAP projection conceptually share the same underlying, statistical data-driven search for the optimal nonlinear mapping function between input and output data sets. We measured the output speckle patterns corresponding to 1000 MNIST digit input images [31], projected with a DMD and repeated this by translating the scattering medium in the transverse plane for 96 different non-overlapping regions. Therefore, each of the 96 repetitions involves completely different microscopic realisations of the random medium, albeit with the same average property, i.e. grit. This data is used to train a U-net ANN [22], following the same architecture explained in detail in Ref. [23].

Examples of ANN image reconstruction of digits (unseen during the training) are shown in Fig. 3(a). The top row shows the ground truth examples followed by the reconstruction with just one diffuser and then for two diffusers from which it can be seen that despite the absence of ME, the ANN is still able to reconstruct hidden images successfully. As shown in Fig. 3(b) the reconstruction mean squared error of this method continues to decrease with the number of disorder configurations used for training. Moreover, the MSE for both 1 and 2 diffusers seems to decrease at the same rate and with the same absolute values. This possibly indicates that, the ME is never playing a major role in the ANN reconstruction, regardless of its presence. This is rather surprising as one might expect a strong linear correlation property to be the dominant feature captured by the high-dimensional interpolation properties of ANNs. Rather, our findings indicate that the ANN is extracting information from the nonlinear correlation described above, not only here but also in previous studies that relied on simpler single-scattering systems [22, 32].

**Conclusions.** We have shown the presence of non-linear statistical dependencies within the scattered fields by applying dimensional reduction to the TM for a system where no linear correlations are possible. The hyperboloidal correlations are also nonlocal in the sense that they connect the TMs across different regions of the diffusers, i.e. for different realisations of the random disorder. These nonlinear correlations allow an ANN to reconstruct images transmitted and randomly scrambled through unseen disorder configurations. Recent work has also shown how these ANN approaches can be extended to imaging through not only dynamic random media but also at different depths and defocus conditions, thus indicating that these results are not specific to a given imaging system [32].

Looking forward, a first obvious extension would be to apply these results to imaging through inherently dynamical and changing scattering media such as living tissue or fog. This leads to further questions such as how these results extend to the case in which one physically modifies the microscopic properties, e.g. average scatterer size, rather than moving the scattering system laterally (thus maintaining the same statistical properties of the diffuser). There are also implications for applications of scattering media for secure encoding and transmission of information. These systems typically rely on the fact that any given random medium is practically unclonable and therefore acts as one-pad encryption key. However, our results seem to imply that knowledge of the statistical properties of the scattering medium (e.g. the distribution of refractive index perturbations) is sufficient to decode scrambled information, with important implications on the security of these encoding approaches [33, 34]. Finally identification of the shape of the nonlinear correlations could provide insight for their in-depth theoretical study in analogy to the theory of linear correlations [21, 24].

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