Bayesian Convolutional Neural Networks with Bernoulli Approximate Variational Inference

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Abstract

We present an efficient Bayesian convolutional neural network (convnet). The model offers better robustness to over-fitting on small data than traditional approaches. This is by placing a probability distribution over the convnet’s kernels (also known as filters). We approximate the model’s intractable posterior with Bernoulli variational distributions. This requires no additional model parameters. Our model can be implemented using existing tools in the field. This is by extending the recent interpretation of dropout as approximate inference in the Gaussian process to the case of Bayesian neural networks. The model achieves a considerable improvement in classification accuracy compared to previous approaches. We finish with state-of-the-art results on CIFAR-10 following our new interpretation.

1 Introduction

Bayesian neural networks (NNs, also known as multilayer perceptrons – MLPs) [1, 2] have been studied extensively in the literature. They offer a probabilistic interpretation of deep learning models by inferring distributions over networks’ weights. They are robust to over-fitting, offer uncertainty estimates, and can easily learn from small datasets. Convolutional neural networks (convnets) are specialised MLP models designed for image processing [3]. Convnets have led to state-of-the-art results on various object recognition and classification tasks [4, 5]. However convnets usually require large amounts of data to avoid model over-fitting. Modelling a distribution over the kernels (also known as filters) of a convnet has never been attempted successfully before, perhaps because of the vast number of parameters and extremely large models commonly used in practical applications.

Performing inference in Bayesian NNs is a difficult task, and approximations to the model posterior are often used. Variational inference is an example tool for obtaining approximate model posteriors. In this approach one would model the posterior using a simple variational distribution such as a Gaussian, and try to fit the distribution’s parameters to be as close as possible to the true posterior. This is done by minimising the Kullback-Leibler divergence from the full model. Many have followed this approach in the past for standard MLP models [6, 7, 8, 9]. But the variational approach used to approximate the posterior in Bayesian NNs can be fairly computational expensive – the use of Gaussian approximating distributions increases the number of model parameters considerably, without increasing model capacity by much. Blundell et al. [9] for example use a Gaussian distribution in a Bayesian NN and have doubled the number of model parameters, yet report the same predictive performance as traditional approaches using dropout. This makes the approach unsuitable for use with convnets as the increase in the number of parameters is too costly.

Instead, we use Bernoulli approximating variational distributions. The use of Bernoulli variables does not require additional parameters for the approximate posteriors, and allows us to obtain a computationally efficient Bayesian convnet implementation. Perhaps surprisingly, we can implement our model using existing tools in the field. Gal and Ghahramani [10] have recently shown that dropout in MLPs can be interpreted as an approximation to a well known Bayesian model, the Gaussian process (GP). Extending on their work, we show that the dropout model can further
be interpreted as performing Bernoulli approximate variational inference in Bayesian NNs. This allows us to use operations such as convolution and pooling in a principled way. To implement our Bayesian convnet we perform dropout after every layer that is modelled with a posterior distribution. In Bayesian NNs often all weight layers are modelled by posterior distributions. Weight layers with no approximating distributions lead to over-fitting. Thus, compared to the standard use of dropout in convnets after the inner-product layers alone, implementing a Bayesian convnet we apply dropout after every convolution layer as well. Following the derivations of [10] we average stochastic forward passes through the model at test time (referred to as Monte Carlo (MC) dropout).

The contributions of this work are numerous. First, we show that dropout in MLPs is equivalent to approximate variational inference in Bayesian NNs, extending on the work of [10]. Second, following insights from this connection we propose new practical convnet structures that are equivalent to Bayesian convolutional neural networks. These models obtain much better test accuracy compared to existing approaches in the field with no additional computational cost during training. Third, we show that the proposed model reduces over-fitting on small datasets compared to standard techniques. Lastly, we demonstrate improved results with MC dropout on existing convnet models in the literature. This suggests that the standard dropout approximation is effective but can be improved in convnets. We give empirical results assessing the number of MC samples required to improve model performance, and finish with new state-of-the-art results on the CIFAR-10 dataset following our insights.

The paper is structured as follows. In section 2 we briefly review the derivation of Gal and Ghahramani [10]. In section 3 we extend this derivation to Bayesian NNs, and discuss the implications of this such as the introduction of convolution operations. In section 4 give the experiment results, which are then analysed in section 5.

2 Background

We review the main results of [10], relating dropout to approximate inference in the Gaussian process. We link these to Bayesian NNs with Bernoulli approximating variational distributions in the next section.

We denote by \( E \) a loss function such as the softmax loss or the euclidean loss. We denote by \( W_i \) weight matrices of dimensions \( K_i \times K_{i-1} \), and by \( b_i \) the bias vectors of dimensions \( K_i \) for each layer \( i = 1, \ldots, L \). We denote by \( \hat{y} \) the outputs of an MLP model and by \( y \) the observed outputs corresponding to inputs \( x \). During MLP optimisation, the loss term is scaled by the learning rate \( r_1 \) and a regularisation term is added. We often use \( L_2 \) regularisation weighted by some weight decay \( r_2 \), resulting in a minimisation objective (often referred to as cost),

\[
L_{\text{dropout}} := r_1 E(y, \hat{y}) + r_2 \sum_{i=1}^{L} (||W_i||_2^2 + ||b_i||_2^2).
\]

With dropout, we sample binary variables for every input point and for every network unit in each layer. Each binary variable takes value 1 with probability \( p_i \) for layer \( i \). A unit is dropped (i.e. its value is set to zero) for a given input if its corresponding binary variable takes value 0. We use the same binary variable values in the backward pass propagating the derivatives to the parameters. The probabilities \( p_i \) might be optimised as well.

The deep Gaussian process [11] is a powerful tool in statistics that allows us to model distributions over functions. Assume we are given a covariance function of the form \( K(x, y) = \int p(w)p(b)\sigma(w^T x + b)\sigma(w^T y + b)dw db \) with some element-wise non-linearity \( \sigma(\cdot) \) and distributions \( p(w), p(b) \). It is straightforward to show that \( K(x, y) \) is a valid PSD covariance function – it is an example of a marginalised covariance function [12]. Gal and Ghahramani [10] have shown that a deep Gaussian process with \( L \) layers and covariance function \( K(x, y) \) can be approximated using the following parametric probabilistic model. Let \( b_i \) be a \( K_{i-1} \) dimensional binary random vector for each layer \( i \). Write \( \hat{M}_i = M_i \text{diag}(b_i) \) with matrices \( M_i \) of dimensions \( K_i \times K_{i-1} \) and let \( \omega = \{ M_i \}_{i=1}^L \). We use a parametrisation for \( \omega \) which is similar to the well known Gaussian distribution re-parametrisation, where \( x \sim N(\mu, \sigma^2) \) is written as \( x = \mu + s \epsilon \) with \( \epsilon \sim N(0, 1) \). When the Gaussian distributions is used in variational inference, \( \mu \) and \( s \) would be treated as a variational

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1The \( \text{diag}(\cdot) \) operator maps a vector to a diagonal matrix whose diagonal is the elements of the vector.
parameter. In our case, \( \mathbf{M}_i \) are treated as variational parameters. The variational distribution, \( q(\omega) \), is defined by

\[
b_{i,j} \sim \text{Bernoulli}(p_i) \quad \text{for} \quad i = 1, \ldots, L, \quad j = 1, \ldots, K_{i-1}
\]

given some probabilities \( p_i \). In effect, \( q(\omega) \) is a distribution over matrices whose columns can randomly be set to zero. The binary variable \( b_{i,j} = 0 \) indicates that unit \( j \) in layer \( i - 1 \) is dropped out as an input to layer \( i \).

Given bias vectors \( \mathbf{m}_i \) of dimensions \( K_i \) for each layer, and observed label \( y \) in one-hot encoding (i.e. only one element of \( y \) takes value 1 while all others take value 0), the approximate model \( q(y|x) \) is defined as

\[
q(y|x) = \int p(y|x, \omega)q(\omega)d\omega
\]

In variational inference we minimise the KL divergence between the approximate model \( q(\omega) \) above and the posterior of the full deep GP – \( p(Y|X) \). The minimisation objective is obtained from this KL divergence by Monte Carlo integration over \( \omega \) with a single sample [10]:

\[
\mathcal{L}_{\text{GP-MC}} \propto \gamma \mathcal{T} \mathbf{E}(y, \tilde{y}) + \gamma \sum_{i=1}^L \left( \frac{p_i}{2} \frac{1}{2} \left| \mathbf{M}_i \right|^2 + \frac{1}{2} \left| \mathbf{m}_i \right|^2 \right)
\]

with \( \mathbf{E}(y, \tilde{y}) = -\log p(y|x, \omega) \) and \( b_{i,j} \) realisations from the Bernoulli distribution. This is identical to [1] for an appropriate setting of the precision hyper-parameter \( \tau \) and the scale parameter \( \gamma \).

We next relate this model to the Bayesian NN model. This allows us to extend this derivation and use convolution and pooling operations which do not necessarily have a mathematical equivalence with the Gaussian process.

3 Dropout as Approximate Variational Inference in Bayesian Convolutional Neural Networks

We now link the approximate model above to variational inference in Bayesian NNs with Bernoulli approximating variational distributions, extending on [10]. This allows us to extend the model beyond the Gaussian process interpretation. We then inspect the representation of convolution and pooling operations in this Bayesian NN interpretation. These do not necessarily have a corresponding GP interpretation, but can be modelled following our Bayesian NN interpretation.

One defines a Bayesian neural network by placing a prior distribution over its weights. Given weight matrices \( \mathbf{W}_i \) and bias vectors \( \mathbf{b}_i \) for layer \( i \), we place Gaussian prior distributions over the weight matrices \( p(\mathbf{W}) \):

\[
\mathbf{W}_i \sim \mathcal{N}(0, I).
\]

We assume a point estimate for the bias vectors. Denote the output of an MLP with weights (\( \mathbf{W}_i \)\)\(^L\)) on input \( x \) by \( \tilde{y}(x, (\mathbf{W}_i)\)\(^L\))\). We assume a softmax likelihood given the MLP’s weights:

\[
p(y_d = 1|x, (\mathbf{W}_i)\)\(^L\)) = \text{Categorical} \left( \frac{\exp(\tau \tilde{y}_d)}{\sum_{d'} \exp(\tau \tilde{y}_{d'})} \right)
\]

with \( \tilde{y} = \tilde{y}(x, (\mathbf{W}_i)\)\(^L\)).
We are interested in finding the posterior over the weights given our observed data $X, Y$:

$$p(W|X, Y).$$

This posterior is not tractable in general, and we use variational inference to approximate it as was done in [6, 7, 8, 9]. We need to define an approximating variational distribution $q(W)$, and then minimise the KL divergence between the approximating distribution and the full posterior:

$$\text{KL}(q(W)||p(W|X, Y)).$$

Defining our approximating variational distribution $q(W_i)$ for every layer $i$ as

$$b_{i,j} \sim \text{Bernoulli}(p_i) \text{ for } j = 1, ..., K_{i-1}$$

with $b_i$ vectors of Bernoulli distributed random variables and variational parameters $M_i$, we recover the model in the previous section [10].

Minimising our optimisation objective in eq. (6) maximises the log likelihood term while minimising the KL divergence term. This KL divergence is between the approximate distribution placed over the weights and the prior distribution over the weights. This encourages the model to explain the data well while keeping it from over-fitting. The two terms are obtained using a Monte Carlo approximation of the integral over the variational distribution $q(\omega)$.

We extend the developments above to convolutional neural networks. Let $k_k \in \mathbb{R}^{h \times w \times K_{i-1}}$ for $k = 1, ..., K_i$ be kernels with height $h$, width $w$, and $K_{i-1}$ channels in the $i$'th convnet layer. The input to the layer is represented as a 3 dimensional tensor $y \in \mathbb{R}^{W_{i-1} \times H_{i-1} \times K_{i-1}}$ with height $H_{i-1}$, width $W_{i-1}$, and $K_{i-1}$ channels. Convolving the kernels with the input with a given stride $s$ is equivalent to extracting patches from the input and performing a matrix product. We extract $h \times w \times K_{i-1}$ dimensional patches from the input with stride $s$ and vectorise these. Collecting the vectors in the rows of a matrix we obtain a new representation for our input $\tilde{y} \in \mathbb{R}^{n \times hwK_{i-1}}$ with $n$ patches. The vectorised kernels form the columns of the weight matrix $W_i \in \mathbb{R}^{n \times K_{i-1} \times K_i}$. The convolution operation is then equivalent to the matrix product $W_i \tilde{y} \in \mathbb{R}^{n \times K_i}$. The rows of the output can be re-arranged to a 3 dimensional tensor $y' \in \mathbb{R}^{W_i \times H_i \times K_i}$. Pooling can then be seen as a non-linear operation on the matrix $\tilde{y}$.

Following our Bayesian NN approximation above, we model the kernels with a Bernoulli variational distribution. We sample vectors of Bernoulli random variables $b_i$ and multiply these by the weight matrix: $W_i \times \text{diag}(b_i)$. This can be seen as randomly setting kernels to zero for different patches. With the dropout interpretation, this is equivalent to applying dropout for each element in the tensor $y'$ before pooling. As a result of this interpretation, we can use dropout after every convolution layer before pooling to get an efficient implementation of our Bernoulli Bayesian convnet.

We next assess the model above with an extensive set of experiments studying its properties.

## 4 Experiments

We evaluate the insights brought above by implementing Bernoulli Bayesian convnets using dropout. We show that a considerable improvement in classification performance can be attained with correct use of dropout on a variety of tasks. We assess the LeNet network structure [8] on MNIST [13] and CIFAR-10 [14] classification tasks with different settings. We inspect model over-fitting by training the model on small random subsets of the MNIST dataset. We test various existing model architectures with MC dropout. We then empirically evaluate the number of samples needed to obtain an improvement in results. We finish with a new state-of-the-art result on CIFAR-10 obtained by an almost trivial change of an existing model. All experiments were done using the Caffe framework [15] with the configuration files available online at [http://mlg.eng.cam.ac.uk/yarin/](http://mlg.eng.cam.ac.uk/yarin/).

We present the experimental results here and analyse these results in the following section.

### 4.1 Bayesian Convolutional Neural Networks

We show that performing dropout after all convolution and weight layers (our Bayesian convnet implementation) in the LeNet convnet results in a considerable improvement in test accuracy on both the MNIST dataset and CIFAR-10 dataset. We refer to the Bayesian convnet implementation with dropout used after every parameter layer as “lenet-all” throughout the experiment. We compare this model to the traditional use of dropout after the fully connected inner-product layers at the end
We refer to this model as “lenet-ip”. Additionally we compare to LeNet as described originally in [3] with no dropout at all, referred to as “lenet-none”. For MNIST we use the LeNet network as described in [3] with dropout probability $0.5$ in every dropout layer. The model used with CIFAR-10 is set up in an identical way, with the only difference being the use of 192 outputs in each convolution layer instead of 20 and 50, as well as 1000 units in the last inner product layer instead of 500.

We ran a stochastic gradient descent optimiser for $1e7$ iterations for all MNIST models and $1e5$ iterations for all CIFAR-10 models. We used learning rate policy base-lr $\times (1 + \gamma \times \text{iter})^{-p}$ with $\gamma = 0.0001$, $p = 0.75$, and momentum 0.9. We used base learning rate 0.01 and weight decay 0.0005. All models were optimised with the same parameter settings.

We evaluate each dropout network structure (lenet-all and lenet-ip) using two testing techniques. The first is using weight averaging, the standard way dropout is used in the literature (referred to as “Standard dropout” throughout the paper). This involves multiplying the weights of the $i$'th layer by $p_i$ at test time. We use the Caffe reference implementation for this [15]. The second testing technique involves averaging $T$ stochastic forward passes through the model, with dropout used in the same way as during training time. We do this following the Bayesian interpretation of dropout derived in [10]. This technique is referred to here as “MC dropout”. This technique has been presented in the literature before as model averaging. In this experiment we average $T = 50$ forward passes through the network. We stress that the purpose of this experiment is not to achieve state-of-the-art results on either dataset, but rather to compare the different models with different testing techniques.

Figure 1 shows classification error on log scale for all three models (lenet-all, lenet-ip, and lenet-none) with the two different testing techniques (Standard dropout and MC dropout) for MNIST (fig. [1a]) and CIFAR-10 (fig. [1b]). It seems that Standard dropout in lenet-ip results in improved results compared to lenet-none, with the results more pronounced on the MNIST dataset than CIFAR-10. When Standard dropout testing technique is used with our Bayesian convnet (with dropout applied after every parameter layer – lenet-all) performance suffers. However by averaging the forward passes of the network the performance of lenet-all supersedes that of all other models (MC dropout lenet-all in both [1a] and [1b]).

### 4.2 Model Over-fitting

We evaluated our model’s tendency to over-fit on training sets decreasing in size. We used the same experiment set-up as above, without changing the dropout ratio for smaller datasets. We randomly subset the MNIST dataset into smaller training sets of sizes $1/4$ and $1/32$ fractions of the full set.
Figure 2: **Test set error of LeNet trained on random subsets of the MNIST dataset decreasing in size.** In green are networks with dropout applied after the last layer alone (lenet-ip) and evaluated with Standard dropout (the standard approach in the field), in blue are networks with dropout applied after every weight later (lenet-all) and evaluated with MC dropout – our Bayesian convnet implementation. Note how lenet-ip starts over-fitting even with a quarter of the dataset. With a small enough dataset, both models over-fit. MC dropout was used with 10 samples.

We evaluated our model with MC dropout compared to lenet-ip with Standard dropout – the standard approach in the field. We did not compare to lenet-none as it is known to over-fit even on the full MNIST dataset.

The results are shown in fig. 2. For the entire MNIST dataset (figs. 2a and 2b) none of the models seem to over-fit (with lenet-ip performing worse than lenet-all). It seems that even for a quarter of the MNIST dataset (15,000 data points) the Standard dropout technique starts over-fitting (fig. 2c). In comparison, our model performs well on this dataset (obtaining better classification accuracy than the best result of Standard dropout on lenet-ip). When using a smaller dataset with 1,875 training examples it seems that both techniques over-fit, and other forms of regularisation or higher dropout probabilities are needed.

### 4.3 MC Dropout in Standard Convolutional Neural Networks

We evaluate the use of Standard dropout compared to MC dropout on existing convnet models previously published in the literature. The recent state-of-the-art convnet models use dropout after fully-connected layers that are followed by other convolution layers, suggesting that improved performance could be obtained with MC dropout.

We evaluate two well known models that have achieved state-of-the-art results on CIFAR-10 in the past several years. The first is Network in network (NIN) [16]. In the original paper the model is described as using MLPs to transform the patches non-linearly, rather than using linear convolution operations. An alternative interpretation is that the model performs convolutions with $1 \times 1$ kernels followed by non-linearities. In practical implementations the model is implemented following this interpretation [15]. The model was extended by [17] who added multiple loss functions after some of the layers – in effect encouraging the bottom layers to explain the data better. The new model was named Deeply supervised network (DSN). The same idea was used in [5] to achieve state-of-the-art results on ImageNet.
### Table 3: Test error using both Standard dropout and MC dropout for the models (NIN, DSN, and Augmented-DSN on the augmented dataset)

| Model            | Test error (Standard dropout) | Test error (MC dropout) |
|------------------|-------------------------------|-------------------------|
| lenet-none       | 23.46                         | —                       |
| lenet-ip         | 23.46                         | 23.47 ± 0.04            |
| lenet-all        | 41.82                         | 16.05 ± 0.07            |
| NIN              | 10.43                         | 10.27 ± 0.05            |
| DSN              | 9.37                          | 9.32 ± 0.02             |
| Augmented-DSN    | 7.95                          | 7.71 ± 0.09             |

Figure 3: Test error on CIFAR-10 with the same networks evaluated using Standard dropout versus MC dropout ($T = 100$, averaged with 5 repetitions and given with standard deviation). MC dropout achieves consistent improvement in test error compared to Standard dropout. The lowest error obtained is 7.51 for DSN-augmented. Note the large difference between lenet-all Standard dropout and lenet-all MC dropout.

We assess these models on the CIFAR-10 dataset, as well as on an augmented version of the dataset for the DSN model [17]. We replicate the experiment set-up as it appears in the original papers, and evaluate the model’s test error using Standard dropot as well as using MC dropout, averaging $T = 100$ forward passes. MC dropout testing gives us a noisy estimate, with potentially different test results over different runs. We therefore repeat the experiment 5 times and report the average test error with the model obtained when optimisation is done (using no early stopping). We report standard deviation as well to see if the improvement is statistically significant.

Test error using both Standard dropout and MC dropout for the models (NIN, DSN, and Augmented-DSN on the augmented dataset) are shown in table 3. Also shown in the table are the results of the models from the previous section using the LeNet convnet structure for comparison. As can be seen, using MC dropout a statistically significant improvement can be obtained for all three models (NIN, DSN, and Augmented-DSN), with the largest increase for Augmented-DSN. It is also interesting to note that the lowest test error we obtained for Augmented-DSN is 7.51.

We observed no improvement on ImageNet [18] using the same models. This might be because of the large number of parameters in the models above compared to the relatively smaller CIFAR-10 dataset size. We speculate that our approach offers better regularisation in this setting. ImageNet dataset size is much larger, perhaps offering sufficient regularisation. However labelled data is hard to collect, and in some applications larger amounts of data are not available. It would be interesting to see if a subset of the ImageNet data could be used to obtain the same results obtained with the full ImageNet dataset with the stronger regularisation suggested in this work. We leave this question for future research.

#### 4.4 MC Estimate Convergence

We give empirical results suggesting that 20 samples are enough to improve performance on some datasets. We evaluated the last model (Augmented-DSN) with MC dropout for $T = 1, ..., 100$. We repeat the experiment 5 times and average the results. In fig. 4 we see that within 20 samples the error is reduced by more than one standard deviation. Within 100 samples the error converges to 7.71 with a small standard deviation.

#### 5 Analysis of the Results

We analyse the results in the previous section, explaining the properties of the models. Further, we attempt to explain the source of the observed improvement.

Dropout is not often used in convnets after convolution layers. This is because empirical results with Standard dropout suggest deteriorated performance (as can also be seen in section 4.1). Standard dropout would approximate model output during test time by weight averaging [19]. The mathematically correct way of using dropout at test time is by Monte Carlo averaging of stochastic forward passes through the model [10, 19]. It has been argued that the Standard dropout approximation results in similar predictive performance to that of MC dropout [19 section 7.5]. Empirical results were given for a simple MLP structure over the MNIST dataset [18], and it seems that most research
Figure 4: **Augmented-DSN test error for different number of forward passes in MC dropout** (blue) averaged with 5 repetitions, shown with 1 standard deviation. In green is test error with Standard dropout. MC dropout achieves a significant improvement (more than 1 standard deviation) after 20 samples.

has followed this approximation. The results we obtained in our experiments suggest the contrary however. Replicating the experiment in [19, section 7.5] with the augmented CIFAR-10 dataset and the DSN convnet model, we showed in section 4.4 that a significant reduction in test error can be achieved. This might be because convnets exhibit different characteristics from standard MLPs. We speculate that the non-linear pooling layer affects the dropout approximation considerably.

In the first experiment (section 4.1) a possible explanation for the improvement we observed is that the additional layers of dropout prevent over-fitting in the convnet’s kernels. This can be seen as a full Bayesian treatment of the model, approximated with MC integration. The stochastic optimisation objective converges to the same limit as the full Bayesian model [20, 21, 22, 23, 24]. Thus the approximate model possess the same robustness to over-fitting properties as the full Bayesian model. The Bernoulli approximating variational distribution is a fairly weak approximation however – a trade-off which allows us to use no additional model parameters. This explains the over-fitting observed with small enough datasets (section 4.2).

Lastly, it is worth noting that our improved results come with a potential price: longer test time. The training time of our model is identical to that of existing models in the field. The test time is scaled by \( T \) – the number of averaged forward passes through the network. However this should not be of real concern in real world applications, as convnets are often implemented on distributed hardware. This allows us to obtain MC dropout estimates in constant time almost trivially. This could be done by transferring an input to a GPU and setting a mini-batch composed of the same input multiple times. In dropout we sample different Bernoulli realisations for each output unit and each mini-batch input, which results in a matrix of probabilities. Each row in the matrix is the output of the dropout network on the same input generated with different random variable realisations. Averaging over the rows results in the MC dropout estimate. Further, many models are tested with multiple crops of the same input. This could be done with stochastic forward passes instead of averaged weights.

### 6 Conclusions and Future Research

We have presented an efficient Bayesian convolutional neural network, offering better robustness to over-fitting on small data by placing a probability distribution over the convnet’s kernels. The model’s intractable posterior was approximated with Bernoulli variational distributions, requiring no additional model parameters. The model implementation uses existing tools in the fields and requires almost no overheads.

Future research includes the study of the Gaussian process interpretation of convolution and pooling. These might relate to existing literature about convolutional kernel networks [25]. Furthermore, it would be interesting to see if a subset of the ImageNet data could be used to obtain the same results with the stronger regularisation suggested in this work. We further aim to study how the learnt filters are affected by dropout with different probabilities.
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