Developing a mathematical model and a computer program for a preliminary design of a transonic axial compressor

A.I. Borovkov¹, Yu.B. Galerkin¹, O.A. Solovieva¹, A.A. Drozdov¹, A.F. Rekstin¹, V.B. Semenovsky¹, P.N. Brodnev¹

R&D Laboratory “Gas dynamics of turbo machines” Peter the Great St.Petersburg Polytechnic University, Polytechnical st. 29, St.Petersburg, Russia

Abstract. This paper presents a mathematical model underlying the program for calculating and designing axial compressors. The process of calculating pressure loss in the elements of the axial compressor’s stage flow path is described. The loss coefficient consists of losses on the limiting surfaces, secondary losses and profile losses. The effect of roughness on the pressure loss is taken into account by introducing the corresponding empirical coefficient. An algorithm for calculating the blades’ and vanes’ angles of the impeller and the guide apparatus is presented by calculating the incidence angle and the lag angle of the flow. The flow lag angle is the sum of the lag angle of the flow on the profile and the lag angle due to viscous flow on the limiting surfaces.

1. Introduction

The complex nature of the course and the absence of adequate computer technology until the 70s of the last century have determined the empirical nature of axial stages and compressors research. The adequacy of the flow in the elementary annular lattice of the impeller blade rows or in the stator elements vane rows greatly simplified obtaining the desired results. The basic information used in the design practice of axial compressors’ blade rows, was obtained as a result of testing elementary flat lattices in wind tunnels. The technique of these experiments is much simpler than, for example, that which is used when testing model stages of axial or centrifugal compressors.

In domestic literature, the principles of profiling axial compressors’ blades and vanes based on testing flat rows were described in [1], in more detail in [2]. A later publication [3], summarizing the results of research by foreign experts, basically describes the same approach. The most complete exposition of these principles can be found in a monograph [4] published in 2003. Results of extensive research and design methods for axial fans are presented in the monograph [5]. Recommendations for the design of industrial axial compressors can be found in technical materials [6] as well as in others. A number of particular design issues have been resolved by domestic researchers and described in a number of publications, for example, in [7].

At later stages, one-dimensional and two-dimensional engineering programs started being developed, allowing for calculation and optimization of both axial stages and compressors in general. Most of these programs were created at manufacturing plants of compressor equipment based on the processing of experimental studies and are a trade secret. Information about them is extremely scarce.
Foreign scientists are working in the same direction. Development of a software package for optimizing the size and shape of axial compressors is presented in [8], [9]. The developed mathematical model takes into account various components of pressure loss in axial compressors and allows to optimize their sizes, the formulas of impeller blades and stator elements vanes, and provides predicted gas-dynamic characteristics of the compressor. The one-dimensional calculation on the midline of the stream is used in the preliminary design and the two-dimensional approach with the calculation of flow parameters and velocity triangles is used on several streamlines along the blade height. This allows to achieve the optimal blade shape along its height and non-incidence flow. The calculation of pressure loss is carried out according to the Liblein formulas. A compressor can be designed with a constant hub ratio, a constant shroud diameter or a constant diameter of the midline of the stream. It is also possible to set a linear law for changing the shroud diameter. The program was identified based on experimental data for a multi-stage axial compressor obtained at NASA (Figure 1).

Figure 1. Comparison of calculated and experimental data for multistage axial compressor [[8]]

Practically the same mathematical models and computer programs are presented in papers [10]-[15]. The R&D laboratory "Gas Dynamics of Turbomachines" has developed its own computer programs designed for calculating and designing axial compressors. Many years of experience of applying programs in the interests of the industry [16], [17] have shown their effectiveness.

During the scientific research of axial compressors, a tool was needed that allowed to create a preliminary design of an axial compressor in the design mode, digital twins of existing compressors and
to determine their parameters in the design mode. To solve these problems, the staff of the R&D Laboratory "Gas Dynamics of Turbomachines" created a specialized tool - the DDAC program.

The program allows to determine the main dimensions, calculate the velocity triangles and flow parameters at 20 radii along the blades and vanes height. It is possible to assess:
- the efficiency of the impeller, stator elements and each stage of the compressor;
- uneven flow rates and the total pressure field at the exit of the impeller and the stage.

2. A mathematical model for calculating flow parameters in compressor control sections

The mathematical model underlying the DDAC program assumes that the calculation is made on 20 streamlines along the blade height. Each streamline takes into account the fact that it has a conical shape rather than a cylindrical shape. This is taken into account by calculating the angle of inclination of the axisymmetric current surface:

$$
\chi_{zi} = \arctg \left( \frac{r_{zi} - r_{zi}}{B_{zi}} \right),
$$

where $B_{zi}$ is axial length of the blade; $r$ is radius.

The hub streamline is initially calculated, then flow parameters on all other streamlines are iteratively determined. The design scheme for one compressor stage is shown in Figure 2.

![Figure 2. Axial compressor stage scheme](image)

The task of calculating flow parameters is solved numerically, for which the blade height $l = r_{shub} - r_{hub}$ is divided by a number of intermediate radii. The mass flow rate through each streamline is assumed to be the same. Velocity triangles and the shape of elementary rows at these radii are determined in accordance with the law of variation of flow parameters along the blades height.
In [1], [3], [4], [18], at the stage of variant calculation, the spatial flow shape is calculated using a number of simplifying assumptions. A potential, inviscid flow at the homogeneous stage, in the gap between rotating blades and stator vanes, is considered. The centrifugal force from the rotation of the flow with \( c_u \) speed is balanced by the pressure gradient along the radius (condition of radial equilibrium):

\[
\frac{\partial p}{\partial r} = \rho \frac{c_u^2}{r}.
\]  

(2)

where \( p \) is pressure; \( \rho \) is gas density; \( c_u \) is tangential component of absolute flow velocity.

From the above mentioned literature it follows that the generally accepted principle of profiling is the uniformity of the loading factor at all radii along the blades’ height:

\[
h_r = f(r) = \psi_r u^2 = \text{const}, \quad \text{or} \quad \psi_r = f(r) = \psi_{r, const} \left( \frac{r_o}{r} \right)^2.
\]  

(3)

where \( h_r \) is Euler head; \( \psi_r \) is the loading factor; \( u \) is blade velocity.

This condition ensures the equality of the mechanical energy of gas particles in each of the control sections. If the mechanical energies are equal, there is no loss of mixing between gas particles with different mechanical energies, which explains the feasibility of designing according to equation (3).

As \( p^* = p + \rho \frac{c_u^2 + c_z^2}{2} = f(r) = \text{const} \) (where \( p^* \) is total pressure; \( c_z \) is axial component of absolute flow velocity), then the condition of radial equilibrium (2) is satisfied with the following ratio between the components of the absolute velocity:

\[
\frac{c_u}{r} + \frac{\partial c_u}{\partial r} + \frac{\partial c_z}{\partial r} = 0.
\]  

(4)

Equation (4) shows that at the homogeneous stage, provided that \( h_r = f(r) = \text{const} \) the nature of the change in velocity along the radius cannot be arbitrary. It is obvious that centrifugal forces appearing due to flow swirling increase the static pressure along the radius. Therefore, the condition of constant total pressure requires a decrease in the absolute velocity in this direction. This nature of the change in speed can be realized with a different ratio between the components \( c_u \) and \( c_z \).

On a hub current line, the Euler head is determined based on a user-defined flow angle \( \beta_{2, hub} \):

\[
h_{2, hub} = (u_{2, hub} - c_{z, 2, hub} \tan \beta_{2, hub}) u_{2, hub} - c_{z, 2, hub} u_{2, hub}
\]  

(5)

The total temperature is calculated through the amount of energy supplied:

\[
T_{2i}^* = T_0^* + \frac{h^n}{c_p}
\]  

(6)

where \( T^* \) is total temperature; \( c_p \) is constant pressure heat capacity.

Lost head is calculated based on the loss coefficient:

\[
h_v = \zeta \cdot 0.5 w^2,
\]  

(7)

where \( \zeta \) is loss coefficient; \( w \) is relative velocity.

Increase in static temperature due to pressure loss (compared with adiabatic expansion temperature):

\[
\Delta T_{w_{li}} = h_{w_{li}} / c_p
\]  

(8)

Static temperature is calculated by the equation:

\[
T_{2i} = T_v \left( \frac{p_{w_{li}}}{p_{li}} \right)^{\frac{k-1}{k}} + \Delta T_{w_{li}}
\]  

(9)

where \( k \) is isentropic coefficient.
One of the main parts of the mathematical model is calculating pressure loss and blades’ and vanes’ angles of the impeller and the stator elements. These parts of the model are described in more detail below.

3. Head loss calculation
Pressure losses are calculated according to the methodology [19]. The calculated parameters that take into account losses at the outlet of the impeller row are listed in Figure 3.

\[\zeta = \zeta_{\text{imp}} + \zeta_{\text{airf}} + \zeta_{\text{profile}} + \zeta_{\text{outlet}}\]  

(10)

The profile loss coefficient for the impeller and stator element, respectively:

\[\zeta_{\text{profile}} = X \cdot \zeta_{\text{profile}} \cdot K_{\text{Re1}} \cdot K_{\text{Re2}};\]  

(12)

\[\zeta_{\text{profile}} = X \cdot \zeta_{\text{profile}} \cdot K_{\text{Re1}} \cdot K_{\text{Re2}};\]  

(13)

The coefficient of airfoil losses at maximum quality for the impeller and stator element, respectively:

\[\zeta_{\text{airf}} = \frac{0.65 + 2 \left( \frac{\theta_{\text{imp}}}{100} \right)^2}{100 \sin \beta_{\text{imp}} \cdot (t / B)_{\text{imp}}};\]  

(14)

\[\zeta_{\text{airf}} = \frac{0.65 + 2 \left( \frac{\theta_{\text{SE}}}{100} \right)^2}{100 \sin \alpha_{\text{SE}} \cdot (t / B)_{\text{SE}}};\]  

(15)

where \(\alpha\) is flow angle with respect to tangent; \(\theta\) is bend angle.

The value \(\zeta_{\text{airf}}\) according to formulas (14) and (15) is greater than the value of the minimum coefficient of airfoil losses for the mode adopted as the calculated one. The difference is small and can be taken into account along with other factors presented by the empirical coefficient.

In accordance with the methodology, the influence of Mach, Reynolds, and roughness criteria was taken into account in [20, 21, 22, 23, 24, 25, 26, 27].
The influence of compressibility is taken into account using the formula:

$$K_{X,i} = 1 + X(4) \cdot \frac{\lambda^{(5)}}{X}.$$  \hspace{1cm} (16)

The correction form for Reynolds – roughness was not specified in [19]; therefore, a technique similar to that which is used for calculating characteristics of centrifugal stages [29] was applied.

The friction calculation is based on the formulas of the friction force coefficient of the plate

$$c_f = \frac{0.0307}{Re^{1/7}}$$ - hydraulically smooth, or

$$c_f = 0.0162 \left( \frac{K_{\text{roug}}}{B} \right)^{1/7}$$ - rough surface.

The ratio between the coefficient of friction and the loss coefficient is directly proportional:

$$\frac{\zeta_{\text{airf}}}{c} = \frac{c_w}{B} \left( \frac{w_{\text{airf}}}{w_i} \right)^3 \frac{1}{\sin \beta_i}$$ \hspace{1cm} [28]. Therefore, the correction factor can be added to loss calculation formulas as an efficient.

The amendment is introduced as:

$$K_{Re} = \frac{c_f}{c_{f(Kou)}}.$$ \hspace{1cm} (17)

For the assessment of $c_{f(Kou)}$, information from [3] was used. Data on the roughness of the blades’ surfaces in the calculations is most often absent, and the tests in [3] were conducted at $Re_w = 5 \times 10^5$. If the surfaces are assumed to be hydraulically smooth, then $c_{f(Kou)\text{h.s.}} = \frac{0.0307}{Re^{1/7}} = 0.00471$.

In order for the same value to be true with a hydraulically rough surface, the height of the roughness must correspond to the ratio $c_f = 0.0162 \left( \frac{K_{\text{roug}}}{B} \right)^{1/7} = 0.00471$.

With a blade chord of 80 mm in [19], their roughness should be equal to 14 micrometers, which is too large. It is obvious that, when blowing rows in [19], a hydraulically smooth flow took place. Then:

$$K_{Re} = \frac{c_f}{0.00471} = 212.3 \cdot c_f.$$  \hspace{1cm} (18)

Thus, finally, with a hydraulically smooth flow $K_{Re} = \frac{6.518}{Re^{1/7}}$. With a rough surface $K_{Re} = 3.44 \left( \frac{K_{\text{roug}}}{B} \right)^{1/7}$.

Loss coefficient on hub and shroud surfaces:

$$\zeta_{\text{imp}} = X(7) \zeta_{\text{airf imp p}} \left( \frac{t}{B} \right)_{\text{mp}} \left( \frac{B_i}{B_{\text{imp}}} \right)^{1/2} \left( 1 + B_z / B_{\text{mp}} \right);$$ \hspace{1cm} (19)

$$\zeta_{\text{imp SE}} = X(7) \zeta_{\text{airf imp p}} \left( \frac{t}{B} \right)_{\text{SE}} \left( \frac{B_i}{B_{\text{SE}}} \right)^{1/2} \left( 1 + B_z / B_{\text{SE}} \right).$$ \hspace{1cm} (20)

Secondary loss coefficient:

$$\zeta_{\text{imp}} = X(8) \cdot 0.1 \left( \frac{\tan \beta_i - \tan \beta_{2i}}{\tan \beta_{2i}} \right) \sin \beta_{2i} \left( \frac{t}{B} \right)_{\text{mp}} \left( \frac{B_i}{B_{\text{mp}}} \right)^{1/2};$$ \hspace{1cm} (21)

$$\zeta_{\text{imp SE}} = X(8) \cdot 0.1 \left( \sin \alpha_{2i} \right) \left( \frac{t}{B} \right)_{\text{SE}} \left( \frac{B_i}{B_{\text{SE}}} \right)^{1/2}.$$ \hspace{1cm} (22)
4. Calculating the blades’ and vanes’ angles of the impeller and stator elements

The DDAC program uses a mathematical model to calculate the optimal incidence angle and the deviation angle. This allows to form the spatial middle surface of the blades. For this surface, a pressure loss is calculated using a mathematical model and taking into account the flow parameters. For both models, the authors of the DDAC program used the recommendations of the Russian researcher A. Komarov, introducing a number of amendments and additions. 7 empirical coefficients were introduced into A. Komarov’s model, which is logical, since the model is based on a generalization of the results of blowing flat rows, which only partially corresponds to the actual operating conditions of the compressor.

The blades’ and vanes’ angles of the stage elements are calculated at the minimum loss mode, taken as the design mode. The inlet angles of the impeller blades and the stator elements vanes are calculated as the sum of the flow angle and the incidence angle:

\[ \beta_{i1} = \beta_{ii} + i_i; \]
\[ \alpha_{i2j} = \alpha_{i2j} + i_{2j}. \]

where \( \alpha_i \) is vane angle; \( \beta_{ii} \) is blade angle; \( i_i \) is incidence angle.

The outlet angles of the impeller blades and the stator elements vanes are respectively calculated as the sum of the flow angle and the deviation angle:

\[ \beta_{f2i} = \beta_{2i} + \Delta \beta_i; \]
\[ \alpha_{f3j} = \alpha_{3j} + \Delta \alpha_i. \]

In a supersonic flow, the incidence angle for flow path elements is zero.

The total deviation angle of the flow in the elements consists of the deviation angle on the impeller profile and the deviation angle due to viscous flow on hub and shroud surfaces. The deviation angle on hub and shroud surfaces is evenly distributed on all streamlines. For impeller blades and stator elements vane, respectively:

\[ \Delta \beta_i = \Delta \beta_{aif} + \Delta \beta_h, \text{deg}; \]
\[ \Delta \alpha_i = \Delta \alpha_{aif} + \Delta \alpha_h, \text{deg}. \]

The deviation angle on the airfoils in the impeller and stator elements, respectively, according to Komarov [19]:

\[ \Delta \beta_{aif} = \left[ 0.26(2\bar{B}_f) \left( \frac{t}{B} \right)_{imp} + 0.18K_{imp} \right] \theta_{imp}; \]
\[ \Delta \alpha_{aif} = \left[ 0.26(2\bar{B}_{f SE}) \left( \frac{t}{B} \right)_{SE} + 0.18K_{SE} \right] \theta_{SE}. \]

The curvature angle of the impeller blades and the stator elements vanes, respectively:

\[ \theta_{imp} = \beta_{2i} - \beta_{ii} + \Delta \beta_i - i_i, \text{deg}; \]
\[ \theta_{SE} = \alpha_{3j} - \alpha_{2i} + \Delta \alpha_i - i_{2j}, \text{deg}. \]

An analysis of the formulas shows that the calculation of the blades’ angles leads to the necessity of applying iterative processes. During the first stage, it is necessary to set the first approximation of the values of the deviation angle and the incidence angle. In the future, these values are refined until the differences between their values at each iteration become minimal.

The relative pitch of the blade row on the streamline is calculated:

\[ \left( \frac{t}{B} \right)_{imp} = \left( \frac{t}{B} \right)_{imp} \frac{r_i}{r_{sh}}. \]

The generalized relative pitch of the blade row:

\[ \left( \frac{t}{B} \right)_{imp} = \left( \frac{t}{B} \right)_{imp}. \]
If \((t / B)\) \(> 1.0\) the generalized relative pitch of the blade row is:

\[
(t / B)_\text{imp}^* = 2 - 1 / (t / B)_\text{imp}.
\] (34)

The coefficient \(K_{\text{imp}}\) included in the equations is calculated by the formulas:

\[
K_{\text{imp}} = 1.0 \text{ at } (t / B)_\text{imp}^* \leq 1.0; \tag{35}
\]

\[
K_{\text{imp}} = 1 / (t / B)_\text{imp}^* \text{ at } (t / B)_\text{imp}^* > 1.0. \tag{36}
\]

The position of the maximum deflection point \(B_{f, \text{imp}}\) is set by the user. In most cases, this value is 0.6.

The deviation angle due to viscous flow on hub and shroud surfaces according to Komarov:

\[
\Delta \beta_h = 2.5 \left( \text{ctg} \beta_i - \text{ctg} \beta_2 \right) \left( \frac{(t / B)_{\text{imp}}}{(t / B)_{\text{imp}}} \right) \sin \beta_2, \text{ deg}; \tag{37}
\]

\[
\Delta \alpha_h = 2.5 \text{ctg} \beta_2 \left( \frac{(t / B)_{\text{SE}}}{(t / B)_{\text{SE}}} \right), \text{ deg.} \tag{38}
\]

The incidence angle at the minimum loss coefficient in the impeller and stator elements, respectively:

\[
i_{\text{i, imp}} = 6 - \frac{1}{3} \theta_{\text{i, imp}} \left( \frac{t / B}{(t / B)_{\text{imp}}} \right) \left[ 1.81 - \left( 2 B_{f, \text{imp}} \right)^2 \right] - 6 \left[ 1 - \left( \frac{1}{60} \right)^2 \right], \text{ deg}; \tag{39}
\]

\[
i_{\text{i, SE}} = 6 - \frac{1}{3} \theta_{\text{i, SE}} \left( \frac{t / B}{(t / B)_{\text{SE}}} \right) \left[ 1.81 - \left( 2 B_{f, \text{SE}} \right)^2 \right] - 6 \left[ 1 - \left( \frac{1}{60} \right)^2 \right], \text{ deg.} \tag{40}
\]

5. Description of the program’s functionality

Mathematical models described above were implemented in the DDAC computer program. To calculate the compressor, gas parameters at the compressor inlet are set (total pressure and temperature, isentropic index, gas dynamic viscosity) as well as its main design parameters (calculated mass flow rate, number of stages and rpm). For each compressor stage, the geometric dimensions of the blades are set: the value of the blades’ chord on the hub and shroud, the axial length of the blades, the position of the maximum arrow deflection of the blades, the number of blades (Figure 5). The measurement of the blades’ chord value along its length is assumed to be linear, respectively, on each stream line the current value of the chord is calculated by the formula (shown through the example of the impeller):

\[
B_{\text{imp, i}} = B_{\text{imp, hub}} + \frac{B_{\text{imp, sh}} - B_{\text{imp, hub}}}{r_{\text{1, sh}} - r_{\text{1, hub}}} (r_{\text{i, sh}} - r_{\text{i, hub}}) \tag{41}
\]

In addition, the hub diameters in the control sections are set. This limits the set of geometrical dimensions of the compressor necessary for its initial design to the specified parameters. If the program is used to create a digital twin of an existing compressor the user sets the shroud diameters in the control sections (Figure 6). The appearance of the compressor’s meridional section is shown to the user in a simplified form.
Figure 4. DDAC program. Defining inlet gas parameters and compressor parameters
### Figure 5. DDAC program. Setting geometrical and gas-dynamic parameters of the compressor stage

In addition to the geometric parameters of the compressor, the user sets a number of gas-dynamic parameters. First of all, these are the parameters that determine the amount of energy supplied to the gas: the angle of the stream at the outlet of the impeller on the hub $\beta_{\text{hub}} \leq 90^\circ$ and the parameters that determine the law of measuring the loading factor along the blade height $\frac{h_{\text{hub}}}{h_{\text{hub}}}$, $m_2$. On each line of the theoretical flow pressure is calculated by the formula:
\[ h_{ri} = h_{rhub} + \left( h_{rhub} \frac{h_{rsh}}{h_{rsh}} - h_{rhub} \right) \left( \frac{r_{li} - r_{lhub}}{r_{lsh} - r_{lhub}} \right)^{m_2} \] (42)

In addition, values of the flow rates in the control sections of the flow path are set. When calculating with a set shroud circuit, the value of the axial component of the velocity in the control sections that provide the required peripheral sizes is iteratively calculated. Parameters \( c_{z0}, c_{z1} / c_{z0}, c_{z2} / c_{z1}, c_{z3} / c_{z2} \) are determined during the calculation and are not set by the user.

The value of the tangential component of the velocity at the exit from the stator elements is set on the hub and the shroud, the law of change in the height of the blade is set by the degree indicator \( m_3 \). Then, on each streamline, the tangential velocity component is calculated by the formula:

\[ c_{u3i} = c_{u3hub} + \left( c_{u3sh} - c_{u3hub} \right) \left( \frac{r_{li} - r_{lhub}}{r_{lsh} - r_{lhub}} \right)^{m3} \] (43)

Figure 6. DDAC program. Defining compressor hub and shroud

6. Conclusion
The presented mathematical model and the computer program created on its basis were used in carrying out the project "Development of technical solutions and prototypes of devices - a combustion chamber, a low-pressure compressor to create competitive gas turbines with a capacity of 25 MW for gas pumping units based on digital twins of the developed devices". In the digital twin mode of the existing compressor, the gas-dynamic parameters of a four-stage axial transonic compressor were simulated in the design mode.

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