Stability of superfluid $^3$He-B in compressed aerogel

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In recent work it was shown that new anisotropic $p$-wave states of superfluid $^3$He can be stabilized within high porosity silica aerogel under uniaxial positive strain. In contrast, the equilibrium phase in an unstrained aerogel is the isotropic superfluid B-phase. Here we report that this phase stability depends on the sign of the strain. For negative strain of $\sim 20\%$ achieved by compression, the B-phase can be made more stable than the anisotropic A-phase resulting in a tricritical point for A, B, and normal phases with a critical field of $\sim 100$ mT. From pulsed NMR measurements we identify these phases and the orientation of the angular momentum.

The natural state of superfluid $^3$He is completely free of impurities. In zero magnetic field it has two phases, one of which has an isotropic order parameter amplitude, the B-phase, and the other, the A-phase, is axially anisotropic. Near the superfluid transition temperature $T_c$, energetics favor the A-phase at pressures greater than 21 bar. However, in the presence of quenched disorder from high-porosity, silica aerogel, theory indicates that isotropic scattering of the $^3$He quasiparticles will tip the stability balance in favor of the isotropic phase. Furthermore, it is predicted that anisotropic scattering will have the opposite effect, stabilizing anisotropic phases like the A-phase. These predictions have been confirmed experimentally for isotropic and anisotropic scattering, where scattering anisotropy was achieved by stretching the aerogel during growth producing a positive uniaxial strain. In this Letter we report our discovery that negative strain from compression, contrary to the theory, stabilizes the isotropic phase and suppresses the A-phase. The competition between the strain and the magnetic field reveals a new critical point. The A-phase. The equilibrium phase boundaries $T_{BA}(H)$ at $P = 26.3$ bar from B to A-phases on warming, are shown in Fig. 2, for all three samples as a function of magnetic field. For the isotropic aerogel, $\varepsilon = 0$ (sample 1, green squares), the extrapolation of $T_{BA}$ for $H^2 \rightarrow 0$ occurs precisely at $T_{BA}/T_c = 1$. This is expected for a quadratic field contribution to the Ginzburg-Landau free energy following exactly the same behavior as is well-established for pure $^3$He below the polycritical point, $P < 21$ bar. However, for the anisotropic sample was then compressed to a strain $\varepsilon = -19.4\%$, directed along the field, $\varepsilon \parallel H$, Fig. 1a. We refer to this sample as sample 2. Sample 3, was grown and characterized in the same way as was sample 1, and was compressed to a strain $\varepsilon = -22.5\%$ oriented with $\varepsilon \perp H$, Fig. 1b.

Pulsed NMR measurements were performed at a pressure $P = 26.3$ bar in magnetic fields ranging from $H = 49.1$ to 196 mT. After an RF pulse that tips the nuclear magnetization of the $^3$He atoms by an angle $\beta$ away from the external field, Fourier transformation of the free induction decay signal was phase corrected to obtain absorption spectra, as shown in the inset to Fig. 1c. The magnetic susceptibility, $\chi$, was determined from the numerical integral of the spectrum. The thermodynamic transition between superfluid states was precisely determined from the susceptibility discontinuity on warming (see Fig. 1c) displayed in phase diagrams in Fig. 2. On cooling the supercooled transitions appear $\approx 300\mu$K below $T_c$ for all samples and magnetic fields, as indicated in Fig. 1c. The frequency shift of the spectrum, $\Delta \omega$, was calculated from the first moment of each spectrum relative to the Larmor frequency, $\omega_L$, given by the resonance frequency in the normal state. From the spectra in the inset of Fig. 1c, it is clear that $\varepsilon \parallel H$ has a positive shift and $\varepsilon \perp H$ has a negative shift in the A-phase. We infer that the orientation of the strain axis relative to the field controls the sign of the frequency shift; this implies specific directions for the angular momentum which we will discuss later in the context of theory.

In sufficiently large magnetic fields, an equal-spin-pairing (ESP) state, $i.e.$ having the same susceptibility as the normal state, is stabilized near $T_c$, Fig. 2. For sample 1 we have established earlier that this ESP phase is the A-phase. The equilibrium phase boundaries $T_{BA}(H)$ at $P = 26.3$ bar from B to A-phases on warming, are shown in Fig. 2, for all three samples as a function of magnetic field. For the isotropic aerogel, $\varepsilon = 0$ (sample 1, green squares), the extrapolation of $T_{BA}$ for $H^2 \rightarrow 0$ occurs precisely at $T_{BA}/T_c = 1$. This is expected for a quadratic field contribution to the Ginzburg-Landau free energy following exactly the same behavior as is well-established for pure $^3$He below the polycritical point, $P < 21$ bar. However, for the anisotropic
aerogels, $\varepsilon < 0$ (sample 2, blue circles and sample 3, red triangles) the B-phase stability is enhanced with respect to the A-phase as a result of a positive offset of this quadratic field dependence described by,

$$1 - \frac{T_{BA}}{T_c} = g_{BA} \left( \frac{H^2 - H_0^2}{H_0^2} \right) + O \left( \frac{H}{H_0} \right)^4,$$

thereby defining a critical field $H_c$ at $T_{BA}/T_c = 1$. Here $H_c$ is 88.6 mT and 97.8 mT for samples 2 and 3 respectively. $H_0$ and $g_{BA}$ are superfluid constants from the GL theory \[3\] [15].

We have calculated the quasiparticle mean free path, $\lambda$, by fitting our results to Eq. 1 using the homogeneous isotropic scattering model \[3\] [13]. For sample 1, the isotropic aerogel, $H_c = 0$ and we find $\lambda = 210$ nm. Similarly, for the compressed aerogel samples 2 and 3 we have calculated $\lambda$ from the slope of the phase line in Fig. 2 to be 250 and 220 nm respectively, all of which are very reasonable values for a 98% porous aerogel \[12\].

The existence of a critical field, $H_c$, is unexpected. It indicates that anisotropic impurity scattering with $\varepsilon < 0$ introduces a new additive term in the free energy which depends on strain. Furthermore, contrary to theory \[3\] [15] [16], it appears that aerogel anisotropy induced by uniaxial compression enhances the B-phase stability relative to the A-phase, countering the effect of an applied magnetic field.

Based on symmetry arguments there are two possibilities for the direction of the angular momentum, \( \hat{l} \), in the presence of a large uniaxial strain which determine the sign of the frequency shift in the anisotropic A-phase. One model requires that the angular momentum be parallel to the strain $||\varepsilon$, called the easy-axis model, which was predicted for $\varepsilon < 0$ by two different theories \[7\] [11]. The other, the easy-plane model, allows the angular momentum to be in a plane perpendicular to the strain, $\perp\varepsilon$, not favored by either theory if $\varepsilon < 0$.

Using NMR frequency shift measurements as a function of temperature and NMR tip angle we have investigated the orientation of the angular momentum for these two models. The dipole energy in the A-phase is \[17\],

$$F_D = -\frac{1}{2} \Omega_A^2 \gamma^2 (\hat{l} \cdot \hat{d})^2$$

where $\Omega_A$ is the longitudinal resonance frequency, $\gamma$ is the gyromagnetic ratio, $\chi_A$ is the nuclear magnetic susceptibility and $\hat{d}$ is a spin-space vector constrained to be perpendicular to the spin angular momentum, $\hat{s}$, while minimizing $F_D$. The relative orientation of the A-phase order parameters, $\hat{l}$ and $\hat{d}$, can be parametrized by two angles $\theta$ and $\phi$, where $\theta$ is the angle between $\hat{l}$ and the magnetic field $\hat{H}$; and $\phi$ is

![Figure 1](image1.png)

**FIG. 1. (Color online).** a) and b) Schematics of the experimental arrangements for samples 2 and 3. Sample 2, $\varepsilon = -19.4\%$, has strain axis parallel to the magnetic field. For sample 3, $\varepsilon = -22.5\%$, the strain axis is perpendicular to the magnetic field. c) Liquid susceptibility normalized to the susceptibility of the normal state on warming (open red circles) versus reduced temperature. d) Liquid susceptibility normalized to the susceptibility of the normal state on warming (open red circles) versus reduced temperature.

![Figure 2](image2.png)

**FIG. 2. (Color online).** Superfluid phase diagram $T_{BA}/T_c$ versus $H^2$ for three samples at a pressure of $P = 26.3$ bar from warming experiments. Two of the three have negative strain, $\varepsilon < 0$, and one is isotropic, $\varepsilon = 0$. All three have the common feature of a quadratic dependence on magnetic field for the B to A-phase transitions. Remarkably, for negative strain there appears to be a critical field, $H_c$. 

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the angle between \( \hat{d} \) and the projection of \( \hat{l} \) in the plane perpendicular to the field. We note that \( \Omega \) should be approximately the same for all three samples since the quasiparticle mean free paths are very similar \[12\]. The dependence of the frequency shift on tip angle, \( \beta \), is given by \[11, 18\]:

\[
\Delta \omega = \frac{\Omega_A^2}{2\omega_L} \left( -\cos \beta + \left( \frac{7}{4} \cos \beta + \frac{1}{4} \right) \langle \sin^2 \theta(r) \rangle \right) \\
- \frac{\Omega_A^2}{2\omega_L} \left( \frac{1}{2} (1 + \cos \beta) \langle \sin^2 \phi(r) \rangle \langle \sin^2 \theta(r) \rangle \right),
\]

where \( \langle \sin^2 \theta(r) \rangle \) and \( \langle \sin^2 \phi(r) \rangle \) are spatial averages over a dipole length \( \xi_D \) at location \( r \). According to the easy-axis model, \( \varepsilon \parallel \hat{l} \), we have \( \theta = 0 \) for sample 2 (\( \varepsilon \parallel \hat{H} \)) and \( \theta = \pi \), \( \phi = 0 \) for sample 3 (\( \varepsilon \perp \hat{H} \)). For these two orientations of the strain relative to the magnetic field Eq. 2 becomes:

\[
\Delta \omega_{\varepsilon \parallel \hat{H}} (\beta) = -\frac{\Omega_A^2}{2\omega_L} \cos \beta, \quad (3)
\]

\[
\Delta \omega_{\varepsilon \perp \hat{H}} (\beta) = \frac{\Omega_A^2}{2\omega_L} \frac{(3 \cos \beta + 1)}{4}. \quad (4)
\]

Our frequency shift data in the small tip angle limit, Fig. 3a and 3c have signs opposite to Eqs. 3 and 4. Furthermore, the magnitude of the shifts observed are smaller than the predictions for the easy-axis model, which are the maximum (minimum) possible shifts for the A-phase often referred to as dipole-locked (unlocked) configurations \[17\], shown as black solid (dashed) curves. Consequently the easy-axis model is incorrect.

For the easy-plane model, \( \hat{l} \perp \varepsilon \), there is a continuous symmetry for the angular momentum in the plane perpendicular to the strain. This will lead to a 2-dimensional (2D) superfluid glass phase similar to the 3D glass state observed for sample 1 \[3\]. In this case, for sample 2, \( \theta = \frac{\pi}{2} \) and \( \langle \sin^2 \theta \rangle = 1 \); whereas for sample 3, \( 0 < \theta < \frac{\pi}{2} \) and \( \langle \sin^2 \phi \rangle = \frac{1}{2} \). To calculate \( \Delta \omega \) we must constrain \( \phi \) and we consider two possible scenarios. In the first scenario, the \( \hat{d} \)-vector is also disordered on a length scale smaller than a dipole length then, \( 0 < \phi < 2\pi \) and \( \langle \sin^2 \phi \rangle = \frac{1}{2} \), and Eq. 2 gives,

![FIG. 3. (Color online). NMR frequency shifts as a function of temperature and tip angle. The magnitude of the shifts presented here has been scaled to a common field of \( H = 196 \) mT, and the tip angle dependence has been scaled to a common temperature of \( T/T_c = 0.8 \) based on our results from sample 1. In all the panels, the dipole-locked configuration, \( \hat{l} \parallel \hat{H} \) is shown by the black solid curves, and the black dashed curves are the dipole-unlocked configuration, \( \hat{l} \perp \hat{H} \), with \( \Omega_A \) taken from the measurements for sample 1 \[2\]. Comparison of frequency shifts in the A-phase: a, b) for \( \varepsilon \parallel \hat{H} \) and c,d) \( \varepsilon \perp \hat{H} \). a) Dependence on reduced temperature with small tip angle, \( \beta < 20^\circ \). a) warming in \( H = 174 \) mT, yellow crosses; cooling in \( H = 174 \) mT, green x; cooling in \( H = 95.6 \) mT, blue circles. c) cooling in \( H = 79.2 \) mT, red circles. b, d) Dependence on tip angle after cooling from the normal state. b) \( T/T_c = 0.87, H = 49.1 \) mT, green squares; \( T/T_c = 0.82, H = 196 \) mT, blue circles. d) \( T/T_c = 0.78, H = 95.6 \) mT, red circles. The blue and red solid curves are calculated for the easy-plane, 2D glass model, see text.](image-url)
\[ \Delta \omega_{\parallel H} (\beta) = \frac{\Omega_A^2}{2 \omega_L} \left( \frac{1}{2} \cos \beta \right). \] (5)

\[ \Delta \omega_{\perp H} (\beta) = \frac{\Omega_A^2}{2 \omega_L} \left( -\frac{1}{4} \cos \beta \right). \] (6)

Our calculations for this case, shown as solid blue and red curves in Fig. 3b and 3d, are in excellent agreement with our measurements as a function of tip angle and coincide with the measured temperature dependences for both orientations of the strain relative to the magnetic field, Fig. 3a and 3c. From this calculation, we obtained the value of $\Omega_A$ at $T/T_c = 0.8$, $\Omega_A = 51.0$, 54.0 and 51.2 kHz for sample 1, 2 and 3 respectively, within the combined measurement errors of 10%. The consistency in the values of $\Omega_A$ strongly supports this scenario.

For the second scenario, the $\mathbf{d}$-vector has a uniform orientation on length scales longer than the dipole length. Then Eq. 5 holds for sample 2; however, for sample 3, $\phi = 0$ and $\langle \sin^2 \phi \rangle = 0$, and from Eq. 2,

\[ \Delta \omega_{\perp H} (\beta) = \frac{\Omega_A^2}{8 \omega_L} (1 - \cos \beta). \] (7)

At small tip angles the frequency shift from Eq. 7 is zero for all temperatures, inconsistent with the data in Fig. 3c. Additionally, the tip angle dependence from Eq. 7, shown as a blue dash-dotted curve in Fig. 3d, is inconsistent with the data indicating that the first scenario better describes the $\mathbf{d}$-vector orientation. We infer that the easy-plane model correctly describes the orientation of the angular momentum to be constrained to a plane perpendicular to the strain-axis for a compressed aerogel, giving rise to a 2D glass phase in that plane.

Recent measurements of the superfluid density, $\rho_s/\rho$, in zero applied field with a torsional oscillator using a similarly prepared aerogel sample with 10% compression \[19\] found evidence of a stable phase just below $T_c$. In the magnetic field-temperature plane, such a stable phase might occupy the shaded area, Fig. 2, resulting in a positive slope of the phase boundary to the B-phase. We cannot make a direct comparison with these results since our NMR experiments have not been performed in sufficiently low magnetic fields. However, according to the Clausius-Clapeyron relation such a stable phase could not be an ESP state.

In summary, we have investigated the nature of superfluid $^3$He-A in two uniformly anisotropic aerogel samples with negative strain achieved with uniaxial compression of $\sim 20\%$ and we have compared them with isotropic aerogel. We discovered a critical field corresponding to a new critical point ($T_c, H_c$) at a pressure of $P = 26.3$ bar and that negative strain enhances the stability of the isotropic B-phase over the A-phase in a magnetic field, in contrast with existing theories \[13, 15, 16\]. Furthermore, the angular momentum in the A-phase is not aligned with the strain axis as has been predicted \[13, 15\], rather it forms a 2D glass phase in the plane perpendicular to the strain.

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