Using multipoles of the correlation function to measure $H(z)$, $D_A(z)$ and $\beta(z)$ from Sloan Digital Sky Survey luminous red galaxies

Chia-Hsun Chuang$^1$† and Yun Wang$^2$

$^1$Instituto de Física Teórica, (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain
$^2$Homer L. Dodge Department of Physics & Astronomy, University of Oklahoma, 440 W Brooks Street, Norman, OK 73019, USA

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ABSTRACT

Galaxy clustering data can be used to measure the cosmic expansion history $H(z)$, the angular diameter distance $D_A(z)$ and the linear redshift-space distortion parameter $\beta(z)$. Here we present a method for using effective multipoles of the galaxy two-point correlation function ($\hat{\xi}_0(s)$, $\hat{\xi}_2(s)$, $\hat{\xi}_4(s)$ and $\hat{\xi}_6(s)$, with $s$ denoting the comoving separation) to measure $H(z)$, $D_A(z)$ and $\beta(z)$, and validate it using LasDamas mock galaxy catalogues. Our definition of effective multipoles explicitly incorporates the discreteness of measurements, and treats the measured correlation function and its theoretical model on the same footing. We find that for the mock data, $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$ captures nearly all the information, and gives significantly stronger constraints on $H(z)$, $D_A(z)$ and $\beta(z)$, compared to using only $\hat{\xi}_0 + \hat{\xi}_2$.

We apply our method to the sample of luminous red galaxies from the Sloan Digital Sky Survey Data Release 7 without assuming a dark energy model or a flat universe. We find that $\hat{\xi}_4(s)$ deviates on scales of $s < 60 \text{ Mpc} h^{-1}$ from the measurement from mock data [in contrast to $\hat{\xi}_0(s)$, $\hat{\xi}_2(s)$ and $\hat{\xi}_6(s)$]; thus, we only use $\hat{\xi}_0 + \hat{\xi}_2$ for our fiducial constraints. We obtain $\{H(0.35), D_A(0.35), \Omega_m h^2, \beta(z)\} = \{79.6^{+8.3}_{-8.7} \text{ km s}^{-1} \text{ Mpc}^{-1}, 1057^{+88}_{-87} \text{ Mpc}, 0.103 \pm 0.015, 0.44 \pm 0.15\}$ using $\hat{\xi}_0 + \hat{\xi}_2$. We find that $H(0.35) r_s(z_d)/c$ and $D_A(0.35)/r_s(z_d)$ [where $r_s(z_d)$ is the sound horizon at the drag epoch] are more tightly constrained: $\{H(0.35) r_s(z_d)/c, D_A(0.35)/r_s(z_d)\} = \{0.0437^{+0.044}_{-0.043}, 0.48^{+0.043}_{-0.043}\}$ using $\hat{\xi}_0 + \hat{\xi}_2$.

Key words: cosmology: observations – distance scale – large-scale structure of Universe.

1 INTRODUCTION

The cosmic large-scale structure from galaxy redshift surveys provides a powerful probe of dark energy and the cosmological model that is highly complementary to the cosmic microwave background (CMB) (Bennett et al. 2003), supernovae (Riess et al. 1998; Perlmutter et al. 1999) and weak lensing (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; Van Waerbeke et al. 2000; Wittman et al. 2000). The scope of galaxy redshift surveys has dramatically increased in the last decade. The PSCz surveyed ~15 000 galaxies using the Infrared Astronomical Satellite (Saunders et al. 2000), the 2dF Galaxy Redshift Survey obtained 221 414 galaxy redshifts (Colless et al. 2001, 2003) and the Sloan Digital Sky Survey (SDSS) has collected 930 000 galaxy spectra in the Seventh Data Release (DR7; Abazajian et al. 2009). WiggleZ has collected 240 000 emission-line galaxies at $0.5 < z < 1$ over 10 000 deg$^2$ (Blake et al. 2009; Parkinson et al. 2012) and BOSS is surveying 1.5 million luminous red galaxies (LRGs) at $0.1 < z < 0.7$ over 10 000 deg$^2$ (Eisenstein et al. 2011). The BOSS data set has been made publicly available recently in the SDSS DR9 (Anderson et al. 2012; Manera et al. 2012; Nuza et al. 2012; Reid et al. 2012; Sanchez et al. 2012; Tojeiro et al. 2012; Samushia et al. 2013). The planned space mission Euclid$^1$ will survey over 60 million emission-line galaxies at $0.7 < z < 2$ over 15 000 deg$^2$ (Cimatti et al. 2009; Wang et al. 2010; Laureijs et al. 2011).

Large-scale structure data from galaxy redshift surveys can be analysed using either the power spectrum or the correlation function. Although these two methods are Fourier transforms of one another, the analysis processes are quite different and the results cannot be converted using Fourier transform directly because of the finite size of the survey volume. The SDSS data have been analysed using both the power spectrum method (see e.g. Tegmark et al. 2004; Hutsi 2005; Blake et al. 2007; Padmanabhan et al. 2007; Percival et al. 2007, 2010; Reid et al. 2010; Montesano, Sanchez & Phleps 2012) and the correlation function method (see e.g. Eisenstein et al. 2011).
The SDSS-III/I/II has observed one-quarter of the entire sky and performs a redshift survey of galaxies, quasars and stars in the five passbands $u$, $g$, $r$, $i$ and $z$ with a 2.5-m telescope (Fukugita et al. 1996; Gunn et al. 1998, 2006). We use the public catalogue, the NYU Value-Added Galaxy Catalog (NYU-VAGC; Blanton et al. 2005), derived from the SDSS-II final public data release, DR7 (Abazajian et al. 2009). We select our LRG sample from the NYU-VAGC with the flag primTarget bit mask set to 32. $K$-corrections have been applied to the galaxies with a fiducial model [A cold dark matter (CDM) with $\Omega_m = 0.3$ and $h = 1$], and the selected galaxies are required to have rest-frame $g$-band absolute magnitudes $-23.2 < M_g < -21.2$ (Blanton & Roweis 2007). The same selection criteria were used in previous papers (Eisenstein et al. 2005; Zehavi et al. 2005; Okumura et al. 2008; Kazin et al. 2010a). The sample we use is referred to as ‘DR7full’ in Kazin et al. (2010a). Our sample includes 87 000 LRGs in the redshift range 0.16–0.44. Spectra cannot be obtained for objects closer than 55 arcsec within a single spectroscopic tile due to the finite size of the fibre. To correct for these ‘collisions’, the redshift of an object that failed to be measured would be assigned to be the same as the nearest successfully observed one. Both fibre collision corrections and $K$-corrections have been made in the NYU-VAGC (Blanton et al. 2005). The collision corrections applied here are different from what has been suggested in Zehavi et al. (2005). However, the effect should not be significant since we are using relatively large scale which are less affected by the collision corrections.

We construct the radial selection function as a cubic spline fit to the observed number density histogram with the width $\Delta z = 0.01$. The NYU-VAGC provides the description of the geometry and completeness of the survey in terms of spherical polygons. We adopt it as the angular selection function of our sample. We drop the regions with completeness below 60 per cent to avoid unobserved plates (Zehavi et al. 2005). The Southern Galactic Cap (SGC) region is also dropped because it consists of three non-contiguous stripes in all, and only half as many mocks are available if we include the SGC in our analysis.

### 3 METHODOLOGY

In this section, we describe the measurement of the multipoles of the correlation function from the observational data, construction of the theoretical prediction, and the likelihood analysis that leads to constraints on dark energy and cosmological parameters.

#### 3.1 Measuring the 2D, two-point correlation function

We convert the measured redshifts of galaxies to comoving distances by assuming a fiducial model, $\Lambda$CDM with $\Omega_m = 0.25$. We use the two-point correlation function (2PCF) estimator given by Landy & Szalay (1993):

$$\xi(\sigma, \pi) = \frac{DD(\sigma, \pi) - 2DR(\sigma, \pi) + RR(\sigma, \pi)}{RR(\sigma, \pi)},$$

where $\pi$ is the separation along the LOS, $\sigma$ is the separation in the plane of the sky, $DD$, $DR$ and $RR$ represent the normalized data–data, data–random and random–random pair counts, respectively, in a distance range. The LOS is defined as the direction from the observer to the centre of a pair. The bin size we use here is $1 \times 1 \ h^{-1} \text{Mpc}$. The Landy & Szalay estimator has minimal variance for a Poisson process. Random data are generated with the same radial and angular selection functions as the real data. One can reduce the shot noise due to random data by increasing the number of random data. The number of random data we use is 10 times that of the real data. While calculating the pair counts, we assign to each data point a radial weight of $1/[1 + n(z)P_w]$, where $n(z)$ is the radial selection function and $P_w = 4 \times 10^4 \ h^{-3} \text{Mpc}^{-3}$ (Eisenstein et al. 2005). The weight function is included to minimize the variance of clustering measurements for an inhomogeneous sample (Feldman, Kaiser & Peacock 1994).

#### 3.2 Theoretical 2D, two-point correlation function

We compute the linear power spectrum by using CAMB (Lewis, Challinor & Lasenby 2000). To include the effect of non-linear structure formation on the BAOs, we first calculate the dewiggled power spectrum

$$P_{dw}(k) = P_{lin}(k) \exp \left( -\frac{k^2}{2k_0^2} \right) + P_{nw}(k) \left[ 1 - \exp \left( -\frac{k^2}{2k_0^2} \right) \right],$$

where $P_{lin}(k)$ is the linear matter power spectrum, $P_{nw}(k)$ is the no-wiggle or pure CDM power spectrum calculated using equation (29) from Eisenstein & Hu (1998) and $k_0$ is marginalized over with a flat prior over the range of 0.09–0.13.
We then use the software package \texttt{HALOFIT} (Smith et al. 2003) to compute the non-linear matter power spectrum:

\begin{equation}
  r_{\text{halofit}}(k) = \frac{P_{\text{halofit},nw}(k)}{P_{\text{ws}}(k)},
\end{equation}

\begin{equation}
  P_{nl}(k) = P_{nl}(k) r_{\text{halofit}}(k),
\end{equation}

where \( P_{\text{halofit},nw}(k) \) is the power spectrum obtained by applying \texttt{HALOFIT} to the no-wiggle power spectrum and \( P_{nl}(k) \) is the non-linear power spectrum. We compute the theoretical real-space two-point correlation function, \( \xi(r) \), by Fourier transforming the non-linear power spectrum \( P_{nl}(k) \).

In the linear regime (i.e. large scales) and adopting the small-angle approximation (which is valid on scales of interest), the 2D correlation function in the redshift space can be written as (Kaiser 1987; Hamilton 1992)

\begin{equation}
  \xi^\ast(\sigma, \pi) = \xi_0(s) P_0(\mu) + \xi_3(s) P_3(\mu) + \xi_4(s) P_4(\mu),
\end{equation}

where \( s = \sqrt{\sigma^2 + \pi^2} \), \( \mu \) is the cosine of the angle between \( s = (\sigma, \pi) \) and the LOS, and \( P_i \) are Legendre polynomials. The multipoles of \( \xi \) could be expressed as

\begin{equation}
  \xi_0(r) = \left( 1 + \frac{2 \beta^2}{3} + \frac{\beta^4}{5} \right) \xi(r),
\end{equation}

\begin{equation}
  \xi_3(r) = \frac{4 \beta^2 + 4 \beta^4}{7} \xi(r) - \xi_0(r),
\end{equation}

\begin{equation}
  \xi_4(r) = \frac{8 \beta^2}{35} \left[ \xi(r) + \frac{5}{2} \xi_0(r) - \frac{7}{2} \xi_2(r) \right],
\end{equation}

where \( \beta \) is the redshift-space distortion parameter and

\begin{equation}
  \xi(r) = \frac{3}{r^2} \int_0^r \xi(r') r'^2 \, dr',
\end{equation}

\begin{equation}
  \bar{\xi}(r) = \frac{5}{r^5} \int_0^r \xi(r') r'^4 \, dr'.
\end{equation}

Next, we convolve the 2D correlation function with the distribution function of random pairwise velocities, \( f(v) \), to obtain the final model \( \xi(\sigma, \pi) \) (Peebles 1980):

\begin{equation}
  \xi(\sigma, \pi) = \int_{-\infty}^{\infty} \xi^\ast(\sigma, \pi - \frac{v}{H(z)\mu(z)}) f(v) \, dv,
\end{equation}

where the random motions are represented by an exponential form (Ratcliffe et al. 1998; Landy 2002):

\begin{equation}
  f(v) = \frac{1}{\sigma_v \sqrt{2\pi}} \exp\left( -\frac{v^2}{2\sigma_v^2} \right),
\end{equation}

where \( \sigma_v \) is the pairwise peculiar velocity dispersion.

The parameter set we use to compute the theoretical correlation function is \{\( H(z), D_s(z), \beta, \Omega_m h^2, \Omega_b h^2, n_s, \sigma_v, k_s \)\}, where \( \Omega_m \) and \( \Omega_b \) are the density fractions of matter and baryons, \( n_s \) is the power-law index of the primordial matter power spectrum, and \( h \) is the dimensionless Hubble constant (\( H_0 = 100 h \, \text{km s}^{-1} \, \text{Mpc}^{-1} \)). We set \( h = 0.7 \) while calculating the non-linear power spectra. On the scales we use for comparison with data, the theoretical correlation function only depends on cosmic curvature and dark energy through parameters \( H(z), D_s(z) \) and \( \beta(z) \), assuming that dark energy perturbations are unimportant (valid in the simplest dark energy models). Thus, we are able to extract constraints from data that are independent of a dark energy model and cosmic curvature.

### 3.3 Effective multipoles of the correlation function

From equations (5) and (11), we define

\begin{equation}
  \hat{\xi}(s) \equiv \int_{-\infty}^{\infty} d\sigma f(\sigma) \xi(\sigma, \pi) \left( \frac{\sigma^2 + \pi^2}{H(z)\mu(z)} \right)^{\frac{1}{2}},
\end{equation}

\begin{equation}
  \xi_0(s) = \frac{2l + 1}{2} \int_{-\infty}^{\infty} d\mu \xi(\sigma, \pi) P_l(\mu),
\end{equation}

\begin{equation}
  \xi_2(s) = \frac{2l + 1}{2} \int_{-\infty}^{\infty} d\theta \sqrt{1 - \mu^2} \xi(\sigma, \pi) P_l(\mu),
\end{equation}

where \( \mu = \cos \theta \) and \( P_l(\mu) \) is the Legendre polynomial \( (l = 0, 2, 4 \) and 6 here). Note that we are integrating over a spherical shell with radius \( s \), while actual measurements of \( \xi(\sigma, \pi) \) are done in discrete bins. To compare the measured \( \xi(\sigma, \pi) \) and its theoretical model on the same footing, we convert the last integral in equation (13) into a sum. This leads to our definition for the effective multipoles of the correlation function:

\begin{equation}
  \hat{\xi}(s) \equiv \frac{\sum_{l=0}^{\infty} (2l + 1) \xi(l, \pi) P_l(\mu) \sqrt{1 - \mu^2}}{\text{Number of bins used in the numerator}},
\end{equation}

where \( \Delta s = 5 h^{-1} \text{ Mpc} \) in this work, and

\begin{equation}
  \sigma = \left( n + \frac{1}{2} \right) h^{-1} \text{ Mpc}, \quad n = 0, 1, 2, \ldots
\end{equation}

\begin{equation}
  \pi = \left( m + \frac{1}{2} \right) h^{-1} \text{ Mpc}, \quad m = 0, 1, 2, \ldots
\end{equation}

\begin{equation}
  \mu = \frac{\pi}{\sqrt{\sigma^2 + \pi^2}}.
\end{equation}

Note that both the measurements and the theoretical predictions for the effective multipoles are computed using equation (14), with \( \xi(l, \pi) \) given by the measured correlation function (see equation 1) for the measured effective multipoles, and equations (5)–(11) for their theoretical predictions. We do not use the conventional definitions of multipoles to extract parameter constraints as they use continuous integrals. Bias could be introduced if the definitions of multipoles are different between measurements from data and the theoretical model.

### 3.4 Covariance matrix

We use the 160 mock catalogues from the LasDamas simulations\(^2\) (McBride et al., in preparation) to estimate the covariance matrix of the observed correlation function. LasDamas provides mock catalogues matching SDSS main galaxy and LRG samples. We use the LRG mock catalogues from the LasDamas gamma release with the same cuts as the SDSS LRG DR7 full sample, \(-23.2 < M_r < -21.2\) and 0.16 < \( z < 0.44 \). We have diluted the mock catalogues to match the radial selection function of the observational data by randomly selecting the mock galaxies according to the number density of the data sample. We calculate the multipoles of the correlation functions of the mock catalogues and construct the covariance matrix as

\begin{equation}
  C_{ij} = \frac{1}{N - 1} \sum_{k=1}^{N} (X_i - X_i^\ast)(X_j - X_j^\ast),
\end{equation}

\(^2\) http://lss.phy.vanderbilt.edu/lasdamas/
where \( N \) is the number of the mock catalogues, \( \vec{X}_m \) is the mean of the \( m \)th element of the vector from the mock catalogue multipoles and \( X_m^{k} \) is the value in the \( m \)th element of the vector from the \( k \)th mock catalogue multipole. The data vector \( X \) is defined by

\[
X = \left\{ \xi^{(1)}_0, \xi^{(2)}_0, \ldots, \xi^{(N)}_0, \xi^{(1)}_2, \xi^{(2)}_2, \ldots, \xi^{(N)}_2, \ldots \right\},
\]

where \( N \) is the number of data points in each measured multipole; \( N = 16 \) in this work. The length of the data vector \( X \) depends on how many multipoles are used.

### 3.5 Likelihood

The likelihood is taken to be proportional to \( \exp(-\chi^2/2) \) (Press et al. 1992), with \( \chi^2 \) given by

\[
\chi^2 = \sum_{i,j=1}^{N_X} \{T^{-1}X_{th,i} - X_{obs,i}\} C^{-1}_{ij} \{T^{-1}X_{th,j} - X_{obs,j}\},
\]

where \( N_X \) is the length of the vectors \( X_{th} \) and \( X_{obs} \), which represent the theoretical model and the observational data, respectively.

As explained in Chuang & Wang (2012), instead of recalculating the observed correlation function for different theoretical models, we rescale the theoretical correlation function to avoid rendering \( \chi^2 \) values arbitrary. The rescaled theoretical correlation function is computed by

\[
T^{-1}(\xi_{th}(\sigma, \pi)) = \xi_{th} \left( \frac{D_A(z)}{D_A^0(z)} \sigma \frac{H^0(z)}{H(z)} \pi \right),
\]

where \( \xi_{th} \) is given by equation (11). Hence, \( \chi^2 \) can be rewritten as

\[
\chi^2 = \sum_{i,j=1}^{N_X} \{T^{-1}X_{th,i} - X_{th,obs,i}\} C^{-1}_{fid,ij} \{T^{-1}X_{th,j} - X_{th,obs,j}\},
\]

where \( T^{-1}X_{th} \) is a vector given by equation (21) with \( \xi_{th} \) replaced by its effective multipoles (defined by equation 14), and \( X_{th,obs} \) is the corresponding vector from observational data measured assuming the fiducial model in converting redshifts into distances. See Chuang & Wang (2012) for a more detailed description of our rescaling method.

### 3.6 Markov chain Monte Carlo likelihood analysis

We use \textsc{CosmoMC} in a Markov chain Monte Carlo (MCMC) likelihood analysis (Lewis & Bridle 2002). The parameter space that we explore spans the parameter set of \[ \{H(0.35), D_A(0.35), \Omega_m h^2, \beta, \Omega_b h^2, n_s, \sigma_8, k_0\} \]. Only \[ \{H(0.35), D_A(0.35), \Omega_b h^2, \beta\} \] are well constrained using SDSS LRGs alone in the scale range of interest. We marginalize over the other parameters, \( \{\Omega_m h^2, n_s, \sigma_8, k_0\} \), with the flat priors, \( \{0.01859, 0.02657\}, \{0.865, 1.059\}, \{0, 500\} \) \( s^{-1} \) km, \( \{0.09, 0.13\} \) Mpc \(^{-1}\), where the flat priors of \( \Omega_b h^2 \) and \( n_s \) are centred on the measurements from WMAP7 and have width of \( \pm 3 \sigma_{\text{WMP}} \) (with \( \sigma_{\text{WMP}} \) from Komatsu et al. 2010). These priors are wide enough to ensure that CMB constraints are not double counted when our results are combined with CMB data (Chuang et al. 2012). We also marginalize over the amplitude of the galaxy correlation function, effectively marginalizing over a linear galaxy bias.

### 4 RESULTS

#### 4.1 Measurement of multipoles

Figs 1, 2, 3 and 4 show, respectively, the effective monopole (\( \hat{\xi}_0 \)), quadrupole (\( \hat{\xi}_2 \)), hexadecapole (\( \hat{\xi}_4 \)) and hexacontatetrapole (\( \hat{\xi}_6 \)) measured from SDSS LRGs, compared with the average effective multipoles measured from the mock catalogues. We use the same scale range as Chuang & Wang (2012) (\( s = 40–120 \) h \(^{-1} \) Mpc) for comparison and the bin size used is 5 h \(^{-1} \) Mpc. The data points from the multipoles in the scale range considered are combined to form a vector, \( X \) (see equation 19).

We find that \( \hat{\xi}_s(s) \) deviates on scales of \( s < 60 \) Mpc \( h^{-1} \) from the measurement from mock data [in contrast to \( \hat{\xi}_s(s) \) and \( \hat{\xi}_s(s) \)]. We note that there are 10 out of 160 mocks which have at least one bin in the range 40 < \( s < 55 \) h \(^{-1} \) Mpc for which the amplitude of \( \hat{\xi}_s(s) \) is smaller than −0.01. Therefore, this deviation could be due to the statistical variance.
A frequently used combination of the monopole and the quadrupole is the normalized quadrupole, defined by
\[
Q(s) = \frac{\hat{\xi}_2(s)}{\hat{\xi}_0(s) - (3/s^2) \int_0^s \hat{\xi}_0(s') s'^2 \, ds'}. \tag{23}
\]

For comparison with previous work, we measure the effective normalized quadrupole defined by
\[
\hat{Q}(s) = \frac{\hat{\xi}_2(s)}{\hat{\xi}_0(s) - (3/s^2) \sum_{0 < s' < s} \hat{\xi}_0(s') s'^2 \Delta s}. \tag{24}
\]

from SDSS LRGs (see Fig. 5). It is in good agreement with the expectation from the LasDamas mocks, as well as with previous work by Samushia et al. (2011).

\[\text{Figure 3. Measurement of the hexadecapole of the correlation function of the SDSS DR7 LRG (diamond data points), compared to the average hexadecapole of the correlation functions of the mock catalogues (red solid line) and the theoretical model with the input parameters of the simulations (green dashed line). The error bars are taken to be the square roots of the diagonal elements of the covariance matrix.}\]

\[\text{Figure 4. Measurement of the hexacontatetrapole of the correlation function of the SDSS DR7 LRG (diamond data points), compared to the average hexacontatetrapole of the correlation functions of the mock catalogues (red solid line) and the theoretical model with the input parameters of the simulations (green dashed line). The error bars are taken to be the square roots of the diagonal elements of the covariance matrix.}\]

\[\text{Figure 5. Measurement of the normalized quadrupole from the SDSS DR7 LRG (diamond data points), compared to the mean measurement from the mock catalogues (red solid line). The error bars are taken to be the square roots of the diagonal elements of the covariance matrix. The green dashed line is the theoretical prediction for } \beta = 0.3 \text{ assuming a linear power spectrum and small-angle approximation.}\]

4.2 Measurement of \( \{ H(0.35), D_A(0.35), \beta(0.35) \} \)

We now present the model-independent measurements of the parameters \( \{ H(0.35), D_A(0.35), \Omega_m h^2, \beta \} \), obtained by using the method described in previous sections. We also present constraints on the derived parameters \( H(0.35) r_z(z_a)/c \) and \( D_A(0.35)/r_z(z_a) \), which are more tightly constrained.

4.2.1 Validation using mock catalogues

We first validate our method using mock catalogues. We have applied it to the first 40 LasDamas mock catalogues (which are indexed with 01a–40a).\(^3\) Again, we apply the flat and wide priors \((\pm 7\sigma_{MAP})\) on \( \Omega_m h^2 \) and \( n_s \), centred on the input values of the simulation \( (\Omega_m h^2 = 0.0196 \text{ and } n_s = 1) \).

Table 1 shows the means and standard deviations of the distributions of our measurements of \( \{ H(0.35), D_A(0.35), \Omega_m h^2, \beta, H(0.35) r_z(z_a)/c, D_A(0.35)/r_z(z_a) \} \) from each monopole + quadrupole \( \{ \hat{\xi}_0, \hat{\xi}_2 \} \) and monopole + quadrupole + hexadecapole + hexacontatetrapole \( \{ \hat{\xi}_0, \hat{\xi}_2, \hat{\xi}_4, \hat{\xi}_6 \} \) of the LasDamas mock catalogues of the SDSS LRG sample. The measurements using monopole + quadrupole + hexadecapole \( \{ \hat{\xi}_0, \hat{\xi}_2, \hat{\xi}_4, \hat{\xi}_6 \} \) are shown in the same table as well. These are consistent with the input parameters, establishing the validity of our method. In addition, we count the number of the measurements that are outside 1\( \sigma \) from the input values of the simulations. The measurements include \( H(0.35), D_A(0.35) \) and \( \Omega_m h^2 \) from all three methods, \( \hat{\xi}_0 + \hat{\xi}_2, \hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4 \) and \( \hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4 + \hat{\xi}_6 \). The average percentage is 0.34, close to 0.32, the value we would expect assuming Gaussian distributions. Note that there is a small difference \((\sim 0.5\sigma)\) between the restored value and the input value for \( D_A(z_a)/r_z(z_a) \); it should be possible to remove this by using a more accurate model for redshift-space distortions, e.g. as described in Reid et al. (2011). However, applying such models is too computationally expensive in our method. We will investigate alternative approaches in future work.

\[^3\] We only use 40 instead of 160 mock catalogues because the MCMC is computationally expensive. However, the covariance matrix is constructed with 160 mock catalogues.
While the constraints from using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$ are significantly tighter than using $\hat{\xi}_0 + \hat{\xi}_2$, the constraints from using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4 + \hat{\xi}_6$ are nearly the same as that from using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$. This indicates that $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$ captures nearly all of the information that can be extracted from the data, given the noise level. Since linear theory predicts that $\hat{\xi}_l = 0$ for $l > 4$, it is not surprising that $\hat{\xi}_0$, $\hat{\xi}_2$ and $\hat{\xi}_4$ capture most of the information from the 2D 2PCF.

In principle, one could obtain better constraints by including more multipoles. However, the trade-off is introducing noise to the covariance matrix which could be a problem, since the number of the mock catalogues used to construct the covariance matrix is not big enough. We also show the measurements of $H(0.35)\xi_{r}(z_d)/c$, $D_{A}(0.35)/\xi_{r}(z_d)$, $\Omega_m h^2$ and $\beta$ of each mock catalogue in Figs 6, 7, 8 and 9, respectively, to show the scattering among different mock catalogues and the deviations among different methods. One can see that the measurements from different methods are consistent for most mock catalogues, but there are still some obvious deviations ($>1\sigma$) for a few mock catalogues.

An important point to note is that since the mock data do not include unknown systematic effects, the mean values of estimated parameters remain nearly unchanged as more multipoles measured from data are added to the analysis and the parameter constraints are tightened with the addition of information.

Table 1. The mean and standard deviation of the distribution of the measured values of $\{H(0.35), D_A(0.35), \Omega_m h^2, \beta, H(0.35)\xi_{r}(z_d)/c, D_A(0.35)/\xi_{r}(z_d)\}$ from each $\hat{\xi}_0 + \hat{\xi}_2, \hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$ or $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4 + \hat{\xi}_6$ of 40 LasDamas mock catalogues (which are indexed with 01a–40a). Our measurements are consistent with the input values within $1\sigma$, where each $\sigma$ is computed from the 40 means measured from the 40 mock catalogues. The unit of $H$ is km s$^{-1}$ Mpc$^{-1}$. The unit of $D_A$ and $\xi_r(z_d)$ is Mpc.

|               | $\hat{\xi}_0 + \hat{\xi}_2$ | $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$ | $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4 + \hat{\xi}_6$ | Input value |
|---------------|-----------------------------|---------------------------------------------|-------------------------------------------------|-------------|
| $H(0.35)$     | 81.1±5.6                    | 80.4±4.9                                    | 80.3±5.1                                        | 81.79       |
| $D_A(0.35)$   | 1017±63                     | 1027±56                                     | 1021±48                                         | 1032.8      |
| $\Omega_m h^2$ | 0.119±0.014                  | 0.116±0.013                                  | 0.116±0.013                                     | 0.1225      |
| $\beta$       | 0.325±0.076                  | 0.327±0.066                                  | 0.324±0.075                                     | --          |
| $H(0.35)\xi_{r}(z_d)/c$ | 0.0436±0.0030                | 0.0435±0.0025                                | 0.0434±0.0026                                   | 0.0434      |
| $D_A(0.35)/\xi_{r}(z_d)$ | 6.29±0.36                    | 6.31±0.31                                    | 6.28±0.26                                       | 6.48        |

Figure 6. Measurements of the means and standard deviation of $H(0.35)\xi_{r}(z_d)/c$ from 40 individual mock catalogues (indexed as 01a–40a). The blue circles show the measurements using $\hat{\xi}_0 + \hat{\xi}_2$. The red diamonds show the measurements using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$. The black crosses show the measurements using the full 2D 2PCF method from our previous work. The black line shows the theoretical value computed with the input parameters of the simulations.

Figure 7. Measurements of the means of $D_A(0.35)/\xi_{r}(z_d)$ from 40 individual mock catalogues (indexed as 01a–40a). The blue circles show the measurements using $\hat{\xi}_0 + \hat{\xi}_2$. The red diamonds show the measurements using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$. The black crosses show the measurements using the full 2D 2PCF method from our previous work. The black line shows the theoretical value computed with the input parameters of the simulations.

Figure 8. Measurements of the means of $\Omega_m h^2$ from 40 individual mock catalogues (indexed as 01a–40a). The blue circles show the measurements using $\hat{\xi}_0 + \hat{\xi}_2$. The red diamonds show the measurements using $\hat{\xi}_0 + \hat{\xi}_2 + \hat{\xi}_4$. The black crosses show the measurements using the full 2D 2PCF method from our previous work. The black line shows the theoretical value computed with the input parameters of the simulations.

Finally, we compare with the work of Kazin, Sanchez & Blanton (2012), who measured $H(z)$ and $D_A(z)$ using the average multipoles of the correlation function from the LasDamas mock catalogues. They assume a larger survey volume (~12 times) by dividing the
c.m. uncertainties of \{H, D_\delta, \beta\} we measure using \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\) is \{1.17 per cent, 0.81 per cent, 4.45 per cent\} which are similar to their results, \{1.42 per cent, 0.76 per cent, 4.95 per cent\} (the numbers are taken from fig. 7 in Kazin et al. 2012). They derive the theoretical multipoles analytically, instead of using the same definition as applied to the observational data. In principle, it could introduce biases to the measurements. However, the effect might be minimized since they construct the theoretical model based on the measured multipoles from the mock catalogues, which is equivalent to computing the theoretical multipoles with the same definition as applied to the observational data.

4.2.2 Measurements from the SDSS DR7 LRG sample

Table 2 lists the mean, rms variance and 68 per cent confidence level (CL) limits of the parameters, \{\(H(0.35)\), \(D_\delta(0.35)\), \(\Omega_m h^2\), \(\beta\), \(H(0.35) r(z_{cd})/c\), \(D_\delta(0.35)/r(z_{cd})\)\}, derived in an MCMC likelihood analysis from the measured \(\hat{\xi}_0 + \hat{\xi}_1\) of the correlation function of the SDSS LRG sample. Table 3 lists the mean, rms variance and 68 per cent CL limits of the same parameter set from the measured \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\) of the correlation function of the SDSS LRG sample for this parameter set. The \(\chi^2\) per degree of freedom (\(\chi^2/\text{d.o.f.}\)) is 1.23 for \(\hat{\xi}_0 + \hat{\xi}_1\) and is 1.06 for \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\). These are independent of a dark energy model, and obtained without assuming a flat universe. There are obvious deviations between the cosmological constraints obtained from the measured \(\hat{\xi}_0 + \hat{\xi}_1\) and \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\) of the correlation function of the SDSS LRG sample, i.e. \{\(\Delta \beta = 0.10, \Delta H(0.35)/r(z_{cd})/c = 0.0037, \Delta D_\delta(0.35)/r(z_{cd}) = 0.31\)\}. To explore how significant these deviations are, we compute the standard deviations of these differences from Figs 6, 7 and 9 and find \{\(\sigma(\Delta \beta) = 0.049, \sigma(\Delta H(0.35)/r(z_{cd})/c) = 0.0024, \sigma(\Delta D_\delta(0.35)/r(z_{cd})) = 0.24\)\}. One can see that the differences from

![Figure 9](https://example.com/figure9.png)

Figure 9. Measurements of the means of \(\beta\) from 40 individual mock catalogues (indexed as 01a–40a). The blue circles show the measurements using \(\hat{\xi}_0 + \hat{\xi}_1\). The red diamonds show the measurements using \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\). The black crosses show the measurements using the full 2D 2PCF method from our previous work.

![Figure 10](https://example.com/figure10.png)

Figure 10. 2D marginalized contours (95 per cent CL) for \(D_\delta(z)/D_\delta^\text{true}\) and \(H(z)/H_0^\text{true}\) for the comparison with fig. 6 in Kazin et al. (2012). The black solid contour is measured using \(\hat{\xi}_0 + \hat{\xi}_1\) and the blue dotted contour is measured using \(\hat{\xi}_0 + \hat{\xi}_1 + \hat{\xi}_4\). Our constraints are similar to the results in Kazin et al. (2012).

| Parameter | Mean | \(\sigma\) | Lower | Upper |
|-----------|------|------------|-------|-------|
| \(H(0.35)\) | 79.6 | 8.8 | 70.9 | 87.8 |
| \(D_\delta(0.35)\) | 1060 | 92 | 970 | 1150 |
| \(\Omega_m h^2\) | 0.103 | 0.015 | 0.088 | 0.118 |
| \(\beta\) | 0.44 | 0.15 | 0.29 | 0.59 |
| \(H(0.35)/r(z_{cd})/c\) | 0.0435 | 0.0045 | 0.0391 | 0.0477 |
| \(D_\delta(0.35)/r(z_{cd})\) | 6.44 | 0.51 | 5.99 | 6.90 |

| Parameter | Mean | \(\sigma\) | Lower | Upper |
|-----------|------|------------|-------|-------|
| \(H(0.35)\) | 87.3 | 6.7 | 80.8 | 93.7 |
| \(D_\delta(0.35)\) | 1095 | 59 | 1037 | 1153 |
| \(\Omega_m h^2\) | 0.107 | 0.015 | 0.093 | 0.122 |
| \(\beta\) | 0.54 | 0.11 | 0.44 | 0.65 |
| \(H(0.35)/r(z_{cd})/c\) | 0.0472 | 0.0033 | 0.0441 | 0.0503 |
| \(D_\delta(0.35)/r(z_{cd})\) | 6.75 | 0.25 | 6.52 | 6.98 |
the measurements are around 1–2σ. Thus, the deviations between Tables 2 and 3 could be due to statistical variance.

Table 4 gives the normalized covariance matrix for this parameter set measured using \( \tilde{\xi}_0 + \tilde{\xi}_2 \). While the measurement of \( \beta \), 0.44 ± 0.15, seems to be higher than what we expect (i.e., \( \beta = 0.325 ± 0.076 \) from the mock catalogues using \( \tilde{\xi}_0 + \tilde{\xi}_2 \)), note that there is a negative correlation between \( \beta \) and \( \Omega_m h^2 \) and the correlation coefficient is −0.2549. Thus, the somewhat high \( \beta \) value is mildly correlated with the somewhat low \( \Omega_m h^2 \) value. In addition, the somewhat high \( \beta \) value is actually still statistically consistent with the measurement from the mock catalogues. The most robust measurements are that of \( \{\langle H(0.35) r(z_d)/c, D_A(0.35)/r(z_d)\rangle\} \), same as in Chuang & Wang (2012). These can be used to combine with other data sets and constraining dark energy and cosmological parameters (see Wang, Chuang & Mukherjee 2012).

Fig. 11 shows 1D and 2D marginalized contours of the parameters, \( \{H(0.35), D_A(0.35), \Omega_m h^2, \beta, H(0.35)r(z_d)/c, D_A(0.35)/r(z_d)\} \), derived in an MCMC likelihood analysis from the measured \( \tilde{\xi}_0 + \tilde{\xi}_2 \) of the SDSS DR7 LRG sample.

### 4.3 Comparison with previous work

While we have developed a general method to measure the dark energy and cosmological parameters that could be extracted from the galaxy clustering data alone, we restrict our method now by fixing some parameters to obtain the results for comparison with previous work.

In our previous paper (Chuang & Wang 2012), we used the full 2D correlation function and measured \( H(z = 0.35) = 82.1^{+1.1}_{-0.7} \) km s\(^{-1}\) Mpc\(^{-1}\) and \( D_A(z = 0.35) = 1048^{+68}_{-58} \) Mpc, which are consistent with this study; note that the full 2D correlation function captures more information than the leading multipoles. Xu et al. (2012) applied the density field reconstruction method on the same data and obtained \( H(z = 0.35) = 84.4 ± 7.1 \) and \( D_A(z = 0.35) = 1050 ± 38 \) Mpc, which are also in excellent agreement with our measurements.

Cabre & Gaztanaga (2009) measure \( \beta \) from SDSS DR6 LRGs using the normalized quadrupole defined by equation (23). To compare with their results, we make similar assumptions, and use the monopole–quadrupole method with fixing \( \Omega_m = 0.25, \Omega_\Lambda = 0.045, h = 0.72, n_s = 0.98, k_c = 0.11 \) and \( \sigma_8 = 0.300 \) km s\(^{-1}\) in the \( \Lambda \)CDM model \( [H(0.35) \) and \( D_A(0.35) \) would also be fixed accordingly]. Considering the scale range \( s = 40–100 h^{-1} \) Mpc, we obtain \( \beta = 0.333 ± 0.055 \), in excellent agreement with their measurement of \( \beta = 0.34 ± 0.05 \). Since the definition of the normalized quadrupole includes an integral of the monopole with the minimum boundary from \( s = 0 \), the advantage of using our effective multipole method instead of the normalized quadrupole method is to avoid the distortion from the small scales where the scale-dependent uncertainties are not well known. However, the distortion might be negligible compared to the statistical uncertainty of current measurements.

Song et al. (2011) split the same galaxy sample (SDSS DR7 LRG) to two redshift slices and obtained \( \beta(z = 0.25) = 0.30^{+0.046}_{-0.048} \) and \( \beta(z = 0.38) = 0.39 ± 0.056 \) without considering the geometric distortions. Their results are in excellent agreement with the values measured by Cabre & Gaztanaga (2009) and us under the same assumptions. In addition, Blake et al. (2011) measured \( H(z) D_A(z)(1 + z)/c = 0.28 ± 0.04 \) and \( 0.44 ± 0.07 \) at \( z = 0.22 \) and 0.41, respectively, from the WiggleZ survey (Blake et al. 2009; Parkinson et al. 2012). Linearly interpolating their results, we find the mean of \( H(z) D_A(z)(1 + z)/c \) to be 0.39 at \( z = 0.35 \), which is in excellent agreement with our measurement of \( H(0.35) D_A(0.35)(1.35)/c = 0.38 ± 0.06 \).

### 5 SYSTEMATICS

Table 5 shows the systematic tests that we have carried out by varying key assumptions made in our analysis. These include the multipoles used, the range of scales used, the bin size used and the minimum of the transverse separation used to calculate the correlation function.

We use the results using \( \tilde{\xi}_0 + \tilde{\xi}_2 \) as our fiducial results. We find that the constraints are stronger for using \( \tilde{\xi}_0 + \tilde{\xi}_2 + \tilde{\xi}_4 \), but using \( \tilde{\xi}_0 + \tilde{\xi}_2 + \tilde{\xi}_4 \) does not improve the constraints significantly. Therefore, it seems that \( \tilde{\xi}_0 + \tilde{\xi}_2 + \tilde{\xi}_4 \) contains most of the information from the 2D 2PCF. Since the measurements of \( \tilde{\xi}_0 + \tilde{\xi}_2 + \tilde{\xi}_4 \) deviate from those of \( \tilde{\xi}_0 + \tilde{\xi}_2 \) by about 1σ, we use the latter as our fiducial results to be conservative.

We vary the scale range chosen and the bin size used and find that the results are basically consistent. However, we find that the measurement of \( D_A(z)/r(z_d) \) is more stable than that of \( H(0.35) r(z_d)/c \). It might indicate the appearance of systematic errors from the measurement of the correlation function in the direction along the LOS. While the observed correlation function along the LOS is noisier and harder to model due to galaxy peculiar velocities, we test the impact of systematic uncertainties along the LOS by removing the data with the transverse separation, \( \sigma \), smaller than 5 or 10 h\(^{-1}\) Mpc. We find that the results are insensitive to this. Thus, our measurement of \( H(0.35) r(z_d)/c \) should not be contaminated by systematic errors along the LOS.

There is possible systematic uncertainty from the radial selection function used to construct the random catalogues. Ross et al. (2012) found that the least biased way is using the ‘shuffled’ method for the SDSS-III/BOSS DR9 CMASS sample. The shuffled method is to assign the redshift of a galaxy of a random catalogue with the redshift of the observed data picked randomly. Samushia et al. (2013) found that using the spline method, which is the same as we use in this study, could obtain a less biased result for the SDSS DR7.
Figure 11. 2D marginalized contours (68 and 95 per cent CL) for \{H(0.35), D_A(0.35), \Omega_m h^2, \beta, H(0.35) r_s(z_d)/c, D_A(0.35)/r_s(z_d)\}. The diagonal panels represent the marginalized probabilities. The unit of $H$ is km s$^{-1}$ Mpc$^{-1}$. The unit of $D_A$ and $r_s(z_d)$ is Mpc.

LRG sample. In fact, the biased effect due to the radial selection function depends on the galaxy sample, survey geometry and scale range studied. For example, for a narrow beam survey, while most of the structure is in the LOS direction, the shuffled method would erase most of the information. Samushia et al. (2013) showed that the spline method has least bias for the SDSS DR7 LRG sample in the scale range we are interested ($s > 40 h^{-1}$ Mpc), and the bias is much smaller than the statistic error. Therefore, we expect the bias to be negligible.

6 CONCLUSION AND DISCUSSION

We have demonstrated the feasibility of using multipoles of the correlation function to measure $H(z)$, $D_A(z)$, $\Omega_m h^2$ and $\beta$ by applying the method to individual mock catalogues from LasDamas in an MCMC likelihood analysis.

The method we developed is modified from Chuang & Wang (2012), which was the first method to include the geometric distortion (also known as the Alcock–Paczynski test, see Alcock &
Table 5. This table shows the systematic errors that vary the combination of multipoles, the scale range, the bin size and the minimum transverse separation used in the analysis. The fiducial results are obtained using $\hat{e}_0 + \hat{e}_2$, the scale range $40 < s < 120 \, h^{-1} \text{Mpc}$, a bin size of $5 \, h^{-1} \text{Mpc}$, and no minimum transverse separation. The other results are calculated with only specified quantities different from the fiducial one. The unit of $H$ is $\text{km s}^{-1} \text{Mpc}^{-1}$. The unit of $D_A$ and $r_s(z_0)$ is Mpc. In the last row, we show the variation between these tests by computing the maximum difference between the mean values divided by the errors of the fiducial measurements.

| $H(0.35)$ | $D_A(0.35)$ | $\Omega_m h^2$ | $\beta$ | $H(0.35) r_s(z_0)/c$ | $D_A(0.35)/r_s(z_0)$ |
|------------|-------------|----------------|--------|----------------------|----------------------|
| $\hat{e}_0 + \hat{e}_2$ (fiducial) | $80.0^{+8.2}_{-8.6}$ | $1063^{+87}_{-85}$ | $0.103 \pm 0.015$ | $0.45^{+0.15}_{-0.14}$ | $0.0437^{+0.0044}_{-0.0043}$ | $6.48^{+0.44}_{-0.43}$ |
| $\hat{e}_0 + \hat{e}_2 + \hat{e}_4$ | $87.3 \pm 6.4$ | $1095 \pm 58$ | $0.107 \pm 0.014$ | $0.54 \pm 0.11$ | $0.0472 \pm 0.0031$ | $6.75^{+0.24}_{-0.23}$ |
| $\hat{e}_0 + \hat{e}_2 + \hat{e}_6$ | $78.5^{+8.7}_{-8.9}$ | $1025^{+88}_{-82}$ | $0.107 \pm 0.016$ | $0.41 \pm 0.14$ | $0.0424^{+0.0044}_{-0.0043}$ | $6.31 \pm 0.49$ |
| $30 < s < 120$ | $85.1^{+7.8}_{-8.2}$ | $1074^{+64}_{-62}$ | $0.115 \pm 0.014$ | $0.38 \pm 0.10$ | $0.0454^{+0.0037}_{-0.0039}$ | $6.71^{+0.31}_{-0.30}$ |
| $50 < s < 120$ | $77.5^{+8.2}_{-8.4}$ | $1034^{+103}_{-109}$ | $0.101^{+0.019}_{-0.018}$ | $0.5^{+0.19}_{-0.18}$ | $0.0425^{+0.0038}_{-0.0040}$ | $6.27^{+0.55}_{-0.61}$ |
| $< 400 < s < 110$ | $73.5^{+6.7}_{-7.0}$ | $1064^{+77}_{-76}$ | $0.107 \pm 0.015$ | $0.35 \pm 0.11$ | $0.0399^{+0.0031}_{-0.0034}$ | $6.54^{+0.41}_{-0.40}$ |
| $40 < s < 130$ | $83.5^{+8.3}_{-8.7}$ | $1085^{+78}_{-75}$ | $0.105 \pm 0.015$ | $0.48 \pm 0.15$ | $0.0454^{+0.0041}_{-0.0043}$ | $6.63^{+0.37}_{-0.35}$ |

$\sigma_{\text{fiducial}} = 0.0031$ using monopole and $\pm 20^\circ$ quadrupole as our fiducial results.

Paczyński 1979) on galaxy clustering data on large scales. We compute the multipoles from the theoretical and observed 2D PCCFs in the same way; thus, the only approximation made is that the distance of any pair of galaxies can be converted with two stretch factors between different models in the redshift range considered.

We have obtained the constraints for the measured and derived parameters, $\{H(0.35), D_A(0.35), \Omega_m h^2, \beta, H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0)\}$, from the multipoles of the correlation function from the sample of SDSS DR7 LRGs which are summarized in Tables 2 and 3.

We find that while the mean values of estimated parameters remain stable (with rare deviations) for the mock data when higher multipoles are used, this is not true for the SDSS DR7 LRG data. We find $H(0.35) r_s(z_0)/c = 0.043^{+0.0044}_{-0.0043}$ using monopole + quadrupole and $H(0.35) r_s(z_0)/c = 0.0472 \pm 0.0031$ using monopole + quadrupole + hexadecapole. This deviation could be caused by statistical variance. In addition, there is some deviation between the LaDamas measurements and the theoretical model for hexadecapole. However, the deviation is small compared to the uncertainties of the measurements. To be conservative, we choose the measurements using monopole + quadrupole as our fiducial results.

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