Implementing backjumping by \texttt{throw/1 and catch/3} of Prolog

Wlodzimierz Drabent 2022-02-07

Institute of Computer Science, Polish Academy of Sciences,
and
Department of Computer and Information Science, Linköping University, Sweden

drabent at ipipan dot waw dot pl

Abstract
We discuss how to implement backjumping (or intelligent backtracking) in Prolog programs by means of exception handling. This seems impossible in a general case. We provide two solutions. One works for binary programs; in a general case it imposes a restriction on where backjumping may originate. The other restricts a class of backjump targets. We also show how to simulate backjumping by means of backtracking and the Prolog database.

KEYWORDS: Prolog, intelligent backtracking, backjumping, exception handling

In this note we first explain the incompatibility between backjumping and the exception handling of Prolog. Then we discuss how to employ the Prolog exception handling mechanism to implement backjumping for definite clause programs (Section 2). We present two approaches of adding backjumping to a Prolog program. The first approach is applicable to a restricted but broad class of cases, including binary programs with arbitrary backjumping. The restriction is on from where the backjumping may originate. In the second approach the class of available backjump targets is restricted, so the resulting backjumping may only be an approximation of that intended. Section 3 presents an example of each approach. The next section discusses backjumping by means of backtracking and the Prolog database. The report is completed by a brief discussion of the related work and conclusions.

1 Backjumping and Prolog exception handling

A Prolog computation can be seen as a depth-first left-to-right traversal of an SLD-tree (LD-tree, when no delay mechanisms are employed). Each node with \emph{i} children is visited \emph{i} + 1 times. Moving from a node to its parent is called backtracking. By backjumping we mean skipping a part of the traversal, by moving immediately from a node to one of its non-immediate ancestors. Intelligent backtracking (Bruynooghe and Pereira 1984) is backjumping in which it is known that there are no successes in the omitted part of the SLD-tree. (More generally, that there are no successes with answers distinct from those already obtained.)

Prolog provides an exception handling mechanism, consisting of built-in predicates \texttt{throw/1} and \texttt{catch/3}. Let us follow the Prolog standard (Deransart et al. 1996) and
explain them in terms of LD-trees. Let $A_c = \text{catch}(Q, s, \text{Handler})$, where $Q$ and $\text{Handler}$ are queries, and $s$ is a term. A node $A_c, N$ of an LD-tree has a single child $Q, N$. A second child may however be created as a result of exception handling. An exception is raised by invoking $\text{throw}(t)$ (formally, by visiting a node $N_t = \text{throw}(t), N'$; such node has no children). This is sometimes called “throwing a ball $t$”. Visiting $N_t$ starts a search along the path from $N_t$ to the root. The search is for a node $N_c = \text{catch}(Q, s, \text{Handler}), N$ with one child such that (a) a freshly renamed copy $t'$ of the ball $t$ is unifiable with $s$, with an $\text{mgu} \theta$, and (b) “the ball is thrown during the execution of” $Q$ (Deransart et al. 1996), in other words – no node between $N_c$ and $N_t$ is an instance of $N$. The first (closest to $N_t$) such node $N_c$ on the path is chosen, and a new child $(\text{Handler}, N)\theta$ of $N_c$ is added to the tree. The new child becomes the next visited node of the tree.

Prolog does not provide any way to directly implement backjumping. It may seem that exception handling is a suitable tool for this task. There is however an important difference. Not all backtrack points can be reached by means of catch/3.

Consider an LD-tree containing a node $Q$ with $k$ children $Q_1, \ldots, Q_k$. We may say that $k$ backtrack points correspond to $Q$. Any of them may be a target for backjumping, but exception handling is able to arrive only at the last one. In particular, for intelligent backtracking it may be necessary that backjumping to $Q$ from a descendant of $Q_1$ is followed by visiting $Q_{i+1}, \ldots, Q_k$ and their descendants. Exception handling would however omit all such nodes. More precisely, if $N = \text{catch}(Q, s, H)$ is employed to catch an exception, then $N$ has a child $Q$, and all its descendants are as above. However catching an exception at $N$ results in omitting all the unexplored descendants of $Q$. The same happens if $N = \text{catch}(Q', s, H), Q''$, where $Q = Q', Q''$. Also, it seems that the omitted part of the tree cannot be explored by reconstructing it by the exception handler $H$, at least in the general case.

A possible solution could be backjumping to the last backtrack point of $Q_t$, instead of the $i$-th backtrack point of $Q$. (This idea is exploited in Approach 1 below.) However, to implement such a backjump by $\text{catch}/3$ in a general case, one requires replacing $Q_t$ by $\text{catch}(Q_t, \ldots)$ (as the backjump may come from any descendant of $Q_t$). In many cases there is no way of adding $\text{catch}/3$ to the original program to obtain such a query.\footnote{Consider $Q = A, B$ and $Q_t = (B_t, B)\theta$; assume that clause $A' \leftarrow B_t$ was used in the resolution step. Adding $\text{catch}$ to the clause results in $B\theta$ not being a part of the argument of $\text{catch}$. Adding $\text{catch}$ elsewhere in the program results in backjumping to another node of the tree.}

This discussion shows that backjumping cannot be, in general, directly implemented by means of Prolog exception handling. It refutes the claim contained in the title of a recent paper [Robbins et al. 2021], which says “backjumping is exception handling”\footnote{We should also mention differences not related to implementing backjumping. In exception handling, after an exception is caught, the exception handler is activated. In backjumping there is nothing similar to an exception handler. Also, in contrast to backjumping, exception handling makes it possible to pass information (an arbitrary term) from the point where the exceptio is raised to the one where it is caught. This is done by means of the argument of $\text{throw}/1$.}

### 2 Implementing backjumping by exception handling

#### 2.1 Approach 1

Now we discuss a way of implementing backjumping by employing Prolog exception handling. Assume that we deal with a definite clause program $P$, which we want to
execute with backjumping. The target of backjumping is to be identified by a term \( \text{id} \). So backjumping is initiated by \( \text{throw}(\text{id}) \).

Assume that the target of backjumping is a node \( A, Q \) of the LD-tree, where \( A \) is an atom. Assume that \( A, Q \) has \( k \) children, \( Q_1, \ldots, Q_k \). Let \( p \) be the predicate symbol of \( A \) and

\[
p(t_1) \leftarrow B_1, \ldots, p(t_n) \leftarrow B_n.
\]

where \( k \leq n \), be the procedure \( p \) of program \( P \) (i.e. the clauses of \( P \) beginning with \( p \)).

Consider backjumping initiated by \( \text{throw}(\text{id}) \) in the subtree rooted in \( Q_i \). The subtree should be abandoned, but the descendants of \( Q_{i+1}, \ldots, Q_k \) should not. Thus we need to restrict the exception handling to this subtree. A way to do this is to replace each \( B_j \) by \( \text{catch}(B_j, \text{id}, \text{fail}) \). Then performing \( \text{throw}(\text{id}) \) while executing \( B_j \) results in failure of the clause body and backtracking to the next child of \( A, Q \), as required. Assume that a query \( \text{btid}(\vec{t}, \text{Id}) \) (backjump target identifier) produces the unique identifier \( \text{id} \) out of the arguments of \( p \). The backjumping is implemented by a transformed procedure consisting of clauses

\[
p(t_j) \leftarrow \text{btid}(t_j, \text{Id}), \text{catch}(B_j, \text{Id}, \text{fail}). \quad \text{for } j = 1, \ldots, n
\]

(2)

where \( \text{Id} \) is a variable.

Transforming a program in this way correctly implements backjumping, however with an important limitation. Speaking informally, backjumping to a node with \( p(t) \) selected must occur while executing \( p(t) \). Otherwise the exception is not caught and the whole computation is abandoned.

An important class of programs which satisfy this limitation are binary logic programs (i.e. programs with at most one body atom in a clause). The approach presented here works for such programs and arbitrary backjumping.

Sometimes (like in Ex. 2 below) it may be determined in advance that, for some \( j \), no exception will be caught by the \( \text{catch/3} \) in (2). So in practice some clauses of (1) may remain unchanged (or a choice between \( B_j \) and \( \text{catch}(B_j, \text{Id}, \text{fail}) \) may be made dynamically, e.g. by modifying the body of (2) into \( \text{btid}(t_j, \text{Id}) \rightarrow \text{catch}(B_j, \text{Id}, \text{fail}) ; B_j \).}

**Approach 1a.** Here we present a variant of Approach 1. Roughly speaking, in the former approach control is transferred to the next clause due to failure of a clause body. So catching an exception causes an explicit failure. Here control is transferred to the next clause by means of an exception, so standard backtracking eventually raises an exception.

To simplify the presentation we assume that in (1) all the clause heads are the same, \( t_1 = \cdots = t_n = \vec{t} \).

Assume first that \( n = 2 \). Backjumping equivalent to that of Approach 1 can be implemented by

\[
p(\vec{t}) \leftarrow \text{btid}(\vec{t}, \text{Id}), \text{catch}(B_1 ; \text{throw}(\text{Id})), \text{Id}, \text{catch}(B_2, \text{Id}, \text{fail} ).
\]

(3)

Invocation of \( B_2 \) is placed in the exception handler, so we additionally raise an exception when \( B_1 \) (the first clause body) fails. For arbitrary \( n \), the transformed procedure (1) is:
Initial queries is similar. In such case, the program contains a clause origin of backjumping is an instance of atomic. We have shown how to implement backjumping to an LD-tree node the execution of approach it is possible to augment backjumping by passing information (from the place where backjump originates to the backjump target).\(^\text{3}\) Such augmenting is impossible in Approach 1 and Approach 2 below.

### 2.2 Approach 2, approximate backjumping

We have shown how to implement backjumping to an LD-tree node \(N = A, Q\) (with atomic \(A\)) from within the execution of \(A\). (Formally: no node between \(A, Q\) and the origin of backjumping is an instance of \(Q\).) It remains to discuss backjumping originating in the execution of \(Q\). Assume that the initial query is atomic; dealing with arbitrary initial queries is similar. In such case, the program contains a clause \(H \leftarrow B_0, B_1\) (where \(B_0, B_1\) are nonempty), such that, speaking informally, the backjumping is from within the execution of \(B_1\), and its target is within the execution of \(B_0\).\(^\text{4}\)

---

3. To pass a term \(t\), one may choose the backjump target identifier to be \(f(X_i)\) for clause \(i\). Then performing \(\text{throw}(f(t))\) while executing \(B_i\) results in binding \(X_i\) to \(t\) when the exception is caught. This makes \(t\) available in those bodies \(B_{i+1}, \ldots, B_n\) that contain \(X_i\). E.g., for \(n = 2\) instead of the body of \(B_i\) we obtain \(\text{catch}( (B_i; \text{throw}(\text{nobj})), f(X_j)), \text{catch}(B_{j+1}, f(X_k), \text{fail})\); constant \text{nobj} (for "no backjumping") is passed when standard backtracking takes place.

4. Let us provide a detailed explanation. Note first that for any (occurrence of an) atom \(A\) in a node \(N\) of an LD-tree, and any ancestor \(N_0\) of \(N\), there exists a unique (occurrence of an) atom \(A_0\) in \(N_0\) such that \(A\) has been derived from \(A_0\) in the resolution steps between \(N_0\) and \(N_1\). We omit a detailed formalization of this correspondence. Let us denote such occurrence (of \(A_0\) in \(N_0\)) by \(\text{pre}(A, N, N_0)\).

Assume that node \(N = A, Q\) is the target of backjumping, and that its origin is \(N'.\) Let \(A'\) be the first atom of \(N'.\) Consider the closest ancestor \(N_0\) of \(N\) such that \(\text{pre}(A, N, N_0) = \text{pre}(A', N', N_0)\).

Let \(A_0 = \text{pre}(A', N', N_0)\). Consider the child \(N_1\) of \(N_0\) that is an ancestor of \(N\). So \(N_0 = A_0, Q_0\), and \(N_1 = (B_0, B_1, Q_0)\), where \(\text{pre}(A, N, N_1)\) occurs in \(B_0\), and \(\text{pre}(A', N', N_1)\) in \(B_1\). Moreover \(N_1\) was obtained by resolving \(N_0\) with a clause \(H \leftarrow B_0, B_1\). (Note that splitting its body into \(B_0\) and \(B_1\) may be not unique.)

Now \(N\) can be represented as \(N = Q_2, (B_1, Q_0)\). No node between \(N_1\) and \(N\) is an instance of \(B_1, Q_0\). So, informally, the backjump target \(N\) is within the execution of \(B_1\).

Note that if a descendant of \(N\) is of the form \(N_2 = A_2, \ldots, (B_1, Q_0)\varphi\) then \(\text{pre}(A_2, N_2, N_1)\) occurs in \(B_0\). Thus \(N\) has a descendant of the form \(N'' = (B_1, Q_0)\varphi\) (otherwise \(N' = A'\ldots\) is of the same form as \(N_2\), thus \(\text{pre}(A', N', N_1)\) is in \(B_0\), contradiction). As \(\text{pre}(A', N', N_1)\) occurs in \(B_1\), no node between \(N_1\) and \(N'\) (including \(N'\)) is an instance of \(Q_0\). Hence \(N''\) is of the form \(A', \ldots, Q_0\psi\), and thus the backjump origin \(N''\) is within the execution of \(B_1\).
Such backjumping exactly to the target does not seem possible to be implemented by means of throw/1 and catch/3. However we may force \( B_1 \) to fail when an exception is thrown. This means backjumping to, speaking informally, the success of \( B_0 \), instead of the original target \( N \). (In the notation of footnote 4, the target of this backjump is \( N'' = B_1\varphi, \ldots \).) This in a sense approximates backjumping to \( N \). In some cases such shorter backjumping may still be useful. It may exclude from the search space a major part of what would be excluded by backjumping to \( N \).

To implement such approximated backjumping we need to change the program, so that the instance \( B_1\varphi \) of \( B_1 \) in node \( N'' \) is replaced by \( \text{catch}(B_1\varphi, Id, \text{fail}) \). To obtain this, the clause

\[ H \leftarrow B_0, B_1 \]

is transformed to

\[ H \leftarrow B_0, \text{btid}(\ldots, Id), \text{catch}(B_1, Id, \text{fail}) \]

where \( \text{btid} \), as previously, is used to obtain the unique identifier for the backjump target.

### 3 Examples

We apply the approaches introduced above to a simple program, a naive SAT solver. It uses the representation of clauses proposed by [Howe and King (2012)](Howe and King 2012). (Note that we deal here with two kinds of clauses – those of the program, and the propositional clauses of a SAT problem.) A conjunction of clauses is represented as a list of (the representations of) clauses. A clause is represented as a list of (the representations of) literals. A positive literal is represented as a pair \( \text{true-}X \) and a negative one as \( \text{false-}X \), where the Prolog variable represents a propositional variable. For instance a formula \((x \lor \neg y \lor z) \land (\neg x \lor v)\) is represented as \([\text{true-}X, \text{false-}Y, \text{true-}Z], [\text{false-}X, \text{true-}V]\). In what follows we do not distinguish literals, clauses, etc from their representations.

Thus solving a SAT problem for a conjunction of clauses \( \text{sat} \) means instantiating the variables of \( \text{sat} \) in such way that each of the lists contains an element of the form \( t\rightarrow t \). This can be done by a program \( P_1 \):

\[
\begin{align*}
\text{sat_cl}([\text{Pol}\rightarrow\text{Pol}][\text{Pairs}]). \\
\text{sat_cl}([H][\text{Pairs}]) \leftarrow \text{sat_cl}(\text{Pairs}). \\
\text{sat_cnf}([\text{}]). \\
\text{sat_cnf}([\text{Clause}][\text{Clauses}]) \leftarrow \text{sat_cl}(\text{Clause}), \text{sat_cnf}(\text{Clauses}).
\end{align*}
\]

and a query \( \text{sat_cnf}(\text{sat}) \). See [Drabent 2018](Drabent 2018) Section 3) for further discussion and a formal treatment of the program.

We add backjumping to \( P_1 \). The intention is that, after a failure of \( \text{sat_cl}(cl) \) (where \( cl \) is the representation of a partly instantiated clause) a backjump is performed to the last point where a variable from clause \( cl \) was assigned a value. This does not correctly implement intelligent backtracking but the purpose is to illustrate the approaches proposed in the previous section.

---

5 E.g. for \((x \lor y) \land (\neg z \lor v) \land (\neg x \lor \neg y) \land (\neg x \lor y \lor z)\) no solution with \( z \) being \text{true} is found. An explanation is that, speaking informally, backjumping from the last clause (with \( x, y, z \) instantiated to \text{true, false, false}) arrives to the previous one (where \( y \) was set to false), this immediately causes backjumping to the first clause.
Example 1
Here we employ Approach 2 to program $P_1$. Speaking informally, the required backjumping originates from within $\text{sat}_\text{cnf}(\text{Clauses})$ in the last clause of the program, and its target is in $\text{sat}_\text{cl}(\text{Clause})$. We approximate this backjumping by a failure of $\text{sat}_\text{cnf}$.

(Nota that in this case the approximation is good, the intended target is a node of the form $\text{sat}_\text{cl}([v-V|t]), \text{sat}_\text{cnf}(t')$ and we implement backjumping to its child $\text{sat}_\text{cnf}(t'\{V/v\})$.)

We augment the values of variables; the value of a variable is going to be of the form $(l,v)$, where $l$ is a number (the level of the variable) and $v$ a logical value $\text{true}$ or $\text{false}$. The level shows at which recursion depth of $\text{sat}_\text{cnf}$ the value was assigned. The levels will be used as identifiers for backjump targets. In such setting, a substitution $\theta$ assigning values to variables makes a SAT problem $\text{sat}$ satisfied when each member of list $\text{sat}\theta$ contains a pair of the form $(l,v)$. This leads to transforming the first clause of $P_1$ to $\text{sat}_\text{cl}([\text{Pol}_-(-,\text{Pol})|\text{Pairs}])$.

We transform $P_1$ into a program $P_2$ which takes levels into account. We add the current level as the second argument of $\text{sat}_\text{cnf}$ and of $\text{sat}_\text{cl}$, and we add a third argument to $\text{sat}_\text{cl}$. The declarative semantics of the new program is similar to that of $P_1$; the answers of $P_2$ are as follows. If the first argument of $\text{sat}_\text{cl}$ is a list then it has a member of the form $t\rightarrow(l,v)$. Also, this condition is satisfied by each element of the list that is the first argument of $\text{sat}_\text{cnf}$.

Operationally, an invariant will be maintained that, whenever $\text{sat}_\text{cl}(cl,l,hl)$ is selected in LD-resolution, $cl$ is a list and $l$ and $hl$ are numbers, $l > hl$ and $l$ is greater than any number occurring in $cl$. List $cl$ is the not yet processed fragment of a clause $cl_0$ (possibly instantiated), $l$ is the current level, and $hl$ is the highest level of those variables that occur in the already processed part of $cl_0$ and have been bound to some values at previous levels; $hl = -1$ when there is no such variable. In case of failure of $\text{sat}_\text{cl}(cl,l,hl)$, an exception will be raised with the ball being the maximum of $hl$ and the levels of the variables occurring in $cl$ (provided the maximum is $\geq 0$).

Checking the value already assigned to a variable must be treated differently from assigning a value to an unbound variable. This leads to two clauses playing the role of the first clause of $P_1$. So procedure $\text{sat}_\text{cl}$ of $P_1$ is transformed into the following procedure of $P_2$:

\begin{verbatim}
  sat_cl( [Pol-V|_Pairs], _L, _HL ) :-
      nonvar(V), V=(_,-,Pol).
  sat_cl( [Pol-V|_Pairs], L, _HL ) :-
      var(V), V=(L,Pol).
  sat_cl( [_-V|Pairs], L, HL ) :-
      new_highest( V, HL, H ),
      sat_cl( Pairs, L, HNew ).
\end{verbatim}

Predicate $\text{new}_\text{highest}$ takes care of updating the highest level of the variables from the already processed part of the clause.

\begin{verbatim}
% new_highest(var, h, hnew) - if var is a Prolog variable then h = hnew
% otherwise var = (l, v) and hnew = max(h, l)
new_highest( V, H, H ) :- var( V ).
new_highest( V, H, H ) :- nonvar( V ), V=(L,Value), H>=L.
new_highest( V, H, L ) :- nonvar( V ), V=(L,Value), H<L.
\end{verbatim}
Procedure $\text{sat\_cnf}$ is transformed into

\[
\begin{align*}
\text{sat\_cnf}( \emptyset, _L ). \\
\text{sat\_cnf}( [\text{Clause}|\text{Clauses}], L ) :- \\
\text{sat\_cl}( \text{Clause}, L, -1 ), \\
\text{Lnew is L+1}, \\
\text{sat\_cnf}( \text{Clauses}, \text{Lnew} ).
\end{align*}
\]

(11) (12)

Program $P_2$ consists of clauses (5) – (12). An initial query $\text{sat\_cnf}(\text{sat}, 0)$ results in checking the satisfiability of a conjunction of clauses $\text{sat}$.

Now we add backjumping to $P_2$. The backjumping has to be triggered instead of a failure of $\text{sat\_cl}$. The latter happens when the first argument of $\text{sat\_cl}$ is $\emptyset$. The new program $P_3$ contains the procedure $\text{sat\_cl}$ of $P_2$, and additionally a clause

\[
\text{sat\_cl}( \emptyset, -, \text{HL} ) :- \text{HL}>=0, \text{throw}( \text{HL} ).
\]

(13)

triggering a backjump. When $\text{HL} < 0$ then there is no target for backjumping, and standard backtracking is performed.

The procedure $\text{sat\_cnf}$ of the new program $P_3$, is constructed out of that of $P_2$ by transforming clause (12) as described in Approach 2:

\[
\begin{align*}
\text{sat\_cnf}( [\text{Clause}|\text{Clauses}], L ) :- \\
\text{sat\_cl}( \text{Clause}, L, -1 ), \\
\text{Lnew is L+1}, \\
\text{catch}( \text{sat\_cnf}( \text{Clauses}, \text{Lnew} ), \\
\text{L}, \\
fail )
\end{align*}
\]

(14)

So backjumping related to the variable with level $l$, implemented as $\text{throw}(l)$, arrives to an instance of clause (14) where $L$ is $l$. The whole $\text{catch}(\ldots)$ fails, and the control backtracks to the invocation of $\text{sat\_cl}$ that assigned the variable. (An additional predicate $\text{btid}$ was not needed, as $L$ is the unique identifier.)

Now program $P_3$ consists of clauses (5) – (11) and (13) – (14). To avoid leaving unnecessary backtrack points in some Prolog systems, each group of clauses with $\text{var}/1$ and $\text{nonvar}/1$ (clause (5) with (6), and (8) with (9) and (10)) may be replaced by a single clause employing $(\text{var}(V) \rightarrow \ldots; \ldots)$ and, in the second case, $(H<L \rightarrow \ldots; \ldots)$. To simplify a bit the initial queries, a top level predicate may be added, defined by a clause $\text{sat}(\text{Clauses}) :- \text{sat\_cnf}(\text{Clauses}, 0)$.

Example 2

Here we transform $P_1$ from Ex.1 to a binary program and apply Approach 1. The binary program $P_b$ is

\[
\begin{align*}
\text{sat\_b}(\emptyset), \\
\text{sat\_b}( [[\text{Pol}-\text{Pol}|-]\text{Clauses}] ) \leftarrow \text{sat\_b}(\text{Clauses}). \\
\text{sat\_b}( [[]\text{Pairs}|\text{Clauses}] ) \leftarrow \text{sat\_b}(\text{Pairs}|\text{Clauses}).
\end{align*}
\]

Note that in Ex.1 the unprocessed part of the current clause was an argument of $\text{sat\_cl}$, now it is the head of the argument of $\text{sat\_b}$. In what follows we do not explain some details which are as in the previous example.
As previously we introduce levels, and represent a value of a variable by \((l,v)\), where \(l\) is a level and \(v\) a logical value. As previously, we first transform \(P_b\) into \(P_{b2}\) dealing with levels, and then add backjumping to \(P_{b2}\). We add two arguments to \(\text{sat}_b\), they are the same as the arguments added to \(\text{sat}_cl\) in Ex. 1. The declarative semantics is similar, the first argument of \(\text{sat}_b\) (in an answer of \(P_{b2}\)) is as the first argument of \(\text{sat}_cnf\) in \(P_2\). An invariant similar to that of Ex. 1 will be maintained by the operational semantics.

Whenever \(\text{sat}_b(cls,l,hl)\) is selected, \(l\) and \(hl\) are numbers, \(l > hl\) and \(l\) is greater than any number occurring in \(cls\). List \(cls\) is a conjunction of clauses (possibly instantiated), and its head, say \(cl\), is the not yet processed fragment of the current clause, say \(cl_0\); number \(l\) is the current level, and \(hl\) is the highest level of variables from the already processed part of \(cl_0\). Now program \(P_{b2}\) is:

\[
\text{sat}_b( [], _L, _HL ).
\]
\[(15)\]

\[
\text{sat}_b( [[Pol-V[_] | Clauses], L, _HL ) :- \text{nonvar}(V),
V=(_, Pol), Lnew is L+1,
\text{sat}_b( Clauses, Lnew, -1 ).
\]
\[(16)\]

\[
\text{sat}_b( [[Pol-V[_] | Clauses], L, _HL ) :- \text{var}(V),
V=(L, Pol), Lnew is L+1,
\text{sat}_b( Clauses, Lnew, -1 ).
\]
\[(17)\]

\[
\text{sat}_b( [[]-V|Pairs] | Clauses], L, HL ) :-
Lnew is L+1,
\text{new_highest}( V, HL, HLnew ),
\text{sat}_b( [Pairs | Clauses], Lnew, HLnew ).
\]
\[(18)\]

Procedure \text{new_highest}/3 is the same as in the previous example. Program \(P_{b2}\) with a query \(\text{sat}_b(sat,0,-1)\) checks satisfiability of the conjunction of clauses \(sat\).

Now we add backjumping to \(P_{b2}\). As previously, backjumping originates when an empty clause is encountered:

\[
\text{sat}_b( [] | _Clauses], _L, HL ) :- HL>=0, \text{throw}( HL ).
\]
\[(19)\]

Let us discuss backjump targets. Assume that the nodes of an LD-tree satisfy the invariant. Consider the descendants of a node \(N = \text{sat}_b(cls,l,hl)\) obtained by first resolving \(N\) with clause \[(16)\] or \[(18)\]. A ball thrown from such a descendant \(N_2\) is not \(l\). So for the backjump target we need to modify only clause \[(17)\]. Following Section 2.1 we obtain:

\[
\text{sat}_b( [[Pol-V[_] | Clauses], L, _HL ) :-
catch(
( \text{var}(V), V=(L, Pol), Lnew is L+1,
\text{sat}_b(\text{Clauses}, Lnew, -1)
),
L,
\text{fail}
).
\]
\[(20)\]

\[^6\] Consider a node \(N = \text{sat}_b(cls,l,hl)\) and its closest descendant \(N’\) of the form \(\text{sat}_b(\ldots)\). So \(N’ = \text{sat}_b(\ldots, l+1, \ldots)\). If a number \(i\) occurs in a node between \(N\) and \(N’\), or in \(N’\), then \(i = l+1\) or \(i\) occurs in \(N\). By induction, if \(i\) occurs in a descendant of \(N\) then \(i\) occurs in \(N\) or \(i > l\). Additionally, if \(N’\) was obtained by first resolving \(N\) with \[(16)\] or \[(18)\], then \(N’\) does not contain \(l\). Thus no descendant of \(N’\) contains \(l\).
The final program $P_{b3}$ consists of clauses (15), (16), (20), (18), (19), and (8) – (10)\(^7\).

4 Another approach

Here we discuss simulating backjumping by means of Prolog backtracking. This requires employing the Prolog database. An example of such approach was presented by Bruynooghe (2004). A backjump is initiated by a failure preceded by depositing in the Prolog database an identifier of the backjump target. At each backtracking step, the database is queried to check if the backjumping target is reached. If not, further backtracking is caused. This is done by some extra code placed at the beginning of the body of each clause involved in backjumping. (In the presented example, there is only one such clause.)

For some programs it may be impossible, or difficult, to statically determine the clauses involved in backjumping. Also, the set of such clauses may differ for various initial queries. In a general case, the idea of Bruynooghe (2004) can be implemented by converting each clause $p(\vec{t}) \leftarrow B$ into

$$ p(\vec{t}) \leftarrow \text{btid}(\vec{t}, Id), \text{catch}(Id), B. $$

where btid/2 is as in Section 2 and catch/1 is a new predicate. An invariant is maintained that the database contains a backjump target only during backjumping. Query catch(t) succeeds immediately, unless during backjumping. In the latter case it fails if t is not unifiable with the backjumping target. Otherwise it removes the target from the database and succeeds (instantiating t in the obvious way)\(^8\).

Note that there are no restrictions in this approach on the origin/target of backjumping, in contrast to those discussed in Section 2.

5 Final comments

Related work. For the work of Bruynooghe (2004), see the previous section. Robbins et al. (2021) present a non-trivial example of using Prolog exception handling to implement backjumping. (The main example is preceded by a simple introductory one.) The program is a SAT solver with conflict-driven clause learning. A learned clause determines the target of a backjump. There is no plain backtracking. The program keeps the learned clauses in the Prolog database, to preserve them during backjumping. Prolog coroutining is is employed in a fundamental way. The program is rather complicated, it seems impossible to view it as some initial program with added backjumping. To understand it one has to reason about the details of the operational semantics.

That paper does not propose any general way of adding backjumping to logic programs. The difference between backjumping and Prolog exception handling discussed here in

---

\(^7\) In (20), the first atom $\text{var}(V)$ from the body of (17) can be moved outside of catch, transforming the body of (20) to $\text{var}(V), \text{catch}(\ldots)$. (This is because $\text{var}(V)$ is deterministic and not involved in backjumping.) Now, similarly as in the previous example, some backtrack points may be avoided by replacing clauses (20) and (16) by a single clause with the body of the form $\text{var}(V) \rightarrow \ldots :: ::$.

\(^8\) In SICStus, it can be defined by $\text{catch}(Id) :- \text{bb_get(target,\ldots)} \rightarrow \text{bb_delete(target,Id)} ; \text{true.}$, and use $\text{bb_put(target,\ldots),fail}$ to cause a backjump.
Section is not noticed. We cannot agree with the claims “backjumping is exception handling” and that “catch and throw [provide] exactly what is required for programming backjumping” (Robbins et al. 2021, the title, and p. 142-143).

Conclusions. The subject of this paper is adding backjumping to logic programs. Additionally, we briefly showed how to simulate backjumping by means of plain backtracking and the Prolog database. We discussed the differences between backjumping and Prolog exception handling, and proposed two approaches to implement the former by the latter. This seems impossible in a general case. The first approach imposes certain restrictions on where backjumping can be started. The second one – on the target of backjumping. The restrictions seem not severe. The first approach is applicable, among others, to binary programs with arbitrary backjumping. For the second approach, the presented example shows that sometimes the difference between the required and the actual target may be unimportant. As every program can be transformed to a binary one (Maher 1988; Tarau and Boyer 1990), the first approach is indirectly applicable to all cases.

References

Bruynooghe, M. 2004. Enhancing a search algorithm to perform intelligent backtracking. Theory Pract. Log. Program. 4, 3, 371–380.

Bruynooghe, M. and Pereira, L. M. 1984. Deduction revision by intelligent backtracking. In Implementations of Prolog, J. A. Campbell, Ed. Ellis Horwood/Halsted Press/Wiley, 194–215.

Deransart, P., Ed-Dbali, A., and Cervoni, L. 1996. Prolog - the standard: reference manual. Springer.

Drabent, W. 2018. Logic + control: On program construction and verification. Theory and Practice of Logic Programming 18, 1, 1–29.

Howe, J. M. and King, A. 2012. A pearl on SAT and SMT solving in Prolog. Theor. Comput. Sci. 435, 43–55.

Maher, M. J. 1988. Equivalences of logic programs. In Foundations of Deductive Databases and Logic Programming, J. Minker, Ed. Morgan Kaufmann, 627–658.

Robbins, E., King, A., and Howe, J. M. 2021. Backjumping is exception handling. Theory Pract. Log. Program. 21, 2, 125–144.

Tarau, P. and Boyer, M. 1990. Elementary logic programs. In Programming Language Implementation and Logic Programming, PLILP’90, P. Deransart and J. Maluszynski, Eds. Lecture Notes in Computer Science, vol. 456. Springer, 159–173.

Most likely this is because the authors have not faced the limitations pointed out here. Backjumping in their program is similar to that of (Approach 1a for n = 2), with throw(Id) dropped (as there is no standard backtracking), and catch(B₂, Id, fail) replaced by B₂ (as there is no backjumping from B₂ with the current Id).