Thermodynamic assessment and performance optimization of solid oxide fuel cell-Stirling heat engine–reverse osmosis desalination

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Abstract

Fuel cells are chemical energy converted to electric energy, which is today a new technology in energy production. Among the existing fuel cells, solid fuel oxide cells have a high potential for use in synthetic and combined production systems due to their high temperature (700–1000°C). The solid oxide fuel cell (SOFC) output acts as a high-temperature source, which can be used for heat engines such as the Stirling engine as a high-temperature heat source. A hybrid system including solid oxide fuel cell and Stirling engine and reverse osmosis desalinating is a cogeneration plant. This system includes two parts for power generation; the first part is power generated in the SOFC, and the second part is that with use of heat rejection of solid oxide fuel cell to generate power in the Stirling engine. Also, due to the water critical situation in the world and the need for freshwater, it is very common to use desalination systems. In this study, important goals such as power density and exergy destruction, and exergy efficiency, have been investigated. In general, the performance of the hybrid system has been investigated. Firstly, a thermodynamic analysis for all components of the system and then multi-objective optimization performed for several objective functions include exergy destruction density, exergy efficiency, fuel cell power and freshwater production rate. The present optimization is performed for two overall purposes; the first purpose is to improve fuel cell output power, exergy efficiency and exergy destruction density, and the second purpose is to improve the exergy efficiency, the amount of freshwater production and exergy destruction density. In this optimization, three robust decision-making methods TOPSIS, LINMAP and FUZZY are used. Two scenarios are presented; the first scenario is covering power, exergy efficiency and exergy destruction density. The output power and exergy efficiency, and exergy destruction density, have optimum values in the TOPSIS method's results. The values are 939.393 (kW), 0.838 and 1139.85 (w/m²) respectively. In the second scenario that includes the freshwater production rate, the exergy destruction density and exergy efficiency, three objective functions are at their peak in the FUZZY results, which are 5.697 (kg/s), 7561.192 (w/m²) and 0.7421 respectively.

Keywords: solid oxide fuel cell; reverse osmosis desalination; Stirling engine; exergy destruction density; hybrid system; multi-objective optimization

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1. INTRODUCTION

The crisis related to environment and energy necessitates the development of clean and efficient energy technologies [1]. Fuel cells as attractive energy technologies are applied to efficiently convert the chemical energy content of the fuels into electricity [2]. These different kinds of fuel cells and solid oxide fuel cells (SOFCs) are one of the most attractive types due to their low emission of greenhouse gases, flexibility in the utilized fuel, affordable catalyst and noticeable rate of electrochemical reaction [3–5]. The studies on the SOFCs have focused on different aspects including the fabrication of novel material as the electrodes and their thermal management [6–9], enhancement in their durability [10, 11], development of advanced and new prototypes [12–13] and modeling of their cell as a single unit [14–16]. The high working temperature of these fuel cells makes it possible to utilize the produced high-grade heat in a bottoming cycle for power generation [17–21]. Applying SOFC in the tri- or co-generation systems can improve the energy and exergy efficiency up to 80 and 60%, respectively [22–24]. Various studies have been performed on the hybrid systems consisting of SOFC fed with different fuels [25–27] and coupled with various bottom cycles [28–32] by employing different analysis methods [33–35]. Liao et al. [28] proposed thermo–photovoltaic cells to efficiently exploit the waste heat from SOFC and compared the proposed hybrid system with some other SOFC-based hybrid systems [28]. Mehrpooya et al. [29] proposed a system composed of the gas turbine, SOFC, Rankine cycle and system work as an absorption refrigeration unit. In this research, energy, exergy and economic criteria were considered to find the optimum working condition. In another study [30], thermodynamic analysis was done on a tri-generation unit by applying a mixture of ammonia–water to use the waste heat of a system composed of SOFC and gas turbine. Elmeter et al. [36] evaluated the application of liquid desiccant air conditioning technology. Pike et al. [37] investigated the potential of synthesis of new Mn/Ti-containing perovskite for use as SOFC electrodes. Tsai et al. [38] evaluated control system design for micro-tubular SOFC which improves the overall efficiency and extends the working life.

In a study done by Winkler et al. [39], they compared which one of two-hybrid systems consists of SOFC with a gas turbine and which one consists of SOFC and the Stirling engine for automotive technology use. It was indicated that the one using the Stirling engine in the system can result in a more appropriate performance in terms of thermal efficiency compared with utilizing gas turbines. Foley [40] suggested using a hydrogen-fed SFOC integrated with the Stirling engine for small-scale applications. It was accepted that the designed configuration can be used to provide power with 5-kW capacity and efficiency equal to 51%. In a study performed by Rokni [41], a system consisting of a Stirling engine and SOFC fed with alternative fuel was thermodynamically analyzed for producing both electricity and heat. The determined efficiency of the system was 60%, and its efficiency increased by a partial decrease in the fuel utilization factor. According to the economic analysis, the plant cost was $2060/kW. In another scientific work [42], energy analysis was done to assess a gasification unit coupled with the SOFC–Stirling engine in order to generate 120 kW electricity. It was concluded that there is an optimal value for the fuel utilization factor which was ~65%. Rokni [43] performed an analysis based on thermodynamic and thermoeconomic aspects of this system and concluded that the electricity and hot water costs are 0.1204 and 0.0214$/kWh, respectively. In another research [44], municipal solid wastes were employed for producing syngas in order to be used in a system composed of SOFC and Stirling engine. It was concluded that the efficiency of the system in electricity generation was 48% and the thermal efficiency of the CHP can reach 95%. Evolutionary algorithms (EAs) have been applied since the 1980s to assist clarifying the multi-objective problems [45]. In general, these algorithms provide a set of acceptable answers which do not have any overlap [46]. Typically, in the optimization with several objectives, different sets of solutions known as Pareto frontier are tested to find the nearest match in the area of the objective function. These algorithms are applied to find the optimum conditions for the operation of the system [47–59]. Beizgadeh et al. [60] investigated a simple approach of a SOFC power system that feeds from natural gas. In the field of hybrid systems, Emin et al. [61] analyzed the environmental performance of a hybrid system consisting of an irreversible SOFC and a Stirling engine. Gholamian et al. [62] analyzed recycled anode–cathode for a hybrid system consisting of SOFC and Stirling engine for air transport plans. Sematic et al. [63] formed a smart grid using the GA-ANN model and predicted the power output of a solar sterling engine. Zhan et al. [64] investigated the optimal performance modes and selected criteria for a high-temperature fuel cell. Various studies have been performed on the hybrid systems consisting of a Bryton CO2 supercritical cycle and molten carbonate fuel cell [65–66]. Ahmadi et al. [67] developed a thermal model to evaluate the performance of a Stirling engine. Khoshbazan et al. [68] evaluated a multi-objective optimization of micro-CHP Stirling and investigated thermoeconomic analysis in Iran. Increasing energy storage and reducing number of emissions are acceptable results which have been obtained. Ahmadi et al. [69] evaluated output power and Atkinson engine thermal efficiency via employing the NSGA-II approach and thermodynamic analysis.

On the performance of reverse osmosis desalination, Zahn et al. [70] conducted a study to study the desalination with a type of low-deposition reverse osmosis carrier AL2O3. In the field of multi-objective optimization, Ahmadi et al. [71] evaluated multi-objective optimization of irreversible molten carbonate fuel cell–Stirling thermal engine–reverse osmosis. Ahmadi et al. [72] investigated a thermodynamic study that optimized the performance of a multi-objective CO2 power cycle guided by solar energy and LNG cooling recycling. Ahmadi et al. [73] performed multi-objective optimization of the irreversible molten carbonate fuel cell–Bryson thermal engine with an approach objective environment. Chen et al. [74–75] investigated progress in field generalized thermodynamic dynamic optimization of irreversible cycles.  

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Ahmadi et al. [76] evaluated the performance optimization of a nano-scale Stirling cryogenic cycle using Maxwell–Boltzmann gas. Maleki et al. [77] investigated harmony search optimization for optimum sizing of hybrid solar schemes that observe results where improved harmony search (HIS) has higher robustness compared with the HS and simulated annealing algorithms. Ghandari et al. [78] evaluated applications of nanofluids containing carbon nanotubes in solar energy systems. Cai et al. [79] investigated harmony search optimization for optimum sizing of hybrid solar schemes that observe results where improved harmony search (HIS) has higher robustness compared with the HS and simulated annealing algorithms. Ghandari et al. [78] evaluated applications of nanofluids containing carbon nanotubes in solar energy systems. Cai et al. [79] investigated harmony search optimization for optimum sizing of hybrid solar schemes that observe results where improved harmony search (HIS) has higher robustness compared with the HS and simulated annealing algorithms.

In the present paper, an irreversible thermodynamic model of a combined SOFC-Stirling heat engine-RO system is developed in which there are not only irreversible losses in SOFC but also heat leakage from Stirling heat engine to the environment. Also, the transfer between the SOFC and the Stirling heat engine is considered. The aim of the considered system is to investigate the effects of irreversibilities on the fuel cell performance for different operating conditions. In addition, parameter optimization evaluation was conducted to obtain the optimal performance of the system. All these analyses were accomplished by MATLAB.

### 2. MODEL DESCRIPTION

Figure 1 schematically shows the entire cycle of SOFC-Stirling heat engine-RO desalination. This cycle includes solid-oxide fuel cell, Stirling engine and reverse osmosis desalination. SFOC is a heat resource of high temperature that generates the required power of the Stirling engine; fuel cell output is calculated as below, where $\Delta h$ is the molar enthalpy change [80]:

$$
\eta_F = \frac{P_F}{-\Delta h} = \frac{n_e F}{m - \frac{k}{RT d} m^2} \quad (1)
$$

$$
P_F = \frac{i A}{n_e F} \left( m - \frac{k}{RT d} m^2 \right) \quad (2)
$$

In current flow, $V_{act,a}$ and $V_{act,c}$ are the activation overpotential of the anode and cathode, $V_{con,c,a}$ and $V_{con,c,c}$ are the concentration overpotential of the anode and cathode and $V_{ohm}$ is the Ohmic overpotential.

$$
V_F = E - V_{act,a} - V_{act,c} - V_{con,c,a} - V_{con,c,c} - V_{ohm} = \frac{1}{n_e F} \left[ -\Delta g^o (T) + RT \ln \left( \frac{P_{H_2}P_{O_2}^{1/2}}{P_{H_2O}} \right) - RT d \right] \quad (3)
$$

In current flow in the system, $F$ is Faraday’s fixation, $(n_e)$ is the number of electrons, $A$ is the page surface area, $R$ is the gas fixation world, $T$ is the fuel cell temperature and $d$ and $m$ are defined as follows:

$$
d = 2n_e \sin h^{-1} \left( \frac{i}{2i_{act,a}} \right) + 2n_e \sin h^{-1} \left( \frac{i}{2i_{act,c}} \right) - \ln \left( 1 - \frac{i}{i_{act,a}} \right) - \ln \left( 1 - \frac{i}{i_{act,c}} \right) + \frac{m R F E_{act}}{\sigma_0 R} \exp \left( \frac{E_{act}}{RT} \right) \quad (4)
$$

$$
m = -\Delta g(T) + RT \ln \left( \frac{P_{H_2}P_{O_2}^{1/2}}{P_{H_2O}} \right) - RT d \quad (5)
$$

$(i_{act,a})$ and $(i_{act,c})$ are the amount of anode and cathode, respectively. $(L_e)$ is the amount of electrode thickness and $(\sigma_0)$, $(P_{H_2})$, $(P_{O_2})$ and $(P_{H_2O})$ are the partial pressures. $(i_{act,a})$ and $(i_{act,c})$ are the partial currents of anode and cathode, $(E_{act})$ activation energy and $(\Delta g)$ is the Gibbs energy change.

The irreversible power of the fuel cell is calculated by the below relation [80]:

$$
P_{rev,F} = -\frac{i A}{n_e F} \Delta g \quad (6)
$$
Exergy loss rate:

\[ Exd_F = P_{\text{rev,F}} - P_F \]  

(7)

The heat transfer rate is defined as follows:

\[ Q_r = K_r (1 - \varepsilon_r) (T - T_0) \]  

(8)

\( (T_0), (K_r) \) and \( \varepsilon_r \) are the environment temperature, heat transfer rate and transfer output, respectively.

Using the first thermodynamic rule, the heat input to the cycle is defined as

\[ Q_H = -\Delta H - P_F - Q_r \]  

(9)

The energy efficiency of the irreversible Stirling cycle is defined as follows [81]:

\[ \eta_S = 1 - \frac{Q_L}{Q_H} = \frac{(1 - y)}{1 + \frac{\alpha(1-y)}{(y-1)\pi x}} \]  

(10)

\( \pi \) the specific thermal ratio, \( \alpha \) is the imperfect regeneration coefficient and \( x \) is the compression ratio \( x = \frac{V_{\text{max}}}{V_{\text{min}}} \) which is the heat offset from the irreversible Stirling engine, defined as follows:

\[ Q_L = Q_H (1 - \eta_S) \]  

(11)

The power of irreversible sterilization, the exergy structure of the Stirling engine and the reversible power of the Stirling engine are defined as follows, where \( T_0 \) is the ambient temperature:

\[ P_S = Q_H - Q_L \]  

(12)

\[ exd_S = T_0 \left( \frac{Q_L}{T_L} - \frac{Q_H}{T} \right) \]  

(13)

\[ P_{\text{rev,S}} = Q_H \left( \frac{T_0}{T} \right) - Q_L \left( \frac{T_0}{T_L} \right) \]  

(14)

Output power, energy efficiency, exergy efficiency and exergy destruction of the hybrid system are defined as follows:

\[ P_h = P_F + P_S \]  

(15)

\[ \eta_h = \frac{P_F + P_S}{-\Delta H} \]  

(17)

\[ \varphi_h = \frac{P_F + P_S}{P_{\text{rev,F}} + P_{\text{rev,S}}} \]  

(18)

\[ Exd_h = Exd_F + Exd_S \]  

(19)

\[ exd_h = \frac{Exd_h}{A} = exd_F + exd_S \]  

(20)

The output from the maximum power output and the minimum environmental effect are defined as follows, where \( b_1 \) and \( b_2 \) are obtained from Eco indicator 95 and Eco indicator 99 [82, 83]:

\[ F = \frac{P_h}{Mb_1 + exd_h b_2 + P_h b_3} \]  

(21)

In this section, we look at the overall system that combines the fuel cell and the Stirling system with the reverse osmosis system, and here we remind all the formulations related to the Stirling fuel cell, then transferring the Stirling power to the reverse osmosis amount. We calculate the flow volume and plot the flow diagram relative to the volume of flow generated by the reverse osmosis desalination. The mathematical model for the RO unit is developed by [84].

\[ \text{Power}_{\text{net}} = P_F + P_S \]  

(22)

\[ \bar{P} = 0.5 (P_F + P_b) \]  

(23)

\[ \pi = 0.5 (\pi_F + \pi_b) \]  

(24)

As shown in Eq. 12, \( P_F \) and \( P_b \) are the flow hydraulic pressure and declining flow, respectively. Also, according to Eq. 13, \( \pi_F, \pi_b \) is the osmosis pressure of the flow and declining flow. In Eq. 13, \( P_p \) and \( \pi_p \) are the hydraulic pressure and osmosis pressure in flow, respectively.

\[ \Delta P = \bar{P} - P_p \]  

(25)

\[ \Delta \pi = \pi - \pi_p \]  

(26)

\[ \pi_F = (RTX_F) \]  

(27)

\[ \pi_b = (RTX_b) \]  

(28)

\[ \pi_p = (RTX_p) \]  

(29)

\( R \) is the gas world fixation and \( T \) is the water temperature. \( X_F, X_b, X_p \) are salt, \( \bar{P} \) and \( \pi \) are the average feed water pressures and average osmotic pressure on the feed side and brine side, respectively.

The net pressure difference across the high pressure pump is defined as follows:

\[ \Delta P_{\text{net}} = \Delta P + \Delta \pi \]  

(30)

\( \Delta P \) and \( \Delta \pi \) are the hydraulic penetration and osmosis pressure, respectively.

\[ m_f = \frac{\text{Power}_{\text{net}} \rho_f \eta_{\text{pump}}}{\Delta P_{\text{net}}} \]  

(31)

In the formula above, \( m_f \) is the mass flow rate of freshwater, \( \rho_f \) is the ratio of the flow dilution density, \( \eta_{\text{pump}} \) is the mechanical impeller of the pump and \( \Delta P_{\text{net}} \) is the difference between the pure pressures of the high-pressure pump.
3. OPTIMIZATION ALGORITHM

Typically, in optimization problems considering various objectives, several sets of answers known as Pareto frontier are provided. These answers indicate the nearest match in the area of the objective function. Researchers have applied multi-objective evolutionary algorithms (MOEAs), in various studies. These types of algorithms are able to couple with the difficulties that exist in the conventional approaches [42]. In the current research, the Pareto frontier is obtained by employing the NSGA-II method. The basics of this approach are completely represented in Refs. [55, 56]. For determination of the optimal solution, a process which is known as ‘decision making’ must be utilized. Various approaches are used as decision-making tools for choosing the final solution from the Pareto front. In order to overcome the possible problems related to the objectives of dissimilar scales, the dimensions are unified. In addition, it is necessary to make the vectors non-dimensionalized in the process of decision making. Some of the most applicable non-dimensionalizing approaches are Linear, Fuzzy and Euclidian. In this article, Fuzzy, TOPSIS and LINMAP are used for decision making. The two former approaches employed Euclidian for non-dimensionalizing, while Fuzzy applies Fuzzy for this purpose. More details of the applications of decision-making methods are represented in the previous studies [45–54].

Three objective functions are utilized in this optimization: Power density, \( \phi \) and \( exd \), described by Eqs. (16), (17) and (19), respectively. Also, three decision variables are considered, T and A.

Although the decision variables might be different in the optimizing plan, they typically need to be fitted in a sensible range. Thus, the objective functions are determined by the limits of decision variables:

\[
7000 \leq i \leq 15000 \text{ A/m}^2 \quad (32)
\]

\[
1073 \leq T \leq 1273 \text{ K} \quad (33)
\]

\[
0.075 \leq A \leq 0.38 \text{ m}^2 \quad (34)
\]

4. RESULTS AND DISCUSSION

In this section, evaluations of parameters on fuel cell performance include the variations in the output power of the cell, the cell voltage and the efficiency of the cell, which are shown by Eqs. (1), (2) and (3).

As shown in Figure 2, at a constant current density by increasing the temperature, the fuel cell voltage also increases, because the fuel cell voltage is directly related to the internal voltage \( (V_{\text{int}}) \) and the internal voltage value is related to the values: partial pressure, temperature and current density, where, by increasing the temperature, the internal voltage \( (V_{\text{int}}) \) increases, and ultimately the fuel cell voltage also increases. Also, at a constant temperature, by increasing the current density, the fuel cell voltage decreases, because the value of the internal voltage \( (V_{\text{int}}) \) is inversely related to the current density; therefore, by increasing the current density, the value of the internal voltage \( (V_{\text{int}}) \) decreases and causes the overall voltage of the fuel cell to decrease.

As shown in Figure 3, at a constant current density, by increasing the temperature, the output power performance also increases, because the fuel cell output power is related to the internal voltage value \( (V_{\text{int}}) \) and the internal voltage value is directly related to the temperature value; therefore, by increasing the temperature, the value internal voltage increases and ultimately the value output power will increase too. Also at a constant temperature, by increasing the current density, the output power of the fuel cell will decrease, because the internal voltage \( (V_{\text{int}}) \) is inversely related to the current density; therefore, increasing the current density causes a decrease in the internal voltage and ultimately reduces the output power of the fuel cell.

As shown in Figure 4, at a constant density by increasing temperature, the value fuel cell efficiency also increases, because fuel cell efficiency is related to the value power output of the fuel cell and the value power output of the fuel cell is directly related to the temperature; therefore, by increasing the temperature, the value power fuel cell increases, and ultimately the value fuel cell efficiency also increases. Also, at a constant temperature by increasing the current density the fuel cell efficiency reduces, because the cell power is inversely related to the current density, and increasing the current density causes reduction in the output.
power and ultimately reduces the fuel cell efficiency. Therefore, it can be concluded to improve fuel cell performance, the fuel cell temperature should be increased.

Another effective parameter affecting different fuel cell performance is the electrolyte thickness ($L_e$) used in the fuel cell. As shown in Figure 5, at a constant current density by increasing the electrolyte thickness ($L_e$) the output power and fuel cell voltage decreases, because the internal voltage ($V_{\text{int}}$) with the electrolyte thickness ($L_e$) is inversely correlated, so the internal voltage ($V_{\text{int}}$) decreases by increasing the electrolyte thickness ($L_e$); ultimately, the output power and fuel cell voltage decreases. Also at a constant value of electrolyte thickness ($L_e$), by increasing the current density, the value power and voltage of the fuel cell decreases. Also at a constant current density by increasing the electrolyte thickness ($L_e$), the value power and voltage of the fuel cell decreases, because the current density is inversely related to the internal voltage ($V_{\text{int}}$) and the internal current ($V_{\text{int}}$) decreases by increasing current density, and ultimately both the value power output and voltage of fuel cell decrease quickly.

As shown in Figure 6, at a constant current density by increasing the electrolyte thickness ($L_e$), the value fuel cell efficiency decreases, because fuel cell efficiency is related to the value power output of the fuel cell, and the value power output is inversely related to the value of electrolyte thickness. Therefore, by increasing the value of electrolyte thickness ($L_e$), the output power of the fuel cell decreases and ultimately the value fuel cell efficiency will decrease. Also, at a constant electrolyte thickness ($L_e$), the fuel cell efficiency decreases by increasing the current density, because the value fuel cell output power is inversely related to the current density; therefore, by increasing the current density, the value power output of the fuel cell decreases and ultimately the value fuel cell efficiency also decreases. In general, it can be concluded that to improve fuel cell performance, the fuel cell temperature ($T$) and fuel cell electrolyte thickness ($L_e$) should be increased and decreased respectively.

$k$, the ratio of internal resistance to leakage resistance, is defined as $k = \left( \frac{R_{\text{int}}}{R_{\text{leak}}^2} \right)$. As shown in Figure 7, at a constant current density by increasing the leakage resistance ($R_{\text{leak}}$), the value fuel cell output power also increases, because the fuel cell output power is directly related to leakage resistance ($R_{\text{leak}}$), so the value fuel cell output power increases by increasing leakage resistance ($R_{\text{leak}}$). Also at a constant value of leakage resistance ($R_{\text{leak}}$), by increasing the current density, the output power decreases, because the output power of the fuel cell is directly related to the internal voltage ($V_{\text{int}}$) and the internal voltage ($V_{\text{int}}$) inversely related to the current density; therefore, by increasing the current density, the value of the internal voltage ($V_{\text{int}}$) decreases, resulting in a decrease in the output power of the fuel cell.

$k$, the ratio of internal resistance to leakage resistance, is defined as $k = \left( \frac{R_{\text{int}}}{R_{\text{leak}}} \right)$. As shown in Figure 8, at a constant current density, the fuel cell efficiency increases by increasing the leakage resistance ($R_{\text{leak}}$), because fuel cell efficiency is directly related.
to leakage resistance \( R_{\text{leakage}} \), so by increasing the leakage resistance \( R_{\text{leakage}} \), the value of fuel cell efficiency increases. Also, at a constant value of leakage resistance \( R_{\text{leakage}} \), the fuel cell efficiency decreases; by increasing the current density, because fuel cell efficiency is inversely correlated with current density, the value of fuel cell efficiency decreases by increasing the current density.

As shown in Figure 9, at a constant current density by increasing the hydrogen molar fraction, the output power of the cell increases, because the output power is directly related to the molar fraction, so by increasing the molar fraction the output power of the fuel cell also increases. Also, at a constant molar fraction, the power output of the fuel cell decreases by increasing the current density, because the output power of fuel cell is inversely correlated with the current density; therefore, the output power of fuel cell decreases by increasing current density.

As shown in Figure 10, at a constant current density by increasing the hydrogen fraction, the fuel cell efficiency also increases, because fuel cell efficiency is related to the value power output of the fuel cell and the value of power is directly related to the molar fraction hydrogen; therefore, by increasing the hydrogen mole fraction, the value power increases, and the value fuel cell efficiency also increases. Also at a constant amount of molar fraction, the fuel cell efficiency decreases by increasing the current density; because the output power of the fuel cell is inversely correlated with the current density, the power output of fuel cell decreases by increasing current density, and ultimately the fuel cell efficiency also reduces.

Figure 11 shows the changes in fuel cell efficiency and power output of the current density fuel cell. According to the figure, by increasing the current density, the value power output of the fuel cell and also the fuel cell efficiency decreases, because the internal density \( V_{\text{int}} \) is inversely related to the current density, and the output power and fuel cell efficiency are directly related to the internal voltage \( V_{\text{int}} \); therefore, by increasing the current density, the value of internal voltage \( V_{\text{int}} \) decreases and then the value power output of the fuel cell and fuel cell efficiency also decreases. As shown in the figure, by increasing the current density, the maximum value output power of the fuel cell decreases gradually, but the maximum value of fuel cell efficiency is a constant value. Also, by increasing the current density, the fuel cell power initially increases and then decreases because by increasing the current density, the loss increases, and by increasing loss, the fuel cell voltage decreases but because the voltage is high initially; therefore, the power of the fuel cell also increases, but the more the voltage decreases, the more fuel cell power decreases. Also, for fuel cell efficiency, by increasing current density and losses, fuel cell efficiency, like fuel cell power, initially increases and then decreases.
In this section, the performance of the parameters on the stirling engine is evaluated. The parameters include the output power of the stirling engine and the exergy destruction density in Eqs. (13) and (14).

The changes in the output power of the stirling engine with the current density have been investigated, as shown in Figure 12. According to the figure at a constant current density $i = 20000 \text{ (A/m}^2\text{)}$, the maximum value power of the stirling engine is $P_{S,\text{max}} = 3700 \text{ (W/m}^2\text{)}$. As can be seen by increasing the current density, the value power output of the Stirling engine first increases and then decreases, because the output power of the stirling engine is inversely related to the current density value. In addition, by increasing the current density, the power of the Stirling engine initially increases and then decreases. Because by increasing the current density, also loss will increase, and by increasing loss, the amount of Stirling engine voltage decreases, but because the voltage has a high value initially, the power of the Stirling engine first increases to achieve a maximum value, but then the Stirling engine voltage decreases more.

The changes of exergy destruction density ($\text{exd}$) with current density have been investigated, as shown in Figure 13. According to the figure, at a constant current density $i = 15000 \text{ (A/m}^2\text{)}$, the maximum value of the exergy destruction density $\text{isexd}_{\text{max}} = 1900 \text{ (W/m}^2\text{)}$. As can be seen, by increasing the current density, the value exergy destruction density ($\text{exd}$) increases, because the current density is directly related to the exergy destruction density of the stirling engine ($\text{exd}$). Also, by increasing the current density, the exergy destruction density increases. All thermodynamic defects on the system are defined exergy destruction density, and increasing the current density also causes it to increase the losses, thus defining a kind of destruction for the Stirling engine. Therefore, increasing the current density causes increase in the exergy destruction density.

In this section, the performance of the parameters on the hybrid system is evaluated, which includes the parameters of output power, efficiency, exergy destruction density and thermo-environmental function density in Eqs. (15), (17) and (20) and (21), respectively.

The changes in the output power of the hybrid system with the current density are shown in Figure 14. According to the figure, at a constant current density $i = 20000 \text{ (A/m}^2\text{)}$, the maximum output power of the hybrid system is $P_{\text{max}} = 4100 \text{ (W/m}^2\text{)}$. As can be seen, by increasing the current density, the value power output of the hybrid system first increases and then decreases, because the current density is inversely related to the output power of the hybrid system. Also according to the figure, the hybrid system power increases by increasing the current density, because by increasing the current density the losses will increase, and by increasing losses, the voltage of the hybrid system decreases. However, because the voltage has a high value at first, so the power of the hybrid system first increases and reaches a maximum value, but then with more decrease in voltage, the power of the hybrid system will decrease.

The changes in the hybrid system efficiency ($\eta$) with current density are shown in Figure 15. According to the figure, at a constant current density $i = 5000 \text{ (A/m}^2\text{)}$, the maximum value of the hybrid system efficiency is $(\eta)_{\text{max}} = 0.57$. As can be seen, the hybrid system efficiency decreases by increasing the current density, because the current density is inversely related to the hybrid system efficiency($\eta$).

The changes of exergy destruction density($\text{exd}$) with the current density are shown in Figure 16. According to the figure at a constant current density $i = 15000 \text{ (A/m}^2\text{)}$, the maximum value of the exergy destruction density ($\text{exd}$)$\text{isexd} = 6500 \text{ (W/m}^2\text{)}$. As can be seen, by increasing the current density, the value of exergy destruction...
The changes of thermo-environmental function density ($F$) with the current density are shown in Figure 18. According to the figure at a constant current density $i = 15000$ (A/m$^2$), the maximum value thermo-environmental function density is $F = 0.035$ (W/m$^2$ m$^3$ kgs$^{-1}$). As can be seen, by increasing the current density, the thermo-environmental function density ($F$) first increases and then reaches its maximum value at a constant current density.

As shown in Figure 19, at a constant current density, by increasing the temperature, the mass flow rate ($m_f$) increases, because the mass flow rate is directly related to the value power output of the fuel cell and the value power output is related to the temperature; therefore, by increasing the temperature, the value power increases and ultimately the mass flow rate ($m_f$) also increases. Also at a value, at constant temperature by increasing the current density, the mass flow rate ($m_f$) decreases, because fuel cell power is inversely correlated with the current density and increases the current density and then causes reduction in the output power and ultimately reduces the mass flow rate ($m_f$). Therefore, it can be concluded that by increasing the temperature, the value mass flow rate also increases.
Figure 20. Optimal beam front for three objective functions (Power, ϕ, exd).

Table 1. Optimal results by decision-making algorithms.

| Decision variable | Power (kW) | ϕ  | exd (w/m²) | I (A/m²) | T (k)  | A (m²) |
|-------------------|------------|----|------------|----------|--------|--------|
| TOPSIS            | 939/393261 | 0/83819664 | 1139/85   | 7367/137 | 1073/183 | 0/290582 |
| LINMAP            | 939/293261 | 0/76982658 | 858/1006  | 7323/969 | 1073/167 | 0/234602 |
| FUZZY             | 939/293261 | 0/76982658 | 858/1006  | 7323/969 | 1073/167 | 0/234602 |

Table 2. Optimal results by decision-making algorithms.

| Decision variable | mf (kg/s) | ϕ  | exd (w/m²) | I (A/m²) | T (k)  | A (m²) |
|-------------------|-----------|----|------------|----------|--------|--------|
| TOPSIS            | 4/7728113 | 0/71428093 | 6664/39   | 13937/84 | 1141/644 | 0/34874 |
| LINMAP            | 3/10963692| 0/60181479 | 4080/088  | 11902/86 | 1208/147 | 0/255684 |
| FUZZY             | 5/69766052| 0/74212518 | 7561/192  | 14470/36 | 1222/069 | 0/375563 |

Figure 21. Optimal beam front for three objective functions (mf, ϕ, exd).

As shown in Figure 20, the Pareto optimal frontier for three objective functions (exd, ϕ, Power) has been shown based on various decision-making methods such as: TOPSIS, FUZZY and LINMAP, the optimal points on the selected graph.

Table 1 shows the optimal outputs achieved by executing LINMAP, TOPSIS and Fuzzy decision-making methods. Also, the object functions (power, ϕ, exd) for the decision variable are (I, T, A).

As shown in Figure 21, the Pareto optimal frontier for the three objective functions (exd, ϕ, mf) has been shown based on various decision-making methods such as TOPSIS, FUZZY and LINMAP, the optimal points on the selected graph.

Table 2 addresses the optimal outputs achieved by executing LINMAP, TOPSIS and Fuzzy decision-making methods. Also, the object functions (mf, ϕ, exd) for the decision variable are (I, T, A).

5. CONCLUSIONS

In this paper, thermodynamic analysis is performed on a hybrid system consisting of a Stirling engine and SOFC used for providing the required power of reverse osmosis desalination units with high-pressure pumps. In this regard, the characteristic curves of the designed cycle are studied and triple-objective optimization is carried out. Three decision-making approaches such as LINMAP, FUZZY and TOPSIS are used to determine the final solutions based on the Pareto fronts which are achieved by employing GA. In the first scenario which includes the power, exergy efficiency and exergy destruction density, the results have a better situation in the TOPSIS method. In the second scenario which includes the fresh water production rate, exergy efficiency and exergy destruction density, the results have a better situation in the FUZZY method.

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