The Leggett-Garg inequality and Page-Wootters mechanism

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Abstract – Violation of the Leggett-Garg inequality (LGI) implies quantum phenomena. In this light we establish that Moreva et al.’s (Phys. Rev. A, 89 (2014) 052122) experiment demonstrating Page-Wootters’ mechanism (Page D. N. and Wootters W. K., Phys. Rev. D, 27 (1983) 2885; Wootters W. K., Int. J. Theor. Phys., 23 (1984) 701). falls in the quantum domain. An observer outside a 2-photons world does not detect any change in the 2-photons state, i.e., there is no time parameter for the outside observer. But an observer attached to one of the photons sees the other photon evolving and this means that there is an “internal” time. The LGI is violated for the clock photon whose state evolves with the internal time as measured by the system photon.

Conditional probabilities in this 2-photons system are computed for both sharp and unsharp measurements. The conditional probability increases for entangled states as obtained by Page and Wootters for both ideal and also unsharp measurements.

Introduction. – An early attempt to quantise gravity was through the Wheeler-DeWitt (WD) equation [1]. This results from the canonical quantisation of the Einstein gravity using Dirac’s constrained formalism [2]. An embodiment of the quantum version of the Hamiltonian constraint using metric variables gives the WD equation which is time independent and thus all observables are constant and the resulting universe is unchanging and boring.

The WD equation is the analogue of the Schroedinger equation for the wave function of the universe. Therefore, solutions of this equation should give the possible quantum states of the universe. However, there is a marked difference from usual quantum-mechanical scenarios. The time parameter does not occur in the WD equation and so there is no possibility for the quantum state of the universe to evolve. Note that the wave function of the universe has to be constructed from the wave functions of all the constituents in the universe. Page and Wootters [3] proposed that such an unchanging universe (as observed by an external observer outside the universe) actually evolves with respect to time as seen by some internal observer within the universe. This is because of quantum correlations (entanglement) between different constituents within the universe.

According to Page and Wootters [3], suppose (for simplicity) that the universe is made up of two constituents only. Consider a direct product of the wave functions of these two entities where each of the wave functions depend on a monotonically increasing parameter as well as other variables. Then an average over this parameter will result in a state (wave function) that cannot be written as a direct product of states (wave functions). This is entanglement. Conversely, entanglement implies the existence of a monotonically increasing parameter present in the wave functions of the constituents making up the product and this parameter is identified with some “time”.

Note that the question of time averaging occurs only when systems evolve. Systems evolve only when there are interactions. Therefore, time averaging is meaningful only when there are interactions. If there are no interactions, there is no evolution and so the question of time averaging does not arise. Page and Wootters used a magnetic field to introduce interactions (their constituents were two electrons). Thus, the presence of entanglement is deeply connected with the presence of interactions. Probabilities can be determined using these entangled states. Alternatively, one may first calculate the probabilities from the direct product of time-dependent states giving time-dependent probabilities. Time-averaging these gives probabilities of lower values than those for the entangled states. So a measure of quantum correlations are the conditional probabilities described above.

Entanglement of states is a non-classical feature of the quantum world. Here Bell’s inequality (BI) [4] provides a
tool to study “quantumness” or specifically “non-locality”. BI sets a bound on a certain combination of correlation functions corresponding to results of measurements on two spatially separated systems. Suitable relative orientations of these measurements exist for which BI is violated by the relevant quantum-mechanical (QM) results for appropriate states of the entangled systems. Numerous experiments \[5\] have proved the empirical violation of BI, consistent with the QM predictions. Subsequently the Leggett-Garg inequality (LGI) \[6,7\] was discovered. This is a temporal analogue of BI in terms of time-separated correlation functions corresponding to successive measurement outcomes for a temporally evolving system. While furnishing a signature of distinctly quantum behaviour, LGI complements BI in providing insight into physical reality manifested by non-classicality of quantum systems. LGI has been used for probing possible limits of quantum mechanics in the macroscopic regime in various scenarios \[8–21\]. For particular mention, ref. \[10\] demonstrates LGI violations over large macroscopic distances.

Consider a two-states system which oscillates between the states 1 and 2 in time. Let $Q(t)$ be an observable taking values $\pm 1$ whenever measured, depending on whether the system is in state 1 or 2. Now consider a collection of runs starting from identical initial conditions such that on the first series of runs $Q$ is measured at times $t_1, t_2$, on the second at $t_2, t_3$, on the third at $t_3, t_4$, and on the fourth at $t_4, t_5$ with $t_1 < t_2 < t_3 < t_4$. The expression $[Q(t_1)Q(t_2) + Q(t_2)Q(t_3) + Q(t_3)Q(t_4) - Q(t_4)Q(t_5)]$ is always $+2$ or $-2$. The temporal correlations $C_{ij} \equiv (Q(t_i)Q(t_j))$ are determined. Replacing individual product terms by their averages over the entire ensemble of such sets of runs, the LGI is \[6,7\]

$$C \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2.$$  

(1)

This is a a Bell-type inequality. The $t_i$’s play the role of apparatus settings. This inequality imposes realistic constraints on time-separated joint probabilities pertaining to oscillations in any two-states system. Violation of this inequality signifies quantum phenomena.

Sometime ago Moreva et al. \[22\] in a remarkable experiment illustrated Page and Wooters' mechanism of “static” time by using an entangled state of the polarisation of two photons. One of the photons, the “clock” photon, measures the evolution of the other photon, the “system” photon as follows: the clock photon is like an internal observer who sees the other photon evolve, while an “external” observer (outside the two-photons world) recording only global properties of the two photons sees no change. The motivation of our present work is to consider this experiment in the light of the LGI (second section). We also seek further evidence for the Page-Wooters mechanism by computing relevant conditional probabilities for both ideal (third section) and unsharp (fourth section) measurements. Concluding remarks are given in the last section.

The Page-Wooters mechanism. — Page and Wooters used a static entangled state $|\psi\rangle$ whose subsystems evolve according to quantum mechanics for an observer who uses one of the subsystems as a clock system $c$ to measure the time evolution of the rest $r$. Subsystems are assumed to be non-interacting. The Hamiltonian of the global system is written as $\mathcal{H} = \mathcal{H}_c \otimes 1_r + 1_c \otimes \mathcal{H}_r$. Here $\mathcal{H}_c, \mathcal{H}_r$ are local Hamiltonians for $c$ and $r$, respectively \[3\]. The state of the “universe” $|\psi\rangle$ is then determined by enforcing the WD equation $\mathcal{H}|\psi\rangle = 0$. So $\psi$ is an eigenstate of $\mathcal{H}$ with eigenvalue zero. The reason for this choice is that by projecting $\psi$ on the states $|\phi(t)\rangle_c = e^{-i\mathcal{H}_c t/\hbar}|\phi(0)\rangle_c$ of the clock, one gets $|\phi(t)\rangle_r$ which is defined to be $|\psi(t)\rangle_r := \langle \phi(t)|\psi\rangle = e^{-i\mathcal{H}_r t/\hbar}|\phi(0)\rangle_r$. This describes an evolution of the subsystem $r$ under its local Hamiltonian $\mathcal{H}_r$, the initial state being $|\psi(0)\rangle_r := |\phi(0)\rangle$. Hence, although globally the system appears static, its components (i.e., locally) exhibit correlations that signify dynamical evolution \[3\].

In \[22\] an entangled quantum state $|\psi\rangle$ of two photons, clock photon $c$ and system photon $r$, is represented as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H_c\rangle V_r - |V_c\rangle |H_r\rangle),$$  

(2)

where $H$ and $V$ denote horizontal and vertical polarisations, respectively.

The Hamiltonian of the global system is $\mathcal{H} = \mathcal{H}_c \otimes 1_r + 1_c \otimes \mathcal{H}_r$ with $\mathcal{H}_c = \mathcal{H}_r = i\hbar \omega (|V\rangle \langle V| - |H\rangle \langle H|)$. Here $\mathcal{H}|\psi\rangle = 0$, i.e., the 2-photons state satisfies the WD equation \[1\]. So an observer outside the 2-photons world does not detect any change in the 2-photons state and so there is no time parameter for this outside observer. But an observer attached to one of the photons (i.e., the observer is inside the 2-photons world) sees the other photon evolving and this signifies the existence of a time parameter. Here we study the quantum-mechanical violation of LGI for the clock photon whose state evolves with the time measured by the system photon.

Let the clock photon be in state $|H\rangle$ at $t = 0$, i.e., $|\phi(0)\rangle_c = |H\rangle$. Denote this initial condition by 1). If the clock photon is in state $|V\rangle$ at $t = 0$, i.e., $|\phi(0)\rangle_c = |V\rangle$ then this will be denoted as initial condition 2). The time-evolved state of the clock photon with initial condition 1) is

$$|\phi(t)\rangle_c = e^{-i\mathcal{H}_c t/\hbar}|\phi(0)\rangle_c = e^{-i\mathcal{H}_c t/\hbar}|H\rangle = (1 - i\frac{\mathcal{H}_c t}{\hbar} + \ldots)|H\rangle = \cos(\omega t)|H\rangle - \sin(\omega t)|V\rangle.$$  

(3)

So after time $t$ the probability of getting horizontal polarization $H$ is $P_{1H}(t) = \cos^2(\omega t)$ and vertical polarization $V$ is $P_{1V}(t) = \sin^2(\omega t)$. The time-evolved state of the clock photon with initial condition 2) is

$$\exp(-i\mathcal{H}_c t/\hbar)|V\rangle = \cos(\omega t)|V\rangle + \sin(\omega t)|H\rangle$$  

(4)

so that now $P_{2V}(t) = \cos^2(\omega t)$; $P_{2H}(t) = \sin^2(\omega t)$, where the suffixes 1, 2 denote the initial conditions.
Now start an experiment with the clock photon in state \(|H\rangle\) at \(t = 0\). Then the joint probability \(P_{1H,1H}(t_1,t_2)\) of finding the clock photon in state \(|H\rangle\) at \(t_1\) and again \(|H\rangle\) at \(t_2\) (where \(0 < t_1 < t_2\)) is

\[
P_{1H,1H}(t_1,t_2) = \cos^2(\omega t_1) \cos^2(\omega(t_2 - t_1)).
\]

Similarly one can calculate \(P_{1H,V}(t_1,t_2)\), \(P_{2V,1H}(t_1,t_2)\) and \(P_{2V,V}(t_1,t_2)\). Now consider an observable quantity \(Q(t)\) such that, whenever measured, it takes values +1 or -1 depending on whether the system is in the \(|H\rangle\) or \(|V\rangle\) state. Then the time correlation function \(C_{12} = \langle Q(t_1)Q(t_2) \rangle\) can be evaluated by using the above-mentioned four joint probabilities to obtain

\[
C_{12} = P_{1H,1H}(t_1,t_2) - P_{1H,V}(t_1,t_2)
\]

\[- P_{2V,1H}(t_1,t_2) + P_{2V,V}(t_1,t_2)
\]

\[= \cos^2(\omega(t_2 - t_1)) - \sin^2(\omega(t_2 - t_1)).
\]

\(C_{12}\) depends on the temporal separation \((t_2 - t_1)\) and the parameter \(\omega\) (which defines the time scale of the system). In general \(C_{ij} = \cos^2[\omega(t_j - t_i)] - \sin^2[\omega(t_j - t_i)], j > i\). Using these equations

\[
C = \cos^2[\omega(t_2 - t_1)] + \cos^2[\omega(t_4 - t_3)] + \cos^2[\omega(t_4 - t_1)]
\]

\[+ \sin^2[\omega(t_2 - t_1)] + \sin^2[\omega(t_4 - t_3)]
\]

\[+ \sin^2[\omega(t_4 - t_1)] - \sin^2[\omega(t_2 - t_1)]
\]

\[= \cos^2(x) + \sin^2(3x) - 3 \sin^2(x) - \cos^2(3x),
\]

where we have chosen \((t_4 - t_3) = (t_3 - t_2) = (t_2 - t_1) = \Delta t\), and defined \(x = \omega \Delta t\). The behaviour of the quantity \(C\) with \(x\) is shown in fig. 1. \(C\) oscillates with time. The maximum upper bound for \(C\) is \(C_{\text{max}} = 2\sqrt{2} = 2.82843\).

Therefore, the results of this section establish the fact that the same photon at different times is autocorrelated and the Moreva et al. [22] experiment falls in the quantum domain with maximum violation of the LGI for the clock photon whose state evolves with the internal time as measured by the system photon.

Conditional probabilities for sharp measurements. We now compute the conditional probabilities following Page and Wootters [3]. We first discuss sharp (i.e., projective or ideal) measurements. At time \(t = 0\), let the clock photon be in the state \(|H\rangle\) and the system photon be in the state \(|V\rangle\). After time \(t\) the clock photon state is \(\cos \omega t |H\rangle_c - \sin \omega t |V\rangle_c\) and the system photon state becomes \(\cos \omega t |V\rangle_r + \sin \omega t |H\rangle_r\). The time-averaged (stationary) state \(|\bar{\psi}\rangle\) of the two-photon system is then

\[
|\bar{\psi}\rangle \propto \int_0^{2\pi/\omega} \left(\cos \omega t |H\rangle_c - \sin \omega t |V\rangle_c\right) \times \left(\cos \omega t |V\rangle_r + \sin \omega t |H\rangle_r\right) dt
\]

\[\propto |H\rangle_c |V\rangle_r - |V\rangle_c |H\rangle_r.
\]

Hence the normalized state of the two-particle system is

\[
|\bar{\psi}\rangle = \frac{1}{\sqrt{2}} (|H\rangle_c |V\rangle_r - |V\rangle_c |H\rangle_r).
\]

If the clock photon is found in the horizontal polarization, then the probability of finding a system photon with vertical polarization will be

\[
P\left(|V\rangle_r \mid |H\rangle_c\right) = \frac{|\langle c(H_r,V)|\bar{\psi}\rangle|^2}{|\langle c(H_c,V)\bar{\psi}\rangle|^2} = 1.
\]

Now consider the time-dependent 2-photons state

\[
|\psi(t)\rangle = (\cos \omega t |H\rangle_c - \sin \omega t |V\rangle_c)
\]

\[\times (\cos \omega t |V\rangle_r + \sin \omega t |H\rangle_r).
\]

With this state the time-averaged probability for a system photon to be vertically polarized is

\[
P\left(|V\rangle_r \mid |H\rangle_c\right) = \frac{\int_0^{2\pi/\omega} \left| \langle c(H_r,V)|\psi(t)\rangle \right|^2 dt}{\int_0^{2\pi/\omega} \left| \langle c(H_c,V)|\psi(t)\rangle \right|^2 dt} = \frac{3}{4}.
\]

The above two conditional probabilities are next determined using density matrix formalism. Consider the stationary state \(|\bar{\psi}\rangle\) (9). The density matrix corresponding to this state is

\[
\rho_{\bar{\psi}} = \frac{1}{2} (|H\rangle_c |V\rangle_r - |V\rangle_c |H\rangle_r) \left(\langle c(H_r,V)|\bar{\psi}\rangle \langle c(H_r,V)\bar{\psi}\rangle^\dagger\right) = \frac{3}{4}.
\]

With this \(\rho_{\bar{\psi}}\) (clock photon horizontally polarized), probability of finding a vertically polarized system photon is

\[
P_{\rho_{\bar{\psi}}} \left(|V\rangle_r \mid |H\rangle_c\right) = \frac{\text{Tr}[\mathcal{P}_{HN} \rho_{\bar{\psi}}]}{\text{Tr}[\mathcal{P}_{HN}]} = 1.
\]

\[
\mathcal{P}_{HN, V_r} = |H\rangle_c |V\rangle_r \langle c(H_r,V)\rangle \langle c(H_r,V)|\text{ is the projection operator where the clock photon is in the state } |H\rangle_c\text{, while the system photon is in the state } |V\rangle_r. \mathcal{P}_{HN} = |H\rangle_c c(H)\text{ is the projection operator for the clock photon in the state } |H\rangle_c.
\]

Next consider the time-dependent state (11). Here the density operator \(\rho_{\psi} = |\psi(t)\rangle \langle \psi(t)|\) contains 16 terms, and \(\text{Tr}[\mathcal{P}_{HN, V_r} \rho_{\psi}] = \cos^2 \omega t\) and \(\text{Tr}[\mathcal{P}_{HN} \rho_{\psi}] = \cos^2 \omega t\). So when the clock photon is horizontally polarized, the time-averaged conditional probability of finding a vertically polarized system photon is

\[
P_{\rho_{\psi}} \left(|V\rangle_r \mid |H\rangle_c\right) = \frac{\int_0^{2\pi/\omega} \text{Tr}[\mathcal{P}_{HN, V_r} \rho_{\psi}] dt}{\int_0^{2\pi/\omega} \text{Tr}[\mathcal{P}_{HN} \rho_{\psi}] dt} = \frac{3}{4}.
\]
So (10), (12), (14), (15) show that for pure states there are no differences between the conditional probabilities calculated using the density matrices or otherwise. This is as it should be.

**Conditional probabilities for unsharp measurements.** – Now consider unsharp (or non-projective, non-ideal) measurements [23]. For a sharp measurement of the polarisation of a single (say i-th) photon the dichotomic observable is (no sum over i) $Q_{i} = |H\rangle_{i}\langle H| - |V\rangle_{i}\langle V| = P_{i+} - P_{i-}$, with outcomes $\pm 1$ for the photon in the state $|H\rangle$ or $|V\rangle$, respectively. Corresponding projection operators are $\mathbf{P}_{i\pm} = \frac{1}{2}(I_{i} \pm Q_{i})$ where $I_{i} = |H\rangle_{i}\langle H| + |V\rangle_{i}\langle V|$. For unsharp measurements a sharpness parameter $\lambda$ is introduced to characterize the precision of a measurement and $\lambda = 1$ means projective (“sharp”) measurement. The unsharp projection operators are now

$$\mathbf{F}_{i\pm} = \frac{1}{2}(I_{i} \pm \lambda Q_{i}) = \lambda_{i} \mathbf{P}_{i\pm} + (1 - \lambda_{i}) \frac{I_{i}}{2}$$

(0 < $\lambda_{i}$ < 1). $\mathbf{F}_{i\pm}$ are mutually commuting operators with non-negative eigenvalues and $\mathbf{F}_{i+} + \mathbf{F}_{i-} = I$. $\lambda_{i} = 1$ corresponds to sharp measurements and $\mathbf{F}_{i\pm}$ reduce to $\mathbf{P}_{i\pm}$. For a sharp measurement of the polarisation of two photons, the projection operator for the clock photon in the $|H\rangle$ state and the system photon in the $|V\rangle$ state is

$$\mathbf{P}_{H,V} = \frac{1}{4}((c_{+} + Q_{c})(I_{c} - Q_{c}) = |H\rangle_{c}|V\rangle_{r,c}\langle H|_{r}\langle V|_{r}.$$  

(17)

For the unsharp measurement this operator becomes

$$\mathbf{F}_{H,V} = \frac{1}{4}(c_{+} + \lambda Q_{c})(I_{c} - \lambda Q_{c}) =$$

$$\frac{1}{4}((1 + \lambda)(1 - \lambda)|H\rangle_{c}|H\rangle_{r,c}\langle H|_{r}\langle H| + (1 + \lambda)(1 - \lambda)|V\rangle_{c}\langle V|_{r}\langle V|$ $\times (1 + \lambda)(1 + \lambda)|H\rangle_{c}|V\rangle_{r,c}\langle H|_{r}\langle V|_{r} = (1 + \lambda)\langle V|_{r}(H| + (1 - \lambda)|H\rangle_{c}|V\rangle_{r,c}\langle V|_{r}|V\rangle.$$

(18)

So for the unsharp measurement on the state (13) (clock photon with horizontal polarization), the probability of finding the system photon with vertical polarization is

$$P_{\rho_{\psi}}(\langle V|_{r} |H\rangle_{c}) = \frac{\text{Tr}[\mathbf{F}_{H,V,\rho_{\psi}}]}{\text{Tr}[\mathbf{F}_{H,\rho_{\psi}}]} = \frac{1}{2}(1 + \lambda_{c}\lambda_{r}).$$

(19)

Now consider unsharp measurements for a *time-dependent* 2-photon state with density matrix $\rho_{\psi} = |\psi(t)\rangle\langle\psi(t)|$, where $\psi(t)$ is given by eq. (11). Here

$$\text{Tr}[\mathbf{F}_{H,V,\rho_{\psi}} = \frac{1}{4}\left[(1 + \lambda)(1 - \lambda) \sin^{2}\omega t \cos^{2}\omega t\right.\times (1 + \lambda)(1 + \lambda) \cos^{2}\omega t + (1 - \lambda)(1 - \lambda)$$

$$\times \sin^{2}\omega t + (1 - \lambda)(1 + \lambda) \sin^{2}\omega t \cos^{2}\omega t\right]$$

(20)

and

$$\text{Tr}[\mathbf{F}_{H,\rho_{\psi}} = \frac{1}{2}(1 + \lambda_{c}\cos 2\omega t),$$

(21)

So now if the clock photon is horizontally polarized, the probability of the vertically polarized system photon is

$$P_{\rho_{\psi}}(\langle V|_{r} |H\rangle_{c}) = \frac{\int_{0}^{2\pi/\omega} \text{Tr}[\mathbf{F}_{H,V,\rho_{\psi}}] dt}{\int_{0}^{2\pi/\omega} \text{Tr}[\mathbf{F}_{H,\rho_{\psi}}] dt} = \frac{1}{4}\left[2 + \lambda_{c}\lambda_{r}\right].$$

(22)

For $\lambda_{c} = \lambda_{r} = 1$, (19) and (22) for unsharp measurements match exactly with the values of ideal measurements, i.e., 1 and $\frac{3}{4}$, respectively. Figure 2 gives the variation of the two conditional probabilities for unsharp measurements (19) and (22) where $0 \leq (\lambda_{c},\lambda_{r}) \leq 1$. The upper graph is for two entangled photons while the lower graph is for two un-entangled (i.e., time-dependent states) photons. The probability for entangled (i.e., time-independent state) photons is always greater than that of the un-entangled photons for all non-zero values of $\lambda_{c},\lambda_{r}$. Figure 3 is for $\lambda_{c} = \lambda_{r} = \lambda$. However, an important point must be remembered. Page and Wootters considered massive spin particles. A massive spin-j particle will have 2j + 1 spin angular momentum eigenstates. We are considering photons which are massless gauge particles. The gauge constraint allows only two independent degrees of freedom, i.e., two polarisations. So our calculated probabilities correspond to two-state systems. In fact, our calculated probabilities are identical to that for electrons which are also two-state
systems \((j = 1/2\) and \(2j + 1 = 2\)). Note that our result is different from their values for massive spin-one particles because of the above reasons.

**Concluding remarks.** – We first consider the Moreva et al. [22] experiment in the light of the LGI (second section). Recall that an “external” observer will see no dynamics of the two photons, i.e., “static” universe. However, an observer attached to one of the photons will see the other evolve if the photons are entangled. So temporal evolution \(\iff\) entanglement/correlations. Each of the two photons (i.e., “clock photon” and “system photon”) individually violates the maximum bound of LGI and this implies that this is a quantum phenomenon. We show that autocorrelations between the same photon at different times are present as revealed by the maximal violation of the LGI. The time variable of the LGI is the internal time as seen by the other photon. So the applicability of the LGI means the existence of an internal time as seen by the other photon. It should be mentioned that repeating the exercise of the second section for unsharp measurements will still give the existence of internal time and the phenomenon will still be in the quantum domain. Only the maximum bound of the LGI will be lower owing to the fact that the noise in the measurement stage reduces the correlations. We next compute the relevant conditional probabilities for ideal and non-ideal measurements (third and fourth sections) in the 2-photon system (photons are massless spin-one particles) and show that the conditional probability increases for entangled states as obtained by Page and Wootters for both ideal and also unsharp measurements.

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