Effect of interactions on the noise of chiral Luttinger liquid systems

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We analyze the current noise, generated at a quantum point contact in fractional quantum Hall edge state devices, using the chiral Luttinger liquid model with an impurity and the associated exact field theoretic solution. We demonstrate that an experimentally relevant regime of parameters exists where the noise coincides with the partition noise of independent Laughlin quasiparticles. However, outside of this regime, this independent particle picture breaks down and the inclusion of interaction effects is essential to understand the shot noise.

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In fractional quantum Hall (FQH) devices at filling factor $\nu = 1/3$ the effect of electron interactions turns the bulk of the system into an incompressible quantum fluid. The edge excitations are gapless, and can be described by the chiral Luttinger liquid (CLL) model $^2$. This model exhibits quasiparticles (QPS) which can be thought of as the chiral Luttinger liquid (CLL) model $^2$. This model exhibits quasiparticles (QPS) which can be thought of as collective excitations of the electronic system. One of the consequences of strong electron interactions is that these QPS have fractional charge $e^* = e \nu$ $^1$. In many other respects however, these QPS do behave like free electrons; for instance, in the absence of a dominant impurity, i.e. a quantum point contact, the right and left moving Laughlin QPS move without scattering along the edges of the sample. However, once a quantum point contact brings the edges close together, QPS can tunnel from one edge to the other near the constriction. In this tunnelling process, they do not behave like independent particles any longer: the tunnelling term has a very confining action, and can, in the strong backscattering limit (SBL), bind them into triplets with the charge of an electron. This difference to the naive picture of non–interacting Laughlin QPS in the bulk of a FQH device has led in our opinion to confusing interpretations of transport experiments for FQH edge states. A particularly elaborated example is the measurement of fractional charge through shot noise $^3$, $^4$, $^5$, $^6$. Most of these measurements have been successfully described by essentially non–interacting formulae, where the effect of electron interactions is usually represented by the fractional charge, and sometimes by an energy–dependent transmission coefficient. From the CLL point of view, however, it seems to be clear that the tunnelling process involves interaction effects which should, in general, not be encompassed by independent particle formulae.

In view of these problems, it has even been suggested that the CLL model is not sufficient to explain the measurements of $e^* = e/3$ and more complex mechanisms need to be taken into account. $^5$ In order to assess the relevance of such mechanisms it is necessary to study further the physical predictions for the noise in tunnelling experiments coming from the model of a CLL with an impurity. This is a nontrivial exercise, as the problem is highly non–perturbative. Although an exact solution exists, this solution is complex, and its physical implications had not been much explored. We revisit this solution in the present paper, and find, surprisingly maybe, that a rather wide regime of parameters does exist, in which Laughlin QPS behave as if they were indeed tunnelling independently. There is, however, a complementary regime of parameters, in particular, at low temperatures, where the effect of interactions is very pronounced and cannot be incorporated in an independent particle model over an extended range of the applied voltage anymore. The latter regime is probably as relevant as the former for experiments, and we expect our work to spur further analysis of data deviating from non–interacting approximations.

The CLL model is described by the Hamiltonian

$$H_0 = \frac{\hbar \nu}{g} \int dx \left( n_R^2 + n_L^2 \right),$$

where the right and left moving densities are related to Bosonic phase fields $\phi_{R/L}$ by $n_{R/L} = \pm \partial_x \phi_{R/L}/2\pi$. $\nu$ is the plasmon velocity, and $g$ the standard Luttinger interaction parameter (see e.g. $^4$). It has been shown that the right and left modes $n_{R/L}$ are mathematically isomorphic to the right and left moving edge states in a FQH bar with filling fraction $\nu = 1/m$ (m odd integer) $^2$ provided one takes $g = \nu$. An impurity at site $x = 0$ in such a system corresponds to adding the dominant $2k_F$ backscattering contribution

$$H_B = \lambda \cos [\phi_L(0) - \phi_R(0)]$$

to the Hamiltonian. This term hops a quasiparticle $e^{i\phi_{R/L}}$, i.e. a right/left mover, between the two modes. The charge transferred in such a hopping process is fractional and given by $e^* = eg$ $^3$. From the latter two Hamiltonians it becomes clear that the right and left
moving edge states in FQH bars are non–interacting in the bulk, but do interact with each other at the quantum point contact. A symmetrically applied external bias \( V \) can be modelled by a third term of the Hamiltonian

\[
H_V = (eV/2) \int dx (n_L - n_R).
\]

We are particularly interested in the effect of interactions on the zero frequency current noise

\[
S = \lim_{\omega \to 0} \int dt e^{i\omega t} \langle \{ \Delta I(t), \Delta I(0) \} \rangle \tag{3}
\]

with the current fluctuation operator \( \Delta I(t) = I(t) - \langle I \rangle \). If uncorrelated electrons in a single channel quantum wire, connected to reservoirs with a voltage bias \( V \), are partly backscattered off an impurity in the wire, the voltage and temperature dependence of the current noise in a two–terminal setup can be written as

\[
S = 2G_0 T (1 - T) eV \coth \left( \frac{eV}{2k_B T} \right) + 4k_B T G_0 T^2, \tag{4}
\]

where \( T \) is the energy–independent transmission coefficient of the impurity,

\[
G_0 = e^2 / h,
\]

and \( T \) the temperature. The simplest question one can ask is whether a formula similar to Eq. 4 could describe (maybe with an appropriate carrier charge) the noise due to tunnelling through a constriction in a FQH bar. This has been assumed in the interpretation of some shot of Laughlin QPS through a constriction in Ref. 12. The exact field theoretic solution of the CLL model with an impurity, carried out in Ref. 12, allows for a non–perturbative calculation of the current noise, even at finite temperature 13. The main steps of this calculation are as follows. A CLL with an impurity in the presence of a DC bias, defined by the Hamiltonian

\[
H = H_0 + H_B + H_V,
\]

can be mapped onto the boundary sine–Gordon model 12. The appropriate excitations of that model are kinks, antikinks, and breathers. In the presence of an external voltage, the pseudoeenergies of the kinks, antikinks (both with charge \( e \)), and breathers (with no charge) are affected by the bias \( V \). At \( \nu = 1/3 \), there is just one kink, one antikink, and one breather. The effect of the impurity is the scattering of these quasiparticles 12. Importantly, the quasiparticle basis is chosen in such a way that they scatter one by one (without particle production) on the boundary. This allows us to do a Landauer–Blüttiker–like transport theory for these quasiparticles. Although, there are just three types of quasiparticles at \( \nu = 1/3 \), the calculation of the current noise is tedious and we briefly sketch it now. For a more detailed description we refer to the original literature 12, 13. In the following, we set \( \hbar = 1 \) and only restore it in the final results. In the field theoretic solution of the CLL model, a system on a circle of large length \( L \) is studied at temperature \( T \). Each quasiparticle has the energy \( \epsilon_i \) (\( i = + \) for the kink, \( i = - \) for the antikink, and \( i = b \) for the breather); in a gapless theory \( \epsilon_i = \pm \hbar \nu \propto e^{\theta} \). The level density \( n_i(\theta) \) \((\theta \text{ is the rapidity and the filling fraction } f_i(\theta) \) are introduced, so that \( L n_i(\theta) f_i(\theta) \) is the number of allowed states for a quasiparticle of type \( i \) in the rapidity range \( (\theta, \theta + d\theta) \). Therefore, the density of occupied states per unit length is given by \( P_i(\theta) = n_i(\theta) f_i(\theta) \). Requiring that the wave functions be periodic in space, this yields the Bethe equations

\[
n_i(\theta) = \frac{dp_i}{d\theta} + \sum_j \int d\theta' \Phi_{ij}(\theta - \theta') P_j(\theta), \tag{7}
\]

where the kernel \( \Phi_{ij}(\theta - \theta') \) is related to the bulk scattering matrix of the quasiparticles 12. Furthermore, pseudoeenergies \( \epsilon_i \) are defined by

\[
f_i = \frac{P_i}{n_i} \equiv \frac{1}{1 + e^{\epsilon_i - \mu_i}}, \tag{8}
\]

where \( \epsilon_i \) and the chemical potentials \( \mu_i \) are scaled by the temperature to make them dimensionless. In our case
\( \mu_+ = eV/2k_BT, \mu_- = -eV/2k_BT, \) and \( \mu_0 = 0 \). In order to determine the equilibrium values of the pseudoenergies, one has to find the configuration which minimizes the free energy. This procedure is known as the thermodynamic Bethe ansatz. After all we are able to calculate the current noise defined by Eq. (4) in this formalism exactly. The final result is given by

\[
S = L \int \int d\theta d\theta' D(\theta, \theta') T(\theta) T(\theta')
+ \int d\theta \left\{ \left[ \bar{T}_+ (\theta) (1 - \bar{T}_- (\theta)) + \bar{T}_- (\theta) (1 - \bar{T}_+ (\theta)) \right] \times T(\theta) (1 - T(\theta)) \right\},
\]

where \( \bar{T} \) indicates thermal equilibrium of \( A \) and the density–density correlator \( D(\theta, \theta') \) reads \( D(\theta, \theta') = \Delta(P_+ - P_-)(\theta) \Delta(P_+ - P_-)(\theta') \). In Eq. (9), we have introduced the energy–dependent transmission coefficient \( T(\theta) \equiv 1/(1 + e^{2(\nu - 1)(\theta - \theta_0)/\nu}) \). \( \theta_B \) parametrizes the impurity strength and is related to \( \lambda \) in Eq. (2) through \( k_B T_B \equiv M \lambda^g \propto \lambda^{1/(1-g)} \) (\( M \) is an arbitrary energy scale that cancels out of all physical results) [12].

Let us now turn back to the relevance of interactions in tunnel experiments with Laughlin QPS; more specifically, we discuss whether Eq. (9) can be approximated by a formula like Eq. (4) with an appropriate adjustment of parameters. In order to do so, we first find the best candidates to approximate Eq. (9). Eqs. (10) and (11) suggest that the WBL and the SBL should be treated by separate approximations. Moreover, the slope of the finite frequency (non–equilibrium) noise around \( \omega = 0 \) [14] indicates that an identification of the dimensionless conductance \( 1/G_\nu dI/dV \) (with \( G_\nu = e^2/h \)) as an energy–dependent transmission coefficient \( T(E) \equiv (1/G_\nu)dI/dV \) is plausible. An appropriate combination of Eqs. (10) and (11) yields the tentative expression

\[
S_B = 2e^* I_{BS} T(E) \coth \left( \frac{e^*V}{2k_BT} \right) + 4k_B T G_\nu T^2(E),
\]

which is expected to satisfyingly approximate \( S \) in the WBL. We checked numerically for \( eV/k_BT \in [0, 12] \) that this is, indeed, the case. A comparison between Eqs. (10) and (11) (under the choice \( e^* = e\nu \) for \( \nu = 1/3 \)) at different values of the zero bias conductance, \( G_\nu = e^2/h \) for different values of \( \theta_B \), is illustrated in FIG. 1. It is clearly visible that \( S_B \) agrees very well with \( S \) over the whole range of \( eV/k_BT \) in the WBL, see e. g. the curve for \( G = 0.97 e^2/3h \). Thus, there does exist a meaningful window of parameters where the effect of interactions can be accounted for by adjusting parameters in the non–interacting expression (10).

In the intermediate scattering regime, e. g. for \( G = 0.44 e^2/3h \) in FIG. 1 the agreement between \( S_B \) with \( e^* = e/3 \) and \( S \) is still quite good (within a few percent). However, in the SBL, the case \( G = 0.015 e^2/3h \) in FIG. 1 Eq. (11) with \( e^* = e\nu \) diverges from the exact solution as \( eV/k_BT \) grows. For that value of \( G \), we have verified that the alternative expression

\[
S_T = 2eI_T(1 - T(E)) \coth \left( \frac{eV}{2k_BT} \right) + 4k_B T G_\nu T^2(E),
\]

almost coincides with the exact solution \( S \) in the full range \( eV/k_BT \in [0, 12] \). This is not very surprising since Eq. (11) could have been guessed as the best approximation to the exact solution in the SBL by combining Eqs. (10) and (11), i. e., roughly, by exchanging Laughlin QPS with electrons.

This is of course not to say that the more sophisticated theory is useless. There does exist as well a regime of parameters, where interaction effects are so pronounced.
Eq. (11) (white points) for solution of the noise Eq. (9) with Eq. (10) (black points) and

**FIG. 3:** Apparent evolution of fractional charge from the SBL to the WBL. The points are obtained by fitting the exact solution of the noise Eq. (8) with Eq. (10) (black points) and Eq. (11) (white points) for $eV/k_BT \in [0, 12]$. The inset shows the $\chi^2$ values of the two fits and $G$ is the zero bias conductance ($G_{1/3} = e^2/3h$). The dashed line corresponds to $e^* = e/3$.

that an analysis of the noise with interacting theories is unavoidable. As realized by the authors of Ref. [8] this is the regime of low temperatures. In that regime, the $dI/dV$ curves show strong nonlinearity and the current noise cannot be fitted by non–interacting formulae anymore (see Figs. 1 and 2 in Ref. [7]). The reason for this is apparent: For $V = 0$ and $T = 0$, the system is in the SBL fixed point. If we now increase $V$ we leave the SBL and drive it into the WBL. Thus, neither of the two approximations [4] and [5] can be the good one in the whole range of the applied voltage. However, as shown in our FIG. 2, the zero temperature shot noise for $g = 1/3$ describes well the behavior of the shot noise as a function of voltage bias reported in FIG. 2(b) of Ref. [7].

Finally, the validity of non–interacting formulae in some of the regimes of parameters, provided the effective charge is adjusted to $e^*$ or $e$, suggests considering the “evolution of the charge of tunnelling objects” with the scattering strength as measured in Ref. [8]. Let us stress that we do not believe that a charge other than $e^* = e\nu$ and $e$ has much significance: While, in the WBL, Laughlin QPS indeed tunnel independently, and, in the SBL, electrons do so, intermediate scattering regions can only be matched by a more complex description than independent particle tunnelling. However, one can always take the field theoretic calculation of the current noise, and extract a value of $e^*$ by fitting the exact solution to Eqs. (10) and (11) for filling fraction $\nu = 1/3$ with $e^*$ being a $\nu$–independent fitting parameter [8]. In FIG. 3, the extracted values of $e^*$ are shown for comparison. Evidently, the fractional charge depends significantly on the choice of the heuristic formula used to extract it, especially in the intermediate scattering regime. We see that the values of $e^*$, which have been obtained through Eq. (11), bear a lot of similarity to the measured values in FIG. 4 of Ref. [6]. This should be due to the fact that Eq. (11) is, although less complex, very similar to Eq. (4) of Ref. [6]. From this analysis we conjecture that the measured noise in Ref. [6] could have been fitted quite well by the exact field theoretic solution for all values of zero bias transmission without any adjustment of $e^*$.

In conclusion, we have clarified the subtle issue of the relevance of interactions of Laughlin QPS at a constriction in a FQH bar by analyzing the noise properties of such a system. For $\nu = 1/3$, we have quantitatively identified the regime of parameters, in which the effect of interactions can be incorporated into non–interacting expressions by an “adjustment of parameters”, and, on the other hand, the regime, in which an appropriate treatment of interactions is crucial. Our findings are of high relevance for the interpretation of measurements of shot noise in FQH systems.

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[1] R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[2] X. G. Wen, Phys. Rev. Lett. 64, 2206 (1990); Phys. Rev. B 41, 12838 (1990).
[3] R. de-Picciotto et al., Nature (London) 389, 162 (1997).
[4] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).
[5] D. C. Glattli et al., Physica E 6, 22 (2000).
[6] T. G. Griffiths et al., Phys. Rev. Lett. 85, 3918 (2000).
[7] Y. C. Chung et al., Phys. Rev. B 67, 201104(R) (2003).
[8] B. Rosenow and B. I. Halperin, Phys. Rev. Lett. 88, 096404 (2002).
[9] M. P. A. Fisher and L. I. Glazman in *Mesoscopic Electron Transport*, Vol. 345 of NATO ASI, edited by L. Kouwenhoven, G. Schön, and L. Sohn (Kluwer, Dordrecht, 1997).
[10] G. B. Lesovik, JETP Lett. 49, 592 (1989); M. Büttiker, Phys. Rev. Lett. 65, 2901 (1990); Th. Martin and R. Landauer, Phys. Rev. B 45, 1742 (1992).
[11] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 72, 724 (1994).
[12] P. Fendley, A. W. W. Ludwig, and H. Saleur, Phys. Rev. Lett. 74, 3005 (1995); 75, 2196 (1995); Phys. Rev. B 52, 8934 (1995).
[13] P. Fendley and H. Saleur, Phys. Rev. B 54, 10845 (1996).
[14] C. Chamon and D. E. Freed, Phys. Rev. B 60, 1382 (1999).
[15] The quasiparticles of the appropriate basis of the boundary sine–Gordon model should not be confused with the Laughlin QPS of the original problem. The relation between these two kinds of quasiparticles is nontrivial.
[16] In Eq. (11), $e$ has to be replaced by $e^*$ and $G_0$ by $G_\nu$ to do a self–consistent fit. Furthermore, $G_\nu$ has to be defined as $G_\nu \equiv (e^*/e)c^2/\hbar$ during the fitting procedure.