MarS-FL: A Market Share-based Decision Support Framework for Participation in Federated Learning

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Federated learning (FL) enables multiple data owners (a.k.a. FL participants (FL-PTs)) to collaboratively train machine learning models without sharing data, thus preserving data privacy and user confidentiality. In horizontal FL scenarios, FL-PTs can be in a competitive market where market shares represent their competitiveness (e.g., multiple banks in a same country). An understanding of the impact of FL on FL-PTs’ market shares plays a key role in their decisions for joining FL. However, there is a lack of such modeling tools to support informed decision-making. In this paper, we bridge this gap in existing FL literature by proposing a market share-based decision support framework for participation in FL (MarS-FL). It adapts a general economic model to the FL context and introduces two notions of $\delta$-stable market and friendliness to measure the viability of FL and the market acceptability of FL.

Further, it addresses related decision-making issues with FL designer and FL-PTs. Firstly, we characterize the process by which each FL-PT joins FL as a non-cooperative game and analyze its dominant strategy. Secondly, as an FL designer, the final model performance improvement of each FL-PT should be bounded, which relates to the market conditions of FL applications. We provide a sufficient and necessary condition $Q$ for maintaining the market $\delta$-stability and quantify the friendliness $\kappa$. The condition $Q$ can be used to derive specific requirements while the performance improvement as a result of FL is allocated among FL-PTs as a form of incentive. In a typical case of oligopoly, closed-form expressions of $Q$ and $\kappa$ are given. Extensive experimental results show the viability of FL in a wide range of market conditions. The proposed approach is useful for identifying the optimal FL participation strategies, the viable operational space of an FL system, and the market conditions under which FL can be especially beneficial. It can be a useful tool for FL ecosystem management.

Additional Key Words and Phrases: Federated Learning, Competitive Market, Game Theory

1 INTRODUCTION

Federated learning (FL) is an emerging form of privacy-preserving collaborative machine learning [3]. It has many commercial applications such as safety monitoring [5], digital banking [6] and healthcare [4, 13]. An individual often has limited data to train its learning model. Multiple FL participants (FL-PTs) can join a federated learning ecosystem to collaboratively improve their model performance in a privacy-preserving manner [17]. Proper incentive mechanisms have to be designed to ensure that all FL-PTs are motivated to join the FL training process [3, 7, 22].

Based on the relationship of the eventual model users and FL-PTs, the use cases of FL can be categorized as follows and the related work is summarized in Table 1. Firstly, the FL-PTs are not the model users. The model users benefit from the model performance improvement, which however has no direct impact on FL-PTs. In this case, monetary rewards are the key to incentivizing FL-PTs to contribute more local resources to the FL training process and several schemes have been proposed to allocate rewards [12, 14, 18–21, 23]. Secondly, the FL-PTs are the model users and they are self-interested. They aim to obtain improved model performance through FL, but do not care about how the improved model impact on other FL-PTs [15].

In this paper, we study the third case in which the FL-PTs are the eventual FL model users and also compete against each other in a given market environment by using the FL model. This is commonly found in horizontal federated learning (HFL) scenarios in which the FL model is used...
Table 1. Related Work Classification

| Relationship | The State of the Art |
|--------------|----------------------|
| model users ≠ FL PTs | [12, 14, 18–21, 23] |
| model users = self-interested FL-PTs | [15] |
| model users = FL-PTs in a competitive market | [7, 8, 22] |

for profit-making activities (e.g., digital banking) [17]. In such scenarios, FL-PTs provide the same services and compete for a same group of customers. Thus, market share is a key indicator of FL-PTs’ market competitiveness (i.e., how well a firm is doing against its competitors) [1, 22]. For example, when multiple digital banks collaboratively train FL models to predict the creditworthiness of customers, larger banks with more data may be reluctant to join FL for fear of benefiting its smaller competitors and eroding its market share. This challenge has been framed as the “free-rider problem” in FL [3] for which monetary incentives are not effective.

This challenge has inspired trust-based FL ecosystems to emerge which are built on the proposition that the FL model performance improvement obtained by each FL-PT is proportional to its contribution [3, 7]. Existing literature determines the distribution of FL model performance among FL-PTs in an intuitive manner without regard on how it affects FL-PTs’ market shares in different market environments. The data owners still face uncertainty regarding potential market share erosion as a result of joining FL, which hinders the adoption of FL in HFL scenarios involving competition.

FL can improve FL-PTs’ model performance which, in turn, results in better products or services and enhances their attractiveness to customers [3, 17]. A clear understanding of the impact of joining FL on each FL-PT’s market share is key for wider adoption of FL. Intuitively, the FL model performance improvement achieved by a given FL-PT is bounded, resulting in bounded variations in market shares. The bound relates to specific market conditions (e.g., original market shares, customer loyalty and switching, market growth). In this paper, we propose an analytical framework for market share-based decision support about participation in FL (MarS-FL) by leveraging game theory and marketing models [10, 11].

- It adapts a general economic model [11] to the FL context and introduce two notions of δ-stable market and friendliness to measure the viability of FL and the market acceptability to FL.
- For each FL-PT, we model the decision process by which it joins FL as a non-cooperative game and theoretically analyze its dominant strategy.
- For the manager of an FL ecosystem, MarS-FL provides the sufficient and necessary condition Q to maintain the market δ-stability and quantify the friendliness κ. The condition Q guides the FL ecosystem manager to allocate final model performance improvements among FL-PTs.
- Collaborative FL model training between business owners from a given domain typically involves negotiations which are much easier between two entities. In the case that two entities wish to engage in FL, we provide closed-form expressions of Q and κ, which can help reveal the basic properties of market factors that govern the minimum requirement of FL model performance improvements and market friendliness to FL.
- Finally, extensive experimental results show the viability of conducting FL in a wide range of market conditions.

The rest of this paper is organized as follows. In Section 2, we introduce the related work. In Section 3, we define the FL market model and our problem formulation. The analytical framework and results of MarS-FL are presented in Section 4. In Section 5, extensive experimental results
illustrate the FL viability in different market environments. Finally, we conclude the paper in Section 6.

2 RELATED WORK

Our work in this paper is broadly related to incentive mechanism design for motivating participation in FL. Existing research in this domain can be broadly divided into two categories [3]: 1) monetary incentive schemes, and 2) non-monetary incentive schemes.

Monetary incentive schemes for FL are generally designed for the first FL use case scenario in which the FL-PTs are not the end users of the FL models. These works focus on providing monetary rewards to FL-PTs in order to motivate them to contribute more high quality local data to FL model training. Song et al. [14] and Zhang et al. [23] propose schemes to select and pay FL-PTs based on reputation and reverse auction. Zeng et al. [20] achieve this by applying multi-dimensional procurement auction theory. Sarikaya et al. [12] model the interaction between the FL server (i.e., the model user) and FL-PTs as a Stackelberg game with the aim of improving the FL model performance by jointly optimizing the commitment of local computational resources and the allocation of the incentive budget. Zhan et al. [21] determine the optimal pricing strategy for the FL server and the optimal training strategies for the FL-PTs via deep reinforcement learning. In [18, 19], the FLI approach has been proposed which supports fair allocation of incentive payout to FL-PTs using future revenues generated by the FL model. These approaches leverage incentive mechanism research results in economics and game theory. As they generally assume that the FL-PTs care only about monetary rewards, the intermediate FL models and the final FL model are generally freely shared during the FL training process.

In the third FL use case scenario in which the FL-PTs are also the end users of the final FL models and competing in the same market, which is the focus of our study, sharing the same FL model to all participants without regards to their contributions has been shown to cause breakdown of collaboration [8]. Non-monetary incentive schemes which assign each FL-PT a different model in each training iteration with performance reflecting its contribution are starting to emerge [7, 8, 16]. However, these existing approaches did not take FL-PTs’ market share into account when allocating different versions of FL models to them. MarS-FL bridges this gap by identifying the optimal FL participation strategies, the viable operational space of an FL system, and the market conditions under which FL can be beneficial for a given FL-PT.

3 THE FL MARKET DYNAMICS

In this section, we provide a detailed analysis of the market dynamics surrounding the third FL use case scenario. We first introduce two typical architectures of FL.

3.1 FL System Architectures

In federated learning, each FL-PT has a local model with a common model structure. The training process iterates for multiple rounds. The collaborative training process among FL-PTs enables each FL-PT to achieve improved model performance in a privacy-preserving manner.

We use $w_i^t$ to denote the parameters of the local model of FL-PT $i$ at round $t \in \{1, 2, \cdots, T\}$. In the centralized FL architecture, there is a central FL server to mediate the training process [15]. Let $w^t$ denote the parameters of the global FL model at round $t$ owned by the FL server and $\hat{w}_i^t$ denotes the model parameters sent from the FL server to FL-PT $i$. In the beginning of round $t$, each FL-PT $i$ downloads $\hat{w}_i^{t-1}$ at the end of the previous round of training. $i$ uses a batch of its local data to train model $\hat{w}_i^{t-1}$ and computes the gradient $\nabla w_i^t$. The updated local model is denoted as $w_i^t = \hat{w}_i^{t-1} - \eta \nabla w_i^t$ where $\eta$ is the learning rate. $i$ then uploads $w_i^t$ (or $\nabla w_i^t$) to the FL server. The
FL server produces the global model $w^t$ by aggregating the received local model updates from all FL-PTs.

In the decentralized FL architecture, the training process iterates without a central FL server [3]. In the beginning of round $t$, each FL-PT $i$ uses a batch of its local data to train model $w_{i}^{t-1}$ and computes the gradient $\nabla w_{i}^{t}$. The updated model is denoted as $w_{i}^{t} = w_{i}^{t-1} - \eta \nabla w_{i}^{t}$. $i$ then shares $\nabla w_{i}^{t}$ with other FL-PTs which have agreed to establish collaborative training partnership with $i$ (based on considerations such as trust [8]). In the meantime, $i$ also downloads local models from other partners. Then, $i$ updates its model by aggregating the received local model updates from its partners.

### 3.2 FL Market Model

Let us consider $n$ firms (i.e., FL-PTs) $C = \{1, 2, \cdots, n\}$ in a given market which can join FL. The market size of FL-PT $i$ is $P_{i}$ (e.g., in terms of the number of customers) and their aggregated market size is denoted as $P$. The (relative) market share of $i$ is $MS_{i} \in (0, 1]$ where

$$P_{i} = MS_{i} \times P. \quad (1)$$

Thus, we have $\sum_{i=1}^{n} MS_{i} = 1$. It is possible that the $n$ firms are only a part of the whole market. Firm $i$ owns a local dataset $D^{i}$ with $D^{i} = |D^{i}|$ samples. The local model performance can be measured by the model loss value, which in turn, affects the quality of service offered by the firm. Smaller loss values imply better quality of service. The main notation of this paper is listed in Table 2.

Let $x_{i} \in [0, D^{i}]$ denote the amount of local data that FL-PT $i$ decides to use for FL training. If $i$ trains its model solely using its local data $D^{i}$ without joining FL, the final loss function $L_{i}$ of the resulting model can be expressed as:

$$L_{i} = L_{i} \left(D^{i}\right).$$

If $i$ joins FL, it can leverage local models from other FL-PTs. After the FL model training process ends, the model loss function $L_{i}'$ of FL-PT $i$ can be expressed as:

$$L_{i}' = L_{i}' \left(\{D^{i}\}_{j=1}^{n}, \{x_{j}\}_{j=1}^{n}, Trad\right).$$

$Trad$ denotes a given model exchange scheme among FL-PTs (e.g., under the centralized FL architecture [9], or the decentralized FL architecture [8]). It determines how much information $i$ obtains during the FL process and thus the value of $L_{i}'$.

In this paper, we consider FL ecosystems with non-monetary incentive schemes in which an FL-PT’s reward is reflected by the final loss function value of the model it obtains. We make a natural assumption that for an FL-PT $i \in C$, its contribution to the FL system increases as it commits more of its local data for FL training, and the loss function value of $i$ decreases as its contribution to FL increases. Formally, the assumption is expressed as follows.

**Assumption 1.** Suppose $D^{t} \geq x_{i}^{(1)} > x_{i}^{(2)} \geq 0$. Then, we have

$$L_{i}' \left(x_{i}^{(1)}\right) < L_{i}' \left(x_{i}^{(2)}\right).$$

After the FL model training, the model performance improvement of FL-PT $i \in C$ is measured by:

$$d_{i} = L_{i} - L_{i}' \geq 0. \quad (2)$$

We aim to study the effect of $\{d_{i}\}_{i=1}^{n}$ on the market shares. $\sum_{j=1}^{n} d_{j}$ denotes the aggregate model performance improvement for all FL-PTs. The relative model performance improvement $Q_{i}$ for
Table 2. Main Notation

| Symbol | Explanation |
|--------|-------------|
| \( P \) | the original market size e.g., the number of customers |
| \( n \) | the number of parties in both the market and the federated learning ecosystem |
| \( i \) | the index of a party where \( i \in \{1, 2, \ldots, n\} \) |
| \( MS_{i} \) | the original market share of party \( i \) where \( MS_{i} \in (0, 1) \) and \( \sum_{i=1}^{n} MS_{i} = 1 \) |
| \( P_{i} \) | the market size of party \( i \) where \( P_{i} = MS_{i} \times P \) |
| \( L_{i} \) | the original loss function value of party \( i \) |
| \( L'_{i} \) | the loss function value of party \( i \) after the FL process |
| \( Q_{i} \) | the relative model performance improvement of party \( i \) after the FL process, defined in (3) |
| \( S'_{i} \) | the service quality improvement of party \( i \) after the FL process, defined in (5) |
| \( S_{i} \) | the relative service quality improvement of party \( i \) after the FL process, defined in (6) |
| \( r_{i} \) | the proportion of customers always loyal to party \( i \) |
| \( v_{i} \) | the proportion of customers that will leave party \( i \) and not join any other party |
| \( MS'_{i} \) | the market share of party \( i \) after the FL process, defined in (12), where \( MS'_{i} \in (0, 1] \) and \( \sum_{i=1}^{n} MS'_{i} = 1 \) |

FL-PT \( i \) is defined as:

\[
Q_{i} = \frac{d_{i}}{\sum_{j=1}^{n} d_{j}}, \quad (3)
\]

where we have:

\[
Q_{i} \in [0, 1] \text{ and } \sum_{i=1}^{n} Q_{i} = 1. \quad (4)
\]

\( \{Q_{i}\}_{i=1}^{n} \) corresponds to \( \{d_{i}\}_{i=1}^{n} \).

The service quality improvement \( S'_{i} \) of FL-PT \( i \) is proportional to the model performance improvement \( d_{i} \) and is defined as:

\[
S'_{i} = \alpha \cdot d_{i} \quad (5)
\]

where the weight \( \alpha > 0 \) is the degree of improvement to service quality brought by model performance improvement. The relative service quality improvement for \( i, S_{i} \), is defined as:

\[
S_{i} = \frac{S'_{i}}{\sum_{j=1}^{n} S'_{j}} \stackrel{(a)}{=} Q_{i} \in [0, 1] \quad (6)
\]

where the equality (a) is a result of Eq. (5). We focus on a competitive market in which each firm serves a group of customers with no overlap. The attractiveness of firms to customers changes based on their relative service quality. The customers of a firm may switch to another firm. If the model performance of the firms improves in general, the entire market becomes more attractive, resulting in increases in the market size. Such market dynamics are detailed in Section 3.3. We use \( MS'_{i} \) to denote the market share of \( i \) after the FL process.

We now define a notion of market stability to measure the viability of FL. Intuitively, each FL-PT would not accept a large reduction in its market share after joining FL. The ideal situation is that the market shares of all participants do not change significantly while the market size increases. Formally, we have:
Fig. 1. Market composition and dynamics when $n = 2$: $\phi_i = 1 - r_i - \nu_i$ for $i \in \{1, 2\}$, $S_1 + S_2 = 1$, CT is short for customer.

Definition 3.1 ($\delta$-Stable Market). After all participants join FL, we say that the market is $\delta$-stable if the market share of every participant $i \in C$ satisfies the following condition:

$$V_i = MS_i (D^i) - MS'_i \left( \{ D^j \}_{j=1}^n, \{ x_j \}_{j=1}^n, \text{Trad} \right) \leq \delta$$

where $\delta$ is an upper bound of the difference of market shares before and after joining FL.

We refer to $V_i$ as the market variance of an FL-PT $i$ as a result of joining FL. If the market is $\delta$-stable, the FL ecosystem is viable and sustainable since no FL-PT experiences a significant decrease in its market share. Otherwise, the FL ecosystem is not viable.

3.3 FL-PT Market Share Dynamics

In this section, we adapt the classic model of Rust and Zahorik [11] to characterize the FL-PTs’ market shares $\{MS'_1, MS'_2, \cdots, MS'_n\}$ after FL training concludes. The market share dynamics depends on the dynamics between new customers and existing customers. Initially, the customers for $i \in C$ (whose number totals to $P_i = MS_i \times P$) can be classified into three types:

- Let $r_i \in [0, 1]$ denote the proportion of customers loyal to $i$. After FL model training, $(1 - r_i)P_i$ customers will leave $i$. They will either exit the market or switch to other FL-PTs.
- Let $\nu_i \in [0, 1 - r_i]$ denote the proportion of customers who leave the market where we have $r_i + \nu_i \in [0, 1]$.

After FL model training, $\nu_iP_i$ customers of $i$ will exit the market.
- The remaining $(1 - r_i - \nu_i)P_i$ customers are still active in the market, but will switch to other FL-PTs. These customers are referred to as “free customers” from $i$.

In the case that $n = 2$, the customer types of FL-PT 1 and FL-PT 2 are illustrated in the red and blue rectangles of Figure 1. The market dynamics are mainly driven by the customers’ loyalty, leaving and switching.

Up to $\theta P$ new customers may join the market and be served by the $n$ firms, where $\theta \geq 0$. After the FL model training process, the customer composition of FL-PT 1 and FL-PT 2 is illustrated by the red and blue CTs of Figure 1 respectively; generally, the market share of FL-PT $i \in C$ consists of the following:
Loyal customers of \( i \). The number of loyal customers of \( i \) is:

\[
P_{l,i} = r_i P_i.
\]  

Free customers from other FL-PTs joining \( i \). We use \( S_i \) defined in Eq. (6) to denote the attractiveness of \( i \) to customers. The fraction of free customers from \( j \) joining \( i \) is affected by \( S_i \). Specifically, the number of customers leaving \( j \) for \( i \) equals to \((1 - r_j - v_j)P_j \left( S_i + \frac{S_j}{n-1} \right)\), which guarantees that all the free customers of \( j \) will switch to the other FL-PTs \( C - \{ j \} \) (i.e., \( \sum_{j \in C - \{ j \}} \left( S_i + \frac{S_j}{n-1} \right) = 1 \)). The total number of free customers from the other \((n-1)\) FL-PTs joining \( i \) is:

\[
P_{f,i} = \sum_{j \in C - \{ i \}} (1 - r_j - v_j)P_j \left( S_i + \frac{S_j}{n-1} \right).
\]  

New customers joining \( i \). \( \theta P \) new customers will be divided among the \( n \) FL-PTs. The number of new customers joining \( i \) is proportional to its attractiveness:

\[
P_{e,i} = S_i \theta P.
\]  

After FL training, \( \sum_{i=1}^{n} v_i P_i \) customers will exit the market; we assume that this amount is small such that the whole market size is still positive and the new market size becomes:

\[
P' = (1 + \theta)P - \sum_{i=1}^{n} v_i P_i > 0.
\]  

Based on Eq. (1) and Eq. (8)–(11), the market share of FL-PT \( i \) becomes:

\[
MS_i' = \frac{r_i S_i + \sum_{j \in C - \{ i \}} (1 - r_j - v_j)MS_j \left( S_i + \frac{S_j}{n-1} \right) + S_i \theta}{(1 + \theta) - \sum_{j=1}^{n} v_j MS_j}.
\]  

### 3.4 Decision Support Tasks

Here, we formulate the decision support tasks to be addressed while understanding the role of FL in a competitive market.

#### 3.4.1 A Non-Cooperative Game

The model of non-cooperative game provides insight into the interaction among decision-makers and predict their behaviors in a competitive market. The notion of Nash equilibrium is used to predict the best course of action by any given player, which often depends on the other players’ actions [10].

The \( n \) FL-PTs are self-interested and compete against each other. Each FL-PT \( i \) has a decision variable \( x_i \) which determine how many local data samples are used for FL training. \( x_i \), in turn, determines the loss function value \( L_i ' \) for \( i \) after the FL model training concludes. This corresponds to a non-cooperative game where the \( n \) FL-PTs are players. \( i \)'s strategy space is \( X_i = \left[ 0, D^i \right] \) and a single strategy is \( x_i \in X_i \). The strategy space of the other \((n - 1)\) FL-PTs is denoted as \( X_{-i} = \{ (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n) | x_i \in X_i, \forall i' \in C - \{ i \} \} \). \( i \) aims to enhance its market status in a competitive market, and its payoff is defined as its market share \( MS_i' \) (or equivalently \( MS_i'(P') \)). We assume that an FL-PT does not have any knowledge of the strategies of other FL-PTs. The objective of an FL-PT \( i \) is to choose the best strategy \( x_i^* \) to maximize its payoff in spite of the strategies of other FL-PTs:

\[
\text{maximize } MS_i'(x_i, x_{-i}) = MS_i' \left( \left\{ D^i \right\}^{n}_{j=1}, \left\{ x_j \right\}^{n}_{j=1}, \text{Trad} \right).
\]

Then, \( x_i^* \) is a dominant strategy of FL-PT \( i \). \( x^* = \left\{ x_i^* \right\}^{n}_{i=1} \) is a Nash equilibrium of the game.
3.4.2 Viability of Federated Learning. Another decision support task of importance to the manager of an FL ecosystem is to understand the viability of FL in a competitive market. As discussed above, the market dynamics are parameterized by $\theta$, $\{r_j\}_{j=1}^n$ and $\{v_j\}_{j=1}^n$. The effect of FL on model performance is parameterized by $\{Q_j\}_{j=1}^n = \{S_j\}_{j=1}^n$. The resulting market shares after the FL model training are functions of these parameters, also denoted by $MS'_i(\{MS_j\}_{j=1}^n, \{Q_j\}_{j=1}^n, \{r_j\}_{j=1}^n, \{v_j\}_{j=1}^n, \theta)$.

As formally analyzed in Section 4.2, for every FL-PT $i \in C$, there is a lower bound of the relative model performance improvement, which is denoted by $Q_i^{\min}$, to avoid an unacceptable market share reduction after the FL model training and thus maintain the market $\delta$-stability. Formally, the market is $\delta$-stable if and only if

$$Q_i \geq Q_i^{\min} \text{ for all } i \in C.$$  \hspace{1cm} (13)

Mathematically, it is possible that $Q_i^{\min} < 0$. This means that the market share reduction of FL-PT $i$ will not exceed $\delta$ even if $i$ does not obtain model performance improvement in the FL model training process.

Market Friendliness towards FL. We now define an index to measure the friendliness of market environments towards FL. In a real FL ecosystem, each FL-PT $i$ can achieve some level of model performance improvement, i.e., $Q_i \geq 0$. Let

$$Q_i = \max \{0, Q_i^{\min}\} + y_i \text{ for all } i \in C.$$  \hspace{1cm} (14)

By Eq. (4) and Eq. (13), we have that a feasible allocation of $Q_1, Q_2, \cdots, Q_n$ that can guarantee the market $\delta$-stability satisfies the following relation:

$$y_i \geq 0 $$  \hspace{1cm} (15)

**Definition 3.2.** The level of market friendliness towards FL, $\kappa$, is defined as:

$$\kappa = 1 - \sum_{i=1}^n \max \{0, Q_i^{\min}\} = \sum_{i=1}^n y_i \leq 1$$

where Equality (a) is due to Eq. (4) and Eq. (14).

The value of $\kappa$ indicates the viability of FL:

- If $\kappa < 0$, then Eq. (15) can not be satisfied. In this case, a high variation in market shares arises, and the $\delta$ stability of the market cannot be maintained under any FL framework.
- If $0 \leq \kappa \leq 1$, there exists a feasible allocation of model performance improvements among FL-PTs such that the $\delta$ stability of the market can be achieved. The value of $\kappa = \sum_{i=1}^n y_i$ determines the decision space size of a FL designer. The larger the value of $\kappa$, the more friendly a market is towards FL. For example:
  - When $\kappa = 1$, then $\sum_{i=1}^n \max \{0, Q_i^{\min}\} = 0$. Further, $Q_i^{\min} \leq 0, \forall i \in C$. This indicates that any FL framework can satisfy the market’s $\delta$-stability, since Eq. (13) holds naturally.
  - When $\kappa = 0$, then $\sum_{i=1}^n y_i = 0$. By (14) and (15), the FL framework has to satisfy $Q_i = \max\{0, Q_i^{\min}\}$ in order to achieve the market $\delta$-stability.
  - In Figures 2(a) and 2(b), the grey triangle denotes the full region in which each point represents a possible pair $(Q_1, Q_2)$. Each colored triangle (blue or green) denotes the feasible region in which each point corresponds to a feasible pair $(Q_1, Q_2)$ that can achieve the market $\delta$-stability. In Figure 2(a), we illustrate the cases with $\kappa = 0.4$, but different $(Q_1^{\min}, Q_2^{\min})$ values. In Figure 2(b), we illustrate the case with a larger $\kappa = 0.8$.

In the next section, we will analyze the FL-PTs’ dominant strategies, the bound $Q_i^{\min}$ for all $i \in C$ and the friendliness $\kappa$. 

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\[ \kappa = 1 - \sum_{i=1}^{2} Q_i^\min = 0.4 \]

\[ \kappa = 1 - \sum_{i=1}^{2} Q_i^\min = 0.8 \]

Fig. 2. Friendliness of market environments to FL \((n = 2)\).

4 MARKET SHARE-BASED FL ANALYTICS

In this section, we present the analytical results produced by the proposed MarS-FL framework.

4.1 FL-PTs’ Dominant Strategies

We first analyze the behaviours of FL-PTs in a Nash equilibrium and give their dominant strategies.

**Proposition 4.1.** For each FL-PT \(i\), its dominant strategy is \(x^*_i = D^i\). Specifically, for any strategy \(x'_i \in X_i\) other than \(x^*_i\) where \(x'_i < x^*_i\), we have

\[ MS'_i(x'_i, x_{-i}) \geq MS'_i(x^*_i, x_{-i}), \forall x_{-i} \in X_{-i}. \]

**Proof.** With abuse of notation, we let \(Q_1, Q_2, \cdots, Q_n\) (resp. \(Q'_1, Q'_2, \cdots, Q'_n\)) denote the relative model performance improvements of the \(n\) participants under the strategy profile \(x^*_i, x_{-i}\) (resp. \(x'_i, x_{-i}\)). Based on Assumption 1, we have

\[ L'_i(x^*_i) < L'_i(x'_i) \]

for all \(j \in C - \{i\}\). The loss function values of other FL-PTs remain unchanged. Thus, from Eq. (2) and Eq. (3), we have:

\[ Qi > Q'_i \quad \text{and} \quad Q_j < Q'_j \quad \text{for all} \quad j \in C - \{i\} \]

\[ \sum_{j=1}^{n} Q_j = \sum_{j=1}^{n} Q'_j = 1 \]

Let

\[ \epsilon_i = Q_i - Q'_i \]

\[ \epsilon_j = Q_j - Q'_j \]

where the inequalities (a) and (b) are due to Eq. (16). From Eq. (17) and Eq. (18), we have:

\[ \epsilon_i = -\sum_{j \in C - \{i\}} \epsilon_j > 0. \]

From Eq. (18) and Eq. (19), we have:

\[ \epsilon_i > -\epsilon_j > 0 \quad \text{for all} \quad j \in C - \{i\}. \]
Finally,

\[
\text{MS}'(x^*_i, x_{-i}) - \text{MS}'(x'_i, x_{-i}) = \frac{\sum_{j \in C-\{i\}} (1 - r_j - v_j)MS_j + \frac{\epsilon_j}{n} + \epsilon_i \theta}{1 + \theta - n \sum_{j=1}^{n} v_j MS_j} \tag{21}
\]

The equality (c) is due to Eq. (6) and Eq. (12); the inequality (d) is due to Eq. (7), Eq. (11) and Eq. (20).

In a competitive market, each FL-PT aims to maximize its market share. By Proposition 4.1, an FL-PT’s market share is maximized when it uses all of its local data for FL training, regardless of others’ strategies. From a system perspective, the system efficiency of an outcome \(x^*\) is measured by the extent to which the market is enlarged (i.e., the market growth parameter \(\theta\)). Ideally, \(\theta\) should be as large as possible. With a natural assumption that the market growth parameter \(\theta\) increases with the overall service quality of all FL-PTs, which increases with the amount of local data they use for FL training, the system efficiency is optimal when each FL-PT adopts its respective dominant strategy.

### 4.2 Market Stability in the General Case

Suppose that the number of FL-PTs, \(n\), is arbitrary. In this subsection, we first define a lower bound that relates to the market factors. Afterwards, we prove that this bound is exactly the minimum model performance improvement required to maintain the \(\delta\)-stability of the FL market.

**A Bound Relating to Market Factors.** Let \(A\) be an \(n\)-by-\(n\) square matrix in which its element at the \(i\)-th row and the \(p\)-th column is:

\[
a_{i,p} = \begin{cases} 
\sum_{j \in C-\{i\}} (1 - r_j - v_j)MS_j + \theta, & \text{if } i = p \\
\frac{1}{n-1}(1 - r_p - v_p)MS_p, & \text{otherwise.}
\end{cases} \tag{22}
\]

**Lemma 4.2.** The matrix \(A\) defined by Eq. (22) is invertible.

**Proof.** For all \(i \in \{1, \ldots, n\}\), we have:

\[
a_{i,i} = \sum_{j \in C-\{i\}} (1 - r_j - v_j)MS_j + \theta \geq \sum_{p \in C-\{i\}} (1 - r_p - v_p)MS_p \frac{1}{n-1}.
\]

where \(1 - r_j - v_j \geq 0\) by Eq. (7) and \(\theta \geq 0\). Thus, \(A\) is diagonally dominant and invertible [2].

Let \(B\) be an \(n\)-by-1 vector in which its element at the \(i\)-th row is:

\[
b_i = (MS_i - \delta) \left(1 + \theta - \sum_{j=1}^{n} MS_j v_j\right) - MS_i r_i. \tag{23}
\]

By Lemma 4.2, the lower bound is defined as follows:

\[
Q_{\text{min}} = A^{-1}B \tag{24}
\]

where \(A^{-1}\) is the inverse of \(A\).

**FL Viability.** Let \(Q\) be an \(n\)-by-1 vector in which its element at the \(i\)-th row is \(Q_i\). Suppose we are given an arbitrary FL training framework in which each FL-PT \(i \in C\) uses \(x_i\) local data samples for FL training. This process determines the value of \(Q\) via Eq. (3). The proposition below shows that
the $\delta$-stability of the market is achieved when $Q$ satisfies a particular relation with the original market status and dynamics.

**Proposition 4.3.** The market stability is achieved if and only if the following condition is satisfied:

$$Q \geq Q_{\text{min}}$$

where $Q_{\text{min}}$ is given in Eq. (24).

**Proof.** To achieve market stability, based on Eq. (12) and Definition 3.1, we have for each FL-PT $i \in C$ that:

$$V_i = MS_i - \frac{r_i MS_i + \sum_{j \in C \setminus \{i\}} (1 - r_j - \nu_j) MS_j \left(S_i + \frac{S_j}{n-T}\right) + S_i \theta}{(1 + \theta) - \sum_{j=1}^n MS_j v_j} \leq \delta. \quad (25)$$

Let $S$ be an $n$-by-1 vector in which its element at the $i$-th row is $S_i$. By Eq. (22), Eq. (23) and Eq. (25), we have:

$$AS \succeq B.$$

By Lemma 4.2, Proposition 4.3 holds where $S = Q$ by Eq. (6).

Proposition 4.3 provides the minimum relative model performance improvements required to maintain market stability. Together with Definition 3.2, we can obtain the friendliness towards FL under any given market environment.

### 4.3 Market Stability in the Oligopoly Case

Here, we derive the closed-from expressions of the minimum requirement of relative model performance improvements $Q_{\text{min}}$ and the friendliness $\kappa$ towards FL, in a market with two FL-PTs. Based on Eq. (12) and Definition 3.1, when $n = 2$, the market variance of FL-PT $i \in C = \{1, 2\}$ is:

$$V_i = MS_i - MS'_i = MS_i - \frac{r_i MS_i + MS_{i'} (1 - r_{i'} - \nu_{i'}) + S_i \theta}{1 + \theta - \sum_{j=1}^n v_j MS_j} \quad (26)$$

where $i' = 2 - i$. Intuitively, an FL-PT with a higher quality of service improvement will obtain a larger market share improvement. To maintain market stability, the relative model performance improvements of the two FL-PTs should be bounded to avoid that most of the new customers are attracted to a single FL-PT, thereby confounding the other FL-PT’s motivation to continue FL model training.

**Proposition 4.4.** $\forall i \in \{1, 2\}$, let

$$Q_i^{\text{min}} = \frac{(MS_i - \delta) \left(1 + \theta - \sum_{j=1}^n v_j MS_j\right) - r_i MS_i - MS_{i'} (1 - r_{i'} - \nu_{i'})}{\theta} \quad (27)$$

where $i' = 2 - i$. The market stability can be achieved if and only if the relative model performance improvement satisfies:

$$Q_i \geq Q_i^{\text{min}}, \forall i \in \{1, 2\} \quad (28)$$

**Proof.** To maintain the market stability, we need to guarantee $V_i \leq \delta$ for both $i \in \{1, 2\}$. From Eq. (26), Proposition 4.4 holds where $S = Q$ by Eq. (6).
Corollary 4.5. In the case that there are two FL-PTs, the friendliness of market environment to FL is:

\[ \kappa = 1 - \max \{0, Q_{\min}^1, Q_{\min}^2, \kappa'\} \]

where

\[ \kappa' = Q_{\min}^1 + Q_{\min}^2 = \frac{\theta - 2\delta \left(1 + \theta - \sum_{j=1}^{2} v_j M_S_j\right)}{\theta}. \]

Proof. It follows from Definition 3.2 and Proposition 4.4. □

5 EXPERIMENTAL EVALUATION

As stated in Section 4.3, there are scenarios where an oligopoly occurs. Our closed-form results shed light on the basic properties of market factors that govern the minimum requirement of FL model performance improvements for market stability, and the market friendliness towards FL. In this section, we report related numerical results and use these closed-form results to analyze the viability of FL in several typical scenarios.

5.1 Experiment Settings

We set \( n = 2 \) and classify firms according to their strengths in the market in terms of their market shares:

**Weak and Strong Firms:** The two firms have very different market shares. In our experiments, we set \( M_S_1 = 0.3 \) and \( M_S_2 = 0.7 \) to reflect this scenario.

**Two Comparable Firms:** The two firms have similar market shares. In our experiments, we set \( M_S_1 = M_S_2 = 0.5 \) to reflect this scenario.

The two firms may be facing different market conditions:

**Fast-Growing Market:** We set the market growth rate variable \( \theta \) to be 0.5.

**Mature Market:** We set \( \theta \) to be 0.1.

The value of \( \delta \) is set to be 0.05. The rate at which customers leave the market is set to be a small value (i.e., \( v_1 = v_2 = 0.02 \)).

We will thus consider four typical market situations and provide the following insights:

- Identify the condition under which FL is feasible (i.e., when the friendliness to FL \( \kappa \) is greater than zero);
- Identify the condition under which the friendliness of market environments to FL is especially high; and
- Observe from an individual firm’s perspective the minimum relative model performance improvement that each FL-PT \( i \in \{1, 2\} \) needs under different market conditions.

As explained in Section 3.4, as long as \( \kappa < 0 \), FL is not feasible. In our subsequent figures that illustrate the value of \( \kappa \), when \( \kappa < 0 \), its value is set as \( \max\{\kappa, -0.1\} \) for better clarity.

5.2 Comparable Firms

Firstly, we study the case in which the two firms are comparable (i.e., \( M_S_1 = M_S_2 \)). From Proposition 4.4, for any FL-PT \( i \in \{1, 2\} \):

\[ Q_{\min}^i (r_i, r_{i'}) = \frac{M_S_i (r_i - r_i) - 0.049 + 0.45\theta}{\theta} \quad (29) \]
where \( i' = 2 - i \). In case one of \( Q_{i}^{\text{min}} \) and \( Q_{2}^{\text{min}} \) is negative and the other is positive (i.e., \( Q_{1}^{\text{min}}Q_{2}^{\text{min}} < 0 \)), we denote \( Q_{i}^{\text{min}} \) as negative. Thus, by Corollary 4.5, we have:

\[
\kappa = \begin{cases} 
1 - Q_{i'}^{\text{min}} = 1 - (\kappa' + |Q_{i}^{\text{min}}|), & \text{if } Q_{1}^{\text{min}}Q_{2}^{\text{min}} < 0 \\
1 - \kappa', & \text{if } Q_{1}^{\text{min}} > 0 \text{ and } Q_{2}^{\text{min}} > 0 \\
1, & \text{if } Q_{1}^{\text{min}} < 0 \text{ and } Q_{2}^{\text{min}} < 0 
\end{cases}
\]

(30)

where \( \kappa' \) is independent of \( r_1 \) and \( r_2 \).

5.2.1 A Mature Market. We first study the case in which two comparable firms are in a mature market, where the market growth rate is small (i.e., \( \theta = 0.1 \)). According to Eq. (29), the main factor that determines \( Q_{1}^{\text{min}} \) and \( Q_{2}^{\text{min}} \) is the difference \( r_1 - r_2 \) of the customer loyalties of the two firms, where we have \( Q_{i}^{\text{min}} = 5(r_i - r_i - 0.08) \). \( \forall i \in \{1, 2\} \), the minimum relative performance improvement \( Q_{i}^{\text{min}} \) is illustrated in Figure 3(a). \( Q_{i}^{\text{min}} \) increases as the customer loyalty \( r_{i'} \) of the other firm \( i' \) increases.

In Figure 3(a), we have \( Q_{1}^{\text{min}} > 0 \) and \( Q_{2}^{\text{min}} > 0 \) if \( r_1 = r_2 \); otherwise, \( Q_{1}^{\text{min}}(r_1, r_2)Q_{2}^{\text{min}}(r_2, r_1) < 0 \). The friendliness of market to FL is illustrated in Figure 4. Together with Eq. (30), we have:

![Figure 3](image1.png)

(a) Mature market  

![Figure 4](image2.png)

(b) Fast-growing market

Fig. 3. The minimum relative improvement \( Q_{i}^{\text{min}} \) required to maintain market stability under different customer loyalty of the competitor firm \( r_{i'} \), when the two firms are comparable. For the four lines in each plot, \( r_1 \) equals 1.0, 0.9, 0.8 and 0.7 from bottom to top.

Fig. 4. The friendliness to FL with two comparable firms in a mature market.
• The friendliness to FL is very high (i.e., close to 1) when the two firms have similar customer loyalty.
• The larger the value of $|r_1 - r_2|$, the less friendly the market is towards FL.

5.2.2 A Fast-Growing Market. We now study the case in which two comparable firms are in a fast-growing market (i.e., $\theta = 0.5$). The numerical results are illustrated in Figure 3(b) and Figure 5. Still, $Q_i^{\min}$ increases as the competitor customer loyalty increases. In this case, the parameter $\theta$ in Eq. (29) plays a key role in determining the sign of $Q_i^{\min}$, where we have $Q_i^{\min} = (r_i - r_1) + 0.352$. It can be observed in Figure 3(b) that $Q_i^{\min}$ is positive. Thus, from Eq. (30), we have $\kappa = 1 - \kappa'$. As illustrated in Figure 5, the friendliness to FL is independent of the customer loyalty of the two firms, and remains constant and positive in the range of $(r_1, r_2)$.

5.3 Weak and Strong Firms

To enhance our understanding, we still exemplify our closed-form results in Section 4.3 in this experiment. Based on Eq. (27), we have:

$$Q_1^{\min} = \frac{\Delta - 0.041 + 0.25\theta}{\theta}$$
$$Q_2^{\min} = \frac{-\Delta - 0.057 + 0.65\theta}{\theta}$$

where $\Delta = MS_1(1 - r_1) - MS_2(1 - r_2)$, $\Delta \times P$ represents the difference of the number of customers leaving FL-PT 1 and the number of customers leaving FL-PT 2. The friendliness of the market to FL is still defined by Eq. (30).

5.3.1 A Mature Market. We first study the case that the weak and strong firms are in a mature market where the market growth rate is low (i.e., $\theta = 0.1$). Under this setting, from Eq. (31), we have $Q_1^{\min} = 10(\Delta - 0.016)$ and $Q_2^{\min} = 10(-\Delta + 0.008)$. The experiment results are illustrated in Figure 6 and Figure 7.

From Figure 7 and Eq. (30), the friendliness presents similar patterns as in the case of two comparable firms in a mature market.

• The friendliness to FL is very high (i.e., close to 1) when $|\Delta|$ is small (i.e., when the number of customers leaving FL-PT 1 is close to the number of customers leaving FL-PT 2).
Fig. 6. The minimum relative improvement $Q_i^{\text{min}}$ required to maintain market stability under different customer loyalty of the competitor firm $r_i$, with one strong firm and one weak firm in a mature market. For the four lines in each plot, $r_i$ equals 1.0, 0.9, 0.8, and 0.7 from bottom to top.

Fig. 7. The friendliness to FL with one strong firm and one weak firm in a mature market.

- The larger the value of $|\Delta|$, the less friendly the market is to FL.
- If $\Delta$ is too large, FL becomes infeasible. For example, when $(r_1, r_2) = (1, 0.7)$, we have $\Delta = -0.21$; then $Q_1^{\text{min}} = -2.26$, $Q_2^{\text{min}} = 2.18 > 1$ and $\kappa < 0$.

**Summary of a Mature Market.** From Figure 4 and Figure 7, we can observe that in a mature market:

- When the two firms are comparable, the market friendliness towards FL is positive under a wider range of market conditions. This implies that FL is viable in a wider range of market conditions.
- When the two firms have very different market shares, the market friendliness towards FL is positive under a smaller range of market conditions under which $|\Delta|$ is small.

5.3.2 **A Fast-Growing Market.** We now study the case in which the weak and strong firms are in a fast-growing market (i.e., $\theta = 0.5$). The term $\theta(MS_i - \delta)$ in Eq. (31) plays a key role in determining the sign of $Q_i^{\text{min}}$ where $MS_i - \delta$ is typically greater than zero. In this setting, based on Eq. (31), we have $Q_1^{\text{min}} = 2(\Delta + 0.084)$ and $Q_2^{\text{min}} = 2(-\Delta + 0.268)$. 
The minimum relative performance improvements required for the two firms are illustrated in Figure 8. The friendliness of the market to FL is illustrated in Figure 9. It can be observed that both $Q_{min}^1(r_1, r_2)$ and $Q_{min}^2(r_2, r_1)$ are positive in a wide range of $(r_1, r_2)$ values:

- From Eq. (30), the market friendliness towards FL is independent of the customer loyalty towards the two firms and remains stable and positive in a wide range of $(r_1, r_2)$ under which $Q_{min}^1$ and $Q_{min}^2$ are greater than zero. The range is such that $|\Delta|$ is close to zero.
- When $|\Delta|$ is large enough, the larger the value of $|\Delta|$, the less friendly the market is towards FL. Nevertheless, the friendliness remains positive over the entire range of $(r_1, r_2)$.

**Summary of a Fast-Growing Market.** Regardless of whether the two firms are comparable or not, the market growth rate $\theta$ is important in keeping both $Q_{min}^1$ and $Q_{min}^2$ positive, as shown in Eq. (29) and Eq. (31). From Eq. (30), the market is friendly towards FL over the full range of $(r_1, r_2)$, which is corroborated in Figure 5 and Figure 9. This shows that FL is generally viable in fast-growing markets.

**6 CONCLUSIONS AND FUTURE WORK**

In this paper, we proposed an analytical framework to help FL decision-makers understand the impact of FL on firms’ market shares under various competitive market settings. For each FL-PT,
we characterize the process by which it joins FL as a non-cooperative game and derive its dominant strategy. For an FL ecosystem manager, we provide a sufficient and necessary condition $Q$ for maintaining market stability and quantify how friendly a given market is towards FL. The results can guide non-monetary FL incentive mechanisms to allocate model performance improvements among FL-PTs in order to encourage larger data owners to overcome their fear of smaller FL-PTs free-riding on them and join FL.

In future, we will consider distributed and decentralized FL architectures respectively and study generic parametric algorithms to determine the allocation of model performance while maintaining the $\delta$-stability of the market.

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