Preheating in Quintessential Inflation

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We perform a numerical study of the preheating mechanism of particle production in models of quintessential inflation and compare it with the usual gravitational production mechanism. We find that even for a very small coupling between the inflaton field and a massless scalar field, $g \gtrsim 10^{-8}$, preheating dominates over gravitational particle production. Reheating temperatures in the range $10^{-2}$ GeV $\lesssim T_r \lesssim 10^{15}$ GeV can be easily obtained.

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I. INTRODUCTION

Inflationary models of the early universe have become a paradigm that can explain the isotropy of the cosmic microwave background, the origin of the small adiabatic perturbations that seed galaxy formation and the flatness of space-time. There is also evidence that the universe has recently entered another stage of mild inflation, responsible for its accelerated expansion. It would be natural to investigate models where the two periods of inflation are related. In particular, models of quintessential inflation explore the possibility that the same scalar field is responsible for both periods of inflation [1,2].

In quintessential inflation models, the inflaton potential has no minimum in which the field could oscillate to produce the matter that would reheat the universe, as in usual chaotic inflation models. Therefore, it is assumed that all matter and energy in the universe will be created by gravitational production associated to changes in the geometry of the universe. However, it is known that this process is not very efficient [3]. In this letter we assess the importance of another mechanism of particle production in the early universe, namely preheating [4], in which particles are produced due to the variation of the classical inflaton field. We find that the introduction of a non-zero coupling between the inflaton and matter fields leads to particle production that can easily overcome the gravitational one.

II. THE MODEL

For definiteness, we study the Peebles and Vilenkin model [1], with a potential for the inflaton field $\Phi$ of the form

$$V(\Phi) = \lambda (\Phi^4 + M^4) \quad \text{for} \quad \Phi < 0, \quad \frac{\lambda M^8}{\Phi^4 + M^4} \quad \text{for} \quad \Phi \geq 0,$$

where $\lambda = 1 \times 10^{-14}$ is required from structure formation [6] and $M = 5 \times 10^8$ GeV in order to produce the present quintessential energy density [1].

Defining adimensional parameters $\phi = \Phi/M_{Pl}$, $q = M/M_{Pl}$ and $\tau = \sqrt{M_{Pl} t}$, the classical equation of motion for the inflaton field becomes independent of the $\lambda$ parameter:

$$\ddot{\phi} + \sqrt{24\pi} \sqrt{\frac{\phi^2}{2} + (\phi^4 + q^4) \phi'} + 4\phi^3 = 0 \quad \text{if} \quad \phi < 0,$$

$$\ddot{\phi} + \sqrt{24\pi} \sqrt{\frac{\phi^2}{2} + \frac{q^8}{(\phi^4 + q^4)} \phi'} - 4\frac{q^8}{(\phi^4 + q^4)^2} \phi^3 = 0 \quad \text{if} \quad \phi \geq 0,$$

where the primes are the derivatives with respect to $\tau$ and we have used the Hubble parameter,

$$H^2(\tau) = \left( \frac{a'(\tau)}{a(\tau)} \right)^2 = \frac{8\pi}{3} \left( \frac{\phi^2}{2} + V(\phi) \right).$$

In order to study preheating, we couple the classical inflaton field to a massless quantum scalar field $\chi$ through the lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} \xi R \chi^2 - \frac{1}{2} g^2 \Phi^2 \chi^2,$$
where $R$ is the scalar curvature, $\xi$ is the gravitational coupling and $g$ is a quartic coupling constant between $\phi$ and $\chi$. Ford [3] calculated the energy density of $\chi$ particles produced gravitationally in the limit $|\xi - 1/6| \ll 1$ due to the transition of the universe from the de Sitter spacetime at the end of inflation. However, this production is not efficient since the ratio of matter and inflaton field energy densities is typically $\rho_m/\rho_\phi \sim 10^{-18}$ [3]. After thermalization of the products it is possible to define a reheating temperature when radiation starts to dominate and in that case it is of the order of $T_{rh} \sim 10^4$ GeV.

When preheating is operative, the picture changes drastically. Taking into account the dilution of gravitational particle production, at the end of inflation, until the beginning of the particle production by the preheating mechanism, the universe is still dominated by the inflaton density energy and this is our starting point.

The equation of motion for each comoving $k$ mode of the $\chi$ field can be written in terms of a new convenient variable $X_k = a^{3/2}\chi_k$ as:

$$X_k'' + \left\{ \frac{1}{\lambda} \left( \frac{k^2}{M_{Pl}^2 a^2(\tau)} + g^2 \phi^2(\tau) \right) - \Delta \right\} X_k = 0. \quad (5)$$

The physical momentum is given by $k/a$ and we have verified that $\Delta = \frac{3}{4} \left( \frac{\omega_k}{a} \right)^2 + \frac{3}{4} \left( \frac{\omega_k'}{a} \right)^2$ is negligible during the production of $\chi$ particles.

This is an equation of motion for an harmonic oscillator with variable frequency, $\omega_k^2 = (k^2/(M_{Pl}^2 a^2(\tau))) + g^2 \phi^2(\tau)/\lambda$. When the adiabaticity condition is broken, that is, $\omega_k^2 \geq \omega_k'^2$, particles in the $k$ mode are produced. Figure 1 illustrates the time interval where adiabaticity is violated, leading to particle creation. This time interval corresponds to the passage of the inflaton field through its origin.

Finally, substituting (6) and the solution of equations (1 - 2) into (5) we obtain $X_k(\tau)$.

We define an adiabatic invariant $n_k$ that can be interpreted as the comoving number density of particles produced with momentum $k$:

$$n_k = \frac{\omega_k}{2} \left( \frac{|X_k'|^2}{\omega_k^2} + |X_k|^2 \right) - \frac{1}{2}. \quad (7)$$

In figure 3 we show the comoving number density of particles created during preheating obtained from the solution of equation (5) with vacuum initial conditions for different values of comoving momentum $k$, for $q = 5 \times 10^{-5}$. One can see that particle production occurs at $\tau \simeq 7$, and quickly stabilizes.

III. PARTICLE PRODUCTION

In order to compute the total amount of particles produced during preheating, we have to solve equation (5). To obtain $\phi(\tau)$ we choose the initial conditions as being $\phi(\tau = 0) = -1$ and $\phi'(\tau = 0) = 0$, since in $\lambda \Phi^4$ model inflation ends when $\Phi \simeq -M_{Pl}$. Then we can evaluate the evolution of the scale factor $a(\tau)$ given by:

$$a(\tau) = a_0 \exp \left[ \int_0^\tau d\tau' H(\tau') \right], \quad (6)$$

where $H(\tau)$ is given in equation (3) and $a_0 = 1$. We show in figure 2 the evolution of the scale factor during the period relevant for particle production.

FIG. 1. $\omega_k^2$ (solid line), $\omega_k'$ (dotted line) and $10 \times \phi$ (dot-dashed line) as a function of $\tau$ for $q = 10^{-5}$ and $k = 5\sqrt{\lambda}M_{Pl}$. Notice that adiabaticity is violated around $\tau \simeq 7$, when the inflaton field goes through zero.

FIG. 2. Evolution of the scale factor as a function of $\tau$. 

FIG. 3. Comoving number density of particles created during preheating.
FIG. 3. Comoving number density of χ particles produced
during preheating as a function of τ for comoving momentum
$k = 0, 100$ and $200 \sqrt{λM_{pl}}$, for $g = 5 \times 10^{-5}$.

In order to illustrate the $k-$ dependence of the den-
sity of particles, we plot in figure 4 the density spectrum
produced during preheating for different values of $λ$ and $g$.

The spectrum is well described by the expression:

$$n_k = \exp \left( -\frac{1}{52} \frac{\sqrt{λ}}{g \lambda M_{pl}} k^2 \right),$$

where $\sqrt{λ}$ is the time of particle production and $|\dot{Φ}_p|$ is the
velocity of the inflaton field at the bottom of the potential.
In our case we have $|\dot{Φ}_p| = 3.6 \times 10^{-2} \sqrt{λM_{pl}^2}$ and
$\rho(τ_p) = 68$, which results in:

$$n_k^{app} = \exp \left[ -\frac{1}{53} \frac{\sqrt{λ}}{g \lambda M_{pl}^2} k^2 \right],$$

an excellent approximation indeed.

The total energy density produced at $τ_p$ is given by:

$$\rho_χ = \int \frac{d^3k}{(2π)^3 a^3(τ_p) a(τ_p)} n_k \simeq 3 \times 10^{-6} \lambda g^2 M_{pl}^4$$

Since the energy density in the inflaton field around
the time of particle production is

$$ρ_ϕ \simeq \frac{\dot{Φ}^2}{2} \simeq 6 \times 10^{-4} \lambda M_{pl}^4,$$

the energy density that is transferred from the inflaton
field to the χ field at the moment of production is given by:

$$\frac{ρ_χ}{ρ_ϕ} \simeq 5 \times 10^{-3} g^2$$

This means that there is an upper limit in energy that can be
extracted from ϕ field, at the moment of production,
$\frac{ρ_χ}{ρ_ϕ} \simeq 5 \times 10^{-3}$. However, once the χ-particles are produced
they rapidly become non-relativistic, since their
effective masses are given by $m_χ = g ϕ$ (see equation(5)).
In such a case, the above comparison becomes

$$\frac{ρ_χ}{ρ_ϕ} \simeq 14 g^{5/2},$$

for $ϕ \simeq 10^{-1} M_{pl}$ which is a good approximation for early
times as shown in figure 1.

In order to prove the initial statement that the diluted
energy density of particles produced gravitationally, $ρ_m$
[3] is in fact much smaller than the inflaton energy
density, $ρ_ϕ$ at $ϕ \sim 0$, when the preheating starts, we take the
ratio between them $^1$:

$$\frac{ρ_m(ϕ = 0)}{ρ_ϕ(ϕ = 0)} \sim \frac{ρ_m(ϕ = -1)/a^{-4}}{ρ_ϕ(ϕ = -1)/a^{-6}} \sim 10^{-22}$$

On the other hand, we can compare $ρ_m$ with our χ production
(equation (11)):

$$\frac{ρ_χ(ϕ = 0)}{ρ_m(ϕ = 0)} \sim 10^{23} g^{5/2}$$

$^1$Although $ρ_ϕ$ is not proportional to $a^{-6}$ through all the in-
terval, it corresponds to a maximum dilution.
and conclude that the latter dominates over the former for $g \gtrsim 6 \times 10^{-10}$.

Finally, let us estimate the reheating temperature. We will make the assumption that the $\chi$ particles decay fast enough to avoid the effects of backreaction over the $\phi$ field. In reference [5] it is shown that backreaction is important only for large times. So, without loss of generality, we will consider the transference of $\chi$ energy to relativistic particles just after the $\chi$ production.

As the inflaton field energy density scales as $a^{-6}$ since it is dominated by kinetic energy whereas the relativistic particles energy density, $\rho_R$, scales as $a^{-4}$, the latter will start to dominate the universe at a time $t_d$ given by:

$$\frac{a^2(\tau_p)}{a^2(\tau_d)} \simeq 14g^{5/2} \tag{17}$$

At this time, the $\rho_R$ will be diluted to the value:

$$\rho_R = 3 \times 10^{-5} \frac{\chi^{3/4} g^{5/2} M_{Pl}^3}{\Phi} \frac{a^4(\tau_p)}{a^4(\tau_d)} \simeq 6 \times 10^{-4} \chi^{3/4} g^{15/2} M_{Pl}^4,$$

and the reheating temperature will be:

$$T_{rh} \sim (\rho_R)^{1/4} \Rightarrow T_{rh} \simeq 10^{-1} \left(\chi^{3/4} g^{15/2}\right)^{1/4} M_{Pl}. \tag{18}$$

For our limits on $g$, $6 \times 10^{-10} \lesssim g \lesssim 1$, the reheating temperature corresponds respectively to:

$$10^{-2} \text{ GeV} \lesssim T_{rh} \lesssim 10^{15} \text{ GeV}, \tag{19}$$

which is a confortable range. Further constraints in $T_{rh}$ imply in new bounds on the coupling constant between the inflaton and the $\chi$ fields.

**IV. CONCLUSIONS**

Quintessential inflation models postulate that the same field is responsible for inflation in the early universe and the accelerated expansion of the universe today. In this type of models, the potential has no minimum and the traditional reheating mechanism of the universe can not be operative. Gravitational production of particles is usually adopted to reheat the universe.

We have demonstrated in this letter that introducing a small coupling $g \gtrsim 10^{-9}$ between the inflaton field and a massless scalar particle $\chi$ leads to particle creation in the preheating mechanism that dominates over the usual gravitational production in quintessential inflation models.

Reheating temperatures in the range $10^{-2}$ GeV $\lesssim T_{rh} \lesssim 10^{15}$ GeV can be easily obtained for $6 \times 10^{-10} \lesssim g \lesssim 1$.

Our result is stronger than the one obtained in the approximation used in [5] because we took into account the dilution of the produced particles between the epochs of gravitational creation and preheating. It is also not fully complete since the gravitational production mechanism could be affected when the $\chi - \phi$ coupling is introduced in the lagrangian. However, the effect would be to inhibit the gravitational production, making our results even stronger.

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[1] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D59 (1999) 063505.
[2] B. Spokoiny, Phys. Lett. B 315 (1993) 40.
[3] L. H. Ford, Phys. Rev. D35 (1987) 2955.
[4] A. Dolgov and D. Kirilova, Sov. Nucl. Phys. 51, (1990) 273; J. Traschen and R. Brandenberger, Phys. Rev. D42 (1990) 2491; L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Lett. 73, (1994) 3195.
[5] G. Felder, L. Kofman and A. Linde, Phys. Rev. D59 (1999) 123523.
[6] A. Linde, Particle Physics and Inflationary Cosmology, Harwood Academic Publishers (1990).