A quantitative analysis of the spatial effects of retaining structure for slender foundation pits

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Abstract. The methods for design of retaining structures of hollow slender foundation pits based on classical earth pressure theories such as Rankine’s and Coulomb’s theories have inevitably drawn fierce criticism for their incapable of considering spatial effects of the structures. Researches in understanding the spatial effects of retaining structures are still in initial qualitative analysis stages. In this paper, three spatial effect ratios including displacement ratio, earth pressure ratio and bending moment ratio of a retaining structure were defined for the sake of analyzing the spatial effects of retaining structures of hollow slender foundation pits. Nonlinear finite element analyses and laboratory model tests of retaining structures in sands were conducted. The spatial effect ratios and the relationships of additional earth pressures vs. wall displacements were obtained from the results of the finite element analyses and model tests, and they all shown significantly variation with the process of excavation, and show a strong tendency in their distribution at different positions. These results are distinct from that based on classical earth pressure theories in general rules, and of importance in developing new design methods that make use of the spatial effects in practice.

1. Introduction

With the rapid social and economy development both in China and other countries, retaining structures for hollow slender foundation pits have been more and more used in municipal engineering. The design philosophy and construction technique of this kind of retaining structures based on classical earth pressure theories, e.g., The Rankine’s and Coulomb’s theories [1], which are suitable for very long retaining walls where plain strain condition exists have inevitably drawn fierce criticism from designers and investors for their obvious deviation in describing the behaviour of the retaining structures due to their incapable of considering spatial effects of the structures.

Ou et al. studied the spatial effects though a three dimensional finite element analysis and investigated the corner effects in terms of the width-to-length ratio of a retaining wall [2] [3]. Lin et al. analyzed the undrained behaviour of a retaining structure with an elastoplastic three dimensional finite difference method and presented an evaluation chart relating the wall dimension ratio, the plane strain ratio and the distance from the corner to a section of a wall under study [4]. Finno et al. reported the values of PSR based on more than 150 finite element simulations with different lengths, widths and excavation depths in clay [5]. Hou YM et al. presented a three dimensional finite element modelling using the program ABAQUS for an oversize deep excavation in Shanghai soft deposits [6]. Zhang F et al. conducted both plane strain and three dimensional finite element analyses for braced excavations in soft clays involving multi-levels of struts [7]. Lee F H et al. utilized finite-element back-analyses of an excavation case showed that corner effects were significant, three-dimensional analysis may be able to offer significantly better predictions of movement than two-dimensional analysis [8]. Though
researches have been made in understanding the spatial effects of the retaining structures, researches in this field have been still in initial qualitative analysis stages.

In this paper, the spatial effects were studied in three aspects. Firstly, the spatial effects were for the first time defined by a set of variable ratios such as displacement ratio, earth pressure ratio and bending moment ratio of a retaining structure in understanding and use the spatial effects of retaining structures for hollow slender foundation pits. Secondly, laboratory model tests of spatial effects of retaining structures for hollow slender foundation pits in sands were conducted. Thirdly, nonlinear three dimensional finite element analyses of the spatial effects of retaining structures for hollow slender foundation pits corresponding to the model tests above mentioned were conducted in investigation the development of additional earth pressure with horizontal displacement of retaining walls. These results are of importance in developing analysis and design methods that make implantation of the spatial effects in practice.

2. Conceptual analysis of spatial effects

2.1. Corner effect
The spatial effects of a retaining structure basically refer to that the spatial of mechanical behaviour of the retaining structure. The corner effects refer the deformation, earth pressure and internal forces near a corner the retaining structure of a foundation pit show different distribution in different corner forms, especially different from that from plane theories of earth pressure and structure analyses.

2.2. Depth effect
For foundation pits with same plane dimensions but different excavation depths, as shown in figure 2, the retaining structures are of different behaviour. Usually the deeper the excavation depth is, the stronger the uneven distribution in structure deformations, earth pressures and internal forces. Therefore, the spatial effect ratios for a retaining structure are of course variable with its depth of excavation.

For a given plane dimension, the shallower the excavation depth, the more alike the plane solutions, especially in the middle part of a side of a pit, while the deeper the excavation depth, the stronger spatial distribution and farther from plane state.

2.3. Definitions of spatial effect ratios
A simple sketch of the structure of a retaining structure for a rectangular foundation pit is shown in figure 1. Where, \( z \) refers to the vertical coordinate for a foundation pit, with its origin at the top elevation of the retaining structure, \( x \) denotes the horizontal coordinate of a point in direction along a side of the retaining structure of the foundation pit. The spatial effect ratios are defined with reference of figure 1.
2.3.1. Displacement Ratio. The displacement spatial effect can be described with uneven distribution of wall displacement in a horizontal plan at a depth. Taking the displacement at the middle point M as an examining point where usually the largest displacement take place, as the referential value, the ratio of the displacement at another examining point D in the plan over the displacement at the middle point M is a adequate index for expressing the spatial effect in displacement, and is given in Equation (1).

\[
R_d(z) = \frac{\delta_D(z)}{\delta_M(z)}
\]  

Where, \(R_d(z)\) is the displacement ratio in a plan at the depth \(z\), \(\delta_D(z)\) is the horizontal displacement at the examining point \(D(x, z)\),\(\delta_M(z)\) is the horizontal displacement at point M of a side of the wall.

2.3.2. Earth pressure ratio. Similarly, the earth pressure ratio is defined as the ratio of earth pressure value at point D over that at point M, as given in Equation (2).

\[
R_p(z) = \frac{p_D(z)}{p_M(z)}
\]  

Where, \(R_p(z)\) is the earth pressure ratio in a plan at the depth \(z\), \(p_D(z)\) is the earth pressure at the point \(D(x, z)\),\(p_M(z)\) is the earth pressure at the middle point M of a side of the structure.

2.3.3. Bending moment ratio. There internal forces in a structure member include three kinds, i.e., axial force, shear force and bending moment. Among all the three internal forces, the bending moment is more significance for practical analysis and design. So, the spatial effect of internal force herein is defined as the ratio of the bending moment at point D over that at point M, as shown in equation (3).

\[
R_m(z) = \frac{m_D(z)}{m_M(z)}
\]  

Where, \(R_m(z)\) is the bending moment ratio in a plan at the depth \(z\), \(m_D(z)\) is the earth pressure at the point \(D(x, z)\), while \(m_M(z)\) is the internal bending moment at the middle point M of a side of the structure.
3. The conception of additional earth pressure

From earlier investigation, the authors realised that the theories of total earth pressures are unable to explain the spatial effects correctly. A suitable way to investigate the spatial effects is to introduce the concept of additional earth pressure [9]. For the purpose of clarity, some concepts concern with the relationship between earth pressure and displacement are stated with reference to figure 2 with an origin at point O.

![Figure 2. Concepts and definitions for relationship between earth pressure and displacement.](image)

In figure 2, the horizontal coordinate represent the displacement $u$ of a point at a depth on a retaining wall, and the vertical coordinate is the earth pressure $p$ at the point. The curved solid line represents the relationship between the total earth pressure $p(u)$ and the displacement $u$. The leveled solid line represents the earth pressure at rest, i.e., $p_0$, which remain unchanged with the variation of the displacement of the point. The conception is demonstrated as follows:

- **Active earth pressure $p_a$:** The earth pressure when the displacement reaches $-u_a$, which induces the active limit equilibrium state at the point, as shown on the left side of Figure 2.
- **Additional active earth pressure $p_{ai}$:** The increment in active earth pressure, which is equal to the difference between earth pressure at rest $p_0$ and the active earth pressure $p_a$.
- **Active earth pressure in progress $p_a(u)$:** The total active earth pressure corresponding to an arbitrary displacement $u$, i.e., the earth pressure changing with the variation of displacement $u$.
- **Additional active earth pressure in progress $p_{ai}(u)$:** The increment in active earth pressure, which is equal to the difference between earth pressure at rest $p_0$ and the active earth pressure in progress $p_a(u)$.

Similarly, the conception demonstration is continued with passive aspect, with the reference of the right side of figure 2.

4. Model test study of spatial effects

4.1. Conditions of model tests

In order to investigate the spatial effects of retaining structures for slender foundation pits, we recently conducted two model excavation tests at Tianjin University. The tests were conducted in a soil box which the effective inner dimensions are 3.0 m in length, 1.0 m in width and a total height of 1.2 m and filled with river sand.

The dimensions of the model retaining structures are shown in table 1. Each of the retaining structures consists of four sides of walls that are at right angles in plane and connected at their corners, as shown in figures 3 and 4. The material of the model retaining structures is a kind of plexiglass; whose parameters of mechanical properties are listed in table 2.
Table 1. The dimensions of the model retaining walls and excavation depths.

| Test type | Wall length \( l \) (mm) | Wall width \( b \) (mm) | Wall height \( h \) (mm) | Wall thickness \( t \) (mm) | Maximum excavation depth \( d \) (mm) |
|-----------|------------------------|------------------------|------------------------|------------------------|------------------------|
| No. 1     | 200                    | 200                    | 700                    | 2                      | 400                    |
| No. 2     | 500                    | 200                    | 700                    | 2                      | 400                    |

Table 2. The mechanical properties of plexiglass for the model retaining structures.

| Young’s Modulus \( E \) (GPa) | Poisson’s ratio \( \nu \) | Thickness \( d \) (mm) |
|-------------------------------|--------------------------|------------------------|
| 3.0                           | 0.3                      | 2                      |

The river sand was filled into the test box and compacted to required density. The density of the sand is controlled with two parameters, i.e., the void ratio and the dry unit weight as given in table 3. The internal friction angle of the sand is 32 degree while the cohesion soil is zero, together with the Young’s modulus and the Poisson’s ratio of the soil are also listed in table 3.

Table 3. The mechanical properties of the sand.

| Young’s modulus \( E \) (MPa) | Poisson’s ratio \( \nu \) | Internal friction angle \( \phi \) (°) | Void ratio \( e \) | Dry unit weight \( \gamma_d \) (g/cm³) |
|-------------------------------|--------------------------|----------------------------------------|-------------------|------------------------------------|
| 60                            | 0.30                     | 32                                     | 0.70              | 1.62                               |

4.2. Main process of model tests
The process of the model tests as follows: 1) Initially sand soil was filled into the test box till to the elevation of wall tips, i.e., 500 mm above the bottom of the box. 2) Installation of the model structures of the retaining wall right on the surface of the soil fill in the first step. 3) Fill the soil layer by layer simultaneously both in and out of the retaining walls, as shown in figure 4(b). The thickness of the each compacted soil layer was taken 100 mm by controlling the density of the sand to the designed value in table 3. 4) Excavate soil layer by layer with the thickness of each layer is 100 mm. And totally four steps of excavation, i.e., 400 mm soil, were conducted. 5) In each step of excavation, the horizontal displacements were measured with dial gauges, earth pressures in both sides, i.e., in front and at back, of walls were monitored with pressure cells, and the internal bending moments were determined from the measured data of wall strain gauges that were adhered on the surfaces in both side of a wall.

![Figure 3](image-url)
Figure 4. The model retaining wall for Test No.2: (a) Finished wall structure with pressure cells and strain gauges installed; (b) The model retaining structure installed and soil filled in the test box.

Figure 5 shows the state of test No. 2 at the end of final step of the excavation process. It can be seen from figure 5 that the horizontal displacement of the longer side of the wall is obviously larger than that of the shorter side of the wall, with a maximum deformation occurred in middle of the side, while the smaller and smaller displacements appeared in other positions. The horizontal displacement even nearly approaches to zero at the corner of the wall. This indicates that strong spatial effects exist in the behaviour of the wall.

4.3. Analyses of spatial effects
In order to analyze the spatial effects quantitatively, three variables are defined herein, the displacement ratio, earth pressure ratio and moment ratio. In the case, the displacement ratio is defined as the ratio of the value of horizontal displacement at a examining point that with a distance of a quarter length of the longer side to the corner of the wall (the examining point D) $u_{l/4}$ over the value of horizontal displacement at midpoint of the longer side of the wall (the examining point M) $u_{l/2}$, as referred to figure 1. Similarly, earth pressure ratio and moment ratio are defined as the ratios of earth pressures and bending moments at the positions, respectively.

It is found from the observations that the three ratios change both with the excavation process and the elevation of the points defined in horizontal plane. So, the ratios are mainly observed and studied at elevations of different depth from ground surface, i.e., the elevations in depth of 0, 25 mm, 75 mm and 200 mm.

Figure 5. A top view of the Test No. 2 at the end of final step of the excavation process.

4.3.1. Horizontal displacements. The deformation is the most obvious indication of the spatial effects of the behaviour of model walls. Figure 6 shows the variation of the displacement ratio $R_d$ with the relative excavation depth $D$, which is ratio of the depth of a step of excavation to the total or final
depth of excavation. It can be seen from figure 6, the displacement ratios at different elevations are different but shows a similar general tendency, i.e., all of the four ratios decrease with the depth of excavation to their lowest value at a relative excavation depth about 0.75, and then show rebound to the end of excavation. It can be detected that, from the concept of the displacement ratio definitions, the smaller the ratio, the stronger the spatial effects.

![Figure 6. Variation of displacement ratios with the relative excavation depth at examining points.](image)

4.3.2. Earth pressures. In the process of excavation, earth pressure on the surface of the wall also shows a strong uneven distribution. The earth pressure ratio $R_p$ is not only different at different depth, but also varies with the process of excavation, as shown in figure 7. In figure 7, the earth pressure ratios generally decrease with the process of excavation, but the earth pressure at shallow depth 25 mm, for example, shows a rebounce after the relative excavation depth reaches 0.75.

4.3.3. Bending moments. The bending moments in vertical plane of the wall are of most importance in design of retaining walls. The moment ratios ($R_m$) vary much complexly with the process of excavation due to the strong deflection occurred with wall depth.

Figure 8 shows the bending moment ratios changes with the process of excavation, i.e., the relative excavation depth at different examining points. The moment ratios exhibit a fluctuation tendency because of the earth pressure changes results from excavation process. Furthermore, the some bending moment ratios change directions during the excavation.
5. Finite element analyses of additional earth pressures

5.1. Condition of the finite element analyses
In order to deeply exam the spatial effects of retaining structure for slender foundation pit, nonlinear finite element analyses were made to the same cases studied in the model tests mentioned above, in other words, the same soil and structure propertied and geometric shapes and dimensions. The internationally famous software ZSOIL-3D 2014 was used in the numerical simulations.

The finite element mesh for the test No. 2 is given in figure 9. Total excavation depth is 400 mm, and the excavation process was simulated in four steps. In each excavation step, soil mass with 100 mm thick was removed by eliminate the elements corresponding to each step. For the Test No.1, the excavation process was simulated in the same way as that for Test No. 2.

The constitutive model of the soil is assumed to be an elastic-prefect plastic, without considering the strain hardening behaviour, and the Mohr-Coulomb’s yield/failure criterions and an associated
flow rule were adopted. In the finite element mesh, the wall was simulated with shell elements and the soil was with 8-node brick elements.

![Figure 9. The finite element mesh for the Test No. 2 at excavation step 4.](image)

5.2. Analysis of Spatial effects: Additional pressure vs. wall displacement

The mechanism of spatial effects of retaining structure can be best explained with the spatial distribution of the additional pressure and its development with structure deformation, especially the horizontal displacement of the walls. For purpose of correct use of the analyzing the deformation of retaining structure, the relations of the development of earth pressure with the wall displacement in deferent depth were investigated both in model tests and corresponding finite element analyses.

The displacement contours simulated by the finite element method in different excavation steps of tests No. 1 and 2 were all generated. Figure 10 shows the horizontal displacement contours for tests No. 1 and 2 in the final excavation stage when excavation reached a depth of 400 mm. From figure 10, it can be obvious seen that spatial effects in deformation are very strong for both tests, and the effects of test No.1 is stronger than that of test No.2.
Figure 10. The horizontal displacement (in meters) contours on the top surfaces when the excavation depth is 400 mm. (a) Tests No. 1; (b) Tests No. 2

From the deformation fields and stress fields of these numerical simulations, the additional active earth pressures and the horizontal displacements of the side walls can be easily obtained. Figure 11 (a) shows that the typical relationships of the additional active earth pressure vs. the horizontal displacement for the examining points in depths of 125 mm in Test No.1 and the numerical simulation of the test, respectively. Similarly, figure 11 (b) gives the relationship for the examining point in depths of 225 mm in Test No.1 and the numerical simulation of Test No.1. The results of other examining points are similar to these, but have been omitted for limited space.

As shown in figure 11, the additional active earth pressure at a point on the wall develops with the horizontal displacement at the point. But, the patterns of the curves at different examining points at deferent depths are also different. This demonstrates that the spatial effect ratios are variable with the process of excavation.

Figure 11. The development of additional earth pressure with the horizontal displacement at examining points in Test No. 1. : (a) \( z = 125 \) mm; (b) \( z = 225 \) mm

Figure 12 (a) shows the typical relationships of the additional active earth pressure vs. the horizontal displacement for the examining points in depths of 125mm in Test No.2 and the numerical simulation of Test No.2, respectively. Similarly, figure 12 (b) gives the relationship for the examining point in depths of 225 mm in Test No.2 and the numerical simulation of Test No.2.
Figure 12. The development of additional earth pressure with the horizontal displacement at examining points in Test No. 2. : (a) \(z = 125\ mm\); (b) \(z = 225\ mm\)

The results of other examining points are similar to these, but have been omitted for limited space. As shown in figure 12, the relationships at different examining points at different depths are also different. This again demonstrates that the spatial effect ratios are variable with the process of excavation.

The relation curves shown in figures 11 and 12 have a similar general tendency that the additional earth pressure increase with the horizontal displacement, only small deviations occurred. There are some difference between the results of tests and finite element analyses, as can be seen in figures 11 and 12. The reason for these errors may be the fallacies in measurement of material parameters, and choose of constitutive models for the test materials. However, the results from both the model tests and the numerical analyses are encouraging in understanding the complexity of the behaviour of spatial effects of retaining structures for slender foundation pits.

6. Conclusions
The spatial effects are of very importance in analyzing and design of three dimensional retaining structures for hollow slender foundation pits where the classical earth pressure theories are not applicable for their severe errors in describing the real behaviour of the retaining structures.

The spatial effect ratios of the retaining structures vary significantly with the process of excavation, which are of basic distinction from general rules based on classical earth pressure theories.

The mechanism of the spatial effects of retaining structures can be described by the development of the additional earth pressure with the displacement of retaining structures.

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References
[1] Terzaghi K, Peck R B and Mesri G 1996 Soil mechanics in engineering practice, 3rd Edition. (New York: John Wiley & Sons, Inc.)
[2] Ou C Y, Chiu D and Wu T 1996 Three-dimensional finite element analysis of deep excavations [J]. Journal of Geotechnical Engineering. ASCE, 122(5):337-345.
[3] Ou C Y, Shiau B Y and Wang I W 2000 Three-dimensional deformation behavior of the Taipei national enterprise center (TNEC) excavation case history. *Canadian Geotechnical Journal*, 37, 438-448.

[4] Lin D G, Chung T C and Phienwej N 2003 Quantitative evaluation of corner effect on deformation behavior of multi-strutted deep excavation in Bangkok subsoil. *Geotechnical Engineering Journal*, South East Asian Geotechnical Society, 34(1), 41-57.

[5] Finno R J, Blackburn J T and Roboski J F 2007 Three-dimensional effects for supported excavations in clay. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 133(1):30–36.

[6] Hou Y M, Wang J H and Zhang L L 2007 Three-dimensional numerical modelling of a deep excavation adjacent to Shanghai metro tunnels. In: Shi Y, van Albada G D, et al (eds) *Computational Science – ICCS 2007. Lecture Notes in Computer Science*, Vol. 4489. (Berlin: Springer )

[7] Zhang F and Gohb T C A 2013 Three dimensional finite element analyses of deep braced excavation in soft clay. *Proc. of Conf. of 18th Southeast Asian Geotechnical Conf. & Inaugural AGSSEA Conf.*, 289-294.

[8] Lee F H, Yong K Y and QUAN K C N 1998 Effect of corners in strutted excavations: Field monitoring and case histories [J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 124(4):339-348.

[9] Wang C H 2014 The methods for analyzing deformation and stability of global retaining structures for hollow slender foundation pits based on spatial effects, *A Research proposal submitted to the National Natural Science Foundation of China*. (Tianjin: Tianjin University.)