NON-ABELIAN EQUILIBRATION IN HEAVY ION COLLISIONS

S.M.H. Wong

Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

Abstract

The uncertainty in the equilibration in heavy ion collisions due to the choice of the value of the coupling is examined. The results of the equilibration are indeed affected by this choice. In particular, a variation of $\alpha_s$ from 0.3 to 0.5 reduces the parton phase of the plasma by as much as 4.0 fm/c and increasing coupling causes a reduction in the generated entropy. As far as equilibration is concerned, larger coupling results in faster equilibration both chemically and kinetically but improvements only happen to the fermions. These are accompanied by more rapid cooling and therefore shortened lifetime. In the light of these results, to choose $\alpha_s$ is almost equivalent to choosing the results. Also because of consistency, any fixed $\alpha_s$ is incompatible with an evolving plasma. The best choice is therefore not to choose at all but let the system makes its own decision. We show a simple recipe how this can be done. With an evolving coupling, equilibration is accelerated with better results at the expense of the duration of the deconfined phase.
1 Introduction

In future heavy ion collision experiments at LHC and at RHIC, a major effort is not only to try to produce deconfined matter, given some of which might have already been created at CERN SPS at present energies, but also to show beyond any doubt that during the collisions that deconfined matter really exists for however brief moment. At AGS and SPS, this latter task would be difficult because their energies might not be sufficiently high or the system might not be large enough to reduce boundary effects on the produced quark-gluon plasma.

In the very violent collisions, many particles will be produced, some of which can escape from the system such as electromagnetic probes, others can only reveal themselves after the final break up of the system such as probes using strange and charm hadrons. Nevertheless, they will all be affected by the evolution of the system. Particle production are not exactly the same during the initial, pre-equilibrium and equilibrium phases, therefore the durations of these periods also play a role in modifying the yield of the final produced particles which will eventually fall into the detectors. It is therefore necessary to study the evolution of the system or equivalently to study the equilibration of the parton plasma. There have already been quite a few works on the equilibration in the parton phase of the plasma, for example chemical equilibration\textsuperscript{1)}, thermalization\textsuperscript{2)} and full equilibration\textsuperscript{3)}\textsuperscript{4)}. Our purpose here is not exactly to repeat these investigations but to concern ourselves on one of the uncertainties which can alter the results of the equilibration. The first and most obvious is caused by the uncertainty in the initial inputs. These can be studied but we will not do this at present. The second uncertainty could have been the infrared screening parameter used in the usual perturbative QCD. However, in a multi-particle system, QCD will generate Debye, quark and gluon medium masses\textsuperscript{5)}. These will effectively screen off the infrared divergences. So this parameter and therefore the uncertainty needs not be present. The third one is the strong coupling constant itself. It is the only remaining free parameter once that of infrared screening is no longer present. We would like to find out how the results will change with the value of $\alpha_s$. There is however a second reason concerning consistency for which we would like to look into the uncertainty arising from the choice of $\alpha_s$. The common choice for these kinds of studies is $\alpha_s = 0.3$ which corresponds to an average momentum transfer of $Q \sim 2.0$ GeV with $\Lambda_{QCD} = 200$ MeV. This choice is reasonable early on after the initial collisions. We show in Eq. (1), the evolution of the average energy for quarks and gluons in our previous investigation into the equilibration at LHC and at RHIC. As can be seen, the average parton energy drops by at least 1.0 GeV over the entire evolution. Since the average momentum transfer has to be related to the average parton energy, and so $\alpha_s = 0.3$ cannot be a good value for the entire duration of the evolution. In fact, any choice of fixed $\alpha_s$ will equally not be good enough. We will show a recipe for obtaining a varying coupling in the next section to overcome this problem of consistency and will show the corresponding results as well as those with other fixed values of the $\alpha_s$. Together they reveal the effects of the coupling on the equilibration.
2 Evolution of a parton plasma with various couplings

To study and to make the effect of $\alpha_s$ manifest, we choose some large values of $\alpha_s = 0.5, 0.8$ to do the evolution in addition to the previous $\alpha_s = 0.3$ case. Now also to overcome the problem of consistency, we introduce, as a solution, a varying coupling which evolves with the system in the following way. Since two incoming partons each carrying the average parton energy $\langle E \rangle = \langle \epsilon_{tot}/n_{tot} \rangle$ can exchange a maximum $Q^2 = 4\langle E \rangle^2$. So combining the $\langle E(\tau) \rangle$ at any moment $\tau$ and the 1-loop running coupling formula, we have a coupling that is entirely determined by the system. With this latter approach, there is no remaining free parameter. This is in fact, with hindsight, a better choice for the coupling as we will see presently. We denote this coupling by $\alpha^v_s$ from now on.

With these choices of the coupling, we show the effects of the coupling in Fig. 2 and Fig. 3. Fig. 2 shows the effects on chemical equilibration in terms of the fugacities as well as the parton estimated temperatures. Fig. 3 shows the pressure and energy density to pressure ratios from which we can deduce information on kinetic equilibration. Let us first look at parton chemical equilibration. We used the same fixed initial conditions as before so as to concentrate only on the coupling. From Fig. 2 we see that with increasing $\alpha_s$, chemical equilibration is definitely faster both for gluons and for quarks, however, only in the case of the fermion, do they show any improvements. For gluons, the end degree of chemical equilibration does not change very much with the coupling.

For kinetic equilibration, we use the pressure to pressure and energy to pressure ratios as a check of the isotropy of the parton momentum distribution. Because of our thermalized initial conditions, these ratios should start at 1.0 or with an isotropic distribution. This isotropy is lost subsequently, as seen in Fig. 3 where all the curves shift downward away from 1.0. This is due to the disruptive effect of the longitudinal expansion. As the net interaction responds by increasing its rate, the expansion effect is later overcome and isotropy is progressively being recovered. This is when all the curves in Fig. 3 rise again. Larger couplings lead to faster equilibration but as in the case of chemical equilibration, only quarks and antiquarks...
Figure 2: With increasing coupling, chemical equilibration is faster but the cooling is also more rapid. As seen here, the fastest increase in the parton fugacities \( l_g \) and \( l_q \) are the curves (dashed) with \( \alpha_s = 0.8 \), the next are those produced with \( \alpha_s = 0.5 \) (dotted) and 0.3 (solid). The curves of \( \alpha_v \) (long dashed) shift across the constant \( \alpha_s \) “contours” and equilibrate definitely better than the solid lines. The shifts of the descending \( T \) curves with \( \alpha_s \) are sizable. As a result, the lifetime of the parton phase of the plasma is controlled by the value of the coupling.

show obvious improvements. Equilibration is clearly faster for all partons in Fig. 2 and Fig. 3. However, this is achieved at the expense of similarly faster cooling. The temperature estimates all decrease more rapidly than before in Fig. 2. Not only that but the time at which they reach the assumed phase transition temperature at \( T_c \sim 200 \) MeV changes by as much as 4.0 fm/c when the \( \alpha_s \) varies from 0.3 to 0.5 at LHC. So the lifetime of the parton phase of the plasma is very sensitive to the value of \( \alpha_s \). Similarly sensitive to the coupling is the generated entropy. It is clear therefore, as we have already mentioned, to have to choose a value for \( \alpha_s \) is not the best choice. With the consistent \( \alpha_v \), we see in Fig. 2 and Fig. 3 these curves start off near \( \alpha_s = 0.3 \) case but shift away across the constant \( \alpha_s \) “contours”. They therefore achieve faster and better equilibration for quark and antiquark than the \( \alpha_s = 0.3 \) case. They equilibrate, in this case, in an accelerated fashion. This is a non-abelian effect not found in the equilibration of ordinary electromagnetic plasma or in other many-body system.

The non-abelian effect mentioned in the previous paragraph can be shown more clearly through the relaxation time approximation. In Fig. 4, we have plotted the evolution of these for the various values of the coupling. From the fixed coupling \( \theta \)’s, their behaviours are similar, i.e. a fast rapid initial decrease and then a slow rise until the end. These can be explained by the short expansion and the net interaction dominated phases. It would be helpful to identify the inverse collision time as
Figure 3: These ratios show the isotropy of the parton momentum distribution and therefore kinetic equilibration with $\tau$. The top (bottom) set of four curves are for energy (pressure) to pressure ratios. Faster thermalization is seen with increasing coupling. The assignment of the coupling to the curves are $\alpha_s = 0.3$ (solid), 0.5 (dotted), 0.8 (dashed) and $\alpha_s^v$ (long dashed). Improvements are, however, reserved only for the fermions.

Figure 4: Evolution of the collision time reflects that of the net interaction rate. The values of the coupling are assigned to the curves in the same way as before. The curves produced with fixed couplings have similar time-dependent behaviours. The case of $\alpha_s^v$ is, however, very distinct. It shows the non-abelian effect of QCD accelerates the equilibration. This is unique to a QCD parton plasma.
the net interaction rate, then the first phase is simply the response of the system to being driven out of equilibrium. As a consequence, the net interaction rate has to increase or $\theta$ has to drop. As equilibrium is approached, the net rate has to slow down so $\theta$ has again to increase but slowly. In the case of $\alpha_s^u$, the slow rise of the net interaction dominated phase is replaced by a continued slow decrease. So the net rate continues to increase albeit at a much slower rate as the initial drop. We have already seen in Fig. 2 and Fig. 3 that equilibration is present in all cases, so what is happening to the $\alpha_s^u$ case is, the increase in the interaction strength compensates for the near equilibrium slowing down of the net rate. The result is the non-abelian accelerated equilibration that we have already seen. This is of course unique to a QCD plasma.

In summary, we have studied the $\alpha_s$ dependence of the equilibration and introduced a simple recipe to solve the consistency problem raised in the introduction. It is found that larger $\alpha_s$ means faster equilibration for all partons but improvements are only for the fermions. Lifetime of the parton phase as well as the entropy are sensitive to the value of $\alpha_s$ therefore the consistent $\alpha_s^u$ is a better choice which gives rise to accelerated equilibration unique to a QCD plasma.

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