Small size pentaquark width: calculation in QCD sum rules

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Abstract

The pentaquark width is calculated in QCD sum rules. The higher dimension operators contribution is accounted. It is shown, that $\Gamma_\Theta$ should be very small, less than 1MeV.

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The status of $\Theta^+$, predicted in 1997 by D.Diakonov, V.Petrov and M.Polyakov [1] in the Chiral Soliton Model, till now is doubtful. Few years ago narrow exotic baryon resonance $\Theta^+$ with quark content $\Theta^+ = uudd\bar{s}$ and mass 1.54 GeV had been discovered by two groups [2, 3]. But the question is open until now, during last two years some groups confirm the pentaquark $\Theta^+$ existence, while other see null signal. Moreover, last year some groups, which have seen pentaquark, in the new experiments with higher statistics reported null result for pentaquark signal (CLAS experiments on hydrogen and deuterium [4], BELLE [5]) but at the same time DIANA [6], and also LEPS, SVD-2 confirm their results with higher statistic(see [7] for the review). (Some theoretical reviews are given in [8, 9]).

So yet one can say only that if pentaquark exist, it should be a narrow state. Experimentally, only an upper limit was found, the stringer bound was presented in [3]: $\Gamma < 9MeV$. The phase analysis of $KN$ scattering results in the even stronger limit on $\Gamma$ [10], $\Gamma < 1MeV$. A close to the latter limitation was found in [11] from the analysis of $Kd \to ppK$ reaction and in [12] from $K + Xe$ collisions data [3]. Also [5] from the negative result of the experiment give the upper limit for pentaquark width less than 640KeV.

In the paper [13, 14] it was shown, that if pentaquark is genuine state it width should be strongly parametrically suppressed. It is necessary to note that this is general statement and does not depend of the choice of the pentaquark current (without derivatives), but at the same time it is significantly based on the assumption, that the size of the pentaquark is not larger, than usual hadronic. The main goal of this paper is to find numerical estimation of pentaquark width by use the method offered in [13,14].
Part 1. In the papers [13], [14] it was shown, that pentaquark width should be suppressed as \( \Gamma_\Theta \sim \alpha_s^2 (0|\bar{q}q|0)^2 \), (for any current without derivatives). Later, in the short paper [15] the first non-vanishing operator (dimension \( d = 3 \)) contribution was calculated and the sum rule was considered numerically. It was shown that pentaquark width is suppressed numerically also and the width of the pentaquark width was estimated to be less 1 Mev. In this paper we will discuss this sum rules in more detail and also the contribution of the operators of the higher dimensions will be accounted. Let us shortly remind the main points of the method. We start from 3-point correlator

\[
\Pi_\mu = \int e^{i(p_1 x - y q)} \langle 0 | \eta_\theta(x) j_5^\mu(y) \eta_n(0) | 0 \rangle
\]

where \( \eta_n(x) \) is the neutron quark current [16], \( \langle 0 | \eta_n | n \rangle = \lambda_n v_n \), \( v_n \) is the nucleon spinor, \( \eta_\theta \) is an arbitrary pentaquark current

\[
\langle 0 | \eta_\theta | \theta^+ \rangle = \lambda_\theta v_\theta \text{ and } j_5^\mu = \bar{s} \gamma_\mu \gamma_5 u \]

is the strange axial current.

As an example of \( \eta_\theta \) one can use the following one (see [17], where it was first offered, and also [18], where the sum 2-point rule analysis for this current was discussed): 

\[
J_A = \varepsilon^{abc} \varepsilon^{def} \varepsilon^{gcf} (u^a T C d^b) (u^d T C \gamma^\mu \gamma_5 d^e) \gamma^\nu c \bar{s} g
\]

and we will use it farther to obtain numerical results. (Of course there is large number of the another currents, for example see [19], where 2-point correlators was analyzed very carefully, taking into account operators up to dimension 13 and direct instanton contribution).

As usual in QCD sum rule the physical representation of correlator (1) can be saturated by lower resonance states plus continuum (both in \( \eta_\theta \) and nucleon channel)

\[
\Pi_{\mu}^{phys} = \langle 0 | \eta_\theta | \theta^+ \rangle \langle \theta^+ | j_\mu | n \rangle \langle n | \eta_n | 0 \rangle \frac{1}{p_1^2 - m_\theta^2} \frac{1}{p_2^2 - m^2} + \text{cont.}
\]

where \( p_2 = p_1 - q \) is nucleon momentum, \( m \) and \( m_\theta \) are nucleon and pentaquark masses.

Obviously, in the limit of massless kaon

\[
\langle \theta^+ | j_\mu | n \rangle = g_{\theta n}^A \bar{v}_n \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2} \right) \gamma^\nu \gamma_5 v_\theta
\]

where axial transition constant \( g_{\theta n}^A \) is just we are interesting in (the width is proportional to the square of this value). Such a method for calculation the width in QCD sum rules is not new, see, e.g. [20]. In the case of massive kaon the only change is in denominator of second term in r.h.s of the eq. (4), i.e.

\[
\langle \theta^+ | j_\mu | n \rangle = g_{\theta n}^A \bar{v}_n \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2 - m_k^2} \right) \gamma^\nu \gamma_5 v_\theta
\]

It is clear that the second term vanishes at small \( q^2 \).

Substituting \( \langle 0 | \eta_n | n \rangle = \lambda_n v_n \), and \( \langle 0 | \eta_\theta | \theta^+ \rangle = \lambda_\theta v_\theta \) in eq (3) and take the sum on polarization one can easily see, that (in the limit of small \( q^2 \)) correlator (1) is proportional to \( g_{\theta n}^A \).
\[ \Pi_{\mu}^{phys} = \lambda_n \lambda_\theta g_{\theta n}^A \frac{1}{p_1^2 - m_\theta^2} \frac{1}{p_2^2 - m^2} (-2\hat{p}_1^\mu \gamma_5 + \ldots) \]  

(6)

where dots in r.h.s mean other kinematic structures (proportional to \( q \) e.t.c). From (6) one can easily obtain sum rule for axial constant \( g_{\theta n}^A \). For our sum rules we will use invariant amplitude just at the kinematical structure \( \hat{p}_1 \hat{p}_1^\mu \), because, as it was discussed in [21], [22], [23] the choice of the kinematic structures with maximal number of momentum lead to better sum rules. So we obtain the following sum rules

\[ \lambda_n \lambda_\theta g_{\theta n}^A e^{-(m_\theta^2 + m_n^2 + m_\theta^2)/M^2} = (-1/2)B_\theta B_n \Pi^{QCD} \]  

(7)

where \( B_\theta, B_n \) mean Borel transformation on pentaquark and nucleon momenta correspondingly, and continuum extraction is supposed.

By use of the equation of motions the eq.(4) close to the mass shell can be rewritten

\[ \langle \theta^+ \mid j_\mu \mid n \rangle = g_{\theta n}^A \bar{v}_n \left( \gamma^\mu + \frac{m_\theta + m_n}{q^2} q^\mu \right) \gamma_5 v_\theta \]  

(8)

At the same time, the second term in (4,8) correspond to the kaon contribution to \( \theta - n \) transition with lagrangian density \( L = ig_{\theta nk} v_n \gamma^5 v_\theta \phi_k \), so one can write

\[ \langle \theta^+ \mid j_5^\mu \mid n \rangle = g_{\theta nk} \frac{q^\mu f_k}{q^2 - m_k^2} \bar{v}_n \gamma^5 v_\theta \]  

(9)

Comparing (9) and (8) one can found (if we for a moment neglect the kaon mass)

\[ g_{\theta nk} f_k = (m_n + m_\theta)g_{\theta n}^A \]  

(10)

This is the analog of the Golderberger-Trieman relation. Of course the accuracy of this relation is about the scale of SU(3) violation but as estimation of the value of \( g_{\theta nk} \) it is enough good. In [13], [14] some general properties of correlator (10) and correspondingly sum rules for \( g_{\theta n}^A \) was obtained. First of all, it was shown, that correlator (1) vanishes in the chiral limit for any pentaquark current without derivatives. That means, that first non-vanishing contribution to sum rules give the operator with dimension \( d = 3 \) (quark condensate), so axial constant \( g_{\theta n}^A \) should be proportional to the quark condensate. An examples of corresponding diagrams are shown on the Fig.1a,b. But as was also shown in these papers, diagrams like those on fig.1a (i.e. without hard gluon exchange) can not contribute to the sum rule. The reason is that such diagrams, as one can easily check, are expressed in terms of the following integrals

\[ \int \frac{d^4 x d^4 y}{((x - y)^2)^n (x^2)^m} \equiv \int \frac{e^{ip_1 x}}{(x^2)^m} \frac{e^{-iqt}}{(t^2)^n} d^4 xd^4t \]  

(11)

It is clear that such integrals have imaginary part on \( p_2^2 \) and \( q^2 \) - the momentum of nucleon and axial current - but there is no imaginary part on \( p_1^2 \) - the momentum of pentaquark. So we come to the conclusion that such diagrams correspond to the case, when there is no \( \Theta^+ \) resonance in the pentaquark current channel (this correspond to background of this decay). (Note, that this conclusion don’t depend on the fact that one ore more of the quark propagators should be replaced by condensate, as we
discuss before). The double imaginary part on \( p_1^2 \), \( p_2^2 \) (i.e. \( \Theta^+ \) resonance and baryon) appears only if one take into account hard gluon exchange, and not arbitrary, but only those, which connect the quark line, going to axial current vertex with those going to an baryon vertex, (so that it provide the moment exchange between these vertexes), as on Fig.1b. So we come to conclusion, that if \( \Theta^+ \) is a genuine 5-quark state (not, say, the \( NK \) bound state), then the hard gluon exchange is necessary, what le ads to additional factor of \( \alpha_s \). We see, that pentaquark width \( \Gamma_{\Theta} \approx \alpha_s^2 (0|\bar{q}q|0)^2 \), i.e., \( \Gamma_{\Theta} \) has strong parametrical suppression.

**Part 2.**

Let us now discuss the sum rules (7). First of all, of course, the contribution of the operator of \( d = 3 \) should be accounted. As we discuss in previous section, only diagrams like those on Fig.1b give the non-vanishing contribution to double dispersion relation. It is necessary to note, that in the sense of discussion before (see [14]) it is quite necessary to keep only those part of these diagrams, which have really imaginary part both on \( p_1^2 \) and \( p_2^2 \) (i.e pentaquark and nucleon 4-momenta square). This mean that double Borel transformation (for \( p_1^2 \) and \( p_2^2 \) independently) in (7) is quite necessary.

In what following, we will denote the result of calculation (i.e. operator \( d = 3 \) contribution to sum rules (7)) as \( R_{d3} \)

\[
R_{d3} = 1/(\lambda_n \lambda_{\Theta})e^{(m_n^2/M_n^2+m_{\Theta}^2/M_{\Theta}^2)}(-1/2)B_n B_\Theta \Pi_{QCD}^{CD} \tag{12}
\]

It is clear, that

\[
g_A^{d3} = R_{d3} + R_{d5} + ...
\]

and if one confine itself only of \( d = 3 \) operator contribution, (as it was done in [15]) then \( g_A^{d3} = R_{d3} \). Let us shortly discuss the results of [15] (the higher order operators contribution will be discussed in the next section).

First of all the calculation of the diagrams Fig.1b is technically enough complicated. One should pay special attention to extract the terms, which have no imaginary part on pentaquark 4-momenta correctly. We perform calculation in the x-representation in Euclid space, using standard exponential representation of propagators and the relation \( B e^{-bp^2} = \delta(b - 1/M^2) \).

We neglect in the calculations the effects of \( s - quark \) mass, which are very small. We also use in calculation the fact, that the ratio \( A1 = M_n^2/M_{\Theta}^2 \), (where \( M_n^2, M_{\Theta}^2 \) are nucleon and pentaquark Borel masses) should be of order of ratio of the corresponding mass square \( m_n^2/m_{\Theta}^2 \), so we can threats \( A1 \) as small parameter. But even at this simplification the analytical answer is enormous large and is expressed in terms of a very large number of different double (and ordinary) integrals and it total size is very large (about some hundred terms). That’s why, unfortunately, we can not write down pure analytical answer, and the last last stage of calculation we prefer to do only numerically . (Of course we check, that all integrals converge if \( Q^2 (= -q^2) \) is not equal to zero.)

Of course, as was discussed before, the result \( (R_{d3}) \) is proportional to quark condensate and strong coupling constant, i.e. \( \alpha_s a \), where \( a = -(2\pi)^2(0|\bar{q}q|0) \). As a characteristic virtuality we chose the Borel mass of nucleon, but one should note, that
because $\alpha_s a^2$ do not depend on normalization point, this choice is rather unessential. We use the value of

$$\alpha_s a^2 = 0.23 GeV^6, \quad \bar{\lambda}_n^2 = \lambda_n^2 \cdot 32 \pi^4 = 3.2 GeV^6, \quad \bar{\lambda}_\theta^2 = \lambda_\theta^2 \cdot (4\pi)^8 = 12 GeV^{12} \quad (14)$$

(see [24], [18]). To extract the continuum contribution, usually one should write down explicitly double dispersion integral, but it is too difficult to do technically in our case, so we estimate continuum contribution by a usual factor $E_0 = 1 - e^{s_0/M^2}$ in both pentaquark and baryon channels. The continuum dependence is not strong, we use standard value $s_0 = 1.5 GeV^2$ for nucleon and $s_0 = 4.5 GeV^2$ for pentaquark current [18]. We estimate the inaccuracy of the value of $R_{d3}$ due to continuum contribution about 30%. In the paper [15] only the contribution $R_{d3}$ to the value $g_{d3}^A$ (eq.(13)) was accounted and the estimation $g_{d3}^A < 1 Mev$ was obtained. In this paper we also account the contribution of the higher dimension operators we are now going to discuss.

**Part 3.**

There are large number of diagrams, corresponding to higher dimension operators contribution (see some examples on figs.2,3,4 for dimension $d = 5, 7, 9$ correspondingly). Numerically the main contribution in our case give diagrams on Fig.4, (operator of the dimension $d = 9$), proportional to $a^3$, where $a = -(2\pi)^2 \langle 0|\bar{q}q|0 \rangle$). It is not surprising, the same situation appear in many similar cases (see [16] for the case of nucleon, or [18] for the case of pentaquark). The reason is clear – each cut quark line lead, from one side, to small factor - the quark condensate, but, from the other side, it decreases the number of loops, so lead to additional factor of order of $4\pi^2$, and this two factors more or less compensate each other. That’s why one can expect, that main contribution of high dimension operator come from diagrams on Fig.4a-4b. When calculating this diagrams, one should also carefully exclude those, which have no double imaginary part both on $p_1^2$ and $p_2^2$. Examples of such diagrams which have no double imaginary part are shown on Fig.5 - one can easily check this just in the same way as was discussed in the previous chapter (for diagrams on Fig.1a). So to estimate dimension $d = 9$ operator contribution $R_{d9}$ one should account only four types of diagrams on Fig.4a,b (of course each of them with all possible combinations). Fortunately, in this case it is possible to write down analytical answer. If we neglect the small terms, proportional to $s$-quark mass, and also for simplicity suppose strange and light quark condensates to be equal, we can write:

$$R_{d9} = -0.209(\bar{\lambda}_n \bar{\lambda}_\theta)^{-1}e^{(m_n^2/M_n^2 + m_\theta^2/M_\theta^2)} \frac{\alpha_s a^3}{\pi} \int_0^{s_\theta} e^{-u/M_\theta^2} du \int_0^{s_n} e^{-s/M_n^2} ds (\rho_1 + \rho_2/v) \quad (15)$$

where

$$\rho_1 = \frac{(-3/8)u\delta(s)}{Q^4}$$

$$\rho_2 = -1 \frac{Q^2 + 2 s^2 - u^2 + Q^4 - 1.5 s Q^2}{Q^2v^2} + 7 u s \frac{u + Q^2 - s}{v^4}$$

$v^2 = (u + Q^2 - s)^2 + 4 Q^2 s$, and $\bar{\lambda}_n, \bar{\lambda}_\theta$ are defined in (14).
In the numerical analysis we suppose again \( M_\theta^2 = 3M_n^2 \), as for \( d = 3 \) operator case in previous section.

On Fig. 6a, b the axial constant \( g_{\theta n}^A = R_{d3} + R_{d9} \), (see eq. (13)), obtained from sum rules, is shown for two values of \( Q^2 \): \( Q^2 = 1.5 Gev^2 \) (Fig. 6a) and \( Q^2 = 2.5 Gev^2 \) (fig. 6b) as a function of Borel mass of nucleon. Thick (upper) line mean the total result for \( g_{\theta n}^A \), thin line and dashed line - \( R_{d3} \) and \( R_{d9} \) contributions correspondingly. One can see, that \( R_{d9} \) is smaller than \( R_{d3} \) in this region and also the Borel mass behavior of \( g_{\theta n}^A \) is very good, so one can suppose, that sum rule are reliable at this range of \( Q^2 \).

From the sense of sum rule, it is clear that we can found axial constant \( g_{\theta n}^A \) only at \( Q^2 \) not close to zero (about 1 Gev\(^2\) or higher). And really, at \( Q^2 = 1 Gev^2 \) \( R_{d9} \) became equal (and even more) than \( R_{d3} \), so at this value sum rules became very doubtful, and at lower values of \( Q^2 \) sum rules does not exists.

On the Fig. 7 the \( Q^2 \) dependence of \( g_{\theta n}^A \) is shown (at \( M_n^2 = 1 Gev^2 \) and \( M_\theta^2 = 3M_n^2 \) - thin (lower) line, and the same for \( M_n^2 = 1.2 Gev^2 \) - thick (upper) line ). One can see, that they are practically identical.

Really we are interest the value \( g_{\theta n}^A \) in the limit \( Q^2 \to 0 \), which can’t be calculated directly from S.R., obviously. But one can see from Fig. 7, that \( Q^2 \) behavior is found to be almost linear so one can extrapolate it to zero. We found averaged (on Borel mass) \( g_{\theta n}^A = 0.034 \) at \( Q^2 = 0 \). Of course the accuracy of this value is not high, and highly depend on the extrapolation (about a factor 1.5-2 for different more or less reasonable extrapolations). Also large inaccuracy appear due to:

a) the method itself has the accuracy of the order of the SU(3) violation
b) accuracy in the value of \( \lambda_\theta \) (about 20-30%)          
c) accuracy of sum rule approach, especially for pentaquark case (see, for example, discussion in [25]).

For all this reason, we estimate the value of axial constant \( g_{\theta n}^A(0) \) with inaccuracy about a factor two, so it can varied from 0.017 to 0.068 with central point \( g_{\theta n}^A = 0.034 \). By use of eq (10) one can easily express the pentaquark width in terms of \( (g_{\theta n}^A)^2 \), and we come to conclusion, that our result for \( \Gamma_\Theta \) can vary in the region from 60KeV to 1MeV. The central value \( g_{\theta n}^A = 0.034 \) correspond to width about 0.25 MeV. Note, that our result for width of the pentaquark (with positive parity) don’t contradict to value 0.75 MeV, obtained in [25] also in sum rules, but by quite differ method.

Our estimation of the pentaquark width also is in the agreement with the result [6] (0.36 MeV with accuracy about 30%), obtained from the ratio between numbers of resonant and non-resonant charge exchange events.

Of course the accuracy of our result is not high, really we predict only the order of magnitude of the width, but it seems, that more precise prediction for pentaquark width in this approach is impossible. That’s why we don’t discuss contribution of operators \( d = 5,7 \) e.t.c, because they are much smaller than those we have accounted and their contribution is within of the our accuracy.

The main conclusion is, that if the \( \theta^+ \) is genuine states then sum rules really predict the very narrow width of pentaquark, less than 1 Mev, (most probably about 0.25 – 0.5 MeV) and the suppression of the width is both parametrical and numerical.

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References

[1] D.Diakonov, V.Petrov and M.Polaykov, Z.Phys. A359, 305 (1997).
[2] T.Nakano et al., Phys.Rev.Lett. 91, 012002 (2003).
[3] V.V.Barmin, A.G.Dolgolenko et al., Yad.Fiz. 66, 1763 (2003). (Phys.At.Nucl. 66, 1715 (2003)).
[4] M. Battaglieri et al (CLAS Coll.) hep-ex/0510061
[5] R. Mizuk et al (BELLE Coll.) Phys Lett. B362, 173, (2006)
[6] Barmin, V.V, A.G.Dolgolenko et al, hep-ex/0603017
[7] Volker D.Burkert, hep-ph/0510309
[8] D.Diakonov, hep-ph/0406043
[9] V.B.Kopeliovich, Uspekhi Fiz.Nauk, 174, 323 (2004)
[10] R.A.Arndt, I.I.Strakovsky and R.L.Workman, nucl-th/0311030
[11] A.Sibirtsev, J.Heidenbauer , S.Krewald and If-G.Meissner, hep-ph/0405099
[12] A.Sibirtsev, J.Heidenbauer, S.Krewald and Ulf-G.Meussner, nucl-th/0407011
[13] B.L.Ioffe and A.G.Oganesian, JETP Lett. 80, (2004) 386
[14] A.G.Oganesian, hep-ph/0410335
[15] Oganesian A.G, hep-ph/0605131
[16] B.L.Ioffe, Nucl.Phys. B188, 317 (1981).
[17] Shoichi Sasaki, Phys. Rev. Lett. 93, 152001 (2004)
[18] Oganesian A.G, hep-ph/0510327
[19] Hee-Jung Lee, N.I.Kochelev and V.Vento, Phys.Rev. D73:014010, (2006)
[20] V.L.Eletsky, B.L.Ioffe and Ja.I.Kogan Phys.Lett. B122, 423 (1983)
[21] V.M.Belyaev and B.L.Ioffe Sov.Phys.JETP 56, (1982) 493
[22] V.M.Belyaev and B.L.Ioffe Nucl.Phys. B310 (1988) 548
[23] B.L.Ioffe and A.V.Smilga Nucl.Phys. B232 (1984), 109
[24] B.L.Ioffe Prog.Part.Nucl.Phys. 56 (2006) 232
[25] Matheus, R.D and Narison, S, hep-ph/0412063
[26] F.S. Navarra, M.Nielsen and R. Rodrigues da Silva hep-ph/0510202
Figure 1: examples of the diagrams for $d = 3$ operator contribution

Figure 2: Examples of diagrams for $d = 5$ operator contribution, dashed line mean vacuum gluon field, wave line -hard gluon exchange, circles mean derivatives
Figure 3: The same as in Fig. 2. for $d = 7$ operators

Figure 4: examples of the diagrams for $d = 9$ operator contribution
Figure 5:

Figure 6: $g_{\alpha n}^A$ dependence on Borel mass for: a) $Q^2 = 1.5GeV^2$, b) for $Q^2 = 2.5GeV^2$
Figure 7: $Q^2$ dependence of $g_{\theta n}^A$ for $M_n^2 = 1.2 GeV^2$ - thick (upper) line and for $M_n^2 = 1. GeV^2$ - thin (lower) line