Lepton Flavor Violation
and Fermion Masses*

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Abstract

Lepton flavor violation may be a signature of “GUT scale” physics, if the messenger scale for SUSY breaking is above the “GUT scale.” We elaborate on the details of this simple statement in the following talk.

Introduction

The minimal supersymmetric standard model [MSSM] is defined by its spectrum and interactions, i.e. the minimal particle spectrum necessary for a self-consistent extension of the standard model, along with R parity so that the only interactions are those in the standard model or supersymmetric extensions thereof. Even with these constraints the theory in principle has many unknown parameters. These are associated with soft SUSY breaking

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parameters defined at a messenger scale, $M$. In minimal supergravity the messenger scale $M = M_{Pl} \sim 10^{18}$ GeV. In this case SUSY breaking occurs in a hidden sector and is transmitted to the visible sector via gravitational interactions. It results in 5 soft SUSY breaking parameters, a universal scalar mass $m_0$, a universal gaugino mass $M_{1/2}$, a supersymmetric Higgs mass parameter $\mu$ (which in some theories is only generated once SUSY is broken), the Higgs scalar mass $B\mu$ and a universal soft trilinear interaction parameter $A$. In gauge-mediated SUSY breaking, on the other hand, the messenger scale is typically much less than $M_{Pl}$. In this case, scalars with common gauge charges are degenerate and $A$ vanishes at tree level. In this talk we consider minimal supergravity SUSY breaking, unless otherwise stated. Finally in the MSSM, as in the standard model, neutrinos are massless.

The MSSM as defined above is a symmetry limit. Individual lepton numbers, $L_e$, $L_\mu$, $L_\tau$, are conserved. Thus processes such as $\mu \to e\gamma$, $\mu \to 3\,e$, $\mu \to e$ conversion or $\tau \to \mu\gamma$ are forbidden. The experimental branching ratios for these processes are bounded by $B(\mu \to e\gamma) \leq 5 \times 10^{-11}$, $B(\mu \to 3\,e) \leq 1 \times 10^{-12}$, $B(\mu \,\not_{22} T_i \to e \,\not_{22} T_i) \leq 4.3 \times 10^{-12}$ and $B(\tau \to \mu\gamma) \leq 4.2 \times 10^{-6}$.

These strong constraints have two significant consequences.

1. **Possible non-universal scalar masses or soft trilinear parameters are severely constrained**\cite{8,7}. For example, define

$$
\delta_{ij}^e \equiv \frac{\Delta_{ij}^e}{m_0^2}
$$

$$
\delta_{ij}^{LR} \equiv \frac{\Delta_{ij}^{LR}}{m_0^2}
$$

(1)

where $\Delta_{ij}^e$ ($\Delta_{ij}^{LR}$) is the off-diagonal mass squared term for right-handed (left-to-right handed) scalar leptons in a superbasis where lepton masses are diagonal ($i,j$ are flavor indices). Then typical constraints\cite{7} are,

$$
\delta_{12}^e < 4.3 \times 10^{-3} \left(\frac{m_0(GeV)}{100}\right)^2
$$

(2)

(from $\mu \to e\gamma$ with $(\frac{m_0}{m_\mu})^2 = 0.3$), or

$$
\delta_{ij}^{LR} < 1.5 \times 10^{-6} \left(\frac{\tilde{m}(GeV)}{100}\right)^2
$$

(3)
2. Lepton flavor violation[LFV] is sensitive to “GUT scale” physics. Although “GUT scale” physics could easily violate flavor symmetries, one might suspect these flavor violations to be suppressed by powers of $1/M_G$. This is not the case however. As shown by Hall et al., flavor violation in the lepton sector can be induced at the GUT scale due to RG running from $M$ to $M_G$. Moreover, this flavor violation enters as a boundary condition in the slepton mass matrices; hence it is not suppressed by inverse powers of $M_G$.

As an illustration of this phenomenon, consider a generic GUT-like theory with heavy states $X, Y, Z$ with mass $M_I \sim M_G$ and the standard model states $F_i = \{Q_i, \bar{u}_i, \bar{d}_i, L_i, \bar{e}_i\}$. Assume some new interactions between the scales $M_I$ and $M$ given by

$$\lambda_{ij} F_i F_j X + k_i F_i Y Z$$  \hspace{1cm} (4)

As a consequence of renormalization group running, we find at $M_I$

$$\delta_{ij}^F \sim -((\lambda^\dagger \lambda)_{ij} + k_i^\dagger k_j) \ln \frac{M}{M_I}$$  \hspace{1cm} (5)

Of course, the numerical value depends on the scale $M_I$ and the magnitude of the Yukawa couplings $\lambda_{ij}, k_i$. In the rest of this talk we consider three different possible contributions to lepton flavor violating interactions emanating from “GUT scale” physics. In all three cases, $M_I$ is the scale where the structure of the fermion mass hierarchy is generated.

A. Adding neutrino masses to the MSSM

B. GUTs and the Third family yukawa couplings

C. Family mass hierarchy and the FN mechanism

A. — Adding neutrino masses to the MSSM

Consider adding to the MSSM some right-handed neutrinos; one for each family. The most general renormalizable superspace potential including the

\footnote{Universal scalar masses at $M$ are assumed in all analyses.}
right-handed neutrinos is given by

\[ W = \lambda_{ij}^\nu \bar{\nu}_i L_j h + M_{ij} \bar{\nu}_i \bar{\nu}_j \]  

(6)

where \( M_{ij} = \delta_{ij} M_1 \) (\( \delta_{ij} \) is a Kronecker delta) and we work in a basis where charged lepton yukawa couplings are diagonal. In this case, the scale \( M_I \) is given by \( M_I = \min\{ M_i \} \gg M_Z \). As long as \( \det M \neq 0 \), this theory results in 3 light majorana neutrinos (predominantly left-handed) and 3 superheavy majorana neutrinos (predominantly right-handed). RG running leads to radiative mass corrections of the form [9, 10]

\[ \delta_{ij}^L \sim -\left( \lambda^\nu \bar{\lambda}^\nu \right)_{ij} \ln \left( \frac{M}{M_I} \right) \]  

(7)

A recent analysis by Hisano et al. [10] takes \( \lambda_{ij}^\nu = \lambda_u^i V_{ij}^{CKM} \) where \( \lambda_u^i \) are the diagonal up quark yukawa couplings and \( V_{ij}^{CKM} \) is the CKM matrix. This form for \( \lambda_{ij}^\nu \) is suggested by SO(10) GUTs. A value for \( M_I \) of order \( 10^{12} \) GeV was also assumed. With this value, the tau neutrino has mass of a few eV and thus it makes a good hot dark matter candidate in a universe with hot + cold dark matter. Branching ratios for LFV processes are obtained which are below the experimental bounds but close enough to be observable in future LFV experiments at Los Alamos or PSI.

B. — GUTs and the Third family yukawa couplings

Quark flavor is not conserved; this is the essence of CKM mixing. Since GUTs relate quarks and leptons, it is not surprising that GUT interactions also violate lepton flavor.

For example, consider a simple SUSY SU(5) model with quarks and leptons in the \( 10_i \supset \{ Q_i, \bar{u}_i, \bar{e}_i \} \), and \( \bar{5}_i \supset \{ \bar{d}_i, L_i \} \). For \( \tan \beta \sim 1 \), the top quark yukawa coupling is the largest yukawa coupling in the theory. It enters the superspace potential in the expression

\[ W \supset \lambda_i^u \ (10_i, 10_i, H) \]  

(8)

where \( H \) is a 5 of SU(5) containing the Higgs doublets as well as their color triplet partners and we work in a basis where the up quark yukawa coupling is diagonal. (Note, this simple SU(5) model with Higgs in the 5 and \( \bar{5} \) representation and only dimension 4 fermion mass operators cannot fit the known
fermion masses. Nevertheless, this is a useful exercise, since in any more realistic theory, the top quark yukawa coupling must still be large.\[1\]

RG running from $M$ to $M_G$ induces lepton flavor violating masses for right-handed sleptons given by\[8, 11, 12\]

$$\delta^\ell_{ij} \sim -\lambda_t^2 \delta_{3i} \delta_{3j} \ln\left(\frac{M}{M_G}\right)$$ \hspace{1cm} (9)

In the effective theory below $M_G$, in a basis where lepton masses are now diagonal, we have

$$\delta^\ell_{ij} \sim -\lambda_t^2 V_{3i}^* V_{3j} \ln\left(\frac{M}{M_G}\right)$$ \hspace{1cm} (10)

where $V = V^{CKM}$. Note, in SU(5), only right-handed sleptons are affected.

The CKM elements mixing the first two families with the third are small. In addition, with only $\delta^\ell_{ij} \neq 0$, LFV is further suppressed due to a subtle cancellation between neutralino and higgsino contributions\[12\]. As a result lepton flavor violating processes are well within experimental bounds and possibly beyond the reach of future experiments.

In SO(10), on the otherhand, both $\delta^L_{ij}$ and $\delta^\ell_{ij}$ are non-zero. Non-vanishing contributions to $\delta^L_{ij}$ occur, even in the limit of small $\tan \beta$, because both $L$ and $\bar{e}$ are contained in a 16 $\supset \{Q, \bar{u}, \bar{d}, L, \bar{e}\}$ of SO(10). The combination of both these terms avoids the accidental cancellation discussed previously when only $\delta^\ell_{ij} \neq 0$\[12, 14\]. In this case, observable flavor violating effects may be expected in future experiments. Moreover, certain regions of parameter space are already ruled out. A study of the large $\tan \beta$ regime has also been carried out, see Ciafaloni et al.\[13\], with results similar to those at low $\tan \beta$.

C. — Family mass hierarchy and the Froggatt-Nielsen mechanism

The problem with the specific GUT models discussed above is that they give unrealistic fermion masses and mixing angles. In order to improve upon this situation within the context of GUTs one needs to either add several Higgs multiplets (with some in higher dimensional representations of the GUT symmetry) or consider the possibility of a simple Higgs sector but with higher dimension effective fermion mass operators. The latter case can provide effective higher dimensional Higgs representations by incorporating
direct products such as $(5 \times 24 \supset 45 + \cdots)$ in SU(5). Moreover, it was shown by Froggatt and Nielsen [14] that these effective fermion mass operators are “natural” in theories with heavy intermediate states and softly broken flavor symmetries.

As an example, consider the renormalizable superspace potential given by

$$W = \psi_3 \bar{\psi}_3 H + \psi_2 \chi H + \bar{\chi} (M_{FN} \chi + \phi \psi_3)$$

(11)

where $\psi_2$ ($\psi_3$) represent the second (third) generation of quarks or leptons, $H$ is the electroweak Higgs, $(\chi, \bar{\chi})$ are heavy Froggatt-Nielsen states with mass $M_{FN}$ and $\phi$ contains a scalar whose vev breaks the FN flavor symmetry at a scale below $M_{FN}$, so that $\epsilon \equiv <\phi>/M_{FN} << 1$. In the effective theory below $M_{FN}$, the FN states $(\chi, \bar{\chi})$ are integrated out; giving the effective superspace potential

$$W = \psi_3 \bar{\psi}_3 H + \epsilon \psi_2 \bar{\psi}_3 H$$

(12)

plus calculable corrections of order $\epsilon^2$. Thus we have a $2 \times 2$ yukawa matrix of the form

$$\lambda = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 1 \end{pmatrix}$$

(13)

In these theories, the scale $M_I = M_{FN}$.

It has been shown by Dimopoulos and Pomarol [13] that the FN mechanism can lead to enhanced flavor violation due large yukawa couplings (of order one) as well as to the mixing of heavy FN scalar states with light squarks and sleptons. Consider the scalar masses

$$L_{soft} \supset \tilde{m}_{\psi_2}^2 |\psi_3|^2 + \tilde{m}_{\chi}^2 |\chi|^2 + \tilde{m}_{\psi_2}^2 |\psi_2|^2 + \cdots$$

(14)

Assume that at the scale $M$ we have universal boundary conditions

$$\tilde{m}_{\psi_3}^2(M) = \tilde{m}_{\psi_2}^2(M) = \tilde{m}_{\chi}^2(M) = m_0^2$$

(15)

After RG running from $M$ to $M_I$ and integrating out $(\chi, \bar{\chi})$ we obtain the scalar mass matrix for these two families given by

$$\tilde{m}^2 \sim \begin{pmatrix} \tilde{m}_{\psi_2}^2(M_I) & 0 \\ 0 & \tilde{m}_{\psi_3}^2(M_I) + \epsilon \tilde{m}_\chi^2(M_I) \end{pmatrix} = \begin{pmatrix} \tilde{m}_2^2 & 0 \\ 0 & \tilde{m}_3^2 \end{pmatrix}$$

(16)
Since the Yukawa couplings in the renormalizable theory above $M_I$ are assumed to be of order one, we have $(\tilde{m}_2^2 - \tilde{m}_3^2)/\tilde{m}_3^2 \sim 1$. When extended to three families, order one flavor splittings between all three families of sleptons are induced. Such large splittings between the third and the first two families has already been discussed in the previous section. It leads to acceptable LFV rates due to the small mixing angles between the third and first two families. Order one splittings between the first and second family, on the other hand, gives unacceptable LFV rates, since Cabibbo like mixing between the first two families is not small. Recently Lucas [16] has calculated the LFV rates in an SO(10) SUSY GUT with realistic fermion masses and mixing angles [17, 18]. LFV interactions place severe constraints on this model [19]. Consistency with present data is only obtained with sufficiently heavy scalars; in particular, sneutrinos can be as light as 800 GeV, but only in a very restricted region of parameter space.

**U(2) family symmetry**

When using the Froggatt-Nielsen mechanism to generate a fermion mass hierarchy, one may also need to suppress large slepton mass mixing between the first and second families. This can be accomplished by a non-abelian family symmetry which is only broken below the FN scale or by lowering the messenger scale below $M_I (= M_{FN})$, such as in gauge-mediated SUSY breaking models [2, 4]. Several such symmetries have been considered in the literature. These include: SU(2), SU(3), S$_3$, U(2), $\Delta(3n^2)$ with $n = 4, 5$.

As an example consider the family symmetry group U(2) [19]. Extensions to include an SU(5) [20] (or SO(10) [21, 22]) GUT have also been considered. In the $SO(10) \times U(2)$ model, the first two families transform as a $(16,2)$ and the third family transforms as a $(16,1)$ (represented by the fields $16_a$, $a = 1, 2$ and $16_3$).

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2 This analysis assumed that the messenger scale is the Planck scale with universal boundary conditions for soft SUSY breaking parameters at $M_{Pl}$.

3 In gauge-mediated SUSY breaking models, the messenger scale may be as small as $O(10^5)$ GeV. The suppression of flavor violating effects is one of the main motivations for these models.
The superspace potential for the fermion mass sector is given by

\[
W = 16_3 \bar{16}_3 10 + 16_a \chi^a 10 \\
+ \bar{\chi}_a (M_{FN} \chi^a + S^{ab} 16_b + A^{ab} 16_b + \phi^a 16_3) \tag{17}
\]

where 10 contains the electroweak Higgs doublets and their color triplet partners, \((\bar{\chi}_a, \chi^a)\) are the massive FN states, and \((S^{ab} = S^{ba}, A^{ab} = -A^{ba}, \phi^a)\) contain the scalars which spontaneously break the FN U(2) symmetry.\[\] The vacuum expectation values of the latter fields determine the small parameters

\[
\begin{align*}
\epsilon &= \frac{<\phi^2>}{M_{FN}} \approx \frac{<S^{22}>}{M_{FN}} \\
\epsilon' &= \frac{<A^{12}>}{M_{FN}} \\
\epsilon' &= \epsilon 
\end{align*}
\tag{18}
\]

This theory results in fermion yukawa matrices schematically given by

\[
\lambda \sim \begin{pmatrix} 0 & \epsilon' & 0 \\
-\epsilon' & \epsilon & \epsilon \\
0 & \epsilon & 1 \end{pmatrix} \tag{19}
\]

with \(\epsilon \sim V_{cb} \sim 0.03\). For more details, see refs. \[21, 22\].

Using a simple operator analysis, the scalar mass are given by\[22\]

\[
\tilde{m}^2 \sim \begin{pmatrix} m_1^2 & 0 & \epsilon \epsilon' m_5^2 \\
0 & m_1^2 (1 + \epsilon^2) & m_4^2 \\
\epsilon \epsilon' m_5^2 & m_4^2 & m_3^2 \end{pmatrix} \tag{20}
\]

Hence \(\delta^e_{12} \sim \delta^L_{12} \sim \epsilon^2 \sim 10^{-3}\); consistent with experimental bounds (see eqn. \[3\]).

**The Bottom Line**

If the messenger scale \(M\) for soft SUSY breaking is above the “structure scale” \(M_I\) for fermion mass hierarchies, then observable *lepton flavor violation*

\[\]

\[4\] Note, in order to fit fermion masses and mixing angles, \(S^{ab}\) transforms as a 45 while \(M_{FN}\) transforms as a direct sum of \(1 + 45\).
is predicted due to the RG running of slepton masses from $M$ to $M_I$. In the examples discussed in this talk, the messenger scale was assumed to be the Planck scale.

- For the “structure scale” of fermion masses three cases were considered:

A. $M_I \sim 10^{12} \text{ GeV}$ — Neutrino masses in the MSSM, consistent with $m_{\nu_e} \sim \text{few eV}$;

B. $M_I = M_G \sim 10^{16} \text{ GeV}$ — GUTs and the third family yukawa couplings;

C. $M_I = M_{FN} \geq 10^{12} \text{ GeV}$ — Family hierarchy described by Froggatt-Nielsen mechanism.

- The results are strongly model dependent, but in most cases LFV effects should be observed in the next generation experiments.

Lepton flavor violation may be a rich goldmine of “GUT scale” physics.

OR

If $M << M_I$, then lepton flavor violation may be suppressed. This would be the case in gauge-mediated SUSY breaking models where $M << M_{Pl}$.

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