IMPACT OF BARYONIC PROCESSES ON WEAK-LENSING COSMOLOGY: POWER SPECTRUM, NONLOCAL STATISTICS, AND PARAMETER BIAS

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ABSTRACT

We study the impact of baryonic physics on cosmological parameter estimation with weak-lensing surveys. We run a set of cosmological hydrodynamics simulations with different galaxy formation models. We then perform ray-tracing simulations through the total matter density field to generate 100 independent convergence maps with a field of view of 25 deg2, and we use them to examine the ability of the following three lensing statistics as cosmological probes: power spectrum (PS), peak counts, and Minkowski functionals (MFs). For the upcoming wide-field observations, such as the Subaru Hyper Suprime-Cam (HSC) survey with a sky coverage of 1400 deg2, these three statistics provide tight constraints on the matter density, density fluctuation amplitude, and dark energy equation of state, but parameter bias is induced by baryonic processes such as gas cooling and stellar feedback. When we use PS, peak counts, and MFs, the magnitude of relative bias in the dark energy equation of state parameter \( w \) is at a level of, respectively, \( \delta w \sim 0.017 \), 0.061, and 0.0011. For the HSC survey, these values are smaller than the statistical errors estimated from Fisher analysis. The bias could be significant when the statistical errors become small in future observations with a much larger survey area. We find that the bias is induced in different directions in the parameter space depending on the statistics employed. While the two-point statistic, i.e., PS, yields robust results against baryonic effects, the overall constraining power is weak compared with peak counts and MFs. On the other hand, using one of peak counts or MFs, or combined analysis with multiple statistics, results in a biased parameter estimate. The bias can be as large as 1σ for the HSC survey and will be more significant for upcoming wider-area surveys. We suggest to use an optimized combination so that the baryonic effects on parameter estimation are mitigated. Such a “calibrated” combination can place stringent and robust constraints on cosmological parameters.

Key words: cosmological parameters – cosmology: theory – gravitational lensing: weak – large-scale structure of universe

1. INTRODUCTION

An array of recent observations of the large-scale structure of the universe such as cosmic microwave background anisotropies (e.g., Hinshaw et al. 2013; Planck Collaboration et al. 2014) and galaxy clustering (e.g., Reid et al. 2010; Beutler et al. 2014) established the standard cosmological model called the \( \Lambda \)CDM model. In the \( \Lambda \)CDM model, the energy content of the present-day universe is dominated by two mysterious components: dark energy and dark matter. Dark energy realizes the cosmic acceleration at present, and dark matter plays an important role of formation of rich structure in the universe. However, we have not understood yet the nature of dark energy and the physical properties of dark matter. In order to reveal the mysterious dark components in the universe, several observational programs are proposed and still under investigation. Such observational programs include Subaru Hyper Suprime-Cam (HSC),4 the Dark Energy Survey,5 and the Large Synoptic Survey Telescope.6 Space missions such as Euclid7 and WFIRST8 are also promising. Gravitational lensing is expected to be the main subject of these future surveys that are aimed at studying the large-scale structure of the universe at present and in the past.

Weak gravitational lensing (WL) by large-scale structure in the universe is the promising probe into properties of dark matter and dark energy (for a review, see Bartelmann & Schneider 2001; Munshi et al. 2008; Kilbinger 2014). WL causes small distortions of images of distant source galaxies, called cosmic shear, which reflects directly the intervening matter distribution along a line of sight. Recent cosmic shear observations have proved the WL measurement to be a powerful tool for studying dark matter distribution in the universe, from which one can extract information on the basic cosmological parameters (e.g., Massey et al. 2007; Kilbinger et al. 2013). Forthcoming weak-lensing surveys are aimed at measuring cosmic shear over a wide area of more than 1000 deg2. These observations will address important questions of dark matter and dark energy at unprecedented precision.

Unfortunately, major statistical methods to make the best use of WL data in upcoming surveys are still under debate. The problem originates from the fact that cosmic shear follows non-Gaussian probability distribution due to nonlinear gravitational growth (Sato et al. 2009). In order to incorporate the nonlinear features accurately into WL statistics, cosmological N-body simulations have been extensively used. Previous numerical studies (Hilbert et al. 2009; Sato et al. 2009, 2011) have already provided important guides for cosmological studies.
with WL statistics. There still remain several possible factors to be examined. One of the uncertainties in such studies is the effects of baryonic physics. Modeling baryonic effects on the WL statistics is difficult because of the overall complexities in galaxy formation. Recent numerical simulations and semianalytic methods successfully reproduce key observational data (Duffy et al. 2010; Martizzi et al. 2012, 2014; Okamoto et al. 2014; Pike et al. 2014; Schaller et al. 2014b, 2014a; Schaye et al. 2014; Velliscig et al. 2014). Some of these studies focus on active galactic nucleus (AGN) feedback, which quenches star formation in massive halos and may solve the overcooling problem (i.e., the overproduction of stars in numerical simulations). These simulations also show that baryonic physics can change significantly the distribution of both dark matter and baryons within a halo. However, since baryonic effects are expected to be weak at large scale, most of the WL studies so far are based on simulations with a dark matter component only. For future “precision cosmology” with WL, neglecting various astrophysical processes may lead to an undesirable bias of cosmological parameter estimation, as has been pointed out by several studies (Jing et al. 2006; Semboloni et al. 2011; Yang et al. 2013; Zennier et al. 2013; Mohammed et al. 2014). Hence, it is crucial and timely to study the effects of baryonic physics on WL statistics in detail.

In this paper, we use several statistics to extract the non-Gaussian information of WL maps. In addition to the power spectrum (PS) of WL convergence, we consider peak counts and the Minkowski functionals (MFs). Kratochvil et al. (2010) show that peak counts on lensing maps can be indeed useful for cosmological parameter estimation. High-σ lensing peaks are likely associated with massive dark matter halos along a line of sight (Hamana et al. 2004, 2012; Yang et al. 2011). MFs are morphological statistics for a given multidimensional field. MFs are one of the good measures of topology in WL and contain the suitable cosmological information beyond two-point statistics (Kratochvil et al. 2012; Munshi et al. 2012; Shirasaki et al. 2012; Shirasaki & Yoshida 2014). Throughout this paper, we use the terms “local statistics” and “nonlocal statistics.” Shear two-point correlation functions and the corresponding PS are measured directly from shear that is obtained from local measurements, i.e., from galaxy ellipticities. Peak counts and MFs are nonlocal statistics because they can be obtained from convergence maps that are derived by integrating shear over the region of interest. In order to clarify the baryonic effects on the WL statistics, we utilize a large set of hydrodynamical N-body simulations including various baryonic processes. We then study how the baryonic effects bias cosmological parameter estimation with WL measurement.

The rest of this paper is organized as follows. In Section 2, we summarize the basics of WL statistics of interest and the implementations to estimate the statistics for given WL data. In Section 3, we describe our simulation set that includes cosmological simulations with baryonic physics. In Section 4, we provide the details of analysis performed in this paper. In Section 5, we show the impact of baryonic physics on WL statistics. We also perform a Fisher analysis to present the expected cosmological constraints in upcoming lensing surveys. We show the results of a χ² analysis to quantify the baryonic effects on parameter estimation with WL statistics. Concluding remarks and discussions are given in Section 6.

2. WEAK-LENSING STATISTICS

We first summarize the basics of gravitational lensing by large-scale structure. The WL effect is characterized by the image distortion of a source object by the following 2D matrix:

\[ A_{ij} = \frac{\partial^2 \varphi}{\partial x_i \partial x_j} \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \]

where the observed position of a source object is denoted by θ, the true position is β, κ is convergence, and γ is shear. In the weak field, where gravitational potential is small compared with c², each component of Aij can be related to the second derivative of the gravitational potential Φ as

\[ A_{ij} = \delta_{ij} - \Phi_{ij}, \]

where \( r(\chi) \) is the angular distance and \( x_i = r \theta_i \) represents physical separation (Bartelmann & Schneider 2001; Munshi et al. 2008). By using the Poisson equation and Born approximation (Bartelmann & Schneider 2001; Munshi et al. 2008), one can express the WL convergence field as

\[ \kappa(\theta, \chi) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_m \int_0^\chi \, d\chi' f(\chi, \chi') \delta \frac{r(\chi') \theta, \chi}{a(\chi)}. \]

The Born approximation yields sufficiently accurate two-point statistics (e.g., Schneider et al. 1998). In the present paper, we take into account the nonlinearity of convergence shown in Equation (3) by performing ray-tracing simulations through the matter density field obtained from cosmological simulations.

2.1. Observables

Here we summarize three different statistics of the WL convergence field, PS, peak counts, and MFs. PS has complete cosmological information only if statistical properties of matter fluctuation follows Gaussian distribution. However, nonlinear structure formation induced by gravity inevitably makes the fluctuation deviate significantly from Gaussian. In order to extract cosmological information, we will also use nonlocal statistics, i.e., peak counts and MFs.

2.1.1. Power Spectrum

PS is one of the basic statistics for modern cosmology. For a convergence field κ, PS is defined by the two-point correlation in Fourier space:

\[ \left\langle \tilde{\kappa}(\ell) \tilde{\kappa}^* (\ell') \right\rangle = (2\pi)^2 \delta_D(\ell - \ell') P_\kappa(\ell), \]

where the multipole ℓ is related to angular scale through \( \theta = \pi/\ell \). By using the Limber approximation (Limber 1954; Kaiser 1992) and Equation (5), one can derive the convergence PS as

\[ P_\kappa(\ell) = \int_0^\chi \, d\chi \frac{W(\chi)^2}{r(\chi)^2} P_k \left( k = \frac{\ell}{r(\chi)}, z(\chi) \right). \]
where $P(k)$ represents the three-dimensional matter PS, $\chi_c$ is the comoving distance to source galaxies, and $W(\chi)$ is the lensing weight function defined as

$$W(\chi) = \frac{3}{2} \left\langle H_0^2 \right\rangle c^2 \Omega_m \frac{r(\chi_c - \chi)r(\chi)}{r(\chi_c)}(1 + z(\chi)).$$

We follow Sato et al. (2009) to estimate the convergence PS from numerical simulations. We measure the binned PS of the convergence field by averaging the product of Fourier modes $|\tilde{\kappa}(\ell)|^2$. We use 20 bins logarithmically spaced in the range of $\ell = 100$–$10^5$. For parameter estimation performed in Section 4, we recompute PS using 10 bins logarithmically spaced in the range of $\ell = 100$–$2000$.

2.1.2. Peak Count

Peaks in convergence maps can be a probe of massive halos (Hamana et al. 2004, 2012) and thus contain cosmological information (Yang et al. 2011).

In practice, peaks on convergence maps are defined by local maxima on the “smoothed” map. We do the map-smoothing because the observed lensing field is significantly contaminated by the intrinsic ellipticities of source galaxies. The contaminant is called shape noise, which is indeed the major contribution to the measured shape of source galaxies. In practice, we use a Gaussian window function in order to reduce the effect of shape noises on WL statistics. The smoothed convergence $\mathbf{K}(\theta; \theta_G)$ can be written as convolution with a filter function of $W_G$:

$$\mathbf{K}(\theta; \theta_G) = \int d^2\phi \ W_G(\theta - \phi; \theta_G)\kappa(\phi),$$

where $\theta_G$ is the smoothing scale and $W_G$ is a Gaussian filter given by

$$W_G(\theta; \theta_G) = \frac{1}{\pi\theta_G^2} \exp\left\{ -\left[ \theta^2_1 + \theta^2_2 \right] / \theta_G^2 \right\}.$$  

One can evaluate the smoothed convergence due to an isolated massive cluster at a given redshift by assuming the universal matter density profile of dark matter halos (e.g., Navarro et al. 1997b). A simple theoretical framework to predict the number density of the $K$ peaks is presented by Hamana et al. (2004) and Maturi et al. (2011). Their calculation yields reasonable results when the signal-to-noise ratio (S/N) of $K$ due to massive halos is larger than ~4 (see Hamana et al. 2004 for details). Fan et al. (2010) also consider more detailed calculation by including the statistical properties of shape noise and the impact of shape noise on peak position.

In order to locate peaks on a discretized map obtained from numerical simulations, we define the peak as a pixel that is higher than eight neighboring pixels. We then measure the number density of peaks as a function of $K$. We exclude the region within $2\theta_G$ from the boundary of the map in order to avoid the effect of incomplete smoothing. In this paper, we divide peaks into two subgroups: “medium peaks (MPs)” and “high peaks (HPs).” We define MPs and HPs by the peak height in a similar manner to Yang et al. (2013). The former is defined by the convergence peak with $1.0 \leq K_{\text{peak}}/\sigma_{\text{noise}} < 3.0$, whereas the latter corresponds to the peak with $3.0 \leq K_{\text{peak}}/\sigma_{\text{noise}} \leq 5.0$. Here $\sigma_{\text{noise}}$ is the rms of shape noise on the smoothed map given by Equation (23). It is important to note that these peaks are thought to have different physical origins. MPs are likely caused by the shape noise and/or several dark matter halos aligned along a line of sight (Yang et al. 2011), whereas HPs are associated with individual massive dark matter halos (e.g., Hamana et al. 2004).

Throughout the paper, we set the number of bins to be 10 for parameter estimation when measuring the peak count of each subgroup.

2.1.3. Minkowski Functionals

MFs are morphological descriptors for smoothed random fields. There are three kinds of MFs for two-dimensional maps. Each MF of $V_0$, $V_1$, and $V_2$ represents the area above the threshold $\nu$, the total boundary length, and the integral of geodesic curvature along the contours. They are given by

$$V_0(\nu) \equiv \frac{1}{A} \int_{Q_0,} da,$$

$$V_1(\nu) \equiv \frac{1}{A} \int_{\theta Q_0} d\ell,$$

$$V_2(\nu) \equiv \frac{1}{A} \int_{\theta Q_0} \frac{1}{2\pi} K d\ell,$$

where $K$ is the geodesic curvature of the contours, $da$ and $d\ell$ represent the area and length elements, and $A$ is the total area. $Q_0$ and $\theta Q_0$ are denoted to be excursion sets and boundary sets for the smoothed field $K(x)$, respectively. They are defined by

$$Q_\nu = \{ x | K(x) > \nu \},$$

$$\theta Q_\nu = \{ x | K(x) = \nu \}.$$  

In particular, $V_2$ is equivalent to a kind of genus statistics and equal to the number of connected regions above the threshold, minus ones below the threshold. Therefore, for high thresholds, $V_2$ is essentially equivalent to the number of peaks.

For a two-dimensional Gaussian random field, the expectation values for MFs can be described by analytic functions as shown in Tomita (1986):

$$V_0(\nu) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{\nu - \mu}{\sigma_0} \right) \right],$$

$$V_1(\nu) = \frac{1}{8\sqrt{2}} \frac{\sigma_1}{\sigma_0} \exp \left( -\frac{(\nu - \mu)^2}{\sigma_0^2} \right),$$

$$V_2(\nu) = \frac{\nu - \mu}{2\pi^2} \frac{\sigma_2}{\sigma_0^2} \exp \left( -\frac{(\nu - \mu)^2}{\sigma_0^2} \right).$$

where $\mu = \langle K \rangle$, $\sigma_0^2 = \langle K^2 \rangle - \mu^2$, and $\sigma_1^2 = \langle |\nabla K|^2 \rangle$. Although MFs can be evaluated perturbatively if the non-Gaussianity of the field is weak (Matsubara 2003, 2010), it is difficult to evaluate MFs of a highly non-Gaussian field (Petri et al. 2013). In this paper, we pay special attention to the non-Gaussian cosmological information obtained from convergence MFs.

For discretized $K$ maps, we employ the following estimators, as shown in, e.g., Kratochvil et al. (2012):

$$V_0(\nu) = \frac{1}{A} \int \Theta(K - \nu) dxdy,$$  

$$V_1(\nu) = \frac{1}{A} \int \partial_\nu \Theta(K - \nu) dxdy.$$
is the Heaviside step function and $0.002 \text{ Mpc}^1$ is a derived parameter. The Dirac $\delta$ function is generated by the linear Boltzmann code MUSIC, which is based on the second-order Lagrangian perturbation theory & Abel2011

Table 1

| Run   | $w$ | $10^3 A_s$ | $\Omega_m$ | $\Omega_a$ | $\sigma_8$ | No. of Sim. | No. of Maps | Explanation               |
|-------|-----|------------|------------|------------|------------|-------------|-------------|---------------------------|
| DM    | -1.0| 2.41       | 0.279      | 0.721      | 0.821      | 10          | 100         | CDM only fiducial model   |
| BA    | -1.0| 2.41       | 0.279      | 0.721      | 0.821      | 10          | 100         | CDM and adiabatic gas     |
| FE    | -1.0| 2.41       | 0.279      | 0.721      | 0.821      | 10          | 100         | CDM and baryonic processes|
| High $\Omega_m$ | -1.0 | 2.41 | 0.302 | 0.698 | 0.872 | 10 | 100 | 1σ higher $\Omega_m$ model |
| Low $\Omega_m$ | -1.0 | 2.41 | 0.256 | 0.744 | 0.767 | 10 | 100 | 1σ lower $\Omega_m$ model |
| High $w$ | -0.8 | 2.41 | 0.279 | 0.721 | 0.766 | 10 | 100 | Higher $w$ model |
| Low $w$ | -1.2 | 2.41 | 0.279 | 0.721 | 0.860 | 10 | 100 | Lower $w$ model |
| High $A_s$ | -1.0 | 2.51 | 0.279 | 0.721 | 0.838 | 10 | 100 | 1σ higher $A_s$ model |
| Low $A_s$ | -1.0 | 2.31 | 0.279 | 0.721 | 0.804 | 10 | 100 | 1σ lower $A_s$ model |

Note. Two parameters ($\Omega_m, 10^3 A_s$) are varied by the 1σ value of the WMAP 9 yr result (Hinshaw et al. 2013), and we also adjust $\Omega_a$ accordingly to keep the universe spatially flat. We also show the normalization of linear matter PS $\sigma_8$, which is a derived parameter.

\[ V_1(\nu) = \frac{1}{4A} \int \delta(\mathcal{K} - \nu) \sqrt{\mathcal{K}^2 + \mathcal{K}_x^2} dxdy, \]  
\[ V_2(\nu) = \frac{1}{2\pi A} \int \delta(\mathcal{K} - \nu) \times \frac{2\mathcal{K}_x \mathcal{K}_y \mathcal{K}_x y - \mathcal{K}^2 \mathcal{K}_x y - \mathcal{K}^2 \mathcal{K}_x y}{\mathcal{K}^2 + \mathcal{K}_x^2} dxdy, \]

where $\Theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The subscripts represent differentiation with respect to $x$ or $y$. The first and second differentiations are evaluated with finite difference. We precompute MFs for 100 equally spaced bins of $\nu' = (\nu - \langle \mathcal{K} \rangle)/\sigma_0$ between $-10$ and 10. For the cosmological parameter estimation, we recalculate values on 10 equally spaced bins in the range $-3.0 \leq \nu' \leq 3.0$ from 100 bins.

3. SIMULATION

3.1. N-body Simulations

We are interested in nonlinear gravitational evolution of large-scale structure. In order to follow the evolution accurately, we run cosmological N-body simulations. We use parallelized tree-PM code Gadget-3 (Springel 2005) with baryonic processes (discussed below) to follow structure formation from an early epoch ($z = 99$) to present ($z = 0$). The initial conditions are generated by the MUSIC code (Hahn & Abel 2011), which is based on the second-order Lagrangian perturbation theory (e.g., Crocce et al. 2006). The transfer function is generated by the linear Boltzmann code CAMB (Lewis et al. 2000). The volume of each simulation box is comoving 240 Mpc/h on a side. We adopt the fiducial cosmological parameters as follows: matter density $\Omega_m = 0.279$, baryon density $\Omega_b = 0.0463$, dark energy density $\Omega\Lambda = 0.721$, Hubble parameter $h = 0.70$, spectral index $n_s = 0.972$, and amplitude of scalar perturbation $A_s = 2.41 \times 10^{-9}$ at the pivot scale $k = 0.002$ Mpc$^{-1}$. These parameters are consistent with the 9 yr Wilkinson Microwave Anisotropy Probe (WMAP) result (Hinshaw et al. 2013).

In this paper we perform three kinds of cosmological simulations: cold dark matter (CDM) simulation (denoted as “DM”), and two baryonic simulations. In order to model the degeneracy between cosmological parameters in WL statistics, we run six CDM-only simulations with one cosmological parameter varied. Cosmological parameters for our models are summarized in Table 1. The number of particles is set to be $512^3$ for CDM-only simulations and $2 \times 512^3$ for baryonic simulations, which consist of both CDM and gas particles. For the fiducial cosmological model, the mass of a particle is found to be $m_p = 7.97 \times 10^9 M_{\odot}/h$, and $m_{\text{cdm}} = 6.65 \times 10^9 M_{\odot}/h$, $m_{\text{gas}} = 1.32 \times 10^9 M_{\odot}/h$ for baryonic simulations. Our baryonic simulations are based on two models denoted as BA and FE. The BA model contains adiabatic gas particles, which exert only adiabatic pressure in a smoothed particle hydrodynamics manner. In the FE model, we employ the galaxy formation “recipes” of Okamoto et al. (2014). Our FE model corresponds to the SN+AGN model in their paper with some modifications, which include star formation, radiative cooling, supernova (SN) feedback, stellar wind feedback, and an ad hoc active AGN feedback. This prescription saves computation time and contains less free parameters than solving fully the evolution of black holes. In their model, the radiative cooling rate is exponentially suppressed when the velocity dispersion within a dark matter halo exceeds some threshold. Our simulations employ a simpler method that switches off radiative cooling when the local velocity dispersion reaches the threshold. Okamoto et al. (2014) report that the sudden change of the cooling function leads to a somewhat artificial increase of the stellar mass, but we expect that this modification does not affect the final results significantly.

In order to check basic statistics of our simulations, we measure the halo mass function for each model and the stellar mass function for the FE model. We run a friends-of-friends (FoF) halo finder SubFind (Springel et al. 2001; Dolag et al. 2009) to identify halos in simulations. For our simulations with baryons, the procedure is different from that for CDM-only simulations. First, using only dark matter particles, we find FoF groups that contain more than 31 particles. Next, we find gas particles for the BA model and gas and star particles for the FE model linked with a dark matter particle that belongs to an FoF group. Finally, we separate each FoF group into a central halo and substructures. In the FE model, the stellar component of a halo is called “galaxy,” and “halo” includes the baryonic components and dark matter. Figures 1 and 2 represent halo and stellar mass functions measured from simulations at $z = 0$. We compare the simulation result with the theoretical prediction of Sheth & Tormen (1999). Figure 1 clearly shows that our simulation results are well described by the theoretical prediction over the typical cluster mass scale, while there are appreciable discrepancies between DM and FE simulations in the halo mass of less than $\sim 10^{13.5} M_{\odot}$. This is
partly induced by SN explosions and/or stellar winds. Energy released by SN explosions or stellar winds expels gas particles from the halo, but massive halos can retain the particles by the deep gravitational potential well. As a result, the mass function for our FE model is slightly smaller at small masses. We also compare the stellar mass functions of our simulation results and the observation of the galaxy stellar mass function in Baldry et al. (2012). The stellar mass distribution in our FE model is in reasonable agreement with the observation at $z \sim 0$. Because we adopt a simpler model than the original implementation of Okamoto et al. (2014) that includes radiation pressure feedback, our FE run produces fewer small-mass galaxies. Also, our simulation results are inaccurate at stellar masses less than $\sim 10^{10} M_\odot$, because such small galaxies contain only a few star particles. These small discrepancies are, however, unimportant in the present paper because we draw our main conclusions through comparisons of multiple sets of simulations with and without baryonic components, rather than through comparisons of different feedback models.

3.2. Ray-tracing Simulations

For ray-tracing simulations of gravitational lensing, we utilize multiple simulation boxes to generate light-cone outputs similarly to White & Hu (2000), Hamana & Mellier (2001), and Sato et al. (2009). Details of the configuration are found in the last reference.

We place the simulation outputs to fill the past light-cone of a hypothetical observer with an angular extent $5^\circ \times 5^\circ$, from $z = 0$ to 1. The angular grid size of our maps is set to be $5^\circ/4096 \sim 0.073$ arcmin. We randomly rotate and shift the simulation boxes in order to avoid the same structure appearing multiple times along a line of sight. In total, we generate 100 independent lensing maps for the source redshift of $z_{\text{source}} = 1$. We show an example of convergence maps obtained from each baryonic model (DM, BA, and FE) in Figure 3. Note that each realization of three models uses the same random seed when we generate initial conditions and multiple planes. In order to make the mock lensing maps more realistic, we add random Gaussian noises as shape noise to the simulated convergence data (e.g., Kratochvil et al. 2010 and Shirasaki et al. 2012):

$$\kappa_{\text{noise}}(x_1, x_2) \approx \sigma_{\text{noise}}(x_1, x_2') = \frac{\sigma_{\text{noise}}^2}{n_{\text{gal}}A_{\text{pix}}} \delta_{x_1, x_1'} \delta_{x_2, x_2'},$$

where $\delta_{x,y}$ is the Kronecker delta symbol, $n_{\text{gal}}$ is the number density of source galaxies, $\sigma_{\text{noise}}$ is the rms of shape noise, and $A_{\text{pix}}$ is the solid angle of a pixel. In the following, we adopt $\sigma_{\text{noise}} = 0.4$, $n_{\text{gal}} = 30$ arcmin$^{-2}$. These values are expected to be typical for the HSC survey.

As described in Sections 2.1.2 and 2.1.3, smoothing is required to measure nonlocal statistics of convergence from noisy WL data. For a given smoothing scale $\theta_G$, the rms of the shape noise after Gaussian smoothing is given by (van Waerbeke 2000)

$$\sigma_{\text{noise}} = 0.0291 \left( \frac{\sigma_{\text{noise}}}{0.4} \right) \left( \frac{\theta_G}{1 \text{arcmin}} \right)^{-1} \left( \frac{n_{\text{gal}}}{30 \text{arcmin}^{-2}} \right)^{-1/2}.$$

4. ANALYSIS

We quantify the effects of the baryonic processes on WL analyses in terms of errors and bias in cosmological parameter estimation. We consider primarily a lensing survey with a sky coverage of 1400 deg$^2$, i.e., the ongoing wide-field survey by HSC. In the following, we describe in detail the calculation of the ensemble averages of three statistics and the covariance matrix in order to derive statistical implications and to estimate cosmological parameters $p = (\Omega_m, w, 10^9 A_s)$.

4.1. Theoretical Model and Covariance of Lensing Statistics

First, we calculate the theoretical template and covariance of the WL statistics using our ray-tracing simulations. The configuration of bins is given in Table 2. The model template and covariance are based on our DM models. Later, we examine the baryonic effects on WL cosmological analysis by comparison with the DM model and two baryonic models.

We represent the data vector as $\mathbf{D} = \{D_1, D_2, \ldots, D_N\}$, the lensing weight function $W(x)$ has a maximum at $z \sim 0.5$. This means that our lensing statistics are not sensitive to the large-scale structure above $z = 0.7$--0.8.
realizations:

\[
\langle N_i(p) \rangle \approx \bar{N}_i(p) \equiv \frac{1}{R} \sum_{r=1}^{R} N_i(r, p). \tag{24}
\]

In order to consider the cosmological parameter dependence on \(N_i\), we employ a linear interpolation based on DM models with seven different cosmologies. Thus, we obtain the theoretical model of \(N_i\) for a given cosmological model as follows:

\[
\bar{N}_i(p) = \bar{N}_i(p^0) + \sum_{\alpha} \left( \bar{N}_i(p^{0+\alpha}_\alpha) - \bar{N}_i(p^{0-\alpha}_\alpha) \right) \left( p_{\alpha} - p^{0}_{\alpha} \right), \tag{25}
\]

where \(\alpha\) runs 1–3 and the superscript 0 means fiducial value \(p^0 = (0.279, -1.0, 2.41)\). \(p^{\pm\alpha}\) represents a vector with one parameter with a higher or lower value; for example, \(p^{1+} = (0.302, -1.0, 2.41)\) (the other parameter values are given in Table 1).

The covariance matrix of the data vector \(N_i\) on 25 deg\(^2\) maps is estimated as

\[
C_{ij}(p) = \frac{1}{R - 1} \sum_{r=1}^{R} \left[ N_i(r, p) - \bar{N}_i(p) \right] \left[ N_j(r, p) - \bar{N}_j(p) \right]. \tag{26}
\]

We ignore the cosmological dependence of \(C(p)\) (see, e.g., Eifler et al. 2009) and hence evaluate the covariance matrix by using the fiducial model, i.e., \(C(p^0)\). For the fiducial HSC survey, we simply scale the covariance matrix by survey area, by multiplying the covariance matrix \(C_{ij}(p^0)\) in Equation (26) by a factor of 25/1400. When calculating the inverse covariance, we include a debiasing correction, the so-called Anderson–Hartlap factor \(\alpha = (R - n - 2)/(R - 1)\) (Hartlap et al. 2007), where \(R\) is the number of realizations and \(n\) is the dimension of the data vector.

4.2. Fisher Analysis

Fisher analysis gives a simple forecast for the statistical confidence level of three cosmological parameters \((\Omega_m, w, 10^n A_s)\) with WL statistics. The Fisher matrix is given by

\[
F_{ij} = \frac{1}{2} Tr \left[ A_i A_j + C^{-1} M_{ij} \right], \tag{27}
\]

where \(A_i = C^{-1} \partial C / \partial p_i\) and \(M_{ij} = 2(\partial N / \partial p_i)(\partial N / \partial p_j)^T\). The first term vanishes when the cosmological dependence is weak (Eifler et al. 2009). To compute the second term, the first derivative is evaluated by the first-order finite difference, which is given by

\[
\frac{\partial N}{\partial p_i} \simeq \frac{N(p^{i+}_i) - N(p^{i-}_i)}{p^{i+}_i - p^{i-}_i}. \tag{28}
\]

Then, the marginalized error over the other two parameters is given by

\[
\sigma_i = \sqrt{(F^{-1})_{ii}}. \tag{29}
\]

4.3. Mock Lensing Surveys

In order to make a realistic forecast in the upcoming HSC survey, we employ the bootstrap method as in Yang et al. (2013). Since our suite of WL maps consist of 100 25 deg\(^2\) maps, we randomly choose 1400/25 = 56 realizations from 100 25 deg\(^2\) WL maps. Repeating this procedure 1000 times, we can get 1000 “mock” HSC WL maps. The resulting maps are based on the fiducial cosmological model, but we also have the same set from simulations with the different baryonic effects. Lensing statistics of interest (i.e., PS, peak counts, and MFs) in HSC surveys are evaluated by averaging each statistic on a 25 deg\(^2\) map over 56 realizations.

We then utilize the HSC observables derived in this way to investigate the impact of baryonic effects on parameter...
estimation. We perform $\chi^2$ minimization to $N_{i,\text{HSC}}$ as follows:

$$\chi^2(r, p, m) = \sum_{i,j} \Delta N_i \left( C^{-1}\right)_{ij} \Delta N_j,$$

(30)

where $\Delta N_i \equiv N_{i,\text{HSC}}(r) - \tilde{N}_i(p)$, $r$ is the index of realizations and $1 \leq r \leq 1000$, $\tilde{N}_i$ represents the theoretical model of $N_i$ for our dark-matter-only (DM) model, and $m$ represents the difference of models of baryonic physics, i.e., BA and FE. Suppose we calculate the $i$th data $N_{i,\text{HSC}}$ from a dark-matter-only convergence map ($m = \text{DM}$); we should then find that the resulting best-fit points distribute around the fiducial point $p^0 = (0.279, -1.0, 2.41)$. However, when one considers the case of $m = \text{BA}$ or FE, the center of the distribution of best-fit values could be biased in the parameter space of $p$ if baryonic effects induce discrepancies between $N_{i,\text{HSC}}$ and $\tilde{N}_i(p^0)$. That means that $\chi^2$ values calculated from Equation (30) do not follow $\chi^2$ distribution for baryonic models because the ensemble mean and the covariance matrix are computed from the fiducial model, not baryonic models.

There is another way to estimate the bias of parameter estimation. The parameter biases of the BA and FE models from fiducial parameters can be computed in the following manner (see, e.g., Huterer et al. 2006):

$$\delta p_{\alpha} = \sum_{\beta} F_{\alpha\beta}^{-1} \sum_{i,j} \left[ N_i^m - \tilde{N}_i(p^0) \right] \left( C^{-1}\right)_{ij} \frac{\partial \tilde{N}_i}{\partial p_{\beta}},$$

(31)

where $F$ is the Fisher matrix given by Equation (27) and $N_i^m$ represents the average of $N_i$ over 100 convergence maps for $m = \text{BA}$ and FE.

In the following, we quantify the baryonic effects on parameter estimation with WL surveys by considering the distribution of $\chi^2$ in Equation (30) and $\delta p_{\alpha}$ in Equation (31) for our BA and FE models. We then compare the bias of parameter estimations with the marginalized error given in Equation (29).

5. RESULTS

5.1. Baryonic Effect on Convergence Statistics

We summarize the main results of the baryonic effects on WL statistics in Figures 4–6.

Figure 4 show the PS from our maps, where DM, BA, and FE models are indicated by blue, green, and red lines, respectively. We also show the theoretical prediction by the cyan line, which is calculated by Equation (7) with the modeling of matter PS, essentially a modified HaloFit, of Takahashi et al. (2012). We plot the shot noise contribution in the right panel as the dashed magenta line. The error bars represent the standard deviations over 100 maps. The left and right panels represent results from the maps with and without noise, respectively. The lower portion in each panel shows the fractional differences of BA and FE models with respect to the DM model.

5.1.1. Power Spectrum

Compared with the DM model, gas pressure suppresses small-scale ($\ell > 10,000$) structures in the BA model. On the other hand, in the FE model feedback processes are efficient at the scales of $2000 < \ell < 10,000$, but cooling processes enhance the convergence power at the smaller scale $\ell > 20,000$. These features by cooling and feedback processes are consistently found by Semboloni et al. (2011) and Mohammed et al. (2014). However, the strength of the baryonic enhancement and suppression is different, because of the details of baryonic physics implementations and also partially because of the difference of cosmological parameters.

5.1.2. Peak Count

Figure 5 shows the peak counts measured from the simulated convergence maps for the three models. We use the same combination of line colors as in Figure 4. The error bars represent standard deviations from 100 maps. The lower portion in each panel shows the fractional difference of BA and FE models with respect to the DM model, and the shaded region indicates the Poisson error of the fiducial model, i.e., the square root of the number of peaks within a bin. In this figure, we employ 5 bins per unit $S/N$ $K_{\text{peak}}/\sigma_{\text{noise}}$ in the peak height of convergence $K_{\text{peak}}$ divided by $\sigma_{\text{noise}}$. For the analytical prediction, we adopt the model of Hamana et al. (2004) with the Sheth–Tormen (Sheth & Tormen 1999) mass function and the Navarro–Frenk–White (NFW) dark matter density profile (Navarro et al. 1996, 1997a). We assume the relation between
Figure 5. Number of peaks per unit square degree. The left and right panels show the result without and with noise, respectively. The solid cyan line shows the model prediction of peaks by Hamana et al. (2004), which assumes that the halo mass function is described by Sheth & Tormen (1999) and the density profile of halos follows the Navarro–Frenk–White profile (Navarro et al. 1996, 1997a). The lower portion in each panel shows the fractional difference from the fiducial model. The blue filled region represents the expected Poisson error of the DM model from 100 1400 deg$^2$ maps.

Figure 6. Three MFs averaged over 100 convergence maps. Panels in the first and second rows show the results without noise, whereas the third and fourth rows are for those with noise. The error bars show the standard deviation of 100 maps. Note that the range of threshold $\nu$ is different between the noise-free and noisy cases. The blue filled region corresponds to the expected statistical error of the DM model from 100 1400 deg$^2$ maps.
the halo mass and concentration parameter as in Hamana et al. (2012),
\[
c(M, z) = 7.26 \left( \frac{M}{10^{12} M_\odot / h} \right)^{-0.086} (1 + z)^{-0.71}.
\] (32)

In Figure 5, MPs correspond to the peaks with \(0.03 \lesssim \kappa_{\text{peak}} \lesssim 0.09\), while HPs are those with \(0.09 \lesssim \kappa_{\text{peak}} \lesssim 0.15\). HPs are typically associated with massive halos with the mass of \(\sim 10^{13} M_\odot\). The number of HPs is less affected by baryonic effects because the baryonic processes do not cause very strong effects to change the number of such massive halos, as seen in Figure 1. MPs often originate from multiple halos with masses of \(10^{12} - 10^{13} M_\odot\) aligned in the line-of-sight direction. Baryonic physics, such as SN and AGN feedback, reduce the mass of these halos (see Section 2.1.2). Hence, the baryonic effects would decrease the number of MPs, but the difference is largely made unimportant in the noisy convergence maps because MPs are affected significantly by shape noise (Yang et al. 2011).

5.1.3. Minkowski Functionals

Figure 6 show the MFs computed from our maps of DM, BA, and FE models. Again we use the same color for each model as in the previous figures. The MFs \(V_0\), \(V_1\), and \(V_2\) are plotted from left to right. The upper (lower) two rows represent results from noise-free (noisy) maps. Panels in the second and fourth row show the difference of BA and FE models from the DM model. The error bars represent standard deviations from 100 maps. We employ 5 bins per unit \(\nu = (\nu - \langle \kappa \rangle) / \sigma_0\). It is useful to calculate the expectation values for a Gaussian random field (Equations (16)–(18)) in order to understand the overall feature of MFs. In Equations (16)–(18), the spectral moments \(\sigma_0\) and \(\sigma_1\) can be calculated from the convergence PS as
\[
\sigma_p^2 = \int \frac{d^2 \ell}{(2\pi)^2} \ell^2 \rho_0(\ell) \left| \tilde{W}_G(\ell; \theta_G) \right|^2,
\] (33)

where \(\tilde{W}_G\) represents the Fourier transform of Equation (10), which is given by \(\exp(-\ell^2 \theta_G^2/4)\). Note that \(\tilde{W}_G\) decreases exponentially at \(\ell \gtrsim 20,000 \times (1 \text{ arcmin} / \theta_G)\). From the result shown in Figure 4, we expect that \(\sigma_0\) of BA and FE models is smaller than that of the DM model, but \(\sigma_1\) would be nearly the same. We confirm this notion by direct measurement of \(\sigma_0\) and \(\sigma_1\) from our 100 maps. It is important to test whether the differences between the models are attributed to \(\sigma_0\). In the above analysis, we measured MFs in terms of normalized threshold \(\nu'\) because, if the difference is largely owing to variation of \(\sigma_0\), the normalization would absorb all or most of the differences. The difference in the MFs becomes indeed slightly smaller with the normalization, but does not vanish completely. Therefore, the baryonic processes apparently affect the MFs. It is necessary to develop further analysis beyond the Gaussian description to explain the difference of MFs between baryonic and fiducial models.

### Table 3

| Data Statistics     | \(\Delta \Omega_m\) | \(\delta_{\text{H0}}\) | \(\delta^{10\%}_{\Lambda}\) |
|---------------------|---------------------|------------------------|-----------------------------|
| PS                  | 0.0275              | 0.276                  | 1.01                        |
| MP                  | 0.00853             | 0.110                  | 0.0557                      |
| HP                  | 0.0129              | 0.118                  | 0.0971                      |
| \(V_0\)             | 0.00678             | 0.0711                 | 0.0831                      |
| \(V_1\)             | 0.00767             | 0.0911                 | 0.0931                      |
| \(V_2\)             | 0.00912             | 0.115                  | 0.0718                      |
| \(V_0 + V_1 + V_2\) | 0.00439             | 0.0513                 | 0.0521                      |
| PS+MP+HP             | 0.00478             | 0.0520                 | 0.0371                      |
| PS+MP+V_0            | 0.00372             | 0.0391                 | 0.0412                      |
| PS+HP+V_0            | 0.00547             | 0.0456                 | 0.0662                      |

| Parameter Bias of BA |                      |                        |                             |
|----------------------|----------------------|------------------------|-----------------------------|
| PS                  | -0.00171             | -0.0172                | 0.0139                      |
| MP                  | -0.00591             | -0.0616                | 0.00536                     |
| HP                  | -0.00668             | -0.0610                | 0.0347                      |
| \(V_0\)             | -0.000951            | -0.00112               | 0.0543                      |
| \(V_1\)             | -0.000559            | 0.00157                | 0.120                       |
| \(V_2\)             | -0.00182             | 0.0135                 | 0.0342                      |
| \(V_0 + V_1 + V_2\) | -0.000412            | 0.0347                 | 0.0481                      |
| PS+MP+HP             | -0.00594             | -0.0587                | 0.0156                      |
| PS+MP+V_0            | -0.00288             | -0.00899               | 0.0544                      |
| PS+HP+V_0            | 0.000589             | 0.0136                 | 0.0241                      |

Note. The marginalized errors of three parameters based on the Fisher forecast of the DM model and the biases of BA and FE models compared with the DM model. Note that these values are not normalized by the fiducial errors.

5.2. Fisher Forecast and Bias of Cosmological Parameters

We show the results of our parameter estimation in Figures 9–11. In each figure, we show two-dimensional error contours marginalized over the other parameter. The blue, green, and red dots indicate the best-fit values for DM, BA, and FE models, respectively. Red dashed lines represent the fiducial values. Note that in these three figures, the plotted parameter range is adjusted according to the size of the forecast circle for the observable. In order to make a comparison between the figures easy, we show the 1σ confidence regions of six observables in the same scale in Figure 8. The biases and marginalized errors of parameters are summarized in Table 3 and Figure 7. In Figure 7, green and red points show the biases divided by the marginalized errors for BA and FE models, respectively. To check the accuracy of our code, we also present results from the fiducial simulation. We plot results obtained from \(\kappa\) maps with shape noise.

In the following, we summarize the baryonic effects on parameter estimation with different convergence statistics.
We compare the 1σ confidence regions calculated from the Fisher forecast using six observables separately. The confidence region from PS (green line) is clearly the largest. The other nonlocal statistics yield substantially smaller confidence regions.

5.2.1. Power Spectrum

The top three panels in Figure 9 show the results of parameter estimation for three (DM, BA, FE) models using only PS. We find only a small bias caused by the baryonic physics. That is because, with realistic ground-based surveys, PS can be measured accurately at a low ℓ of about 2000 at most, which is the maximum multipole used in our estimation. The difference between the fiducial and baryonic models appears at the small scale. The PS amplitude and the shape at ℓ ≲ 2000 are not significantly affected by the baryonic physics. Previous studies (Yang et al. 2013; Mohammed et al. 2014) also examined differences due to the choice of the maximum multipole. In our result, the difference at large scale is quite small, whereas noise dominates at smaller angular scales, where PS cannot be measured accurately. For this reason, we fix the maximum multipole at ℓ = 2000. Then the baryon effects on PS are negligible in cosmological parameter estimation using PS.

5.2.2. Peak Count

The middle (bottom) three panels in Figure 9 show the results using only MPs (HPs). As we have discussed above, medium height peaks are also dominated by intrinsic shape noise but still have cosmological information. HPs have a nearly one-to-one relation with massive halos, whose masses are less affected by the baryonic processes. Interestingly, we find that both MPs and HPs induce biased parameter estimation of w with a level of ~0.56σ and 0.52σ, respectively, for the FE model. The bias may partly originate from the baryonic effect on the shape of massive halos. Massive halos tend to be rounder when baryonic processes are included (e.g., Kazantzidis et al. 2004). Then the height of a peak associated with a single halo is reduced, and so may be the peak counts at K ~ 0.0−0.05.

5.2.3. Minkowski Functionals

Figure 10 shows the results using each of the three MFs, V₀, V₁, V₂. The center of the dots is shifted in the respective parameter space, significant parameter bias can be caused by the baryonic effects. Note that the absolute shift itself is not very large, but that the bias with respect to the error circle is appreciable (see Figure 8 for the relative size of the error circles). When one constructs the theoretical template of MFs without the modeling of baryonic physics, analyses using V₀, V₁, and V₂ cause the biased estimation of w with a level of ~0.016σ, 0.017σ, and 0.12σ, respectively, for the FE model. Although it is difficult to explain the origin of this bias completely, we expect that the following two effects can be responsible for the biased parameter estimation: (1) the change of variance of K (i.e., ⟨σ⟩₀) and (2) the change in halo shape. More detailed modeling with an analytic halo approach would be useful to investigate the relation between the property of dark matter halos and lensing MFs. This is along the line of our ongoing study using a large set of cosmological simulations.

5.2.4. Combined Analysis

Let us consider a combined analysis with multiple observables in order to tighten the error and possibly mitigate the bias due to baryonic effects.

Our combination of the observables are of the following four types: (1) all the MFs; (2) PS and peak counts; (3) PS, MPs, and V₀; and (4) PS, HPs, and V₀. The last two are examples of less biased combinations of the statistics. We propose the two combinations that are expected to cause a small net bias in each parameter on the basis of the result of Equation (31). The basic idea is to find a combination of observables with biases in the opposite directions in parameter space.

Figure 11 shows the results of the above-combined analysis. All of the combined analyses presented here can tighten forecast errors, i.e., they can effectively extract more cosmological information. Some combinations, e.g., PS and peak counts, are largely affected by biases by baryonic effects. This degrades parameter constraints of combined analysis. However, if we adopt, e.g., PS+HPs+V₀, such a combination can mitigate the bias by baryonic effects. When all the statistics
cause biases with the same sign (e.g., for $10^3 A_s$), it is safe to combine those with small biases. On the other hand, there is still a possibility of causing large biases by combined analysis, if one includes statistic(s) whose bias is very large. In general, cosmological parameters are degenerate with each other, and thus it is not trivial to determine the best combination for multiple parameters. The above case focusing on a single parameter may serve as a useful guide for combined analysis when parameter degeneracy is not strong. We further discuss this with a simple example with details in multidimensional parameter space of the formalism in the Appendix.

6. CONCLUSION AND DISCUSSION

We have studied baryonic effects on WL statistics using a suite of cosmological simulations that incorporate galaxy formation processes. Various baryonic processes are implemented in our code, such as gas cooling, star formation, and stellar feedback. These processes themselves are important subjects of research, and the model uncertainties with free parameters are still controversial, but such simulations can be used to quantify the baryonic effect, or at least to compare the WL statistics with those calculated from dark-matter-only simulations. We focus on cosmological parameter estimation.
using WL statistics. To this end, we made realistic mock observations by performing ray-tracing simulations through the nonlinear density fields with the size of the survey region being over 1000 square degrees.

We have studied three statistics, PS, peak counts, and MFs, calculated directly from mock lensing maps with baryons, and compared them with the results from our “fiducial” dark-matter-only simulations. The PS deviates appreciably at $\ell \gtrsim 7000$ from the fiducial model due to the baryonic processes. This feature is also seen in previous studies, e.g., Semboloni et al. (2011). The shape noise dominates, however, over the baryonic effects at the small angular scales, and thus the baryonic effects are not critical in practice in the analysis using PS. Peak counts in convergence maps are also affected because the height of a peak is sensitive to the mass and the density profile of the corresponding halo. Stellar feedback can effectively reduce the mass of small galaxy halos, which results in a decrease of the number of medium-height peaks. HPs are less affected by the stellar feedback effects because the mass distribution in massive, cluster-size halos is not significantly changed by the stellar feedback. Yang et al. (2013) examine the influence on peak counts of the enhancement of the halo concentration parameter, motivated by radiative cooling. They show that the number of HPs is increased accordingly, while the number of MPs is mostly unaffected. We do not see the feature in our HP counts, probably because our simulations include not only radiative cooling but also SN and stellar feedback that can compensate, at least partially, for the condensation of baryons. Finally, MFs are morphological statistics and thus are promising as a probe of the baryonic
Figure 11. Error contours and best-fit points for combined analysis.
effects that change the internal matter distribution of halos. The shape noise again substantially affects MFs owing to the effectively increased \( \kappa \)-variance. Also, the smoother distribution of the gaseous components causes the overall feature of the MF to be smoother, especially in our BA model (see Figure 6).

We have considered three kinds of statistics as probes of cosmological parameters. When we use only one of them, the parameter bias due to the baryonic effects is not significant. Most of the biases are found to be within the 1\( \sigma \) error for the fiducial 1400 deg\(^2\) survey. It is expected that, with the upcoming Subaru HSC survey, cosmological parameters can be determined without being significantly compromised by baryonic processes as long as a single statistic is used. However, because the statistical error itself becomes small, roughly in proportion to the increase of the square root of the survey area, even a very small bias would become critical for future surveys with a half- or an all-sky coverage. The overall bias can appear relatively amplified when a combination of a few or more statistics is used because the expected error of the parameters becomes small and the bias remains the same. For example, when using both PS and peak counts, the parameter bias for \( w \) is over 1\( \sigma \). Clearly, such a bias needs to be well understood before analyzing real observational data. It is also desirable to find a combination of statistics that yields both high precision and small bias. If we consider only one parameter, the best way would be to combine statistics such that their respective biases can cancel each other (see Figure 7). Unfortunately, it is generally nontrivial to estimate parameter bias in a multidimensional space because the degeneracy between multiple parameters leads to complicated dependence on parameters. We suggest to try all possible combinations and study in detail, as has been done in the present paper in the case with only a few statistical measures. Among the combinations we have tested, we find the combination of PS, one of peak counts, and \( V_0 \) gives high precision and yet robust results against the baryonic effects. Furthermore, all of the quantities can be predicted accurately by analytic models. Takahashi et al. (2012) studied nonlinear PS using the HaloFits approach. Hamana et al. (2004) employ a one-to-one relation that the NFW halo corresponds to a peak, but this approach is only valid at high-S/N (\( \geq 4 \)) peaks. For low-S/N peaks, this relation breaks down. Das & Ostriker (2006) studied the probability distribution function of convergence using fitting formulae motivated by the lognormal distribution. This probability distribution function is directly related to \( V_0 \). In recent studies, Fedeli (2014), Fedeli et al. (2014), and Mohammed et al. (2014) study baryonic effects on PS using a halo model. It would be interesting to study the link between other two statistics and properties of halos by extending the halo model approach. We leave it as a future work.

Our results suggest that the nonlocal statistics have a stronger parameter-constraining power than PS, even when baryonic effects are taken into account. However, one needs to be careful in using the nonlocal statistics. Theoretical frameworks to predict PS have been well developed through both numerical and analytic approaches. In realistic observations, the shear measurement itself is clean and local, especially when compared with the other nonlocal statistics.

### APPENDIX

#### COMBINED ANALYSIS IN ONE-DIMENSIONAL PARAMETER SPACE

We present the criterion for the choice of observables for a combined analysis in a simplified case. The parameter bias of a combined analysis is given by

\[
\delta \hat{p}_i = (\hat{F})^{-1}_{a\beta} \sum_{ij} (N_i - \hat{N}_i)(\hat{C})^{-1}_{ij} \frac{\partial N_j}{\partial p_\beta},
\]

(34)

where \( \hat{F} \) and \( \hat{C} \) are the Fisher matrix and the covariance matrix, respectively, in the case of a combined analysis with several observables.\(^{10}\) Assuming that each observable is independent, one can write \( \hat{p}_i \) as

\[
\delta \hat{p}_i = (\hat{F})^{-1}_{a\beta} \sum_{ij,s} (N_i^s - \hat{N}_i^s)(C^s)^{-1}_{ij} \frac{\partial N_j^s}{\partial p_\beta} = (\hat{F})^{-1}_{a\beta} \sum_s (F^s)_{a\beta} \delta p_i^s,
\]

(35)

where \( F^s \) and \( C^s \) are the Fisher matrix and the covariance matrix, respectively, for each observable \( s \), e.g., \( s = \text{PS} \).

For one-dimensional parameter space, we can simplify Fisher matrices,

\[
F^s \rightarrow \sigma_s^{-2}, \quad \hat{F} \rightarrow \sum_s \sigma_s^{-2},
\]

(36)

\(^{10}\) In this appendix, we always take the sums for subscripts of Greek letters that appear twice in the same equation.
where $\sigma$ is the forecast error with the observable $s$. Hence, the expected bias with a combined analysis is given by

$$\delta \hat{p} = \left( \sum_s \sigma^2_s \right)^{-1} \sum_s \delta \hat{p}_s \sigma_s.$$  \hspace{1cm} (37)

When the sum of $\delta \hat{p}_s / \sigma^2_s$ vanishes, the net bias $\delta \hat{p}$ also vanishes. Therefore, a combination that makes the sum small is a good probe with less bias. We show the bias divided by the square of the error for each observable in Figure 12. For multidimensional parameter space, this procedure is not generally applicable because the degeneracy between different parameters gives additional terms to Equation (37). But when the degeneracy is not strong and the bias is not large, this condition gives the effective criterion for a less biased combination of observables in terms of cosmological parameters.

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