On nuclear states of $\bar{K}$ mesons

S. Wycech
National Centre for Nuclear Studies, Warsaw, Poland
E-mail: wycech@fuw.edu.pl

Abstract. The search for deeply bound nuclear states of $\bar{K}$ mesons is presented and the main experimental and theoretical uncertainties are briefly discussed. The need for new experiments is stressed and some atomic experiments are suggested. The involved physics is illustrated on a simple fixed nucleon model which becomes a basis for variational calculations.

1. Introduction

The interest in the search for nuclear states of anti-\(K\) mesons stems from several sources:

- A new branch of nuclear physics emerges. It opens the way for interesting laboratory discoveries with some astrophysical consequences.
- The $\bar{K}$ meson may be deeply bound and create high density clusters inside nuclei.
- A new mechanism of nuclear attraction is met. It is generated by subsequent excitations of nucleons into strange baryons $\Lambda(1405)$ and $\Sigma(1385)$. It differs from the standard nuclear force mechanism that generates attraction by the meson exchanges.

This text presents the main problems. As yet, there are no definite solutions, experiments have not been fully confirmed and their interpretation is frequently questioned. However, this field is very active and it involves several laboratories in Japan, Europe and US. A recent review [1] is recommended.

It has been known since discoveries of $K$-mesic atoms that the $\bar{K}$ is attracted by nuclei. It was also realized that absorption from atomic states was so strong that if nuclear states can be formed their lifetimes would be very short and discovery rather improbable and uninteresting. Already at that time it was well understood that the basic $\bar{K}N$ interaction is strongly energy dependent and it may be different in the nuclear matter.

Later on the experimental possibilities were reduced as most of the low energy kaonic beam facilities were dismantled. As a consequence, the interest in the theories almost disappeared.

New life was given to this field with the calculations of Yamazaki and Akaishi predicting deeply bound states in light systems $\bar{K}NN$ and $\bar{K}NNN$. New experiments performed in Japan indicated positive results. These turned out to be over-optimistic but the impetus was given to further studies.

At this moment there are two experimental indications of a $\bar{K}NN$ state shown in Table 1. The structure of this state might be quite complicated but in the kaonic channel it is a mixture of $K^-pp$ and $K^0np$ commonly named $K^-pp$ which apparently constitutes the dominant component.
Table 1. Experimental measurements of the $K^-pp$ binding energies and half widths $E_B - i\Gamma/2$ [MeV]. FINUDA - [2] and DISTO - [3].

| FINUDA     | DISTO          |
|------------|----------------|
| $115(7) - i67(14)/2$ | $103(8)-i118(18)/2$ |

Discussion of these results and related questions are given below in three sections:

- Section (2) presents basic models of $K^-N$ interactions
- Section (3) characterizes K-mesic atomic and nuclear states
- Section (4) presents a variational model used to calculate the $K^-few$ nucleon levels.

2. $K^-N$ multiple channel system

The scenario for low energy $K^-N$ interaction is determined by a system of coupled channels. These channels and relevant thresholds are given in Table 2. Experiments provide elastic $K^-p \rightarrow K^-p$ and inelastic $K^-p \rightarrow K^0n$, $K^-p \rightarrow \Lambda, \pi, \Sigma, \pi$ scattering cross sections from the threshold up to some 250 MeV/c. These are dominated by $S$ waves amplitudes. For isospin 0 there are two relevant channels $KN, \Sigma\pi$ and for isospin 1 three channels $KN, \Lambda\pi, \Sigma, \pi$. These data, which may be found in ref. [4], are about 40 years old and more precise results are still unavailable. The important contribution of this century has been the measurement of the $1S$ level shift in kaonic hydrogen atom which produces precise $K^-p$ scattering length [5] consistent with the scattering result.

The scattering data are parameterized, for each isospin $I$, in terms of reaction matrix $\hat{K}^I$ related to the scattering matrix $\hat{T}$ by

$$(\hat{T}^I)^{-1} = (\hat{K}^I)^{-1} - i\hat{Q}$$

where $\hat{Q}$ is a diagonal matrix of channel momenta. Unitarity requires $\hat{K}^I$ to be real and symmetric which allows overall nine free parameters, three for $I=0$ and six for $I = 1$. With the best fit values one finds the elastic amplitudes $T^I_{KK}$ in terms of scattering lengths $A^I$

$$T^I_{KK} = A^I / (1 - iQ_{KK}A^I)$$

It turns out that the isospin 0 amplitude extrapolated below $KN$ threshold and into complex energy plane displays a pole. It apparently describes $\Lambda(1405)$ which in this way may be interpreted as $\hat{K}^I$ quasi-bound state decaying into $\Sigma, \pi$ channel. The existence of $\Lambda(1405)$ is apparently the result of short ranged $\hat{K}^I$ interactions. This is not the only possibility. If $\Lambda(1405)$ contains a genuine three quark component, it may be introduced into $\hat{K}$ matrix formalism as an additional pole coupled to the channel states. The best fit to the data does not indicate any
large component of this type [4]. As far as the the nuclear states are concerned the actual origin of Λ(1405) is not crucial. What matters is the actual spectral profile of this state.

Many technical questions arise at this stage. First, the determination of $\overline{K}$ matrices is not complete, the data allows to fix six parameters out of nine allowed. Second, there is some energy dependence of these parameters. These uncertainties affect particularly the subthreshold region rendering sizable uncertainties of the position and widths of Λ(1405). Several methods have been used to fix the Λ(1405) parameters. A natural one is to observe its shape in reactions like $pp \rightarrow K\Lambda(1405)$ with subsequent decay $\Lambda(1405) \rightarrow \Sigma, \pi$. Such an analysis is complicated by several allowed isospin states of $\Sigma, \pi$, uncertain formation mechanism and final state interactions.

Indeed, the very recent progress in this search involving electro and photo-production of Λ(1405) indicates six different locations of the resonant maxima in the $\Sigma, \pi$ charge states. No understanding has so far not been attempted, a brief presentation and references may be found in [6].

For nuclear interactions of $\overline{K}$ mesons it is the elastic amplitude $T_{KK}$ that is of prime importance as it determines the optical potential for this meson. So far the best method to check the subthreshold extrapolation is a mixed use of the reaction matrix and dispersion relations. Lucid presentation of this approach is given in ref.[4]. This method - "the phenomenological approach" - locates the pole in the complex energy plane of $T_{KK}(E)$ at $E_{\text{real}} \approx 1410$ MeV.

Another method recently studied - "the chiral approach" - assumes additional SU(3) symmetry in the reaction matrix introduced within the chiral perturbation theory [7],[8],[9]. The number of free parameters is comparable to that used in the phenomenological approach but the physics is different. One difference is the existence of another pole in the complex energy plane of $T_{KK}(E)$ due to an additional resonance in the $\Sigma, \pi$ channel. However, the basic difference is the position of the main pole related to Λ(1405) arising at $E_{\text{real}} \approx 1420$ MeV. As discussed in next sections the change of the $\overline{K}N$ quasi-bound state position has tremendous impact on the levels of nuclear states of $\overline{K}$.

3. Nuclear states of $\overline{K}$

The dominant attraction comes from the $S$-wave Λ(1405) and an additional contribution is due to Σ(1385) which couples to $\overline{K}N$ in $P$ wave. These states are additional to the scattering states and by the "level repulsion rule" one should expect that at energies above resonances the $\overline{K}$ interaction in nuclear systems is repulsive while below resonances it is attractive. This is seen clearly in K-mesic atoms. The level shifts and widths in such atoms are shifted from electromagnetic values by the effects of strong interactions. The latter may be described by optical potential which in its simplest form is given by

$$V_{\text{optical}}^{K}(r) = -\frac{2\pi}{\mu_{KN}} T_{KK}(E) (-E_B - E_{\text{recoil}}) \rho(r),$$

where $\rho$ is the nuclear density and $\mu$ is the reduced mass. Atomic levels which can be tested involve interactions at a distant nuclear surface where an average binding energies $E_B \approx 15$ and average $KN$ recoil energies $E_{\text{recoil}} \sim 10 - 15$ MeV are calculable. Average $KN$ scattering lengths $A$ are repulsive but for bound nucleons $T_{KK}$ refers to sub-resonant region and becomes attractive. That allows for a semi-quantitative understanding of the level shifts [10], [11], [12]. A number of calculations performed in remote past also indicated sizable dependence of $T_{KK}$ on the density of nuclear matter. In particular, the Pauli principle and self-consistency of the nuclear propagator results in a weaker binding of Λ(1405). In consequence, the optical potential tested in kaonic atoms is not a linear function of the density as the effective "in medium " $T_{KK}$ becomes weaker or even repulsive in the nuclear center. Unfortunately these calculations depend on the details of the self-consistency procedure and have not been tested experimentally. Nevertheless, recent precise studies of the optical potential indicate such an effect [20].
There is a long way from the atomic optical potential tested at nuclear surface for a zero energy meson to an optical potential for a meson deeply bound in a dense nuclear matter. Inspection of Table 2 shows that binding of a hundred MeV closes the dominant Σ, π decay mode and leaves small phase space for the other Λ, π mode. Is such scenario conceivable? If one looks for a solution of the Dyson equation in nuclear matter

\[ T(E) = T_0(E) + T_0(E)(G[E,V(E)] - G_0[E])T(E) \]  

(4)

where \( G_0 \) is the free propagator for \( \bar{K} \)N pair and \( G \) is a propagator in nuclear matter there exists a self-consistent solution at \( E \approx 90 \) MeV with a small \( \text{Im} \ T(E) \). This led to conjecture that narrow (about 10-20 MeV) states may exist in large nuclei [13].

On the other hand, Yamazaki and Akaishi have shown that quasi-bound states may exist also in a few nucleon, in particular in the \( \bar{K}NN \), systems [14]. A number of calculations followed. The contemporary status is given in Table 3. The dominant configuration of the indicated state is \( I_{NN} = 1 \) and the total isospin \( I_{tot} = 1/2 \), but some models predict also other structures. The calculations differ in terms of technical methods used and in terms of the \( \bar{K}N \) input. The first four references use fairly similar phenomenological interactions. The calculations apply molecular technique [14], Faddeev equations with separable forces [15] [16], and a variational technique with realistic NN forces [17]. The latter is discussed in the next section. The first calculation operates with single \( \bar{K}N \) channel the other three introduce the multiple scattering also in the decay states. This accounts for the largest part of difference in the binding energy. On the other hand, the last two calculations [18], [19] use chiral input. The different Λ(1405) position is reflected in much weaker binding energy. Two of the listed works include also \( P \) wave interactions mediated by Σ(1385). In the case of strong binding the energy of \( \bar{K}N \) subsystem is located mostly below the resonance. The binding is enhanced by about 10 MeV and the widths of the state is reduced by the related reduction of the phase space, [17]. In the case of weak binding, the main effect of Σ(1385) is a sizable enhancement of the width [18].

Comparison of Tables 1 and 3 creates an impression that the chiral approach is refuted by the data. This is likely, but not necessarily true. First, the experiments need to be confirmed. Second, the FINUDA measurement has been performed as a capture of \( K^- \) on Li from an atomic state and a subsequent observation of the Λ and proton. Final state interactions are important [23], [22] and may deform the spectrum. On the other hand, the DISTO result obtained in \( pp \rightarrow K^+\Lambda, p \) reaction indicates surprisingly broad state which rises suspicions that the process is not well understood. Proximity of the Σ, π, N threshold may also have some effect. On the theory side, one obvious conclusion from Table 3 is the necessity for further studies of the Λ(1405) structure.

**Table 3.** The \( \bar{K} \) NN system. The second line - calculated binding energies of the lowest state. The third line gives related widths, all in [MeV]. These calculations assume only \( S \)-wave interactions.

|     | [14] | [15] | [16] | [17] | [18] | [19] |
|-----|------|------|------|------|------|------|
| 48  | 55-70|    ≈ 80 |  70  |  20  |  30-40 |
| 61  | 95-110|  ≈ 73 |  85  | 20-40 |  50  |

**3.1. Variational calculations**

This section presents a method [17] which may be used to calculate the \( \bar{K} \)-few-nucleon levels. It is inspired by Brueckner’s [21] calculation of the scattering length of a meson on two nucleons.
Here, it is turned into an eigenvalue problem. An advantage over the Faddeev approach is a simple way to use realistic NN forces. The advantage over the optical potential method is the inclusion of correlations induced by the meson. Realistic calculation requires several steps of complication:

1. The $\bar{K}$-few-N levels are found within the fixed nucleon approximation with a simple $S$ wave $\bar{K}N$ interaction.
2. The nucleon degrees of freedom and NN interactions are introduced and a related Schrödinger equation is solved as a variational problem.
3. The method is extended to multiple channel situations.
4. Both $S$ and $P$ wave $KN$ interactions are allowed.

With up to four nucleons this method is simple to implement. Here it is presented in two nucleon case, with a simplified notation and several shortcuts. Consider scattering of a meson on two identical nucleons fixed at coordinates $x_i (i = 1, 2), r = |x_1 - x_2|$. The wave function is assumed to be

$$\Psi(x, x_1, x_2) = \chi_K(x, x_1, x_2) \chi_{NN}(x_1, x_2), \quad (5)$$

where $x$ is the meson coordinate. The meson wave function $\chi_K$ is given by the solution of the multiple scattering equation

$$\chi_K(x, x_1, x_2) = \chi_K(x^o) - \Sigma_i \int dy \frac{exp[ip |x-y|]}{4\pi |x-y|} U_{KN}(y, x_i) \chi_K(y, x_1, x_2). \quad (6)$$

One looks for solutions of Eq. (6) with no incident wave $\chi_K(x^o)$. The momentum $p$ becomes a complex eigenvalue $p(x_i)$ which determines the energy and width of the system for given nucleon positions $x_i$. The potential $U_{KN} = 2\mu_{KN} V_{KN}$ is chosen in a separable form

$$V_{KN}(x - x_i, x' - x_i) = \lambda \nu(x - x_i) \nu(x' - x_i), \quad (7)$$

where $\nu$ is a form-factor and $\lambda$ is a strength parameter. The eigenvalue equation (6) may be now reduced to matrix equation for wave amplitudes $\psi_i$ defined at each scatterer $i$ by

$$\psi_i = \lambda \int dx \nu(x - x_i) \chi_K(x, x_1, x_2). \quad (8)$$

To find equations for $\psi_i$ one introduces the off-shell $\bar{K}N$ scattering matrices $T$ and matrix elements of the propagator

$$G_{1,2}(x_1, x_2) = \int dy dx \nu(x - x_1) \frac{exp(iP |x - y|)}{4\pi |x - y|} \nu(y - x_2). \quad (9)$$

The diagonal value, $G_{i,i} \equiv G$, determines the meson nucleon scattering matrix $t$ by the well known relation

$$t(E) = (1 + \lambda G)^{-1} \lambda \quad (10)$$

and this yields the full off-shell scattering amplitude $T$

$$T(k, E, k') = -\nu(k) t(E) \nu(k'). \quad (11)$$

Here, $k, k'$ are the initial and final momenta while the form-factor $\nu(k)$ is given by the Fourier transform of $\nu(r)$.

Simple manipulations allow to obtain the standard multiple scattering equations

$$\psi_1 + t G_{1,2} \psi_2 = 0, \quad \psi_2 + t G_{2,1} \psi_1 = 0, \quad (12)$$

which describe the meson bouncing off the two nucleons. When the determinant $D = 1 - (t G_{12})^2$ is put to zero, the binding "momenta" $p(r)$ may be obtained numerically. Two different solutions corresponding to $1 + tG = 0$ or $1 - tG = 0$ may exist. The first solution is symmetric $\psi_2 = \psi_1$. 


and describes the meson in an $S$ wave state with respect to the NN center of mass. The second solution is antisymmetric $\psi_2 = -\psi_1$ and describes a $P$ wave solution. With the rank one separable interaction this latter solution does not exist in the full range of $r$. However, it may with more complicated $S$ wave or $P$ wave interactions.

Eigenvalues corresponding to unstable quasi-bound states are obtained in the second quadrant of complex $p(r) = p_R + ip_I$ plane. In this quadrant the kernel has asymptotic behavior

$$G \sim \exp(-p_I r) \exp(ip_R r)/r$$

as required by the asymptotic form of the bound state wave function $\chi_K$. At short distances $G$ is regularized by the form-factor. The $G(r)$ describes meson nucleon correlations which differ from the correlation inherent to $\Lambda(1405)$. The eigen-value $p(r)$ determines the energy of the meson bound to the fixed NN pair

$$E = p(r)^2/(2\mu_{KN}).$$

The motivation for this definition of $E$ follows from the example discussed below. If the $\bar{K}N$ interaction is dominated by a quasi-bound state, such as $\Lambda(1405)$, then the related pole dominates the scattering amplitude and in some energy region $t = \gamma^2/(E - E^*)$, where $\gamma$ is a coupling constant and $E^* = E_r - i\Gamma_r/2$ is the $\Lambda(1405)$ complex binding energy. The KNN eigenvalue $p(r)$ is given by the equation $1 + tG = 0$, which now takes the form

$$E = E^* - \gamma^2 G(r, p).$$

The solution $E(r) \equiv E_B(r) - i\Gamma(r)/2$ depends on the N-N separation $r$. Since $\text{Re} \ G(r, p)$ close to the resonance is positive, the binding of $K$ to fixed NN is stronger than its binding to a nucleon, $|E_B(r)| > |E_r|$. Increasing the separation $r \to \infty$ one obtains $G \to 0$ and $E(r) \to E^*$, i.e. the $K$ meson becomes bound to one of the nucleons. In the same limit the lifetime of $K$ becomes equal to the lifetime of $\Lambda(1405)$. Hence, the separation energy is understood here as the energy needed to split the $K$-N-N system into the $\Lambda(1405)$-N system. The condition required is $p(\infty) = q_r$ where $q_r$ is the resonant momentum in the KN center of mass system i.e. $q_r^2/2\mu_{KN} = E^*$. Definition (14) fulfils this condition.

The difference between the binding at a given separation $r$ and its asymptotic value generates a potential $V_K(r)$, which contracts the nucleons to a smaller radius. It is defined as

$$\text{Re} V_K(r) = E_B(r) - E_B(\infty)$$

while the corresponding imaginary part is

$$\text{Im} V_K(r) = -\Gamma(r)/2.$$

Typical profiles of $E_B(r)$ are plotted in Figure 1.

In the next step of this calculation the nucleon degrees of freedom are restored. The solution of the full KNN bound state problem is given by equation

$$(-\frac{\Delta_x}{2m} - \frac{\Delta_1}{2M} - \frac{\Delta_2}{2M} + V_{KN1} + V_{KN2} + V_{NN})\Psi = E\Psi.$$  

which with the wave function from Eq. (5) and given $V_{NN}$ may be solved by variational methods.

The explicit description of the decay channels requires more components in the amplitudes $\psi_i$. We refer to paper [17] which includes decay channels and finds a number of KNNN and KNNNN states. It may be interesting to note that a loosely bound state in the $K^-nn$ system is also predicted to exist as a consequence of $\Sigma(1385)$.
4. Conclusions
The search for nuclear states of anti-kaons is a fascinating story of successes and failures. One still has to answer some basic questions:

(a) What is the binding mechanism? How many mesons can be bound in one nucleus?
(b) Are the technical questions under control?
(c) Can the widths be narrow?

Ad (a), it seems clear that the profile of \( \Lambda(1405) \) in the basic \( \bar{K}N \) channel is of prime importance. One way to study it would be a measurement of radiative transitions \( (K^- p)_{\text{atom}} \rightarrow \gamma + \Lambda(1405) \). Another possibility exists in studies of the highest available levels in \( K \)-mesic atoms. In such states the optical potentials of equation (3) involves predominantly valence nucleons and the scattering amplitude may be traced as a function of the separation energy. Figure 2 shows the absorptive amplitude extracted from atomic data. Unfortunately, the data describing the essential region of \( \sim 25 \text{ MeV} \) below the threshold are uncertain. New experiments are needed.

Ad (b), it is not clear whether the optical potential or other average field approximations are a good description in view of strong \( N- \bar{K}N \) and \( \bar{K}N \) correlations.

Ad (c), if experimental results are correct the widths on nuclear states might be determined by two nucleon capture of the kaon. There is no reliable knowledge in this question.
Figure 2. The isospin averaged, $NK$ amplitudes $a = < T_{KK}^I >$ in the sub-threshold region extracted from upper levels of kaonic atoms. To trace the position of resonance one needs to add about 10 MeV of the recoil energy to the values of $E_{sep}$.

Acknowledgements
Financial support from European Hadron 3 - LEANNIS project is acknowledged.

References
[1] J. Zmeskal, Progr.Part.Nucl.Phys. 61,512(2008).
[2] M. Agnello et al. (FINUDA Collaboration), Phys. Rev. Lett. 94(2005)212303.
[3] M. Maggiora et al. (DISTO Collaboration), Nucl. Phys. A835(2010)45.
[4] A.D. Martin Nucl.Phys. B179(1981)33, in Low and intermediate energy kaon-nucleon physics, Ed. E.Ferrari and G.Violini, Reidel Publ.1981.
[5] M. Iwasaki et al. Phys. Rev.Lett. 78(1997)3067 ; G. Beer et al. Phys. Rev. Lett 94(2005) 212302.
[6] S. Wycech, Proceedings of EXA 11, Hyperfine Interactions (2011).
[7] E. Oset, A. Ramos and C. Bennhold, Phys. Lett. B527 (2002)99.
[8] N. Kaiser, P.B. Siegel and W. Weise, Nucl. Phys. A 594,(1995)325.
[9] B. Boraso, U.-G. Meiñner and R. Nißler,Phys. Rev. C74(2006) 055201 (2006).
[10] M. Alberg, E.M. Henley and L. Wilets, Ann. Phys. NY 96 (1976)43.
[11] S.Wycech, Nucl.Phys. B 28(1971)541.
[12] W.Bardeen and E.W.Torigoe, Phys. Rev. C 3(1971)1785.
[13] S. Wycech, Nucl. Phys. A450 (1986) 399c.
[14] Y. Akaishi and T. Yamazaki, Phys. Rev. C 65(2002) 044605.
[15] N. V. Shevchenko, A. Gal, J. Mareš and J. Révai, Phys. Rev. C 76(2007)044004.
[16] Y. Ikeda and T. Sato, Phys. Rev. C 76(2007) 035203.
[17] S. Wycech and A.M. Green Phys. Rev. C79(2009)014001.
[18] T. Hyodo and W. Weise, Phys. Rev. C77(2008) 035204.
[19] M.Bayar, J. Yamagata-Sekihara and E.Oset, Phys. Rev. C 84(2011)015209.
[20] A.Cieply, E. Friedman, A. Gal, D. Gazda and J. Mareš, Phys. Lett. B 702(2011)402.
[21] K. A. Brueckner, Phys. Rev. 89(1953)834.
[22] Grishma Pandejee , N.J. Upadhyay and B.K. Jain, Phys. Rev. C 82(2010)034608.
[23] V. K. Magas, E. Oset, A. Ramos and H. Toki, Phys. Rev. C 74(2006)025206.