JAM mean-field update: mean-field effects on collective flow in high-energy heavy-ion collisions at $\sqrt{s_{NN}} = 2 - 20$ GeV energies

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We consider different implementations of momentum-dependent hadronic mean-fields in the relativistic quantum molecular dynamics (RQMD) framework. First, Lorentz scalar implementation of Skyrme type potential is examined. Then, full implementation of Skyrme type potential as a Lorentz vector in the RQMD approach is proposed. We find that scalar implementation of Skyrme force is too weak to generate repulsion explaining observed data of sideward flows at $\sqrt{s_{NN}} < 10$ GeV, while vector implementation gives collective flows compatible with the data for a wide range of beam energies $2.7 < \sqrt{s_{NN}} < 20$ GeV. We show that our approach reproduces the negative proton directed flow at $\sqrt{s_{NN}} > 10$ GeV discovered by the experiments. We discuss the dynamical generation mechanisms of the directed flow within a conventional hadronic mean-field. A positive slope of proton directed flow is generated predominantly during compression stages of heavy-ion collisions by the strong repulsive interaction due to high baryon densities. In contrast, at the expansion stages of the collision, the negative directed flow is generated more strongly over the positive one by the tilted expansion and shadowing by the spectator matter. At lower collision energies $\sqrt{s_{NN}} < 10$ GeV, the positive flow wins against the negative flow because of a long compression time. On the other hand, at higher energies $\sqrt{s_{NN}} > 10$ GeV, negative flow wins because of shorter compression time and longer expansion time. A transition beam energy from positive to negative flow is highly sensitive to the strength of the interaction.

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I. INTRODUCTION

The phase structure of QCD matter for a wide range of baryon density is of fundamental interest [1]. Understanding the equation of state (EoS) for QCD matter is a primary goal. In particular, a first-order phase transition and a critical point at finite baryon densities are predicted by several effective models [2]. The properties of QCD matter have been explored experimentally using high energy nuclear collisions under various conditions: beam energy, rapidity, centrality, and system size dependence. Currently, high energy heavy-ion experiments is one of the most active areas: heavy-ion experiments are being performed from a few GeV to TeV beam energies at the same time. We now have a vast body of data including different centralities and system sizes from many experiments by the accelerators such as SIS [3] AGS [4], SPS [5], RHIC [6], and LHC [7] and so on. Future experiments as FAIR [8], NICA [9], HIAP [10], and J-PARC-HI [11] are being constructed or are planned to perform high precision measurements.

Anisotropic collective flows are considered to be a good probe to extract EoS of dense QCD matter [12–16]. Non-central collisions create azimuthally asymmetric excited matter, and subsequent collective expansion results in azimuthally asymmetric emission of particles. The distribution can be analyzed from the coefficients in the Fourier expansion of measured particle spectra [17]. The directed flow is defined by the first coefficient $v_1 = \langle \cos \phi \rangle$ and the second coefficient $v_2 = \langle \cos 2\phi \rangle$ is called the elliptic flow, where $\phi$ is the azimuthal angle of an outgoing particle with respect to the reaction plane. These flows have been measured by various experiments, and now we have excitation functions of flows from $\sqrt{s_{NN}} \approx 2$ GeV to 5 TeV. The proton elliptic flow is negative below $\sqrt{s_{NN}} \approx 3$ GeV due to a shadowing of spectator matter (squeezed-out) and it becomes positive at higher beam energies [14–18]. Transport theoretical models describe this sign change of the elliptic flow [14–18]. On the other hand, the data shows that the slope of the proton $v_1(y)$ with respect to rapidity $y$ is positive (normal flow) $dv_1/dy > 0$ up to the beam energy of $\sqrt{s_{NN}} \approx 10$ GeV, and then it becomes negative (anti-flow) above 10 GeV at mid-rapidity [19–21].

It has been argued that the negative directed flow could be an effect of the softening of the EoS, and it may be a signature of a first-order-phase transition [22–24]. Fluid dynamical simulations and microscopic transport models predict that the softening happens at around the beam energy less than 5 GeV, [22–26] which is inconsistent with the experimental data. On the other hand, anti-flow at beam energies above 27 GeV is naturally explained by the transport models [27] by the combination of space-momentum correlations together with the correlation between the position of a nucleon in the nucleus and its stopping [30]. The direct reason for the negative slope is the tilted matter created in non-central collisions, which generates anti-flow predominately over the normal flow during the expansion stage. Color glass condensate model also predicts twisted matter [31]. The tilted source was used in the initial condition of the hydrodynamical evolution to explain negative directed flow at the top RHIC energy [32]. The transport models describe the directed flow below 7.7 GeV or above 27 GeV [27,29]. The three fluid (3FD) model [33] reproduces the rapidity dependence of the directed flow at 11.5 GeV with the crossover and 1st-order phase transition scenario. A transport calculation with attractive trajectory prescription [34] also fit the data at 11.5 GeV. However, they fail to explain the beam energy dependence of the slope. Thus, the question remains: what is the reason for the transition from
positive to negative slope at around 10 GeV? In this work, we will address this question by using a newly developed microscopic transport model JAM2.

To extract information on the properties of excited QCD matter from experimental data, we need to understand the space-time evolution of the matter created in high-energy nuclear collisions. For this purpose, microscopic transport models such as Boltzmann-Uehling-Uhlenbeck (BUU) and quantum molecular dynamics (QMD) approaches and their relativistic versions, RBUU and RQMD, have been developed and successfully employed to understand the collision dynamics of high-energy nuclear collisions. The main two gradients of the microscopic transport model are Boltzmann type collision term and the mean-field interaction. Later, hybrid models have been developed by combining fluid dynamics into a microscopic transport model (in the cascade mode) to describe heavy-ion collisions at high baryon density regions. Here, we employ an RQMD approach to explore the anisotropic flows. It is well known that collective flows are highly sensitive to the mean-field interactions at high baryon density regions. In Ref. [46], we showed that the RQMD/S model describes both directed and elliptic flow for a wide range of beam energies emphasizing the importance of the momentum-dependent potential into the RQMD/S approach. Then, we present EoS using Lorentz scalar and vector potentials, which will be used in the new version of RQMD.

II. EQUATION OF STATE

We start from a short review of equation of state, which is used in our previous model [46], based on the so called simplified quantum molecular dynamics (RQMD/S) approach. Then, we present EoS using Lorentz scalar and vector potentials, which will be used in the new version of RQMD.

A. Non-relativistic potential

We use the Skyrme-type density-dependent potential together with the momentum-dependent potential. The single-particle potential is given by

\[ U(\rho, p) = U_{sk}(\rho) + U_m(p). \]  

(1)

The baryon-density \( \rho \) dependent part \( U_{sk}(\rho) \) is assumed to have the following density dependence

\[ U_{sk}(\rho) = \alpha \left( \rho / \rho_0 \right) + \beta \left( \rho / \rho_0 \right)^\gamma, \]  

(2)

where the normal nuclear density is taken to be \( \rho_0 = 0.168 \text{ fm}^{-3} \). The momentum-dependent part \( U_m(p) \) is assumed to be given as the momentum folding with the Lorentzian form factor

\[ U_m(p) = \frac{C}{\rho_0} \int d^3p' \frac{f(x, p')}{1 + \left[ (p - p')/\mu \right]^2}, \]  

(3)

where \( f(x, p) \) is the single-particle distribution function for nucleon. In the case of symmetric nuclear matter at zero temperature, it is given by

\[ f(x, p) = \frac{g_N}{(2\pi)^3} \theta(p_f - |p|) \]  

(4)

with \( g_N = 4 \) being the degeneracy factor for spin and isospin of nucleons, and \( p_f = \left( \frac{\rho_f^2}{g_N} \right)^{1/3} \) is a Fermi momentum. The energy density at zero temperature is

\[ e = e_{\text{kin}} + \int_0^\rho U_{sk}(\rho')d\rho' + \frac{1}{2} \int d^3p U_m(p)f(x, p) \]  

(5)
where
\[ e_{\text{kin}} = \frac{g_N}{(2\pi)^3} \int_0^{p_f} d^3p \sqrt{m^2 + p^2} \]
\[ = \frac{g_N}{16\pi^2} \left[ 2p_f^2 e_f + m_N^2 e_f p_f - m_N^4 \log \left( \frac{e_f + p_f}{m_N} \right) \right] \]
\[ \approx \left[ \frac{3}{5} \frac{p_f^2}{2m_N} + m_N \right] \rho, \quad (6) \]

where \( m_N \) is a nucleon mass and \( e_f = \sqrt{m_N^2 + p_f^2} \). The total energy per nucleon is obtained by
\[ \frac{E}{A} = \frac{e}{\rho} - m_N \quad (7) \]

We compare the single-particle potential at the normal nuclear density
\[ U_{\text{opt}}(p) = U(\rho_0, p) = \alpha + \beta + U_m(p) \quad (8) \]

with the Schrödinger-equivalent optical potential from the Dirac phenomenology [52]. The parameters of the potentials are fixed by the five conditions. The first three conditions are given by the saturation properties, saturation at normal nuclear density \( \rho = \rho_0 = 0.168 \, \text{fm}^{-1} \), the nuclear matter binding energy \( B = -16 \, \text{MeV} \) at saturation, and the nuclear incompressibility \( K = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{e}{\rho} \right) = 380 \, \text{MeV} \) (Hard) or 210 MeV (Soft). Other two conditions come from the energy dependence of the optical potential. The optical potential is required to take the following values
\[ U_{\text{opt}}(\rho_0, p = 1.7 \, \text{GeV}) = U_h, \quad (9) \]
\[ U_{\text{opt}}(\rho_0, p = p_0) = 0 \, \text{MeV}, \quad (10) \]

where we adopt \( U_h = 60 \, \text{MeV} \) and \( p_0 = 0.65 \, \text{GeV} \) in the non-relativistic implementation. Saturation condition \( P = p^2 \frac{\partial}{\partial p} \left( \frac{e}{\rho} \right) = 0 \) and the saturation energy \( e/\rho - m_N = B \) at \( \rho = \rho_0 \) leads to the Weiskopf relation
\[ \sqrt{m_N^2 + p_f^2} + \alpha + \beta + U_m(p_f) = m_N + B \quad (11) \]

We first fix the parameters in the momentum-dependent potential and \( U_0 = \alpha + \beta \) by using Eq. (9) to Eq. (11). Next the parameters of the density-dependent part (\( \alpha, \beta \) and \( \gamma \)) are fixed by using the saturation density and the incompressibility. The details of the fitting procedure is found in the Appendix [B]

We adopt two range Lorentzian-type momentum-dependent potential [46, 53] for the parameter set MH2 and MS2 with the range parameters \( \mu_1 = 2.02 \, \text{femtometers} \) and \( \mu_2 = 1.0 \, \text{femtometers} \). These parameters lead to \( \gamma < 2 \), which has softer baryon density dependence than the hard EoS without momentum-dependent part (\( \gamma = 2 \)) at high densities. As an alternative parametrization, we assume one range Lorentzian for momentum-dependent potential, which leads to slightly harder EoS (MH1) and softer EoS (MS1) at high densities. Parameters are summarized in Table I. The effective mass for
\[ m^* = p_F/|\epsilon_F| = p_F/|\partial e_f/\partial p_F| = 0.877 m_N \] (formula given by the Ref. [56, 57]), while \( m^* = 0.705 m_N \) for MH1.

The left-upper panel of Fig. [1] compares the energy dependence of the optical potential Eq. (8) with the real part of the global Dirac optical potential [52]. Here we assume that the incident kinetic energy \( E_{\text{lab}} = E - m_N \) is related to the momentum in the argument of the potential by the relation \( (E - U_{\text{opt}})^2 - p^2 = m_N^2 \). It is seen that all of the parameter sets (see Table I) well describe the data except for the momentum-independent parametrization (H and S).

In the left-lower panel of Fig. [1] the baryon-density dependence of the energy per nucleon is shown for hard (\( K = 380 \, \text{MeV} \)) and soft (\( K = 210 \, \text{MeV} \)) EoSs. Parameter set MH2 yields softer EoS than the sets MH4. This behavior can be also confirmed by comparing the values of the \( \gamma \) in Table I as well as the effective mass \( m^*/m_N \). When the effective mass at saturation density is smaller, the EoS becomes harder at high densities. The same trend is observed also in the relativistic mean field theory, which demonstrates that the energy dependence of the optical potential modifies the EoS at high densities even if the incompressibility is fixed.

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B. Lorentz scalar potential

In this section, we consider scalar potentials to construct EoS. We assume the single-particle energy as
\[ e^* = \sqrt{p^2 + m_{\text{sym}}^2}, \quad m_{\text{sym}} = m + U_s + U_m \quad (12) \]

The energy density of the nuclear matter for scalar potential is given by [55, 56]
\[ e = \int d^3p \left( e^* - \frac{1}{2} m_{\text{sym}}^* U_m(p) \right) f(x, p) \]
\[ - \rho_s U_s(\rho_s) + \int_0^{\rho_s} U_s(\rho_s') d\rho_s', \quad (13) \]

where the scalar density is defined as
\[ \rho_s = \int d^3p \frac{m_{\text{sym}}^*}{e^*} f(x, p). \quad (14) \]

The density dependent scalar Skyrme potential is the function of scalar density, not the baryon density. The momentum-dependent part of the potential takes the form
\[ U_m(p) = \frac{C}{\rho_0} \int dp \frac{m_{\text{sym}}^*}{e^*} \frac{f(x, p')}{1 + (|p - p'|/\mu)^2}. \quad (15) \]

The main difference between the previous and present approach is the form of pre-factor \( m_{\text{sym}}^*/e^* \) which is also introduced in the momentum-dependent potential.

In Table I we present EoS parameter sets for scalar implementation of the Skyrme potential. We compare the Dirac optical potential [52] with the single-particle energy of nucleon subtracting kinetic energy [56, 58, 59]:
\[ U_{\text{opt}}(p) = e^* - \sqrt{m_N^2 + p^2} \quad (16) \]
TABLE I. Parameter sets for the Skyrme-type potential in the non-relativistic, scalar, and vector implementations. The range parameters in the momentum dependent part is taken to be $\mu_1 = 2.02$ fm$^{-1}$ and $\mu_2 = 1.0$ fm$^{-1}$ for MH2 and MS2, independent of the implementation scheme. For MH1 and MS1, we adopt $\mu_1 = 3.173$ fm$^{-1}$ (non-relativistic), $\mu_1 = 5.18$ fm$^{-1}$ (scalar), and $\mu_1 = 3.23$ fm$^{-1}$ (vector). Optical potential is controlled by the two parameters, $p_0$ and $U_\infty$ via the relations $U_{\text{opt}}(p_0, p = p_0) = 0$ and $U_{\text{opt}}(p_0, p = 1.7$ GeV $) = U_\infty$.

In the non-relativistic and vector implementations, we take $(p_0, U_\infty) = (0.685$ GeV, 60 MeV) for MH1 and MS1 and $(p_0, U_\infty) = (0.685$ GeV, 50 MeV) for MH2 and MS2.

This optical potential approaches zero in the high momentum limit, since the vector potential is not included. As shown in the middle-upper panel of Fig. [1] the optical potential vanishes at about $E_{\text{lab}} \approx 100$ GeV.

We obtain the EoS MH2 which is slightly softer than that in the non-relativistic approach when $\mu_1$ and $\mu_2$ are fixed to be 2.02 and 1.0 l/fm. We also provide MH1 and MS1 which has a closer density dependence to the original hard and soft EoS without momentum-dependence as depicted in the middle-lower panel of Fig. [1].

C. Lorentz vector potential

Next, we consider the vector implementation of the Skyrme type density dependent mean-field and a Lorentzian type momentum-dependent mean-field. The energy density of the nuclear matter has the following form [53]:

$$e = \int d^3p \left( e^* + U_m^0 - \frac{1}{2} \frac{p_k^*}{e^*} U_{\text{sk}}^0(p) \right) f(p) + \int_0^\infty U_{\text{sk}}^0(p') dp'$$

where the momentum-dependent part of the potential takes the form:

$$U_m^0(p) = \frac{C}{\rho_0} \int d^3p' \frac{p^*_k}{e^*} \frac{f(x, p')}{\Gamma + [(p - p')^2/\mu_k]^2}$$

FIG. 1. Incident energy dependence of the optical potential in comparison with the real part of the global Dirac optical potential [52] (upper panels) and the energy per nucleon as a function of the baryon density (normalized by $\rho_0$) (lower panels) for different parametrizations. Left, middle and right panels show the results in the non-relativistic potential treatment, the scalar potential implementation, and the vector potential implementation, respectively.
We replace the argument of momentum-dependent potential to the relative momentum at the two-body c.m. frame or the rest frame of a particle in the simulation.

We list our parameter set in Table II for vector potential, too. Energy dependence of the optical potential and baryon density dependence of the binding energy per nucleon are plotted in the right-bottom panel of Fig. 1.

In this section, we first give a brief explanation of basics in the RQMD model. Next we give the equations of motion for the RQMD/S approach with the non-relativistic potential and mass-shell constraints in the non-relativistic, scalar, and vector implementations of the potentials.

III. RELATIVISTIC QUANTUM MOLECULAR DYNAMICS

The RQMD model is a non-equilibrium transport model which can simulate a space-time evolution of the N-particles interacting via the potentials based on the constraint Hamiltonian dynamics \[60\].

In this section, we first give a brief explanation of basics in the RQMD model. Next we give the equations of motion for the RQMD/S approach with the non-relativistic potential implementation, and then present the equation of motion for RQMD with scalar-vector implementation of potentials in the on-mass-shell constraints.

A. Preliminaries

In the RQMD approach, \(8N\) phase space variables are reduced by the \(2N\) constraints to obtain the physical \(6N\) phase space, \(\phi_i \approx 0 (i = 1, \ldots, 2N)\) with \(\approx\) representing the weak equality satisfied on the realized evolution path. According to Dirac’s constraint Hamiltonian formalism, Hamiltonian is given by a linear combination of \(2N - 1\) constraints

\[
H = \sum_{i=1}^{2N-1} u_i \phi_i. \tag{19}
\]

Among the \(2N\) constraints, the first \(N (i = 1 \sim N)\) are the on-mass-shell constraints and the latter \((i = N + 1 \sim 2N)\) represents the time fixation of the particles. Since one of the time fixation constraints defines the global evolution temporal parameter, we use \(2N - 1\) constraints in Eq. \(19\). The equations of motion are

\[
\frac{dq_i}{dt} = [H, q_i] \approx \sum_{j=1}^{2N-1} u_j \frac{\partial \phi_j}{\partial p_i}, \tag{20}
\]

\[
\frac{dp_i}{dt} = [H, p_i] \approx - \sum_{j=1}^{2N-1} u_j \frac{\partial \phi_j}{\partial q_i}, \tag{21}
\]

where the Poisson brackets are defined as

\[
[A, B] = \sum_k \left( \frac{\partial A}{\partial q^k} \frac{\partial B}{\partial p^k} - \frac{\partial A}{\partial p^k} \frac{\partial B}{\partial q^k} \right). \tag{22}
\]

The Lagrange multipliers \((u_i)\) are determined by requiring that the constraints are kept to be satisfied. For \(i = 1 \sim 2N - 1\), \(\phi_i\) is assumed not to explicitly contain the time, then we find

\[
\frac{d\phi_i}{dt} = [H, \phi_i] = \sum_{j=1}^{2N-1} C_{i,j} u_j \approx 0, \tag{23}
\]

\[
C_{i,j} = [\phi_j, \phi_i]. \tag{24}
\]

By comparison, \(\phi_2N\) is assumed to explicitly contain \(t\),

\[
\frac{d\phi_{2N}}{dt} = [H, \phi_{2N}] + \frac{\partial \phi_{2N}}{\partial t} = \sum_{j=1}^{2N-1} C_{2N,j} u_j + \frac{\partial \phi_{2N}}{\partial t} \approx 0. \tag{25}
\]

Thus, by defining \(u_{2N} = 0\), the Lagrange multipliers are obtained by solving the following linear equation,

\[
\sum_{j=1}^{2N} C_{i,j} u_j = - \delta_{i,2N} \frac{\partial \phi_{2N}}{\partial t}, \tag{26}
\]

\[
\Rightarrow u_i = - C_{i,2N}^{-1} \frac{\partial \phi_{2N}}{\partial t}. \tag{27}
\]

We adopt the time fixation constraints proposed in Ref. \[45\], which allows us to obtain the Lagrange multipliers analytically.

\[
u_{i+N} = \dot{a} \cdot (q_i - q_N) \approx 0 (i = 1 \sim N - 1), \tag{28}
\]

\[
u_{2N} = \dot{a} \cdot q_N - t \approx 0. \tag{29}
\]

The unit vector \(\dot{a}\) is chosen to be \((1, 0)\) in the reference frame. By replacing \(p_i^0\) in the potential with the kinetic energy, \(\sqrt{\dot{p}_i^2 + m^2}\), the matrix \(C_{i,j}\) and its inverse \(C_{i,j}^{-1}\) is found to have the form,

\[
C_{i,j} = \begin{pmatrix} * & - D \\ D^T & 0 \end{pmatrix}, \quad C_{i,j}^{-1} = \begin{pmatrix} * & (D^{-1})^T \\ - D^{-1} & 0 \end{pmatrix}. \tag{30}
\]

The matrix \(D\) is obtained as \(D_{i,j} = [\phi_i, \phi_{N+j}] = \frac{\partial \phi_i}{\partial p_N} - \delta_{i,N} \frac{\partial \phi_N}{\partial p_N} (j = 1 \sim N - 1)\) and \(D_{i,N} = \delta_{i,N} \frac{\partial \phi_N}{\partial p_N}\). In the case where the on-mass-shell constraint is given as \(u_i = p_i^2 - m^2 - f(p_i, q)\) with \(f\) representing the potential effects, the inverse of \(D\) matrix is found to be \(D_{i,j}^{-1} = \delta_{ij}(1 - \delta_{i,N})/2p_i^0 + \delta_{i,N}/2p_N^0\). Then the Lagrange multiplier is found to be

\[
u_i = - \frac{1}{2p_i^0} \frac{\partial \phi_{2N}}{\partial t} = \frac{1}{2p_i^0} (i = 1 \sim N), \tag{31}
\]

\[
u_i = 0 (i = N + 1 \sim 2N - 1). \tag{32}
\]

In the following subsections, we compare the results of on-mass-shell constraints in the non-relativistic, scalar, and vector implementations of the potentials.

B. Equations of motion for the RQMD/S model

In this section, we present the equations of motion for the RQMD/S model \[45\]. In the RQMD/S model, the EoS in the
non-relativistic implementation is used, and the on-mass-shell constraint is given as \[ \phi_i = p_i^2 - m_i^2 = 0 \]. Thus the one-particle energy for the \( i \)th particle takes the form:

\[
p_i^0 = \sqrt{p_i^2 + m_i^2 + 2m_iV_i}.
\]

Then the above ansatz leads to the equations of motion for \( N \)-particle system in the RQMD/S approach [45]

\[
r_i^\prime = \frac{p_i}{\rho_i} + \sum_{j=1}^N \frac{m_{ij}}{p_j} \frac{\partial V_j}{\partial p_i}, \quad \dot{p}_i = -\sum_{j=1}^N \frac{m_{ij}}{p_j} \frac{\partial V_j}{\partial r_i}. \]

The suppression factor \( m_i/p_i^0 \) appearing in the equations of motion is the direct consequence of scalar implementation of the potential in Eq. (33). Furthermore, we make an ansatz that the potential of the one-particle potential for the \( i \)th particle takes the form [42]:

\[
\rho_i = \sum_{j \neq i}^N \rho_{ij} = \sum_{j \neq i}^N \frac{1}{4\pi L} \exp\left(\frac{q_{ij}^2}{4L}\right),
\]

where \( q_{ij}^2 = q_{ij} - (q_{ij} \cdot u_{ij}) u_{ij} \) is the distance in the two-body center-of-mass frame of particles \( i \) and \( j \), where \( q_{ij} = q_i - q_j \) and \( u_{ij} = (p_i + p_j)/\sqrt{(p_i + p_j)^2} \).

The one-particle potential \( V_i \) in RQMD/S framework is not actually the single particle potential, but the sum of \( V_j \) gives the total potential energy in the non-relativistic limit. Thus we need to consider the \( i \)th particle contribution to the total potential energy. Accordingly, the density and momentum dependence of the one-particle potential for the \( i \)th particle in the RQMD/S approach \( V_i = V_{sk,i} + V_{m,i} \) are taken to be

\[
V_{sk,i} = \frac{\alpha}{2\rho_0} \rho_i + \frac{\beta}{(1 + \gamma)(\rho_0)} \rho_i^2, \quad V_{m,i} = \sum_{j \neq i}^N V_{m,ij} \rho_{ij}, \quad V_{m,ij} = \frac{C}{\rho_0} \frac{1}{1 - (p_{Tij}/\mu)^2}.
\]

We also use the relative momentum \( p_{Tij} = p_{ij} - (p_{ij} \cdot u_{ij}) u_{ij} \) in the center-of-mass frame of two particles in the momentum-dependent potential, where \( p_{ij} = p_i - p_j \). We present corrections to the previous paper [46] regarding the implementation of this equations of motion in the code in the Appendix [46].

C. The equations of motion for scalar-vector potentials

We present both scalar and vector implementation of above phenomenological potentials within the framework of the RQMD approach [42].

We impose on-mass shell condition:

\[
H_i = p_i^2 - m_i^2 = (p_i - V_i)^2 - (m_i - S_i)^2 = 0
\]

for \( i \)th particles, where \( V_i^\mu \) and \( S_i \) are the one-particle vector and scalar interactions. According to Ref. [45] [61], we assume the time fixation constraints which equate the all time coordinate of the particles. Within those constraints together with the assumption that the arguments of the potentials are replaced by the free one, one obtains the equations of motion for \( i \)th particle as

\[
\dot{x}_i = \frac{p_i^0}{\rho_i} + \sum_j \left( \frac{m_j^* \partial m_j^*}{p_j^0 \partial p_i} + v_j^\mu \frac{\partial V_{j\mu}}{\partial p_i} \right),
\]

\[
\dot{p}_i = -\sum_j \left( \frac{m_j^* \partial m_j^*}{p_j^0 \partial r_i} + v_j^\mu \frac{\partial V_{j\mu}}{\partial r_i} \right),
\]

where \( v_j^\mu = p_j^\mu/p_j^0 \). The equations of motion for the kinetic momentum \( p_i^0 = p_i - V_i \) may be obtained by adding the derivative of the vector potential:

\[
\dot{V}_i^\mu = \sum_j \left( \dot{x}_j \frac{\partial V_j^\mu}{\partial x_j^\nu} + \dot{p}_j \frac{\partial V_j^\mu}{\partial p_j^\nu} \right).
\]

In the case of our equal time fixation, potentials does not explicitly depend on the time, then \( \partial V/\partial t = (\partial V/\partial x^\nu)(\partial \rho^\nu/\partial t) = 0 \).

In RQMD, the one-particle potentials \( S_i \) and \( V_i^\mu \) are dependent on the scalar density \( \rho_s \) and the baryon current \( J_i^\mu \), respectively, which are obtained by

\[
\rho_{s,i} = \sum_{j \neq i}^N \frac{m_j^*}{p_j^0} \rho_{ij}, \quad J_i^\mu = \sum_{j \neq i}^N B_j \gamma_j^\mu \rho_{ij}
\]

where \( B_j \) is the baryon number of the \( j \)th particle, and \( \rho_{ij} \) is the so-called interaction density (overlap of density with other hadron wave-packet) which will be specified below. The vector potential is defined by using the baryon current [62] [63]

\[
V_i^\mu = B_i V_i((\rho_{Bi}^s)/\rho_{Bi}) J_i^\mu,
\]

where \( \rho_{Bi} = \sqrt{J_i^\mu J_i^\mu} \) is the invariant baryon density. The momentum-dependent part of the one-particle potential in the vector implementation is given by

\[
V_{m,i}^\mu(p_{Tij}) = \sum_{k=1,2}^{\pm} \frac{C_k}{2\rho_0} \sum_{j \neq i}^N \frac{p_{ij}^\mu}{1 - (p_{Tij}/\mu)^2} \frac{\rho_{ij}}{1 - (p_{Tij}/\mu)^2},
\]

while the scalar implementation is

\[
V_{m,i}(p_{Tij}) = \sum_{k=1,2}^{\pm} \frac{C_k}{2\rho_0} \sum_{j \neq i}^N \frac{m_j^*}{1 - (p_{Tij}/\mu)^2} \frac{\rho_{ij}}{1 - (p_{Tij}/\mu)^2},
\]

where \( p_{Tij} \) is the relative momentum between particle \( i \) and \( j \), which will be specified below. As the equations of motion are obtained by assuming that the argument of the potentials are replaced by the free one, we also replace \( m_j^* \) and \( p_j^\mu \) in the definition of the scalar density and baryon current as well as the momentum-dependent potentials by the free one.

We now discuss the form of the interaction density in the RQMD approach. As a first option, we use the following interaction density

\[
\rho_{ij} = \frac{\gamma_{ij}}{4\pi L^{3/2}} \exp\left(\frac{q_{ij}^2}{4L}\right),
\]
where $q_{T,ij}$ is the distance in the center-of-mass frame of the particle $i$ and $j$,

$$q_{T,ij} = q_{ij} - (q_{ij} \cdot u_{ij})u_{ij}, \quad u_{ij} = P_{ij}/\sqrt{P_{ij}^2},$$

(46)

$$q_{ij} = q_i - q_j, \quad P_{ij} = p_i + p_j$$

(47)

and $\gamma_{ij} = P_{ij}^0/\sqrt{P_{ij}^2}$ is the Lorentz $\gamma$-factor to ensure the correct normalization of the Gaussian [64]. We obtain the RQMD/S approach which follows the original RQMD [42, 43], by replacing the normalization factor $\gamma_{ij}$ by the $\gamma_j = p_j^0/m_j$ to obtain Eq. (35), which is the Lorentz scalar. With this replacement, we lost a correct normalization of the Gaussian, but this would not be a problem as one may adjust the width parameter of the Gaussian as far as we have only scalar potentials. The same approximation was used in Ref. [44] for both the scalar density and the baryon current for low energy heavy-ion collisions $E_{lab} < 2A$ GeV. We found that this approximation overestimates the vector density significantly at relativistic energies. Thus, the predictions from this approach are not reliable at relativistic energies.

Another approach is to use the rest frame of a particle $j$

$$q_{R,ij}^2 = (q_i - q_j)^2 - [(q_i - q_j) \cdot u_{ij}]^2, \quad u_i = p_j/m_j$$

(48)

for the definition of the two-body distance in the argument of the potential. This is used in the relativistic Landau-Vlasov model [65], in which Gaussian shape is used to solve the relativistic Boltzmann-Vlasov equation. In this case, the interaction density takes the form

$$\rho_{ij} = \frac{\gamma_j}{(4\pi L)^{3/2}} \exp(q_{R,ij}^2/4L),$$

(49)

where $\gamma_j = p_j^0/m_j$. When we substitute this interaction density into Eq. (41), the Lorentz factor in front of the Gaussian cancels the factor in the scalar density, and the scalar density becomes manifestly Lorentz scalar, and the baryon current is also covariant vector without loss of the correct normalization of the Gaussian:

$$\rho_{s,i} = \sum_{i \neq j} \rho_{ij}, \quad J^\mu_s = \sum_{i \neq j} B_j u_j \rho_{ij}$$

(50)

where $\rho_{ij} = (1/\sqrt{4\pi L})^{3/2} \exp(q_{R,ij}^2/4L), u_j$ is a 4-velocity:

$$u_j = \gamma_j (1, p_j/p_j^0) = (\gamma_j, p_j/m_j).$$

Numerically, the main difference between the two-body distance at the c.m. of two particles $q_{T,ij}$ and the rest-frame of a particle $q_{R,ij}$ in the estimation of the interaction density $\rho_{ij}$ is to use the different shapes of the Gaussian. So we expect that if a violation of the Lorentz invariance is not significant, two different choices may yield the same results with a possible different choice of the Gaussian width. Numerical study indicates that Eq. (49) needs generally smaller Gaussian width than that of Eq. (45) to obtain a similar result at ultra-relativistic energies as shown below.

D. Numerical implementation

The mean-field models mentioned above have been implemented into the JAM2 Monte-Carlo event-generator. The physics of the collision term in JAM2 is the same as the previous version of JAM [66], in which particle productions are modeled by the resonance (up to 2 GeV) and string excitations and their decays [66, 67–70]. However, there are several improvements: We use Pythia8 event generator [71] to perform string decay as well as the hard scatterings instead of Pythia6 [72]. Resonance excitation cross sections are also changed to improve the threshold behavior by fitting the matrix elements [36, 68–70]. As technical improvements in JAM2, we introduced expanding boxes to reduce the computational time for both two-body collision term and potential interaction. A detailed explanation will be presented elsewhere.

IV. RESULTS

We consider three types of anisotropic flows. The first one is the sideward flow $\langle p_x \rangle$, which is the mean particle transverse momentum projected on to the reaction plane, where angle brackets indicate an average over particles and events. The directed flow $v_1$ is also used, which is defined by

$$v_1 = \langle \cos \phi \rangle = \left( \frac{p_x}{p_T} \right),$$

(51)

where $\phi$ is measured from the reaction plane, and $p_T = \sqrt{p_x^2 + p_y^2}$ is the transverse momentum. $z$-axis is the beam direction. The elliptic flow

$$v_2 = \langle \cos 2\phi \rangle = \left( \frac{p_x^2 - p_y^2}{p_T^2} \right)$$

(52)

reflects the anisotropy of transverse particle emission. These anisotropic flows are sensitive to the pressure built up during the collisions; thus, sensitive to the mean-field in the microscopic transport models [14–62].

It was shown in Ref. [73] that the sensitivity of the sideward flow to the Gaussian width parameter for heavy-ion collisions at the $E_{lab} \approx 1 A$ GeV regime, because the Gaussian width $L$ controls the range and strength of the interaction in the QMD approach. Thus, we will examine width dependence on the flow. IQMD uses $L = 2.165$ fm$^2$ for Au nucleus to obtain stable nuclear density profile [73], while UrQMD [68] uses $L = 1.0$ fm$^2$. The width in the JQMDF model [74] is $L = 2.0$ fm$^2$. Recent QMD model called PHQMD [38] uses $L = 0.54$ fm$^2$.

We first compare RQMD/S with the relativistic quantum molecular dynamics with the scalar potential (RQMDs). Then, new results from the relativistic quantum molecular dynamics with the vector potential (RQMD/v) will be present.

A. Comparison of RQMD/S and RQMDs

In this section, we compare the results from two different RQMD models with scalar potentials. The name RQMDs1 is used when the two-body distance Eq. (46) is used for the argument of potential, while RQMDs2 for Eq. (48).
In the left panel of Fig. [2] we compare the proton sideward flow \( \langle p_x \rangle \) in mid-central Au + Au collisions at \( \sqrt{s_{NN}} = 2.7, 3.3, 3.8, \) and 4.3 GeV \( (E_{lab} = 1.85, 4, 6, 8.4 \text{ GeV}) \). We use \( L = 2.0 \text{ fm}^2 \) for the Gaussian width as used in the previous calculations [46]. The impact parameter range is chosen to be \( 4 < b < 8 \text{ fm} \). In the left panel of Fig. [2] we compare four different approaches: RQMD/S, RQMDs1, and RQMDs2 for the MH2 EoS, and cascade mode, in which only collision term is included and potentials are disabled. As is well known, a cascade model lacks some pressure at AGS energies \((2.3 < \sqrt{s_{NN}} < 5 \text{ GeV})\), and a cascade model significantly underestimates the sideward flow. Both RQMD/S and RQMDs improve the description of the sideward flow due to an additional pressure generated by the mean-field. However, all calculations with scalar potentials predict less flow compared with the experimental data.

All three models with scalar potentials show good agreement with each other. The agreement of RQMDs with RQMD/S may justify the approximations in the RQMD/S model, which significantly simplifies the model compared to RQMDs.

We do not show the results for other parameter sets MS2, MH1, MS1 because their results are almost identical to those from MH2 in RQMD/S and RQMDs. This insensitivity of the sideward flow to the EoS is consistent with our previous finding in Ref. [46]. We argue that the main reason for this insensitivity is to use the scalar potential, which is a function of the scalar density. We have checked that the sideward flow results are not significantly modified with a smaller width \( L = 0.5 \text{ fm}^2 \) at AGS energies in the RQMDs model.

In the previous paper [46], we showed that RQMDs reproduce the flow data with the momentum-dependent mean-field. The reason for the discrepancy between the current result and the previous one is that we overestimate force by a factor of two due to a mistake in the earlier calculations. A comparison of the previous one and the corrected one is provided in the appendix A.

In the left panel of Fig. [3] we compare the rapidity dependence of the \( v_1 \) of protons and pions from cascade mode, RQMD/S, RQMDs1, and RQMDs2 with the STAR [20] and NA49 data [19]. We select the impact parameter range \( 4.6 < b < 9.4 \text{ fm} \) to compare 10-40% central collisions for Au + Au at 7.7, 11.5, and 19.6 GeV, and Pb + Pb collisions at 8.87 and 17.3 GeV. It is seen that the RQMD/S results are in good agreement with the RQMDs1 results for all incident energies, while RQMDs2 show somewhat less \( v_1 \) than the other models. All models predict negative proton \( v_1 \) slope above \( \sqrt{s_{NN}} = 10 \text{ GeV} \) except for cascade results. The negative \( v_1 \) is mainly generated during the expansion stage after two nuclei pass through each other. Additional potential interaction generates more negative flow. However, when we use stronger potential by taking smaller width \( L = 0.5 \text{ fm}^2 \), three models RQMD/S, RQMDs1, and RQMDs2 predict positive \( v_1 \) for protons at 11.5 GeV, which demonstrates the strong sensitivity of the slope of the proton directed flow to the interaction; both weak and strong interaction generate positive proton flow. We will discuss the dynamical origin of a negative flow in our model later.

In the left panel of Fig. [4] the beam energy dependence of the proton elliptic flow \( v_2 \) at mid-rapidity in mid-central Au + Au collisions from the cascade mode, RQMD/S and RQMDs2 are compared with the data. RQMDs1 results are not plotted because it is almost identical to the RQMDs2 result. Elliptic flow is also consistent with each other between models. The calculations with \( L = 2.0 \text{ fm}^2 \) predict less squeeze-out compared with the data below \( \sqrt{s_{NN}} < 5 \text{ GeV} \). We found that RQMDs2 with \( L = 0.5 \text{ fm}^2 \) improves the description of the elliptic flow.

In this section, we have demonstrated that two different implementations of the Skyrme force as a Lorentz scalar into the quantum molecular dynamics approach; RQMD/S and RQMDs yields almost the same results at relativistic energies,
and they improve the description of the data from the cascade model simulations. However, scalar potential does not generate enough pressure to reproduce anisotropic collective flows at AGS energies. In the next section, we will discuss results from the RQMDv model, in which the Skyrme potential is incorporated as a Lorentz vector potential.

B. RQMDv: RQMD with Lorentz vector potential

We shall now present the results of the RQMD model, in which the Skyrme potential is implemented as a Lorentz vector (RQMDv). Similarly to the previous section, the RQMDv1 model refers to the model which uses the two-body distance Eq. (46) for the argument of potential, while RQMDv2 uses Eq. (48).

The right panel of Fig. 2 shows the sideward flow from the RQMDv2 model for mid-central Au + Au collisions at $\sqrt{s_{NN}} = 2.7, 3.3, 3.8$ and 4.3 GeV. It is seen that vector potential predicts stronger flow than the scalar potential, and a good description of the data is obtained for the soft momentum-dependent EoS MS2 for both Gaussian width of $L = 0.5$ and 2.0 fm$^2$ and hard momentum-dependent EoS MH2 with $L = 2.0$ fm$^2$. Hard momentum-dependent MH2 EoS with $L = 0.5$ fm$^2$ overestimates the data. We note that the RQMDs1 results are almost identical to those of RQMDs2 results at AGS energies. We also note that the parameters MH2 is not as hard as the hard EoS at high baryon densities, as the parameter which controls the repulsive part of the potential is $\gamma = 1.67$ in MH2, while it is $\gamma = 2$ in hard momentum-independent EoS. If we compare the parameter set MH4 and MS1, we have slightly stronger EoS dependence than the parameter sets of MH2 and MS2.

The right panel of Fig. 3 shows the directed flow from the RQMDv1 model with $L = 2.0$ fm$^2$ and RQMDv2 with $L = 0.5$ fm$^2$ for protons (left panels) and pions (right panels). The EoS MS2 is used in the calculations. Both the RQMDv1 and RQMDv2 models describe the beam energy dependence of the proton directed flow data at SPS energies. Our models correctly predict the negative pion directed flow for all beam energies, which is due to the shadowing effects by the participant matter. As beam energy is increased, the models predict...
less slope than the data. We may need to include pion potential.

In Fig. 5, we examine the EoS and the Gaussian width dependence on the directed flow for mid-central Au + Au collision at $\sqrt{s_{NN}} = 11.5$ GeV from RQMDv1 (upper panel) and RQMDv2 (lower panel) compared with the STAR data.

In Fig. 5, hard and soft momentum-dependent EoS are compared. It is seen that $v_2$ is not sensitive to the EoS at 11.5 GeV, which is understood by the fact that Au + Au collision at 11.5 GeV does not prove high baryon densities due to the partial stopping of the nuclei.

The right panel of Fig. 4 compares the elliptic flow of protons at mid-rapidity from the RQMDv1 and RQMDv2 model for the mid-central Au + Au and Pb + Pb collisions with the experimental data. The EoS dependence between momentum-dependent soft and hard EoS for the elliptic flow is seen for the beam energy below 5 GeV. The effect of the Gaussian width on the elliptic flow is also seen, which indicates the sensitivity to the interaction strength. The elliptic flow using the smaller width of $L = 0.5$ fm$^2$ is slightly larger above 7 GeV due to a more negligible shadowing effect because of the shorter interaction range than the calculations with $L = 2.0$ fm$^2$. On the other hand, a shorter width predicts a stronger shadowing effect at energies less than 5 GeV due to stronger interaction.

### V. INSPECTIONS OF COLLISION DYNAMICS

We now investigate the collision dynamics how the directed flow is generated within our model. First, let us summarize the role of collision term for the generation of the directed flow.

In Ref. [80], we investigate the effects of spectator and meson-baryon interactions on the flow within a cascade model. When secondary interactions (mainly meson-baryon and meson-meson collisions) are disabled, negative proton flow is generated by the shadowing from the spectator matter at $\sqrt{s_{NN}} < 30$ GeV, while above 30 GeV there is no shadowing effect, and directed flow is not generated by the initial Glauber-type nucleon-nucleon collisions. The effect of the secondary interaction is to generate positive directed flow at $\sqrt{s_{NN}} < 30$ GeV since secondary interactions can start before two nuclei pass through each other. In contrast, negative flow is generated by the secondary interactions at $\sqrt{s_{NN}} > 30$ GeV, since secondary interactions start after two nuclei pass through each other due to a tilted expansion. We note that tilted expansion of the matter created in non-central heavy-ion collisions is highly sensitive to the interaction strength.
collisions is a general feature of the dynamics for a wide range of collision energies.

Let us now investigate the time evolution of the directed flow. In Fig. 6 we show in the upper panel the time evolution of the invariant interaction density $\rho_{B,i} = \sqrt{J_i^2}$, which is used for the density dependent part of the potentials in Eq. (42) for mid-central Au + Au collisions at $\sqrt{s_{NN}} = 11.5$ GeV from the RQMDs2 calculation are shown in the left panels. Right panels show the same but for the beam energy of 4.86 GeV. The solid lines show the results from default calculations. The dotted-dashed lines show the results of the calculations, which include the potential interaction for pre-formed baryons. The dashed lines represent the results of the calculation without interactions of spectator matter.

![Fig. 6](image)

FIG. 6. Time evolution of the invariant interaction density (upper panel) averaged over the central cell of $|x| \leq 3$ fm, $|y| \leq 3$ fm, and $|z| \leq 1$ fm and sign weighted directed flow $v_1^y$ of baryons at mid-rapidity $|y| < 0.5$ (lower panel) for mid-central Au + Au collisions at $\sqrt{s_{NN}} = 11.5$ GeV from the RQMDs2 calculation are shown in the left panels. Right panels show the same but for the beam energy of 4.86 GeV. The solid lines show the results from default calculations. The dotted-dashed lines show the results of the calculations, which include the potential interaction for pre-formed baryons. The dashed lines represent the results of the calculation without interactions of spectator matter.

A general feature of the temporal evolution of the directed flow at mid-rapidity is that it rises within a first few fm/c during the compression stages of the reaction and then decreases at the expansion stages for both 11.5 and 4.86 GeV. Finally, the flow goes up slowly at the very late stages of the collisions. Only formed baryons feel the potentials in the default simulation (solid lines), although the constituent quarks can scatter in the pre-hadrons. At 11.5 GeV beam energy, most of the nucleons are excited to strings, which results in the dip in the interaction density evolution at early times.

To see the effects of a possible potential interaction for the pre-formed baryons, we include the potential interaction for the pre-formed leading baryons that have original constituent quarks with the reduced factor of 1/3 (one quark) or 2/3 (di-quark) [29], which is shown in the dotted-dashed lines. Additional potential interaction generates two times more positive directed flow in the compression stage of the collision. It is also worthwhile to recognize that the directed flow decreases quickly even for the stronger interaction by generating more negative directed flow at expansion stages. We should emphasize that density-dependent interactions (hard or soft) do not predict negative directed flow in our framework because the interaction is weak, and it generates a small amount of negative flow during the expansion stage. When only density-dependent potentials are included, the potential becomes attractive at expansion stages at 11.5 GeV as the baryon density is around the normal nuclear density, which prevents developing the flow and positive flow developed in the collision stage remains positive at freeze-out. In contrast, in the case of momentum-dependent potential, the attractive force is mainly generated by the momentum-dependent part of the potential and, density-dependent part is repulsive. At the expansion stages, momenta of particles are random, and momentum-dependent part of the potential becomes weak, as a result, net effect of the interaction is repulsive, which contributes to generating strong negative flow. The negative proton directed flow at around 11.5 GeV beam energies can only be obtained for an appropriate amount of the interaction strength: weak interaction (including cascade mode) does not generate strong anti-flow, on the other hand, strong interaction generates a very large positive flow at the compression stage.

To investigate the primary mechanism of decreasing directed flow at expanding stages of the collisions, we have checked the effect of the spectator-participant interaction on the directed flow by disabling the interaction between them. Where ‘spectator’ is defined as the nucleons which will not collide in the sense of the Glauber type initial nucleon-nucleon collisions. Specifically, we first compute the number of nucleon-nucleon collisions, and we remove nucleons from the system if they will not interact with other nucleons. It is seen that when ‘spectators’ are not included in the simulation, the directed flow decreases less at expansion stages and the directed flow remains positive at 11.5 GeV. The shadowing effect by the spectator matter is large even at 11.5 GeV.

Besides the shadowing effect, the main dynamical origin of decreasing behavior of the directed flow at mid-rapidity at the expansion stages of the collision is the creation of an ellipsoid titled with respect to the beam axis, which generates anti-flow predominately over the normal flow as discussed in detail in Ref. [23] within a 3FD model. We argue that this mechanism is general; it holds true for all high energy none-central heavy-ion collisions. In Fig. 7, we plot baryon density and local velocity for mid-rapidity $|y| < 5$ at $t = 7$ fm/c for Au + Au collision at $\sqrt{s_{NN}} = 4.86$ GeV at the impact parameter $b = 6$ fm (upper panel), and $t = 3$ fm/c at $\sqrt{s_{NN}} = 11.5$ GeV (lower panel). It is seen that a tilt of the matter distribution is created for both energies. The tilted matter is a general consequence of the collision dynamics for none-central collisions, which is caused by the initial nucleon-nucleon collisions and the degree of nuclear stopping.

At lower energies, the compression time is long enough to create a large directed flow, mainly due to a strong repulsive interaction. At late times where the system starts to expand, ant-flow wins against normal flow. However, interaction is weaker during the expansion as compared to compression stages at lower energies. At very late times, after the tilted
FIG. 7. Baryon density distribution at the time $t = 7 \text{ fm}/c$ in Au + Au collision at $\sqrt{s_{NN}} = 4.86 \text{ GeV}$ at the impact parameter $b = 6 \text{ fm}$ (upper panel) and $t = 3 \text{ fm}/c$ at $\sqrt{s_{NN}} = 11.5 \text{ GeV}$ (lower panel). The thin (red) arrows show the local velocity of anti-flow, and the bold (blue) arrows indicate normal flow for mid-rapidity $|y| < 0.5$.

VI. CONCLUSION

We have developed a new mean-field model by implementing the Skyrme type potential into the RQMD framework as a Lorentz scalar (RQMDs) or vector (RQMDv) potential, including momentum-dependence. The RQMDs and RQMDv models are realized by the event generator JAM2 code. We have studied the mean-field effects on the directed and elliptic flows by using RQMDs and RQMDv. RQMDs does not generate enough pressure at AGS energies and fails to reproduce the flow data, while the RQMDs model describes the directed flow at $\sqrt{s_{NN}} > 10 \text{ GeV}$ where net baryon number at mid-rapidity start to decrease, and scalar potential play a role rather than vector potential. In contrast, the RQMDv approach, in which the Skyrme type potential is implemented as a Lorentz vector, generates a strong pressure at AGS energies and describes the flow data very well. RQMDv also predicts the correct sign change of the proton directed flow. The conventional hadronic mean-field explains the negative directed flow of protons within our approach.

The slope of the directed flow at freeze-out is determined by the delicate cancellation of the positive and negative flow at high energy mid-central heavy-ion collisions. We found that the positive directed flow develops more than the negative flow at compression stages of the collisions, while the more negative flow is developed during expansion stage due to a tilted expansion and shadowing by the spectator nucleon matter. At lower collision energies, a large positive flow can be developed by the strong repulsion due to high baryon density and long compression time. The strength of the interaction is weaker at the late expansion stage because of lower baryon density. A net effect is to have a positive flow at lower energies. On the other hand, at higher energies, where most secondary interactions start after two nuclei pass through each other, there is not enough time to develop positive flow during the short compression time. Contrary to lower energies, expansion time becomes longer due to a large number of produced particles. A net effect is to generate negative flow.

The results strongly depend on the model parameters; thus, we will not rule out the possible softening scenario within the current study. To confirm that our model correctly describes the collision dynamics, we need to perform more systematic data comparisons, e.g., the flow of strangeness particles. This line of work is in progress.

In this work, we examine the effect of scalar or vector potential separately, and we see that negative proton directed flow can be only be obtained with the momentum-dependent potential. It is mandatory to study other approaches such as the relativistic mean-field theory, in which both scalar and vector interactions are included. Recently, a transport approach with a vector potential that includes a first-order transition and a critical point has been formulated [63]. It is interesting to use this potential to see the effects of a phase transition on the flows.

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Appendix A: The correction of the RQMD/S in 2005

In this section, we correct the results of Ref. [46], and compare the corrected results for Au + Au collision at \( E_{\text{lab}} = 1.85\, A\, \text{GeV} \).

The equations of motion Eq.(34) may be evaluated as

\[
\dot{r}_i = \frac{p_i}{p_i^0} + \sum_{j \neq i}^N D_{ij} \frac{\partial^2 r_{T,ij}}{\partial p_i} + \sum_{j \neq i}^N E_{ij} \frac{\partial p_{T,ij}}{\partial p_i}, \tag{A1}
\]

\[
\dot{p}_i = - \sum_{j \neq i}^N D_{ij} \frac{\partial^2 r_{T,ij}}{\partial r_i}. \tag{A2}
\]

where

\[
D_{ij} = \frac{\rho_{ij}}{4L} \left[ \frac{m_i \partial V_{sk,i}}{p_i^0} + \frac{m_j \partial V_{sk,j}}{p_j^0} + \left( \frac{m_i}{p_i^0} + \frac{m_j}{p_j^0} \right) V_{m,ij} \right],
\]

\[
E_{ij} = \rho_{ij} \left( \frac{m_i}{p_i^0} + \frac{m_j}{p_j^0} \right) V_{m,ij} \frac{\partial V_{m,ij}}{\partial p_{T,ij}}. \tag{A4}
\]

We note that Eq.(A25) and (A26) in Ref. [46], which correspond to Eq. (A3) and Eq. (A4), contain mistakes. After correcting the mistake, we found that the results in Ref. [46] was modified; potential effects on the anisotropic flows become relatively smaller in the RQMD/S approach. The results in Ref. [44] are also influenced by this mistake. In this paper, we will present the corrected result.

The first factor in Eq.(A25) is wrong by a factor of two: \( \frac{1}{2L} \) should be \( \frac{1}{2L} \). Furthermore, \( 1/(1 - \rho_{T,ij}/\mu_k)^2 \) in Eq. (A26) should be \( (1 - \rho_{T,ij}/\mu_k)^2 \). In the code, only the first error was found. We will present the explicit expression here:

\[
D_{ij} = \left( \frac{1}{4L} \right) \rho_{ij} \left[ \frac{\alpha}{2p_0} \left( \frac{m_i}{p_i^0} + \frac{m_j}{p_j^0} \right) + \frac{\beta}{1 + \frac{\rho_0}{\rho_0}} \left\{ \frac{m_i}{p_i^0} \rho_1^{-1} + \frac{m_j}{p_j^0} \rho_1^{-1} \right\} \right] + \left( \frac{1}{4L} \right) \frac{1}{2p_0} \rho_{ij} \left( \frac{m_i}{p_i^0} + \frac{m_j}{p_j^0} \right) \sum_{k=1,2} \frac{C_k}{1 - p_{T,ij}/\mu_k^2}, \tag{A5}
\]

\[
E_{ij} = \frac{1}{2p_0} \rho_{ij} \left( \frac{m_i}{p_i^0} + \frac{m_j}{p_j^0} \right) \sum_{k=1,2} \frac{C_k}{1 - p_{T,ij}/\mu_k^2} \tag{A6}
\]

The corrected results show less collective flow than the previous ones. In the left panel of Fig. [8] the sideward flow in mid-central Au + Au collision at \( \sqrt{s_{NN}} = 2.7 \, \text{GeV} \) is shown from RQMD/S. The dotted line corresponds to the result, in which factor two is multiplied in the Skyrme force to show the wrong result in Ref. [46].

![Fig. 8](image-url)

**FIG. 8.** The directed flow (upper panel) and the elliptic flow (lower panel) in Au + Au collision at \( \sqrt{s_{NN}} = 2.7 \, \text{GeV} \). Dotted lines correspond to the results of Ref. [46], in which the factor two larger force is used by mistake. The corrected results are shown in the solid line. Impact parameter range \( 4.0 < b < 8.0 \, \text{fm} \) is used.

Appendix B: EoS parameters

The parameters of the potentials are fixed by the five conditions assuming a nuclear matter binding energy \( B = -16 \, \text{MeV} \) at a nuclear matter saturation density \( \rho_0 = 0.168 \, \text{fm}^{-3} \) and the momentum-dependent parameter sets fulfill the conditions \( U_{\text{opt}}(\rho_0, p = 1700 \, \text{MeV}) = 60 \, \text{MeV} \) and \( U_{\text{opt}}(\rho_0, p = 650 \, \text{MeV}) = 0 \, \text{MeV} \). At the saturation density and \( T = 0 \), we have the Weisskopf relation

\[
\sqrt{m_N^2 + p_f^2} + U(\rho_0) + U_m(p_f) = m_N + B \tag{B1}
\]

where the Fermi momentum is \( p_f = (6\pi^2 \rho_0 / g_N)^{1/3} \) with \( g_N = 4 \) being the spin-isospin degeneracy. Pressure \( P = \rho^2 \partial (\varepsilon / \rho) / \partial \rho|_{\rho_0} \) is zero at the saturation density. At zero temperature, the distribution function takes the form:

\[
f(x, p) = \frac{g_N}{(2\pi)^3} \theta(p_f - p) = g\theta(p_f - p), \tag{B2}
\]

and pressure at zero temperature is given by

\[
P = P_k + \frac{g}{2} \int_0^{p_f} d^3 p U_m(p) + p U(p) - \int_0^p U(p') dp'. \tag{B3}
\]
where

\[ P_k = g \int_0^{p_f} d^3 p \left( \frac{p^2}{3E} + \frac{p \partial U_m(p)}{3 \partial \rho} \right) \]

\[ = g \int_0^{p_f} d^3 p \left( \frac{p^2}{3E} - U_m(p) \right) + \rho U_m(p_f) \quad \text{(B4)} \]

The nuclear incompressibility \( K \) is defined by the second derivative of the energy density with respect to the baryon density:

\[ K = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{\rho} \right) \bigg|_{\rho=\rho_0} = 9\rho \frac{\partial^2 E}{\partial \rho^2} \bigg|_{\rho=\rho_0} \quad \text{(B5)} \]

The first derivative of \( E \) with respective to the baryon density gives the baryon chemical potential:

\[ \mu = \frac{\partial E}{\partial \rho} = \sqrt{m^2 + p_f^2} + U(\rho) + U_m(p_f) \quad \text{(B6)} \]

The incompressibility is now given by

\[ K = 9\rho \frac{\partial \mu}{\partial \rho} = 9 \left( \frac{p_f^2}{3E} + \rho \frac{\partial U}{\partial \rho} + \frac{p_f \partial U_m(p_f)}{3 \partial p_f} \right) \quad \text{(B7)} \]

where we used \( \frac{\partial \mu}{\partial p_f} = 3\rho/\rho_f \). From Eqs. (B1), (B4), and (B7), for the Skyrme potential Eq. (2) and the momentum-dependent potential Eq. (3), we obtain the system of equations:

\[ E_f + \frac{\alpha + \beta}{2} \rho_0 + \beta \gamma \rho_0 + \rho_0 U_m(p_f) - \frac{9}{2} \int_0^{p_f} d^3 p U_m(p) = 0, \quad \text{(B8)} \]

\[ \frac{p_f^2}{3E} + \alpha + \beta \gamma + \frac{p_f \partial U_m(p_f)}{3 \partial p_f} = K \frac{9}{g}, \quad \text{(B10)} \]

\[ \alpha + \beta = U_{\text{opt}}(\rho_0, p \to \infty), \quad \text{(B11)} \]

\[ (E_f + \frac{\alpha + \beta}{2} \rho_0 + \beta \gamma \rho_0 + \rho_0 U_m(p_f) - \frac{9}{2} \int_0^{p_f} d^3 p U_m(p) = 0, \quad \text{(B2)} \]

where \( E_f = \sqrt{m^2 + p_f^2} \) and

\[ P_{\text{kin}} = d \int_0^{p_f} d^3 p \left( \frac{p^2}{3E} + \frac{p \partial U_m(p)}{3 \partial \rho} \right) = \frac{gN}{16\pi^2} \]

\[ \times \left[ \frac{2}{3} E_f p_f^3 - m^2 E_f p_f + m^4 \log \left( \frac{E_f + p_f}{m} \right) \right] \quad \text{(B13)} \]

We solve these equations for \( \alpha, \beta, \gamma, \mu, \) and \( C \).

**Appendix C: scalar implementation**

In the case of the scalar potential, we have

\[ \mu = \frac{\partial e}{\partial \rho} = \sqrt{m^2(p_f) + p_f^2}, \quad \text{(C1)} \]

and the incompressibility is given by

\[ K = 9\rho \frac{\partial \mu}{\partial \rho} = 9 \left( \frac{p_f^2}{3E} + \rho \frac{\partial m^*}{\partial \rho} \right) \quad \text{(C2)} \]

The derivative of the effective mass with respect to the density can be calculated as

\[ \frac{\partial m^*}{\partial \rho} = \frac{\partial U_s}{\partial \rho_s} \frac{\partial \rho_s}{\partial \rho} = \frac{\partial U_s}{\partial \rho_s} \frac{m^*}{\epsilon^*} \frac{\partial m^*}{\partial \rho} \frac{g_N}{(2\pi)^3} \int d^3 p \frac{p^2}{\epsilon^*} \quad \text{(C3)} \]

which yielding \[ \text{[81]} \]

\[ \frac{\partial m^*}{\partial \rho} = \frac{\partial U_s}{\partial \rho_s} m^* \left[ 1 - \frac{\partial U_s}{\partial \rho_s} \frac{g_N}{(2\pi)^3} \int d^3 p \frac{p^2}{\epsilon^*} \right]^{-1} \quad \text{(C4)} \]

**Appendix D: Derivatives of transverse distances**

The calculation of the derivatives for transverse momentum can be done as follows. First, we consider the distance in the two-body c.m.:

\[ q_i^2 = q_i^2 - \frac{(q_i \cdot P_{ij})^2}{s}, \quad \text{(D1)} \]

\[ p_i^2 = p_i^2 - \frac{(p_i^2 - p_f^2)^2}{s^2}, \quad \text{(D2)} \]

where \( q_{ij} = q_i - q_j, p_{ij} = p_i - p_j, P_{ij} = p_i + p_j, \) and \( s = P_{ij}^2 \).

The derivatives are

\[ \frac{\partial q_i^2}{\partial r_i} = -2 \frac{r_i - (q_i \cdot P_{ij})}{s} \quad \text{(D3)} \]

\[ \frac{\partial q_i^2}{\partial p_i} = \frac{2(q_i \cdot P_{ij})}{s} \left[ r_i - P_{ij}^0 \frac{(q_i \cdot P_{ij})}{s} \right] \quad \text{(D4)} \]

\[ \frac{\partial p_i^2}{\partial r_i} = -2 \left[ p_i^2 - (p_i^0 - p_f^0) p_i \right] + P_{ij}^0 \frac{(p_f^2 - p_i^2)^2}{s^2} \quad \text{(D6)} \]

where \( \tilde{v}_{ij} = P_{ij}^0 / P_{ij} - p_f^0 / p_f \).

The two-body distances in the rest frame of a particle \( j \) are

\[ q_{Rij}^2 = q_{Rij}^2 - (q_{Rij} \cdot u_{ij})^2, \quad \text{(D7)} \]

\[ p_{Rij}^2 = p_{Rij}^2 - (p_{Rij} \cdot u_{ij})^2, \quad \text{(D8)} \]

where \( u_{ij} = p_{ij} / m_j \). The derivatives are

\[ \frac{\partial q_{Rij}^2}{\partial r_i} = -2r_i + 2(q_{Rij} \cdot u_{ij}) u_j, \quad \text{(D9)} \]

\[ \frac{\partial q_{Rij}^2}{\partial p_i} = 0 \quad \text{(D10)} \]

\[ \frac{\partial p_{Rij}^2}{\partial r_i} = \frac{2(q_{Rij} \cdot u_{ij})}{m_i} r_{ji} \quad \text{(D11)} \]

\[ \frac{\partial p_{Rij}^2}{\partial p_i} = \frac{2}{m_j} \left[ p_{ij}^0 v_i - p_{ij} - \frac{(p_{ij} u_{ij})}{m_j} p_{ij}^0 v_i \right] \quad \text{(D12)} \]

\[ \frac{\partial p_{Rij}^2}{\partial p_i} = \frac{2}{m_i} \left[ p_{ij}^0 v_i - p_{ij} + \frac{(p_{ij} u_{ij})}{m_i} p_{ij}^0 v_i \right] \quad \text{(D13)} \]
where \( v_i = p_i / p_{i0} \), and \( v_{ij} = v_i - v_j \).

### Appendix E: Equations of motion

#### 1. Equations of motion for RQMDs

The equations of motion for the RQMDs model are

\[
\dot{x}_i = \frac{p^*_i}{p^*_{i0}} + \sum_j m^*_i \frac{\partial m^*_j}{\partial p_i}, 
\dot{p}_i = -\sum_j m^*_i \frac{\partial m^*_j}{\partial r_i}.
\] (E1, E2)

where \( m^*_i = m_i + S_i \) and \( S_i \) is the scalar potential:

\[ S_i = V_i(\rho_{si}) + V_{m,ij}(p^2_{Tij}) \] (E3)

where the scalar density is given by

\[ \rho_{si} = \sum_{i \neq j} f_j \rho_{ij}, \quad f_j = m^*_j / p^*_{j0} \] (E4)

and the density dependent part is

\[ V_i(\rho_{si}) = \frac{\alpha}{2\rho_{0i}} \rho_{si} + \frac{\beta}{(1 + \gamma)} \left( \frac{\rho_{si}}{\rho_{0i}} \right)^\gamma \] (E5)

The momentum-dependent potential is given by Eq. (E4). The equations of motion Eq. (E1) and Eq. (E2) become

\[
\dot{x}_i = \frac{p^*_i}{p^*_{i0}} + \sum_{j \neq i} \left[ D_{ij} \frac{\partial q^2_{ij}}{\partial p_i} + D_{ji} \frac{\partial q^2_{ji}}{\partial p_i} \right], 
\dot{p}_i = -\sum_{j \neq i} \left[ E_{ij} \frac{\partial p^2_{ij}}{\partial p_i} + E_{ji} \frac{\partial p^2_{ji}}{\partial p_i} \right],
\] (E6, E7)

where

\[
D_{ij} = \frac{m_i^*}{p^*_{i0}} f_j \left( \frac{\partial V_i(\rho_{si})}{\partial \rho_{si}} + V_{m,ij} \right) \rho_{ij} / 4L, 
E_{ij} = \frac{m_i^*}{p^*_{i0}} f_j \frac{\partial^2 V_{m,ij}}{\partial \rho_{si}^2} \rho_{ij}, 
V_{m,ij} = \frac{C}{2\rho_{0i} \left( 1 - p^2_{ij}/\mu^2 \right)},
\] (E9, E10, E11)

If \( q_{Tij} \) is defined as the distance in the two-body c.m., \( q_{Tij}^2 = q_{Tij}^2 \). One may need to add the term that comes from the derivative of the factor \( f_j \) in the scalar density

\[
\sum_{j \neq i} m^*_i \frac{\partial S_i}{\partial \rho_{si}} \frac{\partial f_j}{\partial p_i}.
\] (E12)

#### 2. Equations of motion for RQMDv

The equations of motion for RQMDv are

\[
\dot{x}_i = \frac{p^*_i}{p^*_{i0}} + \sum_{j} v_{j}^\mu \frac{\partial V_{ij}^\mu}{\partial p_i}, 
\dot{p}_i = -\sum_{j} v_{j}^\mu \frac{\partial V_{ij}^\mu}{\partial r_i}.
\] (E13, E14)

The equations of motion Eq. (E13) and Eq. (E14) for the vector implementation have the same structure as Eq. (E7) and different \( D_{ij} \) and \( E_{ij} \):

\[
D_{ij} = \frac{\rho_{ij}}{4L} v_{i}^\mu (B_i B_j A_{ij\mu} + u_{j\mu} V_{m,ij}), 
E_{ij} = v_{i}^\mu u_{j\mu} \frac{\partial V_{m,ij}}{\partial p_{ij}}, 
J_{ij}^\mu = \sum_{i \neq j} B_j u_{j\mu} \rho_{ij},
\] (E15, E16, E17)

where \( B_i \) is the baryon number and \( V_i \) is a function of an invariant baryon density \( \rho_{Bi} = \sqrt{J_i^2} \).

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