Nonlinear oscillations of gas in an open tube near the resonance frequency in the shock-free mode

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Abstract. The forced oscillations of gas in an open tube, excited by harmonical oscillations of piston in the shock-free mode were investigated near the first first eigenfrequencies. An expression for the pressure oscillations of gas was obtained for the tube with unrounded end without flange. The amplitude impact of piston displacement on the oscillations of pressure and velocity of the secondary flow of gas was investigated. The comparison of theoretical calculations with experimental data was executed. The effect of secondary flow on the particle drift along the tube axis with acoustic oscillations of gas was shown.

Introduction
Various modern equipment often contain tubing systems with finite length with rest or moving homogeneous or heterogeneous operating environment [1]. Review of studies on nonlinear resonant oscillations of a homogeneous gas in tubes with different conditions at the ends is given in [2].

Focused on the study of nonlinear oscillations of gas in the tube and the external wave field at large excitation amplitudes (of the order of 0.1-0.4 bar) in the shock-wave mode, periodic shock waves was occurred. Gas oscillations in the shock-free mode at low amplitudes of excitation (about 0.01 bar) is not well understood.

The purpose of this work is research of the resonant oscillations of gas occurring by secondary flow in the shock-free mode in a tube with an open end.

1. Basic conditions
Harmonic oscillations of gas $x = l \sin \omega t$ in a long cylindrical tube ($l \ll L_0, L_0/R \gg 1$, where $l$ - the amplitude of the displacement of the piston, $R$ - radius, $L_0$ - length, $\omega$ - the angular frequency) are described by [3]

$$
\bar{p}_i = \eta_i \cos \left( k_i x + \alpha_i + i\beta_1 \right) e^{i(\omega t + \phi_1)}, \quad \bar{u}_i = -i\eta_i \sin \left( k_i x + \alpha_i + i\beta_1 \right) e^{i(\omega t + \phi_1)},
$$

(1)

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Where \( \bar{\rho}_i = \rho_i \sqrt{\rho_v c_s^2} \), \( \bar{u}_i = u_i / c_s \), \( k_i = (\omega/c_o)(1 + \beta_i - j\beta'_i) \) – complex wave number, 
\( \beta_i = \sqrt{\mu_i^2/2\rho_0 \omega [1 + (\kappa - 1)/ \text{Pr}]} / R \) – the dissipation coefficient, \( \kappa = c_p / c_v \) – ratio of specific heats at constant pressure and volume, \( \rho_0 \) – density, \( \text{Pr} \) – Prandtl number, \( c_o \) – speed of sound in the undisturbed gas, \( r_i, \psi_i, \alpha_i, \beta_i \) – constants.

Substituting (1) into the boundary conditions at the ends of the tube [4]

\[
\bar{u}_{i\rho} = M_{\rho} e^{i\omega t}, \quad x = 0; \quad \bar{P}_{iE} = m r_i e^{i(\omega t + \phi)}, \quad m = \frac{2}{3\pi} (1 + k_{mn}), \quad x = L.
\]

(2)

Here, \( m \) contains parameter \( k_{mn} \), which is determined experimentally [5], and depends on the shape of the open end of the tube, \( L = L_0 + \sigma R \) - reduced tube length, \( \sigma R \) - correction of Rayleigh, \( \sigma = 0.6133 \). As a result, we obtain expressions for the constants \( r_i, \psi_i, \alpha_i, \beta_i \)

\[
\begin{align*}
    r_i &\sqrt{\cos^2 \left[k_o L (1 + \beta'_i)\right] + (k_o L (1 + \beta'_i) + m r_i)^2 \sin^2 \left[k_o L (1 + \beta'_i)\right]} = M_{\rho}, \\
    \alpha_i &\left[\frac{\pi}{2} - k_o L (1 + \beta'_i)\right], \quad \beta_i = k_o L (1 + \beta'_i) + m r_i, \quad \psi_i = \arctg \left(\frac{\tan \alpha_i}{\beta_i}\right),
\end{align*}
\]

(3)

2. Experiment

Experimental researches were carried out on an automated equipment, previously used to study the nonlinear oscillations of a homogeneous gas in a closed tube in the shock-free mode [6]. Longitudinal stationary oscillations of gas in a vertical quartz tube (length \( L_0 = 1.06 \) m, inner diameter \( 2R = 0.0365 \) m), created by a sinusoidally oscillating piston. Piston is driven by the vibration generator with power amplifier TV51075. Passive end of the tube was opened. Vibration generator was controlled by piezoelectric IEPE accelerometer controller using a software module SineVIEW (VR610), installed on a computer. Piezoelectric pressure sensor was hostead close to the piston, the signal from sensor was transfered through the three-channel bridge voltage amplifier to the digital oscilloscope and then to the computer, where the value and form of signal were recorded with accurate to 0.3 mV.

3. Results

The theoretical dependence and the experimental data for the dimensionless amplitude of oscillations of the gas pressure are given on the Figure 1, \( \Delta p_i = (\Delta p_i / p_0) \times 10^2 \) (\( \Delta p_i \) – difference of the maximum and minimum values of the pressure for the piston stroke, \( p_0 \) – the equilibrium pressure of of gas) on the relative frequency of excitation \( \nu = \nu_i / \nu_{i\rho} \) (\( \nu_{i\rho} \)–experimentally observed resonance frequency) for a given values of the relative amplitude of the displacement of the piston. There is dotted line represents the resonant frequency of excitation of the gas, calculated from the linear theory [2] \( \nu_i = c_o / 2L = 80.1 \) Hz. Dashed line is a frequency obtained in the presence of losses \( \nu_{i\rho} = c_o / 2L (1 + \beta'_i) = 78.6 \) Hz, which mathes with the value observed in the experiment. The results of theoretical oscillation calculations of gas pressure for the unrounded end of tube without flange \( m = 0.84 \) [5] are somewhat higher than the experimental data. The discrepancy is about 10% for all studied displacement amplitude of the piston. Also Figure 1, b shows a typical waveform recordings with pressure oscillations of gas in the time of the experimentally observed resonant frequency. Pressure diagrams have an uninterrupted view, and no breaks were observed. The form of diagrams is somewhat different from the harmonic (the duration of the leading edge somewhat less than the duration of the trailing edge of the wave).
Figure 1. Dependences of gas pressure near the piston on $\nabla$ (a) and on $t$ (b): $1-T=6.13$, $2-T=9.43$, $3-T=12.73$. Solid lines - theory, dotted - experiment.

To calculate the secondary flows we’ll use the method proposed in [7]. As a result of integration, we have the expressions for the axial and radial components of the velocity of the secondary current in the form

$$
\langle u_z \rangle = \frac{\omega R_i^2}{4} \left[ g_1(0,Pr) \left( 1 - \frac{2r^2}{R^2} \right) + g_1(\eta,Pr) - \frac{\delta}{R} g_3(0,Pr) \left( 1 - \frac{r^2}{R^2} \right) \right] \sin 2\nu + \\
+ \left[ g_0(0,Pr) \left( 1 - \frac{2r^2}{R^2} \right) + g_0(\eta,Pr) + \frac{\delta}{R} g_2(0,Pr) \frac{1}{\beta'_1} \left( 1 - \frac{r^2}{R^2} \right) \right] \sinh 2\nu,
$$

(4)

$$
\langle v_z \rangle = \frac{\omega R_i^2}{8} \left[ 2g_1(0,Pr) \left( \frac{r^3}{R^3} - \frac{r}{R} \right) - \frac{\delta}{R} g_3(0,Pr) \left( \frac{r^3}{R^3} - \frac{2r}{R} \right) - \frac{\delta}{R} g_3(\eta,Pr) \right] \cos 2\nu - \\
- \left[ 2\beta' g_0(0,Pr) \left( \frac{r^3}{R^3} - \frac{r}{R} \right) + \frac{\delta}{R} g_2(0,Pr) \left( \frac{r^3}{R^3} - \frac{2r}{R} \right) - \frac{\delta}{R} g_2(\eta,Pr) \right] \cosh 2\nu,
$$

(5)

where

$$
\nu = \frac{\omega x}{c_0} (1 + \beta_1'), \; \omega = \beta_1 - \frac{\omega x}{c_0} \beta_1', \; g_0(\eta,Pr), \; g_1(\eta,Pr), \; g_2(\eta,Pr), \; g_3(\eta,Pr)
$$

are the functions depending on $\eta = (R - r)/\delta$, $\kappa$ and $Pr$, $\delta = \sqrt{2\mu/\rho_0 \omega}$ - is the thickness of the Stokes’s layer.

The calculations show that the flow in the form of one toroidal vortex with movement direction to the open end of the tube is being observed at the tube axis. In the near-wall region there are vortices with movement direction opposite to the one in the central region. One can note the presence of areas of the vortex movement near the piston and at the open end of the tube with the direction of movement opposite to the main vortices. The flow velocity in the near-wall region is essentially lower than in the stream nucleus (for the axial component of velocity the difference is 10 times, for the radial one – 100 times). This is connected with the presence of the near-wall absorption. While amplitude of the
displacement of the piston increases both axial and radial velocity components increase as well. Figure 2 shows the distribution of the axial velocity component on the axis (with \( r \to 0 \)) along the tube length for different amplitudes of the displacement of the piston. Near the piston and at the open end of the tube the value of the axial component of the velocity of the secondary flow differs from zero and has a negative value. This is connected with the presence of the item with \( \text{sh} \, 2w \) which is aperiodic on \( x \) in the expression for the axial velocity component. With the increase of the amplitude of the displacement of the piston the secondary flow velocity increases on the axis of the tube.

![Figure 2. Dependences of \( \langle u_z \rangle \) on \( \tilde{x} \) with \( \tilde{r} = 0 \): 1 \(-\) \( \tilde{T} = 6.13 \), 2 \(-\) \( \tilde{T} = 9.43 \), 3 \(-\) \( \tilde{T} = 12.73 \)

The presented calculation of the secondary flow velocity can be used to estimate the influence of the acoustic flow on the particle dynamics with the gas oscillations in tube. The Stokes’ force, the Archimedes’ force, the force of friction, the Basset’s force and others, as well as acoustic flows \([8, 9]\) act on a particle in a wave field. At the same time the direction and the drift velocity of the particle depend on the particles parameters and the wave field characteristics \([10, 11]\), which is widely used in the wave technologies of the particles separation.

Consider the data for the experimental investigation of the drift of polystyrene flat particles (diameter 9 mm, thickness 0.4 mm, weight 5.7 mg) strung along the axis of the tube open at gas oscillations on the fishing line. Particle was placed at predetermined distances along the tube axis and after launching the sinusoidal oscillations particle drift started filming. Particle placed near the open end of the tube, moving up to a certain point in the direction of the piston, making oscillations. When placing the particles closer to the piston, reversed movement is observed to the open end of the tube. This behavior of particle is consistent with obtained pattern of secondary flows. Thus, when the relative amplitude is \( \tilde{T} = 16.34 \), the velocity of particle installed near the open end is 0.025 m / s, and installed near the piston - 0.05 m / s. The particle velocity is 2.5 times less than the velocity of the secondary flow on the axis of the tube. This is due to the influence of the other forces drift such as friction forces of the fishing line.

4. Conclusions
The forced oscillations of gas in an open tube, excited by harmonical oscillations of piston in the shock-free mode were investigated near the first eigenfrequencies frequency. The observed resonance frequency has less value than calculated by the linear theory, but coincides with the frequency calculated by taking into account the nonlinear theory of near-wall losses. We found that the maximum axial velocity component is in the middle of the tube, and radial component maximum is in
the ends of the tube. The flow velocity in the boundary region is considerably lower than in the core stream (the difference for an axial velocity component is 10 times, for the radial is 100 times). This is due to the presence of a wall surface absorption. With increase of the amplitude of the piston displacement and intensity oscillations, there is increasing of the axial and radial velocity components. It is shown that the acoustic flow has a significant effect on the particles drift. Particle moves to a certain point in the direction of the piston, making oscillations near the open end of the tube. Reverse movement is observed in the open end of the tube when particle is placing closer to the piston.

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