The thermal emission from boulders on (25143) Itokawa and general implications for the YORP effect

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ABSTRACT

Infrared radiation emitted from an asteroid surface causes a torque that can significantly affect rotational state of the asteroid. The influence of small topographic features on this phenomenon, called the YORP effect, seems to be of utmost importance. In this work, we show that a lateral heat diffusion in boulders of suitable sizes leads to an emergence of a local YORP effect which magnitude is comparable to the YORP effect due to the global shape. We solve a three-dimensional heat diffusion equation in a boulder and its surroundings by the finite element method, using the FreeFem++ code. The contribution to the total torque is inferred from the computed temperature distribution. Our general approach allows us to compute the torque induced by a realistic irregular boulder. For an idealized boulder, our result is consistent with an existing one-dimensional model. We also estimated (and extrapolated) a size distribution of boulders on (25143) Itokawa from close-up images of its surface. We realized that topographic features on Itokawa can potentially induce a torque corresponding to a rotational acceleration of the order $10^{-7}\text{ rad day}^{-2}$ and can therefore explain the observed phase shift in light curves.

Key words: minor planets, asteroid: individual: (25143) Itokawa – methods: numerical.

1 INTRODUCTION

The Yarkovsky–O’Keefe–Radzievskii–Paddack effect is the torque caused by the infrared emission from an asteroidal surface that has an influence on a rotational state of an asteroid (Rubincam 2000). It is now widely recognized as an important factor, affecting the evolution of rotational states of asteroids alongside mutual collisions and tidal forces. The YORP effect helped to explain numerous observed phenomena, such as a spin axis alignment of asteroids in the Koronis family (Vokrouhlický et al. 2003), a non-maxwellian rotational frequency distributions of small main-belt asteroids (Pravec et al. 2008) or significant binary asteroid population among near-Earth objects (Walsh et al. 2012).

Even a direct evidence of a non-gravitational torque has been found. A phase shift in light curves has been measured for a few asteroids that cannot be explained by a solely gravitational model — (1862) Apollo (Kaasalainen et al. 2007), (54509) YORP (Lowry et al. 2007), Taylor et al. (2007), (1620) Geographos (Ďurech et al. 2008), (3103) Eger (Ďurech et al. 2012), and finally, (25143) Itokawa (Lowry et al. 2014). There is no known mechanism except the YORP effect that could explain the quadratic trend in rotational phase observed for these asteroids.

The asteroid Itokawa has been a suitable candidate for a detection of YORP effect for its highly asymmetric shape and its favourable position among near-Earth objects. Vokrouhlický et al. (2004) predicted a measurable acceleration of rotation of the order of $10^{-7}\text{ rad day}^{-2}$, based on the shape model derived by radar ranging. Itokawa was then a target of the Hayabusa spacecraft in 2005 and a state-of-the-art shape model of the asteroid was constructed from silhouette images (Gaskell et al. 2006). The torque computed using the latter model would lead to a significant deceleration $-(1.8 \text{ to } 3.3) \times 10^{-7}\text{ rad day}^{-2}$ (Scheeres et al. 2007). However, the measured phase shift in light curves revealed an acceleration $(0.35\pm0.04) \times 10^{-7}\text{ rad day}^{-2}$ (Lowry et al. 2014). Theoretical models did not predict even the sign of the effect correctly. This discrepancy between the observed and predicted change of the angular frequency is concerning and only one viable explanation has been put forward to date.

The observed rotational acceleration could be attributed to density inhomogeneities in the asteroid. Scheeres and Gaskell (2008) showed that the YORP effect on Itokawa is indeed sensitive to the position of the center of mass.
Based on the measured acceleration, \cite{Lowry2014} computed the required offset between the center of mass and the center of figure to be \(\sim 21\) m. Such offset indicates that the asteroid might consist of two parts with substantially different densities — \((2850 \pm 500)\) kg m\(^{-3}\) and \((1750 \pm 110)\) kg m\(^{-3}\).

The deceleration predicted by \cite{Scheeres2007} was computed from the shape model with \(~ 50000\) facets. Calculations of the effect with a more detailed shape lead to an even bigger deceleration. According to \cite{Breiter2009}, the deceleration does not show any sign of convergence with increasing resolution, implying that even sub-meter sized surface features possibly have a non-negligible influence. For the highest available shape resolution, their model predicted the deceleration \(-5.5 \times 10^{-7}\) rad day\(^{-2}\).

\cite{Lowry2014} employed the advanced thermophysical model \cite{Rozitis2011, Rozitis2012, Rozitis2013}, including the effect of thermal infrared beaming and the global self-heating of the asteroid. By varying the distribution of rough surface in patchy ways, the model showed a change of angular velocity \((-1.80 \pm 1.96) \times 10^{-7}\) rad day\(^{-2}\) (see Fig. 5 in the cited paper). Remarkably, an acceleration can be achieved even without a shift of the center of mass. However, this result was obtained only in 16.5% cases and the roughness distribution corresponding to these cases seems rather artificial.

There is a problem that shapes with surface features of sub-meter sizes cannot be easily included in existing models of the YORP effect. There are several reasons for this limitation. First, numerical YORP models typically assume that temperature changes only in the direction perpendicular to the surface (i.e., a plane-parallel approximation). This assumption allows a solution of the one-dimensional heat diffusion equation for each surface facet independently. It is well justified as long as surface features are significantly larger then the diurnal thermal skin depth, which varies from mm to dm \cite{Vokrouhlicky1999}. This assumption is no longer applicable for a high-resolution shape model and a full three-dimensional solution of the heat diffusion equation is required. Second, no shape is described to the required level of detail. So far, the best shape model is that of the asteroid Itokawa. The model in the best available resolution consists of over 3 million facets, which corresponds to meter-sized surface features \cite{Gaskell2006}.

As \cite{Golubov2012} pointed out, surface features of sizes comparable to the thermal skin depth could potentially have significant influence on the total YORP effect. They considered a stone wall (an idealized boulder) located on the equator of a spherical asteroid and aligned with a local meridian. The wall was assumed to be high enough so that the heat would be mostly conducted in a transverse direction and the heat diffusion equation can be solved using the one-dimensional approximation. They demonstrated that the emission from the surface of the wall can create a torque that will not vanish after averaging over the rotational period. Assuming a large number of such “walls” placed along the equator, the corresponding torque may be comparable to the torque arising from the global-shape asymmetry.

\cite{Golubov2014} generalized the previous model by assuming spherical boulders. They studied a dependence of the torque on a number of free parameters of the problem.

\begin{equation}
Ku + \epsilon u^3 = F
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The domain \(\Omega\) and boundary conditions of our problem. The boulder is located on the top. The strips indicate the boundary \(\Gamma_2\), where the temperature is kept by a Dirichlet boundary condition. The temperature distribution inside the domain and on the surface \(\Gamma_1\) is then computed numerically as a solution of Eq. \ref{eq:heat} with boundary conditions, Eqs. \ref{eq:heat_u} to \ref{eq:heat_n}.}
\end{figure}

As expected, the resulting torque is lower than in a simple one-dimensional model; nevertheless, boulders can still affect rotational dynamics significantly. Authors assumed even mutual shadowing and heating of the boulders.

The goal of this paper is to solve the heat diffusion equation in a realistic boulder. The problem requires a numerical solution in a general three-dimensional domain. We derive a formulation of the numerical problem in Section 2. We discuss the magnitude of the torque induced by a single boulder in Section 3. We estimate the total torque that boulders contribute to the YORP effect on the asteroid Itokawa in Section 4. Finally, the results of our model and the implications are summarized in Section 5.

\section{The Heat Diffusion Equation and a Weak Formulation of the Problem}

Our problem may be specified as follows. We search for a temperature \(u(\vec{r}, t)\) inside the boulder and its surroundings, i.e., an unknown scalar function on a domain \(\Omega\). The differential operator corresponding to the heat diffusion equation (HDE) is:

\begin{equation}
\mathcal{L} \equiv \rho C_\partial u - \nabla \cdot K \nabla u,
\end{equation}

where \(K\) denotes the thermal conductivity, \(\rho\) the density, \(C\) the specific heat capacity of the material. The function \(u\) thus has to fulfill the relation:

\begin{equation}
\mathcal{L}(u) = 0.
\end{equation}

At the same time, we require the boundary conditions to be met at the boundary \(\partial \Omega\) of the domain.

\subsection{Boundary conditions}

The boundary consists of two parts denoted \(\Gamma_1\) and \(\Gamma_2\), as shown in Figure 1. The boundary \(\Gamma_1\) represents the surface of the asteroid, the boundary condition is essentially an energy balance equation:

\begin{equation}
K \partial_n u + \epsilon \sigma u^4 = F,
\end{equation}

\begin{equation}
\nabla \cdot (K \nabla u) - \rho C \partial_t u = 0
\end{equation}

\begin{equation}
\partial_n u = 0
\end{equation}

\begin{equation}
\partial_t u = 0
\end{equation}

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\end{equation}
where $\partial_\alpha$ denotes a derivative along the normal, $\epsilon$ the infrared emissivity, $\sigma$ the Stefan–Boltzmann constant, $F$ the incoming flux absorbed by the surface. The boundary $\Gamma_1$ is a non-convex surface, thus the radiative heat exchange also contributes to the total flux $F$ (i.e. the self-heating effect). We denote the solar flux contribution $F_{\odot}$ and the contribution of flux coming from visible parts of the surface — either the thermally emitted flux or the scattered flux — as $F_{\text{th}}$ and $F_{\text{sc}}$ respectively. The total absorbed flux is the sum $F = F_{\odot} + F_{\text{th}} + F_{\text{sc}}$. These terms can be expressed as:

$$F_{\odot} = (1 - A) \Phi \mu \hat{s} \cdot \hat{n},$$

$$F_{\text{th}} = (1 - A) \int_{\Gamma_1} \epsilon \sigma u^4 \cos \alpha \cos \alpha' \frac{\nu}{\pi(r - r')^2} d\Gamma',$$

$$F_{\text{sc}} = (1 - A) \int_{\Gamma_1} A' \Phi \mu \hat{s} \cdot \hat{n} ' \cos \alpha \cos \alpha' \frac{\nu}{\pi(r' - r^2)} d\Gamma',$$

where $A$ is the hemispherical albedo, $\Phi$ the flux of solar radiation, $\hat{s}$ the body–Sun direction, $\hat{n}$ the outward normal to the surface, $\hat{r}$ the position vector, $\alpha$ the angle between the local normal and the direction vector connecting points $\hat{r}$ and $\hat{r}'$, $\mu$ the shadowing function, $\nu$ the visibility function (see Table 1). The prime denotes a value of a quantity at the point of surface element $d\Gamma'$. We assume the Lambert’s cosine law for the intensity of thermal emission and scattered radiation, hence the $\cos \alpha'$ in equations (5) and (6).

The shadowing function $\mu$ is defined on the surface $\Gamma_1$. The value of $\mu(\hat{r})$ equals 1 if the point $\hat{r}$ is insolated, 0 if it lies in the shadow. The visibility function $\nu$ is defined on $\Gamma_1 \times \Gamma_1$ space. We assign the value of function $\nu(\hat{r}, \hat{r}')$ to 1 if points $\hat{r}$ and $\hat{r}'$ have a visual contact, 0 otherwise. In most cases, the value of $\nu$ is simply $\nu(\hat{r}, \hat{r}') = H(\hat{n} \cdot (\hat{r} - \hat{r}')) H(\hat{n}' \cdot (\hat{r}' - \hat{r}'))$, where $H$ is the Heaviside step function and $\hat{n}, \hat{n}'$ denote the local normal at points $\hat{r}$ and $\hat{r}'$.

The discretized boundary $\Gamma_1$ is defined by a set $S_i$ of triangular facets. Integrals in equations (5) and (6) can be therefore computed as sums, $\int_{\Gamma_1} d\Gamma' \rightarrow \sum_i S_i$. The values of the shadowing function $\mu$ and the visibility function $\nu$ always correspond to whole facets. This restriction gives rise to an error; however, it can be estimated and limited substantially by choosing a high-resolution surface mesh.

We also need to specify conditions on the boundary $\Gamma_2$, which goes through the interior of the asteroid, closing the boundary $\partial \Omega$. It can be selected arbitrarily; we choose a shape corresponding to five walls of a block, which is a convenient choice as we can simply set a zero-flux boundary condition:

$$K \partial_n u = 0. \quad (7)$$

This condition will be met as long as dimensions of the domain $\Omega$ are significantly greater than dimensions of the boulder. The influence of the boulder can be considered negligible at large distances. At sides of the domain, the temperature will only change in the direction perpendicular to the surface, the dot product of the normal vector and the temperature gradient will therefore be null. At great depths, the temperature will be effectively constant, which means the temperature gradient at the bottom of the domain will be null, satisfying the boundary condition (7).

We also need to specify an initial condition as the HDE is an evolution equation. However, we seek for a stationary solution that does not depend on a chosen initial condition. The choice of a condition will affect the speed of convergence only. The best available estimate of the solution is the linearized analytical solution in a half-space domain, derived in Appendix A hence we set:

$$u = u_{\text{theory}}, \quad t = 0. \quad (8)$$

2.2 A discretization

We are going to solve the HDE (Eq. 2) numerically, using a finite-element discretization in space. In this approach, the function $u$ is approximated by (Langtangen 2003):

$$u \doteq \tilde{u} \doteq \sum_{j=1}^{M} u_j N_j, \quad (9)$$

where $N_j$ denote prescribed basis functions, $u_j$ unknown coefficients we search for and $M$ corresponds to the number of vertices defined on the domain. Since $\tilde{u}$ is only an approximation of $u$, applying the operator would generally yield
a non-zero result:
\[ \mathcal{L}(\hat{u}) \neq 0, \quad (10) \]

nevertheless, we require the integral of all residua over the domain to be zero:
\[ \int_{\Omega} \mathcal{L}(\hat{u}) W_i \, d\Omega = 0, \quad (11) \]
where \( W_i \) are suitable weighting (test) functions. This is called a weak formulation of the problem. In the Galerkin method, the test functions are simply the basis functions, \( W_i \equiv N_i \), so that:
\[ \int_{\Omega} \mathcal{L}(\hat{u}) N_i \, d\Omega = 0. \quad (12) \]

Essentially, this constitutes a system of \( M \) equations for \( u_j \) coefficients.

In our case of the HDE (Eq. 2):
\[ \int_{\Omega} \rho C \partial_t \hat{u} N_i \, d\Omega - \int_{\Omega} \nabla \cdot (K \nabla \hat{u}) N_i \, d\Omega = 0. \quad (13) \]
The second term may be rewritten according to the Green formula as:
\[ \int_{\Omega} \nabla \cdot (K \nabla \hat{u}) N_i \, d\Omega = -\int_{\Omega} K \nabla \hat{u} \cdot \nabla N_i \, d\Omega + \oint_{\partial \Omega} K \partial_\nu \hat{u} N_i \, d\Gamma, \quad (14) \]
which enables to incorporate the boundary condition easily, because we can express the normal derivative from boundary conditions [3] and [7], that is \( K \partial_\nu \hat{u} = -\epsilon \sigma \hat{u} + \mathcal{F} \) on \( \Gamma_1 \), \( K \partial_\nu \hat{u} = 0 \) on \( \Gamma_2 \).

Regarding the temporal derivative, we use a finite-difference discretization:
\[ \partial_t \hat{u} \simeq \frac{\hat{u}^n - \hat{u}^{n-1}}{\Delta t} \quad (15) \]
and an implicit Euler scheme, so that we plug \( \hat{u}^n \) in the remaining terms, whenever possible. The only exception is the non-linear radiative term, where we perform a linearization:
\[ (\hat{u}^{n,m})^4 \simeq (\hat{u}^{n,m-1})^3 \hat{u}^{n,m} \quad (16) \]
and we employ an iterative method to find a solution; at the given time-step (denoted by superscript \( n \)) we find a sequence of solutions \( \hat{u}^{n,m} \) to a linear problem, until \( |\hat{u}^{n,m} - \hat{u}^{n,m-1}| \) is sufficiently small. The initial value \( \hat{u}^{n,0} \) can be selected arbitrarily, a common choice is \( \hat{u}^{n,0} = \hat{u}^{n-1} \).

In some cases, the iterative method does not converge and we thus introduce a relaxation parameter \( \zeta \). In each iteration, we find a preliminary solution \( \hat{u}^{n,m} \) of the linear problem and we then assign a new value of \( \hat{u}^{n,m} \) by taking a linear combination of the current and previous solutions:
\[ \hat{u}^{n,m} \equiv \zeta \hat{u}^{n,m} + (1 - \zeta) \hat{u}^{n,m-1}. \quad (17) \]
We achieved a convergence for all considered values of parameters by selecting \( \zeta = 0.6 \).

The final equation is thus:
\[ \int_{\Omega} \frac{\rho C}{\Delta t} \hat{u}^{n,m} N_i \, d\Omega - \int_{\Omega} \frac{\rho C}{\Delta t} \hat{u}^{n-1,m} N_i \, d\Omega + \int_{\Omega} K \nabla \hat{u}^{n,m} \cdot \nabla N_i \, d\Omega + \]
\[ \int_{\Gamma_1} \sigma \epsilon (\hat{u}^{n,m})^4 \hat{u}^{n,m} N_i \, d\Gamma - \int_{\Gamma_1} \mathcal{F} \, d\Gamma = 0. \quad (18) \]
We actually need not to substitute for \( \hat{u} \) from Eq. [9] or express the corresponding matrices, because this is done automatically by the FreeFem++ code [Hecht 2012]. We use a conjugate gradient method for the matrix inversion, which is suitable for sparse linear systems.

After a careful testing of our numerical method (see Appendix [C]), we choose the time step \( \Delta t = 10^{-3} P \), where \( P \) is the period present in the insolation function \( \mathcal{F} \). The spatial step is controlled by the maximum volume \( \Delta \Omega = 10^{-5} \text{m}^3 \) of tetrahedra generated by the TetGen code [Si 2006].

3 THE MEAN TORQUE CAUSED BY AN IRREGULAR BOULDER

The magnitude of a recoil force varies during a rotational period and a revolution of an asteroid around the Sun. A long-term effect of the force is therefore given by its time-averaged value. We follow the assumption of Golubev and Krugly (2012) and consider an asteroid on a circular orbit with zero obliquity. In fact, the eccentricity of Itokawa is \( e = 0.28 \); we address this issue in Section 3.5. Although the YORP effect depends on the obliquity in a non-trivial way (Capek and Vokrouhlický 2004), the zero obliquity allows us to average the recoil force over a rotational period only.

We consider the thermal emission and the scattered radiation. The direct radiation pressure has a negligible influence (Nesvorný and Vokrouhlický 2008). Again, we assume the Lambert’s cosine law for the intensity of scattered and emitted radiation. The recoil force from the surface element \( dS \) is then:
\[ d\vec{f}_{th} = -\frac{2 \epsilon \sigma}{3} \hat{u}^{n,m} \vec{n} \, dS, \quad (19) \]
\[ d\vec{f}_{sc} = -\frac{2 A \Phi}{3} \epsilon \mu (\vec{s} \cdot \vec{n}) \vec{n} \, dS. \quad (20) \]

The total torque caused by the boulder is given by the surface integral over the boulder:
\[ \vec{\tau} = \int_{\Gamma_1} \vec{r} \times d\vec{f}. \quad (21) \]

The direction of the torque is generally different from the axis of rotation \( \vec{e} \). Both the direction and the magnitude of the torque depend on exact shape of the boulder. However, even a symmetric boulder can induce a non-zero torque due to the lateral heat diffusion. The torque is caused by the asymmetry of emission from the eastern side and western side of the boulder. We anticipate the torque will therefore have a direction of the rotational axis \( \vec{e} \).

3.1 The coordinate system and free parameters of the problem

We choose a topocentric coordinate system centered on the studied boulder. The \( z \) axis has therefore a direction of a local normal, \( x \) axis is aligned with a meridian and \( y \) axis completes a right-handed orthogonal Cartesian system.

We introduce quantities that help us to reduce a number of independent parameters of the problem. We define the subsolar temperature:
\[ u_s \equiv \sqrt[4]{\frac{(1 - A) \Phi}{\epsilon \sigma}}, \quad (22) \]
the diurnal thermal skin depth:

\[ L \equiv \sqrt{\frac{2K}{\omega \rho C}}, \quad (23) \]

where \( \omega \) is the angular frequency of the asteroid, and the thermal parameter:

\[ \Theta \equiv \frac{\sqrt{K \omega \rho C}}{4\sqrt{2\pi} - \pi \sigma u_0^2}. \quad (24) \]

Numerical factors in these definitions arise from the derivation of the analytical solution (see Appendix A); however, some authors do not use them (Lagerros 1996; Golubov and Krugly 2012).

If we neglect self-heating terms, the heat diffusion equation (2) and its boundary condition (3) can be rewritten using dimensionless variables \( \xi \equiv r/L, \varphi \equiv \omega t, v \equiv u/u_* \) as:

\[ \frac{1}{2} \Delta_\xi v - \frac{\partial v}{\partial \varphi} = 0, \quad (25) \]

\[ 4\pi^{-\frac{3}{2}} \Theta \vec{n} \cdot \nabla_\xi v + v^4 = \vec{s} \cdot \vec{n}, \quad (26) \]

where \( \nabla_\xi, \Delta_\xi \) is the gradient and the Laplacian with respect to the variable \( \xi \). The only independent parameter in these equations is the thermal parameter \( \Theta \). However, the boundary condition must hold for all \( L \xi \in \partial \Omega \). If \( L \) is the characteristic size of the boulder, then the problem of finding a dimensionless temperature \( v \) has actually two independent parameters — the thermal parameter \( \Theta \) and the dimensionless size \( L \). In the following, we select the characteristic size as the square root of the base area of the boulder, \( L \equiv \sqrt{S} \).

### 3.2 The dimensionless pressure

In our model, the shape of the boulder can be arbitrary. As a special case, we can choose a high wall and compare our results to the model of Golubov and Krugly (2012). We started with such an idealized “boulder” to determine the influence of the self-heating effect and the absorption of radiation on the recoil force (see Appendix B). Nevertheless, we then selected a boulder of realistic irregular shape for our computations, as shown in Figure 2. The shape was obtained by a 3D scanning of a randomly selected boulder. In this case, we do not take into account the self-heating effect nor the influence of absorption though; this decision is justified in Appendix B as well.

We considered different values of the thermal conductivity for the studied boulder and for the surrounding layer of regolith, as demonstrated in Figure 3. Thermal properties of the regolith are taken from Farinella et al. (1998); properties of the boulder are determined by values of thermal parameter \( \Theta \) and the skin depth \( L \).

We should stress the importance of the non-linearity of the problem. We derived a linearized analytical solution of the heat diffusion equation in a half-space domain (see Appendix A), where we deal with the non-linear term \( u^4 \) by substituting \( u^0 + 4u^0_0 \delta u \), where \( u_0 \) is a constant, \( \delta u \) is the change of temperature. The same term appears in the expression for the recoil force (19) from a thermal emission. In case of a symmetric boulder, the complete linearization would lead to identically zero mean torque. Therefore, we solve a non-linear problem by an iterative method, as described in Section 2.

The solution of the HDE is a time-dependent temperature distribution in the boulder, particularly on its surface. It follows that we can determine the recoil force the boulder exerts; the force is given by the formula (19). However, it is convenient to introduce the dimensionless pressure:

\[ \Pi \equiv \frac{2}{3} \int_{\Gamma_L} \frac{u^4}{u_*} n_y d\Gamma, \quad (27) \]

where \( n_y \) is the \( y \)-th component of the local normal, \( S \) is the base area of the boulder. The dimensionless pressure allows us to compare the magnitude of the tangential force for different sizes of the boulder.

The projection of the total torque to the rotational axis is then given by:

\[ \vec{\tau} \cdot \vec{e} = \frac{(1 - A) \Phi}{c} \Pi S r_\perp, \quad (28) \]

where \( r_\perp \) is the distance of the boulder from the rotational axis.

If we consider a wall aligned with a local meridian, which face of area \( S \) has a constant temperature \( u \), our definition (27) is then reduced to:

\[ \Pi = \frac{2}{3} \frac{u^4}{u_*} r_\perp, \quad (29) \]

which is equivalent to the definition of a dimensionless pressure by Golubov and Krugly (2012). Our definition is there-
boulder. It is evident that the limit of very high conductivity (that is $\ell/L \to 0$) leads to a zero dimensionless pressure $\Pi$ for all shapes of a boulder. In such a case, the boulder is isothermal and therefore emits the same flux to the western and eastern directions, resulting in a null torque.

The limit of the mean dimensionless pressure for zero thermal conductivity ($\ell/L \to \infty$) differs from boulder to boulder. The conductive term in the energy balance equation (3) is negligible and the temperature at a given point of surface is determined by the immediate balance between incoming and outgoing radiant flux. Since we solve the HDE in a single boulder only, we need to obtain a torque (as a function of a boulder size) that would represent a set of all boulders on the surface. It is reasonable to assume that the boulders are randomly oriented on the surface. Although some orientations of boulders seems to be preferred on certain parts of the surface of Itokawa (Miyamoto et al. 2007), we anticipate that no orientation prevails on a global scale. The total torque induced by boulders will therefore vanish in the limit of zero conductivity. For that reason, we demand the mean torque $\langle \Pi \rangle$ to approach zero as well. However, in the general case of an asymmetric boulder, the mean dimensionless pressure will approach a non-zero value.

We have several options how to resolve this issue. For instance, we can restrict ourselves to symmetric boulders only. If the boulder is symmetric with respect to the plane of the local meridian, the mean pressure will vanish in the limit case. Nonetheless, we want to keep the universality of our model and use irregular asymmetric boulders. In this case, we can compute the mean pressure $\langle \Pi \rangle$ for several orientations of the boulder and then take the average of these values. Another possibility is to calculate the mean pressure for a single orientation and subtract the pressure in the limit case. Nonetheless, we want to keep the universality of our model and use irregular asymmetric boulders. In this case, we can restrict ourselves to symmetric boulders only. If the boulder is symmetric with respect to the plane of the local meridian, the mean pressure will vanish in the limit case. Nonetheless, we want to keep the universality of our model and use irregular asymmetric boulders. In this case, we can compute the mean pressure $\langle \Pi \rangle$ for several orientations of the boulder and then take the average of these values. Another possibility is to calculate the mean pressure for a single orientation and subtract the pressure in the limit case.

### 3.4 A dependence of the mean pressure on asteroidal latitude

For an asteroid with zero obliquity, the body–Sun vector $\vec{s}$ has Cartesian coordinates:

$$\vec{s} = (\sin \vartheta \cos \mathrm{HA}, -\sin \mathrm{HA}, -\cos \vartheta \cos \mathrm{HA}),$$  \hspace{1cm} (31)

where HA is the hour angle and $\vartheta$ is the asteroidal latitude (defined as $\sin \vartheta = \vec{e} \cdot \vec{n}$). The dependence of the dimensionless pressure $\Pi$ on the hour angle HA vanishes after averaging over a rotational period, the dependence on $\vartheta$ remains.

Assuming we can separate variables $\ell, \vartheta$, we can decompose the mean pressure $\langle \Pi \rangle$ as:

$$\langle \Pi \rangle(\ell, \vartheta) = P(\ell) V(\vartheta),$$  \hspace{1cm} (32)

where $P(\ell) = \langle \Pi \rangle(\ell, 0)$. The function $V(\vartheta)$ constitutes a latitudinal dependence and is normalized such that $V(0^\circ) = 1$. It obviously depends on the shape of a boulder. For this test, we chose an approximately hemispherical boulder, because it is axially symmetric and therefore one latitude is not preferred over other values.

We show the computed values of the function $V(\vartheta)$ in Figure 4 shows the mean dimensionless pressure for several orientations of the studied boulder and the averaged values. We assumed the boulder lies on the equator of an asteroid. We see that the averaged values approach zero in the limit of zero conductivity, as expected.

### 3.3 The total pressure exerted by a set of variously oriented boulders

The mean dimensionless pressure $\langle \Pi \rangle$ as a function of the dimensionless size $\ell/L$ varies significantly for different shapes of the boulder, or even for different orientations of the same boulder. It is evident that the limit of very high conductivity (that is $\ell/L \to 0$) leads to a zero dimensionless pressure $\Pi$ for all shapes of a boulder. In such a case, the boulder is isothermal and therefore emits the same flux to the western and eastern directions, resulting in a null torque.

The limit of the mean dimensionless pressure for zero thermal conductivity ($\ell/L \to \infty$) differs from boulder to boulder. The conductive term in the energy balance equation (3) is negligible and the temperature at a given point of surface is determined by the immediate balance between incoming and outgoing radiant flux. Since we solve the HDE in a single boulder only, we need to obtain a torque (as a function of a boulder size) that would represent a set of all boulders on the surface. It is reasonable to assume that the boulders are randomly oriented on the surface. Although some orientations of boulders seems to be preferred on certain parts of the surface of Itokawa (Miyamoto et al. 2007), we anticipate that no orientation prevails on a global scale. The total torque induced by boulders will therefore vanish in the limit of zero conductivity. For that reason, we demand the mean torque $\langle \Pi \rangle$ to approach zero as well. However, in the general case of an asymmetric boulder, the mean dimensionless pressure will approach a non-zero value.

We have several options how to resolve this issue. For instance, we can restrict ourselves to symmetric boulders only. If the boulder is symmetric with respect to the plane of the local meridian, the mean pressure will vanish in the limit case. Nonetheless, we want to keep the universality of our model and use irregular asymmetric boulders. In this case, we can compute the mean pressure $\langle \Pi \rangle$ for several orientations of the boulder and then take the average of these values. Another possibility is to calculate the mean pressure for a single orientation and subtract the pressure in the limit case. Nonetheless, we want to keep the universality of our model and use irregular asymmetric boulders. In this case, we can restrict ourselves to symmetric boulders only. If the boulder is symmetric with respect to the plane of the local meridian, the mean pressure will vanish in the limit case.

The dimensionless pressure varies during a rotation. We average the mean pressure $\langle \Pi \rangle$ over one rotational period; to this point we introduce the mean dimensionless pressure:

$$\langle \Pi \rangle = \frac{1}{P} \int_0^P \Pi \, dt.$$  \hspace{1cm} (30)

**Figure 4.** Computed values of the mean dimensionless pressure $\langle \Pi \rangle$ as a function of the dimensionless boulder size $\ell/L$ for two different values of the thermal parameter $\Theta = 5.38$ and 1.70 (notice that vertical axes have different scalings). The dark curves correspond to the boulder rotated by 0°, 90°, 180° and 270° around the vertical axis. We notice that all curves approach a zero for $\ell/L \to 0$ and they exhibit a maximum for $\ell \sim L$. For $\Theta = 1.70$ there seems to be an inflection at about $\ell \sim 0.1L$, which is absent for $\Theta = 5.38$. The bright curve is the arithmetic mean of the dark curves. We assumed a simpler “shadowing” model in this case (but cf. Figure 3B).
distance from the Sun as Θ thermal parameter Θ will vary over time; it depends on the separated by a time step equal to 1 less pressure ⟨⟩

model significantly. We thus compute the mean dimension-
clear whether the ellipticity of the trajectory affects our
⟨⟩

Figure 5. It can be approximated by a function cos aθ, where a = 0.653±0.004 is a parameter determined by a least square fit. The mean pressure ⟨⟩ is maximal for the boulders located on the equator and does not drop below 50% of the maximum for θ = 80° latitude. The known dependency of the mean pressure on the latitude allows us to compute the torque induced by a boulder on any given point of the surface, which we shall use in the next section.

3.5 An influence of the elliptical trajectory
So far we assumed the asteroid orbits on a circular trajec-
tory. This assumption allowed us to ignore any seasonal ef-
fects and average the torque over a rotational period only.
However, the eccentricity of Itokawa is e = 0.28; it is not clear whether the ellipticity of the trajectory affects our model significantly. We thus compute the mean dimensionless pressure ⟨⟩ in several “discrete” points on the orbit separated by a time step equal to 1/30 of the orbital period. We assume a high-conductivity case here. Nevertheless, the thermal parameter Θ will vary over time; it depends on the distance from the Sun as Θ ~ r^2, so it will reach Θ = 3.29 in the perihelion and Θ = 7.79 in the aphelion. We restrict ourselves to a single value of the dimensionless size t/L = 1 and we average the result over four different orientations of the boulder, as before. Then, we obtain the time-averaged value by simply taking an arithmetic mean of the computed values.

This approximate average over elliptical orbit is to be compared with the mean dimensionless pressure corresponding to the circular orbit, which we obtained already in Section 3.3. The result can be seen in Figure 6. We see that even for the considerable eccentricity of Itokawa, the difference seems negligible. The value differs from our previous result by less than 5%. Therefore, the approximation of the circular trajectory is well justified.

Figure 6. The values of the mean dimensionless pressure (Π) from the perihelion (t = 0) to aphelion (t = P/2). The values are computed for the high-conductivity case (Θ = 5.38 at the distance of the semimajor axis) and for t = L, averaged over four different orientations of the boulder. One can readily see that the difference between the time-averaged value and the value of corresponding circular orbit is negligible, given other uncertainties of our model.

4 THE ANGULAR ACCELERATION OF ASTEROID (25143) ITOKAWA
In the following, we focus our attention on the asteroid (25143) Itokawa. First, we estimate the total number of boulders on the surface and then compute how the thermal emission from boulders alters the angular acceleration predicted by global-shape models of the YORP effect.

Existing models of the YORP effect usually assume a normal direction of the recoil force. However, for non-convex asteroids the force can be influenced by the absorp-
tion of radiation emitted by the surface (Statler 2009). In the previous section, we demonstrated that a surface feature can alter the recoil force as well. We pointed out that the lat-
eral heat diffusion through boulders leads to an emergence of the tangential component of the recoil force. The pres-
ence of surface features also changes the normal component. While a complete solution would require solving the heat diffusion equation in the whole asteroid, including boulders, we neglect the change in the normal component and we solve for the tangential component separately.

The torque generated by a single boulder was discussed in previous chapter 3. We now place a large number of such boulders on the shape model of Itokawa and calculate the torque they induce. The total YORP torque and correspond-
ing change of the angular velocity of the asteroid is then ob-
tained by adding our result to the result of the global-shape model of the YORP effect.

4.1 The torque induced by boulders
We demonstrated the emergence of the tangential compo-
nent of the force, which is of our interest. Therefore, we consider a direction of the force perpendicular to the loc-
al normal n, regardless of the location on the surface of the asteroid. Although the direction depends on the shape, we discussed that the overall effect corresponds to the force

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perpendicular to the rotational axis $\vec{e}$. In follows that the direction of force is $\vec{e} \times \vec{n}$.

The magnitude of $\vec{e} \times \vec{n}$ is proportional to $\cos \vartheta$. However, we studied the dependence of the torque on asteroidal latitude in Section 3.4. We showed that the mean torque as a function of latitude $\vartheta$ is approximately $\cos 0.653 \vartheta$. The recoil force induced by single boulder is therefore proportional to the term:

$$f \sim \frac{\cos 0.653 \vartheta}{\cos \vartheta} \vec{e} \times \vec{n}. \quad (33)$$

Our goal is to compute the total torque $\vec{T}$ induced by the boulders; to be more precise, its projection $\vec{T} \cdot \vec{e}$ to the rotational axis. Let $N(\ell) d\ell$ be the number of boulders in the size interval $(\ell, \ell + d\ell)$. We can write the size distribution $N(\ell)$ as:

$$N(\ell) d\ell = N_0 g(\ell) d\ell, \quad (34)$$

where $N_0$ is the total number of boulders on the surface and $g(\ell) d\ell$ is the probability that a randomly selected boulder has a size in the interval $(\ell, \ell + d\ell)$. We compute the total torque induced by boulder as follows. We select a facet of Itokawa shape model randomly. Assuming boulders cover Itokawa uniformly, the probability of selecting each facet is proportional to its area. Now, we generate a random size $\ell$ with the probability distribution $g$, using the inverse sampling method. The torque induced by the boulder on the selected facet is added to the total torque. The total number of generated boulders is given by the value $N_0$ from Eq. (34). Because the number of boulders is very high, the total torque basically does not depend on the realization of boulder distribution on the surface.

It is clear that the boulder size distribution $N(\ell) d\ell$ constitutes an important parameter for a calculation of the total torque. We thus discuss the size distribution of boulders on Itokawa in the following section.

### 4.2 The observed size distribution of small boulders

In order to obtain the torque caused by boulders, it is necessary to find out the total number of boulders and their size distribution. The differential size distribution of boulders larger than 5 m on the whole surface of Itokawa can be approximated by a power law [Saito et al. 2006].

$$N(\ell) d\ell \approx 1.3 \times 10^7 \ell^{3.8} d\ell. \quad (35)$$

Surface images taken by the Hayabusa spacecraft revealed that this power law can be extrapolated down to sizes of 0.1 m on certain parts of the surface [Miyamoto et al. 2007], although the slope of a log-log graph falls off significantly for smaller sizes. However, other parts of the surface clearly show a different topography. Furthermore, an extrapolation of the above-mentioned size distribution down to 1 mm is clearly unacceptable; boulders of sizes between 1 mm and 0.1 mm alone would cover about $4 \times 10^6$ m$^2$ of the surface area, but the surface of Itokawa is only $3.93 \times 10^6$ m$^2$ [Demura et al. 2006].

Therefore, we sought for a different size distribution of small pebbles. We estimated their size distribution from close-up images taken by the Hayabusa during its descend, namely the images ST_2563537820_v and ST_2563607030_v from [Saito et al. 2010] dataset. The resolution of these images is 7 mm/pixel and 6 mm/pixel, respectively [Miyamoto et al. 2007], which allows us to find distinct boulders only few centimeters in size. Identified boulders are shown in Figure 7. We constructed a histogram of sizes (see Figure 8) from which we inferred the size distribution:

$$N(\ell) d\ell = (14 \pm 9) \times 10^3 \ell^{-(3.0 \pm 0.2)} d\ell. \quad (36)$$

We assume this power law can be extrapolated to millimeter-sized pebbles. Considering the uncertainties, the area covered by boulders of sizes between 1 mm and 1 m would be between 19 % and 32 % of the total surface area. This finding seems to be viable.

![Figure 7](image_url)

Figure 7. The image ST_2563607030_v [Saito et al. 2010] with highlighted boulders from which we derived their size distribution, used for the computation of the total torque.

![Figure 8](image_url)

Figure 8. The histogram of small boulder sizes on the surface of Itokawa, constructed from images ST_2563537820_v and ST_2563607030_v. The three power-law fits correspond to the direction of force is $\vec{e} \times \vec{n}$. The three power-law fits correspond to the av-
4.3 A comparison of the YORP torque by boulders and by the global shape

The global-shape YORP effect model of the asteroid Itokawa predicts a significant rotational deceleration, which is inconsistent with the observed acceleration. As mentioned above, the lateral heat diffusion through boulders induces an additional torque, which affects the angular velocity.

Let the magnitude of the total torque generated by boulders be $\tau$. The asteroid will undergo the rotational acceleration:

$$\frac{d\omega}{dt} = \frac{d\omega}{dt}_{\text{global}} + \frac{\tau}{I}, \quad (37)$$

where $(d\omega/dt)_{\text{global}}$ is the prediction of the global-shape YORP model, $I \approx 7.77 \times 10^{14} \text{kg m}^2$ is the moment of inertia of Itokawa ([Scheeres et al. 2007]).

The global-shape model of the YORP effect predicts a rotational deceleration $-2 \text{to } 6 \times 10^{-7} \text{ rad day}^{-2}$, depending on the resolution of the shape model ([Breiter et al. 2009]). In order to determine the torque induced by boulders, it is necessary to select values of the parameters — namely the thermal parameter $\Theta$ and the thermal skin depth $L$. We adopted following material properties: $K = 2.65 \text{ W m}^{-1} \text{ K}^{-1}$, $C = 680 \text{ J kg}^{-1} \text{ K}^{-1}$, $\rho = 2700 \text{ kg m}^{-3}$. Together with orbital parameters of Itokawa, this yields the thermal parameter $\Theta = 5.38$ and the thermal skin depth $L = 0.141 \text{ m}$. Utilizing the size distribution of boulders derived in Section 4.2 we obtain the result:

$$\frac{\tau}{T}_{\Theta = 5.38} = (1.20 \pm 0.11) \times 10^{-7} \text{ rad day}^{-2}. \quad (38)$$

As the thermal conductivity seems to be the most uncertain parameter, we also computed the torque for a lower value, $K = 0.26 \text{ W m}^{-1} \text{ K}^{-1}$ (keeping other parameters unchanged). In such a case, the thermal parameter is $\Theta = 1.70$ and the thermal skin depth $L = 0.0446 \text{ m}$. The corresponding torque is then:

$$\frac{\tau}{T}_{\Theta = 1.70} = (4.8 \pm 1.2) \times 10^{-7} \text{ rad day}^{-2}. \quad (39)$$

The probability distribution for both cases is shown in Figure 9.

For the sake of comparison, we can refer to the result of global-shape models of [Lowry et al. 2014], $-1.80 \pm 1.96 \times 10^{-7} \text{ rad day}^{-2}$, or [Breiter et al. 2009], $-(2.5 \text{ to } 5.5) \times 10^{-7} \text{ rad day}^{-2}$. We notice that this torque is of the same order as our result, but has an opposite sign. The torque induced by boulders and the torque from the global asymmetry could effectively cancel out, resulting in the change of angular velocity much smaller than predicted by global-shape models. As the observed angular acceleration of Itokawa is $(0.35 \pm 0.04) \times 10^{-7} \text{ rad day}^{-2}$ ([Lowry et al. 2014], our model presents an alternative explanation of the observed acceleration without any need for a non-uniform density distribution.

5 CONCLUSIONS

In this paper, we presented a detailed numerical model of the local YORP effect induced by a boulder or a set of boulders. The three-dimensional heat diffusion equation in the boulder was solved using the finite element method. Unlike

the finite difference method, the finite elements have basically no restriction on the shape of a domain, allowing us to solve the heat diffusion equation in the boulder of a realistic shape. Furthermore, we assumed the studied boulder has a different thermal conductivity than the surrounding regolith layer.

Our boulder had a general asymmetric shape, so it exhibited a non-zero torque even in the limit of zero thermal conductivity. However, this torque depends on the orientation of the boulder. In order to obtain an average torque representing a set of randomly-oriented boulders, we computed torques for several orientations and then the average of these values. We verified that the averaged torque approaches zero in the zero conductivity limit.

The non-zero torque arises from the asymmetry of the emission (averaged over the rotational period). There are two reasons for the emission asymmetry. The first is the asymmetry of the boulder shape. Indeed, [Rubincam 2000] demonstrated an emergence of the YORP effect on a toy
model of a spherical asteroid with two wedges attached to its equator. The torque created by the emission from the vertical side is greater in magnitude than the torque created by the emission from the inclined side, thus resulting in a non-zero total torque.

The second reason for the emission asymmetry is the lateral heat diffusion through the boulder (as in Golubov and Krugly 2012). We can imagine a boulder on the equator. In the morning, the eastern side of the boulder is heated up and the boulder exerts a recoil force of western direction. Provided the width of the boulder is comparable to the thermal skin depth, the heat diffusion contributes to heating of the western side in the afternoon. The emission from the western side is therefore more intense. The recoil force has an eastern direction and exceeds the force from the eastern side in magnitude, thus creating a non-zero mean force of eastern direction. The corresponding torque causes an angular acceleration of the asteroid.

The global contribution of the shape asymmetry of boulders to the YORP effect is likely to be null, because of the very large number of boulders on the surface. In contrast, the lateral heat diffusion leads to the torque with a direction of the rotational axis, thus accumulating over individual boulders. Even though the torque generated by a single boulder is tiny, the overall effect can be comparable to the global-shape YORP effect, if there is a sufficient amount of boulders, of course.

We showed the maximum pressure is exerted by boulders of sizes comparable to the diurnal skin depth \( L \). Figure 4 shows the dependence of the pressure on the size of the boulder. However, this graph does not reflect the actual contribution of boulders of different sizes to the total torque. The decisive factor is the exponent \( \gamma \) of the power-law size distribution. If \( \gamma > 2 \), smaller boulders will exert a higher torque (compared to the Fig. 1). In our case, the overall torque is generated mostly by boulders of sizes between 0.1 \( L \) and \( L \).

The general approach allowed us to compare our threedimensional model with the one-dimensional model of Golubov and Krugly (2012). In case of a symmetric boulder, we confirmed that the torque vanishes in the limits of high conductivity and zero conductivity. We showed that the maximum torque appears for \( \ell \sim L \).

Unlike Golubov and Krugly (2012), we found positive values of the torque for all parameter ranges in the case of a symmetric boulder. An asymmetric boulder could produce a negative torque, but after averaging over orientations the resulting torque is again strictly positive. Even Golubov and Krugly (2012) realized the torque is mostly positive, and proposed a possibility of an equilibrium between the global-shape torque and the torque induced by boulders, resulting in a null total torque; they suggested this could be the case of the asteroid (25143) Itokawa. However, Lowry et al. (2014) detected a positive change in angular velocity of Itokawa, which means this asteroid is not in such equilibrium state.

Of course, our model contains a number of free parameters that can change the magnitude of the torque significantly. The crucial factor is the total number of boulders and their size distribution. We realized that the size distribution of large boulders on Itokawa of Saito et al. (2006) cannot be extrapolated to centimeter sizes. Having no better alternative, we estimated the size distribution from close-up images of the surface of Itokawa and we extrapolated it for sizes of indiscernible pebbles. We also assumed that other parts of the surface have the same size distribution of boulders. If we neglect an interaction between boulders (mutual shadowing or thermal irradiation), which seems to be reasonable given the separations of boulders seen on Figure 4, the torque is directly proportional to the number of boulders.

The choice of the lower limit of the size distribution might be also a disputable parameter. Although the magnitude of the torque induced by a boulder approaches zero as the size of the boulder approaches zero, even sub-millimeter pebbles could have a non-negligible influence on the total torque. However, it is doubtful whether so small particles can be considered as boulders, or whether they form a uniform layer of matter. We selected the lower limit 1 mm.

The shape of the studied boulder is another key factor of our model. We selected a boulder of a realistic irregular shape, but it was selected ad hoc. There are two particularly important properties of boulders: their height and flatness. The higher the boulder is, the greater torque it likely induces. If the sides of the boulder are perpendicular to the surface, the lever arm of the torque is maximal; the lower the slope of sides, the lower lever arm.

We finally applied our model to the case of the asteroid (25143) Itokawa. We showed that boulders could induce a torque that would cause the angular acceleration of the order \( 10^{-7} \) rad day\(^{-2} \). We realized there was a significant uncertainty; nevertheless, we clearly demonstrated that the emission from boulders is capable of producing the torque comparable in magnitude to the global-shape YORP effect. Our result is consistent with the observed acceleration (Lowry et al. 2014) and presents an alternative and viable explanation of the discrepancy between the observed acceleration and the acceleration predicted by global-shape models.

6 FUTURE WORK

We postpone the following topics for future work. We assumed a rotational axis perpendicular to the orbital plane, therefore we need not consider the orbital movement and the torque is averaged over the rotational period only. Luckily, the obliquity of Itokawa is approximately 178° (Demura et al. 2006), which is very close to the perpendicular state. Considering a general direction of the rotational axis, the insolation will change during the revolution about the Sun and the seasonal changes of temperature will occur. We expect a seasonal variant of the studied effect to appear on boulders whose size is comparable to the seasonal thermal skin depth.

We already discussed that the diurnal torque has a direction of the rotational axis, as it is caused by the asymmetry of emission between the western and the eastern part of the boulder due to the lateral heat diffusion. Following the same principle, the seasonal torque could be caused by the asymmetry between the northern and the southern part of the boulder. The direction of this torque would therefore be perpendicular to the rotational axis. We thus anticipate the seasonal component of the effect will not affect the angular velocity of the asteroid, it will only cause an evolution of the obliquity.
In our model, we assumed a shadow is casted by the boulder; however, we did not take into account shadows casted by global-shape inconvexities of Itokawa. The same goes for the self-heating effect, which is also considered only locally. To resolve this issue, it would be necessary to determine the isolation function for each facet of the shape model separately and then solve many three-dimensional heat diffusion problems.

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APPENDIX A: A LINEARIZED ANALYTICAL SOLUTION OF THE ONE-DIMENSIONAL HEAT DIFFUSION EQUATION

The general three-dimensional heat diffusion equation with a non-linear boundary condition in an irregular domain has no analytical solution. To find the temperature distribution, we must employ a numerical approach such as the finite element method. Nevertheless, it is useful to derive an analytical solution for a simplified case of a half-space domain, which allows to reduce the problem to one spatial dimension only, as in [Čapek[2007]]. The solution can be used as a test for our numerical model and also as a Dirichlet boundary condition (see Figure [1]).

Suppose the Sun illuminates an (infinite) plane $z = 0$ and a half-space $z > 0$ represents the domain $\Omega$. We seek for the temperature $u$ as a function of the depth $z$ and time $t$. 

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solving the heat diffusion equation:
\[ \alpha \partial_z^2 u - \partial_t u = 0, \quad (A1) \]
with boundary conditions:
\[ -K \partial_z u(0, t) + \sigma u^4(0, t) = F(t), \quad (A2) \]
\[ \partial_z u(\infty, t) = 0, \quad (A3) \]
where \( \alpha \equiv \frac{K}{\rho C} \) is the thermal diffusivity, \( K \) the thermal conductivity, \( \rho \) the density, \( C \) the specific heat capacity, \( \sigma \) the Stefan-Boltzmann constant and \( F(t) \) the incoming radiant flux. The first boundary condition is the energy balance equation and the second one is necessary for a uniqueness of the solution — it eliminates solutions where the temperature rises ad infinitum as \( z \to \infty \).

The radiant flux is a periodic function, therefore we can represent it as a Fourier series, \( F(t) = \sum_{n=-\infty}^{\infty} F_n e^{i n \omega t} \). We look for a stationary solution, which we can represent by a sum \( u(z, t) = \sum_{n=-\infty}^{\infty} u_n(z) e^{i n \omega t} \). Substituting into (A1) and applying the constraint (A3) we obtain the solution:
\[ u_0(z) = a_0, \quad (A4) \]
\[ u_n(z) = a_n e^{-(1+i)\beta_n z}, \quad (A5) \]
\[ u_{-n}(z) = a_{-n} e^{(1-i)\beta_n z}, \quad (A6) \]
where \( \beta_n \equiv \frac{\sqrt{\omega_n}}{2\sigma} \). Notice that \( \beta_1 \) is the reciprocal of the thermal skin depth \( L \) introduced in Section 3.1, hence the \( \sqrt{2} \) factor. We determine constants \( a_n \) from the boundary condition (A2). Here we encounter problems with the non-linear term \( u^4 \). A substitution of the Fourier sum for the temperature \( u \) would lead to a set of non-linear equations, which — unsurprisingly — does not have an analytical solution. Nevertheless, we can solve the problem analytically under the assumption that the changes of the temperature are significantly smaller than its absolute value. In such a case, we can linearize the fourth power \( u^4 \approx u_0^4 + 4u_0^3 \sum_{n \neq 0} u_n e^{i n \omega t} \). The boundary condition (A2) then composes a set of linear and separated equations for the coefficients \( a_n \). We immediately see the solution for the constant term:
\[ u_0 = 4 \sqrt{\frac{F_0}{\epsilon \sigma}}. \quad (A7) \]

It is convenient to introduce auxiliary parameters that help us to get rid of complex numbers in the solution. First, we present the thermal parameter \( \Theta_n \) of the \( n \)-th mode:
\[ \Theta_n = \frac{K \beta_n}{4\sigma u_0^3}. \quad (A8) \]
Once again, \( \Theta_1 \) corresponds to the thermal parameter \( \Theta \) defined in Section 3.1. Second, let the phase shift \( \varphi_n \) of the \( n \)-th mode be:
\[ \tan \varphi_n = - \left( \frac{\Theta_n}{\Theta_n + 1} \right) \text{sgn } n. \quad (A9) \]
Since the solution is periodic, we can choose the initial time arbitrarily. Therefore, we choose the insolation function in the form of cosine series:
\[ F(t) \approx (1 - A) \Phi \Xi(\cos \omega t), \]
where \( \Xi(x) = x \) for \( x \geq 0 \), \( \Xi(x) = 0 \) for \( x < 0 \). First four Fourier modes of this function are:
\[ F(t) \approx (1 - A) \Phi \left( \frac{1}{\pi} + \frac{1}{2} \cos \omega t + \frac{2}{3\pi} \cos 2\omega t - \frac{2}{15\pi} \cos 4\omega t \right). \quad (A10) \]

**Figure B1.** The mean dimensionless pressure \( \langle \Pi \rangle \) as a function of the dimensionless width \( d/L \) for four different variants of our model for an idealized boulder: i) shadowing only, ii) factor, iii) self-heating, and iv) a complete model. See text for a detailed explanation.

Now, we can write the solution \( u \) of the problem in a simple form:
\[ u(z, t) = 4 \sqrt{\frac{F_0}{\epsilon \sigma}} + \sum_{n=1}^{\infty} F_n \frac{e^{-\beta_n z} \cos (n\omega t - \beta_n z + \varphi_n)}{\sqrt{2\Theta_n^2 + 2\Theta_n + 1}}. \quad (A11) \]

We see that for each cosine term in the series of the insolation function \( F \) there exists a corresponding cosine term in the solution \( u \), but with some phase shift. For the surface temperature in particular, the offset of the \( n \)-th Fourier mode is equal to \( \varphi_n \), defined above.

**APPENDIX B: VARIANTS OF THE INSOLATION FUNCTION AND A COMPARISON WITH THE EXISTING ONE-DIMENSIONAL MODEL**

The influence of topographic features on rotational dynamics of an asteroid has been studied by Golubov and Krugly (2012). Their model uses a one-dimensional approximation, which implicitly corresponds to a “wall” of an infinite height. Such an infinite domain cannot be used in our numerical model; therefore, we used the wall the height of which is about three times the width. The vertical heat diffusion along the wall is therefore significantly reduced (in comparison to the boulder studied in Section 3). Furthermore, we employed four variants of our model, called i) shadowing, ii) factor, iii) self-heating and iv) complete. They can be understood as sorted by a degree of their completeness.

The first variant is called the “shadowing” model. It takes into account shadows cast by the boulder only. The self-heating effect is ignored, the influence of absorption on the direction of the recoil force is neglected as well. The insolation function is simply:
\[ F = (1 - A) \Phi \mu \mathbf{s} \cdot \mathbf{n}. \quad (B1) \]

The second model, which we call the “factor” model, accounts for the self-heating by including simply the factor 2...
to the solar flux. The corresponding insolation function is therefore:
\[ F = 2(1 - A)\Phi \mu \delta \cdot \vec{n}. \]  
(B2)

This model has been used by [Golubov and Krugly 2012].

The third “self-heating” model computes the self-heating contribution directly by a calculation of the thermal emission and the scattered flux as described in Section A.

\[ F = (1 - A)\Phi \mu \delta \cdot \vec{n} + F_{\text{th}} + F_{\text{sc}}, \]  
(B3)

where \( F_{\text{th}}, F_{\text{sc}} \) are the fluxes defined by Eqs. (5), (6).

We called the fourth model “complete”, as it contains the direct computation of the self-heating, but also the influence of the absorption on the direction and magnitude of the recoil force. We assume that the thermally emitted radiation falling on the surface is absorbed and does not contribute to the torque. The recoil force is then given by:
\[
\frac{d\vec{F}}{ds} = -\frac{\epsilon \sigma \mu}{c} \left( \frac{2}{3} \vec{n} - \int_{\Gamma} \frac{\cos \alpha \cos \alpha'}{\pi (\vec{r} - \vec{r}')^2} \frac{\vec{r}' - \vec{r}}{||\vec{r}' - \vec{r}||^3} d\Gamma \right)
\]  
(B4)

The mean dimensionless pressure (II) (see Section 3.2) as a function of the dimensionless width \( d/L \) of the wall is shown in Figure C1. We notice certain common properties of the functions. All of them have a global maximum at \( d \sim L \), but there is also either a secondary local maximum or an inflection at \( d \sim 0.1L \). These properties are evident for an irregular boulder as well, as we pointed out in Section 3.

The maximal value of the “factor” curve is \( \sim 0.014 \), which is a result consistent with the findings of [Golubov and Krugly 2012]. We also see that this model is a remarkably viable approximation of the “self-heating” model. We thus confirm that the multiplication of the solar flux by the factor 2 leads to a similar outcome as the inclusion of the self-heating effect (which is much more computationally demanding).

Let us compare the “shadowing” and “self-heating” models. We see a notable property of the local YORP effect: the self-heating effect causes an increase of the mean pressure by \( \sim 50\% \) in our model. The YORP effect induced by boulders therefore qualitatively differs from the global-shape YORP effect, where an inclusion of the self-heating effect leads to a decrease of the torque magnitude for most cases [Rozitis and Green 2013].

Finally, we compare the “shadowing” and “complete” model. We observe that the “complete” model is well approximated by the “shadowing” model in the vicinity of the global maximum, although it differs significantly for smaller boulders. Nevertheless, this result allowed us to proceed with the simple “shadowing” model. Regarding the conclusions of this paper, a computation with the complete model would lead to very similar results.

APPENDIX C: THE NUMERICAL UNCERTAINTY OF THE FINITE ELEMENT METHOD

The discretization of both space and time inevitably introduces a numerical uncertainty to the solution. In order to estimate this uncertainty and find its upper bound, we utilize the linearized analytical solution of the HDE, derived in Appendix A. We select a simple block as a domain for

\[
\text{max}(u_i, u_{\text{theory}}) = \max_j |u^j_i - u^j_{\text{theory}}|,  \]  
(C1)

where \( u^j_i \) denotes the temperature in the \( j \)-th time step. Figure C1 shows the dependence of this metric on the tetrahedra volume \( \Delta \Omega \) and the time step \( \Delta t \); the limit \( \delta \) is a constant here. We checked not only the maximum difference of \( u - u_{\text{theory}} \) but also its mean dispersion, nonetheless, the results were very similar. We see that the uncertainty indeed decreases with a refinement of the discretization. We use \( P/\Delta t = 1000 \) and \( \Delta \Omega^{-1} = 100000 \) in our analyses.

For high values of the thermal conductivity \( K \), the changes of temperature are small and the iterative procedure converges quickly — the limit \( \delta = 10^{-4} \) was reached.
after only two or three iterations. For lower values of $K$, the procedure with the same $\delta$ took about 10 iterations. As we foreshadowed in Section 2, the procedure did not converge for the lowest values of considered range of $K$ and we had to solve the problem using the relaxation method.

Moreover, we have two possibilities of a posteriori uncertainty estimates. First, we can compare our results to Golubov and Krugly (2012) (see Appendix B). Although our and their approaches are substantially different, both results are indeed comparable. Second, the mean torque induced by a boulder must vanish in the limit of zero conductivity, $K \to 0$, as we mentioned in Section 3.3. This is especially important as the low-$K$ case cannot be tested with the analytical solution (A11). Luckily, low $K$ effectively corresponds to larger boulders ($\ell/L$) which do not contribute much to the total torque, so we consider this case as being of lesser importance.

This paper has been typeset from a TeX/LaTeX file prepared by the author.