Holographic duality in nonlinear hyperbolic metamaterials

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Abstract

According to the holographic principle, the description of a volume of space can be thought of as encoded on its boundary. Holographic principle establishes equivalence, or duality, between theoretical description of volume physics, which involves gravity, and the gravity-free field theory, which describes physics on its surface. While generally accepted as a theoretical framework, so far there was no known experimental system which would exhibit explicit holographic duality and be amenable to direct experimental testing. Here we demonstrate that nonlinear optics of hyperbolic metamaterials admits such a dual holographic description. Wave equation which describes propagation of extraordinary light through the volume of metamaterial exhibits 2 + 1 dimensional Lorentz symmetry. The role of time in the corresponding effective 3D Minkowski spacetime is played by the spatial coordinate aligned with the optical axis of the material. Nonlinear optical Kerr effect bends this spacetime resulting in effective gravitational interaction between extraordinary photons. On the other hand, a holographic dual theory may be formulated on the metamaterial surface, which describes its nonlinear optics via interaction of cylindrical surface plasmons possessing conserved charges proportional to their angular momenta. Potential implications of this duality for superconductivity of hyperbolic metamaterials are discussed.

Keywords: metamaterial, waveguide, holographic principle

(Some figures may appear in colour only in the online journal)
Lorentz symmetry. The role of time in the corresponding effective 3D Minkowski spacetime is played by the spatial coordinate aligned with the optical axis of the material. Nonlinear optical Kerr effect bends this spacetime resulting in effective gravitational interaction between extraordinary photons [7]. On the other hand, a holographic dual theory may be formulated on the metamaterial surface, which describes its nonlinear optics via interaction of cylindrical surface plasmons (CSPs) possessing conserved charges proportional to their angular momenta. This theory is built based on a field-theoretical description of nonlinear optics of CSPs introduced earlier in [8]. Since the latter theory may be extended to encompass mutual interaction of Kaluza–Klein charges, the developed holographic framework may have potential implications for understanding of superconductivity of hyperbolic metamaterials [9]. While our results provide new and important application of holographic principle to a condensed matter system (where there exist just a few known applications), they are also very important for nonlinear optics of metamaterials. They provide new and explicit tool to solve nonlinear Maxwell equations in a difficult situation where magneto–optical coupling appears to be strong by using well-developed tools of effective field theory.

Let us start by outlining the analog gravity-based ‘volume theory’ of nonlinear optics of wire array hyperbolic metamaterials [7]. Light propagation through hyperbolic metamaterials has attracted much recent attention due to their potential implications for understanding of superconductivity of hyperbolic metamaterials [9]. While our results provide new and important application of holographic principle to a condensed matter system (where there exist just a few known applications), they are also very important for nonlinear optics of metamaterials. They provide new and explicit tool to solve nonlinear Maxwell equations in a difficult situation where magneto–optical coupling appears to be strong by using well-developed tools of effective field theory.

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\[ \varepsilon_{1x} = \varepsilon_{2} < 0. \]

In the linear optics approximation all the non-diagonal components are assumed to be zero. Propagation of extraordinary light in such a metamaterial may be described by a coordinate-dependent wave function \( \varphi_{\omega} = E_{z} \) obeying the following wave equation [20]:

\[ \frac{\omega^{2}}{c^{2}} \varphi_{\omega} = \frac{\delta^{2} \varphi_{\omega}}{\varepsilon_{1} \partial z^{2}} + \frac{1}{\varepsilon_{2}} \left( \frac{\delta^{2} \varphi_{\omega}}{\partial x^{2}} + \frac{\delta^{2} \varphi_{\omega}}{\partial y^{2}} \right) \]  

This wave equation coincides with the Klein–Gordon equation for a massive scalar field \( \varphi_{\omega} \) in 3D Minkowski spacetime:

\[ \frac{\delta^{2} \varphi_{\omega}}{\varepsilon_{1} \partial z^{2}} + \frac{1}{(-\varepsilon_{2})} \left( \frac{\delta^{2} \varphi_{\omega}}{\partial x^{2}} + \frac{\delta^{2} \varphi_{\omega}}{\partial y^{2}} \right) = \frac{\omega_{0}^{2}}{c^{2}} \varphi_{\omega} = \frac{m_{\pi}^{2} c^{2}}{\hbar^{2}} \varphi_{\omega} \]  

in which spatial coordinate \( z = \tau \) behaves as a ‘timelike’ variable. Equation (2) describes world lines of massive particles which propagate in a flat 2 + 1 dimensional Minkowski spacetime.
space-time [19, 20]. The components of metamaterial dielectric tensor define the effective metric $g_{ij}$ of this spacetime: $g_{00} = - \varepsilon_1$ and $g_{11} = g_{22} = - \varepsilon_2$. Similar to our own Minkowski spacetime, the effective Lorentz transformations in the $xz$ planes form the Poincare group together with translations along $x$, $y$, and $z$ axis, and rotations in the $xy$ plane. We should also point out that Lorentz symmetry is generally accepted to be broken at the Planck scale—see for example [21] and references therein. This means that similar to hyperbolic metamaterials, metric coefficients of physical vacuum exhibit temporal and spatial dispersion (metric coefficient dependence on the energy-momentum). Mathematically, this dispersion must be expressed as frequency-dependent complex-valued metric coefficients (due to Kramers–Kronig relationship). One cannot guarantee that the wave equation in hyperbolic metamaterials always coincides with Klein–Gordon equation for physical vacuum at the Planck scale, since the latter is yet unknown. However, qualitative similarity of both equations in the case of hyperbolic wire medium has been illustrated in [21].

When the nonlinear optical effects become important, they are described in terms of various order nonlinear susceptibilities $\chi^{(n)}$ of the metamaterial:

$$D_i = \chi_0^{(1)} E_i + \chi_0^{(2)} E_j E_j + \chi_0^{(3)} E_j E_j E_m + ...$$

(3)

Taking into account these nonlinear terms, the dielectric tensor of the metamaterial (which defines its effective metric) may be written as

$$\varepsilon_i = \chi_0^{(1)} + \chi_0^{(2)} E_j + \chi_0^{(3)} E_j E_m + ...$$

(4)

Equation (4) provides coupling between the matter content (extraordinary photons) and the effective metric of the metamaterial ‘space-time’. Let us find what kind of simplifications of this general framework may lead to a metamaterial model of usual gravity.

In the weak gravitational field limit the Einstein equation

$$R^i_i = \frac{8\pi\delta}{c^4} \left( T^i_i - \frac{1}{2} g_i^i T \right)$$

(5)

is reduced to

$$R_{00} = \frac{1}{c^2} \Delta \phi = \frac{1}{2} \Delta g_{00} = \frac{8\pi\delta}{c^4} T_{00}$$

(6)

where $\phi$ is the gravitational potential [22]. Since in our effective Minkowski spacetime $g_{00}$ is identified with $-\varepsilon_1$, comparison of equations (4) and (6) indicates that all the second order nonlinear susceptibilities $\chi_0^{(2)}$ of the metamaterial must be equal to zero, while the third order terms may provide correct coupling between the effective metric and the energy-momentum tensor. These terms are associated with the optical Kerr effect.

Indeed, detailed analysis performed in [7] indicates that Kerr effect in a hyperbolic metamaterial leads to effective gravity. Since $z$ coordinate plays the role of time, while $g_{00}$ is identified with $-\varepsilon_1$, equation (6) must be translated as

$$-\Delta^{(2)} \varepsilon_1 = \frac{16\pi\delta}{c^2} T_{00} = \frac{16\pi\varepsilon}{c^2} \sigma_{zz}$$

(7)

where $\Delta^{(2)}$ is the 2D Laplacian operating in the $xy$ plane, $\chi^{(3)}$ is the effective ‘gravitational constant’, and $\sigma_{zz}$ is the $zz$ component of the Maxwell stress tensor of the electromagnetic field in the medium:

$$\sigma_{zz} = \frac{1}{4\pi} \left( D_i E_i + H_i B_i - \frac{1}{2} \left( \nabla D + \nabla H \right) \right)$$

(8)

A contribution to $\sigma_{zz}$, which is made by a single extraordinary plane wave propagating inside the metamaterial, may be found by assuming without a loss of generality that the $B$ field of the wave is oriented along $y$ direction, so that the other field components may be found from the Maxwell equations as

$$k_x B_y = \frac{\alpha}{c} \varepsilon_1 E_x, \quad \text{and} \quad k_x B_z = - \frac{\alpha}{c} \varepsilon_1 E_z$$

(9)

Taking into account the dispersion law of the extraordinary wave [23]

$$\omega^2 = \frac{k_x^2}{\varepsilon_1} + \frac{k_z^2 + k_y^2}{\varepsilon_2},$$

(10)

the contribution to $\sigma_{zz}$ from a single plane wave appears to be

$$\sigma_{zz} = - \frac{\varepsilon}{c^2} B^2 k^2$$

(11)

Thus, for a single plane wave equation (7) may be rewritten as

$$-\Delta^{(2)} \varepsilon_1 = -\Delta^{(2)} \left( \varepsilon_1^{(0)} + \varepsilon_{00} \right),$$

(12)

$$= k_x^2 \varepsilon_{00} = - \frac{4\pi\varepsilon}{c^2} B^2 k^2,$$

where we have assumed that nonlinear corrections to $\varepsilon_1$ are small, so that we can separate $\varepsilon_1$ into the constant background value $\varepsilon_1^{(0)}$ and weak nonlinear corrections. These nonlinear corrections do indeed look like the Kerr effect, assuming that the extraordinary photon wave vector components are large compared to $\omega/c$.

$$\delta \varepsilon_1 = - \frac{4\pi\varepsilon}{c^2} B^2 k^2 \approx \frac{4\pi\varepsilon}{c^2} \varepsilon_{00} \varepsilon_1^{(0)} \approx \frac{4\pi\varepsilon}{c^2} \varepsilon_1^{(0)} = \varepsilon^{(3)} B^2$$

(13)

The latter assumption has to be the case indeed if extraordinary photons may be considered as classic ‘particles’. Equation (13) establishes connection between the effective gravitational constant $\varepsilon^{(3)}$ and the third order nonlinear susceptibility $\chi^{(3)}$ of the hyperbolic metamaterial. Since $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_1 > 0$ and $\varepsilon_{zz} = \varepsilon_2 < 0$, the sign of $\chi^{(3)}$ must be negative for the effective gravity to be attractive. For a metal wire array metamaterial shown in figure 1(a) the diagonal components of the dielectric tensor may be obtained using Maxwell–Garnett approximation [23]:

$$\varepsilon_2 = \varepsilon_1 = ne_m + (1 - n)\varepsilon_0$$

(14)
\[ \varepsilon_1 = \varepsilon_{xy} \]
\[ = \varepsilon_m(1 + n) + \varepsilon_d(1 - n) \]
\[ = \varepsilon_{xy}(1 + n) + \varepsilon_{xy}(1 - n) \]  
(15)

where \( n \) is the volume fraction of the metallic phase, and \( \varepsilon_m \) and \( \varepsilon_d \) are the dielectric permittivities of the metal and dielectric phase, respectively. For a typical metal \( -\varepsilon_m \gg \varepsilon_d \), and at small \( n \) equation (15) may be simplified as

\[ \varepsilon_1 \approx \varepsilon_{xy}(1 + n) \sim \varepsilon_d \]  
(16)

Thus, in order to obtain attractive effective gravity the dielectric host medium must indeed exhibit negative (self-defocusing) Kerr effect. Extraordinary light rays in such a medium will behave as 2 + 1 dimensional world lines of self-defocusing Kerr solitons. A dielectric medium must indeed exhibit negative (self-defocusing) Kerr effect. Thus, in order to obtain attractive effective gravity the dielectric host medium must indeed exhibit negative (self-defocusing) Kerr effect.

As a next step, let us follow the general spirit of AdS/CFT duality and arrange for an effective ‘gravitational horizon’ at some radius \( \rho \) from the metamaterial sample center. A general recipe for such an analog horizon has been described in [24]. Near horizon its surface may be considered as almost flat so that a case of constant \( \varepsilon_2 = \varepsilon < 0 \) and finite \( \varepsilon_1(x) = \varepsilon \), which changes sign from \( \varepsilon_1 > 0 \) to \( \varepsilon_1 < 0 \) as a function of \( x \) in some frequency range around \( \omega = \omega_0 \), may be considered as a metamaterial waveguide, which supports electromagnetic modes having divergent electric field \( E_\rho \) on its surface. The field decays exponentially into the anisotropic metal outside the waveguide. Such guided modes are usually called CSPs, which ‘live’ at the \( r = \rho \) interface [8]. Let us demonstrate that nonlinear optics of these CSPs may be formulated as an effective field theory, which is holographic dual to the effective 2 + 1 gravity described above. Such a theory may be formulated similar to the field-theoretical description of nonlinear optics of CSPs living on surfaces of single nanoholes and nanowires [8], which has strong similarity with Kaluza–Klein theories.

Similarity between the nonlinear optics of CSPs and the Kaluza–Klein theories stems from the way in which electric charges are introduced in the original five-dimensional Kaluza–Klein theory (see for example [26]). In this theory the electric charges are introduced as chiral (nonzero angular momentum \( L \)) modes of a massless quantum field, which is quantized over the cyclic compactified fifth dimension. In a similar fashion, nonlinear optics of CSPs may be formulated as a field theory in a curved 2 + 1 dimensional space-time defined by the metal interface which has an extended \( z \)-coordinate and a small ‘compactified’ angular \( \phi \) dimension along the circumference of the cylinder. The resulting 1 + 1 dimensional effective field theory of CSP interaction describes higher order \( (L > 0) \) CSP modes as having quantized effective chiral charges equal to their angular momenta \( L \). These massive slow moving effective charges exhibit long-range interaction via exchange of fast massless CSPs having zero angular momentum.

In order to look similar to Kaluza–Klein theory, the medium should be either optically active, or exhibit magnetic field induced optical activity. Such a medium would be able to discriminate between CSP waves which have opposite angular momenta, and thus expected to have opposite effective Kaluza–Klein charges. The best way to accommodate this requirement is to consider zero angular momentum CSP modes as quanta of the ‘gyration field’ (the field of the gyration vector \( g \)), which relates the \( D \) and \( E \) fields in an optically active medium [27]:

\[ \vec{D} = \vec{E} + i \vec{E} \times \vec{g} \]  
(19)

If the medium exhibits magneto-optical effect, and does not exhibit natural optical activity, \( g \) is proportional to the magnetic field \( H \):

\[ \vec{g} = fH, \]  
(20)

where the constant \( f \) may be either positive or negative. For metals in the Drude model at \( \omega \gg eH/mc \) the magnetic field induced optical activity is defined by

\[ f(\omega) = \frac{4\pi Ne^3}{cm^2\omega^2} = -\frac{ea_0^2}{mc\omega}, \]  
(21)

where \( \omega_p \) is the plasma frequency and \( m \) is the electron mass [27]. Comparison of equations (3), (19), (20) indicates that introduction of gyration field constitutes an alternative way of treating third order nonlinear optical effects responsible for the effective gravity of the bulk theory. Moreover, it is easy to demonstrate that so introduced gyration field would indeed lead to the Coulomb-like interaction of effective ‘chiral charges’.
Let us consider Maxwell equations in the presence of the axial gyration field \( g_\phi = g_\phi (r,z,t) \). In order to illustrate essential physics, let us consider solutions at \( r > \rho \) where the metamaterial may be treated as anisotropic metal, and neglect metal anisotropy for the sake of simplicity. After simple calculations the wave equation in such a case may be written in the form

\[
-\Delta \vec{B} = \frac{\varepsilon}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + i \frac{\partial}{c} \left( \vec{\nabla} \times \left[ \vec{E} \times \vec{g} \right] \right)
\]

(22)

which for the \( z \)-components of a solution proportional to \( \sim e^{iLz} \) may be re-written as

\[
\frac{\partial}{\rho \partial r} \left( r \frac{\partial B_r}{\partial r} \right) + \frac{\partial^2 B_r}{\partial z^2} - \frac{\varepsilon \rho \frac{\partial^2 B_r}{\partial z^2}}{c^2} = \frac{L^2}{r^2} B_r
\]

\[
+ \frac{iLg}{rc} \frac{\partial (iE_z)}{\partial t} + \frac{iL}{cr} \left( \frac{\partial g}{\partial t} \right) (iE_z) = 0
\]

(23)

The latter equation is similar to the Klein–Gordon equation, in which \( L \) and \( g/r \) play the role of the effective charge, and the effective potential, respectively [8] (here it is important to mention that for such higher order CSP modes \( iE_z \sim B_r \), where the coefficient of proportionality is determined by the boundary conditions). Thus, action of gyration field \( g \) on chiral charges is similar to action of electric field on electric charges.

On the other hand, we may solve the nonlinear Maxwell equations and explicitly demonstrate that the higher angular momentum modes (the chiral charges) behave as the sources of gyration field (the field of the fundamental \( L=0 \) CSP mode). Let us search for the solutions of the nonlinear Maxwell equation (22) of the form \( \vec{B} = \vec{B}_0 + \vec{B}_L \) and \( \vec{E} = \vec{E}_0 + \vec{E}_L \), where \( \vec{B}_0 \) and \( \vec{B}_L \) are the fundamental mode, and the \( L > 0 \) guided mode, respectively. The gyration field may be obtained in a self-consistent manner as \( \vec{g} = f(\vec{B}_0 + \vec{B}_L) \). We are interested in the solution for the field \( \vec{B}_0 \) in the limit of small frequencies \( \omega_0 \) in the presence of \( \vec{B}_L \) field, so that in the found solution the \( \vec{B}_L \) field will act as a source of \( \vec{B}_0 \). After neglecting the terms proportional to \( \vec{B}_L^2 \) in equation (22), and taking into account that \( \vec{B}_0 \) and \( \vec{B}_L \) are incoherent solutions of linear Maxwell equations, we obtain

\[
\Delta \vec{B}_0 = \frac{4\pi \omega_0 f}{c^2} \vec{\nabla} \times \vec{S}_L
\]

(24)

where \( \vec{S}_L \) is the Pointing vector of the \( L \)th mode. This equation is similar to the Poisson equation in which the term \( \vec{\nabla} \times \vec{S}_L \) acts as a source. Moreover, using vector calculus we may also derive an analog of the Gauss theorem for the effective chiral charges. Let us consider a cylindrical volume \( V \) around the ‘metamaterial waveguide’ formed by the effective horizon at \( r = \rho \) (see figure 2), such that the side wall of the volume \( V \) is located very far from the waveguide and the electromagnetic field is zero at this wall. If \( S \) is the closed two-dimensional cylindrical surface bounding \( V \), with area element \( da \) and unit outward normal \( \hat{n} \) at \( da \), and \( S_1 \) and \( S_2 \) are the front and the back surfaces of \( V \), we may write the following integral equation for the Pointing vector \( \vec{S}_L \) of the \( L \)th mode:

**Figure 2.** Schematic view of the metamaterial waveguide formed by the effective horizon at \( r = \rho \) from figure 1(b), and the front and back surfaces \( S_1 \) and \( S_2 \) of the auxiliary cylinder used in the derivation of the Gauss theorem for cylindrical surface plasmons treated as ‘chiral charges’ (see equation (25)). Vector \( N \) indicates chosen direction of the metamaterial waveguide.

\[
\int_V \vec{\nabla} \times \vec{S} _L \, dV = \int_{S_2} \vec{N} \times \vec{S}_L \, da - \int_{S_1} \vec{N} \times \vec{S}_L \, da
\]

(25)

where \( \vec{N} \) is the chosen direction of the metamaterial waveguide. Using equation (24) we obtain

\[
\int_V \frac{4\pi \omega_0 f}{c^2} \vec{\nabla} \times \vec{S}_L \, dV = \int_{S_2} \vec{N} \times \left[ \vec{\nabla} \times \vec{B}_L \right] \, da - \int_{S_1} \vec{N} \times \left[ \vec{\nabla} \times \vec{B}_L \right] \, da
\]

(26)

Since \( \vec{N} \times \left[ \vec{\nabla} \times \vec{B}_L \right] = \frac{\partial \omega_0 f}{\partial \omega} \), we see that a ‘chiral charge’ produces a local step in the gyration field. Equations (24) and (26) (which represent effective Poisson equation, and effective Gauss Theorem for chiral charges, respectively) clearly demonstrate that the ‘chiral charges’ interact according to the one-dimensional Coulomb law with the interaction energy growing linearly with distance (in reality this idealized linear growth is cut off by the absorption in the metamaterial). This field-theoretical description of nonlinear optics of chiral charges provides a holographic dual to the effective gravity description discussed earlier. The described duality appears useful for exactly the same reason as the original AdS/CFT duality is useful in high energy physics. Nonlinear optical Maxwell equations are very difficult to study and analyze either numerically or analytically. This task is especially difficult in the case of newly developed sophisticated metamaterials, where the usual experience does not always apply. For example, increased light confinement in hyperbolic metamaterials due to formation of spatial solitons [16–18] is counter-intuitive. It occurs only if a self-defocusing
Kerr medium is used as a dielectric host. On the other hand, this behavior finds simple and natural explanation in terms of analog gravity [7]. Similar to the original AdS/CFT correspondence, the duality between effective gravity and effective field theory of chiral charges described above provides us with a new powerful tool to understand nonlinear optics of metamaterials when the nonlinear interactions are strong. Moreover, similar to AdS/CFT, these dual descriptions work best in the opposite limits: strong chiral interaction in $1 + 1$D space corresponds to weak gravity limit in $2 + 1$D, and vice versa.

It is also interesting to note that due to similar topological origin, theoretical description based on chiral charges may be extended to encompass mutual interaction of Kaluza–Klein electric charges [8]. This approach leads to a picture of strong plasmon-mediated electron-electron interaction in metal nanowire and metal nanohole arrays. Therefore, the developed holographic framework may have potential implications for understanding of superconductivity in metal wire array hyperbolic metamaterials. It appears that similar to a different metamaterial approach discussed in [9], conditions favorable for strong electron-electron interaction arise in the epsilon near zero regime, which corresponds to formation of an effective ‘gravitational horizon’ (see figure 1(b)). This feature of our model is somewhat similar to the holographic approach to superconductivity developed in [3]. Indeed, within the scope of holographic models the superconducting transition is typically linked to classical instability of a black hole horizon in AdS space against perturbations by a charged scalar field. The instability appears when the black hole has Hawking temperature $T = T_c$ [3].

Another interesting application of the developed holographic dual formalisms appears to be consideration of the superconducting state of physical vacuum in a strong magnetic field. As demonstrated by Chernodub [28], strong magnetic field forces vacuum to develop real condensates of electrically charged $\rho$ mesons, which form an anisotropic inhomogeneous superconducting state similar to Abrikosov vortex lattice. As far as electromagnetic field behavior is concerned, this state of vacuum constitutes a hyperbolic metamaterial [29]. Moreover, it was demonstrated in [21] that a well-known ‘additional wave’ solution of macroscopic Maxwell equations describing metamaterial optics of vacuum in the presence of dispersion leads to prediction of $\sim 2$ GeV ‘heavy photons’ in vacuum subjected to a strong magnetic field [2]. These ‘heavy photons’ may be identified with CSPs propagating along individual Abrikosov vortices. Holographic consideration presented above leads to conclusion that heavy photon states which have non-zero angular momenta must behave as interacting ‘chiral charges’. The chiral charge of a heavy photon is equal to its angular momentum $L$. These massive slow moving effective charges exhibit long-range interaction via exchange of fast massless CSPs having zero angular momentum. Effective field theory of these chiral charges may be formulated similar to the effective field theoretical description of nonlinear optics of wire array hyperbolic metamaterials presented above. Thus, our dual holographic description is directly applicable to real physical vacuum subjected to a strong magnetic field.

In summary, we have demonstrated that nonlinear optics of hyperbolic metamaterials admits explicit dual holographic descriptions both in terms of $2 + 1$ dimensional bulk effective gravity, and in terms of $1 + 1$ dimensional surface effective field theory. To our knowledge, this is the first known experimental system which exhibits such an explicit holographic duality and is amenable to direct experimental testing. Such testing may be performed using ferrofluid-based self-assembled hyperbolic metamaterials [30], which exhibit considerable negative Kerr effect (and therefore effective attractive gravity-like interaction [7]) due to thermal expansion of kerosene in the ferrofluid. Gradual variation of the dielectric tensor components $\varepsilon_{xx} = \varepsilon_{yz}$ and $\varepsilon_{xy}$ of the ferrofluid may be achieved due to gradient of external magnetic field and/or self-focusing, leading to appearance of an effective horizon. On the other hand, illumination of ferrofluid with light having non-zero orbital angular momentum [31] will lead to excitation of CSPs having non-zero angular momentum propagating inside the ferrofluid. As demonstrated by equations (23) and (26), such CSPs will exhibit strong 1D Coulomb-like interaction, which will lead to nonlinear dependence of ferrofluid transmission on illuminating power, similar to nonlinear effects in plasmon-mediated nonlinear optical transmission through individual nanoholes [32] and nanohole arrays [33].

References

[1] Maldacena J 1998 The large N limit of superconformal field theories and supergravity Adv. Theor. Math. Phys. 2 231–52
[2] Bouso R 2002 The holographic principle Rev. Mod. Phys. 74 825–74
[3] Hartnoll S A, Kovtun P K, Müller M and Sachdev S 2007 Theory of the nernst effect near quantum phase transitions in condensed matter and in dyonic black holes Phys. Rev. B 76 144502
[4] Donos A and Hartnoll S A 2013 Interaction-driven localization in holography Nat. Phys. 9 649–55
[5] Khveshchenko D V 2013 Simulation of holographic correspondence in flexible graphene Eur. Phys. Lett. 104 47002
[6] Amarit A, Forcella D, Mariotti A and Pollicastro G 2011 Holographic optics and negative refractive index J. High Energy Phys. JHEP04(2011)036
[7] Smolyaninov I I 2013 Analogue gravity in hyperbolic metamaterials Phys. Rev. A 88 033843
[8] Smolyaninov I I 2003 Electron-plasmon interaction in a cylindrical mesoscopic system: important similarities with Kaluza–Klein theories Phys. Rev. B 67 165406
[9] Smolyaninov I I and Smolyaninova V N Is there a metamaterial route to high temperature superconductivity? arXiv:1311.3277
[10] Smith D R and Schurig D 2003 Electromagnetic wave propagation in media with indefinite permittivity and permeability tensors Phys. Rev. Lett. 90 077405
[11] Jakob Z, Alekseyev L V and Narimanov E 2006 Optical hyperlens: far-field imaging beyond the diffraction limit Opt. Express 14 8247–56

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[12] Salandrino A and Engheta N 2006 Far-field subdiffraction optical microscopy using metamaterial crystals: theory and simulations Phys. Rev. B 74 075103
[13] Smolyaninov I I, Hung Y J and Davis C C 2007 Magnifying superlens in the visible frequency range Science 315 1699–701
[14] Liu Z, Lee H, Xiong Y, Sun C and Zhang X 2007 Far-field optical hyperlens magnifying sub-diffraction-limited objects Science 315 1686
[15] Belov P A, Hao Y and Sudhakaran S 2006 Subwavelength microwave imaging using an array of parallel conducting wires as a lens Phys. Rev. B 73 033108
[16] Ye F, Mihalache D, Hu B and Panoiu N C 2011 Subwavelength vortical plasmonic lattice solitons Opt. Lett. 36 1179
[17] Kou Y, Ye F and Chen X 2011 Multipole plasmonic lattice solitons Phys. Rev. A 84 033855
[18] Silveirinha M G 2013 Theory of spatial optical solitons in metallic nanowire materials Phys. Rev. B 87 235115
[19] Smolyaninov I I and Narimanov E E 2010 Metric signature transitions in optical metamaterials Phys. Rev. Lett. 105 067402
[20] Smolyaninov I I and Hung Y J 2011 Modeling of time with metamaterials JOSA B 28 1591–5
[21] Smolyaninov I I 2012 ‘Planck-scale physics’ of vacuum in a strong magnetic field Phys. Rev. D 85 114013
[22] Landau L and Lifshitz E 2004 Field Theory (Amsterdam: Elsevier)
[23] Wangberg R, Elser J, Narimanov E E and Podolskiy V A 2006 Nonmagnetic nanocomposites for optical and infrared negative-refractive-index media J. Opt. Soc. Am. B 23 498–505
[24] Smolyaninov I I, Hwang E and Narimanov E E 2012 Hyperbolic metamaterial interfaces: hawking radiation from rindler horizons and spacetime signature transitions Phys. Rev. B 85 235122
[25] Smolyaninov I I, Hung Y J and Hwang E 2011 Experimental modeling of cosmological inflation with metamaterials Phys. Lett. A 376 2575–9
[26] Smolyaninov I I 2002 Fractal extra dimension in Kaluza–Klein theory Phys. Rev. D 65 047503
[27] Landau L D and Lifshitz E M 1984 Electrodynamics of Continuous Media (New York: Pergamon)
[28] Chernodub M N 2011 Spontaneous electromagnetic superconductivity of vacuum in a strong magnetic field: evidence from the Nambu–Jona–Lasinio model Phys. Rev. Lett. 106 142003
[29] Smolyaninov I I 2011 Vacuum in strong magnetic field as a hyperbolic metamaterial Phys. Rev. Lett. 107 253903
[30] Smolyaninova V N, Yost B, Lahneman D, Narimanov E E and Smolyaninov I I Self-assembled tunable photonic hyper-crystals arXiv:1312.7138
[31] Gibson G, Courtial J, Padgett M J, Vlasov M, Pas’ko V, Barnett S M and Franke-Arnold S 2004 Free-space information transfer using light beams carrying orbital angular momentum Opt. Express 12 5448–56
[32] Smolyaninov I I, Zayats A V, Gungor A and Davis C C 2002 Single-photon tunneling via localized surface plasmons Phys. Rev. Lett. 88 187402
[33] Smolyaninov I I, Zayats A V, Stanishevsky A and Davis C C 2002 Optical control of photon tunneling through an array of nanometer scale cylindrical channels Phys. Rev. B 66 205414