On the justification of applying quantum strategies to the Prisoners’ Dilemma and mechanism design

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The Prisoners’ Dilemma is perhaps the most famous model in the field of game theory. Consequently, it is natural to investigate its quantum version when one considers to apply quantum strategies to game theory. There are two main results in this paper: 1) The well-known Prisoners’ Dilemma can be categorized into three types and only the third type is adaptable for quantum strategies. 2) As a reverse problem of game theory, mechanism design provides a better circumstance for quantum strategies than game theory does.

Keywords: Quantum games; Prisoners’ Dilemma; Mechanism design.

1. Introduction

Game theory and mechanism design are two important branches of economics. Game theory aims to investigate rational decision making in conflict situations, whereas mechanism design just concerns the reverse question: given some desirable outcomes, can we design a game that produces it?

In 1999, Eisert et al.\(^1\) proposed a pioneering quantum-version model of two-player Prisoners’ Dilemma. The novel model showed a fascinating “quantum advantages” as a result of a novel quantum Nash equilibrium. Guo et al.\(^2\) gave a good review on quantum games. In 2010, Wu\(^3\) investigated what would happen if agents could use quantum strategies in the theory of mechanism design. The result was interesting, i.e., by virtue of a quantum mechanism, agents who satisfied a certain condition could combat “bad” social choice rules instead of being restricted by the traditional mechanism design theory.

Despite the aforementioned accomplishments in quantum games, in 2002, van Enk and Pike\(^4\) criticized that a quantum game was indeed a new game that was constructed and solved, not the original classical game. In this paper, we will deeply investigate whether and when quantum strategies are useful for economic society. In Section 2, we will analyze the justification of applying quantum strategies to the Prisoners’ Dilemma. In Section 3, we will investigate the justification of applying quantum strategies to mechanism design. Section 4 draws the conclusions.
2. The justification of applying quantum strategies to the Prisoners’ Dilemma

2.1. Three types of Prisoners’ Dilemma

As is well known, the Prisoners’ Dilemma is a simple model that captures the essential contradiction between individual rationality and global rationality. In the classical Prisoners’ Dilemma, two prisoners are arrested by a policeman. Each prisoner must independently choose to cooperate (strategy $C$) or to defect (strategy $D$). Table 1 shows the payoff matrix of two prisoners, Alice and Bob.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
   & Alice & $C$ & $D$  \\
\hline
$C$ & $(3, 3)$ & $(0, 5)$ & \\
$D$ & $(5, 0)$ & $(1, 1)$ & \\
\hline
\end{tabular}
\caption{The payoff matrix of two prisoners. The first entry in the parenthesis denotes the payoff of Alice and the second stands for the payoff of Bob.}
\end{table}

Suppose the prisoners are rational, then the unique Nash equilibrium is the dominant strategy $(D, D)$, while the Pareto optimal strategy is $(C, C)$. Although the Prisoners’ Dilemma has found widespread applications in many disciplines such as economics, politics, sociology and so on, perhaps not everybody notice that actually there are three different types of Prisoners’ Dilemma:

- **Type-1 Prisoners’ Dilemma:** There is no arbitrator to assign payoffs to the players, i.e., there are only two players in the game, whose payoffs are generated by the players themselves. For example, let us consider two countries (e.g., US and Russia) confronted the problem of nuclear disarmament. The strategy $C$ means “Obeying disarmament”, and $D$ means “Refusing disarmament”. Obviously, in the nuclear disarmament game, there is no arbitrator and the payoffs of two countries are generated by themselves.

- **Type-2 Prisoners’ Dilemma:** There is an arbitrator in the game to assign payoffs to the players. Before the players send strategies to the arbitrator independently, they cannot communicate to each other. For example, if the prisoners are arrested separately, this is the type-2 Prisoners’ Dilemma.

- **Type-3 Prisoners’ Dilemma:** There is an arbitrator in the game to assign payoffs to the players. Before the players send strategies to the arbitrator independently, they can communicate to each other but cannot enter into a binding contract. For example, if the prisoners are arrested in one room and confront a payoff matrix specified by Table 1, this is the type-3 Prisoners’ Dilemma. It should be emphasized that although the rational players all agree that the strategy $C$ will benefit both of them, they will definitely choose the strategy $D$ when they actually send strategies to the policeman independently, because $(D, D)$ is the unique dominant strategy. This is the catch of the dilemma. (Note: If the two players can enter into a binding contract before they send strategies to the arbitrator, then the game will become a cooperative game. We simply ignore this case.)
2.2. Eisert et al’s model

In 1999, the Prisoners’ Dilemma was generalized to a quantum domain. Fig. 1 shows the setup of a two-player quantum game. The timing sequence of the quantum Prisoners’ Dilemma is referred to Ref. 1. Under classical circumstances, people usually do not discriminate the differences among three types of Prisoners’ Dilemma because they correspond to the same payoff matrix (i.e., Table 1). But in the context of quantum domain, only the type-3 Prisoners’ Dilemma is adaptable for quantum strategies. In the following, we explain the three cases in details.

![Fig. 1. The setup of a two-player quantum game.](image)

**Type-1 Prisoners’ Dilemma:** Obviously, it is meaningless to consider the quantum version of type-1 Prisoners’ Dilemma, because in Eisert et al’s model, the payoffs of two players are assigned by an arbitrator, which is inconsistent to the model of type-1 Prisoners’ Dilemma.

**Type-2 Prisoners’ Dilemma:** In Eisert et al’s original paper, it is implicit whether the quantum version of the type-2 Prisoners’ Dilemma is feasible or not. Here, we will analyze this problem deeply. Initially, the two qubits possessed by the prisoners are separable. It is impossible for the prisoners themselves to entangle the two qubits, because they cannot communicate to each other and each prisoner can only perform a local unitary operation on his/her own qubit. Therefore, we have to assume that the policeman entangles the two qubits. However, just as van Enk and Pike have said: “it seems to us counter to the spirit of the game to have an attorney or interrogator be helpful to the prisoners and give them an entangled state”, it is unreasonable to make this assumption.

Furthermore, there are two ways to understand the unreasonableness: 1) Since the Prisoners’ Dilemma is a game with complete information, the policeman knows exactly that once the prisoners possess two entangled qubits, they will certainly choose the quantum strategy \((\hat{Q}, \hat{Q})\) and obtain the Pareto optimal payoffs (3,3). Therefore, if the policeman does help the prisoners perform entangling operation, then why doesn’t the policeman assign the Pareto optimal payoffs (3,3) directly to the prisoners? 2) Since the role of policeman is the arbitrator, it is unreasonable to require the policeman to have an incentive to help the prisoners better off and reach their Pareto optimal payoffs.

Consequently, the Eisert et al’s model is meaningless to the type-2 Prisoners’
Dilemma.

Type-3 Prisoners’ Dilemma: For this case, the two prisoners can communicate to each other. Hence, it is reasonable to assume that the two qubits held by the prisoners can be entangled by themselves, i.e., the quantum game doesn’t need a benevolent attorney or interrogator to give the prisoners an entangled state. The essence of the quantum game is that for the type-3 Prisoners’ Dilemma, the unique dominant strategy is changed from \((\hat{D}, \hat{D})\) to \((\hat{Q}, \hat{Q})\), which results in a Pareto optimal payoff \((3,3)\).

Now, we have categorized the well-known Prisoners’ Dilemma into three different types. It can be seen that Eisert et al’s model is adaptable only to the type-3 Prisoners’ Dilemma. Can we think that the problem about the justification of applying quantum strategies to the Prisoners’ Dilemma has been solved? Not yet, because another serious question arises naturally: Since the policeman also knows the aforementioned three types of Prisoners’ Dilemma, then why does he allow the prisoners to communicate to each other and have a chance to use quantum strategies? For the case of the classical Prisoners’ Dilemma, there is no reason for the policeman to do so.

3. The justification of applying quantum strategies to mechanism design

3.1. Maskin’s classical theorem

In the field of economics, game theory has a reverse problem, i.e., the theory of mechanism design. Game theory aims to predict the outcome of a given game, whereas the theory of mechanism design concerns a “social engineering” question: given some desirable outcomes, can we design a game that produces it?

There are two important notions in mechanism design theory: social choice rule (SCR) and mechanism. An SCR specifies the objects that the designer would like to implement: in each state, she would like to realize some set of outcomes, but unfortunately, she does not know the true state. A mechanism is a representation of the social institution through which the agents interact with the designer and with one another: each agent sends a message to the designer, who chooses an outcome as a function of these strategy choices.

Ref. 5 is a fundamental work in the field of mechanism design. It provides an almost complete characterization of social choice rules (SCRs) that are Nash implementable. The main results of Ref. 5 are the two following theorems: 1) (Necessity): If an SCR \(F\) is Nash implementable, then it is monotonic. 2) (Sufficiency): Let \(n \geq 3\), if an SCR \(F\) is monotonic and satisfies no-veto, then it is Nash implementable.

According to the sufficiency theorem, even if all agents dislike an SCR specified by the designer, as long as it is monotonic and satisfies no-veto, one can always construct a mechanism to implement the SCR in Nash equilibrium.
3.2. Wu’s quantum mechanism

In 2010, Wu investigated what would happen if agents could use quantum strategies in the theory of mechanism design. The setup of a quantum mechanism is depicted in Fig. 1. Working steps of the quantum mechanism are referred to Ref. 3.

According to Ref. 3, when condition $\lambda$ is satisfied, an original Nash implementable “bad” (i.e., Pareto-inefficient) SCR will no longer be Nash implementable in the context of quantum domain. The key point of the quantum mechanism is to carry out a quantum version of $n$-player type-3 Prisoners’ Dilemma. In the end of Section 2, we argue that the type-3 Prisoners’ Dilemma is not meaningful either. However, according to the three reasons listed below, it is natural for us to adopt the quantum version of the type-3 Prisoners’ Dilemma into the field of mechanism design.

1) In the theory of mechanism design, there always exists a designer (i.e., the arbitrator) to assign outcomes to the agents. Hence, the payoffs of the agents are specified by the arbitrator.

2) When Nash implementation is concerned, agents possess complete information. Therefore, it is reasonable to think that agents can communicate to each other before they send their strategies to the designer independently.

3) In the theory of mechanism design, the designer is at an information disadvantage with respect to the agents. He cannot restrict agents from communicating to each other, so the question in the end of Section 2 is solved. In Wu’s quantum mechanism, from the viewpoint of the designer, the interface between agents and the designer is the same as that in the Maskin’s classical mechanism. The designer cannot discriminate whether the underlying principle of agent’s actions is classical or quantum mechanical. Put in other words, the designer cannot restrict the agents from using quantum strategies to reach their Pareto-optimal outcomes.
4. Conclusions

In this paper, we categorize the well-known Prisoners’ Dilemma into three types, and point out that Eisert et al.’s model is adaptable only to the type-3 Prisoners’ Dilemma. Moreover, in the context of game theory, it is still unreasonable that the arbitrator allow the agents to communicate to each other and have a chance to use quantum strategies. Just as Maskin \(^7\) has said: “Mechanism design provides the circumstances perhaps most favorable for Nash equilibrium being a good predictor of human behavior in strategic settings.” \(^8\), in Section 3, we find that mechanism design provides a more favorable circumstance for quantum strategies than game theory does.

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