Huge Quantum Gravity Effects in the Solar System

Don N. Page *
Theoretical Physics Institute
Department of Physics, University of Alberta
Room 238 CEB, 11322 – 89 Avenue
Edmonton, Alberta, Canada T6G 2G7

2010 May 17

Abstract

Normally one thinks of the motion of the planets around the Sun as a highly classical phenomenon, so that one can neglect quantum gravity in the Solar System. However, classical chaos in the planetary motion amplifies quantum uncertainties so that they become very large, giving huge quantum gravity effects. For example, evidence suggests that Uranus may eventually be ejected from the Solar System, but quantum uncertainties would make the direction at which it leaves almost entirely uncertain, and the time of its exit uncertain by about a billion billion years. For a time a billion billion years from now, there are huge quantum uncertainties whether Uranus will be within the Solar System, within the Galaxy, or even within causal contact of the Galaxy.

*Internet address: don@phys.ualberta.ca
The motion of planets in the Solar System is often regarded as the paradigm of a completely deterministic system, say in the nonrelativistic model of Newtonian gravitation for a spherical Sun and spherical planets. Of course it is known that a deterministic classical model is only an approximation for an underlying quantum description, but one might estimate that quantum uncertainties should be quite negligible for the Solar System.

For example, if one has an object with mass $M$ and no external forces on it, one may put its center of mass into a minimum-uncertainty (gaussian) quantum wavefunction with rms position uncertainty $\Delta x$ and rms momentum uncertainty $\Delta p = \hbar/\Delta x$. Then after a time $t$, the motion due to the rms momentum uncertainty will combine with the original rms position uncertainty to give a total rms position uncertainty $\sqrt{\Delta x^2 + [(\hbar t)/(M \Delta x)]^2}$. For fixed $t$, this has the minimal value $\sqrt{2\hbar t/M}$. If we take $t$ to be the age of the Solar System, 4.6 billion years, and $M$ to be the mass of the Earth, $5.972 \times 10^{24}$ kilograms, then the minimal rms position uncertainty is $1.2 \times 10^{-27}$ meters, which for practical purposes is quite negligible. That is, it appears that if one put a planet into a minimal-uncertainty wavepacket with suitable $\Delta x$, the spreading would be negligible over the lifetime of the Solar System.

One can also consider the quantum effect of sunlight scattering off the Earth, which would be expected to give the Earth an rms momentum roughly equal to the momentum of each photon multiplied by the square root of the number of scattered photons. Over the age of the Solar System, this would give a position uncertainty of the order of ten nanometers ($10^{-8}$ m), much larger than the uncertainties without scattered sunlight, but again in practice quite negligible.

Thus it appears that the quantum fluctuations from the Uncertainty Principle applied to the center of mass of the Earth and from the quantum fluctuations in the scattering of sunlight from the Earth are always quite tiny. Such considerations
seem to suggest that quantum effects are very small for the motion of planets in the Solar System.

Nevertheless, there are other effects that can make quantum effects become enormous. In particular, classical chaos in the Solar System \[1, 2, 3, 4, 5, 6, 7, 8, 9\] can exponentially amplify quantum uncertainties \[10, 11, 12, 13\] to become huge uncertainties in the positions of the planets. Because these effects involve both quantum uncertainties and the uncertain gravitational fields produced by the Sun and planets at their uncertain positions, they are truly large quantum gravity effects. Here I shall argue that one set of such uncertainties is the time and velocity at which Uranus is likely to be ejected from the Solar System.

As summarized in \[9\], it was recognized that a point-mass Newtonian model with three or more masses could exhibit both chaotic and regular motion, but up to the 1980s, it was thought that the actual conditions for the Sun and planets of our Solar System would result in regular motion. Therefore, it was a surprise in 1988 when Sussmann and Wisdom \[1\] showed that Pluto’s motion was chaotic, with a Lyapunov time (the timescale for exponential growth of uncertainties) of about 20 Myr. The next year Laskar \[2\] found that the entire Solar System was chaotic, with a Lyapunov time of about 5 Myr, as confirmed in more detail in 1992 by Sussman and Wisdom \[3\], who also found that the outer Solar System by itself was chaotic with a Lyapunov time of about 7 Myr. Nevertheless, in 1994 Laskar \[6\] integrated the motion of all the planets except Pluto for 25 Gyr and found no ejections or collisions, so it seems that nothing drastic may happen to the present planets until the Sun swells up into a red giant and engulfs the inner Solar System out through the earth 7.6 Gyr from now \[15\]. If the Sun did not do that, Laskar found evidence supporting the idea that Mercury might be removed by a collision with the Sun or Venus on a timescale of roughly 2000 Gyr, far later than the Sun would actually swallow it.
Despite the fact that the finite future lifetime of the Sun does not seem to give enough time for the inner Solar System to undergo a drastic transition (e.g., by losing a planet) before it gets destroyed by the Sun’s red giant phase, there can still be huge uncertainties in the relative positions of the planets. Zurek [13], expanding upon ideas of [10, 11, 12], has noted that an upper bound on the timescale for which the evolution of a classically chaotic system becomes flagrantly non-classical (huge amplified quantum uncertainties) is the Lyapunov time multiplied by the logarithm of the action of the system in units of $\hbar$. If we take the action of the Solar System as the angular momentum of the planetary system, which is \[3.148 \times 10^{43} \text{ kg m}^2 \text{s}^{-1} = 2.985 \times 10^{77} \hbar,\] its logarithm is 178.393, so if one uses a Lyapunov time of 5 Myr, one gets that quantum uncertainties should be amplified to huge values by 900 Myr, which is about 20% of the present age of the Solar System. Therefore, one would expect that even if one measured the position and momentum as precisely as possible, the relative positions of the planets would have uncertainties of the order of an astronomical unit after another billion years or so. (The distances between the planets and the Sun are not likely to be uncertain by nearly so much within this timescale of gigayears, but the angles between the planets and the Sun would be expected to have quantum uncertainties of the order of radians.)

On a longer timescale, the ultimate fate of the outer Solar System seems likely to have even much greater uncertainties. It has recently been shown [7, 8, 9] that overlapping three-body resonances among Jupiter, Saturn, and Uranus give a Lyapunov time of 5–10 Myr and an escape time for Uranus of the order of $10^{18}$ years, say from Uranus’ orbit becoming perturbed to pass close to Saturn and gain enough kinetic energy by the slingshot effect to escape the gravitational pull of the Sun. The ratio of the escape time to the Lyapunov time, $10^{11}$ or so, is of the order of a billion times the logarithm of the action, so the details of the escape will have huge quantum uncertainties.
In particular, the time at which Uranus escapes from the Solar System is likely to have quantum uncertainties of the same order of magnitude as the escape time itself, of the order of a billion billion ($10^{18}$) years. With an escape velocity of the order of the orbital velocity of Uranus, $2 \times 10^{-5}c$, and with a Galactic size of the order of 25 kiloparsecs or close to $10^5$ light years, it would take less than $10^{10}$ years for Uranus to move very far across the Galaxy from where the Sun is. Therefore, in $10^{18}$ years, Uranus would have time to make hundreds of millions of orbits around the Galaxy relative to the Sun. Furthermore, the relaxation time for an object to change its orbit in the Galaxy significantly has been given [16] in the solar neighborhood as $2.6 \times 10^6$ years multiplied by the cube of the velocity in kilometers per second, which gives nearly $10^9$ years for the orbital velocity of Uranus. Thus in a time comparable to the escape time from the Solar System of $10^{18}$ years, Uranus would have time for something like a billion ($10^9$) significant changes in its orbit in the Galaxy. This should be plenty of time for Uranus to gain a significant probability of escaping not only from the Solar System, but also from the entire Galaxy.

If the present value of the dark energy persists, so that universe continues to expand exponentially at a timescale of the order of $10^{10}$ years, by a proper time equal to the expected escape time from the Solar System of $10^{18}$ years, Uranus will not only have a significant probability of escaping from both the Solar System and the Galaxy, but also of crossing the cosmological event horizon, so that no signals can thereafter get back from Uranus to the rest of the Solar System. Therefore, whether or not Uranus can even in principle be seen from another planet like Saturn at the expectation value of its escape time will have enormous quantum gravity uncertainties.

Thus quantum gravity effects in the motions of the planets give a huge uncertainty in the time at which Uranus will escape from the Solar System, as well as in the escape direction. At a time corresponding to the expectation value of this
escape time, the position of Uranus could, with significant probabilities, be either within the Solar System, almost anywhere within the Galaxy, or even beyond the cosmological event horizon. Although quantum gravity effects within the Solar System may be small within a human lifetime, over the timescale needed for a planet like Uranus to escape, quantum gravity effects are enormous.

References

[1] G. J. Sussman and J. Wisdom, Science 241, 433-437 (1988).

[2] J. Laskar, Nature 338, 237-238 (1989).

[3] G. J. Sussman and J. Wisdom, Science 257, 56-62 (1992).

[4] I. Peterson, Newton’s Clock: Chaos in the Solar System (Freeman, New York, 1993).

[5] M. Duncan and T. Quinn, Annu. Rev. Astron. Astrophys. 31, 265-295 (1993).

[6] J. Laskar, Astron. Astrophys. 287, L9-L12 (1994).

[7] N. Murray and M. Holman, Science 283, 1877-1881 (1999).

[8] R. Malhotra, M. Holman, and T. Ito, Proc. Natl. Acad. Sci. 98, 12342-12343 (2001).

[9] M. Lecar, R. A. Franklin, M. J. Holman, and N. Murray, Annu. Rev. Astron. Astrophys. 39, 581-631 (2001).

[10] G. P. Berman and G. M. Zaslavsky, Physica 91A, 450-460 (1978).

[11] M. V. Berry and N. L. Balazs, J. Phys. A: Math. Gen. 12, 625-642 (1979).
[12] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. 72, 2508-2511 (1994).

[13] W. H. Zurek, Phys. Scr. T76, 186-198 (1998).

[14] A. N. Cox, ed., Allen’s Astrophysical Quantities, 4th ed. (Springer-Verlag, New York, 2000), p. 293.

[15] K.-P. Schröder and R. C. Smith, Mon. Not. R. Astron. Soc. 386, 155-163 (2008).

[16] C. W. Allen, Astrophysical Quantities, 3rd ed. (Athlone Press, London, 1973), p. 284.