Abstract

Supersymmetric (SUSY) extension of the Standard Model (SM) is a primary candidate for new physics beyond the SM. If SUSY breaking scale is very low, for example, the multi-TeV range, and the SUSY breaking sector, except for the goldstino (gravitino), is decoupled from the low energy spectrum, the hidden sector effect in the minimal SUSY SM (MSSM) is well described by employing the goldstino chiral superfield (X) with the nilpotent condition of $X^2 = 0$. Although this so-called “nonlinear MSSM” (NL-MSSM) provides a variety of interesting phenomenologies, there is a cosmological problem that the lightest superpartner gravitino is too light to be the major component of the dark matter (DM) in our universe. To solve this problem, we propose a minimal extension of the NL-MSSM by introducing a parity-odd SM singlet chiral superfield ($\Phi$). We show that the interaction of the scalar component in $\Phi$ with the MSSM Higgs doublets is induced after eliminating $F$-component of the goldstino superfield and the lightest real scalar in $\Phi$ plays the role of the Higgs-portal DM. With a suitable choice of the model parameters, a successful Higgs-portal DM scenario can be realized while achieving the SM-like Higgs boson mass of 125 GeV from the tree-level Higgs potential through the multi-TeV SUSY breaking effect.
1 Introduction

Although the current experimental data show no plausible evidence of new physics beyond the Standard Model (SM), the minimal supersymmetric (SUSY) extension of the SM (MSSM) is still a primary candidate for new physics. As has been well-known and intensively studied, the MSSM not only provides us with a solution to the gauge hierarchy problem but also offers a variety of interesting phenomenologies, such as the origin of the electroweak symmetry breaking from SUSY breaking, the SM-like Higgs boson mass prediction with soft SUSY breaking parameters, the lightest superpartner (LSP) as a natural candidate for the dark matter (DM) in our universe, and the grand unified theory paradigm with the successful unification of the three SM gauge couplings at a scale of $O(10^{16}$ GeV). Many ongoing and planned experiments will continue searching for the MSSM, or in more general, supersymmetric theories beyond the SM.

In phenomenologically viable models, SUSY is spontaneously broken in the hidden sector and the SUSY breaking effects are mediated to the MSSM sector by a certain mechanism for generating soft SUSY breaking terms in the MSSM. Associated with spontaneous SUSY breaking, a massless fermion called goldstino emerges due to the Nambu-Goldstone theorem, and it is absorbed into the spin-$1/2$ component of the spin-$3/2$ massive gravitino in supergravity. The gravitino mass is characterized by the SUSY breaking order parameter $f$ and the reduced Planck mass of $M_P = 2.43 \times 10^{19}$ GeV as $m_{3/2} \simeq f/M_P$. It is possible that SUSY breaking occurs at a very low energy (see, for example, Ref. [1]). If this is the case, gravitino becomes the LSP and is involved in phenomenology at low energies. For example, if the SUSY breaking scale lies in the multi-TeV range, the LSP gravitino is extremely light with its mass of $O$(meV). Assuming the decoupling of the hidden sector fields except for the light gravitino (or, equivalently, goldstino) the low energy effective theory involving the very light gravitino can be described by employing a goldstino chiral superfield $X$ with the nilpotent condition $X^2 = 0$ [2, 3, 4]. With this formalism, the phenomenology of the MSSM with the goldstino superfield has been studied in detail [5, 6, 7, 8] (see also Ref. [8] for the phenomenology in a more general setup). This framework is the so-called nonlinear MSSM (NL-MSSM). In particular, it has been shown that if the SUSY breaking scale lies in the multi-TeV range, the SM-like Higgs boson receives a sizable contribution to its mass at tree-level after eliminating $F$-component of the goldstino superfield and as a result, the Higgs mass of around 125 GeV can be achieved even without the scalar top-quark quantum corrections.

Although the extremely light gravitino in the NL-MSSM is harmless in the phenomenological point of view (see, for example, Ref. [9]), its relic density is far below the observed dark matter (DM) density. Thus, for the completion of the NL-MSSM, we may consider an extension of the model which can supplement the model with a suitable DM candidate. In this paper, we propose a minimal extension of the NL-MSSM by introducing a $Z_2$-parity odd SM gauge
singlet chiral superfield $\Phi$ and show that the lightest scalar component in $\Phi$ plays the role of the Higgs-portal DM \cite{10,11} through its coupling with the MSSM Higgs doublets induced by the goldstone superfield. With a suitable choice of the model parameters, we can realize a phenomenologically viable Higgs-portal DM scenario while achieving the 125 GeV mass for the SM-like Higgs boson from the Higgs potential at the tree level.

2 NL-MSSM and the Higgs boson mass

We first present the basic formalism of the NL-MSSM and show how the 125 GeV SM-like Higgs boson mass can be achieved in the framework. We begin with the goldstino effective Lagrangian of the form \cite{4}:

$$\mathcal{L}_X = \int d^4 \theta X^\dagger X + \left( \int d^2 \theta f X + \text{h.c.} \right),$$

where $X$ is a goldstino chiral superfield, and $f$ is the SUSY breaking order parameter in the hidden sector. Although the stability of the hidden sector scalar potential needs an extension of the above minimal Kähler potential, this Lagrangian is enough to understand the essence of the formalism. The goldstino chiral superfield is subject to the nilpotent condition \cite{2,3,4},

$$X^2 = 0,$$

which leads us to express the superfield with the components,

$$X = \frac{\psi_X \psi_X}{2F_X} + \sqrt{2} \theta \psi_X + \theta \theta F_X.$$

The scalar component in the goldstino superfield is to be integrated out in the low energy effective theory, and under the nilpotent condition, it is replaced by the bilinear term of the goldstino fields. In fact, substituting Eq. (3) into Eq. (1) and eliminating the auxiliary field $F_X$, we recover the Volkov-Akulov Lagrangian \cite{13}.

In the superfield formalism, the spurion technique is a simple way to introduce the soft SUSY breaking terms to the MSSM Lagrangian. We introduce a dimensionless and SM-singlet spurion field of the form, $Y = \theta^2 m_{\text{soft}}$, where $m_{\text{soft}}$ is a generic notation for the soft terms (denoted $m_{1,2,3}$, $m_\psi$, $m_\lambda$, in the following), and attach it to any SUSY operators in the MSSM. The recipe to obtain the NL-MSSM is to replace the spurion by the goldstino superfield as \cite{4}

$$Y \rightarrow \frac{m_{\text{soft}}}{f} X.$$

We apply this rule and write the NL-MSSM Lagrangian as follows \cite{5}:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_X + \mathcal{L}_H + \mathcal{L}_m + \mathcal{L}_{AB} + \mathcal{L}_g.$$
In the right-hand side, the first term $L_0$ denotes the supersymmetric part of the MSSM Lagrangian given by

$$L_0 = \sum_{\Psi, H_u, H_d} \int d^4 \theta \, \Psi^\dagger e^V \Psi$$

$$+ \left\{ \int d^2 \theta \, \left[ \mu_H D_H H_u + \lambda_u H_u Q U^c + \lambda_d Q D^c H_d + \lambda_e L E^c H_d \right] + \text{h.c.} \right\}$$

$$+ \sum_{a=1}^3 \frac{1}{4 g_a^2 \kappa} \int d^2 \theta \, \text{Tr}[W_a^\alpha W_{a\alpha}] + \text{h.c.},$$

(6)

where $\Psi = Q, U^c, D^c, L, E^c$, the index $a = 1, 2, 3$ denotes the the SM gauge groups $SU(3), SU(2)$ and $U(1)$, $g_a$ is the corresponding gauge couplings, and $\kappa = 1$ for $U(1)$ and $1/2$ for $SU(3)$ and $SU(2)$. The vector superfield $V$ in the Kahler potential for the chiral superfields implies, for example, $V = 2V_3 + 2V_2 + \frac{1}{2}V_4$ for $Q$ etc., where $V_a$ ($a = 1, 2, 3$) denote the vector superfields of the corresponding SM gauge groups. $L_X$ is the hidden sector Lagrangian already introduced in Eq. (1). $L_H$ is the Higgs sector Lagrangian involving the goldstino superfield:

$$L_H = -\frac{m_H^2}{f^2} \int d^4 \theta \, (X^\dagger X) \, H_d^\dagger e^V H_d - \frac{m_d^2}{f^2} \int d^4 \theta \, (X^\dagger X) \, H_u^\dagger e^V H_u.$$  

(7)

The matter field Lagrangian involving the goldstino superfield is given by

$$L_m = -\sum_{\Psi} \left( \frac{m_{\Psi}^2}{f^2} \right) \int d^4 \theta \, (X^\dagger X) \, \Psi^\dagger e^V \Psi.$$  

(8)

The bilinear and trilinear SUSY breaking couplings are given by $L_{AB}$:

$$L_{AB} = \frac{m_f}{f} \int d^2 \theta \, X H_d H_u + \text{h.c.}$$

$$+ \int d^2 \theta \, X \left\{ \lambda_u \left( \frac{A_u}{f} \right) U^c H_u Q + \lambda_d \left( \frac{A_d}{f} \right) D^c H_d Q + \lambda_e \left( \frac{A_e}{f} \right) E^c H_d L \right\} + \text{h.c.}$$  

(9)

The last term $L_g$ denotes the gauge sector Lagrangian given by

$$L_g = \sum_{a=1}^3 \frac{1}{4 g_a^2 \kappa} \frac{2m_{\lambda_a}}{f} \int d^2 \theta \, X \text{Tr}[W_a^\alpha W_{a\alpha}] + \text{h.c.}$$  

(10)

We focus on the Higgs potential in the NL-MSSM, which is read off from $L_0 + L_X + L_H + L_{AB}$:

$$V = V_{\text{SUSY}} + V_{\text{soft}},$$  

(11)

where

$$V_{\text{SUSY}} = \mu_H^2 (|H_u|^2 + |H_d|^2) + \frac{g_2^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2} |H_u^\dagger H_d|^2,$$  

(12)

$$V_{\text{soft}} = \frac{|f + \frac{m_a^2}{f} H_u H_d|^2}{1 - \frac{m_f^2}{f^2} |H_d|^2 - \frac{m_d^2}{f^2} |H_u|^2},$$  

(13)

For a concise review of the MSSM and the standard notation, see for example Ref. [14].
with \( g_Z^2 \equiv g_1^2 + g_2^2 \). We express the up-type Higgs and down-type Higgs doublets as

\[
H_u = \left( \frac{1}{\sqrt{2}}(v_u + R_u + iI_u) \right),
\]

\[
H_d = \left( \frac{1}{\sqrt{2}}(v_d + R_d + iI_d) \right),
\]

where \( v_u = v \sin \beta, \ v_d = v \cos \beta \) with \( v = 246 \text{ GeV} \), \( H^\pm \) are charged Higgs fields, and \( R_u, I_u, R_d, I_d \) are real scalar fields. Substituting them into the Higgs potential, we derive the stationary conditions:

\[
\frac{\partial V}{\partial R_u} \bigg|_0 = \frac{v}{4} \left\{ 4\mu_H^2 \sin \beta - 2M_Z^2 \cos 2\beta \sin \beta - \frac{2Am_2^2 \sin \beta}{B} + \frac{A^2m_2^2 \cos \beta}{B^2} \right\} = 0,
\]

\[
\frac{\partial V}{\partial R_d} \bigg|_0 = \frac{v}{4} \left\{ 4\mu_H^2 \cos \beta + M_Z^2 (\cos \beta + \cos 3\beta) - \frac{2Am_2^2 \sin \beta}{B} + \frac{A^2m_2^2 \cos \beta}{B^2} \right\} = 0,
\]

where \( |0 \) means that all the fields are taken to be zero,

\[
M_Z^2 = \frac{1}{4} g_Z^2 v^2,
\]

\[
A = 4f^2 + m_3^2 v^2 \sin 2\beta,
\]

\[
B = -2f^2 + m_1^2 v^2 \cos^2 \beta + m_2^2 v^2 \sin^2 \beta.
\]

The other stationary conditions such as \( \frac{\partial V}{\partial I_u} \bigg|_0 \) are automatically satisfied. The mass matrix of the CP-even Higgs bosons is given by

\[
\mathcal{M}_{CP-even} = \begin{pmatrix}
\frac{\partial^2 V}{\partial R_u^2} \bigg|_0 & \frac{\partial^2 V}{\partial R_u \partial R_d} \bigg|_0 \\
\frac{\partial^2 V}{\partial R_d \partial R_u} \bigg|_0 & \frac{\partial^2 V}{\partial R_d^2} \bigg|_0
\end{pmatrix},
\]

while the mass matrices for the CP-odd Higgs bosons and the charged Higgs bosons are

\[
\mathcal{M}_{CP-odd} = \begin{pmatrix}
\frac{\partial^2 V}{\partial I_u^2} \bigg|_0 & \frac{\partial^2 V}{\partial I_u \partial I_d} \bigg|_0 \\
\frac{\partial^2 V}{\partial I_d \partial I_u} \bigg|_0 & \frac{\partial^2 V}{\partial I_d^2} \bigg|_0
\end{pmatrix}, \quad \mathcal{M}_{charged} = \begin{pmatrix}
\frac{\partial^2 V}{\partial H_{-} \partial H_{-}} \bigg|_0 & \frac{\partial^2 V}{\partial H_{-} \partial H_{+}} \bigg|_0 \\
\frac{\partial^2 V}{\partial H_{+} \partial H_{-}} \bigg|_0 & \frac{\partial^2 V}{\partial H_{+} \partial H_{+}} \bigg|_0
\end{pmatrix}.
\]

By using the above formulas, we numerically calculate the Higgs boson mass spectra. First we choose appropriate values for \( m_1, m_2, \tan \beta \) and \( \sqrt{f} \) as the input parameters and solve the stationary conditions of Eqs. (16) and (17) to fix the values of \( \mu_H \) and \( m_3^2 \). We then substitute them into the Higgs potential and calculate the Higgs boson mass eigenvalues from Eqs. (19) and (20). Our results are shown in Figs. 1 and 2. The solid line in Fig. 1 shows the mass of the SM-like Higgs boson \( (m_h) \) as a function of \( \sqrt{f} \), where we have fixed \( m_1^2 = 1000^2 \) GeV\(^2\), \( m_2^2 = -(2005)^2 \) GeV\(^2\) and \( \tan \beta = 10 \). As \( \sqrt{f} \) decreases, the SM-like Higgs boson mass increases from the standard MSSM prediction at the three level \( m_h \simeq M_Z \cos 2\beta \) (dashed line)
Figure 1: The SM-like Higgs boson mass \( m_h \) as a function of \( \sqrt{f} \) (solid line), along with the standard MSSM prediction at the tree-level (dashed line) and the (green) horizontal line indicating \( m_h = 125 \) GeV. In this plot, we have taken \( m_1^2 = 1000^2 \) GeV\(^2\), \( m_2^2 = -(2005)^2 \) GeV\(^2\) and \( \tan \beta = 10 \).

in the limit of \( \sqrt{f} \to \infty \). The (green) horizontal line indicates \( m_h = 125 \) GeV. We find that the main contribution for increasing the SM-like Higgs boson mass comes from the quartic coupling \( (m_2^2 |H_u|^2/f)^2 \) in the series of expansion of Eq. \( \text{(13)} \) and the resultant Higgs boson mass is approximately expressed as

\[
m_h^2 \simeq M_Z^2 \cos 2\beta + 2 \left( \frac{m_2^2}{f} \right)^2 v^2 \sin^2 \beta.
\]  

Therefore, if the SUSY breaking scale is low enough, the SM-like Higgs boson mass of 125 GeV is achieved by the Higgs potential at the tree-level. As shown in Fig. 1, we have obtained \( m_h = 125 \) GeV for \( \sqrt{f} = 3990 \) GeV. Fig. 2 shows the masses of the heavy neutral Higgs and the charged Higgs bosons as a function of \( \sqrt{f} \) with the same inputs as in Fig. 1. The solid line depicts to the mass of the heavy CP-even Higgs boson \( m_H \) while the dashed and dotted lines correspond to the CP-odd Higgs boson mass \( m_A \) and the charged Higgs boson mass \( m_{H^\pm} \), respectively.

3 Minimal extension with Higgs-portal dark matter

As we have shown that if the SUSY breaking scale lies in the multi-TeV range, the SM-like Higgs boson mass of 125 GeV can be achieved even at the tree-level. Such a low SUSY breaking scale sets the gravitino mass to be \( \mathcal{O}(\text{meV}) \). Although this extremely light gravitino (goldstino) is harmless in phenomenological point of view, it is unable to be the dominant component of
the DM in our universe and a suitable DM candidate should be supplemented. In order to solve this problem, we propose a minimal extension of the NL-MSSM to incorporate a dark matter candidate, namely, the (scalar) Higgs-portal DM.

The Higgs-portal DM scenario is one of the simplest SM extensions to supplement the SM with a dark matter candidate. In the simplest setup, we introduce an SM-singlet real scalar \( S \) along with a \( Z_2 \) symmetry. The stability of this scalar is ensured by assigning an odd-parity to it, while all the SM fields are \( Z_2 \)-even. At the renormalizable level, the SM gauge and \( Z_2 \) symmetries allow the scalar \( S \) to have only one (non-self) interaction term,

\[
\mathcal{L}_{\text{int}} = -\frac{1}{4} \lambda_{\text{HSS}} (H^1 H) S^2 ,
\]

where \( H \) is the SM Higgs doublet field. The scalar DM \( S \) communicates with the SM sector only through this Higgs-portal interaction and the DM phenomenology in this Higgs-portal DM scenario is controlled by only two free parameters: \( \lambda_{\text{HSS}} \) and the DM mass \( m_S \). Phenomenological constraints on the two free parameters have been intensively studied, and the allowed parameter region has been identified to be consistent with the cosmological observations, the direct/indirect DM particle search results and the Higgs-portal DM search results by the Large Hadron Collider (LHC) experiment. The Higgs-portal DM scenario is phenomenologically viable, but the allowed parameter region is very limited \[12\]: \( m_S \simeq M_H/2 \) with the SM Higgs boson mass \( M_H = 125 \, \text{GeV} \) and \( 10^{-4} \lesssim |\lambda_{\text{HSS}}| \lesssim 10^{-3} \).

Now we introduce an SM-singlet chiral superfield \( \Phi \) along with a \( Z_2 \) symmetry and assign odd-parity to it while even-parity to all the MSSM fields. Hence, the lightest component field in \( \Phi \) is stable and the DM candidate. The SUSY Lagrangian \( \mathcal{L}_0 \) in Eq. (6) is then extended to be

\[
\mathcal{L}_0 \rightarrow \mathcal{L}_0 + \int d^4 \theta \Phi^\dagger \Phi + \left\{ \int d^2 \theta \mu_\Phi \Phi^2 + \text{h.c.} \right\} ,
\]

where \( \mu_\Phi \) is a mass parameter. Similar to \( \mathcal{L}_H \) and \( \mathcal{L}_m \), a new Lagrangian for \( \Phi \) involving the goldstino chiral superfield is given by

\[
\mathcal{L}_\Phi = -\frac{m_\Phi^2}{f^2} \int d^4 \theta \left( X^\dagger X \right) \Phi^\dagger \Phi ,
\]

where \( m_\Phi \) denotes a soft SUSY breaking mass. Finally, \( \mathcal{L}_{AB} \) is extended to be

\[
\mathcal{L}_{AB} \rightarrow \mathcal{L}_{AB} + \left\{ \frac{B_\Phi}{2f} \int d^2 \theta X \Phi^2 + \text{h.c.} \right\} .
\]

In the following, we assume that \( B_\Phi \) is real and positive.

We now read off the scalar potential relevant to the Higgs-portal DM scenario by eliminating the auxiliary fields:

\[
V = V_{\text{SUSY}} + V_{\text{soft}} ,
\]
where
\[ V_{\text{SUSY}} = \mu_H^2 (|H_u|^2 + |H_d|^2) + \mu_{\Phi}^2 |\Phi|^2 + \frac{g_2^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2} |H_u^\dagger H_d|^2, \] (27)
\[ V_{\text{soft}} = \frac{|f + \frac{m_3^2}{f} H_u H_d - \frac{B_{\Phi}}{2 f} \Phi|^2}{1 - \frac{m_3^2}{f^2} |H_d|^2 - \frac{m_1^2}{f^2} |H_u|^2 - \frac{m_2^2}{f^2} |\Phi|^2}. \] (28)

Although the complete form of the scalar potential includes all the sfermions in the MSSM, we have consider the potential terms involving only the MSSM Higgs doublets and the SM-singlet scalar \( \Phi \). This is because the sfermions should be heavy to satisfy the current LHC constraints and their couplings with the Higgs-portal DM have little effects on the DM physics. For the physics of the Higgs-portal DM scenario, only the bilinear terms with respect to \( \Phi \) are important. To extract them from the scalar potential, we expand \( V_{\text{soft}} \) up to the order of \( O(1/f^2) \) and then obtain
\[ V \supset \left( \mu_{\Phi}^2 + m_{\Phi}^2 \right) + \left( \frac{m_3^2}{f^2} H_u H_d + \text{h.c.} \right) + 2 \frac{m_1^2}{f^2} |H_d|^2 + 2 \frac{m_2^2}{f^2} |H_u|^2 \right) m_{\Phi}^2 |\Phi|^2 \]
\[ - \left\{ \left( 1 + \frac{m_3^2}{f^2} H_u H_d + \frac{m_1^2}{f^2} |H_d|^2 + \frac{m_2^2}{f^2} |H_u|^2 \right) \frac{B_{\Phi}}{2} \Phi^2 + \text{h.c.} \right\}. \] (29)

Substituting
\[ \Phi = \frac{1}{\sqrt{2}} (\phi + i \eta) \] (30)
into Eq. (29), we can find the mass spectrum of the real scalars, \( \phi \) and \( \eta \), and their couplings with the Higgs bosons. First, we obtain the mass spectrum to be
\[ m_{\phi/\eta}^2 = \mu_{\Phi}^2 + m_{\Phi}^2 + \left( \frac{m_1^2}{f^2} \cos^2 \beta + \frac{m_2^2}{f^2} \sin^2 \beta + \frac{m_3^2}{f^2} \sin \beta \cos \beta \right) \frac{m_{\Phi}^2}{f} \]
\[ + \left\{ 1 + \left( \frac{m_1^2}{f} \cos^2 \beta + \frac{m_2^2}{f} \sin^2 \beta + \frac{m_3^2}{f} \sin \beta \cos \beta \right) \frac{v^2}{f} \right\} B_{\Phi} \]
\[ \simeq \mu_{\Phi}^2 + m_{\Phi}^2 + B_{\Phi}. \] (31)

In the last expression, we have used \( |m_{1,2,3}|, f \gg v^2 \) and \( m_{\Phi}^2 < f \) from the theoretical consistency. We see that \( m_{\phi} < m_{\eta} \) and thus the real scalar \( \phi \) is the DM candidate.

Since all the Higgs bosons except for the SM-like Higgs boson are heavy, the DM physics is mainly controlled by the coupling of \( \phi \) with the SM-like Higgs boson. For a large \( \tan \beta \) value, such as \( \tan \beta = 10 \) as we have used in Figs. [1] and [2] the up-type Higgs doublet is approximately identified as the SM-like Higgs doublet. By employing this approximation \( H_u \simeq H \), we can easily extract the coupling of \( \phi \) with the SM-like Higgs doublet from Eq. (29) such that
\[ \mathcal{L}_{\text{int}} \simeq \frac{m_{\Phi}^2}{f^2} \left( \frac{m_{\Phi}^2}{2} - \frac{B_{\Phi}}{2} \right) (H^\dagger H) \phi^2. \] (32)
This is the formula to be compared with Eq. (22) with the identification of $S = \phi$. Therefore, in the decoupling limit of the heavy Higgs bosons and all the MSSM sparticles, we have obtained the Higgs-portal DM scenario as the low energy effective theory. In terms of our model parameters, the two parameters $m_S = m_\phi$ and $\lambda_{HSS}$, which control the Higgs-portal DM physics, are approximately expressed as

\begin{align*}
m_S^2 &\simeq \mu_\phi^2 + m_\Phi^2 - B_\Phi, \\
\lambda_{HSS} &\simeq 4 \frac{m_\Phi^2}{f^2} \left( m_\phi^2 - \frac{B_\phi}{2} \right).
\end{align*}

(33)

In Fig. 1, we have fixed $m_\Phi^2$ and $f$ so as to achieve $m_h = 125$ GeV. After the choice, we still have three free parameters, $\mu_\phi$, $m_\phi$ and $B_\phi$ and we can arrange them to satisfy the phenomenological constraints, $m_S \simeq M_h/2$ and $10^{-4} \lesssim |\lambda_{HSS}| \lesssim 10^{-3}$ for the Higgs-portal DM scenario. For example, we may set $\mu_\phi^2 \simeq m_\Phi^2 \simeq B_\phi/2 = \mathcal{O}(1 \text{ TeV}^2)$ but fine-tune their differences so as to reproduce the allowed values of $m_S^2 \ll 1 \text{ TeV}^2$ and $|\lambda_{HSS}| \ll 1$.

4 Conclusion

If SUSY is broken at a low energy, the NL-MSSM with the goldstino chiral superfield is a very useful description for taking the hidden sector effect into account to the MSSM. The NL-MSSM is particularly interesting if the SUSY breaking scale lies in the multi-TeV range. This is because in this case the SM-like Higgs boson mass $m_h = 125$ GeV is achieved by the Higgs potential at the tree-level after eliminating the $F$-component of the goldstino superfield. However, such a low scale SUSY breaking predicts a milli-eV gravitino LSP, which is too light to be the main component of the DM in our universe. Thus, a suitable DM candidate is missing in the NL-MSSM. To solve this problem, we have proposed a minimal extension of the NL-MSSM by introducing the SM-singlet chiral superfield ($\Phi$) along with the $Z_2$ symmetry. The stability of the lightest component field in $\Phi$ is ensured by assigning odd-parity to $\Phi$ while even-parity for all the MSSM superfields. We have shown that in the decoupling limit of the sparticles and heavy Higgs bosons, our low energy effective theory is nothing but the Higgs-portal DM scenario with the lightest $Z_2$-odd real scalar being the DM candidate. With a suitable choice of the model parameters, we can reproduce the allowed parameter region of the Higgs-portal DM scenario while achieving $m_h = 125$ GeV.

Finally, we give a comment on a general property of our model. Since the main point of this paper is to propose the minimal extension of the NL-MSSM to incorporate a suitable DM candidate, we have focused on a special parameter region which derives the Higgs-portal DM scenario with one real scalar at low energies. In general, we have a wide variety of the parameter choice to realize a viable dark matter scenario. For example, we may take a very small value of $B_\Phi$ in Eq. (31) so that the mass splitting between $\phi$ and $\eta$ is negligibly small. In this case,
we identify the complex scalar $\Phi$ with the DM particle. This is a complex scalar extension of the simplest Higgs-portal DM scenario with only one real scalar. Since the MSSM includes two Higgs doublets, our Higgs-portal DM scenario is basically two Higgs doublet extension of the Higgs-portal DM scenario. In general, the heavy Higgs bosons can play an important role for the DM physics, for example, an enhancement of the DM pair annihilations through the heavy Higgs boson resonances. We leave such a general analysis for future work.

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