Squeezed states generation in an array of Linear and Nonlinear Waveguides

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Abstract. We demonstrate the generation of squeezed states of light due to the second harmonic generation and Kerr effect in an array of nonlinear waveguides mediated through a linear one. We characterized the electromagnetic field by a quantum mechanical Hamiltonian and the density operator time evolution is obtained from the Von-Neumann equation of motion. Using the quasiprobability positive P of phase space representation, the classical Fokker-Planck equation is obtained from the master equation and translated to its classical matching set of nonlinear differential equations. We showed that because of the new possibilities of correlation between the linear and nonlinear channel waveguides, highly nonclassical light may be produced.

1. Introduction

To take advantage of quantum technology, it is vital to produce and regulate quantum systems and devices. In this context, guided wave structures that entail the manipulation of light and its interaction with nonlinear media are one of the most well-known systems. The generation and control of highly nonclassical light in an array of guided wave structures can be realized using various configurations such as the manipulation of several modes [1], waveguides [2], or the nonlinear processes of the interactive media such as second harmonic generation (SHG) [3, 4] and Kerr effects [5]. SHG may have fewer absorptive losses and reduce quantum noise more effectively [6]. On the other hand, Kerr effects are a great mechanism for generating quantum states of light since they do not require phase matching and the nonlinear response is very fast [7]. Coupled waveguides arrays without nonlinear effects may not be able to generate nonclassical light on their own, however, they might be utilized for transport and quantum walk of nonclassical light [8-11].

As many novel physical dynamics can emerge from a system with combined nonlinearities [12], it's relevant to look into the potential of generating highly nonclassical light in such a system. This paper investigates the generation of squeezed states in an array of waveguides with both SHG and third-order Kerr nonlinearity mediated by a linear one. When a coherent input field interacts with these nonlinear media, fascinating quantum phenomena such as squeezing can be produced. In this system, the electromagnetic field is characterized by a quantum mechanical Hamiltonian, whereas its propagation is given by the master equation for the density matrix. Using the positive P representation of the phase space [13, 14], the master equation is translated to its classical matching set of nonlinear differential equations, and adequate descriptions of the quantum states may be obtained.
2. Theoretical Aspect

As illustrated in Figure 1, we consider the generation of squeezed states in an array of waveguides due to the SHG and Kerr effect.

![Figure 1. Schematic illustration of the model](image)

The channel waveguides are set near together to allow the evanescent fields to overlap, allowing the coupled modes to exchange energy regularly during wave propagation. The model under consideration can be described by the following effective Hamiltonian,

\[
\hat{H} = \frac{\hbar}{2\pi} \left( \omega (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + \hat{\Lambda}^\dagger \hat{\Lambda}) + \Omega \hat{B}^\dagger \hat{B} + i \frac{\chi}{2} \hat{A}^2 \hat{B} - \hat{\Lambda}^2 \hat{B}^\dagger + g \hat{b}^2 \hat{b}^\dagger \right) \\
+ k_i \left( \hat{a}^\dagger \hat{\Lambda} + \hat{\Lambda}^\dagger \hat{a} \right) + k_j \left( \hat{a}^\dagger \hat{b}^\dagger + \hat{b} \hat{a} \right)
\]

(1)

Herein, \(\hbar\) defines Planck constant, \(\omega\) and \(\Omega\) are the fundamental (FH) and second harmonic (SH) frequencies of the input field, \(\chi\) and \(g\) are the nonlinear coefficients proportional to the quadratic \(\chi^{(2)}\) and cubic \(\chi^{(3)}\) nonlinearities and \(k_i, k_j (i = 1, 2)\) is the evanescent linear coupling between the waveguides. The components \(\hat{a}^\dagger, \hat{b}^\dagger, \hat{\Lambda}^\dagger\) and \(\hat{B}^\dagger\) are the bosonic operators for the FH \((\hat{a}^\dagger, \hat{b}^\dagger, \hat{\Lambda}^\dagger, \hat{B}^\dagger)\) and SH \((\hat{B}^\dagger)\) modes respectively. Appropriate representations of quantum states can be obtained by applying the phase space representation to the current system [15]. In this case, the quantum operator Von-Neumann equation of the density operator must be converted to the corresponding classical Fokker-Planck (FP) equation using positive \(P\) representation. In turn, the classical FP equation of the quasiprobability distribution can be mapped to a set of stochastic partial differential equations using Ito Calculus rules. This yield the following set of:

\[
\alpha = -i \left( \omega \alpha + ik_{\xi} \zeta + ik_{\beta} \beta \right) \\
-\alpha^* = -i \left( \omega \alpha^* + ik_{\xi}^* \zeta^* + ik_{\beta}^* \beta^* \right) \\
\beta = -i \left( \omega \beta + 2g \beta^2 \beta^* + ik_{\alpha} \alpha \right) + \sqrt{-2ig} \beta \eta_i(z) \\
-\beta^* = -i \left( \omega \beta^* + 2g \beta^* \beta^2 + ik_{\alpha}^* \alpha^* \right) - \sqrt{-2ig} \beta^* \eta_j(z) \\
\zeta = -i \omega \zeta + \chi \zeta \zeta - ik_{\alpha} \alpha + \sqrt{\chi} \eta_i(z) \\
\zeta^* = i \omega \zeta^* + \chi \zeta \zeta^* + ik_{\alpha}^* \alpha^* + \sqrt{\chi} \eta_j(z) \\
\xi = -i \Omega \xi - \frac{1}{2} \zeta \zeta \\
\xi^* = i \Omega \xi^* - \frac{1}{2} \zeta^* \zeta^*
\]

(2)

The overdot in equation (2) reflects the derivatives with respect to the spatial distance \(z\), whereas \(\{\alpha, \alpha^*\}, \{\beta, \beta^*\}, \{\xi, \xi^*\}\) and \(\{\zeta, \zeta^*\}\) are independent stochastic variables corresponding to the operators \(\hat{a}^\dagger \hat{a}, \hat{b}^\dagger \hat{b}, \hat{\Lambda}^\dagger \hat{\Lambda}\) and \(\hat{B}^\dagger \hat{B}\). The Gaussian noise \(\eta_i, \eta_j\), satisfies the correlation \(\eta_i(z) = 0\) and \(\eta_j(z) \eta_j(z') = \delta(z-z')\). Squeezing is numerically examined in terms of the propagating field quadrature variances, given by

\[
S_x = 4(\Delta\hat{X})^2 - 1 \leq 0 \quad ; \quad S_y = 4(\Delta\hat{Y})^2 - 1 \leq 0
\]

(3)

where

\[
2\hat{X} = (\hat{\bar{O}} + \hat{O'}) \quad ; \quad 2i\hat{Y} = (\hat{\bar{O}} - \hat{O'})
\]

(4)
\begin{align}
\langle (\Delta X)^2 \rangle &= \frac{1}{4} \left[ \langle O \rangle^2 + \langle O^2 \rangle + 2 \langle O' O \rangle + 1 - \langle O \rangle^2 - \langle O' \rangle^2 - 2 \langle O \rangle \langle O' \rangle \right] \\
\langle (\Delta Y)^2 \rangle &= \frac{1}{4} \left[ -\langle O \rangle^2 - \langle O^2 \rangle + 2 \langle O' O \rangle + 1 + \langle O \rangle^2 + \langle O' \rangle^2 - 2 \langle O \rangle \langle O' \rangle \right]
\end{align}
\tag{5}

and \([\hat{O}, \hat{O}'] = \delta_{ij} \hat{O} \hat{O}' = \hat{a} \hat{a}' \hat{b} \hat{b}' \hat{A} \hat{A}'\). For convenience in numerical simulation, the input parameters have been cast to dimensionless form and are fixed at \(\chi = 0.01\), \(g = 10^{-7}\), \(\omega = 2\) and \(\Omega = 2\omega\) with coherent state initialization in all channels.

3. Discussion
Squeezing reduces the intrinsic quantum noise in the field quadrature to below the standard quantum limit. This allows for more exact measurements than would otherwise be achievable, and it has the potential to advance photonic quantum computing [16, 17]. Using the interpolation data of peak amplitude squeezing and antisqueezing, figure 2 depicts the generation of squeezed states in the \(\chi^{(1)}\) channel at various \(k_i\) states where \(\chi^{(1)}\) is a coefficient proportional to the linear susceptibility of the material. For three channels interaction, the \(\chi^{(1)}\) channel has a linear coupling constant \(k_1 = k_2 = 1\), whereas for \(k_1 = 1\), \(k_2 = 0\), \(k_1 = 0\), \(k_2 = 1\) correspondingly, the linear coupling is simplified to \(\chi^{(1)} - \chi^{(2)}\) and \(\chi^{(1)} - \chi^{(3)}\) interaction. Thus, as a result of quantum correlations between the field modes, the system produces a periodic squeezed state. Within the parameter ranges considered in this study, squeezing due to the SHG provides the least amount of squeezing in the \(\chi^{(1)}\) channel. In early evolution, the quadrature variances for \(\chi^{(1)} - \chi^{(3)}\) interaction exhibits the strongest squeezing. Compared to the \(\chi^{(1)} - \chi^{(3)}\) interaction, squeezing owing to both the SHG and Kerr nonlinearity (\(k_1 = k_2 = 1\)) is particularly weaker at this point, however, when squeezing grew in proportion to the interaction length, the \(\chi^{(1)} - \chi^{(2)} - \chi^{(3)}\) interaction demonstrated the largest amount of squeezing.

Figure 2. Squeezing in the \(\chi^{(1)}\) channel

Figure 3 shows the generation of squeezed states in the \(\chi^{(2)}\) channel using the interpolation data of peak amplitude squeezing and antisqueezing following equation (3). While the SHG and Kerr nonlinearity can improve squeezing in the \(\chi^{(1)}\) channel, the presence of Kerr nonlinearity mediated by the \(\chi^{(1)}\) channel limited the maximal squeezing of the FH field achievable in the \(\chi^{(2)}\) channel on an individual mode basis. When only the \(\chi^{(2)}\) nonlinearity is addressed, maximal squeezing can be generated. Furthermore, squeezed light at higher frequency ranges may be advantageous for applications at different wavelengths. We found that, it is possible to extend squeezing over the FH frequency. Nonetheless, when compared to squeezing at the FH frequency, squeezing at the SH frequency is much weaker, with a tendency for collapse and revival pattern in the quadrature variance evolution. The generation of squeezed states in the \(\chi^{(2)}\) channel at SH frequency is shown in figure 4.
Furthermore, figure 5 shows single-mode squeezing in the $\chi^{(3)}$ channel. These results resemble those obtained in $\chi^{(2)}$ channel, however with varied magnitude and phase. The presence of second-order nonlinearity mediated by the $\chi^{(1)}$ channel might reduce the amount of squeezing in the $\chi^{(3)}$ channel. Squeezing might be better for $\chi^{(1)}-\chi^{(3)}$ consideration, however, the amount is limited. The propagating field in the $\chi^{(3)}$ channel is squeezed to a greater extent when there is no interaction at all between the $\chi^{(2)}$ and $\chi^{(3)}$ channels. Therefore, considering the $\chi^{(1)}-\chi^{(2)}-\chi^{(3)}$ interaction might not be ideal to improve squeezing in this channel.
4. Conclusion
In conclusion, we have investigated the possibility of generating enhanced squeezed states of light in linear waveguide through the combination of SHG and Kerr nonlinearity. The presence of Kerr nonlinearity mediated by the $\chi^{(1)}$ channel limited the maximal squeezing achievable at FH frequency in the $\chi^{(2)}$ channel, however, it is possible to extend squeezing at the SH frequency. The current multichannel system combines both linear and nonlinear waveguides is versatile and potentially useful as squeezed light generator especially in the linear media.

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