A Small Scale Study: The Metacognitive Scaffolding in Mathematics Problem Solving for Senior High Students with High Mathematics Ability

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Abstract. To describe the kinds of metacognitive scaffolding which are necessary given, we carried out a test in the topic of three dimensional space to three senior high school students with high mathematics ability. The metacognitive scaffolding was given when they worked with the problems of which they refer to the four steps of Polya. An interview was also conducted to the students for the data triangulation. The findings suggest that, although the three respondents basically understand the problems, they have difficulties in setting a plan for problem solving as they are unaccustomed. In contrast, they relatively have no difficulties in applying the third step of Polya, i.e. applying the plan of problem solving as well as the fourth step because there was no MS given to them. Concerning the kinds of the given MS, monitoring is the most used, followed by planning, and evaluating. Moreover, based on the Polya’s steps of which the MS given, the most frequently used MS is the second step, followed by consecutively the first step, the fourth step, and the third step.

1. Introduction
Based on the data from a documentation and an interview, it had been reviewed that, the national examination results ranging from 2013 to 2015 of the students in SMAN 3 Makassar, a senior high school in Makassar city, suggested the strenuousness of three dimensional figure problems. It took place presumably due to the lack of logical and analytical ability of students which evoked the researchers’ willingness in assisting mathematics learners in the form of scaffolding metacognitive (SM). Such kind of help included some three dimensional problems within the zone of proximal development (ZPD) of learners. The rationale of applying SM is based on the theoretical review, i.e. exercises within the ZPD of a learner are difficult to solve by himself [1] owing to the absence of sufficient knowledge to process available informations. [2] argued that a statement is a problem depends on the individual and the matter of time. Moreover, scaffolding is necessary for students when they face a problem within their ZPDs. Scaffolding itself was introduced by [3], i.e. help or guidance given by adult or more competent person to support a learner to work within his ZPD. The guidance is given in the beginning and after that, when the learner is perceived able to solve a problem by himself, the help is moderately eliminated. It is asserted by [4] stated that Scaffolding is support for learning and problem solving, the support can be clues, remainders, encouragement, breaking the problem down into the steps, providing an example, or anything else that allows the student’s grow in dependence as a leaner.

[5] considered that children have their own conceptions, however, the conceptions are sometimes not systematic, logical, and rational. For that reason, the guide by capable peer will help them establish
the expected conceptions [6], [7]. The ability of learners in well processing and organizing their thinkings is one of the metacognition skills [8]. [9] described metacognition skills as one’s expertise in managing his cognition. In this study, three metacognitive skills are noticed, i.e. planning, monitoring, and evaluating mathematics problem solving of which their definitions are addressed, one of them, by [10]. The planning skill refers to the thinking activities of students about how, when, and why apply certain processes in solving mathematics problem [11]. The monitoring skill is a monitoring activity by learners toward every step and strategy used in solving problem aimed to identify and modify the steps as being expected. Meanwhile, the evaluating skill is defined as reexamining what has been applied in problem solving process. In specific, the research question for this study is that what kinds of MS given to students when they have obstacles in solving three dimensional figure problems?

2. Research Method

The present study was categorized as a qualitative research as it predominantly gave scaffolding to students while solving mathematics problem. The students becoming the subject of this study have high mathematics ability since the study was aimed to simply focus on the scaffolding necessary to students in that category. Therefore it can be also classified as an exploratory research. The subjects are majored in natural science program study in SMA Negeri 3 Makassar. The school itself is located in urban area with middle-up category in the term of quality compared to the other schools in Makassar.

The researchers was the main instrument as they acted as observers and interviewers for the data collection. The other supporting instruments were a problem solving test in the topic of three-dimensional figure and MS guide which had been validated by some experts and colleagues. The subjects were given the test and interviewed separately which led them to work effectively. The researchers monitored some aspects while the students were working, i.e. observing whether the process and the result are true, identifying the obstacles of students in the problem solving, and seeing through the awareness involvement of the students in each step as well as the obtained result.

Related to the latest aspect which was observed, an interview including several questions referring to problem solving steps [12] was carried out to the students. There were four possibilities of students in solving the problems as shown in the table 1

| Table 1. Possible conditions of Students |
|------------------------------------------|
| Possibility | Metacognition Involvement |
| True | Yes | I | False | III | IV |
| False | No | II | |

As the table 1 suggest, MS would not be given when the students were in the first possibility meaning that they didn’t find any impediment. When the second possibility fitted to the situation, the students were perceived having an obstacle which was so called the obstacle I and given MS. In addition, the third possibility was addressed to the obstacle II and the students were given MS evoking cognitive conflict causing them aware of their errors as well as think to find an appropriate strategies in problem solving. Furthermore, the fourth possibility was designated to the state of the obstacle III in which the students would be given MS. To this end, students find an appropriate strategy and assume the clarity of what has been done.

3. Research Findings

One of the problems given is specified as follows:

A cube $ABCD EFGH$ with sides 4 cm is intersected by a plane figure through $F$, $H$, $I$ dan $J$ where $I$ and $J$ are respectively the mid point of $AD$ and $AB$. Find the area of the plane $FHIJ$!
The following is the description of the given SM to each respondent based on the four steps of Polya. In the first step, the first respondent (RT1) and the second respondent (RT2) find different obstacles to each other. However, both RT1 and RT2 are given MS in the kind of planning and monitoring. Meanwhile, RT3, who has different obstacles to those of RT1 and RT2, plainly needs the MS type monitoring. The obstacles and the given MS are asserted by the following descriptions and interview fragments in the face of discussions.

When the researchers saw through the understanding of RT1 about a cube intersected by a plane figure, RT1 were clueless. In other words, RT1 had the obstacle III, i.e. not answering and realizing what to answer implying the researchers gave him MS type monitoring which is clarified to the following discussion.

"could you recognize several possibilities of the results when a cube intersected by a plane figure?"

Then the RT1 thought for a while and answered: "it could be a triangle, rectangle, and others". That means he well understand and tend to answer with his consciousness. Meanwhile, the RT2 had obstacle II, i.e. perceiving her interpretation of intersecting plane FHIJ was true but the fact was false causing the researchers provided her planning type MS which is elucidated in the following discussion. Researchers asked: "re-read please the question, what is the main problem?". After the RT2 had read the question again, she was still not conscious of her mistake. Then it suggested the researchers to give monitoring type MS as the following question by the researchers points out: "which one the intersecting plane FHIJ?". However, the second scaffolding didn't effectively admonish the RT2 to be aware of her mistake. The researchers then decided to give the monitoring type MS to the RT2 as described in the following discussion, i.e. the question: "where is FHIJ?". Pointing out her misunderstood figure, the RT2 found a dissonance between her initial answer and her new thinking by sighing and saying: FHIJ? (being aware of the plane FHIJ). Till this step, the RT2 were aware of her wrong interpretation and continuing to the correct one.

In addition, the evaluation type MS was not given at all since the three subjects were able to start the strategy and solve the problems.

3.1. The Second Step of Polya (Setting a Plan for Problem Solving)
In the second step of Polya, the three respondents experienced different obstacles. To overcome the obstacles, RT1 and RT3 were given three types of metacognitive scaffolding to lead them well understand every process and result. Whereas RT2 was given planning type and monitoring type MS to lead her understand well and tend to be aware of the process and the result obtained.

3.2. The Third Step of Polya (Applying the Plan)
RT1 and RT3 completed the third step of Polya smoothly and convincingly. Both worked without obstacles by applying the plan in the second step. When the metacognitive involvement was identified for each in performing the process and obtaining the result, both gave responds in a convincing way and showed a tendency to answer with their awareness.

Meanwhile, RT2 experienced the first type of obstacle that is; she answered how to multiply two rooted numbers however she tended to be doubt of what was spoken. She then was given planning type MS described in the discussion as follows

"Please notice this " $\sqrt{3} \cdot \sqrt{2} = 3^{1/2} \cdot 2^{1/2}$

"since both have the same exponent, then their base can be multiplied"

$= (3 \cdot 2)^{1/2}$.

From these brief directions from the researchers, RT2 directly interrupted the researcher's explanation by mentioning the root of thirty-six which is a continuation of the solution. In addition, for another problem solving, RT2 solved the problem not by using the steps that should be used in the application of the Phytagoras proposition, i.e. RT2 wrote the length of FH and IJ without scratching for concept. The answers of RT2 were $\sqrt{6}$ for IJ and $\sqrt{30}$ for FH which were wrong solutions.
Neverthelles, RT2 were keen on assuming the results were correct. The researchers then gave monitoring type MS of which the fragment of the discussion was transcribed as follows.

"Be careful, work carefully!"

The purpose of giving such MS was that students could re-observe what comes from their mind to correct. Apparently after RT2 re-checked it, she realized what she wrote was wrong and immediately crossed her mind to the right result in the right way.

Similarly, when determining the area of a triangle, RT2 wrote the length of the hypotenuse as the height and being sure that the answer was true meaning that RT2 had the obstacle II. Subsequently, the researcher gave monitoring with the following metacognitive questions.

"Be careful, which one is the height of the triangle PIH?"

Since RT2 responded "oh no", and being silent for a moment, it means RT2 already realized for the mistake remaining the true answer which hasn’t been shown. It then implied the researchers to give her the following MS:

"Which one is the right angle?"

Then, RT2 said "At the point P, yes the height is IP", then RT2 wrote the right answer in the right step. This means RT2 has realized that its wrong and has shown the correct answer.

Furthermore, RT2 couldn’t show a simpler way to determine the length of the diagonal of a square. The researcher categorized such impediment as the obstacle III, because she didn’t give any answer and didn’t know what to answer, so she was given a evaluating type MS with the following metacognitive questions.

"Consider, if the length of the sides is 4, then the diagonal is 4, what if the length of the sides is 2?"

Then RT2 gave an aswer as follows.

"2√2, so if the length of the side is a then the diagonal is a√a"

That answer indicates that RT2 understands well and tends to involve her awareness in determining the diagonal length of a square.

From the description of the constraints and the types of MS given to RT2, basically the obstacles and the scaffolding provided could be eliminated, if RT2 was thorough in the work.

3.3. The Fourth Step of Polya (Evaluating the Results)

In the fourth step of Polya, the three respondents experienced different obstacles. To overcome these obstacles, the three respondents were given metacognitive monitoring type MS. This was applied because all of of them made a mistake in the process and the result and required careful monitoring which, after that, made them monitored their works and identified their mistakes with their awareness.

For planning type MS was only addressed to the RT1 owing to the fact that the RT1 encountered the obstacle II that is the answer is wrong but he was sure that it was true. After being monitoring type MS, it turned out the RT1 became silent and couldn’t give an answer. It then implied the researchers continue giving planning type MS with the following questions:

"Take a look at the side of EFGH, and the length of the side is a! find the length of FH?"

Spontaneously RT1 gave a respond:

"a√2, so FH = 4√2 cm"

As the researchers would see through whether the RT1 could also apply the square diagonal formulas in finding the length of IJ, they gave the following metacognitive questions.

"What about IJ?"

RT1 answered “We can apply the same way. IJ = 2√2 cm” which is the correct answer and he tended to use his awareness.

RT2 and RT3 were not given planning type MS, because by simply giving them monitoring type MS, they have been able to overcome the obstacles.

4. Conclusion
This study addressed a research objective, i.e. what kinds of MS necessary given to students with high mathematics ability. The results described in the previous section lead to the following conclusions. Although the students basically understand the problems easily with different characteristics implying the given MSs are various of which the monitoring type is the most frequently given MS followed by planning and evaluating consecutively. The students are reported to have obstacles in setting a plan for problem solving as they are accustomed. Consequently, planning type MS are frequently given to them. Conversely, they have no difficulties in applying the plan for problem solving entailing no MS given. It is relatively the same as evaluating step of which they significantly don’t have any obstacle. Furthermore, the number of each type of the MSs are relatively the same given to each student. If it is seen from the steps of Polya, the second step is the most frequent, followed by the first step, the fourth step, and the third step consecutively.

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