Simulation of the dynamic properties of open cell foams by means of a macroscopic stochastic model

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Open cell foams are widely used in engineering which includes an application in areas, where vibration excitations can act on the structures. Therefore dynamical properties as eigenfrequencies and eigenmodes are of particular interest, which show a dispersion due to the stochastic characteristic of the microstructure. To avoid time consuming measurements a simulation algorithm will be presented which enables to estimate the distribution of the eigenfrequencies. In order to do this the microstructure has to be addressed as well as different weights of several specimen, which have an influence on the computed and measured eigenfrequencies.

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1 Initial Situation

The model for the estimation of the eigenfrequencies is an unsupported EULER-BERNOULLI-Beam (Fig. 1) with stochastic cross section $A(z)$ and second moment of area $I(z)$. It follows the stochastic partial differential equations which are noted below, where Youngs Modulus $E$ and density $\varrho$ of the bulk material are assumed constant with $E = 20$ GPa [1] and $\varrho = 2990$ kg/m$^3$.

$$
\begin{align*}
(\varrho Aw)_z - g Aw_{tt} &= 0 \quad \frac{\partial (\cdot)}{\partial z} = ()_z \\
(\varrho Au_z)_z + g Aw_{tt} &= 0 \quad \frac{\partial (\cdot)}{\partial t} = ()_t
\end{align*}
$$

Fig. 1: Macroscopic model and governing partial differential equations for longitudinal and bending vibrations

To determine the eigenfrequencies stochastic processes for $A(z)$ and $I(z)$ are needed, which show a correlation because both are computed from integrals over the cross section. To address this correlation the simulation of a three dimensional foam is necessary, where the amount of material at a certain position can be refered to as stochastic process. For the purpose of validation a number of real foam specimen is analysed regarding the eigenfrequencies using modal analysis and the material distribution utilizing computed tomography. These specimen show a distribution of the mean amount of material inside the structure which additionally has to be addressed in the simulation of artificial foams.

2 Generation of foam samples

The stochastic process of the material distribution can be divided into a mean value $\bar{\sigma}$ and a variable part $\tilde{\sigma}$. To model the variable part the harmonic synthesis approach can be used, which uses a sum of cosine functions as representation of the stochastic process. In the case of the here considered non gaussian distributed processes an algorithm as published by YAMAZAKI [2] has to be used. Because of convergence issues using computed tomography data an improvement by the author [3] was necessary. Details on the principle of the simulation algorithm can be found in [2]. As input data the power spectral density function (PSD) and cumulative distribution function (CDF) are needed. Both quantities can easily be determined from computed tomography data for single specimen.

To model the mean amount of material $\bar{\sigma}$ a fit of the estimated probability density of the mean values can be applied. Therefore a generalized extreme value distribution with $\mu = 0$, $\sigma = 0$, 0178 and $k = 0.0815$ is used.

An analysis of the PSD and CDF of single specimen under incorporation of the mean material amount shows a dependency between all three quantities which has to be addressed. The influence of mean and standard deviation on the CDF is shown in figure 2 (left). It can be seen that with increasing mean material amount the step height at $\varepsilon = 0$ decreases. With rising standard deviation the right end shifts towards $\varepsilon = 1$. This shows that the simulation of specimen with different mean material amounts is not possible by one CDF. Therefore a normalized CDF from $N$ samples is introduced as it is shown below. The PSD in figure 2 (right) shows the same ring structure for all specimen unless the pore sizes in the foam have nearly the same value. The only difference lies in the contained variance which needs to be the integral over the PSD. Therefore a normalized

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Fig. 2: Plot of the CDF of 50 foam specimen (left) with annotations to emphasize the influence of mean material amount $\varepsilon$ and standard deviation $\sigma$. Surface plot of the three dimensional PSD function (right) with characteristic ring structure.

PSD can be defined as shown in following equation.

$$
\Phi_{\varepsilon_{\text{norm}}} (y_{\text{norm}}) = \frac{1}{N} \sum_{i=1}^{N} \Phi_{\varepsilon i} \left( \frac{y_{\text{norm}}}{\varepsilon_i} \right) - 1
$$

$$
S_{\varepsilon_{\text{norm}}} = \frac{1}{N} \sum_{i=1}^{N} S_{\varepsilon i} \frac{\sigma_i}{\varepsilon_i}.
$$

(1)

With these quantities and the found relation $\sigma^2 = 0.4708 \varepsilon$ a simulation algorithm can be set up as follows. First we draw a mean material amount $\varepsilon$ from the generalized extreme value distribution and compute the corresponding variance $\sigma^2$. Then the input PSD is determined with $S_{\varepsilon \varepsilon_{\text{norm}}} = \sigma^2 S_{\varepsilon_{\text{norm}}}$ while the target CDF can be computed via $\Phi_\varepsilon = 1 + \varepsilon \Phi_{\varepsilon_{\text{norm}}} (\sigma y_{\text{norm}})$.

Subsequently the simulation of the variable part $\tilde{\varepsilon}$ is possible. The sought foam sample reveals after adding the drawn mean value from the first step. From this simulated virtual foam the stochastic processes for cross section $A(z)$ and second moment of area $I(z)$ can be computed and used for the determination of the eigenfrequencies.

3 Eigenfrequencies

Because of the stochastic characteristic of the foams an analytic solution for the angular eigenfrequencies is not available. An approximated solution for bending oscillations can be defined using the RAYLEIGH-quotient:

$$
\omega_{01b}^2 \leq \frac{\sum_{i=1}^{N} \int_{z_0}^{z_1} C \left( \frac{\rho_S}{\varepsilon} \right) EI(z_i) w_{1z,z}^2(z_i) dz_i}{\sum_{i=1}^{N} \int_{z_0}^{z_1} \rho A(z_i) w_{1z}^2(z_i) dz_i}
$$

with $w_1 \ldots 1.$ bending mode for $A = \text{const.}$

$\rho_S \ldots$ foam density.

$C \ldots$ Constant to consider the strut cross section.

(2)

The Youngs Modulus has to be scaled with the foam density $\rho_S$ according to scaling laws (see [4]) to address the divergent deformation behaviour of foam bars regarding to bars with full cross section. The results of the computed angular eigenfrequencies compared to the measurements are visualized in figure 3. As it can be seen there is a difference in the mean angular eigenfrequencies as well as in the standard deviations. The absolute values are $\omega_{01b,\text{meas}} = 6050 \text{ 1/s}$ and $\omega_{01b,\text{sim}} = 6850 \text{ 1/s}$ which gives a deviation of 13%. A possible explanation lies in the geometric dimension in relation to the pore size of the measured foams. The number of pores over the height of the specimen is at around 6. Due to the manufacturing of the specimen there are cutted pores at the outer perimeter which have a smaller stiffness than intact ones. Because of the small number of pores in the specimen the cutted pores can lead to a significant decrease in stiffness, which gives an explanation for the deviation in the seen mean values of the angular eigenfrequencies.

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