Quasiparticle poisoning and Josephson current fluctuations induced by Kondo impurities

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Introduction.— Superconducting circuits based on small Josephson junctions are promising candidates for the implementation of qubits [1, 2, 3, 4, 5] and for the development of a prototype quantum current standard [6]. Unfortunately, the performances of these devices are significantly limited by different types of noise whose sources remain mostly unknown. Particularly dangerous one for Single-Cooper-pair Transistors and Cooper pair boxes is the noise produced by the incoherent tunneling of single quasiparticles into the superconducting island. Each tunneling event changes the island charge, thereby shifting the operation points of the device. An important requirement for the regular operation of these devices is that this tunneling is very rare. Despite significant experimental efforts to reduce quasiparticle poisoning [7, 8, 9, 10, 11, 12], a complete understanding of its microscopic mechanisms is still missing. The goal of this Letter is to show that the mechanism of the charge noise discussed in Ref. [13] might be also responsible for the creation of the low energy quasiparticle traps and provides an explanation of the puzzling features observed in quasiparticle poisoning experiments [14].

The work [13] shows that Kondo-like traps located at the Superconductor Insulator (SI) interface might produce the charge noise in small Josephson charge qubits; similar mechanism might be responsible for the critical current fluctuations in large superconducting contacts [14]. These Kondo traps are impurities with a singly occupied electron level that carry a spin degree of freedom. Each trap is characterized by an effective Kondo Temperature \( T_K \) that depends exponentially on its hybridization with the conducting electrons in the bulk superconductor. In this mechanism both charge and critical current noise originate from the electrons tunneling between those Kondo traps with \( T_K \sim \Delta \), where \( \Delta \) is the superconducting gap. In this Letter we show that Kondo-like traps might also be responsible for quasiparticle poisoning of the superconducting island. Further, we show that such traps located close to the Josephson junction generate additional sources of critical current fluctuations due to their coupling to the superconductors on both sides of the barrier. This mechanism for critical current noise provides the alternative to the conventional picture of fluctuators blocking conducting channels in the insulating barrier. In order to derive these results we introduce a toy model that captures the essential physics of a spin impurity coupled to the superconducting electrons in the superconducting island. By computing the Berry phase that is associated with the superconducting phase rotations in this model, we prove that two different low energy states of the impurity are characterized by a different charge. As a consequence, switching between these two low energy states has the same effect as quasiparticle unpoisoning (poisoning) of the island. Finally, we use this model to study the effect of the motion of electrons between the Kondo-like traps in a Josephson junction and we prove that if one of those traps is coupled to both the lead and the island, these processes result in critical current fluctuations. We begin with the review of the features of the Kondo physics that are relevant for the following and which provides justification of the toy model.

The behavior of a spin-1/2 impurity coupled antiferromagnetically with an exchange constant \( J \) to an electron gas characterized by a constant density of states \( \rho_0 \) within a bandwidth \( D \) is completely different at high and low temperature regimes. In the former, the electrons scatter off the impurity inelastically in a spin-flip process while at low temperatures the impurity is screened by the electrons forming a bound singlet state leaving only...
elastic scattering. The crossover takes place at energy scale of Kondo temperature $T_K \sim D e^{-1/J \rho_0}$. All relevant physics is described by the Anderson Hamiltonian:

$$H = H_{\text{lead}} + H_d + H_{sd},$$

where $H_{\text{lead}} = \epsilon_k c_{\kappa \sigma}^\dagger c_{\kappa \sigma}$ and

$$H_d = \sum_{\sigma} \epsilon_d c_{d\sigma}^\dagger c_{d\sigma} + U n_d^\dagger n_d,$$

$$H_{sd} = \sum_{\kappa \sigma} (V c_{\kappa \sigma}^\dagger c_{d\sigma} + h.c.)$$

(1)

Here $c_{\kappa \sigma}$ ($c_{\kappa \sigma}^\dagger$) is the annihilation (creation) operator for an electron in the band, $c_{d\sigma}$ ($c_{d\sigma}^\dagger$) is the annihilation (creation) operator for an electron on the impurity site. $U$ denotes the Coulomb onsite repulsion of the trap and $n_{d\sigma} = c_{d\sigma}^\dagger c_{d\sigma}$. In the limit of strong onsite repulsion, i.e. $U \gg \Gamma$, where $\Gamma = \pi \rho_0 V^2$ defines the hybridization to the conducting electrons, using the Schrieffer-Wolff transformation [14], we can map the Hamiltonian given in Eq. (1) to the Kondo model [17]:

$$H = H_{\text{lead}} + \sum_{\kappa \kappa'} J_{\kappa \kappa'} \tilde{S} \cdot c_{\kappa \sigma_1}^\dagger \bar{c}_{\sigma_2} c_{\kappa' \sigma_2}$$

(2)

Here $J_{\kappa \kappa'} = J = 8V^2/U$. This mapping neglects the particle hole asymmetry of the original problem, the effects that are small in $T_K/D$ but might have important physical consequences. Notice that in the limit $J/D \rightarrow 1$, the Kondo temperature becomes $T_K \approx J$.

A more complicated problem is presented by the impurity interacting with the conduction electrons in the superconductor. At present, only qualitative arguments and numerical results are available which show that the competition between the Kondo temperature of the trap and the superconducting gap $\Delta$ of the lead results in three different regimes for the system (impurity+superconductor): (i) $T_K \ll \Delta$, where the ground state of the system is a doublet and it is characterized by an odd number of electrons; (ii) $T_K \gg \Delta$ where the ground state of the system is a singlet, the electron of the impurity forms a bound state with the superconducting electrons and the total number of electrons is even; (iii) $T_K \sim \Delta$ where the singlet and doublet states become almost degenerate.

**Toy model and quasiparticle poisoning.**— In order to formulate a simplified model that captures the effects of an impurity interacting with the superconducting electrons, we notice that the Kondo physics can be viewed as a result of the ‘poor-man scaling’ in which the high energy degrees of freedom are gradually integrated out resulting in the logarithmic growth of the effective interaction. In the presence of the superconducting gap the process of integration has to stop at $\Delta$. If at this moment the interaction is comparable with $\Delta$, a bound subgap state can be formed in agreement with the results described above. This shows that the essential physics can be captured by a simple model in which the spin interacts with a single electron mode with the coupling constant $J \sim \Delta$ and energy $\varepsilon < \Delta$ that is described by the BCS Hamiltonian:

$$H_{\text{BCS}} = \varepsilon \sum_{i=k_1 \sim \kappa_1} c_i^\dagger c_i + \Delta \left( e^{i \theta} c_{k_1}^\dagger c_{-k_1} + e^{-i \theta} c_{k_1} c_{-k_1} \right).$$

The interaction between the impurity and the superconducting electrons is described by the spin exchange coupling given in Eq. (2):

$$H_{\text{toy}} = H_{\text{BCS}} + \sum_{k, k'} J_{k, k'} \tilde{S} \cdot c_{\kappa \sigma_1}^\dagger \bar{c}_{\kappa \sigma_2} c_{k \sigma_2}$$

(3)

We assume that the coupling is isotropic: i.e. $J_{k, k'} \equiv J \approx T_K \sim \Delta$. The Hamiltonian (3) can be readily diagonalized. We choose the state basis: \{ket_{k_1, j_1 \sim \kappa_1}, \sigma_{\text{imp}}\} where $i, j = 0, 1$ denotes respectively the absence or presence of the quasiparticle in the single electron mode while $\sigma = \uparrow, \downarrow$ represents the spin configuration up or down of the electron in the trap and find the lowest eigenvalues:

$$E_0 = \varepsilon - \sqrt{\Delta^2 + \varepsilon^2}$$

$$E_1 = \varepsilon - 3/2 \Delta$$

(4)

corresponding respectively to the (non normalized) doublet and singlet states:

$$|D_{\sigma}\rangle = \left[ -\left( \varepsilon + \sqrt{\Delta^2 + \varepsilon^2} \right) / |\Delta| \right] e^{-i \theta} |00\rangle + |11\rangle \right] |\sigma\rangle$$

$$|S\rangle = -|01\rangle \downarrow + |10\rangle \uparrow$$

(5)

As expected, the spin impurity interacting with the conduction electrons in the superconductor leads to the formation of weak Kondo subgap states. Notice that the subgap states have different properties: the doublet is characterized by an odd number of electrons, its degeneracy is due to the spin degree of freedom of the trap while the singlet state is a maximally entangled state with even number of electrons. Depending on the ratio $T_K/\Delta$, the ground state of the system can be either doublet or singlet. At a special value $T_K^c = 2/3 \sqrt{\varepsilon^2 + \Delta^2}$ singlet and doublet states are degenerate while for traps with $T_K \approx T_K^c \sim \Delta$ singlet and doublet states are almost degenerate.

So far, we have reproduced in a simplified manner the main content of the numerical results. We want now to show that singlet and doublet states differ in another important feature: namely, the doublet corresponds to a non zero off-set charge in the superconducting island. For this purpose, we recall that the operator $n$, describing the excess number of Cooper pairs on the island and the operator $\theta$, representing the superconducting phase, are conjugate variables, i.e. $[\theta, n] = i$, and that the Hamiltonian of the island/box $H_{\text{island}}$ is invariant with respect
to the local gauge transformation: $U^{-1}H_{\text{int}}/c_{\text{ox}}U$ where $U = e^{in\theta}$ and $n_g$ is the off-set charge induced in the island in unit $2e$, that plays a role similar to the vector potential appearing in the Hamiltonian of an electron in a magnetic field. One consequence of these observations, is that we can deduce the value of the off-set charge induced by the Kondo impurity in the singlet and doublet states from the value of the Berry phases associated to these states. We find that

$$\int n \cdot d\theta = i \int_0^{2\pi} \langle S| \frac{\partial}{\partial \theta} | S \rangle = 0$$

$$\int n \cdot d\theta = i \int_0^{2\pi} \langle D_o| \frac{\partial}{\partial \theta} | D_o \rangle = \pi \left( 1 + \frac{\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2}} \right)$$

Notice that a Kondo impurity induces a non-zero off-set charge in the superconducting island only when the system (impurity+superconductor) is in the doublet state, while in the singlet state the electron charge is fully screened. Moreover, when $\varepsilon \rightarrow 0$, we find that $n \rightarrow \frac{1}{2}$, i.e. exactly one electron is induced on the superconducting island. Let us now consider two Kondo-traps with $T_K \sim \Delta$ located at the SI interfaces, one on the superconducting island, another on the lead within distance $\xi$ from each other. The electron tunneling process across the junction couples these traps. When the Kondo trap located at the SI interface in the island switches between singlet and doublet, the parity of the island changes from even to odd. For this process to be physically relevant, the energy difference between these states should be smaller than $T$. Thus, the pairs of Kondo subgap states with close energy levels might be responsible for the quasiparticle poisoning in superconducting devices. Notice that if the two traps are located on the same side of the barrier, the switching between singlet and doublet caused by the tunneling of quasiparticles through the superconductor results in charge fluctuations, i.e. the entire process can be viewed as a new type of a charge fluctuator.

We now discuss the implications for the recent experiments where quasiparticle tunneling rates were measured with microsecond resolution in a micrometer-sized island with capacitive gate electrode that was probed by two Josephson junctions. The island charging energy is modulated by the gate as $E_c^{\mu}(n_g) = E_c(n - n_g)^2$, where $E_c = e^2/2C$, $e$ is the electron charge and $C$ is the total island capacitance, $n_g = C_oV_o/e$ is the normalized gate charge and $n$ is the integer number of excess charges on the island. At $n_g = 1$ the electrostatic energy of the system is minimized when unpaired electrons reside in the superconducting island. Thus, at $n_g = 1$ the island is a trap for a quasiparticle with depth $\delta E = E_c - E_f/2 + \Delta_1 - \Delta_2$. Here $\Delta_1$, $\Delta_2$ are the superconducting gap of the island, lead and $E_f$ is the Josephson energy. A model suggested by Aumentado et al. [2] explains many features of quasiparticle poisoning of the island. In this model some unknown non equilibrium source of quasiparticles produces them in the leads. Quasiparticles are able to tunnel onto the island which acts as a trap. Subsequently the trapped quasiparticle is thermally excited (unpoisoning) out of the trap and the island returns to its even state. Relevant implications of this model are that the quasiparticle poisoning can be reduced by putting normal metal leads (QP traps) close to the junctions in order to filter the quasiparticles and by making $\Delta_1 > \Delta_2$, because it works as a barrier, which prevents non equilibrium quasiparticles in the leads from entering the island. Experiments showed that these ideas help to reduce quasiparticle poisoning, but do not eliminate it. In particular, the effect of quasiparticle traps have been recently studied in Ref. [14]. In these experiments, two similar Cooper pair boxes were fabricated with (QT) or without (NT) quasiparticles traps attached to the leads. The island was biased at $n_g = 1$ and the dynamics of the quasiparticles captured by the island was characterized by their incoming $(t_{\text{even}})$ and outgoing $(t_{\text{odd}})$ rates. One expects that the incoming process involves quasiparticle tunneling into the island and its relaxation to the bottom of the well, while the reverse process involves thermal excitation. As a result, the outgoing rate should be smaller by a factor $e^{-\delta E/KT}$ than the incoming rate. This is in contrast with the data that show that the trapping and escape rates are roughly equal and temperature independent below $T \lesssim 200$ mK. However, their values are dramatically different in the devices with or without traps: $t_{\text{even}} \sim t_{\text{odd}} \approx (10^2 - 10^3) \mu$s (QT) and $t_{\text{even}} \sim t_{\text{odd}} \approx (0.1 - 1) \mu$s (NT).

![FIG. 1: Sketches of quasiparticle poisoning and unpoisoning due to quasiparticle tunneling between weak Kondo subgap states located at the lead/island SI interfaces.](image-url)

The presence of subgap states in the lead and the island provides a different scenario where two new processes are present: quasiparticles with energies above $\Delta_1 - E_c + E_f/2$ tunnel from a subgap state in the lead to the continuum in the island while quasiparticles below these energies tunnel between the subgap states in the island and in the lead (see Fig. 1). The rate of the former process is $\Gamma_{sc} = G\delta$, where $G \sim 1$ is the conductance of...
the barrier in the units of $\varepsilon^2/\hbar$ and $\delta = 1/V_1 \nu_{Al}$ is the
typical level spacing in the island. For a typical island of
volume $V_1 = 750nm \times 125nm \times 7nm$ and a typical Al
electron density of states $\nu_{Al} \sim 35/eVnm^3$, we estimate $\Gamma_{sc} \approx 10^{-7}s^{-1}$. The rate of the exchange process between
subgap states is much slower $\Gamma_{ss} \ll \Gamma_{sc}$ because it occurs
between two localized states and it depends on the level
width of the state. In both cases the tunneling does not
involve a significant energy transfer, so we expect these
rates to be temperature independent. In the presence of
quasiparticles with energy larger than $\Delta_1 - E_0 + E_1/2$
the first process dominates and the observable rate is
$\Gamma_{sc}$. The presence of quasiparticle traps attached to the
leads eliminate high energy quasiparticles and the rate
decreases to $\Gamma_{ss}$ in agreement with observations. This
scenario can be checked by fabricating QT devices with
traps in the lead located at different distance $L \sim \xi$.
The presence of these traps at sufficiently small distance
broadens the levels of the subgap states in the leads and
consequently it should increase the tunneling rate into
the island as $\propto e^{-L/\xi}$, where $\xi$ is the coherence length
of the superconductor. The estimates of the rate assume
that only a few subgap states are active at the same time,
large number of these states would make the effective rate
higher; we do not know the density of these states and
their occupation in a realistic system.

Josephson current fluctuations.— Kondo-like traps
with $T_K \sim \Delta$ provide additional source of the critical
current fluctuations when they are located close to the
Josephson junction barrier because in this case the spin of
the Kondo trap is coupled both to the electrons in the island
and in the lead. This can be easily seen by including in
our toy model the electrons of the lead and a tunneling
between the lead and the island. The Hamiltonian becomes $H = H_{Toy}^{(1)} + H_{BCS}^{(2)} + H_{pp}^{Josephson}$
through the junction barrier is given by:

$$H_{Josephson}^{pp} = |T_{k1,k2}| \left[ c_{k1}^\dagger c_{k2}^\dagger + c_{-k1}^\dagger c_{-k2} + \text{h.c.} \right].$$

We assume that the superconducting leads are equal and
we calculate the correction to the lowest energy eigenval-
ues and eigenvectors at the second order in perturbation
theory in the tunneling $|T_{k1,k2}| \approx T$. We find that the
correction to the singlet state depends on the phase dif-
cference $\varphi = \theta_1 - \theta_2$. The dependence on $\varphi$ implies additional
contribution to the Josephson current. A straightforward but lengthy calculation, gives the contribution of the Kondo impurity in the barrier to the Josephson
current:

$$\delta I_c \approx \frac{T^2}{J} \frac{\Delta^2}{\varepsilon^2 + \Delta^2} \sin \varphi$$  \hspace{1cm} (6)

To find the parameters of the toy model that describe
the physical situation in which a many channel junction
couples a small metallic island to a large superconducting
lead, we compare the pairing field induced on the state
$k_1$ in the islands by the superconducting order in the
leads in the realistic situation ($\nu T^2 \sim G\delta$) with the
corresponding quantity in the simplified model ($T^2/\Delta$) and
get $\frac{T^2}{J} = G\delta$. Notice that this reasoning holds for small
superconducting islands whose size is less than the super-
conducting correlation length $\xi$. For larger islands the
impurities at a distance larger than $\xi$ from the junction
are coupled exponentially weakly to the superconductor
on the other side of the barrier.

Conclusions.— We have shown that subgap states
generated by magnetic impurities due to the competition
between superconducting pairing and Kondo effect act
as very efficient quasiparticle traps. We argued that
the presence of such states in a typical Single-Cooper-
pair Transistor and Cooper pair box might explain the
results of recent experiments where unexpected poison-
ing/unpoisoning rates were observed. We have also
shown the same subgap states generate critical current
noise.

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