Recent advances in the application of dynamical supersymmetry to describe atomic nuclei.

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Abstract. In recent years we have investigated further the use of supersymmetry in the Au-Pt region. In the extended supersymmetry which takes account of the neutron-proton degree of freedom, the even-even Hg isotopes $^{196,198}$Hg could be described as two proton fermions coupled to a neutron boson core. We have performed experiments on $^{193}$Au at the 10 MV Tandem accelerator in Cologne using the HORUS and Double ORANGE spectrometers and fast LaBr3(Ce) scintillators to perform $\gamma - \gamma$, $\gamma - e^{-}$ and $e^{-} - e^{-}$ coincidences and on $^{196}$Hg at the Yale accelerator using YRAST Ball. Both level schemes could be considerably extended and corrected. This allowed a detailed comparison with the predictions of the U(6/4) supersymmetry for the positive parity states in $^{193}$Au and $^{196}$Hg.

1. Introduction
In 1980, Iachello introduced dynamical supersymmetries to describe bosonic and fermionic systems [1]. In the following year the U(6/4) supersymmetry was compared to the positive parity-states of Au and Pt isotopes [2, 3], showing for the first time that U(6/4) is able to describe odd gold isotopes. In 1984, Van Isacker et al. [4] introduced the so-called extended supersymmetry by including the proton-neutron degree of freedom and thus were able to describe sets of four neighboring nuclei: even-even $^{196}$Pt, odd-neutron $^{197}$Pt, odd-proton $^{197}$Au, and odd-odd $^{198}$Au. Such a supermultiplet is also called magical quartet or magical square. About 15 years later, experimental evidence was found for a new neighboring magical quartet consisting of $^{194,196}$Pt and $^{195,196}$Au [5, 6, 7].

These magical quartets contain odd Au isotopes on which the experimental knowledge is rather scarce as many were studied in the early 1970’s [8, 9]. It is of interest whether other Au isotopes can be described using Bose-Fermi symmetry and/or supersymmetry and to improve the experimental knowledge on odd Au isotopes. Therefore we started recently a campaign to extend the knowledge on odd Au isotopes. Here, we present results for $^{193}$Au [10, 11].

In 1986 the magic square was further extended to include the even-even Hg isotopes. They are described as two proton fermions coupled to $N_{B\nu}$ neutron boson[12]. Only recently these calculations were tested in $^{198}$Hg[13] and $^{196}$Hg[14].

In the next section the Bose-Fermi symmetry and supersymmetry are shortly reviewed. Section 3 gives an overview of the new results for the $^{196}$Hg isotope obtained from a beta decay experiment performed in Yale. Section 4 describes the experiment on $^{193}$Au performed at the Cologne Tandem accelerator. Then, a common description of the isotones $^{193}$Au and $^{192}$Pt in the supersymmetric U(6/4) scheme is investigated.
2. The $U^B(6) \times U^F(4)$ Bose-Fermi symmetry and the $U(6/4)$ supersymmetry

The model used to describe odd proton Au isotopes, is the Interacting Boson Fermion Model 1 (IBFM) [15] in the $U^B(6) \times U^F(4)$ limit [16]. Hereby, a fermion in the $\pi 2d_3$ orbital is coupled to a system of $N_B$ bosons using the $U^B(6) \otimes U^F(4)$ algebra. Isomorphisms in the sub-algebra structure of $U^B(6) \otimes U^F(4)$ can be found between the boson and fermion algebras. Such an isomorphism exists between $U^B(6) \supset SO^B(6)$ and $U^F(4) \supset SU^F(4) \simeq SO^F(6)$ groups. The generators $g_k$ of these subgroups $SO^B(6)$ and $SO^F(6)$ commute and a linear combination of these generators closes under commutation and thus forms a boson-fermion algebra $O^{BF}(6) = Spin(6)$. The group chain of the Hamiltonian with the quantum numbers generated by the groups is:

$$U^B(6) \otimes U^F(4) \supset SO^B(6) \otimes SU^F(4) \supset O^{BF}(6) \supset O^{BF}(5) \supset SU^{BF}(2)$$

with $N_F=1$. The Hamiltonian written in form of a linear combination of Casimir operators, neglecting constant terms that only contribute to the binding energy, is:

$$H = DC_2(SO^B(6)) + AC_2(SO^{BF}(6)) + BC_2(O^{BF}(5)) + CC_2(SU^{BF}(2)).$$

The corresponding energy eigenfunction of the Hamiltonian can be derived from the eigenfunction of the Casimir operators of the subgroups:

$$E = D\sigma(\sigma + 4) + A(\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + \sigma_3^2) + B(\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)) + C(J(J + 1)).$$

For this Hamiltonian, it is possible to find an embedding superalgebra to the Bose-Fermi symmetry [1, 16]. The generators of this graded Lie algebra now also consist of mixed boson-fermion creation and annihilation operators. While the Bose-Fermi symmetry preserves the boson and fermion numbers separately, the supersymmetry only preserves the total number of particles $N = N_B + N_F$. The embedding algebra of $U^B(6) \otimes U^F(4)$ is:

$$U(6/4) \supset U^B(6) \otimes U^F(4).$$

In case that a fermion is annihilated and a boson created, the number of fermions is $N_F = 0$ and the problem can be described within the Interacting Boson Model (IBM - 1). The eigenvalues of the IBM in the $O(6)$ limit are given by the formula:

$$E = \overline{A}\sigma(\sigma + 4) + B\tau(\tau + 3) + CJ(J + 1),$$

with $\overline{A} = A + D$. In the opposite case a boson is converted into a fermion and we have a system with two fermions and $N - 2$ bosons. While this situation generally would describe so-called broken pair states a different interpretation follows when considering an neutron-proton version of (4):

$$U(6/4) \supset U^B(6) \otimes U^F(4).$$

and atomic nuclei having two proton holes in the $Z=82$ shell and neutron boson holes below $N=126$. This is exactly the case for the even-even Hg isotopes next to the magic squares [12, 13].
3. Beta-decay study of $^{196}$Hg
The beta-decay experiment was performed using the ESTU Tandem Van de Graaff Accelerator of the WNSL of Yale University using a $^{198}$HgS target. With a 35 MeV proton beam the (p,3n) reaction produced $^{196}$Tl which decays to $^{196}$Hg with a half life of 1.5 hours. Gamma-gamma coincidences and angular correlations were measured using the YRAST spectrometer. The main results not found in the in-beam experiment of [14] are the non-existence of a 1,2$^+$ state at 958 and the new determination of the spin 2$^+$ of the 1450 keV state, which before was assigned 0$^+$ [17].

![Figure 1. (Color online) Comparison between experiment and theory for $^{196}$Hg. Also given are the values for the quantum numbers. The dashed experimental states were not confirmed by this experiment and the one described in [14].](image)

In Figure 1 a fit of the experimental positive parity states using eq. (2) is shown. The quantum numbers and reduction rules used are given in [13]. The parameter $D$ was artificially set to a high negative value. The other parameters are (all in keV): $A= 78.3$, $B= 81.4$ and $C= 7.9$. The obtained agreement is good and of similar quality as the one in $^{198}$Hg [13]. However, the parameters differ from those obtained for a common description of the Pt, Au nuclei.

4. In-beam study of $^{193}$Au.
The experiments on $^{193}$Au were performed at the Cologne Tandem accelerator by impinging a 14 MeV proton beam onto a 1.3 and 0.2 mm $^{194}$Pt target. The (p,2n) reaction was used which at this energy yields an angular momentum transfer of about 3.5 $\hbar$. The HORUS spectrometer [19], the Orange spectrometers[10] and LaBr3(Ce) scintillators[20] were used. During the experiments HORUS was equipped with 12 high-purity germanium detectors on the edges and the faces of a cube to detect the $\gamma$ transitions of excited states. The setup of the spectrometer allows the analysis of $\gamma\gamma$ angular correlations. The data were sorted in so-called correlation group matrices, which consist of detector pairs defined by specific angles. The method using the HORUS spectrometer is described in more detail in Refs. [19, 21]. By fitting theoretical angular distributions described in Refs. [23, 22] to the data, spins, multipolarities, multipole mixing
ratios, and eventually parities are obtained. The fit was performed with the computer code CORLEONE [24]. An example of a correlation fit for the 861 keV-258 keV cascade from the 3/2^+ level at 1119 keV in ^{193}Au is shown in Fig. 1. Both, the spin and the mixing ratios, could be clearly determined.

By analyzing the γγ coincidence matrices, numerous new transitions and states in ^{193}Au could be identified. As an example, The spins of all low-lying positive-parity states in ^{193}Au were determined except for a state at 828 keV, where the fits cannot distinguish between spin $\frac{1}{2}$ and $\frac{3}{2}$.

With the ORANGE electron spectrometers and fast LaBr₃(Ce) scintillators γ−γ, γ−e⁻ and e⁻−e⁻ coincidences were performed using the fast timing technique. Using the deconvolution method, the centroid shift technique and the new mirror symmetric centroid difference method [20] several new lifetimes could be measured. Unfortunately, all are of states with negative parity. The results are described in detail in Refs. [10,11] to which we refer the interested reader.

For ^{193}Au it was investigated whether a U(6/4) supersymmetry with ^{192}Pt exists. A common fit of both nuclei yielded $A = -0.0432$, $B = 0.0403$ and $C = 0.0173$ (all in MeV)[11]. It is clear that the fit of ^{193}Au shown in Figure 3 gives only a reasonable description.

5. Summary
We have studied ^{196}Hg after beta decay using the YRAST Ball at Yale university and ^{193}Au using the (p,2n) reaction at the Cologne Tandem accelerator. Using angular correlation and lifetime measurements the experimental knowledge on these nuclei could be considerably extended and corrected. A good fit of ^{196}Hg can be obtained considering coupling two proton fermions to a neutron boson core. A less good but still reasonable description is found using the U(6/4) supersymmetry for ^{193}Au-^{192}Pt.
Figure 3. Fit of $^{193}$Au-$^{192}$Pt using the U(6/4) supersymmetry.

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