We introduce a notion of quantum hair which completely characterizes the state of a D-brane in perturbative string theory. The hair manifests itself as a phase (more generally a unitary matrix in subspaces of degenerate string eigenstates) in the scattering amplitudes of elementary strings on the D brane. As the separation of the D-brane and the string center of mass becomes large, the phase goes to zero if we keep the string excitation level fixed. However, by letting the level number increase with the distance, we can keep the phase constant. We argue that this implies that scattering experiments with highly excited strings can detect the state of a D-brane long after it has “fallen into a black hole”.

Quantum Hair on D-branes and Black Hole Information in String Theory

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1. Quantum Hair and the Information Loss Paradox

The notion of quantum hair was introduced some time ago by Krauss and Wilczek[1] as a proposal for resolving the information loss paradox in Hawking radiation. We will take it to mean the contention that every quantum state of a black hole (and thus of anything that can be thrown into a black hole) is characterized by some set of generalized Aharonov-Bohm fluxes which enable an external observer to detect its presence by some set of interference experiments performed outside the black hole. These fluxes are undetectable classically, and do not contradict the no hair theorems.

In its original context of internal Yang Mills gauge symmetries, it is hard to know what to make of this idea. It is hard to imagine a symmetry large enough to characterize every state of the world. Furthermore, Abelian Aharonov-Bohm fluxes carried by localized objects have a multiplicative character on asymptotic states. That is, the flux carried by well separated objects is a product operator on the asymptotic Hilbert space. If the flux operators commute with the S matrix and indeed characterize each individual state, then the S matrix will be unity.

In the present note we will show that string theory contains a notion of quantum hair that avoids all of these objections. We will demonstrate this explicitly for D-branes, but by various string dualities this implies that there is quantum hair for elementary string states as well. This adds to the growing list of evidence which indicates that string theory avoids the information loss paradox by producing a unitary S-matrix for black hole formation and evaporation[2]. We note that our proposal is somewhat reminiscent of the notion of W-hair introduced in [3]. I do not understand the precise relation between the two ideas, but I am fairly certain that the conclusions I draw are different than those reached in that work. J. Schwarz[4] has previously proposed that the infinite symmetry groups of string theory might provide a source of quantum hair for black holes. This work and a conversation with L. Susskind were the inspiration for the current paper. However, as Schwarz himself pointed out, the duality symmetries which were the focus of his discussion are spontaneously broken. Thus, except at special points in moduli space where a tiny piece of the gauge symmetry is restored, we do not expect them to produce observable phases in the scattering from cosmic strings. We will try to relate D-brane hair to string
gauge symmetries, but the gauge symmetries studied in this paper are more like large versions of the continuous BRST symmetries of the perturbative string. They are not broken by the vacuum state.

2. Hair on Branes

We want to study the scattering of highly excited elementary strings from D-branes in perturbative string theory. For high excitation number, the usual Fock basis for the string oscillator state is extremely unwieldy. It is more convenient to remember that strings are collections of oscillators, and that the WKB approximation becomes exact for high excitation energy. Thus, to any periodic solution of the string equations

\[(\partial^2_\sigma - \partial^2_\tau)X^\mu = 0 \quad (2.1)\]

satisfying the classical Virasoro constraints, there corresponds a Gaussian wave functional in the string Hilbert space which is approximately BRST invariant. As the excitation number increases, this approximation becomes exact.

Now consider the elastic scattering of one of these highly excited strings from a D-brane. It proceeds predominantly via the following mechanism. If the classical configuration of the string intersects the D-brane world volume, there is an (intrinsically quantum mechanical) amplitude for the closed string to split into one or more open strings propagating on the D-brane world volume. For a highly excited essentially classical string, these pieces will also be large classical objects. In leading order string perturbation theory the open strings propagate freely on the world volume until they reconnect to form the outgoing closed string.

For scattering off the D-brane ground state, the open strings simply have Dirichlet boundary conditions. Excited states of the D-brane can be accessed by adding a boundary operator \(\delta L\), to the Lagrangian of the open strings. That is, excited states of the D-brane correspond to boundary vertex operators (of course, for very large excitation of the D-brane this vertex operator description will be very clumsy). The amplitude will then pick up a phase

\[\delta = \sum \int \delta L \quad (2.2)\]
where the sum runs over the trajectories of the open string ends on the world volume. This D-brane state dependent phase is the quantum hair.

The essential message here is that stressed in the penultimate section of [5]: the space of states of string theory simply does not split up into a tensor product over a spacelike hypersurface. States “localized at $x$”, in the sense that their center of mass is in a wave packet concentrated at $x$, are not independent of those at some faraway point.

If, for a fixed string state, we let the distance between the D-brane and the string center of mass go to infinity, then the classical solution will no longer intersect the D-brane world volume, and the phase $\delta$ will vanish. However, if we let the level number increase with the distance, we will always find nontrivial phases. Thus, by doing scattering experiments with sufficiently highly excited string states, we can in principle measure the state of distant D-branes. The root mean square distance from the center of mass of a classical string of level $N$ is $\sqrt{N}$. Since the WKB approximation becomes exact for large excitation number, this is also the average fluctuation of the string position in a typical quantum string state of high excitation.

The last remark is the key to answering a possible objection to our definition of quantum hair. A physicist who believes strongly that string theory is in some sense local might complain that our experiment for measuring the state of a distant D-brane cannot actually be performed in a laboratory far from the D-brane. The classical string configuration that we have described does intersect the D-brane. How can we create it by operations in our laboratory?

To answer this objection, consider the amplitude for scattering $n + 1$ photons off the D-brane. It has the form

$$\int_{\text{Disk}} V_\gamma(0)V_\gamma(z_1)\ldots V_\gamma(z_n)$$

There is a region in moduli space in which all of the incoming photon vertex operators are close to a point $z$ and all outgoing operators close to another point $y$. This is the region in which the incoming photons form a highly excited single string state which then scatters off the D-brane. Note that the question of finding a single highly excited state in multiphoton scattering is independent of the boundary conditions on the D-brane, because it is local in moduli space (we work at tree level).
At tree level, the sum of the squares of the amplitudes for finding strings of level \( N \) in \( k \) photon scattering is just the residue of the pole in the \( k \rightarrow k \) photon scattering amplitude, when the incoming multiphoton invariant mass is \( N \). These residues are easily calculated for two to two scattering and they do not go to zero with the level (at generic values of the Mandelstam variable \( t \)). Presumably it is even easier to produce highly excited string states in multiparticle collisions.

Thus there seems to be no argument in principle against creating highly excited string modes in collisions at sufficiently high energy. Once we have those modes available, our previous argument guarantees the observability of the state of a faraway D-brane. Of course, we will not be able to pick the requisite phases directly out of a two photon plus D-brane to two photon plus D-brane amplitude, even though they in principle contribute to the final result. The scattering with a small number of low level external states probes a complicated average of excited state scattering amplitudes off the D-brane. However, I believe that scattering of large numbers of photons should provide us with enough free parameters to probe individual phases. This deserves further study. If it proves true, then the state of a faraway D-brane will be encoded in complicated phase correlations in multiphoton scattering amplitudes. The excitation level required for a semiclassical string to pick up a state dependent phase in scattering off a D-brane, increases like the square of the distance between the D-brane position and the center of mass of the string. We may expect a corresponding increase with distance in the complexity (and in particular of the number of photons) of a process which can measure these phases using only external photons.

The application of these results to the black hole information problem is straightforward. Consider a large mass black hole in “nice slice” coordinates. These are coordinates which smoothly cross the horizon, but asymptote to Schwarzschild coordinates at large spacelike distances from the horizon. It is in such coordinates that the black hole information paradox is most sharply expressed in quantum field theory. For a black hole of very large mass and a region of spacetime in which the Schwarzschild metric is approximated by the Rindler metric, the spacelike distance between a supported and infalling observer grows linearly in the nice slice time \( T \). Their relative velocity approaches the
speed of light as $1 - v^2 \sim \frac{1}{r^2}$. In field theory one then argues that the supported observer cannot measure the state of the infalling observer because no signal can ever propagate between them. If however, the supported observer has at her disposal a highly excited string (at her disposal means that the center of mass of the string remains forever close to her spacetime position), this argument is no longer valid, as we have seen above. As time goes on, the observer must perform a series of more and more intricate experiments at higher and higher energies in order to extract information about things that fell into the black hole. These experiments are difficult beyond comprehension, but perhaps not more so than those required to extract the quantum information in the debris from an exploding hydrogen bomb. By replacing *impossible* with *extravagantly difficult*, these arguments return black hole information to the ordinary realm of macroscopic phenomena.

The stringy notion of quantum hair that we have outlined above does not imply that the S-matrix is unity. Although we concentrated on elastic scattering above, there will also be processes in which the outgoing closed string state is not the same as the incoming one. For large impact parameter, where we probe only very small momentum transfers, the outgoing state must be degenerate in mass with the incoming state, but the highly excited levels are also highly degenerate. Thus the quantum hair of a D-brane is not a collection of phases but rather a collection of unitary matrices in the space of degenerate string levels. These matrices will not commute with one another. Furthermore, because of the nonlocality of the hair, the matrix for two widely separated D-branes will not be the product of the matrices for the individual branes. Thus, although the system has, in some sense, an infinite number of conservation laws, they are not commuting, and do not act as single particle operators in the space of asymptotic states.

Finally we note that various string duality transformations tell us that elementary strings are really D-branes in another vacuum state of string theory. D-brane hair can be measured in D-brane D-brane scattering as easily as it can in the scattering of elementary strings. Thus elementary strings should have quantum hair. We will get some hint of its nature in the next section.
3. D-brane Hair and Stringy Gauge Symmetries

Quantum hair was originally discovered as the remnant of a spontaneously broken gauge symmetry. String theory is replete with such symmetries, and it is not unreasonable to expect that D-brane hair can be related to them. Indeed, the relationship is fairly straightforward.

Consider adding a total derivative to the lagrangian for a single particle:

$$\delta L = \frac{d}{dt} F$$  \hspace{1cm} (3.1)

If $F$ depends only on the particle coordinate $X^\mu$ this has the form $\dot{X}^\mu \partial_\mu F$, which looks like the response of a charged particle lagrangian to a gauge transformation. In the case of a discrete internal gauge symmetry, there would be no external potentials with nonvanishing field strength, but pure gauge transformations of this type would measure Aharonov-Bohm flux and would be the external evidence for quantum hair. Of course, in the case of a particle we can only claim that this is so after deriving the particle lagrangian from a theory in which there are gauge fields etc.

By contrast, the rule of thumb in string theory is that any string lagrangian which satisfies the conditions of conformal invariance is an allowed classical background of the theory. A lagrangian density which is an exact two form $\delta L = d\omega$, with $\omega$ a one form which is a smooth function of the string coordinates and their derivatives, globally defined on target space, has no effect on perturbative string scattering amplitudes.

To be more precise, we can think of the calculation of amplitudes in the presence of such a term in the lagrangian as the computation of a functional integral with fixed boundary conditions, $x_i^\mu(\sigma)$ (one boundary for each vertex operator), followed by multiplication by functionals of the boundary values representing BRST invariant states and integration over the boundary values. The total derivative will change the boundary path integral by a factor $e^{i \int_B \omega}$, which will induce a unitary transformation on the space of BRST invariant states. The important point is that it is the same transformation on each external leg. So there is no observable consequence of these transformations.

Consider however scattering off a D-brane of spatial dimension $p$. At tree level, the amplitudes are computed by computing vertex operator correlation functions in a super-
conformal field theory with a single boundary with Dirichlet boundary conditions on $10-p$ of the spatial coordinates. This computes the scattering amplitudes off the D-brane ground state. Excited state amplitudes are computed by adding a boundary vertex operator to the lagrangian. This is a term $\int_{B_D} V_B$ where $V_B$ is a one form on the boundary of the disk, constrained by boundary conformal invariance. It is a function of the bulk string coordinates, and so has an extension into the interior of the disk. The extension is not unique, since we can add terms to $V$ which vanish at the position of the D-brane. Call the extension $V$.

Now write

$$\int_{B_D} V_B = - \int_D dV - \int_{C_i} V$$

(3.2)

where the first integral on the right is over the interior of the disk. The $C_i$ are small circles surrounding the punctures where vertex operators are inserted. In general these will give nonzero contributions to the amplitude, because the operator product expansion of $V$ with the vertex operators will contain singularities. The first term on the right is a BRST exact operator $\{Q, \{b_{-1}, V\}\}$ and does not contribute to amplitudes.\(^1\)

We began from a BRST invariant expression, and so the $\int_{C_i} V$ is BRST invariant. Its effect on the vertex operator $O_A$ inside $C_i$ is thus to replace it by a linear combination $H^B_A(V_B)O_B$ of the complete set of BRST invariant vertex operators of the bulk theory. $H$ is the “trivial ”unitary operator we discussed above.

In essence, what we are saying above is that BRST transformations which are trivial in the absence of D-branes are nontrivial when they are present. The simplest example is the diffeomorphism generated by a vector field on target space. This is clearly a trivial transformation in the absence of D-branes, but it does not in general leave invariant the Dirichlet boundary condition. From this example it is clear that all aspects of the D-brane state which are related to its configuration in spacetime can be encoded in such nontrivial BRST transformations. Our claim is that the same is true for any aspect of the D-brane state.

\(^1\) The knowledgeable reader will have recognized that we are merely repeating the derivation of axion hair\(^6\) for more complicated stringy gauge transformations.
The scattering off different D brane states can be related to the amplitude for scattering off any reference state (above we chose the simple Dirichlet state) of the D-brane multiplied by the $H$ matrices. Thus,

$$S_{BB\ a_1...a_n} = H_{a_1}^{b_1}(B) \cdot \ldots \cdot H_{a_n}^{b_n}(B) S_{DD\ b_1...b_n}$$  \hspace{1cm} (3.3)$$

Some of the phases are measurable. Those $H$ matrices corresponding to BRST transformations which leave the D-brane state invariant can still be eliminated simultaneously from the amplitudes with and without a D brane by performing the corresponding transformation on both functional integrals. Those that do change the state of the D-brane constitute its quantum hair. The hair can be measured by for example preparing two different D-brane states, scattering strings from them, and then scattering the outgoing strings from the two experiments off of each other.

This argument does not by itself show that the $H$ matrices do not fall off with distance, but from the previous section we know that this is the case for highly excited external states. More precisely, we must increase the level of excitation with the distance in order to measure stringy quantum hair on D-branes.

The connection with “nontrivial”BRST transformations also gives us a hint for the description of the quantum hair of elementary strings. Elementary string states are in the BRST cohomology. Thus, in old fashioned language, they are gauge transformations with a gauge function that is somehow singular. The resemblance to Aharonov-Bohm fluxes is striking. In fact, Moore has obtained relations quite similar to (3.3) for open elementary string scattering by applying a set of symmetry transformations that are in one to one correspondence with the BRST cohomology of the open string\[7\]. In general, Moore’s transformations change the kinematic invariants in a scattering process, but the lightlike Ward identities give relations between amplitudes with zero momentum transfer. It is possible in this way to relate elastic scattering from any given string state to that from any other. The relations I have found so far have the general form of (3.3) but the analogs of the $H$ matrices are not unitary. I believe that this is a consequence of the fact that Moore studies infinitesimal transformations, while I have discussed finite changes of state, but this is an issue which deserves further study. In particular, Moore’s formalism might
enable us to establish the heuristic semiclassical arguments of the present paper in a much more rigorous fashion.

Finally, it is worth pointing out that the formula (3.3) encodes the information about D brane states in leg pole factors, reminiscent of those of 1 + 1 dimensional string theory. This is the advertised connection with the work of [3]. Again, this suggests a direction for further research on the vexing problem of the formation of black holes in 1 + 1 dimensions.

4. Conclusions

We have argued that perturbative string theory contains a notion of quantum hair for D-brane states. The hair takes the form, for the most part, of Berry matrices in the scattering amplitudes of highly excited degenerate string states from the D-brane. We have argued that the quantum hair carries complete information about the D-brane state, so no information about the D-brane is lost as it falls into a black hole. Since perturbative string states are simply D-strings of another classical string vacuum state, all of these statements must be true of general states in string theory.

There are a large number of points where the argument has been less than rigorous. Our derivations have been highly semiclassical and have neglected subtleties of ghosts and BRST invariance that may well be important to a proper understanding of quantum hair. The work of Moore provides the most likely entry into a fully quantum derivation of these results. It is also likely to be the correct description of quantum hair for elementary string states. Finally, all of our work has relied on perturbation theory and the assumption that highly excited string states were stable. One would like to understand how to go beyond such tree level analysis, but the treatment of unstable particles even in weakly coupled string theory is at the moment beyond our capabilities.

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