Research on the Ranking of University Education based on Grey-TOPSIS-DEA Method

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Abstract—The University is the cradle of the higher education. In the college life, the college students continue to accept the new knowledge and continue to grow. Obviously, the level of the university education will be directly related to the growth for the college students. Therefore, it is an important job to evaluate and order the college education quality. In this paper, we combine the Grey theory, TOPSIS with DEA method. And we propose an improved Grey-TOPSIS-DEA model. Then, we use the model to evaluate the college education quality. Finally, we get the rankings of the college education. In the last part of this paper, we use the method to evaluate the education quality for different colleges. And we verify the validity of the method.

Index Terms—College Education, Evaluation, Ranking.

I. INTRODUCTION

In the modern society, the competition is becoming fierce increasingly. Therefore, if we want to get the better development in this society, we must accept the perfect college education. As the place where we accept the high education, the college education quality plays a very important role. Therefore, it is an important job to evaluate and order the college education quality.

M. Ghanji, Z. Khoshnodifar, S.M. Hosseini and S.M. Mazloumzadeh evaluated the education quality of the agricultural higher education institutions. The results showed that the factors that affecting the quality of the agricultural teaching were as follows. (1) lesson plan (19.52%), (2) teaching skill (17.97%), (3) communication skills (17.93%), (4) expertise related to lesson content (10.59%), and (5) individual capabilities of members (9.15%) respectively[1]. Cai Hongmei studied on the factors that affecting the quality of the agricultural teaching were as follows. (1) lesson plan (9.15%), (2) teaching skill (10.59%), and (5) individual capabilities of members (17.93%), (4) expertise related to lesson content (17.93%), (3) communication skills (10.59%), and (1) lesson plan (9.15%) respectively[2].

II. THE BASIC KNOWLEDGE

A. TOPSIS

We assume that the scheme set and the attribute set of the multi attribute decision problem are

$$ A = \{A_1, A_2, \ldots, A_m\} \text{ and } F = \{f_1, f_2, \ldots, f_n\}. $$

The decision matrix is

$$ B = \{b_{ij}\}_{m \times n}. $$

The scheme A is written as

$$ A = \{b_{i1}, b_{i2}, \ldots, b_{in}\}, \quad i = 1, 2, \ldots, m, \quad b_{ij} \geq 0. $$

The weight vector of the attribute is

$$ W = (\omega_1, \omega_2, \ldots, \omega_n)^T. $$

The step of the traditional TOPSIS method is as follows.

$$ \sum_{j=1}^{n} \omega_j = 1, \quad \omega_j \geq 0, \quad j = 1, 2, \ldots, n. $$
Firstly, we use the vector norm method to construct the decision matrix \( C = (c_{ij})_{m \times n} \). For the benefit attribute,

\[
c_{ij} = \frac{b_{ij}}{\sqrt{\sum_{i=1}^{m} b_{ij}^2}} \tag{1}
\]

For the cost attribute,

\[
c_{ij} = \frac{1}{\sqrt{\sum_{i=1}^{m} \left(\frac{1}{b_{ij}}\right)^2}} \tag{2}
\]

Where, \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \).

Secondly, we weight the normalized decision matrix and get the weighted normalized matrix \( Z = (z_{ij})_{m \times n} \). Where \( z_{ij} = \omega_i c_{ij} \), \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \).

The third step is as follows. After we get the normalized matrix, we need to determine the positive ideal solution \( A^+ \) and the negative ideal solution \( A^- \). Then defining two artificial scheme.

The positive ideal scheme and the negative ideal scheme are as follows.

\[
A^+ = (z^*_1, z^*_2, \ldots, z^*_m)
\]

\[
A^- = (z^-_1, z^-_2, \ldots, z^-_m)
\]

For the benefit attribute,

\[
z^*_j = \max z_{ij}, \quad z^-_j = \min z_{ij}
\]

For the cost attribute,

\[
z^*_j = \min z_{ij}, \quad z^-_j = \max z_{ij}
\]

The fourth step is to calculate the distance from each scheme to the positive ideal solution \( d^+_i \) and the negative ideal solution \( d^-_i \).

\[
d^+_i = ||z_i - A^+|| = \sqrt{\sum_{j=1}^{m} (z_{ij} - z^*_j)^2} \tag{3}
\]

\[
d^-_i = ||z_i - A^-|| = \sqrt{\sum_{j=1}^{m} (z_{ij} - z^-_j)^2} \tag{4}
\]

Where \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \), \( z_i = (z_{i1}, z_{i2}, \ldots, z_{im}) \).

The fifth step is to calculate the relative closeness \( C^+_i \) between each scheme and the positive ideal solution.

\[
C^+_i = \frac{d^-_i}{d^+_i + d^-_i} \tag{5}
\]

Where, \( i = 1, 2, \ldots, m \).

It can be seen if \( z_i = A^+ \), \( C^+_i = 1 \). And if \( z_i = A^- \), \( C^+_i = 0 \). 0 \( \leq C^+_i \leq 1 \). When \( C^+_i \rightarrow 1 \) the scheme \( A_i \rightarrow A^+ \). Lastly, we rank \( C^+_i \) according to the descending order.

B. Grey relation

The grey system theory provides the index quantitative measurements for the project selection decision. We do the grey correlation degree analysis according to the same factors for each scheme. Then we get the numerical relationship of the same factor for each scheme. The step of the grey correlation method is as follows.

Firstly, we establish the index matrix \( A = [x_{ij}]_{m \times n} \). \( m \) is the number of the preferred items. \( n \) is the number of the indexes. The weight of each index is \( W = (w_1, w_2, \ldots, w_n) \).

Secondly, we give the weight on the basis of the index matrix \( A \). And we construct the weighted matrix \( B = AW \).

Thirdly, according to the weighted index matrix, we can get the positive ideal scheme \( a^+ \) and the negative ideal scheme \( a^- \).

The fourth step is to calculate the correlation between each scheme and the positive scheme and the negative scheme.

\[
L_j = \min \min \left[ \frac{|a_{ij} - a^-_j|}{|a^+_j - a^-_j|} + \theta \max \max \left[ \frac{|a_{ij} - a^-_j|}{|a^+_j - a^-_j|} \right] \right] \tag{6}
\]

For \( a^+_i \), \( L_j \) is the correlation coefficient at \( j \). The value range for the correlation coefficient is \([0, 1]\). If expresses that the similar degree of the two factors which are compared at one point. Where, \( \theta \) is the distinguish coefficient. It expresses the distortion that is caused by weakening the biggest absolute difference value. The value range for \( \theta \) is \([0, 1]\). According to the experience value, \( \theta = 0.5 \). \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \). The overall correlation of the scheme is the average of all kinds of the correlation coefficient. That is,

\[
R_i = \frac{1}{n} \sum_{j=1}^{n} L_{ij} \tag{7}
\]

The fifth step is to calculate the relative close degree between each scheme and the ideal scheme.

\[
r_i = \frac{R_i}{R^+ + R^-} \tag{8}
\]

The sixth step is as follows. The optimal scheme is the scheme that the relative degree is bigger. Then we order the alternative scheme according to the size of the close degree.

III. THE IMPROVED GREY-TOPSIS-DEA

We assume that there are \( n \) indexes \( x_{i1}, x_{i2}, \ldots, x_{in} \). Correspondingly, it exists the optimal ideal index set \( X^+ = \{x^+_{11}, x^+_{12}, \ldots, x^+_{1n}\} \) and the worst index set \( X^- = \{x^-_{11}, x^-_{12}, \ldots, x^-_{1n}\} \). The weight of each index is \( \omega_{11}, \omega_{12}, \ldots, \omega_{1n} \). Now, there are \( m \) assessment scheme \( y^i_{11}, y^i_{12}, \ldots, y^m_{1n} \). The index of each scheme is \( \{x_{i1}, x_{i2}, \ldots, x_{in}\} \). 1. When \( C^+_i \rightarrow 1 \) the scheme \( X_i \rightarrow A^- \).

Lastly, we rank \( C^+_i \) according to the descending order.

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\[ \alpha_i = \frac{\min \left[ x_i^j - x_i^* + \rho \cdot \max \max_i \left[ x_i^j - x_i^k \right] \right]}{\max_i \left[ x_i^j - x_i^* + \rho \cdot \max \max_i \left[ x_i^j - x_i^k \right] \right]} \]  
(9)

\[ \beta_i = \frac{\min \left[ x_i^j - x_i^* + \rho \cdot \max \max_i \left[ x_i^j - x_i^k \right] \right]}{\max_i \left[ x_i^j - x_i^* + \rho \cdot \max \max_i \left[ x_i^j - x_i^k \right] \right]} \]  
(10)

We assume that \( S'_i \) is the distance from the \( i \) scheme which is processed by the standardized to the optimal ideal index set. \( S'_i \) is the distance from the \( i \) scheme which is processed by the standardized to the worst ideal index set.

\[ S'_i = \frac{S'_i - S_i^*}{S_i^* - S'_i} \]  
(11)

\[ S'_i = \frac{S'_i - S_i^*}{S'_i - S_i^*} \]  
(12)

Where, \( S'_i \) is the distance from the \( i \) scheme which is not processed by the standardized to the optimal ideal index set and \( S'_i = \sum (\omega_k \cdot \beta'_k) \). \( S_i^* \) is the distance from the \( i \) scheme which is not processed by the standardized to the worst ideal index set.

\[ S^* = \sum (\omega_k \cdot \alpha'_k) \]  
(13)

\[ C'_i \] is the relative closes degree of the \( i \) scheme for the ideal index set.

\[ C'_i = \frac{S'_i}{S'_i - S_i^*} \]  
(13)

Where, \( C'_i \) is the evaluation value that the method gives.

Then, we use DEA method to adjust the result for \( C'_i \).

\[ \delta^*_i \] is the output efficiency and the \( \delta^*_i \) is the input efficiency. We notice that when \( \delta^*_i \) is close to zero or \( \delta^*_i \) is greater than 1, the adjusted degree becomes bigger. When \( \delta^*_i \) is close to 1 or \( \delta^*_i \) is close to 1, the adjusted degree becomes smaller. The adjusted formula for DEA method is as follows.

\[ C_i = C'_i + C'_i \]  
(14)

Where,

\[ C'_i = \frac{1}{2} C_i (1 - \delta^*_i \delta^*_i) \]  
(15)

\( C_i \) is the evaluation value which is adjusted by the DEA method.

The flow chart is shown in the following figure.

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**IV. EXPERIMENT**

In this paper, we rank the universities according to the teaching quality. Firstly, we establish the college teaching quality evaluation system. The college teaching quality evaluation system is as follows.

![Figure 1. The flow chart of the improved Grey-TOPSIS-DEA method](image)

Firstly, we get the weight of each index \( W = (0.21, 0.34, 0.14, 0.23, 0.08) \). Then we rank the teaching quality the six universities \( \{ A_1, A_2, A_3, A_4, A_5, A_6 \} \).

The decision matrix is as follows.

\[
C = \begin{bmatrix}
8 & 6 & 7 & 9 & 7 \\
6 & 6 & 9 & 7 & 8 \\
7 & 8 & 7 & 9 & 8 \\
7 & 9 & 8 & 6 & 7 \\
9 & 6 & 7 & 9 & 8 \\
6 & 7 & 9 & 8 & 6 \\
\end{bmatrix}
\]

We can get the evaluation result.
The college education is very important for a person’s growth. The college is not only the place where we study, but also the temple of the knowledge. To evaluate the college education and get the rankings is a very important job. In this paper, we combine the Grey theory, TOPSIS and DEA method and propose an improved Grey-TOPSIS-DEA model. And we apply the model to the quality evaluation of the college education. The main job of this paper is as follows. Firstly, we introduce the background of the university education. Secondly, we propose the method to the quality evaluation of the college education. The experiment in this paper shows the effectiveness.

TABLE I.
THE RANK FOR THE UNIVERSITY WHICH IS NOT ADJUSTED

| university | C’   | rank |
|------------|------|------|
| A1         | 0.487| 5    |
| A2         | 0.529| 4    |
| A3         | 0.712| 1    |
| A4         | 0.534| 3    |
| A5         | 0.450| 6    |
| A6         | 0.613| 2    |

Then, we use the DEA method to adjust. The results are as follows.

TABLE II.
THE RANK FOR THE UNIVERSITY WHICH IS ADJUSTED

| university | Before adjustment | After adjustment | rank |
|------------|-------------------|------------------|------|
| A1         | 0.487             | 0.465            | 5    |
| A2         | 0.529             | 0.544            | 3    |
| A3         | 0.712             | 0.723            | 1    |
| A4         | 0.534             | 0.528            | 4    |
| A5         | 0.450             | 0.452            | 6    |
| A6         | 0.613             | 0.632            | 2    |

V. CONCLUSIONS

The college education is very important for a person’s growth. The college is not only the place where we study, but also the temple of the knowledge. To evaluate the college education and get the rankings is a very important job. In this paper, we combine the Grey theory, TOPSIS with the DEA method and propose an improved Grey-TOPSIS-DEA method. Then we use the model to evaluate the college education quality and get the rankings of the college education. The main job of this paper is as follows. Firstly, we introduce the background of the university education evaluation. Secondly, we introduce the Grey theory and TOPSIS method. Thirdly, we propose an improved Grey-TOPSIS-DEA model. And we apply the method to the quality evaluation of the college education. The experiment in this paper shows the effectiveness.

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