Gravitational Stability and Renormalization-Group Flow

Kostas Skenderis
Spinoza Institute, University of Utrecht
Leuvenlaan 4, 3584 CE Utrecht
The Netherlands

and

Paul K. Townsend
DAMTP, University of Cambridge
Silver Street, Cambridge CB3 9EW, UK

Abstract

First-order ‘Bogomol’nyi’ equations are found for dilaton domain walls of D-dimensional gravity with the general dilaton potential admitting a stable anti-de Sitter vacuum. Implications for renormalization group flow in the holographically dual field theory are discussed.

*e-mail address: K.Skenderis@phys.uu.nl
†e-mail address: pkt10@damtp.cam.ac.uk
1 Introduction

The strong t’ Hooft-coupling limit of certain non-conformal supersymmetric quantum field theories associated with coincident non-conformal branes has a description in terms of supergravity theory \[1\]. This description involves gauged supergravities admitting domain-wall vacua \[2\]. The Minkowski vacuum of the gauge theory at a given scale is a ‘horosphere’ of a supergravity dilaton domain wall, i.e. a hypersurface in the ‘holographic frame’ anti-de Sitter (adS) metric on which the dilaton is constant \[2\]. The position of the horosphere and the value of the dilaton is directly related to the energy scale of the gauge theory. The domain wall solution itself therefore corresponds in the gauge theory to renormalization-group (RG) flow from one scale to another. The cases considered in \[2\], and similar lower-dimensional cases \[3\], are all ones for which the dilaton potential is a simple exponential. In such cases there is no maximally-supersymmetric adS vacuum but there is a 1/2 supersymmetric linear-dilaton vacuum which can be interpreted as a domain wall. Another type of domain wall, interpolating between adS vacua with different radii of curvature, has been extensively studied in the context of D=4 supergravity \[4, 5\], and similar solutions have recently been found for D=5 supergravity theories \[6, 7, 8, 9\]. These domain walls correspond to RG flow from one superconformal field theory to another. Other examples of RG flows of \(d = 4, \mathcal{N} = 4\) SYM theory that have a description in terms of D=5 supergravity can be found in \[10, 11, 12, 13, 14, 15\]. More recently, the RG flow associated with domain walls has been used in the context of ‘Brane World’ scenarios to explain the origin of mass hierarchies and as a possible explanation for the smallness of the cosmological constant \[16, 17, 18\].

Given these new applications of domain wall spacetimes, it would be helpful to have a model-independent analysis of the possibilities in which basic physical requirements are the only input. Since matter fields other than scalars play no role in domain wall solutions, the general framework is gravity coupled to a scalar field theory in \(D\) spacetime dimensions. The scalar fields will take values in some target space \(\mathcal{M}\) and the model is characterized by the metric on \(\mathcal{M}\), which determines the scalar kinetic terms, and a function \(V\) on \(\mathcal{M}\), which determines the scalar potential. The target space metric must be positive definite for vacuum stability. Intuition from non-gravitational field theory might
lead one to suppose that vacuum stability also requires that $V$ be positive but in gauged supergravity theories $V$ is typically unbounded from below, and the supersymmetric adS vacua are either maxima or saddle points of $V$ \cite{19}. The perturbative stability of these adS vacua is guaranteed by the fact that the eigenvalues of the scalar mass matrix satisfy the Breitenlohner-Freedman bound \cite{20} or its D-dimensional generalization \cite{21}. Non-perturbative stability has also been established in many cases by an extension of the spinorial proof of the positive energy theorem \cite{22} to asymptotically adS spacetimes \cite{23,24}. This method was used in \cite{25,26} to determine the restrictions on $V$ that arise from the requirement that there exist a stable adS vacuum, whether supersymmetric or not. The results imply the perturbative stability bounds of \cite{20,21} but go well beyond them by providing information about the potential away from its critical points.

This information is particularly useful if one supposes that there is only a single scalar field $\phi$, which we shall call the ‘dilaton’. In this case $\mathcal{M} = \mathbb{R}$ so the target space metric is diffeomorphic to a constant and $V$ becomes a function of a single real variable. The general model discussed above reduces to one with Lagrangian density

$$
\mathcal{L} = \sqrt{-\det g} \left[ \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]
$$

(1)

where $g$ is the D-dimensional spacetime metric with ‘mostly plus’ signature. The result of \cite{26} is that vacuum stability requires $V$ to take the form

$$
V = 2(D-2) \left[ (D-2)(w')^2 - (D-1)w^2 \right]
$$

(2)

where $w(\phi)$ is any function admitting at least one critical point and the prime indicates differentiation with respect to $\phi$. We shall call $w(\phi)$ the ‘superpotential’. The restriction to a single scalar field might appear severe but there are many supergravity theories of interest for which there is only one scalar or for which it is natural to consider the truncation to a single scalar. For example, in all effective supergravity theories associated to string theory there is a natural truncation in which only the dilaton survives; hence our choice of terminology. There are also some cases in which the potential depends only on the dilaton even though this is not the only scalar field. In many such cases the potential is given by the above formula with $\log w \propto \phi$ even though this superpotential has no critical point. This suggests that the formula (2) is valid under conditions less
restrictive than those used in its derivation. Notice also that a potential of the form (2) for the multi-scalar case still guarantees gravitational stability \[^{[26]}\] although the converse is not necessarily true, i.e. in the multi-scalar case there may be more general potentials than (2) compatible with gravitational stability. In particular, the potential of a subset of the scalars of the D=5 supergravity used in recent studies \[^{[8, 9, 13]}\] is of the form (2). The potential (2) has a form that is typical in supergravity theories, hence the choice of terminology ‘superpotential’ for \(w\), even though supersymmetry is not an ingredient in its derivation.

In this paper we will investigate general properties of domain wall solutions in the theory with Lagrangian (1) with \(V\) given by (2). Our interest in domain wall spacetimes stems from their connection to the RG flow of the dual field theories. Such models are characterized by their superpotential \(w\). Let us first note that

\[
V' = 4(D - 2)[(D - 2)w'' - (D - 1)w]w'
\]

so that \(V\) has critical points at critical points of \(w\), and at points for which \(w'' = \frac{D-1}{D-2}w\).

In the context of supergravity theories the critical points of \(w\) yield stable adS vacua. The other critical points of \(V\) yield non-supersymmetric (but usually adS) vacua which may or may not be stable. Recall that the positivity of the energy, and hence stability, is established subject to prescribed boundary conditions at infinity, so the fact that \(V\) as given in (2) was derived by requiring the existence of a stable adS vacuum does not imply that all of its adS vacua are stable; each such vacuum requires its own boundary conditions.

In addition to adS vacua there will usually be domain wall solutions. These solutions are possible, and may even be supersymmetric, regardless of whether \(V\) has critical points. If \(V\) does have critical points then some of these domain wall solutions will interpolate between the corresponding adS vacuum and some other solution, possibly another adS vacuum. It is convenient to distinguish between two types of domain wall, the ‘BPS’ ones and the ’non-BPS’ ones. In the supergravity context the BPS walls are supersymmetric, and they interpolate between supersymmetric vacua. Domain walls that interpolate between a supersymmetric vacuum and a non-supersymmetric one, or between two non-supersymmetric vacua, are necessarily non-BPS. Our focus will be on BPS domain walls,
but we shall first consider the general case.

2 Domain walls and the c-function

We begin by making the domain-wall ansatz

\[ ds^2 = e^{2A(r)} ds^2 \left( \mathbb{E}^{(1,D-2)} \right) + dr^2 \]  \hspace{1cm} (4)

with dilaton field \( \phi(r) \). Let us introduce a new radial coordinate \( U = e^A \). The domain-wall spacetime then takes the form

\[ ds^2 = U^2 ds^2 \left( \mathbb{E}^{(1,D-2)} \right) + \frac{1}{(\partial_r A)^2} \frac{dU^2}{U^2}. \]  \hspace{1cm} (5)

At critical points of \( V \) the dilaton is constant, as is \((\partial_r A)\), and the geometry becomes anti-de Sitter with a cosmological constant \( \Lambda \) equal to the value of \( V \) at the critical point:

\[ \Lambda = -\frac{1}{2}(D-1)(D-2)(\partial_r A)^2. \]  \hspace{1cm} (6)

In the dual field theory this corresponds to a conformal fixed point of the RG flow. The variable \( U \) is identified with the renormalization-group scale; \( U = \infty \) corresponds to long distances in the bulk, so UV in the dual field theory, and \( U = 0 \) to short distances in the bulk, so IR in the dual field theory. The RG flow of the coupling constant(s) of the field theory is encoded in the \( U \) dependence of the scalar field(s). At a fixed point the scalar field is constant, and the corresponding \( \beta \)-function vanishes.

The Einstein-dilaton equations for the metric (4) reduce to

\[ (D - 2)(D - 1)(\partial_r A)^2 - (\partial_r \phi)^2 + 2V(\phi) = 0 \]  \hspace{1cm} (6)

\[ 2(D - 2)\partial_r^2 A + (D - 1)(D - 2)(\partial_r A)^2 + (\partial_r \phi)^2 + 2V(\phi) = 0 \]  \hspace{1cm} (7)

\[ \partial_r^2 \phi + (D - 1)\partial_r A \partial_r \phi - V'(\phi) = 0 \]  \hspace{1cm} (8)

where the prime again indicates differentiation with respect to \( \phi \). Not all three equations are independent, however. For instance, one can obtain (7), upon differentiation of (4) and using (3) and (8). These equations imply

\[ \partial_r^2 A = -\frac{1}{D - 2}(\partial_r \phi)^2. \]  \hspace{1cm} (9)
In \([6, 8]\) the function
\[
C(U) = C_0 / [\partial_r A(r)]^{D-2}
\] (10)
was proposed as a c-function, where \(C_0\) is a constant related to the universal coefficient appearing in the “holographic” Weyl anomaly \([27]\) (for odd \(D\)). By definition, a c-function is a positive function of the coupling constant(s) that is non-increasing along the RG flow from the UV to the IR. We can easily establish monotonicity:
\[
U \frac{\partial}{\partial U} C = -(D - 2)C \frac{1}{(\partial_r A)^2} \partial_r^2 A = C \left( \frac{\partial_r \phi}{\partial_r A} \right)^2 \geq 0,
\] (11)
where we have used (9). Thus, as we move from the UV at \(U = \infty\) towards the IR at \(U = 0\), \(C\) is non-increasing. More generally, it was shown in \([8]\) that the function \(C(U)\) is monotonic as a consequence of a ‘weaker’ energy condition on the bulk matter.

3 BPS domain walls

The equations (7)-(8) are the Euler-Lagrange equations for the functional
\[
E[A, \phi] = \frac{1}{2} \int_{-\infty}^{\infty} dr \, e^{(D-1)A} \left[ (\partial_r \phi)^2 - (D - 1)(D - 2)(\partial_r A)^2 + 2V \right].
\] (12)
The integrand is minus the effective Lagrangian obtained by substitution of the domain wall ansatz into the Lagrangian \([1]\), and the functional \(E\) is simply related to expressions for the energy obtained by other means\(^1\).

The functional (12) can be rewritten, à la Bogomol’nyi, as
\[
E = \frac{1}{2} \int_{-\infty}^{\infty} dr \, e^{(D-1)A} \left\{ [\partial_r \phi \mp 2(D-2)w]'^2 - (D - 1)(D - 2)[\partial_r A \pm 2w]^2 \right\}
\pm 2(D - 2)[e^{(D-1)A} w]_{-\infty}^{\infty}.
\] (13)
It follows that \(E\) is extremised by solutions of the following pair of first-order equations:
\[
\partial_r A = \mp 2w(\phi),
\]
\[
\partial_r \phi = \pm 2(D - 2)w'(\phi).
\] (14)
\(^1\)For example, use of the field equations allows \(E\) to be expressed as a surface integral of the second fundamental form, which was shown in \([28]\) to be proportional to the energy.
It is straightforward to verify that solutions of these equations indeed solve the second-order equations \((6)-(8)\). We shall call solutions of these equations ‘BPS domain walls’.

Another way to arrive at the first-order equations \((14)\) is to note that the energy bound established in \([26]\) is saturated by field configurations for which the equations
\[
(D_m + w \Gamma_m)\epsilon = 0, \quad [\Gamma^m \partial_m \phi - 2(D-2)w']\epsilon = 0,
\]
(15)
admit solutions for a non-zero spinor \(\epsilon\), which we shall call a Killing spinor. Such field configurations automatically solve the second-order Einstein-dilaton equations. Substitution of the domain wall ansatz into the equations \((15)\) leads immediately to the equations \((14)\). The Killing spinor is \(\epsilon = e^{A/2}\epsilon_0\) with \(\epsilon_0\) a constant spinor satisfying \(\Gamma_r \epsilon_0 = \pm \epsilon_0\). In the context of supergravity, the domain walls admitting Killing spinors are supersymmetric.

4 Examples

We now consider two examples involving the \(N = 1, D = 7\) supergravity \([29]\) and the \(N = 2, D = 6\) \(F(4)\) supergravity \([30]\). The D=7 theory is obtained by compactification of D=11 supergravity on \(S^4\) \([31]\), and it is associated with the near-horizon limit of the \(M5\) brane with an orbifold projection on the transverse sphere \([32, 33, 34]\). The D=6 theory is obtained by a warped \(S^4\)-compactification of massive IIA supergravity \([35]\), and it is associated with the near-horizon limit of the D4-D8 system \([36, 37]\). In both cases there is a single scalar field \(\phi\), and we may discuss them simultaneously. The potential is given by \((2)\) with
\[
w(\phi) = -\frac{1}{2\sqrt{2}(D-2)} \left( g e^{\sqrt{D-2}\phi} + \frac{m}{\sqrt{D-5}} e^{-\frac{g}{\sqrt{D-2}} \phi} \right)
\]
(16)
Here \(g\) is the coupling constant of the gauge group and \(m\) is a ‘topological’ mass parameter. The potential has two critical points: at \(w' = 0\) and at \(w'' = \frac{D-1}{D-2}w\). The two critical points are
\[
\epsilon^{\sqrt{D-2}\phi} = \frac{m}{g} \sqrt{D-5}, \quad \epsilon^{\sqrt{D-2}\phi} = \frac{m}{g} \sqrt{D-5},
\]
(17)
Only the first \((w' = 0)\) critical point is supersymmetric.

Domain-wall solutions of these supergravity theories, preserving 1/2 supersymmetry,
were found in [38]. In terms of a new radial parameter ρ, they take the form

\[ ds^2 = e^{2B} ds^2(\mathbb{E}^{(1,D-2)}) + e^{2B} d\rho^2, \quad \phi = \frac{\sqrt{D-2}}{(D-4)} \log \rho, \]  

(18)

where

\[ e^{-B} = 2\rho(D-4)\sqrt{D-2} w'(\phi). \]  

(19)

There is an apparent singularity at the critical point \( w' = 0 \) but this is only a coordinate singularity, as one can verify by choosing \( U = e^{B/2} \) as a new radial variable. The metric then becomes

\[ ds^2 = U^2 ds^2(\mathbb{E}^{(1,D-2)}) + \left(\frac{D-3}{\partial_\rho e^{-B}}\right)^2 \frac{dU^2}{U^2}, \]  

(20)

which is non-singular when \( w' = 0 \). The relation

\[ \left[ w'' - \frac{D-3}{D-2} w \right]_{w'=0} = 0 \]  

(21)

is required in order for the domain wall solution to become the supersymmetric \( \text{adS} \) solution as the critical point is approached. This relation turns out to be satisfied. In the new radial coordinate the critical point is at \( U = \infty \), so it corresponds to a UV fixed point of the dual field theory.

We conclude with an example of a superpotential admitting a BPS domain wall but which is not the superpotential of any known supergravity theory (at least not for general \( D \)). A class of solutions of equations (14) is obtained by first considering these equations for complex \( \phi, w(\phi), A(r) \), and then imposing reality conditions\(^2\). As an example we consider the case the superpotential \( w(\phi) \) is equal to the Weierstrass elliptic function, \( w(\phi) = \wp(\phi; g_2, g_3) \). The dilaton \( \phi \) is then the uniformizing variable of the elliptic curve associated to the Weierstrass function. Let us recall some standard facts about the Weiersrass function, \( \wp(\phi; g_2, g_3) \). It satisfies the differential equation,

\[ \wp'^2 = 4\wp^3 - g_2 \wp - g_3. \]  

(22)

It follows that the superpotential has three critical points. The value of the superpotential at the critical points is given by the three roots, \( e_1, e_2, e_3 \), of the cubic polynomial in the

\(^2\) This method has also been recently used in [39] in order to obtain supersymmetric domain wall solutions of \( D=5 \) supergravity.
right hand side of (22). The critical points occur at $\phi = \omega_1$, $\phi = \omega_1 + \omega_2$, $\phi = \omega_2$, where $\omega_1$ and $\omega_2$ are the half periods of $\wp$.

One can easily integrate equations (14). The result is

$$r - r_0 = \pm \frac{1}{8(D - 2)} \left[ \frac{1}{(e_1 - e_2)(e_1 - e_3)} \log(\wp - e_1) + \frac{1}{(e_3 - e_1)(e_3 - e_2)} \log(\wp - e_3) \right]$$

$$A(r(\phi)) - A_0 = -\frac{1}{4(D - 2)} \left[ \frac{e_1}{(e_1 - e_2)(e_1 - e_3)} \log(\wp - e_1) + \frac{e_3}{(e_3 - e_1)(e_3 - e_2)} \log(\wp - e_3) \right]$$

where $r_0$ and $A_0$ are integration constants, which we set to zero so that the critical points occur for $r = \pm\infty$, and $U = 0$ or $U = \infty$.

When $g_2, g_3$ are real one can impose reality conditions on the solution. There are two cases to consider. When the discriminant $\Delta = g_2^2 - 27g_3^2$ is positive, one may choose primitive periods such that $\omega_1$ is real and $\omega_2$ is imaginary. In this case all three roots $e_1$ ($e_1 > e_2 > e_3$, $e_1 > 0$, $e_3 < 0$) are real. The (real) superpotential has one critical point at $\phi = \omega_1$ (the other two critical points occur for complex values of the dilaton). When $\Delta < 0$, one may choose $\omega_1, \omega_2$ to be complex conjugate of each other. The roots $e_1$ and $e_3$ are complex conjugates and $e_2$ is real. The (real) superpotential has one critical point at $\phi = \omega_1 + \omega_2$. When two of the roots coincide, or what is the same, one of the periods becomes infinite, the Weierstrass function reduces to an elementary function.

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