Signatures of CP-violation in $\gamma\gamma \rightarrow H$
using polarized beams

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Abstract

The possibility of observing CP-violation in the process $\gamma\gamma \rightarrow H$ is investigated for masses of the Higgs particle in the interval $M_Z \lesssim m_H \lesssim 2m_t$, using a 0.5TeV tunable linear $e^+e^-$ collider through laser backscattering. The use of polarized beams allows the formation of two different asymmetries sensitive to CP-violating New Physics (NP) interactions among the gauge and Higgs bosons. It is shown that very low values of the corresponding NP couplings can be probed, for a large range of the Higgs mass.

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1 Introduction

High energy $e^-e^+$ linear colliders are crucial in thoroughly investigating the Higgs sector of the Standard Model (SM) and beyond. Their significance is twofold:

Assuming that the Higgs particle really exists and its mass is below 1 TeV, it is generally believed that Higgs production at LHC or in $e^-e^+$ collisions, through $W^+W^-$ fusion or $e^-e^+ \rightarrow HZ$, will give a good signal leading to the discovery of the Higgs boson [1, 2].

Once the Higgs boson is detected and its mass is measured, one would like to test whether its properties are as predicted by SM. This means measuring its width and its interactions with the matter and gauge fields. In this respect, it is necessary to check whether New Physics (NP) beyond SM exists, which induces new Higgs interactions. The $e^-e^+$ Linear Colliders, applied either directly or in their $\gamma\gamma$ mode, provide a very useful machinery for such studies. Since in SM the $\gamma\gamma H$ coupling arises only at the one loop level and it is mediated by loops of all charged particles with non zero mass, measurement of $\gamma\gamma \rightarrow H$ can reveal the existence of possibly new interactions induced by new heavy particles that cannot be directly produced in these next generation colliders [3].

In the present work we assume that no new particles responsible for the New Physics (NP) beyond SM, will be producible in the future colliders. Moreover, we assume that the scale $\Lambda_{NP}$ of NP is sufficiently large and that the Higgs particle really exists. Under these conditions, NP may be described in terms of $\text{dimension} = 6 \ SU(3) \times SU(2) \times U(1)$ gauge invariant operators creating new CP conserving and CP violating couplings [4]. A complete list of such operators inducing purely bosonic CP conserving couplings among the Higgs and the gauge bosons can be looked at [5, 6, 7].

On the other hand, the complete list of the $\text{dim} = 6$ purely bosonic CP-violating and $SU(3) \times SU(2) \times U(1)$ gauge invariant operators, may be represented as

$$\tilde{O}_W = \frac{1}{3!} \epsilon_{ijk} W_{i\mu\nu} W_{j\nu\lambda} \tilde{W}_{k\lambda\mu} \ , \quad (1)$$

$$\tilde{O}_G = \frac{1}{3!} f_{ijk} G_{i\mu\nu} G_{j\nu\lambda} \tilde{G}_{k\lambda\mu} \ , \quad (2)$$

$$\tilde{O}_{WW} = (\Phi^\dagger \Phi) W_{\mu\nu} \cdot \tilde{W}_{\mu\nu} \ , \quad (3)$$

$$\tilde{O}_{BB} = (\Phi^\dagger \Phi) B_{\mu\nu} \cdot \tilde{B}_{\mu\nu} \ , \quad (4)$$

$$\tilde{O}_{GG} = (\Phi^\dagger \Phi) \delta_{ij} G_{i\mu\nu} \cdot \tilde{G}_{j\mu\nu} \ , \quad (5)$$

$$\tilde{O}_{BW} = \frac{1}{2} \Phi^\dagger B_{\mu\nu} \tau \cdot \tilde{W}_{\mu\nu} \Phi \ , \quad (6)$$

where e.g. $\tilde{W}_{\mu\nu} = \frac{1}{2} \epsilon_{\kappa\lambda\mu\nu} W^{\kappa\lambda}$. This list of operators differs from the one in [8, 9] in the respect that we have included the gluonic operators $\tilde{O}_G$ and $\tilde{O}_{GG}$ and omitted $\tilde{O}_{W\Phi}$ and $\tilde{O}_{B\Phi}$.

$$\tilde{O}_{W\Phi} = i (D_{\mu} \Phi)^\dagger \tau \cdot \tilde{W}_{\mu\nu} (D_{\nu} \Phi) \ , \quad (7)$$

$$\tilde{O}_{B\Phi} = i (D_{\mu} \Phi)^\dagger \tilde{B}_{\mu\nu} (D_{\nu} \Phi) \ , \quad (8)$$

\footnote{We assume here that such a $\gamma\gamma$ Collider will some day be feasible.}
since they are related to the operators in (1-6) through

\[ \tilde{\mathcal{O}}_{WW} = \frac{g^4}{4} \tilde{\mathcal{O}}_{WW} + \frac{g'^4}{2} \tilde{\mathcal{O}}_{BW} \]

\[ \tilde{\mathcal{O}}_{BB} = \frac{g'^4}{4} \tilde{\mathcal{O}}_{BB} + \frac{g^4}{2} \tilde{\mathcal{O}}_{BW} \]

imposed by the Bianchi identities \( D^\mu \tilde{\mathcal{W}}_{\mu \nu} = 0 \). In (7, 8), \( D_\nu \) is the usual gauge covariant derivative. Various dynamical scenarios for the arising of the operators (1-6), have been discussed in [10, 7]. Motivated from these we note in particular, that the gluonic operators \( \tilde{\mathcal{O}}_G \) and \( \tilde{\mathcal{O}}_{GG} \) are easily generated whenever the heavy particles inducing NP are coloured.

Concerning the list (1-6), we remark that \( \tilde{\mathcal{O}}_W \) and \( \tilde{\mathcal{O}}_{BW} \) are the only operators involving triple electroweak gauge boson couplings (TGC), while \( \tilde{\mathcal{O}}_{WW}, \tilde{\mathcal{O}}_{BB} \) and \( \tilde{\mathcal{O}}_{GG} \) contain only Higgs NP interactions and no TGC. Instead of this parameterization, another in principle equivalent one could had been given by the list in (1, 2, 5-8). We prefer the first parameterization though, because it clearly separates the triple gauge boson couplings (TGC) possibly induced by NP, from the Higgs involving interactions, which could also be generated. Because of their ability to induce TGC, the operators \( \tilde{\mathcal{O}}_W \) and \( \tilde{\mathcal{O}}_{BW} \) are already quite strongly constrained by the existing information on the neutron and electron electric dipole moments [11]. More precisely, only one combination of these two operators is in fact constrained by the electric dipole moments, but these constraints may be further improved (for both operators) in the future, by \( e^-e^+ \rightarrow WW \) studies through polarized beams in the Next Linear Collider (NLC) at 500GeV and above [12].

Our aim here is to concentrate on possible NP effects involving only the Higgs interactions with the electroweak gauge bosons. We are of course aware of the fact that naturality, together with the use of the list of operators in (1, 2, 5-8) as an NP basis, have been invoked to argue that the present very strong constraints on the CP violating TGC, would suggest that it would be very improbable to have any non-vanishing Higgs-gauge boson NP couplings also [1, 11]. Nevertheless, we feel that a direct measurement of such interactions is still very useful and important. Particularly because in the case studied here, where it is assumed that no new particles would be producible in the future; the study of the underlying nature and interactions of the obscure Higgs particle seems to be a prime candidate for supplying crucial hints on NP.

In \( e^-e^+, \gamma\gamma \) and \( e\gamma \) collisions producing gauge and/or Higgs bosons, both, the CP-conserving and the CP-violating boson interactions contribute [3, 13, 14]. In order to be able to disentangle the CP violating Higgs interactions, suitable processes and polarization effects must be looked at [3, 13, 16, 17]. Such CP violating purely electroweak NP couplings are described above by the operators \( \tilde{\mathcal{O}}_{WW} \) and \( \tilde{\mathcal{O}}_{BB} \). In principle, it should be possible to disentangle these two operators by studying suitable polarization effects and angular dependencies in \( e^-e^+ \rightarrow H\gamma, \ HZ \) [3, 10, 17], as well as in \( \gamma\gamma \rightarrow WW \) [18, 9], and in single Higgs production.

In this paper we study \( \gamma\gamma \rightarrow H \) for suitably polarized laser and \( e^\pm \) beams and construct two asymmetries which are sensitive to CP violating Higgs gauge boson interactions. These asymmetries are analogous to those used in [1] for studying \( \gamma\gamma \rightarrow WW \). It turns
out that they are both sensitive to the same combination of the $\tilde{O}_{WW}$ and $\tilde{O}_{BB}$ couplings. The high photon luminosities expected in these machines though, combined with a reasonably expected adequate understanding of the background, guarantees that a very high sensitivity to this coupling combination, should be possible. Augmenting this information with the one obtained from $e^- e^+ \rightarrow H\gamma$, $HZ$, will allow a thorough study of all possible CP violating Higgs involving interactions, induced at the level of $\text{dim} = 6$ gauge invariant operators.

2 Polarization asymmetries sensitive to CP violation in $\gamma\gamma \rightarrow H$.

Polarization effects in the process $\gamma\gamma \rightarrow H$ provide a very efficient way of disentangling the operators $\tilde{O}_{WW}$ and $\tilde{O}_{BB}$ from the rest of the CP-conserving and the CP-violating $\text{dim} = 6$ $SU(3) \times SU(2) \times U(1)$ gauge invariant interactions; (compare (3, 4)). The effective Lagrangian describing the part of NP induced by these operators is

$$\mathcal{L}_{NP} = \frac{\bar{d}}{v^2} \tilde{O}_{WW} + \frac{\bar{d} B}{v^2} \tilde{O}_{BB},$$

(11)

where $v = 2M_W/g \simeq 246 \text{GeV}$. From this we calculate the NP contribution to the $T_{\mu_1\mu_2}$ amplitude for $\gamma\gamma \rightarrow H$, where $\mu_1$ and $\mu_2$ are the helicities of the two incoming photons\footnote{The phase of $T$ is defined by its connection to the $S$-matrix as $S = 1 + iT$.}. We remark that CP-transformation implies for this amplitude that

$$T_{\mu_1\mu_2} = \pm T_{-\mu_2 -\mu_1},$$

(12)

where the upper (lower) sign is valid for the CP-conserving (CP-violating) part of the amplitude. In Appendix A we give the density matrix $\mathcal{R}$ (in the helicity basis), of the two colliding photons, produced by backscattering of two laser beams from the incoming highly energetic $e^\pm$. Using (A.19), we write

$$\sigma(\gamma\gamma \rightarrow H) = \left\{ \frac{d\mathcal{L}^\gamma(\tau)}{d\tau} \right\}_{\tau = \tau_H} \pi \frac{\pi}{s_{ee} m_H^2} \sum_{\mu_j} T_{\mu_1\mu_2} T_{*\mu_2\mu_1} \langle \rho^{BN}_{\mu_1\mu_1} \bar{\rho}^{\mu_2\mu_2}_{BN} \rangle,$$

(13)

where the $\gamma\gamma$ luminosity has been defined through (A.21, A.8, A.9), $\tau_H \equiv m_H^2 / s_{ee}$, while the normalized density matrices of the two backscattered photons are given by (A.4) as

$$\rho^{BN} = \frac{1}{2} \left( \begin{array}{cc} 1 + \xi_2(x) & -\xi_{13}(x)e^{-2i\varphi} \\ -\xi_{13}(x)e^{+2i\varphi} & 1 - \xi_2(x) \end{array} \right),$$

$$\bar{\rho}^{BN} = \frac{1}{2} \left( \begin{array}{cc} 1 + \bar{\xi}_2(x) & -\bar{\xi}_{13}(x)e^{+2i\bar{\varphi}} \\ -\bar{\xi}_{13}(x)e^{-2i\bar{\varphi}} & 1 - \bar{\xi}_2(x) \end{array} \right).$$

(14)
In the definition of the azimuthal angle $\bar{\varphi}$ for the second photon in the last equation, the sign has been changed to take care of the fact that we choose to define it not around its own momentum, but rather around the momentum of the oppositely moving photon.

The $\gamma\gamma \rightarrow H$ cross section defined in (13) is then given as

$$
\sigma(\gamma\gamma \rightarrow H) = \left\{ \frac{dL^{\gamma\gamma}(\tau)}{d\tau} \right\}_{\tau=\tau_H} \left[ (1 + \langle \xi_2 \bar{\xi}_2 \rangle) \Sigma_{unp} + \langle \xi_{13} \bar{\xi}_{13} \rangle \cos[2(\varphi - \bar{\varphi})] \Sigma_1 + \langle \xi_{13} \bar{\xi}_{13} \rangle \sin[2(\varphi - \bar{\varphi})] \Sigma_2 + \langle \xi_2 + \bar{\xi}_2 \rangle \Sigma_3 \right] ,
$$

(15)

where the averages of (the products of) the $\xi_j$ parameters, determining the density matrices of the backscattered photons, have been defined through (A.22, A.23, A.19). In (15), we have used the definitions

$$
\Sigma_{unp} = \frac{\pi}{4s_{ee}m_H^2} \left\{ |T_{++}|^2 + |T_{--}|^2 \right\} ,
$$

(16)

$$
\Sigma_1 = \frac{\pi}{2s_{ee}m_H^2} \text{Re}(T_{++} T_{--}^*) = \Sigma_{unp} ,
$$

(17)

$$
\Sigma_2 = \frac{\pi}{2s_{ee}m_H^2} \text{Im}(T_{++} T_{--}^*) ,
$$

(18)

$$
\Sigma_3 = \frac{\pi}{4s_{ee}m_H^2} \left\{ |T_{++}|^2 - |T_{--}|^2 \right\} .
$$

(19)

According to (12), only $\Sigma_2$ and $\Sigma_3$ are sensitive to the CP violating NP couplings to linear order. Thus, to 1-loop order in the SM contribution, and to linear order in the CP violating NP couplings of (11), we have

$$
\Sigma_{unp} = \frac{G_F m_H^2 \alpha^2}{16\sqrt{2}\pi s_{ee}} \left| \frac{4}{3} F_t + F_W \right|^2 ,
$$

(20)

$$
\Sigma_1 = \frac{G_F m_H^2 \alpha^2}{16\sqrt{2}\pi s_{ee}} \left| \frac{4}{3} F_t + F_W \right|^2 ,
$$

(21)

$$
\Sigma_2 = -\frac{G_F m_H^2 \alpha}{\sqrt{2}s_{ee}} \text{Re} \left( \frac{4}{3} F_t + F_W \right) (\bar{d}_{sW}^2 + \bar{d}_{BcW}^2) ,
$$

(22)

$$
\Sigma_3 = -\frac{G_F m_H^2 \alpha}{\sqrt{2}s_{ee}} \text{Im} \left( \frac{4}{3} F_t + F_W \right) (\bar{d}_{sW}^2 + \bar{d}_{BcW}^2) ,
$$

(23)

where $F_t$ and $F_W$, which denote the top and $W$ loop SM contributions, can be found in [19, 3, 13].

The expected annual number of events for double laser backscattering in $e^-e^+$ colliders, is obtained by multiplying the cross-section in (13) with the annual luminosity $L_{ee} \simeq 20 fb^{-1}\text{year}^{-1}$ for a 0.5$TeV$ collider. Therefore

$$
N_{\tau_H} = L_{ee} \sigma(\gamma\gamma \rightarrow H) .
$$

(24)
We next construct the two possible CP-odd asymmetries. The first may be observable whenever there are non-vanishing average linear polarization parameters $\xi_{13}$ and $\bar{\xi}_{13}$ for the two colliding photons. It is obtained by performing measurements for two different values of the angle $\chi \equiv \varphi - \bar{\varphi}$ between the linear polarizations of these photons. It is given by

$$\tilde{A}_{\text{lin}} = \frac{|N_{\tau H}(\chi = \frac{\pi}{4}) - N_{\tau H}(\chi = -\frac{\pi}{4})|}{N_{\tau H}(\chi = \frac{\pi}{4}) + N_{\tau H}(\chi = -\frac{\pi}{4})} = \frac{\langle \xi_{13}\bar{\xi}_{13} \rangle}{1 + \langle \xi_{2}\bar{\xi}_{2} \rangle} A_{\text{lin}}, \quad (25)$$

with

$$A_{\text{lin}} = \frac{\Sigma_{2}}{\Sigma_{\text{unp}}}. \quad (26)$$

As seen from (26), $A_{\text{lin}}$ is determined through (20, 22) solely by the CP-violating NP induced quantity $\Sigma_{2}$.

The construction of the second CP violating asymmetry is possible whenever there is a non vanishing average for the sum of the circular polarizations $\langle \xi_{2} + \bar{\xi}_{2} \rangle$ of the two back scattered photons. This requires large average helicities for both, the laser and the $e^\pm$ beams, (see the Appendix). Using (15) and remarking that the sign of $\langle \xi_{2} + \bar{\xi}_{2} \rangle$ changes whenever the signs of $(P_e, P_\gamma)$ and $(\bar{P}_e, \bar{P}_\gamma)$ are simultaneously changed, we construct the asymmetry $\tilde{A}_{\text{circ}}$ by making measurements with two opposite values of these polarization pairs. We thus get

$$\tilde{A}_{\text{circ}} = \frac{|N_{++} - N_{--}|}{N_{++} + N_{--}} = \frac{|\langle \xi_{2} + \bar{\xi}_{2} \rangle|}{1 + \langle \xi_{2}\bar{\xi}_{2} \rangle} A_{\text{circ}}, \quad (27)$$

with

$$A_{\text{circ}} = \frac{\Sigma_{3}}{\Sigma_{\text{unp}}}. \quad (28)$$

Each one of the asymmetries $\tilde{A}_{\text{lin}}$ and $\tilde{A}_{\text{circ}}$ is a product of two factors: The first one originates from the degree of polarization of the two photons building the collider, while the second one depends on the product of an SM 1-loop contribution and another contribution sensitive to the CP violating NP interactions. This second factor is denoted respectively by $A_{\text{lin}}$ and $A_{\text{circ}}$, and satisfies $A_{\text{lin}}(SM) = A_{\text{circ}}(SM) = 0$ in the SM case. To study observability limits for the NP couplings from the use of these asymmetries, we need the expressions for the expected 1-standard deviation statistical uncertainties for each of them in the SM case. These are

$$\delta A_{\text{lin}}(SM) = \frac{|1 + \langle \xi_{2}\bar{\xi}_{2} \rangle|}{\sqrt{(N_{\tau H}(\chi = \frac{\pi}{4}) + N_{\tau H}(\chi = -\frac{\pi}{4})\langle \xi_{13}\bar{\xi}_{13} \rangle)}}, \quad (29)$$

$$\delta A_{\text{circ}}(SM) = \frac{1 + \langle \xi_{2}\bar{\xi}_{2} \rangle}{\sqrt{(N_{++} + N_{--})\langle \xi_{2} + \bar{\xi}_{2} \rangle}}. \quad (30)$$

We also note from (22, 23), that both asymmetries measure the same combination $d_{\gamma W}^2 + d_{B\gamma W}^2$ of the NP couplings called $d_{\gamma \gamma}$ in (17).
3 Testing CP violation in $\gamma\gamma \rightarrow H$ through an $e^-e^+$ collider.

As explained in the Appendix, the polarized photons needed to study the CP violating contributions to $\gamma\gamma \rightarrow H$, are obtained by double laser backscattering from the $e^-, e^+$ beams [20, 14]. The outgoing photons are produced almost in the same direction with $e^-, e^+$. All relevant formulae for the description of the Compton scattering kinematics in this framework, are collected in the Appendix. Here we only note that these are characterized by two dimensionless parameters, $x_0 = 4E\omega_0/m_e^2$ and $x = \omega/E$, where $E$ is the energy of the electron beam, $\omega_0$ is the laser energy and $x$ is the fraction of the $e^-$ beam energy carried away by the final photon. The maximum value of $x$ is determined by $x_0$, through the relation $x_{max} = x_0/(1+x_0)$. Operation of the collider below the $e^-e^+$ pair production threshold sets an upper limit to the value of $x_0$, so that $x_0 \leq 2(1 + \sqrt{2})$. Thus the final photons can take, as much as $\sim 83\%$ of the electron beam energy; compare (A.5).

A lower limit in the value of $x_0$ is also set for each specific process under consideration, by the masses of the particles produced in the process studied [15]. Therefore, the allowed range for $x_0$, for a given $e^-e^+$ center of mass energy is

$$\frac{\sum_{i=1}^{n} m_i}{\sqrt{s} - \sum_{i=1}^{n} m_i} \leq x_0 \leq 2(1 + \sqrt{2}) \ , \quad (31)$$

where the sum includes all produced masses in a $\gamma\gamma$ collision.

There are various general options for operating the photon-photon collider [20, 14]:

- The conversion point (C.P.) where the Compton backscattering occurs, may be a few cm away from the interaction point (I.P.) where the $\gamma\gamma$ collisions take place. Since the most energetic photons are those with the smallest scattering angle, it is only those that finally manage to reach the I.P. Therefore, the further away from the I.P., the conversion occurs, the more monochromatic the photon beam becomes, (but with some loss in the integrated luminosity of course). This particular set up can be very advantageous for processes where production of some resonance occurs, whose mass is a priori known. In this case, the collider may be tuned, so as to operate in a narrow window around the relevant specific value of the invariant $\gamma\gamma$ mass [21, 22]. This way, there is also the possibility of reducing the background in cases where the cross-section of the main process dominates.

- Choosing a configuration where I.P. and C.P. coincide, makes the photon spectrum rather flat in the unpolarized and the $P_eP_\gamma > 0$ cases, which may be useful for searching particles with unknown masses. It may help also in the simultaneous study of more than one processes, dominating at different regions of the invariant $\gamma\gamma$ energies.
We turn now to the specific properties of the process $\gamma\gamma \rightarrow H$ in the Next Linear Collider. Assuming that the mass of the Higgs boson has been measured before, in the $e^+e^-$ mode or perhaps at LHC, it may be possible to tune the parameters of the collider so that it operates, (to some extent), in a narrow window around the Higgs mass. The use of polarized beams in searching for any CP violating new interactions among the gauge and Higgs bosons is investigated in this section [3, 16, 23, 22]. The measurements of the two quantities $A_{\text{lin}}$ and $A_{\text{circ}}$, which mostly require different polarization conditions, are considered separately.

A. Measurement of $A_{\text{lin}}$

As seen from (25), the measurement of $A_{\text{lin}}$ requires laser beams with some linear polarization $P_t$. Then, the resulting photons building the photon-photon collider are also polarized in the same direction. The degree of linear polarization transferred to them, $\xi_{13}$, depends on the parameter $x_0$ (determining the maximum collider energy) and on $P_t$.

This Stokes parameter $\xi_{13}$ is given in (A.11), as a function of its fractional energy $x$ [20]. As explained in the Appendix and shown in Fig.3b, the degree of linear polarization $\xi_{13}$ is very small for low energy photons, while it tends to its maximum value $\xi_{13,\text{max}} = 2(1 + x_0)/[1 + (1 + x_0)^2]P_t$, when $x \rightarrow x_{\text{max}}$. It is obvious from this, that in order to increase the linear polarization transferred from the laser beams to high energy photons, it is best to have a machine design such that $x_0$ is as small as possible. However, when $x_0$ decreases, the highest Higgs mass producible through $\gamma\gamma \rightarrow H$ in a given collider, also decreases. The best choice should therefore be decided by tuning the collider after the Higgs particle discovery and the measurement of its mass.

Before discussing the actual measurement of $A_{\text{lin}}$, we should comment a bit about the background for detection of $\gamma\gamma \rightarrow H \rightarrow X$. In the case that $m_H \lesssim 150\text{GeV}$, in which $H \rightarrow b\bar{b}$ dominates, the main background process is $\gamma\gamma \rightarrow b\bar{b}$. This background however, is strongly peaked in the forward-backward direction, while the Higgs decay is isotropic in the Higgs frame. It may also be interesting to utilize laser beams which are partly circularly and partly linearly polarized, like e.g. $P_\gamma = P_t = 1/\sqrt{2}$; compare (A.1, A.2). Because then the backscattered photons acquire circular as well as linear polarizations, which may be useful in further reducing the $\gamma\gamma \rightarrow b\bar{b}$ background; since its dependence of on the Stokes parameters is very different from the Higgs mediated contribution given in (A.9).

More specifically, the background is strongly suppressed for $\langle \xi_2 \xi_2 \rangle = +1$. Concerning the magnitude of this background, a detailed discussion can be found in [24] where there is also a plot of the minimum and maximum masses for which the number of predicted Higgs signal events (S) and background events (B) are such that $S \gtrsim 10$ and $S/\sqrt{B} \gtrsim 5$. We note that these conditions are satisfied for Higgs masses in the range $100 - 150\text{GeV}$, for all possible degrees of polarization [24].

On the other hand, for the Higgs mass range $200 \lesssim m_H \lesssim 350\text{GeV}$ for which the relevant Higgs decay mode is $H \rightarrow ZZ \rightarrow l^-l^+X$, we note that the background should be very small, since there is no SM tree level contribution to the $\gamma\gamma \rightarrow ZZ$ continuum. Combining this with a $ZZ$ detection efficiency of $\sim 18\%$, it is concluded in [21] that the Higgs study in the above mass range is determined by the absolute event rate only.
Using therefore (partially) linear polarizations we should be able to measure the \( \tilde{A}_{lin} \) asymmetry defined in (25), which in turn determines \( A_{lin} \) and the combination \( \tilde{d}_s^2 + \tilde{d}_B^2 \) of the NP couplings; (compare (26)). To get a feeling on the possible limits that can be established, we plot in Fig.1 the ratio

\[
\text{NSD for } A_{lin} \equiv \frac{A_{lin}}{\delta A_{lin}(SM)},
\]

where (26, 29) should be used. This ratio describes number times the measurable factor \( A_{lin} \) exceeds its expected statistical fluctuation in SM, for various choices of the NP couplings [18, 15, 9]. Fig.1 presents NSD for \( A_{lin} \) for a \( \sqrt{s_{ee}} = 0.5 \text{TeV} \) collider, using the small value of \( x_0 = 0.5 \), as motivated above.

The sensitivity to the NP coupling combination \( \tilde{d}_s^2 + \tilde{d}_B^2 \), which can be reached by studying \( A_{lin} \), depends on the Higgs mass. It can be obtained from Tables I and II, where the NP couplings inducing a 3\( \sigma \) effect are tabulated, using an integrated annual luminosity \( L_{ee} \approx 20fb^{-1}\text{year}^{-1} \) for a 0.5TeV Collider and various values for \( x_0 \) [21]. As can be seen from these tables, the sensitivity to the above NP couplings generally increases as \( x_0 \) decreases.

The results in Table I apply for \( 100 \lesssim m_H \lesssim 150\text{GeV} \) and were derived on the basis of the \( H \to b\bar{b} \) mode for which a 25\% detection efficiency is assumed, [21, 1, 2]. In fact, this is also what determines the overall number of the \((\varphi - \bar{\varphi})\) averaged events in the last column of the Table I. Thus, if \( 100 \lesssim m_H \lesssim 150\text{GeV} \), then limits on the NP coupling \( \tilde{d}_{\gamma\gamma} \equiv \tilde{d}_s^2 + \tilde{d}_B^2 \) are at the level \( 10^{-3} - 10^{-4} \) seem possible. In Table I we give results for \( P_t = 1 \) as well as for \( P_t = 1/\sqrt{2} \) (in parentheses), in order to give a feeling of the possible implications from using laser beams with a partial circular polarization.

On the other hand, the results in Table II apply for \( 200 \lesssim m_H \lesssim 350\text{GeV} \) and were derived on the basis of the \( H \to ZZ \to l^-l^+X \). In both Tables I and II, it is always checked that the number of expected events for the chosen decay channels is of the order of a hundred, for all Higgs masses considered and a 0.5TeV collider.

For the high \( m_H \) part in Table II, we should also remark that as the Higgs mass increases, the sensitivity to the NP CP-violating couplings is reduced. The reason is that the lower limit for the \( x_0 \) parameter also increases and consequently the degree of linear polarization transfer decreases; (compare (31)). On top of this, there is a further reduction of sensitivity, since \( A_{lin} \), being proportional to the real part of SM contribution, decreases as \( m_H \) increases. However, as seen from Table II for Higgs masses in the region 200-250GeV, sensitivity limits on the NP couplings like \( \tilde{d}_s^2 + \tilde{d}_B^2 \) \( \sim 10^{-3} - 10^{-2} \) can be obtained at the 2\( \sigma \) or 3\( \sigma \) level, from measurements of \( A_{lin} \).

B. Measurement of \( A_{circ} \)

Eqs. (27, A.10) indicate that the measurement of \( A_{circ} \) require the existence of circularly polarized photons, which may be obtained by backscattering similarly polarized laser beams from polarized \( e^\pm \). Both, the energy spectrum of the resulting photons and

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3Remember the preceding discussion concerning the \( b\bar{b} \) background.
the degree of the polarization transferred, depend on the way we choose the initial average helicities, as well as the conversion and interaction points; (see Appendix and [20, 18, 14]). If C.P. and I.P. coincide, this spectrum is rather flat for $P_e P_\gamma > 0$, and peaked towards the higher energies for $P_e P_\gamma < 0$; (compare Fig.3c).

As for the $A_{\text{lin}}$ case, the statistical significance of a possibly non vanishing value induced by NP to $A_{\text{circ}}$, is again given the number of times this asymmetry exceeds the expected statistical uncertainty in SM; i.e.

$$\text{NSD for } A_{\text{circ}} = \frac{A_{\text{circ}}}{\delta A_{\text{circ}}(SM)},$$

where (28, 30) should be used. Note that $A_{\text{circ}}(SM) = 0$.

The numerical study of (27) indicates that the measurement of $A_{\text{circ}}$ depends very sensitively on the average polarizations along their momenta of the $e^\pm$ beams $P_e, \bar{P}_e$, and of the laser photons $P_\gamma, \bar{P}_\gamma$. On the other hand, since the Higgs particle has no spin, in order to enhance its production we must choose the same average helicities in both arms of the collider; i.e. $P_e = \bar{P}_e$ and $P_\gamma = \bar{P}_\gamma$. If a configuration is selected in which the conversion and interaction points coincide, then we could choose $P_e P_\gamma = +1$, where the backscattered photons acquire for most of the energy range, a mean helicity of the same sign as the initial ones, and only very near to the maximum energy this helicity changes sign; (see Fig.3c in the Appendix). In this way the best polarization transfer is achieved (nearly 100%) for almost the entire range of the invariant $\gamma\gamma$ masses, where the luminosity is also significant.

If, however, a distance is put between C.P. and I.P., then choosing the polarizations such that $P_e P_\gamma = \bar{P}_e \bar{P}_\gamma = -1$ enhances the production of photons with the highest energies, leading thus to a gain in luminosity. Assuming that the Higgs mass is known and tuning the collider so that the most energetic photons are the ones contributing to Higgs production, allows a most efficient use of their high circular polarization which facilitates the measurement of $A_{\text{circ}}$. This is what is done in Fig.2a below.

It is also important observe from (28, 23), that $A_{\text{circ}}$ is proportional to the imaginary part of the SM contribution to the $\gamma\gamma \rightarrow H$ amplitude, which is very small for $m_H \lesssim 2M_W$. This imaginary part starts becoming appreciable only above the $WW$-threshold. Therefore, a measurement of $A_{\text{circ}}$ can be useful only for $m_H \gtrsim 2M_Z$, where the most useful Higgs decay mode is $H \rightarrow ZZ \rightarrow l^+l^-X$ decay. Thus in Fig.2, we plot NSD for $A_{\text{circ}}$ for $2M_z \lesssim m_H \lesssim 350GeV$ and various values of the NP coupling combination $ds^2_W + d_B c^2_W$.

Fig.2a corresponds to $P_e P_\gamma = -1$, while Fig.2b corresponds to $P_e P_\gamma = +1$. The other parameters in these figures are chosen so that they facilitate the measurement of the $A_{\text{circ}}$ asymmetry. Thus for the case of Fig.2a, where the spectrum of the backscattered photons is peaked towards the high energy side, we use the highest value of $x_0$ possible, namely $x_0 = 4.82$, as well as collider whose energy is tuned to the Higgs mass like $\sqrt{s_{ee}} = m_H/0.75$. On the contrary for Fig.2b, corresponding to a rather flat spectrum of backscattered photons, we do not tune $s_{ee}$ to the Higgs mass, but we just fix $\sqrt{s_{ee}} = 0.5TeV$ and $x_0 = 4$ [21]. The plots in Fig.2ab assume that C.P. and I.P. coincide.
In Tables III and IV we also give the $3\sigma$ sensitivity limits to the above couplings from $A_{circ}$, as a function of $m_H$, using the above polarization choices. In these tables we fix the energy to $\sqrt{s_{ee}} = 0.5\,\text{TeV}$, and vary $x_0$. It can be concluded from these Tables that sensitivity limits like $(\bar{d}s^2_W + \bar{d}Bc^2_W) \sim 10^{-4}$, should be possible in the whole range $2M_z \lesssim m_H \lesssim 350\,\text{GeV}$.

## 4 Final discussion

We have shown in the present work that if the Higgs particle predicted by SM really exists, then the study of the process $\gamma\gamma \to H$ using polarized beams in Next Linear Colliders, provide a very sensitive test for the existence of any CP-violating interactions among the gauge and Higgs bosons. Various polarization configurations for the $e^\pm$ beams and the laser photons give complementary information and provide consistency checks for the study of such couplings. In particular, if the Higgs mass is in the range of (100-150)GeV, then linearly polarized laser beams may be used in a $\sim 0.5\,\text{TeV} e^-e^+$ collider, in order to measure the asymmetry $\tilde{A}_{lin}$ defined above, which is sensitive to CP-violating NP induced interactions. This way sensitivity limits on the CP violating NP coupling $\bar{d}s^2_W + \bar{d}Bc^2_W$ may be obtained at the level of $10^{-3} - 10^{-4}$; (compare Table I).

If the Higgs particle is in the range $2M_Z \lesssim m_H \sim 200\,\text{GeV}$, then the most efficient way to look for CP violating NP interactions, is through measurement of the $\tilde{A}_{circ}$ asymmetry, using circularly polarized beams. In this case, non vanishing average helicities for the electron as well as the laser beams are necessary, in order to have good sensitivity to the anomalous couplings. We have also seen that in case $P_e P_\gamma > 0$, it may be useful to try a tunable Linear Collider. Thus, also for these higher masses, the sensitivity limits on the NP coupling $\bar{d}s^2_W + \bar{d}Bc^2_W$ are again at the $10^{-4}$ level; (see Table III, IV).

In both cases, it should be possible to disentangle the CP violating forces, from the CP conserving ones affecting the same processes.

From the theoretical point of view, a nonzero value for any of these anomalous couplings provides, through unitarity, a hint on the related NP scale $\Lambda_{NP}$. Therefore, it is interesting to translate the above sensitivity limits for the couplings, to corresponding lower bounds on NP scales. Assuming that only one of the operators $\tilde{O}_{WW}$ or $\tilde{O}_{BB}$ acts at a time, the unitarity requirement gives the relations

\begin{equation}
|\bar{d}| \simeq \frac{104.5(M_W/\Lambda_{NP})^2}{1 + 3(M_W/\Lambda_{NP})}, \quad (34)
\end{equation}
\begin{equation}
|\bar{d}_B| \simeq \frac{195.8(M_W/\Lambda_{NP})^2}{1 + 100(M_W/\Lambda_{NP})^2}. \quad (35)
\end{equation}

Using then a non vanishing value for the corresponding NP coupling at the level of the aforementioned sensitivity limits, we calculate from the unitarity relations the scale $\Lambda_{NP}$ where unitarity is first reached. In general $\Lambda_{NP}$ depends on the operator considered. Its value provides a rough estimate of the energy scale where either new strong interactions will develop, or new particles will be produced. From (34) and the above sensitivity limits,
we would conclude that the study of $\gamma\gamma \to H$ at a $0.5 TeV$ collider can probe NP scales in the range of 10-20TeV, in case NP is generated by $\mathcal{O}_{WW}$. Because of (35), this scale increases to 30-50TeV, if we assume that the NP forces are due to the operator $\mathcal{O}_{BB}$.

In summary, using polarized beams for realizing the $\gamma\gamma$ colliders, it is possible to construct observables sensitive only to the CP-violating NP couplings, and thereby distinguish them from the CP conserving ones. This is not attainable if unpolarized beams are only used. The overall conclusion for the $dim = 6$ gauge invariant NP interactions considered above, is that single Higgs production in $\gamma\gamma$ collisions at $0.5 TeV$ tunable linear Collider, can be used to put limits on the NP coupling $\bar{d}s_W^2 + \bar{d}_B c_W^2$ at the $10^{-3} - 10^{-4}$ level, for Higgs masses in the ranges $100 \lesssim m_H \lesssim 150 GeV$ and $200 \lesssim m_H \lesssim 350 GeV$. This information is complementary, and at least an order of magnitude more precise than the one attainable through production of $WW$ pairs in $\gamma\gamma$ collisions, where the same combination of NP couplings is measured [9, 25, 13]. Information on independent combinations of the CP violating couplings at the level of $10^{-2}$ may be obtained by looking at $e^-e^+ \to H\gamma, HZ$ [17, 8, 16]. Thus, a combination of such measurements should be able to constrain separately each of the two CP violating couplings $\bar{d}$ and $\bar{d}_B$ at the $10^{-2}$ level.
Appendix A: Density matrix of backscattered photon.

Following [20], we collect in this appendix the formulae describing the helicity density matrix of the photon produced by backscattering a laser photon from an incoming highly energetic $e^\pm$ beam.

We denote by $E$ the energy of the incoming $e^\pm$ beam, while $P_e = 2\lambda_e$ describes its polarization along its momentum, and $\lambda_e$ is its average helicity. The $e^\pm$ beam is assumed to collide with a laser photon moving along the opposite direction with energy $\omega_0$ and characterized, in the basis of its helicity states, by the normalized density matrix [26]

$$\rho^N_{\text{laser}} = \frac{1}{2} \left( \begin{array}{cc} 1 + P_\gamma & -P_t e^{-2i\phi} \\ -P_t e^{2i\phi} & 1 - P_\gamma \end{array} \right). \quad (A.1)$$

Here $P_\gamma$ describes the average helicity of the laser photon and $P_t$ denotes its maximum average linear polarization occurring of course along a direction perpendicular to the photon momentum, described by the azimuthal angle $\phi$ (with respect to this momentum).

By definition

$$0 \leq P^2_\gamma + P^2_t \leq 1. \quad (A.2)$$

After the Compton scattering of $e^\pm$ from the laser photon, the electron beam looses most of its energy and a beam of "backscattered photons" is produced, moving essentially along the direction of the original $e^\pm$ momentum and characterized, in its helicity basis, by the density matrix

$$\rho^B = \frac{dN}{dx} \rho^{BN}, \quad (A.3)$$

$$\rho^{BN} = \frac{1}{2} \left( \begin{array}{cc} 1 + \xi_2(x) & -\xi_{13}(x) e^{-2i\phi} \\ -\xi_{13}(x) e^{2i\phi} & 1 - \xi_2(x) \end{array} \right), \quad (A.4)$$

where $x \equiv \omega/E$ and $x_0 \equiv 4E\omega_0/m_e^2$; with $\omega$ being the energy of the back-scattered photon, and $\omega_0, E$ have been defined above. These satisfy the kinematical constraints

$$0 \leq x \leq x_{\text{max}} \equiv \frac{x_0}{1 + x_0}, \quad 0 \leq x_0 \leq 2(1 + \sqrt{2}), \quad (A.5)$$

which implies that the backscattered photon can take up to 83% of the $e^\pm$ energy. In analogy to (A.2), the elements of $\rho^{BN}$ also satisfy

$$0 \leq \xi^2_2(x) + \xi^2_{13}(x) \leq 1. \quad (A.6)$$

In (A.3), $dN/dx$ is the number of backscattered photons per unit of $x$, normalized to a unit of flux for the incoming $e^\pm$ beam; while $\rho^{BN}$ is the normalized photon density matrix, $\langle Tr \rho^{BN} \rangle = 1$. We note from (A.3), (A.4), that the azimuthal angles of the maximum average linear polarizations of the backscattered and the laser photons, defined around their respective momenta, are the same. The functions appearing in (A.3), (A.4), which
determine the spectrum of the backscattered photon immediately after its production at the conversion point, are given by [20]

\[
\frac{dN(x)}{dx} = \frac{C(x)}{D(x_0)}, \quad (A.7)
\]

\[
C(x) = f_0(x) + P_e P_\gamma f_1(x), \quad (A.8)
\]

\[
D(x_0) = D_0(x_0) + P_e P_\gamma D_1(x_0), \quad (A.9)
\]

\[
\xi_2(x) = \frac{P_e f_2(x) + P_\gamma f_3(x)}{C(x)}, \quad (A.10)
\]

\[
\xi_{13}(x) = \frac{2r^2 P_t}{C(x)}, \quad (A.11)
\]

where

\[
f_0(x) = \frac{1}{1-x} + 1 - x - 4r(1-r), \quad (A.12)
\]

\[
f_1(x) = \frac{x}{1-x} (1-2r)(2-x), \quad (A.13)
\]

\[
f_2(x) = x_0 r [1 + (1-x)(1-2r)^2], \quad (A.14)
\]

\[
f_3(x) = (1-2r) \left( \frac{1}{1-x} + 1 - x \right), \quad (A.15)
\]

\[
r(x) = \frac{x}{x_0(1-x)}, \quad (A.16)
\]

and

\[
D_0(x_0) = \int_0^{x_{\text{max}}} dx f_0(x) = \left[ 1 - \frac{4}{x_0} - \frac{8}{x_0^2} \right] \ln(1 + x_0) + \frac{1}{2} + \frac{8}{x_0} - \frac{1}{2(1 + x_0)^2}, \quad (A.17)
\]

\[
D_1(x_0) = \int_0^{x_{\text{max}}} dx f_1(x) = \left[ 1 + \frac{2}{x_0} \right] \ln(1 + x_0) - \frac{5}{2} + \frac{1}{1 + x_0} - \frac{1}{2(1 + x_0)^2}, \quad (A.18)
\]

where \(x_{\text{max}}\) is defined in \((A.5)\).

The elements of density matrix of the backscattered photon, for various choices of the \(e^\pm\) polarization \(P_e\), and the laser parameters \(x_0\) and \((P_\gamma, P_t)\), are presented in Fig.3abc. Thus, in Fig.3a, we give the backscattered photon flux \(dN/dx\) as a function of \(x\). As seen from \((A.7)-(A.9)\), \(dN/dx\) depends only on the product \(P_e P_\gamma\) and the parameter \(x_0\) determining the highest value of \(x\) through \((A.5)\). In Fig.3b, the average linear polarization \(\xi_{13}\) of the backscattered photon is shown. As seen from comparing Fig.3ab, the demand of a high linear polarization would favour a small value of \(x_0\), which has the drawback that the highest energy of the back scattered photon decreases. Finally, in Fig.3c, we present the average circular polarization \(\xi_2\) of the back scattered photon as a function of \(x\), for various choices of \(P_e, P_\gamma\) and \(x_0\). As seen in Fig.3c, the \(x\)-dependence of \(\xi_2\) is quite sensitive to the relative sign of \(P_e, P_\gamma\).

If an \(e^-e^+\) Collider is transformed to \(\gamma\gamma\) one by using two identical lasers, then the (unnormalized) density matrix \(\mathcal{R}_{\mu_1\mu_2/\mu'_1\mu'_2}\) of the \(\gamma\gamma\)-pair in their helicity basis, is related
to the $\rho^B$, $\bar{\rho}^B$ matrices by (compare (A.3))

$$\frac{d}{d\tau} R_{\mu_1\mu_2;\mu'_1\mu'_2}(\tau) = \rho^B_{\mu_1\mu_1'} \bar{\rho}^B_{\mu_2\mu_2'} \equiv \int_{x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} \rho^B_{\mu_1\mu_1'}(x) \bar{\rho}^B_{\mu_2\mu_2'} \left( \frac{\tau}{x} \right),$$

$$\equiv \frac{dL^{\gamma\gamma}(\tau)}{d\tau} \langle \rho^B_{\mu_1\mu_1'} \bar{\rho}^B_{\mu_2\mu_2'} \rangle, \quad (A.19)$$

where

$$\tau \equiv \frac{s_{\gamma\gamma}}{s_{ee}}, \quad (A.20)$$

with $s_{ee}$ and $s_{\gamma\gamma}$ being the squares of the c.m. energies of the $e^-e^+$ and $\gamma\gamma$ systems respectively. In the r.h.s. of (A.19), $dL^{\gamma\gamma}/d\tau$ is the overall $\gamma\gamma$ luminosity per unit $e^-e^+$ flux, defined by the convolution of the separate $\gamma$ luminosities appearing in (A.7-A.9). If the conversion points where each of the two photons are produced through laser backscattering, coincide with their interaction point, then

$$\frac{dL^{\gamma\gamma}}{d\tau} = \frac{1}{D^2(x_0)} \int_{x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} C(x) C \left( \frac{\tau}{x} \right). \quad (A.21)$$

We note from (A.8, A.9), that the overall flux of the backscattered photons depends on $P_e$ as well as on the circular polarization of the laser photons used to produce them. The definition of $dL^{\gamma\gamma}/d\tau$ specifies also the definition of the ”averages” $\langle \rho^B_{\mu_1\mu_1'} \bar{\rho}^B_{\mu_2\mu_2'} \rangle$ appearing in the r.h.s. of (A.19) for the two photons. These averages may also be used to define $\langle \xi_i \bar{\xi}_j \rangle$, $\langle \xi_{13} \bar{\xi}_{13} \rangle$, $\langle \xi_2 + \bar{\xi}_2 \rangle$, using the form of (A.4). Thus,

$$\langle \xi_i \bar{\xi}_j \rangle = \frac{(C \xi_i \otimes C \xi_j)}{C \otimes C}, \quad (A.22)$$

$$\langle \xi_2 + \bar{\xi}_2 \rangle = \frac{(C(\xi_2 + \bar{\xi}_2) \otimes C)}{C \otimes C}. \quad (A.23)$$

The above assumption that the conversion and interaction points coincide, may most probably not be imposed when the $\gamma\gamma$ collision experiments will be designed. Even in such a case though, the main modification we would need to make in the preceding formulae is to appropriately increase the lower limit in the integrals in (A.19, A.21). Thus, increasing the distance between the conversion and interaction points, will tend to select only the highest energy part of the spectrum for each of the beams of the two colliding photons.
TABLE I. 3σ upper bounds on CP-violating NP couplings from $A_{lin}$ asymmetry, using $H \rightarrow b\bar{b}$ for the two polarization choices:

$P_t = \bar{P}_t = 1, \ P_\gamma = \bar{P}_\gamma = 0, \ (P_t = \bar{P}_t = P_\gamma = \bar{P}_\gamma = 1/\sqrt{2}, \ P_e = \bar{P}_e = 1)$.

| $m_H$ (GeV) | $x_0$ | upper limit on $d_{BCW}^2 + ds_W^2$ | $(\varphi - \bar{\varphi})$-averaged Events |
|------------|------|-----------------------------------|------------------------------------------|
| 100        | 0.5  | $10^{-3}$ ($2 \times 10^{-3}$)    | 197 (168)                                |
|            | 0.8  | $2 \times 10^{-2}$ ($1.6 \times 10^{-2}$) | 184 (233)                                |
|            | 1    | 0.12 (0.05)                         | 160 (255)                                |
| 120        | 0.5  | $5 \times 10^{-4}$ ($8.5 \times 10^{-4}$) | 200 (160)                                |
|            | 0.8  | $8 \times 10^{-3}$ ($6 \times 10^{-3}$) | 207 (223)                                |
|            | 1    | 0.05 (0.017)                        | 192 (264)                                |
|            | 1.5  | 0.16 (0.11)                         | 175 (306)                                |
| 140        | 0.5  | $4 \times 10^{-4}$ ($6.5 \times 10^{-4}$) | 144 (111)                                |
|            | 0.8  | $5 \times 10^{-3}$ ($3.2 \times 10^{-3}$) | 159 (143)                                |
|            | 1    | 0.03 (0.01)                         | 156 (181)                                |
|            | 1.5  | 0.09 (0.06)                         | 147 (237)                                |
|            | 2    | 0.22 (0.22)                         | 138 (252)                                |
| 150        | 0.5  | $5 \times 10^{-4}$ ($8 \times 10^{-4}$) | 85 (64)                                  |
|            | 0.8  | $4.3 \times 10^{-3}$ ($3 \times 10^{-3}$) | 106 (87)                                 |
|            | 1    | 0.023 (0.008)                       | 107 (113)                                |
|            | 1.5  | 0.08 (0.05)                         | 102 (157)                                |
|            | 2    | 0.2 (0.2)                           | 97 (174)                                 |
| $m_H$ (GeV) | $x_0$ | upper limit on $d_R c_W^2 + d_R^2$ | $(\varphi - \bar{\varphi})$-averaged Events |
|------------|-------|------------------|---------------------------------|
| 200        | 0.8   | $10^{-3}$         | 76                              |
|            | 1     | $1.5 \times 10^{-3}$ | 88                              |
|            | 1.5   | $7 \times 10^{-3}$  | 97                              |
|            | 2     | 0.022             | 102                             |
|            | 2.5   | 0.06              | 102                             |
| 230        | 1     | $1.3 \times 10^{-3}$ | 51                              |
|            | 1.5   | $3.7 \times 10^{-3}$ | 79                              |
|            | 2     | $1.1 \times 10^{-2}$ | 86                              |
|            | 2.5   | 0.027             | 89                              |
| 250        | 1.5   | $2.8 \times 10^{-3}$ | 66                              |
|            | 2     | $7.7 \times 10^{-3}$ | 72                              |
|            | 2.5   | 0.018             | 77                              |
| 280        | 1.5   | $2.7 \times 10^{-3}$ | 41                              |
|            | 2     | $5.2 \times 10^{-3}$ | 55                              |
|            | 2.5   | 0.011             | 60                              |
|            | 4     | 0.074             | 67                              |
| 300        | 2     | $4.8 \times 10^{-3}$ | 44                              |
|            | 2.5   | $9.3 \times 10^{-3}$ | 51                              |
|            | 4     | 0.055             | 59                              |
| 330        | 2     | $1.2 \times 10^{-2}$ | 84                              |
|            | 2.5   | $9.8 \times 10^{-3}$ | 36                              |
|            | 4     | 0.044             | 48                              |
|            | 4.82  | 0.09              | 51                              |
| 350        | 2.5   | 0.024             | 14                              |
|            | 4     | 0.06              | 41                              |
|            | 4.82  | 0.11              | 45                              |
TABLE III. 3σ upper bounds on CP-violating NP couplings from $A_{\text{circ}}$ asymmetry, using $H \rightarrow ZZ \rightarrow l^- l^+ X$ decay and circularly polarized laser beams with $P_e = \bar{P}_e = -P_\gamma = -\bar{P}_\gamma = \pm 1$.

| $m_H$ (GeV) | $x_0$ | upper limit on $d_Bc_W^2 + ds_W^2$ | $\xi_2$-averaged Events |
|------------|-------|--------------------------------|------------------------|
| 200        | 1     | 5.7x10^{-4}                   | 141                    |
|            | 1.5   | 6x10^{-4}                     | 133                    |
|            | 2     | 6.2x10^{-4}                   | 121                    |
|            | 2.5   | 6.5x10^{-4}                   | 112                    |
|            | 4     | 7x10^{-4}                     | 95                     |
|            | 4.82  | 7.3x10^{-4}                   | 89                     |
| 240        | 1     | 6.3x10^{-4}                   | 49                     |
|            | 1.5   | 4.2x10^{-4}                   | 109                    |
|            | 2     | 4.3x10^{-4}                   | 105                    |
|            | 2.5   | 4.4x10^{-4}                   | 99                     |
|            | 4     | 4.7x10^{-4}                   | 85                     |
|            | 4.82  | 5x10^{-4}                     | 80                     |
| 280        | 1.5   | 4.5x10^{-4}                   | 55                     |
|            | 2     | 3.7x10^{-4}                   | 78                     |
|            | 2.5   | 3.7x10^{-4}                   | 78                     |
|            | 4     | 4x10^{-4}                     | 70                     |
|            | 4.82  | 4x10^{-4}                     | 66                     |
| 320        | 2     | 4.8x10^{-4}                   | 31                     |
|            | 2.5   | 3.6x10^{-4}                   | 53                     |
|            | 4     | 3.5x10^{-4}                   | 56                     |
|            | 4.82  | 3.6x10^{-4}                   | 54                     |
| 350        | 2.5   | 6x10^{-4}                     | 15                     |
|            | 4     | 3.4x10^{-4}                   | 45                     |
|            | 4.82  | 3.4x10^{-4}                   | 46                     |
TABLE IV. 3σ upper bounds on CP-violating NP couplings from $A_{\text{circ}}$ asymmetry, using $H \rightarrow ZZ \rightarrow l^-l^+X$ decay and circularly polarized laser beams with $P_e = \bar{P}_e = P_\gamma = \bar{P}_\gamma = \pm 1$.

| $m_H$ (GeV) | $x_0$ | upper limit on $d_Pc_{W}^2 + d_sc_{W}^2$ | $\xi_2$-averaged Events |
|-------------|-------|----------------------------------------|--------------------------|
| 200         | 1     | $7 \times 10^{-4}$                     | 94                       |
|             | 1.5   | $6 \times 10^{-4}$                     | 128                      |
|             | 2     | $6 \times 10^{-4}$                     | 131                      |
|             | 2.5   | $6.1 \times 10^{-4}$                   | 125                      |
|             | 4     | $6.8 \times 10^{-4}$                   | 102                      |
|             | 4.82  | $7.1 \times 10^{-4}$                   | 92                       |
| 240         | 1     | $10^{-3}$                              | 18                       |
|             | 1.5   | $5.3 \times 10^{-4}$                   | 68                       |
|             | 2     | $4.7 \times 10^{-4}$                   | 86                       |
|             | 2.5   | $4.6 \times 10^{-4}$                   | 92                       |
|             | 4     | $4.7 \times 10^{-4}$                   | 85                       |
|             | 4.82  | $5 \times 10^{-4}$                     | 80                       |
| 280         | 1.5   | $8.1 \times 10^{-4}$                   | 17                       |
|             | 2     | $5.3 \times 10^{-4}$                   | 39                       |
|             | 2.5   | $4.6 \times 10^{-4}$                   | 51                       |
|             | 4     | $4.3 \times 10^{-4}$                   | 59                       |
|             | 4.82  | $4.3 \times 10^{-4}$                   | 58                       |
| 320         | 2     | $10^{-3}$                              | 6                        |
|             | 2.5   | $6.3 \times 10^{-4}$                   | 17                       |
|             | 4     | $4.5 \times 10^{-4}$                   | 34                       |
|             | 4.82  | $4.4 \times 10^{-4}$                   | 37                       |
| 350         | 2.5   | $1.6 \times 10^{-3}$                   | 2                        |
|             | 4     | $5.5 \times 10^{-4}$                   | 18                       |
|             | 4.82  | $5 \times 10^{-4}$                     | 22                       |
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Figure 1: NSD for the $A_{lin}$-asymmetry, for various choices of the $(\bar{d}Bc_{W}^{2} + \bar{s}s_{W}^{2})$ combination of couplings, using $P_{t} = 1$ and $x_{0} = 0.5$ in a $\sqrt{s_{ee}} = 0.5 TeV$ Collider.
Figure 2: NSD for the $A_{\text{circ}}$-asymmetry, for various choices of the $(\bar{d}_B c_W^2 + \bar{d}_s^2 W)$ combination using: (a) $P_e = \bar{P}_e = -P_\gamma = -\bar{P}_\gamma = \pm 1$, $x_0 = 4.82$ and $\sqrt{s_{ee}} = m_H/0.75$; (b) $P_e = \bar{P}_e = P_\gamma = \bar{P}_\gamma = \pm 1$, $x_0 = 4$ and $\sqrt{s_{ee}} = 0.5 TeV$.  

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Figure 3: The spectrum of the backscattered photon; (a) overall flux, (b) average linear polarization of the backscattered photon along the direction it is maximized, and (c) average circular polarization.