Collective flow continues to play a central role in ongoing efforts to characterize the transport properties of the strongly interacting matter produced in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) \[1, 10\]. An experimental manifestation of this flow is the anisotropic emission of particles in the plane transverse to the beam direction \[17, 18\]. This anisotropy can be characterized by the even order Fourier coefficients;

\[
v_n = \langle \cos(n(\phi - \Psi_{RP})) \rangle, \quad n = 2, 4, ... ,
\]

(1)

where \(\phi\) is the azimuthal angle of an emitted particle, \(\Psi_{RP}\) is the azimuth of the reaction plane and the brackets denote averaging over particles and events \[19\]. Characterization has also been made via the pair-wise distribution in the azimuthal angle difference \((\Delta \phi = \phi_1 - \phi_2)\) between particles \[17, 21, 22\]:

\[
\frac{dN_{\text{pairs}}}{d\Delta \phi} \propto \left( 1 + \sum_{n=1}^{\infty} 2v_n^2 \cos(n\Delta \phi) \right).
\]

(2)

Anisotropic flow is understood to result from an asymmetric hydrodynamic-like expansion of the medium produced by the two colliding nuclei. That is, the spacial asymmetry of the produced medium drives uneven pressure gradients in- and out of the reaction plane and hence, a momentum anisotropy of the particles emitted about this plane. This mechanistic picture is well supported by the observation that the measured anisotropy for hadron \(p_T < 2 \text{ GeV}/c\), can be described by relativistic hydrodynamics \[5, 10, 12, 14, 17, 22, 31\].

The differential Fourier coefficients \(v_2(N_{\text{part}})\) and \(v_2(p_T)\) have been extensively studied in Au+Au collisions at RHIC \[20, 32, 55\]. One reason for this has been the realization that these elliptic flow coefficients are sensitive to various transport properties of the expanding hot medium \[8, 9, 11, 13, 22, 33, 41\]. Indeed, considerable effort has been, and is being devoted to the quantitative extraction of the specific shear viscosity \(\eta/s\) (i.e.

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Experimental measurements of the eccentricity have not been possible to date. Consequently, much reliance has been placed on the theoretical estimates obtained from the overlap geometry of the collision zone, specified by the impact parameter \(b\) or the number of participants \(N_{\text{part}}\) \[31, 34, 45, 52\]. For these estimates, the geometric fluctuations associated with the positions of the nucleons in the collision zone, serve as the underlying cause of the initial eccentricity fluctuations. That is, the fluctuations of the positions of the nucleons lead to fluctuations of the so-called participant plane (from one event to another) which result in larger values for the eccentricities (\(\varepsilon\)) referenced to this plane.

The magnitude of these fluctuations are of course model dependent, and this leads to different predictions for the magnitude of the eccentricity. More specifically, the \(\varepsilon_2\) values obtained from the Glauber \[52, 53\] and the factorized Kharzeev-Levin-Nardi (fKLN) \[41, 54\] models, (the two primary models currently employed for eccentricity estimates) give results which differ by as much as \(\approx 25\%\) \[54, 57\] – a difference which leads to an approximate factor of two uncertainty in the extracted \(\eta/s\) value \[4, 10\]. Thus, a more precise extraction of \(\eta/s\) re-
I. ECCENTRICITY SIMULATIONS

Monte Carlo (MC) simulations were used to calculate event averaged eccentricities (denoted here as $\varepsilon_n$) in Au+Au collisions, within the framework of the Glauber (MC-Glauber) and fKLN (MC-KLN) models. For each event, the spatial distribution of nucleons in the colliding nuclei were generated according to the Woods-Saxon function:

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/d}},$$

where $R_0 = 6.38 \text{ fm}$ is the radius of the Au nucleus and $d = 0.53 \text{ fm}$ is the diffuseness parameter.

For each collision, the values for $N_{\text{part}}$ and the number of binary collisions $N_{\text{coll}}$ were determined within the Glauber ansatz. The associated $\varepsilon_n$ values were then evaluated from the two-dimensional profile of the density of sources in the transverse plane $\rho_s(r_{\perp})$, using modified versions of MC-Glauber and MC-KLN respectively.

For each event, we compute an event shape vector $S_n$ and the azimuth of the rotation angle $\Psi_n$, for $n$-th harmonic of the shape profile:

$$S_{nx} = S_n \cos(n\Psi_n) = \int dr_\perp \rho_s(r_{\perp}) \cos(n\phi)$$

$$S_{ny} = S_n \sin(n\Psi_n) = \int dr_\perp \rho_s(r_{\perp}) \sin(n\phi)$$

$$\Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{S_{ny}}{S_{nx}} \right),$$

where $\phi$ is the azimuthal angle of each source and the weight $\omega(r_{\perp}) = r_{\perp}^2$ and $\omega(r_{\perp}) = r_{\perp}^n$ are used in respective calculations. Here, it is important to note that the substantial differences reported for $\varepsilon_n$ in Refs. 30, 31, 47, 50–52, 58–60 is largely due to the value of $\omega(r_{\perp})$ employed.

The eccentricities were calculated as:

$$\varepsilon_n = \langle \cos(n(\phi - \Psi_n)) \rangle$$

and

$$\varepsilon_n = \langle \cos(n(\phi - \Psi_m)) \rangle, \ n \neq m.$$
Au+Au collisions. The filled and open symbols indicate the results for odd and even harmonics respectively. For this weighting scheme, $\varepsilon_n$ is essentially the same for $n \geq 3$, and have magnitudes which are significantly less than that for $\varepsilon_2$, except in very central collisions where the effects of fluctuation dominate the magnitude of $\varepsilon_{n,n \geq 3}$. Note the approximate $1/\sqrt{N_{\text{part}}}$ dependence for $\varepsilon_{n,n \geq 3}$ and the smaller magnitudes for $\varepsilon_{n,n \geq 3}$ (with larger spread) apparent in Fig. 1(b), can be attributed to the sharper transverse density distributions for MC-KLN.

Figure 2 shows a similar comparison of $\varepsilon_{n,n\leq 6}$ vs. $N_{\text{part}}$ for calculations performed with the weight $\omega(r_\perp) = r_\perp^n$. This weighting results in an increase in the sensitivity to the outer regions of the transverse density distributions. Consequently, the overall magnitudes for $\varepsilon_{n,n \geq 3}$ are larger than those shown in Fig. 1. This weighting also lead to a striking difference in the relative magnitudes of $\varepsilon_{n,n \geq 2}$ for MC-Glauber (a), MC-KLN (b) and the results for $\omega(r_\perp) = r_\perp^2$ shown in Fig. 1.

II. ECCENTRICITY RATIOS

The magnitudes and trends of the calculated eccentricities shown in Figs. 1 and 2 are expected to influence the measured values of $v_n$. To estimate this influence, we first assume that the resulting anisotropic flow is directly proportional to the initial eccentricity, as predicted by perfect fluid hydrodynamics. Here, our tacit assumption is that a possible influence from the effects of a finite viscosity ($\eta/s$) is small because current estimates indicate that $\eta/s$ is small [4, 6, 7, 9–16, 30, 40, 43, 44] – of the same magnitude as for the conjectured KSS bound $\eta/s = 1/(4\pi)$ [64].

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Figure 1 indicates specific testable predictions for the relative influence of $\varepsilon_{n,n \geq 2}$ on the magnitudes of $v_{n,n \geq 2}$. That is, (i) $\varepsilon_2$ should have a greater influence than $\varepsilon_{n,n \geq 3}$ in non-central collisions, (ii) the respective influence of $\varepsilon_{n,n \geq 3}$ on the values for $v_{n,n \geq 3}$ should be similar irrespective of centrality and (iii) the ratios $v_{4,5,6}/v_3$ should
follow a specific centrality dependence due to the influence of $\varepsilon_{4,5,6}/\varepsilon_3$. Such a dependence is illustrated in Fig. 3, where we show the centrality dependence of the ratios $\varepsilon_{2,4,5}/\varepsilon_3$, obtained for MC-Glauber (a) and MC-KLN (b) calculations. They suggest that, if MC-Glauber-like eccentricities with weight $\omega(r_\perp) = r_\perp^n$, are the relevant eccentricities for Au+Au collisions, then the measured ratio $v_2/v_3$ should increase by a factor $\approx 2$, from central to mid-central collisions ($N_{\text{part}} \sim 350 - 150$). For $N_{\text{part}} \lesssim 150$, Fig. 2(a) shows that the ratio $v_2/v_3$ could even show a modest decrease. The eccentricity ratios involving the higher harmonics suggest that, if they are valid, the measured values of $v_4,v_5,v_6$ should show little, if any, dependence on centrality, irrespective of their magnitudes.

The ratios $\varepsilon_{2,4,5}/\varepsilon_3$ obtained for MC-KLN calculations are shown in Fig. 3(b). While they indicate qualitative trends which are similar to the ones observed in Fig. 3(a), their magnitudes and their detailed dependence on centrality are different. Therefore, if the qualitative trends discussed earlier were indeed found in data, then these differences suggest that precision measurements of the centrality dependence of the relative ratios for $v_2/v_3$, $v_4/v_3$, $v_5/v_3$, ... for several $p_T$ selections, could provide a constraint for aiding the distinction between Glauber-like and Glauber-like initial collision geometries. Specifically, smaller (larger) values of the relative ratios are to be expected for $v_2/v_3$ and $v_4/v_3$ for Glauber-like (KLN-like) initial geometries. Note the differences in the expected centrality dependencies as well.

Figure 4 compares the eccentricity ratios $\varepsilon_{2,4,5}/\varepsilon_3$ obtained for MC-Glauber (a) and MC-KLN (b) calculations with the weight $\omega(r_\perp) = r_\perp^n$. The magnitudes of these ratios and their centrality dependencies are distinct for MC-Glauber and MC-KLN. They are also quite different from the ratios shown in Fig. 3. This suggests that precision measurements of the centrality dependence of the relative ratios $v_2/v_3$, $v_4/v_3$, $v_5/v_3$, ... (for several $p_T$ selections) should not only allow a clear distinction between MC-Glauber and MC-KLN initial geometries, but also a distinction between the the $\omega(r_\perp) = r_\perp^n$ and $\omega(r_\perp) = r_\perp^2$ weighting methods.

A finite viscosity will influence the magnitudes of $v_n$. Thus, for a given $p_T$ selection, the measured ratios for $v_2/v_3$, $v_4/v_3$, $v_5/v_3$, ... will be different from the eccentricity ratios shown in Figs. 3 and 4. Note as well that, even for ideal hydrodynamics, the predicted magnitude of $v_4/v_3$ is only a half of that for $v_2/v_3$. Nonetheless, the rather distinct centrality dependent eccentricity patterns exhibited in Figs. 3 and 4 suggests that measurements of the ratios of these flow harmonics should still allow a distinction between MC-Glauber and MC-KLN initial geometries, as well as a distinction between the two weighting methods.

The ratios $v_3/(v_2)^{3/2}$ and $v_4/(v_2)^2$ have been recently found to scale with $p_T$, suggesting a reduction in the influence of viscosity on them. Thus, the measured ratios $v_n/(v_2)^{n/2}$ could give a more direct indication of the centrality dependent influence of $\varepsilon_n/(\varepsilon_2)^{n/2}$ on $v_n/(v_2)^{n/2}$. The open symbols in Figs. 5 and 6 indicate a substantial difference between the ratios $\varepsilon_3/(\varepsilon_2)^{3/2}$ (a) and $\varepsilon_4/(\varepsilon_2)^2$ (b) for the MC-Glauber and MC-KLN geometries as indicated. Note as well that the ratios in Fig. 6 are substantially larger than those in Fig. 5. The latter difference reflects the different weighting schemes used, i.e. $\omega(r_\perp) = r_\perp^n$ and $\omega(r_\perp) = r_\perp^2$ respectively. Interestingly, the ratios for $\varepsilon_4/(\varepsilon_2)^2$ imply much larger measured ratios for $v_4/(v_2)^2$ than the value of 0.5 predicted by per-
fect fluid hydrodynamics (without fluctuations). However, they show qualitative trends which are similar to those for the measured ratios \(v_4/(v_2)^2\), obtained for \(v_4\) evaluations relative to the \(\Psi_2\) plane. The relatively steep rise of the ratios in Figs. 7 and 8 (albeit steeper for MC-Glauber), can be attributed to the larger influence that fluctuations have on the higher harmonics. Note that these are the same fluctuations which give rise to the “anomalously low” values of \(\varepsilon_4\) evaluated with respect to \(\Psi_2\) in central collisions.

Figures 3–6 suggests that measurements of the centrality dependence of the ratios \(v_3/(v_2)^{3/2}\) and \(v_4/(v_2)^2\), in conjunction with those for \(v_2/v_3, v_4/v_3, v_5/v_3\ldots\) may provide a robust constraint for the role of initial eccentricity fluctuations, as well as an additional handle for making a distinction between Glauber-like and fKLN-like initial geometries. These measurements could also lend insight, as well as place important constraints for the degree to which a small value of \(\eta/s\) and/or the effects of thermal smearing, modulate the higher order flow harmonics [compared to \(v_2\)] as has been suggested.

III. SUMMARY

In summary, we have presented results for the initial eccentricities \(\varepsilon_{n,n}\leq 6\) for Au+Au collisions with different weighting schemes, for the two primary models currently employed for eccentricity estimates at RHIC. The calculated values of \(\varepsilon_{n,n}\leq 6\), which are expected to influence the measured flow harmonics \(v_n\), suggests that measurements of the centrality dependence of \(v_2/(v_2)^2\), \(v_4/(v_2)^2\), etc. could provide stringent constraints for validating the predicted influence of eccentricity fluctuations on \(v_n\), as well as an important additional handle for making a distinction between Glauber-like and fKLN-like initial geometries. Measurements of \(v_n\) and their ratios are now required to exploit these simple tests.

Acknowledgments We thank Wojciech Broniowski for profitable discussions and invaluable model calculation cross checks. This research is supported by the US DOE under contract DE-FG02-87ER4031.A008 and by the NSF under award number PHY-1019387.
Note that the event planes for the eccentricities are specified by the initial state coordinate asymmetry, whereas the experimental event planes for $v_n$ are specified by the final state momentum space anisotropy. The two planes are correlated in ideal hydrodynamics.