Direct detection of the relative strength of Rashba and Dresselhaus spin-orbit interaction: Utilizing the SU(2) symmetry

Jun Li and Kai Chang
SKLSM, Institute of Semiconductors, Chinese Academy of Sciences, P. O. Box 912, Beijing 100083, China
(Dated: March 5, 2010)

We propose a simple method to detect the relative strength of Rashba and Dresselhaus spin-orbit interactions in quantum wells (QWs) without relying on the directional-dependent physical quantities. This method utilizes the asymmetry of critical gate voltages that leading to the remarkable signals of SU(2) symmetry, which happens to reflect the intrinsic structure inversion asymmetry of the QW. We support our proposal by the numerical calculation of in-plane relaxation times based on the self-consistent eight-band Kane model. We find that the two different critical gate voltages leading to the maximum spin relaxation times [one effect of the SU(2) symmetry] can simply determine the ratio of the coefficients of Rashba and Dresselhaus terms. Our proposal can also be generalized to extract the relative strengths of the spin-orbit interactions in quantum wire and quantum dot structures.

PACS numbers: 72.25.Dc, 71.70.Ej, 73.21.Fg, 72.25.Rb

The spin-orbit interaction (SOI), which is a manifestation of the relativistic effect, transforms the electric fields into momentum-dependent effective magnetic fields, coupling the electron spin with electron orbital motion. The SOI provides us an efficient way to control the electron spin with electric fields instead of magnetic fields [2, 3], therefore plays an important role in realizing all-electrical controlled spintronic devices. According to the different origins of SOI in semiconductor quantum structures, the SOI has been distinguished by the Rashba SOI (RSOI) arising from structure inversion asymmetry (SIA) [4] and the Dresselhaus SOI (DSOI) caused by bulk inversion asymmetry (BIA), respectively [5]. These two types of SOI, yielding different effective SO magnetic field, leads to different behaviors of spin-transport properties and spin relaxation. Naturally, the ratio of Rashba and Dresselhaus coefficients (RD ratio) becomes a key parameter for understanding the spin-related phenomena and designing the future spintronic devices. Previously, the RD ratio can be determined by mapping the k-dependent spin photocurrent [6, 7] in-plane spin-relaxation time [8] the spin precession about the effective spin-orbit magnetic fields [9] into the components coming from DSOI and RSOI, or utilizing the anisotropic conductance of quantum wires in the presence of in-plane magnetic fields [10]. Therefore the above methods require exquisite measurement with respect of the crystallographic axis. Although these methods have been successfully used to study the relative strength of the Rashba and Dresselhaus SOIs in two-dimensional quantum well and heterostructures, the obtained RD ratio still holds certain ambiguity as pointed out by the authors [2, 8, 10]. Hence, finding a simple and accurate method to determine the RD ratio in all sorts of systems remains a challenging but important task.

In this Letter, we propose a direct method that can separate the RSOI from DSOI and determine the RD ratio in an asymmetric [001]-oriented zincblende quantum well (QW). Applying a gate voltage across the QW to tune the total RSOI in this structure (see Fig. 1), we can find two different magnitudes of critical gate voltages to restore the exact SU(2) symmetry [11] by strengthening or canceling intrinsic RSOI existing in this QW. The difference between these two critical gate voltages extract the exact information of intrinsic structure inversion asymmetry of this QW, with bulk inversion asymmetry separating apart. Therefore the two critical gate voltages can be used to determine the RD ratio in this asymmetric QW. Compare with the previous works, this proposal does not rely on measurements along specific directions [12] and is robust against all the effects that cannot change the structure inversion asymmetry such as the isotropic impurity scattering. In addition, this proposal offers a general scheme that does not dependent on a specific experimental technology and the dimensionality of the experimental sample, e.g., quantum wires and dots. A series of remarkable physical effects in quantum wells, wires and dots [11] led by the SU(2) symmetry can be used to measure the critical gate voltages, consequently the RD ratio in these quantum structures.

Firstly we give a picture of our proposal based on the single-band model of two-dimensional electron gas (2DEG) with two types of SOI:

\[
H = \frac{\hbar^2 k^2}{2m} + H_R + H_D
\]

where \(H_R = \alpha \langle k_x \sigma_x - k_y \sigma_y \rangle\) is the Rashba spin-orbit interaction term, \(H_D = \beta \langle k_x \sigma_y - k_y \sigma_x \rangle + \gamma \langle k_x^2 k_y \sigma_x - k_y^2 k_x \sigma_y \rangle\) is the Dresselhaus spin-orbit interaction term, and \(k = (k_x, k_y)\) is in-plane wave vector. Here, \(\alpha\) is the linear Rashba coefficient, \(\beta\) and \(\gamma\) are the linear and cubic Dresselhaus coefficients, respectively. The Rashba coefficient \(\alpha\) can be tuned easily by changing the structure inversion asymmetry, for instance, by gate voltage applied perpendicular to the QW plane. \(\beta\) While the Dresselhaus coefficients \(\beta\) and \(\gamma\) can be adjusted by changing the thickness of quantum wells. If we adopt the infinite high barrier model, \(\beta \approx \gamma \langle k_x^2 \rangle = \gamma \langle \frac{\hbar^2}{m} \rangle^2\).

The interplay between the RSOI and DSOI would lead to the anisotropy of optical and transport properties, since...
The DSOI depends sensitively on the crystallographic orientations, while the RSOI shows an isotropic behavior. If we tune gate voltage to satisfy $\alpha = \pm \beta$ (neglecting the cubic Dresselhaus term), the Hamiltonian of 2DEG show the exact SU(2) symmetry.\(^{11}\) The exact SU(2) symmetry is a very unique property of quantum systems that the RSOI and DSOI happen to cancel each other for $k$ along [110] or [110] and is revealed to be robust against spin-independent disorder interactions.\(^{11}\) As a consequence, the exact SU(2) symmetry would lead to a series of remarkable physical effects. For example, there should be a maximum spin life-time for electron spins align along [110] or [110] direction.\(^{13}\) A persistent spin helix could exist in the sample\(^{11,14}\) the diminishing of the weak antilocalization\(^{15,16}\) and the beating pattern of SdH oscillation.\(^{17,18}\) Besides it is worth noticing that in quasi-one-dimensional quantum wire and zero-dimensional quantum dot, the SU(2) symmetry could also induce strong physical effects. Such as the conductance of a quantum wire shows strong anisotropy\(^{19,20}\) and the spin relaxation curve of a quantum dot shows a cusplike structure.\(^{21}\) All these physical effects can be used to determine the critical gate voltages that restore the SU(2) symmetry.

Notice that the SU(2) symmetry could be achieved by applying both positive and negative electric fields, each satisfying $\alpha = \pm \beta$. The total Rashba coefficient of an asymmetric QW with gate voltage tuning the SIA can be viewed as a superposition of two parts $\alpha = \alpha_0 + \alpha_{ex}$. The first part $\alpha_0$ comes from the intrinsic SIA of the sample, e.g., the asymmetric doping or band profile. The second part $\alpha_{ex}$ is introduced by the external electric field of the gate voltage. By sweeping the gate voltage, one can find two values $U_{\pm}$ to meet the SU(2) symmetry conditions: $\alpha_0 + \alpha_{ex} = \pm \beta$. Here we label the external Rashba coefficients that lead to the SU(2) symmetry conditions with $\alpha_{ex}$. If we neglect the difference of the dielectric constant between the well and barrier materials, we can simply assume $\alpha_{ex}$ to be proportional to $U_{\pm}$. From the requirement restoring the SU(2) symmetry, one can find

\[
\frac{\alpha_0}{\beta} = \frac{\alpha_0 + \alpha_{ex}}{\alpha_0 - \alpha_{ex}} = \frac{U_{\pm} + U_{\mp}}{U_{\pm} - U_{\mp}}
\]

Eq. (2) demonstrate that, if there is no intrinsic SIA in the sample, i.e., $\alpha_0 = 0$, we should expect that $U_{\pm} = -U_{\mp}$, while if $\alpha_0 \neq 0$, we get $U_{\pm} \neq -U_{\mp}$. From a symmetry consideration, this conclusion is easy to understand because there would be no difference between $|U_{\pm}|$ unless the [001] and [001] directions of the QW are asymmetric. This consideration guarantee our proposal to be robust against all the effects that does not change the symmetry of [001] and [001] directions, such as the isotropic impurity scattering. This conclusion can be also supported by an eight-band self-consistent calculation, as shown in Fig. 1. In an asymmetrically doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW, the results show that the critical voltages to satisfy $\alpha = \pm \beta$ are 0.4 V and $-0.08$ V respectively [see the panels (b) and (c)]. From the compositions of total effective spin-orbit magnetic field (the panels under the band profile), one can see clearly that the difference between $|U_{\pm}|$ comes from the intrinsic SIA. In Fig. 1(d)-(f), we show the effective spin-orbit magnetic field calculated by eight-band self-consistent calculation, which already takes the cubic Dresselhaus terms into account. Although the existence of cubic Dresselhaus terms might cause a different configuration rather than the exact SU(2) symmetry [see the panels (e) and (f)], the panel (e) still shows the mirror reflection symmetry with the panel (f), indicating that the total Rashba coefficient of these two panels are of the same magnitude (but with opposite signs). The intrinsic SIA still request two asymmetrical critical voltages to achieve the total ROI in panels (e) and (f). So the asymmetrical critical voltages always reflects the intrinsic SIA of QW, even taking account of the cubic Dresselhaus terms.

Next, we will take the in-plane D’yakonov-Perel’ (DP) spin...
relaxation times [22] as an example to demonstrate the validity of our proposal. This calculation is based on a self-consistent eight-band Kane model. [23] The band parameters can be found in Ref. [24], and the BIA Kane parameter $B_0$ is obtained from 14-band effective mass model in Ref. [25].

In Fig. 2(a) and (c) we exhibit the calculated spin relaxation times for electron spin along [110] or [110] (denoted by $\tau_+,$ $\tau_-$ respectively) as a function of gate voltage in 15 nm n-doped GaAs/AlGaAs QWs with different doping conditions (the unit is 10$^{11}$ cm$^{-2}$). (c) $\tau_+$ as a function of gate voltage in Al,Ga$_{1-x}$As/GaAs/Al$_x$Ga$_{1-x}$As QW with different Al composition $x.$ (b) and (d) the RD ratios $\alpha_0/\beta^*$ as a function of the doping concentration and Al composition, respectively, for different thicknesses of QWs.

The two peaks in Fig. 2 (a) correspond to the peaks of spin relaxation times, we introduce the analytical results of $\tau_+$ as a function of gate voltage in n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW ($N_D = 4 \times 10^{11}$ cm$^{-2}$) as a function of gate voltage in 15 nm asymmetrically n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW (Fig. 3). The two peaks in Fig. 2 (a) correspond to the peaks of spin relaxation times, we introduce the analytical results of $\tau_+$ as a function of gate voltage in n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW ($N_D = 4 \times 10^{11}$ cm$^{-2}$) as a function of gate voltage in 15 nm asymmetrically n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW with different Al composition $x.$ The two peaks in Fig. 2 (a) correspond to the peaks of spin relaxation times, we introduce the analytical results of $\tau_+$ as a function of gate voltage in n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW ($N_D = 4 \times 10^{11}$ cm$^{-2}$) as a function of gate voltage in 15 nm asymmetrically n-doped Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW with different Al composition $x.$

In Fig. 2 (a) and (c) we exhibit the calculated spin relaxation times for electron spin along [110] or [110] (denoted by $\tau_+,$ $\tau_-$ respectively) as a function of gate voltage in 15 nm n-doped GaAs/AlGaAs QWs with different doping conditions and Al compositions of barrier (i.e., asymmetric barrier heights), respectively. In order to understand the numerical results of spin relaxation times, we introduce the analytical results of DP spin relaxation times at $T = 0$ K [13]

$$\frac{1}{\tau_s} = \frac{2\tau_1}{\hbar^2} \left[ (\alpha + \beta) k_F^2 \frac{1}{2}\gamma (1 + \frac{1 + \tau_1/\tau_s}{16}) \right],$$

where $k_F$ is the Fermi wave vector of electron, and we simply take a typical momentum scattering time $\tau_1 = \tau_1 = 0.1$ ps in this paper. By sweeping the gate voltage, the total Rashba coefficient $\alpha$ change linearly. So one can find the maximum spin relaxation time $\tau_{s\max}^\alpha = \hbar^2/(k_F^2 \gamma^2 \tau_1)$ when $\alpha = \pm (\beta + 4 k_F^2 \gamma),$ corresponding to the peaks of $\tau_s$ in Fig. 2(a). The two peaks are symmetric with respect to the zero voltage ($\vert U_s \vert = \vert U_{s-} \vert$) for a symmetrical QW [see the solid lines] but asymmetric ($\vert U_s \vert \neq \vert U_{s-} \vert$) [see the dashed, dotted and dashdotted lines] for asymmetrical QWs. Different from the $k$-linear SOI model which neglects the cubic Dresselhaus term, $\tau_s$ would not go infinite and the maximum values do not show at $\alpha = \pm \beta.$ Similar to the analysis on the spin-galvanic effect by Ganichev et al. [4], we can take the $\beta^* = \beta + 4 k_F^2 \gamma$ as the renormalized Dresselhaus coefficient for k-linear SOI model. Therefore we actually get $\alpha_0/\beta^*$ from Eq. (2) rather than $\alpha_0/\beta$ and the difference between $\beta$ and $\beta^*$ comes from the contribution of cubic Dresselhaus term. In Figs. 2(b) and (d) we display $\alpha_0/\beta^*$ determined from the critical gate voltage that lead to the maximum in-plane spin relaxation time. As expected, $\alpha_0/\beta^*$ increase with increasing the asymmetrical doping concentration or the composition difference between the left and right barriers. For the QWs with different doping concentrations, $\alpha_0/\beta^*$ also increase with increasing the well width, since $\beta^* \approx \gamma (\frac{\pi}{\alpha})^2.$ For the QWs with different compositions of barriers, $\alpha_0/\beta^*$ turns to be not sensitive to the change of well width, because $\alpha_0$ are very small in these cases.

In addition to the intrinsic Rashba coefficient $\alpha_0,$ it is also possible to determine the relative strength of total Rashba coefficient $\alpha$ and Dresselhaus coefficient $\beta$ in the QW through the in-plane spin relaxation times. From Eq. (3) we can find the condition for the in-plane spin relaxation times $\tau_s$ to reach a same value $\tau_s$ satisfying $\alpha + \alpha_0 = 2\alpha.$ As shown in Fig. 3 (a), for a $\tau_s < \tau_{s\max}^\alpha,$ we can find the gate voltages $U_s$ corresponding to $\tau_s = \tau_r.$ Usually, there are two different gate voltages for each $\tau_s$ and $\tau_r.$ So we must limit that if $\vert U_s \vert > \vert U_{s-} \vert$ ($\vert U_s \vert < \vert U_{s-} \vert$), we choose $\vert U_s \vert > \vert U_{s-} \vert$ ($\vert U_s \vert < \vert U_{s-} \vert$). Combine these conditions and Eq. (2), we can get

$$\frac{\alpha}{\beta^*} = \frac{(U_s + U_{s-})(U_s - U_{s-})}{(U_s - U_{s+})(U_s + U_{s+})}.$$  

If the QW is inversion symmetric, we should have

$$\frac{\alpha}{\beta^*} = \frac{U_{s+} - U_{s-}}{U_{s+} - U_{s-}} = \frac{U_s^+ - U_s^-}{U_{s+} - U_{s-}}.$$  

In Fig. 3 (b) we plot $\alpha/\beta^*$ as a function of gate voltage in asymmetrically n-doped GaAs/AlGaAs QWs with differ-
ent well widths. The figure shows $\alpha/|\beta|$ increase almost linearly with gate voltage, which is consistent with the analytical results of Rashba coefficient in the previous work [26]. The slope of $\alpha/|\beta|$ for narrow wells are smaller than that of thick wells because $|\beta|$ is larger in narrow wells.

In Fig. 4(a), we investigate the temperature effect on the in-plane spin relaxation time. As we shown in the figure, the in-plane spin life peaks which characterize the emergence of SU(2) symmetry would be gradually smeared out when temperature increases from $T = 4$ K to 150 K or even higher. That is because of the blurring of the Fermi surface with increasing temperature. We may also associate that many of other SU(2) symmetry phenomenons may disappear at high temperature due to the blurring of the Fermi surface. So it is suggested that the SU(2) symmetry phenomenons should be observed at $T < 77$ K. In Fig. 4(b) we compare the RD ratios $\alpha/|\beta|$ obtained by Eq. 2 and that obtained by fitting parameters directly from the spin-splitting. We find that the ratios $\alpha/|\beta|$ agree well with the results fitted from the spin-splitting at low doping concentration and are especially good for narrow QWs. For heavily doped QW with wide wells, $\alpha/|\beta|$ obtained by the spin relaxation time calculation could deviate from that from the fitting of the spin-splitting. The reason is when the doping concentration is high, $\alpha_0$ could be influenced by the change of the internal electric field due to the charge redistribution induced by the external gate voltage. That makes the $\alpha_0$ under the external electric field different from the direct fitting parameters from the zero-field spin-splitting. However, as we see in the Fig. 4(b), the charge redistribution effect is very limited at light doping ($N_D < 4 \times 10^{11}$ cm$^{-2}$) and narrow QWs ($L_n < 15$ nm). $\alpha_0/|\beta|$ obtained by the critical gate voltages measurement are more accurate in these cases. In Fig. 4(b) we also show the difference between $\alpha_0/|\beta|$ and $\alpha_0/|\beta|$. This difference comes from the contribution of the cubic Dresselhaus terms, and is very small (less than 0.2) when $N_D$ is less than $3 \times 10^{11}$ cm$^{-2}$, which demonstrates that the single band model with $k$-linear SOI coefficients are valid at low doping concentration. In Fig. 4(c) we show the calculated RD ratio $\alpha_0/|\beta|$ for different QWs. Though these BIA Kane parameters $B_0$ are still in a big uncertainty today, we can still see that for the narrow bandgap QW, such as InAs/GaInAs and InSb/AlInSb, the relative strength of Rashba SOI are much larger than that of middle bandgap QWs GaAs/AlGaAs, GaInAs/AlInAs. This is because the RSOI comes from the interband coupling of conduction and valence bands, which is much stronger in narrow bandgap materials.

In summary, we proposed a simple and direct method to separate the intrinsic RSOI from DSOI. The relative strength of RSOI and DSOI can be determined by the critical gate voltages that restores the SU(2) symmetry in 2DEG. The SU(2) symmetry leads to a series of characteristic physical effects, such as the maximum in-plane spin relaxation time, the persistent spin helix and so on. Through the in-plane spin relaxation time calculation based on the self-consistent eight-band model, we demonstrate our proposal is valid and can be used to detect the strengths of the SOIs in quantum wells, wires and dots utilizing the SU(2) symmetry. Our proposal offers a general scheme that many experimental techniques could be used to determine this important parameter and facilitate us to manually control the spin degree of freedom.

This work is supported by the NSFC Grant No. 60525405, 10874175 and National Basic Research Program of China(973 Program) (2010CB933700).

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**Table 1**

| Material    | $B_0$ (emu/cm$^3$) |
|-------------|--------------------|
| InAs/GaInAs | 0.2                |
| InSb/AlInSb | 0.3                |

**Figure 4**

(Color online) (a) Calculated in-plane spin relaxation time $\tau_s$ as a function of gate voltage in an 15 nm asymmetrically doped ($N_D = 4 \times 10^{11}$ cm$^{-2}$) Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW at different temperatures. (b) The RD ratio $\alpha_0/|\beta|$ as a function of asymmetric doping concentration in Al$_{0.3}$Ga$_{0.7}$As/GaAs/Al$_{0.3}$Ga$_{0.7}$As QW with different well widths $L_n = 12, 15, 18$ nm. The triangle, diamond and square dots are obtained from Eq. 2. The solid and dashed lines are $\alpha_0/|\beta|$ and $\alpha_0/|\beta|$ obtained by fitting parameters directly from the eight-band spin-splitting, respectively. (c) The RD Ratio $\alpha_0/|\beta|$ obtained from Eq. 2 as a function of asymmetrical doping concentration $N_D$ for different material of QWs.
Some effects of the SU(2) symmetry (mostly are the anisotropic properties) needs to be observed by the directional-dependent quantities, but the values of critical gate voltages and the RD ratio are not sensitive to the directional measurement.

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