Skewed distributions fixed by diagonal partons at small $x, \xi$ and $\gamma^* p \to V p$ at HERA

K. Golec-Biernat$^a$, A.D. Martin$^b$, M.G. Ryskin$^c$ and A.G. Shuvaev$^c$

$^a$H. Niewodniczanski Institute of Nuclear Physics, ul. Radzikowskiego 152, Krakow, Poland
$^b$Department of Physics, University of Durham, DH1 3LE, United Kingdom
$^c$St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia

We show that the skewed parton distributions are completely determined at small $x$ and $\xi$ by the conventional diagonal partons. We study the application to diffractive vector meson production at HERA.

1. Introduction

Data are becoming available for processes which are described by off-diagonal (or skewed) parton distributions. A relevant example is diffractive vector meson production at HERA, $\gamma^* p \to V p$ with $V = \rho, J/\psi$ or $\Upsilon$, where at high $\gamma^* p$ c.m. energy, $W$, the cross section is dominated by the two-gluon exchange diagram

$$\frac{d\sigma}{dt}(\gamma^* p \to V p) \bigg|_{t=0} = \ldots |x_2 g(x_1, x_2; \mu^2)|^2$$

(1)

where $g$ is the off-diagonal ($x_1 \neq x_2$) gluon distribution with

$$x_1 = (Q^2 + M^2_{q\bar{q}})/W^2,$$

$$x_2 = (M^2_{q\bar{q}} - M^2_g)/W^2 \ll x_1,$$

(2)

see ref. [3]. $M_{q\bar{q}}$ is the mass of the $q\bar{q}$ system produced by a photon of virtuality $Q^2$. The relevant scale is $\mu^2 = z(1-z)Q^2 + k_T^2 + m^2_q$ where $z, 1-z$ and $\pm k_T$ specify the momenta of the $q$ and $\bar{q}$. The quadratic dependence on $g$ in (1) shows that these data may offer a sensitive constraint on the gluon. Indeed our aim is to show that the off-diagonal distributions are fixed by the conventional diagonal partons, so that the data can, in principle, be included in a global parton analysis.

2. Ji’s ‘symmetrized’ distributions

We shall use the “off-forward” distributions $H(x, \xi) \equiv H(x, \xi, t, \mu^2)$ with support $-1 \leq x \leq 1$ introduced by Ji [3], with the minor difference that the gluon $H_g = x H^S_g$ [3]. They depend on the momentum fractions

$$x_{1,2} = x \pm \xi$$

(3)

carried by the emitted and absorbed partons at each scale $\mu^2$ and on the momentum transfer variable $t = (p - p')^2$. The variables $t$ and $\xi$ do not change as we evolve the distributions up in the scale $\mu^2$. In the limit $\xi \to 0$ they reduce to the conventional parton distributions

$$H_q(x, 0) = \begin{cases} 
q(x) & \text{for } x > 0 \\
-\bar{q}(-x) & \text{for } x < 0,
\end{cases}$$

$$H_g(x, 0) = x g(x),$$

and satisfy DGLAP evolution. In the limit $\xi \to 1$ they obey ERBL evolution. If we consider $H_q$ at arbitrary values of $\xi$, then for $x > \xi$ and $x < -\xi$ we have DGLAP-like evolution for quarks and antiquarks respectively, while for $-\xi < x < \xi$ we have ERBL-like evolution for the emitted $q\bar{q}$ pair.

On account of the $x_1 \leftrightarrow x_2$ symmetry the distributions $H_q, H_g$ are symmetric in $\xi$. We also have symmetry relations in terms of the $x$ variable

$$H^{NS}_q(x, \xi) = H^{NS}_q(-x, \xi),$$

$$H^S_q(x, \xi) = -H^S_q(-x, \xi),$$

$$H_g(x, \xi) = H_g(-x, \xi),$$

where the superscripts $S$ and $NS$ denote singlet and non-singlet quarks respectively.
3. \(H(x, \xi)\) in terms of conformal moments

The conformal moments\(^\text{1}\) of the off-diagonal distributions,

\[
O_N(\xi, \mu^2) = \int_{-1}^{1} dx R_N(x_1, x_2) H(x, \xi),
\]

(5)

are not mixed by evolution

\[
O_N(\xi, \mu^2) = O_N(\xi, \mu_0^2) \left( \frac{\mu^2}{\mu_0^2} \right)^{\gamma_N},
\]

(6)

where \(\gamma_N\) are the same anomalous dimensions as for diagonal partons. The \(R_N\) are known polynomials of degree \(N\)

\[
R_N = \sum_{k=0}^{N} \binom{N}{k} \left( \frac{N+2p}{k+p} \right) x_1^{k} x_2^{N-k}
\]

(7)

with \(p = 1, 2\) for quarks and gluons respectively. The \(O_N\) reduce to the usual moments in the limit \(\xi \to 0\). For example for quarks

\[
O_N \to M_N = \int_{0}^{1} x^N q(x) dx,
\]

(8)

up to a normalizing factor \(R_N(1, 1)\).

The crucial step is to find the inverse relation to (8). That is to reconstruct \(H(x, \xi)\) from a knowledge of the conformal moments. The result, due to Shuvaev \([7]\), is

\[
H(x, \xi) = \int_{-1}^{1} dx' K(x, \xi; x') f(x')
\]

(9)

where the kernel \(K\) is a known integral \([8]\) and \(f\) is the Mellin transform

\[
f(x') = \int \frac{dN}{2\pi i} (x')^{-N} O_N(\xi)/R_N(1, 1).
\]

(10)

\(f\) reduces to the diagonal distribution for \(\xi^2 \ll 1\). This follows since \([8]\)

\[
O_N(\xi) = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} O_{Nk} \xi^{2k}
\]

\[
\simeq O_{N0} = O_N(0) = M_N R_N(1, 1)
\]

(11)

for small \(\xi^2\). So the off-diagonal distribution \(H\) is completely determined in terms of the diagonal distribution \(f\) via (9).

\(^1\)Conformal moments were introduced in [4] for \(\xi = 1\), and in [7] for \(\xi \neq 1\); see also [6].

4. A good small \(x, \xi\) approximation

We can simplify (8) further if we assume that the diagonal partons have the form

\[
xq(x) = N_q x^{-\lambda_q}, \quad xg(x) = N_g x^{-\lambda_g}
\]

(12)

for very small \(x\). Then the \(x'\) integration can be performed analytically and

\[
H_i(x, \xi) = \xi^{-\lambda_i} F_i \left( \frac{x}{\xi} \right)
\]

(13)

with \(p = 1, 0\) for \(i = q, g\) respectively. A full set of results for the off-diagonal/diagonal ratios,

\[
R_i(x, \xi) = H_i(x, \xi)/H_i(x + \xi, 0),
\]

(14)

can be found in [8]. There, the ratios \(R_q^{S,S} \) and \(R_g\) are plotted as functions of \(x/\xi\) for different values of \(\lambda_i\). The scale dependence of the off-diagonal distributions, \(H_i(x, \xi)\) of [8], and hence of the \(R_i\), is hidden in the \(\mu^2\) dependence of the \(\lambda_i\). Both \(\lambda_q\) and \(\lambda_g\) increase with increasing \(\mu^2\).

5. Application to \(\gamma^* p \to V p\)

The value of the ratio for the gluon distribution at \(x = \xi\) is relevant for diffractive vector meson production, \(\gamma^* p \to V p\), at high energies, see [8].

This ratio is given by\([8]\)

\[
R_g(x = \xi) = \frac{2\lambda_q + 2 \Gamma \left( \lambda_q + \frac{5}{2} \right)}{\sqrt{\pi} \sqrt{\Gamma \left( \lambda_q + 4 \right)}}.
\]

(15)

The cross section formula (12) may then be expressed in terms of the conventional diagonal gluon distribution \(g\),

\[
\left. \frac{d\sigma}{dt}(\gamma^* p \to V p) \right|_{t=0} = \ldots \left[ R_g x_1 g(x_1, \mu^2) \right]^2,
\]

(16)

where all the off-diagonal effects are contained in the known (enhancement) factor \(R_g\). Of course to calculate the cross section properly we must use the unintegrated gluon distribution and integrate over the transverse momenta of the exchanged gluons and of the \(q\) and \(\bar{q}\) forming the vector meson.

\(^2\)This answer checks with the values of the ratio obtained by direct evolution of the off-diagonal and diagonal gluons in [8].
To obtain the scale dependence of $R_g$, we first obtain the $\mu^2$ dependence of $\lambda_g$ of (12) from the behaviour of the gluon found in the global parton analyses. For example, the MRST partons [8] have $\lambda_g = 0.205$ and 0.38 at $\mu^2 = 4$ and 100 GeV$^2$ respectively. The appropriate scale for the diffractive process $\gamma^*(Q^2)p \rightarrow V(q\bar{q})p$ is $\mu^2 \approx m_g^2 + Q^2/4$. In this way, for diffractive $J/\psi$ and $\Upsilon$ photoproduction at HERA we find that the off-diagonal enhancement, $R_g^2$, is $(1.15)^2$ and $(1.32)^2$ respectively. However, for $\Upsilon$ photoproduction, $x$ is not sufficiently small ($\sim 0.01$) and we have to improve the assumption made in (12).

If we take $xg \sim x^{-\lambda_g} (1-x)^6$ and perform the $x'$ integration in (11) numerically, then we find an enhancement of $(1.41)^2$ for $\Upsilon$ photoproduction [11]. Moreover for $\rho$ electroproduction it is found [10] that the enhancement due to off-diagonal effects of the $\gamma^*p \rightarrow \rho p$ cross section $d\sigma/dQ^2$, at the largest $Q^2$ of the HERA data, is more than a factor 2, which is just the enhancement needed to ensure a perturbative QCD description of the data.

6. Discussion

The main conclusion is embodied in eqs. (1)- (3). That is the skewed distribution $H(x, \xi)$, at any scale, is fully determined at small $x, \xi$ by knowledge of the diagonal parton distribution, at the same scale.

To be sure of this result we have checked that the analytic continuation of the conformal moments $O_N$ in $N$ is allowed [8]. A second consideration is that, from a formal point of view, we may add to the off-diagonal distribution any function which exists only in the ERBL-like region, $|x| < \xi$. In [8] we show such a contribution is negligible $O(\xi^2)$ at small $\xi$. So far our distributions allow the calculation of the imaginary part of the amplitude for the process. At small $x$ and $\xi$ it turns out that the real part may be calculated easily using a dispersion relation in the c.m. energy squared, $W^2$, and that the amplitude

$$A = \frac{i\text{Im}A}{1 + e^{-\pi\lambda x}}$$

(17)

where $A \propto (W^2)^{\lambda}$. Finally we note that our result remains valid at NLO, since there is no conformal mixing for $\xi^2 \ll 1$.

We conclude that, at small $x, \xi$, the skewed distributions $H(x, \xi; \mu^2)$ are completely known in terms of conventional partons. Thus data for processes which are described by such distributions can, in principle, be included in a conventional global analysis to better constrain the low $x$ behaviour of the partons.

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