Application of Maximum Entropy Method in Error Statistic of Vertical Launching Device

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Abstract. The installation accuracy of the vertical launcher will have varying degrees of influence on the initial accuracy of the test missile launching. Therefore, it is of great practical significance to study the assembly error characteristics of the launcher. This article will perform ANSYS simulation of the vertical launcher based on the marine force environment, use the maximum entropy method to perform statistical analysis on the extracted simulation error data, and calculate and solve the probability density function and distribution curve of the error data. The analysis results provide guidance for the next step in the accuracy control research of the launcher.

1. Introduction

At present, many ship-borne missile weapon systems of the world navy adopt the vertical launch method. The missile is launched through the vertical launcher. The assembly accuracy and guide rail installation accuracy of the launcher are important tactical and technical indicators in the vertical launching system. The overall missile also affects the launcher. The accuracy of the vertical launcher puts forward very high requirements, so the study of the accuracy error law of the vertical launcher has important practical significance [1].

This article focuses on the statistical analysis of the error data of the vertical launcher through the maximum entropy method, finds out the distribution law of the error, and provides effective guidance for the next step to accurately control the error in the entire process of production, installation and service of the launcher. Entropy was first proposed in 1865. From classical thermodynamics to information entropy, and then to the principle of maximum entropy, related theories continued to develop and grow, and finally to the fields of nature, engineering, mathematics, etc., and occupy a very important position. The principle of maximum entropy not only connects the knowledge of information theory with statistical physics, but is also widely used in various scientific fields other than thermodynamics [2].

Generally, given constraints, the probability distribution with the largest entropy can be regarded as the most unbiased and most reasonable distribution estimate of the distribution. Unlike the traditional method that focuses on the fitting effect of the original sample data, the maximum entropy method pays more attention to whether the data information is used reasonably. This method is a general non-parametric distribution estimation method, there is almost no subjective judgment, and there is no need to make too many assumptions about the type of distribution [3].

In recent years, many researchers at home and abroad have done a lot of work using entropy and maximum entropy principles to derive probability density functions, and applied them to various scientific research fields. In the early days, Singh and Fiorentino [4] enumerated the process of using
the principle of maximum entropy to derive the probability distribution of sample data, calculated the mean and standard deviation of the sample data, and deduced the small deviation normal distribution from a small amount of data. Krstanovic [5] used the principle of maximum entropy to calculate the distribution type of flood sample data. Bohlke [6] and others used the maximum entropy method to roughly estimate the probability distribution of a chemical structure for detailed analysis of its structure. Among domestic researchers, Xu Zongxue [7] analyzed the relationship between the maximum entropy method and the maximum likelihood method based on the principle of maximum entropy, derived the probability density function of the data, and analyzed and discussed the relationship between different distributions. Zhang Guang [8] applied the principle of maximum entropy to reliability analysis. Zheng Junjie [9] transformed the reliability analysis problem into an entropy density function based on the principle of maximum entropy. Usually, they use the maximum entropy method to calculate the distribution function of the fourth moment of the sample data, which is very helpful for them to do data statistics. Based on the principle of maximum entropy, Zhang Ming [10] analyzed the probability density function of geomorphology data. After that, he analyzed the probability distribution of previous flood disasters, and used accelerated genetic algorithm to optimize the distribution parameters of the maximum entropy distribution, and based on this, he carried out case statistics research and analysis.

From the above engineering application research, it is not difficult to see that the principle of maximum entropy has a wide range of application prospects. It has played an indispensable role in solving practical engineering problems. It is being applied step by step in various fields of theoretical research. In particular, the probability density function is calculated by the principle of maximum entropy, and the calculation result is in line with the objective reality of the project and is also the most reasonable. For researchers in this field, the research on deriving probability density functions through the principle of maximum entropy is still expanding and extending.

2. Maximum Entropy Principle

The principle of maximum entropy provides a method to construct the optimal probability distribution under known conditions. At present, in statistics, the maximum entropy method is more widely used in engineering practice. It does not make subjective assumptions and judgments, nor does it need to make too many assumptions about the distribution type of measurement data. Moreover, it is not limited by insufficient data and is often used. In some cases, there are insufficient data available to solve the problem.

In the analysis of the error law, the estimated value of each order moment of the various error data provided can be easily obtained, and the probability density parameter of the error can be obtained by the principle of maximum entropy. This paper will use the method of maximum entropy principle to fit the probability density function of the calculation error, and then study the statistical characteristics of the error.

Suppose the probability density function of random variable X is \( f(x) \), and the distance of the first k-order origin of X is \( i = 1, 2, \ldots, k \), then we have

\[
\int_{-\infty}^{\infty} f(x)dx = 1
\]

\[
\int_{-\infty}^{\infty} x^i f(x)dx = m_i \quad i = 1, 2, \ldots, k
\]

Among them: \( m_i (i = 1, 2, \ldots, k) \) is the \( i \) order origin moment of the random variable X.

According to the principle of maximum entropy, the maximum entropy probability density function of random variable X should be under the constraints of (1) and (2), and the entropy of random variable X is

\[
S = -\int_{-\infty}^{\infty} f(x) \ln f(x)dx
\]
A Lagrangian function $L$ is constructed by the Lagrangian multiplier method, where \( \lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_k \) is the Lagrangian multiplier, and finally the following formula (4) is the analytical form of the probability density function of maximum entropy.

$$f(x) = \exp(\lambda_0 + \sum_{i=1}^{k} \lambda_i x')$$

(4)

The parameters can be roughly estimated based on the sameness of the existing information, and the estimated parameters are bound to be slightly different from the actual parameters. Therefore, in order to obtain more accurate estimates, we generally use the least squares method to make the actual values and The sum of squares of estimated values is as small as possible, and finally the problem is solved through mathematical transformation.

Usually, we can use nonlinear programming to solve the problem of algebraic equations to obtain approximate solutions, then calculate and solve the estimated values, and finally get the probability density function of the maximum entropy distribution under the constraint conditions.

3. Finite element analysis

At present, the finite element analysis method is a common numerical analysis method, which is the result of the matrix method in structural mechanics. Currently, due to the lack of precision error data during the service process, a certain amount of error data can only be obtained through simulation using the finite element model of the hull cabin and the launcher based on the marine mechanical environment. According to the error data obtained by simulation, the maximum entropy estimation method is used to judge its distribution function and probability density curve. This article uses finite element software to build a three-dimensional model of a vertical launcher. The entire structure will be simulated in the form of a combination of shell elements, beam elements and solid elements.

The entire coordinate system adopts the spatial right-hand Cartesian coordinate system, the X axis is the long side direction of the simulation model, the Z axis is the short side direction of the simulation model, and the Y axis is the height direction of the simulation model. Set the simulation model to level 5 sea state, and the corresponding X direction, Z direction, and Y direction will apply the corresponding maximum load according to the marine force environment.

The whole launcher is replaced by beam unit, the deck surface is replaced by shell unit, the auxiliary support is replaced by beam unit, and the flange is replaced by solid unit. The frame body will be made of rectangular hollow section steel, and the deck material is the same as that of the frame body. The material used in the simulation calculation in this paper is Q345. The entire calculation model is simulated by setting a beam element with a larger elastic modulus, and the quality characteristics are ensured by setting an appropriate density value; the bulkhead and the launcher are usually connected by auxiliary support rods, and beam elements are used here. Instead of simulation, the support rod and the launch frame are integrated by a shared node, and the other end of the support rod is fixedly supported to simulate the rigid magazine bulkhead.

In the calculation model, the quality of equipment such as hatch cover opening and closing devices and maintenance channels does not affect the calculation accuracy, so it is ignored here. In the vertical launcher, except for the reinforcement plate at the upper frame deck, all the reinforcement plates and connecting plates on the frame body do not affect the calculation accuracy and are ignored. The structural parts connected together by welding in the frame body are all integrated. The focus of this paper is to propose a maximum entropy method to calculate the probability distribution function and density curve of the fitted data. Therefore, the influence of the deformation of the hull on the launcher is not considered here, and the simplified simulation calculation model does not affect the overall calculation accuracy at all.

This paper will simulate and calculate the overall changes of a vertical launcher according to the external environment at sea, extract the guide rail error data on the launcher for analysis, and calculate the perpendicularity of the guide rail in the XOY plane and the perpendicularity of the guide rail in the YOZ plane.
Only the partial guide rail sample diagram in the vertical launcher is shown here. The calculation model of the guide rail in the vertical launcher in the X direction is shown in Figure 1 and the calculation model in the Z direction is shown in Figure 2.

![Figure 1. X-direction calculation model](image1.png)

![Figure 2. Z-direction calculation model](image2.png)

Extract the error data in the X direction in the calculation model, and arrange the verticality data according to the above error calculation method as shown in Table 1.

| No. 1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 | No. 7 | No. 8 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| -1.20 | 2.59  | 1.07  | -0.69 | 0.46  | 2.22  | 0.26  | 1.09  |
| 1.88  | 0.95  | 1.58  | 1.73  | 1.80  | 2.39  | 1.07  | -1.46 |
| 1.53  | -1.93 | 2.99  | 1.20  | 1.20  | 1.99  | -1.18 | 2.46  |

Extract the error data in the Z direction from the calculation model, and arrange the verticality data according to the above error calculation method as shown in Table 2.

| No. 1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 | No. 7 | No. 8 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| -0.47 | 0.65  | 0.93  | -0.83 | 1.68  | 0.07  | 0.59  | 0.44  |
| 1.27  | 1.58  | -0.10 | -1.98 | 0.19  | 0.93  | -1.03 | -0.18 |
| 0.50  | 0.26  | 0.87  | -0.08 | 0.44  | 1.62  | 1.43  | -0.46 |

4. **Maximum entropy method to determine probability density distribution**

Generally, the maximum entropy is to satisfy the constraint conditions, and the entropy value S has a maximum value by calculating the probability density function $f(x)$, where $m_k$ is the k-th origin moment of X. Determined by existing sample data. We can use the Lagrange multiplier method to calculate the result, the probability density function of maximum entropy is:

$$f(x) = \exp\left(\lambda_0 + \sum_{i=1}^{k} \lambda_i x^i\right)$$  \hspace{1cm} (5)

We can determine the Lagrangian multiplier $\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_k$ value according to the average
value and variance obtained from the simulated sample data, and substitute (5) to obtain the probability density function $f(x)$.

Generally, the maximum entropy does not require high requirements for the order of the sample data, and usually the first 5 moments can be used to obtain satisfactory results. First, we take the verticality data of the X direction of the vertical launcher rail as an example, and establish a maximum entropy model to solve it. We take the 5th order moment constraint, and calculate it through MATLAB simulation, the 5th order origin moments are \[0.4679, 0.2683, 0.1737, 0.1229, 0.0931\].

According to the derivation process of the maximum entropy, the MATLAB program is written to find the optimal solution of the unknown parameter, and the numerical solution of the unknown parameter of the maximum entropy distribution can be obtained as

\[
\lambda_1 = -0.9705, \, \lambda_2 = -0.519, \, \lambda_3 = 0.034, \, \lambda_4 = -0.00878, \, \lambda_5 = 0.00002
\]

Use MATLAB software to design a program to plot its probability density function, and obtain the probability density function curve of the perpendicularity data of the vertical launcher rail in the X direction under the constraint of the fifth moment, as shown in Figure 3.

![Figure 3. X-direction maximum entropy function distribution diagram](image)

Similarly, we can use this method to calculate the verticality data of the Z direction of the vertical launcher rail. The expression of the probability density function of the maximum entropy distribution function is

\[
f(x) = \exp(-1.0422 + 0.2189x - 0.4733x^2 + 0.0096x^3 - 0.0087x^4 + 0.00002x^5)
\]

Similarly, use MATLAB to plot its probability density function to obtain the probability density function curve of the verticality data of the vertical launcher rail in the Z direction under the constraint of the fifth moment, as shown in Figure 4.

![Figure 4. X-direction maximum entropy function distribution diagram](image)

From the above probability density function curve, it can be seen that the accuracy error data distribution of the vertical launcher rail is similar to the normal distribution to a certain extent, which also verifies that the verticality error distribution obeys the normal distribution to a certain extent. It is
of great significance for us to study the regular characteristics of errors. The essence of the maximum entropy method is: when we want to predict the probability distribution of random events, the prediction must meet all known conditions and make no subjective assumptions about the unknown. The biggest difference between it and the histogram is that it does not make subjective inferences. In this case, the calculated probability distribution is the most uniform.

Generally, the maximum entropy solution is detached. When the data is insufficient, the solution must match the known data and make the least assumptions about the unknown part. According to the entropy concentration principle, we can infer that most of the values will be concentrated around the maximum entropy. Therefore, the prediction using the maximum entropy method is very accurate, and the solution obtained by the maximum entropy method meets the consistency requirements. In addition, the maximum entropy can also give a very similar estimate of random phenomena and fit the parameters of the probability density function, which also has important guiding significance for actual data statistical analysis projects.

5. Summary
This paper uses the finite element model that combines the cabin and the launcher. At the same time, according to the force environment on the sea, a certain amount of error data is obtained through ANSYS simulation. At the same time, we have fitted the distribution function and probability density curve of the error data based on the principle of maximum entropy, and verified that the probability density curve of the error data is similar to the normal distribution to a certain extent. This calculation and fitting result is more in line with the objective reality, which has certain guidance for the next step of research on the accuracy control technology of the vertical launcher.

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