Novel Relationship among Spiral Arm Pitch Angles (p) and momentum parameter of the host spiral galaxies

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Abstract. In this study, we have found a novel relationship among spiral arm pitch angles (p) and momentum parameter of the host spiral galaxies. In this study, we measured the momentum parameter for specimen of Spitzer/IRAC 3.6 µm images of 41 spiral galaxies evaluated employing a relation(Mbulge σ*/c) where Mbulge is mass of the bulge and σ* is the stellar velocity dispersion. We have taken velocity dispersions (σ*) from the literature. In order to determine the spiral arm pitch angles. The selection of specimen of nearly face-on spiral galaxies and employ IRAF ellipse to indicate the ellipticity and major-axis position angle so as to deproject the images to face-on, employing 2D Fast Fourier Transform decomposition method. The specified bulge mass (Mbulge) using the virial theorem was include.

1. Introduction

Astronomers observed many types of black holes, which can be classified into three groups: stellar mass black holes, intermediate mass black holes (IMBHs) and supermassive black holes (SMBHs) [1]. The distinction between normal black holes and SMBHs is that normal black holes are believed to be the endpoint of stellar evolution for the most massive stars. The possible end products of stellar evolution are white dwarfs, neutron stars (usually observed as pulsars), and black holes. Stars that have masses greater than around 5 times the mass of the Sun may end up as a black hole [2]. A SMBH is the largest type of black hole [3].

It is becoming significantly clear that the energy output from SMBHs at the centers of galaxies play an important role to know the mechanism of nuclear energy and consistence and the estimation of SMBHs and their steward galaxies [4, 5]. In addition, the energy emitted by rising SMBHs plays a significant turn in the forming characteristic of galaxies shell and the properties of their host galaxy, possibly by being regulated by energy feedback from its center [6, 7, 8, 9].

Supermassive black hole in the center of almost all galaxies strongly correlates with the central stellar velocity dispersion (σ*) (MBH-σ*) [5,10,11] with the total stellar mass or brilliance of bulge (Mbulge), (Lbul) [12,13,14,15], for rotational velocity or dark matter halo [9], with the concentration of a galaxy's light [16], for bulge effective radius [13], the host bulge Sersic index [16,17], via
The conclusions are
demand

Many of previous studies have found the MBH–Mbulσ relevance using sundry independent galaxy specimen, with good positive results, meaning the (MBH–Mbulσ) relevance could be utilized as an indirect measure of the SMBH mass in the center of galaxies [25].

Astronomers have found that spiral arm pitch angle does not rely measurably on image waveband [26, 27].

The aim of this paper is the study of the cited scaling relations that involve bulge properties (MBH–Mσ, MBH–Mσ, Mσ–P and Mσ–P) in images of selected galaxies.

The paper is structured as follows. In the next section we briefly describe our sample, in Sect. 3 the methods are presented to estimate the bulge (stellar and dynamical) masses and estimate the momentum parameter, Section 4 is an analysis and discussion of the results. The conclusions are given in Sect. 5.

2. Sample

The sample in this research consists of selected galaxies using the Spitzer Space Telescope at 3.6µm. The major request was the momentum estimation parameter of conformable host galaxies (Mσ & Mσ) is an estimation of bulge mass and stellar velocity dispersion. The measure the bulge dynamical mass was verified by stratifying the isothermal model [28, 29] and measured bulge stellar mass using the calibration by [50]. The dispersion velocity of galaxy hosts was evaluated from old researchers as depicted in (table 1).

The specimen consists of Hubble types extended from Sa to Sc, so it is conceivable to measure pitch angle for every galaxy with slope (ranging from 25 to 65 degrees) using ELLIPSE in IRAF [27, 30].

Table 1. Galaxies parameters

| Name   | Leda Type | SMBH (Mσ) | P (deg.) | σ (km/sec) | Mσ(M⊙) | Mσ(M⊙) | Mσσ | Mσ | Mσ |
|--------|-----------|-----------|----------|------------|---------|---------|------|----|----|
| Circinus | Sb        | 6.418±0.1 | 26.7     | 75         | 9.67±0.19 | 9.72±0.1 | 3.87±0.2 | 3.88±0.13 |
| IC 2560 | SBB       | 7.469±0.2 | 16.3     | 137        | 11±0.4   | 10.7±0.2 | 5.73±0.4 | 5.43±0.02 |
| NGC 224 | Sb        | 7.79±0.23 | 8.5±1.3  | 160±8      | 10.6±0.5 | 10.7±0.3 | 5.45±0.2 | 5.56±0.03 |
| NGC 613 | Sbc       | 7.309±0.2 | 23.6±1.77 | 125±18.9   | 10.1±0.3 | 10.4±0.5 | 4.75±0.3 | 5.05±0.07 |
| NGC 1022 | SbA       | 6.902±0.3 | 19.8±3.6 | 99         | 10.1±0.17 | 10.10±14 | 4.54±0.3 | 5.41±0.04 |
| NGC 1068 | Sb        | 7.63±0.05 | 17.5±2.2 | 15±4       | 10.4±0.11 | 11.2±0.1 | 5.21±0.3 | 6.01±0.03 |
| NGC 1097 | SBB       | 7.627±0.18 | 16.7±2.62 | 150       | 10.6±0.13 | 10.8±0.4 | 5.40±0.4 | 5.60±0.06 |
| NGC 1300 | Sbc       | 7.56±0.17 | 17.1±1.8 | 145±22     | 10.5±0.08 | 10.6±0.1 | 5.27±0.4 | 5.37±0.09 |
| NGC 1350 | Sab       | 7.251±0.04 | 20.57±5.38 | 120.9±2.08 | 10.3±0.13 | 10.4±0.1 | 4.92±0.4 | 5.02±0.05 |
| NGC 1353 | Sb       | 6.594±0.13 | 36.6±5.4 | 83         | 9.11±0.73 | 9.43±0.3 | 3.39±0.3 | 3.72±0.08 |
| NGC 1357 | Sab       | 7.25±0.03 | 16.1±2.48 | 121±14     | 10.1±0.02 | 10.3±0.2 | 4.72±0.3 | 4.92±0.05 |
| NGC 1365 | Sb       | 7.639±0.07 | 15.4±2.4 | 151±20     | 10.3±0.25 | 10.4±0.2 | 5.1±0.17 | 5.21±0.02 |
| NGC 1398 | SBab      | 8.264±0.08 | 6.2±2.0 | 216±20     | 10.8±0.23 | 10.9±0.1 | 5.92±0.3 | 6.02±0.35 |
| NGC 1433 | SSB       | 6.615±0.05 | 25.8±2.79 | 84±9       | 9.5±0.034 | 10.2±0.4 | 3.79±0.4 | 4.58±0.07 |
| NGC 1566 | SAB       | 6.919±0.07 | 21.3±4.78 | 100±10     | 9.6±0.032 | 9.77±0.3 | 4.05±0.4 | 4.22±0.083 |
| NGC 1672 | Sb        | 7.388±0.14 | 18.2±14.07 | 130±8.2±0.9 | 10.1±0.57 | 9.93±0.3 | 4.79±0.6 | 4.63±0.036 |
| NGC 1808 | Sa        | 7.601±0.11 | 23.6±5.77 | 148       | 9.89±0.3 | 10.2±0.1 | 4.23±0.3 | 4.99±0.025 |
| NGC 2442 | Sbc       | 7.516±0.12 | 14.9±5.42 | 140.7±18.9 | 10.5±0.3 | 10.4±0.4 | 5.25±0.4 | 5.15±0.015 |
| NGC 3031 | Sab       | 7.544±0.04 | 15.4±8.6 | 143±7     | 10.7±0.14 | 10.9±0.1 | 5.57±0.3 | 5.66±0.01 |
| NGC 3227 | SABa      | 7.35±0.16 | 12.9±9 | 128±13     | 10.9±0.06 | 10.7±0.4 | 5.57±0.3 | 5.37±0.15 |
| NGC 3368 | SABa      | 7.267±0.06 | 14±1.4 | 122±(28.24) | 10.5±0.03 | 10.8±0.3 | 5.12±0.1 | 5.42±0.047 |
| NGC 3511 | SABc      | 6.803±0.07 | 28.2±2.27 | 93.5±6.04 | 9.5±0.13 | 9.58±0.1 | 3.90±0.4 | 3.97±0.096 |
| NGC 3521 | SABb      | 7.384±0.05 | 21.8±6.34 | 130±5.7.1 | 10.3±0.07 | 10.4±0.4 | 4.98±0.9 | 5.08±0.073 |
| NGC 3673 | Sb        | 7.2±0.011 | 19.3±4.38 | 117.45±2.07 | 10.3±0.08 | 10.3±0.1 | 4.89±0.2 | 4.89±0.008 |
NGC 3783 SAb 6.83±0.021 22.73±2.58(1) 95±10 9.31±0.03 9.42±0.2 3.72±0.3 3.835±0.35
NGC 3887 Sbc 6.95±0.04 24.4±2.6(2) 102.0±2.05* 9.75±0.08 9.63±0.3 4.22±0.2 4.1±0.051
NGC 4030 Sbc 7.54±0.06 19.8±3.2(2) 122.4±2.1* 10.7±0.08 10.9±0.3 5.33±0.4 5.53±0.037
NGC 4151 SABa 7.69±0.07 11.8±1.8 156±8 10.3±0.07 10.5±0.3 5.14±0.4 5.346±0.07
NGC 4258 SABb 7.58±0.012 7.7±4.2 146±15 10.8±0.18 11.2±0.7 5.58±0.4 5.988±0.08
NGC 4462 SABb 7.579±0.02 17.2±5.42(1) 146±8 10.6±0.07 10.7±0.2 5.38±0.2 5.485±0.08
NGC 4594 Sa 8.448±0.01 6.1 240±12 11.4±0.09 11.3±0.4 6.1±0.4 6.510±0.32
NGC 4699 SABb 8.25±0.05 6.2±2.2(1) 215±10 10.7±0.06 10.8±0.2 5.82±0.5 5.92±0.35
NGC 5054 Sbc 6.996±0.06 25.57±3.73(1) 104.48±2.05* 9.9±0.13 10.2±0.3 4.39±0.2 4.69±0.071
NGC 5055 Sbc 6.937±0.08 14.9±6.9 101±5 9.84±0.37 9.95±0.5 4.30±0.3 4.41±0.043
NGC 6300 SBB 6.81±0.05 24.3±3.8(1) 94±5 9.82±0.04 10±0.053 4.2±0.07 4.40±0.06
NGC 6744 SABb 7.11±0.07 21.2±3.8 112±25 10.3±0.19 10.4±0.5 4.85±0.2 4.858±0.09
NGC 6902 SABb 7.578±0.04 13.71±2.3(3) 145.86±2.1* 10.6±0.84 10.6±0.5 5.38±0.3 5.45±0.35
NGC 7213 Sa 7.993±0.03 7.05±0.28(1) 185±20 11±0.048 10.9±0.4 5.99±0.4 5.984±0.06
NGC 7531 SABb 7.065±0.09 18.3±9.09(1) 108.7±5.6 10.2±0.08 10.2±0.5 4.72±0.6 4.75±0.021
NGC 7582 SBBa 7.469±0.09 14.7±7.44(3) 137±20 10.7±0.08 10.9±0.5 5.43±0.5 5.61±0.082
NGC 7727 SABa 7.955±0.07 15.9±6.39(1) 181±10 10.9±0.06 11.1±0.4 5.87±0.3 6.07±0.079

3. Methods

3.1 Measurement of SMBHs using MBH-σ*

There are a variety of techniques for measuring supermassive black hole masses. For this study we have selected galaxies which have SMBH mass estimates and applied two correlations. We applied the correlation between supermassive black hole mass (M_{BH}) and host-galaxy bulge velocity dispersion (σ*) (M_{BH-σ*}) [10, 11, 18].

The M_{BH}-σ* relation supports the notion of regulated formation mechanisms and co-evolution for the galaxy’s central black hole mass and the bulge velocity dispersion [9, 31, 32].

The M_{BH-σ} relation is one of the most common techniques used to estimate the mass of SMBH at the center of a spiral galaxy [13]. This is done by measuring the velocity dispersion of stars in the galactic bulge. This method was based on the observation that supermassive black hole masses correlate with the dispersion velocity of the surrounding stellar component (bulge) of spiral galaxies [10, 11, 17, 18]. Because BH masses found in late-type spirals and spheroidal galaxies have lower mass, we used the M_{BH-σ} relation since it has the least scatter [33, 34].

The velocity dispersion (σ) of classical bulges and pseudobulges in spiral galaxies were converted to SMBH masses using the following relation [5]:

\[ M_{BH}(σ*) = 10^{8.13±0.06}(\frac{σ}{200} km s^{-1})^{4.0±0.32} \]  

(1)

Using selected galaxies noticed with Spitzer at 3.6μm along with MBH-σ* relation, SMBH masses were determined. Stellar velocity dispersions were obtained in host galaxies from a literature search, and using the correlation between (V_c) and (σ*) (where V_c and σ* measured in (km s^{-1}) [9].

\[ logV_c = (0.84±0.09) logσ + (0.55±0.19) \]  

(2)

This correlation is a good link between SMBHs and dark matter haloes [9].

3.2 Measuring SMBHs using spiral arm pitch angle

One of the more interesting methods to find SMBH masses in late-type galaxies use the relationship between SMBH mass in the nuclei of disk galaxies and spiral arm pitch angle (P) (a measure of the tightness of spiral structure) [30, 35]. Seigar et al. (2008) found that SMBHs are linked by a strong correlation with P. Additionally, a correlation between P and rotation curve shear (S) was discovered [26, 40, 42].

Previous studies described logarithmic spiral in polar coordinates [27, 36, 37, 38, 39, 40, 41, 61]. This is a special kind of spiral curve that describes the arm in disk galaxies:

\[ r = r_0 e^{θ tan(φ)} \]  

(3)

Where r is radius, θ is central angle, r_0 is initial radius when θ = 0, and pitch angle is -90 ≤ φ ≤ 90.
Because the spiral arm pitch angle has been shown to be independent of the wavelength at which it is measured, multi-band images were used to determine it for our sample of spiral galaxies [26].

Spiral arm pitch angles were measured using a two-dimensional fast Fourier transform (2DFFT) decomposition with logarithmic spirals of Spitzer/IRAC 3.6 μm images of 63 galaxies, with inclinations of $30^\circ \leq i \leq 60^\circ$. The 2DFFT program analyzes images of spiral galaxies and categorizes their pitch angles and number of arms. The two-dimensional fast Fourier transform decomposition program is fully described by [45].

The amplitude of each Fourier component is given by:

$$A(m, p) = \frac{\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} I_{i} e^{-i(2\pi m \phi + \pi n r)}}{\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} m(r, \theta)}$$

(4)

where $r$ and $\theta$ are polar coordinates, $I(lnr, \theta)$ is the intensity at position $(lnr, \theta)$, $m$ represents the number of arms or modes, and $p$ is the variable associated with the pitch angle $P$ defined by $P = \arccos b/a$.

IRAF was used to estimate ellipticity values and major-axis position angle in order to deproject 3.6 μm galaxy images to fully face-on by assuming circular outer isophotes. ELLIPSE in IRAF was used to derive inclination angle ($\alpha$); [43, 44], which is defined by:

$$\alpha = \arccos (b/a),$$

Where $a$ is the semi-major axis and $b$ is the semi-minor axis. Where the value $0^\circ$ describes a face-on galaxy and $90^\circ$ describes an edge-on galaxy.

Two-dimensional fast Fourier Transform [45] was then utilized the deprojected 3.6 μm images.

### 3.3 Measurement of the dynamical bulge mass

In this part the methods used to derive the dynamical bulge mass are described. Bulge’s dynamical mass ($M_{dyn}$) were determined utilizing the virial theorem, i.e. the virial bulge mass given by

$$M_{dyn} = \frac{KR_{e} \sigma_{e}^{2}}{G}$$

(5)

Where $K$ is a model dependent dimensionless constant [28] or is function of the Sérsic index n. [29, 46]. $K$ is a constant throughout the galaxy in the isothermal model, and its value is determined numerically, $K=3$ [47], 8/3 [13], 3 [18] and 5 [29], we follow Sani (2011) to use $K = 5$ instead of 8/3 or 3.

Where $K$ is function of the Sérsic index $n$ [13] as in [28, 29], $\sigma_{e}$, $R_{e}$ and $G$ are host-galaxy bulge velocity dispersion, bulge effective radius and gravitational constant respectively.

### 3.4 Estimation of stellar mass ($M_{*}$)

Many researchers [13, 29, 48] have extensively used the J, I, and K band luminosity to compute stellar mass by assuming a mass-to-light ratio $M/L$ [48]. Bell & de Jong (2001, 2003) [48, 49] established a relation between optical colors (e.g., B−R, B−V).

These former studies of optical colors of disk galaxies do not supply $\gamma$ values for the Spitzer/IRAC bands, thus they couldn’t employed. A novel relevance was used to gain $\gamma$ in 3.6-μm Spitzer/IRAC. Such relevance is amidst $\gamma^k$ and $\gamma$ in the 3.6-μm waveband was notified by Se-Heon Oh [50]:

$$\gamma^{3.6} = B^{3.6}x \gamma^k A^{3.6}$$

(6)

Where $A^{3.6} = -0.05$ and $B^{3.6} = 0.92$

A relevance among the $(\gamma^k)$ and optical colors is given by:

$$log_{10}(\gamma^k) = b^k \times \text{optical colors} + a^k$$

(7)

Where $a^k$ and $b^k$ are coefficients.

### 3.5 Measurement of the bulge luminosity ($L_{bulge}$)

The measurement of the bulge luminosity depend on a two-dimensional decomposition program to model Spitzer/IRAC 3.6 μm images [51]. The bulge luminosity was determined for a sample of 41 spiral galaxies by applying the two-dimensional multicomponent decomposition technique. In this style, an exponential function was employed to depict disc:

$$I_d(r) = I_{0d} e^{-r/r_h}$$,
Where $I_d$ is disc central surface density, $h_d$ is disc radial scalength, and $r$ is disc radius. Bulge is determined with Sersic function:

$$I_b(r_b) = I_{0b} \exp \left[ 1 - \left( \frac{r_b}{h_b} \right)^{n_b} \right]$$  \hspace{1cm} (8)

Where $I_{0b}$ is central surface density of bulge, $h_b$ is the scale parameter of the bulge, and $n_b$ is the Sersic index. The effective radius, $r_e$, of bulge was estimated by converting $h_b$, $r_e = (b_n)^{1/2}$.

Where value of $b_n$ is a proportionality constant defined such that $\tilde{A}(2n) = 2\tilde{a}(2n,b_n)$. $\tilde{A}$ and $\tilde{a}$ are the complete and incomplete gamma functions, respectively. Approximation $b_n \approx 2.17n_b - 0.355$ [51] was used, where $n_b$ is the bulge Sersic index.

The bars and ovals are evaluated using Ferrers or Sersic function:

$$I_{\text{bar}}(r_{\text{bar}}) = I_{0\text{bar}} (\left(1 - \frac{r_{\text{bar}}}{a_{\text{bar}}} \right)^{n_{\text{bar}}+0.5})^{n_{\text{bar}}+0.5} \begin{cases} r_{\text{bar}} < a_{\text{bar}} \\ r_{\text{bar}} > a_{\text{bar}} \end{cases}$$

Where $I_{\text{bar}}$ is the central surface brightness of the $\text{bar}$, $a_{\text{bar}}$ is the bar major axis, and $n_{\text{bar}}$ is the exponent of the bar model defining the shape of the bar radial profile.

First, foreground stars were removed and all point sources from the Spitzer 3.6 $\mu$m images were masked out by using SExtractor [52]. In order to change surface brightness units to mag arcsec$^{-2}$, the next form was used:

$$\mu_{3.6} = -2.5 \log_{10} \left[ \frac{S_{3.6 \mu m}x2.35x10^{-5}}{ZP_{3.6}} \right] \cdot 6 \mu m$$  \hspace{1cm} (10)

Where $S_{3.6 \mu m}$ is the flux value of the 3.6 $\mu$m band in units of MJy sr$^{-1}$, $ZP_{3.6 \mu m}$ is the IRAC zero magnitude flux density in Jy and is 280.9 [53].

Apparent magnitude was converted to absolute magnitude was convert using luminosity distance and absorption in the galaxies according to the NED$^4$ database.

4. Results and discussions

(Table 1) lists the SMBH masses, spiral arm pitch angles, the central stellar velocity dispersion, bulge dynamical masses, bulge stellar masses, and the momentum parameter of the dynamical and stellar bulge, respectively.

This work, was studied scaling relations describe trends that are observed between important physical properties (SMBH masses, spiral arm pitch angles, bulge dynamical masses, bulge stellar masses, and the momentum parameter of the dynamical and stellar bulge) of spiral galaxies. in this work, we studied the scaling relations describe trends that are observed between important physical properties (SMBH masses, spiral arm pitch angles, bulge dynamical masses, bulge stellar masses, and the momentum parameter of the dynamical and stellar bulge) of spiral galaxies. The studied relations were:

$$\log_{10} M_{BH} = b + m\log_{10}x$$
$$\log_{10} M_{bul} = b + m\log_{10}x$$

Where $b$ and $m$ are the intercept and the slope of the relation, $x$ is a parameter of the bulge or spiral arm pitch angle. We used ‘equations (1, 4 and 5)’ to predict the values of $M_{BH}$, $M_{bul}$, $M_{bul}\sigma$ in other galaxies once we know the value of $x$.

In this study, the ordinary linear regression of $M_{BH}$, $M_{bul}$, and $M_{bul}\sigma$ on $x$ were performed for the spiral galaxies.

‘Figures 1’ and ‘figure 2’ show the SMBH masses as a function of $M_{dyn}\sigma$ and $M_{\sigma}$, for 41 galaxies respectively. Pearson’s linear correlation coefficient for a correlation between $M_{BH}+M_{dyn}\sigma$ and $M_{BH}-M_{\sigma}$ relations are 0.77 and 0.79, respectively. The slopes of these relations are 0.675 and 0.686.
respectively, meaning that no a considerable variation amidst $M_{\text{BH}}$-$M_{\text{dyn}}\sigma$ relation and $M_{\text{BH}}$-$M_{\text{s}}\sigma$ relation.

The fitting results of the $M_{\text{BH}}$-$M_{\text{dyn}}\sigma$ and $M_{\text{BH}}$-$M_{\text{s}}\sigma$ correlations are presented in (table 4) agrees with [25, 54, 55].

From the results of the Pearson’s linear correlation coefficient and the significance level at which the null hypothesis of zero correlation is disproved is $3\sigma$. This means that the masses of BHs in the nuclei of disk galaxies can be determined directly from a measurement of their momentum parameter.

![Figure 1](image1.png)

**Figure 1.** SMBH masses from $M_{\text{BH}}$-$\sigma$ relation via the momentum parameter ($M_{\text{dyn}}\sigma$).

![Figure 2](image2.png)

**Figure 2.** SMBH masses from MBH-$\sigma$ relation via momentum parameter ($M_{\text{s}}\sigma$).

As Seigar et al. (2008) have shown in [30], the slopes (0.07) and intercepts (8.44) of black hole-pitch angle correlation is very consistent with our results. This result agrees with [30, 35].

In ‘figure 3’ shows of the relation of spiral arm pitch angles and SMBH masses.
According to the results of this study, we can confident that for galaxies the pitch angle of the spiral arms should correlate well to the parts of galaxies coevolve with black hole, bulge luminosity, bulge mass, etc. The fitting result of $M_{BH}$-P correlation is shown in (table 4).

The relation of $M_{BH}$-P in this study agrees with that obtained in [30, 35]:

\[
\log_{10} M_{BH} = (8.44 \pm 0.1) - (0.07 \pm 0.005) P \quad [30]
\]

Figures 4 and ‘figure 5’ depict SMBH masses versus $M_{dyn}$ and $M_s$ for spiral galaxy, where the masses were determined using ‘equations (4)’ and (5). Fitting results of $M_{BH}$-$M_{dyn}$ and $M_{BH}$-$M_s$ correlations are presented in (table 4).

Figure 3. SMBH mass from $M_{BH}$-$\sigma$ relation versus spiral arm pitch angle (P).

Figure 4. SMBH mass from $M_{BH}$-$\sigma$ relation versus bulge dynamical mass ($M_{dyn}$).

The following conclusion can be estimated from these figures that the best fitting line for $M_{BH}$-$M_s$ and $M_{BH}$-$M_{d}$ relations are shown in (table 4). These results contain data on galaxies with two types of bulges (classical bulges and pseudo-bulges).

Pearson's linear correlation coefficient for a correlation between $M_{BH}$-$M_{dyn}$ and $M_{BH}$-$M_s$ are 0.81 and 0.83, respectively, whereas the slope of the $M_{BH}$-$M_d$ and $M_{BH}$-$M_s$ relations are 0.68 and 0.67 respectively. Due to the Difference of $M^*/M_d$ ratio, the establishment of little difference between
values from both relations can be seen, which might be concerned with mass contribution from dark matter [21]. Astronomers shown that dynamical mass of bulges is dominated by the stellar mass, with a negligible contribution of dark matter and gas [56, 57].

‘Figure 4’ and ‘figure 5’ indicate that M_{dyn} and M_s correlated well with (M_{BH}). These results confirmed the result that [25, 55, 58] found a good correspondence between the values from both relations M_{BH}-M_d and M_{BH}-M_s for Spiral galaxies, due spiral galaxies following the same M_{BH}-σ* relation [34].

The fitting results of the relationship between bulge mass (M_{dyn} and M_s) and spiral arm pitch angle by the linear regression method are summarized in (table 4).

The introduce of M_{dyn} - P and M_s - P relations for selected galaxies are illustrated in Figures 6 and 7.

Our data have shown the concerned SMBHs in the evolution, or co-evolution, of their host galaxies (bulge dynamical mass, bulge stellar mass, spiral arm pitch angle,..ect.) [13, 59].

Recently discovered an important relation between the SMBHs mass and the spiral arm pitch angle (M_{BH}–P) relation by [30], whereas [24] found the relation between SMBH mass and M_{dyn}. ‘Figures 8’ and ‘figure 9’ illustrated the momentum parameter of the host spiral galaxies versus spiral arm pitch angle for 41 spiral galaxies. In all the figures, the solid line is fit to spiral galaxies.

The momentum parameter of the host spiral galaxies (M_{dyn}σ and M_sσ) correlate with P. It is very clear that there is a correlation concerning M_{dyn}σ and M_sσ with P. The fitting results of M_{dyn}σ–P, M_sσ–P correlations are shown in (table 4).

The comparison between our result and other in works were shown in (table 2).

**Table 2.** A comparison with previous studies

| Relation       | a          | B          | r  | References |
|----------------|------------|------------|----|------------|
| MBH-Md         | -1.64±2.55 | 0.87±0.25  | 0.68 | [60]       |
| MBH-P          | -9.01±1.96 | 1.58±0.10  | 0.68 | [30, 35]   |
| MBH-Mσ2        | 8.21±0.16  | -0.062±0.009 | -0.81 | 99.7%       |
|                | 4.55±0.8   | 0.75±0.22  | 0.68 | [30]       |

**Table 3.** Regression result for the selected galaxies

| Relation       | b          | m          | r  | |
|----------------|------------|------------|----|---|
| M_BH - M_d     | -1.22 ± 0.03 | 0.68 ±0.05  | 0.76 | |
| M_BH - M_s     | -1.006 ± 0.06 | 0.675 ±0.07 | 0.79 | |
| M_BH - P       | 8.373 ± 0.5  | -0.056 ± 0.002 | -0.68 | |
| M_BH - M_σ     | 1.006 ± 0.04  | 0.675 ± 0.03  | 0.79 | |
| M_BH - M_σ     | 1.22 ± 0.05  | 0.68±0.04  | 0.76 | |
| M_d - P        | 13.76 ± 0.17  | -0.069 ± 0.006 | -0.63 | |
| M_σ - P        | 13.76 ± 0.22  | -0.077 ± 0.003 | 0.71 | |
| M_dσ - P       | 13.76 ± 0.46  | -0.069 ± 0.0049 | -0.63 | |
| M_σ - P        | 13.76 ± 0.044  | -0.077 ± 0.0065 | 0.71 | |

**Table 4.** Scaling relation for log M•= b + mlogx for the selected galaxies

| Relation       | The best fitting line for the relation |
|----------------|---------------------------------------|
| M_BH - M_d     | log_{10} M_{BH} = (1.22 ± 0.03) + (0.68 ± 0.05)log_{10} (M_d) |
| M_BH - M_s     | log_{10} M_{BH} = (1.006 ± 0.06) + (0.675 ± 0.07)log_{10} (M_s) |
| M_BH - P       | log_{10} M_{BH} = (8.373 ± 0.5) + (0.056 ± 0.02)P |
| M_BH - M_σ     | log_{10} M_{BH} = (1.006 ± 0.04) + (0.675 ± 0.03)log_{10} (M_σ) |
Figure 5. SMBH mass from MBH-σ relation versus bulge stellar mass (Ms).

Figure 6. Bulge dynamical masses versus spiral arm pitch angle.
Figure 7. Bulge stellar masses versus spiral arm pitch angle.

Figure 8. Momentum parameter for bulge dynamical masses versus spiral arm pitch angle.
5. Conclusion
From isothermal model and the calibration by Se-Heon Oh [50], we calculated the bulge dynamical and stellar masses in 41 spiral galaxies. We used $M_{\text{dyn}}\sigma$ and $M_\sigma$ to calculate the momentum parameter for the bulge stellar masses and the bulge dynamical mass.

We have gained preferable-fit lines of four relevance. Through them, we have establish $M_{\text{BH}} - M_{\text{dyn}}$, $M_{\text{BH}} - M_\sigma$, $M_{\text{BH}} - P$, $M_{\text{BH}} - M_{\text{dyn}}\sigma$, $M_{\text{BH}} - M_\sigma$, $M_{\text{dyn}} - P$, $M_\sigma - P$, $M_{\text{dyn}} - P$, and $M_\sigma - P$ have a linear correlation coefficient 0.76, 0.79, 0.68, 0.79, 0.76, 0.71, 0.63, 0.63 and 0.71 respectively. Moreover, the momentum parameter in the bulge correlates well with pitch angle too. We conclude that pitch angle is perfect tool to specify indirect measurements of the dynamical bulge mass, stellar bulge mass, and momentum parameter in bulges. As well, these results underscore the importance of further understanding the role of the host galaxies in the growth of SMBHs.

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