Decaying dark matter mimicking time-varying dark energy

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A ΛCDM model with dark matter that decays into inert relativistic energy on a timescale longer than the Hubble time will produce an expansion history that can be misinterpreted as stable dark matter with time-varying dark energy. We calculate the corresponding spurious equation of state parameter, \( \tilde{w}_\phi \), as a function of redshift, and show that the evolution of \( \tilde{w}_\phi \) depends strongly on the assumed value of the dark matter density, erroneously taken to scale as \( a^{-3} \). Depending on the latter, one can obtain models that mimic quintessence (\( \tilde{w}_\phi > -1 \)), phantom models (\( \tilde{w}_\phi < -1 \)) or models in which the equation of state parameter crosses the phantom divide, evolving from \( \tilde{w}_\phi > -1 \) at high redshift to \( \tilde{w}_\phi < -1 \) at low redshift. All of these models generically converge toward \( \tilde{w}_\phi \approx -1 \) at the present. The degeneracy between the ΛCDM model with decaying dark matter and the corresponding spurious quintessence model is broken by the growth of density perturbations.

I. INTRODUCTION

Cosmological data from a wide range of sources including type Ia supernovae \(^1\), the cosmic microwave background \(^2\), baryon acoustic oscillations \(^3\), cluster gas fractions \(^4\) \(^5\) and gamma ray bursts \(^6\) \(^7\) seem to indicate that at least 70% of the energy density in the universe is in the form of an exotic, negative-pressure component, called dark energy. (See Ref. \(^8\) for a comprehensive review). A parameter of considerable importance is the equation of state (EoS) of the dark energy component, defined as the ratio of its pressure to its density:

\[
w = p_{\text{DE}}/\rho_{\text{DE}}.\tag{1}
\]

If the dark energy is due to a cosmological constant, then \( w \) is constant and exactly equal to \(-1 \). A value of \( w = -1 \) is consistent with current observations \(^9\) \(^10\). On the other hand, a variety of models have been proposed in which \( w \) is time varying. Perhaps the simplest model for time-varying \( w \) is to take the dark energy to be due to a scalar field, dubbed "quintessence" \(^11\) \(^12\) \(^13\). As the data continue to improve, it is important to be able to distinguish models (such as quintessence) with a true time-varying \( w \) from ΛCDM models which can effectively mimic such a time variation.

To motivate our investigation, consider first the simple ΛCDM model, with present-day dark matter and cosmological constant densities of \( \rho_{\text{DM0}} \) and \( \rho_\Lambda \), respectively. (Zero subscripts will be used throughout to denote present-day values of cosmological quantities). If one does not know \emph{a priori} the value of \( \rho_{\text{DM0}} \) or the fact that the dark energy is a pure cosmological constant, then part of the dark matter density can be absorbed into the dark energy, producing (erroneously) a time-varying dark energy. In particular, the ΛCDM model with \( \rho_{\text{DM0}} \), \( \rho_\Lambda \) is degenerate with the quintessence model having a present-day dark matter density \( \rho_{\text{DM0}} \neq \rho_{\text{DM0}} \), and a time-varying dark energy component with density \( \rho_\phi = \rho_\Lambda + \rho_{\text{DM}} - \rho_{\text{DM0}} \) and equation of state

\[
\tilde{w}_\phi = -\frac{\rho_\Lambda}{\rho_\Lambda + (\rho_{\text{DM0}} - \rho_{\text{DM}})(1 + z)^3}.\tag{2}
\]

(Since our paper deals with spurious measurements of a time-varying equation of state, we will use tildes throughout to deal with spurious/unphysical/mismeasured quantities; non-tilded variables will refer to true physical quantities). This degeneracy has been exhaustively explored in Refs. \(^20\) \(^23\), who pointed out that it cannot be resolved without independent knowledge of the dark matter density.

On the other hand, if observational data led to an equation of state for the dark energy that mimicked the evolution of \( w_\phi \) given in Eq. \(^24\), Occam’s razor would suggest that the correct model for the universe was actually ΛCDM, with the appropriately different value for the dark matter density. It is of interest to determine if there are any less obvious spurious time-varying equations of state for the dark energy that arise from simple variations on ΛCDM.

One such model is ΛCDM with decaying dark matter. If the dark matter decays into inert radiation, and one interprets the measured behavior of \( H(z) \) under the incorrect assumption that the dark matter is stable, then the best fit to the observations will be a quintessence model with a time-varying equation of state. This occurs because the nonstandard time-variation in both the dark matter and inert radiation densities gets absorbed into time variation of the dark energy. This was first noted by Ziaeepour \(^24\), who argued that it leads, in general, to a mismeasured value of the equation of state parameter satisfying \( \tilde{w}_\phi < -1 \). (Note that similar \( \tilde{w}_\phi < -1 \) behavior occurs when energy can be exchanged between dark matter and dark energy \(^25\), but this is a different class of models than those discussed here).

In this paper we reexamine the behavior of such models, and provide an improved calculation of \( \tilde{w}_\phi(z) \). We show that the behavior of \( \tilde{w}_\phi(z) \) is very sensitive to the assumed value of \( \rho_{\text{DM}} \). Different choices for \( \rho_{\text{DM}} \) can lead to models that mimic quintessence, phantom-like models, or models that cross the phantom divide. In the next section, we present the calculation of \( \tilde{w}_\phi(z) \), and we discuss our results in Sec. III.
II. THE EFFECTIVE EQUATION OF STATE

We consider a ΛCDM Universe which consists of baryonic matter with density \( \rho_B \), dark matter with density \( \rho_{DM} \), and cosmological constant with density \( \rho_\Lambda \). The dark matter is assumed to decay at a rate \( \Gamma \) into invisible relativistic energy with density \( \rho_R \). By “invisible”, we mean that the relativistic decay products are assumed to interact very weakly with ordinary matter. If the decay products did interact, e.g., electromagnetically, they would be easily detectable at the lifetimes of interest here, and the model would already be ruled out.

The equations for the evolution of matter and the decay-produced radiation are the following:

\[
\begin{align*}
\dot{\rho}_{DM} &= -3H\rho_{DM} - \Gamma\rho_{DM}, \\
\dot{\rho}_R &= -4H\rho_R + \Gamma\rho_{DM}, \\
\dot{\rho}_B &= -3H\rho_B,
\end{align*}
\]

where \( H \) is the Hubble parameter given by the Friedman equation

\[ 3H^2 = 8\pi G (\rho_{DM} + \rho_B + \rho_R + \rho_\Lambda), \]

and dots denote time derivatives. We ignore “ordinary” radiation, since it has a negligible effect on the expansion rate in the redshift regime relevant to this problem.

The existence of such decays with lifetimes comparable to the Hubble time can be constrained by their effects on the CMB, large-scale structure, and SN Ia observations \[^{26–29}\]. We will confine our attention to lifetimes satisfying the constraint given in Ref. \[^{28}\]: \( \Gamma^{-1} > 100 \text{ Gyr} \), or \( \Gamma t_0 < 0.15 \), where \( t_0 \) is the present-day age of the universe.

The \( \rho_R \) term in the expression for \( H \), as well as the nonstandard evolution of \( \rho_{DM} \), will clearly lead to an effective equation of state different from the ACDM cosmology. Observers unaware of the decaying nature of the dark matter might try to explain the time-varying equation of state with the help of a quintessence field \( \phi \) in addition to ordinary non-decaying dark matter. In their model, (denoting the density of presumed non-decaying dark matter by \( \tilde{\rho}_{DM} \)) the same expansion rate \( H \) will be given by:

\[ 3H^2 = 8\pi G (\tilde{\rho}_{DM} + \rho_B + \rho_R + \rho_\Lambda), \]

Equating Eq. (6) and Eq. (7), one can readily deduce the energy density of this fictitious scalar field in terms of the densities of matter and decay-produced radiation as:

\[ \tilde{\rho}_\phi = \rho_\Lambda + \rho_R + \rho_{DM} - \tilde{\rho}_{DM}. \]

From this, it is straightforward to determine the effective equation of state of this fictitious dark energy component from the relation \(-3(1 + \bar{w}_\phi) = d\ln \tilde{\rho}_\phi / d\ln a \) (and also using equations (3) and (4)):

\[ \bar{w}_\phi = \frac{\rho_R/3 - \rho_\Lambda}{\rho_\Lambda + \rho_R + \rho_{DM} - \tilde{\rho}_{DM}}. \]

Note that \( \bar{w}_\phi \) does not depend explicitly on \( \rho_B \). However, there is an implicit dependence, since we assume a flat geometry with \( \Omega = 1 \), so that \( \Omega_{DM} + \Omega_\Lambda + \Omega_R = 1 - \Omega_B \). (In this paper, we will use \( \Omega_{DM}, \Omega_\Lambda, \ldots \) to refer only to present-day values, so we drop the zero subscript in these cases).

Consider first the qualitative behavior of \( \bar{w}_\phi \). The solutions to Eqs. (3) and (4) can be written in terms of the scale factor \( a(t) \) as \[^{30}\]

\[
\begin{align*}
\rho_{DM} &= \rho_{DM0} \left( \frac{a}{a_0} \right)^{-3} \exp \left(-\Gamma(t-t_0)\right), \\
\rho_R &= \rho_{DM0} \left( \frac{a}{a_0} \right)^{-4} \times \int_0^t \left( \frac{a(t')}{a_0} \right) \exp \left(-\Gamma t'\right) \Gamma dt',
\end{align*}
\]

while the fictitious non-decaying dark matter density evolves, of course, as

\[ \tilde{\rho}_{DM} = \tilde{\rho}_{DM0} \left( \frac{a}{a_0} \right)^{-3}. \]

Clearly, the behavior of \( \bar{w}_\phi \) depends on the assumed value of \( \tilde{\rho}_{DM0} \). Using SN Ia data alone (or more generally, the behavior of \( H(z) \) alone) there is no best-fit value for the dark matter density, once the equation of state of the dark energy is allowed to be a free function of \( z \), just as in the case of non-decaying dark matter discussed earlier \[^{20–23}\], and the best one can do is to derive the behavior of \( \bar{w}_\phi \) as a function of \( \tilde{\rho}_{DM0} \). Of course, there are other cosmological measurements that constrain the dark matter density. For instance, large-scale structure provides a constraint on the present-day dark matter density, while the CMB constrains the dark matter density at high redshift, but taken together these limits are consistent with a change in the comoving dark matter density as large as 15%, as noted previously \[^{29}\]. However, it is reasonable to assume that an observer erroneously postulating stable dark matter would derive a value for \( \tilde{\rho}_{DM0} \) that lies somewhere between the values obtained by taking \( \tilde{\rho}_{DM} = \rho_{DM} \) at high redshift (\( \Gamma t \ll 1 \)) or by taking \( \tilde{\rho}_{DM} = \rho_{DM} \) at the present day (\( t = t_0 \)). Equating \( \rho_{DM} \) from Eq. (10) and \( \tilde{\rho}_{DM} \) from Eq. (12) at \( t \to 0 \) and \( t = t_0 \) then gives the plausible bounds on \( \tilde{\rho}_{DM0} \):

\[ \rho_{DM0} \leq \tilde{\rho}_{DM0} \leq \rho_{DM0} \exp(\Gamma t_0), \]

where the upper bound corresponds to equality between \( \tilde{\rho}_{DM} \) and \( \rho_{DM} \) at high redshift, while the lower bound assumes equality today.

To parametrize this uncertainty, we will take

\[ \tilde{\rho}_{DM0} = \rho_{DM0} \exp(\Delta \Gamma t_0), \]

where \( 0 \leq \Delta \leq 1 \). Here \( \Delta = 0 \) corresponds to setting the (spurious) stable dark matter density equal to the
decaying dark matter density at the present, while $\Delta = 1$ corresponds to equality $t \to 0$.

To gain some qualitative insight into the behavior of $\bar{w}_\phi$, it is necessary to derive an approximation to the decay-produced radiation density in Eq. (11). Note that we always have $\exp(-\Gamma t) \approx 1 - \Gamma t$ for our observational bound of $\Gamma_0 < 0.15$. This approximation can be used to integrate Eq. (11) in the matter dominated era ($z \gtrsim 1$) to yield

$$\rho_R = \frac{3}{5} \Gamma t \rho_{DM0}(a/a_0)^{-3}. \quad (15)$$

This result becomes progressively less accurate as the cosmological constant begins to dominate at late times, but it will be sufficient to provide qualitative insight into the evolution of $\bar{w}_\phi$.

Substituting the expressions for $\rho_{DM}$, $\bar{\rho}_{DM}$ and $\rho_R$ from Eqs. (10), (12) and (15), respectively, into Eq. (9), and expanding out to lowest order in $\Gamma t$ and $\Gamma_0$ (both of which are $\ll 1$), we obtain

$$\bar{w}_\phi = \frac{-\rho_\Lambda + \frac{1}{5} \Gamma t \rho_{DM0}(1+z)^3}{\rho_\Lambda + [(1-\Delta)\Gamma_0 - \frac{2}{5} \Gamma t] \rho_{DM0}(1+z)^3}. \quad (16)$$

In order to express $\bar{w}_\phi$ as a function of redshift alone, we make the approximation that the expansion law never diverges very far from $\Lambda$CDM (see, e.g., Ref. [17]). In this case, the cosmic time $t$ is related to the redshift as

$$t(z) = t_\Lambda \sinh^{-1} \left[ \frac{\Omega_\Lambda(1-\Omega_\Lambda)^{1/2}}{(1+z)^{3/2}} \right], \quad (17)$$

where $t_\Lambda$ is a constant with units of time that is related to the energy density of the cosmological constant by

$$t_\Lambda = \sqrt{1/(6\pi G \rho_\Lambda)}. \quad (18)$$

Eq. (17) will be a good approximation as long as $\bar{w}_\phi$ is close to $-1$ whenever the dark energy dominates the expansion. In the models we investigate here, $w_\phi$ can significantly diverge from $-1$ only at early times, when the expansion is dark-matter dominated, so we expect Eq. (17) to be sufficiently accurate for our purposes. Then Eq. (16) can be written as

$$\bar{w}_\phi = \frac{-\Omega_\Lambda + (1/5) \Gamma t_0 f(z)(1-\Omega_\Lambda)(1+z)^3}{\Omega_\Lambda + [(1-\Delta) - (2/5)f(z)] \Gamma_0 (1-\Omega_\Lambda)(1+z)^3}. \quad (19)$$

where $f(z)$ is the fractional age of the universe at redshift $z$, given by

$$f(z) \equiv \frac{t(z)}{t_0} = \frac{\sinh^{-1} \left[ \frac{\Omega_\Lambda(1-\Omega_\Lambda)^{1/2}}{(1+z)^{3/2}} \right]}{\tanh^{-1} \left( \sqrt{\Omega_\Lambda} \right)}. \quad (20)$$

In Figs. 1-2, we show the exact evolution of $\bar{w}_\phi$, derived by numerical integration of Eqs. (3) and (11), along with our approximation given by Eqs. (19) and (20), for different values of $\Gamma_0$, $\Delta$, and $\Omega_\Lambda$. We see that our analytic approximation is quite accurate for all of the cases examined.

### III. DISCUSSION

Eqs. (19) and (20) yield several important insights into the behavior of $\bar{w}_\phi$. First, we note that all of our models converge to $\bar{w}_\phi \to -1$ at late times. This is obvious from the fact that $\rho_\Lambda$ is always the dominant component at late times. However, the way in which $\bar{w}_\phi$ evolves with redshift is extremely sensitive to the assumed value of $\bar{\rho}_{DM}$ through its dependence on $\Delta$. For small values of $\Delta$ (i.e., taking $\bar{\rho}_{DM}$ nearly equal to the present-day value of $\rho_{DM}$) the spurious time-varying dark energy evolves as a quintessence component with $\bar{w}_\phi > -1$, with $\bar{w}_\phi$ decreasing with time. In the terminology of Ref. [31], the model mimics a “freezing” quintessence field.

For larger values of $\Delta$, the model crosses the phantom divide at some redshift $z_c$, with $\bar{w}_\phi > -1$ at $z > z_c$ and $\bar{w}_\phi < -1$ at $z < z_c$. An approximate value for $z_c$ can be derived from Eq. (19):

$$1 + z_c = \frac{\Omega_\Lambda}{(1-\Omega_\Lambda) \sinh^2[5(1-\Delta) \tanh^{-1}(\sqrt{\Omega_\Lambda})]}^{1/3}. \quad (21)$$

Note that $z_c$ is independent of $\Gamma$ (which is apparent in Figs. 1-2) and depends only on $\Delta$. The expression for the time $t_c$ at which $\bar{w}_\phi$ crosses $-1$ can be derived from Eq. (16) and takes the particularly simple form

$$t_c/t_0 = 5(1-\Delta). \quad (22)$$

From Eq. (22), it is obvious that these models cross the phantom divide whenever $0.8 < \Delta < 1$. (Strictly speaking, models with $\Delta < 0.8$ also cross the phantom divide, but this crossing takes place in the future, while $\Delta = 0.8$ gives $\bar{w}_\phi$ exactly equal to $-1$ at the present). Finally, for $\Delta = 1$ (i.e., setting $\bar{\rho}_{DM}$ equal to $\rho_{DM}$ at early times) the model behaves like a phantom, with $\bar{w}_\phi < -1$ at all times.

The $\Lambda$CDM model with decaying dark matter can be distinguished from the corresponding quintessence model with the same $H(z)$ using the growth of density perturbations. (For a general discussion of linear perturbation growth as a probe of dark energy, see, e.g., Refs. [14, 32–42]). The equation for the evolution of the density perturbation, $\delta$, in the linear regime ($\delta \ll 1$), well inside the horizon, is

$$\ddot{\delta} + 2H \dot{\delta} - 4\pi G(\rho_{DM} + \rho_B)\delta = 0, \quad (23)$$

where the dot denotes the derivative with respect to time, and we assume the baryons are decoupled from the photon background and can cluster freely. By construction,
FIG. 1: The evolution of the spurious equation of state parameter, $\tilde{w}_\phi$, in a $\Lambda$CDM model ($\Omega_\Lambda = 0.7$) with decaying dark matter, in which the dark matter is erroneously taken to be stable and the corresponding time variation is absorbed into a time-varying equation of state for the dark energy. Here $\Gamma$ is the decay rate of the dark matter, $t_0$ is the age of the universe, and $\Delta$ is determined by the assumed value of the dark matter density, erroneously taken to scale as $a^{-3}$. Broken lines denote the exact evolution and solid lines denote the analytic approximation given by Eqs. (19)-(20).

The growth rate in the spurious quintessence model will
be larger (smaller) than the corresponding growth rate in the \( \Lambda \)CDM model as \( \Gamma_t + (\Delta - 1)\Gamma_t^0 \) is greater than (less than) zero. In the limiting case \( \Delta = 1 \) the growth rate will be larger for the spurious quintessence model than for the decaying dark matter model over the entire evolution history, while the reverse is true for \( \Delta = 0 \). Independent of \( \Delta \), the ratio of the perturbation growth rate at late times to the growth rate at early times is smaller for the decaying dark matter model than for the corresponding spurious quintessence model with the same \( H(z) \). This result follows from the fact that the dark matter density decreases more rapidly with scale factor in the former model.

An obvious question is whether or not these results can ever be relevant. If dark matter really were unstable, with a lifetime much longer than the age of the universe, then it is certainly plausible that a cosmological signature of unstable dark matter would be detected first in some combination of CMB and large-scale structure observations. In this case, the information on decaying dark
matter would simply be incorporated into calculations for the equation of state of the dark energy, rendering our calculations moot. On the other hand, it is also plausible that such effects would be detected first in precision measurements of the dark energy equation of state, in which case our expressions for $\tilde{w}_\phi$ provide a useful guide to the sort of spurious signal produced by decaying dark matter.

These results can be generalized, in an obvious way, to quintessence models (or other dark energy models with a time-varying equation of state) with unstable dark matter.

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