Global quantum discord and quantum phase transition in XY model

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We study the relationship between the behavior of global quantum correlations and quantum phase transitions in XY model. We find that the two kinds of phase transitions in the studied model can be characterized by the features of global quantum discord (GQD) and the corresponding quantum correlations. We demonstrate that the maximum of the sum of all the nearest neighbor bipartite GQDs is effective and accurate for signaling the Ising quantum phase transition, in contrast, the sudden change of GQD is very suitable for characterizing another phase transition in the XY model. This may shed lights on the study of properties of quantum correlations in different quantum phases.

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I. INTRODUCTION

The recent development in quantum information theory [1] has provided much insight into quantum phase transitions [2]. In particular, using the quantum correlations to investigate quantum phase transitions has drawn much attention and has been successful in characterizing a number of critical phenomena of great interest. For example, entanglements measured by concurrence, negativity, geometric entanglement, von Neumann entropy, mutual information and quantum discord are studied in several critical systems [3–6]. From the previous literature, we know that the concurrence shows a maximum at the critical points of the transverse field Ising model and XY model [4], and the von Neumann entropy diverges logarithmically at the critical point [3]. The quantum critical phenomena in the XY model can also be characterized by the divergence of the concurrence derivative or the negativity derivative with respect to the external field parameter [5,6]. Furthermore, recent studies show that entanglement spectra can be used to describing quantum phase transitions [6,11]. The structure of the correlations is shown to be related with the quantum critical phenomena [12,16]. On the other hand, fidelity and the fidelity susceptibility of the ground state can also be used as a good tool for detecting numerous phase transition points in some critical systems [12,14]. Notably, the methods from quantum information may also play a key role for the topology of many-body system and the phase transition of only one spin [17,19].

Additionally, the quantum discord can be used to describing the quantum phase transitions in some critical systems [20,23]. It is worth noting that some of the investigations performed so far have indicated that quantum discord is more sensible than entanglement in revealing quantum critical points [23], even for systems that are not at zero temperature [25]. As we all know, there are several multipartite promotions of quantum discord [26–31]. The global quantum discord (GQD) proposed by Rulli and Sarandy is an widely accepted one [31], which can be seen as a generalization of symmetric bipartite quantum discord. There is an interesting question that if the GQD can be used as a good tool to characterizing the quantum phase transitions in the typical critical systems. On the contrary, there is another interesting question that if our understanding of the quantum phase transitions can tell us some useful information of the behavior of GQD in the critical systems. Fortunately, the answers of these questions are yes.

In this paper, we investigate the relationship between the behavior of global quantum discord and quantum phase transitions in the XY model [32,33]. Considering the local convertibility, the phase diagram of XY model can be divided into three phases, which we label phase 1A, phase 1B and phase 2 [32]. The are two kinds of quantum phase transitions in XY model, the phase transition between phase 1A and phase 1B and the second order phase transition between phase 1 and phase 2. In order to provide a good description of these quantum phase transitions, we analysis the behavior of total global quantum discord (GQD), the sum of all the bipartite GQDs and the residual GQD [34]. We will show that the sum of all the nearest neighbor bipartite GQDs is effective and accurate for signaling the Ising quantum phase transitions between phase 1 and phase 2. Moreover, it is worth noting that the sudden change of GQD is very suitable for characterizing the phase transitions between phase 1A and phase 1B. On the other hand, since GQD can be seen as a kind of physical resource for quantum information processing, the nature of these quantum phase transitions can tell us some useful features of it, such as the maximum points and sudden changes.

The paper is organized as follows: In Sec. II, we introduce the XY model and the definitions of GQD, the sum of all the nearest neighbor bipartite GQDs and the residual GQD. In Sec. III, we study the behavior of these quantum correlations and provide some figures of the three kinds of correlations and the corresponding derivative, it shows that these correlations can be used to characterizing the quantum phase transitions effectively and accurately. At the same time, using the nature

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of quantum phase transitions, we can get some useful features of the behavior of global quantum discord. In Sec. IV, we give some conclusions and discussions.

II. THE BRIEF INTRODUCTION OF THE XY MODEL AND GLOBAL QUANTUM DISCORD

We study quantum phase transitions in one dimensional XY-model \( [35] \) by using the method from quantum information theory. The Hamiltonian for our model is as follows:

\[
H = -\sum_{i=0}^{N-1} \left\{ \frac{J}{2} \left[ (1 + \gamma) \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + (1 - \gamma) \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y \right] + h \hat{\sigma}_i^z \right\},
\]

with \( N \) being the number of spins in the chain, \( \hat{\sigma}_i^m \) the \( i \)th spin Pauli operator in the direction \( m = x, y, z \) and periodic boundary conditions assumed. The XX model and transverse field Ising model thus correspond to the special cases for this general class of models. For the case that \( \gamma \to 0 \), our model reduces to XX model. When \( \gamma = 1 \), the model reduces to transverse field Ising model. For simplify, here we take \( J = 1 \) and the parameter \( h \) is associated with the external transverse magnetic field. For \( h = 1 \), a second-order quantum phase transition takes place for any \( 0 \leq \gamma \leq 1 \). In fact, there exists additional structure of interest in phase space beyond the breaking of phase flip symmetry at \( h = 1 \). It’s worth noting that there exists a circle, \( h^2 + \gamma^2 = 1 \), on which the ground state is fully separable. According to the previous literature, this circle can be seen as a boundary between two differing phases which are characterized by the presence and absence of parallel entanglement \( [30, 39] \). In fact, for each fixed \( \gamma \), the system is only locally non-convertible when \( h^2 + \gamma^2 > 1 \). Now we can divide the system into three separate phases, phase 1A, phase 1B and phase 2, where the ferromagnetic region is now divided into two phases defined by their differential local convertibility. These results are summarized in “phase-diagram” Fig.1. Consideration of differential local convertibility separates the XY model in three phases, which we label phase 1A, phase 1B and phase 2. The phase transition from phase 1 to phase 2 is the second-order phase transition. The green points around the critical line are the critical points \( h_c \) obtained by our method as \( N \to \infty \). Phase 1 has two distinct regions A and B, the boundary is a quarter of a circle, \( h^2 + \gamma^2 = 1 \). The phase transition from phase 1A to phase 1B can be seen as a first-order phase transition when we consider the GQD as a kind of order parameter. The blue points on the boundary are the critical points obtained by our method. It is obvious that our method is very accuracy for characterizing the phase transitions.

For understanding the relationship between the global quantum correlations and quantum phase transitions in XY-model, we study the behavior of global quantum discord carefully. The global quantum discord is a measure of multipartite quantum correlation, which can be seen as a symmetric generalization of bipartite quantum discord to multipartite cases. As a well-defined multipartite quantum correlation, the GQD is always non-negative and symmetric with respect to subsystem exchange. Considering its applications, GQD has been shown to be useful in many areas, such as quantum communication and quantum phase transitions \( [31, 40–43] \). In detail, GQD can play a role in quantum communication, in the sense that its absence means that the quantum state simply describes a classical probability multi distribution. That is to say, it allows for local broadcasting of correlations \( [41] \). On the other hand, the global discord has been proved to be useful in the characterization of quantum phase transitions. In previous literature, the behavior of GQD in some typical critical systems, such as the Ashkin-Teller spin chain, transverse field Ising model, open-boundary XX model and cluster-Ising model have been studied \( [42, 43] \). It is worth noting that both of the transverse field Ising model and XX model can be seen as a special case of XY model, the nature of quantum phase transitions is more complicated in this case. So there are two interesting questions: (1) How the behavior of GQD is like in this model; (2) If the global discord can be used to describing the complicated and richness critical phenomenon in this model. Fortunately, the answers are yes.

In order to study the behavior of global discord in our model, we first review the definition of it. The definition of global quantum discord is a generalization of bipartite symmetric quantum discord. Consider a \( N \)-partite system \( A_1, A_2, \ldots, A_N \) (each of them is of finite dimension), the GQD of state \( \rho_{A_1 A_2 \cdots A_N} \) is defined as follows:

\[
D(A_1 : \cdots : A_N) = \min_{\Phi} \left\{ I(\rho_{A_1 \cdots A_N}) - I(\Phi(\rho_{A_1 \cdots A_N})) \right\},
\]

where \( \Phi(\rho_{A_1 \cdots A_N}) = \sum_k \Pi_k \rho_{A_1 \cdots A_N} \Pi_k \), with \( |\Pi_k = \Pi_{A_1} \otimes \cdots \otimes \Pi_{A_N}^0 | \) representing a set of local measurements and \( k \) denoting the index string \( (j_1, \ldots, j_N) \). In Eq. \( 2 \), the multipartite mutual information \( I(\rho_{A_1 \cdots A_N}) \) and \( I(\Phi(\rho_{A_1 \cdots A_N})) \) are given by

\[
I(\rho_{A_1 \cdots A_N}) = \sum_{k=1}^{N} S(\rho_{A_k}) - S(\rho_{A_1 \cdots A_N}),
\]

\[
I(\Phi(\rho_{A_1 \cdots A_N})) = \sum_{k=1}^{N} S(\Phi(\rho_{A_k})) - S(\Phi(\rho_{A_1 \cdots A_N})),
\]
particles is defined as \( GQD \), we can define two corresponding multipartite correlations \[34\]. The sum of GQDs between two nearest neighbor particles is defined as

\[
D( A_1 : A_2 ) + D( A_2 : A_3 ) + \cdots + D( A_{L-1} : A_L ),
\]

\[5\]

\( L \) is the size of our system. This correlation represents all nearest neighbor bipartite GQDs contained in the critical system. Then, similar to the definition of tangle as a measure of residual multipartite entanglement, we can define the residual GQD corresponding to the second monogamy relation,

\[
D_R^L \equiv D( A_1 : \cdots : A_L ) - \sum_{i=1}^{L-1} D( A_i : A_{i+1} ).
\]

\[6\]

It is a measure for residual multipartite quantum correlation, namely, contributions to quantum correlation beyond pairwise GQD. This measure of residual multipartite quantum correlation describes the total quantum correlation except for all nearest neighbor interaction of quantum correlations. In most cases, since the bipartite GQDs do not increase under the discard of subsystems, the second monogamy relation holds. That is to say, the residual GQD is greater than or equal to zero. In other words, the system contains non-zero long-range correlation.

In order to calculate the global quantum correlations mentioned above, we first reformulate GQD as \[43\]:

\[
D( A_1 : \cdots : A_L ) = \min_{\{I^l\}} \left\{ \sum_{j=1}^{L} \sum_{l=0}^{j-1} \rho_{jl}^ll \log_2 \rho_{jl}^ll - \sum_{k=0}^{L-1} \rho_{kk}^ll \log_2 \rho_{kk}^ll \right\} + \sum_{j=1}^{L} S(\rho_{jl}) - S(\rho_{jl})\]

\[7\]

with \( \rho_{jl}^ll = \langle k| \tilde{R}^l_j \rho_{jl} \tilde{R}^l_j |k\rangle \) and \( \rho_{jl}^ll = \langle l| \tilde{R}^l_j \rho_{jl} \tilde{R}^l_j |l\rangle \), where \( \tilde{R}^l = R^l |k\rangle \langle k| R^l \) are the multi-qubit projective operators. Here \( |k\rangle \) are separable eigenstates of \( \bigotimes_{j=1}^{L} \sigma_j \), and \( R \) is a local \( L \)-qubit rotation: \( \tilde{R} = \bigotimes_{j=1}^{L} \tilde{R}_j(\theta_j, \phi_j) \) with \( \tilde{R}_j(\theta_j, \phi_j) = \cos \theta_j \hat{I} + i \sin \theta_j \cos \phi_j \hat{x} + i \sin \theta_j \sin \phi_j \hat{\sigma}_x \) acting on the \( j \)-th qubit.

This formula greatly reduces the computational efforts needed to evaluate GQD.

III. THE BEHAVIOR OF GLOBAL QUANTUM DISCORD AND QUANTUM PHASE TRANSITIONS IN THE XY MODEL

In this section, we analysis the behavior of the three kinds of multipartite quantum correlations mentioned above in XY model. In order to see the features of these correlations more clearly, we provide some figures of them and their derivatives. It shows that the property of these quantum correlations can be applied to characterizing the two kinds of quantum phase transitions effectively and accurately. From another perspective, since GQD can be considered as a kind of physical resource in
quantum information processing, our knowledge about quantum phase transitions can also help us to understand and predict the behavior of these correlations better. That is to say, the study of quantum critical systems can help us make a better understanding of the quantum correlations and quantum information processing.

Now we consider the nature of these quantum correlations as $h$ changes at the zero temperature similar as [34]. Fig.2 shows the total GQD, the sum of all nearest neighbor bipartite GQDs $D(A_1 : A_2) + D(A_2 : A_3) + \cdots + D(A_{L-1} : A_L)$ and the residual GQD $D(A_1 : \cdots : A_L) - \sum_{i=1}^{L-1} D(A_i : A_{i+1})$ as a function of $h$, when $\gamma = \sin \theta$, ($\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$). The figures in each column correspond to a specific angle; from first column to last column, the $\theta$ increases. The figures in first line are about GQD, the figures in second line show the sum of all nearest neighbor bipartite GQDs, the figures in last line are corresponding to the residual GQD. When $\theta = 90^\circ$, our model reduces to the transverse field Ising model. We study rings with $L = 3, \cdots, 10$, from bottom to top.

First of all, we consider the case that $\gamma = \sin 75^\circ = 0.966$ as shown in the last column of Fig.2. It shows that the sum of all nearest neighbor bipartite GQDs reaches a maximum at nearly the critical point $h = 1$, which is more suited to be used to describing the second-order quantum phase transition than the total GQD. If we consider GQD as a resource for quantum information processing, these figures tell us that we can get the most resource for quantum information and computation tasks when the second-order quantum phase transition occurs. That is to say, in order to obtain more physical resource for quantum information tasks from this system, we just need to adjust the external transverse magnetic field parameter $h$ at the critical point. On the other hand, the phase 1 ($h \leq 1$) can be divided into two phases, phase 1A and phase 1B. The boundary is a circle $h^2 + \gamma^2 = 1$, on which the ground state is fully separable. From these figures, we find an interesting fact that the GQD sudden changes in some points. In fact, the sudden change which occurs at $h = \cos 75^\circ = 0.259$ can characterize the phase transition between phase 1A and phase 1B very accurately, since this point is just on the boundary $h^2 + \gamma^2 = 1$. From another point of view, when we observed a phase transition like this, there must be a sudden change occurs. In other words, we can get a sudden change of these global quantum correlations just by adjusting corresponding parameter to a appropriate value. If we consider GQD as a kind of order parameter, the phase transition can be seen as a first-order phase transition. Now we know that the GQD can be used to detecting both the first-order and second-order phase transitions in our model.

The fourth column of Fig.2 shows the case that $\gamma = \sin 60^\circ = 0.866$. Similar as the case that $\theta = 75^\circ$, the sudden change which occurs at $h = \cos 60^\circ = 0.5$ can characterize the first-order phase transition very perfectly. The $\sum_{i=1}^{L-1} D(A_i : A_{i+1})$ still reaches a maximum at nearly critical point $h = 1$, which detecting the second-order phase transition accurately.

From these figures, we can find that there are many sudden changes of GQD, only the rightmost sudden change characterizes the phase transition between phase 1A and phase 1B. Other sudden changes reflect the level-crossings that redefine the ground state of the system (which are evident from the spectrum of the model). It is obvious that the sudden change will be more apparent when we consider the smaller angle $\theta$ or smaller system size $L$. That is to say, we can use this method to detect the quantum phase transitions easier and more effective in these cases. As $\theta$ decreases, the two kinds of critical points become closer.

In particular, when $\theta = 0^\circ$, our model reduces to XX model. In this case, the two kinds of phase transitions both occur at $h = 1$. For low magnetic field the GQD displays a step-wise behavior, jumps occurring in correspondence of the level-crossings that redefine the ground state of the system (which are evident from the spectrum of the model). That is, GQD tracks the structural changes in the ground state of the spin system as $h$ varies. Comparing all these figures, we find that as $\theta$ decreases, the GQD curves become more complicated. When we consider the case that $\theta$ is large enough, the maximum of the sum of all nearest neighbor bipartite GQDs can characterize the second-order phase transition accurately. On the other hand, the phase transition between phase 1A and phase 1B can be described by the rightmost sudden change of GQD. When $\theta$ is small enough, both phase transitions can be characterized by the rightmost sudden change of GQD. The sudden change will be more apparent when we consider the smaller angle $\theta$ or smaller system size $L$.

To get rid of the finite-size effect in the phase transitions between phase 1A and phase 2, we exemplify the scaling analysis with the cases that $\theta = 45^\circ$ and $\theta = 60^\circ$ in Fig.3. From Fig.3 (a), we find that when $\theta = 45^\circ$, the critical point labeled $h_c$ obtained by our method tends to 0.995 in the thermodynamic limit. The data can be fit to $h_c = -1.033 \times \exp(-L/2.5666) + 0.955$ with accuracy 0.00922. From Fig.3 (b), we find that when $\theta = 60^\circ$, the critical point labeled $h_c$ tends to 1.020 as $N \to \infty$. The data can be fitted exponentially as $h_c = -1.202 \times \exp(-L/2.294) + 1.020$ with accuracy 0.00263. For the large system, the critical point obtained by our method is very accurate.

In order to see the sudden change more clearly, we show the figures of the derivative of the sum of all nearest neighbor bipartite GQDs in Fig.4. We give 6 figures about the cases that $\theta$ equals to 15 degree and 30 degree. The first column is the case that $\theta = 15^\circ$ and the second column is for $\theta = 30^\circ$. For
for system size form 3 to 5. In each case, we give three figures of the derivative of the sum of all nearest neighbor bipartite GQDs. FIG. 4: Derivative of the sum of all nearest neighbor bipartite GQDs.

The first column is the case that \( \theta = 15^\circ \) and the second column is for \( \theta = 30^\circ \). In each column from top to bottom, we investigate the system size form 3 to 5.

each case, we give three figures of \( L = 3, 4, 5 \). The first line is about \( L = 3 \), the second line is about \( L = 4 \), the last line is corresponding to \( L = 5 \). It shows that when the sudden changes of the sum of all nearest neighbor bipartite GQDs occur, the corresponding derivative exhibited bizarre behavior. Based on our discussion above, only the rightmost peak of the derivative characterizes the phase transition between phase 1A and phase 1B, other peaks reflect the level-crossings that redefine the ground state of the system. On the contrary, when we observed a phase transition like this, there must be a peak of the derivative occurs. This result can be used to predicting the sudden change of the corresponding correlation. It is obvious that for the same angle, the peak is more obvious for small \( L \).

In general, when we consider the different angles, the peak is more obvious for small \( \theta \).

IV. CONCLUSIONS AND DISCUSSION

In this paper, we analyzed the global quantum discord and quantum phase transitions in quantum critical systems. We developed a proposal to describe the quantum phase transitions in quantum critical systems by examining the behavior of global quantum discord. We applied this proposal to the study of XY model. For the Ising phase transition between phase 1 and phase 2, we find that the sum of all nearest neighbor bipartite GQDs is effective and accurate for signaling the phase transition point since it always reaches a maximum just around the critical point. From another perspective, since GQD can be considered as a resource for quantum information processing, our result tells us that we can get most resource for quantum information and computation tasks when the second-order quantum phase transition occurs. In other words, in order to obtain more physical resource for quantum information tasks from this system, we just need to adjust the corresponding parameters to appropriate values. On the other hand, for the phase transition between phase 1A and phase 1B, it shows that the sudden change of GQD is very suitable for detecting the critical point of this first-order phase transition. On the contrary, when we observed a phase transition like this, there must be a sudden change occurs. That is to say, we can get a sudden change of these global quantum correlations just by creating a first-order phase transition in this model. In detail, only the rightmost sudden change characterizes the phase transition between phase 1A and phase 1B, other sudden changes reflect the level-crossings that redefine the ground state of the system. It shows that the sudden change will be more apparent when we consider the smaller angle \( \theta \) or smaller system size \( L \). That is to say, this method can be used to detecting the quantum phase transition more easier and effective in these cases. When \( \theta \) is small enough, both phase transitions can be characterized by the rightmost sudden change of GQD. In order to see the sudden change more clearly, we provide some figures of the derivative of the sum of all nearest neighbor bipartite GQDs.

Our proposal may help further understanding both of the complicated phenomena in quantum critical systems and the behavior of global quantum correlations. This paper would initiate extensive studies of quantum phase transitions from the perspective of quantum information processing. On the other hand, it also shows that the nature of quantum phase transitions can be applied to predicting the behavior of corresponding quantum correlations. Since GQD is still meaningful for mixed states, this simple but effective method is able to study finite-temperature phase transitions, which is superior to other measure of quantum correlations. Our method is also worth applying to other quantum critical systems.

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