Electron and Ion Heating during Magnetic Reconnection in Weakly Collisional Plasmas

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Abstract

Gyrokinetic simulations of magnetic reconnection are presented to investigate plasma heating for strongly magnetized, weakly collisional plasmas. For a low plasma beta case, parallel and perpendicular phase mixing strongly enhance energy dissipation yielding electron heating. Heating occurs for a long time period after a dynamical process of magnetic reconnection ended. For a higher beta case, the ratio of ion to electron dissipation rate increases, suggesting that ion heating (via phase-mixing) may become an important dissipation channel in high beta plasmas.

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I. INTRODUCTION

Magnetic reconnection is a commonly observed fundamental process in both astrophysical and fusion plasmas. It allows topological change of magnetic field lines, and converts the free energy stored in the magnetic field into various forms of energy, such as bulk plasma flows, heating of plasmas, or non-thermal energetic particles. One of the key issues of magnetic reconnection is energy partition, yet it has not been widely studied. Here, we focus on plasma heating—how much free magnetic energy is transformed into the thermal energy during magnetic reconnection? To address thermodynamic properties of plasmas, inter-particle collisions are especially important even though plasmas are weakly collisional in many environments where magnetic reconnection occurs.

In weakly collisional plasmas, various kinetic effects play crucial roles. Landau damping is one of the well known examples of such kinetic effects, in which nearly synchronous particles with waves absorb energy from the waves. The phase mixing effect of the Landau damping process creates progressively oscillatory structures in velocity space. Such small scale structures suffer strong collisional dissipation as the collision operator provides diffusion in velocity space. As a result, as long as collisions are sufficiently infrequent, the rate of energy dissipation stays finite and is independent of collision frequency. Similar phase mixing is also induced by a finite Larmor radius (FLR) effect. In strongly magnetized plasmas, particles undergo drifts (dominantly the $E \times B$ drift) in the perpendicular direction to the mean magnetic field. Gyro-averaging of the fields will give rise to spread of the drift velocity of different particles, hence will lead to phase mixing in the perpendicular direction. Unlike linear parallel phase mixing of Landau damping, the FLR phase mixing process causes damping proportional to the field strength, and only appears nonlinearly (nonlinear phase mixing). Linear and nonlinear phase mixing have been studied numerically in electrostatic turbulence. The importance of the phase mixing mechanism for energy dissipation in solar wind turbulence has been verified in. Numerical simulations of magnetic reconnection using a hybrid fluid/kinetic model have shown strong electron heating via linear Landau damping for low-$\beta$ plasmas ($\beta$ is the ratio of the plasma to the magnetic pressure).

In this paper, we present gyrokinetic simulations of magnetic reconnection using AstroGK. We follow to setup a tearing instability configuration, and extend it to the
nonlinear regime. Even though the collision frequencies we choose are sufficiently small so that reconnection occurs in the collisionless regime, we show that significant collisional dissipation resulting in background plasma heating occurs via phase-mixing. We also consider the heating ratio of electrons and ions, and its dependence on plasma $\beta$.

II. PROBLEM SETUP

We consider magnetic reconnection of strongly magnetized plasmas in a two-dimensional doubly periodic slab domain. We initialize the system by a tearing unstable magnetic field configuration (see [9, 10] for details). The equilibrium magnetic field profile is

$$\mathbf{B} = B_{z0} \mathbf{\hat{z}} + B_{eq}^{\parallel}(x) \mathbf{\hat{y}}, \quad B_{z0} \gg B_{eq}^{\parallel},$$

(1)

where $B_{z0}$ is the background guide magnetic field and $B_{eq}^{\parallel}$ is the in-plane, reconnecting component, related to the parallel vector potential by $B_{eq}^{\parallel}(x) = -\partial A_{eq}^{\parallel}/\partial x$, and

$$A_{eq}^{\parallel}(x) = A_{eq}^{\parallel0} \cosh^{-2} \left( \frac{x - L_x/2}{a} \right) S_h(x).$$

(2)

($S_h(x)$ is a shape function to enforce periodicity [9].) $A_{eq}^{\parallel}$ is generated by the electron parallel current to satisfy the parallel Ampère’s law. The equilibrium scale length is denoted by $a$ and $L_x$ is the length of the simulation box in the $x$ direction, set to $L_x/a = 3.2\pi$. In the $y$ direction, the box size is $L_y/a = 2.5\pi$. We impose a small sinusoidal perturbation to the equilibrium magnetic field, $\tilde{A}_\parallel \propto \cos(k_y y)$ with wave number $k_y a = 2\pi a/L_y = 0.8$, yielding a value of the tearing instability parameter $\Delta a \approx 23.2$.

We solve the fully electromagnetic gyrokinetic equations for electrons and ions using AstroGK [9]. The code employs a pseudo-spectral algorithm for the spatial coordinates $(x, y)$, and Gaussian quadrature for velocity space integrals. The velocity space is discretized in the energy $E_s = m_s v^2/2$ ($m_s$ is the mass and $s = i, e$ is the species label) and $\lambda = v_{\perp}^2/(B_{z0} v^2)$.

There are four basic parameters in the system: The mass ratio, $\sigma \equiv m_e/m_i$, the temperature ratio of the background plasma, $\tau \equiv T_0/e/T_0/e$, the electron plasma beta, $\beta_e \equiv n_0 T_0/e/(B_{z0}^2/2\mu_0)$ where $n_0$ is the background plasma density of ions and electrons and $\mu_0$ is the vacuum permeability, and the ratio of the ion sound Larmor radius to the equilibrium scale length $a$, $\rho_{se}/a \equiv c_{se}/(\Omega_{ci} a)$. The ion sound speed is $c_{se} = \sqrt{T_0/e/m_i}$, and the ion cyclotron frequency is $\Omega_{ci} = eB_{z0}/m_i$. Those parameters define the physical scales.
associated with the non-magnetohydrodynamic (MHD) effects:

\[
\rho_i = \tau_{Si}^{1/2} \rho_{Se} \sqrt{2}, \quad d_i = \beta_{ei}^{-1/2} \rho_{Se} \sqrt{2}, \quad \rho_e = \sigma^{1/2} \rho_{Se} \sqrt{2}, \quad d_e = \beta_{ei}^{-1/2} \sigma^{1/2} \rho_{Se} \sqrt{2}. \quad (3)
\]

Collisions are modeled by the linearized, and gyro-averaged Landau collision operator in AstroGK \[11, 12\]. There are like-particle collisions of electrons and ions whose frequencies are given by \(\nu_{ee}\) and \(\nu_{ii}\), and inter-species collisions of electrons with ions given by \(\nu_{ei}\), which is equal to \(\nu_{ee}\) for the current parameters. The ion-electron collisions are subdominant compared with the ion-ion collisions. The electron-ion collisions reproduce Spitzer resistivity for which the electron-ion collision frequency and the resistivity (\(\eta\)) are related by \(\eta/\mu_0 = 380 \nu_{ei} d_e^2\). In terms of the Lundquist number, \(S = \mu_0 a V_A / \eta = 2.63 (\nu_{ei} \tau_A)^{-1} (d_e/a)^{-2}\), where \(V_A \equiv B_{ymax} / \sqrt{\mu_0 n_0 m_i}\) is the Alfvén velocity corresponding to the peak value of \(B_y\), and the Alfvén time \(\tau_A \equiv a/V_A\). We choose the electron collision frequency \(\nu_e = \nu_{ee} = \nu_{ei}\) to be sufficiently small so that macroscopic dynamics (e.g., the tearing mode growth rate) is independent of \(\nu_e\) (i.e., the frozen flux condition is broken by electron inertia, not collisions.).

III. SIMULATION RESULTS

A. Low-\(\beta\) case

As a reference case, we perform a simulation for low-\(\beta\) plasmas (\(\beta_e \sim m_e/m_i\)). In this case, the ion response is essentially electrostatic, and only electron heating matters. We take \(\beta_e = \sigma = 0.01, \sqrt{2} \rho_{Se}/a = 0.25, \tau = 1 (\beta_i = \beta_e)\), yielding \(\rho_i/a = d_e/a = 0.1 d_i/a = 10 \rho_e/a = 0.25\). Collision frequencies are \(\nu_{ee} \tau_A = \nu_{ii} \tau_A = 8 \times 10^{-5} (S \sim 5 \times 10^5)\). The spatial resolutions in the x, y directions are \(N_x = 256, N_y = 128\) subject to the 2/3 rule for dealiassing. The number of the velocity space collocation points \(N_x = N_E = 64\) are determined to resolve fine structures in velocity space.

To estimate plasma heating, we measure the collisional energy dissipation rate,

\[
D_s = - \int \int \left\langle \frac{T_{0s} h_s}{f_{0s}} \frac{\partial h_s}{\partial t} \right\rangle \text{coll} \, d\mathbf{r} d\mathbf{v} \geq 0. \quad (4)
\]

Without collisions, the gyrokinetic equation conserves the generalized energy consisting of the particle part \(E_s^p\) and the magnetic field part \(E_{\parallel,\perp}\)

\[
W = \sum_s E_s^p + E_{\perp}^m + E_{\parallel}^m = \int \left[ \sum_s \int \frac{T_{0s} \delta f_s^2}{2 f_{0s}} d\mathbf{v} + \frac{\left\| \nabla_A A_{\parallel} \right\|^2}{2 \mu_0} + \frac{\left\| \delta B_{\parallel} \right\|^2}{2 \mu_0} \right] d\mathbf{r} \quad (5)
\]
where $\delta f_s = - (q_s \phi / T_0) f_{0s} + h_s$ is the perturbation of the distribution function, $h_s$ is the non-Boltzmann part obeying the gyrokinetic equation, and the generalized energy is dissipated by collisions as $dW/dt = - \sum_s D_s$. The collisional dissipation increases the entropy (related to the first term of the generalized energy), and is turned into heat.

Figure 1 shows time evolutions of the reconnection rate measured by the electric field at the $X$ point $(x, y) = (L_x/2, L_y/2)$, the energy components normalized by the initial magnetic energy, and the collisional energy dissipation rate. The peak reconnection rate is fast $E_X/(V_A B_y^{\text{max}}) \sim 0.2$. During magnetic reconnection the magnetic energy is converted to the particle’s energies reversibly. First, the ion $E \times B$ drift flow is excited, thus the ion energy increases. Then, electrons exchange energies with the excited fields through phase mixing process. Electrons store the increased energy in the form of temperature fluctuations and higher order moments. Collisionally dissipated energy is about 1% of the initial magnetic energy after dynamical process has almost ended ($t/\tau_A = 25$). The energy dissipation starts to grow rapidly when the maximum reconnection rate is achieved. It stays large long after the dynamical stage, and an appreciable amount of the energy is lost in the later time. As has previously been reported in [2, 3], the collisional dissipation is independent of the collision frequency in the collisionless regime indicating phase mixing. The ion dissipation is negligibly small compared with the electron’s for the low-$\beta_e$ case.

To illustrate the phase mixing process, we show the parallel electron temperature fluctuation and electron distribution function in velocity space in Fig. 2 at $t/\tau_A = 10, 20$. We subtract the lower order moments corresponding to the density and parallel flow from $h$ to clearly see phase-mixing structures. The distribution functions are taken near the separatrix denoted by the cross marks in the left panels. In the earlier time of the nonlinear stage ($t/\tau_A = 10$), the distribution function only has gradients in the $v_\parallel$ direction indicating parallel phase mixing. We note that since there is no variation in the $z$ direction, parallel phase...
FIG. 2: Parallel electron temperature fluctuations and velocity space structures of the electron distribution function. Isolines of the magnetic flux are over-plotted with the temperature fluctuation. The distribution functions are taken near the separatrix.

mixing occurs along the perturbed field lines. Later, the perpendicular FLR phase mixing follows to create structures in the \( v_\perp \) direction although the effect seems weak because \( k_\perp \rho_e \) is small. The occurrence of the perpendicular phase mixing highlights the difference from [3]. About 10\% of the initial magnetic energy, which accounts for \( \sim 36\% \) of the released magnetic energy, is dissipated at \( 25 \leq t/\tau_A \leq 40 \). The rest of the released magnetic energy is used to excite electron temperature perturbations.

B. High-\( \beta \) case

In high-\( \beta \) plasmas, compressible fluctuations will be excited which are strongly damped collisionlessly (see, e.g., Sec. 6 of [14]). This may open up another dissipation channel where phase mixing of the ion distribution function ends up in ion heating. We compare the ratio
of the energy dissipation rate for ions and electrons $D_i/D_e$ for $\beta_e = 0.01, 0.1$ in Fig. 3. For $\beta_e = 0.01$ case, the ion dissipation rises prior to the electron dissipation, and the heating ratio peaks at about the same time as the peak reconnection rate. For the high-$\beta_e$ case, the reconnection rate is peaked around $t/\tau_A = 18$ because linear growth is slower, and it sharply drops around $t/\tau_A = 22$ where a small island like structure is generated at the X-point. $D_i/D_e$ for $\beta_e = 0.1$ peaks before the maximum reconnection rate, and reaches twice as high as that for $\beta_e = 0.01$. The ratio increases again from $t/\tau_A = 20$ because the electron dissipation starts to decrease. As anticipated, the heating ratio of ions to electrons increases with increasing $\beta_e$ though it is still small. The ion heating may be relevant only for much higher-$\beta_e$ plasmas.

IV. CONCLUSIONS

We have performed gyrokinetic simulations to investigate plasma heating during magnetic reconnection in strongly magnetized, weakly collisional plasmas. In the collisionless limit where the macroscopic behavior of plasmas is insensitive to collisions, parallel and perpendicular phase mixing create fine structures in velocity space leading to strong energy dissipation. We have shown that a significant amount of the initial magnetic energy is converted into electron heating for low-$\beta_e$ plasmas. The electron heating occurs after the dynamic reconnection process ceased, and continues for longer time. We observe perpendicular phase mixing as well as parallel phase mixing in the nonlinear regime. The low-$\beta_e$ case result is consistent with the previous study using the hybrid fluid/kinetic model in [3] except for the perpendicular phase mixing.

We have also shown a relatively high-$\beta_e$ case. For $\beta_e = \beta_i = 0.1$, the ion heating rate becomes larger compared with the $\beta_e = \beta_i = 0.01$ case although it is still small compared with that of electrons. The ion heating may be important only for much higher-$\beta$ plasmas.
Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 24740373. This work was carried out using the HELIOS supercomputer system at Computational Simulation Centre of International Fusion Energy Research Centre (IFERC-CSC), Aomori, Japan, under the Broader Approach collaboration between Euratom and Japan, implemented by Fusion for Energy and JAEA. NFL was supported by Fundação para a Ciência e Tecnologia (Ciência 2008 and Grant No. PTDC/FIS/118187/2010) and by the European Communities under the Contracts of Association between EURATOM and IST.

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