Neutron-skin thickness of $^{208}$Pb, and symmetry-energy constraints from the study of the anti-analog giant dipole resonance

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INTRODUCTION

There is a renewed interest in measuring the thickness of the neutron skin 1 3, 4, 5 because it constrains the symmetry-energy term of the nuclear equation of state. The precise knowledge of the symmetry energy is essential not only for describing the structure of neutron-rich nuclei, but also for describing the properties of the neutron-rich matter in nuclear astrophysics.

The symmetry energy determines to a large extent, through the Equation of State (EoS), the proton fraction of neutron stars 6, the neutron skin in heavy nuclei 7 and enters as input in the analysis of heavy-ion reactions 8. Furnstahl 9 demonstrated that in heavy nuclei an almost linear empirical correlation exists between the neutron-skin thickness (\( \Delta R_{pn} \)) and the isovector giant dipole resonance (IVGDR) 10, because it constrains the symmetry energy at saturation (\( J = 32.7 \pm 0.6 \text{ MeV} \)) and the slope of the symmetry energy (\( L = 49.7 \pm 4.4 \text{ MeV} \)), resulting in more stringent constraints than most of the previous studies.

The \( ^{208}\text{Pb}(\vec{p}, \vec{n} \gamma) \) \( ^{207}\text{Pb} \) reaction at a beam energy of 30 MeV has been used to excite the anti-analog of the giant dipole resonance (AGDR) and to measure its \( \gamma \)-decay to the isobaric analog state in coincidence with proton decay of IAS. The energy of the transition has also been calculated with the self-consistent relativistic random-phase approximation (RRPA), and found to be linearly correlated to the predicted value of the neutron-skin thickness (\( \Delta R_{pn} \)). By comparing the theoretical results with the measured transition energy, the value of 0.190 \( \pm 0.028 \) fm has been determined for \( \Delta R_{pn} \) of \( ^{208}\text{Pb} \), in agreement with previous experimental results. The AGDR excitation energy has also been used to calculate the symmetry energy at saturation (\( J = 32.7 \pm 0.6 \text{ MeV} \)) and the slope of the symmetry energy (\( L = 49.7 \pm 4.4 \text{ MeV} \)), resulting in more stringent constraints than most of the previous studies.

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The aim of the present work is to determine \( \Delta E_{AGDR-IAS} \) with high precision by measuring the energy of the corresponding \( \gamma \)-transition. The direct \( \gamma \)-branching ratio of the AGDR to the IAS is expected to be similar to that of the isovector giant dipole resonance (IVGDR) to the ground-state (g.s.) in the parent nucleus, which can be calculated from the parameters of the IVGDR 11.

THE ANTI-ANALOG GIANT DIPOLE RESONANCE AND ITS \( \gamma \) DECAY

Due to the isovector nature of the \((\vec{p}, \vec{n})\) reaction, the strength of the E1 excitation is distributed into \( T_0-1, T_0 \) and \( T_0+1 \) components, where \( T_0 \) is the g.s. isospin of...
the initial nucleus. The relevant Clebsch-Gordan coefficients \[14\] show that the $T_0-1$ component (AGDR) is favored compared to the $T_0$ and $T_0+1$ ones by factors of about $T_0$, and $2T_0^2$, respectively. According to the work of Osterfeld \[14\], the non-spin-flip transition is preferred at low bombarding energies below 50 MeV.

Dipole resonances were excited earlier at such low energies in the $^{208}\text{Pb}(p,n)$ reaction by Sterrenburg et al. \[15\], and Nishihara et al. \[16\] at $E_p = 45$ MeV and 41 MeV, respectively. However, it was shown experimentally \[17, 18\] that the observed $\Delta L = 1$ resonance was a superposition of all possible IVSGDR modes and the non-spin-flip dipole AGDR even at these low bombarding energies.

The expected $\gamma$-decay properties of the states excited in $^{208}\text{Bi}$ are shown in Fig. 1 together with the proton-decay branching ratios of the IAS \[19–21\]. The observed $\gamma$-ray branching ratio of the IVGDR to the g.s. of $^{208}\text{Pb}$ is about 1% \[13\]. In contrast, in the investigation of the electromagnetic decay properties of the IVSGDR to the low-lying Gamow-Teller (GT) states by Rodin and Dieperink \[22\] the $\gamma$-decay branching ratio was found in the range of $10^{-3}$.

**EXPERIMENTAL METHODS AND RESULTS**

The experiment, aiming at studying the neutron-skin thickness of $^{208}\text{Pb}$, was performed at the Oslo Cyclotron Laboratory (OCL) with 30 MeV proton beam bombarding a 5.5-ng/cm$^2$ thick, self-supporting metallic $^{208}\text{Pb}$ target and a 1 mg/cm$^2$ thick $^{12}\text{C}$ target for energy calibration.

In the experiment, the proton-decay of the IAS was used as a signature of the de-excitation of the IAS. The $\gamma$-transition from the decay of the AGDR was measured in coincidence with such proton lines. These particle-$\gamma$ coincidences were measured with the SiRi particle telescope and CACTUS $\gamma$-detector systems \[23, 24\]. The SiRi detectors were placed at backward angles, covering an angular range of $\Theta=126^\circ-140^\circ$ relative to the beam axis. The $\Delta E$ and $E$ detectors had thicknesses of 130 $\mu$m and 1550 $\mu$m, respectively. The CACTUS array consists of 28 collimated 5" × 5" NaI(Tl) detectors with a total efficiency of 15.2% for $E_\gamma = 1.33$ MeV.

A typical proton spectrum is shown in Fig. 2. The proton transitions populating the low-lying states in $^{207}\text{Pb}$ are marked by arrows and used for gating the $\gamma$ rays.

![FIG. 2. Proton energy spectrum measured in coincidence with the $\gamma$ rays.](image-url)

The energy of the $\gamma$ rays was measured in coincidence with the protons stemming from the decay of the IAS in $^{208}\text{Bi}$. The random coincidence contribution was subtracted as well as the contribution of the proton decay of the GTR, which represents a broad ($\Gamma \approx 2.9$ MeV) background in the proton spectrum.

The centroid of the $\gamma$ transition was shifted towards lower energies as a result of the decreasing efficiency of the NaI detectors. In order to correct this effect, the spectrum was normalized with the detector response function that was extracted experimentally in Refs. \[23, 24\]. The $\gamma$-ray energy spectrum, as a result of these corrections, is presented in Fig. 3 together with the statistical error.
safely against neutrons produced in the time-of-flight method could not be used to discriminate relatively close (d=22 cm) to the target. Therefore, the neutrons is constant as a function of energy. theoretically, and the response of the NaI detectors for MeV energies the neutron-capture cross section decreases drastically beyond 30 degrees. Since the smallest angle of the NaI detectors of the CACTUS setup was 39° with respect to the beam direction, the ejected neutrons did not disturb the γ-spectrum considerably.

Giant resonances (including the AGDR) decay also by neutrons, which are detected by CACTUS with high efficiency. However, such neutron emission goes to the low-lying states of 207Bi, and therefore such neutrons are not in coincidence with the proton-decay of the IAS in 208Bi. These neutrons contributed to the random coincidences only, which were subtracted.

Since the random coincidences in the proton-gated γ spectrum around $E_γ = 7$ MeV is dominated by neutrons, it can be used to eliminate the neutron-related events from the real coincidences by subtracting it with a weighting factor, which is defined by the ratio of the corresponding time windows. In the resulting $p - γ$ coincidence spectrum, the peak observed at 8 MeV represents γ-rays from the AGDR $→$ IAS transition only.

The energy distribution of the γ rays was fitted by a Gaussian curve and a second-order polynomial background as shown in Fig. 3. The obtained energy and width of the transition are $E_γ = 8.090 ± 0.013$ MeV and $Γ = 2.2$ MeV. However, the energy calibration of the CACTUS spectrometer has been performed with photo-peaks having significantly smaller width than giant resonances. In order to determine the real energy of the resonance, GEANT Monte-Carlo simulations were performed and convoluted with a Gaussian function with the width of the resonance. This analytical procedure caused a reduction of 10% in the position of the peak, which was taken into account when the final energy of the transition was extracted. As a result, the transition energy is $E_γ = 8.90 ± 0.02$ MeV including only the statistical error.

The contribution of the systematical error stems from the uncertainty of the energy calibration, which is estimated to be 1.0%, so the final transition energy is $E_{AGDR} - E_{IAS} = 8.90 ± 0.09$ MeV. The energy and width of the transition agree well with the previously measured values of Refs. [15, 16] but having significantly smaller error bars.

THEORETICAL ANALYSIS

The AGDR and IAS excitation energies are calculated with the self-consistent relativistic proton-neutron random-phase approximation (pn-RRPA) [27, 28] based on the Relativistic Hartree (RH) model [26]. As in our previous studies of the AGDR [10, 11], the calculation is based on family of density-dependent meson-exchange (DD-ME) interactions, for which the constraint on the symmetry energy at saturation density has been systematically varied: $J = 30, 32, 34, 36$ and $38$ MeV, and the remaining model parameters have been adjusted to accurately reproduce nuclear-matter properties (the saturation density, the compression modulus) and the binding energies and charge radii of a standard set of spherical nu-
the measured spectrum of in the energy interval above the IAS that corresponds to
For the excitation energy of the AGDR we take the corresponding RH predictions for the neutron-skin thickness.
the target nucleus, is plotted as a function of the corresponding g.s. neutron-skin thickness. In addition, the relativistic functional DD-ME2 \cite{30} will be also used in the calculation of the excitation energies of the AGDR and the IAS for the target nucleus \(^{208}\text{Pb}\) \cite{20}. These interactions were also used to calculate the electric dipole polarizability and neutron-skin thickness of \(^{208}\text{Pb}\), \(^{132}\text{Sn}\) and \(^{48}\text{Ca}\), in comparison to the predictions of more than 40 non-relativistic and relativistic mean-field effective interactions \cite{2}. From the results presented in that work one can also assess the accuracy of the present calculation.

From the comparison to the experimental result for \(E(\text{AGDR}) - E(\text{IAS})\) we deduce the value of the neutron-skin thickness in \(^{208}\text{Pb}\): \(\Delta R_{pn} = 0.190 \pm 0.028\) fm (including the 10\% theoretical uncertainty). In Table I this value is compared to previous results obtained with a variety of experimental methods.

In parallel with our work the neutron-skin thickness has been extracted from coherent pion photo-production cross sections \cite{32}. The half-height radius and diffuseness of the neutron distribution are found to be 6.77±0.03(stat) fm and 0.55±0.01(stat)±0.03(sys) fm respectively, corresponding to a neutron skin thickness \(R_{pn} = 0.19\pm0.03(\text{stat})\pm0.03(\text{sys})\) fm \cite{32}, which agrees very well with our results.

The very good agreement with all available data supports the reliability of the method employed in the present study.

![FIG. 4. The difference in the excitation energies of the AGDR and the IAS for the target nucleus \(^{208}\text{Pb}\), calculated with the pn-RRPA using five relativistic effective interactions characterized by the symmetry energy at saturation \(J = 30, 32, 34, 36\) and 38 MeV (squares), and the interaction DD-ME2 \((J = 32.3\) MeV) (star). The theoretical values \(E(\text{AGDR}) - E(\text{IAS})\) are plotted as a function of the corresponding g.s. neutron-skin thickness \(\Delta R_{pn}\), and compared to the experimental value \(E(\text{AGDR}) - E(\text{IAS}) = 8.90 \pm 0.09\) MeV.](image)

\[\alpha_D = \frac{8\pi}{9}e^2 m_{-1}\] (directly proportional to the inverse energy-weighted moment \(m_{-1}\)) for \(^{208}\text{Pb}\): \(\alpha_D = 20.8\) fm\(^3\), in agreement with the recently obtained experimental value: \(\alpha_D = (20.1 \pm 0.6)\) fm\(^3\) \cite{4}.

The results of the calculations for \(^{208}\text{Pb}\) are shown in Fig. 4. The difference in the excitation energies of the AGDR and the IAS, calculated with the pn-RRPA based on the RH self-consistent solution for the g.s. of the target nucleus, is plotted as a function of the corresponding RH predictions for the neutron-skin thickness. For the excitation energy of the AGDR we take the centroid of the theoretical strength distribution, calculated in the energy interval above the IAS that corresponds to the measured spectrum of \(\gamma\)-ray energies: \(E_\gamma = 6\) to 14.8 MeV (Fig. 3). A single peak is calculated for the IAS. For the effective interactions with increasing value of the symmetry energy at saturation \(J = 30, 32, 34, 36\) and 38 MeV (and correspondingly the slope of the symmetry energy at saturation \cite{31}), one notices a linear decrease of \(E(\text{AGDR}) - E(\text{IAS})\) with increasing values of the neutron skin \(\Delta R_{pn}\). The value calculated with DD-ME2 \((J = 32.3\) MeV) is denoted by a star.

The uncertainty of the theoretical predictions for the neutron-skin thickness is estimated around 10\%. This uncertainty was adopted for the differences between the neutron and proton radii for the nuclei \(^{116}\text{Sn}, \text{\text{124}}\text{Sn}, \text{\text{208}}\text{Pb}\), when the parameters of the effective interactions with \(J = 30, 32, 34, 36\) and 38 MeV, and DD-ME2 were adjusted \cite{29, 30}. These interactions were also used to calculate the electric dipole polarizability and neutron-skin thickness of \(^{208}\text{Pb}\), \(^{132}\text{Sn}\) and \(^{48}\text{Ca}\), in comparison to the predictions of more than 40 non-relativistic and relativistic mean-field effective interactions \cite{2}. From the results presented in that work one can also assess the accuracy of the present calculation.

![TABLE I. The value of the neutron-skin thickness of \(^{208}\text{Pb}\) determined in the present work compared to available data.](image)

| Method                    | Ref. | Date     | \(\Delta R_{pn}\) (fm) |
|---------------------------|------|----------|-------------------------|
| (\(p,p\)) 0.8 GeV         | \cite{33} | 1980     | 0.14 ± 0.04             |
| (\(p,p\)) 0.65 GeV        | \cite{34} | 1994     | 0.20 ± 0.04             |
| (\(\alpha,\alpha\)) IVGDR 120 MeV | \cite{13} | 1994     | 0.19 ± 0.09             |
| antiproton absorption     | \cite{35} | 2001     | 0.18 ± 0.03             |
| (\(\alpha,\alpha\)) IVGDR 200 MeV | \cite{36} | 2003     | 0.12 ± 0.07             |
| pygmy res.                | \cite{37} | 2007     | 0.180 ± 0.035           |
| pygmy res.                | \cite{38} | 2010     | 0.194 ± 0.024           |
| \((\bar{p},p)\)           | \cite{4}   | 2011     | 0.156 ± 0.025           |
| parity viol. \((e,e)\)    | \cite{1}   | 2012     | 0.33 ± 0.17             |
| AGDR                      | pres. res. | 2013     | 0.190 ± 0.028           |
CONCLUSIONS

In this study we have analyzed the γ decay of the AGDR to the IAS excited in the $^{208}\text{Pb}(p,\gamma\bar{p})^{207}\text{Pb}$ reaction. Using the experimental value obtained for the energy difference of the AGDR and the IAS, and comparing with the results of the RH+pn-RRPA model, we have been able to determine the corresponding neutron skin thickness in $^{208}\text{Pb}$: $\Delta R_{pn} = 0.190 \pm 0.028$ fm. The agreement between the present result and values obtained in previous experiments using different methods is very good. In particular, the value obtained here is in accordance with results of a very recent high-resolution study of electric dipole polarizability $\alpha_D$ in $^{208}\text{Pb}$ [1], the correlation analysis of $\alpha_D$ and $\Delta R_{pn}$ [2], as well as with the Pb Radius Experiment (PREX) that used parity-violating elastic electron scattering at JLAB [3].

The measured energy difference between the AGDR and the IAS has also been used to constrain possible values of the symmetry energy at saturation density (J), and the slope of the symmetry energy (L). We have found good agreement between constraints that result from the AGDR and $\alpha_D$, whereas the discrepancy with the constraint obtained from the pygmy resonance is probably due to the missing strength in PDR experiments [4]. Therefore, measurements of the AGDR might be important not only to constrain possible values of J and L, but also to understand differences between results obtained in various experiments. Since the mean values of J-L constraints obtained from the AGDR and $\alpha_D$ appear in excellent agreement, obviously the two very different collective modes of excitation in nuclei probe the same underlying physical content. The main advantage of the method based on the AGDR compared to the $\alpha_D$ analysis and most of the previous studies is that it provides more stringent constraints on the symmetry energy parameters.

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FIG. 5. Constraints on the slope L and magnitude J of the symmetry energy at saturation density from different experiments compared to our present result (AGDR).
