Abstract. After reviewing how the Borel-Weil-Bott theorem can be interpreted as an index theorem, we present a proof using Kostant’s cubic Dirac operator and the equivariant McKean-Singer formula.

The Borel-Weil-Bott theorem “completes” the representation theory of compact Lie groups by providing a cohomological method to construct each and every irreducible representation of a compact Lie group. The main constituents of this construction are the harmonic forms, that is, the homogeneous solutions to the Hodge-Dolbeault operator. Thus the Borel-Weil-Bott method is essentially an index theorem. This viewpoint underlies the work of Bott [2] and is manifest in Slebarski’s theorem [8, Thm. 1, p. 296]. In this article, we give a brief review on the interpretation of the Borel-Weil-Bott theorem as an index theorem, and prove it using Kostant’s cubic Dirac operator and the equivariant McKean-Singer formula.

1. Preliminaries

Throughout the article we write $X = G/T$, where $G$ is a compact connected Lie group and $T$ is a maximal torus of $G$. The Lie algebra of $G$, that is, the tangent space of $G$ at the identity $e$, shall be denoted by the lowercase black letter $\mathfrak{g}$. We endow $\mathfrak{g}$ with an inner product $\langle \cdot, \cdot \rangle$ by taking the negative of the Killing form. This inner product is invariant under the adjoint action of $G$ on $\mathfrak{g}$.

1.1. Some Basic Facts and Notations Surrounding the Representation Theory of Compact Lie Groups. Consider the conjugation action of $G$ on itself. The elements among $G$ that preserve $T$ constitute the normalizer $N_G(T)$ of $T$ relative to $G$. The quotient $W = N_G(T)/T$ is known as the Weyl group of $G$.

The action of $W$ on $T$ induces a $W$-action on functions on $T$; in particular, the irreducible characters of $T$. The irreducible characters of $T$ constitute the unitary dual $\hat{T}$ of $T$. So the orbit space $\hat{T}/W$ makes sense. A consequence of the Weyl character formula is that there is a one-to-one correspondence between $\hat{T}/W$ and the unitary dual $\hat{G}$ of $G$.

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Let $\theta: T \to \mathbb{C}^\times$ be an irreducible character of $T$. Its derivative $\theta_*: t \to \mathbb{C}$ is a Lie algebra representation. The function $-i\theta_*$ is a linear functional on the real vector space $t$. The image of the map $\hat{T} \to t^*$, $\theta \mapsto -i\theta_*$, forms a lattice $\Lambda_T$ in $t^*$. We have:

$$\Lambda_T = \{ \lambda \in t^* | \lambda(H) \in 2\pi\mathbb{Z} \text{ for all } H \in t \cap \exp^{-1}\{e\} \},$$

where $exp$ denotes the exponential map. The constituents of $\Lambda_T$ are known as the \textit{analytically integral weights} of $G$.

Let $K$ denote the fundamental Weyl chamber of our choice for the $W$-action on $t^*$. Then the one-to-one correspondence between $\hat{G}$ and $\hat{T}/W$ implies a one-to-one correspondence between $\hat{G}$ and $\Lambda_T \cap K$. More precisely:

$$\hat{G} \leftrightarrow \Lambda_T \cap K,$$

$$[V] \mapsto \text{highest weight of } V.$$

For each $\lambda \in \Lambda_T \cap K$, we denote by $V_\lambda$ an irreducible $G$-representation space according to the above correspondence.

Let $w \cdot \lambda$ denote the action of $w \in W$ on $\lambda \in \Lambda_T$. The \textit{shifted action} of $w$ on $\lambda$ is:

$$w \circ \lambda := w \cdot (\lambda + \rho) - \rho,$$

where $\rho$ is the Weyl vector, that is, half the sum of the positive roots. (The positive roots are determined by the Weyl chamber $K$.) Then $\Lambda_T \cap K$ consists of all elements of $\Lambda_T$ that has free $W$-orbit with respect to the shifted $W$-action. In summary, the Weyl character formula implies the following one-to-one correspondence:

$$\hat{G} \leftrightarrow \{ \text{free shifted } W\text{-orbits in } \Lambda_T \},$$

$$[V] \mapsto W \circ \lambda.$$  \hfill (1)

### 1.2. Borel-Weil-Bott Theorem

Let $\mu \in \Lambda_T \cap K$, and let $U_\mu$ denote the complex vector space $\mathbb{C}$ on which $T$ acts by the irreducible character of weight $\mu$. We denote by $\ell(\mu)$ the word length of $\mu$ relative to the fundamental Weyl chamber $K$; it satisfies:

$$\ell(\mu) = \# \{ \alpha \in \Phi_+ | \langle \mu, \alpha \rangle < 0 \}.$$ 

Here $\Phi_+$ denotes the set of positive roots of $G$, and $\langle \cdot, \cdot \rangle$ is the inner product on $\mathfrak{g}$.

Let $G \times_T U_\mu$ denote the space of equivalence classes in $G \times U_\mu$ with respect to the relation $(g, z) \sim (gx^{-1}, x \cdot z)$ for $x \in T$. This is a complex line bundle over $X$. It is diffeomorphic to

$$\mathcal{L}_\mu := G^C \times_B U_\mu,$$

which is a complex line bundle over $G^C/B$, where $G^C$ is the complexification of $G$, and $B$ is a Borel subgroup of $G^C$.

The statement of the Borel-Weil-Bott theorem depends on the selected Borel subgroup $B$. Our convention is as follows. Let $\mathfrak{g}_C$ be the complexification of $\mathfrak{g}$, and take the root space decomposition:

$$\mathfrak{g}_C = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+,$$

where $\mathfrak{n}_\pm$ denotes the positive and negative root spaces, and $\mathfrak{h}$ is the complexification of $\mathfrak{t}$. We set

$$\mathfrak{b} := \mathfrak{h} \oplus \mathfrak{n}_+.$$
Then $B$ is the connected subgroup in $G^C$ with Lie algebra $b$.

Let $\mathcal{O}(\mathcal{L}_\mu)$ be the sheaf of germs of holomorphic sections of $\mathcal{L}_\mu$. The celebrated Borel-Weil-Bott theorem states that the sheaf cohomology $H^*(X; \mathcal{O}(\mathcal{L}_\mu))$ is nontrivial only if $\mu$ has free shifted $W$-orbit, and if that is the case then

$$H^q(X; \mathcal{O}(\mathcal{L}_\mu)) \cong \begin{cases} V_{W \circlearrowleft \mu}, & q = \ell(\mu); \\ 0, & \text{otherwise} \end{cases}$$

Here $V_{W \circlearrowleft \mu}$ denotes the irreducible representation space of $G$ corresponding to the shifted orbit of $\mu$ according to the correspondence (1).

Now consider the twisted Hodge-Dolbeault complex

$$A^p_\mu := \Omega^{0,p}(X) \otimes U_\mu,$$

whose differential is given by the Dolbeault operator

$$\bar{\partial} := d_{0,1} \otimes 1.$$

Owing to the Dolbeault theorem (see [10, Thm. 3.20, p. 63]), the complex $(A^*_\mu, \bar{\partial})$ computes the sheaf cohomology of $\mathcal{O}(\mathcal{L}_\mu)$:

$$H^*(X; \mathcal{O}(\mathcal{L}_\mu)) \cong H^*(\{ (A^*_\mu, \bar{\partial}) \}).$$

Meanwhile, by the Hodge theorem, $H^*\{ (A^*_\mu, \bar{\partial}) \}$ is isomorphic to the kernel of the Dirac operator

$$D := (\bar{\partial} + \bar{\partial}^\dagger)/\sqrt{2},$$

where $\bar{\partial}^\dagger$ is the formal adjoint of $\bar{\partial}$. Since $D$ is $G$-equivariant, the kernel of $D$ is a $G$-representation space; we denote the corresponding virtual representation as $[\ker D]$. Then Borel-Weil-Bott theorem is equivalent to saying:

$$[\ker D] = \begin{cases} [V_{W \circlearrowleft \mu}], & \text{if } W \circlearrowleft \mu \text{ is a free orbit}; \\ 0, & \text{otherwise}; \end{cases}$$

and $\ker D$ is homogeneous in degree equal to $\ell(\mu)$. This form of the Borel-Weil-Bott theorem first appeared in Slebarski [8].

1.3. Borel-Weil-Bott Theorem as an Equivariant Index Theorem.

The complex $A^*_\mu$ is naturally bi-graded by the even and odd forms. Let $D_+$ and $D_-$ denote the restrictions of $D$ onto the even and odd subspaces. We have

$$[\ker D] = [\ker D_+] + [\ker D_-].$$

Since $V_{W \circlearrowleft \mu}$ is irreducible (when $W \circlearrowleft \mu$ is free), Equation (3) can be refined as follows. If $W \circlearrowleft \mu$ is free then:

$$[\ker D] = [\ker D_+] \text{ or } [\ker D] = [\ker D_-].$$

If $W \circlearrowleft \mu$ is not free then:

$$[\ker D] = [\ker D_+] = [\ker D_-] = 0.$$

Now the equivariant index of $D$ is by definition the virtual representation

$$[\text{Ind } D] := [\ker D_+] - [\ker D_-].$$

Owing to what we have just seen above, we have:

$$[\text{Ind } D] = [\ker D_+] = [\ker D] \text{ or } [\text{Ind } D] = -[\ker D_-] = -[\ker D].$$
provided that \( W \odot \mu \) is free; otherwise we have:

\[
[\text{Ind } D] = [\ker D_{\pm}] = [\ker D] = 0.
\]

Thus, the Borel-Weil-Bott theorem implies that \([\text{Ind } D]\) is nontrivial if and only if \( \mu \) has free shifted \( W \)-orbit, and if that is the case then \([\text{Ind } D]\) is equal to \([V_{W \odot \mu}] \) up to sign. In fact, in our proof of the Borel-Weil-Bott theorem, we shall show that:

\[
[\text{Ind } D] = \begin{cases} 
(-1)^{\ell(\mu)}[V_{W \odot \mu}], & \text{if } W \odot \mu \text{ is a free orbit;} \\
0, & \text{otherwise.}
\end{cases}
\]

(4)

Equation (3) then follows with the aid of Equation (16). This index theorem is a refinement of Bott’s result [2, Thm. III, p. 170]. It was first shown by Landweber [7] (for the general case of compact homogeneous space \( G/H \) where \( H \) is a closed subgroup of maximal rank in \( G \)).

1.4. Equivariant McKean-Singer Formula and Kostant’s Cubic Dirac Operator. In obtaining Equation (4), Landweber uses Bott’s equation:

\[
[\text{Ind } D] = i_*( [E] - [F]),
\]

where \( E \) and \( F \) denotes \( T \)-spaces, and \( i_* \) is the induction map \( R(T) \to \hat{R}(G), [E] \to [\Gamma^2(G \times_T E)] \). Our method is to use, in place of Bott’s equation, the equivariant McKean-Singer formula. What we have in mind more precisely is this: A virtual representation can be identified with its image under the character map

\[
\chi: R(G) \to C(G),
\]

which maps an irreducible element \([V]\) to its character \( \chi_V \). For the value of \( \chi_V \) at \( g \in G \), we write

\[
[V]_g := \chi_V(g).
\]

The equivariant McKean-Singer formula then states that \([\text{Ind } D]_g\) is equal to the super trace of the operator \( ge^{t D^2} \) where \( t \) is a positive real number (see Berline, Getzler, and Vergne [1, Prop. 6.3, p. 185]):

\[
[\text{Ind } D]_g = \text{Str}(ge^{t D^2}) = \text{tr}(ge^{t D_- D_+}) - \text{tr}(ge^{t D_+ D_-}).
\]

(5)

(Although the right-hand side seems at first to be an infinite linear combination of irreducible characters, it is actually a finite combination due to the symmetry between the eigenvalues of \( D_+ \) and \( D_- \).)

Instead of directly working with the complex \( \mathcal{A}_\mu^* \) in calculating the super trace, we shall use the isomorphism:

\[
\mathcal{A}_\mu^* \cong (C^\infty(G) \otimes \wedge^*(n_+) \otimes U_\mu)^T.
\]

Here the action of \( T \) is as follows: On \( U_\mu \) it is by the irreducible character of weight \( \mu \); on \( C^\infty(G) \) it is the one induced by right-translations; and on \( \wedge^*(n_+) \) it is that induced by the adjoint action. The \( T \)-action on \( \wedge^*(n_+) \) is related to the spinors constructed out of the orthogonal complement \( p \) of \( t \) in \( g \) by:

\[
[\wedge^*(n_+)] = [S^* \otimes U_\mu] \in R(T).
\]

(7)

Here \( S^* \) is dual of the spinor space \( S \) associated to the Clifford algebra \( \text{Cl}(p) \) generated by \( p \) (see Kostant [6, Prop. 3.6, p. 76]). The action of \( T \) on \( S \) is
provided by taking the homomorphism $T \to \text{SO}(p)$, coming from the adjoint representation of $G$, and lifting it to $T \to \text{Spin}(p)$:

\[
\begin{array}{ccc}
\text{Spin}(p) & \xrightarrow{\text{Ad}} & \text{SO}(p) \\
\text{T} & \xrightarrow{\text{Ad}} & \text{SO}(p)
\end{array}
\]

This lift always exists [3, Cor.1.12, p. 91]. In short, we have:

\[
\mathcal{A}^*_\mu \cong (C^\infty(G) \otimes S^* \otimes U_{\mu+\rho})^T.
\] (8)

Finally, because the equivariant index of a Dirac operator depends only on its symbol (this is easy to check directly, but there is a general theorem by Bott [2, Thm. I, p. 169]), we may use, in place of the Dirac operator (2), Kostant’s cubic Dirac operator:

\[
D := \dim p \sum_{i=1}^{\dim p} Y_i \otimes Y_i + 1 \otimes \frac{1}{3} \sum_{i=1}^{\dim p} Y_i \gamma(Y_i) \in \mathcal{U}(\mathfrak{g}) \otimes \text{Cl}(\mathfrak{p}).
\] (9)

Here $\mathcal{U}(\mathfrak{g})$ is the universal enveloping algebra of $\mathfrak{g}$; $\{Y_i\}_{i=1}^{\dim p}$ is any orthonormal basis for $\mathfrak{p}$; and $\gamma$ is the map $\mathfrak{g} \rightarrow \text{spin}(\mathfrak{p})$ defined by:

\[
\gamma(Z) := -\frac{1}{2} \sum_{i,j=1}^{\dim \mathfrak{p}} \langle Z, [Y_i, Y_j]_{\mathfrak{g}} \rangle Y_i Y_j.
\] (10)

The action of the algebra $\mathcal{U}(\mathfrak{g}) \otimes \text{Cl}(\mathfrak{p})$ on the right-hand side of (8) is trivial on $U_{\mu+\rho}$; the action on $S^*$ comes from the canonical action of $\text{Cl}(\mathfrak{p})$; and the action on $C^\infty(G)$ is solely from $\mathcal{U}(\mathfrak{g})$, which arises from identifying $Z \in \mathfrak{g}$ with the left-invariant vector field it generates on $G$.

The advantage of using the cubic Dirac operator lies in the simple form of its square:

\[
D^2 = -\Omega_{\mathfrak{g}} + \text{diag} \Omega_t + \|\rho\|^2.
\] (11)

Here $\Omega_{\mathfrak{g}}$ denotes the Casimir element in $\mathcal{U}(\mathfrak{g})$, and $\text{diag}$ denotes the algebra homomorphism $\mathcal{U}(t) \rightarrow \mathcal{U}(\mathfrak{g}) \otimes \text{Cl}(\mathfrak{p})$ induced by the map $t \rightarrow \mathcal{U}(\mathfrak{g}) \otimes \text{Cl}(\mathfrak{p})$, $X \mapsto X \otimes 1 + 1 \otimes \gamma(X)$.

To see the effectiveness of Equation (11), decompose the right-hand side of (8) using the Peter-Weyl theorem:

\[
(C^\infty(G) \otimes S^* \otimes U_{\mu+\rho})^T \cong \bigoplus_{[\lambda] \in \hat{G}} V_{\lambda} \otimes (V_{\lambda}^* \otimes S^* \otimes U_{\mu+\rho})^T.
\] (12)

(This is not entirely correct; the isomorphism is true upon taking the norm closures on both sides.) This isomorphism is obtained by identifying $|v\rangle \otimes |w\rangle \in V_{\lambda} \otimes V_{\lambda}^*$ with the function $G \rightarrow \mathbb{C}$, $g \mapsto \langle g \cdot w | v \rangle$. The action of $\mathcal{U}(\mathfrak{g})$ on the right-hand side is the one induced by the Lie algebra representation on each $V_{\lambda}^*$. Now take a summand

\[
H_{\lambda} := V_{\lambda} \otimes (V_{\lambda}^* \otimes S^* \otimes U_{\mu+\rho})^T.
\]

Owing to Schur’s lemma, the action of $\Omega_{\mathfrak{g}}$ on $V_{\lambda}^*$ is constant with the value $- \|\lambda + \rho\|^2 + \|\rho\|^2$. For similar reasons, the action of $\text{diag} \Omega_t$ on $V_{\lambda}^* \otimes S^*$ is
again constant with the value $-\|\mu + \rho\|^2$. In summary, the restriction of $D^2$ on the summand $H_\lambda$ is simply the constant operator

$$D^2_\lambda := \|\lambda + \rho\|^2 - \|\mu + \rho\|^2.$$  

So the super trace of the operator $ge^{-tD^2}$ restricted to $H_\lambda$ is:

$$\text{Str}(ge^{-tD^2}) = [V_\lambda]_g \langle [V_\lambda \otimes S_+] - [V_\lambda \otimes S_-], [U_{\mu + \rho}] \rangle_T e^{-t(\|\lambda + \rho\|^2 - \|\mu + \rho\|^2)},$$

where $\langle \cdot, \cdot \rangle_T$ denotes the nondegenerate paring on $R(T)$ defined by:

$$\langle E, F \rangle_T = \dim \text{Hom}_T(E, F).$$

2. A Proof of the Borel-Weil-Bott Theorem via the Equivariant McKean-Singer Formula

We now derive the Borel-Weil-Bott theorem using the equivariant McKean-Singer formula. As we have explained in Section 1, the Borel-Weil-Bott theorem is equivalent to the following:

**Theorem 1.** Let $D$ be Kostant’s cubic Dirac operator acting on the smooth sections of the twisted vector bundle $G \times_T (S \otimes U_{\mu + \rho})$ over $X = G/T$. The equivariant index of $D$ satisfies:

$$[\text{Ind } D] = \begin{cases} (-1)^{\ell(\mu)} [W]_g, & \text{if } W \otimes \mu \text{ is free;} \\ 0, & \text{otherwise.} \end{cases}$$

Moreover,

$$[\text{Ind } D] = \begin{cases} [\ker D_+], & \text{if } \ell(\mu) \text{ is even;} \\ -[\ker D_-], & \text{if } \ell(\mu) \text{ is odd.} \end{cases}$$

In each case, contributions to $\ker D_\pm$ come from sections whose degree is equal to $\ell(\mu)$.

**Proof.** We begin by invoking the equivariant McKean-Singer formula:

$$[\text{Ind } D]_g = \sum_{[V_\lambda] \in G} \text{Str}(ge^{tD^2_\lambda}).$$

By Equation (14), we have:

$$[\text{Ind } D]_g = \sum_{[V_\lambda] \in G} [V_\lambda]_g \langle [V_\lambda \otimes S_+] - [V_\lambda \otimes S_-], [U_{\mu + \rho}] \rangle_T e^{-t(\|\lambda + \rho\|^2 - \|\mu + \rho\|^2)}.$$  

But the left-hand side is independent of the parameter $t$; hence, the only contribution in the sum occurs from the terms with the exponential factor equal to 1, that is, when $\|\lambda + \rho\| = \|\mu + \rho\|$. Thus we have:

$$[\text{Ind } D] = \sum_{[V_\lambda] \in G, \|\lambda + \rho\| = \|\mu + \rho\|} [V_\lambda] \langle [V_\lambda \otimes S_+] - [V_\lambda \otimes S_-], [U_{\mu + \rho}] \rangle_T.$$  

According to the multiplicity result of Kostant [5, Thm. 4.17, p. 486], we have:

$$\begin{cases} \langle [V_\lambda \otimes S_+], [U_{\mu + \rho}] \rangle_T, \langle [V_\lambda \otimes S_-], [U_{\mu + \rho}] \rangle_T \rangle = \begin{cases} (1, 0), & \text{if } \ell(\mu) \text{ is even;} \\ (0, 1), & \text{if } \ell(\mu) \text{ is odd,} \end{cases} \end{cases}$$

(17)
provided that $\mu \in W \odot \lambda$; this last condition can be satisfied by some $[V_\lambda] \in \hat{G}$ if and only if $W \odot \mu$ is free. As a consequence we have Equation (15). We also find from Equation (17) that a nontrivial contribution to $[\text{Ind} D]$ comes solely from the even or the odd domain according to the parity of $\ell(\mu)$; hence Equation (16) holds. The same multiplicity result of Kostant also implies that such contribution to $[\text{Ind} D]$ comes from elements whose degree is $\ell(\mu)$. This completes the proof. □

Remark. Theorem 1 can be modified so that it holds for more general cases where $T$ may be any closed subgroup $H$ of $G$ that is of maximal rank; the only change necessary is that we replace $U_{\mu+\rho}$ with $U_{\mu+\rho'}$, where

$$\rho' := \frac{1}{2} \sum_{\alpha \in \Phi_+ \setminus \Phi_+} \alpha.$$ 

Here $\Phi_+ (\mathfrak{h})$ denotes the set of positive roots of the Lie algebra $\mathfrak{h}$ of $H$ (the roots are calculated with respect to a common maximal toral subalgebra of $\mathfrak{g}$ and $\mathfrak{h}$). This change is necessary because Equation (7) now takes the form:

$$[\wedge \bullet (n_+)] = [S^* \otimes U_{\rho'}].$$

The formula for $D^2$ also changes to:

$$D^2 = -\Omega_g + \text{diag} \Omega_h + \| \rho \|^2 - \| \rho_h \|^2,$$

where $\rho_h$ is the Weyl vector of $\mathfrak{h}$. But Equation (13) remains unmodified; so the argument we gave for $G/T$ can be repeated word-for-word for $G/H$, and we have the full results of Landweber [7, Thm. 3, p. 471] and Slebarski [9, Thm. 2, p. 509].

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