The Stabilizing Role of Global Alliances in the Dynamics of Coalition Forming

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Abstract

Coalition forming is investigated among countries, which are coupled with short range interactions, under the influence of external fields produced by the existence of global alliances. The model rests on the natural model of coalition forming inspired from Statistical Physics, where instabilities are a consequence of decentralized maximization of the individual benefits of actors within their long horizon of rationality as the ability to envision a way through intermediate losing states, to a better configuration. The effects of those external incentives on the interactions between countries and the eventual stabilization of coalitions are studied. The results shed a new light on the understanding of the complex phenomena of stabilization and fragmentation in the coalition dynamics and on the possibility to design stable coalitions. In addition to the formal implementation of the model, the phenomena is illustrated through some historical cases of conflicts in Western Europe.

Keywords: Social Models, Statistical Physics, Coalition Forming, Coalition Stabilization, Political Instability.

1 Introduction

This work is devoted to the study of stabilization in coalition forming in a collective of individual actors under the influence of external fields. The model rests on the natural model of coalition
forming inspired from the Statistical Physics’ model of Spin Glasses, through which the system of countries is compared to a collection of interacting spins – tiny magnetic dipoles that interact with each other and align themselves in a way to attain the most "comfortable" position, the one that minimizes their energies. While the presentation addresses the coalition forming in an aggregate of countries, the discussion and the results can be applied to any type of political, social or economical collectives where the association of actors takes place based on their bilateral propensities.

This work subscribes to the growing field of modeling complex social situations using Statistical Physics which has started over thirty years ago with . Later, a study of collective decision making combining Social Psychology hypotheses with recent concept of Statistical Physics set the frame of using spin Hamiltonian. Then, the coalition as a form of aggregation among a set of actors (countries, groups, individuals) has been studied using concepts from the theory of Spin Glasses. Various social applications of the model were suggested . The dynamical analogue of this model was introduced in .

The model of coalition forming among countries leans on the existence of strong and static bilateral geographic-ethnic propensities linking the countries. Those propensities have emerged during the ongoing historical interactions between neighbor countries and appear to favor either cooperation or conflict. Their spontaneous and independent evolution have produced an intricate circuit of bilateral bonds which causes contradictory tendencies in the simultaneous individual searches for optimal coalitions. Due to stronger interactions with a common ally, conflicting countries may be brought to cooperate momentarily despite their natural tendency to conflict. Such a situation produces an endeavor of the concerned countries to escape from the unfavorable cooperation leading to instabilities, which in turn produce a break down of the current coalitions inducing the formation of new ones.

The origin of such instability is twofold, either coming from spontaneous fluctuations or directed by external attraction towards a global alliance. The extremely disordered dynamics of coalitions and fragmentation in Western Europe in past centuries belong to the first kind, while the building up of the Soviet and Nato global alliances is of the second kind.

In this work we aim to study the instability of coalition forming among countries, which, in contrast to physical entities, are rational actors that are able to maximize their individual benefits through a series of choices within a decentralized maximization process.

On this basis, coalitions are formed through the short range interactions between the countries – the attraction or repulsion based on the unalterable historical bonds between them. According to the principle that "the enemy of an enemy is a friend", the countries are assumed to ally to one of two competing coalitions.

Allying to the same coalition is unfavorable to the countries which went through historical rejection. As a result, such countries seek to affiliate with the opposite coalitions. Alternatively, allying to the opposite coalition is unfavorable to friendly countries. Countries which belong to
the same coalition are expected to cooperate even if their natural propensity is to conflict. Such a contradiction results in a potential instability.

Our previous study [1] focused on studying the effects of instabilities arising in the coalition forming among rational actors as a function of the bonds structure, the optimal and non-optimal stabilizations as well as the robustness of the stability.

In this work the model is extended to investigate the mechanisms by which the setting of a global alliance produces attraction in an aggregate of individual countries otherwise connected by their natural bilateral propensities. In particular, the focus is on how those new interactions can eventually stabilize the intrinsically unstable process of coalition forming keeping the short range nature of interactions. Global attraction is ensued from a global external field set over the system of countries, which in turn polarizes the countries’ interests and produces incentive unifications under two opposing global alliances. The resulting coalitions are affected by the net bilateral balance between the new motivations and the traditional historical ones.

We focus on how the interactions produced by attraction to global alliances overwhelm the current instability among the countries. The results provide new theoretical tools that enable to measure the efficiency of a global attraction in forming stable alliances, as well as to theoretically design new effective global attractions that can yield stability.

The study of stabilization of coalitions using global alliances was started in [12]. The authors describe spontaneous formation of economic coalitions given a random distribution of propensity bonds, and illustrate new exchanges between the countries incited by the global alliances. Those exchanges, along with an additional parameter of economical and military pressure, are viewed as the ones that produce additional bilateral propensities yielding new stable coalitions.

In the current work, we develops further the research on coalition forming under a global external field. We address the stabilization by unique factors – such as economical, political, social, ecological, as well as by multi-factor stabilization, where the influence of several independent factors is equiprobable. Based on the new formulation, we investigate the remarkable historical cases of conflicts in Western Europe.

The multi-factor stabilization is an innovative concept both in Political Sciences where it explains the complexity of coalition forming, and in Statistical Physics where it illustrates how a stable disorder arises from an anti-ferromagnetic coupling achieved by the interlocking of two opposite ferromagnetic states. Some forms of such mixed phases of ferromagnetism have been studied in [15].

2 Background – The Natural Model and Instability

The Spin Glass model in Statistical Physics is an idealized model of bulk magnetism represented by a collection of interacting spins – atoms acting as a tiny dipole magnet with a mixture of ferromagnetic and anti-ferromagnetic couplings. Those magnets interact with each other seeking...
to align themselves parallel or anti-parallel in order to minimize their energies. The collection of spins forms a disordered material in which the competing interactions cause high magnetic frustration – changes of spins at no energy cost, with a highly degenerate ground state.

The Ising model of a random bond magnetic system can be described as follows. The model consists of $N$ discrete variables $\{S_i\}_{i=1}^{N}$, called spins, that can be in one of two states up or down. Figure (1) shows schematically the case of 8 spins with identical amplitude of the propensity bonds located on a lattice and interacting at most with their nearest neighbors. The spins for which a shift of the state cost no energy are defined as frustrated.

![Figure 1: Ising model of 8-spins with mixed pair interactions. The pair propensity bonds are denoted by + or −, and states of the spins are denoted by the arrows. Frustrated spins are marked by both up and down arrows. This Spin Glass phase yields an unstable disorder.](image)

The natural model of coalition forming is formally identical to the Ising model with pure or mixed anti-ferromagnetic couplings in a particular geometry of the lattice. the model considers a system of $N$ countries whose historical interactions have defined propensity bonds between them, which are either positive (ferromagnetic-like) or negative (anti-ferromagnetic-like). To each country labeled with an index $i$ ranging from 1 to $N$, is attached a discrete variables $S_i$ which can assume one of two state values $S_i = +1$ or $S_i = −1$. The values correspond to the country’s choice between the two possible coalitions. The same choice allies two countries to the same coalition, while different choices separate them into the opposite coalitions.

Combination of states of all the countries $S = \{S_1, S_2, S_3, \ldots, S_N\}$ forms a state configuration that defines an allocation of coalitions. Here, by symmetry, both configuration $S$ and it’s inverse $−S = \{-S_1, -S_2, -S_3, \ldots, -S_N\}$ define the same coalitions.

Bilateral propensities $J_{i,j}$ have emerged from the respective mutual historical experience between the countries $i$ and $j$. The propensities measure the amplitudes and the directions of the exchange between two countries – cooperation or conflict. $J_{i,j}$, which is symmetric, is zero when there are no direct exchanges between the countries.

Product $J_{i,j}S_iS_j$ measures the benefit or the gain from the interactions between both the countries as a function of their choices. Aimed to maximize this measure, the countries seek to ally to the same coalition when $J_{i,j}$ is positive and to the opposing ones, otherwise. Thus, depending on the direction of the primary propensity, the conflict can be beneficial to the same extent as the cooperation.
The sum of the benefits from all the interactions of country \(i\) in the system makes up the net gain of the country:

\[ H_i(S) = S_i \sum_{j \neq i} J_{ij} S_j. \]  

(1)

Thus, a configuration \(S\) that maximizes the gain function defines the country’s most beneficial coalition setting.

For the sake of visualization, we depict the system of countries through a connected weighted graph with the countries in the nodes and the bilateral propensities as the weights of the respective edges (see Figure 2). We take red (dark) color for the +1 choice and blue (light) color for the −1 choice.

![Figure 2: Triangle of three conflicting countries 1, 2, 3 with negative mutual bonds.](image)

The total gain of the system of countries is identical to the Hamiltonian of an Ising random bond magnetic system which represents the energy of the system. For a configuration \(S\), we have for system’s gain:

\[ \mathcal{H}(S) = \frac{1}{2} \sum_i H_i(S). \]  

(2)

In physical systems, the Hamiltonian – the function that determines the physical properties of the spin system, is precisely concerned with minimization of the system’s energy. This physical analogy allows to address the bilateral propensities between the countries as mean of maximization of the countries’ individual gain (minimization of their energy) and as the principal guide in the coalition forming.

A major difference between the model of spins and the model of rational countries is the long horizon rationality of the countries in contrast to the spins, which are only able to foresee only the immediate effect of their shifts. Countries have the ability to maximize their individual benefits through a series of planned changes while assuming possible losses in the intermediate phase.

The Ising model, indeed, can be represented through the natural model where the countries’ rationality is limited to observation of an immediate gain, optimizing only their local maximums.

When the most beneficial coalition configurations of different countries do not coincide, the maximization of individual gains induces competitions for the beneficial associations. Among the countries with complete rationality which are aware of attainability of a better configuration, those competing interactions cause endless instability in the system. However, the system may remain stable when some actors have limited rationality – being unaware of possibility to attain a better
configuration, they are satisfied having reached a local maximum.

Figure (3) shows the triangle of conflict in a configuration where it is stable when the countries 2’s and 3’s rationality is limited to immediate improvements; any change cause a loss in their gain. The triangle is unstable when the countries are fully rational: their most beneficial configurations $S_1 = \{\{+1, -1, -1\}, \{-1, +1, +1\}\}$, $S_2 = \{\{-1, +1, -1\}, \{+1, -1, +1\}\}$, $S_3 = \{\{-1, -1, +1\}, \{+1, +1, -1\}\}$, do not coincide. Here, at any coalition configuration, at least two of the countries improves their gain when the other changes. As a result, aiming in their best configurations and being able to forecast an improvement at any step, the countries may make changes that impair the gain in the immediate steps.

![Figure 3: The triangle of conflict. The triangle is stable when the countries 2's and 3's rationality is limited to immediate improvements; any change cause a loss in their gain. The triangle is unstable when the countries are fully rational: their most beneficial configurations do not coincide and the countries, being able to forecast an improvement, make changes that impair the gain in the immediate steps.](image)

It is interesting to note that for the case of equal propensities over the edges, the triangle of conflict is unstable for any limited rationality actors, including the spins, due to zero-value gain produced in the cyclic geometry resulting in no-cost frustrations.

**Definition 1 (Instability of the System of Countries)** The system of countries is said to be unstable if in any configuration of the countries’ states there is a country which is able to forecast an improvement of its gain.

Negative product on a circle means an unpaired negative coupling where two neighbors are found to be connected both though positive and negative branch in the circle. This creates an everlasting competition between the neighbors for the exclusive arrangement to ally with the positive branch. The countries thereby continuously shift their respective choices producing the instability.

In Statistical physics the necessary condition of instabilities in Spin Glasses (3) reads that

$$\text{the instability implies the existance of a closed circle of spins connected with the bonds on which the product of total bonds is negative.}$$

Indeed, the Spin Glasses’ instability is a result of frustrations, and a negative circle can appear to be stable as soon as a shift increases the spin energy preventing the spin flop.

In contrast to the Spin Glass model where changes are limited to the spontaneous no-cost
fluctuations, in the natural model where the instability is due to the rationality of actors, changes may impair the immediate gain. In the theoretical interpretation of the model where the complete rationality of all countries is assumed, the terms (3) are also the sufficient condition of the instability in the model – the condition of endless competitions among the countries for the beneficial configurations.

Formally, the theoretical terms of instability in the natural model are as follows. Denote a circle of countries by $C$ and the countries composing the circle by $1, 2, \ldots, k$.

\[
\text{If there is a closed circle of countries on which the product of total propensities is negative,} \\
\Pi_{i,j \in C} p_{ij} < 0 \\
then the system is unstable.
\]

(4)

Let us remark that, in the theoretical interpretation of the model where the complete rationality of all countries is assumed, the instability is not value-dependent but is determined by the signs of the propensities – the distribution is such that involves a negative circle. At the same time, any local maximum strictly depends on the propensities’ values.

3 Global Alliance Model Of Coalition Forming

Global alliance model starts from a global principle which represents an external field polarizing the interests of the countries. This leads to the emergence of two opposing global alliances. The countries attach themselves to one or to the other based on their pragmatic interests with respect to the global principle. The new interactions, while favor either cooperation or conflict, stimulates contributions to the countries’ mutual propensities. The new perspectives unify or separate the countries based on the pragmatic motivations which in combination with the historical concerns allow other distributions of coalitions.

Here we address the role of the global alliances in forming of stable coalitions among the countries or other rational actors. Whether the system is unstable or there is an optimal or local maximum stable configuration, the new exchanges between the countries incited by the global alliances impact the stability. For the sake of simplicity of presentation, we assume the extensive rationality of the countries. While in such theoretical interpretation the instability is not value-dependent, the effect from the globally generated additional propensities on the stability is subject to the values of primary propensities. Therefore, in spite of the extensive rationality of the countries, in the presentation we address the model with arbitrary range of values.

Let us define the global alliance model formally. The global alliance unifies the countries that support the global principle, while its opponents are unified under the opposing global alliance. Denote the two alliances by $M$ and $C$. A country’s individual disposition to the alliances is
determined by the countries’ cultural and historical experiences and is expressed through the parameter of natural belonging. The natural belonging parameter of country \( i \) is \( \epsilon_i = +1 \) if the country has natural attraction to alliance \( M \), \( \epsilon_i = -1 \) for \( C \).

By making a choice among the two possible state values \( S_i = +1 \) and \( S_i = -1 \), country \( i \) chooses to belong to either alliance \( M \) or \( C \). The choice of \( +1 \) allies the country to alliance \( M \) and the choice of \( -1 \) allies it to alliance \( C \). Any particular distribution of two countries among the alliances creates new interactions between the countries whose directions depends on the natural disposition of the countries. Namely, if the countries are attracted to the opposing alliances, the exchange will be negative as soon as they ally to the same alliance.

Those new exchanges between any two countries \( i \) and \( j \) define additional propensity between the countries. The propensity is the amplitude of the exchange \( G_{ij} \) in the direction \( \epsilon_i \epsilon_j \) that favors either cooperation or conflict. For the purpose of this presentation we assume that the exchange amplitudes are unchanged.

The overall propensities between the countries, involving both the historical inclinations and the propensities resulting from the new exchanges, are determined as follows

\[
p_{ij} = J_{ij} + \epsilon_i \epsilon_j G_{ij}. \tag{5}
\]

Respectively, the net gain of country \( i \) is

\[
H_i = S_i \sum_{j \neq i} (J_{ij} + \epsilon_i \epsilon_j G_{ij})S_j. \tag{6}
\]

Thus, in the presence of external incentives of the global alliances, the couplings between the countries obtain new guidance. The countries adjust their states to the best benefit with regards to the new propensities. The new choice of coalition is determined by both spontaneous reactions and planned interactions, which enable coupling based on a planned profit.

4 Stabilization Of The System By Additional Factors

Here we address the stabilization of coalition forming in the system where rational countries have no optimal configuration of coalitions, and where, as result, the spontaneous stabilization can not be attained. Global alliances based interactions in such systems enable stable coalitions among the actors even if they remain of short-range nature. Such interactions, however, being a complex superposition of several factors of countries’ objectives, must satisfy particular stability constraints.
4.1 The Uni-Factor Stabilization

Consider two opposing global alliances $M$ and $C$ in a system of $N$ countries. A particular factor of the countries’ interests produces specific dispositions to the global alliances which encourage new exchanges between the countries. The appropriate amplitudes of the exchanges enable the stabilization among the countries, the *uni-factor stabilization*.

With respect of unique factor of stabilization, the necessary and sufficient condition stability (reformulated terms) is that

$$\Pi_{i,j \in C} (J_{ij} + \epsilon_i \epsilon_j G_{ij}) \geq 0.$$  \hspace{1cm} (7)

Now we state the existence of a stable coalition within the global alliance model.

**Statement 1** The presence of global alliances, regardless of the global principle that produced them, enables a stable coalition among countries.

In order to prove this statement, let us first observe that the product of the additional propensities $p_i^\vartheta_{ij} = \epsilon_i \epsilon_j G_{ij}$ on any circle is always positive. Indeed, given circle $C$,

$$\Pi_{i,j \in C} G_{ij} \epsilon_i \epsilon_j = \Pi_{i,j \in C} G_{ij} (\epsilon_1 \epsilon_2 \epsilon_3 \ldots \epsilon_k)^2 = \Pi_{i,j \in C} G_{ij}.$$  

This implies that on any circle, the number of negative couplings produced by the global alliances is even. If the system is unstable, than there is at least one negative circle. We define the new interaction amplitude as follows. For each couple $i, j$ whose $\epsilon_i \epsilon_j < 0$ we take $G_{ij} = 0$ if the primary propensity is negative and $G_{ij} = 2|J_{ij}|$ for the positive original coupling. When $\epsilon_i \epsilon_j > 0$, we take $G_{ij} = 2|J_{ij}|$.

Making the new propensities negative for the negative global couplings and positive for the positive ones, guarantees that there is an even number of negative couplings on the circle. This remains invariant for each circle in the system, which implies that the construction produces non-negative product on any circle in the system. The stability condition (7) holds true which concludes the proof of the statement.

4.1.1 A Case of the England-Spain-France Triangle

A typical example of the uni-factor stabilization is stabilization of the triangle of England, Spain and France during historical events of 1584.

**Example 1 (Stabilization in the ESF Triangle of Conflict by the Religious Factor)** Against the background of sequence of wars in the old Europe, the countries attained stability when in 1584 Catholic Spain and France formed an alliance against Protestant forces, the most
notable of which were settled in England.

In order to illustrate historical example using the global alliance model, we describe the propensities between the countries from "negative" to "positive" through mixed ones. Attaching to them numerical values with respect to their relative strength, taking "neutral" as 0.

Accounting for the historical relationship between England, Spain and France, we take the propensities as "neutral-negative", "negative" and "highly negative". There numerical interpretations, as shown in Figure 4, are arbitrary values that aim to account for a relative strength of the interactions.

Figure 4: Triangle of England (E), Spain (S) and France (F), the ESF-conflicting triangle.

By M and C we denote the two opposing global alliances – the countries in M choose unification into a "European union" and those in C are against the unification. With respect to the religious factor, Catholic Spain and France were naturally associated to M ($\epsilon_S, \epsilon_F = 1$), while Protestant England was associated to C ($\epsilon_E = -1$). Then, $\epsilon_S \epsilon_F = 1, \epsilon_E \epsilon_S = \epsilon_E \epsilon_F = -1$, and the overall propensities between the three countries are:

$p_{SE} = -3 - G_{SE}, p_{EF} = -1 - G_{EF}$ and $p_{SF} = -2 + G_{SF}$.

Solving the inequality

$$(-3 - G_{SE})(-1 - G_{EF})(-2 + G_{SF}) \geq 0$$

(8)

yields the constraint the new interaction amplitudes $G_{SE}, G_{EF}, G_{SF}$ must satisfy in order to stabilize the triangle. Since $G_{EF}, G_{SE}$ and $G_{SF} > 0$, the only root of the respective equality is $G_{SF} = 2$. The solution space is $G_{SF} \geq 2$, as depicted in Figure 4, represents a three-dimensional space of the independent additional propensities.

In the historical example, coalition of Spain and France against England implies that the amplitude of their new interaction belonged to the solution space. The respective stable configuration is $S = (+1, -1, -1)$, as shown in Figure 6 where $G_{EF}, G_{SE}$ are taken to be 0 and $G_{SF}$ to be 3, so that the corresponding total propensities become $-1, -3$ and 1.

It is interesting to observe that:

**Statement 2** Any system of countries in the global alliance model with a unique factor of interests is reducible to a stable system represented in the natural model.
Figure 5: Three-dimensional solution space of the independent additional propensities in the uni-factor stabilization of the ESF-triangle of conflict.

Figure 6: The global alliances model of the ESF-triangle stabilized by the religious factor in configuration $S = (+1, -1, -1)$. Here, $G_{EF} = G_{SE} = 0$, $G_{SF} = 3$, so that the respective resulting total propensities are $-1, -3$ and $1$.

Indeed, given a system in the global alliance model, let us define the new state variable to be $\tau_i = \epsilon_i S_i$. The variable takes a value of $\{+1, -1\}$. Then, the hamiltonian $H_i$ of country $i$ can be written in the terms of the new state variables as

$$H_i = \sum_{i \neq j} (J_{ij} S_i S_j + G_{ij} \epsilon_i \epsilon_j S_i S_j) = \sum_{i \neq j} (J_{ij} \epsilon_i \epsilon_j + G_{ij}) \tau_i \tau_j.$$

Here, since $G_{ij}$ is positive, some choice of $\{G_{ij}\}_{i,j}$ produces the propensities that guarantee a stable system.

### 4.2 The Multi-Factor Stabilization

Taking into account only one factor of countries’ interests would be too restricting – along with religious interests, the global principle may impact economical, ecological, moral, political or any other interest and concern. Distinct interests simultaneously influence the interactions between the countries in different ways. They modify the countries’ propensities by aggregating the corresponding independent interactions – economical, political and others.

Let us define formally the multi-factor form of the global alliance model through two coexisting factors of interests, denoted by $G$ and $K$ respectively. Within each factor, a country has independent natural disposition to the global alliances. Therefore, each country has two independent natural...
belonging parameters associated with the factors. For country $i$, this is $\epsilon_i = +1$ if within factor $G$ the country naturally belongs to $M$. Similarly, $\beta_i = +1$ within factor $K$. For the global alliance $C$, $\epsilon_i = -1$ and $\beta_i = -1$ respectively.

We denote by $G_{ij}$ the amplitude of the exchanges between the countries $i$ and $j$ on factor $G$, and by $K_{ij}$ the amplitude on $K$. Then, the total new propensity between the countries $i$ and $j$ is the superposition of those directed exchanges on the two factors:

$$p_{ij}^G = \epsilon_i \epsilon_j G_{ij} \quad \text{and} \quad p_{ij}^K = \beta_i \beta_j K_{ij}.$$  

The two-factor form of the global alliance model superposes the spontaneous interactions of the natural model with the intended interactions based on the two-dimensional choice among the global alliances:

$$p_{ij} = J_{ij} + \epsilon_i \epsilon_j G_{ij} + \beta_i \beta_j K_{ij}.$$  

The net gain of country $i$ is

$$H_i = S_i \sum_{j \neq i} S_j (J_{ij} + G_{ij} \epsilon_i \epsilon_j + K_{ij} \beta_i \beta_j).$$

In order to illustrate the multi-factor stabilization, we turn again to the Example 1 of stabilization of the ESF-conflicting triangle.

### 4.2.1 Multi-factor Stabilization of the England-Spain-France Triangle

We assume, in addition to the religious factor $G$ in the ESF-conflicting triangle, that there is an economical factor $K$. In this golden age Spain had a pronounced disinclination to any economical unification with its old enemies, while England and France remarked the advantages of such unification. Therefore, the respective parameters of natural belonging on the economical factor $K$ are $\beta_S = -1$ and $\beta_E, \beta_F = 1$. With respect to the economical factor, the overall propensities between the countries are: $p_{SE} = -3 - G_{SE} - K_{SE}$, $p_{EF} = -1 - G_{EF} + K_{EF}$, and $p_{SF} = -2 + G_{SF} - K_{SF}$.

Solutions of inequality

$$\Pi_{i,j \in C} \ p_{ij} = (-3 - G_{SE} - K_{SE})(-1 - G_{EF} + K_{EF})(-2 + G_{SF} - K_{SF}) \geq 0$$

yield the exchange amplitudes that guarantee stability of the ESF-triangle in the multi-factor form. Since $G_{SE} + K_{SE} \geq 0$, the solution must satisfy $-G_{EF} + K_{EF} \geq 1$ and $G_{SF} - K_{SF} \leq 2$, or $-G_{EF} + K_{EF} \leq 1$ and $G_{SF} - K_{SF} \geq 2$ (see Figure 1).

In the historical reality of this period of the countries, the economical factor $K$ could not produce interactions as strong and significant as the exchanges on the religious factor. That is why $K_{EF} < 1 + G_{EF}$ and $K_{SF} < G_{SF} + 2$ which have prevented the ESF-triangle to reach the stability until religion took a secondary place conceding importance to economics. See Figure 8), where $G_{SE}$ is taken to be 0, $G_{EF}$ to be 2 and $G_{SF}$ is 3, so that the respective total propensities become 1, -3, 1.

It worth to notice that in the multi-factor form, a system in the global alliance model can
be no more interpreted as a system in the natural model as soon as the choice of at least two countries differs on at least two factors. Still, the general multi-factor case can be reduced to the two-factor form of the global alliance model: one of the factors unifies the amplitudes of all the positive coupling and the other unifies those of all the negatives ones.

Therefore, with no restriction on the generality, the multi-factor form of global alliance model can be studied within the case of two coexisting factors. This also explains the fact that in the majority of cases, only two camps of opposing concerns play the crucial role in the coalition forming.

5 Physical Interpretation of the Multi-Factor Stabilization

In the context of Statistical Physics, the multi-factor stabilization is equivalent to the superposition of unstable disorder of a spin glass with two stable orders (two factors) of ferromagnetic states which split the spins in two directions (two alliances). Each spin’s absolute direction is the average of those ferromagnetic directions, as shown in Figure (9). Among the two opposite directions, either one of them dominates or the two eliminate each other, thus neutralizing the ferromagnetic
states on the spin. In the figure, thick arrows indicate the absolute directions of the spins, and thin arrows show their ferromagnetic directions.

Figure 9: Ising model of 8-spins, initially mixed negative and positive pair interactions (highlighted by grey color), is stabilized by mixing of two ferromagnetic states. Each spin’s absolute direction (marked by the thick arrows) is the average of those ferromagnetic directions. Among the two opposite directions, either one of them dominates or the two eliminate each other, thus neutralizing the ferromagnetic states on the spin. The Spin Glass phase yields a stable disorder.

The multi-factor stabilization of coalition forming is an innovative concept both in Political Sciences and in Statistical Physics. In the former, it explains the multitude of elements influencing the coalition forming. In the later, it shows how in a frustrated system a stable disorder is achieved from interlocking of two ferromagnetic states of opposite directions with anti-ferromagnetic coupling among them.

6 Multi-factor Stabilization in Western Europe

Here we attempt to illustrate the formation of Italian state within the context of the global alliance model. It is known that, given a system from the reality, it is hard to obtain exact numerical values of the propensities in the system. Once such values are known we can explain the transitions and predict resulting configurations with arguable precision. Having no such values, we still can provide some analysis based on estimated values of the propensities extracted from the historical chronicles. Running the model with those values allows to analyze and explain the transitions and the result configuration. This can not be done based only on the canonical representations of historical events.

Let us illustrate the Italian unification in 1856 - 1858, where four countries were involved: Italy, France, Russia and Austria [18] and [19]. The period from the end of 18th till the middle of 19th century was marked by the series of European wars including the French invasion of Italy where Austrian and Sardinian forces had to face French army in the War of the First Coalition, The War of the Fifth Coalition of Austria against French Empire.

In 1852, the new president of the Council of Ministers in an Italian region Piedmont, Camillo di Cavour, had expansionist ambitions one of which was to displace the Austrians from the Italian peninsula. An attempt to acquire British and French favor was however unsuccessful.
Then, Napoleon III, who had belonged to an Italian family originally, decided to make a significant gesture for Italy. In the summer of 1858, Cavour and Napoleon III agreed to cooperate on war against Austria. According to the agreement, Piedmont would be rewarded with the Austrian territories in Italy (Lombardy and Venice), as well as the Duchies of Parma and Modena, while France would gain Piedmont’s transalpine territories of Savoy and Nice.

Despite the Russian help in crushing the Hungarian Revolt in 1849, Austria failed to support Russia in the Crimean War of the middle of 1850s. Therefore, Austria could not count on Russian help in Italy and Germany. Alexander II has agreed to support France in a fight with Austria for the liberation of the Italians, though only by showing up the army on its borders with Austria. It appeared to be enough to force the Austrians withdrew behind the borders of Venice.

However, the conquest of Venice required a long and bloody mission, which may cause revolts and threaten Napoleon III’s position in France. In the private meeting with Franz Joseph, together they agreed on the principles of a settlement to the conflict according to which the Austrians have to cede Savoy and Nice to the French, yet would retain Venice. The Russian was indignant at this turn of France.

Let us reproduce the historical chronicle presented above with the help of our model. The initial states of the countries with their primary propensities are shown in Figure (10).

The value of propensities indicating the relative strength of primary interactions between the countries are taken from ”negative” to ”positive” through mixed ones with the respective numerical values taking ”neutral” as 0 are shown in Figure 10. Thus, the historical relationships between the two absolutist monarchies Russia and Austria are estimated as ”neutral” with \( J_{RA} = 1 \). Italy and Russia having no noticeable political relationship are ”neutral” to each other. The Franco-Russian relationship built up during the French Revolutionary and the Napoleonic Wars is are rather ”neutral-negative” with \( J_{FR} = -1 \), as well as the interactions of France with Italy and Austria who had experienced series of military conflicts. The opposition between Italy and Austria tied to the mutual territorial claims is estimated to be ”significantly-negative” with \( J_{IA} = -2 \).

![Figure 10: The unstable system of France, Russia, Italy, Austria with their relative primary propensities, 1856-1858.](image)

Figure 10 shows the system of the countries in its natural model. The model has two negative
circles and so is unstable which appears through the historical changes before the rise of the Italian question. The instability originates from the fact that France gets identical benefits from the alliance with Russia and Italy as from the opposing alliance with Austria.

An external field in the model results from the principle of independent state of Italy. The respective opposing global alliances are $M$ which associates the countries that support the independence of Italy, and $C$ which unifies the countries opposing the independence.

Here, two respective factors influencing the historical series of events must be distinguished: external politics with the military goals, and internal politics involving the social concerns of the countries (their governing classes). Denote the two factors by $G$ and $K$ respectively.

With respect to their external goals, Italy and France, as well as Russia, agree to the relevance of an independent state of Italy. Yet, in the social concerns the governing classes of France, Russia and Austria agree in their rejection of socialist ideas springing over all the Italy. Therefore, the respective parameters of the countries’ natural belonging to the alliances are distributed as follows.

With the natural belonging parameter $\epsilon$ referring the external goals and $\beta$ referring the internal social politics, for France $\epsilon_F = +1$ and $\beta_F = -1$, for Italy $\epsilon_I = +1$ and $\beta_I = +1$, for Russia $\epsilon_R = +1$ and $\beta_R = -1$, and for Austria $\epsilon_A = -1$ and $\beta_A = -1$.

The global alliance motivated propensities are given in the following chart:

| Propensity | F-I | I-A | F-R | F-A | R-A |
|------------|-----|-----|-----|-----|-----|
| Primary    | -1  | -2  | -1  | -1  | 1   |
| On $G$     | $G_{FI}$ | $-G_{IA}$ | $G_{FR}$ | $-G_{FA}$ | $-G_{RA}$ |
| On $K$     | $-K_{FI}$ | $-K_{IA}$ | $K_{FR}$ | $K_{FA}$ | $K_{RA}$ |

The historical chronicle of the four countries is concluded in three phases: a phase of no global alliances, or the natural model phase, and two phases of global alliances rose due to the Italian question, where in the first one the external and military concerns come to picture and in the second one the internal social concerns rise over the countries.

As we have seen in Figure 10, the system in its natural model is unstable, where France fluctuates between Russia and Austria.

Let us evaluated the amplitudes of the military exchanges between the countries through numerical values providing the relative magnitudes of interactions. Russia has equally "moderate" interest in military cooperation with both France and Austria, with $G_{FR} = 2$ and $G_{RA} = 2$. Italy and France are "strongly" interested in the conflict having Italian land at stake, $G_{FI} = 4$ and $G_{IA} = 4$, while the interest between Austria and France is "moderately-strong" with $G_{FA} = 3$. A sympathy of Russian to Italian state comes up in the "basic" interest, $G_{RI} = 1$. The new propensities between the countries with respect to the external politics interests are shown in the following table:
As a result of the interactions the system obtains a new shape shown in Figure 11. Here, absence of negative circles allow a perfectly stable coalition of France, Italy and Russia against Austria.

Figure 11: France, Russia, Italy and Austria, 1856-1858, with the new military propensities. It forms a stable system with the coalition of France, Italy and Russia against Austria.

However, the social aspect of the internal politics of the countries dramatically interferes with the stability. The relative amplitudes of the consequent exchanges can be estimated as follows. Due to the political insularity of Russia where serfdom still prevailed over large part of the country, the amplitudes of all its exchanges on the social aspect are "negligible", $K_{FR} = 0$ and $K_{RA} = 0$. France and Austria had a "strong" involvement in the subject, with $K_{FA} = K_{FI} = K_{IA} = 4$. The new propensities between the countries are shown in the table.

| Propensity | F-I | I-A | F-R | F-A | R-A | R-I |
|------------|-----|-----|-----|-----|-----|-----|
| Primary    | -1  | -2  | -1  | -1  | 1   | 0   |
| On $\mathcal{G}$ | 4   | -4  | 2   | -3  | -2  | 1   |
| Total      | 3   | -6  | 1   | -4  | -1  | 1   |

The result system with the French change in favor of cooperation with Austria is shown in Figure 12. As we can see the modified system includes three negative circles. The change of France put Russia in an unfavorable position moving it away from a most beneficial coalition configuration. At the same time, Italy and Austria found themselves in a satisfactory state.

7 Conclusions

Coalitions in a collective of individual rational actors such as countries, when formed spontaneously are rare to stabilize. The probability that the system becomes stable vanishes exponentially with
the size of the system. In reality, stabilization among countries as rational actors is more likely to happen under the external incentive of global alliances and is more practical. The impact of the global principle on the economical, political, social or any other factor of the countries’ interests produces new, intended, interactions between the countries. In contrast to the spontaneous primary interactions, those interactions are intended in the sense that they are based on the directed view of the countries’s needs and interests. Superposed with the spontaneous ones, the interactions guarantee the stabilization once their amplitudes satisfy the constraints of positive circuits of propensities.

One of the interesting directions for further research in the context of the global alliance model is to study the general effect from the global attractions, that is the general interaction amplitudes. While some global attractions represent efficient mediators, the others may be less successful or even harmful with respect to the system’s stability. Because either they become obsolete, or provide no sufficient motivations, or they acts with harmful intentions, the global alliance may fail to stabilize an unstable system, and even may destabilize a stable system. It is interesting to study those effects from general perspective of the system’s total gain (energy), which can be reduced or augmented by the global alliances. The study should help shed the new light on conflicts in post colonial Africa and Middle East which, being under the influence of external fields, continuously cycle in series of contentions.

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