Scheduling the Production of Precast Concrete Elements Using the Simulated Annealing Metaheuristic Algorithm

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Abstract. The problem of optimal scheduling of construction production is particularly important in the case of production processes of concrete precast structural elements. The most important features characterizing this type of construction production are as follows: - the necessity to produce precast elements in factory conditions in order to make the production process independent of the weather conditions, - the repeatability of production processes of precast elements. The paper presents a model for scheduling of the production process of concrete precast structural elements. The model is based on the hybrid flow shop problem (HFS problem), in which it is possible to include more than one device/equipment or group of workers performing one type of production process. The model respects the constraints that are characteristic of the production processes of such elements. The issue of discrete optimization, presented in the paper, is NP-hard. Therefore, the authors proposed the original solution of an optimization task in the model using the simulated annealing metaheuristic algorithm. The application of the presented scheduling model is illustrated by a practical calculation example showing the effectiveness of the used optimization algorithm. The model can be successfully used to solve planning tasks of the production process in plants producing concrete precast elements in order to significantly shorten the duration of production process.

1. Introduction
The production of concrete prefabricated elements is usually carried out in factory conditions. Such conditions allow for the independence of the production process from the influence of atmospheric conditions: rainfall, wind and temperature. This type of production is characterized by considerable repeatability of activities. The production processes of prefabricated elements are usually done by brigades (working groups) consisting of qualified workers serving the necessary equipment. An important issue that occurs during production process planning of such elements is the work schedule of working groups. Production process problems are most often analyzed using various theoretical models of a deterministic or random nature. The so-called method Line of Balance (LOB) [1, 2] and most often the flowshop models are used in the scheduling of the deterministic production process. They have their source in the achievements of job scheduling theory. The flowshop models are usually used in Enterprise Resource Planning systems (i.e. ERP systems) that may not take into account the specifics of the production process of prefabricated elements. Modelling of this type of production process has been presented, among others, in the works [3, 4, 5]. They consider the issues of optimization of production process schedules taking into account various constraints and the objective function, e.g. tardiness of production of elements, penalties for exceeding the dates of production of elements, costs of elements storage, constraints on the quantity of available moulds, devices
accelerating concrete curing. The metaheuristic algorithms were used to solve the discrete optimization tasks, among others the evolutionary algorithm. The paper will present a model for scheduling the production process of elements, which assumes the possibility of doing a given type of works by more than one working group. Unlike in the previous research, the presented model assumes the possibility of changing the order of production of elements for different types of works. This assumption allows for a significant increase in the number of feasible solutions. The duration of production process of a set of prefabricated elements will be the considered objective function.

2. Production technology of the reinforced concrete prefabricated elements
The elements, which are produced in a prefabrication plant, are reinforced concrete elements, i.e.: passenger lift shafts, sanitary cabins, and ventilation blocks. The elements are formed in stationary, for single elements, metal moulds with demountable structure. The mould consists of a hydraulically foldable inner core and four outer sides (walls). One outer side (the so-called “door leaf”) is moved hydraulically, the other three sides are moved by hand using a lever. A working platform with a hydraulic and electric installation is placed around the mould. The production process consists of:
- mould preparation,
- forming of element,
- heat curing,
- mould disassembly.

The preparation of the mould consists in: cleaning the mould (by hand using pneumatic scrapers), then applying anti-adhesive agent to the mould (pneumatically - emulsion sprayer) and, after assembling the reinforcement, assembling the mould (hydraulic assembling of the inner core and then assembling four sides: one (the so-called “door leaf”) hydraulically, three others by hand using a lever). Assembly of the reinforcement takes place from the top – a ready, semi-prefabricated reinforcement basket (prepared in the reinforcement plant) is transported and mounted using a crane with a sling. The concrete mix is laid out from the container (a hopper) suspended by a sling on the crane. The compaction of the concrete mix is performed with vibrators mounted on four sides of the mould. Accelerated maturation of concrete is carried out by the thermal curing - the so-called “contact” method. The heating medium is water vapour, which is supplied to the inner core of the mould (which has a closed cross-section). The element is disassembled by folding the inner core of the mould and removing the outer sides (the one, so-called “door leaf”, hydraulically, the other three by hand using a lever). The element is removed from the mould by lifting using a crane with a sling.

3. Optimization model of the production process of precast elements
The optimization model of the precast production described in the paper is as follows:

Parameters:
- The project consists of a set of precast components:
  \[ Z = \{Z_1, Z_2, Z_3, ..., Z_j, ..., Z_n\} \] (1)
- For the execution of the works in the precast production, there are teams of working groups created, each of which performs one type of work. They form a set:
  \[ B = \{B_1, B_2, B_3, ..., B_k, ..., B_m\} \] (2)
- In each team of working group \( B_k \in B \) there is \( m_k \geq 1 \) of the same working groups (of the same efficiency and composition):
  \[ B_k = \{B_{k1}, B_{k2}, B_{k3}, ..., B_{ki}, ..., B_{km_k}\} \] (3)
- Each precast component \( Z_j \in Z \) requires performance of \( m \) works which create the set:
  \[ O_j = \{O_{j1}, O_{j2}, O_{j3}, ..., O_{jk}, ..., O_{jm}\} \] (4)
- It is assumed that work \( O_{jk} \in O_j \) can be realized by the working group \( B_{ki} \subset B_k \). Work duration time \( O_{jk} \) performed by the group \( B_{ki} \) equals \( p_{jk} > 0 \). The set of duration of \( p_j \) works from the set \( O_j \) is defined by vector:
\[ p_j = \{ p_{j1}, p_{j2}, p_{j3}, \ldots, p_{jm} \} . \]  

Constraints:

- The order of execution of the works resulting from work technology is assumed such that: 
  \[ O_{j,k+1} < O_{j,k} < O_{j,k+1} . \]
- It is assumed that work \( O_{j,k} \in O_j \) is performed without any stops by one working group \( B_k \) from \( B_k \) team in \( p_{jk} > 0 \) time.
- It is assumed that the work \( O_{j,k} \in O_j \) is performed continuously by the working group \( B_k \) in \( p_{jk} > 0 \) time.

Decisive variable is precast components processing order, which is defined by \( m \)-tuple:

\[ \pi = (\pi_1, \pi_2, \ldots, \pi_k, \ldots, \pi_m). \]  

Performing of the work \( k \) by the team of working group \( B_k \) in all precast components is defined by set:

\[ \pi_k = (\pi_{k1}, \pi_{k2}, \ldots, \pi_{ki}, \ldots, \pi_{km}) , \]  

where: \( \pi_{ki} = (\pi_{ki}(1), \pi_{ki}(2), \ldots, \pi_{ki}(l), \ldots, \pi_{ki}(n_{ki})) \). \( \pi_{ki} \) determines the processing order of precast components by working group \( B_k \) from the team of working groups \( B_k \subset B \) of size \( m_k \).

In the presented model of the precast production the task is to find a schedule (\( S, A \)), (where \( S = [ S_{jk} ]_{n \times m} , A = [ a_{jk} ]_{n \times m} \) and \( S_{jk} \) defines earliest start time for carrying out works in precast component \( Z_j \) by team of working groups \( B_k \) and \( a_{jk} \) defines number of working group allocated to carrying out works in unit \( Z_j \) ) in order to minimize or maximize value of objective function, while meeting the accepted constraints.

Criterion (objective function) is a term \( C_{max}(\pi) \) of implementation of all works in all precast components:

\[ C_{max}(\pi) = \max \{ C_{mj} \} . \]  

Optimization task in the model is to find a schedule for the work realization which minimizes the value of the objective function \( C_{max}(\pi) \), satisfying the constraints given above. The considered model can be represented in the form of a disjunctive graph. Any graph for the present model has the property of the critical path of length \( C_{max}(\pi) \). The earliest finish time for works can be determined from the recursive formula:

\[ C_{k,l} = \max \{ C_{k,l-1} \}, C_{k-1,l} \} + p_{k,l} . \]

In the scheduling theory the presented model is a kind of flow shop problem with parallel machines (i.e. hybrid flow shop problem [6]) with the \( C_{max} \) criterion. In literature this problem is strongly NP-hard. From this fact it follows that it is not possible to create a branch-and-bound algorithm for solving optimization problems in the presented model of the precast production. Such an algorithm would enable its users to solve a given discrete optimization task in an accurate way, however, it would require a computational time in the form of very rapidly increasing exponential function along with an increase in the size of the problem. In the analyzed model, the solution to this problem was carried out using the approximate simulated annealing metaheuristic algorithm.

4. Computation example

In the prefabrication plant, it is planned to produce a set of \( n = 9 \) reinforced concrete prefabricated elements. The set includes the following elements: precast component No.1 – 1 piece, precast component No.2 – 2 pieces, precast component No.3 – 3 pieces, precast component No.4 – 2 pieces, precast component No.5 – 1 piece. Each of the prefabricated element requires five different works to
be done on it: A – mould assembly, B – applying anti-adhesive agent to mould and reinforcement assembly, C – laying concrete mix with its compacting, D – thermal curing, E – mould disassembly. Works: A, B, C, E are done by teams of working groups that are specialized to do only one type of work. Work D, i.e. thermal curing, is done without direct participation of working groups and lasts for each element 1005 minutes (16.75 hours). The creation of an appropriate work schedule of working group teams (doing the first three works during one work shift) is the most important problem for the organization of works in the prefabrication plant. Therefore, the optimization task was limited to only the first three works. The following types and quantities of working group teams are available in the prefabrication plant during the work shift: for work A – 2 working groups, for work B – 2 working groups, for work C – 2 working groups. On the basis of the labour intensity of works and the composition and efficiency of working groups, the authors determined the works durations for \( n = 9 \) prefabricated elements, which are presented in table 1.

| number of element type | 1 | 2 | 3 | 4 | 5 |
|------------------------|---|---|---|---|---|
| quantity of elements of a given type | 1 | 2 | 3 | 2 | 1 |
| A. Mould assembly | 55 | 14 | 21 | 19 | 39 |
| B. Applying anti-adhesive agent to mould and reinforcement assembly | 36 | 29 | 13.5 | 31 | 37 |
| C. Laying concrete mix with its compacting | 27.5 | 20 | 9 | 25 | 23 |

It was adopted an initial solution (i.e. reference solution) with the assumption of producing elements according to the order of their numbering, i.e. for the following decision variable \( \pi = (\pi_1, \pi_2, \pi_3) \), where:

\[
\pi_1 = ((1,2,3,4,5), (6,7,8,9)), \\
\pi_2 = ((1,2,3,4,5), (6,7,8,9)), \\
\pi_3 = ((1,2,3,4,5), (6,7,8,9)).
\]

In the above decision variable, it is assumed that for works: A, B, C the first working groups (from the given teams) will do their type of works for the first five elements. The second working groups (from the given teams) will do their type of work for the remaining four elements. The order of their execution was adopted in accordance with their numbering. The duration of production process of \( n = 9 \) prefabricated elements according to the adopted schedule for the accepted decision variable \( \pi \) is \( C_{\text{max}} = 187 \) minutes. The work schedule of working groups teams for the initial solution is presented in figure 1.
5. Method for solving optimization problem

The described optimization problem shown in the above example, belongs to NP-hard discrete optimization problem. For searching the minimum duration of the production process of the precast elements there is the approximate simulated annealing (SA) algorithm proposed, which belongs to the group of metaheuristics. The SA algorithm has been proposed in the work of Kirkpatrick [7]. This algorithm uses analogous to the thermodynamic process of cooling the solid in order to introduce the trajectory of the search of the local extreme. States of solid matter are seen analogously as individual solution to the problem, whereas the energy of the body as the value of the objective function. During the physical process of cooling the temperature is reduced slowly in order to maintain energy balance. The SA algorithm starts with the initial solution, usually chosen at random. Then, in each iteration, according to established rules or randomly, there is solution $\pi'$ selected from the base neighbourhood $\Pi$. It becomes the base solution in the next iteration, if the value of the objective function is better than the current base solution or if it otherwise may become the base solution with the probability of:

$$p = \exp(-\Delta / T_i),$$  \hspace{1cm} (10)

where $\Delta = c(\pi') - c(\pi)$, $T_i$ – the temperature of the current iteration $i$, $c$ – the objective function. In each iteration there are $m$ draws from the neighbourhood of the current basic solution performed. The parameter called the temperature decreases in the same way as in the natural process of annealing. The most frequently adopted patterns of cooling are:

- geometrical $T_{i+1} = \lambda_i T_i$,
- logarithmic $T_{i+1} = T_i / (1 + \lambda_i T_i)$,

where $i = 0, ..., N - 1$, $T_0$ – the initial temperature, $T_N$ – the final temperature, $N$ – number of iterations, $\lambda_i$ – parameter. In the algorithm there are usually parameter values $T_0$, $T_N$, $N$ adopted and parameter $\lambda_i$ is calculated. The relationship $T_0 > T_N$ should take place, whereas $T_N$ should be small, close to zero.

Below, there is presented a general method of SA algorithm used to solve the flow shop problem.

**Step 0.** Determine the initial solution $\pi^0 \in \Pi$. Substitute $\pi^{SA} = \pi^0$, $k = 0$, $T = T_0$.

**Step 1.** Perform steps 1.1 - 1.3 $x$-times.

**Step 1.1.** Substitute $k := k + 1$. Choose random $\pi \in \mathcal{N}(\Pi, \pi^{k-1})$.

**Step 1.2.** If $c(\pi) < c(\pi^{SA})$ then substitute $\pi^{SA} = \pi'$.

**Step 1.3.** If $c(\pi') < c(\pi^{k-1})$ then substitute $\pi^k = \pi'$. Otherwise, accept solution $\pi'$ with a probability of $p = \exp((c(\pi^{k-1}) - c(\pi'))/T_i)$, i.e. $\pi^k = \pi'$, if solution $\pi'$ was not accepted.

**Step 2.** Change the temperature $T$ according to a defined pattern of cooling.

**Step 3.** If $T > T_N$, return to step 1, otherwise STOP.

The following assumptions concerning the form and the parameters of the SA algorithm were adopted:

- neighbourhood $\mathcal{N}_c$ contains permutations generated from $\pi$ with the use of "insert" move,
- Boltzmann function of acceptance was adopted,
- geometric cooling scheme was adopted, i.e. $T_{i+1} = \lambda T_i$ and $T_0 = 60$, $\lambda = 0.99$, the number of considered solutions at a set temperature - $0.5 n$,
- maximum number of iterations of the algorithm SA: $\text{MaxIter} = 10000$.

The algorithm terminates with the condition of completion, which relies in performing $\text{MaxIter}$ of its iterations. SA algorithms are used to solve many optimization problems, including flow shop problems considered in the context of discrete optimization problems [8, 9]. Good results obtained in the applications are result of the fact that SA algorithms are considered to be as one of the most powerful tools in solving such problems.
5.1. Verification of results obtained using the simulated annealing algorithm

For the presented above form of the SA algorithm, the authors evaluated the results which were obtained with it. For this purpose, the dedicated software in the Mathematica system has been created for the model of the problem presented in the paper. The examples from [10] were used to verify the obtained results. They are related to the practical problems of the task scheduling of production line in a company that produces printed circuit boards. This company accepts six orders a day to produce a certain amount of different printed circuit boards. The assembly of each board is a task that always consists of three operations and is performed on three types of machines. Each of the three operations can be performed by a number of machines working in parallel. The problem to solve is to find the order of assembly of boards included in the daily order, so that the duration of work is as short as possible. These orders are test examples that can be modelled as a hybrid problem. The sizes of these examples are the following: \( n \times m = 51 \times 3 \) ("day 1"), \( 38 \times 3 \) ("day 2"), \( 38 \times 3 \) ("day 3"), \( 36 \times 3 \) ("day 4"), \( 40 \times 3 \) ("day 5"), \( 30 \times 3 \) ("day 6"). Each of the considered test example was solved seven times. Then the arithmetic mean of these calculations was related to the value of the lower bound of objective function \( LBC_{max} \), given for these examples in [10], by calculating for each example the mean relative error \( PRD(SA) \) of the SA algorithm:

\[
PRD(SA) = 100\%\frac{(C^{SA} - LBC_{max})}{LBC_{max}},
\]

where: \( C^{SA} \) – the average value of the objective function obtained by the SA algorithm, \( LBC_{max} \) – the value of the estimation of the lower bound of the objective function from [10]. In order to compare the quality of the obtained results, the relative errors of the heuristic algorithm \( W \) were presented as proposed in [10] and the TSAB algorithm, which is an advanced version of the metaheuristic tabu search algorithm given in [11]. The results of calculation of the SA algorithm for the test examples from [10] are presented in Table 2.

| Notation of the example acc. to [10] | Average value \( C^{SA} \) acc. to [10] | \( LBC_{max} \) acc. to [10] - algorithm W | \( C^{W} \) acc. to [10] - algorithm W | \( C^{TSAB} \) acc. to [11] - algorithm TSAB | \( PRD(SA) \) [%] | \( PRD(W) \) [%] | \( PRD(TSAB) \) [%] |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|----------------|----------------|----------------|
| “day1”                              | 784,43                              | 720                                 | 784                                 | 765                                 | 8,95           | 8,89           | 6,25           |
| “day2”                              | 780,86                              | 715                                 | 789                                 | 753                                 | 9,21           | 10,35          | 5,31           |
| “day3”                              | 784,00                              | 694                                 | 785                                 | 760                                 | 12,97          | 13,11          | 9,51           |
| “day4”                              | 792,57                              | 694                                 | 796                                 | 761                                 | 14,20          | 14,70          | 9,65           |
| “day5”                              | 963,00                              | 963                                 | 964                                 | 963                                 | 0,00           | 0,10           | 0,00           |
| “day6”                              | 684,86                              | 584                                 | 686                                 | 661                                 | 17,27          | 17,47          | 13,18          |

The presented above results of verification calculations of the SA algorithm confirm the great usefulness of this algorithm for solving optimization tasks. The results obtained using the SA algorithm used in the paper are comparable or better than the heuristic algorithm \( W \) from [10]. The presented algorithm obtains slightly worse results compared to the results obtained with the advanced TSAB algorithm (the tabu search algorithm) from [11].

5.2. Solution of optimization task of calculation example using the simulated annealing algorithm

Using the software, created in the Mathematica system, the authors have made three attempts to optimize the schedule for the calculation example presented in section 4 of the paper. The best value \( C_{max} = 155,5 \) minutes was obtained for the following decision variable \( \pi = (\pi_1, \pi_2, \pi_3) \), where:

\[
\pi_1 = ((7,8,9,6,5), (2,3,1,4)),
\]

\[
\pi_2 = ((7,8,9,6,5), (2,3,1,4)),
\]

\[
\pi_3 = ((7,8,9,6,5), (2,3,1,4)).
\]
\[ \pi_2 = ((2,8,9,6), (7,3,1,5,4)), \]
\[ \pi_3 = ((2,7,8,9,6,4), (3,1,5)). \]

The optimization done using the SA algorithm allowed shortening the duration of working groups teams by 16.8%. The work schedule of working groups teams for the above suboptimal solution is presented in figure 2.

![Figure 2](image-url)

Figure 2. The work schedule of working groups teams for the found suboptimal solution

Finding a suboptimal solution in the presented calculation example allows also for a significant reduction of breaks during production process of working groups. For the reference solution, the sum of breaks in the work schedule of all working groups is total 61.5 minutes. In the solution obtained using the SA algorithm, the sum of these breaks decreased to 27 minutes, i.e. it decreased by 56.1%.

6. Conclusions

The paper presents a model for scheduling the production process of reinforced prefabricated elements, which uses the assumptions of the NP-hard hybrid flowshop problem (i.e. flowshop problem with the parallel machines). This problem is considered as a problem of the job scheduling theory. The vast majority of problems of discrete optimization discussed in this field of science are also NP-hard. The NP-hard of discrete optimization problems lies in the inability to construct an exact algorithm (finding the optimal solution) that would solve a given problem in a time depending on the polynomial expression. For such problems, it is possible to construct only such exact algorithms, whose calculation time increases exponentially as the size of problems increases. Therefore, even the multiple increases in computing power of computers do not significantly improve the speed of solving such problems with the exact algorithms. At present, for practical tasks, the metaheuristic algorithms are most often used, which provide approximate results, but not too far from optimal ones. The authors propose the use of a simulated annealing algorithm in the paper. It provides results close to the optimal ones, as shown in the verification of results. In addition, it provides results comparable in quality or slightly inferior to other advanced algorithms of discrete optimization. The model presented in the paper can be successfully used in scheduling work of brigades in factory conditions due to the possibility of a significant reduction in the duration of production process of prefabricated elements. An additional benefit of using the model is a significant increase in the continuity of their work by reducing the duration of necessary breaks between works. In the further research the authors plan to apply some additional constraints, other objective functions, which will model the problem of scheduling the production process of reinforced concrete prefabricated elements to a better extent. In addition, it is planned to use other optimization algorithms that can provide better suboptimal solutions than the simulated annealing algorithm used in the paper.
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