Entanglement of individual photon and atomic ensembles

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Abstract

Here we present an experimentally feasible scheme to entangle flying qubit (individual photon with polarization modes) and stationary qubit (atomic ensembles with long-lived collective excitations). This entanglement integrating two different species can act as a critical element for the coherent transfer of quantum information between flying and stationary qubits. The entanglement degree can be also adjusted expediently with linear optics. Furthermore, the present scheme can be modified to generate this entanglement in a way event-ready with the employment of a pair of entangled photons. Then successful preparation can be unambiguously heralded by coincident between two single-photon detectors. Its application for individual photons quantum memory is also analyzed. The physical requirements of all those preparation and applications processing are moderate, and well fit the present technique.

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Quantum entanglement is a fancy correlation between two or more distant subsystems which has no classical analog. Such a correlation has found wide applications in high-precision spectroscopy [1], quantum lithography [2], and quantum information processing including computation, communication and cryptography [3,4]. Up to now individual atoms or photons have been entangled with a linear ion-trap [5], with a spontaneous parametric down converter [6,7], or with a high-$Q$ cavity [8,9]. There is also entanglement between indistinguishable atoms in Bose-Einstein condensates [10] or atomic ensembles [11–14]. In most of those schemes the entangled subsystems are all congeners (the same kind of objects).

From the beginning of information, light has been believed and used as an ideal information carrier. It is well known that the photon polarization modes can be conveniently manipulated with linear optics. Thus individual photon is a perfect chosen for flying qubit in the nearly developed quantum information theory. On the other hand, atomic ensemble has enabled it as a well qualified candidate for stationary and register qubits of quantum information and computation. Its collective excitation modes have the inherent robust to realistic noise and imperfections. Quantum repeater based on atomic ensembles has recently been proposed and shown to be useful in the long distance quantum communications [15]. It has been also proved that the emission of a photon in the forward direction is correlated with an excitation in a symmetric mode of the atomic ensembles [16].

Here we present an idea to entangle two subsystems of different species, individual photon and atomic ensemble, to integrate the features of both flying qubit (individual photon) and stationary qubit (atomic ensemble). The entanglement degree can be also adjusted expediently with linear optics. This entanglement integrate two different congeners can act as a critical element for the coherent transfer of quantum information between flying and stationary qubits. Furthermore, this entangled state can be prepared in a way event-ready, and successful preparation can be unambiguously heralded by coincident between two single-photon detectors. As an example of its various applications, we propose a quantum memory for individual photons based on this novel entanglement.

A quantum memory for individual photons is obviously valuable in many quantum in-
formation processing such as quantum cryptography [3,4,17,18], and secret sharing [19]. Distinct from the previous light memory schemes [20–23], this quantum memory employs a procedure similar to quantum teleportation and is designed to store individual photon modes. As the qubit needed store and the memory qubit don’t directly interact with each other, a higher memory fidelity can be expected in the present quantum memory. As most quantum protocols with atomic ensembles [12–15], the physical requirements of our entanglement preparation and applications are moderate and well fit the present technique.

The basic element of our system is an ensemble of alkali atoms with the relevant level structure shown as Fig. 1. A pair of metastable low states $|g\rangle$ and $|s\rangle$ correspond to Zeeman sublevels of the electronic ground state of alkali-metal atoms. Its experimental realization can be either a room-temperature atomic gas or a sample of cold trapped atoms where long lifetimes for the relevant coherence has been both observed [24–26]. To facilitate enhanced coupling to light, the atomic medium is preferably optically thick along one direction. This can be achieved either by working with a pencil-shaped atomic sample [24–26] or by placing the sample in a low-finesse ring cavity [27,28].

Two identical atomic samples are placed in a beeline with a half-wave plate and a polarization beam-splitter plate (PBS) between them just as shown in Fig. 2. Define an operator $S = (1/\sqrt{N_a}) \sum_{i=1}^{N_a} |g\rangle_i \langle s|$ where $N_a \gg 1$ is the total atom number. All atoms are initially prepared through optical pumping to the ground state $|g\rangle$, which is effectively the vacuum state $|0\rangle_a = \otimes_i |g\rangle_i$ of the operator $S$. Those two atomic samples are illuminated by a short, off-resonant laser pulse in turn that induces atom Raman transitions into state $|s\rangle$.

What particularly interests us is the forward-scattered Stokes light that is co-propagating with the laser. As such scattering events are uniquely correlated with the excitations of the symmetric collective atomic mode $S$, an emission of the single Stokes photon in the forward direction results in the state of atomic ensemble given by $S^\dagger |0\rangle_a = (1/\sqrt{N_a}) \sum_{i=1}^{N_a} |s\rangle_i = |S\rangle$. The excitations in the mode $S$ can be transferred to optical excitations and then detected by single-photon detectors [25,26,29]. Due to the collectively enhanced coherent interaction, the efficiency of such transfer can be very high, which has been demonstrated both in theory
Assume the Stokes photon from this emission is right-handed rotation, and define an effective single-mode bosonic operator $a_R$ for this Stokes pulse with the corresponding vacuum state denoted by $|0\rangle_p$. The $\lambda/2$ plate between the two samples is employed to transform this Stokes photon into left-handed rotation, where $\lambda$ is the wavelength of the Stokes photon. Its function can be denoted by operator $P = |a_L^\dagger\rangle\langle a_R| + |a_R^\dagger\rangle\langle a_L|$ with $L$ ($R$) represents the left (right) handed rotation. We can assume this half-wave plate has no remarkable influence on the interaction efficiency of the pump light and the atomic ensemble. The forward-scattered Stokes pulses are collected and coupled to optical channels (such as fibres) after a filter, which is frequency selective to filter out the pumping light. In the case there is only one Stokes photon, the whole system of the two atomic ensembles and this photon can be written in state

$$|\Phi\rangle_{ap} = (\alpha S_1^\dagger P a_R^\dagger + \beta S_2^\dagger a_R^\dagger) |0\rangle_a |0\rangle_p,$$

(1) where 1 and 2 denotes the two atomic samples respectively. We can assume that the two atomic samples are identical and the phase difference depending on the quantum channel between the two samples is fixed as $\alpha = \beta = 1/\sqrt{2}$. In fact, the two ensemble needn’t be identical and only the parameters $\alpha, \beta$ are affected. But we can adjust those parameters to alter the entanglement degree of the above state with the polarization beam-splitter plate placed between the two atomic ensembles. This polarization plate controls the pass ratio of the left-handed rotation Stokes photon from the first atomic ensemble. Finally the Stokes photons pass through a $\lambda/4$ plate which transforms the circularly-polarized wave to the linearly polarized light. Here $\lambda$ is again the wavelength of the Stokes photon. Then $|\Phi\rangle_{ap}$ can now be written as the maximally entangled EPR state

$$|\Psi\rangle_{ap} = (S_1^\dagger a_L^\dagger + S_2^\dagger a_R^\dagger)/\sqrt{2} |0\rangle_a |0\rangle_p$$

$$= (|S_1\rangle_a |H\rangle_p + |S_2\rangle_a |V\rangle_p)/\sqrt{2}. $$

(2)

Here we have assumed that the light-atom interaction time $t_\Delta$ is short and the mean photon number in the forward-scattered Stokes pulse is much smaller than 1. The probabilities [29] and in experiments [25,26].
for more than one photon $p_0^n$ will be so small that they can be neglected safely, where $n$ represents the number of the Stokes photons. As we will show below, the including of the case that there is no Stokes photon scattered out is insignificance.

Noticeably, the subsystems of this entanglement are of different species, individual photon and atomic ensembles. It is well known that the collective state of atomic ensemble has inherent resilience to noise and imperfections, which makes it well fit to act as quantum memory (stationary qubit). On the other hand, the light is believed to be an ideal carrier (flying qubit) of quantum information and the polarization states of individual photon can be conveniently manipulated with linear optics. Although Raman processing to generate Stokes photons is random, and the probability $p_0$ to prepare the state $|\Phi\rangle_{ap}$ is quite low, various applications can be expected for this novel entanglement combining the features of both flying and stationary qubits. As most entanglement generation protocols, postselection may be needed in those applications.

Alternatively, we can employ a quantum non-demolition measurement device [30,31] to sense the presence of the Stokes photons and wipe off postselection. This device has been shown implementable with an optical interferometer [30]. However, it has shown recently that one can conditionally prepare a pair of photons in maximally entangled polarization state when smaller coefficient higher terms are neglected [32]. The successful preparation can be unambiguously heralded. With a pair of entangled photons, we can further modify the present protocol to generate entanglement state between flying and stationary qubits in an event-ready way.

Assume we have a pair of photons in state $|\Phi\rangle_{AB}^+ = (|HH\rangle_{AB} + |VV\rangle_{AB})/\sqrt{2}$. As shown in Fig. 2, we can incident the Stokes photons generating from the atomic ensembles, and the photons A simultaneously onto a beam-splitter plate. The whole system can then be written in state

$$|\Psi\rangle_{apAB} = (|00\rangle_{ap} + p_0^{1/2} |\Phi\rangle_{ap} + o(p_0)) \otimes |\Phi\rangle_{AB}^+,$$

where $o(p_0)$ represents the term involving more than one Stokes photon. In the expanding
of this state, only the second term with a coefficient $p_0^{1/2}$ totally involve three photons:

$$\langle \Phi \rangle_{ap} \otimes \langle \Phi \rangle_{AB}$$

$$= \left( |S_1\rangle_a |H\rangle_p + |S_2\rangle_a |V\rangle_p \right) / \sqrt{2} \otimes \left( |HH\rangle_{AB} + |VV\rangle_{AB} \right) / \sqrt{2}$$

$$= \frac{1}{2} \left\{ \langle \Phi \rangle^+_p (|H\rangle_B |S_1\rangle_a + |V\rangle_B |S_2\rangle_a) / \sqrt{2} + \langle \Phi \rangle^-_p (|H\rangle_B |S_1\rangle_a - |V\rangle_B |S_2\rangle_a) / \sqrt{2} \right. + |\Psi\rangle^+_p (|V\rangle_B |S_1\rangle_a + |H\rangle_B |S_2\rangle_a) / \sqrt{2} + |\Psi\rangle^-_p (|V\rangle_B |S_1\rangle_a - |H\rangle_B |S_2\rangle_a) / \sqrt{2} \right\} ,$$

where $|\Phi\rangle^\pm_p = (|HH\rangle_{pA} \pm |VV\rangle_{pA}) / \sqrt{2}$, $|\Psi\rangle^\pm_p = (|HV\rangle_{pA} \pm |VH\rangle_{pA}) / \sqrt{2}$ are the four Bell states. When there are coincidence clicks between single-photon detectors placed on each side after the beam-splitter, $D_H$ and $D_V$, or $D_V$ and $D_H'$, the two photons are measured in state $|\Psi\rangle^-_{aB}$. In this case the two atomic samples and photon $B$ are projected into the state $|\Psi\rangle^-_{aB} = (|V\rangle_B |S_1\rangle_a - |H\rangle_B |S_2\rangle_a) / \sqrt{2}$. Similarly, if there are coincidence clicks between single-photon detectors behind the two-channel polarizer (PBS), $D_H$ and $D_V$ or $D_V'$ and $D_H'$, then two photons $A$ and $p$ are measured in state $|\Psi\rangle^+_p$. And the atomic samples are projected into state $|\Psi\rangle^+_a = (|V\rangle_B |S_1\rangle_a + |H\rangle_B |S_2\rangle_a) / \sqrt{2}$. Obviously, this state can be simply transformed into the state $|\Psi\rangle^-_{aB}$. Thus we totally have a probability of $p_0/2$ to generate a maximally entangled state between the two ensembles and one photon. And the successful preparation is unambiguously heralded by coincidence detection of two photons.

As no postselection is needed, this event-ready entangled state between stationary and flying qubits can be used straightforwardly used in various quantum information processing. For example, this entangled state can be explored as a valuable individual photons quantum memory. This can be accomplished just by measuring the photon needed storing $q$ in state $|\Phi\rangle_q = \cos \theta |H\rangle_q + \sin \theta |V\rangle_q$ and the photon $B$ of the state $|\Psi\rangle^-_{aB}$, with the same Bell analyzer as shown in Fig. 2. Then the state of the photon $q$ can be transferred to the two atomic ensembles. The successful probability is $1/2$ as we can only distinguish two Bell states out of four with linear optics.

With the present technique when it transferred to optical excitations [25,26,29] we can conveniently readout the quantum states stored in atomic ensembles. Remarkably quantum state exchange in this quantum memory is between individual photon and atomic ensembles.
This exchange is an important ingredient for the future quantum information networks [20]. It is also crucial for sensitive atomic measurements in optics when quantum limits of accuracy are approached [1]. Distinct from the previous quantum memory [20–23], the present quantum memory has following features: firstly, the polarization states of individual photon is stored, which is used to encode information in many quantum computation and information proceeding. Secondly, the qubit needed storing doesn’t interact with the memory qubit. It is in based on a memory procedure similar to quantum teleportation. Thirdly, there is no complex parameters as interaction time and conditions to effect the memory fidelity. Improving Bell measurement efficiency of the quantum teleportation procedure, we can enhance the memory fidelity which is defined as $|\langle \Phi | \Psi \rangle|^2$ with $|\Phi\rangle$ and $|\Phi\rangle$ represent the states before and after storing. Finally, the physical requirements of the individual photon memory scheme are moderate and well fit the experimental technique.

Finally, we briefly discuss the matters of the experimental implication. As the light-atom interaction time $t_\Delta$ is short and the mean photon number in the forward-scattered Stokes pulse is much smaller than 1, the whole system of the Stokes photon and atomic ensembles is in the mixed

$$\rho_{ap} = \frac{1}{p_0 + 1} (|00\rangle_{ap}\langle 00| + p_0 |\Psi\rangle_{ap}\langle \Psi|).$$

(4)

This mixed state will be purified automatically to the maximally entangled state in the entanglement-based communication schemes with the one Stokes photon coefficient $p_0$ determining the purification efficiency [15]. Then this mixed state can be called as an effective maximally entangled state of individual photon and atomic ensembles. Furthermore, it has been shown that this scheme is capable of event-ready generation of entanglement between stationary and flying qubits with a probability of $p_0/2$. In order to efficiently filter out the pump light from the Stokes photon with the frequency-selective filter, the energy difference between the ground state and the excited should be large enough. Similar to the papers [15,13], the single-bit rotation error (below $10^{-4}$ with the use of accurate polarization techniques for Zeeman sublevels) and the dark count probability (about $10^{-5}$ in a typical
detection time window 0.1\(\mu s\) of single-photon detectors can be both neglected.

Actually, similar entanglement states can be prepared by using only one atomic ensemble with the relevant level structure as shown in Fig. 3. The difficult is the control of the probability amplitudes for the emitted of two kinds of polarized Stokes photons, which may be much easier in the above two separated atomic ensembles case. Supplied with multi-photon entanglement, one can also prepare event-ready multi-party entanglement of flying and stationary qubits in a similar teleportation way as above.

In summary, we have proposed an experimentally feasible scheme to generate entanglement between flying qubit (individual photon with polarization modes) and stationary qubit (atomic ensembles with long-lived collective excitations). This entanglement integrate two different species can act as a critical element for the coherent transfer of quantum information between flying and stationary qubits. Furthermore, the present scheme is modified to generate this entanglement between stationary and flying qubits in a way event-ready. Successful preparation can be unambiguously heralded by coincident between two single-photon detectors. The entanglement degree can be also adjusted expediently with linear optics. As an example of its various applications, we propose a high fidelity quantum memory for individual photons, which is based on a procedure similar to quantum teleportation. As those protocols based on atomic ensembles, the physical requirements of all the preparation and applications of the present scheme are moderate and well fit the present technique. Their experimental implementations can be expectable in near future.

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Figure Captions:

Figure 1: The relevant level structure with $|g\rangle$, the ground state, $|e\rangle$, the excited state, and $|s\rangle$ the metastable state for storing a qubit. The transition $|g\rangle \rightarrow |e\rangle$ is coupled by the classical laser (the pump light) with Rabi frequency $\Omega$, and the forward-scattered Stokes light comes from the transition $|e\rangle \rightarrow |s\rangle$, which is right-handed rotation. For convenience, we assume off-resonant coupling with a large detuning $\Delta$.

Figure 2: Schematic setup-up for the preparation and application for quantum memory of entanglement between the Stokes photon $p$ and the two atomic ensembles $S_1$ and $S_2$. The $\lambda/2$ plate transforms the Stokes photon scattered out from the sample $S_1$ into the left-handed rotation. The polarization beam splitter plate (PBS) between the two atomic ensembles is
employed to adjust the entanglement degree. The frequency-selective filter separate the pump light from the Stokes photon. And the $\lambda/4$ transforms the circularly-polarized wave to the linearly polarized light. This Stokes photon $p$ and the addition photon $A$ or $q$ interfere at a 50%–50% beam splitter BS, with the outputs analyzed by two double-channel polarisers respectively and detected by four single-photon detectors $D_H$, $D_V$, $D_V'$ and $D_{H'}$. In fact this a Bell-state analyzer. The coincidence clicks between $D_H$ and $D_V$, or $D_V$ corresponds $|\Psi\rangle^-_{pq}$ and the coincidence clicks between $D_H$ and $D_V$ or $D_{V'}$ and $D_{H'}$ corresponds $|\Psi\rangle^+_{pq}$.

Figure 3: The relevant level structure with $|g\rangle$, the ground state, $|e\rangle$, the excited state, and $|r\rangle$, $|l\rangle$ the two metastable state for storing a qubit. The transition $|g\rangle \rightarrow |e\rangle$ is also coupled by the classical laser (the pump light) with Rabi frequency $\Omega'$, and the two kinds of forward-scattered Stokes light comes from the transition $|e\rangle \rightarrow |r\rangle$ and $|e\rangle \rightarrow |l\rangle$ which are right-handed and left-handed rotation respectively. For convenience, we assume off-resonant coupling with a large detuning $\Delta$. 

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Fig. 1 Guo

\[ |e\rangle \]

\[ \Delta \]

\[ |s\rangle \]

\[ |g\rangle \]

Fig. 3. Guo

\[ |e\rangle \]

\[ \Delta \]

\[ |r\rangle \]

\[ |l\rangle \]

\[ |g\rangle \]