A Universal Density Profile for Cosmic Voids

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We present a simple empirical function for the average density profile of cosmic voids, identified via the watershed technique in ΛCDM N-body simulations. This function is universal across void size and redshift, accurately describing the entire radial range of scales around void centers with only two free parameters. In analogy to halo density profiles, these parameters describe the scale radius and the central density of voids. While we initially start with a more general four-parameter model, we find two of its parameters to be redundant, as they follow linear trends with the scale radius in two distinct regimes of the void sample, separated by its compensation scale. Assuming linear theory, we derive an analytic formula for the velocity profile of voids and find an excellent agreement with the numerical data as well. In our companion paper, Sutter al. (2014) [1], the presented density profile is shown to be universal even across tracer type, properly describing voids defined in halo- and galaxy distributions of varying sparsity, allowing to relate various void populations by simple rescalings. This provides a powerful framework to match theory and simulations with observational data, opening up promising perspectives to constrain competing models of cosmology and gravity.
8 contiguous logarithmic bins in void radius, to account for the poor statistics of the largest voids. The resulting stacks, along with their standard deviations, are shown as symbols with error bars in the left panel of Fig. 1. As expected, stacked voids are deeply underdense inside, with their central density increasing with void size. In addition, the variance of underdense regions is suppressed compared to overdense ones, yielding the smallest error bars in the centers of the emptiest voids. The profiles all exhibit overdense compensation walls, with a maximum located slightly outside their effective void radius, shifting outwards for larger voids. The height of the compensation wall decreases with void size, causing the inner profile slope to become shallower. This trend divides all voids into being either over- or undercompensated, depending whether the total mass within their compensation wall exceeds or falls behind their missing mass in the center, respectively. Ultimately, at sufficiently large distances to the void center, all profiles approach the mean background density.

We propose a simple empirical formula that accurately captures the properties described above,

$$\frac{\rho_v(r)}{\bar{\rho}} - 1 = \delta_c \frac{1 - (r/r_s)^\alpha}{1 + (r/r_c)^\beta}, \quad (2)$$

where $\delta_c$ is the central density contrast, $r_s$ a scale radius at which $\rho_v = \bar{\rho}$, and $\alpha$ and $\beta$ determine the inner and outer slope of the void’s compensation wall, respectively. The best fits of this four-parameter model to the void density stacks are shown as solid lines in the left panel of Fig. 1. The concordance with the numerical data is exquisite everywhere.

**Velocity profile.**—We estimate the velocity profile of tracer particles around void centers by calculating

$$v_c(r) = \frac{1}{N(r)} \sum_i v_i(r_i) \cdot \frac{r_i}{r_i} V_c(r_i) \Theta(r_i), \quad (3)$$

for every void and then averaging over all void radii in a given bin. Here, $v_i$ is the particle velocity vector, $V_c(r_i)$ the Voronoi cell volume of a particle located at $r_i$ and $N(r) \equiv \sum_i V_c(r_i) \Theta(r_i)$. Using the Voronoi volumes $V_c$ as weights ensures a volumetric representation of the velocity field.

The right panel of Fig. 1 depicts the resulting velocity stacks using the same void radius bins as for the density stacks. Note that a positive velocity implies outflow of tracer particles from the void center, while a negative one denotes infall. As the largest voids are undercompensated (void-in-void), i.e. the total mass in their surrounding does not make up for the missing mass in their interior, they are characterized by outflow in the entire distance range. Tracer velocities increase almost linearly from the void center until they reach a maximum located slightly below the effective void radius of each sample, which indicates the increasing influence of the overdense compensation wall. When passing the latter, tracer velocities are continuously decreasing again in amplitude and approach zero in the large distance limit.

Small voids may exhibit infall velocities, as they can be overcompensated (void-in-cloud). This causes a sign change in their velocity profile around the void’s effective radius beyond which matter is flowing onto its compensation wall, ultimately leading to a collapse of the void. Moreover, because small voids are more underdense in the interior, their velocity profile is more nonlinear and less accurately sampled there. The distinction between over- and undercompensation can directly be inferred from velocities, since only overcompensated voids feature a sign change in their velocity profile, while undercompensated ones do not. Consequently, the flow of tracer particles around precisely compensated voids vanishes already at a finite distance to the void center and remains zero outwards. By slightly shifting the void radius bins, we determined this to be the case for voids with
\( \bar{r}_v \simeq 17.6 h^{-1} \text{Mpc} \) in our sample, which we denote as the compensation scale. It can also be inferred via clustering analysis in Fourier space, as compensated structures do not generate any large-scale power \([34]\). We checked that the compensation scales obtained from these two independent methods agree very accurately, indicating a strong link between the spatial and dynamical characteristics of voids.

In linear theory the velocity profile can be related to the density using \([37]\),

\[
v_v(r) = -\frac{1}{3} \Omega_m^2 H(z) r \Delta(r) ,
\]

where \(\Omega_m\) is the matter content in the Universe, \(\gamma \simeq 0.55\) the growth index of matter perturbations, \(H(z)\) the Hubble constant and \(\Delta(r)\) the integrated density contrast defined as

\[
\Delta(r) = \frac{3}{r^3} \int_0^r \left( \frac{\rho_c(q)}{\bar{\rho}} - 1 \right) q^2 dq .
\]

With Eq. (2), this integral yields

\[
\Delta(r) = \delta_c \left[ 2F_1 \left( 1, \frac{3}{\beta}; \frac{3}{\beta} + 1, -(r/r_v)^\beta \right) + \frac{3 \delta_c (r/r_v)^\alpha}{\alpha + 3} \right] 2F_1 \left( 1, \frac{\alpha + 3}{\beta}; \frac{\alpha + 3}{\beta} + 1, -(r/r_v)^\beta \right) ,
\]

where \(2F_1\) is the Gauss hypergeometric function. When plugged into Eq. (4), we can use this analytic formula to fit the velocity profiles obtained from our simulations, the results are shown as solid lines in the right panel of Fig. 1. As for the density profiles, the quality of the fits is remarkable, especially for large voids. Only the interiors of smaller voids show stronger discrepancy, which is mainly due to the decreasing validity of linear theory, i.e. Eq. (4). We obtain best-fit parameter values that are very similar to the ones resulting from the density stacks above.

In fact, evaluating the velocity profile at the best-fit parameters obtained from the density stacks yields almost identical results, as indicated by the dashed lines in Fig. 1.

With the explicit form for the integrated density profile in Eq. (6) it is straightforward to determine the void’s uncompensated mass, defined as \([34]\)

\[
\delta m = \lim_{r \to \infty} \frac{4\pi}{3} \bar{\rho} r^3 \Delta(r) .
\]

The limit only exists for \(\beta > \alpha + 3\) and yields

\[
\delta m = \frac{4\pi^2 \bar{\rho} r^3 \delta_c}{\beta} [\csc(3\pi/\beta) - (r_v/r_s)^\alpha \csc((\alpha + 3)\pi/\beta)] ,
\]

i.e., compensated voids with \(\delta m = 0\) satisfy the relation

\[
(r_v/r_s)^\alpha = \frac{\sin(3\pi/\beta)}{\sin((\alpha + 3)\pi/\beta)} ,
\]

independently of \(\delta_c\).

**Universality.**—Figure 2 depicts the best-fit parameters for each void density stack, where we plot \(\delta_c, \alpha\) and \(\beta\) against \(r_s/\bar{r}_v\). This representation reveals noticeable correlations among the parameters, which hints at a redundancy of the parameter space. In particular, \(\alpha\) exhibits a linear trend with \(r_s/\bar{r}_v\), while \(\beta\) follows a more complicated behavior. However, dividing the void sample into over- and undercompensated voids at \(\bar{r}_v \approx 17.6 h^{-1} \text{Mpc}\), one can approximate \(\beta\) to also follow a linear trend on either side of the vertical line indicating the compensation scale. This can be seen in Fig. 2, where the solid lines result from a linear regression of the corresponding data points, with their explicit linear relations stated aside. They provide a reasonable fit with respect to the size of the error bars. For compensated voids, \(r_s/\bar{r}_v \approx 0.91\), \(\alpha \approx 2\) and \(\beta \approx 9.5\) attains a maximum. These values precisely satisfy Eq. (9). Even the central density \(\delta_c\) exhibits a noticeable correlation with \(r_s/\bar{r}_v\), but following a more nonlinear behavior. We therefore do not attempt to express \(\delta_c\) as a function of \(r_s/\bar{r}_v\) and leave it as a free parameter.

These results suggest the parametrization of Eq. (2) to be overdetermined, and hence the number of free parameters too large for the entire sample of voids. We can plug the best-fit relations

\[
\alpha(r_s) \simeq -2(r_s/\bar{r}_v - 2) ,
\]

\[
\beta(r_s) \simeq \begin{cases} 
17.5 r_s/\bar{r}_v - 6.5 & \text{for } r_s/\bar{r}_v < 0.91 \\
-9.8 r_s/\bar{r}_v + 18.4 & \text{for } r_s/\bar{r}_v > 0.91 
\end{cases}
\]

from Fig. 2 back into Eq. (2) and repeat the fitting procedure for the void stacks in Fig. 1 with the two remaining free parameters \(\delta_c\) and \(r_s\). This yields fits that are essentially indistinguishable from the original four-parameter model.
Figure 3 examines the redshift dependence of the density and velocity profile of voids. Here we focus on one of the previous bins with fixed comoving void radius $\bar{r}_v = 11.7h^{-1}\text{Mpc}$, representing an overcompensated void. For a first-order approximation, we shall neglect the expansion or contraction of voids that can either leave or enter the bin. As apparent from the left panel, the compensation walls around the void radius grow substantially, while the inner void regions are continuously emptied out, in agreement with theoretical expectations [36]. This is consistent with the evolution of velocities as depicted in the right panel of Fig. 3. Due to its overcompensation, tracer particles outside the void build up higher and higher velocities towards the void center, which causes the compensation wall to grow. Inside the void the outflow also first increases at high redshift, but as the Universe accelerates its expansion after the onset of dark energy domination ($z \sim 1$), this trend reverses and the outflow is attenuated.

Solid lines in Fig. 3 represent the best-fit solutions of Eqs. (2), (4) and (6) to the data, the corresponding parameters for the density profile are shown in the figure inset. The excellent agreement even at higher redshifts indicates a universal behavior of these empirical formulae. We also repeated our analysis based on a WMAP7 cosmology [38], finding fully consistent results. Moreover, in this paper we neglected the impact of redshift-space distortions, since void density profiles can be reconstructed in real space when statistical isotropy is assumed [25]. Nevertheless, we find redshift-space distortions to just mildly affect the profile shapes of voids, barely degrading the quality of our fits.

Discussion.—There are a number of cosmological applications to make use of the presented functional form of the average void density profile. For example, studies of gravitational (anti-) lensing that directly probe the projected mass distribution around voids [39, 42], which in turn may serve as a tool for constraining models of dark matter, dark energy and modified gravity. But also considering galaxy surveys, an accurate model for the void density profile can aid in measuring the Alcock-Paczynski effect [3, 43, 44] and the Integrated Sachs-Wolfe effect [2, 6, 10, 45, 46], for example. This is thanks to the universal nature of Eq. (2), which even describes voids in the distribution of galaxies remarkably well, as demonstrated in [1]. With that, clustering analyses based on the void model [34] can directly make use of the analytical form of the density profile for voids. In [1] it is further pointed out that the impact of tracer sparsity and bias on the definition of voids can be accounted for by simple rescaling of void sizes. These findings corroborate other indications that cosmic voids may indeed offer new and complementary approaches to modeling fundamental aspects of the large-scale structure of our Universe.

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