A locus problem solved by using a mechanism with three dyads and two leading elements

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Abstract. In Geometry there are many types of loci, solved by means of classic geometrical considerations and yielding lines and arcs of circles or conics. Yet more complicated locus can be solved by means of the Theory of Mechanisms. Our research starts from a locus and provides a solution based on the Theory of Mechanisms, finding the equivalent mechanism. The structural and cinematic analysis of the mechanism is made, determining the trajectory of a point representing the locus which presents interest. The mechanism has three dyads and two leading elements, for which the movements were correlated by means of a coefficient q. For various values of q different loci were obtained, similar for close values of q but different for significantly different values of q.

1. Introduction

Many mechanisms generating geometric curves were built in an empiric manner along time, by ingenuity and intuition. Afterward, mathematicians like Pafnuti Cebișev, Evans, Paucellier, Cardan, Reuleaux and many others obtained mechanisms with generating properties by using geometric considerations, relying on the properties of curves and the Theory of Mechanisms. A significant number of contributions are authored by I. I. Artobolevskii [1], who determined techniques to apply some geometric considerations. They enabled him to create many mechanisms which generate mathematic curves [2].

Cinematic possibilities of mechanisms with two and three leading elements, with linearly correlated movements are studied in [3]. Many examples of such mechanisms and examples of plotted curves are provided.

Scale models of mechanisms which move such as to describe loci of some points with aesthetic properties are presented in [4].

Many examples of mechanisms used to generate different curves as geometric curves are presented in [5]. The types of curves fall into well known categories like ellipses, parables, lemniscates etc. Imposed loci are used as starting points in [6] in order to define various mathematic curves. Generating mechanisms which determine such curves are determined. When rotating, they generate aesthetic surfaces.

In [7] graphical constructions of some ordinary mathematic curves are given.
2. The locus problem
Considering the system of axis HOB from figure 1, a parallel to the ordinate is plotted such as to include H. We search for the locus corresponding to the intersection point of two lines which move with their heads following the axes OH, OB and HD.

Figure 1. Geometric construction

3. Study of mechanism
Coulisses are introduced at the ends in order to force the heads of the lines to move always along the axes, figure 2. In order to find the locus of the point E which represents the intersection between the mentioned lines, two coulisses will be introduced. They move along the lines and are related by means of a rotation couple.

Figure 2. Equivalent mechanism of type P-P-RRP-RRP-PRP
Figure 3. The mechanism structure

The mechanism’s degree of mobility is: \( M=3n-2C5-C4=3 \times 2-2 \times 11=2 \). Consequently there are two leading elements \(( S_1 \text{ and } S_6)\), figure 1 reveals that both lines can perform movements independently to each other.

The mechanism’s structure is given in figure 3. The mechanism has two leading elements \(( S_1 \text{ and } S_6)\) and three dyads. Its type is P-P-RRP-RRP-PRP.

The following equations can be written by using the method of contours:

\[
\begin{align}
  x_B &= S_1 + AB \cos \alpha = 0 \\
  y_B &= AB \sin \alpha = S_3 \\
  x_D &= S_4 + CD \cos \beta = const. \\
  y_D &= CD \sin \beta = S_6
\end{align}
\]
\begin{align*}
  x_E &= S_3 + S_5 \cos \beta = S_1 + S_2 \cos \alpha \\
  y_E &= S_5 \sin \beta = S_2 \sin \alpha 
\end{align*}

(3)

From equations (1) one can determine \( S_3 \) and \( \alpha \). \( S_4 \) and \( \beta \) can be determined from equations (2). The next step is to calculate \( S_2 \), \( S_5 \) and the coordinates of the tracing point \( E \) from equations (3).

4. Results

The values used as initial data are: \( AB=68 \), \( CD=70 \), \( x_D=56 \). The movements of the leading elements are considered to be linearly correlated through the equation (4):

\[ S_6 = q \cdot S_1 \]

(4)

Different loci are expected to be obtained for different values of the coefficient \( q \).

It should be mentioned that the length \( AB \) imposes limits over \( S_1 \), which for this case varies within the range \(-68 ... 68\). Similarly the limits for the variation of \( S_4 \) \([-70, 70]\) are imposed by the length \( CD \). These quantities appear in the above equations, as having known values. For certain values of \( "q" \), the values for \( S_6 \) can fall outside the range \(-70...+70\). In this case the mechanism can no longer operate.

Therefore the program includes special sequences to detect these cases and prompts the user to provide other values for \( "q" \).

For \( q=-0.6 \) we obtained the position of the mechanism depicted by figure 4.

![Figure 4](image)

**Figure 4.** Position of the mechanism for \( q=-0.6 \)

For \( q=1 \) there were obtained the locus from figure 5 and the values of the races from figure 6.

This is a curve not known from Geometry. It has a turning point in a specific area and exhibits other properties to be studied with methods from Differential Geometry. Such curve can be used, for example, in technical applications - e.g. for tracing operations or for trajectories imposed by certain points from equipment in the textile industry.
The locus is an open curve, similar to the letter α. The following conclusions can be drawn based on figure 6:
- the curve described by S₆ is linear;
- the curve described by S₅ is similar to a cosine;
- the curves for S₃ and S₄ are similar, with opposite curvatures and present symmetries.
For q=0.6 there was obtained the locus from figure 7 and the values of races from figure 8.

![Figure 7. Locus for q=0.6](image)
![Figure 8. Values of races for q=0.6](image)

In this case the locus is a curve with a shape of flower with two petals. Races similar to those described above were obtained, except for that corresponding to S₆, which is opposite to that corresponding to S₃ now. Different sizes characterize the new curves.
The locus obtained for q=-1 is depicted by figure 9 whilst the values of races are presented in figure 10.

![Figure 9. Locus for q=-1](image)
![Figure 10. Values of races for q=-1](image)

In this case the locus is an interesting curve, with rectilinear portions, similar to a hyperbola with asymptotes, not known from Geometry. The influence of the coefficient “q” is noticeable.
Different loci, obtained for various values of q are depicted by figures 11-15.
The curves are different to each other. For close values of q, similar curves are obtained. On the other side, when the values of q are significantly different, completely different curves are obtained.
All the curves are open. Some of them have complex shapes, whilst others are simple branches, similar to lines. When changing the movement sense of the element 6, when q<0, curves similar to those corresponding to the initial sense are obtained, but their positioning with respect to the system of axes is different.
Figure 11. Locus for q=5

Figure 12. Locus for q=0.8

Figure 13. Locus for q= -3

Figure 14. Locus for q= -0.8

Figure 15. Locus for q= -0.2
Figures 16-18 present successive positions of the mechanism for different values of q.

- The lines CD and AB fall out the system of axes toward negative values;
- For small values of q, the line CD is moving inside a narrow domain;
- The race of the point C toward the left limit of the system of axes is smaller for high values of the race of the point D, because the difference between the line HC and the arc of circle with the radius CD and the center at the left limit of C is small;
- For q<0 identical successive positions to those corresponding to q>0 are obtained for the same absolute values of q.

5. Conclusions
The problem of locus used as starting point was made equivalent to a problem of cinematic synthesis and analysis (positions and trajectories) of an equivalent mechanism. The mechanism was analyzed and the searched locus was determined. It consists of the trajectory of the point representing the intersection of two lines. These trajectories are not known from Mathematics. Their usability in technique for example can be related to the design of some equipment in the textile industry.

Because the movement of both lines were independent, they were correlated by means of a coefficient q in order to prevent the random character of their movement. For various values of q, various loci were obtained. For certain domains corresponding to the variation of q, the obtained curves are similar, but have different sizes. For significantly different values of q, different curves were obtained. All the curves are open and their shapes are either complex, or simple arcs.

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