Ideas whose time has gone

José M. Gracia-Bondía¹,² and Joseph C. Várilly³

¹ Departamento de Física Teórica, Universidad de Zaragoza, Zaragoza 50009, Spain
² Escuela de Física, Universidad de Costa Rica, San Pedro 11501, Costa Rica
³ Escuela de Matemática, Universidad de Costa Rica, San José 11501, Costa Rica

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Abstract

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String-local fields constitute a relatively new tool for solving quantum field theory, stressing and embodying locality and positivity. We examine here their usefulness – as well as some drawbacks. Starting from just the physical masses and charges of the known particles, and bringing string independence in the Bogoliubov–Epstein–Glaser (BEG) theory framework, regarded as a means of discovery, one finds the allowed couplings of quantum fields associated to those particles, and thereby recovers all of the Standard Model (SM) without invoking theoretical prejudices. One of the outcomes is the requirement that the fields be governed by reductive Lie algebras. Another is the need for at least one scalar particle. Yet another is chirality of interactions mediated by massive particles. There is no room in this formulation for “global” gauge invariance as an a priori construct.

Armed with this modern weapon, we reassess here a few classical and recent conundra. In particular, we examine new perspectives in Cosmology from the adoption of string-local fields.

Science flourishes best when it uses freely all the tools at hand [. . .] A new tool always leads to new and unexpected discoveries, because Nature’s imagination is richer than ours.

— Freeman Dyson [4]

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1 Introduction

1.1 The scenario

Modern algebraic quantum field theory (AQFT), with its emphasis on algebras of observables rather than on fields, and the customary theoretical constructions and procedures to deal with the Standard Model (SM) phenomena, in particular the model’s reliance on classical Lagrangians and gauge theory, are sociologically divorced. Since it is hard to dispute that QFT should stand on its own legs, “not on classical crutches”, a review of this sorry state of affairs is permanently on the charts.

Consider in this respect the Wigner classification of elementary particles as unirreps of the restricted Poincaré group. It establishes three types:

1. Massive \((m > 0)\) particles.
2. Massless \((m = 0)\) particles of finite helicity \(h\).
3. Massless \((m = 0)\) particles of unbounded helicity.
Each of these types can be represented in infinitely many ways by quantum fields \([9]\), Chap. 5]. Of course, some of the latter are more useful and/or natural than others – and most work on QFT is done in terms of potential fields. On the other hand, one would expect the descriptions to ensure positivity (leading to a probabilistic interpretation of the formalism), covariance, and causal localization.

As is well known, the picture in terms of the fields is rather more complicated:

1. All massive particles can be described by potential fields that obey positivity, covariance and causal localization\(\text{[1]}\). Their high-energy behaviour grows steadily worse with spin.

2. Massless particles with \(h = 0\) or \(|h| = \frac{1}{2}\) enjoy the same above properties as do the corresponding massive particles. Between them and their massive counterparts, there is a smooth transition of correlation functions as \(m \downarrow 0\).

3. Massless particles of helicity \(|h| \geq 1\) own field descriptions that possess those properties as well; however, the apparently “natural” ones do not. A paradigmatic example is the description of photons: the Maxwell field \(F^{\mu\nu}(x)\) is positive, covariant and local, but the electrodynamic potential \(A^\nu(x)\) is neither. Similarly for gluodynamics. Since violation of positivity conflicts with the probabilistic interpretation of quantum mechanics, to salvage positivity for observables, one is forced to introduce ghost fields and the like.

Moreover, the current theory contains some counter-intuitive negative results, such as the Weinberg–Witten theorem, stating that for \(|h| \geq 2\) no stress-energy-momentum can exist whose zero-components generate the Poincaré momenta. Another somewhat strange result is the discontinuity of scattering amplitudes in the mass \(m\) at \(m = 0\) and \(|h| = 2\) (the Zakharov–van Dam–Veltman gap) \([11],[12]\).

4. On the face of it, fields describing massless particles of unbounded helicity enjoy neither positivity nor causal localization. Long ago Yngvason \([13]\) proved that old algebraic QFT, based on the Wightman axioms for the fields, does not encompass them.

Given this picture, the received wisdom in the SM, that quantum particles are naturally massless and that their masses are ascribable to “spontaneous symmetry breaking” of a “gauge symmetry”, is somewhat suspect. The latter improbable construct is obviously the outcome of trying to fit the scalar particle(s) in the Procrustean bed of gauge theory. But gauge invariance, rather than a physical symmetry, is a formal instrument to extract a physical sub-theory from an unphysical formalism. Nowadays this is recognized even in textbooks: “Gauge invariance is not physical and is not a symmetry of nature . . . [It] is merely a redundancy of description we introduce to be able to describe the theory with a local Lagrangian” \([10]\, Sect. 8.6]. It is hard to see in which sense it can be “broken” – consult \([14]\, App. C]. We shall return to the matter.

The situation as regards the relation between the classes numbered 1 and 3 in the second list above and of the handling of class 4 has drastically changed in relatively recent times, thanks

\(^1\)This is not quite obvious, but can be seen to hold – consult \([10]\, Ch. 8] for the simplest case.
to the emergence of rigorous *string-localized field theory*. Now, string-localized quantum fields (SLFs) are definitely non-exotic. In a heuristic version recognizably similar to the modern one, they were a brainchild of Mandelstam [15]. In a different vein, their pertinence follows from work by Buchholz and Fredenhagen [16] in the eighties, on the construction of particle states in AQFT. At about the same time, SLFs were being further developed by Steinmann [17-19]. The papers by Jordan [20] and Dirac [21] are rightly deemed to be precedents. All these works strive to break free from the shortcomings of the $\mathcal{A}^\nu(x)$-type fields.

The new development was made possible by the advent of *modular localization* and the Bisognano–Wichmann theorem, which most effectively marry geometric with analytic aspects of QFT. For a clear introduction to modular localization, consult [5]. Key articles on what concerns us here were [22-24]. There it is proved that quantum fields connecting the vacuum with the single-particle states, associated to any kind of Wigner particle, can be localized on spacelike cones in Minkowski space. It turns out that the half-line limits of such cones (the strings) are satisfactory for most purposes. We shall denote by $H$ the open set of spacelike vectors. In the subsequent period Yngvason, Mund and Schroer [25-26], were able to construct SLFs, localized in such spacelike half-lines, associated with all irreducible Wigner representations of the Poincaré group, including the particles of our class 4.

Following [27], we summarize a few main traits, advantages, and uses of SLFs.

1. Their improved scaling behaviour. It so happens that the high-energy behaviour of SLFs is always the one corresponding to either $s, h = 0$ or $s, |h| = \frac{1}{2}$, irrespective of their helicity or spin.

2. SL fields satisfy the Bisognano–Wichmann property, relating the modular group associated with a spatial wedge in momentum space to corresponding Lorentz boosts.

3. Smooth transition of the correlation functions as $m \downarrow 0$ is kept, irrespectively of spin. How this works may be illustrated in the case of massive and massless QED. The coupling to the indefinite positive Maxwell potential $A^F$ ($F$ for “Feynman gauge”) is replaced by a coupling $j^\mu A^p_\mu$ to the massive Proca potential $A^p$. This avoids negative-norm states, but the interaction is apparently non-renormalizable because of the UV dimension 2 of the Proca potential. Now a decomposition $A^p_\mu(x) = A_\mu(x, e) - m^{-1}\partial_\mu a(x, e)$ into a string-localized potential and its *escort field* $a$ is brought to bear: $A_\mu(x, e)$ has UV dimension 1 and is regular at $m = 0$. The $\mathcal{UV}$-divergent part of the interaction is carried away by the escort field: $-m^{-1}j^\mu\partial_\mu a(e) = -\partial_\mu(m^{-1}j^\mu a(e))$ is a total derivative and is discarded from the coupling. The remaining string-localized interaction $j^\mu A_\mu(x, e)$ is equivalent to the point-localized one and keeps the good ultraviolet behaviour at $m = 0$.

4. SLF theory is formulated exclusively with physical degrees of freedom. Consequently, the concept of “gauge invariance” as a fundamental principle is absent. A formal similarity to geometric connection theory and classical Lagrangians of the allowed
coupings for SLFs is all that remains. We show in the body of the paper how this overdue banishing of unphysical degrees of freedom comes about.

Before continuing, we call to the reader’s attention the paper by Herdegen [28], where SL fields for photons are compared to radial and almost-radial gauges’ formalism for electrodynamics. There the suggestion is made of using full spacelike lines instead of half-lines.

In general, SLFs offer few calculational advantages. Their role is more of a detective nature: wherever in QFT, and in the SM in particular, one “smells a rat”, they often come in handy. The upshot is that large no-go territories for quantum field theory are now trespassed. A few examples are:

- Chirality of the electroweak sector of the SM is derived without special assumptions.
- All of gluodynamics is established without previous assumptions.
- The need for a scalar particle – say, the Higgs’ particle – in the SM is established without recourse to unobservable mechanisms.
- The separation of helicities in the massless limit of higher spin fields is clarified. The van Dam-Veltman–Zakharov discontinuity [11,12] at the $m \downarrow 0$ limit of massive gravitons is resolved.
- Unimpeachable stress–energy-momentum (SEM) tensors for massless fields of any helicity are constructed [20,30] – allowing for gravitational interaction, and in particular flouting the Weinberg–Witten theorems [31].
- The strong $CP$ (pseudo)problem is exorcised.

1.2 Dispensing with “renormalization”

A recent conceptual step forward, related to the issues of this paper, can be detected in [32]. Its authors argue that (perturbative) QFT ought to be defined by “divergencies-free” methods, resulting in particular that the fashionable “hierarchy problem” is exorcised, all postulated fine-tunings in theories with well-separated energy scales being discarded.

The main tool in [32] is renormalization by Callan–Symanzik-type differential equations. We hold, and set out to show in this paper, that the BEG framework can be used to the same purpose, and generally as a means of discovery. In so doing, we contend that to regard this framework as a mere tool of renormalization is somewhat misleading. Indeed, Epstein and Glaser originally [22] stressed not so much “renormalization”, but rather the pursuit of locality in perturbative quantum field theory. Like Scharf [33], in most instances we shall choose to speak of normalization rather than of “renormalization”, inasmuch as no manipulation of infinities takes place in an Epstein–Glaser approach.

3The present authors are not entirely guiltless in this regard.
1.3 Aims and plan of the paper

In the spirit of algebraic quantum field theory, one contemplates here the particles regarded as elementary in the SM as described simply by their masses (couplings to the $h = \pm 2$ particle) and charges (couplings to charged $h = \pm 1$ particles), as determined by the experiments. All the details of the SM interactions are to be retrieved on this basis. Now, string-localized quantum fields come into the world with a powerful heuristic principle: the string “ought not to be seen”. That is to say, the construction of physical observables and scattering amplitudes cannot depend on the string coordinates. The principle will be used in the BEG framework for scattering theory. Coming to the SM, we consider here essentially full flavourdynamics and gluodynamics (rather than full QCD) in turn. Gleaning from previous work of ours on these subjects, all the main traits of the SM, including the presence of at least one scalar particle, are recovered in a constructive, bottom-up approach.

In Appendix A we describe the structure of propagators for SL fields.

In Appendix B we ponder the so-called “strong CP problem”.

In Appendix C, in a more speculative vein, we deal with another subversive aspect of SLFs, as regards higher-spin fields.

2 On the relation between potentials and field strengths

For a generic free quantum field on Minkowski space we may write

$$\Psi_I(x) = \sum_\sigma \int d\mu_m(p) \left[ u_I(p, \sigma) a(p, \sigma) e^{-i(px)} + v_I(p, \sigma) a^\dagger(p, \sigma) e^{i(px)} \right].$$

We would like this to transform as

$$U(\Lambda) \Psi_I(x) U(\Lambda)^\dagger = \sum_I D_{II}(\Lambda^{-1}) \Psi_I(\Lambda x),$$

with $U(\Lambda)$ denoting the second quantization of the pertinent unirrep of the Lorentz group, and $D$ being some matrix representation of that group. Let $d$ be a unirrep for a little group, say for $p_0$. There are $(\dim D \times \dim d)$ coefficient matrices $u(p), v(p)$ for $p$ in the mass-$m$ hyperboloid, such that $u(\Lambda p) d(R(\Lambda, p)) = D(\Lambda) u(p)$; $v(\Lambda p) d(R(\Lambda, p)) = D(\Lambda) v(p)$, where $R(\Lambda p)$ is the “Wigner rotation” in the little group. For a complex scalar field:

$$\varphi_m(x) = \int d\mu_m(p) \left[ c(p) e^{-i(px)} + d^\dagger(p) e^{i(px)} \right],$$

and these relations trivially and Eq. (11) hold. Were the same be true of the standard potential:

$$A_\mu(p) = \sum_{h=\pm 1} \int d\mu(p) \left[ u^{h\dagger}_\mu(p) a_h(p) e^{-i(px)} + u^{h\dagger}_\mu(p) a^\dagger_h(p) e^{i(px)} \right],$$

where $h$ denotes helicity. But it isn’t. For a messy linear combination $\Gamma$ of creation and annihilation operators:

$$U(\Lambda) A_\mu(p) U(\Lambda)^\dagger = \Lambda^\nu_\mu A_\nu(\Lambda x) + \partial \Gamma(x, \Lambda).$$

No intertwiners $u, v$ avoiding this can be found.
Instead, with the help of polarization vectors $\varepsilon^h_\nu$ (zweiseibeins or dreibeins), one may construct the Faraday covariant tensor fields on Hilbert space,

$$F_{\mu\nu}(x) = \sum \int d\mu(p) \left[ u^h_{\mu\nu}(p)a_h(p)e^{-i(px)} + \overline{u^h_{\mu\nu}(p)}a^\dagger_h(p)e^{i(px)} \right],$$  

with $u^h_{\mu\nu}(p) = -ip_\mu \varepsilon^h_\nu(p)$, where $\Sigma$ denotes $\Sigma_{h=\pm 1}$ in the massless case and $\Sigma_{-1 \leq h \leq 1}$ in the massive case. Then, as is well known,

$$U(\Lambda)F_{\mu\nu}(p)U(\Lambda)^\dagger = \Lambda^\mu_\nu F_{\rho\sigma}(\Lambda x).$$

Consider now semi-infinite strings $S_{x,e} := x + \mathbb{R}^+ e$, and

$$A_\mu(x,e) := I_eF_{\mu\nu}(x)e^\nu = \int_0^\infty ds e^\nu F_{\mu\nu}(x + se).$$

Clearly $I_eF_{\mu\nu} = I_{\lambda e}F_{\mu\nu}$ for $\lambda > 0$. We understand

$$\int_0^\infty dt e^{\pm ix} \equiv \lim_{\epsilon \downarrow 0} \frac{\pm i}{x \pm i\epsilon} = \pm i \Re \frac{x}{x \pm i0}.$$  

Then

$$A_\mu(x,e) = \sum_{h=\pm 1} \int d\mu(p) \left[ u^h_{\mu}(p,e)a_h(p)e^{-i(px)} + \overline{u^h_{\mu}(p,e)}a^\dagger_h(p)e^{i(px)} \right],$$

with $u^h_{\mu}(p,e) := \frac{u^h_{\mu}(p,e)e^\nu}{i[(pe) - i0]} = \varepsilon^h_\mu(p) - \frac{(\varepsilon^h(p)e)}{(pe) - i0} p_\mu$.  

These $u$ are bona fide intertwiners:

$$u_h(\Lambda p, \Lambda e) d(R(\Lambda, p)) = D(\Lambda) u_h(p,e),$$

where $R(\Lambda, p)$ is the “Wigner rotation”.

It is often helpful to invert these formulas in order to obtain the Fock space annihilation and creation operators. Using $(u^h_{\mu}(p,e)u^\nu_{\mu}(p,e')) = (\varepsilon^h(p)\varepsilon^\nu(p')) = -\delta_{hh'}$, one gathers:

$$a_h(p) = -i \int_{x^0=0} d^3x \frac{u^h_{\mu}(p,e)e^{-i(px)}}{pe - i0} \partial_0 A^\mu(x, e)$$

$$a^\dagger_h(p) = \int_{x^0=0} d^3x \frac{u^h_{\mu}(p,e)e^{i(px)}}{pe - i0} \partial_0 A^\mu(x, e).$$

For arbitrary mass and integer spin or helicities, denote the associated field strengths (symmetric under any exchange of pairs $(\mu_j, \nu_j) \leftrightarrow (\mu_k, \nu_k)$) by $F_{[\mu_1, \nu_1] \cdots [\mu_s, \nu_s]}(x)$, often shortened to $F_{[\mu, \nu]}(x)$. The corresponding string-localized potential, symmetric under exchange of its indices, is defined by line integrals in the direction $e$:

$$A_{\mu_1 \cdots \mu_s}(x,e) := \int_0^\infty \cdots \int_0^\infty dt_1 \cdots dt_s F_{[\mu_1, \nu_1] \cdots [\mu_s, \nu_s]} \left( x + \sum_{i=1}^s t_i e \right) e^{\nu_1} \cdots e^{\nu_s}$$

$$=: I_eF_{[\mu_1, \nu_1] \cdots [\mu_s, \nu_s]}(x)e^{\nu_1} \cdots e^{\nu_s}. \tag{4}$$
Using \((e \partial_x) I_e = -1\) as well as the Bianchi identities for the field strength, one can check that \(A_\mu(x, e) \equiv A_{\mu_1 \cdots \mu_s}(x, e)\) is indeed a potential for \(F_{[\mu, \nu]}(x)\). Moreover, \(A_\mu(x, e)\) is axial with respect to the string variable: \(e^{\mu_1} A_{\mu_1 \cdots \mu_s}(x, e) = 0\). Furthermore, in the massless case \(\partial^{\mu_1} A_{\mu_1 \cdots \mu_s}(x, e) = 0\).

By their definitions, in both the massless and massive cases, the string-localized \(A_\mu\) live on the same Hilbert space as the field strengths. Positivity, locality and Poincaré invariance hold:

\[
[A_{\mu_1 \mu_2}(x, e), A_{\mu_1 \mu_2}(x', e')] = 0 \quad \text{if} \quad (x + \mathbb{R} e - x' - \mathbb{R} e')^2 < 0;
U(a, \Lambda) A_{\mu_1 \mu_2}(x, e) U^\dagger(a, \Lambda) = \Lambda^{\kappa_1}_{\mu_1} \Lambda^{\kappa_2}_{\mu_2} A_{\kappa_1 \kappa_2}(\Lambda x + a, \Lambda e);
\]

where \(U(a, \Lambda)\) denotes, as before, the second quantization of the corresponding unirrep of the Poincaré group, and we have taken \(s = 2\) or \(|h| = 2\) to simplify notations. Similarly with anticommutators for fermionic string-localized fields.

For practical purposes, we hasten to introduce translation-invariant two-point functions for SLFs. Given two string-localized fields of the same mass \(m \geq 0\), we define

\[
\langle \langle X(x, e) X'(x', e') \rangle \rangle := \langle \Omega | X(x, e) X'(x', e') \Omega \rangle
= \int \frac{d^3 p}{(2\pi)^3} \delta(p^2 - m^2) \theta(p^0) e^{-i(p(x-x'))} M_{m}^{XX'}(p, e, e'),
\]

where \(\Omega\) denotes the vacuum state and \(M_{m}^{XX'}\) or simply \(M^{XX'}\) will be a polynomial in \(p\). For the electromagnetic field and its massive look-alike, by use of the intertwiners:

\[
M_{\mu \nu p \sigma}^{FF}(p) = g_{\mu \sigma} v_p p_\rho + g_{\nu \sigma} p^\mu p_\rho - g_{\mu \rho} p^\nu p_\sigma - g_{\nu \tau} p^\mu p_\rho.
\]

Also

\[
-M_{\mu \nu p}^{AA}(p, e, e') = g_{\mu \nu} - \frac{e_\rho p_\mu}{(pe) - i\varepsilon} - \frac{e'_\rho p_\mu}{(pe') + i\varepsilon} + \frac{(ee') p_\mu p_\rho}{[(pe) - i\varepsilon][(pe') + i\varepsilon]}
=: E_{\mu \nu}(p, e, e').
\]

Formulas like \([4]\) will require translation into momentum space. With some abuse of notation:

\[
I_e f(p) := \int d^4 x \int_0^\infty ds \, e^{i(px)} f(x + se) = \int d^4 x \int_0^\infty ds \, e^{i((p - se)x)} f(x)
= f(p) \int_0^\infty ds \, e^{-is(pe)} =: -\frac{if(p)}{(pe) - i\varepsilon}.
\]

### 2.1 Escort fields and the \(m \downarrow 0\) process

As indicated above, in the case of a massive spin-1 field, the Proca potential can be retrieved as \(A^\mu_e(x) := A_\nu(x, e) - m^{-1} \partial_\nu a(x, e)\), where the escort field \(a(x, e)\) “carries away” the UV
divergence of $A^0_\mu(x)$ as $m \downarrow 0$ since it contributes a total divergence to the coupling $j^\nu A^0_\nu$. This escort, a scalar SL field, is defined as $a(x, e) := -m^{-1} \partial^\mu A_\mu(x, e)$. For bosons of higher spin, one can define a family of escorts recursively, as follows. For a spin $s$ field, one first declares the string-localized potential $a_\mu^{(s)}(x, e) \equiv A_\mu(x, e) := I^s_e F_\mu(x)$. For $r = s - 1, \ldots, 1, 0$, one defines escort fields $a^{(r)}$ by putting

$$a^{(r)}_{\mu_1 \cdots \mu_r}(x, e) := -m^{-1} \partial^\nu a^{(r+1)}_{\mu_1 \cdots \mu_r \nu}(x, e).$$

Thus a spin-2 potential $A_{\mu \nu}(x, e)$ has two escorts, a vector $a^{(1)}_\mu(x, e)$ and a scalar $a^{(0)}(x, e)$.

These escort fields obey the axial condition and the field equations:

$$e^\mu a^{(r)}_{\mu \kappa} = 0, \quad \partial^\nu a^{(r)}_{\mu \nu \kappa} = -ma^{(r-1)}_{\mu \kappa}, \quad g^{\mu \nu} a^{(r+1)}_{\mu \nu \kappa} = -a^{(r-1)}_{\kappa},$$

where $\kappa = \kappa_2 \cdots \kappa_r$ denotes the other indices. One can recover a point-like potential from $A^0_\mu(x, e)$ and derivatives of its escorts. For instance, for spin 2,

$$A^0_{\mu \nu}(x) := A_{\mu \nu}(x, e) - \frac{1}{m} [(\partial_\mu a^{(1)}_\nu(x, e) + \partial_\nu a^{(1)}_\mu(x, e)) + \frac{1}{m^2} \partial_\mu \partial_\nu a^{(0)}(x, e)].$$

Now, $A^0_{\mu \nu}(x, e)$ and its escorts have well-defined massless limits as $m \downarrow 0$, but do not yield a decoupling of the lower helicities $h = \pm r$ without further modification. Rewrite (6) as an operator in $x$-space:

$$E_{\mu \nu}(e, e) := g_{\mu \nu} + (e_\nu \partial_\mu + e_\mu \partial_\nu) I^e + e^2 \partial_\mu \partial_\nu I^2_e.$$  

On replacing $A^{(2)}_{\mu \nu}(x, e)$ by

$$A^{(2)}_{\mu \nu}(x, e) := A_{\mu \nu}(x, e) + \frac{1}{2} E_{\mu \nu}(e, e) a^{(0)}(x, e),$$

one gets a potential that in the massless limit is traceless, decouples from $a^{(1)}_\mu$ and $a^{(0)}$, and has the correct two-point function $M_0^{A_{(2)} A_{(2)}}$. For higher spins, analogously modified potentials yield the desired decoupling of lower helicities [33].

## 3 The principle of string independence: flavourdynamics

Not so long ago one could read in the short version of the Review of Particle Physics [37] p. 209 that the fermions of the SM are chiral, and that therefore in principle they should be massless. Furthermore, given that they are in fact massive, somehow a “spontaneous symmetry breaking” generating their masses must have happened. Fortunately, these disquisitions appear to have vanished in more recent booklets [38]. For that is to have matters backwards. To begin with, what may or may not be chiral are the interactions; quarks are not schizophrenic, chiral in flavourdynamics and non-chiral in chromodynamics: chirality is not a Platonic property of particles. Secondly, our work together with Mund [34] established that the couplings of the massive vector bosons with fermions are chiral precisely because the former are massive. Our finding is a theorem, rigorously derived from the string independence principle at second order in S-matrix theory: given merely the masses and charges of the interaction carriers and the fermions, all of electroweak dynamics flows from this principle. For lack of space, here we just provide an outline of the construction, leaving the full derivation of the model for [39].
3.1 The intermediate bosons

Formula (2) for a set of four Faraday fields \( F_a = \{ W^\pm, Z, \gamma \} \) is our starting point. It proves useful to consider the spinless string-local escort fields:

\[
\phi_b(x, e) := \sum_r \int d\mu(p) \left[ e^{i(p_x)\frac{\epsilon_r(p) e}{(pe) + i\epsilon}} a^+_{r,b}(p) + e^{-i(p_x)\frac{-i(\epsilon_r(p) e)^*}{(pe) - i\epsilon}} a_{r,b}(p) \right]. \tag{9}
\]

As remarked already, the differences

\[ A_\mu^b(x, e) - \partial^\mu \phi_b(x, e) = A_{\mu, b}^b(x) \]

define pointlike Proca fields, so that \( dA_b^P = F_b \). All these fields live on the same Fock spaces as the \( F_b \) and have the same mass. Moreover:

\[ \phi_b(x, e) = \int_0^\infty A_{b, \Lambda}^P(x + se)e_\Lambda ds. \]

Note the relations \((e \partial \phi_b) = -(eA_b^P)\) and \(\partial_\mu A_\mu^b(x, e) + m_b^2\phi_b(x, e) = 0\), in accordance with (8). The second one follows directly from (2) and (2), since \((p \epsilon_r(p)) = 0\).

Now, a complete account of electroweak theory would start by showing that, when the string independence principle is applied to the physically relevant set of boson SLFs, with their known masses and charges, replacing the standard pointlike fields, plus one physical Higgs particle \( \phi_4(x) \), one recovers precisely the phenomenological couplings of flavourdynamics in the SM, with massive bosons mediating the weak interactions, and the \( U(2) \) structure constants, as expounded for instance in [40]. For want of space, we just summarize here the main conclusions concerning the boson sector.

- Apart from the higgs particle sector, a string-local theory of interacting bosons at first order in the coupling constant \( g \) must be of the form:

\[
S_1^B(x, e) = g \sum_{a,b,c} f_{abc}(m_a^2 - m_b^2 - m_c^2)(A_a(x, e)A_b(x, e)\phi_c(x, e)) - A_a(x, e) \partial \phi_b(x, e)\phi_c(x, e) + g \sum_{a,b,c} f_{abc}F_a(x)A_b(x, e)A_c(x, e), \tag{10}
\]

where we omit the notation : — : for Wick products, and the restricted sum \( \Sigma' \) runs over massive fields only. Here the \( f_{abc} \) denote the (completely skewsymmetric) structure constants of the (reductive) symmetry group of the model; the mass of the vector boson \( A_a \) is denoted \( m_a \), and complete contraction of Lorentz indices is understood. Notice that the escort fields hold a place somewhat analogous to Stückelberg fields.

\[ \text{following Okun and for obvious grammatical reasons, henceforth we refer to a (physical) Higgs boson as a higgs, with a lower-case h.} \]
• It is straightforward to check that the 1-form $\frac{d}{d\tau}S^B_1$, measuring the dependence on the string variable of the vertices in (10), is a divergence: $\frac{d}{d\tau}S^B_1(\tau, i\tau) = (\partial Q^B_1)(\tau, i\tau)$, where $Q^B_{1\mu}$ is given by:

$$2g \sum_{a,b,c} f_{abc}(F_a A_c)_{\mu} w_b + g \sum_{a,b,c} f_{abc}(m^2_a + m^2_c - m^2_b)(A_{a\mu} - \partial_\mu \phi_a) \phi_c w_b.$$  \hspace{1cm} (11)

We did introduce above the form-valued fields $w_a := d_\mu \phi_a$. We need $Q^B_1$ to prove chirality of the couplings to the fermion sector.

• At once we adapt our notation to the one used in the SM. This model has three nonzero masses $m_1 = m_2 < m_3$ and one $m_4 = 0$. Defining the Weinberg angle by $m_1/m_3 := \cos \Theta$, we employ the basis in which

$$f_{123} = \frac{1}{2} \cos \Theta, \quad f_{124} = \frac{1}{2} \sin \Theta, \quad f_{134} = f_{234} = 0,$$

all other $f_{abc}$ following from complete skewsymmetry. They are seen to be the structure constants of (the Lie algebra of) the $U(2)$ determined by the physical particle fields.

We use the standard notations

$$W_\pm \equiv \frac{1}{\sqrt{2}} (W_1 \mp iW_2) := \frac{1}{\sqrt{2}} (A_1 \mp iA_2), \quad Z := A_3, \quad A := A_4$$

and similarly for $\phi_\pm, w_\pm, \phi_Z$ and $w_Z$; with masses $m_W = m_1, m_Z = m_3$ and $m_\gamma = m_4 = 0$.

• With this in hand, we focus on (11), keeping in mind that, although the photon has no escort field, the field $w_4$ exists at the same title as $w_1, w_2, w_Z$. The first summand in (11) yields:

$$2g \sum_{a,b,c} f_{abc}(\partial_\mu A_{a\lambda} - \partial_\lambda A_{a\mu}) A_\lambda^c w_b$$

$$= ig \sin \Theta [ \left( \partial_\mu A_{-\lambda} - \partial_\lambda A_{-\mu} \right) (w_+ W^A_+ - w_4 W^A_4) + (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu}) (w_+ A^4 - w_4 W^A_4)$$

$$+ (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu}) (w_4 W^A_4 - w_4 A^4)]$$

$$+ ig \cos \Theta [ \left( \partial_\mu Z_\lambda - \partial_\lambda Z_\mu \right) (w_+ W^A_+ - w_4 W^A_4) + (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu}) (w_+ Z^A - w_Z W^A_4)$$

$$+ (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu}) (w_Z W^A_4 - w_4 Z^A)].$$

• Our $Q^B_{1\mu}$ above is incomplete, since bosonic couplings involving the higgs sector are not included. They are also derived from the string independence principle. Of those, for our purposes in this analysis we need only

$$\frac{g}{2 \cos \Theta} m_Z (\phi_4 (\partial_\mu \phi_Z - Z_\mu) - \partial_\mu \phi_4 \phi_Z) w_Z;$$

actually these play a pivotal role in our work in [34]. Clearly, terms of this type are suggested by the last group of summands in (11).

• The expected $g^2 AAAAA$ terms and thus the trappings of the geometrical gauge approach are recovered in our formalism from string independence at the level of $S_2$.  

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5This makes sense in the normalized theory [10, Chap. 29].
3.2 On string independence

The general procedure to be followed engages the construction of the Bogoliubov-type functional scattering operator \( \mathbb{S}[g; c] \) dependent on a multiplet \( g \) of external fields and a test function \( c \in \mathcal{D}(H) \) with integral equal to 1 that averages over the string directions \( [31] \). Bogoliubov’s \( \mathbb{S}[g; c] \) obeys the customary conditions of causality, unitarity and covariance. One looks for it as a formal power series in \( g \),

\[
\mathbb{S}[g; c] := 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \prod_{k=1}^{n} \prod_{l=1}^{m} \int d^4x_k \int d^4(e_{k,l}) g(x_k) c(e_{k,l}) S_n(x_1, e_1; \ldots; x_n, e_n). \tag{12}
\]

Only the first-order vertex coupling \( S_1 = S_1(x, e) \), a Wick polynomial in the free fields, is postulated – already under severe restrictions. It depends on an array \( e = (e_1, \ldots, e_m) \) of string coordinates, with \( m \) the maximum number of SLFs appearing in a sub-monomial of \( S_1 \).

For \( n \geq 2 \), the \( S_n \) are time-ordered products that must be recursively constructed. These \( S_n \) will be symmetric with respect to permuting the string coordinates, which are smeared with the same test function \( c \). (This symmetry will be heavily exploited in what follows.) The extension of the \( S_n \)-products across exceptional sets like

\[
\mathcal{D}_2 := \{ (x, e; x', e'): (x + \mathbb{R}^+ e_k) \cap (x' + \mathbb{R}^+ e'_k) \neq \emptyset \text{ for some } k, l \},
\]

and similar ones \( \mathcal{D}_n \), is the normalization problem in a nutshell.

The hypothesis of perturbatively interacting SLF theory is simple enough: physical observables and closely related quantities, particularly the \( \mathbb{S} \)-matrix, cannot depend on the string coordinates. This is the – intrinsically quantum – string independence principle, which here replaces the gauge “principle” and classical Lagrangians with advantage.

3.3 Summary of the proof of chirality

The couplings between interaction carriers and matter currents in a theory with massive or massless vector bosons \( A_{a\mu} \) must be of the form

\[
g \left( b^a A_{a\mu} J^\mu_V + \bar{b}^a A_{a\mu} J^\mu_A + c^a \phi_a S + \bar{c}^a \phi_a S_5 \right); \tag{13}
\]

where

\[
J^\mu_V = \bar{\psi} \gamma^\mu \psi, \quad J^\mu_A = \bar{\psi} \gamma^\mu \gamma^5 \psi, \quad S = \bar{\psi} \psi, \quad S_5 = \bar{\psi} \gamma^5 \psi,
\]

with electric charge conserved in the interaction vertices. Our key assumption is that these \( A^\mu_a \) and \( \phi_a \) above are now given as string-local quantum fields, thus satisfying renormalizability by power counting. There are no other scalar couplings which comply with renormalizability. To wit, Lorentz invariance requires that all cubic terms be of the above form, and renormalizability forbids quartic terms.\(^6\)

\(^6\)Since the two Fermi fields required by Lorentz invariance already have scaling dimension 3, any two further fields would give 5, exceeding the power-counting limit.
The $\psi$ in (13) are ordinary fermion fields – we should not assume chiral fermions \textit{ab initio}, and we do not.

The coefficients $b^a$, $\bar{b}^a$, $c^a$, $\bar{c}^a$ in (13) are to be determined from string independence.

With the above in the bag, one just makes the most general Ansatz of the kind (13). At first order, the string independence requirement amounts to hermiticity, which demands for the couplings with fermions:

$$
S^F_1(x, l) = g \left( b_1 W_{-\mu} \bar{e} \gamma^\mu \nu + b_1 W_{-\mu} \bar{e} \gamma^\mu \gamma^5 \nu + b_1 W_{+\mu} \bar{\nu} \gamma^\mu \mu + b_1 W_{+\mu} \bar{\nu} \gamma^\mu \gamma^5 \nu + b_2 Z \bar{\mu} \bar{\nu} \gamma^\mu \nu + b_3 Z \bar{\mu} \bar{\nu} \gamma^\mu \gamma^5 \nu + b_4 Z \bar{\mu} \bar{\nu} \gamma^\mu \gamma^5 \nu + b_5 A_\mu \bar{e} \gamma^\mu \nu + b_5 A_\mu \bar{e} \gamma^\mu \gamma^5 \nu 
+ i (m_e - m_\nu) \bar{e}_1 \bar{e}_1 + i (m_e + m_\nu) \bar{e}_1 \bar{e}_1 - i (m_e - m_\nu) \bar{e}_1 \bar{e}_1 
+ i (m_e + m_\nu) \bar{e}_1 \bar{e}_1 + 2im_e \bar{e}_1 \bar{e}_1 + 2im_\nu \bar{e}_1 \bar{e}_1 + 2im_\nu \bar{e}_1 \bar{e}_1 + 2im_e \bar{e}_1 \bar{e}_1 + 2im_\nu \bar{e}_1 \bar{e}_1 
+ c_0 \bar{\phi}_4 \bar{\phi}_4 + c_5 \bar{\phi}_4 \bar{\phi}_4 + c_5 \bar{\phi}_4 \bar{\phi}_4 + c_5 \bar{\phi}_4 \bar{\phi}_4 
\right),
$$

where $\Phi_1, \Phi_2$ are escort fields and $\Phi_4$ is the higgs field. Here $e$ stands for an electron or muon or $\tau$-lepton pointlike field or for a suitable combination of the quarks $d, s, b$; and $\nu$ for the neutrinos or for a suitable combination of the quarks $u, c, t$ – it is enough to consider just one generation of leptons.

After harder work, second-order string independence allows for the full determination of the fermionic couplings:

$$
S^F_1 = g \left\{ - \frac{1}{2\sqrt{2}} W_{-\mu} \bar{e} \gamma^\mu (1 - \gamma^5) \nu - \frac{1}{2\sqrt{2}} W_{+\mu} \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu + \frac{1}{4 \cos \Theta} Z_{-\mu} \bar{e} \gamma^\mu \nu 
- \frac{1}{4 \cos \Theta} Z_{+\mu} \bar{\nu} \gamma^\mu \nu + \frac{1}{4 \cos \Theta} Z_{+\mu} \bar{\nu} \gamma^\mu \nu + \sin \Theta A_\mu \bar{e} \gamma^\mu \nu 
+ \frac{1}{2 \sqrt{2}} (\phi_\nu - \phi_4 \bar{\nu} e) - \frac{1}{2 \sqrt{2}} (\phi_\nu \gamma^5 \nu + \phi_4 \bar{\nu} \gamma^5 \nu) 
+ \frac{1}{2 \cos \Theta} \phi_2 \bar{\nu} \gamma^5 \nu + \frac{1}{2 \cos \Theta} \phi_2 \bar{\nu} \gamma^5 \nu + \frac{1}{2 \cos \Theta} \phi_2 \bar{\nu} \gamma^5 \nu 
\right\}.
$$

(As established in [34], too, one can sweep away the escort fields. However, this is not convenient in the present dispensation.)

4 The principle of string independence: gluodynamics

Let us now reconsider QCD from our present viewpoint. Suppose that we are given three massless fields $A_{\mu a}$ ($a = 1, 2, 3$). Essentially following [35], we first show that, for a mutual cubic coupling modulo divergences, string independence \textit{at first order} in the coupling constant enables only the Wick-product combination:

$$
S_1(x, e_1, e_2) = \frac{g}{2} f_{abc} A_{\mu a}(x, e_1) A_{\nu b}(x, e_2) F_{\mu \nu}^c(x),
$$

where the $f_{abc}$ are completely skewsymmetric coefficients. (Subindices that appear twice are summed over, and normal ordering is understood.) For the physics of the model described
That our claims can be sustained was already proved in relation to perturbative QCD in [35]. Thanks to more recent work [44], here we are able to sketch improvements in the technical details. Note first from the simplest version of Eq. (13) that the string differential of the gluon field is a gradient:

\[ d_e A_{\mu b}(x, e) := i \partial_{\mu} \sum_{h=\pm} \int d\mu(p) d\mathcal{E}^\mu \left[ e^{i(p x)} \frac{\mathcal{E}^h_{\frac{b}{e}}}{(pe)} - \frac{\mathcal{E}^h_{\frac{b}{e}}}{(pe)^2} a^*_b(p, h) - e^{-i(p x)} \left( \frac{\mathcal{E}^h_{\frac{b}{e}}}{(pe)} - \frac{\mathcal{E}^h_{\frac{b}{e}}}{(pe)^2} \right)^* a_b(p, h) \right] \equiv \partial_{\mu} u_b(x, e). \] (15)

Let us examine the general cubic Ansatz renormalizable by power counting. With an obvious notation:

\[ S'_1(x, e_1, e_2, e_3) := g f_{abc}^1 A_{\mu a}(x, e_1) A_{\nu b}(x, e_2) \partial^\mu A^\nu_c(x, e_3) =: g f_{abc}^1 A_{\mu a}^1 A_{\nu b}^2 \partial^\mu A^\nu_c^3 := g (d_{abc} + l_{abc}) A_{\mu a}^1 A_{\nu b}^2 \partial^\mu A^\nu_c^3, \]

where \( d_{abc} = d_{acb} \) and \( l_{abc} = -l_{acb} \). Because \( \partial^\mu A_{\mu a}^1 = 0 \), on symmetrizing \( e_2 \leftrightarrow e_3 \) we can drop a total divergence and are left with

\[ S''_1(x, e_1, e_2, e_3) := g f_{abc}^2 A_{\mu a}(x, e_1) A_{\nu b}(x, e_2) \partial^\mu A^\nu_c(x, e_3), \]
where $f_{abc}^2$ is skewsymmetric under $b \leftrightarrow c$. Repeating the procedure, we split $f_{abc}^2$ into a part $f_{abc}^+ = f_{bac}^+$ and a totally skewsymmetric part $f_{abc}^-$, and study

$$S_1^{\mu\nu}(x, e_1, e_2) := \frac{g}{2} f_{abc}^- A_{\mu da}(x, e_1) A_{\nu da}(x, e_2) F_{\mu\nu}^a(x).$$

Now, in view of (15):

$$d_{e_1} S_1^{\mu\nu}(x, e_1, e_2) = \frac{g}{2} f_{abc}^- \partial_{\mu}(u_1 A_{\nu da}^2 F_{\mu\nu}^a);$$

so this part is string-independent in the adiabatic limit. On the other hand, it is easily seen that the $f_{abc}^+$ must vanish identically for the symmetric part to be string-independent. In conclusion, string independence at first order demands complete skewsymmetry of the $f_{abc}$. 

### 4.2 String-independence at second order

As yet the $f_{abc}$ do not make a Lie algebra. For that a Jacobi identity is required. We obtain it from string independence of the functional $S$-matrix at second order in the couplings. Since time-ordering products need to be defined here, that requires a normalization procedure.

Let $y_i = x_i + s_i e_i$ and $y'_j = x'_j + s'_j e'_j$. The second-order tree graph contribution to the $S$-matrix is given by the string integral:

$$T[S_1(x, e_1, e_2) S_1(x', e'_1, e'_2)] \big|_{\text{tree}} = \int_0^\infty ds_1 \cdots ds_2' T[S_1(x, y_1, y_2) S_1(x', y'_1, y'_2)] \big|_{\text{tree}}.$$

Since the gluon propagators are diagonal in the color indices, this simply takes the form

$$T[S_1(x, e_1, e_2) S_1(x', e'_1, e'_2)] \big|_{\text{tree}} = \sum_{\varphi, \chi} \frac{\partial S_1(x_1, y_1, y_2)}{\partial \varphi} \langle \langle T \varphi \chi' \rangle \rangle \frac{\partial S_1'(x_1', y'_1, y'_2)}{\partial \chi'}$$

for the fields $\varphi, \chi$ entering $S_1$; Wick ordering is understood throughout. Inserting the explicit formulas for our problem, we obtain:

$$\frac{1}{4} g(x) g(x') f_{abc} f_{a'b'c'} \left[ \langle \langle T F_{\mu\nu}^a F_{\nu a'}^{\mu a} \rangle \rangle \right] A_{\mu da}^1 A_{\nu da}^2 A_{\nu a}^1 A_{\nu a}^2 A_{\nu a}^1 + \langle \langle T A_{\mu da}^1 F_{\nu a}^{\mu a} \rangle \rangle A_{\nu da}^2 A_{\nu a}^1 A_{\nu a}^2 A_{\nu a}^2 + (e_1 \leftrightarrow e_2)$$

$$+ \langle \langle T F_{\nu a}^{\mu a} A_{\mu da}^1 \rangle \rangle A_{\nu da}^2 A_{\nu a}^1 A_{\nu a}^2 F_{\nu a}^{\mu a} + (e'_1 \leftrightarrow e'_2)$$

$$+ \left( \langle \langle T A_{\mu da}^1 A_{\nu a}^{\mu a} \rangle \rangle A_{\nu da}^2 F_{\nu a}^{\mu a} A_{\nu a}^1 A_{\nu a}^2 + (e_1 \leftrightarrow e_2) \right) + (e'_1 \leftrightarrow e'_2) \right].$$

The propagators in the previous formula are unspecified as yet. For the purpose one naturally considers in the first place the “kinematic” propagators:

$$\langle \langle T_0 \varphi(x, e) \chi(x', e') \rangle \rangle := \frac{i}{(2\pi)^4} \int d^4 p \frac{e^{-i(p(x-x'))}}{p^2 + i0} M^{\varphi\chi}(p, e, e'),$$

(17)
where the $M^{\phi \chi}(p, e, e')$ are given by $M^{\phi \chi}_{\mu \nu, \lambda} := \sum_{\sigma} u_{\sigma, e}^{\phi}(p, e) u_{\sigma, \chi}(p, e'$) for the appropriate spacetime indices $*$, $\bullet$ on the respective intertwiners. Besides (5), one calls for:

$$M^{FA}_{\mu \nu, \lambda}(p, e') = i \left( p_\mu g_{\nu \lambda} - p_\nu g_{\mu \lambda} + p_\lambda \frac{p_\mu e'_\mu - p_\mu e'_\nu}{(pe') + i0} \right),$$

to be found for instance in our [34]. The more singular $M^{AA}$ is considered in Appendix A. Note that in all generality

$$d_e \langle T_0 \varphi(x, e) \chi(x', e') \rangle = \langle \langle T_0 d_e \varphi(x, e) \chi(x', e') \rangle \rangle,$$

and similarly for $d_{e'}$. In general one should admit for the propagators:

$$\langle \langle T F^{\mu \nu}_c F^{\rho \lambda}_d \rangle \rangle = \delta_{cd} \left[ \langle \langle T_0 F^{\mu \nu}_c F^{\rho \lambda}_d \rangle \rangle + c (\eta^{\mu \kappa} \eta^{\nu \lambda} - \eta^{\mu \lambda} \eta^{\nu \kappa}) \delta(x - x') \right];$$
$$\langle \langle T F^{\mu \nu}_c A^{\rho \lambda}_d \rangle \rangle = \delta_{cd} \left[ \langle \langle T_0 F^{\mu \nu}_c A^{\rho \lambda}_d \rangle \rangle + c (\eta^{\mu \kappa} e_1^{\nu} - \eta^{\nu \kappa} e_1^{\mu}) I_e I_{e_1} \delta(x - x') \right];$$
$$\langle \langle T A^{\mu \nu}_a F^{\rho \lambda}_d \rangle \rangle = \delta_{ad} \left[ \langle \langle T_0 A^{\mu \nu}_a F^{\rho \lambda}_d \rangle \rangle + c (\eta^{\mu \kappa} e_1^{\nu} - \eta^{\nu \kappa} e_1^{\mu}) I_a I_{e_1} \delta(x - x') \right];$$
$$\langle \langle T A^{\mu \nu}_a A^{\rho \lambda}_d \rangle \rangle = \delta_{ad} \left[ \langle \langle T_0 A^{\mu \nu}_a A^{\rho \lambda}_d \rangle \rangle + c (\eta^{\mu \kappa} (e_1^{\nu} - e_1^{\mu} e_1^{\nu}) I_a I_{e_1} \delta(x - x') \right],$$

for an indeterminate constant $c$. However, we do not need the non-kinematic terms, which are killed by the string independence principle [27 Sect. 6.1.2].

Looking back at Eq. (16), we summarize what we have obtained so far:

$$T_0 [S_1(x, e_1, e_2) S_1(x', e_1', e_2')]_{\text{sym,tree}} = \frac{1}{4} g(x) g(x') f_{abc} f_{def} \times \left[ \langle \langle T_0 F^{\mu \nu}_c F^{\rho \lambda}_f \rangle \rangle A^{\mu \alpha}_a A^{\nu \beta}_b A^{\rho \gamma}_d A^{\lambda \epsilon}_f + 2 \langle \langle T_0 A^{\mu \nu}_a F^{\rho \lambda}_f \rangle \rangle A^{\nu \beta}_b F^{\rho \lambda}_c A^{\lambda \epsilon}_f \right] + 2 \langle \langle T_0 F^{\mu \nu}_c A^{\rho \lambda}_d \rangle \rangle A^{\mu \alpha}_a A^{\nu \beta}_b A^{\rho \gamma}_d A^{\lambda \epsilon}_f + 4 \langle \langle T_0 A^{\mu \nu}_a A^{\rho \lambda}_d \rangle \rangle A^{\nu \beta}_b F^{\rho \lambda}_c A^{\lambda \epsilon}_f \right]_{\text{sym}}. \quad (18)$$

Now Eq. (15) entails $d_{e_1} \langle \langle T_0 A^{\rho \lambda}_d \rangle \rangle = \partial_{\mu} \langle \langle T_0 u^{\mu \rho}_a \rangle \rangle$. As a consequence, most of the terms contained in (18) are total divergences and can be dropped. In the end [27], one is left with:

$$d_{e_1} T_0 [S_1(x, e_1, e_2) S_1(x', e_1', e_2')]_{\text{sym,tree}} = -\frac{g^2(x)}{2} f_{abc} f_{dec} d_{e_1} \left[ (A^{1 \mu}_{a} A^{2}_{d}) (A^{2}_{b} A^{1}_{c}) \right]_{\text{sym}}$$

$$- \frac{g^2(x)}{6} \left[ f_{abc} f_{dec} + f_{ade} f_{bec} + f_{aeb} f_{bec} \right] (u^{1 \mu}_{a} F^{\rho \lambda}_d A^{1}_{k} A^{2}_{l} + \text{two similar terms}).$$

The first one yields the quartic term in Yang–Mills theory, redefining $S_2$. The second obstruction needs to vanish in order to achieve string independence, yielding the Jacobi identity.

A contemporaneous, even more elegant bottom-up approach without prior prejudices neither about Lagrangians nor classical gauge fields, also leading uniquely to the Yang–Mills type theory, is found in [10] Chap. 25. This one utilizes the spinor-helicity formalism. Far from Yang’s Platonic dictum “symmetry dictates interaction”, we do concur with that approach, in an Aristotelian mould, that interaction dictates symmetry.
Compared with this latter approach, as well as the standard one, the main drawback of string-local field theory is manifest in formulas like (6): calculations quickly become rather complicated by the abundance of propagator terms. At present, tree-graph calculations have been pushed reasonably far [45]; but loop-graph calculations with stringy particles on internal lines are still beyond reach.

5 On the scalar particle(s)

The attentive reader of Section 3 will have learned the important role that the scalar “escort fields” associated to the massive vector bosons of electroweak theory play, as a bridge between the SLF vector fields representing them and the corresponding Proca fields. The higgs field there did inauspiciously make its appearance as a kind of escort field associated to the fourth carrier of the electroweak theory, the photon. There string independence quickly establishes the need for at least one scalar particle whose coupling to the particles of the SM is proportional to their masses. As remarked first by Alejandro Ibarra [34, Sect. 7.2], its presence warrants chirality of the interactions of the charged bosons right away.

One also reads in [37, p. 210]: “The higgs boson couplings are not dictated by any local gauge symmetry”. We already showed the derivative character of the latter concept. String-local theory, on the other hand, recognizes that the higgs’s interactions are on the same footing as those of any other particles.

One finds, too, in a Review of Particle Physics [46, p.182]: “... the SM Higgs couplings to fundamental fermions are linearly proportional to the fermion masses, whereas the couplings to bosons are proportional to the square of the boson masses”. This is painful to read. Unfortunately, this is reproduced verbatim in the newer [47] p. 205).

It isn’t so: if one naturally uses the overall electroweak coupling constant, instead of the “Fermi constant” associated to the intangible “spontaneous symmetry breaking mechanism”, mere dimension counting tells us that the couplings of scalar particle(s) to any others must be proportional to their masses. From the viewpoint of modular localization and Wigner particle theory, as from plain rationality, the above statement in [46,47] makes no sense.

Now, we ought to able to recover all the higgs’ couplings, particularly the self-couplings, from string independence. As it turns out, this requires examination of third-degree couplings. Our task is helped by having done so already in the past [14,44], in the framework of causal gauge invariance. The method and the results, showing again the efficient detective character of the string independence principle, will appear in the review paper [39].

A Microlocal analysis of string-localized theories

In subsection 3.2 the first-order vertex coupling $S_1(x, e) = S_1(x, e_1, \ldots, e_m)$ that enters the Dyson expansion (12) depends on a spacetime point $x$ and several strings $e_l$, one for each

---

7How clever of the higgs, to distinguish between fermions and bosons in this way!
8Should there be more scalars in nature [48], one still learns much about their couplings from SLF theory. But it would no longer be possible to determine those couplings entirely.
string-like field in that Wick polynomial; see \[14\], for instance. One should not try to simplify by taking all the \(e_i\) to be equal, since this may lead to ill-defined products of distributional terms such as \(\langle T \varphi \chi \rangle\).

A careful analysis of the singularities affecting the extension of time-ordered products needed to construct \(S_2\) and higher-order terms has been carried out recently by Gaß in \[41\]. We briefly review what is involved.

Two distributions \(u\) and \(v\), defined on some open set \(X \subseteq \mathbb{R}^n\), can be multiplied if their wave-front sets do not clash. The wave-front set \(WF(u)\) is the set of covectors \((x, p)\) for which \(u\) is not smooth at \(x \in X\) and \(p \neq 0\) is a direction along which the Fourier transform \(\hat{u}\) does not have rapid decay. If there are \((x, p) \in WF(u)\) and \((x, k) \in WF(v)\) such that \(k = -p\), then the distributional product \(uv\) is absent; otherwise, \(uv\) is well defined and \(WF(uv)\) is \(WF(u) \cup WF(v)\) together with all such \((x, p + k)\). For the full story of WF sets and their properties, see \[49\].

An easy pair of examples of wave-front sets, for \(X = \mathbb{R}\), is

\[WF((t \pm i0)^{-1}) = \{ (0, \lambda) : \lambda \gtrless 0 \},\]

since \((t \pm i0)^{-1}\) have the Fourier transforms \(\theta(\pm \lambda)\). From there, WF sets of other distributions, such as \((p^2 \pm i0)^{-1}\) or \((p^2 - m^2 \pm i0)^{-1}\) in \(p\)-space, can be found by pullback operations.

To deal with string-local fields, we put \(u_{\pm}(p, e) := [(pe) \pm i0]^{-1}\) for \((p, e) \in M^4 \times H\), already considered, whose wave-front sets are found to be \[41\]:

\[WF(u_{\pm}) = \{ (p, e; \lambda e, \lambda p) : (pe) = 0, \ e^2 < 0, \ \lambda \leq 0 \} .\]

These signs of \(\lambda\) are opposite to those of \[19\] because we use the mostly-negative metric on \(M^4\). Because of the fixed sign of \(\lambda\), it is seen that there exist tempered distributions \((u_+)^k\) and \((u_-)^k\) for any \(k = 2, 3, \ldots\) but that the product \(u_+ u_-\) does not exist.

In particular, when computing propagators for string-local potentials \(A_\mu(x, e)\), the expression \[6\] contains a term \((ee')p_\mu p_\rho u_- (p, e) u_+ (p, e')\) which is undefined for \(e' = e\). Thus, within \(M^{4\lambda}_{\mu \rho}(p, e, e')\), the fields \(A_\mu(x, e)\) and \(A_\rho(x, e')\) should each have their own string variable.

As noted in \[7\], when computing the integrands \(M^{XX'}\) in propagators in \(p\)-space, each string integration \(I_r\) picks up an extra factor \(-iu_- (p, e)\) for \(X\), and an extra factor \(iu_+ (p, e') = -iu_- (p, -e')\) for \(X'\). The general structure of the integrands is thus of the form

\[u_- (p, e)^k u_+ (p, e')^{k'} (p^2 - m^2 + i0)^{-1} M_m (p)\]

for a suitable point-like polynomial \(M_m (p)\). If this polynomial is, say, homogeneous of degree \(r\), the above expression will be a well-defined distribution on \(M^4 \times H \times H\), provided that \(r - k - k' > -4\) to ensure local integrability at \(p = 0\). (When \(m = 0\), the requirement is that \(r - k - k' - 2 > -4\).) In the end, products of string-like propagators are always well defined away from \(p = 0\).
B Debunking the axion tale

Absence of evidence is not, as the saying goes, the same thing as evidence of absence. But if you continue looking for something intently, and keep failing to find it, you can be forgiven for starting to worry.

— The Economist, 12 March 2022

It should be clear that in the present formulation for QCD there is no room for instantons and the (in)famous $\theta$-vacuum. The contention to the purpose in SLF theory is straightforward. The string-localized vector potentials live on Hilbert space and act cyclically on the vacuum. In fact, every local subalgebra of operators enjoys this property – this is part of the Reeh–Schlieder property [6]. Therefore $\theta$-vacua are not allowed. In plainer language: a string-localized potential for a field strength is always given: roughly speaking, it consists of integrating the latter along a string. So where the field-strength vanishes, the potential remains bounded. What is more, our vector potential for gluons possesses only the two physical degrees of freedom, so “pure gauge” configurations cannot exist, then $\theta$-vacua cannot exist either, and the so-called strong CP problem is solved without invoking a hypothetical particle, the axion – which, 45 years after its inception, has not been found [35].

Now, if the above is the case, it should be possible, if perhaps not quite straightforward, to contend the same within the standard formalism. And indeed, in [50] it was proved by use of the rigorous, covariant Kugo–Ojima formalism [51] that the BRS charge “kills” the physical vacuum, which if cyclic must be unique. But that charge and the antiunitary operator for CPT invariance commute, and this obviously demands the zero (or $\pi$) value for the $\theta$-parameter of the instanton makeup. One could contend that the $\theta$-vacua are non-normalizable (a sign of trouble in itself) and that the physical vacuum is a superposition of them. However, by means of a simple procedure, Okubo and Marshak showed how in that case CPT invariance still guarantees the experimentally measurable value of $\theta$ to be zero. We find truly surprising in this connection that for thirty years their powerful argument, reiterated in [52, Chap. 10], has been systematically ignored. In this respect, the silence of reviews of axion theory – like [53] – strikes us as unaccountable.

Much more recently, two important papers make the same point by a clear, apparently different argument within the standard approaches [54,55].

It is harder still to see where the $\theta$-vacua could come from, when using the spinor formalism – which makes completely explicit the counting of degrees of freedom – to describe the gluons. In the present respect, one still detects here the idea – by now discredited [56] – that the $A(x)$-fields “know” something that the $F(x)$-fields do not. It ain’t so: for instance, AQFT [57] tells us that Haag duality fails to hold on causally incomplete spacetime domains, thus felicitously explaining the Aharonov–Bohm effect without recourse to the potentials.

The moral of the story, again: quantum field theory should stand on its own two feet, rather than on classical crutches.
C String-localized stress-energy-momentum tensors and dark matter

The remorseless ascent of the “neutrino floor” against the searches for dark matter (DM) by assumed “direct detection” particle physics processes raises the distinct possibility that DM does not have interactions with the SM particles, other than through gravitational effects.

In this respect, it has been persuasively argued in recent work by Cheek et al \[58,59\] that DM particles might have been emitted as part of the Hawking radiation from “small” black holes emanating all extant degrees of freedom in nature, in the early stages of the Universe. Many current inflation models do predict the existence of such PBH. Then dark matter would be essentially constituted by a background of inert high-spin and high-helicity particles (\(s, |h| \geq 3\)). Those, interacting with other particles and among themselves only via gravity, could perhaps partially collapse into latter-day black holes (LDBH).

Whereas the elegant solution in \[58,59\] to the current predicaments appears cogent, the suggested path of DM production in the early Universe’s stages still begs the question. This is so because the received wisdom nowadays – well represented in and argued by Porrati’s papers \[60,61\] in particular – holds on the basis of Weinberg–Witten theorems’ stiff no-go rules against interaction with gravitons \[31\].

Now, precisely here SLF theory comes in succour. Perhaps the most momentous phenomenological consequence of its use so far is that Weinberg–Witten-like theorems are falsified: not only are bona fide stress-energy-momentum (SEM) tensors for massive fields of spin \(s \geq 3\) exhibited; but it is shown that for massless fields of helicity \(|h| \geq 3\) SEM tensors do exist after all. We quote below the explicit form of these tensors. For details, including the continuous passage to the massless limit, we refer to the original work \[30,30\].

- For \((m > 0, s)\):

  \[
  T_{\alpha\beta}(x) = (-)^s \left[ -\frac{1}{4} A^\mu_{\mu\kappa\ldots\kappa}(x) \partial_\alpha \partial_\beta A^\nu_{\mu\kappa\ldots\kappa}(x) - \frac{1}{2} s \partial^\mu \left( A^\nu_{\mu\kappa}(x) \partial_\beta A^\rho_{\mu\kappa}(x) + [\alpha \leftrightarrow \beta] \right) \right].
  \]

Here the superindex \(p\) stands for “Proca” and \(k\) for contraction in \(s-1\) indices \(\kappa_2, \ldots, \kappa_s\). The Proca field \(A^\mu_{\mu\kappa\ldots\kappa}\) is completely symmetric. It is interesting that this was derived in \[30\] by using SLF theory, and restoring the above point-localized fields from their string-localized counterparts. This is possible because \(A^\mu\) enjoys positivity.

- For \((m = 0, |h|)\):

  \[
  T_{\alpha\beta}(x, e, e') := (-)^r \left[ -\frac{1}{4} A^\tau_{\mu\nu\kappa}(x, e) \partial_\alpha \partial_\beta A^\rho_{\mu\nu\kappa}(x, e') - \frac{1}{4} r \partial^\mu \left( A^\tau_{\mu\nu}(x, e) \partial_\beta A^\rho_{\mu\nu}(x, e') + [e \leftrightarrow e'] + [\alpha \leftrightarrow \beta] \right) \right].
  \]

In conclusion, the essential idea in \[58,59\] gets a lot of traction from SLF theory. Less clearly, it seems possible that the same would apply to unbounded helicity particles, as suggested by Schroer – consult \[62\] and related earlier papers cited there. Direct detection of either would be well-nigh impossible, however.
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