Monte-Carlo Shell Model calculations of triaxially deformed states around $^{134}$Ba

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Abstract. The nuclear structure of ground states and low-lying excited states of even-even isotopes around $^{134}$Ba is described using the nuclear shell model microscopically. These structures are revealed to show various phenomena of quadrupole collective states, including the transition of spherical, axially symmetric deformed and triaxially deformed shapes and the realization of the critical point symmetry, $E(5)$.

1. Introduction

The nuclear collective motion has been one of the most intriguing and characteristics of many-body physics. It is unique in the sense that a nucleus is a truly quantal system and, at the same time, one can discuss its “shape”. In the region of the nuclear chart around $^{134}$Ba, the structure of even-even nuclei shows various phenomena including the transition between spherical and triaxially deformed shapes, and the realization of the critical point symmetry of $E(5)$ [1]. Several microscopic studies of the triaxial deformation based on the nuclear shell model have been achieved (ex. see Ref.[2]), but its systematic study is lacked because a large amount of computation is required. Because a nucleus consists of a finite number of particles, the phase transition occurs gradually and this phenomenon is quite interesting. We discuss the systematic study of low-lying collective states of these nuclei in terms of the nuclear shell model microscopically utilizing the Monte Carlo Shell Model (MCSM) [3].

2. Systematic study of E2 transition probabilities

In the study of medium-heavy nuclei, the huge dimensions of the Hilbert space of the hamiltonian matrix prevents us from diagonalizing the matrix exactly. In order to overcome such difficulty, we have introduced the MCSM, which enables us to obtain the wave functions of ground states and low-lying excited states directly [3]. The validity of the MCSM was demonstrated in the studies of the shape phase transition from a spherical vibrator to axially symmetric rotor [4] and the anomalously small $B(E2)$ value of $^{136}$Te [5]. In this work, we carried out an MCSM calculation for the nuclear structure of the triaxially deformed states of Ba isotopes.
The model space of the MCSM calculation is taken as one major shell including the intruder orbit. The single-particle energies are taken from experimental excitation energies of $^{131}$Sn and $^{133}$Sb. The two-body interaction is assumed to consist of monopole pairing, quadrupole pairing, and quadrupole-quadrupole interactions for identical particles and a quadrupole-quadrupole interaction for a proton-neutron interaction, and the strengths of the interaction are determined for reproducing the $2^+_1$, $4^+_1$ excitation energies of semi-magic nuclei [6]. This phenomenological interaction is sufficiently complex for describing the quadrupole collective states.

The filled symbols in Fig.1 show $E2$ transition probabilities of Sn, Te, Xe, Ba and Ce isotopes obtained experimentally [7]. The isotopes with $N = 82$, the semi-magic nuclei, show the feature of spherical vibrator and their collectivities are small. With increasing neutron number, the collectivity, and the $B(E2)$ on the same time, are increased and shows the feature of an axially symmetric rotor. On the other hand, with decreasing neutron number, the collectivity and $B(E2)$ are increased and shows the feature of the triaxial deformation.

The MCSM results, which are denoted by the open symbols in Fig.1, well reproduce the experimental values consistently in the whole region, where the nuclear structure shows the transition of various quadrupole collective motions. We focus on the Ba isotopes in the transitional region between spherical vibrator and triaxial deformation in the following sections.

3. Transitional structures of Ba isotopes

In order to discuss the gradual transition, a potential energy surface (PES) is provided by the BCS calculation with an external quadrupole deformed field parametrized by $\beta$, the degree of axial symmetric deformation, and $\gamma$, the degree of triaxial deformation. The external field is given as

$$\epsilon_{\text{ext}} = -1.056M\omega^2 r^2 \beta \cdot \left\{ \cos \gamma Y^2_0 + \frac{\sin \gamma}{\sqrt{2}} (Y^2_2 + Y^{-2}_2) \right\}, \quad (1)$$

where $Y^2_0$ denotes the spherical harmonics with its $z$-component being $M$, and its further detail can be seen in Refs.[8, 9].

The PESs without angular-momentum projection are shown for six Ba isotopes in Fig.2. A horizontal axis in this figure shows $\beta$, and an angle around $\beta = 0$ denotes $\gamma$. We see from the figure that the minimum of the PES of $^{136}$Ba locates at $\beta = 0$, and its structure seems to be well described by the spherical vibrator model, whose hamiltonian has the dynamical symmetry.
of $U(5)$. The PES of $^{134}$Ba has a large flat sector around $\beta = 0$, and it is very similar to the assumption of the $E(5)$ symmetry [10]. It has been discussed that the structure of $^{134}$Ba is a good example of the realization of the $E(5)$ symmetry by examining the level scheme and transition probabilities [11]. The position of the minimum of the PES of $^{132}$Ba indicates prolate deformation, and the PES has a valley in the direction of $\gamma$, or the degree of freedom of triaxial deformation. It seems to have the $\gamma$-unstable feature. Thus we can confirm that the $E(5)$ symmetry can appear in an even narrower region as expected from the origin of the symmetry.

The minima of the PESs of $^{126}$Ba, $^{128}$Ba and $^{130}$Ba locate at $\gamma = 0$ and $\beta \neq 0$ with small valley in the direction of $\gamma$. It means that the picture of axially symmetric deformation with $\gamma$-vibration is suitable for describing these nuclei.

![Figure 2. The potential energy surfaces of Ba isotopes without angular-momentum projection. The crosses denote the positions of the local minima. The pairs of numbers over the figures denote the numbers of valence neutron holes and protons.](image)

![Figure 3. The potential energy surfaces of Ba isotopes with angular-momentum projection to $J = 0$ state.](image)

4. MCSM and GCM calculation for Ba isotopes
The excitation energies of Ba isotopes are discussed in this section. Figure 5 shows the experimental excitation energies of the $2^+_1$, $4^+_1$, $6^+_1$, $2^+_2$ states of the Ba isotopes. The increase of $2^+_1$ excitation energy with increasing neutron number indicates the gradual decrease of collectivity. The yrast levels of $^{126,128,130}$Ba seem to be rotational bands, in which the ratio
of the excitation energies of $4^+_1$ and $2^+_1$ is $10/3$. In addition, the excitation energy of $2^+_2$ state is higher than that of $4^+_1$ state. These features are consistent with the description of an axially symmetric rotor with $\gamma$-vibration.

We performed the MCSM calculation for $^{136,134,132}$Ba for discussing these structures microscopically. However, the wave functions of $^{130,128,126}$Ba have not yet been obtained due to a huge amount of calculation. As a preliminary step for the further microscopic calculation using the MCSM, we perform the generator coordinate method with the BCS wave function in the deformed external field which is discussed in Sect. 3 for these isotopes (GCM+BCS). The generator coordinates of the GCM+BCS are taken as $\beta, \gamma$, and pairing gaps of protons and neutrons.

In order to check the validity of the GCM+BCS, we compare computational results with the same hamiltonian using various methods. Figure 4 shows the level schemes and $B(E2; 2^+_1 \rightarrow 0^+_1)$ of $^{132}$Xe obtained by various methods.

![Figure 4. Level schemes and $B(E2; 2^+_1 \rightarrow 0^+_1)$ of $^{132}$Xe obtained by (1) the HFB assuming axial symmetry, (2) the MCSM assuming the axial symmetry, (3) the angular-momentum projected BCS solution, (4) the GCM+BCS, (5) the fully microscopic MCSM, and (6) the experiments [6, 7]. The effective charges are $(e_\nu, e_\pi) = (1.0e, 1.5e)$. The ground state energy of the MCSM is taken as a zero.](image)

The level schemes (1) and (2) of Fig. 4 are obtained under the assumption of the axial symmetry and cannot reproduce the quasi-$\gamma$ band consisting of $2^+_2$, $3^+_1$, $4^+_2$, $5^+_1$, $6^+_2$ states, which is originally related to the degree of freedom of triaxial deformation. In addition, the $B(E2)$ values of these calculations are slightly smaller than the experimental ones.

The level scheme (3) of Fig. 4 shows the result provided by the angular-momentum projection of one wave function provided by the BCS. It well reproduces some low-lying excited states, not highly-excited states. In addition, the $B(E2; 2^+_1 \rightarrow 0^+_1)$ is too strong due to the shortage of many-body correlation.

The level scheme (4) of Fig. 4 shows the result of the GCM+BCS, which well reproduces some low-lying states and $B(E2; 2^+_1 \rightarrow 0^+_1)$ provided by the MCSM, or fully microscopic calculation. However, it cannot reproduce highly-excited states, because it does not take into account sufficiently the contribution of configuration mixing. It suggests that the GCM+BCS calculation is useful for describing only low-lying states.

The level scheme (5) of Fig. 4 shows the MCSM result, which well reproduces the scheme (6) obtained experimentally.

Figure 6 shows the excitation energies of $^{132,134,136,138}$Ba provided by the MCSM calculation and the ones of $^{126,128,130}$Ba provided by the GCM+BCS. The yrast levels of the MCSM reproduce well the experimental levels in Fig. 5, which show gradual transition from an axially symmetric rotor via triaxial deformation to spherical vibrator with increasing neutrons. We can see the slight decrease of the excitation energy of the $2^+_2$ states from $^{130}$Ba to $^{132}$Ba. This anomalous decrease is due to the difference of the approximation methods. Figure 4 shows that the GCM+BCS calculation is likely to show larger excitation energies of highly excited states.
including $2^+_2$ state than the MCSM. By applying the MCSM to the calculation of $^{126,128,130}$Ba, the anomalous decrease is expected to disappear and excitation energies of $2^+_2$ states are also expected to decrease and approach to the experimental values. It suggests that the MCSM calculation of $^{126,128,130}$Ba is essential to evaluate the validity of the description of an axially symmetric rotor with $\gamma$-vibration, and this calculation is now in progress.

5. Summary
The energy levels and other quantities provided by the MCSM show apparent characteristics of the quadrupole collective motion and agree well with the experimental values. The usefulness of the MCSM calculation has been verified by examining the $B(E2)$ values in the large region of the nuclear chart. The microscopic realization of the $U(5)$, $E(5)$, $O(6)$, and $SU(3)$ dynamical symmetries with a single phenomenological Hamiltonian is well explained by the PESs parametrized by $\beta$ and $\gamma$. We especially showed that the PES of $^{134}$Ba has similar structure to that of the discussion of the $E(5)$ critical point symmetry [10].

Both the PESs and the energy levels of Ba isotopes demonstrate gradual transition of different types of quadrupole collective states with varying the neutron number. The MCSM calculation is performed for $^{132,134,136}$Ba and the GCM+BCS calculation is performed for $^{126,128,130}$Ba as a preliminary step for the microscopic study, where the validity of GCM+BCS calculation is checked by comparing the results of $^{132}$Xe using various approximation methods. It is suggested that the combination of axially symmetric rotor with $\gamma$-vibration is suitable for describing structures of $^{126,128,130}$Ba. The GCM+BCS calculation is shown to be able to provide only low-lying yrast states, and the MCSM calculation for these isotopes will be achieved in near future.

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