Nonlinear Supersymmetry Without the GSO Projection and Unstable D9-Brane

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Abstract

Orientable open string theories containing both bosons and fermions without the GSO projection are expected to have the 10 dimensional $N = 2(A)$ space-time supersymmetry in a spontaneously broken phase. We study the low-energy theorem for the nonlinearly realized $N = 2$ supersymmetry using the effective action for an unstable D9-brane. It is explicitly confirmed that the 4-fermion open string amplitudes without the GSO projection obey the low-energy theorem derived from the nonlinear $N = 2$ supersymmetry. An intimate connection between the existence of the hidden supersymmetry and the open-open string ($s$-$t$) duality is pointed out.
1. Introduction

In uncovering the whole structure of string theory, it seems important to understand not only perturbatively stable vacua but also some important classes of unstable vacua. In particular, the unstable vacua corresponding to nonBPS D9(\(D9\))-branes are considered to be of fundamental importance \([1][2][3]\). In connection with this, the condensation of tachyon by which both types of vacua can be related to each other, has been a focus of much interest recently. From the viewpoint of symmetry structure, we expect that one and the same supersymmetry must govern both the stable and unstable vacua in different ways. This is so even in the presence of tachyons signaling instability in the case of the unstable vacua. It is not, however, evident how the space-time supersymmetry is realized in string perturbation theory with tachyons, and the existence of spontaneously broken supersymmetry in the presence of tachyons has never been explicitly proven. We emphasize that this problem is among several unanswered questions concerning the principles of string theory. We should have some direct formulation of supersymmetries within the intrinsic logic of string theory.

There are only a few works which studied the hidden supersymmetry in unstable vacua. On one hand, Sen \([4]\) has proposed the effective world-volume action for nonBPS D-branes in type II theories. The spontaneous breaking of supersymmetry in this proposal is reflected to the absence of \(\kappa\)-symmetry for the world-volume action. This leads to the correct degrees-of-freedom of massless modes, in which the numbers of bosonic and fermionic on-shell degrees of freedom are different, as expected in general from spontaneously broken supersymmetry. On the other hand, from the viewpoint of open-string theories which are supposed to provide the exact (finite \(\alpha'\)) theory for stable and unstable D-branes, it has been investigated \([5]\) by one of the present authors how the \(N = 2\) supersymmetry is buried in the properties of spectra, vertex operators, the boundary conditions, etc, in both the NSR and GS formalisms. Moreover, a formal construction of the supersymmetry transformation law is given, assuming the framework of Witten’s open string field theory.

The purpose of this note is to present further evidence for the hidden nonlinear supersymmetry without the ordinary GSO projection as a support for the ideas discussed in the above two works. We study the low-energy theorem for the scattering of would-be Goldstone fermions. To the present authors’ knowledge, the fermion scattering of open
strings involving both the GSO and opposite-GSO projected sectors has not been fully studied in the literature. We perform a detailed study of the $s$-channel and $t$-channel duality properties of relevant fermion amplitudes. It is shown that the $s$-$t$ duality is responsible for the emergence of the hidden nonlinear supersymmetry. The low-energy effective action for fermion scattering is constructed on the basis of Sen’s proposal and compared to the zero-slope limit of the 4-point amplitudes of the Goldstone fermions.

In the next section, we construct the low-energy effective action for the 10-dimensional open superstring theory with the $N = 2$ space-time supersymmetry being realized nonlinearly, on the basis of the effective world-volume action for a space-time filling D9-brane. In section 3, we study the structure of four-fermion scattering amplitudes involving both the GSO sectors, using the old results given by Schwarz and Wu and paying special attention on their ($s$-$t$) duality properties. The zero-slope limit of the amplitudes is studied. The result agrees precisely with the behaviors predicted from the supersymmetric effective action and also from the arguments of [5]. The final section is devoted to concluding remarks.

2. Low-energy effective action

The work [5] gave strong evidence for believing that the $N = 2$ space-time supersymmetry is hidden in orientable open string theory containing both bosons and fermions without the GSO projection. In particular, all the massive excitations are shown to participate in essential ways. This means that if only the massless modes are used, the effective theories should exhibit the supersymmetry to all orders in the derivative (or $\alpha'$-) expansion in some nonlinear realization. Unfortunately, however, it is not easy to derive the nonlinear realization directly in this case, since we do not know any complete off-shell superspace representation of the $N = 2$ supersymmetry in 10 dimensions, containing the usual $N = 1$ Maxwell (or Yang-Mills) supermultiplet. If we restrict ourselves to the low-energy limit, however, we can switch to the D9-brane picture in type IIA theory, following the procedure used originally in [7] for the BPS D9-brane of type IIB theory. For sufficiently low energies, we can assume the static gauge condition for fixing the world-volume coordinates and derive the low-energy effective action for open strings in 10 dimensions from the effective world-volume action for D9-brane which is believed to describe the low-energy behavior of open strings coupled to D9-brane. The effective action obtained in this way takes a
form of the Volkov-Akulov type \[8\].

Let us briefly recapitulate the world-volume action for unstable D-branes proposed in \[4\]. The action is nothing but the Dirac-Born-Infeld part of the \(\kappa\)-symmetric action constructed in \[7\] for BPS Dp-branes (\(\tau_p = 1/(g_s\sqrt{2\pi\sqrt{\alpha'}})\)):

\[
S_p = -\tau_p \int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + F_{\mu\nu})},
\]

where

\[
G_{\mu\nu} = \eta_{mn}\Pi^m_{\mu}\Pi^n_{\nu},
\]

\[
F_{\mu\nu} = F_{\mu\nu} - i[\bar{\theta}\Gamma_{11}\Gamma_m\partial_\mu\theta(\partial_\nu X^m - \frac{i}{2}\bar{\theta}\Gamma^m\partial_\nu\theta) - (\mu \leftrightarrow \nu)],
\]

with

\[
\Pi^m_{\mu} = \partial_\mu X^m - i\bar{\theta}\Gamma^m\partial_\mu\theta.
\]

Our conventions for the metric and \(\Gamma\) matrices are \(\eta_{\mu\nu} = (+,+,\ldots,+,\ldots)\), \(\{\Gamma_\mu,\Gamma_\nu\} = 2\eta_{\mu\nu}\). The world volume indices are Greek (\(\mu,\nu,\ldots\)) and the space-time indices are lower-case alphabets (\(m,n,\ldots\)). Note that the spinor field \(\theta\) is a 10-dimensional Majorana spinor with 32 components.‡ We denote the Weyl components by \(\theta^\pm\) which satisfy \(\Gamma_{11}\theta^\pm = \mp\theta^\pm\), \(\bar{\theta}_\pm\Gamma_{11} = \pm\bar{\theta}_\pm\). This action has an \(N = 2\) supersymmetry since the tensors \(G_{\mu\nu}, F_{\mu\nu}\) are separately invariant under the supertransformation

\[
\delta\theta = \epsilon, \quad \delta X^m = i\epsilon\Gamma^m\theta,
\]

\[
\delta A_\mu = i\epsilon\Gamma_{11}\Gamma_m\theta\partial_\mu X^m + \frac{1}{6}(\epsilon\Gamma_{11}\Gamma_m\theta\bar{\theta}\Gamma^{mn}\partial_\mu\theta + \bar{\epsilon}\Gamma_{11}\Gamma_{11}\Gamma_m\Gamma_{11}\partial_\mu\theta).
\]

We now consider a space-time filling D9-brane in type IIA theory. Then, in the low-energy limit, it is legitimate to set the world-volume and space-time coordinates equal, \(\sigma^\mu \rightarrow x^\mu = X^m (\mu = m,\ldots)\), by assuming that configurations with various types of folding do not contribute in the low-energy limit.

\[
G_{\mu\nu} = \eta_{\mu\nu} - i\bar{\theta}(\Gamma_\mu\partial_\nu + \Gamma_\nu\partial_\mu)\theta - \bar{\theta}\Gamma^\alpha\partial_\mu\theta\bar{\theta}\Gamma_\alpha\partial_\nu\theta,
\]

\[
F_{\mu\nu} = F_{\mu\nu} - i[\bar{\theta}\Gamma_{11}\Gamma_\alpha\partial_\mu\theta(\delta^\alpha_\nu - \frac{i}{2}\bar{\theta}\Gamma^\alpha\partial_\nu\theta) - (\mu \leftrightarrow \nu)].
\]

Then we have

\[
G_{\mu\nu} \equiv G_{\mu\nu} + F_{\mu\nu} = \eta_{\mu\nu} + \lambda F_{\mu\nu} + \lambda^2 S^{(2)}_{\mu\nu} + \lambda^4 S^{(4)}_{\mu\nu},
\]

‡ Note that our convention for the spinor field is slightly different from those of refs. \[7\] and \[4\] in that we put the factor \(i = \sqrt{-1}\) in front of \(\bar{\theta}\Gamma_{\mu}\partial_\theta\) to fit to the standard field theory convention.
\[ S_{\mu\nu}^{(2)} = -i \bar{\psi}_+ \Gamma_\nu \partial_\mu \psi_+ - i \bar{\psi}_- \Gamma_\mu \partial_\nu \psi_-, \quad (2.10) \]
\[ S_{\mu\nu}^{(4)} = -\frac{1}{4} (\bar{\psi}_+ \Gamma^\alpha \partial_\mu \psi_+ \bar{\psi} \Gamma_\alpha \partial_\nu \psi + \bar{\psi}_- \Gamma^\alpha \partial_\nu \psi_+ \bar{\psi} \Gamma_\alpha \partial_\mu \psi). \quad (2.11) \]

Here we changed the normalization of the fields by resetting as \( A_\mu \rightarrow \lambda A_\mu, \theta = \lambda \psi / \sqrt{2} = \lambda (\psi_+ + \psi_-) / \sqrt{2} \) such that the final form of the effective action has the standard normalization of field theory. Note also that the fermion bilinears without chirality indices \( \pm \) are the sum of two chiral sectors. The constant \( \lambda \) is the Yang-Mills coupling constant in 10 dimensions and is equal to the square root of the inverse tension: \( \lambda = 1 / \sqrt{\tau_9} = (g_s (2\pi)^3 \alpha'^5) / 2 \).

The supertransformation law is now given as \( \epsilon \rightarrow \epsilon / \sqrt{2} \)
\[ \delta \psi = \frac{1}{\lambda} \epsilon - i \frac{\lambda}{2} (\bar{\epsilon} \Gamma_\mu \psi) \partial_\mu \psi, \quad (2.12) \]
\[ \delta A_\mu = \frac{i}{2} \bar{\epsilon} \Gamma_1 \Gamma_\mu \psi + \frac{\lambda^2}{24} (\bar{\epsilon} \Gamma_1 \Gamma_\nu \Gamma_\mu \psi) \bar{\psi} \Gamma_\nu \partial_\mu \psi + \bar{\epsilon} \Gamma_\nu \psi \bar{\psi} \Gamma_1 \Gamma_\nu \partial_\mu \psi - i \frac{\lambda}{2} (\bar{\epsilon} \Gamma_\nu \psi) \partial_\nu A_\mu - i \frac{\lambda}{2} (\bar{\epsilon} \Gamma_\nu A_\mu \psi) \partial_\nu, \quad (2.13) \]

Note that, in both equations \((2.12)\) and \((2.13)\), the last (two, in case of \((2.13)\),) terms are originated from the reparametrization of the world volume, which is required to compensate the supertransformation of the D-brane coordinates as given in \((2.5)\) in preserving the static gauge condition \( \sigma^\mu = x^\mu = X^\mu \), and also that in \((2.13)\) the right hand side is gauge invariant up to the field dependent gauge transformation \( \partial_\mu \Lambda \) with \( \Lambda = -i \frac{\lambda}{2} (\bar{\epsilon} \Gamma_\nu \psi) A_\nu \).

The effective action
\[ S_{\text{eff}} = -\frac{1}{\lambda^2} \int d^{10}x \sqrt{-\det G_{\mu\nu}} \quad (2.14) \]
and the supertransformation law take quite similar forms as those of the well known Volkov-Akulov action \([8]\) for the simplest nonlinear realization of supersymmetry, except for the presence of the gauge field and the associated quartic fermion terms in \( G_{\mu\nu} \). As a matter-of-fact, we should in general expect higher derivative corrections to this simplest form of the action, being the effective theory for interacting open strings. However, at least in the lowest nontrivial approximation with respect to the derivative expansion, we expect that this action describes the scattering amplitudes of massless Ramond fermions if the open string theory indeed has the hidden supersymmetry without the GSO projection and if the massless Ramond fermions behave as the Goldstone fermions. In this sense,
we can regard the above action as the low-energy theorem for the spontaneously broken \( N = 2 \) supersymmetry which should be buried in open string perturbation theory.

By expanding the action (2.14) up to the order \( \lambda^2 \), we obtain

\[
S_{\text{eff}} + \frac{1}{\lambda^2} \int d^{10}x = \int d^{10}x \left( -\frac{1}{4} F_{\mu\nu}^2 + i \frac{1}{2} (\bar{\psi}_+ \Gamma^\mu \partial_\mu \psi_+ + \bar{\psi}_- \Gamma^\mu \partial_\mu \psi_-) - \frac{\lambda^2}{32} (F_{\mu\nu}^2)^2 + \frac{\lambda^2}{8} F^{\mu\nu} F_{\rho\sigma} F_{\alpha\beta}^\rho \partial_\beta F_{\alpha\nu} - i \frac{\lambda}{2} (\bar{\psi}_+ \Gamma_\nu \partial_\nu \psi_+ + \bar{\psi}_- \Gamma_\nu \partial_\nu \psi_-) F^{\nu\mu} + i \frac{\lambda^2}{2} (\bar{\psi}_+ \Gamma_\mu \partial_\mu \psi_+ + \bar{\psi}_- \Gamma_\mu \partial_\mu \psi_-) F^{\mu\nu} + i \frac{\lambda^2}{8} \bar{\psi} \Gamma^\mu \partial_\mu \psi F^2 \\
+ \frac{\lambda^2}{8} (\bar{\psi}_+ \Gamma_\mu \partial_\mu \psi_+ + \bar{\psi}_- \Gamma_\mu \partial_\mu \psi_-)^2 + \frac{\lambda^2}{8} (\bar{\psi}_+ \Gamma_\mu \partial_\nu \psi_+ \bar{\psi}_+ \Gamma^\nu \partial_\nu \psi_+ + \bar{\psi}_- \Gamma_\mu \partial_\nu \psi_- \bar{\psi}_- \Gamma^\nu \partial_\nu \psi_-) - \frac{\lambda^2}{4} (\bar{\psi}_+ \Gamma_\mu \partial_\nu \psi_+ \bar{\psi}_+ \Gamma^\nu \partial_\nu \psi_+ + \bar{\psi}_- \Gamma_\mu \partial_\nu \psi_- \bar{\psi}_- \Gamma^\nu \partial_\nu \psi_-) \right) + \text{higher orders}
\]

(2.15)

This action reduces to the standard free action in the limit \( \lambda \to 0 \), while the supertransformation law given by (2.12) and (2.13) does not apparently reduce to the free form. In fact, if one rescales the supertransformation parameter by \( \epsilon \rightarrow \lambda \epsilon \), the \( \lambda \to 0 \) limit of the supertransformation is simply \( \delta \psi = \epsilon, \delta A_\mu = 0 \). But then the superalgebra becomes trivial. To preserve the superalgebra, it is not allowed to take the \( \lambda \to 0 \) limit in this way, since the cancellation in the product of the order \( 1/\lambda \) and \( \lambda \) terms give the correct superalgebra in terms of the original unscaled superparameter.

The form of the action and, correspondingly, the supertransformation law are not unique, since there is the ambiguity of field redefinition. The field redefinition for spinor fields in 10 dimensions was studied in ref. [9]. For the purposes of self-containedness of our exposition and of making some small corrections to the latter reference, we present general formulae. Let us define a general form of the action containing quartic fermion terms. Here we consider each Majorana-Weyl spinor, namely, either \( \psi_+ \) or \( \psi_- \), separately, although we use the notation \( \psi \) without subscript \( \pm \).

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + i \frac{\lambda}{2} \bar{\psi} \Gamma_\mu \partial_\mu \psi + a_1 \lambda^2 (F^2)^2 + a_2 \lambda^2 (F)^4 \\
+ b_0 i \lambda \bar{\psi} \Gamma_\mu \partial_\mu \psi F_{\mu\nu} + b_1 i \lambda^2 \bar{\psi} \Gamma_{\mu\nu\rho} \psi \partial_\sigma F^{\sigma\nu \mu} F^{\rho \mu} + b_2 i \lambda^2 \bar{\psi} \Gamma_{\mu\nu\rho} \psi F^{\mu\sigma} \partial_\sigma F^{\nu \rho}
\]
\begin{align}
&+ b_3 i \lambda^2 \overline{\psi} \Gamma^{\mu\rho\sigma} \partial_\tau \psi F_{\mu\rho} F_{\sigma\tau} + b_4 i \lambda^2 \overline{\psi} \Gamma_\mu \partial_\nu \psi F^{\mu\sigma} F_{\sigma\nu} + b_5 i \lambda^2 \overline{\psi} \Gamma_\mu \partial_\nu \psi F^2 \\
&+ c_1 \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\nu \psi)^2 + c_2 \lambda^2 \overline{\psi} \Gamma_\mu \partial_\nu \psi \overline{\psi} \Gamma_\mu \partial_\nu \psi + c_3 \lambda^2 \overline{\psi} \Gamma_\mu \partial_\nu \psi \overline{\psi} \Gamma_\nu \partial_\mu \psi \\
&+ c_4 \lambda^2 \overline{\psi} \Gamma^{\mu\rho} \partial_\sigma \overline{\psi} \Gamma_\mu \partial_\nu \psi + c_5 \lambda^2 \overline{\psi} \Gamma^{\mu\rho} \partial_\sigma \overline{\psi} \Gamma_{\mu\nu\sigma\tau} \partial_\tau \psi.
\end{align}

(2.16)

The most general form, to this order of $\lambda$-expansion, of the field redefinition which preserves chirality is

\[ \psi \to \psi + h_1 \lambda \Gamma_{\mu\nu} \psi F^{\mu\nu} + h_2 \lambda^2 \Gamma_{\mu\nu\rho\sigma} \psi F^{\mu\nu} F^{\rho\sigma} + h_3 \lambda^2 \psi F^2 \]

\[ + h_4 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\rho \psi) \Gamma_{\mu\nu} \partial_\sigma \psi + h_5 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\nu \psi) \psi + h_6 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\nu \psi) \Gamma_{\mu\nu} \psi \]

\[ + h_7 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\nu \psi) \Gamma_{\mu\nu} \psi + h_8 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\nu \psi) \Gamma_{\mu\nu\sigma\tau} \partial_\tau \psi. \]

(2.17)

We also consider the field definition of the gauge field

\[ A_\mu \to A_\mu + h_9 i \lambda^2 (\overline{\psi} \Gamma_\mu \partial_\rho \psi) F^{\mu\rho}. \]

(2.18)

In these definitions, the factor $i$ is inserted so that all the coefficients are purely real.

The following formulae for the transformation of the coefficients under the above field redefinition are obtained by performing Fierz rearrangements appropriately.

\[ b_0 \to b_0 - 2h_1, \]
\[ b_1 \to b_1 - 2h_2 + h_5 + h_1^2, \]
\[ b_2 \to b_2 - 2h_2 + b_0 h_1, \]
\[ b_3 \to b_3 - \frac{1}{2} h_1^2 + h_2, \]
\[ b_4 \to b_4 + 4h_1^2 - 4b_0 h_1, \]
\[ b_5 \to b_5 + h_1^2 + h_3, \]
\[ c_1 \to c_1 - 8h_4 - h_5 + 312h_8, \]
\[ c_2 \to c_2 + 4h_4 + h_6 - 72h_8, \]
\[ c_3 \to c_3 - 8h_4 - h_6 + 120h_8, \]
\[ c_4 \to c_4 - 4h_4 + h_6 + 2h_7 - 24h_8, \]
\[ c_5 \to c_5 + h_7 - 24h_8. \]

We note that, to the present order of the expansion, the field redefinition, performed for each chiral sector separately, does not affect the terms of the action (2.10) which are
the product of two chiral sectors. Since there are only 8 free parameters for the field redefinition (in each chiral sector), there exist three independent combinations of the coefficients that are invariant under these transformations: $I_1 \equiv 4(b_2 + 2b_3) + b_0^2$, $I_2 \equiv b_4 - b_0^2$, $I_3 \equiv c_2 + \frac{2}{3}c_3 - \frac{1}{3}c_4 + \frac{2}{3}c_5$. As long as these invariants are preserved, we can freely choose 8 coefficients out of 11 coefficients in the general form of the action. The nonvanishing coefficients in the original form (2.16) of the action are $a_1 = -1/32, a_2 = 1/8, b_1^+ = \mp 1/2, b_4^+ = 1/2, b_5^+ = 1/8, c_1^+ = c_2^+ = 1/8, c_3^+ = -1/4$. Here the upper indices ± denote the chiralities of the corresponding terms. The values of the invariants are thus $I_1^+ = 1/4, I_2^+ = 1/4, I_3^+ = -1/24$. To facilitate comparison with string scattering amplitudes in the next section, we perform the field redefinition such that there remain only the $c_2$ terms for the quartic fermion terms containing a single chiral sector, by setting $c_1 = c_3 = c_4 = c_5 = 0$ and $c_2 = -1/24$. For the $b$-coefficients, we choose $b_0 = b_1 = 0$ which leads to $b_4 = 1/4$. This choice is convenient in making connection to the usual $N = 1$ supersymmetry of the GSO projected action. Using the remaining degrees of freedom, we can further set $b_2 = 1/16, b_3 = b_5 = 0$. The coefficients of the field redefinition are given as $h_1^+ = \mp 1/4, h_2 = 1/32, h_3 = -3/16, h_4 = -1/48, h_5 = 7/24, h_6 = -1/12, h_7 = h_8 = h_9 = 0$. Note that the values indicated without the chirality indices are common to both sectors.

The effective action now takes the form

$$S_{\text{eff}} + \frac{1}{\lambda^2} \int d^{10} x = \int d^{10} x \left( -\frac{1}{4} F^2_{\mu\nu} + \frac{i}{2} \left( \overline{\psi}_+ \Gamma^\mu \partial_\mu \psi_+ + \overline{\psi}_- \Gamma^\mu \partial_\mu \psi_- \right) \right)$$

$$- \frac{\lambda^2}{32} (F^2_{\mu\nu})^2 + \frac{\lambda^2}{8} F^{\mu\nu} F_{\nu\alpha} F^{\alpha\beta} F_{\beta\mu}$$

$$+ \frac{i \lambda^2}{4} (\overline{\psi}_+ \Gamma^\mu \partial_\mu \psi_+ + \overline{\psi}_- \Gamma^\mu \partial_\mu \psi_-) F^{\mu\alpha} F^\alpha_{\nu} + \frac{i \lambda^2}{16} (\overline{\psi}_+ \Gamma^\mu\rho \psi_+ + \overline{\psi}_- \Gamma^\mu\rho \psi_-) F_{\mu\sigma} \partial^\sigma F_{\nu\rho}$$

$$+ \frac{\lambda^2}{4} (\overline{\psi}_+ \Gamma^\mu \partial_\mu \psi_+ + \overline{\psi}_- \Gamma^\mu \partial_\mu \psi_-) - \frac{\lambda^2}{24} \left( \overline{\psi}_+ \Gamma^\mu \partial_\mu \psi_+ + \overline{\psi}_- \Gamma^\mu \partial_\mu \psi_- \right)$$

$$- \frac{\lambda^2}{4} \overline{\psi}_+ \Gamma^\mu \partial_\mu \overline{\psi}_- \Gamma^\mu \partial^\nu \psi_- + \text{higher orders} \right). \tag{2.19}$$

Because of the field redefinition, the supertransformation law is also changed to

$$\delta \psi \equiv \sum_{n=1} \lambda^n \delta \psi^{(n)}, \quad \delta A_{\mu} \equiv \sum_{n=0} \lambda^n \delta A_{\mu}^{(n)} \tag{2.20}$$

where

$$\delta \psi_\pm^{(-1)} = \epsilon_\pm, \tag{2.21}$$
\[
\delta \psi_\pm^{(0)} = \pm \frac{1}{4} \Gamma^{\mu \nu} \epsilon_\pm F_{\mu \nu}, \tag{2.22}
\]
\[
\delta \psi_\pm^{(1)} = -\frac{i}{2} (\tau \Gamma^\mu \psi) \partial_\mu \psi_\pm + \frac{1}{16} \Gamma^{\mu \nu} \Gamma^\rho \epsilon_\pm F_{\mu \nu} F_{\rho \sigma} - \frac{1}{32} \Gamma^{\mu \rho \sigma} \epsilon_\pm F_{\mu \nu} F_{\rho \sigma} + \frac{3}{16} \epsilon_\pm F^2
\]
\[
\pm \frac{i}{4} \Gamma^{\mu \nu} \psi_\pm (\tau \Gamma^{11} \Gamma^\mu \partial_\mu \psi) + \frac{i}{24} \Gamma^{\mu \nu} \partial_\rho \psi_\pm (\overline{\psi}_\pm \Gamma^{\mu \rho \sigma} \epsilon_\pm)
\]
\[
- \frac{7i}{24} \psi_\pm (\tau \Gamma^{\mu} \partial_\mu \psi_\pm) - \frac{7i}{24} \epsilon_\pm (\overline{\psi}_\pm \Gamma^{\mu} \partial_\mu \psi_\pm)
\]
\[
+ \frac{i}{12} \Gamma^{\mu \nu} \psi_\pm (\tau \Gamma^{\mu} \partial_\nu \psi_\pm) + \frac{i}{12} \Gamma^{\mu \nu} \epsilon_\pm (\overline{\psi}_\pm \Gamma^{\mu} \partial_\nu \psi_\pm), \tag{2.23}
\]
\[
\delta A_{\mu}^{(0)} = \frac{i}{2} \Gamma^{11} \Gamma_\mu \psi, \tag{2.24}
\]
\[
\delta A_{\mu}^{(1)} = -\frac{i}{8} \tau \Gamma^\mu \Gamma^\rho \epsilon_\pm F_{\rho \sigma} - \frac{i}{2} (\tau \Gamma^\mu \psi) \partial_\nu A_\mu - i \frac{\lambda}{2} (\tau \Gamma^\mu \partial_\mu \psi) A_\nu, \tag{2.25}
\]
...etc.

With this choice of the field redefinition, the action does not have the order \(\lambda\) term. Correspondingly, the supertransformation law reduces, by dropping one of the two chiral sectors \(\psi_- = \epsilon_- = 0\), to the ordinary linear transformation in the \(\lambda \to 0\) limit, up to the trivial fermion translation \(\delta^{(-1)} \psi_+\). The action, restricted to a single chiral sector, coincides with the action given in [9], as it should. Equivalently, the supercurrents take the form

\[
S^\mu_\pm = \frac{1}{\lambda} \Gamma^\mu \psi_\pm + \frac{1}{4} \Gamma^{\alpha \beta} \Gamma^\mu \psi_\pm F_{\alpha \beta} + O(\lambda),
\]

where the second term restricted to a single chirality coincides with the ordinary supercurrent of the free linear theory. Note that, in the free theory, both terms of this supercurrent are conserved separately. As is well known [9] [10], even if we confine ourselves to the usual GSO projected sector, the supertransformation law is not linear for nonzero slope parameter \((\lambda \neq 0, \alpha' \neq 0)\) even off-shell. This is somewhat mysterious from the viewpoint of the usual \(N = 1\) supersymmetry, because the mass spectrum after the usual GSO projection exhibits the symmetry at each mass level. From our viewpoint, however, this is not surprising, since the symmetry governing the action is in fact the nonlinear \(N = 2\) supersymmetry rather than the \(N = 1\) supersymmetry. We also note that the order \(\lambda^{-1}\) terms of the supertransformation law and the supercurrents are just consistent with the apparent violation of the conservation of supercharges for matrix elements involving a flip of G-parity, which is, as elucidated in [3], caused by the ‘wrong’ boundary condition in the world-sheet formulations.
3. Four-fermion scattering without the GSO projection

Let us now proceed to investigate the low-energy behavior of string scattering for the would-be Goldstone fermions in open string theory without the GSO projection. Fortunately, the general expression for the open string amplitudes with four fermion external lines was given longtime ago in [6]. Since the GSO projection [12] made the inclusion of both chiral sectors unnecessary, the properties of the four fermion amplitudes involving both chiralities in 10 dimensions have not been fully discussed in the literature.

We start from summarizing the old construction. Up to the normalization, the formula for \((s, t)\) amplitude with 4 massless fermion lines is

\[
A_{1,2,3,4}(s, t) = \int_0^1 dx \, x^{-\alpha' s - 3/2}(1 - x)^{-\alpha' t - 3/2}\sum_{\ell=0}^{10} T^{(\ell)}(1, 2)T^{(\ell)}(3, 4)f_\ell(x)
\]  

where

\[
T^{(\ell)}(1, 2) = \mathbf{\pi}_1 \Gamma^{(\ell)} u_2, \quad T^{(\ell)}(3, 4) = \mathbf{\pi}_3 \Gamma^{(\ell)} u_4,
\]

\[
f_\ell(x) = \frac{1}{32} x^{5-\ell} (1 - \sqrt{1 - x})^{\ell - 5} = \frac{1}{32} x^{\ell} (1 + \sqrt{1 - x})^{5-\ell}.
\]

Apart from the momentum dependent factor \(x^{-\alpha' s}(1-x)^{-\alpha' t}\), the integrand is proportional to the vacuum expectation value of the product of four spin fields \(\langle S_\alpha(\infty)S_\beta(1)S_\gamma(x)S_\delta(0)\rangle\) multiplied by the product of the spinor wave functions \(u_i\delta_u u_{i\beta} u_{4\alpha}\) and by the ghost factor \([x(1-x)]^{-1/4}\). The \(\Gamma^{(\ell)}\)'s (and the lower-index counterpart \(\Gamma^{(\ell)}\)'s) are the normalized antisymmetric products of \(\ell\) Gamma matrices, and the sum with respect to \(\ell\) in (3.1) is over the complete set of \(\Gamma^{(\ell)}\). Note that the Lorentz indices and their contractions are suppressed. With this notation, the completeness relation is expressed as the identity

\[
T^{(0)}(1, 4)T^{(0)}(3, 2) = \frac{1}{32}\sum_{\ell=0}^{10} T^{(\ell)}(1, 2)T^{(\ell)}(3, 4),
\]

which is valid for arbitrary four spinor wave functions \(u_i\ (i = 1, 2, 3, 4)\). The spinor bilinears \(T^{(\ell)}(1, 2)\) have symmetry property, \(T^{(\ell)}(1, 2) = -(-1)^{\frac{1}{2}(\ell+1)}T^{(\ell)}(2, 1)\), under the exchange of external lines. We also have \(T^{(\ell)}(1, 2)T^{(\ell)}(3, 4) = \pm(-1)^{\ell}T^{(10-\ell)}(1, 2)T^{(10-\ell)}(3, 4)\) where \(\pm\) depending on whether the lines 1 and 3 have the same or opposite chiralities.

The Mandelstam variables are defined as \(s = -(k_1 + k_2)^2 = -2k_1k_2, t = -2k_2k_3\) and \(u = -2k_1k_3\).

As a first exercise, let us check the case of a single chirality where the above general formula must give the well known expression which is familiar from textbooks. If all the
four external massless fermions have the same chirality, say, +, the formula is equal to
\[ A_{1+,2+,3+,4+}(s,t) = \int_0^1 dx \, x^{-\alpha's-3/2}(1-x)^{-\alpha't-3/2} \left[ T^{(1)}(1+,2+)(3+,4+)(f_1(x) - f_9(x)) + T^{(3)}(1+,2+)(3+,4+)(f_5(x) - f_7(x)) \right]. \] (3.4)

Here and in what follows we put the indices ± to denote the chirality of the external lines.

By using the identity
\[ T^{(3)}(1+,2+)T^{(3)}(3+,4+) = -2T^{(1)}(1+,2+)T^{(1)}(3+,4+) - 4T^{(1)}(1+,4+)T^{(1)}(3+,2+), \]

it is easy to check that (3.4) reduces to
\[ A_{1+,2+,3+,4+}(s,t) = \frac{1}{2} \frac{\Gamma(-\alpha's)\Gamma(-\alpha't)}{\Gamma(1-\alpha's-\alpha't)} \times \left( -\alpha'tT^{(1)}(1+,2+)(3+,4+) + \alpha'sT^{(1)}(1+,4+)(2+,3+) \right). \] (3.5)

This is the standard form of the 4 massless fermion amplitude of open superstring theory with the ordinary GSO projection. The kinematical factor in the parenthesis in eq. (3.5), which is usually denoted by \(-K(u_1, u_2, u_3, u_4)\) \([1]\), has total antisymmetry in external lines, guaranteeing the s-channel-t-channel duality \(A_{1+,2+,3+,4+}(s,t) = -A_{4+,1+,2+,3+}(t,s)\). The minus sign shows the compatibility of the s-t duality with fermi antisymmetry. In the form (3.4), which is more convenient for identifying the states propagating in each channel, the duality can be expressed by the matrix identity
\[ F^TM = -1, \quad F = \begin{pmatrix} -1/2 & -1/4 \\ -3 & 1/2 \end{pmatrix}, \quad M = \begin{pmatrix} 1/2 & 3 \\ 1/4 & -1/2 \end{pmatrix}, \] (3.6)

where the matrices \(F, M\) are defined, respectively, as
\[ \begin{pmatrix} T^{(1)}(1+,2+)T^{(1)}(3+,4+) \\ T^{(3)}(1+,2+)T^{(3)}(3+,4+) \end{pmatrix} = F \begin{pmatrix} T^{(1)}(1+,4+)T^{(1)}(3+,2+) \\ T^{(3)}(1+,4+)T^{(3)}(3+,2+) \end{pmatrix} \] (3.7)

and, with \(y = 1 - x\),
\[ \begin{pmatrix} f_1(x) - f_9(x) \\ f_3(x) - f_7(x) \end{pmatrix} = M \begin{pmatrix} f_1(y) - f_9(y) \\ f_3(y) - f_7(y) \end{pmatrix}. \] (3.8)

The consistency of the amplitude (3.4) with the effective action (2.19) restricted to the single chiral sector has been checked in \([9]\).

We now consider amplitudes with mixed chiralities. Let us start from the configuration with the ordering \((1+,2+,3-,4-)\).
\[ A_{1+,2+,3-,4-}(s,t) = \int_0^1 dx \, x^{-\alpha's-3/2}(1-x)^{-\alpha't-3/2} \left[ T^{(1)}(1+,2+)(3-,4-)(f_1(x) + f_9(x)) + T^{(3)}(1+,2+)(3-,4-)(f_5(x) - f_7(x)) \right]. \]
\[ + T^{(3)}(1_+, 2_+)T^{(3)}(3_-, 4_-)(f_3(x) + f_7(x)) + T^{(5)}(1_+, 2_+)T^{(5)}(3_-, 4_-)f_5(x) \].

(3.9)

The matrix identity expressing the s-t-duality in this case is

\[
F^T M = 1, \quad F = \begin{pmatrix}
10/16 & 6/16 & 2/16 \\
120/16 & 8/16 & -8/16 \\
252/16 & -28/16 & 12/16
\end{pmatrix}, \quad M = \begin{pmatrix}
1/16 & 27/16 & 42/16 \\
1/16 & 3/16 & -14/16 \\
1/32 & -5/32 & 10/32
\end{pmatrix},
\]

(3.10)

where the matrices F and M are defined by

\[
\begin{pmatrix}
T^{(1)}(1_+, 2_+)T^{(1)}(3_-, 4_-) \\
T^{(3)}(1_+, 2_+)T^{(3)}(3_-, 4_-) \\
T^{(5)}(1_+, 2_+)T^{(5)}(3_-, 4_-)
\end{pmatrix} = F \begin{pmatrix}
T^{(0)}(1_+, 4_-)T^{(0)}(3_-, 2_+) \\
T^{(2)}(1_+, 4_-)T^{(2)}(3_-, 2_+) \\
T^{(4)}(1_+, 4_-)T^{(4)}(3_-, 2_+)
\end{pmatrix},
\]

(3.11)

\[
\begin{pmatrix}
f_1(x) + f_9(x) \\
f_3(x) + f_7(x) \\
f_5(x)
\end{pmatrix} = M \begin{pmatrix}
f_0(y) - f_{10}(y) \\
f_2(y) - f_{8}(y) \\
f_4(y) - f_{6}(y)
\end{pmatrix}.
\]

(3.12)

Thus the amplitude (3.9), in which the s-channel poles are the ordinary GSO projected states, is dual to \(A_{1+,4-,3-,2+}(t, s)\):

\[
A_{1+,2+,3-,4-}(s, t) = A_{1+,4-,3-,2+}(t, s),
\]

(3.13)

\[
A_{1+,4-,3-,2+}(t, s) = \int_0^1 dx x^{-\alpha' t - 3/2}(1 - x)^{-\alpha' s - 3/2}[T^{(0)}(1_+, 4_-)T^{(0)}(3_-, 2_+)(f_0(x) - f_{10}(x))
\]

\[+ T^{(2)}(1_+, 4_-)T^{(2)}(3_-, 2_+)(f_2(x) - f_8(x)) + T^{(4)}(1_+, 4_-)T^{(4)}(3_-, 2_+)(f_4(x) - f_6(x))],
\]

(3.14)

in which the t-channel poles are oppositely GSO projected, as it should be.

On the other hand, for the ordering \((1_+, 3_-, 2_+, 4_-)\), the amplitude is

\[
A_{1+,3-,2+,4-}(u, t) = \int_0^1 dx x^{-\alpha' u - 3/2}(1 - x)^{-\alpha' t - 3/2}[T^{(0)}(1_+, 3_-)T^{(0)}(2_+, 4_-)(f_0(x) + f_{10}(x))
\]

\[+ T^{(2)}(1_+, 3_-)T^{(2)}(2_+, 4_-)(f_2(x) + f_8(x)) + T^{(4)}(1_+, 3_-)T^{(4)}(2_+, 4_-)(f_4(x) + f_6(x))],
\]

(3.15)

We can check that the duality relation is now

\[
A_{1+,3-,2+,4-}(u, t) = A_{1+,4-,2+,3-}(t, u),
\]

(3.16)

\[
A_{1+,4-,2+,3-}(t, u) = \int_0^1 dx x^{-\alpha' t - 3/2}(1 - x)^{-\alpha' u - 3/2}[T^{(0)}(1_+, 4_-)T^{(0)}(2_+, 3_-)(f_0(x) + f_{10}(x))
\]

\[+ T^{(2)}(1_+, 4_-)T^{(2)}(2_+, 3_-)(f_2(x) + f_8(x)) + T^{(4)}(1_+, 4_-)T^{(4)}(2_+, 3_-)(f_4(x) + f_6(x))],
\]

(3.17)
In this case, both \( u \) and \( t \) channel poles are oppositely GSO projected. The matrix identity is

\[
F^T M = 1, \quad F = \begin{pmatrix}
1/16 & 1/16 & 1/16 \\
45/16 & 13/16 & -3/16 \\
210/16 & -14/16 & 2/16
\end{pmatrix}, \quad M = \begin{pmatrix}
1/16 & 45/16 & 210/16 \\
1/16 & 13/16 & -14/16 \\
1/16 & -3/16 & 2/16
\end{pmatrix},
\]

(3.18)

where

\[
\begin{pmatrix}
T^{(0)}(1+, 3-)T^{(0)}(2+, 4-) \\
T^{(2)}(1+, 3-)T^{(2)}(2+, 4-) \\
T^{(4)}(1+, 3-)T^{(4)}(2+, 4-)
\end{pmatrix} = F \begin{pmatrix}
T^{(0)}(1+, 4-)T^{(0)}(2+, 3-) \\
T^{(2)}(1+, 4-)T^{(2)}(2+, 3-) \\
T^{(4)}(1+, 4-)T^{(4)}(2+, 3-)
\end{pmatrix},
\]

(3.19)

\[
\begin{pmatrix}
f_0(x) + f_{10}(x) \\
f_2(x) + f_8(x) \\
f_4(x) + f_{16}(x)
\end{pmatrix} = M \begin{pmatrix}
f_0(y) + f_{10}(y) \\
f_2(y) + f_8(y) \\
f_4(y) + f_{16}(y)
\end{pmatrix}.
\]

(3.20)

The duality relation (3.16) tells us a remarkable fact that the duality symmetry of the configuration \((1+, 3-, 2+, 4-)\) is actually contradictory to fermi antisymmetry under the exchange of \(3-\) and \(4-\) (or of \(1+\) and \(2+\)). The extra minus factor under the exchange is expected from the extra phase factor associated to the OPE of fermion emission vertex operators between the different GSO sectors. This means that the configuration \((1+, 3-, 2+, 4-)\) (or its dual) does not contribute to the total amplitude after the summation over the inequivalent permutations of the external lines, which is, in general, required to make crossing-symmetric scattering amplitudes.

We can provide another interpretation for the phenomenon that the S-matrix for mixed chiralities does not involve the ordering \((1+, 3-, 2+, 4-)\), on the basis of the property of supercharges in the world-sheet formulation. As discussed in ref. \[x\], the necessity of inserting the Goldstone fermions for recovering from the apparent violation of the conservation of supercharges defined on the string world sheet occurs at one of the two ends of an open string. Namely, when we consider the wave function of a \((+)-\)chirality fermion string regarding \((-)-\)chirality fermions as external fields, the violation of the \(N = 2\) supercharges associated to the superparameter \(\epsilon_-\) occurs only at one and the same end (say, \(\sigma = \pi\)) of the open string\[x\]. Thus, the insertion of the Goldstone fermion, treated as external field \(\psi_-\), occurs only at the one end, \(\sigma = \pi\), of the \((+)-\)chirality fermion string. It is natural to assume that this property is satisfied for the S-matrix too. We can then conclude that only the two types of cyclic ordering, \((1+, 2+, 3-, 4-)\) and \((1+, 2+, 4-, 3-)\),

\[\text{See the sentence after the equation (4.9) of }[x] \text{: We have } d\Omega_\alpha / d\tau = -2S_\alpha e^{-\phi/2(\tau - i\pi)}, \text{ shifting to the Euclidean metric on the world sheet.} \]
are possible for 4-fermion scatterings with mixed chiralities. For higher point amplitudes, this implies that the external lines of the same chiralities must always be paired as two adjacent lines. Of course, possible permutations of the external lines allowed under this rule must be taken into account. If we consider the supertransformation associated to the chirality $\epsilon_+$ for the wave function of a + chirality state, the corresponding charge is conserved due to the existence of the ordinary $N = 1$ supersymmetry without the insertion of the Goldstone fermions $\psi_-$. Therefore, the above argument is not applicable. Consequently, the amplitude should be constructed according to the ordinary rule, leading to all the 3 types of cyclic ordering.

Let us next study the low-energy expansion of the above amplitudes. This is easily done by expressing the amplitudes in terms of Gamma functions. Taking into account fermi antisymmetry, the end results to the order $O(\alpha')$ are

$$2A_{1+2+3+,4+}(s,t) - 2A_{1+2+4+,3+}(s,u) - 2A_{3+,2+4+,1+}(t,u)$$
$$= \frac{\pi^2}{2} \left( \alpha' t T(1)(1_+,2_+) T(1)(4_+,3_+) - \alpha' s T(1)(1_+,4_+) T(1)(2_+,3_+) \right), \quad (3.21)$$

$$2A_{1+2+3-,4-}(s,t) - 2A_{1+2+4-,3-}(s,u)$$
$$= \frac{\pi^2}{2} \alpha' (t-u) T(1)(1_+,2_+) T(1)(3_-,4_-). \quad (3.22)$$

Note that both the amplitudes (3.21) and (3.22) are normalized in the same way so that the massless poles of each planar amplitude before the summation over noncyclic permutations have residues of the same strength. Of course, these massless poles corresponding to the gauge boson exchange are canceled by the fermi antisymmetrization in the present U(1) case.

For comparison, we show the result for the ordering $(1+, 3-, 2+, 4-)$:

$$A_{1+3-,2+,4-}(u,t) = A_{1+,4-,2+,3-}(t,u)$$
$$= \left[ - \frac{3\alpha' \pi}{2} s - \frac{5\pi}{16} \left( 1 - \alpha' s K \right) \right] T(0)(1_+,3_-) T(0)(2_+,4_-)$$
$$+ \left[ - \frac{\alpha' \pi}{2} s + \frac{3\pi}{16} \left( 1 - \alpha' s K \right) \right] T(2)(1_+,3_-) T(2)(2_+,4_-)$$
$$- \alpha' s K T(4)(1_+,3_-) T(4)(2_+,4_-), \quad (3.23)$$

where $K = 2(1 - \ln 2)$. The difference of (3.23) from the previous two is that there remain the order $(\alpha')^0$ contact terms corresponding to the 4-fermi interaction terms $(\bar{\psi}\Gamma^{(\ell)}\psi)^2$ ($\ell = \ldots$)
0, 2, 4) with no derivative. Such terms, being nonvanishing in the zero-momentum limit, would contradict the nonlinear supersymmetry. However, as emphasized above, these terms do not contribute to the total amplitude after the summation over all inequivalent (anti)permutations, due to the duality symmetry.

From the viewpoint of ordinary field theory, this phenomena is quite miraculous. Remember that there is the Yukawa interaction $\overline{\psi}\psi\phi$ of fermions and tachyon $\phi$, corresponding to poles in $t$ and $u$ channels in the amplitude $A_{1+,2+,3-,4-}(s,t) - A_{1+,2+,4-,3-}(s,u)$ for nonzero $\alpha'$. The contribution of tachyon exchange to this amplitude is

$$T^{(0)}(1+, 4-)T^{(0)}(2+, 3-)$$

$$\frac{\alpha't + \frac{1}{2}}{\alpha'u + \frac{1}{2}}$$

giving the order $(\alpha')^0$ terms with correct fermi antisymmetry. However, the exact amplitude again does not have the order $(\alpha')^0$ contribution. Clearly, in both this and the case (3.23), the infinite tower of massive modes of the oppositely GSO-projected states guaranteeing the validity of the duality relation (3.18) is responsible for the cancellation of the order $(\alpha')^0$ terms in the low-energy limit. In [5], a conceptually similar phenomenon related to the duality between open and closed string channels was pointed out for the case of the supersymmetric mass formula, expressed as $\text{Tr } [(-1)^F M^{2n}] = 384 \delta_{n,4}/(2\pi\alpha')^4$. These properties should be kept in mind as an indication that it is essential to take into account all the massive modes, when we discuss the hidden supersymmetry for nonBPS D-branes.

Finally, comparing the amplitude(s) (3.22) (and (3.21) which has already been checked in [4]) with the effective action (2.19), we find that the open string amplitudes with the both GSO sectors indeed satisfy the low-energy theorem of the nonlinear $N = 2$ supersymmetry, by relating the necessary overall normalization constant $g^2$ for (3.22) and (3.21) to the Yang-Mills coupling $\lambda$ as $g^2 \alpha'/\pi^2/2 = \lambda^2/4 = g_s(2\pi)^9\alpha'^5/4$. Note that only the last term in the effective action (2.13) contributes to the 4-point on-shell S-matrix elements with mixed chiralities. The validity of the effective action for the other terms involving gauge field is already known, since those terms are essentially the same as in the case with the usual GSO projection.

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\textsuperscript{1} It is a good exercise to explicitly check that the expression (3.23) has the duality symmetry which contradicts the fermi antisymmetry, using the identity (3.19).
4. Concluding remarks

We studied the low-energy effective action for open strings without the GSO projection, by assuming the existence of the nonlinearily realized $N = 2$ supersymmetry and showed that the low-energy behavior of the 4-point amplitudes of would-be Goldstone fermions is perfectly consistent with the hidden $N = 2$ supersymmetry. This provides a further concrete support to earlier discussions in [4] and in [5]. These previous works have analyzed the aspects of the hidden $N = 2$ supersymmetry from mutually complementary standpoints.

There remain many further questions to which we hope to return in future works. For instances: (1) In the present paper, we have only treated the $U(1)$ case of type IIA theory. The type IIA effective action can trivially be changed to type IIB case [4] for a single Dp-brane. On the other hand, the extensions of the effective action to the unstable D9-D9 system of type IIB and to general non-Abelian cases [13] are not so straightforward. On the side of the open-string computation, we can easily extend our results to non-Abelian cases by taking into account the Chan-Paton factors appropriately. (2) Of course, generalizations to higher orders in $\lambda$-expansion and to higher derivatives are also important. It is very desirable to formulate the higher derivative corrections in a systematic geometric manner. (3) We have considered only the open string sector. This is justified in the weak string-coupling region ($g_s \ll 1$), since the gravitational length $\ell_P \sim g_s^{1/4} \sqrt{2\pi\alpha'}$ charactering gravity in string theory is much smaller than the characteristic length $\ell_{\text{susy}} \sim \lambda^{1/5} \sim g_s^{1/10} \sqrt{2\pi\alpha'}$ associated with the supersymmetry breaking in the present effective open-string theory. Nonetheless, it is desirable to extend the present work and the discussions in the previous works [4] including all the supergravity background fields. If the supergravity fields are treated dynamically as closed strings, the Goldstone fermions are expected to be absorbed by gravitino through the super Higgs effect in the wrong tachyonic vacuum. In addition to this, there is a nonvanishing cosmological constant corresponding to the tension of D9-brane. How such effects can affect the dynamics of unstable systems is an interesting question. (4) It is important to discuss the role of the $N = 2$ supersymmetry in the dynamics of tachyon condensation. Within the validity of the lowest nontrivial approximation we are using, the supertransformation law (2.12) implies that the restoration of the full $N = 2$ supersymmetry requires $\langle i\bar{\psi}_\pm \partial_\mu \psi_\pm \rangle = - (\Gamma_\mu (1 \pm \Gamma_{11})/2)_{\alpha\beta}/5\lambda^2$ before the field redefinition. A meaningful ques-
tion we may ask here is how this vacuum expectation value is related to tachyon condensation. One possible approach might be to directly construct the effective action for fermion bilinears starting from the effective action (2.14) without introducing tachyon fields explicitly. As we have emphasized repeatedly, the massive excitations of open strings other than the tachyon play essential roles for the hidden $N = 2$ supersymmetry. We note that a simple addition of tachyon field to the effective action is a rather dubious procedure, in view of the nature of our effective action in which all massive modes are integrated out at equal footing. Unfortunately, the standard methods for treating string field theory, which is the only framework in our hand at present to treat all massive modes democratically, are not particularly convenient for exhibiting supersymmetry and other exact properties of stringy nature such as $s$-$t$, open-closed string dualities and modular invariance. It is desirable to develop more powerful formalisms for investigating the nonperturbative structure of string theory and underlying symmetries.

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