Modeling of rectangular plates by the elastic scheme method

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Abstract. This study presents an approach, based on correction coefficients, by which bending moments in rectangular plates when obtained by the elastic scheme method, can be calculated more accurately. At first, the well-known generalized coefficients, used in the elastic scheme method, and also taken from different literature sources are juxtaposed. The correction coefficients are obtained by the FEM according to the thin elastic plate theory. On the next stage, by looking into the shear deformability of the plate cross-section, the values of these coefficients are provided in a more accurate manner. The research has been done for a quadratic plate, but the aforenamed approach can be used to investigate any general rectangular plate, as well.

1. Introduction
The article presents a methodology based on correction coefficients, as it specifies the expressions for obtaining the bending moments in rectangular plates, calculated by an elastic scheme. Initially, a comparison of the generalized coefficients found from various literature sources was made. Correction coefficients are obtained by the Finite Element Method (FEM) as they refer to a thin elastic plates model. Additionally, their value-expressions have been compared upon calculating the angular deformability of the cross section, i.e. a model of a moderately thick plate. The study was conducted for the geometry of a square plate, but the method can be applied to a rectangular shape, as well.

Modelling is required taking into account the actual behaviour of the structure, including taking into account the rheological properties – more information in [5].

2. Theoretical framework of the method of calculation of slabs while observing the elastic scheme
Bending moments across cross-reinforced slabs have been determined by the principles of elasticity theory or the force-distribution method.

When calculating the slabs under the Elastic Frame Method, most commonly utilized appear to be the tables compiled by Marcus to determine the forces depending on the boundary conditions of the slab and the relationship between the sides of the field, according to (1).

\[
M_x = \frac{1}{m_{x1}}ql_{x}^2, \quad M_y = \frac{1}{m_{y1}}ql_{y}^2
\]  

(1)

where \(m_{x1}\) and \(m_{y1}\) (for being required to determine the bending moments in the field of the slab in both directions) are calculated by replacing each slab with two mutually perpendicular stripes of 1 m
width. By using these coefficients, also known as generalized ones, the interaction between the individual stripes, the boundary conditions, as well as the behaviour of load-transferring towards the supports (supporting structure) may be calculated. The total load $q$ acting upon the slab becomes distributed between the two stripes of $q_{xi}$ and $q_{yi}$, so that at the point of intersection, the two stripes may achieve equal sagging moments. In this way, a continuity of the stripe configuration is simulated.

The hypotheses on which this method is based determine the scope of its applicability. It is well known that the method can only be applied to model rectangular smooth elastic plates loaded with uniformly distributed continuous load, and also supported but continuously on the contour sides. Moreover, the results obtained relate to specific and well-defined points in the scheme.

3. Numerical models

The purpose of this study is to initially find the correction coefficients by the Finite Element Method (FEM) of a model of a thin elastic plate, and then only to compare the results with those obtained from modelling a moderately thick plate, whereupon the angular deformability of the cross section has been reported, as well.

Two types of computational models have been considered as objects of the study of elastic plates:
- type I – thin elastic plate. Conditionally, the computational models have been denoted accordingly, with P1-P10.
- type II – a moderately thick plate (the angular deformability of the cross section is also taken into account). Conditionally, the computational models have been denoted accordingly, with P11- P20.

![Geometric and discretization model](image)

**Figure 1.** Geometric and discretization model

Of the two types of computational models, respectively, ten types of square elastic plates of the following fixed parameters have been studied, Figure 1a:
- plate thickness $h = 0.1 \text{ m}$;
- boundary conditions – free surface support along the contour
- load - evenly distributed load of intensity $q(x, y) = 10kN/\text{m}^2$.

The study is a parametric one with alterations of the dimensions of the plate $a (b)$, in both directions within an interval of $(3 – 12) \text{ m}$ with a step of 1 m.

The Plane Finite-Element Method, shell type, with 12 degrees of freedom was used in order to build and simulate the computational models.

The finite elements dimensionality, having been used for discretization, is 0.1x0.1m.
4. Numerical solutions
As a main indicator for the comparison of the computational models of elastic plates, the bending moment $M_x$ (respectively $M_y$) at node A has been identified, as shown on Figure 1b. SAP2000 is used for FEA analysis.

Throughout Tables 1-6, the obtained numerical results for the selected computational models are seeded in a consecutive order.

Table 1. Numerical results for P1 - P10 models

| Type | $M_x$ [kNm/m] | $\Delta M_x$ [kNm/m] | $\Delta^2 M_x$ [kNm/m] |
|------|---------------|----------------------|------------------------|
| P1   | 3.981         |                      |                        |
| P2   | 7.075         | 3.094                |                        |
| P3   | 11.053        | 3.978                |                        |
| P4   | 15.916        | 4.862                |                        |
| P5   | 21.662        | 5.746                |                        |
| P6   | 28.292        | 6.630                |                        |
| P7   | 35.807        | 7.514                |                        |
| P8   | 44.206        | 8.398                |                        |
| P9   | 53.488        | 9.28                 |                        |
| P10  | 63.654        | 10.167               |                        |

Furthermore, for the calculation models of thin elastic plates, manual calculation was done under the method of the elastic scheme on different values of the $m_x$ coefficient (18.00 and 27.43) [1,2,3,4]. The aim is to obtain the bending moment value of the investigated node A, as well as to make the comparison of its value with the results obtained by the Finite Element Method. The models worked out through the method of the elastic scheme are conditionally denoted as P1' - P10'.

Table 2. Numerical results for models P1' – P10' when $m_x = 18.00$

| Type | $M_x$ [kNm/m] | $\Delta M_x$ [kNm/m] | $\Delta^2 M_x$ [kNm/m] |
|------|---------------|----------------------|------------------------|
| P1'  | 3.981         |                      |                        |
| P2'  | 7.075         | 3.094                |                        |
| P3'  | 11.053        | 3.978                |                        |
| P4'  | 15.916        | 4.862                |                        |
| P5'  | 21.662        | 5.746                |                        |
| P6'  | 28.292        | 6.630                |                        |
| P7'  | 35.807        | 7.514                |                        |
| P8'  | 44.206        | 8.398                |                        |
| P9'  | 53.488        | 9.28                 |                        |
| P10' | 63.654        | 10.167               |                        |

Table 3. Numerical results for $m_yi$ and $m_yj$ coefficients of the P1' – P10' models

| Type | $M_x$ [kNm/m] | $\Delta M_x$ [kNm/m] | $\Delta^2 M_x$ [kNm/m] |
|------|---------------|----------------------|------------------------|
| P1'  | 3.981         |                      |                        |
| P2'  | 7.075         | 3.094                |                        |
| P3'  | 11.053        | 3.978                |                        |
| P4'  | 15.916        | 4.862                |                        |
| P5'  | 21.662        | 5.746                |                        |
| P6'  | 28.292        | 6.630                |                        |
| P7'  | 35.807        | 7.514                |                        |
| P8'  | 44.206        | 8.398                |                        |
| P9'  | 53.488        | 9.28                 |                        |
| P10' | 63.654        | 10.167               |                        |

Throughout Figure 2 to Figure 4, the following numerical results ($M_x, \Delta M_x, \Delta^2 M_x$) for the considered computational models are illustrated graphically.
Figure 2. Modification of $M_x (M_y)$ for different computational models

Figure 3. Modification of $\Delta M_x (\Delta M_y)$ for different computational models

Figure 4. Modification of $\Delta^2 M_x (\Delta M_y)$ for different computational models
Table 4. Numerical results for P11 – P18 models

| Type | $M_x$ [kNm/m] | $\Delta M_x$ [kNm/m] | $\Delta^2 M_x$ [kNm/m] |
|------|---------------|----------------------|-----------------------|
| P11  | 3.989         |                      |                       |
| P12  | 7.083         | 3.094                |                       |
| P13  | 11.061        | 3.978                |                       |
| P14  | 15.924        | 4.862                |                       |
| P15  | 21.670        | 5.746                | 0.884                 |
| P16  | 28.300        | 6.630                |                       |
| P17  | 35.815        | 7.514                |                       |
| P18  | 44.214        | 8.398                |                       |

Table 5. Numerical results for models P11’ – P20’ when $m_x = 18.00$

| Type | $M_x$ [kNm/m] | $\Delta M_x$ [kNm/m] | $\Delta^2 M_x$ [kNm/m] |
|------|---------------|----------------------|-----------------------|
| P11’ | 5.00          | 3.989                |                       |
| P12’ | 8.89          | 7.083                |                       |
| P13’ | 13.89         | 11.061               |                       |
| P14’ | 20.00         | 15.924               |                       |
| P15’ | 27.22         | 21.670               | 2.61                  |
| P16’ | 35.56         | 28.300               |                       |
| P17’ | 45.00         | 35.815               |                       |
| P18’ | 55.56         | 44.214               |                       |
| P19’ | 67.22         | 53.497               |                       |
| P20’ | 80.00         | 63.663               |                       |

5. Conclusions
The article demonstrates an applied methodology for obtaining bending moments across rectangular smooth elastic plates, calculated upon observing an elastic scheme. It is based on generalized corrective coefficients derived under the Finite Element Method, with and without taking into account the angular deformation of the cross section. It can be summarized that the proposed methodology is easily applicable for practical purposes, as it may also facilitate a preliminary assessment of a stress state of a slab.

6. References
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