Unparticle-two scalars mixing beyond the standard model and the muon anomalous magnetic moment

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Abstract

We study the electroweak symmetry breaking induced by the interaction of unparticles and the SM Higgs with an additional complex scalar. Furthermore, we estimate the contribution of the mixing scalars on the muon anomalous magnetic moment. We observe that this contribution is at least two order less than the discrepancy between the experimental and the SM results of the muon anomalous magnetic moment.
The electroweak symmetry breaking is an interesting phenomena which has not understood yet. A possible hidden sector beyond the standard model (SM) can be among the candidates to explain this breaking. Such a hidden sector has been proposed by Georgi \cite{1, 2} as a hypothetical scale invariant one with non-trivial infrared fixed point. It is beyond the SM at high energy level and it comes out as new degrees of freedom, called unparticles, being massless and having non integral scaling dimension $d_u$. The interactions of unparticles with the SM fields in the low energy level is defined by the effective lagrangian (see for example \cite{3}).

Recently, the possibility of the electroweak symmetry breaking from unparticles has been introduced \cite{4} (see also \cite{5} which leads to the origin of the electroweak symmetry breaking in the unparticle physics, that is caused by the mixing between the unparticle and the Higgs boson.). The idea is based on the interaction of the SM scalar sector with the unparticle operator in the form $\lambda (\Phi^\dagger \Phi) O_U$ where $\Phi$ is the SM scalar and $O_U$ is the unparticle operator with mass dimension $d_u$ (see \cite{6, 7, 8, 9, 10, 11}). Since unparticles look like a number of $d_u$ massless particles with mass dimension one, the operator $O_u$ can be considered in the form of $(\phi^* \phi)^{\frac{d_u}{2}}$ which induces the interaction term

$$V \sim \lambda (\Phi^\dagger \Phi) (\phi^* \phi)^{\frac{d_u}{2}},$$

driving the electroweak symmetry breaking in the tree level \cite{4}.

In the present work we introduce a toy model that we extend the scalar sector by considering a shadow Higgs one, the complex scalar $\phi_2$ (see \cite{12}) in addition to the the SM Higgs and we study the mechanism for the electroweak symmetry breaking due to the unparticle-neutral scalars mixing, by following the procedure given in \cite{4}. Furthermore, we estimate the contribution of the mixing scalar spectrum on the anomalous magnetic moment (AMM) of the muon. Here, we expect that this contribution is stronger compared to the case that only one Higgs doublet exists and mixes with the unparticle sector. Now, we will construct the toy model by starting with the scalar potential which is responsible for the unparticle-neutral scalars mixing:

$$V(\Phi_1, \phi_2, \phi) = \lambda_0 (\Phi_1^\dagger \Phi_1)^2 + \lambda_0' (\phi_2^* \phi_2)^2 + \lambda_1 (\phi^* \phi)^2 + 2\lambda_2 \mu^{2-d_u} (\Phi_1^\dagger \Phi_1) (\phi^* \phi)^{\frac{d_u}{2}} + 2\lambda_2' \mu^{2-d_u} (\phi_2^* \phi_2) (\phi^* \phi)^{\frac{d_u}{2}},$$

\footnote{See \cite{13} for the necessity of the radiative corrections for the electroweak symmetry breaking from hidden sector due to the interaction in the form $\lambda (\Phi^\dagger \Phi) \phi^* \phi$.}

\footnote{Here the $U(1)_s$ invariant Lagrangian including the shadow sector and the SM one reads

$$L = L_{SM} - \frac{1}{4} X^{\mu \nu} X_{\mu \nu} + \left| \left( \partial_\mu - \frac{1}{2} g_s X_\mu \right) \phi_2 \right|^2 - V(\Phi_1, \phi_2, \phi)$$

where $g_s$ is the gauge coupling of $U(1)_s$.}
where $\mu$ is the parameter inserted in order to make the couplings $\lambda_2$ and $\lambda'_2$ dimensionless. Here, our aim is to find the minimum of the potential $V$ along the ray $\Phi_i = \rho N_i$ with $\Phi_i = (\Phi_1, \Phi_2, \phi)$ (see [13]) and in unitary gauge we have

$$\Phi_1 = \frac{\rho}{\sqrt{2}} \begin{pmatrix} 0 \\ N_0 \end{pmatrix} ; \phi_2 = \frac{\rho}{\sqrt{2}} N'_0 ; \phi = \frac{\rho}{\sqrt{2}} N_1. \label{3}$$

Since $\vec{N}$ is taken as the unit vector in the field space we have

$$N_0^2 + N'_0^2 + N_1^2 = 1. \label{4}$$

Using the eqs. \eqref{2} and \eqref{3} we get

$$V(\rho, N_i) = \frac{\rho^4}{4} \left( \lambda_0 N_0^4 + \lambda'_0 N'_0^4 + 2 \left( \frac{\hat{\rho}^2}{2} \right)^{-\epsilon} (\lambda_2 N_0^2 + \lambda'_2 N'_0^2) N'^{d_u} + \lambda_1 N_1^4 \right). \label{5}$$

The stationary condition for the potential $V$, $\frac{\partial V}{\partial N_i}|_{\vec{n}}$ along a special $\vec{n}$ direction reads

$$\left( \frac{\hat{\rho}^2}{2} \right)^{-\epsilon} \lambda_2 n_1^{d_u} = -\lambda_0 n_0^2, \label{6}$$

$$\left( \frac{\hat{\rho}^2}{2} \right)^{-\epsilon} \lambda'_2 n'_0^{d_u} = -\lambda'_0 n'_0^2,$$

$$2 \lambda_1 n_1^4 = d_u (\lambda_0 n_0^4 + \lambda'_0 n'_0^4), \label{6}$$

where $\epsilon = \frac{2 - d_u}{2}$ and $\hat{\rho} = \frac{\rho}{\mu}$. This condition and eq.\eqref{4} results in

$$n_0^2 = \frac{\chi}{1 + \chi + \kappa},$$
$$n'_0^2 = \frac{1}{1 + \chi + \kappa},$$
$$n_1^2 = \frac{\kappa}{1 + \chi + \kappa}, \label{7}$$

where

$$\chi = \frac{\lambda'_0 \lambda_2}{\lambda_0 \lambda'_2},$$
$$\kappa = \sqrt{\frac{d_u \chi \lambda_0 (\lambda_2 \chi + \lambda'_2)}{2 \lambda_1 \lambda_2}}. \label{8}$$

By using eq.\eqref{7} the nontrivial minimum value of the potential is obtained as

$$V(\rho, n_i) = -\frac{\rho^4}{4} \left( \lambda_0 n_0^4 + \lambda'_0 n'_0^4 \right) \epsilon. \label{9}$$

Notice that, for $d_u = 2$, the minimum of the potential is trivial, namely $V(\rho, n_i) = 0$. However, the nontrivial minimum can be obtained at tree level for $1 < d_u < 2$ without the need for
CW mechanism (see [13] for details of CW mechanism). The stationary condition fixes the parameter $\rho$ as,

$$\rho = \rho_0 = \left(\frac{-2^\epsilon \lambda_2 n_l^d}{\lambda_0 n_0^2}\right)^{\frac{1}{2\pi}} \mu,$$

(10)

or

$$\rho = \rho_0 = \left(\frac{-2^\epsilon \lambda'_2 n_l'^2}{\lambda'_0 n_0'^2}\right)^{\frac{1}{2\pi}} \mu,$$

(11)

and this parameter should be stabilized as $d_u \rightarrow 2$, since it is responsible for the mass scales of the theory. When one chooses $d_u = 2$ in the stationary conditions (see eq. (6)), the restriction which connects the couplings $\lambda_0$, $\lambda'_0$, $\lambda_2$ and $\lambda'_2$ can be reached naturally:

$$\sqrt{\lambda_0 \lambda'_0 \lambda_1} = -\sqrt{\lambda_0^2 + \lambda_0 \lambda_2^2},$$

(12)

and this is the sufficient condition in order to stabilize the parameter $\rho_0$. If we consider this restriction we get

$$\hat{\rho}_0^2 = 2 \left(\frac{d}{2}\right) \frac{d}{d-2} \left(\frac{-\lambda'_2}{\lambda'_0}\right) \left(1 - \sqrt{\frac{d}{2} \frac{\lambda'_0}{\lambda'_2 \lambda_0}}\right),$$

(13)

and, for $d_u \rightarrow 2$, it converges to

$$\hat{\rho}_0^2 \rightarrow 2 \frac{\lambda'_0 (-\lambda_2) + (-\lambda'_2) \lambda_0 + \lambda_0 \lambda'_0}{\sqrt{\epsilon \lambda_0 \lambda'_0}}.$$ 

(14)

At this stage we consider that the couplings $\lambda_2$, $\lambda'_2$ which drive the strengths of the neutral Higgs boson-the massless scalar field vertices have the same strength. With this choice the constraint eq. (12) becomes

$$\lambda_2 = -\sqrt{\frac{\lambda_0 \lambda'_0 \lambda_1}{\lambda_0 + \lambda'_0}},$$

(15)

and parameters $\chi$ and $\kappa$ read

$$\chi = \frac{\lambda'_0}{\lambda_0},$$

(16)

$$\kappa = \sqrt{\frac{d_u}{2} \frac{\lambda'_0 (\lambda_0 + \lambda'_0)}{\lambda_0 \lambda_1}}.$$

Now we are ready to study the mixing matrix of the scalars under consideration. If one expands the fields $\Phi_1$, $\phi_2$ and $\phi$ around the vacuum as

$$\Phi_1 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ \rho_0 n_0 + h \end{array}\right) ; \phi_2 = \frac{1}{\sqrt{2}} (\rho_0 n_0' + h') ; \phi = \frac{1}{\sqrt{2}} (\rho_0 n_1 + s),$$

(17)
the potential (eq. (2)) reads

\[
V(h, h', s) = \frac{\lambda_0}{4} (\rho_0 n_0 + h)^4 + \frac{\lambda_0'}{4} (\rho_0 n_0' + h')^4 + \frac{\lambda_1}{4} (\rho_0 n_1 + s)^4 + 2^{-\frac{d_u}{2}} \lambda_2 \mu^2 \varepsilon (\rho_0 n_0 + h)^2 (\rho_0 n_1 + s)^{d_u} + 2^{-\frac{d_u}{2}} \lambda_2^2 \mu^2 \varepsilon (\rho_0 n_0' + h')^2 (\rho_0 n_1 + s)^{d_u}.
\]

(18)

Using this potential we get the mass matrix \((M^2)_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \mid \phi_i = 0\) with \(\phi_i = (h, h', s)\) as

\[
(M^2)_{ij} = 2 \rho_0^2 n_0^2 \begin{pmatrix}
\lambda_0 & 0 & -\left(\frac{d_u \lambda_0}{2}\right)^\frac{3}{4} \left(\frac{\lambda_0'}{\lambda_0 + \lambda_0'}\right)^\frac{1}{4} \\
0 & \lambda_0 & -\left(\frac{d_u \lambda_0}{2}\right)^\frac{3}{4} \left(\frac{\lambda_0^2 \lambda_1}{\lambda_0 (\lambda_0 + \lambda_0')}\right)^\frac{1}{4} \\
-\left(\frac{d_u \lambda_0}{2}\right)^\frac{3}{4} \left(\frac{\lambda_0'}{\lambda_0 + \lambda_0'}\right)^\frac{1}{4} & -\left(\frac{d_u \lambda_0}{2}\right)^\frac{3}{4} \left(\frac{\lambda_0^2 \lambda_1}{\lambda_0 (\lambda_0 + \lambda_0')}\right)^\frac{1}{4} & (2 - \frac{d_u}{2}) \sqrt{\frac{d_u}{2}} \left(\frac{\lambda_0 \lambda_1 (\lambda_0 + \lambda_0')}{\lambda_0^2}\right)
\end{pmatrix}.
\]

(19)

The eigenvalues of the matrix are

\[
m_I^2 = 2 \lambda_0 n_0^2 \rho_0^2,
\]

\[
m_{II}^2 = \lambda_0 n_0^2 \rho_0^2 \left(1 + 2 - \frac{d_u}{2}\right) \sqrt{\frac{d_u}{2}} \frac{s_{10} (1 + s_0)}{s_0} - \sqrt{\Delta},
\]

\[
m_{III}^2 = \lambda_0 n_0^2 \rho_0^2 \left(1 + 2 - \frac{d_u}{2}\right) \sqrt{\frac{d_u}{2}} \frac{s_{10} (1 + s_0)}{s_0} + \sqrt{\Delta},
\]

(20)

where

\[
\Delta = d_u \sqrt{\frac{2 d_u s_{10} (1 + s_0)}{s_0}} + \left(1 + \frac{d_u}{2} - 2 \sqrt{\frac{d_u}{2}} \frac{s_{10} (1 + s_0)}{2 s_0}\right)^2.
\]

(21)

Here we used the parametrization

\[
\lambda_0' = s_0 \lambda_0,
\]

\[
\lambda_1 = s_{10} \lambda_0.
\]

(22)

The physical states \(h_I, h_{II}, h_{III}\) are connected to the original states \(h, h', s\) as

\[
\begin{pmatrix}
 h \\
 h' \\
 s
\end{pmatrix} = \begin{pmatrix}
 c_\alpha & -c_\eta s_\alpha & s_\eta s_\alpha \\
 s_\alpha & c_\eta c_\alpha & -s_\eta c_\alpha \\
 0 & s_\eta & c_\eta
\end{pmatrix} \begin{pmatrix}
 h_I \\
 h_{II} \\
 h_{III}
\end{pmatrix},
\]

(23)

where \(c_\alpha(\eta) = \cos \alpha (\eta), s_\alpha(\eta) = \sin \alpha (\eta)\) and

\[
tan 2 \alpha = \frac{2 \sqrt{s_0}}{s_0 - 1},
\]

\[
tan 2 \eta = \left(\frac{d_u}{2}\right)^\frac{3}{4} \frac{2 \left(s_0 s_{10} (1 + s_0)\right)^\frac{1}{2}}{(1 - \frac{d_u}{2}) \sqrt{2 d_u s_{10} (1 + s_0)} - \sqrt{s_0}}.
\]

(24)
When $d_u \to 2$, the state $h_{II}$ is massless in the tree level and it has the lightest mass for $1 < d_u < 2$. $h_I$ and $h_{II}$ can be identified as the SM Higgs boson and heavy scalar coming from the shadow sector, respectively.

Finally, we construct the last restriction by fixing the vacuum expectation value $v_0 = n_0 \rho_0$, by the gauge boson mass $m_W$ as

$$v_0^2 = \frac{4 m_W^2}{g_W^2} = \frac{1}{\sqrt{2} G_F}, \quad (25)$$

where $G_F$ is the Fermi constant. By using eqs. (7) and (13) we get

$$\hat{v}_0^2 = c_0 \frac{s_{10} \sqrt{2 s_0 (1 + s_0)} + s_0 \sqrt{d_u s_{10}}}{\sqrt{d s_0 (1 + s_0) + (1 + s_0) \sqrt{2 s_{10}}}}, \quad (26)$$

with $c_0 = 2 \left( \frac{d_u}{2} \right)^{2 (2 - d_u)}$. The choice of the parameter $\mu$ around weak scale as $\mu = v_0$ results in additional restriction which connects parameters $s_0$ and $s_{10}$ (see eq. (26) by considering $\hat{v}_0^2 = 1$) as

$$s_{10} = \frac{1 + s_0}{c_0^2 s_0}. \quad (27)$$

When $d_u \to 2$, $s_{10} \to \frac{e}{4} \frac{1 + s_0}{s_0}$ and when $d_u \to 1$, $s_{10} \to \frac{1 + s_0}{2 s_0}$. It is shown that the ratios are of the order of one and the choice $\mu = v_0$ is reasonable (see [4] for the similar discussion.)

The effect of the unparticle-neutral scalars mixing on the muon anomalous magnetic moment

The current experimental world average of the muon AMM by the latest BNL experiment [14] has been announced as

$$a_\mu = 116 592 080 (63) \times 10^{-11}. \quad (28)$$

From the theoretical point of view, the muon AMM is written in terms of different contributions in the framework of the SM as

$$a_\mu(SM) = a_\mu(QED) + a_\mu(weak) + a_\mu(hadronic), \quad (29)$$

where $a_\mu(QED) = 116 584 718.09 (0.14) (0.04) \times 10^{-11}$ and $a_\mu(weak) = 152 (2) (1) \times 10^{-11}$. The hadronic contributions are under theoretical investigation and need the forthcoming results from the high precision measurements. With the new data from Novosibirsk, some exclusive
channels from BaBar and the compilation of the $e^+e^-$ data by Michel Davier and collaborators
the numerical value $6908.7 (39) (19) (7) \times 10^{-11}$ (see [15], [16] and references therein) is obtained
for the leading order hadronic vacuum polarization to $a_\mu(SM)$. The higher order contribution
(next to leading contributions) is estimated as $-97.9 (0.9) (0.3) \times 10^{-11}$ (see [17, 18, 19, 20, 21])
and light by light scattering is calculated as $105 (26) \times 10^{-11}$ (see [22, 23, 24, 25, 26, 27]).
Finally, one gets the SM result

$$a_\mu(SM) = 116 591 785 (51) \times 10^{-11},$$

which shows that there is still $3.6 \sigma$ discrepancy between the experimental result and the SM
one. Now, we will study the additional effect on the muon AMM due to the unparticle-neutral
scalars mixing system of our toy model and check whether it is possible to explain the present discrepancy ($\delta$) by playing with the free parameters of the model. By considering the highest
and the lowest experimental and SM values of the muon AMM, $\delta$ is estimated in the range

$$1.810 \times 10^{-9} < \delta < 4.089 \times 10^{-9}.$$

Here, we will check if the new contribution coming from the mixing spectrum\footnote{In the calculation of upper and lower limits of $\delta$ we take the SM electroweak contribution without the SM Higgs one, since we insert our mixing spectrum effect instead. We see that the SM Higgs effect is of the order of $10^{-12}$.} reaches to the values of the order of $10^{-9}$.

The starting point is the SM Higgs-quark (lepton) interaction in the SM

$$\mathcal{L}_Y = \eta_{ij}^U Q_{iL} \Phi_1 U_{jR} + \eta_{ij}^{D(E)} Q_{iL} (\bar{l}_{iL}) \phi_1 D_{jR} (E_{jR}) + h.c.$$  

where $L = \frac{1}{2}(1 - \gamma_5)$ and $R = \frac{1}{2}(1 + \gamma_5)$ denote chiral projections, $\Phi_1$ is the scalar doublet,
$Q_{iL} (\bar{l}_{iL})$ for $i = 1, 2$, are quark (lepton) doublets, and $U_{jR}$, $D_{jR}$ ($E_{jR}$) are right, handed up
type, down type quark singlets (right handed lepton singlets), $\eta_{ij}^{U,D,E}$ is the matrices of the Yukawa couplings.

Our aim is to estimate the effect of the unparticle-neutral scalars mixing on the muon AMM and, therefore, we consider the charged leptons in our calculations. The charged lepton masses are proportional to the vacuum expectation value $v_0 = n_0 \rho_0$ and after the spontaneous breakdown
of the electroweak symmetry the $i^{th}$ charged lepton mass is obtained as $m_i = v_0 \eta_{ii}$, with $v_0 = \frac{2m_W}{\rho_W}$ and, therefore, the coupling $\eta_{ii}$ is $\eta_{ii} = \frac{m_i}{v_0}$.
The effective interaction for the anomalous magnetic moment of the lepton is defined as

\[ \mathcal{L}_{AMM} = a_l \frac{e}{4 m_l} \bar{l} \sigma_{\mu\nu} l F^{\mu\nu}, \]  

(33)

where \( F_{\mu\nu} \) is the electromagnetic field tensor and "\( a_l \)" is the AMM of the lepton "\( l \)", \( l = e, \mu, \tau \). This interaction can be induced by the unparticle-neutral scalars mixing system at loop level in the toy model under consideration (see Fig. 1). Since charged leptons couple to the SM Higgs doublet, in the internal line, only the neutral Higgs \( h \) and, therefore, the physical states \( h_I, h_{II}, h_{III} \) appear since

\[ h = c_\alpha h_I - c_\eta s_\alpha h_{II} + s_\eta s_\alpha h_{III}, \]

(34)

where \( h_I, h_{II} \) and \( h_{III} \) are identified as new neutral scalar boson, scalon and the SM Higgs boson (see eqs. (23) and (24) for the physical states and the parameters \( c_\alpha, c_\eta, s_\alpha \) and \( s_\eta \)). Using the definition of AMM of the lepton \( l \) (eq. (33)), we get

\[ a_{\mu}^{New} = a_{\mu}^{h_I} + a_{\mu}^{h_{II}} + a_{\mu}^{h_{III}}, \]

(35)

where

\[ a_{\mu}^{h_I} = \frac{G_F m_\mu^4 c_\alpha^2}{4 \sqrt{2} \pi^2 m_{h_I}^2} \int_0^1 dx \frac{(2 - x) x^2}{1 + x \left( \frac{m_\mu^2}{m_{h_I}^2} - 1 \right)}, \]

\[ a_{\mu}^{h_{II}} = \frac{G_F m_\mu^4 s_\alpha^2 c_\eta^2}{4 \sqrt{2} \pi^2 m_{h_{II}}^2} \int_0^1 dx \frac{(2 - x) x^2}{1 + x \left( \frac{m_\mu^2}{m_{h_{II}}^2} - 1 \right)}, \]

\[ a_{\mu}^{h_{III}} = \frac{G_F m_\mu^4 s_\alpha^2 s_\eta^2}{4 \sqrt{2} \pi^2 m_{h_{III}}^2} \int_0^1 dx \frac{(2 - x) x^2}{1 + x \left( \frac{m_\mu^2}{m_{h_{III}}^2} - 1 \right)}. \]

(36)

**Discussion**

In the present work we extend the scalar sector by considering a shadow Higgs one with complex scalar and choose a scalar potential responsible for the mixing of neutral scalars. Here, the motivation is to drive the electroweak symmetry breaking at tree level and, for this, the hidden sector is chosen as the unparticle sector propsed by Georgi recently. The

\(^4\)The most general Lorentz-invariant form of the coupling of a charged lepton to a photon of four-momentum \( q_\nu \) can be written as \( \Gamma_\nu = G_1(q^2) \gamma_\nu + G_2(q^2) \sigma_{\mu\nu} q^\nu + G_3(q^2) \sigma_{\mu\nu} \gamma_5 q^\nu \) where \( q_\nu \) is the photon 4-vector and the \( q^2 \) dependent form factors \( G_1(q^2), G_2(q^2) \) and \( G_3(q^2) \) are proportional to the charge, AMM and EDM of the \( l \)-lepton respectively.

\(^5\)Here we do not take charged FC interaction in the leptonic sector due to the small couplings for \( \mu - \nu_l \) transitions.
unparticle sector causes the electroweak symmetry breaking without need for CW mechanism (see [4]) since this sector is scale invariant and the unparticle operator has non-integral scaling dimension. When the scaling dimension reaches to \(d_u = 2\) one gets the trivial minimum and the CW mechanism needs for the electroweak symmetry breaking. On the other hand, when the electroweak symmetry is broken, the scalar fields are mixed and three massive states appear. Therefore, the hidden sector scale invariance is also broken.

There are number of parameters existing in our toy model and they should be restricted by conditions based on mathematical and physical backgrounds. At first, we choose that the couplings \(\lambda_2\) and \(\lambda'_2\) which drive the strengths of the neutral Higgs boson-the massless scalar field vertices have the same strength. Second, we construct the restriction\(^6\) which connects the couplings \(\lambda_0, \lambda'_0, \lambda_2\) naturally (see eq.(12)). The choice of \(\mu\) around weak scale, namely \(\mu = v_0\), results in a new restriction and the couplings \(\lambda_1\) and \(\lambda_0\) are also connected (see eq.(27)). Finally, we have the couplings \(\lambda_0, \lambda'_0, \) and the scaling dimension \(d_u\) as free parameters. Notice that we take the state \(h_I\) as the SM Higgs boson and choose its mass \(m_I\) as a fixed value, which will be hopefully measured as a SM Higgs mass in the forthcoming experiments.

In Fig.2 we plot \(d_u\) dependence of \(s_{10}\) for different values of the parameter \(s_0 = \frac{\lambda'_0}{\lambda_0}\). Here the solid (dashed, dotted) line represents \(s_{10}\) for \(s_0 = 0.1\) \((s_0 = 0.5, s_0 = 0.9)\). \(s_{10}\) is sensitive to \(d_u\), especially for small \(s_0\), and increases with the decreasing values of \(s_0\) and the increasing values of \(d_u\).

Fig.3 represents \(d_u\) dependence of \(\lambda_0\) for different values of the parameter \(s_0\) and the mass \(m_I\). Here the lower-intermediate-upper solid (dashed, dotted) line represents \(\lambda_0\) for \(m_I = 110 - 120 - 130\) \((GeV)\), \(s_0 = 0.1\) \((s_0 = 0.5, s_0 = 0.9)\). We observe that \(\lambda_0\) does not depend on the scaling dimension \(d_u\) and the increasing values of the mass \(m_I\) result in the enhancement of the coupling \(\lambda_0\). On the other hand \(\lambda_0\) increases with the increasing values of \(s_0\).

In Figs. 4 and 5 we show \(d_u\) dependence of the masses \(m_{II}\) and \(m_{III}\) for different values of the mass \(m_{III}\). Fig.4 is devoted to \(d_u\) dependence of the mass \(m_{I}\) for the parameter \(s_0 = 0.1\). Here the solid (dashed, dotted) line represents \(m_{I}\) for \(m_{I} = 130 - 120 - 110\) \((GeV)\). We see that \(m_{II}\) reaches to zero when \(d_u = 2\) at tree level and this is the case that \(h_{II}\) is the pseudo Golstone boson due to the spontaneous symmetry breaking of conformal symmetry. With the increasing values of the mass \(m_I, m_{II}\) also increases. Fig.5 shows \(d_u\) dependence of the mass \(m_{III}\) for the parameter \(s_0 = 0.1\). Here the solid (dashed, dotted) line represents \(m_{III}\) for \(m_{I} = 130 - 120 - 110\) \((GeV)\). The state \(h_{III}\) is the heaviest one and it is appropriate to expect

\(^6\)This restriction is sufficient to stabilize the parameter \(\rho_0\) which plays the role of mass scale of the theory.
that it is the new neutral shadow scalar.

For completeness, in Fig 6 we present $d_u$ dependence of the masses $m_I$, $m_{II}$ and $m_{III}$ without restricting $m_I$. Here the upper-intermediate-lower solid (dashed, dotted) line represents $m_I$ $(m_{II}, m_{III})$ for $s_0 = 0.1 - 0.5 - 0.9$ and for $\lambda_0 = 0.2$. Here we observe that $h_{III}$ is the heaviest state and $h_{II}$ becomes massless when $d_u = 2$ at tree level as expected. $m_{III}$ increases, $m_{II}$ is fixed and $m_I$ decreases with the increasing values of $d_u$. Furthermore, the masses of the scalars increase with the decreasing values of $s_0$.

Now, we start to analyze the effect of the unparticle-neutral scalars mixing spectrum on the muon AMM and we study $d_u$ and $s_0$ dependence of the part of the muon AMM, $AMM_{scl}$, which is driven by the mixed scalar spectrum.

Fig 7 represents $d_u$ dependence of the $AMM_{scl}$ for different values of the parameter $s_0$ and the mass $m_I$. Here the lower-intermediate-upper solid (dashed, dotted) line represents $AMM_{scl}$ for $m_I = 130 - 120 - 110$ (GeV), $s_0 = 0.001$ ($s_0 = 0.1$, $s_0 = 0.9$). It is shown that $AMM_{scl}$ increases with increasing values $s_0$ the scaling dimension $d_u$, especially for large values of $s_0$. The $AMM_{scl}$ reaches to the values of the order of $10^{-12}$ and it is almost three order smaller than the discrepancy between the experimental and the SM results of muon AMM.

In Fig 8 we show the $s_0$ dependence of the $AMM_{scl}$ for different values of the parameter $d_u$ and the mass $m_I$. Here the lower-intermediate-upper solid (dashed, dotted) line represents $AMM_{scl}$ for $m_I = 130 - 120 - 110$ (GeV), $d_u = 1.1$ ($d_u = 1.5$, $d_u = 1.9$). We observe that the $AMM_{scl}$ is relatively sensitive to $s_0$ for its large values.

In summary, the interaction of the SM Higgs doublet with the hidden unparticle sector is a possible candidate to drive the electroweak symmetry breaking at tree level. We study this breaking by considering the SM Higgs doublet and an additional scalar and estimate the contribution of new scalar spectrum on the muon AMM. We observe that this contribution is almost three order less than the discrepancy between the experimental and the SM results of muon AMM.

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Figure 1: One loop diagrams contributing to AMM of $l$-lepton due to the neutral scalars $h_i$ where $i = I, II, III$. Wavy (dashed) line represents the electromagnetic field ($h_i$ fields).
Figure 2: $s_{10}$ as a function of $d_u$. Here the solid (dashed, dotted) line represents $s_{10}$ for $s_0 = 0.1$ ($s_0 = 0.5$, $s_0 = 0.9$).

Figure 3: $\lambda_0$ as a function of $d_u$. Here the lower-intermediate-upper solid (dashed, dotted) line represents $\lambda_0$ for $m_f = 110 - 120 - 130$ (GeV), $s_0 = 0.1$ ($s_0 = 0.5$, $s_0 = 0.9$).
Figure 4: $m_{III}$ as a function of $d_u$ for $s_0 = 0.1$. Here the solid (dashed, dotted) line represents $m_{III}$ for $m_I = 130 - 120 - 110$ (GeV).

Figure 5: $m_{III}$ as a function of $d_u$ for $s_0 = 0.1$. Here the solid (dashed, dotted) line represents $m_{III}$ for $m_I = 130 - 120 - 110$ (GeV).
Figure 6: $m_i$ as a function of $d_u$ for $\lambda_0 = 0.2$. Here the upper-intermediate-lower solid (dashed, dotted) line represents $m_I (m_{II}, m_{III})$ for $s_0 = 0.1 - 0.5 - 0.9$.

Figure 7: $AMM_{sel}$ as a function of $d_u$. Here the lower-intermediate-upper solid (dashed, dotted) line represents $AMM_{sel}$ for $m_I = 130 - 120 - 110$ (GeV), $s_0 = 0.001$ ($s_0 = 0.1$, $s_0 = 0.9$).
Figure 8: $AMM_{scl}$ as a function of $s_0$. Here the lower-intermediate-upper solid (dashed, dotted) line represents $AMM_{scl}$ for $m_I = 130 - 120 - 110$ (GeV), $d_u = 1.1$ ($d_u = 1.5$, $d_u = 1.9$).