Gravitational Collapse in 1+1 Dimensions and Quantum Gravity†

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Abstract

We investigate the quantum theory of 1+1 dimensional dilaton gravity, which is an interesting toy model of the black hole dynamics. The functional measures of gravity part are explicitly evaluated and derive the Wheeler-DeWitt like equations as physical state conditions. In ADM formalism the measures are very ambiguous, but in our formalism they are explicitly defined. Then the new features which are not seen in ADM formalism come out. A singularity appears at $\varphi^2 = \kappa(>0)$, where $\kappa = (N - 51/2)/12$ and $N$ is the number of matter fields. At the final stage of the black hole evaporation, the Liouville term becomes important, which just comes from the measures of the fields. Behind the singularity the quantum mechanical region $\kappa > \varphi^2 > 0$ extends, where the sign of the kinetic term in the Wheeler-DeWitt like equation changes. If $\kappa < 0$, the singularity disappears. We briefly discuss the possibility of gravitational tunneling and the issue of the information loss.

1. Introduction

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Recently many authors investigate the dynamics of the black hole by using an interesting toy model of gravity in 1+1 dimensions.$^{[1-5]}$ It is called the dilaton gravity, which was proposed by Callan, Giddings, Harvey and Strominger.$^{[1]}$ The model has very similar features to the spherically symmetric gravitational system in 3+1 dimensions. The essence of the black hole dynamics appears to be included enough. Really in the semi-classical approximation we can argue the dynamics in completely parallel way to the case of the spherically symmetric black hole. Furthermore the gravitational back-reaction effects can be included systematically.

In this paper we develop the argument to the quantum gravity.$^{[4]}$ The quantum gravity becomes very important at the final stage of the black hole evaporation. It is expected that the issue of the information loss may be resolved in the quantum gravity.

As a quantization method of gravitation, there is Arnowitt-Deser-Misner (ADM) formalism or Wheeler-DeWitt approach. However there are some problems in ADM formalism, the issues of measures and orderings. Here we explicitly evaluate the contributions of measures. Following the procedure of David-Distler-Kawai (DDK)$^{[6]}$ we determine the measures of metrics in conformal gauge. From the gauge fixed theory the physical state conditions are derived. Then the new features which are not seen in ADM formalism appear.

2. Quantum dilaton gravity

The theory of 1+1 dimensional dilaton gravity is defined by the following action.$^{\dagger}$

\[ I_{EH} = \frac{1}{16\pi} \int d^4x \sqrt{-g} R^{(4)} = \frac{1}{4} \int d^2x \sqrt{-\bar{g}} (R\varphi^2 + 2g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 2). \]

The resemblance to the dilaton gravity is manifest. Thus the image of $\varphi = r$ is very convenient when we consider the dynamics of the dilaton gravity.

$^{\dagger}$ Compare with the spherically symmetric gravitational system in 3+1 dimensions. If the metric is restricted as $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + \varphi^2 d\Omega^2$, where $\alpha, \beta = 0, 1$ and $d\Omega^2$ is the volume element of a unit 2-sphere, the Einstein-Hilbert action becomes

\[ I_{EH} = \frac{1}{16\pi} \int d^4x \sqrt{-g} R^{(4)} = \frac{1}{4} \int d^2x \sqrt{-\bar{g}} (R\varphi^2 + 2g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 2). \]
\[ I(g, \varphi, f) = I_D(g, \varphi) + I_M(g, f) , \]
\[ I_D(g, \varphi) = \frac{1}{2\pi} \int d^2x \sqrt{-g}(R\varphi^2 + 4g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 4\lambda^2 \varphi^2) , \]
\[ I_M(g, f) = -\frac{1}{4\pi} \sum_{j=1}^N \int d^2x \sqrt{-g} g^{\alpha\beta} \partial_\alpha f_j \partial_\beta f_j , \]

(1)

where \( \varphi = e^{-\phi} \) is the dilaton field and \( f_j \)'s are \( N \) matter fields. \( \lambda^2 \) is the cosmological constant. \( R \) is the curvature of the metrics \( g \). The classical equations of motion can be solved exactly and one obtains, for instance, the black hole geometry

\[ \varphi^2 = e^{-2\rho} = \frac{M}{\lambda} - \lambda^2 x^+ x^- , \quad f_j = 0, \]

(2)

where \( g_{\alpha\beta} = e^{2\rho} \eta_{\alpha\beta} \), \( \eta_{\alpha\beta} = (-1, 1) \) and \( x^\pm = x^0 \pm x^1 \). \( M \) is the mass of the black hole. More interesting geometry is the gravitational collapse. It is given by

\[ \varphi^2 = e^{-2\rho} = -\frac{M}{\lambda x_0^+} (x^+ - x_0^+) \theta(x^+ - x_0^+) - \lambda^2 x^+ x^- , \]

(3)

where the infalling matter flux is given by the shock wave along the line \( x^+ = x_0^+ \)

\[ \frac{1}{2} \sum_{j=1}^N \partial_+ f_j \partial_+ f_j = \frac{M}{\lambda x_0^+} \delta(x^+ - x_0^+) . \]

(4)

The quantum theory of the dilaton gravity is defined by

\[ Z = \int \frac{Dg(g)Dg(\varphi)Dg(f)}{Vol(Diff.)} e^{iI(g, \varphi, f)} , \]

(5)

where \( Vol(Diff.) \) is the gauge volume. The functional measures are defined from
the following norms

\[ < \delta g, \delta g >_g = \int d^2 x \sqrt{-g} g^{\alpha \gamma} g^{\beta \delta} \delta g_{\alpha \beta} \delta g_{\gamma \delta}, \]

\[ < \delta \varphi, \delta \varphi >_g = \int d^2 x \sqrt{-g} \delta \varphi \delta \varphi, \]

\[ < \delta f_j, \delta f_j >_g = \int d^2 x \sqrt{-g} \delta f_j \delta f_j \quad (j = 1, \cdots N). \]

Let us first discuss the measure of the metrics. We decompose the metrics into a conformal factor \( \rho \) and a background metric \( \hat{g} \) as \( g = e^{2\rho} \hat{g} \). This is the conformal gauge-fixing condition adopted here. The change in the metric is given by the change in the conformal factor \( \delta \rho \) and the change under a diffeomorphism \( \delta \xi \) as

\[ \delta g_{\alpha \beta} = 2 \delta \rho g_{\alpha \beta} + \nabla_\alpha \delta \xi_\beta + \nabla_\beta \delta \xi_\alpha 
= 2 \delta \rho' g_{\alpha \beta} + (P_1 \delta \xi)_{\alpha \beta}, \]

where

\[ \delta \rho' = \delta \rho + \frac{1}{2} \nabla^\gamma \delta \xi_\gamma, \quad (P_1 \delta \xi)_{\alpha \beta} = \nabla_\alpha \delta \xi_\beta + \nabla_\beta \delta \xi_\alpha - g_{\alpha \beta} \nabla^\gamma \delta \xi_\gamma. \]

The variations \( \delta \rho' g_{\alpha \beta} \) and \( (P_1 \delta \xi)_{\alpha \beta} \) are orthogonal in the functional space defined by the norms (6). Therefore the measure over metrics can be decomposed as

\[ D_g(g) = D_g(\rho') D_g(P_1 \xi) \]

\[ = D_g(\rho) D_g(\xi_\alpha) \det_g P_1. \]

The functional integration over \( \xi_\alpha \) cancels out the gauge volume. The Jacobian \( \det_g P_1 \) can be represented by the functional integral over the ghosts \( b, c \). Thus the

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* The definitions of measures in ref. 5 are quite different from ours. Their definitions are mysterious for me, especially the origin of the \( b - c \) ghosts. Thus our quantum theory appears to be quite different from theirs.
The partition function (5) becomes

\[ Z = \int D_g(\rho) D_g(\varphi) D_g(f) D_g(b) D_g(c) \exp\left[ i I_D(g, \varphi) + i I_M(g, f) + i I_{gh}(g, b, c) \right] , \]

(10)

where \( I_{gh} \) is the well-known ghost action (see for example ref.7). The measure \( D_g(\rho) \) is defined from the norm (6) by

\[ < \delta \rho, \delta \rho >_g = \int d^2 x \sqrt{-g} (\delta \rho)^2 = \int d^2 x \sqrt{-\hat{g}} e^{2\rho}(\delta \rho)^2 . \]

(11)

This is not the end of the story. The expression (10) has serious problems. The measure (11) is not invariant under the local shift \( \rho \to \rho + h \) and also the measures of the fields \( \varphi, f, b \) and \( c \) explicitly depend on the dynamical variable \( g = e^{2\rho} \hat{g} \). This is quite inconvenient because we must pick up contributions from the measures when the conformal factor \( \rho \) is integrated. So we will rewrite the measures on \( g \) into more convenient ones defined on the background metric \( \hat{g} \).

We do not repeat the calculation in detail, which was discussed in ref.4. Here we mention the outline of the arguments and list some key relations. First we rewrite the measures of the dilaton, the matter and the ghost fields into the convenient ones. It is realized by using the well-known transformation property for the measures of the matter and the ghost fields (see for example ref.8)

\[ D_{e^{2\rho} \hat{g}}(f) D_{e^{2\rho} \hat{g}}(b) D_{e^{2\rho} \hat{g}}(c) = \exp \left[ i N \frac{1}{12\pi} S_L(\rho, \hat{g}) \right] D_{\hat{g}}(f) D_{\hat{g}}(b) D_{\hat{g}}(c) \]

(12)

and the relation for the measure of the dilaton field

\[ \int D_{e^{2\rho} \hat{g}}(\varphi) e^{i I_D(e^{2\rho} \hat{g}, \varphi)} = \exp \left[ i \frac{c_\varphi}{12\pi} S_L(\rho, \hat{g}) \right] \int D_{\hat{g}}(\varphi) e^{i I_D(e^{2\rho} \hat{g}, \varphi)} \]

(13)

with \( c_\varphi = -1/2 \), which was proved in ref.4. \( S_L(\rho, \hat{g}) \) is what is called the Liouville action defined by

\[ S_L(\rho, \hat{g}) = \frac{1}{2} \int d^2 x \sqrt{-\hat{g}} (\hat{g}^{\alpha\beta} \partial_\alpha \rho \partial_\beta \rho + \hat{R} \rho) . \]

(14)

(Note that the actions of the matter and the ghost fields are invariant under the Weyl rescalings or \( I_M(g, f) = I_M(\hat{g}, f) \) and \( I_{gh}(g, b, c) = I_{gh}(\hat{g}, b, c) \), but the action
of the dilaton part is not so. Pay attention to the \( \rho \)-dependence of each side of (13).) From eqs. (12) and (13) we get

\[
Z = \int D_{e^{2\rho}}(\rho) D_\phi(\phi) D_f(b) D_c(c) \exp \left[ i \frac{c_\phi + N - 26}{12\pi} S_L(\rho, \hat{g}) + i I_D(e^{2\rho} \hat{g}, \phi) + i I_M(\hat{g}, f) + i I_{gh}(\hat{g}, b, c) \right].
\]

(15)

Next we rewrite the measure of \( \rho \). According to the procedure of DDK, \(^6\) we assume the following relation

\[
D_{e^{2\rho}}(\rho) = D_\phi(\rho) \exp \left[ i \frac{A}{12\pi} S_L(\rho, \hat{g}) \right].
\]

(16)

Note that the measure \( D_\phi(\rho) \) is invariant under the local shift of \( \rho \). The parameter \( A \) is determined by the consistency. Since the original theory depends only on the metrics \( g = e^{2\rho} \hat{g} \), the theory should be invariant under the simultaneous shift

\[
\rho \to \rho - \sigma, \quad \hat{g} \to e^{2\sigma} \hat{g}.
\]

(17)

This requirement leads to \( A = 1 \). Finally we get the expression

\[
Z = \int D_\phi(\Phi) e^{i \hat{I}(\hat{g}, \Phi)},
\]

(18)

where \( \Phi \) denotes the fields \( \rho, \phi, f, b \) and \( c \). \( \hat{I} \) is the gauge-fixed action

\[
\hat{I} = \frac{1}{2\pi} \int d^2 x \sqrt{-\hat{g}} \left[ 4 \hat{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + 4 \hat{g}^{\alpha\beta} \phi \partial_\alpha \phi \partial_\beta \rho + \hat{R} \phi^2 + 4 \lambda^2 \phi^2 e^{2\rho} + \hat{\kappa} \left( \hat{g}^{\alpha\beta} \partial_\alpha \rho \partial_\beta \rho + \hat{R} \rho \right) - \frac{1}{2} \sum_{j=1}^N \hat{g}^{\alpha\beta} \partial_\alpha f_j \partial_\beta f_j \right] + I_{gh}(\hat{g}, b, c)
\]

(19)

with

\[
\kappa = \frac{1}{12} (1 + c_\phi + N - 26) = \frac{N - 51/2}{12}.
\]

(20)

Closing this section there are some remarks. We showed that the theory (which includes the measures) is invariant under the simultaneous shift (17). Furthermore
the measure $D_{\hat{g}}(\rho)$ is invariant under the local shift of $\rho$. So the theory is invariant under conformal changes of the background metric $\hat{g}$: $\hat{g} \rightarrow e^{2\sigma} \hat{g}$. More explicitly the Liouville-dilaton part is transformed as

$$\int D_{e^{2\sigma} \hat{g}}(\rho) D_{e^{2\sigma} \hat{g}}(\varphi) \exp \left[ i \frac{\kappa}{\pi} S_L(\rho, e^{2\sigma} \hat{g}) + i I_D(e^{2\rho} e^{2\sigma} \hat{g}, \varphi) \right]$$

$$= \int D_{e^{2\sigma} \hat{g}}(\rho) D_{e^{2\sigma} \hat{g}}(\varphi) \exp \left[ i \frac{\kappa}{\pi} S_L(\rho - \sigma, e^{2\sigma} \hat{g}) + i I_D(e^{2\rho} \hat{g}, \varphi) \right]$$

$$= \exp \left[ -i \frac{N-26}{12\pi} S_L(\sigma, \hat{g}) \right] \int D_{\hat{g}}(\rho) D_{\hat{g}}(\varphi) \exp \left[ i \frac{\kappa}{\pi} S_L(\rho - \sigma, e^{2\sigma} \hat{g}) + i I_D(e^{2\rho} \hat{g}, \varphi) \right]$$

where in the last equality we use the relation for the Liouville action

$$S_L(\rho - \sigma, e^{2\sigma} \hat{g}) = S_L(\rho, \hat{g}) - S_L(\sigma, \hat{g}) .$$

(22)

The extra Liouville action $-i \frac{N-26}{12\pi} S_L(\sigma, \hat{g})$ cancels out with that induced from the measures of the matter and ghost fields (see eq.(12)) so that the partition function is invariant under the conformal change of $\hat{g}$. The exact proof is given in ref.4. This invariance is quite reasonable because the background metric $\hat{g}$ is very artificial. The theory should be independent of how to choose the background metric.

Here there is a question whether the theory (18) is regarded as a kind of conformal field theory (CFT) on $\hat{g}$ or not. Usual definition of CFT is that the action is invariant under the conformal transformation. According to this definition the Liouville theory is not CFT. However, as shown in ref.9, the energy-momentum tensors of the quantum Liouville theory satisfy the Virasoro algebra. So it is considered as a kind of CFT. In the theory (18), if we ignored the coupling between the Liouville field $\rho$ and the dilaton field $\varphi$, the Liouville part would be regarded as CFT with the central charge $c_\rho = 1 - 12\kappa$. In general CFT is described by a set of decoupled fields, while the theory (18) has the non-trivial coupling so that
it is quite different from usual CFT.†

The second remark is that the partition function is a scalar. This is manifest in the definition (5). After rewriting the partition function into the expression (18), however, this invariance is hidden. It is instructive to show that the partition function is really scalar. The Liouville field \( \rho \) is transformed as

\[
\rho'(x') = \rho(x) - \gamma(x), \quad \gamma(x) = \frac{1}{2} \log \left| \frac{\partial x'}{\partial x} \right|^2,
\]

where we only consider the conformal coordinate transformation \( x^\pm = x^\pm'(x^\pm) \) to preserve the conformal gauge and use the notation \( |x|^2 = x^+ x^- \). On the other hand the background metric is not transformed: \( \hat{g}'(x') = \hat{g}(x) \). It is natural because the background metric is not dynamical. Therefore the gauge-fixed action is transformed as \( \hat{I}' = \hat{I} - \frac{2 \pi}{\hbar} S_L(\gamma, \hat{g}) \), where note that \( \hat{R} \) is a scalar, but \( \hat{R}' \) is transformed as \( \hat{R}' = |\frac{\partial x}{\partial x'}|^2 (\hat{R} + 2 \hat{\Delta} \gamma) \). The measures defined on \( \hat{g} \) are also non-invariant under the coordinate transformation. The extra Liouville term \( S_L(\gamma, \hat{g}) \) cancels out with that coming from the measures so that the partition function is invariant.*

3. Physical state conditions

Now we carry out the canonical quantization of the gauge-fixed 1+1 dimensional dilaton gravity. As mentioned in Sect.2 the theory should be independent of how to choose the background metric \( \hat{g} \). Thus the variation of the partition function with respect to \( \hat{g} \) vanishes

\[
0 = \frac{\delta Z}{\delta \hat{g}_{\alpha\beta}} = \int D\hat{g}(\Phi) \frac{\delta \hat{I}}{\delta \hat{g}_{\alpha\beta}} e^{i\hat{I}(\hat{g}, \Phi)} + \int \frac{\delta D\hat{g}(\Phi)}{\delta \hat{g}_{\alpha\beta}} e^{i\hat{I}(\hat{g}, \Phi)}.
\]

The first term of r.h.s. is nothing but \( < \frac{\delta \hat{I}}{\delta \hat{g}_{\alpha\beta}} > \hat{g} \). The second term picks up an

† Furthermore note that CFT generally indicates a theory which is conformally invariant at the classical level, but not at the quantum level by anomalies. On the other hand, in the case of the dilaton gravity, it is meaningless to discuss the invariance under the conformal change of \( \hat{g} \) at the classical level. It is significant only in the quantum gravity.

* Note that after all the invariance under the conformal change of \( \hat{g} \) is in other words the invariance under the coordinate transformation.
anomalous contribution. But if we choose the Minkowski background \( \hat{g} = \eta \), this contribution vanishes. So it is convenient to choose the Minkowski background metric. Thus the physical state conditions are

\[
\left\langle \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha \beta}} \right\rangle_{\hat{g} = \eta} = 0
\]

(25)

or

\[
< \hat{T}_{00} >_{\hat{g} = \eta} = < \hat{T}_{01} >_{\hat{g} = \eta} = 0
\]

(26)

where the energy-momentum tensor \( \hat{T}_{\alpha \beta} \) is defined by \( \hat{T}_{\alpha \beta} = -\frac{2}{\sqrt{-\hat{g}}} \delta \hat{I} / \delta \hat{g}^{\alpha \beta} \mid_{\hat{g} = \eta} \). The condition for \( \hat{T}_{11} \) reduces to the one for \( \hat{T}_{00} \) by using the \( \rho \)-equation of motion. Furthermore we restrict the physical state to the one which satisfies the condition \( < \hat{T}_{\alpha \beta}^{gh} >_{\hat{g} = \eta} = 0 \) because the ghost flux should vanish in the flat space time.

Since the functional measures are defined on the Minkowski background metric, we can set up the canonical commutation relations in usual way. The conjugate momentums for \( \varphi, \rho \) and \( f_j \) are given by

\[
\Pi_{\varphi} = -\frac{4}{\pi} \dot{\varphi} - \frac{2}{\pi} \varphi \dot{\rho}, \\
\Pi_{\rho} = -\frac{\kappa}{\pi} \dot{\rho} - \frac{2}{\pi} \varphi \dot{\varphi}, \\
\Pi_{f_j} = \frac{1}{2\pi} \dot{f}_j,
\]

(27)

where the dot stands for the derivative with respect to the time coordinate. Then the physical state conditions (26) can be expressed as

\[
\left[ \frac{\pi/2}{\varphi^2 - \kappa} \left( \Pi_\rho^2 - \varphi \Pi_{\varphi} \Pi_{\rho} + \frac{\kappa}{4} \Pi_\varphi^2 \right) + \frac{2}{\pi} (\varphi \varphi'' - \varphi \varphi' \rho' - \lambda^2 \varphi^2 \rho'^2) \\
- \frac{\kappa}{2\pi} (\rho'^2 - 2\rho'') + \sum_{j=1}^{N} \left( \pi \Pi_{f_j}^2 + \frac{1}{4\pi} f_j^2 \right) \right] \Psi = 0
\]

(28)

and

\[
(\varphi' \Pi_\varphi + \rho' \Pi_{\rho} - \Pi_\rho' + \sum_{j=1}^{N} \Pi_{f_j} f_j') \Psi = 0
\]

(29)
where $\kappa$ is defined by eq.(20). $\Psi$ is a physical state. The prime stands for the derivative with respect to the space coordinate.

Here we have two remarks. The first is that the fields $\rho$ and $\varphi$ are dynamical variables so that it is significant to consider the equations of motion of $\rho$ and $\varphi$. But $\hat{g}$ is not dynamical. So we should not regard the physical state conditions as the equations of motion of $\hat{g}$. The conditions come from the symmetry of the theory. In this point of view the conditions (28-29) indeed correspond to the “constraints”.

The second remark is that the energy-momentum tensor $\hat{T}_{\alpha\beta}$ is transformed as non-tensor because the Liouville field $\rho$ is transformed as (23) for the conformal coordinate transformation. In the light-cone coordinate we get

$$
\hat{T}_{\pm\pm}'(x') = \left( \frac{\partial x^\pm}{\partial x'^{\pm}} \right)^2 \left( \hat{T}_{\pm\pm}(x) + \frac{\kappa}{\pi} t_\pm(x) \right),
$$

$$
\hat{T}_{+-}'(x') = \left| \frac{\partial x}{\partial x'} \right|^2 \hat{T}_{+-}(x),
$$

where $t_\pm(x)$ is the Schwarzian derivative

$$
t_\pm(x) = \frac{\partial \gamma(x)}{\partial x^\pm} \frac{\partial \gamma(x)}{\partial x^\pm} - \frac{\partial^2 \gamma(x)}{\partial x^{\pm2}}, \quad \gamma(x) = \frac{1}{2} \log \left| \frac{\partial x'}{\partial x} \right|^2.
$$

Therefore the physical state conditions (28-29) correspond to the case of $t_\pm = 0$. To determine what coordinate system corresponds to this case is a physical requirement. It is natural that the coordinate system which is asymptotically Minkowskian is considered as the coordinate system with $t_\pm = 0$.

If we rewrite the canonical momentums as the differential operators

$$
\Pi_\rho = \frac{\delta}{i\delta \rho}, \quad \Pi_\varphi = \frac{\delta}{i\delta \varphi}, \quad \Pi_f = \frac{\delta}{i\delta f_j},
$$

the eqs.(28) and (29) give the differential equations similar to the Wheeler-DeWitt equations\(^\dagger\). The most important difference between the usual Wheeler-DeWitt

\(^\dagger\) See for example ref.10, in which the spherically symmetric gravitational system of 3+1 dimensions is discussed. Application to the 1+1 dimensional dilaton gravity is straightforward.
equations and ours is just the measures of the fields. In our case the commutation relations are explicitly defined on the Minkowski background, but in ADM formalism they are implicitly defined on the curved metric. Therefore at first sight the conditions (28-29) seem to coincide with the usual Wheeler-DeWitt equations at $\kappa = 0$, but this is wrong.

If $\kappa > 0$, there is a singularity at finite $\varphi^2 = \kappa$. The region $\varphi^2 > \kappa$ is the classically allowed region *, whereas the region $\kappa > \varphi^2 > 0$ is called the Liouville region, where the sign of the kinetic term of eq.(28) changes. This is the classically forbidden region. The existence of the Liouville region is mysterious. There may be some possibility of gravitational tunneling through this region. If $\kappa < 0$, the situation drastically changes. In this case the singularity disappears.

4. Black hole dynamics

Until now the arguments are completely non-perturbative. If we can solve the physical state conditions exactly, the solution should include the complete dynamics of black hole. Unfortunately it is a very difficult problem so that we take an approximation. The original action (1) is order of $1/\hbar$, but the Liouville part of $\hat{I}$ is zeroth order of $\hbar$. However, if $|\kappa|$ is large enough, then it is meaningful to consider the “classical” dynamics of $\hat{I}$. This is nothing but the semi-classical approximation, which is valid only in the case of $M \gg 1$ and $N \gg 1$. In the other cases the quantum effect of gravitation becomes important. The classical dynamics of $\hat{I}$ is ruled by the equations $\hat{T}_{\alpha\beta} = 0$ and the dilaton equation of motion

* Here $\hat{I}$ is considered as a classical action
\[-2\partial_+\varphi\partial_+\varphi + 2\varphi\partial_+^2\varphi - 4\varphi\partial_+\varphi\partial_+\rho + \frac{1}{2}\sum_{j=1}^{N}\partial_+f_j\partial_+f_j \]

\[-\kappa(\partial_+\rho\partial_+\rho - \partial_+^2\rho + t_+) = 0 , \]

\[-2\partial_-\varphi\partial_-\varphi + 2\varphi\partial_-^2\varphi - 4\varphi\partial_-\varphi\partial_-\rho + \frac{1}{2}\sum_{j=1}^{N}\partial_-f_j\partial_-f_j \]

\[-\kappa(\partial_-\rho\partial_-\rho - \partial_-^2\rho + t_-) = 0 , \]

\[-2\partial_+\varphi\partial_-\varphi - 2\varphi\partial_+\partial_-\varphi - \lambda^2\varphi^2e^{2\rho} = 0 \] (33)

and

\[4\partial_+\partial_-\varphi + 2\varphi\partial_+\partial_-\rho + \lambda^2\varphi^2e^{2\rho} = 0 . \] (34)

These are nothing but the CGHS equations\[^1\] with the coefficient \(\kappa\) instead of \(N/12\) before the Liouville part. Many authors have solved these equations for \(\kappa > 0\) and derived the dynamics of evaporating black hole.\[^2,3\] Initially the location of the horizon shifts to the out-side of the classical horizon defined by the solution (3) by quantum effects (almost matter’s effects). Then the black hole evaporates and approaches to the singularity\[^\dagger\] asymptotically. As far as the gauge-fixed action is treated classically, the horizon does not seem to cross the singularity. As mentioned before the quantum mechanical region \(\kappa > \varphi^2 > 0\) extends behind the singularity, where the quantum gravitational effects become important.

If \(N\) is small, the “non-anomalous” quantum corrections of gravity part maybe cannot neglect and the approximation becomes bad. Nevertheless we apply the approximation for \(\kappa < 0\) because we hope that some new insights are obtained from the solution. If \(\kappa < 0\), the singularity disappears. The location of the horizon initially shifts to the inside of the classical horizon. If the effective mass of the black hole is defined by \(M_{BH} = \lambda\varphi^2|_{\text{horizon}}\), this means that the initial mass of the black hole is less than the infalling matter flux \(M\). After the black hole is formed, the

\[^\dagger\] The location of the singularity given by solving the equations (33) and (34) coincides with that determined by the physical state conditions. Note that at the singularity the curvature is singular, but the metric is not so.
positive flux comes in through the horizon and the black hole mass increases. It seems that the horizon approaches to the classical horizon asymptotically.

The problem of the information loss seems to come out in the case of $\kappa > 0$. Then the black hole evaporates and the information seems to be lost. However in this case the Liouville region extends behind the singularity. So it appears that there is a possibility that the informations run away through this region by gravitational tunneling. On the other hand, if $\kappa \leq 0$, the Liouville region disappears. But the black hole seems to be stable. In this case the problem of the information loss appears not to exist.

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