Performance Analysis of Signal Detection using Quantized Received Signals of Linear Vector Channel

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Abstract

Performance analysis of optimal signal detection using quantized received signals of a linear vector channel, which is an extension of code-division multiple-access (CDMA) or multiple-input multiple-output (MIMO) channels, in the large system limit, is presented in this paper. Here the dimensions of channel input and output are both sent to infinity while their ratio remains fixed. An optimal detector is one that uses a true channel model, true distribution of input signals, and perfect knowledge about quantization. Applying replica method developed in statistical mechanics, we show that, in the case of a noiseless channel, the optimal detector has perfect detection ability under certain conditions, and that for a noisy channel its detection ability decreases monotonically as the quantization step size increases.

1 INTRODUCTION

In modern wireless communication systems, many processes are performed on digital signals converted by an analog-to-digital (A/D) converter. In code-division multiple-access (CDMA) system which is one of key technologies for third generation cellular phone system, spreading, de-spreading, and code acquisition are all performed on digital signals.

Although there have been many studies on performance analysis of digital communication systems, information-theoretic performance analyses usually assume that the received analog signals are available [1–3]. Previous studies [4, 5],

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considering the quantized effect of received signals, used non optimal signal detection schemes, such as linear filters.

In this paper, we evaluate the performance of optimal signal detection using the quantized received signals of a linear vector channel, which is an extension of CDMA or multiple-input multiple-output (MIMO) channels, in the large system limit, where the dimensions of channel input and output are both sent to infinity while their ratio remains fixed.

2 CHANNEL MODEL AND QUANTIZATION OF RECEIVED SIGNALS

We consider a $K$-input, $N$-output linear vector channel, defined as follows. Let $x_0 = (x_{01}, \ldots, x_{0K})^T \in \{+1, -1\}^K$ denote the channel input vector, and $y = (y_1, \ldots, y_N)^T$ denote the output vector, given a linear transform $Hx_0$ of the inputs, where $H$ is an $N \times K$ channel matrix. Assuming the additive white Gaussian (AWGN) channel (variance $\sigma_0^2$), the outputs are described as

$$y = Hx_0 + \sigma_0 \nu,$$

where the components of $\nu = (\nu_1, \ldots, \nu_N)^T$ follow the normal distribution $N(0, 1)$. To simplify the analysis, we assume a random channel matrix; the elements $\{H_{\mu k}\}$ are independent and identically distributed (i.i.d.) with mean zero and variance $1/N$.

In this study, we consider signal detection using received signals, which are quantized by an A/D converter at the receiver. We assume an A/D converter that quantizes received signal $y$ into integer $n$, which satisfies the condition $(n - 1/2)d < y < (n + 1/2)d$, where $d > 0$ is the quantization step size. The probability distribution of the quantized received signals $n = (n_1, \ldots, n_N)^T$, given the input $x_0$ and channel matrix $H$, is represented as

$$P_0(n|Hx_0) = \prod_{\mu=1}^{N} \rho_0(n_{\mu}|H_{\mu}^T x_0),$$

where $H_{\mu}^T$, which denotes the $\mu$th row of $H$ and $Q(x) = \int_{x}^{\infty} D_t dt$, $D_t = (2\pi)^{-1/2}e^{-t^2/2} dt$. Equation (3) represents the input($x_0$)-output($n$) characteristic of the quantization channel.

The signal detection of channel input $x_0$, given the quantized received signals $n$ and the channel matrix $H$, can be solved by an inference scheme based on
Bayesian inference. We define a prior as $P_0(x_0)$, and assume $P_0(x_0) = 1/2^K$. The
detector assumes the channel model of quantization channel to be $P(n|Hx) = \prod_{\mu=1}^N \rho(n_\mu|H_\mu x)$ and the prior distribution to be $P(x)$. The posterior distribution is represented as

$$P(x|n, H) = \frac{P(n|Hx)P(x)}{\sum_x P(n|Hx)P(x)}.$$  \hspace{1cm} (4)

Maximizer of the Posterior Marginals (MPM) estimation $\hat{x}_k = \arg\max_{x_k} \sum_{x_k \neq k} P(x|n, H)$ is the optimal inference scheme for minimizing the component-wise estimation error probability, when the assumed channel model and prior distribution are matched to the true ones. Hereafter, we call the detector using true channel model and prior distribution as an optimal detector.

3 PERFORMANCE ANALYSIS USING REPLICA METHOD

We evaluate the performance of the optimal detector using the quantized received signals in the large system limit, where $K, N \to \infty$ while the ratio $\beta = K/N$ is kept finite. This paper [6] studies the multiuser detection performance of a CDMA channel with an arbitrary memoryless channel and arbitrary distribution of channel inputs in the large system limit, using the replica method. Since the defined quantized channel is an example of memoryless channels, one can apply the analysis reported in [6] to the performance evaluation of the optimal detector using quantized received signals in the large system limit. Although there is no rigorous justification for the replica method, we assume the validity of the replica method, and related techniques throughout this paper.

In the large system limit, the estimation error probability of the optimal detector

$$P_b = E_{x_0, \nu, H} \left[ \frac{1}{K} \sum_{k=1}^K \frac{1 - \hat{x}_k x_{0k}}{2} \right],$$  \hspace{1cm} (5)

where $E_X[\cdots]$, which denotes the average with respect to $X$, can be evaluated as

$$P_b = Q(\sqrt{E}).$$  \hspace{1cm} (6)

The parameter $E$ is determined by solving the following equations for $\{m, E\}$,

$$m = \int \tanh \left( E + \sqrt{E} z \right) Dz, \hspace{1cm} (7)$$

$$E = \sum_{n=-\infty}^{\infty} \int \frac{p_0(n|\sqrt{\beta m t})^2}{p_0(n|\sqrt{\beta m t})} Dt, \hspace{1cm} (8)$$
where $\bar{\rho}_0(n|\sqrt{\beta m})$ is defined as

$$
\bar{\rho}_0 \left( n \big| \sqrt{\beta m} t \right) = Q \left[ \frac{(n - \frac{1}{2}) d - \sqrt{\beta m t}}{\sqrt{\sigma_0^2 + \beta(1 - m)}} \right] - Q \left[ \frac{(n + \frac{1}{2}) d - \sqrt{\beta m t}}{\sqrt{\sigma_0^2 + \beta(1 - m)}} \right],
$$

and $\bar{\rho}_0'(n|\Delta) = \frac{\partial}{\partial \Delta} \bar{\rho}_0(n|\Delta)$.

For some values of the parameters ($\sigma_0$, $d$, $\beta$), equations (7) and (8) have three solutions, and these solutions can be distinguished by the value of estimation error probabilities $P_b$; we define good, intermediate, and bad solutions from the smallest to the largest value of $P_b$. Furthermore, we define the correct solution as one minimizes the function $F$, which is defined as

$$
F = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \bar{\rho}_0 \left( n \big| \sqrt{\beta m} t \right) \log \bar{\rho}_0 \left( n \big| \sqrt{\beta m} t \right) Dt \\
- \frac{1}{2} EM + \int \log \left[ 2 \cosh \left( E + \sqrt{EZ} \right) \right] Dz.
$$

It is known that the correct solution corresponds to the performance of the optimal detector. Since the exact evaluation of marginal posterior probability is usually computationally infeasible, we have to resort to approximation techniques. It is also known that a bad solution corresponds to the performance of the belief propagation based on iterative algorithm [7], which produces approximate marginal posterior probabilities in low time complexity.

4 RESULTS

4.1 NOISELESS CHANNEL

For a noiseless channel ($\sigma_0 = 0$), the equations (7) and (8) have the perfect detection solution $m = 1$, $E = \infty$, which provides $P_b = 0$ for any values of $d$ and $\beta$. Figure 1 shows the bifurcation diagram. The optimal detector can achieve the perfect detection in the lower left side of the solid curve, and the coexistence of the solutions occurs above the broken curve. Figure 2 shows the estimation error probabilities of the optimal detector.

4.2 NOISY CHANNEL

For a noisy channel ($\sigma_0 > 0$), the optimal detector cannot exhibit perfect detection, due to the effect of quantization noise. Figure 3 shows estimation error probabilities versus $E_b/N_0 = 1/(2\sigma_0^2)$. The detection ability of the optimal detector decreases monotonically as the quantization step size increases.
Figure 1: Bifurcation diagram for the optimal detector over a noiseless channel.

Figure 2: Estimation error probabilities for the optimal detector over a noiseless channel. Solid lines with arrows represent the upper bound of $d/\sqrt{\beta}$, for which the optimal detector can achieve the perfect detection. Broken lines with arrows represent the lower bounds of $d/\sqrt{\beta}$, for which coexistence of the solutions occurs.

Figure 3: Estimation error probabilities versus $E_b/N_0$. (a) $\beta = 1.0$, (b) $\beta = 1.8$. The solid curves represent $d/\sqrt{\beta} = 0.5$ (right), 0.25 (left), and the broken curve represents $d/\sqrt{\beta} = 0$. The arrows represent the values of $E_b/N_0$, below which the bad solution becomes the correct solution.
5 CONCLUSION

In this paper, we have evaluated the performance of an optimal detector using the quantized received signals of a linear vector channel in the large system limit. Applying the replica method, we have shown that, for a noiseless channel, under certain conditions, the optimal detector has a perfect detection ability, i.e., it is not affected by quantization noise. We have also shown that, for a noisy channel, the optimal detector’s detection ability decreases monotonically as the quantization step size increases.

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