On the turbulent energy cascade in anisotropic magnetohydrodynamic turbulence

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Abstract – The problem of the occurrence of an energy cascade for Alfvénic turbulence in solar wind plasmas was historically addressed by using phenomenological arguments based to the sweeping of Alfvénic fluctuations by the large-scale magnetic field and the anisotropy of the cascade in wave vectors space. Here, this paradox is reviewed through the formal derivation of a Yaglom relation from the anisotropic magnetohydrodynamic equation. The Yaglom relation involves a third-order moment calculated from velocity and magnetic fields and involving both Elsässer vector fields, and is particularly useful to be used as far as spacecraft observations of turbulence are concerned.

Introduction. – In a famous paper, Dobrowolny, Mangeney and Veltri [1] (hereafter DMV), raised the question of the existence of an energy cascade in magnetohydrodynamic (MHD) turbulence because of the apparent contradiction between two competing observations within the solar wind turbulence by in situ satellite measurements. In fact, since the oldest space flights, both a well-defined turbulent spectrum, and strong correlations between velocity and magnetic-field fluctuations have been observed [2,3] (for a modern review of turbulence in the solar wind cf. ref. [4]). The apparent contradiction between these observations can be immediately seen by introducing the Elsässer variables

\[ Z^\pm(x,t) = v^\pm \frac{B}{\sqrt{4\pi\rho}}, \] (1)

where \( v \) and \( B \) represent the velocity and magnetic field, respectively, while \( \rho \) is the mass density. These quantities represent Alfvénic fluctuations propagating along the background magnetic field \( B^{(0)} \) in opposite directions. MHD equations can be immediately written in terms of these variables as

\[ \partial_t Z^\pm + (B^{(0)}\partial_\alpha)Z^\pm + (Z^\mp_\alpha \partial_\alpha)Z^\mp = -\partial_i\pi + \lambda^\pm \partial^2_{\alpha\alpha}Z^\pm, \] (2)

where \( \pi = P/\rho \) (\( P \) being the the total pressure), \( \partial_t \) represents the time derivative, while \( \partial_i \) represents the derivative with respect to the spatial variable \( x_i \). The kinematic viscosity \( \nu \) and the magnetic diffusivity \( \mu \) form the dissipative coefficients \( \lambda^\pm = (\nu \pm \mu)/2 \). The third term in eqs. (2) shows that nonlinear interactions only occur between opposite sign fluctuations. Since high correlations between velocity and magnetic fluctuations imply either \( Z^+ = 0 \) or \( Z^- = 0 \), a turbulent energy cascade should be incompatible with the disappearance of one of the Alfvénic fluctuations. The puzzle has been apparently solved by DMV [1]. In the presence of a strong background magnetic field, turbulence becomes anisotropic because a privileged direction can be defined in space, as evidenced both in the solar wind [5] and laboratory plasmas [6]. As a consequence Alfvénic fluctuations are swept away by the large-scale magnetic field (the so-called Alfvén effect) and nonlinear interactions are slowed down by this transport of fluctuations. The usual Kolmogorov phenomenology must
then be modified in favor of the Iroshnikov-Kraichnan (IK) This yields to the fact that the energy transfer rates per unit mass for both pseudo-energies associated to Alfvénic fluctuations must be of the same order, \( \epsilon^+ \sim \epsilon^- \). More precisely, they must have the same scaling laws in the IK phenomenology [7]. Thus, an initial small unbalance between Alfvénic fluctuations is maintained during the cascade, eventually leading to both a turbulent spectrum, and high correlations [1]. This framework is referred to in the literature as Alfvénic turbulence. The above arguments have been criticized [8,9] on the basis of the fact that, at variance with the conjecture in ref. [1], both in closures equations [8] and in direct numerical simulations [9] the energy transfer rates are never the same. A different phenomenological argument has also been introduced [10]. In a strongly anisotropic medium as the solar wind, the energy cascade will eventually develop in the direction transverse to the background magnetic field. Using the hypothesis of a critical balance between the time of transport in the parallel direction and the eddy turnover time for turbulence in the transverse direction, a Kolmogorov spectrum, rather than a Kraichnan spectrum, is expected for transverse wavevectors. In homogeneous and isotropic fluid turbulence, the energy cascade is evidenced by the observation of a well-defined relation between the third-order longitudinal structure function and the energy dissipation transfer rate, namely the well-known 4/5-Kolmogorov law [11]. A similar relation have been derived for MHD, following the Yaglom law for passive scalars, in the framework of homogeneous and isotropic turbulence [12,13]. This law has been recently observed in samples of ecliptic [14] and polar [15] solar wind, thus showing unambiguously that a turbulent cascade is active and can help to solve the problem of solar wind heating [16,17]. These observations solve the apparent paradox raised in the DMV paper, confirming the presence of both strong correlations and turbulent cascade. However, the anisotropic nature of solar wind turbulence occasionally violates the conditions for the observation of the Yaglom law, namely of the cascade. In this letter we analyze the relevance of anisotropy conjecture in deriving the Yaglom scaling law for MHD turbulence.

The Yaglom law for anisotropic and isotropic MHD. – First of all it is worthwhile to remark that, for high enough magnetic Reynolds numbers, the fluctuating magnetic field might be larger than the background magnetic field. Even in this case turbulence is anisotropic because the large-scale magnetic field, at variance to a bulk velocity field, cannot be eliminated by a Galilean transformation. As a consequence it is formally not essential to separate \( B^{(0)}_y \) from \( Z^\pm \) in eqs. (2), so that in the following we formally disregard the second term on the l.h.s. of eqs. (2). Let us consider the anisotropic MHD equations (2) written twice for Elsässer variable \( Z^\pm(x_i) \) at the point \( x_i \), and for \( Z^\pm(x_i + r_i) \) at the independent point \( x'_i = x_i + r_i \). By subtraction, we obtain an equation for the differences \( \Delta Z^\pm_{ij} = (Z^\pm_{ij} - Z^\pm_{ij} \) (here and in the following “primed” variables are intended as calculated on the point \( x'_i \)). Using the hypothesis of independence of points \( x'_i \) and \( x_i \), with respect to derivatives, namely \( \partial_i (Z^\pm_{ij} \) = \( \partial_i' Z^\pm_{ij} \) = 0 (where \( \partial'_i \) represents the derivative with respect to \( x'_i \)), we get

\[
\begin{align*}
\partial_i \Delta Z^\pm_{ij} + Z'_i \partial'_i \Delta Z^\pm_{ij} &= -(\partial'_i + \partial_i)(\Delta P) \\
&\quad + (\partial'_a + \partial_a) \left[ \lambda^\pm \Delta Z^\pm_i + \lambda^\mp \Delta Z^\mp_i \right],
\end{align*}
\] (3)

\((\Delta P = n' - \pi)\). By adding and substituting the term \( \Delta Z^\pm_{ij} \) to (3) we obtain

\[
\begin{align*}
\partial_i \Delta Z^\pm_{ij} + Z'_i \partial'_i \Delta Z^\pm_{ij} + Z'_a \Delta Z^\pm_{ij} (\partial'_a + \partial_a) \Delta Z^\pm_{ij} &= \\
&\quad -(\partial'_i + \partial_i) \Delta P + (\partial'_a + \partial_a) \left[ \lambda^\pm \Delta Z^\pm_i + \lambda^\mp \Delta Z^\mp_i \right].
\end{align*}
\] (4)

We are seeking for an equation for the second-order correlation tensor \( \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle = \left[ \lambda^\pm \Delta Z^\pm_i + \lambda^\mp \Delta Z^\mp_i \right]\). In fact, in a more general approach one should look at a mixed tensor, namely \( \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \), taking into account not only both pseudo-energies but also cross-correlation \( \langle Z^+ \Delta Z^- \rangle \) and \( \langle Z^- \Delta Z^+ \rangle \). However, using the DIA closure by Kraichnan, it is possible to show that these elements are in general poorly correlated [18]. Since we are interested in the energy cascade, we limit ourself to the most interesting equation that describes correlations about Alfvénic fluctuations of the same sign. To obtain the equations for pseudo-energies we multiply eqs. (4) by \( \Delta Z^\pm_{jk} \), then by averaging we get

\[
\begin{align*}
\partial \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle + \langle \Delta Z^\pm_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \rangle = \\
&\quad + \langle Z'_a \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \rangle = \\
&\quad - \langle \Delta Z^\pm_{ij} \partial'_a \langle \Delta P \rangle + \Delta Z^\pm_{ij} \partial'_a \langle \Delta P \rangle \rangle \\
&\quad + \lambda^\pm \langle \Delta Z^\pm_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \rangle + \lambda^\mp \langle \Delta Z^\mp_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\mp_{jk} \rangle \rangle \\
&\quad + \lambda^\pm \langle \Delta Z^\pm_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \rangle + \lambda^\mp \langle \Delta Z^\mp_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\mp_{jk} \rangle \rangle.
\end{align*}
\] (5)

If we consider local homogeneity we have

\[
\begin{align*}
\partial'_a &= \frac{\partial}{\partial(x'_a + r_a)} \sim \frac{\partial}{\partial r_a}, \\
\partial_a &= \frac{\partial}{\partial(x_i + r_a)} \sim - \frac{\partial}{\partial r_a},
\end{align*}
\]
when applied to difference quantities, so that the nonlinear term, using incompressibility, becomes

\[
\begin{align*}
\langle \Delta Z^\pm_{ij} \partial'_a \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle \rangle &= \partial \langle \Delta Z^\pm_{ij} \Delta Z^\pm_{ij} \Delta Z^\pm_{jk} \rangle.
\end{align*}
\] (6)

Note that in eqs. (2) kinematic viscosity is not assumed equal to magnetic diffusivity, and this generates a coupling
between $Z_i^\pm$ and $Z_{ij}^\pm$ not only in the nonlinear term but also in the dissipative term. Kinematic viscosity and magnetic diffusivity, being due to different physical effects, can be very different in real situations, and there could be different dissipation rates which could generate different spectral laws in the inertial interval of turbulence. This is based on the fact that dissipation in MHD equations are usually described through a simple $\nabla^2$-term, as in usual fluid flows. This is not true in solar wind, where kinetic effects are at work to dissipate the energy at small scales, although a finite dissipation rate exists [14–17]. Here, to simplify the algebraic calculations, we come back to the usual assumption that kinematic viscosity is equal to magnetic diffusivity, $\lambda^+ = \lambda^- = \nu$. Then, by using the independence of derivatives with respect to both points and using the local homogeneity hypothesis, the dissipative term becomes

$$\nu(\partial^2_{\alpha\alpha} + \partial^2_{\beta\beta})(\Delta Z_i^\pm \Delta Z_j^\pm) = 2\nu \frac{\partial^2}{\partial r^2_a} (\Delta Z_i^\pm \Delta Z_j^\pm) - \frac{4}{3} \frac{\partial}{\partial r_a} (\epsilon^\pm_{ij} r_a), \quad (7)$$

where we defined the average dissipation tensor

$$\epsilon^\pm_{ij} = \nu((\partial_{ij} Z^\pm)(\partial_{ij} Z^\pm)). \quad (8)$$

Using the above equations in (5), we finally obtain the equation

$$\partial_t (\Delta Z_i^\pm \Delta Z_j^\pm) + \frac{\partial}{\partial r_a} (\Delta Z^\pm_{ij}(\Delta Z_i^\pm \Delta Z_j^\pm)) = -\Pi_{ij} - \Lambda_{ij} + 2\nu \frac{\partial^2}{\partial r^2_a} (\Delta Z_i^\pm \Delta Z_j^\pm) - \frac{4}{3} \frac{\partial}{\partial r_a} (\epsilon^\pm_{ij} r_a). \quad (9)$$

The first term on the r.h.s. of the last equation represents the tensor related to the pressure term $\Pi_{ij} = \langle \Delta Z_i^\pm (\partial_i^r + \partial_i) \Delta P + \Delta Z_j^\pm (\partial_j^r + \partial_j) \Delta P \rangle$. The second term $\Lambda_{ij} = \langle Z^2_{ij}(\partial^r_{ij} + \partial_{ij})(\Delta Z_i^\pm \Delta Z_j^\pm) \rangle$ represents a tensor related to both large-scale inhomogeneities and on the average effect of the sweeping of Alfvénic fluctuations by the large-scale magnetic field, namely the so-called Alfvén effect. Equation (9) is an exact equation for anisotropic MHD equations that links the second-order complete tensor to the third-order mixed tensor through the average dissipation rate tensor.

Using incompressibility and independence of derivatives with respect to both points $x_i$ and $x'_i$, the second term on the r.h.s. can be written as $\Lambda_{ij} = \langle \partial^r_{ij} + \partial_{ij} \rangle \langle Z^2_{ij}(\Delta Z_i^\pm \Delta Z_j^\pm) \rangle$, which vanishes for a globally homogeneous situation, because in this case $\partial_i \equiv 0$. However, when the background magnetic field is large, the sweeping effect is important and cannot be disregarded. In this case the term $\Lambda_{ij}$ cannot be eliminated. The pressure term is more complicated to be managed. Using independence of derivatives and local homogeneity we get

$$\langle \Delta Z_i^\pm \partial_j \Delta P \rangle = \langle \partial_j [\Delta Z_i^\pm \Delta P] \rangle - \langle (\partial_j z_i^\pm) \Delta P \rangle = -\langle (\partial_j [\Delta Z_i^\pm \Delta P]) \rangle - \langle \partial_j (z_i^\pm) \Delta P \rangle \quad (10)$$

from which

$$\Pi_{ij} = \langle \partial_j (z_i^\pm) \rangle \langle \partial_j \Delta P \rangle + \langle \partial_j (z_i^\pm) \partial_j \Delta P \rangle. \quad (11)$$

Then the diagonal terms of the tensor containing the pressure vanish. In fact summing over indices eq. (11) yields $[\partial_j (z_i^\pm) \partial_j \Delta P]$ which is zero for local homogeneity and incompressibility. This means that, assuming global homogeneity and incompressibility, and disregarding the Alfvén effect, the equation for the trace of tensor can be written as

$$\partial_t (\Delta Z_i^\pm |\Delta Z_i^\pm|^2) + \frac{\partial}{\partial r_a} (\Delta Z_i^\pm \Delta Z_i^\pm |\Delta Z_i^\pm|^2) = 2\nu \frac{\partial^2}{\partial r^2_a} (\Delta Z_i^\pm |\Delta Z_i^\pm|^2) - \frac{4}{3} \frac{\partial}{\partial r_a} (\epsilon_i^\pm r_a). \quad (12)$$

This expression is valid even in the anisotropic case, that is fields depend on the vector $r_a$. Moreover by considering only the trace, we ruled out the possibility to investigate anisotropies related to different orientations of vectors within the second-order moment. Note that only the diagonal elements of the dissipation rate tensor, namely $\epsilon_i^\pm$ are positive defined, while in general the off-diagonal elements $\epsilon^\pm_{ij}$ can be in principle also negative. For a stationary state eq. (12) can be written as the divergenceless condition of a quantity involving the third-order correlations and the dissipation rates

$$\frac{\partial}{\partial r_a} \left[ \langle |\Delta Z_i^\pm|^2 \rangle - 2\nu \frac{\partial}{\partial r_a} (\Delta Z_i^\pm |\Delta Z_i^\pm|^2) - \frac{4}{3} (\epsilon_i^\pm r_a) \right] = 0 \quad (13)$$

from which we can obtain the Yaglom relation by projecting eq. (13) along the longitudinal $r_a = \mathbf{r}_e$ direction. This operation involves the assumption that the flow is locally isotropic, that is fields depend locally only on the separation $r$, so that

$$\left( \frac{2}{r} + \frac{\partial}{\partial r} \right) \left[ \langle |\Delta Z_i^\pm|^2 \rangle - 2\nu \frac{\partial}{\partial r} (\Delta Z_i^\pm |\Delta Z_i^\pm|^2) + \frac{4}{3} \epsilon_i^\pm r \right] = 0 \quad (14)$$

The only solution that is compatible with the absence of singularity in the limit $r \to 0$ is

$$\langle |\Delta Z_i^\pm|^2 \rangle = 2\nu \frac{\partial}{\partial r} (\Delta Z_i^\pm |\Delta Z_i^\pm|^2) - \frac{4}{3} \epsilon_i^\pm r \quad (15)$$

which reduces to the Yaglom law for MHD turbulence as obtained by Politano and Pouquet [12,13] in the inertial range when $\nu \to 0$

$$\langle |\Delta Z_i^\pm|^2 \rangle = \frac{4}{3} \epsilon_i^\pm r. \quad (16)$$
Finally, in the fluid-like case where \( z_i^+ = z_i^- = \nu \), we obtain the usual Yaglom law \( \langle \Delta v_{ii} \Delta v_{ij} \rangle = -4/3 (\nu \epsilon) \) (\( \epsilon \) being the dissipation rate) which immediately reduces to the Kolmogorov law \( \langle \Delta v_{ij} \rangle^2 = -4/5 (\nu \epsilon) \) in the isotropic case where \( \langle \Delta v_{ii} \Delta v_{ij} \rangle = \langle \Delta v_{ii} \Delta v_{ij} \rangle = 1/3 \langle \Delta v_{ij} \rangle^2 \) (assuming the separation \( r \) along the streamwise direction \( x \)).

Even if eq. (16) remains formally valid only for incompressible MHD, this cannot completely solve the problem of the energy cascade in MHD. This tensor \( \Pi_{ij} \) is zero only when local anisotropy is assumed. In fact by calculating the divergence with respect to the index \( j \) of \( \Pi_{ij} \), assuming independence of derivatives, we get

\[
\partial_i \langle \Delta Z_i^\pm (\partial_j P - \delta_j^i P^j) \rangle = \langle \Delta Z_i^\pm \partial_i \partial_j P^j \rangle = \partial_j \langle \Delta Z_i^\pm \partial_j P^j \rangle = - \langle \delta_j^i \Delta Z_i^\pm \partial_j P^j \rangle = 0. \tag{17}
\]

By symmetry the divergence with respect to \( j \) also vanishes, that is \( \partial_j^i \Pi_{ij} = 0 \). By using for \( \Pi_{ij} \) the isotropic formula for a generic tensor [19]

\[
\Pi_{ij}(r) = [\Pi_{11}(r) - \Pi_{\alpha\alpha}(r)] \frac{r r_j}{r^3} + \Pi_{\alpha\alpha}(r) \delta_{ij}, \tag{18}
\]

it can be easily shown that \( \Pi_{ij} = 0 \) by local isotropy [19].

**Reduced MHD.** – As we have seen in an anisotropic situation, say when a strong background magnetic field is at work, the tensor \( \Lambda_{ij} \) cannot be disregarded, even in a globally homogeneous situation. The problem is that large-scale magnetic field cannot be eliminated by a Galilean transformation and \( \Lambda_{ij} \) describes a basic phenomenon in MHD, namely the Alfvén effect. This effect can be evidenced in presence of a strong background magnetic field. In this case, under the hypothesis of a strong guide magnetic field, we can use the so called Reduced MHD approximation (RMHD) [20,21]. The sweeping effect of the large-scale magnetic field on Alfvénic fluctuations is explicitly introduced in a simplified approximation in RMHD, namely the dynamics along the parallel and perpendicular directions (with respect to the large-scale average magnetic field \( B(0) \)) are disentangled. Then we can distinguish between derivatives for perpendicular (\( \partial / \partial x_\perp \)) and parallel (\( \partial / \partial x_\parallel \)) coordinates.

In terms of Elsässer variables, the RMHD approximation reads [22]

\[
\frac{\partial Z_i^\pm}{\partial t} + Z_i^\pm \frac{\partial Z_i^\pm}{\partial x_\perp} + B(0) \frac{\partial Z_i^\pm}{\partial x_\parallel} = - \frac{\partial \pi}{\partial x_\perp} + \lambda^\pm \frac{\partial^2 Z_i^\pm}{\partial x_\perp^2} + \lambda^\pm \frac{\partial^2 Z_i^\pm}{\partial x_\parallel^2}, \tag{19}
\]

where \( Z_i^\pm (x_\parallel, x_\perp) \) is the perpendicular component of the Elsässer variables. From this equations, by performing the same calculations as before, and by defining the separations along the perpendicular \( r_\perp \) and parallel \( r_\parallel \) directions, respectively, we obtain the following Kármán-Howarth relation for the stationary state:

\[
\langle \Delta Z_i^\pm \Delta Z_i^\pm \rangle^2 = 2 \nu \frac{\partial}{\partial r_\perp} \langle |\Delta Z_i^\pm|^2 \rangle - 2 \epsilon_\parallel^\pm r_\perp \\
\pm \frac{2}{r_\perp} \int_0^{r_\perp} r_\perp^2 B(0) \frac{\partial}{\partial r_\parallel} \langle |\Delta Z_i^\pm|^2 \rangle \; dr_\parallel. \tag{20}
\]

From this last equation it is evident that, in the limit of vanishing viscosity \( \nu \to 0 \), in general, a Yaglom relation cannot be derived. This can only be the case by assuming that the average pseudo-energies are almost constant along the parallel direction. So, if the third term on the r.h.s. is zero, we can derive a Yaglom relation between the mixed third-order correlation term as a linear function of the transverse scale \( r_\perp \)

\[
\langle \Delta Z_i^\pm \Delta Z_i^\pm \rangle^2 \sim -2 \epsilon_\parallel^\pm r_\perp. \tag{21}
\]

Even if this is only an approximate relation, it is worthwhile to note that, in this last case, by assuming the scaling \( \Delta Z_i^\pm \sim \Delta Z_i^\pm_2 \), relation (21) is compatible with the scaling law \( \Delta Z_i^\pm \sim r_\perp^{2/3} \), which immediately leads to the Kolmogorov spectrum \( E(k_\perp) \sim k_\perp^{5/3} \) predicted for anisotropic MHD turbulence [10].

**Conclusions.** – To conclude, we reviewed the derivation of a general Yaglom equation for MHD turbulence where two fields are coupled. When this equation is satisfied a turbulent cascade is at work. The most general equation (9) that relates the third-order mixed tensor to the dissipation rate tensor, is valid in the anisotropic and nonhomogeneous case. By using homogeneity, we derive eq. (13), that is valid in the presence of anisotropy, while to derive the usual Yaglom law (16) the local isotropy assumption is required. Moreover, the tensor containing the pressure has zero diagonal elements, but is completely zero only when local isotropy is assumed. As far as the local isotropy assumption is considered, while for usual fluid flows the return to isotropy at small scales is assured, even if they are anisotropic at large scales [23], in general the MHD flows cannot return completely to isotropy at small scales [24–26]. This in turn means that if we want to fully investigate the turbulent cascades in anisotropic MHD flows, the off-diagonal elements of the third-order mixed tensor cannot be disregarded. Of course the tensor term related to pressure cannot be eliminated, while if we cannot assume local isotropy the relation (13) cannot reduce to the usual Yaglom law obtained in refs. [12,13] and investigated experimentally in refs. [14,15]. Of course, since the tensor term \( \Pi_{ij} \) cannot be calculated using solar wind data, we cannot have any feeling of the relative importance of this term and the
term containing the dissipation rate tensor in anisotropic MHD turbulence. High-resolution numerical simulations for anisotropic MHD turbulence should be used as a first approach.

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