Gilbert damping in non-collinear magnetic systems

S. Mankovsky, S. Wimmer, H. Ebert
Department of Chemistry/Phys. Chemistry, LMU Munich, Butenandstrasse 11, D-81377 Munich, Germany

(Dated: May 30, 2018)

The modification of the magnetization dissipation or Gilbert damping caused by an inhomogeneous magnetic structure and expressed in terms of a wave vector dependent tensor $\alpha(q)$ is investigated by means of linear response theory. A corresponding expression for $\alpha(q)$ in terms of the electronic Green function has been developed giving in particular the leading contributions to the Gilbert damping linear and quadratic in $q$. Numerical results realistic systems are presented that have been obtained by implementing the scheme within the framework of the fully relativistic KKR (Korringa-Kohn-Rostoker) band structure method. Using the multilayered system (Cu/Fe$_{1-x}$Co$_x$/Pt)$_n$ as an example for systems without inversion symmetry we demonstrate the occurrence of non-vanishing linear contributions. For the alloy system bcc Fe$_{1-x}$Co$_x$ having inversion symmetry, on the other hand, only the quadratic contribution is non-zero. As it is shown, this quadratic contribution does not vanish even if the spin-orbit coupling is suppressed, i.e. it is a direct consequence of the non-collinear spin configuration.

PACS numbers: 71.15.-m, 71.55.Ak, 75.30.Ds

I. INTRODUCTION

The magnetization dissipation in magnetic materials is conventionally characterized by means of the Gilbert damping (GD) tensor $\alpha$ that enters the Landau-Lifshitz-Gilbert (LLG) equation \[ \dot{\vec{m}} = -\gamma \vec{m} \times \vec{H} - \alpha(q) \vec{q} \times \vec{m} + \alpha(q) \partial \vec{m} / \partial t. \] This positive-definite second-rank tensor depends in general on the local character of GD tensor in such systems has to be taken into account [2–4]. This implies that the dissipative torque on the magnetization should be represented by the expression of the following general form [12]:

$$ \tau_{GD} = \dot{\vec{m}}(\vec{r}, t) \times \int d^3\vec{r}' \alpha(q - \vec{r}') \frac{\partial}{\partial t} \dot{\vec{m}}(\vec{r}', t). \quad (1) $$

In the case of a magnetic texture varying slowly in space, however, an expansion of the damping parameter in terms of the magnetization density and its gradients [11] is nevertheless appropriate:

$$ \alpha_{ij} = \alpha_{ij} + \alpha_{ij}^{kl} m_k m_l + \alpha_{ij}^{klp} \frac{\partial}{\partial \vec{r}^l} m_p + ... \quad (2) $$

where the first term $\alpha_{ij}$ stands for the conventional isotropic GD and the second term $\alpha_{ij}^{kl} m_k m_l$ is associated with the magneto-crystalline anisotropy (MCA). The third so-called chiral term $\alpha_{ij}^{klp} \frac{\partial}{\partial \vec{r}^l} m_p$ is non-vanishing in non-centrosymmetric systems. The important role of this contribution to the damping was demonstrated experimentally when investigating the field-driven domain wall (DW) motion in asymmetric Pt/Co/Pt trilayers [13].

As an alternative to the expansion in Eq. (2) one can discuss the Fourier transform $\alpha(q)$ of the damping parameter characterizing inhomogeneous magnetic systems, which enter the spin dynamics equation

$$ \frac{\partial}{\partial t} \vec{m}(q) = -\gamma \vec{m}(q) \times \vec{H} - \vec{m}(q) \times \alpha(q) \partial \vec{m}(q) / \partial t. \quad (3) $$

In this formulation the term linear in $q$ is the first chiral term appearing in the expansion of $\alpha(q)$ in powers of $q$. Furthermore, it is important to note that it is directly connected to the $\alpha_{ij}^{klp} m_k \frac{\partial}{\partial \vec{r}^l} m_p$ term in Eq. (2).

By applying a gauge field theory, the origin of the non-collinear corrections to the GD can be ascribed to the emergent electromagnetic field created in the time-dependent magnetic texture [14, 15]. Such an emergent
electromagnetic field gives rise to a spin current whose divergence characterizes the change of the angular momentum in the system. This allows to discuss the impact of non-collinearity on the GD via a spin-pumping formulation \[ \vec{\alpha}(\vec{q}, \omega) \]. Some details of the physics behind this effect depend on the specific properties of the material considered. Accordingly, different models for magnetisation dissipation were discussed in the literature \[ 6, 12, 14, 17 \]. Non-centrosymmetric two-dimensional systems for which the Rashba-like spin-orbit coupling plays an important role have received special interest in this context. They have been discussed in particular by Akosa et al. \[ 19 \], in order to explain the origin of chiral GD in the presence of a chiral magnetic structure.

The fourth term on the r.h.s. of Eq. (2) corresponds to a quadratic term of an expansion of \( \vec{\alpha}(\vec{q}) \) with respect to \( q \). It was investigated for bulk systems with non-magnetic \[ 20 \] and magnetic impurity atoms, for which the authors have shown on the basis of model consideration that it can give a significant correction to the homogeneous GD in the case of weak metallic ferromagnets. In striking contrast to the uniform part of the GD this contribution does not require a non-vanishing spin-orbit interaction.

To our knowledge, only very few ab-initio investigations on the Gilbert damping in non-collinear magnetic systems along the lines sketched above have been reported so far in the literature. Yuan et al. \[ 21 \] calculated the in-plane and out-of-plane damping parameters in terms of the scattering matrix for permalloy in the presence of Néel and Bloch domain walls. Freimuth et al. \[ 22 \] discuss the properties of a \( q \)-dependent Gilbert damping \( \vec{\alpha}(\vec{q}) \) calculated for the one-dimensional Rashba model in the presence of the Néel-type non-collinear magnetic exchange field, demonstrating different GD for left-handed and right-handed DWs. Here we extend the formalism developed before to deal with the GD in ferromagnets \[ 6 \], to get access to non-collinear systems. The formalism based on linear response theory allows to expand the GD parameters with respect to a modulation of the magnetization expressed in terms of a wave vector \( \vec{q} \). Corresponding numerical results will be presented and discussed.

II. GILBERT DAMPING FOR NON-COLLINEAR MAGNETIZATION

In the following we focus on the intrinsic contribution to the Gilbert damping, excluding spin current induced magnetization dissipation which occurs in the presence of an external electric field. For the considerations on the magnetization dissipation an adiabatic variation of the magnetization is assumed. Moreover, it is assumed that the magnitude of the local magnetic moments is unchanged during a change of the magnetization, i.e. the exchange field should be strong enough to separate transverse and longitudinal parts of the magnetic susceptibility. With these restrictions, the non-local Gilbert damping can be determined in terms of the spin susceptibility tensor

\[
\chi_{\alpha\beta}(-\vec{q}, \omega) = i \frac{1}{V} \int_0^\infty dt \langle \hat{S}_\alpha(\vec{q}, t) \hat{S}_\beta(-\vec{q}, 0) \rangle e^{i(\omega-\delta)t} ,
\]

where \( \hat{S}_\alpha(\vec{q}, t) \) is the \( \vec{q} \)- and \( t \)-dependent spin operator and reduced units have been used \((h = 1)\). With this, the Fourier transformation of the real-space Gilbert damping can be represented by the expression \[ 23, 24 \]

\[
\alpha_{\alpha\beta}(\vec{q}) = \frac{\gamma}{M_0 V} \lim_{\omega \to 0} \frac{\partial \chi_{\alpha\beta}(-\vec{q}, \omega)}{\partial \omega} .
\]

Here \( \gamma = g \mu_B \) is the gyromagnetic ratio, \( M_0 = \mu_{tot} \mu_B / V \) is the equilibrium magnetization and \( V \) is the volume of the system. In order to avoid the calculation of the dynamical magnetic susceptibility tensor \( \chi(\vec{q}, \omega) \), which is the Fourier transformed of the real space susceptibility \( \chi(\vec{q}, \omega) \), it is convenient to represent \( \chi(\vec{q}, \omega) \) in Eq. (10), in terms of a correlation function of time derivatives of \( \hat{S} \). As \( \hat{S} \) corresponds to the torque \( \vec{T} \), that may include non-dissipative and dissipative parts, one may consider instead the torque-torque correlation function \( \chi(\vec{q}, \omega) \) \[ 24, 27 \].

Assuming the magnetization direction parallel to \( \hat{z} \) one obtains the expression for the Gilbert damping \( \vec{\alpha}(\vec{q}) \)

\[
\alpha(\vec{q}) = \frac{\gamma}{M_0 V} \lim_{\omega \to 0} \frac{\partial \chi(\vec{q}, \omega)}{\partial \omega} \cdot \vec{q} .
\]

Here \( \vec{\alpha} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \) is the transverse Levi-Civita tensor. This implies the following relationship of the \( \vec{\alpha} \) tensor elements with the elements of the torque-torque correlation tensor \( \vec{\alpha} \rightarrow -\pi_{yy} \) and \( \alpha_{xx} \sim -\pi_{xy} \) \[ 24 \].

Using Kubo’s linear response theory in the Matsubara representation and taking into account the translational symmetry of a solid the torque-torque correlation function \( \pi_{\alpha\beta}(\vec{q}, \omega) \) can be expressed by (see, e.g. \[ 25 \]):

\[
\pi_{\alpha\beta}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{pm} (\vec{T}^\alpha \vec{G}(\vec{k}+\vec{q}, i\omega_n+i\omega_p)) e c ,
\]

where \( \vec{G}(\vec{k}, ip_m) \) is the Matsubara Green function and \( \langle ... \rangle_c \) indicates a configurational average required in the presence of any disorder (chemical, structural or magnetic) in the system. Using a Lehman representation for the Green function \[ 25 \]

\[
G(\vec{k}, ip_m) = \int_{-\infty}^{+\infty} dE \frac{G^+(\vec{k}, E)}{\pi} \frac{1}{ip_m - E} ,
\]

with \( G^+(\vec{k}, E) \) the retarded Green function and using the relation

\[
\frac{1}{\beta} \sum_{pm} \frac{1}{ip_m + i\omega_n - E_1} \frac{1}{ip_m - E_2} = \frac{f(E_2) - f(E_1)}{i\omega_n + E_2 - E_1}
\]

with

\[
f(E_2) - f(E_1) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{dE}{E_1 - E} \frac{1}{E_2 - E} \frac{1}{ip_m + i\omega_n - E_1} \frac{1}{ip_m - E_2} .
\]
for the sum over the Matsubara poles in Eq. (7), the torque-torque correlation function is obtained as:

\[
\pi_{\alpha\beta}(\bar{q}, i\omega_n) = \frac{1}{\Omega BZ} \int d^3k \int_{-\infty}^{+\infty} \frac{dE_1}{\pi} \int_{-\infty}^{+\infty} \frac{dE_2}{\pi} \text{Tr} \left[ T^\alpha \Im G(\bar{k}, E_1) T^\beta \Im G(\bar{k}, E_2) \frac{f(E_2) - f(E_1)}{i\omega_n + E_2 - E_1} \right] .
\]  

(9)

Performing finally the analytical continuation \(i\omega_n \rightarrow \omega + i\delta\) one arrives at the expression

\[
\Gamma_{\alpha\beta}(\bar{q}, \omega) = -\frac{\pi}{\Omega BZ} \int d^3k \int_{-\infty}^{+\infty} \frac{dE_1}{\pi} \int_{-\infty}^{+\infty} \frac{dE_2}{\pi} \text{Tr} \left[ T^\alpha \Im G(\bar{k} + \bar{q}, E_1) T^\beta \Im G(\bar{k}, E_2) \right] (f(E_2) - f(E_1)) \delta(\omega + E_2 - E_1)
\]

\[
= -\frac{\pi}{\Omega BZ} \int d^3k \int_{-\infty}^{+\infty} \frac{dE}{\pi} \text{Tr} \left[ T^\alpha \Im G(\bar{k} + \bar{q}, E) T^\beta \Im G(\bar{k}, E + \omega) \right] (f(E) - f(E + \omega))
\]  

(10)

for the diagonal part of the correlation function with \(\Gamma_{\alpha\beta}(\bar{q}, \omega) = -\pi \Im \pi_{\alpha\beta}(\bar{q}, \omega)\). Accordingly one gets for the diagonal elements of Gilbert damping tensor the expression

\[
\alpha_{\alpha\alpha}(\bar{q}) = \frac{\gamma}{M_0 V} \lim_{\omega \rightarrow 0} \frac{\partial}{\partial \omega} \left[ \Gamma(\bar{q}, \omega) \cdot \bar{q} \right]_{\alpha\alpha}
\]

\[
= \frac{\gamma \pi}{M_0 V} \lim_{\omega \rightarrow 0} \frac{1}{\Omega BZ} \int d^3k \int_{-\infty}^{+\infty} \frac{dE}{\pi^2} (f(E + \omega) - f(E)) \text{Tr} \left[ T^\beta \Im G(\bar{k} + \bar{q}, E) T^\beta \Im G(\bar{k}, E + \omega) \right] \delta(E - E_F)
\]

\[
= \frac{\gamma}{M_0 V} \frac{1}{\Omega BZ} \int d^3k \int_{-\infty}^{+\infty} \frac{dE}{\pi} \delta(E - E_F) \text{Tr} \left[ T^\beta \Im G(\bar{k} + \bar{q}, E) T^\beta \Im G(\bar{k}, E) \right] \delta(E - E_F)
\]

\[
= \frac{1}{4} \left[ \alpha_{\alpha\alpha}(\bar{q}, G^+, G^+) + \alpha_{\alpha\alpha}(\bar{q}, G^-, G^-) - \alpha_{\alpha\alpha}(\bar{q}, G^+, G^-) - \alpha_{\alpha\alpha}(\bar{q}, G^-, G^+) \right]
\]  

(11)

where the index \(\beta\) of the torque operator \(T^\beta\) is related to the index \(\alpha\) according to Eq. (6) and the auxiliary functions

\[
\alpha_{\alpha\alpha}(\bar{q}, G^\pm, G^\pm) = \frac{\gamma}{M_0 V} \frac{1}{\Omega BZ} \int d^3k \text{Tr} \left[ T^\beta G^\pm(\bar{k} + \bar{q}, E_F) T^\beta G^\pm(\bar{k}, E_F) \right] \delta(E - E_F)
\]  

(12)

expressed in terms of the retarded and advanced Green functions, \(G^+\) and \(G^-\), respectively.

To account properly for the impact of spin-orbit coupling when dealing with Eqs. (11) and (12) a description of the electronic structure based on the fully relativistic Dirac formalism is used. Working within the framework of local spin density formalism (LSDA) this implies for the Hamiltonian the form

\[\hat{H}_D = c\bar{\alpha} \cdot \bar{p} + \beta mc^2 + V(\bar{r}) + \beta \bar{\sigma} \cdot \hat{m} B_{xc}(\bar{r}) .\]

(13)

Here \(\alpha_i\) and \(\beta\) are the standard Dirac matrices, \(\bar{\sigma}\) denotes the vector of relativistic Pauli matrices, \(\bar{p}\) is the relativistic momentum operator [30] and the functions \(V(\bar{r})\) and \(B_{xc} = \bar{\sigma} \cdot \hat{m} B_{xc}(\bar{r})\) are the spin-averaged and spin-dependent parts, respectively, of the LSDA potential [31] with \(\hat{m}\) giving the orientation of the magnetisation.

With the Dirac Hamiltonian given by Eq. (13), the torque operator may be written as \(\hat{T} = \beta[\bar{\sigma} \times \hat{m}] B_{xc}(\bar{r})\). Furthermore, the Green functions entering Eqs. (11) and (12) are determined using the spin-polarized relativistic version of multiple scattering theory [29, 32] with the real space representation of the retarded Green function given by:

\[
G^\pm(\bar{r}, \bar{r}', E) = \sum_{\Lambda \Lambda'} Z_{\Lambda}(\bar{r}, E) \tau_{\Lambda \Lambda'}(\bar{r}, \bar{r}') Z_{\Lambda'}^\dagger(\bar{r}', E) + \delta_{nm} \sum_{\Lambda} \left[ Z_{\Lambda}(\bar{r}, E) J_{\Lambda \Lambda'}(\bar{r}, \bar{r}') \Theta(r_n - r_m) \right].
\]

(14)
Here \( \vec{\tau}, \vec{\tau}' \) refer to atomic cells centered at sites \( n \) and \( m \), respectively, where \( Z_\Lambda^n(\vec{\tau}, E) = Z_\Lambda(\vec{r}_n, E) = Z_\Lambda(\vec{r} - \vec{R}_n, E) \) is a function centered at the corresponding lattice vector \( \vec{R}_n \). The four-component wave functions \( Z_\Lambda^n(\vec{\tau}, E) \) are regular (irregular) solutions to the single-site Dirac equation labeled by the combined quantum numbers \( \Lambda = (\kappa, \mu) \), with \( \kappa \) and \( \mu \) being the spin-orbit and magnetic quantum numbers \([30]\). Finally, \( \tau_{\Lambda \Lambda}^m(E) \) is the so-called scattering path operator that transfers an electronic wave coming in at site \( m \) into a wave going out from site \( n \) with all possible intermediate scattering events accounted for.

Using matrix notation with respect to \( \Lambda \), this leads to the following expression for the auxiliary damping parameters in Eq. (12):

\[
\alpha_{\alpha\alpha}(\vec{q}, G^\pm, G^\pm) = \frac{\gamma}{M_0 V \pi \Omega_{\text{BZ}}} \frac{1}{2} \int d^3k Tr \left( T^\beta_\delta \tau_\beta(\vec{k}, E_F^\pm) T^\gamma_\delta \tau_\gamma(\vec{k}, E_F^\pm) \right)_c.
\]

(15)

In the case of a uniform magnetization, i.e. for \( q = 0 \) one obviously gets an expression for the Gilbert damping tensor as it was worked out before \([7]\). Assuming small wave vectors, the term \( \tau(\vec{k} + \vec{q}, E_F^\pm) \) can be expanded w.r.t. \( \vec{q} \) leading to the series

\[
\tau(\vec{k} + \vec{q}, E_F) = \tau(\vec{k}, E) + \sum_{\mu} \frac{\partial \tau(\vec{k}, E)}{\partial k_\mu} q_\alpha + \frac{1}{2} \sum_{\mu\nu} \frac{\partial^2 \tau(\vec{k}, E)}{\partial k_\mu \partial k_\nu} q_\mu q_\nu + \ldots
\]

(16)

that results in a corresponding expansion for the Gilbert damping:

\[
\alpha(\vec{q}) = \alpha + \sum_{\mu} \alpha^{\mu\alpha} q_\mu + \frac{1}{2} \sum_{\mu\nu} \alpha^{\mu\nu} q_\mu q_\nu + \ldots
\]

(17)

with the following expansion coefficients:

\[
\alpha^{\mu\pm\pm}_{\alpha\alpha} = \frac{g}{\pi \mu_{\text{tot}}} \frac{1}{\Omega_{\text{BZ}}} \text{Trace} \int d^3k Tr \left( T^\beta_\delta \tau(\vec{k}, E_F^\pm) T^\gamma_\delta \tau(\vec{k}, E_F^\pm) \right)_c
\]

(18)

\[
\alpha^{\mu\pm\pm}_{\alpha\alpha} = \frac{g}{\pi \mu_{\text{tot}}} \frac{1}{\Omega_{\text{BZ}}} \text{Trace} \int d^3k Tr \left( \frac{\partial^2 \tau(\vec{k}, E_F^\pm)}{\partial k_\mu \partial k_\nu} T^\gamma_\delta \tau(\vec{k}, E_F^\pm) \right)_c
\]

(19)

\[
\alpha^{\mu\pm\pm}_{\alpha\alpha} = \frac{g}{\pi \mu_{\text{tot}}} \frac{1}{2 \Omega_{\text{BZ}}} \text{Trace} \int d^3k Tr \left( \frac{\partial^2 \tau(\vec{k}, E_F^\pm)}{\partial k_\mu} T^\gamma_\delta \tau(\vec{k}, E_F^\pm) \right)_c
\]

(20)

and with the g-factor \( 2(1 + \mu_{\text{orb}}/\mu_{\text{spin}}) \) in terms of the spin and orbital moments, \( \mu_{\text{spin}} \) and \( \mu_{\text{orb}} \), respectively, and the total magnetic moment \( \mu_{\text{tot}} = \mu_{\text{spin}} + \mu_{\text{orb}} \). The numerically cumbersome term in Eq. (20), that involves the second order derivative of the matrix of \( \vec{k} \)-dependent scattering path operator \( \tau(\vec{k}, E) \), can be reformulated by means of an integration by parts:

\[
\frac{1}{\Omega_{\text{BZ}}} \int d^3k T^\alpha_\beta \tau(\vec{k}, E_F) T^\beta_\gamma \frac{\partial^2 \tau(\vec{k}, E_F)}{\partial k_\mu \partial k_\nu} = \left[ \int dk_\beta dk_\gamma T^\beta_\gamma \tau(\vec{k}, E_F) T^\gamma_\delta \frac{\partial \tau(\vec{k}, E_F)}{\partial k_\mu} \right]_{k_\mu = 0}^{k_\mu = k_F

\]

leading to the much more convenient expression:

\[
\alpha^{\mu\nu\pm\pm}_{\alpha\alpha} = -\frac{g}{2 \pi \mu_{\text{tot}}} \int d^3k Tr \left( T^\beta_\delta \frac{\partial \tau(\vec{k}, E_F^\pm)}{\partial k_\mu} T^\gamma_\delta \frac{\partial \tau(\vec{k}, E_F^\pm)}{\partial k_\nu} \right)_c.
\]

(21)

III. RESULTS AND DISCUSSIONS

The scheme presented above to deal with the Gilbert damping in non-collinear systems has been implemented within the SPR-KKR program package \([32]\). To exam-
ine the importance of the chiral correction to the Gilbert damping a first application of Eq. (19) has been made for the multilayer system (Cu/Fe_{1-x}Co_x/Pt)_n seen as a non-centrosymmetric model system. The calculated zero-order (uniform) GD parameter \( \alpha_{xx} \) and the corresponding first-order (chiral) \( \alpha_{xx}^x \) correction term for \( \vec{q} \parallel \hat{x} \) are plotted in Fig. 1 top and bottom, respectively, as a function of the Fe concentration \( x \). Both terms, \( \alpha_{xx} \) and \( \alpha_{xx}^x \), increase approaching the pure limits w.r.t. the Fe_{1-x}Co_x alloy subsystem. In the case of the uniform parameter \( \alpha_{xx} \), this increase is associated with the dominating breathing Fermi-surface damping mechanism. This implies that the modification of the Fermi surface (FS) induced by the spin-orbit coupling (SOC) follows the magnetization direction that slowly varies with time. An additional contribution to the GD, having a similar origin, occurs for the non-centrosymmetric systems with helimagnetic structure. In this case, the features of the electronic structure governed by the lack of inversion symmetry result in a FS modification dependent on the helicity of the magnetic structure. This implies a chiral contribution to the GD which can be associated with the term proportional to the gradient of the magnetization. Obviously, this additional modification of the FS and the associated mechanism for the GD does not show up for a uniform ferromagnet. As \( \alpha \) is caused by the SOC one can expect that it vanishes for vanishing SOC. This was indeed demonstrated before [5]. The same holds also for \( \alpha^x \) that is caused by SOC as well.

Another system considered is the ferromagnetic alloy system bcc Fe_{1-x}Co_x. As this system has inversion symmetry the first-order term \( \alpha^u \) should vanish. This expectation could also be confirmed by calculations that account for the SOC. The next non-vanishing term of the expansion of the GD is the term \( \propto q^2 \). The corresponding second-order term \( \alpha_{xx}^{xx} \) is plotted in Fig. 2 (bottom) together with the zero-order term \( \alpha_{xx} \) (top). The bottom panel shows in addition results for \( \alpha_{xx}^{xx} \) that have been obtained by calculations with the SOC suppressed. As one notes the results for the full SOC and for SOC suppressed are very close to each other. The small difference between the curves for that reason have to be ascribed to the hybridization of the spin-up and spin-down subsystems due to SOC. As discussed in the literature [9, 17, 20] a non-collinear magnetic texture has a corresponding consequence but a much stronger impact here. In contrast to the GD in uniform FM systems where SOC is required to break the total spin conservation in the system, \( \alpha_{xx}^{xx} \) is associated with the spin-pumping effect that can be ascribed to an emergent electric field created in the non-uniform magnetic system. In this case magnetic dissipation occurs due to the misalignment of the electron spin following the dynamic magnetic profile and the magnetization orientation at each atomic site, leading to the dephasing of electron spins [10].

IV. SUMMARY

To summarize, expressions for corrections to the GD of homogeneous systems were derived which are expected to contribute in the case of non-collinear magnetic systems. The expression for the GD parameter \( \alpha(\vec{q}) \) seen as a function of the wave vector \( \vec{q} \) is expanded in powers of \( q \). In the limit of weakly varying magnetic textures, this leads to the standard uniform term, \( \alpha^u \), and the first- and second-order corrections, \( \alpha^m \) and \( \alpha^{mm} \), respectively. Model calculations confirmed that a non-vanishing value
for $\alpha_{\mu}$ can be expected for systems without inversion symmetry. In addition, SOC has been identified as the major source for this term. The second-order term, on the other hand, may also show up for systems with inversion symmetry. In this case it was demonstrated by numerical work, that SOC plays only a minor role for $\alpha_{\mu\nu}$, while the non-collinearity of the magnetization plays the central role.

V. ACKNOWLEDGEMENT

Financial support by the DFG via SFB 1277 (Emergente relativistische Effekte in der Kondensierten Materie) is gratefully acknowledged.

[1] T. L. Gilbert, IEEE Transactions on Magnetics 40, 3443 (2004)
[2] V. Kambersky, Can. J. Phys. 48, 2906 (1970), URL http://www.nrcresearchpress.com/doi/abs/10.1139/p70-389
[3] M. Fähnle and D. Steiauf, Phys. Rev. B 73, 184427 (2006)
[4] K. Gilmore, Y. U. Idzerda, and M. D. Stiles, Phys. Rev. Lett. 99, 027204 (2007), URL http://link.aps.org/doi/10.1103/PhysRevLett.99.027204
[5] S. Mankovsky, D. Ködderitzsch, G. Woltersdorf, and H. Ebert, Phys. Rev. B 87, 014440 (2013), URL http://link.aps.org/doi/10.1103/PhysRevB.87.014440
[6] A. A. Starikov, P. J. Kelly, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Phys. Rev. Lett. 105, 236601 (2010), URL http://link.aps.org/doi/10.1103/PhysRevLett.105.236601
[7] H. Ebert, S. Mankovsky, D. Ködderitzsch, and P. J. Kelly, Phys. Rev. Lett. 107, 066603 (2011), http://arxiv.org/abs/1102.4519v1, URL http://link.aps.org/doi/10.1103/PhysRevLett.107.066603
[8] I. Turek, J. Kudrnovský, and V. Drchal, Phys. Rev. B 92, 214407 (2015), URL http://link.aps.org/doi/10.1103/PhysRevB.92.214407
[9] Y. Tserkovnyak, E. M. Hankiewicz, and G. Vignale, Phys. Rev. B 79, 094415 (2009), URL http://link.aps.org/doi/10.1103/PhysRevB.79.094415
[10] K. M. D. Hals, K. Flensberg, and M. S. Rudner, Phys. Rev. B 92, 094403 (2015), URL http://link.aps.org/doi/10.1103/PhysRevB.92.094403
[11] K. M. D. Hals and A. Brataas, Phys. Rev. B 89, 064426 (2014), URL http://link.aps.org/doi/10.1103/PhysRevB.89.064426
[12] N. Umetsu, D. Miura, and A. Sakuma, Phys. Rev. B 91, 174440 (2015), URL http://link.aps.org/doi/10.1103/PhysRevB.91.174440
[13] E. Jue, C. K. Safeer, M. Drouard, A. Lopez, P. Balint, L. Buda-Prajbeanu, O. Boulle, S. Auffret, A. Schuhl, A. Manchon, et al., Nature Materials 15, 272 (2015), URL http://dx.doi.org/10.1038/nmat4518
[14] S. Zhang and S. S.-L. Zhang, Phys. Rev. Lett. 102, 086601 (2009), URL http://link.aps.org/doi/10.1103/PhysRevLett.102.086601
[15] G. Tatara and N. Nakabayashi, Journal of Applied Physics 115, 172609 (2014), http://dx.doi.org/10.1063/1.4870919, URL http://dx.doi.org/10.1063/1.4870919
[16] Y. Tserkovnyak and M. Mecklenburg, Phys. Rev. B 77, 134407 (2008), URL https://link.aps.org/doi/10.1103/PhysRevB.77.134407
[17] J. Zang, M. Mostovoy, J. H. Han, and N. Nagaosa, Phys. Rev. Lett. 107, 136804 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.107.136804
[18] T. Chiba, G. E. W. Bauer, and S. Takahashi, Phys. Rev. B 92, 054407 (2015), URL https://link.aps.org/doi/10.1103/PhysRevB.92.054407
[19] C. A. Akosa, I. M. Miron, G. Gaudin, and A. Manchon, Phys. Rev. B 93, 214429 (2016), URL https://link.aps.org/doi/10.1103/PhysRevB.93.214429
[20] E. M. Hankiewicz, G. Vignale, and Y. Tserkovnyak, Phys. Rev. B 78, 020404 (2008), URL https://link.aps.org/doi/10.1103/PhysRevB.78.020404
[21] Z. Yuan, K. M. D. Hals, Y. Liu, A. A. Starikov, A. Brataas, and P. J. Kelly, Phys. Rev. Lett. 113, 266603 (2014), URL https://link.aps.org/doi/10.1103/PhysRevLett.113.266603
[22] F. Freimuth, S. Blügel, and Y. Mokrousov, Phys. Rev. B 96, 104418 (2017), URL https://link.aps.org/doi/10.1103/PhysRevB.96.104418
[23] Z. Qian and G. Vignale, Phys. Rev. Lett. 88, 056404 (2002), URL http://link.aps.org/doi/10.1103/PhysRevLett.88.056404
[24] E. M. Hankiewicz, G. Vignale, and Y. Tserkovnyak, Phys. Rev. B 75, 174434 (2007), URL https://link.aps.org/doi/10.1103/PhysRevB.75.174434
[25] H. Mori and K. Kawasaki, Progr. Theor. Phys. 27, 529 (1962), URL http://ptp.ipj.jp/link?PTP/27/529/
[26] G. Mahan, Journal of Physics and Chemistry of Solids 31, 1477 (1970), ISSN 0002-938X, URL http://www.sciencedirect.com/science/article/pii/0002938X70900028
[27] M. Hasegawa, Journal of the Physical Society of Japan 31, 649 (1971), https://doi.org/10.1143/JPSJ.31.649, URL https://doi.org/10.1143/JPSJ.31.649
[28] G. D. Mahan, Many-particle physics, Physics of Solids and Liquids (Springer, New York, 2000), ISBN 978-1-4757-5714-9, URL https://openlibrary.org/works/OL4474340W/Many-particle_physics
[29] H. Ebert, in Electronic Structure and Physical Properties of Solids, edited by H. Dreyssé (Springer, Berlin, 2000), vol. 535 of Lecture Notes in Physics, p. 191.
[30] M. E. Rose, Relativistic Electron Theory (Wiley, New York, 1961), URL http://onlinelibrary.wiley.com/doi/10.1002/9781118675938.ch6
[31] A. H. MacDonald and S. H. Vosko, J. Phys.: Condens. Matter 12, 2977 (1979), URL http://iopscience.iop.org/1022-3719/12/15/007
[32] H. Ebert, J. Braun, D. Ködderitzsch, and S. Mankovsky, Phys. Rev. B 93, 075145 (2016), URL https://link.aps.org/doi/10.1103/PhysRevB.93.075145
[33] H. Ebert et al., The Munich SPR-KKR package, version 7.7, URL http://olymp.cup.uni-muenchen.de/ak/ebert/SPRKKR
[34] A. Manchon et al., Nature Materials 8, 575 (2009), URL http://www.nature.com/nmat/doi/10.1038/nmat2440
[35] H. Ebert, J. Braun, and D. Ködderitzsch, Phys. Rev. B 91, 075131 (2015), URL https://link.aps.org/doi/10.1103/PhysRevB.91.075131