The Meaning of Decoherence*

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Abstract. The conceptual and dynamical aspects of decoherence are analyzed, while their consequences on several fundamental applications are discussed. This mechanism, which is based on a universal Schrödinger equation, is furthermore compared with the phenomenological description of open systems by means of 'quantum dynamical maps'.

1 Ensembles, Entanglement, and Zwanzig Projections

Decoherence is usually defined as the practically irreversible disappearance of certain nondiagonal elements from the density matrix of a bounded but open system. It can be explained by the latter’s unavoidable interaction with its environment according to the Schrödinger equation for a global system under certain initial conditions. This delocalization of phase relations seems to form the most ubiquitous irreversible process in nature — similar to (but in general far more efficient than) the Boltzmann equation in classical statistical physics. In order to understand its importance correctly, one has to analyze carefully the meaning of the quantum mechanical density matrix, since its usual introduction by means of ‘quantization rules’ would be insufficient for this purpose. As will be explained below, there are indeed two quite different interpretations of the density matrix. Their confusion would lead to a common ‘naive’ misinterpretation of the concept of decoherence as describing a collapse of the wave function.

If the wave function (that is, the quantum mechanical state) of a physical system is assumed to be physically defined, but not completely known, one may often describe this incomplete knowledge by an ensemble consisting of wave functions $\psi_\alpha$ with probabilities $p_\alpha$. In this ensemble, the probabilities $p_\alpha$ (rather than a density matrix $\rho(q,q')$ that would arise from the formal quantization rules) form the analog of the classical probability distribution on phase space, $\rho(p,q)$. The meaning of the density matrix can only be appreciated when considering ensemble expectation values of observables $A$, that is, mean values of expectation values with respect to the various states $\psi_\alpha$.

* To be published in the proceedings of the Bielefeld conference on “Decoherence: Theoretical, Experimental, and Conceptual Problems”, edited by P. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.-O. Stamatescu (Springer 1999). This contribution is based on Sects. 4.2 - 4.4 of Zeh (1999).
which form the ensemble:

$$\langle A \rangle := \sum_\alpha p_\alpha \langle \psi_\alpha | A | \psi_\alpha \rangle = \text{Trace}\{ A \rho \} = \sum_n a_n \langle \phi_n | \rho | \phi_n \rangle \quad , \quad (1a)$$

with

$$\rho := \sum_\alpha | \psi_\alpha \rangle p_\alpha \langle \psi_\alpha | \quad \text{and} \quad A := \sum_n | \phi_n \rangle a_n \langle \phi_n | \quad . \quad (1b)$$

The symbol \( \langle A \rangle \) denotes here a twofold mean: with respect to the ensemble of quantum states \( \psi_\alpha \) with their probabilities \( p_\alpha \), and with respect to the quantum mechanical indeterminism of measurement results \( a_n \) with their probabilities \( |\langle \phi_n | \psi_\alpha \rangle|^2 \), valid for given quantum states \( \psi_\alpha \). In this way, the concept of a density matrix depends on the probability interpretation of the wave function — although not on any specific kinematical concept (such as hidden or classical variables) that would characterize the objects to which these probabilities apply, or where and how they might dynamically arise.

This ensemble interpretation of the density matrix according to \( \rho = \sum_\alpha | \psi_\alpha \rangle p_\alpha \langle \psi_\alpha | \) does not require the members \( \psi_\alpha \) of the ensemble of wave functions to be mutually orthogonal. They may in general even form an over-complete set. Therefore, the ensemble cannot be uniquely recovered from the density matrix. Von Neumann’s entropy,

$$S := -k \text{Trace}\{ \rho \ln \rho \} \quad , \quad (2)$$

would represent an ensemble entropy in the form \(-k \sum p_\alpha \ln p_\alpha \) only for the special ensemble consisting of the orthonormal eigenstates of \( \rho \). Its conservation describes dynamical determinism, provided the inner products between the states \( \psi_\alpha \) are also conserved. This requires the unitarity of the Schrödinger equation, since the formal density matrix does not distinguish between norm and probability of a wave function.

The mapping of ensembles of wave functions onto those which diagonalize their density matrix is an idempotent, information-reducing operation. Nonetheless, one may derive the von Neumann equation,

$$i \frac{\partial \rho}{\partial t} = [H, \rho] =: \hat{L} \rho \quad , \quad (3)$$

from the ensemble interpretation and the further assumption that all wave functions defining the ensemble satisfy a Schrödinger equation \( i \partial \psi_\alpha / \partial t = H \psi_\alpha \) with one and the same Hamiltonian \( H \). Although similar assumptions are used in classical statistical mechanics, presuming the Hamiltonian to be ‘given’ does not appear as an entirely consistent procedure while regarding states as incompletely known: the exact Hamiltonian of a bounded classical system would in general essentially depend on the uncontrollable state of its environment.

Rather than describing an ensemble of wave functions, a density matrix may also represent the local or ‘reduced’ perspective of entangled quantum
systems. The generic quantum state of any two combined systems (with variables \( x \) and \( y \), say) may be written as

\[
\psi(x, y) = \sum_{m,n} c_{mn} \phi_m(x) \Phi_n(y).
\]  

(4)

For spatially distinct subsystems, this entanglement represents the fundamental (kinematical) quantum nonlocality. All partial measurements at one of the subsystems (\( \phi(x) \), say) can then be characterized by expectation values of observables \( A \phi \),

\[
\langle A \phi \rangle := \text{Trace} \{ A \phi \rho_{\text{total}} \} = \text{Trace}_\phi \{ A \phi \rho_\phi \}.
\]  

(5)

Here, \( \rho_\phi \) is defined as a partial trace, \( \rho_\phi := \text{Trace}_\phi \{ \rho_{\text{total}} \} \), while \( \rho_{\text{total}} \) may still represent a wave function (or pure state), \( \rho_{\text{total}} := |\psi\rangle\langle\psi| \). This new density matrix \( \rho_\phi \) is explicitly defined by the expansion coefficients \( c_{mn} \) of the total state (4),

\[
(\rho_\phi)_{mn} := \langle \phi_m | \rho_\phi | \phi_{m'} \rangle = \sum_n c_{mn} c_{m'n}^* ,
\]  

(6)

rather than by a statistical distribution \( p_\alpha \) according to \( \sum_\alpha c_{\alpha m} p_\alpha c_{\alpha m'}^* \). Although it cannot be distinguished by local operations from the density matrix describing an ensemble of quantum states, it does here evidently not represent such an ensemble. Mere incomplete information would mean that one definite member of the ensemble described reality. Therefore, this ‘apparent ensemble’ or ‘improper mixture’ (d’Espagnat 1966) must not be used to explain the probability interpretation on that it has been based in (5). The fundamental difference between proper and improper mixtures cannot be overcome (though possibly obscured) by applying the formal limit of an infinity of degrees of freedom (cf. Hepp 1972). Statistical operators for all subsystems (as widely used in the phenomenological quantum theory of open systems) are therefore insufficient, as they neglect nonlocal quantum correlations. This formalism remains blind to the measurement problem (see below).

Any density matrix, such as \( \rho_\phi \) or \( \rho_\phi := \text{Trace}_\phi \{ \rho_{\text{total}} \} \), is hermitian, and can therefore be diagonalized in the form \( \rho_\phi = \sum_n |\hat{\phi}_n\rangle p_n \langle \hat{\phi}_n| \) that defines its eigenbasis \{\( \hat{\phi}_n \)\}. This form represents an (in general apparent) ensemble of orthogonal states. By using this diagonal form and (6) one observes that all eigenvalues \( p_n \) are non-negative. Phenomenological dynamical maps (Sect. 3) must therefore be chosen ‘completely positive’ by hand, that is, they have to conserve this property for all density matrices (including those of their subsystems).

For an entangled state such as (4), the eigenbasis of both subsystem density matrices defines the Schmidt canonical form,

\[
\psi(x, y) = \sum_k \sqrt{p_k} \hat{\phi}_k(x) \hat{\Phi}_k(y) ,
\]  

(7)
which, in contrast to (4), is a single sum (Schmidt 1907, Schrödinger 1935). For given subsystems, this representation (and therefore also its time dependence — see Kübler and Zeh 1973) is completely defined by the state $\psi$ of the total system (except for degenerate probabilities). All phase factors which could multiply the roots of the formal probabilities $\sqrt{p_k}$ in (7) have here been absorbed into the orthonormal states $\tilde{\phi}_k$ or $\tilde{\Phi}_k$. Since indistinguishable particles cannot be used to define subsystems, entanglement is not understood to include the formal correlations which describe symmetrization or antisymmetrization of the wave function. These pseudo-correlations are merely an artifact from using classical particle concepts.

Complete neglect of all correlations between two subsystems can be formally described by a Zvanzig projection (an idempotent mapping of density matrices),

$$\hat{P}_{\text{sep}} \rho := \rho_\phi \otimes \rho_\Phi \quad .$$

Such operators $\hat{P}$ on density matrices, with $\hat{P}^2 = \hat{P}$, form a convenient tool in deriving master equations for the ‘relevant’ part $\rho_{\text{rel}} := \hat{P}\rho$ of $\rho$, such as

$$\left\{ \frac{\partial \rho_{\text{rel}}}{\partial t} \right\}_{\text{master}} := \frac{\hat{P} e^{-i\hat{L} \Delta t} \rho_{\text{rel}} - \rho_{\text{rel}}}{\Delta t}$$

They reduce information contained in the density matrix (and thus raise the entropy) if they are genuine projections (linear and hermitian — see Sect. 3). The stronger Zvanzig projection of locality, $\hat{P}_{\text{local}} \rho = \prod_k \rho_{\Delta V_k}$, leading to a density matrix that factorizes with respect to small volume elements $\Delta V_k$ in space, would be required in order to arrive at the approximate concept of an entropy density $s(r)$. In quantum mechanics, a Zvanzig projection that neglects certain interference terms,

$$\hat{P}_{\text{semidiag}} \rho := \sum_n P_n \rho_{P_n} \quad ,$$

is often useful. Here, $\{P_n\}$ is a complete set of projectors on mutually orthogonal Hilbert subspaces. A master equation requires that the irrelevant part, $\rho_{\text{irrel}} := (1 - \hat{P})\rho$, is dynamically irrelevant in the future (similar to the particle correlations neglected in Boltzmann’s Stoßzahlansatz).

A reduction of information less than by $\hat{P}_{\text{sep}}$ is obtained by the relevance concept of classical correlations only,

$$\hat{P}_{\text{classical}} (|\psi\rangle \langle \psi|) := \sum_k p_k |\tilde{\phi}_k\rangle \langle \tilde{\phi}_k| \otimes |\tilde{\Phi}_k\rangle \langle \tilde{\Phi}_k| \quad ,$$

here again written in the Schmidt-canonical basis. The Zvanzig projection $\hat{P}_{\text{classical}}$ retains all ‘statistical’ correlations (based on incomplete information) while dropping all quantum correlations (entanglement). In the Schmidt representation the latter would require a twofold sum over $k$ and $k'$ in the density matrix.
A reduced density matrix does in general not obey a von Neumann equation any more if the total wave function $\psi$ evolves according to a Schrödinger equation. Its dynamics cannot be autonomous. Although it can be explicitly formulated (Kühler and Zeh 1973, Perle 1979), its solution would in general require solving the Schrödinger equation for the total system. Indeed, $\rho_\phi$ multiplied by the unit operator in $\Phi$-subspace represents another Zwanzig projection,

$$\hat{P}_{\text{sub}}\rho_{\text{total}} := \rho_\phi \otimes 1_\Phi .$$

Phenomenological master equations for $\rho_\phi$ are referred to as ‘open systems quantum dynamics’ (see Sect. 3). They are often derived by assuming a heat bath as an (uncorrelated) environment (Favre and Martin 1968, Davies 1976). However, from a fundamental point of view, master equations should explain the presence of heat baths (that is, canonical ensembles described by a temperature parameter) rather than presuming their existence.

The expectation values (1a) and (5), which led to the concept of a density matrix, refer to probabilities for the outcomes of quantum measurements. Von Neumann (1932) introduced his dynamical concept of ideal measurements (or measurements of the first kind) as unitary interactions between microscopic systems and measurement devices. They represent forks of causality (spreading information — see Zeh 1999), defined by the transition $\phi_n \Phi_0 \rightarrow t \phi_n \Phi_n$, where $\phi_n$ is an eigenstate of an observable $A = \sum_n |\phi_n\rangle a_n \langle \phi_n|$. $\Phi_0$ is the initial state of the apparatus, and $\Phi_n$ its ‘pointer position’ corresponding to the result $n$. The observable $A$ is defined by this interaction between wave functions up to the scale factor $a_n$ if the states $\phi_n$ are orthogonal. If the microscopic system is initially in one of the eigenstates, it does not change during an ideal measurement, while the apparatus evolves into the corresponding pointer state $\Phi_n$. (Non-ideal measurements could be similarly described by replacing $\phi_n$ with a different final state $\phi'_n$.)

However, for a general initial state, $\sum_n c_n \phi_n$, one obtains for the same interaction and for the same initial state of the apparatus

$$\left( \sum_n c_n \phi_n \right) \Phi_0 \rightarrow t \sum_n c_n \phi_n \Phi_n =: \psi .$$

The rhs is an entangled state, while an ensemble of different measurement results (that is, of states $\phi_n \Phi_n$ with probabilities $|c_n|^2$), would require this fork of causality to be replaced by a fork of indeterminism (leading to different potential states). The formal ‘plus’ characterizing the superposition would have to be replaced with an ‘or’. This discrepancy represents the quantum
measurement problem. The subsystem density matrices resulting from these two types of fork are identical, since there is no way of distinguishing these different situations operationally by a local observation. However, as emphasized above, this argument does not explain the fork of indeterminism that is at the heart of the probability interpretation. If the pointer states $\Phi_n$ are also orthogonal (as will have to be assumed for a measurement), both sides of (13) are Schmidt-canonical.

This measurement problem exists regardless of the complexity of the measurement device (which may give rise to thermodynamically irreversible behavior), and of the presence of fluctuations or perturbations caused by the environment, since the states $\Phi$ may be defined to describe this complexity completely, and even include the whole ‘rest of the world’. The popular argument that the quantum mechanical indeterminism might, in analogy to the classical situation, be due to distortions (such as uncontrollable ‘kicks’) during a measurement (cf. Peierls 1985, for example) is incompatible with a universal unitary dynamics. It would require the existence of an initial ensemble of microscopic states that in principle had to determine the outcome. However, the entropy of the rhs of (13) does not characterize an ensemble as it would be required for this purpose.

If both systems in (13) are microscopic, the dynamics representing the fork of causality can even be practically reversed in order to demonstrate that all arising components still exist. They may then contribute to individual observable consequences that depend on all of them, and on their relative phases. This excludes the interpretation of (13) as representing a fork of indeterminism (a fork between mere possibilities), even though von Neumann’s fork of causality is defined in a classical configuration space (in terms of branching wave packets). If the transition from quantum to classical were completely understood, it would have to explain why the arena for the wave function appears as a space of ‘classical’ configurations in many situations.

It should be kept in mind, however, that the local concepts of relevance, such as $\hat{P}_{\text{sep}}$, $\hat{P}_{\text{local}}$ and $\hat{P}_{\text{classical}}$, appear ‘natural’ only to our classical prejudice. In the unusual situation of EPR/Bell type experiments, quantum correlations become relevant even for local observers. The locality of the dynamics, in field theory described by means of point interactions, for example, merely warrants the dynamical consistency of this concept of relevance, such as the approximate validity of autonomous master equations for $\hat{P}_{\text{local}}\rho$.

2 Decoherence: Examples

Using the terminology of Zwanzig projections introduced above, decoherence may be defined as the dynamical justification of a specific $\hat{P}_{\text{semidiag}}$ for a given system by presuming the relevance of the corresponding ‘local perspective’ that is formally represented by $\hat{P}_{\text{sub}}$. If this $\hat{P}_{\text{semidiag}}$ is valid under all normal circumstances, its eigenspaces are labeled by classical properties of the local
(bounded but open) systems. In this way, ‘quasi-classical’ concepts (not just quasi-classical dynamics for presumed classical objects such as particles) *emerge* through unavoidable interaction with the environment.

Equation (13) formulates the interaction of a microscopic system $\phi$ with appropriate controllable ‘pointer states’ $\Phi$ of a measurement device. Its asymmetry in time represents a fact-like arrow (leading from factorizing to entangled states). It can possibly be reversed (with sufficient effort) if both subsystems are microscopic. For genuine measurements, the states $\Phi_n$ must be quasi-classical. Assume now that the pointer positions $\Phi_n$ in (13) are replaced with uncontrollable (such as thermal) states of the unavoidable environment of the system that is described by the states $\phi$, while this ‘system’ may even represent the macroscopic pointer states (being ‘measured’ themselves by their environment). Then this interaction cannot practically be reversed, and measurements by means of macroscopic instruments can thus not be undone. This (inter)action is clearly analogous to Boltzmann’s *Stoßzahlansatz* in creating correlations which propagate away, while it describes the specific quantum aspect of delocalizing phase relations (‘decoherence’). Its time arrow may therefore be referred to as *quantum causality*. The resulting local (reduced) density matrix $\rho_\phi$ in the sense of (5) describes an *apparent* ensemble of quasiclassical states.

This interaction with the environment, which leads to *ever-increasing entanglement*, is practically unavoidable for most systems in all realistic situations (Zeh 1970, 1971, 1973, Leggett 1980, Zurek 1981, 1982, Joos and Zeh 1985). It is this quantitative aspect that seems to have been greatly overlooked when classical systems were unsuccessfully described by a Schrödinger equation. Decoherence is efficient, since it does not require the environment to *act* on the system (as it would have to do in order to induce ‘distortions’ or Brownian motion). It does neither depend on the concepts of momentum and energy, nor include any transfer of heat or the presence of an environmental heat bath, since it may occur very far from equilibrium.

Decoherence is effective on a much shorter time scale than thermal relaxation or dissipation (see Joos and Zeh 1985, Zurek 1986). Its most important message is that *there are no approximately closed macroscopic systems* (save the whole universe). On the other hand, systematic decoherence requires a ‘normal’ environment that monitors *certain* properties (represented by subspaces). The latter may then appear as ‘classical facts’, which exist regardless of their observation, while their superpositions never occur (locally). While this situation is the basis of Zurek’s (operationally understood) *existential interpretation* (Zurek 1998), it is here evidently derived from the assumption of an ‘absolutely existing’ universal wave function (of which it forms a dynamically autonomous branch — or a ‘consistent history’ in terms of wave functions). It appears indeed strange that most physicists prefer to accept certain conceptual inconsistencies (often referred to as ‘complementarity’) which allow them to believe in the existence only of what they can ‘see’,
even though their successful and consistent theory tells them that there must be a lot more (unless this theory is deliberately changed where it cannot be confirmed).

Some important applications of decoherence will now be discussed.

2.1 Trajectories

In an imagined two-slit interference experiment with ‘bullets’ (or slightly more realistically with small dust particles or large molecules), not only their passage through the slits, but their whole path would unavoidably be measured by scattered air molecules or photons under all realistic circumstances. For macroscopic objects we could simply confirm this fact by opening our eyes. Therefore, no interference fringes could ever be observed — even if the resolution of our reading and registration devices were fine enough. Macroscopic objects resemble alpha particles in a cloud chamber (Mott 1929), since they can never be regarded as being isolated in a vacuum (as it can be arranged for microscopic objects). Their unavoidable entanglement with their environment leads to a reduced density matrix that is equivalent to an ever-increasing ensemble of narrow wave packets following slightly stochastic trajectories.

For a continuous variable, such as the center of mass position of a macroscopic particle, decoherence competes with the dispersion of the wave packet that is reversibly described by the Schrödinger equation. Even the apparently small scattering rate of photons or atoms off small dust particles in intergalactic space would suffice to suppress all coherent spreading of the wave packets (Joos and Zeh 1985). An otherwise free particle, for example, is then dynamically described by the master equation

$$i \frac{\partial \rho(x, x', t)}{\partial t} = \frac{1}{2m} \left( \frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2} \right) \rho - i \lambda (x - x')^2 \rho \quad ,$$

which can be derived from a universal Schrödinger equation by assuming the future irrelevance of all initial correlations with the environment (cf. Joos’ Chap. 3 and App. 1 of Giulini et al. 1996). The coefficient $\lambda$ is here determined by the scattering rate and its efficiency in orthogonalizing states of the environment. One does not have to postulate a fundamental semigroup in order to describe open quantum systems (Sect. 3). If the environment represents a heat bath, (14) corresponds to the infinite-mass limit of quantum Brownian motion (cf. Caldeira and Leggett 1983, Zurek 1991, Hu, Paz and Zhang 1992, Omnès 1997). This demonstrates that even for entirely negligible recoil (which is responsible for noise and friction) there remains an important effect that is based on quantum nonlocality. Although Brownian fluctuations are required in a thermal environment, they describe much smaller effects on the density matrix of macroscopic degrees of freedom than decoherence.
Classical properties (e.g. shape and position of a droplet) thus emerge from the wave function (and are maintained) in an irreversible manner. Particle aspects (such as tracks in a bubble chamber) are described by the reduced density matrix because of unavoidable interactions with the environment according to a universal Schrödinger equation. The disappearance of interference between partial waves during a welcher Weg experiment (Scully, Englert and Walther 1991, Dürr, Nomn and Rempe 1998) does not require a (fundamental) wave-particle ‘complementarity’. Similarly, there is no superluminal tunnelling (see Chiao 1998) in a consistent quantum description, since all parts of a wave packet propagate (sub-) luminally, while its group velocity does not represent propagation of a physical object in the absence of a fundamental particle concept.

Master equations for open systems, such as (14), can also be derived by means of Feynman path integrals (Feynman and Vernon 1963, Mensky 1979), that is, by using a decoherence functional. The path integral is here used for calculating the propagation of wave functions of systems together with their environments, while the latter are then dynamically traced out. However, only because of the decoherence contained in this procedure may superpositions of different paths with their physically meaningful phase relations (that is, propagating wave functions) appear to a local observer as representing ensembles of trajectories.

All classical phenomena, even those representing ‘reversible’ classical mechanics, are based on this (for all practical purposes) irreversible decoherence, with a permanent production of physical entropy that may be macroscopically negligible, although it is large in terms of bits. These ‘measurements’ by the environment according to (13) must be irreversible in this sense in order to avoid the possibility of ‘quantum erasing’ the information from the environment, and thus to restore coherence (just as for measurements proper). Recoherence would mean that every scattered particle were completely and coherently recovered in order to relocalize the initial phase relations. The terminology of quantum erasing is therefore misleading; conventional erasing is understood as the destruction (that is, dissipation) of information — corresponding to an increase of entropy —, while the dissipation of phase relations would just warrant perfect decoherence (rather than undoing it). Experimental realizations of quantum erasers in certain microscopic systems (cf. Kwiat, Steinberg and Chiao 1992) do not always recover the whole initial superposition — they may partly rebuild it.

2.2 Molecular Configurations as Robust States

Chiral molecules, such as right- or left-handed sugar, represent another simple property controlled by decoherence. A chiral state is described by a wave function, but in contrast to the otherwise analogous spin-3 state of an ammonia molecule, say, not by an energy eigenstate (see Zeh 1970, Primas 1983,
Woolley 1986). The reason is that it is chirality (but not parity) that is continuously ‘measured’, for example by scattered air molecules (for sugar under normal conditions on a decoherence time scale of the order $10^{-9}\text{sec}$ — see Joos and Zeh 1985). Measurements of the parity of sugar molecules, or their preparation in energy eigenstates, are therefore practically excluded, since this would require an even stronger coupling to the corresponding device.

As a dynamical consequence, each individual molecule in a bag of sugar must then retain its chirality, while a parity state — if it had come into existence in a mysterious or expensive way — would almost immediately ‘collapse’ into an apparent mixture of both chiralities with equal probabilities. Parity is thus not conserved for sugar molecules, while chirality could be confirmed ‘without demolition’ when measured again. (A mixture of chiralities would be formally identical to a mixture of parities only in the pathological case of exactly equal probabilities.)

This dynamical robustness of certain properties under the influence of the environment seems to characterize what we usually regard as ‘real classical facts’ (in the operational sense), such as spots on the photographic plate, or any other ‘pointer states’ of a measurement device. The concept of robustness is compatible with a (regular) time dependence, as exemplified in the previous section for the center of mass motion of macroscopic objects. Since entropy production by interaction with the environment is least for a density matrix already diagonal in terms of robust states, this property has been called a ‘predictability sieve’, and proposed as a definition of classical states (Zurek, Habib and Paz 1993).

Robustness also gives rise to quasi-classical ‘consistent histories’ in terms of wave packets, and it is required for the physical concept of memory, as in DNA, brains or computers — with the exception of quantum computers, which are extremely vulnerable to decoherence (Haroche and Raimond 1996, Ekert and Jozsa 1996 — see also Zurek 1998). In contrast to robust properties, which can be assumed to exist as ‘facts’ regardless of their measurement, potentially measurable quantities are called ‘counterfactual’ if they may occur in superpositions. They must then not be assumed to possess definite though unknown values. However, all situations can be described and distinguished by means of decoherence in terms of a (f)actual universal wave function.

Chemists know that atomic nuclei in large molecules have to be described classically (for example by rigid configurations, which may vibrate or rotate in a time-dependent manner), while the electrons have to be represented by stationary or adiabatically comoving wave functions. This asymmetry is then often attributed to a Born-Oppenheimer approximation in terms of the mass ratio. However, this argument is insufficient, since the same approximation can be applied to small molecules for calculating the stationary energy eigenstates with their discrete energy bands. This insufficient argument is now also found in quantum gravity, where it is claimed to explain classical spacetime by merely employing a Born-Oppenheimer approximation with respect to the
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inverse Planck mass. The argument cannot be improved by means a WKB
approximation for the massive variables, since this would still not exclude broad
wave functions (instead of narrow wave packets following quasi-trajectories).
The WKB approximation may only explain the quasi-classical propagation of
wave fronts according to geometric optics, and therefore the stability of wave
packets once they have formed.

The formation of quasi-classical wave packets for the atomic nuclei in
large molecules or for the gravitational field can instead be explained by
decoherence (Joos and Zeh 1985, Unruh and Zurek 1989, Kiefer 1992 — see
also Sect. 2.4). For example, the positions of atomic nuclei in large molecules
are permanently monitored by scattering of (other) molecules. But why only
the nuclei (or ions), and why not even they in very small molecules? The
answer can only be quantitative, and it is based on a delicate balance between
internal dynamics and interaction with the environment, whereby the density
of states plays a crucial role (Joos 1984). Depending on the specific situation,
one will either obtain an approximately unitary evolution, a master equation
(with time asymmetry arising from quantum causality), or complete freezing
of the motion (quantum Zeno effect). The situation becomes simple only for a ‘free’ massive particle, which is described by (14).

2.3 Charge Superselection

Gauß law, \( q = \frac{1}{4\pi} \int \mathbf{E} \cdot d\mathbf{S} \), tells us that every local charge is correlated with
its Coulomb field on a sphere at any distance. A superposition of different
charges,

\[
\sum_q c_q \psi_q^{\text{total}} = \sum_q c_q \chi_q \psi_q^{\text{field}} = \sum_q c_q \chi_q \psi_q^{\text{near}} \psi_q^{\text{far}} = : \sum_q c_q \chi_q^{\text{dressed}} \psi_q^{\text{far}},
\]

would therefore represent an entangled state of the charge and its field. Here,
\( \chi_q \) describes the bare charge, while \( \psi_q^{\text{field}} = \psi_q^{\text{near}} \psi_q^{\text{far}} \) is the wave functional
of its complete field, symbolically written as a tensor product of a near-
field and a far-field. The dressed (physical) state of a charge would then be
described by a density matrix of the form

\[
\rho_{\text{local}} = \sum_q |\chi_q^{\text{dressed}}\rangle c_q^2 \langle \chi_q^{\text{dressed}}| \quad (16)
\]

if the states of the far field are mutually orthogonal (uniquely distinguishable)
for different charge \( q \). The charge is decohered by its own Coulomb field, and
no charge superselection rule has to be postulated in a fundamental way (see
Giulini, Kiefer and Zeh 1995). The dressing of a charged particle by its near-
field (including reversible polarization of surrounding matter) would formally
decohere the bare charge, but remain observationally irrelevant, since only the dressed particle can be used in experiments. While this result is satisfactory from a theoretical point of view, a more practical question is, at what distance and on what time scale a point charge in a superposition of two different locations (such as an electron during an interference experiment) would be decohered by the corresponding dipole field. Classically, the retarded Coulomb field on the forward light cone would contain complete information about the path of the charge. However, since interference between different electron paths has been observed over distances of the order of millimeters (Nicklaus and Hasselbach 1993; see also Hasselbach’s contribution to this conference), one has to conclude that in QED Coulomb fields contribute to decoherence by their monopole component only.

This consequence, which appears surprising from a classical point of view, may be readily understood in terms of quantized fields, since photons with infinite wave length (representing static Coulomb fields) cannot ‘see’ position at all (even though their number may diverge). Static dipole (or higher) moments do not possess any far-fields, which are defined to decrease with \(1/r^2\) only. Therefore, only the ‘topological’ Gauss constraint \(\partial_\mu F^{\mu0} = 4\pi j^0\) remains of the Coulomb field in QED. This requires that the observed (retarded) Coulomb field is completely described by transversal photons, corresponding to the vector potential \(A\), with \(\text{div} A = 0\) in the Coulomb gauge, and in states obeying the Gauss constraint. According to this picture, only the ‘positions’ of electric field lines — not their total number or flux — represent dynamical variables that have to be quantized. In this sense, charge decoherence has been regarded as kinematical, although it may as well be interpreted as being dynamically caused in the usual way by the retarded Coulomb field of the (conserved) charge in its past. However, the absence of a dynamical Coulomb field, which may also eliminate the need for renormalizing the mass of a charged particle by its Coulomb field, is incompatible with the concept of a Hilbert space spanned by direct products of local states.

Dipole moments (defining position differences of a point charge), can thus be ‘measured’ either by the ‘real’ emission of transversal photons, or by the irreversible polarization of nearby matter (Kübler and Zeh 1973, Zurek 1982a, Anglin and Zurek 1996). Emission of photons requires acceleration. For example, a transient dipole of charge \(e\) and maximum distance \(d\), existing for a time interval \(t\), involves accelerations \(a\) of at least the order \(d/t^2\). According to Larmor’s formula, the intensity of radiation is at least \(\frac{2}{3}e^2a^2\). In order to resolve the dipole, the radiation has to consist of photons with energy greater than \(\hbar c/d\) (that is, wave lengths smaller than \(d\)). The probability that information about the dipole is radiated away (by one photon, at least) is then very small (of the order \(\alpha Z^2(d/ct)^3\), where \(\alpha\) is the fine structure constant and \(Z\) the charge number). In more realistic cases, such as interference experi-

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1 This conclusion emerged from discussions with E. Joos.
ments with electrons, stronger accelerations may be involved, although they would still cause negligible decoherence in most situations.

The gravitational field of a mass point is similar to the Coulomb field of a charge. Superpositions of different mass should therefore be decohered by the quantum state of spatial curvature, and give rise to a mass superselection rule. However, superpositions of slightly different mass evidently exist. They form the time-dependent states of local systems, which would otherwise be excluded. The quantitative aspects of this situation do not yet appear to be sufficiently understood.

There would be no Coulomb field at all if the total charge of the quantum universe vanished (cf. Giulini, Kiefer and Zeh 1995). The gravitational counterpart of this complete disappearance of a classical quantity is the absence of time from a closed universe in quantized general relativity (see Sect. 6.2 of Zeh 1999).

2.4 Fields and Gravity

Not only are quantum states of charged particles decohered by their fields, quantum field states may also be decohered in turn by the sources on which they (re)act (see Kiefer’s Chap. 4 of Giulini et al. 1996, and his contribution to these proceedings). In this case, ‘coherent states’, that is, Schrödinger’s time-dependent but dispersion-free Gaussian wave packets for the amplitudes of classical wave modes (eigenmodes of coupled oscillators), have been shown to be robust for similar reasons as chiral molecules or the wave packets describing the center of mass motion of quasi-classical particles (Kübler and Zeh 1973, Kiefer 1992, Zurek, Habib and Paz 1993, Habib et al. 1996). This explains why macroscopic states of neutral boson fields usually appear as classical fields, and why superpositions of macroscopically different ‘mean fields’ or different vacua (see Sect. 6.1) have never been observed. In particular, quantum (field) theory must not and need not be reduced to a mere description of scattering processes with their unproblematic probability interpretation in terms of asymptotically isolated fragments.

These coherent (minimum uncertainty) harmonic oscillator states are defined for each mode $k$ as the (overcomplete) eigenstates of the nonhermitian photon annihilation operators $a_k$ with complex eigenvalues $\alpha_k$ (that is, $a_k |\alpha_k\rangle = \alpha_k |\alpha_k\rangle$). They represent Gaussian wave packets centered at a time-dependent mean field $\alpha_k(t) = \alpha_0^k e^{i\omega t}$, where $Re(\alpha_k)$ and $Im(\alpha_k)$ are analogous to the mean position and momentum of a mechanical oscillator wave packet. Since the Hamiltonian that describes the interaction with charged sources is usually linear in the field operators $a_k$ or $a_k^\dagger$, these coherent states form a robust ‘pointer basis’ under normal conditions: they cause negligible entanglement with their environment (here their ‘sources’).

In contrast to these superpositions of many different photon numbers (that is, oscillator quantum numbers), one-photon states resulting from the decay of different individual atoms (or even the n-photon states resulting
from the decay of a different number of atoms) are unable to interfere with one another, since they are correlated with mutually orthogonal final states (different atoms being in their ground state). Two incoherent components of a one-photon state may then appear (using Dirac’s language) as ‘different’ photons, although photons are not even conceptually distinguishable from another. A coherent macroscopic (‘collective’) excited state of the source would instead react negligibly (judged by means of the inner Hilbert space product) when as a whole emitting a photon. It would thus be able to produce the coherent (‘classical’) field states discussed above (see Kiefer 1998). In his textbook, Dirac (1947) discussed also states of two (or more) photons which are entangled with one source state containing two or more decayed atoms (and which may be described by a symmetrized product of one-photon wave functions). Although these n-photon states form coherent components of QED, their (two or more) one-photon probabilities add again without interference (except for the exchange terms, which give rise to the Hanbury Brown-Twiss effect).

Superpositions of different quasi-classical fields, \(c_1|\alpha_1\rangle + c_2|\alpha_2\rangle\), have recently been produced and maintained for a short time as one-mode laser fields in a cavity (Monroe et al. 1996). Their smooth decoherence has also been observed (Brun et al. 1996), as reported at this conference by Haroche.

Similar arguments as used above for electromagnetic fields apply to space-time curvature in quantum gravity (Joos 1986, Demers and Kiefer 1996 — for applications to quantum cosmology see Chap. 6 of Zeh 1999). One does not have to know its precise form (that may be part of an elusive unified quantum field theory) in order to conclude that the quantum states of matter and geometry (as far as this distinction remains valid) must be entangled and give rise to mutual decoherence. The classical appearance of spacetime geometry with its fixed light cone structure (that is presumed in conventional quantum field theory) is thus no reason not to quantize gravity. The beauty of Einstein’s theory can hardly be ranked so much higher than that of Maxwell’s to justify its exemption from established theory. An exactly classical gravitational field interacting with a quantum particle would be incompatible with the uncertainty relations — as is known from the early Bohr-Einstein debate. The resulting density matrix (functional) for the gravitational tensor field as a ‘quantum system’ must therefore be expected to represent an apparent mixture of different quasi-classical curvature states (to which even the observer is correlated).

Moreover, the entropy and thermal radiation characterizing a black hole or an accelerated Unruh detector are consequences of the entanglement between relativistic vacua on two half-spaces separated by an event horizon (see Gerlach 1988, and Sect. 5.2 of Zeh 1999). This entropy measures the same type of ‘apparent’ ensembles as the entropy produced according to the master equation (14) for a macroscopic mass point. An event horizon need not be different from any other quasi-classical property. Nonetheless, the dis-
appearance of coherence behind (even a virtual) horizon has been regarded as a \textit{fundamental} irreversible process of decoherence. This does in no way appear justified.

2.5 Quantum Jumps

Quantum particles are often observed as flashes on a scintillation screen, or heard as ‘clicks’ of a Geiger counter. These macroscopic phenomena are then interpreted as being caused by point-like objects passing through the observing instrument during a short time interval, while this is in turn understood as evidence for the discontinuous decay of an excited state (such as an atomic nucleus). Using a \textit{rate} for stochastic ‘decay events’ is equivalent to a master equation. A \textit{constant} rate would describe an exponential decay law. Discrete quantum jumps between two energy eigenstates have also been monitored for single atoms in a cavity when strongly coupled to an observing device (Nagourney, Sandberg and Dehmelt 1986, Sauter et al. 1986).

The Schrödinger equation, on the other hand, describes a wave function (usually with angular distribution according to a spherical harmonic) smoothly leaking out of a decaying system (such as a ‘particle’ in a potential well). This contrast between observation and the Schrödinger dynamics is clearly the empirical root of the probability interpretation \textit{in terms of discrete classical concepts}, such as particles and events. Since its norm is conserved, a wave function can disappear exponentially only from a bounded open region (usually an expanding sphere of size determined by the history of the decaying object and the speed of the decay fragments). This wave function represents a \textit{superposition} (rather than an ensemble) of different decay times. Their interference and the dispersion of the outgoing wave lead to further deviations from an exponential law. Although they are too small to be observed in free decay, they have been confirmed as ‘coherent state vector revival’ for photons emitted into reflecting cavities (Rempe, Walther and Klein 1987).

The appearance of ‘particles’ as local objects following tracks in a cloud chamber has been described in Sect. 2.1 in terms of an apparent ensemble of narrow wave packets. If droplets forming a condensate in the cloud chamber, or similar phenomena, such as spots on a photographic plate and clicks of a counter, appear at certain times, this is interpreted as indicating ‘quantum events’. However, the same decoherence which describes localization in space may, in the same sense, also explain localization in time. Neither particles nor quantum jumps are required as fundamental concepts (Zeh 1993). Whenever decay fragments (or the decaying systems) interact strongly with their environment, any interference between ‘decayed’ and ‘not yet decayed’ disappears on a very short (though finite) decoherence time scale, similar to Schrödinger’s cat superpositions described in the previous section. This time scale is in general far shorter than the time resolution in genuine measurements. If decoherence is even faster than relaxation into exponential behavior,
decay may be strongly suppressed (‘quantum Zeno effect’ — see Joos 1984 for a non-phenomenological discussion of its dynamics).

Decoherence thus leads to an apparent ensemble of decay histories consisting of a succession of events. The environment ‘monitors’ the decay status (in general uncontrollably) at all times with a resolution defined by the decoherence time scale. A decaying system is then more appropriately described by a decay rate than by a Schrödinger equation. Its time dependence would be exactly exponential, while this master equation represents only an approximate local consequence of the global Schrödinger equation. Similarly, distinct decay energies forming an initial superposition would usually be absorbed into mutually orthogonal final states of the environment. Microscopic systems (with their distinct energy levels) must therefore decohere into the eigenstates of their Hamiltonian. This consequence of robust numbers of emitted photons (Sect. 2.4) explains why stationary states characterize the atomic world, and von Neumann spoke of an Eingriff (intervention) required for their change.

It seems that this situation of continuously monitored decay has led to the myth of quantum theory as a stochastic theory for fundamental quantum events (cf. Jadczyk 1995). For example, Bohr (1928) remarked that “the essence” (of quantum theory) “may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality . . . ” (my italics). If this were true, there could be no lasers, superconductors, or similar macroscopic superpositions. Heisenberg and Pauli similarly emphasized that their preference for matrix mechanics originated in its (as it now seems misleading) superiority in describing discontinuities. However, according to the Schrödinger equation and recent experiments, the underlying entanglement processes are smooth. The short decoherence time scale mimics jumps between energy eigenstates or, depending on the situation, into narrow wave packets which in the Heisenberg-Bohr picture are interpreted as particles (with classical properties restricted in validity by the uncertainty relations in order to comply with the Fourier theorem).

While this new description may now appear as a consistent picture in terms of wave functions, an important question remains open: how do the probabilities which were required to justify the concept of a density matrix in Sect. 1 have to be understood if they do not describe quantum jumps or the spontaneous occurrence of classical properties through fundamental ‘events’. These interpretational problems are discussed in Sect. 4.6 of Zeh (1999), but we have here to conclude that, from an external point of view, the ensembles of wave functions derived by decoherence are apparent ones.

3 Quantum Dynamical Maps

The phenomenological description of open quantum systems by means of semi-groups offers some novel possibilities which go beyond a global Schrödinger equation. For example, quantum dynamical maps have been used to formu-
late von Neumann’s ‘first intervention’ (the reduction of the wave function) as part of the dynamics (cf. Kraus 1971). This is possible, since semigroups cannot only describe the transition from pure states to ensembles, but also the ‘selection’ of an individual member. Otherwise they are equivalent to an entropy-enlarging Zwanzig-type master equation with respect to $P_{\text{sub}}$ (or its equivalent in terms of path integrals — Feynman and Vernon 1963). Although the ‘irrelevant’ correlations with the environment, which would arise according to the exact global formalism, represent quantum entanglement, they are here usually not distinguished from classical statistical correlations (defined for ensembles only) when it comes to applications. This attitude is equivalent to a popular but insufficient ‘naive’ interpretation of decoherence, which pretends to derive genuine ensembles.

Quantum theory is sometimes even defined as describing open systems by means of a dynamical semigroup, that is, as a time-asymmetric local statistical theory. (Hence the term ‘statistical operator’ for the density operator.) However, this ‘minimal statistical interpretation’ is insufficient as a fundamental theory, as it neglects the essential difference between genuine and apparent ensembles, and thus all consequences of entanglement beyond the considered systems (quantum nonlocality). The superposition principle has even been claimed to be derivable (cf. Ludwig 1990), although it must then be re-introduced in a different way (for example by changing the laws of statistics — in conflict with any ensemble interpretation).

Semigroups are certainly mathematically elegant and powerful. Therefore, they would represent candidates for new (fundamental) theories if conventional (Hamiltonian) quantum theory should prove wrong as a universal theory. The question is whether mathematical elegance here warrants physical relevance or is merely convenient within a certain approximation. To quote Lindblad (1976): “It is difficult, however, to give physically plausible conditions . . . which rigorously imply a semigroup law of motion for the subsystem. . . Applications . . . have led some authors to introduce the semigroup law as the fundamental dynamical postulate for open (non-Hamiltonian) systems.” Such a law would fundamentally introduce an arrow of time, but it would depend on the choice of systems (and in some cases contradict experiments that have already been performed).

The simplest quantum systems (such as spinors) are described by a two-dimensional Hilbert space. Their density matrix may be written by means of the Pauli matrices $\sigma_i$ ($i = 1, 2, 3$) in the form

$$\rho = \frac{1}{2} (1 + \sigma \cdot \pi), \quad (17)$$

where the (mathematically) real polarization vector $\pi = \text{Trace}\{\sigma \rho\}$ — that is, the expectation value of all spin components — completely defines $\rho$ as a general hermitian $2 \times 2$ matrix of trace 1. The latter is in turn equivalent to a (genuine or apparent) ensemble of orthogonal states (a spinor basis). The length of $\pi$ is a measure of purity, since $\text{Trace}\{\rho^2\} = (1 + \pi^2)/2$, with
\[ \pi^2 \leq 1. \] A pure state corresponds to a unit polarization vector, while an arbitrary density matrix (a general ‘state’ in the language of mathematical physics) is characterized by the mean value \[ \pi = \sum \alpha p_{\alpha} \pi_{\alpha} \] of all unit vectors \( \pi_{\alpha} \) in an ensemble of spinors that may represent this density matrix.

A general trace-preserving linear operator \( \hat{P} \) on \( \rho \) must be defined on 1 and \( \sigma \) in order to be completely defined:

\[ \hat{P} 1 := 1 + \pi_0 \cdot \sigma \quad \hat{P} \sigma := A \cdot \sigma, \]

with a real vector \( \pi_0 \) and a linear vector transformation \( A \). \( \hat{P} \) is idempotent (a Zwanzig ‘projector’) if \( A^2 = A \) and \( \pi_0 \cdot A = 0 \) (\( A = 0 \), for example). If \( \pi_0 \neq 0 \), \( \hat{P} \) creates information — even from the unit matrix.

Dynamical combination of the projection \( \hat{P} \) with a Hamiltonian evolution (rotation of \( \pi \)) in the form of a master equation leads to the Bloch equation for the vector \( \pi(t) \),

\[ \frac{d\pi}{dt} = \omega \times (\pi - \pi_0) - \sum \gamma_i (\pi^i - \pi_0^i) e_i \]

in a certain vector basis \( \{e_i\} \) (cf. Gorini, Kossakowski and Sudarshan 1976). Values of \( \gamma_i < 0 \) or \( |\pi_0| > 1 \) would violate the positivity of the density matrix at some \( t > 0 \) (cf. Sect. 4.2), and have thus to be excluded. The second term on the rhs describes anisotropic damping towards \( \pi_0 \). This formation of new information may describe very different situations — for example equilibration with a stationary external heat bath of given temperature, or evolution towards a certain measurement result. However, hermiticity of \( \hat{P} \) (corresponding to a genuine projection operator) would require \( \pi_0 = 0 \) and \( A = A^\dagger \), that is, a projection of vectors \( \pi \) in space.

If the two-dimensional Hilbert space describes something else than spin, such as isotopic spin or a \( K, \bar{K} \) system, the polarization vector lives in an abstract three-dimensional space, with environmental conditions that cannot practically be ‘rotated’. The abstract formalism can also be generalized to \( n \)-dimensional Hilbert spaces. For this purpose the Pauli matrices have to be replaced with the \( (n^2 - 1) \) hermitean generators of \( SU(n) \), while the real ‘coherence vectors’ (the generalizations of the polarization vector \( \pi \)) now live in the vector space spanned by them. For example, \( SU(3) \) gives rise to the ‘eight-fold way’. The most important difference is that there are now more than one (in fact, \( n - 1 \)) commuting hermitean generators. They may contain a nontrivial subset that is decohered under all realistic environmental conditions, and thus may form the center of a phenomenological set of observables (the set of ‘classical observables’ — cf. Sect. 2).

\[ \text{As mentioned before, all subsystem density matrices remain positive under a global Hamiltonian dynamics, and even under a collapse of the global state vector. This property of ‘complete positivity’ has to be separately postulated for phenomenological quantum dynamical maps (cf. Kraus 1971), thus further illustrating that these maps do not describe a fundamental quantum concept.} \]
In the infinite-dimensional Hilbert space of quantum mechanics, the Wigner function

\[ W(p, q) := \frac{1}{\pi} \int e^{2ipx} \rho(q + x, q - x) \, dx \]

\[ = \frac{1}{2\pi} \int \int \delta \left( q - \frac{z + z'}{2} \right) e^{ip(z - z')} \rho(z, z') \, dz \, dz' =: \text{Trace}\{\Sigma_{p,q}\rho\} \quad (20) \]

(written in analogy to \( \pi = \text{Trace}\{\sigma\rho\} \)) assumes the role of the Bloch vector. Evidently, \( \Sigma_{p,q}(z, z') := \frac{1}{2\pi} e^{ip(z - z')} \delta(q - \frac{z + z'}{2}) \) is the generalization of the Pauli matrices (with ‘vector’ index \( p, q \)). Therefore, the Wigner function is a continuous set of expectation values, which form the components (one for each point in phase space) of a generalized coherence vector. This ‘vector’ of expectation values characterizes the density matrix \( \rho \) again completely, and regardless of its interpretation according to Sect. 1. It does neither represent a quantum state nor a probability distribution on phase space (as is evident from its possibly negative values), even though it allows one to calculate all expectation values in the form of an ensemble mean, \( < F > = \int f(p, q)W(p, q)dpdq \).

Lindblad (1976) was able to generalize the Bloch equation to infinite-dimensional Hilbert spaces. He wrote it (in its form applicable to the density matrix) as

\[ i\frac{\partial \rho}{\partial t} = [H, \rho] - \frac{i}{2} \sum_k \left( L_k L_k \rho + \rho L_k L_k + 2L_k \rho L_k^\dagger \right) \]  

(21)

with arbitrary generators \( L_k \) in Hilbert space. It represents creation (localization) of information in the considered system, that is, a decrease of the corresponding von Neumann entropy (such as described by \( \pi_0 \) in (18)), precisely if some generators do not commute with their hermitian conjugates \( L_k^\dagger \).

This can be demonstrated by applying the non-Hamiltonian terms of (20) to the unit matrix \( \rho = 1 \). Otherwise it describes information loss (a genuine Zwanzig projection). This can also be seen from the general representation of a Zwanzig projector in quantum mechanical Hilbert space, \( \hat{P}_\rho = \sum_k V_k \rho V_k^\dagger \), which is analogous to the square root of a positive operator in its eigenbasis for \( V_k = V_k^\dagger \). If \( L_k^\dagger = L_k \), the Lindblad terms can be written in the form of a double commutator, \( L^2 \rho + \rho L^2 - 2L\rho L = [L, [L, \rho]] \). For \( L = \sqrt{2\lambda x} \) one recovers (14), that is, decoherence in the \( x \)-basis, as it could be derived from unitary interaction with the environment (and shown to be practically unavoidable for macroscopic variables).

\[ ^3 \text{On a finite interval of length } L, \Sigma_{p,q} \text{ would require an additional term} \]

\( -\frac{1}{\pi L} \text{exp}[ip(z - z')] \) in order to remain traceless. In (20), \( W(p, q) \) is then accordingly replaced with \( W(p, q) - \frac{1}{L} \int W(p, q) \, dq \) as the generalized Bloch vector. (Note that in Zeh (1999) the factor \( \text{exp}[ip(z - z')] \) of this additional \( \frac{1}{L} \)-term has erroneously been replaced by the \( \delta \)-function from (20).)
One may similarly describe other ‘unread measurements’ and their corresponding loss of phase relations. However, ‘damping’ towards a pure state (a semigroup proper) according to the second term of (17) with a unit vector $\pi_0$ allows one even to describe dynamically the transition from the initial state vector into a (freely chosen) definite measurement outcome (a ‘collapse’ — in contrast to a local or global Schrödinger dynamics). This can then readily be combined with a stochastic formalism representing an appropriate dice (or random number generator) that selects pure states according to the Born-von Neumann probabilities (Bohm and Bub 1966, Pearle 1976, Gisin 1984, Belavkin 1988, Diósi 1988). If applied continuously, such as by means of the Itô process, this formalism describes measurements phenomenologically as a smooth indeterministic process (that does not distinguish between ensembles and entanglement).

Many explicit models have been proposed in the literature (see Stamatescu’s Chap. 8 of Giulini et al. 1996). Some of them merely replace the apparent ensemble arising through decoherence for a bounded open system with a genuine one (Gisin and Percival 1992). The system is then assumed always to possess its own (yet unknown) state $\psi(t)$ that follows an indeterministic trajectory in its Hilbert space according to a quantum Langevin equation — in conflict with the exact global dynamics that leads to entanglement. Therefore, this ‘quantum state diffusion model’ is essentially equivalent to what I have called above the ‘naive interpretation’ of decoherence. It may serve as an intuitive picture (or tool) for many practical purposes if (and insofar as) it selects the dynamically robust wave packets described in Sect. 2.2. However, it would be severely misleading if this formalism (based on the concept of a density matrix) gave rise to the impression of deriving a real collapse by just taking into account the interaction with the environment. If real physical states are described by wave functions, there are only two possibilities: deviations from the Schrödinger equation or the Everett interpretation.

Many contributions in the literature remain ambiguous about their true intentions, or simply disregard the difference between genuine and apparent ensembles (proper and improper mixtures). In particular, the quantum state diffusion model is not appropriate to define a fundamental dynamical process, since the resulting pure states $\psi(t)$ of a system would in general not define states for any of its subsystems (which could as well have been chosen as the system, and thus have led to a different stochastic evolution). The picture of a trajectory of states $\psi(t)$ for a macroscopic system that is not the whole universe is simply in drastic conflict with quantum nonlocality.

Other models therefore attempt to reproduce the observed statistical aspects of quantum theory (as they occur in measurements, for example) by means of dynamical laws which may be truly fundamental. Since they cannot remove all entanglement, they cannot describe trajectories of wave functions $\psi(t)$ for all ‘systems’. Measurements are special applications of this general stochastic quantum dynamics that describes an increase of ensemble entropy.
Explicit modifications of a universal Schrödinger equation were originally suggested in the form of stochastic ‘hits’, assumed to act in addition to the unitary evolution in order to suppress coherence with growing distance (Ghirardi, Rimini and Weber 1986). They were postulated to occur rarely for individual particles, but sufficiently often for entangled aggregates of many particles in order to describe quasi-classical behavior. This proposal was later formulated as a continuous process as indicated above (Pearle 1989, Ghirardi, Pearle and Rimini 1990). Since these models lead to novel predictions, they can be distinguished from a universally valid Schrödinger equation. In their original form they would either be completely camouflaged by environmental decoherence (Joos 1986, Tegmark 1993), or are ruled out by existing experiments (Pearle and Squires 1994). They cannot be excluded in general, however, if one allows them to occur clearly after environmental decoherence has occurred in the observational chain of interactions. Their precise form would then be hard to guess in the absence of any empirical hints.

Several authors have suggested to find the root of a fundamental quantum indeterminism in gravity. Their main motivation is the apparently classical nature of spacetime curvature. However, it has been indicated in Sect.2.4 that spacetime need not be classical. Collapse models along these lines have been proposed in a more or less explicit form (Penrose 1986, Károlyházy, Frenkel and Lukácz 1986). They regard the quantum state of the gravitational field either as an environment to matter in a specific quantum state diffusion model (as criticized above — see Diósi 1987), or they are using an arising event horizon as a ‘natural’ boundary to cut off entanglement (Hawking 1987, Ellis, Mohanty and Nanopoulos 1989). Even though this boundary between ‘systems’ may appear natural, this procedure would still not define an objective fundamental process (see also Myers 1997). In particular, the horizon depends on the complete history of motion of the observer. Under no circumstances would this proposal justify the replacement of apparent ensembles with genuine ones, unless explicitly postulated so in an invariant form as a modification of unitary quantum dynamics.

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