A gluon condensate term in a heavy quark mass

V.V. Kiselev

Russian State Research Center "Institute for High Energy Physics",
Protvino, Moscow Region, 142281, Russia.
Fax: +7-0967744739

We investigate a connection between a renormalon ambiguity of heavy quark mass and the gluon condensate contribution into the quark dispersion law related with a virtuality defining a displacement of the heavy quark from the perturbative mass-shell, which happens inside a hadron.

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An Operator Product Expansion (OPE) is among the most powerful tools in the heavy quark physics. In this respect it is usually applied in the form of series in the inverse heavy quark mass, determining the characteristic energy scale, say, in sum rules or for decays etc. [1]. It is well recognized that the Wilson coefficients standing in front of quark-gluon operators can contain the uncertainty caused by the factorization of perturbative contribution and the nonperturbative matrix elements of composite operators. In this case the restriction on internal virtualities in Feynman diagrams has to be introduced to control the dependence on an “infrared” energy scale \( \lambda \). Usually, the gluon propagator is modified by replacement: \( 1/k^2 \to 1/(k^2 - \lambda^2) \) or the cut off the gluon momenta is performed as \( k^2 > \lambda^2 \) [2]. The calculation results depend on these parameters. Say, a peculiar behaviour at \( \lambda^2 \to 0 \) appears in physical quantities. For example, a perturbative correlator of two heavy quark currents acquires a power correction like \( \lambda^4/m^4 \), where \( m \) is the heavy quark mass [3]. Physically, it means that the OPE can be valid if we sum the perturbative and nonperturbative parts with the vacuum expectation of gluon operator which has the same low energy scale dependence: the gluon condensate \( \sim \lambda^4 \). Then the \( \lambda \)-dependent term can be adopted by an appropriate definition of OPE with the condensates. Another case takes place for the uncertainty in the heavy quark mass, where the perturbative calculation of self-energy with the gluon virtuality cut off leads to the linear term in \( \lambda \). However, there is no appropriate operator whose vacuum expectation is proportional to the first power of low energy scale [1]. It was shown that the mentioned uncertainty proportional to the powers of factorization scale \( \lambda \) can be related with the perturbative summation of higher order diagrams, which in the limit of infinitely large number of flavors has the divergency of series in \( \beta_0 \alpha_s \), where \( \beta_0 \) denotes the first coefficient of Gell-Mann–Low function in QCD. The Borel transform of such series has some peculiar points, which provide the uncertainty in the inverse transformation. This uncertainty, related with the divergency of perturbative series is called the renormalon [4], since the physical contents of such fact is clarified by the representation, where the series are combined in the running of QCD coupling constant dependent of the gluon virtuality. The coupling has the singularity, which is the indication of confinement. In this way, the uncertainty in powers of \( \Lambda_{QCD} \) appears again. Modern studies on the renormalon applications can be found in [5]. These facts imply that the OPE for fixed values of physical quantities (say, partial widths or coupling constants in the sum
rules) in terms of perturbative heavy quark mass results in the heavy quark mass, whose value extracted from the data, strongly depends on the order of calculation in $\alpha_s$-series \([1]\): the mass value is significantly changed from order to order.

Thus, the heavy quark quantities have the renormalon uncertainties connected to the infrared confinement in QCD. Some of them can be eaten by the appropriate definition of OPE with condensates. The heavy quark mass is of a special interest, since its infrared uncertainty cannot be straightforwardly adopted by the vacuum expectation of an operator with the dimension 1 in the energy scale.

In present paper we evaluate the gluon condensate contribution to the dispersion law of heavy quark. We find that the corresponding operator is divided by the third power of quark virtuality, which results in the appropriate dimension of term in the heavy quark mass. We discuss how this fact can be used to cancel the infrared uncertainty of mass.

We perform the calculation of diagram shown in Fig.1 in the technique of fixed-point gauge \([6]\) with the NRQCD propagators of heavy quarks \([7]\).

![Diagram](image.png)

**FIG. 1:** The diagram with the gluon condensate contribution to the two-point effective action of heavy quark.

The covariant form of two-point heavy quark effective action $\bar{h}_v \Gamma h_v$ can be represented as

$$\Gamma = p \cdot v - \frac{(p \cdot v)^2 - p^2}{2m} + \frac{\pi^2}{24} \frac{\alpha_s G_{\mu \nu}^2}{\pi} \left[ \frac{(p \cdot v)^2 - p^2}{m^2} \left( \frac{1}{p \cdot v - \frac{(p \cdot v)^2 - p^2}{2m}} \right)^3 + \frac{1}{m} \frac{1}{p \cdot v - \frac{(p \cdot v)^2 - p^2}{2m}} \right],$$

(1)

where $v$ denotes the four-velocity of hadron containing the heavy quark. The validity of (1) holds under the certain condition on the region of kinematical variables: the gluon condensate term in the dispersion law of quark is less than the leading contribution.

In the rest frame of hadron $v = (1, \mathbf{0})$ we have

$$p \cdot v - \frac{(p \cdot v)^2 - p^2}{2m} = p_0 - \frac{p^2}{2m} = \Delta E,$$

where $\Delta E$ denotes a heavy quark virtuality inside the hadron. The perturbative mass-shell is defined by the following expression:

$$\Delta E = 0.$$
It is quite clear that the confined quark cannot reach the mass-shell and there is a minimal
displacement from the surface of free quark motion, which is a nonperturbative quantity.
So, we suppose that
\[ \Delta E \sim \Lambda_{QCD}. \]

In what follows we apply the model with the quark dispersion law determined by the form
dictated by the account of gluon condensate in (1):
\[ p_0 = \omega_0 + \frac{p^2}{2\tilde{m}}, \] (2)

where again \( \omega_0 \sim \Lambda_{QCD} \) and \( \tilde{m} \) denotes the effective heavy quark mass, which differs from the
perturbative pole mass due to the contribution of gluon condensate. In the nonrelativistic
rest frame we have\(^1\)
\[ \Gamma = p_0 - \frac{p^2}{2\tilde{m}} + \frac{\pi^2}{24} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \left[ \frac{p^2}{m^2 \Delta E^3} + \frac{1}{m \Delta E^2} \right]. \] (3)

Then, we can derive that
\[ \tilde{m} = m + \frac{\pi^2}{12} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \frac{1}{\Delta E^3}. \] (4)

Eq.(4) shows that at \( \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \sim \Lambda_{QCD}^4 \) the contribution of gluon condensate to the heavy
quark mass is about \( \Lambda_{QCD} \), i.e. it is linear in the infrared scale of energy, when the operator
determining this term is of the fourth power in the scale.

Note, that the second term independent of \( p^2 \) in the gluon condensate contribution shown
in (3) results in the correction to the static energy of heavy quark, so that
\[ \delta\omega_0 = \frac{\pi^2}{24} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \frac{1}{m \Delta E^2}. \] (5)

Furthermore, the gluon condensate contributes to \( \omega_0 \) in two ways: the first one is explicitly
given by (4), the second is related with the redefinition of heavy quark mass \( (m \rightarrow \tilde{m}) \).
Indeed, in this case we have to redefine the “large” momentum of heavy quark by the
substitution for \( mv \) by \( \tilde{m}v \) and so on, which means that the resulting change of static
energy is given by
\[ \Delta\omega_0 = \tilde{m} - m + \delta\omega_0 \sim \Lambda_{QCD} \left( 1 + \kappa \frac{\Lambda_{QCD}}{2m} \right), \quad \kappa \sim 1. \]

Then, we can see that after the account for the gluon condensate the displacement of static
energy can be basically adopted in the mass \( \tilde{m} \).

Furthermore, we can write down the following relations for the perturbative dependence
of heavy quark quantities on the scale \( \lambda \):
\[ \frac{d m_{\text{pert}}}{d\lambda} = \frac{d\omega_0}{d\lambda} = \frac{d\Delta E}{d\lambda}, \] (6)

\(^1\) In NRQCD, where \( |p|/m < 1 \), the gluon condensate correction to the heavy quark action \( \Gamma \) tends to zero
at large virtualities \( Q = \Delta E \) as \( O(1/Q^2) \) and \( O(1/Q^3) \) for the static and dynamic terms, respectively.
However, the correction remains small even at lower scales.
where in the second equality we neglect the dynamical term and remain the static energy. Then the linear dependence on \( \lambda \) in \( m \) appears in two ways: the first is the direct calculation of self-energy diagram for the heavy quark, which results in

\[
\frac{dm^{(1)}}{d\lambda} = C_m \alpha_s(\lambda),
\]

and the second is contributing from the gluon condensate term due to the \( \Delta E \) dependence according to (4) and (6) (the vacuum condensate of gluon operator has the higher power: \( \lambda^4 \)), so that

\[
\frac{dm^{(2)}}{d\lambda} = -\frac{\pi^2}{4} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \frac{1}{\Delta E^4} C_m \alpha_s(\lambda).
\]

Then, we see that at \( \Delta E \approx \omega_0 \) the heavy quark mass can be physically independent on the introduction of factorization scale \( \lambda \), i.e.

\[
\frac{dm}{d\lambda} = \frac{dm^{(1)}}{d\lambda} + \frac{dm^{(2)}}{d\lambda} = 0,
\]

if

\[
\omega_0^4 = \frac{\pi^2}{4} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle.
\]

At \( \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle \approx (0.37 \text{ GeV})^4 \), the evaluation gives

\[
\omega_0 \approx 0.46 \text{ GeV}.
\]

Neglecting the dynamical term in the heavy quark virtuality we obtain the following estimate of displacement for the heavy quark mass due to the gluon condensate:

\[
\Delta m \approx \frac{1}{3} \omega_0 \approx 0.15 \text{ GeV},
\]

which can serve as the constrain of maximal value, since we expose the minimal virtuality.

Thus, the main statement on the nonperturbative displacement of heavy quark masses remains the following: it is about the confinement scale. However, we can get some definite estimates for these values.

The validity of above consideration is determined by the condition for the formation of hadron containing the heavy quark. Indeed, the time for the binding of the heavy quark, i.e. for the formation of its wavefunction, in general depends on the hadron contents. So, in the heavy-light hadron \( H_Q \) with a single heavy quark, the static energy for light degrees of freedom is of the order of \( \Lambda_{QCD} \), and we get the estimate

\[
\tau[H_Q] \sim \frac{1}{\Lambda_{QCD}},
\]

which is comparable with the characteristic time for the heavy quark interaction with the gluon condensate. In the doubly heavy hadron \( H_{QQ} \), the formation of wavefunction is determined by the average size of the doubly heavy system divided by the heavy quark velocity

\[
\tau[H_{QQ}] \sim \frac{r_{QQ}}{v_Q} \sim \frac{1}{m_Q v_Q^2},
\]

and it depends on the inverse kinetic energy in the doubly heavy subsystem. So, the calculated contribution by the gluon condensate in the heavy quark mass would be inapplicable.
for the quarks heavier than 20 GeV. However, in the systems composed by charmed and beauty quarks the kinetic energy is about $\Lambda_{QCD}$, and in the reality we deal with the situation, when the effects connected with the formation of wavefunction for the hadron containing the heavy quark and the gluon condensate term are competitive. So, for instance, the energy shift due to the interaction of coulomb doubly heavy system with the gluon condensate is determined by the following expression:

$$\Delta E_{QQ} = \frac{\pi^2}{18} \left( \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right) \frac{n^2 m}{(mE_n)^2} \epsilon_{nl}, \quad E_n = -\frac{1}{4n^2} \left( \frac{2}{3} \alpha_s \right)^2 m,$$

(8)

where $\epsilon_{nl}$ is a rational factor depending on the principal and radial quantum numbers $n$ and $l$. Comparing (8) with (4), we see that despite different approaches these two equations can be in agreement with each other, if we substitute of $\Delta E \sim E_n$, i.e. if the virtuality is determined by the bound energy of the heavy quark in the heavy quarkonium system. This fact implies that the heavy quarkonium represents a specific case of the general consideration applied to the coulomb system, wherein the virtuality is prescribed to a concrete value. As for the numerical estimates, we have to take into account, that one should use (4) instead of (8), if the virtuality of the heavy quark in the quarkonium is less than the value following from the general form of dispersion law for the quark, i.e. it is less than 0.46 GeV. Otherwise, the quark is heavy enough in order to use the coulomb approximation of (8).

To conclude, we have shown that the Operator Product Expansion including the gluon condensate results in the following dispersion law for the heavy quark:

$$p_0(p) = \omega_0 + \frac{p^2}{2m},$$

where the correction to the heavy quark mass is given by

$$\Delta m = \frac{\pi^2}{12} \left( \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right) \frac{1}{\omega_0^4},$$

and the infrared ambiguity in the mass caused by the corresponding renormalon, can be cancelled at

$$\omega_0^4 = \frac{\pi^2}{4} \left( \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right).$$

Of course, the conclusion is drawn to the given, linear order in $\alpha_s$, and the well known divergency of heavy quark pole mass with the increase of $\alpha_s$-order probably can be removed, if the higher order corrections to the Wilson coefficient of gluon condensate as well as the higher condensates will be included into the consideration in the same manner.

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