The equilibrium of dense plasma in a gravity field

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March 19, 2022

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PACS: 71.10.-w

Abstract

The equilibrium of dense plasma in a gravity field and problem of a gravity-induced electric polarization in this matter are discussed. The calculation for metals performed before shows that both - the gravity-induced compressive strain and the gravity-induced electric field - are inversely proportional to their Young moduli. The calculation for high dense plasma, where Young modulus is equal to zero, shows that there is another effect: each cell of this plasma inside a celestial body in own gravity field obtains the small positive electric charge. It happens as heavy ions sag on to light electron clouds. A celestial body stays electrically neutral as a whole, because the negative electric charge concentrates on its surface. The gravity-induced positive volume charge is very small, its order of magnitude equals to $10^{-18}e$ per atom only. But it is sufficient for the complete counterbalancing of the gravity force.


1 The action of gravity on a metal and ultra-high density plasma

The action of gravity on atomic substances, particularly on metals, produces compressive strains which is inversely proportional to their Young moduli $Y$. The equilibrium equation of an atomic substance is

$$\gamma \vec{g} + \nabla P = 0$$

Where $\nabla P$ is the gradient of gravity-induced pressure, $\gamma$ is the density of a metal, $\vec{g}$ is the gravity acceleration.

The gravity-induced electric polarization in metal has often been a subject for investigation before (see, for example [2]).

The basic result of those investigations may be reduced to the statement that gravity induces inside a metal an electric field with an intensity of

$$\vec{E} \simeq \frac{nE_F m_i}{Y e} \vec{g},$$

Where $m_i$ is the mass of an ion, $e$ is an electron charge, $n$ and $E_F$ is the density and Fermi-energy of electrons.

Under conditions of the Earth, this field is so small that it is not possible to measure it experimentally.

It is a direct consequence of the presence of an ion lattice inside metal. This lattice is deformed under the action of gravity, depending on its Young modulus, and then the electron gas adapts its density to this deformation, depending on its compressibility $nE_F$.

As under the action of ultra-high pressure, all substances transform into high density electron-nuclear plasma. For this state Young modulus $Y = 0$ and the action of gravity on plasma demands a special consideration.

First, let us mentally divide plasma into spherical cells. The volume of a cell must be equal to a volume of plasma related to an ion. The radius $r_s$ of such a spherical cell in plasma with the mass density $\gamma$ and the density of electrons are given by

$$\left(\frac{\gamma}{m_i}\right)^{-1} = \frac{4\pi}{3} r_s^3 = \frac{Z}{n}.$$
Where $Z$ and $A$ are the charge and atomic number of nucleus, $m_i = A m_p$ is the mass of nucleus, $m_p$ is the mass of proton, and the density of electrons

$$n = \frac{3Z}{4\pi r_s^3}. \quad (4)$$

Now let us write down the equilibrium condition of plasma. Here, in nuclear subsystem, the direct interaction between nuclei is absent. Therefore the equilibrium of nuclear subsystem of $eN$-plasma (at $T=0$) looks like

$$\mu_i = Z e \Phi + m_i \Psi = \text{const.} \quad (5)$$

Where $\Psi$ is the potential of a gravitational field, $\Phi$ is the potential of electric field.

The direct action of gravitation on electrons can be neglected due to their small mass. Therefore, the equilibrium condition in electron gas obtains the form

$$\mu_e = \varepsilon F = \frac{p_F^2(r)}{2m} - (e - \delta q) \Phi = \text{const.} \quad (6)$$

By introducing the charge $\delta q$, we take into account that the charge of an electron cloud inside a cell can differ from $Ze$. A small number of electrons can stay on the surface of a plasma body where the electric potential is absent. As a result, the charge of a cell, subjected to the action of the electric potential, is effectively decreased by a small value $\delta q$. If the radius of a star $R_0$ is approximately $10^{10}$ cm, one can expect that this mechanism gives an order of magnitude $\delta q_e \approx \frac{r_s}{R_0} \approx 10^{-18}$.

The electric potential inside each cell consists both of the potential of a considered cell $\varphi(r)$ and the potential, which is induced by other cells $\varphi(R)$:

$$\Phi = \varphi(r) + \varphi(R). \quad (7)$$

The electrostatic potential of the arising field is determined by the Gauss law

$$\frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d}{dr} \Phi \right] = -4\pi [Ze\delta(r) - en(r)]. \quad (8)$$

where the position of nuclei is described by the function $\delta(r)$.\ 3
2 The Thomas-Fermi approximation

According to the Thomas-Fermi method, the density of electrons is approximated by

\[ n(r) = \frac{8\pi}{3h^3p_F^3}(r). \]  

(9)

With this substitution, Eq.(8) is converted into a nonlinear differential equation for \( \Phi \), which for \( r > 0 \) is given by

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi}{dr} \right) = 4\pi \left[ \frac{8\pi}{3h^3} \right] [2m_e(\mu_e + (e - \delta q)\Phi)]^{3/2}. \]  

(10)

It can be simplified by introducing the following variables [3]

\[ \mu_e + (e - \delta q)\Phi = Ze^2 u \]  

(11)

and \( r = ax \).

Where \( a = \left( \frac{9\pi^2}{128Z} \right)^{1/3} a_0 \),

and \( a_0 = \left( \frac{\hbar^2}{me} \right) \) is the Bohr radius.

With an allowance for Eq.(11)

\[ Ze^2 \frac{u}{r} = \text{const} - \frac{m_i}{Z} - \delta q\Phi. \]  

(12)

then Eq.(10) is transformed to

\[ \frac{d^2 u}{dx^2} = \frac{u^{3/2}}{x^{1/2}}. \]  

(13)

In terms of \( u \) and \( x \), the electron density within a cell is given by

\[ n_x = \frac{8\pi}{3h^3p_F^3} = \frac{32Z^2}{9\pi^3a_0^3} \left( \frac{u}{x} \right)^{3/2}. \]  

(14)

3 The gravity-induced electric charge of a cell of ultra-high density plasma

The full charge of a cell under absence of gravitation is zero. Under influence of gravitation, the charge of the electron gas in a cell becomes equal to
\[ Q_e = 4\pi e \int_0^{r_s} n(r)r^2dr = \frac{8\pi e}{3h^3} \left[ 2m \frac{Ze^2}{a} \right]^{3/2} 4\pi a^3 \int_0^{r_s} x^2dx \left[ \frac{u}{x} \right]^{3/2}. \] (15)

Using Eq. (13), we obtain

\[ Q_e = Ze \int_0^{x_s} xdx \frac{d^2u}{dx^2} = Ze \int_0^{x_s} dx \frac{d}{dx} \left[ x \frac{du}{dx} - u \right] = Ze \left[ x \frac{du}{dx} \bigg|_{x_s} - u(x_s) + u(0) \right]. \] (16)

At \( r \to 0 \) the main part of electric potential is due to nuclei alone \( \Phi \to \frac{Ze}{r} \). It means that \( u(0) \to 1 \) and each cell of plasma obtains a small charge

\[ \delta q = Ze \left[ x \frac{du}{dx} \bigg|_{x_s} - u(x_s) \right] = Ze x_s^2 \left[ \frac{d}{dx} \left( \frac{u}{x} \right) \right]_{x_s}. \] (17)

If a radius of a cell can be a function of some parameters, Eq. (17) transforms to

\[ \delta q = Zer_s^2 \frac{d}{dr_s} \left( \frac{u(r_s)}{r_s} \right). \] (18)

When the charge of a cell depends on its location inside a star

\[ \delta q = Zer_s^2 \left[ \frac{d}{dR} \left( \frac{u(r_s)}{r_s} \right) \right] \left[ \frac{dR}{dr_s} \right]. \] (19)

Because inside a spherically symmetric star the gravity acceleration is

\[ \vec{g} = -\frac{d\Psi}{dR} \frac{\vec{R}}{R} \] (20)

and the electric field intensity is

\[ \vec{E} = -\frac{d\Phi}{dR} \frac{\vec{R}}{R} \] (21)

from Eq. (19)

\[ \frac{dr_s}{dR} \frac{\vec{R}}{R} = \frac{r_s^2}{e\delta q} \left[ m_i \vec{g} + \delta q \vec{E} \right]. \] (22)
This equation has the following solution

\[
\frac{dr_s}{dR} = 0 \quad (23)
\]

and

\[
\frac{m_i}{Z} \vec{g} + \delta q \vec{E} = 0. \quad (24)
\]

As

\[
div \vec{g} = -4\pi G n m_i \quad (25)
\]

and

\[
div \vec{E} = 4\pi n \delta q \quad (26)
\]

the gravity-induced electric charge in a cell is very small

\[
\delta q = \sqrt{G} \frac{m_i}{Z} \simeq 10^{-18} e, \quad (27)
\]

where \( G \) is the gravitation constant.

In plasma the equilibrium value of the electric field on the nuclei is approximately equal to \( gm_i/e \). This value is, like that in a metal, very small. But there is one additional effect in plasma. Simultaneously with the confinement of nuclei in equilibrium, each cell obtains an extremely small electric charge. However, because the size of bodies is very large, the electric field intensity may be very large as well

\[
\vec{E} = \frac{\vec{g}}{\sqrt{G}} \quad (28)
\]

Thus, a celestial body is electrically neutral as a whole, because the positive volume charge is concentrated inside the charged core and the negative electric charge exists on its surface and so one can infer gravity-induced electric polarization of a body.

In accordance with Eqs. (23)-(24) the action of gravity here is completely compensated by the electric force and the pressure gradient is absent. Instead of Eq. (1) which is correct for atomic substances, these equations describe the equilibrium into celestial bodies which consist of high dense plasma.
References

[1] Landau L.D. and Lifshits E.M. - Theory of Elasticity, 3rd edition, Oxford: Pergamon, 1984.

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[3] Leung Y.C. - Physics of Dense Matter, 1984, Science Press/World Scientific, Beijing and Singapore.