The Detectability of Radio Auroral Emission from Proxima b

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Abstract

Magnetically active stars possess stellar winds whose interactions with planetary magnetic fields produce radio auroral emission. We examine the detectability of radio auroral emission from Proxima b, the closest known exosolar planet orbiting our nearest neighboring star, Proxima Centauri. Using the radiometric Bode’s law, we estimate the radio flux produced by the interaction of Proxima Centauri’s stellar wind and Proxima b’s magnetosphere for different planetary magnetic field strengths. For plausible planetary masses, Proxima b could produce radio fluxes of 100 mJy or more in a frequency range of 0.02–3 MHz for planetary magnetic field strengths of 0.007–1 G. According to recent MHD models that vary the orbital parameters of the system, this emission is expected to be highly variable. This variability is due to large fluctuations in the size of Proxima b’s magnetosphere as it crosses the equatorial streamer regions of dense stellar wind and high dynamic pressure. Using the MHD model of Garraffo et al. for the variation of the magnetosphere radius during the orbit, we estimate that the observed radio flux can vary nearly by an order of magnitude over the 11.2-day period of Proxima b. The detailed amplitude variation depends on the stellar wind, orbital, and planetary magnetic field parameters. We discuss observing strategies for proposed future space-based observatories to reach frequencies below the ionospheric cutoff (∼10 MHz), which would be required to detect the signal we investigate.

Key words: planet–star interactions – planets and satellites: magnetic fields – stars: low-mass – stars: winds, outflows

1. Introduction

All planets with substantial magnetic fields in our solar system strongly emit coherent low-frequency radio cyclotron emission associated with auroral activity. This radio emission is produced by magnetospheric electrons propagating around magnetic field lines (Gurnett 1974; Zarka 1992) and is proportional to the magnetic and kinetic energy around the radius of the planetary magnetosphere. Although radio emission from planetary aurora is well-studied in the solar system, it has yet to be detected for exoplanetary systems (Grießmeier et al. 2007; Luger et al. 2017). Detecting radio emission from exoplanets would allow for further constraints on their orbital parameters (Luger et al. 2017), open up new detection methods at radio wavelengths (Farrell et al. 1999), and allow us to measure exoplanet magnetospheres for the first time (Shkolnik et al. 2008; Vidotto et al. 2010; Kisl’yakova et al. 2014). Furthermore, measurements of magnetic fields via auroral emission would also shed light on exoplanet habitability, plate tectonics, and atmospheric composition (Lundin et al. 2007; Grießmeier et al. 2016; Luger et al. 2017).

The discovery of the exosolar planet Proxima b orbiting in the potential habitable zone of our nearest stellar neighbor Proxima Centauri (Anglada-Escudé et al. 2016) presents the closest (1.3 pc away) candidate for possibly detecting an exo-aurora. Proxima b is estimated to be at least 1.3 Earth masses (with an inclination-corrected mass of $M \sin(i) = 1.3 M_\oplus$) and has an orbital period of 11.2 days. Its orbit has a semimajor axis of 0.049 au, 20 times closer to Proxima Centauri than the Earth is to the Sun (Anglada-Escudé et al. 2016). Due to its 1.3 pc distance from our solar system, Proxima b could be directly observable by the next generation of space telescopes such as the Extremely Large Telescope (ELT), WFIRST, and JWST (Barnes et al. 2016; Kreidberg & Loeb 2016; Luger et al. 2017), and has been the subject of studies examining its likely irradiation history, possible climate, and evolution in relation to potential habitability (Barnes et al. 2016; Meadows et al. 2016; Turbet et al. 2016; Ribas et al. 2017).

Garraffo et al. (2016) studied the space weather of Proxima b using numerical MHD simulations of the stellar wind from Proxima Centauri. The authors examined the wind conditions (densities, pressures, and magnetic field strength) over different plausible orbits of Proxima b. They found that the planet is subject to stellar wind pressures of more than 2000 times those experienced by Earth from the solar wind. Magnetic activity can be enhanced due to interactions between close-in planets and their host stars, similar to the interaction of the Io-Jupiter system (Crary & Bagenal 1997). Based on MHD models and solar system examples, it is therefore likely that, if Proxima b has an Earth-like magnetic field, auroral emission could be detectable from low-frequency space-based radio arrays such as the proposed Radio Observatory on the Lunar Surface for Solar studies (ROLSS; Lazio et al. 2011; Zarka et al. 2012). Future ground-based telescopes, such as an ELT, could possibly observe the exo-aurora of Proxima b using optical emission lines (Luger et al. 2017).

In this paper we examine the detectability of radio auroral emission from Proxima b. Using the radiometric Bode’s law, we estimate the expected radio flux and frequency emitted from Proxima b and compare that to previous estimates for other exoplanets in Section 2. In Section 3 we calculate the expected radio light curves for Proxima b based on the space weather modeling of Garraffo et al. (2016) and for varying orbital parameters. Finally, in Section 4 we discuss our results, followed by a summary of our conclusions in Section 5.
2. The Radiometric Bode’s Law: Stellar-wind-driven Cyclotron Emission

First, we address the expected order-of-magnitude radio flux density from low-frequency radio observations of the Proxima b system. To estimate the radio power being emitted from the interaction of the stellar wind and Proxima b’s magnetosphere we use the radiometric Bode’s law for the solar system, where the planetary radio power, \( P_{\text{radio}} \), can be estimated from the power released by the dissipation of stellar wind kinetic or magnetic energy (Desch & Kaiser 1984). Several variants on Bode’s law exist in the literature and provide an empirical scaling relation for observed radio power to planetary quantities such as planetary mass \( m \), magnetic moment \( \mathcal{M} \), rotation speed \( \omega \), planetary radius \( r \), and planetary distance from the host star \( d \).

While the radiometric Bode’s law is an empirical result derived for the solar system, its application for detecting radio emission from exoplanets has been discussed extensively (Desch & Kaiser 1984; Zarka 1992; Farrell et al. 1999, 2004; Lazio et al. 2004). The radio auroral power for a planetary magnetosphere increases in proportion to the size of the magnetosphere cross section, which depends on the planetary magnetic moment \( \mathcal{M} \). Assuming the magnetic moment scales with planetary mass and rotation speed, e.g., via so-called Blackett’s law (Blackett 1947), \( \mathcal{M} \propto \omega^2 \). The radiometric Bode’s law based on solar system scalings proposed by Zarka (1992) and Lazio et al. (2004) can be written as

\[
P_{\text{radio}} \approx 4 \times 10^{11} \left( \frac{\omega}{\omega_j} \right)^{0.79} \left( \frac{m}{M_j} \right)^{1.33} \left( \frac{d}{d_j} \right)^{-1.6} \text{W},
\]

with all planetary qualities normalized to Jupiter values.

We adopt fiducial values for Proxima b of \( d = 0.049 \) au and a mass of \( m = 2M_{\oplus} \). Proxima b is most likely tidally locked due to its close-in orbit, therefore we assume the rotation period is the same as the orbital period of \( \omega = 11.2 \) days. Using these values, Equation (1) gives a predicted radio power for Proxima b of \( P_{\text{radio,prox}} = 1 \times 10^{13} \) W.

In order to calculate the observed radio flux we consider a plausible range of planetary magnetic field values for Proxima b \( B_P \) within an order of magnitude of Earth’s magnetic field. Ultimately, the magnetic field of Proxima b is unknown, and furthermore, the location of auroral radio emission may vary strongly with latitude and distance from the planet (Zarka 1998). For these reasons we consider \( B_P = 0.007–1 \) G. The strongest field strength we consider may be possible for Proxima b but is likely to be transient (Driscoll & Barnes 2015).

Our chosen range implies cyclotron frequencies of \( f = eB/2\pi m_e = 0.02–2.8 \) MHz.

The radio flux density is given by

\[
\Phi = \frac{P_{\text{radio}}}{4\pi d^2} \text{W m}^{-2},
\]

where \( \delta f = f/2 \) and the emission cone\(^1\) is in \( 4\pi \text{sr} \). Using magnetic field values of \( B_P = 1–0.007 \) G yields a flux density of \( \Phi_{\text{proxima}} = 41 \) mJy to over 1 Jy, respectively, for Proxima b.

\[\text{Equation (2) is written in the form as it appears in Lazio et al. (2004) for comparison with their Figure 1. We note that planetary radio emission is typically more focused in} \]

\[\text{in either two-lobe or hollow cone patterns, which can increase the radio flux. However, this makes the orientation of the dipole field and orbital inclination relative to the observer more crucial (Tilley et al. 2016).}\]

\[\text{We note that currently there is no consensus on whether the generation of strong, dipole magnetic fields occurs in “super-Earths” due to uncertainty in mantle convection/tectonic processes (Korenaga 2012; Driscoll & Barnes 2015), hence Bode’s law may not be applicable to these planets.}\]
expected average $\sim 600$ G surface magnetic field strength of Proxima Centauri. They found that Proxima b’s magnetopause standoff distance, i.e.,

$$r_m = \left( \frac{B_p^2}{4\pi P_W} \right)^{1/6} R_p,$$

(3)

where $P_W$ is the ram pressure of the stellar wind, $R_p$ is the planetary radius ($R_{p,\text{prox}} = 1.1 R_{\odot}$), and $B_p$ is the planetary magnetic field strength, undergoes sudden and periodic changes by a factor of 25 and is bombarded with a solar wind pressure and density of the order of $10^4$ times that of Earth. The exact amplitude change depends on the choice of the input $B_p$ as well as the orbital inclination, $i$, between the orbit of the planet and the rotational axis of the star. These rapid changes in $r_m$ occur twice each orbit as the planet passes through the equatorial streamer regions of the dense wind and high dynamic pressure on a timescale as short as a day (see e.g., Figure 5 of Garraffo et al. 2016). We therefore adopt $r_m$ fluctuations during Proxima b’s orbit to make a more physically realistic assessment of the auroral radio flux.

Due to the high dynamic pressure of the stellar wind and relatively weak planetary magnetic field, the net dissipated power on the planet dayside of Proxima b is expected to be larger than that in the reconnection region of the magnetotail on the planet’s nightside by more than an order of magnitude (Varela et al. 2016). For exoplanets that experience a stellar wind of lower dynamic pressure and have more intense magnetic fields, the configuration can be the opposite, leading to stronger radio emission from the reconnection region of the magnetotail and weaker radio emission from the planet’s dayside.

For the purposes of the estimations in this paper, we will consider that most of the radio emission is coming from the dayside of Proxima b. Figure 2 shows the basic schematic view of the system.

The dissipated magnetic and kinetic powers of the impacting wind on the planet are approximated as

$$P_B = \frac{B_p^2 v_w^2}{4\pi},$$

(4)

$$P_k = \rho_w v_w^3 R_m^2,$$

(5)

where $v_w$ is the relative velocity between the wind and the Keplerian velocity of the planet, $\rho_w$ is the local wind density, and $B_p$ is the interplanetary magnetic field component perpendicular to the solar wind flow impinging on the planet’s magnetosphere (Zarka 2007; Vidotto & Donati 2017).

In order to estimate the radio power being emitted from the interaction of the stellar wind and Proxima b’s magnetosphere, we adopt the model of Garraffo et al. (2016) for the variation of $r_m$ as a function of orbital period and again apply the radiometric Bode’s law for the solar system as given in Vidotto et al. (2010) an Vidotto & Donati (2017). The planetary auroral radio power ($P_{\text{radio}}$) can be estimated from the power released by the dissipation of stellar wind kinetic power, $P_k$, and/or the power released from the dissipation of magnetic power, $P_B$, of the wind (Farrell et al. 1999; Zarka et al. 2001; Zarka 2007; Vidotto et al. 2010). We consider both here for completeness; however, we expect that the magnetic power is likely to be dominant for converting flow power to energetic particles (Zarka 2010; Vidotto & Donati 2017).

The radiometric Bode’s law is then given as

$$P_{\text{radio},k} = \nu_k P_k,$$

(6)

and

$$P_{\text{radio},B} = \nu_B P_B,$$

(7)

where $\nu_k$ and $\nu_B$ are efficiency ratios whose values we adopt from the solar system data (Zarka 2007; Vidotto & Donati 2017), $\nu_k = 1 \times 10^{-5}$ and $\nu_B = 2 \times 10^{-3}$.3

Based on the strong field model of Garraffo et al. (2016), we assume $v_w = 1600$ km s$^{-1}$, $\rho_w/m_p = 1000$ cm$^{-3}$ and $B_p = 0.01$ G (representative of a 1200 Gauss dipole field at the surface of a star with radius of 0.2 $R_{\odot}$ at a distance of Proxima b). In agreement with Garraffo et al. (2016), our parameters produce a wind dynamic pressure in excess of the wind magnetic pressure by about an order of magnitude. We investigate four possible values of Proxima b’s magnetic field, $B_p = 0.007, 0.1, 0.3, 1$ G, and scale the values of $r_m$ versus orbital phase given in Garraffo et al. (2016) according to Equation (3). We then apply Equations (4)–(7) and calculate the expected radio fluxes as viewed from Earth.

Figures 3 and 4 show the variation in the observed radio flux versus orbital phase for orbital inclination, $i$, between the orbit of the planet and the rotational axis of the star. The top panels show a planetary eccentricity of $e = 0$ and the bottom panels show $e = 0.2$. While there is a dependency on orbital eccentricity, the dominant dependency of the light curve on orbital parameters is on the inclination between the orbit and the stellar rotation axis. Figure 4 adopts $i = 10$, while Figure 3 assumes $i = 60$. Magnetic flux (Equation (4)) is shown in the left panel and kinetic flux (Equation (5)) is shown in the right panel. The overall radio flux output from the magnetic power

3 We stress that these efficiency coefficients are based on data from our solar system and could be different for M-dwarf stars. However, the overall estimated mass-loss rate from Proxima Centauri is estimated to be no more than an order of magnitude higher than that of the Sun (Wood et al. 2001).
the star of Garaffo et al. varies over the planetary orbital period of 11.2 days. We use the MHD model and

\[
\alpha_i \propto n^{-3/2} \nu^{-2} \eta^2 \text{ cm}^{-1},
\]

where \( n \) is the interstellar density in \( \text{cm}^{-3} \), \( T \) is the temperature in K, and \( \nu \) is the frequency of the radiation in Hz. Figure 5 shows the free–free absorption from the local ISM integrated along the line of sight to Alpha Centauri (1.3 pc) as a function of observing frequency for two different values of ISM density and \( T = 7000 \text{ K} \). We also include the absorption by the interplanetary medium at a temperature of \( T = 1 \times 10^6 \text{ K} \) and a density of \( n = 1000 \text{ cm}^{-3} \) integrated over 1 au around the Proxima system. Lower-frequency emission (corresponding to lower values of magnetospheric magnetic field strength) is more likely to be absorbed, owing to the scaling of \( \alpha_i \propto n^{-3/2} \nu^{-2} \eta^2 \). Due to the proximity of the Alpha Centauri system, free–free absorption is not problematic for frequencies above 0.3 MHz. However, for more distant planetary systems (e.g., the TRAPPIST-1 system at a distance of 10 pc) low-frequency observations will be absorbed by the ISM.

The Low Frequency Array (LOFAR)\(^4\) provides the possibility of studying weak radio emission from exoplanets with high sensitivity and resolution for frequencies between 10 and 240 MHz. This corresponds to cyclotron emission for magnetic fields down to strengths of 4 G. For lower magnetic field strengths, including the probable field strengths of Earth-like planets such as Proxima b, even arrays operating at lower frequency will be required. However, Earth’s ionosphere significantly impedes any ground-based observations below 10 MHz, the plasma frequency cutoff of the ionosphere.

In order to avoid attenuation by the ionosphere, extremely low-frequency observations for detection of Earth-like magnetic fields from exoplanets must be done from space. A variety of observatory concepts have been proposed for ultra-low-frequency arrays on the lunar surface (Zarka et al. 2012). One such proposal is ROLSS (Lazio et al. 2011). Alternate proposals for the lunar surface include the Sun Radio Imaging Space Experiment Mission (Lazio et al. 2017) and other large arrays of CubeSats with dipole electric field antennas (Rajan et al. 2016). Our study provides additional motivation for the construction of a space-based low-frequency array with very long baselines. For Proxima b, our study shows that the main objective should not necessarily be to resolve the emission but rather to have the sensitivity to observe the variability in the radio flux, which would indicate the presence of a varying magnetosphere under the influence of the stellar wind of Proxima Centauri. Additional scientific goals of a space-based low-frequency array include studies of the dark ages and epoch of re-ionization via detection of the redshifted 21 cm line (Lazio et al. 2011; Fialkov & Loeb 2013; Rajan et al. 2016).

While observations of radio emission for exoplanets below 10 MHz are not feasible with current observatories, auroral signatures could also be detectable via optical emission. Toward this goal, Luger et al. (2017) examined the feasibility of detecting auroral emission from Proxima b to constrain the presence and composition of its atmosphere in addition to determining the planet’s eccentricity and inclination. Luger et al. (2017) searched the Proxima b HARPS data for any oxygen or nitrogen lines near 5577 Å but found no signal, indicating that the OI auroral line contrast must be lower than 2 \( \times 10^2 \), which was consistent with their model predictions. Optical studies of Proxima b’s possible auroral signatures therefore will also need to wait for future observatories. A space-based coronagraphic telescope or a ground-based ELT with a coronagraph could push sensitivity down to line

\(^4\) http://www.lofar.org/
The magnetic energy dissipation and hence the amount of radio power that will be observed from auroral reconnection sites is partially controlled by the orientation of the interplanetary magnetic field. We have assumed 100% reconnection efficiency on the dayside, which is the case for only the most optimistic configurations. Nevertheless, we have shown that the radio flux can be substantial due to the varying size of Proxima b’s magnetosphere as it crosses regions of high stellar wind pressure and density. Using the MHD model of Garraffo et al. (2016) for the variation of the magnetosphere radius during the orbit, we estimate that the observed radio flux can vary by an order of magnitude over the 11.2-day period of Proxima b.

3. The amplitude of the variation depends on orbital parameters as well as the parameters of the stellar wind and planetary magnetic field. This implies that these parameters can be discerned from the detection of auroral variations and detailed MHD modeling.

4. We discuss a number of caveats regarding the detection of radio emission from Proxima b, namely that ultra-low-frequency observatories must be constructed in space in order to avoid blocking by the ionosphere. We show that systems to relativistic speeds ($v \approx 0.2c$). This will enable the spacecraft to reach the nearest stars, like Proxima Centauri, within a human lifetime. Precise orbital measurements, possibly provided by the detection of optical or radio auroral emission, of Proxima b will be important for realizing a close fly-by of this planet that includes images and direct measurements of its magnetic field and atmospheric properties.

5. Conclusions

In this work we have used the radiometric Bode’s law for the solar system to investigate the possibility of detecting radio auroral emission coming from Proxima b, the closest known exoplanet orbiting our nearest neighboring star, Proxima Centauri. We have found the following.

1. Due to its proximity both to Earth and its host star, Proxima b could produce radio fluxes of 100 mJy or more in a frequency range of 0.02–3 MHz for planetary magnetic field strengths of $B = 0.007–1$ G.

2. Based on recent MHD models, the auroral emission should also be highly variable due to the varying size of Proxima b’s magnetosphere as it crosses regions of high stellar wind pressure and density. Using the MHD model of Garraffo et al. (2016) for the variation of the magnetosphere radius during the orbit, we estimate that the observed radio flux can vary by an order of magnitude over the 11.2-day period of Proxima b.

3. The amplitude of the variation depends on orbital parameters as well as the parameters of the stellar wind and planetary magnetic field. This implies that these parameters can be discerned from the detection of auroral variations and detailed MHD modeling.

4. We discuss a number of caveats regarding the detection of radio emission from Proxima b, namely that ultra-low-frequency observatories must be constructed in space in order to avoid blocking by the ionosphere. We show that

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5 https://breakthroughinitiatives.org/Initiative/3
free–free absorption by the local ISM will hinder detection for frequencies below 0.3 MHz.

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