CONSISTENT INTERACTIONS BETWEEN GAUGE FIELDS: THE COHOMOLOGICAL APPROACH

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Abstract. The cohomological approach to the problem of consistent interactions between fields with a gauge freedom is reviewed. The role played by the BRST symmetry is explained. Applications to massless vector fields and 2-form gauge fields are surveyed.

1. Introduction

The BRST symmetry was originally discovered in a purely quantum context [1, 2]. It was realized only later that it has also a useful classical interpretation. This was done first in the Hamiltonian context developed by Fradkin and his school [3, 4, 5, 6], where it was shown that the BRST cohomology can be related to the Hamiltonian reduction of the system [7, 8, 9, 10, 11, 12, 13]. This cohomological understanding of the BRST symmetry enabled one to provide as a by-product complete proofs of the existence of the BRST generator for an arbitrary gauge system (subject to definite regularity conditions) [12, 14], paying due account to the global phase space features (the original proofs [7, 15, 16], based on some particular local representation of the constraint surface, were only local).

More recently, it has been observed in the Lagrangian context that the classical problem of consistently introducing interactions in a gauge theory can also be usefully reformulated in terms of the BRST differential and the BRST cohomology [17] (see also [18]). The use of cohomological ideas systematizes the search for all possible consistent interactions and, moreover, relates obstructions to deforming a gauge-invariant action to precise cohomological classes of the BRST differential.

The purpose of this review article is to survey the BRST approach to the problem of consistent interactions. Applications of the general theory to massless vector fields and 2-form gauge fields are also reviewed. Other applications (with references to the original literature) are listed at the end.

Key words and phrases: Consistent interactions, BRST cohomology, Deformation theory
2. The problem of consistent interactions between gauge fields

2.1. Consistent interactions for a set of $U(1)$-gauge fields

We shall develop the theory mostly by means of examples. Consider a set of $N$ massless abelian vector fields $A^a_\mu$ described each by the familiar Maxwell action. The free action is thus

$$I_0 = -\frac{1}{4} \int d^n x F^a_{\mu\nu} F^a_{\mu\nu}, \quad a = 1, \ldots, N$$  \hfill (2.1)

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu.$$  \hfill (2.2)

The action (2.1) is invariant under the gauge transformations

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a,$$  \hfill (2.3)

which close according to an abelian algebra. The gauge symmetry of (2.1) is quite important since it removes the unphysical (longitudinal and temporal) degrees of freedom. The free equations of motion are

$$\frac{\delta I_0}{\delta A^a_\mu} = \partial_\nu F^a_{\nu\mu} = 0.$$  \hfill (2.4)

We shall assume that the spacetime dimension is strictly greater than 2 (in 2 dimensions, the theory has no local degree of freedom).

The question addressed here is whether one can add interaction terms to the action (2.1) in a manner that maintains the number, but not necessarily the form, of the gauge transformations. In other words, we want to deform the free action by adding to it interaction terms

$$I_0 \to I_0 + g I_1 + g^2 I_2 + \ldots$$  \hfill (2.5)

and to deform simultaneously the gauge symmetries

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a + g \Delta^a_\mu \epsilon^b + O(g^2)$$  \hfill (2.6)

in such a way that the deformed action is invariant under the deformed gauge transformations, at each order in the “coupling constant” $g$. The expansions (2.5) and (2.6) are a priori formal power series in $g$ and we shall not worry about convergence questions here. In most cases, however, the series terminate or can be made to terminate upon introduction of appropriate auxiliary variables. It is required that each term in the expansion (2.5) be a local functional, i.e., be the integral of a function of the vector potentials and their derivatives up to some finite order.

We insist that the deformed action should have the same number of gauge symmetries as the original, free, action, because the decoupling of the temporal and longitudinal modes, guaranteed by gauge invariance, appears to be essential for consistency. Although there is no theorem stating that this is the only possibility leading to a consistent theory, we shall consider only this case here. Other additional criteria may be imposed on the deformation (e.g., causal propagation, or preservation of some specific rigid symmetry\(^1\)) but we shall

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\(^1\)The inclusion of rigid symmetries can actually be also performed along BRST lines, see [19].
focus here only on gauge invariance. We shall call throughout “consistent deformations” the deformations that preserve gauge invariance (possibly in a deformed way).

Consistent interactions are easily constructed by taking for the interaction terms $I_i$ ($i > 0$) functions of the curvatures $F_{\mu\nu}^a$ and their derivatives. An example is given by Born-Infeld theory [20]. Because these terms are gauge-invariant under the gauge transformations (2.3) of the undeformed theory, they do not deform the gauge symmetry: the full action (2.5) is invariant under the original gauge symmetries.

More interesting deformations (from the algebraic point of view) are those that deform not only the action, but also the gauge transformations and their algebra. A well-known example is the Yang-Mills gauge theory in which the abelian symmetry (2.3) is replaced by a non-abelian one. We shall see below to what extent the Yang-Mills construction is unique.

2.2. Consistent Interactions For A Set Of Free Exterior 2-Forms

The same problem can be addressed for any gauge system. In particular, one may start with the free action describing a system of exterior 2-forms $B_{\mu
u}^A$ instead of (2.1)

$$I_0 = -\frac{1}{2 \cdot 3!} \int d^n x H_{\mu\nu\rho}^A H_A^{\mu\nu\rho}, \quad A = 1, \ldots, N$$

with

$$H_{\mu\nu\rho}^A = \partial_\mu B_{\nu\rho}^A + \partial_\nu B_{\rho\mu}^A + \partial_\rho B_{\mu\nu}^A.$$  

The gauge transformations are now

$$\delta \epsilon B_{\mu\nu}^A = \partial_\mu \epsilon_{\nu}^A - \partial_\nu \epsilon_\mu^A$$

and the equations of motion read

$$\frac{\delta I_0}{\delta B_{\mu\nu}^A} = \frac{1}{2} \partial_\lambda H_{\lambda\mu\nu}^A = 0.$$  

The new feature of this model, however, is that the gauge transformations cease to be independent. Indeed, if one takes as gauge parameters pure gradients,

$$\epsilon_\mu^A = \partial_\mu \lambda^A$$

one gets gauge variations of the fields that are identically zero.

Since the deformed theory should have the same number of independent gauge symmetries as the original one, we require that the deformation should also preserve the number of reducibility identities. That is, the deformed gauge symmetries should reduce to zero (at least on shell) when the gauge parameters are given by some definite deformation of (2.11),

$$\epsilon_\mu^A = \partial_\mu \lambda^A + g \Sigma_{\nu B} \lambda^B + O(g^2)$$

For the 2-form model, we shall assume that the spacetime dimension is strictly greater than 3 (in 3 dimensions, the theory has no local degree of freedom).
Again, one may easily construct consistent interactions for the system by adding terms that involve only the curvature components $H^{A}_{\lambda\mu\nu}$ and their derivatives. These interactions do not modify the gauge symmetries (2.9) since they are strictly gauge invariant. It turns out that contrary to the vector case, these are actually the only consistent interactions in spacetime dimensions $\geq 5$ (with, possibly, Chern-Simons terms that do not modify either the gauge symmetries, see below).

2.3. General Equations

To write down the general equations and to convey succinctly the main qualitative ideas, it is convenient to adopt condensed notations. The fields will be collectively denoted by $\phi^i$. So, in the vector case, $\phi^i \equiv A^a_{\mu}$, while in the 2-form case, $\phi^i \equiv B^A_{\mu\nu}$. The undeformed action $I_0[\phi^i]$ is a local functional of the fields, and the Euler-Lagrange equations are

$$\frac{\delta I_0}{\delta \phi^i} = 0.$$  (2.13)

The gauge symmetries of the undeformed theory are given by

$$\delta \epsilon \phi^i = R^{(0)i}_\alpha \epsilon^\alpha$$  (2.14)

where there is an implicit integration over spacetime in (2.14) – besides the summation over the index $\alpha$ –, and where $R^{(0)i}_\alpha$ is linear in $\delta(x, y)$ and (a finite number of) its derivatives (locality of the gauge transformations). Thus, (2.14) really stands for

$$\delta \epsilon \phi^i(x) = \int d^n y R^{(0)i}_\alpha(x, y) \epsilon^\alpha(y)$$  (2.15)

with

$$R^{(0)i}_\alpha(x, y) = r^{(0)i}_\alpha \delta(x - y) + r^{(0)i}_{\alpha \mu} \delta_{\mu}(x - y) + \cdots + r^{(0)i}_{\alpha \mu_1 \cdots \mu_s} \delta_{\mu_1 \cdots \mu_s}(x - y).$$  (2.16)

The coefficients $r^{(0)i}_\alpha, \ldots, r^{(0)i}_{\alpha \mu_1 \cdots \mu_s}$ are local functions. The invariance of the action under the gauge transformations (2.14) is equivalent to the so-called Noether identities

$$\frac{\delta I_0}{\delta \phi^i} R^{(0)i}_\alpha = 0$$  (2.17)

(identically), where again there is an implicit integration over spacetime ($\int dxd\delta I_0/\delta \phi^i(x) R^{(0)i}_\alpha(x, y) = 0$).

The deformations of the action and the gauge symmetries are given by

$$I_0 \rightarrow I \equiv I_0 + gI_1 + g^2I_2 + \ldots,$$

$$R^{(0)i}_\alpha \rightarrow R^{i}_\alpha \equiv R^{(0)i}_\alpha + gR^{(1)i}_\alpha + g^2R^{(2)i}_\alpha + \ldots.$$  (2.18, 2.19)

The same locality assumptions are made for the interacting model, namely, each term in the expansion of the action is a local functional, and each term $R^{(k)i}_\alpha$ in the expansion of the
gauge symmetries is linear in $\delta(x, y)$ and (a finite number of) its derivatives, with coefficients that are local functions.

Consistency of the deformations holds if and only if the Noether identities are fulfilled at each order in the deformation parameter $g$

$$\frac{\delta I}{\delta \phi^i} R^i_\alpha = 0.$$  \hspace{1cm} (2.20)

By expanding this condition in powers of $g$, one gets an infinite number of equations on the deformations of the action and the gauge symmetries.

If the original theory is reducible, i.e., if there exist choices of the gauge parameters $\epsilon^\alpha$, say $\epsilon^\alpha = Z_A^{(0)\alpha} \lambda^A$, such that the gauge variations of the fields are on-shell trivial,

$$Z_A^{(0)\alpha} R^{(0)i}_\alpha = \mu^{(0)ij}_A \delta I_0, \quad \mu^{(0)ij}_A = -\mu^{(0)ji}_A.$$  \hspace{1cm} (2.21)

then, the deformation should preserve this reducibility. Accordingly, one should be able to deform both $Z_A^{(0)\alpha}$ and $\mu^{(0)ij}_A$,

$$Z_A^{(0)\alpha} \rightarrow Z_A^{\alpha} = Z_A^{(0)\alpha} + gZ_A^{(1)\alpha} + g^2 Z_A^{(2)\alpha} + \ldots,$$

$$\mu^{(0)ij}_A \rightarrow \mu^{ij}_A = \mu^{(0)ij}_A + g\mu^{(1)ij}_A + g^2 \mu^{(2)ij}_A + \ldots$$  \hspace{1cm} (2.22)

in such a way that reducibility identities of the form (2.21) hold for the full theory,

$$Z_A^\alpha R^i_\alpha = \mu^{ij}_A \frac{\delta I}{\delta \phi^i}.$$  \hspace{1cm} (2.24)

Again, by expanding this condition in powers of $g$, one gets an infinite number of equations on the deformations of the “structure functions” $Z_A^\alpha$ and $\mu^{ij}_A$.

2.4. Algebra Of The Gauge Transformations

In the course of the deformation, the algebra of the gauge transformations may of course get also deformed. The new gauge transformations may only close on-shell, even if the original transformations formed a true algebra. So, one has

$$R^i_\alpha \frac{\delta R^i_\beta}{\delta \phi^j} - R^i_\beta \frac{\delta R^i_\alpha}{\delta \phi^j} = C^{\gamma}_{\alpha\beta} R^\gamma_i + M^{ij}_{\alpha\beta} \frac{\delta I}{\delta \phi^i}, \quad M^{ij}_{\alpha\beta} = -M^{ji}_{\alpha\beta}.$$  \hspace{1cm} (2.25)

with

$$C^{\gamma}_{\alpha\beta} = C^{(0)\gamma}_{\alpha\beta} + gC^{(1)\gamma}_{\alpha\beta} + O(g^2)$$  \hspace{1cm} (2.26)

$$M^{ij}_{\alpha\beta} = M^{(0)ij}_{\alpha\beta} + gM^{(1)ij}_{\alpha\beta} + O(g^2).$$  \hspace{1cm} (2.27)

The $C$’s and $M$’s can depend on the fields. The structure relations (2.25) are actually consequences of the Noether identities and of the fact that the gauge transformations are assumed to form a complete set (see e.g. [14]).

In the abelian models considered above, the structure functions $C^{(0)\gamma}_{\alpha\beta}, M^{(0)ij}_{\alpha\beta}$ (and $\mu^{(0)ij}_A$ in (2.21)) all vanish, but they may no longer vanish in the deformed theory. We thus allow a priori for the most general deformation compatible with the existence of gauge symmetries and do not impose any restriction on the deformed structure functions.
3. Trivial Deformations

The above equations on the deformations always admit solutions of a particular type. These solutions are simply obtained by making $g$-dependent redefinitions of the field variables,

$$ \phi^i \rightarrow \phi^i + g k^i + 0(g^2) \tag{3.1} $$

where $k^i$ can depend on the fields and their derivatives. Under such a redefinition, the action and the gauge transformations are in general modified. For instance, the action becomes

$$ I_0[\phi^i] \rightarrow I[\phi^i] = I_0[\phi^i + g k^i + 0(g^2)] = I_0[\phi^i] + g \frac{\delta I_0}{\delta \phi^i} k^i + O(g^2) \tag{3.2} $$

and a similar redefinition holds for the gauge transformations. Such transformations are, however, rather trivial, and will be regarded in the sequel as “fake” deformations. Our interest lies in the determination of the non-trivial deformations of the action, i.e., in the deformations that do not arise from a (local) redefinition of the field variables.

It should be noted that even if the fields are unchanged, there is some freedom in the description of the gauge transformations. Indeed, one may always redefine the $R^{(0)i}_\alpha$ as

$$ R^{(0)i}_\alpha \rightarrow \epsilon^\beta_\alpha R^{(0)i}_\beta + t^{ij}_\alpha \delta I_0 \delta \phi^j \tag{3.3} $$

where $\epsilon^\beta_\alpha$ and $t^{ij}_\alpha$ are local functions with $\det \epsilon^\beta_\alpha \neq 0$ and $t^{ij}_\alpha = -t^{ji}_\alpha$. Expanding $\epsilon^\beta_\alpha$ and $t^{ij}_\alpha$ in powers of $g$ yields

$$ \epsilon^\beta_\alpha = \delta^\beta_\alpha + g \epsilon^{(1)i\beta}_\alpha + O(g^2), \tag{3.4} $$

$$ t^{ij}_\alpha = 0 + g t^{(1)ij}_\alpha + O(g^2), \tag{3.5} $$

$$ R^{(0)i}_\alpha \rightarrow R^{(0)i}_\alpha + g (\epsilon^{(1)i\beta}_\alpha R^{(0)i}_\beta + t^{(1)ij}_\alpha \frac{\delta I_0}{\delta \phi^j}) + O(g^2). \tag{3.6} $$

Because of this ambiguity, the deformation of the gauge symmetries is not unique (for a fixed deformation of the action).

It is of particular interest to examine the deformations of the action that truly deform the gauge transformations, i.e., such that there is no redefinition of the field variables and of the $R$’s that brings the deformed gauge functions $R^{i}_\alpha$ to the original form $R^{(0)i}_\alpha$. Among the interactions that deform non trivially the gauge transformations, it is customary to distinguish between those that do not deform the gauge algebra ($C^{\gamma}_{\alpha\beta} \text{ unchanged}$), and those that (non trivially) do ($C^{\gamma}_{\alpha\beta} \neq C^{0\gamma}_{\alpha\beta}$).

Finally, we note that there is also some ambiguity in the reducibility functions (when there is reducibility) which are defined, for a fixed set of field variables and gauge functions $R^{i}_\alpha$, up to

$$ Z^{\alpha}_A \rightarrow t^{B}_A Z^{\alpha}_B + k^{\alpha}_A \frac{\delta I_0}{\delta \phi^i}. \tag{3.7} $$

A non trivial deformation of the reducibility functions is one that cannot be brought back to the original form by means of the allowed redefinitions.
4. Analysis Of The Equations - Overview

The theoretical problem of determining consistent deformations of a given gauge invariant action is of course not a new one and has been much studied in the context of consistent interactions for massless, higher spin, particles. It has been formulated in general terms in [21, 22]. The usefulness of the deformation point of view (but not in the general framework of the antifield formalism, which allows off-shell open deformations of the algebra) has been advocated in [23].

The equations determining the consistent deformations are rather intricate because they are nonlinear and involve simultaneously not only the deformed action, but also all the deformed structure constants. The problem is further complicated by the fact that one has to sort out the trivial deformations from the non eliminable ones.

In practice, one first determines the consistent first-order deformations $I_1$, which may or may not exist. If they do, the crucial test is then to determine whether the deformation can proceed to the next order.

The cohomological approach systematises the recursive construction and relates the consistent interactions to cocycles of the BRST differential. Furthermore, trivial deformations are also trivial in the cohomological sense, i.e., BRST-coboundaries. Thus, the two aspects involved in the construction of consistent interactions (consistency conditions and elimination of trivial solutions) have a clear cohomological counterpart and are just the two familiar aspects involved in computing cohomology (impose coboundary condition and eliminate trivial solutions, i.e., coboundaries).

Of course, the cohomological approach ultimately deals with the same equations as the standard approach, but it organizes them in a rather appealing way. Furthermore, it clearly exhibits the obstructions to deforming the given action to non-trivial BRST-cohomological classes. Finally, by reformulating the problem of consistent interactions as a cohomological problem, one can bring in the powerful tools of homological algebra.

5. Equations to First Order

In order to explain the cohomological approach, we shall first write explicitly the conditions on the deformation to first order in the coupling constant $g$. We shall also write the triviality condition to the same order. The Noether identity (2.20) reads, to zeroth and first orders in $g$,

$$O(g^0) : \frac{\delta I_0}{\delta \phi^i} R^{(0)i}_\alpha = 0 \quad \text{(5.1)}$$

$$O(g^1) : \frac{\delta I_1}{\delta \phi^i} R^{(0)i}_\alpha + \frac{\delta I_0}{\delta \phi^i} R^{(1)i}_\alpha = 0. \quad \text{(5.2)}$$

The first condition (5.1) is nothing but the Noether identity (2.17) of the free theory and is fulfilled by assumption. The second condition states that the gauge variation (for the undeformed gauge symmetries) of the first-order deformation should vanish when the (free) equations of motion hold,

$$\delta_e I_1 \approx 0. \quad \text{(5.3)}$$
Solutions of (5.3) are called “observables”. First order deformations are therefore observables. The relevant observables are of course “integrated observables”, i.e., spacetime integrals of local functions.

Similarly, it follows from (3.2) that a first order-deformation is trivial if and only if it vanishes on-shell (for the free theory),

\[ I_1 \text{ is trivial if and only if } I_1 \approx 0. \] (5.4)

We thus see that to first order in the deformation parameter \( g \), the problem of finding the consistent deformations is equivalent to that of finding the (integrated) observables of the undeformed theory, with the understanding that two observables that coincide on-shell are equivalent – as it is usually implied in the definition of “observables”.

It should be noted that the equations (5.3) and (5.2) are equivalent. This is because any function that is zero on-shell can be written as a combination of the equations of motion (we assume the necessary regularity conditions that guarantee this). Thus, to first order in \( g \), it is only necessary to find the deformation \( I_1 \) of the action. The deformation of the gauge symmetry follows from (5.3), (5.2) and can be read off from the coefficients of the equations of motion in the variation of \( I_1 \). Similarly, (5.4) imply (3.2) to first order in \( g \).

Now, it is well known that the observables of a gauge theory can be described cohomologically in terms of the BRST differential. To correctly implement the equivalence relation implied by the equations of motion, it is necessary to include the antifields (Zinn-Justin “sources for the BRST variations” [24]) and to work within the antifield formalism developed by Batalin and Vilkovisky [25] along lines initiated in [26, 27].

More precisely, one of the main points of the cohomological approach to the construction of consistent interactions is that there exists a differential \( s \), the so-called “BRST differential”, whose cohomology in “ghost number zero” is precisely given by the space of observables, on-shell vanishing observables being BRST exact. Thus, the non-trivial first-order deformations are described by the group \( H^0(s) \) (in the space of local functionals).

The description of the observables of a theory involves two ingredients: one is the gauge-invariance condition; the other is the fact that an on-shell vanishing function should be identified with zero. As shown in [28, 29, 14], there corresponds a separate differential for each of these ingredients. The first is the longitudinal differential \( \gamma \) along the gauge orbits, which implements the gauge invariance condition. The second is the Koszul-Tate differential \( \delta \) associated with the stationary surface, which takes the equations of motion into account. The BRST differential combines these two differentials into a single object (in the simplest cases, it is just the sum). We refer the reader to [28, 29, 14] for the details. Locality is taken care of in [30]. It is precisely because the BRST differential contains the Koszul-Tate piece – something that does not appear to be always properly appreciated – that the above on-shell relations are enforced when going to the BRST cohomology [28, 29, 14]. The associated antifields, initially introduced in order to keep an hand on the renormalization of the BRST symmetry, have also the extremely important homological interpretation of being the generators of the “stationary” Koszul-Tate complex.

We shall give here only the form of \( s \) for the specific models given above. In both these models, \( s \) is simply given by the sum of \( \delta \) and \( \gamma \), because the gauge symmetries close off-shell.
In the case of reducible theories, one must impose in addition to (5.2) the condition that the gauge symmetries should remain reducible in the deformation, i.e., that (2.24) holds in the deformed theory to order $g$. This is, however, a consequence of (2.24) at this order, so that the requirement that $I_1$ be an observable of the free theory is the sole independent requirement at order $g$. Indeed, if one contracts (5.2) with $Z_\alpha^{(0)}A$, uses the reducibility identity at order zero, and recalls that the gauge transformations of the free theory are assumed to form a complete set, one easily finds that there exist functions $Z_\alpha^{(1)}A$ and $\mu^{(1)ij}$ such that (2.24) holds up to order $g$ (included) [14]. Thus, the only condition at order $g$ is (5.3), so that the non trivial deformations are parametrized by the cohomogy group $H^0(s)$ (in the space of local functionals) also in the reducible case.

6. BRST Differential

We shall from now on give up the condensed notations where a summation over repeated indices involved also an integral. Spacetime integrals will always be explicitly written and the objects that we shall manipulate ("local functions") will be ordinary functions of the variables (original fields, ghosts, antifields) and their derivatives up to some finite order, without $\delta$-function or derivatives of it. We shall also deal with local spacetime forms, i.e., forms with coefficients that are local functions.

The appropriate mathematical framework for dealing with local forms is the one of jet bundles and it is straightforward to formulate the general BRST cohomological construction in this language [31]. The reader may consult [32, 33, 34, 35] for information on jet bundles.

6.1. BRST Differential For a Set of $U(1)$-Gauge Fields

By following the general prescriptions of the antifield formalism [25, 29, 14, 36], one finds that the BRST differential for a set of $U(1)$-gauge fields is given by

$$s = \delta + \gamma$$

(6.1)

where the Koszul-Tate differential reads

$$\delta A^a_\mu = \delta C^a = 0, \; \delta A^*_\mu = \partial_\nu F^\mu_{\nu}, \; \delta C^*_a = \partial_\mu A^*_a$$

(6.2)

while the exterior derivative along the gauge orbits is

$$\gamma A^a_\mu = \partial_\mu C^a, \; \gamma C^a = 0, \; \gamma A^*_\mu = 0, \; \gamma C^*_a = 0.$$

(6.3)

In these relations, the $C^a$ are the ghosts, the $A^*_\mu$ are the antifields conjugate to the vector potentials, while the $C^*_a$ are the antifields conjugate to the ghosts.

The action of $\delta$ and $\gamma$ is extended to the derivatives of the variables by demanding $\delta \partial_\mu = \partial_\mu \delta$ and $\gamma \partial_\mu = \partial_\mu \gamma$. One then extends the action of $\delta$ and $\gamma$ to products of variables by using the Leibnitz rule so that they are (anti)derivations. It is easy to check that

$$\delta^2 = 0, \; \gamma^2 = 0, \; \delta \gamma + \gamma \delta = 0.$$
Hence, \( s \) is also a differential,
\[
s^2 = 0 \quad (6.5)
\]

The variables are conveniently assigned the following gradings:

**Antighost number:**
\[
antigh(A^\mu_\mu) = 0, \quad antigh(C^\mu_a) = 0, \quad antigh(A^{*\mu}_a) = 1, \quad antigh(C^{*}_a) = 2 \quad (6.6)
\]

**Pure ghost number:**
\[
puregh(A^\mu_\mu) = 0, \quad puregh(C^\mu_a) = 0, \quad puregh(A^{*\mu}_a) = 0, \quad puregh(C^{*}_a) = 0. \quad (6.7)
\]

The (total) ghost number is the difference between the pure ghost number and the antighost number. Furthermore, the \( A^\mu_\mu \) and \( A^{*\mu}_a \) are even, while \( C^\mu_a \) and \( A^{*\mu}_a \) are odd (anticommuting).

The Koszul-Tate differential \( \delta \) decreases the antighost number by one unit and does not modify the pure ghost number. The longitudinal derivative \( \gamma \) increases the pure ghost number by one unit and does not modify the antighost number. Accordingly, all three differentials \( s, \delta \) and \( \gamma \) increase the total ghost number by one unit.

### 6.2. BRST Differential For a Set of Free Exterior 2-Forms

The rules for reducible systems yield the following BRST differential \( s = \delta + \gamma \) \[25, 14, 36\]
\[
d\rho^\mu A_\nu - d\rho^\nu A_\mu = \partial_\mu H^{A\nu\lambda}, \quad dC^\mu A = \partial_\mu B^{*\mu\nu} A, \quad d\rho_A = \partial_\mu C^{*\mu}_A \quad (6.8)
\]

and
\[
\gamma B^\mu_\nu = \partial_\mu C^\nu A - \partial_\nu C^\mu A, \quad \gamma C^\mu A = \partial_\mu \rho^A, \quad \gamma \rho^A = 0, \quad \gamma B^{*\mu\nu} = 0, \quad \gamma C^{*\mu}_A = 0, \quad \gamma \rho^{*}_A = 0. \quad (6.9)
\]

The \( C^\mu_A \) are the “ghosts” while the \( \rho^A \) are the “ghosts of ghosts”. The \( B^{*\mu\nu}_A \) are the antifields conjugate to the \( B \)’s, the \( C^{*\mu}_A \) are those conjugate to the ghosts \( C^\mu_A \), while the \( \rho^{*}_A \) are the antifields conjugate to the ghosts of ghosts \( \rho^A \).

The ghost number assignments are this time

**Antighost number:**
\[
antigh(B^\mu_\mu) = 0, \quad antigh(C^A_\mu) = 0, \quad antigh(\rho^A) = 0, \quad (6.10)
\]
\[
antigh(B^{*\mu\nu}_A) = 1, \quad antigh(C^{*\mu}_A) = 2, \quad antigh(\rho^{*}_A) = 3 \quad (6.11)
\]

**Pure ghost number:**
\[
puregh(B^\mu_\mu) = 0, \quad puregh(C^A_\mu) = 1, \quad puregh(\rho^A) = 2, \quad (6.12)
\]
\[
puregh(B^{*\mu\nu}_A) = 0, \quad puregh(C^{*\mu}_A) = 0, \quad puregh(\rho^{*}_A) = 0. \quad (6.13)
\]

The total ghost number \( gh \) is again the difference between the pure ghost number and the antighost number. One has \( gh(s) = 1 = gh(\delta) = gh(\gamma) \).

By extending \( \delta \) and \( \gamma \) to derivatives and products as above, one finds again that these are anticommuting differentials, so that \( s \) is also a differential.
As we have pointed out, the set of observables is isomorphic to the zeroth cohomology group of the BRST differential. Since we are interested in integrated observables, we must consider the cocycles and coboundaries of the BRST differential that are also given by integrals of local densities (local \( n \)-forms).

If one works with the integrands \( a \) of the integrated observables \( A = \int a \), which is more convenient, then one finds that these should be in the so-called cohomology of \( s \) modulo \( d \) in form degree \( n \) (since \( a \) is a local \( n \)-form). That is, the cocycle condition \( sA = 0 \) reads, in terms of \( a \)

\[
sa + db = 0 \tag{7.1}
\]

for some \((n-1)\)-form \( b \), while a solution of (7.1) is trivial if and only if it can be written as \( a = sm + dl \), for some \( m, l \).

The proof that the set of integrated local observables is isomorphic with \( H^0(s|d) \) is given in [30, 37]. A crucial step in the proof is the acyclicity of the differential \( \delta \) in the space of local functionals with positive antighost number and positive pure ghost number [30].

We shall not work out explicitly here the BRST cohomology \( H^0(s|d) \) for the two models described above. We shall merely report the results and refer to the literature for the detailed proofs. It is rather remarkable that the cohomological approach gives the complete list of all first-order consistent interactions for these (and other) models. [That the vertices listed below are consistent to first-order is rather obvious; that there are no other vertices is perhaps not as straightforward and appears to be a definite pay-off of the cohomological method].

### 7.1. First-Order Consistent Interactions For a Set of \( U(1) \)-Gauge Fields

We shall classify the interactions according to whether they deform non-trivially or not the gauge algebra, which is abelian at order \( g^0 \).

Consistent interactions of a given gauge theory may be classified into three categories: (i) those that do not modify the gauge transformations; (ii) those that modify the gauge transformations without changing their algebra; and (iii) those that modify both the gauge transformations and their algebra. For the first type, the gauge variation \( \delta I_1 \) of the vertex \( V \) vanishes (up to a surface term) off-shell and not just on-shell. For the second and third types, \( \delta I_1 \) vanishes only on-shell,

\[
\delta \epsilon I_1 = \int b^a_\mu \frac{\delta I}{\delta A^a_\mu} d^n x \tag{7.2}
\]

with \( b^a_\mu \neq 0 \). The modification of the gauge transformations is given, to first order in the coupling constant \( g \), by

\[
\delta^{NEW}_\epsilon A^a_\mu = (de^a_\mu)_\mu - gb^a_\mu \tag{7.3}
\]

since then, the gauge variation \( \delta^{NEW}_\epsilon (I_0 + gI_1) \) vanishes to order \( g^2 \). If \( b^a_\mu \) is gauge invariant, the second variation \( \delta^{NEW}_\epsilon \delta^{NEW}_\epsilon A^a \) is of order \( g^2 \) and the interaction does not modify the gauge algebra to order \( g \).
Interactions of each type exist for a set of free vector fields $A^a_\mu$. Let us start with the interactions that truly deform (to first order in $g$) not only the gauge transformations, but also their algebra. In the antifield language, these are the cohomological classes of $H^0(s|d)$ for which all representatives necessarily involve the antifields $C^*_{a}$ [17]. As shown in [38, 39], the only such interactions are given by the familiar Yang-Mills cubic vertex proportional to

$$f_{abc}F^{a\mu\nu}A^b_\mu A^c_\nu$$

(7.4)

where the constants $f_{abc}$ are completely antisymmetric but otherwise arbitrary at this stage. The modification of the gauge transformations induced to first order in $g$ is the familiar transformation of a non-abelian gauge connection

$$\delta\epsilon A^a_\mu = \partial_\mu \epsilon^a + gf^a_{bc}A^b_\mu \epsilon^c.$$  

(7.5)

The new gauge transformations involve explicitly the vector potentials and no longer commute. Their algebra is the well-known Yang-Mills algebra. Any gauge algebra deforming interaction differs from (7.4) by terms that do not deform the algebra. It is in this sense that (7.4) is the unique vertex deforming the algebra.

Let us now turn to the interactions that deform non trivially the gauge transformations, but do not deform their algebra. These are exhausted by the terms of the form

$$A^a_\mu j^\mu_a$$

(7.6)

where $j^\mu_a$ are gauge-invariant conserved “currents” (for the free theory), constructed out of the potentials and their derivatives. Since one has

$$\partial_\mu j^\mu_a = t^b_{a\mu} \delta I_0 A^b_\mu$$

(7.7)

for some gauge-invariant functions $t^b_{a\mu}$, the gauge transformations become

$$\delta\epsilon A^a_\mu = \partial_\mu \epsilon^a + gt^a_{b\mu} \epsilon^b$$

(7.8)

and clearly remain abelian to order $g$ since $t^a_{b\mu}$ is gauge-invariant. [One could have terms involving the derivatives of the field equations in (7.7) with the same conclusions].

As a free theory, the system of abelian gauge fields possesses an infinite number of conserved currents. Thus, there are in general an infinite number of first-order-consistent interactions that do not deform the algebra but do deform the gauge transformations. However, in 4 dimensions, according to a result due to Torre, there is no conserved current $j^\mu_a$ that transforms as a Lorentz vector and does not involve explicitly the coordinates [40]. Thus, there is no Lorentz-invariant interaction of the type (7.6) in 4 dimensions. It seems plausible to extrapolate this result to all spacetime dimensions, except 3, where the gauge-invariant currents

$$g_{abc} \epsilon^{\mu\beta\gamma} F^a_\beta \star F^b_\gamma$$

(7.9)

are conserved and transform as Lorentz-vectors. The corresponding interaction vertex

$$g_{abc} \epsilon^{\mu\beta\gamma} A^a_\mu \star F^b_\beta \star F^c_\gamma,$$

(7.10)
is known as the Freedman-Townsend vertex [41]. Here, \( *F^b_{\beta} \) is the one-form dual to the two-form \( F^b_{\mu\nu} \).

Finally, the interactions that do not deform the gauge transformations are given by the functions of the curvatures \( F^a_{\mu\nu} \equiv \partial_{\mu}A^a_{\nu} - \partial_{\nu}A^a_{\mu} \) and their derivatives, as well as by the Chern-Simons terms

\[
g_{a_1...a_k} A^{a_1} \wedge F^{a_2} \wedge \cdots \wedge F^{a_k} \tag{7.11}
\]

in odd spacetime dimensions \( n = 2k - 1 \) [42]. Here, \( g_{a_1...a_k} \) is completely symmetric.

The most general Lorentz-invariant, first-order consistent, interaction vertex for a set of free abelian vector fields is a linear combination of the Yang-Mills cubic vertex and of the Lorentz-invariant, gauge-invariant functions (plus the Freedman-Townsend vertex in 3 dimensions and the Chern-Simons terms in odd dimensions). Higher-order consistency will be discussed below.

### 7.2. First-Order Consistent Interactions For a Set of Free exterior 2-Forms

Consistent interactions for a set of exterior 2-forms can also be classified according to whether they modify the gauge transformations or the gauge algebra. However, in the 2-form case, the situation is much simpler than for 1-forms. Indeed, there is no interaction that truly deforms the gauge algebra. Furthermore, there is even no interaction that deforms non-trivially the gauge transformations, except in 4 dimensions, where the Freedman-Townsend vertex is the only possibility [43],

\[
f_{ABC} \epsilon^{\mu\nu\beta\gamma} B_{\mu\nu} A^A - H^B_\beta \epsilon^C_\gamma.
\]

Here, \( *H^B_\beta \) is the one-form dual to the three-form \( H^B_{\mu\nu\lambda} \). The Freedman-Townsend vertex deforms the gauge transformations as follows

\[
\delta B^A_{\mu\nu} = D_{\mu} \epsilon^A_{\nu} - D_{\nu} \epsilon^A_{\mu}, \tag{7.13}
\]

where

\[
D_{\mu} \epsilon^A_{\nu} = \partial_{\mu} \epsilon^A_{\nu} + g f_{BC} H^B_{\mu} \epsilon^C_\nu.
\]

The new gauge transformations (7.13) are clearly still reducible, but only weakly so,

\[
\delta_{i} B^A_{\mu\nu} \approx 0 \tag{7.15}
\]

for

\[
\epsilon^A_{\mu} = D_{\mu} \Lambda^A. \tag{7.16}
\]

The deformed coefficient \( \mu_{ij}^A \) occurring in the reducibility identities are non-zero.

Thus, except in four dimensions, the only available first-order consistent interactions do not deform the gauge transformations and are given by gauge-invariant functions of the field strength components and their derivatives, as well as by Chern-Simons terms

\[
f_{A_1...A_k} B^{A_1} \wedge H^{A_2} \wedge \cdots \wedge H^{A_k} \tag{7.17}
\]
in 2 mod 3 dimensions. Here, \( f_{A_1...A_k} \) is completely antisymmetric. In particular, there is no analog of the Yang-Mills cubic vertex coupling for 2-forms. One does not need to invoke Lorentz invariance to reach this result. It is a direct consequence of the cohomological analysis alone. The gauge symmetries of exterior forms of degree \( > 1 \) are extremely rigid. This was anticipated in [44]

### 8. Consistent Deformations: Higher Orders

The BRST cohomology plays a central role to first order in the deformation parameter because it gives, at ghost number zero, the first order terms of the consistent deformations. It also plays a central role at higher order in \( g \), because it is the group \( H^1(s|d) \) that controls the obstructions to the existence of the higher order deformation terms [17].

The most expedient way to analyse this problem is through the master equation [24, 25]. We refer the reader to [17] for the details and sketch here only the main idea.

The space of fields, ghosts and antifields is naturally equipped by an “antibracket” structure, in which the antifields are conjugate to the corresponding fields or ghosts. This antibracket structure induces an antibracket in the local BRST cohomological classes \( H^k(s|d) \). It is such that the antibracket of one element of \( H^k(s|d) \) with one element of \( H^j(s|d) \) is an element of \( H^{k+j+1}(s|d) \). In particular, the antibracket of one element of \( H^0(s|d) \) with one element of \( H^1(s|d) \) is an element of \( H^2(s|d) \). Now, a first-order consistent deformation – that is, an element of \( H^0(s|d) \) – is non obstructed to second order if and only if its antibracket with itself is BRST-exact, i.e. is the zeroth element of \( H^1(s|d) \). This follows from a direct analysis of the deformed solutions of the master equation,

\[
(S, S) = 0 \tag{8.1}
\]

with

\[
S = S^{(0)} + gS^{(1)} + g^2S^{(2)} + O(g^3). \tag{8.2}
\]

We recall that \( S \) contains all the information about the action, gauge symmetries, gauge algebra ... of the theory [29, 14]. From (8.1), one gets at order \( g^2 \) the condition

\[
(S^{(1)}, S^{(1)}) + 2(S^{(0)}, S^{(2)}) = 0. \tag{8.3}
\]

Since \( S^{(0)} \) generates the BRST symmetry of the undeformed theory through the antibracket, one sees that \( S^{(2)} \) exists if and only if the cocycle \((S^{(1)}, S^{(1)})\) is trivial in \( H^1(s|d) \). If \((S^{(1)}, S^{(1)})\) is not BRST exact, there is no \( S^{(2)} \) and the deformation gets obstructed at order \( g^2 \). It is thus the cohomological group \( H^1(s|d) \) that controls the obstructions to the existence of the second order terms, as it was announced above.

It turns out that the obstructions to the existence of the higher order terms besides \( S^{(2)} \) can also be expressed in terms of \( H^1(s|d) \). Thus, the problem of consistent deformations is entirely governed by the BRST cohomological groups \( H^0(s|d) \) (for the first-order deformations) and \( H^1(s|d) \) (for the obstructions to continuing the deformation to higher orders). In particular, if \( H^1(s|d) \) vanishes, no first-order deformation can be obstructed.

The analysis of the higher-order consistency conditions for the Yang-Mills deformation of the abelian \( U(1)^N \) gauge theory leads to the conclusion that the \( f_{abc} \) should fulfill the Jacobi
identity and are thus the structure constants of a Lie algebra. If they do not fulfill the Jacobi identity, the first order deformation (7.4) is obstructed at second order because the bracket of the corresponding BRST cohomological class with itself is then a non trivial element of $H^1(s|d)$ [39]. If the Jacobi identity is fulfilled, the bracket is zero (in cohomology) and $S^{(2)}$ is the familiar Yang-Mills quartic contact term.

If one adds also gauge invariant terms or Chern-Simons terms to the action, these get modified at higher order to the corresponding non-abelian gauge-invariant or Chern-Simons terms. [There is, for the Chern-Simons terms, a second-order condition that the coefficients $g_{a_1...a_k}$ should be invariant tensors of the deformed Lie algebra].

We have seen that in three dimensions, the Freedman-Townsend vertex is another candidate deformation. By itself (or in the presence of gauge invariant /Chern-Simons terms), this vertex is not obstructed at higher order provided the $g$'s defining it fullfill also the Jacobi identity. The higher order deformations exist and the full interacting action is non polynomial (in this second order form) [41].

If one considers simultaneously, in three dimensions, the Yang-Mills vertex and the Freedman-Townsend vertex, one finds further conditions on the $f$'s and the $g$'s which have been examined in [45].

For a set of free abelian 2-forms, one finds that the Freedman-Townsend interaction, which exists only in four dimensions, is not obstructed only if the $f$'s fulfill the Jacobi identity, as in three dimensions [43]. The resulting theory is non-polynomial. In other spacetime dimensions one finds of course that the gauge-invariant interactions, which do not deform the gauge symmetries, are not obstructed at higher order since the sum $I_0 + gI_1$ is in that case fully consistent to all orders.

9. Other Models

The cohomological approach has been used in the analysis of other models. Although we have often called above the undeformed theory the “free theory”, nowhere was it used that its Lagrangian was quadratic. Accordingly, the formalism applies equally well to the investigation of the rigidity of an already interacting theory. An important application is perturbative renormalization theory where the deformation parameter is $\hbar$, which can be couched in the antifield language [46, 47, 48].

Various models have been studied. The local BRST cohomology of Yang-Mills theory has been worked out in [49, 38], following the antifield-independent work of [50, 51, 52, 53]. The cohomological approach has enabled one to rigorously establish previously unproved conjectures on the renormalization of gauge invariant operators [54, 55].

The rigidity of Einstein theory of gravity against deformations that would modify its gauge symmetries has been established in [56] (see also [57]). The study applies also to higher derivative models, whose quantum properties have been investigated in [58].

Similarly $D = 4, N = 1$ supergravity has been analysed in [59] and the cohomological investigation of $D = 11$-supergravity [60] has been started in [61]. In that model, the impossibility of introducing a cosmological constant along conventional lines [62, 63] has been explicitly proved and related to the fact that the mass term for the spin $3/2$-field, which must accompany the cosmological term, is not an observable.
Other recent applications include couplings of $p$-forms [64, 65, 66, 67] and two-dimensional models [68, 69]. In particular, the cohomological approach enables one to give the exhaustive list of all the consistent interactions among exterior form gauge fields of different degrees $\geq 2$. It would be of interest to investigate the long-standing problem of interactions for massless higher spin particles (e.g. spin 3) [21, 22, 70, 71, 72] along the cohomological lines. It is planned to pursue this question in the future [73].

10. Conclusions

In this review, we have described the cohomological approach to the problem of consistently deforming a gauge invariant action. We have indicated that the consistent deformations are controlled by the local cohomological groups $H^0(s|d)$ and $H^1(s|d)$ of the BRST differential. The first group gives the first-order deformations. The second contains the obstructions to higher-order deformations. We have also given references to articles where explicit models are completely analysed along the cohomological lines.

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