Fractional pilot reuse and max k-cut based pilot decontamination scheme for multi-cell TDD massive MIMO systems

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Abstract
In multi-cell massive multiple-input multiple-output (MIMO) cellular networks, pilot contamination (PC) caused by ineluctable pilot reuse severely restricts the system performance in the effectiveness and reliability of transmission. To deal with this problem, an efficient pilot assignment scheme based on fractional pilot reuse and max k-cut (FPR-MKC) is therefore proposed in this study. Specifically, first, the measure of susceptibility to interference for each user is designed, and based on this, an innovative boundary is determined for properly selecting the edge users who could employ the unique pilot sequences. Subsequently, the process of allocating reused pilots is innovatively treated as the partition of the vertices of a graph. Building on this idea, an edge-weighted undirected graph is constructed to present the potential interference intensity among users, and eventually, the max k-cut (MKC) strategy is executed, which aims to partition centre users into a desired number of mutually exclusive subsets with maximizing the total weight of the edges between the disjoint subsets. Compared with some existing pilot assignment schemes, the proposed FPR-MKC scheme can adequately mitigate the PC with significantly enhanced quality of service for edge users, which is verified by theoretical analysis and numerical simulation results.

1 INTRODUCTION

Massive multiple-input multiple-output (MIMO) has been considered as one of the most promising technologies of fifth-generation (5G) wireless communication [1, 2]. A massive MIMO system is typically defined as a system that is equipped with a very large number, that is, hundreds or thousands, of antennas at the base station (BS) side and simultaneously serves a cell with multiple terminals [1, 3]. Scaling up the number of antennas always has substantial potential to obtain greatly high sum spectral efficiencies and energy efficiencies, which meets the continuously increasing demand for wireless data connectivity and satisfies the rising expectations of service quality [3, 4]. Compared to the existing cellular network technologies, the massive MIMO is capable of offering the following basic advantages: multiplexing gain, energy efficiency, spectral efficiency, increased robustness and reliability, reduced latency, simple low-complexity linear processing, and extensive use of low-cost radio frequency (RF) power components [1–6].

To achieve these benefits in practical implementation of the Massive MIMO, sufficiently timely and accurate channel state information (CSI) is indispensable at the BS for downlink beam forming and uplink detection. Typically, the CSI may be obtained either by estimation with the assistance of pilot signals or by feedback from the receiver to the transmitter. In time-division duplexing (TDD) mode, the legitimate estimate of the downlink channel can be obtained automatically by exploiting reciprocity of the propagation channel, wherein the amount of resources needed for pilots only depends on the number of simultaneously served terminals. Consequently, compared with frequency-division duplexing (FDD) mode, the TDD mode has a natural advantage with Massive MIMO [4]. Despite that, the system capacity still has an ultimate bottleneck. Due to the limited length of the channel coherence interval, the orthogonal
pilot is generally viewed as a kind of scarce resource in wireless communications, which cannot always meet the large user number. As a result, the orthogonal pilots ineluctably need to be reused across cells. This would make the estimate of the desired CSI correlated with the users in other cells and incur inter-cell interference, which is referred to as the pilot contamination (PC) [1, 7]. The phenomenon of PC, which would lead to the degraded achievable rates and spectrum efficiency in the networks, has been shown to be a predominant constraint on the performance of multi-cell multi-user massive MIMO systems [3, 7].

In the literature, a plethora of approaches to mitigating the PC have been proposed from three perspectives, that is, interference-rejecting precoding, clever channel estimation, and optimized pilot assignment, whereof many have complementary benefits. Improving the existing precoding algorithm is an effective way to reduce the CSI estimation error caused by pilot pollution. A precoding approach based on multi-cell minimum mean square error (MMSE) was presented in [8]. It focuses to minimize the sum of square errors of desired user signals and interference user signals, however, at the cost of high mathematical complexities. Another research direction to effectively reduce PC effect focuses on the channel estimation method assisted by users’ statistical information [9, 10], such as the angle of arrival (AOA) or channel covariance matrix. This method generally adopts the spatial correlation of the channel to differentiate users from different geospatial directions. However, the difficulty of obtaining second-order statistics (channel covariance matrix) is the main obstacle to the implementation of this method. Besides, it relies on the presupposition that the AOAs of desired users do not overlap with that of interfering users, which is not always valid in actual wireless systems. Normally, massive MIMO systems exploit synchronous pilots or aligned pilots, which means that the uplink pilots for all users are transmitted in the same time slot. To avoid mutual interference between users who reuse the same pilot, a time-shift pilot structure was constructed for massive MIMO TDD systems in [11, 12]. According to this time-shift approach, the uplink pilot transmission phase of users who reuse the same pilot is performed in non-overlapping time slots, and the obtained result proves it as an effective solution to alleviate the impact of PC and improve system spectral efficiency. Although promising, two challenges would be faced in practice. First, a control mechanism is necessary to dynamically avoid pilot overlap, and secondly, a high computational complexity process is needed to alleviate the mutual interference between pilot and data signals in the non-asymptotic state.

Switching the pilot reuse factor appropriately was often considered as another suitable solution. The fractional pilot reuse (FPR) [13–15] and soft pilot reuse (SPR) [16, 17] scheme were proposed to further enhance the quality of service and mitigate the inter-cell interference for edge users. In these schemes, all the cells were divided into cell centre region and cell edge region based on a threshold radius, wherein the edge users suffered from severe interference can employ the orthogonal pilots. However, the degree of vulnerability to the interference of one user depends not only on its location in the cell but also on the density of interfering users relative to its anchor BS. Conventionally, most of the actual massive MIMO systems assigned the available pilot sequences to different users in a random way and ignored that the exact pilot-user assignment scheme would make a large difference. Quite recently, considerable attention has been paid to the pilot assignment optimization. Many publications tried to find a low complexity strategy for a reasonable allocation of limited orthogonal pilot sequences and minimize system pilot contamination as much as possible. The pilot assignment optimization is essentially a combinational optimization issue, and consequently, many proposed combined-optimization-algorithm-based pilot schemes have been proved significant gains, such as a smart pilot assignment [18] and a coalitional game-based pilot allocation algorithm [19]. Especially, inspired by the graph theory, the graph-colouring-based pilot allocation (GCPA) scheme [20] and weighted-graph-colouring-based (WGC) pilot scheme [21] brought novel enlightenment to solve the PC problem. The results demonstrated the graph-colouring method could handle conflict between users. In addition, the case that pilot reuse in the same cell merits consideration, which could save more pilot overhead and enhance spectral efficiency under certain circumstances.

To improve spectral efficiency and link reliability with consideration of both the complexity and adaptability, we propose an innovative fractional pilot reuse and max k-cut based (FPR-MKC) pilot assignment scheme. In the proposed scheme, we firstly develop a typical multi-cell TDD massive MIMO system model and derive an expression for users’ data rates in the PC regime. To enhance the quality of service of edge users, we define a novel measure of users’ susceptibility to interference and formulate the rule of selecting edge users who could utilize a special pilot group in advance. And then, according to the innovative ideas that considering the pilot allocation process as the partition of the vertices of a graph, a heuristic max k-cut (MKC) regulation is applied for avoiding collision and interference of centre users to the utmost extent. Specifically, the contributions of this study are summarized as follows.

- The issue of pilot decontamination is first formulated as an MKC problem, which is addressed in polynomial time by a proposed heuristic algorithm. Fundamentally differing from a graph-colouring method whose object is minimizing the number of pilots in use and preferentially allocating pilots for users with the higher potential mutual interference [20, 21], the MKC method aims at achieving approximate optimal system performance under the constraint of the predetermined pilot resource. In addition, we focus attention on the aggressive interference across user-sets, and convert minimizing the sum of the PC within all user-sets to maximizing interference between all user-sets, which is capable of allocating the users into the optimum subset for the global pilot decontamination.
- A better combination of the FPR and MKC is investigated in this study to take advantage of both two schemes for the pilot decontamination. On the one hand, the communication quality for edge users would be enhanced by the reasonable sacrifice of pilot resources, on the other hand, the interference suppression for centre users would have a further
improvement by the MKC scheme. We have defined a new measure of the user’s potential susceptibility to interference, and regard it as the main basis for judgments of identifying edge users. In addition, the adjustable threshold parameter of FPR would play a crucial role in optimizing the trade-off between both schemes in diverse massive MIMO scenarios.

- Theoretically, we analyse the formation mechanism and influence of the PC and derive the expression of achievable rates. The proposed scheme is contingent upon not the signal-to-interference-plus-noise-ratio (SINR) but the large-scale fading coefficients. This would address the practical challenge that the precise and global SINR information cannot be available before pilot allocation. Furthermore, pilot reuse in the same cell has been taken into account in this study to achieve the results that reducing the pilot overhead and improving spectral efficiency.

The remainder of this study is organized as follows. In Section 2, the general multi-cell TDD massive MIMO system model and the effect of PC are introduced, whereas Section 3 proposes the FPR-MKC pilot assignment scheme. In Section 4, the numerical simulation results verifying the performance of our proposals are given. At last, the conclusions with some suggestions for future works are presented in Section 5.

The conjugate transpose is denoted with superscripts $H$. The operator $E \{ \cdot \}$ denotes the expectation of a random variable, the notation $\| \cdot \|$ stands for the Euclidean norm, and $\text{diag} ( \cdot )$ denotes the diagonal matrix.

## 2 System Model and Pilot Contamination

As illustrated in Figure 1, an $L$ hexagonal cells massive MIMO network which operates according to a synchronous TDD protocol is considered. In each cell, one BS equipped with $M$ antennas is located at the centre and $K (K < M)$ single-antenna terminal users are distributed randomly.

In the TDD scheme, the channel responses are reciprocal. That is, the channel responses are the same in both the uplink and downlink directions. Therefore, the downlink channel response can be efficiently estimated at the BS by only uplink pilots instead of feedback. Thanks to this, the pilot burden is only dependent on the number of simultaneous service terminals, which renders that the number of BS antennas can be scalable to the desired extent. Thus, the TDD operation is clearly ideal for a massive MIMO system. The synchronous structure TDD multicarrier modulation scheme of a massive MIMO network is shown in Figure 2. In this figure, the time-frequency plane is divided into several coherence blocks. And in each coherence block, the channel response can be approximated as time-invariant and frequency-flat. The coherence interval is naturally divided into three subintervals: uplink pilot transmission, uplink data transmission, and downlink data transmission, and these parts of all terminal users occur synchronously.

For convenience, the symbol $U \langle j,k \rangle$ is used to represent the $k$th user in the $j$th cell, $B < l_m >$ is used to represent the $m$th antenna of the BS in the $j$th cell, and $g_{l,j,k} \langle \langle l,m \rangle \rangle$ represents the complex channel response from $U \langle j,k \rangle$ to $B < l_m >$. In the canonical TDD massive MIMO system, assuming the uncorrelated fading model, $g_{l,j,k} \langle \langle l,m \rangle \rangle$ is determined by both large-scale fading and small-scale fading and can be modelled as

$$
\bar{g}_{l,j,k} (j,k) = \sqrt{\bar{\beta}_{l,j,k} (j,k)} h_{l,j,k} (j,k,\langle \langle l,m \rangle \rangle)
$$

where, $h_{l,j,k} \langle \langle l,m \rangle \rangle$ denotes the small-scale fading coefficient between $U \langle j,k \rangle$ and $B < l_m >$, which obeys independent and identically distributed (i.i.d.) complex-valued Gaussian random variables with zero-mean and unit variance, i.e. $h_{l,j,k} \langle \langle l,m \rangle \rangle \sim \mathcal{CN} (0,1)$. And $\bar{\beta}_{l,j,k} (j,k)$ denotes the large-scale fading coefficient embodying range-dependent path loss and shadow fading. The large-scale fading coefficient is virtually independent of the index of BS antennas and frequency, and it typically is assumed a priori known to the BS. The large-scale fading coefficient $\bar{\beta}_{l,j,k} (j,k)$ can be calculated as

$$
\frac{\bar{\beta}_{l,j,k} (j,k)}{R^{\alpha}} = \frac{\bar{\beta}_{l,j,k} (j,k)}{r_{l,j,k} (j,k)^{\alpha}}
$$


![](image1.png)

**FIGURE 1** Hexagonal cells massive MIMO network

![](image2.png)

**FIGURE 2** The synchronous structure TDD multicarrier modulation scheme in a massive MIMO network
loss exponent of links and \( R \) denotes the radius of the hexagonal cell. Thus, the channel propagation matrix \( \mathbf{G}_{j,l} \in \mathbb{C}^{M \times K} \) between users in \( j \)th cell and BS in \( l \)th cell can be expressed as

\[
\mathbf{G}_{j,l} = \mathbf{H}_{j,l} D_{j,l}^{1/2} = \begin{bmatrix}
\mathbf{g}(j,1)_l & \ldots & \mathbf{g}(j,K)_l \\
\vdots & \ddots & \vdots \\
\mathbf{g}(j,1)_{l,M} & \ldots & \mathbf{g}(j,K)_{l,M}
\end{bmatrix}
\]  

(3)

where, \( D_{j,l} \in \mathbb{C}^{K \times K} = \text{diag} (\varphi_1, \varphi_2, \ldots, \varphi_K) \) and matrix \( \mathbf{H}_{j,l} \in \mathbb{C}^{M \times K} \) is represent as

\[
\mathbf{H}_{j,l} = \begin{bmatrix}
\mathbf{h}(j,1)_l & \ldots & \mathbf{h}(j,K)_l \\
\vdots & \ddots & \vdots \\
\mathbf{h}(j,1)_{l,M} & \ldots & \mathbf{h}(j,K)_{l,M}
\end{bmatrix}
\]  

(4)

According to the law of large numbers and central limit theorem, if the number of BS antennas \( M \) approaches infinity, we can get

\[
\left( \frac{\mathbf{H}_{j,l}^{H} \mathbf{H}_{j,l}}{M} \right)_{M \to \infty} = \begin{cases} I, & j_1 = j_2 \\
0, & j_1 \neq j_2
\end{cases}
\]  

(5)

Thus, under the favourable propagation conditions in the massive MIMO system, the column-vectors of the channel propagation matrix are asymptotically orthogonal, and we have

\[
\left( \frac{\mathbf{G}_{j,l}^{H} \mathbf{G}_{j,l}}{M} \right)_{M \to \infty} = \sqrt{\frac{\mathbf{H}_{j,l}^{H} \mathbf{H}_{j,l}}{M} \left( \frac{1}{M} \right)} \approx \mathbf{D}_{j,l}
\]  

(6)

Acquiring the CSI is a critical operation and the BS estimates the channels generally with the assistance of pilots. Ideally, the terminal users must be assigned mutually orthogonal pilot sequences to guarantee that the intra-cell and inter-cell interference be totally eliminated. However, the signal dimensions in each scheduling slot are restricted due to the limited channel coherence time, which consequently results in the upper-bounded number of available orthogonal pilots. Assuming the number of available orthogonal pilots is \( S \) and the available orthogonal pilot group can be expressed as \( \Psi \in \mathbb{C}^{S \times K} = [\psi_1, \psi_2, \ldots, \psi_S]^{T} \) satisfying \( \Psi \Psi^{H} = I \) (\( I \) is the size-\( S \) identity matrix). During the uplink pilot transmission, the received pilot signals at \( B < l_m > \) can be represented as

\[
\mathbf{y}(l,m) = \sum_{j=1}^{L} \sum_{k=1}^{K} \sqrt{\rho} \mathbf{g}(j,k)_{l,m} \psi(j,k) + \mathbf{w}(l,m)
\]  

\[
= \sum_{k=1}^{K} \sqrt{\rho} \mathbf{g}(k)_{l,m} \psi(k) + \mathbf{w}(l,m)
\]  

(7)

where, \( \rho \) denotes the normalized signal-to-noise ratio (SNR) of each pilot symbol, \( \psi_{<j,k>} \) denotes the pilot that \( U < j,k > \) applies and \( \mathbf{w}(l,m) \) denotes the additive Gaussian white noise (AWGN). Note that the other possible sources of the PC such as hardware impairment and non-reciprocal transceivers are out of the question in this study.

Based on the received pilot sequences, the BS can estimate the uplink CSI of \( U < j,k > \) applying a conventional channel estimation method as

\[
\mathbf{\hat{g}}(j,k)_{l,m} = \frac{1}{\sqrt{\rho}} \mathbf{y}(l,m) \psi(j,k)
\]  

\[
= \mathbf{g}(j,k)_{l,m} + \sum_{U'(l',m') \in \Omega(j,k)} \mathbf{g}(l',m') \psi(j,k) + \frac{1}{\sqrt{\rho}} \mathbf{w}(l,m) \psi(j,k)
\]  

(8)

where, \( \Omega_{<j,k>} \) denotes the set of other users employing the same pilot as \( U < j,k > \). From Equation (8), it is clear that the service antenna array correlates the desired channel with other terminals. Actually, the desired CSI will be corrupted by channels from other users that share the same pilot, and the erroneous channel estimate will be obtained at BS. This negative effect is termed pilot contamination.

During the subinterval of uplink data transmission, the data received at \( B < l_m > \) can be expressed as

\[
\mathbf{y}(l,m) = \sum_{j=1}^{L} \sum_{k=1}^{K} \sqrt{\rho} \mathbf{g}(j,k)_{l,m} \hat{x}_{(j,k)} + \mathbf{w}(l,m)
\]  

\[
= \sqrt{\rho} \mathbf{g}(j,k)_{l,m} \hat{x}_{(j,k)} + \mathbf{w}(l,m)
\]  

(9)

where, \( \rho \) indicates the normalized transmission SNR and \( x_{<j,k>} \) denotes the symbol transmitted from the \( U < j,k > \).

By using the matched filter (MF) as the detector, the detected data signal of \( U < j,k > \) can be obtained at \( j \)th BS as

\[
\hat{x}_{(j,k)} = \sum_{s=1}^{M} \mathbf{g}_{(j,k)}^{H} \mathbf{g}_{(j,k),l,m} \mathbf{\Psi}_{(j,k)}^{(j,k)} + \mathbf{g}_{(j,k),l,m} \mathbf{\Psi}_{(j,k)}^{(j,k)} \psi_{\text{desired signal}}
\]  

\[
+ \sigma_{(j,k)}^{\text{noise}}
\]  

(10)

where, \( \mathbf{y}_{<j,m>} \) denotes the received signal of \( B < l_m > \), \( \mathbf{g}_{<j,k>,l,m} \in \mathbb{C}^{1 \times K} \) denotes the channel vector from \( U < j,k > \) to the BS in \( j \)th cell, and \( \sigma_{<j,k>} \), represents the uncorrelated interference and noise.
Thus, the SINR of \( U < j, k > \) can be calculated as

\[
\text{SINR}_{(j,k)} = \frac{\left\| \mathbf{h}_{(j,k),j}^H \mathbf{g}_{(j,k),j} \right\|^2}{\sum_{U(j',k') \in O_{(j,k)}} \left\| \mathbf{h}_{(j',k'),j}^H \mathbf{g}_{(j',k'),j} \right\|^2 + \sigma^2_{(j,k)} / \rho}
\]

In the presupposition of pilot-assistance channel estimation, the PC is not really specific but much profound to massive MIMO systems. From Equation (11), it leads to the following observations: (a) With the number of BS antennas \( M \) increasing without bound and the pilot reuse factor is one, the effect of the additive noise, fast-fading, and intra-cell interference vanishes. However, the interference caused by PC constitutes an ultimate limit on system performance. (b) In a massive MIMO system, the effective value of SINR is determined by the square value of the large-scale fading coefficients, and \( \beta_{<j,k>} \) plays an important factor in spectral efficiency. (c) It is not indeed beneficial that devoting more power to transmission as the number of base station antennas \( M \) increases. (d) An appropriate \( O_{<j,k>} \) perhaps contributes to a higher SINR for \( U < j, k > \).

Subsequently, the corresponding achievable rate of \( U < j, k > \) can be formulated as

\[
C_{(j,k)} = (1 - \mu_j) E \left[ \log_2 \left(1 + \text{SINR}_{(j,k)} \right) \right]
\]

where, \( \mu_j \) denotes the loss of spectral efficiency caused by pilot transmission, which is the ratio of pilot length to channel coherence time.

Evidently, with simple linear signal processing, the PC introduces a finite SINR to the network, which will in turn cause saturation effect, that is, the system throughput would not grow linearly with the number of BS antennas \( M \). This limited SINR would be translated into achievable rates based on different system model parameters, and eventually leads to an observable side effect on the throughput of a cellular network.

\[3 \quad \text{PROPOSED FPR-MKC PILOT ASSIGNMENT SCHEME} \]

As discussed, the acquisition of the CSI undergoes a significant deviation caused by the PC. In order to mitigate this impact, we aim to find an appropriate pilot assignment scheme to maximize the total achievable rate. And initially, the problem of pilot assignment is formulated by an optimization problem \( P_1 \) as:

\[
\text{arg max}_{P} \left\{ (1 - \mu_j) \sum_{j=1}^{L} \sum_{k=1}^{K} \log_2 \left(1 + \text{SINR}_{(j,k)} \right) \right\}
\]

where \( P \) denotes all pilot assignment schemes.

For conveniently displaying the pilot assignment result, there the user-set of one pilot sequence is defined as:

\[
P(i) = \left\{ U(j, k) : \psi_{(j,k)} = \psi \right\}
\]

And all user-sets \( P(i) (s \in \{1, \ldots, S\}) \) are subject to the following two constraints:

\[
\begin{align*}
\forall s \in \{1, 2, \ldots, s\} - \{j\}, & \quad P(i) \cap P(j) = \emptyset \\
\bigcup_{i=1}^{S} P(i) &= \bigcup_{j=1, k=1} \{U(j, k)\}
\end{align*}
\]

Note that the first constraint states that any two user-sets are mutually exclusive so that each user can be assigned into at most one user-set, and the last constraint guarantees that each user is allocated into at least one user-set. Thus, each of the users is surely in only one user-set \( P(i) \).

As for \( P_1 \), however, it is not practical to obtain the perfect knowledge of SINR before pilot allocation. Actually seen in Equation (11), the SINR of target user \( U < j, k > \) depends largely on the square of large-scale fading coefficients of the target user and interference users. Fortunately, the large-scale fading coefficients can be tracked easily at BS with low complexity. They are always applied to realize the handover process among adjacent cells for users in practical mobile-cellular networks. In addition, approaches on estimating and tracking these coefficients have been investigated in pieces of works \( [22, 23] \). Thus, as the number of base station antennas \( M \) trends to be unlimited, the \( P_1 \) can be approached by \( P_2 \) where no precise SINR information is required, that is,

\[
P_2 : \max_{P_2} \left\{ (1 - \mu_j) \sum_{j=1}^{L} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\beta^2_{(j,k),j}}{\sum_{U(j',k') \in O_{(j,k)}} \beta^2_{(j',k'),j}} \right) \right\}
\]

The above-mentioned pilot assignment optimization problem is essentially a combinatorial optimization problem, and the exhaustive law that searches all kinds of pilot assignment results is the most direct method. But the high computational complexity of this scheme is intolerable in most massive MIMO systems, particularly that with the expensive servicing area and the large user number. The traditional pilot allocation scheme randomly allocates orthogonal pilots to users in one cell. However, this random scheme ignores the fact that it generates varying levels of the PC when the same pilots are assigned to different users.

\[3.1 \quad \text{Fractional pilot reuse} \]

The users at the cell edge region would not only suffer from severe PC but also cause particularly detrimental interference to neighbours. The FPR scheme always has the ability to enhance
the quality of service for edge users, wherein the edge users and centre users are treated separately. Specifically, a fraction of users reuse the same pilot across the whole system and the rest that are at the cell edge region could be assigned into exclusive user-sets $P(j)$. It is shown to provide promising gains that providing the extra pilot resources for segregating the side effect of the edge users.

The traditional FPR [13] and SPR [16] schemes classify one user according to its physical proximity or the square of large-scale fading coefficient to its anchor BS. However, it is often ignored that the density of the interfering users in other cells relative to the anchor BS is also a key factor affecting the user’s quality of the service. Seen in Equation (11), the SINR is inversely proportional to the sum of squares of large-scale fading coefficients of interference users. It means that the SINR also get remarkable related to the relative position of other interfering users. When the large-scale fading coefficient between the interfering user and the target BS is close or even higher than that between the target user and the target BS, it can produce a high level of PC.

An illustrative example of users’ susceptibility to interference is shown in Figure 3. We use the colour shades to indicate the strength of the susceptibility to interference in the region. On the one hand, in a cell, the service quality of one user is related to the distance between it and its anchor base station. Users at the cell edge region are more likely to be disturbed than those at cell centre region, such as user A and user B. On the other hand, the overall interference level of the cell depends on the density of the interference users, like user A and user C. Even user A and user C are with the same distance from their anchor BS, the potential interference intensity of user A is stronger than that of user C, because the interference users in $Cell2$ and $Cell4$ are located rather more concentratedly around the BS in $Cell1$ than the BS in $Cell3$. Therefore, these cells make a difference in overall interference level, and simply using the physical proximity of the users to their anchor BS to classify the cell edge region or cell centre region is not accurate enough.

In this study, we define the innovative measure of users’ susceptibility to interference as:

$$\xi_{(j,k)} = \eta_j \times \chi_{(j,k)}$$  \hspace{1cm} (17)

where, $\chi_{<j,k>}$ denotes the location interference coefficient of $U < j,k >$, which can be represented as the square of large-scale fading coefficient between $U < j,k >$ to its anchor BS. And $\eta_j$ denotes global interference intensity coefficient of $j$th cell, which can be calculated as:

$$\eta_j = \frac{1}{\sum_{j=1}^{m} \sum_{k=1}^{N_{\text{BS}}} \beta_{(j,k),j}^{2}}$$ \hspace{1cm} (18)

Then, the users are divided into two groups according to the following rules,

$$\begin{align*}
\xi_{(j,k)} > \lambda & \rightarrow \text{center} \rightarrow U \langle j, k \rangle \in O_c \\
\xi_{(j,k)} \leq \lambda & \rightarrow \text{edge} \rightarrow U \langle j, k \rangle \in O_e
\end{align*}$$ \hspace{1cm} (19)

where, $O_c$ and $O_e$ denote the edge user set and centre user set separately, and the value of threshold $\lambda$ can be set before simulation or be obtained an estimated value based on the actual cellular network. In this case, the judgment basis for identifying user types not only depends on its own location in the cell but also gets related to the density of the other interfering users relative to the anchor BS.

### 3.2 Weighted pilot contamination graph

Constructing the pilot contamination graph $G = (V, E)$ that corresponds to the network topology is the main step in the proposed pilot scheme, where $V = \{ U < j,k > : U < j,k > \in O_i \}$ denotes the vertex set that represents the centre users and $E$ denotes the weighted edge associated with the potential strength of PC.

From Equation (11), we can see the large-scale fading coefficients contain useful information related to the interference between users. Specifically, the PC between $U < j,k >$ and $U < j',k' >$ with the same pilot are directly bound up with the ratios $\beta_2 < j',k' > / \beta_2 < j,k >$ and $\beta_2 < j,k > / \beta_2 < j',k' >$. Thus, we employ the metric in [21] to present the potential strength $w_{<j,k>,<j',k>}'$ of pilot contamination between any two centre $U < j,k >$ and centre $U < j',k' >$ as:

$$w_{(j,k),(j',k')'} = \frac{\beta_2^{2} < (j,k),j >}{\beta_2^{2} < (j',k'),j >} + \frac{\beta_2^{2} < (j,k),j >}{\beta_2^{2} < (j',k'),j >}$$ \hspace{1cm} (20)

Note that the edge weight $w_{<j,k>,<j',k>}'$ is a positive value and it is associated with the locations of $U < j,k >$, $U < j',k' >$, $j$th BS, and $j'$th BS. We also have $w_{<j,k>,<j',k>}' = w_{<j',k'>,<j,k>}$.

Obviously, the larger edge weight between two users implies more serious interference if they apply the same pilot. In a practical multicellular massive MIMO network, the weight of an intra-cell edge is regularly large, and the sum of the weights related to the edge users is always larger than that related to the centre users. Additionally, we would not neglect the case that the pilot can be reused in the same cell, equivalently, we
take the intra-cell weighted edge into consideration. And in this study, the number of pilots for pilot reuse $S_{\text{reuse}}$ is predetermined according to the number of available orthogonal pilots $S$.

Figure 4 shows a simple schematic of undirected weighted pilot contamination with two cells and four users. In fact, four levels of the interference intensity between any two users can be inferred as follows.

- **Level 1:** Intra-cell interference. Two users are within the same cell. User A and user B are an example. This would incur Intra-cell interference if the two users are in the same pilot set. The condition of the existence of this situation is that the pilot reuse in the same cell is applied. Generally, the weight value $w_{AB}$ associated with intra-cell interference is so large that pilot reuse in the same cell is generally avoided in the most pilot scheme.

- **Level 2:** Inter-cell interference 1. Two users are in different cells, where both two users are closed to the cell edge region which is associated with the two cells. User A and user C are an example. This case suggests that the transmission of one user is highly vulnerable to interference from the other.

- **Level 3:** Inter-cell interference 2. Two users are in different cells, where only one user is located in the cell edge area which is associated with the two cells. To cite an example, user A in Cell1 has a high physical proximity to the BS in Cell2, while user D in Cell2 is far away from the BS in Cell1.

- **Level 4:** Inter-cell interference 3. Two users are in different cells, where neither two users are closed to the cell edge region which is associated with the two cells, like user B and user D.

Generally, the four levels of the interference intensity can be ranked as Level 1 $>$ Level 2 $>$ Level 3 $>$ Level 4. Using the method of calculating weights as Equation (20), Table 1 presents the weight value of edge in Figure 4, where the cell radius is $R = 500$ m; the path loss exponent is $\alpha = 3.8$ and the log-normal shadowing fading is $\sigma = 8$ dB. The coordinates of the two base stations are [0,0] and [0,1000], and the coordinates of the four users A, B, C, D are set as [20,470], [−30,−200], [−60,600], [300,1100]. To avoid the weight value fluctuating due to shadow fading, we perform 10,000 times simulations under the set conditions and take the average value.

### 3.3 Max k-cut based pilot assignment scheme

The process of allocating users into separate user-sets $P(j)$ is innovatively treated as the partition of the vertices of a graph. Figure 5 considers an illustrative example of the pilot assignment of three scenarios with four users. The scenarios in Figure 5(a–c) are $S = 1$, $S = 2$ and $S = 4$. Obviously, if an edge $w_{<j,k>,<j',k'>}$ that characterize the potential strength of PC is within one user-set $P(j)$, the interference related to them would really exist in practice. In contrast, if the edge $w_{<j,k>,<j',k'>}$ that crosses two different user-sets, it implies that the mutual PC between the two users would be eliminated after the pilot assignment, which is indicated by the dotted line in Figure 5.

In the first case shown in Figure 5(a), all users are allocated into a single user-set $P(j)$ and all edges $w_{<j,k>,<j',k'>}$ are within the user-set. This means that each user would be interfered with by other users. In this case, the pilot overhead is the smallest but the PC of the whole system is the largest. While the most ideal scenario for PC is illustrated in Figure 5(c). It is shown there exists a one-to-one correspondence between users and pilots. The pilot assignment scheme optimization would become trivial in this case, because the amount of pilot resource that is available for assignment is equal to the number of users (i.e. $S = K$) such that all edges in the weighted pilot contamination graph are cut by the boundary of user-sets. As a result, the overall system will be interference-free.

After the aforementioned observation, we are led to the following discussion.

First, more user-sets $P(j)$ mean a larger number of edges cut after the pilot assignment. For instance, with the number of user-sets is $S = 1$, $S = 2$ and $S = 4$ in Figure 5(a–c), the number of edges across two different user-sets is 0, 4, and 6, respectively. From this figure, a positive correlation was found between the number of pilots and the number of edges across user-sets. In other words, creating more orthogonal pilot sequences can provide more boundaries to segregate interference, which can suppress the PC to a great extent. Due to the finite pilot budget, in a multi-cell system, there is an important trade-off of pilot resource and interference strength.

### Table 1 The weight value of edge in Figure 4

| Edge | Level | Weight |
|------|-------|--------|
| $w_{AB}$ | Intra-cell interference | 12.4571 |
| $w_{AC}$ | Inter-cell interference 1 | 2.9320 |
| $w_{AD}$ | Inter-cell interference 2 | 1.0560 |
| $w_{BC}$ | Inter-cell interference 2 | 0.4725 |
| $w_{BD}$ | Inter-cell interference 3 | 0.0132 |
| $w_{CD}$ | Intra-cell interference | 8.2836 |
Algorithm 1 Proposed FPR-MKC scheme

Input:
- System parameters: $K, I_s, \bar{\gamma}_{\text{max}}$, and $\lambda$.
- Large-scale fading coefficient: $\beta_{\xi,k,j}$.

1. Initialization:
   2. $O_i, O_0, K_s = K_i = 0$; $P(u) = \emptyset$.
   3. for $j = 1$ to $I_o$.
      4. for $k = 1$ to $K$.
         5. Calculate susceptibility to interference as (17).
         6. if $\xi_{i,j,k} \leq \beta_{\xi,k,j}$ then
            7. $O_i = O_i \cup \{U_{<j,k}\}$; $K_s = K_s + 1$.
            8. else
               9. $O_i = O_i \cup \{U_{<j,k}\}$; $K_s = K_s + 1$.
            10. $P(S_{\text{max}} + K_s) = O_o U_{<j,k}$.
         end if
      end for
   end for
   14. for $U_{<j,k} \in O_i$.
      15. for $U_{<j',k'} \in O_i$ & $U_{<j,k} \neq U_{<j',k'}$.
         16. Calculate $w_{\xi,k,j}',\xi,j'/' as (20).
      end for
   end for
   19. $O_{\text{max}} = O_i$.
   20. Arbitrarily assign $S_{\text{max}}$ users $U_{<j,k} \in O_o$ to $S_{\text{max}}$ user-sets $P(u)$ for $s = 1, \ldots, s_{\text{max}}$.
   21. $O_{\text{max}} = O_{\text{max}} - \{U_{<j,k} \in \text{selected in Line 20}\}$.
   22. for $U_{<j_0,k_0} \in O_{\text{max}}$.
      23. for $s = 1$ to $s_{\text{max}}$.
         24. Calculate the increased intra-set weight using
            \( \sum_{U_{<j,k'} \in P(u)} w_{\xi,k,j}(j',k') \).
            \( P'(u) = P(u) U_{<j_0,k_0} \) with
            \( j' = \arg\min_{U_{<j,k'} \in P(u)} w_{\xi,k,j}(j',k') \).
         25. $O_{\text{max}} = O_{\text{max}} - \{U_{<j_0,k_0}\}$.
      end for
      end for
   end for
   29. for $s = 1$ to $K_s + K_i$.
      30. Return all user-sets $P(u)$.
   end for

Second, the process of assigning users into finite user-sets is equivalent to partitioning the vertices into several subsets of vertices in graph theory. To further optimize the pilot scheme with the fixed and predetermined pilot overhead, an effective way is to find a segmentation rule that makes the summation of weights of the edges with end point in different subsets as large as possible. Thus, we can find a significantly close relationship between MKC in graph theory and the pilot assignment in massive MIMO networks.

Methodologically, given an undirected edge-weighted graph $G = (V,E)$ with $n$ vertexes, nonnegative edge costs $w_{ij}$ ($i,j \in (1,n)$) and an integer $k \in (2,n)$, the MKC is to find a partition of $V$ into $k$ disjoint sets $C = \{C_1, C_2, \ldots, C_k\}$ with the maximized total weight of the edges between the disjoint sets. In graph theory, the size of the cut is the sum weights of the edges that cross the cut. This issue can be formulated as

\[
(P_3) : \max_C \sum_{1 \leq p < q \leq k} \sum_{\mathbb{P}_{C_p}, \mathbb{P}_{C_q}} w_{pq} \tag{21}
\]

Max k-cut is one of many polynomial-time (NP) hard graph theory problems in combinatorial optimization, which has attracted many researchers over the years because of its theoretical significance and large application potential. Though there is almost no hope in finding a polynomial-time algorithm for the max k-cut problem, various heuristics, or a combination of optimization and heuristic methods have been developed to solve this problem. A simple linear time heuristic algorithm for this max k-cut problem with low computational complexity has been proposed in [24], of which the idea is to iteratively assign nodes to the cluster with the goal of minimizing the increased intra-cluster weight. This algorithm is proven to achieve an absolute ratio of $(1-1/k)$ for a general k-cut problem [24]. In other words, the algorithm can yield a cut in which the inter-subset weight sum is at least $(1-1/k)$ times the optimal cut.

The complexity of this algorithm with $n$ vertexes and $k$ subset is $O(n^2/2 + n/2 + k)$, which is proportional to the sum of the number of edges, vertexes, and subsets in the graph.

Using the max k-cut to eliminate the PC for centre users, the result tends to separate the users that have high PC into different user-sets or, equivalently, place weak interfered users into one same user-set $P(u)$. Here, each cluster corresponds to a user-set $P(u)$ and the node corresponds to a centre user. Inspired by the heuristic algorithm in [24], the proposed max k-cut based pilot assignment scheme for centre users proceeds by the following steps:

![Figure 5](image-url)
3.4 Proposed FPR-MKC scheme

The pseudocode of the proposed FPR-MKC scheme for all users is provided in Algorithm 1 with three assumptions, that is, (1) $s_{\text{reuse}}$ and $\lambda$ are predetermined, (2) the large-scale fading coefficient is a priori known to the BS, and (3) the pilots can be reused in one same cell.

Therein, Lines 1–2 initialize some necessary parameters. Lines 3–5 calculate users’ susceptibility to interference as Equations (17) and (18). Lines 6–13 separate all users into either the edge group or centre group according to the rule seen in Equation (19), and according to the results of grouping, assign the extra pilot resources for each edge user. Lines 14–18 evaluate the potential strength of pilot contamination for all centre users according to Equation (20). Line 19 defines the user set $O_{\text{res}}$, in which the users are not in any user-set $P(s)$. Lines 20–28 execute the max k-cut heuristic algorithm to reuse the pilots allocated to centre users. So far, we have completed the program of assigning all users to users-sets. At last, Lines 29–31 output all user-sets $P(s)$ ($s \in \{1, \ldots, s_{\text{reuse}} + K_s\}$) as a result of the pilot assignment.

4 NUMERICAL RESULT AND DISCUSSION

In this section, we investigate the performance of the proposed FPR-MKC scheme compared with the following three schemes: conventional random scheme, WGC scheme [21], and SPR scheme [16], in terms of the SINR and the achievable rate by using a set of Monte Carlo simulations. Moreover, in Figures 9 and 10, we analyse the gap between the proposed scheme and the optimal solution using the exhaustive algorithm. A typical multi-cellular TDD massive MIMO network with $L$ hexagonal cells illustrated in section II is considered. In each cell, a BS with $M$ antennas is located in the centre and $K$ single-antenna users are randomly distributed. And the corresponding fixed system parameters are summarized as follows: the number of cells is $L = 7$; the cell radius is $R = 500$ m; the carrier frequency is $f = 2$ GHz; the system bandwidth is $B = 10$ MHz; the loss of spectral efficiency is $\mu_s = 0.1$; the path loss exponent is $\alpha = 3.8$; and the log-normal shadowing fading is $\sigma = 8$ dB.

Figure 6 shows the cumulative density function (CDF) of SINR with (a) $s_{\text{reuse}} = 16$ (b) $s_{\text{reuse}} = 12$, where the number of users in a cell is $K = 12$, the number of BS antennas is $M = 2^8$; the transmission SNR is $\rho = 15$ dB and the threshold adjustment parameter is $\lambda = 0.03$. The results presented by 1000 random simulation trials obviously show that the proposed FPR-MKC scheme can obtain a higher SINR than other schemes. Another notation is that the zero-forcing (ZF) detector always has a more desirable performance in attaining a higher SINR than the MF detector. Furthermore, we could see that the WGC scheme is superior to the SPR scheme when $s_{\text{reuse}} = 16$ in Figure 6(a), on the contrary, the result in Figure 6(b) indicates the SPR scheme is better than the WGC scheme, this illustrates that the WGC scheme is more suitable for the scene that $s_{\text{reuse}} > K$.

Figure 7 shows the CDF of the average achievable rate with (a) $K = 12$, $s_{\text{reuse}} = 16$ (b) $K = 12$, $s_{\text{reuse}} = 12$ (c) $K = 8$, $s_{\text{reuse}} = 8$, where the number of base station antennas is $M = 2^8$; the transmission SNR is $\rho = 15$ dB and the threshold adjustment parameter is $\lambda = 0.03$, which is presented by 1000 random simulation trials. The results show that the average achievable rate in the proposed FPR-MKC scheme is higher than that in other schemes because the channel estimation error is reduced most efficaciously in the FPR-MKC scheme. And consequently, the highest spectral efficiency can be obtained in it. This result confirms the rationality and effectiveness of the proposed FPR-MKC scheme for a massive MIMO system. Moreover, the possibility of the extremely low average achievable rate in FPR-MKC scheme is much lower than other schemes, it implies that the quality of service can be remarkably enhanced for edge users in the proposed scheme. Similarly, the ZF detector always outperforms the MF detector by about 2 bps Hz < $sp$ > - 1 < $sp$ > per user. Comparing Figure 7(a–c), some observations could be obtained: (1) The WGC scheme performs better in improving the average achievable rate than the SPR scheme in Figure 7(a), and yet the result is converse in Figure 7(b). This result can also be observed in Figure 6. Equation (2) The performance superiority of the proposed FPR-MKC scheme and the SPR is more significant with $K = 8$ than that with $K = 12$. This implies that the SPR scheme and the WGC scheme would be adapted to different practical scenarios, and the SPR scheme prefers the case that $K$ is smaller. The SPR scheme needs higher pilot overheads to enhance the quality of service for the edge users and will lose more spectral efficiency, which is always unbearable when $K$ is large. The SPR scheme and the WGC scheme approach the PC problem from different angles, and accordingly, the proposed FPR-MKC scheme combines the advantages of both two schemes, and it would improve the adaptability in different settings by adjusting the threshold parameter $\lambda$.

Figure 8 illustrates the average achievable rate for the average number of edge users per cell and the number of pilots for reuse $s_{\text{reuse}}$ using the proposed FPR-MKC scheme with (a) $K = 8$ (b) $K = 12$, where the number of base station antennas is $M = 2^8$ and the transmission SNR is $\rho = 15$ dB. The data cursors labeled in the figure mark the maximum value of the average achievable rate for all different $s_{\text{reuse}}$. It is obvious that the average achievable rates in Figure 8(a) are larger than that in Figure 8(b), since the number of users in a cell $K$ is smaller in Figure 8(a) and this leads to less inter-cell interference. From this figure, it is interesting to find that the efficacy of FPR scheme becomes
Figure 6 shows the CDF of SINR for the proposed FPR-MKC scheme, the random scheme, the WGC scheme and the SPR scheme with (a) $S_{\text{reuse}} = 16$ (b) $S_{\text{reuse}} = 12$, where the number of users in a cell is $K = 12$, the number of BS antennas is $M = 28$, the transmission SNR $\rho = 15$ dB and the threshold adjustment parameter is $\lambda = 0.03$. The plots compare the performance of these schemes under different reuse schemes and show the advantage of the proposed FPR-MKC scheme in terms of outage probability and SINR distribution.
unsatisfactory as the $S_{\text{reuse}}$ increases, and a similar phenomenon can be seen in Figure 7. Even so, adjusting $\lambda$ to separate a small number of edge users would obtain more spectral efficiency when $S_{\text{reuse}}$ is close to $K$ in spite of the extra pilot overheads for edge users, and the optimal value of the threshold parameter would increase as the $K$ is reducing. In this condition, the combination of FPR and MKC would outperform more significantly than any single. In addition, the most optimal average number of edge users per cell obtained by simulation trials can be regarded as an important reference for calculating the threshold $\lambda$.

Figure 9 demonstrates the average achievable rate against the number of antennas $M$, where the number of users in a cell is $K = 10$; the number of pilots for reuse is $S_{\text{reuse}} = 12$, the transmission SNR is $\rho = 15$ dB and the threshold adjustment parameter is $\lambda = 0.03$. As $M$ further increases, the average achievable rate of the three algorithms increases, which is due to the spectral efficiency obtained by configuring a large number of antennas on the base station side in a massive MIMO system, while the conventional scheme that assigns pilot in a random manner performs unsatisfactorily. Observed in Figure 9, apparently, the average achievable rate in the FPR-MKC scheme appears to be close to the optimal and be always higher than that in other schemes and the performance advantage of the proposed pilot scheme expands by increasing the number of the base station antennas $M$. For the example of Figure 9, when the number of BS antennas $M$ is 512, 1024, and 2048, the average achievable rate of the proposed FPR-MKC scheme is about 0.693 bps Hz$^{-1}$, 0.945 bps Hz$^{-1}$ and 1.225 bps Hz$^{-1}$ higher than that of the random scheme, respectively. This observation verifies the performance advantage of the proposed scheme in a very large MIMO system.

Figure 10 shows the average achievable rate with $M = 2^9$ and $\lambda = 0.03$ against the transmission SNR $\rho$, where the number of users in a cell is $K = 10$; the number of pilots for reuse is $S_{\text{reuse}} = 12$. From the figure, we can see that the transmission-power-scaling approach can contribute to achieving a higher average achievable rate in cellular networks since increasing the transmit power can be interpreted as a rise in SNR. However, the average achievable rate cannot increase indefinitely with transmission-power, this is due to the fact that both the desired signal and the interfering signal boost their transmit powers and the network will be pushed eventually into a limited regime completely determined by the strength of the interference. As shown in the figure, the gap of the proposed FPR-MKC scheme and the optimal solution is no more than 0.1 bps Hz$^{-1}$ Meanwhile, the proposed scheme can remarkably outperform the traditional random scheme, WGC scheme, and SPR scheme, and for the example of Figure 10, when the transmission SNR is 2, 10, and 20 dB, the average achievable rate of the proposed FPR-MKC scheme is higher than that of the random scheme by about 0.42 bps Hz$^{-1}$, 0.647 bps Hz$^{-1}$ and 0.715 bps Hz$^{-1}$ respectively. The results indicate that the improvement brought from the proposed FPR-MKC scheme grows more remarkable in the case that the transmission power is higher.
CONCLUSION

In this study, the FPR-MKC pilot scheme has been proposed for a multi-cell TDD massive MIMO system, which is able to alleviate the PC and enhance the achievable rate for both centre users and edge users. In principle, the FPR-MKC scheme maps the pilot assignment issue to the MKC in graph theory and proposes an adaptive combination of the FPR and the MKC. By properly providing the extra pilot resources for edge users and maximizing the summation of weights of the edges that are across different user-sets, the proposed scheme can efficaciously suppress the PC incurred by pilot reuse. As a consequence, the SINR and the average achievable rate can be significantly raised compared with the conventional random scheme, particularly in the cases with increasing the scale of antennas or the transmission power.

Due to its remarkable effectiveness and low complexity, it is believed that the pilot scheme proposed in this study would
FIGURE 9  The average achievable rate against the number of antennas $M$, where the number of users in a cell is $K = 10$; the number of pilots for reuse is $S_{\text{reuse}} = 12$; the transmission SNR is $\rho = 15$ dB and the threshold adjustment parameter is $\lambda = 0.03$.

FIGURE 10  The average achievable rate with $M = 2^9$ and $\lambda = 0.03$ against the transmission SNR $\rho$, where the number of users in a cell is $K = 10$; the number of pilots for reuse is $S_{\text{reuse}} = 12$. 
provide some useful hints in the practical design of the TDD massive MIMO networks. Finally, building on the works in this study, some suggestions for future works are given as follows: (1) An optimal design to determine pilot resource, (2) the further improved method based on group intelligent optimization algorithm for max k-cut, (3) a synthesis of FPR-MKC scheme and transmission power control mechanisms [25], [26] and (4) the advanced channel estimate method and precoding can be integrated into the pilot assignment.

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REFERENCES
1. Elijah, O., et al.: A comprehensive survey of PC in massive MIMO–5G system. IEEE Commun. Surveys Tuts. 18(2), 905–923 (2016)
2. Larsson, E.G., et al.: Massive MIMO for next generation wireless systems. IEEE Commun. Mag. 52(2), 186–195 (2014)
3. Marzetta, T.L.: Noncooperative cellular wireless with unlimited numbers of base station antennas. IEEE Trans. Wireless Commun. 9(11), 3590–3600 (2010)
4. Lu, L., et al.: An overview of massive MIMO: Benefits and challenges. IEEE J. Sel. Topis Signal Process. 8(5), 742–758 (2014)
5. Rusek, F., et al.: Scaling up MIMO: Opportunities and challenges with very large arrays. IEEE Signal Process. Mag. 30(1), 40–60 (2013)
6. Ngo, H.Q., Larsson, E.G., Marzetta, T.L.: Energy and spectral efficiency of very large multiuser MIMO systems. IEEE Trans.Commun. 61(4), 1436–1449 (2013)
7. Hoydis, J., ten Brink, S., Debbah, M.: Massive MIMO in the UL/DL of cellular networks: How many antennas do we need? IEEE J. Sel. Areas Commun. 31(2), 160–171 (2013)
8. Jose, J., et al.: Pilot contamination and precoding in multi-cell TDD systems. IEEE Trans. Wireless Commun. 10(8), 2640–2651 (2011)
9. Yin, H., et al.: A coordinated approach to channel estimation in large-scale multiple-antenna systems. IEEE J. Sel. Areas Commun. 31(2), 264–275 (2013)
10. You, L., et al.: Pilot reuse for massive MIMO transmission over spatially correlated Rayleigh fading channels. IEEE Trans. Wireless Commun. 14(6), 3352–3366 (2015)
11. Malyshev, W.A.W., Martin, P.A., Smith, P.J.: Performance of synchronized and unsynchronized pilots in finite massive MIMO systems. IEEE Trans. Wireless Commun. 14(12), 6763–6776 (2015)
12. Kong, D., Qu, D., Luo, K.: Channel estimation under staggered frame structure for massive MIMO system. IEEE Trans. Wireless Commun. 15(2), 1469–1479 (2016)
13. Arzeni, I., Arnau, J., Debbah, M.: Fractional pilot reuse in massive MIMO systems. In: Proceedings of IEEE International Conference on Communication Workshop (ICCW), London, England, pp. 1030–1035 (2015)
14. Parida, P., Dhillon, H.S.: Stochastic geometry-based uplink analysis of massive MIMO systems with fractional pilot reuse. IEEE Trans. Wireless Commun. 18(3), 1651–1668 (2019)
15. Fan, J., Li, W., Zhang, Y.: Pilot contamination mitigation by fractional pilot reuse with threshold optimization in massive MIMO systems. Digit. Signal Process. 78, 197–204 (2018)
16. Zhu, X., et al.: Soft pilot reuse and multicell block diagonalization precoding for massive MIMO systems. IEEE Trans. Veh. Technol. 65(5), 3285–3298 (2016)
17. Chang, W., Chan, H.-W., Hua, Y.K.: Weighted graph coloring based softer pilot reuse for TDD massive MIMO systems. IEEE Trans. Veh. Technol. 67(7), 6272–6285 (2018)
18. Zhu, X., et al.: Smart pilot assignment for massive MIMO. IEEE Commun. Lett. 19(9), 1644–1647 (2015)
19. Zhi, H., Huang, Z.J., Wang, F.Y.: Flexible pilot allocation scheme for massive MIMO two-tier heterogeneous networks. IET Commun. 14(2), 219–233 (2020)
20. Zhu, X., Dai, L., Wang, Z.: Graph coloring based pilot allocation to mitigate pilot contamination for multi-cell massive MIMO systems. IEEE Commun. Lett. 19(10), 1842–1845 (2015)
21. Zhu, X., et al.: Weighted-graph-coloring-based pilot decontamination for multicell massive MIMO systems. IEEE Trans. Veh. Technol. 66(3), 2829–2834 (2017)
22. Chen, K.F., Liu, Y.C., Y.T. Su: On composite channel estimation in wireless massive MIMO systems. In: Proceedings of 2013 IEEE Globecom Workshops, Atlanta, Georgia, pp. 135–139 (2013)
23. Li, K., et al.: An improved multicell MMSE channel estimation in a massive MIMO system. Int. J. Antennas Propag. 2014(2), 1-9 (2014)
24. Sahni, S., Gonzalez, T.: P-complete approximation problems. J.ACM 23(3), 555–565 (1976)
25. Dao, H.T., Kim, S.: Pilot power allocation for maximising the sum rate in massive MIMO systems. IET Communications 12(11), 1367–1372 (2018)
26. Liu, P., et al.: Pilot power allocation through user grouping in multi-cell Massive MIMO systems. IEEE Trans. Commun. 65(4), 1561–1574 (2017)

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