Theory of transverse spin dynamics in a polarized Fermi liquid and an itinerant ferromagnet

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The linear equations for transverse spin dynamics in a weakly polarized degenerate Fermi liquid with arbitrary relationship between temperature $T$ and polarization $\gamma H$ are derived from Landau-Silin phenomenological kinetic equation with general form of two-particle collision integral. Unlike the previous treatment where Fermi velocity and density of states have been taken as constants independent of polarization here we made derivation free from this assumption. The obtained equations are applicable for description of spin dynamics in paramagnetic Fermi liquid with finite polarization as well in an itinerant ferromagnet. In both cases transverse spin wave frequency is found to be proportional to $k^2$ with complex constant of proportionality (diffusion coefficient) such that the damping has a finite value at $T = 0$. The polarization dependence of the diffusion coefficient is found to be different for a polarized Fermi liquid and for an itinerant ferromagnet. These conclusions are confirmed by derivation of transverse spin wave dispersion law in frame of field theoretical methods from the integral equation for the vortex function. It is shown that similar derivation taking into consideration the divergency of static transverse susceptibility also leads to the same attenuating spin wave spectrum.

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I. INTRODUCTION

Since it appearence in the pioneering papers of V.P.Silin and A.J.Leggett the theory of spin dynamics in spin polarized Fermi liquid has a long history mainly concerned with the problem of the zero-temperature transverse spin-wave attenuation. The calculations of transverse spin-diffusion coefficient in dilute degenerate Fermi gas with arbitrary polarization was done for the first time in the papers by W.Jeon and W.Mullin where the low temperature saturation of corresponding relaxation time has been established. About the same time A.Meyerovich and K.Musaelyan have derived the spin kinetics in the polarized Fermi liquid from microscopic theory and also came to the same conclusion. A derivation and an exact solution of the kinetic equation in the s-wave scattering approximation for dilute degenerate Fermi gas with arbitrary polarization at $T = 0$ and for a small polarization at $T \neq 0$ have been obtained also in the papers by D.Golosov and A.Ruckenstein. For the treatment of this problem in a Fermi liquid the Matthiessen-type rule arguments and simple relaxation-time approximation for the collision integral have been used. More recently, the derivation of transverse spin dynamics in a spin-polarized Fermi liquid from the Landau-Silin kinetic equation with general form of a two-particle collision integral has been performed. The existence of zero-temperature damping of transverse spin waves has been established. At low temperatures and polarizations $\gamma H$ it proves to be proportional to the rate of collisions between quasiparticles

$$
\frac{1}{\tau} \propto (\gamma H)^2 + (2\pi T)^2.
$$

(1)

Experimentally the saturation of the transverse spin wave diffusion constant at temperatures about several millikelvin has been registered by the spin-echo technique (see, for instance). On the other hand, the spin wave experiments demonstrate the behaviour characterized rather by the absence of transverse spin wave damping in the same temperature region. The latter seem to be a confirmation of the point of view of I.Fomin who has argued for the dissipationless form of transversal spin wave spectrum derived from the correction to the system energy due to the gauge transformation into the coordinate system where the magnetization vector is constant. The calculation of the generalized susceptibility coefficient in the expression for the spin current found in has not been performed, just the reference on such calculation has been given. Indeed, one can calculate susceptibility using a similar procedure. However, in the case of polarized Fermi liquid one must use the Green functions with the finite imaginary self-energy parts due to collisions between quasiparticles as it was done in, that inevitably leads to the spin waves attenuation.

The derivation of linear on transversal deviations spin wave dynamics has been undertaken in the papers at finite polarization. However, all the Fermi liquid characteristic parameters have been taken as constants independent of polarization. The derivation partly free of this assumption (taking in consideration the polarization dependence of Fermi velocity and the density of states) is proposed in the present article (Section II). It results not only in equations for the time-space variations of spin and spin current densities with more general expressions for all the coefficients but reveals the possibility to find the distinctions for the spin dynamics in spin polarized paramagnetic Fermi liquid and in ferromagnetic Fermi liquid with spontaneous magnetization. In both systems
transverse spin wave frequency is found to be proportional to \(k^2\) with complex constant of proportionality such that the damping has a finite value at \(T = 0\).

It is well known that the phenomenological Landau Fermi liquid theory has well established foundations based on microscopic theory. Namely, the transport equation for the vibrations of a Fermi liquid was derived from an integral equation for vertex function and general relation between the amplitude of forward scattering and the Fermi liquid interaction parameters was found. On the other hand there were known several publications where the kinetic equation and field theoretical methods based on Landau Fermi liquid theory have been applied to the treatment of itinerant isotropic ferromagnet. In particular the derivation of dissipationless (up to the terms of order \(\sim k^4\)) spin waves spectrum has been announced. In the present article in frame of microscopic theory we reconsider the problem of transverse spin waves in spin-polarized Fermi liquid (Section III) and in itinerant ferromagnet taking into account the divergency of static susceptibility (Section IV). It is shown that in the both cases the microscopic derivation leads to the same spin wave spectrum. Along with the dissipationless part it contains also the finite zero-temperature damping. The polarization dependence both dissipative and reactive part of diffusion constant corresponds to dependences found by means of kinetic equation with two-particle collision integral (Section II).

II. SPIN WAVE DISPERSION

The quasiparticle distribution function as well as quasiparticle energy are given by 2 × 2 matrix in spin space,

\[
\hat{n}_k(r,t) = n_k(r,t)\hat{I} + \sigma_k(r,t)\hat{\sigma},
\]

(2)

\[
\hat{\varepsilon}_k(r,t) = \varepsilon_k(r,t)\hat{I} + h_k(r,t)\hat{\sigma}.
\]

(3)

Here \(\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)\) are Pauli matrices. As long as we consider the small deviations of the magnetization direction from its equilibrium direction the equation for the scalar part of the distribution function \(n_k(r,t)\) decouples from the equation for the vector part of distribution function \(\sigma_k(r,t)\) and we may put \(n_k\) equal to its equilibrium value, namely, usual Fermi function. Hence, the equation for the \(\sigma_k(r,t)\) has the form

\[
\frac{\partial \sigma_k}{\partial t} + \varepsilon_k \frac{\partial \sigma_k}{\partial x_i} - h_k \frac{\partial n_k}{\partial k_i} = 2(h_k \times \sigma_k)
\]

\[
= \left(\frac{\partial \sigma_k}{\partial t}\right)_{coll}. \quad (4)
\]

We divide all matrices in equilibrium and nonequilibrium parts,

\[
\hat{n}_k = \hat{n}_k^0 + \delta\hat{n}_k,
\]

(5)

\[
\hat{\varepsilon}_k = \hat{\varepsilon}_k^0 + \delta\hat{\varepsilon}_k,
\]

(6)

where

\[
\hat{n}_k^0 = \hat{n}_0(\varepsilon_k)\hat{I} + \frac{1}{2}\Delta n_0(\varepsilon_k)(im\hat{\sigma})
\]

(7)

is the equilibrium distribution function of polarized Fermi liquid and

\[
\hat{\varepsilon}_k^0 = \varepsilon_k\hat{I} - \frac{1}{2}\gamma(B\hat{\sigma})
\]

(8)

is the equilibrium quasiparticle energy. Here, the functions

\[
\hat{n}_0(\varepsilon_k) = \frac{1}{2}(n_0^+ + n_0^-)
\]

(9)

and

\[
\Delta n_0(\varepsilon_k) = n_0^+ - n_0^-
\]

(10)

are determined through two Fermi distribution functions

\[
n_0^\pm(\varepsilon_k) = \frac{\exp\left(\frac{\varepsilon_k \mp \varepsilon_0}{2}\right)}{1 + \exp\left(\frac{\varepsilon_k \mp \varepsilon_0}{2}\right)}
\]

(11)

shifted on the value of polarization \(\gamma H/2\), \(\gamma\) is the gyromagnetic ratio, Planck constant \(\hbar\) is throughout the paper, the polarization direction is determined by the unit vector \(\mathbf{m} = \mathbf{H}/H\).

We have introduced two magnetic fields \(\mathbf{H}\) and \(\mathbf{B}\) and shall assume that they are parallel each other. The field \(\mathbf{H}\) determining the shift of the quasiparticle distribution function corresponds in paramagnetic Fermi liquid to the magnetization created by the external magnetic field \(\mathbf{H}_0\) and by the pumping. The pumped part in view of very long time of longitudinal relaxation should be considered as equilibrium part of magnetization. In a ferromagnetic Fermi liquid \(\mathbf{H}\) is spontaneous magnetic field existing even in absence of an external field and pumping. The field \(\mathbf{B}\) determines the shift in energy of quasiparticles consisting of an external magnetic field \(\mathbf{H}_0\) and the Fermi-liquid molecular field. To define \(\mathbf{B}\) we must consider the equilibrium distribution matrix \(\sigma\) and equilibrium energy matrix \(I\) as corresponding matrices for nonpolarized Fermi liquid,

\[
\hat{n}_k^0 = n_0(\varepsilon_k)\hat{I} + \delta\hat{n}_k^0,
\]

(12)

\[
\hat{\varepsilon}_k^0 = \varepsilon_k\hat{I} - \frac{1}{2}\gamma(B\hat{\sigma})
\]

\[
= \varepsilon_k\hat{I} - \frac{1}{2}\gamma(H_0\hat{\sigma}) + \frac{1}{2}Sp\int d\tau f_{kk'}\sigma_{\sigma'}\delta\hat{n}_{k'}^0,\quad (13)
\]

where \(d\tau = 2dk/(2\pi)^3\) and \(f_{kk'}\sigma_{\sigma'}\) is the Fermi-liquid interaction matrix.

As it was discussed in for the finite polarization and the general form of \(f_{kk'}\sigma_{\sigma'}\) the vector \(\mathbf{B}\) proves to be energy dependent. This in its turn leads to impossibility
of the spin dynamics description in terms of two closed equations for spin and spin current densities. To circumvent these difficulties as in the paper we assume the independence functions $f_{kk'} e^{-\sigma'} = f_{kk'} e^{-\sigma} \tilde{I} + |f_0 + f_1(\tilde{K})| \tilde{\sigma} \tilde{\sigma}'$.\(14\)

Now, from (13), (14) we obtain an equation for $B$ determination

$$\gamma B = \gamma H_0 - \mathbf{m} f_0 a \int dt \Delta n_0.$$\(15\)

For small polarizations and taking for simplicity $T = 0$ one can rewrite \((16)\) as

$$B = H_0 - H F_0 a \left(1 - \frac{1}{6} \left(\frac{\gamma H}{4 \mu}\right)^2\right).$$\(16\)

Here $F_0 a = N_0 f_0 a$, $N_0 = m^* k_F / \pi^2$ is the density of states at zero polarization. In the absence of a pumped magnetization the field $B = H$ and \((16)\) is just the self-consistency equation for the field $H$ determination as the function of an external field $H_0$ giving in the lowest order

$$H = \frac{H_0}{1 + F_0 a}.$$\(17\)

As a particular case one can consider also a ferromagnetic state realized at $F_0 a = -1 - \delta$ when the solution of the equation \((16)\)

$$\frac{1}{6} \left(\frac{\gamma H}{4 \mu}\right)^2 = 1 + F_0 a$$\(18\)

exists even in the absence of an external field.

When the part of magnetization is created by pumping, $H$ presents an independent value and the total energy shift $\gamma \langle \mathbf{B} \tilde{\sigma} \rangle / 2$ is determined by means of two fields: external $H_0$ and "effective" $H$.

We discuss the only perpendicular deviations from the initial equilibrium state,

$$\delta \tilde{n}_k = \delta \tilde{\sigma}_k (r, t) \tilde{\sigma}, \quad (\mathbf{m} \delta \tilde{\sigma}_k) = 0.$$\(19\)

Then the energy deviation matrix has the form

$$\delta \varepsilon_k = \delta h_k \tilde{\sigma}, \quad \delta h_k = \int dr' f_{kk' a} \delta \sigma_k.$$\(20\)

and the kinetic equation \((4)\) can be rewritten as

$$\frac{\partial \delta \sigma_k}{\partial t} + \frac{\partial \varepsilon_k}{\partial x} \frac{\partial \delta \sigma_k}{\partial x_i} - \frac{\partial \tilde{n}_0}{\partial x_i} \frac{\partial \delta h_k}{\partial x_i} \left(\frac{1}{2} \mathbf{m} \Delta n_0 + \delta \tilde{\sigma}_k\right) = \left(\frac{\partial \tilde{\sigma}_k}{\partial t}\right)_{\text{coll}}.$$\(21\)

To derive the closed system of equations for the spin density $\mathbf{M}$ and the spin current density $\mathbf{J}$, in the case of finite polarization we make an assumption which is plausible for weakly polarized Fermi liquid that the energy dependence of $\delta \tilde{\sigma}_k (r, t)$ is factorized from the space and direction of $\tilde{\mathbf{K}}$ dependences:

$$\delta \tilde{\sigma}_k (r, t) = \mathbf{A} (r, t) \alpha (\varepsilon) + \mathbf{B} (r, t) \hat{k} \beta (\varepsilon).$$\(22\)

In terms of these functions one can write the spin density

$$\mathbf{M} (r, t) = \frac{1}{2} \int d\tau \delta \tilde{\sigma}_k = \frac{1}{2} \mathbf{A} (r, t) \int d\tau \alpha (\varepsilon),$$\(23\)

and spin current density

$$\mathbf{J}_i (r, t) = \frac{1}{2} \int d\tau \left[ \nabla \delta \tilde{\sigma}_k - \frac{\partial \tilde{n}_0}{\partial x_i} \delta \mathbf{h}_k \right] = \frac{1}{2} \psi \int d\tau v_i \delta \tilde{\sigma}_k$$\(24\)

Making the integrations of kinetic equation \((21)\) \(\int d\tau / 2\) and \(\int d\tau v_i / 2\) we obtain

$$\frac{\partial \mathbf{M}}{\partial t} + \frac{\partial \mathbf{J}_i}{\partial x_i} - \mathbf{M} \times \gamma H_0 = 0,$$\(26\)

$$\frac{\partial \mathbf{J}_i}{\partial t} + \frac{w^2}{3} \frac{\partial \mathbf{M}}{\partial x_i} - \mathbf{J}_i \times \gamma H_0 + \mathbf{J}_i \times \mathbf{C} = \frac{\psi}{2} \int d\tau v_i \left(\frac{\partial \tilde{\sigma}_k}{\partial t}\right)_{\text{coll}}.$$\(27\)

Here

$$w^2 = \psi \left[\int d\tau v_i^2 (\varepsilon) \alpha (\varepsilon) - f_0 a \int d\tau v_i^2 (\varepsilon) \frac{\partial \tilde{n}_0}{\partial \varepsilon}\right]$$\(28\)

and

$$\mathbf{C} = \mathbf{m} \left[ f_0 a \int d\tau \Delta n_0 (\varepsilon) - \frac{f_1 a}{3} \int d\tau \frac{\partial \varepsilon}{\partial \varepsilon} \int d\tau \beta (\varepsilon) \Delta n_0 (\varepsilon) \right].$$\(29\)

The equations \((26), (27)\) have the same form as in the case of vanishingly small polarization. The equation \((27)\) is transformed to the analogous equation for vanishingly small polarization

$$\frac{\partial \mathbf{J}_i}{\partial t} + \frac{1}{3} v_F^2 (1 + F_0 a) (1 + F_1 a) \frac{\partial \mathbf{M}}{\partial x_i} - \mathbf{J}_i \times \gamma H_0 + \frac{4}{N_0} (F_0 a - F_1 a) (\mathbf{J}_i \times \mathbf{M})$$

$$= \frac{1}{2} \left(1 + F_1 a / 3\right) \int d\tau v_i \left(\frac{\partial \tilde{\sigma}_k}{\partial t}\right)_{\text{coll}},$$\(30\)

$$\Delta n_0 (\varepsilon) = \int d\tau v_i \left(\frac{\partial \tilde{\sigma}_k}{\partial t}\right)_{\text{coll}}.$$\(31\)
\[ M^l = \frac{\gamma N_0}{4} H, \quad F_i^a = N_0 f_i^a \]  

(31)

if we put

\[ \alpha(\varepsilon) \propto \beta(\varepsilon) \propto \Delta n_0(\varepsilon). \]  

(32)

Thus, one can work with eqn (27) taking more specific definitions for

\[ \psi = \frac{\int d\tau v(\varepsilon)[\Delta n_0(\varepsilon)]}{\int d\tau v(\varepsilon)\Delta n_0(\varepsilon)}. \]  

(33)

\[ w^2 = \psi \left[ \frac{\int d\tau v^2(\varepsilon)\Delta n_0(\varepsilon)}{\int d\tau \Delta n_0(\varepsilon)} - f_0^a \int d\tau v^2(\varepsilon) \frac{\partial n_0}{\partial \varepsilon} \right]. \]  

(34)

and

\[ C = \frac{m}{N_0}(F_0^a - \frac{F_1^a}{3}) \int d\tau \Delta n_0(\varepsilon). \]  

(35)

At last, using the calculations of the paper\(^8\) for the collision integral in weakly polarized liquid we come to the equation for the spin current density

\[ \frac{\partial J_i}{\partial t} + \frac{w^2}{3} \frac{\partial M}{\partial x_i} = J_i \times \gamma H_0 + J_i \times C = -\frac{J_i}{\tau} \]  

(36)

where the current relaxation time is

\[ \frac{1}{\tau} = \frac{m^3}{6(2\pi)^3} \left( \frac{2W_1 + W_2}{(2\pi T)^2 + (\gamma H)^2} \right). \]  

(37)

The dispersion law of the transversal spin waves following from equations \(26, 36\) is (see for instance\(^\text{22}\))

\[ \omega = \omega_L + (D'' - iD')k^2, \]  

(38)

where \(\omega_L = \gamma H_0\) is the Larmor frequency,

\[ D' = \frac{w^2 \tau}{3(1 + (C\tau)^2)} \cong \frac{w^2}{3C^2 \tau} \]  

(39)

is the dissipative part of diffusion coefficient and

\[ D'' = C\tau D' \cong \frac{w^2}{3C} \]  

(40)

is its reactive part. Here the second approximative values of \(D'\) and \(D''\) correspond to the limit \(C\tau \gg 1\).

For a weakly polarized fluid \(C = (F_0^a - F_1^a/3)\gamma H\) and \(\psi = 1 + F_1^a/3\). The expression for \(w^2\) depends of state of liquid. One can find it analytically in the case of weak polarization. In a paramagnetic Fermi liquid it is

\[w^2 = v_F^2(1 + F_0^a)(1 + F_1^a/3)\]  

(41)

where \(v_F\) is the Fermi velocity in nonpolarized liquid. In a ferromagnetic Fermi liquid (if an external field is smaller than spontaneous) we find from eqn (34) with help of eqn (15)

\[ w^2 = -\frac{v_F^2(1 + F_0^a)}{3} \left( \frac{\gamma H}{4\mu} \right)^2 \]  

(42)

Thus, the reactive part of diffusion coefficient in paramagnetic state at \(T = 0\) proves to be inversely proportional to magnetization

\[ D'' = \frac{v_F^2(1 + F_0^a)(1 + F_1^a/3)}{3(F_0^a - F_1^a/3)\gamma H} \]  

(43)

whereas in ferromagnetic state it is directly proportional to magnetization

\[ D'' = -\frac{v_F^2(1 + F_1^a/3)\gamma H}{3(F_0^a - F_1^a/3)(4\mu)^2}. \]  

(44)

The latter is in correspondence with known result obtained in frame of Stoner-Hubbard model\(^12\).

The dissipative part of diffusion coefficient given by eqn (39) at \(T = 0\) in paramagnetic state is polarization independent, whereas in ferromagnetic state it is proportional to square of magnetization.

So, the transverse spin waves frequency in polarized paramagnetic Fermi liquid as well as in a Fermi liquid with spontaneous magnetization is found to be proportional to \(k^2\) with complex diffusion coefficient such that the damping at \(C\tau \gg 1\) has a finite value proportional to the scattering rate of quasiparticles at \(T = 0\). As it was pointed out in previous the paper, the latter is in formal analogy with ultrasound attenuation in collisionless regime. It is worth noting, however, that in neglect of processes of longitudinal relaxation the parameter \(\gamma H\tau\) has no relation to the local equilibrium establishment.

The results \(38-44\) are valid both in hydrodynamic \(Dk^2\tau \ll 1\) and in collisionless regime \(Dk^2\tau \gg 1\) so long

\[ Dk^2 \ll \gamma H \]  

(45)

that is the condition of two moment approximation \(22\) for the solution of the kinetic equation\(^\text{22}\). This behavior of polarized Fermi liquid contrasts with the behavior of Heisenberg ferromagnet in hydrodynamic regime\(^2\) where the transverse spin wave attenuation appears in terms proportional to \(k^4\).

### III. MICROSCOPIC DERIVATION OF SPIN WAVE SPECTRUM IN POLARIZED FERMI LIQUID

The Landau-type derivation of transverse spin dynamics in a weakly spin-polarized Fermi-liquid from microscopic theory has been performed in the paper\(^8\). Here we make a similar derivation with the purpose to stress the conditions it needs to be valid, to compare the answer with that obtained from kinetic equation at nonzero
temperatures, and to juxtapose this with the derivation for ferromagnetic Fermi-liquid which we also reproduce afterwards.

As in the original paper by Landau we consider a system of fermions at $T = 0$, with arbitrary short range interaction forces. The presence of polarization means that subsystems of spin-up and spin-down particles have different chemical potentials $\mu_{\pm} = \mu \pm \gamma B/2$ and the distribution functions with different Fermi momenta $p_{\pm} = p_0 \pm \gamma H/2v_F$. The polarization in general is nonequilibrium and, as in previous section, we shall distinguish the fields $H$ and $B$. Here, we shall not take in mind the polarization dependence of the Fermi velocity and density of states and obtain results relating to weakly polarized paramagnetic Fermi liquid. The ferromagnetic case shall be discussed in the next section. So, the Fermi velocity is $v_F = \frac{\partial\epsilon(p)}{\partial p}\big|_{p=p_0}$ and $p_0 = \frac{p_+ + p_-}{2}$. The Green functions near $|p| = p_{\pm}$ and $\varepsilon(p) = \mu_{\pm}$ have the form

$$G_{\pm}(p,\varepsilon) = \frac{a}{\varepsilon - \varepsilon(p) + \ii v_F^2(p_{\pm} - p_{\mp})[p - p_{\pm}]}.$$  

(46)

We use a weak polarization $v_F(p_{\pm} - p_{\mp}) \ll \varepsilon_F$ and also assume that both the Fermi distributions are characterized by the same Landau Fermi liquid parameters.

We introduce here the general form of imaginary part of self-energy which is quadratic function of the difference $(p_{\mp} - p_{\pm})$ and changes its sign at $p = p_{\pm}$ correspondingly. The assumption of small polarization means in particular that $G_+$ is given by the expression (46) not only near $|p| = p_{\pm}$ and $\varepsilon(p) = \mu_{\pm}$ but in the whole intervals $p_{\mp} < p < p_{\pm}$ and $\mu_{\mp} < \varepsilon(p) < \mu_{\pm}$ and also near $|p| = p_{\pm}$ and $\varepsilon(p) = \mu_{\mp}$. The same is true for $G_-$.

Following Landau, let us write the equation for the vortex function describing scattering of two particles with opposite spin directions and a small transfer of 4-momentum $K = (k,\omega)$

$$\Gamma(P_1, P_2, K) = \Gamma_1(P_1, P_2) - \ii \frac{i}{(2\pi)^2} \ii \Gamma_1(P_1, Q)$$

$$\times G_+(Q)G_-(Q + K)G(Q, P_2, K)d^4Q$$

(47)

If $K$ is small and polarization is also small, the poles of two Green functions are close to each other. Let us assume that all other quantities in the integrand are slowly varying with respect to $Q$: their energy and momentum scales of variation are larger than $\max\{|H, \omega\}$ and $\max\{|H/v_F, k\}$ correspondingly. Then one can perform the integration in (47) at fixed values of $q = p_0$, $\varepsilon = 0$ in the arguments of $\Gamma$ and $\Gamma_1$ functions. In other words, one can substitute in (47)

$$G_+(Q)G_-(Q + K) = G_+(q, \varepsilon)G_-(q + k, \varepsilon + \omega)$$

$$= \frac{2\pi a^2}{v_F} \delta(\varepsilon)\delta(|q| - p_0)$$

$$\times \frac{\gamma H + kv_F}{\omega - \omega_L + \gamma HF_0^a + \ii b(\gamma H)^2/2 - kv_F + i\gamma \omega kv_F} + \Phi_{\text{reg}}.$$  

(48)

For eliminating $\Gamma_1$ from (2) we shall rewrite this equation in the operator form

$$\Gamma = \Gamma_1 - \ii \Gamma_1(i\Phi + \Phi_{\text{reg}})\Gamma,$$  

(49)

where product is interpreted as integral, and $i\Phi$ denotes the first term from right-hand side eq. (48). In equation (49), we transpose the term involving $\Phi_{\text{reg}}$ to the left-hand side, and then apply the operator $(1 + \ii \Gamma_1 \Phi_{\text{reg}})^{-1}$, obtaining

$$\Gamma = \Gamma^\omega + \Gamma^\omega \Phi \Gamma,$$  

(50)

where

$$\Gamma^\omega = (1 + \ii \Gamma_1 \Phi_{\text{reg}})^{-1}\Gamma_1.$$  

(51)

As it is known, $\Gamma^\omega(H = 0)$ is directly related to the function determining the Fermi liquid interaction,

$$\Gamma^\omega(H = 0) = \Gamma((|k|/\omega) \rightarrow 0, H = 0) = \frac{F_{nn'}}{a^2 N_0}.$$  

(52)

At finite $H$ the $\Gamma^\omega$ function can be expanded over the polarization as

$$a^2 N_0 \Gamma^\omega = F_{nn'} + \ii b\gamma H C_{nn'} + O(H^2).$$  

(53)

From eqns (50) and (53), we come, according to a well known procedure, to kinetic equation

$$\left(\omega - \omega_L + \gamma HF_0^a + \frac{\ii b(\gamma H)^2}{2} - \text{knv}_F \right.$$

$$+ \ii b\text{knv}_F \gamma H \nu(n') \right) \nu(n) = (\gamma H + \text{knv}_F) \ii \frac{dn'}{4\pi} (F_{nn'} + \ii b\gamma H C_{nn'}) \nu(n').$$  

(54)

We limit ourself to the first two harmonics in the Landau interaction function $F_{nn'} = F_0^a + (nn')F_1^a$ and $C_{nn'} = C_0 + (nn')C_1$. To obtain the spectrum of the spin waves (see below) obeying the Larmor theorem: the system of spins in a homogeneous magnetic field executes the precessional motion with the Larmor frequency $\omega_L = \gamma H_0$, the coefficient $C_0$ has to be chosen equal to $1/2$.

Introducing the expansion of the distribution function $\nu(n)$ over spherical harmonics of direction $n = v_F/v_F$, one can find from (53) that the ratio of amplitudes of the successive harmonics with $l \geq 1$ is of the order of $kv_F/\gamma H$. Hence if this ratio is assumed to be a small parameter one can work with distribution function taken in the form $\nu(n) = \nu_0 + (nk)\nu_1$. The functions $\nu_0$ and $\nu_1$ obey the following system of linear equations:

$$\left(\omega - \omega_L\right)\nu_0 - \frac{\text{knv}_F}{3} \left(1 + \frac{F_1^a}{3} - \ii b\left(1 - \frac{C_1}{3}\right)\gamma H\right) \nu_1 = 0,$$

(55)

$$- \frac{\text{knv}_F}{3} \left(1 + \frac{F_1^a}{3} - \ii b\frac{\gamma H}{2}\right) \nu_0$$

$$\left(\omega - \omega_L + (F_0^a - \frac{F_1^a}{3})\gamma H + \ii b\left(\frac{C_1}{3}\right)\gamma H^2\right) \nu_1 = 0.$$  

(56)
Vanishing of the determinant of this system gives the spin waves dispersion law. At long enough wave-lengths when the dispersive part of $\omega(k)$ dependence is much less than $\omega_L$ and neglecting the terms $\sim (b\gamma H)^2$, we have

$$\omega = \omega_L + (D'' - iD')k^2,$$  \hspace{1cm} (57)

where

$$D'' = \frac{bv_F^2(1 + F_0^a)(1 + F_1^a/3)}{3(F_0^a - F_1^a/3)\gamma H}$$  \hspace{1cm} (58)

is a reactive part of the diffusion coefficient, and

$$D' = \frac{bv_F^2[(1 - C_1/3)(1 + F_0^a)^2 - (1 + F_1^a/3)^2/2]}{3(F_0^a - F_1^a/3)^2}$$  \hspace{1cm} (59)

is a dissipative part of the diffusion coefficient. We derived eqns (58) and (59) in the assumption of $F_0^a$ being a dissipative part of the diffusion coefficient. We denote $F_1^a$ as a reactive part of the diffusion coefficient, and we treat the spin waves attenuation in polarized Fermi liquid. The value of $D''$ is inversely proportional to polarization.

The expressions for $D''$ and $D'$ have been obtained first by the same method by A.Meyerovich and K.Musaelyan. The former is literally coincides with that found in this paper, the latter has the same parametric dependence but depends in different way from Fermi liquid parameters. The reason for this is not clear at the moment. These expressions reproduce the corresponding diffusion constants obtained from phenomenological Landau-Silin kinetic equation with two-particle collision integral (see previous Section) at arbitrary relation between polarization and temperature if we put in the latters constants obtained from phenomenological Landau-Silin kinetic equation with two-particle collision integral (see previous Section) at arbitrary relation between polarization and temperature if we put in the latters constants obtained from phenomenological Landau-Silin kinetic equation with two-particle collision integral (see previous Section) at arbitrary relation between polarization and temperature if we put in the latters.

Thus, the general microscopic derivation confirms the statement about the existence of zero-temperature spin waves attenuation in polarized Fermi liquid. The value of the dissipative part of spin diffusion $D'$ is determined by the amplitude $(b'v)^2$ of the imaginary part of self-energy. It originates from collisions between quasiparticles.

IV. MICROSCOPIC DERIVATION TAKING INTO ACCOUNT THE TRANSVERSE STATIC SUSCEPTIBILITY DIVERGENCY

There are several known investigations of an isotropic itinerant ferromagnetic state as some peculiar type of Fermi liquid. This subject was discussed first phenomenologically by A.A.Abrikosov and I.E.Dzyaloshinskii and then microscopically by P.S.Kondratenko. They did not include in the theory a finite scattering rate between quasiparticles and as result they obtained the dissipationless transverse spin wave dispersion law as it seemed to be in isotropic ferromagnet. The derivation was critisized by C.Herring who pointed out on the existence of finite scattering rate. Later I.E.Dzyaloshinskii and P.S.Kondratenko rederived the spin-wave dispersion law in ferromagnets. Making use as the starting point the Landau equation for the vertex function for the scattering of two particles with opposite spin direction and a small transfer of 4-momentum they have redefined the product of two Green functions $G_+G_-$ in such a manner that its resonant part was taken equal to zero at $\omega = 0$. This trick gives a possibility to use the $1/k^2$ divergency of transverse static susceptibility, which is an inherent property of degenerate systems and occurs both in an isotropic ferromagnet and in spin polarized paramagnetic Fermi-liquid. The latter of course is true in the absense of interactions violating total magnetization conservation. As in the previous papers the authors did not introduce a scattering rate in the momentum space between the Fermi surfaces for the particles with opposite spins.

Let us see now what kind of modifications appear if we reproduce the derivation proposed in taking into account the finite quasiparticle scattering rate in the whole interval $p_- < p < p_+$. We discuss an isotropic ferromagnet at equilibrium $B = H$ first in the absence of external field. Following we write:

$$G_+(Q)G_-(Q + K) = G_+(q, \varepsilon)G_-(q + k, \varepsilon + \omega)$$

$$= \frac{2\pi a^2}{v_F}\delta(\varepsilon)\delta(|q| - p_0) \times \omega - \gamma H + i(b(\gamma H)^2/2 - kv_F + i\beta kv_F \gamma H + \Phi_{\text{reg}}.$$  \hspace{1cm} (60)

Now the eqn (17) is written as

$$\Gamma = \Gamma_1^k + \Gamma^k \Phi,$$  \hspace{1cm} (62)

where $i\Phi$ denotes the first term from right-hand side eq. (19). The equivalent form of this equation is

$$\Gamma = \Gamma^k \Phi,$$  \hspace{1cm} (63)

The isotropic part of $\Gamma^k$ is proportional to the static transverse susceptibility. Hence it has a singular form:

$$\Gamma^k \propto -\frac{1}{N_0(c \kappa)^2}.$$  \hspace{1cm} (63)

Here, $c$ is a parameter with the dimensions of length. One can show by direct calculation of static transverse susceptibility in ferromagnet that $c$ is polarization independent

$$c \sim \frac{1}{p_0}.$$  \hspace{1cm} (65)

At the same time the similar calculations for polarized paramagnetic Fermi liquid gives the value of $c$ inversely proportional to polarization

$$c \sim \frac{v_F}{\gamma H}.$$  \hspace{1cm} (66)
such that the divergency $\gamma H \to 0$.

Substitution of eqn (64) into eqn (62) gives the transverse spin wave dispersion law

$$\omega = \gamma H (ck)^2 (1 - \frac{ib\gamma H}{2})$$

(67)

which proves to be attenuating. One can take into consideration a static external field, by working in the rotating with Larmor frequency coordinate frame that is equivalent to the substitution $\omega \to \omega - \omega_L$ (see also $\gamma H \to 0$). As a result, we obtain the dispersion law

$$\omega = \omega_L + \gamma H (ck)^2 (1 - \frac{ib\gamma H}{2})$$

(68)

that has the same form as (67). Taking into account the relations $\gamma H \to 0$ and $\gamma H \to 0$, one can make sure that the polarization dependences of reactive and dissipative part of diffusion constant in ferromagnetic Fermi liquid and in polarized paramagnetic Fermi liquid coincide with those described at the end of the Section II.

V. CONCLUSION

In conclusion we stress once again that the transverse spin wave dispersion in polarized paramagnetic Fermi liquid as well as in a Fermi liquid with spontaneous magnetization is found to be attenuating. The spin wave frequency is proportional to $k^2$ with complex diffusion coefficient such that the damping at $T = 0$ has a finite value proportional to the scattering rate of quasiparticles. This behavior of polarized paramagnetic or feromagnetic Fermi liquid contrasts with the behavior of Heisenberg ferromagnet in hydrodynamic regime $20$ where the transverse spin wave attenuation appears in terms proportional $k^4$.

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