The fluxes of CN neutrinos from the Sun in case of mixing in a spherical layer in the solar core

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Abstract. The results of the calculation are presented for the fluxes of CN neutrinos from the Sun in case of mixing in a spherical layer in the solar core, consistent with the seismic data and with the measured solar neutrino fluxes. It is shown that a substantial increase of the flux of $^{13}$N neutrinos can be gained in this case. The possible implications for experiment are discussed.

Keywords: solar and atmospheric neutrinos, neutrino experiments

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Solar neutrinos and helioseismology are complimentary tools in studying solar structure. The measured fluxes of solar neutrinos [1] - [7] and the found parameters of neutrino oscillations [8] are in a good agreement with the predictions of the standard solar model [9]. However, still there is ”solar abundance problem” which represents the fact that best available photospheric abundances of high Z elements are at odds with the helioseismic measurements [10]. This stimulates the search for some modifications of the solar model.

Because few acoustic modes penetrate to the nuclear active region where the solar energy is generated, there is still very scarce experimental data for the region 0.1 – 0.2 \( R_\odot \) which could illuminate the dynamics of the solar core [11]. In the article [12] the attempt has been made to set the stringent limit for the average mean molecular weight in the inner 20% of the Sun by radius \( \mu_c \) (mean molecular weight of the solar core). In other paper [13] it was suggested that the variation in the sound speed in the solar core might be also caused by a partial mixing. In the framework of a new approach (Linear Solar Model [14]) the constraints on opacity (and composition) were discussed in [15]. Unfortunately, there are still no direct experimental determinations of these quantities and it would be very interesting to get some direct experimental data for the solar core. The flux of \( ^{13}N \) neutrinos is very sensitive to the tiny variations of the abundance of \( ^{12}C \) across the solar core while the flux of \( ^{15}O \) neutrinos is practically stable by any variation of abundances. This might be the key for solving the problem of the dynamics of the solar core emphasizing the measured ratio of the fluxes of \( ^{13}N \) and \( ^{15}O \) neutrinos as the parameter directly characterizing the mixing in the solar core. The increase of accuracy in the measurements of the effect from \( \nu - e^- \) scattering in electronic detectors promises in the future the precise measurement of this effect not only beyond the upper bound of \( ^7Be \) neutrinos, but also beyond the upper bound of \( ^{13}N \) neutrinos, thus facilitating the finding of this ratio. This would be the next step in the solar neutrino research.

In this article we follow previous works [16], [17] where the calculation of the effect of mixing on the fluxes of solar neutrinos were presented for the mixing within all central zone. It was shown that a substantial increase of the flux of \( ^{13}N \) neutrinos can be obtained in this case while this model with mixing still is consistent with the seismic data and the measured solar neutrino fluxes. This effect is due to the large gradient of concentrations of \( ^{12}C \) on solar radius so that even very mild mixing can increase substantially the abundance of \( ^{12}C \) in a nuclear active region, where \( ^{13}N \) is produced. Here the question is addressed what effect can be obtained if the mixing occurs only in a spherical layer within the radiative zone. Two parameters are used for the layer: the radius of the bottom of the layer \( R_m \) and the depth
$\Delta R_m$ in units of Solar radius so that the top of the layer is at the radius $R_m + \Delta R_m$. The mixing is conceived as a mild process running during a time interval, short in comparison with the age of the Sun. The effect of this mixing would be the homogeneous distribution of all elements within full spherical layer while the temperature gradient still is preserved of the one of the standard solar model. The limited geometry of the mixing in this case should not change drastically the parameters fixed by helioseismology while the effect on the flux of neutrinos from the decay of $^{13}N$ can be large while all other fluxes of solar neutrinos will be left practically unchanged. This is the main point of our approach.

It is worth to note that mixing changes the profile of the mean molecular weight along the radius of the Sun and consequently it changes also the opacity profile. This, in turn, affects the whole solar structure, as it was shown in [15]. However, as it was noted, for example, in [18] so far there are no direct seismic determinations of these quantities in the region of the solar core. The most precise parameter characterizing this region is a mean molecular weight of the solar core. This value has been found in [12] with the uncertainty 1% at the level of 95.4% (2$\sigma$) C.L. Thus, the parameters of mixing should not result in the change of the mean molecular weight of the solar core exceeding 1% in comparison with the standard model. It is also worth to note that the effects produced by variations of opacity in distinct zones of the Sun may compensate each other, as it was noted in [15]. In our case the change of the mean molecular weight along the radius of the Sun has different signs: in lower layer of the mixing zone $\delta \mu < 0$ and in upper one: $\delta \mu > 0$. Thus, the changes of opacity in these two layers due to the changes of the mean molecular weight are partially compensated. We did not go into details of how the sound speed profile will be changed after mixing limiting ourselves only by the mean molecular weight of the solar core values.

Being the most sensitive probe provided by helioseismology the sound speed profile in case of mixing demands a further study. The detailed consideration of this question should probably be addressed somewhere else as a next step of our approach.

2 Calculation.

Let's suggest that during a time interval, short in comparison with the age of the Sun, the solar matter in a certain layer $\Delta R_m$ has got mixed. As a result of this mixing the abundance of all elements has been averaged within all this layer (Figure 1).

Let's also suggest that profile of temperature and density across the Sun has not been changed due to the partial compensation of the changes of $\delta \mu$ in the lower and upper layers of the mixed zone. If we divide the matter of the Sun on thin spherical layers so that inside these layers the temperature and the density can be taken to be constant, then the evolution of the abundance of the elements in each layer can be described by the set of differential equations ([17]).

\[
\begin{align*}
\frac{dX_1}{dt} &= -X_1^2\frac{a_{11}}{a_{11} + a_{11}'} + X_3^2\frac{a_{33}}{a_{33} + a_{33}'} - X_3 X_4 \frac{a_{34}}{a_{34} + a_{34}'} \\
\frac{dX_3}{dt} &= X_3^2\frac{a_{33}}{a_{33} + a_{33}'} - X_3 X_4 \frac{a_{34}}{a_{34} + a_{34}'} \\
\frac{dX_4}{dt} &= X_3^2\frac{a_{33}}{a_{33} + a_{33}'} + X_3 X_4 \frac{a_{34}}{a_{34} + a_{34}'} \\
\frac{dX_{12}}{dt} &= -X_1 X_2 \alpha_{12} + X_1 X_4 \frac{\alpha_{12}}{\alpha_{115}} \\
\frac{dX_{13}}{dt} &= -X_1 X_3 \alpha_{13} + X_1 X_4 \frac{\alpha_{12}}{\alpha_{115}} \\
\frac{dX_{14}}{dt} &= -X_1 X_4 \alpha_{14} + X_1 X_4 \frac{\alpha_{13}}{\alpha_{113}} \\
\frac{dX_{15}}{dt} &= -X_1 X_5 \alpha_{15} + X_1 X_4 \frac{\alpha_{14}}{\alpha_{114}}
\end{align*}
\]

(2.1)

Here the first three equations describe the variations of the abundance of elements $X_i$ of the atomic weight $i$ in the reactions of $pp$-chain ($^1H$, $^3He$ and $^4He$), and the latter four – in the
The profile for the abundance of $^1H$, $^3He$, $^4He$, $^{12}C$ and $^{14}N$ before (dashed line) and after (solid line) mixing.

Figure 1: The profile for the abundance of $^1H$, $^3He$, $^4He$, $^{12}C$ and $^{14}N$ before (dashed line) and after (solid line) mixing.

reactions of CNO-cycle ($^{12}C$, $^{13}C$, $^{14}N$ and $^{15}N$). Here $\alpha_{ij} = \rho f_{ij}$, where $f_{ij}$ – the rates of the reaction $A_i + A_j$ from [19] with the known S-factor [20], [21] ($\alpha_{11}$ corresponds to the reaction $^1H + ^1H + e^-$), $\rho$ – the density in the given layer. The small contribution in the first equation of the reactions of the type $X_1X_{15}\alpha_{115}$ were neglected, as well as the processes of gravitational settling and diffusion.

Similar to [16], the set (2.1) was resolved by 4th order Runge-Kutt method. As the initial abundance of elements the present one observed on the surface of the Sun has been used. The time interval has been taken equal to the age of the Sun (4.6 billion years). Thus, by resolving this set of equations we obtain the abundance of elements in each layer at the present time. In the same way the abundance of elements and the corresponding fluxes of solar neutrinos are calculated in case of mixing. The total flux of neutrino from the Sun is determined by the reaction rates and the abundance of elements. For the flux of neutrinos from the decay of $^{13}N$:

$$F_{13}(t) = 4\pi N_A R_\odot^3 \int_0^1 \rho(r)X_1(r,t)\frac{X_{12}(r,t)}{12}\alpha_{112}(r)dr$$

(2.2)

Similar expressions can be written for the neutrino fluxes $F_{15}$ and $F_7$ from the decay of $^{15}O$ and $^7Be$. The neutrino fluxes will be changed by mixing in a certain layer with the lower boundary $R_m$ and the thickness $\Delta R_m$. The parameters fixed by helioseismology will be changed also, particularly, the mean molecular weight of the solar core $\mu_c$ [12].

We have calculated these values before and after mixing which happened at certain time. The resulting relative changes of the neutrino fluxes and the mean molecular weight of the solar core $\mu_c$ are presented on Figure 2. At Figure 3 the results obtained are presented as contours (with the exclusion of $F_{15}$ which has been changed marginally in comparison with $F_{13}$).

One can see that the allowed region for the parameters used is mainly determined by the uncertainty in the determination of the flux of beryllium neutrinos (5% in Borexino...
Figure 2. Relative changes of the neutrino fluxes $F_{13}$, $F_{15}$, $F_7$ and of the mean molecular weight of the solar core $\mu_c$ (in %) by mixing.

Let’s look how the neutrino fluxes are changed after mixing in a layer. Figure 4 shows the evolution of the fluxes of beryllium and $^{13}$N-neutrino in case of mixing ($F_{\text{mix}}$ – the flux after mixing in a layer, $F$ – the flux without mixing at the same moment). Figure 4 shows, that the flux of beryllium neutrino recovers more quickly the stationary value than the flux of $^{13}$N-neutrino. This is explained by higher rate of the reaction $^3\text{He} + ^4\text{He} \to ^7\text{Be} + \gamma$ (determining the neutrino generation in the reaction $^7\text{Be} + e^- \to ^7\text{Li} + \nu_e$) than the one of the reaction $^{12}\text{C} + p \to ^{13}\text{N} + \gamma$ (determining the neutrino generation in the reaction $^{13}\text{N} \to ^{13}\text{C} + e^+ + \nu_e$). Therefore beryllium neutrino flux returns to the stationary value faster than the flux of $^{13}$N-neutrino. Let’s see how the picture presented on Figure 3 is changed in a certain time, for example, in 10 million years. The results are presented on Figure 5. We see that the allowed change of the flux of $^{13}$N-neutrino is limited only by the mean molecular weight of the solar core fixed by helioseismology and may exceed a few hundreds percent.

A recent publication of Borexino experiment [6] gives an upper limit for CNO neutrino flux $7.7 \cdot 10^8 \text{ cm}^{-2}\text{s}^{-1}$ what corresponds to less then 1.5 for the ratio between Borexino measurements and predictions of the High Metallicity (GS98) SSM [10]. Because the energy of the end-point of $^{13}$N-neutrino is lower than the one of $^{15}$O-neutrino the weight of $^{13}$N-neutrino in the total flux of CNO neutrino in Borexino experiment is approximately 5 times lower than the weight of $^{15}$O-neutrino [22]. Thus, the change of the flux of $^{13}$N-neutrino on the level of a few hundreds percent still can be considered as compatible with this latest experiment [5]).
Figure 3. Contours for the relative changes of the neutrino fluxes $F_{13}$, $F_{15}$, $F_7$ and of the mean molecular weight of the solar core $\mu_c$ (in %) by mixing within the layer. The painted regions correspond to the changes of $\mu_c$ and of the flux of beryllium neutrino greater than $1\sigma$ (0.5% and 5% correspondingly). Lines marked by line-dots correspond to $2\sigma$.

Figure 4. The evolution of the flux of (a) $^7$Be- and (b) $^{13}$N-neutrino after mixing (the boundary of the mixed layer 0.12, the thickness of the layer 0.06, 0.08 and 0.1 of the radius of the Sun)

results of Borexino experiment.

3 Conclusions.

The results of the calculation show that there is the possibility of mixing in a spherical layer compatible with the measured fluxes of solar neutrinos, with the parameters fixed by helioseismology and at the same time producing substantial increase of $^{13}$N neutrinos. The flux of $^{15}$O neutrinos is changed only marginally in comparison with the predicted by standard solar model. In this model the ratio of the fluxes of $^{13}$N and $^{15}$O neutrinos is equal to 1.40 with the uncertainty of about 0.05 for different parameters of solar model, see, for example, Table 3 of [10]. As one can see from Figure 5 the mixing can bring this ratio to many sigmas
above this value. This constitutes the clear signature of mixing in the solar core and presents the motivation for future experiment provided it is capable to measure both neutrino fluxes of $^{13}N$ and $^{15}O$ neutrinos. The increase of accuracy in the measurements of the effect from $\nu - e^-$ scattering in electronic detectors promises in the future the precise measurement of the effect not only beyond the upper bound of $^7Be$ neutrinos, but also beyond the upper bound of $^{13}N$ neutrinos, thus facilitating the finding of this ratio. This would be the next step in the solar neutrino research.

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