Suppression of phase mixing in drift-kinetic plasma turbulence

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Transfer of free energy from large to small velocity-space scales by phase mixing leads to Landau damping in a linear plasma. In a turbulent drift-kinetic plasma, this transfer is statistically nearly canceled by an inverse transfer from small to large velocity-space scales due to “anti-phase-mixing” modes excited by a stochastic form of plasma echo. Fluid moments (density, velocity, and temperature) are thus approximately energetically isolated from the higher moments of the distribution function, so phase mixing is ineffective as a dissipation mechanism when the plasma collisionality is small. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4958954]

Kinetic turbulence in weakly collisional, strongly magnetized plasmas is ubiquitous in magnetic-confinement-fusion experiments1–3 and in astrophysical settings.4,5 Like fluid turbulence, kinetic turbulence may be described as the injection (e.g., by a plasma instability), cascade to small scales, and dissipation of a quadratic invariant, viz., free energy. On spatial scales larger than the ion Larmor radius, kinetic turbulence incorporates two mechanisms for dissipating free energy into heat. The first is a fluid-like nonlinear cascade from large to smaller, sub-Larmor, spatial scales (where the free energy is dissipated eventually by collisions6–8). The second is parallel phase mixing, a linear process that transfers free energy from the fluid moments (density, fluid velocity, and temperature) to the kinetic (higher-order) moments by creating perturbations in the velocity distribution on ever finer scales in velocity space, perturbations which are also then dissipated by collisions. In a linear plasma, this is known as Landau damping and the free energy is dissipated at a rate independent of collision frequency.9,10

The macroscopic properties of the turbulence (such as heat and momentum transport) are directly affected by the two energy dissipation channels; yet, while each is understood in isolation, how they interact is not clear. Recent work leads to some disquieting observations. First, a fluid-like theory for the nonlinear cascade,11 predicts power-law spectra for the electrostatic potential in good agreement with those found in gyrokinetic simulations, but its derivation neglects free-energy transfer by phase mixing,12 contrary to what might be expected on the basis of linear theory. Including a constant flux of free energy into velocity space leads to non-universal spectra that tend to be steeper than what might be expected on the basis of linear theory.12–17 Some simulations show a significant proportion of injected free energy cascading and dissipating in velocity space,18 albeit with a slower transfer rate than in the linear case, and with a dissipation rate that depends on collision frequency.19,20 These observations suggest a complicated relationship between parallel phase mixing and the nonlinear cascade; there is as yet no complete picture of free-energy flow and dissipation in phase space.

In this letter, we propose the outlines of such a picture for electrostatic drift-kinetic turbulence. We show that the net transfer of free energy from fluid to kinetic modes is strongly inhibited in a turbulent plasma, compared to a “linear plasma.” This is due to a stochastic version of the classic plasma-echo phenomenon.21,22 The nonlinearity excites “anti-phase-mixing” modes that transfer free energy from small to large velocity-space scales, leading to statistical cancellation of the free-energy flux. The significance of this effect depends on the relative rates of phase mixing and nonlinear advection.23,32 We identify regions of wavenumber space where either the echo effect dominates or phase mixing occurs at the usual linear rate. Most of the free energy contained in fluid moments is at wavenumbers that lie within the echo-dominated region. Therefore, there is very little net free-energy transfer to fine velocity-space scales via linear phase mixing. Consequently, Landau damping is strongly suppressed as a dissipation mechanism.

We study electrostatic ion-temperature-gradient (ITG) driven drift-kinetic turbulence in an unsheared slab with kinetic ions and Boltzmann electrons. The equations are the drift-kinetic equation for ions

\[ \frac{\partial g}{\partial t} + v_\parallel \nabla_\parallel (g + \varphi F_\parallel) + u_\perp \cdot \nabla_\perp g = C[g] + \chi, \]

(1)

and the quasineutrality condition24,33,34

\[ \varphi = \frac{Ze\phi}{T_i} = \int_{-\infty}^{\infty} \mathrm{d}v_\parallel g, \quad \frac{Ze}{T_i} \equiv \frac{ZT_e}{T_i}. \]

(2)

Here, \( g = \langle n_i \rangle \int d^2v_\perp \delta f \) is the perturbed ion distribution function integrated over perpendicular velocity space, with...
n, the mean ion density, \( \phi \) the electrostatic potential, \(-e\) the electron charge, \( Ze \) the ion charge, and \( T_i \) and \( T_e \) the mean ion and electron temperatures; 
\[ F_0(v_{th}) = e^{-v_t^2/m_e} \] is the one-dimensional Maxwellian, with \( v_t \) the parallel velocity, 
\( v_{th} = \sqrt{2T_i/m_i} \) the ion thermal velocity, and \( m_i \) the ion mass; 
\( u_\perp \equiv (\rho v_{th}/2)\hat{z} \times \nabla \phi \) is the \( E \times B \) velocity with ion gyroradius \( \rho_i \), and \( \hat{z} \) the unit vector in the direction of the magnetic field line. The perpendicular directions are \( x \) and \( y \). The energy-injection term due to a mean ITG in the negative \( x \) direction is

\[ \chi = -\rho v_{th} \sigma \partial_\theta \left( \frac{v_{th}^2}{2} \right) F_0, \quad \frac{1}{L_T} = -\frac{d \ln T_i}{dx}. \quad (3) \]

The collision operator \( C[\varrho] \) will be described shortly.

The system (1) and (2) conserves the free energy 
\[ W = \int d^3r \rho^2/2x + \int d^3r \int_\infty^\infty d\varrho \rho^2/2F_0 \],

except for injection by the ITG and dissipation by collisions

\[ \frac{dW}{dt} = \int d^3r \int_\infty^\infty d\varrho \frac{g \varrho}{F_0} + \int d^3r \int_{-\infty}^\infty d\varrho \frac{g C[\varrho]}{F_0}. \quad (4) \]

We study the saturated state of drift-kinetic turbulence in a box with parallel and perpendicular lengths \( L_0 \) and \( L_1 \). We solve Equations (1) and (2) with SpectrOK, a phase-space-spectral code designed to capture discrete free-energy conservation exactly. We use a Fourier representation in physical space, \( r \), with \( 128 \times 128 \times 256 \) wavenumbers \( k = (k_x, k_y, k_z) \). We damp the finest resolved scales in physical space by adding a hyperviscous term \( -\nu_0 (k_\perp/k_{\perp, max})^8 + (k_\parallel/k_{\parallel, max})^8 \varrho \) to the right-hand side of (1), with \( k_{\parallel} = \sqrt{k_x^2 + k_y^2} \) and \( v_{th} = 1000 \), a damping rate chosen to provide sufficient dissipation to absorb the nonlinear cascade without reflection. The exponent 8 concentrates dissipation at the finest scales, allowing an inertial range with a power-law spectrum \( \propto k^{-3} \), so \( 2^\gamma \) is the free-energy injection scale. While this arrangement does not inject free energy in a realistic fashion, we are able to study the key features of its transfer and dissipation in drift-kinetic turbulence.

While we solve fully spectrally, it is convenient for presentation to write equations in a form spectral in velocity and the parallel spatial direction only. Equation (1) becomes

\[ \frac{\partial \hat{\varrho}_m}{\partial t} + \frac{ik_v v_{th}}{\sqrt{2}} \varrho \delta_{m1} + \sum_{p_1+q_1=k_1} \hat{u}_\perp(p_1) \cdot \nabla \hat{\varrho}_m(q_1) = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

\[ \frac{\partial \hat{\varrho}_m}{\partial t} + \frac{ik_v v_{th}}{\sqrt{2}} \varrho \delta_{m1} + \sum_{p_1+q_1=k_1} \hat{u}_\perp(p_1) \cdot \nabla \hat{\varrho}_m(q_1) = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

\[ \frac{\partial \hat{\varrho}_m}{\partial t} + \frac{ik_v v_{th}}{\sqrt{2}} \varrho \delta_{m1} + \sum_{p_1+q_1=k_1} \hat{u}_\perp(p_1) \cdot \nabla \hat{\varrho}_m(q_1) = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

where hats denote functions in \( \hat{r}_1, k, m \) space, and \( \chi = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \). Equation (5) exhibits the two mixing mechanisms present in the turbulence: the “fluid” cascade due to the \( \hat{u}_\perp \cdot \nabla \hat{\varrho}_m \) nonlinearity, and linear phase mixing by coupling with Hermite modes \( m \leq 1 \).

We first formulate the free-energy transfer between fluid and kinetic modes. Equation (5) implies

\[ \frac{\partial \| \hat{\varrho}_m \|^2}{\partial t} + \frac{\gamma_m}{2} - \Gamma_m - \Gamma_{m-1} + \text{Im} \left( \hat{\varrho}_m \hat{\varrho}_* \delta_{m1} \right) + N_m = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

\[ \frac{\partial \| \hat{\varrho}_m \|^2}{\partial t} + \frac{\gamma_m}{2} - \Gamma_m - \Gamma_{m-1} + \text{Im} \left( \hat{\varrho}_m \hat{\varrho}_* \delta_{m1} \right) + N_m = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

\[ \frac{\partial \| \hat{\varrho}_m \|^2}{\partial t} + \frac{\gamma_m}{2} - \Gamma_m - \Gamma_{m-1} + \text{Im} \left( \hat{\varrho}_m \hat{\varrho}_* \delta_{m1} \right) + N_m = -e^{-v_t^2/m_e} / \sqrt{2\pi} \varrho \delta_{m1} \]

where \( N_m = \text{Re} \left( \sum_{p_1+q_1=k_1} \hat{u}_\perp(p_1) \cdot \nabla \hat{\varrho}_m(q_1) \right) \) is the free-energy transfer in \( \hat{r}_1, k_1 \) space due to the nonlinearity, and \( \Gamma_m = k_{\parallel} v_{th} \sqrt{(m+1)/2} \text{Im}(\hat{\varrho}_m \hat{\varrho}_* \delta_{m1}) \) is the free-energy transfer from Hermite mode \( m \) to \( m+1 \) due to phase mixing. 29 Since we focus on this transfer between Hermite modes \( m \), rather than the nonlinear cascade in \( k \), we omit the \( m \)-independent hyperviscous dissipation at large \( k \). A full theoretical treatment that includes both transfers may be found in Ref. 12.

Let \( W = \int d^3r_1 \sum_{k} |W_{\text{fluid}}(r_1, k_1)|^2 + |W_{\text{kin}}(r_1, k_1)|^2 \)

\[ \frac{\partial W_{\text{fluid}}}{\partial t} = M + S - T, \quad \frac{\partial W_{\text{kin}}}{\partial t} = N + T - C, \quad (7) \]
where \( S(r_\perp, k_\parallel) = \text{Re}(\delta\tilde{T}_\parallel^*\tilde{u}_s)/(2T_l\lambda T) \) is the source due to the ITG, \( C(r_\perp, k_\parallel) = \nu \sum_{m=3}^\infty m^2|\tilde{g}_m|^2 \) is the sink due to collisions, \( A(r_\perp, k_\parallel) = -[(1+z)N_s + N_1 + N_2] \) and \( \mathcal{N}(r_\perp, k_\parallel) = -\sum_{m=3}^\infty N_m \) are free-energy transfers due to the nonlinearity, and \( T(r_\perp, k_\parallel) = k_\parallel v_{th} \sqrt{3 \text{Im}(\tilde{g}_m^*\delta\tilde{T}_\parallel)/(2\tilde{T}_l)} = \Gamma_2 \) is the transfer of free energy from fluid to kinetic moments by the streaming term \( v_0 \nabla_x \Phi \) in (1). Streaming is linear and reversible, so \( T \) may be positive or negative; however, setting \( T = T_L \equiv |k_\parallel|v_{th}\sqrt{3/2}|\tilde{g}_2|^2 \) (more generally, \( \Gamma_m = \Gamma_m^* \equiv |k_\parallel|v_{th} \sqrt{(m+1)/2|\tilde{g}_m|^2} \)) amounts to an effective Landau-fluid-style closure that captures free-energy dissipation by Landau damping. While not exact,\(^{10}\) it yields spectra in excellent agreement with linear drift-kinetic simulations.\(^{25}\)

We now consider the saturated state of the turbulence, in which the time averages \( \langle dW_{\text{fluid}}/dt \rangle = \langle dW_{\text{kin}}/dt \rangle = 0 \), so \( \langle \mathcal{N} \rangle + \langle T \rangle = \langle C \rangle \geq 0 \). For linear phase mixing (and Landau damping) to play a similar role in a turbulent plasma, we must introduce a continuous approximation for the finite dimensional cutoff.\(^9,10\) For \( T_L \) would have to be similar to its linear value, \( \langle T \rangle \sim T_L \). In Fig. 1, we plot the ratio \( \langle T/T_L \rangle \) as a function of \( (k_\perp, k_\parallel) \) (it is isotropic in \( k_\perp \)) for a time average taken over a time window of length \( 2L_j/v_{th} \) in saturated turbulence. The transfer is almost completely suppressed compared to the linear case, \( \langle T/T_L \rangle \ll 1 \), across a large range of wavenumbers that we will shortly characterize.

The suppression of free-energy transfer associated with phase mixing is a nonlinear, kinetic effect. To understand the suppression mechanism, we decompose the distribution function into propagating modes in Hermite space, by writing \( \tilde{g}_m = (-isgn k)_m^* \hat{g}_m^\pm + (-1)^m \hat{g}_m^\mp \), where \( \hat{g}_m^\pm = \frac{1}{2} (\pm isgn k)_m \). The “phase-mixing mode,” \( \hat{g}_m^\pm \), propagates forward from low to high \( m \), the “anti-phase-mixing mode,” \( \hat{g}_m^\mp \), propagates backward from high to low \( m \).\(^{10,12,26}\)

For \( m \geq 3 \), \( |\hat{g}_m^\pm|^2 \) evolves according to

\[
\frac{\partial}{\partial t} \left( \frac{|\hat{g}_m^\pm|^2}{2} \right) + \frac{|k_\parallel|v_{th}}{\sqrt{2}} \frac{\partial}{\partial \mathcal{N}} \left( \sqrt{m}|\hat{g}_m^\pm|^2 \right) + \nu m \mathcal{P} |\hat{g}_m^\pm|^2 = -\text{Re}\left\{ \sum_{p_l+q_l=k_\parallel} \left[ \hat{g}_{m^*}^\pm(q_l) + \hat{g}_m^\mp(q_l) \right] \right\},
\]

where \( \delta_{k_\parallel q_l} = \frac{1}{2} \left[ 1 \pm \text{sgn}(k_\parallel, q_l) \right] \) so \( \delta_{k_\parallel q_l}^\pm \) picks out \( k_\parallel \) and \( q_l \) that have the same sign, and \( \delta_{k_\parallel q_l}^\mp \) the opposite sign. We have introduced a continuous approximation for the finite difference in \( m \), valid because \( \hat{g}_m^\pm \) are smooth in the sense that \( \hat{g}_m^\pm \approx \hat{g}_m^\pm_{m+1} \) to the lowest order at large \( m \).\(^\text{12}\) In terms of \( \hat{g}_m^\pm \), the normalized free-energy transfer from \( m \) to \( m+1 \) is

\[
\Gamma_m = \frac{k_\parallel v_{th} \sqrt{(m+1)/2\text{Im}(\hat{g}_m^\pm)\hat{g}_m^\mp)}}{|k_\parallel|v_{th} \sqrt{(m+1)/2|\hat{g}_m|^2}} \approx \frac{|\hat{g}_m^\pm|^2 - |\hat{g}_m^\mp|^2}{|\hat{g}_m^\pm|^2 + |\hat{g}_m^\mp|^2},
\]

FIG. 1. \( \langle T/T_L \rangle \), the free-energy transfer from fluid to kinetic moments in the saturated turbulent state, normalized to its value in a linear plasma, expressed in Fourier space. We also show the line of critical balance \( \tau_{\text{nl}} = \tau_s \), as defined in the text.
Second, since the fluid and kinetic moments are decoupled, the free energy is nearly energetically decoupled from other scales, so, in particular, the fluid and kinetic moments are decoupled. First, each velocity scale region \( \frac{k}{k_{z}} = 1 \) contains the majority of the free energy, as seen in the theoretically predicted \(^{12}\) and numerically measured \(^{25}\) spectra. This has two important consequences. First, each velocity scale \( m \) is, statistically, very nearly energetically decoupled from other scales, so, in particular, the fluid and kinetic moments are decoupled. Second, since \( \sum_k \int d^2r_L \langle \mathcal{N} \rangle = 0 \), the time average of (7) implies that the collisional dissipation rate, \( \sum_k \int d^2r_L \langle \mathcal{C} \rangle = \sum_k \int d^2r_L \langle T \rangle \), is also strongly suppressed, so collisional dissipation at fine velocity-space scales is a far less effective dissipation channel than in the case of linear Landau damping.

We now confirm this suppression of collisional dissipation by considering the Hermite spectra. The different free-energy transfer behaviors in the phase-mixing-dominated and the nonlinearity-dominated regions give rise to two different Hermite spectra, plotted in Fig. 3 for fixed wavenumbers and averaged over the same time window of length \( 2L \), used in Figs. 1 and 2. In the phase-mixing-dominated region \( (\Gamma_m = 1) \), we observe the linear \( m^{-1/2} \) spectrum given in (10) for \( m \ll \tau_e \). In the nonlinearity-dominated region \( (\Gamma_m \ll 1) \), we observe the steep \( m^{-5/2} \) spectrum predicted by Schekochihin et al.\(^{12}\) Both spectra exhibit a \(-(1)^m\) zig-zag behavior that arises from a superposition of both \( g_m^+ \) and \( g_m^- \) modes in \( |g_m|^2 = |g_m^+|^2 + |g_m^-|^2 + 2(1)^m \text{Re}(g_m^+ g_m^-^*) \). The spectrum in the phase-mixing region gives rise to free-energy dissipation at the usual Landau damping rate: for a fixed Fourier mode in this region, \( \int_0^\infty dm vmm^{-1/2} \sim |k|v_{th} \). This remains finite as \( \nu \to 0^+ \). In contrast, the dissipation rate for the \( m^{-5/2} \) spectrum observed in the nonlinear region is \( \int_0^\infty dm vmm^{-5/2} \sim |k|^{-1/2}v_{th}^{-1/3} \to 0 \) as \( \nu \to 0^+ \). Landau damping is thus suppressed in the nonlinear region. As most of the free energy is contained in this region, \(^{12, 25}\) the total dissipation via phase mixing to collisional scales in \( v_{th} \) tends to zero as \( \nu \to 0^+ \). The vast majority of free energy cascades nonlinearly to dissipate at fine physical-space scales instead.

These spectra and dissipation patterns explain other numerical observations\(^{19, 20}\) that the Hermite spectrum summed over all Fourier modes is much steeper than the linear \( m^{-1/2} \) spectrum, and that free-energy dissipation via collisions in the inertial range decreases as \( \nu \) decreases. Those observations may be understood as the result of aggregating the behaviors of the two distinct regions of Fourier space identified above. It is possible to prove\(^{12}\) that the aggregate Hermite spectrum is \( m^{-7/2} \), consistent with the reported numerical scalings.\(^{19, 25}\)

In this letter, we have shown that linear phase mixing and the nonlinear cascade are strongly interdependent. The nonlinear cascade in the inertial range excites anti-phase-mixing modes, suppressing the net transfer of free energy into kinetic modes. This has both theoretical and practical implications. Theoretically, these results profoundly change our understanding of the way in which free energy is cascaded and dissipated in phase space. As there is only a small net free-energy flux out of fluid modes in the inertial range, it is probably legitimate to neglect parallel phase mixing when deducing physical-space spectra from Kolmogorov arguments, as was done in earlier work.\(^{11}\) Since the Hermite spectrum at energetically dominant scales is a steep \( m^{-5/2} \) power law, these scales experience no free-energy dissipation via Landau damping as \( \nu \to 0^+ \), and almost all free energy cascades to perpendicular spatial sub-Larmor scales. The steep Hermite spectrum also means that free-energy dissipation via linear phase mixing is not independent of collision frequency. This has the important practical implication that enlarged collision frequencies cannot necessarily be used to compensate for low \( v_{th} \) resolution in weakly collisional simulations.

In this work, we have studied electrostatic ITG drift-kinetic turbulence. However, our focus is on inertial-range physics, which does not depend on the details of the energy injection. The approach presented here should be applicable to other kinetic systems, where a nonlinearity coexists with particle streaming. Indeed, similar suppressions of phase mixing have already been observed due to a different nonlinearity in the Vlasov–Poisson system,\(^{26}\) and in kinetic passive scalar simulations.\(^{26}\) We also expect this work be relevant in electromagnetic plasmas, though these have the complication
that particle streaming along the perturbed magnetic field line also contributes to the nonlinear term. This will be a subject of future work.

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