Clusteringogenesis: from Light to Heavy Primordial Black Holes

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We show that heavy primordial black holes may originate from much lighter ones if the latter are strongly clustered at the time of their formation. While this population is subject to the usual constraints from late-time universe observations, its relation to the initial conditions is different from the standard scenario and provides a new mechanism to generate massive primordial black holes even in the absence of efficient accretion, opening new scenarios, e.g. for the generation of supermassive black holes.

Introduction. Multiple detections of gravitational waves (GWs) coming from black hole binary mergers [1–4] have revived the interest in the physics of Primordial Black Holes (PBHs) [5–8]. Indeed, some of the LIGO/Virgo/KAGRA data may be of primordial origin [9–16] and future GW experiment will shed light on the possible existence of PBHs [17–22].

PBHs in the early universe are commonly born in the radiation-dominated phase (see Ref. [7] for a review on the various formation mechanisms). Given our ignorance of the production mechanism giving rise to PBHs, if any, we do not know if they are born randomly distributed or with a strong correlation among them. In the standard scenario where PBHs are generated by the collapse of large overdensities created during inflation on small scales [5], PBHs are Poisson distributed in space [23–26]. However, large initial correlations are conceivable and not ruled out, unless PBHs abundance in the universe (normalised to the dark matter) \( f_{\text{PBH}} \) is larger than \( \mathcal{O}(0.1) \) in the stellar mass range [27].

In this paper we propose a new mechanism to generate heavy PBHs from light ones making use of the large initial PBH clustering, a mechanism that we dub “clusteringgenesis”. The idea is quite simple: if PBHs are born close to each other, the strong gravitational interactions among them may result in the collapse of this clump into a more massive PBH, even in the radiation dominated phase of the early universe. This mechanism provides a novel way to increase the mass of light PBHs in the primordial epochs, which is alternative to the more standard process relying on baryonic mass accretion, which is efficient only for PBHs with mass larger than \( \mathcal{O}(10) M_\odot \) [28] at lower redshifts.

In the following we will describe the basics of this idea and discuss some of its possible implications.

Collapse of large overdensities in the radiation-dominated era. Independently from the formation mechanism and being discrete objects, the most generic initial two-point correlator for the PBH density contrast \( \delta_{\text{PBH}} = \delta \rho_{\text{PBH}} / \rho_{\text{PBH}} \) acquires the form [23]

\[
\langle \delta_{\text{PBH}}(\vec{r}) \delta_{\text{PBH}}(0) \rangle = \frac{1}{\pi_{\text{PBH}}} \delta_D(r) + \xi_{\text{PBH}}(r),
\]

in terms of their distance \( r \), where

\[
\pi_{\text{PBH}} \approx 30 f_{\text{PBH}} \left( \frac{M_{\text{PBH}}}{M_\odot} \right)^{-1} \text{kpc}^{-3}
\]

is the average PBH number density per comoving volume for a monochromatic PBH population with mass \( M_{\text{PBH}} \).

We suppose that, at the time of formation, such two-point correlator is dominated by the reduced correlation function \( \xi_{\text{PBH}}(r) \) up to some comoving clustering scale \( r_{cl} \), while on larger scales the Poisson shot noise, arising from the discrete nature of PBHs, dominates. For simplicity and for the sake of the argument, we assume an approximately constant in space and large reduced two-point correlation function up to \( r_{cl} \),

\[
\xi_{\text{PBH}}(r) \approx \begin{cases} 
\xi_0 & \text{for } r \lesssim r_{cl}, \\
0 & \text{otherwise},
\end{cases}
\]

We will be agnostic in the following regarding the origin of such correlations. Let us just point out that large clustering appears if the local properties of the PBH overdensity field are space dependent. This effect might either come from an actual field different from the overdensity field, or from a long wavelength modulation of the overdensity field itself, resulting from a self-coupling of long and short scales as happens, e.g., in local models of non-Gaussianity [29]. Alternatively, large correlation may arise if PBHs are formed by bubble collisions in first-order phase transitions [30–32] or for PBHs generated thanks to long-range scalar forces [33]. Once rescaled to the total dark matter density \( \delta \rho_{\text{PBH}} / \rho_{\text{DM}} \sim \xi_0 f_{\text{PBH}} \), the corresponding density contrast is suppressed by the PBH abundance, which we will assume to be tiny in the following in order to avoid bounds coming from CMB anisotropies on isocurvature perturbations [34–36].
The key point is the subsequent evolution of such large
density PBH clumps during the radiation phase. The
common lore is that they do not grow till the matter-radiation
equality epoch is reached, because of the counteraction
of the radiation pressure [37]. However, this is true only
at the linear level. In the full non-linear theory, the self
gravity of these large non-linear fluctuations may become
important before the equality time, and consequently give
rise to their collapse and production of very dense clus-
ters, after they decouple from the general expansion and
virialize [38].

Adopting the spherical collapse model, one can write
down the evolution equation for the parameter \( R \), de-
scribing the deviation of the motion of each collapsing
shell from the uniform Hubble flow of the background
Friedmann Universe [38]

\[
x(1 + x) \frac{d^2 R}{dx^2} + \left(1 + \frac{3}{2} x^2\right) \frac{dR}{dx} + \frac{1}{2} \left(1 + \xi_0 \frac{R^2}{R} - R\right) = 0,
\]

as a function of the rescaled scale factor \( x = a/a_{eq} \), in
terms of the one at the epoch of matter-radiation equality
\( a_{eq} \). This equation assumes that the statistical distribu-
tion of the initial overdensities (or clumps) is determined
by the correlation function \( \xi_{PBH} \), see the appendices of
Ref. [27] for details on the relation between the profile of
initial clumps and the correlation function. In particular,
at scales smaller than the clump separation, the density
profile of the clusters is directly related to the correla-
tion function and its evolution depends on its amplitude
\( \xi_0 \) [39].

An analytic approximation for the solution can be ob-
tained with a power expansion in the rescaled scale factor
\( x \), giving [38]

\[
R \simeq 1 - \frac{\xi_0 x}{2} - \frac{\xi_0^2 x^2}{8} + \mathcal{O}(x^3),
\]

from which one can show that the decoupling occurs ap-
proximately at \( a_{eq} = a_{eq}/\xi_0 \). The corresponding timescale
is approximately given by the free-fall time [40]

\[
\tau_{eq} = \sqrt{\frac{3\pi}{32G\rho_{eq}}} \simeq 1.2 \cdot 10^4 \xi_0^{-2} \left(\frac{C}{200}\right)^{-1/2} \text{yr}, \quad (6)
\]

expressed in terms of the average density of such clusters
after relaxation \( \rho_{eq} = C\rho_{eq}(a_{eq}) = C\rho_{eq}\xi_0^4 \) [38, 41], where
\( C = \mathcal{O}(1 \div 10^2) \) is a constant that describes the overdensity
amplitude. Their mass and physical radius are given by [27]

\[
M_{cl} \simeq 1.3 \cdot 10^2 f_{PBH} \xi_0 \left(\frac{r_{cl}}{\text{kpc}}\right)^3 M_\odot,
\]

\[
r_h \simeq 4 \cdot 10^{-5} f_{PBH} \xi_0^{-1} \left(\frac{C}{200}\right)^{-1/3} \left(\frac{r_{cl}}{\text{kpc}}\right) \text{kpc}. \quad (7)
\]

Notice that the mass of the cluster \( M_{cl} \) does not depend on
the individual PBH mass \( M_{PBH} \) because of two competing
effects that cancel out. Indeed, for a given correlation
function, a fixed PBH abundance may either result into
heavier PBHs with a smaller number density, or into
lighter and more abundant ones.

**Formation of heavy PBHs by initial clustering.** The key
point of this paper is that the PBH clusters may collapse
into PBHs of mass \( \approx M_{cl} \) if the final halo is more compact
than a BH, giving rise to a population of heavy PBHs. 
These objects are initially Poisson distributed on scales
larger than \( r_{cl} \), as the perturbation of their number density
induced by their discreteness dominates over the small
residual correlation, thus evolving subsequently along
what described in Refs. [42, 43], see Fig. 1 for a pictorial
representation.

FIG. 1. Pictorial representation of the clusteringenesis mechanism. The red line denotes the growing cosmological horizon \( H^{-1} \).
More in details, according to the hoop conjecture \cite{44}, this happens if

\[ r_b \lesssim 2GM_{\odot}, \]  

(8)

which translates into a requirement on the correlation function to be

\[ \xi_0 \gtrsim 6 \cdot 10^4 f_{\text{PBH}}^{-1/3} \left( \frac{C}{200} \right)^{-1/6} \left( \frac{r_b}{\text{kpc}} \right)^{-1}. \]  

(9)

We also expect that, due to frequent BH encounters during the cluster’s collapse, strong GW emission may occur, inducing an even more efficient clustering.

Since PBH clusters follow a Poisson distribution on large scales and they are characterised by a small physical size, they may dynamically evaporate \cite{40} before effectively collapsing into heavy PBHs. This occurs if the evaporation timescale is smaller than the characteristic free-fall time shown in Eq. (6). The evaporation time of a system of \( N_{\text{cl}} = M_{\text{cl}}/M_{\text{PBH}} \) PBHs clustered in a region of size \( r_b \) and subject to the gravitational force is given by \cite{40}

\[ t_{\text{ev}} \approx 14 \frac{N_{\text{cl}}}{\log N_{\text{cl}}} \frac{r_b}{v_b} \approx 10^{11} \left( \frac{N_{\text{cl}}}{10^9} \right)^{1/2} \left( \frac{M_{\text{PBH}}}{M_{\odot}} \right)^{-1/2} \left( \frac{r_b}{\text{pc}} \right)^{3/2}. \]  

(10)

By imposing that evaporation is slower than free-fall, \( t_{\text{ev}} \gtrsim \tau_{\text{cl}} \), one then gets

\[ N_{\text{cl}} \gtrsim 6 \cdot 10^{-6} \xi_0^2 \left( \frac{\tau_{\text{cl}}}{\text{yr}} \right) \left( \frac{C}{200} \right)^{1/2}, \]  

(11)

such that using \( N_{\text{cl}} \sim \pi_{\text{PBH}} r_b^3 \xi_0 \) one gets the condition to avoid cluster evaporation

\[ \text{Eva} : \quad \xi_0 \gtrsim 5.5 \cdot 10^{-4} f_{\text{PBH}}^{-1} \left( \frac{M_{\text{PBH}}}{M_{\odot}} \right) \left( \frac{r_b}{\text{kpc}} \right)^{-3}. \]  

(12)

This bound has to be interpreted to be conservative since relaxation may occur on a timescale larger than free-fall, and thus make evaporation even less efficient. Moreover, the initial collapse might produce a BH before the cluster size relaxes to \( r_b \) with a smaller compactness parameter \( C \).

By combining this constraint with the requirement of heavy PBHs formation in Eq. (9) and assuming that, for definiteness, each PBH cluster contains at least three PBHs \cite{27}

\[ N_{\text{cl}} \gtrsim 3 : \quad \xi_0 \gtrsim 2.3 \cdot 10^{-2} f_{\text{PBH}}^{-1} \left( \frac{M_{\text{PBH}}}{M_{\odot}} \right) \left( \frac{r_b}{\text{kpc}} \right)^{-3}, \]  

(13)

one can show in Fig. 2 the allowed parameter space on the clustering model to have the formation of heavy PBHs.

From the figure it is clear that the bound coming from the hoop conjecture sets the strongest constraint on the heavy PBH mass, which is then found to be at least

\[ M_{\text{HypBH}} \gtrsim 2.4 \cdot 10^{16} \xi_0^{-2} \left( \frac{C}{200} \right)^{-1/2} M_{\odot}. \]  

(14)

It is important to stress that the parameter space of these newly formed heavy PBHs is constrained by the same set of observational bounds that apply to the standard PBHs that are formed in the radiation-dominated epoch, see Ref. [6] for a review. However, as their formation is induced by different initial conditions with respect to the standard scenario, their relation to indirect probes.

**Implications.** The production of heavy PBHs from the gravitational collapse of initially clustered PBH seeds provides a novel scenario for PBH formation and has several implications, some of which we described below.

In the standard scenario where the PBHs are created by the collapse of large overdensities created during inflation when they reenter the Hubble radius in the radiation-dominated phase, there is a standard correlation between the PBH mass, as a fraction of the mass enclosed in the cosmological horizon at the time of horizon crossing, and
the corresponding comoving momentum $k$ \[5\]

$$M_{\text{PBH}} \simeq 2.45 \cdot 10^6 \left(\frac{k}{\text{kpc}}\right)^{-2} M_\odot.$$ \(15\)

In the clustering genesis this relation does not hold anymore.

There are two straightforward implications of this fact. In the standard scenario, the power spectrum of the curvature perturbation to generate PBHs must be of the order of $\mathcal{O}(10^{-2})$. Therefore, using Eq. (15), the current constraints on the $\mu$-distortions would rule out PBHs in the mass range $(10^4 \div 10^4) M_\odot$ [36, 45, 46]. On the other hand, within our mechanism, PBHs with such masses may be generated through large curvature perturbations at much smaller scales, thus evading the bounds.

The second point is related to the generation of the stochastic gravitational wave background (SGWB) associated to PBH production due to the nonlinear nature of gravity (see Ref. [47] for a review). In the standard scenario of PBH formation one expects a relation between the PBH mass and the peak frequency of the SGWB $f_{\text{SGWB}}$ as [48]

$$f_{\text{SGWB}} \simeq 0.4 \left(\frac{M_{\text{PBH}}}{M_\odot}\right)^{-1/2} \text{nHz.}$$ \(16\)

On the other hand, if PBHs are initially clustered and gravitationally collapse to create an heavier PBHs, the observation of a SGWB today with a characteristic frequency would then be totally decoupled to the mass of the initially formed PBHs. Furthermore, a large spatial correlation at formation time and the dynamics of cluster collapse may eventually induce a GW emission due to the presence of putative time dependent quadrupolar anisotropies [49–52], whose characteristic frequency would depend on the clustering parameter space.

Another possible application of PBH clustering genesis is related the generation of Supermassive BHs (SMBHs). Such BHs are believed to sit at the center of galaxies and they have been recently observed at high redshifts $z \gtrsim 6$, providing a challenge to standard formation mechanisms [53, 54]. Furthermore, SMBHs currently account for about $10^{-5}$ the dark matter density in the universe [55, 56]. Using the Schechter function to describe the SMBH mass function at high-redshifts, it has been assessed that the mean SMBH mass is approximately of the order of $10^{10} M_\odot$ [57, 58], which might be the final result after a phase of efficient baryonic mass accretion starting from SMBH seeds with masses $M_{\text{SMBH}} \approx 10^4 M_\odot$ [45, 59].

Scenarios involving the presence of PBH seeds able to generate SMBHs invoke an efficient phase of baryonic mass accretion during their cosmological evolution, and require values of the PBH abundance smaller than $f_{\text{PBH}} \lesssim 10^{-9}$ to avoid bounds coming from CMB data [59]. Within the clustering genesis scenario, on the other hand, one can efficiently produce heavy PBHs with masses comparable to a SMBH seed at formation time. In particular, the dashed line in Fig. 2 shows the values for the clustering parameters which efficiently give rise to a SMBH seed with mass $M_{\text{SMBH}} \approx 10^4 M_\odot$. Let us also notice that the SMBHs generated with this mechanism, with low abundance and masses larger than about $10^{10} M_\odot$, may accelerate the formation of bright and massive galaxies at very high redshifts, as recently suggested in Ref. [60].

Conclusions. Given our ignorance of the initial clustering of PBHs, we have shown that heavy PBHs may be generated, even during the radiation phase, during the collapse of a halo composed by previously generated lighter PBHs. This happens if the hoop conjecture inequality is satisfied, provided that the PBH clusters do not dynamically evaporate away. We have discussed possible implications of this scenario, in particular providing a new explanation to the generation of the supermassive black holes observed at the center of galaxies and pointing out that constraints on the curvature perturbation at a given scale may be uncorrelated with the mass of the PBHs.

It will be interesting to investigate other possible implications of the clustering genesis scenario, for instance characterising in more details the mass distribution of the heavy PBHs coming from the collapse, extending the standard Press-Schechter formalism to initial non-linear densities and thresholds.

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