Shortcuts to Adiabaticity with Inherent Robustness and without Auxiliary Control

Yiyao Liu\(^1\) and Zhen-Yu Wang\(^1,2\)

\(^1\)Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China

\(^2\)Frontier Research Institute for Physics, South China Normal University, Guangzhou 510006, China

We develop a framework to construct a new class of shortcuts to adiabaticity for realization of adiabatic processes in short times. In contrast to previous strategies for accelerating adiabatic protocols, our approach speeds up the adiabatic process by applying the adiabatic Hamiltonian at discrete points of the parameter space, thereby avoiding the use of any counterdiabatic control that can be experimentally unfeasible, skipping undesired points of the adiabatic path, and retaining the intrinsic robustness of adiabatic control. We exemplify our general theory by construction of fast and noise-resilient control protocols for bosonic modes, three-level systems, and interacting qubits. Our framework offers a new route to design robust and fast control methods for general quantum systems.

Introduction. — Due to its intrinsic robustness against control imperfections, quantum adiabatic control has a broad range of applications from quantum information processing to quantum sensing, such as stimulated Raman adiabatic passage (STIRAP) for state transfer in three-level systems [1-14] and adiabatic quantum computing [3]. However, these quantum adiabatic control methods are extremely slow following the traditional adiabatic condition. This slowness can be a big problem because decoherence and noise could have enough time to spoil the desired states.

Shortcuts to adiabaticity (STA) [9-8] are alternative fast processes to realize the same final state evolution and have attracted much attention due to practical and fundamental interests [9-19]. In existing STA methods, the total Hamiltonian \(H = H + H_{\text{CD}}\) is different from the original time-dependent Hamiltonian \(H\) of adiabatic control. The addition of the counterdiabatic Hamiltonian \(H_{\text{CD}}\) ensures that an eigenstate of \(H(t = 0)\) will evolve to the corresponding eigenstate of \(H(T)\) at a later time \(T\). However, implementation of \(H_{\text{CD}}\) can be challenging as it may require forbidden transitions or unavailable experimental resources [9]. While for some cases it is possible to derive experimentally feasible STA control using a suitable unitary transformation [20], the procedure is complicated even for the case of two- and three-level systems [20-26]. Moreover, although a larger \(H_{\text{CD}}\) lessens the effects of dissipation and decoherence by shortening the evolution time, \(H_{\text{CD}}\) generally adds additional sources of control errors and changes the eigenstates of the total Hamiltonian \(H'\). The latter point implies that the evolution of an initial ground state does not follow the ground-state evolution of the total Hamiltonian. Therefore, the robustness of STA is not guaranteed. Indeed, a recent comparative study on the control of two-level systems showed that STA performs worse to typical control errors such as amplitude and detuning errors. See Fig. 1a). Designing STA protocols robust against control errors is a non-trivial problem [8 19 28 29]. In Refs. [30 31], it was found that for the case of two-level systems one can, without the use of any \(H_{\text{CD}}\), speed up the quantum adiabatic evolution and retain the inherent robustness of adiabatic process, by utilizing the necessary and sufficient quantum adiabatic condition [32]. This shed new light on designing STA protocols in a new way for general systems.

In this Letter, we develop a general theory to construct a new type of STA control. This STA by modulation (STAM) method is achieved by dynamically modulating the functional forms of the parameters in the Hamiltonian for quantum adiabatic evolution. The resulting protocols are simple, do not require additional control, retain the robustness of adiabatic processes, and can avoid the control and interactions that are not feasible in experiments. We illustrate the idea and show superior performances of STAM by applying it to bosonic systems.

![Figure 1](https://example.com/figure1.png)

**FIG. 1.** (a) Schematic trajectories (defined by the eigenstates of the total Hamiltonian) to the target state (indicated by the blue dashed circle) for an infinitely slow adiabatic process (solid blue), an ideal STA (solid green), a STA with control errors (dashed light green), and a STAM control (orange crosses). STA can be sensitive to control errors \(\delta\). STAM and adiabatic control are robust and have the same instantaneous eigenstates of the total Hamiltonian. (b) A sketch of the function \(F(\lambda)\). The integral for the regions of the same color is zero. (c) Nodes \(n\) and \(m\) that are not connected have \(g_{n,m} = 0\) in STAM. The colors denote the types of the nodes. (d) An example of (c) for a bosonic mode.
three-level systems, and interacting qubits.

\[ H(\lambda) = \sum_n E_n |n_\lambda\rangle \langle n_\lambda|, \]

(1)

where \( E_n \) are the eigenenergies, and the instantaneous eigenstates \(|n_\lambda\rangle\) depend on a monotonic function \( \lambda = \lambda(t) \) which starts from an initial path point \( \lambda(0) = 0 \). In the adiabatic limit of an infinitely slow change of \( H \), the time-ordered evolution \( U = \mathcal{T} e^{-i \int_0^t H(t) dt} \) becomes the adiabatic evolution operator \( U_{\text{adia}} = \sum_n e^{-i \varphi_n(\lambda)} |n_\lambda\rangle \langle n_\lambda| \).

Since the choice of gauge does not change the Hamiltonian and its evolution operator \( \mathcal{U} \), for convenience we use the Born-Fock gauge \( g \) where \( g \) transfers the original state \(|n_\lambda\rangle\) to \(|\varphi_n(\lambda)\rangle\).

\[ g_{n,m} = \langle n_0| e^{iG \lambda} | m_0 \rangle = \langle n_0| G | m_0 \rangle \]

(5)

for the eigenstates \(|n_\lambda\rangle\) of the Hamiltonian \( H(\lambda) \) using a constant Hermitian operator \( G \). In this manner each

\[ g_{n,m} = \langle n_0| e^{iG \lambda} \frac{d}{d\lambda} e^{-iG \lambda} | m_0 \rangle = \langle n_0| G | m_0 \rangle \]

becomes a constant, by using Eq. (2). This also reduces the bound of the non-adiabatic error \(|D|\) in Eq. (3) since \( g_{\text{max}}' = 0 \). The second step is to modulate the functional forms of the dynamic phases with respect to \( \lambda \) for all pairs of \(|n, m\rangle\) with \( g_{n,m} \neq 0 \) such that \( e^{i\varphi_n(\lambda) - i\varphi_m(\lambda)} = \mathcal{F}(\lambda) \), where \( F(\lambda) \equiv e^{i \lambda} \) when \( \lambda \in [\lambda_j, \lambda_{j+1}) \), see Fig. 1b.

For other pairs of \(|n, m\rangle\) that \( e^{i\varphi_n(\lambda) - i\varphi_m(\lambda)} \neq \mathcal{F}(\lambda) \), we set \( g_{n,m} = 0 \). In this way, \( W(\lambda) = \mathcal{F}(\lambda) G \) commutes with itself, and from Eq. (3) \( U_D(\Theta_j) = e^{i \int_0^\lambda W(\lambda) d\lambda} = I \), where \( \Theta_j = 2 \sum_{k=1}^j (-1)^{k+1} \pi k \), \( j = 1, 2, \ldots, N \), see Fig. 1b. Therefore, the controlled evolution \( U = U_{\text{adia}} \) perfectly realizes the target evolution at \( \Theta_j \). For simplicity we assume equally spaced points \( \lambda_j = \frac{\Theta}{2 \pi} (2j - 1) \) and \( \Theta_j = j \Theta N / N \).

While other ways are possible to achieve the desired functional forms of \( \{\varphi_n(\lambda)\} \) for \( \mathcal{F}(\lambda) \), here we apply a sequence of control \( H(\lambda) \) with \( \lambda = \lambda_1, \lambda_2, \ldots, \lambda_N \). In contrast to traditional adiabatic control where \( H(\lambda) \) varies slowly over a continuous range of \( \lambda \), applying \( H(\lambda) \) only at \( \{\lambda_j\} \) can avoid unwanted points \( \lambda \notin \{\lambda_j\} \) that could be challenging or not feasible in experiments. We note that some of \(|n(\lambda)\rangle\) could be chosen as dark states because only relative energies are important.

Any given initial and target states can be connected via Eq. (5) with a suitable choice of \( G \). This can be shown by an explicit example that \( G = \sum_{n=0}^{L_{\text{dim}}} g_{n,1} | n_0 \rangle \langle n_0 | + \mathcal{H}. \) In this case, we may choose \( \varphi_1 = E_1 = 0 \), while \( n \neq 1 \), \( E_n = E, \varphi_n(\lambda) = j \pi \) when \( \lambda \in [\lambda_j, \lambda_{j+1}) \). The sequence, where each \( H(\lambda_j) \) is applied for a pulse duration of \( \pi / E \), transforms an arbitrary initial state \(|1_{\text{ini}}\rangle\) to a general superposition state \(|1_{\text{fin}}\rangle = \cos(g(\Theta N)|1_0\rangle + i \sin(g(\Theta N)|\sum_{n=2}^{L_{\text{dim}}} g_{n,1} | n_0 \rangle) \) with \( g_{n,1} = g_{n,1}/g = g = \sqrt{\sum_{n=1}^{L_{\text{dim}}}|g_{n,1}|^2} \).

To find which \( g_{n,m} \) can be nonzero, we consider the clusters where the nodes marked by state labels \( n \) and \( m \) are connected (disconnected) if \( g_{n,m} \neq 0 \) (\( g_{n,m} = 0 \), as illustrated in Fig. 1c). Since for \( g_{n,m} \neq 0 \) the difference of dynamic phases \( \varphi_n - \varphi_m \) has to be changed by an amount of \( (2k_{nm} + 1) \pi \) (with \( k_{nm} \) being an integer) at each point \( \lambda_j \), each cluster \( C \) of a size larger than one is required to have exactly two types of nodes, \( A \) and \( B \). If \( n \in A \) and \( m \in B \), \( e^{i\varphi_n(\lambda) - i\varphi_m(\lambda)} = \mathcal{F}(\lambda) \), while for the pair \( \{n, m\} \in A \) or \( \{n, m\} \in B \) the nodes \( n \) and \( m \) cannot be linked (i.e., \( g_{n,m} = 0 \)).

Fast preparation of coherent states.— A simple example of choosing the values of \( g_{n,m} \) is illustrated for a bosonic mode in Fig. 1i(d), where we choose \( g_{n+1,m} = i \alpha \sqrt{n + 1} \) in the Fock state basis \(|n\rangle\) \( (n \geq 0) \). Therefore \( G = i\alpha a^\dagger - i^* a \), where \( a \) is the annihilation operator. \( e^{-iG} \) is a displacement operator which transforms the Hamiltonian \( \omega a^\dagger a \) of a harmonic oscillator to \( H(\lambda) = e^{-iG} \omega a^\dagger a = e^{G} \omega a^\dagger a - \omega \lambda (a^\dagger + a) - |\omega \lambda|^2 \). Applying \( H(1/2) \) for a duration of \( \pi / \omega \) transfers the original
ground state $|0\rangle$ to the ground state of $H(1)$, i.e., a coherent state $|\alpha\rangle = e^{-i\delta}\hat{G}_1^j|0\rangle$. This is much faster than adiabatically varying the control $H(\lambda)$.

**Intrinsic robustness.**— A general, rigorous theory of the robustness for standard adiabatic control can be seen from Eq. (4). When the control becomes adiabatic, $\epsilon_{\text{ave}}$ is small (due to the averaging by $e^{i(|\varphi_\lambda(\lambda)\rangle - \varphi_m(\lambda))}$) and hence the fidelity $F$ is high [30, 32]. It is obvious that $\epsilon_{\text{ave}}$ remains small even in the presence of a positive error on $|\varphi_\lambda(\lambda)\rangle - \varphi_m(\lambda)\rangle$. Thus a more adiabatic control is more robust against control amplitude errors. For STAM, a relative error $\delta_\epsilon$ between $\varphi_n(\lambda)$ induces $\epsilon_{\text{ave}} = e^{i|\delta_\epsilon|^2/2}$ when $\lambda \in [\lambda_j, \lambda_{j+1})$. In [33], we obtain $\epsilon_{\text{ave}} = \Theta_{N}(1 + |\delta_\epsilon| / \delta_\epsilon^*)$, which remains small when $N$ is large. This implies that the robustness is stronger when the distance $\lambda_{j+1} - \lambda_j$ between subsequent points is smaller. Furthermore, the robustness of STAM is enhanced by the constraint $\delta'_\text{max} = 0$ because it gives a smaller deviation $|D(\lambda)|$.

For STAM there is another mechanism to enhance the robustness against fast fluctuating errors. In contrast to traditional adiabatic approach that $\lambda$ changes in time, in STAM $H(\lambda)$ is applied at a fixed $\lambda = \lambda_j$ for a pulse duration $\tau_p$. At the same point $\lambda_j$, the random energy shifts $\beta(t)$ at different moments accumulate as a pulse area error $\delta = \int_{t_j}^{t_j + \tau_p} \beta(t')dt'$. A fluctuating drift $\beta(t)$ with a zero mean implies that the expectation value of $\delta$ is $\delta = 0$. This is the same mechanism of robustness as the geometric gate in Ref. [37], because fast fluctuation at the boundary (\theta) hardly changes the area ($\delta$). The variance $\overline{\delta^2} = \int_{t_j}^{t_j + \tau_p} \int_{t_j}^{t_j + \tau_p} dt_1 dt_2 \beta(t_1) \beta(t_2)$ is negligible when the correlation time of the correlation function $\beta(t)\beta(0)$ is small.

**Adaptive control for A-systems.**— To demonstrate STAM, consider a $\Lambda$-type three-level system where there is no direct coupling between the two qubit states $|0\rangle$ and $|1\rangle$, see Fig. (2a). For this system, the adiabatic jumping protocol in [39] and the STA protocols [8] which require direct coupling between $|0\rangle$ and $|1\rangle$ cannot be used. To realize robust and fast state transfer between $|0\rangle$ and $|1\rangle$ by a dark state $|\lambda\rangle = e^{-i\lambda}\lambda(1) = \cos(\lambda)\langle 1|\langle 0\rangle + \sin(\lambda)\langle 0\rangle|0\rangle$ with $E_1 = 0$, we choose $g_{1,2} = i\sin \xi$, $g_{1,3} = i\sin \xi$, $g_{2,3} = 0$, and the initial eigenstates $|\lambda_{\epsilon=0}\rangle = \cos(\xi)\langle 0| - \sin(\xi)|\epsilon\rangle$, $|\lambda_{\epsilon=0}\rangle = \sin(\xi)\langle 0| + \cos(\xi)|\epsilon\rangle$. See [33] for more details. The requirement $\langle 0|H|1\rangle = 1$ gives $(E_3 - E_2)\cos(2\xi) = (E_3 + E_2)$ and the Hamiltonian of STAM, $H(\lambda) = \Omega(|B\rangle\langle \epsilon| + |\epsilon\rangle\langle B\rangle) + \Delta|\epsilon\rangle\langle \epsilon|$, where the bright state $|B\rangle = \sin(\lambda)\langle 1| + e^{i\delta}\cos(\lambda)|0\rangle$, the detuning $\Delta = E_2 + E_3$, and the overall coupling strength $\Omega = \sqrt{-E_2 E_3}$. To meet the condition for $f(\lambda)$, $E_2 = -(2k_2 + 1)\pi/t_p$ and $E_3 = (2k_3 + 1)\pi/t_p$ with the integers $k_2, k_3 \geq 0$. The amplitudes of the Stokes ($\Omega_m = 2\Omega\cos\lambda$) and pump ($\Omega_p = 2\Omega\sin\lambda$) pulses are illustrated in Fig. (2b). Applying $H(\lambda)$ according to the sequence $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_N$ realizes a universal single-qubit quantum gate,

$$U = \begin{pmatrix}
\cos \Theta_N & (-1)^N e^{-i\phi} \sin \Theta_N \\
-e^{i\phi} \sin \Theta_N & (-1)^N \cos \Theta_N
\end{pmatrix},$$

in the basis $\{|\epsilon\rangle, |\epsilon\rangle\}$ of the qubit subspace. For even $N$ the gate is a rotation of $2\Theta_N$ along a direction in the $x-y$ plane of the Bloch sphere, while for odd $N$ it is a $\pi$ rotation along an arbitrary direction.

It is interesting that the value of $\Delta$ is adjustable for optimal performance. For the resonant case $\Delta = 0$, $t_p = \pi/|\Omega|$ reaches the quantum speed limit [36, 37] and the protocol for $N = 1$ coincides with the geometric gate in Ref. [33]. On the other hand, a larger $\Delta$ can be used to reduce the effect of dissipation on the intermediate state $|\epsilon\rangle$. The traditional method to suppress this dissipation is the adiabatic elimination of $|\epsilon\rangle$ via a slow, second-order control in the regime $\Omega \ll \Delta$. This slowness is a large obstacle in the presence of dephasing noise since STIRAP is very sensitive to the two-photon detuning (equivalent to dephasing noise on the qubit subspace) [10]. In contrast, STAM can accelerate the control in the interme-

![FIG. 2. Adaptive control for three-level systems. (a) Indirect coupling of $|0\rangle$ and $|1\rangle$ qubit states via two driving fields with a single-photon detuning $\Delta$ in a $\Lambda$-type system. (b) The time envelopes of Stokes $\Omega_\lambda(t)$ (blue line) and pump $\Omega_p(t)$ (red pulse) of the STAM protocol have simple pulse shapes. The cases of $N = 2, 3, 4$ are shown. (c) The gate fidelity (yellow triangles) for $N = 1$ and $\Theta_N = \frac{\pi}{2}$ in Eq. (5) within the qubit subspace at different detuning $\Delta$, by including a relaxation rate $\Gamma_\epsilon = 1.5|\Omega|/2\pi$ for decay of the intermediate state $|\epsilon\rangle$ into the qubit subspace and a dephasing rate $\Gamma_\text{deph} = 0.05|\Omega|/2\pi$ on the qubit states. Blue circles (green rectangles) are the result of setting $\Gamma_\text{deph} = 0$ ($\Gamma_\epsilon = 0$). (d) Robustness of the transfer efficiency as a function of the amplitude and detuning errors (measured in terms of $\Omega$) for STAM with different $N$. (e) As (d) but for comparison between the superadiabatic transitionless driving (SATD) developed in Refs. [24, 26] and the STAM. Both protocols have the same maximal value of control field for $\Omega_\lambda$, $\Omega_\epsilon$, and the same length of control time $T = 2\pi/|\Omega|$.
diate regime $\Delta \sim \Omega$ to overcome this obstacle, offering a higher gate fidelity in the presence of both dephasing and dissipation [see Fig. 2(c)].

Comparing with existing robust STA for $\Lambda$ systems [24 20], the robustness of STAM is much better and can be further enhanced by using a larger $N$ [see Fig. 2(d),(e)].

Application to coupled qubits.—As another example, consider the Hamiltonian for adiabatic quantum computation,

$$H = (1 - s)H_0 + sH_1,$$

where $H_0 = -E_x (\sigma^{(1)}_x + \sigma^{(2)}_x)$ and $H_1 = -E\sigma^{(1)}_z \sigma^{(2)}_z$ ($E > 0$) [38]. Starting from the ground state of $H_0$, $|11\rangle = |1\rangle \otimes |1\rangle$, slowly increasing the value of $s = s(t)$ from $s = 0$ to $s = 1$ is supposed to adiabatically prepare the target solution state $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$. However, $H$ has a gap closing at the final point $s = 1$, with another ground state $|\psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, see Fig. 3(a).

Therefore, a local fluctuation (e.g., a weak perturbation $\epsilon_x \sigma^{(1)}_x$ with $\epsilon_x > 0$) breaks the parity symmetry of $H$ and leads to a wrong result (e.g., $|\psi_x\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle + |\psi_-\rangle)$ in the adiabatic limit, as shown in Fig. 3.

STAM avoids the use of the Hamiltonian at the degenerate point $s = 1$ and is much faster. For this case we use $g_{1,2} = g_{2,1} = -1$ (other $g_{n,m} = 0$) and $|2_0\rangle = i|00\rangle$ such that $|1_\lambda\rangle$ connects the initial ground state $|1_0\rangle = |11\rangle$ and the target ground state $|\psi_+\rangle$.

The other initial eigenstates are the general superposition states $|3_0\rangle = \cos \beta e^{i\xi} |01\rangle - \sin \beta |10\rangle$ and $|4_0\rangle = \sin \beta e^{i\xi} |01\rangle + \cos \beta |10\rangle$. Setting $\xi = 0$, $\beta = \pi/4$, $E_1 = -E_2 = E$, $E_3 = -E_4 = E \sin(2\lambda)$, we obtain the Hamiltonian $H(\lambda) = \cos(2\lambda)H_0 + \sin(2\lambda)H_1$ as Eq. (7) but with a different interpolation. As shown in Fig. 3, compared with standard adiabatic control with a linear interpolation of $s = t/T$, STAM with $N = 1$ achieves a higher state fidelity using a shorter time $\pi/(2E)$ and is much more robust to local fluctuations because the degenerate point $\lambda = \pi/4$ (i.e., $s = 1$) is avoided.

We can also use a different Hamiltonian, say, $H(\lambda) = \cot(2\lambda)H_0 + H_1$, which has similarities to the model of Landau-Zener-Stückelberg transitions [39]. In contrast to the adiabatic control that continuously varies $\lambda$ from $\lambda = 0$ to $\lambda = \pi/4$, our control is bounded because the divergent point $\lambda = 0$ is avoided. Because the interaction $H_1$ is a constant and $\cot(2\lambda)H_0$ can be varied by the detuning of control field, this control is faster and is easy to realize in systems such as a nitrogen-vacancy (NV) center in diamond and nuclear spins [40 43].

Conclusion.—We developed a general theory for general quantum systems to construct a new kind of STA protocols that speed up the adiabatic evolution only by the use of interactions in the original adiabatic Hamiltonian.

Without the requirement of counterdiabatic control or its equivalence using unitary transformations, the resulting STAM protocols are more experimentally feasible and can avoid obstacles due to degenerate points, unbounded values of control fields, and unavailable control resources. In addition to the ability to retain the intrinsic robustness of quantum adiabatic control, STAM protocols have other mechanisms to suppress the effects of control errors and noise from the environment.

Our results provide a new route to design robust and high-speed control methods, as demonstrated by the examples of preparation of coherent states, reliable evolution to the solution state in an adiabatic quantum computing model, as well as a three-level system control protocol which has adjustable values of detuning for optimal performance in the presence of both dephasing noise and excited state decay. In future work, the pulse-sequence like STAM could be modified to have the ability of coherence protection as dynamical-decoupling (DD) pulse sequences [44 45] and may be incorporated in DD-based sensing with adiabatic control [46 47].

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* zhennyu.wang@m.scnu.edu.cn
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