STRONG FIELD EFFECTS ON EMISSION LINE PROFILES: KERR BLACK HOLES AND WARPED ACCRETION DISKS

YAN WANG1,2 AND XIANG-DONG LI1,2

1 Department of Astronomy, Nanjing University, Nanjing 210093, China
2 Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

ABSTRACT

If an accretion disk around a black hole is illuminated by hard X-rays from non-thermal coronae, fluorescent iron lines will be emitted from the inner region of the accretion disk. The emission line profiles will show a variety of strong field effects, which may be used as a probe of the spin parameter of the black hole and the structure of the accretion disk. In this paper, we generalize the previous relativistic line profile models by including both the black hole spinning effects and the non-axisymmetries of warped accretion disks. Our results show different features from the conventional calculations for either a flat disk around a Kerr black hole or a warped disk around a Schwarzschild black hole by presenting, at the same time, multiple peaks, rather long red tails, and time variations of line profiles with the precession of the disk. We show disk images as seen by a distant observer, which are distorted by the strong gravity. Although we are primarily concerned with the iron K-shell lines in this paper, the calculation is general and is valid for any emission lines produced from a warped accretion disk around a black hole.

Key words: accretion, accretion disks – black hole physics – line: profiles

Online-only material: color figures

1. INTRODUCTION

Accretion power serves as an important source of energy in astrophysics, especially in galactic X-ray binaries (XRBs) and active galactic nuclei (AGNs). In these systems, black holes are possible candidates for the central accreting compact objects. Accretion onto both stellar-mass black holes in XRBs and supermassive black holes (106–109 M☉) in AGNs usually proceeds through an accretion disk, which may be truncated at the innermost stable circular orbit (ISCO) because of the general relativistic effect of the black hole. Due to viscosity, plasma elements in the disk gradually spiral inward to the black hole while losing their angular momenta until reaching the ISCO; they then plunge into the black hole, if there is no external force acting on them.

The quasi-thermal spectra of the black hole accretion disks are relatively cold, which peak at the optical/UV band in AGNs and the soft X-ray band in XRBs (Frank et al. 2002). If point-like hot coronae, above and below the center of the accretion disk (as in the lamp-post model; Matt et al. 1991), illuminate the inner region of the disk by hard X-ray photons, then emit iron K-shell photons centered at 6.40–6.97 keV, depending on the ionization state) through the fluorescent mechanism. Iron K-shell lines may be the strongest emission lines in the X-ray observations of XRBs (Barr et al. 1985, Miller et al. 2002a for Cygnus X-1; Miller et al. 2002b, Miniutti et al. 2004, Rossi et al. 2005 for XTE J1650-500) and AGNs (Tanaka et al. 1995, Wilms et al. 2001, Brenneman & Reynolds 2006, Miniutti et al. 2007 for the Seyfert-1 MCG-6-30-15, the most important source to date). Observations suggest that some two-horn broadened iron K-shell lines originate from the innermost area of the accretion disk. The iron lines could be a powerful tool to study the strong gravity within a gravitational radius rg = GM/c2 and a probe of the black hole spin parameter (see reviews by Reynolds & Nowak 2003 and Miller 2007).

Fabian et al. (1989) modeled the relativistic line profile from a standard thin disk (see Shakura & Sunyaev 1973 for Newtonian disk model; Novikov & Thorne 1973 and Page & Thorne 1974 for general relativistic effects) orbiting around a Schwarzschild black hole. Laor (1991) and Bromley et al. (1997) calculated the line profile from a standard thin disk orbiting around an extreme Kerr black hole. Fanton et al. (1997) calculated the line profile from a standard thin disk orbiting around a Kerr black hole with spin parameter a = cJ/GM (where J is the angular momentum and M is the mass of the black hole) as a free parameter, in their calculation the complication of computing azimuthal angle ϕ has been avoided due to the axisymmetry of both the Kerr metric and the accretion disk. Thereafter, the line profiles from accretion disks with different structures have been investigated: for a warped accretion disk orbiting around a Schwarzschild black hole (Hartnoll & Blackman 2000; Wu et al. 2008) and for a flat accretion disk with spiral patterns orbiting around a non-rotating black hole (Hartnoll & Blackman 2002) or a rotating black hole (Karas et al. 2001). In these calculations, the accretion disks are optically thick and geometrically thin; furthermore, it was assumed that the plasma elements within ISCO contribute little to the line profiles, because they are hot and almost fully ionized (Young et al. 1998; Brenneman & Reynolds 2006). In contrast with the thin disk, Wu & Wang (2007) explored the role of the disk self-shadowing effect due to significant geometrical thickness. Fuerst & Wu (2007) investigated the influence of geometrical thickness of the relativistic accretion torus on the line profiles. Line profiles emitted from two-component accretion disks have also been studied (Fragile et al. 2005; Dexter & Fragile 2011).

It is interesting to model the relativistic iron K-shell lines emerging from the warped disks orbiting around Kerr black holes. Bardeen & Petterson (1975) showed that for Kerr black holes the dragging of inertial frames can lead to coupling between the spin of the black hole and the orbital angular momentum of the disk, and that the component of the viscous torque, which is in the disk plane and perpendicular to the line...
of nodes, will tend to force the inner part of the disk to lie on the equatorial plane of the Kerr black hole. Nevertheless, misalignment of the inner part of the disk may take place even in the presence of dissipation. Demianski & Ivanov (1997) and Ivanov & Illarionov (1997) showed that the misaligned disk adopts a steady warped shape in which the tilt angle is an oscillatory function of radius. Lubow et al. (2002) considered the time evolution of the warped disk under the condition that the warping propagates in a wave-like manner and concluded that, in a low-viscosity disk around a Kerr black hole, the warping may be induced by non-axisymmetric radiation pressure (Pringle 1996; Maloney et al. 1998) or tidal interaction (Terquem & Bertout 1993; Larwood et al. 1996). In observations, evidence of warped disks in XRBs includes the 35 day periodic variation of X-ray flux of Her X-1 (Tananbaum et al. 1972; Katz 1973) and the precessing relativistic jet in SS 433 (Margon 1984); for AGNs disk warping can explain the radial dependence of declination between the high-velocity maser and systemic maser in NGC 4258 (Miyoshi et al. 1995; Martin 2008) and the misalignment between the angular momentum of the radio jets and the host galactic disks (Kinney et al. 2000; Schmitt et al. 2002).

The motivations for this work are numerous. First, the iron K-shell lines carry the information about the spacetime around a rotating black hole; fitting line profiles from the observational data with better spectral resolution and signal-to-noise ratio can explain the radial dependence of declination between the relativistic jet in SS 433 (Margon 1984); for AGNs disk warping can explain the radial dependence of declination between the high-velocity maser and systemic maser in NGC 4258 (Miyoshi et al. 1995; Martin 2008) and the misalignment between the angular momentum of the radio jets and the host galactic disks (Kinney et al. 2000; Schmitt et al. 2002).

This paper is organized as follows. In Section 2 we review the properties of photon trajectories in the Kerr spacetime and summarize the relevant equations needed in our calculation. Next, we describe in Section 3.1 the geometric formalism for the warped disk and in Section 3.2 the kinematic structure. These considerations are employed in Section 4, where we show our main results, including the emission line profiles and the images of the warped disk around a Kerr black hole observed from a distant place. We summarize the main results and discuss their possible implications in Section 5. To allow the main ideas of the paper to be as clear as possible, several sets of details have been relegated to the Appendix, in which we give the detailed procedure for deriving the expression of the ratio \( g \) for the warped disk. The relativistic calculations presented in this paper use the notational conventions of the text by Misner et al. (1973); in particular, we use \( c = G = 1 \).

2. PHOTON TRAJECTORIES IN THE KERR SPACETIME

When calculating the relativistic line profiles, one needs to trace the trajectories of the photons in the Kerr spacetime. In the standard Boyer–Lindquist coordinates, the Kerr metric can be written as

\[
ds^2 = -\Delta dt^2 + \Sigma dr^2 + \sin^2 \theta \Delta^{-1} (d\phi - \omega dr)^2 + \Sigma d\theta^2,
\]

where

\[
\Sigma = r^2 + a^2 \cos^2 \theta,
\]

\( \Delta = r^2 + a^2 - 2Mr \),

\[
A = (r^2 + a^2)^2 - a^2 \sin^2 \theta \Delta = \Sigma (r^2 + a^2)^2 + 2Ma^2 \sin^2 \theta,
\]

\[
\omega = 2aMr \Delta^{-1}.
\]

This metric contains two parameters, the mass of the black hole \( M \) and the specific angular momentum or the spin parameter \( a = J/M \). The greater root \( r_* = M + \sqrt{M^2 - a^2} \) of \( \Delta = 0 \) corresponds to the event horizon which is an one-way membrane in the Kerr spacetime. We note that the right-hand sides of Equations (2)–(5) are always positive outside the event horizon.

The dragging of inertial frame forbids any observer remaining static within the static limit \( r_\text{in} = M + \sqrt{M^2 + a^2 \cos^2 \theta} \).

Given a spacetime background, the motion of a photon obeys the null geodesic equation \( \nabla_p \cdot \mathbf{p} = 0 \), where \( \mathbf{p} \) is the 4-momentum of the photon and \( \nabla_p \) is the directional derivative operator along \( \mathbf{p} \). This is a set of second-order ordinary differential equations, which can be solved in principle by the Runge–Kutta method with given initial position and momentum of the photon. Alternatively, in Kerr spacetime one can take advantage of its axisymmetric property (the corresponding constants of motion include the energy of the photon \( E \), the angular momentum along the spin axis \( L_z \), and the norm of the 4-momentum \( \delta \)) and solve the trajectory of a photon in the equatorial plane by the Lagrangian approach. For the general null geodesics in the Kerr spacetime, Carter (1968) demonstrated the separability of the Hamilton–Jacobi equation and discovered the existence the fourth constant of motion (i.e., Carter constant \( Q \)). To obtain the equations governing the motion of the photons, one can take the partial derivatives of Hamilton’s principle function

\[
S = \frac{1}{2} \delta \lambda - Et + L_z \phi + \int_0^r \frac{\sqrt{R(r)}}{\Delta} dr + \sum_0^\theta \sqrt{\Theta(\delta, \xi, \eta)},
\]

with respect to \( E, L_z, \delta, \) and \( \Theta \) then set them to zero. After some manipulations (see Chandrasekhar 1983 for details), one can find that the equation governing the motion of the photon (\( \delta = 0 \)) in \( r \) and \( \theta \) is

\[
\int_0^r \frac{dr}{\sqrt{R(r, \xi, \eta)}} = \int_0^\theta \frac{d\theta}{\sqrt{\Theta(\delta, \xi, \eta)}} = P,
\]

where

\[
R(r, \xi, \eta) = r^4 + (a^2 - \xi^2 - \eta^2)r^2 + 2M(\eta + (\xi - a)^2)r - a^2 \eta,
\]

\[
\Theta(\delta, \xi, \eta) = \eta + a^2 \cos^2 \theta - \xi^2 \cos^2 \theta,
\]

and \( P \) is the parameter along the photon trajectory. Here we have defined two dimensionless impact parameters \( \xi = L_z/E \) and \( \eta = \Theta/E^2 \). The equations of motion for other coordinates (including the affine parameter \( \lambda \)) can be expressed as combinations of the integrals about \( r \) and \( \Theta \):

\[
\lambda = \int_0^r \frac{r^2dr}{\sqrt{R(r, \xi, \eta)}} + \int_0^\theta \frac{a^2 \cos^2 \theta d\theta}{\sqrt{\Theta(\delta, \xi, \eta)}},
\]

\[
t = E\lambda + 2M \int_0^r \frac{r(Er^2 - aL_z + a^2E)dr}{\Delta \sqrt{R(r, \xi, \eta)}},
\]

where

\[
\Delta = r^2 + a^2 - 2Mr,
\]

\[
A = (r^2 + a^2)^2 - a^2 \sin^2 \theta \Delta = \Sigma (r^2 + a^2)^2 + 2Ma^2 \sin^2 \theta,
\]

\[
\omega = 2aMr \Delta^{-1}.
\]
The Astrophysical Journal, 744:186 (12pp), 2012 January 10

WANG & LI

3. GEOMETRIC AND KINEMATIC STRUCTURE OF THE WARPED ACCRETION DISK

3.1. Geometric Structure of the Disk

In our calculations, we use a series of concentric rings with increasing radii to describe the shape of the geometrically thin warped disk, as shown in Figure 1. The spherical $(r, \theta, \phi)$ and cylindrical $(\rho, \phi, h)$ coordinate systems both appear in our calculations, and they can be transformed from each other by

$$h = r \sin \beta \cos \phi = \rho \tan \beta \cos \psi,$$

and

$$\rho = r (1 - \sin^2 \beta \cos^2 \phi)^{1/2} = r \left(1 - \frac{\sin^2 \beta}{1 + \tan^2 \psi \cos^2 \beta}\right),$$

where we use $\rho \cos \psi = r \cos \phi \cos \beta$, $\tan \phi = \tan \psi \tan \beta$, and $\psi = \phi - \gamma$ in the derivation above. The formalism of the warped disk has been discussed by Terquem & Bertout (1993), Hartnoll & Blackman (2000), and Wu et al. (2008). Here we adopt the function expression of the two Eulerian angles used by Wu et al. (2008), and the warping of disk is in an oscillation shape:

$$\gamma = \gamma_0 + n_1 \exp \left(n_2 \frac{r_{in} - r}{r_{out} - r_{in}}\right),$$

$$\beta = n_3 \sin \left(\frac{\pi}{2} \frac{r - r_{in}}{r_{out} - r_{in}}\right).$$

We further specify that the inner radius of the disk $r_{in}$ is always fixed at the ISCO, and the outer radius $r_{out}$ at 50$r_s$. The parameters $n_1$, $n_2$, and $n_3$ are used to describe the magnitude of warping. $\gamma_0$ measures the azimuthal viewing angle of the observer relative to the warped disk. Choosing different values for $\gamma_0$ is equivalent to changing the observational time during the disk precession. Therefore, we can study the time variations of the line profiles and the disk images.

3.2. Kinematic Structure of the Disk

The kinematic structure of the warped disk carries the information about the motion of emitting particles in the vicinity of the Kerr black hole. It mainly contributes to the Doppler shifts (the only contributor to blueshift) in the line profiles. We consider that each plasma element follows a quasi-Keplerian orbit in a concentric ring, which means the radial velocity is much less than the transverse velocity as in the standard thin disks, and the angular velocity of the spatially circular orbit can be written as

$$\Omega = (a + \epsilon \sqrt{r^3/M})^{-1},$$

where $\epsilon = 1$ corresponds to the prograde orbit and $\epsilon = -1$ to the retrograde orbit with respect to the spinning of the black hole. This angular velocity is modified by the relativistic effect of the black hole by the inclusion of an additional spin parameter $a$. Strictly speaking, the circular orbit exists only in the equatorial plane. However, we assume that the tilted angle is small so that Equation (17) is a good approximation for the warped disk. For clarity, in this paper we simply list the 4-velocity of particles below and relegate the detailed derivation to the Appendix:

$$u^0 = \left(\Sigma \Delta^{-1} \sin^2 \theta \Lambda^{-1} \left(\phi - \omega\right)^2 - \Sigma \Omega^2\right)^{-1/2},$$

$$u^\prime = 0,$$

$$u^0 = \dot{\theta} u^\theta = \Omega u^0 \left(-\cos \psi \cos \beta \sin \phi \cos \theta + \sin \psi \cos \phi \cos \theta + \sin \beta \sin \phi \sin \theta\right),$$

$$u^\phi = \dot{\phi} u^\phi = \frac{\Omega u^0}{\sin \theta} (\sin \psi \cos \beta \sin \phi + \cos \psi \cos \phi).$$

4. EMISSION LINE PROFILES AND WARPED DISK IMAGES

4.1. Emission Line Profiles

The emission line profile produced by an accretion disk around a black hole is influenced by the Doppler shift, gravitationnal redshift, beaming effect, and gravitational lensing effect.
The changing of photon energy along a bundle of photon trajectories (geodesic congruence) can be characterized by the ratio $g$ of the energy $E_{\text{obs}}$ measured by a local inertial (rest) observer at asymptotic infinity to the emitted energy $E_{\text{em}}$ measured by an observer comoving with the plasma element on the accretion disk,

$$g = \frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{(u^\mu p_\mu)_{\text{obs}}}{(u^\mu p_\mu)_{\text{em}}} = (1 + z)^{-1}, \quad (22)$$

where $z \equiv (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}}$ is the redshift as usually defined, $u^\mu$ in the denominator and numerator are the 4-velocity of the emitting plasma element and the observer at infinity, respectively, and $p_\mu$ is the four-momentum of the photon which propagates from the plasma element to the observer along a null geodesic. The total observed flux $F_{\text{obs}}$ is determined by integrating the specific flux $dF_{\text{obs}}$ over all the plasma elements on the accreting disk. For the line profile, the specific flux $dF_{\text{obs}}(E_{\text{obs}})$ at observed energy $E_{\text{obs}}$ can be expressed as

$$dF_{\text{obs}}(E_{\text{obs}}) = I_{\text{obs}}(E_{\text{obs}}) d\Omega_{\text{obs}}, \quad (23)$$

where $I_{\text{obs}}$ is the observed specific intensity and $d\Omega_{\text{obs}}$ is the observed solid angle subtended by each plasma element. Using Liouville’s theorem

$$I_{\text{obs}}/v_{\text{obs}}^3 = I_{\text{em}}/v_{\text{em}}^3 \quad (24)$$

we find

$$I_{\text{obs}} = I_{\text{em}} v_{\text{obs}}^3 / v_{\text{em}}^3 = I_{\text{em}} g^3 = I_{\text{em}} (1 + z)^{-3}. \quad (25)$$

And the observed solid angle $d\Omega_{\text{obs}}$ is naturally the size of the pixels in the observer’s screen. The detailed derivation of the factor $g$ for the warped disk is shown in the Appendix. We ignore the intrinsic line width and assume $I_{\text{em}} = \varepsilon \delta (E_{\text{em}} - E_0)$, where $E_0$ is the rest energy of photons, $\varepsilon$ is the surface emissivity, and $q$ is the emissivity index (Fabian et al. 1989).

In the actual computation, we trace every null geodesic from the observer’s screen backward toward the central black hole and search for the interactions of the trajectories with the surface of the disk. The shadowing from the observer due to the warping of the disk is taken into account; we ignore the shadowing from the corona, since in our calculation we assume that the scale height of the corona is large compared to the size of the disk of our interest, therefore this shadowing may not be important. We calculate the observed specific flux $dF_{\text{obs}}$ from each plasma element and add its contribution into a energy bin according to the observed energy of photons ($E_{\text{obs}} = E_{\text{em}} g^3$). Then we can obtain a line profile by plotting count number $dF_{\text{bin}}/E_{\text{bin}}$ versus $E_{\text{bin}}$. The number of bins we choose determines the spectrum resolution. We assume an optically thick disk, thus we consider

![Figure 2. Line profiles from a warped disk with $n_1 = 0$, $n_2 = 1$, $n_3 = 0.95$, $r_{in} = 4.23r_g$, and $r_{out} = 50r_g$. The spin parameter $a$ of the Kerr black hole is 0.5M. The emissivity index $q$ is taken to be $-2$. The horizontal axis is the $g$ factor, i.e., the observed photon energy per unit iron K-shell photon energy; the vertical axis is flux in arbitrary units. In each panel, three colored line profiles represent inclination angles $\theta$ of 10° (red), 50° (blue), and 70° (black) measured from the spin axis of the black hole, respectively. Panels (a)–(h) contain the line profiles seen from different azimuthal angles $\phi$ of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°. Panel (a) shows the $\phi = 0°$ case which is defined as the observer viewing from the direction furthest from the lowest point of the disk. Panel (i) shows the line profile from a standard flat disk for comparison.](https://example.com/figure2.png)
Figure 3. Images of a geometrically thin and optically thick warped disk around a Kerr black hole. The spin parameter $a$ is 0.5 $M$, and the observer’s inclination angle $\theta$ is 50° measured from the spin axis. The warping parameters are the same as previous figures ($n_1 = 0, n_2 = 1, n_3 = 0.95$). The inner and outer radii are $4.23 r_g$ and $50 r_g$. $\alpha$ is the horizontal axis of the observer’s photographic plate and $\beta$ is the vertical axis. The false color contour maps show the ratio of the observed energy to the emitted energy. The blue shaded areas represent the regions of the disk where photons emitted are blueshifted, while the red shaded areas represent the regions where photons are redshifted. The large areas colored with cyan and green represent the regions where photon energies are weakly blueshifted and redshifted. The white areas are the zero-shift regions, where the gravitational redshift is balanced with the Doppler blueshift. Panels (a)–(h) contain the line profiles seen from different azimuthal angles $\phi$ of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°. Panel (a) shows the $\phi = 0°$ case which is defined as the observer viewing from the direction furthest from the lowest point of the disk. Panel (i) shows the disk image from a standard flat disk for comparison.

(A color version of this figure is available in the online journal.)

only the direct photons and neglect the photons which would circle around the black hole and strike the disk.

4.1.1. Changing of the Spin Parameter $a$

The overall properties of the line profiles can be understood by carefully examining the contribution of the specific flux from each of the narrow rings of the disk. They are determined by the spin parameter, the warping parameters, the emissivity index, shadowing, and orientation of the disk relative to a distant observer (both the inclination angle and azimuthal angle).

Figure 2 shows the line profiles from the warped accretion disk with the warping parameters $n_1 = 0, n_2 = 1$, and $n_3 = 0.95$. The inner radius $r_m$ of the prograde disk ($\epsilon = 1$) locates at the ISCO, which is $4.23 r_g$ for the black hole spin parameter $a = 0.5 M$. The outer radius is chosen as $r_{out} = 50 r_g$. From Equation (16) we know that the tilt angles are 0° and 54° for the narrow rings of the disk at the inner radius and the
outer radius, respectively. This warped disk is twist-free which means that the twist angle $\gamma$ does not depend on the radius ($n_1 = 0$). The emissivity index $\gamma$ is taken to be $-2$. The horizontal axis is the $g$ factor, i.e., the observed photon energy per unit iron K-shell photon energy, while the vertical axis gives flux in arbitrary units at different observed photon energies. In each panel, there are three colored line profiles representing the inclination angles $\theta$ of $10^\circ$ (red), $50^\circ$ (blue), and $70^\circ$ (black) measured from the spin axis of the black hole, respectively. Panels (a)–(h) contain the line profiles seen from different azimuthal viewing angles, for the retrograde precession of the warped disk about the spin axis of the black hole from $0^\circ$ to $315^\circ$ with uniform angular intervals of $45^\circ$. Panel (a) shows the $\phi = 0^\circ$ case, which is defined as the observer viewing from the direction furthest from the lowest point of the disk. Panel (i) shows the line profiles from the standard flat disk with the same inner and outer radii for comparison.

The line profile is sensitive to the inclination angle of the observer. For the low inclination angle $\theta = 10^\circ$ (red), the line profile in panels (a)–(h) tends to be a single broad peak with the same endpoints of the red and blue tails, and almost all of the photons are redshifted due to strong gravitational redshift and weak Doppler blueshift. Those line profiles are similar to the flat disk line profile in panel (i). With an increase of the inclination angle (blue and black), the two-peak or multi-peak structures start to emerge. Compared with the flat disk case where the two peaks are shown (the blue peak is always brighter than the red one), the line profiles from the warped disk present considerable changes of their features, for example, the red peak can be comparable or stronger than the blue one in some cases. The deviations are produced by the warping structure and shadowing effect, which can only be seen from the images of the disk in Figure 3 (for $\theta = 50^\circ$).

For the large inclination angles ($\theta = 50^\circ$ and $\theta = 70^\circ$), the shape of the line profiles is also sensitive to the azimuthal angle. This is because the observable regions of the disk change with the precession of the disk over a long period (see images in Figures 3 for $\theta = 50^\circ$). It appears that the line profiles are less sensitive to the low inclination angles ($\theta = 10^\circ$). At a low inclination, the inner region of the disk is almost face-on (for the twist-free warped disk the innermost area coincides with the equatorial plane) and the shadowing effect due to the warping is less important. We note that the black lines in panel (a) ($\phi = 0^\circ$) and panel (e) ($\phi = 180^\circ$) for $\theta = 70^\circ$ are similar, and the relatively narrower tails for $\phi = 0^\circ$ are due to the fact that part of the strongly blueshifted and redshifted area is shadowed.

Figure 4 shows the line profiles from the accretion disk with the same warping parameters $n_1$, $n_2$, and $n_3$ as in Figure 2 but for a more relativistic system, where the black hole has an extreme spin parameter $a = 0.998M$ (Thorne 1974). The inner radius $r_{in}$ for the accretion disk is only $1.23r_g$. The general features mentioned for Figure 2 hold here. In addition, the effect of the gravitational redshift and light bending is more significant. Some photon trajectories which are initially retrograde may be forced to turn back to be prograde by the frame dragging effect.
In this more relativistic system, the red tails are rather long and extend to as far as $g = 0.1$, while the blue tails end at about the same places as in Figure 2, although they are cut off more sharply for largest inclinations ($\theta = 70^\circ$). However, the line profiles around $g = 1$ (the rest energy) are very similar to Figure 2, since the contribution to the flux here is mainly from the rings in the outer region of the disk ($r > 10r_g$, where the frame dragging effect is much smaller ($w \propto r^{-3}$).

Figure 5 shows the line profiles from the accretion disk with the same warping and spin parameters as in Figures 2, except we choose the disk to rotate around the black hole in a retrograde orbit ($\epsilon = -1$). The retrograde orbiting of the disk will increase its inner radius $r_m$ to a larger value $7.55r_g$, which reduces the contribution from the innermost part of the disk so that the red and blue tails become slimmer than in Figure 2. In addition, the patterns are anti-symmetric about $180^\circ$ compared to Figure 2.

4.1.2. Changing of the Emissivity Index $q$

The detailed irradiation law is unknown and the emissivity of the iron line in the inner region of the disk is poorly understood so far. For simplicity, we assume that the emissivity is a power-law function of radius with the emissivity index as the power-law index (i.e., $\epsilon \sim (r/M)^q$).

Figure 6 shows the line profiles from the warped disk with the same warping parameters and the spin parameter as in Figure 2. The difference is that the inclination angle $\theta$ in all cases here is chosen as $70^\circ$ measured from the black hole spin axis. In each panel, the three colored lines represent the line profiles with the emissivity index $q$ being $-2$ (red), $-2.5$ (blue), and $-3$ (black), respectively. Panels (a)–(d) contain the line profiles seen from different azimuthal viewing angles as in Figures 2–5, and panel (i) shows the line profiles from the standard flat disk for comparison. All the line profiles are normalized to unity.

Generally speaking, smaller emissivity index indicates a larger contribution from the inner region of the disk, if we do not consider the shadowing effect which prevents part of the inner region to be seen in some cases. The inner regions contribute mostly to the flux at blue and red tails while the outer region to that around $g = 1$ (the rest energy). Therefore, for smaller emissivity index (black than red), the flux in the red/blue tails is more intense than that for larger emissivity index. However, the shadowing effect is pronounced in some cases. For example, in panel (a) ($\phi = 0^\circ$), part of the most blueshifted region is unobservable due to the shadowing, the blue tails for three different $q$ are almost identical. Moreover, they are less blueshifted ($g \lesssim 1.2$) and are not as steep as others. A similar feature appears in panel (b) ($\phi = 45^\circ$) and panel (h) ($\phi = 315^\circ$) too. In panel (e) ($\phi = 180^\circ$), the disk is fully observed (least shadowing), the main feature of the line profile is similar to the flat disk where there is no shadowing at all. In panels (c)–(g), where the shadowing effect is less important, the blue tails for smaller emissivity index are steeper, and it is a common feature shared by both the warped disk and the flat disk. Since
the nature of the central black hole, the dynamic structure of
the disk, and the orientation to the observer are the same for
the profiles in each panel, the blue and red tails representing
different emissivity index end at the same places.

Figure 7 shows the line profiles from the warped disk with the
same warping parameter and inclination angle as in Figure 6,
but with the spin parameter \( \alpha \) taken to be \(-2\). The horizontal axis is the \( g \) factor, i.e., the observed photon energy per unit iron K-shell photon energy; the vertical axis is flux in
arbitrary units. The inclination angle \( \theta \) is 70° measured from the spin axis of the black hole. In each panel, three line profiles represent emissivity index \( q \) of \(-2\) (red),
\(-2.5\) (blue), and \(-3\) (black), respectively. Panels (a)–(h) contain the line profiles seen from different azimuthal angles \( \phi \) of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and
315°. Panel (a) shows the \( \phi = 0° \) case which is defined as the observer viewing from the direction furthest from the lowest point of the disk. Panel (i) shows the line
profiles from a standard flat disk for comparison.

(A color version of this figure is available in the online journal.)

5. DISCUSSION AND CONCLUSIONS

To date there are two practical methods for measuring the spin
of astrophysical black holes, namely, X-ray continuum fitting
and relativistic iron lines. The continuum-fitting method so far
can only be applied to stellar-mass black holes (McClintock
et al. 2011), whereas relativistic iron lines can be used as a
probe for both the spins of stellar-mass black holes in XRBs
and supermassive black holes in AGNs. Furthermore, if the
accretion disks in these systems are warped, it is possible that
the relativistic iron line profiles may serve as a way to diagnose
the structures of the disks.

In this paper, we have developed a method for calculating
the relativistic iron line profiles from the warped accretion disks
orbiting around rotating black holes. It essentially generalizes
previous calculations by including the black hole spin, the disk
warping, and the shadowing effect at the same time. We have
also presented the disk images which can help us understand the
general features of the line profiles from different disk and black
hole systems. The detailed calculation concerns the computation
of the trajectories of the photons emitted from the disk and
the velocity field of the plasma elements orbiting around the
center black hole. The formalism described above makes no
assumption about the emitting energy of the photon. Therefore,
it can be applied to any emission lines which could be detected in the immediate vicinity of the black hole, such as carbon, nitrogen, and oxygen lines although they are not as prominent as iron lines.

We assumed a set of warping parameters for the geometry of the twist-free warping disk and Keplerian orbit for the motion of the plasma elements as examples. Even for this simplified system we have found a number of interesting phenomena. (1) Different from the two-peak feature generally found in flat disks, the line profiles from the warped disk show multiple peaks. (2) The line profiles for some cases present sharper red tails and/or softer blue tails, which may provide an alternative explanation to observations for some Seyfert 1 galaxies (Nandra et al. 1997) and Seyfert 2 galaxies (Turner et al. 1997). (3) When the disks are orbiting around a highly spinning black hole, a rather long red tail is a common feature shared between flat disks and warped disks, if the shadowing effect is not important. (4) At low inclinations, the line profiles from different azimuthal angles differ very little from each other, indicating that the warping has little influence on the line profile and it may not provide much information for diagnosing the disk structure. (5) The line profile is also sensitive to the illumination law: smaller emissivity index \( q \) generally indicates stronger red and blue tails. In some extreme cases, red bumps may grow in the red tails. (6) It may not be true that a sharp blue tail is a common feature in all axisymmetric disk models, and is expected for time-integrated profiles even if the disk has strong asymmetry or inhomogeneities (Bromley et al. 1998). The shadowing we encounter here may cause a soft blue tail in the integrated profile. (7) With the precession of the warped disk, time variations of line profiles may be present and may be a possible observational signature for the warped disk. (8) Photometric variations of the line fluxes may be due to the varying amount of area of the disk facing the observer.

It is nontrivial to constrain the model parameters by fitting the theoretical line profiles to observations. However, from the above results and discussions we can see that generally the red tails of the line profiles provide the information about the spin of the black hole, while the blue tails the inclination angle. And the warping would cause complicated multiple peaks (which depends on the warping parameters) and time variation of line profiles with the precession of the disk. In the large inclination cases, the warping would cause significant disk shadowing which could prevent all part of the inner area to be seen. Smaller emissivity index \( q \) normally indicates more intensive red and blue tails in the line profiles. Our present work is an initiative of a systematic investigation about the impact of disk warping and shadowing on the line profile. It can be considered as an extension of previous work on line profiles for warped disk (Hartnoll & Blackman 2000; Wu et al. 2008). Our line profiles distinguish from the line profiles from non-rotating black hole systems by presenting longer red tails which could extend to \( g = 0.1 \) for highly spinning cases. And the overall line feature is smooth and has a similarity to the flat disk if the shadowing is not significant. These results are different from Hartnoll & Blackman (2000), in which there are abrupt spike-like structures.

Figure 7. Line profiles as in Figure 6 with \( \theta = 50^\circ \), \( r_{in} = 4.23 r_g \), and \( r_{out} = 50 r_g \). (A color version of this figure is available in the online journal.)
in some cases. We provide an alternative explanation to the variation of line profile which is complementary to the picture of Karas et al. (2001), who investigated the non-axisymmetric patterns in the disk surface. However, the non-axisymmetric patterns are unlikely to produce multiple peak structure. Due to the axisymmetry, both the thick disk (Wu & Wang 2007) and accretion torus (Fuerst & Wu 2007) would not show the time-variant of line profile with precession and the line profiles are usually single-peaked and double-peaked.

Long-period photometric X-ray variation is observed for many accreting systems. The precession of warped disk may be an explanation for this phenomenon. The timescale characterizing the precession induced by tidal force of the companion star in XRBs (Wijers & Pringle 1999) is

\[ t_{\Omega_p} = \frac{2\pi}{\Omega_p} \approx 6.7 \left( \frac{P_{\text{orb}}}{1d} \right)^2 \left( 1 + \frac{q}{M} \right)^{1/2} \times \left( \frac{R}{10^{11}\text{cm}} \right)^{-3/2} \text{d}, \]  

(26)

where \( P_{\text{orb}} \) is the orbital period of the binary and \( q \) is the ratio of the mass of the companion star to the central accreting object. The timescale characterizing the precession induced by radiation can be found in Maloney et al. (1996). The typical precession period for an AGN may be much longer than that of an XRB. The purpose of showing this formula is to point out that, apart from the time-averaged line profile (with typical explosion time longer than \( 10^6 \) s), it is possible to study the time variation of the iron line profiles on the precession timescale with the next generation space X-ray observatories which have sufficiently large collecting area.

With the advance of the millimeter/submillimeter Very Long Baseline Interferometer (VLBI), the proposed Earth-based Event Horizon Telescope (EHT; Doeleman et al. 2009), and the on-going space-VLBI (such as VSOP\(^3\) and RadioAstron\(^4\)) promise to provide microarcsecond imaging resolution, which is sufficient to resolve the event horizons and the inner regions of the accretion disks around a handful of supermassive black holes. The calculated disk images may help understand the possible asymmetric intensity profile, blueshift/redshift, and other observational signatures caused by the warping of the accretion disk and the rotating of the black hole. In our next work we will try to constrain the model parameters by fitting the theoretical profiles of the predicted spectra to actual data.

Although we have given results for only one set of warping parameters, our formalism can be easily extended to more general disks. Work is underway on investigating systems with different warping parameters. By using the method of moments, we can explore the large warping parameter space \((n_1, n_2, n_3)\) and study the distribution of the warping parameters in the moment space. In practice, it may be helpful to compare the positions of the prominent line peaks in the warped disks

---

\(^3\) http://www.vsop.isas.ac.jp/
\(^4\) http://www.asc.rssi.ru/radioastron/
with the corresponding peaks in the flat disks. And it would be easier than fitting the overall line profiles (Murphy et al. 2009; Sochora et al. 2011). Through this approaches we may connect the calculated line profiles with the current and/or future observations. 

We are grateful to an anonymous referee for constructive comments. We also thank Xinlian Luo, Teviet Creighton, and Frederick Jenet for helpful discussions and comments, and Richard Price for directing us to several related publications and reviewing the manuscript. This work was supported by the Natural Science Foundation of China (under grant number 10873008) and the National Basic Research Program of China (973 Program 2009CB824800).

APPENDIX

EXPRESSION OF g FOR THE WARPED DISKS

This section describes how to derive the expression of g, the ratio of observed photon energy to the emitted energy for the warped disk.

A.1. 4-velocity of the Particle

In each concentric ring, we let the angular position of each plasma element be $\varphi = \Omega t$, with $t$ being the coordinate time in the Kerr metric and the plasma particle's velocity components are

$$v' = dx_i / dt = v(-\sin \varphi, \cos \varphi, 0), \quad (A1)$$

where $v = r\Omega = r(a + \epsilon \sqrt{r^3/M})^{-1}$, $\epsilon = 1$ is for corotating/prograde orbiting, and $\epsilon = -1$ is for counter-rotating/retrograde orbiting. In addition, we need to transfer the velocity into the coordinate frame in which the $z$ axis is aligned with the black hole spin. In Figure 1, the inclined $x'y'$ plane is the orbital plane, and the $y'$ axis coincides with the line of nodes and points at the ascending node. The $xy$ plane is the equatorial plane which has a tile angle $\gamma$ relative to the $x'$ axis. The projection of the $z'$ axis on the $xy$ plane has a twist angle $\gamma$ relative to the $x$ axis. We find the components $v_i$ in the $xyz$ coordinate basis using $v_i = T_{ij}v'_j$, where

$$T_{ij} = \begin{pmatrix} \cos \gamma \cos \beta & -\sin \gamma & -\cos \gamma \sin \beta \\ \sin \gamma \cos \beta & \cos \gamma & -\sin \lambda \sin \beta \\ \sin \beta & 0 & \cos \beta \end{pmatrix}, \quad (A2)$$

from which we get

$$v_x = v(-\cos \gamma \cos \beta \sin \varphi - \sin \gamma \cos \varphi)$$
$$v_y = v(-\sin \gamma \cos \beta \sin \varphi + \cos \gamma \cos \varphi)$$
$$v_z = v(-\sin \beta \sin \varphi). \quad (A3)$$

We then calculate the velocity components in spherical coordinates by projecting the velocity $v$ onto the spherical basis $e_\varphi, e_\theta, e_\varphi$:

$$v_\varphi = 0$$
$$v_\theta = v(-\cos \psi \cos \varphi \cos \beta \sin \varphi + \sin \psi \cos \varphi \cos \psi + \sin \beta \sin \varphi \sin \varphi)$$
$$v_\varphi = v(\sin \psi \cos \beta \sin \varphi + \cos \psi \cos \varphi), \quad (A4)$$

where $\psi = \phi - \gamma$ is the azimuthal angle of the plasma element measured from the projection of the $x'$ axis on the $xy$ plane.

In the Kerr spacetime, if we set $\tau$ as the proper time of the plasma element, it satisfies

$$-d\tau^2 = ds^2 = -\Sigma A^{-1} \frac{dt^2}{dt} + \sin^2 \theta \Sigma A^{-1} (d\phi - \omega dt)^2 + \Sigma A^{-1} dr^2 + \sigma d\theta^2, \quad (A5)$$

and we divide both sides by $dt^2$

$$-\frac{d\tau^2}{dt^2} = -\Sigma A^{-1} + \sin^2 \theta \Sigma A^{-1} (\phi - \omega)^2 + \Sigma \delta^2. \quad (A6)$$

Therefore, we can write the 4-velocity of the plasma element as follows:

$$u^0 = \frac{dt}{d\tau} = (\Sigma A^{-1} - \sin^2 \theta A^{-1}(\phi - \omega)^2 - \Sigma \delta^2)^{-1/2},$$
$$u^r = \frac{dr}{d\tau} = \frac{dr}{dt} \frac{dt}{d\tau} = \dot{r}u^0 = 0,$$
$$u^\theta = \frac{d\theta}{d\tau} = \frac{d\theta}{dt} \frac{dt}{d\tau} = \dot{\theta}u^0 = \frac{u^0 v_\theta}{r},$$
$$u^\varphi = \frac{d\varphi}{d\tau} = \frac{d\varphi}{dt} \frac{dt}{d\tau} = \dot{\varphi}u^0 = \frac{u^0 v_\varphi}{r \sin \theta}, \quad (A7)$$

which are identical to Equation (18) in Section 3.2 if we insert Equation (A4) into Equation (A7). We note that the radial component of velocity is zero because we have assumed that the particle follows a Keplerian orbit. And the term under the square root of Equation (A7) is always positive if $r > \text{ISCO}$.

A.2. Expression for $g$

The 4-velocity of the particle is expressed by the Boyer–Lindquist coordinates in Equation (A7). We then derive the expression for $g$ as

$$g = \frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{(u^\mu p_\mu)_{\text{obs}}}{(u^\mu p_\mu)_{\text{em}}} = (1 + z)^{-1}. \quad (A8)$$

The observer at rest at infinity has 4-velocity $p_\mu = (-1, 0, 0, 0)$, thus in the numerator $(u^\mu p_\mu)_{\text{obs}} = E$; while in the denominator

$$(u^\mu p_\mu)_{\text{em}} = -Eu^0 \pm Eu^0 \sqrt{\Sigma v_\theta / r} + Lz u^0 v_\varphi / \sin \theta, \quad (A9)$$

therefore we get

$$g = \frac{1}{u^0(-1 \pm \sqrt{\Sigma v_\theta / r + \xi v_\varphi / \sin \theta})}, \quad (A10)$$

where the sign is determined by the zenithal emitting direction.

REFERENCES

Abramowicz, M. A., Jaroszyński, M., Kato, S., et al. 2010, A&A, 521, A15
Bardeen, J. M., & Petterson, J. A. 1975, ApJ, 195, L65
Bardeen, W. R., & Reynolds, C. S. 2009, ApJ, 696, 1616
Bromley, B. C., Chen, K., & Miller, W. A. 1997, ApJ, 475, 57
Bromley, B. C., Miller, W. A., & Pariev, V. I. 1998, Nature, 391, 54
Carter, B. 1968, Phys. Rev., 174, 1559
Chandrasekhar, S. (ed.) 1983, The Mathematical Theory of Black Holes (Oxford: Oxford Univ. Press)
Dennissi, M., & Ivanov, P. B. 1997, A&A, 324, 829
Dexter, J., & Agol, E. 2009, ApJ, 696, 1616
Dexter, J., & Fragile, P. C. 2011, ApJ, 730, 36
Doeleman, S., Agol, E., Backer, D., et al. 2009, astro2010: The Astronomy and Astrophysics Decadal Survey, 68
Fabian, A. C., Rees, M. J., Stella, L., & White, N. E. 1989, MNRAS, 238, 729
Fanton, C., Calvani, M., de Felice, F., & Cadez, A. 1997, PASJ, 49, 159
Fragile, P. C., Miller, W. A., & Vandermost, E. 2005, ApJ, 635, 157
Frank, J., King, A., & Raine, D. J. 2002, Accretion Power in Astrophysics
(3rd ed.; Cambridge: Cambridge Univ. Press)
Fuerst, S. V., & Wu, K. 2007, A&A, 474, 55
Hartnoll, S. A., & Blackman, E. G. 2000, MNRAS, 317, 880
Hartnoll, S. A., & Blackman, E. G. 2002, MNRAS, 332, L1
Ivanov, P. B., & Illarionov, A. F. 1997, MNRAS, 285, 394
Karas, V., Martocchia, A., & Subr, L. 2001, PASJ, 53, 189
Katz, J. I. 1973, Nature, 246, 87
Kinney, A. L., Schmitt, H. R., Clarke, C. J., et al. 2000, ApJ, 537, 152
Laor, A. 1991, ApJ, 376, 90
Larwood, J. D., Nelson, R. P., Papaloizou, J. C. B., & Terquem, C. 1996,
MNRAS, 282, 597
Lubow, S. H., Ogilvie, G. I., & Pringle, J. E. 2002, MNRAS, 337, 706
Maloney, P. R., Begelman, M. C., & Nowak, M. A. 1998, ApJ, 504, 77
Maloney, P. R., Begelman, M. C., & Pringle, J. E. 1996, ApJ, 472, 582
Margon, B. 1984, ARA&A, 22, 507
Martin, R. G. 2008, MNRAS, 387, 830
Matt, G., Perola, G. C., & Piro, L. 1991, A&A, 247, 25
McClintock, J. E., Narayan, R., Davis, S. W., et al. 2011, Class. Quantum Grav.,
28, 114009
Miller, J. M. 2007, ARA&A, 45, 441
Miller, J. M., Fabian, A. C., Wijnands, R., et al. 2002a, ApJ, 578, 348
Miller, J. M., Fabian, A. C., Wijnands, R., et al. 2002b, ApJ, 570, L69
Miniutti, G., Fabian, A. C., Anubaki, N., et al. 2007, PASJ, 59, 315
Miniutti, G., Fabian, A. C., & Miller, J. M. 2004, MNRAS, 351, 466
Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation (San Francisco,
CA: Freeman)
Miyoshi, M., Moran, J., Herrnstein, J., et al. 1995, Nature, 373, 127
Murphy, K. D., Yaqoob, T., Karas, V., & Dovčiak, M. 2009, ApJ, 701, 635
Nandra, K., George, I. M., Mushotzky, R. F., Turner, T. J., & Yaqoob, T.
1997, ApJ, 477, 602
Novikov, I. D., & Thorne, K. S. 1973, in Black Holes (Les Astres Occlus), ed.
C. Dewitt & B. S. Dewitt (Paris: Gordon and Breach), 343
Page, D. N., & Thorne, K. S. 1974, ApJ, 191, 499
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (ed.)
1992, Numerical recipes in FORTRAN. The Art of Scientific Computing
(Cambridge: Cambridge Univ. Press)
Pringle, J. E. 1996, MNRAS, 281, 357
Rauch, K. P., & Blandford, R. D. 1994, ApJ, 421, 46
Reynolds, C. S., & Nowak, M. A. 2003, Phys. Rep., 377, 389
Rossi, S., Homan, J., Miller, J. M., & Belloni, T. 2005, MNRAS, 360, 763
Schmitt, H. R., Pringle, J. E., Clarke, C. J., & Kinney, A. L. 2002, ApJ, 575,
150
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Sochora, V., Karas, V., Svoboda, J., & Dovčiak, M. 2011, MNRAS, in press
(arXiv:1108.0545)
Tanaka, Y., Nandra, K., Fabian, A. C., et al. 1995, Nature, 375, 659
Tananbaum, H., Gursky, H., Kellogg, E. M., et al. 1972, ApJ, 174, L143
Terquem, C., & Bertout, C. 1993, A&A, 274, 291
Thorne, K. S. 1974, ApJ, 191, 507
Turner, T. J., George, I. M., Nandra, K., & Mushotzky, R. F. 1997, ApJ, 488,
164
Wijers, R. A. M. J., & Pringle, J. E. 1999, MNRAS, 308, 207
Wilms, J., Reynolds, C. S., Begelman, M. C., et al. 2001, MNRAS, 328, L27
Wu, S., Wang, T., & Dong, X. 2008, MNRAS, 389, 213
Wu, S.-M., & Wang, T.-G. 2007, MNRAS, 378, 841
Young, A. J., Ross, R. R., & Fabian, A. C. 1998, MNRAS, 300, L11