Learning Efficient Multi-agent Communication: An Information Bottleneck Approach

Rundong Wang, Xu He, Runsheng Yu, Wei Qiu, Bo An, Zinovi Rabinovich
Nanyang Technological University
{rundong001, xuhe0002, qiuw0008}@e.ntu.edu.sg, {runsheng.yu, boan, zinovi}@ntu.edu.sg

Abstract
Many real-world multi-agent reinforcement learning applications require agents to communicate, assisted by a communication protocol. These applications face a common and critical issue of communication’s limited bandwidth that constrains agents’ ability to cooperate successfully. In this paper, rather than proposing a fixed communication protocol, we develop an Informative Multi-Agent Communication (IMAC) method to learn efficient communication protocols. Our contributions are threefold. First, we notice a fact that a limited bandwidth translates into a constraint on the communicated message entropy, thus paving the way of controlling the bandwidth. Second, we introduce a customized batch-norm layer, which controls the messages’ entropy to simulate the limited bandwidth constraint. Third, we apply the information bottleneck method to discover the optimal communication protocol, which can satisfy a bandwidth constraint via training with the prior distribution in the method. To demonstrate the efficacy of our method, we conduct extensive experiments in various cooperative and competitive multi-agent tasks across two dimensions: the number of agents and different bandwidths. We show that IMAC converges fast, and leads to efficient communication among agents under the limited-bandwidth constraint as compared to many baseline methods.

Introduction
Multi-agent reinforcement learning (MARL) has long been a go-to tool in complex robotic and strategic domains (RoboCup 2019; OpenAI 2019). A key difficulty, faced by a group of learning agents in such domains, is the need to efficiently and timely exploit the available communication resources. Communicated information enables action and belief correlation that benefits the group’s activity. While initially only pre-constructed communication protocols (Melo, Spaan, and Witwicki 2011; Maravall, de Lope, and Dominguez 2013; Zhang and Lesser 2013) were used, the advent of deep learning allows to make the communication aspects a part of the learning process (Foerster et al. 2016; Sukhbaatar, Fergus, and others 2016; Jiang and Lu 2018; Singh, Jain, and Sukhbaatar 2018).

Nonetheless, the limited bandwidth constraint remains a common issue for multi-agent communication to apply MARL algorithms to the real-world multi-agent systems, where agents need to carefully choose what, when and with whom to communicate. In fact, in existing works (Zhang and Lesser 2013; Mao et al. 2019; Kim et al. 2019), the limited bandwidth constraint in MARL is not even well-defined, and is simply represented either by the topology of the agents’ relationship (Zhang and Lesser 2013; Mao et al. 2019) or by the bits used in the messages (Kim et al. 2019). This simplicity would be of a definite detriment in dynamic or limited-bandwidth scenarios such as autonomous driving (Shalev-Shwartz, Shammah, and Shashua 2016), search and rescue (Nagatani et al. 2013; Yuan, Liu, and Zhang 2017), and space/deep sea exploration (Gao and Chien 2017; Cressey 2015). In all of the above, the contextual content and the impact of the transmitted information matter.

To address this quality of information requirement, inspired by the variational information bottleneck method (Tishby, Pereira, and Bialek 2000; Alemi et al. 2016), we propose a general regularization method for learning informative communication protocols under the limited-bandwidth constraint, named Informative Multi-Agent Communication (IMAC). First, due to vagueness of the limited-bandwidth constraint definition in existing works, we clarify it by showing that limited bandwidth translates into a constraint on the communicated message entropy. In more detail, derived from source coding theorem (Shannon 1948) and Nyquist criterion (Freeman 2004), we state that in a noiseless channel, when a $K$-ary communication system with bandwidth $B$ transmits $n$ symbols/messages per second, the entropy of the messages $H(M)$ is limited by the bandwidth according to $H(M) \leq \frac{2B \log K}{n}$. Thus, agents should generate low-entropy messages to satisfy the limited-bandwidth constraint.

Additionally, due to the variety of the real-life communication source coding methods and communication protocols, bitstream can carry different amount of information in different situations. Hence we utilize the entropy as a general measurement and clip the messages’ variance to simulate the limited-bandwidth constraint.

IMAC utilizes the information bottleneck method to control the entropy of the messages. Specifically, IMAC applies the variational information bottleneck to the communication channel by viewing the messages as latent variables and ap-
approximating its posterior distribution. By regularizing the mutual information between the channel’s inputs (the internal features extracted from agents) and the channel’s outputs (the messages), we constrain the content of messages to learn informative communication protocols, which convey low-entropy and useful messages.

We conduct extensive experiments in chosen environments: cooperative navigation and predator-prey. Results show that IMAC can convey low-entropy messages, enable effective communication among agents under the limited-bandwidth constraint, and lead to faster convergence as compared with MADDPG, MADDPG-M and AMP.

Related Work
There are two lines of related works: communication in deep multi-agent reinforcement learning and the information bottleneck method in reinforcement learning.

Methods for solving large scale communication or limited-resource communication are also explored. For instance, learning the communication protocols by utilizing specific networks as communication channels or scheduling multi-agent communication via various mechanisms. However, these methods are susceptible to failure under the limited-bandwidth constraint. The former such as DIAL (Foerster et al. 2016), CommNet (Sukhbaatar, Fergus, and others 2016), BiCNet (Peng et al. 2017), AMP (Peng, Zhang, and Luo 2018), in particular, fails to “extract valuable information for cooperation” as the analysis in (Jiang and Lu 2018) shows. As the number of agents and the size of a single message grow, communication participants are overwhelmed by the message flow. The latter, scheduling mechanism methods like ATOC (Jiang and Lu 2018), IC3Net (Singh, Jain, and Sukhbaatar 2018), SchedNet (Kim et al. 2019), MADDPG-M (Kilinc and Montanta 2018), and GACML (Mao et al. 2019) cannot constrain the message content either. Rather, these methods limit the number of agents who can communicate to comply with the limited-bandwidth constraint. Concretely, ATOC proposes attention gating units, while IC3Net extends the work of CommNet by using a gating mechanism with MLP structure. However, these two methods are inflexible to real-world multi-agent systems because they are only for homogeneous agents. MADDPG-M generates a score for each pair of agents, allows the two agents in every pair to communicate. However, MADDPG-M constrains the number of agents with whom each agent can communicate, thus restrains potential cooperation. SchedNet allows a group of agents to send messages at every step, according to top-k algorithm or sampling. GACML introduces a dynamic threshold with bandwidth and their controlled parameters, like k in SchedNet and the threshold in GACML. These parameters need to be handcrafted or learned in different environments to satisfy specific bandwidth conditions.

The combination between the information bottleneck method and reinforcement learning has brought a few applications in the last few years, especially in imitation learning (Peng et al. 2018), inverse reinforcement learning (Peng et al. 2018) and exploration (Goyal et al. 2019). Among them, Goyal et al. (2019) mention the multi-agent communication in their appendix, showing a method to minimize the communication by penalizing the effect of one agent’s messages on another one’s policy. However, it does not consider the limited-bandwidth constraint.

Multi-agent Communication with Limited Bandwidth
In this section, we introduce the multi-agent communicative MDP, clarify the limited bandwidth constraint as well as prove that limited bandwidth restricts message entropy, and finally discuss its implementation.

Multi-agent Communicative MDP
Multi-agent reinforcement learning can be formalized in the framework of DEC-POMDP. An n-agent multi-agent MDP is an n-agent DEC-POMDP, which is described by a tuple \( \langle n, S, A, r, P, O, \Omega, \gamma \rangle \), where \( n \) represents the number of agents, \( S \) represents the set of all agents’ states, \( A = \{ A_i \}_{i=1, \ldots, n} \) denotes the sets of actions available to the agents, \( O = \{ O_i \}_{i=1, \ldots, n} \) denotes the sets of observation for each agent, \( P : S \times A \rightarrow S \) denotes the state transition probability function. All agents share the same reward as a function of the states and agents’ actions \( r : S \times A \rightarrow \mathbb{R} \) with a new component \( \gamma \) that can generate messages to help with finding a policy to maximize the expected return.

In our work, we mainly consider the following scenario: \( n \) agents interact with each other by sending messages and performing actions in a shared environment \( S \). We assume that each agent has a bounded observation \( O \), and the observation and action of each agent can affect each other. The reward function \( r \) measures the expected return of state-action pairs, and action-value function \( Q^{\pi}(s, a) = \mathbb{E}_{\pi}[R_t|S=s, A=a] \) assesses the expected return of a state-action pair \( (s, a) \). So the objective of agent \( i \) can be written as:

\[
J_i = \mathbb{E}_{s \sim p^\pi_i, a \sim \pi_i} [R_t] = \mathbb{E}_{s \sim p^\pi_i} [V^\pi_i(s)]
\]

We extend multi-agent MDP to a communicative one. Compared with multi-agent MDP, multi-agent communicative MDP introduces a new component, i.e., messages. Unlike actions that directly change the world state, messages only affect the receiving agents’ individual strategy. Consequently, we define communicative multi-agent MDP as a tuple, \( \langle n, S, A, r, P, O, \Omega, M, \gamma \rangle \), with a new component \( M = \{ M_i \}_{i=1, \ldots, n} \) which represents the sets of messages received by agents. The objective is slightly different from MARL: not only find an optimal policy but also learn optimal communication protocols \( \tau_{\text{protocol}}(m_i|o) : O \rightarrow M_i \) that can generate messages to help with finding a policy to maximize the expected return.
Limited Bandwidth

The process of communication (Figure 1) consists of three stages, coding, transmission, and decoding. In the coding phase, communication system maps the messages $M_i$ of agent $i$ to a bitstream $C_i$. Transmission means that the bitstream $C_i$ is transmitted through a channel, and becomes another bitstream $D_i$ due to some distortion in the channel. Then, decoding is the inverse operation of coding. We focus on the first two stages by assuming no loss of information in the decoding stage.

**Remark 1** (Source Coding Theorem (Shannon 1948)). Assume a set of $n$ symbols is to be transmitted through the communication channel. These symbols can be treated as $n$ independent samples of a random variable $X$ with entropy $H(X)$. Let $L$ be the average number of bits to encode the $n$ symbols. The minimum $L$ satisfies $H(X) \leq L < H(X) + \frac{1}{n}$.

**Remark 2** (The Maximum Rate Data Rate (Freeman 2004)). The maximum data rate $R_{\text{max}}$ (bits per second) over a noiseless channel satisfies: $R_{\text{max}} = 2B \log_2 K$, where $B$ is the bandwidth (Hz) and $K$ is the number of signal levels.

Remark 1 shows that when we map $n$ samples of a random variable to binary bits, the code rate $R_{\text{code}}$ (bits per symbol) is larger than $H(X)$, i.e., $R_{\text{code}} \geq H(X)$ (Proof can be seen in the supplementary materials). Remark 2 is derived from the Nyquist criterion (Freeman 2004) and specifies that a communication system can only transmit data at a rate $R_{\text{trans}} \leq R_{\text{max}}$ for reliable transmission in the noiseless and limited-bandwidth condition (Proof can be seen in the supplementary materials). Based on these two remarks, we show how the limited-bandwidth constraint affects the multi-agent communication.

**Proposition 1.** In a noiseless channel, the bandwidth of channel limits the entropy of the messages.

*Proof of Proposition 1.* Given the message $M$ as an i.i.d continuous random variable with differential entropy $H(M)$ and entropy $H(M) + \log \Delta$ with quantization, its time series $M_1, \ldots, M_n$, · · · , the communication system’s bandwidth $B$, as well as the signal level $K$, the communication system transmits $n$ symbols per second. So the transmission rate $R_{\text{trans}} = n \frac{\text{bit}}{\text{second}} \cdot R_{\text{code}}(\frac{\text{bit}}{\text{symbol}}) \geq n(\frac{\text{symbol}}{\text{second}}) \cdot H(M) + \log \Delta(\frac{\text{bit}}{\text{symbol}})$. According to Remark 2, $R_{\text{trans}} \leq R_{\text{max}} = 2B \log_2 K$. Consequently, we derive $H(M) \leq \frac{2B \log_2 K}{n} - \log \Delta$.

Proposition 1 means that if $M$ satisfies $H(M) \leq \frac{2B \log_2 K}{n}$, when sending a series of messages $M_1, \ldots, M_n, \ldots$, we can achieve reliable transmission under the limited bandwidth. A limited-bandwidth constraint equals to enforcing an upper bound $H_c$ to the message entropy $H(M) \leq H_c$.

**Implementation of the Limited-bandwidth Constraint**

We focus on the measurement of $H(M)$ as well as how to implement the limited-bandwidth constraint. Given the messages $M$ as an i.i.d variable with a certain distribution, we find a quantity to measure the information of the messages because the exact distribution of $M$ is unknown to us.

**Proposition 2.** When we have a historical record of the messages to estimate the messages’ mean $\mu$ and variance $\sigma$, the information of the messages can be measured and represented by $\frac{1}{2} \log(2\pi e \sigma^2)$.

*Proof of Proposition 2.* The message $M$ follows a certain distribution, and we are only certain about its mean and variance. According to the principle of maximum entropy (Jaynes 1957), the Gaussian distribution has maximum entropy relative to all probability distributions covering the entire real line $\langle -\infty, \infty \rangle$ but having a finite mean and finite variance (proof see (Cover and Thomas 2012)). We notice that if a random variable $X \sim N(\mu, \sigma)$, then its entropy is $H(X) = \frac{1}{2} \log(2\pi e \sigma^2)$. In short, with the message’s mean $\mu$ and variance $\sigma^2$, $H(M) \leq H(X) = \frac{1}{2} \log(2\pi e \sigma^2)$, where $X \sim N(\mu, \sigma)$.

We conclude that $\frac{1}{2} \log(2\pi e \sigma^2)$ offers an upper bound to approximate $H(M)$. So Gaussian distribution can be viewed as a good approximation to the messages’ distribution.

Due to the variety of the real-life communication source coding methods, like Huffman coding, and communication protocols, like TCP/UDP, bitstream can carry different amount of information in different situations. As a result, we utilize entropy as a general measurement and clip the messages’ variance to simulate the limited-bandwidth constraint. More specifically, we use a batch-normalization-like layer which records the messages’ mean and variance during training, and it normalizes messages during inferring.

The purpose of our normalization layer is to simulate the external limited-bandwidth constraint only in inference (which is implemented by a clip on the variance of messages). It is customized and different from standard batch normalization (Ioffe and Szegedy 2015). Specifically, our normalization layer records the mean and variance of the messages in training, shifts and scales with predefined parameters in inference. The messages have more powerful representation when having a mean and variance according to VAE (Kingma and Welling 2013). The mean of the messages is trained to approach zero, while the variance is important because under Gaussian distribution assumption, the variance reflects the messages’ entropy, which is one of the regularization objective.

For example, the maximum bandwidth of a 4-ary communication system is 100 bit/s, if we want to achieve reliable transmission at the rate of $10^3$ messages per second. Then we can determine the equivalent variance $\sigma^2 \approx 3.2$ according to $\frac{1}{2} \log(2\pi e \sigma^2) = \frac{2B \log_2 K}{n}$. During the training stage, we record the agent’s message variance, which is 5. However, at the inference stage, the bandwidth requires the message entropy not to exceed 3.2. We, therefore, decrease the variance from 5 to 3.2 by using the specific normalization layer.

**Informative Multi-agent Communication**

As stated in the previous section, agents should generate low-entropy messages to satisfy the limited bandwidth constraint.
The main idea behind our proposed method is to compress the messages of agents as much as possible to satisfy the limited-bandwidth constraint, and meanwhile to maintain agents’ performance. There is a trade-off between the degree of compression and the performance: when agents get all uncompressed messages from others’ information, they can reach a good performance with taking others into consideration; when the messages suffer a lossy compression, agents will perform poorly in cooperative tasks due to the incompleteness of shared information. We can alleviate the effect of incompleteness of shared information by using the information bottleneck method, which encourages the model to focus only on the most informative component in the shared information.

The Model of Informative Multi-agent Communication

Before introducing our model, we clarify there are two schemes in multi-agent communication: intra-step communication and inter-step communication. Illustration figures can be found in the supplementary material. In the intra-step communication, agents generate, send and receive messages in every single step, while in the inter-step communication, agents send messages along with agents’ actions as next-step agents’ input. The view of intra-step communication is widely discussed in most previous methods such as the CommNet, ATOC, AMP, SchedNet, GACML, etc. Hence we follow them and focus on the intra-step communication.

Our proposed model is shown in Figure 2. We divide each agent’s decision epoch into three phases: pre-communication, communication, and post-communication phases. Formally, consider n agents, we model these three phases for each agent: the pre-communication functions as $f_1^i, f_2^i, \cdots, f_m^i$, the channel functions as $f_1^c, f_2^c, \cdots, f_m^c$, and the post-communication functions as $f_1^p, f_2^p, \cdots, f_m^p$. The pre-communication function $f_1^i(o_i) = h_i$ takes as input the observation $o_i$ of agent i and outputs the internal features $h_i$ of agent i. The channel function $f_m^c(h_i, h_2, \cdots, h_n) = m_i$ takes as input the internal features of all agents and outputs the messages $m_i$ for agent i. The post-communication function $f_m^p(h_i, m_i) = a_i$ takes as input the messages of agent i and outputs the action of agent i.

The channel function plays a core role in the multi-agent communication. However, the channel function is different from the channel in the communication model (Figure 1). The former one actually acts like a coordinator to aggregate all agents’ internal features and to extract the messages $m_i$ for each agent $i$. In the real-world applications, the channel function can be deployed in an agent who has stronger capability of computation.

Variational Information Bottleneck for Learning Protocols

We propose our variational information bottleneck solution in multi-agent communication for learning informative protocols in bandwidth-limited scenarios. Consider a scenario with n agents with policies parameterized by $\theta = \{\theta_i\}_{i=1,\cdots,n}$, and let $\pi = \{\pi_i\}_{i=1,\cdots,n}$ be the set of all agents’ policies. The optimal policy $\pi_i$ of agent $i$ can be determined according to $\max_{\pi_i} E_{x_a \sim \pi_i} [r_i]\{\pi_i\}$. We utilize the centralized training such as MADDPG (Lowe et al. 2017). The objective of agent $i$ is:

$$J(\theta_i) = E_{a \sim \pi_i} \left[ Q^\pi_i (x, a_1, \cdots, a_n) \right]$$

where $p^\pi$ is the state distribution of the overall policy $\pi$ and $Q^\pi_i (x, a_1, \cdots, a_n)$ is a centralized action-value function that takes as input the actions of all agents, in addition to the observations of all agents, $x = (a_1, \cdots, a_n)$, and outputs the Q-value for agent $i$.

The information bottleneck method is used to encourage the model to focus only on the most informative features (Alemi et al. 2016). Base on the same principle, we incorporate a variational information bottleneck by viewing the channel function as an encoder that maps the internal features $h = (h_1, h_2, \cdots, h_n)$ to a stochastic messages $m_i \sim p(m_i|h)$. From the perspective of agent $i$, the bottleneck can be incorporated by enforcing an upper bound $I_c$ on the mutual information between the messages and the internal features $I(h, m_i)$. In the information bottleneck framework, $I(h, m_i)$ can be viewed as a penalty term that restricts the complexity of channel function. Then, the objective with the regularization of information bottleneck can be written as:

$$J(\theta_i) = E_{a \sim \pi_i} \left[ Q^\pi_i (x, a_1, \cdots, a_n) \right]$$

s.t. $I(h, m_i) \leq I_c$.

Practically, we propose to maximize the following objective using the information bottleneck Lagrangian:

$$J'(\theta_i) = E_{a \sim \pi_i} \left[ Q^\pi_i (x, a_1, \cdots, a_n) \right] - \beta I(h, m_i)$$

where the $\beta$ is the Lagrange multiplier. The mutual information is defined according to:

$$I(h, m_i) = \int p(h, m) \log \frac{p(h, m)}{p(h)p(m)} dh dm$$

$$= \int p(h)p(m|h) \log \frac{p(m|h)}{p(m)} dh dm$$

where $p(m)$ is the probability of the message $m_i$, $p(h)$ is the probability of the internal features of all agents $h$, $p(h, m)$ is the joint probability of $h$ and $m_i$, and $p(m|h)$ is the conditional probability of $m_i$ given $h$. However, computing the
marginal distribution \( p(m) = \int p(m|h)p(h)dh \) can be challenging since we do not know the prior distribution of hidden states \( p(h) \) and condition probability \( p(m|h) \). So, we turn to a variational lower bound. We view channel function as multiple variational encoders \( m_i = f^h_i(h) \), where \( m_i \) becomes a random variable with learned means and variances. So, \( f^h_i \) actually represents the probability \( p(m|h) \) that maps the features \( h \) to a latent message distribution over \( m \), and then use an approximation \( z(m) \) of the prior distribution of messages.

Since \( D_{KL}[p(m)||z(m)] \geq 0, \int p(m) \log p(m)dm \geq \int p(m) \log z(m)dm, \) an upper bound on the mutual information \( I(h, m_i) \) can be obtained via the KL divergence:

\[
I(h, m_i) \leq \int p(h)p(m|h) \log \frac{p(m|h)}{z(m)} dh \cdot dm
\]

This provides a lower bound \( J(\theta) \) on the regularized objective that we maximize:

\[
J'(\theta_i) \geq J(\theta_i) = \mathbb{E}_{a \sim \pi, h \sim p(h)} \left[ Q^\pi_i(x, a_1, \cdots, a_n) \right]
\]

\[
-\beta \mathbb{E}_{h \sim p(h)} [D_{KL}[p(m|h)||z(m)]]
\]

Consequently the objective’s derivative is:

\[
\nabla_{\theta_i} J(\pi_i) = \mathbb{E}_{a \sim \pi, x \sim p(x), h \sim p(h)} \left[ \nabla_{\theta_i} \log (\pi_i(a_i|x)) \right]
\]

\[
Q^\pi_i(x, a_1, \cdots, a_n) - \beta \nabla_{\theta_i} D_{KL}[p(m|h)||z(m)]
\]

Note that, with the regulation of \( D_{KL}[p(m|h)||z(m)] \),

\[
p(m) = \sum_h p(m|h)p(h) \approx \sum_h z(m)p(h) = z(m) \sum_h p(h) = z(m).
\]

We can easily control the messages to satisfy different bandwidth conditions with different distribution of \( z(m) \) in the training stage. That is, our method provides a direct relationship between the bandwidth and the parameters. Our approach is summarized in Algorithm 1.

**Algorithm 1: Informative Multi-agent Communication**

Initialize the network parameters \( \theta_m, \theta_e, \theta_a, \) and \( \theta_Q \)

Initialize the target network parameters \( \theta'_m, \theta'_e, \theta'_a, \) and \( \theta'_Q \)

for episode \( \leftarrow 1 \) to num_episodes do

Reset the environment for \( t \leftarrow 1 \) to num_step do

Get internal features \( h_i = f^h_i(o_i) \) for each agent \( i \)

Each agent \( i \) gets messages from channel \( m_i = f^f_i(h_1, h_2, \cdots, h_n) \)

Each agent \( i \) selects action based on features and messages \( a_i = f^a_i(h_i, m_i) \)

Execute actions \( a = (a_1, \cdots, a_n) \), and observe reward \( r \) new observation \( o_i \) for each agent \( i \)

Store \( (o_i, a, r, o_{i+1}) \) in replay buffer \( D \)

if episode\%update threshold == 0 then

Sample a random mini-batch of \( S \) samples \( (o, a, r, o') \) from \( D \)

Obtain the internal features \( h'_i = f^h_i(o'_i) \)

and the messages \( m'_i = f^m_i(h'_1, \cdots, h'_n) \) for each agent \( i \)

Set \( y_j = r_j + \gamma Q^\pi_j(x, a_1, \cdots, a_n') |_{a_i = f^a_i(h_i', m'_i)} \)

Update Critic by minimizing the loss

\[
L(\theta) = \frac{1}{2} \sum_j (Q(x, a_1, \cdots, a_n) - y_j)^2
\]

Update Policy using the sampled policy gradients \( \nabla_{\theta_i} J(\pi_i) \) for each agent \( i \)

Update all target networks’ parameters for each agent \( i: \theta'_i = \tau \theta_i + (1 - \tau) \theta'_i \)

end

end

end

As for training details, we set the number of agents as 3,5,10 respectively. We use MLP with hidden layer size of 64 as basic module as before communication model, after communication model. We use the ADAM as optimizer with learning rate of 0.01. Since the environment of cooperative navigation does not send “terminal/done” to agents, we set each episode with a maximal steps of 25. The reward of each agent is identical, which equals to the sum of distances between agents to their nearest landmark. It means that agents are required not only to approach its nearest landmark, but also share information with other for a common goal. We use the same hyper-parameter as MADDPG of openAI’s version.

**Cooperative Navigation**

In this scenario, \( n \) agents cooperatively reach \( k \) landmarks with avoiding collisions. Agents observe the relative positions of other agents and landmarks, and are rewarded with a shared credit based on the sum of distances between agents to their nearest landmark, while it is penalized when colliding with other agents. Agents learn to infer and occupy the landmarks without colliding with other agents based on their
As the number of agents increases (from left to right), IMAC can still improve agents’ performance and converge faster.

Figure 3: Learning curves comparing IMAC to other methods for cooperative navigation. Here the bandwidth is not constrained. We evaluate algorithms by checking sample efficiency, thus converging fast. (More analysis can be seen in the supplementary materials)

Comparison to the baselines. We compare IMAC with baselines in the 3-agent scenario. Figure 3(a) shows the learning curve of 100,000 episodes in terms of the mean episode reward over a sliding window of 1000 episodes. We can see at the end of training, agents trained with communication have higher mean episode reward. According to (Lowe et al. 2019), “increase in reward when adding a communication channel” is sufficient to effective communication. Additionally, IMAC outperforms other baselines along the process of training, i.e., IMAC can reach upper bound of performance early. By using the information bottleneck method, agents have a better sample efficiency, thus converging fast. (More analysis can be seen in the supplementary materials)

Increasing the number of agents. We investigate agents’ performance when the number of agents increases. We made a slight modification on environment about agents’ observation. According to (Jiang and Lu 2018), we constrain that how much information is used in these algorithms (The detailed results can be seen in the supplementary materials)

Limited bandwidth. We evaluate algorithms by checking agents’ performance under different limited-bandwidth constraints during the inference stage. Figure 4 shows density plot of episode reward per agent during the inference stage. Figure 4(a) IMAC with different prior distributions against MADDPG with communication. As depicted in Figure 4(a), IMAC with different prior distributions can reach the same outcome as MADDPG with communication. Consequently, the entropy of agents’ messages satisfies the bandwidth constraints. Also, we train MADDPG with communication for comparison. Then, in the inference stage, we constrain these algorithms into different bandwidths. As depicted in Figure 4(b), MADDPG with communication fails in the limited-bandwidth environment. From Figure 4(c) and (d), we can see that the same bandwidth constraint is less effective in IMAC compared with MADDPG with communication. Results here demonstrate that IMAC discards useless information without impairment on performance. We also evaluate how much information is used in these algorithms (The detailed results can be seen in the supplementary materials)

Predator Prey

In this scenario, $m$ slower predators chase $n$ faster preys around an environment with $l$ landmarks impeding the way. As same as cooperative navigation, each agent observes the
relative position of other agents and landmarks. Predators share common rewards, which are assigned based on the collision between predators and preys, as well as the minimal distance between two groups. Preys are penalized for running out of the boundary of the screen. In this way, predators would learn to approach and surround preys, while preys would learn to feint to save their teammates.

We set the number of predators as 4, the number of preys as 2, and the number of landmarks as 2. We use the same architecture and hyper-parameter as configuration in cooperative navigation. We trained our agents by self-play for 100,000 episodes and then evaluate performance by cross-comparing between IMAC and the baselines. We average the episode rewards across 1000 rounds (episodes) as scores.

**Comparison to baselines.** Table 1 represents the cross-comparing between IMAC and the baselines. Each cell consists of two numbers which denote the mean episode rewards of predator and prey respectively. The larger the score is, the better the algorithm is. We first focus on the mean episode rewards of predator row by row. Facing the same prey, IMAC has higher scores than the predators of all the baselines and hence are stronger than other predators. Then, the mean episode rewards of prey column by column shows the ability of prey to escape. We can see that IMAC has higher scores than the preys of most baselines and hence are stronger than other preys. We argue that IMAC leads to better cooperation than the baselines even in competitive environments and the learned policy of IMAC predators and preys can generalize to the opponents with different policies.

**Limited bandwidth.** Similar to the cooperative navigation, we evaluate algorithms by showing the performance under different limited-bandwidth constraints during inference. The detailed results can be seen in the supplementary materials. We can see that with the limited-bandwidth constraint, MADDPG with communication and IMAC suffer a degradation of performance. However, IMAC outperforms MADDPG with communication in respect of resistance to the effect of limited bandwidth.

**Ablation**

We investigate the effect of (1) the effect of limited bandwidth; (2) the effect of $\beta$ on multi-agent communication on the performance of agents.

**The effect of limited bandwidth.** Figure 5(a) shows the learning curve of IMAC with different prior distributions. IMAC with $z(m) = N(0, 1)$ achieves the best performance. When the variance is smaller or bigger, the performance suffers some degradation. It is reasonable because a smaller variance means a more lossy compression, leading less information sharing, and a variance which is larger than the variance without regulation must bring about redundant information, thus leading to slow convergence.

**The effect of $\beta$.** $\beta$ controls the degree of compression between $h$ and $m_i$ for each agent $i$: the larger $\beta$, the more lossy compression. Figure 5(b) shows a similar result to the ablation on limited-bandwidth constraint. The reason is the same: a larger $\beta$ means a more strict compression while a smaller $\beta$ means a less strict one.

The ablation shows that as a compression algorithm, the information bottleneck method extracts the most informative elements from the source. A proper compression rate is good for multi-agent communication, because it can not only avoid lose much information caused by higher compression, but also resist much noisy caused by lower compression.

**Conclusions**

In this paper, we have proposed an informative multi-agent communication method in the limited-bandwidth environment, where agents utilize the information bottleneck to learn an informative protocol. We have given a well-defined explanation of the limited-bandwidth constraint: limited bandwidth restrains the entropy of the messages. We introduce a customized batch-norm layer, which controls the messages’entropy to simulate the bandwidth constraint. Inspired by the information bottleneck method, our proposed IMAC algorithm learns informative protocols, which convey low-entropy and useful messages. Empirical results and an accompanying ablation study show that IMAC improves the agents’ performance under limited-bandwidth constraint and leads to faster convergence.

| Predator | Prey | AMP            | IMAC | MADDPG | MADDPG w/ com |
|----------|------|----------------|------|--------|---------------|
| AMP      | 1.06 | -20.42         | 25.13| 6.39   | 44.62         |
| IMAC     | 27.29| -22.27         | 32.32| -4.26  | 34.33         |
| MADDPG   | 10.09| -24.93         | 5.98 | -19.39 | 28.63         |
| MADDPG w/ com | 21.02 | -21.52 | 28.63 | -15.60 | 20.85 | -37.48 | 16.87 | -13.09 |
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Source Coding

Source Code: A source code $C$ is a mapping from the range of a random variable or a set of random variables to finite length strings of symbols from a $K$-ary alphabet.

Expected length of a source code denoted by $L(C)$ is given as follows: $L(C) = \sum_{x \in X} p(x)l(x)$, where $l(x)$ is the length of codeword $c(x)$ for a symbol $x \in X$, and $p(x)$ is the probability of the symbol.

Intuitively, a good code should preserve the information content of an outcome. Since information content depends on the probability of the outcome (it is higher if probability is lower, or equivalently if the outcome is very uncertain), a good codeword will use fewer bits to encode a certain or high probability outcome and more bits to encode a low probability outcome. Thus, we expect that the smallest expected code length should be related to the average uncertainty of the random variable, i.e., the entropy.

Source coding theorem states that entropy is the fundamental limit of data compression; i.e., $\forall C : L(C) \geq H(X)$. Instead of encoding individual symbols, we can also encode blocks of symbols together. A length $n$ block code encodes $n$ length strings of symbols together and is denoted by $C(x_1, \ldots, x_n) =: C(x^n)$.

Proof. Consider the optimization problem:

$$\min_{l(x)} \sum_{x \in X} p(x)l(x) \quad \text{such that} \quad \sum_{x \in X} 2^{-l(x)} \leq 1$$

The above finds the shortest possible code length subject to satisfying the Kraft inequality. If we relax the the codelengths to be non-integer, then we can obtain a lower bound. To do this, the Lagrangian is:

$$\mathcal{L} = \sum_{x \in X} p(x)l(x) + \lambda \left( \sum_{x \in X} 2^{-l(x)} - 1 \right)$$

Taking derivatives with respect to $l(x)$ and $\lambda$ and setting to 0, leading to:

$$p(x) + \ln 2 \lambda 2^{-l(x)} = 0 \quad \sum_{x \in X} 2^{-l(x)} - 1 = 0$$

Solving this for $l(x)$ leads to $l(x) = \log \frac{1}{p(x)}$, which can be verified by direct substitution. This proves the lower bound.

Theoretical analysis can be seen in (Shannon 1948).

Maximum Data Rate

We first introduce the Nyquist ISI criterion:

**Proposition 3** (Nyquist ISI criterion). If we denote the channel impulse response as $h(t)$, then the condition for an ISI-free response can be expressed as:

$$h(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$$

for all integers $n$, where $T_s$ is the symbol period. The Nyquist ISI criterion says that this is equivalent to:

$$\frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H(f - k/T_s) = 1 \quad \forall f$$

where $H(f)$ is the Fourier transform of $h(t)$.

We may now state the Nyquist ISI criterion for distortionless baseband transmission in the absence of noise: The frequency function $H(f)$ eliminates intersymbol interference for samples taken at interval $T_s$ provide that it satisfies Equation 3.

The simplest way of satisfying Equation 3 is to specify the frequency function $H(f)$ to be in the form of a rectangular function, as showing by

$$H(f) = \begin{cases} 1/2W, & -W < f < W \\ 0, & |f| > W \end{cases}$$

$$= \frac{1}{2W} \text{rect} \left( \frac{f}{2W} \right)$$

where $\text{rect}(f)$ stands for a rectangular function of unit amplitude and unit support centered on $f = 0$, and the overall system bandwidth $W$ is defined by

$$W = \frac{1}{2T_s} = \frac{R_b}{2}$$

The special value of the bit rate $R_b = 2W$ is called the Nyquist rate, and $W$ is itself called the Nyquist bandwidth.

The key here is that we have restricted ourselves to binary transmission and are limited to $2W$ bits/s no matter how much we increase the signal-to-noise ratio. The way to attain a higher $R_b$ value is to replace the binary transmission system with a multilevel system, often termed an $K$-ary transmission system, with $K > 2$. An $K$-ary channel can pass $2W\log_2 K$ bits/s with an acceptable error rate.

Thus, we conclude that the bit rate $R_b \leq R_{max} = 2W\log_2 K$.

Figure 6: Intra-step Communication

The Information Bottleneck Method

Background

The information bottleneck method provides a principled way to extract information that is present in one variable that is relevant for predicting another variable. Consider $X$ and
We discuss why information bottleneck works in multi-agent compression or minimality term in information bottleneck, a learning perspective, we discuss the role of $I(X; T)$ of the data. Notice that taking the identity encoding always ensures a sufficient statistic if $I(X; T)$ is the information constraint. Equivalently, with $X \in \mathcal{X}$, the problem is to learn an encoding that is maximally informative about the target $Y$, which is measured by the mutual information between $Y$ and $Z$, where

$$I(Z, Y; \theta) = \int p(z, y|\theta) \log \frac{p(z, y|\theta)}{p(z|\theta)p(y|\theta)} \, dx \, dy \quad (1)$$

Figure 7: Inter-step Communication

Notice that taking the identity encoding always ensures a maximally informative representation if only with the above objective, but it is not a useful representation obviously. It is evident to constrain on encoding’s complexity if we want the best representation, i.e., $Z$. (Tishby and Zaslavsky 2015) proposed the information bottleneck that expresses the trade-off between the mutual information measures $I(X, Z)$ and $I(X, Y)$. This suggests the objective:

$$\max_{\theta} I(Z, Y; \theta) \quad \text{s.t.} \quad I(X, Z; \theta) \leq I_c \quad (2)$$

where $I_c$ is the information constraint. Equivalently, with the introduction of a Lagrange multiplier $\beta$ we can maximize the objective function:

$$L(\theta) = I(Z, Y; \theta) - \beta I(X, Z; \theta) \quad (3)$$

where $\beta$ controls the trade-off. Intuitively, the first term encourages $Z$ to be predictive of $Y$; the second term encourages $Z$ to “forget” $X$. Essentially it forces $Z$ to act like a minimal sufficient statistic of $X$ for predicting $Y$.

Why IMAC works?

We discuss why information bottleneck works in multi-agent communication.

We first introduce minimal sufficient statistics: a transformation $T(X)$ of the data $X$ is a minimal sufficient statistic if $T(X) \in \arg \min_S I(X, S(X))$, where $S(X)$ is s.t. $I(\theta, S(X)) = \max_{T'} I(\theta, T'(X))$.

Information bottleneck principle generalizes the notion of minimal sufficient statistics and suggests using a summary of the data $T(X)$ that has least mutual information with the data $X$ while preserving some amount of information about an auxiliary variable $Y$.

According to (Shamir, Sabato, and Tishby 2010), from a learning perspective, we discuss the role of $I(X; T)$, the compression or minimality term in information bottleneck, as a regularizer when maximizing $I(Y; T)$.

Reinforcement learning learns a optimal action given a state. If we know the optimal action in advance, like imitation learning, then we would maximize the mutual information between the state and its corresponding optimal action, which is a straightforward application of supervised learning. Here, $X$ represents the states, $T$ represents the messages, $Y$ represents the actions. Note that without regularization, $I(Y; T)$ cannot be estimated efficiently from a sample of a reasonable size; It means that more samples are needed in reinforcement learning. In another words, methods with regularization on $I(X; T)$, e.g., IMAC, can accelerate convergence.

Experimental Details and Results

Cooperative navigation

In this environment, agents must cooperate through physical actions to reach a set of landmarks. Agents observe the relative positions of other agents and landmarks, and are collectively rewarded based on the proximity of any agent to each landmark. In other words, the agents have to `cover' all of the landmarks. Further, the agents occupy significant physical space and are penalized when colliding with each other. Our agents learn to infer the landmark they must cover, and move there while avoiding other agents.

Table 2 shows that MADDPG without communication tends to use high-entropy messages, while IMAC can convey low-entropy messages. Combined with the performance in Figure 4, we can see that under limited-bandwidth constraint, IMAC learns informative communication protocols.

Predator and prey

In this variant of the classic predator-prey game, some slower cooperating agents must chase some faster adversaries around a randomly generated environment with some large landmarks impeding the way. Each time the cooperative agents collide with some adversaries, the agents are rewarded while the adversary is penalized. Agents observe the relative positions and velocities of the agents, and the positions of the landmarks.

Table 3 shows the performance under different limited-bandwidth constraints during inference in the environment of predator and prey. We can see with limited-bandwidth constraint, MADDPG with communication and IMAC suffer a degradation of performance. However, IMAC outperforms MADDPG with communication in respect of resistance to the effect of limited bandwidth.
Figure 8: Illustration of the experimental environments. (a) Cooperative navigation of 3 agents (b) Cooperative navigation of 10 agents (c) Predator and Prey

| $\sigma^2$ | None | 1   | 5   | 10  |
|-----------|------|-----|-----|-----|
| $H(M_i) = \frac{1}{2} \log(2\pi e \sigma^2)$ | -    | 1.419 | 2.223 | 2.570 |
| maddpg w/ com | 3.480±0.042 | 1.530±0.038 | 3.147±0.088 | 3.891±0.059 |
| IMAC train w/ bw=1 | 2.24±0.028 | - | - | - |
| IMAC train w/ bw=5 | 2.227±0.002 | 1.383±0.003 | - | - |
| IMAC train w/ bw=10 | 2.763±0.215 | 1.695±0.044 | 3.017±0.026 | - |

Table 2: Entropy of messages in different limited bandwidths (number in cell represents $H(M_i) = \frac{1}{2} \log(2\pi e \sigma^2)$, which is calculated based on running variance)

| Predator \ Prey | MADDPG_c1 | MADDPG_c5 | IMAC | IMAC_s5_c1 | IMAC_s10_c1 | IMAC_s10_c5 |
|-----------------|------------|-----------|------|------------|------------|------------|
| MADDPG_c1      | 18.01 \-14.22 | 24.15 \-29.88 | 22.38 \-16.91 | 47.59 \-45.64 | 34.25 \-27.68 | 50.81 \-43.62 |
| MADDPG_c5      | 26.32 \-20.48 | 15.67 \-11.59 | 29.06 \-22.16 | 27.07 \-22.89 | 23.44 \-20.41 | 32.24 \-26.46 |
| IMAC           | 51.24 \-42.56 | 37.37 \-45.52 | 44.64 \-36.49 | 49.12 \-42.65 | 36.63 \-30.03 | 35.42 \-28.82 |
| IMAC_s5_c1     | 38.86 \-32.06 | 34.54 \-35.03 | 9.97 \-3.11 | 26.25 \-21.06 | 11.80 \-7.55 | 38.32 \-32.28 |
| IMAC_s10_c1    | 26.67 \-21.41 | 34.99 \-35.02 | 9.71 \-4.11 | 9.82 \-6.92 | 9.82 \-6.92 | 37.30 \-31.30 |
| IMAC_s10_c5    | 43.88 \-38.27 | 26.39 \-35.42 | 11.31 \-9.12 | 30.02 \-27.41 | 29.08 \-25.66 | 22.25 \-16.51 |

Table 3: Cross-comparison in different bandwidths on predator-prey