Lorentz violation at high energy: concepts, phenomena and astrophysical constraints

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Abstract

We consider here the possibility of quantum gravity induced violation of Lorentz symmetry (LV). Even if suppressed by the inverse Planck mass such LV can be tested by current experiments and astrophysical observations. We review the effective field theory approach to describing LV, the issue of naturalness, and many phenomena characteristic of LV. We discuss some of the current observational bounds on LV, focusing mostly on those from high energy astrophysics in the QED sector at order $E/M_{\text{Planck}}$. In this context we present a number of new results which include the explicit computation of rates of the most relevant LV processes, the derivation of a new photon decay constraint, and modification of previous constraints taking proper account of the helicity dependence of the LV parameters implied by effective field theory.

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1 Introduction

The discovery of Lorentz symmetry was one of the great advances in the history of physics. This symmetry has been confirmed to ever greater precision, and it powerfully constrains theories in a way that has proved instrumental in discovering new laws of physics. Moreover the mathematical structure of
the Lorentz group is compellingly simple. It is natural to assume under these circumstances that Lorentz invariance is a symmetry of nature up to arbitrary boosts. Nevertheless, there are several reasons to question exact Lorentz symmetry. From a purely logical point of view, the most compelling reason is that an infinite volume of the Lorentz group is (and will always be) experimentally untested since, unlike the rotation group, the Lorentz group is non-compact. Why should we assume that exact Lorentz invariance holds when this hypothesis cannot even in principle be tested?

While non-compactness may be a logically compelling reason to question Lorentz symmetry, it is by itself not very encouraging. However, there are also several reasons to suspect that there will be a failure of Lorentz symmetry at some energy or boost. One reason is the ultraviolet divergences of quantum field theory, which are a direct consequence of the assumption that the spectrum of field degrees of freedom is boost invariant. Another reason comes from quantum gravity. Profound difficulties associated with the “problem of time” in quantum gravity [1,2] have suggested that an underlying preferred time may be necessary to make sense of this physics, and general arguments suggest radical departures from standard spacetime symmetries at the Planck scale [3]. Aside from general issues of principle, specific hints of Lorentz violation have come from tentative calculations in various approaches to quantum gravity: string theory tensor VEVs [4], cosmologically varying moduli [5], spacetime foam [6], semiclassical spin-network calculations in Loop QG [7,8], non-commutative geometry [9,10,11,12], some brane-world backgrounds [13], and condensed matter analogues of “emergent gravity” [14].

None of the above reasons amount to a convincing argument that Lorentz symmetry breaking is a feature of quantum gravity. However, taken together they do motivate the effort to characterize possible observable consequences of LV and to strengthen observational bounds. Moreover, apart from any theoretical motivation, significant improvement of the precision with which fundamental symmetries are tested is always desirable.

The study of the possibility of Lorentz violation is not new, although it has recently received more attention because of both the theoretical ideas just mentioned and improvements in observational sensitivity and reach that allow even Planck suppressed Lorentz violation to be detected (see e.g. [15] for an extensive review). A partial list of such “windows on quantum gravity” is

- sidereal variation of LV couplings as the lab moves with respect to a preferred frame or directions
- cosmological variation of couplings
- cumulative effects: long baseline dispersion and vacuum birefringence (e.g. of signals from gamma ray bursts, active galactic nuclei, pulsars, galaxies)
- new threshold reactions (e.g. photon decay, vacuum Čerenkov effect)
• shifted existing threshold reactions (e.g. photon annihilation from blazars, GZK reaction)
• LV induced decays not characterized by a threshold (e.g. decay of a particle from one helicity to the other or photon splitting)
• maximum velocity (e.g. synchrotron peak from supernova remnants)
• dynamical effects of LV background fields (e.g. gravitational coupling and additional wave modes)

The possibility of interesting constraints (or observations) of LV despite Planck suppression arises in different ways for the different types of observations. In the laboratory experiments looking for sidereal variations, the enormous number of atoms allow variations of a resonance frequency to be measured extremely accurately. In the case of dispersion or birefringence, the enormous propagation distances would allow a tiny effect to accumulate. In the new or shifted threshold case, the creation of a particle with mass $m$ would be strongly affected by a LV term when the momentum becomes large enough for this term to be comparable to the mass term in the dispersion relation. Finally, an upper bound to electron group velocity, even if very near the speed of light, can severely limit the frequency of synchrotron radiation. We shall discuss examples of all these phenomena.

The purpose of the present paper is twofold. First, we aim to give an introductory overview of some of the important issues involved in the consideration of Lorentz violation, including history, conceptual basis and problems, observable phenomena, and the current best constraints in certain sectors. Second, we present a number of new results, including computations of rates of certain LV processes and the derivation of a new photon decay constraint. We also analyze the modifications of previous constraints that are required when the helicity dependence of the LV parameters is properly taken into account. In order to make the paper most useful as an introduction to LV we have placed much material, including most of the technical details of the new results, in appendices.

The structure of this paper is as follows. The next section gives a historical summary of LV research, while section 3 introduces the framework for parameterizing LV, together with the conceptual issues this raises. In section 4 we focus on the phenomenology of dimension 5 LV in QED, and section 5 presents the current constraints on such LV. Constraints on other sorts of LV are surveyed in section 6, focusing on ultra-high energy cosmic rays, and we close in section 7 with a discussion of future prospects. Appendix A presents the analysis behind the LV synchrotron constraint, appendix B derives the LV threshold configuration results, and appendix C includes the computation of rates and thresholds for some LV processes.
2 A brief history of some LV research

We present at this point a brief historical overview of research related to Lorentz violation, mentioning some influential work but without trying to be complete. For a more complete review see Ref. [15].

The idea of cosmological variation of coupling constants goes back at least to Milne and Dirac beginning in the 1930’s [16], and continues to be of interest today. This would be associated with a form of LV since the spacelike surfaces on which the couplings are assumed constant would define a local preferred frame. It has recently been stimulated both by the string theoretic expectation that there are moduli fields, and by controversial observational evidence for variation of the fine structure constant $\alpha$ [17]. A set of related ideas goes under the generic name of “varying speed of light cosmologies” (VSL), which includes numerous distinct formulations [18,19]. New models and observational constraints continue to be discussed in the literature (see e.g. [20,21]).

Suggestions of possible LV in particle physics go back at least to the 1960’s, when several authors wrote on that idea [22]. The possibility of LV in a metric theory of gravity was explored beginning at least as early as the 1970’s with work of Nordtvedt and Will [23]. Such theoretical ideas were pursued in the ’70’s and ’80’s notably by Nielsen and several other authors on the particle theory side [24], and by Gasperini [25] on the gravity side. A number of observational limits were obtained during this period [26].

Towards the end of the 80’s Kostelecky and Samuel [27] presented evidence for possible spontaneous LV in string theory, and motivated by this explored LV effects in gravitation. The role of Lorentz invariance in the “trans–Planckian puzzle” of black hole redshifts and the Hawking effect was emphasized in the early 90’s [28]. This led to study of the Hawking effect for quantum fields with LV dispersion relations commenced by Unruh [29] and followed up by others. Early in the third millennium this line of research led to work on the related question of the possible imprint of trans–Planckian frequencies on the primordial fluctuation spectrum [30]. Meanwhile the consequences of LV for particle physics were being explored using LV dispersion relations e.g. by Gonzalez-Mestres [31].

Four developments in the late nineties seem to have stimulated a surge of interest in LV research. One was a systematic extension of the standard model of particle physics incorporating all possible LV in the renormalizable sector, developed by Colladay and Kostelecký [32]. That provided a framework for computing in effective field theory the observable consequences for many experiments and led to much experimental work setting limits on the LV parameters in the Lagrangian [33]. On the observational side, the AGASA
experiment reported ultra high energy (UHE) cosmic ray events beyond the GZK proton cutoff [34,35]. Coleman and Glashow then suggested the possibility that LV was the culprit in the possibly missing GZK cutoff [36], and explored many other high energy consequences of renormalizable, isotropic LV leading to different limiting speeds for different particles [39]. In the fourth development, it was pointed out by Amelino-Camelia et al. [6] that the sharp high energy signals of gamma ray bursts could reveal LV photon dispersion suppressed by one power of energy over the mass $M \sim 10^{-3} M_P$, tantalizingly close to the Planck mass.

Together with the improvements in observational reach mentioned earlier, these developments attracted the attention of a large number of researchers to the subject. Shortly after Ref. [6] appeared, Gambini and Pullin [7] argued that semiclassical loop quantum gravity suggests just such LV. Following this work, a very strong constraint on photon birefringence was obtained by Gleiser and Kozameh [42] using UV light from distant galaxies. Further stimulus came from the suggestion [43] that an LV threshold shift might explain the apparent under-absorption on the cosmic IR background of TeV gamma rays from the blazar Mkn501, however it is now believed by many that this anomaly goes away when a corrected IR background is used [44].

The extension of the effective field theory (EFT) framework to incorporate LV dispersion relations suppressed by the ratio $E/M_{\text{Planck}}$ was performed by Myers and Pospelov [45]. This allowed $E/M_{\text{Planck}}$ LV to be explored in a systematic way. The use of EFT also imposes certain relations between the LV parameters for different helicities, which strengthened some prior constraints while weakening others. Using EFT a very strong constraint [46] on the possibility of a maximum electron speed less than the speed of light was deduced from observations of synchrotron radiation from the Crab Nebula. However, as discussed here, this constraint is weakened due to the helicity and particle/anti-particle dependence of the LV parameters.

1 Remarkably, already in 1972 Kirzhnits and Chechin [37] explored the possibility that an apparent missing cutoff in the UHE cosmic ray spectrum could be explained by something that looks very similar to the recently proposed “doubly special relativity” [38].

2 Some later work supported this notion, but the issue continues to be debated [40,41]. In any case, the dynamical aspect of the theory is not under enough control at this time to make any definitive statements concerning LV.
3 Parametrization of Lorentz violation

A simple approach to a phenomenological description of LV is via deformed dispersion relations. This approach was adopted in much work on the subject, and it seems to afford a relatively theory-independent framework in which to explore the unknown possibilities. On the other hand, not much can really be predicted with confidence just based on free particle dispersion relations, without the use of both conservation laws and interaction dynamics. Hence one is led to adopt a more comprehensive LV model in order to deduce meaningful constraints. In this section we discuss these ideas in turn, ending with a focus on the use of effective field theory, which provides a well-motivated and unambiguous hypothesis that can be tested.

3.1 Deformed dispersion relations

If rotation invariance and analyticity around \( p = 0 \) are assumed the dispersion relation for a given particle type can be written as

\[
E^2 = p^2 + m^2 + \Delta(p),
\]

where \( E \) is the energy, \( p \) is hereafter the magnitude of the three-momentum, and

\[
\Delta(p) = \tilde{\eta}_1 p^1 + \tilde{\eta}_2 p^2 + \tilde{\eta}_3 p^3 + \tilde{\eta}_4 p^4 + \cdots
\]

Since this relation is not Lorentz invariant, the frame in which it applies must be specified. Generally this is taken to be the average cosmological rest frame, i.e. the rest frame of the cosmic microwave background. ³

Let us introduce two mass scales, \( M = 10^{19} \text{GeV} \approx M_{\text{Planck}} \), the putative scale of quantum gravity, and \( \mu \), a particle physics mass scale. To keep mass dimensions explicit we factor out possibly appropriate powers of these scales, defining the dimensionful \( \tilde{\eta} \)'s in terms of corresponding dimensionless parameters. It might seem natural that the \( p^n \) term with \( n \geq 3 \) be suppressed by \( 1/M^{n-2} \), and indeed this has been assumed in many works. But following this pattern one would expect the \( n = 2 \) term to be unsuppressed and the \( n = 1 \) term to be even more important. Since any LV at low energies must be small,

³ There are attempts to interpret such deformed dispersion relations as Casimir invariants of a new relativity group which incorporates two invariant scales, \( c \) and \( M_{\text{Pl}} \), instead of just the speed of light like in Special Relativity (SR). These theories are generally called Doubly (or Deformed) Special Relativity (DSR) [38].
such a pattern is untenable. Thus either there is a symmetry or some other mechanism protecting the lower dimension operators from large LV, or the suppression of the higher dimension operators is greater than $1/M^{n-2}$. This is an important issue to which we return in section 3.4.

For the moment we simply follow the observational lead and insert at least one inverse power of $M$ in each term, viz.

\[
\tilde{\eta}_1 = \eta_1 \frac{\mu^2}{M}, \quad \tilde{\eta}_2 = \eta_2 \frac{\mu}{M}, \quad \tilde{\eta}_3 = \eta_3 \frac{1}{M}, \quad \tilde{\eta}_4 = \eta_4 \frac{1}{M^2},
\]

(3)

In characterizing the strength of a constraint we refer to the $\eta_n$ without the tilde, so we are comparing to what might be expected from Planck-suppressed LV. We allow the LV parameters $\eta_i$ to depend on the particle type, and indeed it turns out that they must sometimes be different but related in certain ways for photon polarization states, and for particle and antiparticle states, if the framework of effective field theory is adopted. In an even more general setting, Lehnert [47] studied theoretical constraints on this type of LV and deduced the necessity of some of these parameter relations.

The deformed dispersion relations are introduced for individual particles only; those for macroscopic objects are then inferred by addition. For example, if $N$ particles with momentum $p$ and mass $m$ are combined, the total energy, momentum and mass are $E_{\text{tot}} = NE(p)$, $p_{\text{tot}} = Np$, and $m_{\text{tot}} = Nm$, so that $E_{\text{tot}}^2 = p_{\text{tot}}^2 + m_{\text{tot}}^2 + N^2 \Delta(p)$. Although the Lorentz violating term can be large in some fixed units, its ratio with the mass and momentum squared terms in the dispersion relation is the same as for the individual particles. Hence, there is no observational conflict with standard dispersion relations for macroscopic objects.

This general framework allows for superluminal propagation, and spacelike 4-momentum relative to a fixed background metric. It has been argued [48] that this leads to problems with causality and stability. In the setting of a LV theory with a single preferred frame, however, we do not share this opinion. We cannot see any room for such problems to arise, as long as in the preferred frame the physics is guaranteed to be causal and the states all have positive energy.

3.2 The need for a more complete framework

Various different theoretical approaches to LV have been taken. Some researchers restrict attention to LV described in the framework of effective field theory (EFT), while others allow for effects not describable in this way, such
as those that might be due to stochastic fluctuations of a “space-time foam”. Some restrict to rotationally invariant LV, while others consider also rotational symmetry breaking. Both true LV as well as “deformed” Lorentz symmetry (in the context of so-called “doubly special relativity”[38]) have been pursued. Another difference in approaches is whether or not different particle types are assumed to have the same LV parameters.

The rest of this article will focus on just one of these approaches, namely LV describable by standard EFT, assuming rotational invariance, and allowing distinct LV parameters for different particles. This choice derives from the attitude that, in exploring the possible phenomenology of new physics, it is useful to retain enough standard physics so that clear predictions can be made, and to keep the possibilities narrow enough to be meaningfully constrained. Furthermore, of the LV phenomena mentioned in the Introduction, only dispersion and birefringence are determined solely by the kinematic dispersion relations. Analysis of threshold reactions obviously requires in addition an assumption of energy-momentum conservation, and to impose constraints the reaction rates must be known. This requires knowledge of matrix elements, and hence the dynamics comes into play. We therefore need a complete (at least at low energy) theory to properly derive constraints. Only EFT currently satisfies this requirement.

This approach is not universally favored (for an example of a different approach see [49]). Therefore we think it is important to spell out the motivation for the choices we have made. First, while of course it may be that EFT is not adequate for describing the leading quantum gravity phenomenology effects, it has proven very effective and flexible in the past. It produces local energy and momentum conservation laws, and seems to require for its applicability just locality and local spacetime translation invariance above some length scale. It describes the standard model and general relativity (which are presumably not fundamental theories), a myriad of condensed matter systems at appropriate length and energy scales, and even string theory (as perhaps most impressively verified in the calculations of black hole entropy and Hawking radiation rates). It is true that, e.g., non-commutative geometry (NCG) can lead to EFT with problematic IR/UV mixing, however this more likely indicates a physically unacceptable feature of such NCG rather than a physical limitation of EFT.

It is worth remarking that while we choose EFT so as to work in a complete and well motivated framework, in fact many constraints are actually quite insensitive to the specific dynamics of the theory, other than that it obeys energy-momentum conservation. As an example, consider photon decay. In ordinary Lorentz invariant physics, photon decay into an electron/positron pair is forbidden since two timelike four-momenta (the outgoing pair) cannot add up to the null four momentum of the photon. With LV the photon four momentum can be timelike and therefore photons above a certain energy can
be unstable. Once the reaction is kinematically allowed, the photon lifetime is extremely short (≪ 1 sec) when calculated with standard QED plus modified dispersion, much shorter for example than the required lifetime of 10^{11} seconds for high energy photons that reach us from the Crab nebula. Thus we could tolerate huge modifications to the matrix element (the dynamics) and still have a photon decay rate incompatible with observation. Hence, often the dynamics isn’t particularly important when deriving constraints.

The assumption of rotational invariance is motivated by the idea that LV may arise in QG from the presence of a short distance cutoff. This suggests a breaking of boost invariance, with a preferred rest frame\(^4\) but not necessarily an observable breaking of rotational invariance. Note also that it is very difficult to construct a theory that breaks rotation invariance while preserving boost invariance. (For example, if a spacelike four-vector is introduced to break rotation invariance, the four-vector also breaks boost invariance.) Since a constraint on pure boost violation is, barring a conspiracy, also a constraint on boost plus rotation violation, it is sensible to simplify with the assumption of rotation invariance at this stage. The preferred frame is assumed to coincide with the rest frame of the CMB since the Universe provides this and no other candidate for a cosmic preferred frame.

Finally why do we choose to complicate matters by allowing for different LV parameters for different particles? First, EFT for first order Planck suppressed LV (see section 3.3) requires this for different polarizations or spin states, so it is unavoidable in that sense. Second, we see no reason \textit{a priori} to expect these parameters to coincide. The term “equivalence principle” has been used to motivate the equality of the parameters. However, in the presence of LV dispersion relations, free particles with different masses travel on different trajectories even if they have the same LV parameters [52,53]. Moreover, different particles would presumably interact differently with the spacetime microstructure since they interact differently with themselves and with each other. For an explicit example see [54]. Another example of this occurs in the braneworld model discussed in Ref. [13], and an extreme version occurs in the proposal of Ref. [55] in which only certain particles feel the spacetime foam effects. (Note however that in this proposal the LV parameters fluctuate even for a given kind of particle, so EFT would not be a valid description.)

\(^4\) See however [50] for an example where (coarse grained) boost invariance is preserved in a discrete model, and [51] for a study of how discreteness may be compatible with Lorentz symmetry in a quantum setting.
3.3 Effective field theory and LV

In this subsection we briefly discuss some explicit formulations of LVEFT. First, the (minimal) standard model extension (SME) of Colladay and Kostelecký [32] consists of the standard model of particle physics plus all Lorentz violating renormalizable operators (i.e. of mass dimension \( \leq 4 \)) that can be written without changing the field content or violating the gauge symmetry. For illustration, the leading order terms in the QED sector are the dimension three terms

\[
- b_a \bar{\psi} \gamma_5 \gamma^a \psi - \frac{1}{2} H_{ab} \bar{\psi} \sigma^{ab} \psi
\]

and the dimension four terms

\[
- \frac{1}{4} k^{abcd} F_{ab} F_{cd} + i \bar{\psi} (c_{ab} + d_{ab} \gamma_5) \gamma^a D^b \psi
\]

where the dimension one coefficients \( b_a, H_{ab} \) and dimensionless \( k^{abcd}, c_{ab}, \) and \( d_{ab} \) are constant tensors characterizing the LV. If we assume rotational invariance then these must all be constructed from a given unit timelike vector \( u^a \) and the Minkowski metric \( \eta_{ab} \), hence \( b_a \propto u_a, H_{ab} = 0, k^{abcd} \propto u^a \eta^b[c_u d], c_{ab} \propto u_a u_b, \) and \( d_{ab} \propto u_a u_b \). Such LV is thus characterized by just four numbers.

The study of Lorentz violating EFT in the higher mass dimension sector was initiated by Myers and Pospelov [45]. They classified all LV dimension five operators that can be added to the QED Lagrangian and are quadratic in the same fields, rotation invariant, gauge invariant, not reducible to a combination of lower and/or higher dimension operators using the field equations, and contribute \( p^3 \) terms to the dispersion relation. Just three operators arise:

\[
- \frac{\xi}{2M} u^m F_{ma} (u \cdot \partial)(u_n \tilde{F}^{na}) + \frac{1}{2M} u^m \bar{\psi} \gamma_m (\zeta_1 + \zeta_2 \gamma_5) (u \cdot \partial)^2 \psi
\]

where \( \tilde{F} \) denotes the dual of \( F \), and \( \xi, \zeta_{1,2} \) are dimensionless parameters. The sign of the \( \xi \) term in (6) is opposite to that in [45], and is chosen so that positive helicity photons have \( +\xi \) for a dispersion coefficient (see below). Also the factor 2 in the denominator is introduced to avoid a factor of 2 in the dispersion relation for photons. All of the terms (6) violate CPT symmetry as well as Lorentz invariance. Thus if CPT were preserved, these LV operators would be forbidden.
As discussed above in subsection 3.1, if LV operators of dimension $n > 4$ are suppressed as we have imagined by $1/M^{n-2}$, LV would feed down to the lower dimension operators and be strong at low energies [39,45,56,57], unless there is a symmetry or some other mechanism that protects the lower dimension operators from strong LV. What symmetry (other than Lorentz invariance, of course!) could that possibly be?

In the Euclidean context, a discrete subgroup of the Euclidean rotation group suffices to protect the operators of dimension four and less from violation of rotation symmetry. For example [58], consider the “kinetic” term in the EFT for a scalar field with hypercubic symmetry, $M^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. The only tensor $M^{\mu\nu}$ with hypercubic symmetry is proportional to the Kronecker delta $\delta^{\mu\nu}$, so full rotational invariance is an “accidental” symmetry of the kinetic operator.

If one tries to mimic this construction on a Minkowski lattice admitting a discrete subgroup of the Lorentz group, one faces the problem that each point has an infinite number of neighbors related by the Lorentz boosts. For the action to share the discrete symmetry each point would have to appear in infinitely many terms of the discrete action, presumably rendering the equations of motion meaningless.

Another symmetry that could do the trick is three dimensional rotational symmetry together with a symmetry between different particle types. For example, rotational symmetry would imply that the kinetic term for a scalar field takes the form $(\partial_t \phi)^2 - c^2 (\partial_i \phi)^2$, for some constant $c$. Then, for multiple scalar fields, a symmetry relating the fields would imply that the constant $c$ is the same for all, hence the kinetic term would be Lorentz invariant with $c$ playing the role of the speed of light. Unfortunately this mechanism does not work in nature, since there is no symmetry relating all the physical fields.

Perhaps under some conditions a partial symmetry could be adequate, e.g. grand unified gauge and/or super symmetry. In fact, some recent work [59,60] presents evidence that supersymmetry can indeed play this role. Supersymmetry (SUSY) here refers to the symmetry algebra that is a ‘square root’ of the spacetime translation group. The nature of this square root depends upon the Minkowski metric, so is tied to the Lorentz group, but it does not require Lorentz symmetry. Nibbelink and Pospelov showed in Ref. [59], using the superfield formalism, that the SUSY and gauge symmetry preserving LV operators that can be added to the SUSY Standard Model first appear at dimension five. This solves the naturalness problem in the sense of having Planck-suppressed dimension five operators without accompanying huge lower dimension LV operators. However, it should be noted that in this scenario the
$O(p^3)$ terms in the particle dispersion relations are suppressed by an additional factor of $m^2/M^2$ compared to (2,3).

In a different analysis, Jain and Ralston [60] showed that if the Wess-Zumino model is cut off at the energy scale $\Lambda$ in a LV manner that preserves SUSY, then LV radiative corrections to the scalar particle self-energy are suppressed by the ratio $m^2/\Lambda^2$, unlike in the non-SUSY case [57] where they diverge logarithmically with $\Lambda$. This is another example of SUSY making approximate low energy Lorentz symmetry natural in the presence of large high energy LV.

Of course SUSY is broken in the real world. LV SUSY QED with softly broken SUSY was recently studied in [61]. It was found there that, upon SUSY breaking, the dimension five SUSY operators generate dimension three operators large enough that the dimension five operators must be suppressed by a mass scale much greater than $M_{\text{Planck}}$. In this sense, the naturalness problem is not solved in this setting (although it is not as severe as with no SUSY). If CPT symmetry is imposed however, then all the dimension five operators are excluded. After soft SUSY breaking, the dimension six LV operators generate dimension four LV operators that are currently not experimentally excluded. Hence perhaps this latter scenario can solve the naturalness problem. But again, the SUSY LV operators considered in [61] do not give rise to the type of dispersion corrections we consider, hence the astrophysical bounds discussed here are not relevant to that SUSY model.

At this stage we assume the existence of some realization of the Lorentz symmetry breaking scheme upon which constraints are being imposed. If none exists, then our parametrization (3) is misleading, since there should be more powers of $1/M$ suppressing the higher dimension terms. In that case, current observational limits on those terms do not significantly constrain the fundamental theory.

4 Phenomenology of QED with dimension 5 Lorentz violation

We now discuss in general terms the new phenomena arising when the extra dimension five operators in (6) are added to the QED lagrangian. This lays the groundwork for the specific constraints discussed in the next section.

Free particles and tree level interactions can be analyzed without specifying the underlying mechanism that (we assume) protects approximate low energy Lorentz symmetry, hence we restrict attention to these. In principle radiative loop corrections cannot be avoided, but their treatment requires some commitment as to the specific mechanism.
The appearance of higher time derivatives in the Lagrangian brings with it, if treated exactly, an increase in the number of degrees of freedom. In the EFT framework however it is natural to truncate the theory via perturbative reduction to the degrees of freedom that exist without the higher derivative terms (see e.g. [62]). Although not discussed explicitly, it is this truncation we have in mind in what follows.

4.1 Free particle states

4.1.1 Photons

In Lorentz gauge, $\partial^\mu A_\mu = 0$, the free field equation of motion for $A_\mu$ in the preferred frame from (6) is

$$\Box A_\alpha = \frac{\xi}{2M} \epsilon_{\alpha\beta\gamma\delta} u^\beta (u \cdot \partial)^2 F^{\gamma\delta}. \quad (7)$$

For a particle travelling in the $z$-direction, the $A_0, A_3$ equations are the usual $\Box A_0, A_3 = 0$ and hence the residual gauge freedom can still be applied to set $A_0 = A_3 = 0$, leaving two transverse polarizations with dispersion [45]

$$\omega^2_\pm = k^2 \pm \frac{\xi}{M} k^3. \quad (8)$$

The photon subscripts $\pm$ denote right and left polarization, $\epsilon^\mu_\pm = \epsilon^\mu_x \pm i \epsilon^\mu_y$, which have opposite dispersion corrections as a result of the CPT violation in the Lagrangian.

4.1.2 Fermions

We now solve the modified free field Dirac equation for the electron and positron eigenspinors and find the corresponding dispersion relations. We shall see that there exists a basis of energy eigenstates that also have definite helicity $\hat{p} \cdot \mathbf{J}$, hence helicity remains a good quantum number in the presence of the LV dimension five operators.

Beginning with the electron, we assume the eigenspinor has the form $u_s(p) e^{-ip \cdot x}$ where $p$ is the 4-momentum vector, assumed to have positive energy, and $s = \pm 1$ denotes positive and negative helicity. The field equation from (6) in the chiral basis implies

$$A_R u_R = m u_L, \quad A_L u_L = m u_R \quad (9)$$
where

$$A_{R,L} = E \mp \mathbf{p} \cdot \sigma - \eta_{\pm} E^2 / 2M,$$  \hspace{1cm} (10)$$

with \( \eta_{\pm} = 2(\xi_1 \pm \xi_2) \) (the upper signs refer to \( A_R \)) and \( \sigma \) are the usual Pauli spin matrices. If the spinors are helicity eigenspinors we can replace \( \mathbf{p} \cdot \sigma \) with simply \( s \). The dispersion relation is then \( A_R A_L = m^2 \), or

$$E^2 = m^2 + p^2 + \frac{\eta_+}{2M} E^2 (E + sp) + \frac{\eta_-}{2M} E^2 (E - sp) - \frac{\eta_+ \eta_-}{4M^2} E^4$$  \hspace{1cm} (11)$$

Moreover, the spinors \( u_R \) and \( u_L \) are proportional to each other, with ratio 
\( u_L/u_R = \sqrt{A_R/A_L} \). The eigenspinors for the electron can thus be taken as

$$u_s(p) = \left( \frac{\sqrt{A_R} \chi_s(p)}{\sqrt{A_L} \chi_s(p)} \right) = \left( \frac{\sqrt{E - sp - \frac{\eta_+ E^2}{2M}} \chi_s(p)}{\sqrt{E + sp - \frac{\eta_- E^2}{2M}} \chi_s(p)} \right)$$  \hspace{1cm} (12)$$

where \( \chi_s \) is the two-component eigenspinor of \( \mathbf{p} \cdot \sigma \), with eigenvalue \( s \). Note that the dispersion relation \( A_R A_L = m^2 \) implies that either \( A_R \) and \( A_L \) are both positive, or they are both negative. The definition (10) shows that when the energy \( E \) is positive and much smaller than \( M \), at least one of the two is always positive, hence they are both positive. Then from (9) we see that \( u_R \) and \( u_L \) are related by a positive real multiple, implying that the square roots in (12) have a common sign.

The normalization of this spinor is 
\( u_s^\dagger(p) u_s(p) = A_R + A_L = 2E - 2\xi_1 E^2 / M \).

If the usual factor \((2E)^{-1/2}\) is included in the momentum integral in the field operator, the spinors should be normalized to \( 2E \) to ensure the canonical commutation relations hold with the standard annihilation and creation operators assumed. Since we work only at energies \( E \ll M \), the correction is small and may be neglected.

The constraints we shall discuss arise from processes in which the energy satisfies \( m \ll E \ll M \). Then to lowest order in \( m \) and \( E^2 / M \) the dispersion relation (11) for positive and negative helicity electrons takes the form [45]

$$E_{\pm}^2 = p^2 + m^2 + \eta_\pm \frac{p^3}{M},$$  \hspace{1cm} (13)$$

where we have replaced \( E \) by \( p \) in the last term, which is valid to lowest order.
At $E \gg m$ the helicity states take the approximate form

$$u_+(p) \simeq \sqrt{2E} \left( \frac{m}{2E} \chi^+(p) \right), \quad u_-(p) \simeq \sqrt{2E} \left( \frac{m}{2E} \chi^-(p) \right). \quad (14)$$

These are almost chiral, with mixing still controlled by the mass as in the usual Lorentz invariant case.

To find the dispersion relation and spinor wavefunctions for positrons we can use the hole interpretation. A positron of energy, momentum, and spin-angular momentum $(E, p, J)$ corresponds to the absence of an electron with $(-E, -p, -J)$. This electron state has the same helicity as the positron state, since $(-\hat{p}) \cdot (-J) = \hat{p} \cdot J$. Hence the positron dispersion relation is obtained by the replacement $(E, s) \rightarrow (-E, s)$ in (11). This is equivalent to the replacement $\eta_\pm \rightarrow -\eta_\mp$ [63], from which we conclude that the LV parameters for positrons are related to those for electrons by

$$\eta_{\text{positron}}^\pm = -\eta_{\text{electron}}^\mp. \quad (15)$$

The spinor wavefunction of the $(-E, -p, -J)$ electron state—which is also the wavefunction multiplying the positron creation operator in the expansion of the fermion field operator—has the opposite spin state compared to the $(E, p, J)$ electron, so in place of $\chi_s$ one has $\chi_\bar{s}$. Since the energy $-E$ is negative, both $A_R$ and $A_L$ are now negative. Hence the positron eigenspinors take the form

$$v_s(p) = \left( \frac{\sqrt{|A_R|} \chi_\bar{s}(p)}{-\sqrt{|A_L|} \chi_\bar{s}(p)} \right) = \left( \frac{\sqrt{E + sp + \frac{E^2}{2M}} \chi_\bar{s}(p)}{-\sqrt{E - sp + \frac{E^2}{2M}} \chi_\bar{s}(p)} \right). \quad (16)$$

At high energy these take the approximate form

$$v^+(p) \simeq \sqrt{2E} \left( \frac{\chi^-(p)}{-\frac{m}{2E} \chi^-(p)} \right), \quad v^-(p) \simeq \sqrt{2E} \left( \frac{\frac{m}{2E} \chi^+(p)}{-\chi^+(p)} \right). \quad (17)$$

This completes our analysis of the free particle states.
4.2 LV signatures: free particles

4.2.1 Vacuum dispersion and birefringence

The photon dispersion relation (8) entails two free particle phenomena that can be used to look for Lorentz violation and to constrain the parameter \( \xi \):

(i) The propagation speed depends upon both frequency and polarization, which would produce dispersion in the time of flight of different frequency components of a signal originating at the same event.

(ii) Linear polarization direction is rotated through a frequency-dependent angle, due to different phase velocities for opposite helicities. This vacuum birefringence would rotate the polarization direction of monochromatic radiation, or could depolarize linearly polarized radiation composed of a spread of frequencies.

4.2.2 Limiting speed of charged particles

The electron or positron dispersion relations (13) imply that if the \( \eta \)-parameter is negative for a given helicity, that helicity has a maximum propagation speed which is strictly less than the low frequency speed of light. This LV phenomenon would limit the frequency of synchrotron radiation produced by that helicity. This is not strictly a free particle effect since it involves acceleration in an external field and radiation, but the essential LV feature is the limiting speed. In the Appendix section A we review the EFT analysis of this phenomenon.

4.3 LV signatures: particle interactions

There are a number of LV effects involving particle interactions that do not occur in ordinary Lorentz invariant QED. These effects include photon decay \( (\gamma \to e^+e^-) \), vacuum Cerenkov radiation and helicity decay \( (e^- \to e^-\gamma) \), fermion pair emission \( (e^- \to e^-e^+e^-) \), and photon splitting \( (\gamma \to n\gamma) \). Additionally, the threshold for the “photon absorption” \( (\gamma\gamma \to e^+e^-) \) is shifted away from its Lorentz invariant value.

All but photon splitting have nonvanishing tree level amplitudes, hence can be considered without getting involved in the question of the UV completion of the theory. Once loop amplitudes are considered, the UV completion becomes important, and as discussed in section 3.4 can produce large effects at low energy unless there is fine tuning or a symmetry protection mechanism.
The threshold effects can occur at energies many orders of magnitude below the Planck scale. To see why, note that thresholds are determined by particle masses, hence if the $O(E/M)$ term in Eqs. (8) or (13) is comparable to the electron mass term, $m^2$, one can expect a significant threshold shift. For LV coefficients of order one this occurs at the momentum

$$p_{\text{dev}} \sim \left( \frac{m^2 M}{M} \right)^{1/3} \approx 10 \text{ TeV},$$

(18)

which gives a rough idea of the energies one needs to reach in order to put constraints of order one on the Lorentz violations considered here.  

To use the anomalous decay processes for constraining LV one needs to know the threshold energy (if any) and how the rate depends on the incoming particle energy and the LV parameters. The details of computing these thresholds and rates are relegated to the Appendix sections B and C. Here we just cite the main results.

4.3.1 Photon decay

A photon with energy above a certain threshold can decay to an electron and a positron, $\gamma \to e^+e^-$. The threshold and rate for this process generally depend on all three LV parameters $\xi$ and $\eta_{\pm}$. Earlier use of the photon decay constraint with the dispersion relation (8) did not allow for helicity and particle/antiparticle dependence of the LV parameters. To obtain constraints on just two parameters, but consistent with EFT, we can focus on processes in which only either $\eta_+$ or $\eta_-$ is involved, namely reactions in which the positron has opposite helicity to the electron. For example, if the electron has positive helicity then its LV parameter is $\eta_+$, and according to (15) the LV parameter for the negative helicity positron is $-\eta_+$. 

At threshold, the final particle momenta are all aligned (cf. Appendix B). Since the incoming photon has nonzero spin, angular momentum cannot be conserved if the electron and positron have opposite helicities. However, the momenta above threshold need not be aligned, so that angular momentum can be conserved with opposite helicities at any energy above threshold. In this case, the rate vanishes at threshold and is suppressed very close to the threshold, but above the threshold the rate quickly begins to scale as $E^2/M$, where $E$ is the photon energy.  

For a 10 TeV photon, this corresponds to a decay time of order $10^{-8}$ seconds (where we have taken into account the extra suppression coming from overall numerical factors, see section C.2 in

\footnote{See also [53] for a generalization to arbitrary order of Lorentz violation and different particle sectors.}

\footnote{This corrects a prior assertion [64,53] that the rate scales as $E$.}
the Appendix). The rapidity of the reaction (in comparison to the required lifetime of $10^{11}$ seconds for an observed Crab photon) implies that a threshold constraint, i.e. a bound on LV coefficients such that the reaction does not happen at all, will be extremely close to the constraint derived by requiring the lifetime to be longer than the observed travel time.

4.3.2 Vacuum Čerenkov radiation and helicity decay

The process $e^- \rightarrow e^- \gamma$ can either preserve or flip the helicity of the electron. If the electron helicity is unchanged we call it vacuum Čerenkov radiation, while if the helicity changes we call it helicity decay. Vacuum Čerenkov radiation occurs only above a certain energy threshold, and above that energy the rate of energy loss scales as $p^3/M$, which implies that a 10 TeV electron would emit a significant fraction of its energy in $10^{-9}$ seconds. Above the threshold energy, constraints derived simply from threshold analysis alone are again reliable.

In contrast, if the positive and negative helicity LV parameters for electrons are unequal, say $\eta_- > \eta_+$, then decay from negative to positive helicity can happen at any energy, i.e. there is no threshold. However it can be shown (see Appendix C.7) that assuming generic order unity values of the LV parameters, the rate is extremely small until the energy is comparable to the threshold for Čerenkov radiation. Above this energy, the rate is $\sim e^2 m^2/p$ independent of the LV parameters, which (taking into account all the numerical factors) implies that a 10 TeV electron would flip helicity in about $10^{-9}$ seconds. Thus, above this “effective threshold”, constraints on helicity decay derived from the value of the effective threshold alone are reliable.

4.3.3 Fermion pair emission

The process $e^- \rightarrow e^- e^- e^+$ is similar to vacuum Čerenkov radiation or helicity decay, with the final photon replaced by an electron-positron pair. This reaction has been studied previously in the context of the SME [48,65]. Various combinations of helicities for the different fermions can be considered individually. If we choose the particularly simple case (and the only one we shall consider here) where all electrons have the same helicity and the positron has the opposite helicity, then according to Eq. 15 the threshold energy will depend on only one LV parameter. In Appendix C.6 we derive the threshold for this reaction, finding that it is a factor $\sim 2.5$ times higher than that for soft

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7 The lack of a threshold can easily be seen by noting that the dispersion curves $E(p)$ of the two helicity electrons can be connected by a null vector for any energy. Therefore there is always a way to conserve energy-momentum with photon emission (the photon four-vector is almost null even with LV).
vacuum Čerenkov radiation. The rate for the reaction is high as well, hence constraints may be imposed using just the value of the threshold.

### 4.3.4 Photon splitting

The photon splitting processes $\gamma \rightarrow 2\gamma$ and $\gamma \rightarrow 3\gamma$, etc. do not occur in standard QED. Although there are corresponding Feynman diagrams (the triangle and box diagrams), their amplitudes vanish. In the presence of Lorentz violation these processes are generally allowed when $\xi > 0$. However, the effectiveness of this reaction in providing constraints depends heavily on the decay rate.

Aspects of vacuum photon splitting have been examined in [53,66]. An estimate of the rate, independent of the particular form of the Lorentz violating theory and neglecting the polarization dependence of the photon dispersion relation, was given in Ref. [53]. It was argued that a lower bound on the lifetime is $\delta^{-4}E^{-1}$, where $\delta$ is a Lorentz violating factor which for LV from the Myers-Pospelov lagrangian is $\delta \sim \xi E/M$. It was implicitly assumed in this argument that no large dimensionless ratios occur in the rate. However, a recent paper [67] shows that such a large ratio can indeed occur.

Ref. [67] analyzes the case where there is no Lorentz violation in the electron-positron sector. Then the (Lorentz-invariant) one-loop Euler-Heisenberg Lagrangian characterizes the photon interaction, even in the “rest frame” of the incoming photon. Using this interaction it is argued that the lifetime is much shorter, by a factor $(m_e/E_\gamma)^8\delta^{-1}$, than the lower bound mentioned in the previous paragraph. The extra factor of $\delta$ in the rate is compatible with the analysis of [53] since there only the minimum number of factors of $\delta$ was determined. However, the possible appearance of a large dimensionless factor like $(E_\gamma/m_e)^8$ in the rate was overlooked in [53].

Using 50 TeV photons from the Crab nebula, about $10^{13}$ seconds away, the analysis of Ref. [53] would have implied that the constraint on $\xi$ can be no stronger than $\xi \lesssim 10^4$, and even this is not competitive with the other constraints. However, for a 50 TeV photon the extra factor in the lifetime from the analysis of Ref. [67] is of order $10^{-50}$, which would produce a bound $\xi \lesssim 10^{-3}$. Note however that this analysis so far neglects the helicity dependence of the photon dispersion, and assumes no Lorentz violation in the electron-positron sector.

### 4.3.5 Photon absorption

A process related to photon decay is photon absorption, $\gamma\gamma \rightarrow e^+e^-$. Unlike photon decay, this is allowed in Lorentz invariant QED. If one of the photons
has energy \( \omega_0 \), the threshold for the reaction occurs in a head-on collision with
the second photon having the momentum (equivalently energy) \( k_{\text{LI}} = m^2 / \omega_0 \).
For \( k_{\text{LI}} = 10 \text{ TeV} \) (which is relevant for the observational constraints) the soft
photon threshold \( \omega_0 \) is approximately 25 meV, corresponding to a wavelength
of 50 microns.

In the presence of Lorentz violating dispersion relations the threshold for this
process is in general altered, and the process can even be forbidden. Moreover,
as noticed by Kluźniak [68], in some cases there is an upper threshold beyond
which the process does not occur. The lower and upper thresholds for photon
annihilation as a function of the two parameters \( \xi \) and \( \eta \), were obtained in [53],
before the helicity dependence required by EFT was appreciated. As the soft
photon energy is low enough that its LV can be ignored, this corresponds to
the case where electrons and positrons have the same LV terms. The analysis
is rather complicated. In particular it is necessary to sort out whether the
thresholds are lower or upper ones, and whether they occur with the same or
different pair momenta.

The photon absorption constraint, neglecting helicity dependent effects, came
from the fact that LV can shift the standard QED threshold for annihilation
of multi–TeV \( \gamma \)-rays from nearby blazars, such as Mkn 501, with the ambient
infrared extragalactic photons [68,70,71,53,72,73,74]. LV depresses the rate of
absorption of one photon helicity, and increases it for the other. Although
the polarization of the \( \gamma \)-rays is not measured, the possibility that one of the
polarizations is essentially unabsorbed appears to be ruled out by the observ-
ations which show the predicted attenuation [74]. The threshold analysis has
not been redone to allow for the helicity and particle/anti-particle dependence
of \( \eta \). The motivation for doing so is not great since the constraint would at best
not be competitive with other constraints, and the power of the constraint is
limited by our ignorance of the source spectrum of emitted gamma rays.

5 Constraints on LV in QED at \( O(E/M) \)

In this section we discuss the current observational constraints on the LV
parameters \( \xi \) and \( \eta_\pm \) in the Myers-Pospelov extension of QED, focusing on
those coming from high energy processes. The highest observed energies occur
in astrophysical settings, hence it is from such observations that the strongest
constraints derive.

The currently most useful constraints are summarized in Fig. 1. The allowed

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8 A detailed investigation of upper thresholds was carried out by the present authors
in [53,69]. Our results agree with those of [68] only in certain limiting cases.
Fig. 1. Constraints on LV in QED at $O(E/M)$ on a log-log plot. For negative parameters minus the logarithm of the absolute value is plotted, and region of width $10^{-10}$ is excised around each axis. The constraints in solid lines apply to $\xi$ and both $\eta_{\pm}$, and are symmetric about both the $\xi$ and the $\eta$ axis. At least one of the two pairs $(\eta_{\pm}, \xi)$ must lie within the union of the dashed bell-shaped region and its reflection about the $\xi$ axis. The IC and synchrotron Čerenkov lines are truncated where they cross.

Parameter space corresponds to the dark region of the Figure. These constraints strongly bound Lorentz violation at order $O(E/M)$ in QED, assuming the framework of effective field theory holds. While the natural magnitude of the photon and electron coefficients $\xi, \eta$ would be of order unity if there is one power of suppression by the inverse Planck mass, the coefficients are now restricted to the region $|\xi| \lesssim 10^{-4}$ by birefringence and $|\eta_{\pm}| \lesssim 10^{-1}$ by photon decay. The narrower bell-shaped region bounded by the dashed lines is determined by the combination of synchrotron and Čerenkov constraints and applies only to one of the four pairs $(\pm \eta_{\pm}, \xi)$. Equivalently, the union of this bell-shaped region with its reflection about the $\xi$ axis applies to one of the two pairs $(\eta_{\pm}, \xi)$. We shall now briefly discuss how each constraint is obtained, leaving the details for the Appendices.

5.1 Photon time of flight

Photon time of flight constraints [75] limit differences in the arrival time at Earth for photons originating in a distant event [76,6]. Time of flight can vary with energy since the LV term in the group velocity is $\xi k/M$. The arrival time
difference for wave-vectors $k_1$ and $k_2$ is thus

$$\Delta t = \xi (k_2 - k_1) d/M,$$  \hspace{1cm} (19)

which is proportional to the energy difference and the distance travelled. Constraints were obtained using the high energy radiation emitted by some gamma ray bursts (GRB) and active galaxies of the blazars class. The strength of such constraints is typically $\xi \lesssim O(100)$ or weaker [75].

A possible problem with the above bounds is that in an emission event it is not known if the photons of different energies are produced simultaneously. If different energies are emitted at different times that might mask a LV signal. One way around this is to look for correlations between time delay and redshift, which has been done for a set of GRB’s in [77]. Since time of flight delay is a propagation effect that increases over time, a survey of GRB’s at different redshifts can separate this from intrinsic source effects. This enables constraints to be imposed (or LV to be observed) despite uncertainty regarding source effects. While bounds derived in this way are, at present, weaker than the bound presented here, they are also more robust. One might also think that the source uncertainties in GRB’s could be mitigated by looking at high energy narrow bursts, which by definition have nearly simultaneous emission. However, for high energy narrow bursts the number of photons per unit time can be very low, thereby limiting the shortest detectable time lag [78].

The simultaneity problem may also be avoided if one uses the EFT dispersion relation (8) for photons of the same energy. Then one can consider the velocity difference of the two polarizations at a single energy [63]

$$\Delta t = 2|\xi| k/M.$$  \hspace{1cm} (20)

In principle this leads to a constraint at least twice as large as the one arising from energy differences which is also independent of any intrinsic time lag between different energy photons. [It is possible, of course, that there is an intrinsic helicity source effect, however this seems unlikely.] In Fig. 1 we use the EFT improvement of the constraint of Biller et al. [75], obtained using the blazar Markarian 421, an object whose distance is reliably known, which yields $|\xi| < 63$ [63]. Note however that this bound assumes that both polarizations are observed. If for some reason (such as photon decay of one polarization) only one polarization is observed, then the bound shown in is Fig. 1 weakened by a factor of two. In any case the time of flight constraint remains many orders of magnitude weaker than the birefringence constraint so is rather irrelevant from an EFT perspective.
5.2 Birefringence

The birefringence constraint arises from the fact that the LV parameters for left and right circular polarized photons are opposite (8). The phase velocity thus depends on both the wavevector and the helicity. Linear polarization is therefore rotated through an energy dependent angle as a signal propagates, which depolarizes an initially linearly polarized signal comprised of a range of wavevectors. Hence the observation of linearly polarized radiation coming from far away can constrain the magnitude of the LV parameter.

In more detail, with the dispersion relation (8) the direction of linear polarization is rotated through the angle

$$\theta(t) = (\omega_+ (k) - \omega_- (k)) t / 2 = \xi k^2 t / 2M$$

(21)

for a plane wave with wave-vector $k$ over a propagation time $t$. The difference in rotation angles for wave-vectors $k_1$ and $k_2$ is thus

$$\Delta \theta = \xi (k_2^2 - k_1^2) d / 2M,$$

(22)

where we have replaced the time $t$ by the distance $d$ from the source to the detector (divided by the speed of light). Note that the effect is quadratic in the photon energy, and proportional to the distance travelled. The constraint arises from the fact that once the angle of polarization rotation differs by more than $\pi/2$ over the range of energies in a signal, the net polarization is suppressed.

This effect has been used to constrain LV in the dimension three (Chern-Simons) [79], four [80] and five terms. The strongest currently reliable constraint on the dimension five term was deduced by Gleiser and Kozameh [42] using UV light from distant galaxies, and is given by $|\xi| \lesssim 2 \times 10^{-4}$. The much stronger constraint $|\xi| \lesssim 2 \times 10^{-15}$ was derived [63,81] from the report [82] of a high degree of polarization of MeV photons from GRB021206. However, the data has been reanalyzed in two different studies and no statistically significant polarization was found [83].

5.3 The Crab nebula

Apart from the two constraints just discussed, all the others reported in Fig. 1 arise from observations of the Crab nebula. This object is the remnant of a supernova that was observed in 1054 A.D., and lies only about 1.9 Kpc from Earth. It is characterized by the most energetic QED processes observed (e.g.
it is the source of the highest energy gamma rays) and is very well studied. In contrast to the distant sources desirable for constraints based on the cumulative effects of Lorentz violation, the Crab nebula is nearby. This proximity facilitates the detection of the low fluxes characteristic of the emission at the highest energies, which is particularly useful for the remaining constraints. Before undertaking a discussion of the constraints we first summarize the nature of the Crab emission.

The Crab nebula is a bright source of radio, optical, X-ray and gamma-ray emission. It exhibits a broad spectrum characterized by two marked humps. This spectrum is consistently explained by a combination of synchrotron emission by a high energy wind of electrons and positrons, and inverse Compton scattering of the synchrotron photons (plus perhaps 10% other ambient photons) by the same charges [84,85]. No other model for the emission is under consideration, other than for producing some fraction of the highest energy photons. For the constraints discussed here we assume that this standard synchrotron/inverse Compton model is correct. In contrast to our previous work, here we take fully into account the role of the positrons, as well as the helicity and particle/anti-particle dependence of the LV parameters. This complicates matters and changes the nature of the constraints, weakening some aspects and strengthening others.

The inverse Compton gamma ray spectrum of the Crab extends up to energies of at least 50 TeV. The synchrotron emission has been observed to extend at least up to energies of about 100 MeV [84], just before the inverse Compton hump begins to contribute to the spectrum. In standard Lorentz invariant QED, 100 MeV synchrotron radiation in a magnetic field of 0.6 mG would be produced by electrons (or positrons) of energy 1500 TeV. The magnetic field in the emission region has been estimated by several methods which agree on a value between 0.15–0.6 mG (see e.g. [86] and references therein). Two of these methods, radio synchrotron emission and equipartition of energy, are insensitive to Planck suppressed Lorentz violation, hence we are justified in adopting a value of this order for the purpose of constraining Lorentz violation. We use the largest value 0.6 mG for $B$, since it yields the weakest constraint for the synchrotron radiation.

5.4 Photon decay

The observation of 50 TeV gamma rays emitted from the Crab nebula implies that the threshold for photon decay for at least one helicity must be above 50 TeV. By considering decays in which the electron and positron have opposite helicity we can separately constrain both $(\eta_+, \xi)$ and $(\eta_-, \xi)$ (cf. section 4.3.1 and Appendix C). The allowed region is the union of those for which positive
and negative helicity photons do not decay. This yields the constraint shown in Figure 1. The complete form of the allowed region was determined numerically. However in the region $|\xi| < 10^{-4}$ defined by the birefringence constraint, $\xi$ can be neglected compared to $\eta_{\pm}$, in which case the constraint takes the analytic form $|\eta_{\pm}| < 6\sqrt{3}m^2M/k_{th}^3$. With $k_{th} = 50$ Tev this evaluates to $|\eta_{\pm}| \lesssim 0.2$.

5.5 Vacuum Čerenkov—Inverse Compton electrons

The inverse Compton (IC) Čerenkov constraint uses the electrons and positrons of energy up to 50 TeV inferred via the observation of 50 TeV gamma rays from the Crab nebula which are explained by IC scattering. Since the vacuum Čerenkov rate is orders of magnitude higher than the IC scattering rate, that process must not occur for these charges [39,53] for at least one of the four charge species (plus or minus helicity electron or positron), and for either photon helicity. The excluded region in parameter space is thus symmetric about the $\eta$-axis, but applies only to one of the four parameters $\pm \eta_{\pm}$. That is, there is a constraint that must be satisfied by either the pair $(|\eta_{+}|, |\xi|)$ or the pair $(|\eta_{-}|, |\xi|)$.

The absence of the soft Čerenkov threshold up to 50 TeV produces the dashed vertical IC Čerenkov line in Fig. 1 (see Appendix C.5 for a more detailed discussion of the threshold). One can see from (C.21) that this yields a constraint on $\eta$ of order $(11$ TeV$/50$ TeV$)^3 \sim 10^{-2}$. By itself this imposes no constraint at all on the parameters $\eta_{\pm}$, since one of the two parameters $\pm \eta_{\pm}$ always satisfies it, as does one of $\pm \eta_{-}$. However, this is not the whole story.

For parameters satisfying both $\xi < -3 \eta$ and $\xi < \eta$ the Čerenkov threshold is “hard”, since it involves emission of a high energy photon [53,72]. This constraint depends upon both $\xi$ and $\eta$, and is a curve on the parameter space. We do not indicate this constraint in Fig. 1 since it is superseded by the synchrotron–hard Čerenkov constraint discussed below. It imposes a lower bound on one of $|\eta_{\pm}|$ once $|\xi|$ is large enough.

5.6 Fermion pair emission

If we knew that all electrons and positrons were stable at 50 TeV, then the threshold for all of them to undergo fermion pair emission would necessarily be over 50 TeV. Using the processes $e^- \rightarrow e^- e^- e^+$ and $e^+ \rightarrow e^+ e^+ e^-$ with the helicities chosen so that only one of $\eta_{\pm}$ is involved in the reaction, electron/positron stability would lead to the constraint $|\eta_{\pm}| \lesssim 0.16$. This is a factor of 16 higher than that from the soft Čerenkov threshold, and roughly the same as the photon decay constraint. However, at least without further
detailed analysis of the Crab spectrum, all we can say is that at least one of the fermion types is able to produce the 50 TeV IC radiation. By itself this imposes no constraint at all, since for each helicity either the electron or the positron of the opposite helicity is stable to this particular pair emission process.

5.7 Synchrotron radiation

An electron or positron with a negative value of $\eta$ has a maximal group velocity less than the low energy speed of light, hence there is a maximal synchrotron frequency $\omega_{c}^{\text{max}}$ (A.3) that it can produce, regardless of its energy [46] (for details see Appendix A). Thus for at least one electron or positron helicity $\omega_{c}^{\text{max}}$ must be greater than the maximum observed synchrotron emission frequency $\omega_{\text{obs}}$. This yields the constraint

$$\eta > -\frac{M}{m} \left(\frac{0.34 eB}{m\omega_{\text{obs}}}\right)^{3/2}.$$  \hspace{1cm} (23)

The strongest constraint is obtained in the case of a system that has the smallest $B/\omega_{\text{obs}}$ ratio. This occurs in the Crab nebula, which emits synchrotron radiation up to 100 MeV and has a magnetic field no larger than 0.6 mG in the emitting region. Thus we infer that at least one of $\pm \eta_{\pm}$ must be greater than $-7 \times 10^{-8}$. This constraint is shown as a dashed line in Fig. 1. As with the soft Čerenkov constraint, by itself this imposes no constraint on the parameters, but as discussed in the next subsection it plays a role in a combined constraint using also the Čerenkov effect.

5.8 Vacuum Čerenkov effect—synchrotron electrons

The existence of the synchrotron producing charges can be exploited to extend the vacuum Čerenkov constraint. For a given $\eta$ satisfying the synchrotron bound, some definite electron energy $E_{\text{synch}}(\eta)$ must be present to produce the observed synchrotron radiation. (As explained in Appendix A, this is higher for negative $\eta$ and lower for positive $\eta$ than the Lorentz invariant value [46].) Values of $|\xi|$ for which the vacuum Čerenkov threshold is lower than $E_{\text{synch}}(\eta)$ for either photon helicity can therefore be excluded [63]. This is always a hard photon threshold, since the soft photon threshold occurs when the electron group velocity reaches the low energy speed of light, whereas the velocity required to produce any finite synchrotron frequency is smaller than this. The corresponding constraint is shown by the dashed line labelled “Synch. Čerenkov” in Fig. 1. This constraint improves on the current birefringence
limit on $\xi$ in the parameter region $\eta \lesssim 10^{-4}$. At least one of the four pairs $(\pm \eta_\pm, |\xi|)$ must satisfy both this constraint and the synchrotron constraint discussed above. This amounts to saying that one of the two pairs $(|\eta_\pm|, |\xi|)$ must satisfy this combined constraint or its reflection about the $\xi$ axis. As with the hard Čerenkov constraint discussed above, this imposes a lower bound on one of $|\eta_\pm|$ once $|\xi|$ is large enough.

The synchrotron and Čerenkov constraints can further be linked, to obtain an upper bound on one of $|\eta_\pm|$ as follows. We know that (automatically) at least one of the four parameters $\pm \eta_\pm$ satisfies the synchrotron constraint (a negative lower bound), and at least one of them (automatically) satisfies the IC Čerenkov constraint (a positive upper bound). In fact, at least one of the four parameters must satisfy both of these constraints. The reason is that otherwise the synchrotron charges would violate the Čerenkov constraint, hence their energy would necessarily be under 50 TeV, which is thirty times lower than the Lorentz invariant value of 1500 TeV for the highest energy synchrotron charges. By itself this is not impossible since, as explained in Appendix A, if $\eta$ is positive a charge with lower energy can produce high frequency synchrotron radiation. However, the Crab spectrum is well accounted for with a single population of charges responsible for both the synchrotron radiation and the IC $\gamma$-rays. If there were enough charges to produce the observed synchrotron flux with thirty times less energy per electron, then the charges that do satisfy the Čerenkov constraint would presumably be at least equally numerous (since their $\eta$ is smaller so helicity decay and source effects would, if anything, produce more of them), and so would produce too many IC $\gamma$-rays [63]. Thus at least one of $\pm \eta_\pm$ must satisfy together the synchrotron, synchrotron–Čerenkov, and the IC Čerenkov constraints. That is, one of the four pairs $(\pm \eta_\pm, \xi)$ must fall within the dashed bell-shaped boundary in Fig. 1. This amounts to the statement that one of the pairs $(\eta_\pm, \xi)$ must fall within the union of the bell-shaped boundary and its reflection about the $\xi$ axis. This imposes the two-sided upper bound $|\eta_\pm| < 10^{-2}$ on one of the two parameters $\eta_\pm$.

5.9 Helicity decay

The constraint $\Delta \eta = |\eta_+ - \eta_-| < 4$ on the difference between the LV parameters for the two electron helicities was deduced by Myers and Pospelov [45] using a previous spin-polarized torsion pendulum experiment [87].$^9$ Using the photon decay bound $|\eta_\pm| < 0.2$ discussed here, we infer the stronger bound

$^9$ They also determined a numerically stronger constraint using nuclear spins, however this involves four different LV parameters, one for the photon, one for the up-down quark doublet, and one each for the right handed up and down quark singlets. It also requires a model of nuclear structure.
\[ \Delta \eta < 0.4. \] In this section we discuss the possibility of improving on this constraint using the process of helicity decay.

If \[ \eta_- > \eta_+ \], negative helicity electrons are unstable to decay into positive helicity electrons via photon emission. (We assume for this section that \( \eta_- > \eta_+ \), the opposite case works similarly.) This reaction has no kinematic threshold. However, the rate is very small at energies below an effective threshold \( (m^2 M/\Delta \eta)^{1/3} \approx 10 \text{ TeV} \) (see Appendix C.7 for explicit rate calculations). The decay lifetime is minimized at the effective threshold. Below that it is longer than at least \( (\Delta \eta)^{-3}(p/10 \text{ TeV})^{-8} \times 10^{-9} \text{ seconds} \), while above it is given by \( \sim (p/10 \text{ TeV}) \times 10^{-9} \text{ seconds} \), independent of \( \Delta \eta \).

While accelerator energies are well below the effective threshold if \( \Delta \eta \) is \( O(1) \), in principle one might still get a bound by looking at fractional loss for a large population of polarized accelerator electrons. In practice this is probably impossible due to other polarizing and depolarizing effects in storage rings. But just to see what would be required to improve the current bound suppose (optimistically) that flipping of one percent of the electrons stored for \( 10^4 \) seconds could in be detected. The experimental exclusion of this phenomenon would require that the lifetime be greater than \( 10^6 \) seconds. Using our overestimate of the decay rate (C.30) this would yield the constraint \( \Delta \eta < 10^3(p/10 \text{ GeV})^{-8/3} \).

To improve on the photon decay constraint would then require electron energies at least around 200 GeV. When it was running LEPII produced 100 GeV beams, but the currently highest energy electrons in storage rings are around 30 GeV at HERA and 10 GeV at BABAR and the KEK B factory. Hence even if the non-LV polarization effects could be somehow factored out, a useful helicity decay bound from accelerators does not appear to be currently attainable.

A helicity decay bound can also be inferred using the charged leptons in the Crab nebula. We have just argued that at least one of the two parameters \( \eta_\pm \) must lie within the dashed, bell-shaped region together with its reflection about the \( \xi \) axis. Let us assume that the pair \((\eta_-, \xi)\) is inside the allowed region. We can divide the region into three parts, A where \( \eta_- > 7 \cdot 10^{-8} \), B where \( |\eta_-| < 7 \cdot 10^{-8} \), and C where \( \eta_- < -7 \cdot 10^{-8} \). In the first case negative helicity electrons can produce the synchrotron radiation, but positive helicity positrons (which have LV parameter \( -\eta_- \)) are below the synchrotron bound, so cannot. Now, if \( \eta_+ < -0.01 \) neither positive helicity electrons or negative helicity positrons can be responsible for the observed synchrotron radiation, since the electrons cannot emit the necessary frequencies and the positrons lose energy too rapidly via vacuum Cerenkov emission. Thus, in this case, of the four populations of leptons only the negative helicity electrons can produce the synchrotron radiation. However, an \( \eta_+ \) this low would allow rapid helicity decay (since the particle energy must be above 50 TeV which is around the effective helicity decay threshold when \( \Delta \eta = 0.01 \)) from negative to positive...
helicity electrons, leaving no charges to produce the synchrotron. So if $\eta_-$ is in region A (which violates Lorentz symmetry) we infer the lower bound $\eta_+ > -0.01$. Similarly, if $\eta_-$ is in region C, the same logic implies the upper bound $\eta_+ < 0.01$. If $\eta_-$ is in region B, then negative helicity electrons and positive helicity positrons are both able to produce the synchrotron radiation. Whatever the value of $\eta_+$, at least one of $\pm(\eta_- - \eta_+)$ is positive, hence at least one of these two species is always stable to helicity decay, so no helicity bound can be presently inferred with one parameter in region B. If the Crab spectrum could be modelled and observed precisely enough to know that both (or all four) species must contribute to the synchrotron radiation, a helicity bound could be obtained in case B.

6 Other types of constraints on LV

In this section, we briefly summarize and point to references for various constraints on LV effects besides those associated with $O(E/M)$ effects in QED.

6.1 Constraints on dimension 3 and 4 operators

For the $n = 2$ term in (2,3), the absence of a strong threshold effect yields a constraint $\eta_2 \lesssim (m/p)^2(M/\mu)$. If we consider protons and put $\mu = m = m_p \sim 1$ GeV, this gives an order unity constraint when $p \sim \sqrt{mM} \sim 10^{19}$ eV. Thus the GZK threshold (see the following subsection), if confirmed, can give an order unity constraint, but multi-TeV astrophysics yields much weaker constraints. The strongest laboratory constraints on dimension three and four operators for fermions come from clock comparison experiments using noble gas masers [88]. The constraints limit a combination of the coefficients for dimension three and four operators for the neutron to be below $10^{-31}$ GeV (the dimension four coefficients are weighted by the neutron mass, yielding a constraint in units of energy). This corresponds to a bound on $\eta_1$ of order $10^{-12}$ in the parametrization of (3) with $\mu = 1$ GeV. For more on such constraints see e.g. [33,89]. Astrophysical limits on photon vacuum birefringence give a bound on the coefficients of dimension four operators of $10^{-32}$ [80].

6.2 Constraints at $O(E/M)$ from UHE cosmic rays

In collisions of ultra high energy (UHE) protons with cosmic microwave background (CMB) photons there can be sufficient energy in the center of mass
frame to create a pion, leading to the reaction \( p + \gamma_{\text{CMB}} \rightarrow p + \pi^0 \). The so-called GZK threshold \([34]\) for the proton energy in this process is

\[
E_{\text{GZK}} \approx \frac{m_p m_\pi}{2E_\gamma} \approx 3 \times 10^{20} \text{eV} \times \left(\frac{2.7 \text{K}}{E_\gamma}\right)
\] (24)

To get a definite number we have put \( E_\gamma \) equal to the energy of a photon at the CMB temperature, 2.7K, but of course the CMB contains photons of higher energy. This process degrades the initial proton energy with an attenuation length of about 50 Mpc. Since plausible astrophysical sources for UHE particles are located at distances larger than 50 Mpc, one expects a cutoff in the cosmic ray proton energy spectrum, which occurs at around \( 5 \times 10^{19} \text{ eV} \), depending on the distribution of sources \([90]\).

One of the experiments measuring the UHE cosmic ray spectrum, the AGASA experiment, has not seen the cutoff. An analysis \([91]\) from January 2003 concluded that the cutoff was absent at the 2.5 sigma level, while another experiment, HiRes, is consistent with the cutoff but at a lower confidence level. (For a brief review of the data see \([90]\).) The question should be answered in the near future by the AUGER observatory, a combined array of 1600 water Čerenkov detectors and 24 telescopic air fluorescence detectors under construction on the Argentine pampas \([92]\). The new observatory will see an event rate one hundred times higher, with better systematics.

Many ideas have been put forward to explain the possible absence of the GZK cutoff \([90]\), one being Lorentz violation. According to equation (24) the Lorentz invariant threshold is proportional to the proton mass. Thus any LV term added to the proton dispersion relation \( E^2 = p^2 + m_p^2 \) will modify the threshold if it is comparable to or greater than \( m_p^2 \) at around the energy \( E_{\text{GZK}} \). Modifying the proton and pion dispersion relations, the threshold can be lowered, raised, or removed entirely, or even an upper threshold where the reaction cuts off could be introduced (see e.g. \([53]\) and references therein).

If ultra-high energy cosmic rays (UHECR) are (as commonly assumed) protons, then strong constraints on \( n = 3 \) type dispersion can be deduced from a) the absence of a vacuum Čerenkov effect at GZK energies and b) the position of the GZK cutoff if it will be actually found.

a) For a soft emitted photon with a long wavelength, the partonic structure of a UHECR proton is presumably irrelevant. In this case we can treat the proton as a point particle as in the QED analysis. With a GZK proton of energy \( 5 \times 10^{19} \text{ eV} \) the constraint from the absence of a vacuum Čerenkov effect is \( \eta < O(10^{-14}) \) \([53]\). Since the helicity of cosmic rays is not observed, one can say only that this constraint must be satisfied for at least one helicity.
For a hard emitted photon, the partonic nature of the proton is important and the relevant mass scale will involve the quark mass. The exact calculation considering the partonic structure for $n = 3$ has not been performed. Treating the proton as structureless, the threshold region would be similar to that in [53]. The allowed region in the $\eta - \xi$ plane would be bounded on the right by the $\xi$ axis (within a few orders of magnitude of $10^{-14}$) and below by the line $\xi = \eta$ [53]. This constraint applies to both photon helicities, but only to one proton helicity, since the UHECR could consist all of a single helicity. In principle, however, what one can really constrain is some combinations of the various quark dispersion parameters. This approach has been worked out in detail using parton distribution functions in [93].

b) If the GZK cutoff is observed in its predicted place, this will place limits on the proton and pion parameters $\eta_p$ and $\eta_\pi$. For example, if the GZK cutoff is eventually observed to be somewhere between 2 and 7 times $10^{19}$ eV then there are strong constraints of $O(10^{-11})$ on the relevant $\eta_p$ and $\eta_\pi$ [53]. (Allowing for helicity dependence, no set of parameters allowing long distance propagation (forbidding the vacuum Čerenkov effect) should modify the GZK cutoff.)

As a final comment, an interesting possible consequence of LV is that with upper thresholds, one could possibly reconcile the AGASA and Hi-Res/Fly’s Eye experiments. Namely, one can place an upper threshold below $10^{21}$ eV while keeping the GZK threshold near $5 \times 10^{19}$ eV. Then the cutoff would be “seen” at lower energies but extra flux would still be present at energies above $10^{20}$ eV, potentially explaining the AGASA results [53]. The region of parameter space for this scenario is terribly small, however, again of $O(10^{-11})$.

6.3 Constraints on dimension 6 operators

As previously mentioned, CPT symmetry alone could exclude the dimension five LV operators in QED that give $O(E/M)$ modifications to particle dispersion relations. Moreover, the constraints on those have become quite strong. Hence we close with a brief discussion of the constraints that might be possible at $O(E^2/M^2)$. Such LV effects arise for example from dimension 6 operators. Note that helicity dependence of the LV parameters is not required in this case, and on the other hand it can occur without violating CPT. Without any particular theoretical prejudice, one should keep in mind that constraints will generally only limit the parameters for some helicity species, while they might be evaded for other helicities.

As previously discussed we can estimate that the LV modification of the dispersion relation becomes important when comparable with the mass term,
\( \eta_4 p^4 / M^2 \lesssim m^2 \), which yields

\[
\eta_4 \lesssim \left( \frac{m}{1 \text{ MeV}} \frac{10^{17} \text{ eV}}{p} \right)^4.
\] (25)

Thus, for electrons, an energy around \( 10^{17} \text{ eV} \) is needed for an order unity constraint on \( \eta_4 \), and we are probably not going to see any effects directly from such electrons.

For protons an energy \( \sim 10^{18} \text{ eV} \) is needed. This is well below the UHE cosmic ray energy cutoff, hence if and when Auger [92] confirms the identity of UHE cosmic rays as protons at the GZK cutoff, an impressive constraint of order \( \eta_4 \lesssim 10^{-5} \) will follow from the absence of vacuum Čerenkov radiation for \( 10^{20} \text{ eV} \) protons. From the fact that the GZK threshold is not shifted, a constraint of order \( \eta_4 \gtrsim 10^{-2} \) will follow, assuming equal \( \eta_4 \) values for proton and pion.

In fact, if one assumes the cosmic rays already observed near but below the GZK cutoff are hadrons, one obtains a strong bound [93]. Depending on the species and helicity dependence of the LV coefficients, bounds of order \( 10^{-2} \) or better can be placed on \( \eta_4 \). The bounds claimed in [93] are actually two sided, and come about in a manner somewhat analogous to (but more complicated than) the photon decay constraint (see section 4.3.1). They are derived by using a parton model for particles where the LV coefficients apply to the constituent partons. By considering many different outgoing particle spectra from an incoming hadron in combination with the parton approach the authors of [93] are able to find sets of reactions that yield two sided bounds. Hence, the parton approach is quite useful, as it dramatically increases the number of constraints that can be derived from a single incoming particle.

One might think that impressive constraints can also be obtained from the absence of neutrino vacuum Čerenkov radiation: putting in 1 eV for the mass in (25) yields an order unity constraint from 100 TeV neutrinos, but only if the Čerenkov rate is high enough. The rate will be low, since it proceeds only via the non-local charge structure of the neutrino. Recent calculations [94] have shown that the rate is not high enough at that energy, even for cosmogenic neutrinos. However, for \( 10^{20} \text{ eV} \) UHE neutrinos, which may be observed by the proposed EUSO (Extreme Universe Space Observatory) [95] and/or OWL (Orbiting Wide-angle Light collectors) [96] satellite observatories, the rate will be high enough to derive a strong constraint as long as the neutrinos are cosmogenic, and perhaps even if they originate closer to the earth. The specific value of the constraint would depend on the exact value of the rate, which has not yet been computed. For a gravitational Čerenkov reaction, the rate (which is lower but easier to compute than the electromagnetic rate) would be high enough for a \( 10^{20} \text{ eV} \) neutrino from a distant source to radiate provided \( \eta_4 \gtrsim 10^{-2} \). Hence in this case one might obtain a constraint of order \( \eta_4 \lesssim 10^{-2} \).
from gravitational Čerenkov. If Čerenkov constraints apply only when the observed UHE neutrino originates from a distant source, one would need to either identify an astrophysical object as the source or somehow otherwise rule out local generation of the neutrino.

A time of flight constraint at order $(E/M)^2$ might be possible [97] if gamma ray bursts produce UHE ($\sim 10^{19} \text{ eV}$) neutrinos, as some models predict, via limits on time of arrival differences of such UHE neutrinos vs. soft photons (or gravitational waves). Another possibility is to obtain a vacuum birefringence constraint with higher energy photons [81], although such a constraint would be less powerful since EFT does not imply that the parameters for opposite polarizations are opposite at order $(E/M)^2$. If future GRB’s are found to be polarized at $\sim 100 \text{ MeV}$, that could provide a birefringence constraint $|\xi_{4+} - \xi_{4-}| \lesssim 1$.

7 Future prospects

In the last decade or so the old dogma that all our observations are insensitive to Planck scale effects has been shown to be quite wrong if Lorentz symmetry is violated. A large number of impressively strong constraints have been obtained on LV Planck scale effects arising from operators of mass dimension three, four, five, and even six.

From the conceptual point of view, the most burning issue is naturalness of small low energy LV. We have explained in this article both the rationale for adopting an effective field theory parametrization of LV, and the “naturalness” problem of preserving approximate low energy Lorentz invariance if LV is to exist in the UV theory. As discussed in section 3.4, currently the best prospect for resolving this issue in favor of LV is via supersymmetry, but perhaps other ideas could work. In the Lorentz violating, softly broken SUSY QED framework of Ref. [61], dimension five (CPT violating) LV operators are already strongly constrained by current QED experiments, but the dimension six LV operators are not yet constrained at the $M_{\text{Planck}}^2$ level. However the authors of that reference indicate that Planck level constraints on dimension six LV quark operators in the standard model may be possible. We emphasize that high energy LV dispersion of the sort discussed in the present paper does not occur in the scenario of Ref. [61].

Constraints continue to improve with advances in observational technology. In this concluding section we examine some of the future prospects for improving the constraints on high energy LV using astrophysical observations. To begin, we note that while photon “time of flight” constraints are not competitive with other current constraints, they enjoy a special status in being less dependent
on assumptions about the underlying theoretical framework. Given that time of flight constraints are strongest with a high energy signal that has structure on short time scales, gamma ray bursts are probably the best objects for improving these constraints. Experiments like GLAST (Gamma-ray Large Area Space Telescope) [98] may be helpful in improving bounds from time of flight; they might lead to a better understanding of GRB emissions and provide far better photon counts and time resolution (less than $10\mu$sec). These improvements might help overcome some of the problems involving GRB time of flight measurements discussed in section 5.1. See e.g. [78] for a detailed discussion of these issues.

Here we focused on LV in the dimension five, CPT violating sector of QED, characterized by the photon parameter $|\xi|$ bounded at $O(10^{-4})$ and the electron parameters $|\eta_\pm|$ bounded collectively at $O(10^{-1})$, while at least one of them is bounded at $O(10^{-2})$. The most promising prospect for significant improvement in these bounds would be a closer study of the effect of this sort of LV on the complete electromagnetic spectrum of the Crab nebula and or other plerions. We have been very conservative in imposing constraints, by allowing for the possibility that with LV the synchrotron radiation could be produced by much lower energy charges than required in the usual Lorentz invariant case. We have also tolerated the possibility of four different populations of charges behaving differently (two helicities for electrons and positrons). By modelling more completely the production of the electron-positron wind and electromagnetic spectrum in the presence of such LV it may be possible to infer that charges of energy 1500 TeV are in fact required as in the Lorentz invariant case. If so, the constraints on $\eta_\pm$ might be improved by a factor of order $\sim (50/1500)^3 \sim 10^{-4}$ to $|\eta_\pm| \lesssim 10^{-6}$, and that on $|\xi|$ by a factor of order $\sim 10^{-2}$ to $|\xi| \lesssim 10^{-6}$ as well. This would seem to be rather definitive so is a useful goal. To achieve it would perhaps require not only better modelling of the LV effects but more precise observations of the Crab nebula. In this regard we can look forward to the observations to be made with GLAST which should start its observations in 2007.

The birefringence bound, currently of order $|\xi| \lesssim 10^{-4}$ from UV polarization observations of a distant galaxy, might be improved in various ways. Since the LV effect scales as the square of the energy, the most dramatic improvement would be if polarized gamma rays from a distant source could be observed, for example from a GRB. (The observation reported in [82] appears now to be unreliable.) This might be possible with RHESSI or via the IBIS camera of the INTEGRAL satellite [99], using the so-called Compton mode that was used in the previous RHESSI analyses. Less dramatically, it might instead be possible to improve by an order of magnitude the constraint reported in [42], obtained using UV light from the radio galaxy 3C 256, via specifically targeted observations possibly using FORS1 [100] at the ESO Very Large Telescope (VLT), and/or by observing more distant sources of polarized UV light.
Bounds on LV effects on threshold reactions for ultra-high energy cosmic rays can be significant even when suppressed by two powers of the Planck mass, i.e. at order $E^2/M^2$. This is important since CPT symmetry alone could preclude effects at order $E/M$. We already discussed the currently available bounds. The possibility of improvement lies in experiments which will be capable of reaching high sensitivities at the small fluxes associated with the highest energy particles. The Auger detector will be the next big thing, with first release of data expected in summer 2005.

It is conceivable that ultra-high energy neutrinos could provide extremely stringent constraints on LV due to their small mass. The vacuum Čerenkov effect occurs at a given energy with a minimum LV coefficient that scales with mass as $m^2$. On the other hand the rate for the reaction is very low, so it is not clear whether in practice this possibility will pan out. UHE neutrinos might be observed using IceCube [101], or planned detectors such as the cited satellites EUSO [95] and OWL [96] (the former being already in an advanced stage of development). Also promising are upcoming experiments like the sea based detector NESTOR (Neutrino Extended Submarine Telescope with Oceanographic Research) [102] which is now starting its activity, or the balloon-borne detector ANITA (Antarctic Impulse Transient Antenna) [103] whose first flight is planned for the end of 2006. Finally also very effective could be some proposed experiments like the underground salt based detector SalSA (Salt-dome Shower Array) [104].

In closing, we have been motivated by considerations of quantum gravity to look for evidence of, and bounds on, Lorentz violation. These considerations include suggestions of LV from various viewpoints, as well as simple desperation for observational guidance. The constraints discussed here and elsewhere are mounting up to a significant limitation on the possibility of LV in quantum gravity, at least insofar as would appear in low energy effective field theory. It seems very worthwhile to push on, even if the ultimate result will only be extremely restrictive constraints. In this way we will have solidified the observational foundations of Lorentz symmetry, and acquired valuable constraints on future speculations on the nature of quantum gravity.

A LV cutoff of synchrotron radiation frequency

Cycling electrons in a magnetic field $B$ emit synchrotron radiation with a spectrum that sharply cuts off at a frequency $\omega_c$ given in the Lorentz invariant case by the formula

$$\omega_c = \frac{3}{2} eB \gamma^2(E) \frac{E}{E},$$  \hspace{1cm} (A.1)
where $\gamma(E) = (1 - v^2(E)/c^2)^{-1/2}$. Here $v(E)$ is the electron group velocity, and $c$ is the usual low energy speed of light. In standard relativistic physics, $E = \gamma m$, so the energy dependence in (A.1) is entirely through the factor $\gamma^2$, which grows without bound as the energy grows. Arbitrarily large synchrotron frequencies are therefore possible.

In the Lorentz violating case with negative values of $\eta$, electrons have a maximal group velocity strictly less than the low energy speed of light, hence there is a maximal synchrotron frequency that can be produced, regardless of the electron energy \[46].\footnote{Since we have LV in the photon sector as well, there is no “speed of light” per se. However, the emitted frequencies of synchrotron radiation are much lower than the energy of the source particles in the Lorentz invariant case. Effectively the LV violation in the photon sector can be ignored.} As we shall argue below, Eq. (A.1) still holds in this case, assuming the framework of effective field theory. Therefore the maximal frequency is obtained by maximizing $\gamma^3(E)/E$ with respect to the electron energy. Using the difference of group velocities

$$c - v(E) \simeq \frac{m^2}{2E^2} - \eta \frac{E}{M},$$

obtained from the electron dispersion relation (13), we find that this maximization yields

$$\omega_c^{\text{max}} = 0.34 \frac{eB}{m} (-\eta m/M)^{-2/3}.$$  

This maximum frequency is attained at the energy $E_{\text{max}} = (-2m^2M/5\eta)^{1/3} = 10 \left(-\eta\right)^{-1/3}$ TeV. This is higher than the energy that produces the same cutoff frequency in the Lorentz invariant case, but only by a factor of order unity.

Note that if $\eta$ is positive, then the effect is the opposite: an electron can produce a given frequency of synchrotron radiation with an energy less than in the Lorentz invariant case. The electron speed can even exceed the low frequency speed of light, at which point $\gamma(E)$ diverges. This corresponds to the soft Čerenkov threshold discussed in section C.5.

We shall now justify the derivation of (A.1) in the LV case, adding some details to previously published work\[46]. The first step is a purely kinematical analysis that does not assume Lorentz invariance, which follows the standard heuristic derivation \[105\]. A cycling electron of energy $E$ emits radiation in a cone of some opening angle $\delta(E)$. The cone sweeps past a distant observer as the electron moves on a circle of radius $R(E)$ through an angle $\delta(E)$. The electron travels at a speed (group velocity) $v(E)$ so the time it takes to orbit through
the angle $\delta(E)$ is $\Delta t = R(E)\delta(E)/v(E)$. The light from the leading edge of the cone travels a distance $c(\omega)\Delta t$ while the electron travels the distance $v(E)\Delta t$ toward the observer. Hence the spatial width of the pulse seen by the observer is approximately $(c(\omega) - v(E))\Delta t$, which arrives at the observer over a time interval equal to this distance divided by the speed of light. The cut off frequency of the synchrotron pulse is roughly the inverse of this time interval,

$$\omega_c = \frac{3}{4} \frac{1}{R(E)\delta(E)} \frac{1}{c(\omega_c) - v(E)}, \quad (A.4)$$

where we have used the fact that the electron and photon speeds are very close to the low energy speed of light $c$, which is set equal to unity. In the Lorentz invariant case the radius is given by $R(E) = E/eB$ and the opening angle is $\delta(E) \sim \gamma^{-1}(E)$. The numerical constant in (A.4) is chosen so that when these values are substituted the correct relativistic result (A.1) is obtained.

Under the assumption that the Lorentz violation is described by effective field theory, we will argue that $R(E)$ and $\delta(E)$ are the same in the LV case as in the Lorentz invariant case. Moreover, since the emitted photons have relatively low energy compared to the electrons, it turns out that $\xi$ can be neglected in the relevant region of parameter space. (As shown in [46] the photon energy is low enough to neglect any possible LV correction as long as $|\xi| \lesssim 10^{11}(-\eta)^{4/3}$.) Thus $c(\omega_c)$ in (A.4) can be replaced by $c$, so the reciprocal of the difference of group velocities is well approximated by $2\gamma^2(E)$, as in the Lorentz invariant case. This yields (A.1) where now all the dependence on the LV is in how the gamma factor depends on $E$.

The radius $R(E)$ is determined by the equation of motion for the electron in a magnetic field. All Lorentz violating terms in the equation of motion are suppressed by the ratio $E/M$. From this it is clear that the trajectory determined by a given initial position and momentum is not much affected by the Lorentz violation, as long as $E$ remains much smaller than the Planck energy. \textsuperscript{11}

This conclusion about $R(E)$ can be more explicitly verified by reference to the modified Hamilton’s equations, with the Hamiltonian given in terms of the momentum by the dispersion relation between energy and momentum. The leading high energy corrections come from modifications to the minimal coupling terms. As usual the minimal coupling is incorporated replacing the momentum by $p - eA$, where $A$ is a vector potential for the magnetic field.

\textsuperscript{11} A subtlety here is that this argument fails if we refer to the velocity rather than to the momentum. The reason is that tiny differences in the speed make a huge difference in the momentum and energy when the speed is close to the speed of light.
This yields the equation of motion $a = [1 + 3\eta E/2M](e/E)\mathbf{v} \times \mathbf{B}$, where we have kept only the lowest order term in $\eta$ and assumed relativistic energy $E \gg m$. Since $E \ll M$, the presence of the Lorentz violation makes very little difference to the orbital equation, hence we conclude that to a very good approximation the radius is related to the magnetic field and the energy of the electron by the standard formula $R(E) = E/eB$ (where again the speed of the electron has been set equal to unity). This result was independently confirmed by [106] from the starting point of the Dirac equation.

The angle $\delta(E)$ scales in the Lorentz invariant case as $\gamma^{-1}(E)$. Since $R(E)$ and hence the charge current is nearly unaffected by the Lorentz violation, any significant deviation in $\delta(E)$ could only come from the modified response of the electromagnetic field to the given current. The Lagrangian in the presence of the dimension five LV operator for the EM field in (6) takes the form

$$L = \frac{1}{4} F_{ab} F^{ab} + \frac{\xi}{M} G(A) + A_a j^a,$$

(A.5)

where $\xi G(A)/M$ is the LV operator and $j^a$ is the given current. The equations of motion are

$$\partial_a F^{ab} - \frac{\xi}{M} \frac{\partial G(A)}{\partial A_b} = j^b.$$  

(A.6)

If we expand $A^a = A^a_0 + A^a_1$, where $A^a_0$ is the field in the Lorentz invariant case, then to lowest order the LV correction $A^a_1$ satisfies

$$\partial_a F^{ab}_1 = \frac{\xi}{M} \frac{\partial G(A^a_0)}{\partial A_a}.$$  

(A.7)

The correction $A^a_1$ is suppressed by at least one power of $M$. In terms of Fourier components, this suggests that the relative size of the LV correction is of order $\xi \omega/M$, which is extremely small for even the peak photon energies $\approx 10^8$ eV present in the synchrotron radiation from the Crab nebula. Therefore near the source current the amplitude of the electromagnetic field modification is negligible in comparison to the zeroth order synchrotron radiation (which is still present), and hence $\delta(E)$ scales with $\gamma^{-1}(E)$ in the usual way. Note that this does not mean that the relative size of the correction is small everywhere. The source of $A^a_1$ on the right hand side of (A.7) is non-vanishing not just where the current $j^b$ lies but also wherever the emitted radiation propagates. It can therefore produce a cumulative effect that can be large. In fact, this must clearly happen, since the group velocity is modified by the LV term, so that after enough propagation the relative correction must become of order unity so that it can shift the support of the radiation to a disjoint location.
A detailed analysis of synchrotron radiation in this LV QED theory was carried out in Ref. [107]. Our heuristically obtained results were confirmed there, and certain regimes were identified in which the LV solution deviates strongly from the Lorentz invariant one. Our interpretation of such deviations is what was mentioned at the end of the previous paragraph, i.e. that these are cumulative effects. A claim of significant deviations was made in Ref. [108] however we do not currently understand the basis of that claim.

B Threshold configurations

Threshold configurations and new phenomena in the presence of LV dispersion relations were systematically investigated in [39,69] (see also references therein). We give here a brief summary of the results. We shall consider reactions with two initial and two final particles (results for reactions with only one incoming or outgoing particle can be obtained as special cases). Following our previous choice of EFT we allow each particle to have an independent dispersion relation of the form (1) with \( E(p) \) a monotonically increasing non-negative function of the magnitude \( p \) of the 3-momentum \( p \). While the assumption of monotonicity could perhaps be violated at the Planck scale, it is satisfied for any reasonable low energy expansion of a LV dispersion relation. EFT further implies that energy and momentum are additive for multiple particles, and conserved.

Consider a four-particle interaction where a target particle of 3-momentum \( p_2 \) is hit by a particle of 3-momentum \( p_1 \), with an angle \( \alpha \) between the two momenta, producing two particles of momenta \( p_3 \) and \( p_4 \). We call \( \beta \) the angle between \( p_3 \) and the total incoming 3-momentum \( p_{in} = p_1 + p_2 \). We define the notion of a threshold relative to a fixed value of the magnitude of the target momentum \( p_2 \). A lower or upper threshold corresponds to a value of \( p_1 \) (or equivalently the energy \( E_1 \)) above which the reaction starts or stops being allowed by energy and momentum conservation.

We now introduce a graphical interpretation of the energy-momentum conservation equation that allows the properties of thresholds to be easily understood. For given values of \( (p_1, p_2, \alpha, \beta, p_3) \), momentum conservation determines \( p_4 \). Since \( p_3 \) and \( p_4 \) determine the final energies \( E_3 \) and \( E_4 \), we can thus define the final energy function \( E_f^{\alpha, \beta, p_3}(p_1) \). (Since \( p_2 \) is fixed we drop it from the labelling.) Energy conservation requires that \( E_f \) be equal to \( E_i(p_1) \), the initial energy (again, we do not indicate the dependence on the fixed momentum \( p_2 \)).

Now consider the region \( \mathcal{R} \) in the \( (E, p_1) \) plane covered by plotting \( E_f^{\alpha, \beta, p_3}(p_1) \) for all possible configurations \( (\alpha, \beta, p_3) \). An example is shown in Figure B.1. The region \( \mathcal{R} \) is bounded below by \( E = 0 \) since the particle energies are
assumed non-negative, hence it has some bounding curve $E_B(p_1)$. Similarly one can plot $E_i(p_1)$. The reaction threshold occurs when $E_i(p_1)$ enters or leaves $\mathcal{R}$, since it is precisely in $\mathcal{R}$ that there is a solution to the energy and momentum conservation equations.

This graphical representation demonstrates that in any threshold configuration (lower or upper) occurring at some $p_1$, the parameters $(\alpha, \beta, p_3)$ are such that the final energy function $E_f^{\alpha, \beta, p_3}(p_1)$ is minimized. That is, the configuration always yields the minimum final particle energy configuration conserving momentum at fixed $p_1$ and $p_2$. From this fact, it is easy to deduce two general properties of these configurations:

1. All thresholds for processes with two outgoing particles occur at parallel final momenta ($\beta = 0$).
2. For a two particle initial state the momenta are antiparallel at threshold ($\alpha = \pi$).

These properties are in agreement with the well known case of Lorentz invariant kinematics. Nevertheless, LV thresholds can exhibit new features not present in the Lorentz invariant theory, in particular upper thresholds, and asymmetric pair creation.
Figure B.1 clearly shows that LV allows for a reaction to not only to start at some lower threshold but also to end at some upper threshold where the curve $E_i$ exits the region $\mathcal{R}$. It can even happen that $E_i$ enters and exits $\mathcal{R}$ more than once, in which case there are what one might call “local” lower and upper thresholds.

Another interesting novelty is the possibility to have a (lower or upper) threshold for pair creation with an unequal partition of the initial momentum $p_{in}$ into the two outgoing particles (i.e. $p_3 \neq p_4 \neq p_{in}/2$). Equal partition of momentum is a familiar result of Lorentz invariant physics, which follows from the fact that the final particles are all at rest in the zero-momentum frame at threshold. This has often been (erroneously) presumed to hold as well in the presence of LV dispersion relations.

A reason for the occurrence of asymmetric LV thresholds can be seen graphically, as shown in Figure B.2. Suppose the dispersion relation for a massive outgoing particle $E_{out}(p)$ has negative curvature at $p = p_{in}/2$, as might be the case for negative LV coefficients. Then a small momentum-conserving displacement from a symmetric configuration can lead to a net decrease in the final state energy. According to the result established above, the symmetric configuration cannot be the threshold one in such a case. A lower $p_1$ could satisfy both energy and momentum conservation with an asymmetric final configuration. A sufficient condition for the pair-creation threshold configuration to be asymmetric is that the final particle dispersion relation has negative curvature at $p = p_{in}/2$. This condition is not necessary however, since it could
happen that the energy is locally but not globally minimized by the symmetric configuration.

C Rates and thresholds of LV processes

In this appendix we first derive a general expression for reaction rates. We then discuss the rates and thresholds for three reactions of interest: photon decay, vacuum Čerenkov, and helicity decay.

C.1 General rate expressions

The first step in deriving constraints from particle interactions is to derive the rate $\Gamma$ for various processes. As it turns out, the most useful constraints are derived from reactions where a single particle decays into two particles. If we label the incoming particle momentum and helicity/polarization by $p, s$ and the final particles by $p’, s’, p”, s”$ then the rate for fixed helicities is given by [109]

$$\Gamma(p, s, s', s'') = \int \frac{1}{8E_{p,s}} \frac{d^3p’d^3p''}{(2\pi)^2E_{p’,s'}E_{p'',s''}} |M(p, s, p’, s’, p'', s’’)|^2$$

\[\times \delta(E_{p,s} - E_{p’,s'} - E_{p'',s’’})\delta^3(p - p’ - p’’),\]  

where $M(p, s, p’, s’, p'', s’’)$ is the matrix element. As mentioned in section 4.1.2 our fermion spinors are not quite normalized to $2E$. As a result (C.1) is not quite correct. However, as long as we work in the preferred frame, this correction is small and can be neglected since it is only a factor in overall normalization.

If we choose the initial particle to be travelling in the $z$-direction, then we can integrate over $p''$ and apply the axial symmetry to get

$$\Gamma(p, s, s', s'') = \int \frac{1}{16\pi E_{p,s}} \frac{dp’_z dp’}{E_{p’,s'}E_{p’,s’’}E_{p – p’, s’’}} |M|^2 \delta(E_{p,s} - E_{p’,s'} - E_{p – p’, s’’}),\]  

Integrating over $p’_z$ then yields

$$\Gamma(p, s, s', s'') = \int \frac{p’_z dp’}{16\pi E_{p,s}E_{p’,s'}E_{p’ - p’, s’’}} |M|^2 \left| \frac{\partial(E_{p’,s’'} + E_{p’ - p’, s’’})}{\partial p’_z} \right|^{-1}.\]  

(C.3)
This expression and those that follow are evaluated at the solution of the energy conservation equation, \( p_\perp' = p_\perp'(p'_z, p, s, s', s'') \). The limits of integration \( p'_{z_1}, p'_{z_2} \) are the bounds between which such a solution exists. We are mostly interested in cases where there is no solution unless \( p \) is greater than some threshold momentum \( p_{th} \).

For reactions that do not occur in the Lorentz-invariant limit, the phase space must close up as we approach Lorentz invariance, i.e. the magnitude of \( p'_\perp \) vanishes as \( E_{pl} \rightarrow \infty \). Hence \( p'_\perp \) must be small relative to \( p'_z \) and \( p - p'_z \) if \( p \ll E_{pl} \). The exception to this is if either \( p'_z \approx 0 \) or \( p'_z \approx p \). This region is, however, a negligible amount of phase space and so we ignore it. Thus \( p'_\perp \) can be treated as an expansion parameter.

Expanding the derivative of \( E'_{p',s} + E''_{p-p',s''} \) with respect to \( p'_\perp \) gives

\[
|\partial(E'_{p',s} + E''_{p-p',s''})|^{-1} \approx \frac{E'_{p',s}E''_{p-p',s''}}{p'_\perp E_{p,s}}, \tag{C.4}
\]

and substituting into (C.3) gives

\[
\Gamma(p, s, s', s'') = \int_{p'_{z_1}}^{p'_{z_2}} \frac{1}{16\pi} \frac{dp'_{z_2}}{E^2_{p,s}} |M|^2. \tag{C.5}
\]

To compute an energy loss rate for the Čerenkov effect a \( p'_z \) must be inserted in the integrand to reflect the energy carried off by the emitted photon.

\section*{C.2 Photon decay matrix element}

We now show that, if the incoming momentum \( p \) is above the threshold \( p_{th} \), then the rate for photon decay is very high.

The QED matrix element for photon decay is

\[
iM = ie\bar{u}_s(p')\gamma^\pm\alpha_\ell\gamma^\alpha v_s(p'') \tag{C.6}
\]

where \( \bar{u}(p') \) is the outgoing electron spinor and \( v(p'') \) is the outgoing positron spinor. We consider the case where the electron and positron have positive and negative helicity respectively, so only the coefficient \( \eta_+ \) is involved. The LV term in the electron dispersion is \( \eta_+ p^3 \) (with \( u^+ \) wavefunction (12)) while that in the positron dispersion is \( -\eta_+ p^3 \) (with \( v^- \) wavefunction (16)). The result
is directly transferable to the $\eta_-$ case. We will also relabel $E(p)$ as $\omega(k)$ since the incoming particle is a photon.

With these choices the dominant contribution to the matrix element at high energy is straightforwardly evaluated as

$$iM = i e \sqrt{2E' \sqrt{2E''}} [\chi_+(p') \hat{\varepsilon}_+ \cdot \hat{\sigma} \chi_+(p'')]$$

which is equal to

$$i e \sqrt{2E' \sqrt{2E''}} \left[ \cos \left( \frac{\theta''}{2} \right) \sin \left( \frac{\theta'}{2} \right) (1 \pm 1) - \sin \left( \frac{\theta''}{2} \right) \cos \left( \frac{\theta'}{2} \right) (1 \mp 1) \right]$$

where $\theta', \theta''$ are the opening angles of the electron and positron and the $\pm$ reflects the initial photon polarization. (The overall phase depends on the phase convention for the spinors and does not affect the result.) Note that the matrix element vanishes in the threshold configuration when the momenta are all parallel. This is because we chose the electron and positron helicities to be opposite. Nevertheless, above threshold the rate is high enough to obtain a useful constraint, as we now argue.

The opening angles are small, since the perpendicular momenta are Planck suppressed. Therefore we can expand (C.8) to first order in $\theta', \theta''$, yielding

$$iM = i e \sqrt{E' \sqrt{E''}} \left[ \frac{p'_\perp}{p'_z} (1 \pm 1) - \frac{p''_\perp}{p''_z} (1 \mp 1) \right],$$

where $p'_\perp$ is the magnitude of the perpendicular momentum (which is the same for the electron and positron). Recall that $p'_\perp$ is determined from the energy conservation equation (where $k$ is the initial photon momentum),

$$\pm \frac{\xi k^2}{M} = m^2 \frac{p'_z}{p'_z} + \frac{(p'_\perp)^2}{M} + \eta_+(p'_z)^2 + \frac{m^2}{k - p'_z} + \frac{(p''_\perp)^2}{k - p''_z} - \frac{\eta_+(k - p'_z)^2}{M}.$$ (C.10)

In (C.10) we have already cancelled the 0th order terms using momentum conservation, and we have neglected higher order terms in $p'_\perp/p_z$. We have also assumed that the $z$-components of the electron and positron momenta are both positive. This is not necessarily the case, but when $m \ll k \ll M$ (which has also been assumed) we are only missing a tiny fraction of the phase space by making this restriction.

Defining $z = p'_z/k \in [0, 1]$, $p'_\perp$ is given by

$$p'_\perp^2 = [\pm \xi - \eta_+(2z - 1)] z (1 - z) \frac{k^3}{M} - m^2.$$ (C.11)
If the reaction is to happen at all, then the RHS of (C.11) must be positive. The points where \( p'_\perp = 0 \) give the upper and lower bounds \( p'_{z1}, p'_{z2} \) for the integral (C.5). Well above threshold the mass term becomes negligible, and the longitudinal phase space is determined by those \( z \) values for which \( \pm \xi - \eta_+ (2z - 1) \) is positive in the interval \( z \in [0, 1] \).

Using these results in the expression (C.5) for the decay rate we find that, well above threshold, the rate for decay of a positive helicity photon to a positive helicity electron and negative helicity positron is given by

\[
\Gamma \approx \frac{e^2 k^2}{48 \pi M} \left[ (2\xi + \eta_+) \Theta(\xi + \eta_+) + \frac{(\xi - \eta_+)^4}{16 |\eta_+|^3} \Theta(|\eta_+| - |\xi|) \right].
\] (C.12)

where \( \Theta(x) \) is the Heaviside step function. This agrees up to a numerical factor with the estimate in [67] in the case where \( \eta_+ = 0 \).

For the purposes of imposing constraints, it is important to know how rapidly the decay rate becomes large above threshold. We will return to this question in section C.4, after we derive the photon decay threshold.

### 3.3 Photon decay threshold

The photon decay threshold is found by setting \( p'_\perp = 0 \) and solving for the minimum value of \( k \) for which there is a solution for some \( z \). From the threshold analysis described in section B we know that the outgoing momenta are parallel, hence \( 0 < z < 1 \) at threshold, so the minimum must have \( z \) within this range.

Let us specialize to the case of a positive helicity photon (the negative helicity case works similarly). In terms of the variable \( x = 1 - 2z \), the condition that (C.11) vanish then becomes

\[
(1 - x^2)(\xi + \eta_+ x) = \frac{4m^2 M}{k^3}.
\] (C.13)

The range of \( x \) is \(-1 < x < 1\), so we see that if \( |\eta_+| \) is sufficiently large for a given \( \xi \) the conservation equation has a solution. Hence the threshold depends on \( \eta_+ \) only through its absolute value (the equation is invariant under \( \eta_+, x \to -\eta_+, -x \)). Physically, this corresponds to the fact that since the positron and electron have opposite dispersion, one is always subluminal. By depositing most of the outgoing momentum in the subluminal particle we can magnify its LV while minimizing the LV of the superluminal partner. Hence for either sign of \( \eta_+ \) we can find a solution.
Since the RHS of (C.13) decreases as $k$ increases, the threshold will occur when the LHS is at its maximum in $x$. Let us consider a few special cases. First, if $\eta_+ = 0$, then the LHS has a positive maximum only if $\xi > 0$. This occurs at $x = 0$ (equal electron and positron momenta) and is equal to $\xi$. In this case the threshold $k_{th}$ satisfies $\xi = 4m^2M/k_{th}^3$. If instead $\xi = 0$, then the maximum of the LHS occurs at $x = 1/\sqrt{3}$, and is equal to $2|\eta_+|/3\sqrt{3}$. In this case the threshold $k_{th}$ satisfies $|\eta_+| = 6\sqrt{3}m^2M/k_{th}^3$. If $\xi < -|\eta_+|$, the LHS is never positive in the range $-1 < x < 1$, so photon decay does not occur for such parameters. More generally, the maximum of the LHS occurs at

$$x_{max} = \frac{-\xi + \sqrt{\xi^2 + 3\eta_+^2}}{3\eta_+}. \quad (C.14)$$

Substituting $x_{max}$ into (C.13) allows us to find the maximum of the LHS, but the expression is a bit complicated so we don’t bother to display it here. One can use it to solve for $\xi$ as a function of $\eta_+$ and $k_{th}$ and thus derive a constraint.

### C.4 Photon decay rate near threshold

We now return to the question of reaction rate above threshold. If particle lifetimes are short slightly above threshold, then we are justified in using the absence of a threshold to establish constraints on the parameters. As a simple example of the rapidity with which reaction rates increase above threshold, let us analyze photon decay with $\xi = 0$, a positive helicity incoming photon, and $\eta_+ > 0$ (other parameter choices and the Cerenkov effect work similarly).

We expand the initial photon momentum as $k = (1 + \alpha)k_{th}$, where $k_{th}^3 = 6\sqrt{3}m^2M/|\eta_+|$ and $\alpha \ll 1$. At threshold the entire phase space consists of one point, $x = x_{max} = 1/\sqrt{3}$, and $p'_\perp$ vanishes at this point. To calculate the rate using Eqns. C.5 and C.9 we need to know the range in $x$ slightly above threshold and the value of $p'_\perp$ in this range to first order in $\alpha$. The rate is then given by

$$\Gamma(p) = \frac{c^2}{16\pi k} \int_{x_{max} - \Delta x}^{x_{max} + \Delta x} dx \frac{1 + x}{1 - x} p'^2_\perp(x), \quad (C.15)$$

where we have rewritten everything in terms of $x$. $\Delta x$ is the spread around $x_{max}$ as we move above threshold. The fact that $x_{max}$ is an extremum implies that spread around $x_{max}$ is symmetric, and just above threshold the spread is small, $\Delta x \ll x_{max}$. Expanding $p'^2_\perp$ about $x_{max}$ as a parabola, we find Eq. C.15
is approximately given by

\[ \Gamma(p) = \frac{e^2}{12\pi k_{th}} \frac{1 + x_{max}}{1 - x_{max}} p'_\perp(x_{max}) \Delta x. \]  

(C.16)

Now \( p'_\perp(x_{max}) \) is given by Eq. C.11 at \( k = k_{th}(1 + \alpha) \), i.e.

\[ p'_\perp(x_{max}) = \frac{1 - x_{max}^2}{4} k_{th}^3 (1 + \alpha)^3 - m^2. \]  

(C.17)

From the definition of \( k_{th} \) we know that the zeroth order term in \( \alpha \) vanishes. With this fact we can easily solve for \( p'_\perp(x_{max}) = 3\alpha m^2 \) (to first order in \( \alpha \)).

All that remains to evaluate is \( \Delta x \). At the endpoints of integration in Eq. C.5 we know that \( p'_\perp = 0 \), which implies that we have the relation

\[ \eta_+(x_{max} \pm \Delta x) \frac{1 - (x_{max} \pm \Delta x)^2}{4} k_{th}^3 (1 + \alpha)^3 - m^2 = 0. \]  

(C.18)

Expanding Eq. C.18 to lowest order in \( \Delta x \) and \( \alpha \) yields

\[ x_{max}(1 - x_{max}^2) + 3\alpha x_{max}(1 - x_{max}^2) - 3x_{max}(\Delta x)^2 = \frac{4m^2}{k_{th}^3 \eta_+}. \]  

(C.19)

There is no linear term in \( \Delta x \) since \( x_{max} \) is an extremum of Eq. C.18. The first term on the left hand side is equal to the right hand side by definition of \( k_{th} \) so we obtain \( (\Delta x)^2 = 2\alpha/3 \). Substituting \( \Delta x \) and \( p'_\perp(x_{max}) \) into Eq. C.16 yields our final expression for the rate as a function of \( \alpha \),

\[ \Gamma = \frac{e^2 m^2}{4\pi k_{th}} \frac{12}{\sqrt{2/3(2 + \sqrt{3})}} \alpha^{3/2} \]  

(C.20)

To see how quickly the reaction starts to happen above threshold, consider an incoming photon of energy 1% above threshold. The lifetime of the photon is equal to \( 10^{-8} k_{th}/10 \) TeV seconds, short enough that on any relevant astrophysical timescale a population of photons of multi-TeV energies will almost completely decay. As this example demonstrates, since the lifetime is extremely short only slightly above threshold, we are justified in using threshold values to derive constraints.
C.5 Vacuum Čerenkov threshold and rate

The vacuum Čerenkov process $e \rightarrow e\gamma$ is forbidden by angular momentum conservation in the threshold configuration. However, as with photon decay to opposite helicities, the rate becomes large above threshold. In fact the rate calculation is very close to that for photon decay.\(^{12}\) The only significant difference is that a factor of the outgoing photon energy $\omega(k)$ must be inserted into (C.5) since we are concerned with the energy loss rate. The net result is that $dE/dt \sim p^3/M$ above threshold, which implies that a 10 TeV electron would emit a significant fraction of its energy in $10^{-9}$ seconds. Hence in this case as well a threshold analysis will give accurate results.

We now summarize the results of the threshold analysis. To begin with, if the photon dispersion is unmodified and the electron parameter $\eta$ (for one helicity) is positive, then the electron group velocity $v_g = 1 - \left(\frac{m^2}{2p^2}\right) + \left(\frac{\eta p}{M}\right) + \cdots$ exceeds the speed of light when

$$p_{th} = (m^2/M/2\eta)^{1/3} \approx 11 \text{ TeV } \eta^{-1/3}. \quad (C.21)$$

This turns out to be the threshold energy for the vacuum Čerenkov process with emission of a zero energy photon, which we call the soft Čerenkov threshold. There is also the possibility of a hard Čerenkov threshold\(^{[53,72]}\). For example, if the electron dispersion is unmodified and the photon parameter $\xi$ is negative then at sufficiently high electron energy the emission of an energetic positive helicity photon is possible. This hard Čerenkov threshold occurs at $p_{th} = (-4m^2M/\xi)^{1/3}$, and the emitted photon carries away half the incoming electron momentum. It turns out that the threshold is soft when both $\eta > 0$ and $\xi \geq -3\eta$, while it is hard when both $\xi < -3\eta$ and $\xi < \eta$. The hard threshold in the general case is given by $p_{th} = (-4m^2M/\xi)^{1/3}$, and the photon carries away a fraction $(\xi - \eta)/2(\xi + \eta)$ of the incoming momentum. In the general case at threshold, neither the incoming nor outgoing electron group velocity is equal to the photon phase or group velocity, so the hard Čerenkov effect cannot simply be interpreted as being due to faster than light motion of a charged particle.

\(^{12}\) An extensive investigation of vacuum Čerenkov radiation was carried out in Ref. [110], focusing on the case of a Maxwell-Chern-Simons, dimension three LV operator for the photon. Unfortunately, that form of LV is different from what we study here, hence we cannot make direct use of those results.
C.6 Fermion pair emission threshold

The threshold for the process $e^- \rightarrow e^-e^e^+$ involves only one LV parameter when the electrons all have the same helicity and the positron has the opposite helicity. In the threshold configuration this is allowed by angular momentum conservation, and the form of the spinors (14,17) shows that the amplitude is large as well, so rate will be high above threshold. It thus suffices to determine the threshold.

Suppose the electron has positive helicity, and suppose that $\eta_+ > 0$, so the reaction will proceed in this configuration. Let the incoming electron have momentum $p$, and let the positron have momentum $zp$. Since the electron dispersion relation then has positive curvature everywhere, as discussed in Appendix B the two final state electrons will have the same momentum, $(1 - z)p/2$. Writing out the conservation of energy in the threshold configuration, and keeping as usual just the first order terms in $m^2$ and $\eta_+$, one finds the relation

$$p^3 = \frac{2m^2M}{\eta_+z(1 - z)} \quad \text{(C.22)}$$

The right hand side is minimized when $z = 1/2$, yielding the threshold formula

$$p_{\text{th}} = (8m^2M/\eta)^{1/3}, \quad \text{(C.23)}$$

which is a factor $16^{1/3} \sim 2.5$ above the soft Čerenkov threshold (C.21).

C.7 Helicity decay rate

A variation on the Čerenkov effect that has received almost no attention in the literature is “helicity decay”. If $\eta_+$ and $\eta_-$ are unequal, say $\eta_- > \eta_+$, then a negative helicity electron can decay into a positive helicity electron and a photon, even when the LV parameters do not permit the vacuum Čerenkov effect. In this process, the large $R$ or small $(O(m/E)) L$ component of a positive helicity electron can decay into the small $R$ or large $L$ component of a negative helicity electron respectively.\footnote{Note that the relative size of $L$ and $R$ components of the fermion spinors is still controlled by the mass, as can be seen from Eq. 9.} Such helicity decay has no threshold energy, so whether this process can be used to set a constraint is solely a matter of the decay rate. As we shall see however, the rate is maximum at an “effective threshold” energy below which it is very strongly suppressed, and
above which it decreases as $1/E$. Hence in practice there is a threshold for helicity decay as well.

The helicity decay rate involves all three LV parameters $\eta_\pm$ and $\xi$. However, since the birefringence constraints limit $\xi$ to a level far beyond where helicity decay is sensitive, we set $\xi = 0$ for the purposes of evaluating the helicity decay rate here.

We specialize to the case of a negative helicity electron decaying into a positive helicity electron, the opposite case works identically. Again, the first step is to calculate the matrix element for the reaction, given by

$$i \mathbf{M} = i e \bar{u}_+(p') \epsilon_\alpha^{\pm*} \gamma^\alpha u_-(p)$$ \hspace{1cm} (C.24)

where $p, p'$ are the incoming and outgoing electron momenta, respectively. The calculation proceeds along the lines of photon decay, and the matrix element can be evaluated approximately as

$$i \mathbf{M} = i e m \left( \sqrt{\frac{E}{E'}} - \sqrt{\frac{E'}{E}} \right).$$ \hspace{1cm} (C.25)

No angular dependence occurs in this lowest order approximation for $\mathbf{M}$ since the helicity flip allows angular momentum to be conserved when all momenta are parallel.

There are two different regimes for the size of the matrix element and rate, separated by the momentum $p_{th} \approx (m^2 M/|\eta_+ - \eta_-|)^{1/3}$. To see this, we must turn to the conservation equation. We will be interested in the amount of longitudinal phase space available to the reaction, i.e. the bounds $p'_{z1}$ and $p'_{z2}$ in Eq. C.5. The energy conservation equation without any transverse momenta is

$$2p + \frac{m^2}{p} + \eta_ - \frac{p^2}{M} = 2|p'| + \frac{m^2}{|p'|} + \eta_+ \frac{p'^2}{M} + 2|k|$$ \hspace{1cm} (C.26)

where $p'$ and $k$ are the longitudinal components of the final electron and photon momenta. We introduce the variable $z$ by $k = pz, p' = p(1 - z)$. The photon energy is $|k|$, so energy conservation implies $|z| < 1$. Since this is not a threshold situation, the final momenta can be anti-parallel, so negative values of $z$ are permitted. In terms of $z$, Eq. C.26 becomes

$$\frac{m^2}{p^2} = \left[ \frac{p}{M}(\eta_- - \eta_+(1 - z)^2) - 2(|z| - z) \right] \frac{1 - z}{z}. \hspace{1cm} (C.27)$$
If \( z < 0 \) then the lowest order solution (using \( m^2/p^2 \ll 1 \) and \( p/M \ll 1 \)) is \( z = -(\eta_- - \eta_+)p(4M)^{-1} \). This is negative only if \( \eta_- > \eta_+ \) and is a solution for any \( p \). Hence, there is no absolute threshold. In practice, \( p/M \) is so small that we can neglect the difference between this value of \( z \) and zero. We shall therefore set the lower bound on our rate integral for helicity decay to zero.

Now consider \( z > 0 \). Eq. C.27 can in this case be rewritten as

\[
\frac{m^2 M}{p^3} = \left[ \eta_- - \eta_+(1 - z)^2 \right] \frac{1 - z}{z}.
\] (C.28)

At low energies (\( m^2M/p^3 \gg (\eta_- - \eta_+) \)), the value of \( z \) that solves Eq. C.28 is also very small, \( z \approx (\eta_- - \eta_+)p^3/(m^2M) = \bar{z} \). The corresponding bounds on the integral (C.5) are therefore zero and \( \bar{z} \), so there is only a very small amount of phase space available for the reaction. Furthermore, since \( z \) is small, \( E' \approx E \) and (C.25) is doubly suppressed, once by the mass and once by \( \Delta E = E - E' \).

Evaluating \( M \) explicitly at \( z = \bar{z} \) gives

\[
iM \approx ie(\eta_- - \eta_+)\frac{p^3}{mM}.
\] (C.29)

We can overestimate the rate at low energies by substituting (C.29) and the bounds on \( z \) into (C.5). It is an overestimate since at \( z = \bar{z} \) the photon has its maximum possible energy, so the difference between \( E \) and \( E' \) is maximized, hence we are using the largest possible value of \( M \) in the allowed region of phase space. Upon integration the decay rate is at most

\[
\Gamma_1 = \frac{e^2}{16\pi} \frac{(\eta_- - \eta_+)^3p^8}{m^4M^3}.
\] (C.30)

If \( p \gg (m^2M/(\eta_- - \eta_+))^{1/3} \) then the situation is much different. The solution to the conservation equation (C.28) has \( z \approx 1 \) so the entire longitudinal phase space has opened up. Therefore \( E' \neq E \) and the only suppression is by the mass. The resulting decay rate in this case is approximately

\[
\Gamma_2 = \frac{e^2m^2}{16\pi p} \times O(1),
\] (C.31)

which roughly coincides with \( \Gamma_1 \) at the transition momentum \( p_{tr} = (m^2M/(\eta_- - \eta_+))^{1/3} \) (as it must). Note that the decay rate actually decreases with increasing momentum. This is because the electrons become more and more chiral as momentum increases, thereby reducing their ability to flip helicity via the QED vertex.
For $\eta_- - \eta_+$ of $O(1)$, $p_\nu$ is approximately 10 TeV. Hence a 1 TeV negative helicity electron has a decay rate given by $\Gamma_1$ which yields a lifetime of about one second. In contrast a 50 TeV electron with a decay rate of $\Gamma_2$ has a lifetime of approximately $10^{-9}$ seconds, making a simple “threshold” analysis applicable for deriving constraints if 50 TeV negative helicity electrons are required to exist for the longer time scales relevant in a system such as the Crab nebula.

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References

[1] C. J. Isham, “Structural issues in quantum gravity”, arXiv:gr-qc/9510063; J. Butterfield and C. J. Isham, “On the emergence of time in quantum gravity”, arXiv:gr-qc/9901024.

[2] K. V. Kuchar, “Time And Interpretations Of Quantum Gravity”, In “Winnipeg 1991, Proceedings, General relativity and relativistic astrophysics”, 211-314.

[3] L. J. Garay, “Quantum gravity and minimum length,” Int. J. Mod. Phys. A 10, 145 (1995) arXiv:gr-qc/9403008.

[4] V. A. Kostelecky and S. Samuel, “Spontaneous Breaking Of Lorentz Symmetry In String Theory,” Phys. Rev. D 39, 683 (1989).

[5] T. Damour and A. M. Polyakov, “The String dilaton and a least coupling principle,” Nucl. Phys. B 423, 532 (1994) arXiv:hep-th/9401069.

[6] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, “Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity,” Int. J. Mod. Phys. A 12, 607 (1997) arXiv:hep-th/9605211; G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, “Potential Sensitivity of Gamma-Ray Burster Observations to Wave Dispersion in Vacuo,” Nature 393, 763 (1998) arXiv:astro-ph/9712103.
[7] R. Gambini and J. Pullin, “Nonstandard optics from quantum spacetime,” Phys. Rev. D 59, 124021 (1999) [arXiv:gr-qc/9809038].

[8] J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, “Quantum gravity corrections to neutrino propagation,” Phys. Rev. Lett. 84, 2318 (2000) [arXiv:gr-qc/9909079];
J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, “Loop quantum gravity and light propagation,” Phys. Rev. D 65, 103509 (2002). [arXiv:hep-th/0108061].

[9] M. Hayakawa, “Perturbative analysis on infrared aspects of noncommutative QED on $R^4$,” Phys. Lett. B 478, 394 (2000) [arXiv:hep-th/9912094];
M. Hayakawa, “Perturbative analysis on infrared and ultraviolet aspects of noncommutative QED on $R^{4*4}$,” [arXiv:hep-th/9912167].

[10] I. Mocioiu, M. Pospelov and R. Roiban, “Low-energy limits on the antisymmetric tensor field background on the brane and on the noncommutative scale,” Phys. Lett. B 489, 390 (2000) [arXiv:hep-ph/0005191].

[11] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane and T. Okamoto, “Noncommutative field theory and Lorentz violation,” Phys. Rev. Lett. 87, 141601 (2001) [arXiv:hep-th/0105082].

[12] A. Anisimov, T. Banks, M. Dine and M. Graesser, “Comments on non-commutative phenomenology,” Phys. Rev. D 65, 085032 (2002) [arXiv:hep-ph/0106356].

[13] C. P. Burgess, J. Cline, E. Filotas, J. Matias and G. D. Moore, “Loop-generated bounds on changes to the graviton dispersion relation,” JHEP 0203, 043 (2002) [arXiv:hep-ph/0201082].

[14] Artificial Black Holes, Ed. M. Novello, M. Visser and G. Volovik, World Scientific, Singapore;
G. E. Volovik, “Superfluid analogies of cosmological phenomena,” Phys. Rept. 351, 195 (2001) [arXiv:gr-qc/0005091];
The Universe in a Helium Droplet (Oxford University Press, 2003);
C. Barcelo, S. Liberati and M. Visser, “Analogue gravity,” arXiv:gr-qc/0505065.

[15] D. Mattingly, “Modern tests of Lorentz invariance,” arXiv:gr-qc/0502097.

[16] J. D. Barrow, “Varying g and other constants,” arXiv:gr-qc/9711084.

[17] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater and J. D. Barrow, “Evidence for time variation of the fine structure constant,” Phys. Rev. Lett. 82, 884 (1999) [arXiv:astro-ph/9803165];
J. K. Webb et al., Phys. Rev. Lett. 87, 091301 (2001) [arXiv:astro-ph/0012539];
M. T. Murphy, J. K. Webb, V. V. Flambaum and S. J. Curran, Astrophys. Space Sci. 283, 577 (2003) [arXiv:astro-ph/0210532].

[18] J. Magueijo, “New varying speed of light theories,” Rept. Prog. Phys. 66, 2025 (2003) arXiv:astro-ph/0305457.
[19] G. F. R. Ellis and J. P. R. Uzan, “'c' is the speed of light, isn’t it?,” Am. J. Phys. 73, 240 (2005) [arXiv:gr-qc/0305099].

[20] B. A. Bassett, S. Liberati, C. Molina-Paris and M. Visser, “Geometrodynamics of variable speed of light cosmologies,” Phys. Rev. D 62, 103518 (2000) [arXiv:astro-ph/0001441].

[21] O. Bertolami, R. Lehnert, R. Potting and A. Ribeiro, “Cosmological acceleration, varying couplings, and Lorentz breaking,” Phys. Rev. D 69, 083513 (2004) [arXiv:astro-ph/0310344].

[22] See, e.g. P. A. M. Dirac, “Is there an aether?,” Nature 168, 906–907 (1951); J. D. Bjorken, “A dynamical origin for the electromagnetic field,” Ann. Phys. 24, 174 (1963); P. Phillips, “Is the graviton a Goldstone boson?,” Physical Review 146, 966 (1966); D.I. Blokhintsev, Usp. Fiz. Nauk. 89, 185 (1966) [Sov. Phys. Usp. 9, 405 (1966)]; T. G. Pavlopoulos, “Breakdown Of Lorentz invariance,” Phys. Rev. 159, 1106 (1967); L.B. Rédei, “Validity of special relativity at small distances and the velocity dependence of the muon lifetime,” Phys. Rev. 162, 1299-1301 (1967).

[23] C.M. Will and K. Nordtvedt, Jr., “Conservation Laws and Preferred Frames in Relativistic Gravity. I. Preferred-Frame Theories and an Extended PPN Formalism,” Astrophys. J. 177, 757 (1972); K. Nordtvedt, Jr. and C.M. Will, “Conservation Laws and Preferred Frames in Relativistic Gravity. II. Experimental Evidence to Rule Out Preferred-Frame Theories of Gravity,” Astrophys. J. 177, 775 (1972); R.W. Hellings and K. Nordtvedt, Jr., “Vector-metric theory of gravity,” Phys. Rev. D7, 3593 (1973).

[24] H. B. Nielsen and M. Ninomiya, “Beta Function In A Noncovariant Yang-Mills Theory,” Nucl. Phys. B 141, 153 (1978); S. Chadha and H. B. Nielsen, “Lorentz Invariance As A Low-Energy Phenomenon,” Nucl. Phys. B 217, 125 (1983); H. B. Nielsen and I. Picek, “Lorentz Noninvariance,” Nucl. Phys. B 211, 269 (1983) [Addendum-ibid. B 242, 542 (1984)]; J. R. Ellis, M. K. Gaillard, D. V. Nanopoulos and S. Rudaz, “Uncertainties In The Proton Lifetime,” Nucl. Phys. B 176, 61 (1980). A. Zee, “Perhaps Proton Decay Violates Lorentz Invariance,” Phys. Rev. D 25, 1864 (1982).

[25] See, for example, M. Gasperini, “Singularity prevention and broken Lorentz symmetry”, Class. Quantum Grav. 4, 485 (1987); “Repulsive gravity in the very early Universe”, Gen. Rel. Grav. 30, 1703 (1998); and references therein.

[26] See M. Haugan and C. Will, Physics Today, May 1987; C.M. Will, Theory and Experiment in Gravitational Physics (Cambridge Univ. Press, 1993), and references therein.

[27] V. A. Kostelecky and S. Samuel, “Spontaneous Breaking Of Lorentz Symmetry In String Theory,” Phys. Rev. D 39, 683 (1989).

[28] T. Jacobson, “Black hole evaporation and ultrashort distances,” Phys. Rev. D 44, 1731 (1991).
[29] W. G. Unruh, “Dumb holes and the effects of high frequencies on black hole evaporation,” Phys. Rev. D 51, 2827-2838 (1995), arXiv:gr-qc/9409008.

[30] See e.g. J. Martin and R. Brandenberger, “On the dependence of the spectra of fluctuations in inflationary cosmology on trans–Planckian physics,” Phys. Rev. D 68, 063513 (2003) arXiv:hep-th/0305161 and references therein.

[31] L. Gonzalez-Mestres, “Lorentz symmetry violation and high-energy cosmic rays,” arXiv:physics/9712005.

[32] D. Colladay and V. A. Kostelecky, “Lorentz-violating extension of the standard model,” Phys. Rev. D 58, 116002 (1998), arXiv:hep-ph/9809521.

[33] V. A. Kostelecky, Proceedings of the Second Meeting on CPT and Lorentz Symmetry, Bloomington, USA, 15-18 August 2001. Singapore, World Scientific (2002).

[34] K. Greisen, “End To The Cosmic Ray Spectrum?,” Phys. Rev. Lett. 16, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, “Upper Limit Of The Spectrum Of Cosmic Rays,” JETP Lett. 4, 78 (1966) [Pisma Zh. Eksp. Teor. Fiz. 4, 114 (1966)].

[35] M. Takeda et al., “Extension of the cosmic-ray energy spectrum beyond the predicted Greisen-Zatsepin-Kuzmin cutoff,” Phys. Rev. Lett. 81, 1163 (1998) arXiv:astro-ph/9807193.

[36] S. R. Coleman and S. L. Glashow, “Evading the GZK cosmic-ray cutoff,” arXiv:hep-ph/9808446. For previous suggestions relating LV to modification of the GZK cutoff see Kirzhnits and Chechin[37] and Gonzalez-Mestres[31].

[37] D.A. Kirzhnits and V.A. Chechin, “Ultra-High-Energy Cosmic Rays and a Possible Generalization of Relativistic Theory,” Yad. Fiz. 15, 1051 (1972) [Sov. J. Nucl. Phys. 15, 585 (1972)].

[38] G. Amelino-Camelia, “Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale,” Int. J. Mod. Phys. D 11, 1643 (2002) arXiv:gr-qc/0210063.

[39] S. R. Coleman and S. L. Glashow, “High-energy tests of Lorentz invariance,” Phys. Rev. D 59, 116008 (1999) arXiv:hep-ph/9812418.

[40] C. N. Kozameh and M. F. Parisi, “Lorentz invariance and the semiclassical approximation of loop quantum gravity,” Class. Quant. Grav. 21, 2617 (2004) arXiv:gr-qc/0310014.

[41] J. Alfaro, M. Reyes, H. A. Morales-Tecotl and L. F. Urrutia, “On alternative approaches to Lorentz violation in loop quantum gravity inspired models,” arXiv:gr-qc/0404113.

[42] R. J. Gleiser and C. N. Kozameh, “Astrophysical limits on quantum gravity motivated birefringence,” Phys. Rev. D 64, 083007 (2001) arXiv:gr-qc/0102093.
[43] R. J. Protheroe and H. Meyer, “An infrared background TeV gamma ray crisis?,” Phys. Lett. B 493, 1 (2000) arXiv:astro-ph/0005349.

[44] A. K. Konopelko, A. Mastichiadis, J. G. Kirk, O. C. de Jager and F. W. Stecker, “Modelling the TeV gamma-ray spectra of two low redshift AGNs: Mkn 501 and Mkn 421,” Astrophys. J. 597, 851 (2003) arXiv:astro-ph/0302049.

[45] R. C. Myers and M. Pospelov, “Experimental challenges for quantum gravity,” Phys. Rev. Lett. 90, 211601 (2003) arXiv:hep-ph/0301124.

[46] T. Jacobson, S. Liberati and D. Mattingly, “A strong astrophysical constraint on the violation of special relativity by quantum gravity,” Nature 424, 1019 (2003) arXiv:astro-ph/0212190.

[47] R. Lehnert, “Threshold analyses and Lorentz violation,” Phys. Rev. D 68, 085003 (2003) arXiv:gr-qc/0304013.

[48] V. A. Kostelecky and R. Lehnert, “Stability, causality, and Lorentz and CPT violation,” Phys. Rev. D 63, 065008 (2001) arXiv:hep-th/0012060.

[49] G. Amelino-Camelia, “Improved limit on quantum-spacetime modifications of Lorentz symmetry from observations of gamma-ray blazars,” arXiv:gr-qc/0212002; “A perspective on quantum gravity phenomenology,” arXiv:gr-qc/0402009.

[50] F. Dowker, J. Henson and R. D. Sorkin, “Quantum gravity phenomenology, Lorentz invariance and discreteness,” Mod. Phys. Lett. A 19, 1829 (2004) arXiv:gr-qc/0311055.

[51] E. R. Livine and D. Oriti, “About Lorentz invariance in a discrete quantum setting,” JHEP 0406, 050 (2004) arXiv:gr-qc/0405085.

[52] E. Fischbach, M. P. Haugan, D. Tadic and H. Y. Cheng, “Lorentz Noninvariance And The Eotvos Experiments,” Phys. Rev. D 32, 154 (1985).

[53] T. Jacobson, S. Liberati and D. Mattingly, “Threshold effects and Planck scale Lorentz violation: Combined constraints from high energy astrophysics,” Phys. Rev. D 67, 124011 (2003) arXiv:hep-ph/0209264.

[54] J. Alfaro, “LIV dimensional regularization and quantum gravity effects in the standard model,” arXiv:hep-th/0501129.

[55] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, “Spacetime foam may violate the principle of equivalence,” arXiv:gr-qc/0312044.

[56] A. Perez and D. Sudarsky, “Comments on challenges for quantum gravity,” Phys. Rev. Lett. 91, 179101 (2003) arXiv:gr-qc/0306113.

[57] J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, “Lorentz invariance: An additional fine-tuning problem,” Phys. Rev. Lett. 93, 191301 (2004) arXiv:gr-qc/0403053.
See e.g. M. Creutz, *Quarks, gluons and lattices* (Cambridge Univ. Press, 1985); G. Moore, “Informal Lectures on Lattice Gauge Theory,” http://www.physics.mcgill.ca/~guymoore/latt_lectures.pdf.

S. G. Nibbelink and M. Pospelov, “Lorentz violation in supersymmetric field theories,” Phys. Rev. Lett. 94, 081601 (2005) [arXiv:hep-ph/0404271]; P. A. Bolokhov, S. G. Nibbelink and M. Pospelov, “Lorentz Violating Supersymmetric Quantum Electrodynamics,” [arXiv:hep-ph/0505029].

P. Jain and J. P. Ralston, “Supersymmetry and the Lorentz fine tuning problem,” [arXiv:hep-ph/0502106].

P. A. Bolokhov, S. G. Nibbelink and M. Pospelov, “Lorentz Violating Supersymmetric Quantum Electrodynamics,” [arXiv:hep-ph/0505029].

T. C. Cheng, P. M. Ho and M. C. Yeh, “Perturbative approach to higher derivative and nonlocal theories,” Nucl. Phys. B 625, 151 (2002) [arXiv:hep-th/0111160]. T. C. Cheng, P. M. Ho and M. C. Yeh, “Perturbative approach to higher derivative theories with fermions,” Phys. Rev. D 66, 085015 (2002) [arXiv:hep-th/0206077].

T. A. Jacobson, S. Liberati, D. Mattingly and F. W. Stecker, “New limits on Planck scale Lorentz violation in QED,” Phys. Rev. Lett. 93, 021101 (2004) [arXiv:astro-ph/0309681].

T. Jacobson, S. Liberati and D. Mattingly, “TeV astrophysics constraints on Planck scale Lorentz violation,” Phys. Rev. D 66, 081302 (2002) [arXiv:hep-ph/0112207].

A. A. Andrianov, P. Giacconi and R. Soldati, JHEP 0202, 030 (2002) [arXiv:hep-th/0110279].

C. Adam and F. R. Klinkhamer, “Photon decay in a CPT-violating extension of quantum electrodynamics,” Nucl. Phys. B 657, 214 (2003) [arXiv:hep-th/0212028]; V. A. Kostelecky and A. G. M. Pickering, “Vacuum photon splitting in Lorentz-violating quantum electrodynamics,” Phys. Rev. Lett. 91, 031801 (2003) [arXiv:hep-ph/0212382]; C. Adam and F. R. Klinkhamer, “Comment on ’Vacuum photon splitting in Lorentz-violating quantum electrodynamics’,” [arXiv:hep-ph/0312153].

G. Gelmini, S. Nussinov and C. E. Yaguna, “On photon splitting in theories with Lorentz invariance violation,” [arXiv:hep-ph/0503130].

W. Kluzniak, “Transparency Of The Universe To Tev Photons In Some Models Of Quantum Gravity,” Astropart. Phys. 11, 117 (1999).

D. Mattingly, T. Jacobson and S. Liberati, “Threshold configurations in the presence of Lorentz violating dispersion relations,” Phys. Rev. D 67, 124012 (2003) [arXiv:hep-ph/0211466].
[70] G. Amelino-Camelia and T. Piran, “Planck-scale deformation of Lorentz symmetry as a solution to the UHECR and the TeV-gamma paradoxes,” Phys. Rev. D 64, 036005 (2001) [arXiv:astro-ph/0008107].

[71] F. W. Stecker and S. L. Glashow, “New tests of Lorentz invariance following from observations of the highest energy cosmic gamma rays,” Astropart. Phys. 16, 97 (2001) [arXiv:astro-ph/0102226].

[72] T. J. Konopka and S. A. Major, “Observational limits on quantum geometry effects,” New J. Phys. 4, 57 (2002) [arXiv:hep-ph/0201184].

[73] T. Jacobson, S. Liberati and D. Mattingly, “Comments on ‘Improved limit on quantum-spacetime modifications of Lorentz symmetry from observations of gamma-ray blazars’,” [arXiv:gr-qc/0303001].

[74] F. W. Stecker, “Constraints on Lorentz invariance violating quantum gravity and large extra dimensions models using high energy gamma ray observations,” Astropart. Phys. 20, 85 (2003) [arXiv:astro-ph/0308214].

[75] S. D. Biller et al., “Limits to quantum gravity effects from observations of TeV flares in active galaxies,” Phys. Rev. Lett. 83, 2108 (1999) [arXiv:gr-qc/9810044]; B. E. Schaefer, “Severe Limits on Variations of the Speed of Light with Frequency,” Phys. Rev. Lett. 82, 4964 (1999) [arXiv:astro-ph/9810479]; P. Kaaret, “Pulsar radiation and quantum gravity,” Astronomy and Astrophysics, 345, L32-L34 (1999) [arXiv:astro-ph/9903464]; S. E. Boggs, C. B. Wunderer, K. Hurley and W. Coburn, “Testing Lorentz Non-Invariance with GRB021206,” [arXiv:astro-ph/0310307], Ap. J. Letters, to appear.

[76] See Pavlopoulos in [22].

[77] J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, Astrophys. J. 535, 139 (2000) [arXiv:astro-ph/9907340]; J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, Astron. Astrophys. 402, 409 (2003) [arXiv:astro-ph/0210124].

[78] T. Piran, “Gamma Ray Bursts as Probes of Quantum Gravity,” [arXiv:astro-ph/0407462].

[79] S. M. Carroll, G. B. Field and R. Jackiw, “Limits On A Lorentz And Parity Violating Modification Of Electrodynamics,” Phys. Rev. D 41, 1231 (1990).

[80] V. A. Kostelecky and M. Mewes, “Cosmological constraints on Lorentz violation in electrodynamics,” Phys. Rev. Lett. 87, 251304 (2001) [arXiv:hep-ph/0111026]; “Signals for Lorentz violation in electrodynamics,” Phys. Rev. D 66, 056005 (2002) [arXiv:hep-ph/0205211].

[81] I. G. Mitrofanov, “A constraint on canonical quantum gravity”, Nature 426, 139 (2003).

[82] W. Coburn and S. E. Boggs, “Polarization of the prompt gamma-ray emission from the gamma-ray burst of 6 December 2002,” Nature 423, 415 (2003) [arXiv:astro-ph/0305377].
[83] R. E. Rutledge and D. B. Fox, “Re-Analysis of Polarization in the Gamma-ray flux of GRB 021206,” arXiv:astro-ph/0310385; S. E. Boggs and W. Coburn, “Statistical Uncertainty in the Re-Analysis of Polarization in GRB021206,” arXiv:astro-ph/0310515. C. Wigger et al., “Gamma-Ray Burst Polarization: Limits from RHESSI Measurements,” arXiv:astro-ph/0405525.

[84] F.A. Aharonian, A.M. Atoyan, “On the mechanisms of gamma radiation in the Crab Nebula”, Mon. Not. R. Astron. Soc. 278, 525 (1996).

[85] de Jager, O. C. et. al., “Gamma-Ray Observations of the Crab Nebula: A Study of the Synchro-Compton Spectrum,” Astrophysical J. 457, 253-266 (1986).

[86] A. M. Hillas et al., “The Spectrum of TeV gamma rays from the Crab nebula,” Astrophysical J., 503, 744 (1998).

[87] B. R. Heckel et al., “Torsion balance test of spin coupled forces,” Proceedings of the International Conference on Orbis Scientiae, 1999, Coral Gables, (Kluwer, 2000); B. R. Heckel, http://www.npl.washington.edu/eotwash/publications/cpt01.pdf.

[88] D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecky and C. D. Lane, “Limit on Lorentz and CPT violation of the neutron using a two-species noble-gas maser,” Phys. Rev. Lett. 85, 5038 (2000) [Erratum-ibid. 89, 209902 (2002)] arXiv:physics/0007049.

[89] R. Bluhm, “Lorentz and CPT tests in matter and antimatter,” Nucl. Instrum. Meth. B 221, 6 (2004) arXiv:hep-ph/0308281.

[90] F. W. Stecker, “Cosmic physics: The high energy frontier,” J. Phys. G 29, R47 (2003) arXiv:astro-ph/0309027.

[91] D. De Marco, P. Blasi and A. V. Olinto, “On the statistical significance of the GZK feature in the spectrum of ultra high energy cosmic rays,” Astropart. Phys. 20, 53 (2003) arXiv:astro-ph/0301497.

[92] http://www.auger.org/

[93] O. Gagnon and G. D. Moore, “Limits on Lorentz violation from the highest energy cosmic rays,” Phys. Rev. D 70, 065002 (2004) arXiv:hep-ph/0404196.

[94] D. Mattingly and B. McElrath, unpublished.

[95] http://www.euso-mission.org/

[96] http://owl.gsfc.nasa.gov/

[97] G. Amelino-Camelia, “Proposal of a second generation of quantum-gravity-motivated Lorentz-symmetry tests: Sensitivity to effects suppressed quadratically by the Planck scale,” Int. J. Mod. Phys. D 12, 1633 (2003) arXiv:gr-qc/0305057.

[98] http://glast.gsfc.nasa.gov/

[99] http://isdc.unige.ch/Outreach/Integral/integral.html

59
[100] http://www.eso.org/instruments/fors1/

[101] http://icecube.wisc.edu/

[102] http://www.nestor.org.gr/

[103] http://www.ps.uci.edu/~anita/

[104] P. Gorham, D. Saltzberg, A. Odian, D. Williams, D. Besson, G. Frichter and S. Tantawi, Nucl. Instrum. Meth. A 490, 476 (2002) [arXiv:hep-ex/0108027].

[105] Jackson, J. D. *Classical Electrodynamics*, 3rd edn, 671 (Wiley & Sons, New York, 1998).

[106] J. R. Ellis, N. E. Mavromatos and A. S. Sakharov, “Synchrotron radiation from the Crab Nebula discriminates between models of space-time foam,” Astropart. Phys. 20, 669 (2004) [arXiv:astro-ph/0308403].

[107] R. Montemayor and L. F. Urrutia, “Synchrotron radiation in Myers-Pospelov effective electrodynamics,” Phys. Lett. B 606, 86 (2005) [arXiv:hep-ph/0410143].

[108] P. Castorina, A. Iorio and D. Zappala, “Violation of Lorentz invariance and dynamical effects in high energy gamma rays,” Nucl. Phys. Proc. Suppl. 136, 333 (2004) [arXiv:hep-ph/0407363].

[109] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, (Addison Wesley, 1995).

[110] R. Lehnert and R. Potting, “Vacuum Cerenkov radiation,” Phys. Rev. Lett. 93, 110402 (2004) [arXiv:hep-ph/0406128]; “The Cerenkov effect in Lorentz-violating vacua,” Phys. Rev. D 70, 125010 (2004) [Erratum-ibid. D 70, 129906 (2004)] [arXiv:hep-ph/0408285].