Application of active vibration control on single stage spur gear

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Abstract. Vibrations in a mechanical system can cause power losses or vibrational fatigue which leads to failure in the material. In the current scenario automotive vehicles majorly use passive vibrational control techniques for vibration attenuation. Passive vibrational control tend to be heavy in nature and can only control frequencies in the range of 100 – 200 Hz. Conversely, active vibrational control is the potential alternative to passive vibrational control, they consist of strain sensor, piezoelectric actuator, a control system and a power amplifier to draw the control actuator. The application of active vibrational control is flourishing due to the demand in critical vibrational control in extreme conditions (underwater, polar, space). This paper provides an overview of the applications of active vibration control on single stage spur gear. The vibrations emitted in a single stage spur gear is optimised by active vibration control.

1. Introduction

Gears are one of the most widely used mechanical transmission systems due to their ability of transmitting large power and efficiency and have a longer service life than belt drive or chin drive. The spur gear drive transmits power between two parallel shafts which gives it the highest efficiency compared to all the other gears (bevel, helical, worm, cycloid, and hypoid) of about 94% to 98%. The power is lost in the form of vibration and noise. Vibrations in the single stage spur gear system are measured by modal analysis and harmonic analysis of the system. The contact stress and bending stresses are induced in the gear due to the tangential force acting on the gears. If the contact stresses on the gear is more than the wear resisting power of material, then the gear failure might take place this process is called as wear or pitting failure of the gear. This is primarily discussed in Patil and Chappar [1].

Bryan and Mark [2] discussed the development of an active system for vibration control over the chassis and reduce noise in the passenger cabin and adaptive filter theory. Further, identified optimal transducer locations to help in reduction of vibrations. Bianchini [3] discussed the implementation of a cost effective active vehicle control system which was incorporated into the steering wheel column, which specifically eliminates abundant idle engine vibrations transmitted to the steering wheel. Further, presented the numerical analysis of the actuator placement and sizing. The optimal actuator location for each mode of vibration was the one that exhibits the highest localized strain at particular frequency of interest. Tzou et. al., [4], discussed the various materials used in small and microscale transducers. It also provided an overview of the smart materials and strutonic systems available in the market. Sam et. al., [5] did a comparative study of active and passive suspensions focusing on ride quality improvement by using Linear Quadratic Regulator (LQR) control for the actuator in active suspension. Active suspension increases tire to road contact, makes vehicle more stable. Timo et. al., [6] developed a multi-
channel adaptive feed forward controller based on LMS algorithm, controller uses commuted motor as an actuator. Experiments showed that a housing vibration reduction of 31 dB at second gear mesh frequency the controller reduces the structure borne sound by 18 dB and airborne sound by 15 dB. Arjon [7] presented an active engine mount system that reduces the transmission of engine vibration to the chassis and the noises in the passenger cabin. The adaptive feed forward control enabled the actuator to reduce the force transmitted to chassis. Simulation results showed significant reduction in transmission of vibrations and the response time of control signal by improving the inherent delayed control signal. Dogruer and Pirosoltan [8] showed the periodic time-varying mesh stiffness and elastic modes of spur gear drive by using a non-linear controller which had full access to the absolute angles of spur gear pair from a reference point. The non-linear controller cancels the nonlinear force that acts on gear and pinion teeth. The system becomes a linear one, which opened up a room for the application of linear control techniques. Hatch [9] explained state space equations of various models using ANSYS and MATLAB. Khot et al., [10] discussed the extraction of full and reduced models of a cantilever beam by using ANSYS which helped to reduce the computational time. Narayankar and Mangrulkar [11] showed the importance of considering contact stress and bending stress during the mating of gears as a parameter of gear design. Further, validated with analytical results which showed well agreement. Ali et al., [12] presented the appropriate methodology in order to conduct a modal analysis and hence compare the performance of a conventional spur gear with a modified involute spur gear. Chouhan and Lodwa [13] shown the importance of finding the natural frequencies and mode shapes of a two stage spur gear system as a design parameter by using mathematical formulation. Kowelczyk et al., [14] discussed the advancements in active vibration control systems used in automotive industries. Further, gave an overview of the developments and research behind this systems. Adel et al., [15] developed a dynamic model for a composite cantilever beam and used this to create a feedback active vibration control loop using a PID controller in a Simulink environment. Maslan et al., [16] formulated a transfer function for a piezo electric patch with appropriate sizing and location by using a system identification technique. Yanjun et al., [17] used finite element method (FEM) in order to obtain the natural frequencies of gear and rack power transmission system at varying working speeds in order to avoid resonance. Hassan et al., [18] evaluated the dynamic responses and natural frequencies of a section of spur gear drive using a two dimensional finite element model. Further, evaluated the optimum operating speed. Ramesh Babu et al., [19] developed a PID controller loop with a negative feedback mechanism for vibration control in composite beams which is being used extensively in many engineering fields. Gabbert and Ringwelski [20] discussed the use of a design approach for smart-lightweight structures that were made of sheets of metal or fibre which was reinforced with plastics and equipped with thin piezo-electric wafers that acted as actuators and sensors that aided in controlling vibrations and noise. Validation of the results was done by comparing the simulation output with experimental data. Experimental and numerical results showed good agreement. With a velocity feedback control attenuations up to 4dB in the vibration level were achieved at the resonance frequency regions of the most dominant modes.

In this paper, the source of vibration within transmission systems such as single stage spur gear is considered and its noise levels are measured. Further, simulate an open active vibration control system with feed forward strategy with the help of Simulink module in MATLAB with piezoelectric material as actuators that can help reduce transmission error in a single stage spur gear pair. Also, performs modal analysis of the spur gear system with ANSYS, results are validated with analytical results and there is good agreement is observed. Further, the reduction in vibration amplitudes is observed after usage of piezoelectric patches and actuators.

2. Modelling

2.1. Gear drive modelling

A spur gear drive with gear ratio 1:3 is designed using the following dimensions on SOLIDWORKS, material used for the gear drive is grey cast iron. The distance between both the centres of the gears is
40 mm. The complete gearbox model is shown in figure 1 and the dimensions of the drive are shown in table 1.

| Parameter                  | Gear      | Pinion    |
|----------------------------|-----------|-----------|
| Module                     | 1         | 1         |
| Pressure angle             | 20 degrees| 20 degrees|
| Pitch diameter             | 60 mm     | 20 mm     |
| Number of tooth            | 60        | 20        |
| Width                      | 12 mm     | 12 mm     |
| Radius of the inner hole   | 8 mm      | 4 mm      |

This gear system is placed inside a gear casing also made of grey cast iron with stub axial diameter for gear and pinion of 17 mm and 8 mm respectively. Four ball bearings are also attached on both either sides of the gear and pinion with thickness 9 mm and 4 mm respectively. The gears are placed in a closed casing but the figure shows the gear placed in only the bottom casing. Meshing is an important part of the analysis, for an accurate result fine mesh is necessary. The finer the mesh, higher the accuracy but with more analysis run time. The SOLIDWORKS model is imported into ANSYS workbench for the meshing and further analysis procedures. The meshing of the spur gears is shown in the figure 2.

Any component kept in space has 6 Degrees of Freedom (DOF). Three DOF are translation along X, Y, Z axes and three rotation along the same axes. The pinion gear is given fixed support for analysis as shown in figure 3. In ANSYS, fixed support constraints all the DOF. Frictionless support is applied to the gear model as the force is applied in a tangential direction and the surface body is allowed to rotate and move freely [1]. A moment is applied to the part where frictionless support is applied. Due to this moment or torque the gears start rotating.

2.2. Modal analysis
A modal analysis is performed in ANSYS workbench in order to determine the frequencies at each mode. The boundary conditions and constraints are set accordingly. Torque of 300 Nm is applied. The first three mode shapes are shown in figure 4, 5 and 6. The values of first three natural frequencies are presented in table 2.
Figure 3. Fixed support.

Figure 4. 1st mode.

Figure 5. 2nd mode shape of spur gear drive.

Figure 6. 3rd mode shape of spur gear drive.

Figure 7. Meshing of the shaft.

Figure 8. Simulink modal of piezoelectric patch actuator.

Table 2. Frequency at each mode shape of the gear.

| Frequency number | Hertz | Rad/sec |
|------------------|-------|---------|
| 1                | 82.2  | 516.4   |
| 2                | 686.1 | 4310.8  |
| 3                | 2176.0| 13672.2 |
The modal analysis of the shaft also performed in ANSYS APDL environment, with the material properties of grey cast iron, and the modelled as 10 sections as shown in figure 7. After meshing the model the modal analysis is performed. The first three natural frequencies at particular modes are extracted using the Block Lanczos Solver method and presented in Table 3. Further, a MATLAB code to extract the Eigen values and Eigen vectors to form the state space matrix.

### Table 3. Frequency at each mode shape of the shaft.

| Frequency number | Hertz | Radians/sec |
|------------------|-------|-------------|
| 1                | 2.3   | 14.5        |
| 2                | 69.5  | 436.4       |
| 3                | 545.7 | 3428.5      |

### 3. State space formulation

The results of the FEM analysis are then converted into Eigen values and saved, which are readable by MATLAB. The state space of the model is represented as follows.

\[
\dot{X} = AX + BU \quad (1)
\]

\[
Y = CX + DU \quad (2)
\]

where A is system Matrix, B is input Matrix, C is output Matrix and D is direct transmission Matrix. X represents the column vector that in turn represents the state of the given system, \( X^k \) represents the differentiation of column vector, Y is the output matrix and U is the input matrix. The equations of motion of a multi degree of freedom [10] system acted on by external loads is represented by

\[
[m]x^{(\ddot{\cdot})}+[c]x^{(\cdot)}+[k]x = F \quad (3)
\]

The above equation leads to ‘n’ coupled second order differential equations. This equation can be uncoupled by solving the eigenvalue problem for (3). Eigen vectors \( x(1), x(2), \ldots \) are obtained. This forms a modal matrix

\[
[x]_n = \{x_1, x_2, x_3, \ldots, x_n\} \quad (4)
\]

The solution of equation (3) is expressed as a linear combination of the normal modes as

\[
x(t) = [x]_n x_p(t) \quad (5)
\]

Here, \( x_p(t) \) is known as the displacement in principle coordinates.

### 4. Validation – Hertz contact stress formula

Hertzian contact stress is something that tackles studies of mating parts in mechanical engineering and tribology. This is considered to be a stress formed within mating parts. If not accounted for properly it can usually cause serious problems to machinery. Here, the contact stress between two spheres of different sizes is considered. The contact stress equation is as follows:

\[
\sigma_c = \left( \frac{W_t}{\pi} \times F \cos \phi \times \left[ \frac{1}{r_1} + \frac{1}{r_2} \right] \times \left[ \frac{1}{E_1} \left( 1 - v_1^2 \right) + \frac{1}{E_2} \left( 1 - v_2^2 \right) \right] \right)^{\frac{1}{2}} \quad (6)
\]
where, \( W_i = \frac{2T}{D_p} \), \( r_f = r_2 = \frac{D_p \sin \phi}{2} \), \( F = \) Face width \( = 12 \text{ mm} \), \( \phi = \) Pressure angle \( = 20^o \), \( D_p = \) pitch diameter \( = 21.5 \text{ mm} \), \( \nu = \) Poisson’s ratio \( = 0.21 \), \( E = \) Young’s modulus \( = 1.8 \times 10^5 \text{ MPa} \), \( T = \) Torque \( = 250 \text{ N-m} \), \( W_f = \) load.

By substituting the above values in the equation (6), from the calculation it is found that the contact stress comes to about 2914.98 MPa.

| Table 4. Validation of contact stress values. |
|-----------------------------------------------|
| Materials          | ANSYS (MPa) | SOLIDWORKS (MPa) | Hertz Contact Stress | Percentage of deviation |
| Grey Cast Iron     | 2732        | 2934             | 2915                | 6%                     | 1%                     |

From the table 4, the error percentage is quite less when the results are compared and therefore the analytical formula validates the results obtained.

5. Results and discussion – Simulink control loop

It is a graphical programming domain where dynamic systems can be modelled and analysed. Simulink provides many features which help us to develop control loops which can be used in many real time practical situations. Simulink library provides most commonly used blocks in the control environment. PID controller expels the steady-state error and reduces framework settling times while keeping up a sensible transient reaction. Trial and error method is integrated for tuning of PID. The proportional, integral and derivative are used together to find the output of the controller. \( K_p, K_i \) and \( K_d \) represents proportional gain, integral gain and derivative gain respectively. The transfer function should be modelled out of the parameters piezoelectric patch and is given below:

\[
G(s) = \frac{Y(s)}{U(s)}
\]  \hspace{1cm} (7)

Here G represents the Laplace transform. U(s) is the input and Y(s) is the output. The transfer function of the Piezo electric patch is given below:

\[
G(s) = \frac{0.03747s^2 - 0.7066s + 0.03319}{0.4882s^2 - 1.492s + 0.0039}
\]  \hspace{1cm} (8)

These functions i.e. equation (7) and (8) will be inputted in the Simulink environment for further study. The actuator (piezoelectric patch) to be perfectly bonded to the casing of the spur gear pair [3]. The control force acts on the nodes in the X-direction. It acts at the end of the actuator. The output displacement and the exciting force are at points of maximum stiffness, calculated as Eigen values at each mode shape, these acts in the Z-axis. Using Simulink, a feedback loop is created that will help cancel out the transmission vibration error due to meshing issues. A feedback loop is shown in figure 9.

The PID controller used here is tuned according to the output signal of the spur gear pair. The PID controller accordingly corrects the error in the signal with the specified \( K_p, K_i \) and \( K_d \) values. Tuning the PID controller helps in providing the controlled vibrations by optimizing the parametric values through selecting targeted frequency ranges. The plots from Figure 8 helps to give a detailed analysis and what parameters are required to control it. The PID controller is once again tuned according to get an optimized values as shown in table 5.
Figure 9. Simulink modal of DC motor actuator loop

Table 5. Piezoelectric patch PID.

| Parameters | Values |
|------------|--------|
| $K_p$      | 5472   |
| $K_i$      | 1535   |
| $K_d$      | 538    |
| $N$        | 6434   |

Figure 10. Amplitude versus time graph for Piezoelectric patch without PID.  
Figure 11. Amplitude versus time graph for Piezoelectric patch with PID.

The plot in figure 10 and 11 shows a decent decrease in the curve of the amplitude versus time graph, showing how the PID controller helps in reducing the vibrations. This shows that there is a significant reduction in vibrations when a PID controller is used. Steady state is achieved much faster with lesser frequency changes. The input is given to the system as a step block which simulates the vibrations in the gear pair.

6. Conclusion

In this paper, a feedback control loop that can be used to reduce the vibrational noise produced by the transmission errors within a spur gear drive is created. Using the modal analysis of the spur gear system which is incorporated into the Simulink environment through state space form to obtain the desired results. From the graphs it can be inferred that using piezoelectric patch covers on a large surface area and therefore can cover more strain points. The piezoelectric patch provides a low overshoot than any other actuator making the patch more efficient to use. Use of a PID controller is necessary to create an
output signal that can have lesser vibration patterns and therefore do a better job than using an actuator alone. The controller helps to achieve the steady state in a short time, therefore decreasing the settling time of the control system. This project helps in understand the importance of the behaviour of a system at various mode shapes and how it effects the system overall. This paper also realises the importance of appropriate tuning of PID controller to obtain a rectifying signal that helps attenuate the unwanted noise. This work can helps the manufacturers to make machines more reliable, efficient and effective by explaining the need for a tuneable controller that can help generate better results when combined with an equally good actuator. It also portrays the power of the Simulink environment and its ease of use. It also explains the need to study modal analysis of systems in order to understand the behaviour of systems and make modelling more effective and reliable.

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