Spinning loop black holes

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Abstract

In this paper, we construct four Kerr-like spacetimes starting from the loop black hole (LBH) Schwarzschild solutions and applying the Newman–Janis transformation. In previous papers, the Schwarzschild LBH was obtained replacing the Ashtekar connection with holonomies on a particular graph in a minisuperspace approximation which describes the black hole interior. Starting from this solution, we use a Newman–Janis transformation and restrict our study to two different and natural complexifications inspired from the complexifications of the Schwarzschild and Reissner–Nordström metrics. We show explicitly that the spacetimes obtained in this way are singularity free and thus there are no naked singularities. We show that the transformation moves, if any, the causality violating regions of the Kerr metric far from $r=0$. We study the spacetime structure paying particular attention to the shape of the horizons. We conclude the paper with a discussion on a regular Reissner–Nordström black hole derived from the Schwarzschild LBH and then apply again the Newmann–Janis transformation.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The quantization of gravity is one of the still open issues in theoretical physics. Several researchers with several different approaches are trying to achieve this target. From this point of view loop quantum gravity \cite{1} is one of the most conservative, being based on the Dirac quantization and not relying on any exotic idea. However, in this context it is difficult to study the semiclassical regime of the theory and the classical limit. While progress has been made in the context of spin foams, a lot of work is still needed in order to control the theory \cite{2}.
However, one of the successes of general relativity is its geometric content, and it could be useful to keep a way to deal with the quantum using old and well-known techniques.

In the context of polymeric quantization this point of view is truly the dominant one. In loop quantum black holes (inspired by loop quantum cosmology (LQG) [3]), for instance, the quantum relies on a strong energy condition violating an effective stress–energy tensor. The same techniques of LQG have been used in the context of polymeric black holes where we have a rich literature [4–6]. In this paper, in particular, we concentrate on the regular Schwarzschild metric that has been found in [4] within the minisuperspace approximation. This metric has several interesting properties, first of all the resolution of the singularity, the non-expected self-duality property and the stability of the Cauchy horizon. It would be then interesting at this point to go a step forward and study black holes which possesses an asymptotic notion of angular momentum. Within general relativity such black hole has been found by Roy Kerr in the 1960s, and today it is well known as the Kerr spacetime; its electrically charged counterpart is known as the Kerr–Newman solution. These solutions are usually considered as not physically acceptable inside horizon. The reason is the presence of closed time-like curves (CTCs) close to the ring singularity. In the context of quantum corrected black holes, we can ask if such CTCs are still present when addressing the resolution of the Kerr ring-singularity problem. Unfortunately, it is known that finding such type of solution is not an easy task. In the context of loop black holes (LBHs) not even some sort of simplified equations of motion are known in the cylindrical case. Thus, we are facing the exceptional task of finding a solution for a spacetime without even having the differential equations to solve in order to find it. Being such the situation, we can try to use a trick to find a solution that would be otherwise impossible to obtain. Such trick has long been known in general relativity as the Newman–Janis transformation. As we will see one of the steps of this transformation, the complexification step, is totally arbitrary. However, with a particular choice of the complexification, the algorithm gives both the Kerr and the Kerr–Newman solutions starting from the Schwarzschild and the Reissner–Nordström, respectively. It is then not surprising that when applied to the polymeric Schwarzschild black hole the solutions turn into an ordinary Kerr metric if we set the polymeric parameters and $l_P$ (the Planck length) to zero. We will use the Newman–Janis algorithm to construct four different rotating spacetimes.

In this paper, we consider two spherically symmetric spacetimes that we call semi-polymeric and full-polymeric. In [4], LBH solutions were obtained replacing the Ashtekar connection with holonomies on a particular graph in a minisuperspace approximation which describes the black hole interior. We call semi-polymeric the solution which is polymeric only in the radial component of the connection and full-polymeric the solution obtained replacing all the connection components with holonomies. The two LBH solutions are

**SEMI-POLYMERIC**:

$$ds^2 = \frac{r^3}{r^4 + a_o^2} (r - 2m) dt^2 - \frac{dr^2}{r^4 - 2mr} - \left( r^2 + \frac{a_o^2}{r^2} \right) d\Omega^2,$$

**FULL-POLYMERIC**:

$$ds^2 = \frac{(r - r_*)(r - r_*)(r + r_*)^2}{r^4 + a_o^2} dr^2 - \frac{dr^2}{(r - r_*)(r - r_*)(r + r_*)^2} - \left( r^2 + \frac{a_o^2}{r^2} \right) d\Omega^2,$$

where we used signature (+, −, −, −). In sections 2 and 4, we will give a few more details about the solutions.

We will start from the semi-polymeric metric and choose the two most natural complexifications of the metric compatible with the complexification of the Schwarzschild metric. We will discuss the same procedure in the full-polymeric case.
We would like to stress the second aim of the paper even if this is repetitive. The Kerr solution is considered as non-physical inside the horizon due to the presence of CTC when the metric is geodetically extended in the negative radial regions. For this reason, long ago Penrose considered the possibility of a cosmic censorship avoiding the creation of naked singularities in general relativity. Being that the CTCs are close to the ring singularity of the Kerr metric, it is interesting to study geometries that are asymptotically Kerr-like but resolve the ring-singularity problem. Recently, Smailagic and Spallucci [7] found that the equivalent of the Kerr black hole in the non-commutative geometry inspired a black hole scenario introduced by Nicolini and Spallucci [8]. Such solution has no ring singularity, no superluminal motion and no CTC, and so can be considered as a physical solution. The ring singularity is replaced by a rotating classical string which has the effect of frame dragging on the whole spacetime, as in the Kerr case. It is then interesting for us to study the spacetimes obtained from the LBH with the Newmann–Janis algorithm and check if there are still causality-violating regions.

The structure of the paper is as follows. In section 2, we briefly recall the properties of the Schwarzschild LBH. In section 3, we review the Newman–Janis transformation. In section 4, we apply the transformation to the semi-polymeric Schwarzschild LBH and we study the CTCs for these two metrics. In section 5, we apply the transformation to the full-polymeric Schwarzschild metric. In section 6, we introduce the electric charge in the spherically symmetric LBHs and we apply again the Newmann–Janis transformation to construct the Kerr–Newmann spacetime. In section 7, conclusions follow. In this paper, we use natural units \( c = G = \hbar = 1 \).

2. The regular Schwarzschild metric

Let us first summarize the regular black hole metric [4] that will be the starting point in the following. The solution is obtained from the canonical quantization of Einstein’s equations written in terms of the Ashtekar variables, that is in terms of an \( SU(2) \) three-dimensional connection \( A \) and a triad \( E \). The result is that the basis states of LQG are closed graphs whose edges are labeled by irreducible \( SU(2) \) representations and the vertices by \( SU(2) \) intertwiners. Physically, the edges represent quanta of area with the area \( \gamma l_P^2 \sqrt{j(j+1)} \), where \( j \) is the representation label on the edge (a half-integer), \( l_P \) is the Planck length and \( \gamma \) is a parameter of order 1 called the Immirzi parameter. Vertices of the graph represent quanta of 3-volume.

The important observation to make here is that the area is quantized and the smallest possible quanta of area takes the value \( \sqrt{3}/2 \gamma l_P^2 \).

The regular black hole metric that we will be using is derived from a simplified model of LQG [4]. To obtain this simplified model, we make the following assumptions. First of all, the number of variables is reduced by assuming spherical symmetry. Then, instead of all possible closed graphs, a regular lattice with edge lengths \( \delta_b \) and \( \delta_c \) is used. The solution is then obtained dynamically inside the homogeneous region (inside the horizon where space is homogeneous but not static).

One can reduce the two free parameters by analytically continuing the solution outside the horizon and by demanding that the minimum value the area can take equals the minimum value of the area given in LQG. The one remaining unknown constant \( \delta_b \) is a parameter of the model determining the strength of deviations from the classical theory, and would have to be constrained by experiment. With the plausible expectation that the quantum gravitational corrections become relevant only when the curvature is in the Planckian regime, corresponding to \( \delta_b < 1 \), outside the horizon, the solution is the Schwarzschild solution up to negligible Planck-scale corrections which allows us to believe the legitimacy of the analytical extension
outside the horizon. The analytical extension is supported by a rigorous analysis explained in detail in [4].

This quantum gravitationally corrected Schwarzschild metric can be expressed in the form

$$ds^2 = G(r) \, dt^2 - \frac{dr^2}{F(r)} - H(r) \, d\Omega^2,$$

with $d\Omega = d\theta + \sin^2 \theta \, d\phi$ and

$$G(r) = \frac{(r - r_+)(r - r_-(r + r_+)^2}{r^4 + a_o^2},$$
$$F(r) = \frac{(r - r_+)(r - r_-)^4}{(r + r_+)^2(r^4 + a_o^2)},$$
$$H(r) = r^2 + a_o^2.$$

Here, $r_+ = 2m$ and $r_- = 2mP^2$ are the two horizons, and $r_\star = \sqrt{r_+ r_-} = 2mP$. $P$ is the polymeric function, $P = (\sqrt{1 + \epsilon^2} - 1)/(\sqrt{1 + \epsilon^2} + 1)$, with $\epsilon \ll 1$ being the product of the Immirzi parameter ($\gamma$) and the polymeric parameter ($\delta_b$). Also with this, $P \ll 1$, such that $r_-$ and $r_\star$ are very close to $r = 0$. The area $a_o$ is equal to $A_{\text{min}}/8\pi$, $A_{\text{min}}$ being the minimum area gap of LQG.

Note that in the metric (3), $g_{\theta\theta}$ takes only asymptotically the standard $r^2$ form. This choice of coordinates however has the advantage of revealing with ease the properties of this metric, as we will see. But first, most importantly, in the limit $r \to \infty$, the deviations from the Schwarzschild solution are of order $M^2/2r$, where $M$ is the usual ADM mass:

$$G(r) \to 1 - \frac{2M}{r} (1 - \epsilon^2),$$
$$F(r) \to 1 - \frac{2M}{r},$$
$$H(r) \to r^2.$$

The ADM mass is the mass inferred by an observer at flat asymptotic infinity; it is determined solely by the metric at asymptotic infinity. The parameter $m$ in the solution is related to the mass $M$ by $M = m(1 + P^2)$.

If one now makes the coordinate transformation $R = a_o/r$ with the rescaling $t = t r_\star^2/a_o$, and simultaneously substitutes $R_\pm = a_o/r_\pm$ and $R_\star = a_o/r_\star$, then one finds that the metric in the new coordinates has the same form as in the old coordinates and thus exhibits a very compelling type of self-duality with dual radius $r = \sqrt{a_o}$. Looking at the angular part of the metric, one sees that this dual radius corresponds to a minimal possible surface element. It is then also clear that in the limit $r \to 0$, corresponding to $R \to \infty$, the solution does not have a singularity, but instead has another asymptotically flat Schwarzschild region.

The metric in equation (3) is a solution of a quantum gravitationally corrected set of equations which, in the absence of quantum corrections $\epsilon, a_o \to 0$, reproduces Einstein’s field equations.

### 3. The Newman–Janis algorithm

In this section, we review the Newman–Janis transformation for a generic spherically symmetric spacetime [9]. Roughly speaking, the algorithm starts with a non-rotating spacetime and at the end of the steps the spacetime has an asymptotic notion of angular momentum.
We stress that one of the steps is arbitrary but can be constrained by the classical limit. The starting point is a spherically symmetric spacetime. In its most general form, the metrics are of the following form:

\[ ds^2 = e^{2\Phi(r)} dt^2 - e^{2\lambda(r)} dr^2 - H(r) d\Omega^2 \]

which defines the functions $\Phi(r)$ and $\lambda(r)$ used in the literature with $G(r)$ and $F(r)$. The first step of the transformation is a change of coordinates. This step requires the advanced null coordinates \{u, r, \theta, \phi\}, where

\[ u = t - r^* \]

and $dr^* = dr/\sqrt{GF}$. The line element above then becomes

\[ ds^2 = G(r) du^2 + 2\sqrt{G(r)F(r)} du dr - H(r) d\Omega^2, \]

while the non-zero components of the inverse metric are

\[ g_{\mu\nu} = e^{-\Phi(r)} - \lambda(r), g_{\phi\phi} = - [H(r) \sin^2 \theta]^{-1}, \]

\[ g_{\theta\theta} = -H(r)^{-1}, \quad g_{rr} = -e^{-2\lambda(r)}. \]  

The second step of the algorithm is to find the null tetrads for the inverse matrix as follows:

\[ g^{\mu\nu} = l^\mu l^\nu + l^\mu m^\nu - m^\mu \bar{m}^\nu - \bar{m}^\mu m^\nu \]

\[ l^\mu = \delta^\mu_1 \]

\[ n^\mu = \sqrt{\frac{F}{G}} \delta^\mu_u - \frac{1}{2} F \delta^\mu_r, \]

\[ m^\mu = \frac{1}{\sqrt{2H}} \left( \delta^\mu_\theta + i \sin \theta \delta^\mu_\phi \right) \]

where the vectors satisfy the relations $l_\mu l^\mu = m_\mu m^\mu = n_\mu n^\mu = l_\mu m^\mu = n_\mu m^\mu = 0$ and $l_\mu n^\mu = -m_\mu \bar{m}^\nu = 1$ ($\bar{x}$ is the complex conjugate of the general quantity $x$). The main step of the procedure is the combination of two operations. The first is a complex transformation in the $r–u$ plane which is as follows:

\[ r \rightarrow r' = r + i a \cos \theta, \]

\[ u \rightarrow u' = u - i a \cos \theta \]

together with a complexification of the functions $F$, $G$ and $H$ of the metric, under which the null tetrads become

\[ l^\mu = \delta^\mu_1 \]

\[ n^\mu = \sqrt{\frac{\tilde{F}(r')}{\tilde{G}(r')}} \tilde{\delta}^\mu_u - \frac{1}{2} \tilde{F}(r') \delta^\mu_r, \]

\[ m^\mu = \frac{1}{\sqrt{2\tilde{H}(r')}} \left( ia \sin \theta (\delta^\mu_\theta - \delta^\mu_\phi) + \delta^\mu_\phi + i \frac{1}{\sin \theta} \delta^\mu_\theta \right), \]

where $\tilde{F}$, $\tilde{G}$ and $\tilde{H}$ are the real functions on the complex domain. This step of the procedure is in principle completely arbitrary. In fact in the original paper Newman and Janis could not give a true explanation of the procedure, if not that it works for the Kerr metric with a particular choice of the complexifications. The situation was improved by Drake and Szekeres.
in [9], in which they proved that the only Petrov D spacetime generated by the Newman–Janis algorithm with a vanishing Ricci scalar is the Kerr–Newman spacetime. Once we have applied the transformation using the tetrads (8), the non-zero components of the inverse metric (7) can be rewritten as

\[
\begin{align*}
g_{uu} &= -\frac{a^2 \sin^2(\theta)}{H(r, \theta)}, \\
g_{\phi\phi} &= -\frac{1}{H(r, \theta) \sin^2 \theta}, \\
g_{RR} &= -\frac{1}{H(r, \theta)}, \\
g_{u\phi} &= -\frac{a}{H(r, \theta) \sin^2 \theta}, \\
g_{\phi\theta} &= \frac{a}{H(r, \theta)}, \\
g_{ur} &= \frac{a^2 \sin^2(\theta)}{H(r, \theta) \sin^2 \theta} + e^{-\frac{\Phi_1(r, \theta)}{r}} - \frac{\lambda(r, \theta)}{r^2},
\end{align*}
\]

(11)

Let us now apply the procedure to the classical Schwarzschild example. In this case, the metric has \( G = F \) or \( \lambda = -\Phi \) and \( H = r^2 \). The metric reads, in usual Eddington–Finkelstein coordinates,

\[
ds^2 = G(r) \, du^2 + 2 \, du \, dr - r^2 \, d\Omega^2
\]

where \( G(r) = 1 - \frac{2m}{r} \) and we see that \( H(r) = r^2 \). If we apply the Newman–Janis algorithm as prescribed above, we have to choose a complexification of the \( r^2 \) and \( 1/r \) terms. In general, this prescription is not unique. However, since we know what the Kerr solution is, we know that if we take the following complexification:

\[
\begin{align*}
r^2 &\rightarrow r' \bar{r}', \\
\frac{1}{r} &\rightarrow \frac{1}{2} \left( \frac{1}{r'} + \frac{1}{\bar{r}'} \right),
\end{align*}
\]

(12)

then this trick works well. This is the same as complexifying in the following way the functions \( G(r) \) and \( H(r) \):

\[
\begin{align*}
\tilde{G}(r') &= 1 - m \left( \frac{1}{r'} + \frac{1}{\bar{r}'} \right) = 1 - \frac{2mr}{r^2 + a^2 \cos^2 \theta}, \\
\tilde{H}(r') &= r' \bar{r}' = r^2 + a^2 \cos^2 \theta,
\end{align*}
\]

(13)

in the classical Schwarzschild metric. The final metric is the Kerr metric in Kerr–Schild coordinates. Without using Einstein’s equations, we could not have been able to know which particular complexification is favored over the other. This situation is even worse for Reissner–Nordström. In fact, in this last case the function \( G(r) \) is of the form

\[
G(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2},
\]

(14)

where \( Q \) is the electric charge of the black hole. In this case, the two most natural complexification of the last term in (14) is

\[
\frac{1}{r^2} \rightarrow \frac{1}{r' \bar{r}'}.
\]

(15)

Now we can apply the procedure to the LBHs.
4. Semi-polymeric spinning LBH

The line element for the semi-polymeric black hole can be obtained from (3) setting $r_+ = 0$ and it depends only on the mass and the minimum area $a_o \propto l_P^2$, where $l_P = \sqrt{G \hbar} / \kappa$ is the Planck length, (but can also be obtained from a simpler Hamiltonian constraint [4]) as

$$ds^2 = \frac{r^3 (r - 2m)}{r^4 + a_o^2} dr^2 - \frac{dr^2}{r^2 (r - 2m)} - \frac{a_o^2}{r^2} dt^2,$$

$$H(r) = r^2 + a_o^2 r^2.$$

This metric is regular everywhere and reproduces the Schwarzschild metric in the limit $a_o \to 0$. We can work with a more general form of the metric leaving the physical radius of the two-sphere implicit

$$ds^2 = \frac{r^2 (1 - \frac{2m}{r})}{H(r)} dr^2 - \frac{dr^2}{r^2 (1 - \frac{2m}{r})} - \frac{a_o^2}{r^2} \frac{dt^2}{H(r)} \Omega^2,$$

and in Eddington–Finkelstein coordinates because $G = F$ (or $\Phi = -\lambda$) reduces to

$$ds^2 = \frac{r^2 (1 - \frac{2m}{r})}{H(r)} du^2 + 2 du dr - \frac{a_o^2}{r^2} \frac{dt^2}{H(r)} \Omega^2.$$

Now we complexify the functions $G$ and $H$ appearing in the metric. It is easy to understand that, in $G$, the term of the form $(1 - 2m/r)$ must be complexified as (12) for compatibility with the Schwarzschild metric in the limit $a_o \to 0$. On the same footage the $r^2$ term in $H(r)$ must be complexified in such a way that the $a_o \to 0$ limit is compatible with the Kerr solution. This means that in $G$ the $r^2$ term must be complexified as $r^2 \to r' \bar{r}'$ for compatibility with the Kerr metric in the limit $a_o \to 0$. Thus, we are left only with the complexification of the $a_o$ term in $H$, which represent the quantum correction of the metric. The two most natural complexifications of the term proportional to $a_o$ in $H(r)$ are

$$\text{type 1 : } \frac{a_o^2}{r^2} \to \frac{a_o^2}{(r' + \bar{r}')^2/4},$$

or, as the Reissner–Nordström case suggests,

$$\text{type 2 : } \frac{a_o^2}{r^2} \to \frac{a_o^2}{r' \bar{r}'}.$$

In the following, we refer to complexifications (19) and (20) as types 1 and 2, respectively.

4.1. Type 1 complexification

As explained earlier, we proceed to complexify the components of the metric as

$$G(r) = \frac{r^2 (1 - \frac{2m}{r})}{H(r)} \to \tilde{G}(r') := G(r', \theta),$$

$$G(r, \theta) = r' \bar{r}' \left[ 1 - 2m \frac{1}{2} \left( \frac{1}{r'} + \frac{1}{\bar{r}'} \right) \right] \frac{1}{H(r')}.$$
by the coordinate transformations $d$ inside the metric and we write down the line element explicitly in the B-L coordinates defined

In the original Kerr coordinates the metric reads

In Kerr coordinates the metric is regular everywhere contrary to the classical one. There is no singularity on the event horizons and it is simpler to show the regularity of this spacetime. By a coordinate transformation the metric can be written in the Boyer–Lindquist (B-L) coordinates:

In order to simplify the notation, we introduce the following quantities:

and $\rho^2(r, \theta) := r^2 + a^2 \cos^2 \theta$.

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In Kerr coordinates the metric is regular everywhere contrary to the classical one. There is no singularity on the event horizons and it is simpler to show the regularity of this spacetime. By a coordinate transformation the metric can be written in the Boyer–Lindquist (B-L) coordinates:

In order to simplify the notation, we introduce the following quantities:

inside the metric and we write down the line element explicitly in the B-L coordinates defined by the coordinate transformations $du = dt + g(r) dr$ and $d\phi = d\phi' + h(r) dr$ (we will omit the dependence on $\theta$ and $r$ in the function $\Delta$, $\Sigma$), where

are valid for general functions $\Phi$ and $\lambda$. The B-L metric reads

The first quantity we study is the Ricci scalar that classically is zero in empty space. However, in the present context, due to quantum geometry effects, it is different from zero and takes the form

$$R(r, \theta) = \frac{8a_o^2}{(a^2 r^2 \cos(2\theta) + a^2 r^2 + 2a_o^2 + 2r^4)^3} \times \left[ 3a^4 r^2 + a^2 \cos(2\theta)(3r^2(a^2 - 2mr) + a_o^2) + 3a^2(a_o^2 - 2mr^3 + 6r^4) - 4r(a_o^2 + 3r^3)(2m - r) \right].$$
which is zero for $a_o \to 0$. On the equatorial plane ($\theta = \pi/2$) and $r = 0$, $R(0, \pi/2) = 2a^2/a_o^2$. The plots in figure 1 show that the Ricci scalar is non-singular and peaked in $\theta \approx \pi/2$, $r \approx \sqrt{a_o}$. To complete the singularity resolution analysis we check the regularity properties of the Kretschmann invariant tensor $K := R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$. This quantity has been found analytically, is given in appendix A and plotted in figures 2 and 3. It is evident that the metric is regular everywhere and the value of the curvature in $3 = 0$, $\theta = \pi/2$ is $K(0, \pi/2) = 4a^4/a_o^4$.

The component $g_{tt}$ of the metric changes sign on the surfaces defined by

$$g_{tt} = 0 \Rightarrow \Delta(r) - a^2 \sin^2 \theta = 0,$$

Figure 1. Plot of the Ricci scalar for $m = 10$, $a = 5$ when the radial coordinate assume only positive values and for $r \in [-2, 2]$ in the second plot. Type 1 complexification.

Figure 2. Plot of the Kretschmann invariant for $(m, a) = (5, 3)$ and $r \geq 0$ in Planck units. Type 1 complexification.
Figure 3. Plot of the Kretschmann invariant for $(m, a) = (5, 3)$ and positive and negative values of $r$ in Planck units. Type 1 complexification.

or more explicitly in

$$r = m \pm \sqrt{m^2 - a^2 \cos^2 \theta},$$

that defines the ergosphere of the classical Kerr spacetime.

The horizons are null hypersurfaces that split the spacetime into two regions where in one no time- or light-like path can escape to infinity. In other words, they are defined by

$$\left( \frac{dr}{dt} \right)^2 = 0 \; \forall \; d\theta, d\phi.$$  

In our case, this happens only where $\Delta(r) = 0$ or

$$r = r_{\pm} := m \pm \sqrt{m^2 - a^2} \quad \text{for} \quad a < m.$$  

The event horizon is a null surface and a Killing surface as we are going to show. The surface $S(t, r, \theta, \phi) = \text{const}$ is a null surface if the normal $n_i = \partial S / \partial x^i$ is a null vector or satisfies the condition $n_i n^i = 0$. The last identity says that the vector $n^i$ is on the surface $S(t, r, \theta, \phi)$ itself, in fact $dS = dx^i \partial S / \partial x^i$ and $dx^i | n^i$. The norm of the vector $n_i$ is

$$n_i n^i = g^{ij} \frac{\partial S}{\partial x^j} \frac{\partial S}{\partial x^i} = 0.$$  

In our case, (32) reduces to

$$g^{rr} \frac{\partial S}{\partial r} \frac{\partial S}{\partial r} + g^{\theta \theta} \frac{\partial S}{\partial \theta} \frac{\partial S}{\partial \theta} = 0.$$  

and this equation is satisfied where $g^{rr}(r) = 0$ if the surface is independent of $\theta$, $S(r, \theta) = S(r)$. The points, where $g^{rr} = 0$, are $r_-$, $r_+$ and $r = 0$ but only $r_-$ and $r_+$ are horizons. The metric not only admits two Killing vectors $t^\mu = \partial_t$ and $\phi^\mu = \partial_\phi$, but also any linear combination of them is a Killing vector. In particular,

$$\xi^\mu = t^\mu + \Omega \phi^\mu.$$  

is a Killing vector and for
\[ \Omega_H = \frac{ar^2}{r^2(a^2 + r^2) + a^2} \]  
(35)
it is null on the event horizon \((r_+, \xi^\mu \xi_\mu)|_{r_+} = 0\); this concludes the proof that the event horizon is a Killing horizon. We can calculate also the surface gravity on \(r_+\) and \(r_-\). It is defined in terms of the Killing vector \((34)\) by
\[ \kappa^2 = -\frac{1}{2} \nabla^\mu \xi^\nu \nabla_\mu \xi_\nu \]
(36)
and the result is
\[ \kappa_+ = \frac{r^2_+(r_+ - r_-)}{2 (a^2 r^2_+ + a^2 + r^4_+)} \]
(37)
\[ \kappa_- = \frac{r^2(r_+ - r_-)}{4 (a^2 r^2 + a^2 + r^4)} \].

To conclude, the event horizon area is
\[ A_H = 4\pi \left( \frac{r^2_+ + 2a^2 + a^2_0}{r^2_+} \right) \].
(38)

When \(\theta\) and \(\phi\) are constants, the metric reduces to
\[ ds^2 = \left( \Delta - a^2 \sin^2 \theta_0 \right) \frac{dr^2}{\Sigma} - \frac{\Sigma dr^2}{\Delta} \]
(39)
and the tortoise coordinate for \(\theta = 0\) is
\[ r^* = r - \frac{a^2}{rr_+} + \frac{a^2_0 (r_+ - r_0)}{r^2_+} \log |r|
+ \frac{(a^2 r^2 + a^2_0 + r^4)}{r^2_+ (r_+ - r_-)} \log |r - r_-| + \frac{(a^2 r^2 + a^2_0 + r^4)}{r^2_+ (r_+ - r_-)} \log |r - r_+| \].
(40)

We can introduce first the coordinates \(u = t - r^*\) and \(v = t + r^*\) and then \(U^\pm = \mp \exp(\mp \kappa r)/\kappa \) and \(V^\pm = \pm \exp(\pm \kappa r)/\kappa \) for \(r > r_-\) and \(r < r_-\), respectively. Looking to \(U^+ V^- = - \exp(-2\kappa r)/\kappa^2\), we see that \(U^+ V^- \to 0\) for \(r \to r_+\) and \(U^+ V^- \to -\infty\) for \(r = 0\). In figure 4, we are plotting a Penrose diagram when \(r > 0\). Despite the position of the line \(r = 0\) in the diagram it is not an event horizon as can be seen solving \((30)\). A maximal extension to negative values of \(r\) is obtained following the analysis in [10].  

The result is given in figure 5.  

As we showed studying the Ricci scalar and in particular the Kretschmann invariant the spacetime is regular everywhere. If we plot the Kretschmann invariant for the case \(a > m\), we obtain plots similar to those in figures 2–4. In other words, we do not have naked singularities. The tortoise coordinate for \(a > m\) and \(\theta = 0\) is
\[ r^* = r - \frac{a^2}{a^2 r} + \frac{2a^2 m \log |r|}{a^4} + \frac{(2a^4 m^2 - a^2 a^2_0 + 2a^2 m^2)}{a^4 \sqrt{a^2 - m^2}} \arctan \left( \frac{a^2 - m^2}{a^2 - m^2} \right)
+ \frac{m(a^2 - a^2_0)}{a^4} \log \left( a^2 - 2mr + r^2 \right) \].
(41)

In this case, to understand the causal structure of the spacetime, we can also introduce coordinates \((u, v)\) and then a single couple of new coordinates \((U, V)\) because there is just one coordinate singularity in \(r = 0\). The result is a block of spacetime which extends from \(+\infty\) to \(r = 0\). Following again [10] the maximal extension of the spacetime is given in figure 6.
For the extremal cases $m = a$ and $\theta = 0$, the tortoise coordinate is
\[
    r^* = r - \frac{a^2}{rr_o^2} - \frac{a^2r_o^2}{rr_o^2(r - r_o)} + \frac{2a^2}{r_o^3} \log |r| + \frac{2(r_o^4 - a_o^2)}{r_o^3} \log |r - r_o| \tag{42}
\]
and the Penrose diagram is in figure 7.

Following the analysis of the classical Kerr metric [10], we focused our attention to the axis of symmetry because it is easier to study. Nevertheless, it seems natural, as in the classical
Figure 5. Maximal extension of type 1 spacetime for $a < m$ and $\theta = 0$. The surface $r = 0$ is a null surface but it is not a horizon beside $g_{tt}$, and $g_{rr}$ does not change sign. A block of the same color has to be identified to have the maximal extension of the spacetime to negative values of $r$.

case, that the basic topological properties of the four-dimensional spacetime are essentially the same.

4.2. Type 2 complexification

In this section, we consider a different complexification, but still starting from the same spherically symmetric metric (18) in Eddington–Finkelstein coordinates:

$$ds^2 = r^2 \left(1 - \frac{2m}{r}\right) du^2 + 2 du dr - H(r) d\Omega^2.$$  \hspace{1cm} (43)

As explained the complexification (21) changes only in the factor $H(r)$:

$$H(r) \mapsto \Sigma(r, \theta) = r^2 + \frac{\rho^2}{\rho^2} = \rho^2 + a^2 \cos^2 \theta.$$  \hspace{1cm} (44)

This small modification is sufficient to make the metric harder to study. In Kerr coordinates, the metric is regular everywhere contrary to the classical one and is given in (22). There is no singularity on the event horizons and is easier to show the regularity of the improved spacetime metric. Again, the first quantity we study is the Ricci scalar. The plots in figures 8 and 9 show that the Ricci scalar is non-singular. To complete the singularity resolution analysis, we should analyze the Kretschmann-invariant tensor $K := R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$. In this case $K(r, \theta)$ is very involved, more than that in the previous case of type 1 metric. $K(r, \theta)$ will show its regularity properties by 3D plots, given in figures 10–12.
Given the tortoise coordinate and using the same analysis of the previous section for type 1 metric, we can obtain the Penrose diagrams for type 2 metric. In the $\theta = 0$ case the diagrams are exactly the same as that of the classical Kerr metric for $a < m$, $a = m$ and $a > m$. 

$$r^* = r + \frac{a^2 (r_+ + r_+)}{2 (a^2 + r_+^2) (a^2 + r_+^2)} \log (a^2 + r^2) \nonumber$$

$$- \frac{a^2 (a^2 - r_- r_+)}{a (a^2 + r_+^2) (a^2 + r_+^2)} \arctan (r/a) \nonumber$$

$$- \frac{(a^4 + 2 a^2 r_+^2 + a_+^2 + r_+^4)}{(a^2 + r_+^2) (r_+ - r_-)} \log |r - r_-| \nonumber$$

$$+ \frac{(a^4 + 2 a^2 r_+^2 + a_+^2 + r_+^4)}{(a^2 + r_+^2) (r_+ - r_-)} \log |r - r_+|. \quad (45)$$
There is no naked singularity and for \( \theta = \pi/2 \) the diagram for the case \( a > m \) looks like the Minkowski spacetime diagram, as in the classical case, but with an extension to negative values of \( r \).

4.3. Closed time-like curves

Since neither of the metrics obtained in the previous section (nor type 1 nor type 2) have ring singularities, we can now check if the CTCs disappear. In the Kerr case such CTCs are present in the extended spacetime sector for \( r < 0 \).

In order to study the CTC’s problem we study the norm of the Killing vector along the \( \phi \) direction. This vector has norm \( \phi^\mu \phi_\mu = g_{\phi \phi} \), and we evaluate this norm for the classical metric and for types 1 and 2 metrics. We consider the norm near the point \( r = 0, \theta = \pi/2 \). Let \( r/a = \delta \) (small and negative) and consider \( \theta = \pi/2 + \delta \). Then, classically we find

\[
\phi_\mu \phi^\mu = g_{\phi \phi} = -\frac{am}{\delta} - a^2 + \mathcal{O}(\delta),
\]

(46)
Figure 8. Plot of the Ricci scalar for $m = 10$, $a = 5$ in Planck units ($r > 0$).

Figure 9. Plot of the Ricci scalar for $m = 10$, $a = 5$ in Planck units and $r \in [-\infty, +\infty]$. The behavior at the origin is $\lim_{\theta \to \pi/2} \lim_{r \to 0} R(r, \theta) = \lim_{\theta \to \pi/2} \lim_{r \to 0} R(r, \theta) = 2a^2/a_0^2$. 

16
which is positive for sufficiently small and negative $\delta$. For types 1 and 2 LBHs instead we find

$$\phi_\mu \phi^\mu = -\frac{a^2}{a^2 \delta^2} + \text{const} + O(\delta).$$

(47)

that are always negative for small values of $\delta$. We conclude that there are no CTCs in the region around $r \approx 0$ and $\theta \approx \pi/2$ contrary to the classical Kerr spacetime.
However, for negative values of $r$ and arbitrary values of $\theta$ the norm of the Killing vector can change sign as showed in figure 13 and we can still have CTCs. The lump region in the plot is a time machine region [11].

5. The full-polymeric spinning LBH

In this section, we apply the Newmann–Janis [9] transformation to the LBH metric in its full-polymeric form, (3). This metric is more complicated than the semi-polymeric one; thus, we restrict our analysis to features independent of the complexification of the function $H(r)$. Moreover, we restrict ourselves to a particular complexification of the functions, suggested by the one of the Reissner–Nordström metric. The full-polymeric line element can be rewritten as follows:

$$ds^2 = g_{tt} \, dt^2 + g_{rr} \, dr^2 + g_{\Omega} \, d\Omega^2,$$

$$g_{tt} := e^{2\phi(r)} = \left(1 - \frac{r_+ + r_-}{r} + \frac{r_+ r_-}{r^2}\right) \left(1 + \frac{r_s}{r}\right)^2 \frac{r^2}{H(r)},$$

$$g_{rr} := -e^{2\lambda(r)} = -\frac{(1 + \frac{r_s}{r})^2}{\left(1 - \frac{r_+ r_-}{r^2}\right)} \frac{H(r)}{r^2},$$

$$g_{\Omega} := -H(r) = -\left(r^2 + \frac{a^2}{r^2}\right).$$

where $r_+ = 2m$, $r_- = 2mP^2$ and $r_s = 2mP$ as already defined in section 1 of this paper. The terms of the form $1/r$, $1/r^2$ and $r^2$ are naturally complexified as the Reissner–Nordström
terms, leaving the choice, as for the semi-polymeric case, to complexify the function $H(r)$:

$$
\begin{align*}
\frac{1}{r} &\mapsto 2 \left( \frac{1}{r'} + \frac{1}{\rho} \right), \\
\frac{1}{r^2} &\mapsto \frac{1}{r'\rho'}, \\
r^2 &\mapsto r'^2.
\end{align*}
$$

(49)

The line element in Kerr coordinates is

$$
\begin{align*}
ds^2 &= e^{2\Phi} \, du^2 + 2 e^{\lambda+\Phi} \, du \, dr - 2a \sin^2 \theta \, e^{\lambda+\Phi} \, dr \, d\phi \\
&\quad - \Sigma \, d\theta^2 - \sin^2 \theta (\Sigma + a^2 \sin^2 \theta (2 e^{\lambda+\Phi} - e^{2\Phi})) \, d\phi^2 \\
&\quad + 2a \sin^2 \theta (e^{\lambda+\Phi} - e^{2\Phi}) \, du \, d\phi,
\end{align*}
$$

(50)

where $\Sigma$ comes from the complexification of $H(r)$ and

$$
\begin{align*}
e^{2\Phi(r,\theta)} &= e^{\lambda(r,\theta)+\Phi(r,\theta)} \left( \rho^2(r, \theta) - (r + r_+)r + r_+r_- \right), \\
e^{\lambda(r,\theta)+\Phi(r,\theta)} &= \left( 1 + \frac{rr_+}{\rho^2(r, \theta)} \right)^2,
\end{align*}
$$

(51)

and $\rho^2 := r^2 + a^2 \cos^2 \theta$ is the same function introduced in the semi-polymeric case.

We show now the regularity of the solution considering type 1 complexification of $H(r) \rightarrow \Sigma(r, \theta)$. We can rewrite the metric (51) in a conformal shape where the conformal factor is $\exp(\Phi + \lambda) := \exp(2\sigma)$. The metric reads

$$
\begin{align*}
g_{\mu\nu} = e^{\Phi+\lambda} \tilde{g}_{\mu\nu} := e^{2\sigma} \tilde{g}_{\mu\nu},
\end{align*}
$$

(52)
where $\bar{g}_{\mu\nu}$ is regular $\forall r \geq 0$ (this is very simple to see for type 1 complexification because the components of the metric $\bar{g}_{\mu\nu}$ never diverge for $\theta = \pi/2$) and presents the bounce of the two-sphere in $r = 0$. Now, we consider the Ricci scalar which can be written in the following way:

$$R = e^{-2\sigma} (\bar{R} + 6(\bar{\nabla}\sigma)), \quad (53)$$

where $\bar{R}$ and $\bar{\nabla}$ are defined by $\bar{g}_{\mu\nu}$. When we replace the components of the metric in (53) for $\theta = \pi/2$, we find the following leading term:

$$R \approx e^{-2\sigma} 6(\bar{\nabla}\sigma) \approx \frac{6a^2}{a_o}, \quad (54)$$

which shows that the Ricci invariant does not diverges on the equatorial plane for $r = 0$. The behavior of the Ricci scalar (53) is a strong argument in favor of the regularity of the metric $g_{\mu\nu}$ for $\theta \approx \pi/2$ and $r \geq 0$. Another argument pro-regularity of the spacetime comes from the radial geodesic analysis (for $r \approx 0$) in the B-L coordinates we are going to introduce. Having introduced such coordinates we will return to this point.

The components of the loop-improved Kerr metric can be written in the B-L coordinates applying the transformation (25). The result is

$$g_{tt} = \frac{(\Delta(r) + a^2 \cos^2 \theta)(\rho^2 + rr_*)^2}{\rho^4 \Sigma},$$

$$g_{rr} = -\frac{\rho^4(\Delta(r) + a^2 \cos^2 \theta) + a^2 \sin^2 \theta(\rho^2 + rr_*)^2}{\Sigma \rho^4},$$

$$g_{t\phi} = \frac{a \sin^2 \theta (\rho^2 + rr_*)^2 [\Sigma - (\Delta(r) + a^2 \cos^2 \theta)]}{\Sigma \rho^4},$$

$$g_{\phi\phi} = -\frac{\Sigma}{\rho^4} \left[ \Sigma + a^2 \sin^2 \theta (\rho^2 + rr_*)^2 (2\Sigma - (\Delta(r) + a^2 \cos^2 \theta)) \right], \quad (55)$$

where $\Sigma(r, \theta)$ is the complexification of $H(r)$ and we introduced the notation

$$\Delta(r) = r^2 - (r_+ + r_-)r + r_+ r_- \quad (56)$$

The ergosphere is quite similar to the classical one and is defined by the surface

$$g_{tt} = 0 \rightarrow \Delta(r) + a^2 \cos^2 \theta = 0 \quad (57)$$

or more explicitly

$$r = \frac{r_+ + r_- - \sqrt{(r_+ - r_-)^2 - 4a^2 \cos^2 \theta}}{2} = m(1 + P^2) \pm m^2(1 - P^2)^2 - a^2 \cos^2 \theta. \quad (58)$$

The event horizon is defined by (30) and such relation for the full-polymeric metric reads

$$\rho^4(\Delta(r) + a^2 \cos^2 \theta) + a^2 \sin^2 \theta(\rho^2 + rr_*)^2 = 0. \quad (59)$$

The definition of black hole horizon we are using here is the following: it is a surface within all light-like paths and hence all paths in the forward light cones of particles within the horizon are warped so as to fall farther into the hole. Equation (59) defines a null surface as it is easy to see.

Contour plots of the six-order equation (59) are given in figures 14 and 15. On the $x$ and $y$ axes there are $r$ and $\theta$, and on the $z$ axis there is the angular momentum $a$. The horizontal plane
Figure 14. Contour plot for the geometric surface where equation (59) is satisfied. The intersecting plane is $a = \text{const}$. This plot refers to the case of two event horizons one inside the other for $m = 100$ in Planck units and $P = 0.1$.

at constant $a$ shows explicitly that for small values of $a$, we have two quasi-spherical event horizons (figure 15), one inside the other, but for $a$ sufficiently large we have two horizons separated from each other (figure 14). For the second configuration we have the following picture: from an observer in the outside region of the hole, the black hole, the region splits along the symmetry axes into two distinct black holes regions both with ellipsoidal horizon. In the presence of two horizons, one inside the other, or no horizons, the Penrose diagrams (for $\theta = 0$) are the same as those of type 1 or type 2, respectively. In the case of two topologically distinct event horizons, the Penrose diagram representation is not well define. Indeed, in this case we have two black holes each one with a single event horizon but both inside the same ergosphere.

Another elegant way to verify the non-existence of singularity in $r = 0$ for $\theta = \pi/2$ is to study the geodesic in the equatorial plane. We then have a singularity problem only when the proper time to arrive in $r = 0$ is finite. For the metric (55) on the plane $\theta = \pi/2$ orbits are parametrized by the conserved energy per unit of mass, $E$, and the angular momentum per unit of mass along the symmetry axis. The radial geodesic of a massive particle can be obtained from the norm of the 4-velocity and the energy. The conserved quantities associated with the symmetries of the metric

$$E = t^\mu U_\mu g_{\mu\nu},$$

$$\ell = -\phi^\mu U_\mu g_{\mu\nu},$$

$$U^\mu U_\mu = 1,$$  

(60)
where we recall the Killing vectors $t^\mu = (1, 0, 0, 0)$ and $\phi^\mu = (0, 0, 0, 1)$. The second equation of (60) for a general value of $E$ and $\ell$, and a small value of $r$ reads

$$\frac{1}{2} \left( \frac{dr}{d\tau} \right)^2 \approx -\frac{a^2 r^2}{2a_o^2} - \frac{(aE\ell + r - r)}{2a^2} r^4.$$  \quad (61)

Since the leading term is negative for any value of $E$ and $\ell$, there is a barrier which prevents us to get in $r = 0$, for both values of $r$, positive and negative ones. In the classical case a test particle can get to $r = 0$ only from $r < 0$ because $r^2 \propto -1/r^5$ ($r$ is the derivative with respect to the proper time $\tau$). Any test particle will move around the ring $(r = 0, \theta = \pi/2)$ without ever reaching it. Moreover, for $\theta = 0$ any particle arrives at $r = 0$ in finite time and then there is a natural analytical extension to negative values of the coordinate $r$.

This is a strong argument in favor of the regularity of the spacetime at $r = 0$ but it does not exclude the possibility of having singularities for negative values of $r$. We can indeed reach the region $r < 0$ starting from $\theta \neq \pi/2$. This result is independent of types 1 or 2 complexification because for $\theta = \pi/2$ the two different complexifications of $\Sigma$ coincide. On the other hand, for $\theta \neq \pi/2$ there is no singularity because of the presence of the angular momentum $a$ in $\Sigma$ and $\rho^2$.

5.1. Violation of causality

We showed in the previous section that any observer on the equatorial plane never reaches the origin $r = 0$. This result is important in relation to the CTCs because those curves exist...
only for negative values of \( r \) which in this metric are not part of the maximal extension for \( \theta = \pi/2 \). We can conclude that there are no CTCs when an observer arrives in \( r = 0 \) if he traveled on the plane \( \theta = \pi/2 \) because the maximal extension does not involve \( r < 0 \). What about the region \( r \approx 0 \) and \( \theta \approx \pi/2 \)? We consider the Killing vector along the \( \phi \) direction. This vector has the norm \( \phi^\mu \phi_\mu = g_{\phi\phi} \). We compare such norm in the classical case with the new regular metric. Let \( r/a = \delta \) (small) and consider \( \theta = \pi/2 + \delta \). Then, classically

\[
\phi_\mu \phi^\mu = g_{\phi\phi} = -\frac{am}{\delta} - a^2 + O(\delta),
\]

(62)

which is positive for sufficiently small and negative \( \delta \). For the LBH and types 1 and 2 complexifications, respectively, we find

\[
\phi_\mu \phi^\mu = -\frac{2a^2}{2a^2\delta^2} - \frac{2ar}{\delta} + \text{const} + O(\delta)
\]

(63)

which are always negative for small values of \( \delta \). We conclude that there are no CTCs in the region around \( r \approx 0 \) and \( \theta \approx \pi/2 \) contrary to the classical Kerr spacetime.

Also for the full-polymeric metric like for the semi-polymeric one we have a region, for negative values of \( r \), where the norm \( \phi^\mu \phi_\mu \) changes sign. We have a good improvement with respect to the classical metric but we still have CTCs.

5.2. Horizon transitions

The metric we have introduced shows that there is a relation between the critical points of the surface \( C(r, \theta; a) = 0 \) and the transitions of the null surface. This can be seen if we align the surface with the \( a \) axis. In the following we will refer to this surface as critical surface. In this section, we will try to make this relation precise on a more general setting, relying only on the properties of the surface for a generic perturbation of the critical surface. In particular, we assume that there is a situation similar to the one of the previous section: a rotating black hole not in vacuum according to the Einstein equations but that in the limit of some parameters going to zero (and that we consider small in general); the metric we obtain is Kerr. The previous example catches general peculiarities enough to generalize it. The only assumption is that such corrections of the Kerr metric do not break the main symmetries, that is, the spacetime has a symmetry axis along which rotations keep the metric invariant.

The first thing to note is that the surface is embedded in three dimensions and that can be Taylor expanded around the critical points:

\[
C(r \approx r_c, \theta \approx \theta_c; a \approx r_c, \theta \approx \theta_c)) = C(r_c, \theta_c; a(r_c, \theta_c)) + c_1 r^2 + c_2 \theta^2 + O(r^3, \theta^3),
\]

(64)

where \( c_1 \) and \( c_2 \), normalized to \( \pm 1 \), are called the Morse indices of the surface critical point. In general, however, we have to extend the notion of criticality also to points that cannot have an expansion of this form, such as the points lying on \( \theta = 0 \) or \( \theta = \pi \), so to the notion of absolute minimum and absolute maximum. Of course, if there are local critical points, we have three distinct possibilities: maxima, minima and saddle points corresponding to \((c_1 = -1, c_2 = -1),(c_1 = 1, c_2 = 1)\) and \((c_1 = 1, c_2 = -1)\), respectively. These indices are fundamental to study the topological properties of surfaces in Morse theory. However, in our case the problem is slightly different due to the invariance of this surface with respect to the \( \phi \) variable, which corresponds to a global \( M \times S \) topology, where \( M \) is the manifold given by the intersection of the \( a = \text{const} \) surface and the critical surface, as in figure 16 and \( S \) is a circle.
In the following we will refer to as tori and spheres the surfaces having the topology of tori and spheres, respectively. It is easy to understand that in general local maxima coincide with shrinking, saddle points with splitting, and local minima create spherical null surfaces, as the angular momentum, the parameter \( a \), increases. Absolute minima and maxima occurring at \( \theta_0 = 0 \) and thus, \( \theta_0 = \pi \) are slightly different as we explain now. In fact, the critical properties of the null surface are the global one; we have to translate the criticality of \( M \) to the criticality of \( M \times S \). The points \( \theta = 0 \) and \( \theta = \pi \) are different because they have to be identified, since they lie on the axis of symmetry; moreover due to the axial symmetry, critical points at an angle \( 0 < \theta_0 < \pi \) have an identical critical points at \( \theta_0' = \pi - \theta_0 \). By the Weierstress theorem two local maxima coincide at least a minimum that, in this case, coincide with a saddle point in between the two local maxima. In general, local saddle points correspond to splits: when the surface \( a = \text{const} \) hit a saddle point, the null surface splits into two parts, at the angle where there is the saddle point, the null surface bends, and when the \( a = \text{const} \) surface hits the saddle point, the null surface separates. The way how this happens depends on the nature of the critical surface. We can apply the same analysis for local maxima. The surface at constant angular momentum cuts local maxima of the critical surface on circles\(^3\) if these are not at the extremal points, which correspond to a tori for the null surface, being \( S \times S \). Thus, local maxima in general correspond to shrinking of tori or spheres if the maxima are at the extremal at increasing angular momentum. When the surface \( a = \text{const} \) hits the critical point, the tori shrinks to a point and the null surface disappears. The same happens for the maxima occurring at the extremal points. However, a local minimum at the extremal points of the critical surface is tricky. In fact, when it occurs, the surfaces of the inner and outer horizons bend and touch the axis. If, for example, the local minima at the extremal points are the first occurring, the overall topology assumes a toroidal shape, transforming the inner and outer horizons in a torus. Local minima instead are the opposite of local maxima; when the surface \( a = \text{const} \) hit them, they create spherical null surfaces. This analysis complete the picture, in the approximation of little corrections to the Kerr metric, of transitions from the inner–outer horizon to null-surface free spacetime.

6. Toward spinning LBH with charge

In this section, we consider a black hole with spin and electric charge. First we introduce the generalization of the Schwarzschild LBHs (semi-polymeric and full-polymeric) to the Reissner–Nordström LBHs, then we apply again the Newmann–Janis complex transformation to obtain rotating and charged LBHs.

6.1. Reissner–Nordström LBH

It is easy to extend the spherically symmetric LBHs to the case of a black hole with charge. We consider first the semi-polymeric case and then the full-polymeric case.

6.1.1. Semi-polymeric case. We recall the semi-polymeric metric for the spherically symmetric case without charge:

\[
 ds^2 = \frac{r^2 - 2mr}{H(r)} dt^2 - \frac{dr^2}{\frac{r^2 - 2mr}{H(r)}} - H(r) d\Omega^2, \\
 H(r) = r^2 + \frac{a^2}{r^2}. 
\]

\(^3\) With the topology of a circle.
Figure 16. Contour plot for the geometric surface where the event horizon equation (59) is satisfied together with the ergosphere surface. The event horizon surface is always inside the ergosphere surface and the two surfaces meet at the poles $\theta = 0, \pi$. In this plot $m = 10$ in Planck units and $P = 0.5$ but it is true for any value of the parameters.

It is very simple to introduce the charge and to obtain a regular metric with the correct classical limit. This can be done by the replacement [12]

$$2mr \rightarrow 2mr - e^2$$

in (65), where $e$ is the electric charge. The metric is very close to the Reissner–Nordström but with a bounce of the $S^2$ sphere on a minimum area $a_o$ which solve the singularity problem. It is easy to show, going through the analysis in [4], that the regularity of the metric for any value, positive, negative or zero, of the radial coordinate.

6.1.2. Full-polymeric case. The generalization of the full-polymeric LBH to a charged black hole is also very simple. We recall again the metric

$$ds^2 = g_{tt} \, dt^2 + g_{rr} \, dr^2 - H(r) \, d\Omega^2,$$

$$g_{tt} = \frac{(r^2 - (r_+ + r_-)r + r_+ r_-)}{H(r)} \left(1 + \frac{r_+}{r}\right)^2,$$

$$g_{rr} = -\frac{H(r)}{(r^2 - (r_+ + r_-)r + r_+ r_-)} \left(1 + \frac{r_+}{r}\right)^2,$$

$$H(r) = r^2 + \frac{a_o^2}{r^2}.$$
In this case, we replace

\[(r_+ + r_-)r \rightarrow (r_+ + r_-)r - e^2.\]  

The horizons are now located at

\[\tilde{r}_\pm = \frac{(r_+ + r_-) \pm \sqrt{(r_+ + r_-)^2 - 4(e^2 + r_+ r_-)}}{2}.\]  

It is easy to show, going again through the analysis in [4], the regularity of the metric for any value of the radial coordinate. A radial geodesics analysis shows that we cannot reach \(r = 0\) in finite time-like in case of \(e = 0\).

For \(m(1 - P)^2 \geq e\), we can express the metric in the following way:

\[g_{tt} = \frac{(r^2 - (\tilde{r}_+ + \tilde{r}_-)r + \tilde{r}_+ \tilde{r}_-)}{H(r)} \left(1 + \frac{\tilde{r}_+}{r}\right)^2,\]

\[g_{rr} := -\frac{H(r)}{(r^2 - (\tilde{r}_+ + \tilde{r}_-)r + \tilde{r}_+ \tilde{r}_-)} \left(1 + \frac{\tilde{r}_+}{r}\right)^2,\]

\[H(r) = r^2 + \frac{a^2}{r^2},\]  

where we have introduced \(\tilde{r}_+ = \tilde{r}_+ \tilde{r}_-\) if we want to keep the duality property of the metric. It is easy to see that the Reissner–Nordström full-polymeric LBH has exactly the same shape of the Schwarzschild full-polymeric LBH with \(r_+\), \(r_-\) and \(r_\ast\) being replaced by \(\tilde{r}_+\), \(\tilde{r}_-\) and \(\tilde{r}_\ast\) at least for \(m(1 - P)^2 \geq e\); this makes very easy to derive the full-polymeric Kerr–Newmann spacetime.

### 6.2. Kerr–Newmann LBH

In this section, we apply the Newmann–Janis complexification to the Reissner–Nordström LBH in its semi-polymeric and full-polymeric forms. The following derivation is justified by the decoupling between polymerization of the space and the electric field. This is easy to see in the semi-polymeric case but \(a\) is conjecture in the full-polymeric one.

#### 6.2.1. Semi-polymeric case

The complexification is straightforward and following section 4.1, the natural choices of \(G\) and \(H\) are

\[G(r) \rightarrow \frac{\rho^2 - 2mr + e^2}{H(r)},\]

\[H(r) \rightarrow \rho^2 + \frac{a^2}{r^2}\]  

for type 1,

\[H(r) \rightarrow \rho^2 + \frac{a^2}{\rho^2}\]  

for type 2,

where \(\rho^2 = r^2 + a^2 \cos^2 \theta\). The Kerr–Newmann LBH in the B-L coordinates reads

\[ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \, dt^2 - \frac{\Sigma}{\Delta} \, dr^2 + 2a \sin^2 \theta \left(1 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma}\right) \, dt \, d\phi - \Sigma \, d\phi^2 \]

\[- \sin^2 \theta \left[\Sigma + a^2 \sin^2 \theta \left(2 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma}\right)\right] \, d\phi^2.\]  

(72)
where $\Delta(r)$ is

$$\Delta(r) = r^2 - 2mr + e^2 + a^2. \quad (73)$$

The rest of the analysis exactly follows sections 4.1 and 4.2 but with the new $\Delta(r)$ function defined in (73). Particularly for type 1 complexification, $\Omega_1$, $\kappa_1$, and the event horizon area have the same shape as those calculated in section 4.1 but with being $r_\pm$ replaced with the roots of the new equation $\Delta(r) = 0$ defined in (73).

### 6.2.2. Full-polymeric case.

This section is very short since it is identical to section 5 if we replace everywhere $r_+$, $r_-$ and $r_\ast$ with $\tilde{r}_+$, $\tilde{r}_-$ and $\tilde{r}_\ast$, when $m(1 - P)^2 \geq e$. If $m(1 - P)^2 < e$, we still define $\tilde{r}_\ast^2 = \tilde{r}_+ \tilde{r}_- = e^2 + r_+ r_- \ast$ but we apply the Newmann–Janis transformation directly to (67) with the replacement (68).

### 7. Conclusions

In this paper we used the Newman–Janis algorithm to construct regular spinning black holes from the Schwarzschild LBH. We used constraints coming from the classical limits and arguments from the Newman–Janis algorithm applied in the past to the Schwarzschild metric and the Reissner–Nordström metric. We found Kerr-like geometries without ring singularity. These results, while not definitive, hints in the direction that the polymeric quantization inspired by loop quantum gravity could solve the singularity problem also for the Kerr spacetime. We started considering two different spherically symmetric spacetime obtained in [4] that we called semi-polymeric and full-polymeric. The first metric can be obtained from the latter in an appropriate limit. We studied the semi-polymeric one for reasons of pure simplicity since such metric has all the good properties of regularity. We introduced also the notations types 1 and 2 to indicate the two complexifications we used. For the semi-polymeric spinning LBH, we showed explicitly that the Ricci scalar and the Kretschmann invariant are regular in $r = 0$ and $\theta = \pi/2$. (For the semi-polymeric type 1 LBH, the reader can find the explicit formula for the Ricci scalar and the Kretschmann invariant.) The structure of the event horizon and of the ergosphere is the same as that of the classical Kerr metric, and the causal spacetime structure is given in the text for each case in terms of Penrose diagrams. The full-polymeric spinning LBH has a more rich structure. The ergosphere surfaces are very similar to the classical ones but the horizon surfaces, which are very similar to the classical ones for small values of the angular momentum, change topology for a large value of the angular momentum compared to that of the mass. The singularity here is also cured but in a more elegant way. Any observer in the equatorial plane ($\theta = \pi/2$) can never reach the point $r = 0$ starting from positive or negative values of the radial coordinate. Of course, the Ricci scalar and the Kretschmann invariants are regular.

For the first time we introduced the Reissner–Nordström LBH metric and extended the Newmann–Janis transformation to this one to obtain the Kerr–Newmann LBH with spin and electric charge. The properties of the spinning LBHs are shared by the spinning and charged LBHs. For all semi-polymeric cases studied, there are no naked singularities for any value of the angular momentum.

We studied the presence of CTCs in the region near $r \approx 0$ and $\theta \approx \pi/2$ and we have shown that CTCs disappear in all the new metrics. In particular, for the full-polymeric since each observer can never arrive in $r = 0$, there is no physical reason to extend the spacetime to negative values of $r$ where classically the CTCs are located. This result does not exclude the existence of other CTC’s regions for negative values of $r$ where $g_{\phi\phi}$ changes sign.
In this paper we did not solve the equations of motion coming from a fundamental theory but we simply introduced the angular momentum in spherically symmetric solutions by the Newmann–Janis transformation. However, we can always see spherically symmetric LBHs to be solutions of the Einstein theory with an effective energy tensor: \( G_{\mu\nu} = 8\pi T_{\mu\nu}^{QG} \), where \( T_{\mu\nu}^{QG} \) summarizes the loop corrections. The spinning LBHs obtained in this paper are actually solutions of the Einstein equations with a stress–energy tensor obtained from the spherically symmetric one applying the Newmann–Janis transformation properly. The effective stress–energy tensor is a function of two or three parameters depending on the semi-polymeric or full-polymeric nature of the LBH:

\[
G_{\mu\nu} = 8\pi \begin{cases} 
T_{\mu\nu}^{QG}(a, P), & \text{SEMI-POLYMERIC}, \\
T_{\mu\nu}^{QG}(a, P, a_0), & \text{FULL-POLYMERIC}.
\end{cases}
\]

We conclude the paper summarizing the metrics obtained.

**SEMI-POLYMERIC:**

\[
ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \, dr^2 - \Sigma \, dt^2 - \Sigma \, d\theta^2 + 2a \sin^2 \theta \left( 1 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) \, dt \, d\phi - \sin^2 \theta \left[ \Sigma + a^2 \sin^2 \theta \left( 2 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) \right] \, d\phi^2,
\]

\[
\Delta(r) = r^2 - 2mr + a^2.
\]

**FULL-POLYMERIC:**

\[
ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\rho^4 \Sigma} \left( \rho^2 + rr_+ \right)^2 \, dt^2 - \frac{\Sigma(\rho^2 + rr_+)^2}{\rho^4(\Delta - a^2 \sin^2 \theta) + a^2 \sin^2 \theta(\rho^2 + rr_+)^2} \, dr^2 - \Sigma \, d\theta^2 + 2a \sin^2 \theta(\rho^2 + rr_+)^2 \left[ \Sigma - (\Delta - a^2 \sin^2 \theta) \right] \, dt \, d\phi - \sin^2 \theta \left[ \Sigma + a^2 \sin^2 \theta \left( \frac{(\rho^2 + rr_+)^2(2 \Sigma - (\Delta - a^2 \sin^2 \theta))}{\Sigma \rho^4} \right) \right] \, d\phi^2,
\]

\[
\Delta = r^2 - (r_+ + r_-)r + r_+ r_- + a^2.
\]

Type 1: \( \Sigma = r^2 + a^2 \cos^2 \theta + \frac{a_0^2}{r^2} \),

Type 2: \( \Sigma = r^2 + a^2 \cos^2 \theta + \frac{a_0^2}{r^2 + a^2 \cos^2 \theta} \).

The function \( \Sigma \) is the same for both the metrics.

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Appendix A. Kretschmann invariant for type 1 semi-polymeric LBH

The Kretschmann invariant for type 1 complexified metric reads

\[
K(r, \theta) = \frac{16}{(2r^4 + a^2 r^2 + a^2 \cos(2\theta))} \times (192m^2 r^{18} - 1440a^2 m^2 r^{16} + 1536a^4 m r^{14} \\
+ 1080a^4 m^2 r^{14} - 3072a^2 m^2 r^{14} - 2880a^2 a^2 m r^{13} + 4032a^4 r^{12} \\
- 60a^6 m^2 r^{12} + 7296a^2 a^2 m^2 r^{12} - 6a^8 m^2 \cos(6\theta) r^{12} \\
- 17408a^4 m r^{11} - 1536a^4 a^2 m r^{11} + 5328a^4 r^{10} \\
+ 18816a^2 m^2 r^{10} - 288a^4 m^2 r^{10} - 10464a^4 m r^9 \\
- 312a^6 m^2 r^9 + 12a^6 a^2 m \cos(6\theta) r^9 - 2176a^8 r^8 \\
+ 3192a^4 a^2 r^8 - 160a^2 a^2 m^2 r^8 + 7168a^4 m r^7 \\
- 3360a^4 a^2 m r^7 - 672a^2 a^2 r^7 + 1002a^2 a^2 r^6 - 4608a^4 m^2 r^6 \\
+ 336a^4 a^2 m^2 r^6 + 3a^4 a^2 m \cos(6\theta) r^6 - 448a^2 a^2 m r^5 \\
- 396a^4 a^2 m r^5 - 18a^4 a^2 m \cos(6\theta) r^5 + 704a^4 r^4 \\
+ 144a^4 a^2 m r^4 + 144a^2 a^4 r^4 + 64a^2 a^4 m r^4 + 9a^4 a^2 \cos(6\theta) r^4 \\
- 1536a^4 m r^3 + 88a^4 a^2 m r^3 - 176a^4 a^2 r^2 + 48a^6 a^2 r^2 \\
+ 960a^8 m^2 r^2 - 32a^2 a^8 m r + 34a^4 a^2 + a^2 (207a^4 r^4 a^6 \\
+ 3 (-30m^2 r^2 - 132a^2 m r^9 + 3^2 (133r - 62m) r^5 \\
+ 16a^2 r^5)^2 a^4 + 8(180m^2 r^4 - 48a^2 m (m + 4r) r^4 \\
+ 2a^2 (28m^2 - 216m + 123r^2) r^6 + 4a^2 (m - 3r) r^3 + 5a^2 ) a^2 \\
- 16r (90m^2 r^5 + 12a^2 m (21r - 38m) r^1 + a^2 (10m)^2 \\
+ 26r^2 - 27r^2 + 7) r^7 - 2a^2 (2m^2 + 2m + r^2) r^3 \\
+ a^2 (30m - 19r) \cos(6\theta) + 2a^2 (18m^2 (10r^2 - a^2) r^12 \\
- 12a^2 m (3m^2 + 4mr) r^9 + a^2 (36a^2 + 99r^2 - 90mr) a^2 \\
+ 4r^2 (14m^2 - 12r m + 9r^2) r^4 - 4a^2 a^2 (7m - 6r) r^3 + 11a^2 \cos(6\theta)). \tag{A.1}
\]

This quantity is regular and finite everywhere and in particular

\[
\lim_{r \to 0} \left( \lim_{\theta \to \pi/2} K(r, \theta) \right) = \lim_{r \to 0} \left( \lim_{\theta \to 0} K(r, \theta) \right) = \frac{4a^4}{a^6}. \tag{A.2}
\]

Appendix B. Tortoise coordinates for the full-polymeric metric

The case of two distinct horizons with type 1 complexification:

\[
r^* = r - \frac{a^2}{r r_2 r_1} - \frac{(a^2 r_2^2 + a^2 + r_2^2)}{r_2^2 (r_1 - r_2)} \log |r - r_2| \\
+ \frac{(a^2 r_1^2 + a^2 + r_1^2)}{r_1^2 (r_1 - r_2)} \log |r - r_1| + \frac{(a^2 r_1^2 - a^2 r_1^2)}{r_1^2 r_2^2} \log |r|, \tag{B.1}
\]

where \(r_{1,2}\) are the bigger and the smaller horizons for \(\theta = 0\) and then coincide with the roots in (58).
The case of no horizons at all and type 1 complexification:

\[
r^+ = r - \frac{a_0^2}{r(a^2 + r_- + r_+)} + \frac{a_0^2 \log(r)(r_- + r_+)}{(a^2 + r_- + r_+)^2}
+ \frac{(r_- + r_+)(a^4 + 2a^2r_+ - a_0^2 + r_-^2)}{2(a^2 + r_- + r_+)^2} \log(a^2 + (r - r_-)(r - r_+))
+ \arctan\left[\frac{2r - r_- - r_+}{\sqrt{4a^2 - (r_- - r_+)^2(a^2 + r_- + r_+)^2}}\right]
+ \sqrt{4a^2 - (r_- - r_+)^2(a^2 + r_- + r_+)^2}\left[a^4(r_-^2 + r_+^2) + a^2(2r_-r_+(r_-^2 + r_+^2) - 2a_0^2) + (r_-^2 + r_+^2)(a_0^2 + r_-^2 + r_+^2)\right].
\] (B.2)

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