Research on Optimal Algorithm for Time-domain Synthesis of Shock Response Spectrum Based on Continuous Second Derivative

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Abstract. Aerospace power supply and distribution products must predict the shock response based on the shock boundary conditions to determine the mechanical design safety factor. In this paper, the SRS of a typical product is used as an input condition, and a damped-sine discrete wave is used to synthesize an equivalent discrete time-domain sequence for simulation calculation. In order to improve the calculation efficiency, a low-sampling frequency time-domain sequence optimization algorithm based on continuous second derivative is proposed; through calculation comparison and verification, the algorithm can effectively improve the simulation operation efficiency, and the response prediction accuracy can meet the allowable requirements, which provides a new idea for products’ SRS calculation.

1. Introduction
Satellite power supply and distribution technology is a key technology in the aerospace field, and it is the lifeline of aerospace products. The power supply and distribution products will distribute the power output from the satellite primary power supply to each electrical equipment or system according to the needs, and carry out real-time on-off control of the electrical equipment according to the command of the satellite data management system or the command of the ground measurement and control station. In order to realize the above functions, the electrical part of power supply and distribution products is equipped with a large number of mechanical sensitive devices represented by relays to realize state control. Relay is an electromechanical combination device composed of moving mechanism and electromagnetic structure. It is a vibration shock sensitive device, which can withstand impact during using and transportation. The impact often makes it excite forced vibration and natural frequency response, resulting in different degrees of damage to its performance and structural strength and even failure; the reliability and life of relay in mechanical environment largely determines the safety of power supply and distribution products, if the relay fails, it may even endanger the safety of the whole spacecraft in orbit.

In order to improve the safety of mechanical sensitive devices, shock response simulation must be carried out according to the mechanical boundary conditions of products at the beginning of design to determine the safety margin of mechanical design. One or more equivalent discrete time series of shock spectrum are generated as input curves. Chen et. adopted wavelet synthesis method; Liu et. adopted rectangular wave synthesis method; Yang et. adopted wavelet superposition Hanning window synthesis method in literature. Although the results of time-domain sequence synthesized by above methods can m...
et the requirements of shock spectrum tolerance, however, in the process of simulation calculation, due to the relationship between the sampling frequency and the calculation error of shock spectrum, it is necessary to greatly increase the sampling frequency to at least 10 times of the highest frequency of the shock response spectrum, which will make the subsequent impact simulation volume larger and longer; for example, a product containing 87653 elements and 423117 nodes, when the sampling frequency is 51200, it takes about 112 hours to complete a shock response spectrum calculation, which greatly restricts the design efficiency.

In this paper, an optimization algorithm is proposed by combining the discrete time-domain sequence of equivalent shock spectrum synthesized by attenuating sine wave and continuous interpolation of second-order difference quotient, which can reduce the dimension of sampling frequency, improve the simulation calculation speed of shock response and ensure the calculation accuracy does not exceed the established index.

2. Synthesis method of shock spectrum damped sine wave and its error

The product shock spectrum is generally given by segmented spectral lines, including two parts. The common shock simulation method is to transform the shock spectrum into its equivalent time-domain curve by a certain selected method, and set the curve as the product load and load it on the product to calculate its response. In this paper, the attenuated sine wave is selected as its basic wave to synthesize the equivalent time-domain curve.

2.1. Typical shock response spectrum

The shock spectrum of typical products is shown in Table 1. The acceleration is expressed as $+6 \text{ dB/oct}$ in the rising frequency range of $100 \sim 600\text{Hz}$. The maximum frequency of the shock spectrum is recorded as $f_{\text{max}}$. The acceleration is constant at $600\text{g}$.

| Frequency $(f \text{ Hz})$ | Shock spectrum acceleration | tolerance $(dB)$ |
|---------------------------|-----------------------------|------------------|
| 100~600                   | $+6\text{ dB/oct}$          | ±3               |
| 600~3000                  | $600\text{g}$              |                  |

The shock spectrum is shown in Fig. 1.

![shock spectrum of typical products](image.jpg)

**Figure 1.** Shock spectrum of typical products.
2.2. synthesis method of attenuated sine wave

The equation of single damped sine wave is as follows:

\[
W_n(t) = A_n \cdot e^{[-\xi_n \cdot \omega_n \cdot (t-t_{d_n})]} \cdot \sin[\omega_n \cdot (t-t_{d_n})]
\]  

(1)

When \( t < t_{d_n} \), \( W_n(t) = 0 \);

Among them:
- \( W_n(t) \) is the acceleration of damped sine wave mat time \( t \);
- \( A_n \) is the acceleration can be positive or negative;
- \( \xi_n \) is the damped sine damping ratio;
- \( \omega_n \) is the natural frequency;
- \( t_{d_n} \) is time delay for damped sine wave.

Since the waveform has only slight damping, it is generally assumed that the natural frequency of damping is consistent with that of the system.

The equivalent time-domain values of acceleration at any time of the synthesis of damped sine waves can be expressed as follows:

\[
f(t) = \sum_{i=1}^{n} W_i(t)(0 \leq t \leq t_m)
\]  

(2)

\( t_m \) is the duration of the composite discrete time series.

General damped sine wave synthesis can be carried out according to the steps listed in Table 2.

![Figure 2. flow chart of discrete acceleration time series synthesis.](image-url)
0. Determine the input conditions: determine the target SRS Tolerance $Tol_u$ and $Tol_d$, continuous $t_m$ Sampling frequency and $f_s$;

1. Randomization initial parameter: generate random parameter for each damped sine wave: amplitude $A_n$, damping ratio $\xi_n$ and delays $t_{dn}$; natural frequency $\omega_n$ in the unit of frequency multiplication, the number of times $1/W$, which is generally selected as $1/6.1/12$ perhaps $1/24$;

The relationship between adjacent frequency doubling is as follows:

$$\frac{f_n(j+1)}{f_n(j)} = 2^{\frac{1}{W}} \quad (3)$$

2. Generating the initial discrete time series: synthesizing the initial acceleration time series according to the randomly generated parameters;

3. Initial SRS calculation: calculate the shock response spectrum of composite discrete time series;

4. Calculate the main frequency amplitude scale factor $\theta$: combining the response spectrum of the target with the shock spectrum $SRS$, a scale factor is formed for each main frequency parameter

$$\theta = \frac{A_i}{A_{mi}} \quad (4)$$

Among them, $A_i$ is the corresponding amplitudes of main frequencies which is calculated; $A_{mi}$ is target amplitudes corresponding to main frequencies

5. Modify the amplitude of main frequency: scaling damping sine wave amplitude;

6. Generate modified discrete time series: generate modified acceleration discrete time series;

7-8. Iterative calculation: repeat steps 3-6 and 1-7 for cyclic iteration until the error of SRS converges to $\pm 3db$;

9. Determine the output waveform: select the waveform with minimum error to meet the specified requirements SRS

2.3. The relationship of sampling frequency and equivalent SRS error

Step 3 of table 2 is based on the determined sampling frequency $f_s$. In general, SRS is calculated based on the slope invariant method, and the acceleration amplitude error based on this method is analyzed $\varepsilon(f)$. The corresponding relationship of maximum dominant frequency $f$ and sampling frequency $f_s$ is shown as follows:

$$\varepsilon(f) = 1 - \left[ \sin \left( \frac{\pi \cdot f}{f_s} \right) \right]^2 \quad (5)$$
The curve in Figure 3 shows that the slope invariant method can only be used when the frequency value concerned is far lower than the sampling frequency. If the upper limit of the frequency range calculated by this algorithm reaches 10% of the sampling frequency, the maximum error is 8%; if the upper limit of the frequency range is 5% of the sampling frequency, the maximum error is 2%. Generally, the sampling frequency should be at least 10 times of the highest frequency component in the input signal.

3. Interpolation optimization method

In order to meet the accuracy requirements of the calculation results, the interval of each discrete time \( \Delta T \) must be set one or \( n \) internal points. At least \( Z = n \cdot t_m \cdot f_s, n = 1,2,\ldots \) sampling points must be set in \( t_n \). When \( f_s \) is larger, the calculation will be more accuracy, but it will lead to the huge calculation model and reduce the simulation efficiency. When \( t_m = 0.05 \cdot f_s = 5120 \), at least 2560 points are needed in the simulation process. When \( f_s \) is smaller, such as \( t_m = 0.05 \cdot f_s = 640 \), although only 320 output points are needed, the calculation efficiency is improved by 87.5%, but the corresponding theoretical error will be 54.33%. In order to solve the above problems, an optimization algorithm based on the combination of sampling frequency dimensionality reduction and discrete time series interpolation optimization is proposed to reduce the sampling frequency, simplify the simulation process and ensure the calculation accuracy.

3.1. Dimension reduction of sampling frequency

The basic principle of dimension reduction of sampling frequency is to selected between \( [2 \cdot f_{\text{max}}, 10 \cdot f_{\text{max}}] \). In this paper, the lower limit 6400 is selected as the reduced dimension sampling frequency.

3.2. Interpolation optimization algorithm

In order to improve the accuracy of SRS acceleration amplitude calculation after reduced sampling frequency, the discrete time series must be interpolated.
If \( a = x_0 < x_1 < \ldots < x_n = b \), order \( S(x) \in C^2[a, b] \), \( S(x_i) = f(x_i), x_i (i = 0, 1 \ldots n) \), then called \( S(x) \) by \( f(x) \) (in the interval \([a, b]\)) is the guarantee function interpolation of \( S(x) \) in the of interval \([a, b]\). Suppose the inner second order difference quotient of \( S(x) \) is continuous of inner node \( x_i (i = 0, 1 \ldots n - 1) \), the continuity conditions shall be met at:

\[
S^k(x_i^-) = S^k(x_i^+), (k = 0, 1)
\]

\( S(x) \), in this paper, we choose cubic polynomial as interpolation algorithm. In the interval \([x_i, x_{i+1}]\), \( S(x) \) can be written as follows:

\[
S(x) = a_i \cdot x^3 + b_i \cdot x^2 + c_i \cdot x + d_i
\]

\((i = 0, 1 \ldots n - 1)\) (7)

Solve the undetermined constants in the above formula \( a_i \sim d_i \). In this paper, the three moment method is selected to solve the problem

\[
\gamma_{i-1} \cdot M_{i-1} + 2 \cdot M_i + \alpha_{i+1} \cdot M_{i+1} = \beta_i
\]

Among them:

\[
h_i = x_i - x_{i-1}
\]

\[
y_i = S(x_i)
\]

\[
\alpha_i = \frac{h_{i+1}}{h_i + h_{i+1}}
\]

\[
\beta_i = \frac{6}{h_i + h_{i+1}} \left( \frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i} \right)
\]

\[
\gamma_i = 1 - \alpha_i
\]

(12)

According to the solution method of class II boundary conditions of cubic interpolation function, the method is determined as \( S'(x_0) = m_0 \) as well as \( S'(x_n) = m_n \) to solve the boundary conditions for it.

\[
m_0 = f'(t_a) = \sum_{i=1}^{n} W_i(t_a)
\]

\[
m_n = f'(t_b) = \sum_{i=1}^{n} W_i(t_b)
\]

(13) (14)

4. Example

Take the target shock spectrum shown in Table 1 as an example, select \( t_{sa} = 0.05s \), then \( f_s \) will be 51200 and 6400 respectively, their curves are shown in Fig. 4 (a). When \( f_s \) is 51200, the maximum error is
shown in the sequence diagram $\text{error}_{\text{max}}$, the average error is 0.528 dB. As shown in Fig. 5, when $f_s$ is 6400, the maximum error of the resultant acceleration amplitude is obtained $\text{error}_{\text{max}}$ 6.34 dB and the average error was 4.38 dB. After interpolation optimization for $f_s=6400$ according to the method shown in Section 2.2, the maximum error of acceleration amplitude is optimized $\text{error}_{\text{max}}$ 1.23 dB, and the average error was 0.83 dB. Comparing to $f_s$ 51200, the consistency is very good, and the error distribution is within the tolerance range.

Figure 4. discrete time series curve.
5. Conclusions

In order to improve the prediction efficiency of shock response in high-precision shock simulation of products and save computing resources, starting from the equivalent calculation method of shock response spectrum, a discrete time-domain sequence synthesis method based on damped sine wave is proposed. The relationship between sampling frequency and calculation error of shock response spectrum is given, and combined with continuous interpolation optimization of second-order difference quotient, the results show that the efficiency of simulation can be improved by more than 85%, and the accuracy tolerance of simulation curve is within the allowable range.

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