Vanishing Contribution to quark electric dipole moment in the 2HD model with CKM CP violation

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Abstract

In the standard model (SM) of electroweak interactions, CP noninvariance arises from the nonzero phase in the CKM matrix. Its contribution to the quark electric dipole moment (EDM) vanishes surprisingly at two loop order. This makes the quark EDM extremely small in the SM. In this paper, we consider the two Higgs doublet extension of the SM and assume that CP noninvariance is still encoded in the CKM matrix. We calculate the charged Higgs boson contribution to the quark EDM which naively should be of order $eG_F\delta(4\pi)^{-4}m_{u(d)}m_{\tilde{g}}m_{H}^2$ for the up (down) quark with possible enhancement factors of $\tan^2\beta$. Here $\delta$ is the rephasing invariant of CP violation. However, contrary to the above naive expectation, we find that the charged Higgs boson contribution vanishes strictly at two loop order. We show explicitly how this comes about and explains how it is related to the general form of Yukawa couplings in a spontaneously broken gauge theory.

Keywords: electric dipole moment, charged Higgs boson, CP violation

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I. INTRODUCTION

CP noninvariance in any models with CPT symmetry will in general induce P- and T-violating electric dipole moments (EDM) for elementary particles through quantum effects. The discovery of these moments would be direct evidence of CP noninvariance outside the scope of the neutral kaon system and would help us identify the origin of CP noninvariance \[1\]. Of special interest among these moments is the neutron EDM. The current experimental upper bound for the neutron is \[|d(N)| < 1.1 \times 10^{-25}\text{ e cm} \#2\] and this limit is hopefully to be improved by several orders of magnitude in the near future \#3.

From theoretical point of view, any calculation of the neutron EDM proceeds in two steps. In the first step, one writes down all relevant operators which break the CP symmetry and involve only light degrees of freedom. These operators are usually defined at a high energy scale where CP violation occurs. They include the electric or chromo-electric dipole moments of the light quarks and gluons \#4, four quark operators \#5 and possibly others. In the second step, they are evolved down to the typical hadronic scale and their effects on the neutron EDM are then calculated. While naive dimensional analysis or hadronic models have to be invoked in the second step, the first step can be implemented unambiguously once the model of CP violation is specified. In this work, we will be concerned with this first step calculation, in particular, the calculation of the quark EDM.

In the standard model (SM) of electroweak interactions, CP noninvariance arises from the nonzero phase in the CKM matrix. The quark EDM vanishes trivially at one loop order since only moduli of the matrix are involved in the relevant amplitude. At two loop order, the flavour structure is rich enough which could in principle allow for a CP-violating EDM \#6; however, the final contribution to the quark EDM vanishes surprisingly when the sum over internal virtual flavours is taken \#7 \#8. This circumstance also appears in the $W^\pm$ EDM \#9 \#10. Although there are attempts to understand the vanishing result \#11 \#12, it seems clear by now that it does not result from any symmetry which would dictate the zero EDM automatically at the lowest nontrivial order. Furthermore the vanishing result is accidental in some sense, as a result of specific Lorentz and flavour structure. First, the $W^\pm$ boson still acquires a nonvanishing, P- and T-violating magnetic quadrupole moment at two loop order \#13 though its EDM vanishes at the same order. Second, the quark EDM does not vanish any more when QCD corrections are included \#14 \#11 \#13. Of course, this makes the quark EDM extremely small in the SM. If the light quark EDM is one of the important contributions to the neutron EDM, it is then hopeless to observe the neutron EDM in the near future. Considering this, we would like to investigate how the quark EDM could be enhanced beyond the SM. In this paper we study the two Higgs doublet extension of the SM \#14. We will assume conservatively that CP noninvariance is still encoded in the CKM matrix so that the charged Higgs boson $H^\pm$ is the only other particle besides $W^\pm$ that mediates CP violation. Since the Yukawa couplings between $H^\pm$ and quarks generally involve the relevant quark masses and are thus less universal as compared to the gauge couplings between $W^\pm$ and quarks, we would expect naively that the $H^\pm$ exchange will make a contribution to the quark
EDM which should be of order \( d(u(d)) \sim eG_F^2 \delta(4\pi)^{-4} m_{u(d)}^2 m_{t}^2 m_{H}^{-2} \) for the up (down) quark. [Here \( \delta \) is the rephasing invariant of CP violation \(^{17}\).] As we will display later on, \( d(d) \) could even be enhanced by a large factor of \( \tan^2 \beta \) which would make \( d(d) \) easily reach the level of \( 10^{-31} \text{ e cm} \) for a charged Higgs mass of 200 GeV. However, a detailed calculation shows that the above naive expectation is actually not realized in the final result: the \( H^\pm \) contribution vanishes strictly at two loop order when the sum over internal virtual quark flavours is taken. We show explicitly how this null result comes about as a consequence of the general form of Yukawa couplings in a spontaneously broken gauge theory.

The following sections are organized as follows. In section 2, we first discuss the renormalization of one loop elements to be used in the complete two loop calculation. Then, we present respectively the contributions from exchanges of two charged Higgs bosons, one charged Higgs boson and one \( W^\pm \), for the general form of Yukawa couplings. In passing we also give the result from exchanges of two \( W^\pm \) which was previously calculated in the SM. The naive expectation for \( d(u) \) and \( d(d) \) is then verified. In section 3, we first show that the contribution involving exchanges of \( H^\pm \) or \( W^\pm \) vanishes when we sum over internal quark flavours. We also indicate a difference of the cancellation mechanism in the present case and in the SM case as computed in the unitarity gauge. Then we examine generally how it could be possible to have such a vanishing result. Section 4 is a recapitulation of our result.

**II. EXPLICIT RESULT OF CHARGED HIGGS BOSON CONTRIBUTIONS**

We shall calculate in this section the quark EDM arising from exchange of \( H^\pm \) and \( W^\pm \) in the two Higgs doublet extension of the SM. The Feynman diagrams at the lowest two loop order are depicted in Fig. 1. We shall denote the internal up-type quarks by Greek letters \( \alpha, \beta \) etc., the internal down-type quarks by Latin letters \( j, k \) etc., and the external up- or down-type quark by \( e \). Within this section, the flavours of internal quarks are fixed. We shall examine in the next section what will happen when summation over flavours is done.

To set up our notation, we first list the relevant Feynman rules. The Feynman rule for the \( \bar{u}_\alpha d_j W_{\mu}^+ \) vertex is

\[
i \frac{g}{2\sqrt{2}} V_{\alpha j} \gamma_\mu (1 - \gamma_5),
\]

where \( g \) is the \( SU(2) \) weak coupling constant and \( V_{\alpha j} \) is the entry \( (\alpha, j) \) of the CKM matrix. Then the vertex \( \bar{d}_j u_\alpha W^-_{\mu} \) is \( i \frac{g}{2\sqrt{2}} V_{\alpha j}^* \gamma_\mu (1 - \gamma_5) \). The Feynman rule for the \( \bar{u}_\alpha d_j H^+ \) vertex is parametrized as

\[
i \frac{g}{2\sqrt{2}m_W} V_{\alpha j} (C_{\alpha j} + C'_{\alpha j} \gamma_5),
\]
where $C_{\alpha j}$ and $C'_{\alpha j}$ are real constants and may depend on the masses of $u_\alpha$ and $d_j$. The vertex $\bar{d}_j u_\alpha H^-$ is then $i g \frac{V_{\alpha j}^*}{2 \sqrt{2} m_W} (C_{\alpha j} - C'_{\alpha j} \gamma_5)$. We emphasize again that $C_{\alpha j}$ and $C'_{\alpha j}$ are assumed to be real in our calculation; i.e., CP noninvariance occurs only in the CKM matrix. If they are complex numbers, it will be completely another story [18].

The ordinary couplings in the Minimal Supersymmetric Standard Model (MSSM) are recovered by setting

$$C_{\alpha j} = u_\alpha \cot \beta + d_j \tan \beta, \quad C'_{\alpha j} = -u_\alpha \cot \beta + d_j \tan \beta,$$

where $\tan \beta$ is a parameter measuring the ratio of the vacuum expectation values of the two Higgs doublet fields. From now on, we always use the names of quarks to denote their masses. The vertices involving the would-be Goldstone bosons, $\bar{u}_\alpha d_j G^+$ and $\bar{d}_j u_\alpha G^-$, also arise as a special case:

$$C_{\alpha j} = u_\alpha - d_j, \quad C'_{\alpha j} = -u_\alpha - d_j.$$  

To simplify the computation of diagrams involving $W^\pm$ exchange, we shall use the background field gauge [19] [20] [21] (or the nonlinear $R_\xi$ gauge [22]) with $\xi = 1$. There will be no mixed $W^\pm G^\mp A$ vertex [A is the external electromagnetic field], and the Feynman rule for the $W^+ W^- A_\mu$ vertex is

$$-i e \left[ (k_0 - k_+) \sigma g_{\mu \rho} + (k_+ - k_-) \mu g_{\rho \sigma} + (k_- - k_0) \rho g_{\sigma \mu} \right],$$

where $k_0, k_+, k_-$ are incoming momenta for the fields $A_\mu, W^+_\rho, W^-_\sigma$. Finally, to avoid any ambiguity, we define the effective EDM interaction as

$$\mathcal{L}_{\text{eff}} = id \bar{\psi} \gamma_5 \sigma_{\mu \nu} \psi F^{\mu \nu},$$

where $d$ is the EDM of the fermion $\psi$ and it is real by Hermiticity. The Feynman rule for the effective vertex $\bar{\psi} \psi A_\mu$ is

$$-d \left[ \gamma_{\mu}, q \right] \gamma_5,$$

where $q$ is the outgoing momentum of the photon.

A. Renormalization of one loop elements

We shall be mainly concerned with the EDM of the up-type quark. The result for the down-type quark will be obtained by simple substitutions at the end. To induce an EDM for the quark $u_e$, the imaginary part of the CKM matrix must be involved so that the flavours $j$ and $k$ in Fig. 1 are different. Therefore, we need to renormalize the off-diagonal self-energy $-i \Sigma^{kj}(\ell)$ and the vertex with the photon $ie \Gamma^{kj}_\mu(\ell, \ell + q; q)$. Denote the bare one loop contribution by a hat and the counter-term one by a tilde, so that the renormalized quantities are

$$\Sigma^{kj} = \hat{\Sigma}^{kj} + \tilde{\Sigma}^{kj}, \quad \Gamma^{kj}_\mu = \hat{\Gamma}^{kj}_\mu + \tilde{\Gamma}^{kj}_\mu.$$
\[ \hat{\Sigma}^{kj}(\ell) = \ell [A(\ell^2) + B(\ell^2)\gamma_5] + u_\alpha [C(\ell^2) + D(\ell^2)\gamma_5]. \]  

The off-diagonal self-energy is renormalized by requiring that there be no mixing when either of \( d_j \) and \( d_k \) is on-shell:

\[ d_k(\ell)\Sigma^{kj}(\ell)|_{q = d_k} = 0, \quad \Sigma^{kj}(\ell)d_j(\ell)|_{q = d_j} = 0. \]  

The counter-term is then determined to be

\[ \hat{\Sigma}^{kj}(\ell) = \frac{1}{d_k - d_j} \left[ (\ell - d_k) \left( d_j A(d_j^2) + u_\alpha C(d_j^2) \right) - \left( d_k A(d_k^2) + u_\alpha C(d_k^2) \right) (\ell - d_j) \right] 
+ \frac{1}{d_k + d_j} \left[ (\ell - d_k)\gamma_5 \left( -d_j B(d_j^2) + u_\alpha D(d_j^2) \right) + \left( d_k B(d_k^2) + u_\alpha D(d_k^2) \right) \gamma_5 (\ell - d_j) \right]. \]  

The renormalization of the vertex \( \Gamma^{kj}_\mu \) is not independent but related to that of the self-energy by the Ward identity,

\[ q^\mu ie\Gamma^{kj}_\mu(\ell, \ell + q; q) = ieQ_d \left[ \Sigma^{kj}(\ell) - \Sigma^{kj}(\ell + q) \right]. \]

It may be explicitly checked that the bare one loop quantities satisfy the above identity so that the latter must also be separately satisfied by the counter-term quantities. In this way, we find

\[ \hat{\Gamma}^{kj}_\mu(\ell, \ell + q; q) = Q_d \gamma_\mu \left\{ \frac{1}{d_k - d_j} \left[ (d_k A(d_k^2) - d_j A(d_j^2)) + u_\alpha \left( C(d_k^2) - C(d_j^2) \right) \right] 
+ \frac{1}{d_k + d_j} \left[ (d_k B(d_k^2) + d_j B(d_j^2)) + u_\alpha \left( D(d_k^2) - D(d_j^2) \right) \right] \gamma_5 \right\}. \]

Since we shall present separate contributions from exchanges of \( H^\pm \) and \( W^\pm \) in the subsequent subsections, we give below the functions \( A, B, C, D \) arising from exchanges of \( H^\pm, W^\pm \) and \( G^\pm \). We work in \( n = 4 - 2\epsilon \) dimensions to regularize ultraviolet divergences. For the \( H^\pm \) exchange,

\[ A(\ell^2) = -(4\pi)^{-2}G_F/\sqrt{2}V_{ak}V_{aj}(C_{ak}C_{aj} + C'_{ak}C'_{aj}) f_1(\ell^2, u_\alpha^2, m_H^2), \]
\[ B(\ell^2) = -(4\pi)^{-2}G_F/\sqrt{2}V_{ak}V_{aj}(C_{ak}C'_{aj} + C'_{ak}C_{aj}) f_1(\ell^2, u_\alpha^2, m_H^2), \]
\[ C(\ell^2) = -(4\pi)^{-2}G_F/\sqrt{2}V_{ak}V_{aj}(C_{ak}C_{aj} - C'_{ak}C'_{aj}) f_0(\ell^2, u_\alpha^2, m_H^2), \]
\[ D(\ell^2) = -(4\pi)^{-2}G_F/\sqrt{2}V_{ak}V_{aj}(C_{ak}C'_{aj} - C'_{ak}C_{aj}) f_0(\ell^2, u_\alpha^2, m_H^2). \]

For the \( W^\pm \) exchange in the \( \xi = 1 \) gauge,

\[ A(\ell^2) = -B(\ell^2) = +(4\pi)^{-2}g^2V_{ak}V_{aj} \frac{1}{4} (2 - n) f_1(\ell^2, u_\alpha^2, m_W^2), \]
\[ C(\ell^2) = D(\ell^2) = 0. \]

The \( G^\pm \) exchange in \( \xi = 1 \) gauge is a special case of the \( H^\pm \) exchange, i.e., \( m_H^2 \to m_W^2 \), and with couplings \( C \) and \( C' \) substituted by values in Eqn.(4). The functions \( f_1 \) and \( f_0 \) are given in the Appendix.
B. Double $H^\pm$ exchanges

The momentum arrangement for external quarks and photon is shown in Fig. 1. To pick out the EDM, we expand to the linear order term in the photon momentum $q$. One should be careful in dropping terms that are superficially of zero order in $q$, since some of them are actually of linear order when the equation of motion is applied, and thus may contribute to the EDM. Notice that the final $\gamma_5$ in the effective EDM vertex can only come from Yukawa vertices since there would be no P violation if no $\gamma_5$ were involved in these vertices. To simplify the expression, the equation of motion for external quarks is freely used and only those terms that can finally contribute are kept. After a tedious computation, the Feynman diagrams in Fig. 1 sum up to the following structure with a common coefficient, $(4\pi)^{-2}eG_F^2/2\nu V_{ek}V_{\alpha j}V_{\alpha i}^* q^\nu \gamma_5$:

\[
+C_1(e\alpha; kj) [F_{1,\mu\nu}(k) - F_{1,\mu\nu}(j)] + C_2(e\alpha; k) [F_{2,\mu\nu}(k) - F_{2,\mu\nu}(j)] \\
+C_3(e\alpha; kj) [F_{3,\mu\nu}(k) - F_{3,\mu\nu}(j)] + C_4(e\alpha; kj) [F_{4,\mu\nu}(k) - F_{4,\mu\nu}(j)] \\
+C_5(e\alpha; kj) [F_{5,\mu\nu}(k) - F_{5,\mu\nu}(j)],
\]

(16)

where $C_i$ are combinations of Yukawa couplings and quark masses $d_k$ and $d_j$:

\[
C_1(e\alpha; kj) = \left[ (C_{ek}C_{ej} - C_{ek}^\prime C_{ej}')(C_{ek}C_{\alpha j} - C_{ak}C_{\alpha j}^\prime) \\
- (C_{ak}C_{\alpha j} - C_{ak}^\prime C_{\alpha j}')(C_{ek}C_{ej} - C_{ek}C_{ej}^\prime) \right] \frac{d_j d_k}{d_k^2 - d_j^2},
\]

\[
C_2(e\alpha; kj) = \left[ (C_{ek}C_{ej} - C_{ek}^\prime C_{ej}')(C_{ak}C_{\alpha j} - C_{ak}C_{\alpha j}^\prime) \\
+ (C_{ak}C_{\alpha j} - C_{ak}^\prime C_{\alpha j}')(C_{ek}C_{ej} - C_{ek}C_{ej}^\prime) \right] \frac{1}{d_k^2 - d_j^2},
\]

\[
C_3(e\alpha; kj) = \left[ -(C_{ek}C_{ej} - C_{ek}^\prime C_{ej}')(C_{ak}C_{\alpha j} - C_{ak}C_{\alpha j}^\prime) \frac{1}{d_k + d_j} \\
- (C_{ak}C_{\alpha j} - C_{ak}^\prime C_{\alpha j}')(C_{ek}C_{ej} - C_{ek}C_{ej}^\prime) \frac{1}{d_k - d_j} \right] d_k,
\]

(17)

\[
C_3(e\alpha; kj) = C_5(e\alpha; kj)|_{j\leftrightarrow k},
\]

\[
C_4(e\alpha; kj) = \left[ -(C_{ek}C_{ej} - C_{ek}^\prime C_{ej}')(C_{ak}C_{\alpha j} + C_{ak}C_{\alpha j}^\prime) \frac{1}{d_k + d_j} \\
+ (C_{ek}C_{ej} - C_{ek}C_{ej}')(C_{ak}C_{\alpha j} + C_{ak}C_{\alpha j}^\prime) \frac{1}{d_k - d_j} \right],
\]

\[
C_5(e\alpha; kj) = C_4(e\alpha; kj)|_{\alpha\leftrightarrow e}.
\]

The functions $F_{i,\mu\nu}$ are complicated loop momentum integrals which are too lengthy to be displayed here. For brevity, we only indicate their dependence on the internal quark mass $d_k$ or $d_j$ although they depend as well on the external momentum $p$, the internal quark mass $u_\alpha$ and the charged Higgs mass $m_H$.

The above results are obtained without using any approximations. The discussions in the next section will be based on these general results. We notice from Eqn.(17) that the coupling combinations $C_1$, $C_2$, $C_4$ and $C_5$ are symmetric with respect to $k$ and $j$ such that the structure in Eqn.(16) is antisymmetric with respect to $k$ and $j$. Therefore, the mirror-reflected diagrams corresponding to interchange of $k$ and $j$ in Fig.1 are simply
related by \( V_{ek}V_{ak}^*V_{e j}V_{ej}^* \rightarrow -V_{ej}V_{aj}V_{ak}V_{ek}^* = -\left(V_{ek}V_{ak}^*V_{e j}V_{ej}^*\right)^* \), so that in their sum the \( \text{Re}(V_{ek}V_{ak}^*V_{e j}V_{ej}^*) \) term is cancelled while the \( \text{Im}(V_{ek}V_{ak}^*V_{e j}V_{ej}^*) \) term is doubled. This is essential, as emphasized for the SM case in Ref. [12], to guarantee that the EDM is a real number as required by the Hermiticity of the effective action. The results for the down-type quark \( d_e \) are obtained by the following substitutions:

\[
\begin{align*}
V_{ek}V_{ak}^*V_{e j}V_{ej}^* & \rightarrow V_{\beta e}V_{\beta i}V_{\alpha i}V_{\alpha e}, \\
Q_u & \leftrightarrow Q_d, \\
C'_{ek} & \rightarrow C'_{\beta e}, C'_{ak} \rightarrow C'_{\beta i}, C'_{aj} \rightarrow C'_{\alpha i}, C'_{ej} \rightarrow C'_{\alpha e}, \\
J_{\alpha j} & \rightarrow J_{\beta e}, J_{\beta d} \rightarrow J_{\beta i}, J_{\alpha e} \rightarrow J_{\alpha i}, J_{\alpha d} \rightarrow J_{\alpha i}, \\
J_{\alpha j} & \rightarrow J_{\alpha j}, J_{\alpha d} \rightarrow J_{\alpha d}, \\
\alpha_a & \rightarrow d_i, \ d_j \rightarrow u_\alpha, \ u_\beta \rightarrow u_\beta, \ u_\epsilon \rightarrow d_e.
\end{align*}
\]  

\[ (18) \]

Before concluding this subsection, we would like to get some idea of how the quark EDM looks like. For this purpose, let us specialize to the case of the \( u \) and \( d \) quarks in the MSSM. We may use then the small external mass approximation (SEMA). In this approximation, only terms linear in the external mass are kept while higher order terms are safely ignored. [At least one factor of external mass is involved due to the chirality flip feature of the EDM operator.] The formula simplifies considerably. Fig. 1 along with its mirror reflection gives for the \( u \) quark,

\[
d(u) = +eG_F^2 \text{Im}(V_{uk}V_{ak}^*V_{\alpha j}V_{\alpha j}^*)(4\pi)^{-2}2u_u^2[F(k) - F(j)],
\]

\[
F(k) = +Q_u i \int \frac{d^4\ell}{(2\pi)^4} \left[ \frac{m_{H_d}^2 m_{H_u}^2}{D_H^2 D_k^2} \left( I_1(k) \cot^2 \beta + I_0(k) \right) + \frac{\ell^2}{D_H D_k} \left( J_{1,1} \cot^2 \beta + J_{1,1} \right) \right],
\]

where \( D_H = \ell^2 - m_H^2 \). For later discussion we should mention that the whole contribution from Fig. 1(a) is given by the term involving \( D_H^2 \) upon setting \( Q_u \) to 1. The \( I \) and \( J \) functions arise from the inner loop integration and their explicit forms are given in the Appendix. The relevant feature at the moment is that the \( J \) functions depend only on \( \ell^2 \) and the inner loop masses \( m_{H_d}^2 \) and \( u_u^2 \) while the \( I \) functions depend also on \( d_i^2 \) or \( d_j^2 \) as indicated above. For the \( d \) quark,

\[
d(d) = +eG_F^2 \text{Im}(V_{\beta d}V_{\beta i}V_{\alpha d}V_{\alpha i}^*)(4\pi)^{-2}4d_d^2[F(\beta) - F(\alpha)],
\]

\[
F(\beta) = +Q_d i \int \frac{d^4\ell}{(2\pi)^4} \left[ \frac{m_{H_d}^2 m_{H_d}^2}{D_H^2 D_{\beta}^2} \left( I_1(\beta) \tan^2 \beta + I_0(\beta) \right) + \frac{\ell^2}{D_H D_{\beta}} \left( J_{1,1} \tan^2 \beta + J_{1,1} \right) \right],
\]

where \( D_{\beta} = \ell^2 - u_u^2 \), \( J_{1,1} \) and \( J_{2,1} \) depend on \( \ell^2 \), \( d_i^2 \) and \( m_{H_d}^2 \) while \( I_0(\beta) \) and \( I_1(\beta) \) depend on \( u_u^2 \) as well. Suppose that \( u_\alpha, d_k = d_i \) and \( d_j \) are respectively the top, bottom and down quarks, we find that, up to logarithms,
\[ d(u) \sim eG_F^2 \bar{\delta}(4\pi)^{-4}m_u m_t^2 \frac{m_d^2}{m_H^2} \cdot \left(1 + \cot^2 \beta \right), \]
\[ d(d) \sim eG_F^2 \bar{\delta}(4\pi)^{-4}m_d m_b^2 \frac{m_u^2}{m_H^2} \cdot \left(1 + \tan^2 \beta \right), \]

where \( \bar{\delta} = \text{Im}(V_{ub}V_{td}^* V_{ub}V_{td}^*) \). For numerical estimate we take the following input parameters, \( G_F \sim 10^{-5} \text{ GeV}^{-2} \), \( |\bar{\delta}| \sim 5 \cdot 10^{-5} \), \( m_u \sim m_d \sim 5 \text{ MeV} \), \( m_t \sim 170 \text{ GeV} \), \( m_b \sim 4.5 \text{ GeV} \), \( m_H \sim 200 \text{ GeV} \) and \( \tan \beta \sim 30 \), then,
\[ |d(u)| \sim 10^{-34} \text{ e cm}, \quad |d(d)| \sim 10^{-31} \text{ e cm}. \]

For comparison, we quote the three loop result in the SM [15], \( |d(u)| \sim 0.35 \cdot 10^{-34} \text{ e cm}, \quad |d(d)| \sim 0.15 \cdot 10^{-34} \text{ e cm} \). Therefore, if not for the cancellation mechanism to be discussed in the next section, the result for the light quark EDM in the two Higgs doublet model would be very different from that in the SM.

C. Outer loop \( W^\pm \) plus inner loop \( H^\pm \) exchanges

Since we work in the background field gauge, we present the separate results from \( W^\pm \) and \( G^\pm \) exchanges. For the \( W^\pm \) exchange, Fig. 1(a) does not contribute to the EDM and Fig. 1(b) is completely cancelled by corresponding terms in diagrams (c)-(e). The counter-term diagrams do not contribute either. Diagrams (c)-(e) with their mirror reflection then give for the external \( u_e \) quark,
\begin{align*}
&+eg^2 \sqrt{2} G_F \text{Im}(V_{ek}V_{ak}^* V_{aj}V_{ej}^*)(4\pi)^{-2} u_a \left[ \frac{C_{ak}C_{aj} - C'_{ak}C'_{aj}}{d_k + d_j} - \frac{C_{ak}C'_{aj} - C'_{ak}C_{aj}}{d_k - d_j} \right] \\
&\frac{1}{2} q^\gamma_5 i \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{D_W} \left( \frac{1}{D_k} - \frac{1}{D_j} \right) (Q_u J_{0,1} - Q_d J_{1,1}) (\ell_\mu \gamma_\nu - \ell_\nu \gamma_\mu),
\end{align*}

where \( D_W = (\ell^2 - m_W^2) \). The result for the external \( d_e \) quark is obtained by substitutions. In the SEMA, we have
\begin{align*}
d(u_e) &= +eg^2 G_F / \sqrt{2}(4\pi)^{-2} \text{Im}(V_{ek}V_{ak}^* V_{aj}V_{ej}^*) \frac{1}{4} u_a u_e \left[ \frac{C_{ak}C_{aj} - C'_{ak}C'_{aj}}{d_k + d_j} - \frac{C_{ak}C'_{aj} - C'_{ak}C_{aj}}{d_k - d_j} \right] \\
&i \int \frac{d^4 \ell}{(2\pi)^4} \frac{\ell^2}{D_W^2} \left( \frac{1}{D_k} - \frac{1}{D_j} \right) (Q_u J_{0,1} - Q_d J_{1,1}),
\end{align*}

where now \( D_W = \ell^2 - m_W^2 \). The \( G^\pm \) contribution is a special case of the double \( H^\pm \) exchange; i.e., we only need to replace the couplings \( C_{ek}, C_{ej}, C'_{ek}, C'_{ej} \) by their values in Eqn.(4) and \( D_H \) by \( D_W \). We display here the coupling combinations which are relevant to discussions in the next section:
\[ C_1(e\alpha; kj) = 2u_\alpha d_k d_j \left[ \frac{C_{ek} C_{ej} - C'_{ek} C'_{ej}}{d_k + d_j} + \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right], \]
\[ C_2(e\alpha; kj) = -2u_\alpha \left[ \frac{C_{ek} C_{ej} - C'_{ek} C'_{ej}}{d_k + d_j} - \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right], \]
\[ C_3(e\alpha; kj) = 2u_\alpha d_k [C_{ek} C_{ej} - C'_{ek} C'_{ej}] - (C_{ek} C'_{ej} - C'_{ek} C_{ej}), \]
\[ C_4(e\alpha; kj) = -2u_\alpha (C_{ek} - C'_{ek})(C_{ej} - C'_{ej}), \]
\[ C_5(e\alpha; kj) = 2 \left[ (C_{ek} C_{ej} - C'_{ek} C'_{ej}) \frac{d_k^2 - d_j d_j}{d_k + d_j} + (C_{ek} C'_{ej} - C'_{ek} C_{ej}) \frac{d_k^2 + d_k d_j}{d_k - d_j} \right]. \]

**D. Outer loop \( H^\pm \) plus inner loop \( W^\pm \) exchanges**

The contribution from \( W^\pm \) exchange in Fig. 1 and its mirror reflection is

\[ +g^2\sqrt{2}G_F \text{Im}(V_{ek} V_{ak}^* V_{aj} V_{ej}^*)(4\pi)^{2} \left[ \frac{C_{ek} C_{ej} - C'_{ek} C'_{ej}}{d_k + d_j} - \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right] \]
\[ \frac{1}{4} q^\nu \gamma_5 [H_{\mu\nu}(k) - H_{\mu\nu}(j)], \]

where \( H_{\mu\nu} \) is another chain of loop momentum integrals. In the SEMA, we have for the \( u_e \) quark,

\[ d(u_e) = +g^2\sqrt{2}G_F (4\pi)^{-2} \text{Im}(V_{ek} V_{ak}^* V_{aj} V_{ej}^*), \]
\[ \frac{1}{4} \left[ \frac{C_{ek} C_{ej} - C'_{ek} C'_{ej}}{d_k + d_j} - \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right] [H(k) - H(j)], \]
\[ H(k) = +Q_u i \int \frac{d^4 \ell}{(2\pi)^4} \frac{m_H^2 d_k^2}{D_H D_k} I_1(k) + Q_d i \int \frac{d^4 \ell}{(2\pi)^4} \frac{d^2 d_k^2}{D_H D_k} I_1(k) \]
\[ + \frac{1}{2} i \int \frac{d^4 \ell}{(2\pi)^4} \frac{D_H D_k}{\ell^2} [Q_d J_{2,1} - Q_u J_{1,1}] \]
\[ - (Q_u - Q_d) i \int \frac{d^4 \ell}{(2\pi)^4} \frac{\ell^2}{D_H D_k} J_{2,0}, \]

where \( D_H = \ell^2 - m_H^2 \). The \( G^\pm \) contribution arises as a special case of the double \( H^\pm \) exchange: \( m_H^2 \) in the \( I^- \)– and \( J^- \)–functions is replaced by \( m_W^2 \) and the coupling combinations are given by the following ones,

\[ C_1(e\alpha; kj) = -2u_\alpha d_k d_j \left[ \frac{C_{ek} C_{ej} - C'_{ek} C_{ej}}{d_k + d_j} + \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right], \]
\[ C_2(e\alpha; kj) = -2u_\alpha \left[ \frac{C_{ek} C_{ej} - C'_{ek} C_{ej}}{d_k + d_j} - \frac{C_{ek} C'_{ej} - C'_{ek} C_{ej}}{d_k - d_j} \right], \]
\[ C_3(e\alpha; kj) = 2u_\alpha d_k \left[ (C_{ek} C_{ej} - C'_{ek} C_{ej}) \frac{d_k - d_j}{d_k + d_j} - (C_{ek} C'_{ej} - C'_{ek} C_{ej}) \frac{d_k + d_j}{d_k - d_j} \right], \]
\[ C_4(e\alpha; kj) = 2 \left[ (C_{ek} C_{ej} - C'_{ek} C_{ej}) \frac{u^2_e - d_k d_j}{d_k + d_j} + (C_{ek} C'_{ej} - C'_{ek} C_{ej}) \frac{u^2_e + d_k d_j}{d_k - d_j} \right], \]
\[ C_5(e\alpha; kj) = -2u_\alpha (C_{ek} - C'_{ek})(C_{ej} - C'_{ej}). \]

**E. Double \( W^\pm \) exchanges**
For completeness, we present in this subsection the result from double $W^\pm$ exchanges, i.e., the SM result. Since we work in the background field gauge, we may separate four kinds of contributions: double $W^\pm$ exchanges, double $G^\pm$ exchanges, outer loop $W^\pm$ plus inner loop $G^\pm$ exchanges, and outer loop $G^\pm$ plus inner loop $W^\pm$ exchanges. For double $W^\pm$ exchanges, Fig. 1(a) does not contribute and the contributions from (b)-(e) are completely cancelled. The counter-term diagrams do not contribute either. The case of double $G^\pm$ exchanges is recovered from subsection B by $m^2_H \to m^2_W$ and by evaluating couplings in terms of Eqn.(4); i.e., the coupling combinations become

\begin{align}
C_1(e\alpha; k j) &= 0, \quad C_2(e\alpha; k j) = +8u_\alpha u_e d_k, \\
C_4(e\alpha; k j) &= -8u_\alpha^2 u_e d_k, \quad C_5(e\alpha; k j) = -8u_\alpha u_e^2.
\end{align}

The contribution from outer loop $W^\pm$ plus inner loop $G^\pm$ exchanges is obtained from Eqn.(23) by $m^2_H \to m^2_W$ and replacing the coupling combination in the square parentheses by $-4u_\alpha$. Similarly, for outer loop $G^\pm$ plus inner loop $W^\pm$ exchanges, we replace the coupling combination of Eqn.(26) by $-4u_e$.

### III. ANALYSIS OF CANCELLATION MECHANISM

The SM result for the quark EDM was presented in the subsection 2E. For fixed internal quark flavours $\alpha, j, k$ and external quark $u_e$ and upon summing the pair of reflection-related diagrams, it has the following structure,

\begin{equation}
\begin{aligned}
\delta(u_e) &= \text{Im}(V_{ek}V_{ak}^* V_{aj}V_{ej}^*) [H(k) - H(j)], \\
\text{where the two terms depend exclusively on flavours } d_k \text{ and } d_j \text{ respectively.}
\end{aligned}
\end{equation}

When we evaluate the contribution involving exchange of charged Higgs bosons in the MSSM by using the couplings in Eqn.(3), we find that the above structure is also preserved. Summing over the three down-type flavours $i, j, k$ while fixing the up-type flavour $\alpha$, we arrive at

\begin{equation}
\begin{aligned}
\delta(u_e) &= +\text{Im}(V_{ek}V_{ak}^* V_{aj}V_{ej}^*) [H(k) - H(j)] + \text{Im}(V_{ei}V_{ai}^* V_{ak}V_{ek}^*) [H(i) - H(k)] \\
& \quad + \text{Im}(V_{ej}V_{aj}^* V_{ai}V_{ei}^*) [H(j) - H(i)] \\
& = 0,
\end{aligned}
\end{equation}

where the second equality is due to unitarity of the CKM matrix; e.g., the $H(k)$ term is

\begin{equation}
\begin{aligned}
& \left[ \text{Im}(V_{ek}V_{ak}^* V_{aj}V_{ej}^*) - \text{Im}(V_{ei}V_{ai}^* V_{ak}V_{ek}^*) \right] H(k) \\
& = \text{Im} \left[ V_{ek}V_{ak}^* (\delta_{ae} - V_{ak}V_{ek}^*) \right] H(k) \\
& = 0.
\end{aligned}
\end{equation}

The above cancellation occurs actually for any number of generations. However, it should be emphasized that the antisymmetric structure itself in Eqn.(16) and others does not guarantee the above cancellation. The crucial point is that the dependence on quark
flavours $d_k$ and $d_j$ is completely separate. We believe that this point is also responsible for the strong cancellation witnessed in the three loop QCD corrections in the SM. We also notice in passing that this cancellation is weaker than in the SM case where it occurs even before summation over flavours if one works in the unitarity gauge: Fig. 1(a) vanishes automatically and others cancel among themselves due to simple equalities [12]. In the present case however, Fig. 1(a) always contributes, e.g. as indicated in the subsection 2B, and there are no similar equalities which would demand the cancellation before summation over flavours is taken. Below we examine how this separate structure could be possible for general couplings $C_{ai}$ and $C'_{ai}$. In other words, we want to determine what kinds of couplings are allowed for the separate structure to occur.

Let us begin with the case of double $H^\pm$ exchanges. It is natural to require from Eqn.(16) that $C_1(\alpha; kj)$, $C_2(\alpha; kj)$, $C_4(\alpha; kj)$ and $C_5(\alpha; kj)$ be independent of $d_k$ and $d_j$ and that $C_3(\alpha; kj)$ can only depend on $d_k$ and $C_3(\alpha; jk)$ only on $d_j$. It is reasonable to assume that $C_{ak}$ and $C'_{ak}$ are universal as functions of quark masses $d_k$ and $u_\alpha$, and that these masses do not appear as denominators in $C_{ak}$ and $C'_{ak}$. Then, we must have $C_1(\alpha; kj) = 0$, so that

$$C_2(\alpha; kj) = 2(C_ekC_{ej} - C'_ekC'_{ej})(C_{ak}C_{aj} - C'_akC'_{aj}) \frac{1}{d_k^2 - d_j^2},$$

$$C_3(\alpha; kj) = -d_k^2C_2(\alpha; kj),$$

$$C_3(\alpha; jk) = C_3(\alpha; kj)_{j\leftrightarrow k}.$$

The independence in $C_2$ of $d_k$ and $d_j$ along with assumptions about $C_{ak}$ and $C'_{ak}$ implies,

$$C_{ak}C_{aj} - C'_akC'_{aj} = \eta_\alpha(d_k + d_j), \quad C'_{ak}C_{aj} - C_{ak}C'_{aj} = \delta_\alpha(d_k - d_j),$$

(34)

where $\eta_\alpha$ and $\delta_\alpha$ are independent of $d_k$ and $d_j$ and may depend on $u_\alpha$. Furthermore, from the above equations, we have

$$(C_{ak} + C'_{ak})(C_{aj} - C'_{aj}) = \eta_\alpha(d_k + d_j) + \delta_\alpha(d_k - d_j).$$

(35)

The factorized dependence on $d_k$ and $d_j$ on the left-hand side means that we may have two choices,

Case (I) : $\eta_\alpha = +\delta_\alpha,$

$$C_{ak} + C'_{ak} \propto d_k, \quad C_{ak} - C'_{ak} \text{ independent of } d_k;$$

(36)

Case (II) : $\eta_\alpha = -\delta_\alpha,$

$$C_{ak} - C'_{ak} \propto d_k, \quad C_{ak} + C'_{ak} \text{ independent of } d_k.$$

Then,

$$C_4(\alpha; kj) = \delta_\alpha(C_{ak} \mp C'_{ak})(C_{aj} \mp C'_{aj}),$$

$$C_5(\alpha; kj) = \eta_\alpha(C_{ek} \mp C'_{ek})(C_{ej} \mp C'_{ej}).$$

(37)

where the upper sign corresponds to the Case (I) and the lower sign to the Case (II). Therefore $C_4(\alpha; kj)$ and $C_5(\alpha; kj)$ are both independent of $d_k$ and $d_j$. As one may
have realized, MSSM falls into the Case (I). Generally, the Case (I) corresponds to a left-handed theory in the sense that the Higgs bosons are doublets under $SU(2)_L$, while the Case (II) corresponds to a right-handed theory under $SU(2)_R$. As far as the contribution from double $H^\pm$ exchanges is concerned, the two cases are equivalent up to a sign. This two-fold ambiguity is dismissed when contributions from mixed exchanges of $H^\pm$ and $W^\pm$ are considered, since $W^\pm$ couples only to the left-handed current. Consider for example the case of the subsection 2C. The coupling combination in Eqn.(23) is $(\eta_\alpha + \delta_\alpha)$ while those in Eqn.(25) become

$$
C_1(e\alpha; k j) = 2u_\alpha d_k d_j (\eta_\alpha - \delta_\alpha), \\
C_2(e\alpha; k j) = -2u_\alpha (\eta_\alpha + \delta_\alpha), \\
C_3(e\alpha; k j) = 2u_\alpha d_k[\eta_\alpha(d_k + d_j) + \delta_\alpha(d_k - d_j)], \\
C_4(e\alpha; k j) = -2u_\alpha (C'_{\alpha k} - C''_{\alpha k})(C'_{\alpha j} - C''_{\alpha j}), \\
C_5(e\alpha; k j) = 2[\eta_\alpha(u_e^2 - d_k d_j) + \delta_\alpha(u_e^2 + d_k d_j)].
$$

(38)

The previous requirements on these combinations then single out the Case (I) as the only choice, otherwise the contribution in the subsection 2C cannot be cancelled upon summing over quark flavours and the quark EDM already arises at two loop order from mixed exchanges of $H^\pm$ and $W^\pm$. The subsection 2D does not give further constraints.

The Case (I) may be parametrized as follows,

$$
C_{\alpha k} = x_\alpha d_k + y_\alpha, \quad C'_{\alpha k} = x_\alpha d_k - y_\alpha, \quad (39)
$$

where $x_\alpha$ and $y_\alpha$ are independent of $d_k$ but may depend on $u_\alpha$. A similar analysis may be repeated for the EDM of the down-type quark $d_e$, following the prescriptions in Eqn.(18). The cancellation of contributions from both double $H^\pm$ exchanges and mixed $H^\pm - W^\pm$ exchanges requires,

$$
C_{\alpha i} - C'_{\alpha i} \propto u_\alpha, \quad C_{\alpha i} + C'_{\alpha i} \text{ independent of } u_\alpha. \quad (40)
$$

The constraints from Eqns.(39) and (40) then demand,

$$
C_{\alpha i} = xd_i + yu_\alpha, \quad C'_{\alpha i} = xd_i - yu_\alpha, \quad (41)
$$

where $x$ and $y$ are real constants that depend on the detail of the models and cannot be determined from this analysis. We see that in order to have a separate structure in the contribution to the quark EDM, the $H^\pm$ couplings with quarks cannot be arbitrary, but have to be in a form that is required by spontaneous symmetry breaking and the Yukawa couplings.

**IV. CONCLUSIONS**

We have presented an explicit two loop calculation of the quark EDM in the two Higgs doublet extension of the SM where CP noninvariance is assumed to be encoded in the nonzero phase of the CKM matrix. Naively we would expect a contribution which is much
larger than the SM result arising at three loop order. However, this large contribution is not realized in practice. Our detailed calculation shows that the contribution involving exchange of charged Higgs bosons vanishes completely due to a cancellation mechanism. We found that two factors are responsible for this complete cancellation. One is the unitarity of the CKM matrix, the other is the separate dependence in the relevant amplitude on the masses of internal quarks which are weak doublet partners of the external quark considered. We noticed that the antisymmetric structure itself of the amplitude in internal quark flavours is sufficient to guarantee the reality of the EDM, but is not to guarantee its complete cancellation. To analyse the cancellation mechanism in more detail, we examined the inverse problem of what kinds of couplings between $H^\pm$ and quarks are allowed for the separate structure to occur. We found that the couplings must be in a form that is dictated by spontaneous symmetry breaking and the Yukawa coupling. This is consistent with our assumption that the origin of the CP noninvariance resides in the complex CKM matrix.

¿From the vanishing result of the quark EDM at the lowest nontrivial order, it is safe to conclude that it should be difficult to detect the CP noninvariance through the EDM of the neutron or the leptons if the CP noninvariance is of the CKM origin.

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APPENDIX A: SOME FUNCTIONS APPEARING IN ONE LOOP DIAGRAMS

The functions $f_i$ ($i = 0$, 1) appearing in renormalization of one loop elements are

\[
 f_i(\ell^2, m^2, M^2) = \Gamma(\epsilon)(4\pi\mu^2)^\epsilon \int_0^1 dx \ x^i [h(\ell^2, m^2, M^2; x)]^{-\epsilon},
\]

\[
 h(\ell^2, m^2, M^2; x) = xM^2 + (1 - x)m^2 - x(1 - x)\ell^2,
\]

(A1)

where $m$ and $M$ are respectively the masses of the quark and the boson in the loop. The on-shell subtraction then produces the functions $I_0$ and $I_1$ which depend on $d_k^2$ or $d_j^2$, as well as on $\ell^2$, $m^2$, $M^2$. These functions generally involve the function $h$ to the power of $-\epsilon$; but in the SEMA, it is sufficient to expand them to the zero order in $\epsilon$ ($i = 0$, 1):

\[
 I_i(k) = I_i(\ell^2, m^2, M^2; d_k^2) = \int_0^1 dx \ x^i \ln \left| \frac{h(\ell^2, m^2, M^2; x)}{h(d_k^2, m^2, M^2; x)} \right| + O(\epsilon).
\]

(A2)

The functions $J_{i,j}$ ($i = 0$, 1, 2; $j = 0$, 1) arise from differentiation with respect to $\ell^2$, $m^2$ or $M^2$ of the functions $f_i$:

\[
 J_{i,j} = J_{i,j}(\ell^2, m^2, M^2) = \int_0^1 dx \ \frac{x^i(1 - x)^j}{h(\ell^2, m^2, M^2; x)} + O(\epsilon).
\]

(A3)
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Figure Captions

Fig. 1 Diagrams that contribute to the EDM of the up-type quark $u_e$. The wavy lines represent the electromagnetic fields and the dashed lines the $H^\pm$ or $W^\pm$ fields. Diagrams for the down-type quark $d_e$ are similar, with the replacements: $\alpha \rightarrow i$ and $j$, $k \rightarrow \alpha$, $\beta$. 
Figure (1)