Controlled splitting of an atomic wave packet

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We propose a simple scheme capable of adiabatically splitting an atomic wave packet using two independent translating traps. Implemented with optical dipole traps, our scheme allows a high degree of flexibility for atom interferometry arrangements and highlights its potential as an efficient and high fidelity atom optical beam splitter.

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Controlled manipulation of atomic motion is an active frontier in atom optics [1], an emerging field built upon the close analogy between matter waves and optical fields [2]. In recent years, the field of atom optics has received a significant boost from atomic Bose-Einstein condensates, the matter wave analogy of a laser field. The unavoidable dispersion associated with a massive field has limited the performance of various atom optical elements, such as atomic mirrors and beam splitters. An innovative approach involves trapped or guided atoms, as a recent experiment by Hänsel et al. has demonstrated, splitting and uniting a trapped rubidium cloud in a chip trap [3].

Atomic beam guiding, or confined propagation, is a well-studied topic in atom optics. A variety of setups, involving a single current carrying wire [3] to multiple wires [3], hollow optical fibers [6, 7], and far off-resonance optical traps (FORT) [8, 9] have been thoroughly investigated. Several schemes for coherent large angle splitting of a guided atomic wave packet have also been realized experimentally [10]. In this paper, we outline a simple scheme capable of coherently splitting an atomic wave packet using two identical translating traps. As illustrated in Fig. 1(a), our system involves a trap with an atom and an identical trap with no atom. By adiabatically translating the empty trap towards the trap with the atom, overlapping them and then passing over to the other side, the atom will be coherently split into the two traps depending on the magnitude of the relative velocity. Such a situation can be easily implemented using optical dipole traps as in Ref. [11] or micro-traps in simple conveyor belt configurations [3, 13, 14]. In addition to the above translating trap system, our model also applies to situation (b) of Fig. 1 where an atomic wave packet propagates through an X-cross beam splitter.

In the following, we carefully describe our scheme. For simplicity, the system is taken to be one dimensional along the z-axis, which is easily realizable through tight confinement in the other two orthogonal directions. In the center of massive motion frame, we take the origin to be at middle point of the two traps $V(z - d/2)$ and $V(z + d/2)$, centered respectively at $z = d/2$. The two traps start at a large distance $d$ away. They approach each other, overlap, then pass on, and move away from each other. Because of the symmetric arrangement, parity is conserved at any time $t$, thus can be used to characterize the eigen-structure of our system. For large negative/positive values of $d$, when the trap at $z = d/2$ is at far left/right, the associated Hilbert space is rather simple and consists of doubly degenerate states of the single trap energy levels $\epsilon_n$. The corresponding single trap eigen-state $\phi_n(z)$ satisfies

$$\left[\frac{p^2}{2M} + V(z)\right] \phi_n(z) = \epsilon_n \phi_n(z). \quad (1)$$

With both traps, $V_2(z, d) = V(z - d/2) + V(z + d/2)$, we find

$$\left[\frac{p^2}{2M} + V_2(z, d)\right] \psi_n^{(p)}(z, d) = E_n^{(p)}(d) \psi_n^{(p)}(z, d), \quad (2)$$

with an eigen-energy $E_n^{(p)}(d)$ and an eigen-function $\psi_n^{(p)}(z, d)$ parametrically dependent on $d$. The spectrum is $E_n = \epsilon_n$ for large separations $d$ when quantum tunnelling is practically absent. The double degeneracy is due to the presence of both an even/odd parity ($p = e, o$) state for each level with $\epsilon_n$, i.e.,

$$E_n^{(e/o)}(z, d) \xrightarrow{[d] \to \infty} \epsilon_n,$$

$$\psi_n^{(e/o)}(z, d) \xrightarrow{[d] \to \infty} \frac{1}{\sqrt{2}} [\phi_n(z + d/2) \pm \phi_n(z - d/2)]. \quad (3)$$

FIG. 1: (a) Illustration of the proposed splitting scheme with two moving traps; this is in fact the familiar type X-cross splitting for an guided atom as in (b).

\[ \text{time} \]

\[ \text{z} \]

\[ \text{(a)} \]

\[ \text{(b)} \]
When $d = 0$, the eigen-structure of Eq. 2 is simply that of an atom in a single trap $2V(z)$ (or two completely overlapped single traps) and consists of alternating even and odd parity states with increasing energies. Since the parity is conserved for a symmetric single trap $V(-z) = V(z)$, the $d$-dependent adiabatic energy level diagrams is obtained simply by connecting energy levels in the limiting cases for $d = -\infty$, 0, and $\infty$. In ascending order, they are $E_{0}^{(e)} \leq E_{0}^{(o)} < E_{1}^{(e)} < E_{1}^{(o)} < E_{2}^{(e)} \ldots$.

Our splitting scheme can now be easily presented. Assuming an atom is initially in the ground motional state $|\phi_{0}(z - d/2)\rangle$ of the left trap at a large negative value of $d$, e.g., at point I in Fig. 2, its state is simply in an equal superposition of states $|\psi_{0}^{(e)}(z, d)\rangle$ and $|\psi_{0}^{(o)}(z, d)\rangle$, or

$$
\Psi(z, d) \xrightarrow{d \rightarrow \text{large}} \frac{1}{\sqrt{2}} \left[ |\psi_{0}^{(e)}(z, d)\rangle + |\psi_{0}^{(o)}(z, d)\rangle \right].
$$

(4)

When the right trap is adiabatically translated towards the left trap and eventually passed over to the other side, the two states of the superposition follow the adiabatic energy levels $E_{0}^{(e)}$ or $E_{0}^{(o)}$, i.e., along the curves $I \rightarrow A \rightarrow F$ or $I \rightarrow B \rightarrow F$, respectively. In the end when $d$ takes on a large positive value, corresponding to the initial right trap now on the far left side of the left trap, the atomic wave packet is split. Depending on the relative phases of states $|\psi_{0}^{(e)}(z, d)\rangle$ and $|\psi_{0}^{(o)}(z, d)\rangle$, the superposition for atomic wave packets in the two traps can be controlled. We find that

$$
\Psi(z, d) \xrightarrow{d \rightarrow \text{large}} \frac{1}{\sqrt{2}} \left[ e^{-i\int E_{0}^{(e)}[d(\tau)]d\tau} |\psi_{0}^{(e)}(z, d)\rangle + e^{-i\int E_{0}^{(o)}[d(\tau)]d\tau} |\psi_{0}^{(o)}(z, d)\rangle \right],
$$

(5)

where $E_{n}^{(p)}[d(\tau)]$ denotes the parametric time dependence of eigen-energies on $d$ and the integration is over the elapsed time. The splitting is controlled by the relative phase between the two amplitudes; or,

$$
2\theta(t) = \frac{1}{\hbar} \int \left[ E_{0}^{(o)}[d(\tau)] - E_{0}^{(e)}[d(\tau)] \right] d\tau.
$$

(6)

For our scheme, this is simply related to the area enclosed by the closed loop $I \rightarrow A \rightarrow F \rightarrow B \rightarrow I$; or with $dA$ as the surface integration measure,

$$
2\theta_{C} = \frac{1}{\hbar} \oint_{I \rightarrow A \rightarrow F \rightarrow B \rightarrow I} dA,
$$

(7)

where a constant velocity $v$ is assumed. Although not exactly a topological invariant, it is important to note that Eq. 7 does not depend sensitively on the details of the splitting process. In the end, we find that the atomic wave packet is described by

$$
\Psi_{\text{split}}(z, d) \approx e^{i\theta_{C}}\Psi_{\text{unsplit}}(z, d).
$$

Our scheme can be contrasted with the works of Refs. 15, 16, where interferometry for a trapped atom were discussed based on the splitting of a single trap into two separate traps. In fact, the level diagram of negative $d$, replicated as the long dashed lines to the right part of Fig. 2 was shown in Fig. 4 of Ref. 15 and in Fig. 1(c) of Ref. 16. In the interference scheme proposed in Ref. 15, a ground state atom is split from a splitting of the single trap (at point A) into two separate traps (at point F) following adiabatic parametric motion along the $d$-axis. A phase shift between the two separate traps then appears as a finite population in the first excited state (point B) upon recombing into a single trap after moving back to $A'$. The first half involves atomic splitting, the time phase of the adiabatic eigenstate can be neglected assuming the atom starts and remains in the ground state. In the second half when the double traps are reunited, the relative time phase between the excited state (odd parity state along the $F \rightarrow B'$ curve) and the ground state (even parity state along the $F \rightarrow A'$ curve) may become significant. This will potentially hinder an accurate determination of the small differential phase accumulated (at point F) between the two parts of the atomic wave packet in the two separate traps. In the guided interference scheme of two connected Y-shaped beam splitters, the atom again starts at $d = 0$ or the trunk of the Y-shaped guide. It splits into two wave packets along the two branches upon parametric propagation into the cross (regions of large $d$). The second (inverted) Y-shaped guide then reverses the above and the two parts of the guided wave packets reunite and interfere at $d = 0$. Both works 15, 16 involve an open-ended parametric path for atomic splitting and

FIG. 2: The adiabatic energy levels in the center of mass frame. The long dashed part to the right is a replica of negative $d$ on the left. This figure should be compared to the analogous Fig. 4 of Ref. 15 and Fig. 1(c) of Ref. 16.
recombination and neither attempts an active utilization of the relative time phase difference between different adiabatic energy levels. In contrast, our scheme makes use of a closed path in the adiabatic energy level diagram (Fig. 2). We start with two separate traps when the lowest two adiabatic energy levels are degenerate. The splitting is accomplished through the crossing of the two traps and is controlled by the total time phase difference. As a result, our scheme does not depend on the details of the exact path, provided the parametric motion is adiabatic and the parity of the system is conserved.

We now consider a realistic example easily implemented with two optical traps. Assume each trap is an inverted Gaussian formed by a red detuned focused laser beam, i.e., we take \( V(z) = -M\omega^2\sigma^2 e^{-z^2/(2\sigma^2)} \), which is approximately harmonic near the bottom \( \sim M\omega^2 z^2/2 \) with a ground state size \( \sim \sqrt{\hbar/(M\omega)} \). The width of the trap, \( \sigma \), is another independent parameter. For \( d \gg \sigma \), there are two symmetrically located minimums approximately at \( \pm d/2 \); the merging of the two traps with decreasing \( d \) leads eventually to the merging of the two minimums into just a single one located at the middle between the two traps. This occurs at \( d = \pm 2\sigma \), as simply obtained from \( d^2V_2(z, d)/dz^2|_{z=0} = 0 \).

Over a wide range of realistic parameters, we find the splitting proceeds exactly as our scheme predicts, provided adiabaticity is maintained. In Figure 3 we illustrate the time dependent splitting as obtained from numerically solving the time dependent Schroedinger equation with translating traps. The vertical and oblique solid straight lines denote the centers of the two traps as in Fig. 4. The dotted oblique straight line in between denotes the middle point between the two trap centers and the thickened middle section denotes the single minimum of the combined traps when \( |d| \leq 2\sigma \). The thick wavy line denotes the center of mass motion of an ground state atom initially in the left trap. It does not follow the middle dashed line because of the unbalanced splitting for the particular choice of \( v = 0.15 \). It displays clear tunnelling oscillations in the central region when the two traps are close, especially where only a single minimum exists. With increased speed \( v \), the splitting actually becomes easier to simulate and control as the tunnelling oscillation disappears. To help visualize the splitting, we have over-plotted the atomic wave-packet densities in dashed lines at selected times. For a \(^{87}\)Rb atom, this simulation corresponds to simply take \( \sigma = \sqrt{\hbar/(M\omega)} \). With \( \omega = (2\pi)10 \) (kHz), we find \( \sigma \approx 0.108 \) (\( \mu \)m), and the unit of time is \( \approx 0.016 \) (ms). The trap speed of \( v = 0.15 \) corresponds to \( \approx 1 \) (mm/s).

To have a desirable control over the splitting requires \( \theta C \sim \pi \). This can be easily estimated from the total area of \( A \sim \omega \sigma \), which gives rise to a highest velocity value of \( v \sim 1 \), or close to 1 (cm/s), for the physical parameters considered above. In Fig. 4, we display the predicted \( 1/v \) dependence of the relative phase Eq. 4. The squares and circles denote results from numerical simulations with 500 and 1001 spatial grid points. Clearly, we see both convergence and agreement. We have also attempted to optimize, a minimization of the time required for the adiabatic splitting using a more general time dependence for \( d(t) \), e.g., with the use of Blackman pulses 3 or other optimal control schemes 17. They did not seem to significantly improve our result.

Intuitively, the surprising resistance of our scheme to nonadiabatic transitions can be simply summarized as follows: 1) parity conservation forbids transitions between the two states \( |\psi^{(e)}\rangle \) and \( |\psi^{(o)}\rangle \) in the ground state manifold, as they are of opposite parities; and, 2) transitions to high lying states with the same parities are also suppressed due to the increased energy gaps.

We now estimate the adiabaticity requirement. The parametric motion of \( d(t) \) leads to a time dependent Hamiltonian \( H(t) \) with eigen-energy \( E_n^{(p)}(t) \) and state \( \psi_n^{(p)}(z, d(t)) \). Nonadiabatic transitions can be neglected only if

\[
\left| \langle \psi_i^{(p)}(z, d(t)) | \frac{d}{dt} | \psi_j^{(p)}(z, d(t)) \rangle \right| \frac{E_i^{(p)}(t) - E_j^{(p)}(t)}{E_i^{(p)}(t) - E_j^{(p)}(t)} \ll 1. \tag{8}
\]

As was pointed out in Ref. 13, nonzero matrix elements can only exist among states with same parity because \( d/dt \) does not change parity. Further, because \( \langle \psi_i^{(p)}(z, d(t)) | dH/dt | \psi_j^{(p)}(z, d(t)) \rangle = \langle \psi_i^{(p)}(z, d(t)) | dH/dt \rangle | \psi_j^{(p)}(z, d(t)) \rangle / | E_i^{(p)}(t) - E_j^{(p)}(t) | \), Eq. 8 can be easily estimated. For eigenstates with fixed parity \( p \), we find \( \langle \psi_i^{(p)}(z, d(t)) | d/dt | \psi_i^{(p)}(z, d(t)) \rangle = 0 \) as \( \psi_i^{(p)}(z, d(t)) \) can always be chosen as a real function.

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The non-inertial force term $Maz$ breaks parity conservation, thus will cause non-adiabatic transitions between different parity states. This can seriously affect the splitting as the two opposite parity states within the ground state family are always closely spaced in the large $|d|$ limit. We have performed detailed studies on the asymmetries between the two traps and on the effect of fluctuations of $d(t)$. The effect of asymmetries are found to influence the splitting only in the second order. The tolerable levels for imperfections, e.g., fluctuations and asymmetries, seem to be well within the current experimental capabilities. These results as well as an interesting extension of our scheme to more atoms and atomic condensates will be presented elsewhere.

The quality performance of our splitting scheme resembles the earlier adiabatic passage protocol with a pair of counter-propagating Raman pulses in a temporally counter-intuitive order \cite{18, 20, 21}. The Raman protocol involves only one adiabatic state, or the famous dark state \cite{20}, thus it is only a beam splitter, not an interferometer as only one adiabatic path is involved. Our protocol on the other hand contains two interfering parts: the symmetric and anti-symmetric states, thus does form a complete interferometer. However, it is not an universal interferometer because the amplitudes for the two paths are always equal to $1/\sqrt{2}$ \cite{23}, and the interference is completely controlled by their relative phase. In both cases, the restrictions from being an universal interferometer give rise to higher fidelities provided strict adiabaticity is maintained.

In conclusion, we have studied the splitting of an trapped atomic wave-packet by moving one trap with respect to a second trap. Through a careful analysis of the dependence of the single atom energy levels on the trap separation, we have shown that our splitting scheme makes use of a "topologically invariant" like quantity. Thus, our suggested approach is capable of demonstrating a high level of fidelity against imprecise or imperfect control of external parameters. To our knowledge, our results have not been previously seen in other studies of optical or atomic splitting/guiding systems. In view of the active experimental efforts in this area \cite{24, 25, 26, 27}, our scheme shines new light on atomic splitting with a time dependent trap.

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\[ \text{FIG. 4: An illustration of our proposed atom splitter where the probability of the atom to remain in the stationary trap is plotted against the relative velocity.} \]

\[ i \hbar \frac{\partial}{\partial t} \psi_n^{(p)}(z, d) = \left[ \frac{\hbar^2}{2M} + V_2(z, d) + Maz \right] \psi_n^{(p)}(z, d). \]  

\[ |d| \]

\[ \psi_n^{(p)}(z, d) = \frac{1}{\sqrt{\pi}} e^{-z^2/2} \int_0^d e^{-y^2/2} dy. \]  

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