On non existence of tokamak equilibria with purely poloidal flow

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Abstract

It is proved that irrespective of compressibility tokamak steady states with purely poloidal mass flow can not exist in the framework of either magneto-hydrodynamics (MHD) or Hall MHD models. Non-existence persists within single fluid plasma models with pressure anisotropy and incompressible flows.
I. Introduction

Motivation of the present study was a proof that ideal magnetohydrodynamics (MHD) steady states of magnetically confined plasmas with purely poloidal incompressible flows and magnetic fields having toroidal and poloidal components can not exist [1, 2]. The inconsistency relates to the toroidicity because in cylindrical geometry equilibria with purely poloidal (azimuthal) flows exist without restriction on the direction of the magnetic field [3]. The aforementioned statement of non-existence includes tokamak equilibria; only field reversed configurations with purely poloidal incompressible flows which by definition have only poloidal magnetic fields are possible. Also, magnetic dipolar steady states with purely poloidal compressible flows parallel to the magnetic field were studied in Ref. [4]. In tokamaks, however, poloidal sheared flows play an important role in the transition from low to high confinement mode (L-H transition). In fact, in many experiments no toroidal rotation in connection with the transition is reported (see for example Ref. [5]). It is therefore interesting to theoretically examine whether alternative or additional physical input to incompressibility can remove the incompatibility.

The present study aims at examining whether (i) compressibility (ii) two fluid effects and (iii) pressure anisotropy can give rise to the existence of tokamak steady states with purely poloidal flows. It turns out that in cases (i) and (ii) as well as in case (iii) for incompressible flows the non-existence statement keeps hold. It should be noted that there is a number of papers on equilibria with flow within the framework of MHD [2]-[4], [6]-[17], the two fluid model [7, 14, 17], [18]-[21] and for anisotropic pressure [22]-[25]. Certain of the derivations and equations therein are related to the present work. We will prefer, however, to present the study in a self contained way because explicit reference to the content of the aforementioned papers for the three models employed would make a common notation difficult and the manuscript inconveniently readable. Compressible axisymmetric MHD equilibria is the subject of Sec. II. In Sec. III two fluid effects are examined in the framework of Hall MHD model with electron temperatures uniform on magnetic surfaces and either ion temperatures uniform thereon or incompressible ion flows. Pressure anisotropy for incompressible flows is considered in Sec. IV. The conclusions are summarized in Sec. V.
II. Compressibility

The ideal MHD equilibrium states of a magnetically confined plasma are governed in standard notation and convenient units by the following set of equations:

\[ \nabla \cdot (\rho \mathbf{v}) = 0, \]  

(1)

\[ \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{j} \times \mathbf{B} - \nabla P, \]  

(2)

\[ \nabla \times \mathbf{E} = 0, \]  

(3)

\[ \nabla \times \mathbf{B} = \mathbf{j}, \]  

(4)

\[ \nabla \cdot \mathbf{B} = 0, \]  

(5)

\[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0. \]  

(6)

An energy equation or equation of state needed to close the set of Eqs. (1)-(6) is not adopted from the beginning; it will be specified when necessary later. On account of axisymmetry and Ampere’s law (4) the divergence-free fields, i.e. the magnetic field \( \mathbf{B} \), the current density \( \mathbf{j} \), and the momentum of the ion fluid element \( \rho \mathbf{v} \) can be expressed in terms of scalar functions \( \psi(R,z) \), \( I(R,z) \), \( F(R,z) \) and \( \Theta(R,z) \) as

\[ \mathbf{B} = I(R,z)\nabla \phi + \nabla \phi \times \nabla \psi(R,z), \]  

(7)

\[ \mathbf{j} = \Delta^* \psi \nabla \phi - \nabla \phi \times \nabla I(R,z), \]  

(8)

\[ \rho \mathbf{v} = \Theta(R,z)\nabla \phi + \nabla \phi \times \nabla F(R,z). \]  

(9)

Here, \((R, z, \phi)\) are cylindrical coordinates with \( z \) corresponding to the axis of symmetry; the functions \( \psi \) and \( F \) label the magnetic and velocity surfaces, respectively; \( \Delta^* = R^2 \nabla \cdot (\nabla / R^2) \). Expressing the electric field in terms of the electrostatic potential, \( \mathbf{E} = -\nabla \Phi \), the component of Ohms law along \( \nabla \phi \) implies that function \( F \) is uniform on magnetic surfaces:

\[ F = F(\psi). \]  

(10)

The electrostatic potential is also a surface quantity

\[ \Phi = \Phi(\psi), \]  

(11)
as it follows by projecting (6) along $\mathbf{B}$. Two other integrals are identified in terms of surface quantities by the components of (6) along $\nabla \psi$, and (2) along $\mathbf{B}$, respectively:

$$\frac{1}{\rho R^2} (IF' - \Theta) = \Phi',$$

$$I \left( 1 - \frac{(F')^2}{\rho} \right) + R^2 F' \Phi' \equiv X(\psi).$$

Eqs. (12) and (13) can be solved for the functions $I$ and $\Theta$ associated with the toroidal components of $\mathbf{B}$ and $\mathbf{v}$ to yield

$$I(\psi, R) = \frac{X - R^2 \Phi'}{1 - M_p^2},$$

$$\Theta(\psi, R) = \frac{XF' - R^2 \rho \Phi'}{1 - M_p^2}.$$

Here $M_p^2$ is the square of the Mach function of the poloidal velocity with respect to the Alfvén velocity:

$$M_p^2 = \left( \frac{v_p}{v_{pA}} \right)^2 = \frac{(F')^2}{\rho}.$$

In the present study we will consider both compressible and incompressible flows. For typical tokamak poloidal velocities and temperatures ($\text{max} v_p \approx 10^4 \text{ m/sec}$ and $\text{max} kT_i \approx 10 \text{ keV}$) the incompressibility condition

$$\left| \frac{v_p}{v_{th,i}} \right| \ll 1,$$

where $v_{th,i}$ is the ion thermal velocity, is satisfied and therefore incompressible flows are of relevance. Also, it may be noted that incompressibility is a good approximation for small flows lying within the first elliptic regime of the equilibrium differential equations. In MHD this regime is

$$0 < M_p < \beta$$

(see for example Ref. [9]), where the maximum value of $\beta$, defined as the ratio of the plasma pressure to the total (plasma and magnetic field) pressure, is about 0.35 (taking into account the particularly high values of $\beta$ which have
been obtained in spherical tokamaks). Note that (17) sets nearly the same bounds on $v_p$ as (16) as can be seen by using the definitions of $M_p$ and $\beta$.

On the basis of (14) and (15) let us first recover the non-existence of equilibria with purely poloidal incompressible flows and magnetic fields having poloidal and toroidal components. On account of incompressibility, $\nabla \cdot v = 0$, (1) implies that $\rho = \rho(\psi)$. Then, for $\Theta = 0$, (15) can not be satisfied, because in addition to surface quantities it contains $R$ explicitly, unless either $F' = \Phi' = 0$ or $X = \Phi' = 0$. The former case corresponds to a static equilibrium ($v = 0$). The latter implies that $v$ is parallel to $B$ and therefore $B_\phi = 0$ as it also follows from (14). Therefore, tokamak equilibria with purely poloidal incompressible flows are not possible. Note that, inversely, the magnetic field is purely poloidal ($I = 0$) inspection of Eqs. (14) and (15) implies that the flow is either purely poloidal or purely toroidal. Therefore in field reversed configurations coexistence of toroidal and poloidal incompressible flows is not admitted.

In the case of compressibility the density can vary on magnetic surfaces. For purely poloidal flows, however, Eqs. (14) and (15) imply $I = X(\psi)$ and restrict $\rho$ to be of the form

$$\rho = \frac{XF'}{\Phi'R^2}. \tag{18}$$

Evaluation of $\nabla \rho$ on magnetic axis, on which $\nabla \psi = 0$, by (18) yields

$$\nabla \rho_0 = -2F_0^3X_0F'_0\Phi'_0 e_R. \tag{19}$$

Therefore,

$$\nabla \rho_0 \neq 0, \tag{20}$$

unless $X_0 = 0$ or $F'_0 = 0$ which would imply $\rho_0 = 0$ by (18). To proceed further we need an energy equation or equation of state. Since thermal conductivity along $B$ is very large in high temperature plasmas, isothermal magnetic surfaces is an appropriate equation of state for tokamaks. If $T = T(\psi)$, using the ideal gas law $P = \alpha\rho T$ the component of (2) along $B$ yields for $\Theta = 0$

$$B \cdot \nabla \left(\frac{v^2}{2} + \alpha T \ln \rho \right) = 0. \tag{21}$$

or

$$\frac{v^2}{2} + \alpha T \ln \rho \equiv H(\psi). \tag{22}$$
Evaluation of $\nabla \rho$ on axis from (22) after multiplying it by $\rho$ and using (9) for $v$, $\Theta = 0$ and (18) for $\rho_0$ yields

$$[H_0 - \alpha T_0(1 + \ln \rho_0)] \nabla \rho_0 = 0. \quad (23)$$

If $H_0 = \alpha T_0(1 + \ln \rho_0)$ it follows from (22) that $T_0 = 0$. Therefore, (23) implies $\nabla \rho_0 = 0$ which contradicts (20). As can be shown along the same lines the non unique definition of $\nabla \rho$ on axis persists if alternative equations of state are adopted such as isentropic magnetic surfaces or barotropic plasmas [$P = P(\rho)$]. Thus, we can conclude that the non existence of tokamak equilibria with purely poloidal flows is extended to the compressible regime.

The above local proof of non existence can be extended near the magnetic axis. Indeed, on account of (18), Eq. (22) takes the form

$$\left(\frac{RF'}{G^2}\right)^2 \left[\left(\frac{\partial \psi}{\partial R}\right)^2 + \left(\frac{\partial \psi}{\partial z}\right)^2\right] + \alpha T \ln \left(\frac{G}{R^2}\right) = H(\psi), \quad (24)$$

where $G(\psi) \equiv X F'/\Phi'$. The $R$-derivative of (24) on axis yields

$$\left(\frac{R_0 F'}{G_0^2}\right)^2 \left[\frac{\partial \psi}{\partial R} \frac{\partial^2 \psi}{\partial R \partial R^2}\right]_{|_{0}} = -2\alpha T_0 R_0^3 \frac{R_0^3}{G_0}. \quad (25)$$

Note that the term involving $\frac{\partial^2 \psi}{\partial R \partial z}|_0$ is not included in (25) because this derivative vanishes as it follows from a Mercier expansion based consideration of the generalized Grad-Shafranov equation [Eq. (27) below] around the magnetic axis. Since the RHS of (25) is finite, $\frac{\partial^2 \psi}{\partial R^2}|_0$ must tend to infinity. Consequently, the validity of the statement follows on the basis of a Mercier expansion of any solution of (24) near the magnetic axis:

$$\psi(x, y) = \psi_0 + \frac{1}{2} \frac{\partial^2 \psi}{\partial R^2}\bigg|_{z_0, R_0} x^2 + \frac{1}{2} \frac{\partial^2 \psi}{\partial z^2}\bigg|_{z_0, R_0} y^2 + \ldots \quad (26)$$

Here, $(x, y)$ are Cartesian coordinates defined by $R = R_0 + x$ and $z = z_0 + y$ and $\psi_0$ refers to the magnetic axis. Also, the current density on axis becomes singular. Furthermore, it may be noted that projection of (2) onto $\nabla \psi$ yields the generalized Grad-Shafranov equation

$$\nabla \cdot \left[\left(1 - M_p^2\right)\frac{\nabla \psi}{R^2}\right] + \frac{F' F''}{\rho} \left|\nabla \psi\right|^2 + \frac{IX'}{R^2} + \rho H' + a \rho \left(1 - \ln \rho\right) T' = 0. \quad (27)$$
Therefore, \( \psi \) should satisfy the two differential equations (21) and (27) containing the surface quantities \( T(\psi) \), \( H(\psi) \), \( F(\psi) \) and \( X(\psi) \). As will be shown in the next section for incompressible flows the solution of these equations is irrelevant to tokamaks.

III. Two fluid effects

We will employ the Hall MHD, a simple two-fluid model in the approximation of very small electron mass. Consequently, the electron momentum equation can be put in the Ohm’s law form

\[
\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{h}{\rho} (\mathbf{j} \times \mathbf{B} - \nabla P_e),
\]

(28)

where \( \mathbf{v} \) is the ion fluid element velocity, \( \rho = nM \) and \( h \equiv M/e \). The right hand side (RHS) of (28) contains the Hall and electron pressure gradient terms. Eq. (28) replaces (6) which is formally recovered in the limit of \( h \to 0 \). The other equations of the model are identical with (1), (2-5) on the understanding that \( \mathbf{v} = \mathbf{v}_i \). The momentum equation (2) is derived by a superposition of the electron and ion momentum equations neglecting the electron convective velocity term because of the very small electron mass. As in Sec. II certain integrals will be identified in the form of conserved quantities on magnetic surfaces by projecting (6) and (2) onto \( \nabla \phi \), \( \mathbf{B} \) and \( \nabla \psi \). The axisymmetric representations (7)-(9) of the divergence free fields will be employed in the derivations to follow which up to Eq. (33) below hold for velocities of arbitrary direction. Assuming that the electron temperature is uniform on magnetic surfaces and using \( \mathbf{E} = -\nabla \Phi \) and \( P_e = \alpha \rho T_e(\psi) \) the component of (6) along \( \mathbf{B} \) yields

\[
\mathbf{B} \cdot \nabla (\Phi - h\alpha T_e \log \rho) = 0
\]

(29)

or

\[
\Phi - h\alpha T_e \log \rho \equiv \Xi(\psi).
\]

(30)

Eq. (28) then becomes

\[
\mathbf{v} \times \mathbf{B} = \frac{h}{\rho} \mathbf{j} \times \mathbf{B} + \frac{d\Xi}{d\psi} \nabla \psi - h\alpha \frac{dT_e}{d\psi} (1 - \log \rho) \nabla \psi.
\]

(31)

Projecting (13) along \( \nabla \phi \) yields another integral:

\[
F + hI \equiv f(\psi).
\]

(32)
Eq. (32) implies that in general the velocity surfaces depart from magnetic surfaces. The fact that \( v \) and \( B \) share the same surfaces in MHD [Eq. (10)] is recovered for \( h \to 0 \). Furthermore, the component of (31) along \( \nabla \psi \) leads to the elliptic differential equation

\[
h \Delta^* \psi = \rho R^2 \left[ \frac{d\Xi}{d\psi} - h\alpha \frac{dT_e}{d\psi} (1 - \log \rho) \right] + \Theta - \frac{df}{d\psi} I. \tag{33}
\]

For \( h \to 0 \) (33) reduces to the algebraic MHD equation (12). Purely poloidal ion flows will be further considered either incompressible or compressible on an individual basis as follows.

*Incompressible ion flow*

It is convenient to introduce the generalized vorticity

\[
\Omega \equiv B + h \nabla \times v. \tag{34}
\]

For purely poloidal flows the \( \Omega \)-surfaces coincide with the magnetic surfaces. Since \( \Omega \) is divergence free it can be expressed by

\[
\Omega = N(R, z) \nabla \phi + \nabla \phi \times \nabla \psi. \tag{35}
\]

Using (34), \( P = P_e + P_i \), \( E = -\nabla \Phi \) and the identity

\[
(v \cdot \nabla) v = \nabla \frac{v^2}{2} - v \times \nabla \times v,
\]

Eq. (2) can be cast in the form

\[
\rho \nabla \tilde{W} = \rho v \times \Omega - h \nabla P_i, \tag{36}
\]

where

\[
\tilde{W} \equiv h \frac{v^2}{2} + \Phi. \tag{37}
\]

The component of (36) along \( \nabla \phi \) yields

\[
F = F(\psi). \tag{38}
\]

Eq. (38) implies that \( v \) lies on magnetic surfaces and therefore \( \rho = \rho(\psi) \) because of incompressibility. Note that the electron fluid element velocity lies
on magnetic surfaces too whatever is the direction of $v_e$ as it follows from the electron momentum equation with the $(v_e \cdot \nabla) v_e$ term being neglected. From (30) and (32) then it follows $\Phi = \Phi(\psi)$ and $I = I(\psi)$. Also, substituting the expressions for $B$ and $v$ from (7) and (9) into (35) leads to the following expression for $N$:

$$N = I + h\nabla\left(\frac{F'}{\rho}\right) \cdot \nabla \psi + h\frac{F'}{\rho} \Delta^* \psi.$$  

(39)

Projecting (36) onto $\Omega$ and $\nabla \psi$, respectively, and using (39) for $N$ furnishes

$$hP_i = hP_{is}(\psi) - \rho \tilde{W},$$  

(40)

$$hM_p^2(\psi)\Delta^* \psi + \frac{1}{2} \frac{dM_p^2}{d\psi} |\nabla \psi|^2 = R^2 \left( \frac{dP_{is}}{d\psi} - \frac{d\rho}{d\psi} \Phi(\psi) \right) - I(\psi) \frac{dF}{d\psi}.$$  

(41)

Here, the surface quantity $P_{is}(\psi)$ for vanishing flow coincides with the ion pressure and $M_p^2 = (F')^2/\rho$. Eqs. (41) and (33) for $\Theta = 0$ can be cast in the forms

$$\Delta^* \psi = -f(\psi) - R^2g(\psi),$$  

(42)

$$|\nabla \psi|^2 = 2\left[i(\psi) + R^2j(\psi)\right],$$  

(43)

where $f$, $g$, $i$ and $j$ are known functions of $\rho$, $F'$, $T_e$, $P_{is}$ $\Phi$ and $I$. The forms of Eqs. (42) and (43) indicate that the magnetic surfaces are identical in shape with those of the Palumbo solution [26] which, however, can not describe tokamak equilibria ($\psi$ contours of this solution are provided in Figures 1 and 2 of Ref. [27]). Note that the pressure is not uniform on magnetic surfaces and therefore the equilibrium is not isodynamic. Only under the additional assumption $P = P(\psi)$, (40) implies $B^2 = B^2(\psi)$. Recapitulating, two fluid effects in the frame of Hall-MHD model result in non pertinent to tokamaks isodynamic like equilibria with purely poloidal incompressible flows.

**Compressible ion flow**

In connection with tokamak equilibria we will assume that additionally to the electron temperature the ion temperature is uniform on magnetic surfaces. Some of the derivations in the previous part of this section, i.e. Eqs. (30), (32), (33) and (36), remain valid. Also, for purely poloidal flows irrespective of compressibility the velocity surfaces coincide with the magnetic
surfaces \([F = F(\psi)]\) and therefore \(I = I(\psi)\) by (32). Using \(P_i = \alpha \rho T_i(\psi)\) the component of (36) along \(B\) yields

\[
\tilde{W} + h \alpha T_i \ln \rho \equiv \Lambda(\psi).
\]  

(44)

From (30), (37) and (44) eliminating the functions \(\Phi\) and \(\tilde{W}\) we obtain for \(\Theta = 0\)

\[
h \frac{\left(F' \right)^2 |\nabla \psi|^2}{\rho^2} = \Lambda(\psi) - \Xi(\psi) - h \alpha (T_e + T_i) \ln \rho.
\]  

(45)

Acting the gradient operator on (45) and evaluating the resulting equation on magnetic axis leads to

\[
\frac{T_{i0} + T_{e0}}{\rho_0} \nabla \rho_0 = 0
\]

and therefore \(\nabla \rho_0 = 0\). On the other side, on account of (32) and \(I = I(\psi)\), (33) for \(\Theta = 0\) becomes

\[
h \Delta^* \psi = \rho R^2 \left[\Xi' - \alpha T_e'(1 - \log \rho)\right] - (F' + h I') I.
\]  

(46)

Evaluating the gradient of (46) on magnetic axis leads to

\[
R_0^2 \left(\Xi'_0 + h \alpha T_{e0}' \ln \rho_0\right) \nabla \rho_0 = -2 R_0 \rho_0 \Xi'_0 e_R + h \left[\nabla \Delta^* \psi_0 + 2 \alpha R_0 T'_{e0} \rho_0 \left(1 - \ln \rho_0\right) e_R\right].
\]  

(47)

A prerequisite that (47) is compatible with \(\nabla \rho_0 = 0\) is that the RHS of this relation vanishes. The RHS of (47), however, consists of the first large MHD-like term and the second term on the order of \(h\) which for tokamaks is small. Therefore, these terms can not cancel each other. Note that the first term involving the product \(\rho_0 \Xi'_0\) remains always finite because if \(\Xi'_0\) becomes very small, this in the MHD-like limit of \(h \to 0\) would imply very large densities by (18) which keeps valid in this limit. Therefore, it follows from (47) that \(\nabla \rho_0 \neq 0\). It may be noted that the assumption of weak enough two-fluid equilibrium effects leading to non-unique definition of \(\nabla \rho_0\) on axis is fulfilled in tokamaks particularly in the major interior part of the plasma. The inconsistency in connection with \(\nabla \rho_0\) persists if alternative ion equations of state are adopted such as barotropic \([P_i = P_i(\rho)]\) or isentropic ion velocity surfaces. Therefore, as in MHD, tokamak equilibria with purely poloidal compressible flows in the framework of Hall-MHD model can not exist.
IV. Pressure anisotropy

We will employ an one fluid model with the momentum equation (2) replaced by
\[ \rho (v \cdot \nabla) v = j \times B - \nabla \cdot \mathcal{P}, \] (48)
where the tensor
\[ \mathcal{P} = P_\parallel I + \sigma BB \] (49)
is associated with the pressure anisotropy, a measure of which is the quantity
\[ \sigma \equiv \frac{P_\parallel - P_\perp}{B^2}. \] (50)
The other equations (1) and (3)-(6) remain unchanged. Double adiabatic equilibria can be obtained in the framework of the Chew Goldberger Low equations of state [30]: \( P_\parallel = S_\parallel(\psi)\rho^3/B^2 \) and \( P_\perp = S_\perp(\psi)\rho B \) where \( S_\parallel \) and \( S_\perp \) are arbitrary surface quantities. Also, adiabatic MHD equilibria with isotropic pressure can be recovered by setting \( P_\parallel = P_\perp = S(\psi)\rho^\gamma \), where \( S \) is the specific entropy. Here we will consider incompressible flows without other specification of \( P_\parallel \) and \( P_\perp \). The integrals (10)-(12) stemming from the Ohm’s law remain valid. The component of (48) along \( B \) yields
\[ B \cdot \nabla \left[ I \left( 1 - M_p^2 - \sigma \right) + R^2 F' \Phi' \right] = 0 \] (51)
or
\[ I \left( 1 - \frac{(F')^2}{\rho} - \sigma \right) + R^2 F' \Phi' \equiv X(\psi). \] (52)
For isotropic pressure \( [(\sigma = 0)] \), (52) reduces to (13). As in the isotropic case (12) and (52) can be solved for \( I \) and \( \Theta \) to yield
\[ I = \frac{X - R^2 \Phi'}{1 - M_p^2 - \sigma}, \] (53)
\[ \Theta = \frac{XF' - R^2 \rho \Phi'(1 - \sigma)}{1 - M_p^2 - \sigma}. \] (54)
For purely poloidal flows (53) and (54) imply
\[ \rho = \frac{XF'}{R^2 \Phi'(1 - \sigma)} \] (55)
and

\[ I = \frac{X}{1 - \sigma}. \]  (56)

If \( \sigma = \sigma(\psi) \) it follows from (55) that equilibria with purely poloidal incompressible flows are not possible (when \( B_\phi \neq 0 \) and \( B_p \neq 0 \)) because then these flows are not compatible with \( \rho = \rho(\psi) \). Static equilibria with \( \sigma = \sigma(\psi) \) were investigated in Refs. [28, 29]. Compatibility of (55) with \( \rho = \rho(\psi) \) restricts \( \sigma \) to be of the form

\[ \sigma = 1 - \frac{f(\psi)}{R^2}, \]  (57)

where \( f(\psi) \) is an arbitrary surface quantity. Eq. (56) then implies \( I = R^2X(\psi)/f(\psi) \). Apparently, possible equilibria of this kind are irrelevant to tokamaks in which the toroidal magnetic field, \( B_\phi = I/R \), must have a vacuum component proportional to \( R^{-1} \).

V. Conclusions

We have shown that the non-existence of ideal MHD tokamak equilibria with purely poloidal incompressible flow can be extended to the cases of (i) compressible MHD flows (ii) Hall-MHD incompressible and compressible flows and (iii) one fluid equilibria with pressure anisotropy and incompressible flows. The non-existence relates to the toroidicity. Specifically, for incompressible flows an inconsistency appears in terms of relations which in addition to surface quantities have an explicit dependence on \( R \) or only isodynamic-like equilibria are possible and for compressible flows the density gradient does not have unique definition on the magnetic axis. Also, for MHD compressible flows the proof can be extended near the magnetic axis.

The existence of this kind of equilibria on account of additional physical input, e.g. in plasmas with pressure anisotropy and compressible flows, or other models remains an open question. Experimental clarification on whether an even small toroidal velocity is necessary for the L-H transition to occur would be helpful. If not necessary, the existence of tokamak equilibria with purely poloidal flows from the theoretical point of view should remain an even more challenging problem.
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