Light Qubit Storage and Retrieval using Macroscopic Atomic Ensembles

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We present an experimentally feasible protocol for the complete storage and retrieval of arbitrary light states in an atomic quantum memory using the well-established Faraday interaction between light and matter. Our protocol relies on multiple passages of a single light pulse through the atomic ensemble without the impractical requirement of kilometer long delay lines between the passages. Furthermore, we introduce a time dependent interaction strength which enables storage and retrieval of states with arbitrary pulse shapes. The fidelity approaches unity exponentially without squeezed or entangled initial states, as illustrated by explicit calculations for a photonic qubit.

Faithful storage and retrieval of an unknown quantum state of traveling light pulses plays an important role in quantum information protocols, such as long distance communication [1, 2] and quantum computation [3]. Several theoretical proposals for deterministic quantum memory for light have been put forward using single atoms coupled to high finesse cavities [4], as well as atomic ensembles in free space coupled to light [5, 6, 7]. Recently [8], the first quantum storage of weak coherent light pulses with a higher fidelity than achievable classically was demonstrated using the off-resonant Faraday rotation of light passing through a macroscopic atomic ensemble. The subsequent experimental challenge is to demonstrate retrieval of the quantum state back to a light pulse following for example the protocol [5]. This protocol shows better performance, i.e., higher fidelities, than achievable classically, but in order to become truly applicable for future quantum information protocols, fidelities close to unity must be achieved. This is possible with the protocol, but only if one is able to prepare highly squeezed initial states, i.e., spin squeezed atomic states and squeezed input light beams for mapping and retrieval respectively. The difficulty in producing such squeezed states limits the realistically achievable fidelity of the quantum memory. In this Letter we present a new protocol which can reach unity fidelity without any squeezing. The requirement for this protocol to reach unit fidelity is sufficient interaction strength, which can be met by a sufficient number of atoms and photons taking part in the interaction. Unlike previous proposals our theoretical analysis considers a long light pulse which is reflected and redirected so that it is on its way through the sample along different directions at the same time, see Fig. 1. This dramatically changes the dynamics of the whole system, and the evolution is no longer governed by the Quantum Non Demolition (QND) coupling used in the direct mapping protocol [5] and considered in various atom-light interface protocols discussed in Refs. [6, 9, 10]. These protocols have assumed sequential passage of the light through the sample which for pulses of millisecond duration as required in [5] means that delay lines of hundreds of kilometers would be necessary. In addition to presenting a protocol which works for long pulses, we demonstrate that it is possible to tailor the interaction strength such that the quantum memory may store and retrieve incoming light pulse with arbitrary pulse shapes. Finally, we note that, although being an effectively continuous variable system a macroscopic atomic ensemble can be used to store a single qubit and we shall show explicitly that our protocol enables high fidelity storage and retrieval of qubit states in the form of superposition states of 0 and 1 photons.

We investigate a setup with a macroscopic atomic spin angular momentum oriented along the x-direction [11] or, equivalently, two samples with opposite macroscopic spin placed in a constant magnetic field [3]. See Fig. 1. As described in [8, 12] the total angular momentum along the x-direction, $J_x$, is effectively a classical parameter and the two remaining components can be rescaled to form operators, $x_A = J_y/\sqrt{J_z}$ and $p_A = J_z/\sqrt{J_z}$ with canonical commutation relation $[x_A, p_A] = i$. The light state is composed of a strong coherent component linearly polarized in the x-direction and the weak quantum mechanical signal to be stored in the y-polarization. For this system the Stokes vector component $S_x$ is a macroscopic classical quantity and we can define the operators $x_L = S_y/\sqrt{S_x}$ and $p_L = S_z/\sqrt{S_x}$ with commutation relation $[x_L, p_L] = i$.

As shown in Fig. 1 our protocol begins with a light passage along the z-direction. The off-resonant interaction Hamiltonian for this process is given by $H \propto p_{APL}$ [8]. After a 90 deg. rotation in $x_L-p_L$ space the light is sent through the atomic sample along the y-direction, creating a $H \propto x_A x_L$ interaction. To describe the effect of this interaction we first review the case where the two passes of the beam occur one after the other [3]. The output operators are then given by $p_{L}^{\text{out}} = (1-\kappa^2)p_{L}^{\text{in}}-\kappa x_{L}^{\text{in}}$, $x_{L}^{\text{out}} = x_{L}^{\text{in}}+\kappa p_{L}^{\text{in}}$, $p_{A}^{\text{out}} = (1-\kappa^2)p_{A}^{\text{in}}-\kappa x_{A}^{\text{in}}$, $x_{A}^{\text{out}} = x_{A}^{\text{in}}+\kappa p_{L}^{\text{in}}$, where $\kappa$ is the integrated interaction strength. $\kappa^2$ is proportional to the total number of atoms and the total number of photons in the pulse [12, 13]. The protocol both

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Figure 1: a) Schematic of the multi-pass protocol. The light is sent through the atomic sample along the z- and then along the y-direction. After the second passage, we may reflect the light back onto the sample to improve the performance by two further passages of the light beam. b) Schematic of an implementation if the atomic system consists of two atomic samples with oppositely oriented mean spin in a homogeneous magnetic field as in [8].

maps the light properties on the atoms and the atomic properties on the outgoing light pulse. For $\kappa = 1$, mean values are faithfully stored whereas uncanceled contributions from $x_A^\text{in}$ or $x_A^\text{in}$ will limit the fidelity of storage or retrieval of e.g. a coherent state to 88% [6]. As discussed in the introduction this deficiency can be remedied by initially squeezing these variables (e.g. $x_A \rightarrow x_A/\sqrt{\tau}$, $p_A \rightarrow p_A\sqrt{\tau}$), in which case the fidelity may be increased towards 100%. This however, represents an unachievable limit due to the insurmountable difficulty in creating arbitrarily squeezed states. Furthermore, as mentioned above, the requirement of hundreds of kilometer long optical pulses cannot be avoided for the experimental realization of Ref. [8].

To circumvent this, we propose simultaneous passage of the light pulses through the atomic medium in two perpendicular directions. We treat this case by splitting the incoming light pulse into segments of duration $\tau$ as discussed in [14]. The interaction can then be treated sequentially using the coarse grained Hamiltonian $H_{\tau,i} = \kappa \tau KP_{i\tau}A_{x L,i}$ and $H'_{\tau,i} = \kappa \tau x A x L,i$. The crossing of beams in the atomic sample gives rise to standing wave interference patterns. Since these will either be stationary or move with a speed of $c/\sqrt{2}$ in the 45 degree directions with respect to the beams, atoms with a thermal velocity distribution will effectively average over these interferences and only experience the traveling wave components. By taking the $\tau \rightarrow 0$ limit, differential equations are obtained, which can be solved under the physically reasonable assumption that the atomic state evolves slowly compared to the time it takes for light to travel between the two passages [14]:

$$x_A(t) = x_A(0) + \int_0^t dt' \tilde{\kappa}(t') P_{L}^{\text{in}}(t')$$

$$p_A(t) = p_A(0)e^{-\int_0^t du \tilde{\kappa}(u) + \int_0^t dt' \tilde{\kappa}(t') X_L^{\text{out}}(t')}$$

$$X_L^{\text{out}}(t) = X_L^{\text{in}}(t) + 2\tilde{\kappa}(t)p_A(t)$$

$$P_L^{\text{out}}(t) = P_L^{\text{in}}(t) - \tilde{\kappa}(t)x_A(t),$$

where for future use we have defined a time dependent interaction rate $\tilde{\kappa}(t)$ which is characterized by $\kappa_{\tau,\text{tot}}^2 = \int_0^\infty \tilde{\kappa}^2(t)dt$ (time dependence of $\tilde{\kappa}$ is implicit in the integrals). $X_L(t)$ and $P_L(t)$ are the position and momentum operators for the light segment arriving at time $t$ with commutation relation $[X_L(t), P_L(t')] = i\hbar(t-t')$. As can be seen the solution turns out to be very similar to the simple QND case reviewed earlier. In particular we recognize an asymmetry in which only the initial quantum fluctuations of one of the atomic quadratures are damped, which is exactly the reason why squeezing of the initial states is required.

The result in itself is noteworthy, because it gives the first indication that retrieval of stored states is experimentally feasible for the system of Ref. [8]. Indeed we have devised a protocol, which uses this two pass retrieval in combination with the single pass storage demonstrated in [8] to achieve an ideal quantum memory if infinitely squeezed states are available [14]. Our main purpose here is to show that one may devise a protocol which does not require squeezing, and we shall not go into more detail with this direct mapping/two-pass retrieval ("1+2") protocol here, except for a comparison of its performance with the main novel protocol of this paper in which the light is reflected back through two further passages of the atomic medium, see Fig 4. In the resulting four pass protocol the light will experience an interaction sequence $pp, xx, xx$, and $pp$, which effectively restores the symmetry between the atomic quadratures. As in the two pass case a differential equation for the time evolution of the atomic and light variables can be formulated and solved to obtain [14]:

$$x_A(t) = x_A(0)e^{-\int_0^t du \tilde{\kappa}(u) + \int_0^t dt' \tilde{\kappa}(t') P_{L}^{\text{in}}(t')}$$

$$p_A(t) = p_A(0)e^{-\int_0^t du \tilde{\kappa}(u) + \int_0^t dt' \tilde{\kappa}(t') X_L^{\text{in}}(t')}$$

$$X_L^{\text{out}}(t) = X_L^{\text{in}}(t) + 2\tilde{\kappa}(t)p_A(t)$$

$$P_L^{\text{out}}(t) = P_L^{\text{in}}(t) - \tilde{\kappa}(t)x_A(t),$$

By introducing the annihilation operators $\hat{a} = (x + i\hat{p})/\sqrt{2}$ and $\hat{A} = (X + i\hat{P})/\sqrt{2}$ the entire interaction can be written as a beam splitter relation:

$$\hat{a}_A(t) = \hat{a}_A(0)e^{-\int_0^t du \tilde{\kappa}(u) + \int_0^t dt' \tilde{\kappa}(t') \hat{A}(t')}$$

To quantify the quantum memory performance, we calculate the combined storage and retrieval fidelity $F = \langle \Psi_{\text{ideal}} \rangle \rho \langle \Psi_{\text{ideal}} \rangle$, where $\Psi_{\text{ideal}}$ is the ideal state and $\rho$ is the retrieved density matrix. We consider the storage and retrieval of an unknown qubit stored in the zero and one photon subspace of the light field, and calculate the fidelity averaged over all orientations on the Bloch sphere. For the "4+4" protocol, this can be calculated directly from Eq. (3), by noting that imperfect operation results in admixture of vacuum noise into the mode.
we are interested in, and the effect (or an actual source of loss or decoherence) is therefore fairly easily determined, e.g., by noting that within the 0 and 1 photon subspace \( \kappa \) corresponds to the relation for a decaying two-level system. The calculations for protocols involving either a single or a two-pass component are significantly more complicated involving orthogonal mode decomposition and appropriate differentiations of the Husimi Q-function and we shall not go into details here [14]. In Fig. 2 we show two examples of the behavior of the fidelity as a function of the total interaction strength \( \kappa_{\text{tot}}^2 \) for constant \( \kappa \). Experimentally \( \kappa \) is proportional to the amplitude of the classical light field, and hence the total number of photons is given by the time integral of \( \kappa^2 \). As discussed below the noise introduced by spontaneous emission each time the light passes through the sample is proportional to the number of photons. Hence, in order to be able to compare protocols involving different number of passes we compare their performance as a function of total numbers of photon passes which, e.g., for four pass storage followed by four pass retrieval is proportional to \( \kappa_{\text{tot}}^2 = 8 \int dt\kappa^2(t) \). The crosses show the result of the direct mapping protocol with \( \kappa_{\text{tot}}^2 = 2 \) and retrieval of light with a two pass interaction (“1+2”) [14]. Both the initial atomic state and the light of the retrieval pulse are initially squeezed by a factor \( \epsilon = 4 \). The solid curve shows the fidelity for the "4+4" protocol. The table summarizes the optimal combined storage and retrieval fidelities at different squeezing factors for the "1+2" and "1+4" protocols. In each case the total accumulated interaction strength is also given. We note that with a moderate amount of squeezing, fidelities well above the classical boundary of 2/3 can be achieved. We also note that the "4+4" protocol does not require any squeezing, and with a moderate \( \kappa_{\text{tot}}^2 = 4.8 \) we get \( F = 88% \) with constant coupling strength.

As a departure from all previous proposals we propose to increase the achievable degree of fidelity and at the same time facilitate the storage of light pulses with arbitrary temporal profile by appropriately varying the interaction constant throughout the pulse. This is readily achieved since the time dependent interaction strength \( \kappa(t) \) is proportional to the amplitude of the strong x-polarized driving field, which can be varied experimentally using e.g., an electro-optic modulator (EOM). In order to store an incoming light pulse with a temporal mode function \( f(t) \) so that it is described by the annihilation operator \( \hat{a}_{\text{in}}^{\text{in}} = \int_0^T dt f(t)\hat{A}_L(t) \), we want \( \hat{a}_A(T) = \hat{a}_{\text{in}}^{\text{in}} \) which is equivalent to

\[
\frac{1}{\kappa} \frac{d\kappa}{dt} + 2\kappa^2 = \frac{1}{f} \frac{df}{dt} \tag{4}
\]

If we are interested in storing the state of a light pulse described by any (real) function \( f(t) \) this differential equation can be solved to obtain the appropriate shape of the interaction strength \( \kappa \). Hence by suitable tailoring of \( \kappa \) the protocol allows for ideal storage (and retrieval) of an incoming light state of arbitrary shape without any initial squeezing. Note, that the shape of \( \kappa \) for storage and retrieval may be different so that the retrieved pulse may be transformed into any desirable shape. As a specific example let us consider a constant field mode of duration \( T \) with \( f(t) = 1/\sqrt{T} \). In the case of mapping, the initial atomic state is damped exponentially but so is the early part of the input light pulse. This can be counteracted by an increased interaction strength for the front part of the pulse, and Eq. (4) gives \( \kappa(t) = \frac{1}{2\sqrt{t}} \) for the optimal mapping interaction. When retrieving a stored state the rear end of the light pulse reads out a damped atomic state and the divergence needs to be placed at this end of the pulse.

Another interesting result can be obtained if we let \( \kappa(t) = \frac{1}{2\sqrt{t+\epsilon}} \), where \( T \) is the pulse length. In this case we get a 50/50 beam splitter between the light and atoms. This adds to the growing toolbox of interesting interactions between atomic ensembles and light pulses, which pave the way for a number of hybrid light-atom protocols, e.g., for teleportation and entanglement swapping.

Our protocol for optimum storage and retrieval of photonic states requires divergences in the coupling strength, which cannot be achieved experimentally. To quantify the effect of finite \( \kappa \), we assume that we truncate \( \kappa \) by making it constant close to the divergences, \( 1/\sqrt{T} \) and \( 1/\sqrt{T-\epsilon} \) for the mapping and retrieval respectively, e.g., for the retrieval \( \kappa = \min[1/2\sqrt{T-\epsilon}, \phi] \) where \( \phi \) is a chosen constant, see inset of Fig. 3. The fidelity for the combined storage and retrieval can be calculated analytically[14], and for large interaction the fidelity approaches unity rapidly:

\[
F \rightarrow 1 - \frac{8\sqrt{3} - 4e}{3} \exp(-\kappa_{\text{tot}}^2/2) , \quad \kappa_{\text{tot}} \gg 1 . \tag{5}
\]

In Fig. 3 we compare the "1+2" and the "4+4" protocols, both with time-dependent \( \kappa \). For the "1+2" we include curves for which both \( x_A^\eta \) and \( \hat{a}_{\text{in}}^{\text{in}} \) are squeezed with a factor \( \epsilon = 1.4 \) . We see that fidelities of the order of 93 % can be achieved with experimentally feasible
squeezing. As expected the four pass protocol without squeezing approaches unit fidelity already for experimentally achievable values of $\kappa_{\text{tot}}^2$, whereas significant squeezing is necessary for the "1+2" protocol. We thus see that the "4+4" protocol solves the challenge of feasible high fidelity quantum memory. Note, that this protocol involves a significant amount of passages so if the atomic sample is contained within glass cells as in light reflection loss at each interface would decrease the fidelity. This decrease, however, can be reduced by adding anti reflection coating to the glass cells or by having the additional mirrors within the vacuum chamber in a trapped atom setup.

The effect of spontaneous emission of light by the atoms should also be addressed. For a single pass through a gas with resonant optical density $\alpha$ the spontaneous emission probability per atom $\eta$ is given by $\eta = \kappa_{\text{tot}}^2/\alpha$. For several passes the probability of spontaneous emission is proportional to the number of passes, which is why we multiply the single passage $\kappa_{\text{tot}}^2$ by the number of passes when comparing different schemes. Because of the spontaneous emission it is important to keep $\kappa_{\text{tot}}^2$ fairly small. The effect of spontaneous emission will reduce the fidelity of the protocol by an amount proportional to $\eta$, so that in combination with Eq. 6 the total error $(1 - F)$ is of the order of $\exp(-\alpha \eta) + \eta$. Optimizing this expression with respect to $\eta$ we find that the error scales as $\log(\alpha)/\alpha$. It is thus advantageous to make $\alpha$ as large as possible, something which is readily achieved in trapped cold samples. Here large optical depths are achieved for ultra compact samples of atoms, thus preserving the prospect of future miniaturization.

The present protocols can also be extended to employ two atomic memory units (ensembles) $A$ and $B$ to store or retrieve a qubit originally represented by a single photon in two modes $L$ and $M$, $\psi_{LM} = \alpha|0\rangle_{LM} + \beta|1\rangle_{LM}$. This is important since this encoding is used in a vast majority of current experiments on optical quantum information processing with discrete variables. The fidelity of storage and/or retrieval can be calculated using similar techniques as for the single-mode qubit.

In conclusion, existing protocols for using the off-resonant Faraday interaction to retrieve an unknown light state stored in a macroscopic atomic sample are very impractical because they assume sequential passage of the light beam which requires prohibitively long delay lines. Furthermore, even if the protocols could be realized experimentally they predict unity fidelity only in the limit of infinitely squeezed input states.

Our solution to both of these problems involves three novel features. First of all, we have solved the dynamics arising when the probe pulse travels through the atomic sample in two orthogonal directions simultaneously. This we applied to a novel scheme involving four passages of the light. Combined with the third new component: a time varying interaction strength, this protocol can be used for storage or retrieval symmetrically and works with fidelity exponentially approaching unity with increased interaction strength. No squeezing is required and since the interaction strength in the Faraday interaction depends on the number of atoms and the number of photons there is no fundamental limit to the achievable fidelity for this protocol. As a final remark, we note that the feasibility of the protocol is accentuated by the fact that all basic components are in place already. In experiments like one only has to add eight mirrors as illustrated in Fig. 1.

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