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Quantum Spin Ice Dynamics in the Dipole-Octupole Pyrochlore Magnet $\text{Ce}_2\text{Zr}_2\text{O}_7$

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Neutron scattering measurements on the pyrochlore magnet $\text{Ce}_2\text{Zr}_2\text{O}_7$ reveal an unusual crystal field splitting of its lowest $J = 5/2$ multiplet, such that its ground state doublet is composed of $m_J = \pm 3/2$, giving these doublets a dipole - octupole (DO) character with local Ising anisotropy. Its magnetic susceptibility shows weak antiferromagnetic correlations with $\theta_{CW} = -0.4(2)$ K, leading to a naive expectation of an All-In, All-Out ordered state at low temperatures. Instead our low energy inelastic neutron scattering measurements show a dynamic quantum spin ice state, with suppressed scattering near $|Q|=0$, and no long range order at low temperatures. This is consistent with recent theory predicting symmetry enriched U(1) quantum spin liquids for such DO doublets decorating the pyrochlore lattice. Finally, we show that disorder, especially oxidation of powder samples, is important in $\text{Ce}_2\text{Zr}_2\text{O}_7$ and could play an important role in the low temperature behaviour of this material.

The rare-earth pyrochlore oxides $\text{R}_2\text{B}_2\text{O}_7$, where $\text{R}^{3+}$ and $\text{B}^{4+}$ consist generally of rare earth and transition-metal ions respectively, display a wealth of both exotic and conventional magnetic ground states. Their $\text{R}^{3+}$ ions decorate a network of corner-sharing tetrahedra, one of the archetypes for geometrical frustration in three dimensions. Due to strong crystal electric field (CEF) effects, the nature of the magnetic interactions in such materials are strongly influenced by their single-ion physics [1–3]. A naive theoretical description of the magnetic interactions in rare-earth pyrochlores is generally performed by introducing an ad hoc effective single-ion term in addition to Heisenberg exchange interactions. For example, Heisenberg antiferromagnetism with an effective Ising anisotropy leads to non-frustrated All-In, All-Out (AIAO) magnetic order, as seen in several heavy rare earth iridate pyrochlores [4, 5] and illustrated in the insert to Fig. 1(a). Heisenberg ferromagnetism and an effective Ising anisotropy gives rise to a classical spin ice ground state [6], as seen in (Ho,Dy)$_2\text{Ti}_2\text{O}_7$ [7, 8] and illustrated as the $\text{I}_2\text{O}$ local structure in the inset to Fig. 1(a). However, the magnetic interactions should be projected into pseudo-spin operators acting solely on the low energy CEF states [3, 9–13]. This procedure has been applied for example in the Yb$^{3+}$ [11, 14, 15] and Er$^{3+}$ [12, 16–18] XY pyrochlores where CEF effects give rise to effective $S = 1/2$ quantum degrees of freedom that interact via anisotropic exchange interactions.

More recently, it has been realized that the precise composition of the ground state crystal field doublets in rare-earth pyrochlores is crucial in determining the form of the microscopic Hamiltonian, and in itself, diversifies the possibility of quantum magnetic states [3, 19]. This has been appreciated for some time in the case of non-Kramers doublets, based on magnetic ions with an even number of electrons such as the $4f^2$ configuration in $\text{Pr}^{3+}$. Only the local $z$-component of the spin operators transforms as a dipole, with the transverse components transforming as quadrupoles [20–22]. This restricts the form of the effective spin Hamiltonian and can stabilize quadrupolar phases that are not present in the phase diagram for dipolar doublets [23, 24]. For Kramers ions with an odd number of electrons, such as $4f^5$ in $\text{Ce}^{3+}$, $4f^3$ in $\text{Nd}^{4+}$ and $4f^6$ in $\text{Sm}^{3+}$, a crystal field ground state doublet with DO character can be realized where the local $z$ and $x$ components transform as a dipole, but the local $y$ component transforms as an octupole [19, 25–27]. After a rotation of the pseudo-spins about the $y$ axis, the DO exchange Hamiltonian on the pyrochlore lattice can be reduced to an XYZ model with three independent exchange parameters $(J_x, J_y, J_z)$ [19, 25]. This Hamiltonian allows for multiple phases to emerge such as an AIAO order, octupolar ordered phases and also for moment fragmentation, as observed in $\text{Nd}_2\text{Zr}_2\text{O}_7$, where static AIAO order co-exists.
with dynamic spin ice fluctuations [19, 25, 28, 29]. In the limit of dominant antiferromagnetic interactions and strong easy-axis exchange anisotropy, a dipolar quantum spin ice is stabilized so long as the easy-axis is along one of the dipolar components of the DO doublet (\( J_2 >> J_\alpha J_\beta \)). An octupolar quantum spin ice is favored if the easy-axis is along the octupole component (\( J_\beta >> J_2 J_\delta \)) [19, 25].

A promising family of candidate materials for dipolar or octupolar quantum spin physics originating from DO doublets are the cerium pyrochlores Ce\(_2\)Zr\(_2\)O\(_7\). The Ce\(^{3+}\) ions in the pyrochlore Ce\(_2\)Sn\(_2\)O\(_7\) are believed to have a DO CEF ground state and to interact via dominant antiferromagnetic interactions, but do not magnetically order down to \( T = 20 \) mK [25, 30]. The low energy spin dynamics of the cerium pyrochlores remains unexplored and their characterization is key in determining the nature of their possible spin liquid states. In this letter, we report new inelastic neutron scattering experiments on powder and single crystal samples of Ce\(_2\)Zr\(_2\)O\(_7\). Using high energy inelastic neutron scattering, we first confirmed the DO nature of the Ce\(^{3+}\) single ion ground state wave functions in Ce\(_2\)Zr\(_2\)O\(_7\). We also present low energy inelastic neutron scattering measurements performed on a single crystal of Ce\(_2\)Zr\(_2\)O\(_7\) and observe diffuse, inelastic magnetic scattering that emerges at low temperatures. The \( Q \) dependence of this diffuse scattering is consistent with a symmetry-enriched U(1) quantum spin ice state at low but finite temperatures. Furthermore, we show the quantum spin-ice correlations remain dynamic down to at least 60 mK with no sign of static magnetic order. Our results demonstrate \( Q \) signatures of a dynamic quantum spin ice ground state in Ce\(_2\)Zr\(_2\)O\(_7\), with associated emergent quantum electrodynamics and elementary excitations based on magnetic and electric monopoles as well as emergent photons [31–34].

Single crystal and powder samples of Ce\(_2\)Zr\(_2\)O\(_7\) have been grown using floating zone techniques and solid state synthesis. Stabilizing the Ce\(^{3+}\) oxidation state in Ce\(_2\)Zr\(_2\)O\(_7\) is not simple, and requires growth and annealing in strong reducing conditions to minimize Ce\(^{4+}\) [35]. As discussed in the Supplemental Material (SM), which includes Refs. [36–43], this is a serious issue, especially in powder samples, where oxidization is observed to occur in powders exposed to air on a time scale on the order of minutes, complicating the exact characterization of the material’s stoichiometry. The oxidization process can be tracked through high resolution x-ray diffraction measurements of the lattice parameter, and it is much slower for single crystal samples. There we can make an estimate of the stoichiometry of the single crystal used in our experiments as Ce\(_2\)Zr\(_2\)O\(_7\) with \( \delta \sim 0.1 \).

We first present high energy inelastic neutron scattering measurements, which probe the single-ion properties of the Ce\(^{3+}\) ions. To do so, we used theSEQUOIA high resolution inelastic chopper spectrometer [44] at the Spallation Neutron Source of Oak Ridge National Laboratory and employed neutrons with incident energies (\( E_i \)) of 150 and 500 meV. The \( E_i = 150 \) meV instrument setting was chosen to resolve the crystal electrical field (CEF) states that belong to the spin-orbit ground state manifold (\( J = 5/2 \)). The CEF interaction lifts the Ce\(^{3+}\) spin-orbit ground state degeneracy into three different eigenstates that are each doubly degenerate. We also estimated a CEF Hamiltonian for Ce\(_2\)Zr\(_2\)O\(_7\) using a scaling procedure based on the Er\(^{3+}\) pyrochlore CEF scheme [45]. This predicts two CEF excited states near 80 and 100 meV with similar inelastic neutron scattering intensity at \( T = 5 \) K.
The powder-averaged As seen in Fig. 1(b), this scenario is in qualitatively good we conclude the features at 56 and 112 meV are the CEF perturbation yields a small fraction of the spectral weight and in holmium and terbium pyrochlores [47, 48]. In any case, this unidentified contribution to the inelastic scattering is due to the influence of Ce$^{3+}$ in defective sites [46], on residual Ce$^{3+}$, or on the possible presence of hybridized phonon - crystal field excitations known as vibronic bound states, as has been recently observed in holmium and terbium pyrochlores [47, 48]. In any case, this unidentified contribution to the inelastic scattering yields a small fraction of the spectral weight and we conclude the features at 56 and 112 meV are the CEF excitations corresponding to the main Ce$^{3+}$ site.

The details of the crystal field analysis determining the full set of eigenvalues and eigenfunctions for Ce$^{3+}$ are summarized in Fig. 1(c), and further discussed in the SM. The key conclusion is that the ground state Kramers doublet appropriate to Ce$^{3+}$ is well separated from all excited crystal field states (by $\sim$56 meV), and is composed of pure $m_J = \pm 3/2$ states. A large CEF gap is consistent with the high temperature heat capacity of Ce$_2$Zr$_2$O$_7$ measured in ref. [49] where no Schottky anomaly is observed between 5 and 300 K. These pure $m_J = \pm 3/2$ states have a dipole-octupole character with a dipolar moment whose anisotropy is purely Ising and whose magnitude must be 1.286 $\mu_B$. This result does not originate from a fine-tuning of the CEF parameters, but is instead a property protected by the point-group symmetry of the A-site in the pyrochlore lattice.

As seen in Fig. 1(b), this scenario is in qualitatively good agreement with our 150 meV inelastic neutron experimental spectra where two strong magnetic excitations are observed at $\sim 56$ and $\sim 112$ meV. The relative scattered intensity of these CEF transitions can be obtained giving an energy resolution of $\sim$0.09 meV at the elastic line. One experiment was performed on a $\sim 6$ gram powder sample and a second one was performed on a $\sim 5$ gram single crystal, which was mounted with its $\{100\}$ plane coincident with the horizontal plane of the spectrometer. Figure 2(a) shows the DCS measurements on our powder, where the integration in $|Q|$ is 0.35 Å$^{-1}$ to 0.85 Å$^{-1}$. This integration in momentum transfer $|Q|$ corresponds to integrating over the $|Q| = |\{001\}|$ position ($\sim 0.59$ Å$^{-1}$), where quantum spin ice correlations are expected to be strongest [34]. A build up of inelastic spectral weight below $\sim 0.4$ meV is observed on decreasing the temperature.

Low energy inelastic neutron scattering from our single crystal is shown in Fig. 2(b, c, d) and Fig. 3(a). All this data was acquired using the same $E_i = 3.27$ meV instrument configuration of DCS, and Fig. 2(b, c, d) shows...
FIG. 3. Comparison of the measured low energy inelastic neutron scattering from (a) an annealed single crystal sample of Ce$_2$Zr$_2$O$_7$ with the calculated quasi-elastic neutron scattering for (b) the classical near-neighbor spin ice model at $T = 0$ K and (c) a quantum spin ice at finite $T$. Data in (a) is the symmetrized difference between inelastic scattering at $T = 0.06$ K and $T = 2$ K, integrated between 0 and 0.15 meV. Both (b) and (c) are simulations taken from Benton et al. [34]. The lack of intensity around $Q = (000)$ and the fact that the ring of diffuse inelastic scattering peaks along $(00L)$ provides evidence for Ce$_2$Zr$_2$O$_7$ displaying a dynamic quantum spin ice state at these low temperatures. Also, the observed diffuse inelastic scattering at $Q = (003)$ is more pronounced than that at $Q = (\frac{2}{3} \frac{2}{3} \frac{1}{3})$, again consistent with the expectations of quantum spin ice, and not consistent with classical near-neighbor spin ice. Note the extra features centred at the Bragg peak positions such as (111) likely originate from leakages of the structural Bragg peaks, due to the subtraction of two large intensities.

powder-averaged single crystal data. Figure 2(b) shows the full powder-averaged spectrum at $T = 0.06$ K with a $T = 2$ K data set subtracted from it. This result shows enhanced inelastic scattering at low temperature, which peaks up at $|Q| \sim 0.59 \text{ Å}^{-1}$, that is the magnitude of the $Q = (001)$ position in reciprocal space. This is explicitly shown via the $|Q|$-cut of the data presented in Fig.2(c). Importantly, Fig.2(c) shows no enhancement of the low energy inelastic scattering around $|Q| = 0$, consistent with expectations for a U(1) quantum spin ice. Finally, Fig.2(d) shows energy cuts through the full difference spectrum shown in (b), taken by integrating in $|Q|$ from 0.35 to 0.85 Å$^{-1}$, so around $|Q| = |(001)|$, and also well away from $|Q| = |(001)|$, integrating from 1.3 to 1.8 Å$^{-1}$. This clearly shows the quantum spin ice correlations to be dynamic in nature, characterized by an energy less than $\sim 0.15$ meV.

With the energy range of the dynamic quantum spin ice correlations identified, we can look explicitly at this scattering from the single crystal, but now comparing $Q$ maps of these correlations to the expectations of both classical, near-neighbor spin ice (without dipolar interactions) and a U(1) quantum spin ice. Fig.3(a) shows $T = 0.06$ K - $T = 2$ K data integrated between 0 and 0.15 meV, folded into a single quadrant of the [HHL] map and further symmetrized. The details of this data symmetrization are in the SM. For reference, a theoretical simulation of the structure factor expected for classical near-neighbor spin ice [34] is shown in Fig.3(b), and that for a U(1) quantum spin ice at low but finite temperature [34] is shown in Fig.3(c). While these theoretical predictions have similarities, the structure factor for U(1) quantum spin ice has minima in intensity near $Q = 0$, while the intensity of the structure factor is maximal there for classical near-neighbor spin ice.

Clearly, the measured dynamic $S(Q)$ shows a qualitatively stronger resemblance to the expectations of the symmetry enriched U(1) quantum spin ice [31–34]. The quantum spin ice ground-state is one of various spin liquids that are supported by a model of well isolated DO CEF doublets on the pyrochlore lattice [19, 25]. A similar dynamic $S(Q)$ is expected in the case of classical dipolar spin ice (here dipolar refers to long range dipolar interactions between magnetic dipoles), which also shows the suppression of diffuse scattering near $|Q| = 0$ [50, 51]. Although a definitive conclusion can only be reached once a full spin Hamiltonian is parametrized, the Ce$^{3+}$ ions in Ce$_2$Zr$_2$O$_7$ have a moment of 1.286 $\mu_B$, which is roughly a factor 8 smaller than those associated with Ho$^{3+}$ or Dy$^{3+}$ in the classical dipolar spin ices Ho$_2$Ti$_2$O$_7$ and Dy$_2$Ti$_2$O$_7$. The resulting long range dipole terms are expected to be $\sim 64$ times weaker in Ce$_2$Zr$_2$O$_7$ making such a scenario unlikely. This suggests the spin-ice correlations in Ce$_2$Zr$_2$O$_7$ originate from quantum effects. An octupolar ordered state is also consistent with the lack of magnetic dipole order in Ce$_2$Zr$_2$O$_7$. However, the neutron scattering spectra associated with such an octupolar ordered phase has yet to be calculated, thus we cannot compare our data in Fig.3(a) to it.

The effect of disorder in Ce$_2$Zr$_2$O$_7$ is still an open question as we are aware that our single crystals have some low levels of oxidation. Furthermore, stuffing [52–55] (site-mixing) is expecting to be important in Ce$_2$Zr$_2$O$_7$, because both undesired Ce$^{4+}$ and Zr$^{3+}$ ions are chemically stable. It is known that small amounts of disorder can have a drastic impact on the physical properties of
frustrated pyrochlore magnets [52, 53, 55]. It will then be important to further optimize the growth procedure and annealing techniques of Ce$_2$Zr$_2$O$_7$. However, we believe that our inelastic neutron scattering results rule out the scenario of a sensitive AIAO order. Indeed, the conventional impact of quenched disorder on a pyrochlore antiferromagnet would be spin glass physics with diffuse scattering peaked for $Q$’s corresponding to the Bragg positions of the AIAO state. Here, we observe strong diffuse scattering at $Q = (001)$, which is not only strictly zero for an AIAO state, but also forbidden for all $k = 0$ long-range ordered magnetic structures allowed by symmetry of the pyrochlore lattice. We thus conclude that our work demonstrates Ce$_2$Zr$_2$O$_7$ to be one of a very few candidates for quantum spin ice physics. Other candidates for quantum spin ice physics are based on Pr$^{3+}$ and Tb$^{3+}$ pyrochlores [56–60]. However, in contrast to Pr$^{3+}$ and Tb$^{3+}$, Ce$^{3+}$ is a Kramers ion and its magnetism is thus further protected against disorder, which in and of itself, can drive a spin liquid state for non-Kramers doublets [22, 61–63]. Furthermore, Tb$^{3+}$ and Pr$^{3+}$ pyrochlores display low lying CEF field states, which complicate their theoretical understanding due to multipolar interactions [24, 64, 65]. For all these reasons, the cerium pyrochlores are an excellent theoretical and experimental template to investigate quantum spin ice physics.

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