A high-significance detection of non-Gaussianity in the WMAP 3-yr data using directional spherical wavelets

J. D. McEwen,1⋆ M. P. Hobson,1 A. N. Lasenby1 and D. J. Mortlock2

1Astrophysics Group, Cavendish Laboratory, J. J. Thomson Avenue, Cambridge CB3 0HE
2Blackett Laboratory, Imperial College of Science, Technology and Medicine, Prince Consort Road, London SW7 2BW

Accepted 2006 June 15. Received 2006 June 15; in original form 2006 April 13

ABSTRACT

We repeat the directional spherical real Morlet wavelet analysis, used to detect non-Gaussianity in the Wilkinson Microwave Anisotropy Probe (WMAP) 1-yr data, on the WMAP 3-yr data. The non-Gaussian signal previously detected is indeed present in the 3-yr data, although the significance of the detection is reduced. Using our most conservative method for constructing significance measures, we find that the significance of the detection of non-Gaussianity drops from 98.3 ± 0.4 to 94.9 ± 0.7 per cent; the significance drops from 99.3 ± 0.3 to 97.2 ± 0.5 per cent using a method based on the χ² statistic. The wavelet analysis allows us to localize most likely sources of non-Gaussianity on the sky. We detect very similar localized regions in the WMAP 1-yr and 3-yr data, although the regions extracted appear more pronounced in the 3-yr data. When all localized regions are excluded from the analysis, the 3-yr data are consistent with Gaussianity.

Key words: methods: data analysis – methods: numerical – cosmic microwave background.

1 INTRODUCTION

Recent measurements of the cosmic microwave background (CMB) anisotropies, in particular those made by the Wilkinson Microwave Anisotropy Probe (WMAP), provide data of unprecedented precision with which to study the origin of the Universe. Such observations have lent strong support to the standard cosmological concordance model. Nevertheless, many details and assumptions of the concordance model are still under close scrutiny. One of the most important and topical assumptions of the standard model is that of the statistics of the primordial fluctuations that give rise to the anisotropies of the CMB. In the simplest inflationary models, primordial perturbations seed Gaussian temperature fluctuations in the CMB that are statistically isotropic over the sky. However, this is not necessarily the case for non-standard inflationary models or various cosmic defect scenarios.

The assumptions of Gaussianity and isotropy have been questioned recently, with many works highlighting deviations from Gaussianity in the WMAP 1-yr data (WMAP1; Bennett et al. 2003), calculating measures such as the bispectrum and Minkowski functionals (Komatsu et al. 2003; Magueijo & Medeiros 2004; Land & Magueijo 2005a; Medeiros & Contaldi 2005), the genus (Colley & Gott 2003; Eriksen et al. 2004), correlation functions (Gaztanaga & Wagg 2003; Eriksen et al. 2005; Tojeiro et al. 2006), low-multipole alignment statistics (de Oliveira-Costa et al. 2004; Copi, Huterer & Starkman 2004; Copi et al. 2006; Schwarz et al. 2004; Slosar & Seljak 2004; Weeks 2004; Land & Magueijo 2005b, c,d,e; Bielewicz et al. 2005; de Oliveira-Costa & Tegmark 2006), structure alignment statistics (Wiaux et al. 2006), phase associations (Chiang, Naselsky & Verkhodanov 2003; Chiang & Naselsky 2004; Coles et al. 2004; Dineen, Rocha & Coles 2005), local curvature (Hansen et al. 2004; Cabella et al. 2005), the higher criticism statistic (Cayón, Jin & Treaster 2005), hot and cold spot statistics (Larson & Wandelt 2004, 2005), fractal statistics (Sadegh Movahed et al. 2006) and wavelet coefficient statistics (Vielva et al. 2003; Mukherjee & Wang 2004; McEwen et al. 2005a,c; Cruz et al. 2005, 2006a). Some statistics show consistency with Gaussianity, whereas others provide some evidence for a non-Gaussian signal and/or an asymmetry between the northern and southern Galactic hemispheres. Although the recently released WMAP 3-yr data (WMAP3; Hinshaw et al. 2006) are consistent with the WMAP1 data, a more thorough treatment of beams, foregrounds and systematics in the 3-yr data, in addition to a further two years of observing time, means that the WMAP3 data provide a more reliable data set on which to confirm or refute previous results. A Gaussianity analysis is performed on the WMAP3 data by Spergel et al. (2006), using the one-point distribution function, Minkowski functionals, the bispectrum and the trispectrum. No evidence is found for non-Gaussianity; however, the authors do not re-evaluate the large number of statistical tests that have been used to detect non-Gaussianity in the WMAP1 data. Indeed, deviations from Gaussianity and isotropy have recently been detected in the WMAP3 data, using measures such as anisotropy statistics (Helling, Schupp & Tesileanu 2006; Bernui et al. 2006), phase associations (Chiang, Naselsky & Coles 2006) and wavelet coefficient statistics (Cruz et al. 2006b), with little change in the significance.
levels obtained by each technique for the 1- and 3-yr data. Although the departures from Gaussianity and isotropy detected in the WMAP1 and WMAP3 data may simply highlight unremoved foreground contamination or other systematics, which itself is of importance for cosmological inferences drawn from the data, if the source of these detections is of cosmological origin then this will have important implications for the standard cosmological model.

In this Letter we focus on the significant detection of non-Gaussianity that we made previously in the WMAP1 data using directional spherical wavelets (McEwen et al. 2005a), to see if the detection is still present in the WMAP3 data. The remainder of this Letter is organized as follows. In Section 2 we briefly review the analysis procedure and discuss the data maps considered. Results are presented and discussed in Section 3, before concluding remarks are made in Section 4.

2 NON-GAUSSIANITY ANALYSIS

We repeat on the WMAP3 data our non-Gaussianity analysis performed previously on the WMAP1 data (McEwen et al. 2005a), focusing only on the most significant detection of non-Gaussianity made previously. We refer the reader to our previous work (McEwen et al. 2005a) for a detailed description of the analysis procedure and present here only a very brief overview.

We apply a spherical wavelet analysis to probe the WMAP data for non-Gaussianity. Wavelets are an ideal tool to search for deviations from Gaussianity because of the scale and spatial localization inherent in a wavelet analysis. To perform a wavelet analysis of full-sky CMB maps, we apply our fast continuous spherical wavelet transform (CSWT: McEwen et al. 2005b), which is based on the spherical wavelet transform developed by Antoine, Vandergheynst and colleagues (Antoine & Vandergheynst 1998, 1999; Antoine, Demanet & Jacques 2002; Antoine et al. 2004; Wiaux, Jacques & Vandergheynst 2005) and the fast spherical convolution developed by Wandelt & Górski (2001). We use only the real Morlet wavelet in this analysis since it gave the most significant detection of non-Gaussianity in the WMAP1 data (McEwen et al. 2005a).

To minimize the contribution of foregrounds and systematics to CMB anisotropy measurements, the WMAP assembly contains a number of receivers that observe at a range of frequencies (Bennett et al. 2003). In this analysis we consider the signal-to-noise ratio enhanced co-added map constructed from the WMAP3 data. This map is constructed by the same procedure as described generally by Komatsu et al. (2003) and described in the context of our non-Gaussian analysis by McEwen et al. (2005a). We use the foreground reduced sky maps and apply the KP0 mask to remove residual Galactic emission and known point sources. The foreground maps and mask are available from the Legacy Archive for Microwave Background Data Analysis (LAMBDA) website.

To quantify the statistical significance of the detected deviation from Gaussianity, we consider two techniques. The first technique involves comparing the deviation of the observed statistic to all statistics computed from the simulations. This is a very conservative means of constructing significance levels. The second technique involves performing a $\chi^2$ test. In both of these tests we relate the simulated observations for each receiver are then combined to give a co-added map.

To probe the WMAP3 data for deviations from Gaussianity, the skewness of the real Morlet wavelet coefficients is examined over a range of scales and orientations [the scales and orientations considered are defined in McEwen et al. (2005a)]. Any deviation from zero is an indication of non-Gaussianity in the data. An identical analysis is performed on the 1000 Gaussian simulations to quantify the significance of any deviations.

3 RESULTS

The skewness of the real Morlet wavelet coefficients of the co-added WMAP3 map is displayed in Fig. 1, with confidence intervals constructed from the 1000 WMAP3 Monte Carlo simulations also shown. Only the plot corresponding to the orientation of the maximum deviation from Gaussianity is shown. The non-Gaussian signal present in the WMAP1 data is clearly present in the WMAP3 data. In particular, the large deviation on scale $a = 550$ arcmin and orientation $\gamma = 72^\circ$ is almost identical (although it is in fact very marginally lower in the WMAP3 data).

Next we consider in more detail the most significant deviation from Gaussianity on scale $a = 550$ arcmin and orientation $\gamma = 72^\circ$. Fig. 2 shows histograms of this particular statistic constructed from the WMAP1 and WMAP3 Monte Carlo simulations. The measured statistic for the WMAP1 and WMAP3 data is also shown on the plot, with the number of standard deviations that each observation deviates from the mean of the appropriate set of simulations. The distribution of this skewness statistic is not significantly altered between simulations that are consistent with WMAP1 or WMAP3 data. The observed statistics for the WMAP1 and WMAP3 data are similar but the slightly lower value for WMAP3 is now more apparent.

To quantify the statistical significance of the detected deviation from Gaussianity, we consider two techniques. The first technique involves comparing the deviation of the observed statistic to all statistics computed from the simulations. This is a very conservative means of constructing significance levels. The second technique involves performing a $\chi^2$ test. In both of these tests we relate the

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Real Morlet wavelet coefficient skewness statistics ($\gamma = 72^\circ$). Points are plotted for the WMAP1 data (solid, green, squares), the WMAP3 data (solid, blue, circles) and the WMAP3 data with localized regions removed (dashed, blue, triangles). Confidence regions obtained from 1000 WMAP3 Monte Carlo simulations are shown for 68 per cent (red), 95 per cent (orange) and 99 per cent (yellow) levels, as is the mean (solid white line).
Figure 2. Histograms of real Morlet wavelet coefficient skewness ($a_{11} = 550$ arcmin; $\gamma = 72^\circ$) obtained from 1000 Monte Carlo simulations. Histograms are plotted for simulations in accordance with WMAP1 (green) and WMAP3 (blue) observations. The observed statistics for the WMAP1 and WMAP3 maps are shown by the green and blue lines respectively. The number of standard deviations by which these observations deviate from the mean of the appropriate set of simulations is also displayed.

Figure 3. Histograms of normalized $\chi^2$ test statistics computed from real Morlet wavelet coefficient statistics obtained from 1000 Monte Carlo simulations. Histograms are plotted for simulations in accordance with WMAP1 (green) and WMAP3 (blue) observations. The $\chi^2$ values computed for the WMAP1 and WMAP3 maps are shown by the green and blue lines respectively. The significance of these observations, computed from the appropriate set of simulations, is also displayed.

Figure 4. Real Morlet spherical wavelet coefficient maps and thresholded versions ($a_{11} = 550$ arcmin; $\gamma = 72^\circ$). To localize most likely deviations from Gaussianity on the sky, the coefficient map is thresholded so that only those coefficients above $3\sigma$ (in absolute value) remain. All sky maps (here and subsequently) are illustrated in Galactic coordinates, with the Galactic Centre in the middle.

---

Although we recognize the distinction between skewness and kurtosis, there is no reason to partition the set of test statistics into skewness and kurtosis subsets. The full set of test statistics must be considered.
pronounced, in the sense that the peaks are larger, in the WMAP3 data. To investigate the impact of localized regions on the initial detection of non-Gaussianity, the analysis is repeated with the WMAP3 localized regions excluded from the analysis. The resulting skewness statistics are shown by the dashed line in Fig. 1. Interestingly, the highly significant detections of non-Gaussianity are eliminated when these localized regions are removed.

In our previous non-Gaussianity analysis (McEwen et al. 2005a) we also performed a preliminary noise analysis and found that noise was not atypical in the localized regions that we detect. The localized regions have not changed in the WMAP3 data, hence we do not expect this finding to change. In an additional work of ours (McEwen et al. 2006) that investigated a Bianchi VIIh component as a possible source of non-Gaussianity – which, incidentally, we found not to be the predominant source of non-Gaussianity – we performed a preliminary analysis of foregrounds and systematics. We concluded that foregrounds or systematics were not the likely source of the detected non-Gaussianity. Again, we do not believe this finding to change in the WMAP3 data since both foregrounds and systematics are treated more thoroughly.

4 SUMMARY AND DISCUSSION

We have repeated on the WMAP3 data the directional spherical real Morlet wavelet analysis used to make a significant detection of non-Gaussianity in the WMAP1 data (McEwen et al. 2005a). The non-Gaussian signal previously detected is indeed present in the WMAP3 data, although the significance of the detection is reduced. Using our first very conservative method for constructing significance measures, the change in the detection of non-Gaussianity drops from 98.3 ± 0.7 to 97.2 ± 0.5 per cent. Using our second technique for constructing significance measures, we find that the significance of the detection of non-Gaussianity drops from 98.3 ± 0.4 to 94.9 ± 0.7 per cent.

The most likely sources of non-Gaussianity were also localized on the sky. We detect the same regions in the WMAP3 data as found in the WMAP1 data, although the localized regions extracted appear slightly more pronounced in the WMAP3 data. When all localized regions are excluded from the analysis the data are consistent with Gaussianity. An interesting structure is extracted in the upper-left regions are excluded from the analysis the data are consistent with Gaussianity. An interesting structure is extracted in the upper-left.

Figure 5. Thresholded real Morlet spherical wavelet coefficient map with localized regions inset (\(\alpha = 550\) arcmin; \(\gamma = 72^\circ\)).

ACKNOWLEDGMENTS

JDM thanks the Association of Commonwealth Universities and the Cambridge Commonwealth Trust for the support of a Commonwealth (Cambridge) Scholarship. Some of the results in this paper have been derived using the HEALPix package (Górski et al. 2005). We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science.

REFERENCES

Antoine J.-P., Vanderheyden P., 1998, J. Math. Phys., 39, 3987
Antoine J.-P., Vanderheyden P., 1999, Appl. Comput. Harmonic Anal., 7, 1
Antoine J.-P., Demanet L., Jacques L., 2002, Appl. Comput. Harmonic Anal., 13, 177
Antoine J.-P., Murenzi R., Vanderheyden P., Ali S. T., 2004, Two-dimensional wavelets and their relatives, Cambridge Univ. Press, Cambridge
Bennett C. L. et al. (WMAP team), 2003, ApJS, 148, 1
Bernui A., Mota B., Reboucas M. J., Tavakol R., 2006, preprint (astro-ph/0511666)
Bielewicz P., Eriksen H. K., Banday A. J., Górski K. M., Lilje P. B., 2005, ApJ, 635, 750
Cabella P., Liguori M., Hansen F. K., Marinucci D., Matarrese S., Moscardini L., Vittorio N., 2005, MNRAS, 358, 684
Cayón L., Jin J., Treaster A., 2005, MNRAS, 362, 826
Chiang L.-Y., Naselsky P. D., 2004, preprint (astro-ph/0407395)
Chiang L.-Y., Naselsky P. D., Verkhodanov O. V., 2003, ApJ, 590, 65
Chiang L.-Y., Naselsky P. D., Coles P., 2006, ApJL, submitted (astro-ph/0603662)
Coles P., Dineen P., Earl J., Wright D., 2004, MNRAS, 350, 989
Colley W. N., Gott J. R., 2003, MNRAS, 344, 686
Copi C. J., Huterer D., Starkman G. D., 2004, Phys. Rev. D, 70, 043515
Copi C. J., Huterer D., Schwartz D. J., Starkman G. D., 2006, MNRAS, 367, 79
Cruz M., Martinez-González E., Vielva P., Cayón L., 2005, MNRAS, 356, 57
Cruz M., Tucci M., Martinez-González E., Vielva P., 2006a, MNRAS, 369, 57
Cruz M., Cayón L., Martinez-González E., Vielva P., Jin J., 2006b, MNRAS, submitted (astro-ph/0603859)
de Oliveira-Costa A., Tegmark M., 2006, Phys. Rev. D, submitted (astro-ph/0603369)
de Oliveira-Costa A., Tegmark M., Zaldarriaga M., Hamilton A., 2004, Phys. Rev. D, 69, 63516
Dineen P., Rocha G., Coles P., 2005, MNRAS, 358, 1285
Eriksen H. K., Novikov D. I., Lilje P. B., Banday A. J., Górski K. M., 2004, ApJ, 612, 64
Eriksen H. K., Banday A. J., Górski K. M., Lilje P. B., 2005, ApJ, 622, 58
Gaztanaga E., Wagg J., 2003, Phys. Rev. D, 68, 213026
Górski K. M., Hivon E., Banday A. J., Wandelt B. D., Hansen F. K., Reinecke M., Bartelmann M., 2005, ApJ, 622, 759
Hansen F. K., Cabella P., Marinucci D., Vittorio N., 2004, ApJ, 607, 67
Helling R. C., Schupp P., Tesileanu T., 2006, preprint (astro-ph/0603594)
Hinshaw G. et al. (WMAP team), 2006, preprint (astro-ph/0603451)
Komatsu E. et al., 2003, ApJS, 148, 119
Land K., Magueijo J., 2005b, MNRAS, 359, 994
Land K., Magueijo J., 2005c, Phys. Rev. Lett., 95, 071301
Land K., Magueijo J., 2005d, MNRAS, 362, 838
Land K., Magueijo J., 2005e, Phys. Rev. D, 72, 101302
Larson D. L., Wandelt B. D., 2004, ApJ, 613, 35
Larson D. L., Wandelt B. D., 2005, Phys. Rev. D, submitted (astro-ph/0505046)
McEwen J. D., Hobson M. P., Lasenby A. N., Mortlock D. J., 2005a, MNRAS, 359, 1583
APPENDIX A: ERRORS ON SIGNIFICANCE LEVELS

In this appendix we derive the standard deviation of a significance level determined from Monte Carlo (MC) simulations. Suppose that we perform $n$ independent MC simulations. Let $p$ denote the probability that an MC simulation chosen at random has a value for some test statistic that is larger than the corresponding value derived from the real data (hence $p$ is the underlying significance that we attempt to estimate). Choosing an MC simulation at random and defining whether it has a test statistic greater than that of the data thus corresponds to a Bernoulli trial with a probability of success equal to $p$.

Suppose that we observe $x$ successes in the $n$ MC simulations. The likelihood for $x$ is

$$\Pr(x \mid p) = nC_x p^x (1 - p)^{n-x}.$$  

The maximum likelihood (ML) estimate $\hat{p}$ of $p$ is most easily given by maximizing the log-likelihood:

$$\frac{\partial \ln \Pr(x \mid p)}{\partial p} \bigg|_{p=\hat{p}} = 0 \quad \Rightarrow \quad \hat{p} = \frac{x}{n},$$

which recovers the intuitive result. Approximating the shape of the likelihood near its peak by a Gaussian, we may approximate the standard deviation of $\hat{p}$ by

$$\sigma_{\hat{p}} = \left[ -\frac{\partial^2 \ln \Pr(x \mid p)}{\partial p^2} \bigg|_{p=\hat{p}} \right]^{-1/2} = \sqrt{\frac{x(n-x)}{n^3}}.$$