Robust sliding-mode control of a MEMS optical switch

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Abstract. Over the last few years interests have emerged for application of MEMS in telecommunications. The use of MEMS for optical switching has turned to be the most attractive since this application could revolutionize fiber optic telecommunications. In this paper a robust control strategy based on sliding-mode control theory is developed for a MEMS optical switch, considering electrical, mechanical, and optical models. Sliding-mode control enables compact realization of a robust controller tolerant of device characteristics variation, non-linearities, and types of inherent instabilities. Robustness of proposed control scheme against disturbances is proved by Lyapunov second method and demonstrated through simulations. In addition, the presented control scheme is simple to implement in practical application.

1. Introduction
The main attraction of optical switch is that it enables routing of optical data signals without the need for conversion to electrical signals, and therefore is independent of data rate and data protocol. Since there is no need for lots of expensive and power-hungry high-speed electronics, transmitters, and receivers, the system becomes less expensive; meanwhile, the reduction of complexity improves reliability and reduces the footprint of all-optical switches. There are numerous applications for optical switches that require precision positioning of micro-actuators. The analog nature of MEMS actuators and their device characteristic uncertainties due to the manufacturing tolerances make this either impractical and or requires costly calibrations unless a closed-loop robust control scheme is employed. In addition the nonlinear characteristics of MEMS actuator particularly, damping modeling could result in instability over an extended actuation range in open-loop operation. Mechanical structures of MEMS, consist of moving parts controlled by electronics, and hence usually show oscillating transitions specially if they work in vacuum environment. Small changes such as external disturbances and unmodeled dynamics of the system will also have adverse effects and produce undesired results. In this paper, we review the application of sliding-mode control for non-linear optical switch that addresses challenges involved in MEMS actuator control. Sliding-mode control is a particular type of Variable Structure Control (VSC) that are characterized by a suite of feedback control law and a decision rule known as switching function. The main advantages of sliding-mode control are robustness, computation speed, compact implementation, controller order reduction, disturbance rejection, and insensitivity to parameter variations.
2. Mathematical Model

A general optical switch structure consisting of an electrostatic comb drive, the body of the device, and a blade or shuttle is shown in Fig. 1 [1]. A voltage applied to the comb drive actuator generates a force that moves the shuttle and attached micro-mirror that cuts a light beam exiting a transmitting fiber and being collected in a receiving and modulating its density.

In order to derive a mathematical model of system dynamics it is needed to determine parameters of the relevant differential equation that describes forces acting on the shuttle. It is assumed that the shuttle has one degree of freedom and moves only in one direction. It is important to mention that there might be other degrees of freedom, like rotation around the main body axes, translational along them, as well as different vibrational modes. The influence of additional degrees of freedom should be addressed through modal analysis, as they can influence the dynamical behavior of the switch in the main degree of freedom. Subsequent control system design procedure should filter out harmonics that corresponds to those modes. They can be determined either analytically or with the aid of finite element modal analysis. However, only the main degree of freedom will be considered in this paper and related materials for modeling purpose are referred to [2].

![Fig. 1. SEM image of a MEMS optical switch [1]](image)

The mathematical model of switch consists of three parts; electrical, mechanical and optical. Altogether, the system can be described with a second order nonlinear differential equation as

\[ m \ddot{x} + d(x, x) + k(x) = f(V, x), \quad P = h(x) \]

(1)

where \( m \) is the effective moving mass of the shuttle, \( d \) is a function describing losses such as damping and friction, \( k \) is the stiffness of the suspension, \( f \) is the electrostatic force acting on the model, \( P \) is intensity of light, and \( x \) is the shuttle position.

The system exact parameters \( m, d, k, \) and \( f \) are not easy to obtain and we will go step by step to determine all of these parameters. First, the electrical model is built and followed by mechanical model.

2.1. Electrical Model

The electrical part of the model considers generation of the electrostatic force by applying voltage to the terminals of the comb drive electrodes. The capacitance of the comb drive as a function of position should be determined first. Capacitance of the comb drive can be calculated as a sum of all capacitance among pairs of its movable interdigitated fingers. Each two fingers form one parallel plate capacitor. Capacitance is given as a function of position as

\[ C(x) = \varepsilon_0 A / d_G = 2n \varepsilon_0 F(x + x_0) / d_G \]

(2)
where $\varepsilon_0 = 8.854\times10^{-12} \text{ Fm}^{-1}$ is the dielectric constant of vacuum, $n$ is the number of the movable comb fingers ($n=150$), $T$ is thickness of the structural layer ($T = 35\mu m$), $d_G$ is the gap between fingers ($d_G = 2.6\mu m$) and $x_0$ is the overlapped length of fingers when no voltage is applied ($x_0 = 15\mu m$). At rest position, the capacitance of the comb drive is $C(x=0, x_0 = 15\mu m) = 0.27pF$, which increases as force is applied and the fingers move closer. Generally, the electrostatic force of the capacitor is given as a product of squared voltage and change of capacitance with respect to position as

$$f(V, x) = 0.5V^2 \frac{\partial C}{\partial x} \quad (3)$$

where $V$ is voltage applied over the electrodes. By combining (2) and (3) electrostatic force can be calculated as

$$f(V, x) = n\varepsilon_0 TV^2 / d_G = k_V V^2 \quad (4)$$

where $k_V$ is defined as the input gain to the system with the value of $k_V = 17.8nN/V^2$.

It is interesting to note that capacitance (2) depends linearly on the position over a wide range of deflections. It is one of the most important characteristics of the comb drive. Generally, for other configurations, this is not the case and capacitance is a higher nonlinear function of position $x$. Consequently, electrostatic force (4) depends only on voltage across the capacitor not on position. It should be noted that the linear relationship does not hold for extreme deflections and may cause considerable undesired results that necessitate using a robust control scheme to meet such those unmodeled disturbances.

2.2. Mechanical Model

In order to get the mechanical model of the system three parameters, have to be determined. The first one is the so-called effective moving mass, the second one is damping coefficient, and the third one is the stiffness of the employed suspension. Available models have been developed for typical types of suspensions based on measurements and FEA. Effective mass for the switch can be expressed as

$$m = m_{\text{mirror}} + 0.5m_{\text{rigid}} + 2.74m_{\text{beam}} \quad (5)$$

When calculated, the effective mass of the system is $m = 2.39\times10^{-9} \text{ kg}$.

Stiffness is generally a nonlinear function of position $f=k(x)$. For most metals and for silicon spring-like structures [8], it can be described as $k(x) = k_x x + k_{x^3} x^3$. For the suspension given in this paper stiffness is assumed to be linear and its coefficient is given as $k_x = k = 0.46 \text{ N/m}$.

Damping, or energy dissipation, is the parameter that is the most difficult to determine analytically, even through FEA. The reason lies in the number of different mechanisms that cause it, including friction, viscous forces, drag, etc [9]. We will consider viscous forces as a primary reason that causes damping. Four different mechanisms could contribute to damping, Couette flow, Poiseuille flow, Stokes flow, and Squeeze film damping [10]. Generally, they can be summarized as $f_d = (d_x x + d_0)\dot{x}$. When actual parameters are substituted, damping is expressed as $d(x, \dot{x}) = 0.0363(x + 15\times10^{-6})\dot{x}$.

2.3. Optical Model

The optical model is simply a function that connects the intensity of light to the position of the blade. Setup is shown on Fig. 2. Light beam is intercepted by the blade, increasing and decreasing the throughput of light. The Rayleigh-Sommerfeld model is based on a Gaussian distribution of the intensity across the light beam.
Transmitted power can be described as

\[ P = 0.5 \left[ 1 - \text{erf} \left( \frac{\sqrt{2}(x - \eta_0)}{w_1} \right) \right] \]  \hspace{1cm} (6)

where \( w_1 = 10.9 \mu m \) and \( \eta_0 = 11.2 \mu m \).

Consequently, integrating created model for each section and applying procedures done for increasing accuracy of the model in [2], results in the nonlinear mathematical model of the switch as

\[ 2.35 \times 10^{-9} \dddot{x} + (0.0363x + 4.5 \times 10^{-5}) \dot{x} + 0.6x = 1.9 \times 10^{-8} V^2 \]  \hspace{1cm} (7)

3. Sliding-Mode Control

One particular approach to robust controller design of achieved non-linear model, considering discrepancies due to unmodeled dynamics, variation in system parameters or the approximation of complex behavior by the straightforward model is the so-called sliding-mode control methodology. Sliding-mode control is a particular type of Variable Structure Control (VSC) that are characterized by a suite of feedback control law and a decision rule known as switching function.

In sliding mode control, VSC systems are designed to drive and then constrain the system state to lie within a neighborhood of the switching function. There are two main advantages to this approach. Firstly, the dynamics of the system may be tailored by the particular choice of switching function. Secondly, the closed-loop response becomes totally insensitive to uncertainties. The sliding-mode design approach consists of two components. The first involves the design of a switching function so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law which will make the switching function attractive to the system state.

3.1. Controller Design

There are two kinds of problems encountered in the controller design process [2].

1) the quadratic term in voltage as input of the system (\( u = V^2 \))
2) limited voltage available for control (0-35V)

It will be shown that proposed control scheme satisfies these constraints without any additional treatment.

The state space representation of Eq. (7) can be rewritten as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left[ 1.9 \times 10^{-8} u - 0.6x_1 - (0.0363x_1 + 4.5 \times 10^{-5})x_2 \right]/(2.35 \times 10^{-9})
\end{align*}
\]  \hspace{1cm} (8)

Let us consider the sliding surface (\( s = 0 \)) using states error as

\[ s = \dot{e} + \lambda e \]  \hspace{1cm} (9)

where \( e = x - x_d \) and \( \dot{e} = \frac{de}{dt} \). This expression can be rewritten in the form of

\[ s = x_2 - \dot{x}_d + \lambda(x_1 - x_d) = 0 \]  \hspace{1cm} (10)

To ensure that the states of the system approach the sliding-mode, \( \dot{s} = 0 \) should be satisfied.

\[ \dot{s} = \ddot{x}_2 - \dot{x}_d + \dot{\lambda}(x_1 - x_d) = 0 \]  \hspace{1cm} (11)
Substitution of states from (8), results in the following expression for equivalent control, \( u_{eq} \).

\[
\begin{align*}
  u_{eq} &= \frac{1}{1.9 \times 10^{-8}} \left[ 0.6 x_1 + (0.0363 x_1 + 4.5 \times 10^{-3}) x_2 + 2.35 \times 10^{-9} \left[ \dot{x}_d - \lambda (x_2 - \dot{x}_d) \right] \right] \\
\end{align*}
\]

Finally, the sliding-mode controller will be as

\[
\begin{align*}
  u &= \frac{1}{1.9 \times 10^{-8}} \left[ 0.6 x_1 + (0.0363 x_1 + 4.5 \times 10^{-3}) x_2 + 2.35 \times 10^{-9} \left[ \dot{x}_d - \lambda (x_2 - \dot{x}_d) - \rho \text{sgn}(s) \right] \right] \\
\end{align*}
\]

where \( \rho \) is a constant parameter depending on disturbances exerted on the system and reaching time.

### 3.2. Robustness Analysis

Let us consider bounded disturbance \( d(t) \) exerted on the system as

\[
\begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= \left[ 1.9 \times 10^{-8} V^2 - 0.6 x_1 - (0.0363 x_1 + 4.5 \times 10^{-3}) x_2 \right] / (2.35 \times 10^{-9}) + d(t) \\
\end{align*}
\]

where we assume that upper limit of \( d(t) \) is known as \( d_{\text{max}} \). By using the second method of Lyapunov, let the Lyapunov function be \( V = \frac{1}{2} s^2 \). Differentiating \( V \) with respect to time results in

\[
\begin{align*}
  \dot{V} &= s[-\rho \text{sgn}(s) + d(t)] \\
  &\leq s[-\rho \text{sgn}(s) + d_{\text{max}}] \\
\end{align*}
\]

Obviously, if \( \rho \geq |d_{\text{max}}| \) is satisfied, then \( \dot{V} < 0 \) is sufficiently ensured. In practice, the signum function, in Eq. (13) can be replaced by a continuous function \( s / |s| + \gamma \) to alleviate chattering, where \( \gamma \) is a positive scalar constant. The controller then becomes

\[
\begin{align*}
  V^2 &= \frac{1}{1.9 \times 10^{-8}} \left[ 0.6 x_1 + (0.0363 x_1 + 4.5 \times 10^{-3}) x_2 + 2.35 \times 10^{-9} \left[ \dot{x}_d - \lambda (x_2 - \dot{x}_d) - \rho \frac{s}{|s| + \gamma} \right] \right] \\
\end{align*}
\]

### 4. Simulation Results

Suppose the desired position is \( x_d = 5 \mu m \) and \( 23 \mu m \). Consider a disturbance exerted on the system as \( d(t) = D \sin(\omega t) \), where \( D = 10^{-6} \) and \( \omega = \pi \). The responses of the system for short and long displacement of actuator are shown in Fig. 3 and Fig. 4. \( x_d, x, \) and \( V \) are desired and actual positions, and applied voltage, respectively.

![Fig. 3. position, phase diagram, and control input](image-url)
Fig. 4. position, phase diagram, and control input $x_d = 23\mu m$, $\rho = 5\times 10^{-6}$, $\lambda = 2$, and $\gamma = 10^{-7}$

5. Conclusion

A robust sliding-mode control scheme is proposed for control of a nonlinear MEMS optical switch to achieve robustness against disturbances and terminal accuracy. The proposed control requires disturbance limit, and therefore exact information of disturbance is not necessary. The effectiveness of the presented method is proved by second method of Lyapunuv and the robustness of the controller against disturbances demonstrated by simulation results. Furthermore, the presented controller is simple to implement in practical applications.

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