On the Evolution of Cosmological Type Ia Supernovae and the Gravitational Constant

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There are at least three ways in which a varying gravitational constant $G$ could affect the interpretation of the recent high-redshift Type Ia supernovae results. If the local value of $G$ at the space-time location of distant supernovae is different, it would change both the thermonuclear energy release and the time scale of the supernova outburst. In both cases the effect is related to a change in the Chandrasekhar mass $M_{\text{Ch}} \propto G^{-3/2}$. Moreover the integrated variation of $G$ with time would also affect cosmic evolution and therefore the luminosity distance relation. Here we investigate in a consistent way how these different effects of a varying $G$ could change the current interpretation of the Hubble diagram of Type Ia supernovae. We parametrize the variation of $G$ using scalar-tensor theories of gravity, such as the Jordan-Brans-Dicke theory or its extensions. It is remarkable that Dirac’s hypothesis that $G$ should decrease with time can qualitatively explain the observed $\Delta m \simeq 0.2$ mag decrease at $z \simeq 0.5$ (with respect to a decelerating universe) and, at the same time, reduce the duration of the risetimes of distant Type Ia supernovae as recently reported.

I. INTRODUCTION

Type Ia supernovae (SNeIa) are supposed to be one of the best examples of standard candles. This is because, although the nature of their progenitors and the detailed mechanism of explosion are still the subject of a strong debate, their observational light curves are relatively well understood and, consequently, their individual intrinsic differences can be easily accounted for. Therefore, thermonuclear supernovae are well suited objects to study the Universe at large, especially at high redshifts ($z \sim 0.5$), where the rest of standard candles fail in deriving reliable distances, thus providing an unique tool for determining cosmological parameters or discriminating among different alternative cosmological theories.

Using the observations of 42 high redshift Type Ia supernovae and 18 low redshift supernovae (Riess et al. 1998; Perlmutter et al. 1999), both the Supernova Cosmology Project and the High-$z$ Supernova Search Team found that the peak luminosities of distant supernovae appear to be $\sim 0.2$ magnitude fainter than predicted by a standard decelerating universe ($q_0 > 0$). Based on this, the Supernova Cosmology Project derived $\Omega_M = 0.28_{-0.12}^{+0.14}$ at 1$\sigma$, for a flat universe, thus forcing a non-vanishing cosmological constant. However this conclusion lies on the assumption that there is no mechanism likely to produce an evolution of the observed light curves over cosmological distances. In other words: both teams assumed that the intrinsic peak luminosity and the time scales of the light curve were exactly the same for both the low-$z$ and the high-$z$ supernovae.

More recently Riess et al. (1999a,b) have found evidences of evolution between the samples of nearby supernovae and those observed at high redshifts by comparing their respective risetimes, thus casting some doubts about the derived cosmological parameters. In particular Riess et al. (1999a,b) find that the sample of low-$z$ supernovae has an average risetime of $19.98 \pm 0.15$ days whereas the sample of high-$z$ supernovae has an average risetime of $17.50 \pm 0.40$ days. The statistical likelihood that the two samples are different is high (5.8$\sigma$). Riess et al. (1999b) also analyze several potential alternatives to produce, within a family of theoretical models, an evolution with the observed properties:

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distant supernovae should be intrinsically fainter and at the same time should have smaller risetimes. All the families of models studied so far have the inverse trend: decreasing peak luminosities correspond to longer risetimes.

On the other hand, and from the theoretical point of view, it is easy to show that a time variation of the gravitational constant, in the framework of a Scalar-Tensor cosmological theory, can reconcile the observational Hubble diagram of SNeIa with an open $\Omega_\Lambda = 0$ universe. The starting point is simple: assume that all thermonuclear supernovae release the same amount of energy ($E$). In a simple model of light curve (Arnett 1982) the peak luminosity is proportional to the mass of nickel synthesized, which in turn, to a good approximation, is a fixed fraction of the Chandrasekhar mass ($M_{\text{Ni}} \propto M_{\text{Ch}}$), which depends on the value of gravitational constant: $M_{\text{Ch}} \propto G^{-3/2}$. Thus we have $E \propto G^{-3/2}$, and if one assumes a slow decrease of $G$ with time, distant supernovae should be dimmer. Moreover, the time scales of supernovae also depend on the Chandrasekhar mass. Let us elaborate on this last point. According to the analytic model of light curve of Arnett (1982), the width of the peak of the light curve of SNeIa is given by:

$$
\tau \propto \left( \frac{M_{\text{ej}}}{M_{\text{inc}}} \right)^{1/4}
$$

(1)

where $M_{\text{ej}}$ is the ejected mass and $M_{\text{inc}}$ is the incinerated mass. Within our current knowledge of the mechanisms of explosion of SNeIa both masses can be considered proportional to the Chandrasekhar mass, and therefore we have $\tau \propto M_{\text{Ch}}^{1/2}$ or, equivalently, $\tau \propto G^{-3/4}$. Since the risetime for distant supernovae is obtained from semi-empirical models, that is a template light curve which takes into account the decline rate and the width of the peak, one can then also assume this dependence on $G$ for the risetime. This expression has the right properties since distant supernovae have smaller peak luminosities and, at the same time, smaller risetimes, as required by observations.

II. THE EFFECTS OF A VARYING $G$

Despite the beauty and successes of the simplest version of General Relativity (GR), the possibility that $G$ could vary in space and/or time is well motivated. Its study can shed new light into fundamental physics and cosmology and it seems natural in Scalar-Tensor theories of gravity (STTs) such as Jordan-Brans-Dicke (JBD) theory or its extensions.

To make quantitative predictions we will consider cosmic evolution in STTs, where $G$ is derived from a scalar field $\phi$ which is characterized by a function $\omega = \omega(\phi)$ determining the strength of the coupling between the scalar field and gravity. In the simplest JBD models, $\omega$ is just a constant and $G \propto \phi^{-1}$, however if $\omega$ varies then it can increase with cosmic time so that $\omega = \omega(z)$. The Hubble rate $H$ in these models is given by:

$$
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi \rho}{3\phi} + \frac{1}{a^2 R^2} + \frac{\Lambda}{3} + \frac{\omega \phi^2}{6 \phi^2} - H \frac{\dot{\phi}}{\phi},
$$

(2)

this equation has to be complemented with the acceleration equations for $a$ and $\phi$, and with the equation of state for a perfect fluid: $p = (\gamma - 1)\rho$ and $\dot{\rho} + 3\gamma H \rho = 0$. The structure of the solutions to this set of equations is quite rich and depends crucially on the coupling function $\omega(\phi)$ (see Barrow & Parsons 1996). Here we are only interested in the matter dominated regime: $\gamma = 1$. In the weak field limit and a flat universe the exact solution is given by:

$$
G = \frac{4 + 2\omega \phi^{-1}}{3 + 2\omega \phi^{-1}} = G_0 (1 + z)^{1/(1+\omega)}.
$$

(3)

In this case we also have that $a = (t/t_0)^{(2\omega+2)/(3\omega+4)}$. This solution for the flat universe is recovered in a general case in the limit $t \to \infty$ and also arises as an exact solution of Newtonian gravity with a power law $G \propto t^n$ (Barrow 1996). For non-flat models, $a(t)$ is not a simple power-law and the solutions get far more complicated. To illustrate the effects of a non-flat cosmology we will consider general solutions that can be parametrized as Eq.(3) but which are not simple power-laws in $a(t)$. In this case, it is easy to check that the new Hubble law given by Eq.(2) becomes:

$$
H^2(z) = H_0^2 \left[ \Omega_M (1 + z)^{3+1/(1+\omega)} + \Omega_R (1 + z)^2 + \Omega_\Lambda \right]
$$

(4)

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1While we were writing this paper we became aware of a similar idea independently proposed by Amendola et al. (1999).
where $\hat{\Omega}_M, \hat{\Omega}_R$ and $\hat{\Omega}_\Lambda$ follow the usual relation: $\hat{\Omega}_M + \hat{\Omega}_R + \hat{\Omega}_\Lambda = 1$ and are related to the familiar local ratios ($z \to 0$): $\hat{\Omega}_M = 8\pi G_0 \rho_0/(3H_0^2)$, $\hat{\Omega}_R = 1/(\rho_0 H_0 R^2)$ and $\hat{\Omega}_\Lambda = \Lambda/(3H_0^2)$ by:

$$\hat{\Omega}_M = \frac{\Omega_M}{g} \left( \frac{4 + 2\omega}{3 + 2\omega} \right) ; \quad \hat{\Omega}_\Lambda = \frac{\Omega_\Lambda}{g} ; \quad \hat{\Omega}_R = \frac{\Omega_R}{g} \tag{5}$$

$$g = 1 + \frac{1}{1 + \omega} - \frac{1}{6} \frac{\omega}{(1 + \omega)^2} \tag{6}$$

Thus the GR limit is recovered as $\omega \to \infty$. The luminosity distance $d_L = d_L(z, \Omega_M, \Omega_\Lambda, \omega)$ is obtained as usual from the (line-of-sight) comoving coordinate distance: $r(z') = \int dz'/H(z')$, with the trigonometric or the hyperbolic sines to account for curvature (Peebles 1993). In the limit of small $z$ we recover the usual Hubble relation: $y = H_0 r = z - (1 + \hat{q}_0)z^2/2$ where a new deceleration $\hat{q}_0$ parameter is related to the standard one by:

$$\hat{q}_0 = \frac{q_0}{g} + \frac{\hat{\Omega}_M}{2(1 + \omega)} \tag{7}$$

One can see from this equation that even for relative small values of $\omega$ the cosmological effect is small. For example for $\Omega_M \simeq 0.2$ and $\Omega_\Lambda \simeq 0.8$ we have $q_0 \simeq -0.7$ while $\hat{q}_0$ is around $\hat{q}_0 \simeq -0.4$ for $\omega \simeq 1$. Note nevertheless that this effect, although small, tends to decrease the acceleration and therefore it partially decreases the effect in the peak luminosity of SNeIa caused by an increasing $G$. In summary, Eq.\ref{eq:omega} parametrizes the change in $G$ as a function of $\omega$ while Eqs.\ref{eq:omega} and \ref{eq:omega} parametrize the corresponding cosmic evolution.

As mentioned in the introduction, we are assuming that thermonuclear supernovae release a similar amount of energy $E \propto G^{-3/2}$. Thus using Eq.\ref{eq:omega}, we have:

$$\frac{E}{E_0} = \left( \frac{G}{G_0} \right)^{-3/2} ; \quad M - M_0 = \frac{15}{4} \log \left( \frac{G}{G_0} \right) = \frac{15}{4(1 + \omega)} \log (1 + z), \tag{8}$$

were $M$ is the absolute magnitude and the subscript 0 denotes the local value. Therefore we have the following Hubble relation:

$$m(z) = M_0 + 5 \log d_L(z, \Omega_M, \Omega_\Lambda, \omega) + \frac{15}{4(1 + \omega)} \log (1 + z) \tag{9}$$

which reduces to the standard relation as $\omega \to \infty$. From the last term alone we can see that $\omega \simeq 5$ can reduce the apparent luminosity by $\Delta m \simeq 0.2$, which is roughly what is needed to explain the SNeIa results without a cosmological constant. For illustrative purposes figure \ref{fig:omega} shows the above relation for two representative cosmological models, including the effects of $\omega$ in $d_L$, for $\omega = \pm 5$ (dotted lines) and the standard ($\omega = \infty$) case (solid line).

The effect of a varying $G$ on the time scales of SNeIa can be obtained from Eq.\ref{eq:omega}. Since $\tau \propto G^{-3/4}$, the ratio of the local time scale, $\tau_0$, to the faraway one is:

$$\left\langle \frac{\tau}{\tau_0} \right\rangle \simeq \left( \frac{G}{G_0} \right)^{-3/4} = (1 + z)^{-3/4} \tag{10}$$

and, to make some quantitative estimates, we can use the mean evolution found by Riess et al. (1999a,b). From their figure 1 we obtain the following widths of the light curve when the supernova is 2.5 magnitudes fainter than the peak luminosity: $\tau_0 = 45.0 \pm 0.15$ (at $z \simeq 0$) and $\tau = 43.8 \pm 0.40$ (at $z \simeq 0.5$), were the errors in the widths have been ascribed solely to the errors in the risetimes. Thus, from Eq.\ref{eq:omega} we obtain $\omega \simeq 10.25^{+3.22}_{-3.65}$ (2σ errors). Therefore, a very small variation of the gravitational constant can account for the reported differences in the SNeIa time scales. However these limits on $\omega$ should be considered as weak, in the sense that since most SNeIa are discovered close to its peak luminosity the width of the light curve is poorly determined. These values are shown as horizontal dashed (1σ) and continuous (2σ) lines in Fig.\ref{fig:omega} where the confidence contours (at the 99%, 90%, 68% — solid lines — 5% and 1% confidence level — dotted lines) in the $(\omega, \Omega_\Lambda)$ plane for a flat $\Omega_R = 0$ universe (left panel) and in the $(\omega, \Omega_M)$ plane for the case $\Omega_\Lambda = 0$ (right panel) are shown.

III. DISCUSSION AND CONCLUSIONS

In astrophysics and cosmology the laws of physics (and in particular the simplest version of GR) are extrapolated outside its observational range of validity. It is therefore important to test for deviations of these laws at increasing
cosmological scales and times (redshifts). SNeIa provide us with a new tool to test how the laws of gravity and
cosmology were in faraway galaxies ($z \simeq 0.5$). In particular, current limits on the (parametrized) Post Newtonian
formalism mostly restrict to our very local Universe (see Will 1993). The observational limits on $\dot{G}/G$ come from quite
different times and scales (see Barrow & Parsons 1996 for a review), but mostly in the local and nearby environments
at $z \simeq 0$ (solar system, binary pulsars, white dwarf cooling, neutron stars) typical bounds give $\dot{G}/G \lesssim 10^{-11} - 10^{-12}$
$\text{yr}^{-1}$, or $\omega \gtrsim 10 - 100$. However, STTs predict $\omega = \omega(\phi)$. That is, $\omega$ is not required to be a constant, so that $\omega$
can increase with cosmic time, $\omega = \omega(z)$, in such a way that it could approach the GR predictions ($\omega \to \infty$) at
present time and still give significant deviations at earlier cosmological times. In this sense bounds from primordial
nucleosynthesis could provide an important test. Current bounds on $\omega$ from nucleosynthesis are comparable to the
local values but these bounds are model dependent and also involve very large extrapolations.

Our analysis indicates that if we adopt the constraints derived from the width of the light curves of SNeIa then
our best fit to the data requires $\omega \simeq 10$ (or equivalently $\dot{G}/G \sim 10^{-11} \text{yr}^{-1}$ or $\sim 10\%$ in $G$). This value is slightly
smaller than some of the the current constraints at $z \simeq 0$, but it corresponds to higher redshifts $z \simeq 0.5$ and could be
accommodated in STTs with $\omega = \omega(\phi) = \omega(z)$. If this is the case, at the $2\sigma$ confidence level we obtain $0.0 \lesssim \Omega_A \lesssim 1.0$
and the Hubble diagram of SNeIa poorly constrains $\Omega_M \lesssim 1$. At the $1\sigma$ confidence level we obtain $0.2 \lesssim \Omega_A \lesssim 0.8$
and $\Omega_M \lesssim 0.7$. If we do not take into account the restrictions derived from the width of the light curves then
our conclusions are much weaker: the observational data and the theory can be reconciled in the framework of a
cosmological theory with a varying $G$ with no cosmological constant ($\Omega_A = 0$) only if $\omega \gtrsim 1.5$. If we further require a
flat $\Omega_R = 0$ universe then $1.5 \lesssim \omega \lesssim 3.0$ is needed.

Obviously more work is needed both regarding other observational consequences of STTs and on the physics of
supernovae. In particular, an improvement of our knowledge of the physics of thermonuclear supernovae would
provide us with an unique tool to test fundamental laws of physics over cosmological distances. In addition it should
be stressed that new observations of distant supernovae, or other standard candles, at higher redshifts ($z > 1$) could
constrain even more the current limits on the variation of the fundamental constants.

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FIG. 1. Hubble diagram for the high-redshift SNe.
FIG. 2. Confidence contours in the plane $(\omega, \Omega_\Lambda)$ for a flat case $\Omega_R = 0$ (left panel) and in the plane $(\omega, \Omega_M)$ for the case $\Omega_\Lambda = 0$ (right panel).