SIGN-CHANGING MULTI-BUMP SOLUTIONS
FOR CHOQUARD EQUATION
WITH DEEPENING POTENTIAL WELL

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Abstract. In this paper, we are concerned with the existence of sign-
changing multi-bump solutions for the following nonlinear Choquard equa-
tion

\[ -\Delta u + (\lambda V(x) + 1)u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N, \]

where \( I_\alpha \) is the Riesz potential, \( \lambda \in \mathbb{R}^+, (N - 4)^+ < \alpha < N, 2 \leq p < (N + \alpha)/(N - 2) \), and \( V(x) \) is a nonnegative continuous function with a poten-
tial well \( \Omega := \text{int}(V^{-1}(0)) \) which possesses \( k \) disjoint bounded com-
ponents \( \Omega_1, \ldots, \Omega_k \). We prove the existence of sign-changing multi-bump
solutions for (0.1) if \( \lambda \) is large enough.

1. Introduction

We study the following nonlinear Choquard equation

\[ -\Delta u + (\lambda V(x) + 1)u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N, \]

where \( \lambda \in \mathbb{R}^+, V(x) \in C(\mathbb{R}^N, \mathbb{R}) \) is a potential function, \( 2 \leq p < (N + \alpha)/(N - 2) \), \( I_\alpha : \mathbb{R}^N \to \mathbb{R} \) is the Riesz potential defined at each point \( x \in \mathbb{R}^N \setminus \{0\} \) by

\[ I_\alpha(x) = \frac{A_\alpha}{|x|^{N-\alpha}}, \quad A_\alpha = \frac{\Gamma((N - \alpha)/2)}{\Gamma(\alpha/2)\pi^{N/2}2^\alpha}, \]

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