Ensemble averaged coherent state path integral for disordered bosons with a repulsive interaction and derivation of a nonlinear sigma model

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Abstract
A coherent state path integral is considered for bosons with an ensemble average of a random potential and with an additional, repulsive interaction in the context of BEC under inclusion of specially prepared disorder. The essential normalization of the coherent state path integral, as a generating function of observables, is obtained from the non-equilibrium time contour for 'forward' and 'backward' propagation so that a time contour metric has to be taken into account in the ensemble average with the random potential. Therefore, the respective symmetries for the derivation of a nonlinear sigma model follow from the involved time contour metric which leads to a coset decomposition Sp(4) / U(2) ⊗ U(2) of the symplectic group Sp(4) with the subgroup U(2) for the unitary invariance of the density-related vacuum or ground state; the corresponding spontaneous symmetry breaking gives rise to anomalous- or 'Nambu'-doubled field degrees of freedom within self-energy matrices which are finally regarded by remaining coset matrices. The notion of a 'return probability', according to the original 'Anderson-localization', is thus naturally contained within coherent state path integrals of a non-equilibrium contour time for equivalent 'forward' and 'backward' propagation.

Keywords: Bose-Einstein condensation, ensemble averages for random potentials, coherent state path integral, many-particle physics, non-equilibrium or Keldysh time contour.
PACS 03.75.Nt , 03.75.Kk , 03.75.Hh

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1 Coherent state path integral and averaging method for disorder

1.1 Ensemble averages for static and dynamic disorder

The original concept of 'Anderson localization' is combined to a 'return probability' of a wave packet, which starts out to propagate from an initial space point and which is scattered back by the impurities and the disordered potential, for a measure of its 'localization' [1]. This 'return probability' can therefore be specified by the propagation of ensemble averaged, retarded and advanced Green functions [2]. Our presented approach in terms of coherent state path integrals, adapted to a system of BEC in an external trap potential, naturally contains this concept by a time contour metric with 'forward' and 'backward' propagation [3]–[14]. The ensemble average of a random potential involves the time contour metric which thus determines the symmetries for the derivation of a nonlinear sigma model [15, 16]. The coset decomposition Sp(4)/U(2) ⊗ U(2) of an anomalous-doubled self-energy gives rise to a spontaneous symmetry breaking (SSB) with the subgroup U(2) for an invariant state of density-related field degrees of freedom. The remaining coset matrices Sp(4)/U(2) finally comprise the anomalous- or 'Nambu'-doubled field degrees of freedom from the off-diagonal block parts of the total self-energy which is accomplished by a Hubbard-Stratonovich transformation (HST) of anomalous-doubled, dyadic products of boson fields with incorporation of 'hinge' fields from the SSB [17, 18, 19].

We consider two models \( \hat{H}_{I} \) with Hamiltonians \( \hat{H}_{I}(\psi , \hat{\psi}, V_{I}) \) and \( \hat{H}_{II}(\psi , \hat{\psi}, V_{II}) \) in normal ordering of bosonic creation and annihilation operators and with static and dynamic disorder potentials \( V_{I}(\vec{x}), V_{II}(\vec{x}, t) \) in parallel. Both Hamiltonians (1.1) contain the same one-particle part \( \hat{h}(\vec{x}) \) with the kinetic energy and with an external trap-potential \( u(\vec{x}) \), which is shifted by a reference energy \( \mu_{0} \) as a chemical potential. Apart from the same one-particle part \( \hat{h}(\vec{x}) \), we take the identical repulsive interaction with parameter \( V_{0} > 0 \) for a quartic contact interaction of bosonic operators \( \hat{\psi}_{x}^{\dagger} \hat{\psi}_{x}^{\dagger} \hat{\psi}_{x} \hat{\psi}_{x} \) (1.2). Furthermore, even- and complex-valued, spatially local source fields \( \hat{j}_{\psi}(\vec{x}, t) \), \( \hat{j}_{\psi}(\vec{x}, t) \) are included in common in order to allow for a SSB with a coherent, macroscopic wavefunction and with anomalous (or 'Nambu') paired bosons, respectively. These source fields can also be used for the determination of observables from differentiating various manners of the generating function. According to a presupposed, spatially spherical symmetry of \( u(\vec{x}) \), we normalize the spatial summations \( \sum_{\vec{x}} \ldots \) by the spherical system volume \( V^{(D)} \) of \( 'D' \) dimensions and by the spatial unit cell \( (\Delta x)^{D} \) which yields a parameter \( N_{x} = V^{(D)}/(\Delta x)^{D} \) to be applied for various approximations, as e.g. in a saddle point computation. The time parameter \( t_{p} \) is restricted to the range \( 0 \ldots + T_{0} \) with the discrete intervals \( \Delta t > 0 \) so that one is limited by the order of maximum energy \( h\Omega_{max} = \hbar/\Delta t \) within the propagation of the two Hamiltonians

\[
\hat{H}_{I, II}(\hat{\psi}^{\dagger}, \hat{\psi}, V_{I, II}) = \sum_{\vec{x}} \hat{\psi}_{x}^{\dagger} (\hat{h}(\vec{x}) + (V_{I}(\vec{x}) or V_{II}(\vec{x}, t)) + V_{0} \hat{\psi}_{x}^{\dagger} \hat{\psi}_{x}) \hat{\psi}_{x} + \]

\[
\hat{h}(\vec{x}) = -\frac{\hbar^{2}}{2m} (\frac{\partial}{\partial \vec{x}} \cdot \frac{\partial}{\partial \vec{x}}) + u(\vec{x}) - \mu_{0};
\]

\[
\sum_{\vec{x}} \ldots = \sum_{\vec{x}} (\Delta x)^{D} V^{(D)} \ldots = \int_{|\vec{x}|<L} d^{D}x \ldots V^{(D)} = \frac{S^{(D)}}{D} L^{D};
\]

\[
S^{(D=1)} = 1; \quad S^{(D=2)} = 2\pi; \quad S^{(D=3)} = 4\pi; \quad N_{x} = \frac{V^{(D)}}{(\Delta x)^{D}};\]

\[
\Omega_{max} = \frac{1}{\Delta t}; \quad 0 < t_{p} < +T_{0}; \quad T_{0} = T_{max}.
\]

The two Gaussian distributions for random potentials \( V_{I}(\vec{x}), V_{II}(\vec{x}, t) \) are determined by the second moments \( (1.4, 1.5) \) for static and dynamic disorder, respectively, and vanishing mean values. Both distributions are delta-function like, concerning the spatially 'contact' Kronecker-delta and concerning the white-noise delta-function of time. Moreover, we emphasize the two different normalizations of second moments in \( (1.4, 1.5) \) which are important in subsequent transformations and derivations for a proper, finite scaling of energy ranges within the nonlinear sigma models (cf. the second moments and their normalization in random matrix theories). Therefore, there occur two different disorder parameters \( R_{I}, R_{II} \).
of physical dimensions [energy × time] and [energy × (time)^1/2] in the two moments \(1.1 \div 1.6\). We apply the normalized generating functions \(Z[\hat{\alpha}, V_I, Z_{II}[\hat{\alpha}, V_{II}]\) \(1.1.7\) with the time development operators \(1.1.6\) which are composed of the prevailing Hamiltonian 'I' or 'II', each having an additional source matrix \(\hat{\alpha}\) for calculating observables by differentiating the coherent state path integral. The latter path integral \(1.1.6\) results into proper normalization of unity \(1.1.8\) for vanishing source matrix \(\hat{\alpha}\) = 0 and identical values \(1.1.9\) of symmetry breaking fields \(\psi; x(t), \bar{\psi}; \bar{x}(t)\) on the two branches of propagating time development operators in forward and backward direction.

\[
\begin{align*}
V_I(\vec{x}_1)V_I(\vec{x}_2) &= \frac{R^2 \Omega_{max}^2}{N_x} \delta_{\vec{x}_1, \vec{x}_2}; \text{ static disorder} \\
V_{II}(\vec{x}_1, t_1)V_{II}(\vec{x}_2, t_2) &= R^2_{II} \delta_{\vec{x}_1, \vec{x}_2} \delta(t_1 - t_2); \text{ dynamic disorder} \\
Z[\hat{\alpha}, V_{I,II}] &= \langle 0 | \hat{U}_{I,II}(t = 0, +T_0; V_{I,II}; \hat{\alpha}) \hat{U}_{I,II}(+T_0, t = 0; V_{I,II}; \hat{\alpha}) | 0 \rangle; \\
\hat{U}_{I,II}(t, 0; V_{I,II}; \hat{\alpha}) &= \exp \left\{ -\frac{i}{\hbar} \int_0^t dt \hat{H}_{I,II}(\hat{\psi}^0, \hat{\bar{\psi}}, V_{I,II}; \hat{\alpha}) \right\}; \\
Z[\hat{\alpha} = 0, V_{I,II}] &\equiv_{\{\hat{\psi}, \hat{\bar{\psi}}\}} 1; \\
\ldots &\equiv_{\{\hat{\psi}, \hat{\bar{\psi}}\}} \langle 0 | \hat{U}_{I,II}(t = 0, +T_0; V_{I,II}; \hat{\alpha}) \hat{U}_{I,II}(+T_0, t = 0; V_{I,II}; \hat{\alpha}) | 0 \rangle_{\{\hat{\psi}, \hat{\bar{\psi}}\}}. 
\end{align*}
\]

In accordance to the forward '+' and backward '-' propagation of time development operators, we introduce a contour time \(t_{p=\pm}\) and contour integral \(\int_C dt_p \ldots\) with a contour time metric \(\eta_{p=\pm} = \pm 1\) in order to regard the changing sign of phases in the exponent of the time evolution operators \(1.1.1 \div 1.1.4\) \(20 \div 27\). In the remainder we will also briefly outline the various steps of transformations to a nonlinear sigma model with incorporation of the precise time steps and intervals within the coherent state path integral which is specified by the normal ordering of creation and annihilation operators. These precise time steps, with additional time shift \(\Delta t_{p}\) in the complex conjugated coherent state fields, are usually omitted for brevity in literature, but are ubiquitous in many-particle physics; this problem of proper time steps does not prevent transformations and derivations to sigma models with SSB and a coset decomposition and has necessarily been omitted for brevity in literature, but are ubiquitous in many-particle physics; this problem of proper time steps does not prevent transformations and derivations to sigma models with SSB and a coset decomposition and has necessarily to be regarded as soon as quantum mechanical field integration variables at neighbouring, but still different time \(t_{p}\) and \(t_{p} + \Delta t_{p}\) are considered within normal ordered coherent state path integrals. We briefly hint at the problem of the appropriate, precise time steps by defining a slightly modified time contour \(1.1.10 \div 1.1.14\) for simplified representation of the exact time steps (cf. also additional boundary conditions of coherent state fields in \(2.2.2 \div 2.2.5\)).

\[
\begin{align*}
\int_C dt_p \ldots &= \int_0^{+T_0} dt_+ \ldots + \int_0^{-T_0} dt_- \ldots = \int_0^{+T_0} dt_+ \ldots - \int_0^{-T_0} dt_- \ldots; \\
\int_C dt_p \ldots &= \sum_{p=\pm} \int_0^{+T_0} dt_p \eta_p \ldots; \quad \eta_p = \left\{ \eta_+ = +1; \eta_- = -1 \right\}; \\
\int_C dt_p \ldots &= \int_{-\Delta t}^{+T_0 + \Delta t} dt_+ \ldots + \int_{-\Delta t}^{-T_0} dt_- \ldots = \int_{-\Delta t}^{+T_0 + \Delta t} dt_+ \ldots - \int_{-\Delta t}^{-T_0} dt_- \ldots; \\
\int_C dt_p \ldots &= \sum_{p=\pm} \int_0^{+T_0} dt_p \eta_p \ldots; \quad \hat{T}_+ = -\Delta t; \quad \hat{T}_+ = T_0; \quad \hat{T}_- = 0; \quad \hat{T}_- = T_0 + \Delta t.
\end{align*}
\]

Since coherent state path integrals allow for the exact time sequence of coherent state fields with proper, additional time shifts \(\Delta t_p\) of the corresponding complex-conjugated fields, one can also investigate other kinds of coherent state path integrals as those of Eqs. \(1.1.10 \div 28 \div 29 \div 30\). Apart from the presented problem, we have also applied coherent state path integrals to a trace representation of delta functions with maximal commuting sets of symmetry operators \(1.1.5 \div 1.1.10\). As these symmetry operators \(1.1.17 \div 1.1.20\) are given in terms of normal ordered creation and annihilation
operators, one achieves a similar coherent state path integral on time contours as

\[ \varrho_{j,m_c}(E, n_0) = \text{Coherent state path integral representation of the trace} \]

\[ := \Tr \left[ \delta(E - \hat{H}) \delta(n_0 - \hat{N}) \delta(h^2 j(j + 1) - \hat{J} \cdot \hat{J}) \delta(h m_z - \hat{J}_z) \right] ; \]

\[ \hat{H}, \hat{N}, \hat{J}, \hat{J}_z := \text{maximal commuting set of second quantized operators according to symmetries}; \]

\[ \hat{H}(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{\vec{x},s} \hat{\psi}_{\vec{x},s}^\dagger \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \vec{x}^2} + \frac{n_0 e^2}{|\vec{x}|} + \sum_{\vec{x}',\vec{s}' \neq \vec{x},s} |\vec{x} - \vec{x}'| \hat{\psi}_{\vec{x}',\vec{s}'}^\dagger \hat{\psi}_{\vec{x}',\vec{s}'} \right) \hat{\psi}_{\vec{x},s} ; \]

\[ \hat{N}(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{\vec{x},s} \hat{\psi}_{\vec{x},s}^\dagger \hat{\psi}_{\vec{x},s} ; \]

\[ \hat{J}(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{\vec{x},s_1,s_2} \hat{\psi}_{\vec{x},s_2}^\dagger \left( \vec{x} \times \vec{p} + \frac{\hbar}{2} \hat{\sigma} \right) \hat{\psi}_{\vec{x},s_1} ; \]

\[ \hat{J}_z(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{\vec{x},s_1,s_2} \hat{\psi}_{\vec{x},s_2}^\dagger \left( (\vec{x} \times \vec{p})_z + \frac{\hbar}{2} \hat{\sigma}_z \right) \hat{\psi}_{\vec{x},s_1} . \]

This is accomplished by the application of the Dirac identity (1.21) to the principal value 'P' and delta function of the symmetry operators (1.22, 1.23) so that propagation with exponentials is also implied on two branches of a 'disconnected' time contour for the integral representation of delta functions. Therefore, one can also perform ensemble averages (1.24) of trace relations with delta-functions of symmetry operators in their representation with coherent state path integrals, very similar to (1.4, 1.5). However, it is important to distinguish between one-particle (1.19, 1.20) and two-particle operators (1.17, 1.26) in the various transformations to a nonlinear sigma model with inclusion of a coset decomposition for a SSB with a HST

\[ \lim_{y \rightarrow 0^+} \frac{1}{E - \hat{H}(\hat{\psi}^\dagger, \hat{\psi}) - \hbar^2 \epsilon \tau^+} = \frac{\varPsi}{\hbar} \frac{1}{x + \hbar \tau} \delta(x) ; \]

\[ \lim_{y \rightarrow 0} \frac{1}{E - \hat{H}(\hat{\psi}^\dagger, \hat{\psi}) - \hbar^2 \epsilon \tau^+} = \frac{\varPsi}{\hbar} \frac{1}{x + \hbar \tau} \delta(E - \hat{H}(\hat{\psi}^\dagger, \hat{\psi})) ; \]

\[ \lim_{y \rightarrow 0} \frac{1}{\epsilon - \hat{D}(\hat{\psi}^\dagger, \hat{\psi}) - \hbar^2 \epsilon \tau^+} = \frac{\varPsi}{\hbar} \frac{1}{x + \hbar \tau} \delta(\epsilon - \hat{D}(\hat{\psi}^\dagger, \hat{\psi})) ; \]

\[ \delta(\epsilon - \hat{D}(\hat{\psi}^\dagger, \hat{\psi})) = \sum_{p = \pm \hbar} \int_{0}^{+\infty} \frac{dt_p}{2\pi \hbar} \exp \left\{ -i \frac{\epsilon}{\hbar} t_p \right\} \exp \left\{ -i \frac{\epsilon}{\hbar} t_p \right\} \right\} ; \]

\[ \hat{D}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{H}(\hat{\psi}^\dagger, \hat{\psi}), \hat{N}(\hat{\psi}^\dagger, \hat{\psi}), \hat{J}(\hat{\psi}^\dagger, \hat{\psi}), \hat{J}(\hat{\psi}^\dagger, \hat{\psi}), \hat{J}_z(\hat{\psi}^\dagger, \hat{\psi}) ; \]

\[ \varrho_{j,m_c}(E_1, E_2, n_0) = \Tr \left[ \delta(E_1 - \hat{H}(\hat{\psi}^\dagger, \hat{\psi})) \delta(E_2 - \hat{H}(\hat{\psi}^\dagger, \hat{\psi})) \delta(n_0 - \hat{N}(\hat{\psi}^\dagger, \hat{\psi})) \times \delta(h^2 j(j + 1) - \hat{J}(\hat{\psi}^\dagger, \hat{\psi}), \hat{J}(\hat{\psi}^\dagger, \hat{\psi})) \delta(h m_z - \hat{J}_z(\hat{\psi}^\dagger, \hat{\psi})) \right] ; \]

\[ \hat{J}(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{J}(\hat{\psi}^\dagger, \hat{\psi}) = \left( \sum_{\vec{x},s_1,s_2} \hat{\psi}_{\vec{x},s_2}^\dagger \left( \vec{x} \times \vec{p} + \frac{\hbar}{2} \hat{\sigma} \right) \hat{\psi}_{\vec{x},s_1} \right) \cdot \left( \sum_{\vec{x}',s_1',s_2'} \hat{\psi}_{\vec{x}',s_2'}^\dagger \left( \vec{x}' \times \vec{p}' + \frac{\hbar}{2} \hat{\sigma}' \right) \hat{\psi}_{\vec{x}',s_1'} \right) . \]

\[ 2 \text{ Ensemble averages in model I and II for a normal-ordered Hamiltonian} \]

\[ 2.1 \text{ Precise time steps with shifts } \Delta t_p \text{' of the complex conjugated fields } \psi_{\vec{x},s}^{\dagger}(t_p + \Delta t_p) \text{'} \]

The ensemble averages of random potentials involve an additional, mathematical aspect which concerns the combination of fields of the two different branches \( (p = \pm) \) of the time contour. This formal aspect is also implied by the original formulation of Anderson localization where one examines a 'return' probability of a test wave-packet (as an initial delta-spike at a particular space point). This 'return' probability measures the 'forward' propagation of the wave-packet,
away from an initial space point, and the corresponding ‘backward’ propagation of remaining wave-packet parts, back again to the same initial space point. Localization of wave-packets within a disordered potential is specified by a ‘finite’ ‘return’ probability which approaches vanishing values towards delocalization. According to this physical picture, we have to combine the doubling of bosonic coherent state fields \( \psi_X(t_+ = \psi_{X,+}(t), \psi_{X,-}(t) \) apart from their usual anomalous or ‘Nambu’ doubling \( (\psi_X(t_+), \psi_X(t_-) \) of ordered systems with solely hermitian operators, with the two distinct branches of contour time. Consequently, there occur two different kinds of ‘Nambu’ doubling which we term ‘anomalous-doubled ordering’ (2.1,2.4) and ‘contour time ordering’ (2.5,2.8), the latter being marked by a bar under the doubled fields \( \bar{\Psi}_p(t_p) (\bar{\Psi})_p \) (2.5), \( \bar{\Psi}'_p(t_p) (\bar{\Psi}')_p \) (2.7), \( \bar{\Psi}''_p(t_p) (\bar{\Psi}'')_p \) (2.9). The ‘anomalous-doubled’ ordering (2.1,2.4) groups the fields \( \bar{\Psi}_p(t_p) = (\psi_{X,p=\mp}(t) \) (a = 1) \( \psi_{X,p=\mp}(t) \) (a = 2) \) according to complex conjugation regardless of the branches of contour time \( t_p=\pm \) whereas the ‘contour time ordering’ (2.5,2.8) comprises fields of identical contour metric sign \( \eta_p \), regardless of complex conjugation. Aside from the ‘equal time’ and the hermitian conjugation of ‘equal time’ ‘Nambu’ doubled fields \( \bar{\Psi}_p(t_p) \) \( \bar{\Psi}_p(t_p) \) (2.1,2.2) \( \bar{\Psi}_p(t_p) \) (2.8), one has also to introduce time shifted versions of anomalous doubled fields \( \bar{\Psi}_p(t_p) \) \( \bar{\Psi}_p(t_p) \) (2.3,2.4), \( \bar{\Psi}_p(t_p) \) (2.8), \( \bar{\Psi}_p(t_p) \) (2.2,2.8) for the exact proper sequence of time steps within the considered ‘quantum’ problem which is represented by the coherent state path integrals of normal ordered :

\[
\text{hermitian} \text{– conjugation} \text{'} of \text{’equal time’}, \text{anomalous–doubled field}:
\]

\[
(\hat{n})_{a_1}^\dagger \bar{\Psi}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}^+(t) \\
\bar{\psi}_{X,-}^+(t)
\end{array} \right)_{a_1}^T ; \quad (\hat{n})_{a_2}^\dagger \bar{\Psi}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}^- (t) \\
\bar{\psi}_{X,-}^- (t)
\end{array} \right)_{a_2}^T
\]

\[
\text{contour time ordering}:
\]

\[
t_p = \Delta t \frac{1-\eta_p}{2}, \ldots, T_0 - \Delta t \frac{1+\eta_p}{2} ;
\]

(1) : ‘equal time’, anomalous-doubled field :

\[
\bar{\Psi}^{"(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{a_1}^T ; \quad \bar{\Psi}^{"(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}^-(t) \\
\bar{\psi}_{X,-}^-(t)
\end{array} \right)_{a_2}^T
\]

(2) : ‘time shifted’ \( \Delta t_p \), anomalous-doubled field denoted by ‘"’:

\[
\bar{\Psi}^{\dagger(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{a_1}^T + \Delta t \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{a_2}^T
\]

(3) : ‘hermitian–conjugation’ ‘\( \dagger \)’ with ‘time shift correction’ \( \Delta t_p \) in the complex part :

\[
\bar{\Psi}^{\dagger(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{a_1}^T - \Delta t \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{a_2}^T
\]

(4) : ‘contour time ordering’ :

\[
t_0 = \Delta t \frac{1-\eta_p}{2}, \ldots, T_0 - \Delta t \frac{1+\eta_p}{2} ;
\]

(1) : ‘equal time’, contour time doubled field :

\[
\bar{\Psi}^{(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{p=+}^T ; \quad \bar{\Psi}^{(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{p=-}^T
\]

(2) : ‘hermitian–conjugation’ ‘\( \dagger \)’ of ‘equal time’, contour time doubled field :

\[
\bar{\Psi}^{(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{p=+}^T ; \quad \bar{\Psi}^{(1/2)}_p(t_p) = \left( \begin{array}{c}
\bar{\psi}_{X,+}(t) \\
\bar{\psi}_{X,-}(t)
\end{array} \right)_{p=-}^T
\]
2 Ensemble Averages in Model I and II for a Normal-Ordered Hamiltonian

\[ (1) \quad \text{'}time shifted' \ \Delta t_p, \ \text{contour time doubled field} ' \wedge' : \]
\[
\Psi^{a(1/2)} \left( t_p(\pm) \right) = \begin{pmatrix}
\psi^{-}(t), & \psi^{+}(t + \Delta t) ; & \psi^{+}(t), & \psi^{-}(t - \Delta t) \n\end{pmatrix}^T;
\]

\[ \text{contour time ordering} \]

\[ (2) \quad \text{'}hermitian-conjugation' \ ^{\dagger} \text{'} with 'time shift correction' \ \Delta t_p \ \text{in the complex part} : \]
\[
\Psi^{a(1/2)} \left( t_p(\pm) \right) \overset{'}{\rightarrow} \psi \Psi^{a(1/2)} \left( t_p(\pm) \right) = \begin{pmatrix}
\psi^{+}(t + \Delta t), & \psi^{-}(t) ; & \psi^{-}(t - \Delta t), & \psi^{+}(t) \n\end{pmatrix} .
\]

The Gaussian distributions (1.4.15) with random potentials \( V_I(x), \ V_{II}(x,t) \) lead to the averaged coherent state path integrals \( Z_I[\mathcal{B}], \ Z_{II}[\mathcal{B}] \) which are composed of the same functional part (2.11) with the doubled source fields \( J_{\psi;x}(t_p) \) and source matrices \( \tilde{J}^a_{\psi;x}(t_p) \) (2.12) and source matrices \( \tilde{J}^a_{\psi;x}(t_p) \) (2.13) \( (\Delta t_p = \eta_p \Delta t, \ \Delta t_q = \eta_q \Delta t, \ \Delta t > 0) \)

\[
\overline{Z_I[\mathcal{B}]} = \left\langle F[\psi^*, \psi; J, \dot{J}; \mathcal{B}] \right\rangle \exp \left\{ - \frac{R^2 \Omega_{\text{max}}}{2h^2 \mathcal{A}_x} \sum_x \left( \int_C dt_p \ \psi^{\dagger}_x(t_p + \Delta t_p) \ \psi_x(t_p) \left( \int_C dt' \ \psi_x^{\dagger}(t' + \Delta t') \ \psi_x(t') \right) \right) \right\}.
\]

\[
\overline{Z_{II}[\mathcal{B}]} = \left\langle F[\psi^*, \psi; J, \dot{J}; \mathcal{B}] \right\rangle \exp \left\{ - \frac{R^2}{2h^2} \int_0^{+T} dt \sum_{p,q=\pm} \left( \psi_x^{\dagger}(t_p + \Delta t_p) \ \eta_p \ \psi_x(t_p) \right) \left( \psi_x^{\dagger}(t_q + \Delta t_q) \ \eta_q \ \psi_x(t_q) \right) \right\} \left\langle F[\psi^*, \psi] \right\rangle.
\]

\[
J_{\psi;x}^{a(1/2)} \left( t_p(\pm) \right) = \begin{pmatrix}
\dot{J}_{\psi;x}^{(1)}(t_+), & \dot{J}_{\psi;x}^{(2)}(t_-) ; & \dot{J}_{\psi;x}^{(1)}(t_-), & \dot{J}_{\psi;x}^{(2)}(t_+) \n\end{pmatrix}^T.
\]

\[
\left\langle \text{ordering for anomalous-doubled source field} \right\rangle \quad \text{'} : \quad (2.12)
\]

\[
J_{\psi;x}^{ab} \left( t_p \right) = \begin{pmatrix}
0 & 0 & \dot{J}_{\psi;x}^{(1)}(t_+); & 0 \\
0 & 0 & \dot{J}_{\psi;x}^{(2)}(t_-); & 0 \\
\dot{J}_{\psi;x}^{(1)}(t_+); & 0 & 0 & 0 \\
\dot{J}_{\psi;x}^{(2)}(t_-); & 0 & 0 & 0 
\end{pmatrix};
\]

\[
\left\langle \text{ordering for anomalous-doubled source matrix of pair condensates} \right\rangle \quad \text{'} : \quad (2.13)
\]

\[
\tilde{J}_{\psi;x}^{ab} \left( t_p \right) = \begin{pmatrix}
0 & 0 & \dot{J}_{\psi;x}^{(1)}(t_+); & 0 \\
0 & 0 & \dot{J}_{\psi;x}^{(2)}(t_-); & 0 \\
\dot{J}_{\psi;x}^{(1)}(t_+); & 0 & 0 & 0 \\
\dot{J}_{\psi;x}^{(2)}(t_-); & 0 & 0 & 0 
\end{pmatrix};
\]

\[
\text{I} \quad \text{'} : \quad \text{J}^{\tilde{J}_{\psi;x}}(t_p) = \delta_{pq} \ \eta_p \ \delta(t_p - t'_q) \ \delta_{x,\bar{x}} \ \tilde{J}_{\psi;x}^{ab}(t_p).
\]

\[
\text{II} \quad \text{'} : \quad \text{J}^{\tilde{J}_{\psi;x}}(t_p) = \delta_{pq} \ \eta_p \ \delta(t_p - t'_q) \ \delta_{x,\bar{x}} \ \tilde{J}_{\psi;x}^{ab}(t_p).
\]
2.1 Precise time steps with shifts \( \Delta t_p \) of the complex conjugated fields \( \psi^*_x(t_p + \Delta t_p) \)

The one-particle part of (2.11) is listed in relations (2.16, 2.17) with Kronecker-deltas of time for precise time steps following from properly normal ordered Hamiltonians 'I' and 'II'. We also give a 'laxed' kind of the one-particle part in relations (2.16, 2.17) which is appropriate for a classical approximation, as a first order variation of fields, within the exponentials of the path integrals, but fails to result into the correct quantum expressions if integrations of coherent state fields and their complex conjugates have to be performed.

\[
\int \mathcal{D}x \sum_x \psi^*_x(t_p + \Delta t_p) \hat{H}_p(\vec{x}, t_p) \psi_x(t_p) =
\]

\[
= \int \mathcal{D}x \sum_x \left[ -i \hbar \psi^*_x(t_p + \Delta t_p) \frac{1}{\Delta t_p} \left( \hat{h}(\vec{x}) - i \hat{\varepsilon}_p \right) \psi_x(t_p) \right]
\]

\[
= \int \mathcal{D}x \sum_x \psi^*_x(t'_p + \Delta t'_p) \delta_{qp} \eta_{q} \left( -i \hbar \frac{\delta \rho_{t'_p} - \delta \rho_{t_p} + \Delta \rho_{t_p} \delta \rho_{t'_p + \Delta t'_p}}{\delta \rho_{t_p}} + \left( \hat{h}(\vec{x}) - i \hat{\varepsilon}_p \right) \delta \rho_{t'_p} \psi_x(t_p) \right)
\]

\[
= \int \mathcal{D}x \sum_x \psi^*_x(t'_p + \Delta t'_p) \delta_{qp} \eta_{q} \left( -i \hbar \frac{\delta \rho_{t'_p} - \delta \rho_{t_p} + \Delta \rho_{t_p} \delta \rho_{t'_p + \Delta t'_p}}{\delta \rho_{t_p}} + \left( \hat{h}(\vec{x}) - i \hat{\varepsilon}_p \right) \delta \rho_{t'_p} \psi_x(t_p) \right)
\]

\[
\hat{H}_p(\vec{x}, t_p) = -\hat{E}_p - i \hat{\varepsilon}_p + \hat{\Delta}(\vec{x}) = -i \hbar \frac{\partial}{\partial t_p} - i \hat{\varepsilon}_p + \frac{\vec{p}^2}{2m} + u(\vec{x}) - \mu_0 ;
\]

\[
\hat{H}_{x, x'}(t_p, t'_q) = \hat{H}_p(t_p, \vec{x}) \delta_{pq} \eta_p (t_p - t'_q) \delta_{x, x'} ; \quad \varepsilon_p = \eta_p \in \{ \varepsilon > 0 ; \ \eta \pm = \pm 1 \}.
\]

According to the anomalous doubling, one can attain order parameter matrices (2.18-2.21) which are nonlocal in time for case 'I' of static disorder and which are local in time for case 'II' of dynamic disorder. This corresponds to the described picture of a 'return' probability in Anderson localization for case 'I' without an additional white-noise distribution in time.

We can use the given order parameter matrices (2.18-2.21) with their various forms for the anomalous doubled self-energy with 'equal time' doubling of fields (2.18-2.20) and for the 'Nambu'-terms extended density matrices with 'time shifted' doubling of fields (2.19-2.21). However, we can simplify to the spatially local case due to the contact disorder of spatial Kronecker-deltas with the second moments of the two Gaussian distributions (2.31, 2.35).

\[
\hat{\Phi}^{(ab)}_{x, x'; q}(t, t') = \Psi_{\vec{x}; q}(t_p) \otimes \Psi_{\vec{x}; q}(t'_q) \rightarrow \text{anomalous doubled self-energy of static disorder} ;
\]

\[
\hat{\Phi}^{(ab)}_{x, x'; q}(t, t') = \hat{\Psi}_{\vec{x}; q}(t_p) \otimes \hat{\Psi}_{\vec{x}; q}(t'_q) \rightarrow \text{density matrices with anomalous terms for static disorder} ;
\]

\[
\hat{\Phi}^{(ab)}_{x, x'; q}(t) = \Psi_{\vec{x}; q}(t_p) \otimes \Psi_{\vec{x}; q}(t'_q) \rightarrow \text{anomalous doubled self-energy of dynamic disorder} ;
\]

\[
\hat{\Phi}^{(ab)}_{x, x'; q}(t) = \hat{\Psi}_{\vec{x}; q}(t_p) \otimes \hat{\Psi}_{\vec{x}; q}(t'_q) \rightarrow \text{density matrices with anomalous terms for dynamic disorder} .
\]

In order to achieve the precise, subsequent time steps at the integration boundaries, we have to introduce a modified time contour integration '[\mathcal{C}', \int \mathcal{D}t_p \ldots '] which is extended to the parameter end points \( t' = -\Delta t \frac{1+\eta_p}{2} \) and \( \hat{T}_p = T_0 + \Delta t \frac{1+\eta_p}{2} \). We thus take integrations with the extended doubled coherent state fields which are restricted to a single field (\( a \equiv 1 \)) at the values \( t' = -\Delta t \frac{1+\eta_p}{2} \) and \( \hat{T}_p = T_0 + \Delta t \frac{1+\eta_p}{2} \). One therefor has the particular field values

\[
\hat{\Psi}_{\vec{x}, +}^{(a\equiv 1)}(t = -\Delta t) = \psi^*_{\vec{x}, +}(t = 0) ; \quad \hat{\Psi}_{\vec{x}, -}^{(a\equiv 1)}(t = 0) = \psi^*_{\vec{x}, -}(t = 0) ;
\]

\[
\hat{\Psi}_{\vec{x}, +}^{(a\equiv 1)}(t = -\Delta t) = \psi^*_{\vec{x}, +}(t = 0) ; \quad \hat{\Psi}_{\vec{x}, -}^{(a\equiv 1)}(t = 0) = \psi^*_{\vec{x}, -}(t = 0) ;
\]

\[
\hat{\Psi}_{\vec{x}, +}^{(a\equiv 1)}(t = +T_0) = \psi^*_{\vec{x}, +}(t = +T_0) ; \quad \hat{\Psi}_{\vec{x}, -}^{(a\equiv 1)}(t = T_0 + \Delta t) = \psi^*_{\vec{x}, -}(t = +T_0) ;
\]

\[
\psi^*_{\vec{x}, +}(t = +T_0) = \psi^*_{\vec{x}, -}(t = +T_0) ; \quad \psi^*_{\vec{x}, +}(t = +T_0) = \psi^*_{\vec{x}, -}(t = +T_0) ,
\]

which have to be combined with anomalous doubled matrices \( \hat{M}^{ab}_{\vec{x}, \vec{x'}}(t'_q, t_p) \) (2.20) for the proper exact sequence of time steps in order to obtain the correct propagation with the ensemble averaged generating functions (2.9, 2.11). We list the various notations (2.20, 2.27) of extended contour time integrations for the presented cases of disorder which are
straightforwardly generalized from the solely hermitian cases of interactions with the already described precise time steps and intervals in one of our earlier references

\[
\int_{\mathcal{C}} dt_p dt'_q \sum_{\bar{x},\bar{x}'} a, b = 1, 2 \sum_{\bar{x},\bar{x}' \sigma, \bar{p}, \bar{q} = \pm} \tilde{\Psi}^b_{\bar{x}, \bar{x}'}(t'_q) \tilde{M}^{ba}_{\bar{x}, \bar{x}'}(t'_q, t_p) \tilde{\Psi}^a_{\bar{x}'}(t_p) = \int_0^{+T} dt \int_0^{+T'} dt' \sum_{\bar{x}, \bar{x}' p, q = \pm} \tilde{\Psi}^b_{\bar{x}, \bar{x}'}(t') \tilde{M}^{ba}_{\bar{x}, \bar{x}'}(t, t') \eta_q \tilde{\Psi}^a_{\bar{x}'}(t).
\]

Concerning presentation and notation, the case 'II' of dynamic disorder turns out to have simpler relations than the case 'I' because the static disorder involves the combination of two distinct time parameters \( t, t' \) in the self-energy matrices as in the already given order parameter matrices \( \tilde{\Phi}^{(1)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t, t') \) (2.18) and \( \tilde{\Phi}^{(2)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t, t') \) (2.19). The dynamic disorder with a white-noise time distribution restricts to a single time parameter in the self-energy matrices, as in the order parameter matrices \( \tilde{\Phi}^{(1)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t) \) (2.20), \( \tilde{\Phi}^{(2)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t) \) (2.21), but still incorporates the two metric sign labels \( p, q = \pm \) as additional field degree of freedom for a 'disorder' quasi-particle. However, both cases 'I', 'II' of disorder allow for analogous treatment of HST's and coset decompositions of nonlinear sigma models, as one only considers the relevant reduction to stationary time states of fields with a single frequency '\( \omega \)' in case 'I' of static disorder.

So exclusively stationary states of time in case 'I' result in the analogous transformations and derivations to nonlinear sigma models of case 'II', if one replaces the single time parameter '\( t' \)' in \( \tilde{\Phi}^{(1)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t) \) (2.20), \( \tilde{\Phi}^{(2)ab}_{\bar{x}, \bar{x}, \bar{q}, \bar{q}'}(t) \) (2.21) with a single frequency parameter '\( \omega \)' in case 'I' of static disorder. After the corresponding Fourier transformation to a frequency (or energy) contour \( \mathcal{C}_\omega \), according to (2.28) the generating function \( Z_I[\hat{\beta}] \) (2.30) which contains two independent, non-stationary energy contour integrations '\( \hbar \omega_p \)' and '\( \hbar \omega_q \)' in the ensemble averaged Gaussian part. As we reduce to stationary states or to a single frequency parameter \( \omega = \omega' \), we introduce the approximation (2.31) with various contour labels \( p, q \) very similar as in \( Z_{II}[\hat{\beta}] \) (2.10)

\[
\Omega_{\text{max}} = \frac{1}{\Delta t}; \quad 0 < t_p < +T_0; \quad T_0 = T_{\text{max}};
\]

\[
0 < \omega_p < -\Omega_0; \quad \Omega_0 = \Omega_{\text{max}} = \frac{1}{\Delta t}; \quad \Delta \omega = \frac{2\pi}{T_{\text{max}}}; \quad N_\omega = T_{\text{max}}/\Delta t;
\]

\[
\int_{\mathcal{C}_\omega} \frac{d\omega_p}{\Delta \omega} \cdots = \cdots \int_{-\Delta \omega}^{+\Omega_0} \frac{d\omega_p}{2\pi T_{\text{max}}} \cdots + \int_{+(\Omega_0 + \Delta \omega)}^{+(\Omega_0 + \Delta \omega)} \frac{d\omega_p}{2\pi T_{\text{max}}} \cdots = \int_{-\Delta \omega}^{+\Omega_0} \frac{d\omega_p}{2\pi T_{\text{max}}} \cdots + \int_{0}^{+(\Omega_0 + \Delta \omega)} \frac{d\omega_p}{2\pi T_{\text{max}}} \cdots - \int_{0}^{+(\Omega_0 + \Delta \omega)} \frac{d\omega_p}{2\pi T_{\text{max}}} \cdots = \sum_{p=\pm} \int_{0}^{+(\Omega_0 + \Delta \omega)} \frac{d\omega_p}{2\pi T_{\text{max}}} \eta_p \cdots;
\]

\[
\mathcal{Z}_I[\hat{\beta}] = \left\langle F[\psi^*, \psi; \eta, \bar{J}; \bar{\psi}, \bar{\eta}] \times \exp \left\{ \int \frac{R^2}{2\hbar^2 N_\xi} \sum_{\xi} \left( \int \frac{d\omega_p}{2\pi T_{\text{max}}} \right) \psi_{\bar{x}, \bar{x}'}^*(\omega_p) e^{i \omega_p \Delta_t \psi_{\bar{x}}(\omega_p)} \left( \int \frac{d\omega'_q}{2\pi T_{\text{max}}} \right) e^{i \Delta t \eta_q \left( \psi_{\bar{x}, \bar{x}'}(\omega_q) \right)} \right\} \right\rangle [\psi^*, \psi];
\]

\[
\mathcal{Z}_I[\hat{\beta}] \approx \left\langle F[\psi^*, \psi; J', \bar{J}; \bar{\psi}; \bar{\psi}] \times \exp \left\{ \int \frac{R^2}{2\hbar^2 N_\xi} \sum_{\xi} \left( \int \frac{d\omega_p}{2\pi T_{\text{max}}} \right) \left( \psi_{\bar{x}, \bar{x}'}^*(\omega_p) e^{i \omega_p \Delta_t \psi_{\bar{x}}(\omega_p)} \eta_p \psi_{\bar{x}, \bar{x}'}(\omega_p) \right) \right\} \right\rangle [\psi^*, \psi].
\]

The 'time-shift' correction of complex conjugated fields implies additional phases \( e^{i \omega \Delta t_p} \) in the static disorder case whose consideration also leads to the exact, proper frequency or energy steps within the propagation of the time development
operators as in case 'II' of dynamic disorder. It is even possible to substitute the parameter $R_{II}^2$, according to relation (2.32), and the contour time integrals by contour frequency integrals $\int \frac{\omega_p}{(2\pi/T_{max})} \ldots$ (2.29) so that the nonlinear sigma model of case 'II' straightforwardly generalizes to case 'I' of static disorder.

$$R_{II}^2 \approx R_{II}^2 \frac{\alpha^2 \Omega_{max}}{N_\eta}.$$  (2.32)

In the following sections 3, 4 we can therefore concentrate on the dynamic disorder case with a single time variable in the self-energy matrix and can then transfer the result of case 'II' to the stationary case with a single frequency variable 'ω' in the self-energy matrices for case 'I' of static disorder. The approximation to a single frequency $\omega_p$ for static disorder can also be attained at very later steps of transformations to a nonlinear sigma model, e.g. as one simplifies the two-time or two-frequency dependent, nonlocal disorder self-energy matrix to a single dependence with contour frequency $\omega_p$.

3 HST for dynamic disorder and repulsive interaction with 'hinge' fields

3.1 Anomalous doubling of the one-particle part

We perform the anomalous doubling (3.1) of the bosonic fields with inclusion of the contour time metric $\eta_p$, due to the ensemble average with a dynamic disorder. This defines a 'Nambu' metric tensor $\tilde{K}_{pq}^{ab}$ for the anomalous doubled fields $\tilde{\Psi}_{x,a}^{ab}(t)$, $\tilde{\Psi}_{x,p}^{ab}(t)$ whose dyadic product determines the density matrices $\tilde{R}_{x,pq}^{ab}(t)$ (3.3) with anomalous extended parts in the off-diagonal blocks $a \neq b$. The quartic disorder interaction of bosonic fields is therefore equivalent to the trace relation (3.4) with the density matrices $\tilde{R}_{x,pq}^{ab}(t)$ (3.3) which are modified by the 'Nambu' metric tensor (3.2) with a final multiplication of a factor $\frac{1}{4}$ corresponding to two dyadic product operations of (3.1) with anomalous doubling of relation (2.21)

$$\tilde{\Psi}_{x,a}^{ab}(t) \tilde{\Psi}_{x,b}^{ab}(t) \eta_p \tilde{\Psi}_{x,p}^{ab}(t) = \frac{1}{2} \left( \tilde{\Psi}_{x,a}^{ab}(t) \eta_p \tilde{\Psi}_{x,b}^{ab}(t) + \tilde{\Psi}_{x,b}^{ab}(t) \eta_p \tilde{\Psi}_{x,a}^{ab}(t) \right) = \frac{1}{2} \tilde{\Psi}_{x,a}^{ab}(t) \tilde{K}_{pq}^{ab} \tilde{\Psi}_{x,b}^{ab}(t); \quad (3.1)$$

$$\tilde{R}_{x}^{ab}(t) = \tilde{R}_{x,pq}^{ab}(t) = \tilde{\Psi}_{x,a}^{ab}(t) \otimes \tilde{\Psi}_{x,b}^{ab}(t) = \tilde{\Psi}_{x,a}^{ab}(t) \otimes \tilde{\Psi}_{x,b}^{ab}(t) \quad (3.3)$$

This anomalous doubling has also to be taken for the one-particle part (2.15) so that we accomplish relation (3.5) with the anomalous doubled fields $\tilde{\Psi}_{x,a}^{ab}(t)$, $\tilde{\Psi}_{x,b}^{ab}(t)$ and the doubled one-particle operator $\tilde{R}_{x,pq}^{ab}(t)$ (3.6) whose lower
block diagonal part $\tilde{H}_{\vec{x},\vec{p}}'(t', t)$ follows by transposition from the upper block diagonal part $\tilde{H}_{\vec{x},\vec{p}}(t', t)$.

Corresponding to propagation with the exact, precise time steps, we outline these two one-particle parts in Eqs. (3.8), with the exact Kronecker deltas of time and also describe a ‘laxed’ kind of one-particle operators in (3.9), only applicable in classical approximations or equations

$$\int_{\mathcal{C}} dt_p \sum_{\vec{x}} \psi^*_p (t_p + \Delta_p) \hat{H}_p(\vec{x}, t_p) \psi_p (t_p) = \int_{\mathcal{C}} dt_p \sum_{\vec{x}} \psi^*_p (t_p) \hat{H}_p^T(\vec{x}, t_p) \psi^*_p (t_p + \Delta_p) =$$

$$= \int_{\mathcal{C}} dt_p dt'_q \sum_{\vec{x}, \vec{p}} \frac{1}{2} \tilde{g}_{\vec{x},\vec{p}}^b (t'_q) \tilde{H}_{\vec{x},\vec{p}}^a (t'_q, t_p) \tilde{g}_{\vec{x},\vec{p}}^b (t'_q) ;$$

$$\tilde{H}_{\vec{x},\vec{p}}'(t'_q, t_p) \equiv \text{diag} \left( \tilde{H}_{\vec{x},\vec{p}}(t'_q, t_p) ; \tilde{H}_{\vec{x},\vec{p}}'(t'_q, t_p) \right) ;$$

$$\tilde{H}_{\vec{x},\vec{p}}(t'_q, t_p) = \left( -i \hbar \frac{\delta(t'_q - \Delta_p) - \delta(t'_q - \Delta_p)}{\Delta_p} + \left( \tilde{h}(\vec{x}') - \iota \tilde{\varepsilon}_p \right) \delta(t'_q - t_p) \right) \delta_{qp} \tilde{\eta} \delta_{\vec{x},\vec{p}} ;$$

$$\tilde{H}_{\vec{x},\vec{p}}'(t'_q, t_p) = \left( \tilde{H}_{\vec{x},\vec{p}}(t'_q, t_p) \right)^T = \left( -i \hbar \frac{\delta(t'_q - \Delta_p) - \delta(t'_q - \Delta_p)}{\Delta_p} + \left( \tilde{h}(\vec{x}') - \iota \tilde{\varepsilon}_p \right) \delta(t'_q - t_p) \right) \delta_{qp} \tilde{\eta} \delta_{\vec{x},\vec{p}} ;$$

$$\tilde{H}_{\vec{x},\vec{p}}'(t'_q, t_p) \equiv \text{diag} \left( \tilde{H}_+(\vec{x}', t'_q) - \tilde{H}_-(\vec{x}', t'_q) ; \tilde{H}_+^T(\vec{x}', t'_q) - \tilde{H}_-^T(\vec{x}', t'_q) \right) \delta_{\vec{x},\vec{p}} \delta_{qp} \delta(t'_q - t_p) ;$$

$$\tilde{H}_p(\vec{x}, t_p) = -i \hbar \frac{\partial}{\partial t_p} - \iota \tilde{\varepsilon}_p + \frac{\vec{p}^2}{2m} + u(\vec{x}) - \mu_0 ; \quad \tilde{H}_p^T(\vec{x}, t_p) = +i \hbar \frac{\partial}{\partial t_p} - \iota \tilde{\varepsilon}_p + \frac{\vec{p}^2}{2m} + u(\vec{x}) - \mu_0 .$$

### 3.2 Anomalous doubled self-energies and their coset decomposition

In the following we convey the results of our earlier articles to the presented case 'II' of dynamic disorder with the additional contour time metric tensor $\tilde{\eta}_p$ and have to use a modified 'Nambu' metric tensor $\tilde{K}_{pq}^{ab}, \tilde{K}_{pq}^{ab}$ (3.14) which changes the anomalous parts in the self-energy to anti-hermitian relations

$$\tilde{K}_{pq}^{ab} = \left( \tilde{\eta}_p^{11} \right)_{ab}^{pq} \equiv \delta_{ab} \tilde{\eta}_p \tilde{\eta}_p \tilde{\eta}_p^{-1} = \delta_{ab} \tilde{\eta}_p \tilde{\eta}_p \tilde{\eta}_p^{-1} \quad \text{diag} \left\{ \begin{array}{ll} +1 & -1 \\ -1 & +1 \end{array} \right\} ;$$

$$\tilde{K}_{pq}^{ab} = \delta_{ab} \text{ diag} \left\{ \begin{array}{ll} +1 & -1 \\ -1 & +1 \end{array} \right\} .$$

The final HST is achieved by one half with the self-energy density variable $\sigma_{R_{ij}}^{(0)}(\vec{x}, t)$ (3.13) and by one half with the 'Nambu' parts $\tilde{\Sigma}_{\vec{x},pq}^{11}(t), \tilde{\Sigma}_{\vec{x},pq}^{22}(t)$ (3.13) of the anomalous doubled self-energy matrix $\tilde{\Sigma}_{\vec{x},pq}^{ab}(t)$ (3.14) whose block diagonal, hermitian self-energy density parts $\tilde{\Sigma}_{\vec{x},pq}^{11}(t), \tilde{\Sigma}_{\vec{x},pq}^{22}(t)$ (3.15) are related by transposition and are only used as 'hinge' fields in a SSB with a coset decomposition. The off-diagonal, 'Nambu' matrix parts $\tilde{\Sigma}_{\vec{x},pq}^{11}(t), \tilde{\Sigma}_{\vec{x},pq}^{22}(t)$ (3.16) of $\tilde{\Sigma}_{\vec{x},pq}^{ab}(t)$ (3.14) are symmetric matrices and are related by hermitian conjugation. One thus acquires following real and complex parameters $\delta B_{\vec{x},+}(t), \delta B_{\vec{x},-}(t)$ and $\delta B_{\vec{x},+}(t), \delta B_{\vec{x},-}(t) = \delta B_{\vec{x},-}(t)$ and the solely complex field variables $\delta c_{\vec{x},+}(t), \delta c_{\vec{x},-}(t), \delta c_{\vec{x},-}(t) = \delta c_{\vec{x},+}(t)$ in the corresponding anomalous doubled part (3.16) of the total self-energy (3.14)

$$\sigma_{R_{ij}}^{(0)}(\vec{x}, t) \in \mathbb{R} ; \quad \delta \Sigma_{\vec{x},pq}^{ab}(t) ; \delta \Sigma_{\vec{x},pq}^{11}(t) ; \delta \Sigma_{\vec{x},pq}^{22}(t) ; \delta \Sigma_{\vec{x},pq}^{21}(t) \text{ in analogy to } \tilde{K}_{pq}^{ab}(t) ;$$

$$\delta \Sigma_{\vec{x},pq}^{11}(t) = \left( \begin{array}{cc} \delta B_{\vec{x},+}(t) & \delta B_{\vec{x},-}(t) \\ \delta B_{\vec{x},-}(t) & \delta B_{\vec{x},+}(t) \end{array} \right) ;$$

$$\delta \Sigma_{\vec{x},pq}^{21}(t) = \left( \begin{array}{cc} \delta B_{\vec{x},+}(t) & \delta B_{\vec{x},-}(t) \\ \delta B_{\vec{x},-}(t) & \delta B_{\vec{x},+}(t) \end{array} \right) ;$$

$$\delta \Sigma_{\vec{x},pq}^{22}(t) = \left( \begin{array}{cc} \delta B_{\vec{x},+}(t) & \delta B_{\vec{x},+}(t) \\ \delta B_{\vec{x},+}(t) & \delta B_{\vec{x},+}(t) \end{array} \right) ;$$

$$\delta \Sigma_{\vec{x},pq}^{22}(t) = \delta \Sigma_{\vec{x},pq}^{11}(t) ; \delta B_{\vec{x},+} \in \mathbb{R} ; \quad \delta B_{\vec{x},-} \in \mathbb{C} ; \delta B_{\vec{x},+} = \delta B_{\vec{x},-} ;$$
\[ \delta \hat{\Sigma}^{12}_{x,pp}(t) = \begin{pmatrix} \delta c_{x,++}(t) & \delta c_{x,+-}(t) \\ \delta c_{x,+-}(t) & \delta c_{x,--}(t) \end{pmatrix} ; \quad \delta \hat{\Sigma}^{21}_{x,pp}(t) = \begin{pmatrix} \delta c^*_x(t) & \delta c_{x,+-}(t) \\ \delta c_{x,+-}(t) & \delta c_{x,--}(t) \end{pmatrix} ; \]  

\[ \delta \Sigma^{ab}_{x,pp}(t) = \delta \Sigma^{ab,\bar{a}\bar{b}}_{x,pp}(t) = \delta \Sigma^{ab}_{x,pp}(t) ; \quad \delta c_{x,+-}(t) \in \mathbb{C} ; \quad \delta c_{x,--}(t) = \delta c_{x,+-}(t). \]  

We combine the self-energy density variable \( \sigma^{(0)}_{R_{II}}(\vec{x},t) \) and the anomalous doubled self-energy matrix \( \delta \Sigma^{ab}_{x,pp}(t) \) into block diagonal parts \( \Sigma^{0}_{x,pp}(t) \) and introduce the modified self-energy matrix \( \delta \Sigma^{ab}_{x,pp}(t) \) with 'Nambu' metric tensor \( \tilde{K} \) and anti-hermitian related, off-diagonal, anomalous doubled blocks \( \delta \Sigma^{ab,\bar{a}\bar{b}}_{x,pp}(t) = \iota \delta \Sigma^{ab}_{x,pp}(t) \). This allows to perform a coset decomposition into densities and bosonic parts according to \( \text{Sp}(4) \rightarrow \text{Sp}(4)/U(2) \otimes U(2) \)  

\[ \left( \begin{array}{c} \Sigma^{11}_{x,pp}(t) \\ \delta \Sigma^{12}_{x,pp}(t) \\ -\Sigma^{22}_{x,pp}(t) \end{array} \right) = \sigma^{(0)}_{R_{II}}(\vec{x},t) \tilde{K} + \begin{pmatrix} \delta \Sigma^{0}_{x,pp}(t) \\ \delta \Sigma^{12}_{x,pp}(t) \\ -\delta \Sigma^{22}_{x,pp}(t) \end{pmatrix} ; \]  

\[ \delta \Sigma^{ab}_{x,pp}(t) \tilde{K} = \begin{pmatrix} \delta B_{x,++}(t) & \delta B_{x,+-}(t) \\ \delta B_{x,+-}(t) & \delta B_{x,--}(t) \end{pmatrix} \begin{pmatrix} \delta c_{x,++}(t) & \delta c_{x,+-}(t) \\ \delta c_{x,+-}(t) & \delta c_{x,--}(t) \end{pmatrix} \]  

As one includes a further, derived 'Nambu' metric tensor \( \hat{I}^{ab}_{pq} \) so that various kinds of self-energy terms with hermitian or anti-hermitian off-diagonal block parts can be transformed into each other \( \tilde{K} \), one finally succeeds into the coset decomposition \( \Sigma^{ab}_{D,pp}(\vec{x},t) \tilde{K} \) similar to \( \sigma^{(0)}_{R_{II}}(\vec{x},t) \) and coset matrices \( \tilde{K} \) consit of the generator \( \hat{Y}^{ab}_{pq}(\vec{x},t) \) with sub-generators \( \hat{X}_{pq}(\vec{x},t) \), \( -\hat{X}^*_pq(\vec{x},t) \) as the 'Nambu' or anomalous doubled field degrees of freedom  

\[ \hat{I}^{ab}_{pq} = \delta_{ab} \delta_{pq} \text{ diag}\{+1, +1 ; +i, +i\} ; \]  

\[ \Sigma^{ab}_{x,pp}(t) = \hat{I} \begin{pmatrix} \Sigma^{11}_{x,pp}(t) \\ \delta \Sigma^{12}_{x,pp}(t) \\ -\Sigma^{22}_{x,pp}(t) \end{pmatrix} \hat{I}^{ab}_{pq} \]  

\[ \Sigma^{ab}_{x,pp}(t) \tilde{K} = \sigma^{(0)}_{R_{II}}(\vec{x},t) \hat{1}_{4 \times 4} + \delta \Sigma^{ab}_{x,pp}(t) \]  

\[ \hat{Y}^{ab}_{pq}(\vec{x},t) = \begin{pmatrix} (\hat{X}_{pq}(\vec{x},t))^{11}_{pq} \\ (\hat{X}^*_pq(\vec{x},t))^{22}_{pq} \end{pmatrix} ; \]  

\[ \hat{X}_{pq}(\vec{x},t) = \begin{pmatrix} -\delta c_{D,++}(\vec{x},t) \\ \delta c_{D,--}(\vec{x},t) \end{pmatrix} ; \quad -\eta_p \hat{X}^*_pq(\vec{x},t) \eta_q = \begin{pmatrix} \delta c_{D,++}(\vec{x},t) & \delta c_{D,--}(\vec{x},t) & \delta c_{D,+-}(\vec{x},t) \end{pmatrix}. \]  

It is further possible to diagonalize the various block diagonal density parts \( \delta \Sigma^{pq}_{D,pp}(\vec{x},t) \tilde{K} \) as in \( \Sigma^{ab}_{D,pp}(\vec{x},t) \) and the various 'Nambu' generators \( \hat{Y}^{ab}_{pq}(\vec{x},t), \hat{X}_{pq}(\vec{x},t) \) as in \( \hat{X}^*_pq(\vec{x},t) \). In the case of the density parts we have the
diagonal eigenvalue elements (3.29) with diagonalizing ‘rotation’ matrices (3.30) so that the lower block diagonal ‘22’ part is related by transposition to the upper ‘11’ part

$$\delta \tilde{\Sigma}^{aa}_{D,pq}(\vec{x},t) \bar{K}^{aa} = \hat{Q}^{-1}_{pq}^{aa}(\vec{x},t) \delta \hat{\lambda}^{aa}_{p}(\vec{x},t) \hat{Q}^{aa}_{pq}(\vec{x},t)$$

$$\delta \hat{\lambda}^{aa}_{p}(\vec{x},t) = \text{diag}\{-(\delta \tilde{\lambda}^{+}_{p}(\vec{x},t), \delta \tilde{\lambda}^{-}_{p}(\vec{x},t))\} = \delta \hat{\lambda}^{aa}_{p}(\vec{x},t) \bar{K}^{aa}_{pp};$$

$$\delta \tilde{\lambda}^{aa}_{p}(\vec{x},t) = \text{diag}\{-(\delta \tilde{\lambda}^{+}_{p}(\vec{x},t), \delta \tilde{\lambda}^{-}_{p}(\vec{x},t))\}; \quad \delta \tilde{\lambda}^{aa}_{p}(\vec{x},t) \in \mathbb{R};$$

$$\hat{Q}^{11}_{pq}(\vec{x},t) = \left( \exp\left\{ i \hat{B}_{D}(\vec{x},t) \right\} \right)_{pq} \hat{Q}^{22}_{pq}(\vec{x},t) = \left( \exp\left\{ i \hat{B}^{T}_{D}(\vec{x},t) \right\} \right)_{pq};$$

$$\hat{B}_{D}(\vec{x},t) = \begin{pmatrix} 0 & -\hat{B}^{+}_{D}(\vec{x},t) \\ \hat{B}^{+}_{D}(\vec{x},t) & 0 \end{pmatrix}; \quad \hat{Q}^{11}_{pq}(\vec{x},t) = \hat{Q}^{11}_{pq}(\vec{x},t);$$

$$\delta \Sigma^{aa}_{D,pq}(\vec{x},t) \bar{K}^{aa}_{pq} = \hat{Q}^{-1}_{pp}^{aa}(\vec{x},t) \delta \hat{\lambda}^{aa}_{p}(\vec{x},t) \hat{Q}^{aa}_{pq}(\vec{x},t); \quad \hat{Q}^{aa}_{pq}(\vec{x},t) = \left( \begin{pmatrix} 0 \\ \hat{Q}^{22}_{pq}(\vec{x},t) \end{pmatrix} \right);$$

$$\left( \delta \hat{\Sigma}^{11}_{D}(\vec{x},t) \right) = \hat{Q}^{11}_{pq}(\vec{x},t) \left( p \cdot \delta \hat{\lambda}^{aa}_{p}(\vec{x},t) \right) \hat{Q}^{11}_{pq}(\vec{x},t);$$

$$\left( \delta \hat{\Sigma}^{22}_{D}(\vec{x},t) \right) = \hat{Q}^{22}_{pq}(\vec{x},t) \left( -p \cdot \delta \hat{\lambda}^{aa}_{p}(\vec{x},t) \right) \hat{Q}^{22}_{pq}(\vec{x},t);$$

A similar diagonalization (3.30) is achieved for the off-diagonal block parts with sub-eigenvalue parts $\hat{X}_{D,pq}(\vec{x},t)$ (3.38) and ‘eigenvector’ matrices $\hat{P}^{11}_{2\times 2}(\vec{x},t)$, $\hat{P}^{22}_{2\times 2}(\vec{x},t)$ (3.39) in such a manner that the symmetry relations of (3.29) to (3.27) are still retained. This involves the complex parameters $\tau^{+}_{+}(\vec{x},t)$, $\tau^{-}_{-}(\vec{x},t)$ as eigenvalues and $\epsilon^{*}_{D,+}(\vec{x},t)$, $\epsilon^{*}_{D,-}(\vec{x},t)$ as the angular parameters of the rotation for the off-diagonal elements $\delta \epsilon^{*}_{D,+}(\vec{x},t)$, $\delta \epsilon^{*}_{D,-}(\vec{x},t)$ within $\hat{X}_{pq}(\vec{x},t)$, $-\eta_{p} \hat{X}^{\dagger}_{pq}(\vec{x},t) \eta_{q}$ (3.37)

$$\hat{Y}_{pq}(\vec{x},t) = \hat{P}^{-1}_{4\times 4}(\vec{x},t) \hat{Y}_{D:\times 4}(\vec{x},t) \hat{P}_{4\times 4}(\vec{x},t);$$

$$\hat{Y}_{D:\times 4}(\vec{x},t) = \left( \begin{pmatrix} 0 \\ \hat{X}_{D,pq}(\vec{x},t) \end{pmatrix} \right); \quad \hat{P}_{4\times 4}(\vec{x},t) = \begin{pmatrix} \hat{P}^{11}_{2\times 2}(\vec{x},t) & 0 \\ 0 & \hat{P}^{22}_{2\times 2}(\vec{x},t) \end{pmatrix};$$

$$\hat{X}_{D,pq}(\vec{x},t) = \begin{pmatrix} -\tau^{+}_{+}(\vec{x},t) \\ 0 \end{pmatrix} \hat{X}_{D,pq}(\vec{x},t); \quad -\hat{X}_{D,pq}^{\dagger}(\vec{x},t) = \begin{pmatrix} \tau^{+}_{+}(\vec{x},t) \\ 0 \end{pmatrix} \hat{X}_{D,pq}(\vec{x},t);$$

$$\hat{P}^{11}_{2\times 2}(\vec{x},t) = \hat{P}_{2\times 2}(\vec{x},t) \hat{E}_{D,2\times 2}(\vec{x},t); \quad \hat{P}^{22}_{2\times 2}(\vec{x},t) = \exp\left\{ i \hat{E}_{D,2\times 2}(\vec{x},t) \right\};$$

$$\hat{E}_{D,2\times 2}(\vec{x},t) = \hat{E}_{D,pq}(\vec{x},t) = \begin{pmatrix} 0 \\ -\epsilon^{*}_{D,+}(\vec{x},t) \end{pmatrix} \hat{E}_{D,pq}(\vec{x},t); \quad \hat{P}^{22}_{2\times 2}(\vec{x},t) = \hat{P}^{11}_{2\times 2}(\vec{x},t);$$

$$\epsilon^{*}_{D,+}(\vec{x},t) := |\epsilon^{*}_{D}(\vec{x},t)| \exp\{ i \gamma_{D}(\vec{x},t) \};$$
3.3 HST transformations with 'hinge'-fields

Eventually, we can collect the various parameters $\delta B_{\pm,\pm}(t)$, $\delta B_{\pm,-}(t)$, $\delta B_{\pm,-}(t)$, $\delta B_{\pm,\pm}(t)$ for the density parts and $\delta c_{\mp pq}(t)$, $\delta c_{\pm pq}(t)$ within the 'Nambu' terms in (3.45) in order to determine the HST (3.46).

$$\sigma^{(0)}_{R_1}(\vec{x}, t) \in \mathbb{R} \quad \text{ 'hinge' functions : } \delta \Sigma^{11}_{\pm pq}(t), \delta \Sigma^{22}_{-pq}(t)$$

(3.41)

$$\delta c^{(0)}_{\mp pq}(t) \in \mathbb{C}; \quad \delta B_{\pm,\pm}(t), \delta B_{\pm,-}(t) \in \mathbb{R}; \quad \delta B_{\pm,-}(t) \in \mathbb{C}$$

(3.42)

$$\delta \Sigma^{11}_{\pm pq}(t) = \left( \begin{array}{c} \delta B_{\pm,\pm}(t) \\ \delta B_{\pm,-}(t) \\ \delta B_{\pm,-}(t) \end{array} \right)$$

(3.43)

$$\delta \Sigma^{22}_{-pq}(t) = \left( \begin{array}{c} \delta B_{\pm,\pm}(t) \\ \delta B_{\pm,-}(t) \\ \delta B_{\pm,-}(t) \end{array} \right) \left( \begin{array}{c} \delta \Sigma^{22}_{pq}(t) \end{array} \right)^T = \delta \Sigma^{11}_{pq}(t)$$

(3.44)

$$\delta \Sigma^{12}_{\pm pq}(t) = \left( \begin{array}{c} \delta c_{\pm,\pm}(t) \\ \delta c_{\pm,-}(t) \\ \delta c_{\pm,-}(t) \end{array} \right) \left( \begin{array}{c} \delta \Sigma^{21}_{pq}(t) \end{array} \right)^T = \delta \Sigma^{12}_{\pm pq}(t)$$

(3.45)

The HST of a quartic interaction of fields can be taken in various manners; we consider the case where one half of the quartic interaction (with correspondingly reduced pre-factor $R_{II}^2/(4\hbar^2)$ in the exponent) is transformed by the real self-energy density variable $\sigma^{(0)}_{R_1}(\vec{x}, t)$ as invariant vacuum or ground state in a SSB and where the other half of the quartic interaction (also with pre-factor $R_{II}^2/(4\hbar^2)$ in the exponent) remains within the anomalous or off-diagonal blocks of the Gaussian transformations. This can be verified as one decomposes the trace inside the exponential of the last line of (3.46). As we remove the trace and dyadic product inside the exponent of the last line in (3.46), one notes that terms with the block diagonal self-energy density $\delta \Sigma^{11}_{\pm pq}(t)$, $\delta \Sigma^{22}_{-pq}(t)$ cancel and only the self-energy variable $\sigma^{(0)}_{R_1}(\vec{x}, t)$ and anomalous parts $\delta \Sigma^{12}_{\pm pq}(t)$, $\delta \Sigma^{21}_{pq}(t)$ couple to the anomalous doubled, bilinear fields $\hat{\Psi}^a_{\pm pq}(t) \ldots \hat{\Psi}^a_{\pm pq}(t)$

$$\exp \left\{ - \frac{R_{II}^2}{2\hbar^2} \int_0^T dt \sum_{p,q=\pm} \left( \psi^*_\mp(t_p + \Delta t_p) \eta_p \psi_\pm(t_p) \right) \left( \psi^*_{\pm}(t_q + \Delta t_q) \eta_q \psi_{\pm}(t_q) \right) \right\} =$$

(3.46)

$$= \int d[\sigma^{(0)}_{R_1}(\vec{x}, t)] \exp \left\{ - \frac{1}{4} \frac{R_{II}^2}{\hbar^2} \int_0^T dt \sum_{x} \sigma^{(0)}_{R_1}(\vec{x}, t) \sigma^{(0)}_{R_1}(\vec{x}, t) \right\} \times$$

$$\times \int d[\delta \Sigma^{ab}_{\pm pq}(t) \vec{K}] \exp \left\{ - \frac{1}{8} \frac{R_{II}^2}{\hbar^2} \int_0^T dt \sum_{x} \text{Tr} \left[ \delta \Sigma^{ab}_{\pm pq}(t) \vec{K} \delta \Sigma^{ba}_{pq}(t) \vec{K} \right] \right\} \times$$

$$\times \exp \left\{ - \frac{i}{\hbar} \int_0^T dt \sum_{x} \text{Tr} \left[ \left( \delta R^{11}_{\pm pq}(t) \delta R^{22}_{\pm pq}(t) \right) \left( \begin{array}{c} \eta_0 \eta_0 \end{array} \right) \left( \delta \Sigma^{11}_{\pm pq}(t) - \delta \Sigma^{22}_{\pm pq}(t) \right) \left( \begin{array}{c} \eta_0 \eta_0 \end{array} \right) \right] \right\}.$$
the one-particle part $\tilde{\mathcal{F}}$

\begin{equation}
\sigma_x^{11}(t_p) = \sigma_{V_0}^{(0)}(x, t_p) + \delta \sigma_x^{11}(t_p), \quad \delta \sigma_x^{11}(t_p), \quad \delta \sigma_x^{22}(t_p) \in \mathbb{R};
\end{equation}

\begin{equation}
\sigma_x^{22}(t_p) = -\sigma_{V_0}^{(0)}(x, t_p) + \delta \sigma_x^{22}(t_p), \quad \sigma_{V_0}^{(0)}(x, t_p) \in \mathbb{R};
\end{equation}

\begin{equation}
\delta \sigma_x^{12}(t_p) = \delta \sigma_{ab}^{a b}(t_p); \quad \delta \sigma_x^{21}(t_p) = i \delta \sigma_x^{ab}(t_p) (a \neq b);
\end{equation}

\begin{equation}
\delta \sigma_x^{12}(t_p) \in \mathbb{C}; \quad (\delta \sigma_x^{21}(t_p))^* = \delta \sigma_x^{12}(t_p); \quad \delta \sigma_x^{11}(t_p) = \delta \sigma_x^{22}(t_p).
\end{equation}

3.4 Removal of the ’hinge’-fields from the generating function

As we insert the two HST’s (3.46, 3.47) of the quartic, non-hermitian disorder term and the repulsive contact interaction into (3.46, 3.47), we achieve the ensemble averaged path integral $\overline{Z_{II}[\tilde{\mathcal{F}}]}$ (3.52) with only linear and bilinear anomalous doubled fields $\tilde{\Psi}_{p,q}^a(t', \mathbf{x})$, $\tilde{\Psi}_q^a(t, \mathbf{x})$. Aside from the linear coupling to the doubled source fields $J_{\psi, \mathbf{x}}^a(t_p)$, $J_{\psi, \mathbf{x}}^a(t_q)$, we abbreviate the bilinear term of fields $\tilde{\Psi}_{p,q}^a(t') \cdots \tilde{\Psi}_q^a(t, \mathbf{x})$ by introducing the matrix $\tilde{\mathcal{N}}_{p,q}^{ab}(t', t_q)$ (3.53) which consists of the one-particle part $\tilde{H}_{\psi, \mathbf{x}}(t_q, t_p)$ (3.54), the source matrix $\tilde{\mathcal{D}}_{p,q}^{ab}(t_q, t_p)$ for generating bilinear observables of bosonic fields, the condensate seed field $\tilde{J}_{\psi, \mathbf{x}}^a(t_q)$ (3.49, 3.50), and the various self-energy variables $\sigma_{V_0}^{(0)}(\mathbf{x}, t_p)$ and $\sigma_{V_0}^{(0)}(\mathbf{x}, t_q)$ and matrices $\delta \mathcal{N}_{\psi, \mathbf{x}, \mathbf{p}}(t)$, $\delta \mathcal{D}_{\psi, \mathbf{x}, \mathbf{p}}(t_p)$.\n
\[
\overline{Z_{II}[\tilde{\mathcal{F}}]} = \int [d\tilde{\mathcal{N}}_{p,q}^{ab}(t) \tilde{K}] \exp \left\{ -\frac{1}{8} R_{II}^{11} \int_0^{+T} dt \sum_{x, p, q} \mathrm{Tr} \left[ \delta \mathcal{N}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t) \tilde{K} \delta \mathcal{N}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t) \tilde{K} \right] \right\}
\times \int [d(\tilde{\sigma}_{x, t}^{ab}(t_p) \tilde{K})] \exp \left\{ \frac{i}{4h} \int dt_p \sum_{x, p} \mathrm{tr} \left[ \delta \mathcal{D}_{p, q}^{ab}(t_p) \tilde{K} \delta \mathcal{D}_{p, q}^{ab}(t_p) \tilde{K} \right] \right\}
\times \int [d(\tilde{\sigma}_{V_0}^{(0)}(\mathbf{x}, t_p) \tilde{K})] \exp \left\{ \frac{i}{4h} \int dt_p \sum_{x} \frac{\tilde{\sigma}_{V_0}^{(0)}(\mathbf{x}, t_p)}{V_0 - i \varepsilon_p} \right\}
\times \int [d(\tilde{J}_{\psi, \mathbf{x}}^{(0)}(\mathbf{x}, t) \tilde{K})] \exp \left\{ -\frac{1}{4} \frac{1}{R_{II}^{11}} \int_0^{+T} dt \sum_{x, p} \tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t) \tilde{J}_{\psi, \mathbf{x}}^{ab}(t_q) \tilde{K} \right\}
\times \exp \left\{ -\frac{i}{2h} \int dt_p \sum_{x, p, q} \frac{\tilde{J}_{\psi, \mathbf{x}}^{ab}(t_p) \tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t_q)}{N_x} \right\};
\end{equation}

\[
\tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t' \mathbf{x}, t_p) = \tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t', t_p) + K \frac{\tilde{\mathcal{D}}_{p, q}^{ab}(t', t_p)}{N_x} \tilde{K} +
\delta(t - t') \delta_{x, x'} \delta_{q, q'} \eta_p \left( \tilde{J}_{\psi, \mathbf{x}}^{ab}(t_q) \frac{1}{2} \left( \tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t) + \frac{1}{2} \delta \mathcal{D}_{p, q}^{ab}(t) \right) \right) +
\delta(t - t') \delta_{x, x'} \frac{1}{2} \tilde{K} \left( \tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t) + \frac{1}{2} \delta \mathcal{D}_{p, q}^{ab}(t) \right) \right) \frac{\tilde{J}_{\psi, \mathbf{x}}^{ab}(t_q)}{N_x} \tilde{K}.
\end{equation}

After integration over the bilinear, anomalous doubled fields in (3.52), one attains the inverse square root of the determinant of $\tilde{\mathcal{N}}_{\psi, \mathbf{x}, \mathbf{p}}^{ab}(t_q)$, which is transformed to a $\frac{1}{8}$ trace $\ln[ \cdots ]$ in an exponential with normalizing term $h \Omega_{\text{max}} N_x$ and
to a propagator part $\tilde{\mathcal{M}}^{-1,ba}_{\xi,\tilde{\epsilon},\tilde{\epsilon}_{p},(t',t)}$ between bilinear, 'Nambu' doubled source fields $J^{b}_{\psi;x}(t') \ldots J^{a}_{\psi;x}(t_{p})$

$$
\tilde{Z}_{II}[\hat{\beta}] = \int d[\hat{\Sigma}_{x}^{ab}(t) \, \hat{K}] \exp \left\{- \frac{1}{8 R_{II}^{4}} \int_{0}^{+T} dt \sum_{x} \text{Tr}_{a,b,p,q} \left[ \delta \hat{\Sigma}_{x}^{ab}(t) \, \hat{K} \, \delta \hat{\Sigma}_{x}^{ba}(t) \, \hat{K} \right] \right\}
$$

(3.54)

$$
\times \int d[\delta \hat{\sigma}_{x}^{ab}(t_{p}) \, \hat{K}] \exp \left\{ \frac{i}{4h} \int_{C} dt \sum_{x} \text{Tr} \, \left[ \delta \hat{\sigma}_{x}^{ab}(t_{p}) \, \hat{K} \, \delta \hat{\sigma}_{x}^{ba}(t_{p}) \, \hat{K} \right] \right\}
$$

$$
\times \int d[\sigma_{x}^{(0)}(\vec{x},t_{p})] \exp \left\{ \frac{1}{2h} \int_{C} dt \sum_{x} \delta \sigma_{x}^{(0)}(\vec{x},t_{p}) \right\}
$$

$$
\times \int d[\sigma_{R_{II}}^{(0)}(\vec{x},t_{p})] \exp \left\{ - \frac{1}{4 R_{II}^{4}} \int_{0}^{+T} dt \sum_{x} \delta \sigma_{R_{II}}^{(0)}(\vec{x},t_{p}) \right\}
$$

$$
\exp \left\{ \frac{1}{2} \sum_{p} \hbar \Omega_{max}^{2} V_{x} \text{Tr}_{a,b,p,q} \ln \left[ \tilde{\mathcal{M}}_{x}^{pq}(t_{p}',t_{p}) \right] \right\}
$$

$$
\times \exp \left\{ \frac{1}{4} \sum_{x,x'} \sum_{p,q=\pm} J^{b}_{\psi;x}(t_{p}') \, \tilde{\mathcal{M}}^{-1,ba}_{\xi,\tilde{\epsilon},\tilde{epsilon},x}(t',t) \, J^{a}_{\psi;x}(t_{p}) \right\}
$$

(3.54)

In order to simplify the self-energy matrices in a coset decomposition, we shift the matrix $\delta \hat{\Sigma}_{x}^{ab}(t)$ and self-energy variable $\sigma_{V_{x}}^{(0)}(\vec{x},t_{p})$, according to (3.55, 3.56) and (3.57), respectively, and perform the transformations (3.58) of $\tilde{M}_{x}^{ab}(t_{p}',t_{p})$ (3.59) with the 'Nambu' metric tensors (3.24, 3.11, 3.22) to a modified matrix $\tilde{\mathcal{M}}^{ab}_{x}(t_{p}',t_{p})$ (3.59, 3.60). This does not alter the value of the determinant (3.61) and allows for the factorization (3.63, 3.66) into density-related self-energy matrices and coset matrices $\tilde{T}_{pq}^{ab}(\vec{x},t)$ (3.25, 3.27). Similar transformations (3.62) are also accomplished for the propagator part from $\tilde{\mathcal{N}}^{-1,ab}_{x,x'}(t_{p}',t_{p})$ to $\tilde{\mathcal{N}}^{-1,ab}_{x,x'}(t_{p}',t_{p})$ with $\delta \tilde{\Sigma}_{x}^{ab}(t_{p},t_{p}) \, \hat{K}$ (3.63) for the coset decomposition $\text{Sp}(4)/U(2) \otimes U(2)$.

$$
\delta \hat{\Sigma}_{x}^{ab}(t_{p}) \rightarrow \delta \hat{\Sigma}_{x}^{ab}(t_{p}) - 2 \delta \hat{\sigma}_{x}^{ab}(t_{p}) \; ;
$$

(3.55)

$$
\delta \hat{\sigma}_{x}^{ab}(t_{p}) \rightarrow \delta \hat{\sigma}_{x}^{ab}(t_{p}) - 2 \delta \hat{\sigma}_{x}^{ab}(t_{p}) \; ;
$$

(3.56)

$$
\sigma_{V_{x}}^{(0)}(\vec{x},t_{p}) \rightarrow \sigma_{V_{x}}^{(0)}(\vec{x},t_{p}) - \frac{1}{2} \delta \sigma_{R_{II}}^{(0)}(\vec{x},t_{p}) \; ;
$$

(3.57)

$$
\tilde{\mathcal{N}}^{ab}_{x,x'}(t_{p},t_{p}') \rightarrow \hat{K} \hat{I}^{-1} \left( i \hat{K} \, \tilde{\mathcal{N}}^{ab}_{x,x'}(t_{p},t_{p}') \, \hat{K} \hat{I} \right) \hat{I}^{-1} \hat{K}
$$

(3.58)

$$
\rightarrow \hat{K} \hat{I}^{-1} \left( i \hat{K} \, \tilde{\mathcal{N}}^{ab}_{x,x'}(t_{p},t_{p}') \, \hat{K} \hat{I} \right) \hat{I}^{-1} \hat{K}
$$

(3.59)

$$
\tilde{\mathcal{N}}^{ab}_{x,x'}(t_{p},t_{p}')
$$

$$
= \hat{K} \hat{I} \hat{K}
$$

(3.59)

$$
\hat{K} \hat{I} \hat{K}
$$

(3.59)
\[
\delta \hat{\Sigma}^{ab}_{D,pq}(x, t) = \left( \delta \hat{\Sigma}^{11}_{D,pq}(x, t) \right)_{pq}^{aa} ; \\
\delta \hat{\Sigma}^{11}_{D,pq}(x, t) \eta_q = \left( \delta B_{D:++}(x, t) \right)_{pq}^{\mp \mp} - \delta B_{D:+-}(x, t) ; \\
-\delta \hat{\Sigma}^{22}_{D,pq}(x, t) \eta_q = \left( -\delta B_{D:++}(x, t) \right)_{pq}^{\mp \mp} - \delta B_{D:+-}(x, t) .
\]

Since the shift of the matrix \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \) and self-energy variable \( \sigma^{(0)}_{V_0}(x, t_p) \) has removed the self-energy matrix \( \delta \hat{\Sigma}^{ab}_{x, pq}(t_p) \) and variable \( \sigma^{(0)}_{R_1}(x, t) \) from \( \hat{\Sigma}^{ab}_{x, pq}(t_p, t'_p) \) or \( \hat{\Sigma}^{ab}_{x, pq}(t_p, t'_p) \) (compare with \( \delta \hat{\Sigma}^{ab}_{x, pq}(t_p, t'_p) \)), we can completely eliminate latter self-energies by integration of Gaussian identities which involves the appearance of a new parameter \( \mu^{(II)}_{p} \) with the ratio \( R^2/(hV_0) \) in the denominator.

\[
\int d[\sigma^{(0)}_{R_1}(x, t)] \exp \left\{ -\frac{1}{4 R^2_II} \int_0^{+T} dt \sum_x \frac{\sigma^{(0)}_{R_1}(x, t) \sigma^{(0)}_{R_1}(x, t)}{\sigma^{(0)}_{V_0}(x, t_p) - \frac{1}{2} \sigma^{(0)}_{R_1}(x, t) \sigma^{(0)}_{R_1}(x, t) V_0 - \epsilon_p} \right\} = \\
\int d[\sigma^{ab}_{x, pq}(t)] \exp \left\{ -\frac{1}{8 R^2_II} \int_0^{+T} dt \sum_x Tr \frac{\delta \hat{\Sigma}^{ab}_{x, pq}(t) - 2 \delta_{pq} \eta_p \left( \delta \hat{\Sigma}^{ab}_{x, pq}(t) + \hat{J}^{ba}_{x,y}(t) \right) \tilde{K}}{\sigma^{(0)}_{V_0}(x, t_p) - \frac{1}{2} \sigma^{(0)}_{R_1}(x, t) \sigma^{(0)}_{R_1}(x, t) V_0 - \epsilon_p} \right\} \times \\
\exp \left\{ -\frac{1}{8 R^2_II} \int_0^{+T} dt \sum_x \sum_{p=q=\pm} (1 - \delta_{pq} \mu^{(II)}_{p}) \frac{1}{\tilde{K}} \right\} \times \\
\exp \left\{ \frac{1}{2 R^2_II} \int_0^{+T} dt \sum_x \sum_{p=\pm} \eta_p (1 - \mu^{(II)}_{p}) \right\} \times \\
\exp \left\{ \frac{1}{2 R^2_II} \int_0^{+T} dt \sum_x \sum_{p=\pm} (1 - \mu^{(II)}_{p}) \right\} \times \\
\mu^{(II)}_{p} = \frac{1}{\left( 1 - (i/2) \eta_p \left( R^2_II/(hV_0) \right) \right)} .
\]

After insertion of the Gaussian identities \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \) into the path integral \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \) which is determined by the self-energy variable \( \sigma^{(0)}_{V_0}(x, t_p) \) and the self-energy matrix term \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \tilde{K} \) for the coset decomposition as remaining field degrees of freedom.

\[
\mathcal{Z}_{II}[-\eta] = \exp \left\{ -\frac{1}{2 R^2_II} \int_0^{+T} dt \sum_x \sum_{p=\pm} (1 - \mu^{(II)}_{p}) \right\} \times \\
\int d[\sigma^{(0)}_{V_0}(x, t_p)] \exp \left\{ -\frac{1}{4 (hV_0)^2} \int_0^{+T} dt \sum_x \left( \sigma^{(0)}_{V_0}(x, t_p) - \sigma^{(0)}_{V_0}(x, t_p) \right)^2 \right\} \times
\]

\( \mathcal{Z}_{II}[-\eta] \) is expanded into the path integral \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \) which is determined by the self-energy variable \( \sigma^{(0)}_{V_0}(x, t_p) \) and the self-energy matrix term \( \delta \hat{\Sigma}^{ab}_{x, pq}(t) \tilde{K} \) for the coset decomposition as remaining field degrees of freedom.
\[ \times \exp \left\{ \frac{1}{2\hbar} \int \frac{dt_p}{\mathcal{C}} \sum_{\vec{x}} \frac{\sigma_{0}\left(\vec{x},t_p\right) \sigma_{0}\left(\vec{x},t_p\right)}{V_0 - t_{\varepsilon_p}} \right\} \times \int d[\delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K}] \times \]
\[ \times \exp \left\{ \frac{1}{2} \int \frac{dt_p}{\mathcal{C}} \sum_{\vec{x}} \sum_{p,q=\pm} \left( 1 - \delta_{pq} \mu_{p}^{(II)} \right) \text{tr}_{a,b} \left[ \delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K} \delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K} \right] \right\} \times \]
\[ \times \exp \left\{ \frac{1}{2} \int \frac{dt_p}{\mathcal{C}} \sum_{\vec{x}} \sum_{\eta_{p,q=\pm}} \left( 1 - \mu_{p}^{(II)} \right) \text{tr}_{a,b} \left[ \delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K} \delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K} \right] \right\} \times \]
\[ \times \exp \left\{ \frac{1}{2} \int \frac{dt_p}{\mathcal{C}} \sum_{\vec{x}} \sum_{\eta} h \Omega_{\text{max}} N_x \text{tr}_{a,b} \left[ \tilde{N}^{ab}_{\vec{x},\bar{x}}(t',t_p) \tilde{I} \tilde{K} \tilde{N}^{-1,ba}_{\vec{x},\bar{x}}(t',t_p) \tilde{I} \tilde{J}_{\psi,\bar{x}}^{a}(t_p) \right] \right\} . \]

Finally, we factorize the self-energy matrix \( \delta \tilde{\Sigma}_{\varepsilon_{pq}}(t) \tilde{K} \) inside \( \tilde{N}^{ab}_{\vec{x},\bar{x}}(t, t'_{q}) \) (3.71) corresponding to the coset decomposition \( \text{Sp}(4)/U(2) \otimes U(2) \) (3.22, 3.23) and apply this factorization inside the action \( A_{DET}[\tilde{T}, \delta \tilde{\Sigma}_{D}, \sigma_{0}; \hat{\tilde{\Sigma}}] \) (3.73) from the determinant and within \( A_{J^{a}_{\varepsilon},J_{\varepsilon}}[\tilde{T}, \delta \tilde{\Sigma}_{D}, \sigma_{0}; \hat{\tilde{\Sigma}}] \) (3.74) from the propagator part of the source fields \( J_{\psi,\bar{x}}^{a}(t_p), J_{\psi,\bar{x}}^{b}(t_{q}) \).

In subsequent steps we straightforwardly outline how to factorize the matrix \( \tilde{N}^{ab}_{\vec{x},\bar{x}}(t, t'_{q}) \) (3.70) with the coset matrices \( \tilde{T}^{pq}_{\vec{x},\bar{x}}(t, t'_{q}) \) (3.22, 3.23) and with the various 'Nambu' metric tensors (3.2, 3.11, 3.12, 3.22) into the modified matrices \( \tilde{N}^{ab}_{\vec{x},\bar{x}}(t, t'_{q}; D, \tilde{K}) \) (3.77), \( \tilde{N}^{ab}_{\vec{x},\bar{x}}(t, t'_{q}; I - 1, D) \) (3.78), \( \tilde{N}^{ab}_{\vec{x},\bar{x}}(t, t'_{q}) \) (3.81) where the first two kinds of matrices (3.77, 3.78) do still contain the block diagonal self-energy density matrices \( \delta \Sigma_{D}^{11}(\vec{x}, t), \delta \Sigma_{D}^{22}(\vec{x}, t) \) as 'hinge' fields in a SSB with a coset decomposition \( \text{Sp}(4)/U(2) \otimes U(2) \).
so that the determinant and propagator part are changed back to the bilinear, 'Nambu' doubled term of bosonic fields after substitution of above separating relations into self-energy densities into the path integral (3.70), we can 're-introduce' integrals of bosonic, anomalous doubled fields \( \tilde{\Psi} \) and has introduced a new gradient term

\[
\delta \hat{\mathcal{H}}_{ab}(\tilde{x}, \tilde{t}) = \left( \hat{T}_{q'}^{ab'}(\tilde{x}', \tilde{t}') \hat{H}_{p'}(\tilde{x}', \tilde{t}') \hat{T}_{p'}^{a}(\tilde{x}, \tilde{t}) - \hat{H}_{p}(\tilde{x}', \tilde{t}') \delta_{ab} \delta_{p', \tilde{x}'} \delta(\tilde{t}' - \tilde{t}) \right) ,
\]

so that we can use this separation into 'hinge' density-related self-energy terms and 'Nambu' gradient terms for the actions \( A_{DET}[\tilde{T}, \delta \Sigma_D, \sigma_v(0); \tilde{\mathcal{J}}] \) and \( A_{J_{\psi}, J_{\psi}^e}[\tilde{T}, \delta \Sigma_D, \sigma_v(0); \tilde{\mathcal{J}}] \). Since the determinant and propagator of the matrices \( \tilde{\mathcal{H}}_{ab}(\tilde{x}, \tilde{t}) \) without 'hinge' fields are related by

\[
\text{DET} \left( \tilde{N}_{\tilde{x}, \tilde{t}}^{ab}(t_p, t_q') \right) = \text{DET} \left( \tilde{\mathcal{H}}_{\tilde{x}, \tilde{t}}^{ab}(t_p, t_q') \right) ,
\]

we can considerably simplify the actions \( \tilde{\mathcal{H}}_{ab}(\tilde{x}, \tilde{t}) \) by following transformations

\[
\tilde{N}_{\tilde{x}, \tilde{t}}^{-1} \tilde{\mathcal{H}}_{ab}(\tilde{x}, \tilde{t}) \tilde{N}_{\tilde{x}, \tilde{t}}^{ab}(t_p, t_q') = \tilde{T}_{p'}^{a'}(\tilde{x}, \tilde{t}) \tilde{K}_{ab} \tilde{N}_{\tilde{x}, \tilde{t}}^{-1} \tilde{\mathcal{H}}_{p'}(\tilde{x}', \tilde{t}') \delta_{ab} \delta_{p', \tilde{x}'} \delta(\tilde{t}' - \tilde{t}) ,
\]

After substitution of above separating relations into self-energy densities \( \delta \hat{\mathcal{H}}_{ab}(\tilde{x}, \tilde{t}) \) and gradient term \( \tilde{K} \) of coset matrices into the path integral (3.70), we can 're-introduce' integrals of bosonic, anomalous doubled fields \( \tilde{\Psi}_D(t_p) \ldots \tilde{\Psi}_D(t_p) \) so that the determinant and propagator part are changed back to the bilinear, 'Nambu' doubled term of bosonic fields.
3.4 Removal of the 'hinge'-fields from the generating function

\[ Z_{II}[\bar{\beta}] = \exp \left\{ -\frac{1}{2} R_{II}^{2} \int_{0}^{+T} dt \sum_{p=\pm} \sum_{x} \left( 1 - \mu_{p}^{(I)} \right) \text{tr}_{a,b} \left[ \tilde{\sigma}_{V_{0};x}^{ab}(t)p \tilde{\sigma}_{V_{0};x}^{ba}(t) \right] \right\} \times \]

\[ \times \int d[\sigma_{V_{0}}(\bar{x},t_{p})] \exp \left\{ -\frac{1}{2} \frac{R_{II}^{2}}{(hV_{0})^{2}} \int_{0}^{+T} dt \sum_{x} \left( \sigma_{V_{0}}^{(0)}(\bar{x},t) - \sigma_{V_{0}}^{(0)}(\bar{x},t_{-}) \right)^{2} \right\} \times \]

\[ \exp \left\{ \frac{1}{2h} \int_{C} dt_{p} \sum_{x} \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \right\} \times \int d[\sigma_{\Sigma^{ab}_{pq}}(t)] K \]

\[ \times \exp \left\{ -\frac{1}{2} \frac{R_{II}^{2}}{8 \hbar} \int_{0}^{+T} dt \sum_{x, p, q = \pm} \left( 1 - \delta_{pq} \mu_{p}^{(I)} \right) \text{tr}_{a,b} \left[ \delta_{\Sigma^{ab}_{pq}}(t) K \delta_{\Sigma^{ba}_{pq}}(t) \right] \right\} \times \]

\[ \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left[ K \left. \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right] + \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right] \right\} \times \]

\[ \exp \left\{ -\frac{1}{2h} \int_{C} dt_{p} \sum_{x, x'} J_{\Psi^{+b}_{x'}(t_{p})} \left[ \tilde{J}^{b}_{\Psi^{+b}_{x'}(t_{p})} \left( \tilde{T}^{(2)}(\bar{x},t) \tilde{\Sigma}^{(2)}(\bar{x},t) \right) \right] \Psi_{x}(t_{p}) \right\} \]

\[ \times \exp \left\{ -\frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right\} \right\} \]

\[ \times \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right\} \right\} \]

However, the presented factorization of \( \tilde{N}_{x}^{ab}(t_{p},t_{q}) \) with the various 'Nambu' metric tensors and coset matrices has modified the original path integral (3.70) (3.71) or (3.76) (3.77) to (3.80) (3.81) or (3.90) in such a manner that the bilinear coupling of doubled, Bose fields \( \Psi_{x}^{+b}(t_{p}) \ldots \Psi_{x}^{+a}(t_{p}) \) with the self-energy densities \( \delta_{\Sigma^{ab}_{pq}^{(g)}}(\bar{x},t) \), \( \delta_{\Sigma^{ba}_{pq}^{(g)}}(\bar{x},t) \) cancels in (3.90) and restricts the corresponding exponential (3.91) to the value of unity

\[ \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right\} \right\} \]

\[ \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right\} \right\} \]

\[ = 1 \].

The above transformations with 'Nambu' metric tensors and factorization of the total self-energy \( \delta_{\Sigma^{ab}_{pq}^{(g)}}(t) \tilde{K} \) have resulted into a projection onto the anomalous doubled field degrees of freedom with the coset matrices \( T_{pq}^{ba}(\bar{x},t) \) and the self-energy variable \( \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \) as the invariant vacuum or ground state in a SSB. We thus obtain the path integral (3.92) with the operator \( \hat{O}_{x}^{ab}(t'_{q}, t_{p}) \) (3.93), being composed of gradient \( \delta \bar{K}(\tilde{T}^{-1}, \tilde{T}) \) (3.81) and density part \( \tilde{H}_{p}^{ab}(t_{p}) + \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \), where the block diagonal 'hinge' densities \( \delta_{\Sigma^{ab}_{pq}^{(g)}}(\bar{x},t) \) are only contained within Gaussian factors and the traces of \( \delta_{\Sigma^{ab}_{pq}^{(g)}}(t) \tilde{K} \) or \( \delta_{\Sigma^{ab}_{pq}^{(g)}}(t) \tilde{K} \) so that these 'hinge' fields can be removed by Gaussian like integrations

\[ Z_{II}[\bar{\beta}] = \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} dt_{q} \sum_{x, x'} N_{x} \Psi^{+b}_{x'}(t_{q}) \left( \delta_{\Sigma^{ba}_{pq}}^{(g)} \right)_{x, x'} \delta_{\Sigma^{ba}_{pq}}(t_{q},t_{p}) \right\} \]

\[ \times \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} \sum_{x} \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \right\} \times \]

\[ \times \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} \sum_{x} \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \right\} \times \int d[\delta_{\Sigma^{ab}_{pq}}^{(g)}(t)] \tilde{K} \]

\[ \times \exp \left\{ -\frac{1}{2} \frac{1}{2h} \int_{C} dt_{p} \sum_{x} \sigma_{V_{0}}^{(0)}(\bar{x},t_{p}) \right\} \times \int d[\delta_{\Sigma^{ab}_{pq}}^{(g)}(t)] \tilde{K} \]
\[ \times \exp \left\{ \frac{1}{2R_{II}^2} \int_0^{+\tilde{T}} dt \sum_{p=\pm} \eta_p \left( 1 - \mu_p^{(II)} \right) \text{tr} \left[ \delta \tilde{\Sigma}_{x;pq}^{ab}(t) \tilde{K} \tilde{J}_{\psi;\tilde{x}}^{ba}(t_p) \tilde{K} \right] \right\} \]

\[ \times \exp \left\{ -\frac{1}{2} \int_C \frac{dt_p}{h} \eta_p \sum_{x} h_{\Omega_{\text{max}}N_x} \text{tr} \left[ \delta \tilde{\Sigma}_{\tilde{x};t'_q}^{ba}(t'_q, t_p) \right] \right\} \]

\[ \times \exp \left\{ \frac{i}{2} \Omega_{\text{max}} \frac{\sum_{b=1,2}}{h} \sum_{p,q=\pm} \sum_{x,\tilde{x}} N_{x} \left( J^{+b}_{x;\tilde{x}}(t'_q) + \hat{J}_{x;\tilde{x}}^{b}(t'_q) \right) \hat{K} J_{x,\tilde{x}}^{a}(t', t'_q) + \hat{K} J_{x,\tilde{x}}^{a}(t, t'_q) + \hat{K} J_{x,\tilde{x}}^{a}(t', t'_q) \right\} \]

\[ \hat{O}_{\tilde{x};\tilde{x}}^{ba}(t'_q, t_p) = \left( \hat{H}_{\tilde{x}}^{b}(t'_q) + \sigma_{\tilde{V}_0}^{(0)}(\tilde{x}', t'_q) \right) \delta_{ba} \delta_{\tilde{q}p} \delta_{\tilde{x}',\tilde{x}} \delta(t'_q - t_p) + \left( \hat{T}^{-1} \hat{I} \frac{3}{N_\sigma} \hat{I} \hat{K} \hat{T} \right)^{ba}_{x,\tilde{x}}(t'_q, t_p) + \]

\[ + \frac{1}{2} \delta_{\tilde{x}^2}(\hat{T}^{-1}, \hat{T}) \] (3.93)

### 3.5 The change of integration measure for anomalous and density-related parts

In order to integrate over the density parts of the Gaussian-like factors, we determine the change of integrate measure from the ‘flat’ Euclidean, total self-energy $\delta \Sigma_{x;pq}^{ab}(t) \hat{K}$ to the density-related and anomalous doubled parts; we use the factorization of latter two different kinds of various blocks ‘$a = b$’ and ‘$a \neq b$’ and furthermore separate a background averaging functional ($\ldots \sigma_{\tilde{V}_0}$) from the path integral (3.93, 3.93) with the coset field degrees of freedom

\[ d[\delta \tilde{\Sigma}_{x;pq}^{ab}(t) \hat{K}] = N_{\delta \Sigma} \hat{K} \left( \prod_{\{x,t\}} d(\delta \lambda_{+}(\tilde{x}, t)) d(\delta \lambda_{-}(\tilde{x}, t)) \left( \delta \lambda_{+}(\tilde{x}, t) \right)^2 \right) \times \]

\[ \times \left( \left( \delta \lambda_{+}(\tilde{x}, t) \right)^2 - \left( \delta \lambda_{-}(\tilde{x}, t) \right)^2 \right)^2 \times d(\mid B_D(\tilde{x}, t)\mid) \text{ sinh}(4|\text{B}_D(\tilde{x}, t)|) \] d$\beta_D(\tilde{x}, t)$ \times \]

\[ \times \left( \prod_{\{x,t\}} d(\mid \tau_{+}(\tilde{x}, t)\mid) \text{ sinh}(4|\text{B}_D(\tilde{x}, t)|) \right)^2 d\varphi_{+}(\tilde{x}, t) \times \left( \prod_{\{x,t\}} d(\mid \tau_{-}(\tilde{x}, t)\mid) \text{ sinh}(4|\text{B}_D(\tilde{x}, t)|) \right)^2 d\varphi_{-}(\tilde{x}, t) \]

\[ d(\mid \sigma_D(\tilde{x}, t)\mid \sigma_D(\tilde{x}, t)) d(\mid \tau_D(\tilde{x}, t)\mid \tau_D(\tilde{x}, t)) = N_{\delta \Sigma} \hat{K} \left( \prod_{\{x,t\}} d(\delta \lambda_{+}(\tilde{x}, t)) d(\delta \lambda_{-}(\tilde{x}, t)) \right) \text{ cosh}(4|\text{B}_D(\tilde{x}, t)|) \] \text{ cosh}(4|\text{B}_D(\tilde{x}, t)|) \] d$\gamma_D(\tilde{x}, t)$ \times \]

\[ \times \left( \frac{1}{2} d\left( \text{ cosh}(4|\text{B}_D(\tilde{x}, t)|) \right) \right) \times \frac{1}{2} d\left( \text{ sin}(4|\tau_D(\tilde{x}, t)|) \right) d\varphi_{+}(\tilde{x}, t) \times \frac{1}{2} d\left( \text{ sin}(4|\tau_D(\tilde{x}, t)|) \right) d\varphi_{-}(\tilde{x}, t) \] (3.94)
\[ \exp \left\{ \frac{i}{\hbar} \sum_{a,b'=1}^{p+q=2} N_{a,b} (t_{pq}') \hat{J}_{a,b}(t_{pq}') \hat{I} \right\} \]

\[ = \int_{C} \int_{0}^{+\hbar} dt' \sum_{p,q=\pm} N_{a,b} \left\{ \hat{J}_{a,b}(t_{pq}) \right\} \left[ \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} \right] \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} + \]

\[ + (\hat{J}^{-1} \hat{I}_{K} \hat{I} \hat{J}) \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} (t_{pq}' \rightarrow t_{pq}, \hat{J}_{a,b}(t_{pq}') \hat{I} \hat{J}_{a,b}(t_{pq}')) \phi_{V_0}. \]

\[ \left\langle \coset fields and \hat{\sigma}_{V_0} \right\rangle = \int_{C} \int_{0}^{+\hbar} dt \sum_{p,q=\pm} \left\{ \hat{J}_{a,b}(t_{pq}) \right\} \left[ \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} \right] \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{-1} (t_{pq}' \rightarrow t_{pq}, \hat{J}_{a,b}(t_{pq}')) \phi_{V_0}. \]

We hint again to following abbreviations, already specified in (3.60,3.81)

\[ \left( \hat{J} + \hat{\sigma}_{V_0} \right)^{ab}_{\tilde{x},\tilde{x}'} (t_{pq}', t_{pq}) = \left( \hat{J}_{a,b}(t_{pq}) + \sigma_{V_0}(t_{pq}) \right) \delta_{a,b} \delta_{pq} \delta(t_{pq}' - t_{pq}) ; \]

\[ \delta_{V_0}^{a,b}_{\tilde{x},\tilde{x}'} (\hat{T}^{-1}, \hat{T}, t_{pq}) = \left( \hat{T}_{p,0}^{-1} \hat{a}_{a}^{\dagger}(\tilde{x}, t) \right) \left( \hat{T}_{q,0}^{-1} \hat{a}_{b}^{\dagger}(\tilde{x}', t') \right) \hat{V}_{q,p} \hat{a}_{b}^{\dagger} \hat{a}_{a}(\tilde{x}, \tilde{x}') - \delta_{V_0}^{a,b}_{\tilde{x},\tilde{x}'} (t_{pq}', t_{pq}). \]

Further simplification of above path integrals (3.94, 3.95) with coset field degrees of freedom arises from taking a separate saddle point approximation of the background averaging functional (3.96) with respect to a variation with \[ \delta_{V_0}^{a,b}_{\tilde{x},\tilde{x}'} \]

This results in definite, complex-valued functions \( \langle \sigma_{V_0}^{a,b}_{\tilde{x},\tilde{x}'} \rangle \) of space and time where the imaginary part of \( \langle \sigma_{V_0}^{a,b}_{\tilde{x},\tilde{x}'} \rangle \) has to comply with the original, already introduced, convergence generating \( -i \hat{\varepsilon}_{p} \) terms for proper convergence and analytic properties of Green functions.

### 4 Summary and conclusion

#### 4.1 Transformation to the case 'I' with static disorder reduced to stationary states

At the end of section 2 we have already specified the path integral \( Z_{I} \) (3.34) with the approximation to stationary states in frequency space which can also be attained from simplifying the two frequency dependence of the corresponding total disorder self-energy \( \delta \Sigma_{ab}^{\omega_{q,p}}(\omega, \omega') \rightarrow \delta_{\omega, \omega'} \delta \Sigma_{ab}^{\omega_{q,p}}(\omega) \) at very later steps of transformations to a nonlinear sigma model

\[ Z_{I} \simeq \left\langle F[\psi^{*}, \psi, J_{\psi}, \hat{I}_{\psi}; \hat{\sigma}] \right\rangle \times \]

\[ \times \exp \left\{ -\frac{R^{2}_{p} N^{2}_{p}}{2 \hbar^{2}} \sum_{p,q=\pm} \left( \psi_{p,q}^{*}(\omega) e^{i \omega \Delta t_{p}} \eta_{p,q} \psi_{p,q}(\omega) \right) \right\} \phi_{[\psi^{*}, \psi]} ; \]

\[ \Omega_{\max} = \frac{1}{\Delta t} ; \quad 0 < t_{p} < +T_{0} ; \quad T_{0} = T_{\max} ; \]

\[ 0 < \omega_{p} < +\Omega_{0} ; \quad \Omega_{0} = \Omega_{\max} = \frac{1}{\Delta t} ; \quad \Delta \omega = \frac{2\pi}{T_{\max}} \quad N_{\Omega} = T_{\max}/\Delta \omega ; \quad N_{t} = T_{\max}/\Delta t ; \]

\[ \int_{C_{\omega}} d\omega_{-} \Delta \omega \cdots = \int_{-\Omega_{0}}^{+\Omega_{0}} \frac{d\omega_{+}}{(2 \pi)_{\max}} \cdots + \int_{+\Omega_{0}+\Delta \omega}^{+\Omega_{0}} \frac{d\omega_{+}}{(2 \pi)_{\max}} \cdots = \]

(4.3)
\[ R_{II}^2 \approx R_I^2 \frac{\Omega_{\text{max}}}{N_x}. \]
4.1 Transformation to the case 'I' with static disorder reduced to stationary states

\[ Y_{pq}(\mathbf{x}, \omega) = \left( \begin{array}{cc} 0_{11}^{pq} & (\hat{X}_{pq}(\mathbf{x}, \omega))^{12} \end{array} \right) \left( \begin{array}{c} (\hat{X}_{pq}(\mathbf{x}, \omega))^{12} \end{array} \right) \]  

\[ \hat{X}_{pq}(\mathbf{x}, \omega) = \left( \begin{array}{cc} -\delta C_{D;++}(\mathbf{x}, \omega) & -\delta C_{D;+–}(\mathbf{x}, \omega) \\ -\delta C_{D;+–}(\mathbf{x}, \omega) & -\delta C_{D;––}(\mathbf{x}, \omega) \end{array} \right); \eta_{pq} \hat{X}_{pq}(\mathbf{x}, \omega) \eta_{pq} = \left( \begin{array}{cc} -\delta C_{D;++}(\mathbf{x}, \omega) & -\delta C_{D;+–}(\mathbf{x}, \omega) \\ -\delta C_{D;+–}(\mathbf{x}, \omega) & -\delta C_{D;––}(\mathbf{x}, \omega) \end{array} \right); \]  

\[ \hat{Y}_{pq}(\mathbf{x}, \omega) = \hat{P}_{4\times 4}^{22}(\mathbf{x}, \omega) \hat{Y}_{D;4\times 4}(\mathbf{x}, \omega) \hat{P}_{4\times 4}(\mathbf{x}, \omega); \]  

\[ \hat{Y}_{D;4\times 4}(\mathbf{x}, \omega) = \left( \begin{array}{cc} 0_{11}^{pq} & (\hat{X}_{pq}(\mathbf{x}, \omega))^{22} \end{array} \right); \hat{P}_{4\times 4}(\mathbf{x}, \omega) = \left( \begin{array}{cc} 0_{11}^{pq} & (\hat{P}_{2\times 2}^{11}(\mathbf{x}, \omega))^{22} \end{array} \right); \]  

These similarities also hold for the block diagonal density-related parts so that one can immediately state following relations in place of [3.23],[3.35]

\[ \delta \hat{\Sigma}_{pq}(\mathbf{x}, \omega) K = \hat{Q}_{pq}^{-1}(\mathbf{x}, \omega) \delta \hat{\Lambda}_{pq}(\mathbf{x}, \omega) \hat{Q}_{pq}(\mathbf{x}, \omega) \]  

\[ \delta \hat{\Lambda}_{pq}(\mathbf{x}, \omega) = \text{diag} \{ \delta \lambda_{++}(\mathbf{x}, \omega), \delta \lambda_{––}(\mathbf{x}, \omega) \}; \quad \delta \hat{\lambda}_{pq}(\mathbf{x}, \omega) = \text{diag} \{ \delta \lambda_{++}(\mathbf{x}, \omega), \delta \lambda_{––}(\mathbf{x}, \omega) \}; \]  

\[ \hat{Q}_{pq}^{22}(\mathbf{x}, \omega) = \left( \begin{array}{c} \exp \{ \hat{\mathcal{B}}_{D;4\times 4}(\mathbf{x}, \omega) \} \end{array} \right)_{pq} \]  

However, the real self-energy variable \( \sigma_{0}(\mathbf{x}, \omega) \) for the hermitian contact interaction takes a dependence on the difference of two frequencies \( \Delta \omega_{p} = \omega_{p} - \omega_{p}^{0} \) which therefore reduces in the limit \( \omega_{p} \rightarrow \omega_{p}^{0} \) for stationary states to the zero frequency...
mode of a real-valued self-energy variable $\sigma^{(0)}_{V_0}(\vec{x}, \Delta \omega_p \equiv 0)$

$$
\sigma^{(0)}_{V_0}(\vec{x}, t_p) \rightarrow \sigma^{(0)}_{V_0}(\vec{x}, \Delta \omega_p = \omega_p - \omega_p') \overset{\Delta \omega_p=0}{\longrightarrow} \sigma^{(0)}_{V_0}(\vec{x}, \omega_p') \in \mathbb{R}.
$$

(4.31)

Consequently, we can directly convey the integration measure (3.94) and path integral $Z_{I[\bar{\sigma}]}$ (3.95) with background averaging functional \((3.56)\) to the case $Z_{I[\bar{\sigma}]}$ of static disorder for the restriction to stationary states; we do not outline the Fourier transformations of source fields and source matrices in detail and simply introduce their results for brevity

$$
d[\delta\Sigma^b_{\bar{x}pq}(\omega) \bar{K}] = N_{\delta\Sigma}\bar{K} \left( \prod_{\{\bar{x}, t\}} d(\delta\lambda_+(\vec{x}, \omega)) d(\delta\lambda_-(\vec{x}, \omega)) \left( \delta\lambda_+(\vec{x}, \omega) \right)^2 \left( \delta\lambda_-(\vec{x}, \omega) \right)^2 \times \right.
$$

(4.32)

$$
\times \left. \left( \delta\lambda_+(\vec{x}, \omega) \right)^2 \left( \delta\lambda_-(\vec{x}, \omega) \right)^2 \times d(\alpha_D(\vec{x}, \omega)) \left( \sinh(4\alpha_D(\vec{x}, \omega)) \right) d\beta_D(\vec{x}, \omega) \times \right.
$$

$$
\times \left. \sum_{\{\bar{x}, t\}} d(\cos^2(4\alpha_D(\vec{x}, \omega))) d\gamma_D(\vec{x}, \omega) = N_{\delta\Sigma}\bar{K} \left( \prod_{\{\bar{x}, t\}} d(\delta\lambda_+(\vec{x}, \omega)) d(\delta\lambda_-(\vec{x}, \omega)) \det \left[ \begin{array}{cc}
\delta\lambda_+\bar{K} & \delta\lambda_-(\vec{x}, \omega) \\
\delta\lambda_-(\vec{x}, \omega) & \delta\lambda_+\bar{K}
\end{array} \right] \times \right.ight.
$$

$$
\times \left. \frac{1}{2} d \left( \cos^2(4\alpha_D(\vec{x}, \omega)) \right) d\beta_D(\vec{x}, \omega) \times \left( \prod_{\{\bar{x}, t\}} \frac{1}{2} d \left( \cos^2(4\alpha_D(\vec{x}, \omega)) \right) d\gamma_D(\vec{x}, \omega) \times \right. \right.
$$

$$
\left. \frac{1}{2} d \left( \delta\lambda_+(\vec{x}, \omega) \right) d\beta_-(\vec{x}, \omega) \times \frac{1}{2} d \left( \delta\lambda_-(\vec{x}, \omega) \right) d\beta_-(\vec{x}, \omega) \right) ;
$$

(4.33)

$$
\int d[\delta\Sigma^b_{\bar{x}pq}(\omega) \bar{K}] = \left\langle \exp \left\{ -\frac{1}{2 R^2 \sum_{N_x}} \sum_{\bar{x}, t} \left( 1 - \mu_p^{(f)} \right) \left[ J_{ab}^\psi \bar{K} \bar{K}_{ab} \bar{K} \right] \right\} d[\delta\Sigma^b_{\bar{x}pq}(\omega) \bar{K}] \right\rangle
$$

(4.34)
The corresponding Green function term of (3.97) and gradient operator (3.98) has a similar form in frequency space under the simplifying restriction to stationary states

\[
\langle \tilde{\delta} + \tilde{\sigma}^{(0)} \rangle_{x',x,\omega}^{ab} = \left( \tilde{H}^{+}_{P}({\omega}_p) + \tilde{\sigma}^{(0)}(x) \right) \delta_{ab} \delta_{x',x} \delta(\omega_p - \omega'_q) ;
\]

(4.35)

\[
\delta \tilde{\delta}_{x',x}^{ab}(\tilde{T}^{-1}, \tilde{T}; \omega_p, \omega'_q) = \tilde{T}^{-1;ab'}(\tilde{x}, \omega) \tilde{\delta}^{q',a}_{x'}(\omega'_{p'}, \omega'_q) \tilde{T}_{pp}^{0;ba}(\tilde{x}, \omega') - \tilde{\delta}^{a}_{x',x}^{x'}(\omega_p, \omega'_q).
\]

(4.36)

Although it is straightforward to integrate over the density-related variables \(d|B_D(x, \omega)|, d\beta_D(x, \omega)\) and \(d(\delta \lambda_{x}(x, \omega))\) within the Gaussian factors of \(Z_{I|\beta} (x, \omega)\) in a direct manner, there occurs a further simplifying fact which concerns the parameter \(\mu_P^{(I)}\); as one takes the limit \(N_x = (T_{max}/\Delta t) \rightarrow \infty\) for an infinite number of discrete time steps, the parameter \(\mu_P^{(I)}\) approaches zero for an even simpler calculation of the density-related Gaussian factors within \(Z_{I|\beta}\).

Considering this simplifying aspect, we finally attain the reduced path integral (4.37) for the case of static disorder with the appropriate scale of the disorder parameter and the parameter \(N_x\) for the number of spatial points which allows for a saddle point approximation in the limit \(N_x \rightarrow \infty\).

\[
\langle \tilde{\delta}^{ab}(x, \omega) \rangle_{x,\omega}^{x'} = \exp \left\{ \frac{1}{2} \frac{T_{max}}{\hbar} \sum_{x,p=q=\pm} \eta_p \tilde{\sigma}^{(0)}(x) \tilde{\sigma}^{(0)}(x) - \int_{\mathcal{C}_\omega} \frac{d\omega_p}{2\pi} \eta_p \sum_{x} N_x \text{tr} \ln \left[ \left( \tilde{\delta}^{11} + \tilde{\sigma}^{(0)}(x) \right)^{11}_{x'=x',p=q}(\omega, \omega') = t \right] \right\}
\]

\[
\times \exp \left\{ \frac{N_x^2}{\hbar} \int_{0}^{+\omega} \frac{d\omega}{2\pi} \left( \frac{d\omega}{2\pi} \right) \sum_{p,q=\pm} N_x T_{max}^2 \left( \frac{d\omega}{2\pi} \right) \sum_{p,q=\pm} \eta_p \sum_{x} N_x \text{tr} \ln \left[ \left( \tilde{\delta}^{11} + \sigma^{(0)}_{V_0}(x) \right)^{11}_{x'=x',p=q}(\omega, \omega') = t \right] \right\}
\]

\[
\times \left( \text{coset fields and } \tilde{\sigma}^{(0)}(x) \right).
\]

4.1 Transformation to the case 'I' with static disorder reduced to stationary states

Although it is straightforward to integrate over the density-related variables \(d|B_D(x, \omega)|, d\beta_D(x, \omega)\) and \(d(\delta \lambda_{x}(x, \omega))\) within the Gaussian factors of \(Z_{I|\beta}\) in a direct manner, there occurs a further simplifying fact which concerns the parameter \(\mu_P^{(I)}\); as one takes the limit \(N_x = (T_{max}/\Delta t) \rightarrow \infty\) for an infinite number of discrete time steps, the parameter \(\mu_P^{(I)}\) approaches zero for an even simpler calculation of the density-related Gaussian factors within \(Z_{I|\beta}\). Considering this simplifying aspect, we finally attain the reduced path integral (4.37) for the case of static disorder with the appropriate scale of the disorder parameter and the parameter \(N_x\) for the number of spatial points which allows for a saddle point approximation in the limit \(N_x \rightarrow \infty\).
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