A New Look at Composition of Authenticated Byzantine Generals

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Abstract

The problem of Authenticated Byzantine Generals (ABG) aims to simulate a virtual reliable broadcast channel from the General to all the players via a protocol over a real (point-to-point) network in the presence of faults. We propose a new model to study the self-composition of ABG protocols. The central dogma of our approach can be phrased as follows: Consider a player who diligently executes (only) the delegated protocol but the adversary steals some private information from him. Should such a player be considered faulty? With respect to ABG protocols, we argue that the answer has to be no.

In the new model we show that in spite of using unique session identifiers, if $n < 2t$, there cannot exist any ABG protocol that composes in parallel even twice. Further, for $n \geq 2t$, we design ABG protocols that compose for any number of parallel executions. Besides investigating the composition of ABG under a new light, our work also brings out several new insights into Canetti’s Universal Composability framework. Specifically, we show that there are several undesirable effects if one deviates from our dogma. This provides further evidence as to why our dogma is the right framework to study the composition of ABG protocols.

Keywords: Protocol composition, Authenticated Byzantine Generals, Universal composability, Unique session identifiers.

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1 Introduction

The goal of an Authenticated Byzantine Generals (ABG) protocol is to simulate a virtual reliable broadcast channel from the General to all the players via a protocol over a real (point-to-point) network in the presence of faults. In general, one may wish to simulate a virtual network which may not only involve some reliable/secure channels but also reliable/secure nodes (like a Trusted Third Party). Traditionally, the notion of faults in the network is captured via a fictitious entity called adversary that can choose to actively corrupt up to any \( t \) of the \( n \) players. A player is said to be non-faulty if he executes only the delegated protocol code and does no more. In contrast, the computation performed by the faulty players is chosen by the adversary.

It is well known that protocols which correctly simulate the desired virtual network may not remain correct when run in presence of same/other protocols [Can01]. For most real life networks such as the Internet, a protocol is seldom executed in a stand alone setting. Composition of protocols aims to study the correctness/security of protocols when several protocols are run concurrently.

Over the past few decades security of cryptographic protocols in stand alone settings has been studied fairly well. Canetti extended these studies to arbitrary unknown environments by introducing the framework of Universal Composability (UC). Ever since, several researchers have built on it to prove many exciting results. Besides providing a rigorous and an elegant mathematical framework for proving security guarantees of protocols that run in arbitrary environments, UC is a classic notion that has seemingly brought secure function evaluation and multi-party computation ever close to practice.

1.1 Prior Work

Byzantine Generals Problem (BGP) and Byzantine Agreement (BA) were first introduced by Pease et al. [LSP82, PSL80]. It is well known that BGP (likewise BA) over a completely connected synchronous network is possible if and only if \( n > 3t \) [PSL80, LSP82]. Later on, the problem was studied in many different settings, giving both possibility (protocols) and impossibility results. Some of the prominent settings are incomplete networks [Dol81], probabilistic correctness [Rab83], asynchronous networks [FLP85], partially synchronous networks [DDS87], mobile adversaries [Gar94], non-threshold adversarial model [FM98], mixed adversarial model [GP92, AFM99], hypergraphs [FM00] to name a few.

Pease et al. [PSL80, LSP82] introduced the problem of authenticated Byzantine Generals (ABG). Here, the players are augmented with Public Key Infrastructure (PKI) for digital signatures to authenticate themselves and their messages. Pease et al. proved that in such a model tolerability against a \( t \)-adversary can be amazingly increased to \( n > t \), which is a huge improvement over \( n > 3t \). Being a reasonably realistic model and because of its high fault tolerance, ABG is an important and popular variant of BGP and hence, has been fairly well studied. Dolev [DS83] proved that any ABG protocol over a completely connected synchronous network of \( n \) nodes tolerating a \( t \) Byzantine adversary will require \( t + 1 \) rounds of communication. Further, he proposed algorithms that takes \( O(t + 1) \) rounds and \( O(nt) \) messages. Authenticating every message being sent by a player can be an expensive. Some works have explored cost cutting by considering alternatives to authentication and limiting the use of signatures. Specifically, Borcherding [Bor95, Bor96b] explored the possibility of using signatures in only some rounds and not all. An alternative line of thought was suggested by Srikanth and Toueg [ST87] wherein authenticated messages are simulated by non-authenticated sub-protocols. In another work, Borcherding [Bor96a] studied different levels and styles of authentication and its effects on the agreement protocols. This work focuses on understanding the properties of authentication scheme their impact on building faster algorithms.
for BGP. Gong et al. [GLR95] study the assumptions needed for the authentication mechanism in protocols for BGP that use signed messages. They propose protocols for BA that add authentication to oral message protocols so as to obtain additional resilience due to authentication. Schmid and Weiss [SW04] study ABG under hybrid failure model of node and communication failures. Katz et al. [KK09] propose expected constant round ABG protocols in the case \( n > 2t \). Gupta et al. [GGBS10] study ABG in a model where in the adversary can corrupt some players actively and some more players passively. Further, they require the passively corrupt player to be consistent with the honest players. They show that their model unifies the results of \( n > 3t \) (BPG) and \( n > t \) (ABG). [GKKY10] too explores ABG in a partially compromised signature setting. Bansal et al. [BGG+11] extend the studies of [GGBS10] to the case arbitrarily connected (undirected) networks.

Security of protocols under composition was first investigated in [Ore87, MR91, GO94, GK96]. Owing to its impact on the modular approach of constructing cryptographic protocols, composition of protocols has been well studied in literature. Goldreich and Krawczyk [GK96] studied sequential and parallel composition zero-knowledge protocols. They proved that zero-knowledge and strong formulation of zero-knowledge (e.g. black box simulation) are not closed under parallel execution. Richardson and Kilian [RK99] examined the concurrent composition of zero-knowledge proofs. Canetti et al. [CKPR01] proved that Black-box concurrent zero-knowledge requires \( \omega(\log n) \) rounds. Dwork et al. [DNS04] show that under the assumption of a restricted adversary (they call it \((\alpha,\beta)\) constraint) there exists perfect concurrent zero-knowledge arguments for every language in class \( NP \). Canetti [Can00] proposed generic definitions of security for multi-party cryptographic protocols and proved that the security under these definitions continue to hold under the natural composition operation [MR91]. Canetti [Can01] introduced UC to study the security/correctness of protocols when run with arbitrary unknown protocols. [CF01, CK02] study commitments and key exchanges under composition. Canetti et al. [CLOS02] show that any two-party and multi-party functionality is closed under universal composition, irrespective of the number of corrupted players. Canetti and Rabin [CR03] initiated the study of universal composition with joint state. Ben-Or et al. [BOHL+05] took up the study of universal composability in quantum key distribution. Some of the recent papers on protocol composition are [Lin03, PS04, HUMQ09, CKS11, CH11, RO12].

Lindell et al. [LLR02] studied the properties of self composition of ABG. They proved that, over a completely connected synchronous network of \( n \) players in presence of a \( t \)-adversary, if \( n \leq 3t \), then there does not exist any ABG protocol that self-composes in parallel even twice. Further, for \( n > 3t \), they designed ABG protocols that self-compose in parallel for any number of executions. Thus, proving the bound of \( n > 3t \) to be tight. In the same work, they also show that if one assumes additional facility of unique session identifiers, fault tolerance for ABG under parallel self-composition can be restored back to \( n > t \).

The work closest (yet, incomparable) to our line of thought is the model considered by Canetti and Ostrovsky [CO99]. They use a slightly different perspective as to what a fault “means”. In particular, they operate under a model where in all the parties (even uncorrupted ones) may deviate from the protocol but under the sole restriction that most parties do not risk being detected by other parties as deviating from the protocol.

**Organization of the Paper.** In Section 2, we present our case as to why the study of self composition of ABG protocols needs a new approach. This renders the model used in the extant literature (to study the protocol composition) inappropriate for, at least, a few problems such as ABG and necessitates the formulation of a better model. In Section 3, we propose a new model to study protocol composition. We prove our results in Section 4.
2 The Central Dogma

Consider a player who is concurrently executing several protocols which run as processes. Clearly, the player is faulty if any one of these processes deviates from the originally designated protocol code. However, there can be faulty players wherein some of the processes continue to diligently execute the delegated protocol code. Should such processes be deemed faulty? The answer can be no since they diligently execute the delegated protocol, hence, they are certainly not Byzantine faulty. On the other hand, the answer can be yes because the player is faulty and therefore, the data private to such processes can always be accessed by the adversary\(^1\).

All of literature on composition of protocols and in particular, the most recent one on ABG [LLR02] has considered such processes to be Byzantine faulty. However, in Section 2.1 and 2.2 we argue as to why it is better to model such processes as passively corrupt (similar to honest but curious parties).

2.1 To Make Them Faulty or Not?

The extant literature on (unauthenticated) reliable broadcast requires all non-faulty players to agree on the same value [PSL80, LSP82]. Consequently, a player can be non-faulty in the following two ways – (i) The adversary is absent and (therefore) the player follows the delegated protocol. (ii) The adversary is present, but allows the player to diligently follow the delegated protocol and therefore, by the virtue of diligently following the protocol the player is non-faulty.

With respect to ABG, the answer does not reveal itself automatically. The issue with ABG is more subtle because ABG is spawned by interests across various disciplines. In particular, ABG has continuously drawn inspiration from cryptography and in particular secure multi-party computation (SMPC). So, in the case of ABG, any attempt to settle the question must consider the cryptographic viewpoint. When it comes to defining faults in SMPC, recall that, Ber-Or et al. [BGW88] define a player as faulty if and only if the player deviates from the designated protocol. Therefore, w.r.t ABG, we have a choice – (i) To punish the player by labelling him as faulty, if the adversary steals any private information (such as digital signature) from the player, despite the player diligently executing the designated protocol. (ii) To reward the player for diligently following the protocol and pay him back for his efforts by not labelling him as faulty (remember, that this player has certainly helped in the simulation of broadcast channel by routing several crucial messages). In continuation, it is natural to brand all processes, whose private information is stolen by the adversary during a cryptographic protocol, as faulty. However, we believe w.r.t. ABG, the answer has to be the other way around. One may argue that reliable broadcast is essentially a primitive in distributed computing and that authentication was introduced only as a tool. Rather, authentication was only a means to facilitate the end (reliable broadcast). This is, however, hardly any reason. To find the answer, one must journey to the very heart of every protocol for Byzantine Generals (BG).

The purpose of any BG (likewise ABG) protocol is to simulate a (virtual) reliable broadcast channel over a point-to-point network. Consider a scenario wherein the General is connected to all the players via an actual physical broadcast channel. All the players including those under the adversary’s control will always receive the same message from the General. The adversary can make the players under his influence to discard this message and deviate from the protocol. However, if the adversary chooses not to do so for any of the player(s) under his control, then such a player(s) will be in agreement with the group of players who were honest. Therefore, any ABG

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\(^1\)Any process with administrative privileges can always read the data internal to any other process within the same system.
A protocol aiming to truly simulate a broadcast channel must ensure consistency between all the players who follow the designated protocol.

We now elaborate the implications of our dogma on the composition of ABG protocols. It is well known that in the stand alone execution model, a $t$-adversary is free to corrupt up to any $t$ players. With respect to parallel composition of protocols, a $t$-adversary is free to choose any set of $\leq t$ players and corrupt them in all or only some of the executions. This permits the adversary to corrupt different players in different executions i.e. a $t$-adversary may as well corrupt, say $t_1$ players ($t_1 < t$) in some of the parallel execution(s) and a different set of $t_2$ players ($t_2 < t$) in the remaining execution(s). As long as $t_1 + t_2 \leq t$, w.r.t composition, such an adversary is a valid $t$-adversary.

The above leads to an interesting observation w.r.t composition of ABG protocols – by Byzantine corrupting a player in some and not all parallel executions, the adversary can forge messages on behalf this player even in those executions wherein this player is uncorrupted. We facilitate the same with the help of the following simple scenario: Consider a player $P$ running two parallel executions, say $E_1$ and $E_2$, of some (correct) ABG protocol (Figure 1). Further, $P$ uses distinct authentication keys, say $k_1$ and $k_2$ in the executions $E_1$ and $E_2$ respectively. The adversary corrupts $P$ in Byzantine fashion only in $E_1$. Consequently, the adversary can forge messages on behalf of $P$ in $E_2$ even though $P$ is non-faulty in $E_2$. This is because in $E_1$ the adversary can delegate that code to $P$ which can read the key $k_2$ (or for that matter any private data) from the process $E_2$ (Figure 2).

The above observation can be extended to any number parallel executions $E_1, E_2, \ldots, E_k$. It is easy to see that the above observation holds good even if $P$ uses a distinct authentication key in each of the parallel executions $E_1, E_2, \ldots, E_k$.

### 2.2 New insights into current model

Besides providing a new model for composing ABG protocols, our work also draws attention to a few (not so serious, yet interesting) grey areas in the existing popular models of protocol composition. These issues deserve to be discussed in greater detail and we deem that these aspects must be studied in depth before one can unleash the paradigm of composability into the real world. The following can also be viewed as the ill-effects of modelling the non-faulty processes within a faulty player as Byzantine faulty:

1. **Proving incorrect protocols as correct:** A very popular paradigm used in the literature to capture the security/correctness requirements of a cryptographic protocol is the ideal-world/real-world simulation paradigm [Can01]. Informally, in this paradigm, a protocol is said to be correct if for every real world adversary $\mathcal{A}$ there exists an ideal world adversary $\mathcal{S}$ that can
match the views of the players and the adversary in the two worlds. We now show that if the non-faulty processes within a faulty player are treated as Byzantine faulty, then, using the ideal-world/real-world simulation paradigm, the protocols which are incorrect in the stand-alone settings can be proven to be correct under composition!

Example: Consider the problem of secure addition – players \{P_a, P_b, P_c, P_d\} start with input values \{v_a, v_b, v_c, v_d\} respectively and wish to find the sum of their combined input values without revealing their input value to any other player. Consider a protocol \(\xi\) which gives a random value as the answer. Clearly, in a stand-alone setting \(\xi\) is an incorrect protocol. However, one can prove \(\xi\) to be a correct protocol under composition as follows: Let \(E_1\) and \(E_2\) be two concurrent executions of \(\xi\). The real world adversary \(A\) corrupts player \(P_a\) actively only in \(E_1\). In \(E_2\), \(P_a\) executes the code exactly as per the specifications of \(\xi\). If \(P_a\) is treated as Byzantine faulty in \(E_2\), then in the ideal world execution the ideal world adversary \(S\) can actively corrupt \(P_a\) in \(E_2\) as well. Thus, in the ideal world execution of \(E_2\), \(S\) sends that input value to the TTP (Trusted Third Party) which ensures that the view of the players and the adversary in the ideal world and real world is same. Specifically, if in \(E_2\), the protocol \(\xi\) gives the answer as \(v_a + v_b + v_c + v_d + r\) where \(r\) is some positive random number. Then, in the corresponding ideal world execution, \(S\) on behalf of \(P_a\) sends \(v_a + r\) as the input value to the TTP. This will ensure that the views in the two worlds are same.

2. Basing security on internal communication: Let \(\Lambda\) be a secure composable protocol for some problem \(\zeta\). From \(\Lambda\) one can always construct a new protocol \(\Lambda'\) as follows: \(\Lambda'\) is exactly same as \(\Lambda\) except for the following two changes – (i) Every process in \(\Lambda'\) sends all it’s data (including its private data) to all the concurrent processes within the same player. (ii) Every process in \(\Lambda'\) ignores this incoming data from any of the fellow concurrent process within the same player. Clearly, if \(\Lambda'\) is secure then so is \(\Lambda\). However, is \(\Lambda'\) secure given \(\Lambda\) is secure? The answer can be no if, in \(\Lambda'\), the faulty processes chose not to ignore the incoming data.

This implies that the security definition is dependent on the internal communication between the processes within a player. Clearly, one will prefer to have a security definition which does not depend on such intricate details. As highlighted by Canetti [Can00], this preference stems from the need and benefits of a simple, intuitive and workable security definition.

In essence, our dogma is the following: All non-faulty processes, i.e. the processes within a non-faulty player that execute the delegated protocol diligently and do no more, are considered honest. All Byzantine faulty processes in a Byzantine faulty player are considered corrupt. All non-faulty processes within a Byzantine faulty player are treated as passively corrupt. We aim to study self composition of ABG protocols in this new paradigm.

3 Model

We are now ready to present our model. Our model is same as the one used in the extant literature [Can01, LLR02] except for the following (small but important) changes – (i) All non-faulty processes within a non-faulty player are considered as honest. (ii) All Byzantine faulty processes within a Byzantine faulty player are considered as corrupt. (iii) All non-faulty processes within a Byzantine faulty player are treated as passively corrupt. (iii) follows from the observations made in Section 2). Here, a process is said to be non-faulty if it exactly executes the delegated protocol code and does no more. Further, a process is said to be Byzantine faulty if it executes the program code of adversary’s choice.
We consider a set of \( n \) players, \( P = \{ p_1, p_2, \ldots, p_n \} \), over a completely connected synchronous network. Any protocol in this setting is executed in a sequence of rounds where in each round, a player can perform some local computation, send new messages to all the players, receive messages sent to him by other players in the same round, (and if necessary perform some more local computation), in that order. The notion of faults in the system is captured by a virtual entity called adversary. During the execution, the (polynomial-time) adversary\(^2\) may take control of up to any \( t \) players and make them behave in any arbitrary fashion. Such an adversary is called as a \( t \)-adversary. Further, the players can invoke multiple parallel executions of any protocol. We model this via players running multiple processes in parallel. We assume that the communication channel between any two players is perfectly reliable and authenticated. We also assume existence of a (signature/authentication) scheme via which players authenticate themselves. This is modelled by all the players having an additional setup-tape that is generated during the preprocessing phase. Note that keys cannot be generated within the system itself. Similar to [LLR02], it is assumed that the keys are generated using a trusted system and distributed to players prior to running of the protocol. Typically, in such a preprocessing phase, signatures and verification keys are generated. That is, each player gets his own private signature key, and in addition, public verification keys for all the other players. No player can forge any other player’s signature and the receiver can uniquely identify the sender of the message using the signature. However, the adversary can forge the signature of all the \( t \) players under its control. The adversary can inject forged messages, on behalf passively corrupt processes, via Byzantine corrupt processes. Further, we assume that each run of a protocol is augmented with unique session identifiers (USIDs).

3.1 Defining Composable ABG

We use the well established ideal/real process simulation paradigm to define the requirements of ABG. Both the ideal process and the real process have the set \( P \) of \( n \) players including the General \( G \) as common participants. Apart from these, the ideal process has a TTP (Trusted Third Party) and an ideal process adversary \( S \) whereas the real process has a real process adversary \( A \). We start by defining the ideal process for ABG.

**Ideal process** (\( \Psi_{ideal} \)): (1) \( G \) sends his value \( v \) to TTP. (2) TTP forwards the same to all the \( n \) players and \( S \). (3) All honest players output \( v \). \( S \) determines the output of faulty players.

We assume that all message transmissions in the above protocol are perfectly secure.

Let \( IDEAL_{TTP,S}(v, r_S, \vec{r}) \) denote a vector of outputs of all \( n \) players running \( \Psi_{ideal} \) where \( G \) has input \( v \), \( S \) has random coins \( r_S \) and \( \vec{r} \) where \( \vec{r} = r_1, r_2, \ldots, r_n, r_{TTP}; r_1, r_2, \ldots, r_n \) and \( r_{TTP} \) are the random coins of \( n \) players and the TTP respectively. \( IDEAL_{TTP,S}(v) \) denotes the random variable describing \( IDEAL_{TTP,S}(v, r_S, \vec{r}) \) when \( r_S \) and \( \vec{r} \) are chosen uniformly at random. \( IDEAL_{TTP,S} \) denotes the ensemble \( \{ IDEAL_{TTP,S}(v) \}_{v \in \{0,1\}} \).

**Real life process** (\( \Psi_{real}(\Pi) \)): Unlike in the ideal process, here the players interact among themselves as per a designated protocol \( \Pi \) and the real process adversary \( A \). Specifically: (1) Every honest player proceeds according to the protocol code delegated to him as per \( \Pi \). (2) The adversary \( A \) may send some arbitrary messages (perhaps posing as any of the corrupt players) to some/all

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\(^2\)Digital signatures based authentication necessitates the assumption of a polynomial-time adversary. Our impossibility proofs do not need this assumption but our protocols require a “magical” means to authenticate if the adversary is unbounded.
of the players. (3) Honest players output a value as per $\Pi$. $A$ determines the output of faulty players.

Let $REAL_{\Pi,A}(v, r_{A}, \bar{r})$ denote a vector of output of all $n$ players running $\Psi_{\text{real}}(\Pi)$ where $G$ has input $v$, and $r_{A}, \bar{r} = r_{1}, r_{2}, \ldots, r_{n}$ are the random coins of the adversary and $n$ players respectively. Let $REAL_{\Pi,A}(v)$ denote the random variable describing $REAL_{\Pi,A}(v, r_{A}, \bar{r})$ when $r_{A}$ and $\bar{r}$ are chosen uniformly at random. Let $REAL_{\Pi,A}$ denote the ensemble $\{REAL_{\Pi,A}(v)\}_{v \in \{0,1\}}$.

We directly adopt the definitions of Lindell et al.[LLR02].

**Definition 1 (ABG)** $\Pi$ is an ABG protocol tolerating a $t$-adversary if for any subset $I \subseteq \mathbb{P}$ of cardinality up to $t$ (that is, $|I| \leq t$), it holds that for every probabilistic polynomial-time real process adversary $A$ that corrupts the players in $I$ in $\Psi_{\text{real}}(\Pi)$, there exists a probabilistic polynomial-time ideal process adversary $S$ in $\Psi_{\text{ideal}}$ that corrupts the players in $I$, such that the ensembles $\text{IDEAL}_{TTPS, S}$ and $REAL_{\Pi,A}$ are computationally indistinguishable.

**Definition 2 (Composable ABG [LLR02])** Let $\Pi$ be an ABG protocol. $\Pi$ is said to remain secure under parallel composition if for every polynomial time adversary $A$, the requirements for ABG (which is elaborated in Definition 1) hold for $\Pi$ for every execution within the following process: Repeat the following process in parallel until the adversary halts:

1. The adversary $A$ chooses the input $v$ for the General $G$.
2. All players are invoked for an execution of $\Pi$ (using the strings generated in the preprocessing phase and an unique session identifier for this execution). All the messages sent by the corrupted players are determined by the adversary $A$, whereas all other players follow the instructions of $\Pi$.

Furthermore, as noted by Lindell et al., Definition 2 implies stateless composition i.e. all honest players are oblivious to the other executions taking place in parallel. In contrast, the adversary $A$ can coordinate between the parallel executions, and the adversary’s view at any given time includes all the messages received in all the executions.

### 3.2 Our Results

Recall that in the absence of unique session identifiers, ABG is not self-composable even twice if $n \leq 3t$ [LLR02]. We prove that unique session identifiers aid in improving the fault-tolerance of ABG protocols (that compose in parallel) but from $n > 3t$ only to $n \geq 2t$. We, now, present the main theorem of this paper:

**Theorem 1 (Main Theorem)** ABG over $n$ players, tolerating a $t$-adversary, can be self-composed in parallel for any number of executions if and only if $n \geq 2t$.

To put things in perspective, one can achieve the bound of $n > t$ for (a simplified variant of) ABG [LLR02] if one makes the following assumption: *Only those non-faulty processes that run in non-faulty players need to be consistent, others need not*, where a process is faulty if it deviates from the designated protocol.
4 Complete Characterization

The aim of this section is to prove aforementioned Theorem 1. We begin with a few definitions:

Definition 3 (Adversary Structure) An adversary structure $\mathcal{Z}$ for the player set $\mathcal{P}$ is a collection of plausible sets of players which can be corrupted by the adversary. Formally, $\mathcal{Z} \subseteq 2^\mathcal{P}$, where all subsets of $\mathcal{Z}$ are in $\mathcal{Z}$ if $Z \in \mathcal{Z}$.

Definition 4 (Adversary Basis) For an adversary structure $\mathcal{Z}$, $\bar{\mathcal{Z}}$ denotes the basis of the structure, i.e. the set of the maximal sets in $\mathcal{Z}$:

$$\bar{\mathcal{Z}} = \{ \mathcal{Z} \in \bar{\mathcal{Z}} : \nexists \mathcal{Z}' \in \bar{\mathcal{Z}} : \mathcal{Z} \subset \mathcal{Z}' \}$$

4.1 Qualifiers

We first prove that there does not exist any ABG protocol that self-composes in parallel even twice over a network of 3 players, $\mathcal{P} = \{A, B, C\}$, tolerating an adversary basis $\bar{\mathcal{A}} = \{(C), (A); (A), (\emptyset); (B), (A))\}$. Here, $(x, y)$ represents a single element of the adversary basis such that the adversary can Byzantine corrupt $x$ and $y$ in the first and second parallel execution, respectively. For the rest of this paper $\Pi_k$ (likewise $\Delta_k$) denotes an ABG protocol $\Pi$ ($\Delta$) that remains correct upto $k$ parallel self-compositions.

Before presenting the proof, we make a few comments on the proof style. As was with Lindell et al. (the base case of their work draws inspiration from [FLM85]), we establish ours on [GGBS10]. We, however, note that the overlap ends there. We remark that this is not a serious concern and if at all everything but a testimony to the impact of [FLM85].

Theorem 2 There does not exist any $\Pi_2$ over a network of 3 nodes, $\mathcal{P} = \{A, B, C\}$, tolerating an adversary basis $\bar{\mathcal{A}} = \{(C), (A); (A), (\emptyset); (B), (A))\}$.

Proof: Our proof demonstrates that the real process adversary (characterized by $\bar{\mathcal{A}}$) can make the non-faulty processes in one of the parallel executions of any ABG protocol to have an inconsistent output. In contrast, in the corresponding ideal world execution the non-faulty processes are guaranteed to have a consistent output. It then follows that there does not exist any ideal world adversary $\mathcal{S}$ that can ensure that the output distributions are similar. This violates Definition 2, hence the theorem.

To prove that $\bar{\mathcal{A}}$ can ensure that the non-faulty processes in one of the parallel executions do not have a consistent output, we assume otherwise and arrive at a contradiction. We accomplish the same using the ideas from the proof technique developed by Fischer et al. [FLM85].

Formally, assume for contradiction that there exists a protocol $\Pi_2$ over $\mathcal{N}$ (Figure 3), $\mathcal{P} = \{A, B, C\}$, tolerating the adversary basis $\mathcal{A} = \{(C), (A); (A), (\emptyset); (B), (A))\}$. Using $\Pi_2$, we create a protocol $\Pi' \ [Definition 5]$ in such a way that existence of $\Pi_2$ implies existence of $\Pi'$ (Proposition 5.1). We, then, combine two copies of $\Pi'$ to construct a system $\mathcal{L}$ (Figure 4) and show that $\mathcal{L}$ must exhibit contradictory behaviour. It then follows that the assumed protocol $\Pi_2$ cannot exist.

We do not know what system $\mathcal{L}$ solves. Formally, $\mathcal{L}$ is a synchronous system with a well defined behaviour. That is, for any particular input assignment $\mathcal{L}$ exhibits some well defined output distribution. We obtain a contradiction by showing that for a particular input assignment, no such well defined behaviour is possible. No player in $\mathcal{L}$ is aware of the complete system, rather...
each player is aware of only his immediate neighbours. In reality, a player may be connected to either \( A \) or \( A' \), but he cannot distinguish between the two. He knows his neighbours only by their local name, in this case, \( A \). Further, all players in \( \mathcal{L} \) are oblivious to the fact that there are duplicate copies of the nodes in \( \mathcal{L} \). Specifically, for all \( X \in \{A, B, C\} \), \( \mathcal{L} \) is constructed in a manner such that the in-neighbourhood of any node \( X \) (or \( X' \)) in \( \mathcal{L} \) is same as the in-neighbourhood of the corresponding node \( X \) in \( \mathcal{N} \).

Let the players in \( \mathcal{L} \) start with input values as indicated in Figure 4; and \( \alpha \) be the resulting execution. All the players in \( \alpha \) are honest and diligently follow \( \Pi' \). Further, let \( E_1 \) and \( E_2 \) be two parallel executions of \( \Pi_2 \) over \( \mathcal{N} \). We, now, define three distinct scenarios – \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \):

- \( \alpha_1 \): In \( E_1 \), \( A \) is the \textit{General} and starts with input value 0. \( \bar{A} \) Byzantine corrupts \( C \) in \( E_1 \). In \( E_2 \), \( \bar{A} \) Byzantine corrupts \( A \).
- \( \alpha_2 \): In \( E_1 \), \( A \) is the \textit{General}. Further, in \( E_1 \), \( \bar{A} \) corrupts \( A \), interacts with \( B \) as if \( A \) started with input value 0 and interacts with \( C \) as if \( A \) started with input value 1.
- \( \alpha_3 \): In \( E_1 \), \( A \) is the \textit{General} and starts with input value 1. \( \bar{A} \) Byzantine corrupts \( B \) in \( E_1 \). In \( E_2 \), \( \bar{A} \) Byzantine corrupts \( A \).

We claim that, in execution \( \alpha \), both \( A \) and \( B \) decide on 0. Similarly, we claim that both \( C \) and \( A' \) in \( \alpha \) decide on 1. Further, we claim that in \( \alpha \) both \( B \) and \( C \) must decide on the same output (Figure 5). [To avoid clutter we defer the proof of these claims to Lemma 4, 6 and 8.] However, in \( \alpha \), players \( B \) and \( C \) have already decided on 0 and 1, respectively. Hence, \( \mathcal{L} \) exhibits a contradictory behaviour.

To complete the above proof, we now define the protocol \( \Pi' \) [Definition 5] and show that existence of \( \Pi_2 \) implies existence of \( \Pi' \) (Proposition 5.1). We then prove the left out claims via Lemma 3 - Lemma 8.

**Definition 5 (**\( \Pi' \)**) For player \( B \), every statement in \( \Pi_2 \) of the kind “\( B \) sends message \( m \) to \( A \)” is replaced by “\( B \) multicasts message \( m \) to all instances of \( A' \)” (i.e. \( A, A' \)) in \( \Pi' \). Similarly, every

\( ^3 \)A and \( A' \) are independent copies of \( A \) with the same authentication key.
statement of the kind “C sends message m to A” in Π₂ is replaced by “C multicasts message m to all instances of A” (i.e. A, A’) in Π’. Rest all statements in Π’ are exactly same as those in Π₂.

**Proposition 5.1** If Π₂ exists, then so does Π’.

**Proof**: Follows directly from Definition 5. Given Π₂, one can always construct Π’ by making appropriate changes in Π₂ as per the definition of Π’.

Rest of this section focuses on proving Lemma 3 - Lemma 8. The proofs are conceptually simple however, owing to the “topology” of system L and presence of authenticated messages, some of the proofs (Lemma 3, 5 and 7) have tedious details. A reader interested in the proof of our main theorem can directly jump to Section 4.2.

We begin by introducing the terminology used in the proofs. Let $msg^Ω_{i}(x,y)_x$ denote the message sent by player $x$ to player $y$ in round $i$ of execution $Ω$. The $x$ in the subscript refers to the last player who authenticated this message. W.l.o.g, we assume that every player always authenticates every message sent by him. Further, let $V^Ω_{x,i}$ denote view of player $x$ at the end of round $i$ in execution $Ω$. Intuitively, $V^Ω_{x,i}$ consists of everything that player $x$ ever “sees” from round 1 until the end of round $i$ in execution $Ω$. For our setting this includes (w.r.t execution $Ω$) – (i) Input value (if any) of $x$ in round $i$ of execution $Ω$: $I^Ω_{x,i}$. (ii) Secret key used by $x$ for authentication: $SK^Ω_{x,i}$. (iii) Protocol code executed by $x$: $θ^Ω_{x,i}$. (iv) Set of all the messages sent by $x$ until the end of round $i$: $∀z ∈ P$, $∀k ∈ (1, i)$, $⋃k(msg^Ω_{k}(z,x)_x)$. (v) Set of all the messages received by $x$ until the end of round $i$: $∀z ∈ P$, $∀k ∈ (1, i)$, $⋃k(msg^Ω_{k}(z,x)_z)$. Formally:

$$V^Ω_{x,i} = [I^Ω_{x,i}, SK^Ω_{x,i}, θ^Ω_{x,i}, ⋃k(msg^Ω_{k}(x,z)_x), ⋃k(msg^Ω_{k}(z,x)_z)]; ∀z ∈ P, ∀k ∈ (1, i) \tag{1}$$

Since the messages sent by player $x$ in round $i$ of $Ω$ is a function of $x$’s view until the end of round $i - 1$ (i.e. $V^Ω_{x,i-1}$), Equation 1 can be rewritten as:

$$V^Ω_{x,i} = [I^Ω_{x,i}, SK^Ω_{x,i}, θ^Ω_{x,i}, ⋃k(msg^Ω_{k}(z,x)_z)]; ∀z ∈ P, ∀k ∈ (1, i) \tag{2}$$

Our proofs will often have statements of the following form – view of player $x$ until round $i$ of execution $γ$ is same as the view of player $y$ until round $i$ of execution $δ$ (dubbed as $V^γ_{x,i} ∼ V^δ_{y,i}$). In order to prove such statements we will use the following observation:-
\[ V_{x,i}^\gamma \sim V_{y,i}^\delta \text{ if and only in the following conditions} \]

(i) \[ T^\gamma_x = T^\delta_y \]
(ii) \[ SK^\gamma_x = SK^\delta_y \]
(iii) \[ \theta^\gamma_x = \theta^\delta_y \]
(iv) \[ \forall z \in \mathbb{P}, \forall k \in (1, i), \ msg_k^\gamma(z, x)_z = msg_k^\delta(z, y)_z \]

Through out our proofs conditions (i), (ii) and (iii) will be trivially satisfied. Thus, our proofs will focus on proving condition (iv). For brevity we say:

\[ V_{x,i}^\gamma \sim V_{y,i}^\delta \iff \forall z \in \mathbb{P}, \forall k \in (1, i), \ msg_k^\gamma(z, x)_z = msg_k^\delta(z, y)_z \] (3)

Let \( V_x^\gamma \) denote view of \( x \) at the end of execution \( \gamma \). Then,

\[ V_x^\gamma \sim V_y^\delta \iff \forall k > 0, \ V_{x,k}^\gamma \sim V_{y,k}^\delta \] (4)

Combining (3) and (4), we get

\[ V_x^\gamma \sim V_y^\delta \iff \forall z \in \mathbb{P}, \forall k > 0, \ msg_k^\gamma(z, x)_z = msg_k^\delta(z, y)_z \] (5)

Though conceptually simple, proving the right hand side of Equation 5 can be a tedious task. This is because the use of authentication limits the adversary’s ability to send forged messages. Hence, one must formally establish that the adversary can indeed ensure that the right hand side of Equation 5 holds true. To facilitate the same, we introduce the notion of execution trees. To understand the utility of execution trees, let us revisit the scenarios \( \alpha \) and \( \alpha_1 \) as defined in the proof of Theorem 2. Recall that the proof requires us to show that \( \forall z \in \{A, B, C\}, \ \forall k > 0, \ msg_k^\gamma(z, A)_z = msg_k^{E_1: \alpha_1}(z, A)_z \). Note that what \( A \) receives in round \( i \) of \( \alpha \) (likewise \( E_1 : \alpha_1 \)) depends on what \( B \) and \( C \) send him in round \( i \) of \( \alpha \) (likewise \( E_1 : \alpha_1 \)). So, we need to argue that these messages, sent in round \( i \) of \( \alpha \) and \( E_1 : \alpha_1 \) respectively, are either same or can be made same by the adversary. The messages sent by \( B \) and \( C \) in round \( i \) of \( \alpha \) (likewise \( E_1 : \alpha_1 \)) depend on what they themselves receive in round \( i - 1 \). This in turn depends on what \( A \) and \( C \) (likewise, \( A \) and \( B \)) send to \( B \) (\( C \)) in round \( i - 2 \) of \( \alpha \) (\( E_1 : \alpha_1 \)). Thus, we need to argue that the adversary can ensure that whatever messages \( A, C \) (likewise, \( A \) and \( B \)) send to \( B \) (\( C \)) in round \( i - 2 \) of \( \alpha \) is same as whatever messages \( A, C \) (\( A \) and \( B \)) send to \( B \) (\( C \)) in round \( i - 2 \) of \( E_1 : \alpha_1 \). Note that this continues in a recursive manner until the recursion stops at round 1. The entire recursion can be visualized as trees, \( T^\alpha_A \) and \( T^{E_1: \alpha_1}_A \), rooted at \( A \) for executions \( \alpha \) and \( E_1 : \alpha_1 \) respectively, as shown in Figure 6.

An execution tree can be regarded as a “visual” analogue of Equation 5. Formally, \( T^\Omega_X \) is an n-ary tree i.e. a node can have up to \( n \) children, where \( n \) is the number of players \( (\mathbb{P}) \) participating in execution \( \Omega \). Each node has a label \( l \in \mathbb{P} \). The root node of \( T^\Omega_X \) has label \( X \). The levels of the tree are named in a bottom up manner. The lower most level is 1, the one immediately above it is 2 and so on. A node \( x \) is a child of node \( y \) if and only if \( x \) is in the in-neighbourhood of \( y \) in execution \( \Omega \). Thus the number of children of any node \( z \) is same as the size of the in-neighbourhood of \( z \). An edge from node \( y \) at level \( j \) to node \( x \) at level \( j + 1 \) in the tree represents the message that \( y \) sends to \( x \) in round \( j \) of \( \Omega \). All the edges in the tree are directed from child to parent and are between adjacent levels only. Let \( T^\Omega_{x,i} \) denote the execution tree of \( x \) until round \( i \) in execution \( \Omega \).

\(^4\) [FLM85] referred to these conditions as Locality Axiom. In the case of ABG, the secret key used by a player is definitely a part of his view. Hence, we added condition (ii).

\(^5\) We remark that \( \theta^\gamma_x \) can differ from \( \theta^\delta_y \). However, as long as long as they generate the same message for any player \( z \) i.e. \( \forall x, y, z \ msg^\gamma_{x}(x, z)_z = msg^\delta_{y}(y, z)_y \), it suffices for our proof.
We note the following – to show that a player, say \( x \), receives same messages in two different executions, say \( \gamma \) and \( \delta \), it suffices to show that execution trees \( T_x^\gamma \) and \( T_x^\delta \) are similar. To prove this similarity, we will use induction on the heights of \( T_x^\gamma \) and \( T_x^\delta \).

As a prelude to proving Lemma 3, we present adversary’s strategy in \( E_1 : \alpha_1 \). Recall that we defined scenario \( \alpha_1 \) as: In \( E_1 \) \( A \) is the General and starts with input value 0. \( \bar A \) Byzantine corrupts \( C \) in \( E_1 \). In \( E_2 \) \( \bar A \) Byzantine corrupts \( A \).

Adversary’s (\( \bar A \)) strategy in \( E_1 : \alpha_1 \) is as follows:

1. **Send outgoing messages of round \( i \):** Based on the messages received during round \( i - 1 \), \( \bar A \) decides on the messages to be sent in round \( i \). For round 1, \( \bar A \) sends to \( B \) what an honest \( C \) would have sent to \( B \) in execution \( E_1 : \alpha_2 \). For \( i \geq 2 \), \( \bar A \) authenticates \( msg_{i-1}^{E_1:\alpha_1}(B,C)_B \) using \( C \)'s key and sends it to \( A \). For \( msg_{i-1}^{E_1:\alpha_1}(A,C)_A \), \( \bar A \) examines the message. If the message has not been authenticated by \( B \) even once, it implies that the message has not yet been seen by \( B \). Then, \( \bar A \) sends a message to \( B \) which is same as what \( C \) would have sent to \( B \) in round \( i \) of execution \( E_1 : \alpha_2 \). Formally, \( \bar A \) constructs \( msg_{i-1}^{E_1:\alpha_1}(A,C)_A \) such that \( msg_{i-1}^{E_1:\alpha_1}(A,C)_A = msg_{i-1}^{E_1:\alpha_1}(A,C)_A \). Note that this is possible because \( A \) is actively corrupt in \( E_2 : \alpha_1 \) and therefore \( \bar A \) can forge messages on behalf of \( A \) in \( E_1 : \alpha_1 \). \( \bar A \) then authenticates \( msg_{i-1}^{E_1:\alpha_1}(A,C)_A \) using \( C \)'s key and sends it to \( B \). However, if the message has been authenticated by \( B \) even once, then \( \bar A \) simply authenticates \( msg_{i-1}^{E_1:\alpha_1}(A,C)_A \) using \( C \)'s key and sends it to \( B \).

2. **Receive incoming messages of round \( i \):** \( \bar A \) obtains messages \( msg_i^{E_1:\alpha_1}(A,C)_A \) and \( msg_i^{E_1:\alpha_1}(B,C)_B \) via \( C \). (These messages are sent by \( A \) and \( B \) respectively to \( C \) in round \( i \)). Similarly via \( A \), \( \bar A \) obtains messages \( msg_i^{E_1:\alpha_1}(B,A)_B \) and \( msg_i^{E_1:\alpha_1}(C,A)_C \). (These are also round \( i \) messages sent by \( B \) and \( C \) respectively to \( A \). Players respectively compute these messages according to their respective view until round \( i - 1 \)).

**Lemma 3** \( \bar A \) can ensure \( V_A^\alpha \sim V_A^{E_1:\alpha_1} \) and \( V_B^\alpha \sim V_B^{E_1:\alpha_1} \).

**Proof:** Using induction on \( i \), we show that for any round \( i \), \( T_A^{\alpha_i} \sim T_A^{E_1:\alpha_1} \). It then follows that \( T_A^{\alpha_k} \sim T_A^{E_1:\alpha_1} \). Combining this with Equation 5 gives \( V_A^\alpha \sim V_A^{E_1:\alpha_1} \).

Note that owing to the topology of \( \mathcal{L} \), only nodes present in \( T_A^{\alpha} \) are \( A, B, C \) and \( A' \). \( B' \) and \( C' \) do not occur in \( T_A^{\alpha} \). Hence, \( A' \) has an outgoing directed edge only and only to \( C \). Likewise, \( A \)
has an outgoing directed edge only to $B$. Corresponding nodes present in $T_A^{E_1;\alpha_1}$ are $A$, $B$, $C$ and $A$ respectively. We analyse $T_A^\alpha$ and $T_A^{E_1;\alpha_1}$ in a bottom up manner.

**Base Case:**

$i = 1$

Consider round 1 of executions $\alpha$ and $E_1 : \alpha_1$. Corresponding execution trees $T_A^\alpha$ and $T_A^{E_1;\alpha_1}$ are shown in Figure 7. Now, $B$ starts with same input, secret key and executes same code in $\alpha$ and $E_1 : \alpha_1$ i.e. $T_B^\alpha = T_B^{E_1;\alpha_1}$, $SK_B^\alpha = SK_B^{E_1;\alpha_1}$ and $\theta_B^\alpha = \theta_B^{E_1;\alpha_1}$ (the last equality follows from our definition of $\Pi$ [Definition 5]). Thus, $B$ will send same messages to $A$ in round 1 of $\alpha$ and $E_1 : \alpha_1$ i.e. $msg_1^\alpha(B, A)_B = msg_1^{E_1;\alpha_1}(B, A)_B$. Since $C$ is Byzantine corrupt in $E_1 : \alpha_1$, $\bar{A}$ can ensure that $msg_1^\alpha(C, A)_C = msg_1^{E_1;\alpha_1}(C, A)_C$. Thus, $T_A^\alpha \sim T_A^{E_1;\alpha_1}$.

![Figure 7: $T_A^\alpha$ and $T_A^{E_1;\alpha_1}$](image)

$i = 2$

We show that the similarity holds for round 2 as well. Consider $T_A^\alpha$ and $T_A^{E_1;\alpha_1}$ as shown in Figure 8. Now, $T_A^\alpha = T_A^{E_1;\alpha_1}$, $SK_A^\alpha = SK_A^{E_1;\alpha_1}$, $\theta_A^\alpha = \theta_A^{E_1;\alpha_1}$ and $T_B^\alpha = T_B^{E_1;\alpha_1}$, $SK_B^\alpha = SK_B^{E_1;\alpha_1}$, $\theta_B^\alpha = \theta_B^{E_1;\alpha_1}$. Thus, $msg_1^\alpha(A, B)_A = msg_1^{E_1;\alpha_1}(A, B)_A$ and $msg_1^\alpha(B, C)_B = msg_1^{E_1;\alpha_1}(B, C)_B$. Since $C$ is Byzantine corrupt in $E_1 : \alpha_1$, $\bar{A}$ can ensure that $msg_1^\alpha(C, B)_C = msg_1^{E_1;\alpha_1}(C, B)_C$.

![Figure 8: $T_A^\alpha$ and $T_A^{E_1;\alpha_1}$](image)

Now, $T_A^\alpha \neq T_A^{E_1;\alpha_1}$, thus $msg_1^\alpha(A', C)_A' \neq msg_1^{E_1;\alpha_1}(A, C)_A$. However, since $A$ is Byzantine corrupt in $E_2 : \alpha_1$, $\bar{A}$ can forge messages on behalf of $A$ in $E_1 : \alpha_1$ (follows from observations made in Section 2.1). $\bar{A}$ can use this to simulate $C$ having received same messages in round 1 in $E_1 : \alpha_1$ and $A$.

Specifically, as $C$ is Byzantine faulty in $E_1 : \alpha_1$, $\bar{A}$ can construct $msg_2^{E_1;\alpha_1}(A, C)_A$ in a way such that $msg_1^{E_1;\alpha_1}(A, C)_A = msg_2^\alpha(A', C)_A$. Now $B$ receives same round 1 messages in $E_1 : \alpha_1$ and $A$ and $B$ has same input value, secret key and executes same code, thus $msg_2^{E_1;\alpha_1}(B, A)_B = msg_2^\alpha(B, A)_B$. Thus, edge $BA$ (between levels 2 and 3) in $T_A^{E_1;\alpha_1}$ is same as the corresponding edge $BA$ in $T_A^\alpha$. Since, $C$ is Byzantine corrupt $\bar{A}$ can ensure that $msg_2^{E_1;\alpha_1}(C, A)_C = msg_2^\alpha(C, A)_C$. Therefore, $\bar{A}$ can ensure that the edge $CA$ (between levels 2 and 3) in $T_A^{E_1;\alpha_1}$ is same as the corresponding edge $CA$ in $T_A^\alpha$. Thus, $T_A^{E_1;\alpha_1} \sim T_A^\alpha$.

**Induction hypothesis:** Let it be true for all rounds upto $k$ i.e. $\forall i, i \leq k, T_A^\alpha \sim T_A^{E_1;\alpha_1}$. Likewise $\forall i, i \leq k, T_B^\alpha \sim T_B^{E_1;\alpha_1}$.
Induction step: We now prove that the similarity holds for round $k + 1$ as well i.e. $T_{A,k+1}^\alpha \sim T_{A,k+1}^{E_1:\alpha_1}$.

Consider $T_{A,k+1}^\alpha$ and $T_{A,k+1}^{E_1:\alpha_1}$ as shown in Figure 9. Consider the edges between level $k$ and $k + 1$. From the induction hypothesis, we have $\forall j \leq k$, $T_{A,j}^\alpha \sim T_{A,j}^{E_1:\alpha_1}$. Further, $T_A^\alpha = T_A^{E_1:\alpha_1}$, $SK_A^\alpha = SK_A^{E_1:\alpha_1}$, $\theta_A^\alpha = \theta_A^{E_1:\alpha_1}$. Thus, $A$ sends same messages to $B$ in round $k$ of both the executions i.e. $msg_k^{E_1:\alpha_1}(A, B)_A = msg_k^\alpha(A, B)_A$. Thus, edge $AB$ (between levels $k$ and $k + 1$) is same in both the trees. Likewise, from the induction hypothesis we have $\forall j \leq k$, $T_{B,j}^\alpha \sim T_{B,j}^{E_1:\alpha_1}$. Therefore, $msg_k^{E_1:\alpha_1}(B, C)_B = msg_k^\alpha(B, C)_B$. Hence, edge $BC$ (between levels $k$ and $k + 1$) is same in both the trees.

Now, consider the nodes - $A'$ at level $k$ in $T_A^\alpha$ and the corresponding node $A$ in $T_A^{E_1:\alpha_1}$. For time being assume\(^6\) that $\forall j \leq k$, $T_{A,j}^\alpha \sim T_{A,j}^{E_1:\alpha_1}$. We claim that $\tilde{A}$ can simulate $C$ at level $k + 1$ in $T_A^{E_1:\alpha_1}$ to have received messages from $A'$ exactly same as the messages received by $C$ at level $k + 1$ in $T_A^\alpha$. This is because $A$ is Byzantine corrupt in $E_2 : \alpha_1$, thus $\tilde{A}$ can forge messages on behalf of $A$ in $E_1 : \alpha_1$. Formally, $\tilde{A}$ constructs $msg_k^{E_1:\alpha_1}(A', C)_{A'}$ such that $msg_k^{E_1:\alpha_1}(A', C)_{A'} = msg_k^\alpha(A, C)_A$. Thus, $\tilde{A}$ can ensure that the edge $A'C$ (one between levels $k$ and $k + 1$) in $T_{A,k+1}^{E_1:\alpha_1}$ is same as the corresponding edge $AC$ in $T_{A,k+1}^\alpha$.

Now, $T_C^\alpha = T_C^{E_1:\alpha_1}$, $SK_C^\alpha = SK_C^{E_1:\alpha_1}$, $\theta_C^\alpha = \theta_C^{E_1:\alpha_1}$ Thus, $C$ sends same round $k + 1$ messages to $A$ in $\alpha$ and $E_1 : \alpha_1$ i.e. $msg_{k+1}^{E_1:\alpha_1}(C, A)_C = msg_{k+1}^\alpha(C, A)_C$. Thus, edge $CA$ (one between levels $k + 1$ and $k + 2$) in $T_{A,k+1}^{E_1:\alpha_1}$ is same as the corresponding edge $AC$ in $T_{A,k+1}^\alpha$. Similarly one can argue that the edge $BA$ (one between levels $k + 1$ and $k + 2$) in $T_{A,k+1}^{E_1:\alpha_1}$ is same as the corresponding edge $BA$ in $T_{A,k+1}^\alpha$. Thus, $\tilde{A}$ can ensure that $T_{A,k+1}^\alpha \sim T_{A,k+1}^{E_1:\alpha_1}$. Since it is true for all values of $k$, we have $T_A^\alpha \sim T_A^{E_1:\alpha_1}$.

The proof for $V_B^\alpha \sim V_B^{E_1:\alpha_1}$ follows on very similar lines, we omit the details. \(\blacksquare\)

**Lemma 4** In execution $\alpha$, players $A$ and $B$ output 0.

**Proof:** From Lemma 3, it follows that player $A$ cannot distinguish execution $E_1$ of scenario $\alpha_1$ from execution $\alpha$ (dubbed as $E_1 : \alpha_1 \sim A$). Similarly, to player $B$ execution $E_1$ of scenario $\alpha_1$ is indistinguishable from $\alpha (E_1 : \alpha_1 \sim B)$. In $E_1 : \alpha_1$, as per the definition of ABG [Definition 2] both

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\(^6\) Using induction on the value of $j$, one can show that the assumption is true.
A and B will decide on 0. Since, $E_1 : \alpha_1 \sim A \alpha$ and $E_1 : \alpha_1 \sim B \alpha$, in $\alpha$ too A and B will decide on 0. (We are able to make claims about the output of A and B in $\alpha$ as they cannot distinguish $E_1 : \alpha_1$ from $\alpha$. Thus, by analysing their output in $E_1 : \alpha_1$, we can determine their output in $\alpha$.)

Adversary’s strategy in $E_1 : \alpha_2$ –

(Recall that adversary Byzantine corrupts A only in $E_1 : \alpha_2$)

1. **Send outgoing messages of round i**: Based on the messages received during round $i - 1$, $\bar{A}$ decides on the messages to be sent in round i. For round i, $i \geq 1$, $\bar{A}$ sends to B what an honest A would have sent to B in execution $E_1 : \alpha_1$. Formally, $\bar{A}$ constructs $msg_i^{E_1:\alpha_2}(A, B)_A$ such that $msg_i^{E_1:\alpha_2}(A, B)_A = msg_i^{E_1:\alpha_1}(A, B)_A$. Likewise, $\bar{A}$ sends to C what an honest A would have sent to C in execution $E_1 : \alpha_3$. Formally, $\bar{A}$ constructs $msg_i^{E_1:\alpha_2}(A, C)_A$ such that $msg_i^{E_1:\alpha_2}(A, C)_A = msg_i^{E_1:\alpha_3}(A, C)_A$. Since A is Byzantine faulty in $E_1 : \alpha_2$, $\bar{A}$ can always send the above stated messages.

2. **Receive incoming messages of round i**: $\bar{A}$ obtains messages $msg_i^{E_1:\alpha_2}(B, A)_B$ and $msg_i^{E_1:\alpha_2}(C, A)_C$ from $\bar{A}$.

**Lemma 5**: $\bar{A}$ can ensure $V_B^\alpha \sim V_B^{E_1:\alpha_2}$ and $V_C^\alpha \sim V_C^{E_1:\alpha_2}$.

**Proof**: Using induction on i, we show that for any $i$, $T_{B,i}^\alpha \sim T_{B,i}^{E_1:\alpha_2}$. This implies $T_B^\alpha \sim T_B^{E_1:\alpha_2}$. From Equation 5 it then then follows that $V_B^\alpha \sim V_B^{E_1:\alpha_2}$.

**Base Case**:

$i = 1$

Consider $T_{B,1}^\alpha$ and $T_{B,1}^{E_1:\alpha_2}$ are shown in Figure 10. C does not have any input in either $\alpha$ or $E_1 : \alpha_2$. Thus, $T_C^\alpha = T_C^{E_1:\alpha_2}$ holds trivially. Further, C starts with same secret key and executes same code in $\alpha$ and $E_1 : \alpha_2$ i.e $SK_C^\alpha = SK_C^{E_1:\alpha_2}$ and $\theta_C^\alpha = \theta_C^{E_1:\alpha_2}$. Thus, it will send same messages to B in round 1 of $\alpha$ and $E_1 : \alpha_2$ i.e. $msg_1^{E_1:\alpha_2}(C, B)_C = msg_1^\alpha(C, B)_C$. Since A is Byzantine corrupt in $E_1 : \alpha_2$, $\bar{A}$ can ensure that $msg_1^{E_1:\alpha_2}(A, B)_A = msg_1^\alpha(A, B)_A$. Thus, $T_{B,1}^\alpha \sim T_{B,1}^{E_1:\alpha_2}$.

![Figure 10: $T_{B,1}^\alpha$ and $T_{B,1}^{E_1:\alpha_2}$](image)

$i = 2$

We now argue that the similarity holds for round 2 as well. Consider $T_{B,2}^\alpha$ and $T_{B,2}^{E_1:\alpha_2}$ as shown Figure 11. Now, B does not have any input in either $\alpha$ or $E_1 : \alpha_2$, thus $T_B^\alpha = T_B^{E_1:\alpha_2}$ is trivially true. Further, $SK_B^\alpha = SK_B^{E_1:\alpha_2}$ and $\theta_B^\alpha = \theta_B^{E_1:\alpha_2}$. Hence, $msg_1^{E_1:\alpha_2}(B, A)_B = msg_1^\alpha(B, A)_B$ and $msg_1^{E_1:\alpha_2}(B, C)_B = msg_1^\alpha(B, C)_B$. Likewise, C does not have any input in $\alpha$ and $E_1 : \alpha_2$, hence, $T_C^\alpha = T_C^{E_1:\alpha_2}$. Also, $SK_C^\alpha = SK_C^{E_1:\alpha_2}$ and $\theta_C^\alpha = \theta_C^{E_1:\alpha_2}$. Hence, $msg_1^{E_1:\alpha_2}(C, A)_C = msg_1^\alpha(C, A)_C$. Since, A is Byzantine corrupt in $E_1 : \alpha_2$, $\bar{A}$ can ensure that $msg_1^{E_1:\alpha_2}(A', C)_A' = msg_1^{E_1:\alpha_2}(A, C)_A$. On similar lines, one can get $msg_2^{E_1:\alpha_2}(C, B)_C = msg_2^\alpha(C, B)_C$. Since, A is Byzantine corrupt in $E_1 : \alpha_2$, $\bar{A}$ can ensure that $msg_2^{E_1:\alpha_2}(A, B)_A = msg_2^\alpha(A, B)_A$. Thus, $T_{B,2}^\alpha \sim T_{B,2}^{E_1:\alpha_2}$.
Induction hypothesis: Let it be true for all rounds upto \(k\) i.e. \(\forall i, \, i \leq k, \, T^\alpha_{B,i} \sim T^{\alpha_1}_{B,i} \). Likewise \(\forall i, \, i \leq k, \, T^\alpha_{C,i} \sim T^{\alpha_1}_{C,i} \).

Induction step: We now show that the similarity holds for round \(k + 1\) too i.e. \(T^\alpha_{B,k+1} \sim T^{\alpha_1}_{B,k+1} \).

Consider \(T^\alpha_{B,k+1} \) and \(T^{\alpha_1}_{B,k+1} \) as shown in Figure 12. Consider the edges between level \(k\) and \(k + 1\). From the induction hypothesis, we have \(\forall j \leq k, \, T^\alpha_{B,j} \sim T^{\alpha_1}_{B,j} \). Now, \(T^\alpha_{B} = T^{\alpha_1}_{B} \) and \(SK^\alpha_B = SK^{\alpha_1}_B \). Thus, \(B\) sends same messages to \(A\) in round \(k\) of both the executions i.e. \(msg^E_{k}(B,A)_{B} = msg^\alpha_{k}(B,A)_{B}\). Thus, edge \(BA\) (between levels \(k\) and \(k + 1\)) is same in both the trees. Likewise, \(B\) sends same messages to \(C\) in round \(k\) of both the executions i.e. \(msg^E_{k}(B,C)_{B} = msg^\alpha_{k}(B,C)_{B}\). Thus, edge \(BC\) (between levels \(k\) and \(k + 1\)) is same in both the trees.

From the induction hypothesis we have \(\forall j \leq k, \, T^\alpha_{C,j} \sim T^{\alpha_1}_{C,j} \). Since, \(T^\alpha_{C} = T^{\alpha_1}_{C} \) and \(SK^\alpha_C = SK^{\alpha_1}_C \), therefore, \(C\) sends same messages to \(A\) in round \(k\) of both the executions i.e. \(msg^E_{k}(C,A)_{C} = msg^\alpha_{k}(C,A)_{C}\). Hence, edge \(CA\) (between levels \(k\) and \(k + 1\)) is same in both the trees.

Using same arguments as in preceding two paragraphs, we get \(C\) sends same messages to \(B\) in round \(k + 1\) of both the executions i.e. \(msg^E_{k+1}(C,B)_{C} = msg^\alpha_{k+1}(C,B)_{C}\). Hence, edge \(CB\) (between levels \(k + 1\) and \(k + 2\)) is same in both the trees. Now, given \(A\) is Byzantine faulty in \(E_1 : \alpha_2\), \(A\) sends that \(msg^E_{k+1}(A,B)_{A}\) which ensures \(msg^E_{k+1}(A,B)_{A} = msg^\alpha_{k+1}(A,B)_{A}\). Thus, the edge \(AB\) (between levels \(k\) and \(k + 1\)) is same in both the trees.

The proof for \(V^\alpha_C \sim V^{\alpha_1}_C\) is nearly a repetition of the above arguments. Details omitted.

**Lemma 6** In execution \(\alpha\), output of \(B\) will be same output of \(C\).
Proof: From Lemma 5, we get that player B cannot distinguish execution $E_1: \alpha_2$ from $\alpha$ (dubbed as $E_1: \alpha_2 \sim B \sim \alpha$). Similarly, to player C execution $E_1: \alpha_2$ is indistinguishable from $\alpha$ ($E_1: \alpha_2 \sim B \sim \alpha$). Since, the General $(A)$ is Byzantine corrupt in $E_1: \alpha_2$, from the definition of ABG [Definition 2], in $E_1: \alpha_2$, B and C must have the same output. Then, so must $B$ and $C$ in $\alpha$.

Adversary’s strategy in $E_1: \alpha_2$ –
(Recall that in $\alpha_3$, A Byzantine corrupts $B$ in $E_1$ and $A$ in $E_2$.)

1. **Send outgoing messages of round $i$**: Based on the messages received during round $i-1$, $\mathcal{A}$ decides on the messages to be sent in round $i$. For round 1, $\mathcal{A}$ sends to $C$ what an honest $B$ would have sent to $C$ in execution $E_1: \alpha_2$. For $i \geq 2$, $\mathcal{A}$ authenticates $msg_{i-1}^{E_1: \alpha_3}(C, B)_C$ using $B$’s key and sends it to $A$. For $msg_{i}^{E_1: \alpha_3}(A, B)_A$, $\mathcal{A}$ examines the message. If the message has not been authenticated by $C$ even once, it implies that the message has not yet been seen by $C$. Then, $\mathcal{A}$ authenticates and sends a message to $C$ which is same as what $B$ would have sent to $C$ in round $i$ of execution $E_1: \alpha_2$. Formally, $\mathcal{A}$ constructs $msg_{i-1}^{E_1: \alpha_3}(A, B)_A$ such that $msg_{i}^{E_1: \alpha_3}(A, B)_A \sim msg_{i-1}^{E_1: \alpha_3}(A, B)_A$. $\mathcal{A}$ then authenticates $msg_{i}^{E_1: \alpha_3}(A, B)_A$ using $B$’s key and sends it to $C$. If the message has been authenticated by $B$ even once, $\mathcal{A}$ simply authenticates $msg_{i}^{E_1: \alpha_3}(A, B)_A$ using $B$’s key and sends it to $C$.

2. **Receive incoming messages of round $i$**: $\mathcal{A}$ obtains messages $msg_{i}^{E_1: \alpha_3}(A, B)_A$ and $msg_{i}^{E_1: \alpha_3}(C, B)_C$ via $B$. Likewise, via $A$, $\mathcal{A}$ obtains messages $msg_{i}^{E_1: \alpha_3}(B, A)_B$ and $msg_{i}^{E_1: \alpha_3}(C, A)_C$.

Lemma 7 $\mathcal{A}$ can ensure $V_{\mathcal{C}}^{\alpha} \sim V_{\mathcal{C}}^{E_1: \alpha_3}$ and $V_{\mathcal{A}}^{\alpha} \sim V_{\mathcal{A}}^{E_1: \alpha_3}$.

Proof: Owing to the symmetry of $\mathcal{L}$, the proof is very similar to the proof of Lemma 3. Details omitted.

Lemma 8 In execution $\alpha$, players $C$ and $A'$ output 1.

Proof: From Lemma 7, we have $E_1: \alpha_3 \sim \alpha$ and $E_1: \alpha_3 \sim \alpha$. From Definition 2, $A$ and $C$ will output 1 in $E_1: \alpha_3$. Then, so must $A'$ and $C$ in $\alpha$.

4.2 Finale

We now proceed to proving the main result of this work.

**Theorem 9 (Main Theorem)** ABG over $n$ players, tolerating a $t$-adversary, can be self-composed in parallel for any number of executions if and only if $n \geq 2t$.

Proof: By combining Lemma 10 and Lemma 11.

Lemma 10 If $n < 2t$, then there does not exist any ABG protocol that self-composes in parallel even twice ($\Delta_2$) over a network of $n$ players, tolerating a $t$-adversary.

Proof: Our proof demonstrates that if $n \leq 2t_1 + \min(t_1, t_2)$, $t_2 > 0$, then there does not exist any $\Delta_2$, over a network ($\mathcal{N}'$) of $n$ players tolerating a $(t_1, t_2)$-adversary. Here, $t_1$ and $t_2$ are the number of players that the adversary can corrupt in the two parallel executions $E_1$ and $E_2$, respectively, of $\Delta_2$ such that $t_1 + t_2 \leq t$ (dubbed as $(t_1, t_2)$-adversary). Substituting $t_1 = t - 1$ and $t_2 = 1$ in $n \leq 2t_1 + \min(t_1, t_2)$, gives the impossibility of $\Delta_2$ when $n < 2t$. Hence, the Lemma.
To show the impossibility of $\Delta_2$ when $n \leq 2t_1 + \min(t_1, t_2)$, $t_2 > 0$, we assume otherwise and arrive at a contradiction. For the purpose of contradiction, we assume the existence of $\Delta_2$ over $\mathcal{N}'$ such that $n \leq 2t_1 + \min(t_1, t_2)$, $t_2 > 0$, tolerating a $(t_1, t_2)$-adversary. Using $\Delta_2$, we construct a protocol $\Pi_2$ over a network $\mathcal{N}$ of $3$ nodes, $\mathbb{P} = \{A, B, C\}$, that tolerates an adversary basis $\mathcal{A} = \{(C), (A); (A), (\emptyset); (B), (A)\}$. But this contradicts Theorem 2, hence the assumed protocol $\Delta_2$ cannot exist.

Construction of $\Pi_2$ from $\Delta_2$ is as follows: partition the $n$ players in $\mathcal{N}'$ into three, mutually disjoint, non-empty sets $I_A$, $I_B$ and $I_C$ such that $|I_A| \leq \min(t_1, t_2)$, $|I_B| \leq t_1$ and $|I_C| \leq t_1$. Since $n \leq 2t_1 + \min(t_1, t_2)$, such a partitioning is always possible. The edges in $\mathcal{N}'$ can then be considered as bundle of edges between the sets $I_A$, $I_B$ and $I_C$. Let $E_1$, $E_2$ be two parallel executions of $\Delta_2$. Since $\Delta_2$ tolerates $(t_1, t_2)$-adversary, then $\Delta_2$ will tolerate an adversary, $\mathcal{A}$, characterised by adversary basis $\{(I_C), (I_A); (I_A), (\emptyset); (I_B), (I_A)\}$. Let the corresponding parallel executions of $\Pi_2$ be $E_1'$ and $E_2'$. Player $i, i \in \{A, B, C\}$, in execution $E_i'$, $l \in \{1, 2\}$, simulates all the players in set $I_i$ in execution $E_l$. W.l.o.g, let the honest, passively corrupt and Byzantine faulty players in $E_1'$ simulate the honest, passively corrupt and Byzantine faulty players respectively in $E_l$.

Player $i$ in $E_i'$ simulates players in $I_i$ in $E_l$ as follows: player $i$ keeps a track of the states of all the players in $I_i$. It assigns its input value to every member of $I_i$ and emulates the steps of all the players in $I_i$ as well as the messages communicated between every pair of players in $I_i$. If any player in $I_i$ sends a message to any player in $I_j$, $j \in \{A, B, C\}, i \neq j$, then player $i$ sends exactly the same message to player $j$. If any player in $I_i$ terminates then so does player $i$. If any player in $I_i$ decides on a value $v$, then so does player $i$.

We, now, show that if $\Delta_2$ satisfies Definition 2 tolerating $\mathcal{A}$, then so does $\Pi_2$ tolerating $\mathcal{A} = \{(C), (A); (A), (\emptyset); (B), (A)\}$. Let $i$ and $j$, ($i \neq j$), be two non-faulty players (honest or passively corrupt but not Byzantine faulty) in execution $E_i'$ of $\Pi_2$. Player $i$ (likewise $j$) simulates at least one player in $I_i$ ($I_j$) in execution $E_i$. Since both $i, j$ are non-faulty in $E_i'$, then so are all the players in $I_i, I_j$ in $E_i$. If the General is non-faulty in $E_i'$ and starts with a value $v$, then in $E_i$ too, the General is non-faulty and starts with a value $v$. Hence, as per the definition of ABG [Definition 2], all the players in $I_i, I_j$ in execution $E_i$ must decide on value $v$. Then, so should players $i, j$ in $E_i'$. If the General is faulty in $E_i'$, then so is the General in $E_i$. As per the definition of ABG all the players in $I_i, I_j$ in execution $E_i$ must have the same output. Then, so should players $i, j$ in $E_i'$. This implies $\Pi_2$ satisfies Definition 2 tolerating $\mathcal{A}$, contradicting Theorem 2. 

We now show that the bound of $n < 2t$ is tight. For this we present a protocol – $EIGPrune^+$ (Figure 13) and prove that if $n \geq 2t$, then $EIGPrune^+$ remains a valid ABG protocol under any number parallel self-compositions (Lemma 11). $EIGPrune^+$ is based on a sequence of transformations on EIG tree [BNDDS87]. [Lyn96, page 108] gives an excellent discussion on the construction of EIG tree. $EIGPrune^+$ is essentially same as $EIGPrune$ protocol [GGBS10], barring two minor additions – (i) Each concurrent execution of the protocol is augmented with a Unique Session Identifier (USID). (ii) Non-faulty players in any concurrent execution reject any message that does not carry a valid USID. We remark that like $EIGPrune$, $EIGPrune^+$ is also exponential in the number of messages. However, owing to the simplicity and intuitive appeal of the protocol we present the same as it makes the discussion very lucid. The exponential nature of the protocol is not a serious concern as using well known techniques in literature [BNDDS87], it can be converted into an efficient protocol.

**Definition 6 (Prune(EIG))** It takes an EIG tree as an input and deletes subtrees say $s_j^i$ (subtree
In every concurrent execution:

1. The General $G$ send his value to every player.

2. On receiving this value from $G$, every player assumes it be his input value and exchanges messages with others as per EIGStop protocol [Lyn96, page 103] for $t+1$ rounds.

3. At the end of $t+1$ rounds of EIGStop protocol, player $p_i$ discards any messages that does not have a valid authentication or USID and invokes Prune(EIG) [Definition 6].

4. Player $p_i$ applies the following decision rule – take majority of the values at the first level (i.e. all the nodes with labels $l$ such that $l \in \mathcal{P}$) of its EIG tree. If a majority exists, player $p_i$ decides on that value; else, $p_i$ decides on default value, $v_0$.

Figure 13: EIGPrune$^+$ Protocol

rooted at a node whose’s label is $j$ in EIG tree of player $i$) as given in the sequel. For each subtree $s_j^1$, where label $j \in \mathcal{P}$, a set $W_j$ is constructed that contains all distinct values that ever appears in $s_j^1$. If $|W_j| > 1$, $s_j^1$ is deleted and modified EIG tree is returned.

Our proof of correctness of EIGPrune$^+$ is based on the idea developed by Lindell et al. [LLR02], wherein, the security of a protocol in the concurrent setting is reduced to the security of the protocol in the stand alone setting. In our case, to ensure that this reduction is correct, we must account for the possibility of the players being (implicitly) passively corrupt in an execution (from the observation made in section 2.2). For this reason, we use a variant of ABG - christened as ABG$_{mix}$, proposed by Gupta et al. [GGBS10], as the stand alone setting for our reduction.

Gupta et al. studied (stand alone) ABG in the presence of a mixed adversary that can corrupt up to any $t_b$ players actively and up to another $t_p$ players passively (dubbed as $(t_b,t_p)$-adversary). The adversary, thus, can forge the signatures of all $t_b + t_p$ players. Further, Gupta et al. require all the passively corrupt players to always output a value same as the output of the honest players. They prove that (stand alone) ABG$_{mix}$ over a completely connected synchronous network of $n$ players tolerating a $(t_b,t_p)$-adversary, $t_p > 0$, is solvable if and only if $n > 2t_b + \min(t_b,t_p)$. It is easy to see that like EIGPrune, EIGPrune$^+$ is also a correct protocol for ABG$_{mix}$. By substituting $t_b = t - 1$ and $t_p = 1$, their result can be extended to achieve a bound of $n \geq 2t$.

Lemma 11 EIGPrune$^+$ over $n$ players, tolerating a $t$-adversary, is a valid ABG protocol that self-composes for any number of parallel executions if $n \geq 2t$.

Proof: Let $\Psi(id_1), \Psi(id_2), \ldots, \Psi(id_l)$ be $l$ parallel executions of EIGPrune$^+$. For the purpose of contradiction assume that there exists an adversary, $A$, that attacks these $l$ parallel executions and succeeds in execution $\Psi(id_i)$, for some $i \in (1,l)$. Using $A$ we construct an adversary $A'$ that is bound to succeed in the (stand alone) execution, $\varphi$, of EIGPrune for ABG$_{mix}$. This contradicts the results of Gupta et al., hence the Lemma.

Our construction of $A'$ requires $A'$ to internally simulate the parallel executions $\Psi(id_1), \Psi(id_2), \ldots, \Psi(id_l)$. For this to happen, $A'$ must be able to simulate the signatures generated by the honest players in these executions. To facilitate the same we assume $A'$ is given access to all the oracles $S_{-id}(sk_1, \cdot), \ldots, S_{-id}(sk_n, \cdot)$. Further, to ensure that the above simulation is perfect we augment

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8 $\Psi(id_i)$ denotes the use of USID in the $i$th concurrent execution of $\Psi$.

9 Players use the signature scheme, ($(Gen, S_{id}, V_{id}), S_{-id}$), developed by Lindell et al. [LLR02]. We present a brief overview of the same in Appendix B.
the stand alone execution, ϕ, of EIGPrune for ABGmix with USID – ϕ(id). It is easy to see that this augmentation has no bearing on the correctness of EIGPrune. Formally, for ABGmix, if ϕ is a correct execution of EIGPrune, then so is ϕ(id).

Construction of A′ is as follows: A′ internally incorporates A and attacks ϕ(id) as given in the sequel. A′ selects an execution i, i ∈ (1, l), and sets id=id_i. Then, A′ invokes A and simulates the concurrent executions of Ψ(id_1)…Ψ(id_l) for A. A′ does this by playing the roles of the non-faulty players in all the executions but Ψ(id). Since A′ has access to the signing oracles S¬id(sk_1, ·), . . . , S¬id(sk_n, ·), it can generate signature on behalf of honest players in all the executions Ψ(id_j), j ≠ i. In Ψ(id), A′ externally interacts with the non-faulty players and passes messages between them and A. A′ interacts with the players in Ψ(id_i) in exactly the same manner as A interacts with players in Ψ(id_i). Note that this is possible because if A forges messages on behalf of some player in Ψ(id_i) by active corrupting this player in Ψ(id_j), j ≠ i; then A′ can do the same by passively corrupting this particular player in Ψ(id). Since, A′ never queries the oracle for messages whose prefix is id. Therefore, the emulation by A′ of the concurrent executions for A is perfect. Thus, if A succeeds in breaking Ψ(id), then A′ will break ϕ(id).

5 Closing Remarks

In this paper we argue for the need of a better model for studying self composition of ABG protocols. We propose a new model to study composition of ABG protocols and show that, in this model, unique session identifiers aid in improving the fault-tolerance of ABG protocols (that compose in parallel) but from n > 3t only to n ≥ 2t. Note that, in the stand alone setting, ABG is possible for n > t. Thus surprisingly, USID’s may not always achieve their goal of truly separating the protocol’s execution from its environment to the fullest extent. However, for most functionalities, USID’s indeed achieve their objective, as is obvious from Universal Composability (UC) theorem [Can01].

Besides proving (im-)possibility results for self composition of ABG, our work also brings to forefront a few minor, yet interesting and undesirable properties of UC framework. It will nice to see if one can fine tune UC framework to this end. Further, with respect to composition of ABG protocols, we show that the worst-case adversary (with respect to a given execution) is not the one that corrupts players at full-throttle across all protocols running concurrently in the network. There may be several other problems apart from ABG wherein similar anomaly holds. It is an intriguing open question to characterize the set of all such problems. Further, from our results of n ≥ 2t, it appears that studying self composition of ABG protocols over general networks will be interesting in its own right.

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A  \textit{EIGPrune} Protocol [GGBS10]

For the benefit of the reader, we now reproduce the protocol proposed by Gupta et al. (Figure 14). It is obtained by a sequence of transformations on EIG tree [BNDDS87]. [Lyn96, page 108] gives an excellent discussion on the construction of EIG tree. All the messages exchanged during the protocol are signed/authenticated. We only give the protocol here. The proof of its correctness in stand alone setting for the problem of \textit{ABG}$_{mix}$ can be found in [GGBS10]

\begin{quote}
\textbf{EIGPrune Algorithm}

General $G$ send his value to every player. Every player assumes this value from the $G$ as his input value and exchanges messages with others as per \textit{EIGStop} protocol [Lyn96, page 103] for $t_b + t_p + 1$ rounds.

At the end of $t_b + t_p + 1$ rounds of \textit{EIGStop} protocol, player $p_i$ invokes \textbf{Prune}(EIG) [Definition 7]. Player $p_i$ applies the following decision rule – take majority of the values at the first level (i.e. all the nodes with labels $l$ such that $l \in \mathbb{P}$) of its EIG tree. If a majority exists, player $p_i$ decides on that value; else, $p_i$ decides on default value, $v_0$.

Figure 14: \textit{EIGPrune} algorithm
\end{quote}

\begin{definition}(Prune(EIG)) This method that takes an EIG tree as an input and deletes subtrees say subtree$_j$ (subtree$_j$ refers to a subtree in i’s EIG tree such that the subtree is rooted at node whose label is $j$) of i’s EIG tree as given in the sequel. For each subtree subtree$_j$, where label $j \in \mathbb{P}$, a set $W_j$ is constructed which contains all distinct values that ever appears in subtree$_j$. If $|W_j| > 1$, subtree$_j$ is deleted and modified EIG tree is returned.
\end{definition}

B  Lindell \textit{et al.’s} Signature Scheme

We present an overview of the signature scheme ($(Gen,S_{id},V_{id}),S_{-id}$) developed by Lindell \textit{et al.} [LLR02]. They define a signature scheme as $(Gen,S,V)$ where $S,V$ are are algorithms for signing and verification of any message. $Gen$ is used to generate signature and verification keys for a particular player (say $P_k$) and defined as a function: $(1)^n \rightarrow (vk, sk)$. A signature scheme is said to be a valid one if honestly generated signatures are almost always accepted. Formally, with non negligible probability, for every message $m$, $V(vk,m,S(sk,m)) = 1$, where $(vk, sk) \leftarrow (1)^n$. They model the valid signatures that adversary $\mathcal{A}$ can obtain in a real attack via a signing oracle $S(sk, \cdot)$. $\mathcal{A}$ is defined to succeed in generating a forged message $m^*$ if $\mathcal{A}$ given $vk$, access to oracle $S(sk, \cdot)$ can generate a pair $(m^*, S^*)$ such that if $Q_m$ is the set of oracle queries made by $\mathcal{A}$ then $V(vk,m^*,S^*) = 1$ holds true if $m^* \notin Q_m$. A signature scheme is said to be existentially secure against chosen-message attack if $\mathcal{A}$ cannot succeed in forging a signature with greater than non-negligible probability. They further model any information gained by $\mathcal{A}$ from any query with another oracle Aux$(sk, \cdot)$. However, this oracle cannot generate any valid signature but provides any other auxiliary information about the query. They assume some scheme say $(Gen,S,V)$ to be secure against chosen-message attack and show how to construct a secure scheme $(Gen,S_{id},V_{id})$ from it where $S_{id}(sk,m) = S(sk, id \circ m)$ and $V_{id}(vk,m,\sigma) = V(vk, id \circ m, \sigma)$. For the new scheme they define the oracle Aux$(sk, \cdot) = S_{-id}(sk,m)$ where $S_{-id}(sk,m) = S(sk,m)$ if the prefix of $m$ is not
id else $S_{-id}(sk, m) = \bot$. Further, they assume $\pi$ to be a secure protocol for ABG using signature scheme $(Gen, S, V)$. They define modified protocol $\pi(id)$ to be exactly same as $\pi$ except that it uses signature scheme $(Gen_{id}, V_{id})$ as defined above. They further prove as to why $(Gen_{id}, V_{id}, S_{-id})$ is secure against chosen-message attack. Intuition behind the proof is the fact that if the prefix of $m \neq id$, then $S_{-id}(sk, m) = S(sk, m)$ which is of no help to the adversary as any successful forgery must be prefixed with $id$ and all oracle queries to $S_{-id}$ must be prefixed with $id' \neq id$. 

26