A Novel Hybrid Mgstar-Rnn Model for Forecasting Spatio-Temporal Data

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Abstract. Time series data have a time dimension and space dimension called Spatio-temporal data. GSTAR is one of the models that can be used to analyze spatio-temporal data. One of development for this model is the Multivariate Generalized Spaced-Time Autoregressive (MGSTAR). The MGSTAR model has limitations of not being able to model a nonlinear time series, this problem can be overcome by applying a hybrid model on MGSTAR. This research aims to propose a hybrid MGSTAR-RNN model, where the MGSTAR model as a linear component and RNN as a nonlinear component. This research focused on a simulation study to evaluate the performance of MGSTAR and MGSTAR-RNN model. Several scenarios be experimented, i.e simulation in data with different variable component and location. The result show that hybrid MGSTAR-RNN model is better than individual MGSTAR model. In general, it is in line with the latest results of the 2018 M4 forecasting competition show that combined models or hybrid models tend to provide more accurate forecast performance than forecast results with individual models.

Keywords : Hybrid, MGSTAR, RNN, Simulation, Spatio-temporal

1. Introduction

Time series data is not only time-dimensional but also has a spatial dimension that is commonly called as Spatio-temporal data. One of the models that commonly used is the Generalized Space-Time Autoregressive (GSTAR). The GSTAR model was first developed by Ruchjana which is an extension of the STAR model that was first introduced by Pfeifer and Deutsch [1,2]. The assumption of the GSTAR model is the model parameters will change for each location [2]. GSTAR models have limitations i.e. this model has not been able to perform for observations in various locations with several different variables in the same location. A multivariate model of Spatio-temporal has better performance than other model alternatives [3]. However, a time series data in life not just has linear or nonlinear patterns [4].

Hybrid models can forecast time series data that contain linear and nonlinear patterns. In Spatio-temporal data research with hybrid models of VAR-NN and GSTAR-NN can improve forecasting accuracy [5]. The frequently used nonlinear method is the Artificial Neural Network (ANN). There are various types of ANN methods, such as Feed Forward Neural Network (FFNN), Deep Learning Neural Network (DLNN), and Recurrent Neural Network (RNN).

RNN is a neural network that has a feedback connection. This model is similarities to multi-layer perceptron with one or more context layers [6]. There are 2 kinds of RNN including Elman Recurrent Neural Network (ERNN) and Jordan Recurrent Neural Network (JRNN). Neurons in the JRNN context
layer originate from the output layer. In the Elman RNN context layer, the neurons originate from the hidden layer. In a study using JRNN for short-term forecasting electricity consumption resulted that JRNN’s forecasting accuracy was better than the FFNN model [7]. One of the ERNN model ever undertaken is to predict short-term electricity consumption indicating that the ERNN models can process non-linear data properly [8].

In this paper conducted simulated studies with three ANN architectures to know the accuracy of forecasting models. The hybrid MGSTAR-FFNN model with one hidden layer, the MGSTAR-DLNN model with two hidden layers and the last model is MGSTAR-RNN that developed in this study. The purpose of this research is to evaluate the performance of the MGSTAR-RNN hybrid model in the forecasting of Spatio-temporal multivariate data by using the Root Mean Square Error (RMSE) criterion.

2. Methods

2.1. Multivariate GSTAR

The Generalized Space-Time Autoregressive (GSTAR) model is an extension of STAR model that can accomplish the assumptions of heterogeneity in location, so that the parameters for each location are different. Multivariate GSTAR (MGSTAR) models are the development of GSTAR and VAR models. This model is used for Spatio-temporal data with more than one observation variable in a single observation site. Equation of the matrix of the MGSTAR model with the time and spatial order 1 and there are three locations with two observation variables in each location are as follows:

\[
Z_{i,t} = \Phi_{\theta} Z_{i,t-1} + \Phi_{\phi} Z_{i,t-2} + \Phi_{\psi} Z_{i,t-3} + \Phi_{\tau} a_{i,t}
\]

or we can write as,

\[
\begin{bmatrix}
Z_{1,t} \\
Z_{2,t} \\
Z_{3,t} \\
\end{bmatrix} = \begin{bmatrix}
\phi_{10}^i \\
\phi_{20}^i \\
\phi_{30}^i \\
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
Z_{1,t-1} \\
Z_{2,t-1} \\
Z_{3,t-1} \\
\end{bmatrix} + \begin{bmatrix}
\phi_{11}^i \\
\phi_{21}^i \\
\phi_{31}^i \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-2} \\
Z_{2,t-2} \\
Z_{3,t-2} \\
\end{bmatrix} + \ldots + \begin{bmatrix}
\phi_{1k}^i \\
\phi_{2k}^i \\
\phi_{3k}^i \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-3} \\
Z_{2,t-3} \\
Z_{3,t-3} \\
\end{bmatrix} + \ldots + \begin{bmatrix}
\phi_{1k}^i \\
\phi_{2k}^i \\
\phi_{3k}^i \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-k} \\
Z_{2,t-k} \\
Z_{3,t-k} \\
\end{bmatrix} + \begin{bmatrix}
a_{11} \\
a_{22} \\
a_{33} \\
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
\phi_{11}^{k*} \\
\phi_{21}^{k*} \\
\phi_{31}^{k*} \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-k} \\
Z_{2,t-k} \\
Z_{3,t-k} \\
\end{bmatrix} + \ldots + \begin{bmatrix}
\phi_{1k}^{k*} \\
\phi_{2k}^{k*} \\
\phi_{3k}^{k*} \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-3} \\
Z_{2,t-3} \\
Z_{3,t-3} \\
\end{bmatrix} + \ldots + \begin{bmatrix}
\phi_{1k}^{k*} \\
\phi_{2k}^{k*} \\
\phi_{3k}^{k*} \\
\end{bmatrix} \begin{bmatrix}
w_{12} \\
w_{23} \\
w_{31} \\
\end{bmatrix} \begin{bmatrix}
Z_{1,t-k} \\
Z_{2,t-k} \\
Z_{3,t-k} \\
\end{bmatrix} + \begin{bmatrix}
a_{1k} \\
a_{2k} \\
a_{3k} \\
\end{bmatrix}
\]

(2)

where \( \phi_{ij}^k \) and \( \phi_{ij}^{k*} \) is a parameter that describes the relation of lag between the same variable in the same location and the same variable in different locations. For \( \phi_{ii}^k \) and \( \phi_{ii}^{k*} \) is a parameter that explains the relation between different variables in the same location and the different variables in different locations, with \( i = 1, 2, 3 \) and \( k = 1, 2 \).
2.2. Recurrent Neural Network
The Artificial Neural Network (ANN) is a mathematical model inspired by the neural networks of living beings. In ANN there is an architecture term consisting of a layer of modeling consisting of layer inputs, hidden layers, and layer outputs. ANN has several types of models including Feed Forward Neural Network (FFNN), Deep Learning Neural Network (DLNN), and Recurrent Neural Network (RNN).

The RNN is a neural network that has a feedback connection. There are two types of RNN including the Elman Recurrent Neural Network (ERNN) and the Jordan Recurrent Neural Network (JRNN). ERNN is a neural network that has a simple, feedback connection from a hidden layer to the context layer consisting of the unit time delay operators \[9\]. In ERNN the number of context layer neurons equals the number of neurons in the hidden layer and each of the context layer neurons is connected to each neuron in the hidden layer. The ERNN model equation with \(p\) unit input and \(q\) unit hidden is as follows.

\[
\hat{Y}_t = f^0 \left[ b^0 + \sum_{j=1}^{q} w^j f^h \left( b^j + \sum_{i=1}^{p} w^i z_{t-i} + \sum_{i=1}^{q} w^i h_{i(t-1)} \right) \right]
\]

While JRNN has a feedback connection from the output layer to the context layer \[6\]. The result of the layer output will be copied to the context layer so that the unit weight in the context layer will stay one. Then the value of the context layer will be entered in the network again \[10\]. Here’s the equation for JRNN model.

\[
\hat{Y}_t = f^0 \left[ \sum_{j=1}^{q} w^j h^j \left( \sum_{i=1}^{p} w^i I_{i(t)} + u^j C_{i(t)} \right) \right]
\]

2.3. Hybrid Model
Zhang first introduced the hybrid model to increase the level of accuracy of a forecast. Zhang \[4\] developed this model for several reasons. First, a difficulty for the selection of forecasting methods that can meet the unique situation of the data. Secondly, a time series data in an impure life not just has linear or nonlinear patterns and the third lot of research states that there is not yet a good method of use for every situation. Mathematically hybrid models that combine two components i.e. linear and nonlinear components can be written as follows:

\[Y_t = L_t + N_t\] (5)

where \(L_t\) is a linear component and \(N_t\) is a nonlinear component. A nonlinear component of a data can be known by modeled the residual by using the RNN. In this research, hybrid modeling will be done with two stages. The first stage is to model the data using the MGSTAR models so that the residual obtained. The next stage is to model the residual result of the first stage with RNN models.

2.4. Model Evaluation Criteria
The best model selection uses the Root Mean Square Error (RMSE) criterion by comparing the smallest RMSE value. The RMSE value of a model can be obtained through the following calculations \[11\], with \(M\) is a forecast period.

\[RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (Z_i - \hat{Z}_i)^2} \] (6)

3. Data
This research used simulation studies to determine the performance of the MGSTAR-RNN model in forecasting time series data. The simulated study used is data generation that follows the Vector Autoregressive (VAR) model with the model equation for three locations with two variables as follows:
The data that is used have 4.2. A combination on the smallest RMSE value in each replication.

The result of the parameter is less than 1. MGS 4.1. 4. There are 2 scenarios created for this simulation study. First scenario uses data where the same variables with different locations interconnected. While second scenario employed data interconnected when the variable is different but is in the same location. Data that is used have $a \sim MN(0, \Sigma)$ with matrix variance-covariance as follows:

$$
\Sigma_1 = 
\begin{bmatrix}
1 & 0.65 & 0.50 & 0 & 0 & 0 \\
0.65 & 1 & 0.45 & 0 & 0 & 0 \\
0.50 & 0.45 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0.55 & 0.40 \\
0 & 0 & 0 & 0.55 & 1 & 0.35 \\
0 & 0 & 0 & 0.40 & 0.35 & 1
\end{bmatrix}
$$

$$
\Sigma_2 = 
\begin{bmatrix}
1 & 0 & 0 & 0.35 & 0 & 0 \\
0 & 1 & 0 & 0 & 0.50 & 0 \\
0 & 0 & 1 & 0 & 0 & 0.25 \\
0.35 & 0 & 0 & 1 & 0 & 0 \\
0 & 0.50 & 0 & 0 & 1 & 0 \\
0 & 0 & 0.25 & 0 & 0 & 1
\end{bmatrix}
$$

4. Result

4.1. MGSTAR Model

Modeling in this simulated study uses the MGSTAR (1,1) as in equations (2) using uniform weights that assume simulated data between locations is homogeneous. The parameter coefficient employed in the MGSTAR model is in accordance with the stationary requirements of the MGSTAR parameter, i.e. the eigen value of the parameter is less than 1. MGSTAR model in scenario I can be written as follows.

a. Model for variable 1 in location 1

$$Z_{1,j}^1 = 0.401Z_{1,j-1}^1 + 0.247Z_{1,j-1}^2 + 0.436(w_{12}Z_{2,j-1}^1 + w_{13}Z_{3,j-1}^1) - 0.327(w_{12}Z_{2,j-1}^2 + w_{13}Z_{3,j-1}^2) + a_j^1$$

b. Model for variable 1 in location 2

$$Z_{2,j}^1 = 0.196Z_{2,j-1}^1 + 0.076Z_{2,j-1}^2 + 0.603(w_{21}Z_{1,j-1}^1 + w_{23}Z_{3,j-1}^1) - 0.213(w_{21}Z_{1,j-1}^2 + w_{23}Z_{3,j-1}^2) + a_j^1$$

c. Model for variable 1 in location 3

$$Z_{3,j}^1 = 0.409Z_{3,j-1}^1 + 0.198Z_{2,j-1}^2 + 0.418(w_{31}Z_{1,j-1}^1 + w_{32}Z_{2,j-1}^1) - 0.238(w_{31}Z_{1,j-1}^2 + w_{32}Z_{2,j-1}^2) + a_j^1$$

d. Model for variable 2 in location 1

$$Z_{1,j}^2 = 0.224Z_{1,j-1}^2 + 0.196Z_{1,j-1}^1 + 0.256(w_{12}Z_{2,j-1}^2 + w_{13}Z_{3,j-1}^2) - 0.250(w_{12}Z_{2,j-1}^1 + w_{13}Z_{3,j-1}^1) + a_j^2$$

e. Model for variable 2 in location 2

$$Z_{2,j}^2 = 0.373Z_{2,j-1}^2 + 0.229Z_{2,j-1}^1 + 0.521(w_{21}Z_{1,j-1}^2 + w_{23}Z_{3,j-1}^2) - 0.276(w_{21}Z_{1,j-1}^1 + w_{23}Z_{3,j-1}^1) + a_j^2$$

f. Model for variable 2 in location 3

$$Z_{3,j}^2 = 0.333Z_{3,j-1}^2 + 0.233Z_{2,j-1}^1 + 0.629(w_{31}Z_{1,j-1}^2 + w_{32}Z_{2,j-1}^2) - 0.234(w_{31}Z_{1,j-1}^1 + w_{32}Z_{2,j-1}^1) + a_j^2$$

4.2. MGSTAR-FFNN Model and MGSTAR-DLNN Model

The number of neurons in MGSTAR-FFNN determine 1 until 10. The best architecture is selected based on the smallest RMSE value in each replication. For the MGSTAR-DLNN model uses 2 hidden layers with a combination number of neurons in the first and second hidden layers. The number of neurons in the first hidden layer is 1 to 10 while the second hidden layer consists 1 to 5 neurons.
Table 1. The average of RMSE value for MGSTAR-FFNN and MGSTAR-DLNN on Scenario I

|        | MGSTAR-FFNN | MGSTAR-DLNN |
|--------|-------------|-------------|
| V1L1   | 0.951       | 0.933       |
| V1L2   | 0.940       | 0.922       |
| V1L3   | 0.940       | 0.927       |
| V2L1   | 0.946       | 0.931       |
| V2L2   | 0.930       | 0.910       |
| V2L3   | 0.925       | 0.907       |

Based on Table 1 known that the RMSE values of the MGSTAR-DLNN model are smaller than the MGSTAR-FFNN for each variable and different location. Both RMSE value of the model is around 1 and tends to be similar. Likewise, the value of RMSE in table 2 is less than 1 with a small difference for both models.

Table 2. The average of RMSE value for MGSTAR-FFNN and MGSTAR-DLNN on Scenario II

|        | MGSTAR-FFNN | MGSTAR-DLNN |
|--------|-------------|-------------|
| V1L1   | 0.944       | 0.932       |
| V1L2   | 0.968       | 0.958       |
| V1L3   | 0.927       | 0.922       |
| V2L1   | 0.941       | 0.928       |
| V2L2   | 0.941       | 0.931       |
| V2L3   | 0.935       | 0.926       |

4.3. MGSTAR-RNN Model
In this study the hybrid MGSTAR-RNN model was divided into two namely the MGSTAR model coupled with the Elman RNN hereinafter called the MGSTAR-ERNN and Jordan RNN models called the MGSTAR-JRNN model. The number of neurons in the hidden layer is determine 1 to 10 and selected the best architecture in each replication.

4.3.1 MGSTAR-ERNN Model
The inputs in MGSTAR-ERNN are 24 parameters of MGSTAR model like in equation (2). This model is applied activation function hyperbolic tangent for hidden layer and linear for output layer. Figure 1 shows the architecture of MGSTAR-ERNN model using \( q \) number of neurons in hidden layer.

Figure 1. Architecture of MGSTAR-ERNN
where,

\[
N'_{i,j-1} = \begin{bmatrix}
N'_{i,j-1} \\
0 \\
0 \\
0
\end{bmatrix}, \quad W'_{j,i-1} = \begin{bmatrix}
w_{i,j-1}N'_{i,j-1} + w_{i,j-1}N'_{i,j-1} \\
0 \\
0 \\
0
\end{bmatrix}, \quad W_{2,i,j-1} = \begin{bmatrix}
0 \\
0 \\
0 \\
w_{i,j-1}N_{i,j-1} + w_{i,j-1}N_{i,j-1}
\end{bmatrix}, \quad \hat{N}_j = \begin{bmatrix}
\hat{N}_1 \\
\hat{N}_2 \\
\hat{N}_3 \\
\hat{N}_4 \\
\hat{N}_5
\end{bmatrix}
\]

The \(N'_{i,j-1}\) is input which show relation between the same variables and locations; \(W'_{j,i-1}\), and \(W_{2,i,j-1}\) are inputs which show relation between the same variables and different locations. While \(N_{i,j-1}\) show relation between the different variables with same locations; \(W'_{j,i-1}\), and \(W_{2,i,j-1}\) show relation between different variables and locations. The MGSTAR-ERNN model equation in scenario 2 with 5 neurons in hidden layer, is as follows.

\[
\hat{N}_1 = 0.84f_1^h(z) - 1.03f_2^h(z) + 1.16f_3^h(z) - 0.87f_4^h(z) + 1.05f_5^h(z)
\]  \( \text{(9)} \)

where,

\[
f_1(z) = \tanh\left(-3.57N_{i,j-1} + 8.33W_{j,i-1} - 0.33N_{i,j-1} - 0.12W_{j,i-1} + \cdots + 1.05W_{2,i,j-1} - 5.46C_i + \cdots - 3.73C_5\right)
\]

\[
f_2(z) = \tanh\left(-3.67N_{i,j-1} - 6.70W_{j,i-1} - 0.95N_{i,j-1} + 0.65W_{j,i-1} + \cdots + 4.05W_{2,i,j-1} + 0.92C_i + \cdots + 0.20C_5\right)
\]

\[
f_3(z) = \tanh\left(3.17N_{i,j-1} + 3.77W_{j,i-1} - 0.95N_{i,j-1} - 0.15W_{j,i-1} - \cdots - 6.79W_{2,i,j-1} - 0.82C_i + \cdots + 2.85C_5\right)
\]

\[
f_4(z) = \tanh\left(-3.62N_{i,j-1} - 3.89W_{j,i-1} - 0.38N_{i,j-1} - 0.24W_{j,i-1} + \cdots + 7.49W_{2,i,j-1} + 6.60C_i + \cdots - 3.21C_5\right)
\]

\[
f_5(z) = \tanh\left(1.00N_{i,j-1} + 0.30W_{j,i-1} + \cdots + 0.68N_{i,j-1} - 0.72W_{j,i-1} + \cdots + 5.64W_{2,i,j-1} - 7.25C_i + \cdots - 0.30C_5\right)
\]

4.3.2 MGSTAR-JRNN Model

The hybrid MGSTAR-JRNN employed same input as hybrid MGSTAR-ERNN with same activation function that used in this model. The different one is the architecture that shows in figure 2 with \(q\) number of neurons in hidden layer.
Each replication in both scenarios result different best number of neuron hidden layer according to RMSE value. The MGSTAR-JRNN model equation in scenario 1 with 3 neurons in hidden layer is as follows.

\[
\hat{N}_t = 0.95 f_1^h(z) - 1.72 f_2^h(z) + 0.85 f_3^h(z)
\]  

(10)

where,

\[
f_1^h(z) = \tanh\left( (4.97 + 0.90)N_{t-1}^i + (0.12 + 0.90)W_{t-1}^i + \cdots + (0.96 + 0.90)N_{t-1}^{i+3} + (6.85 + 0.90)W_{t-1}^{i+3} + \cdots + (1.17 + 0.90)W_{t-1}^{i+3} + 0.90a_{t-1} \right)  
\]

\[
f_2^h(z) = \tanh\left( (-1.23 - 0.90)N_{t-1}^i - (0.26 + 0.90)W_{t-1}^i + \cdots - (0.33 + 0.90)N_{t-1}^{i+3} - (2.40 - 0.90)W_{t-1}^{i+3} + \cdots - (0.55 - 0.90)W_{t-1}^{i+3} - 0.90a_{t-1} \right)  
\]

\[
f_3^h(z) = \tanh\left( (3.64 - 3.07)N_{t-1}^i + (1.25 - 3.07)W_{t-1}^i + \cdots + (2.61 - 3.07)N_{t-1}^{i+3} - (2.62 + 3.07)W_{t-1}^{i+3} + \cdots - (0.55 + 3.07)W_{t-1}^{i+3} - 3.07a_{t-1} \right)  
\]

4.4. Best Model Selection

Figure 3 shows model performance out of 10 replications based on RMSE in each variable and locations. For a variable 2 in location 1, one of 10 replications represent that hybrid MGSTAR-FFNN model has the smallest RMSE value, 4 replications are the MGSTAR-DLNN and 5 replications result that MGSTAR-ERNN has smallest RMSE value. So that we can say for variable 2 in location 1 the best performance model is MGSTAR-ERNN. In scenario II model with the smallest RMSE is the MGSTAR-ERNN. It can be seen from Figure 3b which shows that MGSTAR-ERNN from 10 replication mostly has smaller RMSE value than the other model for each variable and location.

Figure 3. Comparison of the Best Models in the (a) Scenario I and (b) Scenario II

The average value of RMSE in Figure 4 shows that the hybrid MGSTAR-ANN model has lower RMSE value when compared to the MGSTAR model. Although in the scenario I the average RMSE
value of MGSTAR-JRNN model is higher than the MGSTAR model, for the hybrid MGSTAR-FFNN, MGSTAR-DLNN, and the MGSTAR-ERNN show a smaller average value.

![Figure4](image)

**Figure 4.** The average value of each RMSE Model in scenario (a) I and (b) II

The four models of the MGSTAR-ANN hybrid in scenario II have a smaller RMSE average value compared to the MGSTAR model. The MGSTAR-ERNN Model with the smallest RMSE average value in each variable and location can be said to be the best model for scenario II. So the overall model of the MGSTAR-ANN hybrid is better than the MGSTAR model.

5. Conclusion

This study proposed a method that combined linear and non-linear model to improve accuracy of MGSTAR model. The non-linear approach used is RNN. There were 2 different architecture of RNN that want to compare i.e. ERNN and JRNN. The results shown that the hybrid MGSTAR-RNN produce better accuracy than individual MGSTAR model. However, MGSTAR-JRNN model had smaller accuration than MGSTAR-ERNN due to its simple architecture [12]. Hence, it could be said the forecasting hybrid model tends to be better compared with the forecasting MGSTAR model in accordance with the conclusion of the M4-competition results [13].

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