Forecasts on CMB lensing observations with AliCPT-1

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Received July 7, 2022; accepted July 29, 2022; published online September 2, 2022

AliCPT-1 is the first Chinese cosmic microwave background (CMB) experiment aiming for the high-precision measurement of CMB B-mode polarization. The telescope, currently under deployment in Tibet, will observe in two frequency bands centered at 90 and 150 GHz. We forecast the CMB lensing reconstruction, lensing-galaxy, and lensing-cosmic infrared background (CIB) cross-correlation signal-to-noise ratio (SNR) for AliCPT-1. We consider two stages with different integrated observation times, namely “4 module*yr” (first stage) and “48 module*yr” (final stage). For lensing reconstruction, we use three different quadratic estimators, namely temperature-only, polarization-only and minimum-variance (MV) estimators, using curved sky geometry. We take into account the impacts of inhomogeneous hit counts and mean-field bias due to incomplete sky coverage. In the first stage, our results show that the 150 GHz channel can measure the lensing signals at 15σ significance with the MV estimator. In the final stage, the measurement significance will increase to 31σ. We also combine the two frequency data in the harmonic domain to optimize the SNR. Our results show that the coadding procedure can significantly reduce the reconstruction bias in the high multiple range. Owning to the high quality of the polarization data in the final stage of AliCPT-1, the EB estimator will dominate the lensing reconstruction in this stage. We also estimate the SNR of cross-correlations between AliCPT-1 CMB lensing and other tracers of the large scale structure of the universe. For its cross-correlation with Dark Energy Spectroscopic Instrument (DESI) galaxies/quasars, we report the cross-correlation SNR = 10-20 for the four redshift bins at 0.05 < z < 2.1. In the first stage, the total SNR is approximately 32. In the final stage, the lensing-galaxy cross-correlation can reach SNR = 52. For lensing-CIB cross-correlation, in the first stage, the cross-correlations between AliCPT-1 lensing and Planck CIB 353, 545 and 857 GHz channels are approximately SNR = 18, 19, and 23, respectively. In the final stage, the cross-correlations can reach SNR

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1 Introduction to CMB Lensing

When primary cosmic microwave background (CMB) photons approach us from the last scattering surface, they are deflected by the intervening large scale structures that distort the observed pattern of CMB anisotropies. This effect is called CMB lensing [1]. The characteristic deflection angle is approximately 2 arcmin. Its coherence angular scale is about a few degrees, roughly corresponding to the peak scale of the matter power spectrum at \(1 \leq \ell \leq 4\), where the dominant lensing contribution arises. On the one hand, CMB lensing carries fruitful information on the underlying matter distribution. On the other hand, it also converts some portions of primary B-mode (even parity) polarization into B-mode (odd parity) polarization, generating lensing B-modes [2–4], which contaminate the measurement of the primordial gravitational waves. Hence, CMB lensing is a useful probe of large-scale structures, which can be used to explore the nature of dark energy, dark matter and neutrinos. However, for primordial B-mode detection, the lensing B-modes are an unavoidable intrinsic source of noise, with typical amplitude 5 \(\mu K\) arcmin in B-mode maps. We recommend refs. [5, 6] for a review.

Because a gravitational interaction neither creates nor destroys photons, the total number of CMB photons is conserved during the lensing process. Lensing just remaps the original spatial distributions of CMB photons from the direction \(\hat{n}\) into a new direction \(\hat{n} + d(\hat{n})\), where \(d(\hat{n})\) is the deflection angle with characteristic amplitude of 2 arcmin. Up to the leading order, the deflection angle can be expressed in terms of the gradient of the lensing potential \(d(\hat{n}) = \nabla \phi(\hat{n})\). The CMB lensing potential field is defined as:

\[
\phi(\hat{n}) = -2 \int_{\chi_{0}}^{\infty} d\chi \frac{\chi - \chi_{0}}{\chi} \Psi(\chi, \eta_{0} - \chi),
\]

where \(\chi\) is the conformal distance and \(\chi_{0} \approx 14\) Gpc is the conformal distance between the present and the CMB last scattering surface. \(\Psi(\chi, \eta_{0} - \chi)\) is the Weyl potential at a conformal distance \(\chi\) along the direction \(\hat{n}\) at a conformal time \(\eta\) (the conformal time today is denoted as \(\eta_{0}\)). Here, we explicitly assume a flat spatial geometry. For the temperature field, we have

\[
T(\hat{n}) = T(\hat{n} + d(\hat{n})) = T(\hat{n}) + \sum_{i} \nabla^{i} \phi(\hat{n}) \nabla_{i} T(\hat{n}) + \mathcal{O}(\phi^{2}).
\]

For polarization fields, we have

\[
\tilde{Q} = \mathcal{O}(U) \tilde{\hat{n}} - \mathcal{O}(U) \tilde{\hat{n}},
\]

\[
\mathcal{Q} = \mathcal{O}(U) \tilde{\hat{n}} + \sum_{i} \nabla^{i} \phi(\hat{n}) \nabla_{i} \mathcal{Q} = \mathcal{O}(U) \tilde{\hat{n}} + \mathcal{O}(\phi^{2}),
\]

where the quantities with and without tilde correspond to the lensed and primary CMB, respectively.

A first attempt at measuring the CMB lensing signals was made via the cross-correlations between WMAP 1-year temperature data and Sloan digital sky survey (SDSS) luminous red galaxies [7]. However, the statistical evidence is not significant enough to claim a detection. The first CMB lensing detection (3.4 \(\sigma\) significance) was performed by cross-correlating the WMAP result with radio galaxy counts from the NRAO VLA sky survey [8]. Subsequently, several teams have attempted to reconstruct the lensing signals using other large-scale structure tracers [9, 10]. At a low signal-to-noise ratio (SNR), the lensing signals have been internally detected with WMAP7-year temperature data [11, 12]. Afterward, a series of ground-based CMB experiments have measured the lensing signals with higher SNR. The Atacama cosmology telescope (ACT) collaboration in 2011 first presented a 4.0 \(\sigma\) detection of CMB lensing signals internally with their temperature map [13]. It was then updated in 2013 with a 4.6 \(\sigma\) detection [14]. The south pole telescope (SPT) collaboration reported their first lensing power spectrum reconstruction result in 2012 [15]. Later on, the first detection of lensing B-modes was made by the SPT in 2013 by using the cross-correlation between maps of CMB polarization and sub-millimeter maps of galaxies from Herschel-SPire [16]. By using 500 deg\(^2\) of SPTpol data from 95 and 150 GHz channels, the collaboration measured the BB power spectrum in the multipole range 52 < \(\ell\) < 2301 with 18.1 \(\sigma\) detection, among which lensing B-modes were detected with 8.7 \(\sigma\) significance [17]. The POLARBEAR collaboration detects the
lensing signals at the 4.2σ confidence level from the 30 deg^2 polarization map [18] and at the 4.0σ (2.3σ for lensing B-modes) confidence level from the cross-correlation with the Herschel cosmic infrared background [19]. Owning to the excellent sensitivity (∼ 3 μK arcmin), BICEP2 and the Keck array can measure the lensing signals at the 5.8σ level, with a modest angular resolution (∼ 0.5°) [20].

The first reconstructed lensing potential map on the nearly full sky was obtained by the Planck Collaboration in 2013 [21] with 25σ significance. This lensing map was reconstructed from the 15-month temperature data alone. In 2015, the Planck Collaboration updated this reconstruction by adding another 15 months of temperature and 30 months of full-mission polarization data [22]. These additional data help increase the lensing reconstruction significance up to the 40σ level. The final full-mission analysis in 2018 used essentially the same data as those in 2015, but improved the foreground masking in the simulations. It helped increase the significance of the detection of lensing in the polarization maps alone from 5σ to 9σ [23]. Furthermore, the collaboration demonstrated the delensing technique with the final full-mission data. A decrease in the power of the B-mode polarization after delensing was detected at 9σ. To the best of the authors’ knowledge, this 10.1σ detection from SPTpol [24] and 9σ detection from Planck 2018 [23] represent the state-of-the-art measurement of the lensing signals from polarization data. We can treat this number as the benchmark for lensing reconstruction from polarization data. In the rest of this paper, one can see that the polarization data from AliCPT-1 has the capability of significantly improving this number. This is one of the extraordinary science cases for the AliCPT-1 project.

CMB lensing is also physically correlated with LSS tracers such as galaxy and galaxy cluster distribution, and cosmic shear. On the one hand, these cross-correlations are immune to certain systematics in the auto-correlation, such as the additive errors in both CMB lensing and cosmic shear. On the other hand, they provide essential information, such as the redshift information, to improve the cosmological applications of CMB lensing. The cross-correlation of CMB lensing with galaxies [25-29], galaxy groups and clusters [30-33] have already been detected. As the AliCPT sky area is fully covered by the DESI footprint [34], the AliCPT CMB lensing-DESI galaxy cross-correlation is a natural method of enhancing the science return of AliCPT. For this reason, we estimate the cross-correlation signal between AliCPT and DESI. We find that the total SNR is ∼ 32, which will provide a useful constraint on the structure growth rate.

The cosmic infrared background (CIB) carries the integrated history of star formation between the redshift 1 ≤ z ≤ 3, which highly overlaps with the CMB lensing signals. The first detection of this cross-correlation was reported by the Planck Collaboration at 2013 [35] by correlating CMB lensing with the CIB measured in the 353, 545, and 857 GHz channels. As an external tracer, the CIB can also be used for delensing. Via the delensing technique, the Planck 2018 result [23] showed the primary CMB peak sharpening at 11σ from the lensing reconstruction alone and 15σ on further combination with the CIB.

2 AliCPT-1 mock datasetup

AliCPT-1 is the first Chinese CMB experiment. Its main scientific goal is to constrain the primordial gravitational wave signal with high precision. The telescope has two frequencies, namely 90 and 150 GHz. The full focal plane can accommodate 19 modules. Each of the 19 modules contains 1704 transition edge sensor (TES) detectors, among which half are in 90 GHz and the other half are in 150 GHz.

In this section, we introduce the mock data for each single frequency band. According to the scanning strategy, we determine the number of hits per pixel. The noise level in each frequency band is proportional to the ratio of noise equivalent temperature (NET) and the square root of the hit counts, i.e.,

\[ \text{noise} = \frac{\text{NET}}{\sqrt{\text{hits}}}. \]  

In our simulations, we assume that there is no correlation between the AliCPT temperature and polarization noise. The NET for polarization is a factor \( \sqrt{2} \) higher than that of the temperature because the observation time is spread between two polarization signals, the Q and U Stokes parameters. Because the number of detectors in each frequency is evenly distributed in the modules, the noise spatial distributions in the two frequencies are almost the same up to an overall normalization factor. Here, we explicitly assume the noise to be uncorrelated between different pixels.

We consider two different accumulated module numbers for the nominal and final AliCPT-1 experiment. The first stage uses 4 modules observing for 1 year, which is dubbed hereafter the "4 module*yr" stage. The preliminary roadmap for the final mission is as follows: The first year observes with 4 modules; the second year adds 6 modules; the third year adds another 5 modules; and finally the last 4 modules are added in the focal plane for the last year). Hence, the total integrated observation time will be "48 module*yr" after four years of observation. In this paper, we forecast

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1) The focal plane can assemble 19 modules in total.
the lensing reconstruction, lensing-galaxy and lensing-CIB cross-correlations based on the two integrated observation times. In practice, the actual time of observation is less, because of bad weather and various data cuts. This is taken into account in the simulations.

In Figure 1, we show the noise standard deviation maps for the "4 module*yr" stage. From now on, we simply consider the statistical noise, where the noise standard deviation in the "4 module*yr" stage can be obtained by rescaling the "4 module*yr" one with an overall normalization factor of $1/\sqrt{2}$. The left and right panels of Figure 1 are the noise standard deviation maps in the 90 and 150 GHz channels, respectively. These were obtained by adopting the present "deep patch" scanning strategy, which covers 14% sky area. The square roots of the harmonic mean of the noise variance in "4 module*yr" case are approximately 11 μK arcmin for 90 GHz and 17 μK arcmin for 150 GHz.

The mock datasets in each frequency channel contain 301 sets of simulations, among which 240 sets are used for covariance estimation, 60 sets are used for mean field subtraction and 1 set represents the "real" data. For each of the 301 datasets, we include the unlensed primary CMB maps $(T,E,B)$, a lensing potential map $(\phi)$ to lens the primordial maps, and a noise realization according to the noise variance map shown in Figure 1. All of the datasets are generated following the same procedure but with independent Gaussian random seeds. The CMB maps, lensing potential maps, and the noise maps among different datasets are independent.

The primary CMB and lensing potential maps were generated via CAMB [36] and HEALPix [37] with a resolution parameter $N_{\rm side} = 1024$. The lensed CMB maps are obtained by combining the unlensed primary CMB with the lensing potential maps via lenspyx code [38]. Then, we convolved the beam function $(B_{\ell})$ and the HEALPix transfer function $(H_{\ell})$ with the lensed CMB maps in the pixel domain. The full widths at half maximum (FWHM) of the beam function are 19 arcmin for the 90 GHz channel, and 11 arcmin for the 150 GHz channel. Finally, we added the noise in the pixel domain directly on top of the lensed CMB signals convolved by the Gaussian beams.

In this paper, we adopted the curved sky quadratic estimators developed by Okamoto and Hu [39] in 2003. This estimator was first introduced by Hu and Okamoto [40] in 2001 with a flat sky approximation. Because AliCPT-1 has a 14% sky coverage ($\sim 5600$ deg$^2$), to handle the lower multipoles better, we used the mathematically more complicated curved-sky quadratic estimator, in the slightly modified version of the sub-optimal quadratic estimator by ref. [41] by considering the TE cross-correlation.

3 Lensing reconstruction

In this section, we present the quadratic lensing reconstruction methodology and results.

3.1 Method

Lensing induces correlations between different multipoles. The off-diagonal term of the covariance matrix of the CMB fields reads

$$\langle X_{\ell_1 m_1}, Z_{\ell_2 m_2} \rangle = \sum_{LM} (-1)^M \left( \begin{array}{ccc} \ell_1 & \ell_2 & L \\ m_1 & m_2 & -M \end{array} \right) W^{XZ}_{\ell_1 \ell_2 M} \phi_{LM},$$

where the fields $X_{\ell m}, Z_{\ell m} \in \{T_{\ell m}, E_{\ell m}, B_{\ell m}\}$. The expression of the covariance response function $W^{XZ}_{\ell_1 \ell_2 L}$ for all the possible field combinations can be found in Okamoto and Hu [39]. The big parenthesis denotes the Wigner 3-j symbol due to

Figure 1 (Color online) Noise standard deviation map for the temperature mode in 90 and 150 GHz under the "4 module*yr" stage. The noise standard deviation for the polarization mode can be obtained by multiplying a factor $\sqrt{2}$.

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2) We refer the details of the "deep patch" scanning strategy in the simulation paper of this series.

3) https://github.com/caronj/lenspyx.
the coupling of different angular momentum modes. $\phi_{LM}$ is the lensing potential spherical harmonics, namely our reconstruction goal. Owing to the nearly Gaussian nature of the primary CMB signal at recombination, there is no correlation between different multipoles of primordial components. Hence, the lensing contribution appears in the leading order in the off-diagonal terms. The basic idea of the quadratic estimator is to utilize these induced correlations in an (almost) optimal and unbiased way. In the following, we will list a few essential steps of our reconstruction recipe.

3.1.1 Filtering

In this paper, we followed the Planck 2018 lensing paper\(^4\)\(^5\) [23] formalism, which is close to the original Hu-Okamoto formalism [39, 40]. The basic idea is to rewrite the lensing estimator in terms of pairs of filtered maps [42]. One leg of the pair is the inverse-variance filtered CMB fields, and the other leg is the Wiener-filtered CMB fields. As the formulation is more easily operated in a spin-weighted harmonic space, we first convert the traditional $T$, $E$, $B$ fields into $X^{\ell m}(\hat{n}) \in \{\mathbf{T}^\text{dat}, \mathbf{T}^\text{dat}, \mathbf{P}^\text{dat}\}$,

$$
\begin{pmatrix}
\mathbf{T}^\text{dat} \\
\mathbf{E}^\text{dat} \\
\mathbf{B}^\text{dat}
\end{pmatrix} = \mathbf{B} \mathbf{Y} 
\begin{pmatrix}
T \\
E \\
B
\end{pmatrix} + \text{noise},
$$

where matrix $\mathbf{B}$ accounts for the real-space operations of the beam and pixel convolution. The matrix $\mathbf{Y}$ contains the appropriate (spin-weighted) spherical harmonic functions to map from multipoles to the sky. Then, the inverse-variance filter operation is defined as:

$$
\hat{X}(\hat{n}) = \mathbf{B}^\dagger \text{CoV}^{-1} \begin{pmatrix}
\mathbf{T}^\text{dat} \\
\mathbf{E}^\text{dat} \\
\mathbf{B}^\text{dat}
\end{pmatrix}.
$$

CoV is the pixel-space covariance defined as $\text{CoV} = \mathcal{T} \mathcal{C}^\text{sd} \mathcal{T}^\dagger + \mathcal{N}$, where $\mathcal{C}^\text{sd}$ denotes the spherical harmonic space covariance matrix consisting of the fiducial lensed CMB power spectra; $\mathcal{N}$ denotes for the pixel-space noise covariance matrix, which is approximated as a diagonal, and $\mathcal{T} = \mathbf{B} \mathbf{Y}$ denotes the complete transfer function from multipoles to the pixelized sky. The Wiener-filtered CMB fields, $X^{\text{WF}}(\hat{n}) \in \{W^{\text{WF}}, W^{\text{WF}}, B^{\text{WF}}\}$, are defined as:

$$
\begin{pmatrix}
W^{\text{WF}} \\
E^{\text{WF}} \\
B^{\text{WF}}
\end{pmatrix} = \mathcal{C}^\text{sd} \mathcal{T} \text{CoV}^{-1} \begin{pmatrix}
\mathbf{T}^\text{dat} \\
\mathbf{E}^\text{dat} \\
\mathbf{B}^\text{dat}
\end{pmatrix}.
$$

Here, we filtered the temperature and polarization data independently, which means that we neglected the TE correlation in the covariance matrix of eq. (8). As demonstrated in PL18, neglecting $C_\ell^{\text{TE}}$ in the Wiener filter process will cause approximately 3% noise increase for the multipoles $L < 400$, and less in our case. The inversion of the covariance matrix was computed via a conjugate-gradient inversion method with a multi-grid preconditioner [8].

3.1.2 Quadratic estimator

Then, one can construct a (yet unnormalized) lensing deflection angle (spin-1 field) estimate based on the inverse-variance filtered and Wiener-filtered fields

$$
j_{1}(\hat{n}) = - \sum_{\ell=0,\pm 1} \delta X(\hat{n}) \left[ \delta, X^{\text{WF}}(\hat{n}) \right],
$$

where $\delta$ is the spin-raising operator, and the pre-subscript $s$ on the field indicates a spin. The explicit expression of the spin-raising operator can be found in PL18. The deflection angle $j_{1}(\hat{n})$ can be decomposed into its gradient-like $(g)$ and curl-like $(c)$ component via the spin-1 harmonic transformation. The expected curl-like component (corresponding to the field rotation [5, 43]) is safely negligible for our purposes, and the gradient term directly traces the lensing potential $\phi$

$$
j_{1}(\hat{n}) = \sum_{LM} \left( \frac{\delta_{LM} \mp i \gamma_{LM}}{\sqrt{L(L+1)}} \right)_{1} Y_{LM}(\hat{n}).
$$

Following PL18, we calculated three estimators, namely temperature-only (T-only) ($s = 0$), polarization-only ($s = \pm 2$) (P-only), and minimum-variance (MV) ($s = 0, \pm 2$), rather than the traditional full set TT, TE, TB, EE, and EB of Hu-Okamoto estimators. In the T-only and P-only estimators, we analyzed the temperature and polarization data independently and did not need to include the TE cross-correlation. For the MV estimator, we did include TE correlation in the fiducial spectra matrix $C^{\text{sd}}$ in eq. (8) (resulting in the generalized MV estimator of ref. [41]\(^5\)).

3.1.3 Mean-field subtraction

The presence of a mask and foreground residuals introduces extra statistical anisotropies in the absence of lensing signals. Hence, they will bias the estimation of lensing potential. Because these effects are hard to model analytically, we calculated their contributions via Monte Carlo simulations. That is, in each of the simulation set, we varied simultaneously multiple ingredients, such as the primary CMB, lensing potential and instrumental noise. Then, we calculated the aver-

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\(^4\) Hereafter, we will refer to this paper as PL18.

\(^5\) In the sub-optimal quadratic estimator, the TE cross-correlation is neglected.
age value (mean-field) of the quadratic estimator. This average will be the most faithful representation of the contribution from the extra statistical anisotropy source. Hence, we can subtract it from the original estimator. This operation is called mean-field subtraction. Accordingly, our lensing potential estimate becomes

$$
\hat{\phi}_{LM} = \frac{1}{\mathcal{R}_L^2} \left[ \hat{\phi}_{LM} - \left( \hat{\phi}_{LM} \right)_0 \right],
$$

(11)

where $\mathcal{R}_L^2$ is the isotropic normalization, which is called response function and is calculated analytically [22]

$$
\mathcal{R}_L^2 = \frac{1}{2(2L+1)} \sum_{t_1,t_2} W^{\alpha \beta}_{t_1,t_2} M^{\alpha \beta}_{t_1,t_2} P^{\alpha \beta}_{t_1,t_2},
$$

(12)

where $W^{\alpha \beta}_{t_1,t_2}$ is the covariance off-diagonal response function defined in eq. (5), $M^{\alpha \beta}_{t_1,t_2}$ the $XZ$-estimator weighting function and $P^{\alpha \beta}_{t_1,t_2}$ is the isotropic Wiener filter

$$
P^{\alpha \beta}_{t_1,t_2} = \frac{C^{\alpha \beta}}{C^{\alpha \alpha} + N^{\alpha \beta}}.
$$

(13)

### 3.1.4 Lensing potential power spectrum and statistical bias

The raw power spectrum of the estimated lensing potential reads

$$
C_{\ell}^{\phi \phi} = \frac{1}{(2L+1) f_{\delta d}^2} \sum_{M=-L}^{L} \hat{\phi}_{LM} \hat{\phi}_{LM},
$$

(14)

where $\hat{\phi}_{LM}, \hat{\phi}_{LM}$ are (possibly different) estimates of the lensing potential, eq. (11). The quadratic estimator spectrum contains not only the sought-after signal, but also unavoidably the Gaussian reconstruction noise sourced by the CMB and instrumental noise ($N^{\delta d}$) and the non-primary couplings of the connected 4-point function [44] ($N_{ij}$ bias). After subtracting these biases, we obtain the final estimated power spectrum

$$
C_{\ell}^{\phi \phi} = C_{\ell}^{\phi \phi} - \Delta C_{\ell}^{\phi \phi} \text{RDN0} - \Delta C_{\ell}^{\phi \phi} |_{N_{ij}} - \cdots,
$$

(15)

where the dots denote other possible contamination biases which are not considered here, such as point sources, "RDN0" means realization-dependent $N^{\delta d}$ bias, which is designed to subtract the primary CMB contamination in the most faithful manner. For further detailed expressions, we refer to the Appendix A of the Planck 2015 lensing paper [22].

In Figure 2, we show several noise spectra in the reconstruction in the "4 module*yr" and "48 module*yr" stages. We highlight the MV-N0 source spectrum in the 150 GHz channel (red solid curve). For the "4 module*yr" case, the signal (black bold curve) is higher than the noise curve for each multipole in the range $10 \leq L \leq 70$. For the "48 module*yr" cases, we can extend this maximum multipole to $L \approx 200$.

#### 3.1.5 Binning and multiplicative correction and SNR estimation

After binning the multipoles, the band power of the lensing potential reads

$$
C_{Lb}^{\phi \phi} = \left( \frac{\mathcal{S}_{L}^{\phi \phi} \mathcal{C}_{Lb}^{\phi \phi}}{\mathcal{S}_{L}^{\phi \phi} \mathcal{C}_{Lb}^{\phi \phi}} \right),
$$

(16)

where the binning function is defined as:

$$
\mathcal{S}_{L}^{\phi \phi} = C_{Lb}^{\phi \phi} \frac{C_{L}^{\phi \phi}}{\mathcal{S}_{L}^{\phi \phi}} V_{L}^{-1} \leq L \leq L_{\max}.
$$

(17)

This binning method is designed to produce the minimum variance with optimal weights, which reads

$$
V_{L}^{-1} \propto (2L+1)f_{\delta d}^2 \mathcal{R}_{Lb}^2.
$$

(18)

$L_{b}$ is the band power index,

$$
L_{b} = \frac{\sum L \mathcal{S}_{L}^{\phi \phi}}{\sum \mathcal{S}_{L}^{\phi \phi}}.
$$

(19)

The final ingredient of the power spectrum reconstruction is the second parenthesis on the right hand side of eq. (16), namely "multiplicative correction". It corrects for the various isotropic and simplified approximations. The $\left( \mathcal{C}_{Lb}^{\phi \phi} \right)_0$ in the denominator is calculated with the much cheaper Monte Carlo $N^{\delta d}$ estimation, rather than the RDN0 method.

In this paper, we estimated the SNR via the Fisher matrix method

$$
\text{SNR} = \sqrt{\sum_{\ell \ell'} C_{\ell} C_{\ell'}^{-1} C_{\ell'}},
$$

(20)

where the $C_{\ell}$'s in the numerator are chosen to be the theoretical one instead of the reconstructed one. The latter (reconstructed spectrum) has unavoidable random scatter, which may affect the final SNR prediction. To make a stable prediction, here, we chose the former one (theoretical spectrum). $C_{\ell} C_{\ell'}$ is the covariance matrix obtained from 240 simulation sets, which reads

$$
C_{\ell} C_{\ell'} = \frac{1}{N-1} \sum_{s=1}^{N=240} \left[ \left( C_{\ell}^{\phi \phi} - C_{\ell}^{\phi \phi} \right) \times \left( C_{\ell'}^{\phi \phi} - C_{\ell'}^{\phi \phi} \right) \right],
$$

(21)

where $C_{\ell}^{\phi \phi}$ is the averaged lensing potential power spectrum based on the simulation sets.
3.2 Results

In this subsection, we summarize our lensing reconstruction forecast results based on the mock data presented in sect. 2 and the methodology reviewed in sect. 3.1.

In Figure 3, we show the input (left panel) and MV reconstructed (middle and right panels) deflection angle amplitude under the 4 module*yr and 48 module*yr scenarios. To highlight the signals, we show the results after Wiener filtering. Because the noise on the edge drastically increases, the filtered signal on the edges becomes significantly suppressed. Owing to the sufficiently long enough integrated observation time, in the center of the field, we recovered the large-scale features of the original lensing deflection field.

In Figure 4, we show the reconstructed lensing potential spectra in the “4 module*yr” stage. In the 90 GHz channel, the SNRs are 3.5, 5.1, 9.2 for T-only, P-only, and MV estimators, respectively. In the 150 GHz channel, the SNRs are 8.3, 6.6, 15.4 for T-only, P-only, and MV estimators, respectively. For the MV estimator (purple boxes), we can get SNR higher than unity in each of the multiple bands in the range of $L < 400$ (90 GHz) and $L < 700$ (150 GHz). This is the main result of this paper.

In Figure 5, we show the reconstructed lensing potential spectra under the “48 module*yr” stage. In the 90 GHz channel, SNR are 4.8, 21.7, 24.3 for the T-only, P-only, and MV estimators. In the 150 GHz channel, SNR are 8.0, 25.4, 31.1 for T-only, P-only, and MV estimators. For the MV estimator (purple boxes), we can obtain an SNR higher than unity in each of the multiple bands in the ranges $L < 900$ (90 GHz) and $L < 1100$ (150 GHz). We summarize the reconstruction SNRs in Table 1. For the T-only estimator, the SNR under the “48 module*yr” stage are similar to those in the “4 module*yr”6.

One can see this also from the noise

Figure 2 (Color online) Noise power spectra in lensing reconstruction under the “4 module*yr” (a) and “48 module*yr” (b) stages. The green, blue and red curves are the $N^{(0)}$ bias for the T-only, P-only, and MV estimators, respectively. Orange curves are the $N^{(1)}$ biases in the MV estimators. Solid and dotted curves are for 150 and 90 GHz, respectively. The solid gray curve denotes for the mean-field bias in the 150 GHz MV estimator. The bold solid black curve denotes for the theoretical lensing potential spectrum.

Figure 3 (Color online) Wiener-filtered deflection angle amplitude, $\delta_{WF} = \sqrt{L(L+1)\Delta L}\frac{C_{L}^{\alpha\beta\gamma} + \Delta \delta_{L}^{\alpha\beta\gamma} + \gamma^{(0)}_{L}\epsilon_{L}^{(0)} + \gamma^{(1)}_{L}\epsilon_{L}^{(1)}}{C_{L}^{\alpha\beta\gamma}}$. The left panel is the input. The middle and right panels are the MV-reconstructed results in the 4 module*yr and 48 module*yr cases, respectively.

6) Due to the random fluctuations in the realizations, for the 150 GHz channel, the SNR of the T-only estimator from the “48 module*yr” stage is even a bit lower than that from the “4 module*yr” stage.
power spectra level, Figure 2. The accumulation of the temperature data does not significantly help in reconstructing the lensing signals. For polarization, the case is much better. The integration of the polarization data greatly improves the lensing reconstruction results.

We further analyzed the contributions from the TE and EB estimators. Taking the 150 GHz channel as an example, in the “4 module*yr” stage, the SNR from the TE (SNR = 6.4) is higher than that of the EB (SNR = 3.1). Once we accumulated the data and stepped into the “48 module*yr” stage, the EB estimator contributed significantly, with SNR = 21.4. In this stage, the TE estimator becomes sub-dominant, SNR = 13.8. This result demonstrates the important role of polarization data in the AlI/CP lensing analysis. This is another major result of this paper.

Furthermore, we tested the coadd results by combining 90 and 150 GHz CMB maps in the harmonic domain with the inverse minimum-variance weighting and of 11 arcmin resolution. This method has basically the same idea as the “SMICA weights” in the Planck 2015 and 2018 lensing papers [22, 23]. We show the MV estimator coadd results in the “4 module*yr” and “48 module*yr” stages in Figure 6. Comparing the yellow (150 GHz) and purple (90 + 150 GHz) boxes, one can see that the coadding procedure can significantly reduce the systematic bias in the high multipole bins. For the error bars, we find that the improvement by combining the two frequencies is very limited and can be

Table 1 Lensing reconstruction SNR

| Frequency (GHz) | Estimator | 4 module*yr | 48 module*yr |
|-----------------|-----------|-------------|--------------|
| 90              | T-only    | 3.5         | 4.8          |
|                 | P-only    | 5.1         | 21.7         |
|                 | MV        | 9.2         | 24.3         |
| 150             | T-only    | 8.3         | 8.0          |
|                 | P-only    | 6.6         | 25.4         |
|                 | MV        | 15.4        | 31.1         |
Figure 6 (Color online) Coadding result in the “4 module*yr” (left panel) and “48 module*yr” (right panel) stages. Gray, yellow, and purple boxes are the MV estimator reconstructions from 90, 150, and 90 + 150 GHz, respectively. Compared with the 150 GHz channel, the coadding procedure does not reduce the error bars but remove the systematic bias in the high multiple range.

4 Cross-correlations

In this section, we will present the cross-correlation signals that can be detected by AliCPT CMB lensing together with the external large scale structure tracers, such as galaxy number counts and CIB.

4.1 Cross-correlations with galaxies

It is convenient to work with the lensing convergence $\kappa$, instead of potential $\phi$, for the lensing-galaxy cross-correlation

$$\kappa(\theta) = \frac{3\Omega_m H_0^2}{2c^2} \int_0^\chi_{\star} d\chi (\chi_{\star} - \chi) \frac{\delta_m(\chi, \theta)}{\chi_{\star}} a$$

$$= \int_0^\chi_{\star} d\chi W^\kappa(\chi) \delta_m(\chi, \theta).$$

(22)

Here, $W^\kappa(\chi)$ is the $\kappa$ kernel defined by eq. (25), $\delta_m$ is the density fluctuation, $\Omega_m$ is the total matter density today, $H_0$ is the Hubble constant today, and $a \equiv 1/(1 + z)$ is the scale factor.

The galaxy overdensity and CMB lensing convergence $\kappa$ are both projections of 3D density fields, expressed as line-of-sight integrals over their respective projection kernels. The angular cross-correlation power spectrum, adopting the Limber approximation [45], is

$$C_{\ell}^{\kappa g} = \int_0^\chi_{\star} d\chi W^\kappa(\chi) W^g(\chi) \frac{1}{\chi_{\star}^2} P_m(k) \left( k = \frac{\ell + 1/2}{\chi_{\star}} z \right).$$

(23)

The Limber approximation is inaccurate for $\ell < 10$, but such very large-scale modes are excluded from our analysis, due to the poor SNR. The above expression assumes spatial flatness. Here $W^\kappa$ and $W^g$ are the projection kernels for $\kappa$ and the galaxy number density fields, respectively

$$W^g(z) = n(z) = \frac{c}{H(z)} W^g(\chi),$$

(24)

$$W^\kappa(z) = \frac{3}{2c^2 \Omega_m H_0^2} (1 + z) \frac{\chi_{\star}(\chi_{\star} - \chi)}{\chi_{\star}^2} = \frac{c}{H(z)} W^\kappa(\chi).$$

(25)

Here $\chi_{\star}$ is the comoving distance to redshift $z$ and $\chi_{\star} = \chi (z \approx 1089)$ is the distance to the surface of the last scattering. $n(z)$ is the normalized redshift distribution of galaxies. $P_m$ is the 3D cross spectrum between the matter and galaxy number overdensity. We define the galaxy bias through $b_g \equiv P_m/P_{mm}$, where $P_{mm}$ is the matter power spectrum. $b_g$ is approximately scale-independent at the large scales of interest. The galaxy bias is redshift-dependent in our analysis, and the bias distribution of four galaxy populations is given by ref. [46].

The lensing-galaxy cross-correlation measurement $S/N$ relies on both the survey masks of the CMB lensing map and galaxies, and measurement noises. We followed the procedure in measuring the cross-correlation between Planck CMB lensing and DESI imaging survey galaxy groups [33]. First, we filtered the simulated lensing maps with the Wiener filter, and then used HEALPix to obtain the real-space lensing map of $n_{side} = 1024$. Then, we applied the combined lensing and galaxy masks and downgraded it to $n_{side} = ...
512. Naive = 512 was adopted given both the consideration of the lensing $S/N$ distribution and the sparse DESI galaxy distribution in each redshift bin. The Wiener filter was adopted to reduce the aliasing effect, which amplifies the lensing reconstruction noise in the low resolution map. For the galaxy overdensity map, we applied the same combined masks. We used HEALPix to calculate the spherical harmonic coefficients $\hat{\delta}_{2m}$ and $\hat{\kappa}_{2m}$, which are raw estimations of the true $\delta_{2m}$ and $\kappa_{2m}$, respectively. All maps presented in the galaxy-x cross-correlation use $\text{Naive} = 512$ and all relevant spherical harmonic transforms use $\ell_{\text{max}} = 1024$. For simplicity, we assumed that DESI footprints completely cover the AliCPT sky, and therefore we only used the binary mask of AliCPT’s “deep path” in this calculation. The resulting cross-power spectrum reads

$$
C^{\ell}_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \hat{\delta}_{2m} \hat{\kappa}_{2m}^*.
$$

(26)

The cross-power spectrum estimated above contains not only the Wiener filter, but also the survey masks.

For the galaxy mocks, we followed the DESI consortium. The DESI science paper [46] provides the redshift and bias distribution for the bright galaxies (BGGs), luminous red galaxies (LRGs), emission line galaxies (ELGs), and quasistellar objects (QSOs). The redshift ranges of the BGS, LRG, ELG, and QSO targets will cover $0.05 < z < 0.4$, $0.4 < z < 1.0$, $0.6 < z < 1.6$, and $z < 2.1$, respectively. For QSOs, they use the samples as direct tracers of the dark matter in the redshift range $0.9 < z < 2.1$, but not including the foreground neutral-hydrogen absorption systems that make up the Ly-α forest at higher redshifts. The characteristics of the baseline samples for each of the target classes are summarized in Table 3.1 in ref. [46]. Meanwhile, LRGs, ELGs, and QSOs assume values for the ratios of galaxy clustering to dark matter clustering, commonly referred to as the large-scale structure bias. On large scales this may be approximated as a function of the redshift, which is scale-independent, $b(z)$. Except BGSs, the DESI consortium assumes that fiducial biasses follow the constant $b(z)D(z)$, where $D(z)$ is the linear growth factor normalized by $D(z = 0) \equiv 1$. They assume a bias of the form $b_{\text{LRG}}(z)D(z) = 1.7$ for LRGs, $b_{\text{ELG}}D(z) = 0.84$ for ELGs, and $b_{\text{QSO}}D(z) = 1.2$ for QSOs. For BGSs, the bias is given in a numerical form [46]. In the second redshift bin, the redshifts of LRGs and ELGs overlap. Therefore, the effective bias $\bar{b} = f_{\text{LRG}}b_{\text{LRG}} + f_{\text{ELG}}b_{\text{ELG}}$, where $f_{\text{LRG}}$, $f_{\text{ELG}}$ are the galaxy number density fractions of LRGs and ELGs at $0.4 < z < 1.0$, respectively. The increases in the bias with redshifts for various galaxy types are in agreement with observations. These forms keep the observed clustering amplitude of each individual trace constant with redshifts.

The evaluation of the theoretical models involves a computation of the linear theory power spectrum. We used the core cosmology library (CCL) [47] for the theoretical spectrum calculations, with a fiducial Planck 2018 cosmology [48]: $\Omega_0 = 0.315$, $\Omega_\Lambda = 0.685$, $n_s = 0.965$, $h = \text{H}_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.674$, and $\sigma_8 = 0.811$. Given the redshift distribution, bias distribution, and baseline sample sizes of four DESI galaxy types, we can calculate the galaxy auto-power spectra $C_l^{gg}$ and white noise spectra $N_l^{gg} \equiv 4\pi f_{\text{sky}} N_{\text{gal}}$ for each redshift bin. We adopted the sky area overlapped by AliCPT-1 and DESI, which is approximately 5600 deg$^2$. We generated 300 galaxy mocks from the total galaxy power spectra $C_l^{gg} + N_l^{gg}$ by function synfast of HEALPix. The CMB lensing mocks were constructed from 300 simulations of the 150 GHz MV estimator assuming “4 module*yr” and “48 module*yr” stage configurations. As the reference, the corresponding spectrum can be found from the right panel of Figures 4 and 5.

We measured the uncertainty of cross spectra from the 300 simulated galaxy maps and 300 simulated AliCPT-1 CMB lensing reconstructions. The covariance matrix reads

$$
\text{Cov}_{\ell\ell} = \frac{1}{N - 1} \sum_{n=1}^{N-300} \left[ (C^{\ell}_{\ell}) - C^{\ell}_{\ell} \right] \left[ (C^{\ell}_{\ell}) - C^{\ell}_{\ell} \right]^T.
$$

(27)

Here $C^{\ell}_{\ell}(\ell)$ are calculated according to eq. (26) and $C^{\ell}_{\ell}(\ell)$ is the average cross spectrum. The lensing map was reconstruction noise dominated. The galaxy clustering-CMB lensing cross-correlation coefficient is much smaller than unity, due to the mismatch in their redshift distribution. For the above two reasons, cosmic variance arising from the galaxy-CMB lensing cross-correlation signal is negligible in the covariance matrix. Figure 7 shows the theoretical cross spectra in four redshift bins. The LRGs and ELGs overlap in the redshift range $0.4 < z < 1.0$. Remarkably, the theoretical cross spectra were modeled with the AliCPT-1 survey mask and Wiener filter for CMB lensing. As a result, the shapes of the power spectra are not like the conventional ones.

The detections are expected to be significant at all redshift bins. We quantify the detection significance of SNR, which is data-driven, and describes the detection significance of

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7) The noise power spectrum in the reconstructed lensing map keeps increasing toward small angular scales (Figure 2). This condition leads to amplified noise in the downgraded maps due to the aliasing effect. This problem can be avoided by applying the Wiener filter in the high-resolution map first [33].

8) https://github.com/LSSTDESC/CCL.
non-zero signal. Here,

\[ S/N \equiv \frac{1}{\sqrt{\sum_{\ell} C_{\ell}^{\text{bb}} \text{Cov}^{-1}_{\ell} C_{\ell}^{\text{bb}}}}. \]  

(28)

The four redshift bins achieve SNR = 10-20 (Table 2 column "4 module*yr"). As the covariance between different redshift bins is negligible, the total SNR combining all redshift bins is

\[ \left( \frac{S}{N} \right)_{\text{total}} = \sqrt{\sum_{\beta} \left( \frac{S}{N} \right)_{\beta}}. \]

(29)

Here \( \beta = 1, \ldots, 4 \) denotes the four redshift bins.

The measured cross-correlation has rich cosmological applications, such as constraining \( \sigma_8(z) \) and the dark energy equation of state. Here we demonstrate its power in testing general relativity (GR). The cross-correlation, in combination with the galaxy auto-correlation, allows measuring the linear growth function \( D(z) \propto C_{gg}/\sqrt{C_{bb}} \), independent of the galaxy bias. By the time of the Allegro observation, other cosmological data (e.g., Planck CMB and DESI BAO) will determine the shape of the matter power spectrum and the distance-redshift relation to high accuracy. The combination \( C_{gg}/\sqrt{C_{bb}} \) then depends only on \( D \). Usually the fractional measurement error in \( C_{gg} \) is much larger than that in \( C_{bb} \), we then have

\[ \frac{\sigma_D}{D} \approx \left( \frac{S}{N} \right)^{-1}. \]

(30)

Here \( S/N \) is the SNR at each redshift interval that listed in Table 2.

In GR, \( D(z) \) is completely specified by the expansion history even in the presence of dark energy. The linear growth rate, \( f(a) \), is related to the linear growth function \( D(a) \), and in GR is given by a good approximation of \( \Omega_m(z) \),

\[ f \equiv \frac{d \ln D}{d \ln a} = \frac{a d D}{D d a} \approx \Omega_m(z), \]

(31)

where \( \gamma \) is the growth index, approximately equal to 0.55 in GR. \( \Omega_m(z) \) is the fraction of the total matter density at redshift \( z \) in the critical density units. In the alternative gravity theories, a widely adopted parameterization of the modified growth rate is to alter the growth index \( \gamma \). The uncertainty of

| \( z \) | Targets | 4 module*yr | 48 module*yr |
|-------|---------|-------------|-------------|
| 0.05-0.4 | BGS | 13.4 | 18.9 |
| 0.4-1.0 | LRG + ELG | 18.7 | 29.7 |
| 1.0-1.6 | ELG only | 19.4 | 33.5 |
| 1.6-2.1 | QSO | 10.6 | 21.4 |
γ is given by
\[
\sigma^2_\gamma = \left( \frac{\partial \ln D / \partial \ln \gamma}{(\sigma_D / D)^2} \right)^{-1}. \tag{32}
\]

We can constrain γ to σ_γ = 0.16, which is approximately 29% relative error in the growth index determination. This constraint will be complementary to other constraints (e.g., the DESI forecast σ_γ = 0.04 [49]).

All the above results were obtained by assuming the "48 module*yr" stage configuration. For the "48 module*yr" stage configuration, the galaxy clustering-CMB lensing cross-correlation SNR reaches 53, and σ_γ = 0.10, namely the relative error in the growth index is about 18%. In this analysis, we have neglected the cosmic variance caused by the cross-correlation signal. We checked that, including the cosmic variance only reduces the signal-to-noise slightly (e.g., 53.1 to 52.2 for "48 module*yr").

### 4.2 Cross-correlations with cosmic infrared background

The CIB is a far-infrared relic emission of the galaxies during their formation and evolution processes. Produced by the heated dust within the galaxies, the CIB mainly consists of an integrated emission from unresolved dusty star-forming galaxies (DSFGs). Therefore, it contains a wealth of information about the DSFG distribution at high redshifts. Moreover, with the extraordinary redshift depth of CIB observations, the CIB anisotropies are thus an excellent tool to trace the underlying dark matter halos in which the galaxies reside and to probe the connection between luminous matter and dark matter.

In refs. [35, 50], a strong correlation (approximately 80%) was observed between the CIB anisotropies and a lensing-derived projected mass map. Therefore, we can write the CIB-lensing cross-spectra as:
\[
C^\nu_\ell = \int_0^\chi_0 d\chi \frac{1}{\chi^2} W^\nu W^\delta P_{\text{mg}}(k = \ell + \frac{1}{2})/x, z, \tag{33}
\]

where \( \phi \) is the lensing potential and \( \chi \) represents the CIB intensity at frequency \( \nu \); the integral is over \( \chi \), the comoving distance along the line of sight; \( \chi_0 \) is the comoving distance to the last scattering surface; and \( P_{\text{mg}}(k = \ell + \frac{1}{2})/x, z \) is the cross-correlation between the dark matter and dusty galaxies using Limber approximation. \( W^\nu \) and \( W^\delta \) are the redshift weights for the CIB and lensing potential, respectively
\[
W^\nu = a j(\nu, \chi), \\
W^\delta = -3 \frac{\Omega_m H_0}{a} \frac{H_\Lambda^2}{c^2 k^2} \left( \frac{x - \chi}{x \chi_0} \right) = -3 \frac{\Omega_m H_0}{a} \frac{H_\Lambda^2}{c^2 k^2} \left( \frac{x - \chi}{x \chi_0} \right). \tag{34}
\]

Here, \( j(\nu, \chi) \) is the CIB emissivity at a frequency \( \nu \), \( a \) is the scale factor, \( H_0 \) is the Hubble parameter today, and \( \Omega_m \) is the matter density today in the critical density unit. To make the results consistent with the CIB auto and CIB-lensing cross-spectra of Planck multi-frequency measurements, we adopted the halo occupation distribution (HOD) model from Planck papers [35, 51] to calculate \( P_{\text{mg}} \). The details can be found in ref. [52].

We also performed the error estimation of the CIB-lensing potential cross-power spectrum from AlcCPT and Planck. Considering only the Gaussian statistical errors, we calculated the SNR using a simple Fisher matrix prescription [35]

\[
\left( \frac{S}{N} \right)^2 = \sum_{\nu, \nu'} \sum_{\ell, \ell'} C^\nu_\ell C^\nu_{\ell'} \text{CoV}_{\ell, \ell'}(\nu, \nu'), \tag{35}
\]

\[
\text{CoV}_{\ell, \ell'}(\nu, \nu') = \left( \frac{C^\nu_\ell (C^\nu_{\ell'} + N^\nu_{\ell'}) (C^\nu_{\ell'} + N^\nu_{\ell'})}{(2\ell + 1) f_{\text{sky}} \Delta \ell} \right). \tag{36}
\]

Here, \( \Delta \ell \) is the width of the ell bins; \( C_\ell^\nu \) and \( C_\ell^{\delta \delta} \) are the auto-power spectrum of the CIB intensity and lensing potential, respectively; and the corresponding noise spectra are written as \( N_\ell^\nu \) and \( N_\ell^{\delta \delta} \). Furthermore, \( C_\ell^\nu \) and \( N_\ell^\nu \) are the measured CIB intensity cross-correlation signal and noise spectra between different frequencies, respectively. The observed data \( (C_\ell^\nu, N_\ell^\nu, N_\ell^{\delta \delta}) \) were taken from the published Planck legacy archive\(^9\) [53]. To maintain consistency, we applied a binning strategy similar to the one used in ref. [35], namely \( \Delta \ell = 64 \), \( N_\ell^{\delta \delta} \) takes the 150-GHz channel MV-N0 noise spectrum.

In Figure 8, we show the forecasted lensing-CIB cross spectra. The purple, green and red shaded boxes denote the 1σ errors for Planck 857, 545, and 353 GHz channels. Black curves denote for the theoretical predictions by assuming the Planck cosmology. The Fisher matrix based SNRs are 18.2, 19.3, 23.1 for 353, 545, 857 GHz in the "48 module*yr" stage, respectively. Under the "48 module*yr" scenario, the SNRs are 25.1, 33.2, 42.2 for 353, 545, and 857 GHz, respectively. Following eqs. (35) and (36), one can calculate the total SNR. The corresponding results are listed in the last row of Table 3. Due to the strong correlations among different frequencies in the covariance, the combined SNRs are enhanced in a very limited manner as

\[\text{Table 3: Lensing-CIB cross-correlation SNRs} \]

| Frequency (GHz) | 4 module*yr | 48 module*yr |
|----------------|-------------|--------------|
| 353            | 18.2        | 25.1         |
| 545            | 19.3        | 33.2         |
| 857            | 23.1        | 42.2         |
| Total          | 23.3        | 43.1         |

---

9) https://lambda.gsfc.nasa.gov/product/planck/curv/planck_sp_lens_get.cfm.
compared to the 857 GHz channel, which is the highest SNR channel. The final total cross-correlation SNRs are 23.3 and 43.1 for the AliCPT-1 first and final stages, respectively.

5 Conclusions

The AliCPT-1 project is the first Chinese CMB experiment aiming for high-precision measurements of CMB polarization. The experiment observes at 90 and 150 GHz frequencies with an intermediate spatial resolution (FWHM = 19 and 11 arcmin for 90 and 150 GHz, respectively) and a 3rd generation CMB experiment noise level. The harmonic means of the noise variance in the “4 module*yr” case are approximately 11 μK arcmin at 90 GHz and 17 μK arcmin at 150 GHz.

In this paper, we investigated the ability of AliCPT-1 to measure the CMB lensing signals. We forecasted the lensing reconstruction, lensing-galaxy cross-correlation and the lensing-CIB cross-correlation in AliCPT-1. In detail, we considered two mission designs, namely the first (“4 module*yr”) and final (“48 module*yr”) stages. We adopted the technically mature quadratic estimator for the lensing reconstruction. In the first stage, the SNRs for the T-only estimator are 3.5 and 8.3 in 90 and 150 GHz channels, respectively. For the P-only estimator, the SNRs are 5.1 for the 90 GHz channel and 6.6 for the 150 GHz channel. For the MV estimator, the SNRs are 9.2 and 15.4 for the 90 and 150 GHz channels, respectively. In the final stage, for T-only estimator the SNR are 4.8 and 8.0 in 90 and 150 GHz channel, respectively. For P-only estimator, the SNR are 21.7 for 90 GHz and 25.4 for 150 GHz. For MV estimator, the SNR are 24.3 and 31.1 for 90 and 150 GHz channel, respectively. Unlike Planck data, the polarization data in AliCPT-1 play an essential role in the lensing reconstruction due to the excellent noise level in the polarization pattern. Furthermore, the EB estimator will dominate the lensing reconstruction once we accumulate the data and arrive at the final stage. In this work, we focus on the statistical noise according to the “deep patch” scanning strategy. We did not consider the systematic effects and foreground residual contamination to the lensing reconstruction. We leave these effects for the future studies.

For lensing-galaxy cross-correlation, we cross-correlated the AliCPT-1 lensing (150 GHz MV estimator) with BGS samples, LRGs, ELGs, and QSOs. In the first stage, we report the cross-correlation SNR = 10-20 for the redshift bins arranged from z = 0.05 to 2.1. The total SNR can reach 32. Furthermore, we show the constraint on the growth index γ as an example for the cosmology application of the cross-correlation signals. We found that σ_γ = 0.16, which has an approximately 29% relative error in the growth index determination. In the final stage, the total cross-correlation SNR can reach 52 and the growth index determination error can be further reduced to the σ_γ = 0.10 level.

For the lensing-CIB cross-correlation, we forecast the AliCPT-1 lensing (150 GHz MV estimator) cross-correlating with the Planck CIB from the 353, 545, and 857 GHz channels. In the first stage, we report SNR = 18.2 for the 353 GHz channel, SNR = 19.3 for 545 GHz and SNR = 23.1 for the 857 GHz channel. In the final stage, these three numbers are SNR = 25.1, 33.2, and 42.2. The final total lensing-CIB cross-correlation by combining the three frequencies in Planck CIBs are SNRs = 23.3 and 43.1 for the AliCPT-1 first and final stages, respectively.

Owing to the excellent detector noise performance, the AliCPT-1 mission can measure the lensing signals with a fairly good significance, especially via the polarization data.

Figure 8 (Color online) Forecasted lensing CIB cross-correlation. In the “4 module*yr” configuration, the SNRs are: 18.2, 19.3, 23.1 for Planck 353, 545, 857 GHz, respectively. In the “48 module*yr” configuration, the SNRs are: 25.1, 33.2, 42.2 for Planck 353, 545, 857 GHz, respectively.
Here, we present a comprehensive study on the ability of AliCPT-1 to measure lensing signals and the relevant cosmology applications.

This work was supported by the National Key R&D Program of China (Grant Nos. 2020YFC2201603, 2020YFC2201601, and 2020YFC2201600), National Natural Science Foundation of China (Grant No. 11635033), 111 Project (Grant No. B20019). We acknowledge Zhaihui Fan for various comments on the preliminary version of the draft.
