Vibration suppression of cantilever plate using nonlinear energy sink

G Liu¹, W Zhang²

¹ College of Mechanical Engineering, Beijing University of Technology, Beijing, 100124, PR China
² College of Mechanical Engineering, Beijing University of Technology, Beijing, 100124, PR China
liugen1991@hotmail.com

Abstract. Cantilever structure is widely used in the field of aerospace engineering. Due to the complex working environment and the excitation effect, the vibration suppression of the structure is extremely important. The Nonlinear Energy Sink (NES), which is characterized as light weight, targeted energy transfer and high damping efficiency, provides the demanding for the design of vibration suppression of aerospace structures. In this paper, the NES is used to study the vibration reduction of cantilever rectangular plates. Based on the Kirchhoff hypothesis, the coupling dynamic equations of the between the thin plate and the NES are established, and the response of the structure in the first order transverse bending is studied by modal truncation. The suppression effect of the NES under different parameters is analyzed which can provide some theoretical support for the cantilever structure in engineering application.

1. Introduction

Cantilever structure is widely used in the research of aerospace engineering. The aerofoil, compressor blade, mechanical arm and solar wing of artificial satellite are all typical cantilever components. Due to the complex working environment of these structures, it is easy to the vibration of the structure under various loads, thus affecting the performance of the overall system [1-3]. Then necessary vibration reduction means are worth considering.

With the complexity of the working environment of the structure, the traditional methods of vibration reduction can no longer meet the needs of the project. Nonlinear energy sink (NES), compared with traditional shock absorbers, is a nonlinear method of vibration reduction. It is not only featured by its small added mass, but inhibiting frequency band width, and also energy targeted transmission. It is now a hotspot of research on structural vibration reduction. Targeted energy transfer for discrete and linear continuous systems using nonlinear energy sink has been studied since the beginning of 2000. In the early studies, the researchers attached the components with cubic nonlinear stiffness to different discrete structures to form a coupling system, and used the nonlinear energy sink to achieve the passive vibration reduction of the structure with broadband. Gendelman[4] found the targeted energy transfer for the first time in the study of two-degree-of-freedom linear oscillator for a transient dynamics problem with cubic nonlinear stiffness and damping element accessories. Then Vakakis and Gendelman, Bergman, McFarland, Kerschen and other scholars [5-10], broaden the NES on the vibration research of structure from discrete system to the continuous system, from simple finite
degree of freedom linear spring-mass system to generalized continuum beam slab structure. They studied not only the theory of the suppression effect of nonlinear energy sink, but got experiment validation, which laid a solid foundation of the application of structural vibration.

In this paper, the cantilever rectangular thin plate is taken as the research object. We investigated the vibration suppression problem of cantilever plate structure under transient excitation by using NES. The influence of NES placed at different positions and different system parameters on the vibration reduction effect was analyzed by considering the low-order modal mode of the cantilever plate which has been disturbed by mode truncation, which can provide theoretical support for the vibration reduction design of the cantilever thin plate structure.

2. Theoretical model

Due to the boundary condition of the thin cantilever plate, when the structure is subjected to transverse disturbance, the transverse bending vibration wing occurs. It is necessary to establish the dynamic equation of the thin plate in transverse vibration. Based on Kirchhoff’s hypothesis, the partial differential equation of transverse vibration of the thin plate is:

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\rho h}{D} \frac{\partial^2 w}{\partial t^2} = \frac{q}{D} (x, y, t) \tag{1}
\]

Considering the influence of damping on the structure and the effect of transverse transient excitation on the plate, the differential equation of the thin plate with damping effect and transverse transient load is obtained:

\[
D \frac{\partial^4 w}{\partial t^2} + m \frac{\partial^2 w}{\partial t^2} + \lambda \frac{\partial w}{\partial t} = F(t) \delta(x - a_x, y - b_y) \tag{2}
\]

Where, \(\lambda\) is the proportional damping coefficient and \(D\) is the bending stiffness of the thin plate.

\[
D = \frac{Eh^3}{12(1-\nu^2)} \\nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{3}
\]

NES accessories with cubic nonlinear stiffness are attached to the structure of rectangular thin plate. The rectangular thin plate is fixed on one short side and the other three sides are free, as shown in Fig.1

![Fig. 1. The cantilever thin plate attached NES](image)

NES is arranged on the midpoint of each string line of the cantilever plate. \(a\) is the span length of the cantilever plate, \(b\) is the width of the cantilever plate, and \(h\) is the thickness of the cantilever plate. \(F\) is the transverse excitation of cantilever plate in the \(z\) direction. The shaded part is NES attachment, \(m\) is the mass of NES, \(k\) is the stiffness of cubic nonlinear spring, and \(\lambda\) is the damping coefficient of NES attachment. The coupled partial differential equations of thin-plate with NES accessories are obtained:
\[ \begin{align*}
D^4 w + m \frac{\partial^2 w}{\partial t^2} + \lambda \frac{\partial w}{\partial t} + \left\{ K \left[ w(x, y, t) - v(t) \right] + \lambda \left[ \tilde{w}(x, y, t) - \tilde{v}(t) \right] \right\} \\
\cdot \delta(x - d_x, y - d_y) = F(t) \delta \left( x - a_x, y - a_y \right) \quad (4-1)
\end{align*} \]

\[ \varepsilon \ddot{v}(t) + K [v(t) - \tilde{v}(d_x, d_y)] + \lambda [v(t) - \tilde{w}(d_x, d_y)] = 0 \quad (4-2) \]

Dimensionless parameters are introduced to get dimensionless transformation of equation (4):

\[ \tau = \frac{D}{m}, \quad \varepsilon_i = \frac{e_i}{m}, \quad \lambda_i = \frac{\lambda_i}{m}, \quad f(x, \tau) = \frac{F(x, t)}{Dm} \]

The dimensionless partial differential equations of the system is obtained:

\[ \nabla^4 w + \frac{\partial^2 w}{\partial \tau^2} + \lambda_i \frac{\partial w}{\partial \tau} + \left\{ \tilde{K} \left[ w(\tau) - v(\tau) \right] + \lambda_i \left[ \tilde{w}(\tau) - \tilde{v}(\tau) \right] \right\} \]

\[ \cdot \delta(x - d_x, y - d_y) = f(\tau) \delta \left( x - a_x, y - a_y \right) \quad (5-1) \]

\[ \varepsilon_i \ddot{v}(\tau) + \tilde{K} [v(\tau) - \tilde{w}(d_x, d_y)] + \lambda_i [v(\tau) - \tilde{w}(d_x, d_y)] = 0 \quad (5-2) \]

The transverse displacement of cantilever thin plate can be expressed as:

\[ w = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} w_{mn}(\tau) X_n(x) Y_m(y) \quad (6) \]

The modal function of the cantilever plate is selected as follows:

\[ X_i(x) = \text{ch} \frac{\alpha_i}{a} x - \text{cos} \frac{\alpha_i}{a} x + \mu_i \left( \text{sh} \frac{\alpha_i}{a} x - \text{sin} \frac{\alpha_i}{a} x \right) \quad (i = 1, 2, 3, \ldots) \quad (7-1) \]

\[ Y_i(y) = 1 \quad (7-2) \]

\[ Y_i(y) = \sqrt{3} \left( 1 - 2 \frac{x}{l} \right) \quad (7-3) \]

\[ Y_j(y) = \text{ch} \frac{\beta_j y}{b} + \text{cos} \frac{\beta_j y}{b} - \mu_j \left( \text{sh} \frac{\beta_j y}{b} + \text{sin} \frac{\beta_j y}{b} \right) \quad (j = 3, 4, 5, \ldots) \quad (7-4) \]

Where

\[ \begin{align*}
\cos \alpha_i a \cosh \alpha_i a - 1 &= 0 \quad (8-1) \\
\cos \beta_m b \cosh \beta_m a - 1 &= 0 \quad (8-2) \\
\mu_i &= \frac{\cosh \alpha_i - \cos \alpha_i}{\sinh \alpha_i + \sin \alpha_i} \quad (8-3) \\
\mu_m &= -\frac{\cosh \beta_m - \cos \beta_m}{\sinh \beta_m + \sin \beta_m} \quad (8-4) 
\end{align*} \]

Substituting the modal functions(8) to the partial differential equation of the system, the ordinary differential equation of the system is obtained as follows:

\[ \ddot{w}_p + \lambda_1 \dot{w}_p + \omega_p^2 w_p + \left\{ K \sum_{n=1}^{\infty} w_{mn}(\tau) X_n(x) Y_m(y) - v(\tau) \right\} \]

\[ + \lambda_2 \left[ \sum_{n=1}^{\infty} \dot{w}_{mn}(\tau) X_n(x) Y_m(y) - \tilde{v}(\tau) \right] \cdot X_p(d_x) Y_p(d_y) = f(\tau) X_p(a_x) Y_p(b_y) \quad (9-1) \]
$$e_i \ddot{v}(\tau) + k \left[ v(\tau) - \sum_{r=1}^{\infty} w_{r} X_n(x) Y_n(y) \right]^3 + \lambda_i \left[ \ddot{w}(\tau) - \sum_{r=1}^{\infty} \ddot{w}_{r} X_n(x) Y_n(y) \right] = 0 \quad (9-2)$$

3. Transient response analysis of cantilever plate with NES attachment

In the previous section, we obtained the partial differential equation of the system of cantilever thin-plate coupled NES accessories. By modal truncation, we can analyze the dynamic response of the system under various modes. Considering the effect of external excitation on the thin plate in the transverse direction, combined with the NES, we study the first order bending vibration mode of the thin plate in the transverse direction as the larger transverse direction than the chord direction, and investigate the vibration reduction effect of NES when it is placed in different positions along the line to the center of each chord in the transverse direction.

The position of excitation is selected at the free end of the cantilever beam and is located on the line of each chord midpoint. The excitation is given in the form of amplitude, \(d = 5.6, \ldots, 10\), and other dimensionless parameters are selected as \(c = 0.1\), \(\epsilon = 10\), \(k = 1000\), \(\lambda_i = 0.05\), and \(F = 2\).

Fig. 2 - 3 show the response of the structure.

![Fig. 2. The response of the system at d=7 and d=8](image)

![Fig. 3. The response of the system at d=9 and d=10](image)

It can be seen from the vibration response of the structure in above Fig. 2-3 that the placement position of NES has a significant influence on the vibration attenuation of the structure when the first-order transverse vibration of the cantilever thin plate is considered. Because the vibration amplitude of cantilever plate structure gradually increases from the root to the free end, NES's sensitivity to amplitude is also gradually reflected. It can be said that vibration reduction effect is obvious when NES is placed at a place with a large amplitude of the structure.

![Fig. 4. The response of the cantilever plate at d=7 and d=8](image)
Fig. 5. The response of the cantilever plate at d=9 and d=10
Fig.4-5 show the response attenuation of the cantilever plate from 0 seconds to 20 seconds under the effect of NES. When NES is located at the place with the largest structural response, the displacement of the structure has been reduced to 1.75% of the initial displacement response within 20 seconds.

Fig. 6. The response of the structure with f=1,2

Fig. 7. The response of the structure with f=3,4
The amplitude of external excitation affects the initial response of the structure, and the sensitivity of NES to the amplitude of external excitation is particularly prominent. Fig.6-7 show the structure response when external excitation is gradually increased. Not only the initial displacement of cantilever plate but also the NES displacement increases with the increase of excitation. $a_i^* (i = 1, 2, \ldots, 4)$ is the amplitude of structural response at 10 second. When the excitation is large, the NES amplitude is large. The energy dissipated by NES damping has a good effect, and the structure amplitude attenuates significantly.

Fig. 8. The response of the structure with TMD (left) and NES (right) at f=2
Compared with linear stiffness damping damper, NES passive damping effect has obvious advantages. Fig.8 select the response attenuation of the structure when the excitation is located at the free end. And the excitation amplitude $f^* = 2$, the linear damping damper and NES accessories are arranged. Due to
the coupling effect of nonlinear system, the initial response of NES cantilever structure is larger than that of linear damper coupling structure, but the attenuation effect of NES is obvious than the TMD. Comparing the initial and afterwards amplitude $a^*$, $b^*$ of TMD(left) in Fig.8 and $c^*$, $d^*$ in Fig.8 of NES(right), it is obvious that structure amplitude attenuation of NES reached a significant level after 5 seconds, which embodies a good vibration suppression effect of NES.

4. Conclusion

In this paper, the transient amplitude suppression of NES attachments for cantilever thin plate structures is studied. Based on Kirchhoff hypothesis, the coupling dynamic equation between the thin plate and NES was established. The structural response of the first order transverse bending of the thin plate was studied through modal truncation, and the vibration suppression effect of NES under different parameters was analyzed. The influence of the position of NES on vibration attenuation was studied by selecting different NES placement positions. Compared with NES at different positions, the amplitude attenuation effect of cantilever thin plate in a certain period of time was compared.

Compared with the amplitude suppression effect of linear stiffness-mass-damping suppression accessories, it can be seen that NES has a higher efficiency and obvious advantage than TMD in amplitude inhibition under transient excitation. It is expected to provide theoretical support for NES’s application in vibration reduction of engineering structures.

References

[1] Lv Shufeng, Zhang Wei. Nonlinear analysis of deploying laminated composite cantilever plates[J]. Journal of dynamics and control, 2015,13(4): 288-292.
[2] Yang Jiahui, Zhang wei, Yao Minghui. Bifurcations of a composite laminated cantilevered plate under voltage excitation[J]. Journal of dynamics and control, 2017, 15(6): 489-493.
[3] Wang Yu, Hao Yuxin. Free vibration of homogeneous cantilever thin plate with geometric imperfection[J]. Journal of dynamics and control, 2017, 15(6): 525-531.
[4] Gendelman O V. Transition of energy to a nonlinear localized mode in a highly asymmetric system of two oscillators[J]. Nonlinear dynamics, 2001, 25(1-3): 237-253.
[5] Gendelman O, Manevitch L I, Vakakis A F, et al. Energy pumping in nonlinear mechanical oscillators: Part I—Dynamics of the underlying Hamiltonian systems[J]. Journal of Applied Mechanics, 2001, 68(1): 34-41.
[6] Vakakis A F, Gendelman O. Energy pumping in nonlinear mechanical oscillators: part II—resonance capture[J]. Journal of Applied Mechanics, 2001, 68(1): 42-48.
[7] Vakakis A F. Inducing passive nonlinear energy sinks in vibrating systems[J]. Journal of Vibration and Acoustics, 2001, 123(3): 324-332.
[8] Gendelman O, Manevitch L I, Vakakis A F, et al. A degenerate bifurcation structure in the dynamics of coupled oscillators with essential stiffness nonlinearities[J]. Nonlinear Dynamics, 2003, 33(1): 1-10.
[9] Vakakis A F, Rand R H. Non-linear dynamics of a system of coupled oscillators with essential stiffness non-linearities[J]. International Journal of Non-Linear Mechanics, 2004, 7(39): 1079-1091.
[10] Musienko A I, Lamarque C H, Manevitch L I. Design of mechanical energy pumping devices[J]. Journal of vibration and control, 2006, 12(4): 355-371.
[11] Zhang Y W, Zhang Z, Chen L Q, et al. Impulse-induced vibration suppression of an axially moving beam with parallel nonlinear energy sinks[J]. Nonlinear Dynamics, 2015, 82(1-2): 61-71.
[12] Zhang Y W, Yuan B, Fang B, et al. Reducing thermal shock-induced vibration of an axially moving beam via a nonlinear energy sink[J]. Nonlinear Dynamics, 2017, 87(2): 1159-1167.
[13] Kani M, Khadem S E, Pashaei M H, et al. Vibration control of a nonlinear beam with a nonlinear energy sink[J]. Nonlinear Dynamics, 2016, 83(1-2): 1-22.