Standing accretion shock instability: numerical simulations of core-collapse supernova

N Ohnishi\textsuperscript{1,2}, W Iwakami\textsuperscript{2}, K Kotake\textsuperscript{3,4}, S Yamada\textsuperscript{5,6}, S Fujioka\textsuperscript{7} and H Takabe\textsuperscript{7}

\textsuperscript{1} Center for Research Strategy and Support, Tohoku University, Sendai, Miyagi, Japan
\textsuperscript{2} Department of Aerospace Engineering, Tohoku University, Sendai, Miyagi, Japan
\textsuperscript{3} National Astronomical Observatory Japan, Tokyo, Japan
\textsuperscript{4} Max-Planck-Institut f"ur Astrophysik, Garching, Germany
\textsuperscript{5} Science and Engineering, Waseda University, Tokyo, Japan
\textsuperscript{6} Advanced Research Institute for Science and Engineering, Waseda University, Tokyo, Japan
\textsuperscript{7} Institute of Laser Engineering, Osaka University, Suita, Osaka, Japan

E-mail: ohnishi@rhd.mech.tohoku.ac.jp

Abstract. Standing accretion shock instability (SASI) is one of the candidates to solve the mystery of why we cannot reproduce the explosion with the present core-collapse supernova models. We have studied this phenomenon with including neutrino heating and realistic EOS and found that SASI may enhance neutrino heating. Although \(g\)-mode of proto-neutron star may enhance the SASI growth, the simulations just including the pressure perturbation as a mimic of \(g\)-mode induced sound wave reveal no significant effect on the shock dynamics. Moreover, we discuss the required conditions toward the possible laboratory experiment of SASI.

1. Introduction

Supernova explosion phenomena are not fully understood despite of vigorous studies by many astronomical researchers. Computational studies tell us that spherical simulations cannot reproduce any successful explosions even if the simulations are carried out with canonical condition by state-of-the-art computational models including general relativity, microphysics, and/or neutrino physics [1, 2]. Recently, a hydrodynamical instability of a stalled shock wave formed in supernova cores has been found out and remarked as one of the strong candidates which produce the multi-dimensional motion. The instability called as standing accretion shock instability (SASI) was firstly studied for an accretion flow around a black hole by Foglizzo [3]. Blondin et al., however, adopted it to the context of an accretion flow in supernova cores [4]. They found that low-mode (\(\ell = 1, 2\)) perturbations of a shock surface exponentially grows in a spherically accreting flow by axisymmetric two-dimensional simulations. In their recent studies, it is suggested that spiral modes induced by SASI may produce the pulsar kick of the proto-neutron star (PNS) as a consequence of three-dimensional simulations [5].

Ohnishi et al. investigated SASI growth in a non-adiabatic accreting flow to confirm the effect of SASI on the mechanism of the core-collapse supernova with a two-dimensional code including neutrino heating and realistic EOS [6]. They found that the linear growth of the shock surface perturbation and the following nonlinear regime exist even in the non-adiabatic flow. In addition to it the simulation results indicated that the turbulent motion induced by SASI enhances the
neutrino absorption and can help the revival of the stalled shock wave toward a successful explosion. They also investigated that the additional heating of the inelastic neutrino-Helium scatterings is appeared around the oscillating shock wave surface while it is negligible in the unperturbed spherical flow [7].

Although SASI may have a great impact on the explosion scenario of the core-collapse supernova, the mechanism of SASI is not so clear at the moment. Foglizzo proposed the vortex-acoustic cycle for the evolution of the shock instability [3], and Ohnishi et al. obtained the supporting results with frequency analysis in the linear regime [6] though Blondin et al. claims that the pressure wave is only responsible for the instability [8]. This issue is still controversial and should be verified from various aspects through not only simulations but also theoretical works.

In this paper, we present a preliminary work for g-mode of PNS, which may enhance SASI growth and a possible experiment with intense lasers as a new tool to study the shock instability in supernova cores.

2. Numerical methods for standing accretion shock

In order to simulate the SASI growth, we should obtain a steady-state solution of an accreting flow with a standing shock and reproduce in two-dimensional stable code. We employ the ZEUS-2D code [9] as a base code and modified it with adding neutrino heating and replacing EOS to more sophisticated one [6]. This code is constructed with the finite difference manner and the von Neumann & Richtmyer artificial viscosity. Although resolving the discontinuities is inferior to the modern shock capturing schemes such as approximate Riemann solvers, the standing shock solution is relatively easy to be retained. However, in three-dimensional computation we have to introduce a tensor-type artificial viscosity for keeping the standing shock. Readers should refer to Ref. [10] for more detail.

The numerical methods employed in this paper are essentially the same as those used in our previous paper [6]. The following equations describe the compressible accretion flows of matter attracted by the proto-neutron star and irradiated by neutrinos emitted from the neutrino sphere.

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad (1)
\]

\[
\rho \frac{d\mathbf{v}}{dt} = -\nabla P - \rho \nabla \Phi, \quad (2)
\]

\[
\rho \frac{d\left(\frac{e}{\rho}\right)}{dt} = -P \nabla \cdot \mathbf{v} + Q_E, \quad (3)
\]

\[
\frac{dY_e}{dt} = Q_N, \quad (4)
\]

\[
\Phi = -\frac{G M_{\text{in}}}{r}, \quad (5)
\]

where \(\rho, \mathbf{v}, e, P, Y_e, \Phi,\) and \(r\) are density, velocity, internal energy, pressure, electron fraction, gravitational potential, and radius, respectively. The constant \(G\) is the gravitational constant. The self-gravity of matter in the accretion flow is ignored. The parameters of \(Q_E\) and \(Q_N\) are related with the interactions of neutrinos and free nucleons (see also Ref. [6]). \(M_{\text{in}}\) is the mass of the central object and set to be \(1.4 M_\odot\) in this paper.

Spherical coordinates are used. No equatorial symmetry is assumed and the computation domain covers the whole meridian section with 60 angular mesh points. We use 300 radial mesh points to cover \(r_{\text{in}} \leq r \leq r_{\text{out}} = 2000 \text{ km}\), where \(r_{\text{in}}\) is the inner boundary and chosen to be roughly the radius of neutrino sphere. The initial conditions are prepared in the same manner as in Ref. [6] with neutrino luminosity of \(3.0 \times 10^{52} \text{ erg/s}\). The steady state solutions obtained
by [11] for a fixed density at the inner boundary, $\rho_{in} = 10^{11}$ g cm$^{-3}$, and an accretion rate of $1 M_\odot$ s$^{-1}$ are utilized. To induce the non-spherical instability, we have added $\ell = 1$ velocity perturbations of 1% to the initial state mentioned above.

3. Effect of $g$-mode on SASI growth

Burrows et al. pointed out that the sound wave induced by $g$-modes of PNS travels to the shock surface and is efficiently absorbed by the shock wave as the black body for sound wave [12]. There is a possibility that the emitted sound wave from PNS assists the shock revival. Yoshida et al. suggested that $g_2$-mode is most dominated among $g$-modes for $\ell = 1$ under the condition of the accreting flow in supernova cores [13]. We have conducted the simulations with the pressure perturbation at the inner boundary (a mimic of PNS surface), which has a $g$-mode frequency given by Ref. [13] and might enhance the SASI growth.

In the present simulations, the $\ell = 1$ pressure perturbation is imposed with the amplitude of 100% of the initial inner boundary value and the sinusoidal oscillation which has the angular frequency of $2.925 \omega_0$, $1.432 \omega_0$, and $1.031 \omega_0$ for $g_1$, $g_2$, and $g_3$ modes, respectively, where $\omega_0 = 1074.1$ Hz. Figure 1 shows the $\ell = 1$ mode growth of shock surface. One can see that the linear growth by $\sim 100$ ms and the following nonlinear phase exist. This is a typical feature of SASI as found in the previous works [6]. There is also a characteristic oscillation driven by SASI. On the other hand, no new appearance is found despite adding the $g$-mode pressure perturbation, and all the models fail the explosion. The reason why the $g$-mode perturbations cannot affect on the shock dynamics may come from the mismatch among the frequencies of $g$-modes and the shock oscillation.

If the $g$-mode originated sound wave affect on the shock dynamics, the viscous heating rate reveals the oscillation property characterized by the $g$-mode frequencies. Figure 2 shows the Fourier spectra of viscous heating estimated by the artificial viscous term, where the arrows in the figure denote the frequencies of $g$-modes and their higher harmonics. The frequencies of fundamental $g$-modes are far from the characteristic ones of SASI, and we cannot find any distinctive spectra associated to the $g$-mode frequency.

Since the shock oscillation frequency becomes higher with lower neutrino luminosity [6], $g$-mode sound wave may interact with the shock wave more effectively in lower neutrino luminosity models. However, with lower luminosity the stalled shock wave is difficult to revive. Although at the moment, a scenario that the $g$-mode sound wave assists the supernova explosion is not so feasible, PNS $g$-mode oscillation can be excited by SASI and then pushes back the shock wave. This feedback cycle may yield a significant outcome. So we cannot draw any conclusive claims without the simulations including the PNS dynamics.

4. Summary and requirement for possible experiment

We have conducted the numerical studies on the shock instability in core-collapse supernovae focusing on its contribution to the explosion mechanism. We have investigated the SASI-triggered explosion due to the interaction with the neutrino heating and the effect of neutrino-Helium inelastic scattering. Although the preliminary results with pressure perturbation

![Figure 1. Normalized amplitude of $\ell = 1$ mode of shock surface.](image)
originated from PNS $g$-mode indicate that $g$-mode sound wave is difficult to affect on the SASI motion, further investigation is necessary to claim the conclusion of this matter. We have discussed the possibility of the laboratory experiment with intense lasers toward the better understanding of SASI. When we design the SASI experiment, we have to consider at least the following things. In order to observe the SASI growth, the accretion flow should continue in the duration of $\tau_{\text{SASI}} \sim 10 \cdot 2\pi/\omega_{\text{cyc}}$, where $\omega_{\text{cyc}}$ is the frequency of vortex-acoustic cycle [6]. The frequency $\omega_{\text{cyc}}$ is not so different from the value of $\omega_{\text{adv}}$ which is determined by advection flow speed, that is, $\omega_{\text{cyc}} \sim \omega_{\text{adv}} \sim 2\pi u/(R_S - R_\nu)$, where $R_S$ and $R_\nu$ are the shock radius and the neutrino sphere radius, respectively. Therefore, if we can create an accretion flow with an implosion of a shell target as shown in Fig. 3, the shell thickness $d$ is required to be $d \sim u\tau_{\text{SASI}} \sim 10(R_S - R_\nu)$. The bow shock is formed by the central object and becomes unstable while interacting with converging flow with the same mechanism of SASI. However, we should taking care of a reflected shock at the center, which is usually strong enough to break the quasi-steady bow shock. We are now looking for the best condition in this framework.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures}
\caption{Fourier spectra of viscous heating rate.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{example_experiment}
\caption{Example of possible experiment with intense lasers.}
\end{figure}

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