Recent Work on Gravitational Waves From a Generic Standard Model-like Effective Higgs Potential

John Kehayias\textsuperscript{a} \textsuperscript{*}

\textsuperscript{a}Physics Department, University of California Santa Cruz, Santa Cruz, CA, US

I present recent work on gravitational waves (GWs) from a generic Standard Model-like effective potential for the electroweak phase transition. We derive a semi-analytic expression for the approximate tunneling temperature, and analytic and approximate expressions for the two GW parameters $\alpha$ and $\beta$. A quick summary of our analysis and general results, as well as a list of some specific models which easily fit into this framework, are presented. The work presented here has been done in collaboration with Stefano Profumo \textsuperscript{1}.

1. Introduction

As the temperature is lowered in the finite temperature quantum field theory description of the electroweak Higgs sector (for a review, see e.g. \textsuperscript{2}), it is possible to have a first order phase transition through quantum mechanical tunneling. A degenerate vacuum state develops at $T_c$, and what began as the true vacuum of the theory can become unstable at a lower temperature $T_{\text{dest}}$. A potential barrier separates this state from the true vacuum, and tunneling to the lower energy state is probable at a temperature $T_t$, where $T_{\text{dest}} \leq T_t < T_c$.

Gravitational waves (GWs) can arise from a strongly first-order phase transition through both turbulence and bubble nucleation. Here, a bubble is an area of the universe which has transitioned to the true, electroweak symmetry breaking, vacuum. Some fraction of the energy released in these processes gives rise to a stochastic background of gravitational waves.

It has been known for some time that the electroweak phase transition in the minimal version of the Standard Model (SM) is not strongly first order, given the experimental bounds on the Higgs mass. However, many models of physics beyond the SM, including supersymmetry, can enhance the phase transition and produce a GW spectrum which might be experimentally observed in the near future. While many models have been studied extensively, there has not been much work done on a general, model-independent analysis.

In this note I will very briefly summarize work soon to be submitted on studying generic effective potentials for the electroweak phase transition. The potential is very similar in form to the SM Higgs potential, and the general results are applicable to several models beyond the SM.

2. Analysis of a Generic Effective Higgs Potential

We consider a potential for the Higgs which mirrors that of the (finite temperature, one loop, high temperature expansion) SM case of the following form:

$$V_{\text{eff}}(\phi, T) = \frac{\lambda(T)}{4} \phi^4 - (ET - e)\phi^3 + D(T^2 - T_0^2)\phi^2. \quad (1)$$

In the SM $e = 0$.

A semi-analytic expression for the three dimensional Euclidean action, which is the important quantity for finite temperature tunneling, was found in \textsuperscript{3}. The tunneling temperature, $T_t$, is defined as the temperature when the probability to nucleate a bubble in a horizon volume is $O(1)$, a condition that is well approximated by

$$S_{EA}/T_t \sim 140, \quad (2)$$
where we have assumed the temperature scale is \( \mathcal{O}(100\text{GeV}) \) (see, e.g. [4]). Using the results of [3], we can thus derive an approximation for \( T_t \).

At \( T_c \) for our potential the expression for \( S_{E3} \) from [3] has a singularity. It also decreases very rapidly as the temperature is lowered, to 0 at \( T_0 \). This implies that \( T_t \) will be very close to \( T_c \), and so we expand in powers of \( \epsilon \): \( T \to T_c - \epsilon \). The final lowest order expression for \( S_{E3}/T \) has all the parameters of the potential and is proportional to \( 1/\epsilon^2 \). The singularity as \( \epsilon \to 0 \) remains, and \( \epsilon \) is solved for by setting \( S_{E3}/T = 140 \). Our approximation for the tunneling temperature is then

\[
T_t \approx T_c - \epsilon, \tag{3}
\]

with \( \epsilon \ll 1 \).

From our potential it is possible to calculate the exact GW parameters, \( \alpha \) and \( \beta \). \( \alpha \) characterizes the energy change of the vacuum transition, while \( \beta \) characterizes the bubble nucleation rate per unit volume. These are evaluated at \( T_t \), which we now have an approximation for, but they are rather lengthy expressions. However, from our approximation for \( S_{E3}/T \), a simple expression for \( \beta \) is obtained, which, to lowest order, is proportional to \( (T_c - \epsilon)/\epsilon^3 \).

3. The Parameter Space and Models

We enforce that the potential of eq. 11 describes electroweak symmetry breaking with a Higgs boson. This constrains the vev of \( \phi \) to be the usual \( v \approx 246 \text{ GeV} \), which must be a stable minimum, and furthermore that the mass of the Higgs is above the current experimental bound of 114 GeV. The signs of the parameters (except for \( \epsilon \) are fixed through this and the potential considered in [3] (for general stability, etc.), and we then also have \( T^2_\beta = v(3\epsilon + \lambda \nu)/2D \), which is similar to the SM form. There are constraints on \( \epsilon \) based on its sign, and \( \lambda \) is set based on \( e, v \), and the Higgs mass \( m_h \). We also want the theory to be perturbative, so \( \lambda < 1 \), giving us a mass range of 115 GeV \( \leq m_h < 348 \text{ GeV} \) (set by the SM case of 0.11 \( \leq \lambda < 1 \)).

Besides varying \( \lambda \) we also vary one other parameter at a time. For parameters besides \( \epsilon \), which has constraints on its range, we vary up to two orders of magnitude larger and smaller than the SM value. In plotting the \( \alpha - \beta \) plane we describe how the various terms in the potential affect the GW spectrum parameters. This covers as much as 12 orders of magnitude for both \( \alpha \) and \( \beta \). The most remarkable enhancement for \( \alpha \), which would greatly contribute to observing a signal with future experiments, comes at \( \epsilon < 0 \).

Several models can provide changes to the parameters in the potential from their SM values. There has been much work (e.g. [5,6]) on adding non-renormalizable terms to the SM or MSSM which can enhance \( E \), adding to the strength of the phase transition. The addition of an \( SU(2)_L \) triplet (see, e.g. [7]) provides a contribution to \( \lambda \). Since we find that the additional parameter \( \epsilon \) can greatly enhance \( \alpha \), models which add this term to the effective potential are particularly interesting. One such model is the addition of a gauge singlet, arising as a solution to the \( \mu \) problem in supersymmetry, for instance. The phenomenology of this model has been studied extensively (e.g. [8]), and it has been shown that it is possible to find evidence of such an additional singlet at the LHC, allowing us to draw an interesting connection between collider and GW physics.

REFERENCES

1. J. Kehayias and S.Profumo, [arXiv:0911.0687 [hep-ph]].
2. M. Quiros. [arXiv:hep-ph/9901312].
3. F. C. Adams, Phys. Rev. D 48, 2800 (1993) [arXiv:hep-ph/9302321].
4. R. Apreda, M. Maggiore, A. Nicolis and A. Riotto, Nucl. Phys. B 631, 342 (2002) [arXiv:gr-qc/0107033].
5. C. Delaunay, C. Grojean and J. D. Wells, JHEP 0804, 029 (2008) [arXiv:0711.2511 [hep-ph]].
6. K. Blum and Y. Nir, Phys. Rev. D 78, 035005 (2008) [arXiv:0805.0097 [hep-ph]].
7. P. Fileviez Perez, H. H. Patel, M. J. Ramsey-Musolf and K. Wang, [arXiv:0811.3957 [hep-ph]].
8. S. Profumo, M. J. Ramsey-Musolf and G. Shaughnessy, JHEP 0708, 010 (2007) [arXiv:0705.2425 [hep-ph]].