The possibility to convert the photon beam linear polarization into circular one at photon energies of hundreds GeV with the use of crystals is considered. The energy and orientation dependencies of refractive indexes are investigated in case of diamond, silicon and germanium crystal targets. To maximize the values for figure of merit, the corresponding crystal optimal orientation angles and thickness are found. The degree of circular polarization and intensity of photon beam are estimated and possibility of experimental realization is discussed.

The method to convert the linear polarization into circular one at high energies using the single crystals (similar to the quarter-wave plate in optics) was proposed by N. Cabibbo [1] since sixties and its experimental verification is not yet found. In the last decade the interest to this problem has increased [2] and there have been proposals to verify it experimentally [3]. This interest is connected with planned experiments with circular polarized photon beams at energies of tens and hundreds GeV on investigation of fundamental problems of theory. It was considered the possibility of using the circularly polarized photon beams in order to measure G, the polarized gluon distribution in nucleons, which is necessary for understanding the spin crisis problem. The investigation of this problem is of great importance since the results of EMC Collaboration experiment show that only 30% of nucleon spin is carried out by quarks [4]. The problem of spin crisis can be investigated by the processes of production of jets and heavy quarks via photon-gluon fusion [5,6,7], production of high
transverse momentum mesons [8], production of $J/\psi$ [9] and $\rho^0$-mesons by circularly polarized photons. In particular for processes of polarized photon fusion with gluons of polarized nucleon target the asymmetry in produced charm-anticharm quark pairs rates for opposite polarization of the target will appreciate the gluon contribution to the proton spin. According to theoretical calculations and existing proposals [6,10] a large asymmetry, of $\sim 40\%$, is anticipated. The estimated value of $\Delta G/G$ obtained recently in the HERMES experiment (HERA, DESY) at the photon energy of 27.5 GeV is in order 0.41 which corresponds to asymmetry in order of 0.28 [11].

Circular polarized photon beams can be produced by the longitudinal polarized electron beams, however the energy of electrons at modern accelerators is not sufficiently high to realize of above mentioned experiments. Therefore it is nessesary to continue the works on production of circular polarized photon beams at high energy proton accelerators (CERN, Fermilab). The first stage of the experiment NA59 devoted to production of linear polarized photon beam was performed at CERN on the 180 GeV energy SPS electron beam, using the 1.5 cm silicon radiator. The 40\% of the average linear polarization for the photons at energy range of 90-140 GeV was obtained. The second stage of the experiment on conversion of the photon beam linear polarization is planned to realize in this year. In this connection is actual to carry out calculations on investigation of the problem in different crystals to choose more convenient one for polarization conversion experiment and to estimate the crystal plate optimal thickness.

In present work the problem of photon polarization conversion at energies of 100-300 GeV for usually used C, Si and Ge crystals is investigated. The energy and orientation dependencies of real parts of refractive indexes are calculated as well as the optimal thickness’ of crystal plates, which provide the production of photon beams with maximal value of figure of merit (FOM) in sense of the degree of circular polarization and beam intensity, are estimated.

The photon beam orientation with respect to the crystal axes is defined in the following way. Let the chosen three orthogonal axes of cubic crystal are (1,2,3). The beam orientation is defined by angle $\theta$ between the axis 3 and the direction of incident photon beam $\vec{n}$ and by the angle $\alpha$ between the projection of $\vec{n}$ on the plane (1,2) and the axis 1. The photon beam polarization direction is defined with respect to the
incidence plane containing photon direction $\vec{n}$ and axis 3 and the indexes $\parallel$ and $\perp$ correspond to cases of $\varphi_0 = 0$ and $\varphi_0 = \pi/2$ (Fig.1). The coordinate system (x, y, z) connected with beam direction is chosen as shown in Fig.1. The projections $(g_x, g_y, g_z)$ of reciprocal lattice vector $\vec{g} = \vec{g}_1n_1 + \vec{g}_2n_2 + \vec{g}_3n_3$ are equal:

\[
\begin{align*}
g_{\parallel} &= g_z = \theta(G_1 \cos \alpha + G_2 \sin \alpha), \\
g_x &= G_1 \cos \alpha + G_2 \sin \alpha, \\
g_y &= -G_1 \sin \alpha + G_2 \cos \alpha,
\end{align*}
\]

(1)

where $\vec{g}_i$ are the vectors along the crystal basis axes ($|\vec{g}_i| = 2\pi/\alpha$) and $G_1, G_2, G_3$ are projections of $\vec{g}$ on the axes 1, 2, 3.

When high energy photon beam propagates through medium the photons are absorbed mainly due to the pair production mechanism on the medium atoms and photon beam attenuates with penetrating depth. Hence the refractive index is a complex quantity and his imaginary part is connected with pair production cross section by equation: $\text{Im}n = W/2\omega$ where $W = N\sigma$ is the absorption coefficient, $n$ is density of atoms and $\sigma$ is pair production cross section. In crystals the cross section (and accordingly refractive index) depends upon photon beam polarization direction with respect to the crystal axes. The real parts of refractive indexes defined via imaginary parts by dispersion relations [1]. This interesting assumption for photon birefringence at high energies is not so obviously and its experimental verification presents itself of great importance.

Let us consider the linear polarized photon beam incidents upon crystal as shown in Fig.1. The polarization of photon beam is described by Stocks parameters $\xi_1 = P_0 \sin 2\varphi_0$, $\xi_3 = P_0 \cos 2\varphi_0$, where $P_0$ and $\varphi_0$ are the degree and direction of polarization. The beam intensity and Stocks parameters beyond the crystal plate of thickness $l$ are defined by formulae [12]:

\[
\begin{align*}
I(l) &= I(0)[\cosh(al) + P_0 \cos(2\varphi_0) \sinh(al)]e^{-Wl}, \\
\xi_1(l) &= \frac{P_0 \sin(2\varphi_0) \cos(bl)}{\cosh(al) + P_0 \cos(2\varphi_0) \sinh(al)}, \\
\xi_2(l) &= \frac{P_0 \sin(2\varphi_0) \sin(bl)}{\cosh(al) + P_0 \cos(2\varphi_0) \sinh(al)},
\end{align*}
\]

(2)
\[ \xi_3(l) = \frac{P_0 \cos(2\varphi_0) \cosh(al) + \sinh(al)}{\cosh(al) + P_0 \cos(2\varphi_0) \sinh(al)}, \]

where \( W = (W_\parallel + W_\perp)/2 \), \( a = (W_\parallel - W_\perp)/2 \), \( b = \omega \text{Re}(n_\perp - n_\parallel) \).

As it’s follows from (1)

\[ \xi_1^2(l) + \xi_2^2(l) + \xi_3^2(l) = 1 + \frac{P_0^2 - 1}{\cosh(al) + P_0 \cos(2\varphi_0) \sinh(al)} \quad (3) \]

and in case of completely linearly polarized incident photon beam there is conservation of polarization:

\[ \xi_1^2(l) + \xi_2^2(l) + \xi_3^2(l) = \xi_1^2(0) + \xi_3^2(0) \quad (4) \]

and this condition can be used for determination of circular polarization after measuring the linear polarization of survived photon beam.

The formulae (2) permits to calculate the parameters of surviving photon beam in dependence of orientation angle \( \theta \) and incident beam polarization direction \( \varphi_0 \). From expressions for \( \xi_2(l) \) and \( I(l) \) it is clear that crystal must be oriented at angles corresponding to maximum values of real parts differences in order to decrease the beam attenuation or the thickness of crystal plate. The results of calculations for C, Si and Ge crystals at energies 100, 200 and 300 GeV are shown in Fig. 1-3. The photon beam is oriented parallel to the plane \((1\bar{1}0)\) and makes angle \( \theta \) with axis \([110]\). The absorption coefficients and, respectively, differences of real parts are calculated by coherent theory [13], the validity of which is not questionable since the results obtained by this theory are in good agreement with experimental data at considered angles [12].

The criterion of crystal efficiency is the maximal value of the figure of merit (FOM) \( \text{FOM} = \xi_2^2(l)I(l) \) in dependence of orientation angle \( \theta \) and crystal thickness \( l \). For determination of \( \theta_{\text{opt}} \) and \( l_{\text{opt}} \) the calculations are carried out at \( \varphi_0 = 45^0 \). Such choice of incident beam polarization direction is due to the fact, that at considered energies the condition \( al \ll 1 \) is fulfilled and \( \xi_2(l) \) receives its maximum value at that angle.

The results given in Table 1 show, that optimal angles \( \theta_{\text{opt}} \) decrease with increasing of photon energy being significantly higher than characteristic angle of quasiclassical theory at considered energies [12]. The value of FOM is greatest for diamond but silicon crystal is more suitable for conversion experiment because of the silicon can be grown of required thickness and does not need cooling as germanium crystal.
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**Figure Captions**

Fig.1. Crystal orientation with respect to the polarized photon beam.

Fig.2. Curves for diamond, silicon and cooled germanium crystals for photon beam energy 100 GeV. Solid curves for Si, dashed curves for Ge and dotted curves for C.

a - the differences of refractive indexes real parts as a function of $\theta$.

b and c - photon beam circular polarization degree and intensity as a functions of crystal plate thickness respectively.

Fig.3. The curves as in Fig. 2. for photon beam energy 200 GeV.

Fig.4. The curves as in Fig. 2. for photon beam energy 300 GeV.
| Crystal | \( \omega \) (GeV) | FOM | \( \theta_{\text{opt}} \) (mrad) | \( l_{\text{opt}} \) (cm) | \( I(l_{\text{opt}})/I(0) \) | \( \xi_2(l_{\text{opt}}) \) |
|---------|------------------|-----|-----------------|-----------------|-----------------|-----------------|
| C       | 100              | 3.38| 1.50            | 5.32            | 0.21            | 0.73            |
|         | 200              | 3.56| 0.75            | 2.76            | 0.22            | 0.75            |
|         | 300              | 3.62| 0.50            | 1.86            | 0.23            | 0.76            |
| Si      | 100              | 2.21| 2.29            | 9.43            | 0.17            | 0.54            |
|         | 200              | 2.63| 1.14            | 5.50            | 0.18            | 0.61            |
|         | 300              | 2.79| 0.76            | 3.84            | 0.19            | 0.64            |
| Ge (100°K) | 100              | 1.83| 2.38            | 2.25            | 0.16            | 0.46            |
|         | 200              | 2.18| 1.19            | 1.32            | 0.17            | 0.53            |
|         | 300              | 2.32| 0.79            | 0.93            | 0.17            | 0.56            |
Fig. 1
Fig. 2

(A) Re$(n_{\parallel} - n_{\perp}) \times 10^{17}$

(B) $P_{\text{circ}}$

(C) Fraction of surviving photons
Fig. 3
Fig. 4

(a) $\text{Re}(n_r - n_i) \times 10^{-17}$ vs $\theta$ (mrad)

(b) $P_{\text{circ}}$ vs $l$ (cm)

(c) Fraction of surviving photons vs $l$ (cm)