Fully Hyperbolic Graph Convolution Network for Recommendation

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ABSTRACT

Recently, Graph Convolution Network (GCN) based methods have achieved outstanding performance for recommendation. These methods embed users and items in Euclidean space, and perform graph convolution on user-item interaction graphs. However, real-world datasets usually exhibit tree-like hierarchical structures, which make Euclidean space less effective in capturing user-item relationship. In contrast, hyperbolic space, as a continuous analogue of a tree-graph, provides a promising alternative. In this paper, we propose a fully hyperbolic GCN model for recommendation, where all operations are performed in hyperbolic space. Utilizing the advantage of hyperbolic space, our method is able to embed users/items with less distortion and capture user-item interaction relationship more accurately. Extensive experiments on public benchmark datasets show that our method outperforms both Euclidean and hyperbolic counterparts and requires far lower embedding dimensionality to achieve comparable performance.

CCS CONCEPTS

- Computer systems organization → Embedded systems; Redundancy; Robotics; Networks → Network reliability.

KEYWORDS

recommendation system, graph neural networks, hyperbolic space

1 INTRODUCTION

In information era, recommendation systems have been widely adopted to perform personalized information filtering [7, 25]. Even though there are many recommendation paradigms, collaborative filtering [8, 24] which generates recommendations by utilizing available historical interactions, remains a fundamental and challenging task. The core idea behind collaborative filtering is to learn compact user/item embeddings and infer a user’s preference to an item according to the distance between their embeddings.

From the perspective of graph, user-item interactions can be viewed as a bi-partite graph [3], where nodes represent users/items and edges represent their interactions. As a powerful tool of analyzing graph-structured data, Graph Neural Networks (GNNs) [9, 13, 23] have recently demonstrated great success across various domains, including recommendation systems. Employing multiple layers of neighborhood aggregation, GNN-based methods [11, 24] have achieved the state-of-the-art performance on diverse public benchmarks.

Although GNN-based methods have achieved outstanding performance, it might not be appropriate to adopt Euclidean space to embed users and items. In real-world scenarios, user-item bipartite graphs usually exhibit tree-like hierarchical structures [2], in which the number of a node’s neighbors grows exponentially with respect to the number of hops. Ideally, neighbors of node $v$ should be embedded in the ball centering at $v$’s embedding, and the distance between embeddings should reflect the number of hops between nodes. Nevertheless, in Euclidean space, the volume of a ball only grows polynomially as a function of radius. Hence, embedding exponentially-growing number of neighbors into polynomially-growing size of volume would make distance between embeddings less accurate to reflect distance between nodes in the graph, and this is called distortion [6]. This kind of distortion makes it difficult to infer a user’s preference to a target item according to the distance between their embeddings.

In contrast, hyperbolic space [6] in which the volume of a ball grows exponentially with radius offers a promising alternative. Compared with Euclidean space, hyperbolic space is more suitable for modeling user-item interaction graphs which exhibit strong tree-like hierarchical structures. Accordingly, it is a natural choice to conduct user/item embedding and graph convolution in hyperbolic space for recommendation. To the best of our knowledge, the only work adopting a similar idea is HGCF [19]. However, resorting to tangent space to realize graph convolution makes the performance of HGCF inferior for the two following reasons. On the one hand, tangent space is only local linear approximation of hyperbolic space. During the process of message propagation, errors caused by approximation accumulate and spread to the whole graph. As a result, influence from high-order neighbors cannot be captured accurately. On the other hand, tangent space is actually Euclidean...
space according to its definition \[5\]. Hence, the advantage of hyperbolic space in modeling user-item interaction relationship with less distortion cannot be fully utilized.

To overcome the limitation of Euclidean space and obtain more accurate user/item embeddings, we design a novel fully hyperbolic GCN framework specially for collaborative filtering. All operations are conducted in hyperbolic space, more specifically, in the Lorentz model, and we name it Lorentz Graph Collaborative Filtering (LGCF).

The main contributions of this work are summarized as follows:

- We propose a fully hyperbolic graph convolution network for recommendation.
- We conduct extensive experiments on multiple public benchmark datasets, and the results demonstrate the superiority of our method.

2 PRELIMINARIES

Problem Formulation. In this paper, the standard collaborative filtering set-up is considered. Let \(U = \{u_1, u_2, \ldots, u_n\}\) be the set of \(n\) users, and \(I = \{i_1, i_2, \ldots, i_m\}\) be the set of \(m\) items. Historical interactions between users and items are represented as a binary matrix \(B\), where \(B_{ui} = 1\) if the \(i\)-th user interacts with the \(j\)-th item, otherwise 0. Given historical interactions \(B\), the goal is to predict potential interactions.

GNN-based Recommendation Methods. GNN-based recommendation methods have received increasing attention for their ability to learn rich node representations. In collaborative filtering setting, GCN has replaced matrix factorization \[17\] and shown leading performance. In order to capture the influence from high-order neighbors, Wang et al. \[24\] proposed NGCF, a GCN framework performing on user-item interaction bi-partite graphs. Subsequently, He et al. \[11\] empirically found that two most common designs in GCNs — feature transformation and nonlinear activation, contribute little to the performance of collaborative filtering. Hence, they proposed a simplified architecture – LightGCN \[11\].

3 OUR METHOD

As illustrated in Figure 1, there are three components in LGCF: (1) an embedding layer that provides and initializes user/item embeddings in hyperbolic space; (2) multiple graph convolution layers that propagate user/item embeddings over the graph; and (3) a prediction layer that estimates a user’s preference to an item by computing the distance between their embeddings.

3.1 Embed Users/Items in Hyperbolic Space

Existing GNN-based recommendation methods usually embed users and items in the same Euclidean space. To accurately model user-item interaction relationship, we investigate the utilization of hyperbolic space. There are several models of hyperbolic space, such as the Lorentz model, the Klein model and the Poincaré ball model. In this paper, we choose the Lorentz model due to its simplicity and numerical stability. Formally, the Lorentz model of \(d\)-dimensional hyperbolic space is defined as:

\[
\mathcal{L} = \{x = [x_0, x_1, \ldots, x_d] \in \mathbb{R}^{d+1} : (x, x)_{\mathcal{L}} = -1, x_0 > 0\},
\]

where \((x, y)_{\mathcal{L}}\) is Lorentz inner product and is defined as \((x, y)_{\mathcal{L}} = -x_0 y_0 + \sum_{i=1}^{d} x_i y_i\). In addition, at an arbitrary point \(x \in \mathcal{L}\), hyperbolic space can be locally approximated by a linear Euclidean space. And this approximated Euclidean space is termed as tangent space \(T_x \mathcal{L}\) in which norm \(|x|_{\mathcal{L}} = (x, x)_{\mathcal{L}}\) is well defined. In LGCF, both users and items are embedded in the same Lorentz model of hyperbolic space.

It is well known that random initialization can have a significant impact on optimization in training \[20\]. A common practice in Euclidean space is Gaussian distribution initialization. Similarly, we design an initialization strategy for embeddings based on Wrapped Normal Distribution \[15\] which is a generalization of Gaussian distribution to hyperbolic space.

3.2 Graph Convolution Layer

The basic idea of GCN-based recommendation models is learning representations for users and items by aggregating neighbors’ information iteratively over the interaction graph. In order to apply GCN for recommendation where users and items are embedded in hyperbolic space, we design graph convolution layers specially since naïve generalization will drive embeddings out of hyperbolic space. Before that, we give a brief review of existing graph convolution layers in Euclidean space.

In Euclidean space, a graph convolution operation is composed of three steps: feature transformation, neighborhood aggregation and non-linear activation. Among them, feature transformation is performed through linear transformation. In LGCF, we discard the feature transformation operation for two reasons. On the one hand, different from attributed graphs (e.g., citation networks) where nodes bring rich feature information, nodes in user-item interaction graphs contain no semantics but one-hot IDs. In this case, feature transformation may provide no benefits, and could bring difficulties to training. On the other hand, linear transformation (matrix-vector multiplication) is not well-defined in hyperbolic space since it is not a vector space.

Neighborhood Aggregation. Existing neighborhood aggregation of the \(l\)-th layer can be summarized as:

\[
e^{(l)}_i = \sum_{j \in \hat{N}(i)} w_{ij} e^{(l-1)}_j, \tag{2}
\]

in which \(e^{(l-1)}_j\) represents the \(j\)-th node’s embedding and \(\hat{N}(i)\) denotes the set consisting of node \(i\) and its neighborhood nodes. A natural generalization of mean to hyperbolic space is Einstein Normal Distribution \[15\] which is a generalization of Gaussian distribution to hyperbolic space. The main contributions of this work are summarized as follows:

- We propose a fully hyperbolic graph convolution network for recommendation.
- We conduct extensive experiments on multiple public benchmark datasets, and the results demonstrate the superiority of our method.

\[
F(x) = \frac{[x_1, x_2, \ldots, x_d]}{x_0}, \tag{3}
\]

in which \(x = [x_0, x_1, \ldots, x_d] \in \mathcal{L}\) and \(k \in \mathcal{K}\).

Hence, we propose a neighborhood aggregation strategy utilizing the Klein model as an intermediate bridge. Specifically, neighborhood aggregation can be divided into three steps. First, current...
embeddings \{e_1, e_2, \ldots, e_n\} are mapped from the Lorentz model to the Klein model by \(k'_i = F(e_i)\). Then, neighborhood aggregation is conducted in the Klein model as follows:

\[
y_i = \frac{1}{\sqrt{1 - \|k'_i\|^2}}, \quad k'_i = \frac{\sum_{j \in N(i)} y_j k_j}{\sum_{j \in N(i)} y_j}.
\]  

(4)

Here, we obtain aggregated user/item embeddings in the Klein model. Last, aggregated embeddings \(k'_i\) are mapped back to the Lorentz model through \(z_i = F^{-1}(k'_i)\).

**Nonlinear Activation Layer.** In Euclidean space, nonlinear activation has proven to be a key component in modern neural networks. However, direct adoption will drive the computation result out of hyperbolic space. To fix this problem, we design a calibration strategy following general activation. Formally, let \(x_1 = [x_0, x_1, \ldots, x_d] = \sigma(z_i)\) be the output of general activation function \(\sigma\), e.g., ReLU. The first element of \(x\) is calibrated while the other elements remain unchanged to pull the activated embedding back to hyperbolic space:

\[
e'_i = f_k(x_i) = \sqrt{1 + \sum_{j=1}^{d} x_j^2} x_1, x_2, \ldots, x_d.
\]  

(5)

### 3.3 Prediction Layer

After propagating with \(L\) graph convolution layers, we obtain multiple representations for users and items. Representations generated in different layers emphasize messages passed through different connections, and they reflect users’ preference or items’ attributes from different perspectives. Recommendation models operating in hyperbolic space usually estimate a user’s preference to a target item according to the distance or similarity metric between their representations. In the Lorentz model, the generalization of a straight line in Euclidean space is geodesics which gives the shortest distance between two points \(x, y\). Formally, the geodesics distance between \(x\) and \(y\) is defined as:

\[
d_L(x, y) = \text{arcosh}( -\langle x, y \rangle_L ).
\]  

(6)

Hence, in LGCF, we infer the preference of user \(u\) to item \(i\) based on the geodesics distance between their corresponding representations. Further, to utilize different semantics captured by different layers, we take representations learned by different layers into consideration simultaneously. In summary, LGCF estimates the preference of user \(u\) to target item \(i\) as:

\[
y(u, i) = \frac{1}{\sum_{l=1}^{L} d_L^2(e'_u^{(l)}, e'_i^{(l)})},
\]  

(7)

in which \(e'_u^{(l)}\) and \(e'_i^{(l)}\) are representations generated by the \(l\)-th layer. Even though there are multiple choices for layer aggregation, such as weighted average, max pooling, LSTM, etc., we find simple summation adopted here works well empirically.

### 3.4 Margin Ranking Loss

Margin ranking loss [21] has been a competitive choice for distance-based recommendation systems. Since it encourages positive and negative user-item pairs to be separated up to a given margin. Once the difference between a negative user-item pair and a positive one is greater than the margin, these two user-item pairs will make no contribution to the loss. In this way, hard pairs violating the margin are focused all the time, making optimization much easier. We extend margin ranking loss to hyperbolic space based on geodesics distance. Given a sampled positive user-item pair \((u, i)\) and a negative one \((u, j)\), geodesics margin loss is defined as:

\[
\ell_g(u, i, j) = \frac{1}{L} \sum_{l=1}^{L} \min \left\{ d_L^2(e_u^{(l)}, e_j^{(l)}) - d_L^2(e_u^{(l)}, e_i^{(l)}) - m, 0 \right\},
\]  

(8)

where \(m\) is a non-negative hyper-parameter. Note that representations obtained by different layers contribute to the loss simultaneously. This not only makes it possible to utilize different semantics captured by different layers, but also decreases the difficulty of optimization due to the residual connection [10].

### 3.5 Optimization

The only parameter of LGCF is the embedding matrix of users and items. These embeddings lie in the Lorentz model of hyperbolic space which is out of the range of common optimization algorithms.
such as SGD [18] and Adam [12]. Hence, we employ RGSD [4], a generation of SGD to hyperbolic space, which mimics SGD’s behavior while taking into account the geometry of hyperbolic space.

4 EXPERIMENTS

4.1 Set Up

Datasets and Baselines. Following HGCF [19], we employ Amazon-CD [16], Amazon-Book [16] and Yelp2020 [1] datasets. Dataset statistics are provided in Table 1. Each dataset is split into 80-20 train and test sets. Multiple competitive baseline methods from three categories are compared: BPRMF [17], NGCF [24], LightGCN [11], HVAE [14], HGCF [19]. Among them, BPRMF optimizes matrix factorization by Bayesian personalized ranking (BPR) loss [17]. NGCF and LightGCN employ GNN in Euclidean space. HVAE combines variational auto-encoder (VAE) with hyperbolic geometry. Last, HGCF applies the latest Hyperbolic GCN [6] to recommendation systems.

Implementation. For a fair comparison, the embedding dimensionality is set to 50 for all methods, and the same negative sampling strategy is adopted. For all baseline methods, suggested settings in original papers are followed. Grid search for hyper-parameters are conducted following HGCF [19]. For LGCF, the number of GCN layers is set to 3. We set the learning rate to 0.001 and weight decay to 0.005. And model is trained for 1000 epochs. Hyper-parameter margin $m$ is tuned from $[0.01, 2]$.

Table 1: Dataset statistics.

| Dataset         | #User | #Item | #Interactions |
|-----------------|-------|-------|---------------|
| Amazon-CD       | 22,947| 18,395| 422,301       |
| Amazon-Book     | 52,406| 41,264| 1,861,118     |
| Yelp2020        | 91,174| 45,063| 1,940,014     |

Table 2: Recall results for all datasets.

| Datasets          | Metrics | BPRMF | NGCF | LightGCN | HVAE | HGCF | LGCF |
|-------------------|---------|-------|------|----------|------|------|------|
| Amazon-CD         | R@10    | 0.0779| 0.0578| 0.0929   | 0.0781| 0.0962| 0.0996|
|                   | R@20    | 0.1200| 0.1150| 0.1400   | 0.1147| 0.1455| 0.1503|
| Amazon-Book       | R@10    | 0.0611| 0.0658| 0.0799   | 0.0774| 0.0867| 0.0899|
|                   | R@20    | 0.0794| 0.1059| 0.1248   | 0.1125| 0.1318| 0.1360|
| Yelp2020          | R@10    | 0.0325| 0.0458| 0.0522   | 0.0421| 0.0543| 0.0573|
|                   | R@20    | 0.0536| 0.0764| 0.0866   | 0.0691| 0.0884| 0.0916|

Table 3: NDCG results for all datasets.

| Datasets          | Metrics | BPRMF | NGCF | LightGCN | HVAE | HGCF | LGCF |
|-------------------|---------|-------|------|----------|------|------|------|
| Amazon-CD         | N@10   | 0.0610| 0.0591| 0.0726   | 0.0629| 0.0751| 0.0780|
|                   | N@20   | 0.0974| 0.0718| 0.0881   | 0.0749| 0.0909| 0.0945|
| Amazon-Book       | N@10   | 0.0594| 0.0665| 0.0780   | 0.0778| 0.0869| 0.0906|
|                   | N@20   | 0.0971| 0.0791| 0.0938   | 0.0991| 0.1022| 0.1063|
| Yelp2020          | N@10   | 0.0283| 0.0465| 0.0641   | 0.0371| 0.0458| 0.0485|
|                   | N@20   | 0.0512| 0.0513| 0.0582   | 0.0465| 0.0585| 0.0612|

4.2 Overall Results

Recall and NDCG results for all datasets are reported in Table 2 and Table 3 respectively. We can see that LGCF consistently outperforms other methods on all the three datasets. Compared with LightGCN, LGCF achieves an improvement up to 16.15% in NDCG@10 on Amazon-Book dataset, demonstrating the superiority of hyperbolic space over Euclidean space in modeling real-world user-item interactions.

Among the baseline methods, HGCF is the most competitive counterpart of LGCF. Even though HGCF adopts hyperbolic space, it resorts to tangent space to conduct aggregation operations, which brings inevitable distortion. In contrast, LGCF performs all graph convolution operations in hyperbolic space. LGCF outperforms HGCF with wide margins on all the datasets, showing the information loss introduced by tangent space in HGCF.

4.3 Ablation Study

To further analyze the effect of fully hyperbolic graph convolution network, we conduct an ablation study on Yelp2020, the largest one among the three datasets. Since simply replacing fully hyperbolic graph convolution with regular Euclidean graph convolution would drive user/item embeddings out of hyperbolic space, we conduct convolution operations in tangent space, and name this model variant as LGCF-tangent. From experimental results shown in Table 4, we can observe that there is a wide margin between the performance of LGCF and LGCF-tangent. This is because, in LGCF-tangent, errors caused by tangent space approximation accumulate and spread to the whole graph. As a result, influence from neighbors, especially high-order neighbors, cannot be captured accurately.

4.4 Effect of Embedding Dimensionality

In order to validate the advantage of hyperbolic space to learn compact representations, we compare the results of LGCF and LightGCN with different values of embedding dimensionality. From Figure 2, we can observe that LGCF outperforms LightGCN consistently at
all dimensionality values, and the greatest margin occurs at lower dimensionality. LGCF requires far lower embedding dimensionality to achieve comparable performance to its Euclidean analogue. This reflects that LGCF’s advantage is more prominent when the embedding dimensionality cannot be large due to limited computing and storage resources.

5 CONCLUSION

In this paper, we propose LGCF, a fully hyperbolic GCN model for recommendation. Utilizing the advantage of hyperbolic space, LGCF is able to embed users/items with less distortion and capture user-item interaction relationship more accurately. Extensive experiments on public benchmark datasets show that LGCF outperforms both Euclidean and hyperbolic counterparts and requires far lower embedding dimensionality to achieve comparable performance.

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