Tracing Magnetic Fields with the Gradient Technique: Spatial Filtering and Use of Interferometers

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ABSTRACT

Probing magnetic fields in astrophysical environments is important but challenging. The Gradients Technique (GT) is a new tool in tracing the magnetic fields, which is rooted in the properties of MHD turbulence and turbulent magnetic reconnection. In this work, we study the performance of multiple gradients obtained from synchrotron emission and spectroscopic data, when low spatial frequencies are removed. Using synthetic observations obtained from MHD simulations, we demonstrate the improved accuracy of GT to trace magnetic fields in the absence of low spatial frequencies. We apply the low-spatial frequency filter to a diffuse neutral hydrogen region selected from the GALFA-H I survey. We report the increased alignment between the magnetic fields inferred from GT and the Planck 353 GHz polarization measurements. We confirm that the usage of the interferometric data independent of single-dish observations provides a unique way to accurately trace the magnetic fields with GT.

Keywords: Interstellar medium (847); Interstellar magnetic fields (845); Interstellar dynamics (839)

1. INTRODUCTION

Magnetic fields are key parts of understanding our universe. (Larson 1981; Elmegreen & Scalo 2004; Ballesteros-Paredes et al. 2007; McKee & Ostriker 2007; Chepurnov & Lazarian 2009). In the interstellar medium (ISM), magnetic fields are involved in regulating star formation, particle acceleration, and galaxy evolution (Armstrong et al. 1995; Widrow 2002; Chepurnov & Lazarian 2010; Crutcher 2012; Galli et al. 2006; Kronberg 1994; Caprioli & Spitkovsky 2014; Li & Henning 2011). However, the magnetic field is the least-observed physical quantity involved in such processes. The primary ways of observing the magnetic fields are divided into the measurement of the plane-of-the-sky (POS) component through polarized dust emission (Andersson et al. 2015; Lazarian 2007) and synchrotron emission (Clarke & Ensslin 2006; Planck Collaboration et al. 2016; Dickey et al. 2019), and the line-of-sight (LOS) component from the Faraday rotation (Simard-Normandin et al. 1981; Haverkorn et al. 2006; Oppermann et al. 2012) and the Zeeman splitting (Heiles 1976; Crutcher et al. 2010; Crutcher 2012). The study of molecular line polarization using Goldreich-Kulafis polarization (Goldreich & Kylafis 1981, 1982) is another way to trace magnetic fields. Line emission and absorption of atoms with fine and hyperfine structure of ground or metastable levels, i.e. Ground State Alignment (GSA, see Yan & Lazarian 2006, 2007, 2008, 2015; Zhang & Yan 2018) presents another promising way of exploring magnetic fields in diffuse media.

In spite of having different ways to probe magnetic fields, the studies of magnetic fields remain very challenging. The measurement of dust polarization is based on the fact that aligned dust grains will produce polarized emission perpendicular to the projection of the magnetic field onto the POS (Lazarian & Hoang 2007; Andersson et al. 2015). There are, however, difficulties when studying the magnetic fields through dust polarimetry. For instance, the limited sensitivity of the instruments makes it difficult to measure dust polarization unless column densities of matter are sufficiently large. This presents a challenge for tracing magnetic fields at high galactic latitudes.

To get higher resolution the generally accepted way is to use interferometers. Polarimetry measurements using interferometers are problematic, however (Ekers & Rots 1979; Rau & Cornwell 2011; Pingel et al. 2018). An interferometer array can only resolve angular size θ within the range of: λ/B<sub>max</sub> < θ < λ/B<sub>min</sub>, where λ is the observation wavelength, B<sub>max</sub> is the longest baseline, and B<sub>min</sub> is the minimum separation between apertures. As for the single-dish measurement, it delivers low-spatial frequencies from zero to its characteristic angular scale approximately λ/D, where D < B<sub>min</sub> is the diameter of the telescope. Therefore to restore the image, e.g. recover the directions of magnetic field using polarization observations with interferometer, it is important to combine the interferometer and single dish observations. This arrangement is not always available.

In this paper we discuss a way to study magnetic fields that both synergetic to existing ways of magnetic field study...
and has a number of advantages. In particular, this technique can utilize interferometric data with low spatial frequencies missing, i.e. use the result of interferometric measurements without single dish data. This way of magnetic field tracing is the Gradients Technique (GT). This technique is recently proposed as a promising way for studying the magnetic fields in the universe (González-Casanova & Lazarian 2017; Yuen & Lazarian 2017a; Lazarian & Yuen 2018a; Hu et al. 2018).

The GT is developed on the basis of the advancements in understanding of MHD turbulence theory (Goldreich & Sridhar 1995) and turbulent reconnection theory (Lazarian & Vishniac 1999). Due to fast turbulent reconnection, MHD turbulence can be presented as a superposition of turbulent eddies rotating in the direction of local magnetic field of the eddy. This property ensures both gradients of velocity and magnetic field are perpendicular to the local magnetic field\(^1\). Therefore, the magnetic field orientation can be inferred from the velocity gradients rotated by 90°.

Due to symmetry of velocity and magnetic fluctuations in Alfvén turbulence, the gradients of magnetic fluctuations are also perpendicular to local magnetic field. As synchrotron radiation arises from relativistic electrons spiraling along magnetic field lines (Clarke & Ensslin 2006; Planck Collaboration et al. 2016; Dickey et al. 2019), the anisotropic property is also entailed by the synchrotron emission (Lazarian & Pogosyan 2012). This gave rise to the development of the magnetic field tracing based on synchrotron intensity gradients (SIGs, Lazarian et al. 2017) and synchrotron polarization gradients (SPGs, Lazarian & Yuen 2018b). SIGs can provide a direct measurement of the POS magnetic field without the correction of Faraday rotation. SPGs, on the other side, opens a new way to study the 3D magnetic field in the Milky Way and nearby galaxies (see Lazarian & Yuen 2018b for a detailed discussion).

The ability of gradients to trace the magnetic field demonstrated in the original papers (see Lazarian et al. 2017; Yuen & Lazarian 2017a; Lazarian & Yuen 2018a; Hu et al. 2018; Lazarian & Yuen 2018b) showed the promise of the new approach of magnetic field studies with the GT. More recent results of tracing magnetic field with the GT include modeling the galactic foreground polarization using neutral hydrogen data (Hu et al. 2020a; Lu et al. 2020; Hu & Lazarian 2020a), tracing distribution of the Plane-of-Sky (POS) magnetic fields in molecular clouds (Hu et al. 2019a,b; Alina et al. 2020; Liu et al. 2022; Zhao et al. 2022), the Central Molecular Zone (Hu et al. 2022a,b), and nearby galaxies (Hu et al. 2022c; Tram et al. 2022), and predicting the magnetic field morphology in galaxy clusters (Hu et al. 2020c). In particular, González-Casanova & Lazarian (2019) used the GT to obtain the first 3D map of the galactic disk POS magnetic field distribution and Hu et al. (2019a) presented the first magnetic field morphology of the high-velocity Smith cloud.

We mentioned earlier that to restore magnetic field maps with the traditional techniques, i.e. with polarimetry, all spatial frequencies are required. In Lazarian et al. (2020a), however, it shown that the best alignment between rotated velocity/density/magnetic gradients and the local magnetic fields starts at spatial frequency \(k = 1/L_{\text{block}}\), where \(L_{\text{block}}\) is sub-block size for doing averaging of gradients (see § 4 for the averaging method). The GT, therefore, shows the possibility of restoring the magnetic field morphology using only high-spatial-frequencies measurements. In fact, the theory in Lazarian et al. (2020a) suggests that filtering low spatial frequencies can improve the accuracy of magnetic field tracing with gradients.

The theoretical findings above motivates our numerical study in this paper on how the removal of low-spatial-frequencies affects the performance of the GT. The earlier analysis of the GT’s performance in the absence of low-spatial-frequencies was numerically performed in Lazarian et al. (2017) for SIGs and in Lazarian & Yuen (2018a) for VChGs. It was confirmed that the GT can successfully restore the magnetic field using only high-spatial-frequencies measurements. In this work, we provide an extensive study of the GT performance when low-spatial-frequencies are removed from the images analyzed and address the question of whether such spatial filtering can, in fact, improve the accuracy of the magnetic fields traced by the GT. Our study extends to multiple types of gradients, including the Intensity Gradients (IGs, see Hu et al. 2019c), Velocity Centroid Gradients (VCGs, see González-Casanova & Lazarian 2017), Velocity Channel Gradients (VChGs, see Lazarian & Yuen 2018a), SIGs (Lazarian et al. 2017), and SPGs (Lazarian & Yuen 2018b). Different from IGs and SPGs which utilize the synchrotron radiation, IGs, VCGs, and VChGs enable the usage of spectroscopic data to study the magnetic fields in several scenarios.

In what follows, we illustrate the theoretical foundation of the GT in terms of MHD turbulence in § 2. We give details about the numerical simulations used in this work in § 3. In § 4, we describe the full algorithm in probing the magnetic fields through GT. In § 5, we analyze examine the effects of removing low spatial frequency in GT. In § 6 we discuss the advantage of applying the GT to interferometric data. In § 7 and § 8, we give our discussion and conclusions.

2. THEORETICAL CONSIDERATION

2.1. MHD turbulence, reconnection and gradients

The GT employs the properties of MHD turbulence, in particular, the anisotropy of turbulent motions with respect to the magnetic field. It is due to the alignment of turbulent eddies with the local direction of the magnetic field that GT can accurately trace the magnetic field in astrophysical settings.

The subject has been studied over several decades (Montgomery & Turner 1981; Matthaeus et al. 1983; Shebalin et al. 1983; Higdon 1984) with the important breakthrough...
given by Goldreich & Sridhar (1995), denoted as GS95 later. There, to characterize the anisotropy, the critical balance condition was suggested, namely:

\[ k_\parallel \propto (k_\perp)^{2/3}, \]

which is known as GS95 anisotropy scaling. Here \( k_\parallel \) and \( k_\perp \) are wavenumbers perpendicular and parallel to the magnetic field, respectively. There is an important caveat, however. The GS95 scaling was derived using closure relations in the global mean magnetic field reference frame. Unfortunately, this scaling is not correct in this frame of reference. This was demonstrated, for instance, in Cho & Vishniac (2000) as well as in Maron & Goldreich (2001). Instead, MHD simulations demonstrate that in the mean magnetic field system of reference the anisotropy is dominated by the largest eddies and is scale-independent (see Cho et al. 2002) as opposed to the prediction given by Eq. 1.

The scale-dependent anisotropy similar to GS95 can be obtained, however, if one uses the local system of reference associated with the eddies. The eddy description of turbulence is possible due to fast 3D turbulent magnetic reconnection introduced in Lazarian & Vishniac (1999) (henceforth LV99). Indeed, LV99 demonstrated that the turbulent motions of strongly magnetized fluid can be still eddy-like if the eddies axes are aligned with local magnetic field direction in the eddy direct vicinity. This type of motion was not considered earlier, but LV99 showed that the reconnection process reconnects the magnetic field through such an eddy within one eddy turnover. Therefore, the magnetic field cannot constrain this type of eddy motions which, in fact, provides the path of least resistance for the turbulent cascade.

Using the eddy description of Alfvénic turbulence it is easy to obtain its scalings. Indeed, the motions perpendicular to the local magnetic field is Kolmogorov due to the absence of the back-reaction of the magnetic field. The condition for the critical balance is the trivial consequence of the equality of the time scales for the eddy turnover of the eddy \( l_\perp/v_\parallel \) and the period of the Alfvén wave \( l_\parallel/v_\perp \) induced by the eddies’ motion. Here \( l_\parallel \) and \( l_\perp \) are the perpendicular and parallel size of eddies in respect to the local magnetic field. From the Kolmogorov scaling of velocities \( v_\parallel \sim l_\parallel^{1/3} \) and the critical balance it is possible to get the anisotropy relation for the magnetic eddies (LV99):

\[ l_\parallel \simeq L_{\text{inj}}\left(\frac{l_\perp}{L_{\text{inj}}}\right)^{2/3}M_A^{-4/3}, \]

where \( M_A \) is the ratio of the injection velocity \( v_{\text{inj}} \) to the Alfvén speed \( v_A \) and \( L_{\text{inj}} \) is the injection scale of turbulence. The universal scale-dependent anisotropy of Alfvénic turbulence in the local magnetic field reference frame has been demonstrated in numerical simulations (Cho & Vishniac 2000; Cho et al. 2002; Maron & Goldreich 2001).

The picture above implies that the eddies predominate to rotate in the direction aligned with the local magnetic field. It means that we can determine the magnetic field direction by detecting eddy’s preferential rotation. For a rotating anisotropic eddy, the velocity gradients determine the direction perpendicular to the rotation axis, i.e., the velocity gradient is perpendicular to the local magnetic fields. With the aforementioned Kolmogorov scaling of perpendicular motions of the eddies, the gradients of velocities scale as \( v_\parallel/l_\perp \sim l_\perp^{-2/3} \), which make the smallest resolved eddies the most prominent in the gradient signal. Those, as we discussed earlier, reveal the local magnetic field direction at the smallest scales.

The considerations above constitute the theoretical foundation for the Gradient Technique (GT) as applied for spectroscopic data revealing velocities of the turbulent media. Because of the symmetry of magnetic and velocity fluctuation in Alfvénic turbulence, the gradients of magnetic fields are also perpendicular to the local direction of the magnetic field. The latter is the basis for using synchrotron emission within the GT framework (Lazarian et al. 2017; Lazarian & Yuen 2018b).

2.2. Localization of information at small scales

The detailed theoretical discussion of velocity and magnetic field gradients is presented in Lazarian et al. (2020a). There it is shown that due to the fact that the gradients provide the direction of the local magnetic field direction, the information of the magnetic field is contained at the high-spatial-frequencies of the map. This is a very non-trivial conclusion and it provides the foundations for this numerical study.

To understand this conclusion one should consider the fact that the gradient directions are determined by the high-spatial-frequencies of the image. Therefore the knowledge of only high-spatial-frequencies is sufficient for recovering the direction of the magnetic field at a given turbulent volume. This local direction of the magnetic field, however, can be presented as an outcome of the superposition of the magnetic field from different scales. Therefore by using only the high-spatial-frequencies, one can restore the total magnetic field in the underlying volume Lazarian et al. (2020a). The theoretical considerations suggest that the removal of the low-spatial-frequencies does not decrease the ability of the GT to trace the magnetic field. It can improve this tracing by removing the contribution of the interfering large scales.

The property of gradient alignment with the local magnetic field was already used in Lazarian et al. (2017) and Lazarian & Yuen (2018a), where it was shown that magnetic fields can be successfully restored from gradients obtained with the maps with missing low-spatial-frequencies. In fact, in Lazarian et al. (2017) it was demonstrated that magnetic field directions can be improved for super Alfvénic turbulence by filtering out the low-spatial-frequencies.\(^2\)

\(^2\) In super Alfvénic turbulence the motions are hydrodynamic-type at scales larger than \( L_{\text{inj}}M_A^{-3} \), where \( L_{\text{inj}} \) is the injection scale and \( M_A \) is the Alfvén Mach number. The latter is the ratio of the injection velocity \( v_{\text{inj}} \) and the Alfvén velocity \( v_A \). Evidently, our considerations related to the alignment of the gradients and magnetic field direction are not valid for the motions at the scales at which the magnetic forces are subdominant. Therefore filtering out of the contributions from such scales is natural.
2.3. Density gradients

The situation with density fluctuations in turbulent media is somewhat more complicated (see Kowal et al. 2007). The density fluctuations in subsonic turbulence passively mixed by velocity fluctuation. Therefore, in these settings, the density gradient is also perpendicular to the magnetic fields. However, the physics becomes different in super-sonic turbulence, where shocks tend to change the density gradient’s direction by 90° (see more discussion in Yuen & Lazarian 2020). The density gradient is also perpendicular to the magnetic fields.

Explicitly, the velocity gradient and density gradient scale as (Yuen & Lazarian 2020):

\[ \nabla \rho_l \sim \frac{\rho_l}{c} \approx \frac{\rho_0}{c} \theta^{-1}(|\hat{k} \cdot \hat{\zeta}|) \nabla v_l, \]

\[ \nabla v_l \sim v_l \approx \frac{v_{inj}}{L_{inj}} \left( \frac{l}{L_{inj}} \right)^{-\frac{2}{3}} M_S^2 \beta, \]  

(3)

because the anisotropic relation indicates \( l_{\perp} \ll l_{\parallel} \). Here \( \rho_0 \) is the mean density, \( \hat{\zeta} \) is the unit vector for the Alfvénic mode, fast mode, or slow mode, \( c \) is the propagation speed of corresponding mode. In this case, the eddies in small scale provide the most important contribution for the gradients.

Table 1. Description of our MHD simulations. \( M_S \) and \( M_A \) are the instantaneous values at each the snapshots are taken. The compressibility of turbulence is characterized by \( \beta = 2 \left( \frac{M_A}{M_S} \right)^2 \).

| Model | \( M_S \) | \( M_A \) | Resolution | \( \beta \) |
|-------|---------|---------|-------------|-------|
| A1    | 0.66    | 0.12    | 792^3       | 0.07  |
| A2    | 0.63    | 0.34    | 792^3       | 0.58  |
| A3    | 0.62    | 0.56    | 792^3/512^3/480^3 | 1.63 |
| A4    | 0.60    | 0.78    | 792^3       | 3.38  |
| A5    | 0.60    | 1.02    | 792^3       | 5.78  |
| A6    | 1.27    | 0.50    | 792^3       | 0.31  |
| A7    | 5.96    | 0.31    | 792^3       | 0.005 |
| A8    | 10.81   | 0.23    | 792^3       | 0.001 |
| A9    | 11.11   | 0.37    | 792^3       | 0.002 |
| A10   | 10.53   | 0.46    | 792^3       | 0.004 |
| A11   | 10.61   | 0.64    | 792^3       | 0.007 |

3. NUMERICAL DATA

We perform 3D MHD simulations through ZEUS-MP/3D code (Hayes et al. 2006), which solves the ideal MHD equations in a periodic box:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \]

\[ \frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v} + (p + \frac{B^2}{8\pi}) \vec{I}) - \frac{\vec{B} \cdot \vec{B}}{4\pi} = f, \]  

\[ \frac{\partial \vec{B}}{\partial t} - \nabla \times \left( \vec{v} \times \vec{B} \right) = 0, \]  

(4)

where \( f \) is a random large-scale driving force, \( \rho \) is the density, \( \vec{v} \) is the velocity, and \( \vec{B} \) is the magnetic field. We also consider a zero-divergence condition \( \nabla \cdot \vec{B} = 0 \), and an isothermal equation of state \( p = c_s^2 \rho \), where \( p \) is the gas pressure. We use single fluid, operator-split, solenoidal turbulence injections at spatial scale \( k = 2 \), and staggered grid MHD Eulerian assumption.

To emulate a part of an interstellar cloud, we use the barotropic equation of state, i.e., these clouds are isothermal with temperature \( T = 10.0 \) K, sound speed \( c_s = 187 \) m/s and cloud size \( L = 10 \) pc. The sound crossing time \( t_s = L/c_s \) is \( \sim 52.0 \) Myr, which is fixed owing to the isothermal equation of state. We vary the sonic Mach number \( M_S = v_{inj}/c_s \) and Alfvén Mach number \( M_A = v_{inj}/v_A \) to explore different physical conditions. Here \( v_{inj} \) is the injection velocity and \( v_A \) is the Alfvénic velocity. The turbulence is highly magnetized when the magnetic pressure of plasma is larger than the thermal pressure, i.e., the compressibility of turbulence \( \beta = 2 \left( \frac{M_A}{M_S} \right)^2 \) is less than one. When \( M_S > 1 \), the velocity of fluctuation is larger than the sound speed so that shocks begin appearing. We refer to the simulations in Tab. 1 by their model name or parameters. For instance, in Fig. 1 we plot the energy spectrum of velocity using simulations A3, A5, A6, A7, and A8. The energy spectral within the inertial range follow the Kolmogorov scaling \( E_s(k) \propto k^{-5/3} \) for sub-sonic, trans-sonic, and super-sonic conditions. The dissipation scale is approximately at \( k = 80 \) in our simulations. Similar simulations have been used in Yuen & Lazarian (2017b) and Hu et al. (2021a).

4. METHODOLOGY

4.1. The Gradients Technique

The Gradients Technique consists of multiple types of gradients: intensity gradients (IGs, see Hu et al. 2019c), velocity centroid gradients (VCGs, see González-Casanova & Lazarian 2017), velocity channel gradients (VChGs, see Lazarian
satisfies:

\[ v_0 \] is the velocity corresponding to the maximum intensity in PPV’s spectral and \( \Delta v \) is the channel width \( \Delta v \) satisfies:

\[ \Delta v < \sqrt{\delta(v^2)} \]  

where \( \delta(v^2) \) is the velocity dispersion on the scales that turbulence is studied.

The IGs, VCGs, VChGs are usually calculated from the spectroscopic data, for instance, the atomic H ı 21 cm emission line, the molecular CO emission lines etc. This implementation is based on the theory of intensity fluctuations in Position-Position-Velocity (PPV) space pioneered in Lazarian & Pogosyan (2000) and later in Lazarian & Pogosyan (2008). The theory explored the possibility of using the statistics of intensity fluctuations in PPV cubes to study turbulence. The comparison of the moment-0, moment-1, moment-2 maps calculated from PPV cubes were performed in Hu et al. (2020d). The moment-0 map (also called intensity map, I(x,y)) is used to calculate IGs and the moment-1 map (also called velocity centroid map, C(x,y)) is used to calculate VCGs:

\[
I(x, y) = \int \rho(x, y, v) dv,
\]

\[
C(x, y) = \int \frac{v \rho(x, y, v) dv}{\rho(x, y, v) dv}, \tag{5}
\]

where \( \rho \) is the gas volume density, \( v \) is the radial velocity along LOS. The earlier numerical and analytical studies of centroids are performed in Esquivel & Lazarian (2005) and Kandel et al. (2016), respectively.

As for the VChGs, the calculation involves the velocity thin channel map Ch(x,y):

\[
Ch(x, y) = \int_{v_0-\Delta v/2}^{v_0+\Delta v/2} \rho(x, y, v) dv, \tag{6}
\]

where \( v_0 \) is the velocity corresponding to the maximum intensity in PPV’s spectral and \( \Delta v \) is the channel width \( \Delta v \) satisfies:

\[
\Delta v < \sqrt{\delta(v^2)}, \tag{7}
\]

where \( \delta(v^2) \) is the velocity dispersion on the scales that turbulence is studied. Eq. 7 was proposed in LP00 as an crucial criterion to distinguish the dominance of velocity fluctuation in PPV cubes. Once the channel width satisfies with Eq. 7, the intensity fluctuations in PPV cubes could arise due to turbulent velocities along the LOS. This is called the velocity caustics effect, which must inevitably present in the thin velocity channel maps, as a natural result of non-linear mapping from the real space to the PPV space. In what follows, we use the description in LP00 that is provided for the central channels. In a real scenario, the PPV cubes of emission lines are measured in terms of radiation temperature \( T_R(x, y, v) \) rather than gas volume density. The gas volume density used in Eq. 5 and Eq. 6 should be replaced by \( T_R(x, y, v) \) accordingly. In the case of optically thick emission, Eq. 5 and Eq. 6 only sample outskirt of the cloud.

As for SIGs and SPGs, the calculation utilizes the intensity of synchrotron emission \( S(x, y) \) and the polarized synchrotron intensity \( P(x, y) \) given by Stokes parameters \( Q(x, y, z) \) and \( U(x, y, z) \):

\[
S(x, y) \propto \int B_1^2(x, y, z) dz
\]

\[
P(x, y) = |\int (Q + iU) dz|, \tag{8}
\]

\[
Q(x, y, z) \propto pm_e (B_x^2 - B_y^2),
\]

\[
U(x, y, z) \propto pm_e (2B_x B_y),
\]

where \( B_\perp = \sqrt{B_x^2 + B_y^2} \) corresponds to the magnetic field component perpendicular to the LOS, i.e., the z-axis. \( p \) is the polarization fraction, which is to be assumed constant, and \( n_e \) is the relativistic electron density. We used \( \gamma = 2 \) instead of the actual fractional power of the index appealing to the quantitative theory in (Lazarian & Pogosyan 2012). There the relations between the power spectra and structure functions of synchrotron emission for arbitrary \( \gamma \) were related to the statistics obtained for \( \gamma = 2 \) through the function of \( \gamma \) defined there. Therefore results obtained for \( \gamma = 2 \) can trivially be generalized for an arbitrary \( \gamma \). Similar to the case of molecular emission, \( S(x, y) \) measures the full magnetic field structure when the emission is optically thin.

For the sake of simplicity, while addressing the SPGs, we discuss only a very special case of no Faraday rotation influencing the arriving polarized signal. This corresponds, for instance, to high, e.g. tens of GHz, frequencies of radio emission. The value of the SPGs, as they are introduced in Lazarian & Yuen (2018b), is that the SPGs can study magnetic fields at any frequency and combining different frequencies one can restore the 3D structure of magnetic field. We will consider the effects of spatial filtering on SPGs in a general case within a separate study.

The pixelized gradient map \( \psi_g(x, y) \) of each 2D map is calculated from the convolution with 3 \times 3 Sobel kernels \( G_x \) and \( G_y \):

\[
\nabla_x f(x, y) = G_x \ast f(x, y),
\]

\[
\nabla_y f(x, y) = G_y \ast f(x, y),
\]

\[
\psi_g(x, y) = \tan^{-1}\left(\frac{\nabla_y f(x, y)}{\nabla_x f(x, y)}\right), \tag{9}
\]

where \( f(x, y) \) can be replaced by I(x,y), C(x,y), Ch(x,y), \( S(x, y) \), or \( P(x, y) \). \( \nabla_x f(x, y) \) and \( \nabla_y f(x, y) \) are the x and y components of gradient respectively. \( \ast \) denotes the convolution.

4.1.1. Sub-block averaging

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Figure 2. The plot of the magnetic field orientation probed by GT (red) and synthetic dust polarization (blue) with sub-block size 33 pixels. The plot is overlaid on the intensity maps (the 1st column), velocity centroid maps (the 2nd column), and velocity channel maps (the 3rd column). The gradients are calculated within full spatial frequency range (the 1st row), \(k>10\) (the 2nd row), and \(k<10\) (the 3rd row), respectively.

Eq. 9 produces the raw gradient map \(\psi(x, y)\). However, as the anisotropy of turbulent eddies concerning the local magnetic field is a statistical concept, the individual raw gradient is not necessarily required to have any relation to the local magnetic field direction. The perpendicular relative orientation of gradients and magnetic field only appears when the gradient sampling is enough. To extract the anisotropy, the critical step after getting \(\psi(x, y)\) is taking the average of gradients’ orientation within a sub-block of interest, called the sub-block averaging method (Yuen & Lazarian 2017a).

The sub-block averaging method is implemented as: (i) draw the histogram of gradients’ orientation within a sub-region; (ii) fit the histogram with the Gaussian curve and take the expectation value of the Gaussian fitting. The resulting expectation value \(\psi(x, y)\) of the Gaussian distribution reflects the statistical most probable orientation of gradients. By rotating \(\psi(x, y)\) with 90°, we output the predicted the lo-
The Gradient Technique: Improving tracing accuracy with spatial filtering

4.1.2. K-space filter

Several other approaches of improving the GT accuracy have been also proposed. For instance, Hu et al. (2018) utilized the Principal Component Analysis (PCA) to extract crucial velocity components in PPV cubes. Lazarian & Yuen (2018a) used the Moving Window method and Hu et al. (2020c) proposed the Rotational Image test to correct abnormal gradients. Here we study the contribution of different spatial frequencies to the GT and explore the possibility of improving the accuracy further.

The filtering of low spatial frequencies in order to improve the accuracy of the GT was done (Lazarian et al. 2017). There it was shown that this filtering can allow the correct representation of magnetic field in super-Alfvénic turbulence. More studies of the GT magnetic field tracing while removing low-spatial frequencies are presented in Lazarian & Yuen (2018a).

Better theoretical understanding of the localization of gradient information at high wave numbers in Lazarian et al. (2020a) provides the motivation for the current extensive study of K-filtering. We view this work as an integral part of the research aimed at improving the performance of the GT technique.

The K-space filter is implemented as follows: (i) apply the Fast Fourier Transform (FFT) to a 2D map; (ii) remove the intensity values corresponding to a wavenumber \( k \) (or a range of \( k \)) of interests; (iii) apply the inverse FFT to the processed map and then perform the gradients’ calculation.

4.1.3. Alignment measure

To quantify the relative alignment between the magnetic fields and rotated gradient angle, González-Casanova &
Figure 4. The correlation of AM and \( k_{\text{max}} \), \( k_{\text{max}} \) (the x-axis) indicates the spatial frequency from \( k = 0 \) to \( k = k_{\text{max}} \) is removed. The AM is calculated from IGs (the 1st column)/VCGs (the 2nd column)/VChGs (the 3rd column) and the magnetic field inferred from dust polarization. The dashed lines represents corresponding the \( k \) values of the selected sub-block sizes.
Lazarian (2017) introduced the Alignment Measure (AM):

$$\text{AM} = 2\left( \langle \cos^2 \theta_r \rangle - \frac{1}{2} \right)$$ \hspace{1cm} (10)

where $\theta_r$ is the angular difference of two vectors, while $\langle \ldots \rangle$ denotes the average within a region of interests. In the case of a perfect global alignment between rotated gradients and the POS magnetic field gradient, we get AM = 1, while AM = -1 indicates global gradients are perpendicular to the POS magnetic field. In this study, we define AM = 0.70 as a threshold of rough alignment ($\theta_r \approx 22.5^\circ$) and AM = 0.90 as a good alignment ($\theta_r \approx 12.5^\circ$). The standard error of the mean gives the uncertainty $\sigma_{AM}$ which is negligible for sufficient sampling.

5. RESULT

5.1. Analysis of IGs, VCGs, and VChGs

In this section, we analyze the effect of spatial frequency on the accuracy of IGs, VCGs, and VChGs. First of all, we test the case of removing low-spatial-frequency (from $k = 0$ to $k_{\text{max}}$) and removing high-spatial-frequency (from $k_{\text{min}}$ to $k = 792$).

Fig. 2 is an example illustrating the role of spatial frequency on 2D images. We make a comparison of the raw image, the high-spatial-frequency image ($k < 10$ is removed), and the low-spatial-frequency image ($k > 10$ is removed). We can see the high-spatial-frequency components correspond to small scale structures and low-spatial-frequency components correspond to large scale structures. We also calculate the alignment between the gradients and the magnetic fields weighted by density. For the raw images (full spatial frequency range), IGs, VCGs, and VChGs give AM = 0.70, 0.72, and 0.85, respectively. After the low-spatial-frequency components are removed, the AM of IGs and VCGs get increased to 0.83 and 0.85. When the high-spatial-frequency components are removed, however, the AM decreased to 0.19 (IGs), 0.19 (VCGs), and 0.15 (VChGs). It means the high-spatial-frequency components, i.e., small scale structures, dominate the alignment of gradients and the magnetic field.

5.1.1. Removing low-spatial frequency

In Fig. 3, we plot the AM value as a function of $k_{\text{max}}$, which means the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. We vary the sub-block size from 22 pixels to 55 pixels. In the case of sub-sonic and trans-sonic turbulence, the removal of low-spatial-frequencies significantly increases the AM values. In particular, for IGs and VCGs of sub-Alfvénic simulations ($M_A = 0.56$ and $M_A = 0.50$), the AM gets boosted to approximately 0.9 in the range of $10 < k_{\text{max}} < 100$ around. Also, the AM of IGs and VCGs get saturated when $k_{\text{max}}$ is larger than the corresponding $k$ values of the selected sub-block sizes, but the AM drops when $k_{\text{max}}$ is larger than 100 which is approximately the 3D dissipation scale of turbulence. It means the optimal sub-block should cover the minimum scale of turbulent fluctuations that can be resolved. In terms of the sup-Alfvénic and sub-sonic case ($M_S = 0.60, M_A = 1.02$), the initial AM values of IGs and VCGs are 0.2 and 0.3, respectively. After removing the low-spatial-frequencies, the AMs are improved to 0.75 around. This improvement comes from the fact that the turbulent eddies are less anisotropic in weak magnetic fields environment (Lazarian 2006).

As for sub-sonic/trans-sonic VChGs, the situation is slightly different. As shown Fig. 2, the structures in the raw thin velocity channel map is much more filamentary than the intensity map and the velocity centroid map, due to the velocity caustic effect (Lazarian & Pogosyan 2000). These filamentary structures are dominating by the velocity fluctuations and are better aligned with the magnetic fields. Therefore, raw VChGs (full spatial frequency range) always give a higher AM value than raw IGs and raw VCGs. The removal of low-spatial-frequencies still slightly improves the alignment (see Fig. 3). However, being different from IGs and VCGs, the curve of VChGs is more flat. Also similarly, the AM drops off considerably after AM gets its maximum value. The drop-off scale is around $k_{\text{max}} = 200$, which is larger than the dissipation scale in 3D. This can be the consequence of non-linear spectroscopic mapping from real position space to PPV space which creates more small-scale structures. In 2D maps, the correlation fading of turbulence at small separations is compensated by the correlations of along the LOS when the LOS of two pixels are closer than the dissipation scale. Therefore, we can see the alignment at a 2D scale smaller than the dissipation scale.

As for supersonic turbulence, the presence of shocks complicates the picture of turbulent density. For instance, Beresnyak et al. (2005) revealed that the density spectrum of supersonic turbulence becomes shallower because shocks accumulate the fluid into the local and highly dense structures. In terms of gradients, their direction flips by 90° in front of shocks so that the AM gets decreased (Hu et al. 2019c, 2020d).

In Fig. 4, here for the IGs of super-sonic simulations $M_S = 10.81$ and $M_S = 11.11$, we see the AM shows a depression range until $k_{\text{max}} \approx 100$ which is larger than the block-size scale. Different from the sub/trans-sonic cases, when $k_{\text{max}}$ is larger than the block-size scale, the AM gets negative or zero improvement. We expect this comes from high-density contrast associated with shocks produced by strong fluctuations in this highly supersonic simulation. The scale of high-density contrast is smaller than the block-size scale. As a result, the high-density contrasts keep contributing negative AM even when $k_{\text{max}}$ is larger than the block-size scale. For the IGs, the depression of AM is also related to the $M_A$. When $M_A$ increases to 0.64, the depression is more significant and gets no improvement at larger $k_{\text{max}}$. The depression of AM also appears in VCGs and VChGs, but it is less sig-

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4 The effect of thermal broadening is not considered here as we assume that we can use the heavy tracers, e.g., heavy molecules, in the flow. Otherwise the procedures of the thermal broadening removal are described in Yuen & Lazarian (2020).
Figure 5. **Left:** The plot of the magnetic field orientation probed by GT (red) and synthetic dust polarization (blue) with sub-block size 33 pixels. The plot is overlaid on the synchrotron intensity maps (left), synchrotron polarization maps (right). The gradients are calculated within full spatial frequency range (the 1st row), \(k > 10\) (the 2nd row), and \(k < 10\) (the 3rd row), respectively. **Right:** The correlation of AM and \(k_{\text{max}}\). \(k_{\text{max}}\) (the x-axis) indicates the spatial frequency from \(k = 0\) to \(k = k_{\text{max}}\) is removed. The AM is calculated from SIGs (the 1st column)/SPGs (the 2nd column) and the magnetic field inferred from dust polarization. The dashed lines represent corresponding the \(k\) values of the selected sub-block sizes.

Significant than IGs. Recall that in the number of cases, e.g. in particular for low Mach numbers, the turbulent velocity contribution is dominating over the density contribution in the velocity centroid map and the thin velocity channel map (see LP00, Esquivel & Lazarian 2005). This was recently demonstrated, for instance, in Hu et al. (2020d). The shocks therefore have limited effect on VCGs and VChGs. In any case, the alignment drops off at scale smaller than the dissipation scale. The explanation is similar to the sub-sonic case above.

In Appendix A, we study the effect of noise on the performance of GT. We introduce Gaussian noise to 2D maps \(I(x,y), C(x,y), \) and \(CH(x,y)\) processed by the low-spatial frequency filter. The noise’s amplitude in each map is 10% of their mean intensity. We find when the sub-block size is small \((\approx 22\) pixels) the noise degrades the maximum value of AM. However, the noise’s contribution can be canceled out by increasing the sub-block size. Also, we analyze the numerical effect coming from the finite resolution of the simulation. We use simulation A3 \((M_S = 0.62, M_A = 0.56)\) with resolution 792\(^3\), 512\(^3\), and 480\(^3\). We find in the cases of low resolution, the AM of gradients, and the magnetic field weighted by density is still improved by filtering out the low-spatial frequencies. The numerical effect has a significant effect on the dissipation scale so that the AM drops off at the smaller \(k_{\text{max}}\) when the resolution is low.

In Appendix B, we test also the effect of self-gravity which potentially can break up the turbulence’s picture. Our studies showed that in the case of gravitational collapse, as the acceleration induced by the collapse is along the magnetic field direction, density and velocity gradients are expected to change their orientation from perpendicular to magnetic fields to align with magnetic fields (Yuen & Lazarian 2017b; Hu et al. 2019a, 2020b). In this case, the rotated gradients become perpendicular to the magnetic field resulting in a negative AM value. The effect of self-gravity is scale-dependent, which means the gravitational collapse can happen at a small local region where the gravitational energy surpasses the ki-
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Figure 6. Red: plot of the maximum AM getting from removing the spatial frequencies as a function of $M_A$. The low spatial frequencies are removed from $k = 0$ to $k_{max}$. Blue: the correlation of the $k_{max}$ and $M_A$. The $k_{max}$ here corresponds to the AM value is 0.05 lower than the maximum AM. It indicates the AM starts getting saturated when the scale is larger than the $k_{max}$. The AM is calculated from the gradients and the magnetic field inferred from dust polarization using sub-block size 22 pixels.

5.2. Analysis of SIGs and SPGs

Fluctuations of synchrotron emission can also be used to study properties of the magnetic field (Lazarian & Pogosyan 2012). For instance, Lazarian et al. (2017) and Lazarian & Yuen (2018b) proposed to trace the magnetic field directions using synchrotron intensity gradients (SIGs) or synchrotron polarization gradients (SPGs). SIGs can directly measure the POS magnetic field direction getting rid of the correction of Faraday rotation.

In Fig. 5, we give an example of tracing magnetic field through the SIGs and SPGs. Here we did not consider the effect of Faraday rotation on synchrotron polarization, which will be studied elsewhere. We make a comparison of the raw image, the high-spatial-frequency image ($k < 10$ is removed), and the low-spatial-frequency image ($k > 10$ is removed). We make a comparison of the raw image, the high-spatial-frequency image ($k < 10$ is removed), and the low-spatial-frequency image ($k > 10$ is removed). The AM of three cases decreased to 0.30 around. It means the high-spatial-frequency components, i.e., small scale structures, dominate the alignment of gradients and the magnetic field. The SIGs, which utilize the intensity of synchrotron emission, therefore provide a reliable measurement of the POS magnetic fields. As the SIGs are not subject to Faraday rotation, it does not require multiple frequency measurements to compensate for Faraday effect (Lazarian et al. 2017). In addition, the effect of missing spatial frequencies in SIGs is earlier studied by Lazarian et al. (2017). There the focus was on the magnetic field tracing in high Alfvén Mach number media.

For low-frequency synchrotron radiation, the effect of Faraday depolarization allows to use SPGs at multiple frequencies and this way to provide the 3D magnetic field tracing (Lazarian & Yuen 2018b), (see also Ho et al. 2019). This
effect provides the wavelength-dependent LOS depth to the polarized synchrotron intensity. In this case, only the regions close to the observer contribute to the measured polarization, while unpolarized synchrotron radiation comes from more distant regions. The condition for decorrelation of Faraday rotation is given by (Lazarian & Pogosyan 2016):

\[ \lambda^2 \Phi = 0.81 \lambda^2 \int_0^{L_{\text{eff}}} n_e T B_z dz \approx 1, \]  

(11)

where \( n_e T \) and \( B_z \) are the thermal electron density and the LOS magnetic fields. \( \lambda \) represents the wavelength of measurements. The effective depth \( L_{\text{eff}} \) represents the distance to which the polarized radiation is collected effectively. The effect of removing the low-spatial frequencies in SPGs with the presence of Faraday depolarization will be studied elsewhere.

5.3. The dependence on \( M_A \)

When the magnetic field is weak, the motion of turbulence at the injection scale is more similar to the hydrodynamic type due to the relatively weak back-reaction of the magnetic field. The turbulent eddies are isotropic in large scales instead of anisotropic. Therefore, it is important to study the dependence of GT on \( M_A \).

We select a set of sub-sonic simulations (\( M_S \approx 0.60 \)) with various \( M_A \). The analysis of removing low-spatial frequencies from \( k = 0 \) to \( k = k_{\text{max}} \) is repeated for all types of gradients. In Fig. 6, we identify the maximum AM value of gradients and the magnetic field weighted by density, which can be obtained through the removal of low-spatial-frequencies. We see the maximum AM values is negatively proportional to \( M_A \). Nevertheless, the AM values are always larger than 0.60 when \( M_A \approx 1 \). It means globally the gradients are still aligned with the weak magnetic fields, although relatively the alignment is less than the cases of strong magnetic fields. Also, we determine the \( k_{\text{max}} \) value above which the AM is saturated as shown in Fig. 3. The saturation threshold of AM is defined as 0.05 lower than the maximum value. We find the saturation scale \( k_{\text{max}} \) is increasing with the increment of \( M_A \). In general, when the \( k_{\text{max}} \) is larger than 15, the gradients show the best alignment with the magnetic field. This means only high spatial frequency (\( k > 15 \)) matters when tracing the magnetic fields.

5.4. Application to GALFA-H I data

To test the GT in observation, we use the high spatial and spectral resolution neutral hydrogen data from Data Release 2 (DR2) of the Galactic Arecibo L-Band Feed Array H I survey (GALFA-H I) with the Arecibo 305 m radio antenna (Peek et al. 2018). GALFA-H I has a gridded angular resolution of \( 1' \times 1' \) per pixel, a spectral resolution of 0.18 km/s, and a brightness temperature noise of 40 mK RMS per 1 km/s.

For illustration, we select a H I cloud, which stretches from R.A. \( \approx -0.08^\circ \) to \( 8.26^\circ \) and Dec. \( \approx 6.09^\circ \) to \( 26.09^\circ \), as an example. This region spans from galactic latitude \( b \approx -30^\circ \) below the Galactic Plane to \( b \approx -60^\circ \), i.e., close to the Galactic south pole. We analyze the H I data within velocity range \( -23.0 \text{ km/s} < v < 14.0 \text{ km/s} \), which contains the majority of the H I emission. In Fig. 7, we give an example of tracing the magnetic field morphology using VChGs.

We make a comparison of VChGs and the Planck 353GHz polarized dust signal data from the Planck 3rd Public Data Release (DR3) 2018 of High-Frequency Instrument (Planck Collaboration et al. 2020b). The Planck observations provide Stokes parameter maps \( I, Q, \) and \( U \), so the plane-of-the-sky magnetic field orientation angle \( \theta \) can be derived from the Stokes parameters: \( \theta = \frac{1}{2} \tan^{-1}(U/Q) - \frac{1}{2} \). In this work, we smooth the \( Q, U \) maps through a Gaussian filter with the FWHM \( \approx 10 \) arcsec. The gradients are smoothed in a similar way by constructing the pseudo-Stokes parameters \( Q_g \) and \( U_g \):

\[ Q_g(x, y) = I(x, y) \cos(2\psi(x, y)), \]
\[ U_g(x, y) = I(x, y) \sin(2\psi(x, y)). \]  

(12)

The Gaussian filter is applied to both \( Q_g \) and \( U_g \). The resulting gradient vectors are calculated from \( \psi'(x, y) = \frac{1}{2} \tan^{-1}(U_g/Q_g) \).

As shown in Fig. 7, when we use the H I data within the full spatial frequency range, VChGs give good alignment with the Planck polarization is the low-intensity region (approximately \( I \leq 1200 \text{ K m/s} \)). As for the high-intensity region, there are more negative alignments, i.e., the rotated gradients are nearly perpendicular to the Planck polarization (see also Lu et al. 2020). One possible reason for the anti-alignment is the shock effect (Yuen & Lazarian 2017b; Hu et al. 2019c, 2020d). Globally, we have AM = 0.73 for the VChGs and the Planck polarization. Similar to the numerical analysis above, we then remove the low-spatial frequency keeping only \( k > 50 \) components in the velocity channel map. After removing low-spatial frequencies, we find the global AM between VChGs and the Planck polarization increases to 0.90. The majority of anti-alignments seen in high-intensity regions is suppressed. Also, in Fig. 8, we plot the correlation of AM and \( k_{\text{max}} \) which indicates the spatial frequency from \( k = 0 \) to \( k = k_{\text{max}} \) is removed. For IGs, VCGs, and VChGs, we see the AM keeps a flat curve around value 0.70 until \( k_{\text{max}} \approx 15 \). When \( k_{\text{max}} \geq 15 \), the AM starts increasing to it maximum value \( \approx 0.90 \) at \( k_{\text{max}} \approx 50 \). The AM gets drops significantly if \( k_{\text{max}} > 100 \). The results agree with our numerical studies above. To sum up, the most significant contribution to gradients comes from small-scale structures. The accuracy of GT can be improved by filtering out low-spatial frequencies.

6. APPLYING GT TO INTERFEROMETERIC DATA

The measurement of magnetic fields is a hot topic in modern astrophysics. Owing to advances in dust grain alignment theory and synchrotron radiation theory, a comprehensive picture of POS magnetic field orientations has become more accessible using either dust polarization from background starlight or linearly polarized thermal dust emission or synchrotron emission. A number of these observations
Figure 7. The plot of the magnetic field orientation probed by VChGs (red) and Planck polarization (blue) with resolution $\approx 0.5^\circ$. The plot is overlaid on the H I intensity map integrated from the velocity range of -23.0 km/s $< v <$ 14.0 km/s. Top: the VChGs are calculated from the GALFA-H I data within full spatial frequency $k$ range. Bottom: the VChGs are calculated from the GALFA-H I data with $k > 50$.

have been achieved by the ground telescopes/arrays (Barvainis et al. 1988), the balloon-borne telescopes (BLAST-Pol: Galitzki et al. 2014), the airborne-platforms telescopes (SOFIA: Santos et al. 2019), and the space-satellite telescope (Planck: Planck Collaboration et al. 2020a), spanning from millimeter, submillimeter to far-infrared (FIR) wavebands.

However, the measured spatial frequencies from interferometers are constrained by the baseline. In many cases, the single-dish observations necessary for obtaining low-spatial frequencies are missing. For instance, in Gaensler et al. (2011), the radio observations observed with the Australia Telescope Compact Array (ATCA) at a frequency of 1.4 GHz did not include single-dish measurements. Jelić et al. (2015) observed the 3C 196 field with the Westerbork Synthesis Radio Telescope (WSRT) at 350 MHz. However, for the WSRT 350 MHz observations, a single-dish measurement at the same frequencies is not possible. Also, the radio data from LOw-Frequency ARray (LOFAR) suffers the loss of low-spatial-frequencies still (Jelić et al. 2014).

The missing of low-spatial-frequencies in interferometers, however, benefits the Gradients Technique (GT) in terms of probing the magnetic fields. The GT is established on a basis of solid theory, i.e., anisotropic turbulence theory GS95 and turbulent LV99 reconnection. The theoretical prediction based on this theory is that the magnetic field tracing is the best when only the spatial frequencies starting with
The price of using GT to obtain magnetic field tracing is the accuracy of intensity gradients (Yuen & Lazarian 2017b; Hu et al. 2019c). As the alignment of turbulent eddies takes place with respect to the local magnetic field, rather than the mean magnetic field, the gradients associated with the smallest resolved turbulent eddies reveal the detailed magnetic field structure. On top of this, the theoretical description of the GT in Lazarian et al. (2020a) reveals that the alignment of gradients with the local magnetic field brings about another remarkable property of gradients, i.e. the gradients obtained using only high spatial frequencies, represent correctly the underlying total magnetic field.

The accepted procedure for calculating gradients involves averaging over blocks of data $L_{\text{block}} \times L_{\text{block}}$ (Yuen & Lazarian 2017a). The theoretical consideration in Lazarian et al. (2020a) means that only high spatial frequencies starting with $1/L_{\text{block}}$ should be taken into account. The smaller spatial frequencies are not useful and can, in fact, distort the procedure of magnetic field tracing. The latter fact was reported in Lazarian et al. (2017) where it was shown that to correctly recover magnetic field structure from using SIGs obtained with the high Alfvén Mach number turbulent data, it is advantageous to remove the low-spatial-frequencies. Our present study demonstrates that removing low-spatial-frequencies is, in fact, advantageous for different Alfvén Mach numbers.

The present work is important for two major reasons. First of all, it introduces an additional procedure improving the quality of magnetic field tracing using the GT. Moreover, it confirms that GT allows interferometers to successfully restore the magnetic field maps in the absence of sing-dish observation. This is a unique property of GT which is not present for obtaining magnetic field maps using polarimetric observations with interferometers. In the latter case, all spatial frequencies are required for restoring the magnetic field map.

The price of using GT to obtain magnetic field tracing is that the magnetic field maps are coarse-grained compared to the original intensity maps. The use of the interferometers can significantly increase the resolution of the initial data. The fact that only high-spatial-frequencies are required simplifies the GT interferometric studies significantly.

7.2. Recovering 3D magnetic field structure

A full 3D magnetic field model is crucial in many astrophysical problems, for instance, the removal of the galaxy

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{The correlation of AM and $k_{\text{max}}$. $k_{\text{max}}$ (the x-axis) indicates the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. The AM is calculated from IGs (red)/VCGs (blue)/VChGs (green) and the magnetic field inferred from Planck polarization. The gradients are calculated from the GALFA-H I data.}
\end{figure}
Figure 9. The correlation of AM and $k_{\text{max}}$. $k_{\text{max}}$ (the x-axis) indicates the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. The AM is calculated from IGs (the 1st column)/VCGs (the 2nd column)/VChGs (the 3rd column) and the magnetic field inferred from dust polarization. The dashed lines represent the corresponding $k$ values of the selected sub-block sizes. We used sub-sonic simulations $M_S = 0.60$ and $M_A = 0.78$ with the absence of noise (the 1st row) and the presence of noise (the 2nd row).

foreground, star formation, cosmic ray transportation. Such a 3D magnetic field model can be constructed by the GT. For instance, the 3D galactic magnetic fields can be achieved through the galactic rotation curve and the measured atomic hydrogen distribution. González-Casanova & Lazarian (2019) produce the first 3D map of the galactic disk plane-of-the-sky magnetic field distribution using the velocity gradients. The authors successfully tested the resulting 3D model by comparing it with the measured stellar polarization. Also, Hu et al. (2019b) demonstrated the possibility of studying the 3D magnetic field POS distribution using multiple molecular tracers and their corresponding gradients (see also Hu & Lazarian 2021). For example, by stacking the gradient maps calculated from $^{12}$CO, $^{13}$CO, and $^{18}$O data, one could get the magnetic fields over three different optical depths providing spectroscopic information on gas dynamics up to a certain LOS depth. The Gradient Technique, therefore, give new insight into the readily available large-scale molecular line surveys, such as CHAMP (Stolle et al. 2006) and ThRuMMs (Nguyen et al. 2015), or the neutral hydrogen atom distribution survey (HI4PI Collaboration et al. 2016), and the COMPLETE survey (Ridge et al. 2006).

Other important measurements are polarized synchrotron radiation and Faraday rotation (Fletcher et al. 2011; Beck & Wielebinski 2013; Oppermann et al. 2015; Lenc et al. 2016; Van Eck et al. 2017), which can reveal the magnetic fields of the Milky Way and neighboring galaxies. In addition, Lazarian & Yuen (2018b) proposed to trace the 3D magnetic fields using SPGs. Due to the effect of Faraday depolarization, the measured synchrotron polarization can only come from the regions close to the observer. By varying the measurement wavelength, one can obtain the LOS depth of the synchrotron polarization (see Eq. 11). Similar to the multiple-tracers tomography discussed above, by stacking the SPGs maps corresponding to different LOS depth, a 3D magnetic field model can still be established (Lazarian & Yuen 2018b; Ho et al. 2019). Also, SPGs show the possibility to trace the 3D magnetic fields in nearby galaxies. The Milky Way foreground distorts the direction of synchrotron polarization through the Faraday rotation. However, it does not contribute to the value of total polarization arising from an external galaxy. As a consequence, SPGs are not affected by the Faraday rotation foreground. The new instrument LOFAR (Jelić et al. 2014) provides an abundant low-frequency synchrotron data set, for which Faraday depolarization may be important. Also, the Square Kilometer Array (SKA) will provide detailed maps of diffuse emission with unprecedented resolution (Johnston-Hollitt et al. 2015). These new measurements of synchrotron emission definitely pave the way to study the magnetic fields using the GT.

7.3. Comparison with other studies

As an output of modern MHD turbulence theories, the GT has been developed as a new method to trace the magnetic fields. The first attempt to trace the magnetic fields using velocity centroid gradients (VCGs) was carried out by González-Casanova & Lazarian (2017). The subsequent studies enable the usage of SIGs (Lazarian et al. 2017), IGs
(Yuen & Lazarian 2017b; Hu et al. 2019c), SPGs (Lazarian & Yuen 2018b), and VChGs (Lazarian & Yuen 2018a). This multiple-gradients ecosystem paves the way to study the magnetic fields orientation in transparent atomic gas (Yuen & Lazarian 2017a; González-Casanova & Lazarian 2019; Hu et al. 2020a, 2021b; Lu et al. 2020), dense molecular gas (Hu et al. 2019a,b; Alina et al. 2020), circumstellar medium (González-Casanova & Lazarian 2019), and intracluster medium (Hu et al. 2020c). In addition to tracing the magnetic field direction, the GT also can be used to estimate the magnetization level (Lazarian et al. 2018a), measure the sonic Mach number (Yuen & Lazarian 2020), distinguish shocks (Yuen & Lazarian 2017b; Hu et al. 2019c), and identify gravitational collapsing regions (Hu et al. 2020b). In this work, we show the accuracy of the GT can be improved by removing low-spatial-frequencies, which is advantageous in using the interferometric data.

Based on the alignment of H I filaments with the magnetic fields, Clark et al. (2014) proposed the Rolling Hough Transform (RHT) to study the magnetic field in the diffuse interstellar region. However, the RHT does not distinguish the perpendicular structures and the parallel structures with respect to the magnetic field. It is therefore incapable of revealing the magnetic field direction in shock regions (Majinen et al. 2016). Furthermore, the output from RHT depends on three input parameters: smoothing kernel diameter, window diameter, and intensity threshold (Clark et al. 2014; Alina et al. 2020). The adjustment of these inputs is empirically optimal when the reference polarization measurement is available. On the contrary, the GT does not require any adjustable parameters and can handle the shock effect (Hu et al. 2019c).

In particular, the VChGs is theoretically founded on the effect of velocity caustics (Lazarian & Yuen 2018a). This effect produces fluctuations from the field of velocity within thin channel maps, When the channel is sufficiently thin (see Eq. 7), the turbulent velocity fluctuations dominate over intensity fluctuations arising from density clumping Lazarian & Pogosyan (2000). In this case, the filamentary structures in thin velocity channels follow the statistics of velocity rather than intensity. Clark et al. (2019), however, relate the H I filamentary structures to the two-phase nature of H I, while the role of non-linear spectroscopic mapping (i.e., the velocity caustics effect) was totally ignored. This interpretation limits the alignment of filaments in the thin channel maps with the magnetic fields to only H I. However, the alignment has been reported by several studies in single-phase molecular clouds (Hu et al. 2019a,b; Heyer et al. 2020). The corresponding reply is presented in Yuen et al. (2019). The discussion of the H I filaments is out the scope of this work and we will answer it elsewhere.

In terms of practical applications, both the RHT and the GT approaches were used for tracing magnetic fields and predicting CMB foreground polarization arising from dust. For the latter task, the comparison of the performances of the two techniques was performed in Lu et al. (2020) and Hu & Lazarian (2020a). There it was shown that the GT can reproduce Planck polarization slightly better than RHT, while not using any adjustable parameters. Our present study shows that the performance of the GT can be significantly improved, consolidating the lead of the GT approach. We note that the GT is a technique founded on theory and this provides the basis for further improvements.

8. CONCLUSION

In this work, we studied numerically the effect of removing low spatial frequencies on the accuracy of the GT. Our results can be briefly summarized in the following way:

1. For the sub-sonic environment, we show that only high spatial frequencies are crucial for tracing the magnetic field through the GT.

2. We show numerically that the accuracy of the GT in tracing the magnetic field can be improved by removing low-spatial frequencies. In particular, for supersonic cases, the low-spatial frequency larger than the scale of high-density contrasts associated with shocks should be removed.

3. Our application of the low K spatial filtering to a diffuse H I from GALFA-H I survey reveals a significant improvement in correspondence of GT predictions with the Planck 353 GHz polarization.

4. We demonstrate numerically the advantages of using the GT with the interferometric data to study the magnetic fields galactic and extragalactic magnetic field with high resolution.

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Software: Julia (Bezanson et al. 2012), ZEUS-MP/3D (Hayes et al. 2006)
The Gradients Technique: Improving Tracing Accuracy with Spatial Filtering

Figure 10. Panel a: The correlation of AM and $k_{max}$. $k_{max}$ (the x-axis) indicates the spatial frequency from $k = 0$ to $k = k_{max}$ is removed. The AM is calculated from IGs (left)/VCGs (middle)/VChGs (right) and the magnetic field inferred from dust polarization. Panel b: Left: plot of the AM as a function of Gaussian kernel width. The analysis is doing for SIGs and SPGs in the full spatial range using simulation $M_S = 0.60$ and $M_A = 0.56$ in the absence of noise. Middle: the correlation between the Gaussian kernel width which outputs maximum AM value and the noise level with respect to mean intensity value. Right: the correlation of AM and $k_{max}$ which means the spatial frequencies from $k = 0$ to $k_{max}$ is removed.

APPENDIX

A. THE EFFECT OF NOISE AND RESOLUTION

In Fig. 9, we examine the performance of GT in the presence of noise. We produce 2D maps $I(x,y)$, $C(x,y)$, and $Ch(x,y)$ from sub-sonic simulations $M_S = 0.60$ and $M_A = 0.78$ and process them with the low-spatial frequency filter. Then we introduce Gaussian noise to the processed 2D maps. The noise’s amplitude in each map is 10% of their mean intensity. We find when the sub-block size is small ($\approx 22$ pixels) the noise degrades the maximum value of AM. However, the noise’s contribution can be canceled out by increasing the sub-block size. Additionally, the VChGs is less sensitive to noise than IGs and VCGs.

As the dissipation of turbulence comes the numerical effect here, a simulation with high resolution usually guarantees a large dissipation scale $k_{diss}$. The scale $k_{diss}$ is crucial for the implementation of GT. Here then we analyze the effect coming from the finite resolution in numerical simulation. We vary the resolution of simulation A3 ($M_S = 0.60$ and $M_A = 0.56$) from $792^3$, $512^3$, and $480^3$ and repeat the removing the low spatial frequency from $k = 0$ to $k_{max}$ for IGs, VCGs, and VChGS. The results are presented in Fig. 10. We see that the removal of low spatial frequencies can also improve the alignment between gradients and the magnetic field weighted by density. The crucial numerical effect appears when $k_{max}$ is larger than the drop-off scale above which the AM starts decreasing. As discussed in § 5, the scale is approximately the 2D dissipation scale of turbulence. Also, the maximum value of AM is slightly lower when the resolution is poor.

The effect of missing spatial frequencies in SIGs was first studied by Lazarian et al. (2017). There a gradual decrease in the AM when low-spatial frequencies are removed, as a consequence of the Gaussian filtering applied to the numerical data. Indeed, the operation of a Gaussian filter with kernel width 4 equivalent to convoluted the images with a $(4^2 + 1) \times (4^2 + 1)$ matrix.
Figure 11. The correlation of AM and $k_{max}$. $k_{max}$ (the x-axis) indicates the spatial frequency from $k = 0$ to $k = k_{max}$ is removed. The AM is calculated from IGs (the 1st column)/VCGs (the 2nd column) / VChGs (the 3rd column) and the magnetic field inferred from dust polarization. The dashed lines represent the corresponding $k$ values of the selected sub-block sizes. We used supersonic simulations $M_S = 5.96$ and $M_A = 0.31$ with the presence of self-gravity here. $t$ represents the evolution time of the simulation after the self-gravity module is switched on.

The structures occupying less than 17 pixels are smoothed as a result. In Fig. 10, we analysis the correlation of SIGs’ AM and the Gaussian kernel width using simulation $M_S = 0.60$ and $M_A = 0.56$, in the absence of noise. The AM here is calculated in terms of the magnetic field weighted by density. We find the AM is negatively proportional to the kernel width. It agrees with our consideration that only small scale structures are crucial for the alignment but the Gaussian filter removes these structures.

Additionally, we introduce noise to the synchrotron intensity map. Noise level 10% means the Gaussian noise’s magnitude is 10% of the mean intensity value. We repeat the analysis of AM and the kernel width for different noise levels and find out the
optimal kernel width outputting maximum AM. The result is shown in Fig. 10. We find that when the noise is about $10\% - 30\%$, Gaussian kernel width 3 is a good choice to improve accuracy. Once the noise is more intensive than $30\%$, a larger kernel width is required. Also, we apply a Gaussian filter with kernel width 4 to the synchrotron intensity map and then plot the correlation between AM and $k_{\text{max}}$. $k_{\text{max}}$ means the spatial frequencies from $k = 0$ to $k = k_{\text{max}}$ is removed. Theoretically structures smaller than 17 pixels will be removed by the filter, which corresponds to $k \approx 30$ in our simulations. In Fig. 10, we observe that AM starts decreasing from $k_{\text{max}} \approx 45$, which agrees with the results shown in Lazarian et al. (2017).

**B. THE EFFECT OF SELF-GRAVITY**

Initially, we consider non-self-gravitating conditions for all simulations. After the simulation is running for two sound crossing times, we switch on the self-gravity for simulation A7. We employ a periodic Fast Fourier Transform Poisson solver for the self-gravitating module and keep driving both turbulence and self-gravity until the evolution time $t \approx 0.43 t_{\text{ff}}$, where $t_{\text{ff}} \approx 1.88$ Myr is the free-fall time. The total mass $M_{\text{tot}}$ in the simulated cubes A7 is $M_{\text{tot}} \approx 8430.785 M_\odot$, the magnetic Jean mass is $M_{\text{JB}} \approx 86.95 M_\odot$, average magnetic field strength is $B \approx 30.68 \mu G$, mass-to-flux ratio is $\Phi \approx 1.11$, and volume density is $\rho \approx 315 \text{cm}^{-3}$ (Hu et al. 2020b). We repeat the analysis of removing low spatial frequency for the simulation at $t = 0.2$ Myr, 0.4 Myr, 0.6 Myr, and 0.8 Myr. We analysis only IGs, VCGs, and VChGs here. Once self-gravity takes place and the volume density increases, one would expect that most of the gas becomes molecular. In this case, the synchrotron emission would not be a good tracer for the GT.
Figure 13. The correlation of AM and $k_{\text{max}}$. $k_{\text{max}}$ (the x-axis) indicates the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. The AM is calculated from IGs (left)/VCGs (middle)/ VChGs (right) and the magnetic field weighted by density. The legend "two injections" indicates two numerical cubes are superimposed along the LOS, while "one injection" means only one numerical cube is used.

Similar to §. 5, in Fig. 11, we plot the AM value as a function of $k_{\text{max}}$, which means the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. We vary the sub-block size from 22 pixels to 55 pixels. At the beginning stage $t = 0.2$ Myr of self-gravitating, the AM values of IGs, VCGs, VChGs perform similarly to the non-self-gravitating case (see Fig. 4) as the turbulence is still dominating there. At $t = 0.4$ Myr, the AM of IGs starts decreasing in the range of $10 \leq k_{\text{max}} \leq 100$. VChGs also has similar behaviors. At later stages, $t = 0.6$ Myr and 0.8 Myr of self-gravitating, the decreasing trend of AM is more dramatic concerning $k_{\text{max}}$. The AM of VCGs also starts decreasing when $10 \leq k_{\text{max}}$. This anti-correlation of AM and $k_{\text{max}}$ comes from the self-gravity effect. Hu et al. (2020b) showed that in the case of gravitational collapse, as the acceleration induced by the collapse is along the magnetic field direction, density and velocity gradients are expected to change their orientation from perpendicular to magnetic fields to align with magnetic fields. In this case, the rotated gradients become perpendicular to the magnetic field resulting in a negative AM value. The effect of self-gravity is scale-depended, which means the gravitational collapse can happen at a small local region where the gravitational energy surpasses the kinetic and magnetic field energy. Therefore, high-spatial frequency components give a lower or even negative AM value.

C. INCLINATION ANGLE BETWEEN THE LOS AND THE MAGNETIC FIELDS

In the simulations, the mean magnetic field is initially perpendicular to the LOS. However, when the magnetic fields are inclined with respect to the LOS, we have to consider the projection effect for the projected magnetic field structures, which can get more irregular in the POS. Here we examine the effect introduced by the inclination angle to the k-space filter (see § 4). We rotate the simulation cube so that the mean magnetic field has an inclination angle $\gamma = \pi/3, \pi/4, \text{and } \pi/6$ with respect to the LOS.

In Fig. 12, we remove the spatial frequencies from $k = 0$ to $k = k_{\text{max}}$ for the simulation $M_S = 1.27$ and $M_A = 0.50$. For IGs and VCGs, the AM values are still improved until $k_{\text{max}}$ achieves approximately the 3D dissipation scale $\kappa \approx 80$. When $k_{\text{max}} > 80$, the AM significantly drops as the scale smaller than dissipation scale contributes little to the alignment. The AM of VChGs is also improved when $k_{\text{max}} \leq 200$ and then starts significantly decreasing. As explained in § 5, a larger drop-off scale of VChGs comes from the fact that the thin velocity channel has a flatten spectrum and a larger 2D dissipation scale. Nevertheless, we can see the alignment can be improved even the inclination angle is less than $\pi/2$.

D. MULTIPLE INJECTIONS ALONG THE LOS

Here we examine the performance of the gradients when there are multiple injections along the LOS. Initially, the simulations are set so that there is only one injection scale on $k = 2$. We then take two snapshots of simulation cube in the same physical conditions ($M_S = 0.62, M_A = 0.56$, and resolution $= 512^3$) but at different evolution time. The two snapshots are superimposed along the LOS keeping the mean magnetic field still parallel to the POS.

We repeat the analysis of $k_{\text{max}}$ as above, i.e., calculating the AM of IGs, VCGs, and VChGs after the spatial frequency from $k = 0$ to $k = k_{\text{max}}$ is removed. We make a comparison of single turbulence injection and two turbulence injections along the LOS. The results are presented in Fig. 13. We can see the resulting AM exhibits a similar trend for the two cases as expected. Since the injection scales of the two snapshots are both at $k = 2$, the resulting gradients are expected to be added up in a random walk manner. As for the case of multiple injection scales along the LOS will be studied elsewhere.

E. REMOVING HIGH-SPATIAL FREQUENCY

In this section, instead of removing low-spatial frequency, we test the effect of removing high-spatial frequency. In Fig. 14, we plot the AM value as a function of $k_{\text{min}}$, which means the spatial frequency from $k = k_{\text{min}}$ to $k = 792$ which corresponds to 1 pixel is removed.
Figure 14. The correlation of AM and $k_{\text{min}}$. $k_{\text{min}}$ (the x-axis) indicates the spatial frequency from $k = k_{\text{min}}$ to $k = 792$ is removed. The AM is calculated from IGs (the 1st column)/VCGs (the 2nd column)/VChGs (the 3rd column) and the magnetic field inferred from dust polarization. The dashed lines represents corresponding the dissipation scale $k_{\text{diss}} = 80$. 

$M_i = 0.62$  
$M_i = 0.60$  
$M_i = 0.30$  
$M_i = 1.27$  
$M_i = 5.96$  
$M_i = 11.11$
Regarding of both sub-sonic and supersonic turbulence, when $k_{min}$ is less then around 20, the AM values of IGs, VCGs, and VChGs are smaller than 0.2 with some variations. It means the low spatial frequency components, i.e., large scale structures, has less contribution to the alignment of gradients and the magnetic fields. In the range of $20 \leq k_{min} \leq 80$, the AM values are monotonically increasing until the dissipation scale. When $100 \leq k_{min}$ which means the scales smaller then the dissipation scale ($\approx 80$) are removed, the AM values get saturated.

To sum up, spatial frequency in the range of $0 \leq k \leq 15$ has less contribution to the alignment of gradients and magnetic fields. The spatial frequency in the range of $20 \leq k \leq k_{diss}$, where $k_{diss}$ is the dissipation scale, is most crucial for tracing the magnetic field through gradients.
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