Numerical analysis of unsteady cavitating flow by using a modification based on an assumption of apparent phase equilibrium

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Abstract. The prediction accuracy of cavitation by CFD is still not so high even in a simplest flow field around a single hydrofoil especially in transient condition at higher angle of attack, which is common problem in both commercial software and in-house solvers. In the transient condition, unsteady cavitation occurs, in which sheet cavity breaks off and cloud cavity sheds downstream periodically. At that time, the sheet cavity length tends to be underestimated in usual CFD. In the present study, modification for the phase change model is suggested, which is based on an idea of apparent phase equilibrium on gas-liquid interface with unsteady and disturbed flow. At first, a preliminary experiment has been done for evaporation on two gas-liquid interfaces with and without flow, the result contributes the evidence of the idea of apparent phase equilibrium with flow. In the result, the pressure around gas-liquid interface with flow was higher than that without flow on the occasion of evaporation, it means flow accelerates evaporation. I treat the gap of the pressure as a gap of phase equilibrium pressure macroscopically. Then, numerical simulation of cavitating flow around a hydrofoil is performed with a modification of phase change model in the transient condition at higher angle of attack which is most difficult to predict by the present solvers. In the modification, the gap of the pressure with and without flow is taken into account according to a value of a local variation of velocity in the cavitating flow filed. The formulation is similar to the PDF model for phase change model in cavitation by Singhal. The numerical results by the present modification are compared among few pressure variation components which are assumed to accelerate the evaporation in transient cavitation.

1. Introduction
For the last dozen years or so, several cavitation models for numerical simulation have been developed and some commercial software has been put on the market which is implementing the cavitation model. But the prediction accuracy is regrettably still not so high. Especially in a simplest calculation of transient cavitation around a single hydrofoil at higher angle of attack, it was reported that even time averaged lift and drag can’t be predicted by the commercial software and several in-house code [1]. At that time, sheet cavity length tends to be underestimated in common. So, the advancement of prediction accuracy of transient cavitation at higher angle of attack is a remaining problem in a research field of numerical simulation.
In almost of cavitation models for numerical simulation evaporation and condensation are estimated by that local static pressure takes lower or higher value than the local saturated vapour pressure. On the other hand, it is well known that cavity does not occur from lowest pressure point \cite{2}, and that inception cavitation number changes according to change of Reynolds number or air content \cite{3}. So, it may be considered that the evaporation and condensation in unsteady and disturbed cavitating flow field does not be estimated by only saturated vapour pressure which is static and equilibrium thermal property. Additionally, it was reported that pressure fluctuation inside the cavity bubble is several times higher than that in free stream \cite{3}. Then, especially in numerical method of homogeneous approach, in which cavity bubble does not be considered, it may be difficult to estimate phase change rate by that the local pressure of gas-liquid mixture takes higher or lower value than saturated vapour pressure.

In the present study, at first, preliminary simple experiment is performed about evaporation on water-vapour interface with flow. It is shown that the pressure in the vapour in contact with gas-liquid interface with flow is higher than that in vapour without flow. It is the evidence of apparent phase equilibrium pressure in phase non-equilibrium condition in cavitating flow. Next, numerical simulations of unsteady cavitating flows around single hydrofoils are performed by using the present modification for phase change in homogeneous medium which is based on the assumption of apparent phase equilibrium. The flowfield is around single NACA0015 hydrofoil which is well known for benchmark simulation of cavitating flow, in which the most difficult condition for prediction is selected. Through the comparison between some kinds of component of the velocity variation in the apparent phase equilibrium pressure, effectively of the present apparent phase equilibrium model and directionality of modification of cavitation model are considered.

2. Preliminary Experiment

2.1 Experimental setup

At first, preliminary simple experiment is done in order to show that vapor pressure is different or not between with and without flow on a vapor–liquid interface. The experimental set up is shown in Fig. 1, in which two glass flasks are connected by manometer. The experimental procedure is that; 480 ml purified water is putted in each flask, remaining air is expelled by vacuum pump to that evaporation starts, the two flasks are separated, vapor liquid interface in one side flask is rotated by magnetic stirrer, then, the pressure difference of vapor between two flasks is measured. The flasks are put in thermostat bath and keep the temperature 20°C in the present experiment. Initial oxygen solubility is about 90 % in the temperature. Magnetic sticks are put in both flasks. Because the whole glass setup has to be completely sealed from atmosphere for long time by using vacuum pump, digital pressure sensor cannot be used. Then, only the difference of vapor pressure is measure in the present experiment, so it means vapor pressure with flow is be measured directory.

2.2 Experimental result

The experimental result is shown in Fig. 2, which is time evolution of vapour pressure difference between with and without flow in the condition of 200 rpm and 300 rpm of stirrer. From the figure, it is shown that vapour pressure with flow is higher than that without flow. And the maximum difference increases according to increase flow speed. The value is not pressure difference from saturated vapour pressure without flow on interface but from vapour pressure on a way of evaporation on interface without flow. Therefore, it is shown that evaporation speed is higher with flow than that without flow. Also, it is although implication, because the pressure difference remains somewhat long time, vapour pressure at the end of evaporation which is phase equilibrium pressure may be higher with flow than that without flow.

In the conventional phase change models in numerical simulation of cavitation, evaporation starts when local static pressure decreases under saturated vapour pressure, the phase change shifts to
condensation when that increases upper saturated vapour pressure, and phase equilibrium is at saturated vapour pressure. The phase change models were derived from static flat gas-liquid interface or spherically bubble dynamics. There are various shape in term of the phase change rate, but these are proportional to \((p_{\text{sat}} - p)\) or \((p_{\text{sat}} - p)^{1/2}\), where \(p_{\text{sat}}\) is saturated vapour pressure and \(p\) is static pressure. Although, as shown in Fig. 2, phase equilibrium pressure in phase change may be higher with flow than saturated vapour pressure. In the present study, the difference of vapour pressure is assumed as apparent phase equilibrium pressure with flow, and apparent phase equilibrium model for numerical simulation based on homogeneous model is proposed.

![Figure 1](image1.png)

**Figure 1.** Snapshot of experimental setup of evaporation on gas-liquid interface with flow

![Figure 2](image2.png)

**Figure 2.** Pressure difference between vapour with and without flow

3. **Apparent Phase Equilibrium Model**

In the present study, the apparent phase equilibrium pressure with flow which is shown in the preliminary experiment is modeled for modification of phase change model in numerical method of cavitation based on homogeneous model. In the numerical method, saturated vapor pressure \(p_{\text{sat}}\) is replaced by the apparent phase equilibrium pressure \(p_{\text{v,app}}\), which is in the term of phase change rate in source term in mass conservation equation of gas phase. The value of \(p_{\text{v,app}}\) takes distribution according to local condition in cavitating flow field, then start of evaporation and phase equilibrium are shifted.
to higher pressure condition than saturated vapor pressure according to local flow condition. The $p_{v}^{*}$ are expressed by following equation.

\[ p_{v}^{*} = p_{sat} + f(\alpha)(p_{sat} + p_{turb}^*) \quad (1) \]

\[ f(\alpha) = \frac{\tanh((0.5 - \alpha)2n)}{2 \tanh(n)} + 0.5 \quad (2) \]

Here, $p_{turb}^*$ is pressure variation term which changes according to local unsteady flow condition and $f(\alpha)$ is a function of void fraction $\alpha$. The function $f(\alpha)$ takes a value from 1 at $\alpha = 0$ (liquid) to 0 at $\alpha = 1$ (vapor), and according to the constant $n$ in the function, the value of $f(\alpha)$ changes as shown in Fig. 3. The function expresses that threshold pressure of evaporation increases in early stage of evaporation which is shown in the preliminary experiment and the vapor pressure becomes saturated vapor pressure inside fully developed cavity which is well known characteristics. The concept is shown in Table 1. In addition, in Singhal’s PDF model \[4\] which is similar modification of phase change pressure, the phase equilibrium pressure takes higher value regardless as void fraction, then it was shown in the previous study \[5\] that vapor pressure inside the cavity takes higher value although cavity volume increase and it seems to be improved.

![Figure 3. The variation of function $f(\alpha)$](image)

| $\alpha$ | Evaporation | Phase-equilibrium pressure |
|----------|--------------|----------------------------|
| 0        | Starting     | $p_{sat} + p_{turb}^*$    |
| 1        | Ending       | $p_{sat}$                  |

Table 1. Concept of apparent phase equilibrium in unsteady cavitating flow field

In the present study, numerical experiment is performed, in which some kinds of $p_{turb}^*$ are compared as pressure variation component according to local unsteady flow, because unsteady cavitation accompanies locally complex flow variation and disturbance related vorticity or density variation. The following four kinds of variation components are compared.
Model 1: \[ p_{\text{turb}} = C\rho \bar{u}_i \]  
Model 2: \[ p_{\text{turb}} = C\rho \bar{u}_j \]  
Model 3: \[ p_{\text{turb}} = -CL(U/p\bar{u}_i \bar{u}_j) \frac{\partial \bar{u}_i}{\partial x_j} \]  
Model 4: \[ p_{\text{turb}} = C\rho \left( \frac{1}{\rho^2} \nabla (\rho \nabla p) \right)^2 \]  

Above four models base on turbulent energy, Reynolds stress, source term of turbulent energy transport equation, and source term of vorticity transport equation (Boroclinic torque), respectively. Model 1 corresponds to Singhal’s model \([4]\). \( L \) is representative length which is used to adopt the dimension and \( C \) is a control parameter for each value.

4. Numerical Method

4.1 Governing equations

In the present study, we use an in-housed numerical method based on a compressible gas–liquid two-phase locally homogeneous medium model \([6]\). The present method is classified as a method of the equation-of-state type in the two-phase one-fluid method. The advantage of our numerical method is that, by taking into account compressibility in liquid, our model can capture pressure waves in liquid when cloud cavitation collapses \([7]\). In the locally homogeneous medium, the mixture phase of gas and liquid is treated like a pseudo-single phase, then local equilibrium in temperature and pressure is assumed between the gas and liquid phases. Then, the equation of state of the homogeneous medium is derived as follows by using the mass fraction of the gas phase \( Y \):

\[ \rho = \frac{\rho(p + p_c)}{K_1(1-Y)p(T + T_c) + R_g Y (p + p_c) \rho Y} \]  

The above equation of state can reproduce a pure liquid phase \((Y = 0)\), a gas–liquid mixture phase \((0 < Y < 1)\), and a pure gas phase \((Y = 1)\), all using a single equation. Here, \( K_1 \) is the liquid constant, \( R_g \) is the gas constant, and \( p_c \) and \( T_c \) are the pressure and temperature constants of the liquid. The governing equation of the present numerical method is expressed by the following simple form, which takes into account compressibility in both the liquid and gas phases.

\[ \frac{\partial Q}{\partial t} + \frac{\partial (E_j - E_Y)}{\partial x} = S \]

where \( Q \) is the phase change term, \( E_j \) is the energy equation, and \( E_Y \) is the energy equation due to the temperature change of the homogeneous medium, \( S \) is the source term, \( \Gamma \) is the phase-change term, and \( \tau_q \) is the instantaneous equilibrium evaporation model [7]. The working fluid in the present numerical study is room-temperature water, for which the temperature distribution is known to be almost uniform. Then, the energy conservation equation can be omitted from the governing equations given in Eq. (8), and the computational cost is reduced. The equation of state in Eq. (7) provides a closed-form solution that satisfies the governing equation in Eq. (8).

4.2 Numerical Scheme

In this study, the governing equations given in Eq. (8) are solved using the finite difference method. Because it is necessary to stably simulate the discontinuities of a large density jump at the gas–liquid
interface in a cavitating flow field, the TVD scheme, which ensures the monotonicity of the solution, is used. Specifically, the explicit TVD–MacCormack scheme with second-order accuracy in time and space is used for time integration and evaluation of the convection and viscous terms. The present numerical method was validated through comparisons with experimental data about time-averaged lift and drag in non-cavitation and cavitating conditions around a single hydrofoil and the time-averaged pressure distribution on a cascade hydrofoil in non-cavitation conditions. Additionally, the availability of an existing RANS turbulent model of single-phase flow for the CFD of a cavitating flow was discussed, but a clear advantage was not observed, particularly not in a cavitating flow. Consequently, we decided not to use a turbulence model in the present study in order to reduce computational costs. However, the macroscopic disturbance in the flow field, which is caused by cavity volume variation, can be reproduced in the present numerical simulation. Then the values which are shown in Eqs. (3) to (6) can also be obtained in the present numerical simulation.

5. Results and Discussions

5.1 Previous phase change model without apparent phase equilibrium model

Comparison between the numerical result by previous phase change model without the present apparent phase equilibrium model and experimental data of time averaged lift variation about ClarkY 11.7% and NACA0015 hydrofoils are shown in Fig. 4. The time averaged lifts and the drop points agree with the experiment at lower angle of attack in both hydrofoils. But, in higher angle of attack condition, the lift drop points predicted at earlier cavitation number region in both hydrofoils. In the condition of lift drop point at higher angle of attack, cavitation becomes strong unsteady condition as transient cavitation from attached sheet cavitation to quasi-steady super cavitation. The numerical results of time evolution of aspect of sheet cavitation on NACA0015 is shown in Fig 5 at the higher angle of attack condition in the cavitation number \( \sigma = 1.4 \), the condition is most difficult to predict. At that time, the lift was considerably underestimated as shown in right figure in Fig. 4. By comparing with experimental snapshot in Fig. 6 in the same condition, it is found that predicted sheet cavity length is significantly short. Additionally, the released cloud cavity immediately separates from the hydrofoil surface and disappears in the numerical result, although the rolling up cloud cavity flows downstream with contacting the surface in experiment. Then, the direction of the modification of the previous numerical method is considered to increase phase change rate in such strong unsteady cavitating flow.

![Figure 4](image-url)

*Figure 4. Time averaged lift variation by previous model (left: ClarkY 11.7%, Re = 0.7x10^6, right: NACA0015, Re = 1.2x10^6) [5]*
Figure 5. Time evolutions of void fraction (left) and pressure (right) distributions without modification (NACA0015, $\alpha_m = 8\text{deg}$, $\sigma = 1.4$, Time interval = 6 ms)\(^{(5)}\)

Figure 6. Experimental snapshot\(^{(8)}\) (NACA0015, $\text{Re} = 1.2 \times 10^6$, $\alpha_m = 8\text{deg}$, $\sigma = 1.4$)

5.2 Pressure variation component in apparent phase equilibrium model

Before applying the present modification based on the concept of apparent phase equilibrium to the previous numerical method, the distribution in each pressure variation components in Eqs (3) to (6) are compared in the flowfield without modification which is shown in previous section 5.1. The time evolution of distribution of the four types of pressure variation components $p_{\text{turb}}$ in each models in Eqs. (3) to (6) is shown in Fig.7, where, the time zone (1) to (4) corresponds to that in void fraction distribution in Fig.4. The model constants $C$ in Eqs. (3) to (6) are all set as 1.0 in order to compare the order of the values each other. From the distributions in Fig.7, the difference of the distribution of each $p_{\text{turb}}$ can be seen in the unsteady cavitating flowfield. In Model 1 and 2, $p_{\text{turb}}$ distributes always widely in wake region of the cavity. In Model 3, higher value region concentrates in the sheet cavity region, and the value in cloud cavity region is not so high. In Model 4, higher value region scatters in the cavity surface region in both sheet and cloud cavities.
5.3 Apparent phase equilibrium model

The present modification based on the concept of apparent phase equilibrium is applied to the numerical simulation of cavitation in the transient condition at higher angle of attack which is most difficult to predict by the previous numerical method. In this paper, the results Model 1 and Model 2 is compared. In both calculations, control parameter C in Eqs. (3) and (4) is 1.0 and constant in Eq. (2) $n$ is 5. The resulting flow fields of Model 1 and 2 are shown in Fig. 8 and 9, respectively. In each results, cavity volumes slightly increases compared with that without the modification in Fig 5, and shedding cloud cavity is also slightly increased, but it is still smaller than that in experiment. Though, more improvement of the cavity development is expected by controlling the parameter C. Also the distributions of apparent phase equilibrium pressure $p_v^*$ are shown in right hand sides in Figs. 8 and 9. In Model 1, $p_v^*$ does not take higher value in sheet cavity region in the stage of development of the sheet cavity in the time (1) and (5) in Fig. 8, and the value increases around shedding cloud cavity in the time (3) and (4). On the other hand in Model 2, although it is small difference, $p_v^*$ increases inside sheet cavity in developing stage in the time (5) in Fig. 9, evaporation in sheet cavity region is considered to be promoted. Then, not so large difference was observed between Model 1 and Model 2. In future work, I want to compare Model 3 and 4.
Figure 8: Time evolution of distributions of void fraction (left) and apparent phase equilibrium pressure (right) with modification Model 1 ($C = 1.0$ and $n = 5$) (NACA0015, $Re = 1.2 \times 10^6$, $\alpha_{in} = 8\text{deg}$, $\sigma = 1.4$, Time interval = 5 ms)

Figure 9: Time evolution of distributions of void fraction (left) and apparent phase equilibrium pressure (right) with modification Model 2 ($C = 1.0$ and $n = 5$) (NACA0015, $Re = 1.2 \times 10^6$, $\alpha_{in} = 8\text{deg}$, $\sigma = 1.4$, Time interval = 5 ms)
Next, time averaged pressure distribution on the hydrofoil compared with the experimental distribution. Flow condition is same as previous section in which numerical prediction is most difficult. In Fig. 10, time averaged pressure distributions are shown in which the numerical result of present modification of Model 1 and Model 2, numerical result without the modification and experimental data, additionally the numerical result using Singhal’s PDF model \([4]\) are shown. In the result without modification, the pressure recovery point around rear edge of the time averaged cavity locates upstream compared with that in experiment. The distribution corresponds the result of under estimation of cavity volume shown in Fig. 5. In the result of Singhal’s model, the low pressure region around rear end of time averaged cavity expands to downstream region and the cavity is considered successively developed. But the pressure inside the cavity considerably increases compared to experimental value, then the time averaged lift is considered to be still under estimated. In the present modification based on the concept of apparent phase equilibrium, improvement is not seen in Model 1 because pressure recovery point does not change from that without modification and the pressure inside the cavity increases. On the other hand in Model 2, although the pressure inside the cavity increase, the pressure increase is suppressed around leading edge region and the pressure decrease in the rear edge region of the cavity. Therefore, improvement tendency can be seen in Model 2 in the pressure distribution.

In the future direction of modification is considered that the pressure around rear edge of sheet cavity in order to promote the development of the cavity remaining the pressure inside the cavity at saturated vapor pressure. Then including consideration of Model 3 and Model 4, control parameter \(C\) and constant \(n\) in the present apparent phase equilibrium model will be examined.

\[\text{Figure 10: Time averaged pressure distribution on the hydrofoil surface with the present four modifications} \]

\[(\text{NACA0015, Re} = 1.2\times10^6, \alpha_{\text{in}} = 8\text{deg}, \sigma = 1.4)\]

6. Conclusions

In the present study, in order to improve the prediction accuracy of transient cavitation on a hydrofoil in higher angle of attack condition, preliminary experiment of evaporation of gas-vapor interface with flow was performed and modification of phase change model based on the concept of apparent phase equilibrium was proposed for numerical method of cavitation based on homogeneous model. The results are summarized as follows.

- From preliminary experiment, it was shown that vapor pressure with flow on vapor-liquid interface is higher than that without flow in evaporation.
It is shown that the cavity volume and time averaged lift are under estimated in the condition of transient cavitation at higher angle of attack in conventional phase change model.

In the condition of underestimation of lift, the predicted sheet cavity length was shorter than that in experiment, although the transient flowfield can be reproduced.

By applying the present modification based on the concept of apparent phase equilibrium, cavity volume is slightly increased and pressure distribution on a hydrofoil is also slightly improved, although another model or examination of parameters are remained in the present modification.

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