Uncertainty Quantification and Global Sensitivity Analysis of Low-Reynolds-Number Airfoil for Future Mars Airplane

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The effects of uncertainty in flow conditions, namely angle of attack, Reynolds number, and freestream Mach number, on airfoil characteristics in the low-Reynolds-number regime are evaluated. The Ishii airfoil, a thin–cambered airfoil known to have high aerodynamic performance in this regime, is analyzed. The NACA0012 airfoil is also analyzed as a comparative study. The results for two sets of nominal flow conditions are compared to comprehensively characterize performance in the low-Reynolds-number regime. Statistical quantities of aerodynamic coefficients are computed by coupling the stochastic spectral projection method based on polynomial chaos expansion with two-dimensional flow simulations. The relative contribution of the uncertainty of each flow parameter to the variance of outputs is computed using Sobol’s global sensitivity analysis. It is shown that, for the Ishii airfoil, the lift coefficient is highly sensitive to the uncertainty, while the lift-to-drag ratio has a high statistical mean and the pitching moment coefficient has low sensitivity. This indicates that, for thin–cambered airfoils, attention should be given to rapid degradation due to unexpected variation in the angle of attack.

Key Words: Low-Reynolds-Number Flow, Aerodynamic Characteristics, Uncertainty Quantification, Global Sensitivity Analysis, Computational Fluid Dynamics

Nomenclature

- $a$: speed of sound
- $c$: chord length
- $C_l$, $C_d$: lift and drag coefficients, respectively
- COV: coefficient of variance [%]
- $C_p$: pressure coefficient
- $E$: expectation
- $L/D$: lift-to-drag ratio
- $M$: Mach number
- $P$: truncation order of polynomial chaos expansion
- $p(\xi)$: probability density function of random vector $\xi$
- $Q$: total number of terms of polynomial chaos expansion
- $Re_c$: Reynolds number based on chord length
- $s$: partial sensitivity index
- $S$: total sensitivity index
- $u$: streamwise velocity
- $U(a, b)$: uniform probability distribution with lower bound $a$ and upper bound $b$
- $V$: variance
- $w_i$: $i$-th weight of numerical quadrature
- $\alpha$: angle of attack
- $\beta_i$: $i$-th coefficient of polynomial chaos expansion
- $\psi_i$: $i$-th orthogonal polynomial basis for polynomial chaos expansion
- $\xi$: random variable vector
- $\xi_i$: $i$-th node of numerical quadrature in random vector
- $\Omega$: random space
- $(fg)$: inner product of $f$ and $g$, defined as $\int_{\Omega} f g(\xi) d\xi$
- Subscripts
  - $\infty$: freestream

1. Introduction

The use of airplanes for Mars exploration\textsuperscript{1,2} is an attractive approach because it allows larger regions to be explored compared to rovers and higher-resolution data to be obtained compared to orbiting satellites. However, the design and development of such airplanes are challenging.

One challenge comes from the flight conditions on Mars that are quite different from those on Earth. For example, air density on Mars is approximately $1/100$ of that on Earth. In addition, a Mars airplane must be small because it must be stored in an aeroshell during delivery to the planet. Consequently, the flight Reynolds number based on the chord length of a Mars airplane is around $O(10^3–10^5)$, called the low-Reynolds-number regime, whereas that of typical commercial airplanes on Earth is approximately $O(10^7–10^9)$, called the high-Reynolds-number regime. In addition to Mars airplanes, micro-air vehicles and high-altitude airplanes are subjected to flight conditions that correspond to the low-Reynolds-number regime, and thus it is important to understand the aerodynamic characteristics of airfoils in this regime.

Airfoil aerodynamic characteristics in the low-Reynolds-number regime are different from those in the high-Reynolds-number regime.\textsuperscript{3,4} For example, it is difficult to achieve a high lift-to-drag ratio in the low-Reynolds-number regime.
regime. Many previous studies, therefore, have focused on the airfoil characteristics in this regime. One main result of those studies is that thin and cambered airfoils produced a higher lift-to-drag ratio than thick and symmetric airfoils in the low-Reynolds-number regime.\(^5\)–\(^7\)

On the other hand, previous studies also showed that the airfoil characteristics in the low-Reynolds-number regime are more sensitive to the freestream disturbance level,\(^5,\)\(^9\) airfoil geometry shape,\(^10,\)\(^11\) and Reynolds number\(^12,\)\(^13\) than those in the high-Reynolds-number regime. In addition, there are many uncertain factors in Mars airplane missions. For example, the climate and atmospheric conditions on Mars cannot be exactly known in the design process; it is known that wind gusts occur frequently on Mars; the airfoil geometrical shape might be varied due to aeroelastic deformation during operation and manufacturing tolerances. Considering these sensitivities and uncertainties, quantitative evaluations of airfoil performance robustness under uncertain flow conditions in the low-Reynolds-number regime are necessary to increase the probability of a successful Mars airplane mission. Especially, a comparison between thin–cambered and thick–symmetric airfoils is necessary from the viewpoint of uncertainty effects, even though the thin–cambered airfoils show higher performance under nominal flow conditions.

Methods for uncertainty quantification (UQ) have been developed and combined with computational fluid dynamics (CFD) to quantify the effects of uncertainty on a given prediction.\(^14,\)\(^15\) Several studies on the UQ of airfoil characteristics have been performed. For example, Chassaing and Luco\(^16\) performed a study on the propagation of freestream Mach number and angle of attack uncertainties by applying a transonic flow around the NACA0012 airfoil, and Wu et al.\(^17\) investigated the effects of airfoil geometric uncertainties applying transonic flow conditions. Most previous UQ studies on airfoil characteristics, however, focused exclusively on the uncertainty effects in the high-Reynolds-number regime—no studies have considered the stochastic behavior of an airfoil in the low-Reynolds-number regime.

The objective of this study is to evaluate the effects of uncertainty in flow conditions on airfoil characteristics at low Reynolds numbers. Among airplane types that cruise in the low-Reynolds-number regime, the main focus of this study is a Mars airplane. The UQ results are compared between thin–cambered and thick–symmetric airfoils to clarify the characteristics of thin–cambered airfoils. In this study, the Ishii\(^40\) and NACA0012 airfoils are utilized as representatives of the thin–cambered and thick–symmetric airfoils, respectively. The Ishii airfoil was shown to have high aerodynamic performance in the low-Reynolds-number regime,\(^40\) and was adopted as the main wing airfoil of a Mars airplane used in a high-altitude flight test called the Mars Airplane Balloon Experiment-1 (MABE-1), which was conducted by the Japanese Mars Airplane Research Group.\(^18\) In this study, two sets of flight settings corresponding to the flow conditions applied for MABE-1 are used to facilitate a comprehensive evaluation. To this end, an efficient UQ method based on nonintrusive polynomial chaos expansion (PCE),\(^19\) the global sensitivity analysis proposed by Sobol’,\(^20\) and two-dimensional (2D) flow simulations are used.

The rest of this paper is organized as follows. Section 2 describes the problem setting. Section 3 describes the computational methods for UQ, global sensitivity analysis, and flow simulations. The effects of uncertainty in flow conditions are discussed in Section 4. Finally, the conclusions are presented in Section 5.

2. Problem Statement

The stochastic behavior of aerodynamic performance is investigated under uncertainty of the following three parameters: angle of attack \(\alpha\), Reynolds number based on chord length \(Re_c\), and freestream Mach number \(M_\infty\). These parameters could have some uncertainties due to unpredictable operational factors (e.g., atmospheric conditions, instrumentation errors, and airfoil geometric variation due to manufacturing tolerances or aerodynamic loads). \(\alpha\) and \(M_\infty\) have been conventionally employed as uncertain inputs for the UQ of airfoils.\(^16,\)\(^21\) In addition to \(\alpha\) and \(M_\infty\), \(Re_c\) is introduced as an uncertain input here because it can significantly affect aerodynamic performance in the low-Reynolds-number regime.

These parameters are defined as independent random variables that have uniform probability density functions (PDF). The two sets of nominal conditions of \(Re_c\) and \(M_\infty\) are as follows:
- Case A: \(Re_c = 3.3 \times 10^4\) and \(M_\infty = 0.28\), and
- Case B: \(Re_c = 7.0 \times 10^4\) and \(M_\infty = 0.57\).

The nominal conditions for Cases A and B correspond to the flight conditions of MABE-1,\(^18\) respectively, as shown in Fig. 1. The limits of the uncertainty of \(Re_c\) and \(M_\infty\) in each case are \(\pm 1.0 \times 10^4\) and \(\pm 0.05\) from the nominal values, respectively. The variation interval of \(M_\infty\) is the typical operating ranges used for the study of robust shape optimization problems.\(^22\) The variation intervals of \(Re_c\) for Cases A and B correspond to the flight history at 9.0–13.0 s and 20.6–23.0 s of MABE-1, respectively. The nominal conditions of \(\alpha\) are set so that the lift-to-drag ratio of each airfoil takes the max-

![Fig. 1. Reynolds and Mach numbers for MABE-1\(^18\) and ranges for Cases A and B.](image-url)
imum value at each nominal \( R_{e} \) and \( M_{\infty} \). The limit of the uncertainty of \( \alpha \) is \( \pm 0.5 \) deg from the nominal values in each case. This interval of \( \alpha \) corresponds to the amount of installment error estimated for the main wing. The uncertain flow conditions set in this study are summarized in Table 1.

3. Computational Setup

The nonintrusive spectral projection method based on PCE\(^{19}\) is coupled with 2D flow simulations. The contributions of the uncertainty of each flow parameter to the variance in airfoil characteristics are quantified using Sobol’s sensitivity indices.\(^{20}\)

### 3.1. Methods for uncertainty quantification

Although the typical method for UQ is direct Monte Carlo simulation, the computational cost associated with the numerous analysis evaluations of outputs can be prohibitive for complicated problems such as those involving CFD. The nonintrusive spectral projection method based on PCE is thus applied to efficiently analyze uncertainty.

In PCE, the computationally expensive analysis of output function \( f(\xi) \) for random variable vector \( \xi \) is replaced by the following approximation:

\[
f(\xi) \approx \sum_{j=0}^{Q-1} \beta_j \psi_j(\xi).
\]

Here, \( Q \) is the total number of PCE terms that is defined as follows:

\[
Q = \frac{(N + P)!}{N!P!}.
\]

where, \( N \) and \( P \) are the dimension of \( \xi \) and the truncation order of polynomials, respectively, and \( N \) is three based on the problem definition. \( \psi_j \) \((j = 0, \ldots, Q - 1)\) represents the coefficients corresponding to the polynomial basis \( \psi_j \) \((j = 0, \ldots, Q - 1)\). \( \psi_j \) satisfies the following orthogonality relation:

\[
\langle \psi_i \psi_j \rangle = (\psi_i^2) \delta_{ij}.
\]

where, \( \delta_{ij} \) is Kronecker’s delta and the inner product \( \langle \psi_i \psi_j \rangle \) is defined as follows:

\[
\langle \psi_i \psi_j \rangle = \int_\Omega \psi_i \psi_j \rho(\xi) d\xi.
\]

The weight function \( \rho(\xi) \) is the PDF of the random variable vector \( \xi \) and \( \Omega \) is the random space.

The spectral projection method utilized in this study is based on the orthogonality of the basis function \( \psi_j \) to determine the coefficients \( \beta_j \). \( \beta_j \) is determined from the inner product of \( f(\xi) \) and \( \psi_j \). This inner product involves an integral in \( \Omega \). Numerical integration is then required:

\[
\beta_j = \frac{\langle f(\xi) \psi_j(\xi) \rangle}{\langle \psi_j^2 \rangle} = \frac{1}{\langle \psi_j^2 \rangle} \int_\Omega f(\xi) \psi_i(\xi) \rho(\xi) d\xi
\]

\[
\approx \frac{1}{\langle \psi_j^2 \rangle} \sum_{i=1}^{N_q} w_i f(\xi_i^j),
\]

where, \( N_q \) is the number of nodes of a quadrature scheme, and \( w_i \) and \( \xi_i^j \) are the \( i \)-th node and \( j \)-th spacing of the quadrature scheme, respectively. Thus, the output function \( f(\xi) \) is evaluated only at these nodes to quantify uncertainty in this method.

Corresponding to the uniform probability distributions of the uncertain flow parameters defined in this study, the Legendre polynomials are used as the basis of PCE. The truncation order of polynomials is set to \( P = 4 \) on the basis of the convergence assessment provided in the Appendix. The Gauss–Legendre quadrature formula extended to multi-dimensional integration by the full tensor product is utilized to calculate the PCE coefficients. Based on polynomial accuracy, the number \( N_q \) of quadrature points is \( (P + 1)^N = 125 \) per case in this study.

### 3.2. Sobol’s global sensitivity analysis

In this study, the relative contribution of the uncertainty of each input parameter to the variance of the output of interest is determined using Sobol’s global sensitivity analysis.\(^{20}\) In Sobol’s approach, the total variance of the output is decomposed into the contributions of each parameter alone and the interaction between the parameters. A brief overview of Sobol’s approach is as follows. At first, any \( N \)-variate function \( f(\xi) = (\xi_1, \ldots, \xi_N) \), representing the output of interest, is decomposed as follows:

\[
f(\xi) = f_0 + \sum_{m=1}^{N} \sum_{i_1, \ldots, i_m} f_{i_1 \ldots i_m}(\xi_{i_1}, \ldots, \xi_{i_m}),
\]

where, \( f_{i_1 \ldots i_m}(\xi_{i_1}, \ldots, \xi_{i_m}) \) is determined to satisfy the following equation:

\[
\int_\Omega f_{i_1 \ldots i_m}(\xi_{i_1}, \ldots, \xi_{i_m}) d\xi_k = 0 \quad \text{for} \quad k = i_1, \ldots, i_m.
\]

Taking the variance of Eq. (6) leads to

\[
\text{V}[f] = \sum_i V_i + \sum_{i < j} V_{ij} + \cdots + V_{1 \ldots N},
\]
where, $V[f]$ is the variance of $f(\xi)$ and 
\[
V_i = V[E[f(\xi_i)1], \\
V_{ij} = V[E[f(\xi_i, \xi_j)1] - V_i - V_j, \ldots 
\]
Here, $E[f(\xi_i)$, $E[f(\xi_i, \xi_j)$, ... are the conditional expectations. $V_i$ is the variance due to the variation of only parameter $\xi_i$. Similarly, $V_{ij}$ is the variance due to the interaction of parameters $\xi_i$, $\xi_j$, ... . Dividing Eq. (8) by the total variance $V[f]$ leads to the (partial) Sobol’ indices, defined as follows:
\[
s_{i_1 \ldots i_m} = \frac{V_{i_1 \ldots i_m}}{V[f]}, (9)
\]
\[
\sum_i s_i + \sum_{i < j} s_{ij} + \ldots + s_{1 \ldots N} = 1. (10)
\]
Sobol’ indices represent the relative contribution of each factor, and thus, the uncertain input parameter with a high Sobol’ index is the dominant one for the output of interest. The total sensitivity indices $S_i$ are defined to evaluate the total effect of each input parameter. As described in Ref. 23), the total sensitivity indices are defined as the sum of all partial sensitivity indices $s_{i_1 \ldots i_m}$ involving the parameter $i$:
\[
S_i = \sum_{j \in J_i} s_{i_1 \ldots i_m}, (11)
\]
where,
\[
J_i = \{(i_1, \ldots , i_m) : \exists k, 1 \leq k \leq m, i_k = i\}.
\]
All Sobol’ indices reported in this paper are total Sobol’ indices $S_i$.

3.3. Numerical methods for fluid analysis

For evaluating airfoil characteristics, 2D flow simulations are utilized. It is reasonable to use such simulations because we consider a low-angle-of-attack flow that does not exhibit large-scale separation of the boundary layer. The computational solver LANS3D developed by Fujii et al.\(^{24,25}\) is used, which has been validated for many applications including those to airfoil flows\(^{26}\) performed by the Institute of Space and Astronautical Science/Japan Aerospace Exploration Agency (ISAS/JAXA). The governing equations are 2D compressible Navier–Stokes equations. The convective terms are evaluated using third-order monotonic upsteam-centered scheme for conservation law (MUSCL)\(^{27}\) and simple high-resolution upwind scheme (SHUS).\(^{28}\) The viscous terms are evaluated using a second-order central difference scheme. For time integration, a second-order ADI-SGS implicit scheme\(^{29}\) is applied. For Case A, because the flow field is assumed to be fully laminar, no turbulence model is utilized. This laminar simulation has been shown by Lee et al.\(^{30}\) to predict airfoil characteristics and qualitative behaviors on the separation and reattachment of boundary layers with satisfactory accuracy for a flow field without large-scale separation at the Reynolds numbers applied in Case A. For Case B, the flow field is assumed to be turbulent, and the Spalart–Allmaras turbulence model\(^{31}\) is utilized. The flow simulations in this study were performed on the JAXA Supercomputer System Generation 2 (JSS2).

Figure 2 shows the computational grids around the Ishii and NACA0012 airfoils. C-type grids are utilized. The outer boundaries of the grids are extended to 25 times the chord length. The maximum grid spacing values in the chordwise $\Delta x^+$ and wall-normal $\Delta y^+$ directions based on the wall units and the number of grid points in the chordwise $N_x$ and wall-normal $N_y$ directions are listed in Table 2. These grids were used by Lee et al.\(^{30}\) who showed that the computational setups in this study can be used to evaluate qualitative aerodynamic characteristics, except for high-$\alpha$ flows. Considering that this study focuses mainly on comparative discussions regarding the effects of uncertainty, the results and discussions in this study are sufficiently reliable.

4. Results and Discussion

4.1. Case A — with nominal conditions of $Re_x = 3.3 \times 10^4$ and $M_{\infty} = 0.28$

4.1.1. Statistics of aerodynamic coefficients

The statistical results of lift $C_l$ and drag $C_d$ coefficients, lift-to-drag ratio $L/D$, and pitching moment coefficient $C_m$ are around $s/c = 0.25$ are listed in Table 3. In this table, deterministic results obtained at the nominal conditions, statistical means, standard deviations, and coefficients of variance (COV [%]) are compared between the Ishii and

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**Table 2.** Maximum grid spacing values based on the wall units and number of grid points in the computational grid.

| Airfoil   | $\Delta x^+$ | $\Delta y^+$ | $N_x$ | $N_y$ | Total points |
|-----------|--------------|--------------|-------|-------|--------------|
| Ishii     | 23.0         | 0.477        | 693   | 179   | 124,047      |
| NACA0012  | 16.6         | 0.567        | 693   | 179   | 124,047      |
NACA0012 airfoils. The COV is defined as the ratio of standard deviation to absolute value of statistical mean.

Table 3 shows that the statistical means of $C_l$, $C_d$, and $L/D$ are worse than the nominal values, except for $C_d$ of the NACA0012 airfoil. This means that the uncertainty in flow conditions frequently degrades the aerodynamic performance of airfoils from the viewpoint of stochastic behavior. The reason for the degradation is found in the PDFs of $C_l$ and $C_d$, which are shown in Figs. 3 and 4. These figures show that the probability density is more distributed for values degraded from the nominal results, except for $C_d$ of the NACA0012 airfoil (Fig. 4 b)) that shows almost symmetric PDF.

Table 3 also shows that the Ishii airfoil has a higher COV of $L/D$ than the NACA0012 airfoil while having a higher statistical mean of $L/D$. This is clearly due to the higher COVs of $C_l$ and $C_d$. Especially, the probability density of $C_l$ of the Ishii airfoil (Fig. 3 a)) is distributed for $C_l < 0.50$, as well as around the statistical mean, which is attributed to the higher COV. The higher COVs indicate that the Ishii airfoil shows larger unexpected variation in aerodynamic performance in the presence of uncertainty in flow conditions. On the other hand, the standard deviation of $C_m;_0$ of the Ishii airfoil is smaller than that of the NACA0012 airfoil, i.e., the stability for the flight control of the Ishii airfoil is less affected by the uncertainty.

4.1.2. Surface pressure distribution

Figure 5 shows the statistics of $C_p$ distributions. As shown in the figure, the large error bars of the $C_p$ distribution for both types of airfoil are mainly concentrated in the range of $0.2 \lesssim x/c \lesssim 0.7$ on the upper surface, where a laminar separation bubble (LSB) is formed. In fact, the $C_p$ distributions of statistical mean plateau in the range of $0.2 \lesssim x/c \lesssim 0.5$, and rapid pressure recovery occurs in the range of $0.5 \lesssim x/c \lesssim 0.7$. Besides the behaviors of $C_p$ around the LSB that will be discussed later, the enlarged views in Fig. 5 show that the 95CIs near the suction peak...
tend to extend to $C_P$ values higher than the nominal results for both airfoils. This could be attributed to the degradations of the aerodynamic coefficients observed in Section 4.1.1.

Figure 6 shows the PDFs of surface pressure $C_P$ at $x/c = 0.6$ on the upper surface, where the LSB is formed. The PDF of the Ishii airfoil in Fig. 6 a) exhibits a clear double peak while that of the NACA0012 airfoil in Fig. 6 b) does not. The existence of such a double peak indicates a rapid transition between two dominant flow structures. Accordingly, the $C_P$ distribution of the Ishii airfoil abruptly changes through this transition, which is considered to have caused the higher COVs of the aerodynamic coefficients. In contrast, only a single peak in Fig. 6 b) corresponds to one flow structure, which causes the lower COVs of the aerodynamic coefficients of the NACA0012 airfoil. These flow structures will be explained in Section 4.1.4.

![Fig. 5. Statistics of surface pressure distributions for a) Ishii and b) NACA0012 airfoils for Case A.](image)

![Fig. 6. PDFs of surface pressure coefficient at $x/c = 0.7$ on the upper surface of a) Ishii and b) NACA0012 airfoils for Case A.](image)

| Airfoil | $S_{C_T}$ | $S_{Re_c}$ | $S_{M_\infty}$ |
|---------|----------|------------|---------------|
| $C_T$   | Ishii    | 0.811      | 0.090         | 0.183         |
|         | NACA0012 | 0.277      | 0.724         | 0.118         |
| $C_d$   | Ishii    | 0.508      | 0.460         | 0.062         |
|         | NACA0012 | 0.159      | 0.841         | 0.071         |

4.1.3. Sensitivity results

The contribution of the uncertainty of each uncertain flow parameter to the variation in aerodynamic coefficients can be evaluated using the total Sobol’ sensitivity indices. The Sobol’ indices with respect to $\alpha$, $Re_c$, and $M_\infty$, denoted as $S_\alpha$, $S_{Re_c}$, and $S_{M_\infty}$, respectively, are listed in Table 4 for $C_T$ and $C_d$. Higher Sobol’ indices mean larger contributions to the statistical variance of output. $S_{M_\infty}$ is the lowest because the nominal $M_\infty$ is relatively low in this case. For both $C_T$ and
Rec means of Fig. 7 a), the dominant to LSBs and TE separations, respectively. As shown in variations of are shown in Figs. 8 and 9. These

\[ C_d \]

fl

Cd

fl

Fig. 7. Time-averaged fields around a) Ishii and b) NACA0012 airfoils for Case A when Reynolds number and angle of attack are varied at a fixed \( M_\infty \) of 0.28.

The contour shows the streamwise velocity fields.

\[ C_if \]

follow conditions frequently degrades the aerody-

\[ \alpha \]

\[ C_m \]

\[ L/D \]

\[ 1/D \]

\[ C_m,0.25 \]

NACA0012

COV

\[ \frac{1}{C_0} \]

Table 5. Statistical results of aerodynamic coefficients for Case B.

| Airfoil | Nominal | Mean | Standard deviation | COV [%] |
|---------|---------|------|--------------------|---------|
| \( C_f \) Ishii | 0.588 | 0.588 | 0.0402 | 6.83 |
| NACA0012 | 0.601 | 0.587 | 0.0234 | 3.98 |
| \( C_d \) Ishii | 2.04e−2 | 2.11e−2 | 0.254e−2 | 12.03 |
| NACA0012 | 2.91e−2 | 3.03e−2 | 0.534e−2 | 17.64 |
| \( L/D \) Ishii | 28.44 | 28.02 | 1.836 | 6.55 |
| NACA0012 | 20.66 | 20.02 | 3.594 | 17.96 |
| \( C_m,0.25 \) Ishii | −4.50e−2 | −4.53e−2 | 0.155e−2 | 3.42 |
| NACA0012 | 0.84e−2 | 0.75e−2 | 0.275e−2 | 36.51 |

\[ Re_\infty \times 10^4 \]

\[ u/\alpha \]

\[ 0.0 \]

\[ 0.4 \]

\[ 3.5 \]

\[ 4.0 \]

\[ 4.5 \]

\[ \alpha, \text{deg.} \]

\[ 3.0 \]

\[ 3.3 \]

\[ 3.5 \]

\[ 4.0 \]

\[ 4.5 \]

\[ 5.5 \]

\[ 6.0 \]

\[ 6.5 \]

\[ \alpha, \text{deg.} \]

\[ \text{Ishii airfoil} \]

\[ \text{NACA0012 airfoil} \]

\[ \text{a)} \]

\[ \text{b)} \]

\[ \text{Fig. 7. Time-averaged fields around a) Ishii and b) NACA0012 airfoils for Case A when Reynolds number and angle of attack are varied at a fixed } M_\infty \text{ of 0.28. The contour shows the streamwise velocity fields.} \]

4.1.4. Physical interpretation

Some typical flow fields are shown in Fig. 7 with the variations of \( \alpha \) and \( Re_\infty \), taking the sensitivity results above into account. Two dominant flow structures are observed around the Ishii airfoil in Fig. 7 a); namely, LSBs at high \( \alpha \) and trailing-edge (TE) separations at low \( \alpha \). Here, the LSBs and TE separations are the closed and opened regions, respectively, where streamwise velocity is negative on the airfoil upper surface. In contrast, a single dominant flow structure, namely LSB, is observed around the NACA0012 airfoil in Fig. 7 b).

This suggests that the two dominant flow structures (i.e., LSBs and TE separations) around the Ishii airfoil correspond to the double peak in the PDF of \( C_d \) in Fig. 6 a). The peaks at \( C_d \approx −0.35 \) and \( C_d \approx −0.18 \) in Fig. 6 a) correspond to LSBs and TE separations, respectively. As shown in Fig. 7 a), the dominant flow structure of the Ishii airfoil does not vary at fixed \( \alpha \): the rapid transition between the two dominant flow structures is mainly caused by the variation in \( \alpha \), which is consistent with the results of the sensitivity analysis in Section 4.1.3 that \( \alpha \) is the significant factor for the Ishii airfoil.

4.2. Case B — with nominal conditions of

\[ Re_\infty = 7.0 \times 10^4 \] and \( M_\infty = 0.57 \)

4.2.1. Statistics of aerodynamic coefficients

The statistical results for Case B of \( C_f \), \( C_d \), \( L/D \), and \( C_m,0.25 \) are listed in Table 5. Table 5 shows that the statistical means of \( C_f \), \( C_d \), and \( L/D \) are worse than the nominal values, except for \( C_f \) for the Ishii airfoil. Namely, the uncertainty in flow conditions frequently degrades the aerodynamic performance of airfoils from the viewpoint of stochastic behavior, which is similar to Case A. PDFs of \( C_f \) and \( C_d \) are shown in Figs. 8 and 9. These figures show that the probability density is more distributed for values degraded from the nominal results, except for \( C_f \) for the Ishii airfoil (Fig. 8 a)), which shows a nearly symmetric PDF.

Table 5 also shows that the Ishii airfoil has a lower COV of \( L/D \) than the NACA0012 airfoil, as well as a higher statistical mean of \( L/D \). This indicates that the Ishii airfoil shows desirable aerodynamic performance compared to the NACA0012 airfoil. It should be noted that, however, the Ishii airfoil shows a higher COV of \( C_f \), which is similar to Case A. The difference in the COVs of \( L/D \) between the two types of airfoil can be attributed to the difference in the stochastic behavior of \( C_d \). Especially, the probability density of \( C_d \) for the NACA0012 airfoil (Fig. 9 b)) is distributed for \( C_d > 0.040 \) as well as around the statistical mean, which causes the higher COV of \( C_d \) shown in Table 5. On the other hand, the standard deviation of \( C_m,0.25 \) of the Ishii airfoil is smaller than that of the NACA0012 airfoil, which is similar to Case A.

4.2.2. Surface pressure distribution

The statistics of \( C_p \) distributions are shown in Fig. 10. The enlarged views in Fig. 10 show that the 95CLs near the suction peak of the Ishii airfoil are almost symmetric around the nominal results, which could be attributed to the symmetric PDF of \( C_f \). On the other hand, the 95CLs near the suction peak of the NACA0012 airfoil tend to be extended to \( C_p \) values higher than the nominal results, which could be attributed to the degradations of the aerodynamic coefficients observed in Section 4.2.1. It is also observed that, while non-negligible error bars for the Ishii airfoil are concentrated in the range of \( 0.0 \lesssim x/c \lesssim 0.5 \) on the airfoil upper surface, those for the NACA0012 airfoil are distributed near the TE as well. This difference in the distribution of error bars could cause the difference in the COVs of the aerodynamic coefficients. Specifically, the larger variation of \( C_p \) near the TE is attributed to the higher COV of \( C_d \) for the NACA0012 airfoil. The physical interpretation for this will be presented in Section 4.2.4.

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Fig. 8. PDFs of $C_l$ of a) Ishii and b) NACA0012 airfoils for Case B.

Fig. 9. PDFs of $C_d$ of a) Ishii and b) NACA0012 airfoils for Case B.

Fig. 10. Statistics of surface pressure distributions for a) Ishii and b) NACA0012 airfoils for Case A.
Figure 11 shows the PDFs of surface pressure $C_p$ at $x/c = 0.3$ on the upper surface of a) Ishii and b) NACA0012 airfoils for Case B. In this case, no double peak appears in the PDF of either airfoil; however, the variation range of $C_p$ for the Ishii airfoil is wider than that for the NACA0012 airfoil. This indicates that the $C_p$ distribution in the LSB of the Ishii airfoil is more sensitive to uncertainty than that of the NACA0012 airfoil, which can be attributed to the higher COV of $C_l$ for the Ishii airfoil.

### 4.2.3. Sensitivity results

The total Sobol' indices $S_{C_l}$, $S_{Re_c}$, and $S_{M_{\infty}}$ are listed in Table 6 for $C_l$ and $C_d$, $S_{M_{\infty}}$ for Case B is higher than those for Case A because the nominal $M_{\infty}$ is relatively high. For both $C_l$ and $C_d$, the most significant factors are $\alpha$ for the Ishii airfoil and $M_{\infty}$ for the NACA0012 airfoil.

### 4.2.4. Physical interpretation

Some typical flow fields with the variations of $\alpha$ and $M_{\infty}$ taking the above-mentioned sensitivity results into account are shown in Fig. 12. A single dominant flow structure (i.e., LSB) is observed around the Ishii airfoil in Fig. 12 a). On the other hand, two dominant flow structures are observed around the NACA0012 airfoil in Fig. 12 b); that is, the onset of TE separation occurs at high $M_{\infty}$. This suggests that the onset of TE separation causes the non-negligible error bars of $C_l$ near the TE (Fig. 10 b)) and the higher COV of $C_d$ for the NACA0012 airfoil. As shown in Fig. 12 b), the onset of TE separation is caused by the variation in $M_{\infty}$, which is consistent with the results of the sensitivity analysis in Section 4.2.3, confirming that $M_{\infty}$ is the significant factor for the NACA0012 airfoil.

### 4.3. Comprehensive discussions

Table 7 compares the results of the sensitivity analysis between the Ishii and NACA0012 airfoils for Cases A and B. For Case A, the Ishii airfoil shows a higher $L/D$ sensitivity than the NACA0012 airfoil due to the higher sensitivity of $C_l$ and $C_d$. This is attributed to the abrupt transition between the two dominant flow structures (LSB and TE separation) for the Ishii airfoil, while a single dominant flow structure (LSB) is retained for the NACA0012 airfoil. For Case B,
the Ishii airfoil shows lower $L/D$ sensitivity than the NACA0012 airfoil due to the lower sensitivity of $C_d$. This is because the single dominant flow structure (LSB) is retained for the Ishii airfoil, while the onset of TE separation occurs for the NACA0012 airfoil. Although the comparison of $L/D$ sensitivity highly depends on the case, the Ishii airfoil is preferable in the sense that the significant factor $\alpha$ is common to Cases A and B. This clearly identifies the factor that should be consistently taken into account during the design and development process.

The results of this paper are summarized as follows. From the stochastic viewpoint, during the presence of uncertainty in flow conditions, the advantages of the Ishii airfoil over the NACA0012 airfoil in both Cases A and B are:

- the higher statistical mean of $L/D$,
- the significant factor $\alpha$ that is common to Cases A and B, and
- the lower sensitivity of $C_m$.0.25.

On the other hand, the disadvantage of the Ishii airfoil in both Cases A and B is the higher sensitivity of $C_l$. Hence, for the Ishii airfoil, attention should be given to the rapid degradation of aerodynamic performance due to unexpected variation, especially in $\alpha$, which can be caused in the development process and/or by aeroelastic deformation during operation.

### 5. Conclusions

This study evaluated the effects of uncertainty in flow conditions, namely the angle of attack $\alpha$, the Reynolds number $Re_f$, and the freestream Mach number $M_\infty$, on airfoil characteristics in the low-Reynolds-number regime. The Ishii and NACA0012 airfoils, representing thin–cambered and thick–symmetric airfoils, respectively, were used to make comparisons to clarify the characteristics of thin–cambered airfoils. The nominal flow conditions set in this study corresponded to the flight conditions of a high-altitude flight test of a Mars airplane (MABE-1). Statistical quantities such as the probability density function (PDF), statistical mean, and standard deviation of the outputs of interest were obtained by coupling the stochastic spectral projection method with 2D flow simulations. The relative contribution of the uncertainty of each factor to the variation in outputs was computed using Sobol's global sensitivity analysis.

It was found that, compared to the thick–symmetric airfoil, the thin–cambered airfoil has a higher lift-to-drag ratio $L/D$ statistical mean and lower pitching moment coefficient $C_m$.0.25 sensitivity. The thin–cambered airfoil, however, can show higher $L/D$ sensitivity, especially at lower Reynolds and freestream Mach numbers. The significant factor for the thin–cambered airfoil is the angle of attack; therefore, during the design and development process, attention should be given to the rapid degradation of aerodynamic performance due to unexpected variation in the angle of attack caused by manufacturing tolerances and/or aeroelastic deformation during operation.

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Appendix: Convergence Study of Truncation Order for PCE

Figure 13 shows the PDFs of $C_l$ and $C_d$ of the Ishii airfoil for Case B when varying $P$ from two to five. A sufficient convergence property for both $C_l$ and $C_d$ is confirmed at $P = 4$, which is thus employed in this study.

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Associate Editor