A New World Average Value for the Neutron Lifetime

A.P. Serebrov*, A.K. Fomin

Petersburg Nuclear Physics Institute, Russian Academy of Sciences, Gatchina, Leningrad District, 188300, Russia

*Corresponding author: A.P. Serebrov

A.P. Serebrov
Petersburg Nuclear Physics Institute
Gatchina, Leningrad district
188300 Russia
Telephone: +7 81371 46001
Fax: +7 81371 30072
E-mail: serebrov@pnpi.spb.ru
Abstract

The analysis of the data on measurements of the neutron lifetime is presented. A new most accurate result of the measurement of neutron lifetime [Phys. Lett. B 605 (2005) 72] 878.5 ± 0.8 s differs from the world average value [Phys. Lett. B 667 (2008) 1] 885.7 ± 0.8 s by 6.5 standard deviations. In this connection the analysis and Monte Carlo simulation of experiments [Phys. Lett. B 483 (2000) 15] and [Phys. Rev. Lett. 63 (1989) 593] is carried out. Systematic errors of about −6 s are found in each of the experiments. The summary table for the neutron lifetime measurements after corrections and additions is given. A new world average value for the neutron lifetime 879.9 ± 0.9 s is presented.

I. Introduction

The recent neutron lifetime experiment [1] has provided the value 878.5 ± 0.8 s. It differs by 6.5 standard deviations from the world average value 885.7 ± 0.8 s quoted by the particle data group (PDG) in 2006 [2]. The experiment employed a gravitational trap with low-temperature fluorinated oil (fomblin) coating, which provides several advantages with respect to previous experiments. First of all, a small loss factor of only 2·10⁻⁶ per collision of UCN with trap walls results in a low loss probability of only 1% of the probability of neutron β-decay. Therefore the measurement of neutron lifetime was almost direct; the extrapolation from the best storage time to the neutron lifetime was only 5 s. In these conditions it is practically impossible to obtain a systematical error of about 7 s. The quoted systematical error of the experimental result [1] was 0.3 s.

In determination of the world average value of the neutron lifetime there is rather dramatic situation. On the one hand a new value of neutron lifetime from work [1] cannot be included in the world average value because of the big difference of results. On the other hand until this major disagreement is understood the present world average value for the neutron lifetime must be suspect. So the situation on PDG page devoted to the neutron lifetime is formulated [2] in view of this controversy.

The only way out of the present situation is to carry out new more precise experiments. More detailed analysis of the previous experiments and search of possible systematic error is also reasonable.

Table 1 and Fig. 1 show dynamics of developing events. Before carrying out measurements [1] using “Gravitrap” installation the world average neutron lifetime was mainly determined by the result of work [3]. At that time the consistent world average value 885.7 ± 0.8 s was obtained. Occurrence of a new precise measurement of neutron
lifetime in 2004 led to the described above controversy. It became more obvious in 2007 after obtaining measurements of neutron lifetime with UCN magnetic trap [4]. It is easy to see that the experiment [3] is one of the most precise experiments in Table 1. Not only does it give the main contribution to the world average value obtained until 2004, but it also gives the main contribution to the discrepancy between the results of earlier and new measurements.

Table 1. Progress of neutron lifetime measurements till 2007.

| $\tau_n$, s | Author(s), year, reference |
|-------------|-----------------------------|
| 878.2 ± 1.9 | V. Ezhov et al. 2007 [4]   |
| 878.5 ± 0.7 ± 0.3 | A. Serebrov et al. 2005 [1]|
| 886.3 ± 1.2 ± 3.2 | M.S. Dewey et al. 2003 [6]|
| 885.4 ± 0.9 ± 0.4 | C. Arzumanov et al. 2000 [3]|
| 889.2 ± 3.0 ± 3.8 | J. Byrne et al. 1996 [7]   |
| 882.6 ± 2.7 | W. Mampe et al. 1993 [8]   |
| 888.4 ± 3.1 ± 1.1 | V. Nesvizhevski et al. 1992 [9]|
| 893.6 ± 3.8 ± 3.7 | J. Byrne et al. 1990 [10]  |
| 887.6 ± 3.0 | W. Mampe et al. 1989 [5]   |
| 872 ± 8 | A. Kharitonov et al. 1989 [11]|
| 878 ± 27 ± 14 | R. Kossakowski et al. 1989 [12]|
| 877 ± 10 | W. Paul et al. 1989 [13]   |
| 891 ± 9 | P. Spivac et al. 1988 [14]  |
| 876 ± 10 ± 19 | J. Last et al. 1988 [15]   |
| 870 ± 17 | M. Arnold et al. 1987 [16] |
| 903 ± 13 | Y.Y. Kovintsev et al. 1986 [17]|
| 937 ± 18 | J. Byrne et al. 1980 [18]  |
| 881 ± 8 | L. Bondarenko et al. 1978 [19]|
| 918 ± 14 | C.J. Christensen et al. 1972 [20] |
Fig. 1. Progress of the neutron lifetime measurements. a: before “Gravitrap” measurement in 2003; b: after “Gravitrap” measurement in 2004; c: after magnetic trap measurement in 2007.
We cannot find by any means an error in 7 s in our measurements [1] where extrapolation of UCN storage time to the neutron lifetime is only 5 s. Therefore we have examined the analysis of the experiment [3] where extrapolation is 100-120 s and at the same time it is affirmed that it is done with systematic error 0.4 s. It is this point that causes obvious doubts. A detailed analysis of the experiment [3] performed by means of Monte Carlo simulation is made below in paragraph II. In paragraph III the analysis of experiment MAMBO I [5] is given. In this experiment effect of quasi-elastic UCN scattering on Fomblin oil was taken into account. In paragraph IV the analysis of the summary table for the neutron lifetime measurements after corrections and additions is made. A new world average value for the neutron lifetime is also presented.

II. A detailed analysis of the experiment [3]

A. Scheme and method of the experiment [3]

Below we reproduce a short description of the experiment [3] using mainly the text of the article. The setup is shown in Fig. 2. The storage vessel (7), (8) is composed of two coaxial horizontal cylinders made of aluminium of 2 mm thickness. The cylinder walls were coated with a thin layer of Fomblin oil which has very low UCN losses. In order to maintain this oil layer on the surface, the cylinder walls were first coated by a layer of Fomblin grease of about 0.2 mm thickness.

![Fig. 2. The scheme of the experimental setup. 1 - UCN guide, 2 - shutters, 3 - UCN detector, 4 - polyethylene shielding, 5 - cadmium housing, 6 - entrance shutter of the inner vessel, 7 - inner storage vessel, 8 - outer storage vessel, 9 - cooling coil, 10 - thermal neutron detector, 11 - vacuum housing, 12 - oil puddle, 13 - entrance shutter of the gap vessel, 1a - oil puddle, 2a - slit.](image)

The inner cylinder (7) was 33 cm in diameter and 90 cm long, while the dimensions of the outer one (8) were larger by a gap of 2.5 cm. The shutter (6) connects
the inner cylinder to the intermediate chamber which has connections (i) to the neutron guide (1) of the TGV UCN source by the entrance shutter and (ii) to the UCN detector (3) by shutter (2). The shutter (13) connects the inner cylinder to the volume of the annular gap between both cylinders.

The inner cylinder had a long slit (2a) of a special form (see Fig. 2) along a cylinder surface. The edges of the slit were dipped into a Fomblin oil puddle (1a) with level (12) when the slit was situated at the bottom position during storage. The construction allowed to rotate the cylinders in common about its horizontal axis without a vacuum break to refresh the oil layers on the cylinder walls.

The storage vessel was placed inside the vacuum housing (11). The vessel volume was hermetically sealed from the housing. The housing was formed by two coaxial cylinders of stainless steel. The outer surface of the inner cylinder had a serpent tube (9) to cool the bottles. The cooling system stabilized the bottle temperature which could be set in the range +20°C ± – 26°C.

The set-up was surrounded by the thermal neutron detectors comprising a set of 24 counters of the SNM-57 type (10), each counter being a 3He filled tube of 3 cm diameter and 100 cm long. The UCN detector was a 3He loaded proportional counter (3) with an Al entrance window of 100 μm thickness.

The whole installation was placed inside the shielding (5) of 1 mm thick Cd and the shielding (4) of 16 cm thick boron polyethylene.

The construction permitted to store UCN either in the inner cylinder or in the annular space between the inner and outer cylinder, thereby changing the UCN loss rate by a factor of about 5 without breaking the vacuum.

The experiment was carried out using the following sequence of procedures.

1. **Filling.** The chosen vessel, annular or central, was filled for 200 s. For filling only the central vessel the shutter 13 was closed. For the annular vessel shutter 13 was open and the UCN removed from the central vessel in the following step.

2. **Cleaning.** The trapped neutron spectrum in the storage vessel was given time to clean during $t_{cl}$ (200 s to 1000 s). This procedure was necessary as the UCN source provided a rather broad neutron spectrum. During the cleaning time $t_{cl}$ UCN with velocity exceeding the limiting velocity of Fomblin escaped from the vessel. When the annular vessel was chosen the shutter 6 and the shutter to the UCN detector were opened during $t_{cl}$ to empty the central vessel.
3. **Emptying.** The UCN were emptied to the detector from the chosen vessel and counted for 200 s yielding the initial quantities $N_i$ and $n_i(t)$, where $n_i(t)$ denotes the counting rate in the UCN detector during the emptying time $t$ and $N_i$ the integral over $n_i(t)$. On emptying the inner vessel both its shutters were opened to make the emptying conditions more equal for the two vessels.

4. Steps 1. and 2. were repeated to fill the chosen vessel and to clean the UCN spectrum before the storage period. Due to the stable intensity of the UCN source the initial conditions were essentially identical.

5. **Storing.** After the cleaning time the UCN were further stored in the chosen vessel for the time $T$ and the inelastically scattered and leaked neutrons were counted during that interval in the thermal neutron and UCN detector, respectively.

6. Recording of the final UCN quantity $N_f$ and $n_f(t)$ by counting for 200 s (same procedure as step 3).

7. The background of the detectors was measured during 150 s after all UCN have left the vessel.

All abovementioned procedures of the experiment are shown in Fig. 3.

Basic idea of the experimental method for a monoenergetic UCN spectrum is the following. The number of neutrons $N(t)$ in the trap changes exponentially during the storage time, i.e. $N(t) = N_0 e^{-\lambda t}$. The value $\lambda$ is the total probability per unit time for the disappearance of UCN due to both the beta-decay and losses during UCN-wall collisions. In turn, losses are equal to the sum of the inelastic scattering rate constant $\lambda_{ie}$, and that for the neutron capture at the wall, $\lambda_{cap}$:

$$\lambda = \lambda_n + \lambda_{loss} = \lambda_n + \lambda_{ie} + \lambda_{cap}$$  \hspace{1cm} (1)

The ratio $\lambda_{cap}/\lambda_{ie}$ is to a good approximation equal to the ratio of the UCN capture and inelastic scattering cross sections for the material of the wall surface since both values are proportional to the wall reflection rate of UCN in the trap. Hence $\sigma_{cap}/\sigma_{ie}$ and the value

$$a = \frac{\lambda_{loss}}{\lambda_{ie}} = 1 + \frac{\lambda_{cap}}{\lambda_{ie}} = 1 + \frac{\sigma_{cap}}{\sigma_{ie}}$$  \hspace{1cm} (2)

is constant for the given conditions, i.e. same wall material and temperature. During storage the upscattered neutrons are recorded with an efficiency $\epsilon_{in}$ in the thermal neutron detector surrounding the storage trap. The corresponding counting rate is given by
Hence the total counts in the time interval $T$ are equal to

$$J = \varepsilon_{\text{th}} \lambda c (N_0 - N_f) / \lambda$$

(4)

$$\lambda_{sc} = \frac{J \lambda}{N_i - N_f} \frac{\varepsilon}{\varepsilon_{\text{th}}}$$

(5)

$$\lambda = \frac{1}{T} \ln\left(\frac{N_i}{N_f}\right)$$

(6)

Here $N_0$ and $N_f$ are the UCN populations in the trap at the beginning and the end of the storage time $T$, respectively. The UCN themselves are measured with an efficiency $\varepsilon$ such that the detected UCN at the beginning (normalisation measurement) and the end of the storage time are equal to $N_i = \varepsilon N_0$ and $N_f = \varepsilon N_f$ respectively. We have then

$$\lambda_{sc} = \frac{J \lambda}{N_i - N_f} \frac{\varepsilon}{\varepsilon_{\text{th}}}$$

(5)

$$\lambda = \frac{1}{T} \ln\left(\frac{N_i}{N_f}\right)$$

(6)
The experiment is repeated with a different value for the wall loss rates. The ratio of the two corresponding \( \lambda \) values are built following Eq. (1) and including Eq. (2) with constant value \( a \). Thus \( \tilde{\lambda}_n \) is given by

\[
\tilde{\lambda}_n = \frac{\xi \lambda^{(1)} - \lambda^{(2)}}{\xi - 1}
\]

(7)

where

\[
\xi = \frac{\lambda^{(2)}_{ie}}{\lambda^{(1)}_{ie}}
\]

(8)

The indices refer to the two measurements with different \( \lambda_{\text{loss}} \). The expression Eq. (7), (8) contains then only the directly measured quantities \( J, N_i, N_f \) following Eqs. (5), (6) since the efficiencies of the neutron detection cancel. The value for \( \lambda_{\text{loss}} \) can be varied by changing the ratio of the surface to the volume of the bottle and hence the reflection rate with the walls. In order to keep the value \( a \) constant the (monoenergetic) energy of the UCN and the specification of the wall (temperature, type of wall, etc.) must be the same.

Description of this method for a broad UCN spectrum and more experimental details can be found in [3].

B. The Analysis and Monte Carlo Simulation of the experiment [3]

Processing of results of a method of work [3] for extrapolation to the neutron lifetime is presented by Eqs. (7), (8). For descriptive reasons (Fig. 4a) it is possible to suggest the graphic solution, using Eqs. (1), (2).

From Eqs. (1), (2) we can write:

\[
\lambda = \tilde{\lambda}_n + a\lambda_{ie}.
\]

(9)

Accordingly for two measurements in different geometry:

\[
\lambda^{(1)} = \tilde{\lambda}_n + a\lambda^{(1)}_{ie},
\]

(10)

\[
\lambda^{(2)} = \tilde{\lambda}_n + a\lambda^{(2)}_{ie}.
\]

(11)

Excluding \( a \) from the system of equations:

\[
\tilde{\lambda}_n = \frac{\lambda^{(1)} - \lambda^{(2)}}{\lambda^{(1)}_{ie} - \lambda^{(2)}_{ie}} = \frac{\xi \lambda^{(1)} - \lambda^{(2)}}{\xi - 1},
\]

(12)

where \( \xi = \frac{\lambda^{(2)}_{ie}}{\lambda^{(1)}_{ie}} \), i.e. we derive Eq. (7) of work [3].
Fig. 4. (a) diagram showing influence of various effects for measured value of neutron lifetime; (b) correction of neutron lifetime due to effect of not full emptying of the inner vessel during cleaning while working with the annular vessel: simulations for neutron guide length in front of the detector of 0.8 m (curve 1) and 1 m (curve 2); (c) correction of neutron lifetime due to effect of heating of neutrons by the shutters; (d) correction of neutron lifetime due to effect of not equal thermal neutron detection efficiencies for different vessels: simulations without capture and scattering in materials (curve 1) and with capture and scattering in materials (curve 2).
It is quite obvious that for absence of systematic in a method of work [3] it is necessary to have full equivalence of parameters $\lambda$ and $\lambda_{ie}$ for two different vessels. We will consider possible distinctions for $\lambda$ and $\lambda_{ie}$ which arise at change of geometry of experiment.

MC simulation of the experiment [3] was performed using a code capable of taking into account gravity. The code was written by A.K. Fomin especially for simulations with UCN. This code starts with an initial distribution of neutrons and calculates the track of each particle analytically until it reaches a material boundary. At each wall collision the loss and reflection probability is calculated, resulting in a new direction to calculate the trajectory until the next boundary is reached. The code uses specula and diffusion reflections with walls.

The geometry of setup and time intervals were chosen the same as in the experiment. After each simulation we have values of $N_i$, $N_f$, $J$, $j(t)$ and $n(t)$. We evaluate the obtained data in the same way as in the experiment. In all our simulations neutron lifetime was fixed to a definite value. Repeating experimental procedure we obtain the extrapolated neutron lifetime values and comparing it with initial one we get correction to the experimental result.

The percentage values of diffusion reflections by walls were set to reproduce an experimental emptying process, i.e. time dependence of UCN detector count in the course of registration. It is the most detailed information which can be found in work [3]. Neutron reflection by walls was approximated by 50% specular and 50% diffusion reflections for the inner and annular vessels. Such a factor seems to be reasonable since the surface of vessels has been covered by a layer of Fomblin grease before being covered with Fomblin oil. Neutron reflection by walls was approximated by 90% specular and 10% diffusion reflections for the neutron guides. That corresponds to quality of electropolished neutron guides.

MC simulation was done for the temperature $-26^\circ$C because most of the experimental data was obtained at this temperature.

We studied three effects in MC simulations: (1) not full emptying of the inner vessel during cleaning while working with the annular vessel; (2) heating of neutrons by shutters; (3) not equal thermal neutron detection efficiencies for different vessels.

1. **Effect of not full emptying of the inner vessel during cleaning while working with the annular vessel.** One can see from Fig. 3 that process of UCN emptying to the detector after holding in the inner and the annular vessels is different.
Emptying after holding in the inner vessel occurs directly to the detector through neutron guide system. However, after holding in the annular vessel neutrons at first pass through the inner vessel. The authors of work [3] try to make conditions of emptying more identical and on emptying the inner vessel both its shutters were opened to make the emptying conditions more equal for the two vessels. The question arises how perfect emptying the inner vessel will be released before opening of the shutter 13 for emptying the annular vessel after cleaning. For an estimation of a possible systematic error in this process we have done MC simulation of the process taking into account geometry of experiment [3].

The shutter 6 and the shutter of UCN detector are opened during $t_{cl}$ when we work with the annular vessel. It is necessary to empty the inner vessel from UCN during holding in the annular vessel. If this time is not enough for the inner vessel there are still neutrons which are added to neutrons from the annular vessel during its emptying. It gives higher value of $N_i$ and correspondingly higher value of $\lambda$ and lower value of $\lambda_{e}$ for the annular vessel:

$$\lambda_{e} = \frac{J \lambda}{(N_{i} + \Delta N_{i}) - N_{f} \epsilon_{th}},$$  \hspace{1cm} (13)$$

$$\lambda = \frac{1}{T} \ln \left( \frac{(N_{i} + \Delta N_{i})}{N_{f}} \right),$$  \hspace{1cm} (14)$$

where $\Delta N_{i}$ is number of UCN in the inner vessel after cleaning in the annular vessel.

The arrow (1) in Fig. 4a shows the direction of changed position of point for the annular vessel after correction. It gives negative correction for measured value of neutron lifetime. The values of $t_{cl}$ for MC simulation are taken from Table 1 [3]. The results of extrapolations to neutron lifetime are shown in Fig. 4b for different $t_{cl}$ and different neutron guide length in front of the detector which has not been strictly defined in the data of geometry of experiment. By results of simulation it is possible to draw a conclusion that the effect of an incomplete emptying has not been found out, though uncertainty of an estimation of this process is at level of 1 s.

2. Effect of heating of neutrons by shutters. The following non-equivalence of measurements for different vessels is observed at emptying. Before release of neutrons to the detector the shutters 6 and 13 are open. At shutter movement in volume with UCN there is either heating or cooling of UCN depending on a direction of movement of the shutter in relation to UCN gas. In case of emptying from the inner vessel shutters move into a vessel with UCN. There is mainly heating of UCN. In case of emptying
from the annular vessel there is mainly UCN cooling since the shutter escapes from UCN flux. It is necessary to notice that this effect was observed experimentally. The peaks of heated neutrons are visible in the graphs of emptying process (Fig. 5) presented in [21,22]. Unfortunately, the effect has not been considered. It is neither discussed in work [3], nor in detailed work on this experiment [22]. These peaks are connected with UCN heating by shutters and are present only in case of emptying from the inner vessel. Unfortunately, it is not obviously possible to make a numerical estimation from the graphs. Therefore the given process was simulated.

Fig. 5. Effect of heating of neutrons by the shutters.

When we work with the inner vessel the shutters 6 and 13 heat the trapped neutron spectrum after holding. Some part of UCN is lost due to this process. It gives lower value of \( (N_i - N_f) \) and correspondingly higher value of \( \lambda_{\text{ne}} \) for the inner vessel:

\[
\lambda_{\text{ne}} = \frac{J \lambda}{(N_i - N_f)(1 - \delta)} \frac{\varepsilon}{\varepsilon_{\text{sh}}},
\]

(15)

\[
\lambda = \frac{1}{T} \ln \frac{N_i (1 - \delta)}{N_f (1 - \delta)},
\]

(16)

where \( \delta \) is part of neutrons heated by the shutters. The calculations were done with the shutter velocities (\( v_{\text{sh}} \)) of 1 and 2 m/s; the shutter course (\( l_{\text{sh}} \)) of 5, 10 and 15 cm. The arrow (2) in Fig. 4a shows the direction of a changed point position for the inner vessel.
after correction. It gives negative correction for measured value of neutron lifetime. The results of this simulation are shown in Fig. 4c. The correction for effect of UCN heating by shutters is \(-2.9\) s for the shutter velocity of 1 m/s and the shutter course of 10 cm. As there are no detailed data on the shutters we cannot estimate uncertainty of this effect better than 2 s. Thus this correction is \(-2.9\) s with uncertainty of initial data of 2 s.

3. **Effect of not equal thermal neutron detection efficiencies for different vessels.** Another obvious non-equivalence of measurements for different vessels is observed at thermal neutrons detection. The matter is that counters of thermal neutrons do not cover all external surface of the installation. They are absent at the installation end faces. For this reason processes of inelastic scattering occurring at the end faces of traps are registered with geometrical efficiency of about 50%. When neutrons are stored in the inner volume we have 2 end faces (on the left and on the right). But when neutrons are stored in the annular vessel there are 4 end faces (2 on the left and 2 on the right). In addition, the annular vessel is longer than the inner vessel and its end faces are more put forward. Unfortunately the value of this effect in work \([22]\) is underestimated and wrongly considered with an opposite sign. For the estimation of non-equivalence effect in thermal neutrons the simulation of detection process has also been made.

The thermal detector efficiency is lower for the annular vessel because of 4 end faces. It gives lower value of \(J\) and correspondingly lower value of \(\lambda_{\text{th}}\) for the annular vessel:

\[
\lambda_{\text{th}} = \frac{(J - \Delta J) \frac{\lambda}{N_i - N_f} \frac{\varepsilon}{\varepsilon_{\text{th}}}}{N_i - N_f} \varepsilon_{\text{th}},
\]

where \(\Delta J\) is number of not detected thermal neutrons for measurement with the annular vessel. We used mean values for the capture and scattering cross sections of materials of the setup from tables \([23]\). The simulation was done for the thermal neutron detector lengths \((L_{\text{th}})\) of 90, 100 and 110 cm. The arrow (3) in Fig. 4a shows the direction of changed position of point for the annular vessel after correction. It gives negative correction for the measured value of neutron lifetime. The results of this simulation are shown in Fig. 4d.

Geometrically the length of the detector is 100 cm, however its working area, apparently does not exceed 90 cm because of edge effects where devices of fastening of a thread are located. We choose the result of calculation for working length of the detector of 90 cm and for a case of capture and scattering of neutrons in an installation material. In this section we should notice that in work \([22]\) effect of non-equivalence
has been calculated, but the correction (+0.6 s) has appeared underestimated and with the wrong sign. Therefore we have to correct this error. Thus, the correction on effect of not equal thermal neutron detection efficiencies for different vessels is –2.1 s with uncertainty of initial data of 1 s.

Fig. 4a shows that each effect gives negative correction for the measured value of neutron lifetime. The summary table of corrections is shown in Table 2. We assume that after taking into account MC correction and uncertainty the result of work [3] for neutron lifetime could be \(879.9 \pm 0.9_{\text{stat}} \pm 2.4_{\text{syst}}\) s. The resulting corrected value for the neutron lifetime is in agreement with the result \(878.5 \pm 0.8\) s of the work [1].

### Table 2. MC correction on the neutron lifetime result of the experiment [3].

| Effect                                                                                           | Correction, s | Uncertainty, s |
|-------------------------------------------------------------------------------------------------|---------------|----------------|
| Not full emptying of the inner vessel during cleaning while working with the annular vessel     | 0             | 1              |
| Effect of heating of neutrons by the shutters                                                  | –2.8          | 2              |
| Effect of not equal thermal neutron detection efficiencies for different vessels                | –2.1          | 1              |
| Effect of not equal thermal neutron detection efficiencies for different vessels (correction in the experiment is +0.6 s) | –0.6          |                 |
| **Total**                                                                                       | **–5.5**      | **2.4**        |

### III. A detailed analysis of the experiment MAMBO I [5]

#### A. Scheme and method of the experiment [5]

Below we reproduce a short description of the experiment [5]. The setup is shown in Fig. 6. The UCN storage volume is a rectangular box, with constant height 30 cm and width 40 cm but variable length \(x < 55\) cm. The side walls and the roof of the box were made of 5-mm float-glass plates. The oil spray head is mounted on the metal base plate and the assembly is immersed in a 1-mm-deep lake of oil. The movable rear wall, composed of two glass plates with a 1-mm oil-filled gap in between, has a 0.1-mm play with respect to the neighboring walls, except for the base plate where it dips into the oil. The surface of the rear wall was covered with 2-mm-deep, 2-mm-wide sinusoidal
corrugations. For half the surface the wave crests were horizontal, and for the other half vertical. This arrangement transforms within a few seconds the forwardly directed incoming neutron flux into the isotropic distribution essential for the validity of the mean-free-path formula \( \lambda = 4V/S \). The UCN inlet and outlet shutters situated 8 cm above floor level are sliding glass plates with 65-mm holes matching holes in the front wall (Fig. 6). More experimental details can be found in [5].

![Fig. 6. Sketch of the apparatus MAMBO I.](image)

The main idea of the experiment was to deduce the neutron lifetime from an extrapolation, by variation of the mean free path of UCN between wall collisions:

\[
\tau_{st}^{-1} = \tau_{n}^{-1} + \mu(v)v = \tau_{n}^{-1} + \mu(v)v\lambda, \quad (1)
\]

where \( \tau_{st}^{-1} \) is the inversed storage time constant, \( \mu(v) \) is the neutron velocity dependent UCN loss factor per collision, \( v(v) \) is the frequency of collisions, \( \mu(v)v(v) \) is the probability of UCN losses, \( \lambda \) is the mean free path, and the relation \( \lambda = 4V/S \) holds for an isotropic and homogeneous particle population in a trap of volume \( V \) and surface area \( S \). Formula (1) shows that the inverse storage time is a linear function of inverse mean free path. The extrapolation of \( \tau_{st}^{-1} \) to zero frequency of collisions will give the probability of neutron \( \beta \)-decay.

Unfortunately, a correct extrapolation is impossible for the case of a wide UCN spectrum, which changes its form during the storage process due to the velocity
dependence of $\mu(v)$ and $\nu(v)$. To reduce the influence of this spectral dependence, it
was proposed in [5] to fix the number of collisions for different trap sizes by a suitable
choice of the UCN holding time. This leads to the following scaling relations,

$$\frac{t_2(i)}{t_2(j)} = \frac{t_1(i)}{t_1(j)} = \frac{\lambda(i)}{\lambda(j)} = \frac{t_2(i) - t_1(i)}{t_2(j) - t_1(j)},$$  \hspace{1cm} (2)

where $t_1$ and $t_2$ are two different UCN holding times used to determine $\tau_{\text{UT}}^{-1}$, and the
indices $(i, j)$ correspond to different volumes. Unfortunately, even when the scaling
conditions are fulfilled, the extrapolation is faked due to the gravitational field. The

Quasi-elastic scattering of UCN on the surface of liquid fomblin changes the UCN
spectrum and therefore also has to be taken into account separately. A similar problem
arises due to above-barrier neutrons. In this article we demonstrate the importance of
this additional correction which was not taken into account in [5].

B. Monte Carlo simulation of the experiment [5]

The relative importance of a particular effect (above-barrier neutrons or quasi-
elastic scattering) to the final result can be investigated by switching it off in the
simulation. Lacking the knowledge of the temperature at which the experiment [5] was
carried out, we have performed our full analysis for 10°C, for which an analytical
description of the model for quasi-elastic scattering of UCN is available in [24].

The calculations were done on computing clusters, in total lasting for about
several months. In the following all detailed results are given for a reference volume
with length $x = 55$ cm (see previous section) unless stated otherwise. Neutron reflection
by the corrugated surface was approximated by 50% specular and 50% diffuse
reflections from flat walls, if not stated differently. The time intervals of UCN holding
were chosen the same as in the experiment. In all our simulations neutron lifetime was
fixed to a definite value. The calculated corrections to the neutron lifetime extrapolated
from our MC simulation were found to depend only weakly on the chosen value for $\tau_n$.

Fig. 7 shows for illustration some simulated data in comparison with experimental
ones. It demonstrates that our model shows dependence similar to the experimental.
Fig. 2 from article [5] shows typical behavior only, without specified experimental
conditions (for example, the surface structure of the movable wall). Without detailed
experimental information we cannot reach full agreement with Fig. 2 from [5]. It should
be explained that the most important results are the extrapolated neutron lifetime values.

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Therefore it is more correct to compare the behavior of the extrapolated \( \tau_n \) as function of holding time intervals.

![Graph showing measured inverse bottle lifetimes as a function of the bottle inverse mean free path and for different holding intervals. Experimental data from [5] are shown as dotted lines, while results of simulation are shown by solid lines.](image)

**Fig. 7.** Measured inverse bottle lifetimes as a function of the bottle inverse mean free path and for different holding intervals (dotted lines). Experimental data stem from [5], which were quoted there as typical. Results of simulation are shown by solid lines.

Fig. 8 is shown as a benchmark test of the extrapolated \( \tau_n \). One can see that we can reproduce the behavior of the extrapolated \( \tau_n \) from holding time intervals for different type of the surface of the movable wall due to changes of specular reflectivity. The main part of measurements of [5] has been carried out with corrugated surface because in Table 1 of [5] just this data is shown. In our future analysis we use 50% specular and 50% diffuse reflections that describes reasonably the corrugated surface of the movable wall.

Next we investigated the dependence of simulation results on the initial UCN spectrum, which experimentally was known only poorly. Fig. 9 demonstrates that this dependence is rather weak, particularly for the most important points with long holding time (but except for low holding time which anyway had negligible statistical weight in the result presented in [5], see also Table 3 further below). The corrections at the short
holding time are proportional to amplitude of spectrum at critical energy 108 neV. In our future calculations we used the spectrum 4 shown by solid line in Fig. 9.

The effect of the spectral changes during the storage process is shown in Fig. 10. One can see that quasi-elastic scattering changes the form of the UCN spectrum considerably. Such changes are important as they can cause a systematic error. Above-barrier neutrons can be stored for a long time, particularly if the energy is near the critical one.

The results of extrapolations to neutron lifetime are shown in Fig. 11 for different settings of the simulation in- or excluding different effects. Excluding above-barrier neutrons and quasi-elastic scattering (curve 1) we can study the gravitational correction separately. Bigger volumes have larger relative area of the bottom plate and hence more UCN collisions with higher energy due to gravity, resulting in lower values of extrapolated neutron lifetime. The gravitational correction is practically independent from the UCN holding time. The extrapolated neutron lifetime is found lower than the
neutron lifetime by $7.5 \pm 0.3$ s. This result is similar to the gravitational correction introduced in the work [5].

![Graph showing Simulation Results](image)

Fig. 9. Results of simulations with different initial UCN spectra in the trap. $E_c$ is critical energy of fomblin oil (108 neV).

The next simulation shown in Fig. 11 was done including above-barrier neutrons in the UCN spectrum but without quasi-elastic scattering (curve 2). One can see that for short holding time the extrapolated neutron lifetime is much higher in comparison with the previous case for the gravitational correction (curve 1), but for long holding time the extrapolated neutron lifetime comes rather close to it. However, note again that the
contribution of results with short holding time in the final result of [5] is very small because of poor statistical accuracy of these measurements. The points with a holding time of (900-1800) s and (1800-3600) s bring the main contribution.

Fig. 10. UCN spectra in the trap after different holding intervals without taking into account quasi-elastic scattering (dotted lines), and taking into account quasi-elastic scattering (solid lines). $E_c$ is critical energy of fomblin oil (108 neV).

The next simulation shown in Fig. 11 was done taking into account quasi-elastic scattering but without above-barrier neutrons (curve 4). One can see that with increasing of holding time the extrapolated $\tau_n$ is increased due to appearance of new above-barrier neutrons.

The next simulation shown in Fig. 11 was done taking into account quasi-elastic scattering and above-barrier neutrons (curve 3). One can see that due to appearance of new above-barrier neutrons from quasi-elastic scattering the extrapolated $\tau_n$ cannot reach curve 1 at the long holding times. As result curve 3 has independence from the long holding times. This independence was interpreted in [5] that process of cleaning from above-barrier neutrons is finished and extrapolated $\tau_n$ at the long holding times can be accepted as a correct value. Unfortunately, it is not true due to effect of quasi-elastic scattering.
Fig. 11. Results of MC simulations of the extrapolated neutron lifetime for different holding intervals. (1) without quasi-elastic scattering and without above-barrier neutrons, (2) without quasi-elastic scattering and with above-barrier neutrons, (3) with quasi-elastic scattering and with above-barrier neutrons, (4) with quasi-elastic scattering and without above-barrier neutrons. The difference between the curves 1 and 3 provides the correction due to above-barrier neutrons and quasi-elastic scattering which was not taken into account in work [5].

The difference between curves 1 and 3 is the total effect due to above-barrier neutrons and quasi-elastic scattering. These effects were not taken into account in the work [5]. In the work [5] two corrections were introduced: gravitational correction (+0.6%) and correction connected with the small differences in the initial spectra depending on the bottle size (+0.3%). Table 3 quotes data from the work [5] with our additional corrections due to above-barrier neutrons and quasi-elastic scattering effects. In this table we use the data for long holding time intervals which bring the main contribution and do not depend from entry conditions (the form of initial spectrum, diffusion of a covering). As total correction we find $-6.0 \pm 1.6$ s. Our correction is negative one and roughly compensates corrections from [5]. The systematic uncertainty
in [5] is estimated about 3 s. It can cover substantially a lack of the information on experiment details. The resulting corrected value for the neutron lifetime would agree with the result 878.5 ± 0.8 s of the work [1].

Table 3. Results for neutron lifetime $\tau_n$ obtained from different holding intervals: $\tau_n$ is the result from work [5], $\Delta\tau[5]$ is the correction from [5], $\Delta\tau$(this article) is the correction due to above-barrier neutrons and quasi-elastic scattering calculated in this work, and $\tau'_n = \tau_n$ (corrected [5]) + $\Delta\tau$ (this article).

| holding interval, s | $\tau_n$, s (uncorrected [5]) | $\Delta\tau$, s [5] | $\tau_n$, s (corrected [5]) | $\Delta\tau$, s (this article) | $\tau'_n$, s |
|---------------------|--------------------------------|---------------------|-----------------------------|-------------------------------|-------------|
| 112.5-225           | 893(10)                        | ~−2                 | 891(10)                     | −7.84 (0.87)                   | 881.36 (2.65) |
| 225-450             | 885.0(4)                       | +3.5                | 888.5(4)                    | −5.29 (0.70)                   | 881.71 (1.65) |
| 450-900             | 881.2(2.5)                     | +8                  | 889.2(2.5)                  | −5.54 (0.87)                   | 881.56 (2.74) |
| 900-1800            | 878.0(1.5)                     | +9                  | 887.0(1.5)                  |                               |             |
| 1800-3600           | 878.5(2.6)                     | +8.6                | 887.1(2.6)                  |                               |             |

$\tau_n$=887.6(1.1) s  $\tau'_n$=881.6(1.2) s

IV. Conclusion

Now it is necessary to show a new table of results of the neutron lifetime measurements taking into account the correction of work [3], and also works [5] and [25]. We also included in the table the result of the experiment MAMBO II [26] based on the experiment MAMBO I. It uses UCN spectrum with a cutoff below the Fomblin critical energy. Because of this the systematic error of MAMBO I experiment was suppressed. Work [9] can be withdrawn from the list since a new much more accurate result has been obtained on this installation using low-temperature Fomblin rather than solid oxygen. The difference between an earlier result and a new one is 2.9 standard deviations. It is reasonable to withdraw the previous result due to obtaining a new more accurate result. Finally, it is necessary to include result [27]. Then after corrections and additions the table of experimental results for neutron lifetime looks as follows (Table 4, Fig. 12). The standard error of average value from Table 4 is 0.6 s, but the standard deviation of experimental results is 0.9 s. Thus, it will be expedient to accept as the world average value for the neutron lifetime 879.9 ± 0.9 s.
Table 4. The table of the experimental results for the neutron lifetime after corrections and additions.

| $\tau_n$, s | Author(s), year, reference |
|-------------|----------------------------|
| 881.5 ± 2.5 | V. Morozov et al. 2009 [27] |
| 878.2 ± 1.9 | V. Ezhov et al. 2007 [4] |
| 878.5 ± 0.7 ± 0.3 | A. Serebrov et al. 2005 [1] |
| 886.3 ± 1.2 ± 3.2 | M.S. Dewey et al. 2003 [6] |
| 879.9 ± 0.9 ± 2.4 | C. Arzumanov et al. 2000 [3] |
| 881.0 ± 3 | A. Pichlmaier et al. 2000 [26] |
| 889.2 ± 3.0 ± 3.8 | J. Byrne et al. 1996 [7] |
| 882.6 ± 2.7 | W. Mampe et al. 1993 [8] |
| 893.6 ± 3.8 ± 3.7 | J. Byrne et al. 1990 [10] |
| 881.6 ± 3.0 | W. Mampe et al. 1989 [5,25] |
| 872 ± 8 | A. Kharitonov et al. 1989 [11] |
| 878 ± 27 ± 14 | R. Kossakowski et al. 1989 [12] |
| 877 ± 10 | W. Paul et al. 1989 [13] |
| 891 ± 9 | P. Spivac et al. 1988 [14] |
| 876 ± 10 ± 19 | J. Last et al. 1988 [15] |
| 870 ± 17 | M. Arnold et al. 1987 [16] |
| 903 ± 13 | Y.Y. Kosvintsev et al. 1986 [17] |
| 937 ± 18 | J. Byrne et al. 1980 [18] |
| 881 ± 8 | L. Bondarenko et al. 1978 [19] |
| 918 ± 14 | C.J. Christensen et al. 1972 [20] |

Finally, we have to mention that analysis of neutron $\beta$-decay with new world average neutron lifetime demonstrates reasonable agreement in frame of Standard Model. Fig. 13 shows this analysis which is discussed in detail in [28,29]. Fig. 13 shows a dependence of the CKM matrix element on the values of the neutron lifetime and the axial coupling constant $g_A$. The value $|V_{ud}| = 0.9743(7)$, calculated for the new world average value for the neutron lifetime 879.9(9) s and $g_A = 1.2750(9)$ [30], agrees with both $|V_{ud}| = 0.97419(22)$ from the unitarity of the CKM matrix elements [2] and $|V_{ud}| = 0.9738(4)$, measured from the superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$-decays, caused by pure Fermi transitions only [30,31].
One can see that the value $|V_{ud}| = 0.9711(6)$, calculated for the old world average value for the neutron lifetime 885.7(8) s, is ruled out by the experimental values $|V_{ud}| = 0.9738(4)$ and $|V_{ud}| = 0.97419(22)$.

Fig. 12. Distribution of results of measurements for the neutron lifetime, giving average value of 879.9 ± 0.9 s.

Fig. 13. The dependence of the CKM matrix element $|V_{ud}|$ on the values of the neutron lifetime and the axial coupling constant $g_A$. Determination of $|V_{ud}|$: (1) from neutron β-decay data, (2) from unitarity, (3) from $0^+ \rightarrow 0^+$ nuclear transitions.
Besides, it should be mentioned that detailed analysis of the nucleosynthesis process in the early stages of the formation of the Universe was made [32]. They analyzed the effect of the new value of the neutron lifetime on the consistency of data on the initial abundances of D and $^4$He isotopes and the data on baryon asymmetry $\eta_0$. The use of the new value of the neutron lifetime improves the agreement between the data on the initial abundances of deuterium and helium, and those on baryon asymmetry. Although the accuracy of the cosmological data is much lower than that of measurements of the neutron lifetime, the shift of $\tau_n$ from the world average value to the new value has a certain effect on the verification of the nucleosynthesis model in the early stages of the formation of the Universe.

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