Numerical modeling of electric arc motion in external constant magnetic field

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Abstract. The paper generalizes to the three-dimensional case the previously proposed two-dimensional mathematical model of the motion of a plasma formation in a constant transverse magnetic field. The complete system of single-fluid magnetogasdynamic approximation of the equations of the hydrodynamic method for plasma description under conditions of local thermodynamic equilibrium is simplified in the electromagnetic part. Instead of solving the complete system of Maxwell's equations, Ohm's integral law is applied to the conducting region with usage of empirical data. Earlier studies in a two-dimensional version of this model as applied to the problem of describing the motion of an arc showed its effectiveness and efficiency for the class of problems under consideration. The simulation results with proposed extended model are compared with previously obtained calculated and experimental data. Main integral and local features of flow structure are highlighted.

1. Introduction

Earlier, in [1], [2], experimental and computational studies of a plasma actuator were carried out, using a moving electric arc in an external magnetic field in air at atmospheric pressure to control the flow in the boundary layer. Various mathematical flow models based on the source approach and two-dimensional thermally equilibrium magnetogasdynamic approximation with a simplified version for the electromagnetic field in the form of Ohm's law were also constructed there. The global goal of these works was the development of effective plasma actuators that could be used to control the flow in the boundary layer, including in aircraft engines, for a wide range of flow rates. The calculation approach used in these works, however, has a number of fundamental limitations and is not universal. Therefore, in the present work, an attempt was made to improve the mathematical model describing the plasma formation, more precisely, generalize it to the three-dimensional case, and validate it on the results already obtained. A more detailed description of the experimental data used, measurement procedures, and also the statement of the problem of arc motion can be found in [1, 2].
The topic of modeling a moving electric arc has been the subject of many works; here are some of them [3-7]. In these works, as in many others, various forms of Maxwell's equations are used to model the arc. The use of Maxwell's equations, even in a reduced form, is associated with the need to describe the near-electrode layers, the mechanisms of attachment/reattachment of the arc to the electrodes, and many other phenomena. In many ways, these problems can be successfully solved, but not always for a specific class of problems, the complication of the computational model is justified. In this regard, the previously proposed model is a compromise combining simplicity and sufficient accuracy for the applications under consideration. Which primarily include plasma actuators for internal and external flows in technical devices. Additional data on the models used in the field under consideration and the chronology of their development can be found in [1, 2], as well as in references compiled there.

The work is structured as follows: in the first section, a mathematical model of the process of arc motion is formulated, based on the MHD approximation. Moreover, it is assumed in the work that the main assumptions of the applicability of the MHD approximation for the problem under consideration are fulfilled. In the second section, methods of generalizing the previously developed model to the three-dimensional case are described. Data on the thermophysical properties of air in the approximation of local thermodynamic equilibrium (LTE) used in this work can be found in [1]. The third section presents the results of calculations using the developed model, their comparison with experimental data and their discussion. At the end, the main conclusions of the work are formulated.

2. Governing equations

2.1. Gas dynamic part of full MHD approximation system of equations

The most rigorous derivation of the equations of the MHD approximation of the plasma description is based on the Boltzmann kinetic equations for each type of particles (charged, non-charged, electrons) or on Liouville's theorem. One can find details of derivation procedure in any classical textbooks, for example, [8]. The resulting MHD approximation system is quasilinear and has a degenerate parabolic type. Moreover, the second derivatives in the system responsible for parabolicity describe dissipative processes - thermal conductivity, viscosity and non-zero electrical resistivity. The system of equations can also be obtained as a generalization of the Navier-Stokes equations for a conducting medium in an electromagnetic field. In this case, a source term appears in the momentum equation, which describes the Ampere force acting on the conducting region. A source term appears in the energy equation describes the Joule-Lentz heating of the conductive region. Then the gas-dynamic part of the MHD approximation system in a conservative form has the following form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0,$$

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \otimes u) = -\nabla p + \nabla \cdot \tau + J \times B,$$

$$\frac{\partial p e_0}{\partial t} + \nabla \cdot (\rho e_0 u) = \nabla \cdot (\lambda \nabla T) - \nabla \cdot (p u) + \nabla \cdot (u \cdot \tau) + \chi_{eff} \sigma_{SB} T^4 + J \cdot E,$$

where \(u = u(r, t)\) is the velocity vector, \(r \in \Omega, t \in [0, T]\), computational domain denoted by \(\Omega, [0, T]\) is the finite integration time interval for system (1)-(3), \(p, \rho, T > 0\) are pressure, density and temperature, \(J\) is the current density vector, \(B\) is the magnetic induction vector, \(e_0 = e + 0.5 u^2\) is the specific total energy, \(\lambda = \lambda(p, T)\) is the thermal conductivity coefficient, \(E\) is the electric strength vector, \(\chi_{eff} = \chi_{eff}(p, T)\) is the absorption coefficient, \(\sigma_{SB}\) is the Stefan-Boltzmann constant, \(\tau\) is the viscous stress tensor, including the dynamic viscosity \(\mu = \mu(p, T)\). To close the system (1-3), in addition to the relations for \(E\) and \(J\), we must add the equation of state, in this paper this equation of state of an ideal gas \(p = \rho R T M^4\), where \(R\) is the universal gas constant, \(M\) is the molar mass of the mixture. Radiation losses are taken
into account in the energy equation using the source term. When deriving equations (1) - (3), a large number of different assumptions are used, a fairly complete list of which can be found, for example, in [8]. Here we point only on the assumption of isotropy of dissipative processes, which allows scalar rather than tensor quantities to be used for transport coefficients.

2.2. The electromagnetic part of the MHD approximation system of equations

Estimates of the magnetic Reynolds number show that for laboratory scales of the length and intensities of the current density vector, the intrinsic magnetic field generated by charges in the computational domain can be neglected. Thus, when writing the Maxwell equations in the form of potentials, we can only consider the elliptic equation with respect to the scalar electric potential. Earlier, in a two-dimensional formulation, a number of simplifying assumptions were applied to the problem in question in the electromagnetic part, such as: \( \mathbf{B} \perp \mathbf{J}, |\mathbf{B}| = \text{const}, \, \mathbf{E} \uparrow \uparrow \mathbf{J}, \) \( |\mathbf{E}| = E_i = E(t), \) here \( E_i \) is the component of electric strength, perpendicular to the considered plane. The only nonzero projections of the vectors \( \mathbf{E}, \mathbf{J} \) can be obtained from experimental data using the one of the simplest differential form of Ohm's law for the conducting volume:

\[
I(t) = \int_S \mathbf{J} \cdot d\mathbf{S} = E_i \int_S \sigma d\mathbf{S},
\]

(4)

here \( \sigma = \sigma(p, T) \) is the scalar electrical conductivity of the medium, the form of the function \( I(t) \) can be determined from experiment. In general case, Ohm law includes many other terms, which is neglected in (4). For the regime and conditions from [1-2], \( I(t) = I_0 \sin(\pi T/3) \), where \( I_0 = 47 \) A is the amplitude of the current pulse, \( T = 130 \) \( \mu \)s is the pulse duration, \( I(t=T)=0 \), and also \( |\mathbf{B}| = 0.33 \) T. Moreover, for the two-dimensional case, \( \mathbf{E} \) does not depend on the coordinates. Thus, one of the possible options for extending the model to the three-dimensional case is a generalization of the integral on the right-hand side of (4). It is implicitly assumed that the conductive region remains single connected.

2.3. Calculation of electric strength

Let introduce the coordinate system so that its axis \( OZ \) is aligned with the vectors \( \mathbf{J} \) and \( \mathbf{E} \), and the cross section of the arc lies in the XOY plane (see Fig. 2). To calculate the conductivity (resistance), the surface integral is used for a two-dimensional statement. In this case, the boundary of the hot conducting region can be determined by the condition \( \sigma(x, y) > \sigma_0 \), here \( \sigma_0 \) is small enough, for example, the temperature isosurface \( T = 2000 \) K at \( p=1 \) atm corresponds to \( \sigma = 1.06 \times 10^6 \) S/m for air in LTE. That is, the area of integration can be defined as \( \Pi^1 = \{(x, y, z): \sigma > \sigma_0\} \). We construct a set of parallel planes \( P_i \parallel XOY \), here \( i = 1.. N_s \). \( N_s \) is the number of planes and introduce two-dimensional cross sections of the hot region \( \Pi_i = \Pi^1 \cap P_i \), obviously for each of them \( \Pi_i = \{(x, y, z): \sigma > \sigma_0\} \). Let also the length of the arc (hot region) in the \( OZ \) direction be equal to \( L \). Consider the possible ways of representing the generalization of the integral \( S = \iiint \sigma d\mathbf{S} \) to the three-dimensional case:

- Quasiplane option. Assuming that the conductivity does not change along \( OZ \), the value of the integral \( S \) does not depend on the position of the cross section. And in any section \( P_j \parallel XOY \), the integral \( S \) can be obtained from the relation:

\[
V = \iiint \sigma d\mathbf{V} = \iiint \sigma d\mathbf{z} dS = \int_0^{q_L} dz \iiint \sigma d\mathbf{S} = L \iiint \sigma d\mathbf{S} = L \cdot S,
\]

\[
S = \frac{1}{L} \iiint \sigma d\mathbf{V},
\]

(5)
Integration over sections. If we assume that $\sigma$ depends on $z$, then for each section the value of the integral may not coincide, $E_z = E_z(z)$. In this case, to move from the surface integral to the volume one, we can take a sufficiently thin layer, inside which we can assume that $\sigma$ is independent of $z$. Moreover, it follows that if the domain $\Omega$ is discretized for numerical integration (1)-(3), then as layers it is convenient to take the sets of elements of the partition $\{E_j\}$, $E_j \cap P_i \neq \emptyset$. Then, the calculation of the integral is reduced to a set of quasi-plane variant expression elements for each layer $j$, that is, the integral

$$S_j \approx \frac{1}{\Delta z_j} \sum_i \sigma(x_{i,j}, y_{i,j}, z_{i,j}) V_i,$$

(6)

here $\Delta z_j$ is the thickness of the layer $j$ in the OZ direction, $i$ is the index of the element $E_i \in \Pi_j$, $V_i$ is the volume of the element $E_i$. Using the value of the integral $S_j$ in each layer, $E_z$ is calculated, assuming that the modulus of the current density vector in each section is constant.

Let us consider the definition of strength on two consecutive time layers corresponding to $t_n$ and $t_{n+1}$, near the boundary of the region $\Pi^3$. If the energy flow through the common face of two neighboring elements $E_i$ and $E_k$, $i \neq k$ is small and $T_{n+1} > T_{k,n}$, then $T_{n+1} > T_{k,n+1}$ and therefore $\sigma_{n+1} > \sigma_k$ and $\sigma_{n+1} > \sigma_{k,n+1}$, since $\sigma(T)$ is a monotonically increasing function of $T$ in the temperature range under consideration. Continuing further the chain, if neighboring cells are in the same layer, then for $\sigma_{n+1} > \sigma_k$, the integral $S_{n+1,j}$ can be either greater than $S_j$, $n$ or less. However, if the elements $E_i$ and $E_{j+1}$ are located in two adjacent sections $j$ and $j+1$ on the boundary of the region, then from $\sigma_{j+1} < 0$, $\sigma_j > 0$ it follows:

$$\left( \iint_{E_j} \sigma_i dS \right)_{n+1} < \left( \iint_{E_{j+1}} \sigma_i dS \right)_{n+1},$$

(7)

which directly implies that $(E_j)_{n+1} < (E_{j+1})_{n+1}$. That is, in the direction perpendicular to the planes $P_j$ in the boundary cells, an abrupt increase in $E_z$ will be observed from near zero values to $\sim (S_{j,n})^{-1}$ regardless of the size of the time step. This jump is associated with discontinuous of initial conditions (determination of a hot region at $t = 0$) and an algebraic relation for strength. In order to mitigate this drawback in the calculation of strength, a limiter was implemented based on (7), conditions (7) were

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**Figure 1.** Hot channel sections scheme.
checked on each time layer in the boundary layers of the elements, and when condition (7) was fulfilled, \( S_{j,n+1}=0 \), that is, in the direction perpendicular to sections \( \{P\} \) the arc could propagate only under conditions \( T_{i,n}>T_{k,n}, T_{i,n+1}<T_{k,n+1} \).

2.4. Computational domain and initial/boundary conditions

The computational domain was a rectangular parallelepiped \( \Omega^3: \{ (x,y,z) \mid -H \leq x \leq H, 0 \leq y \leq H, 0 \leq z \leq 0.5H \} \), here \( H = 0.04 \text{ m} \). A diagram of the computational domain and the directions of the main vector quantities are shown in Fig. 2. Initially, the area is filled with quiet air under atmospheric conditions. In the experiments, a constant magnetic field is created by a permanent magnet placed under the surface with electrodes, the direction of magnetic field induction \( \mathbf{B} \) is shown in Fig. 2. It is assumed that up to the time \( t=0 \), between the parallel electrodes \( E_1, E_2 \) mounted on the lower surface, a voltage difference is applied and a breakdown is organized. Moreover, at time \( t=0 \), the hot channel after breakdown is formed and has the shape of a cylinder. Further, under the action of the Ampere force, \( \mathbf{F}_A \), the conducting region moves in the positive direction of the \( \text{X} \) axis.

![Diagram](image)

**Figure 2.** Computational domain scheme, not to scale.

To solve the initial-boundary-value problem for equations (1-3), the initial conditions of the hot region (channel) were set, that is, the position of the arc immediately after the breakdown of the gap between the electrodes:

\[
\begin{align*}
\mathbf{u}(r,0) &= 0, r \in \Omega, \\
 p(r,0) = \begin{cases} 
 p_1, & |r - r_0| \leq R_0, \\
 p_0, & |r - r_0| > R_0, 
\end{cases} \\
 T(r,0) = \begin{cases} 
 T_1, & |r - r_0| \leq R_0, \\
 T_0, & |r - r_0| > R_0, 
\end{cases}
\end{align*}
\]

here \( r_0=\langle -0.013, 0.00025, z \rangle, R_0=0.00025 \text{ m}, p_0=101325 \text{ Pa}, p_1=30p_0, T_1=7000 \text{ K}, T_0=293 \text{ K} \) and Dirichlet boundary conditions for pressure \( p \) and velocity \( \mathbf{u} \) and Neumann for temperatures \( T \) were applied:

\[
\begin{align*}
 p(r,t) &= p_0, r \in \partial\Omega \setminus \Gamma, \\
 \mathbf{u}(r,t) &= 0, r \in \Gamma, \\
 T &= T_0, r \in \Gamma.
\end{align*}
\]
For numerical solution of gas dynamic part of the full system ANSYS Fluent software was used. Electromagnetic part were implemented as additional UDF C library.

3. Results and discussion
Calculations in a three-dimensional formulation were performed for the same regime as in [1,2]. Figure 3 shows the evolution of the longitudinal coordinate of the hottest point of the hot channel and the strength for two-dimensional, three-dimensional calculation and experimental data. For three-dimensional calculation, only the results of the variant with integration over the sections are shown; two-dimensional calculations performed by various authors are indicated by the numbers "1" and "2". As can be seen from the figure, the three-dimensional calculation slightly overestimates the coordinate value for the second half of the current pulse, both with respect to the experimental data and with respect to two-dimensional calculations. Which, most likely, is connected with the assumption used about the constancy of the direction of the current density vector in all sections. At the same time, three-dimensional results give values of tension comparable with two-dimensional calculations.

![Figure 3](image3.png)  
**Figure 3.** Dependence of the longitudinal coordinate of elemental fluid volume with maximum temperature on the time: simulations and experiment.

![Figure 4](image4.png)  
**Figure 4.** Evolution of the electric strength (average): simulations and experiment.

In [2], an analysis of the grid convergence for a two-dimensional problem was carried out and it was shown that starting from the cell size \( l=60\ldots70 \text{ μm} \), the local flow characteristics almost doesn’t change. In order to achieve such resolution in the three-dimensional case, a grid with \( l \approx 20 \text{ million cells} \) and \( l=70 \text{ μm} \) was used (the computational costs for a single calculation regime is \( \approx 25,000 \text{ core-hours} \), single core is \( \approx 80 \text{ GFlops LINPACK} \)). Calculations on coarser grids show a rather large dependence of the local flow parameters on the cell size. At the same time, integrated flow characteristics, such as the velocity of the hottest point with good accuracy, up to 15%, can be obtained on coarser grids with \( l=100\ldots110 \text{ μm} \).

As a comparison of calculations using a two-dimensional model, a three-dimensional model, and experimental data, Fig. 5 shows density fields and experimental flow images for time \( t=50 \text{ μs} \). For the three-dimensional case, the field is shown in the middle plane, which most closely corresponds to the two-dimensional calculation. As can be seen from the figure, the flow structure in the wake behind the conducting region inside the thermal cavity for the two-dimensional and three-dimensional formulation is slightly different. At the same time, in a three-dimensional formulation, the structure of this region is closer to the experimental one shown in the schlieren image. Thus, perturbations are manifested, propagating along direction perpendicular to the plane of the figure, completely neglected in a two-dimensional formulation.
A generalization to the three-dimensional case of a simplified mathematical model of the motion of a plasma formation in a constant transverse magnetic field is proposed [1,2]. The results of modeling using a three-dimensional model with previously obtained two-dimensional calculated and experimental data show that according to the integral characteristics, such as the speed of the arc (elementary volume with maximum temperature) and the strength, the proposed generalization has an error comparable with two-dimensional calculations. At the same time, three-dimensional calculations overestimate the speed of the arc relative to experimental measurements. Moreover, the local structure of the conducting region is similar for all calculation options. It is shown that due to the use of the algebraic relation for strength and discontinuous initial data, in calculations in the three-dimensional case, abrupt changes in flow parameters can occur due to spatial non-isotropy introduced by integrating the relation for the electrical conductivity of the hot layer. Simple procedure, based on limiter approach, was introduced to avoid this drawback of strength calculation.

A further development of the proposed model may be to take into account the spatial heterogeneity of the directions of elementary vectors of current density along the OZ direction and in each section of the arc.

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