Beliaev damping of quasi-particles in a Bose-Einstein condensate

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We report a measurement of the suppression of collisions of quasi-particles with ground state atoms within a Bose-Einstein condensate at low momentum. These collisions correspond to Beliaev damping of the excitations, in the previously unexplored regime of the continuous quasi-particle energy spectrum. We use a hydrodynamic simulation of the expansion dynamics, with the Beliaev damping cross-section, in order to confirm the assumptions of our analysis.

In a Bose-Einstein condensate (BEC) collisions with distinguishable excitations (impurities) and collisions with indistinguishable excitations (quasi-particles) differ profoundly. These differences follow from the quantum exchange symmetry of the indistinguishable excitations, and from a different excitation spectrum.

Impurity collisions within a BEC have been measured previously. Using Raman spectroscopy the microscopic onset of superfluidity was measured and found to be in general agreement with prediction [1]. The collisional dynamics and interaction between two distinguishable slowly moving condensates was measured [2] and found to agree with simulation [3]. Macroscopic superfluid behavior has also been demonstrated, involving the interaction of the condensate with large-scale optical dipole-potential structures [4], [5].

The case of identical particle collisions has been extensively studied using many-body theory, starting with [6]. Recently, these results have been applied to BEC explicitly [7], [8], [9], [10]. In this letter we present a measurement of collisions between quasi-particles and the BEC, at velocities near and above the superfluid critical velocity $v_c$.

According to the Fermi golden rule, the rate of scattering within a homogenous BEC is given by [11]:

$$n\sigma \nu_k = 2^{7/2}n a^2 v_c \int dq d\Omega q^2 |A_{q,k}|^2 \delta(E_i - E_f) \tag{1}$$

where $a$ is the s-wave scattering length. The wavenumber $k$ of the excitation is in units of $\xi^{-1} = \sqrt{8\pi na}$, the inverse healing length of the condensate. The free particle velocity of the excitations is $v_k = \hbar k \xi^{-1}/m$, where $m$ is the mass of the BEC atoms, $n$ is the density of the condensate. The superfluid critical velocity is $v_c = \sqrt{\mu/m}$, where $\mu = gn$ is the chemical potential of the BEC, and $q$ is $4\pi\hbar^2 a/m$. The integral is over all possible momentum transfers $q$, and all angles $\Omega$. The $\delta$-function requires energy conservation between the initial energy $E_i$ and the final energy $E_f$ after collision, where we express energy in units of $\mu$. The factor $A_{q,k}$ is the $q$-dependent, momentum conserving, scattering matrix element of an excitation of wavenumber $k$, which may include suppression or enhancement of the collision process due to many-body effects.

For impurity scattering within a condensate [1], $|A_{q,k}|^2$ is given by the structure factor $S_q$ [12]. The integral energy $E_i = E^0_k$ where $E^0_k = k^2$ is the impurity dispersion relation, for impurities with mass $m$. The final energy for this process is $E_f = E^0_{k-q} + E^B_q$, where $E^B_q = \sqrt{q^4 + 2q^2}$ is the recently measured Bogoliubov dispersion relation [13].

Below $v_c$, collisions between the impurity and condensate are completely suppressed (see Fig. 1) by the $\delta$-function requiring conservation of energy and momentum.

![FIG. 1. Cross-section for collisions in a homogenous condensate. The momentum is in units of the inverse healing length, $\xi^{-1} = \sqrt{8\pi na}$. The cross-section is in units of the free particle scattering cross-section for identical particles $8\pi a^2$. The solid line is the theoretical curve, from Eq. (3) for quasi-particles travelling through the condensate. The dashed line is the cross-section for impurity scattering within the condensate [1], which vanishes below the superfluid critical velocity $v_c$. The inset shows the allowed momentum manifold for quasi-particle collisions due to conservation of energy and momentum. The $q_\parallel$ and $q_\perp$ axes correspond to the parallel and orthogonal components of the scattered momentum respectively, where $tan(\theta) = q_\perp/q_\parallel$. The manifolds represent the experimental k’s 4.09 (outermost), 2.63, 1.73 and 1.06 (innermost).](image)
tion \( k \) and the scattered direction \( q \), to be (see inset of Fig. 1):

\[
\cos(\theta) = (2kq)^{-1} \left[ k^2 + q^2 + 1 - \sqrt{1 + (E_k^B - E_q^B)^2} \right]^{-1}
\]

(2)

This result differs in a qualitative way from impurity scattering, since Eq. (2) has solutions for any finite \( k \). There is no longer any well-defined critical velocity at which collisions are completely suppressed. However, not all angles are allowed. At a given \( k \) we find that the maximal allowed angle is \( \cos(\theta_{\text{max}}) = \sqrt{(k^2 + 2)/2}/(k^2 + 1) \). At the limit of small \( k \), this angle approaches zero, and collisions are allowed only for \( q \) parallel to \( k \).

The appropriate suppression term \( |A_{q,k}|^2 \) for quasi-particles has been calculated \[7, 8\]. In this work we expect mainly Beliaev processes which involve creation of lower energy excitations. The Landau damping rate is expected to be an order of magnitude slower than the observed Beliaev collision process \[8\].

We start with the atomic interaction Hamiltonian

\[
H = \frac{\hbar^2}{2V} \sum_{j,l,m,n} a_j^\dagger a_j a_m a_n \delta_{j+l-m-n}, \quad \text{where} \quad V \text{ is the volume of the BEC}, \quad a_j^\dagger \text{ and } a_j \text{ are the atomic creation and annihilation operators at wavenumber } j.
\]

We approximate \( a_j^\dagger \approx a_0 \approx \sqrt{N_0} \), where \( N_0 \) is the number of atoms in the condensate. We take the Bogoliubov transform \( a_j^\dagger = (u_j b_j^\dagger - v_j b_j) \), with \( u_j \) and \( v_j \) the appropriate quasi-particle amplitudes, which were recently measured \[15\]. We are interested in terms of the form \( b_{k} b_{k-q} b_{q}^\dagger \), that remove a quasi-particle of wave number \( k \), and create two in its stead. Calculating the matrix element prefactor of this term in the atomic interaction Hamiltonian, we arrive at

\[
A_{q,k} = \frac{1}{V} (S_q + 3S_kS_{k-q} + S_{k-q} - S_k)/\sqrt{S_kS_{k-q}}.
\]

This result can be viewed as the explicit zero temperature limit of more general calculations \[10\].

Applying Eq. (2) and \( |A_{q,k}|^2 \) to Eq. (1), and using the Feynman relation \( S_q = E_q^B/E_q^B \), we arrive at the rate of excitation-condensate collisions:

\[
n\sigma_k^B v_k = \frac{8\pi a^2}{2k^2} \int_0^k dq |A_{q,k}|^2 \frac{E_k^B - E_q^B}{\sqrt{1 + (E_k^B - E_q^B)^2}}
\]

(3)

The effective cross-section \( \sigma_k^B \) for the quasi-particles, is shown in Fig. 1 (solid line). For large \( k \), \( \sigma_k^B \) approaches \( 8\pi a^2 \), compared to \( 4\pi a^2 \) for impurities (dashed line). This enhancement by a factor of 2 is due to the boson quantum mechanical exchange term. In Eq. (3), for small \( k \), we verify that the scattering rate indeed scales as \( k^3 \), which is the classic result \[8\], \[8\]. In particular, it remains finite even for \( v_k < v_c \), in contrast with impurity scattering.

In \[15\], the identical particle collision cross section for large \( k \) was measured to be \( 2.1(\pm 0.3) \times 4\pi a^2 \). Scattering rates in four-wave mixing experiments in BEC \[1\] were also shown to agree with the high-\( k \) limit of Eq. (3) \[18\].

In the opposite regime of extremely low wavenumber, where the energy levels are discrete. Beliaev damping was observed for the scissors mode of a BEC \[14\]. The discrete energy levels were tuned so that Beliaev damping of the initial mode to exactly one mode of half the energy was achieved. Eq. (3) did not apply, since there was no need to integrate over various scattering modes.

Our experimental apparatus is described in \[13\]. Briefly, a nearly pure (\( > 95\% \)) BEC of \( 10^5 \) \( ^8 \)Rb atoms in the \( |F, m_f\rangle = |2, 2\rangle \) ground state, is formed in a QUIC type magnetic trap \[20\]. The trap is cylindrically symmetric, with radial \( (\hat{r}) \) and axial \( (\hat{z}) \) trapping frequencies of \( 2\pi \times 220 \) Hz and \( 2\pi \times 25 \) Hz, respectively. Thus \( \xi = 0.24 \mu m \) via averaging in the local density approximation (LDA) \[13\].

![FIG. 2. Absorption TOF images of excited Bose-Einstein condensates. (a) Absorption image for \( k = 2.63 \), with the large cloud at the origin corresponding to the unperturbed BEC. A clear halo of scattered atoms is visible between the BEC and the cloud of unscattered outcoupled excitations. (b) Absorption image for \( k = 1.06 \). For this value of \( k \) the distinction between scattered and unscattered excitations is not clear, since both types of excitations occupy the same region in space.](image-url)
an on-resonance absorption beam, perpendicular to the z-axis. Fig. 2a shows the resulting absorption image for k = 2.63, with the large cloud at the origin corresponding to the BEC. A halo of scattered atoms is visible between the BEC and the cloud of unscattered outcoupled excitations. No excitations with energy greater than that of the unscattered excitations are observed, confirming our low estimate of the Landau damping rate. Fig. 2b shows the absorption image for k = 1.06. For this k value the distinction between scattered and unscattered excitations is not clear in the image, since both types of excitations occupy the same region in space.

At a given k the number of excitations is varied by scanning $\Delta \omega$ around the resonance frequency $\omega_k^B$. The number of excitations $N_{\text{mom}}$ is measured by determining the total momentum (in units of the recoil momentum $\hbar k$) contained in the outcoupled region outside the unperturbed BEC. This region includes all the scattered and unscattered excitations, in the direction of k. Thermal effects are removed by subtracting the result of an identical analysis over the other side. The results are shown in Fig. 3a.

In order to quantify the amount of collisions, despite the lack of separation between scattered and unscattered excitations, we take the ratio between $N_{\text{mom}}$ and $N_{\text{count}}$ (both defined in the text). Every collision outcouples more atoms, increasing $N_{\text{count}}$ but leaving $N_{\text{mom}}$ almost unchanged.

At a given k we define the overall probability for an excitation to undergo the first collision $p_k$. If we ignore secondary collisions the result is $N_{\text{mom}}/N_{\text{count}} = 1/(1 + p_k)$, since each collision outcouples, after TOF, an additional particle [24]. Using this relation we infer $p_k^{\text{exp}}$ for the various measured k’s.
We also set the collision rate artificially to zero in the simulation, and find \( N_{\text{mom}} / N_{\text{count}} \) to be unity within 2\%, for all \( k \). This implies that there are no significant other mean-field repulsion effects along the \( z \)-axis \([12]\), confirming our assumption of momentum conservation.

In conclusion, we report a measurement of the suppression of identical particle collisions. Scattering cross-section is in arbitrary units. The momentum is in units of the inverse healing length after LDA averaging, \( \xi = 0.24 \mu \text{m} \). The theoretical curve (solid line) is a LDA average of Eq. (3) \([25]\). The assumptions of our analysis are tested using hydrodynamic simulations, and found to agree with Beliaev damping theory in the experimental regime (dashed line).

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FIG. 4. Suppression of identical particle collisions. Scattering cross-section is in arbitrary units. The momentum is in units of the inverse healing length after LDA averaging, \( \xi = 0.24 \mu \text{m} \). The theoretical curve (solid line) is a LDA average of Eq. (3) \([25]\). The assumptions of our analysis are tested using hydrodynamic simulations, and found to agree with Beliaev damping theory in the experimental regime (dashed line).

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[23] For \( k = 1.06 \), there are atoms in front and behind the BEC that should be counted in our integration. We use computerized tomography \([22]\), to find the correct atomic distribution. The systematic error caused by naively integrating the absorption image, can be as large as 10\%, which is unacceptable for our purposes. At higher \( k \), these systematic integration effects are verified to be negligible.
[24] hydrodynamical simulations (described below) indicate that there are few higher order multiple collisions for the experimental parameters. We also find that nearly all the collision products are indeed located in the counting region after TOF, confirming our momentum and atom number counting procedures.
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