THE RELATIVITY OF SPACE-TIME-PROPERTY

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We describe a geometrical way to unify gravity with the other natural forces by adding fermionic Lorentz scalar variables, characterising attribute or property, to space-time location. [With five such properties one can accommodate all known leptons and quarks.] Using just one property, viz. electricity, the general relativity of such a scheme and its superscalar curvature automatically produces the Einstein-Maxwell Lagrangian and a cosmological term. By adding more properties we envisage the geometrical unification of the standard model with gravitation.

Keywords: Anticommuting coordinates; field properties; unified models.

Preamble

It is a privilege and a delight to be able to celebrate with all of you here Professor Freeman Dyson’s life and achievements on the occasion of his 90th year. Little does Freeman know, but he has exerted a profound influence on my own career. Of course I was perfectly well aware of Dyson’s ground-breaking work on electrodynamics which was one of the first graduate courses we were taught (by John C Taylor and Tom Kibble at Imperial College); what really shaped my own career path was a chance remark made by a physicist, namely Hal Lewis, at Wisconsin University when I was a postdoc during 1963/64. The story goes that Freeman would bowl over at Summer Schools, such as the ones in Madison or Berkeley, and ask what were the most significant problems in physics that year, in order of priority; he would then go through the list and spend his time solving them! It revealed a person who was unafraid to tackle the broadest range of subjects in his own inimitable way – someone who had a panoramic view of science and who was prepared to think ‘outside the square’, without getting enmeshed in minutiae of one particular subarea. As if to reinforce that trait and the breadth of his interests, Freeman visited us in Tasmania in April 1979 and gave us a stimulating talk on how to mitigate carbon dioxide emissions by concerted planting of new trees; he would be well pleased that in Tasmania today a good one third of the state is being preserved as native forest.

The work I am going to talk about was carried out with Mr Paul Stack, a brilliant PhD student with a great aptitude for physics, mathematics and computing: a rare combination. The research has its genesis in the early days of supersymmetry
(SUSY). At that time I was tinkering with the idea of generalizing the auxiliary spinorial coordinate of SUSY to include internal symmetry labels (which now goes under the heading of \textit{N}-extended supersymmetry). Gell-Mann was interested in my attempts and, at a conference in London, he had a look at this proposal. It did not take him long to say that the idea would not fly as it would lead to spin state proliferation of an unacceptable kind, and he was perfectly correct. So I put the idea on a back-burner and moved on to other more fertile areas. Then, when I migrated to the backwoods of Tasmania, a long way from the madding research crowd, I drew inspiration from Freeman Dyson to resist following the bandwagon – a hopeless enterprise at my separation from the heartthrob of particle physics (internet notwithstanding) – and try to do my own thing. Dyson’s approach to science taught me not to keep up with the Jones’ but rather seek to be a Jones myself. Over the last few years I have tried resurrecting the idea of including particle attributes or properties mathematically with the aim of unifying gravity with the other natural forces., but in a way which differs very radically from spinorial SUSY. You can be the judge as to whether we have succeeded in this goal.

1. Events

A static universe is a contradiction in terms. Everything is inertial and non-interacting, so we would not even be aware of its existence! On the contrary, the universe evolves and its evolution is punctuated by series of events, defined by

- **WHERE and WHEN** - location \((x,y,z)\) and time \((t)\)
- **WHAT** - \((\text{ex})\)change of property \((\zeta = \xi + i\eta)\)

My emphasis will be on the ‘what’ and the overriding question is how to incorporate property or attribute mathematically. I shall do so using anticommuting complex numbers, of which there are only a \textsc{finite} number, and by the same token the composition of these numbers remains finite. Let us remind ourselves about such \(a\)-numbers.

Anticommuting operators were formulated in the 1920s but anticommuting numbers, invented earlier by Grassmann for treating differential geometry, have been used in physics only since about 1970, when BRST and SUSY came into prominence. \(c\)-\textsc{nos.} and \(a\)-\textsc{nos.} are of course intimately connected with \textsc{be} and \textsc{fd} statistics; here is a little table which emphasizes their similarities and differences (I shall have more to say about the last entry presently).

\[
\begin{array}{|c|c|}
\hline
\text{Bose-Einstein} & \text{Fermi-Dirac} \\
\hline
xy = +yx & \xi\eta = -\eta\xi \\
[a,a^\dagger] = 1 & \{a,a\} = 1 \\
(e^{\alpha + \beta E} - 1)^{-1} & (e^{\alpha + \beta E} + 1)^{-1} \\
\int e^{cd^2c} d\sigma \propto |\det D|^{-1} & \int e^{cd^2c} d^2c \propto |\det D|^{+1} \\
O(2n) \sim \text{Sp}(-2n) & \text{Sp}(2n) \sim O(-2n) \\
\hline
\end{array}
\]
Let me also remind you that BRST uses Lorentz scalar and vector a-numbers (pairs for ghosts and ghosts) but that SUSY uses Lorentz spinors. BRST is good for proving renormalizability and gauge invariance of gauge models; there is no violation of the spin-statistics theorem for physical fields because asymptotic states are ghost-free. On the other side, SUSY also conforms to spin-statistics but uses all states in the asymptotic limit. Despite SUSY’s great allure, its applicability has turned out to be problematic: nature (even the LHC) shows no signs of supersymmetric partner particles or states (photinos, squarks, gravitino,..) and this has proved a great disappointment to me and to many others.

It is worth recapitulating the main theoretical attractions of SUSY:-

- consistent nontrivial higher group incorporating Poincare: \( \{Q_\alpha, Q_\beta\} = (\gamma.PC)_{\alpha\beta} \),
- unifies bosons and fermions,
- cancellation of \( \infty \)s between boson and fermion loops, better renormalizability,
- allows supersymmetric generalization of gravity (SUGRA),
- its extension allows ‘internal symmetries’ to be incorporated via generators \( Q^n_\alpha \),
- can be generalized to string/brane theory.

These are the reasons why so many physicists have persisted with investigating SUSY, regardless and sometimes oblivious of experiment.

2. Negative dimensions

The most significant aspect of SUSY is that fermions and bosons act ‘oppositely’ to one another. Since \( (\int e^{D^2}cDc)^{AB}(\int e^{D\zeta}d\zeta d\zeta)^{nF} \propto D^{nF-nB} \), where \( D \) is the Dirac operator, we can construe \( c \)-coordinates as adding to dimension and \( a \)-coordinates as subtracting dimension. This is also confirmed by group theory associated with \( O(2n) \) and \( Sp(2n) \) where an alteration in sign of \( n \) allows one to continue Casimirs and dimensions of certain classes of representation from one Lie algebra to the other.

“The Lord giveth and the Lord taketh away”

It does not really matter if the \( a \)-numbers are spinorial or scalar and I would like to take advantage of that fact. By matching \( c \)-coordinates with \( a \)-coordinates (and likewise BE fields with FD fields) we get may end up with zero nett dimensions (= \# dimensions of universe before the BIG BANG?). Therefore to the four \( x^m \) of space-time I will add four \( \zeta^n \) – or equivalently add five \( \zeta^n \), but only take half the states to get the statistics correct. I shall associate these Lorentz scalar \( a \)-nos. with ‘property’ or ‘attribute’, leading to natural internal symmetries between these coordinates.
To obtain a sensible fundamental particle spectrum I have found it necessary to make the following charge $Q$ and fermion number $F$ assignments:

$$Q(\zeta^{0,1,2,3,4}) = (0, 1/3, 1/3, 1/3, -1); \ F(\zeta^{0,1,2,3,4}) = (1, -1/3, -1/3, -1/3, 1).$$

The $\zeta$ coordinates have been labelled from 0 to 4. Crudely we can regard

- label 0 as ‘neutrinity’
- labels 1 - 3 as (antidown) ‘chromicity’
- label 4 as ‘electricity’

Other properties are to be considered composites of these and it helps to regard such attributes as the ingredients of a recipe. As you will recall, Rabi was heard to say “Who ordered that?” when the muon was discovered. Well, in our scheme the muon, tauon and more generally particle families are an integral part of the menu.

From these extended coordinates, superfields (functions of space-time and property) may be constructed. Since the product of two $a$-nos. is a (nilpotent) $c$-no., a Bose superfield $\Phi$ should be a Taylor series in even powers of $\zeta, \bar{\zeta}$ and a Fermi superfield $\Psi$ a series in odd powers of $\zeta, \bar{\zeta}$ — up to the 5th:

$$\Phi(x, \zeta, \bar{\zeta}) = \sum_{\text{even } r+\bar{r}} (\bar{\zeta}^r \phi_{r}(\zeta)^r);$$
$$\Psi_\alpha(x, \zeta, \bar{\zeta}) = \sum_{\text{odd } r+\bar{r}} (\bar{\zeta}^r \psi_{\alpha}(\bar{\zeta})^r).$$

When forming actions as products of superfields we should integrate over all space-time as well as property to cover all possibilities and attribute changes.

The above expansions produce too many states, $\psi_\alpha$ and $\phi$, viz. 256, so they need cutting down. (Had we only used four $\zeta$ we would not have been able to accommodate three generations as a matter of fact.) Now a primary way to halve the number of states essentially is to impose self-conjugation whereby

$$\psi^{(r)}(r) = \psi^{(\bar{r})}(\bar{r}),$$

corresponding to reflection along the main diagonal in an $r, \bar{r}$ magic square. Secondary impose (anti) self-duality corresponding to reflection about the cross diagonal, specifically

$$\psi(r, (\bar{r}) = -\psi^{(\bar{r})}(r);$$

noting that the dual ($\times$) of a field term has exactly the same $Q$ and $F$ as the field. For example,

$$\langle \tilde{\zeta}^A \zeta_M \zeta_N \rangle^X = \frac{1}{3!} \epsilon_{JKLMN} \zeta^J \zeta^K \zeta^L \frac{1}{4!} \epsilon^{ABCDEF} \zeta_B \zeta_C \zeta_D \zeta_E \zeta_f.$$

Apart from diminution of components through antiduality, we are able to exorcize some unpleasant attribute combinations. Thus $\zeta^0 \zeta_1 \zeta_2 \zeta_3$ and $\zeta^4 \zeta_0 \zeta_1 \zeta_2 \zeta_3$ are self-dual, so imposing antiduality eliminates these nasties, since they have $F = 3$ and $Q = -2$ respectively.
If we place these states in a kind of chessboard of dimensions 6 × 6, the fermions fall in the black squares and bosons in the white squares. The thing to note is that this scheme contains all known quarks and leptons, with indications of a fourth generation. There are significant differences with the standard model however:

- There are more than 3 leptons/D-quarks. These must necessarily be heavy.
- The first and second generations are electroweak doublets, but the third generation is a triplet! Specifically \( (i, j, k) \) which run from one to three are color labels below,
  
  \[
  U_{1k} \sim \zeta^i \zeta^j \zeta^0, \quad U_{2k} \sim \zeta^i \zeta^j \zeta^0 (\zeta^4 \bar{\zeta}^4), \quad U_{3k}^0 \sim \zeta^k \bar{\zeta}^4 \zeta^0 \\
  D_{1k} \sim \zeta^i \zeta^j \zeta^4, \quad D_{2k} \sim \zeta^i \zeta^j \zeta^4 (\zeta^0 \bar{\zeta}^0), \quad D_{3k}^0 \sim \zeta^k (\bar{\zeta}^0 \zeta^0 - \bar{\zeta}^4 \zeta^4) \\
  X_{3k} \sim \zeta^k \bar{\zeta}^0 \zeta^4.
  \]

- The CKM matrix will not be exactly unitary due to \( X_3 \) (charge \(-4/3\)); the best place to search for an \( X_3 \) is presumably in electron-positron annihilation.

The mass matrix which affects quarks as well as leptons is due to a set of chargeless Higgs field’s expectation values; there are nine possibilities having \( F = Q = 0 \) but associated with only one SU(5) invariant Yukawa coupling:

- one \( \phi_{(0)(0)} = \langle \phi \rangle \),
- one \( \phi_{(0)(4)} = \langle \phi_{1234} \rangle \) (standard Higgs doublet expectation value),
- three \( \phi_{(1)(1)} = \langle \phi_0^0, \phi_1^1, \phi_1^i \rangle \)
- four \( \phi_{(2)(2)} = \langle \phi_{04}^0, \phi_{0k}^k, \phi_{4k}^k, \phi_{ij}^i \rangle \),

others being related by duality. This scheme is therefore more constrained than the standard model. All this is by way of entree. I wish to offer the main course now.

3. Graded General Relativity for Time-Space-Property

You must be wondering where the gauge fields reside in such a scheme? It has probably already occurred to you that one may mimic SUSY and supergauge the massless free action for \( \Psi \), but without added complication of spin. And indeed one can: in the time-honoured way, using the substitution rule for covariant derivatives whereby the generators of internal symmetry or shape-shifting property transformations are given by

\[
T_A^B = \zeta_A \frac{\partial}{\partial \zeta_B} - \zeta_B \frac{\partial}{\partial \zeta_A}.
\]

But there is a more compelling way, which is fully geometrical and has the benefit of incorporating gravity!

We will construct a fermionic version of Kaluza-Klein (KK) theory, this time without needing to handle infinite modes which arise from compressed normal
bosonic coordinates. These are the significant points about the enlarged metric involving the extended coordinate \( X = (x, \zeta, \bar{\zeta}) \):

- One must introduce a fundamental length \( \ell \) for the enlarged \( X \), because property \( \zeta \) has no dimensions; this is tied to the gravity scale \( \kappa = \sqrt{8\pi G N} \);
- Gravity (plus gauge field products) fall within the \( x - x \) sector, gauge fields in \( x - \zeta, x - \bar{\zeta} \) and the Higgs scalars must form a matrix in \( \zeta - \bar{\zeta} \);
- The maximal gauge group is connected with the number of \( \zeta \), so this is \( SU(5) \) in our scheme, (although nature seems only to gauge the standard subgroup for some unknown reason);
- Gauge transformations are property rotations, dependent on space-time;
- There is no place for a gravitino as spin is absent (\( \zeta \) are Lorentz scalar);
- There are necessarily a small finite number of modes.

To carry out this programme I need to introduce some basic notational niceties first. With the extended coordinate \( X^M \), let \( M = m \) (Roman) correspond to space-time \( x \) and let \( M = \mu \) (Greek) correspond to property \( \zeta, \bar{\zeta} \). Set \([m]=0\) (no grading) and \([\mu]=1\) (grading). Thus \( V^M A^N = (-1)^{[M][N]} A^N V^M \). Now we can revisit general relativity taking great care with ordering and sign factors! Our rule is always to take left derivatives, like \( \partial_{X^M} \), but we have to reconcile this with traditional GR notation, \( A^N_{,M} \), which is ingrained but back to front! This introduces sign factors and we have to live with that.

Transformation laws for vectors \( V \) & tensors \( T \) (such as the metric \( G \)), etc. read

\[ V'^M = V^N \frac{\partial X'^M}{\partial X^N}, \quad V'_M = \frac{\partial X'^N}{\partial X^M} V_N, \quad \text{so} \quad V^N V_N \text{ is invariant,} \]

\[ T'_{MN} = (-1)^{[R][[S]+[N]]} \frac{\partial X^R}{\partial X'^M} \frac{\partial X^S}{\partial X'^N} T_{RS}, \quad \text{so} \quad d\bar{s}^2 = dX^N dX^M G_{MN} \text{ is invariant.} \]

We can use the metric tensor and its inverse to raise and lower indices:

\[ V^M G_{MN} \equiv V_N, \quad G^{MN} V_N = V^M, \quad T^N_{,M} = (-1)^{[M][N]+[L]} G^{NL} T_{ML}, \]

with

\[ G^{MN} G_{NL} = \delta^M_L = (-1)^{[M]} \delta_L^M, \quad (-1)^{[N]} G_{MN} G^{NL} = \delta^L_M \]

and noting that \( G \) and its inverse are graded symmetric:

\[ G_{MN} = (-1)^{[M][N]} G_{NM}, \quad G^{LM} = (-1)^{[L][M]} G^{ML}. \]

4. Covariant derivatives and the Riemann supertensor

With these conventions one can establish the rules for covariant differentiation:

\[ A_{MN;L} = (-1)^{[M][N]} A_{MN,L} - \Gamma_{MN}^L A_L, \]

\[ A^M_{;N} = (-1)^{[M][N]} [A^M_{,N} + A^L \Gamma_{LN}^M], \]
\[ T_{LM;N} = (-1)^{[N]}((-L)+[M])\left[T_{LM,N} - \Gamma_{NL}^K T_{KM} - (-1)^{[L]}([M]+[K]) \Gamma_{NM}^K T_{LK}\right], \]
eq \text{etc. where the Christoffel connection is given by}
\[ \Gamma_{MN}^K = \left((-1)^{[M][N]}G_{ML,N} + G_{N,L,M} - (-1)^{[L]}([M]+[N])G_{MN,L}\right)(-1)^{[L]}G^{LK}/2. \]

From this may be derived the generalized Riemann tensor \( \mathcal{R} \):
\[ (-1)^{[J]}A_J \mathcal{R}^J_{KLM} = A_{K;l;M} - (-1)^{[L]}[M]A_{K;M;l} \]
whereupon \( \mathcal{R} \) can be expressed in terms of the connections.
\[ \mathcal{R}^J_{KLM} = (-1)^{[J][K]+[L]+[M]}\left((-1)^{[K][L]}\Gamma_{KM,l}^{J} - (-1)^{[K][L]}\Gamma_{L,M}^{J}\right) + (-1)^{[L][M]}\Gamma_{KM}^R \Gamma_{RL}^{J} - \Gamma_{KL}^R \Gamma_{RM}^{J}. \]

As a further check, we may derive the lowered tensor
\[ \mathcal{R}_{JKLM} = (-1)^{([I]+[J])([K]+[L]+[M])}\mathcal{R}^J_{KLM}G_{IJ}, \]
and check its symmetry properties,
\[ \mathcal{R}_{KJLM} = (-1)^{[J][K]}\mathcal{R}_{JKLM}, \]
\[ \mathcal{R}_{JKML} = (-1)^{[L][M]}\mathcal{R}_{JKLM}, \]
\[ \mathcal{R}_{LMJK} = (-1)^{([J]+[K])([L]+[M])}\mathcal{R}_{JKLM}. \]
as well as the cyclicity property (first Bianchi identity):
\[ (-1)^{KM}\mathcal{R}_{JKLM} + (-1)^{ML}\mathcal{R}_{JMKL} + (-1)^{LK}\mathcal{R}_{JLMK} = 0. \]
The second (differential) Bianchi identity can also be established,
\[ (-1)^{[L][N]}\mathcal{R}_{JKLM;N} + (-1)^{[N][M]}\mathcal{R}_{JKLN;M} + (-1)^{[M][L]}\mathcal{R}_{JKMN;L} = 0. \]

The \textit{Ricci} tensor is arrived at via the contraction:
\[ \mathcal{R}_{KM} = (-1)^{[K][L]}G^{LK}\mathcal{R}_{JKLM}, \text{ with } \mathcal{R}_{KM} = (-1)^{[K][M]}\mathcal{R}_{MK}, \]
and the full superscalar curvature is obtained as \( \mathcal{R} = G^{MK}\mathcal{R}_{KM}. \) Finally to get the \textit{Einstein} tensor \( \mathcal{G} \) and its vanishing covariant divergence, contract out the second Bianchi identity. One finds automatically:
\[ \mathcal{R}_{;N} = 2(-1)^{[M][N]}\mathcal{R}^{M}_{N;M}, \]
or \( G^{M}_{N;M} = 0 \) where
\[ G^{M}_{N} = \mathcal{R}^{M}_{N} - \delta^{M}_{N}\mathcal{R}/2. \]
5. Frame vectors and metric

Now let me focus on just one property, namely electricity, so there is only one \( \zeta \) and we need not bother with indices on \( \zeta \). In flat space

\[
ds^2 = dX^A dX^B \eta_{BA} = dx^a dx^b \eta_{ba} + \zeta d\bar{\zeta} \eta_{\zeta \zeta} + \bar{\zeta} d\zeta \eta_{\zeta \zeta},
\]

where \( \eta_{\zeta \zeta} = -\eta_{\bar{\zeta} \bar{\zeta}} = \ell^2/2 \) and \( \eta_{ba} \) is Minkowskian. To curve the space, let us be guided by Kaluza-Klein and introduce frame vectors \( E \), allowing for property curvature coefficients \( c_i \):

\[
G_{MN} = (-1)^{[B]+[N]} \epsilon_M^A \eta_{AB} \epsilon_N^B ,
\]

with

\[
E_m^a = (1 + c_1 \zeta \bar{\zeta}) e_m^a, \quad E_m^\zeta = -ie \bar{\zeta} A_m, \quad E_m^{\bar{\zeta}} = -ie A_m \zeta,
\]

\[
E_\zeta^a = 0, \quad E_\zeta^{\bar{\zeta}} = 0, \quad E_\bar{\zeta}^\zeta = (1 + c_2 \zeta \bar{\zeta}), \quad E_\bar{\zeta}^{\bar{\zeta}} = 0,
\]

Putting this all together results in the following metric:

\[
G_{MN} = \begin{pmatrix}
G_{mn} & G_{m\zeta} & G_{m\bar{\zeta}} \\
G_{\zeta m} & 0 & G_{\zeta \zeta} \\
G_{\bar{\zeta} m} & G_{\bar{\zeta} \zeta} & 0
\end{pmatrix}
\]

where

\[
G_{mn} = g_{mn}(1 + 2c_1 \zeta \bar{\zeta}) + e^2 \ell^2 A_m A_n \zeta \bar{\zeta},
\]

\[
G_{m\zeta} = G_{\zeta m} = -ie \ell^2 A_m \bar{\zeta} / 2,
\]

\[
G_{m\bar{\zeta}} = G_{\bar{\zeta} m} = -ie \ell^2 A_m \zeta / 2,
\]

\[
G_{\zeta \zeta} = G_{\bar{\zeta} \bar{\zeta}} = \ell^2 (1 + 2c_2 \zeta \bar{\zeta}) / 2).
\]

A few pertinent observations about the covariant metric \( G \) are appropriate.

- the charge coupling \( e \) accompanies the e.m. potential \( A \);
- the constants \( c_i \) provide phase invariant property curvature – like mass in the Schwarzschild metric. They may be expectation of Higgs fields or possibly be associated with dilatons;
- there is an intrinsic length scale, which will be tied to gravity;
- there are no gravitinos; such fields carry a spinor index, but \( \zeta \) are scalar so any gravitinos would spoil Lorentz invariance and cannot appear in the metric;
- one might consider including a term like \( C_m(1 + \alpha \zeta \bar{\zeta}) \) to the \( x - \zeta \) frame vector where \( C_m \) is an anticommuting vector, as one meets in quantized gravity (Feynman’s vector ghosts). Similarly we could add \( \bar{C} \zeta + \zeta \bar{C} \) where \( \bar{C} \) are scalar ghosts in the purely property sector, connecting with BRST for gauge vector quantization. We have not done so, since we are confining ourselves to classical gravity/e.m.
- otherwise the metric is as general as it can be;
fermions are distinct – they carry a spinor index – must be treated separately.

We can similarly determine the elements of the inverse metric:

\[ G_{mn} = g_{mn}(1 - 2c_1 \bar{\zeta} \zeta), \]
\[ G_{m\bar{\zeta}} = G_{\bar{\zeta}m} = i e A_m \bar{\zeta}, \]
\[ G_{m\bar{\zeta}} = G_{\bar{\zeta}m} = -i e A_m \bar{\zeta}, \]
\[ G^{\zeta \bar{\zeta}} = -G^{\bar{\zeta} \zeta} = 2(1 - 2c_2 \bar{\zeta} \zeta)/ \ell^2 - e^2 A_m A_m \bar{\zeta} \zeta, \]

and go on to determine the Christoffel connections. I shall not bother to list these; some are contained in earlier papers.

Before we can get the action we will also need the superdeterminant of the metric:

\[ s \det(G_{MN}) = \frac{4}{\ell^4} \det(g_{mn}) \left[ 1 + (8c_1 - 4c_2) \bar{\zeta} \zeta \right] \]

or for short,

\[ \sqrt{-G..} = \frac{2}{\ell^2} \sqrt{-g..} \left[ 1 + (4c_1 - 2c_2) \bar{\zeta} \zeta \right]. \]

The absence of the gauge potential should be noted (because \( E \) is triangular).

In connection with the super-determinant, one can establish that \((\sqrt{G..})_M = \sqrt{G..}(-1)^{|N|} \Gamma_{MN}^N\) and, from the definition of the Christoffel symbol, show that the scalar curvature can be reduced to the analogue of the Palatini form:

\[ \sqrt{G..} R \rightarrow (-1)^{|L|} \sqrt{G..} G^{MK} \left[ (-1)^{|L|}{\Sigma}^{MN}_{KL} \Gamma_{KL}^N \Gamma_{MN}^L - \Gamma_{KM}^M \Gamma_{NL}^L \right]. \]

6. Gauge changes as property transformations

Make a spacetime dependent \( U(1) \) phase transformation in the property sector:

\[ x' = x; \quad \zeta' = e^{i \theta(x)} \bar{\zeta}; \quad \bar{\zeta}' = e^{-i \theta(x)} \bar{\zeta}. \]

From the general transformation rules of \( G_{m\bar{\zeta}} \) we find

\[ e A'_m = e A_m + \partial_m \theta, \]

showing the field \( A_m \) acts as a gauge field under variations in electric phase. [This can be checked for all components of the metric \( G_{MN} \).] But \( G^{mn} \) remains unaffected and thus is gauge-invariant.

The same comments apply to \( R_{mn} \) and \( R^{mn} \); the former varies with gauge but the latter does not.
7. The Ricci tensor and gravitational–e.m. action

From $G$ and the evaluation of $\Gamma$ we can determine the full Riemann curvature supertensor $R_{JKLM}$ and the Ricci tensor $R_{KM} = (-1)^{|K||L|}G^{LJ}R_{JKLM}$; whence the superscalar $R = G^{MK}R_{KM}$. It is all a matter of cranking the handle, but an algebraic computer program which works out these quantities with no errors is of enormous assistance and Paul Stack has been instrumental in developing such a program using Mathematica.

To see how electromagnetism emerges geometrically ignore spacetime curvature initially but not property curvature. The spacetime component of contravariant Ricci reduces to the gauge independent pair:

$$R_{mn} = 4g^{mn}c_1[1 + (2c_2 - 6c_1)\bar{\zeta}\zeta]/\ell^2 - \ell^2 F^{ml}F_{ln}\bar{\zeta}\zeta/2,$$

and the curvature superscalar collapses to

$$R = 8[4c_1 - 3c_2 + c_1(8c_2 - 10c_1)\bar{\zeta}\zeta]/\ell^2 - \ell^2 F^{ml}F_{nl}\bar{\zeta}\zeta/4.$$

Including the super-determinant the total Lagrangian density for electromagnetic property emerges:

$$L = \int \sqrt{-G}d\zeta d\bar{\zeta} R \propto -\frac{1}{4}F_{mn}F^{mn} + \frac{48(c_1 - c_2)^2}{e^2 \ell^4},$$

and the Einstein tensor in flat spacetime reduces to:

$$\int d\zeta d\bar{\zeta} \sqrt{-G}(R^{km} - \frac{1}{2}G^{km}) \propto [48c_2(c_1 - c_2)g^{km}/e^2 \ell^4 - (F^{kl}F_{lm} - F_{ln}F^{lm}g^{km}/4)].$$

The familiar expression for the e.m. stress tensor, $T^{km} \equiv F^{kl}F_{lm} + F_{nl}F^{nl}g^{km}/4$, becomes part of the geometry.

Now include gravity by curving spacetime ($\eta_{mn} \rightarrow g_{mn}(x)$).

$$L = \int \sqrt{-G}d\zeta d\bar{\zeta} R = 2e^2 \sqrt{-g}..\left[\frac{2(c_1 - c_2)R}{e^2 \ell^2} - \frac{F_{mn}F^{mn}}{4} + \frac{48(c_1 - c_2)^2}{e^2 \ell^4}\right].$$

Spot the Newtonian constant and the cosmological term,

$$16\pi G_N \equiv \kappa^2 = e^2 \ell^2 / 2(c_1 - c_2), \quad \Lambda = 12(c_2 - c_1)/\ell^2.$$
$e^2\ell^2/4(c_1 - c_2) > 0$. The result is to make the cosmological term go negative and, what is probably worse, it has a value which is inordinately larger than the tiny experimental value found by analyses of supernovae! (All cosmological terms derived from particle physics, except for exactly zero, share the same problem). Numerically speaking, $\kappa \simeq 5.8 \times 10^{-19} \text{ (GeV)}^{-1}$ means $\ell \sim 10^{-18} \text{ (GeV)}$ is Planckian in scale. Of course the magnitude of the miniscule cosmological constant $\Lambda \sim 4 \times 10^{-84} \text{ (GeV)}^2$ is at variance with Planckian expectations by the usual factor of $10^{-120}$, which is probably the most mysterious natural ratio. So far as our scheme is concerned, we are disappointed but not particularly troubled by the wrong sign of $\Lambda$ because it can readily be reversed by extra property curvature coefficients when we enlarge the number of properties (as we have checked when enlarging the number of properties to at least two). The magnitude of $\Lambda$ is quite another matter because it will require some extraordinary fine-tuning, even after fixing the sign.

Note added: since giving the talk, Stack and I have extended our work to two properties and derived the SU(2) Yang-Mills Lagrangian, united geometrically to gravity. A few more steps will take us to electroweak theory, so stay tuned.

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