Masers with selective excitation of Talbot-type supermode

Yu S Oparina¹, A V Savilov¹ and D Yu Schegolkov¹
¹Institute of Applied Physics of RAS, 46 Uliyanova st., Nizhny Novgorod, Russia

E-mail: oparina@appl.sci-nnov.ru

Abstract. The concept of an electron maser based on the excitation of the single supermode formed by a fixed set of transverse eigenmodes is proposed. A possible approach to form a high-Q supermode inside a simple cavity is the use of the Talbot effect, namely, periodic reproduction of the transverse structure of a multimode wave field in an oversized waveguide.

1. Introduction
There is growing interest in the creation of sources operating in the terahertz (THz) and sub-THz frequency ranges with a high power of the output radiation. Powerful THz sources are required, for instance, for heating and current drive or diagnostic systems of fusion installations of the next generation, such as DEMO, and for many actively developing areas, such as high-gradient THz acceleration, sub-THz wave undulators for short-wavelength free-electron lasers, and various plasma physics applications.

There are several problems to generate powerful coherent THz radiation in masers based on relativistic electron beams (free electron masers, FEMs). At first, such devices have to operate on excitation of a one chosen far-from-cutoff transverse mode of the operating cavity (waveguide), as a rule, this is the lowest possible transverse mode (this is to solve the problem of the mode selection). Application of this approach to THz frequency range encounters natural difficulties. Evidently, the operating waveguide in this case should be oversized [1,2]; this means that the characteristic transverse size should be much greater than that of the operating wavelength. This is necessary for several reasons, namely, transportation of the relativistic high-current beam, the problem of breakdown of the field of high-power radiation inside the cavity, ohmic heating of the cavity walls, etc. The second problem is the difficulty in providing selective single-mode feedback in an oversized system [3].

We propose the concept of selective excitation of a THz operating wave in a high-power relativistic electron maser with an oversized microwave system fed by a high-current relativistic electron beam. Our idea is to give up working in a fixed transverse mode. Instead, we propose to work in a supermode, which is formed by a fixed spectrum of several transverse modes of an oversized waveguide, figure 1.

2. The main idea
Wave packet possessing a fixed frequency and propagating in a waveguide can be represented as a sum of fields of partial transverse eigenmodes of this waveguide (figure 1):

$$E_\Sigma = Re \sum_n C_n E_n(r_L) \exp(i\omega t - ih_n z),$$

(1)

here, $n$ is the transverse mode number, $E_n(r_L)$ describes transverse structures of the $n^{th}$ mode, $h_n = \sqrt{k^2 - k^2_{1,n}}$ are the axial wavenumbers, $k = \omega/c = 2\pi/\lambda$, and $k_n$ are the transverse wavenumbers. We
consider the situation of a highly oversized system, \( D/\lambda \gg 1 \). In this situation, a great numbers of partial transverse modes can be involved in the formation of the wave packet, and most of the modes are very far from the cutoff, \( k_{\perp,n} \ll k \). Therefore, their axial wavenumbers can be approximated as follows:

\[
h_n \approx k - \frac{k_{\perp,n}^2}{2k} = k - \frac{\pi \lambda n^2}{4D^2},
\]  

(2)

![Figure 1. Excitation of a supermode formed by a set of several transverse modes.](image)

The phase incursion of the mode at its trip along the waveguide of the length \( L \) is equal to

\[
\varphi_n = h_n L \approx kL - \frac{\pi n^2}{4} \times \frac{\lambda L}{D^2},
\]  

(3)

Consider transverse distribution of the wave field, \( E(x) = E(D - x) \), is formed by modes having odd indices, \( n = 2m - 1 \), here \( m \) is an integer positive number. In this case, field is a “spot” concentrated to the center at the input and the output of the cavity (see figure 1, a). For these modes,

\[
\varphi_n = kL - \frac{\pi}{4} \times \frac{\lambda L}{D^2} - \pi m(m - 1) \times \frac{\lambda L}{D^2},
\]

obviously, that \( m(m - 1) \) is an even number, so transverse distribution of the wave field is reproduced in the waveguide (figure 1),

\[
E_2(x, z + L) = E_2(x, z),
\]

with a period described by the following formula:

\[
L = \frac{D^2}{\lambda},
\]  

(4)

The phenomenon of repetition of the transverse wave structure is well known as the Talbot effect [4]. In paper [5] the use of this effect is proposed as a way to create an oversized microwave system of an electron maser that provides a high Q-factor for a supermode formed by several transverse modes. If relatively narrow mirrors at the input and the output in the centers of the corresponding transverse cross-sections will be include in the system, the field of a high-Q supermode, \( E_2(x) \), is concentrated only in the region of the mirror. If the length of the cavity satisfies formula (4) and the Talbot effect is executed ideally, then this transverse profile of the total wave field is reproduced exactly at the output transverse cross-section (\( z = L \)). In this case, this wave field is reflected completely by the output mirror. The counter-propagation of the reflected wave back to the input mirror is completely analogous to the direct propagation of the supermode. Therefore, the input mirror completely reflects the counter-propagating wave into the forward wave and closes the feedback circuit. Thus, a simple cavity provides a high Q-factor for any supermode, which electric field \( E_2(x) \) in the input/output transverse cross-sections of the cavity is concentrated in the regions of the mirrors, and, therefore, almost totally reflected back to the cavity. This high-Q supermode is a specific set of transverse waveguide modes, (figure 1, a). Another scheme of a Talbot-type cavity can be formed by “shifting” the scheme shown in the figure 1, a along the z-axis by length \( L/2 \). In this case, the supermode field is two “spots” concentrated close to the walls at the input and the output of the cavity (figure 1, b). One more attribute of the Talbot effect is multiplication of the wave beam. This means that, in the middle of the cavity shown in (figure 1, a), the
transverse distribution of the supermode field represents two wave beams concentrated close to the waveguide walls. Therefore, such a supermode can be excited effectively by two electron beams injected in the regions close to the walls. In the case shown in the figure 1, b, a supermode is excited effectively by a single electron beam in the center. Therefore, the problem of separation of the electron beam and the input and output mirrors is easily solved. A natural analogue for cylindrical geometry of the scheme is a waveguide of the circular cross-section. There are several distinctive features caused by non-equidistance of the spectrum of transverse modes [6].

3. Talbot-type supermodes of the planar 2-D cavity

We start from a simple 2-D model of the cavity in the form of a planar waveguide, figure 1, a, terminated by two mirrors at the input and the output. We assume that the system is homogeneous along the y-direction; this implies that the waveguide fields are independent of the y-coordinate. In this case, any TE wave packet includes only y-component of the electric field. Since this field is equal to zero at the both walls, the total field of any wave packet can be represented as a sum over all transverse modes, the spectrum of this wave packet includes transverse modes with only odd numbers, \( n = 2m - 1 \), here m is positive integer number, figure1, a. Figure 2 illustrates the first three supermodes in the case of the mirror size \( d = D/3 \). This picture is obtained by expansion of the supermode field in the cross section \( z = 0 \) by the partial transverse waveguide modes. In the considered above approximation, the structure of any supermode is determined only by the ratio between the mirror size and the waveguide transverse size, \( d / D \), and does not depend on the oversizing factor, \( D / \lambda \).

![Figure 2](image)

Let’s consider eq. 3 in higher accuracy. The expansion of the phase incursion of the transverse mode in the point \( z = L \) with accuracy up to the third term has the form

\[
\varphi_n \approx kL + \frac{\pi n^2}{4} + \frac{\pi n^4}{64} \times \frac{\lambda^2}{D^2}.
\]

The Talbot effect continues to run as long as the additional (i.e., provided by the third term in this formula) foray of the phase remains sufficiently small,

\[
\frac{\pi n^4}{64} \times \frac{\lambda^2}{D^2} \ll \frac{\pi}{4},
\]

Therefore, the maximal index of the partial transverse mode present in the spectrum of supermode is determined by the factor of the transverse oversizing of the system:
There are several high Q-factor modes. For the lowest supermode with the single variation of the wave field on the mirrors, $F_1(x, z)$, this index is of the order of the ratio between the mirror size and the waveguide transverse size, $n_{\text{max}} \approx D / d$. For the next two supermodes with three and five variations on the mirrors, $F_2(x, z)$ and $F_3(x, z)$, the characteristic indexed of partial modes in their spectrum are three and five times bigger, respectively. Actually, in the case of $D / d = 3$, $n_{\text{max}} \approx 3 \times 3 = 9$ for the supermode $F_2$ and $n_{\text{max}} \approx 3 \times 5 = 15$ for the supermode $F_3$ (figure 2). Therefore, we can state that if the following relation is true,

$$\frac{D}{d} \approx \frac{2}{3} \sqrt{\frac{D}{\lambda}},$$  \hspace{1cm} (8)$$

then only the lowest supermode possess a high Q-factor in the cavity with a fixed geometry. This means than at factor of the transverse oversizing of the system, $D / \lambda >> 1$, it is possible to find the proper mirror size, $d / D$, when the cavity provides a high Q-factor only for the lowest supermode.

Naturally, in general, presence of losses of the wave power on the mirrors makes incorrect (or at least inaccurate) the approximate approach used above. The set of orthogonal supermodes and their structure should be found with taking into account the limited and different Q-factors of different supermodes.

4. Talbot-type supermodes of the cylindrical cavity

Accurately, the Talbot effect occurs only in systems with an equidistant spectrum of transverse eigenmodes. In a waveguide with a circular cross section, the mode spectrum is quasi-equidistant (the spectrum of eigen modes with the azimuthal index “1” and various radial indexes); this is true only for sufficiently high transverse modes. Accordingly, the Talbot effect occurs only approximately. Nevertheless, the formation of a high-quality supermodel is possible. As an example, in simulation we considered Cylindrical waveguide of diameter D. Oversize parameter $D / \lambda \approx 70$, a mirror size is 36% (it means, that mirror occupies space $0.18 \times R_w$). Red curve in the figure 3, a illustrates the dependence of the diffraction losses of the highest-Q supermode on the frequency. The mode spectrum of this supermode depends on the frequency. In region of the first TE peak (lower frequencies), this is formed mainly by high-order TE partial modes, see figure 3, a. In region of the second peak in the figure 3, a at slightly higher frequencies, we see the high-Q supermode forms basically through the high-order TM modes, see figure 3, b. Interestingly, in the intermediate region of frequencies, we see the supermode formed by relatively low TE and TM modes, see figure 3, c. Since for such modes the equidistance of the spectrum is strongly disturbed, the Q factor of this supermode is significantly lower, namely, for each mirror $R \approx 75\%$ and, therefore, about 50% of its power is lost at each round trip of the wave through the cavity.

The Q factor of the “next” supermode, blue curve in the figure 3, a, at the frequency corresponding to the maximum of the highest Q mode is significant lower.
5. Modelling of a 2 THz high-current FEM oscillator

As a possible application of the proposed approach, we consider the possibility for the realization of a THz frequency-range FEM oscillator on the basis of the LIU accelerator being created at the Budker Institute for Nuclear Physics [7]. The goal of this work is to provide the formation of a high-current (up to 2 kA) electron beam with electron energy up to 20 MeV and an electron pulse duration of 200–300 ns. The expected thickness of the electron beam is several mm. Therefore, the diameter of the operating waveguide should be at least \( D = 1 \text{ cm} \). In the case of an operating frequency close to 2 THz, the factor of transverse oversizing, \( D/\lambda \), is over 60.

In the model, we consider excitation of a cavity in the form of a piece of waveguide with a circular cross section terminated by two mirrors, see figure 4 (a). We assume that a helically polarized undulator, together with the axis-encircling electron beam, is used. This means that electrons move along a helix around the axis of the operating waveguide. The reason to choose this configuration is an improved selectivity of the system, namely, the axis-encircling electron beam interacts only with TE\(_{1,\pi}\) and TM\(_{1,\pi}\) modes, with an azimuthal index equal to one. These modes also have circular polarization, and the direction of their rotation coincides with the rotation direction of a circularly polarized field of the undulator.

Figure 3. (a): Calculated power reflection coefficient of the input and output mirrors for the highest Q supermode (red curve) and the next high Q mode (blue curve) versus the frequency. Amplitude spectra of the supermodes excited in regimes with the predominance of TE (b), TM (c), and mostly TE (d) partial modes.

Figure 4. (a): High-current 2 THz FEM oscillator. Operating cavity and structure of the excited supermode in the steady-state regime (TE – peak, the operating current is 2 kA). (b): Calculated power reflection coefficient of the input and output mirrors for the high-Q supermode in the “cold” approximation and the starting current versus the frequency.

Figure 4 (b) illustrates the dependence of the power reflection coefficients from the input and output mirrors (which are the same in the “cold” approximation) on the frequency. Obviously, that the high-Q supermode exists in a quite narrow frequency band. We consider undulator factor, \( K_u = 0.5 \), and the electron energy 7 MeV, the undulator period should be close to 5.5 cm. In the case of \( D = 1 \text{ cm} \) and
\( \lambda \approx 0.015 \, cm \), the Talbot formula (4) leads to \( L = 65 \, cm \) (this case is close to the example considered in the previous section). Another curve in the figure 4 (b) illustrates the dependence of the starting current on the frequency. In the considered case the lowest starting current (approximately 1 kA) is achieved in region, corresponding to the TE-type supermode. If the operating current does not exceed 2 kA, then only such a type of supermode can be selectively excited in this system inside a narrow frequency band (less than 0.01 THz). This justifies the use of the single-frequency approximation of the electron-wave interaction. At higher operating current, we should solve a multifrequency problem to describe competition from different supermodes.

In the case of a close-to-starting electron current, 2 kA, the electron efficiency in the steady-state regime amounts to approximately 6%. This corresponds to an output power close to 1 GW at a frequency of 2 THz. The typical duration of the transition process is about 30 round trips of the wave through the cavity, which corresponds to about 100 ns.

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