Deeply virtual Compton scattering at small $x_B$
and the access to the GPD $H$

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Abstract

We give a partonic interpretation for the deeply virtual Compton scattering (DVCS) measurements of the H1 and ZEUS Collaborations in the small-$x_B$ region in terms of generalized parton distributions. Thereby we have a closer look at the skewness effect, parameterization of the $t$-dependence, revealing the chromomagnetic pomeron, and at a model-dependent access to the anomalous gravitomagnetic moment of nucleon. We also quantify the reparameterization of generalized parton distributions resulting from the inclusion of radiative corrections up to next-to-next-to-leading order. Beyond the leading order approximation, our findings are compatible with a ‘holographic’ principle that would arise from a (broken) SO(2,1) symmetry. Utilizing our leading-order findings, we also perform a first model-dependent “dispersion relation” fit of HERMES and JLAB DVCS measurements. From that we extract the generalized parton distribution $H$ on its cross-over line and predict the beam charge-spin asymmetry, measurable at COMPASS.

Keywords: deeply virtual Compton scattering, generalized parton distributions

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1 Introduction

The electron/positron-proton collider experiments H1 and ZEUS at the HERA ring in DESY improved not only the quantitative understanding of inclusive processes, e.g., by pinning down the small-$x$ behavior of parton distribution functions (PDFs) [1, 2], but also led to new insights into the proton structure. Two decades ago, it was mostly unforeseen that at high energies the deep inelastic scattering (DIS) cross section steeply rises, that the proton remains intact in almost one third of all scattering events, and that even exclusive processes become measurable, contributing considerably to the total cross section. The reader may find comprehensive reviews in Refs. [3–5].

These exclusive processes, e.g., the electro- or photo-production of a vector meson or a photon, were extensively studied in the small-$x_B$ kinematics by the H1 and ZEUS Collaborations [6–26]. The amplitude of the subprocesses,

$$\gamma^*(q_1) p(P_1) \rightarrow V(q_2) p(P_2), \quad V = \gamma, \rho, \omega, \phi, J/\Psi, \Upsilon,$$

is necessarily dominated by $t$-channel exchanges that carry the quantum numbers of the vacuum. The large amount of HERA data calls for a phenomenological description and it challenges the theoretical understanding of the nucleon in terms of its partonic substructure. Needless to say, a quantitative understanding of the parton dynamics will be crucial at the frontier of exploration of the structure of matter at LHC [27, 28]. In this context, it is worth noting that exclusive Higgs production via gluon fusion is a rather clean channel [29, 30]; however, for cross section estimates the gluonic content of the proton must be quantified.

One might have hoped to master the phenomenology of such exclusive processes in the framework of Regge theory. Unfortunately, for an ‘incoming’ virtual photon $S$-matrix theory is not applicable. Thus, Regge theory loses the theoretical foundation and might possibly be replaced by a pragmatic Regge phenomenology [31]. Consequently, firm conclusions valid for on-shell scattering, like the one that unitarity requires that the rightmost singularity in the complex angular momentum plane belongs to a $J = 1$ exchange, might not be appropriate for off-shell processes. In fact, one of the lessons of H1/ZEUS experiments is that cross sections, effectively parameterized as

$$\frac{d\sigma_{\gamma^* p \rightarrow V p}}{dt} \propto \left( \frac{W^2}{W_0^2} \right)^{2(\alpha(t)-1)}, \quad W^2 = (P_1 + q_1)^2,$$

rise steeply, contrarily to what is implied by the pomeron ($J = 1$) trajectory

$$\alpha_P(t) = 1 + 0.25 \frac{t}{\text{GeV}^2}. \quad (3)$$

In addition, the effective trajectory $\alpha(t)$ varies with the photon virtuality $Q^2 = -q_1^2$. The corresponding effective Regge pole in the complex angular momentum plane might also be understood as a convenient implementation of cuts.
Inspired by QCD, a phenomenological description of cross sections at small $x_B$ has been achieved in terms of the color dipole model \cite{32,33}, making direct contact to the high energy approximation (BFKL) of scattering amplitudes \cite{34,35}. The physical picture might be set up in the rest frame of the proton in which the highly energetic virtual photon fluctuates into a quark-antiquark pair. The quark-antiquark pair forms a small color dipole, spatially distributed in transverse direction, that interacts with the proton by a gluonic exchange. Finally, the quark-antiquark pair forms a meson or annihilates into a photon. The physical amplitude is given as convolution (with respect to longitudinal momentum fraction and transverse separation) of the color dipole spectral function (cross section) with the corresponding wave functions, describing the transition of the initial photon into a quark-antiquark pair or the quark-antiquark pair into the final state.

A perturbative QCD framework, applicable for longitudinally polarized photons, is founded on factorization theorems \cite{36}. Here, in setting up the partonic space-time picture, one may prefer a ‘brick wall’ frame. The proton is viewed as a bunch of partons travelling along the $z$-direction close to the light-cone, while the virtual photon goes in the opposite direction and may have zero energy. During the scattering process, e.g., a quark is knocked out by the photon, it picks up an antiquark and they form a meson, which travels in the direction of the photon. Another subprocess is when the photon knocks out a quark-antiquark pair arising from the fluctuation of gluonic components in the proton wave function. In this picture the quark-antiquark pair moves almost collinearly and its transition into the final state is described by a collinear distribution amplitude. The partonic content of the proton is encoded in generalized parton distributions (GPDs) \cite{37–39}, which can be interpreted as a probability amplitude to emit and absorb a parton that moves along the light cone or, equivalently, as a $t$-channel exchange \cite{40}. For comprehensive reviews on GPDs see \cite{41,42}.

Obviously, the two space-time pictures are related by a boost along the $z$-axis and in an exact QCD treatment both approaches must yield the same results. The rise of the cross section \cite{2} at large $W$ requires a pomeron-like $t$-channel exchange, encoded either in the color dipole spectral function or in GPDs. The difference between the two approaches, e.g., for the case of longitudinally polarized photons in hard exclusive meson production, is mainly in the treatment of transverse degrees of freedom and in the view on partons, exchanged in the $t$-channel. This partonic exchange is in the color dipole model approach commonly viewed as entirely gluon dominated, while the collinear perturbative framework provides an implicit separation, defined by the factorization scheme, of the quark and gluonic content of the proton. The correspondence of the objects in the two frameworks is obvious, and the color dipole spectral function is often expressed in terms of so-called $k_{\perp}$-unintegrated GPDs or, equivalently, so-called quantum phase space dis-
tributions[43]. We add that in the two-gluon exchange color dipole model one may integrate out the $k_\perp$-dependence in the target related part, which provides then ‘hybrid’ models that contain preasymptotic corrections due to the transverse quark-antiquark pair fluctuation, elaborated, e.g., in Refs. [44–46].

In this article we restrict ourselves to DVCS at small $x_B$, which has been measured at DESY on the electron/positron-proton collider HERA experiments H1 and ZEUS [8, 9, 7, 10]. This has been previously studied in the spirit of the aligned-jet model [47], from high-energy/Regge perspective [48–50], in color dipole model [51–55], and in collinear factorization approach at leading order (LO) [56–58], next-leading order (NLO) [59–61], and next-to-next-leading order (NNLO) [62]. As said above, the view on the partonic content of the proton varies with the approach. In DVCS the different points of view are obvious. In the collinear DVCS approach one starts with the hand-bag diagram, while in a color dipole model one would draw a $t$-channel gluon ladder. In the collinear factorization approach one usually stays with the resolution conventions which are set in the standard PDF analysis of deep inelastic scattering (DIS) at a given fixed order in the coupling. Whether one expresses now the sea quark content in terms of gluons, as in the color dipole approach, or stays with a large amount of sea quarks, might be considered as a matter of taste. However, one should bear in mind that what we shall call gluons is not the same object that appears in a model approaches. Let us also add that a model-dependent evaluation of power suppressed corrections and their interpretation as higher-twist corrections in the collinear factorization approach is likely an oversimplified view on the dynamics of QCD.

We analyze the DVCS data of the H1 and ZEUS Collaborations within the collinear factorization approach, going along the lines of previous work [62]. To set up non-perturbative GPD models, we are to some extent motivated by the Regge phenomenology and we employ its language. However, to avoid any misunderstanding, we enclose in quotation marks Regge-theory terms, appearing in our modelling. For instance, “pomeron” denotes a $t$-channel exchange with vacuum quantum numbers responsible for a steep rise of cross sections, associated with an exchange of a colorless quark-antiquark or gluon pair. A $Q^2$-dependence of our “pomeron trajectory” will be solely induced by evolution [63]. Using a least square fitting procedure, we aim for a model-dependent access to both quark and gluon GPDs at the LO of perturbation theory and beyond. There are two reasons to update our previous findings. First, the H1 Collaboration provided new data from the HERA II run including a significantly improved measurement of the $t$-dependence of the cross section [9] and a preliminary result for the beam charge asymmetry (BCA) [10]. The second reason is that, in contrast to our previous ad hoc model study, we utilize here for the first time flexible GPD models, allowing us to describe DVCS also in the LO (hand-bag) approximation. Such GPDs can then be used in a simple GPD description of fixed target experiments, as
pointed out in Ref. [64].

The outline of the paper is as follows. In Sect. 2 we present the observables, relevant for our DVCS analysis in the small-$x_B$ kinematics. In Sect. 3 we give a short overview of popular GPD models that have been employed for small-$x_B$ phenomenology. Thereby, we adopt the terminology of quantum mechanics for both CFF and GPD models, i.e., we distinguish between GPD model and its representation. A model refers to a specific GPD which can be given in different equivalent representations, e.g., within double distribution representation, some version of conformal partial wave expansion, directly in some SO(3) partial wave expansion or in the light-cone wave function overlap representation. We shall also emphasize that the claim, that at small $x$ GPDs should be rigidly tied to PDFs, cannot be mathematically justified and, moreover, its partonic/physical content is rather speculative. We will then set up our model for integral (conformal) Mellin moments. Then we give a short insight into the technicalities of the (conformal) Mellin-Barnes integral representation of the DVCS amplitude. In Sect. 4 we present then a detailed analysis of H1 and ZEUS DVCS data and provide a GPD interpretation. In particular, we shall discuss the failure of ad hoc small-$x$ models at LO, and we shall give a detailed analysis of the skewness effect, the parameterization of $t$-dependence, the model-dependent access of the anomalous gravitomagnetic moment, and the reparameterization effect of radiative corrections. In Sect. 5 we include our LO GPD findings in a global “dispersion relation” fit to DVCS data for unpolarized proton target, which includes the measurements of HERMES, CLAS, and HALL A Collaborations. Thereby, we aim for a first model-dependent extraction of the dominant GPD $H$. This will allow us to predict the beam charge-spin asymmetry for COMPASS kinematics. Finally, we summarize and conclude.

2 Deeply virtual Compton scattering at small $x_B$

In the deeply virtual electroproduction of photons both the Bethe-Heitler (BH) bremsstrahlung and the DVCS process contribute to the cross section. In the small-$x_B$ region the DVCS process dominates which allows one to extract the DVCS cross section by a subtraction procedure. There the integration over the azimuthal angle $\phi$ projects on the transverse DVCS cross section and guarantees that the contamination by the interference term is negligibly small. In this way, both the H1 [8, 9] and ZEUS [7] Collaborations measured the cross section of DVCS on an unpolarized proton in dependence of the photon virtuality $Q^2$, the center-of-mass energy $W$, and the momentum transfer squared $t$.

\footnote{In the literature specific GPD models are often associated with representations. We have experienced that this indiscriminate use of language can lead to confusion.}
On the theoretical side, the unpolarized DVCS cross section in the small $x_B$-kinematics,

$$
\frac{d\sigma^{\text{DVCS}}}{dt}(W, t, Q^2) \approx \frac{4\pi\alpha^2}{Q^4} \frac{W^2t^2}{W^2 + Q^2} \left[ |\mathcal{H}|^2 - \frac{t}{4M_p^2} |\mathcal{E}|^2 \right] (\xi, t, Q^2) \bigg|_{\xi = \frac{Q^2}{2W^2 + Q^2}},
$$

is primarily given in terms of two Compton form factors (CFFs), $\mathcal{H}$ and $\mathcal{E}$. Here $\alpha$ is the electromagnetic fine structure constant and $M_p$ is the proton mass. Note that we consider CFFs as functions of the symmetric scaling variable $\xi$, which can also be expressed in terms of the Bjorken scaling variable $x_B$:

$$
\xi = \frac{Q^2}{2W^2 + Q^2} = \frac{x_B}{2} + O(x_B^2).
$$

Both CFFs belong to the parity- and charge-even sector and, caused by an effective “pomeron exchange”, they might rise steeply at small $x_B$. In the collinear factorization approach their dominant contribution arises from the twist-two GPDs $H$ and $E$. In the cross section (4) we neglected contributions of parity-odd CFFs $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$, which are supposed to be suppressed by $O(x_B)$, for details see, e.g., Refs. [56, 62]. Owing to the integration over the azimuthal angle $\phi$, there are no interferences of twist-two with twist-three or gluon transversity contributions. The squares of the both latter and possible twist-four and higher contributions are all considered negligible.

Furthermore, the possibility to have both electrons and positrons as probes in the HERA experiments allows direct access to the BH-DVCS interference term. This serves as an experimental consistency check of the aforementioned subtraction procedure and additionally allows the measurement of the beam charge asymmetry (BCA)

$$
A_{\text{BCA}}(\phi) = \frac{d^+\sigma - d^-\sigma}{d^+\sigma + d^-\sigma},
$$

reported in Ref. [10]. This asymmetry has a more intricate azimuthal angular $\phi$-dependence, particularly when one integrates over a restricted phase space. For unbinned data, it can be expressed in terms of the BH and DVCS amplitudes and approximated to twist-two accuracy by the first and third harmonics. Here and in the following we take the conventions of Ref. [56] and find

$$
A_{\text{BCA}}(\phi) = -\frac{T_{\text{BH}}^*T_{\text{DVCS}}^* + T_{\text{BH}}^*T_{\text{DVCS}}}{|T_{\text{BH}}|^2 + |T_{\text{DVCS}}|^2} \frac{F_1 \Re \mathcal{H} - \frac{t}{4M_p^2} F_2 \Re \mathcal{E}}{N(\phi)} x_B \cos(\phi) + x_B \frac{F_1 \Re (\mathcal{E}_T + 2\tilde{\mathcal{H}}_T) - F_2 \Re (\mathcal{H}_T - \frac{t}{2M_p^2} \tilde{\mathcal{H}}_T)}{N_T(\phi)} \cos(3\phi),
$$

\footnote{Let us add that similarly to polarized parton densities the Regge phenomenology of the CFFs $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$ might not be so well understood. The CFF $\xi \tilde{\mathcal{E}}$ contains also a pion pole contribution which yields a constant real part. However, it cannot compete with $\mathcal{H}$ in the DVCS cross section. Hence, for small $x_B$, we can safely neglect these CFFs in $\tilde{\mathcal{H}}$.}
where $F_1(t)$ and $F_2(t)$ are the Dirac and Pauli form factors of the proton. We could again neglect the $\tilde{\mathcal{H}}$ contribution in the first harmonic. The third harmonic is induced by a photon helicity flip of two units. It is perturbatively tied to gluon transversity and might be contaminated by twist-four (quark) contribution \cite{65}. The normalization factors,

$$\mathcal{N}_T^{-1} \approx -\frac{t-t_{\text{min}}}{4M_p^2} \frac{1-y}{2-2y+y^2} \mathcal{N}^{-1}, \quad y = \frac{1}{x_B} \frac{Q^2}{s},$$

and

$$\mathcal{N} \approx \frac{yQ}{8 \sqrt{1-y(2-2y+y^2)}} \sqrt{t_{\text{min}}-t} \left[ \sum_{n=0}^{3} c_{n}^{\text{BH}} \cos(n\phi) + \frac{t}{Q^2} \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) x_B^2 \sum_{n=0}^{2} c_{n}^{\text{DVCS}} \cos(n\phi) \right],$$

moderately induce a further azimuthal angle $\phi$-dependence. Thereby, the DVCS Fourier coefficients $c_{n}^{\text{DVCS}}$ depend of course on the CFFs. Although a good approximation of the normalization factors is obtained by restricting to the zeroth harmonics in the BH and DVCS amplitude squared terms,

$$c_{0}^{\text{DVCS}} = 2(2-2y+y^2) \left[ |\mathcal{H}|^2 - \frac{t}{4M_p^2} |\mathcal{E}|^2 \right],$$

a residual $\phi$-dependence is induced by the product $\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)$ of BH propagators. At twist-three level the zeroth and second harmonics of the interference term complete the azimuthal angular dependencies in the BCA \cite{60}. Note that this zeroth harmonic is dominated by the same twist-two CFF combination as the first harmonic, displayed in Eq. (6).

For the HERA kinematics we expect from the model studies in Ref. \cite{56}, see discussion in Sect. 7.1 there, a moderate twist-two modulation ($\sim 10 - 15\%$) of the BCA with a small twist-three admixture ($\sim 2 - 5\%$), which is mainly governed by the twist-two CFF $\mathcal{H}$. We emphasize that the $\cos(3\phi)$ harmonic, related to gluon transversity, is theoretically and phenomenologically uncharted. To reveal the dominant twist-two $\cos(\phi)$ harmonics of the interference term from a BCA measurement, one might utilize the Fourier decomposition of the charge asymmetry

$$A_{\text{BCA}}(\phi) = p_0 + p_1 \cos(\phi) + p_2 \cos(2\phi) + p_3 \cos(3\phi) + \cdots,$$

given as an infinite sum, and extract the dominant amplitude $p_1$ from a fit. In this way both twist-three and possible gluon transversity contributions diminish and we could on the theoretical side safely employ the approximation for the amplitude

$$p_1 \approx \frac{8 \sqrt{1-y(2-2y+y^2)}}{yQ} \sqrt{t_{\text{min}}-t} \cdot \frac{x_B}{c_{0}^{\text{BH}}} \left[ F_1 \Re \mathcal{H} - \frac{t}{4M_p^2} F_2 \Re \mathcal{E} \right] + \frac{t}{Q^2} \mathcal{P}_{12}^{(0)} 2(2-2y+y^2) x_B^2 \left[ |\mathcal{H}|^2 - \frac{t}{4M_p^2} |\mathcal{E}|^2 \right].$$

\[7\]
where $P_{12}^{(0)}$ denotes the zeroth harmonic of the product $P_1(\phi)P_2(\phi)$. Obviously, the (unbinned) observable $p_1$ is suppressed at large $Q^2$ and small $-t$ and vanishes for $-t \rightarrow -t_{\text{min}} \approx 0$. Note also that twist-two contributions to other harmonics might arise from the residual $\phi$-dependence in the normalization factor as well.

From cross section measurements one extracts in the first place the CFF $H$, since $E$ is kinetically suppressed by a factor $-t/4M_p^2$, see Eq. (4). Both CFFs enter in a different combination in the first harmonic of the BCA (11), where $\Re E$ is now accompanied by $-F_2(t)t/4M_p^2$. A model study, in which $E$ was varied with fixed $H$, has shown that it might be possible in a BCA measurement, supplemented by the DVCS cross section measurement, to access the CFF $E$ [56].

To understand clearly which GPD degrees of freedom are accessible in a DVCS measurement [66, 64], we remind that the CFFs satisfy “dispersion relations”, see, e.g., Refs. [67, 68, 66, 62, 69, 70]. Literally, we have to distinguish between the physical dispersion relations that follow from general considerations and the “partonic” ones that are in one-to-one correspondence [68, 66, 62, 69, 70] with the leading approximation of CFFs in the perturbative framework. For finite photon virtuality the physical and partonic dispersion relations differ by higher twist contributions, see, e.g., Refs. [62, 70]. According to our goal, we employ the set that is consistent with the perturbative framework at leading twist and denote them with quotation marks. For instance, $H$ and $E$ have even signature and their subtracted “dispersion relations” might be written in the following form [62]:

$$\Re \left\{ \frac{H}{E} \right\}(\xi,t,Q^2) \approx \frac{1}{\pi} \text{PV} \int_0^1 dx \frac{2x}{\xi^2 - x^2} \Im \left\{ \frac{H}{E} \right\}(x,t,Q^2) + C(t,Q^2),$$

where we set the threshold $\xi_{\text{th}} = 1 + O(1/Q^2)$ equal to one. Since the real part is determined by the imaginary part, one might choose to extract it from the experiment indirectly, by means of the “dispersion relation” [64]. Then, when fitting the data, one needs to model only the imaginary part and the subtraction constant. It has been pointed out in Refs. [66, 62, 70] that the subtraction constant is entirely related to a GPD term that completes polynomiality. In the representation of Ref. [71] it is the so-called $D$-term, see also discussion in Ref. [72].

The functional form for the CFFs in the small-$x_B$ region might be borrowed from the Regge theory. Thereby, we assume that the “Regge trajectory” is linear in $t$ and is same for both DIS and DVCS. The imaginary part of CFF can then be written as:

$$\Im \left\{ \frac{H}{E} \right\}(\xi,t,Q^2) = \pi \xi^{-\alpha(t,Q^2)} \left\{ h_\alpha \right\}(t,Q^2) + \cdots.$$  

For on-shell scattering, i.e., $Q^2 = 0$, one would take for $\alpha(t) \equiv \alpha(t,Q^2 = 0)$ the soft pomeron trajectory (3) as the leading one. It appears in the (target) helicity conserved CFF $H$ and we shall assume that it couples also to the helicity-flip CFF $E$. This is related to the so-called
chromomagnetic pomeron, studied in Ref. [73] in the instanton approach. Interestingly, that study suggests that the chromomagnetic pomeron should play a dominant role in polarization phenomena at high-energy and so one might expect that the CFF $E$ is sizable. In principle, the pomeron might also couple to the set of transversity CFFs, which would be visible as a sizeable $\cos(3\phi)$ harmonics in the BCA [6].

In accordance with phenomenology and perturbative QCD the “pomeron trajectory” depends on the photon virtuality $Q^2$. Note that perturbation theory states that in the double log approximation, for large $Q^2$ and small $\xi$, the “pomeron” in the Regge-inspired ansatz [13] is replaced by a more intricate functional dependence [63]. Also for this reason, we understand the parameterization [13] as an effective one. Next, we have non-dominant “Reggeon exchanges”, e.g., $\alpha(t) \approx 1/2 + t/\text{GeV}^2$, which we shall neglect in the small-$x_B$ kinematics.

Plugging the Regge-like parameterization [13] of the imaginary part into the “dispersion relation” (12), we of course recover the known Regge formula for the small-$x_B$ region:

\[
\left\{ \begin{array}{c} \mathcal{H} \\ \mathcal{E} \end{array} \right\}(\xi, t, Q^2) = \pi \left[ i - \cot \left( \frac{\pi \alpha(t, Q^2)}{2} \right) \right] \xi^{-\alpha(t, Q^2)} \left\{ \begin{array}{c} h_{\alpha} \\ e_{\alpha} \end{array} \right\}(t, Q^2) + \cdots .
\]

(14)

Here the ellipsis stands for non-dominant contributions of “Reggeon exchanges” with typically $\alpha_{\text{Reg}}(t \sim 0.2 \text{ GeV}^2) \sim 0.3$, for a constant [3], and for further terms that vanish as $\xi \to 0$. In the considered kinematics the “pomeron trajectory” has a typical value of

$\alpha(t \sim 0.2 \text{ GeV}^2, Q^2 \sim 4 \text{ GeV}^2) \sim 1.1 - 1.2$.

Hence, in contrast to on-shell forward Compton scattering at large energies, where the classical pomeron intercept $\alpha(0) = 1$ yields a vanishing real part, we encounter in DVCS with $\alpha(0) > 1$ a positive real part that should even dominate the negative “Reggeon” contributions and subtraction constant [56]. This already predicts that for H1/ZEUS kinematics the sign of the leading $\cos \phi$ harmonic in the BCA [10] is positive and even provides an estimate of its size, quoted above, which is consistent with preliminary measurements of the H1 Collaboration [10].

Footnote 3: From the viewpoint of the “dispersion relation” [12] this constant, i.e., the so-called $J = 0$ pole, contains contributions from the subtraction constant and further contributions induced by both “pomeron” and non-leading “Regge exchanges”.

9
3 Models for the flavor singlet GPDs at small $x$

In the collinear factorization approach the CFFs $F = \{H, E\}$ are given as convolution of a hard scattering part with GPDs $F = \{H, E\}$, which to LO accuracy reads

$$F(\xi, t, Q^2)_{\text{LO}} = \sum_{q=u,d,s,\cdots} e_q^2 \int_{-1}^{1} dx \left[ \frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0} \right] F_q(x, \eta = \xi, t, Q^2),$$  \hspace{1cm} (15)$$

where $e_q$ is the fractional quark charge. Note that this is nothing else but the so-called hand-bag approximation, where only quarks are resolved in a hard Compton scattering process. Obviously, the imaginary part of the CFFs is given by the GPD on the cross-over line ($\eta = x$)

$$\Im F(\xi, t, Q^2)_{\text{LO}} = \pi \sum_{q=u,d,s,\cdots} e_q^2 \left[ F_q(x = \xi, \eta = \xi, t, Q^2) - F_q(x = -\xi, \eta = \xi, t, Q^2) \right].$$  \hspace{1cm} (16)$$

The GPD at negative momentum fraction $x = -\xi$ corresponds to the negative antiquark contribution. The real part of the amplitude can be calculated either by means of the “dispersion relation” \cite{12} or from the convolution formula \cite{15}. The equality of the two results is ensured by construction \cite{68, 62}, i.e., by the polynomiality or support properties of the GPD \cite{66, 62}. For fixed $Q^2$ the LO perturbative approach is completely equivalent to the (approximated) “dispersion relation” \cite{12} and the only information that can be accessed at LO is the GPD on the cross-over line.

Perturbation theory also predicts the evolution of GPDs, e.g., their change on the cross-over line in the flavor singlet sector is

$$Q^2 \frac{d}{dQ^2} F(x, x, t, Q^2) = \int_x^1 \frac{dy}{x} V(1, y/x; \eta = x, \alpha_s(Q)) F(y, x, t, Q^2),$$  \hspace{1cm} (17)$$

where the flavor singlet quark ($\Sigma$) and gluon ($G$) GPDs are collected in the column vector

$$F(x, \eta, t, Q^2) = \left( \begin{array}{c} F_{\Sigma} \\ F_G \end{array} \right)(x, \eta, t, Q^2), \quad F_{\Sigma}(x, \cdots) = \sum_{q=u,d,s,\cdots} \left[ F_q(x, \cdots) - F_q(-x, \cdots) \right].$$  \hspace{1cm} (18)$$

and the evolution kernel is a two-dimensional matrix which leads to the mixing of the quark and gluon GPDs. $F_{\Sigma}(x, \eta, \cdots)$ and $F_G(x, \eta, \cdots)$ are antisymmetric and symmetric in $x$, respectively, and both are symmetric in $\eta$. One realizes from the evolution equation \cite{17} that the scale change

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4 Of course, this is a simplified partonic view, since an infinite number of longitudinal gluons, radiated from the struck quark, are included in the GPD. The radiation of a transverse gluon is taken into account in genuine twist-three contributions.

5 It is common to adopt the convention in which the gluon GPD reduces in the forward limit to $xG(x)$. In such a case the non-diagonal entries in the evolution matrix are accompanied by $1/\eta$ or $\eta$, which are set equal to $x$, see Ref. [71].
of the GPD on the cross-over line is governed by its value in the outer region \((x \geq \eta)\). Moreover, the evolution kernel in the gluon-gluon channel has a “pomeron”-like singularity. Hence, as in the forward case, the evolution in the small-\(x\) region is driven by the gluons and can be solved in the double log approximation. There one finds that the “pomeron intercept” \(\alpha(0)\) effectively increases and the slope \(\alpha'\) decreases with growing \(Q^2\). We note that the change of the residue function in \([13]\) under evolution cannot be derived for the general case, since it is determined by the GPD in the outer region.

Beyond LO accuracy, the imaginary part of CFFs is given by a convolution of the hard-scattering part with the GPDs, analogous to the evolution equation. As in DIS, the momentum fraction integration runs from \(x = \xi\) to \(x = 1\). However, there is an important difference. Namely, the GPD itself depends on the scaling variable \(\xi\). Hence, the deconvolution cannot be performed and the outer GPD region can only be accessed through evolution effects, see discussion in Ref. \([75]\).

According to all this, a parameterization of the outer GPD region is sufficient for a description of DVCS via imaginary part of CFFs. Then, using simple Regge-motivated small-\(x_B\) parameterization of imaginary part of CFFs from \([13]\), and in accordance with the LO relation \([16]\), we write a rather flexible GPD ansatz in the small-\(x\) region at a given input scale:

\[
F(x \geq \eta, \eta, t) = n_F x^{-\alpha(t)} r(\vartheta|F) \beta(t|F); \quad \vartheta \equiv \eta/x. \tag{19}
\]

Here we factorized the GPD into the Regge part \(x^{-\alpha(t)}\), the skewness function \(r(\eta/x|F)\), the residue function \(\beta(t|F)\), and the residue \(n_F\) of the GPD in the forward limit. Hence, in this limit, i.e., \(\eta \to 0\) and \(t \to 0\), both \(r(\eta/x|F)\) and \(\beta(t|F)\) are normalized to one. Note that skewness- and \(t\)-dependence might not necessarily factorize. Since present data do not allow to address their possible functional interplay, we stick here to the most convenient model. Also, we would like to add that in this GPD approach restoring of the GPD in the central region \((x \leq \eta)\) is a mathematical problem, which is solved in the short distance operator product expansion framework by construction \([68, 62, 64]\). Thereby, the Mellin moments of \(F(x, x\vartheta, t)\) are expanded in a Taylor series around the point \(\vartheta = 0\), i.e., \(r(\vartheta|F)\) possesses certain analytic properties.

### 3.1 Survey of GPD models at small \(x\)

GPD modelling can be done in different representations. It is popular to employ either double distribution (DD) representation \([37, 70]\) or some version of conformal partial wave (PW) expansion \([40, 80]\), adopted from the description of mesons \([81, 84]\). Mathematically, there is a one-to-one mapping between GPDs in two different representations. Since this mapping might be cumbersome, it has been sometimes only partly worked out. In these circumstances, one might hope to gain some new physical/partonic insight in the small-\(x\) region just by a change of the
representation. However, it is clear that all popular models finally lead in the small-$x$ region to the GPD of the form \((19)\), where, however, the skewness function might be rather restricted. Surely, the small-$x$ GPD properties, described in momentum fraction representation, can be equivalently set up in any other representation, too.

Our effective Regge-motivated ansatz \((19)\) is supported by a diagrammatic analysis from the \(t\)-channel view \([67]\), where it was found that the “Regge trajectory” is skewness-independent; however, the residue function, i.e., the skewness function \(r(\vartheta|F)\) in our ansatz \((19)\), depends on it. One arrives at the same conclusion from an \(s\)-channel view, if one adopts the ideas of Refs. \([85, 86]\) and convolutes a spectator quark model, expressed in the DD representation, with a constituent quark mass spectral function. Of course, in Regge-inspired DVCS models one starts with such a point of view, where the skewness dependence of the impact form factors has to be modelled. We already stated that Regge behavior is included in the popular GPD models, too.

Such arguments and agreement with experiment, see also Sect. \([1]\) led to a broad acceptance of effective Regge behavior for off-shell processes, even in the absence of a firm theoretical foundation. Assuming this, the remaining task for GPD phenomenology is the determination of the

i. skewness function \(r(\vartheta|F)\) and

ii. residue function \(\beta(t|F)\).

Certainly, one can address these questions by fitting to the experimental measurements using one’s favorite GPD representation. The problem remains always the same, namely, to have appropriate functional GPD ansaezts.

The value of the skewness function at \(\vartheta = 1\), see Eq. \((19)\), is an important characteristic of a GPD model. For the GPD \(H\), with \(r(\vartheta) \equiv r(\vartheta|H)\) and \(r \equiv r(1)\), it can be expressed as the ratio of the GPD at the cross-over line to the corresponding PDF, given as the GPD at \(\eta = 0\) for \(t = 0\). Since we rely on the universality of the “pomeron trajectory”, this skewness ratio is for small \(x\) mostly independent of \(x\). For the quark GPD it reads

\[
r^x(Q^2) = \frac{H^\Sigma(x, \eta = x, t = 0, Q^2)}{\Sigma(x, Q^2)}, \quad \Sigma(x, Q^2) = H^\Sigma(x, \eta = 0, t = 0, Q^2),
\]

and for gluons we use the convention:

\[
r^G(Q^2) = \frac{H^G(x, \eta = x, t = 0, Q^2)}{xG(x, Q^2)}, \quad xG(x, Q^2) = H^G(x, \eta = 0, t = 0, Q^2).
\]

In an LO DVCS analysis the quark skewness \((20)\) is given as the ratio of \(3m\mathcal{H}(x, t = 0, Q^2)/\pi\) and the transverse unpolarized DIS structure function \(F_T(x, Q^2)/x\); therefore, it can be almost directly measured, while the gluonic one \((21)\) can be accessed only by utilizing a large lever arm in \(Q^2\).
The aligned-jet model considerations in Ref. [47] provide estimates for both the DIS structure
function, given as imaginary part of the virtual forward Compton amplitude, and for the imaginary
part of DVCS amplitude. In an LO approximation resulting quark skewness ratio turns out to
be $r^\Sigma(Q^2 \sim 2.5 \text{ GeV}^2) \approx 1$. We note that this model has been used to predict the size of the
DVCS cross section, which was afterwards experimentally confirmed, see Ref. [7]. The model was
then generalized for the deeply virtual Compton amplitude with two virtual photons, providing a
prediction for the quark skewness function [61]:

$$r^\Sigma_{\text{McDS}}(\vartheta) = \frac{(1 + \vartheta)^{1 - \alpha(0)}}{2\vartheta} \left(1 + \frac{M_0^2}{Q_0^2}\right) \ln \left(\frac{1 + \vartheta}{1 - \frac{Q_0^2 - M_0^2}{Q_0^2 + M_0^2} \vartheta}\right), \quad M_0^2 \sim 0.5 \text{ GeV}^2, \quad Q_0^2 \sim 2.5 \text{ GeV}^2. \quad (22)$$

In the double distribution (DD) representation a GPD that has support in the interval $-\eta \leq x \leq 1$ is expressed as follows [37, 76]:

$$F(x, \eta, t) = \int_0^1 dy \int_{-1+z}^{1-z} \delta(x - y - \eta z) f(y, z, t). \quad (23)$$

Note that in this specific representation polynomiality is not completed for $H$ and $E$. This
artifact is not crucial for the small-$x$ application and it can be cured in various ways, see, e.g.,
Refs. [71, 87, 72, 88]. It is quite popular among phenomenologists to utilize Radyushkin’s DD
ansatz (RDDA) [89, 90]. Here the DD at $t = 0$ is factorized in a PDF and a profile function, namely,

$$h(y, z, t = 0) = \frac{\Gamma(3/2 + b)}{\Gamma(1/2)\Gamma(1 + b)} \frac{g(y)}{1 - y} \left(1 - \frac{z^2}{(1 - y)^2}\right)^b. \quad (24)$$

The profile function is chosen to be convex and its width is controlled by the $b$ parameter. The
skewness function can be easily evaluated in terms of hypergeometric functions:

$$r^\Sigma_{\text{RDDA}}(\vartheta) = 2F_1\left(\frac{\alpha/2, (1 + \alpha)/2}{3/2 + b} | \vartheta^2\right), \quad r^G_{\text{RDDA}}(\vartheta) = 2F_1\left(\frac{(\alpha - 1)/2, (\alpha)/2}{3/2 + b} | \vartheta^2\right). \quad (25)$$

Here and in the following we use a shorthand for the intercept $\alpha \equiv \alpha(t = 0)$. Setting $\vartheta = 1$ in the
skewness functions (25), the values of the skewness ratios at the cross-over line $\eta = x$ follow:

$$r^\Sigma_{\text{RDDA}} = 2^{2b - \alpha} \frac{\Gamma(3/2 + b)\Gamma(1 + b - \alpha)}{\Gamma(3/2)\Gamma(2 + b - \alpha)}, \quad r^G_{\text{RDDA}} = 2^{1+2b - \alpha} \frac{\Gamma(3/2 + b)\Gamma(2 + b - \alpha)}{\Gamma(3/2)\Gamma(3 + 2b - \alpha)}. \quad (26)$$

Taking the originally proposed value $b = 1$, one finds for $\alpha = 1.2$:

$$r^\Sigma_{\text{RDDA}} \approx 1.8, \quad r^G_{\text{RDDA}} \approx 1.04. \quad (27)$$

The authors define quantity $R^{\text{FFS}}$ as ratio of imaginary DIS and DVCS amplitudes. In LO approximation this
means $r = 2^{-\alpha(0)}/R^{\text{FFS}} \sim 2/R^{\text{FFS}}$. 
For growing $b$ the quark skewness ratio decreases and rapidly approaches the value $r^\Sigma = 1$, which corresponds to setting GPD equal to a $t$-decorated PDF. For the gluon GPD we see that the ratio $r^G = 1$ is almost realized for $b = 1$. A skewness ratio $r < 1$ requires to give up the convexity of the profile function.

It is also rather popular to expand GPDs in tree-level conformal partial waves. In some versions the conformal partial wave amplitudes are then additionally mapped to forward-like functions. In such expansions the evolution operator is diagonalized at LO. Unfortunately, such representations involve certain mathematical subtleties, which have been understood only in the last few years [91, 92, 40, 93]. Originally, based on an oversimplified view on the inverse integral transformation [94], it has been erroneously stated that for small $x$ the GPD transform reduces to a PDF. Thus, the authors of Ref. [94] claim that the GPDs in the small-$x$ region are at $t = 0$ tied to PDFs by the following ratio:

$$
 r^\Sigma_{\text{con}} = \frac{2^{\alpha} \Gamma(3/2 + \alpha)}{\Gamma(3/2) \Gamma(2 + \alpha)} , \quad r^G_{\text{con}} = \frac{2^{1+\alpha} \Gamma(3/2 + \alpha)}{\Gamma(3/2) \Gamma(3 + \alpha)} .
$$

This ratio is in fact a Clebsch-Gordan coefficient, e.g., occurring in the conformal partial wave expansion of the product of two currents (it can be seen, e.g., by taking a “Regge pole” value for complex conformal spin, $j = \alpha - 1$, in Eq. (65) below). Therefore, we shall call it conformal ratio. Although the small-$x$ claim, which corresponds to a specific double distribution model [95], is not based on general grounds, the arguments were repeated and refined in a more recent version of the claim, where it was assumed that the “pomeron pole” is the rightmost singularity in the complex conformal spin plane [96]. Below we shall spell out, what is already (implicitly) said in the literature, e.g., Refs. [95, 91, 92, 40, 93], namely, that the claim in Refs. [94] and [96] is based on certain mathematical simplifications. As long as the partonic content of the conformal PW expansion is not understood, we consider this claim as not necessary applicable for GPD phenomenology.

We would like to mention that GPDs can also be described by a $t$-channel SO(3) partial wave expansion, formulated within complex-valued angular momentum $J$. Of course, assuming effective “Regge poles”, this description is fully equivalent to our Regge-motivated ansatz [19] in momentum fraction space. The GPD representations are somehow the analogue of a Legendre and power expansion of amplitudes at high energies. We recall that both expansions are equivalent [97] and the latter became more popular in Regge phenomenology in the sixties of the last century.

A combination of $t$-channel SO(3) and conformal partial wave expansion has been proposed [79], where the difference of conformal spin $j + 2$ and $t$-channel angular momentum $J$, is used as a discrete variable:

$$
 \rho = j + 1 - J = \{0, 2, 4, \ldots \} ,
$$

(29)
which is always even. This GPD representation is set up with forward-like functions $Q_\rho(z)$ of momentum fraction-like variable $z$ and has been called “dual” parameterization. Below we employ essentially the same model within a conformal partial wave expansion in terms of a Mellin-Barnes integral. Taking in such a model only the leading contribution, where $j + 1 = J$, one finds the conformal skewness function and ratio \(|28\). Taking into account that the singular behavior of the non-leading contributions $Q_\rho(z)$ increases by $z^{-\rho}$, one can obtain other values for the skewness ratio \(|93\). Hence, $\rho$ cannot be a priori considered as an expansion parameter. Note that in the minimal “dual” model of Ref. \([57]\) and in its corrected version \([58]\) the second forward-like function $Q_2(z)$ has the same small-$z$ behavior as the leading one $Q_0(z)$. Hence, this model possesses the conformal skewness ratio \(|28\).

The inverse transform in the “dual” parameterization has not been derived so far. Nevertheless, it has been exemplified within model studies, where $\rho$ can be effectively employed as an expansion parameter, that “dual” and DD representation can be effectively mapped into each other \([99, 100]\).

In the RDDA the conformal ratio \(|28\) arises when choosing $b = \alpha$, and, moreover, the conformal skewness function coincides then with the RDDA one, given in Eq. \((25)\). Since the value $\alpha \approx 1.2$ is rather close to the originally proposed value for $b$, the conformal ratios

$$r_{\Sigma}^{\text{con}} \approx 1.65, \quad r_{G}^{\text{con}} \approx 1.03,$$

are rather close to the RDDA ones \(|27\). In the “dual” model of Ref. \([57]\) both the leading $Q_0(z)$ and next-leading $Q_2(z)$ forward-like functions have been taken into account; however, both of them have the same $z$ behavior. Hence, in this model the skewness ratio also has the conformal value \(|28\) and it is in the flavor singlet sector not very different from the RDDA, see also Ref. \([57]\) for numerical examples. Therefore, both GPD models contradict the aligned-jet model estimate and so also the experiment \([61, 58]\).

To convert the predictions of the aligned-jet model into the GPD language, the authors of Ref. \([61]\) set the GPD at small $x$ and low input scale $\mu_0^2$ equal to the PDF:

$$H(x, x, t = 0, \mu_0^2) = q(x, \mu_0^2) \quad \text{for small } x \text{ and low } \mu_0^2.$$  \((31)\)

This is nothing but the RDDA model in the limit $b \rightarrow \infty$; practically, a large value $b \gg 1$ is sufficient. This ansatz implies that the skewness function is set $r(\eta/x) = 1$ for all $x$. With such an initial condition, evolution, starting at a rather low input scale, will rapidly lead to an increase of the $r$-ratio. Thus, this GPD model fails to describe data, too. We will not go into details here,

\footnote{Usage of term “dual” was motivated by the fact that in dual models \([98]\) the $s$-channel amplitude is described by the $t$-channel exchanges. We add that this feature is more general and arises from crossing and the Sommerfeld-Watson transform of the $t$-channel SO(3) partial wave expansion. In Regge theory/phenomenology the resummation of $t$-channel exchanges is encoded in the Regge trajectory.}
however, we would just like to point out that the skewness function determines the evolution of the skewness ratio. In RDDA ansatz its functional dependence ensures that the conformal ratio will be approached with increasing $Q^2$, as numerically demonstrated in Ref. [101].

Let us only shortly discuss the functional form of the residual $t$-dependence. Historically, it is common in Regge phenomenology to model it by an exponential ansatz, i.e,

$$\beta(t) = \exp \{Bt\}.$$ (32)

In Regge framework it is clear that this is an effective description for the small $-t$ region. An exponential $t$-dependence of GPDs arises in the overlap wave function representation within wave functions that have an exponential fall-off in the transverse momentum. It is questionable that in such GPD models the GPD spectral properties, ensuring polynomiality, can be implemented. On the other hand, power-like wave functions allow us to represent GPDs having both properties. With power-like wave functions one can easily evaluate DDs and they meet the field theory inspired view. In this framework a simple spectator quark GPD model was set up and it was found that at small $x$ the $t$-dependence indeed factorizes and that a power-like behavior arises:

$$\beta(t) = \left(1 - \frac{t}{M^2}\right)^{-p}.$$ (33)

Certainly, in such models Regge-like behavior is not implemented. As stated above, this can be achieved by convolution of such model with an appropriate constituent mass spectral function. However, it remains unclear to us how should the $t$-dependence be treated, so that common Regge behavior arises and, simultaneously, general positivity constraints are satisfied; see Ref. [108] for GPD integral representations. Hence, we have no definite ansaetze for the residue function at smaller values of $t$ at hand; however, a residual $t$-dependence as in Eq. (33) at a (very) low input scale and smaller values of $x$ looks to us rather plausible.

### 3.2 Modelling of integral conformal GPD moments

Our goal is to have a first empirical look at a more flexible GPD modelling, where we rely on the physical Regge-inspired picture. We shall introduce three different GPD models for the small-$x$ region, each having different skewness function. The models are defined by their conformal Mellin moments. This is foremostly a technical point, allowing us to employ existing stable numerical code that includes radiative corrections at NNLO level. Analogously to the singlet quark and gluon GPDs in Eq. (18), we collect their conformal moments in a two-dimensional vector

$$H_j(\eta, t, \mu^2) = \begin{pmatrix} H_j^\Sigma \\ H_j^G \end{pmatrix}(\eta, t, \mu^2).$$ (34)
These moments are given by the expectation values of local operators which are the lowest state in conformal towers; they are labelled by integral conformal spin \( j + 2 \), and they have spin projection \( j + 1 \). They can be defined by convolution of GPDs with Gegenbauer polynomials

\[
H_j(\eta, t, \mu^2) = \frac{\Gamma(3/2) \Gamma(j + 1)}{2 \Gamma(j + 3/2)} \frac{1}{2} \int_{-1}^{1} dx \eta^j \left( \begin{array}{cc} C^3_{j/2} & 0 \\ 0 & C^3_{j-1} \frac{1}{\eta} \end{array} \right) H(x, \eta, t, \mu^2),
\]

where polynomial indices \( \nu = \{3/2, 5/2\} \) are group-theoretically determined. Since \( C^\nu_j(-x) = (-1)^j C^\nu_j(x) \), our GPDs (18) have only odd \( j \) moments. For the GPD \( E \) we adopt the analogous decomposition. If polynomiality is completed, the integral conformal moments (35) are polynomials in \( \eta \) of order \( j + 1 \). Note that the highest order terms provide the subtraction constant in the "dispersion relation" [66, 62].

The normalization in (35) is chosen in such a way that the basic property of GPDs \( H \), namely, that they reduce in the forward limit to the PDFs, translates into similarly simple integer-\( j \) moment-space relation

\[
H_j(\eta, t, \mu^2) \longrightarrow q_j(t, \mu^2) \equiv \left( \sum_j \frac{G_j}{x_j} \right) (t, \mu^2), \quad \left( \sum_j \frac{G_j}{x_j} \right) (t, \mu^2) \equiv \int_0^1 dx x^j \left( \sum_j \frac{G_j}{x_j} \right) (x, t, \mu^2),
\]

where zero-skewness GPDs \( \Sigma \) and \( G \) at \( t = 0 \) are given by the flavor singlet quark and gluon PDFs, respectively. For the overall normalization we shall use the PDF momentum fraction averages \( N^{\Sigma,G} \), that are given by the first Mellin moments and satisfy the following sum rule

\[
N^{\Sigma}(\mu^2) + N^{G}(\mu^2) = 1 \quad \text{with} \quad N^{\Sigma} = \Sigma_1(t = 0, \mu^2) \quad \text{and} \quad N^{G} = G_1(t = 0, \mu^2).
\]

In our modelling we are going along the lines pointed out in the momentum fraction space. The dominant small-\( x \) behavior arises from the "pomeron exchange" that is for zero-skewness GPD related to sea quark contributions defined via anti-quarks \( \bar{q} \)

\[
\Sigma = q^{\text{sea}} + q^{\text{val}}, \quad q^{\text{sea}} = 2\bar{q}, \quad q^{\text{val}} = \sum_{q=u,d,s,\ldots} [q - \bar{q}].
\]

The difference of quark and anti-quark PDFs we denote as valence-like flavor singlet contributions. They are related to "Reggeon exchanges" with an intercept \( \alpha \approx 1/2 \). Thus, they can be neglected for the small-\( x_B \) kinematics. The standard DIS terminology [38] we adopt for GPDs, too. First we set up the zero-skewness GPD, including the \( t \)-dependence, and afterwards we skew it, i.e., model the skewness function.

We start with the standard ansatz for PDF Mellin moments at a given input scale \( \mu_0 \)

\[
q(x) = N \frac{x^{-\alpha}(1 - x)^\beta}{B(2 - \alpha, \beta + 1)} \quad \text{Mellin transform} \quad q_j = N \frac{B(1 - \alpha + j, \beta + 1)}{B(2 - \alpha, \beta + 1)},
\]
for both quark and gluon PDFs. Here $\alpha \gtrsim 1$ is the intercept of an effective “pomeron trajectory”, while $\beta$ parameterizes the large $x$ or $j$ behavior. The normalization is given by the momentum fraction average $N$, see Eq. (37).

Then, to obtain the model for the zero-skewness GPDs (36), we decorate the PDF Mellin moments (39) with $t$-dependence by (i) extending the “Regge intercept” $\alpha(t) = \alpha + \alpha' t$, where we only introduce the leading pole, and then (ii) by multiplying with a residue function $\beta(t)$, having either exponential (32) or power-like (33) functional form. In accordance with our $t$-factorized ansatz at small $x$, we neglect the dependence of the slope $B$ or cut-off mass $M$ on $j$. All parameters can be separately adjusted for sea quarks and gluons, even the effective “pomeron trajectories” might slightly differ. Thus, our ansatz for the Mellin moments of the zero-skewness GPDs reads

$$\Sigma_j(t) = N_{\text{sea}} \frac{B(1 - \alpha_{\text{sea}}^\alpha + j, \beta_{\text{sea}}^\beta + 1)}{B(2 - \alpha_{\text{sea}}^\alpha, \beta_{\text{sea}}^\beta + 1)} \frac{\beta_{\text{sea}}^\beta(t)}{1 - \frac{1}{(m_j^{\text{sea}})^2}} + \cdots, \quad (m_j^{\text{sea}})^2 = \frac{1 + j - \alpha_{\text{sea}}^\alpha}{\alpha_{\text{sea}}'^\alpha}, \quad (40)$$

$$G_j(t) = N_G \frac{B(1 - \alpha^G + j, \beta^G + 1)}{B(2 - \alpha^G, \beta^G + 1)} \frac{\beta^G(t)}{1 - \frac{t}{(m_j^G)^2}}, \quad (m_j^G)^2 = \frac{1 + j - \alpha^G}{\alpha^G'} . \quad (41)$$

Here the ellipsis in the quark singlet sector indicates valence-like contributions, cf. (38). The “pomeron pole” is written as a monopole form factor, where the cut-off mass squared $m_j^2$ depends on $j$. The pole is located at

$$m_j^2 = t \quad \Rightarrow \quad j = \alpha(t) - 1 .$$

Note that at $t = 0$ these pole contributions are included in the beta functions, appearing in Eqs. (40, 41).

The skewing is achieved by relating the conformal GPD moments for given conformal spin to the forward Mellin moments by a linear transformation that depends on $\eta$, e.g., by

$$H_j(\eta, t) = r_j(\eta) \left( \begin{array}{c} q_j^{\text{sea}} \\ G_j \\ \end{array} \right) (t) + \cdots, \quad r_j(\eta) = \left( \begin{array}{cc} r_j^{\text{sea}}(\eta) & 0 \\ 0 & r_j^G(\eta) \end{array} \right) . \quad (42)$$

It is required that for integral conformal spin $r_j^{\text{sea,G}}(\eta)$ are even polynomials in $\eta$ of order $j + 1$, which are at $\eta = 0$ normalized to one. They are moment-space analogue of the skewness functions $r(\eta/x)$.

Here, we should already point out a crucial property of $r_j(\eta)$, given as a function of two variables $j$ and $\eta$. On the first naive glance one might expect that normalization $r_j(0) = 1$ implies skewness ratio $r(\theta = 1) = 1$; however, this is not the case. The inversion of Mellin transform, such

\begin{footnote}
This is also sufficient from the pragmatical point of view because such a dependence anyway cannot be constrained by fitting small-$x_B$ DVCS data [62].
\end{footnote}
as \((35)\), involves an integration over complex-valued \(j\) with certain weight, e.g., the GPD on its cross-over line is given by \((40)\)

\[
H(x, \eta = x, t) = \frac{1}{2\pi i} \int_{c - i\infty}^{c + i\infty} dj \left(\frac{x}{2}\right)^{-1-j} \frac{\Gamma(5/2 + j)}{\Gamma(3/2)\Gamma(3 + j)} \begin{pmatrix} 1 & 0 \\ 0 & 2x/(3 + j) \end{pmatrix} H_j(\eta = x, t).
\] (43)

If one extends \(j\) to complex values and assumes a smooth forward limit of \(r_j(\eta)\) and uses the pole value \(j = \alpha - 1\) in Eq. \((43)\), one is immediately led to the conclusion that the skewness ratio is equal to its conformal value \((28)\), e.g., for sea quarks,

\[
\lim_{\eta \to 0} r^{\text{sea}}_j(\eta) = 1 \quad \Rightarrow \quad r^{\text{sea}}(\vartheta = 1) = \frac{2^\alpha \Gamma(3/2 + \alpha)}{\Gamma(3/2)\Gamma(2 + \alpha)}.
\] (44)

This is a naive thinking, too, because assuming that the function \(r_j(\eta)\) for complex-\(j\) has the same \(\eta \to 0\) limit as its integer-\(j\) analogue is unwarranted in the case of a singularity at \(\eta = 0\). We will show in Sect. \(3.3\) that indeed a branch point can develop there. Precisely this feature ensures the equivalence of representations and will enable us to construct flexible GPD models, not constrained by the conformal skewness ratio \((28)\).

We proceed with our modelling for integer \(j\). To find the convenient functional form for \(r_j(\eta)\), it is helpful to start by evaluating it from a known GPD at \(t = 0\). A simple example has been given in Ref. \(40\), where the following quark-antiquark unsymmetrized GPD has been evaluated from a toy DD \(f^{\text{toy}}(y, z) = y^{-\alpha}\), taken to be equal to its “Regge” piece:

\[
H^{\text{toy}}(x, \eta) = \frac{1}{1 - \alpha} \theta(-\eta \leq x) \frac{1}{\eta} \left(\frac{x + \eta}{1 + \eta}\right)^{1-\alpha} + \eta \to -\eta.
\] (45)

At small \(x\) this toy GPD provides in the forward limit the realistic PDF behavior \(x^{-\alpha}\) and its skewness function at small \(\eta\) is

\[
r^{\text{toy}}(\vartheta) = \frac{(1 + \vartheta)^{1-\alpha}}{2\vartheta} - \frac{1}{\alpha - 1} \left[\left(\frac{1 - \vartheta}{1 + \vartheta}\right)^{1-\alpha} - 1\right],
\] (46)

which is reminiscent of the aligned-jet model one \((22)\). Since this expression arises from a DD that does not vanish on the support boundaries, it is ill-defined at \(\vartheta = 1\) for \(\alpha \geq 1\) and needs a regularization, e.g., by using a profile function that vanishes at the support boundaries. Note that for RDDA \((24)\) the GPD behavior at the end-point \(x = -\eta\) and cross-over point \(x = \eta\) is governed by both the Regge behavior of the PDF and the end-point behavior of the profile function.

Nevertheless, we can use this model as a guide for modelling conformal GPD moments. The conformal moments of the GPD \((45)\) can be straightforwardly calculated by means of a so-called beta transform of Gegenbauer polynomials into the hypergeometric function \(3F_2\). We write the
resulting conformal moments in the form of a transformation \([42]\) and read off:

\[
\begin{aligned}
r_{j}^{\text{toy}}(\eta) &= \frac{(1 + j - \alpha)(2 + j - \alpha)}{2(1 - \alpha)(2 - \alpha)} \frac{\Gamma(3/2)\Gamma(3 + j)}{\Gamma(3/2 + j)} \\
&\times \left[ \frac{\eta - 1}{2\eta} \left( \frac{\eta}{2} \right)^j \right]_3 \frac{\Gamma(3/2 \pm j)}{\Gamma(3 \pm j)} \left\{ \eta \to -\eta \right\}.
\end{aligned}
\]

Note that for odd \(j\) these moments exist even for \(\alpha = 1\). The divergent behavior of the model \([45]\) at \(\vartheta = 1\) shows up in this space as convergence problem for large \(j\). Hence, we can employ the skewness moments \([47]\) only if the zero-skewness GPD decreases faster than \(1/j^2\) at \(j \to \infty\), which is satisfied for realistic values of \(\beta\) in \([40]\) and \([41]\). As already said, below we shall continue the odd integer \(j\) conformal moments to complex-valued \(j\). Here we only emphasize that \(r_{j}^{\text{toy}}(\eta)\) has a zero at \(j = \alpha - 1\) which will cancel the “pomeron pole” contribution of zero-skewness GPD. This obviously demonstrates that the normalization condition for odd \(j\) will not necessarily imply the limit \([44]\) and so the conformal ratio \([28]\) cannot be taken for granted.

In our modelling we also follow the suggestion of \([109, 79]\) and expand conformal GPD moments in SO(3) \(t\)-channel partial waves \([62]\)

\[
F_j(\eta, t) = \sum_{J = \text{even}}^{j+1} F_{j,J}(t) \eta^{J+1-J} \hat{d}_x^J(\eta),
\]

labelled by the angular momentum \(J\), where \(\eta = -1/\cos \theta\) is expressed in terms of the \(t\)-channel center-of-mass scattering angle. The PW amplitude \(F_{jj+1}\) with angular momentum \(J = j + 1\) is nothing but the Mellin moment of the zero-skewness GPD, while the PW amplitudes with \(J\) smaller than \(j + 1\) are suppressed by a factor \(\eta^\rho\) \((\rho = j + 1 - J)\). The SO(3) partial waves \(\hat{d}_x^J\), normalized to one at \(\eta = 0\), are the crossed version of Wigner’s reduced rotation matrices \(d_{0,\lambda,-\lambda'}^J\) \([41]\). For the \(t\)-channel helicity conserved ‘electric’ \((\lambda = \lambda' = 1/2)\) and helicity flip ‘magnetic’ \((\lambda = -\lambda' = 1/2)\) GPD moments

\[
H_j + \frac{t}{4M^2} E_j, \quad \text{and} \quad H_j + E_j,
\]

respectively, they are expressed by Legendre (Gegenbauer with index \(\nu = 1/2\)) polynomials,

\[
\hat{d}_x^J(\eta) \propto \eta^J d_{0,0}^J(1/\eta)
\]

for \(\mathcal{F} = \mathcal{H} + \frac{t}{4M^2} \mathcal{E}\),

\[
\hat{d}_x^J(\eta) \propto \frac{\Gamma(1/2)\Gamma(J + 1)}{2J\Gamma(1/2 + J)} \eta^J C^{1/2}_j(1/\eta),
\]

\footnote{We are here in the first place interested in the assignment of quantum numbers. Thus, we neglect some corrections which die out either in the Bjorken limit or by setting the proton velocity in the c.o.m. frame to one. These corrections appear in the partial waves and their amplitudes. However, after completing the Sommerfeld-Watson transform they will partly cancel each other in the small-\(x\) region. Hence, it is for our application more convenient to neglect them from the beginning.}

\(20\)
with \( J_{\text{min}} = 0 \) and Gegenbauer polynomials with index \( \nu = 3/2 \),

\[
d_\eta^J(\eta) \propto \eta^J z \sqrt{J(J+1)} d_{0,1}^J(z) \bigg|_{z=1/\eta} \frac{\Gamma(1/2)\Gamma(J)}{2^J\Gamma(J+1/2)} \eta^{J-1} C_{J-1}^{3/2}(1/\eta),
\]

for \( \mathcal{F} = \mathcal{H} + \mathcal{E} \),

with \( J_{\text{min}} = 2 \). Although we here consider integral \( J \), it is already appropriate to think in terms of the Regge language, providing us a guideline for the resummation of (48). The quantum numbers associated with our conformal GPD moments are associated with “pomeron” and parity- and charge conjugation-even “Reggeon exchanges” [110, 41], where the leading ones belong to the \( f \)-meson trajectory, generically given as \( \alpha(t) \sim 1/2 + t/\text{GeV}^2 \). Each term in the sum (48) should therefore have a leading pole at \( J = \alpha(t) \) rather than at \( j = \alpha(t) - 1 \). Note that the former scenario, well established from the physical point of view, yields a non-trivial skewness ratio, while the latter, more formal and vague scenario, yields the conformal ratio (44). Polynomiality is completed, i.e., the “magnetic” and “electric” GPD moments are polynomials of order \( j + 1 \) and \( j - 1 \), respectively, where \( j \) is odd. Terms proportional to \( \eta^{j+1} \) contribute to the so-called \( D \)-term, which finally builds up the subtraction constant in the “dispersion relation”. To this constant not only the \( J = 0 \) \( \sigma \)-meson contributes, but rather all exchanges with \( J \leq j + 1 \). Still, in our small-\( x_B \) DVCS application the “pomeron trajectory” dominates these “Reggeon exchanges”, including the subtraction constant, and so we shall hereafter ignore them in our small-\( x \) GPD modelling.

As one realizes from Eq. (48), for \( t = 0 \) the proper partial waves for \( H_j \) are Legendre polynomials (50), while \( E_j \) can be represented as a sum of \( -H_j \) and an addendum, expanded with respect to Gegenbauer polynomials (51). For the sake of simplicity we take these assignments also for \( t \neq 0 \). Moreover, in the small-\( \eta \) kinematics the different \( \text{SO}(3) \) partial waves have (for complex valued \( j \)) the same asymptotic behavior. Hence, in the following we will relax the requirement of the complete polynomiality.

The \( \text{SO}(3) \) PW expansion (48) can be written in the form of the transformation (42). One immediately reads off that the \( \text{SO}(3) \) PW expansion of the skewness matrix is given by

\[
r_J(\eta|F) = \sum_{J=J_{\text{min}}}^{j+1} \frac{F_{jj+1}}{F_{jJ}} \eta^{j+1-J} d_\eta^J(\eta).
\]

We now construct three different models for the skewness dependence, which will be employed for fitting of small-\( x_B \) DVCS data. Showing also the corresponding “dual” model we have:

i. a leading \( \text{SO}(3) \) partial wave (l-PW), i.e., minimalist “dual” \( (\rho = 0) \) model,

ii. a leading and next-leading \( \text{SO}(3) \) partial wave (nl-PW), i.e., the minimal “dual” \( (\rho = 0, 2) \) model,
iii. a model-dependent resummation of SO(3) partial waves (Σ-PW).

i. l-PW model: Taking in the expansion (48) only the leading SO(3) PW $J = j + 1$ into account and representing the Legendre polynomials $C_{j+1}^1(z)$ by hypergeometric functions, we write this model in the form:

$$H_j^{(l)}(\eta, t) = r_j^{(l)}(\eta) q_j(t), \quad r_j^{(l)}(\eta) = \frac{\Gamma(1/2)\Gamma(j + 2)}{2^{j+1}\Gamma(3/2 + j)} \eta^{j+1/2} F_1 \left( \begin{array}{c} -j - 1, j + 2 \\ \eta \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ \frac{\eta}{2} & 1 \end{array} \right),$$

(53)

where the forward moments $q_j$ are specified in Eqs. (40) and (41). This is the most restrictive model, and its skewness ratio is fixed by the conformal value (28), as claimed in Refs. [94, 96].

ii. nl-PW model: To build a flexible model we include also the next-leading partial wave amplitude with $J = j - 1$. For simplicity, we express this PW in terms of the leading one, where, however, the conformal spin is shifted by two units:

$$H_j^{(nl)}(\eta, t) = H_j^{(l)}(\eta, t) + \eta^2 \theta(j \geq 3) S H_{j-2}^{(l)}(\eta, t), \quad S = \left( \begin{array}{cc} s^{\text{sea}} & 0 \\ 0 & s^G \end{array} \right).$$

(54)

The $\eta^2$-proportional term, having a pole at $J = j - 1 = \alpha(t)$, is crucial for adjustment of the normalization of the DVCS amplitude, controlled by the entries $s^{\text{sea}}$ and $s^G$ in the matrix $S$. For complex conformal spin the pole is at $j = 2 + \alpha - 1$ within the constraint $\Re j \geq 3$ and might thus be denoted as spurious.

iii. Σ-PW model: It turns out that the sum (48) of SO(3) partial waves with physically-motivated effective “Regge poles”,

$$F_{j\alpha} \propto \frac{1}{J - \alpha},$$

yields after analytic continuation in $j$ the same small-$\eta$ asymptotic as implemented in our toy skewness moments (47), see below Eq. (69). To have a flexible skewness ratio, we take the l-PW model and add the difference of the toy model (47) and the l-PW one:

$$H_j^{(\Sigma)}(\eta, t) = H_j^{(l)}(\eta, t) + S_j(\eta, t|\alpha) q_j(t),$$

(55)

where the entries of the skewness matrix

$$S_j(\eta, t) = \left( \begin{array}{cc} s^{\text{sea}} S_j(\eta|\alpha^{\text{sea}}(t)) & 0 \\ 0 & s^G S_j(\eta|\alpha^G(t)) \end{array} \right).$$

(56)

Note that this second term will die out in the forward limit, and so it is invisible in the zero-skewness GPD. Of course, such a trick can be used in any representation.
are defined in terms of the skewness moments \(47\) and \(53\):

\[
S_j(\eta|\alpha(t)) = \frac{\Gamma(7 + \alpha)}{\Gamma(7 + \alpha(t))} \left( r_j^{\text{toy}}(\eta|\alpha(t)) - r_j^{(1)}(\eta) \right). \tag{57}
\]

For convenience, we included an additional factor \(\Gamma(7 + \alpha)/\Gamma(7 + \alpha(t))\) that makes the \(t\)-dependence of skewness effect more flat.

A few comments are in order. Skewness parameters are denoted by the same symbols, \(s_{\text{sea}}\) and \(s^G\), in both nl-PW and \(\Sigma\)-PW models; however, their normalizations are not related in an obvious way. Still, for both models positive (negative) values of these parameters imply increase (decrease) of the normalization of the DVCS amplitude at \(t = 0\), compared to the conformal ratio situation \(28\) of the I-PW model. The existence of both these flexible models explicitly shows that the claim of Ref. \[94\] about conformal ratio cannot be derived from a conformal PW expansion (or equivalently from the integral transformation \[78\]). The assumption of Ref. \[96\], namely, that singularities in the complex conformal \(j\) plane with \(\Re j > \alpha - 1\) (where \(\alpha \sim 1\)) are absent would exclude the chosen representation, including the constraint \(\Re j > 3\), for the nl-PW model. To weaken the constraint, we will change the Mellin-Barnes integral representation for CFFs (or GPDs) by a variable shift \(j \rightarrow j' - 2\), as it is done below in Eq. \(67\). Then the spurious pole will be moved to the “right” position \(J = j' + 1 = \alpha(t)\). This is in accordance with our assumption that the leading poles are associated with the angular momentum \(J\) rather than with the conformal spin \(j\). Moreover, to have a mathematically consistent representation of the \(\Sigma\)-PW model we will perform below the analytic continuation by means of Carlson’s theorem and then the spurious right half-plane poles in the complex \(j\) plane are absent anyway in the conformal moments \(66\). Utilizing the small-\(\eta\) expansion, we will also illuminate that spurious poles should be considered as an artifact of this expansion.

Finally, we specify the models for the GPD \(E\). Since in the small-\(\eta\) region the SO(3) partial waves \(50\) and \(51\) behave rather smoothly and they can be safely approximated by one, we do not need to be careful about specific choices of expansion polynomials. Hence, for simplicity, we can assume that the conformal moments of \(E_j\) are proportional to those of \(H_j\):

\[
E_j = \left( \left( \frac{B_{\text{sea}}}{N_{\text{sea}}} \right) H_{j}^{\text{sea}} + \cdots \right), \tag{58}
\]

where the ellipsis stands for valence contributions. The normalization of the GPD moment \(E_j\) is for \(j = 1\) given by the anomalous gravitomagnetic moments, see, e.g., Refs. \[111\] \[113\], of flavor singlet quarks and gluons

\[
B^S = B_{\text{sea}} + B_{\text{val}} \quad \text{and} \quad B^G.
\]
Ji’s decomposition\footnote{A overview on the common spin decomposition schemes can be found in Sect. 5 of Ref. \cite{114}.} of the proton spin \cite{115} reads then

\[ \mathcal{J}^\Sigma + \mathcal{J}^G = \frac{1}{2}, \quad \mathcal{J}^\Sigma = \frac{1}{2} (N^\Sigma + B^\Sigma), \quad \mathcal{J}^G = \frac{1}{2} (N^G + B^G). \tag{59} \]

As a consequence of both momentum \footnote{\cite{57}} and angular momentum conservation \cite{59}, the sum rule

\[ B^\Sigma + B^G = 0, \tag{60} \]

states that the anomalous gravitomagnetic moment of the nucleon vanishes. Note, however, that the residue of \( E_j \) at the “pomeron pole” \( j = \alpha - 1 \) depends besides the normalization \( B \) also on the functional form with respect to \( j \).

The flavor decomposition of the anomalous gravitomagnetic moment is an important phenomenological goal, which we will address below in Sect. \cite{4.5} within the ansatz \cite{58} in an ad hoc model-dependent manner. Let us set up a few scenarios. Generic thoughts within simplified models, utilizing large \( x \) counting rules, isospin symmetry, and the values of the nucleon magnetic moment, suggest that the sum of valence \( u \) and \( d \) contributions provide an almost vanishing valence-like anomalous gravitomagnetic moment \cite{116}. With this taken literally, we would have the scenario:

\[ B^G = -B^{sea}, \quad B^{val} \equiv B^{uval} + B^{dval} = 0. \tag{61} \]

On the other hand, lattice simulations \cite{117, 118} suggest the scenario

\[ B^\Sigma = 0 \quad \Rightarrow \quad B^{val} = -B^{sea}, \quad B^G = 0, \tag{62} \]

which would suggest that in our model the chromomagnetic “pomeron” should be absent. However, disconnected contributions, which are related to a gluonic \( t \)-channel exchange, are not taken into account in present lattice measurements \cite{117, 118}, and one might be tempted to reinterpret the lattice results as valence dominated, i.e., \( B^{val} = 0 \), supporting our generic thoughts \cite{61}.

### 3.3 Numerical evaluation of CFFs

As motivated above, we approximate the CFFs \( \mathcal{F} = \{\mathcal{H}, \mathcal{E}\} \) in the small-\( x_B \) region by their flavor singlet part

\[ \mathcal{F}(\xi, t, Q^2) \approx e_S^2 S\mathcal{F}(\xi, t, Q^2), \quad e_S^2 = \frac{1}{N_f} \sum_{q=u,d,s,\cdots} e_q^2, \tag{63} \]
where the (averaged) fractional squared charge $e_S^2$ is for $N_f$ active quarks. In the factorization formula for the CFFs we will set both renormalization and factorization scales equal to the photon virtuality $Q$. Instead of the momentum fraction representation, we utilize the conformal partial wave expansion for complex-valued conformal spin. The flavor singlet part of our signature even CFFs is then evaluated via a Mellin-Barnes integral

$$\left\{\mathcal{S}_H \mathcal{E}\right\}(\xi, t, Q^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \xi^{-j-1} \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \left[ \mathcal{C} \otimes \mathcal{E} \right]_j(\alpha_s(Q), \alpha_s(Q_0)) \left\{ \mathcal{H}_j \mathcal{E}_j \right\}(\xi, t, Q_0^2).$$

(64)

The coefficient functions $\mathcal{C}$ and the evolution operator $\mathcal{E}$ are evaluated in perturbation theory. For instance, in LO (hand-bag) approximation the hard scattering amplitude reads

$$\mathcal{C}_j^{\text{LO}} = \frac{2j+1}{\Gamma(3/2)\Gamma(j+3)} \Gamma(j+5/2) (1,0),$$

(65)

while the evolution operator is the same as in unpolarized DIS. The radiative corrections to the coefficient functions $\mathcal{C}$ and the evolution operator $\mathcal{E}$ are presented in Ref. [62] for the standard minimal subtraction ($\overline{\text{MS}}$) scheme in NLO and for a special conformal scheme ($\overline{\text{CS}}$) up to NNLO. We add that in the momentum fraction representation the corresponding expressions are known for the $\overline{\text{MS}}$ scheme in NLO approximation.

A few comments are in order. In the $\overline{\text{CS}}$ scheme conformal symmetry is manifest, up to the breaking by the trace anomaly, also beyond LO. Thus, the NNLO corrections can be obtained by conformal mapping from the corresponding forward quantities, elaborated in DIS; for details we refer to Ref. [62] and references therein (for integral conformal spin see also Refs. [123, 124]). In the $\overline{\text{CS}}$ scheme the convolution $\left[ \mathcal{C} \otimes \mathcal{E} \right]_j$ reduces to a simple multiplication, while in the $\overline{\text{MS}}$ scheme an additional summation over the conformal spin must be included.

In the evolution operator we resum only the leading logs and perturbatively expand the non-leading ones. In the small-$\xi$ region the perturbative expansion of the universal evolution operator is getting unstable, so that the difference to an evolution operator in which also non-leading logs are resummed is of some numerical importance. This difference is implicitly absorbed by a reparameterization of sea quark and gluon PDFs or GPDs at the given input scale.

The continuation of coefficient functions and anomalous dimensions from integral to complex-valued $j$ is, as in DIS, straightforward. This is also the case for zero-skewness GPDs, already given

\footnote{For clarity, the perturbative quantities which were denoted in [62] as $\mathcal{C}$ and $\mathcal{E}$, are renamed in the present paper into $\mathcal{C}$ and $\mathcal{E}$.}

\footnote{Since even at NLO a diagrammatic calculation of evolution kernels has not been performed yet, except for the two simplest ones in the quark sector [119, 120], we do not expect that the NNLO results in the $\overline{\text{MS}}$ scheme will become available in near future.}
in Eqs. (40, 41) as ratios of $\Gamma$ functions. The continuation of the skewness moments, as polynomials of $\eta$ is more intricate. It is required that these moments obey certain bounds for $j \to \infty$ with $|\arg(j)| \leq \pi/2$, see Ref. [40]. Then Carlson’s theorem tells us that their analytic continuation is unique. The conformal moments (53) of our l-PW model, expressed by hypergeometric functions $\, _2F_1$, and their descendants, i.e., the hypergeometric functions $\, _3F_2$ appearing in the skewness moments (47) with argument $(\eta - 1)/2\eta < 0$, satisfy the required condition. However, their $\eta$-symmetrized counterparts, with argument $(1 + \eta)/2\eta > 0$, are problematic. Such functions are now evaluated on the branch cut in the complex $\eta$ plane. The correct treatment is to take the discontinuity over the cut, and to add a term that restores polynomiality for odd $j$:

$$\begin{align*}
3F_2\left(-j, 3 + j, 2 - \alpha \left| \frac{1 + \eta}{2\eta} \right. \right) & \Rightarrow \\
\left. 3F_2 \right| \frac{1+\eta}{2\eta} + i\epsilon - 3F_2 \left| \frac{1+\eta}{2\eta} - i\epsilon \right. & + \frac{2\eta}{1+\eta} \frac{2-\alpha}{\Gamma(1+j)\Gamma(3-\alpha)\Gamma(1+\alpha+j)} \frac{\Gamma(1+j)\Gamma(3-\alpha)\Gamma(1+\alpha+j)}{\Gamma(3+j)\Gamma(\alpha)\Gamma(3-\alpha+j)} \cdot
\end{align*}$$

In the continuation procedure for odd $j$ moments the factor $(-\eta)^j$ is replaced by $-\eta^j$. The Mellin moments, obtained by this continuation procedure, do not possess singularities for $\Re\arg j > \alpha - 1$ with $\alpha \sim 1$ and are unproblematic for $\alpha < 2$. It has been already numerically checked for $\alpha < 1$ in Ref. [40] that our toy model in the conformal Mellin space is equivalent to its momentum fraction space representation. We convinced ourselves that for $\eta \neq \xi$ this holds true also for $\alpha \sim 1.2$. As already mentioned, the singular behavior of the toy GPD at the cross-over line is absent in our $\Sigma$-PW model.

For the numerical evaluation of the Mellin-Barnes integral (64) we choose as integration path a straight line segment, parallel to the imaginary axis. Intercept $c$ of this line and the real axis is taken to be $c \approx 0.35$, so that the contour lies to the right of the “pomeron pole” and to the left of the poles of tangent function in (64). For the nl-PW model the polynomiality condition of the $\eta^2$-proportional term in Eq. (54) requires us to take a separate Mellin-Barnes integral where the integration path is shifted by two units to the right, i.e., the intercept of the integration path is now $c + 2$. Shifting now the integration variable, $j \to j - 2$, we combine both pieces into a single Mellin-Barnes integral, schematically written as

$$\begin{align*}
\left\{ S_H, S_E \right\} = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1} \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \left[ [C \otimes E]_j + [C \otimes E]_{j+2} S \right] \left\{ H^{(l)}_j, E^{(l)}_j \right\},
\end{align*}$$

for our nl-PW model. Obviously, the $\eta^2$-suppressed term in integral conformal moments contributes to the leading Regge behavior and influences the value of the residue function. We emphasize again that the spurious pole in the nl-PW model is now moved to the “right” position $J = j + 1 = \alpha$ and that the new integration variable can now be viewed as the (shifted) $t$-channel angular momentum, i.e., $j = J - 1$.  

26
For the small-$\xi$ kinematics, we are interested in, we can speed up numerics by expanding the conformal GPD moments in the vicinity of $\eta = 0$. For l-PW model (63), using well-known expansion of Legendre functions [125] one gets

$$r_j(\eta) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} [1 + O(\eta^2)] + O(\eta^{2j+3}) \Rightarrow H_j^{(l)}(\eta, t) = q_j(t). \quad (68)$$

With our choice for the integration path, i.e., $\Re j = c > 0$, the $O(\eta^{2j+3})$ term is negligibly small and the conformal GPD moments reduce to the Mellin moments of zero-skewness GPDs. The same approximation we can use in Mellin-Barnes integral (67) for the nl-PW model. The small-$\eta$ expansion of the skewness moments (47) for the Σ-PW model, for complex valued $j$, obtained within the prescription (66), was worked out in Ref. [40]:

$$r_j^{(toy)} = 1 - \left(\frac{\eta}{2}\right)^{j+1-\alpha} \frac{\Gamma(1/2)\Gamma(1+j)\Gamma(1-\alpha)\Gamma(1+\alpha+j)}{\Gamma(3/2+j)\Gamma(3/2+j)} \frac{2[1 + O(\eta^2)]}{1 + \tan\left(\frac{j\pi}{2}\right)\tan\left(\frac{\pi\alpha}{2}\right)} + O(\eta^{2j+3}). \quad (69)$$

This expression has a series of “conformal sibling poles” in the left half-plane at $j = -\alpha - 1 - k$ with $k = 0, 1, 2, \cdots$. The series of spurious poles in the right half-plane at $j = \alpha + 1 + 2k$ with $k = 0, 1, 2, \cdots$ are absent in the exact result, and they appear here as an artifact of the small-$\eta$ expansion. The function $r_j(\eta)$ has also a zero at $j = \alpha - 1$ that cancels the “pomeron pole” in the Mellin moment; however, the corresponding CFF still grows as $\xi^{-\alpha}$ at small $\xi$. We have numerically checked that the exact and approximate results agree well even for larger values of $\eta \sim 0.2$; an example for $\alpha = 1/2$ was presented in Ref. [40]. In contrast to “Regge poles”, the “conformal siblings” will move to the right with increasing $-t$. Fortunately, as long as the condition

$$-t < \frac{2 + \alpha}{\alpha'} \quad (70)$$

is satisfied, it is guaranteed that they all lie to the left of the lowest integral value $j = 1$. All of the ‘conformal’ trajectories contribute to the residuum of the leading “Regge pole” at $J = \alpha(t)$ and this provides the possibility to adjust the normalization of CFFs.

Plugging the approximations (68) and (69) into the $S$-function (57) of the Σ-PW model, provides us with the expression convenient for numerical treatment

$$S_j(\eta|\alpha(t)) = -\left(\frac{\eta}{2}\right)^{j+1-\alpha(t)} \frac{\sqrt{\pi}\Gamma(1+j)\Gamma(1-\alpha(t))\Gamma(1+\alpha(t)+j)}{\Gamma(3/2+j)\Gamma(3/2+j)\Gamma(3/2+j)} \frac{2\Gamma(7+\alpha)/\Gamma(7+\alpha(t))}{1 + \tan\left(\frac{\pi}{2}\right)\tan\left(\frac{\pi\alpha(t)}{2}\right)}. \quad (71)$$

Note that the factor $\xi^{j+1-\alpha}$ for $\eta = \xi$ is partially cancelled in the Mellin-Barnes integral (67) by $\xi^{-j-1}$, which leads to the $\xi^{-\alpha}$ behavior of CFFs. Hence, the Regge behavior in conformal space is
not entirely related to a $j = \alpha - 1$ pole. This is natural from our point of view that the “Regge” behavior is associated with a pole in the complex $J$ plane, and, consequently, the residue is then expanded in terms of conformal PWs.

Finally, we have numerically checked that the small-$\eta$ approximation of the skewness moments works rather well, i.e., for $\xi < 0.01$ ($\xi \sim 0.1$) we find a per mil (below 1%) effect for the $\Sigma$-PW model. The resulting CFFs satisfy also the “dispersion relation” (12) for $0 < \xi < 0.01$ ($\xi \sim 0.1$) on the per mil (4%) level. As expected, the medium- and large-$\xi$ regions are unimportant in the dispersion integral for the real part at small $\xi$. The small-$\eta$ approximation of the SO(3) PWs has been often considered, e.g., in Ref. [62], and is even more unproblematic than the approximation of the $\Sigma$-PW model.

4 GPD interpretation of DVCS data in collider kinematics

We present now a detailed model study of the small-$x_B$ DVCS measurements of the H1 and ZEUS Collaborations [8, 9, 7, 10] by means of least square fits at LO, NLO in the $\overline{\text{MS}}$ and $\overline{\text{CS}}$, and at NNLO in the $\overline{\text{CS}}$ scheme. The fitting procedure is described in Sect. 4.1. Thereby, we aim for a partonic or GPD interpretation; however, the nature of the DVCS process precludes a coverage of the whole space of possible GPD models and, therefore, we are unable to determine the subspace of models that correctly describe the DVCS data. Instead, we use three representative models defined in Sect. 3.2 in an attempt to illuminate all facets of the problem. In Sect. 4.2 we give specific emphasis to LO fits, since the outcome might be conveniently employed in studies of fixed target experiments. GPD reparameterization effects due to radiative corrections are considered in Sect. 4.3. In Sect. 4.4 we extract the transverse distribution of partons from our fits. Finally, we address in Sect. 4.5 the question of whether measurements of beam charge asymmetry allow one to access the GPD $E$ and, consequently, to get a handle on the chromomagnetic “pomeron” and the anomalous gravitomagnetic moment. The lessons from the fits are listed in Sect. 4.6.

4.1 Fitting strategies and parameters

Our numerical studies are based on the twist-two approximation of the DVCS cross section (4) and BCA (6) without gluon transversity, the models from Sect. 3.2, the Mellin-Barnes integral (64), and on the least square fitting routine MINUIT [126]. We fit to the DVCS cross section and to the DIS $F_2$ structure function data, where we add statistical and systematical errors in quadrature and look for the values of GPD model parameters that minimize the $\chi^2$ function. For
instance, for the DVCS cross section $\sigma$ we have

$$\chi^2 = \sum_d \frac{(\sigma^\text{exp}_d - \sigma^\text{the}_d)^2}{(\delta\sigma_d)^2}, \quad \delta\sigma_d = \sqrt{(\delta\text{sys}\sigma_d)^2 + (\delta\text{stat}\sigma_d)^2},$$  

(72)

where $d$ enumerates the experimental data points. The quality of fits is assessed by the $\chi^2$ value divided by the number of degrees of freedom (d.o.f.) and by probability of such and larger $\chi^2$ values. As bad fits we consider those where $\chi^2$/d.o.f is considerably larger than one, or, more precisely, where its probability is smaller than 0.2. We will also separately present the $\chi^2$ values contributed by $t$-, $W$- and $Q^2$-dependence. Since there is no one-to-one correspondence between individual fit parameters and the particular kinematic parameters of the CFF, we cannot determine the exact number of the d.o.f. Therefore, we simply give then $\chi^2$ per number of data points (n.o.p.). The difference is small for large number of points; so this should provide good idea about the quality of the description of dependence on the individual variables. An error analysis of the resulting parameters is an intricate task, which we postpone.

We set the input scale, at which the GPD models and parameters are specified, to

$$Q^2_0 = 4 \text{ GeV}^2.$$  

(73)

We consider then the charm quark as massless and set $N_f = 4$ and $e_S^2 = 5/18$. Numerical effects of $b$ quarks, known to be small in DIS, will be neglected here. Alternatively, the production of heavy quarks might be described within perturbation theory [127], worked out at NLO [91]. The flavor scheme choices we consider to be foremostly related to different partonic interpretations. The value of the QCD running coupling is specified as

$$\alpha_s(Q_0 = \sqrt{2.5 \text{ GeV}}) = \begin{cases} 0.0606 \\ 0.0518 \\ 0.0488 \end{cases} \quad \text{for} \quad \begin{cases} \text{LO} \\ \text{NLO} \\ \text{NNLO} \end{cases}$$  

(74)

and corresponds to its phenomenological value $\alpha_s(M_{Z^0} = 91.18 \text{ GeV}) = 0.114$ at the reference scale $M_{Z^0}$, using the standard evolution prescription [128]. However, we perform forward evolution over the $Q^2$ intervals of interest keeping a fixed number of four active quarks.

Let us recall that our GPD $H$ models are parameterized by the averaged momentum fraction $N$, “pomeron trajectory” $\alpha(t)$, large-$j$ parameter $\beta$, skewness parameter $s$, and one parameter that controls the residue function $\beta(t)$, e.g., the $\Sigma$-PW model reads

$$H^I_j(\eta) = N^I \left[ 1 + s^I S^I_j(\eta) \right] \frac{B(1 - \alpha^I + j, \beta^I + 1)}{B(2 - \alpha^I, \beta^I + 1)} \frac{1 - \alpha^I + j}{1 - \alpha^I(t) + j} \beta^I(t)$$  

(75)

for $I = \{\text{sea, G}\}$. In our previous studies [62] we relied on the ad hoc l-PW model ($s^I = 0$) and performed a simultaneous fit to DIS and DVCS data, where the momentum sum rule was not...
imposed. We utilize now this constraint to fix the gluon normalization,

$$N^G = 1 - N^{\text{sea}} - N^{\text{val}}, \quad N^{\text{val}} = 0.4,$$

where the momentum fraction for valence quarks $N^{\text{val}}$ is compatible with values obtained from standard PDF fits. Our flexible GPD models could be fitted just to DVCS data, where PDF normalization and “Regge intercepts”

$$N^{\text{sea}}, \quad \alpha^{\text{sea}}, \quad \alpha^G$$

at the input scale would be taken from some of the standard parameterizations.\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} However, instead of this, we mostly perform two-step fits. First we fit to the DIS data to extract the PDF parameters\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} \footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}. There the $\beta$ parameters for sea quark and gluon PDFs are taken as

$$\beta^{\text{sea}} = 8, \quad \beta^G = 6,$$

which is slightly larger than their canonical values from counting rules, accounting thus for the increase with resolution scale. Inclusion of DIS fit in our procedure is convenient because it ensures identical scheme conventions and approximations for the evolution operator in DIS and DVCS.

The DIS data used come from H1 data set\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} for the DIS structure function $F_2$ in the region

$$2.5 \text{ GeV}^2 \leq Q^2 \leq 90 \text{ GeV}^2.$$  

The choice of these 85 data points is sufficient for our studies and at the input scale\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} our resulting PDFs are consistent with standard parameterizations, as demonstrated at LO and NLO accuracy in Fig.\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} Note that the PDFs in the regions $10^{-5} \leq x \lesssim 10^{-4}$ and $10^{-2} \lesssim x$ (mainly) arise from extrapolation within our functional pomeron-like ansatz and that we use our conventions, e.g., perturbative NLO corrections are consequently expanded in powers of $\alpha_s$\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}.  

The remaining free model parameters are the skewness parameters, the cut-off masses and power behavior in the residue function\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}, \footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.} or exponential slope parameters in\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}, and the “Regge slopes”

$$\alpha_s, \quad \alpha_s'$$

which control the normalization, $t$-dependence, and the shrinkage of the diffractive forward peak in the DVCS amplitude, respectively. These parameters are to be determined by fits to DVCS data in the second step of the procedure. However, it turns out that from our fits we cannot simultaneously pin down the cut-off masses and powers $p$ in the ansaetze\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}, so we fix the latter.\footnote{Due to the small-$x_B$ collider kinematics, we neglect the “Reggeon” contributions to the flavor singlet part.}
Figure 1: Parton densities at LO (up) and NLO (down) are shown for quarks (left) and gluons (right) at the input scale $Q^2 = 4 \text{ GeV}^2$ (solid). For comparison we also show Alekhin’s parameterization [130] with errors (vertically dashed band) and some other standard PDFs [131, 132] (dotted, dash-dotted).

Large $-t$ counting rules would suggest to take for sea quarks $p^{\text{sea}} = 4$ and for gluons $p^{G} = 3$. To have the possibility of direct comparison of the fitting results with the characteristic size of the proton, given by the cut-off mass in the dipole parameterization of the Sachs form factors, we choose a dipole ansatz:

$$p^{\text{sea}} = p^{G} = 2.$$  \hspace{1cm} (80)

Moreover, since gluons are not directly accessible and a full separation of quarks and gluons through the evolution effects is not yet possible, we must further reduce the parameter space. In our previous ad hoc model studies we learned that fitting routine tends to use the gluonic $t$-slope to adjust the normalization of the total DVCS amplitude. Here we get rid of this unwanted complication by fixing the gluonic cut-off mass (or slope) and taking its value from the analysis of elastic $J/\psi$ production. According to Ref. [44], this process is dominated by the two-gluon $t$-channel exchange and the charm quark mass already provides an internal hard scale that translates
to an effective $J/\psi$ photoproduction scale of $Q_{\text{eff}}^2 \approx 3 \text{GeV}^2$. Therefore, one may even view the $J/\psi$ photoproduction as a hard process with access to the gluon GPD \cite{27}. From the differential cross section measurements \cite{23,26} the “pomeron trajectory” (here defined with respect to $W$ rather than $\xi$) and the residual $t$-slope were extracted by a fit with an exponential ansatz \cite{26}:
\[
\alpha(t) = 1.224 \pm 0.010 \pm 0.012 + (0.164 \pm 0.028 \pm 0.030) t/\text{GeV}^2,
\]
\[
b = 2B = (4.630 \pm 0.060^{+0.043}_{-0.163}) /\text{GeV}^2.
\] (81)

Also, the measurements strongly favor an exponential $t$-dependence of the differential cross section, where the $t$-slope $b$ is quoted in Eq. (81) for $W = 90 \text{ GeV}$. Still, at the GPD level,\footnote{Whether the measurement of the longitudinal part of the $J/\psi$ electroproduction cross section, known to much smaller accuracy than the photoproduction one \cite{28}, is then consistent with the collinear factorization approach, elaborated in Ref. 133 to NLO, is to our best knowledge not yet investigated.} we consider it more natural to take a power-like form for the residual $t$-dependence, as argued in Sect. 3.1. Therefore, the dipole ansatz will be used for the majority of our fits, with some fits using exponential ansatz performed for comparison. The value of the “Regge slope” parameter $\alpha'$ is not fully pinned down. The measurements of the ZEUS Collaboration \cite{24} give a slightly lower mean value as quoted in Eq. (81) and the electroproduction measurements \cite{25,26} indicate that $\alpha'$ might be compatible with zero at a larger resolution scale. The photoproduction fit (81) serves us to fix the slope of the gluonic residue function, while for the “Regge slope” at the input scale (73) we will consider two values. To maximize model differences, we choose the combinations,
\[
\{M^G = \sqrt{0.7} \text{ GeV, } \alpha_G' = 0.15/\text{GeV}^2\} \quad \text{and} \quad \{B^G = 2.32/\text{GeV}^2, \alpha_G' = 0\},
\] (82)
for dipole and exponential ansatz, respectively. Here, the values of $M^G$ and $B^G$ correspond to each other for $\alpha_G' = 0$.

Finally, we equate the quark and gluon “Regge slope” parameters, $\alpha'_{\text{sea}} = \alpha_G'$. Then our parameter sets for the DVCS fits read
\[
\{s^{\text{sea}}, s^G, M^{\text{sea}}\}, \quad \text{with fixed} \quad M^G = \sqrt{0.7} \text{ GeV, } \alpha_{\text{sea}}' = \alpha_G' = 0.15/\text{GeV}^2,
\] (83)
for the dipole, and
\[
\{s^{\text{sea}}, s^G, B^{\text{sea}}\}, \quad \text{with fixed} \quad B^G = 2.32/\text{GeV}^2, \alpha_{\text{sea}}' = \alpha_G' = 0,
\] (84)
for exponential ansatz.

The small-$x_B$ DVCS cross section measurements of the H1 and ZEUS Collaborations are published in Refs. \cite{8,9} and \cite{7}, respectively. The kinematics covers the intervals
\[
3 \text{ GeV}^2 \lesssim Q^2 \lesssim 80 \text{ GeV}^2, \quad 45 \text{ GeV} \lesssim W \lesssim 145 \text{ GeV}, \quad 0.1 \text{ GeV}^2 \lesssim -t \lesssim 0.8 \text{ GeV}^2.
\]
Figure 2: Two-step fit at LO to DVCS [8, 9, 7] and DIS [129] data with the Σ-PW model and the dipole ansatz [88]: (a) differential DVCS cross section versus $-t$ for three values of $Q^2$ at $W = 82$ GeV [9], (b) total DVCS cross section versus $Q^2$ at $W = 82$ GeV (circles, solid) [8, 9] and at $W = 89$ GeV (triangles, dash-dotted) [7], (c) total DVCS cross section versus $W$ at the same $Q^2$ values as on (a) [9], and (d) DIS structure function $F_2(x_B, Q^2)$ versus $Q^2$ for $x_B = \{8 \cdot 10^{-3}, 3.2 \cdot 10^{-3}, 1.3 \cdot 10^{-3}, 5 \cdot 10^{-4}\}$ [129].

We do not include in our fits one ZEUS data set [16]. H1 data versus $x_B$, and model-dependent extractions of skewness ratio and $t$-slope given by those Collaborations. Note that the more recent H1 measurement [9], providing a larger data set on the $t$-dependence, was not used in our previous GPD study [62]. Altogether, the H1 and ZEUS measurements provide us with 101 DVCS data points.

We give a LO fitting example for the Σ-PW model in Fig. 2. Here the first panel shows the $t$-dependence, the second and third the $W$- and $Q^2$-dependence, respectively, and the fourth one our fit to the DIS structure function $F_2$.

\[\text{The excluded data is for the cross section, integrated over } t, \text{ versus } W \text{ for fixed } Q^2 = 9.6 \text{ GeV}^2. \text{ Including this set and the analogous one from H1 would unavoidably increase the } \chi^2 \text{ value of our fit. These two data sets are mutually compatible; however, we give here preference to H1 data set, since it possesses a smoother } W\text{-dependence.}\]
4.2 Leading order fits to DVCS cross section measurements

To simplify our analysis of DVCS cross section measurements we will neglect the helicity non-conserved CFF $E$. According to the formula (4) for the DVCS cross section, this is justified if the condition

$$\frac{-t}{4M^2} |E|^2 \ll |H|^2$$

holds true. Since the average value of $-t$ is $\sim 0.2 \text{ GeV}^2$, there will be at the most a few percent contamination by $E$ as long as the modulus of this CFF is comparable to $|H|$. In the case that $|E|$ turns out to be considerably larger than $|H|$, our findings, or better to say our interpretation, will become questionable.

In Sect. 3.1 we already mentioned that a reasonable description of DVCS data at LO accuracy in the small-$x_B$ region could not be achieved in previous ad hoc GPD model studies [60, 62, 58]. On the other hand, one expects that this should be possible using more flexible GPD models. These

| model   | $\alpha_s$ | $\chi^2$/d.o.f. DIS | $\chi^2$/d.o.f. DVCS | $\chi^2$/n.o.p. $t$ | $\chi^2$/n.o.p. $W$ | $\chi^2$/n.o.p. $Q^2$ |
|---------|------------|----------------------|----------------------|---------------------|---------------------|---------------------|
| 1, dipole | LO         | 49.7/82              | 280./100             | 181./56             | 63.6/29             | 36.2/16             |
| 1, exp.  | LO         | 49.7/82              | 316./100             | 192./56             | 79./29              | 44.9/16             |
| nl, dipole | LO        | 49.7/82              | 95.9/98              | 53.2/56             | 27./29              | 15.8/16             |
| nl, exp. | LO         | 49.7/82              | 97.9/98              | 49.1/56             | 31.2/29             | 17.7/16             |
| $\Sigma$, dipole | LO      | 49.7/82              | 101./98              | 57.7/56             | 27.4/29             | 16./16              |
| $\Sigma$, exp. | LO    | 49.7/82              | 102./98              | 51./56              | 32.3/29             | 18.6/16             |

Table 1: $\chi^2$ values and their individual contributions, coming from experimental data on $t$, $W$, and $Q^2$-dependence, for various models with a residual dipole [83] and exponential [84] $t$-dependence for two-step fits (first DIS, then DVCS; first six rows), and for simultaneous fit (DIS+DVCS; last row). Boldface numbers indicate bad fits, as defined in Sect. 4.1.

statements are quantified in Table 1 where for various models we list the total DIS and DVCS $\chi^2$ over the number of d.o.f., as well as the partial contributions to $\chi^2$ from various subsets of data, over the corresponding n.o.p. Our ad hoc model (l-PW) is with $\chi^2$/d.o.f. $\approx 3$ highly disfavored at LO accuracy in both fitting strategies (two-step and simultaneous), while flexible (nl- and $\Sigma$-PW) GPD models correctly describe both DIS and DVCS data (for them we display only results of two-step fits). For both flexible models, and with either dipole [83] or exponential [84] residual $t$-dependence, we have $\chi^2$/d.o.f. $\approx 1$, and partial $\chi^2$ values indicate a good description for the $t$, $W$, and $Q^2$-dependence. The values of our fitting parameters are listed in Table 2.

In Section 4.2.1 we reveal the reason for the failure of ad hoc models. Then, in Section 4.2.2 we
Table 2: Model parameters, as obtained by two-step fits from Table 1, together with quark GPD $H_t$-slope $B_{\text{sea}}$ at $x = 10^{-3}$ and $Q^2 = 4 \text{ GeV}^2$, CFF $H_t$-slope $b_{\text{eff}}$ at $W = 82 \text{ GeV}$ and $Q^2 = 10 \text{ GeV}^2$, both in $\text{GeV}^{-2}$, and resulting BCA (6). For the dipole and exponential (exp.) $t$-dependence the fixed variables are given in Eq. (83) and (84), respectively. Again, boldface numbers arise from bad fits.

| model | $\alpha_s$ | $N_{\text{sea}}$ | $\alpha_{\text{sea}}(0)$ | $(M_{\text{sea}})^2$ | $s_{\text{sea}}$ | $\alpha^G(0)$ | $s^G$ | $B_{\text{sea}}$ | $b_{\text{eff}}$ | BCA |
|-------|------------|------------------|-----------------|-----------------|----------------|----------------|-----|----------------|---------------|-----|
| l, dipole LO | 0.152 | 1.158 | 0.062 | 1.247 | 33. | 5.7 | 0.19 |
| l, exp. LO | 0.152 | 1.158 | 1.247 | 29. | 5.1 | 0.23 |
| nl, dipole LO | 0.152 | 1.158 | 0.48 | -0.15 | 1.247 | -0.81 | 4.8 | 5.5 | 0.13 |
| nl, exp. LO | 0.152 | 1.158 | -0.18 | 1.247 | -0.86 | 3.1 | 5.8 | 0.14 |
| $\Sigma$, dipole LO | 0.152 | 1.158 | 0.42 | -11. | 1.247 | -32. | 5.4 | 5.5 | 0.14 |
| $\Sigma$, exp. LO | 0.152 | 1.158 | -13. | 1.247 | -34. | 3.1 | 5.8 | 0.15 |

4.2.1 The failure of the small-$x$ conformal skewness ratio at LO

A whole class of GPD models has the same small-$x$ behavior as the l-PW model, characterized by their skewness ratio being equal to its conformal value (28). As explained in Sect. 3.1, for the “pomeron” case, of interest here, we can also include the “dual” model of Ref. [58] and the original RDDA ($b = 1 \approx \alpha$) in this class. We explained in Sect. 3.2 that, contrarily to the claim of Refs. [94, 96], the conformal ratio (28) cannot be a general GPD property. It is important to clarify the phenomenological status of this small-$x$ claim. We shall now have a closer look at its failure by utilizing the l-PW model.

The l-PW model implies a normalization of the CFF that generally overshoots the experimental data, which is also manifested in the large skewness effect (28), i.e., $r \sim 1.6$. To compensate for the too large normalization in the fitting process the $t$-slope gets increased via a very low cut-off mass parameter $(M_{\text{sea}})^2 \approx 0.05 \text{ GeV}^2$ or a very large slope parameter $B_{\text{sea}} \approx 30/\text{GeV}^2$, see Table 2. As a consequence, the $t$-dependence of the cross section, with $\chi^2/n.o.p. \approx 3$, is particularly badly described. A simultaneous DIS/DVCS fit also does not help, see Ref. [62] or last row of Table 1. We add that in such a fitting strategy the structure function $F_2$ is well described, where the gluon PDF comes out slightly softer than in a two-step fit.

We exemplify now that the failure of the l-PW model cannot be cured by modifying the $t$-dependence. From the second line of Table 1 we see that a purely factorized $t$-dependent ansatz, namely, the exponential one (84), is with $\chi^2/d.o.f. \approx 3.2$ even more disfavored than the dipole
ansatz (83). Such an exponential ansatz was also employed in the “dual” model of Ref. [57]. Our poor description of the DVCS data contradicts the statement of Ref. [57] that such a model is compatible with the DVCS data of the HERA I run [8, 7]. We repeated the analysis of Ref. [57] taking a pure Regge ansatz with no residual $t$-dependence, but with large “Regge slope” instead:

$$\alpha'_\text{sea} = 0.9/\text{GeV}^2 \quad \text{and} \quad \alpha'_G = 0.5/\text{GeV}^2.$$  \hspace{1cm} (85)

One might expect that such a choice lowers the normalization of the CFF,

$$\frac{\mathcal{H}_\text{Regge}(\xi, t)}{\mathcal{H}_\text{exp}(\xi, t)} \approx \frac{\Gamma(3/2 + \alpha(t))\Gamma(2 + \alpha)}{\Gamma(3/2 + \alpha(t))\Gamma(2 + \alpha(t))} \exp\left\{ -|t|(\alpha'\ln(2/\xi) - B) \right\} \lesssim 1 \quad \text{for small} \; \xi,$$

in such a way that DVCS data are better described. In contrast to this expectation, we have found total disagreement with the data, e.g., $\chi^2$/d.o.f. = 2100/101. Fortunately, the inconsistency of our findings with those of Ref. [57] has been resolved in Ref. [58] and our statement that l-PW models (or minimalist “dual” model and the small-$x$ claim [94, 96]) are disfavored at LO holds true for a pure Regge ansatz (85), too. Note that the value of $\alpha'_G$ as large as in Eq. (85) is already excluded by the H1 and ZEUS electroproduction measurements of vector mesons, which is dominated by two-gluon exchanges.

We also recall that in a previous LO investigation [61], performed in momentum fraction space, both the quark and gluonic $r$-ratios were taken to be one at the input scale. What happens then is that the large amount of gluons, with their constant skewness function, drives the quark GPD to rapidly approach its conformal skewness ratio with increasing $Q^2$, see Ref. [101]. The failure of this model shows that the skewness ratio for gluons is also not equal to the conformal value $r^G \approx 1$. To investigate this some more, we used the l-PW model for gluons, where the skewness ratio is close to one, see Eq. (28), and allowed a flexible parameterization of the sea quark GPD, e.g., the $\Sigma$-PW model with dipole ansatz. We indeed obtained a bad fit

$$\chi^2/\text{d.o.f.} = 278/99; \quad \chi^2_{\text{n.o.p.}} = 168/56; \quad \chi^2_{W}/\text{n.o.p.} = 70/29; \quad \chi^2_{Q^2}/\text{n.o.p.} = 40/16,$$ \hspace{1cm} (86)

verifying our expectations.

It is popular in LO model descriptions of hard exclusive vector meson production [45, 46, 134, 135] to reduce the amount of gluons by taking the NLO PDF parametrization rather the LO one, since NLO gluon PDFs are by a factor of two or so smaller than LO ones, see error bands in Fig. 1(b). This recipe has been also used in Refs. [134, 135] to conclude that the exclusive $J/\Psi$ production is phenomenologically consistent with the LO small-$x$ claim of Ref. [94]:

$$R^G = 2^\alpha G^{-1} r^G > 1.$$ \hspace{1cm} (87)

In our opinion this finding does not necessarily support the small-$x$ claim, obtained from a LO analysis. The recipe is only justified in the phenomenological context of modelling the hard
exclusive production of vector mesons \cite{45, 46, 134, 135}; however, in a simultaneous description of DIS and DVCS data this recipe is obviously in conflict with the counting scheme of the collinear factorization approach. Moreover, if we take at the input scale NLO PDFs instead of LO ones in the model of the previous paragraph, we found that although

\[
\chi^2/d.o.f. = 154/99, \quad \chi^2_{	ext{n.o.p.}}/n.o.p. = 95/56, \quad \chi^2_{W}/n.o.p. = 38/29, \quad \chi^2_{Q^2}/n.o.p. = 21/16 \quad (88)
\]

improve, compared to Eq. (88), the DVCS fits are still unacceptable.

4.2.2 Skewness ratio, \(t\)-dependence, and their cross-talk

Our flexible models resolve the normalization problem and fits are unproblematic in both two-step and simultaneously fitting strategies, see Table 1 and Fig. 2. As expected, we find negative values of the \(s\)-parameters, listed in Table 2. This can be also viewed as the reduction of the large skewness effect of the l-PW model. We recall that the definition of these \(s\)-parameters does not directly allow to read off from them the size of the skewness effect, or to compare their size in two models.

We quantify the skewness effect in the momentum fraction space by the \(r\)-ratio \cite{20}. An analogous quantity, emphasizing the physical rather than partonic aspect, can be defined as the ratio of the imaginary parts of amplitudes for DVCS and forward Compton scattering, which can be expressed in terms of the differential DVCS cross section at \(t = 0\) and the DIS cross section \(\sigma_T(\gamma^*p \rightarrow X)\) for transversally polarized photon exchange, respectively. Assuming an exponential \(t\)-dependence, the skewness effect is revealed by utilizing the total DVCS cross section \cite{2}:

\[
R(W, Q^2) = \frac{\sqrt{16\pi\sigma_{\text{DVCS}} b(Q^2)/(1 + \rho^2)}}{\sigma_T(\gamma^*p \rightarrow X)} \bigg|_{\text{LO}} \frac{H_{\text{sea}}(x, \eta = x, t = 0, Q^2)}{q_{\text{sea}}(X, Q^2)} \bigg|_{X = 2x/(1+x)}. \quad (89)
\]

Here the \(t\)-slope \(b(Q^2)\) is extracted from a fit with \(\alpha' = 0\) \cite{4}:

\[
b(Q^2) = A \left[ 1 - B \log(Q^2/2 \text{GeV}^2) \right], \quad A = 6.98 \pm 0.54 \text{GeV}^{-2}, \quad B = 0.12 \pm 0.03, \quad (90)
\]

and the ratio of real to imaginary part of the DVCS amplitude might be set to \(\rho = -\cot(\alpha(Q^2)\pi/2)\). At LO this \(R\)-ratio can also be expressed in terms of sea quark GPD, where the momentum fraction for the PDF (or zero-skewness GPD at \(t = 0\)) is \(X = 2x/(1 + x)\). The relation between skewness ratios \cite{20} and \cite{89}, considered now as a function of \(x\) rather than \(W\), follows from their definitions:

\[
R(x, Q^2) \bigg|_{\text{LO}} \propto 2^{\alpha(Q^2)} r(Q^2) \quad \text{for small } x, \quad (91)
\]

where we set \(q_{\text{sea}}(x, Q^2)/q_{\text{sea}}(X, Q^2) = 2^{\alpha(Q^2)}\). In the kinematical region considered here \(\alpha \sim 1.2\) and so \(R \sim 2r\).
Figure 3: (a) skewness ratio $R$ for $W = 82$ GeV$^2$ compared to H1 data and (b) quark skewness ratio $r$ for $x = 10^{-3}$. Model parameters, obtained from fits as in Fig. 2, are specified in the main text and in Table 2 for l-PW (dotted), nl-PW (dot-dashed), Σ-PW (solid) with dipole ansatz and Σ-PW with exponential $t$-dependence (dashed).

In panels (a) and (b) of Fig. 3 we display $R$ and $r$ for fixed $W$ and for fixed $x$, respectively, as functions of $Q^2$. Needless to say, the l-PW model, obeying the small-$x$ claim [94, 96] of conformal ratio for quarks, is in conflict with experimental measurements (dotted curves). The experimentally measured $R$-ratio is well reproduced by both the nl-PW (dot-dashed) and the Σ-PW (solid) models within a dipole ansatz [83], which are barely distinguishable. One sees that the $R$-ratio is slightly larger than 2, while the $r$-ratio is generically $r \approx 1$, with their ratio as expected from Eq. (91). This experimental value of $R \sim 2$ is sometimes interpreted as a large skewness effect; however, from the GPD perspective, i.e., $r \sim 1$, it is much more appropriate to consider it as a manifestation of a small or zero skewness effect.

We also display the result for a Σ-PW model with exponential [84] $t$-dependence (dashed). Compared to the dipole ansatz [83], it provides only a slightly smaller skewness ratio, i.e., the modulus of its negative $s$-parameters is slightly larger. In other words, the normalization of CFF

$$\mathcal{H}(\xi, t = 0, Q_0^2)$$

at $t = 0$ is larger for CFF with a dipole $t$-dependence in order to compensate for initial faster decrease with $t$. This can be also observed by comparing the parameters $B_{sea}$ and $s$ in the third and fourth (or fifth and sixth) rows of Table 2. Thus, the quite drastic correlation of skewness effect and $t$-dependence for the disfavored l-PW model, observed in the previous section, appears also for flexible GPD models, but in a much milder form. Note that the CFF

$$\mathcal{H}(\xi, t \approx 0.2 \text{ GeV}^2, Q_0^2)$$
at the mean value $\langle t \rangle \approx 0.2 \text{ GeV}^2$, and the effective slope over the accessible $t$-interval, see Eq. (92) below, are described equally good with both ansaetze.

Let us also have a closer look at the gluonic skewness effect. The large negative skewness parameters $s^G$ in Table 2 indicate that the gluonic $r$-ratio is much smaller than one. This is seen in Fig. 4(a) for both flexible models, which are again barely distinguishable, and whose gluon skewness is even negative at the input scale. If the skewness ratio at the input scale is constrained to be positive, we could not obtain acceptable LO fits. Although the GPD on the cross-over line $\eta = x$ has no probabilistic interpretation, and negative values are thus not forbidden, a gluonic GPD model with a zero is suspicious. We consider this zero as an artifact related to an improper modelling of the skewness function $r(\vartheta)$, defined in Eq. (19), at the initial scale. The fact that with increasing $Q^2$ the gluonic $r$-ratios of both models approach in the same way the conformal ratio $r_G \approx 1$, reveals that their skewness functions are almost the same, see Fig. 4(b), although their conformal moments look quite different. We add that the same turns out to be true for the quark GPD models. Hence, finally one realizes that the $\Sigma$-PW model is effectively equivalent to the $nl$-PW one.

Within our models, the $t$-dependence is well described at LO accuracy, too, and we cannot discriminate between the two types of functional dependence. As argued in Sect. 3.1, we prefer the dipole ansatz (83) with a small value of $\alpha'_{\text{sea}} = \alpha'_G = 0.15$. In panel (a) of Fig. 2 a fitting example is shown and the $\chi^2$ values, listed in Table 1, confirm a good description of the DVCS data set.

To convince the reader in a more obvious way that our dipole GPD parameterization (83) is compatible with the exponential fit (90) of the H1 Collaboration, we evaluate the effective

Figure 4: Gluonic skewness ratio $r^G$ versus $Q^2$ for $x = 10^{-3}$ (a) and skewness function $r(\vartheta)$ (b). Models are the same as in Fig. 3.
Thereby, we consider the experimentally accessible interval, i.e., we set 
\[ t_1 = -0.1 \text{ GeV}^2 \quad \text{and} \quad t_2 = -0.8 \text{ GeV}^2 \]
and evaluate \( b^{\text{eff}} \) from our GPD models that are fitted only to the cross section measurements.

In Fig. 5 we show the effective slope \( b^{\text{eff}} \) versus \( Q^2 \) for \( W = 82 \text{ GeV} \) and versus \( W \) for \( Q^2 = 10 \text{ GeV}^2 \). One clearly realizes that the solid and dash-dot-dotted curves, which result from the \( \Sigma \)- and nl-PW model with a dipole ansatz (83), respectively, describe the experimental H1 data very well. In the left panel the \( Q^2 \)-evolution of \( b^{\text{eff}} \) follows the experimental data and so the perturbative evolution of these GPD models is fully compatible with the measurements. Also the flatness of the \( W \)-dependence of the data, indicating the absence of a shrinkage of the diffractive forward peak, is well reproduced with the input values \( \alpha'_{\text{sea}} = \alpha'_{G} = 0.15 \text{ GeV}^2 \), see Fig. 5(b)
Consequently, one cannot conclude that the DVCS measurements indicate that \( \alpha' \) is zero at a lower scale.

The quark dipole cut-off mass parameter \( M_{\text{sea}} \approx 0.67 \text{ GeV} \) in both of our models is rather similar and somewhat smaller than the fixed gluonic cut-off mass \( M_{G} \approx 0.85 \text{ GeV} \). They essentially coincide with the characteristic scale of the nucleon, as it appears in the dipole parameterization of its electromagnetic form factors. Note that the dipole masses and the resulting quark slope \( B_{\text{sea}} \) at the input scale obviously differ 10 – 15% between the two flexible GPD models, compare third and fifth rows in Table 2. This essentially reflects the residual \( t \)-dependence of the skewness effect at the input scale and establishes a cross-talk between \( t \)-dependence and skewness.
Alternatively, we can also fit the experimental data set by an exponential $t$-dependence with $\alpha'_\text{sea} = \alpha'_G = 0$. As already mentioned, the quality of fits are equally good, compare third (fifth) with forth (sixth) line in Table 1. Our exponential $t$-parametrization resembles the one used by the H1 Collaboration in the fit of the differential DVCS cross section measurements. Indeed, at the input scale $Q = 2\,\text{GeV}$ our LO $\Sigma$-PW fit gives $b = 6.2/\text{GeV}^2$ which is consistent with the H1 value, $b = (6.4 \pm 0.5)/\text{GeV}^2$. As in the H1 fit, the evolution to higher scales decreases the slope parameter. However, in our GPD parameterization this reduction is slightly weaker than in the H1 fit, displayed below in Fig. 10 together with the NLO prediction of the $\Sigma$-PW. Nevertheless, we conclude that the decoration of the exponential $t$-slope parameter in GPDs by an additional $Q^2$ dependence as in Refs. [60, 57, 58] is a redundant complication, which spoils the perturbative evolution. The exponential ansatz describes the measurements well, too, where the small offset between GPD description and H1 measurements is a bit larger than for the dipole ansatz, see also H1 fit below in Fig. 10.

Let us finally have a closer look at the “Regge slope” parameters. A small value of $\alpha'_G < 0.25/\text{GeV}^2$ can be deduced from $J/\Psi$ data. However, one might wonder whether a large value of $\alpha'_\text{sea} \sim 1/\text{GeV}^2$, as considered in Ref. [101] and utilized in Ref. [57], plays some role for H1/ZEUS kinematics. From the Regge theory point of view, see discussion in Sect. 3.2, such a large slope parameter belongs to “Reggeon exchanges”, that are suppressed relatively to the “pomeron” one by $\xi^\lambda$ with $\lambda \gtrsim 1/2$. Therefore, we expect that our fits should disfavor a large $\alpha'$ value for the quarks.

To investigate this, we have performed a series of fits with the $\Sigma$-PW model, accompanied by the dipole ansatz, with different fixed value of $\alpha'_\text{sea}$ parameter. Probability of the $\chi^2$ of these fits is plotted in Fig 5. One notices that the preferred value is indeed around $\alpha'_\text{sea} \approx 0.15\,\text{GeV}^{-2}$, while values beyond $0.5\,\text{GeV}^{-2}$ are excluded at 95% confidence level. This confirms our model assumption that “Reggeon” contributions are not dominant in the H1/ZEUS kinematics. Note also that the choice $\alpha'_G = 0$, shown as dashed curve, is disfavored in the dipole ansatz.

4.3 Beyond leading order fits

We shall now pursue the model dependence and GPD reparameterization in DVCS fits beyond LO. At NLO the gluons become a part of the hard-scattering process, and they may induce large radiative corrections. The inclusion of radiative corrections in a DVCS fit is compensated by the reparameterization of the GPDs at the initial scale. Within a flexible GPD parameterization we have good fits beyond LO in both the $\overline{\text{MS}}$ and $\overline{\text{CS}}$ schemes. In Table 3 we list the $\chi^2$ values for the dipole ansatz.

It has been demonstrated in Ref. [62] that beyond LO a good description of HERA I DVCS
Figure 6: Probability of (such and larger) $\chi^2$ obtained by fitting the $\Sigma$-PW model and the dipole ansatz with $\alpha_G' = 0.15/\text{GeV}^2$ (solid) and $\alpha_G' = 0$ (dashed), with respect to the “Regge slope” parameter $\alpha_{\text{sea}}'$, which is held fixed during fitting procedure.

| model | $\alpha_s$ | $\chi^2$/d.o.f DIS | $\chi^2$/d.o.f DVCS | $\chi_t^2$/n.o.p | $\chi_W^2$/n.o.p | $\chi_{Q^2}^2$/n.o.p |
|-------|------------|---------------------|---------------------|------------------|------------------|------------------|
| 1 NLO(\overline{\text{MS}}) | 71.6/82 | 148./100 | 77.6/56 | 36.8/29 | 33.9/16 |
| 1 NLO(\overline{\text{CS}}) | 71.6/82 | 105./100 | 62.9/56 | 25.1/29 | 17./16 |
| nl NLO(\overline{\text{MS}}) | 71.6/82 | 102./98 | 60.2/56 | 23.9/29 | 17.5/16 |
| nl NLO(\overline{\text{CS}}) | 71.6/82 | 104./98 | 61.4/56 | 24.9/29 | 18.1/16 |
| $\Sigma$ NLO(\overline{\text{MS}}) | 71.6/82 | 101./98 | 60./56 | 23.9/29 | 17.5/16 |
| $\Sigma$ NLO(\overline{\text{CS}}) | 71.6/82 | 104./98 | 61.5/56 | 24.9/29 | 18.1/16 |

Table 3: $\chi^2$ values as in Table 1 of NLO DIS and DVCS fits for various models in the $\overline{\text{MS}}$ and $\overline{\text{CS}}$ scheme.

After including the HERA II DVCS data \cite{62} with many more data points for the $t$-dependence, it is, however, not necessarily true that the l-PW model still works. In particular for the $\overline{\text{MS}}$ scheme $\chi^2$ is significantly large, see first line in Table 3 while in the $\overline{\text{CS}}$ scheme such a model fit is acceptable within the dipole ansatz. In contrast, within an exponential ansatz the fits in the $\overline{\text{MS}}$ scheme are acceptable, however, in the $\overline{\text{CS}}$ they are now disfavored:

$$\chi^2/\text{d.o.f.} = \begin{cases} 95/100 \\ 155/100 \end{cases}, \quad \chi_t^2/\text{n.o.p.} = \begin{cases} 50/56 \\ 108/56 \end{cases} \quad \text{for} \quad \begin{cases} \overline{\text{MS}} \\ \overline{\text{CS}} \end{cases},$$

where in both cases $\chi_W^2/\text{n.o.p.}$ and $\chi_{Q^2}^2/\text{n.o.p.}$ values are fine.

These findings illustrate the intricate interplay of the functional form of a given ad hoc ansatz and radiative corrections. If one would have relied on the claim of Ref. \cite{94} about the conformal...
skewness ratio, one might have wondered whether the outcome of the NLO fitting favors the l-PW model with the exponential ansatz in the $\overline{\text{MS}}$ scheme or with the dipole ansatz in the $\overline{\text{CS}}$ scheme, see, e.g., the discussion in Ref. [57]. We should recall here that the difference between the $\overline{\text{MS}}$ and $\overline{\text{CS}}$ schemes is entirely related to ‘non-diagonal’ rotation effects\textsuperscript{17} that vanish in the $\eta = 0$ case. In other words, this scheme rotation is nothing but a skewness effect and we observed in fact an interplay of skewness and $t$-dependence, discussed in the previous section in context of flexible models. So we emphasize again that an ‘overinterpretation’ of fit results within an ad hoc GPD model is inappropriate.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig7.pdf}
\caption{Quark skewness $r$ (20) (a) and gluon skewness $r_G$ (21) (b) versus $Q^2$ for $x = 10^{-3}$, extracted from a two-step fit. Dash-dot-dotted (green) line is l-PW model at NLO($\overline{\text{MS}}$), while other lines are for $\Sigma$-PW model at LO (dotted, black), NLO($\overline{\text{MS}}$) (dash-dotted, green), NLO($\overline{\text{CS}}$) (dashed, blue) and NNLO (solid, red). In all models we employ the dipole ansatz (33).}
\end{figure}

The discussion of the previous paragraph becomes superfluous, if one utilizes flexible GPD models. Then we can provide also in the $\overline{\text{MS}}$ scheme good two-step fits and we can reveal the skewness effect. The extracted NLO fitting parameters for the $\overline{\text{MS}}$ and $\overline{\text{CS}}$ schemes are listed in Table 4 for the dipole ansatz. If one goes from LO to NLO, the most drastic changes appear in the skewness parameters. They mutate from large negative values to moderate positive ones in the $\overline{\text{MS}}$ scheme. This qualitative jump is also illustrated in Fig. 7, compare LO fit (dotted curve) with the others, where we show the skewness ratios for $x = 10^{-3}$ versus $Q^2$. (This ratio for a fixed $Q^2$ is rather flat over the experimentally accessible interval $x \in \{10^{-4}, 10^{-2}\}$.)

\textsuperscript{17}Obviously, this rotation is numerically significant in the DVCS kinematics, i.e., $\eta = \xi$. This shows that the statement of Ref. [96], namely, that such $\eta$-proportional effects are negligibly small and so the LO claim [94] about the conformal skewness ratio remains valid at NLO, is amiss, too. The differences between both schemes have also been studied in Ref. [62].

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Table 4: Model parameters as specified in Table 2 obtained by fits from Table 3.

| model | $\alpha_s$ | $N_{\text{sea}}$ | $\alpha_{\text{sea}}(0)$ | $(M_{\text{sea}})^2$ | $s_{\text{sea}}$ | $\alpha^G(0)$ | $s^G$ | $B_{\text{sea}}$ | $b_{\text{eff}}$ | BCA |
|-------|-----------|-----------------|-----------------|-----------------|----------------|----------------|------|----------------|----------------|------|
| 1 NLO (MS) | 0.168 | 1.128 | 0.71 | 1.099 | 3.5 | 5.0 | 0.10 |
| 1 NLO (CS) | 0.168 | 1.128 | 0.57 | 1.099 | 4.2 | 5.7 | 0.09 |
| nl NLO (MS) | 0.168 | 1.128 | 0.59 | 0.04 | 1.099 | 0.02 | 4.0 | 5.6 | 0.09 |
| nl NLO (CS) | 0.168 | 1.128 | 0.58 | -0.01 | 1.099 | -0.01 | 4.1 | 5.6 | 0.09 |
| $\Sigma$ NLO (MS) | 0.168 | 1.128 | 0.60 | 3.10 | 1.099 | 1.10 | 4.0 | 5.7 | 0.09 |
| $\Sigma$ NLO (CS) | 0.168 | 1.128 | 0.58 | -0.42 | 1.099 | -0.58 | 4.1 | 5.6 | 0.09 |

Table 5: $\chi^2$ values as in Table 4 for NNLO DIS and DVCS fits within various models in the CS scheme.

| model | $\alpha_s$ | $\chi^2$/d.o.f DIS | $\chi^2$/d.o.f DVCS | $\chi^2$/n.o.p | $\chi^2$/n.o.p | $\chi^2$/n.o.p |
|-------|-----------|-----------------|-----------------|----------------|----------------|----------------|
| 1 NNLO (CS) | 81./82 | 80.1/100 | 49.2/56 | 19.1/29 | 11.9/16 |
| nl NNLO (CS) | 81./82 | 78.8/98 | 46.7/56 | 18.9/29 | 13.2/16 |
| $\Sigma$ NNLO (CS) | 81./82 | 78.8/98 | 46.7/56 | 18.9/29 | 13.2/16 |

Table 6: Model parameters as specified in Table 2 obtained by fits from Table 5.

| model | $\alpha_s$ | $N_{\text{sea}}$ | $\alpha_{\text{sea}}(0)$ | $(M_{\text{sea}})^2$ | $s_{\text{sea}}$ | $\alpha^G(0)$ | $s^G$ | $B_{\text{sea}}$ | $b_{\text{eff}}$ | BCA |
|-------|-----------|-----------------|-----------------|-----------------|----------------|----------------|------|----------------|----------------|------|
| 1-PW NNLO (CS) | 0.172 | 1.125 | 0.56 | 1.104 | 4.2 | 5.6 | 0.12 |
| nl-PW NNLO (CS) | 0.172 | 1.125 | 0.57 | -0.01 | 1.104 | -0.04 | 4.2 | 5.6 | 0.12 |
| $\Sigma$-PW NNLO (CS) | 0.172 | 1.125 | 0.57 | -0.89 | 1.104 | -1.80 | 4.2 | 5.6 | 0.12 |

In the CS scheme the skewness parameters $s^G_{\text{sea},G}$ are small and negative for the dipole ansatz and so all models are compatible with the conformal ratio, obtained at LO. This is displayed for the $\Sigma$-PW model by the dashed curves in Fig. 7. The same holds true if we include in this scheme NNLO corrections, shown as solid curve. Table 5 states that all models provide similar and good $\chi^2$ values. The values of the $s$-parameters, listed in Table 6, are small and negative and with a slightly larger modulus than at NLO for the CS scheme, cf. Table 4. Hence at

NNLO in the CS scheme, same as at NLO, non-leading SO(3) partial waves do not play essential role. The differences between the three models are so small that we obtain the same values for the effective and partonic $t$-slope parameters.

The reparameterization of sea quark and gluon GPDs can be directly read off from Fig. 8.
Here the GPDs on the cross-over line are shown for the Σ-PW model with dipole ansatz as they arise from our fits at various orders and schemes. It is again obvious that the most drastic effect appears if we go from the LO (dotted) to NLO description. Quark GPDs at NLO, compared to the LO ones, are enhanced by a factor of two or so while the LO gluons suffers from our model artifact. The sizable reparameterization effects for quarks of about 100% or even more are in view of the quoted corrections in Ref. [62] surprising. Naively, we would have expected as in DIS moderate correction on the level of 20% or so. The very large NLO corrections in the quark sector might be connected to the artifacts of our gluon model, discussed above, and we would here not exclude the possibility that a fully flexible GPD model possess in the quark sector only moderate NLO corrections. Consequently, for such a model the quark skewness ratio must then be ~ 1 also beyond LO, i.e., much smaller then the conformal ratio $r_{\text{sea}} \approx 1.6$. The difference at NLO between the $\overline{\text{MS}}$ (dash-dotted) and $\overline{\text{CS}}$ (dashed) scheme are clearly visible for the sea quark GPD, while this skewness-induced effect is tiny for gluons. This simply reflects the properties of the off-diagonal scheme transformation, which is set by conventions. As already observed in Ref. [62] within the l-PW model and a simultaneous fit, the NNLO corrections lead only to a slight change of the parameters, obtained at NLO in the $\overline{\text{CS}}$ scheme, compare the dashed and solid curves in Figs. 8 and also 7 or the corresponding entries in Table 4 with Table 6.

One might wonder whether the observation that the $\overline{\text{CS}}$ (N)NLO skewness ratio approaches the LO conformal value (28) is a definite model independent feature or simply an accident. Naively, one might expect that a reduction of gluonic skewness ratio will also decrease the quark one, since

\footnote{There a given l-PW model has been employed to estimate the radiative corrections for the CFF $\mathcal{H}$.}
they contribute with a different sign to the CFF. In this way one might realize a quark skewness ratio which would be closer to the LO findings. However, taking a more negative value of $s^G$, like that occurring in LO fits, provides bad NLO fits. As in our unsuccessful attempt in the previous section (to have a positive gluon GPD at the cross-over line at LO), we consider also this finding as a model artifact. Namely, our models do not allow adjustment of the scale dependence of the skewness ratio in a flexible way.

### 4.4 Transverse distribution of partons

We would now like to deliver a partonic interpretation of our analysis of the $t$-dependence of the DVCS cross section. We stick to the probabilistic interpretation of zero-skewness GPDs in the infinite momentum frame \[136\]. In this frame the proton might be viewed as a disc with a radius of

$$\sqrt{\frac{4}{d} \ln F_1(t)\bigg|_{t=0} \approx 0.6 \text{ fm},}$$

arising from the parameterization of the Dirac form factor in terms of the common dipole expressions for the Sachs form factors within a cut-off mass of 0.71 GeV$^2$. We add that the electric charge radius $\sqrt{\langle r^2 \rangle} = 0.875$ fm of the proton, quoted in the Review of Particle Physics \[137\], is about ten percent larger than that from the dipole parameterization. Hence, also our proton disc radius \[93\] would then increase to about 0.66 fm.

The transverse width of parton distribution, i.e., the average distance $\sqrt{\langle \vec{b}^2 \rangle}$ of the struck parton from the proton center, is directly given by the $t$-slope of the zero-skewness GPD

$$\langle \vec{b}^2 \rangle(x, Q^2) = 4 \frac{d}{dt} \ln H(x, \eta = 0, t, Q^2)\bigg|_{t=0}. \tag{94}$$

Let us first consider the LO interpretation, where we take the $\Sigma$-PW model as it is specified in Table 2. As discussed in Sect. 4.2.2, the skewness effect will talk back to the $t$-slope parameters on the 10% – 15% level, which implies a $\lesssim 7\%$ model uncertainty for the transverse width. The rigidity, still present in our flexible models, does not allow us to reveal from LO fits to the DVCS data the transverse distribution of gluons. Therefore, we take the parameters as before for the dipole \[83\] and exponential \[84\] $t$-dependence, where the gluon $t$-slope parameters are fixed from the $J/\psi$ photoproduction.

The value of the transverse width, obtained from fitted GPDs, also depends on the functional form of the residual $t$-dependence, which defines the extrapolation from the accessible $t$ interval to $t = 0$. For the exponential ansatz our value for gluons is quoted in Ref. \[138\] and coincide with the proton disc radius of 0.6 fm. For $\alpha' = 0$, the dipole ansatz would provide a 10% larger value than the exponential one. Also a non-vanishing value of $\alpha'$ leads to an increase of the transverse
width. Taking $\alpha' = 0.15$ in exponential ansatz, the gluonic transverse width increases, e.g., to $\approx 0.72$ fm for $x = 10^{-3}$, coming rather close to the one in our dipole ansatz (83) with $\approx 0.77$ fm. We might conclude that even for an exponential $t$-dependence, the experimental uncertainties of the $\alpha'$ measurement in $J/\psi$ production do not exclude a gluon transverse width at small $x$ that is larger than the proton disc radius (93).

The gluon transverse width, obtained at the input scale, is depicted in the right panel of Fig. 9 as thin solid (dipole) and thin dashed (exponential) curves. Our LO fit states then that the quark transverse width at the input scale is slightly larger than for gluons, where the characteristic $x$-dependence, related to the partonic shrinkage effect, appears. We infer that our LO quark interpretation strongly reflects our assumptions and that we cannot discriminate, e.g., for $x \approx 10^{-3}$ and $Q^2 = 4$ GeV$^2$, between

$$\sqrt{\langle \vec{b}^2 \rangle}_{\text{sea}} \approx \begin{cases} 0.9 \\ 0.7 \end{cases} \text{ fm}, \quad \sqrt{\langle \vec{b}^2 \rangle}_G \approx \begin{cases} 0.8 \\ 0.6 \end{cases} \text{ fm} \quad \text{for} \quad \begin{cases} \text{dipole (83)} \\ \text{exponential (84)} \end{cases} \text{ ansatz.} \quad (95)$$

That at the input scale (thin curves) gluons are more centralized than sea quarks has been also found in Ref. [63] by a fine-tuning procedure. Thereby, the latter were dynamically generated within the double log approximation at LO from gluons with a (soft) pomeron trajectory (3). We emphasize that there the differences between a tripole and an exponential ansatz were hardly visible in impact space for distances smaller than 1.5 fm and that the larger value of the transverse width for a tripole ansatz arose due to its long tail in impact space, see illustrative examples in Ref. [63].

At larger resolution scale, e.g., $Q^2 = 10$ GeV$^2$, the partonic shrinkage effect is practically washed out, see the flatness of the thick solid curves in Fig. 9. This indicates that $\alpha'$ rapidly

Figure 9: Transverse width $\sqrt{b^2}$ of sea quark (a) and gluon (b) distributions versus $x$ for $Q^2 = 4$ GeV$^2$ (thin) and $Q^2 = 10$ GeV$^2$ (thick), resulting from the LO fit of $\Sigma$-PW model with dipole (solid) and exponential (dashed) $t$-dependence ansatz, specified in Eqs. (83) and (84), respectively.
approaches zero with growing $Q^2$. Also the initial differences between the transverse widths of quarks and gluons diminish by a slight decrease of the former and increase of the latter. However, the ‘typical’ value associated with the particular $t$-dependence ansatz is robust. Finally, our model findings for $Q^2 = 10 \text{ GeV}^2$,

$$\sqrt{\langle \vec{b}^2 \rangle_{\text{sea}}} \approx \sqrt{\langle \vec{b}^2 \rangle_{G}} \approx \begin{cases} 0.9 \\ 0.65 \end{cases} \text{ fm \ for \ \{ dipole (83) \}} \quad \text{for \ \{ exponential (84) \}}$$

(96)

are compatible with those of Ref. 63, where, e.g., a value of $\approx 0.85 \text{ fm}$ was quoted for a tripole ansatz.

We would also like to emphasize that for the exponential ansatz the gluon transverse width is at the input scale the same as quoted in Ref. 27. It perturbatively evolves to a slightly larger value with increasing scale. In Fig. 9 we also extrapolate the transverse widths obtained from fitted GPDs to larger $x \in [10^{-2}, 10^{-1}]$. This extrapolation is not supported by the interpretation of $J/\psi$ data from fixed target experiments in Ref. 27, where within a dipole ansatz a value of $\approx 0.53 \text{ fm}$ for $x \sim 0.1$ was quoted. Interestingly, the increase of this value by about $20\% - 30\%$ to $\sim 0.6 \text{ fm}$ has been explained with chiral dynamics 138. We add that in realistic GPD models the $t$-dependence dies out at $x \to 1$, e.g., seen in lattice simulations of Ref. 118, and so the partons are entirely concentrated in the center of the proton. This feature can be simply implemented in our models by decorating the cut-off mass (or the exponential slope parameter) with $j$-dependence.

Next we shall show that our LO findings with fixed gluon slope parameters are only moderately influenced by perturbative corrections and scheme conventions, or even by the release of the gluon slope parameters at NLO (or NNLO). Note that this was not the case in our previous investigations, where we employed the 1-PW model 62. We mainly work in the $\overline{\text{MS}}$ scheme to NLO accuracy, which is appropriate for a possible global GPD analysis of deeply exclusive electroproduction processes in momentum fraction space, and we use the $\Sigma$-PW model. We might have employed the nl-PW model as well, however, beyond LO there is only very little model uncertainty induced by the skewness dependence, see $B_{\text{sea}}$ values in Tables 3 and 4.

We first demonstrate in Fig. 10 that at NLO the description of the measured $t$-slope is for fixed gluon slope parameters fully analogous to our LO findings in Fig. 5. Again, the effective slope parameter (92) is evaluated from the outcome of our fits to the DVCS cross section with the dipole (dot-dashed) and exponential (dotted) ansatz, where the NLO ($\overline{\text{MS}}$) parameters are given in Table 4. Comparing the corresponding curves in both figures one can barely see a difference. We also display the H1 fit (90) as solid curve. Obviously, only if experimental errors could be very drastically reduced, one might be able to discriminate between the three curves — for a more detailed discussion see Sect. 4.2.2.

A simultaneous release of all four $t$-related parameters $(B_{\text{sea},G}^2, \alpha_{\text{sea},G}^2)$ leads to a non-convergent search for the minimal $\chi^2$. Therefore, to pin down also the “Regge slope” parameters, we use a
three-step fitting procedure (fitting to DIS data being the first step). We first release the gluonic residue \( t \)-slope parameter, which yields for the exponential \( t \)-dependence
\[
B_{\text{sea}} = 2.85 \text{ GeV}^{-2}, \quad s_{\text{sea}} = 0.71, \quad B_{\text{G}} = 2.71 \text{ GeV}^{-2}, \quad s_{\text{G}} = 0.36.
\]
Now releasing the \( \alpha'_{\text{sea}} = \alpha'_{\text{G}} = 0 \) parameters (thick dashed line in Fig. 11), we find that they essentially stay at zero. Compared with our three-parameter DVCS fit ansatz, \( \frac{M}{\bar{s}} \) photoproduction, the gluonic \( t \)-slope only slightly increases (17\%). Hence, we consider the two-gluon GPD models, obtained from DVCS and \( J/\Psi \) photoproduction, as compatible. Moreover, the small values of the skewness parameters indicate that the exponential ansatz in the \( \overline{\text{MS}} \) scheme is mostly compatible with the \( l \)-PW model, as pointed out in Sect. 4.3.

For the dipole ansatz the parameter set after the second fitting step reads
\[
(M_{\text{sea}})^2 = 0.54 \text{ GeV}^2, \quad s_{\text{sea}} = 5.24, \quad (M_{\text{G}})^2 = 0.47 \text{ GeV}^2, \quad s_{\text{G}} = 8.71
\]
with \( \alpha'_{\text{sea}} = \alpha'_{\text{G}} = 0.15/\text{GeV}^2 \). We realize that the quark skewness and cut-off mass parameters only slightly change, cf. Table 4. The gluon cut-off mass moderately decreases and the skewness parameter increases, compensating each other in the CFF and so the cross section is well described. Now releasing the \( \alpha' \) parameters we essentially find that for the dipole ansatz (thick solid) they slightly reduce for quarks and more significantly for gluons:
\[
\alpha'_{\text{sea}} = 0.15 \rightarrow 0.12, \quad \alpha'_{\text{G}} = 0.15 \rightarrow 0.08.
\]
The quality of fits, compared to those shown in Table 3 for fixed gluon slope parameters, does not change.

![Graphs showing quark and gluon transverse distribution widths](image)

Figure 11: Quark and gluon transverse distribution widths $\sqrt{\langle \vec{b}^2 \rangle}$ with respect to $Q^2$ for $x = 10^{-3}$ (a, b) and with respect to $x$ for $Q^2 = 10 \text{ GeV}^2$ (c, d), obtained from NLO MS DVCS fits of Σ-PW model with dipole (solid) and exponential (dashed) $t$-dependence. Thin lines correspond to parameter choice from (83) or (84), while for thick lines first the gluonic residual $t$-slope parameter $M^G$ or $B^G$ and then both $\alpha'_\text{sea}$ and $\alpha'_G$ parameters were released.

Fig. 11 summarizes our NLO (MS) findings for the quark and gluon transverse width $\sqrt{\langle \vec{b}^2 \rangle}$, defined in Eq. (94). We note that for fixed gluon slope parameters (thin lines) the gluonic transverse width, shown in the right panel, would fully coincide with our LO curves. The only moderate difference appears in the dipole ansatz (thin solid), where the quark transverse width decreases from $\approx 0.9 \text{ fm}$, quoted in Eqs. (95, 96), to $\approx 0.8 \text{ fm}$. If we take the gluonic parameters from the three-step fit, we see that the gluonic transverse width for both the dipole (solid) and exponential (dashed) ansatz increases from $\approx 0.7 \text{ fm}$ and $\approx 0.6 \text{ fm}$ to $\approx 0.85 \text{ fm}$ and $\approx 0.65 \text{ fm}$, respectively. However, this moderate difference for the gluons affects only slightly the quark transverse width,
see the left panel. At NLO we observe already at the input scale that quark and gluon transverse width are mostly the same. The values we can quote

$$\sqrt{\langle \vec{b}^2 \rangle_{\text{sea}}} \approx \sqrt{\langle \vec{b}^2 \rangle_{\text{G}}} \approx \left\{ \begin{array}{l} 0.75 - 0.80 \\ 0.60 - 0.65 \end{array} \right\} \text{fm for} \quad \left\{ \begin{array}{l} \text{dipole} \text{ ansatz} \\ \text{exponential} \text{ ansatz} \end{array} \right\} \quad (100)$$

are stable under evolution. Our results are fully robust with respect to scheme conventions or radiative NNLO corrections.

![Graph showing transverse profile functions](image)

Figure 12: Quark (a) and gluon (b) transverse profile function for $Q^2 = 4 \text{GeV}^2$ and $x = 10^{-3}$ is obtained from a six parameter DVCS fit, as in Fig. 11.

We conclude that within our GPD models, having a flexible skewness ratio and the same functional form of $t$-dependence for sea quarks and gluons, the resolution of the transverse distribution of partons is robust and that at small-$x$ (sea) quarks and gluons have the same transverse width. For the exponential ansatz the DVCS result $\approx 0.65 \text{ fm}$ is compatible with the one quoted in Ref. [27], namely 0.65 fm. For the dipole ansatz, we find a larger value $\approx 0.8 \text{ fm}$, which simply reflects the functional form we have employed. It is illustrated in Fig. 12 that in impact space, the (normalized) transverse profile function

$$\rho(b, x, Q^2) = \frac{\int_{-\infty}^{\infty} d^2 \vec{\Delta} e^{i \vec{\Delta} \cdot \vec{b}} H(x, \eta = 0, t = -\vec{\Delta}^2, Q^2)}{\int_{-\infty}^{\infty} d^2 \vec{\Delta} H(x, \eta = 0, t = -\vec{\Delta}^2, Q^2)} \quad (101)$$

for dipole and exponential $t$-dependence mainly differ for distances larger than the disc radius of the proton, i.e., for $b > 0.6 \text{ fm}$. Hence, the larger value of the transverse width for the dipole ansatz arises from the long range tail of the profile function, see the solid curve. Note that the model uncertainty in the extrapolation of the GPD to $t = 0$ corresponds to the uncertainty in the long range tail.
4.5 Is the anomalous gravitomagnetic moment accessible?

Let us first adopt the “classical” Regge point of view in which the chromomagnetic “pomeron” is absent, i.e., we can neglect as above the CFF $E$. Hence, the BCA (6) is already predicted by the outcome of our cross section fits from the previous sections. A preliminary BCA measurement of the H1 Collaboration with uncorrected acceptance effects has been reported in Ref. [10] for the kinematics

$$0.05 \text{GeV}^2 \leq |t| \lesssim 1 \text{GeV}^2, \quad \langle W \rangle = 82 \text{GeV}, \quad \text{and} \quad \langle Q^2 \rangle = 8 \text{GeV}^2.$$

(102)

To evaluate the BCA (6), we integrate over the $t$-interval and take the given mean values. In Fig. 13(a) we confront our LO prediction (solid curve) for the Σ-PW model (83) with the preliminary BCA measurement. Having in mind that we neglected the $\cos(3\phi)$ harmonics, induced by gluon transversity, and the presumable small twist-three $\cos(2\phi)$ harmonics, we can safely state that our prediction is fully compatible with the preliminary H1 measurement.

As explained in Sect. 2, accessing the twist-two CFFs through the $\cos(\phi)$ harmonic in the Fourier decomposition (10) is in the considered kinematics relatively clean. We recall that this method can only diminish the contamination from other CFFs. The preliminary H1 fit,

$$p_1 = 0.17 \pm 0.3 \pm 0.5,$$

(103)

is shown in Fig. 13(a) as error band and agrees well with our prediction (dashed curve), evaluated from the expression (6). The deviation of the dashed curve from the solid one is of kinematical origin and arises from the BH terms. In the absence of large genuine twist-three contributions and
gluon transversity or its twist-four contamination, all other harmonics in the Fourier decomposition \( \text{(10)} \) of the BCA are predicted to be small \([56]\):

\[
0 < p_0 \lesssim 10^{-2}, \quad p_2 \sim -5 \cdot 10^{-2}, \quad 0 < p_3 \lesssim 10^{-2},
\]

where higher harmonics for \( i \geq 4 \) can be considered as negligible. We emphasize that a significant deviation from these numbers would indicate that genuine twist-three and gluon transversity (or twist-four corrections) are addressable in the small-\( x_B \) kinematics, where it is assumed that electromagnetic radiative corrections will not alter the angular dependence of the BCA \([5] \).

To have a closer look to the parameter dependence of our prediction, we write the approximation \( \text{(11)} \) for \( t \)-integrated data (valid for \( |t_1| \gg -t_{\text{min}} \approx x_B^2 M_p^2 \)) as

\[
p_1 \sim n \frac{\xi \int_{t_1}^{t_2} \frac{dt}{\sqrt{-t}} F_1 \text{Re} \mathcal{H}}{h + \xi^2 \int_{t_1}^{t_2} dt |\mathcal{H}|^2} \bigg|_{W=82 \text{ GeV} \atop Q^2=8 \text{ GeV}^2}, \quad \text{with} \quad t_1 = -0.05 \text{ GeV}^2, \quad t_2 = -1 \text{ GeV}^2,
\]

where \( n \) and \( h \) are two kinematical factors. Apart from the skewness effect, the size of \( p_1 \) is essentially governed by the “pomeron trajectory”

\[
p_1 \propto \cot \left( \pi \alpha \left( \langle t \rangle, \langle Q^2 \rangle \right)/2 \right) \approx -\frac{\pi}{2} \left( \alpha \left( \langle Q^2 \rangle \right) - 1 \right),
\]

which for the mean values \( \langle -t \rangle \approx 0.2 \text{ GeV}^2 \) and \( \langle Q^2 \rangle = 8 \text{ GeV}^2 \) can be replaced by the intercept \( \alpha \left( \langle Q^2 \rangle \right) \). The justification for neglecting the \( \alpha' \) parameter can also be read off from the last column in Table \( \text{2} \) which shows that both ansaetze, with \( \alpha' = 0.15/\text{GeV}^2 \) and \( \alpha' = 0 \), yield almost the same value for the BCA. Hence, in our Regge-inspired GPD framework the BCA prediction arises to a great extent from the LO DIS fit. This is also accompanied by a large uncertainty, namely, a small absolute error of the DIS intercept \( \alpha - 1 \) will induce a large relative one for the BCA. Certainly, it would not be appropriate to quote here the width of \( \chi^2 \) minimization curve as an error estimate, since our model simplifications, e.g., neglecting “Reggeon” contributions, might influence the fit result for \( \alpha - 1 \), too. At NLO (NNLO) we observe that the BCA decreases to \( p_1 = 0.9(0.12) \), see last column in Table \( \text{4} \) \([\text{6}] \), which corresponds to a 1-σ deviation from the preliminary H1 mean value \([\text{103}] \). The relative drop of the BCA size at NLO, compared to LO, seems indeed to be correlated with that of the intercept, cf. corresponding columns in Tables \( \text{2} \) and \( \text{4} \).

We mentioned in Sect. \( \text{2} \) that a measurement of

\[
p_1 \sim n \frac{\xi \int_{t_1}^{t_2} \frac{dt}{\sqrt{-t}} \left( F_1 \text{Re} \mathcal{H} - \frac{t}{4M_p^2} F_2 \text{Re} \mathcal{E} \right)}{h + \xi^2 \int_{t_1}^{t_2} dt \left( |\mathcal{H}|^2 - \frac{t}{4M_p^2} |\mathcal{E}|^2 \right)} \bigg|_{W=82 \text{ GeV} \atop Q^2=8 \text{ GeV}^2},
\]

supplemented by the DVCS cross section measurement, should in principle allow to separate the CFFs \( \mathcal{H} \) and \( \mathcal{E} \). It is obvious from the formula that a negative real part in \( \mathcal{E} \) will more strongly influence our BCA “prediction” than a positive one.
To quantify the sensitivity of the BCA in dependence on $B^{\text{sea}}$, we adopt the GPD $E$ model and rely on the scenario $B^G = -B^{\text{sea}}$. We vary $B^{\text{sea}}$ at the input scale from $-0.4 \cdots 0.4$, which corresponds to a variation of $-0.13 \leq J^{\text{sea}} \leq 0.28 \quad (0.43 \geq J^G \geq 0.02)$ with $J^{\text{val}} = 0.2$.

In the right panel of Fig. 13 we show the dependence of the BCA (at $\phi = 0$) on the $B^{\text{sea}}$ parameter for the $\Sigma$-PW model at LO accuracy. We refitted for each given value of $B_{\text{sea}}$ the model parameters of the GPD $H$ so that the DVCS cross section is described. As one realizes, $p_1$ depends only slightly on the parameter $B^{\text{sea}}$; its relative change is

$$\delta p_1 / p_1 \sim -15\% \cdots +5\% \quad \text{for} \quad B^{\text{sea}} = -0.4 \cdots 0.4.$$ 

We obtain rather similar findings for a $B^G = 0$ scenario and within the nl-PW model. Contrarily to our hopes, we conclude that both the model uncertainties of $H$ and the experimental errors are too large to obtain a bound for $B^{\text{sea}}$.

### 4.6 Lessons from fits

We would like now to summarize the lessons from our fits, presented in the previous sections. We recall that our DVCS description relied on Regge-inspired GPD models, set up at a (low) input scale, and the collinear factorization framework at leading twist-two in LO and beyond. Both the factorization and renormalization scale were set equal to the virtuality of the incoming photon.

Concerning strategies for fitting to the unpolarized DIS structure function and DVCS cross section measurements we have seen that

- GPD models with a flexible skewness ratio describe well the small-$x_B$ DVCS cross section measurements at LO and beyond,

- fitting parameters obtained from simultaneous (DVCS+DIS) and two-step (first DIS, then DVCS) fits are rather similar,

- and that the partonic shrinkage effect can be addressed by a three stage fitting strategy.

Our Regge-inspired modelling within the collinear factorization framework, including the corresponding approximations, is consistent with experimental measurements:

- small “Regge slope” parameters $\alpha' < 0.25/\text{GeV}^2$ are favored and large ones for “pomeron” related quarks, e.g., $\alpha' \sim 1/\text{GeV}^2$, are excluded,
the scale dependence of both the residual $t$-dependence and “Regge slope” are compatible with pQCD,

• and the real part of the amplitude, following from DIS and DVCS cross section fits, predicts the preliminary BCA measurement.

Relying on the assumption that the CFF $\mathcal{H}$ is dominant and that higher twist contributions are unimportant, we conclude from our fits in Sect. 4.2 that realistic GPD models at LO have the following properties:

• quark GPD models possess a skewness ratio $r_{\text{sea}}^{\approx 1}$

• and to ensure this for a large $Q^2$ lever arm, gluon GPDs have a skewness ratio $r_G < 1$.

We emphasize that our finding $r_{\text{sea}}^{\approx 1}$ was predicted by the aligned-jet model and it is realized in an RDDA for $b_{\text{sea}} \gg 1$. However, all popular gluonic GPD models, possessing $r_G \approx 1$, are incompatible with the small-$x_B$ DVCS data (and presumably also in a pure LO description of data on hard vector meson electroproduction, see, e.g., the dashed curves in Fig. 5 of Ref. [46] that refer to the collinear factorization prediction and are included there for illustration). The negative skewness ratio of our gluonic GPD models at the input scale and the feature that it approaches the conformal ratio for growing $Q^2$ indicate that the parameterization of the skewness function is rigid. We can safely state that

• the small-$x$ claim of Refs. [94, 96] that the skewness ratio is equal to its conformal value is ruled out at LO

• and a flexible parameterization of evolution has not been achieved so far.

The reader might be surprised by this second conclusion, which contradicts opinions and model-dependent findings in the literature. Nevertheless, we are not aware of a distinct investigation that addresses the evolution of the residue function in dependence of the initial condition, i.e., the skewness function in momentum fraction space, the series of conformal PWs in $J$ space, the series of forward-like functions $Q_\rho(z)$ in the “dual model”, or the series of “conformal sibling poles” in Mellin space.

Including radiative corrections, we observed in Sect. 4.3 that NLO GPDs qualitatively differ from the LO ones:

• the quark skewness ratio in the $\overline{\text{MS}}$ scheme can be larger than the conformal one,

• the quark skewness ratio in the $\overline{\text{CS}}$ scheme matches the conformal one,
- the scheme dependence at NLO is absorbed by a sizeable GPD reparameterization,
- and the gluonic skewness ratio approaches in both schemes the conformal one.

The large reparameterization effects, in particular for quarks, might be a consequence of the remaining rigidity of the skewness function in our flexible models. We expect that this can be overcome by taking into account two effective non-leading SO(3)-partial waves rather than one.

We would also like to emphasize that the scheme dependence at NLO, we pursued in our studies, is entirely related to a ‘non-diagonal’ rotation, which shows that

- the small-\(x\) claim \([94, 96]\) of conformal skewness ratio can be sustained beyond LO only in special schemes, however, presumably not in the \(\overline{\text{MS}}\) one.

We add that our \(\overline{\text{MS}}\) fit at NLO for GPD models with an exponential \(t\)-dependence is consistent with the LO conformal skewness ratio. In our opinion this fact does not necessarily support the “logic” of the small-\(x\) claim, since it relies on tree-level conformal symmetry \([94, 96]\) that is beyond LO not explicitly implemented in the \(\overline{\text{MS}}\) scheme. Hence, the “predicted” skewness ratio in the \(\overline{\text{MS}}\) scheme at NLO differs from the LO one.

The stability of the perturbative approach can be presently studied only in the \(\overline{\text{CS}}\) scheme, and there

- the inclusion of NNLO corrections yields only a small change of fitting parameters.

Thereby, the skewness ratio in the \(\overline{\text{CS}}\) scheme is essentially given by the conformal ratio \([28]\). Thus, in this scheme beyond LO one might be tempted to conjecture that a GPD is indeed tied to the corresponding PDF by conformal mapping. One might wonder whether it is accidental that this hypothetical ‘holographical principle’ is realized within the group SO(2,1) \([64]\).

The functional form of \(t\)-dependence cannot be pinned down from present DVCS data. However, we can definitely state that:

- “Regge slope” parameters are small at the input scale, however, do not necessarily vanish,
- non-vanishing “Regge slope” parameters rapidly decrease with growing scale,
- both residual dipole and exponential \(t\)-dependence are compatible with present DVCS data,
- within our models, there is a cross-talk between skewness and \(t\)-dependence at LO,
- and beyond LO the skewness and \(t\)-dependence start to decouple in our flexible models.
In a three-step fitting strategy we also observed a correlation between the functional form of the residue function and the partonic shrinkage effect. Namely, an exponential $t$-dependence of the former is quite consistent with a zero shrinkage effect, i.e., $\alpha'_{\text{sea}} = \alpha'_{\text{G}} = 0$.

Our partonic interpretation of the transverse degrees of freedom, given in Sect. 4.4, is rather robust. Relying on the decoupling of $t$- and skewness-dependence, we have found for dipole (exponential) ansatz that

- sea quarks and gluons have roughly the same transverse width $\sim 0.75(0.65) \text{ fm}$,
- the transverse width is rather stable under evolution,
- and a possible partonic shrinkage effect at a lower scale is rapidly washed out with growing scale.

Unfortunately, we saw in Sect. 4.5 that both theoretical uncertainties and experimental errors do not allow to address

- the chromomagnetic “pomeron” or angular momentum of sea quarks,
- and gluon transversity.

5 Small-$x_B$ fit results as input for “dispersion relation” fits

The reader might wonder why have we repeatedly stressed that the claim [94, 96] that at small-$x$ skewness ratio has a conformal value (28) is excluded at LO, when this value seems to be realized in the CS scheme beyond LO. However, this is entirely related to the ambiguities present in our factorization conventions and partonic interpretations. For instance, in a DVCS scheme, where the gluons are not resolved in the hard-scattering subprocess, above LO findings at the input scale would not be essentially altered by radiative corrections, which only modify the pQCD evolution predictions.

Such an interpretation is favored among model builders and also in the GPD phenomenology of fixed target kinematics. We now utilize our LO analysis of the small-$x_B$ data for a “dispersion relation” fit in fixed target kinematics, where real part of CFF is taken to be determined by imaginary part and subtraction constant, as described in Sect. 2 (see also Ref. [64] for more sophisticated strategies and Ref. [139] for an alternative approach). This substantially reduces the model uncertainties in common GPD model description. For fixed target kinematics, where the $Q^2$ lever arm is rather limited, one may additionally rely on the so-called scaling hypothesis,
i.e., on the assumption that the GPD does not evolve under the change of the photon virtuality. The primary goal of such fits is to reveal the shape of the dominant GPD $H$ on its cross-over line ($\eta = x$) from DVCS measurements on unpolarized proton target.

The framework, as described in Sect. 2 for small $x_B$ can be easily adapted. It is beyond the scope of this paper to present a detailed description of our fits; however, we would like to demonstrate that GPD phenomenology for DVCS can be straightforwardly set up. We consider now three active quarks and write the partonic decomposition of $\Im H$ as

$$\Im H(\xi, t) = \pi \left[ \frac{4}{9} H^{u_{val}}(\xi, \xi, t) + \frac{1}{9} H^{d_{val}}(\xi, \xi, t) + \frac{2}{9} H^{sea}(\xi, \xi, t) \right],$$  \hspace{1cm} (105)$$

taken at an “input scale” of $Q^2 = 2\text{ GeV}^2$. We model the GPD on the cross-over line using the DD representation \cite{23},

$$F(x, x, t) = \frac{2}{1 + x} \int_0^1 du f \left( \frac{ux}{1 + x}, \frac{1 - 2u + x}{1 + x}, t \right),$$  \hspace{1cm} (106)$$

and take the $t$-dependence from the quark spectator model \cite{88}. This suggests the following functional form:

$$H(x, x, t) = \frac{n r}{1 + x} \left( \frac{2x}{1 + x} \right)^{-\alpha(t)} \left( \frac{1 - x}{1 + x} \right)^b \left( 1 - \frac{1}{1 + t/M^2} \right)^p.$$  \hspace{1cm} (107)$$

Here $n$ is the residue normalization of the PDF, which can be taken from PDF fits, $r$ is the skewness ratio at small $x$, $\alpha(t)$ is the “Regge trajectory”, which can be borrowed from Regge phenomenology, $b$ controls the large $x$ behavior, which according to counting rules \cite{140} should be different from that of PDFs, and both $p$ and $M$ control the $t$-dependence. The functional dependence on $t$ and $x$ in the $t$-dependent part of the ansatz is specifically motivated by the spectator model \cite{88}. The small-$x$ behavior of the sea quarks is taken from our small-$x_B$ DVCS fit at LO, evolved backwards. The corresponding GPD is described by the model (107) within the parameters

$$\alpha^{sea}(t) = 1.13 + 0.15 t/\text{GeV}^2, \quad n^{sea} = 1.5, \quad r^{sea} = 1, \quad (M^{sea})^2 = 0.5 \text{ GeV}^2, \quad p^{sea} = 2.$$  \hspace{1cm} (108)$$

For valence quarks, in accordance with our Regge-inspired modelling, which implies that we take the $\rho$ and $\omega$ trajectories, and standard PDF parameterizations, e.g., Ref. \cite{130}, we fix quark parameters to be:

$$\alpha^{val}(t) = 0.43 + 0.85 t/\text{GeV}^2, \quad n^{val} = 1.35, \quad (M^{val})^2 = 0.64 \text{ GeV}^2, \quad p^{val} = 1.$$  \hspace{1cm} (109)$$

i.e., we take a monopole ansatz with fixed cut-off mass for the residual $t$-dependence. Let us argue that our model for the GPD on the cross-over line is generically consistent with some full valence-
quark GPD model that satisfies the form factor sum rule. Our choice $p^\text{val} = 1$ is motivated by the dimensional counting rule for the Dirac form factor, which states that this form factor behaves as $(-t)^{-2}$ for $-t \to \infty$. Loosely spoken, the missing power here would arise in our GPD model from the $x$-integration for fixed $\eta$, e.g., $\eta = 0$, providing us another monopole with cut-off mass

$$\sqrt{1 - \alpha^\text{val} / \alpha^\text{val}} \approx M^\text{val} = 0.8 \text{ GeV}.$$ 

This choice is consistent with the characteristic nucleon scale and our ansatz is roughly associated with an effective dipole parameterization for the Dirac form factor with a cut-off mass of $M^\text{val} \approx 0.8 \text{ GeV}$. Such an effective parameterization of the Dirac form factor is for small $-t \lesssim 0.5 \text{ GeV}^2$ region, on which we are interested, at the 20% level consistent with the standard dipole parameterization, used above in Eq. (93).

Furthermore, we take into account the pion-pole contribution, displayed in Eq. (153) of Ref. [56], which, however, is a non-dominant contribution in DVCS. More important is the subtraction constant

$$C(t) = \frac{C}{(1 - t / (M^\text{sub})^2)^2},$$

where the normalization $C$ and dipole cut-off mass are fitting parameters. According to the “dispersion relation”, we also include this constant in the non-dominant CFF $\mathcal{E} = C(t)$, however, set its imaginary part as well as the CFF $\tilde{\mathcal{H}}$ to zero.

The five fitting parameters,

$$b^\text{sea}, \quad r^\text{val}, \quad b^\text{val}, \quad C, \quad M^\text{sub},$$

control the large- and small-$x$ behavior of the imaginary and the real part of $\mathcal{H}$. Note that by a “dispersion relation” fitting procedure one accesses even the large, experimentally inaccessible $x$ region. We also recall that the subtraction constant can be evaluated from the so-called $D$-term and it should not be confused with a so-called $J = 0$ (fixed) pole in the context of merging parton model and Regge phenomenology, see footnote 3, the discussion below Eq. (51) in Sect. 3.2 and for more details Refs. [62, 64].

In the fitting procedure we utilize the formula set of Ref. [56], which are based on $1/Q^2$ approximation for the squared amplitude. Unfortunately, this approximation is inappropriate for JLAB kinematics; still, it can be improved in the unpolarized target case by a ‘hot fix’, taken from

\footnote{Note that in popular Regge-inspired GPD models, used in DVCS phenomenology, the form factor constraint, which is already lost on generic level, is not implemented. Unfortunately, such models are sometimes employed to “constrain” quark angular momentum.}
Figure 14: Description of data by global model fits to HERMES [145] and CLAS [146] data (squares, dashed line) and additionally including the HALL A measurements (circles, solid line): (a) BCA $A^{(1)}_{CA}$ from HERMES versus $t$ (diamonds), (b) BSA $A(\phi = \pi/2)$ from CLAS versus $Q^2$ for $t \sim -0.15$ GeV$^2$ (upside triangles) and $t \sim -0.3$ GeV$^2$ (triangles), (c) helicity-dependent and (d) unpolarized differential cross section from HALL A for $x_B = 0.36$, $-t = 0.33$ GeV$^2$, and $Q^2 = 2.3$ GeV$^2$ (diamonds). Note that in (d) the cos$(2\phi)$ harmonic of the interference term is not included.

A refined evaluation for a scalar target [143]. The available fixed target data for unpolarized proton are the BCA from HERMES [144, 145], the beam spin asymmetry (BSA) from CLAS [146], and the electroproduction cross section measurements from HALL A [147]. The covered kinematics in $\xi (= x)$ is depicted below in panel (a) of Fig. 15.

Within the hypothesis that $H$ is the dominant CFF we can describe HERMES and CLAS data, where in the later case only small $-t \ll Q^2$ data were taken into account. Altogether we included 36 data points and found with

$$\chi^2/d.o.f. \approx 32/31$$

an acceptable fit, see squares in panels (a) and (b) of Fig. 14. The fit also predicts a preliminary BSA measurement from the HERMES Collaboration [148] (not shown) as well as, to some extent,
the helicity-dependent cross section\textsuperscript{20} measurement from the Hall A Collaboration, however, not the unpolarized (helicity-independent) one at \(-t = 0.33 \text{ GeV}^2\), displayed by dashed curves in panels (c) and (d) of Fig. 14 respectively. It provides the following parameter set for our GPD model:

\[ b_{\text{sea}} = 3.09, \quad r_{\text{val}} = 0.95, \quad b_{\text{val}} = 0.45, \quad C = 0.24, \quad M_{\text{sub}} = 0.5 \text{ GeV}. \]  

(111)

The \( b \) values are smaller than the corresponding \( \beta \) ones for PDFs, which is in accordance with Ref. \[140\]. The skewness ratio is smaller than the conformal one, which is in this case \( \approx 1.2 \). However, one should bear in mind that we did not include “Reggeon exchanges” in the flavor singlet sector. The parameter of \( C \) is here smaller than suggested by chiral quark soliton model \[150–153\] or lattice \[118\] calculations and the \( t \)-dependence of the subtraction constant is rather steep.

The hypothesis of the \( H \)-dominance is within our model (107) no longer valid for the cross section measurements of HALL A [147], performed at large \( x_B = 0.36, Q^2 = 2.3 \text{ GeV}^2 \), and for four \( t \) values,

\[ -t = \{ 0.17 \text{ GeV}^2, 0.23 \text{ GeV}^2, 0.28 \text{ GeV}^2, 0.33 \text{ GeV}^2 \} . \]

To describe the full set of these HALL A data, it is required that the real part of the DVCS amplitude steeply varies in this \( t \) interval and becomes large at \( -t = 0.33 \text{ GeV}^2 \). Both of these features are not implemented in common GPD models [154, 155]. We will now explore whether these features of the real part arise from the inclusion of the GPD \( \tilde{H} \). To be specific, we take a model for the CFF

\[ \Im \tilde{H}(\xi, t) = \pi \left( \frac{4}{9} + \frac{1}{9} \right) \frac{\tilde{n}}{1 + \xi} \left( \frac{2 \xi}{1 + \xi} \right)^{-n_{\text{val}}(t)} \frac{1}{1 - \frac{1 - \xi}{1 + \xi} t} \left( \frac{1 - \xi}{1 + \xi} \right)^{3/2} , \]

where for simplicity we employ the parameter set (109) and fixed \( b = 3/2 \). Hence, only the normalization \( \tilde{n} \) appears as a new fitting parameter. If we take this parameter of order one, our ansatz is to some extent consistent with standard GPD models.

In our second global fit we take the DVCS data set from above and include four values of the azimuthal angle asymmetry

\[ \int_0^{2\pi} dw \cos(\phi) \frac{d^4\sigma(\phi, \cdots)}{d\phi dx_B dt dQ^2} / \int_0^{2\pi} dw \frac{d^4\sigma(\phi, \cdots)}{d\phi dx_B dt dQ^2} , \]

which are obtained from the four unpolarized cross section measurements of the HALL A Collaboration. Here, the integral measure \( dw \), defined in Eq. (103) of Ref. [56], compensates the \( \phi \)

\textsuperscript{20}We adopt here the terminology of HALL A [147], where this observable is defined as half of the cross section difference for positive and negative electron beam helicity.
dependence of the Bethe-Heitler propagator product. In other words, in our fit only the relative changes of the unpolarized cross section with respect to a \( \cos \phi \) modulation, dominated on amplitude level by twist-two CFFs, is essential. Within such a strategy the absolute normalization of the cross section is irrelevant.

Compared to the previous global DVCS fit, this provides us four data points and one degree of freedom (normalization of \( \tilde{H} \)) more. The quality of fit

\[
\chi^2 / \text{d.o.f.} \approx 33 / 34
\]

is good and the parameters for the GPD \( H \) are

\[
l_{\text{sea}} = 4.6, \quad r_{\text{val}} = 1.11, \quad b_{\text{val}} = 2.4, \quad C = 6.0, \quad M_{\text{sub}} = 1.5 \text{ GeV}.
\]

(112)

The value of the subtraction constant agrees now qualitatively with the original chiral quark soliton model estimate [150], quoted as \( C \approx 5 \). For the normalization of \( \tilde{H} \) we find the value \( \tilde{n} \approx 3 \). This corresponds to a huge skewness effect \( r_{\tilde{H}} \sim 5 \). We consider this finding as an effective parameterization of some GPD contribution that reveals our lack of partonic understanding of the unpolarized cross section measurement of the HALL A Collaboration. We also like to stress that the real part of the DVCS amplitude at larger values of \( x_B \) is an intricate quantity that cannot be simply related to a specific GPD property. Note also that a closer look reveals that essentially only the HALL A data point at \(-t = 0.33 \text{ GeV}^2\) forces us to explore the scenario of non-standard GPD modelling.

In this way, our model (112) describes present DVCS data, including the helicity-dependent cross section [147] and preliminary BSA [148] data which have not been employed in the fits. This is exemplified by the circles and the solid curves in Fig. 14. Note that in panel (d) the deviation of our fit result from the data points might be attributed to a \( \cos(2\phi) \) harmonic in the interference term. However, the extraction of GPDs from a DVCS measurement is highly nontrivial and we relied here on model assumptions and employed a simple least square fit. Thus, we like to spell out a clear warning, namely, the result we presented does not exclude a successful description of the data within a rather different set of parameters:

- a literal interpretation of these (first) model-dependent fit results is not appropriate.

Nevertheless, the parameters (112) look rather reasonable from the generic point of view, as stated above, and they might be used to set up an \( H \) GPD model in any favored representation.

The resulting GPDs on the cross-over line are shown in the left panel of Fig. 15. It is obvious that the two different fits provide results which are rather different in the large-\( x \) region, but approach each other at smaller \( x \). Essentially, large difference for JLAB kinematics simply
Figure 15: (a) Global GPD $H$ fits \[111\] and \[112\] at $t = -0.3\text{GeV}^2$ (thick) and $t = 0$ (thin) are displayed as dashed and solid lines, respectively. (b) Prediction of the BCSA asymmetry \[113\] for COMPASS kinematics ($E_\mu = 160\text{GeV}$, $Q^2 = 2\text{GeV}^2$, $t = -0.2\text{GeV}^2$) versus $\varphi = \pi - \phi$.

indicates that BSA measurements can be described with two qualitatively different GPD model scenarios, where the dashed curve represents the common GPD models with a, let us say, moderate DVCS amplitude, while the solid curve belongs to GPD models that provide an enhanced DVCS amplitude, here induced by $\tilde{H}$. Whether this is a realistic scenario is a open problem, which should be addressed in future studies.

We can now employ our model fits to deliver a prediction for the COMPASS experiment. In such a fixed target experiment one would scatter positively or negatively charged muons with helicity $-\frac{1}{2}$ and $+\frac{1}{2}$ on a proton target. The preferred observable is the beam charge-spin asymmetry (BCSA),

$$A_{\text{BCSA}}(\phi) = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}},$$

which is essentially related to the real part of the interference term. It is therefore rather sensitive to details of the spectral function $\Im m H$. We recall that the sign of the BCA and, thus, also of the real part of the CFF $H$ changes somewhere between HERMES and H1/ZEUS kinematics. Knowing the position of this zero in dependence on the kinematical variables would be crucial for pinning down GPDs. For the mean values at COMPASS, we expect a sizeable asymmetry, the sign of which is fixed by valence-like quarks (or “Reggeon exchange”), where the subtraction constant (fixed pole) plays a role, too. We display our predictions resulting from our two fits for the BCSA \[113\] in the right panel of Fig. 15 versus the azimuthal angle $\varphi = \pi - \phi$ (i.e., within a so-called Trento convention). Therefore, the COMPASS kinematics $x \sim [10^{-2}, 10^{-1}]$ is certainly well suited to explore, in the LO DVCS interpretation, the transition area between sea and valence-like quarks domination. In the NLO interpretation this translates into the hope of
exploring the interplay between quarks and gluons.

6 Summary and conclusions

In this paper we demonstrated that for the description of small-$x_B$ DVCS data the choice of representation, used to set up GPD models, does not matter too much. We employed different “languages” and found that, although explicit transformation formulae are only partly available, one can easily translate main results. The problem remains always the same, namely, to find a flexible parameterization of the skewness function and the skewness ratio. As a side remark, we explained that the small-$x$ claim, stating that the GPD is rigidly tied to the corresponding PDF by the conformal skewness ratio, is based on unjustified mathematical assumptions. Moreover, we demonstrated in detail that this claim is ruled out at LO. We also revealed that common gluon GPD models (either RDDA, $t$-decorated PDF, leading PW or minimalist “dual” model) possess the conformal ratio. We showed then that this is the very feature which is responsible for the failure of previous attempts to describe DVCS at LO, even if the quark GPD model is flexible. The “workaround recipe” to take NLO gluon PDFs in an LO description fails for our PDF parameterizations in DVCS. It also creates an obvious inconsistency because the same amplitude is described differently in DVCS than in its DIS limit.

We introduced two flexible GPD parameterizations with respect to the skewness ratio, namely, one model was set up by adding a next-leading SO(3) PW and the other one by a model-dependent resummation of SO(3) PWs. Thereby, we relied on the simplest assumptions, e.g., (almost) decoupled skewness and $t$-dependence, and dressed the gluon GPD with the $t$-dependence from the $J/\Psi$ production analysis. However, it turned out that these two models are rather similar and possess a rigid parametrization of the skewness function. Nevertheless, within these Regge-inspired models we could describe small-$x_B$ DVCS cross section data from the H1/ZEUS Collaborations at LO and beyond. Thereby, the gluonic LO GPD on its cross-over line becomes negative at lower values of $Q^2$ and, moreover, we had a rather large reparameterization of the skewness effect at NLO for both the quark and gluon GPDs. Both findings are not expected and we consider them as model artifacts. So we conclude that the effective nl-PW, or the minimal “dual”, or any similar model is not flexible enough to control the evolution over a large lever arm in $Q^2$, and so also the control of the skewness ratio at larger values of $Q^2$ is lost. Also, the fact that at LO the skewness and $t$-dependence are still correlated in our model fits reveals that an effective nl-SO(3) PW parameterization still suffers from rigidity.

We have demonstrated that at NLO the abovementioned small-$x$ claim within the HERA-II DVCS data also does not necessarily hold true. Within a dipole $t$-ansatz, however, we have
confirmed our previous results that claim is to a great extent valid in the CS scheme, at NLO and NNLO. Thereby, we observed very small NNLO corrections, namely, below 1%. This might indicate that the ‘holographic’ principle, which ties GPD and PDF, arises from a broken SO(2,1) symmetry. We stress that we do not consider such ‘holographic’ principle reliable enough to employ it in a fitting procedure, e.g., to reveal the $t$-dependence of GPDs.

We utilized our flexible models to deliver the transverse distribution of quarks and gluons and found that they are quite robust with respect to the change of perturbative order and scheme conventions. We only observed a slight difference between LO and beyond LO results, which is attributed to the different fitting strategies and the remaining rigidity of our models. The results moderately differ from our previous findings, where we employed the l-PW model (conforming to the small-$x$ claim). In contrast to those findings, we observed now that within our flexible GPD models there is no significant difference between quark and gluon transverse width. The perturbative prediction that a partonic shrinkage effect should be washed out at larger scale $Q^2 \sim 10 \text{ GeV}^2$ is in agreement with experimental findings, however, its existence at the lower scale cannot be excluded. We employed a three-step fitting procedure to pin down the $t$-dependence of the GPD. Within an exponential $t$-dependence GPD model there is no shrinkage effect and the partonic transverse width is about $\sim 0.6 - 0.65$ fm. This value is compatible with the one extracted from the $J/\Psi$ photoproduction and with the radius of the proton disc. A dipole $t$-dependence ansatz is accompanied with a small shrinkage effect and the partonic transverse width is now $0.75 - 0.8$ fm. The quoted difference of widths arises essentially from the different extrapolation of the measured $t$-interval to $t = 0$ and essentially provides an uncertainty in the long range tail of the profile function in impact space.

Based on the DVCS cross section fits, we refined a previous model prediction for the beam charge asymmetry. Our prediction is compatible with preliminary measurements from the H1 Collaboration, which supports our Regge-inspired GPD modelling. Unfortunately, both theoretical uncertainties and experimental errors do not allow us to access the chromomagnetic “pomeron” and so neither the anomalous gravitomagnetic moment (or angular momentum) of sea quarks. An immense reduction of both sources of error is needed before one might be able to address these questions. An improvement of the experimental result might lead to some clarification of the presence of a third azimuthal angular harmonic, induced by gluon transversity.

Based on the double distribution representation, we have also built a simple model for the GPD at the cross-over line, where we implemented the small-$x$ behavior, extracted from H1/ZEUS data. Using this model and relying on the scaling hypothesis, we presented a first “dispersion relation” fit to observables for fixed target DVCS experiments on unpolarized proton target. Assuming some unexpected properties of $\tilde{H}$, we were able to describe all available small $-t$ data in a first
global fit. The parameters we found match our generic expectations coming from Regge behavior, large-\(x\) counting, and quark soliton model estimates. Utilizing this GPD as an input, we predict beam charge-spin asymmetry as measurable at COMPASS. Thereby, it becomes obvious that COMPASS is needed to reveal the GPD \(H\) further. This experiment might also be important for pinning down the transverse distribution of partons in the transition region between “pomeron” dominance and “Reggeon” behavior.

Let us finally emphasize that one can have different partonic interpretations of our findings. Namely, one can either state that gluons are very important in DVCS and should be perturbatively resolved or one sticks to the quark picture (DVCS scheme) in which gluons only enter the evolution equation. This is a matter of taste that so far simply provides two qualitatively different GPD parameterizations and also points in two different directions for future DVCS studies. We emphasize that in any case an LO description is a convenient starting point for the phenomenological description of the growing amount of experimental data. It offers the possibility to pin down GPD models within rather straightforward fitting strategies. Such models are phenomenologically valuable for making contact with dynamical models and lattice simulations, since in the latter case matching of lattice and perturbative renormalization is for GPD moments only done at LO.

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Note added: After our manuscript was finalized, we have noticed the new DVCS measurements from the ZEUS Collaboration [156]. We have convinced ourselves that these data will only mildly influence our fits and, thus, we do not update here our results. In other words, we might view our findings as predictions for the new ZEUS data. After the experimental DVCS analysis of HERA II run will be finished, we plan to include them in a forthcoming update.
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sea quarks

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