Optomechanically induced transparency with nonlinear effect

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(Dated: December 24, 2019)

Optomechanically induced transparency is an important quantum phenomenon in cavity optomechanics. Here, we study the properties of optomechanically induced transparency in the simplest optomechanical system (consisting of one cavity and one mechanical resonator) considering the nonlinear effect that was ignored in previous works. With the nonlinear effect, we find the ideal optomechanically induced transparency dip can be easily achieved, and the width of optomechanically induced transparency dip can become very narrow especially in unresolved sideband regime. Finally, we study the properties of optomechanically induced gain, and give the analytic expression about the maximum value of gain.

PACS numbers: 42.50.Gy, 03.65.Ta, 42.50.Wk

I. INTRODUCTION

Cavity optomechanics [1] exploring the interaction between macroscopic mechanical resonators and light fields, has received increasing attention for the broad applications in testing macroscopic quantum physics, high-precision measurements, and quantum information processing [1–5]. Various experimental systems existing such interactions are proposed and investigated, such as Fabry-Perot cavities [6, 7], whispering-gallery microcavities [8, 10], membranes [11–14], and superconducting circuits [15, 16]. In these optomechanical systems, the motion of mechanical oscillator can be effected by the radiation pressure of cavity field, and this interaction can generate various quantum phenomena, such as ground-state cooling of mechanical modes [17–21], quantum entanglement [22–27], nonclassical mechanical states [28, 29], normal mode splitting [30–32], and nonreciprocal optical transmissions [33–35], etc.

Optomechanically induced transparency (OMIT) is an interesting and important phenomenon. It was theoretically predicted by Agarwal and Huang [36] and experimentally observed in a microrod system [37], a superconducting circuit cavity optomechanical system [10], and a membrane-in-the-middle system [14]. More recently, the study of OMIT has attracted much attentions [38–60]. For instance, Huang studied OMIT in a quadratically coupled optomechanical systems where two-phonon processes occur [49]. Jing et al. studied OMIT in a parity-time symmetric microcavity with a tunable gain-to-loss ratio [50]. Lü et al. studied OMIT in a spinning optomechanical system [51], and also studied OMIT at exceptional points [52]. Ma et al. studied OMIT in the mechanical-mode splitting regime [53]. Dong et al. studied the transient phenomenon of OMIT [54] and the Brillouin scattering induced transparency in a high-quality whispering-gallery-mode optical microresonator [55]. Ma et al. studied tunable double OMIT in a hybrid optomechanical system with Coulomb coupling [56]. Kronwald et al. studied OMIT in the nonlinear quantum regime [57]. Xiong et al. studied OMIT in higher-order sidebands [58], and the review articles on OMIT can be found in Refs. [53, 60].

The most prominent application of OMIT is light delay and storage [41, 47, 59] due to the abnormal dispersion accompanied with the narrow transparency window. Hence, having both a large depth and a small width at the transparency window is important for OMIT. However, increasing the power of the control field can lead to the increase of transparency depth, meanwhile, the width of the transparency window also increases. In addition, the ideal depth of the transparency window cannot be achieved due to the nonzero mechanical damping rate. While these problems can be resolved if we consider a nonlinear effect in the response of the optomechanical system to the probe field.

In this paper, we mainly study OMIT in the simplest optomechanical model, described in Fig. (1), considering a nonlinear effect which was ignored in previous works. The Hamiltonian of the system is nonlinear and we can solve the nonlinear Heisenberg-Langevin equations using the perturbation method due to the probe field much weaker than the driving field. Note that if we linearize the nonlinear Heisenberg-Langevin equations as usual linearization procedure [17, 18, 27], then the key nonlinear term will not exist in the response of the optomechanical system to the probe field. Considering the nonlinear term, we obtain the conditions for OMIT and find it has a strong impact on the absorptive and dispersive behavior of the optomechanical system to the probe field. First, the ideal depth of OMIT can be achieved easily even with nonzero mechanical damping rate, and there is only one suitable driving strength that can make the ideal OMIT occur. Secondly, the width of the transparency window depends only on three parameters of the system, and can become very narrow for small mechanical damping rate especially in unresolved sideband regime. And thirdly, if the driving strength continues to increase, the system will exhibit optomechanically induced gain, and the gain will become very large in unresolved sideband regime.

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II. SYSTEM AND EQUATIONS

We consider the standard optomechanical model in which one cavity with length $L$ and frequency $\omega_0$ is coupled to a mechanical resonator with frequency $\omega_m$ via radiation pressure effects (see Fig. 1). The cavity annihilation (creation) operator is denoted by $c (c^\dagger)$ with the commutation relation $[c, c^\dagger] = 1$. Momentum and position operators of the membrane oscillator with mass $m$ are represented by $p$ and $q$, respectively. The cavity is driven by a strong coupling field with frequency $\omega_c$ (amplitude $\varepsilon_c$) and a weak probe field with frequency $\omega_p$ (amplitude $\varepsilon_p$). We define $\nu_c$ and $\nu_p$ as the input powers of relevant fields and $\kappa$ as the cavity decay rate, then the field amplitudes can be described as $\varepsilon_c = \sqrt{2\kappa\nu_c/(\hbar\omega_c)}$ and $\varepsilon_p = \sqrt{2\kappa\nu_p/(\hbar\omega_p)}$. The optomechanical coupling between the cavity field and the movable resonator can be described by interaction Hamiltonian $-\chi_0 c^\dagger c q$ with $\chi_0 = \hbar\omega_0/L$ being the optomechanical coupling constant. Thus, the Hamiltonian of the system can be written as

$$H = \hbar\omega_0 c^\dagger c + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + i\hbar\varepsilon_c (c^\dagger e^{-i\omega_c t} - ce^{i\omega_c t})$$
$$+ i\hbar (c^\dagger \varepsilon_pe^{-i\omega_p t} - c\varepsilon_pe^{i\omega_p t}) - \chi_0 c^\dagger c q. \quad (1)$$

In this paper, we deal with the mean response of the system to the probe field in the presence of the coupling field, hence we do not include quantum fluctuations. We use the factorization assumption $\langle qc \rangle = \langle q \rangle \langle c \rangle$ and also transform the cavity field to a rotating frame at the frequency $\omega_c$, the mean value equations are then given by

$$\langle \dot{q} \rangle = \frac{\langle p \rangle}{m}, \quad (2)$$
$$\langle \dot{p} \rangle = -m\omega_m^2\langle q \rangle + \chi_0 \langle c^\dagger \rangle \langle c \rangle - \gamma_m \langle p \rangle,$$
$$\langle \dot{c} \rangle = -[\kappa + i(\omega_0 - \omega_c - \chi_0 \langle q \rangle/\hbar)] \langle c \rangle + \varepsilon_c + \varepsilon_p e^{-i\delta t}.$$  

Here, $\delta = \omega_p - \omega_c$ is the detuning between probe field and coupling field. Equations (2) are nonlinear, and therefore some of nonlinear effects will be omitted if we linearize Eq. (2) by the usual linearization method. Considering that the probe field $\varepsilon_p$ is much weaker than the coupling field $\varepsilon_c$, we can solve Eq. (2) by the perturbation method attaining its steady-state solutions just to the first order in $\varepsilon_p$, i.e., $\langle s \rangle = s_0 + \varepsilon_p e^{-i\delta t} s_+ + \varepsilon_p^* e^{i\delta t} s_- \quad (s = q, p, c)$. The solutions, such as $s_0 = \varepsilon_c/(\kappa + i\Delta)$ with the effective detuning $\Delta = \omega_0 - \omega_c - \chi_0 \langle q \rangle/\hbar$, can be easily obtained. We will not list them one by one, because here we just care about the field with frequency $\omega_p$ in the output field.

According the input-output relation [36, 61], the quadrature of the optical components with frequency $\omega_p$ in the output field can be defined as $\varepsilon_T = 2\kappa\varepsilon_p/\kappa - ix + \beta$. The real part $\text{Re}[\varepsilon_T]$ and imaginary part $\text{Im}[\varepsilon_T]$ represent the absorptive and dispersive behavior of the optomechanical system to the probe field, respectively. Because it is known that the coupling between the cavity and the resonator is strong at the near-resonant frequency, here we consider $\delta \sim \Delta \sim \omega_m$ and set $x = \delta - \omega_m$ in the following. In this paper, we focus on the most studied regime of optomechanics where $\gamma \ll \omega_m, \kappa$. In this case, the result of $\varepsilon_T$ can be obtained as

$$\varepsilon_T = \frac{2\kappa}{\kappa - ix + \beta} \quad (3)$$

where

$$N = -\frac{\beta}{\kappa - 2i\omega_m}, \quad (4)$$
$$\beta = \frac{\chi_0}{2m\omega_m\hbar(\kappa^2 + \omega_m^2)}. \quad (5)$$

The term $N$ is the key term which will not exist in the subfracion of Eq. (3) if we adopt the usual linearization method to solve Eq. (2). Hence, the effects of this term should be nonlinear effects and we call $N$ the nonlinear term in the following.

With the nonlinear term, the conditions of ideal OMIT dip can be easily obtained. It can be obviously seen from Eq. (3) that the location of the pole in the subfracion of Eq. (3) can give the conditions. According to the location of the pole, setting $\frac{1}{2} - ix + N = 0$, the conditions can be obtained as

$$x = x_o \equiv \frac{\gamma\omega_m}{\kappa}, \quad (6)$$
$$\beta = \beta_o \equiv \frac{\gamma(\kappa^2 + 4\omega_m^2)}{2\kappa}. \quad (7)$$

Equation (6) gives the concrete location $x_o$ where the ideal optomechanically induced transparency dip appears, and Equation (7) gives the suitable $\beta_o$ (corresponding to the suitable amplitude $\varepsilon_c$ of coupling field according to Eq. (5)) that can make the ideal OMIT dip occur. The depth of OMIT dip will become shallow when $\beta < \beta_o$, and the optomechanically induced gain will occur when $\beta > \beta_o$. Next, we first study the properties of OMIT with $\beta = \beta_o$ and then the properties of optomechanically induced gain with $\beta > \beta_o$. For the sake of certainty, we set the mechanical quality factor as $Q = \omega_m/\gamma = 10^4$ in the following.
III. OPTOMECHANICALLY INDUCED TRANSPARENCY

From the above analysis, the natures of OMIT are determined by only three parameters, i.e., $\gamma$, $\omega_m$, and $\kappa$. The width $\Gamma_{\text{OMIT}}$ (full width at half maximum) of the transparency window is an important index in OMIT, and the analytical expression of the width $\Gamma_{\text{OMIT}}$ can be obtained as

$$
\Gamma_{\text{OMIT}} = \gamma(1 + \frac{4\omega_m^2}{\kappa^2}).
$$

(8)

According to Eqs. (7) and (8), when optomechanically induced transparency occurs, we have

$$
\Gamma_{\text{OMIT}} = \frac{2\beta_o}{\kappa}.
$$

(9)

Note that Eqs. (8) and (9) are true for both resolved sideband and unresolved sideband regime. We will discuss the properties of OMIT in the case of resolved sideband and unresolved sideband regime respectively in the following.

A. Resolved sideband regime

In the resolved sideband regime, i.e., $\kappa \ll \omega_m$, The width $\Gamma_{\text{OMIT}}$ in Eq. (8) will become

$$
\Gamma_{\text{OMIT}} = \frac{4\gamma\omega_m^2}{\kappa^2}.
$$

(10)

It can be seen from Eq. (10) that the width $\Gamma_{\text{OMIT}}$ is much larger than mechanical damping rate $\gamma$ in resolved sideband regime. In Fig. (2), we plot the real part of $\varepsilon_T$ vs. the normalized frequency detuning $x/\gamma$ with $\beta = \beta_o$ according to Eq. (7) and with resolved sideband parameters $\omega_m = 5\kappa$ which are similar to those in an optomechanical experiment on the observation of the normal-mode splitting [30]. According to Eq. (10), the width $\Gamma_{\text{OMIT}} = 100\gamma$ for $\omega_m = 5\kappa$, which shows an excellent agreement with the numerical result in Fig. (2).

The dispersive behavior, represented by $\text{Im}[\varepsilon_T]$, is related to slow light effects of the optomechanical system to the probe field. In Fig. (3), we plot the imaginary part of $\varepsilon_T$ vs. the normalized frequency detuning $x/\gamma$ with $\beta = \beta_o$ according to Eq. (7) and with the same parameters in Fig. (2). It can be seen from Fig. (3) that the steepest dispersion occurs at the point $x_o$ where the OMIT dip appears. The expression of dispersion curve slope is too long to be reported here, but the negative maximum value of the dispersion curve slope can be obtained as

$$
K_{\text{max}} = -\frac{4\kappa^2}{\gamma(\kappa^2 + 4\omega_m^2)}.
$$

(11)

Note that Eq. (11) is true for both resolved sideband and unresolved sideband regime. From Eq. (8) and (11), we have

$$
K_{\text{max}} \times \Gamma_{\text{OMIT}} = -4
$$

(12)

FIG. 2: (Color online) The real part of $\varepsilon_T$ vs. normalized frequency detuning $x/\gamma$ with $\beta = \beta_o$ according to Eq. (7) and with resolved sideband parameters $\omega_m = 5\kappa$.

FIG. 3: (Color online) The imaginary part of $\varepsilon_T$ vs. normalized frequency detuning $x/\gamma$ with the same parameters in Fig. (2).

which means that the narrower the width $\Gamma_{\text{OMIT}}$ is, the steeper the dispersion curve becomes.

B. Unresolved sideband regime

Compared with the case of resolved sideband regime, according to Eq. (8), the width $\Gamma_{\text{OMIT}}$ will become more narrower in unresolved sideband regime. In Fig. (4), we plot the real part of $\varepsilon_T$ vs. the normalized frequency detuning $x/\gamma$ with $\beta = \beta_o$ according to Eq. (7) and with unresolved sideband parameters $\kappa = 2\omega_m$ (blue line) and $\kappa = 5\omega_m$ (red line). According to Eq. (8), the width $\Gamma_{\text{OMIT}} = 2\gamma$ for $\kappa = 2\omega_m$ and $\Gamma_{\text{OMIT}} = 1.16\gamma$ for $\kappa = 5\omega_m$. These results are consistent with the numerical results in Fig. (4).

The imaginary part of $\varepsilon_T$ vs. normalized frequency detuning $x/\gamma$ is plotted in Fig. (5) with the same parameters in Fig. (4). From Fig. (5), it can be clearly seen that the dispersion curve becomes steeper with larger ratio of $\kappa/\omega_m$ and the negative maximum value of the dispersion...
FIG. 4: (Color online) The real part of $\varepsilon_T$ vs. normalized frequency detuning $x/\gamma$ with $\beta = \beta_o$ according to Eq. (7) and with unresolved sideband parameters $\kappa = 2\omega_m$ (blue line), $\kappa = 5\omega_m$ (red line).

FIG. 5: (Color online) The imaginary part of $\varepsilon_T$ vs. normalized frequency detuning $x/\gamma$. The parameters are the same in Fig. (4).

curve slope is still given by Eq. (11).

IV. OPTOMECHANICALLY INDUCED GAIN

From the above analysis, the phenomenon of optomechanically induced gain ($\text{Re}[\varepsilon_T] < 0$) will occur when $\beta > \beta_o$. It means that OMIT can still occur at some points where $\text{Re}[\varepsilon_T] = 0$. However, we are not going to discuss such OMIT because the dispersion curve has no specificity at the points.

When $\beta > \beta_o$, $\text{Re}[\varepsilon_T]$ will take negative value, and the maximum negative value occurs at the point

$$x = x_g \equiv -\frac{2\beta\omega_m}{\kappa^2 + 4\omega_m^2}.$$  

However, the gain does not always increase with the increase of $\beta$. The reason is that the negative value of $\text{Re}[\varepsilon_T]$ approaches zero as $\beta \to \infty$ according to Eq. (3).

In Fig. (6), we plot $\text{Re}[\varepsilon_T]$ vs. $\beta/\kappa^2$ with $x = x_g$ according to Eq. (13) and with parameter $\kappa = 4\omega_m$. It can be clearly seen from Fig. (6) that there exist an optimum value $\beta_g$, defined as $\beta_g$, that makes $\text{Re}[\varepsilon_T]$ take the maximum negative value $\mathcal{G}_{\text{max}}$.

With Eq. (3) and Eq. (13), the numerical result of the optimum value $\beta_g$ and the corresponding maximum negative value $\mathcal{G}_{\text{max}}$ can be easily found out. However, the approximate analytic expression of $\beta_g$ and $\mathcal{G}_{\text{max}}$ can be obtained as

$$\beta_g = \frac{(\kappa^2 + 4\omega_m^2)\sqrt{\gamma}/\omega_m}{2},$$  
$$\mathcal{G}_{\text{max}} = -\frac{2\kappa^2}{4\omega_m^2 + \kappa\sqrt{\gamma}\omega_m},$$

if $\kappa \gg \sqrt{\gamma}\omega_m$.

According to Eq. (15), we have the maximum negative value $\mathcal{G}_{\text{max}} \approx -\kappa^2/2\omega_m^2 \ll 1$ in resolved sideband regime. However, the maximum negative value $\mathcal{G}_{\text{max}}$ can become
very large in unresolved sideband regime. In Fig. (7), we plot the real part of $\varepsilon_T$ vs. the normalized frequency detuning $\varepsilon/\gamma$ with $\beta = \beta_g$ according to Eq. (14) and with $\kappa = \omega_m$ (black line), $\kappa = 2\omega_m$ (blue line), and $\kappa = 4\omega_m$ (red line). According to Eq. (15), the maximum negative value $G_{\max} = -1.990$ for $\kappa = 2\omega_m$ and $G_{\max} = -7.921$ for $\kappa = 4\omega_m$. These results are consistent with the numerical results in Fig. (7).

V. CONCLUSIONS

In summary, we have theoretically studied the properties of optomechanically induced transparency in the simplest optomechanical system (consisting of one cavity and one mechanical resonator) with nonlinear effect that was ignored in previous works. We attain the conditions where the system can exhibit perfect optomechanically induced transparency, and obtain the expression of the width of optomechanically induced transparency dip. From these crucial expressions, we can draw three important conclusions: (1) there exist only one suitable driving strength that can make the ideal optomechanically induced transparency dip occur, and the properties of optomechanically induced transparency are determined by only three system parameters ($\gamma$, $\kappa$ and $\omega_m$): (2) the width of optomechanically induced transparency dip can become very narrow in unresolved sideband regime, and the product of the width and the dispersion slope at the transparency window is a constant; (3) the maximum value of optomechanically induced gain is very small in resolved sideband regime, while it can become very large in unresolved sideband regime. We believe these results can be used to control optical transmission in quantum information processing.

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