Impact characteristics of railway bridge with double-block supported track considering carriage oscillation loads

Yuchen Wang¹, Zhi Sun²

¹ Department of Bridge Engineering, Tongji University, Shanghai, China, wangyuchen_5698@126.com;  
² State Key Laboratory on Disaster Reduction in Civil Engineering, Shanghai, China, sunzhi1@tongji.edu.cn.

Abstract. In this study, the impact characteristics of a typical railway bridge with ballastless double-block supported track under moving train considering carriage oscillation load is investigated. The train load is modelled as a constant force superimposed with a harmonic force whose frequency is the heaving oscillation frequency of the carriage. The bridge-track system is discretized and modelled using the double-beam elements connected using the elastic springs and viscous dampers. Numerical computations are conducted to investigate the bending moment impact characteristics of the supporting bridge considering the variations of moving speed and oscillating frequency of the moving load. The result shows that the harmonic loads due to the carriage heaving oscillation will induce some important bending moment impact amplification and peak response offset for the studied bridge in operation.

1. Introduction
The double-block supported track with no ballast and sleepers is generally adopted as the track type for railway bridges. The double-beam model is a reasonable idealization of this bridge-track system under the moving train load. For the double-beam structure systems, they are generally of two category of vertical bending modes, the in-phase modes and anti-phase breath modes. The anti-phase breath modes may cause excessive deformation and damage of the fastener in the railway bridge, which is worthy of study.

For the train load, although the moving concentrated force sequence is the main component. The harmonic loads due to the carriage oscillation may also induce some important impact amplification. Some related analytical works have been reported. Onisyczuk [1] presented some analytical expressions for the undamped free and forced vibrations of an elastically connected double-beam system. Vu [2] presented an exact method for solving the vibration of a double-beam system subject to harmonic excitation, and pointed out that, to decouple the governing partial differential equations, the flexural rigidity and masses of the beams must be identical, and the boundary condition of the same side of the system must be the same. Gurgoze and Erol [3] presented a method of obtaining the exact solution for the forced vibrations of elastic rods coupled by distributed spring and damper. Abu-Hilal [4] investigated the dynamics response for a double-beam system resting on a continuously viscoelastic foundation due to moving constant load. Palmeri and Adhikari [5] investigated the transverse vibrations of two elastic beams continuously joined by an inner viscoelastic layer. As an example of multiple-layer beam problem, Kelly and Srinivas [6] developed a general theory for the determination of natural frequencies and mode shapes for a set of elastically connected axially loaded
beams. Wu and Gao [7] derived the analytical solution for simply supported viscously damped double-beam system under moving harmonic loads. In their study, the double beams considered is of the same cross-section properties.

In this paper, the impact characteristics of a typical railway bridge with ballastless double-block supported track under moving train considering the carriage oscillation load is investigated. The bridge-track system is discretized and modelled using the double-beam elements connected using the elastic springs and viscous dampers. Numerical integrations are conducted to compute bridge and fastener response impact characteristics for varying operation working conditions.

2. Modelling
For the studied railway bridge with ballastless double-block supported track, the double-beam structure system model can be adopted for a simplified bridge response analysis. The governing equation of motion is:

\[
\begin{align*}
E_I \frac{\partial^4 y_1}{\partial x^4} + \rho A \frac{\partial^2 y_1}{\partial t^2} + C \frac{\partial(y_1 - y_2)}{\partial t} + K(y_1 - y_2) &= \sum_{i=1}^{N_o} F_i \delta(x - vt_i) \\
\alpha E_I \frac{\partial^4 y_2}{\partial x^4} + \beta \rho A \frac{\partial^2 y_2}{\partial t^2} + C \frac{\partial(y_2 - y_1)}{\partial t} + K(y_2 - y_1) &= 0
\end{align*}
\]

in which \(E_I\) is the flexural rigidity of the track; \(\rho A\) is the line mass density of the track; \(\alpha\) is the flexural rigidity ratio of the bridge and the track; \(\beta\) is the line density ratio of the bridge and the track; \(K\) is the stiffness per meter of the connecting block and fastener; \(C\) is the damping per meter of the connecting block and fastener; \(y_i\) is the abbreviation of \(y_i(x, t)\) which denotes the vertical displacement of the \(i\)-th beam from the left end \(x\) meters at time \(t\); \(F_i\) is the \(i\)-th axle load of the moving train.

![Figure 1. Mechanical graph for a railway bridge with double-block supported track carrying moving train carriage](image)

Figure 2 shows the simplified load model of the train carriage. As shown, the carriage has four axles and two bogies. The bogie spacing is 17.38 m. The axle spacing is 2.5 m. The carriage length is 24.78 m. The vehicle mass parameters and heaving mode frequency are listed in Table 2. In this load model, each axle load considered is the summation of a concentrated force and an oscillating harmonic force. The constant concentrated force is used to model the force produced by the train weight acting on the rail, while the harmonic load is used to model the additional force caused by the heaving motion of the train carriage during the train running. The frequency of the harmonic load is the modal frequency of the heaving motion of the vehicle. The magnitude of the harmonic force is the multiplication of the carriage mass with its vertical acceleration. According to the evaluation criteria of passenger vibration comfort in railway vehicles (ISO2631), the maximum acceleration which the passenger can bear is 0.1g. It can be estimated according to the parameters in table 1 that the amplitude of the harmonic force accounts for 7.18% of the constant force.
Table 1. Vehicle parameters considered

| Parameter | Value     |
|-----------|-----------|
| Heaving freq. (Hz) | 1.73     |
| Mass of car body (t) | 39.755   |
| Mass of bogie (t)   | 3.490    |
| Mass of wheel axle (t) | 2.155   |

Figure 2. The simplified single carriage load model

For simplification, the following dimensionless parameters are defined:

\[
\tilde{t} = \frac{vt}{L}, \quad \tilde{y}_i = \frac{y_i(t)}{y_0} (i = 1,2), \quad \tilde{\omega} = \frac{\omega}{\omega_1}, \quad \tilde{v} = \frac{v}{v_{cr}}
\]

where \(\tilde{t}, \tilde{y}_i, \tilde{\omega}, \tilde{v}\) are the dimensionless time, displacement, oscillation frequency and moving speed of the harmonic load respectively; \(y_0\) is the peak static deflection at the mid-span of the beam when the four axle forces move on the beam; \(\omega_1\) is the fundamental frequency of the bridge; \(v_{cr} = \omega_1 L / (2\pi)\); \(\omega\) and \(v\) are the harmonic oscillation frequency of load and speed of load respectively.

3. Bridge impact factors computation

Numerical case studies are conducted to investigate the impact factors of a typical 24-meter-long railway bridge under the moving train harmonic load considering the variation of the moving speed and oscillating frequency. The cross-section properties of the rail and the bridge section are shown in Table 2. According to these parameters in Table 1 and Young’s modulus of steel and concrete, which are \(E_s = 2.06 \times 10^{11} Pa\) and \(E_c = 3.45 \times 10^{10} Pa\) respectively, the parameters in Equations (1) and (2) can be obtained. \(EI = 1.325 \times 10^7 \text{Nm}^2, \rho A = 120 \text{kg/m}, \alpha = 32930, \beta = 208.3\).

Table 2. Section characteristics of the bridge

| Parameter            | Value     |
|----------------------|-----------|
| Type                 | Steel rail|
| Area (m²)            | 1.549 \times 10^{-2} |
| Moment of inertia (m⁴) | 6.434 \times 10^{-5} |
| Line density (kgm⁻¹) | 120       |
| Concrete beam        | 10.10     |
|                      | 12.65     |
|                      | 25000     |

3.1. Natural frequencies and mode shapes

The FEM model is established and the eigenvalue analysis is conducted. Fig. 2 presents the computed first ten modal frequencies and the corresponding mass-normalized mode shapes. In each pair of the mode shape curves, the solid curve is the mode shape of the track, and the dashed curve is the mode shape of the bridge. As shown, the modes of the bridge-track system can be classified into two classes. One class is the in-phase modes, such as the 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\), 6\(^{th}\), 7\(^{th}\), 8\(^{th}\), and 10\(^{th}\) modes. Another class is the breath modes such as the 4\(^{th}\), 5\(^{th}\), and 9\(^{th}\) modes.
Figure 3. Natural frequencies and mode shapes for the first six modes of the bridge

3.2. Impact factor computation under moving train loads

Figure 4 shows the peak response impact factor and occurrence position for bridge deflection considering carriage heaving oscillation load, where \( \mu_{y2} \) denotes the peak dynamic deflection after the normalization to the maximum static deflection. As shown, the peak deflection of the bridge almost always occurs at the mid-span of the single-span bridge with the variation of the moving speed of the train. That means the bridge deflection response is governed by the fundamental mode. The contribution of the higher modes are too small to offset the peak deflection occurrence position. The fluctuation of the \( \mu_{y2} \) is mainly due to the amplification of the fundamental mode responses. Considering of the multi-harmonic essence of the moving load problems, the super-harmonic amplification will induce some bulges of the curve in the low speed region. The amplification of the first harmonic will induce the important bulges of the curve in the high-speed region.
Figure 4. Peak response impact factor and occurrence position curves for bridge deflection considering carriage heaving oscillation load.

Figure 5 shows the peak response impact factor and occurrence position for bridge bending moment considering carriage heaving oscillation load, where $\mu_B$ denotes the peak dynamic bending moment.

Figure 5. Peak response impact factor and occurrence position curves for bridge bending moment considering carriage heaving oscillation load.
impact factor. As shown, the \( \mu_B \) curve is of the similar shape as the \( \mu_{y2} \) curve. However, the peak response occurrence position curves are quite different. Important fluctuations are observed on the peak bending moment occurrence position curves with the variation of the moving speed of the train. That means for the bridge bending moment response, the higher modes contribution is important and cannot be neglected. The super-harmonic amplification of the fundamental mode in the low speed region and the super-harmonic amplification of the higher mode in the high speed region may account for these fluctuations in different regions.

![Graph showing peak response impact factor for fastener forces considering carriage heaving oscillation load.](image)

**Figure 6.** Peak response impact factor for fastener forces considering carriage heaving oscillation load

Figure 6 presents the peak response impact factor for fastener forces considering the carriage heaving oscillation load, where \( \mu_f \) denotes the related impact factor. As shown, an up-to 11 times of fastener forces impact amplification is observed when \( \bar{v} \approx 1.27 \). This important amplification implies the notable contribution of the anti-phase breath mode of the bridge-track structure system at the corresponding working condition.

4. Conclusions

This study investigates the impact characteristics of a typical railway bridge with ballastless double-block supported track under moving harmonic loads due to the oscillation of a six-carriage train. The computation results verify the super-harmonic amplification occurrence in the low speed region and the higher mode contributions to bridge bending moment responses. The results also show that the concerned moving loads will induce important impact amplification on the fastener forces. The super-harmonic resonances of the breath mode of the bridge system may accounts for those amplifications. Considering the interaction of the bridge multiple modes with the train carriage, different types of parametric resonances [8, 9] and mutual synchronization resonances will occur, especially in the low
speed region. The related impact amplification phenomenon on bridge stress resultant and fastener system need to be further investigated.

5. References

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