Effective String Theory of Dual Superconductivity

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We show how an effective field theory of long distance QCD, describing a dual superconductor, can be expressed as an effective string theory of superconducting vortices. We use the semiclassical expansion of this effective string theory about a classical rotating string solution in any spacetime dimension $D$ to obtain the semiclassical meson energy spectrum. For $D = 26$, it formally coincides with the energy spectrum of the open string in classical Bosonic string theory. However, its physical origin is different. It is a semiclassical spectrum of an effective string theory valid only for large values of the angular momentum. For $D = 4$, the first semiclassical correction adds the constant $1/12$ to the classical Regge formula for the angular momentum of mesons on the leading Regge trajectory.

1 The Dual Superconductor Mechanism of Confinement

In the dual superconductor mechanism of confinement\cite{1,2,3}, a dual Meissner effect confines color electric flux to narrow tubes connecting a quark-antiquark pair. In the confined phase monopole fields $\phi$ condense to a value $\phi_0$, and dual potentials $C_\mu$ acquire a mass $M = g\phi_0$ via a dual Higgs mechanism. The dual coupling constant is $g = 2\pi/e$, where $e$ is the Yang-Mills coupling constant. Quarks couple to dual potentials via a Dirac string connecting the quark-antiquark pair along a line $L$, the ends of which are sources and sinks of color electric flux. The color field of the pair destroys the dual Meissner effect near $L$ so that $\phi$ vanishes on $L$. At distances transverse to $L$ greater than $1/M$ the monopole field returns to its bulk value $\phi_0$ so that the color field is confined to a tube of radius $a = 1/M$ surrounding the line $L$.\cite{5,6}

As a result, for quark-antiquark separations $R$ greater than $a$, a linear potential develops that confines the quarks in hadrons. At small separations, on the other hand, the color field generated by the quarks expels the monopole condensate from the region between them, and the potential becomes perturbative one gluon exchange. The dual theory thus gives a potential that evolves smoothly from the large $R$ confinement region to the short distance perturbative domain and provides an effective theory of long distance QCD, with short distances cut off at the flux tube radius. It does not describe QCD at shorter distances where radiative corrections due to asymptotic freedom and a running coupling constant are important.

The dual theory must have classical vortex solutions which for short distance reduce to perturbative QCD, and it must not contain massless particles. For $SU(N)$ gauge theory this can be done by coupling dual non-Abelian potentials $C_\mu$ to 3 multiplets of scalar monopole fields $\phi$ in the adjoint representation, so that in the confined phase the dual $SU(N)$ gauge symmetry is "spontaneously broken" to $Z_N$. The gauge bosons $C_\mu$ then all acquire a mass, and there are $Z_N$ electric flux tube excitations\cite{5,6} confining quark-antiquark pairs attached to their ends. Here we do not use a specific form for the action $S[C_\mu, \phi]$ of the dual theory.
We want to use this theory to obtain the energy levels of mesons having large angular momentum. Under such circumstances the distance between quarks is much larger than the flux tube radius and we must include the contribution of flux tube fluctuations to the interaction between the quark and antiquark. The fluctuating vortex line $L$ sweeps out a world sheet $\tilde{x}^\mu$ whose boundary is the loop $\Gamma$ formed from the world lines of the moving quark-antiquark pair. Their interaction is determined by the Wilson loop $W[\Gamma]$,

$$W[\Gamma] = \int D\xi D\phi D\phi^* e^{iS[\phi, \phi^*]}.$$  \hspace{1cm} (1)

The path integral (1) goes over all field configurations for which the monopole field $\phi(x)$ vanishes on some sheet $\tilde{x}^\mu$ bounded by the loop $\Gamma$.

### 2 The Effective String Theory

The Wilson loop $W[\Gamma]$ describes the quantum fluctuations of a field theory having classical vortex solutions. We want to express the functional integration over fields as a path integral over the vortex sheets $\tilde{x}^\mu$ to obtain an effective string theory of these vortices. To do this we carry out the functional integration (1) in two stages:

1. We integrate over all field configurations in which the vortex is located on a particular surface $\tilde{x}^\mu$, where $\phi(\tilde{x}^\mu) = 0$. This integration determines the action $S_{\text{eff}}[\tilde{x}^\mu]$ of the effective string theory.

2. We integrate over all vortex sheets $\tilde{x}^\mu(\xi)$, $\xi = \xi^a$, $a = 1, 2$. This integration goes over the amplitudes $f^1(\xi)$ and $f^2(\xi)$ of the two transverse fluctuations of the world sheet $\tilde{x}^\mu(\xi)$ and gives $W[\Gamma]$ the form

$$W[\Gamma] = \int Df^1 Df^2 \Delta_{FP} e^{iS_{\text{eff}}[\tilde{x}^\mu]},$$  \hspace{1cm} (2)

where $\Delta_{FP}$ is a Faddeev-Popov determinant.

The path integral (1) over fields has been transformed into the path integral (2) over vortex fluctuations. The factor $\Delta_{FP}$ in (2) arises from writing the original field theory path integral as a ratio of path integrals of two string theories and reflects the field theory origin of the effective string theory. This ratio is anomaly free. The path integral (2) goes over fluctuations in the shape of the vortex sheet with wavelengths greater than the flux tube radius.

The presence of the determinant $\Delta_{FP}$ makes the path integral (2) invariant under reparameterizations $\tilde{x}^\mu(\xi) \rightarrow \tilde{x}^\mu(\xi^{\prime})$ of the vortex world sheet. The resulting parameterization invariant measure in the path integral (2) is independent of the explicit form of the dual field theory. On the other hand, the action $S_{\text{eff}}[\tilde{x}^\mu]$ of the effective string theory is not universal and depends upon the parameters of the field theory. However, for wavelengths of the string fluctuations greater than the flux tube radius $a$ it can be expanded in powers of the extrinsic curvature of the world sheet. In mesons of angular momentum $J$ the expansion parameter is of order $1/J$. For mesons having large angular momentum the action $S_{\text{eff}}$ can then be approximated by the first term in this expansion, namely by the Nambu–Goto action,

$$S_{\text{eff}}[\tilde{x}^\mu] \approx S_{NG} \equiv -\sigma \int d^4x \sqrt{-g},$$  \hspace{1cm} (3)

where $\sqrt{-g}$ is the square root of the determinant of the induced metric $g_{ab}$,

$$g_{ab} = \frac{\partial \tilde{x}^\mu}{\partial \xi^a} \frac{\partial \tilde{x}^\mu}{\partial \xi^b},$$  \hspace{1cm} (4)

and the string tension $\sigma$ is determined by the parameters of the underlying dual field theory. In the next section we use the results of a semiclassical expansion of this effective string theory to calculate Regge trajectories of mesons.
Figure 1: The string coordinate system

3 Regge trajectories of Mesons

Consider a quark-antiquark pair rotating with uniform angular velocity $\omega$ (See Fig. 1). The quarks have masses $m_1$ and $m_2$, move with velocities $v_1 = \omega R_1$ and $v_2 = \omega R_2$, and are separated by a fixed distance $R = R_1 + R_2$. To describe the string connecting this pair we use parameters $\xi = (t, r)$ which are the time and the coordinate $r$, which runs from $-R_1$ to $R_2$. We evaluate $W[\Gamma]$ in the leading semiclassical approximation about a classical rotating straight string solution $\bar{x}^\mu(r, t)$. The amplitudes $f^1(\xi)$ and $f^2(\xi)$ of the transverse fluctuations are the spherical coordinates $\theta(r, t)$ and $\phi(r, t)$ of a point on the string. The angle $\theta(r, t)$ is the fluctuation perpendicular to the plane of rotation, and the angle $\phi(r, t)$ is the fluctuation lying in the plane of the rotating string. The classical metric $\bar{g}_{ab} = g_{ab}[\bar{x}^\mu]$ and classical action $S_{NG}[\bar{x}^\mu]$ are independent of the time $t$, so that $W[\Gamma]$ has the form

$$W[\Gamma] = e^{iTL^{\text{string}}(R_1, R_2, \omega)}, \quad (5)$$

where $T$ is the elapsed time.

We calculate $L^{\text{string}}$ in $D$ spacetime dimensions. The fluctuation $\theta(r, t)$ is replaced by $D-3$ fluctuations perpendicular to the plane of rotation and there is still just 1 fluctuation in the plane of rotation. The effective Lagrangian for the quark–antiquark pair, obtained by adding quark mass terms to $L^{\text{string}}$, is the sum of a classical part and a fluctuating part,

$$L_{\text{eff}}(R_1, R_2, \omega) = L_{\text{cl}} + L^{\text{fluc}}_{\text{string}}(R_1, R_2, \omega), \quad (6)$$

where

$$L_{\text{cl}} = -\sum_{i=1}^{2} m_i \sqrt{1 - v_i^2} - \sigma \int_{-R_1}^{R_2} dr \sqrt{1 - r^2 \omega^2}, \quad (7)$$

and the expression for $L^{\text{fluc}}_{\text{string}}$ is obtained from the semiclassical calculation of $W[\Gamma]$. It contains terms which are quadratically, linearly, and logarithmically divergent in the cutoff $M$. The quadratically divergent term is a renormalization of the string tension, the linearly divergent term is a renormalization of the quark mass, and the logarithmically divergent term is a renormalization of a term in the boundary action called the geodesic curvature. After absorbing these divergent terms into renormalizations, we obtain an expression for $L^{\text{fluc}}_{\text{string}}$ which remains finite in the massless quark limit,

$$L^{\text{fluc}}_{\text{string}}(\omega)|_{m_1=m_2=0} = \frac{\omega(D - 2)}{24} + \frac{\omega}{2}. \quad (8)$$

The first term in (8) is the negative of Lüscher potential with the length $R$ of the string replaced by its proper length $\pi/\omega$. It is the contribution of $D-2$ transverse fluctuations in the background
of a flat metric. The $\omega^2$ term accounts for the curvature of the classical background metric $\bar{g}_{ab}$ generated by the rotating string. (For massless quarks, the ends of the string move with the velocity of light and singularities appear in $L^{\text{string}}_{\text{fluc}}$. The scalar quark mass term in $L_{\text{eff}}$ serves as an infrared regulator, and in the limit of massless quarks gives no contribution to $L^{\text{string}}_{\text{fluc}}$.

To take into account the fluctuations of the positions $\vec{x}_1(t)$ and $\vec{x}_2(t)$ of the quarks at the ends of the rotating string, we extend the functional integral (2) to include a path integral over $\vec{x}_1(t)$ and $\vec{x}_2(t)$. Using the methods of Dashen, Hasslacher and Neveu to carry out a semiclassical calculation of the path integral gives the WKB quantization condition,

$$J = \frac{dL_{\text{eff}}(\omega)}{d\omega} = l + \frac{1}{2}, \quad l = 0, 1, 2, \ldots.$$  (9)

Furthermore, for massless quarks, there is no contribution to $L_{\text{eff}}(\omega)$ arising from the fluctuations in the motion of the ends of the string, so that $L_{\text{fluc}}(\omega) = L^{\text{string}}_{\text{fluc}}(\omega)$.

Eqs. (7) and (8) give $L_{\text{fluc}}(\omega)/L_{\text{cl}}(\omega) \sim \omega^2/\sigma \sim 1/J$, so that for large $J$, $L_{\text{fluc}}(\omega)$ can be treated as a perturbation. The meson energy is then

$$E(\omega) = E_{\text{cl}}(\omega) - L_{\text{fluc}}(\omega) = \frac{\pi \sigma}{\omega} - \frac{D-2}{24} \omega - \frac{\omega}{2}.$$  (10)

The energies $E_n(\omega)$ of the excited states of the rotating string (light hybrid mesons) are obtained by adding to (10) the energies $k\omega$ of the excited vibrational modes.

$$E_n(\omega) = \frac{\pi \sigma}{\omega} - \frac{D-2}{24} \omega - \frac{\omega}{2} + n\omega.$$  (11)

Since there are many combinations of string normal modes which give the same $n$ (e.g., a doubly excited $k=1$ mode and a singly excited $k=2$ mode each give $n=2$), the spectrum is highly degenerate.

The value of $\omega$ is given as a function of $J$ through the classical relation $\omega = \sqrt{\frac{\pi \sigma}{2J}}$. Squaring both sides of (11) and using the WKB quantization condition (9) yield

$$E^2_n = 2\pi \sigma \left( l + n - \frac{D-2}{24} + O\left(\frac{n^2}{l}\right)\right).$$  (12)

Setting $D = 4$ in (12) and solving for $l$ we obtain the the Regge trajectories,

$$l = \frac{E^2}{2\pi \sigma} + \frac{1}{12} - n, \quad n = 0, 1, 2, \ldots.$$  (13)

The first semiclassical correction to the leading Regge trajectory, $n=0$, adds the constant $1/12$ to the classical Regge formula. For $D = 26$, Eq. (13) yields the spectrum

$$E^2 = 2\pi \sigma \left( l + n - 1 + O\left(\frac{n^2}{l}\right)\right).$$  (14)

The spectrum of energies (14), valid in the leading semiclassical approximation, coincides with the spectrum of open strings in classical bosonic string theory.

4 Summary and Conclusions

1. We have seen how the dual superconducting model of confinement leads to an effective string theory of long distance QCD. We have evaluated, in the semiclassical large angular momentum domain, where this theory is applicable, the contribution of string fluctuations to Regge trajectories of mesons containing massless scalar quarks. The quark degrees of
freedom did not contribute to the meson energy. (With D. Gromes we are now investigating Regge trajectories of mesons formed by coupling spin 1/2 quarks to the string. We find that, in the limit that the quark masses go to zero, there are no finite energy states. This is another indication that chiral symmetry must be broken in the confined phase of QCD.)

2. The derivation of the effective string theory made no use of the details of the effective field theory from which it was obtained. Furthermore, L. Yaffe has given arguments based on the work of ’t Hooft showing that the confined phase of a non-Abelian gauge theory is characterized by a dual order parameter, which vanishes in regions of space where “dual superconductivity” is destroyed. This generic description of the confined phase of QCD leads to the effective string theory of long distance QCD described here, which provides a concrete picture of the QCD string.

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