Dual technicolor with hidden local symmetry

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Abstract

We consider a dual description of the technicolor-like gauge theory within the D4/D8 brane configuration with varying confinement and electroweak symmetry breaking scales. Constructing an effective truncated model valid below a certain cut-off, we identify the particle spectrum with Kaluza-Klein modes of the model in a manner consistent with the hidden local symmetry. Integrating out heavy states, we find that the low-energy action receives nontrivial corrections stemming from the mixing between Standard Model and heavy gauge bosons which results in reduction of oblique parameters.
1. **Top-down holographic technicolor.** Unravelling the correct mechanism for electroweak symmetry breaking (EWSB) is one of the most important problems facing particle physics. The Standard Model (SM) Higgs boson, endowing gauge bosons and fermions with masses, introduces at the same time the gauge hierarchy problem between the electroweak and Planck scales in the theory. This makes the Higgs mass unstable under radiative effects thus making the construct undesirable theoretically. An intriguing alternatives explored for quite some time, which bypasses this problem, is the dynamical EWSB by condensation of new kinds of fermions coupled by a strongly interacting gauge sector [1, 2]. These technicolor models are hard to tackle theoretically, however, and attempts to use scaled-up versions of QCD, where the phenomenon of chiral symmetry breaking mimics the one of EWSB, led to predictions inconsistent with precision measurements [3, 4]. Thus the new strongly coupled sector, if realized in Nature, is not QCD-like. Versions of walking [5], conformal [6], etc., technicolor were proposed but all plagued by the same calculability problem. Therefore, it appears that the quest for a consistent model of technicolor is not over.

Recent advances in gauge/string dualities open a window for construction of calculable models of dynamical EWSB. In this framework, a way to analyze strong coupling region of gauge theory via weakly-coupled gravitational description enables one to treat non-perturbative dynamics of technicolor theory in a perturbative fashion. Applied to the problem at hand, the holographic dual description explores the regime where the probe branes describe the action below the scale of techniquark condensation. The D-brane configuration in flat space background of type IIA string theory which realizes a technicolor scenario as an effective theory on the D-branes is based on the embedding of \( N_f \) D8-D\( \bar{8} \) branes [7] in the background of \( N \) D4 branes [8] and intersecting in four-dimensional space-time. The gauge fields on the D8 (D\( \bar{8} \)) branes possess \( SU_L(N_f) \) (\( SU_R(N_f) \)) gauge symmetry while techniquark strings live on D4-D8 (D4-D\( \bar{8} \)) intersections. This construction is dual to a confining gauge theory with massless fermions and realizes the \( SU_L(N_f) \times SU_R(N_f) \) non-Abelian global symmetry broken down to the diagonal subgroup \( SU_D(N_f) \). In its original incarnation, it was used for modelling chiral symmetry breaking in QCD. Its adaptation to models of holographic technicolor is achieved by setting \( N_f = 2 \) and interpreting the gauge symmetry in the bulk as a weakly gauged symmetry of the four-dimensional gauge theory, as was done earlier in Refs. [9,10,11]. This ensures the custodial global \( SU(2) \) symmetry for the boundary gauge theory which protects isospin observables from receiving large corrections [12]. To uncover the Standard Model at low energies, the \( SU_R(2) \) group is broken down to \( U(1) \) on the boundary by an adjoint scalar living on \( D\bar{8} \) with a divergent vacuum expectation value, which translates to the Dirichlet boundary condition imposed on the corresponding components the gauge fields.

While attempting to resolve the gauge hierarchy problem, these higher-dimensional theories also address the issue of unitarity of scattering amplitudes of massive gauge bosons, which in the SM is cured by the Higgs boson, by means of the exchange of towers of massive Kaluza-Klein (KK) gauge bosons [13,14].

The starting point for the analysis is the near-horizon geometry of \( N \) coincident D4-branes [8],

\[
\text{ds}^2 = -\left( \frac{u}{R} \right)^{3/2} [\eta_{\mu\nu}dx^\mu dx^\nu - f(u)d\tau^2] + \left( \frac{R}{u} \right)^{3/2} \left[ f^{-1}(u)du^2 + u^2 d\Omega_4^2 \right],
\]

with \( R = \pi g_s N(\alpha')^{3/2} \) and \( f(u) = \left( 1 - \frac{\alpha'_K}{u} \right) \). They extend into the four-dimensional Minkowski space-time with the (mostly negative) metric tensor \( \eta_{\mu\nu} \) and are compactified on a circle in the \( \tau \)-direction with radius \( m_K^{-1} \) in order to avoid conical singularities. Introducing \( N_f \ll N \) D8 branes
into this background, these can be treated in the probe approximation and their embedding is determined by the \( u = u(\tau) \) profile, a solution to \( [15] \)

\[
(u')^2 = \left( \frac{u}{R} \right)^3 \frac{f(u)^2 \left[ u^8 f(u) - u_0^8 f(u_0) \right]}{u_0^8 f(u_0)} .
\]  

(2)

With \( u' \) vanishing at \( u_0 \), the D8 and \( \overline{D8} \) branes are smoothly connected at \( u_0 \leq u_K \), admitting a U-shaped form. This configuration geometrically realizes the dynamical \( SU_L(N_f) \times SU_R(N_f) \) symmetry breaking in the dual gauge theory. To distinguish the D8 and \( \overline{D8} \) branches of the resulting solution, a new variable is particularly convenient

\[
u^3 = u_0^3 \left( 1 + z^2 \right),
\]

(3)

which goes along \( \overline{D8} \) branch for \( -z_R \leq z \leq 0 \) and D8 for \( 0 \leq z \leq z_L \) with cut-offs \( z_{L,R} \) reflecting the finite volumes of the electroweak branes. Then the probe D8-brane Dirac-Born-Infeld (DBI) action encodes the EWSB which endows the SM gauge bosons, identified with the lowest modes in the KK expansions of the brane gauge field, and other technimesons with masses. Ignoring entirely the towers of modes with nonvanishing angular momentum on \( S^4 \) as well as all gauge fields along these transverse directions as being heavier than KK states with respect to the compact \( z \)-direction, we can rewrite the quadratic part of the five-dimensional DBI action in the unitary \( A_z = 0 \) gauge as

\[
S = -\frac{1}{g_5^2} \text{tr} \int d^4x dz \left\{ \frac{1}{2} w_1(z) F_{\mu\nu}^2 - \varepsilon^{-1/3} m_K^2 w_2(z) (\partial_z A_\mu)^2 \right\},
\]

(4)

where \( \varepsilon \equiv (u_K/u_0)^3 \leq 1 \) and

\[
g_5^{-2} = \frac{2}{3} u_0^{-1/2} T_8 V g_s^{-1} (2\pi\alpha')^2 R^{9/2}, \quad m_K^2 = \frac{g_5^2 u_K^3}{4 R^3}
\]

(5)

are the five-dimensional coupling and the compactification scale, respectively. The weights stemming from the warped metric are

\[
w_1(z) = (1 + z^2)^{2/3} w_2^{-1}(z) = (1 + z^2)^{-1/3} \left( 1 + (1 - \varepsilon) \left( \frac{1 + z^2}{z^2} \right)^{5/3} - 1 \right)^{-1/2}.
\]

(6)

The modes diagonalizing the above action obey the eigenvalue equation

\[
\partial_z (w_2(z) \partial_z \psi_n) + \varepsilon^{1/3} \lambda_n w_1(z) \psi_n = 0.
\]

(7)

To have dynamical fields in the ultraviolet on the D8 and \( \overline{D8} \), the boundary conditions are imposed as follows:

\[
\partial_z A_\mu^\pm(z_L) = 0, \quad \partial_z A_\mu^3(z_L) = 0, \quad A_\mu^\pm(-z_R) = 0, \quad \partial_z A_\mu^3(-z_R) = 0.
\]

(8)

There are different ways to identify field content of the model with physical particle spectra. In the present note we will employ a residual gauge freedom of the five-dimensional gauge fields describing the fluctuation of the D-branes to identify (axial) vector mesons differently to the Callan-Coleman-Wess-Zumino (CCWZ) formulation \([16]\). This hidden local symmetry (HLS) \([17, 18]\) introduces a kinetic mixing among the light gauge bosons and their KK excitations.
Integrating out the heavy modes from the spectrum will define a low-energy theory with non-
Standard Model interactions encoded in the renormalization factors. Coupling SM fermions to
five-dimensional gauge bosons, we compute the oblique parameters \cite{4, 19} in this model and
demonstrate that KK axial towers tend to reduce the value of the $S$ parameter, which is of order
one in typical technicolor models.

2. CCWZ vs. HLS. Within the CCWZ approach the fields in the KK decomposition

$$A^a_\mu(x, z) = \sum_{n \geq 0} \psi^a_n(z) A^{a(n)}_\mu(x),$$

(9)

with $a = \pm, 3$, corresponds to observed particles. Here the lowest components are identified
with the photon $A_\mu^{3(0)} = B_\mu^0$ and neutral $A_\mu^{3(1)} = Z_\mu^0$ and charged $A_\mu^{\pm(0)} = W_\mu^\pm$ gauge bosons,
respectively, while the rest with heavier mass modes. Then, the diagonalized quadratic part of
the DBI action written in terms of mass eigenstates reads

$$S = -\frac{1}{g_5^2} \sum_{a=0,\pm} \sum_{n \geq 0} N^a_n \int d^4x \left\{ \frac{1}{4} (F^{a(n)}_{\mu\nu})^2 - \frac{1}{2} m^2_{n,a} (A^{a(n)}_\mu)^2 \right\},$$

(10)

where the masses and normalization constants are

$$m^2_{n,a} = \lambda_{n,a} m_K^2, \quad N_n = \int_{z_L}^{z_R} dz \, w_1(z) (\psi_n(z))^2.$$

Notice however that in the unitary gauge, the gauge field $A^a_\mu(x, z)$ may transform under
a residual gauge transformation $h(x) = U(x, 0)$, independent of $z$ variable, which leaves the
condition $A_z = 0$ invariant. However, while Neumann boundary conditions are consistent with it,
the Dirichlet boundary conditions acquire a nontrivial right-hand side for the gauge transformed
variables. This is a reflection of the well-known fact that the physical gauges, like axial, light-like,
etc., require boundary conditions imposed on fields to fix the gauge symmetry completely and
get rid of residual degeneracies in gauge theories which prohibit their consistent quantization.
For the present setup, this implies that while the third isovector component $A_3^\mu$ of the gauge field
allows for hidden local symmetry transformations \cite{17, 18}, the $A_\mu^\pm$ ones do not, i.e.,

$$A_\mu^\pm(x, z) \to h(x) A_\mu^\pm(x, z) h^\dagger(x)$$

$$A_\mu^3(x, z) \to h(x) A_\mu^3(x, z) h^\dagger(x) + i h(x) \partial_\mu h^\dagger(x).$$

(12)

The general KK decomposition is then

$$A^\pm_\mu(x, z) = \sum_{n \geq 0} W^{\pm(n)}_\mu(x) \phi^\pm_n(z),$$

(13)

$$A^3_\mu(x, z) = \sum_{n \geq 0} \left\{ L^{(n)}_\mu(x) \phi_{L,n}(z) + R^{(n)}_\mu(x) \phi_{R,n}(z) \right\},$$

(14)

where the neutral field is decomposed into left and right modes reflecting the symmetry of the
boundary conditions imposed on it and emphasizing the fact that, contrary to the charged vector,
either of the boundaries will yield light dynamical modes identified with the photon and Z-boson.
As a consequence, while the charged modes transform homogeneously under the HLS and we can identify

$$\phi^\pm_n = \psi^\pm_n,$$

(15)
where $\psi^\pm_n$ are solutions to Eq. (17), the neutral bosons are inhomogeneous,
\begin{equation}
(L^{(n)}\mu(x), R^{(n)}\mu(x)) \to h(x) \left( L^{(n)}\mu(x), R^{(n)}\mu(x) \right) h^\dagger(x) + \text{i} h(x) \partial_\mu h^\dagger(x),
\end{equation}
(16)
The consistency between Eqs. (12) and (16) implies that
\begin{equation}
\sum_{a=L, R} \sum_{n \geq 0} \phi_{a,n}(z) = 1,
\end{equation}
where the modes $\phi_{a,n}$ are not the eigenmodes of the equation of motion but rather their linear combinations which we are about to construct. First, to resolve the consistency condition we introduce the following linear combinations
\begin{align}
\phi_k^a &= \psi^k_n - \psi^{k+1}_n, \quad \text{for} \quad k = 0, \ldots, M - 1, \quad \phi_M^a = \psi^M_n, \quad (18)
\end{align}
whose unusual form is used to restore the conventional normalization of the gauge fields in the decomposition of the left and right vector fields in Eqs. (22) and (23) below. Here the wave functions $\psi_n$ on the right-hand side are indeed the eigenstates of Eq. (17). The lowest mixing angle $\theta_0$ has to be chosen to coincide with the Weinberg angle $\theta_W$. Finally, the gauge eigenstates $B^{(n)}_\mu$ and $Z^{(n)}_\mu$, which transform in- and homogeneously, respectively, with respect to the HLS
\begin{equation}
B^{(n)}_\mu(x) \to h(x)B^{(n)}_\mu(x)h^\dagger(x) + \text{i} h(x)\partial_\mu h^\dagger(x), \quad Z^{(n)}_\mu(x) \to h(x)Z^{(n)}_\mu(x)h^\dagger(x),
\end{equation}
(21)
arise in the decomposition of the left and right gauge fields as
\begin{align}
L^{(n)}_\mu &= B^{(n)}_\mu - \sum_{k=1}^n \left( \sin^2 \theta_k \cot \theta_{k-1} - \cos^2 \theta_k \tan \theta_{k-1} \right) Z^{(k-1)}_\mu - \cot \theta_n Z^{(n)}_\mu, \quad (22)
\end{align}
\begin{align}
R^{(n)}_\mu &= B^{(n)}_\mu - \sum_{k=1}^n \left( \sin^2 \theta_k \cot \theta_{k-1} - \cos^2 \theta_k \tan \theta_{k-1} \right) Z^{(k-1)}_\mu + \tan \theta_n Z^{(n)}_\mu. \quad (23)
\end{align}
Here a linear combination of homogeneous $Z^{(k<n)}_\mu$-fields was absorbed into the KK vector modes $B^{(n)}_\mu$ so as to eliminate the mixing between the vector and axial modes.

Substituting these results into the decomposition (13), we obtain the final KK expansion
\begin{align}
A^{3}_\mu(x, z) &= B^{(0)}_\mu(x)\psi_0(z) + Z^{(0)}_\mu(x)\psi_1(z)
+ \sum_{k=1}^M \left[ B^{(k)}_\mu(x) - B^{(k-1)}_\mu(x) \right] \psi_{2k}(z) + \sum_{k=1}^M \left[ Z^{(k)}_\mu(x) - f_k Z^{(k-1)}_\mu(x) \right] \psi_{2k+1}(z),
\end{align}
(24)
with self-obvious HLS properties. From the above definitions, it follows that $\psi_0(z) = 1$ and

$$f_k = \frac{\sin \theta_k \cos \theta_k}{\sin \theta_{k-1} \cos \theta_{k-1}},$$

(25)

with $f_0 = 1$. The four-dimensional neutral gauge boson action then splits into two orthogonal sectors and reads

$$\mathcal{S}_n = -\frac{1}{2g_5^2} \int \frac{d^4p}{(2\pi)^4} \sum_{j,k=0}^{M} \left\{ B_{\mu}^{(j)}(p) \nu^{jk}_{\mu\nu}(p) B_{\mu}^{(k)}(-p) + Z_{\mu}^{(j)}(p) A_{\mu\nu}^{jk}(p) Z_{\mu}^{(k)}(-p) \right\},$$

(26)

where

$$\nu^{jk}_{\mu\nu}(p) = P_{\mu\nu}(p) \left\{ (\alpha_j + \alpha_{j+1}) \delta_{jk} - \alpha_j \delta_{j-1,k} - \alpha_{j+1} \delta_{j+1,k} \right\} - \eta_{\mu\nu} \left\{ (\mu_j^2 + \mu_{j+1}^2) \delta_{jk} - \mu_j^2 \delta_{j-1,k} - \mu_{j+1}^2 \delta_{j+1,k} \right\},$$

(27)

$$A_{\mu\nu}^{jk}(p) = P_{\mu\nu}(p) \left\{ (\beta_j + f_{j+1}^2 \beta_{j+1}) \delta_{jk} - \beta_j f_j \delta_{j-1,k} - \beta_{j+1} f_{j+1} \delta_{j+1,k} \right\} - \eta_{\mu\nu} \left\{ (\nu_j^2 + f_{j+1}^2 \nu_{j+1}^2) \delta_{jk} - f_j \nu_j^2 \delta_{j-1,k} - f_{j+1} \nu_{j+1}^2 \delta_{j+1,k} \right\},$$

(28)

with the kinetic projection operator being

$$P_{\mu\nu}(p) = p^2 \eta_{\mu\nu} - p_{\mu} p_{\nu}.$$  

(29)

The mixing matrices (27) and (28) are defined in terms of the normalization constants for even $\alpha_k = N_{2k}^0$ and odd $\beta_k = N_{2k+1}^0$ modes and corresponding mass parameters $\mu_k^2 = \alpha_k m_{2k}^2$ and $\nu_k^2 = \beta_k m_{2k+1}^2$, respectively. Notice that $\mu_0^2 = 0$. At the same time, the charged boson action is diagonal from the outset due to identification (15) driven by the boundary conditions (8) such that the lowest component is indeed the physical $W$, however with unconventional normalization $\gamma_{0,\pm} = N_{0,\pm}^\pm$ of the kinetic term and mass $\mu^-_{0,\pm} = \gamma_{0,\pm} m_{0,\pm}^2$.

As a next step, we find the eigenstates of the mass matrix and integrate out all KK modes yielding an effective Lagrangian for the lightest modes only in each sector. As a result, the photon $B_{\mu}$ and $Z$-boson $Z_{\mu}$ mass eigenstates are given by linear combinations of the gauge eigenstates $B_{\mu}^{(k)}$ and $Z_{\mu}^{(k)}$,

$$B_{\mu} = \frac{1}{M+1} \sum_{k=0}^{M} B_{\mu}^{(k)}, \quad Z_{\mu} \simeq \frac{1}{D_M} \sum_{k=0}^{M} \left( \prod_{n=1}^{k} f_n \right) Z_{\mu}^{(k)},$$

(30)

where the last expression is quoted to $\mathcal{O}(\delta)$ accuracy only with $\delta = \max \nu_j^2 / \nu_{k>j}^2$. Here we introduced the following notation for the function of mixing angles

$$D_k \equiv \sum_{m=0}^{k} \prod_{n=0}^{m} f_n^2.$$  

(31)

The ratio of the five-dimensional cut-offs $z_R/z_L$ is correlated with the value of the model’s Weinberg angle $\theta_0$. The lowest states in the KK towers, i.e., photon, $Z$ and $W$, are ultralight (UL) and separated hierarchically from the rest of the KK spectrum by a gap typical to gap-like metrics [20, 21],

$$\frac{m_{UL}}{m_K} \sim \left( \int_{z_L}^{z_R} dz \, w_1(z) \right)^{-1/2}.$$  

(32)
Summarizing our findings, the low-energy action including the charged-boson sector then reads
\[
S = -\frac{1}{2g_5^2} \int \frac{d^4p}{(2\pi)^4} \left\{ P_{\mu\nu}(p) \left[ B_\mu(p) B_\nu(-p) K_B + Z_\mu(p) Z_\nu(-p) K_Z + 2W_\mu(p) W_\nu^*(-p) K_W \right] \right. \\
\left. - \nu_0^2 Z_\mu(p) Z_\nu(-p) - 2\mu_{0,\pm}^2 W_\mu(p) W_\nu^*(-p) \right\}, \quad (33)
\]
where the photon remains massless and the kinetic terms are
\[
K_B = \alpha_0 + \mathcal{O}(p^2), \quad (34)
\]
\[
K_Z = \beta_0 \left( 1 - 2 \sum_{k=1}^{M} \frac{\nu_k^2}{\nu_k^2} \left( 1 - \frac{D_{k-1}}{D_M} \right) \prod_{m=1}^{k} \frac{f_m^{-2} + \mathcal{O}(\delta^2)}{f_m^{-2} + \mathcal{O}(\delta^2)} \right) + \mathcal{O}(p^2), \quad (35)
\]
\[
K_W = \gamma_{0,\pm}. \quad (36)
\]
Here we did not display \( \mathcal{O}(p^2) \) terms since they do not affect Peskin-Takeuchi parameters.

3. Spectrum and oblique parameters. The mass spectrum emerges from the eigenvalue equation and can be computed numerically\(^1\). To calculate precision electroweak corrections we have to find a scheme in which all corrections will be oblique, i.e., the effective four-dimensional Lagrangian describing the coupled gauge boson-fermion system only gets corrections in the gauge boson sector, but none the fermionic sector. There are a couple of ways to achieve this. Either, with the bulk gauge kinetic terms normalized by \( g_5^{-2} \), a very simple convention is to set the gauge boson wave functions to one at the location of the fermion \(^{22}\) and require that the couplings of localized fermions reproduce exactly the leading order relations between them. Or, by choosing the canonical normalization for the four-dimensional gauge boson kinetic terms, channel all new physics into gauge-fermion couplings \(^{19}\). In this work, we choose the latter.

The coupling of gauge fields through the covariant derivative to the SM fermions, localized in the ultraviolet, has the structure\(^2\)
\[
\bar{\lambda}(x)\gamma^\mu \frac{1}{2} \sigma_\alpha A_\mu^a(x, z_L) \lambda(x), \quad (37)
\]
where the strength of the interaction is included into the five-dimensional gauge field as is exhibited by the normalization of its kinetic term. Integrating out the heavy KK modes, the gauge

\(^1\)Here is a simple routine written in Mathematica to solve the boundary value problem Eq. (7) by reducing it to an initial value problem with a variable boundary condition that the software environment can handle with built-in commands

\[
sys[s_, k_]:=\{D[w2[z]*F1[z], z]+eps^(1/3)*k*w1[z]*F[z]==0, D[F[z], z]==F1[z], F[zR]==VR, F1[zR]==s\};
solIV[s_?NumericQ, k_?NumericQ]:=\{F, F1\} /. NDSolve[sys[s, k], \{F, F1\}, \{z, zL, zR\}][[1]]; FlatzL[s_?NumericQ, k_]:=solIV[s, k][[2]][zL]; sFromzL[k_]:=FindRoot[FlatzL[s, k]==VL, \{s, s0min, s0max\}, MaxIterations -> 100]; solBV[k_?NumericQ, z_?NumericQ]:=solIV[sFromzL[k][[1, 2]], k][[1]][z];
\]
While this program is used for the calculation of the \( W \)-tower KK eigenspectrum, its modification to accommodate Neumann-Neumann boundary conditions is self-obvious.

\(^2\)Obviously, since we are allowing for KKs to couple to light fermions, the model induces non-oblique corrections as well.
Table 1: The KK scales and masses (in GeV) of the Z-boson KK tower for \( z_L = 10^7 \) and different values of \( \varepsilon \).

| \( \varepsilon \) | \( m_K \) | \( \nu_1 \) | \( \nu_2 \) | \( \nu_3 \) | \( \nu_4 \) | \( \nu_5 \) | \( \nu_6 \) | \( \nu_7 \) | \( \nu_8 \) | \( \nu_9 \) | \( \nu_{10} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3593.6 | 4512.7 | 7681.4 | 10812.6 | 13932.9 | 17048.2 | 20160.9 | 23272.0 | 26382.1 | 29491.5 | 34154.7 |
| \( \frac{\pi}{12} \) | 1597.3 | 4454.3 | 7638.6 | 10762.3 | 13867.9 | 16966.8 | 20062.8 | 23157.2 | 26250.8 | 29343.7 | 32436.2 |
| \( \frac{\pi}{12} \) | 1128.2 | 4453.6 | 7638.0 | 10761.6 | 13867.0 | 16965.6 | 20061.4 | 23155.6 | 26248.9 | 29341.6 | 32433.9 |

fields are effectively replaced by \( A_\mu^\pm(x, z_L) \rightarrow W_\mu^\pm(x)\psi_0^\pm(z_L) \) and

\[
A_\mu^3(x, z_L) \rightarrow B_\mu(x)\psi_0(z_L)
\]

\[
+ Z_\mu(x)\psi_1(z_L) \left\{ 1 - \sum_{k=1}^{M} \frac{v_0^2}{v_k^2} \left( 1 - \frac{D_{k-1}}{D_M} \right) \prod_{m=1}^{k} f_m^{-1} \left[ \left( 1 - \frac{D_{k-1}}{D_M} \right) \prod_{m=1}^{k} f_m^{-1} - \frac{\psi_{2k+1}(z_L)}{\psi_1(z_L)} \right] \right\},
\]

where the result is valid to order \( \delta^2 \) and we ignored four-fermion contributions.

Since the HLS requires \( \psi_0 = 1 \), and the electromagnetic group remains unbroken, we can impose the standard normalization on the photon kinetic term. This eliminates oblique corrections from the photon sector and yields the relation

\[
g_\gamma^2 = \alpha_0 e^2,
\]

where \( e = \sqrt{4\pi\alpha_{em}} = g \sin \theta_W \) is the electric charge, expressed in terms of the \( SU(2) \) coupling \( g \).

Rescaling the photon field, \( B_\mu \rightarrow eB_\mu \), the strength of the interaction migrates to the photon-fermion terms. The standard normalization for the SM massive bosons is achieved by rescaling the \( Z- \) and \( W- \)fields \( Z_\mu \rightarrow g_5 K_Z^{-1/2} Z_\mu, W_\mu \rightarrow g_5 K_W^{-1/2} W_\mu \), such that the bosons masses and boson-fermion interaction couplings read

\[
m_W = m_{0,\pm}, \quad m_Z \simeq m_1 \left( 1 + \sum_{k=1}^{M} \frac{v_0^2}{v_k^2} \left( 1 - \frac{D_{k-1}}{D_M} \right) \prod_{m=1}^{k} f_m^{-2} \right)
\]

and

\[
g_{cc} = e \psi_1^\pm(z_L) \left( \frac{\alpha_0}{\gamma_{0,\pm}} \right)^{1/2},
\]

\[
g_{nc} \simeq e \psi_1(z_L) \left( \frac{\alpha_0}{\beta_0} \right)^{1/2} \left( 1 + \sum_{k=1}^{M} \frac{\psi_{2k+1}(z_L)}{\psi_1(z_L)} \frac{v_0^2}{v_k^2} \left( 1 - \frac{D_{k-1}}{D_M} \right) \prod_{m=1}^{k} f_m^{-1} \right),
\]

respectively.

The oblique corrections are obtained by inverting the matrix of lepton-boson gauge coupling constants and the \( W \) mass \[19\], yielding the result

\[
\begin{pmatrix}
\alpha_{em} (S - 2c_W^2c_T) \\
\alpha_{em} U \\
\Delta
\end{pmatrix}
= \begin{pmatrix}
4c_W^2(c_W^2 - s_W^2) & -4c_W^2 & 2c_W^2 \\
8s_W^2 & -8s_W^2 & 0 \\
0 & 2 & -1
\end{pmatrix}
\begin{pmatrix}
1 - \frac{\sin \theta_W}{m_W^2} g_{nc} \\
1 - \frac{\sin \theta_W}{m_W^2} g_{cc} \\
1 - \frac{m_W^2}{c_W^2 m_Z^2}
\end{pmatrix}.
\]

Since we are not introducing an additional \( U(1) \) coupling (to avoid additional model dependence) to obtain correct hypercharges, we will not be able to separate \( S \) and \( T \) parameters in the
combination $S - 2c_W^2 T$. However, the fact that the model enjoys the custodial symmetry in the bulk implies that the $T$ parameter is small and thus the above combination is dominated by $S$.

Below, we choose unit normalizations for all wave functions, i.e., $N_n^a = 1$, except for $\alpha_0$ which cannot be altered due to its constrained form by HLS. The value of $\psi_W$ at $z_L$ is very weakly dependent on $z_R$ which, to the accuracy that we worked with, could be ignored. Since the overall phase of the KK wave functions is not automatically fixed, we choose the same sign for their asymptotic values as for the ultralight modes, i.e., $\psi_{2k+1}(z_L)/\psi_1(z_L) > 0$. The values of the KK wave functions, localized in the vicinity of $z = 0$, are suppressed stronger at the cut-off $z_L$ compared to the ultralight modes, by a factor $O(10^{-1})$. The holographic description, which we advocated here, is valid in the strong coupling regime on the gauge theory side, i.e., when the ’t Hooft coupling $\lambda \equiv g_{YM}^2 N \gg 1$. Its value can be estimated from the following equation

$$\lambda = \frac{27 \pi^2}{N \alpha_{em} \alpha_0},$$

which imposes an upper limit on the cut-off parameters $z_L$ for reliable applicability of holography.

We analyzed the mass spectra and oblique corrections as functions of the cut-offs, $z_{L,R}$ and parameter $\varepsilon$ separating the confinement and electroweak symmetry breaking scale fixing two parameters of the model to the $Z$ mass $m_Z$ and the fine structure constant $\alpha_{em}$ at the $Z$ pole. For $z_{L,R} < 10^6$, we found that the $S$ parameter is greater than one and this part of the parameter space is excluded by precision electroweak measurements. For $z_L = 10^6$, we varied the right cut-off in the interval $30 \leq z_R/z_L \leq 40$. Ignoring the contribution of KK towers, i.e., setting $M = 0$ in Eq. (41), one finds that the mixing angle $s_0^2 = 1 - \mu_{0,\pm}^2/\nu_0^2$, the KK scale $m_K$ (in GeV)

Figure 1: The oblique parameters as a function of $\log \varepsilon^{-1}$ for increasing number of KK modes $M$ for the cut-off $z_L = 10^7$. 


and the $S$-parameter vary in the intervals

\[
0.24355 \leq s_0^2 \leq 0.22626, \quad 2421.06 \leq m_K \leq 2448.57, \quad 0.719 \leq (S - 2c_W^2T) \leq 0.623, \\
0.24328 \leq s_0^2 \leq 0.22599, \quad 1075.79 \leq m_K \leq 1088.01, \quad 0.687 \leq (S - 2c_W^2T) \leq 0.596, \\
0.24327 \leq s_0^2 \leq 0.22599, \quad 759.864 \leq m_K \leq 768.494, \quad 0.683 \leq (S - 2c_W^2T) \leq 0.601,
\]

for $\varepsilon = 1, \frac{1}{64}, \frac{1}{512}$, respectively. For $z_L = 10^7$, we fixed $s_0$ to the experimental value $s_0^2 = s_W^2 = 0.23108$, such that the tree-level $\rho = 1$, which corresponds to the choices $z_R/z_L = 36.835, 36.772, 36.771$ for $\varepsilon = 1, \frac{1}{64}, \frac{1}{512}$, respectively. The resulting neutral KK mass spectra are shown in the Table [1] and the oblique corrections, with the lowest 10 KK modes included with equal mixing angles, implying that $f_k \simeq 1$, are displayed in Figure [1]. One finds that in this regime the 't Hooft coupling is moderate $\lambda \simeq 13/N$ (with a very weak dependence on $\varepsilon$) while higher values of $z_L$, though yield smaller $S$ compatible with precision electroweak measurements, start falling outside of the strong-coupling regime. While the contribution of KKs into the oblique corrections definitely reduces the uncorrected $S$-parameter, it is still large (for $z_L = 10^7$) compared to the experimental values. Though direct comparison of the two values is not straightforward (see, e.g., [21] for the most recent discussion) since the subtraction of the Higgs boson and accompanying vector boson loops was not done, a mechanism for further reduction of the Peskin-Takeuchi parameter can be achieved by coupling the localized fermions to the gauge bosons at $z \ll z_L$ where the KK wave functions are larger or by using delocalized fermions in the bulk [23]. Yet understanding of $1/N$-corrections in the gauge dual can be done by means of incorporation of the meson one-loop effects stemming from the DBI actions. Due to HLS the power counting is well-defined in this framework [24]. These questions will be addressed elsewhere.

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References

[1] S. Weinberg, Phys. Rev. D 13 (1976) 974; Phys. Rev. D 19 (1979) 1277; L. Susskind, Phys. Rev. D 20 (1979) 2619.

[2] K. Lane, Two lectures on technicolor, [arXiv:hep-ph/0202255]; C.T.Hill, E.H. Simmons, Phys. Rept. 381 (2003) 235; (E) 390 (2004) 553; F. Sannino, Dynamical stabilization of the Fermi scale, [arXiv:0804.0182] [hep-ph]; Conformal dynamics for TeV physics and cosmology, [arXiv:0911.0931] [hep-ph].

[3] B. Holdom, J. Terning, Phys. Lett. B 247 (1990) 88; M. Golden, L. Randall, Nucl. Phys. B 361 (1991) 3.

[4] M.E. Peskin, T. Takeuchi, Phys. Rev. D 46 (1992) 381.

[5] B. Holdom, Phys. Lett. B 150 (1985) 301; T.W. Appelquist, D. Karabali, L.C.R. Wijewardhana, Phys. Rev. Lett. 57 (1986) 957; K. Yamawaki, M. Bando, K. Matumoto, Phys. Rev. Lett. 56 (1986) 1335.
[6] M.A. Luty, T. Okui, J. High Ener. Phys. 0609 (2006) 070.
[7] T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843.
[8] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505.
[9] C.D. Carone, J. Erlich, M. Sher, Phys. Rev. D 76 (2007) 015015.
[10] T. Hirayama, K. Yoshioka, J. High Ener. Phys. 0710 (2007) 002.
[11] O. Mintakevich, J. Sonnenschein, J. High Ener. Phys. 0907 (2009) 032.
[12] K. Agashe, A. Delgado, M. May and R. Sundrum, J. High Ener. Phys. 0308 (2003) 050.
[13] R. S. Chivukula, D. A. Dicus, H. J. He, Phys. Lett. B 525 (2002) 175.
[14] C. Csaki, C. Grojean, H. Murayama, L. Pilo, J. Terning, Phys. Rev. D 69 (2004) 055006;
    C. Csaki, C. Grojean, L. Pilo, J. Terning, Phys. Rev. Lett. 92 (2004) 101802.
[15] O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322 (2007) 1420.
[16] C.G. Callan, S.R. Coleman, J. Wess, B. Zumino, Phys. Rev. 177 (1969) 2247.
[17] M. Bando, T. Kugo, K. Yamawaki, Prog. Theor. Phys. 73 (1985) 1541; Nucl. Phys. B 259
    (1985) 493; Phys. Rept. 164 (1988) 217;
    M. Bando, T. Fujiwara, K. Yamawaki, Prog. Theor. Phys. 79 (1988) 1140;
    M. Harada, K. Yamawaki, Phys. Rept. 381 (2003) 1.
[18] R. Casalbuoni, S. De Curtis, D. Dominici, R. Gatto, Nucl. Phys. B 282 (1987) 235.
[19] C.P. Burgess, S. Godfrey, H. Konig, D. London, I. Maksymyk, Phys. Rev. D 49 (1994) 6115.
[20] J. Hirn, V. Sanz, Phys. Rev. D 76 (2007) 044022.
[21] Y. Cui, T. Gherghetta, J.D. Wells, J. High Ener. Phys. 0911 (2009) 080.
[22] C. Csaki, J. Erlich, J. Terning, Phys. Rev. D 66 (2002) 064021;
    G. Cacciapaglia, C. Csaki, C. Grojean, J. Terning, Phys. Rev. D 70 (2004) 075014.
[23] G. Cacciapaglia, C. Csaki, C. Grojean, J. Terning, Phys. Rev. D 71 (2005) 035015.
[24] M. Harada, S. Matsuzaki, K. Yamawaki, Phys. Rev. D 74 (2006) 076004.