Computing the Filled Julia Set

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Abstract

The Julia set is one of the most remarkable and exciting fields of dynamic mathematics. Because it contains very beautiful graphics.

Through this study we will highlight how the filled Julia set is computed. It is worth noting that there is an integrative work between computers and mathematical calculations to get great results to complete Julia Set filling process.

Keywords: Filled Julia set, Orbit, Escape, Infinity.

1. Introduction

It is known that there has been a tremendous development in the field of complex analytic dynamics in the last four decades. And some kind of update has been added in 1980 to this field. This great development was represented by a group of the best scientists obtaining graphics images of Julia set by using the computer. Among these scientists, Hubbard, Sullivan, Douady, and others.

There are amazing computer graphics images arise in this field which are called the Julia Set. One of our goals in this research is to explain what these images mean. However in this study we will deal with only quadratic functions that has the form

\[ Q_d(m) = m^2 + d, \]

where both \( d \) and \( m \) are complex numbers.

To understand the Julia set, suppose we have a special complex \( d \) value. For \( m^2 + 2 \) the filled Julia set is the summation of every seeds under \( m^2 + 2 \) whose orbit does not elopment to \( (\infty) \). So, for each \( d \) value we will get various filled Julia set.
2. Computing the Filled Julia Set

In this part of this study we will use the computer to determine different forms of filled Julia set [1,2].

The best way to compute the filled Julia, is by using the definition of $L_d$, which is:

If there exist $L$ and $|Q_d^u(m)| < L$, the orbit of $m$ under $Q_d$ is bounded for all $u$. Except that the orbit is unbounded.

We consider group of points in some region in the plane. For every point in this group we will compute the corresponding orbit, depending on this orbit tends to infinity or not. If the orbit does not tends to $\infty$, then this main point is in $L_d$ in this case we will color the main point black. If the orbit tends to $\infty$, so we leave the main point white. The important question remains here is how can we diagnosed whether the orbit tends to $\infty$. The answer to this question is in the following theory:

**Theorem:** let $|m| \geq |d| > 2$. Then we have $|Q_d^u(m)| \rightarrow \infty$ as $u \rightarrow \infty$.

Now, to find algorithm for the filled Julia set we will choose a maximum number of iterations, $N$, for every point $z$ in the group, compute the first $B$ points on the orbit of $m$. If $|Q_d^j(m)| > \max \{ |d|, 2 \}$ for some $j \leq B$, then stop iteration and color $m$ white. If $|Q_d^j(m)| \leq \max \{ |d|, 2 \}$ for all $j \leq B$, then color $m$ black. The black point have orbit that do not tends to $\infty$, while white points do. So the black points will be the points within the filled Julia set [1,3].

To clarify the process of orbit escaping to infinity or not escaping and remaining somewhere, we will take the following example:

Suppose we have: $f(m) = m^2 + d$, let $d = 0$,

\[
\begin{align*}
  f(0) &= (0)^2 + 0 = 0, \\
  f(0) &= (0)^2 + 0 = 0.
\end{align*}
\]

We conclude here, the orbit do not escape to $\infty$ for $d = -1$,

\[
\begin{align*}
  f(0) &= (0)^2 + (-1) = -1, \\
  f(-1) &= (-1)^2 + (-1) = 0.
\end{align*}
\]

This orbit will remain somewhere and will not escape to $\infty$.

Now let $d = 2i$,

\[
\begin{align*}
  f(0) &= (0)^2 + 2i = 2i, \\
  f(2i) &= (2i)^2 + 2i = -4 + 2i, \\
  f(-4 + 2i) &= (-4 + 2i)^2 + 2i = 12 - 14i, \\
  f(12 - 14i) &= (12 - 14i)^2 + 2i = -52 - 336i, \\
  f(-52 - 336i) &\rightarrow \text{big} \rightarrow \text{biger} \rightarrow \text{tends to } \infty.
\end{align*}
\]
3. Important Remarks

Remark (1)
For every new \( d \) values, \( L_d \) assumes that there are many different shapes. Frequently, \( L_d \) contains of a many connected set in the plane. Some of these areas you can watch it in figure (1) [1,4].

\[
\begin{align*}
\frac{1}{g_{1856}} &= -1.037 + 0.17i \\
\frac{1}{g_{1856}} &= -0.52 + 0.57i \\
\frac{1}{g_{1856}} &= 0.295 + 0.55i \\
\frac{1}{g_{1856}} &= 0.295 + 0.55i
\end{align*}
\]

Figure (1)

Remark (2)
The appearance of the the Julia set for \( Q_d \) is typically a small and refined form, figure (2) illustrates this fact.
4. Conclusion

This research focused on providing a practical and rapid method to compute the filled Julia set through mathematical calculations and with the help of a computer to draw different shapes for Julia Set. After reviewing this research, the reader will be able to perform the calculations to find out Points that belong to or do not belong to the filled Julia set. In addition to sweating on the various shapes of Julia set.

REFERENCES

1. Robert L Devaney,(1992), “A First Course in Chaotic Dynamical Systems Theory and Experiment”, Perseus Books Publishing, L.L.C., USA, Boston.
2. Robert L. Devaney, (1994), “Complex Dynamical Systems”, Proceedings of Symposia in Applied Mathematics, vol. 49.

3. Mark Braereman, Michael Yampolsky, (2009), “Computability of Julia Sets”, Springer, Berlin.

4. Alan F. Beardon, (1991), “Iteration of Rational Function”, Springer-Verlag, New York.