A New, Simple Model for Black Hole High Frequency QPOs

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Abstract. Observations of X-ray emissions from binary systems have always been considered important tools to test the validity of General Relativity in strong-field regimes. The pairs and triplets of high frequency quasi-periodic oscillations observed in binaries containing a black hole candidate, in particular, have been proposed as a means to measure more directly the black hole properties such as its mass and spin. Numerous models have been suggested over the years to explain the QPOs and the rich phenomenology accompanying them. Many of these models rest on a number of assumptions and are at times in conflict with the most recent observations. We here propose a new, simple model in which the QPOs result from basic $p$-mode oscillations of a non-Keplerian disc of finite size. We show that within this new model all of the key properties of the QPOs: $a)$ harmonic ratios of frequencies even as the frequencies change; $b)$ variations in the relative strength of the frequencies with spectral energy distribution and with photon energy; $c)$ small and systematic changes in the frequencies, can all be explained simply given a single reasonable assumption.

A number of models have been proposed to explain the high frequency quasi-periodic oscillations (HFQPOs) seen in accreting black hole systems [1, 2] and two of these seem particularly promising in our view. The first model proposed, the discoseismic model [3], asserts that $g$-modes should become trapped in the potential well of a Keplerian (i.e. geometrically thin) disc in a Kerr potential. The size of the region where the modes are trapped depends on both the mass and the spin of the accreting black hole. Additional frequencies of oscillation should be expected from pressure modes and corrugation modes. The predictions of the model are well summarised by Kato [4]. Given pairs of high frequency QPOs and a proper identification of the frequencies with the particular modes, one can measure both the black hole mass and spin to relatively high accuracy [5]. The discoveries of three systems where the QPOs show a harmonic structure with relatively strong peaks seen in integer ratios [6, 2] $1:2$, $2:3$, or $1:2:3$ seem to cast some doubt upon this model. However, the model remains viable for the intermediate frequency QPOs in GRS 1915+105, seen at 67 Hz [7] and 40 Hz [6]. The second model proposed, the parametric resonance model [8], asserts that a harmonic relationship in the QPO frequencies can be produced as a result of resonances and was motivated by the small number integer ratios of some of the QPOs seen from black holes candidates. In particular, the parametric resonance model suggests that an initial turbulence spectrum is amplified at a radius where the (radial) epicyclic frequency is in resonance with the (azimuthal) orbital frequency, with the two frequencies being in (small) integer ratios. These annuli tend to be close to the black hole event horizon for the observed frequencies and, hence, general relativistic effects can be important. Given a sufficiently accurate mass estimate for the black hole and a proper identification of the ratio of the frequencies in resonance, again, a black hole spin can be measured fairly accurately. It should be noted that given the observed frequencies of 162 and 324 Hz in GRS 1915+105 it is not possible to produce the 40 and 67 Hz QPOs with a discoseismic model and the higher frequency QPOs with a resonance model while retaining the same values for the black hole mass and spin [9].

A potential problem for both models is the observational evidence for a frequency jitter in the HFQPOs of XTE J1550-564, i.e. for small variations of about 10% in frequencies for about 15% of the time. This difficulty could be particularly severe for the parametric resonance model, whose relevant frequencies confine the resonance to a narrow region in radial coordinates (This situation is worsened for the 1:3 resonance for which the radial variation of the frequencies is even more rapid [2]).

We here propose an explanation for the high frequency QPOs in black hole candidates that has distinct novel features and is based on a single assumption, namely that a non-Keplerian disc orbits around a rotating black hole. More specifically, we propose a model based on the oscillation properties of a geometrically thick disc

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1 This work was in collaboration with S’i. Yoshida, T. J. Maccarone and O. Zanotti
located in the vicinity of a Kerr black hole. A more detailed discussion of this model can be found in [10].

In contrast to a Keplerian disc, a non-Keplerian disc (i.e. a torus) is confined to a finite-size region determined uniquely by the distribution of the specific angular momentum and by the pressure gradients. In similarity with a star, however, restoring forces can be used to classify different modes of oscillation in the disc. A first restoring force is the centrifugal force, which is responsible for inertial oscillations of the orbital motion of the disc and hence for epicyclic oscillations. A second restoring force is the gravitational field in the direction vertical to the orbital plane and which will produce a harmonic oscillation across the equatorial plane if a portion of the disc is perturbed in the vertical direction. A third restoring force is provided by pressure gradients and the oscillations produced in this way are closely related to the sound waves propagating in a compressible fluid. In a geometrically thick disc, the vertical and horizontal oscillations are in general coupled and more than a single restoring force can intervene for the same mode. Referring the reader to [4] for a more detailed discussion, it is here sufficient to remind that c modes are essentially controlled by the vertical gravitational field, that g modes are mainly regulated by centrifugal and pressure-gradient forces, and that all of the restoring forces discussed above play a role in the case of p modes. It is also useful to underline that because we are here interested in modes with a prevalent horizontal motion and frequencies above the epicyclic one, we are essentially selecting “inertial-acoustic” modes having centrifugal and pressure gradients as only restoring forces. Hereafter we will refer to these simply as p modes.

Following a recent investigation of the dynamics of perturbed relativistic tori [11], we have analysed the global oscillation properties of such systems and, more specifically, we have performed a perturbative analysis of axisymmetric modes of oscillation of relativistic tori in the Cowling approximation. The eigenvalue problem that needs to be solved to investigate consistently the oscillation properties of fluid tori is simplified considerably if the vertical structure is accounted for by an integration of the gravitational field in the vertical direction. Doing so removes one spatial dimension from the problem and yields to simple ordinary differential equations. While an approximation, this simpler model captures most of the features relevant to the discussion of p-modes in relativistic tori and is a first-step in a research field which is in great part unexplored.

We have solved the eigenvalue problem for a number of vertically-integrated relativistic tori which have been perturbed through the introduction of arbitrary pressure perturbations. The tori are modelled as made of a perfect fluid obeying a polytropic equation of state for an ultrarelativistic gas of electrons (other polytropic indices have also been considered). The equilibrium models span a large parameter space in which both the sizes and the physical properties (e.g. pressure and density profiles, distributions of specific angular momentum, polytropic indices) are varied considerably [12]. The eigenfunctions and eigenfrequencies found in this way correspond to global p-modes, have been computed for the fundamental mode of oscillation as well as for the first few overtones, and all have been found to be in a harmonic sequence $2 : 3 : 4 : \ldots$ to within a few percent.

Figure 1 shows a typical example of the solution of the eigenvalue problem for a black hole with mass $M = 10 M_\odot$ and its comparison with the observations of XTE J1550-564 (other sources could equally have been used). In particular, we have plotted the value of the different eigenfrequencies found versus the radial extension $L$ of the torus, expressed in units of the gravitational radius $r_g \equiv GM/c^2$. The sequences have been calculated for a distribution of specific angular momentum following a power-law [10], keeping the position of maximum density in the torus at $r_{\text{max}} = 3.489$, and for a black hole with dimensionless spin parameter $a \equiv J/M^2 = 0.94$ to maximize the value for $L$. Indicated with a solid line are the fundamental frequencies $f$, while the first overtones $a_1$ are shown with a dashed line; each point on the two lines represents the numerical solution of the eigenvalue problem. The asterisks represent the frequencies of the HFQPOs detected in XTE J1550-564 at 184 and 276 Hz, respectively. Note that the two plotted eigenfrequencies are close to a $2 : 3$ ratio over the full range of $L$ considered (this is shown in the inset) and that while they depend also on other parameters in the problem (e.g. the position of $r_{\text{max}}$, the angular momentum distribution, the polytropic index), these dependences are very weak so that the frequencies depend effectively on $M$, $a$ and $L$. As a result, if $M$ and $a$ are known, the solution of the eigenvalue problem can be used to determine the dimension of the oscillating region $L$ accurately.

On the basis of these results, we suggest that the HFQPOs observed in black hole candidate systems can be interpreted in terms of p-mode oscillations of a small torus [10]. Note that all the properties discussed so far depend simply on the existence of non-Keplerian disc orbiting in the vicinity of a black hole. Such a configuration could be produced in a variety of ways and basically whenever an intervening process [e.g. large viscosity, turbulence, hydrodynamical and magnetohydrodynamical (MHD) instabilities] modifies the Keplerian character of the flow.

In what follows, we discuss how the predictions of such a model can match the observed phenomenology and can be used to extract astrophysical information. (i) The harmonic relations between the HFQPO frequencies in black hole candidates are naturally explained within this model. In a sufficiently small torus, in fact, p modes effectively behave as trapped
sound waves with allowed wavelengths that are \( \lambda = (2/2)L, (3/2)L, (4/2)L, \ldots \). The frequencies of these standing waves would be in an exact integer ratio only if the sound speed were constant. In practice this does not happen, but the eigenfrequencies found are in a sequence very close to 2 : 3 : 4.

(ii) Being global modes of oscillation, the same harmonicity is present at all radii within the torus. This removes the difficulty encountered in the resonance model and provides also a larger extent in radial coordinates where the emissivity can be modulated.

(iii) Because the radial epicyclic frequency represents the upper limit for the disc eigenfrequencies, these scale like \( 1/M \). This is in agreement with the observations made of XTE J1550-564 and GRO J1655-40 as long as the spins are similar [13]. On the other hand, a rather narrow range of black hole spins has been suggested as a possible explanation for the narrow range of radio-to-X-ray flux ratios in the Galactic X-ray binary systems [14].

(iv) The frequency jitter can be naturally interpreted in terms of variations of the size of the oscillating cavity \( L \). The frequencies may also drift over a large range with the harmonic structure preserved.

(v) The observed variations in the relative strength of the peaks can be explained as a variation in the perturbations the torus is experiencing. (This has been reproduced with numerical simulations [11]). Furthermore, while the low frequency overtones are energetically favoured and the corresponding eigenfunctions possess less nodes, any number of harmonics could in principle be observed.

(vi) The evidence that an overtone can be stronger than the fundamental in the harder X-ray bands [1], can be explained simply given that the overtone is an oscillation preferentially of the innermost (and hottest) part of the accretion flow (see [12] for the eigenfunctions).

A few remarks are worth making at this point. The first one is about the existence of a non-Keplerian orbital motion: this is required only very close to the black hole (the inner edge of the torus can be in principle be located at the marginally bound orbit) and beyond this region the fluid motion can be Keplerian. Stated differently, the HFQPOs observed could be produced by the inner parts of an otherwise standard, nearly-Keplerian, geo-
metrical thin accretion disc, where a variety of physical phenomena can introduce pressure-gradients (3D numerical simulations in MHD seem to indicate the formation of these tori near the black hole [15]). The second remark is about the stability of these tori to non-axisymmetric oscillations. It is well-known that a stationary (i.e. non-accreting) perfect fluid torus flowing in circular orbits around a black hole is subject to a dynamical instability triggered by non-axisymmetric perturbations [16]. It is less well-known, however, that the instability can be suppressed if the flow is non-stationary. Stated differently, a fluid torus around black could be stable to non-axisymmetric perturbations if mass-accretion takes place [17, 18, 19]. Because the tori discussed here are assumed to be the terminal part of standard accretion discs, we expect them to be stable to non-axisymmetric perturbations as long as magnetic fields are unimportant.

Note that this model also offers a simple way of extracting astrophysically relevant information from the observation of HFQPOs. We recall that the fundamental \( p \)-mode frequency tends to the radial epicyclic frequency \( \kappa_2 \) at \( r_{\max} \), in the limit of a vanishing torus size, and that for a Kerr black hole this frequency is a function of its mass and spin only. Exploiting this property, a first estimate of the black hole spin and of the size of the oscillating torus can be obtained once the lower frequency in the HFQPOs is measured accurately. In this case, in fact, given a black hole candidate with measured mass \( M_\ast \), a lower limit on the black hole spin \( a \) can be deduced as the value of \( a \) at which the maximum epicyclic frequency is equal to the lower observed HFQPO frequency [12]. Because for small tori \( L \sim r_{\max} \), once \( a \) has been determined, the radial position \( \hat{r} \) of the maximum of the epicyclic frequency provides an upper limit on the size of the torus, thus providing an estimate difficult to obtain through direct observations. These two estimates should be considered first approximations only and can be further refined through the eigenvalue problem.

A final comment will be reserved to the possibility of finding support to this model from the observational data. There are now extensive observations of these black hole HFQPOs and interesting spectral and flux correlations are beginning to be found (see, for instance, [2], and references therein). These observations could be used to deduce the presence of a small torus in the terminal part of the accretion discs.

Being based on a single assumption, our model for HFQPOs is simply constructed, but, equally simply, it can be confuted. Two observational constraints, if not met, will cast serious doubts about this model. The first one requires the lower HFQPO frequency to be always less than the maximum possible epicyclic frequency for a black hole of mass \( M_\ast \), i.e. lower HFQPO frequency \( \leq \max \{ \kappa_2(\vec{a} = 1, M_\ast) \} \). The second constraint, instead, requires that even if the HFQPOs frequencies change as a result of the change in size of the torus, they should nevertheless appear in a harmonic ratio to within 5-10%. Both of these observational requirements are sufficiently straightforward to assess and therefore provide a simple and effective way of falsifying this model.

Clearly, more work is needed to assess whether \( p \)-mode oscillations in a small-size torus represent the correct interpretation of the black hole QPO phenomenology. A first step in this direction will be made when a convincing evidence is provided that \( p \) modes can be responsible for the modulated X-ray emission in black-hole spacetimes; work is now in progress in this direction [20]. We leave our final remark on the complex phenomenology associated to HFQPOs in BHCs to a well-known quote, whose view we feel to share: \"essentia non sunt multiplicandae praeter necessitatem\" [21].

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