A COMPREHENSIVE OVERVIEW AND SURVEY OF RECENT ADVANCES IN META-LEARNING

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ABSTRACT

This article reviews meta-learning which seeks rapid and accurate model adaptation to unseen tasks with applications in image classification, natural language processing and robotics. Unlike deep learning, meta-learning uses few-shot datasets and concerns further improving model generalization to obtain higher prediction accuracy. We summarize meta-learning models in three categories: black-box adaptation, similarity based method and meta-learner procedure. Recent applications concentrate upon combination of meta-learning with Bayesian deep learning and reinforcement learning to provide feasible integrated problem solutions. We present performance comparison of recent meta-learning methods and discuss future research direction.

Keywords Meta-Learning · Few-Shot Learning · Meta-Reinforcement · Meta-Imitation · Meta-Learner

1 Introduction

1.1 Background

Meta-learning is known as learning-to-learn models which allow rapid and precise adaptation to unseen tasks [1]. In [2], meta-learning and transfer learning are regarded as synonyms. Learning-to-learn gains attention under the research topic of continual learning. Similar to lifelong learning which accumulates knowledge and builds one model applicable to all, meta-learning aims to develop a general framework that can be used in a wide variety of tasks.

A most important application of meta-learning is few-shot learning [3, 4, 5, 6, 7, 8, 9, 10, 11]. In image classification, few-shot learning refers to tasks where there are only less than ten images within each category. Human is capable of grasping new concepts from few demonstrations and utilizing past knowledge to identify the category of a new object efficiently. In few-shot learning, the goal is to develop human-like machines that can make fast and accurate classification based upon few images. For ancient languages where we only have few observations, and for dialects with only a small group of people, we resort to few-shot meta-learning for fast and accurate predictive modeling.

Meta-learning models how we learn. By learning how to learn, meta-learning introduces a flexible framework that applies to different tasks. For example, Model-Agnostic Meta-Learning (MAML) [12] is applicable to all tasks which can be solved with gradient descent. On the other hand, based upon pre-trained deep models, meta-learning adapts to new tasks rapidly without precision loss [13]. In meta-learning, both reinforcement learning [14] and neural network [15] can be applied to search for an optimizer autonomously. In both cases, a general representation of an optimizer search space should be defined explicitly.

Recently meta-learning focuses upon integration with other frameworks such as meta-reinforcement learning [16, 17, 18, 19, 20, 21, 22, 23] and meta-imitation learning [24, 11] which are closely associated with robotics research.

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Reinforcement learning estimates optimal actions based upon given policy and environment [25]. Imitation learning evaluates the reward function from observing behaviors of another agent in the same environment [26]. Few-shot learning helps an agent to make predictions based upon only few demonstrations from other agents [12]. Recent application of meta-learning integrates reinforcement learning, imitation learning and few-shot meta-learning for robots to learn basic skills and react to rare situations [27].

A typical assumption behind meta-learning is that tasks share similarity structure so that they can be solved under the same meta-learning framework. To relax this assumption and improve model generalization, one proposal is the integration of statistical models into meta-learning framework. Statistical models are less prone to over-fitting and robust to model misspecification. Machine learning models are data-driven, highly integrated and applicable to real-life problems. Under the framework of meta-learning, we can combine statistical models and machine learning for fast and accurate adaptation.

Structure of our article is as follows. Section 1.2 presents history of meta-learning research. Section 1.3 provides an outline of datasets and formulation. Section 2 summarizes meta-learning models. Section 2.4 surveys Bayesian meta-learning methods. Section 3.1 briefly reviews meta-reinforcement learning and section 3.2 surveys meta-imitation learning. Section 3.3 briefly introduces online meta-learning and section 3.4 reviews methods in unsupervised meta-learning. Section 3 and section 4 summarize main applications of meta-learning and discuss future research direction.

1.2 History

In [28], evolutionary algorithm in self-referential learning has the flavor of "learning-to-learn". Metareasoning [29] lays out early framework for reinforcement learning, causal learning and meta-learning. [30] lays out the framework of learning how to learn and proposes autonomous self-improvement of policy. Combination of meta-learning and reinforcement learning dates back to [31] where reinforcement learning with self-modifications in policy is proposed.

Earlier research of meta-learning concerns hyper-parameter optimization, as in [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]. In [33], neural net is trained simultaneously using distinct but related tasks in order to improve generalization. In [34], differentiation of a cross-validation loss function is used for optimization of several hyper-parameters simultaneously. In [16], meta-learning is integrated in RL to tune hyper-parameters such as learning rate, exploration-exploitation tradeoff and discount factor of future reward.

In addition, meta-learning can be applied to conduct autonomous model selection [44, 45, 40, 46] such as neural architecture optimization [36, 38, 47, 48]. In [44], meta-learning is applied to ranking and clustering, where algorithms are trained on meta-samples and an optimal model with highest prediction accuracy is selected. [45] considers using feedforward neural network, decision tree or support vector machine as learner model. Then it selects the class of models with the best performance on time series forecast. In [40], meta-learning is applied to select parameters in support vector machine (SVM), which demonstrates superior generalization ability in data modeling. Multi-objective particle swarm optimization is integrated with meta-learning to solve parameter selection.

Another research line is based upon learning how to learn by finding the optimal optimizer. [14] considers all first-order and second-order optimization methods under the framework of meta-imitation learning and minimizes the distance between predicted and target actions. Since a neural net can approximate any function, search space of optimizers is defined to be the set of all neural nets. Policy update per iteration can be approximated using neural nets where weight parameters are estimated jointly with step direction and step size. By learning optimizer autonomously, algorithms converge faster and outperform gradient descent in most iterations.

Meta-learning concentrates upon model adaptation between tasks which share similarity structure. For out-of-distribution tasks, we can extract the most similar experience from a large memory, and build predictive models based upon few data collected in a new situation. Classification of recent meta-learning methods is not exact since models tend to be more flexible and integrated in recent development. But with classification, we can roughly outline recent research directions for later mixing. Recent meta-learning methodology can be categorized into three classes which are model-based, metric-based and optimization-based methods, as in [49]. MANN (Memory-augmented neural networks) [50] belongs to the model-based category. It stores all model training history in an external memory and loads the most relevant model parameters from external memory every time a new task is present. Second, convolutional Siamese neural network [3] is within the metric-based category. A metric refers to the similarity between tasks. Siamese network designs a metric that is the similarity measure between convolutional features of different images. Matching networks [4], relation network [8] and prototypical network [6] are all metric-based methods.

Optimization-based technique includes a learner for model estimation at task level and a meta-learner for model generalization across tasks. In [5], a meta-learner updates parameters in learner on different batches of training data and validation data. For learners optimized with gradient descent, a meta-learner can be specified to be a long short term
memory model correspondingly. MAML (Model-Agnostic Meta-Learning) proposed in [12] does not impose any model assumption and is applicable to any learner model optimized with gradient descent. First-order meta-learning algorithm in [52] is also optimization-based, where iterative updates on parameters are designed to be the difference between previous estimate and new sample average estimate.

From another perspective, meta-learning can be formulated under the probabilistic framework of Bayesian inference. In [53], a Bayesian generative model is combined with deep Siamese convolutional network to make classification on hand-written characters. In [54], a Bayesian extension of MAML is proposed, where gradient descent in MAML is replaced with Stein variational gradient descent (SVGD). SVGD offers an efficient combination of MCMC and variational inference. In [5], amortization network is used to map training data onto weights in linear classifier. Amortization network is also used to map input data to task-specific stochastic parameter for further sampling. It utilizes an end-to-end stochastic training to compute approximate posterior distributions of task-specific parameters in meta-learner and labels on new tasks.

Recent applications concentrate upon robotics, where meta-imitation learning [26, 24, 11, 27] and meta-reinforcement learning [55, 56, 57, 18, 58, 59, 21] are of primary interest. Human beings can learn basic movements from few demonstrations so that researchers hope robots can do the same through meta-imitation learning. Imitation of action, reward and policy is achieved by minimization of regret function which measures the distance between current state and imitation target. In [26], MAML for one-shot imitation learning is proposed. Minimization of cloning loss leads to closely mimick target action that robots try to follow. It estimates a policy function that maps visual inputs to actions. [11] also integrates MAML into one-shot imitation learning. It collects one human demonstration video and one robot demonstration video for robots to imitate. The objective here is to minimize behavioral cloning loss with inner MAML parameter adaptation. It also considers domain adaptation with generalization to different objects or environment in the imitation task.

Meta-reinforcement learning (Meta-RL) is designed for RL tasks such as reward-driven situations with sparse reward, sequential decision and clear task definition [19]. RL considers the interaction between agent and environment through policy and reward. By maximizing reward, robots select an optimal sequential decision. In robotics, meta-RL is applied in cases where robots need rapid reaction to rare situations based upon previous experiences. [56] provides an overview of meta-RL models in multi-bandit problems. Meta-learned RL models demonstrate better performance than RL models from scratch. [21] it constructs a highly integrated meta-RL method PEARL which combines variational inference and latent context embedding in off-policy meta-RL. In addition, reward-driven neuro-activities in animals can be explained with meta-RL. In [19], phasic dopamine (DA) release is viewed as reward and meta-RL explains well the DA regulations in guiding animal behaviors with respect to the changing environment in animal experiment.

Besides meta-RL and meta-imitation learning, meta-learning can be flexibly combined with machine learning models for applications in real-life problems. For example, unsupervised meta-learning conducts rapid model adaptation using unlabelled data. Online meta-learning analyzes streaming data and performs real-time model adaptation. First, unsupervised meta-learning [61, 18, 10, 62, 63, 64] is for modelling unlabelled data. Unsupervised clustering methods such as adversarially constrained autoencoder interpolation (ACAI) [65], bidirectional GAN (BiGAN) [66], DeepCluster [66] and InfoGAN [67] are applied to cluster data and estimate data labels [65]. Afterwards meta-learning methods are used on unlabelled data and predicted labels obtained through unsupervised clustering. It is mentioned in [63] that unsupervised meta-learning may perform better than supervised meta-learning. Another combination of unsupervised learning and meta-learning is in [64]. It replaces supervised parameter update in inner loop with unsupervised update using unlabelled data. Meta-learner in the outer loop applies supervised learning using labeled data to update the unsupervised weight update rule. It demonstrates that this unique combination performs better in model generalization.

Second, online meta-learning analyzes streaming data so that the model should respond to changing conditions rapidly using a small batch of data in each model adaptation. [68, 59, 69] proposes a Bayesian online learning model ALPaCA where kernel-based Gaussian process (GP) regression is performed on the last layer of neural network for fast adaptation. It trains an offline model to estimate GP regression parameters which stay fixed through all online model adaptations. [59] applies MAML to continually update the task-specific parameter in prior distribution so that the Bayesian online model adapts rapidly to streaming data. [69] integrates MAML into an online algorithm follow the leader (FTL) and creates an online meta-learning method follow the meta-leader (FTML). MAML updates meta-parameters which are inputs into FTL and this integrated online algorithm generalizes better than previously developed methods.

Meta-learning algorithms are hybrid, flexible and can be combined with machine learning models such as Bayesian deep learning, RL, imitation learning, online algorithms, unsupervised learning and graph models. In these combinations, meta-learning adds a model generalization module to existing machine learning methods.
1.3 Datasets and Formulation

Few-shot datasets used as benchmarks for performance comparison in meta-learning literature are reviewed in [70]. Many meta-datasets are available at [https://github.com/google-research/meta-dataset](https://github.com/google-research/meta-dataset). Commonly used meta-learning datasets are briefly listed as follows.

- **Omniglot** [71] is available at [https://github.com/brendenlake/omniglot](https://github.com/brendenlake/omniglot). Omniglot is a large dataset of hand-written characters with 1623 characters and 20 examples for each character. These characters are collected based upon 50 alphabets from different countries. It contains both images and strokes data. Stroke data are coordinates with time in milliseconds.

- **ImageNet** [72] is available at [http://www.image-net.org/](http://www.image-net.org/). ImageNet contains 14 million images and 22 thousand classes for these images. Large scale visual recognition challenge 2012 (ILSVRC2012) dataset is a subset of ImageNet. It contains 1,281,167 images and labels in training data, 50,000 images and labels in validation data, and 100,000 images in testing data.

- **miniImageNet** [4, 73] is a subset of ILSVRC2012. It contains 60,000 images which are of size 84×84. There are 100 classes and 600 images within each class. [74] splits 64 classes as training data, 16 as validation data, and 20 as testing data.

- **tieredImageNet** [74] is also a subset of ILSVRC2012 with 34 classes and 10-30 sub-classes within each. It splits 20 classes as training data, 6 as validation data and 8 as testing data.

- **CIFAR-10/CIFAR-100** [75, 76] is available at [https://www.cs.toronto.edu/~kriz/cifar.html](https://www.cs.toronto.edu/~kriz/cifar.html). CIFAR-10 contains 60,000 colored images which are of size 32×32. There are 10 classes, each contains 6,000 images. CIFAR-100 contains 100 classes, each includes 600 images. CIAFR-FS [77] is randomly sampled from CIFAR-100 for few-shot learning in the same mechanism as miniImageNet. FC100 [76] is also a few-shot subset of CIFAR-100. It splits 12 superclasses as training data, 5 superclasses as validation data and 5 superclasses as testing data.

- **Penn Treebank (PTB)** [78] is available at [https://catalog.ldc.upenn.edu/LDC99T42](https://catalog.ldc.upenn.edu/LDC99T42). PTB is a large dataset of over 4.5 million American English words, which contain part-of-speech (POS) annotations. Over half of all words have been given syntactic tags. It is used for sentiment analysis and classification of words, sentences and documents.

- **CUB-200** [70, 79] is available at [http://www.vision.caltech.edu/visipedia/CUB-200.html](http://www.vision.caltech.edu/visipedia/CUB-200.html). CUB-200 is an annotated image dataset that contains 200 bird species, a rough image segmentation and image attributes.

- **CelebA** (CelebFaces Attributes Dataset) is available at [http://mmlab.ie.cuhk.edu.hk/projects/CelebA.html](http://mmlab.ie.cuhk.edu.hk/projects/CelebA.html). CelebA is an open-source facial image dataset that contains 200,000 images, each with 40 attributes including identities, locations and facial expressions.

- **YouTube Faces** database is available at [https://www.cs.tau.ac.il/~wolf/ytfaces/](https://www.cs.tau.ac.il/~wolf/ytfaces/). YouTube Faces contains 3,425 face videos from 1,595 different individuals. Number of frames in each video clip varies from 48 to 6070.

Among these datasets, **miniImageNet** [4, 73], **tieredImageNet** [74] and CelebA are the most difficult few-shot classification datasets. They are used to compare performances of meta-learning methods.

Dataset concepts used in meta-learning are outlined in figure [1]. Within each task, there are train data \(D_{\text{tr}}\), validation data \(D_{\text{val}}\) and test data \(D_{\text{test}}\). Support set \(S\) is the set of all labelled data. Train data and validation data are randomly sampled from support set. Query set \(Q\) is the set of all unlabelled data and test data are randomly sampled from query set. Meta-learning datasets include non-overlapping meta-train data \(D_{\text{meta-train}}\), meta-validation data \(D_{\text{meta-val}}\) and meta-test data \(D_{\text{meta-test}}\) which consists of tasks.

In supervised meta-learning, input is labelled data \((x, y)\), where \(x\) is an image or a feature embedding vector and \(y\) is label. Data model is \(y = h_\theta(x)\) parameterized by meta-parameter \(\theta\). As in [25], a task is defined as

\[ T = \{p(x), p(y|x), \mathcal{L}\}, \]

where \(\mathcal{L}\) is a loss function, \(p(x)\) and \(p(y|x)\) are data-generating distributions of inputs and labels. Task follows a task distribution \(T \sim p(T)\). K-shot N-class learning is a typical problem setting in few-shot meta-learning, where there are \(N\) classes each with \(K\) examples.
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Figure 1: Upper part shows data in each task: train data, validation data and test data. Lower part shows meta-train data, meta-validation data and meta-test data that consists of tasks. Support set is the set of all labelled data. Query set is the set of all unlabelled data.

2 Meta-learning

For fast and accurate adaptation to unseen tasks with meta-learning, we need to balance exploration and exploitation. In exploration, we define a complete model search space which covers all algorithms for the task. In exploitation, we optimize over the search space, identify the optimal learner and estimate learner parameter. For example, learning-optimizers method proposed in [14, 80] defines an extensive search space of optimizers for model exploration. In [81], mean average precision defined as precision in predicted similarity is a proposed loss function used for model exploitation.

On the other hand, meta-learning models can combine offline deep learning and online algorithms. In offline modeling, we aggregate past experiences by training a deep model on large historical datasets. In online algorithms, we continually adapt a deep offline model to conduct predictive analysis on few-shot datasets from novel tasks. For instance, memory-based meta-learning model in [82] stores offline training results in memory so that they can be retrieved efficiently in online model adaptation. Online Bayesian regression in [68] uses offline training results to initiate task-specific parameters in prior distributions and update these parameters continually for rapid adaptation to online streaming data. Based upon pre-trained deep learning models, meta-learning methods adapt to new tasks efficiently.

A typical assumption behind meta-learning is that tasks share similarity structure, and model generalization between tasks can be performed efficiently. Degree of similarity between tasks depends upon the similarity function which often constitutes the meta-learning objective function. Reliable adaptation between different tasks relies upon identifying the similarity structure between them. In meta-learning research, primary interest lies in relaxing the requirements upon degree of similarity between tasks and improving model adaptivity.

In this section, we briefly summarize meta-learning frameworks that emerge in recent literature into three categories: black-box adaptation [25], similarity-based approach, learner and meta-learner procedure. This classification of meta-learning frameworks is not exact and the boundaries are vague between different classes. It roughly points out research directions of meta-learning methods.

2.1 Black-Box Adaptation

Hyperparameter optimization can be achieved through random grid search or manual search [41]. Model search space is usually indexed by hyperparameters [14]. In adaptation to novel tasks, hyperparameters are re-optimized using data from the novel task. Optimizers can be approximated with neural networks or reinforcement learners [80]. Neural networks can approximate any function with good convergence results. By using neural networks, the optimizer search space represents a wide range of functions that guarantee better potential optima.

In [14], optimization is through guided policy search and neural network is used to model policy which is the gradient descent. Policy update is formulated as

\[
\Delta x \leftarrow \pi(f, \{x_0, \cdots, x_{i-1}\}) = -\gamma \sum_{j=0}^{i-1} \alpha^j \nabla_{x_j} f(x_j),
\]

where \( f \) is a neural network, \( \gamma \) is the step size and \( \alpha \) is the discount factor. In this case, policy update is approximated with a neural network and is continually adapted using task data.
Another approach is the adaptation of a pre-trained neural network from offline deep model to unseen tasks, as in figure 2. In deep neural network, weights and activations are highly correlated so that we can use a few parameters to predict the others. In unseen tasks, we estimate a few parameters rapidly and use the pre-trained predictive mapping to estimate the output directly. [33] proposes using a feedforward pass that maps activations to parameters in the last layer of a pre-trained deep neural network. It applies to few-shot learning where the number of categories is large and the sample size per category is small.

Figure 2: Black-box adaptation. Pre-trained model is an offline deep learning model on a large historical dataset. Black-box adaptation is applied to adapt the pre-trained deep model to a lightweight novel task.

In offline pre-trained deep model, a deep neural network is trained on a large dataset $D_{large}$ with categories $C_{large}$. Few-shot adaptation model is trained on a small dataset $D_{few}$ with categories $C_{few}$. Denote the activations before fully connected layer as $a(x)$. Denote the set of activations for label $y$ as $A_y = \{ a(x) | x \in D_{large} \cap D_{few}, Y(x) = y \}$. Denote the mean of activations in $A_y$ as $\bar{a}_y$. Denote the parameter for category $y$ in fully connected layer as $w_y$. The pre-trained mapping from activations to parameters in deep neural network is

$$
\phi : \bar{a}_y \rightarrow w_y.
$$

Activation $s_y$ is sampled from $A_y \cup \bar{a}_y$ with probability $p$ to be $\bar{a}_y$ and $1-p$ to be uniform in $A_y$. Define $S_{large} = \{ s_1, \cdots, s_{|C_{large}|} \}$. The pre-trained mapping $\phi$ is estimated by minimizing loss function:

$$
\mathcal{L}(\phi) = \sum_{(x,y) \in D_{large}} E_{S_{large}}[-\phi(s_y)a(x)] + \log \sum_{k \in C_{large}} \exp \{ \phi(s_k)a(x) \}.
$$

In few-shot adaptation of pre-trained mapping $\phi$, define $C = C_{large} \cup C_{few}$ and $D = D_{large} \cup D_{few}$. Define $S = \{ s_1, \cdots, s_{|C|} \}$. Category prediction in few-shot data is represented as the probability of $x$ in category $y$:

$$
P(y|x) = \frac{\exp \{ E_S[\phi(s_y)a(x)] \}}{\sum_{k \in C} \exp \{ E_S[\phi(s_k)a(x)] \}}.
$$

Pre-training of a deep neural network consumes time, memory and electricity. The degree of similarity between tasks should be reasonable for precise adaptation of the pre-trained mapping. In image classification tasks, prediction accuracy is high implying that pre-training the offline deep learner is worthwhile.

In order to adapt pre-trained deep neural network, we can parameterize it with meta-parameter and task-specific parameters so that the network itself allows for rapid adaptation. Network proposed in [13] specifies neural network neurons to be conditionally shifted neurons (CSN), which contain activations with task-specific parameters from a memory module. Activation of CSN is defined to be

$$
h_l = \begin{cases} 
\sigma(a_l) + \sigma(\phi_l), & l \neq M, \\
\text{softmax}(a_l + \phi_l), & l = M,
\end{cases}
$$

where $M$ is the index of output layer in neural network, $a_l$ is the pre-activation and $\phi_l$ is the layer-wise task-specific conditional shift parameter. For ResNet block, we can use CSN as activations and stack these blocks to construct a deep adaptive ResNet model (adaResNet). Similarly, we can also stack LSTM layers with CSN activations to build a deep adaptive LSTM model. We may re-design CNN with CSN activations to be a deep adaptive CNN model (adaCNN).

For task $T$, training data is denoted as $D^T_{tr} = \{(x_i, y_i)\}_{i=1}^n$ and validation data is denoted as $D^T_{val} = \{(x_i^*, y_i^*)\}_{i=1}^m$. Base learner is a mapping from input $(x_i, y_i)$ to label $\hat{y}_i$. Information in neural network can be specified as a layer-wise amortized error gradient or a direct feedback (DF) which measures layer-wise contribution to the current difference between predicted output label and true label. Meta-learner $g_0$ is an MLP with $r$th neuron in $r$th layer conditional upon information $I_{l,i}$. Task-specific parameter $\phi_l$ in CSN activation represents the task-specific conditional shift:

$$
V_{l,i} = g_0(I_{l,i}), \quad \alpha_i = \text{softmax}(\cos(f(x_i^*), f(x_i))), \\
\phi_l = [\alpha_1, \cdots, \alpha_n]^T[V_{l,1}, \cdots, V_{l,n}],
$$
Table 1: Testing accuracy of black-box adaptation meta-learning methods on 5-way 5-shot miniImageNet classification.

| Method                                      | Accuracy       |
|---------------------------------------------|----------------|
| Activation to parameter with neural network | 67.87 ± 0.20%  |
| with wide residual network WRN              | 73.74 ± 0.19%  |
| AdaCNN with DF                              | 62.00 ± 0.55%  |
| AdaResNet with DF                           | 71.94 ± 0.57%  |

where $I_{l,i}$ is the conditioning information for each neuron, $f$ is an MLP with a linear output layer for memory query, and $\cos$ is the cosine distance. Neural network is trained by minimizing the cross-entropy loss for task $T$: $L_T = \sum_j L(\hat{y}_j, y_j)$. It is computationally efficient with high prediction accuracy in few-shot image classification.

Black-box adaptation applies deep learners at the meta-level aggregating all task-specific information and provides a direction for model adaptation to out-of-distribution tasks. Table 1 presents testing accuracy of these methods on 5-way 5-shot miniImageNet. In miniImageNet classification, wide deep residual network performs better than neural network and CNN. We can see that performance highly depends upon deep model specifications and the fine tuning of pre-trained deep learner. Performance of black-box adaptation is comparable to similarity-based meta-learning models in the next section.

2.2 Similarity-Based Method

In this section, we review a class of meta-learning models which depend upon the similarity measure between unseen tasks and previously learned models. As in figure 3, the distances between an unseen task and the prototypes of all trained models are compared and the most similar experiences are applied directly to the unseen task. The pre-requisite is that unseen tasks share similarity structure with previous tasks. Otherwise unseen tasks are not identifiable under the framework of similarity-based meta-learning methods.

A subclass of methods resort to an additional memory module with an efficient critical information retrieval mechanism. MANN (Memory-augmented neural networks) in [50] lies within this category. Memory mod proposed in [82] adds a memory module $M$ to any classification network. Memory module stores model training results and has two properties: an efficient nearest neighbor search mechanism and a memory update rule. Memory module $M$ consists of three components: a matrix of memory keys $K_{|M| \times \text{key-size}}$, a vector of memory values $V_{|M|}$ and a vector of memory item ages $A_{|M|}$. The nearest neighbor of query $q$ in $M$ is

$$\text{NN}(q, M) = \arg\max_i \cos(q, K[i]),$$

where $\cos$ is the cosine distance and $K[i]$ is the key value at location $i$. The $k$ nearest neighbor of query $q$ in $M$ is defined as $\text{NN}_k(q, M) = (n_1, n_2, \cdots, n_k)$, where $K[n_i]$ is the key with $i$th highest similarity to query $q$. Memory update finds items with longest ages and writes to these item locations randomly. Memory module can be added to any neural network model such as CNN, LSTM and ResNet. By adding a memory module with memory update and efficient nearest neighbor search, it enables all classification networks to rapidly and accurately adapt to novel tasks.

Another subclass of methodology is to apply an embedding mapping $f$ from input $x$ to feature $z$, which are then used to compute the similarity measure between tasks. Embedding mapping $f$ performs dimension reduction for high-dimensional input $x$. Similarity measure can be output from an attention block, distance between query input and prototype of each class, or complex similarity score. Choices of embedding mapping function and similarity measure may affect performance of these methods. For each dataset, options of feature extractor and distance function should be explored.

Figure 3: Similarity-based meta-learning. $c_i$ is the prototype of class $i$. Distances between a novel task and prototypes are compared.
SNAIL (Simple Neural Attentive Meta-Learner) proposed in [84] applies temporal convolutions in a dense block to extract features from images. An attention block using soft attention function is applied to identify critical features from past experiences and make classification. In sequence-to-sequence tasks such as translation, inputs for task $T_i$ are denoted as $\{x_s\}_{s=1}^{H_i}$. Loss function is $L_i(x_s, a_s)$, where input feature $x_s \sim P_i(x_s|x_{s-1}, a_{s-1})$ and output from attention block $a_s \sim \pi(a_s|x_1, \cdots, x_s; \theta)$. Meta-parameter in attention block $\theta$ is updated by minimizing loss function:

$$
\min_{\theta} \mathbb{E}_{T_i \sim P(T)} \left[ \sum_{s=1}^{H_i} L_i(x_s, a_s(x_1, \cdots, x_s; \theta)) \right].
$$

In addition, RN (Relation Network) [8] also belongs to this subclass. Denote the embedding function as $f_\phi$ and relation function as $g_\theta$. Relation score $r_{i,j}$ measures the similarity between query $x^*_j$ and training data $x_i$:

$$
r_{i,j} = g_\theta(f_\phi(x_i), f_\phi(x^*_j)).
$$

Higher relation score indicates greater similarity between query and training data. Parameters in the embedding function and relation function are jointly estimated through the minimization of loss function:

$$
\min_{\phi, \theta} \sum_{i=1}^{n} \sum_{j=1}^{m} \{r_{i,j} - \mathbb{I}(y_i = y_j^*)\}^2
$$

where $\mathbb{I}(y_i = y_j^*)$ is 1 if $y_i = y_j^*$ and 0 otherwise.

Similarly, prototypical network proposed in [6] is based upon similarity and clustering. Prototype in class $k$ is the centroid $c_k$:

$$
c_k = \frac{1}{|S_k|} \sum_{(x_i, y_i) \in S_k} f_\phi(x_i),
$$

where $S_k$ is the set of all data in class $k$. Distance function $g$ measures the distance between data and centroid of each class. Softmax from distance to class probability is

$$
p_\phi(y = k|x) = \frac{\exp(-g[f_\phi(x), c_k])}{\sum_{k'} \exp(-g[f_\phi(x), c_{k'}])}.
$$

Finally, we minimize the negative log-probability loss function $J(\phi) = -\log p_\phi(y = k|x)$ via SGD. [85] also combines similarity-based metric learning and meta-learning. The setup of [85] is similar to prototypical network [6], where prototypes of clusters are benchmarks in classification. [85] is the first work which applies Mahalanobis distance to make classification rather than Euclidean distance. Mahalanobis distance is defined as

$$
g(x_i, x_j) = \frac{\|x_i - x_j\|^2}{\sigma^2},
$$

where cluster-wise variance $\sigma^2$ is estimated and updated in the adaptation process. In Mahalanobis distance, both Euclidean distance and cluster-wise variance are considered to make classification. In feature extraction process, [85] proposes a relative-feature extractor in addition to absolute deep-feature extractor $f_\phi$. To avoid over-fitting, dimension of relative-feature extractor does not exceed sample size in few-shot novel task.

Lastly, TADAM [76] is also a typical method based upon embedding function and similarity measure. On mimilImageNet 5-shot 5-way, TADAM shows testing accuracy of 76%, greater than SNAIL, RN and prototypical network. Similarly, distance function $g$ is defined and a softmax function is applied to compute class probability

$$
p_{\phi, \alpha}(y = k|x) = \text{Softmax}(-\alpha g[f_\phi(x), c_k]),
$$

where $\alpha$ is the temperature parameter. Parameters $\phi$ and $\alpha$ are jointly estimated by minimizing the class-wise cross-entropy loss function $J_{k}(\phi, \alpha)$:

$$
\sum_{x_i} \left\{ \alpha g[f_\phi(x_i), c_k] + \log \sum_{j} e^{-\alpha g[f_\phi(x_i), c_j]} \right\}.
$$

By introducing an additional parameter $\alpha$ in softmax function and through joint optimization of embedding parameter $\phi$ and hyperparameter $\alpha$, prediction accuracy of TADAM on 5-way 5-shot mimilImageNet is 8% greater than prototypical network.
Other subclasses of methods design new similarity measures or embedding functions to improve model adaptivity. \cite{86} proposes a dynamic few-shot learning algorithm which consists of a ConvNet-based classification model, a few-shot classification weight generator and a cosine-similarity based classifier. Compared with former similarity-based methods, it contains an additional classification weight generator which adapts ConvNet parameters to novel tasks. It combines black-box adaptation with similarity-based approach.

Training data contains $N_{tr}$ base categories each with $K$ examples. Training data is denoted as $D^{tr} = \cup_{b=1}^{N_{tr}} \{ x_{b,i} \}_{i=1}^{K}$. Testing data contains $N_{ts}$ novel categories each with $K$ examples. Testing data is denoted as $D^{test} = \cup_{n=1}^{N_{ts}} \{ x_{n,i} \}_{i=1}^{K}$.

Embedding feature $z_{n}^* = \{ z_{n,i} \}_{i=1}^{K}$, where $z_{n,i} = f_{\phi}(x_{n,i}).$ Denote $W_{tr}$ as the classification weight of base categories. Generate a classification weight vector for novel categories $W_{n}^* = G(z_{n}^*, W_{tr}(\theta))$ using testing data features and base categories weights. Denote $W_{ts}^* = \{ W_{n,i}^* \}_{n=1}^{N_{ts}}$ as a classification weight vector of novel categories. Let $W^* = W_{tr} \cup W_{ts}^*$.

Classification weight is $W^* = (w_{1}^*, w_{2}^*, \cdots, w_{N}^*)$. Raw classification score of query $z$ is based upon cosine similarity: $g_{k} = \tau \cos(z, w_{k}^*)$, where $\tau > 0$. Probability of class $i$ is $p_{k} = \text{Softmax}(g_{k})$. Embedding parameter $\phi$ and parameter in classification weight generator $\theta$ are jointly estimated through the minimization of loss function:

$$
\min_{\phi, \theta} \frac{1}{N_{tr}} \sum_{b=1}^{N_{tr}} \frac{1}{K} \sum_{i=1}^{K} \mathcal{L}_{\phi, \theta}(x_{b,i}, b),
$$

where $x_{b,i}$ is the $i$th training example in category $b$. Feature extractor options and classification weight generators may be explored to achieve higher adaptation performance.

In addition, distance measure may be designed to estimate similarity structure between tasks, as in \cite{81}. It proposes a similarity ranking based measure mAP (mean Average Precision). Support set is denoted as $S = \{ (x_{1}, y_{1}), \cdots, (x_{N}, y_{N}) \}$. Define $B = \{ x_{1,1}, \cdots, x_{N,1} \}$. Let $R^{x_{i}} = \{ x_{j} \in B : y_{j} = y_{i} \}$ be the set of all points in class $y_{i}$. Let $O^{x_{i}}$ be a ranking of the predicted similarity between $x_{i}$ and the other points in $B$, where $O^{x_{i}}_{j}$ is the $j$th element in $O^{x_{i}}$ with the $j$th highest similarity to $x_{i}$. Let $P(j, x_{i})$ be the proportion of points that are truly relevant to $x_{i}$ in the first $j$ items in $O^{x_{i}}$.

Average Precision (AP) of this ranking is defined as

$$
\text{AP}^{x_{i}} = |R^{x_{i}}|^{-1} \sum_{j: O^{x_{i}}_{j} \in R^{x_{i}}} P(j, x_{i}),
$$

where $P(j, x_{i}) = j^{-1} |\{ k \leq j : O^{x_{i}}_{k} \in R^{x_{i}} \}|$.

Mean Average Precision (mAP) is the mean of AP for all points: $m\text{AP} = |B|^{-1} \sum_{i \in B} \text{AP}^{x_{i}}$. Let $\phi_{\phi}(x_{i}, x_{j})$ be the cosine similarity between $x_{i}$ and $x_{j}$; $\phi_{\theta}(x_{i}, x_{j}) = \cos(f_{\phi}(x_{i}), f_{\phi}(x_{j}))$. Denote $T^{x_{i}}$ as the set of all points with predicted similarity to $x_{i}$. Denote $N^{x_{i}}$ as the set of all points without predicted similarity to $x_{i}$. Indicator $y_{k,j}^{x_{i}}$ is 1 if $\phi_{\phi}(x_{i}, x_{k}) > \phi_{\phi}(x_{i}, x_{j})$ and 0 otherwise. For each query $x_{i}$, score function is defined as

$$
F_{\phi}(B, y) = \sum_{x_{i} \in B} F_{\phi}^{x_{i}}(B, y),
$$

$$
F_{\phi}^{x_{i}}(B, y) = \frac{1}{|R^{x_{i}}|} \sum_{k \in T^{x_{i}} \setminus i} \sum_{j \in N^{x_{i}}} \left( \phi_{\phi}(x_{i}, x_{k}) - \phi_{\phi}(x_{i}, x_{j}) \right).
$$

Indicator $p^{x_{i}}_{j}$ is 1 if $x_{i,j}$ is truly relevant to query $x_{i}$ and 1 otherwise. AP loss for query $x_{i}$ is formulated as

$$
\mathcal{L}^{x_{i}}(p^{j}, \tilde{p}^{j}) = 1 - \frac{1}{|P^{x_{i}}|} \sum_{j, \tilde{p}^{j} = 1} P(j, x_{i}).
$$

Loss-augmented label prediction for query $x_{i}$ is a linear combination of score function and AP loss:

$$
\max_{y_{i}} \left\{ F_{\phi}^{x_{i}}(B, y) - \epsilon \mathcal{L}^{x_{i}}(p^{j}, \tilde{p}^{j}) \right\},
$$

where $\epsilon > 0$.

From Table 2 which presents testing accuracy of similarity-based methods on 5-way 5-shot miniImageNet, we can see that TADAM shows highest accuracy. On average, performance of similarity-based methods is slightly better than black-box adaptation which requires heavy pre-training computation. Similarity-based methods are more flexible and allow choices of feature extractor, similarity measure, loss function and hyperparameters. Similarity-based methods require fine tuning on these choices for best adaptation performance.
Table 2: Testing accuracy of similarity-based meta-learning methods on 5-way 5-shot miniImageNet classification.

| Method                                      | Accuracy       |
|---------------------------------------------|----------------|
| Matching Net                                | 60.0%          |
| SNAIL                                       | 68.88 ± 0.92%  |
| Relation Net                                | 65.32 ± 0.70%  |
| Prototypical Net                            | 68.20 ± 0.66%  |
| PVRT                                        | 70.91 ± 0.85%  |
| TADAM with α, AT and TC                     | 76.7 ± 0.3%    |
| TADAM without tuning                        | 74.2 ± 0.2%    |
| Dynamic few-shot with C128F feature extractor| 73.00 ± 0.64%  |
| with ResNet feature extractor               | 70.13 ± 0.68%  |
| with cosine classifier and attention based weight generator | 74.92 ± 0.36%  |
| with cosine classifier and no weight generator | 72.83 ± 0.35%  |
| mAP-SSVM                                    | 63.94 ± 0.72%  |
| mAP-DLM                                     | 63.70 ± 0.70%  |

2.3 Learner and Meta-Learner Procedure

In [87], author provides an overview of two-level few-shot meta-learning models which consist of a base model that learns rapidly from few-shot data, and a meta-model that optimizes the base learner across few-shot tasks. As in figure 4, base learner is designed for each task. Meta-learner updates task-specific parameters in base learner so that base learner generalizes to different tasks. This meta-learning framework separates model generalization from estimation and allows the combination of different variations in adaptation specifications and estimation procedures. Meta-LSTM [51], Meta Network (MetaNet) [15], MetaOptNet [73], Meta-SGD [7] and MAML [12] are all methods under this meta-learning framework. Learner can be specified as a machine learning algorithm or a statistical model. In MetaOptNet [73], learner is specified to be a convex linear classifier. Meta-learner is a neural network. MetaOptNet shows superior prediction performance in miniImageNet classification.

MAML (Model-Agnostic Meta-Learning) [12] is applicable to any learner that is optimized with SGD. It is the most important meta-learning method in recent literature. We will review Bayesian extensions of MAML and integrated learning frameworks based upon MAML. Base learner is denoted as $h_\theta$ parameterized by meta-parameter $\theta$. Task-specific parameter is denoted as $\phi$. Task $T_i$ follows task distribution $p(T)$. In the inner loop of MAML, task-specific parameter $\phi$ is updated using

$$\phi_i = \theta - \alpha \nabla_\phi \mathcal{L}_{T_i}(h_\theta),$$

where $\alpha$ is the step size. In the outer loop, meta-parameter $\theta$ is updated using

$$\theta \leftarrow \theta - \beta \nabla_\theta \sum_{T_i \sim p(T)} \mathcal{L}_{T_i}(h_{\phi_i}),$$

where $\beta$ is the step size. MAML applies to RL (reinforcement learning) tasks as well. Base learner updates task-specific parameter $\phi$ using task data $T_i$ and meta-learner updates meta-parameter $\theta$ based upon all task data. In MAML, both base learner and meta-learner are SGD optimizers which ensure efficient adaptation to novel tasks.
Meta-SGD [7] has the same structure as MAML but it tunes the step size $\alpha$ jointly with meta-parameter $\theta$. In the meta-learner, step size $\alpha$ and meta-parameter $\theta$ are jointly updated using

$$
(\theta, \alpha) \leftarrow (\theta, \alpha) - \beta \nabla_{(\theta, \alpha)} \sum_{T_i \sim p(T)} L_{T_i}(h_{\phi_i}).
$$

On 5-way 5-shot miniImageNet, testing accuracy of meta-SGD is 64% which is 0.9% greater than MAML.

In addition, Reptile [88] also has the same structure as MAML but the update on task-specific parameter is based upon first-order approximation. Meta-parameter $\theta$ is estimated through the minimization of expected loss function $\min_{\theta} \mathbb{E}_T \left[ \nabla_{\theta} L_T(h_{\theta}) \right]$. Sampled tasks are denoted as $T_i, i = 1, 2, \cdots, J$. Task-specific parameters $\phi_i$ are neural network weights and they are updated using SGD:

$$
\phi_i = \text{SGD}(L_{T_i}, \theta, k),
$$

where $k$ is the number of gradient steps taken to update neural network weights. In the meta-learner, meta-parameter $\theta$ is updated using

$$
\theta \leftarrow \theta + \frac{1}{k} \sum_{i=1}^{J} (\phi_i - \theta),
$$

where $\epsilon > 0$.

Different from previous methods, Meta-LSTM [53] uses a deep neural network classifier as base learner and an LSTM as meta-learner. Meta-learner supplies a task-independent initial parameter that contains the shared information among tasks. Base learner takes this parameter as input and converges faster on novel tasks. Main gradient-based optimization techniques in LSTM include momentum [89], Adagrad [90], Adadelta [91] and ADAM [92]. In convex optimization, convergence speed is guaranteed. In non-convex optimization such as genetic programming and simulated annealing, convergence holds under assumptions as well.

In base learner, task-specific parameters in neural network are updated with MAML. For task $T_i$,

$$
\phi_i = \phi_{i-1} - \alpha_i \nabla_{\phi_{i-1}} L_{T_i}(h_{\phi_{i-1}}).
$$

Neural network weights $\phi_t$ for task $T_i$ in base learner are updated with loss gradients corresponding to cell state update in LSTM:

$$
c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t,
\quad c_t = \phi_t,
\quad \tilde{c}_t = \nabla_{\phi_{t-1}} L_{T_t},
\quad i_t = \sigma(W_f [\nabla_{\phi_{t-1}} L_{T_t}, L_{T_t}, \phi_{t-1}, i_{t-1}] + b_f),
\quad f_t = \sigma(W_F [\nabla_{\phi_{t-1}} L_{T_t}, L_{T_t}, \phi_{t-1}, f_{t-1}] + b_F).
$$

Due to the correspondence between SGD and LSTM cell state update, LSTM can be applied to provide task-specific parameter update in base model adaptation.

In few-shot meta-learning, there are nested loops, where the inner loop is a base learner from each individual task and the outer loop is a meta-learner from all available tasks. Usually, between-task adaptation depends upon the quality of similarity measure. [77] lays out base learner and meta-learner double-layer framework, in which base learner concentrates upon model fitting and meta-learner focuses upon model adaptation. [77] specifies base learner as an efficient and differentiable learner preferably with an explicit solution. Under this specification, over-fitting is reduced in base learner and generalization is faster. In R2D2 (Ridge Regression Differentiable Discriminator) [77], ridge regression is specified to be base learner. In LR-D2 (Logistic Regression Differentiable Discriminator), iteratively reweighted least squares (IRLS) derived from logistic regression is applied as base learner. As in [93], neural nets with low Kolmogorov complexity exhibit high generalization capability. Efficient lightweight base learner demonstrates improvement in generalization.

MetaOptNet [23] is developed under base learner and meta-learner double-layer framework [77], where base learner is formulated as a regularized linear classifier. This idea of base learner specification is similar to that in [77], where an efficient and differentiable statistical base learner is preferred. MetaOptNet is an extension of [77] in the sense that it explores more options of base learner specification under similar framework as in [77]. Base learner options for convex optimization include linear classifiers such as kNN classifier, SVM classifier with nonlinear kernels and ridge regression with hinge loss. Quadratic programming (QP) can be applied for convex optimization where Karush-Kuhn-Tucker
(KKT) condition is derived and then back-propagation using implicit function theorem is performed. Parameter $\theta$ is estimated using

$$\min_{\theta} \mathcal{L}_{\text{base}}(D^{tr}; \theta, \phi) + R(\theta),$$

where $\mathcal{L}_{\text{base}}$ is a loss function of base learner and $R(\theta)$ is an $l_2$ regularization. Meta-learning objective is to learn an embedding model $f_\phi$ such that base learner generalizes well across tasks

$$\min_{\phi} \mathbb{E}_{T \sim p(T)}[\mathcal{L}\text{meta}(D^{test}; \theta, \phi)],$$

where $\mathcal{L}\text{meta}$ is a loss function of meta-learner. For example, meta-loss function can be specified as

$$\mathcal{L}\text{meta}(D^{test}; \theta, \phi, \gamma) = \sum_{(x, y) \in D^{test}} [-\gamma y f_\phi(x) + \log \sum_k \exp(\gamma_k f_\phi(x))].$$

Note that $\phi$ is meta-parameter and $\theta$ is task-dependent in MetaOptNet. We may choose statistical models as base learners since they are less prone to over-fitting. For example, in MetaOptNet-SVM, base learner can be specified as an SVM classifier.

TPN (Transductive Propagation Network) [93] learns adaptation to novel classification tasks using transductive inference in few-shot learning. Transductive inference predicts on whole test data simultaneously, which is different from inductive inference which predicts test data one-by-one. TPN maps directly from meta-training data to meta-testing data, constructs a joint network model on support set $S$ and query set $Q$ to adapt labels between them. Transductive meta-learning framework has two components: a deep CNN feature embedding function $f_\phi$ and a learned manifold structure of novel class space from $S \cup Q$. The cross-entropy loss is computed using feature embedding and graph parameter with end-to-end updates on all parameters through back-propagation.

Support set is denoted as $S = \{(x_1, y_1), \cdots, (x_n, y_n)\}$. Query set is denoted as $Q = \{(x_1^*, y_1^*), \cdots, (x_m^*, y_m^*)\}$. The graph edge weight is defined as Gaussian similarity measure

$$W_{ij} = \exp \left(-\frac{g[f_\phi(x_i), f_\phi(x_j)]}{2\sigma^2}\right),$$

Only the $k$ largest edge weights in each row are retained to create a $k$-nearest neighbor graph. $W$ is a row-normalized matrix of all weights in graph. Indicator $Y_{ij}$ is 1 if $x_i$ has label $y_i = j$ and 0 otherwise. Label propagation in the graph learned from $S$ and $Q$ is specified to be

$$F_{t+1} = \alpha WF_t + (1 - \alpha)Y,$$

where $F_t$ is the predicted labels at timestamp $t$ and the sequence of $\{F_t\}$ converges to $F^* = (I - \alpha W)^{-1}Y$. Loss function is defined as

$$J(\phi, \theta) = \sum_{i=1}^m \sum_{j=1}^N -Y_{ij} \log P(\hat{y}_i = j | x_i),$$

where $P(\hat{y}_i = j | x_i) = \exp(F^*_{ij})/\sum_{j=1}^N \exp(F^*_{ij}),$ $y_i$ and $\hat{y}_i$ are the true and predicted label of $x_i$, respectively.

LEO (latent embedding optimization) [95] is also for K-shot N-class few-shot learning. LEO consists of an encoding process and a decoding process, where encoding process is a feed-forward mapping followed by a relation network, and decoding process is a softmax classifier. Then parameters in encoder-decoder framework are updated using base learner and meta-learner procedure.

Dimension reduction encoder network is denoted as $f_{\phi_e} : \mathcal{R}^{n_x} \rightarrow \mathcal{R}^{n_k}$, where $n_x > n_h$. Training data for class $n$ is denoted as $D^{tr}_n = \{x^n_k, y^n_k\}_{k=1}^K$. Relation network is denoted as $g_{\phi_r}$. Denote $z_n$ as a dimension reduced feature embedding from original input $D^{tr}_n$. Distribution of $z_n$ is a Gaussian distribution:

$$z_n \sim q(z_n|D^{tr}_n) = \mathcal{N}(\mu_n^c, \text{diag}(\sigma_n^{c2})),
$$

where $\mu_n^c, \sigma_n^{c2} = \frac{1}{NK^2} \sum_{k_n=1}^K \sum_{m=1}^N \sum_{k_m=1}^K x_n^{k_n}, x_m^{k_m}$.
Decoder function is denoted as \( f_{\phi_d} : \mathcal{Z} \rightarrow \Theta \) where \( \mathcal{Z} \) is of lower dimension than \( \Theta \). Denote \( w_n \) as a sample from the decoded distribution, which is also a Gaussian distribution:

\[
w_n \sim p(w|z_n) = \mathcal{N}(\mu_n^d, \text{diag}(\sigma_n^d))
\]

where \( \mu_n^d, \sigma_n^d = f_{\phi_d}(z_n) \).

In the base learner within inner loop, loss function of LEO for task \( T_i \) is

\[
\mathcal{L}_{T_i}^{lr} (\phi_i) = \sum_{(x, y) \in D_{tr}} \left[ -w_y \cdot \mathbf{z} + \log \left( \sum_{j=1}^{N} e^{w_y \cdot \mathbf{x}_j} \right) \right]
\]

where \( \phi_i = (\phi_{ei}, \phi_{di}, \phi_{ri}) \). Apply MAML in inner loop to update feature embedding \( z_n \)

\[
z_n' = z_n - \alpha \nabla_{z_n} \mathcal{L}_{T_i}^{lr}
\]

In the meta-learner within outer loop, use meta-training data to update the parameters in encoder, relation net and decoder:

\[
\min_{\phi_e, \phi_r, \phi_d} \sum_{T_i \sim p(T)} [\mathcal{L}_{T_i}^{val}(\phi_i) + \beta D_{KL}(q(z_n|D_{tr})||p(z_n))] + \gamma \left[ \text{stopgrad}(z_n') - z_n \right]^2 + \lambda_1 \left[ \| \phi_r \|^2 + \| \phi_d \|^2 \right] + \lambda_2 |C_d - I|_2
\]

where \( C_d \) is the correlation matrix of rows in decoder parameter \( \phi_d \). LEO is a highly integrated meta-learning method based upon base learner and meta-learner.

Table 3: Testing accuracy of base learner and meta-learner double-layer meta-learning methods on 5-way 5-shot miniImageNet classification.

| Method          | Accuracy       |
|-----------------|----------------|
| [12] MAML       | 63.11 ± 0.92%  |
| [12] Meta-SGD   | 64.03 ± 0.94%  |
| [88] Reptile without transduction | 61.98 ± 0.69% |
| [88] Reptile with transduction   | 66.00 ± 0.62%  |
| [5] Meta-LSTM   | 60.00 ± 0.71%  |
| [73] MetaOptNet-RidgeReg | 77.88 ± 0.46% |
| [73] MetaOptNet-SVM   | 78.63 ± 0.46%  |
| [73] MetaOptNet-SVM-trainval | 80.00 ± 0.45% |
| [17] R2-D2     | 68.4 ± 0.2%    |
| [17] LR-D2 with 5 iterations | 68.7 ± 0.2%     |
| [17] LR-D2 with 1 iteration    | 65.6 ± 0.2%     |
| [94] TPN       | 69.86 ± 0.65%  |
| [95] LEO       | 77.59 ± 0.12%  |

Table 3 summarizes testing accuracy of base learner and meta-learner double-layer meta-learning methods on 5-way 5-shot miniImageNet classification. We can see that MetaOptNet shows highest accuracy, better than black-box adaptation methods and similarity-based methods. Prediction accuracy of highly integrated LEO is next to MetaOptNet. Combination of statistical models and machine learning methods improves model generalization. More integrated meta-learning frameworks also demonstrate better prediction accuracy in image classification.

2.4 Bayesian Meta-learning

From the frequentist perspective, meta-parameter \( \theta \) and task-specific parameter \( \phi \) are regarded as fixed. Under the Bayesian framework, both \( \theta \) and \( \phi \) are treated as random variables. In Bayesian meta-learning, we compute the posterior distributions of model parameters and provide inference upon the predictions on out-of-distribution tasks.

Human learns new concepts through creative thinking and meta cognition. Generative model simulates innovative imagination in human mind to learn new concepts. This subclass of methodology focuses upon integration of a generative model into the existing meta-learning framework. In [53], a generative model BPL (Bayesian program
learning) is proposed to classify hand-written characters using images and strokes data. A Bayesian generative model is integrated into deep Siamese convolutional network. Generated hand-written characters are indistinguishable from real hand writings using visual checks.

Character type is denoted as \( \theta \). A set of \( M \) parameters from these types is denoted as \( \Phi = (\phi_1, \ldots, \phi_M) \) corresponding to binary images \( \mathcal{I} = (I_1, \ldots, I_M) \). The joint distribution of character types, image-specific parameters and images is

\[
P(\theta, \Phi, \mathcal{I}) = P(\theta) \prod_{m=1}^{M} P(I_m|\phi_m)P(\phi_m|\theta).
\]

Based upon the joint distribution, we compute a discrete approximation to the joint posterior distribution \( P(\theta, \phi_m|I_m) \) which is updated for adaptation to different tasks.

Neural Statistician method [96] also contains a generative model. For task \( T_j \), a task-specific generative model \( \hat{p}_i \) is estimated. The generative distribution is defined as \( \hat{p}_i = p(\cdot|c_j) \), where \( c_j \) is the task-specific context. Then the approximate posterior of task-specific context \( q(c|D) \) is computed using variational autoencoder (VAE).

In VAE, decoder with meta-parameter \( \theta \) is denoted as \( p(D|c; \theta) \) and encoder with task-specific parameter \( \phi \) is denoted as \( q(c|D; \phi) \). For adaptation to different tasks, inference network \( q(c|D; \phi) \) is updated. The standard variational lower bound is

\[
\mathcal{L} = \mathbb{E}_{q(c|D; \phi)}[\log p(D|c; \theta)] + \alpha D_{KL}(q(c|D; \phi)||p(c)).
\]

In VAE, we maximize standard variational lower bound to estimate \( \theta \) and \( \phi \). The instance encoder \( f \) is a feedforward neural network such that \( z_i = f(x_i) \). In exchangeable instance pooling layer, the mapping is from encoded features to a pooled vector: \( [z_1, \ldots, z_k] \rightarrow \nu \). In post-pooling network, the mapping is from a pooled vector to a decoded distribution: \( v \rightarrow \text{a diagonal Gaussian (decoder)} \).

The following three papers extend MAML to Bayesian framework. First, [97] proposes LLAMA (Lightweight Laplace Approximation for Meta-Adaptation) which formulates MAML as probabilistic inference in a hierarchical Bayesian model. It considers first-order and second-order Laplace approximation of log likelihood function to construct inference with quadratic curvature estimation.

In task \( T_j \), training data is denoted as \( \mathbf{x}_j = (x_{j1}, \ldots, x_{jn}) \), and validation data is denoted as \( \mathbf{x}_j^* = (x_{j1}^*, \ldots, x_{jm}^*) \). For probabilistic inference, marginal likelihood of observed data \( \mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_j) \) is written as

\[
p(\mathbf{x} | \theta) = \prod_{j=1}^{J} \left\{ \int p(\mathbf{x}_j | \phi_j) p(\phi_j | \theta) d\phi_j \right\}.
\]

In meta-learner, meta-parameter \( \theta \) is estimated using maximum likelihood \( \max_{\theta} p(\mathbf{x} | \theta) \). In base learner, task-specific parameter \( \phi_j \) for task \( T_j \) can be estimated with MAML

\[
\hat{\phi}_j = \theta + \alpha \nabla_{\theta} \log p(\mathbf{x}_j | \theta).
\]

We can also estimate \( \phi_j \) using maximum a posteriori (MAP) estimator. From Bayesian perspective, posterior distribution of \( \phi_j \) is \( p(\phi_j | \mathbf{x}_j, \theta) \propto p(\mathbf{x}_j | \phi_j) p(\phi_j | \theta) \), where \( p(\phi_j | \theta) \) is the prior distribution of task-specific parameter. Maximum a posteriori estimate of \( \phi_j \) is the global mode of \( p(\phi_j | \mathbf{x}_j, \theta) \). Based upon first-order Laplace approximation, log likelihood is approximated with

\[
\log p(\mathbf{x} | \theta) \approx \sum_{j=1}^{J} \left\{ \log p \left( \mathbf{x}_j | \hat{\phi}_j \right) \right\},
\]

where approximate log likelihood for task \( T_j \) is given by \( \mathbb{E}_{\mathbf{x}_j^*} [\log p(\mathbf{x}_j^* | \phi_j)] \approx m^{-1} \sum_{d=1}^{m} \log p(x_{jd}^* | \hat{\phi}_j) \). Second-order Laplace approximation to likelihood function of task \( T_j \) data is

\[
p(\mathbf{x}_j | \theta) = \int p(\mathbf{x}_j | \phi_j) p(\phi_j | \theta) d\phi_j
\approx p(\mathbf{x}_j | \phi_j) p(\phi_j | \theta) \text{det}(H_j/2\pi)^{-1/2},
\]

where \( H_j \) is the Hessian of log likelihood function

\[
H_j = \nabla_{\phi_j}^2 \log p(\mathbf{x}_j | \phi_j) + \nabla_{\theta}^2 \log p(\mathbf{x}_j | \theta).
\]

From second-order Laplace approximation, first-order approximation \( \log p(\mathbf{x}_j | \phi_j) \) is replaced with second-order approximation \( \log p(\mathbf{x}_j | \phi_j) - \eta \log \text{det}(H_j) \). Both first-order and second-order Laplace approximation of log likelihood function can be applied to update task-specific parameters in fast model adaptation to unseen tasks.
Another Bayesian extension of MAML is BMAML (Bayesian Model-Agnostic Meta-Learning) proposed in [54], where stochastic gradient descent (SGD) in base learner is replaced with Stein variational gradient descent (SVGD) method [98] [99]. SVGD is an efficient sampling method which is a combination of MCMC and variational inference. We first draw a sample $\Theta_0 = \{\theta_0^i\}_{i=1}^n$ from target distribution $p(\theta)$ as the initial particles. In the $i$th iteration, particles $\Theta_i = \{\theta_i^j\}_{j=1}^n$ are updated using

$$g(\theta) = n^{-1} \sum_{j=1}^n \left\{ k(\theta_j^i, \theta) \nabla \theta_j^i \log p(\theta_j^i) + \nabla \theta_j^i k(\theta_j^i, \theta) \right\},$$

$$\theta_{i+1}^j \leftarrow \theta_i^j + \epsilon_i g(\theta_i^j), \text{ for } i = 1, \ldots, n,$$

where $\epsilon_i$ is the step size, $k(\cdot, \cdot)$ is a positive kernel function and $\nabla \theta_j^i k(\theta_j^i, \theta)$ is a repulsive force so that $\theta_{i+1}^j \neq \theta_i^j$. During meta-training process, we update these particles to update posterior distribution of task-specific parameters.

In base learner, Bayesian Fast Adaptation (BFA) is performed, where we update task-specific parameter particles ensemble $\Phi_i = \{\phi_i^m\}_{m=1}^M$ for task $T_i$ using SVGD based likelihood approximation. Objective for task $T_i$ is to maximize

$$p(D_i^{val}(\Theta_i, D_i^{tr}) = \int p(D_i^{val}|\Theta_i, \Phi_i) p(\Phi_i|\Theta_i, D_i^{tr}) d\Phi_i \approx M^{-1} \sum_{m=1}^M p(D_i^{val}|\Theta_i, \phi_i^m),$$

where $\phi_i^m \sim p(\Phi_i|\Theta_0, D_i^{tr})$ and $\Theta_0$ is the initial particle ensemble to generate $\Phi_i$ using SVGD. Over all tasks, meta-objective in BFA can be written as

$$\log p(D^{val}|\Theta_0, D^{tr}) = \sum_{i=1}^J \log \mathcal{L}_{BFA}(\Phi_i; D_i^{val}), \text{ where } \mathcal{L}_{BFA}(\Phi_i; D_i^{val}) \approx \log \left\{ M^{-1} \sum_{m=1}^M p(D_i^{val}|\phi_i^m) \right\}.$$ 

For task $T_i$, task-specific particle ensemble $\Phi_i$ is updated using $\Phi_i \leftarrow \text{SVGD}(\Theta_0; D_i^{tr}, \alpha)$, where $\alpha$ is the step size. Meta-parameter particle initialization $\Theta_0$ is evaluated using

$$\Theta_0 \leftarrow \Theta_0 - \beta \nabla \theta_0 \left[ \sum_{i=1}^J \mathcal{L}_{BFA}(\Phi_i; D_i^{val}) \right].$$

BMAML consists of two components: a chaser loss with SVGD and BFA for adaptation. Meta-objective function is specified as the dissimilarity between approximate and true posterior distributions of task-specific parameter $\Phi_i$. Posterior of task-specific parameter after $n$ SVGDs from initial model is denoted as $p_i^n = p_n(\Phi_i|D_i^{tr}; \Theta_0)$, from which we randomly sample $\Phi_i^n$. True task-specific parameter posterior is $p_i^\infty = p(\Phi_i|D_i^{tr} \cup D_i^{val})$, from which we randomly sample $\Phi_i^\infty$. BMAML seeks the minimization of the dissimilarity between estimated approximate posterior and true posterior:

$$\min_{\Theta_0} \sum_{i=1}^J d_p(p_i^n || p_i^\infty) \approx \min_{\Theta_0} \sum_{i=1}^J d_s(\Phi_i^n || \Phi_i^\infty),$$

where $d_p$ and $d_s$ are the distance measures between two distributions and two particle samples respectively. For definition of chaser loss, true posterior $\Phi_i^\infty$ is approximated with a posterior estimate after $s$ more iterations $\Phi_i^{n+s}$, where $s > 0$. Chaser is defined as $\Phi_i^\infty = \text{SVGD}_n(\Theta_0; D_i^{tr}, \alpha)$. Leader is defined as $\Phi_i^{n+s} = \text{SVGD}_s(\Phi_i^n; D_i^{tr} \cup D_i^{val}, \alpha)$. Meta-objective based upon chaser loss is

$$\mathcal{L}_{BMAML}(\Theta_0) = \sum_{i=1}^J d_s(\Phi_i^n || \Phi_i^{n+s}).$$

Finally, meta-parameter $\Theta_0$ is updated using $\Theta_0 \leftarrow \Theta_0 - \beta \nabla \theta_0[\mathcal{L}_{BMAML}(\Theta_0)]$. Prediction accuracy of BMAML is slightly better than LLAMA in 5-way 1-shot minilimageNet classification. However, LLAMA is more lightweight than BMAML.

In addition, PLATIPUS (Probabilistic LATent model for Incorporating Priors and Uncertainty in few-Shot learning based upon MAML) [100] is proposed by creator of MAML. It utilizes variational inference to update task-specific parameters and meta-parameter. Training data is $\{(\textbf{x}_1, y_1), \ldots, (\textbf{x}_n, y_n)\}$. Validation data is $\{(\textbf{x}_1^*, y_1^*), \ldots, (\textbf{x}_m^*, y_m^*)\}$. For task $T_i$, inference network is $q_i(\phi_i|\theta) = q_i(\phi_i|\theta_i)$. Conditional distribution is specified as $q_i(\phi_i|\theta_i) = q_0(\phi_i|\theta_i, \textbf{x}_i, y_i, \textbf{x}_i^*, y_i^*)$ and prior distribution of $\theta$ is $q_0(\theta) = q_0(\theta|\textbf{x}_i, \textbf{x}_i^*, y_i^*)$. Both components in inference network are parameterized by $\psi$. Variational lower bound of log likelihood is

$$\log p(y_i^*|\textbf{x}_i^*, \textbf{x}_i, y_i) \geq \mathbb{E}_{\theta, \phi_i \sim q_0} \left[ \log p(y_i^*|\textbf{x}_i^*, \phi_i) + \log p(y_i^*|\phi_i) + \log p(\phi_i|\theta) + \log p(\theta) \right] + \mathcal{H}(q_0(\phi_i|\theta, \textbf{x}_i, y_i, \textbf{x}_i^*, y_i^*)) + \mathcal{H}(q_0(\theta|\textbf{x}_i, \textbf{x}_i^*, y_i^*)),$$
where inference networks are posited to be Gaussian distributions

\[ q_\phi(\theta|x_i, y_i, x_i^*, y_i^*) = \mathcal{N}(\mu_\theta + \gamma_p \nabla_\mu_\theta \log p(y_i|x_i, \mu_\theta) + \gamma_q \nabla_\mu_\theta \log p(y_i^*|x_i^*, \mu_\theta), v_q). \]

In PLATIPUS, we first initialize all meta-parameters \( \Theta = \{\mu_\theta, \sigma_\theta^2, v_q, \gamma_p, \gamma_q\} \) in inference networks. Based upon these inference networks, we sample parameters \( \theta \sim q_\phi = \mathcal{N}(\mu_\theta - \gamma_q \nabla_\mu_\theta \mathcal{L}(\mu_\theta, D_{\text{test}}), v_q) \). Based upon MAML, task-specific parameter in the inner loop is updated using \( \phi_i = \theta - \alpha \nabla_\theta \mathcal{L}(\theta, D_{\text{tr}}) \). Gaussian inference network is specified as \( p(\theta|D_{\text{tr}}) = \mathcal{N}(\mu_\theta - \gamma_p \nabla_\theta \mathcal{L}(\theta, D_{\text{tr}}), \sigma_\theta^2) \). Finally, meta-parameters \( \Theta \) are updated using ADAM

\[ \nabla_\theta \left\{ \sum_i \mathcal{L}(\phi_i, D_{\text{test}}) + D_{\text{KL}}(q(\theta|D_{\text{test}})||p(\theta|D_{\text{tr}})) \right\}. \]

Prediction accuracy of PLATIPUS in 5-way 1-shot miniImageNet classification is comparable to LLAMA and BMAML which are also Bayesian extensions of MAML.

VERSA (Versatile and Efficient Amortization of Few-Shot Learning) uses Bayesian decision theory (BDT) for variational inference. Training data is \( \{(x_k, y_k)\}_{k=1}^n \). Validation data is \( \{(x_k^*, y_k^*)\}_{k=1}^m \). Task data is the union of training data and validation data denoted by \( \{(x_k, y_k)\}_{k=1}^N \). Meta-parameter is \( \theta \) and task-specific parameter for task \( T_i \) is \( \phi_i \). Joint distribution is

\[ p\left( \{y_i, \phi_i\}_{i=1}^N \mid \{x_i\}_{i=1}^N, \theta \right) = \prod_{i=1}^N p(\phi_i|\theta) \prod_{j=1}^m p(y_j|x_j, \phi_i, \theta) \cdot \prod_{k=1}^m p(y_k^*|x_k^*, \phi_i, \theta). \]

For neural network base learners, task-specific parameters \( \phi_i = \{W_i, b_i\} \) are composed of weights and biases. In testing data, predicted label is \( \hat{y} \) and unknown true label is \( \tilde{y} \). Label is predicted with

\[ \min_{\hat{y}} \mathcal{L}(\tilde{y}, \hat{y})p(\hat{y}|D_i)d\hat{y}, \]

where \( \mathcal{L}(\tilde{y}, \hat{y}) \) is the distance between predicted and true label. Posterior distribution of unknown true label is

\[ p(\hat{y}|D_i) = \int p(\hat{y}|\phi_i)p(\phi_i|D_i)d\phi_i, \]

where \( p(\phi_i|D_i) \) is the posterior distribution of task-specific parameter. In distributional BDT, true label \( \tilde{y} \) follows distribution \( q(\tilde{y}) \) and the objective is to estimate a predictive distribution of unknown label:

\[ \min_{q \in \mathcal{Q}} \int \mathcal{L}(\tilde{y}, q(\tilde{y}))p(\tilde{y}|D_i)d\tilde{y}, \]

where predictive label distribution \( q \) lies within a pre-specified distribution family \( \mathcal{Q} \). Amortized variational training is applied to make estimation based upon distributional BDT. Distribution of unknown label is specified as \( q(\tilde{y}) = q_\phi(\tilde{y}|D) \) where \( \phi \) is the unknown parameter of interest. Parameter estimator in predictive label distribution is

\[ \min_{\phi} \mathbb{E}_{p(D, \tilde{y})}\mathcal{L}(\tilde{y}, q_\phi(\tilde{y}|D)). \]

Loss function \( \mathcal{L} \) is specified as negative log likelihood:

\[ \mathcal{L}(q_\phi) = \mathbb{E}_{p(D, \tilde{y})}[-\log q_\phi(\tilde{y}|D)]. \]

Variational lower bound used to estimate unknown label is

\[ \mathbb{E}_{p(D, \tilde{y})}[D_{\text{KL}}\{p(\hat{y}|D)||q_\phi(\hat{y}|D)\}] + \mathcal{H}[q_\phi(\tilde{y}|D)]. \]

Prediction based upon distributional Bayesian decision theory and amortized variational inference is precise.

Table presents testing accuracy of meta-learning methods on 5-way 1-shot miniImageNet classification. We can see that performance of Bayesian meta-learning methods is comparable to black-box adaptation, similarity-based models and base learner meta-learner double-layer approach. Among Bayesian meta-learning methods, BMAML and VERSA show best performance. Among all meta-learning models reviewed, MetaOptNet shows highest testing accuracy.
Table 4: Testing accuracy of meta-learning methods on 5-way 1-shot miniImageNet classification.

| Method                                                                 | Accuracy         |
|------------------------------------------------------------------------|------------------|
| LLAMA                                                                  | 49.40 ± 1.83%    |
| BMAML                                                                  | 53.8 ± 1.46%     |
| PLATIPUS                                                               | 50.13 ± 1.80%    |
| VERSA                                                                  | 53.40 ± 1.82%    |
| Activation to parameter with neural network                           | 54.53 ± 0.40%    |
| wide residual network WRN                                              | 59.60 ± 0.41%    |
| AdaCNN with DF                                                         | 48.34 ± 0.68%    |
| AdaResNet with DF                                                      | 56.88 ± 0.62%    |
| Matching Net                                                           | 46.6%            |
| Prototypical Net                                                       | 46.61 ± 0.78%    |
| Activation to parameter with neural network                           | 52.68 ± 0.51%    |
| Relation Net                                                           | 50.44 ± 0.82%    |
| SNAIL                                                                  | 45.1%            |
| TADAM with $\alpha$, AT and TC                                        | 58.5 ± 0.3%      |
| TADAM without tuning                                                   | 56.5 ± 0.4%      |
| Dynamic few-shot with C128F feature extractor                         | 55.95 ± 0.84%    |
| with ResNet feature extractor                                          | 55.45 ± 0.89%    |
| with cosine classifier and attention based weight generator            | 58.55 ± 0.50%    |
| with cosine classifier and no weight generator                         | 54.55 ± 0.44%    |
| mAP-SSVM                                                               | 50.32 ± 0.80%    |
| mAP-DLM                                                                | 50.28 ± 0.80%    |
| MAML                                                                   | 48.7 ± 1.84%     |
| MetaNet                                                                | 49.21 ± 0.96%    |
| Reptile                                                                | 49.97 ± 0.32%    |
| Meta-SGD                                                               | 50.47 ± 1.87%    |
| Meta-LSTM                                                              | 43.44 ± 0.77%    |
| R2-D2 with 5 iterations                                               | 51.8 ± 0.2%      |
| LR-D2 with 1 iteration                                                | 51.0 ± 0.2%      |
| TPN                                                                    | 55.51 ± 0.86%    |
| LEO                                                                    | 61.76 ± 0.08%    |
| MetaOptNet-RidgeReg                                                   | 61.41 ± 0.61%    |
| MetaOptNet-SVM                                                        | 62.64 ± 0.61%    |
| MetaOptNet-SVM-trainval                                               | 64.09 ± 0.62%    |

3 Applications of Meta-learning

First application of meta-learning is in robotics. Robots acquire basic skills from few demonstrations using meta-imitation learning. Robots can also perform decision making and make optimal response to different tasks through meta-RL. Similarly, for trading robot, with meta-imitation learning, robots can imitate human trading and learn complex trading decision making scheme. With meta-RL, trading robots can autonomously adapt decision making mechanism to fast changing market conditions.

Second application of meta-learning is in drug discovery. Sample size in medical studies is usually small especially for rare disease or gene SNPs. A task is regarded as finding an effective treatment for a virus which might mutate frequently. Computation for drug candidate is expensive thus adapting previously trained models to new tasks should be considered to save time and expense in drug discovery.

Third application is in translation of rarely used languages. Frequently used words constitute only a small proportion of all words. Most words appear less frequently and their translation depends upon a relatively smaller sample. Recent meta-learning models are applicable to few-shot datasets where translation can be learned using adaptation of pre-trained models. Though sample size is small for ancient or rare languages, a few-shot meta-learning model can be trained to translate these scripts.
Another application is to handle unusual situations in financial market. In sudden market crash, which is rare in the past, proper trading strategies to minimize expected loss can be developed under meta-learning framework. This scheme can be used as a fail-safe and integrated into the current trading machines.

For applications where an explicit definition of task and label can be clearly specified, meta-learning provides a feasible plan for problem-solving. Meta-learning framework is flexible and can be conveniently integrated with most machine learning algorithms to provide feasible real-life solutions. For tasks which generally require heavy computation, meta-learning presents the option of aggregating or adapting previous results to save computation.

3.1 Meta-Reinforcement Learning

Reinforcement learning (RL) is based upon markov decision process (MDP) $\langle S, A, p, r, \gamma, \rho_0, H \rangle$ where $S$ is the set of all states, $A$ is the set of all actions, $p(s'|s,a)$ is the transition process, $r(s,a)$ is a reward function, $\gamma$ is the discount factor on future reward, $\rho_0(s)$ is the initial state distribution, and $H$ is the time horizon $[23]$. A trajectory between timestamp $i$ and timestamp $j$ is denoted as $\tau(i,j) \equiv (s_i, a_i, s_{i+1}, a_{i+1}, \ldots, s_j, a_j, s_{j+1})$. Policy is defined as $\pi: S \to A$. Objective is to identify an optimal policy that maximizes the summation of all rewards from a trajectory. An RL task $[12]$ is defined as

$$T_i \equiv (S_i, A(s), p_i(s), p_i(s'|s,a), r_i(s,a)),$$

where $S_i$ is the state space, $A(s)$ is the action space, $p_i(s)$ is the initial state distribution, $p_i(s'|s,a)$ is the transition process and $r_i(s,a)$ is the reward process. In an RL task, we observe states, actions and find an optimal policy to maximize the estimated sparse reward function.

Meta-reinforcement learning (meta-RL) considers the interaction between agent and changing environment, as in figure 5. Main objective of meta-RL is to train robots to handle unusual situations in reward-driven tasks with only few demonstrations. Meta-RL concerns data-efficient fast adaptation to out-of-distribution RL tasks after only few demonstrations $[20]$.

![Figure 5: Meta-reinforcement learning. Robots interact with environment to get state and reward. Environment is different after interaction with robots. Environmental conditions vary in different tasks.](Image 277x381 to 335x426)

SSA (Success-Story Algorithm) $[31]$ with SMP (Self-Modifying Policy) is for lifelong RL. Action checkpoints along lifetime are denoted by $c_1, \ldots, c_H$, which constitute a solution path for sequential decision making. At each checkpoint, SMP-modification candidates are generated using GP (Genetic Programming) and evaluated to see whether they accelerate reward production. GP evolves slowly and is not aggressive for identifying useful SMP-modifications, making it stable and ideal for lifelong RL. SSC (Success-Story Criterion) is

$$\frac{R(t)}{t} < \frac{R(t) - R(c_1)}{t - c_1} < \frac{R(t) - R(c_2)}{t - c_2} < \cdots < \frac{R(t) - R(c_{V_1})}{t - c_{V_1}},$$

where $V_i$ is the set of all checkpoints up to time $t$. Ratio of reward and time is a measure of RL algorithm efficiency. Useful SMP-modifications meet SSC and are kept in RL algorithm. As in $[101]$, Gődel machines based upon useful incremental self-improvement theoretically achieve global optimum and is never trapped in any local optimum. SSA with SMP is a computationally stable Gődel machine that is suitable for lifelong RL.

All meta-learning models reviewed in section 2 can be extended to RL tasks. $[102]$ and $[103]$ lay out RNN-based meta-RL frameworks. An extension of MAML to RL tasks is presented in $[12]$, where the loss function of task $T_i$ is rewritten as

$$\mathcal{L}_{T_i}(f_\phi) = -\mathbb{E}_{S_t,A_t} \left\{ \sum_{t=1}^{H} \gamma^t r_i(s_t,a_t) \right\},$$

where $f_\phi$ is a model specified for policy, and $\phi$ is the task-specific policy parameter of interest. For RL tasks, we minimize the loss function $\mathcal{L}_{T_i}(f_\phi)$ to estimate an optimal policy $f_\phi$ and seek fast adaptation to novel tasks with changing environment.

Another application of MAML in meta-RL is E-MAML $[104]$ which adds an exploratory term to MAML. E-MAML considers the influence of sampling $\pi_\theta$ upon future rewards $R(\tau)$, where $\tau \sim \pi_{U(\theta)}$. MAML applies stochastic gradient of $\int R(\tau) \pi_{U(\theta)}(\tau) d\tau$. E-MAML uses stochastic gradient of $\int \int R(\tau) \pi_{U(\theta)}(\tau) \pi_\theta(\tau) d\tau d\tau$, where $\tau \sim \pi_\theta$. 


Meta-RL in CNN architecture optimization is surveyed in [17]. Evolutionary algorithms provide candidates for hyperparameter search. We may utilize a goal-based Q-learning model to update parameters in adaptation. Reward function is denoted as

\[ V^\pi(s) = \mathbb{E}(r|s, \pi). \]

Bellman update on Q function is written as

\[
Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma \max_{a} Q(s_{t+1}, a) - Q(s_t, a_t)].
\]

The optimal action that maximizes Q-function is the best CNN architecture design.

PEARL (Probabilistic Embeddings of Actor-critic RL) [21] defines loss function with actor-critic RL and applies MAML for few-shot update on parameters in adaptation. Actor-critic RL contains policy estimation (actor) and value estimation (critic). First, variational inference is applied to infer hidden state \( z \). Inference network \( q_\phi(z|c) \) is specified as

\[
q_\phi(z|c_1, \ldots, c_N) \propto \prod_{n=1}^N \Psi_\phi(z|c_n),
\]

where context \( c_n = (s_n, a_n, s'_n, r_n) \) consists of state-action pairs, \( \Psi_\phi(z|c_n) = N(f_\phi^m(c_n), f_\phi^n(c_n)) \) is a Gaussian distribution and \( f_\phi \) is a neural network that predicts \( \mu_n \) and \( \sigma_n \) for each context \( c_n \). Variational lower bound optimized to estimate \( z \) is

\[
\mathbb{E}_T \mathbb{E}_z [R(T, z) + \beta D_{KL}(q_\phi(z|c)||p(z))],
\]

where \( T \sim p(T) \), \( z \sim q_\phi(z|c), p(z) \) is a prior distribution over \( z \) and \( R(T, z) \) is a task-specific objective. This meta-RL algorithm is based upon soft actor-critic (SAC) method where we consider the joint optimization of inference network \( q_\phi(z|c) \), actor \( \pi_\theta(a|s, z) \) and critic \( Q_\xi(s, a) \). Critic loss function is defined to be

\[
L_{\text{critic}} = \mathbb{E}_{s,a,r,s',z} \left[ Q_\xi(s, a, z) - (r + \hat{V}(s', z)) \right]^2,
\]

where \( Q_\xi(s, a) \) is the value of current state-action pair, \( \hat{z} \) is the average condition, \( r \) is current reward and \( \hat{V}(s', z) \) is the optimal target value derived from all possible future states. Critic loss minimizes the value difference between current state-action and optimal target value. Actor loss function [105] is defined as

\[
L_{\text{actor}} = \mathbb{E}_{s \sim \mathcal{Z}_\theta} D_{KL} [\pi_\theta(a|s, \hat{z}) || \hat{Z}_\theta(s)],
\]

where \( \mathcal{Z}_\theta(s) = \{ \exp(Q_\xi(s, a, \hat{z})) \} da \) is a normalizing constant. Actor loss function minimizes the distance between policy \( \pi_\theta(a|s, \hat{z}) \) and value-directed action weighted specified as \( \exp(Q_\xi(s, a, \hat{z})) / \hat{Z}_\theta(s) \) which assigns more weight to action with higher value.

Joint estimation of parameters in inference network, policy and value function is conducted with MAML. First, sample RL task \( b_i = \{ s_j, a_j \}_{j=1}^N \sim \mathcal{B}^i \) and context \( c^i \sim S_i(B^i) \) where \( c^i = \{ s_j, a_j, s'_j, r_j \}_{j=1}^N \). Then sample hidden state \( z \sim q_\phi(z|c^i) \). Compute loss functions: \( L_{\text{actor}} = \mathbb{E}_{\pi_\theta, z} [L_{\text{actor}}^i], L_{\text{critic}} = \mathbb{E}_{\pi_\theta, z} [L_{\text{critic}}^i], L_{KL} = \beta D_{KL}(q_\phi(z|c^i)||r(z)) \).

MAML updates on parameters are as follows:

\[
\phi \leftarrow \phi - \alpha_1 \nabla_\phi \sum_i (L_{\text{critic}}^i + L_{KL}^i),
\]

\[
\theta \leftarrow \theta - \alpha_2 \nabla_\theta \sum_i L_{\text{actor}}^i,
\]

\[
\xi \leftarrow \xi - \alpha_3 \nabla_\xi \sum_i L_{\text{critic}}^i.
\]

Actor loss function is used to update policy parameter and critic loss function is used to update inference network and value function.

MetaGenRL [106] improves generalization to vastly different environments through combining multiple agents to train a meta-learner. MetaGenRL considers DDPG (Deep Deterministic Policy Gradient) actor-critic model [107, 108] which applies DQN (deep Q network) to continuous action domain. [106] demonstrates that DDPG supports generalization to vastly different environments. MetaGenRL explicitly separates adaptation of policy \( \pi_\theta \) and value \( Q_\xi \) model which contributes to generalization improvement. Application of second-order gradients improves sampling efficiency in RL. Critic loss is

\[
L_{\text{critic}} = \sum_{s,a,r,s'} [Q_\xi(s, a) - (r + \gamma \hat{V}(s', \pi_\theta(s'))) ]^2.
\]

Critic loss is minimized to estimate value parameter \( \xi \). Policy parameter \( \theta \) can be updated by taking the gradient of \( \mathbb{E}_T \sum_{i=1}^H \log \pi_\theta(a|s) A(\tau, V, t) \), where \( A(\tau, V, t) \) is a GAE (generalized advantage estimate) and \( V : S \rightarrow \mathbb{R} \) is a
value function. In MetaGenRL, meta-objective function is specified as a neural network \( L_\alpha(\tau, \pi_\theta, V) \), where \( \pi_\theta \) is an auxiliary input and \( \alpha \) is neural network parameter. During adaptation, \( \xi \) is updated by minimizing critic loss \( L_{\text{critic}} \):
\[
\theta' \leftarrow \theta - \nabla_\theta L_\alpha(\tau, x(\theta), V), \\
\alpha' \leftarrow \alpha + \nabla_\alpha Q_\xi(s, \pi_\theta(s)).
\]

Robots learn to survive dynamic environments through continual meta-RL method GrBAL (Gradient-Based Adaptive Learner) \[23\]. Transition probability estimator is \( \hat{p}_\theta(s'|s, a) \) where \( \theta \) is a maximum likelihood estimate. In meta-learning procedure, a general equation of task-specific parameter update is
\[
\phi_T = u_{\psi}(D^T_T, \theta),
\]
where \( \theta \) is meta-parameter, and \( u_{\psi}(D^T_T, \theta) = \theta - \nabla_\theta L(D^T_T, \theta) \) in MAML. Parameters are estimated by minimizing the meta-loss function:
\[
\min_{\theta, \psi} E_{\tau \sim \rho(T)}[L(D^T_T, \phi_T)].
\]
In total, \( M \) trajectories are sampled from a novel task. We use the experience of \( K \) time steps as training data to predict next \( K \) time steps which is testing data. Let \( \mathcal{E} \) denote the set of all dynamic real-world environments. Under these concrete settings, meta-objective is
\[
\min_{\theta, \psi} E_{\tau \sim \rho(T)}[L(D^T_T, \phi_T)],
\]
where \( \tau_{\mathcal{T}}(t - M, t + K) \) is the task trajectories data between timestamps \( t - M \) and \( t + K \). \( \tau_{\mathcal{T}}(t, t + 1) \) is the task validation trajectories data between \( t \) and \( t + 1 \) and \( \tau_{\mathcal{T}}(t - M, t - 1) \) is the task training trajectories data. Meta update on the parameter is written as
\[
\phi_{\mathcal{T}} = u_{\psi}(\tau_{t - M}, t - 1, \theta).
\]
Meta-loss function is
\[
L(\tau_{\mathcal{T}}(t, t + K), \phi_{\mathcal{T}}) \equiv -\frac{1}{K} \sum_{k=t}^{t+K} \log \hat{p}_{\phi_{\mathcal{T}}}(s_{k+1}|s_k, a_k).
\]
Here MAML update on parameter can be written as GrBAL:
\[
\phi_{\mathcal{T}} = \theta_{\mathcal{T}} + \psi \nabla_{\theta_{\mathcal{T}}} \frac{1}{M} \sum_{k=t-M}^{t-1} \log \hat{p}_{\theta_{\mathcal{T}}}(s_{k+1}|s_k, a_k),
\]
where \( \psi \) is the step size. Recurrence-Based Adaptive Learner (ReBAL) update parameters \( \psi \) using a recurrent model.

ReBAL allows us to make continual improvement in the meta-training process. Another meta-RL framework for continual self-improvement is SSL (Self-Supervised Learning) \[102\]. SSL performs supervised learning augmented by artificial curiosity and boredom. Meta-loss minimization creates dynamic attention which guides algorithm exploration in artificial curiosity. Boredom refers to avoidance of environments which have been perfectly solved. Base learner and meta-learner framework in section 2.3 can be applied in meta-RL setting. Internal feedback corresponding to base learner comes from stepwise interaction between agents and known environments under the influence of agent actions. External feedback corresponding to meta-learner is from all interactions between agents and unknown environments, which shapes the whole embedded credit assignment mechanism.

Coevolution \[109\] involves cooperation and competition between agents and environments. In POET (Paired Open-Ended Trailblazer) \[110\], more complex environment is generated from coevolution between agent and environment, in which POET trains an accurate RL model for a difficult environment which cannot be solved otherwise by training from scratch. In the \( i \)th sampled trajectory, reward from interaction between agent and environment is \( R(\theta_i) \) parametrized by \( \theta_i \sim \rho_{\theta, \sigma}(\theta_i) = N(\theta, \sigma^2) \). Optimization to estimate hyperparameter \( \theta \) is
\[
\max_{\theta, \sigma} E_{w \sim \rho_{\theta, \sigma}(w)} R(w) \approx \frac{1}{n} \sum_{i=1}^{n} R(\theta_i).
\]
To simplify, variance in sampled trajectories \( \sigma \) can be set to be 1. POET consists of three components: generation of more complex environment \( R(\theta) \) from coevolution between agent \( \theta \) and environment \( R \), train an RL model for current agent and environment pair, and adaptation of current agent to varying environment. POET belongs to AI-GA (AI-generating algorithm) \[111\] for self-supervised learning \[102\]. In general, AI-GA consists of three components: meta-learning architecture, learner algorithms and generation of more complex environment.
A similar meta-imitation model for the case where robots learn from both human and robot demonstrations is proposed without expert action to imitate, imitation loss function is written as

$$L_{\pi_t}(f_{\theta}) = \sum_{t \sim p(T)} \sum_{i=1}^{H} \| f_{\phi}(a_{t}^{(j)}) - a_{t}^{(j)} \|_2^2,$$

where $\pi_t^*$ is an expert policy we target to imitate, and $L$ is an imitation loss function. Loss function is mean squared error if actions are continuous, and cross-entropy otherwise. Generally imitation loss function is specified as behavior-cloning loss:

$$L_{BC}(f_{\phi}, d) = \sum_{t \sim D_T} \| Wf_{\phi}(s_t) + b - a_t \|_2^2.$$

Then MAML update on meta-parameter is

$$(\theta, \psi) \leftarrow (\theta, \psi) - \beta \nabla_{\theta, \psi} L_{BC}(\phi_T, d^r).$$

### 3.2 Meta-Imitation Learning

In complex unstructured environments where there is no explicit task or reward definition, we can learn through imitation learning. With only one or few demonstrations, meta-learning is introduced to achieve fast adaptation of imitation model to changing environment or varying target policy, as in figure 6. Meta-imitation learning applies similar past experiences to a novel task in sparse-reward situations. Applications of meta-imitation learning concentrate upon robotics where robots acquire skills from one visual demonstration. [26] proposes a one-shot visual imitation learning model built upon MAML. Few-shot meta-imitation learning is realized through MAML updates on parameters in novel settings. Demonstration trajectory data is denoted as $\tau = \{o_1, a_1, \cdots, o_H, a_H \}$ which consists of observation-action pairs. Similarly, an imitation task is defined as

$$T_i = \{ \tau \sim \pi_t^*, L(a_1, \cdots, a_H, \hat{a}_1, \cdots, \hat{a}_H), H \},$$

where $H$ is the finite time horizon, $\pi_t^*$ is an expert policy we target to imitate, and $L$ is an imitation loss function. Loss function is mean squared error if actions are continuous, and cross-entropy otherwise. Generally imitation loss function is specified as behavior-cloning loss:

$$L_{BC}(f_{\phi}, d) = \sum_{t \sim D_T} \sum_{i=1}^{H} \| Wf_{\phi}(s_t) + b - a_t \|_2^2.$$

After MAML update on parameters, model imitates one demonstration from a new expert policy. Additionally, in cases without expert action to imitate, imitation loss function is written as

$$L_{\pi_t}(f_{\phi}) = \sum_{t \sim p(T)} \sum_{i=1}^{H} \| Wf_{\phi}(a_t^{(j)}) - a_t^{(j)} \|_2^2.$$

Policy CNN $f_{\phi}$ is directly estimated from one demonstration video.

A similar meta-imitation model for the case where robots learn from both human and robot demonstrations is proposed in [11]. Human demonstration data is denoted as $D_T^h$. Robot demonstration data is $D_T^r$. Human video $d^h \sim D_T^h$. Robot video $d^r = \{o_1, \cdots, o_H, s_1, \cdots, s_H, a_1, \cdots, a_H \} \sim D_T^r$. MAML update on task-specific policy parameter is

$$\phi_T = \theta - \alpha \nabla_{\theta} L_{\psi}(\theta, d^h),$$

where $L_{\psi}$ parameterized by $\psi$ is the adaptation objective. Behavioral cloning loss function is

$$L_{BC}(\phi, d^r) = \sum_{t} \log \pi_\phi(a_t|o_t, s_t).$$

Meta-objective is the combination of MAML policy parameter adaptation and behavioral cloning loss function:

$$\min_{\theta, \psi} \sum_{T \sim p(T)} \sum_{d^h \in D_T^h} \sum_{d^r \in D_T^r} L_{BC}(\phi_T, d^r).$$

Then MAML update on meta-parameter is

$$(\theta, \psi) \leftarrow (\theta, \psi) - \beta \nabla_{\theta, \psi} L_{BC}(\phi_T, d^r).$$

Figure 6: Meta-imitation learning. Find a policy such that robots act like demonstration. Interaction between robots and environment is the same as in demonstration. Robots clone policy and reward in demonstration.
From the probabilistic perspective, $\phi_T$ is an MAP estimate of $\phi$ based upon approximate inference on $\log p(\phi|D_T^T, \theta)$ [7].

High-fidelity imitation model MetaMimic proposed in [112] stores memory of off-policy RL and replays memory for imitation learning. Policy model is specified to be a wide deep neural network with batch normalization. Usually larger deeper neural network policy model in off-policy RL brings better adaptation to novel tasks.

Experience replay memory is denoted as $\mathcal{M}$. Memory items are constructed in the following procedure. Action $a_t \sim \pi(a_t, d_{t+1})$, where $d_{t+1}$ is the demonstration video that leads the imitation. Execute $a_t$. Then observe $o_{t+1}$ and task reward $r_t^{task}$. Compute imitation reward $r_t^{imitate} = g(o_{t+1}, d_{t+1})$, which measures the similarity between learned observation and video demonstration. The following item $(o_t, a_t, o_{t+1}, r_t^{task}, r_t^{imitate}, d_{t+1})$ is stored in memory $\mathcal{M}$.

One demonstration is denoted as $d = \{d_1, \cdots, d_H\} \sim p(d)$. Imitation policy is specified as $\pi_\theta$ parameterized by $\theta$. Action $a_t = \pi_\theta(o_t, d)$. Environment render $\mathcal{E}(\pi)$ maps action to observation $\mathcal{E}(\pi) = \{o_1, \cdots, o_H\}$. Policy parameter $\theta$ is estimated by maximizing imitation reward:

$$\max_\theta \mathbb{E}_{d \sim p(d)} \left[ \sum_{t=1}^H \gamma^t g (d_t, \mathcal{E}(\pi_\theta(o_t, d))) \right],$$

where $\gamma > 0$ is the discount factor, and $g$ is a similarity measure. By maximizing imitation reward, similarity between learned observation and demonstration is maximized. Imitation reward is the Euclidean distance between learned observation and demonstration

$$r_t^{imitate} = \beta_1 \exp(-\|o_{t+1}^{image} - d_{t+1}^{image}\|_2^2) + \beta_2 \exp(-\|o_{t+1}^{body} - d_{t+1}^{body}\|_2^2),$$

where $d_{t}^{body}$ is the body position coordinate and velocity, and $d_{t}^{image}$ is the image pixel capturing interactions between body and objects.

WTL (Watch-Try-Learn) [27] is a meta-imitation learning (MIL) model which combines MIL with trial-and-error RL. It includes a trial procedure which generates a policy from one demonstration and a retrial process that compares with trial procedure: to identify a successful policy. Identifiability issue may arise in which policy is not uniquely determined by one demonstration.

A trajectory from RL task $T_t$ is denoted as $\tau_t = [(s_1, a_1), r_t(s_1, a_1), \cdots, (s_H, a_H, r_t(s_H, a_H))]$. Demonstration trajectory is denoted as $d = [(s_1, a_1), \cdots, (s_H, a_H)]$. Training demonstrations $\{d_{i,k}\}$ are sampled from $\mathcal{D}$. Trial policy $\pi_\theta(a|s, \{d_{i,k}\})$ is proposed based upon one demonstration. Trial objective to estimate trial policy $\pi_\theta$ is

$$\mathcal{L}(\theta, \mathcal{D}_t) = \mathbb{E}_{(d_{i,k}) \sim \mathcal{D}_t, \mathcal{E}(d_{i,k}^{test}) \sim \mathcal{D}_t \setminus \{d_{i,k}\}} \mathbb{E}_{(s, a_t) \sim \mathcal{D}_{test}} \left[ -\log \pi_\theta(a_t|s, \{d_{i,k}\}) \right].$$

MAML is applied to update $\theta$ with trial objective:

$$\theta \leftarrow \theta - \alpha \nabla_\theta \mathcal{L}(\theta, \mathcal{D}_t).$$

Trial trajectories are generated from trial policy $\{\tau_{i,l}\} \sim \pi_\theta(a|s, \{d_{i,k}\})$. Demonstration-trial trajectory pairs $\{(d_{i,k}), \{\tau_{i,l}\}\}$ are sampled from $D^*_t$. A retrial policy integrates generated trials with demonstration to learn a successful policy $\pi_\phi(a|s, \{d_{i,k}\}, \{\tau_{i,l}\})$. Retrial objective to estimate a successful policy $\pi_\phi$ is

$$\mathcal{L}(\phi, \mathcal{D}_t, D^*_t) = \mathbb{E}_{(d_{i,k}, \{\tau_{i,l}\}) \sim \mathcal{D}_t, \mathcal{E}(d_{i,k}^{test}) \sim \mathcal{D}_t \setminus \{d_{i,k}\}} \mathbb{E}_{(s, a_t) \sim \mathcal{D}_{test}} \left[ -\log \pi_\phi(a_t|s, \{d_{i,k}, \{\tau_{i,l}\}\}) \right].$$

MAML is applied to update $\phi$ with retrial objective:

$$\phi \leftarrow \phi - \beta \nabla_\phi \mathcal{L}(\phi, \mathcal{D}_t, D^*_t).$$

### 3.3 Online Meta-learning

Online learning is applied to streaming data where a small batch of data is obtained each time and fast adaptation is required for timely reaction to continually changing environment. In online meta-learning, a small batch of streaming data obtained each time is regarded as a task dataset, as in figure [7]. In [69], FTML (follow the meta leader) is constructed by integrating a classical online algorithm follow the leader (FTL) with MAML. FTL is proved to guarantee strong performance for smooth convex loss functions.
For task $T_i$, MAML updates parameter $\theta$ using

$$U_i(\theta) = \theta - \alpha \nabla_{\theta} \mathcal{L}_{T_i}(h_\theta).$$

Regret function from FTL measures the distance between current value and optimal target value

$$R(\theta) = \sum_{i=1}^{J} \mathcal{L}_{T_i}(h_{U_i(\theta)}) - \min_\theta \sum_{i=1}^{J} \mathcal{L}_{T_i}(h_{U_i(\theta)}).$$

By estimating optimal task-specific parameter that minimizes regret function, current value is closest to optimum. Since optimum is a fixed target, minimization of regret function is equivalent to minimizing first component in equation (3).

Tasks are streaming data and follow a natural time order so that FTML updates task-specific parameters by minimizing the summation of task-loss up to task $T_i$:

$$\phi_{i+1} = \arg\min_\theta \sum_{k=1}^{i} \mathcal{L}_{T_k}(h_{U_k(\theta)}).$$

In order to solve this optimization online, we apply practical online gradient computation:

$$d_k(\theta) = \nabla_{\theta} \mathbb{E}_{k \sim \nu} \mathcal{L}(D_{val}^k, U_k(\theta)),$$

where $U_k(\theta) \equiv \theta - \alpha \nabla_{\theta} \mathcal{L}(D_{tr}^k, \theta)$, and $\nu$ is a sampling distribution from tasks $T_1, \cdots, T_i$.

Figure 7: Online meta-learning. Each data batch in streaming data is regarded as a task dataset. Base learner is applied to each task and meta-learner updates base learner across tasks.

In robotics, robots need continually rapid and precise adaptation to online streaming data especially when simulated mechanical failures or extreme environmental conditions suddenly occur. In such out-of-distribution tasks, fast adaptation of trained problem-solving model is required. In [59], deep online meta-learning and nonparametric Bayes are integrated to provide a feasible solution for robots to survive. This method regards tasks to be countably infinitely many.

Task distribution is $P(T_i = T_i|\mathbf{x}_t, y_t)$, where the set of all tasks is a countable infinite set. Expectation-maximization (EM) is applied to maximize log likelihood:

$$\mathcal{L} = \mathbb{E}_{T_i \sim p(T_i|\mathbf{x}_t, y_t)}[\log p_{\phi_i}(T_i)(y_t|\mathbf{x}_t, T_i)],$$

where $\phi_i(T_i)$ is the task-specific parameter. Posterior distribution of task is

$$p(T_i = T_i|\mathbf{x}_t, y_t) \propto p_{\phi_i}(T_i)(y_t|\mathbf{x}_t, T_i = T_i)P(T_i = T_i).$$

Nonparametric Bayesian prior Chinese restaurant process (CRP) is used as the prior distribution on task:

$$P(T_i = T_i) = \frac{\sum_{k=1}^{i-1} P(T_k = T_i)}{1 + \alpha},$$

$$P(T_i = T_{new}) = \frac{\alpha}{1 + \alpha},$$

where $\alpha > 0$. Task prior distribution CRP is

$$P(T_i = T_i) = \left\{ \sum_{k=1}^{i-1} P(T_k = T_i) + \delta(T_k = T_{new})\alpha \right\}.$$
Meta-parameter $\theta$ is the prior of task-specific parameter $\phi_i$. One gradient update of $\phi_i$ from prior $\theta$ is built upon FTML in [69] where loss function depends upon all trained tasks before $t$

$$\phi_{t+1}(T_i) = \theta - \beta \sum_{k=0}^{t} [P(T_k = T_i | x_k, y_k)$$

$$\cdot \nabla_{\phi_k(T_i)} \log P_{\phi_k(T_i)}(y_k | x_k)].$$

MAML update on task-specific parameter $\phi_i(T_i)$ is

$$\phi_{t+1}(T_i) = \phi_{t}(T_i) - \beta P(T_i = T_{t} | x_t, y_t)$$

$$\cdot \nabla_{\phi_i(T_i)} \log P_{\phi_i(T_i)}(y_t | x_t).$$

MAP estimator from posterior task distribution is

$$\mathcal{T}^* = \arg\max_{T_i} P_{\theta_{t+1}(T_i)}(y_t | x_t, T_{t} = T_{t}).$$

Prediction is made based upon task-specific parameter $\phi_{t+1}(T^*)$.

Another application of meta-online learning is fast online adaptation of neural network model. [68] proposes ALPaCA (Adaptive Learning for Probabilistic Connectionist Architectures). The last layer of a neural network is specified as supervised meta-learning methods.

Oracle+ProtoNets achieves 62.29% accuracy on 5-way 5-shot miniImageNet, comparable to the performance of supervised meta-learning methods.
4 Conclusion and Future Research

Our survey provides an overview upon meta-learning research including black-box adaptation, similarity-based approach, base learner and meta-learner model, Bayesian meta-learning, meta-RL, meta-imitation learning, online meta-learning and unsupervised meta-learning. There are more branches of meta-learning covering integration with causal reasoning, change-point estimation, Bayesian deep learning and real-life applications to explore in the future.

Meta-learning concentrates upon adaptation to out-of-distribution tasks. For in-distribution tasks, similarity structure between tasks is more explicit. Models for adaptation to in-distribution tasks should emphasize upon mining similarity structure to facilitate accurate identification. Methods developed for out-of-distribution tasks should maximize model generalization ability. There exists a tradeoff between model fitting and model generalization. Base learner focuses upon fitting and meta-learner concentrates upon generalization, which offer a flexible combination to strike a balance in-between.

In terms of application, meta-learning presents an opportunity for improving model adaptivity across changing environments. For theoretical research, meta-learning provides a flexible framework of integrating different methods combining advantages while complementing disadvantages of each other. Statistical models are less prone to over-fitting and can be combined with machine learning methods under meta-learning framework. Meta-learning allows us to partition tasks into levels, apply base learner, meta-learners and other learners at different levels constructing an integrated solution for a complex mission. Communication between base learner and meta-learners at different levels is not limited to initial values, aggregating gradients, objectives and learned optimizers. More complex model specifications can be designed to suit the need of real-life applications.

Bandit algorithms under piecewise-stationary environment have been delved into thoroughly in literature. With meta-RL, stationarity assumption can be relaxed and algorithms can be extended to non-stationary environment. Ideally meta-learning can be combined with current theorems to relax assumptions, enlarge the set of all applicable cases, and improve adaptivity of methods.

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