Study on Thermo-Mechanical Behaviors of Shape Memory Polymer Beam

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Abstract. Shape memory polymer (SMP) is a new smart material and has attracted much attention of people due to its properties and advantages. SMP can fix various shapes under the action of external environment and recover under specific stimulation, such as heat [1], light [2], electricity [3], water [4] and so on. SMP has many advantages, like low cost, large deformation, deform easily and so on, thus, SMP are being used more and more widely, like medicine field, aviation field, electronic field and so on. The beam structure, as an important structure of practical application, is one of the main application areas of SMP and it is meaningful to study its thermo-mechanical behaviors.

Keywords. Shape memory polymer, beam theory, constitutive model, kinematic equation, thermodynamic behaviors.

1. Introduction
Shape memory polymer (SMP) is a new smart material after shape memory alloy and it has attracted a lot of attention of people due to its properties and advantages. SMP can fix various shapes under the action of external environment and recover under specific stimulation, such as heat [1], light [2], electricity [3], water [4] and so on. SMP has many advantages, like low cost, large deformation, deform easily and so on, thus, SMP are being used more and more widely, like medicine field, aviation field, electronic field and so on. The beam structure, as an important structure of practical application, is one of the main application areas of SMP and it is meaningful to study its thermo-mechanical behaviors.

M Baghani, et al. [5] used a SMP constitutive model to study the behaviors of SMP Euler Bernoulli beam under different external conditions. Molaaghaie-Roozbahani, et al. [6] established the deflection governing equation of SMP beam which is under the different stages of thermodynamic cycle with using the equilibrium equation. Choi M J et al. [7] developed a configuration design optimization method for SMP beam and synthesize lattice structures which can achieve target negative. Zhou, et al. [8] proposed a constitutive model to study the size effect of SMP Euler-Bernoulli beam based on the SMP constitutive model and the modified couple stress theory.

In this paper, a new constitutive model of SMP beam is established to study the thermo-mechanical behaviors of SMP beam and the structure of this paper is as follows. First, based on the Zhou’s work [9], the constitutive equation of SMP is further derived and the material parameter equation is introduced. Then, according to the Euler-Bernoulli beam theory, the displacement components are introduced. Next, according to the relationship among the strain, stress, displacement and the material parameter equation,
the constitutive model of SMP beam is established. Finally, take the simply supported beams for example, the thermo-mechanical behaviors and some mechanical properties are studied.

2. Kinematic Equation of SMP

According to the mechanical properties of SMP, the strain of SMP can be divided into three parts: elastic strain $\varepsilon_{ij}^e$, viscous strain $\varepsilon_{ij}^v$ and thermal strain $\varepsilon_{ij}^{th}$, we can express the relationship as

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^v + \varepsilon_{ij}^{th}. \quad (1)$$

Based on the generalized Hooke law, the elastic strain $\varepsilon_{ij}^e$ can be described of

$$\varepsilon_{ij}^e = \frac{1}{E}[(1 + \nu)\sigma_{ij} - v\sigma_{ik}\delta_{ij}], \quad (2)$$

where the $E$ is the elasticity modulus and the $\nu$ is the Poisson ratio of SMP. According to the literature [8], the viscous strain rate of SMP can be written as

$$\varepsilon_{ij}^v = \frac{1}{\eta}[(1 + \nu)\sigma_{ij} - v\sigma_{ik}\delta_{ij}] - \frac{1}{\tau}[(1 + \nu)\varepsilon_{ij} - v\varepsilon_{ik}\delta_{ij}], \quad (3)$$

where the $\eta$ is the viscosity coefficient of SMP and the $\tau$ is the delay time of SMP. Based on the thermodynamic theories, the thermal strain can be described as

$$\varepsilon_{ij}^{th} = \alpha T \delta_{ij}, \quad (4)$$

where the $\alpha$ is the thermal coefficient of expansion.

According to the equation (1), equation (2), equation (3) and equation (4), and due to that in the initial stage, the viscous stress and strain are 0, the total strain expression can be derived as

$$\varepsilon_{ij} = \frac{1}{E}[(1 + \nu)\sigma_{ij} - v\sigma_{ik}\delta_{ij}] + \frac{1}{\eta}[(1 + \nu)\varepsilon_{ij} - v\varepsilon_{ik}\delta_{ij}] + \alpha T \delta_{ij}. \quad (5)$$

![Figure 1. Bernoulli-Euler beam structure.](image)

In figure 1, the Bernoulli-Euler beam structure is shown, according to the Bernoulli-Euler beam theory, the three displacement components in the $x$, $y$ and $z$ directions can be written as

$$u_i = -3w'(x), \quad u_2 = 0, \quad u_3 = w(x). \quad (6)$$

Based on theory of solid mechanics, the relationship between displacement and strain can be described as

$$\varepsilon_y = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (7)$$
According to the equation (5), equation (6) and equation (7), the expressions of stress components can be derived as follows

\[
\sigma_{ij} = \frac{(\mu + E t)(u_{i,j} + u_{j,i}) + \lambda u_{k,k} \delta_{ij} - \frac{E \alpha}{1 - 2\nu} \Delta T \delta_{ij}}{1 + \frac{E t}{\eta}},
\]

(8)

where \( \lambda \) and \( \mu \) are the Lame constants,

\[
\lambda = \frac{E v}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}.
\]

(9)

Based on the properties of SMP, according to the literature \[9\], the relationship between the material parameters, the elastic modulus \( E \), the viscosity coefficient \( \eta \), and the delay time \( \tau \), and temperature can be described as

\[
X(T) = (X_s - X_p)\left(\frac{T - T_p}{T_S - T_p}\right)^2 + X_p \quad T_s \leq T \leq T_p
\]

\[
X(T) = (X_f - X_p)\left(\frac{T - T_p}{T_f - T_p}\right)^2 + X_p \quad T_p \leq T \leq T_f
\]

(10)

where the \( X \) can represent material parameters and \( T_s, T_p, T_f \) are the start temperature, glass transition temperature and the finish temperature of glass transition. The material parameters in Equation 10 are shown in table 1.

| Table 1. Material parameters of SMP. |
|--------------------------------------|
| \( E_s \) [Mpa] | \( E_p \) [Mpa] | \( E_f \) [Mpa] | \( \eta_s \) [GPs] | \( \eta_p \) [GPs] | \( \eta_f \) [GPs] | \( \alpha \) [1/°C] |
|-----------------|----------------|----------------|-----------------|----------------|----------------|----------------|
| 1500            | 700            | 300            | 116             | 79             | 2.03           | 11.6×10⁻⁵     |
| \( \lambda_s \) [s] | \( \lambda_p \) [s] | \( \lambda_f \) [s] | \( T_s \) [°C] | \( T_p \) [°C] | \( T_f \) [°C] | \( \nu \) |
| 2840            | 521            | 111            | 27              | 55             | 80             | 0.25          |

We will take the simply supported beam subjected to concentrated force for example to study the thermo-mechanical behaviors of SMP beam. As is shown in figure 2, we assume that concentrated force is \( P \), the length, width and height of the beam are \( L, b \) and \( h \), respectively.

**Figure 2.** The simply supported beam subjected to concentrated force.
According to the theory of mechanics of materials, the displacement curve can be described as follows:

\[ w(x) = \frac{P}{12EI} x^3 - \frac{P}{24LEI} x^4 + \frac{PL^2}{24EI} x. \]  

(11)

where the \( I \) is the moment of inertia.

In this model, we combined the displacement functions of beam, the constitutive equation which are further derived and material parameter equation of SMP. We can use the model to study the thermo-mechanical behaviors of SMP beam, and this method can also be extended to other structures. According to the literature [10], many finite element models have been presented for the analysis of SMP and most models required complicated user-defined material subroutines and a standard finite element software package, while, in this model, we can simulate and analyze the thermo-mechanical behaviors of SMP beam easily.

3. Numerical Simulation and Analysis

In this section, we will take the simply supported beam for example to study on the thermo-mechanical behaviors of SMP beam with using the constitutive model established in this paper. We assume that the \( P \) is 150 N, \( l=17.6 \text{ mm} \), is a constant, and the length \( L=20l \), the width \( b=l \).

In figure 3, the stress-time curves and the strain-time curves with different height are shown. As can be seen, with the loading time increasing, the stress and strain decrease, and in the initial stage, the rate of descent is faster, and then flattens out. It is because that with the loading time increasing, the viscous properties of SMP beam are shown and tend to stabilize. It also can be seen that with the height increasing, the changes in the curves are becoming less and less obvious, and this is because with the height increasing, the stiffness increases with constant viscosity.

![Figure 3](image-url)

Figure 3. The stress-time curves (a) and strain-time curves (b) with different height.

In figure 4, the shape memory effect curves with different height are shown. As can be seen, the process of S1 is the loading process at high temperature, the relationship of stress and strain is non-linear, and this is because with the loading time increasing, the viscous properties of SMP are represented. The process of S2 is the stress freezing process, the stress increase with the temperature decreasing. The reason is that with the temperature decreasing, the strain is a constant, while the material parameters of SMP will increase. The process of S3 is the process of unloading process at low temperature, in this process, because the material parameters is bigger than S1, thus, the most strain will be converted to residual strain. The process of S4 is the process of shape recovery process, with the temperature increasing, the material parameters will be recovered, and the residual strain will gradually disappear. The relationship between strain and temperature is non-linear because of the viscous properties. It also
can be seen that with the height increasing, the shape memory effect will be smaller and smaller, this is also because with the height increasing, the stiffness increases with constant viscosity.

![Figure 4](image.png)

**Figure 4.** The shape memory effect curves with different height. (a) stress-strain curves, (b) stress-temperature and (c) strain-temperature curves.

4. Conclusion

In this paper, the constitutive model which can study the thermo-mechanical behaviors of SMP beam is established. Based on the work of Zhou et al. [9], the SMP’s expressions of strain and stress are derived, and the material parameters equations are introduced. According to the beam theory, the displacement components are introduced, and based on the relationship of displacement components, strain components and stress components, the constitutive equation of SMP beam is established. Take the simply supported beam subjected to concentrated force for example, the thermo-mechanical behaviors of SMP beam are simulated and studied with using the constitutive model established in this paper. The simulation results show that

1. With the load fixed and the loading time increasing, the strain and stress will gradually decrease and flatten out.
2. The higher the height of beam, the more rigid it is, and the less viscous it is.
3. In the shape memory process, the smaller the height of SMP beam and the more obvious the shape memory effect.

The model can provide some theoretical guidance for the application of SMP beams and can be helpful for beam application.

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