Baryon and Lepton Number Violation
with Scalar Bilinears

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Abstract

We consider all possible scalar bilinears, which couple to two fermions of the standard model. The various baryon and lepton number violating couplings allowed by these exotic scalars are studied. We then discuss which ones are constrained by limits on proton decay (to a lepton and a meson as well as to three leptons), neutron-antineutron oscillations, and neutrinoless double beta decay.
Grand unified theories (GUTs) appear to be the most natural extensions of the standard model at the very high scale. In GUTs the gauge group is unified, so there is only one single gauge coupling constant to explain all the forces of the quarks and leptons, which are also treated at the same footing. However the scale of unification is much too high to be directly tested in the laboratory. Thus indirect searches become very important. Many predictions of GUTs were studied, but there is no supporting experimental evidence to date, with the possible exception of neutrinoless double beta decay [1, 2]. These GUT predictions also include new particles such as leptoquarks or diquarks and new interactions which violate baryon and lepton numbers. The latter are also required for an understanding of the present baryon asymmetry of the Universe. An observed Majorana neutrino mass also requires lepton number violation. In the context of these issues we present in this paper a general analysis of baryon and lepton number violation with scalar bilinears.

The scalar bilinears we consider are generalizations of the usual Higgs scalar doublet. They couple to two fermions of the standard model and are present in many of its extensions [3, 4, 5]. The quantum numbers of these particles are thus fixed, but their masses and couplings could be arbitrary. Low-energy particle physics phenomenology (such as the non-observation of baryon number violation) could be used to constrain their masses and couplings [6, 7]. Baryogenesis with scalar bilinears has also been studied [8]. Recently a general analysis of the neutrinoless double beta decay has been performed by constructing effective higher dimensional operators [9]. Most of these operators may be realized in models where scalar bilinears are included, a few examples of which were demonstrated [9].

In Table 1 we list all the scalar bilinears allowed in any extension of the standard model. We then write down the different baryon and lepton number violating couplings of these scalars and what kind of interactions these different couplings may induce, leading to a simple classification of all the different possible baryon and lepton number violating interactions.
We include the usual Higgs scalar doublet of the standard model in these interactions.

| Representation | Notation | $qq$ | $\bar{q}\bar{l}$ | $q\bar{l}$ | $ll$ |
|----------------|----------|-----|-----------------|----------|-----|
| $(1, 1, -1)$   | $\chi^-$ |     | $\times$        | $\times$ |     |
| $(1, 1, -2)$   | $L^{--}$ |     | $\times$        | $\times$ |     |
| $(1, 3, -1)$   | $\xi$    |     |                 | $\times$ |     |
| $(3^*, 1, 1/3)$| $Y_a$   | $\times$ |     | $\times$ |     |
| $(3^*, 3, 1/3)$| $Y_b$   | $\times$ |     | $\times$ |     |
| $(3^*, 1, 4/3)$| $Y_c$   | $\times$ |     | $\times$ |     |
| $(3^*, 1, -2/3)$| $Y_d$ |     | $\times$ |     |     |
| $(3, 2, 1/6)$ | $X_a$    |     | $\times$        |         |     |
| $(3, 2, 7/6)$ | $X_b$    |     | $\times$        |         |     |
| $(6, 1, -2/3)$ | $\Delta_a$ | $\times$ |     |         |     |
| $(6, 1, 1/3)$ | $\Delta_b$ | $\times$ |     |         |     |
| $(6, 1, 4/3)$ | $\Delta_c$ | $\times$ |     |         |     |
| $(6, 3, 1/3)$ | $\Delta_L$ | $\times$ |     |         |     |
| $(8, 2, 1/2)$ | $\Sigma$ |     |                 |         |     |

Table 1: Exotic scalar particles beyond the standard model and their couplings to combinations of two fermions.

For the scalar bilinears, it is not possible to define definite baryon or lepton number in some cases. However, depending on their couplings to the combinations of two fermions, it is possible to assign baryon and lepton numbers separately. The different assignments are given in Table 2. For any particular interaction it will be possible to specify what processes it would mediate and what is the change in baryon and lepton numbers. This will allow us to classify them.

The quadratic terms with the scalars are trivial. For any quadratic term $H_1H_2$ to be invariant under the gauge interactions, either both of them are singlets (i.e. neutral under all gauge factors) or else $H_1^\dagger = H_2$. Since all the scalars we are discussing are non-singlets,
none of the quadratic terms can have any $B - L$ quantum number, although some of them may contribute to baryon or lepton number violation. Consider for example, $Y_a^+Y_a$. Since $Y_a$ can couple to $\bar{q}\bar{l}$ as well as to $qq$, this term may induce a $\Delta B = 1, \Delta L = 1$ process, but $B - L$ remains conserved.

The trilinear couplings are the most interesting ones. It was shown [8] that there are 38 possible trilinear couplings with the scalar bilinears. Although baryon and lepton numbers are not always uniquely specified, all of them carry a $B - L$ quantum number $\pm 2$. As we shall see next, these trilinear operators can mediate four possible processes and hence can be classified in four categories

(i) $B = 1, L = -1$: In this case both baryon and lepton numbers are changed by 1, so all these couplings should give only proton decay, which violates $(B - L)$ [10].

(ii) $B = 0, L = 2$: This gives lepton number violating processes, without affecting the baryon number. So this contributes to the Majorana masses of the neutrinos. Some of them will contribute to the neutrinoless double beta decay [11], but others would

| Two-fermion combination | $qq$ | $\bar{q}\bar{l}$ | $ql$ | $ll$ |
|--------------------------|-----|-----------------|-----|-----|
| Baryon number            | $2/3$ | $-1/3$ | $1/3$ | $0$ |
| Lepton number            | $0$ | $-1$ | $-1$ | $2$ |
| $B - L$                  | $2/3$ | $2/3$ | $4/3$ | $-2$ |

Table 2: Baryon and lepton numbers of the the scalars coupling to different combinations of two fermions.
involve neutrinos of different generations.

(iii) $B = 1, L = 3$: These operators give a three-lepton decay mode of the proton [10]. In some models these processes could dominate over the usual proton decay modes.

(iv) $B = 2, L = 0$: In this case baryon number is changed by two unit with no change in lepton number. This corresponds to neutron-antineutron oscillations.

Some of the operators may contribute to two process types. There are no trilinear couplings, which conserve $B - L$ (which includes of course the case $B = 0$ and $L = 0$).

There is only one trilinear coupling which can give proton decay into a lepton and a meson (or mesons) and is not involved in any other process. It is thus possible to assign $B = 1$ and $L = -1$ to this operator:

\[
O_1 = \mu_1 X_a \phi Y_d
\]

(1)

In the following $B - L$ quantum numbers for the trilinear couplings are not mentioned explicitly since all of them have $B - L = \pm 2$. All other couplings which contribute to the proton decay, can also mediate $n - \bar{n}$ oscillations or the neutrinoless double beta decay.

There are only two operators which can give rise to the three-lepton decay mode of the proton:

\[
\begin{align*}
(B = 0, \ L = -2) & \quad (B = 1, \ L = -1) \quad \text{and} \quad (B = -1, \ L = -3) \\
O_2 = \mu_2 Y_a Y_c^f \chi^+ & \quad O_3 = \mu_3 Y_b Y_c^f \xi^+/5
\end{align*}
\]

(2)

These operators will simultaneously allow the single-lepton decay mode of the proton ($B = 1, L = -1$) and neutrinoless double beta decay ($B = 0, L = 2$). There is no trilinear coupling which contributes to the three-lepton decay mode of the proton and no other
process.

There are four trilinear couplings which carry $B = 2, L = 0$ and contribute only to $n - \bar{n}$ oscillations and not to any other process:

\[
( B = 2, \quad L = 0 ) \\
O_4 = \mu_4 \Delta_a \Delta_b \Delta_b \\
O_5 = \mu_5 \Delta_c \Delta_a \Delta_a \\
O_6 = \mu_6 \Delta_c Y_d Y_d \\
O_7 = \mu_7 \Delta_L \Delta_L \Delta_a
\]  

(3)

Another four trilinear interactions contribute to both $n - \bar{n}$ oscillations and proton decay:

\[
( B = 2, \quad L = 0 ) \quad \text{and} \quad ( B = 1, \quad L = -1 ) \\
O_8 = \mu_8 Y_c Y_d \\
O_9 = \mu_9 \Delta_L Y_b Y_d \\
O_{10} = \mu_{10} \Delta_b Y_a Y_d \\
O_{11} = \mu_{11} \Delta_a Y_d Y_c
\]  

(4)

There are several operators which contribute only to neutrinoless double beta decay:

\[
( B = 0, \quad L = 2 ) \\
O_{12} = \mu_{12} \phi \phi \chi^- \\
O_{13} = \mu_{13} \phi \phi \xi \\
O_{14} = \mu_{14} \chi^- \chi^- L^{++} \\
O_{15} = \mu_{15} \xi \xi L^{++} \\
O_{16} = \mu_{16} X_b X_a \chi^- \\
O_{17} = \mu_{17} X_b X_a \xi \\
O_{18} = \mu_{18} \Delta_b X_a X_a \chi^- \\
O_{19} = \mu_{19} \Delta_c X_a \chi^+ \\
O_{20} = \mu_{20} \Delta_L \chi^+ \chi^+ \\
O_{21} = \mu_{21} \Delta_b \chi^+ \\
O_{22} = \mu_{22} \Delta_a \Delta_b \Delta_c \\
O_{23} = \mu_{23} \Delta_b \Delta_c \chi^- \\
O_{24} = \mu_{24} \Delta_b \chi^+ \\
O_{25} = \mu_{25} \Delta_a \Delta_c \chi^-
\]  

(5)

There are few more operators contributing to neutrinoless double beta decay, which also allow proton decay:

\[
( B = 0, \quad L = 2 ) \quad \text{and} \quad ( B = -1, \quad L = 1 ) \\
O_{26} = \mu_{26} Y_a Y_a \chi^- \\
O_{27} = \mu_{27} Y_b Y_b \xi \\
O_{28} = \mu_{28} Y_c Y_c \chi^- \\
O_{29} = \mu_{29} X_a X_b Y_a \chi^- \\
O_{30} = \mu_{30} X_a \phi \phi Y_a \\
O_{31} = \mu_{31} X_a \phi \phi Y_b \\
O_{32} = \mu_{32} X_a X_a Y_a \chi^- \\
O_{33} = \mu_{33} X_a X_b Y_b \phi \phi \\
O_{34} = \mu_{34} X_b Y_d \phi \phi
\]  

(6)

The proton decay constraints would then restrict the couplings of these operators making
them very much suppressed. However, if some of the couplings could be avoided with discrete symmetries, then these operators could also contribute to the neutrinoless double beta decay significantly. Finally there are four more operators which contribute to neutrinoless double beta decay, proton decay, and $n - \bar{n}$ oscillations simultaneously:

\[
(B = 0, \quad L = -2), \quad (B = 1, \quad L = -1) \quad \text{and} \quad (B = 2, \quad L = 0)
\]

\[
\mathcal{O}_{35} = \mu_{35} Y_d Y_a Y_a \quad \mathcal{O}_{36} = \mu_{36} Y_d Y_b Y_b \quad \mathcal{O}_{37} = \mu_{37} \Delta_a Y_a Y_a \quad \mathcal{O}_{38} = \mu_{38} \Delta_a Y_b Y_b
\]

(7)

The quartic terms belong to two broad classes, one with $B - L = 0$ and the other with $B - L = 4$. Again they can be categorized under certain subclasses. Those which are products of the usual quadratic couplings of the form $\Phi^\dagger \Phi$ belong to the trivial category with $B = 0, L = 0$ (and thus $B - L = 0$), which we shall not list here. All the quartic couplings involving only the dileptons ($\chi^-, L^{--}$, and $\xi$), the usual Higgs doublet $\phi$, and the octet $\Sigma$ also fall into this trivial category with $B = 0, L = 0$. There are also some other quartic couplings involving the diquarks and leptoquarks, which can be classified under this trivial category. These are

\[
B - L = 0 \quad (B = 0, \quad L = 0)
\]

\[
\tilde{O}_1 = \chi^+ \chi^+ \xi \xi \quad \tilde{O}_2 = \phi \phi \chi^+ L^{--} \quad \tilde{O}_3 = \Sigma \Sigma \chi^+ L^{--} \\
\tilde{O}_4 = \phi ^\dagger \phi ^\dagger \xi L^{++} \quad \tilde{O}_5 = \Sigma \Sigma \xi L^{--} \quad \tilde{O}_6 = \phi ^\dagger \phi ^\dagger \Sigma \Sigma \\
\tilde{O}_7 = \Delta^+_a X_a X_a \chi^- \quad \tilde{O}_8 = \Delta^+_b X_b X_b \xi \quad \tilde{O}_9 = \Delta^+_b X_b L^{--} \\
\tilde{O}_{10} = \Delta^+_a X_b X_b \chi^- \quad \tilde{O}_{11} = \Delta^+_a X_b X_b \xi \quad \tilde{O}_{12} = \Delta^+_L X_b X_b L^{--} \\
\tilde{O}_{13} = \Delta^+_a X_a X_b L^{--}
\]

(8)

There are several $B - L = 0$ quartic couplings which mediate baryon number violating processes. The simplest of them have $B = 2$ and $L = 2$. The hydrogen-antihydrogen ($p + e^- \rightarrow \bar{p} + e^+$) oscillations and double proton decay into two positrons ($p + p \rightarrow e^+ + e^+$) are typical examples of such processes. There are 7 such interactions

\[
B - L = 0 \quad (B = 2, \quad L = 2)
\]
\( \bar{O}_{14} = \Delta_a \Delta_c \Delta_c L^{-} \quad \bar{O}_{15} = \Delta_b \Delta_b \Delta_b \chi^{-} \quad \bar{O}_{16} = \Delta_b \Delta_b \Delta_L \xi \)
\( \bar{O}_{17} = \Delta_b \Delta_L \Delta_L \chi^{-} \quad \bar{O}_{18} = \Delta_b \Delta_L \Delta_L \xi \quad \bar{O}_{19} = \Delta_L \Delta_L \Delta_L \chi^{-} \quad \bar{O}_{20} = \Delta_L \Delta_L \Delta_L \xi \)

(9)

There are other quartic couplings also, which allow these processes with \( B = 2 \) and \( L = 2 \). They involve \( Y_a, Y_b, \) and \( Y_c \), which couple to two quarks as well as to an antiquark and an antilepton. Hence some of these interactions can simultaneously allow \( B = 1 \) and \( L = 1 \) proton decay processes such as \( p \rightarrow e^+ \) or \( n \rightarrow \bar{\nu} \) associated with a neutral meson and a lepton-antilepton pair. In all these interactions, the constraints from proton decay will not allow processes with \( B = 2 \). These operators are

\[
B - L = 0 \quad (B = 2, \quad L = 2) \quad \text{and} \quad (B = 1, \quad L = 1)
\]

\[
\bar{O}_{21} = \Delta_c Y_a Y_d \chi^{-} \quad \bar{O}_{22} = \Delta_c Y_b Y_d \xi
\]

(10)

\[
B - L = 0 \quad \left( \begin{array}{c} B = 2 \\ L = 2 \end{array} \right), \quad \left( \begin{array}{c} B = 1 \\ L = 1 \end{array} \right) \quad \text{and} \quad \left( \begin{array}{c} B = 0 \\ L = 0 \end{array} \right)
\]

\[
\bar{O}_{23} = \Delta_a Y_a Y_c \chi^{-} \quad \bar{O}_{24} = \Delta_a Y_b Y_c \xi \quad \bar{O}_{25} = \Delta_b Y_a Y_c \xi^{-} \quad \bar{O}_{26} = \Delta_b Y_b Y_c \xi
\]

\[
\bar{O}_{27} = \Delta_b Y_a Y_b \chi^{-} \quad \bar{O}_{28} = \Delta_b Y_b Y_b \xi \quad \bar{O}_{29} = \Delta_b Y_b Y_b \xi
\]

\[
\bar{O}_{30} = \Delta_c Y_a Y_a L^{-} \quad \bar{O}_{31} = \Delta_c Y_a Y_a L^{-} \quad \bar{O}_{32} = \Delta_c Y_b Y_b \chi^{-} \quad \bar{O}_{33} = \Delta_L Y_a Y_b \chi^{-} \quad \bar{O}_{34} = \Delta_L Y_b Y_b \chi^{-}
\]

(11)

\[
\bar{O}_{35} = \Delta_L Y_a Y_b \chi^{-} \quad \bar{O}_{36} = \Delta_L Y_b Y_b \xi \quad \bar{O}_{37} = \Delta_L Y_b Y_b \xi
\]

\[
\bar{O}_{38} = \Delta_L Y_b Y_b \chi^{-} \quad \bar{O}_{39} = Y_b Y_c Y_d \xi \quad \bar{O}_{40} = Y_b Y_d Y_d L^{-}
\]

\[
B - L = 0 \quad \left( \begin{array}{c} B = 2 \\ L = 2 \end{array} \right), \quad \left( \begin{array}{c} B = 1 \\ L = 1 \end{array} \right), \quad \left( \begin{array}{c} B = 0 \\ L = 0 \end{array} \right) \quad \text{and} \quad \left( \begin{array}{c} B = -1 \\ L = -1 \end{array} \right)
\]

\[
\bar{O}_{41} = Y_a Y_a Y_c \chi^{-} \quad \bar{O}_{42} = Y_a Y_b Y_c \xi \quad \bar{O}_{43} = Y_a Y_a Y_c L^{-}
\]

\[
\bar{O}_{44} = Y_a Y_b Y_b \chi^{-} \quad \bar{O}_{45} = Y_a Y_b Y_b \xi \quad \bar{O}_{46} = Y_a Y_b Y_b L^{-}
\]

\[
\bar{O}_{47} = Y_b Y_b Y_b \xi \quad \bar{O}_{48} = Y_b Y_b Y_c L^{-}
\]

(12)

Another class of quartic couplings can give rise to \( B - L = 4 \) processes. Some of them can produce only \( B = 2 \) and \( L = -2 \) processes like \( n + n \rightarrow e^- e^- \pi^+ \pi^+ \) or \( n - \bar{n} \) oscillation
associated with charged leptons and charged pions. The couplings which can give rise to these processes are

\[
B - L = 4 \quad (B = 2, \quad L = -2) \\
\hat{O}_1 = Y_d Y_d L^{++} \quad \hat{O}_2 = \Delta_a Y_d L^{++} \\
\hat{O}_3 = \Delta_a \Delta_a \Delta L^{++} \quad \hat{O}_4 = \Delta_a \Delta_a \Delta b \chi^+
\]

(13)

The other subclass of \( B - L = 4 \) processes have \( B = 1 \) and \( L = -3 \), which give rise to \( p \rightarrow \nu \nu \nu \pi^+ \) for example. There are two such operators,

\[
B - L = 4 \quad (B = 1, \quad L = -3) \\
\hat{O}_5 = X_a X_a X_a \phi^\dagger \quad \hat{O}_6 = X_a X_a X_a \Sigma^\dagger
\]

(14)

There are also some quartic couplings with \( B - L = 4 \), which allow both of the above two interactions. There are three such terms

\[
B - L = 4 \quad (B = 1, \quad L = -3) \quad \text{and} \quad (B = 2, \quad L = -2) \\
\hat{O}_7 = Y_a Y_d Y_d \chi^+ \quad \hat{O}_8 = \Delta_a Y_d \chi^+ \quad \hat{O}_9 = \Delta_a Y_b \xi^\dagger
\]

(15)

In summary, we have made a general model-independent study of the possible baryon and lepton number violations beyond the standard model with scalar bilinears. Depending on the processes under consideration, they can be classified into only a few categories. We list all the trilinear and quartic couplings and discuss the processes to which these operators contribute.

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References

[1] H. V. Klapdor-Kleingrothaus et al., Mod. Phys. Lett. A16, 2409 (2001).

[2] H. V. Klapdor-Kleingrothaus, A. Dietz and I.V. Krivosheina, Particle and nuclei, Letters 110, 57 (2002); Foundations of Physics 32, 1181 (2002).

[3] G. B. Gelmini and M. Roncadelli, Phys. Lett. 99B (1981) 411; C. Wetterich, Nucl. Phys. B187 (1981) 343; G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B181 (1981) 287; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D23 (1981) 165; E. Ma and U. Sarkar, Phys. Rev. Lett. 80 (1998) 5716; G. Lazarides and Q. Shafi Phys. Rev. D 58 (1998) 071702; W. Grimus, R. Pfeiffer and T. Schwetz, Eur. Phys. J. C 13 (2000) 125; E. Ma, T. Hambye and U. Sarkar, Nucl. Phys. B 602 (2001) 23; E. Ma, M. Raidal and U. Sarkar, Eur. Phys. J. C 8 (1999) 301; Phys. Rev. D 60 (1999) 076005.

[4] A. Zee, Phys. Lett. B 93 (1980) 389; L. Wolfenstein, Nucl. Phys. B 175 (1980) 93.

[5] R.N. Mohapatra and J.D. Vergados, Phys. Rev. Lett. 47 (1981) 1713; M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko Phys. Rev. D 54 (1996) R4207; Phys. Lett. B 378 (1996) 17.

[6] M. Leurer, Phys. Rev. D 49 (1994) 333; S. Davidson, D. Bailey and B.A. Campbell, Zeit. Phys. C 61 (1994) 613; F. Cuypers and S. Davidson, Eur. Phys. J. C2 (1998) 503; J.P. Bowes, R. Foot and R.R. Volkas, Phys. Rev. D54 (1996) 6936.

[7] E. Ma, M. Raidal, and U. Sarkar, Eur. Phys. J. C8, 301 (1999).

[8] E. Ma, M. Raidal, and U. Sarkar, Phys. Rev. D60, 076005 (1999).

[9] K. Choi, K.S. Jeong and W.Y. Song, hep-ph/0207180.
[10] J.C. Pati, A. Salam and U. Sarkar, Phys. Lett. B 133 (1993) 330; J.C. Pati, Phys. Rev. D 29 (1984) 1549; P.J. O’Donnell and U. Sarkar, Phys. Lett. B 316 (1993) 121.

[11] One of them was considered in, E. Ma and B. Brahmachari, Phys. Lett. B 536 (2002) 259.