Damage evaluation in metal structures subjected to high energy deposition due to particle beams

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Abstract. The unprecedented energy intensities of modern hadron accelerators yield special problems with the materials that are placed close to or into the high intensity beams. The energy stored in a single beam of LHC particle accelerator is equivalent to about 80 kg of TNT explosive, stored in a transverse beam area with a typical value of 0.2 mm×0.2 mm. The materials placed close to the beam are used at, or even beyond, their damage limits. However, it is very difficult to predict structural efficiency and robustness accurately: beam-induced damage for high energy and high intensity occurs in a regime where practical experience does not exist. The interaction between high energy particle beams and metals induces a sudden non uniform temperature increase. This provokes a dynamic response of the structure entailing thermal stress waves and thermally induced vibrations or even the failure of the component. This study is performed in order to estimate the damage on a copper component due to the impact with a 7 TeV proton beam generated by LHC. The case study represents an accidental case consequent to an abnormal release of the beam: the energy delivered on the component is calculated using the FLUKA code and then used as input in the numerical simulations, that are carried out via the FEM code LS-DYNA.

1. Introduction

Particle accelerators [1] act as microscopes for such a complex research; these large machines accelerate charged elementary particles (electrons, protons or ionized atoms) to high kinetic energies. A high energy particle beam can be brought into collision against a fixed target or against another beam and from this encounter a multitude of short life sub-atomic particles is originated. The investigation of matter in these extreme conditions can be compared with the status of the universe in the first moments after the so-called ‘Big Bang’. The higher the energy of the colliding beams and the event rate, the wider the spectrum of the generable sub-atomic particles. It is in this perspective that the project of building the Large Hadron Collider (LHC) [2] at the European Organization for Nuclear Research (CERN, Geneva) was approved.

The LHC [3] is a circular accelerator with a 26.659 km circumference situated at the border between Switzerland and France at an average depth of 100 m underground. This machine mainly provides the collision between two counter-circulating proton beams. Each proton beam consists of $3\times10^{14}$ protons at 7 TeV, so when the protons collide the collision energy is 14 TeV. Under nominal operating conditions, the beam has 2808 bunches each having $1.11\times10^{11}$ protons. The bunch length is 0.5 ns and the time between two successive bunches is 25 ns, so the duration of the entire beam is...
about 72 µs.

The total energy stored in each beam at maximum energy is about 350 MJ, two orders of magnitude beyond the achievements in the Tevatron or HERA. This is enough energy to melt 500 kg of copper. This large amount of energy is potentially destructive for accelerator equipments having direct interaction with particles in case of uncontrolled beam loss, so everything is done to ensure that this never happens. Besides, it is important to know what will be the damage in case of the LHC malfunction. It is in this perspective that a thermo-mechanical analysis becomes relevant. However, it is very difficult to predict structural efficiency and robustness accurately: beam-induced damage for high energy and high intensity occurs in a regime where practical experience does not exist.

The interaction between high energy particle beams and solids can be considered from a structural point of view as an energy deposition inducing a sudden non uniform temperature increase. In function of which part of material is investigated the behaviour is different (figure 1).

![Figure 1. Pressure distribution in case of impact between the high energy beam and a solid target (a); damage on a metal target due to the impact (b).](image)

In the material part closest to the beam, the pressure and temperature increase and the material could arrive at its melting temperature or vaporize. The material response in this condition is correctly described only using an equation of state that is able to describe the hydrodynamic behaviour, while in this portion of material the deviatoric stress is totally negligible. On the other hand, the remaining part of the material is characterized by high values of plastic strain, strain-rate and temperature, so the response is related with the strength material model used.

From these considerations it is clear what is the complexity of the problem: in order to correctly simulate the thermo-mechanical response of the hit material it is needed to take into account both the hydrodynamic behaviour using a dedicated equation of state (EOS) and the deviatoric behaviour using a dedicated material model.

The numerical simulations are performed using the commercial FEM code LS-DYNA [4]. For the simulations the chosen equation of state is a polynomial EOS, in which the coefficients are obtained fitting a three-phase tabular equation of state, and the material model is the Johnson–Cook model.

The evaluation of thermal loads on the hit material is performed using a statistical code, called FLUKA [5], based on the Monte-Carlo method. FLUKA returns an energy map on a particular geometry taking into account all the particles in the cascade generated by the interaction between the proton beam and the target. Finally the FLUKA results are used as input for thermo-structural studies.

As mentioned before, the material involves in such high energy and high intensity impacts operates under extreme conditions, in which the possibility to perform experimental tests is limited. For this reason the importance of developing a reliable methods and accurate models that could be efficiently applied to estimate the damage occurring during an impact is therefore evident.
2. Equation of state and strength material model

In FEM codes there are a lot of material models implemented for the description of the elasto-plastic behaviour. The classification model makes a distinction between empirical, semi-empirical and phenomenological models. The empirical models have not physical basis, but are obtained by interpolation of experimental data. On the other hand the phenomenological models are obtained starting from the transformation in the material occurring during a deformation process.

Models such those proposed by Johnson-Cook (J-C) [6] and Cowper-Symonds (C-S) [7] are purely empirical models and they are the most widely used. An example of semi-empirical model is the Steinberg-Cochran-Guinan-Lund (S-C-G-L) model [8], which was first developed for the description of high strain-rate behaviour, and after extended to low strain-rate. A completely phenomenological model is the Zerilli-Armstrong (Z-A) model [9], that is obtained on the basis of the dislocation mechanics theory and presents a different formulation for BCC and FCC materials. A more complex dislocation based model is the Mechanical Threshold Stress (MTS) model [10]. The material model chosen in these analyses is Johnson-Cook material model implemented in LS-DYNA code [4]. It is a purely empirical model that allows to take into account the effects of plastic strain, strain-rate and temperature. This model expresses the flow stress as

\[
\sigma_y = \left( A + B\varepsilon_p^n \right) \left( 1 + C \ln \dot{\varepsilon}^* \right) \left( 1 - T^* \right)
\]  

(1)

where \( A, B \) and \( n \) are the material strain hardening constants, \( C \) is the strain-rate parameter, \( m \) is the temperature sensitivity, \( \varepsilon_p \) is the effective plastic strain, \( \dot{\varepsilon}^* \) is the effective total strain-rate normalized by quasi-static threshold rate (\( \dot{\varepsilon}_0 \)) and defined as

\[
\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}
\]  

(2)

and finally \( T^* \) is the homologous temperature defined by the relation

\[
T^* = \frac{T - T_r}{T_m - T_r}
\]  

(3)

in which \( T_r \) and \( T_m \) are the reference temperature and the melting temperature, respectively. It can be notice that the Johnson-Cook model, in contrast with other model like the Steinberg-Lund one, does not include in its formulation the influence of the pressure on the flow stress.

It is important to note that when the temperature reaches the value of the melting temperature the mechanical strength of the material becomes zero. This means that the material loses its shear resistance and starts to be considered like a fluid.

In LS-DYNA the Johnson-Cook material model includes also a fracture model in which the strain at fracture is given by

\[
\varepsilon_f = \left( D_1 + D_2 \varepsilon_p^{b,\sigma^*} \right) \left( 1 + D_4 \ln \dot{\varepsilon}^* \right) \left( 1 + D_5 T^* \right)
\]  

(4)

where \( D_1, D_2, D_3, D_4 \) and \( D_5 \) are the failure parameters and \( \sigma^* \) is the ratio between the pressure and the effective stress. The material fracture occurs when the damage parameter

\[
D = \sum \frac{\Delta \varepsilon_p}{\varepsilon_f}
\]  

(5)

reaches the value of 1. Besides the J-C material implemented in LS-DYNA offers also a spall model to represent the failure under hydrostatic tensile loads. In LS-DYNA there are three type of spallation criteria. The first one (pressure limit model) limits the minimum pressure and if a more tensile pressure is calculated the pressure is reset to the minimum value. The second spall model (maximum
principal stress model) detects spall if the maximum principal stress exceeds a limiting value. The last spall model (hydrostatic tension spall model), as the first one, limits the minimum pressure value, but if spall is detected the deviatoric stresses and the pressure are set to zero.

The other fundamental aspect to consider is the implementation of an equation of state to simulate the hydrodynamic material behaviour. An EOS is a constitutive relation between state variables and describes the state of the matter. Usually it expresses a thermodynamic variable (such as pressure) in function of two other independent state variables (such as density and internal energy). In general the equations of state can be divided into two categories: analytical EOS and tabular EOS.

In LS-DYNA [4] the equations of state for metals are in general partitioned into two terms as in equation (6): a cold pressure and a thermal pressure. The first terms $P_C(\mu)$ is a function of the density and is hypothetically evaluated along a 0 K isotherm, while the second one $P_T(\mu,E)$ depends on both the density and the internal energy per unit volume of the system

$$P(\mu, E) = P_C(\mu) + P_T(\mu, E)$$

(6)

where $\mu = \rho / \rho_0 - 1$. It is clear that for a compressed material $\mu > 0$, for an expanded material $\mu < 0$ and if no load is applied to the material $\mu = 0$. More precisely the EOS implemented have the general form

$$P(\mu, E) = A(\mu) + B(\mu)E$$

(7)

so the pressure is a general function of the density but linearly depending by the energy per unit volume.

Figure 2. Tabular EOS of copper: pressure in function of energy varying the density (a); pressure in function density varying the temperature (b) [11].

Another possible form of an equation of state is the tabular form [11][12]. For the construction of the tabular EOS several codes are developed: the EOS includes solid and fluid phases (liquid, vapour, gas and plasma) and the dependent variable (i.e. pressure or internal energy) is defined as function of independent variables (temperature and density), as shown in figure 2.

3. Case study

In the present work the case study is the impact of 8 bunches at 7 TeV on a cylindrical copper bar that is 1 m long and 5 mm radius. Since this model represents the irradiated part of a bigger component, the external surface is modelled with a no reflection boundary in order to simulate the presence of other material.

The case study is an abnormal situation in which the beam impacts directly against the solid target perpendicular to the base of the cylinder. The energy deposition on the hit material is calculated by FLUKA code [5] on the same geometry for a solid copper target. FLUKA provides the distribution on
a 2D axisymmetric model counting 50 elements both in radial and longitudinal directions for the interaction with a single proton. In figure 3 the specific energy deposition due to the impact with one bunch is reported. As mentioned before, in each bunch there are $1.11 \times 10^{11}$ protons and each bunch is 0.5 ns long and the time between two consecutive bunches is 25 ns so the total duration of the impact (8 bunches) is about 204 ns.

3.1. FEM model
The numerical model used in this simulation is a 2D axisymmetric model with the same geometry and mesh on which the FLUKA calculation is performed: a cylinder of radius 5 mm and length 1 m with 50 elements both in radial and longitudinal directions (element dimension is 0.1 mm $\times$ 20 mm). One point integration elements are used. The influence of the mesh density was analyzed in [13].

In LS-DYNA the only way to deposit external energy on an element is passing it through the equation of state (defining an initial energy value or a power vs time history) and this implies creating a part for each energy value to be deposited. With this aim, the energy distribution per unit bunch calculated by FLUKA code and shown in figure 3.a is split in 5000 levels (linearly spaced between the maximum and the minimum values) and deposited on the component according with the figure 3.b, in which the different model parts are highlighted by different colours. In more details, the adopted energy levels division method entails the creation of wider energy bands with the radius increase. On the other hand, in the central zone (impacted zone) a different model EOS is assigned to each numerical model element.

![Figure 3. FLUKA energy distribution (GJ/m$^3$) for a single bunch (a); numerical model (b): different colours correspond to different energy levels (parts).](image)

3.1.1. Equation Of State. In LS-DYNA code there is not the possibility to use a tabular equation of state in the same form of that is calculated in [11]. To solve this problem, the code offers the possibility to implement an user-defined EOS, and this solution was adopted, for example in [14]. In this work a standard polynomial EOS is used to fit the tabular data with the relation

$$P = C_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + \left( C_4 + C_5 \mu + C_6 \mu^2 \right) E$$  \hspace{1cm} (8)$$

Different solutions for the interpolation were analyzed in order to obtain the set of parameters correspondent to the best fit. In this work a dedicated interpolation is done for each model part (or energy level). Besides, each interpolation is performed limiting the range of interest of the entire tabular data in pressure, density and energy. The purpose is to neglect in the interpolation the areas of the tabular equation of state that are physically unreachable for each element mesh according to the energy delivered by the impact with the particle proton beam.

Figure 2.a shows the tabular equation of state in the $P$-$E$ plane for different density values. It is clear that the hypothesis of linearity in $E$ is not so true over all the range of interest identified by the minimum and maximum value of energy deposited on the component after the impact of 8 bunches.

The figure 4 explains the idea of the interpolation focusing the attention on the constraints imposed in density, pressure and energy. The variables limits are set according to both the results presented in
and preliminary numerical simulations performed with LS-DYNA [13]. The minimum density considered in the interpolation is 4000 kg/m$^3$ and the maximum value is 10000 kg/m$^3$, that are suitable values reached by the component after 1 µs from the impact. The maximum pressure is limited to 150% of the maximum value reached after the arrive of the first bunch in the element in which there is the maximum energy deposited. For what concerns the minimum value of pressure it is necessary distinguished in two regions based on the density. For density greater than 8000 kg/m$^3$ the material is still able to support tensile load (like a solid). So the limitation is the maximum value of hydrostatic tensile load (-1.2 GPa [15]) applicable to the material without failure. In the numerical model, to avoid that the material overcomes this limit a spall model is introduced. Otherwise, under the density of 8000 kg/m$^3$ the material is not still solid, so it is not able to support this kind of load. For this reason the limit has to be of 0 Pa. However, in the numerical model the pressure limit is set equal to 0.001 GPa, that in any case is negligible in comparison with the pressure values reached in the simulation. This limitation coupled in pressure and density is taken into account in the simulations with a condition of erosion (deletion) of the element if this limits are violated. Finally, the last limitation is about the energy delivered on each element. In particular, the fit is performed in the range between the energy deposited after the arrive of 1 bunch and the value correspondent to the arrive of 8 bunches. In figure 4 there are shown two possible situations: $E_{I}$ and $E_{VIII}$ are the limits in energy for an element quite far from the beam impact zone, while $E_{II}$ and $E_{VIII}$ are the limits calculated for an element situated near the impact zone.

![Figure 4. Tabular equation of state: definition of the limits set for the interpolation with the polynomial EOS.](image)

From these considerations it is clear that for each element, or more precisely, for each part, the interpolation of the tabular data with the polynomial EOS is calculated in a smaller areas in which the hypothesis of linearity of the pressure with the energy is more reasonable.

Starting from these considerations, first of all a linear interpolation of the $P-E$ curve is done for each density, calculating a value of slope and intercept. Then, the slopes are fitted by a polynomial of II order obtaining the coefficients of the thermal part $P_T(\mu,E)$ and the intercepts by a III order polynomial to extract the coefficients of the cold curve $P_C(\mu,E)$ according with the equation (8).

3.1.2. Strength material model. The material model used in the simulations is the Johnson-Cook material model [6], and the input parameters are summarized in the table 1.

As mentioned before, the introduction of the damage parameters allows to simulate the cumulative
damage in the component but only where it is still solid. Besides, it is necessary to introduce a spall model in order to correctly simulate the failure under hydrostatic tensile stresses. In this work the pressure limit \((P_{\text{cut-off}})\) for the spallation is set to -1.2 GPa and once spall is detected, the deviatoric stresses are set to 0 and if hydrostatic tension is calculated, then the pressure is reset to 0.

Finally, as discussed before, it is necessary to avoid that the elements that reach the density value of 8000 kg/m\(^3\) could be subjected to pressure lower than 0.001 GPa. In this sense, an element erosion criterion is implemented with the condition that if at the same time both the pressure and the volume strain (that is correlated to the density) become lower than the corresponding imposed limits then the element is deleted.

### Table 1. Johnson-Cook material model parameters

| Parameter | Value | Unit | Parameter | Value | Unit |
|-----------|-------|------|-----------|-------|------|
| \(\rho_0\) | 8937.5 | kg/m\(^3\) | \(T_{\text{melt}}\) | 1356 | K |
| \(G\) | 47 | GPa | \(T_{\text{room}}\) | 300 | K |
| \(E\) | 111 | GPa | \(C_p\) | 383 | J/(kg K) |
| \(\nu\) | 0.3406 | - | \(P_{\text{cut-off}}\) | 1.2 | GPa |
| \(A\) | 90 | MPa | \(D_1\) | 0.54 | - |
| \(B\) | 292 | MPa | \(D_2\) | 4.89 | - |
| \(n\) | 0.31 | - | \(D_3\) | -3.03 | - |
| \(C\) | 0.025 | - | \(D_t\) | 0.014 | - |
| \(\tilde{\varepsilon}_0\) | 1.00 | s\(^{-1}\) | \(D_{t}\) | 1.15 | - |
| \(m\) | 1.09 | - |

3.1.3. Energy deposition. The proton particle beam circulating in LHC is divided into bunches and each of them counts about \(1.11 \times 10^{11}\) protons at 7 TeV. Each bunch lasts 0.5 ns and the time interval between two consecutive bunches is 25 ns. Since in this work the study is performed on the accidental case in which 8 bunches could impact against the component, the total duration of the impact is about 204 ns.

As mentioned before in LS-DYNA the only way to deposit external energy is through the definition of the equation of state. The EOS used in this work is the linear polynomial with energy leak in which the time evolution energy deposition has to be defined as deposition rate (power). So, the bunch profile are represented by the sequence of constant power step of 0.5 ns followed by 25 ns of null power. In particular, since the deposition phase is very short (0.5 ns) it can be considered as an isochoric transformation and this allows to consider the energy of the first bunch as an initial condition.

### 4. Results

Figure 5 shows the time evolution of the thermodynamic (pressure, density and temperature) and structural (Von Mises stress) properties obtained from the numerical simulation. It is clear that the proton beam impact produces a pressure shock waves generated in the centre of the component and travelling in the radial direction. During the deposition phase there is the increase of the pressure due to the external energy contribution. Then the expansion of the impacted zone becomes more relevant and in this part of the component the pressure and the density drop and the density values are proper of a fluid state (liquid, gas or plasma). On the other hand, the density grows in the material behind the shock pressure front. From this considerations it is clear that the proton beam impact against a solid structure, in terms of the evolution of thermodynamic and structural quantities, is comparable with the results obtained in case of a confined explosion. Both cases are complex and multidisciplinary subjects that involve a large number of physical parameters.
Figure 5. Results of the numerical simulation in terms of pressure, density, temperature (limited at 3000 K) and Von Mises stress for four time (after the first bunch deposition, at the end of the deposition and during the expansion).
In figure 6 the temperature is limited to 3000 K. This is because the temperature computation is done by the J-C material routine in order to compute the thermal softening. The calculation is done starting from the internal energy value with the heat capacity and density of the solid material. For this reason the temperature values are realistic only in the solid material part of the component. In the melted part of the component the Von Mises stress is zero and the behaviour is purely hydrostatic. On the other hand it reaches the maximum value behind the shock wave profile.

In order to evaluate the damage in the component it is necessary taking into account different methods of damage: the exceeding the melting energy, the element erosion, the spallation and the accumulation of the damage in the part of the component that is still solid. In figure 6 there is the time evolution of the different type of damage.

For what concerns the melting energy, it is important to underline that for simplicity the evaluation of the fluid zone is done under the hypothesis that the melting temperature (1356 K) is constant with the density. The energy intensity deposited on the central part of the component is such that the temperature overcomes the melting temperature. In this case the shear strength of the element is set to 0 and the material starts to behave like a fluid. In more detail, the results show that after a 1 μs from the impact the central part on the cylindrical component is melted over the 1 m length, while in the radial direction the melted areas is about 4 mm.

Besides, in the central part, the expansion leads to a significant reduction in density and pressure such that the erosion criteria are satisfied and the elements are deleted for all the longitudinal direction on the symmetry axis and until a radius of about 2 mm for the longitudinal coordinate correspondent to the maximum deposition values. The part of the component that is behind the area of impact could be subjected to considerable values of tensile hydrostatic stresses caused by the propagation of the pressure shock waves. In this part, if the tensile load exceeds the $P_{\text{cut-off}}$ and the material is solid, it spalls and loses its strength. In the remaining part of the material, it is possible to calculate also the evolution of the damage of the component: no element fails since the damage parameter is less than unity during all the simulation.

Until 1 μs after the impact, the principal damage source is represented by the material change of state (vaporizing and melting), while spallation and ductile damage are missing. The reason is that the part of the component taken into account in the analysis (5 mm of radius) is prevalent subjected to compressive stress that does not produce any cumulative damage or significant spallation: this will probably happen as soon as the shock wave will be reflected by a free surface.

5. Conclusions
The aim of this work is the analysis and the evaluation of the damage in a cylindrical copper component with a length of 1 m and a radius of 5 mm consequent to the impact of 8 protons bunches.
at 7 TeV. The energy deposited on the component is calculated by the Monte-Carlo based FLUKA code and it is used as input in the numerical simulation performed with LS-DYNA FEM code.

The material model used for the description of the mechanical strength behaviour of the material is the Johnson-Cook model in which a damage criteria and a spall model are introduced.

The external energy is passed to the component through the equation of state as time dependent energy rate. The EOS used in the numerical simulations is a polynomial equation of state linear in energy. The coefficients are calculated fitting a multi-phase tabular EOS for each energy level. Besides, in order to neglect from the fit the areas that are physically unreachable during the evolution, each interpolation is done limiting the entire tabular data in density, pressure and energy.

The evaluation of the damage in the component is estimated in different ways according to the time evolution of the history of each element. The typologies of damage evaluated in this work are: the fusion of the material if the temperature exceeds the melting reference value, the erosion of the element if the value of density is such that the material can not support tensile load, the spallation of the element if the calculated hydrostatic tensile stress grows beyond a fixed limit and finally the calculation of the cumulative damage in the part of the component that is still solid. The results show that the component is damaged until a maximum radius of about 4 mm while in the longitudinal direction the protons beam impact causes the perforation of the material.

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