Synthesis and analysis of discriminators under influence of broadband non-Gaussian noise

V M Artyushenko¹ and V I Volovach²

¹ Technological University, 42, Gagarin str., Korolev, 141070, Russia
² Volga Region State University of Service, 4, Gagarin str., Togliatty, 445017, Russia

E-mail: volovach.vi@mail.ru

Abstract. We considered the problems of the synthesis and analysis of discriminators, when the useful signal is exposed to non-Gaussian additive broadband noise. It is shown that in this case, the discriminator of the tracking meter should contain the nonlinear transformation unit, the characteristics of which are determined by the Fisher information relative to the probability density function of the mixture of non-Gaussian broadband noise and mismatch errors. The parameters of the discriminatory and phase characteristics of the discriminators working under the above conditions are obtained. It is shown that the efficiency of non-linear processing depends on the ratio of power of FM noise to the power of Gaussian noise. The analysis of the information loss of signal transformation caused by the linear section of discriminatory characteristics of the unit of nonlinear transformations of the discriminator is carried out. It is shown that the average slope of the nonlinear transformation characteristic is determined by the Fisher information relative to the probability density function of the mixture of non-Gaussian noise and mismatch errors.

1. Introduction
The most important element of any tracking meter is a discriminator. The issues of synthesis and analysis of discriminators are described in a great number of works [1, 2, etc.]. Studies show that nonlinear inertial units of transformation of instantaneous values or envelopes of processed signals play the most significant role in signal processing algorithms under the influence of non-Gaussian noise. Discriminators are the most important among such devices. Theory and practice of implementation of discriminators is fairly well developed. However, when considering these issues primarily a form of signal was taken into account, but hardly ever the probability density function (PDF) of the influencing noise $W_n(n)$ and mismatch error $W(\varepsilon)$ were counted for.

2. The synthesis of the discriminators of tracking meters when exposed to non-Gaussian broadband noise
First, confirm that you have the correct template for your Let us consider the synthesis and analysis of the discriminators of tracking meters under the influence of additive broadband noise with the non-Gaussian distribution [3–9].

Given the input of the receiving device receives samples $\{y_h\}$ of an additive mixture of carrier signal $s(\lambda_h, t_h)$ and non-Gaussian broadband noise $n_h$

$$y_h = s(\lambda_h, t_h) + n_h, \quad h \in [1, H],$$

where $\lambda_h$ is the information parameter of the signal.
Provided that the demodulation is accurate, using the results in [10], we write equations for estimation that meet the criterion of maximum a posteriori probability:

\[ \hat{\lambda}_h = \hat{\lambda}_{c,h} + \hat{\sigma}^2_{\lambda,h} \left[ \hat{\lambda}_{c,h}, y_h - s(\hat{\lambda}_{c,h}) \right] B'_{\lambda,h}; \]

\[ \hat{\sigma}^2_{\lambda,h} \left[ \hat{\lambda}_{c,h}, y_h - s(\hat{\lambda}_{c,h}) \right] = \left[ \frac{\partial^2}{\partial \lambda_h^2} \ln W \left( \hat{\lambda}_h | \hat{\lambda}^{h-1} \right) - B_{\lambda,h}^* \right]^{-1}, \]

where \( \hat{\sigma}^2_{\lambda,h} \left[ \hat{\lambda}_{c,h}, y_h - s(\hat{\lambda}_{c,h}) \right] \) is the variance of the posterior distribution of the estimation; \( \hat{\lambda}_{c,h} \) is the extrapolated value \( \lambda \) for the \( h \)-th step;

\[ B'_{\lambda,h} = s' \left( \hat{\lambda}_{c,h} \right) z(n_{\Sigma,h}) \]  

(1)

is the output effect of the discriminator; \( s' \left( \hat{\lambda}_{c,h} \right) \) is the derivative of the signal with respect to a measured parameter \( \lambda \); \( z(n_{\Sigma,h}) \) is a characteristic of the nonlinear transformation unit (NLTU), depending on the type of the PDF of noise \( W(n) \) and mismatch error \( W(e) \)

\[ z(n_{\Sigma,h}) = -\frac{d}{dn_{\Sigma,h}} \ln W_n \left[ y_h - s(\hat{\lambda}_{c,h}, t_h) \right] = -\frac{d}{dn_{\Sigma,h}} \ln W_n \left[ s(\hat{\lambda}_{c,h}, t_h) + n_{\Sigma,h} s(\hat{\lambda}_{c,h}, t_h) \right] =
\]

\[ = -\frac{d}{dn_{\Sigma,h}} \ln W_n (n_{\Sigma,h} + n_h) = -\frac{d}{dn_{\Sigma,h}} \ln W_n (n_{\Sigma,h}); \]

where \( n_{\Sigma,h} = s(\lambda_h, t_h) - s(\hat{\lambda}_h, t_h) \) is the discrepancy between the received signal and reference signal, which can be both deterministic or random; \( B_{\lambda,h}^* \) is the derivative \( B'_{\lambda,h} \) with respect to the information parameter \( \lambda \).

Note that this difference also carries the information on the information parameter of the signal.

A device that computes \( B'_{\lambda,h} \left[ y_h - s(\hat{\lambda}_{c,h}) \right] \) is called the discriminator. Its block diagram is shown in Figure 1.

![Figure 1. The block diagram of the discriminator of a tracking meter exposed to non-Gaussian broadband noise.](image-url)
It should be noted that the optimum discriminator can fairly accurately operate only when tracking precision is high. Initially we will analyze this particular case which is practically important though, and then we consider a more general case, when detuning is arbitrary.

Expanding the function $z(n_{\Sigma,h})$ in a Taylor series at the point $\hat{h}_{c,h}$ and taking only two first components, which is justified with high precision tracking, we will get:

$$z(n_{\Sigma,h}) \approx z(n_{n,h}) + \varepsilon_n s_n' \left( \hat{h}_{c,h} \right) z_n' \left( n_{n,h} \right),$$

where $\varepsilon_n = \lambda_n - \hat{\lambda}_{c,h}$.

Substituting this ratio into the expression determining the output effect of the discriminator (1), we get

$$B' \left( n_{\Sigma,h} \right) \approx s_n' \left( \hat{h}_{c,h} \right) z_n' \left( n_{n,h} \right) + \varepsilon_n s_n' \left( \hat{h}_{c,h} \right) z_n' \left( n_{n,h} \right) = \alpha \left( t_h \right) + \beta \left( \varepsilon \right).$$

The obtained expression shows that the output effect of the discriminator consists of two random components $\alpha(t_h)$ and $\beta(\varepsilon)$.

The component $\beta(\varepsilon) = \varepsilon_n s_n' \left( \hat{h}_{c,h} \right) z_n' \left( n_{n,h} \right)$ describes the signal part of the output effect and is called the discriminatory characteristic (DC) of the discriminator.

The component $\alpha(t_h) = s_n' \left( \hat{h}_{c,h} \right) z_n' \left( n_{n,h} \right)$ describes the noise part of the output effect and is called the fluctuation characteristics (FC) of the discriminator.

Let us consider and analyze these characteristics in detail.

Take the mathematical expectation $\beta(\varepsilon)$ obtained by set averaging and time averaging as the useful signal at the output of the discriminator. In this case provided the precision is high we get

$$\tilde{\beta} \left( \varepsilon \right) = \tilde{I}_{F,n} \left[ s_n' \left( \hat{h}_{c,h} \right) \right] \varepsilon = \tilde{K}_d \varepsilon,$$

where $\tilde{K}_d = \tilde{I}_{F,n} \left[ s_n' \left( \hat{h}_{c,h} \right) \right] \varepsilon = I_{F,n} W_\varepsilon$ is the slope of the DC obtained by set and time averaging of the signal component; $I_{F,n} = \int z_n' \left( n_{n,h} \right) W_n \left( n_{n,h} \right) dn_{n,h}$ is the amount of Fisher information relative to the noise with the density of probability distribution $W_0(n_{n,h})$; $W_\varepsilon = \lim_{H \to \infty} \frac{1}{H} \sum_{h=1}^{H} \left[ s_n' \left( \hat{h}_{c,h} \right) \right]^2$.

Note that hereinafter, wavy line above a variable means time averaging, while straight line above a variable means set averaging.

When noise is Gaussian,

$$I_{F,n} = \sigma_n^{-2},$$

where $\sigma_n^2$ is the variance of affecting noise, which coincides with the known results [11].

As it can be seen from the expression (2), DC has a linear section.

In contrast to the case of Gaussian noise, the slope of DC is determined not only by characteristics (form) of a processed signal and variance of noise, but also by the amount of Fisher information $I_{F,n}$ relative to the noise $W_0(n_{n,h})$.

We define the variance of the fluctuation component at the output of the discriminator by averaging $\alpha(t_h)$ over the set

$$\sigma_{e,F}^2 = \alpha^2 \left( t_h \right) = \left[ s_n' \left( \hat{h}_{c,h} \right) \right]^2 \sigma_n^2 \left( n_{n,h} \right) = W_\varepsilon I_{F,n} = \tilde{K}_d^2.$$

(3)
The obtained relation confirms the well-known result that it also characterizes the variance of the fluctuation error at the output of the discriminator [12].

The obtained relation confirms the well-known result that it also characterizes the variance of the fluctuation error at the output of the discriminator

Dividing the noise component by \( \tilde{K}_d^2 \) we determine its equivalent value \( \sigma_n^2 = \tilde{K}_d^2 \).

With non-Gaussian broadband noise and arbitrary discrepancies, the process at the output of the discriminator is the result of a complex nonlinear transformation of the sum of two random processes \( n_{\varepsilon} \) and \( n_{h} \).

In accordance with the above, we determine the slope of DC by averaging the derivative of the output effect of a measured parameter of the discriminator over the set and the time

\[
\tilde{K}_{d,n,\Sigma}^2 = \tilde{K}_n^2 \left[ z_n(n_{\Sigma,h}) \right] = \int_{-\infty}^{\infty} \left[ z_n'(n_{\Sigma,h}) \right] ^2 (n_{\Sigma,h}) W_n(n_{\Sigma,h}) dn_{\Sigma,h} = I_{F,n,\Sigma} W',
\]

where \( W_n(n_{\Sigma,h}) = \int_{-\infty}^{\infty} W_{\varepsilon}(n_{\Sigma,h}) W_{h}(n_{\Sigma,h} - n_{\Sigma,h}) dn_{\Sigma,h} \) of total error defined by resultant.

In contrast to the previous case, when detuning is arbitrary, the slope of DC is determined by the amount of Fisher information relative to the PDF or the error of the mixture of noise and mismatch errors.

It is known that if the PDF \( W_n(n_{\Sigma,h}) \) is a resultant with an arbitrary density, then [11]

\[
I_{F,n,\Sigma} \leq I_{F,n}.
\]

Consequently, the slope of the DC in the case when mismatch error is taken into account, satisfies the inequality

\[
\tilde{K}_{d,n,\Sigma}^2 \leq \tilde{K}_d^2.
\]

Time averaging is performed on the processing interval \( T \). When processing random processes, this interval exceeds the interval of correlation of the information process as a rule.

As \( z(n_{\Sigma,h}) \) is an odd function of the argument, then \( \tilde{\beta}(\varepsilon) = 0 \). This means that the DC is not biased.

Noise component at the output of the discriminator can be written as

\[
\alpha(t_{h}) = B'_{h} - \tilde{\beta}(\varepsilon) = B'_{h}.
\]

As \( \tilde{\beta}(\varepsilon) = 0 \), the variance can be defined as follows:

\[
\sigma_{e}^2 = (\tilde{B}_h')^2.
\]

Taking into account that

\[
\left[ \frac{\partial \ln W(x)}{\partial x} \right]^2 - \left[ \frac{\partial^2 \ln W(x)}{\partial x^2} \right] = I_{F,x},
\]

we can finally get

\[
\sigma_{e}^2 = I_{F,n,\Sigma} W'.
\]
Taking into account the expression (4) and mismatch errors $\sigma^2_e < \sigma^2_{e,G}$, the equation (3) can serve as an upper estimate of the variance of the fluctuation component at the output of the discriminator.

To get a quantitative estimate when comparing relative efficiency of non-linear discriminators in non-Gaussian noise with their efficiency in Gaussian noise, it is convenient to use the ratio

$$
\mu_{e,d} = \frac{\beta_{d,nG}^2(\sigma) \sigma_{e,nG}^2}{\beta_{d,G}^2(\sigma) \sigma_{e,G}^2} = -I_{F,n} \sum \sigma_n^2, \quad (5)
$$

where $\beta_{d,nG}^2$, $\beta_{d,G}^2(\sigma)$, $\sigma_{e,nG}^2$, $\sigma_{e,G}^2$ are respectively, average values and variances of the output signal of the discriminator with non-Gaussian and Gaussian noise.

This ratio shows that if absolute values of the average signal of mismatch error are equal, when we compare two discriminators, more effective will be the one which provides the maximum signal-to-noise ratio or the smaller value of $\sigma^2_e$, which is the same.

For a particular case when the characteristic of the discriminator is described by the linearized ratio

$$
\mu_{e,d}^2 = I_{F,n} \sigma_n^2 = \mu_{0,a}^2,
$$

the efficiency of non-linear processing in the discriminator (5) coincides with the efficiency of the nonlinear transformation units $z(n_h)$ obtained in the theory of asymptotically optimal reception [11].

(PDF) of the influencing noise $W_n(n)$ and mismatch error $W(e)$ were counted for.

3. Analysis of information loss caused by linear section of nonlinear transformation unit

As known, there are three sections in the characteristic of the discriminator. If the mismatch error $e_h = \lambda_h - \hat{\lambda}_e h$ (where $\lambda_h$ is the information parameter of the signal at the $h$-th step; $\hat{\lambda}_e h$ is extrapolated estimation value $\lambda$ at the $h$-th step) is described by the Gaussian PDF, the middle section of discrimination characteristic (DC) of the discriminator is linear. Steepness and length of this section is determined by the variance of the mismatch error $\sigma^2_e$ and it changes in the process of operation of the demodulator. The nature of the end sections of the DC is determined by the form and parameters of the PDF of additive noise $W_n(n)$ and steepness of these sections is determined by the variance of the noise $\sigma^2_n$.

Figure 2 shows an example of the characteristic $z(n_{z,h})$ of the discriminator for the case when the PDF of the total error $z(n_{z,h})$ can be described by bimodal distribution

$$
W_n(n_{z,h}) = \sum_{i=1}^2 n_i N(m_i, \sigma_{2n}^2 + \sigma^2_e).
$$

![Figure 2](image-url)
Let us consider and analyze the information loss connected with the linear section of the characteristic of the NLTU of the discriminator.

Taking into account mismatch errors leads to the necessity of introducing the linear section to the characteristics of the NLTU, which therefore reduces the efficiency of nonlinear processing.

The boundaries and slope of the linear section change in the process of operation and are determined by the value of current a posteriori variance $\sigma^2_{\epsilon,h}$.

To estimate the information loss caused by the presence of the linear section of the characteristic of the NLTU, the stationary mode of operation of the meter has to be considered when $\sigma^2_{\epsilon,h} = \sigma^2_{\epsilon,h-1} = \sigma^2_{\epsilon}$.

In this case, the amplitude of the NLTU can be represented as:

$$z_n(n) = \begin{cases} \frac{n}{\sigma^2_{\epsilon}}, & |n| \leq \Delta; \\ \frac{W'_{n,\Delta}(n)}{W_{n,\Delta}(n)}, & |n| > \Delta, \end{cases}$$

where $\Delta$ is the boundary of linear section, which can be estimated by the value $k_\Delta \sigma^2_{\epsilon}$; $k_\Delta$ is a constant coefficient.

We estimate the average slope of the characteristic of NLTU before the introduction of the linear section

$$K_d = - \int_{-\infty}^{\infty} \frac{d}{dn} z_n(n) W_n(n) dn = \int_{-\infty}^{\infty} z^2_{\epsilon,n}(n) W_n(n) dn = I_{F,n},$$

where $I_{F,n}$ is Fisher information relative to noise with the density of probability distribution $W_n(n)$.

The increment in the signal/noise ratio (SNR) at the output of such NLTU is estimated by

$$\mu^2 = I_{F,n} \sigma^2_{\epsilon},$$

which is called the ratio of the amplitude noise suppression [12].

With the introduction of the linear section the average slope of the NLTU changes and can be estimated by the expression

$$K_{d1} = - \int_{-\infty}^{\infty} z'_{\epsilon,n}(n) W_n(n) dn + \sigma^2_{\epsilon} \int_{-\Delta}^{\Delta} W_n(n) dn + \int_{-\Delta}^{\Delta} z^2_{\epsilon,n}(n) W_n(n) dn = I_{F,n} - I_{F,1},$$

where

$$I_{F,1} = \int_{-\Delta}^{\Delta} z'_{\epsilon,n}(n) W_n(n) dn - \sigma^2_{\epsilon} \int_{-\Delta}^{\Delta} W_n(n) dn ,$$

characterizes Fisher information loss relative to noise due to the introduction of linear section in the characteristic of the NLTU.

In general, the ultimate loss of Fisher information due to errors can be estimated by the expression [13]

$$I_{F,1} = \int_{-\infty}^{\infty} z'_{\epsilon,n}(n) W_n(n) dn - \int_{-\infty}^{\infty} z'_{\epsilon,n}(n) W_n(\Delta) dn_{\Delta} = \int_{-\infty}^{\infty} z^2_{\epsilon,n}(n) W_n(n) dn - \int_{-\infty}^{\infty} z^2_{\epsilon,n}(n) W_n(u - n) W_n(u) du dn.$$

The slope of the characteristic of the NLTU with taking into account mismatch errors will be determined by the expression
\[
K_{d_\varepsilon} = \int_{-\infty}^{\infty} d\xi \; z_n(n_\varepsilon) W_c(u) W_n(u-n_\varepsilon) du d\xi.
\]

The losses mentioned above lead to the reduction of SNR at the output of the NLTU by

\[
\mu^2 = I_{F_1} \sigma_n^2.
\]

As a result the \( \mu^2_{0,a} \) value should be considered as ultimate, which is achieved only in the absence of errors of tracking.

We estimate the value \( I_{F_1} \) for rather general case when the PDF can be described by poly-Gaussian distribution [14]

\[
W_n(n) = \sum_{m=1}^{M} p_m N\left(m_{n,m}, \sigma_{n,m}^2\right); \sum p_m = 1.
\]

In this case

\[
z_n(n) = -W'_n(n)/W_n(n); \quad z'_n(n) = -\left[W'_n(n)W_n(n) - \left(W'_n(n)\right)^2\right]W_n^{-2}(n).
\]

Substituting (7) and (8) in expression (6), after simple but cumbersome transformations, we get

\[
I_{F_1} = \int_{-\Delta}^{\Delta} \left(\frac{W'_n(n)}{W_n(n)}\right)^2 \; dn - W_n(n) \left|\int_{-\Delta}^{\Delta} W_n(n) \; dn\right|,
\]

or

\[
I_{F_1} = I_1 - \sum_{m=1}^{M} \frac{p_m}{\sqrt{2\pi\sigma_{n,m}^2}} \left(\frac{\Delta - m_{n,m}}{\sigma_{n,m}}\right) \exp\left(-\frac{(\Delta - m_{n,m})^2}{2\sigma_{n,m}^2}\right) + \left(\Delta - m_{n,m}\right) \exp\left(-\frac{(\Delta - m_{n,m})^2}{2\sigma_{n,m}^2}\right) - \\
- 0.5 \left[ \text{erf}\left(\frac{\Delta - m_{n,m}}{\sqrt{2}\sigma_{n,m}}\right) + \text{erf}\left(\frac{\Delta + m_{n,m}}{\sqrt{2}\sigma_{n,m}}\right) \right];
\]

\[
I_1 = \int_{-\Delta}^{\Delta} \left(\frac{W'_n(n)}{W_n(n)}\right)^2 \; dn; \quad \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\xi^2) d\xi,
\]

where \( \text{erf}(x) \) is the error function (probability integral).

4. Conclusion
Thus, the synthesis and analysis of the discriminator of the tracking meter operating in conditions of exposure to broadband non-Gaussian noise is carried out. It is shown that in case of exposure to broadband non-Gaussian noise the discriminator the tracking meter contains the nonlinear transformation unit, the characteristic \( z(n_{\varepsilon,h}) \) of which is determined by the form of the PDF \( W_\varepsilon(n_\varepsilon) \) of noise and mismatch errors \( W_\varepsilon \).

Characteristics of the discriminators \( \tilde{K}^2_\varepsilon \) and \( \sigma_\varepsilon^2 \) are determined by the amount of Fisher information \( I_{F,n_\varepsilon} \) relative to the PDF of the mixture of non-Gaussian noise \( n_{n,h} \) and mismatch errors \( n_{\varepsilon,h} \).

The main calculated ratios are given, which allows us to estimate the efficiency of signal processing in the discriminators in presence of non-Gaussian broadband noise, taking into account mismatch errors. The parameters of DC and FC of discriminators operating under these conditions are found.
It is shown that the efficiency of non-linear processing increases with decreasing mismatch error and when \( n_{c,h} \to 0 \), it is estimated by the amount of Fisher information relative to influencing noise \( I_{F,n} \).

Numerical analysis of the expression (9) shows that the expansion of the boundaries of the linear section of the DC leads to information loss increase and when \( \Delta \to 0 \), \( I_{F,l} \to I_{F,n} \). It means that the efficiency of nonlinear transformation is zero and DC degenerates into linear one.

Note that to estimate the efficiency of some real NLTUs, characteristics of which differ from the optimal ones, the above ratio of real amplitude suppression of noise can be used.

References

[1] Ellingson S W 2016 Radio System Engineering (Cambridge: Cambridge University Press)
[2] McClaning K and Vito T 2001 Radio Receiver Design (London: Noble Publishing Corporation)
[3] Kassam S A 1989 Signal detection in non-Gaussian noise (New York: Springer Verlag)
[4] Park J, Shevlyakov G and Kim K 2012 Distributed Detection and Fusion of Weak Signals in Fading Channels with Non-Gaussian Noises IEEE Communications Letters 16(2) pp 220–3
[5] Bandiera F Dodde V and Ricci G 2015 Radar detection and range estimation of a point-like target in non-Gaussian noise Proc. IEEE Radar Conf. (RadarCon) DOI: 10.1109/RADAR.2015.7131051
[6] Yang J Cheng Y Wang H Li Y and Hua X 2015 Unknown stochastic signal detection via non-Gaussian noise modelling 2015 IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC)
[7] Palahina E and Palahin V 2016 Signal detection in additive-multiplicative non-Gaussian noise using higher order statistics Proc. 26th Int. Conf. Radioelektronika (RADIOELEKTRONIKA)
[8] Artyushenko V M, Volovach V I and Shakurskiy M V Analysis of influence of uncorrelated additive non-Gaussian noise on accuracy of motion parameters measurement in short-range radio systems 2015 Proc. IEEE Int. Siberian Conf. Control and Communications (SIBCON-2015) 7147279
[9] Artyushenko V M and Volovach V I Measuring information signal parameters under additive non-Gaussian correlated noise 2016 Optoelectronics, Instrumentation and Data Processing 59(6) pp 22–8
[10] Artyushenko V M and Volovach V I 2016 The demodulation signal under the influence of additive and multiplicative non-Gaussian noise Proc. IEEE East-West Design & Test Symp. (EWDTS’2016)
[11] Novoselov O N and Fomin A F 1991 Bases of the theory and calculation of information and measuring systems (Moscow: Mechanical engineering) (in Russian)
[12] Kuznetsov P I, Stratonovich R L and Tikhonov V I 1965 Non-Linear Transformations of Stochastic Processes (Oxford: Pergamon Press)
[13] Valeyev V T and Sosulin Y G Detection of weak, coherent signals in correlated non-Gaussian noise 1969 Radio technology and electronics 14(2) (in Russian)
[14] Tihonov V I and Harisov V N 1991 Statistical analysis and synthesis of radio engineering devices and systems (Moscow: Radio and communication) (in Russian)