Students’ Perception towards Mathematics using APOS Theory: A Case Study

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Abstract. Students’ perceptions towards mathematics seem to be shaped by how they define mathematics and the role of mathematics in their life. Most students in Malaysia lack experience in dealing with mathematics, thus their achievement in mathematics are not as good as non-Malaysian students (Liew & Pong, 2004). Therefore, the purpose of this study is to investigate students’ perceptions in learning mathematics and how these perceptions may lead to problems in understanding the mathematics itself among students in selected Malaysian secondary schools. A set of questionnaire was distributed to 200 students in Kuantan, Pahang in order to gauge their perceptions in learning mathematics. The researcher also assesses the relationship that exists between students’ perceptions towards mathematics and their academic achievement by using APOS Theory. The respondents generally reported a positive perception towards mathematics. With regard to the correlation analysis, there was a significant statistical relationship between students’ perceptions towards mathematics and their UPSR mathematics grade. However, there was no significant difference between students’ perception towards mathematics and recent mathematics grade. These results warrant further researches using more advanced statistical tools in order to obtain more accurate and relevant findings. Finally it is hoped that the study will open new paths that can benefit mathematics in education system and how mathematics are perceived by students in the classroom.

Keywords: mathematics perception, achievement in mathematics, APOS Theory

1. Background of the Study

Problems related to the process of learning mathematics in the classroom and students’ low achievement in mathematics have been extensively studied. Mathematical anxiety and low confidence level in learning mathematics among students result in confused thinking, incompetence, low level of interest towards the subject, and non-participation in class [1][2]. Students’ attitude towards mathematics seems to be shaped by how they define mathematics in their mind and the importance that they attach to the role of mathematics in their life. A student’s perception towards mathematics as a bunch of symbols and procedures tends to treat his/her mathematics concepts as a set of fact memory. This student will not put any effort in understanding the “whys” questions involved in the mathematics equation that he or she has learnt. A student who thinks mathematics is not important to his/her future life and work will not regard his/her mathematics course worthy enough to spend his/her valuable time on. For these students, their motivations to learn mathematics would be very low thus will result in
the difficulty to learn mathematics meaningfully. Therefore, addressing students’ attitudes and behaviours prior to the introduction of mathematical concepts is very crucial; before expecting any meaningful learning process in the classroom.

Therefore, this study attempts to integrate a well-known mathematical theory called the APOS Theory, into students’ perceptions and mathematics knowledge in answering and explaining the problems that occur during the process of learning mathematics in the classroom. It is hoped that through this study, students and teachers can apply and utilise the principles in APOS Theory to better understand the subject and they will be able to propose solutions to the existing problems of learning mathematics in the classroom. Therefore, the results of this study may pave the way for improvements in the process of teaching and learning mathematics in general and more specifically may help in improving teachers’ teaching methods and enhancing students’ academic performance in mathematics.

2. Statement of the Problem

Students’ perceptions about mathematics are related to their perception in learning mathematics. Mathematics perception is defined as how students personally view mathematics as a subject, and how they feel and think about learning the subject. The perceptions of the students regarding mathematics and mathematics teaching have been considered as a very significant factor underlying their school experiences and achievements [3][4]. However, there is evidence that shows the lack of research exploring mathematics perception, mathematics skills, and mathematics achievement among Malaysian secondary school students [5]. Therefore, more investigations on students’ perceptions about mathematics and its impact towards students’ overall academic achievement in school and later on in their future careers need to be conducted [5].

The major notion or perception that mathematics is a very difficult subject to learn, together with mathematics being labelled negatively among students in schools, makes this study even more vital. Various studies have been conducted to address the problems of mathematics anxiety, yet the results are mixed.

3. Objective of the Study

The main objective of this study is to investigate students’ perceptions towards learning mathematics and how these perceptions may lead to the problems in understanding the subject among students at selected secondary schools in Malaysia.

4. Research questions

Based on the objectives, this study aimed to answer the research question:

1. How can students’ perceptions towards learning mathematics and students’ academic achievement in mathematics be explained through the use of APOS Theory?

5. The significance of the study

The result from this study can be beneficial to better understand students’ performance in mathematics via the various levels of mathematics achievement that they obtain. This study may also play an important role in enhancing students’ understanding on mathematics at schools.

Besides, the findings of this research would also be useful and valuable in educational setting, namely for structuring course design, teaching instructions, teaching approach, and students’ assessment or evaluation in mathematics. Teachers should recognise their students’ perceptions towards mathematics to plan the best methods in teaching the subject. Teachers should also apply students’ evaluation in a variety of ways in order to build a more constructive and positive perception towards mathematics. Having this objective achieved, students’ mathematics performance will be enhanced regardless of any discipline they are majoring in.

In addition, it is important to determine whether mathematics perception affects students’ ability in mathematics, especially in problem solving skills. Students’ perception towards mathematics is
essential as it may influence the way they receive the knowledge and how they organise and define tasks at the cognitive level.

6. Definition of Terms

6.1 Students’ Mathematics Perception
Students’ perception can be described as students’ beliefs and emotions regarding their knowledge of mathematics and their competence in learning mathematics. It also includes how they view the subject and the way they respond to the learning process of the subject.

6.2 Students’ Mathematics Academic Achievement
This term specifically refers to students’ mathematics achievement such as mathematics grade in UPSR and their most recent mathematics grade from school tests.

6.3 APOS Theory
APOS Theory, which is an acronym for Action, Process, Object, and Schema, is the theory of Ed Dubinsky that takes account of the initial objects on which the actions operate. APOS theory begins with base Objects on which the individual performs Actions, coordinates into Processes, represents by symbols that have meaning as Mental Objects, within a wider Schema. Further elaborations and details of this APOS Theory are explained below:

- **Action**: An Action is a transformation of objects perceived by an individual as essentially external and as requiring, either explicitly or from memory, step-by-step instructions on how to perform the operation. When an action is repeated and the individual reflects upon it, he or she can make an internal mental construction called a Process, which the individual can regard as performing the same kind of action, but no longer with the need of external stimuli. An individual can think of performing a process without actually doing it, and therefore can think about reversing it and composing it with other processes. An **Object** is constructed from a process when the individual becomes aware of the process as a whole and realises that transformations can affect it.

- **Process**: Finally, a **Schema** for a certain mathematical concept is an individual’s collection of actions, processes, objects, and other schemas that are linked by some general principles in order to form a framework in the individual’s mind that may be brought up to solve a problem in a situation involving that concept.

This framework must be coherent in the sense that it gives, explicitly or implicitly, way in determining which phenomena are in the scope of the schema and which are not. Thus, this theory considers the notion of meaningful understanding and all mathematical entities can be represented in terms of actions, processes, objects, and schemas.

7. Mathematical Concepts
In learning Mathematics, there are some concerns regarding mathematical concepts such as functions; various topics in abstract algebra including binary operations, groups, subgroups, co-sets, normality and quotient groups; topics in discrete mathematics such as mathematical induction, permutations, symmetries, existential and universal quantifiers; topics in calculus including limits, chain rule, graphical understanding of the derivative and infinite sequences of numbers; topics in statistics such as mean, standard deviation, and central limit theorem; topics in elementary number theory topics such as place value in base \( n \) numbers, divisibility, multiples, and conversion of numbers from one base to another; and fractions.

8. APOS as A Tool in Mathematics
According to Dorman & Knightley, some studies such as on fractions show that the APOS Theory, which is a theory developed for advanced mathematical thinking, is also a useful tool in studying students’ understanding on more basic mathematical concepts. Dubinsky & McDonald report that
“the totality of this body of work, much of it done by researchers in Undergraduate Mathematics Education Community (RUMEC) members involved in developing the theory, but an increasing amount done by individual researchers having no connection with RUMEC or the construction of the theory, suggests that APOS Theory is a tool that can be used objectively to explain students’ difficulties with a broad range of mathematical concepts and to suggest ways that students can used to learn these concepts” (p.9) \[9\]. Besides, Dubinsky & McDonald also claim that “APOS Theory can point us towards pedagogical strategies that lead to marked improvement in student learning of complex or abstract mathematical concepts and students’ use of these concepts to prove theorems, provide examples, and solve problems” (p.9)\[9\].

APOS theory helps us to analyse data and by using the theory to explain the data or results can lead to changes in the theory. Usually, the genetic decomposition in the original theoretical analysis is revised and refined as a result of the data. In rare cases, it may be necessary to enhance the overall theory. An important example of such a revision is the incorporation of the triad concept of Piaget and Garcia, which leads to a better understanding of the construction of schemas [10]. The enhancement of the theory was introduced by Clark, et al. who reported on students’ understanding of the chain rule, and it is being further elaborated upon in three other studies namely sequences of numbers, the chain rule and its relation to composition of functions, and the relations between the graph of a function and properties of its first and second derivatives [11][12][13][14].

9. Research Design

This quantitative study aimed to assess students’ perception towards mathematics together with the application of APOS Theory. This study was an explanatory type of study, which made use of a survey, and also included correlation as a part of the research design. According to Creswel, correlational study provides an opportunity to see relationships between the relevant variables [15]. This type of study works when the researcher wants to relate two or more variables and thus useful in predicting the outcomes. This present study attempted to understand the notion of students’ perception towards mathematics and to analyse the intertwined factors between mathematics achievement and mathematics problem solving. In doing so, this study also attempted to identify how perceptions of students towards mathematics can affect their academic achievement. Hence, the study used a survey which was designed to obtain the precise information from the current state of phenomenon and to get valid conclusion. A set of questionnaire was used as an instrument for the collection of data needed from the participants selected in the study.

10. Research Findings

10.1 Students’ perception towards Mathematics & APOS Theory

| Table 1. Descriptive Statistics on Students’ Perception: Action |
|---------------------------------------------------------------|
| N | Minimum | Maximum | Mean | Std. Deviation |
|-----------------------------------------------|
| Total score (Action) | 200 | 2.11 | 5.00 | 3.9239 | .57704 |
| Valid N (listwise) | 200 | | | |

Table 1 shows the mean and standard deviation for students’ perception in terms of Action (n=9). Based on the table, overall, the respondents had a fairly good perception towards Mathematics in terms of Action since the total mean score is 3.9239 and the standard deviation is 0.57704.
Table 2. Descriptive Statistics on Students’ Perception: Process

|                  | N  | Minimum | Maximum | Mean    | Std. Deviation |
|------------------|----|---------|---------|---------|----------------|
| Total Score (Process) | 200 | 2.36    | 5.00    | 3.9623  | .54127         |
| Valid N (listwise) | 200 |         |         |         |                |

Table 2 shows the mean and standard deviation for students’ perception in terms of Process (n=11). Based on the table, overall, the respondents had a fairly positive perception towards Mathematics in terms of Process since the total mean score is 3.9623 and the standard deviation is 0.54127.

Table 3. Descriptive Statistics on Students’ Perception: Object

|                  | N  | Minimum | Maximum | Mean    | Std. Deviation |
|------------------|----|---------|---------|---------|----------------|
| Total Score (Object) | 200 | 2.22    | 5.00    | 3.6978  | .59405         |
| Valid N (listwise) | 200 |         |         |         |                |

Table 3 shows the mean and standard deviation for students’ perception in terms of Object (n=9). Based on the table, overall, the respondents had a fairly positive perception towards Mathematics in terms of Object since the total mean score is 3.6978 and the standard deviation is 0.59405.

Table 4. Descriptive Statistics on Students’ Perception: Schema

|                  | N  | Minimum | Maximum | Mean    | Std. Deviation |
|------------------|----|---------|---------|---------|----------------|
| Total Score (Schema) | 200 | 1.86    | 5.00    | 4.1336  | .50798         |
| Valid N (listwise) | 200 |         |         |         |                |

Table 4 shows the mean and standard deviation for students’ perception in terms of Schema (n=7). Based on the table, overall, the respondents had a good perception towards Mathematics in terms of Schema since the total mean score is 4.1336 and the standard deviation is .50798.

11. Conclusion
The study generally aimed to measure students’ perception towards Mathematics with the application of APOS Theory.

Students’ perception towards Mathematics
The participants in this study generally showed positive perception towards Mathematics. It means that majority of the respondents were able to obtain the knowledge towards Mathematics in a good way. The perception also included four components of APOS Theory, namely Action, Process, Object, and Schema. Based on the results, all the four components of APOS Theory showed fairly positive score on perception towards Mathematics. This result implies that most of the students had an average attitude towards Mathematics and they can comprehend this subject in their learning process.
For the component of Action, the respondents showed fairly positive perception towards Mathematics. In other words, the respondents can perceive the transformation of objects as essentially external and as requiring, either explicitly or from memory, step-by-step instruction on how to perform the operation according to the problem given [16].

Furthermore, the same outcome was found for students’ perception towards Mathematics in the component of Process. The result reported that the students had fairly positive perception towards Mathematics in terms of Process. From this result, the researcher concluded that students can get the concept in Mathematics by solving the mathematical problems or any related works ideally and somehow the students can even think about the result of the process without actually solving it step by step and, in particular, imagining reversing the process.

The next component of APOS Theory is Object. The researcher found that the students had a fairly positive perception towards Mathematics in terms of Object. A student who has an object conception of a mathematical idea can imagine it as a totality and, in particular, can act on it with higher-level actions or processes.

Lastly, in the component of Schema, it was found that the students’ had a strong positive perception towards Mathematics in terms of Schema. The components of Schema among respondents were well coordinated with actions, processes, objects, and other schemas, which were linked by some general principles to form a framework in the students’ mind. This component of Schema scored the highest mean score among all the four components of APOS.

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