Optical absorption in the soliton-lattice state of a double-quantum-well system

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(October 12, 2021)

When the separation between layers in a double-quantum-well system is sufficiently small, the ground state of the two-dimensional electron gas at filling factor $\nu = 1$ has an interwell phase coherence even in the absence of tunneling. For non-zero tunneling, this coherent state goes through a commensurate-incommensurate transition as the sample is tilted with respect to the quantizing magnetic field at $\nu = 1$. In this article, we compute the optical (infrared) absorption spectrum of the coherent state from the commensurate state at small tilt angle to the soliton-lattice state at larger tilt angle and comment on the possibility of observing experimentally the distinctive signature of the soliton lattice.

Pacs: 73.21.Fg, 73.43.Lp, 78.30.Fs

I. INTRODUCTION

At strong magnetic fields and for sufficiently small separation between the wells, the two-dimensional electron gas (2DEG) in a double quantum well system (DQWS) can have a broken symmetry ground state with a non zero interlayer phase coherence even in the absence of tunneling. At filling factor $\nu = 1$, the quantum Hall effect is observed in such coherent state when the separation between the wells does not exceed some critical value $d_c$ above which the coherence is lost and the 2DEG becomes compressible. In DQWS systems, the interlayer phase coherence gives rise to a rich variety of quantum and finite-temperature phase transitions as well as to some exotic topological excitations such as merons and bimerons. Some of these various phases and excitations are reviewed in details in Refs. [1], [2], and [3].

One convenient way to describe the coherent ground states of the 2DEG in a DQWS is by using a mapping to an equivalent spin-1/2 system. In these states, the real spins are assumed completely frozen and, in the pseudospin language, an electron in the left(right) layer is equated with an up(down) pseudospin. Quantum mechanics allows for any linear superposition of these states and the corresponding pseudospin can point in any direction in space. An interesting phase transition, first reported by Murphy et al. [4], occurs when, at filling factor $\nu = 1$, the sample is tilted with respect to the quantizing magnetic field. A study of the behavior of the activation gap as the sample is tilted shows evidence for a phase transition between two competing quantum Hall ground states. Yang et al. [5] explained this change of behavior as a transition between a commensurate and an incommensurate ground states. This transition can be briefly described in the following way. In a suitable gauge, the tunneling amplitude modified by the parallel component of the magnetic field acts as an effective magnetic field for the pseudospins. This effective field rotates in space with a wavevector given by $Q = d/\ell^2$ where $d$ is the separation between the wells and $\ell^2 = \hbar c/eB_\parallel$ is the magnetic length for the parallel magnetic field. For $d \neq 0$, the pseudospins that are forced to lie in the plane of the wells to minimize the capacitive energy will locally align with this effective magnetic field. This is the commensurate (C) state. As the period of the field increases, however, the gain in tunneling energy obtained by aligning with the field is opposed by the cost in exchange energy of having non parallel pseudospins. At a critical field, the exchange energy exceeds the tunneling energy and the pseudospins cease to rotate with the effective field, behaving almost as if $t = 0$. But, this incommensurate (I) state is never the ground state of the system. Above some critical parallel field or wavevector $Q_c$, the C state has defects in the form of sine-Gordon solitons where the phase of the pseudospin slips by $2\pi$. For $Q \geq Q_c$, the ground state of the systems is a lattice of these kinks, a soliton-lattice state (SLS).

Apart from the original experiment of Murphy et al. [4], there has been no other experimental signature of the SLS. On the theoretical side, Hanna et al. [6] have discussed the possibility of detecting this lattice by surface acoustic wave technique or by measuring the small contribution of the SLS to the parallel-field magnetization of the DQWS. Some possible ways of detecting the pseudospin phase solitons in a DQWS have also been described by Kyriakidis et al. [7]. Read [8] has described in details the behavior of the energy gap near the C-I transition. Some details of the absorption spectrum but in the absence of a parallel magnetic field has already been worked out by Joglekar et al. [9]. In this work, we explore another possible signature of the SLS. Building on earlier work [10], where we derived the collective excitations of the SLS ground state, we compute the signature of these collective modes in an absorption experiment. Our basic idea is the following. The dispersion relation of the collective mode in the SLS has many branches. The lowest-energy branch corresponds to the Goldstone mode that restores the broken translational symmetry. The higher-energy branches have non-zero frequencies at zero wavevector. They (as well as the Goldstone mode) involve motion of the $z$ component of the pseudospins, a
component that is related to changes in the charge density balance between the two layers. These modes can be excited by an external electromagnetic wave. A nice feature of the SLS is that simply changing the magnitude of the parallel magnetic field (i.e., tilting the sample) modifies the period of the lattice and, consequently, the set of frequencies at zero wavevector. In principle, it could be possible to track the complete dispersion relation of the pseudospin collective modes by measuring the absorption as the parallel magnetic field is changed.

This paper is organized as follows. In section II, we relate the electromagnetic absorption to the polarisation tensor of the DQWS. In section III, we derive an expression for this polarisation tensor in terms of the z component of the pseudospin. Section IV gives a brief review of the commensurate-incommensurate transition. In section V and VI, we compute the absorption in the C.I and SL states and comment on the possibility of experimentally observing the resulting spectrum.

II. ABSORPTION OF LIGHT IN A DQWS

The coherent ground states of the 2DEG in a QDWS are characterized by spatial modulations of one or several of their order parameters. If we specify a pseudospin configuration by the spherical-coordinate fields \( \theta(x, y) \) which describes the difference in charge density between the layers, and \( \varphi(x, y) \), which describes the relative phase of electrons in the right and left wells, then the SLS ground state has spatial modulations in \( \varphi(x, y) \) only and \( \theta(x, y) = \pi/2 \) everywhere. Other states such as a lattice of bimerons [11] in a DQWS would have spatial modulations in both the phase and the relative occupation of the two wells and so would the coherent charge-density-wave recently studied by Brey and Fertig [12]. The approach we develop can be applied to these later two cases as well. In all these states, the response functions are non local in space and must be described by tensors of the form \( A_{\nu,\mu}(r, r', \omega) \) where \( \nu, \mu = x, y, z \). In this section, we derive a relation between the electromagnetic absorption and the polarisation response tensor in the coherent states.

In a stationary regime, the energy absorbed per unit time in a system is given by the Joule heating term integrated over all the volume, \( V \), of this system. This energy must also be equal to the difference between the incident and transmitted or diffused energy which is given by an integral of the Poynting vector over the surface \( S_V \) of the sample with outward normal \( \hat{n} \):

\[
P(t) = \int_V dV \mathbf{E}(r, t) \cdot \mathbf{j}(r, t) = \frac{c}{4\pi} \int_{S_V} dS \left[ \mathbf{E}(r, t) \times \mathbf{B}(r, t) \right] \cdot \hat{n}.
\]

We remark that the absorbed energy does not, in general, correspond to the difference between the transmitted and incident energy of the electromagnetic wave since there will be diffusion of the light in an heterogeneous state. For harmonically varying current and electric field, the average power dissipated per unit volume at frequency \( \omega \) is given by

\[
P(\omega) = \frac{1}{2} \text{Re} \left[ \int_V dV \mathbf{E}(r, \omega) \cdot \mathbf{j}(r, \omega) \right] = \frac{1}{2V} \sum_q \text{Re} \left[ \mathbf{E}^*(q, \omega) \cdot \mathbf{j}(q, \omega) \right],
\]

where we have defined the Fourier transform of the current and of the electric field by

\[
\mathbf{j}(r, \omega) = \frac{1}{V} \sum_q \mathbf{j}(q, \omega) e^{iqr}, \quad \mathbf{E}(r, \omega) = \frac{1}{V} \sum_q \mathbf{E}(q, \omega) e^{iqr}.
\]

In Eq. (3), \( \mathbf{E} \) is the electric field in the sample.

We can relate \( P(\omega) \) to the conductivity tensor by using the general relation between conductivity and electric field

\[
\mathbf{j}(q, \omega) = \frac{1}{V} \sum q' \sigma(q, q', \omega) \cdot \mathbf{E}(q', \omega).
\]

Alternatively, we can also formally relate the current to the external field by

\[
\mathbf{j}(q, \omega) = \frac{1}{V} \sum q' \tilde{\sigma}(q, q', \omega) \cdot \mathbf{E}_e(q', \omega),
\]

where, in contrast to \( \sigma \), the tensor \( \tilde{\sigma} \) is directly related to the full current-current response function (see, for example Ref. [13]). \( \sigma \) can be considered as an unscreened response function, a response to the total electric field that includes screening corrections, while \( \tilde{\sigma} \) is the screened response to the bare electric field. We have defined the Fourier transform of the conductivity tensor by

\[
\sigma(r, r', \omega) = \frac{1}{V^2} \sum_{q, q'} e^{i(q\cdot r' - q'\cdot r)} \sigma(q, q', \omega).
\]

With Eq. (3), the absorbed power is then

\[
P(\omega) = \frac{1}{2V^2} \sum_{q, q'} \text{Re} \left[ \mathbf{E}^*(q, \omega) \cdot \tilde{\sigma}(q, q', \omega) \cdot \mathbf{E}_e(q', \omega) \right].
\]

Maxwell’s equations can be used to relate the total and external electric fields

\[
\mathbf{E}(q, \omega) = \mathbf{E}_e(q, \omega) - \left( \frac{4\pi i}{\omega} \right) \frac{1}{V} \sum q' K(q, \omega) \cdot \tilde{\sigma}(q, q', \omega) \cdot \mathbf{E}_e(q', \omega),
\]

\[
K(q, \omega) = \frac{1}{2V} \sum_r \hat{n} \cdot \mathbf{E}(r, \omega) \times \mathbf{B}(r, \omega).
\]
where the tensor $K(q, \omega)$ is defined by

$$K(q, \omega) = \frac{\hbar q^2}{1 - e^{2\hbar q^2/\omega^2}} (I - \eta q).$$

(10)

Inserting Eq. (9) in Eq. (8), we get (one can show that the second term in Eq. (9) does not contribute to the real part of the expression in Eq. (8))

$$P(\omega) = \frac{1}{2V^2} \sum_{q,q'} \text{Re} \left[ E_c(q, \omega) \cdot \tilde{\sigma}(q, q', \omega) \cdot E_c(q', \omega) \right].$$

(11)

an expression that relates the absorbed power to the external electric field. All local field corrections are included in $\tilde{\sigma}$. For an external electromagnetic field in the form of a plane wave with amplitude $E_0$, unit polarisation vector $\xi$, and wavevector $q_0$, we have

$$P(\omega) = \frac{|E_0|^2}{2V^2} \text{Re} \left[ \xi \cdot \tilde{\sigma}(q_0, q_0, \omega) \cdot \xi \right].$$

(12)

Now, from the relation between dielectric and polarisation tensors

$$\varepsilon(q, q', \omega) = I \delta_{q,q'} + \frac{4\pi i}{\omega} \sigma(q, q', \omega)$$

$$= I \delta_{q,q'} + \frac{4\pi \chi(q, q', \omega)}{\omega},$$

(13)

we have

$$\sigma(q, q', \omega) = -i \omega \chi(q, q', \omega),$$

(14)

or alternatively

$$\tilde{\sigma}(q, q', \omega) = -i \omega \tilde{\chi}(q, q', \omega),$$

(15)

where $\tilde{\chi}$ is the response function that takes into account all local field corrections. The absorption is thus related to the imaginary part of the polarisation tensor by

$$P(\omega) = \frac{|E_0|^2}{2V^2} \text{Im} \left[ \xi \cdot \tilde{\chi}(q_0, q_0, \omega) \cdot \xi \right].$$

(16)

For non-interacting electrons, $P(\omega)$ of Eq. (16) is equivalent to the usual absorption formula given by the Fermi golden rule.

III. POLARISATION RESPONSE FUNCTION OF THE DQWS

We consider a symmetric DQWS where $d$ is the separation between the wells measured from center to center. This DQWS is placed in a strong quantizing magnetic field directed along the growth axis $z$. The magnetic field can be tilted from towards the plane of the wells, but its perpendicular component must be such as to maintain a total filling factor $\nu = 1$ in the case of the SLS. We write the total field as $B = B_{\|} y + B_{\perp} z$. In the Landau gauge, the vector potential is $A = (0, B_{\perp} x, -B_{\|} x)$.

In this work, we consider only the case of an unbiased DQWS so that the electric charge is equally distributed between the two layers in the ground state. At low temperature and in the strong magnetic field limit, we keep only one electric subband in each well and one Landau level ($n = 0$) in the description of the electronic states. The non-interacting wavefunctions for each well taken separately are given by

$$\phi_{i,\alpha}(r) = \frac{1}{\sqrt{\ell_y^{(i)}}} e^{-ix y/\ell_y} e^{-i(x-x)^2/2\sigma} \chi_i(z),$$

(17)

where $\ell_y^{(i)} = \hbar c/eB_{\perp}$ defines the magnetic length for the perpendicular component of the magnetic field and $\chi_i(z)$ with $i = R, L$ are the envelope wave functions of the lowest-energy states centered on the right or left well. $X$ is the guiding center quantum number. The degeneracy of each Landau level is given by $N_\phi = S/2\pi \ell_y^{(i)}$ where $S$ is the area of the two-dimensional electron gas. With $N$ electrons in the DQWS, the total filling factor is $\nu = N/N_\phi$. We define another magnetic length associated with the parallel component of the magnetic field by $\ell_z^{(i)} = \hbar c/eB_{\|}$. For simplicity, we describe the DQWS in a narrow well approximation i.e. we assume that the width, $b$, of the wells is small ($b << d$) and treat inter-layer hopping in a tight-binding approximation.\[10]\n
Taking the charge of the electron to be $-e$ and measuring all positions with respect to the center of the DQWS, the polarisation density operator in second quantization is

$$P(r, z) = -e \sum_j \left[ (r + z \hat{z}) \Psi_j^\dagger(r, z) \Psi_j(r, z) \right].$$

(18)

In this expression, $r$ is a vector in the plane of the 2D gas. The Fourier transformed polarisation operator is given by

$$P(q, z) = \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \int dz e^{-i\mathbf{q} \cdot \mathbf{z}} P(r, z)$$

$$= -e \sum_j \left[ \hat{z} \Gamma_{1,j}(q) n_j(q) + i \Gamma_{0,j}(q) \left[ \nabla_q n_j(q) \right] \right],$$

(19)

where the matrix elements $\Gamma_{m,j}(q)$ with $m = 0, 1$ are given by

$$\Gamma_{m,j}(q) = \int dz e^{-i\mathbf{q} \cdot \mathbf{z}} \chi_j^m(z),$$

(20)

and $q$ is now redefined as a vector in the plane of the 2DEG. $n_j(q)$ is the Fourier-transformed density operator for well $j$. For propagation of the wave along the growth axis or in the plane of the 2DEG, we can assume that $q_z \approx 0$. (In the later case, the period of the SLS can always be made much smaller than the wavelength of the light wave by appropriately tilting the sample i.e. by avoiding the region too close to the C-SLS transition). Then,

$$P(q) = P(q, z = 0) = -ed\hat{z} \Lambda(q) - ie \Theta(q),$$

(21)
where
\[
\Lambda (\mathbf{q}) = \frac{n_R (\mathbf{q}) - n_L (\mathbf{q})}{2},
\]
\[
\Theta (\mathbf{q}) = \nabla_q \left[ n_R (\mathbf{q}) + n_L (\mathbf{q}) \right].
\]

The operator \( \Lambda (\mathbf{q}) \) is related to fluctuations of the relative electronic populations in the two wells while the operator \( \Theta (\mathbf{q}) \) is related to fluctuations in the total density of electrons.

The retarded polarisation response function of the inhomogeneous state is defined, at \( T = 0 \) K by
\[
\tilde{\chi} (\mathbf{q}, \mathbf{q}', \omega) = -\frac{i}{\hbar} \left\langle [\mathbf{p} (\mathbf{q}, t), \mathbf{p} (-\mathbf{q}', t')] \mathbf{R} (t - t') \right\rangle \omega,
\]
where \([...]\omega\) stands for a Fourier transform in time. In terms of the operators \( \Lambda \) and \( \Theta \), we have
\[
\tilde{\chi} (\mathbf{q}, \mathbf{q}', \omega) = e^{2} d^{2} \tilde{\chi}_{\Delta} (\mathbf{q}, \mathbf{q}', \omega) + i e^{2} d \tilde{\chi}_{\Delta} (\mathbf{q}, \mathbf{q}', \omega)
+ i e^{2} \tilde{\chi}_{\Delta} (\mathbf{q}, \mathbf{q}', \omega) \tilde{\mathbf{z}} - e^{2} \tilde{\chi}_{\Delta} (\mathbf{q}, \mathbf{q}', \omega),
\]
and we can finally write for the absorption in the modulated coherent states:
\[
P (\omega) = \frac{e^{2} \omega |E_{0}|^{2}}{2V^{2}} \lim_{q \to 0} \text{Im} \left\langle d^{2} (\xi \cdot \tilde{\mathbf{z}})^{2} \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \right\rangle
\]
\[
= \frac{e^{2} \omega |E_{0}|^{2}}{2V^{2}} \lim_{q \to 0} \text{Im} \left[ \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \right]
+ \frac{e^{2} \omega |E_{0}|^{2}}{2V^{2}} \lim_{q \to 0} \text{Re} \left[ (\xi \cdot \tilde{\mathbf{z}}) \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \right]
+ \frac{e^{2} \omega |E_{0}|^{2}}{2V^{2}} \lim_{q \to 0} \text{Re} \left[ (\xi \cdot \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \cdot \xi \cdot \tilde{\mathbf{z}}) \right].
\]

More specifically, this expression gives the absorption for an incident electromagnetic wave linearly polarised along \( \xi \) and propagating in the direction \( \mathbf{Q} = (\mathbf{q}, q_{z}) \).

Exception made of the first term in Eq. (26), all terms involve the scalar product of the polarization vector with a vector in the plane of the 2D gas. The calculation of the absorption for an arbitrary propagation direction of the incoming wave is complicated because of the need to solve for the response functions \( \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \). To avoid these complications, we will consider an experimental situation where the electromagnetic wave propagates in the plane of the 2D gas with its polarisation vector pointing in the \( z \) direction. This imposes severe restrictions to an absorption experiment because of the small area that is covered by the DQWS! We believe, however, that our conclusions will not change qualitatively if the light wave makes a small angle with respect to the normal to the growth axis. In fact, as we showed in Ref. [10], the pseudospin-charge coupling is extremely small in the SLS so that the neglected term are probably very small. With \( \xi \) close to \( \tilde{\mathbf{z}} \), we have
\[
P (\omega) = \frac{1}{2V^{2}} e^{2} \omega |E_{0}|^{2} \lim_{q \to 0} \text{Im} \left[ \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \right]
\]
and
\[
\Phi (\omega) = \frac{1}{2V^{2}} e^{2} \omega |E_{0}|^{2} \lim_{q \to 0} \text{Im} \left[ \chi_{\Delta} (\mathbf{q}, \mathbf{q}, \omega) \right]
\]
For normal incidence, \( \xi \parallel \tilde{\mathbf{z}} \), the absorption is related to the density correlation function only (more precisely, the gradient with respect to wavevector of the density which is also the polarisation function in the plane of the wells).

In the extreme quantum limit where only one Landau level is occupied, it is convenient to characterize the various ground states by the average value of the operator \( \rho_{i,j} (\mathbf{q}) \) defined by
\[
\rho_{i,j} (\mathbf{q}) = \frac{1}{N_{\phi}} \sum_{X} e^{-i q_{x} X - i q_{z} \ell_{z}^{2} / 2} c_{i,X}^{\dagger} c_{j,X} c_{j,X+q_{z} \ell_{z}^{2}},
\]
The diagonal elements of this operator are related to the density of electrons in the right \( \rho_{R,R} (\mathbf{q}) \) or left well \( \rho_{L,L} (\mathbf{q}) \). The off-diagonal terms, \( \rho_{R,L} (\mathbf{q}) \), \( \rho_{L,R} (\mathbf{q}) \) describe coherence between the two wells. If the separation between the wells is smaller than some critical value, it is possible for these coherence terms to be non-zero even if the tunneling term itself is zero.

In the pseudospin representation, the total density and pseudospin density operators are given by
\[
\rho (\mathbf{q}) = \rho_{RR} (\mathbf{q}) + \rho_{LL} (\mathbf{q})
\]
\[
S_{z} (\mathbf{q}) = \frac{1}{2} \left[ \rho_{RR} (\mathbf{q}) - \rho_{LL} (\mathbf{q}) \right]
\]
Any superposition of the \( |R \rangle \) and \( |L \rangle \) states can be mapped into an eigenstate of the pseudospin operator. In particular, the operator \( \rho_{R,L} \) and \( \rho_{L,R} \) can be mapped in to the pseudospin raising, \( S_{+} \), and lowering, \( S_{-} \), operators
\[
S_{+} (\mathbf{q}) = S_{x} (\mathbf{q}) + i S_{y} (\mathbf{q}) = \rho_{RL} (\mathbf{q})
\]
\[
S_{-} (\mathbf{q}) = S_{x} (\mathbf{q}) - i S_{y} (\mathbf{q}) = \rho_{LR} (\mathbf{q})
\]
In the pseudospin representation, the z-component of the polarisation operator takes the form
\[
p_{z} (\mathbf{q}) \equiv -edN_{\phi} e^{-q_{z}^{2} / 2} S_{z} (\mathbf{q}).
\]
If we keep only the fluctuations in \( S_{z} \), then the absorption is given by
\[
P (\omega) = \frac{N_{\phi}}{2V^{2}} e^{2} \omega |E_{0}|^{2} \lim_{q \to 0} \text{Im} \left[ \chi_{S_{z},S_{z}} (\mathbf{q}, \mathbf{q}, \omega) \right]
\]
where the pseudospin response functions \( \chi_{S_{z},S_{z}} \) is computed from an analytic continuation of the finite temperature, Matsubara two-particle Green’s function
\[
\Gamma_{S_{z},S_{z}} (\mathbf{q}, \Omega_{n}) = -N_{\phi} \left( T \delta S_{z} (\mathbf{q}, \tau) \delta S_{z} (-\mathbf{q}, 0) \right) \Omega_{n}
\]
where \( \delta S_{z} = S_{z} - \langle S_{z} \rangle \) and \( \Omega_{n} \) is a Matsubara bosonic frequency.
IV. COMMENSURATE-INCOMMENSURATE TRANSITION IN A DQWS

In the pseudospin description, the Hartree-Fock energy per particle for the 2DEG in a DQWS subjected to an in-plane magnetic field can be written, at $\nu = 1$, as

$$E_{HF} = \frac{d}{\ell_{\perp}} \langle S_z(0) \rangle^2$$

$$-2\sqrt{R}\text{Re} \left[ \langle S_+(\mathbf{Q}) \rangle \right]$$

$$+ \frac{1}{4} \sum_{\mathbf{q}} \varphi(\mathbf{q}) \langle \rho(-\mathbf{q}) \rangle \langle \rho(\mathbf{q}) \rangle$$

$$+ \sum_{\mathbf{q}} J_z(\mathbf{q}) \langle S_z(-\mathbf{q}) \rangle \langle S_z(\mathbf{q}) \rangle$$

$$+ \sum_{\mathbf{q}} J_{\perp}(\mathbf{q}) \langle S_{\perp}(-\mathbf{q}) \rangle \cdot \langle S_{\perp}(\mathbf{q}) \rangle.$$  \hspace{1cm} (36)

All energies in this equation are in units of $e^2/}\kappa\ell_{\perp}$. The tunneling amplitude is given by

$$\tilde{t} = te^{-d/\ell_{\perp}} \equiv te^{-Q^2}\ell_{\perp}/4.$$ \hspace{1cm} (37)

In the gauge we are using, the parallel component of the magnetic field is responsible for the introduction of a guiding-center-dependent phase factor that depends on the $X$ quantum number only. \textit{i.e.} $e^{-iXd/\ell_{\perp}} \equiv e^{-iQX}$ where $Q$ is defined by

$$Q \equiv \frac{d}{\ell_{||}} \propto B||.$$ \hspace{1cm} (38)

This is why $\langle S_+(\mathbf{Q}) \rangle$ appears in Eq. (36), instead of $\langle S_+(\mathbf{0}) \rangle$. In Eq. (37), we have defined

$$J_z(\mathbf{q}) = V_a(\mathbf{q}) - V_b(\mathbf{q}) - V_c(\mathbf{q}),$$ \hspace{1cm} (39)

and

$$\varphi(\mathbf{q}) = V_a(\mathbf{q}) - V_b(\mathbf{q}) - V_c(\mathbf{q}),$$ \hspace{1cm} (40)

and

$$J_{\perp}(\mathbf{q}) = -V_d(\mathbf{q}).$$ \hspace{1cm} (41)

where $V_a$ and $V_c$ are the Hartree intra and inter-well interactions and $V_b$ and $V_d$ are the exchange (Fock) intra and inter-well interactions. These interactions are defined by

$$V_a(\mathbf{q}) = \left(\frac{1}{q^2} \right) e^{-q^2\ell_{\perp}^2/2},$$ \hspace{1cm} (42)

$$V_b(\mathbf{q}) = \int_0^\infty dq' \ell_{\perp} J_0(qq'\ell_{\perp}^2) e^{-q^2\ell_{\perp}^2/2},$$ \hspace{1cm} (43)

$$V_c(\mathbf{q}) = \left(\frac{1}{q^2} \right) e^{-q^2\ell_{\perp}^2/2} e^{-qd},$$ \hspace{1cm} (44)

$$V_d(\mathbf{q}) = \int_0^\infty dq' \ell_{\perp} J_0(qq'\ell_{\perp}^2) e^{-q^2\ell_{\perp}^2/2} e^{-qd}.$$ \hspace{1cm} (45)

We now give a brief summary of the commensurate-incommensurate transition. In this transition, the DQWS is kept at $\nu = 1$ while the sample is inclined by an angle $\alpha$. The commensurate phase that appears when $\alpha < \alpha_c$ is described by the order parameters

$$\{\langle S_z(\mathbf{q}) \rangle \} = 0, \{\rho(\langle \mathbf{q} \neq 0 \rangle) \} = 0, \rho(\langle 0 \rangle) = 1,$$ \hspace{0.5cm} (46)

$$< S_+(Q,0) > = \frac{1}{2}.$$ \hspace{1cm} (47)

The last equation indicates that

$$\langle S_+(\mathbf{X}) \rangle = < c_{R,X}^+ c_{L,X} > = \frac{1}{2} e^{i\varphi(\mathbf{X})},$$ \hspace{1cm} (48)

with $\varphi(\mathbf{X}) = QX$. Hence, the pseudospin rotates in space according to $\langle (S_x(\mathbf{X}), S_y(\mathbf{X})) \rangle = (\cos(QX), \sin(QX))$. At this point, the analysis is simplified if we define a new phase $\bar{\varphi}(\mathbf{X})$ by

$$\bar{\varphi}(\mathbf{X}) = \varphi(\mathbf{X}) - QX,$$ \hspace{1cm} (49)

so that in the commensurate state, $\bar{\varphi}(\mathbf{X}) \equiv 0$. If we use a tilde to denote the operators in the "rotating frame", we have

$$< \tilde{S}_+(q_x) > \equiv < S_+(q_x + Q) > = \frac{1}{2N^2} \sum_{\mathbf{X}} e^{-iq_x X} e^{i\bar{\varphi}(\mathbf{X})},$$ \hspace{1cm} (50)

and so in the commensurate phase

$$< \tilde{S}_+(q_x) > = < \tilde{S}_x(q_x) > = \frac{1}{2} \delta_{q_x,0}.$$ \hspace{1cm} (51)

When the magnetic field is tilted above a critical angle, the energy to create defects in the form of solitons becomes negative. These solitons corresponds to slips of $2\pi$ in the pseudospin texture. In the rotating frame and in the so-called gradient approximation summarized in Ref. [10], they are kinks given by

$$\bar{\varphi}(\mathbf{X}) = 4 \tan^{-1} \left[ e^{-\sqrt{\pi/2}} \right],$$ \hspace{1cm} (52)

where $\rho_s$ is the spin stiffness of the system (which is basically due to the Fock inter-well interaction given above). Because of the repulsive interaction between solitons, there is, at each value of the parallel magnetic field, an optimal density of solitons. In the ground state, these solitons condense into a crystal with a period $L_s$ so that $e^{i\varphi(\mathbf{X}+L_s)} = e^{i\bar{\varphi}(\mathbf{X})}$. This soliton lattice is described by the set of order parameters

$$\{\langle S_z(\mathbf{q}) \rangle \} = 0,$$ \hspace{1cm} (53)

$$\{\rho(\langle \mathbf{q} \neq 0 \rangle) \} = 0, \rho(\langle 0 \rangle) = 1,$$ \hspace{1cm} (54)

$$< S_+(\mathbf{q},0) > \neq 0,$$ \hspace{1cm} (55)

where

$$Q_s = 2\pi \frac{I_s}{L_s}.$$ \hspace{1cm} (56)
is the wavevector of the soliton lattice and is a function of the parallel component of the magnetic field. The energy of the commensurate state is

$$E_C = -\tilde{t} - \frac{1}{4} \left( \frac{e^2}{\kappa \ell} \right) V_d(Q).$$

(57)

It increases monotonically with the magnetic field. In the limit of strong parallel magnetic fields, the energy of the soliton-lattice state becomes equal to the energy of an incommensurate state described by $\psi(X) = 0$ i.e., $\tilde{\psi}(X) = -QX$ or, equivalently, $< \tilde{\rho}_{RL}(-Q) > = \frac{1}{2} \chi$ (all other parameters being zero). In this limit, $Q_s \rightarrow Q$ so that the solitons are spaced by $2\pi/Q$. The energy of the I state is given by

$$E_I = -\frac{1}{4} \left( \frac{e^2}{\kappa \ell} \right) V_d(0),$$

(58)

and is clearly independent of the parallel magnetic field and of the tunneling term. These energies are plotted in Fig. 1.

V. ABSORPTION IN THE COMMENSURATE AND INCOMMENSURATE STATES

In Ref. [10], we computed the density and pseudospin response functions in the C, I and SL states in the time-dependent Hartree-Fock approximation (TDHFA). In the commensurate state, an analytical solution for the pseudospin response is

$$\chi_{s_x,s_z}(\mathbf{q},\mathbf{q}',\omega) = -\left( \frac{b(\mathbf{q})}{2} \right) \frac{1 + \cos \left( \frac{Qyq_y^2}{2} \right)}{(\omega + i\delta)^2 - \omega_C^2(\mathbf{q})},$$

(59)

where the frequency of the collective mode is given by

$$\omega_C^2(\mathbf{q}) = 4a(\mathbf{q})b(\mathbf{q}),$$

(60)

with

$$a(\mathbf{q}) = t_R + \frac{1}{2} \left[ V_a(\mathbf{q}) - V_b(\mathbf{q}) - V_c(\mathbf{q}) \cos \left( \frac{Qyq_y^2}{2} \right) \right],$$

(61)

$$b(\mathbf{q}) = t_R - \frac{1}{4} \left[ V_d(\mathbf{q} + Q\tilde{x}) + V_d(\mathbf{q} - Q\tilde{x}) \right],$$

(62)

and

$$t_R = \tilde{t} + \frac{1}{2} V_d(Q).$$

(63)

In the limit $\mathbf{q} \rightarrow 0$, the absorption is then proportional to

$$P_C(\omega) \sim \omega |E_0|^2 \lim_{\mathbf{q} \rightarrow 0} \text{Im} \chi_{s_x,s_z}(\mathbf{q},\mathbf{q},\omega) \propto \delta(\omega - \omega_C(q = 0))$$

(64)

and

$$P_I(\omega) = 0 \propto \delta(\omega - \omega_I(q = 0))$$

(65)

where

$$\omega_C(\mathbf{q} = 0) = 2 \sqrt{t \left( \tilde{t} + \frac{d}{2\ell} - \frac{1}{2} \sqrt{\frac{\pi}{2} \frac{1}{2} V_d(Q)} \right)}$$

(66)

gives the gap in the dispersion relation of the pseudospin wave in the C phase. This gap is shifted from its noninteracting value, $2\tilde{t}$ because of many-body exchange and vertex corrections. In the absence of vertex corrections (i.e. in the Hartree-Fock approximation), the SAS gap is renormalized to $2t_R$. The vertex corrections produce a substantial reduction of the Hartree-Fock gap.

From the expression of $P_C(\omega)$, we see that there is no absorption in the absence of tunneling when the pseudospin wave mode given by Eq. (64) is gapless. The absorption is non zero, however, in the absence of the parallel magnetic field, if $t \neq 0$ and for $\xi || \tilde{z}$.

In the I state, we have

$$\chi_{s_x,s_z}(\mathbf{q},\mathbf{q}',\omega) = \frac{[V_d(\mathbf{q}) - V_d(0)]\delta_{\mathbf{q},\mathbf{q}'}(\omega + i\delta)^2 - \omega_I^2(\mathbf{q})},$$

(67)

where

$$\omega_I(\mathbf{q}) = \sqrt{(V_d(0) - V_d(\mathbf{q}))}$$

(68)

$$\times \sqrt{(V_a(\mathbf{q}) - V_b(\mathbf{q}) - V_c(\mathbf{q}) + V_d(\mathbf{q}))}.$$ 

There is no signal in the absorption spectrum in this case: $P_I(\omega) = 0 \propto \delta(\omega - \omega_I(q = 0))$.

VI. ABSORPTION IN THE SOLITON LATTICE STATE

Fig. 1 shows the energy of the C, I and SL states calculated in the Hartree-Fock approximation with the parameters $t / (e^2/\kappa \ell) = 0.01, d/\ell = 1.0$ that we used for all the other results presented in this work. The inset in Fig. 1 shows how the SL wavevector $Q_s$ evolves with the parallel magnetic field from the parallel magnetic field wavevector $Q_c$ at the transition from the C to SL states. ($Q_c \approx 0.62$ for our choice of parameters). The SLS extends from approximately $Q = 0.62$ to $Q = 2.0$ where its energy is nearly indistinguishable from that of the I state.

It is not possible to solve analytically for the response functions in the SLS. They must be obtained numerically. The procedure to obtain these dispersions is explained in Ref. [13]. The SLS sustain many branches of collective excitations that are all periodic along the $x$ direction (the parallel magnetic field is applied along $y$). Figs. 2 show the low-energy part of the dispersion in the first Brillouin zone of the SLS for wavevectors $Q \ell = 0.64, 1.0, 2.0$. When the parallel field is strong (Fig. 2(c)), the branches of collective excitations are exactly given by $\omega_C(q = k+nQ,\tilde{x})$ where $n = 0, \pm 1, \pm 2, ...$.
Energy (units of $e^2/\kappa l$)

0.5 1.0 1.5 2.0
-0.170 -0.169 -0.168 -0.167 -0.166 -0.165 -0.164 -0.163

FIG. 1. Energy of the commensurate, incommensurate and soliton-lattice states as a function of the parallel magnetic field. The inset shows the dependence of the soliton-lattice wavevector on the parallel field.

FIG. 2. Low-energy dispersion branches of the collective modes in the SLS for $t/(e^2/k\ell_\perp) = 0.01$, $d/\ell_\perp = 1.0$ and (a) $Q\ell_\perp = 0.64$, (b) $Q\ell_\perp = 1.0$, (c) $Q\ell_\perp = 2.0$. The full lines in (c) are given by $\omega_I(q = k + nQ_s\hat{x})$ with $n = 0, \pm 1, \pm 2$.

$k$ is a vector restricted to the first Brillouin zone of the SL. In this high-parallel field limit, the pseudospin modulations in the SLS results in a folding of the collective modes of the $t = 0$ ground state inside the first Brillouin zone. As the parallel field is decreased, the coupling between different modes increases and gaps open up in the dispersion. As $Q \rightarrow Q_c$, the dispersion becomes increasingly different from $\omega_I(k + nQ_s\hat{x})$.

When the parallel magnetic field is large, the folding of the modes inside the Brillouin zone results in a set of $\omega_{I_0}(0)$ for $k = 0$ modes that correspond to the frequencies $\{\omega_I(nQ_s\hat{x})\}$. As the parallel field decreases, these frequencies evolves into a series of curves represented in Fig. 3. This set of curves is a distinctive feature of the SLS and would be a good signature of its existence. Unfortunately, the absorption spectrum does not capture all of these excitations. In fact, very few branches survive in $P(\omega)$ as is clear from Fig. 4. The absorption spectrum consists of a broad peak near the transition at $Q_c$ (at a frequency close to the renormalized tunneling energy) that further spreads into a number of well-defined peaks as the parallel magnetic field is increased. Very rapidly, however, only two of the peaks (corresponding to the lowest two branches in Fig. 3) survive. At larger parallel field, only the lowest-energy branch has significant weight in the absorption spectrum. For still larger
fields, when the system asymptotically approaches the incommensurate state, the absorption disappears completely. Below the transition to the SLS, the absorption spectrum consists of only one peak at the renormalized value of the gap energy as we have shown above. Fig. 4 shows that there is a definite signature of the SLS in the absorption spectrum although it is not as pronounced as we might first have expected.

From Fig. 4, we also see that the large peak in the absorption spectrum occurs near the renormalized gap energy given approximately by $\omega_C(0)$ of Eq. (66). A large fraction of this gap comes from many-body corrections. For parameters appropriate to the weak-tunneling sample of Murphy et al. [4], i.e. $n_s = 1.26 \times 10^{11}$ cm$^{-2}$, $d = 211$ Å with a small tunneling energy of $t = 0.4$ K, we have $d/\ell = 1.87$ and $t/(e^2/\kappa \ell) = 0.003$ so that $\hbar \omega_C(0) \approx 0.1 \,(e^2/\kappa \ell)$ just as for the parameters we choose in this paper. An energy of $\hbar \omega = 0.1 \,(e^2/\kappa \ell)$ corresponds to a frequency of approximately $2.6 \times 10^{11}$ Hz (if it were observable in the absorption, the highest-energy branch would correspond to a maximal frequency of approximately $1.6 \times 10^{12}$ Hz). This places the interesting features in the absorption spectrum in the far-infrared region of the electromagnetic spectrum, a difficult region to investigate with available laser sources. It is possible to increase $\omega_C(0)$ by a factor of 5 or more using a DQWS with a stronger tunneling gap or by modifying the other parameters of the sample (provided this choice of parameters does not place the sample outside the region of stability of the coherent state) but observation would still remain difficult.

**VII. CONCLUSION**

We have computed the absorption spectrum of the 2DEG in a DQWS when the sample is gradually tilted with respect to the quantizing magnetic field. At filling factor $\nu = 1$, the commensurate-incommensurate transition driven by the parallel field is reflected in a change of behaviour of the absorption spectrum. The soliton lattice that is the ground state of the 2DEG above the transition has a distinctive set of collective excitations. Some of these excitations can be seen, in principle, in the absorption spectrum in a small region above the C-I transition. Experimental observation of this behaviour, however, is expected to be difficult.

**VIII. ACKNOWLEDGMENTS**

This research was supported by a grant from the Natural Sciences and Engineering Research Council of Canada (NSERC). The author want to thank S. Charlebois and J. Beerens for helpful discussions.
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