From response functions to cross sections in neutrino scattering off the deuteron and trinucleons

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(Dated: March 9, 2022)

Abstract

Response functions, differential cross sections and total cross sections for several (anti)neutrino induced reactions on $^2$H, $^3$He and $^3$H are calculated in momentum space for (anti)neutrino energies up to 160 MeV, using the AV18 nucleon-nucleon potential and a single-nucleon weak current operator. This work is a continuation of our investigations presented in J. Golak et al. [Phys. Rev. C98, 015501 (2018)].

PACS numbers: 23.40.-s, 21.45.-v, 27.10.+h
I. INTRODUCTION

Neutrinos interactions with atomic nuclei are important not only for nuclear physics but also for other domains like particle physics and astrophysics. Nuclei serve as neutrino detectors in experiments focusing on neutrino properties, such as oscillation measurements, as well as in experiments where neutrinos from the interior of stars or from supernova explosions carry important information. That is why a deep understanding of neutrino induced processes on nuclei is necessary both for the interpretation of current experiments and for planning of new undertakings [1, 2]. For example, the use of the deuteron in heavy water detectors in the Sudbury Neutrino Observatory (SNO) for solar neutrinos motivated the theoretical efforts by Nakamura et al. [3, 4], Shen et al. [5], Baroni and Schiavilla [6] to provide accurate predictions for inclusive neutrino scattering off the deuteron.

The results of Ref. [3], a large part of the results given in Ref. [4] and more recent predictions by Shen et al. [5] were obtained within the so-called ”standard nuclear physics approach” [7], using the AV18 nucleon-nucleon (NN) force [8] and augmenting the single-nucleon current with two-nucleon (2N) contributions linked to this potential. The latest calculations in this group, by Baroni and Schiavilla [6], were in contrast fully based on chiral effective field theory (χEFT) input. The results of all these calculations performed in coordinate space were quite similar, which suggested that the theoretical results had a very small uncertainty in the low-energy neutrino regime.

We could confirm these findings by performing independent calculations in momentum space [9]. Namely we investigated two- and three-nucleon reactions with (anti)neutrinos in the framework very close to the one of Ref. [5] but with the single nucleon current operator. For all the studied reactions on the deuteron we presented results for the total cross sections, however, restricting ourselves to the lower (anti)neutrino energies. We found that the few percent deviations between our strictly nonrelativistic results and the predictions presented in Ref. [5] originate from the relativistic kinematics, especially the phase space factor, employed in Ref. [5]. Thus our calculations for the reactions with the deuteron passed a necessary test before embarking on three-nucleon (3N) calculations.

In Ref. [9] we collected important references, which dealt with calculations for neutrino scattering on heavier than $A = 2$ nuclei and related processes like muon capture or the triton beta decay [10–17]. Here we mention only early calculations of the $\bar{\nu}_e + ^3\text{He} \rightarrow e^- + ^3\text{H}$ and $\bar{\nu}_\mu + ^3\text{He} \rightarrow \mu^- + ^3\text{H}$ processes by Mintz et al., who used an elementary particle model [18], dealing with non-breakup reactions, and especially work by Gazit et al., who performed a number of calculations for neutrino induced break-up reactions with the $^3\text{H}$, $^3\text{He}$ and $^4\text{He}$ nuclei [19–21], in which final state interactions were included via the Lorentz integral transform method [22]. Heavier light nuclei, including $^{12}\text{C}$, were investigated with the Green’s function Monte Carlo method [23–25] and using an extended factorization scheme in the spectral function formalism [26].

Our calculations in Ref. [9] for the (anti)neutrino-$^3\text{He}$ and (anti)neutrino-$^3\text{H}$ inelastic scattering were limited only to examples of the essential nuclear response functions - we did not calculate any total cross sections. In the present paper we continue our work for the (anti)neutrino reactions with the trinucleons. Within the same framework as described in Ref. [9] we performed several thousands of Faddeev calculations to gather information necessary to compute the differential (with respect to the lepton arm) and total cross sections. This information was stored in the form of the response functions calculated on a sufficiently dense two dimensional grid defined by the internal nuclear energy and the magnitude of the
three-momentum transfer. The very time consuming calculations of the response functions allowed us later to calculate, essentially in no time at all, other observables of interest. To this end simple two-dimensional interpolations were used. This method was of course first carefully tested for the reactions on the deuteron, where the results of the direct calculations exist and then applied to the trinucleons. The results of our calculations are available upon request.

The paper is organized in the following way. We defined all the elements of our formalism in Ref. [9] so in Sec. II we remind only necessary kinematical definitions and in Sec. III we provide the connection between the response functions and the cross sections. In the following two sections we show selected results for neutrino reactions on the deuteron and the trinucleons. In particular we discuss the properties of the 2N and 3N weak response functions and the resulting differential and total cross sections. Finally, Sec. VI contains concluding remarks and outlook.

II. KINEMATICS

In Ref. [9] we presented our treatment of kinematics for the (anti)neutrino induced processes with the deuteron: \( \bar{\nu}_e + \text{C} \rightarrow e^+ + n + n \), \( \nu_e + \text{C} \rightarrow e^- + p + p \), \( \bar{\nu}_e + \text{C} \rightarrow \bar{\nu}_e + \text{C} \), \( \nu_e + \text{C} \rightarrow \nu_e + p + n \), and \( \nu_e + \text{C} \rightarrow \nu_e + p + n \). Specifically in Sec. II.A of Ref. [9] we discussed the differences stemming from the relativistic and nonrelativistic formulas and introduced the internal kinetic energy of the two-nucleon system \( E_{2N} \), which entered the corresponding two-nucleon dynamical equations:

\[
E_{2N} = E + M_d - 2M - \sqrt{M_e^2 + k'^2} - \frac{(k - k')^2}{4M}.
\]

In Eq. (2.1) \( M_d \) is the deuteron mass, the initial electron (anti)neutrino three-momentum is denoted by \( k \), the final lepton three-momentum by \( k' \) and its mass by \( M_e \). The energy of the initial (anti)neutrino is represented by \( E \) and for the massless antineutrino \( E = |k| \). While for the charge-current (CC) driven reactions \( M \) is equal either the proton mass \( M_p \) or the neutron mass \( M_n \), in the case of the neutral-current (NC) induced processes the “nucleon” mass \( M = \frac{1}{2}(M_p + M_n) \) and \( M_e = 0 \).

Our approach to the corresponding processes of \( ^3\text{He} \) and \( ^3\text{H} \) disintegration is quite analogous. Since we can generally deal with a two-body and three-body breakup process, we outline our kinematical formulas for the case of the CC induced disintegration of \( ^3\text{He} \), where a massive lepton appears in the final state. Our aim is to calculate the maximal final lepton energy for a given lepton scattering angle \( \theta \) and express the internal energy of the 3N system \( E_{3N} \) in terms of \( E, \theta \) and the final positron energy \( E' \equiv \sqrt{M_e^2 + k'^2} \). This information is necessary to evaluate the differential and total cross sections for the (anti)neutrino induced \( ^3\text{He} \) and \( ^3\text{H} \) disintegration processes.

We start with the two-body breakup of \( ^3\text{He} \), \( \bar{\nu}_e + \text{C} \rightarrow e^+ + n + d \), and the exact relativistic form of the energy and momentum conservation in the laboratory frame:

\[
E + M_{\text{He}} = \sqrt{M_e^2 + k'^2} + \sqrt{M_n^2 + p_n^2} + \sqrt{M_d^2 + p_d^2},
\]

\[
k = k' + p_n + p_d,
\]

where \( p_n \) and \( p_d \) stand for the final neutron and deuteron momenta, respectively.
The maximal energy of the emerging lepton under a given scattering angle \( \theta \), where \( \cos \theta = \hat{k} \cdot \hat{k}' \), can be obtained by considering the square of the total four momentum of the nuclear system \( s_{\text{nuc}} \):

\[
s_{\text{nuc}} \equiv \left( \sqrt{M_n^2 + p_n^2} + \sqrt{M_d^2 + p_d^2} \right)^2 - (p_n + p_d)^2. \tag{2.3}
\]

Note that as for the reactions with the deuteron there is no restriction on the scattering angle \( \theta \). The invariant quantity \( s_{\text{nuc}} \) can be evaluated in any reference frame and through the four momentum conservation expressed in terms of the lepton momenta \( k \) and \( k' \). Considering a system, where the total three momentum of the neutron-deuteron system is zero, we find that

\[
s_{\text{nuc}} = \left( E + M^{\text{He}}_3 - E' \right)^2 - (k - k')^2 \geq \left( M_n + M_d \right)^2. \tag{2.4}
\]

The condition for \( (E')^{nd}_{\text{max}} \) in the two-body breakup case reads thus

\[
s_{\text{nuc}} = \left( M_n + M_d \right)^2 \tag{2.5}
\]

and can be obtained analytically, providing the reference result for the approximation discussed below. The corresponding equation for the maximal positron energy in the three-body breakup case, \( (E')^{nnp}_{\text{max}} \), has a similar form, namely

\[
s_{\text{nuc}} = \left( 2M_n + M_p \right)^2. \tag{2.6}
\]

The relativistic expression for the two reactions can be written in the following form

\[
(E')_{\text{max}} = \frac{1}{2} \left( -E^2 E_{\text{ini}} + E_{\text{ini}}^3 + E_{\text{ini}} M_e^2 - E_{\text{ini}} M_{\text{tot}}^2 + E \cos \theta \left( E^4 + E_{\text{ini}}^4 + (M_e^2 - M_{\text{tot}}^2)^2 \right) - 2E^2 \left( E_{\text{ini}}^2 + M_e^2 - 2M_e^2 \cos^2 \theta - M_{\text{tot}}^2 \right) - 2E_{\text{ini}}^2 \left( M_e^2 + M_{\text{tot}}^2 \right) \right)^{1/2}, \tag{2.7}
\]

where \( E_{\text{ini}} \equiv E + M^{\text{He}}_3 \) and \( M_{\text{tot}} \equiv M_n + M_d \) (\( M_{\text{tot}} \equiv 2M_n + M_p \)) for the two-body (three-body) breakup reaction.

Since our dynamical equations are solved within a nonrelativistic framework, employing some further kinematical simplifications, we prepare a consistent set of kinematical conditions. We start again with the energy conservation, using nonrelativistic formulas in the nuclear sector

\[
E + M^{\text{He}}_3 = E' + M_n + M_d + \frac{p_n^2}{2M_n} + \frac{p_d^2}{2M_d}
\approx E' + 3M - |B_2| + \frac{p_n^2}{2M} + \frac{p_d^2}{4M}, \tag{2.8}
\]

where \( M \) is the “nucleon” mass and \( B_2 \) is the deuteron binding energy. We require that the kinetic energy of the nuclear system in its total momentum zero frame \( E_{\text{nd}} \) must be
non-negative. With \( M_{\text{He}} = 3M - |B_{\text{He}}| \), where \( B_{\text{He}} \) is the \(^3\text{He}\) binding energy, one obtains:

\[
E_{\text{nd}} = E - E' - |B_{\text{He}}| + |B_{2\text{H}}| - \frac{(k - k')^2}{6M} \geq 0. \tag{2.9}
\]

Similarly, for the full breakup we find

\[
E_{3N} = E - E' - |B_{\text{He}}| - \frac{(k - k')^2}{6M} \geq 0. \tag{2.10}
\]

Here we have introduced also the three-nucleon internal energy \( E_{3N} \), which will be used together with the magnitude of the three momentum transfer \( Q \equiv |k - k'| \) to label the nuclear response functions. The nonrelativistic values of the maximal positron energies in the final state \( (E')_{\text{max}}^{\text{nd}} \), and \( (E')_{\text{max}}^{\text{mp}} \), can be obtained from the conditions \( E_{\text{nd}} = 0 \) and \( E_{3N} = 0 \), respectively. Each of these two conditions can be cast in the form of a forth degree equation, which we chose to solve numerically. As a starting value in the numerical search one can equally well take two analytically known results: the relativistic expressions from Eq. (2.7) or the nonrelativistic formulas stemming from Eqs. (2.9)–(2.10) with \( M_e = 0 \):

\[
(E')_{\text{max}}^{\text{nd}} \approx \sqrt{9M^2 + 6M(|B_{2\text{H}}| - |B_{\text{He}}| + E - E \cos \theta) - E^2 \sin^2 \theta + E \cos \theta - 3M},
\]

\[
(E')_{\text{max}}^{\text{mp}} \approx \sqrt{9M^2 + 6M(- |B_{\text{He}}| + E - E \cos \theta) - E^2 \sin^2 \theta + E \cos \theta - 3M}. \tag{2.11}
\]

We compared the results for the exact relativistic formulas with their approximate nonrelativistic analogues and found that for up to \( E \leq 160 \text{ MeV} \) the maximal relative difference between these values did not reach 1\% for the whole allowed range of \( \theta \) angles.

It is clear that the kinematics of the other processes of (anti)neutrino induced breakup of the trimucleons can be analyzed in the same way. In particular, for the NC driven reactions, where massless neutrinos appear in the final state, the analytical formulas from Eq. (2.11), derived for the nonrelativistic approximation we employ in the paper, become exact.

### III. THE CROSS SECTIONS AND RESPONSE FUNCTIONS

The formalism of neutrino scattering off nuclei is well established, see for example [27]. For the CC induced processes it stems directly from the Fermi theory but it has to be modified to include additionally the NC based processes. For the lowest order processes the transition matrix element can be written as a contraction of the nuclear part \( N^\lambda \) and the leptonic part \( L_\lambda \), where the latter is expressed in terms of the Dirac spinors and gamma matrices so we can focus on matrix elements

\[
N^\lambda = \langle \Psi_f P_f m_f | j_W^\lambda | \Psi_i P_i m_i \rangle \tag{3.1}
\]

of the nuclear weak charged or neutral current \( j_W^\lambda \) between the initial \( |\Psi_i\rangle \) and final \( |\Psi_f\rangle \) nuclear states, where the total initial (final) nuclear three-momentum is denoted by \( P_i (P_f) \), \( m_i \) is the initial nucleus spin projection and \( m_f \) is the set of spin projections in the final state. As explained in Ref. [9] it is advantageous to assume a system of coordinates, where
\[ Q \equiv k - k' \parallel \hat{z} \quad \text{and} \quad \hat{y} = \frac{k \times k'}{|k \times k'|}, \quad \text{so} \]
\[ k_x' = k_x = |k||k'| \sin \theta / |Q|, \]
\[ k_y' = k_y = 0, \]
\[ k_z = |k| (|k| - |k'| \cos \theta) / |Q|, \]
\[ k_x' = |k'| (-|k'| + |k| \cos \theta) / |Q|, \]
\[ |Q| = \sqrt{k'^2 + k'^2 - 2 |k||k'| \cos \theta}. \quad (3.2) \]

Further, the essential part of the square of the transition matrix element \(|L_{\lambda N^\lambda}|^2\) can be written as
\[ |L_{\lambda N^\lambda}|^2 = V_{00} |N^0|^2 + V_{MM} |N_{-1}|^2 + V_{PP} |N_1|^2 + V_{ZZ} |N_z|^2 + V_{Z0} \text{Re}(2 N_z (N^0)^*) \], \quad (3.3) \]
where the \(V_{ij}\) functions arise from the leptonic arm and the spherical components are used for \(N^\lambda\) \[9\]. By “essential part” we mean the part, which contributes to the cross sections in the case, where information about the nuclear sector is integrated over and only the final lepton momentum is known. We restrict ourselves only to such cases in the present work.

For the neutrino induced reactions we get
\[ V_{00} = 8 (k' \cdot k + E E') \]
\[ V_{MM} = 8 (E + k_z) (E' - k_z') \]
\[ V_{PP} = 8 (E - k_z) (E' + k_z') \]
\[ V_{ZZ} = 8 (-k' \cdot k + E E' + 2k_z k_z') \]
\[ V_{Z0} = -8 (E k_z' + E' k_z) \]. \quad (3.4) \]

and for reactions with antineutrinos the corresponding \(\bar{V}_{ij}\) functions are easily obtained from the previous set:
\[ \bar{V}_{00} = V_{00}, \]
\[ \bar{V}_{MM} = V_{PP}, \]
\[ \bar{V}_{PP} = V_{MM}, \]
\[ \bar{V}_{ZZ} = V_{ZZ}, \]
\[ \bar{V}_{Z0} = V_{Z0}. \] \quad (3.5) \]

The standard steps, which take also into account the normalization of the Dirac spinors and nuclear states, lead to the final form of the cross section. For the CC neutrino induced reactions we obtain
\[ \frac{d^3\sigma}{dE' d\Omega'} = \frac{G_F^2 \cos^2 \theta_C}{(2\pi)^2} F(Z, E') \left| \frac{k'}{8E} \right|^2 \left( V_{00} R_{00,CC} + V_{MM} R_{MM,CC} + V_{PP} R_{PP,CC} + V_{ZZ} R_{ZZ,CC} + V_{Z0} R_{Z0,CC} \right), \quad (3.6) \]
where the value of the Fermi constant, \(G_F = 1.1803 \times 10^{-5} \text{GeV}^{-2}\), and \(\cos \theta_C = 0.97425\) are taken from Ref. \[3\]. The Fermi function \(F(Z, E')\) \[23\] is introduced to account for the distortion of the final lepton wave function by its Coulomb interaction with more than one proton in the final state and is not needed otherwise. For the NC driven processes the
Fermi function $F(Z, E')$ and $\cos^2 \theta_C$ are dropped in Eq. (3.6). Our predictions for all the CC induced reactions are valid only for the electron flavor but for the NC reactions results are the same for all the three flavors.

The essential dynamical ingredients in the inclusive cross section are the nuclear response functions originating from the integration of various products of the nuclear matrix elements over the whole nuclear phase space available for the fixed final lepton momentum:

$$R_{AB,CC} = \sum_{m_i, m_f} \int df \delta(E_{3N} - E_f) \langle \Psi_f \mid j_A^{3N} \mid \Psi_i \rangle \left( \langle \Psi_f \mid j_B^{3N} \mid \Psi_i \rangle \right)^* ,$$

(3.7)

where $AB=00, MM, PP, ZZ$ and $Z0$, $m_i$ and $m_f$ represent the whole sets of the initial and final spin magnetic quantum numbers, respectively, while the $df$ integral denotes the sum and the integration over all final 3N states with the fixed energy $E_{3N}$. The 3N bound state wave functions are calculated using the method described in Ref. [29]. The direct integration would allow one to evaluate contributions from any part of the phase space. In particular, for the reactions on $^3$He and $^3$H it would be possible to obtain contributions from the two- and three-body breakup channels. However, the numerical cost of such calculations needed for the total cross section, which is the main objective of the present paper is very high. Thus we decided to compute the 3N response functions in a much more economical way, using closure and employing the special Faddeev scheme [30, 31]. In Ref. [9] we compared results based on these two quite different approaches and obtained a very good agreement. The results for all the 3N response functions presented in this paper are obtained with the second, cheaper method. This closure-based scheme could be formulated also for the 2N system but in that case the integration over the phase space is well under control and perfectly practical as will be explained in the following. Thus all the 2N response functions are obtained as in Ref. [9], by direct integrations.

It is very important to realize that while the differential cross section $\frac{d^3 \sigma}{dE' d\Omega'}$ depends on three kinematical variables, $E, \theta$ and $E'$, the response functions are defined in terms of the internal nuclear energy ($E_{2N}$ or $E_{3N}$) and the magnitude of the three momentum transfer ($Q$). We will use this feature to facilitate the calculations.

Before we discuss our results, we remind the reader of the most important features of our momentum space framework. We follow the path paved by Refs. [3–6] whose authors investigated inclusive neutrino scattering on the deuteron with configuration space methods. Those very advanced investigations were performed with traditional and chiral NN potentials and included weak nuclear current operators with a one-body part and two-body contributions, adjusted to the NN force. Here we continue our work from Ref. [9] with the standard AV18 NN potential [8] and the single nucleon current operator defined in Ref. [15]. This form and parametrization of the single nucleon current was previously employed for example in Refs. [11, 15]. Since we restrict ourselves to the low (anti)neutrino energies, $E \leq 160$ MeV, where our nonrelativistic approach is fully justified, we expect, based on the results of Ref. [5], that 2N contributions in the current operators would lead to effects smaller than $2 - 4\%$.

IV. RESULTS FOR (ANTI)NEUTRINO SCATTERING ON $^2$H

Calculating the total cross section for (anti)neutrino induced breakup reactions for many initial (anti)neutrino energies, starting for each energy anew, could lead in fact to a waste
FIG. 1. The rectilinear grid of \((E_{2N}, Q)\) points used to store the response functions for the \(\nu_e + ^2\text{H} \rightarrow e^- + p + p\) reaction (tiny dots) and the actual \((E_{2N}, Q)\) points used to evaluate the total cross section (circles) in the triangle-like area for the initial neutrino energy \(E = 50\) MeV (a), 100 MeV (b) and 160 MeV (c). Lines, which separate the physical region for a given \(E\) are obtained from approximate Eqs. (4.1)-(4.3). For \(E = 160\) MeV the border lines for the two smaller energies are also shown.

of computer resources. As we show in Fig. 1 for the \(\nu_e + ^2\text{H} \rightarrow e^- + p + p\) reaction, while calculating the total cross section for increasing initial antineutrino energies, the dynamical information in the form of response functions is taken from a part of the \((E_{2N}, Q)\) domain, which necessarily overlaps with the region corresponding to lower energies. This is also true for the reactions with the trinucleons. That is why calculating response functions on a sufficiently dense grid and using stored values for interpolations to integral \((E_{2N}, Q)\)-points appears to be advantageous. The same stored response functions can be used not only to generate the total cross sections but also to calculate the intermediate differential cross sections \(d^3\sigma/(dE'd\Omega')\) and \(d\sigma/d\theta\). Yet another advantage of storing response functions becomes clear for the NC induced processes, where the same response functions are used for the neutrino and antineutrino induced reactions.

Before embarking on 3N calculations, this approach was tested in the 2N system, where results of the direct calculations of the total cross sections as defined for example in Eq. (2.22) of Ref. [9] and predictions based on the response functions’ interpolations could be easily compared. The various sets of response functions should be prepared with great care, taking into account the character of their dependence on \(E_{2N}\) and \(Q\). We decided to use simple rectilinear grids, which forced us to use many points on the whole grid, even if a sharp maximum was strongly localized, leading however to very accurate predictions. This feature of our calculations is clearly visible in Fig. 1.

In Figs. 2-4 we show the three sets of the response functions obtained for the \(\bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n\), \(\nu_e + ^2\text{H} \rightarrow e^- + p + p\) and \(\nu_e(\bar{\nu}_e) + ^2\text{H} \rightarrow \nu_e(\bar{\nu}_e) + p + n\) reactions. We use different \(E_{2N}\) and \(Q\)-ranges in the figures to display the particular features of the response functions. Note that the figures are not drawn with all calculated points, so the actual grids for two dimensional interpolations are in fact much denser. All the response functions have a maximum in the vicinity of the \((0, 0)\) point but their shapes and heights are very different. The response functions \(R_{00}\) and \(R_{Z0}\) stemming at least partly from the \(N^0\) nuclear matrix elements are, for all the three reactions, dominated by the response functions \(R_{MM}, R_{PP}\) and \(R_{ZZ}\), which are by two orders of magnitude more pronounced. Note that for each \(E_{2N} > 0\) there is an interval \([0, Q_{\min}]\) which cannot be physically realized for any initial neutrino energy and for which the values of the response functions are set to zero. The exact expression for \(Q_{\min}\) is quite complicated in the case when the final lepton is massive so we give here only approximate expressions, assuming that the electron (or positron) mass...
can be neglected:

\[ Q_{\text{min}} = 2 \sqrt{M \left( -2 \sqrt{-M(E_{2N} - M_d + M)} - E_{2N} + M_d \right)} . \]  

(4.1)

The corresponding maximal value of \( Q \) for given \( E_{2N} \) depends also on \( E \):

\[ Q_{\text{max}} = 2 \sqrt{M \left( -2 \sqrt{M(2E - E_{2N} + M_d - M)} + 2E - E_{2N} + M_d \right)} \]  

(4.2)

and the maximal value of \( E_{2N} \) for given \( E \) is obtained from the condition \( Q_{\text{min}} = Q_{\text{max}} \) and reads

\[ (E_{2N})_{\text{max}} = \frac{-E^2 + 4EM + 4M_dM - 8M^2}{4M} . \]  

(4.3)

From the response functions it is straightforward to compute the differential and total cross sections. To this end one interpolates the response functions in two dimensions over the \((E_{2N}, Q)\) grid points to the particular \((\tilde{E}_{2N}, \tilde{Q})\) value resulting from the \((E, \theta, E')\) set. 

We used three different methods to interpolate the response functions. While the two first methods employed consecutive cubic splines (from Ref. [32] or from Ref. [33]) interpolations, first along the \( Q \) direction and then along the \( E_{2N} \) direction, the third method was a straightforward bilinear interpolation. In this way we could control the quality of interpolations, since we required that results for all considered observables, obtained by the three methods, did not deviate from the average by more than 1%. Additional points were added to the grid, when that criterion was not met. Since the 2N calculations are relatively easy, we could consider grids which contained from 7200 to 17200 points. This procedure was especially important for the 3N case, where we did not calculate cross sections directly but fully relied on response functions’ interpolations. In the following we show results based on the interpolation scheme from Ref. [32].

The triple differential cross section, \( d^3\sigma / (dE' d\Omega') \), for a fixed lepton scattering angle is a function of the final lepton energy \( E' \). In Fig. [5] we show examples of such cross sections for the initial (anti)neutrino energy \( E = 100 \) MeV and for three different lepton scattering angles \( \theta = 27.5^\circ, \theta = 90^\circ \) and \( \theta = 152.5^\circ \). These results can be compared with middle panels of Figs. 6 and 9 in Ref. [5] and show that the cross sections rise very rapidly with the final (anti)lepton energy, changing in the allowed energy range by several orders of magnitude.

For all the four studied reactions we show also in Fig. [6] the angular distributions of the cross sections, \( d\sigma / d\theta \), which are given as

\[ \frac{d\sigma}{d\theta} = 2\pi \sin \theta \int_{(E')_{\text{min}}}^{(E')_{\text{max}}} dE' \frac{d^3\sigma}{dE' d\Omega'} ; \]  

(4.4)

where \( (E')_{\text{min}} = M_e (0) \) for the CC (NC) induced reactions and the factor \( 2\pi \) arises from the integration over the azimuthal angle \( \phi \). Clearly, the angular distributions rise with the incident energy. For the smallest \( E = 50 \) MeV they are all almost symmetric with respect to \( \theta = 90^\circ \). This symmetry is roughly preserved for the higher energies \( E = 100 \) and 150 MeV in the case of the two neutrino induced reactions but for the two other reactions the angular distributions become asymmetric and their maxima are shifted towards forward angles. This behaviour is most evident for the \( \bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n \) process.
FIG. 2. The nuclear inclusive response functions $R_{00,CC}$ (a), $R_{MM,CC}$ (b), $R_{PP,CC}$ (c), $R_{ZZ,CC}$ (d) and $R_{Z0,CC}$ (e) for the $\bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n$ reaction as a function of the internal 2N energy $E_{2N}$ and the magnitude of the three-momentum transfer $Q$. The results are obtained with the AV18 NN potential and the single nucleon CC operator, which contains the relativistic corrections, employing the nonrelativistic kinematics.

By the final integration over the scattering angle $\theta$ we arrive at the total cross section

$$\sigma_{tot} = \pi \int_0^\pi d\theta \frac{d\sigma}{d\theta}. \quad (4.5)$$

These important observables were presented in Refs. [3–6] and we remind the reader that our momentum space based results [9] agree very well with the predictions presented in [5]. Despite the distinct treatment of kinematics the differences for none of the reactions for
$E \leq 150$ MeV exceed 2% for the single-nucleon current calculations and 6% for calculations including additionally two-nucleon currents. In Fig. 7 we display a comparison of directly obtained results for the total cross sections with the predictions based on the response functions’ interpolations. The agreement is very good for all the four reactions and for all considered (anti)neutrino energies, justifying our “economical” approach to calculations of the cross sections.

V. RESULTS FOR (ANTI)NEUTRINO SCATTERING ON $^3$HE AND $^3$H

We follow the same path for the trimucleons as for the calculations with the deuteron target. That means that also in this case we calculate the response functions on a grid of $(E_{3N}, Q)$ points. As already mentioned, one can evaluate the response functions by
introducing explicit integrations over the available phase space, in particular differentiating between the two- and three-body reaction channels. It is also possible to evaluate the response functions without any resort to explicit final-state kinematics [9, 30, 31]. These two approaches were used and compared successfully in Ref. [9] for a small number of \((E_{3N}, Q)\) points. Since we wanted to produce full grids of response functions, we decided to employ the second scheme. Each 3N grid comprised roughly 2000 points. Even if some points on the rectilinear grids lied in the nonphysical region, where no calculations are necessary and where the response functions are just zero, the actual number of the computations was high. In order to efficiently deal with so many calculations, we prepared a special computational framework to distribute the calculations among several desktop computers. Our calculations were performed with the AV18 NN potential, neglecting the 3N force, and with the same
FIG. 5. The triple differential cross section \( \frac{d^3\sigma}{dE'd\Omega'} \) for the \( \bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n \) (dashed line), \( \nu_e + ^2\text{H} \rightarrow e^- + p + p \) (dash-dotted line), \( \bar{\nu}_e + ^2\text{H} \rightarrow \bar{\nu}_e + p + n \) (solid line) and \( \nu_e + ^2\text{H} \rightarrow \nu_e + p + n \) (dotted line) reactions for the initial (anti)neutrino energy \( E = 100 \text{ MeV} \) at three laboratory scattering angles: \( \theta = 27.5^\circ \) (a), \( \theta = 90^\circ \) (b) and \( \theta = 152.5^\circ \) (c) as a function of the final lepton energy \( E' \). The results are obtained with the AV18 potential and with the single nucleon current, employing the nonrelativistic kinematics.

FIG. 6. The differential cross section \( \frac{d\sigma}{d\theta} \) for the \( \bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n \) (a), \( \nu_e + ^2\text{H} \rightarrow e^- + p + p \) (b), \( \bar{\nu}_e + ^2\text{H} \rightarrow \bar{\nu}_e + p + n \) (c) and \( \nu_e + ^2\text{H} \rightarrow \nu_e + p + n \) (d) reactions as a function of the laboratory scattering angle \( \theta \) for initial (anti)neutrino energy \( E = 50 \text{ MeV} \) (dotted line), \( 100 \text{ MeV} \) (dashed line) and \( 150 \text{ MeV} \) (solid line). The results are obtained with the AV18 potential and with the single nucleon current, employing the nonrelativistic kinematics.
The total cross section $\sigma_{\text{tot}}$ for the $\bar{\nu}_e + ^2\text{H} \rightarrow e^+ + n + n$ (dashed line), $\nu_e + ^2\text{H} \rightarrow e^- + p + p$ (dash-dotted line), $\bar{\nu}_e + ^2\text{H} \rightarrow \bar{\nu}_e + p + n$ (solid line) and $\nu_e + ^2\text{H} \rightarrow \nu_e + p + n$ (dotted line) reactions as a function of the initial (anti)neutrino energy $E$ calculated directly (symbols) or from the interpolated response functions (lines) as explained in the text. The results are obtained with the AV18 potential and with the single nucleon current, employing the nonrelativistic kinematics. The inset focuses on the results for $E \leq 40$ MeV.

We start presenting our results with the 3N weak response functions, shown in Figs. 8–11. It is clear that the 3N response functions are much broader and extend towards higher $E_{3N}$ and $Q$ values than the corresponding 2N observables, which are very localized. The differences between various response functions are not so strong as in the 2N case. The response functions for the CC electron antineutrino disintegration of $^3\text{He}$ and $^3\text{H}$ (Figs. 8–9) have similar shapes and roughly scale according to the number of protons in a nucleus. This seems to reflect the fact that the process described by the single nucleon current involves only protons.

In the case of the NC response functions the proton and neutron contributions to the single nucleon NC operator are comparable. This leads to similar results for the $^3\text{He}$ and $^3\text{H}$
FIG. 8. The total inclusive CC response functions $R_{00,CC}$ (a), $R_{MM,CC}$ (b), $R_{PP,CC}$ (c), $R_{ZZ,CC}$ (d) and $R_{Z0,CC}$ (e) for the CC electron antineutrino disintegration of $^3$He as a function of the internal 3N energy $E_{3N}$ and the magnitude of the three-momentum transfer $Q$. The results are obtained with the AV18 NN potential and the single nucleon CC operator, which contains the relativistic corrections.

NC response functions displayed in Figs. [10][11].

As in the 2N cases, the response functions are the key ingredients of the cross sections, where only total or partial information about the final lepton is retained. The triple differential cross section $d^3\sigma/(dE'd\Omega')$ for the CC electron antineutrino disintegration, NC electron antineutrino disintegration, and NC electron neutrino disintegration of $^3$He just for one initial (anti)neutrino energy $E= 100$ MeV at three scattering angles $\theta = 27.5^\circ$, $\theta = 90^\circ$ and $\theta = 152.5^\circ$ are displayed in Fig. [12] as a function of the final lepton energy $E'$. All the cross sections soar with increasing $E'$ and they are pulled down only in the vicinity of
We give only sample results but it is clear that similar calculations can be used to plan experimental investigations of the NC and CC (anti)neutrino induced reactions.
As a last step before discussing the total cross section we show in Fig. 14 the six angular distributions of the cross sections, which can be now easily obtained from the response functions. We do it again for the same incoming (anti)neutrino energies as in Fig. 6 for the reactions on the deuteron. The curves are less symmetric compared to the predictions from Fig. 6 which is clearly visible for the two higher $E$ values. There is a clear similarity between the results shown in parts (a) and (b) for the antineutrino induced CC processes on $^3$He and $^3$H, which can be traced back to the scaling properties of the corresponding response functions. For the highest energy $E= 150$ MeV the maxima for all the cross sections with antineutrinos are shifted towards forward angles; only for the neutrino induced NC processes displayed in panels (d) and (f) the maxima are reached for $\theta > 90^\circ$.

Finally we arrive at the most important results - the total cross sections for the studied...
(anti)neutrino reactions with the trimucleons. They can be found in Fig. [15] for $^3$He and in Fig. [16] for $^3$H. In the $^3$He case the cross section for CC electron antineutrino disintegration takes the highest values in the whole investigated energy range. It is followed by the cross section for NC electron neutrino disintegration. The cross section for NC electron antineutrino disintegration is approximately two times smaller than the cross section for the corresponding CC driven process.

The picture is different for $^3$H, where the cross section for NC electron neutrino disintegration is roughly two times larger than the predictions for the two antineutrino induced reactions, which are close to each other for all the initial (anti)neutrino energies.
FIG. 12. The triple differential cross section $d^3\sigma/(dE'd\Omega')$ for the CC electron antineutrino disintegration of $^3$He (dashed line), NC electron antineutrino disintegration of $^3$He (solid line) and NC electron neutrino disintegration of $^3$He (dotted line) for the initial (anti)neutrino energy $E = 100$ MeV at three laboratory scattering angles: $\theta = 27.5^\circ$ (a), $\theta = 90^\circ$ (b) and $\theta = 152.5^\circ$ (c) as a function of the final lepton energy $E'$. The results are obtained with the AV18 potential and with the single nucleon current, employing the nonrelativistic kinematics.

FIG. 13. The same as in Fig 12 for the reactions on $^3$H.

VI. SUMMARY

We extended our studies of (anti)neutrino scattering off the deuteron and trinucleons from Ref. [9], where we presented our momentum space framework and obtained predictions for the total cross sections only for the reactions on the deuteron. For the reactions on the trinucleons we could perform in Ref. [9] only feasibility studies, employing two different methods to calculate the essential response functions. In the present paper we provide information about the cross sections for CC electron antineutrino disintegration, NC electron antineutrino disintegration and NC electron neutrino disintegration of $^3$He and $^3$H.

The material presented in this paper is based on tens of thousands of 2N and several thousands of 3N scattering calculations, which were necessary to fill dense two dimensional grids, from which essentially in no time other observables: three fold differential cross sections, angular distributions of the cross sections and, most importantly, the total cross sections can be obtained. The results of our calculations in the form of the tabulated response functions are available to the interested reader. This whole procedure was first carefully tested for the reactions on the deuteron, where the observables had been calculated directly and where accurate predictions obtained in coordinate space were available.

Our calculations leave room for improvement: they have been performed with the single nucleon current operator and without a 3N force, neglecting additionally the Coulomb force between two final protons for one of the studied reactions. Nevertheless our predictions are obtained with the fully realistic AV18 nucleon-nucleon potential [8] and are restricted to the
FIG. 14. The same as in Fig. 6 for the inclusive CC electron antineutrino disintegration of $^3$He (a), CC electron antineutrino disintegration of $^3$H (b), NC electron antineutrino disintegration of $^3$He (c), NC electron neutrino disintegration of $^3$He (d), NC electron antineutrino disintegration of $^3$H (e) and NC electron neutrino disintegration of $^3$H (f).

(anti)neutrino energy region, where two-nucleon current and three-nucleon force effects are not expected to be very important and should not exceed 10%. Thus we provide important information about (anti)neutrino interactions with very light nuclei.

A consistent framework for the calculations of neutrino induced processes on $^2$H, $^3$He, $^3$H and other light nuclei is still a challenge, despite the recent progress in this field. There are many models of the nuclear interactions and weak current operators linked to these forces, but full compatibility has not been achieved yet. We hope that the work on the regularization of the 2N and 3N chiral potentials as well as consistent electroweak current operators will be completed in the near future. This will allow us to repeat the calculations
of the response functions and related observables within a better dynamical framework. We believe, however, that the results presented in this paper constitute an important step towards a consistent framework for the calculations of several neutrino induced processes on $^2$H, $^3$He, $^3$H and other light nuclei.

**ACKNOWLEDGMENTS**

This work is a part of the LENPIC project and was supported by the Polish National Science Centre under Grants No. 2016/22/M/ST2/00173 and 2016/21/D/ST2/01120. The numerical calculations were partially performed on the supercomputer cluster of the JSC, Jülich, Germany.

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FIG. 16. The same as in Fig. [15] for three inclusive (anti)neutrino reactions with $^3$H.
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