Evaluating Defaults

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Abstract

We seek to find normative criteria of adequacy for nonmonotonic logic similar to the criterion of validity for deductive logic. Rather than stipulating that the conclusion of an inference be true in all models in which the premises are true, we require that the conclusion of a nonmonotonic inference be true in “almost all” models of a certain sort in which the premises are true. This “certain sort” specification picks out the models that are relevant to the inference, taking into account factors such as specificity and vagueness, and previous inferences. The frequencies characterizing the relevant models reflect known frequencies in our actual world. The criteria of adequacy for a default inference can be extended by thresholding to criteria of adequacy for an extension. We show that this avoids the implausibilities that might otherwise result from the chaining of default inferences. The model proportions, when construed in terms of frequencies, provide a verifiable grounding of default rules, and can become the basis for generating default rules from statistics.

Keywords: probability, frequency, default logic

1 Introduction

Non-monotonic reasoning, for example default logic [Reiter, 1980], models the intuitive process of making non-deductive inferences in the face of certain supportive but not conclusive evidence. Given a default theory \( \Delta = (D, F) \), we can obtain its extensions by following a prescribed set of steps. However, on what grounds do we employ a particular default rule? Some writers would regard this as an inappropriate question, since they take as their goal the representation of human inference. To this end defaults represent rules that we take to be intuitively appropriate. But then when we apply these rules, we may be led to counterintuitive results [Reiter and Criscuolo, 1981, Lukaszewicz, 1988, Poole, 1989]. The underlying principle seems to be circular: the original default rules are “intuitively good” at first glance, but when we discover that they do not give rise to the desired results, we tweak the rules until they give us those results. It seems that we have to know what results we want first before constructing the default theory, rather than having the default theory tell us what conclusions are warranted. This is precisely the reason why we need an independent measure of validity for default rules and default extensions. We think of nonmonotonic logic as sharing the normative character of other logics. From this point of view default rules require some defense. We will concentrate on default logic here, though much of what we have to say will apply to other nonmonotonic approaches as well. Much of the work on nonmonotonic logic has concerned the syntactic manipulation of the nonmonotonic rules, rather than their basic justification.

1.1 Selective Preference

For a default rule \( d = \alpha ; \beta_1, \ldots, \beta_n \rightarrow \gamma \), \( \alpha \) is the prerequisite, \( \beta_1, \ldots, \beta_n \) are the justifications, and \( \gamma \) is the consequence of \( d \). Loosely speaking, the rule conveys the idea that if \( \alpha \) is provable, and \( \neg \beta_1, \ldots, \neg \beta_n \) are each not provable, then by default we conclude that \( \gamma \) is true. A default theory is an ordered pair \( (D, F) \), where \( D \) is a set of default rules and \( F \) is a set of “facts”. A theory extended from \( F \) by applying the default rules in \( D \) is known as an extension of the default theory.
Consider the following canonical example.

**Example 1** We have a default theory $\Delta = \langle D, F \rangle$, where

\[
D = \{ R(x) \rightarrow T(x), S(x) \rightarrow \neg T(x) \}, \\
F = \{ R(a), S(a) \}. 
\]

We get two extensions, one containing $T(a)$ and the other containing $\neg T(a)$. If we take “$R(x)$” to mean that $x$ is a bird, “$S(x)$” to mean that $x$ is a penguin, and “$T(x)$” to mean that $x$ flies, then we would like to reject the extension containing “$T(a)$” (a flies) in favor of the extension containing “$\neg T(a)$” (a does not fly). However, if we take “$S(x)$” to mean that $x$ is an animal instead and keep “$R(x)$” and “$T(x)$” the same, we would want to reverse our preference. Now the extension containing “$T(a)$” (a flies) seems better.

Note that each of the default rules involved in the example above is intuitively appealing when viewed by itself against our background knowledge: birds fly; penguins do not fly; and animals in general do not fly either. Moreover, both instantiations (penguins and animals) are syntactically identical. Thus, we cannot base our decision to prefer one default rule over the other by simply looking at their syntactic structures.

It is the interaction between the default rules and evidence that gives rise to the selective preference above. We have the evidence that “$a$ is a bird”. If in addition we also have “$a$ is a penguin”, we prefer the penguin rule. If instead we have “$a$ is an animal”, we prefer the bird rule.

There are several approaches to circumventing this conceptual difficulty. The first is to revise the default theory so that the desired result is achieved [Reiter and Criscuolo, 1983]. We can amend the default rules by adding the exceptions as justifications, for example $\frac{B(x) \rightarrow P(x)}{P(x)}$ and $\frac{A(x) \rightarrow \neg P(x)}{\neg P(x)}$. With this approach we have to constantly revise the default rules to take into account additional exceptions. We have little guidance in constructing the list of justifications except that the resulting default rule has to produce the “right” answer in the given situation.

Another approach is to establish some priority structure over the set of defaults. For example, we can refer to a specificity or inheritance hierarchy to determine which default rule should be used in case of a conflict [Touretzky, 1984; Horyt et al., 1987]. The penguin rule is more specific than the bird rule, when both are applicable, and therefore we use the penguin rule and not the bird rule. However, conflicting rules do not always fit into neat hierarchies (for example, adults are employed, students are not, how about adult students? [Reiter and Criscuolo, 1983]).

It is not obvious how we can extend the hierarchical structure without resorting to explicitly enumerating the priority relations between the default rules [Brewka, 1989; Brewka, 1994].

The third approach is to appeal to probabilistic analysis. Defaults are interpreted as representing properties of conditional probabilities. For example, the conditional probability of $a$ being able to fly given that $a$ is a bird is “high” [Pearl, 1988; Pearl, 1990] or increases from the prior probability [Neufeld et al., 1990], while the conditional probability of $a$ being able to fly given that $a$ is a penguin is “low” or decreases. This approach provides a probabilistic semantics for default rules, but in a way which does not represent the fact that the conclusions are accepted. The default conclusion is “Tweety flies,” not “Probably Tweety flies.” This is in contrast to the spirit of nonmonotonic reasoning: default conclusions should be accepted as new facts, and we should be able to chain default rules and build upon the conclusions of previous default applications to obtain further conclusions.

### 1.2 Justifying Nonmonotonic Inference

The justification of beliefs is a long standing issue in epistemology. There is not much that is problematic about the justification of beliefs obtained by deductive inference (though there are plenty of problems that surround deduction — see Kyburg, 1958; Haack, 1976; Dummett, 1978, not to mention the voluminous literature on paraconsistent logic [Priest, 1984]). The reason is that we can show that the ordinary rules of deduction lead from premises to conclusions that are true in every model in which the premises are true. This is exactly what is not true of ampliative inference, and it is what has led some writers (e.g., Morgan, 1998) to deny that there is any such thing as a nonmonotonic logic. This has been disputed in Kyburg, 2001.

But other kinds of justifications of beliefs have been proposed. Isaac Levi [Levi, 1967; Levi, 1980; Levi, 1999] has argued for many years that the way to understand ampliative (inductive, nonmonotonic) argument is in terms of decision theory: we choose (decide) to accept a hypothesis in a given context provided that the expected epistemic utility of doing so in that context is greater than the expected utility of any other epistemic act, such as suspending belief totally, or accepting a stronger hypothesis.
Levi’s approach employs a rich and detailed structure for acceptance, and allows drawing many important distinctions. This structure requires three things that make it less than perfect as a vehicle for ordinary nonmonotonic inferential systems. First, in keeping with a long tradition in pragmatism Dewey, 1938; Peirce, 1903; James, 1948 the context of inquiry must be tied to a specific problem: We need the answer to a question. Second, the epistemic expectation of an answer is the expected value of the information contained in that answer. Thus we need to presuppose an information measure on the language of our inquiry [Levi, 1996, p. 169]. Third, we need to have available a credal or inductive probability, based on a measure (or convex set of measures) on the sentences of the language, in terms of which a conditional probability (or convex set of conditional probabilities) can be defined [Levi, 1985, p. 52].

It is our belief both that in some contexts in which we might wish to use nonmonotonic mechanisms, this overhead is unnecessary, and perhaps itself difficult to justify, and that we would like to be able to explicate the justification of inference in a less context dependent way.

Another approach that has attracted considerable attention in the philosophical community in recent years is that of “reliabilism” whose best known exponent is Alvin Goldman [Goldman, 1979]. According to this view, what justifies a belief is the fact that it is obtained by a “reliable cognitive process...” [Goldman, 1979, p. 20] Of course there are a number of additional hedges to the view that are required for philosophical accuracy, and even with those hedges there remains a certain vagueness in the view. These details need not detain us, since we are seeking inspiration rather than philosophical precision.

What does “reliable” mean? We will construe reliability in terms of frequency or propensity to yield truth when applied. Specifically, we will say that the belief $\phi$ is nonmonotonically justified by a default rule if the rule would frequently lead to truth and rarely to error, given what we know — given our background knowledge.

A deductive argument is justified (valid) if its conclusion is true in every model of its premises. We will attempt to provide an analog of the justification of deductive rules: a default argument is justified if its conclusion is true in a high proportion of the relevant models in which its premises are true. To make this idea precise requires an excursion into model theory.

2 Model Theory

We will suppose that the underlying object language is a first order language that does not involve such intentional predicates as “know” or “believe.” A number of nonmonotonic formalisms (specifically autoepistemic [Moore, 1983]) do involve such locutions within the object language, but they can be dispensed with in default logic. The default rule $\overline{\alpha;\beta;\ldots}$ can be read in terms of the nonmembership of $\Gamma \vdash \neg \beta_i$ in a specified set of expressions $\Gamma$. In original default logic, $\Gamma$ would just be an extension.

There are a number of immediate problems associated with the idea of looking at the “proportion” of models. The least of them is choosing a level at which to regard the evidence as adequate. Should we require that the proportion be 0.95? Or 0.99? Or 0.995? This is just the sort of question that arises in statistical hypothesis testing or in confidence interval estimation. We shall suppose that in a given context there is some agreed-upon level of security $\delta$; we will accept a conclusion if the proportion of models in which we could be committing an error is no greater than $\delta$.

This approach is to be contrasted with those of Adams [Adams, 1975, Adams, 1966], Pearl [Pearl, 1988] and Bacchus et al. [Bacchus et al., 1993]. Adams requires that for $A$ to be a reasonable consequence of the set of sentences $S$, for any $\epsilon$ there must be a positive $\delta$ such that for every probability function, if the probability of every sentence in $S$ is greater than $1 - \delta$, then the probability of $A$ is at least $1 - \epsilon$ [Adams, 1966, p. 274]. Pearl’s approach similarly involves quantification over possible probability functions. Bacchus et al again take the degree of belief of a statement to be the limiting proportion of first order models in which the statement is true. All of these approaches involve matters that go well beyond what we may reasonably suppose to be available to us as empirical enquirers. Our $\delta$, on the other hand, serves much like the $\alpha$ of statistical testing.

We must restrict the number of models under consideration to a finite number so that the idea of looking at proportions makes sense.

We will be taking account of statistical information, and to this end will want each model to have a finite domain. Roughly speaking, we take as a model of our language one in which the domain of empirical individuals is of finite cardinality. This may be regarded as problematic (it entails the

1 We could, instead, seek to develop a way of proceeding to a limit; this still would require restrictions to arrive at a countable number of models, and would entail a large expository cost for little gain in plausibility.
falsity of “every person has two parents and nobody is his own ancestor”) but with reasonable spatial and temporal bounding it can be rendered plausible.

Even so, to ensure that the set of models is finite we must restrict the empirical domain even further. Not only must it consist of a finite set of physical entities, but this same set of physical entities D must be taken to be the empirical domain of every model.

We assume that it is possible to express statistical knowledge in this language. For example, if “B(x)” is the predicate “is a bird” and “F(x)” is “can fly”, we can express the fact that between 85% and 95% of birds fly by the formula 

\[ 0.85 < \frac{|\{x : B(x) \land F(x)\}|}{|\{x : B(x)\}|} < 0.95. \]

Employing the notation of [Kyburg and Teng, 2001] we write this as “\% \geq x(F(x), B(x), 0.85, 0.95).” This renders “\%” a variable binding operator on 4-sequences of expressions: two formulas and two fractions.

We distinguish, as do Pearl and Geffner [Pearl and Geffner, 1990, p. 70] between immediate evidence, represented by a finite set of sentences E concerning particular facts (to be distinguished from the general body of factual knowledge F invoked by classical default logic), and a finitely axiomatizable set of sentences K representing general background knowledge. What defaults are plausible depends, of course, on background knowledge. If it were not for what we take to be the typical (or natural, or frequent) behavior of birds, the world’s best known example of a default rule would not be plausible. On the other hand no one has proposed the default rule

\[ \text{fish(a): mackerel(a).} \]

Thus in general we will represent the set of default rules of a default theory as \( \Delta_K \) rather than \( D \), since we take them to be a function of our body of general knowledge K. Given an error tolerance \( \delta \), we will take a default rule to be \( \delta \)-valid if, for every set of possible input sentences E consistent with K, the application of the rule to E leads to a false conclusion in a proportion of at most \( \delta \) of the relevant models. More precisely, a default rule is \( \delta \)-valid if and only if for every set of input sentences E consistent with K to which the rule is applicable, the proportion of models of \( E \cup K \) in which the conclusion of the rule is false is no more than \( \delta \).

To fix our ideas, let us begin with a simple example. Suppose K includes a statement to the effect that at least \( 1 - \delta \) and not more than \( 1 - \epsilon \) of birds fly and nothing else; that is, “\% \geq x(F(x), B(x), 1 - \delta, 1 - \epsilon).” Consider the rule \( \frac{B(x): F(x)}{P(x)} \). This rule is “applicable” to immediate evidence E only if \( E \cup K \) entails a sentence of the form \( \forall B(a) \) and no corresponding sentence of the form \( \forall \neg F(a) \).

Our models have a single domain D of finite cardinality. We will write “\( I_m(\phi) \)” for the interpretation of \( \phi \) in the model \( m \). The constraint imposed by K is that for every model \( m \) the proportion of objects in \( I_m(B) \) that are also in \( I_m(F) \) lies in \([1 - \delta, 1 - \epsilon]\).

There are three cases. First, suppose \( E \cup K \) does not entail a sentence of the form \( \forall B(a) \). Then the rule is inapplicable. Second, suppose that for some term \( a \), \( E \cup K \) entails \( B(a) \) and also entails \( \neg F(a) \). The rule is again inapplicable, because it is blocked by the failure of a justification. Third, suppose for some term \( a \), \( E \cup K \) entails \( B(a) \) but not \( \neg F(a) \). Then \( I_m(a) \in I_m(B) \). There are \( |I_m(B)| \) interpretations of \( a \) that make \( E \cup K \) true; of these at least \( 1 - \delta \) make \( F(a) \) true. We have said nothing about interpreting the rest of the language, but however many interpretations there are (we have seen to it that there are only a finite number) the proportion that renders \( F(a) \) true will remain unchanged; it will be at least \( 1 - \delta \). Thus, given the background knowledge that we have posited, the rule is \( \delta \)-valid: if it is applicable it will lead to error no more than \( 1 - \delta \) of the time.

Now let us consider a somewhat more complex example: Suppose we know that typically birds fly, and that typically penguins don’t. If that is in our background knowledge K, as well as “\( \forall x(F(x) \supset B(x)) \)”, then the flying default becomes \( B(x): F(x), \neg F(x) \). F(x), and we also have the default \( \frac{B(x): F(x), \neg F(x)}{P(x)} \). If E entails \( F(a) \), only the second default is applicable. In no more than \( \delta \) of the models of E will a fly, unless \( E \cup K \) entails that a can fly.

Another example: Suppose K contains vague information about the frequency with which red birds fly (perhaps because we have encountered few red birds). Say that we know the frequency to be between 0.50 and 1.0. Since the interval for birds in general \([1 - \delta, 1 - \epsilon]\) is included in \([0.5, 1.0]\), this additional piece of information should not interfere with our inference of flying ability. There is no conflict between the two intervals, just less precision in one. The rule about birds in general can be applied to red birds. However, if K contains the knowledge that between 0.5 and r of red birds fly, where r is less than \( 1 - \epsilon \), then this information should interfere. In this case the general rule should be so construed that it does not apply to red birds. If b is a red bird and not a penguin, no conclusion about flying ability is justified.

We can arrange this by judiciously adding or deleting justifications in the general bird rule, in accordance
with the statistical information in $K$: in the first case we allow red birds; in the second we must require that we do not know $a$ is red: “–$R(x)$” must be among the justifications of the rule. This statistical approach provides exactly the normative guidance that is lacking in the ad-hoc approach of tweaking default rules in order to arrive at the “intuitive” results.

More generally, we can give recipes for constructing $\delta$-valid defaults for conclusions of the form $\Gamma \phi$ from background knowledge $K$ and immediate evidence $E$. Let $K$ contain $\Gamma \% x(\phi(x), \psi(x), p, q) \vdash \Gamma \% x(\phi(x), \psi'(x), p', q') \vdash$. We consider three cases:

1. $K$ entails $\Gamma \% x(\psi(x) \supset \psi'(x)) \vdash$. There are three subcases according to the relation among $p$, $p'$, $q$, $q'$:
   
   (a) $(p \leq p'$ and $q \leq q')$ or $(p' \leq p$ and $q' \leq q) \quad \frac{\psi(x) \phi(x)}{\phi(x)}$ and $\frac{\psi(x) \overline{-\psi(x)} \phi(x)}{\phi(x)}$ are candidate defaults.
   
   (b) $p \leq p'$ and $q \leq q' \quad \frac{\psi(x) \phi(x)}{\phi(x)}$ is the only candidate default, since the justification $\overline{-\psi'(x)}$ of $\frac{\psi(x) \phi(x)}{\phi(x)}$, $\overline{-\psi'(x)}$ is inconsistent with the prerequisite $\psi(x)$ and $K$.
   
   (c) $p' \leq p$ and $q \leq q' \quad \frac{\psi(x) \phi(x)}{\phi(x)}$ and $\frac{\psi(x) \overline{-\psi(x)} \phi(x)}{\phi(x)}$ are candidate defaults.

2. $K$ entails $\Gamma \% x(\psi'(x) \supset \psi(x)) \vdash$. This is symmetrical to case 1.

3. $K$ entails neither $\Gamma \% x(\psi(x) \supset \psi'(x)) \vdash$ nor $\Gamma \% x(\psi'(x) \supset \psi(x)) \vdash$. Again there are three subcases:
   
   (a) $(p \leq p'$ and $q \leq q')$ or $(p' \leq p$ and $q' \leq q) \quad \frac{\psi(x) \overline{-\psi(x)} \phi(x)}{\phi(x)}$ and $\frac{\psi(x) \phi(x)}{\phi(x)}$ are candidate defaults.
   
   (b) $p \leq p'$ and $q \leq q' \quad \frac{\psi(x) \phi(x)}{\phi(x)}$ and $\frac{\psi'(x) \phi(x)}{\phi(x)}$ are candidate default rules.
   
   (c) $p' \leq p$ and $q \leq q'$: This is symmetrical to case 3(b).

Having generated a list of candidate default rules based on our background knowledge $K$, we delete those rules derived from statistics with a lower measure less than $1 - \delta$. The remainder is the set of defaults $\Delta_K$.

We have not taken account of relations among default conclusions that may be entailed by $K$. If $K$ contains $\Gamma \% x(\phi(x) \equiv \phi'(x)) \vdash$ then the default conclusion $\Gamma \phi(a) \vdash$ behaves just like the default conclusion $\Gamma \phi'(a) \vdash$. If $K$ contains $\Gamma \% x(\phi(x) \supset \phi'(x)) \vdash$, then since $\Gamma \phi(x) \vdash$ is equivalent to $\Gamma \phi(x) \land \phi'(x) \vdash$ and $\Gamma \phi'(x) \vdash$ is equivalent to $\Gamma \phi(x) \lor \phi'(x) \vdash$ we can make use of the obvious entailment relations.

Soundness of a system of deductive logic requires that the conclusion of any inference be true in every model in which the premises are true. Clearly nonmonotonic inference should not be sound. But there is a property that is like soundness that applies to default inference. It is the property that the conclusion is false in at most a fraction $\delta$ of the models of the premises $K \cup E$.

**Theorem 1 (Default Soundness)** For every set of observations $E$, if $d \in \Delta_K$ is applicable to $E$, the proportion of models of $E \cup K$ in which the conclusion of $d$ is false is less than $\delta$.

The proof of this theorem is provided by the soundness theorem for evidential probability [Kyburg and Teng, 2001, p. 241], since the rules for deriving defaults are a subset of the rules for computing evidential probabilities. □

### 3 Interactions within an Extension

Having determined which default rules are justified with respect to the background knowledge, the next step is to investigate the interaction between default rules in generating an extension. A default extension is a minimal deductively closed set that contains the given facts and the consequents of all applicable default rules. Given an evidence set, we need to determine how to control the compound effects of multiple defaults in an extension.

Take for example, a default version [Poole, 1986] of the probabilistic lottery paradox [Kyburg, 1961]. There are $n$ species of birds, $S_1, \ldots, S_n$. We can say that penguins are atypical in that they cannot fly; hummingbirds are atypical in that they have very fine motor control; parrots are atypical in that they could talk; and so on. If we apply this train of thought to all $n$ species of birds, there is no typical bird left, as for each species there is always at least one aspect in which it is atypical. A parallel scenario is formulated below.
Example 2 $K$ contains

\[
B(x) \equiv S_1(x) \lor \ldots \lor S_n(x) \\
\text{[an exhaustive list of bird species]}
\]

$S_i(x) \supset \neg S_j(x)$, for all $j \neq i$  
[species are mutually exclusive]

$\%\{(S_i(x), B(x), \epsilon_i, \delta_i)\}$, for $1 \leq i \leq n$  
[the proportion of each $S_i$ species of birds is “small”]

From $K$ we can derive\(n \delta^*\)-valid default rules for $\Delta_K$:

\[
d_i = \frac{B(x): \neg S_i(x)}{-S_i(x)}, \text{ for } 1 \leq i \leq n
\]

where $\delta^*$ is the maximum of $\delta_1, \ldots, \delta_n$.

Now consider the evidence set $E = \{B(a)\}$. In the original formulation of default logic, we would get $n$ extensions, each one containing one $S_i(a)$ and the negations of all the other $S_j(a)$’s. Thus, for each extension, we would conclude that $a$ is a particular species of bird, which seems to be an over commitment, considering we have $\%\{(S_i(x), B(x), \epsilon_i, \delta_i)\}$ in $K$. □

Note that each of the $n$ default rules is $\delta$-valid when considered individually, but in an extension the rules interact to sanction a set of conclusions that when taken together seems implausible according to our knowledge of model frequencies. The definition of an extension dictates that we must keep applying rules until all “applicable” ones are exhausted. The “applicability” condition is based on maximizing logical strength: for $d = \frac{\alpha_1 \beta_1 \ldots \beta_m}{\gamma}$, as long as $\alpha$ is derivable, and the $\beta$’s are consistent with the extension, we must apply $d$ and add $\gamma$ to the extension. Thus, for each of the extensions above, we have to keep applying the rules until we have drawn $n - 1$ conclusions: $\neg S_i(a)$ for all $j \neq i$. Then the consistency requirement blocks the last default rule $d_i$, as $B(x) \supset S_1(x) \lor \ldots \lor S_n(x)$ together with $\neg S_i(a)$ for all $j \neq i$ gives us $S_i(a)$, contradicting the $\beta$ of $d_i$. From $\%\{(S_i(x), B(x), \epsilon_i, \delta_i)\}$, we know the proportion of models in which $S_i(a)$ is true, and thus the proportion of models satisfying this extension, given $E$, is at most $\delta_i$, a small ratio.

3.1 Sequential Thresholding

The validity criteria for individual default rules can be extended to extensions resulted from the application of a chain of default rules. We can think of the task of regulating the compound effect of multiple default rules as adjusting the set of relevant models by taking into account the default conclusions of all previously applied rules in the chain of reasoning.

One way to accomplish this is by sequential thresholding [Teng, 1997]. The applicability condition of a default rule $\alpha \rightarrow B(x)$ in an extension can be modified to take into account the validity of the rule. In addition to requiring that $\alpha$ is provable and that none of $\neg \beta_1, \ldots, \neg \beta_n$ are provable, we require that the default rule be “above threshold”, that is, the proportion of relevant models satisfying the consequent $\gamma$ be greater than a threshold $1 - \epsilon^*$.

The set of relevant models shrinks in a stepwise fashion. We start out with all the models satisfying the background knowledge and evidence we have. As default rules are applied sequentially, the consequent of the applied rule at each step is taken as true in all subsequent steps. The relevant models at a particular step are then those that are consistent with the given facts and all the consequents of the rules applied in the previous steps. A default rule, even if it is $\delta$-valid with respect to the background knowledge, would be blocked from application if it does not satisfy the thresholding criterion.

In [Teng, 1997], the thresholding metric is based on a simple probability measure of possible worlds. We can easily extend this metric to employ the same measure as that used for evaluating the $\delta$-validity of default rules.

Example 3 Reconsider Example 2. Let us take $\epsilon^* \geq \delta^*$. We start out with the set $\mathcal{M}$ of all models satisfying $K$ and $E$. From $\%\{(S_1(x), B(x), \epsilon_1, \delta_1)\}$ we know that $d_1$ is above threshold, and it satisfies the other conditions for applicability. Therefore we apply $d_1$ and conclude $\neg S_1(a)$.

Now consider $d_2$. The set $\mathcal{M}'$ of relevant models at this point is a subset of $\mathcal{M}$; it contains only those models in $\mathcal{M}$ that satisfy our new conclusion $\neg S_1(a)$ as well. We have eliminated the models in which $S_1(a)$ is true. Since $S_1(x) \supset \neg S_2(x)$, and $S_1(x) \supset B(x)$, all the models eliminated satisfy $B(a)$, and none satisfies $S_2(a)$. Thus, in $\mathcal{M}'$, the number of models satisfying $S_2(a)$ is the same as in $\mathcal{M}$. However, the number of models satisfying $B(a)$ is lower in $\mathcal{M}'$ as a result of the addition of $\neg S_1(a)$. This gives rise to a higher proportion of models satisfying $S_2(a)$ in $\mathcal{M}' (\delta'_2)$ than in $\mathcal{M} (\delta_2)$. If $\delta'_2 \leq \epsilon^*$, $d_2$ is still above threshold after the application of $d_1$, and we can apply it to obtain $\neg S_2(a)$. Otherwise, $d_2$ is below threshold, and we cannot apply it even though it was above threshold before the application of $d_1$.

After each step of applying a rule, the set of relevant models shrinks, and the proportion of $S_i(a)$ of any unapplied rule $d_i$ increases. After a number of steps,
all the remaining rules would be below threshold, and we thus obtain an extension containing only a portion of the conclusions that would otherwise be present in the non-thresholding version of the extension. □

Note that the size of $\epsilon^*$ determines how much risk is tolerated in an extension. The higher the $\epsilon^*$, the more of the rules can be applied and the longer they can stay above threshold. Reiter’s non-thresholding version corresponds to the case when $\epsilon^* = 1$; that is, every rule whose associated proportion is above 0 is allowed, and logical consistency alone determines the rule’s admissibility.

4 Concluding Remarks

We have developed a notion of validity for default inference based on model proportions. A rule is $\delta$-valid if the proportion of models in which the consequent of the rule is satisfied is greater than $1 - \delta$ in the relevant models picked out by the background knowledge, the evidence, and the applicability conditions of the default rule. Given a body of background knowledge $K$, we can systematically generate candidate default rules and determine which ones are $\delta$-valid based on the statistical facts known in $K$. Conflicts between default rules stemming from multiple inheritance are resolved as a consequence of the validation process. The result is a set of $\delta$-valid default rules which are “pre-compiled” for a given background knowledge base, and can be reused for different evidence sets without change.

This idea of evaluating the validity of a default rule using model proportions is extended to extensions generated by a combinations of rules. The compound effect is regulated by a sequential thresholding process, which blocks the rules whose associated model proportions with respect to the “current” (shrinking) set of models fall below a particular comfort threshold. This allows us to use a more reasonable “closure condition” for extension than the usual maximal logical strength: we can refrain from applying rules that would make the extension satisfiable in only a small set of models, even if the consequent of the rule is logically consistent with the extension.

Grounding the justification of default rules in model proportions provides a way to validate the rules empirically, and is a first step towards automating the learning of default rules from (statistical) data. One might ask why we need the default rules when we can reason with the statistics directly. Default rules provide a succinct and more understandable characterization of the import of the data, as well as a smooth articulation of the information that may exist in the knowledge base.

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