Recent Developments in Radiative Corrections at HERA

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Abstract

We describe several numerical results for radiative corrections for deep inelastic $ep$ scattering at HERA which are calculated using the \texttt{HECTOR} package. We present radiative corrections for ten different choices of kinematical variables for unpolarized neutral and charged current deep inelastic scattering. Radiative corrections for neutral current scattering off polarized protons are calculated in leptonic variables and compared to those obtained by the \texttt{POLRAD} code for the kinematic regime of the HERMES experiment.

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Abstract: We describe several numerical results for radiative corrections for deep inelastic \textit{ep} scattering at HERA which are calculated using the \textsc{hector} package. We present radiative corrections for ten different choices of kinematical variables for unpolarized neutral and charged current deep inelastic scattering. Radiative corrections for neutral current scattering off polarized protons are calculated in leptonic variables and compared to those obtained by the \textsc{polrad} code for the kinematic regime of the HERMES experiment.

1 Introduction

The precise knowledge of QED radiative corrections is indispensable in the determination of nucleon structure functions. The forthcoming high statistics measurements of $F_2(x, Q^2)$, $F_L(x, Q^2)$ and $F_{c\bar{c}}(x, Q^2)$ at H1 and ZEUS require knowing the radiative corrections at the $\%$ level. In some of the measurements particularly, the range of high $y$ is essential. Here the radiative corrections turn out to be large for some choices of the kinematical variables and higher order corrections can be necessary.

In the present note we summarize the status reached in the calculation of the QED radiative corrections. In section 2, we present a short description of main features of the recently released code \textsc{hector} \cite{1}.

For the first time also polarized nucleon structure functions can be measured at HERA with the HERMES experiment Also here the radiative corrections are large. Recently a dedicated new calculation \cite{2} was performed including both $\gamma$ and $Z$-boson exchange and accounting for all twist-2 contributions to the structure functions contributing to scattering cross sections both for the case of longitudinally and transverse polarized nucleons.

In section 3, we present a discussion of numerical results summarized in a collection of figures. A particular emphasis is given on the high $y$ range by presenting and discussing the results for ten different choices of the kinematical variables. A first comparison between the results of \textsc{hector} and earlier results of \textsc{polrad} \cite{3}–\cite{5} is presented.
HECTOR 1.00 and its recent upgrade

The code HECTOR was created at DESY–Zeuthen in 1995. Version 1.00, November 1995, accumulates and comprises results collected over the course of 20 years (1975-1994) by the Dubna-Zeuthen Radiative Correction Group (DZRCG), based on a semi-analytic, model-independent (MI) approach and results by J. Blümlein (1990-1994), based on an inclusive leading logarithmic approach (LLA).

The branches of HECTOR include earlier codes for treatment of selected parts of radiative corrections:

- **HELIOS** – an inclusive LLA treatment of leptonic QED radiative corrections including second order initial state radiation, $O((\alpha L)^2)$, and soft-photon exponentiation to all orders for a variety of measurements: leptonic, mixed, Jaquet-Blondel, double angle variables, the Σ method and others;

- **TERAD** – a complete $O(\alpha)$ MI treatment of leptonic QED radiative corrections for several types of measurements, for a detailed description see [5];

- **DISEP** – a complete $O(\alpha)$ quark-parton model treatment of QED radiative corrections and one-loop electroweak radiative corrections [9] for leptonic and mixed variables;

- **TERADLOW** – a MI treatment of leptonic QED radiative corrections in the photoproduction region for leptonic variables [8].

HECTOR makes use of extensive access to existing libraries of the structure functions and parton densities, both via the PDF-library [10] and directly, and to recent low $Q^2$-libraries.

The QCD corrections are implemented in the framework of different factorization schemes, as the $\overline{MS}$ and DIS schemes, in order to ensure a proper use of available parton densities. The LO option is also available.

Currently one may access ten different choices of kinematical variables for neutral and charged current deep inelastic scattering, see figures 2a-j.

Simple kinematical cuts are possible within the complete $O(\alpha)$ MI approach.

The upgrade of HECTOR, version 1.11, will contain the following additions:

- The option to calculate radiative corrections for neutral current deep inelastic polarized lepton – polarized nucleon scattering has been incorporated [2]. It includes both $\gamma$ and $Z$-boson exchange. The Born cross section contains all twist-2 contributions to the polarized structure functions – for the cases of longitudinal and transverse proton polarization.

- The radiative corrections for a tagged photon measurement based on a mixture of complete MI, deterministic and LLA approaches are being incorporated [11].

Version 1.11 will be released by the end of 1996.
3 Numerical Results

3.1 QED radiative corrections at high $y$

The radiative corrections (RC) at high $y$ are presented in two sets of figures, 1 and 2, at HERA collider energies. Here for the structure functions, we used the CTEQ3M LO parametrization \[12\].

In figures 1a-d, we show the comparison between complete $\mathcal{O}(\alpha)$ MI calculations and those in LLA, for 4 types of measurements for which the complete results are available.

In \textit{leptonic} variables at small $x$ and high $y$, where the correction is big, the difference between complete and LLA calculations reaches tens of percent.

In \textit{mixed} variables we registered an almost constant, $x,y$-independent shift between the two calculations, which is quite small, $\leq 0.5\%$.

An interesting phenomenon is observed in Jaquet-Blondel and \textit{hadronic} variables. There the difference between the two calculations grows with growing $y$, reaching several percent for $y \approx 1$, i.e. in the soft photon corner of hadronic $y$. This could be a reflection of the fact that in these variables the final state radiation leading log correction is absent and non-logarithmic terms can be important.

So, one can conclude, that although the LLA approximates the gross features of radiative corrections in all 4 variables, its precision is not sufficient if one aims at an accuracy of measurement of the order of 1%.

In figures 2a-j, we show the comparison between lowest order and higher order LLA calculations of radiative corrections for ten measurements: eight – for neutral current (NC) and two – for charged current (CC). Although we have presented figures for all ten choices of measurements available in \texttt{HECTOR}, we will discuss only several of the most popular kinematic variables.

In \textit{leptonic} variables at small $x$ and high $y$, the higher order corrections reach tens of percent. Since LLA qualitatively describes the lowest order corrections, one may trust the reliability of higher order corrections estimation.

In \textit{Jaquet-Blondel}, \textit{mixed}, and \textit{hadronic} variables, the higher order corrections exhibit very similar properties. They grow with increasing $x$ and $y$, reaching $1-2\%$ at high $x$ and high $y$, i.e. in the soft photon corner.

The constant positive shift, growing with increasing $x$, is distinctly seen in \textit{double angle} and $\Sigma$ variables. It may reach $1\%$ for $x = 0.1$ and goes down rapidly with decreasing $x$.

In the $e\Sigma$ method, the higher order corrections are surprisingly large, but this method is not so popular.

From figures presented, we may conclude that higher order corrections are in general rather important for the precision measurement of deep inelastic scattering at HERA.

Both sets of figures prove that a realistic radiative corrections procedure must take into account both the complete lowest order calculations and higher order corrections, at least within LLA. This is exactly the strategy that the \texttt{HECTOR} code follows.
3.2 Comparison of HECTOR and POLRAD15 for Polarized Deep Inelastic Scattering

In this section we compare the results of the codes HECTOR and POLRAD15. We refer to the kinematic range of the HERMES experiment and consider only leptonic corrections in leptonic variables for scattering off polarized protons. Both the cases of longitudinal and transverse polarizations were studied. We used the parametrizations of Schäfer’88 [13] and GRSV’96 [14] to describe the polarized structure functions.

To simplify the first comparison between the two codes, we neglect $Z$-boson exchange, account only for one structure function ($g_{1}^{\gamma\gamma}$) for the case of longitudinal proton polarization, and at most for two ($g_{1,2}^{\gamma\gamma}$) for the case of transverse polarization. Furthermore, we neglect the vacuum polarization correction (the running $\alpha$), higher order radiative corrections, hadronic corrections and electroweak corrections.

The comparison for unpolarized, longitudinal and transverse parts of deep inelastic scattering (DIS) radiative corrections normalized by corresponding Born cross sections is presented in figures 3a-d. They are denoted in figures as UNPOL, LONG, TRAN, correspondingly. Further details may be found in [2].

The results of the comparison may be summarized as follows:

- Very good agreement was found for all values of $x$ and $y$ in the unpolarized-case.
- We registered some disagreement for small $x$ and high $y$, i.e. in the Compton region, for longitudinal- and transverse-cases.
- We observed an amazing agreement between complete and LLA calculations in the considered set-up. This suggests the use of a fast, LLA code for HERMES measurements in leptonic variables.

Further work on the comparison, aimed at the resolution of the above-mentioned disagreement, is in progress.

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References

[1] A. Arbuzov, D. Bardin, J. Blümlein, L. Kalinovskaya, T. Riemann, *Computer Phys. Commun.* **94** (1996) 128.

[2] D. Bardin, J. Blümlein, P. Christova, L. Kalinovskaya, DESY 96–189.

[3] T. Kukhto and N. Shumeiko, *Nucl. Phys.* B**219** (1983) 412.

[4] N. Shumeiko, In Proceedings of the 1992 Zeuthen Workshop on Elementary Particle Theory: Deep Inelastic Scattering, Teupitz/Brandenburg, Germany 6-10 April 1992; J. Blümlein and T. Riemann Eds, *Nucl. Phys. B (Proc. Suppl.)* **29A** (1992) 236.

[5] I. Akushevich and N. Shumeiko, *J. Phys.* G**20** (1994) 513.

[6] In the field of deep inelastic scattering, the following physicists worked within DZRCG: A. Akhandov, D. Bardin, C. Burdik, P. Christova, O. Fedorenko†, L. Kalinovskaya, T. Riemann. N. Shumeiko participated in the long first period of work in this direction.

[7] J. Blümlein, *Z. Physik* C**47** (1990) 89; *Phys. Letters* B**271** (1991) 267; *Z. Physik* C**65** (1995) 293.

[8] A. Akhundov, D. Bardin, L. Kalinovskaya and T. Riemann, *Fortschr. Phys.* **44** (1996) 373.

[9] D. Bardin, C. Burdik, P. Christova and T. Riemann, *Z. Physik* C**42** (1989) 679; *Z. Physik* C**44** (1989) 149.

[10] H. Plothow–Besch, *Computer Phys. Commun.* **75** (1993) 396.

[11] D. Bardin, L. Kalinovskaya and T. Riemann, in preparation.

[12] H.L. Lai, J. Botts, J. Huston, J.G. Morfin, J.F. Owen, J.W. Qiu, W.K. Tung and H. Weerts, *Phys. Rev.* D**51** (1995) 4763.

[13] A. Schäfer, *Phys.Lett.* B**208** (1988) 175.

[14] M. Glück, E. Reya, M. Stratmann and W. Vogelsang, *Phys. Rev.* D**53** (1996) 4775.

[15] S. Wandzura and F. Wilczek, *Phys. Lett.* B**72** (1977) 195.

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Fig. 1a: A comparison of complete and leading log calculations of RC for NC DIS at HERA for leptonic variables.

Fig. 1c: The same as Fig.1a but for mixed variables.

Fig. 1b: The same as Fig.1a but for Jaquet-Blondel variables.

Fig. 1d: The same as Fig.1a but for hadronic variables.
Fig. 2a: A comparison of lowest order with higher order leading log calculations of RC for NC DIS at HERA for leptonic variables.

Fig. 2b: The same as Fig.2a but for Jaquet-Blondel variables.

Fig. 2c: The same as Fig.2b but for CC DIS.

Fig. 2d: The same as Fig.2a but for mixed variables.
Fig. 2e: The same as Fig. 2a but for the **double angle** method.

Fig. 2f: The same as Fig. 2a but for the **lepton angle and** $y_{3B}$ **method.**

Fig. 2g: The same as Fig. 2a but for **hadronic variables.**

Fig. 2h: The same as Fig. 2b but for **leptonic variables.**
Fig. 2i: The same as Fig. 2a but for the $\Sigma$ method.  
Fig. 2j: The same as Fig. 2a but for the $\epsilon \Sigma$ method.
Fig. 3a: A comparison of RC between HECTOR and POLRAD for NC unpolarized DIS for leptonic variables.

Fig. 3b: The same as Fig.3a but for longitudinal DIS.

Fig. 3c: The same as Fig.3a but for transverse DIS.

Fig. 3d: The same as Fig.3c but for another assumption on $g_2$. [15]
$\delta_i^\text{tran} [\%]$  
$g_1$ and $g_2 = -g_1$  
Schafer, 1988  

Fixed target DIS  
$E_s = 27.5$ GeV  

$x = 10^{-2}$  
$x = 10^{-1}$  

0.5  
0.9  

HECTOR Complete  
HECTOR LLA  
POLRAD Complete  

$y$