Tetramixing of vector and pseudoscalar mesons: A source of intrinsic quarks

Tao Peng

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Bo-Qiang Ma

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China and Center for High Energy Physics, Peking University, Beijing 100871, China

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Abstract

The tetramixing of pseudoscalar mesons $\pi$-$\eta$-$\eta'$-$\eta_c$ and vector mesons $\omega$-$\rho$-$\phi$-$J/\psi$ are studied in the light-cone constituent quark model, and such mixing of four mesons provides a natural source for the intrinsic charm $c\bar{c}$ components of light mesons. By mixing with the light mesons, the charmonium states $J/\psi$ and $\eta_c$ could decay into light mesons more naturally, without introducing gluons or a virtual photon as intermediate states. Thus, the introduction of light quark components into $J/\psi$ is helpful to reproduce the new experimental data of $J/\psi$ decays. The mixing matrices and the $Q^2$ behaviors of the transition form factors are also calculated and compared with experimental data.

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*Electronic address: mabq@pku.edu.cn
The mixing of mesons has been widely investigated since the 1960s, when the concept of a mixing state of the $\rho$-$\omega$ mesons was proposed [1] by considering that the electromagnetic interaction does not conserve isospin. Later, the $\omega$-$\phi$ mixing and $\eta$-$\eta'$ mixing were introduced [2] to explain the deviation of the meson mass from the Gell-Mann-Okubo mass formula [3, 4]. It was also pointed out that the difference between the $u$ and $d$ quark masses introduces the $\pi$-$\eta$ mixing [5]. Then, the trimixing of $\pi$-$\eta$-$\eta'$ [6, 7] and $\rho$-$\omega$-$\phi$ [7, 8] were proposed, and their effects were studied in different methods. On the other hand, the $c\bar{c}$ contribution to the $\eta$ and $\eta'$ mesons was considered [9], and the trimixing $\eta$-$\eta'$-$\eta_c$ was studied [10]. As a further extension in this paper, we try to combine the above two types of trimixing by considering the tetramixing of pseudoscalar mesons $\pi$-$\eta$-$\eta'$-$\eta_c$. The mixing of gluon component $gg$ and $\eta$-$\eta'$ were also studied [11, 12]. As the mixing of $\eta$ and $\eta'$ is still not completely clear right now, we think that the charm and gluon components may both be possible to mix with these mesons, and it is worthwhile to study both of them carefully.

The recent CLEO experiment [13] of the charmonium decays $J/\psi \rightarrow \gamma\pi$, $\gamma\eta$, and $\gamma\eta'$ also motivates us to extend our tetramixing to the vector mesons $\omega$-$\rho$-$\phi$-$J/\psi$. According to the pure valence $c\bar{c}$ structure of charmonia in the naive quark model, these decay modes of charmonium $\psi(nS)$ must happen via the annihilation of the heavy quark constituents into gluons or a virtual photon [13, 14], because of the Okubo-Zweig-Iizuka rule, which postulates a suppression of transitions between hadrons without valence quarks in common [14]. Moreover, the mechanisms of these decays are not completely clear yet, and there are various ways to describe them, such as $\psi(nS) \rightarrow \gamma gg \rightarrow \gamma P$, $\psi(nS) \rightarrow ggg \rightarrow q\bar{q}\gamma$, and so on [13]. However, with the model of $\omega$-$\rho$-$\phi$-$J/\psi$ mixing, the above-mentioned decays of $J/\psi$ could occur more naturally through the direct transition from its light quark components to light mesons such as $\pi$, $\eta$, or $\eta'$ without introducing intermediate gluons or a virtual photon, and the $c\bar{c}$ components of these light pseudoscalar mesons also allow $J/\psi$ to decay to them. The mixing of $\omega$-$\rho$-$\phi$-$J/\psi$ is thus helpful to reproduce the new experimental data of $J/\psi$ decays.

For the light vector mesons $\omega$, $\rho$, and $\phi$, the existence of $c\bar{c}$ states in them may be interpreted as a support to the theory of intrinsic charm [15] in these mesons. Different from the extrinsic quarks, which are generated on a short time scale in a reaction process with large momentum transfers, the intrinsic quarks are intrinsic nonperturbatively to the hadron wave function and exist over a time scale independent of any probe momentum [15, 16].
postulation of intrinsic $c\bar{c}$ components in $\rho$ and $\pi$ offers a possible solution of the "$\rho\pi$ puzzle" by allowing direct transitions between $J/\psi(\psi')$ and $\rho(\pi)$ through the rearrangement of the valence and the intrinsic $c\bar{c}$ components of $\rho(\pi)$ \[14\]. Now the tetramixing of $\omega$-$\rho$-$\phi$-$J/\psi$ introduces the intrinsic $c\bar{c}$ components into all three light vector mesons $\omega$, $\rho$, and $\phi$, and $J/\psi$ can decay to them in a similar way, without annihilation of the quark constituents. This applies to the pseudoscalar mesons $\pi$, $\eta$, and $\eta'$, too, as they mix with the charmonium $\eta_c$ in our model. The intrinsic $c\bar{c}$ component in $\eta'$ was also studied in Refs. \[17, 18\], in which the intrinsic charm content of the $\eta'$ meson $f^{\text{c}}_{\eta'}$ was evaluated, and we shall compare our result of $f^{\text{c}}_{\eta'}$ with previous results in Refs. \[17, 18\] and other works at the end of this paper.

We adopt the light-cone constituent quark model \[19–21\] to study the mixing of mesons. The light-cone constituent quark model is a convenient and effective model to treat the nonperturbative aspect of QCD, and the mixing of mesons in this model has been studied \[22, 23\].

The mixing of pseudoscalar mesons and vector mesons could be described by two $SO(4)$ rotation matrices $M_v$ and $M_s$, respectively:

$$
\begin{pmatrix}
\omega \\
\rho \\
\phi \\
J/\psi
\end{pmatrix} = M_v
\begin{pmatrix}
\omega_I \\
\rho_I \\
\phi_I \\
J/\psi_I
\end{pmatrix},
\begin{pmatrix}
\pi \\
\eta \\
\eta' \\
\eta_c
\end{pmatrix} = M_s
\begin{pmatrix}
\pi_I \\
\eta_q \\
\eta_s \\
\eta_{c0}
\end{pmatrix},
$$

(1)

in which the unmixed states are

$$
\omega_I = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi_{\omega_I}, \quad \rho_I = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\varphi_{\rho_I}, \quad \phi_I = -s\bar{s}\varphi_{\phi_I}, \quad J/\psi_I = c\bar{c}\varphi_{J/\psi_I},
$$

$$
\pi_I = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\varphi_{\pi_I}, \quad \eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi_{\eta_q}, \quad \eta_s = s\bar{s}\varphi_{\eta_s}, \quad \eta_{c0} = c\bar{c}\varphi_{\eta_{c0}},
$$

(2)

where $\varphi$ is the momentum space wave function of the corresponding meson.

Since the rotation group $SO(4) = SO(3) \otimes SO(3)$, the $SO(4)$ mixing matrix $M$ can be written as

$$
M = R_+ R_-,
$$

(3)

where the matrices $R_+$ and $R_-$ are generated by the $SO(3)$ generators, and each of them
could be parameterized by three independent rotation angles as

\[
R_+(\theta_1, \theta_2, \theta_3) = \begin{pmatrix}
\cos \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} & \frac{\alpha}{2} \sin \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} \\
\frac{\alpha}{2} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} & \frac{\alpha}{2} \sin \frac{\alpha}{2} \\
\frac{\alpha}{2} \sin \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} \\
\frac{\alpha}{2} \sin \frac{\alpha}{2} & -\frac{\alpha}{2} \sin \frac{\alpha}{2} & \frac{\alpha}{2} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2}
\end{pmatrix},
\]

(4)

\[
R_-(\theta_4, \theta_5, \theta_6) = \begin{pmatrix}
\cos \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} \\
-\frac{\beta}{2} \sin \frac{\beta}{2} & \cos \frac{\beta}{2} & -\frac{\beta}{2} \sin \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} \\
-\frac{\beta}{2} \sin \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} & \cos \frac{\beta}{2} & -\frac{\beta}{2} \sin \frac{\beta}{2} \\
-\frac{\beta}{2} \sin \frac{\beta}{2} & -\frac{\beta}{2} \sin \frac{\beta}{2} & \frac{\beta}{2} \sin \frac{\beta}{2} & \cos \frac{\beta}{2}
\end{pmatrix},
\]

(5)

where

\[
\alpha = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}, \quad \beta = \sqrt{\theta_4^2 + \theta_5^2 + \theta_6^2},
\]

(6)

and thus the mixing matrix \( M \) is parameterized as six independent rotation angles \((\theta_1, \theta_2, \ldots, \theta_6)\). Our detailed procedure of obtaining the matrix form of \( R_+ \) and \( R_- \) [Eqs.(4) and (5)] is given in Appendix A.

When referring to the mixing of specific types of mesons, \( M \) stands for \( M_v \) or \( M_s \), and the parameters change to \((\theta_1^v, \theta_2^v, \ldots)\) or \((\theta_1^s, \theta_2^s, \ldots)\) correspondingly.

During the numerical calculation, we also used a more compact form of \( M \) with eight real parameters under constraints, and the detailed procedure is given in Appendix B.

The decay constants and transition form factors also mix as

\[
\begin{pmatrix}
f_\omega \\
f_\rho \\
f_\phi \\
f_{J/\psi}
\end{pmatrix}
= M_v
\begin{pmatrix}
f_{\omega_1} \\
f_{\rho_1} \\
f_{\phi_1} \\
f_{J/\psi_1}
\end{pmatrix},
\]

\[
\begin{pmatrix}
F_{\pi \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta' \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta_{c} \rightarrow \gamma \gamma^*}(Q^2)
\end{pmatrix}
= M_s
\begin{pmatrix}
F_{\pi \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta' \rightarrow \gamma \gamma^*}(Q^2) \\
F_{\eta_{c} \rightarrow \gamma \gamma^*}(Q^2)
\end{pmatrix},
\]

(7)
The above decay constants and transition form factors are defined as \[23, 24\]

To calculate them, we use the light-cone quark model with the Fock state expansions of the only the quark-antiquark valence states of the unmixed mesons into consideration.

and to simplify the problem, we adopt the lowest order of the above expansions, which takes only the quark-antiquark valence states of the unmixed mesons into consideration.

To calculate them, we use the light-cone quark model with the Fock state expansions of the unmixed mesons [the right-hand side of Eq. (1)]:

\[ \langle 0 | j_\mu | V(p, S_z) \rangle = M_V f_V e_\mu (S_z), \] (9)

\[ \langle \gamma(p - q) | J^\mu | P(p, \lambda) \rangle = ie^2 F_{P \rightarrow \gamma^*} (Q^2) \varepsilon^{\mu \rho \sigma} p_\rho e_\nu (p - q, \lambda) q_\sigma, \] (10)

\[ \langle P(p') | J^\mu | V(p, \lambda) \rangle = ieF_{V \rightarrow \gamma} (Q^2) \varepsilon^{\mu \rho \sigma} \epsilon_\nu (p, \lambda) p'_\rho p_\sigma. \] (11)

The wave function of an unmixed meson in the light-cone formalism is \[19, 25\]:
where $\varphi$ is the momentum space wave function, described by the Brodsky-Huang-Lepage prescription [19, 25]:

$$
\varphi(x, k_\perp) = \varphi_{\text{BHL}}(x, k_\perp) = A \exp \left[ -\frac{1}{8\beta^2} \left( \frac{m_1^2 + k_\perp^2}{x} + \frac{m_2^2 + k_\perp^2}{1-x} \right) \right],
$$

(A and $\beta$ are the parameters of the meson, and $m_1$ and $m_2$ are masses of the constituent quarks), and $\chi_M^T(x, k_\perp, \lambda_1, \lambda_2)$ is the light-cone spin wave function, which is related to the instant-form spin wave function by the Melosh-Wigner rotation [26–28]

$$
\chi_1^\uparrow(T) = w_i[(k_i^+ + m_i)\chi_1^\uparrow(F) - k_i^R\chi_1^\uparrow(F)]; \quad \chi_1^\downarrow(T) = w_i[(k_i^+ + m_i)\chi_1^\downarrow(F) + k_i^L\chi_1^\uparrow(F)],
$$

where $w_i = 1/\sqrt{2k_i^0(m_i^0 + m_i)}$, $k_i^{R,L} = k^0 \pm k^\perp$, $k^+ = k^0 + k^3 = x\mathcal{M}$, and $\mathcal{M} = \sqrt{(k_\perp^2 + m_i^2)/x + (k_\perp^2 + m_i^2)/(1-x)}$. The Melosh-Wigner rotation is an important ingredient of the light-cone quark model and plays an essential role in explaining the “proton spin puzzle” [28, 29]. The detailed formulas for calculating the decay constants and transition form factors of mesons were listed in Ref. [23], and the examples of applying them to set meson parameters and to calculate the decay constants and transition form factors numerically can be found in Ref. [30].

The values of the meson parameters $A$, $\beta$, $m_1$, and $m_2$ and the parameters of the mixing matrices $(a_v, b_v, ...)$ and $(a_s, b_s, ...)$ can be chosen by fitting the light-cone constituent quark model results of the meson decay constants and transition form factors (at $Q^2 = 0$) to experimental data. The $Q^2 \to \infty$ limiting behavior of $Q^2 F_{P \to \gamma \gamma^*}$ is also considered as a constraint to set the parameters [21, 31]:

$$
\lim_{Q^2 \to \infty} Q^2 F_{P \to \gamma \gamma^*}(Q^2) = 2c_P f_P = \frac{2c_P^2}{4\pi^2 F_{P \to \gamma \gamma^*}(0)},
$$

where $c_P = (c_{\pi I}, c_{\eta I}, c_{\eta_s}) = (1, 5/3, \sqrt{2}/3)$. With the similar method as Ref. [17], we also obtain $c_{\eta_0} = 4\sqrt{2}/3$. All these requirements are taken as constraints to determine the parameters of mesons and parameters of the mixing matrices. During our calculation, we first use the decay constants and the radii of $\pi^+$ and $K^+$ as the constraints to locate the values of $A_\pi$, $\beta_\pi$, $m_u$, and $m_s$, assuming that the wave function parameters of $\pi^\pm$ are the same as those of $\pi_I$, and the constituent quark mass $m_d \approx m_u$ (their difference could be ignored compared with $m_c$) [7]. The other parameters are then determined under the left constraints.
Our numerical calculation gives the mixing matrices of vector and pseudoscalar mesons:

\[
M_v = \begin{pmatrix}
0.9886 & -0.0122 & -0.1429 & 0.0076 \\
0.0299 & 0.9910 & 0.1221 & -0.0025 \\
0.1400 & -0.1250 & 0.9808 & 0.0258 \\
-0.0111 & 0.0058 & -0.0239 & 0.9986
\end{pmatrix}, \tag{17}
\]

\[
M_s = \begin{pmatrix}
0.9895 & 0.0552 & -0.1119 & 0.0342 \\
-0.1082 & 0.8175 & -0.5614 & -0.0259 \\
0.0590 & 0.5696 & 0.8160 & 0.0452 \\
-0.0395 & -0.0065 & -0.0478 & 0.9960
\end{pmatrix}. \tag{18}
\]

We see that some of the entries of the mixing matrices are small; for example, one of the entries in the first row of \(M_v\) is 0.0076. But this nonzero entry means a charm component in the \(\omega\) meson, which allows \(\eta_c\) to decay to \(\omega\) directly in our model. Other entries of the mixing matrices have the same meaning, and it is such entries that are helpful to reproduce the experimental decay data of \(J/\psi\) and other meson decays.

The results of fitting light-cone constituent quark model results to experimental data are shown in Table I. The fourth column contains the results of tetra-mixing model \(\pi-\eta-\eta'\) and \(\rho-\omega-\phi\), while \(J/\psi\) and \(\eta_c\) do not mix with other meson states, with the values of their parameters (MeV) set as \(A_{J/\psi} = 31.1660\), \(\beta_{J/\psi} = 0.9777\), \(A_{\eta_c} = 125.7935\), and \(\beta_{\eta_c} = 0.7524\) to fit the experimental data. The most apparent differences between tetramixing and trimixing results are in the last four rows, which show that the trimixing formalism do not explain the nonzero decay width of \(J/\psi \rightarrow \pi\), \(\eta\), and \(\eta'\), while the tetramixing formalism can well reproduce these experimental decay data. The parameters of the mesons and the mixing matrices determined during the fitting process are listed in Tables II and Table III.

The \(Q^2\) behaviors of the form factors of \(\pi\), \(\eta\), and \(\eta'\) are shown in Figs. 1-3, and we see that they are generally in agreement with the experimental data. The \(Q^2\) behavior of the form factor of \(\eta_c\) is shown in Fig. 4, and, by comparing with theoretical data from another model, the calculated curve fits well in most of the lower \(Q^2\) region. We can also obtain the \(Q^2\) behavior of the transition form factors in the timelike region, either by making the substitution \(q_\perp \rightarrow iq_\perp\) or by parameterizing the transition form factors as explicit functions of \(q^2\) in the spacelike region and then extending them through analytic continuum
TABLE I: Experimental data and the light-cone constituent quark model fitting results of the meson decay constants and transition form factors. The experimental data (unmarked) are from Ref. [32], and the experimental data (marked with daggers) are from Ref. [13]. The data in the fourth column (unmarked) are from Ref. [7], and the data (marked with stars) are calculated assuming that $J/\psi$ and $\eta_c$ do not mix.

| Decay Constants or Form Factors | Experimental Data (GeV) | Theoretical Fitting of tetramixing (GeV) | Theoretical Fitting of trimixing (GeV) |
|--------------------------------|-------------------------|----------------------------------------|----------------------------------------|
| $F_{\pi \rightarrow \gamma \gamma^*}(0)$ | 0.2744 ± 0.0082 | 0.2909 | 0.279 |
| $F_{\eta \rightarrow \gamma \gamma^*}(0)$ | 0.2726 ± 0.0074 | 0.2891 | 0.277 |
| $F_{\eta' \rightarrow \gamma \gamma^*}(0)$ | 0.3423 ± 0.0101 | 0.3187 | 0.334 |
| $F_{\eta_c \rightarrow \gamma \gamma^*}(0)$ | 0.0806 ± 0.0004 | 0.0568 | 0 | 0.0810* |
| $f_\omega(\omega \rightarrow e^+e^-)$ | 0.0466 ± 0.0005 | 0.0502 | 0.0456 |
| $f_\rho(\rho \rightarrow e^+e^-)$ | 0.1549 ± 0.0009 | 0.1815 | 0.1603 |
| $f_\phi(\phi \rightarrow e^+e^-)$ | 0.0758 ± 0.0005 | 0.0729 | 0.075 |
| $f_{J/\psi}(J/\psi \rightarrow e^+e^-)$ | 0.2768 ± 0.0044 | 0.2734 | 0.2749* |
| $F_{\omega \rightarrow \pi \gamma^*}(0)$ | 2.2978 ± 0.0403 | 2.4639 | 2.382 |
| $F_{\omega \rightarrow \eta \gamma^*}(0)$ | 0.4494 ± 0.0197 | 0.4285 | 0.454 |
| $F_{\eta' \rightarrow \omega \gamma^*}(0)$ | 0.4260 ± 0.0355 | 0.4528 | 0.461 |
| $F_{\eta_c \rightarrow \omega \gamma^*}(0)$ | ? | -0.0895 | 0 |
| $F_{\rho \rightarrow \pi \gamma^*}(0)$ | 0.8237 ± 0.0549 | 0.9207 | 0.84 |
| $F_{\rho \rightarrow \eta \gamma^*}(0)$ | 1.5687 ± 0.0525 | 1.6124 | 1.50 |
| $F_{\eta' \rightarrow \rho \gamma^*}(0)$ | 1.3175 ± 0.0327 | 1.3818 | 1.39 |
| $F_{\eta_c \rightarrow \rho \gamma^*}(0)$ | ? | -0.0537 | 0 |
| $F_{\phi \rightarrow \pi \gamma^*}(0)$ | 0.1331 ± 0.0032 | 0.1301 | 0.132 |
| $F_{\phi \rightarrow \eta \gamma^*}(0)$ | -0.6937 ± 0.0071 | -0.7106 | -0.677 |
| $F_{\phi \rightarrow \eta' \gamma^*}(0)$ | 0.7153 ± 0.0125 | 0.7261 | 0.727 |
| $F_{\eta_c \rightarrow \phi \gamma^*}(0)$ | ? | -0.0404 | 0 |
| $F_{J/\psi \rightarrow \pi \gamma^*}(0)$ | 0.0006 ± 0.000 03† | 0.0006 | 0 |
| $F_{J/\psi \rightarrow \eta \gamma^*}(0)$ | 0.0035 ± 0.000 07† | 0.0035 | 0 |
| $F_{J/\psi \rightarrow \eta' \gamma^*}(0)$ | 0.0085 ± 0.0002† | 0.0083 | 0 |
| $F_{J/\psi \rightarrow \eta_c \gamma^*}(0)$ | 0.6583 ± 0.0787 | 0.5991 | 0.6545* |
TABLE II: The meson parameters $A$ and $\beta$ (GeV), and the masses (GeV) of constituent quarks determined from the fitting process.

| $A_{\omega I}$ | $A_{\rho I}$ | $A_{\phi I}$ | $A_{J/\psi I}$ | $A_{\pi I}$ | $A_{\eta q}$ | $A_{\eta s}$ | $A_{\eta c}$ | $m_{u(d)}$ | $m_s$ |
|---------------|-------------|-------------|---------------|-------------|-------------|-------------|-------------|-----------|-------|
| 41.4712       | 38.1430     | 63.1638     | 31.1724       | 47.3635     | 38.7860     | 95.4496     | 125.8099    | 0.198     | 0.556 |

| $\beta_{\omega I}$ | $\beta_{\rho I}$ | $\beta_{\phi I}$ | $\beta_{J/\psi I}$ | $\beta_{\pi I}$ | $\beta_{\eta q}$ | $\beta_{\eta s}$ | $\beta_{\eta c}$ | $m_c$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|
| 0.4319          | 0.4318          | 0.4757          | 0.9781          | 0.4112          | 0.4887          | 0.4887          | 0.7373          | 1.270 |

TABLE III: Parameters of the mixing matrices $M_v$ and $M_s$ determined from the fitting process.

| $\theta_1^v$ | $\theta_2^v$ | $\theta_3^v$ | $\theta_4^v$ | $\theta_5^v$ | $\theta_6^v$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $-0.2181^\circ$ | $7.9190^\circ$ | $-7.6665^\circ$ | $-2.6511^\circ$ | $8.4010^\circ$ | $-6.5877^\circ$ |

| $\theta_1^s$ | $\theta_2^s$ | $\theta_3^s$ | $\theta_4^s$ | $\theta_5^s$ | $\theta_6^s$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $-7.8710^\circ$ | $4.6505^\circ$ | $32.4605^\circ$ | $2.1461^\circ$ | $5.8085^\circ$ | $36.7524^\circ$ |

to the timelike region $[34]$. The results are shown in Figs. 5-10, among which Figs. 5 and Fig. 6 are compared with the experimental data, while Figs. 7-10 could be considered as our predictions of the $Q^2$ behaviors of $J/\psi$ transition form factors.

We can further use our results to learn the properties of the intrinsic $c\bar{c}$ component in the light pseudoscalar mesons. With the $F_{P \rightarrow \gamma\gamma^*}(0)$ (where $P = \pi, \eta, \eta', \eta_c$) in Table I and the mixing matrix $M_s$, we obtain the transition form factors of unmixed mesons $F_{P_I \rightarrow \gamma\gamma^*}(0)$ (where $P_I = \pi_I, \eta_q, \eta_s, \eta_c^0$). Taking them into Eq. (16), we have the values of $f_\pi, f_q, f_s,$ and $f_c$:

$$f_\pi = 0.0984 \text{ GeV},$$
$$f_q = 0.0976 \text{ GeV},$$
$$f_s = 0.1298 \text{ GeV},$$
$$f_c = 0.4874 \text{ GeV}.$$

Then we have [7].
FIG. 1: The $Q^2$ behavior of the form factor $Q^2 F_{\pi \to \gamma \gamma^*}(Q^2)$ compared with experimental data [42, 43].

\[
\begin{pmatrix}
  f_\pi^2 & f_\pi^q & f_\pi^s & f_\pi^c \\
  f_\eta^2 & f_\eta^q & f_\eta^s & f_\eta^c \\
  f_\eta'^2 & f_\eta'^q & f_\eta'^s & f_\eta'^c \\
  f_\eta_c^2 & f_\eta_c^q & f_\eta_c^s & f_\eta_c^c
\end{pmatrix}
= M_s
\begin{pmatrix}
  f_\pi & 0 & 0 & 0 \\
  0 & f_q & 0 & 0 \\
  0 & 0 & f_s & 0 \\
  0 & 0 & 0 & f_c
\end{pmatrix}
= \begin{pmatrix}
  0.0974 & 0.0054 & -0.0145 & 0.0168 \\
  -0.0106 & 0.0798 & -0.0729 & -0.0127 \\
  0.0058 & 0.0556 & 0.1059 & 0.0219 \\
  -0.0039 & -0.0006 & -0.0062 & 0.4855
\end{pmatrix}.
\]

We see that $f_{\eta'}^c = 0.0219$ GeV = 21.9 MeV. It is compared with previous results in Table IV. $f_{\eta'}^c$ could be considered as the reflection of the intrinsic charm content of the $\eta'$ meson [17], and we see from Table IV that our result of $f_{\eta'}^c$ is in the similar region with most of the previous results.

In summary, we use the light-cone constituent quark model to study the tetramixing of pseudoscalar mesons $\pi-\eta-\eta'-\eta_c$ and vector mesons $\omega-\rho-\phi-J/\psi$. The parameters of mixing matrices and meson parameters are determined by fitting our theoretical model results of
FIG. 2: The $Q^2$ behavior of the form factor $Q^2 F_{\eta \rightarrow \gamma \gamma^*}(Q^2)$ compared with experimental data [42, 43].

TABLE IV: The value of $f_{\eta'}^c$ (MeV) in different models.

|            | Our model | Feldmann and Kroll [35] | Halperin and Zhitnitsky [36] | Cheng and Tseng [37] and Ma [17] | Cao, Cao, Huang and Yuan and Chao [38] |
|------------|-----------|-------------------------|-----------------------------|----------------------------------|----------------------------------------|
| $f_{\eta'}^c$ | 21.9      | -65−15                  | 50−180                      | -50                              | Around -15                             |

the meson decay constants and transition form factors (at $Q^2 = 0$) to the experimental data. We also calculate the $Q^2$ behaviors of the meson transition form factors, and these results are generally in agreement with the experimental data or results from other models. Our results of the $Q^2$ behaviors of transition from factors of $J/\psi$ decaying into pseudoscalar mesons could be regarded as the predictions of our model, as there are no experimental data at present. The introduction of light quark components in $J/\psi$ and $\eta_c$ not only allows them to decay into the light mesons directly without intermediate gluons or virtual photon but is also helpful for us to understand the structures of charmonium states better. Considering that the mixing introduces a $c\bar{c}$ component into the light mesons, and such a $c\bar{c}$ component is intrinsic to the wave functions and exists over a time scale independent of any probe momentum, we could naturally interpret it as the intrinsic charm of these mesons. Our
FIG. 3: The $Q^2$ behavior of the form factor $Q^2 F_{\eta'\rightarrow\gamma\gamma}(Q^2)$ compared with experimental data [42–44].

result of the intrinsic charm content of the $\eta'$ meson $f_{\eta'}^c$ is also comparable with predictions from other models.

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Appendix A:

The SO(4) group elements can be written in terms of the SO(3) group generators ($A_k$ and $B_k$):

$$M = R_+ R_-, \quad (A1)$$
$$R_+ = e^{-i\theta_k A_k}, \quad R_- = e^{-i\theta_k + 2B_k}, \quad (k = 1, 2, 3). \quad (A2)$$

The generators $A_k$ and $B_k$ obey the commuting relations of SO(3) generators [39]:

$$[A_i, A_j] = i\varepsilon_{ijk} A_k, \quad [B_i, B_j] = i\varepsilon_{ijk} B_k, \quad [A_i, B_j] = 0. \quad (A3)$$
FIG. 4: Prediction of the $Q^2$ behavior of the form factor $Q^2 F_{\eta_c \rightarrow \gamma \gamma^*} (Q^2)$, compared with the predictions in the leading order of the perturbative approach [45]. The dotted curve of the perturbative approach indicates the $Q^2$ region where QCD corrections may alter the predictions slightly.

We see that the groups have the relation $SO(4) = SO(3) \otimes SO(3)$, and the generators $A_k$ (as well as $B_k$) $(k=1,2,3)$ could be seen as the angular momentum operators in each of the three directions.

One form of $A_k$ and $B_k$ is [40]

\[
A_1 = \frac{i}{2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad A_2 = \frac{i}{2} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad A_3 = \frac{i}{2} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad (A4)
\]

\[
B_1 = \frac{i}{2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad B_2 = \frac{i}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad B_3 = \frac{i}{2} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \quad (A5)
\]

Then

\[
R_+ = e^{-i\theta_k A_k} = e^{-i\alpha \cdot A} = e^{-i\alpha A_n}, \quad (A6)
\]
FIG. 5: The $Q^2$ behavior of the form factor $Q^2 F_{\omega\to\pi\gamma^*}(Q^2)$ compared with experimental data [46, 47].

where

$$\alpha = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}, \quad n = \frac{1}{\alpha} (\theta_1, \theta_2, \theta_3).$$

(A7)

From the matrix form of $A_k$, we have

$$A_n = \frac{\theta_k A_k}{\alpha} = \frac{i}{2\alpha} \begin{pmatrix} 0 & -\theta_1 & -\theta_2 & -\theta_3 \\ \theta_1 & 0 & -\theta_3 & \theta_2 \\ \theta_2 & \theta_3 & 0 & -\theta_1 \\ \theta_3 & -\theta_2 & \theta_1 & 0 \end{pmatrix}. \tag{A8}$$

$A_n$ is the angular momentum component of the direction $n$. In fact, the matrix form of $A_n$ in Eq. (A8) can be diagonalized as

$$A'_n = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \tag{A9}$$

which is the expression of angular momentum operator $A_n$ in its eigenstate representation, with the eigenvalues of $A_n$ being $(-1/2, -1/2, 1/2, 1/2)$. The two matrix forms are related
FIG. 6: The $Q^2$ behavior of the form factor $Q^2 F_{\phi \to \eta'\gamma^*}(Q^2)$ compared with experimental data [48].

as $A'_n = S^\dagger A_n S$, with the transformation matrix

$$
S = \frac{1}{\sqrt{2\alpha}} \begin{pmatrix}
\frac{\theta_1 \theta_2 + i\theta_3 \alpha}{\sqrt{\theta_2^2 + \theta_3^2}} & i\theta_2 & \frac{\theta_1 \theta_2 - i\theta_3 \alpha}{\sqrt{\theta_2^2 + \theta_3^2}} & -i\theta_2 \\
\frac{\theta_1 \theta_3 - i\theta_2 \alpha}{\sqrt{\theta_2^2 + \theta_3^2}} & i\theta_3 & \frac{\theta_1 \theta_3 + i\theta_2 \alpha}{\sqrt{\theta_2^2 + \theta_3^2}} & -i\theta_3 \\
0 & \alpha & 0 & \alpha \\
\sqrt{\theta_2^2 + \theta_3^2} & -i\theta_1 & \sqrt{\theta_2^2 + \theta_3^2} & i\theta_1
\end{pmatrix}.
$$

(A10)

In the eigenstate representation of angular momentum $A_n$, the matrix form of $R_+$ is

$$
R'_+ = e^{-i\alpha A_n} = \begin{pmatrix}
e^{\frac{i\alpha}{2}} & 0 & 0 & 0 \\
0 & e^{\frac{i\alpha}{2}} & 0 & 0 \\
0 & 0 & e^{-\frac{i\alpha}{2}} & 0 \\
0 & 0 & 0 & e^{-\frac{i\alpha}{2}}
\end{pmatrix},
$$

(A11)

and then we have

$$
R_+ = S R'_+ S^\dagger = \begin{pmatrix}
\cos \frac{\alpha}{2} & -\frac{\theta_1}{\alpha} \sin \frac{\alpha}{2} & -\frac{\theta_2}{\alpha} \sin \frac{\alpha}{2} & -\frac{\theta_3}{\alpha} \sin \frac{\alpha}{2} \\
\frac{\theta_2}{\alpha} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} & -\frac{\theta_1}{\alpha} \sin \frac{\alpha}{2} & -\frac{\theta_3}{\alpha} \sin \frac{\alpha}{2} \\
\frac{\theta_1}{\alpha} \sin \frac{\alpha}{2} & \frac{\theta_2}{\alpha} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} & -\frac{\theta_3}{\alpha} \sin \frac{\alpha}{2} \\
\frac{\theta_1}{\alpha} \sin \frac{\alpha}{2} & -\frac{\theta_2}{\alpha} \sin \frac{\alpha}{2} & -\frac{\theta_3}{\alpha} \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2}
\end{pmatrix},
$$

(A12)
FIG. 7: Prediction of the $Q^2$ behavior of the form factor $Q^2 F_{J/\psi \to \pi \gamma^*}(Q^2)$

which is the expression of $R_+$ given in Eq. (4). Following the same procedure, we also obtain the expression of $R_-$ given in Eq. (5).

Appendix B:

During the numerical calculation, we write $M$ in a more compact form with eight real parameters ($a$, $b$, $c$, $d$, $p$, $q$, $r$, and $s$) under the constraints $a^2 + b^2 + c^2 + d^2 = 1$ and $p^2 + q^2 + r^2 + s^2 = 1$:

$$M = \begin{pmatrix}
a - b - c - d \\
b a - d c \\
c d a - b \\
d - c b a
\end{pmatrix}\begin{pmatrix}
p - q - r - s \\
q p s - r \\
r - s p q \\
s r - q p
\end{pmatrix}$$

$$= \begin{pmatrix}
ap - bq - cr - ds & -aq - bp + cs - dr & -ar - bs - cp + dq & -as + br - cq - dp \\
bp + aq - dr + cs & -bq + ap + ds + cr & -br + as - dp - cq & -bs - ar - dq + cp \\
cp + dq + ar - bs & -cq + dp - as - br & -cr + ds + ap + bq & -cs - dr + aq - bp \\
dp - cq + br + as & -dq - cp - bs + ar & -dr - cs + bp - aq & -ds + cr + bq + ap
\end{pmatrix}.$$
FIG. 8: Prediction of the $Q^2$ behavior of the form factor $Q^2 F_{J/\psi \to \eta \gamma^*}(Q^2)$

These parameters are related to the six rotation angles as

$$ a = \cos \frac{\alpha}{2}, \quad b = \frac{\theta_1}{\alpha} \sin \frac{\alpha}{2}, \quad c = \frac{\theta_2}{\alpha} \sin \frac{\alpha}{2}, \quad d = \frac{\theta_3}{\alpha} \sin \frac{\alpha}{2}, \quad (B3) $$

$$ p = \cos \frac{\beta}{2}, \quad q = -\frac{\theta_4}{\beta} \sin \frac{\beta}{2}, \quad r = -\frac{\theta_5}{\beta} \sin \frac{\beta}{2}, \quad s = -\frac{\theta_6}{\beta} \sin \frac{\beta}{2}. \quad (B4) $$

When referring to the mixing of specific types of mesons, the parameters ($a$, $b$, ...) change to ($a_v$, $b_v$, ...) or ($a_s$, $b_s$, ...) correspondingly.

[1] S.L. Glashow, Phys. Rev. Lett. 7, 469 (1961).
[2] J.J. Sakurai, Phys. Rev. Lett. 9, 472 (1962).
[3] M. Gell-Mann, *The eightfold way*, W.A. Benjamin, NY (1961).
[4] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); S. Okubo, Progr. Theoret. Phys. 27, 949 (1962).
[5] D.J. Gross, S.B. Treiman, and F. Wilczek, Phys. Rev. D 19, 2188 (1979).
[6] D. Gusbin, Phys. Rev. D 24, 797 (1981).
[7] W. Qian and B.-Q. Ma, Eur. Phys. J. C 65, 457 (2010).
[8] M. Benayoun and H.B. O’Connell, Eur. Phys. J. C 22, 503 (2001).
[9] H. Harari, Phys. Lett. B 60, 172 (1976).
FIG. 9: Prediction of the $Q^2$ behavior of the form factor $Q^2F_{J/\psi \to \eta' \gamma^*}(Q^2)$

[10] H. Fritsch and J.D. Jackson, Phys. Lett. B 66, 365 (1977).
[11] C. Di Donato, G. Ricciardi, and I. Bigi, hep-ph/1105.3557v1 (2011).
[12] H.-W. Ke, X.-Q. Li, and Z.-T. Wei, Eur. Phys. J. C 69, 133 (2010).
[13] T.K. Pedlar et al, (CLEO Collaboration), Phys. Rev. D 79, 111101 (2009).
[14] S.J. Brodsky and M. Karliner, Phys. Rev. Lett. 78, 4682 (1997).
[15] S.J. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, Phys. Lett. B 93, 451 (1980).
[16] S.J. Brodsky, C. Peterson, N. Sakai, Phys. Rev. D 23, 2745 (1981).
[17] J. Cao, F.G. Cao, T. Huang, and B.-Q. Ma, Phys. Rev. D 58, 113006 (1998).
[18] T. Huang and X.-G. Wu, Eur. Phys. J. C 50, 771 (2007).
[19] S.J. Brodsky, T. Huang, and G.P. Lepage, in Quarks and Nuclear Forces, edited by D. Fries and B. Zeitnitz (Springer, Tracts in Modern Physics, Vol. 100) (Springer, New York, 1982); in Particles and Fields-2, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), p.143.
[20] S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, Phys. Rep. 301, 299 (1998).
[21] G.P. Lepage and S.J. Brodsky, Phys. Rev. D 22, 2157 (1980).
[22] B.-W. Xiao and B.-Q. Ma, Phys. Rev. D 71, 014034 (2005).
FIG. 10: Prediction of the $Q^2$ behavior of the form factor $Q^2 F_{J/\psi \rightarrow \eta_c \gamma^*}(Q^2)$

[23] W. Qian and B.-Q. Ma, Phys. Rev. D 78, 074002 (2008).
[24] H.-M. Choi and C.-R. Ji, Nucl. Phy. A 618, 291 (1997).
[25] T. Huang, B.-Q. Ma, and Q.-X. Shen, Phys. Rev. D 49, 1490 (1994).
[26] L.A. Kondratyuk and M.V. Terentev, Sov. J. Nucl. Phys. 31, 561 (1980) [Yad. Fiz. 31, 1087 (1980)].
[27] H.J. Melosh, Phys. Rev. D 9, 1095 (1974); E. Wigner, Ann. Math. 40, 149 (1939).
[28] B.-Q. Ma, J. Phys. G 17, L53 (1991); B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58, 479 (1993); B.-Q. Ma, Z. Phys. A 345, 321 (1993); B.-Q. Ma and I. Schmidt, Phys. Rev. D 58, 096008 (1998).
[29] B.-Q. Ma, Phys. Lett. B 375, 320 (1996); B.-Q. Ma and A. Schäfer, Phys. Lett. B 378, 307 (1996); B.-Q. Ma, I. Schmidt, and J. Soffer, Phys. Lett. B 441, 461 (1998); B.-Q. Ma, I. Schmidt, and J.-J. Yang, Eur. Phys. J. A 12, 353 (2001).
[30] B.-W. Xiao, X. Qian, and B.-Q. Ma, Eur. Phys. J.A, 15, 523 (2002); B.-W. Xiao and B.-Q. Ma, Phys. Rev. D 68, 034020 (2003); J. Yu, B.-W. Xiao and B.-Q. Ma, J. Phys. G 34, 1845 (2007); J. Yu, T. Wang, C.-R. Ji, and B.-Q. Ma, Phys. Rev. D 76, 074009 (2007).
[31] F.-G. Cao and A.I. Signal, Phys. Rev. D 60, 114012 (1999).
[32] C. Amsler, et al. (Particle Data Group), J. Phys. G: Nucl. Part. Phys. 37,075021 (2010).

[33] H.-M. Choi and C.-R. Ji, Nucl. Phys. A 679, 735 (2001).

[34] T. Wang, D.-X. Zhang, and B.-Q. Ma, hep-ph/1004.4274.

[35] T. Feldmann and P. Kroll, Eur. Phys. J. C 5, 327 (1998).

[36] I. Halperin, A. Zhitnitsky, Phys. Rev. D 56, 7247 (1997). For further development, see, E.V. Shuryak and A.R. Zhitnitsky, Phys. Rev. D 57, 2001 (1998).

[37] H.Y. Cheng and B. Tseng, Phys. Lett. B 415, 263 (1997).

[38] F. Yuan and K.T. Chao, Phys. Rev. D 56, 2495 (1997).

[39] W. Pauli, Continuous Groups in Quantum Mechanics, Ergeb. Exak. Naturwiss. 37, 85 (1965).

[40] Z.-Q. Ma and X.-Y. Gu, in Problems and Solutions in Group Theory for Physicists (World Scientific, Singapore, 2004), p.416.

[41] L. van Elfrinkhof, in Eene eigenschap van de orthogonale substitutie van de vierde orde, Handelingen van het zesde Nederlandsch Natuur- en Geneeskundig Congres, Delft, 1897, p. 237-240.

[42] H.-J. Behrend, et al. (CELO Collaboration), Z. Phys. C 49, 401 (1991).

[43] J. Gronberg, et al. (CLEO Collaboration), Phys. Rev. D 57, 33 (1998).

[44] M. Acciarri, et al. (L3 Collaboration), Phys. Lett. B 418, 399 (1998).

[45] T. Feldmann and P. Kroll, Phys. Lett. B 413, 410 (1997).

[46] R. Arnaldi, et al. (NA60 Collaboration), Phys. Lett. B 677, 260 (2009).

[47] L.G. Landsberg, Phys. Rep. 128, 301 (1985).

[48] A.N. Achasov, et al., Phys. Lett. B 504, 275 (2001).