Search for low–dimensional chaos in a reversed field pinch plasma

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Abstract

An analysis of experimental data from the RFX (Reversed Field eXperiment) reversed field pinch machine [L. Fellin, P. Kusstatscher and G. Rostagni, Fusion Eng. Des. 25, 315 (1995)] is carried out to investigate the possible existence of deterministic chaos in the edge plasma region. The mathematical tools used include Lyapunov exponents, Kaplan–Yorke dimension, minimum embedding dimension estimates and nonlinear forecasting. The whole analysis agrees in ruling out the possibility of low–dimensional chaos: The dimension of the underlying dynamical system is estimated to be $> 10$. From a critical re–reading of the literature it emerges that the findings of this work are likely to be common to all reversed field pinches.

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I. INTRODUCTION

Fusion machines are complex systems possessing a huge number of degrees of freedom where phenomena involve many scales of length and time. Such devices are therefore natural candidates for checking of nonlinear theories. In the past years, as soon as the experimental instrumentation made it possible, some effort has been exerted to determine if the temporal behaviour of measured quantities in fusion plasmas, displaying a fluctuating behaviour, could be modelled using some low–dimensional deterministic theory or were the manifestation of truly random processes [1–8]. The results of this search for deterministic chaos are contrasted: while the results of numerical simulations [1] show the existence of low–dimensional chaos, experimental works find positive [2–5] as well as negative results [7,8].

In this paper we present a study on this subject done on experimental data taken from the RFX experiment: a large (R = 2m, a = 0.457m) toroidal device for the magnetic confinement of plasmas in reversed field pinch (RFP) configuration [9] built in Padova (Italy) and designed to reach a plasma current of 2 MA. The data refer to measurements made in the outer region of the plasma, inside the last closed magnetic surface. A number of mathematical tools has been applied to these data to detect traces of deterministic chaos. Furthermore, a critical discussion of the existing literature on the subject is done, in the light of the results found, and some general conclusions are drawn.

In the next section we illustrate the mathematical apparatus used, then a brief description of the diagnostics used to collect the data is given. Section IV presents the results of the analysis and finally some comments are given.
II. NUMERICAL TECHNIQUES

Several analysis techniques have been developed to identify the presence of a low dimensional attractor in a dynamical system from its time series. In this section we describe the mathematical tools used in this work. Some of them are very recent and only recently have been applied to real experimental data. Therefore there is a further interest in verifying how they perform in this situation. All the techniques presented herein apply to a scalar time series of recorded data \( S = (x_1, x_2, \ldots, x_N) \). From the scalar series \( S \) one may construct \( q \)-dimensional vectors in delay–coordinates,

\[ X_i(q) = [x_i, x_{i+\tau}, \ldots, x_{i+(q-1)\tau}] . \]

The \( q \)-dimensional space of delay vectors plays a central role in this kind of analysis. The integer \( \tau \) is known as the time lag. It can be larger than 1, i.e. data may be sampled at a frequency lower than the experimental one. This is done since choosing \( \tau \) smaller than the autocorrelation time \( t_c \) of the data would introduce spurious correlations. On the other hand \( \tau \) should neither be too large. The standard choice is to put \( \tau \approx 2 \div 3 t_c \). In our units, depending upon the data, \( t_c = 1 \div 2 \). To test the reliability of our results, \( \tau \) was varied in the test from 1 to 7 times \( t_c \). Conclusions were unaffected by the choice.

A. Estimate of the minimum embedding dimension

In experiments, the true dimension of the phase space of the system under study (the embedding dimension \( m \)) is usually unknown. A reasonable guess for it is however essential for any analysis. It has been proven by Takens [10] that for a chaotic dynamical system the time recording of a single variable is sufficient to reconstruct the relevant dynamics—and in particular the dimension \( d \) of the attractor—provided that \( m > 2d + 1 \) (in actual cases it is known
that this condition may be slightly relaxed).

In this work we have implemented the method proposed by Cao [11] to estimate the minimum embedding dimension from a scalar time series. This method uses the time–delay vectors (with \( \tau \) set to one for simplicity)

\[
y_i(m) = [x_i, x_{i+1}, x_{i+2}, \ldots, x_{i+(m-1)}], \quad i = 1, 2, \ldots, N - (m - 1)
\]

(2)

(where \( m \) is the guessed embedding dimension) to build the function

\[
a(i, m) = \frac{||y_i(m + 1) - y_{n(i,m)}(m + 1)||}{||y_i(m) - y_{n(i,m)}(m)||}, \quad i = 1, 2, \ldots, N - m,
\]

(3)

where \( || \cdot || \) is a norm and \( n(i, m) \) is an integer such that \( y_{n(i,m)}(m) \) is the nearest neighbour of \( y_i(m) \) in the \( m \)-dimensional space, according to the \( || \cdot || \) norm. The actual functional form of \( || \cdot || \) does not appear to be of importance. We have used the maximum norm:

\[
||Y - Z|| = \max_i |Y_i - Z_i|, \quad i = 1, \ldots, m.
\]

(4)

If \( m \) is the true embedding dimension, any two points which are close together in the \( m \)-dimensional reconstructed space, will stay close also in the \( m + 1 \) space. Points which satisfy this condition are called true neighbours, otherwise false neighbours [12]. Starting from a low value for \( m \) and approaching the correct value (hereafter referred to as \( \nu \)) the number of false neighbours should decrease to zero, or equivalently \( a(i, m) \) should reach a constant value. Cao [11] suggests to use averages of this quantity:

\[
E(m) = \frac{1}{N - m} \sum_{i=1}^{N-m} a(i, m)
\]

(5)

and

\[
E1(m) = E(m + 1)/E(m)
\]

(6)

which allow to obtain results independent upon the sample data chosen. \( E1(m) \) should stop changing when \( m \) becomes greater than some value \( \nu \),
if the time series has a finite dimensional attractor. In the case of random
data \( E1 \) will never saturate but when dealing with real data it is difficult to
distinguish if it has attained a constant value or is slowly increasing, therefore
in Ref. [11] it is recommended to also compute the function

\[
E2(m) = E^*(m+1)/E^*(m) \quad ,
\]

\[
E^*(m) = \frac{1}{N-m} \sum_{i=1}^{N-m} |x_{i+m} - x_{n(i,m)+m}| \quad ,
\]

where the meaning of \( n(i, m) \) is the same as above. For random data \( E2 \) will
stay close to 1: the \( x \)'s are now independent random variables, and therefore
their average distance will be the same regardless of the space dimension
\( m \). For deterministic data, there will be a certain correlation between them
which makes \( E2 \) a function of \( m \). As an illustration of the method, we plot
\( E1, E2 \) in Figure 1 for two sets of data: one time series is computed using the
Mackey–Glass equation

\[
\frac{dx(t)}{dt} = -0.1x(t) + \frac{0.2x(t - \Delta)}{1 + x(t - \Delta)^10}
\]

with \( \Delta = 30 \), which is known to describe the dynamics of a chaotic system
with an attractor dimension of about 3.6 [13]. The other time series is gen-
erated using a random number generator. In the random data series \( E2 \) is
always very close to one and \( E1 \) slowly converges to the same value. The plot
of the deterministic map shows instead that both \( E1 \) and \( E2 \) reach the same
value at \( D \approx 6 \); furthermore the behavior of \( E1 \) is not that of an asymptotic
convergence to 1, but is more akin to the reaching of a threshold value.

B. Lyapunov exponents

The Lyapunov exponents, measuring the average divergence or conver-
gencc of orbits in phase space, are among the most frequently used quantities
to ascertain the presence of chaos. Given a map $x(t) = f_{x_0}(t)$, $f : \mathbb{R}^m \to \mathbb{R}^m$, the Lyapunov exponents are defined as

$$\lambda^{(k)} = \lim_{t \to \infty} \frac{1}{t} \ln \left| \frac{\partial f_{x_0}(t)}{\partial x^{(k)}} \right| = \lim_{t \to \infty} \frac{1}{t} \left| J_t^{(k)} \right| , \quad k = 1, \ldots, m ,$$

where $\left| J_t^{(k)} \right|$ is the $k$th eigenvalue of the $m$-dimensional Jacobian. A necessary condition for a system to be chaotic is to have at least one positive exponent.

Several well established techniques exist to compute the Lyapunov exponents for a system whose dynamical evolution is analytically known. Extracting them from an experimentally determined data set is much more difficult due to the limited length of the sample and to the presence of noise. Existing algorithms usually fit the experimental points to an analytical map $g : \mathbb{R}^m \to \mathbb{R}$ such that

$$g([x_i, \ldots, x_{i+m-1}]) = x_{i+m}$$

or, which is the same, to the map

$$\tilde{g}([x_i, \ldots, x_{i+m-1}]) = [x_{i+1}, \ldots, x_{i+m}] \quad , \quad \tilde{g} : \mathbb{R}^m \to \mathbb{R}^m .$$

Under quite general conditions, the largest $\nu$ Lyapunov exponents of $f$ and $\tilde{g}$ are the same [14,15]. Therefore one estimates the $\lambda$'s using standard techniques on $g, \tilde{g}$. In this work we have used two codes for estimating Lyapunov exponents. The former is the code developed by Watts [6,7], based upon the method by Briggs [14,16], and already used on data from a magnetically confined plasma. In this method the time series is embedded in a delay space of given dimension, a number of nearest neighbours is found, and their trajectory is fit to an analytical function (usually a polynomial: in our runs we have used a polynomial of order 2). Then the Jacobian (10) may be obtained by analytical differentiation. The second code has been developed by one of us
Based upon a method by Gencay and Davis Dechert [15]. Here, a single global fit is done by using logistic maps, \( i.e. \)

\[
g(X_i) = \sum_l v_l = \sum_l \frac{\beta_l}{1 + \exp(-b_l - w_l \cdot X_i)}
\]

(13)

where \( L \) is the number of functions \( v \) and each \( w \) is an \( m \)-dimensional array. The \( b_l \)'s, \( \beta_l \)'s and \( w_l \)'s are fitting parameters. Logistic maps have some interesting features: they may fit arbitrarily well any analytical function as well as its derivative. In Ref. [15] it is stated that this choice of functions has some advantages over the local polynomial approach in terms of stability of results in presence of noise and of fewer needed data points. On the other hand it requires a nonlinear fitting which is computationally more demanding.

### C. The correlation and Kaplan–Yorke dimensions

The correlation dimension \( D_c \) gives another estimate of the embedding dimension of the system. \( D_c \) is defined as

\[
C_d(r) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i \neq j = 1}^{N} H(r - |X_i - X_j|)
\]

(14)

where \( H \) is the Heaviside function and the \( X \)'s are defined in Eq.(1). For \( m > \nu \), \( \lim_{r \to 0} C(r) \approx r^\nu \).

The correlation dimension has been and currently is a favourite tool to diagnose the presence of chaos in fusion plasmas [4,8].

Instead of \( D_c \) we have computed, using Watts’s code [7], the Kaplan–Yorke (or Lyapunov) dimension

\[
D_{KY} = j + \frac{\sum_{i=1}^{j} \lambda_i}{-\lambda_{j+1}}
\]

(15)

where \( j \) is the largest integer such that \( \sum_{i=1}^{j} \lambda_i > 0 \), with the \( \lambda \)'s ordered as \( \lambda_1 > \lambda_2 > \ldots \). There exist some conjectures [8] according to which the uguagliance \( D_{KY} \approx D_c \) holds.
$D_{KY}$ is much easier to compute than $D_c$, since it is only necessary to know the Lyapunov exponents, previously computed. This strength is at the same time a weakness when there may exist uncertainties about the correct value of the exponents. This is our case, however our analysis does not rely just on this single parameter and as we shall see, all results corroborate the indications from $D_{KY}$.

D. Nonlinear forecasting

Given a time series of finite length representative of a chaotic dynamical system governed by the map $f : \mathbb{R}^m \to \mathbb{R}^m$, an inverse problem consists in finding a smooth map $\tilde{g} : \mathbb{R}^m \to \mathbb{R}^m$, or its projection $g : \mathbb{R}^m \to \mathbb{R}$ such that $\tilde{g}$ be an accurate approximation of $f$. In the case $\tilde{g}$ is obtained, it may be used to predict accurately further data points. Otherwise, if the system is not governed by a finite dimensional map, or if the guessed $m$ is too low, the forecasting will be unreliable after few predictions. A review about the subject may be found in Ref. [13].

We have used as fitting function the logistic maps (Eq. [13]): the original time series $S$ of length $N$ has been divided into two parts of lengths $N_1$ and $N_2 = N - N_1$. The first $N_1$ data have been used to fit Eq. (13) and the predictive power has been tested on the remaining $N_2$ points. The measure of goodness is the predictive error:

$$\sigma^2 = \frac{1}{N_2} \sum_{n=N_1+1}^{N} \frac{(x_n - g(X_{n-m}))^2}{\sigma_x^2}$$

where $\sigma_x^2$ is the variance of the time series (in this work all data have been normalized so to have $\sigma_x^2 = 1$). The increase of the trial embedding dimension—provided that enough fitting parameters are allowed—will give a slight decrease of $\sigma^2$ until $m \approx \nu$, when a sudden decrease to much smaller values is expected.
To provide the reader with an example, in Figure 2 we have plotted $\sigma^2$ *versus* $m$ for the Mackey–Glass map where is clearly visible the decrease of more than two orders of magnitude of $\sigma$ when the dimension is $> 4$. For comparison we have estimated the predictive power of the method against a time series generated from a normal distribution. As expected the possibility of any forecast is null, with $\sigma^2$ always greater than the variance of the original data.

It is worth mentioning that estimating the Lyapunov exponents and correlation dimensions has been attempted in all of this kind of studies concerning fusion plasmas. The forecasting of the data, however, has been applied, to our knowledge, only to data from the Madison Symmetric Torus (MST) reversed field pinch device [19]. The estimate from the time series itself of the embedding dimension, finally, is applied in this work for the first time.

Some words must be spent about the confidence which may be assigned to the algorithms. Two crucial topics affecting their performances are: the number of data available and their quality, *i.e.* how much they are polluted with noise. As far as the first point is concerned, usually the more data one can elaborate the larger may be the dimension of the system which may be correctly estimated. We had available records of some thousands values, about as many as used by MST group. The authors of all algorithms used here claim them to be able to reconstruct the correct dynamics of a low–dimensional system (*i.e.* four or five–dimensional) using few hundreds data, with the possible exception of the Watts’ code (see [6,7]). In conclusion, and by comparison with the MST group’s estimates, we may assert that dimensions up to or just below 10 may be correctly detected by our techniques.
Signals coming from three different diagnostic techniques have been analysed in search of low dimensional chaos features:

(a) Floating potential ($V_f$): This is the potential of an electrically insulated conducting probe immersed in the edge plasma. It is known to be related to the local plasma potential $V_p$ and to the local electron temperature $T_e$ (in eV) through the relationship $V_f = V_p - \alpha T_e$ where $\alpha$ is a constant which for the RFX edge plasma is approximately equal to 2.5. The probe was a graphite pin housed in a boron nitride structure which protected it from the unidirectional superthermal electron flow commonly observed in RFP edge plasmas. Data were sampled at 1 MHz. The measurements were collected during the experimental campaign described in [20].

(b) Time derivative of the radial magnetic field ($dB_r/dt$): It was measured with a pick-up coil housed in the same boron nitride structure as the $V_f$ measuring pin. This is a local measurement, like the previous one. Data were sampled at 1 MHz.

(c) Density fluctuations at the edge: Collected by a reflectometer. The RFX reflectometer has been especially designed to deal with high-frequency fluctuations. It is an homodyne reflectometer [21] operating at a maximum sweep rate of 4 GHz/µs in the range 34-38 GHz. The collected signal is of the form

\[ s(t) = A(t) \cos(\Delta\phi(t)) \] (17)

with $A$ amplitude and $\Delta\phi$ phase difference of the reflected radiation at a fixed microwave frequency. In this work we have analyzed the temporal behaviour of the amplitude.

Figure 3 displays the plasma current waveform and typical waveforms for
the three signals considered. The data were collected in discharges having a plasma current ranging between 350 and 600 kA.

IV. RESULTS

Figure 4 displays the quantities $E_1, E_2$ (Eqns. 6, 7) for the three analyzed signals. The data are taken for three different shots (shot 8422 for magnetic fluctuations, shot 7999 for potential measures, and shot 7852 for reflectometer data). A number of data points ranging up to about 10000 has been used. The resemblance with random data (Fig. 1) is impressive, which clearly suggests the existence of a very high dimensional phase space.

A further confirmation is obtained by plotting the Kaplan–Yorke dimension (Fig. 5): no sign of saturation is obtained and over the whole explored range $D_{KY}(m) \approx m$.

In Figure 6 we plot the predictive error (Eq. 16). The value of $\sigma$ is always close to one, meaning that no real accurate prediction may be done. Some very small differences may be seen among the three signals, even if they may well be just subjective impressions: the magnetic fluctuations data show the worst predictive power, suggesting perhaps that some different mechanism from the other two diagnostics is at work. This could be the case if magnetic fluctuations are mostly due to magnetohydrodynamics tearing modes whereas potential and density fluctuations are mainly caused by some electrostatic instabilities localized in the edge region. For the numerical aspect, the calculation of this quantity turned out to be rather reliable, with limited variations between runs.

Finally, in Figure 7 we plot the largest Lyapunov exponents. Since the system does not appear to be dominated by low-dimensional chaos, the usefulness of these coefficients is rather limited, however it may be interesting
to compare the two approaches. The estimation of the $\lambda$’s from real data is known to be a difficult task; we did not obtain stable results varying the parameters: indeterminacies of 50% are quite likely, so the values shown are to be considered as representative of the general trend. We found that Watts’s code is rather sensitive to the choice of the input parameters such as the order of the polynomial and the number of neighbours to be used in the fitting. Our code is more stable from this point of view since it needs very few input data (essentially, the embedding dimension $m$ and the number of logistic map $L$). However, it is difficult to perform the fit over a large number of data points (600 is the maximum used), so the results suffer of the scarce statistics.

In Watts’s work it is emphasized that, in order for the previous analysis to hold, the physical system must be in a stationary state. The breaking down of this assumption may translate to an overestimate of the attractor dimension or even make it non measurable. Watts quotes two possible causes for the lack of non–stationarity: (a) The plasma may not reach a state of equilibrium; (b) Even if equilibrium is reached, random perturbations (Watts cites as an example the influx of impurity ions from the walls) may destroy it. In RFX discharges true flat–top periods lasting some tens of milliseconds are reached. In our study we have considered both discharges where this flat–top period was reached, and others where instead plasma parameters were slowly changing. Conclusions are unaffected by the discharge chosen. We are therefore confident that any lack of stationarity due to this causes may just very slightly modify the results. Point (b) is by nature uncontrollable: however, since the same results are obtained from different discharges it may be hoped that these random events have not affected the final results.

All our results agree in clearly pointing out that no low–dimensional chaos appears at the RFX edge. This is well consistent with findings of Watts et al. [6,7] for MST. When limiting to RFP’s the only other research within this
field is that done in HBTX1A [22] by Gee and Taylor [2], where traces of low-dimensional chaos were found in magnetic-field oscillations. This sharp discrepancy puzzled us, therefore we resorted to check conclusions of Gee and Taylor against their own data. In Ref. [2] the attractor dimension is estimated using the correlation dimension technique (Eq. [14]); their figure 1 shows the behaviour of $\log C(r)$ versus $\log r$. Gee and Taylor claim that the slope of these lines saturate in correspondence of a dimension $\approx 7$. We could not check this since in their figure 1 only dimensions from 1 to 6 are plotted. However, from the data available, we could not find any clue of such a saturation: the slope of the straight line for $m = 6$, as estimated by visual inspection of the plot, appears close to 8. Even allowing for the large error induced by our gross way of estimating, the true result cannot be much smaller than 6, which is a necessary condition to speak about a saturation of the slope [23].

Up to this point, our work has focussed almost entirely on RFP plasmas. Some interesting considerations may be drawn by comparison with those tokamak plasmas where the fingerprints of deterministic chaos have been found. We refer to the works [3,4]. From Ref. [3], using data of TOSCA [24] and the Joint European Torus (JET) [25] tokamaks, it appears that a small value of the dimension of the dynamical system is more likely to be found if the level of turbulence (measured for example by $\delta B/B$) is small, which is not what happens in RFP’s. Barkley’s data [4] are a bit difficult to interpret since their analysis are done on filtered data, i.e. by selecting the wave number components $k$. Their finding is that the dimension increases with $k$. However, Barkley’s data are chord averaged, so the central plasma plays a dominant role. In our work, and in the others studied, only edge quantities have been considered. It is quite possible that different mechanisms be at work in the two zones.
V. SUMMARY AND CONCLUSIONS

In this work four statistical tools have been applied to the signals of some plasma turbulence measurements (magnetic field–, electrostatic potential–, and density fluctuations) of the RFX experiment, addressing the question of the existence of low dimensional deterministic chaos in them. The methods of analysis are well established (correlation dimension) as well as more recent (predicting errors and minimum embedding dimension estimates). All the conclusions are strikingly in accordance in ruling out that the dynamics of the edge plasma in RFP’s may be significantly affected by any low–dimensional process [26]. An estimate of a lower bound for the dimension of the system may be given by Figure 6, which shows that this value must be greater than 10. This conclusion is further strengthened by a critical re–examination of previous results [2] conflicting with ours, which has shed some doubt about their validity.

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[23] Notice that also Watts (see Ref. [6]) did a critical re-reading of most of the papers dealing with chaos in fusion devices, reaching the conclusion that the results shown in many of them suffered of an unappropriate elaboration of the data. Gee and Taylor’s results, however, were not explicitly questioned by Watts.

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[26] This is not exactly the same as asserting that no low-dimensional processes exist. Actually, the signals may be composed by a contribution coming from these processes together with
another due to turbulence. What we can say is that the latter is overwhelming.
FIGURES

FIG. 1. Estimate of the minimum embedding dimension using Cao’s method (section II A).

FIG. 2. Diamonds, predictive error for the Mackey–Glass equation (Eq. 9) versus embedding dimension as estimated by the method of section II D; Squares, the same for a series randomly generated from a normal distribution.

FIG. 3. Examples of the signals used. Smooth curves are the plasma current ($I_p$), the wildly fluctuating ones are the signals. Note that we are referring to three different discharges.

FIG. 4. $E_1$, $E_2$ (see section II A) for the three signals. Here and in all the following plots $V$ stands for potential fluctuations; $B$, magnetic field; $n_e$, density fluctuations. Diamonds, $E_1$; Stars, $E_2$.

FIG. 5. Kaplan–Yorke dimension for the three signals.

FIG. 6. The largest Lyapunov exponent versus embedding dimension. Stars are results from Watts’ code, diamonds from our code.

FIG. 7. Predictive error versus embedding dimension.
