Higgsinoless Supersymmetry and Hidden Gravity

Michael L. Graesser\textsuperscript{a}, Ryuichiro Kitano\textsuperscript{b} and Masafumi Kurachi\textsuperscript{a}

\textsuperscript{a}Theoretical Division T-2, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
\textsuperscript{b}Department of Physics, Tohoku University, Sendai 980-8578, Japan

Abstract

We present a simple formulation of non-linear supersymmetry where superfields and partnerless fields can coexist. Using this formalism, we propose a supersymmetric Standard Model without the Higgsino as an effective model for the TeV-scale supersymmetry breaking scenario. We also consider an application of the Hidden Local Symmetry in non-linear supersymmetry, where we can naturally incorporate a spin-two resonance into the theory in a manifestly supersymmetric way. Possible signatures at the LHC experiments are discussed.
1 Introduction

Technicolor is an attractive idea in which electroweak symmetry is dynamically broken by a strong dynamics operating around the TeV energy scale \[1, 2\]. The big hierarchy, \(m_W \ll M_{\text{Pl}}\), is elegantly explained by the very same reason as \(\Lambda_{\text{QCD}} \ll M_{\text{Pl}}\). However, it is well-known that there are two phenomenological difficulties in this idea. One is that the electroweak precision measurements seem to prefer scenarios with a weakly coupled light Higgs boson \[3\]. Another is the difficulty in writing down the Yukawa interactions to generate fermion masses in the Standard Model.

After the LEP-I experiments, supersymmetry (SUSY) has become very popular as a natural scenario for the light weakly coupled Higgs boson. However, with the experimental bound on the lightest Higgs boson mass from the LEP-II experiments, parameters in the minimal SUSY standard model (MSSM) are required to be more and more fine-tuned, at least in the conventional scenarios \[4, 5, 6\].

In this situation, it may be interesting to (re)consider a hybrid of technicolor and SUSY along the similar spirit of the early attempts of SUSY model building \[7, 8\]. We assume that strong dynamics breaks SUSY at the multi–TeV energy scale (which we call the scale \(\Lambda\)), with electroweak symmetry breaking triggered by the dynamics through direct couplings between the Higgs field and the dynamical sector. This scenario has several virtues: (1) the Yukawa interactions can be written down by assuming an existence of elementary Higgs fields in the UV theory, which mix with (or remain as) the Higgs field to break electroweak symmetry at low energy \[9, 10, 11\]; (2) the hierarchy problem, \(\Lambda \ll M_{\text{Pl}}\), is explained by dynamical SUSY breaking \[12\]; (3) one can hope that the little hierarchy, \(m_W \sim m_h \ll \Lambda\), is explained by either SUSY or some other mechanisms such as the Higgs boson as the pseudo-Nambu-Goldstone particle in the strong dynamics \[13\]; (4) the cosmological gravitino problem is absent \[14\]; (5) one can expect additional contributions to the Higgs boson mass from the SUSY breaking sector, with which the mass bound from the LEP-II experiments can be evaded \[15\]; and (6), there is an interesting possibility that the LHC experiments can probe the SUSY breaking dynamics directly. SUSY is phenomenologically motivated from the point (1) (and also (6)) in this framework in addition to the connection to string theory.

Although the TeV-scale SUSY breaking scenario is an interesting possibility, an explicit model realizing this scenario will not be attempted here. In this paper, we take a less ambitious approach and construct an effective Lagrangian for the scenario...
without specifying (while hoping for the existence of) a UV theory responsible for SUSY breaking and its mediation.

In constructing the effective Lagrangian, we take the following as organizing principles: (1) the Lagrangian possesses non-linearly realized supersymmetry; (2) the quarks/leptons and gauge fields are only weakly coupled to the SUSY breaking sector, so that the typical mass splitting between bosons and fermions are $O(100)$ GeV (in other words, the matter and gauge fields are introduced as superfields which transform linearly under SUSY); and (3) the Higgs boson is introduced as a non-linearly transforming field because it is assumed to be directly coupled to the SUSY breaking sector. The Higgsino field is absent in the minimal model.

In this Higgsinoless model, the Higgs potential receives quadratic divergences from loop diagrams with the gauge interactions and the Higgs quartic interaction although the top-quark loops can be cancelled by the loops of the scalar top quarks as usual. The rough estimate of the correction to the Higgs boson mass is of the order of $(\alpha/4\pi)\Lambda^2$ and $(k/16\pi^2)\Lambda^2$ with $k$ being the coupling constant of the Higgs quartic interaction. By comparing with the quadratic term needed for electroweak symmetry breaking, $m_H^2 = k\langle H \rangle^2/2$, naturalness suggests $\Lambda \lesssim 4\pi\langle H \rangle \sim (a \text{ few}) \times \text{ TeV}$. Precision electroweak constraints, on the other hand, obtained from the LEP-II and SLC experiments do not generically allow such a low scale without fine-tuning [16]. The dynamical scale may therefore have to be larger, $\Lambda \simeq O(6 - 10 \text{ TeV})$. To obtain a light Higgs boson at this larger scale either requires fine-tuning, or some new weakly coupled new physics below $\Lambda$. (Or simply the Higgsino appears around a few TeV.) It is also true that the direct coupling to the dynamical sector generically gives the Higgs boson mass to be $O(\Lambda)$. We may therefore need to assume that the Higgs boson is somewhat special in the dynamics, e.g., a pseudo-Goldstone boson. In this paper we simply ignore the issue because its resolution depends on the UV completion, and here we only concerned with the effective theory below the TeV scale.

The stop potential also receives quadratic divergences, in this case from a loop diagram involving the Higgs boson (and proportional to $\lambda_t^2$). This divergence is not cancelled, simply because the Higgsinos are not present in the low energy theory. One therefore expects the stops to have a mass no smaller than a loop factor below the scale $\Lambda$.

The Lagrangian we construct needs to contain interaction terms among superfields
and also partnerless fields such as the Higgs boson. Since these two kinds of fields are defined on different spaces – one superspace and the other the usual Minkowski space – one needs to convert the partnerless fields into superfields or vice-versa. One approach is to utilize established formulations for constructing superfields out of partnerless fields \[17, 18, 19\] where the Goldstino field is also promoted to a superfield. In this paper, we present a simple manifestly supersymmetric formulation where we do not try to convert partnerless fields into superfields, although it is totally equivalent to the known formalisms. The essence is to prepare two kinds of spaces: the superspace and the Minkowski space, on which superfields and partnerless fields are defined. By embedding the Minkowski space into the superspace by using a SUSY invariant map, one can define a Lagrangian density on a single space-time. By using the formalism, one can write down a SUSY invariant Lagrangian, in particular the Yukawa interactions, only with a single Higgs field. We also find that the coupling constant of the Higgs quartic interaction can be a free parameter, unrelated to the gauge coupling constant. Therefore, the Higgs boson mass can be treated as a free parameter in this model.

As a related topic, a model in which the MSSM is only partly supersymmetric has been proposed in Ref. \[20\]. There SUSY is broken explicitly at the Planck scale, and only the Higgs sector is remained to be supersymmetric which is made possible by a warped extra-dimension (or a conformal dynamics). Our philosophy is opposite to that and is, relatively speaking, closer to Ref. \[21\] by the same authors, where SUSY is broken on the IR brane (or equivalently by some strong dynamics at the \(O(\text{TeV})\) scale).

As a possible signature of the TeV-scale dynamics, we construct a model “Hidden Gravity,” which is an analogy of the Hidden Local Symmetry \[22, 23, 24, 25, 26\] in the chiral Lagrangian. The Hidden Local Symmetry is a manifestly chiral symmetric model to describe the vector resonance (the \(\rho\) meson) as the gauge boson of the hidden vectorial SU(2) symmetry (the unbroken symmetry of the chiral Lagrangian). When we apply this technique to SUSY, we obtain a supersymmetric Lagrangian for a massive spin-two field which is introduced as a graviton associated with a hidden general covariance because the unbroken symmetry is the Poincaré symmetry.

One can consistently incorporate the resonance as a non-strongly coupled field for a range of parameters and small range of energy. Indeed, we show that there is a sensible parameter region where we can perform a perturbative calculation of the resonant single-graviton production cross section. At energies not far above the graviton mass
the effective theory becomes strongly coupled and incalculable. If the graviton is much lighter than the cut-off scale, new physics is required to complete the theory up to \( \Lambda \), another direction not pursued here. We discuss signatures of this graviton scenario at the LHC.

2 Non-linear SUSY and invariant Lagrangian

In this section we present a method to construct a Lagrangian invariant under the non-linearly realized global supersymmetry. We will introduce the Higgs boson as a non-linearly transforming field (which we call a non-linear field) and also matter and gauge fields as superfields. We therefore need a formulation to write down a supersymmetric Lagrangian where both kinds of fields are interacting. Roček [17], Ivanov and Kapustnikov [18], and Samuel and Wess [19] have established a superfield formalism of non-linear SUSY by upgrading the Goldstino fermion and other non-linear fields to constrained superfields. (See [27] for a recent work.) Although the formalism is somewhat complicated, using superfields is motivated there as a first step towards embedding the theory into supergravity. As we are not interested in supergravity in this paper, we will use a simpler formalism where the Goldstino field remains as a non-linearly transforming field. We will also use results from earlier work by Ivanov and Kapustnikov [28] that establishes the correspondence between superfields and non-linear fields.

2.1 Convention and superfields

We use the metric convention: \( \eta_{ab} = \text{diag}(+---) \). The SUSY algebra is

\[
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^a_{\alpha\dot{\beta}}P_a. \tag{1}
\]

Under a group element,

\[
g = e^{ic^aP_a+i\eta Q+i\bar{\eta}\bar{Q}}, \tag{2}
\]

the superspace coordinate \((x^a, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})\) transforms as [30]

\[
x^a \rightarrow x'^a = x^a + c^a + \Delta^a(\eta, \theta), \quad \theta_\alpha \rightarrow \theta'_\alpha = \theta_\alpha + \eta_\alpha, \quad \bar{\theta}_{\dot{\alpha}} \rightarrow \bar{\theta}'_{\dot{\alpha}} = \bar{\theta}_{\dot{\alpha}} + \bar{\eta}_{\dot{\alpha}}, \tag{3}
\]

*The generalization of this relationship to local supersymmetry can be found in Ref. [23].
where the $\Delta^a$ factor is defined by

$$\Delta^a(\eta, \xi) \equiv i\eta\sigma^a\bar{\xi} - i\xi\sigma^a\bar{\eta}. \quad (4)$$

A superfield $\Psi(x, \theta, \bar{\theta})$ transforms as

$$g\Psi(x, \theta, \bar{\theta})g^{-1} = r(g^{-1})\Psi(x, \theta, \bar{\theta}) = \Psi(x', \theta', \bar{\theta}') \quad (5)$$

under SUSY. The operation $r(g^{-1})$ is a representation of $g^{-1}$ on superfields defined by the second equality.

### 2.2 Non-linear SUSY

The non-linear transformation under $g$ in Eq. (2) is defined by Volkov and Akulov in Ref. [31]. It is

$$\tilde{x}^\mu \to \tilde{x}^\nu' = \tilde{x}^\mu + c^\mu + \Delta^\nu(\eta, \lambda(\tilde{x})), \quad (6)$$

$$\lambda_\alpha(\tilde{x}) \to \lambda'_\alpha(\tilde{x}') = \lambda_\alpha(\tilde{x}) + \eta_\alpha, \quad (7)$$

$$\bar{\lambda}_\dot{\alpha}(\tilde{x}) \to \bar{\lambda}'_\dot{\alpha}(\tilde{x}') = \bar{\lambda}_\dot{\alpha}(\tilde{x}) + \bar{\eta}_\dot{\alpha}. \quad (8)$$

The fields $\lambda$ and $\bar{\lambda}$ are the Goldstino fermion and its complex conjugate, respectively. The transformation above satisfies the algebra in Eq. (1). Note that a global SUSY transformation induces a general coordinate transformation in Eq. (6) on the $\tilde{x}$ space.

One can construct the Maurer-Cartan 1-forms [32]:

$$A^a_\mu = \eta^a_\mu - i\lambda^a\partial_\mu\bar{\lambda} + i\partial_\mu\lambda^a\bar{\lambda}, \quad (9)$$

$$\nabla_\alpha\lambda = (A^{-1})^a_\mu\partial_\mu\lambda, \quad (10)$$

$$\nabla_\alpha\bar{\lambda} = (A^{-1})^a_\mu\partial_\mu\bar{\lambda}. \quad (11)$$

The matrix $A$ transforms as the vielbein under $g$:

$$A^a_\mu(\tilde{x}) \to A'^a_\mu(\tilde{x}') = \frac{\partial \tilde{x}^\nu}{\partial x^\mu}A^a_\nu(\tilde{x}). \quad (12)$$

\(^1\)Throughout this paper we will use the shorthand notation $\lambda = \lambda(\tilde{x})$ with mass dimension $-1/2$, unless indicated otherwise.
whereas $\nabla_a \lambda$ and $\nabla_a \bar{\lambda}$ are invariant.

Matter fields $\phi(\tilde{x})$ can be introduced on the $\tilde{x}$ space. The SUSY transformation on operators is defined by [28]

$$g \phi(\tilde{x}) g^{-1} = \tilde{r}(h^{-1}(g, \lambda)) \phi(\tilde{x}) = \phi(\tilde{x}'),$$

(13)

where $\tilde{r}(h^{-1})$ is the representation of the space-time translation acting on the $\tilde{x}$ space defined in Eq. (6). A supersymmetric action for $\phi(\tilde{x})$ can be obtained simply by writing an invariant action under the general coordinate transformation in Eq. (6) by using the vielbein in Eq. (9).

Superfields and non-linear fields are living in different spaces $x$ and $\tilde{x}$ which we cannot identify as the same space at this stage since their SUSY transformations are different. In order to write down an interaction term between $\phi(\tilde{x})$ and superfields, we need a “converter” which transforms a field in the $\tilde{x}$ space into a superfield in the superspace $(x, \theta, \bar{\theta})$.

The discussion is completely parallel to the formalism of Callan-Coleman-Wess-Zumino (CCWZ) for internal global symmetries [33]. (See also [25] for a review.) There a global symmetry $G$ is spontaneously broken down to a subgroup $H$. A Lagrangian which is invariant under the global $H$ transformation can be upgraded to a $G$ invariant one by making the Lagrangian invariant under a local $H$ transformation where the Maurer-Cartan 1-form (projected onto the unbroken generators) can be used as the gauge connection.

In the CCWZ formalism, a linear representation of a group element $\xi(x) \equiv e^{i\pi(x)} \in G$ plays a role of the converter between the $G$ and $H$ indices by defining the transformation of $\xi$ to be $\xi(x) \rightarrow g \xi(x) h^{-1}(g, \pi)$. We can follow a similar prescription here by taking the converter $r(\Xi)$ with $\Xi = e^{iQ \lambda + i\bar{Q}\bar{\lambda}}$. For example, a superfield $\Phi(x, \theta, \bar{\theta})$ can be constructed from a non-linear field $\phi(\tilde{x})$ by

$$\Phi(x, \theta, \bar{\theta}) \equiv r(\Xi) \phi(x) = \phi(x - \Delta(\lambda(\tilde{x}), \theta)).$$

(14)

At this stage, $\Phi$ is not defined yet because the last expression still contains $\tilde{x}$. The appropriate identification is found to be

$$\tilde{x}^\mu = x^\mu - \Delta^\mu(\lambda(\tilde{x}), \theta),$$

(15)

‡In terms of the same notation in Eqs. (3) and (4), the transformation of the classical field (or expectation values of the field) is: $\phi(\tilde{x}) \rightarrow \phi'(\tilde{x}') = \phi(\tilde{x})$. For a classical superfield, $\Psi(x, \theta, \bar{\theta}) \rightarrow \Psi'(x', \theta', \bar{\theta}') = \Psi(x, \theta, \bar{\theta})$. The transformation laws remain unchanged for fields with Lorentz indices.
which consistently defines the superfield $\Phi$ [28]. (See also [17, 19, 34, 35, 36, 37] for constructing superfields out of non-linear fields). There is still a little bit of complication because the above equation is non-linear in $\tilde{x}$. It is possible to iteratively solve $\tilde{x}$ in terms of $x, \lambda(x)$ and $\theta$, but the solution involves many terms although the iterations will be terminated at finite steps. Nonetheless, one can explicitly check that $\Phi(x, \theta, \bar{\theta})$ is a superfield, i.e.,

$$\Phi(x', \theta', \bar{\theta}') = \phi(\tilde{x}')$$

because

$$\tilde{x}' \mu = x' \mu - \Delta^\mu(\lambda'(\tilde{x}'), \theta'). \quad \text{(17)}$$

In general, any function of

$$\phi(\tilde{x}), \quad \theta - \lambda(\tilde{x}), \quad \bar{\theta} - \bar{\lambda}(\tilde{x}), \quad \text{with } \tilde{x} \text{ defined by Eq. (15) is a superfield.} \quad \text{(18)}$$

As an equivalent formulation, one can construct a supersymmetric action using the supersymmetric invariant

$$1 = \int d^4 \tilde{x} \det X \delta^4(x^\mu - \tilde{x}^\mu - \Delta^\mu(\lambda(\tilde{x}), \theta)), \quad \text{(19)}$$

in the superspace integral. The Jacobian matrix $X$ is

$$X^a_\mu = \eta^a_\mu - i \theta^a \partial_\mu \bar{\lambda} + i \partial_\mu \lambda^a \bar{\theta}, \quad \text{(20)}$$

which transforms in the same way as $A$, and is equal to $A$ at $\theta = \lambda(\tilde{x}), \bar{\theta} = \bar{\lambda}(\tilde{x})$. With the delta function, one can treat $x, \theta, \bar{\theta}$, and $\tilde{x}$ as independent variables in constructing the Lagrangian. The invariant action can be written down as

$$S = \int d^4x d^4 \theta d^4 \tilde{x} \delta^4(x^\mu - \tilde{x}^\mu - \Delta^\mu(\lambda(\tilde{x}), \theta)) \times K \left[ \Psi(x, \theta, \bar{\theta}), \phi(\tilde{x}), \theta - \lambda, \bar{\theta} - \bar{\lambda}, \nabla_\mu \lambda, \nabla_\mu \bar{\lambda}, A, X, \cdots \right], \quad \text{(21)}$$

where $\Psi$ and $\phi$ represents arbitrary superfields and non-linear fields, respectively. The function $K$ must be real and scalar under the general coordinate transformation about the $\tilde{x}$ coordinate. As an example of the invariant action, we can take $K = \Psi(x, \theta, \bar{\theta})$ which gives the supersymmetric action,

$$S = \int d^4x d^4 \theta \Psi(x, \theta, \bar{\theta}). \quad \text{(22)}$$
If we take $\mathcal{K} = \delta^4(\theta - \lambda) \cdot (-f^4/2)$, we obtain the Volkov-Akulov action \[31\]

$$S = -\frac{f^4}{2} \int d^4\tilde{x} \det A.$$

(23)

This contains the kinetic term for the Goldstino. The parameter $f$ is the decay constant which represents the size of the SUSY breaking. Note that naive dimensional analysis \[38\] implies a cutoff $\Lambda \sim \sqrt{4\pi f}$.\[39\]

One may generalize the Volkov-Akulov action. From the invariance of $\nabla_a \lambda$,

$$S = -\frac{f^4}{2} \int d^4\tilde{x} \det A \, F(\nabla_a \lambda, \nabla_b \bar{\lambda})$$

(24)

is SUSY invariant for any $F$ that forms a Lorentz invariant out of $\nabla_a \lambda$ and/or $\nabla_b \bar{\lambda}$ \[34\] \[32\]. Another possibility is to consider the “metric”

$$G_{\mu\nu} \equiv A_{\mu}^a A_{\nu}^b \eta_{ab} \tag{25}$$

which transforms as a covariant tensor and can be used to build invariant actions. For instance,

$$\int d^4\tilde{x} \det A \, R[G] \tag{26}$$

is invariant under global SUSY transformations. The leading term begins at $O(\partial^3)$ and involves two Goldstinos. Terms involving four Goldstinos have $O(\partial^4)$ and so on, with the last term involving 8 Goldstinos and 6 derivatives.

Lagrangian densities with a single space-time coordinate $x$ or $\tilde{x}$ are obtained by performing one of the space-time integrals, i.e., of the form:

$$S = \int d^4 x L(x) + \int d^4 \tilde{x} \tilde{L}(\tilde{x}). \tag{27}$$

This action is of course identical to

$$S = \int d^4 x \left( L(x) + \tilde{L}(x) \right). \tag{28}$$

\[\text{§}\] Although the Volkov-Akulov action involves many terms with different numbers of derivatives, the momentum expansion still makes sense once we fix the number of external lines in each amplitude. For example, the lowest order (tree) amplitudes with $d$ external Goldstinos is $O(p^d)$ and $n$-loop corrections to that are $O(p^{d+4n})$. When comparing with other terms in the action, we should count the number of derivatives with a fixed number of the Goldstino fields. We can easily see that terms in Eq. (24) and (26) contain more derivatives than the Volkov-Akulov action.
We now have a Lagrangian in a single space-time.

Superpotential-like terms can also be constructed as

\[
S = \int d^4y d^2\theta d^4\tilde{x} \det Y \delta^4(y^\mu - \tilde{x}^\mu - i\lambda\sigma^\mu\bar{\lambda} + 2i\theta\sigma^\mu\bar{\lambda})
\times \mathcal{W}[\Psi(y, \theta), \phi(\tilde{x}), \theta - \lambda, \nabla_a \lambda, \nabla_a \bar{\lambda}, A, \cdots] + \text{h.c.},
\]

with

\[
y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta},
\]

and

\[
Y_{\mu}^a = \eta_{\mu}^a + i\partial_\mu \lambda \sigma^a \bar{\lambda} + i\lambda \sigma^a \partial_\mu \bar{\lambda} - 2i\theta\sigma^a \partial_\mu \bar{\lambda}.
\]

There is an intuitive picture for this construction. We can imagine a set-up where a 3-brane is embedded into a superspace. The Goldstino field \(\lambda(\tilde{x})\) defines the map from a point on the brane to a point in the superspace. One can write down a usual superspace Lagrangian as well as a brane localized action. The brane action should be invariant under the general coordinate transformation because SUSY, which is a translation in the superspace, induces a coordinate transformation (which depends on \(\lambda(\tilde{x})\)) on the brane. The interaction terms between superfields (bulk fields) and brane fields can be written down by using a delta function.

### 2.3 Gauge invariance

It is now possible to write down an interaction term between a chiral superfield \(\mathcal{O}(y, \theta)\) and a non-linear field \(\phi(\tilde{x})\):

\[
S = \int d^4\tilde{x} d^2\theta \det A \mathcal{O}(\tilde{x}^\mu + i\lambda\sigma^\mu\bar{\lambda} - 2i\theta\sigma^\mu\bar{\lambda}, \theta)\phi(\tilde{x}) + \text{h.c.},
\]

from

\[
\mathcal{W} = \mathcal{O}(y, \theta)\phi(\tilde{x}).
\]

By taking \(\mathcal{O}\) as a bilinear of the quarks/leptons superfields and \(\phi\) as the Higgs boson, this gives a supersymmetric Yukawa interaction term.
However, if $O$ and $\phi$ are charged under some gauge symmetry (as it is true in the Standard Model), we need to modify the interaction term since it is not gauge invariant. The gauge transformation is defined by

$$O \rightarrow e^{i\Lambda^a(y,\theta)\tilde{T}^a}O(y,\theta),$$  

(34)

and

$$\phi \rightarrow e^{ia^a(\tilde{x})T^a}\phi(\tilde{x}),$$

(35)

where $(\tilde{T}^a)_{ij} = -(T^a)_{ji}$. Under this transformation, the action is clearly not invariant.

In order to maintain gauge invariance, we write the action as

$$S = \int d^4\tilde{x} d^4\theta d^4\bar{\theta} \det X \delta^4(x - \tilde{x} - \Delta(\lambda, \theta)) \delta^4(\theta - \bar{\lambda}) \left(\frac{1}{2} e^{2gV} D^2e^{-2gV}O\right)\phi(\tilde{x})$$

$$= \left.\left(\frac{1}{2} e^{2gV} D^2e^{-2gV}O\right)\right|_{x=\tilde{x},\theta=\lambda,\bar{\theta}=\bar{\lambda}}\phi(\tilde{x}),$$

(36)

where $V = V^a\tilde{T}^a$. The gauge transformation of the vector superfield is

$$e^{-2gV} \rightarrow e^{i\Lambda^a} e^{-2gV} e^{-i\Lambda}.$$  

(37)

The derivative operators are defined by

$$D_a = \frac{\partial}{\partial \theta^a} - i(\sigma^a\bar{\theta})_a \frac{\partial}{\partial x^a}, \quad \bar{D}_a = -\frac{\partial}{\partial \bar{\theta}^a} + i(\theta\sigma^a)_a \frac{\partial}{\partial \bar{x}^a}. $$

(38)

By defining the gauge transformation of $\phi(\tilde{x})$ with

$$\alpha^a(\tilde{x}) = \Lambda^a(y, \lambda) = \Lambda^a(\tilde{x}, \lambda, \bar{\lambda}),$$

(39)

the interaction term in Eq. (36) is gauge invariant. Note, however, that the function $\alpha(\tilde{x})$ defined above can be complex valued unlike the usual gauge transformation (it is real in Wess-Zumino gauge).

It is possible to define a covariant derivative to write down a kinetic term for $\phi(\tilde{x})$. We define gauge superfields

$$g\mathcal{A}_a(x, \theta, \bar{\theta}) \equiv \frac{1}{4} \bar{D} e^{2gV} \sigma_a D e^{-2gV},$$

(40)

and

$$g\mathcal{A}_a(x, \theta, \bar{\theta}) \equiv e^{2gV} D_a e^{-2gV}. $$

(41)
The gauge transformations of these superfields are

\[ A_a \rightarrow e^{i\Lambda} A_a e^{-i\Lambda} + \frac{i}{g} e^{i\Lambda} \frac{\partial}{\partial x^a} e^{-i\Lambda}, \]  

\[ A_\alpha \rightarrow e^{i\Lambda} A_\alpha e^{-i\Lambda} + \frac{1}{g} e^{i\Lambda} D_\alpha e^{-i\Lambda}. \]  

(42)

(43)

We defined \( V = V^a T^a \) and \( \Lambda = \Lambda^a T^a \) this time. By using these superfields, the “covariant” derivative is constructed as

\[ D_a \equiv \nabla_a - ig A_a + g(\nabla_a \Lambda^\alpha) A_\alpha. \]  

(44)

This derivative operator is not covariant under the gauge transformation at this stage. However, under the \( \delta \)-functions, \( \delta^4(x - \bar{x} - \Delta) \) and \( \delta^4(\theta - \lambda) \), one can confirm that it behaves as a covariant derivative: \( D_a \phi(\bar{x}) \rightarrow e^{ia} D_a \phi(\bar{x}). \)

Then the kinetic term can be written as

\[ K_{\text{kin.}} = \delta^4(\theta - \lambda) \left[ (D_a \phi(\bar{x}))^\dagger e^{-2gV} D^a \phi(\bar{x}) \right]. \]  

(45)

The potential terms are

\[ K_{\text{pot.}} = \delta^4(\theta - \lambda) \left[ -m_2 \phi(\bar{x})^\dagger e^{-2gV} \phi(\bar{x}) - \frac{k}{4} (\phi(\bar{x})^\dagger e^{-2gV} \phi(\bar{x}))^2 \right]. \]  

(46)

Both are gauge invariant under the delta functions with complex-valued \( \alpha \) satisfying Eq. (39). The quartic coupling \( k \) is unrelated to the gauge coupling constant in contrast to the prediction of the MSSM.

## 3 Higgsinoless SUSY

We are now ready to construct a Lagrangian. For quarks/leptons and gauge superfields, one can simply write down the MSSM Lagrangian in the superspace. Soft SUSY breaking terms can be written down by using delta functions \( \delta^2(\theta - \lambda) \) and \( \delta^4(\theta - \lambda) \):

\[ K_{\text{soft}} = -\delta^4(\theta - \lambda) \cdot m_2^2 \Psi^\dagger \Psi \]

\[ \Rightarrow \ S \ni - \int d^4 \bar{x} \det A m_2^2 \Psi^\dagger \Psi(\bar{x}, \lambda, \bar{\lambda}), \]  

(47)
\[ W_{\text{soft}} = -\delta^2(\theta - \lambda) \cdot \frac{m_1/2}{2} W^\alpha W_\alpha \]

\[ \Rightarrow S \ni - \int d^4\tilde{x} \det A \frac{m_1/2}{2} W^\alpha W_\alpha(\tilde{x}, \lambda, \bar{\lambda}) + \text{h.c.} \quad (48) \]

These are the same as the spurion method for the soft SUSY breaking terms. The appearance of the Goldstino interactions makes these terms manifestly supersymmetric. One can also add hard breaking terms by using covariant derivatives. We assume that such soft and hard breaking terms are somewhat suppressed because the quarks/leptons and gauge fields are not participating the SUSY breaking dynamics.

We introduce the Higgs field as a non-linear field on the \( \tilde{x} \) space, \( h(\tilde{x}) \), motivated by an assumption that the SUSY breaking dynamics at the cut-off scale \( \Lambda \) has something to do with the origin of electroweak symmetry breaking. The way to construct interaction terms has been discussed already in the previous subsection. The Yukawa interactions for up-type quarks are

\[ K_{\text{up}} = \delta^4(\theta - \lambda) \left[ y_{ij}^u h(\tilde{x}) \cdot \left( \frac{1}{2} D^2_{(\text{cov})} U^c_j Q_i \right) \right]. \quad (49) \]

For down-type quarks and leptons,

\[ K_{\text{down}} = \delta^4(\theta - \lambda) \]

\[ \times \left[ y_{ij}^d h(\tilde{x})^\dagger e^{-2gV} \left( \frac{1}{2} D^2_{(\text{cov})} D^c_j Q_i \right) + y_{ij}^e h(\tilde{x})^\dagger e^{-2gV} \left( \frac{1}{2} D^2_{(\text{cov})} E^c_j L_i \right) \right]. \quad (50) \]

Here we have used the covariant derivative:

\[ D^2_{(\text{cov})} \equiv e^{2gV} D^2 e^{-2gV}. \quad (51) \]

It is not necessary to introduce two kinds of Higgs fields for the Yukawa interactions. The \( A \)-terms can also be written down by taking

\[ W_A = \delta^2(\theta - \lambda) \left[ A_{ij}^u h(\tilde{x}) \cdot (U^c_j Q_i) \right], \quad (52) \]

and

\[ K_A = \delta^4(\theta - \lambda) \left[ A_{ij}^d h(\tilde{x})^\dagger e^{-2gV} (D^c_j Q_i) + A_{ij}^e h(\tilde{x})^\dagger e^{-2gV} (E^c_j L_i) \right]. \quad (53) \]
Since the quartic coupling of the Higgs boson in Eq. (46) is a free parameter, the Higgs boson mass is not related to the Z-boson mass. It is not a very obvious result that we could write down a Lagrangian with a single Higgs boson with the enlarged gauge invariance. For example, in Ref. [35] it has been necessary to introduce an extra Higgs boson, and that is claimed to be a general requirement for constructing a realistic model with non-linear SUSY.

4 Hidden Gravity

A SUSY transformation in the $\tilde{x}$ space is realized as a local coordinate transformation in Eq. (6). This local translation allows us to introduce a metric in the $\tilde{x}$ space having a local transformation law under the global SUSY. This provides a description of a composite spin-two field in the SUSY breaking dynamics analogous to the $\rho$ meson in QCD. We further elaborate on this comparison towards the end of this section.

Specifically, we introduce a second “metric” whose transformation under $g$ is

$$g_{\mu\nu}(\tilde{x}) \to g'_{\mu\nu}(\tilde{x}') = \frac{\partial \tilde{x}'^\mu}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}'^\sigma}{\partial \tilde{x}^\sigma} g_{\sigma\rho}(\tilde{x}),$$

where $\tilde{x}'$ is given in Eq. (6). Note that this is a global SUSY transformation, and one should not be confused with the actual general coordinate transformation on the $x$-space. The space-time is always flat. The deviation of $g_{\mu\nu}$ from the Minkowski metric describes the spin-two field.

The invariant action having the Fierz-Pauli form is

$$S = \int d^4\tilde{x} \left[ -\frac{f^4}{2} \det A - \frac{m_p^2}{2} \sqrt{g} R(g) - \frac{m^2}{8} \sqrt{gg_{\mu\nu}g_{\alpha\beta}} (H_{\mu\alpha}H_{\nu\beta} - H_{\mu\nu}H_{\alpha\beta}) \right]$$

where

$$H_{\mu\nu} = g_{\mu\nu} - G_{\mu\nu}$$

An attempt to describe a spin-two resonance in QCD as a massive graviton can be found in Ref. [40], whose supergravity extension is discussed in Ref. [41]. A more ambitious attempt to formulate Einstein gravity as a composite of the Goldstino fermions can be found in Ref. [42]. The appearance of a massive bound-state graviton in open string field theory can be found in Ref. [43].

We could have instead defined the spin-two field to transform as a scalar under $g$: $g_{ab}(\tilde{x}) \to g'_{ab}(\tilde{x}') = g_{ab}(\tilde{x})$. This is an equivalent formulation, since the two definitions are related by multiplying by the vielbein, $g_{ab}(\tilde{x}) \equiv A_{\mu}^a A_{\nu}^b g_{\mu\nu}(\tilde{x})$. We will not pursue this formulation any further.
and

\[ G_{\mu\nu} = A^a_\mu A^b_\nu \eta_{ab}. \] (57)

\(G_{\mu\nu}\) is a covariant tensor, defined previously. The \(H_{\mu\nu}\) field is therefore a SUSY covariant tensor. The scale \(m_P\) is a mass parameter of \(O(\text{TeV})\), unrelated to the four-dimensional Planck mass \(G_N^{-1/2}\) of Einstein gravity. With

\[ g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{m_P} h_{\mu\nu}, \] (58)

one has

\[ H_{\mu\nu} = \frac{2}{m_P} h_{\mu\nu} + \left( i\lambda\sigma_\mu \partial_\nu \bar{\lambda} - i\partial_\nu \lambda\sigma_\mu \bar{\lambda} + (\mu \leftrightarrow \nu) \right) + \text{(four-fermion terms)} \] (59)

Note the relative coefficient (of \(-1\)) between the two terms appearing in the definition of \(H_{\mu\nu}\) is fixed by requiring that the Fierz-Pauli mass term not introduce a tadpole for the graviton. The last term in the action gives a mass \(m\) to the spin-two field in a global SUSY invariant way.

There are other invariant terms involving \(h_{\mu\nu}\) and up to two derivatives, such as

\[ \sqrt{g}, \ \det A \cdot R(g), \ \text{etc.} \] (60)

but these are forbidden by the Lorentz invariance of the vacuum and the absence of ghosts and tachyons. That is, the Einstein action with Fierz-Pauli mass term is the unique tachyon and ghost-free action for a spin-two field \([45, 46]\). Although loop corrections will not preserve this form, the ghost pole is harmless since its effect is pushed to the cutoff \([56]\).

Other interactions, such as

\[ \sqrt{g} \cdot R(H), \ \det A \cdot R(H) \] (61)

begin at higher than quadratic order in \(h_{\mu\nu}\).

In the chiral Lagrangian of QCD one can construct SU(2) vector- and axial-type 1-forms \(j_{V,A}\) out of the pion fields. A chirally invariant Lagrangian can be constructed only out of \(j_A\), since \(j_V\) transforms inhomogeneously under the chiral SU(2). By introducing an SU(2) vector boson \(V_\mu\), the term \(\text{Tr} \tilde{j}_V \tilde{j}_V\), with \(\tilde{j}_V = j_V - V\), is made chirally invariant and can be added to the action. This gives a mass to the vector
boson, but no kinetic term; it can be trivially integrated out. The key assumption of [22, 23, 24, 25, 26] is that this vector boson is dynamical (and describes the \( \rho \) vector meson). The action obtained in this way coincides with the spontaneously broken SU(2)\(_{V}\) gauge theory in the unitary gauge, which obviously can have a sensible description up to some high energy scale.

The analogy of the massive spin-two field as formulated here to the Hidden Local Symmetry (HLS) of QCD can now be drawn more closely, though imprecisely. Here the Maurer-Cartan 1-forms \( A_\mu^a \) are analogous to the \( j_V \) in QCD. By introducing a spin-two field having a local and inhomogeneous transformation under the global symmetry, it is then possible to introduce the 1-forms into the action, in the form of the Fierz-Pauli mass term\(^{\text{**}}\). Further assuming a Ricci scalar term in the action for the spin-two field is equivalent to the physical assumption in HLS that the gauge boson is dynamical.

### 4.1 Perturbative Unitarity

A question to be addressed here is whether this new spin-two resonance can be consistently introduced in a weakly coupled regime, in which perturbative calculations make sense at energies of \( O(m) \). We first check this by looking at the elastic scattering of two Goldstinos with the same helicity. Then we require that the spin-two field is not strongly coupled at threshold.

In supergravity, the amplitudes of the elastic scattering of two gravitinos have been calculated in Ref. [47] and it has been shown that the scalar partners of the Goldsino fermion unitarize the amplitudes if they are light enough. We show in this subsection that a spin-two field, instead of the scalar fields, can also partially cancel the growth of the scattering amplitudes. This is analogous to the discussion of the \( WW \) scattering in the Standard Model. The SU(2)\(_{L}\) partner of the Goldstone boson, the Higgs boson, can unitarize the \( WW \) scattering amplitude if it is light enough. But alternatively, it has been known from the analysis of the Higgsless model [48, 49] that a massive vector boson can also partially cancel the amplitude, and the theory can remain perturbative up to some high energy scale above the Kaluza-Klein scale [48, 50]. The massive vector boson is indeed identified with the one in HLS once the Higgsless model is formulated as a four-dimensional theory [51, 52, 53, 54] by using the technique of deconstruction [55].

The amplitude \( \mathcal{M}_{\lambda\lambda} \) for \( \lambda\lambda \rightarrow \lambda\lambda \) receives contributions from both the Volkov-

\(^{\text{**}}\)The other invariant is \( \det A \), since the coordinates also transform.
Akulov action and the action for the spin-two field. Specifically, one obtains

\[ M_{\lambda\lambda} = M_{\lambda\lambda}^{(\det A)} + M_{\lambda\lambda}^{(HH)}, \]  
(62)

\[ M_{\lambda\lambda}^{(\det A)} = \frac{2s^2}{f^4}, \]  
(63)

\[ M_{\lambda\lambda}^{(HH)} = -\frac{5m_P^2 m^2 s^2}{f^8} \left[ \frac{m^2}{t-m^2} \left( \frac{5}{2} + \frac{3}{2} \cos \theta \right) + \frac{m^2}{u-m^2} \left( \frac{5}{2} - \frac{3}{2} \cos \theta \right) \right]. \]  
(64)

The contribution from the Volkov-Akulov action is given in Eq. (63), and those from the spin-two action in Eq. (64). The production angle \( \theta \) is defined in the center-of-mass frame.

The contributions from the spin-two action deserve further comment. The second term in the RHS of Eq. (64) is the contribution from \( t \) and \( u \) channel exchange of the massive graviton, arising from the Goldstino-Goldstino-graviton coupling in the Fierz-Pauli mass term. The Fierz-Pauli mass term however also has a contact four-point interaction involving the Goldstinos, giving the first term in the RHS of Eq. (64). By inspection, in the low-energy limit these two contributions to \( M_{\lambda\lambda}^{(HH)} \) exactly cancel. That is, at low energies one obtains the same \( \lambda\lambda \) scattering amplitude as in the theory without a massive graviton. Therefore, the decay constant \( f \) appearing in the action in Eq. (62) is the same as the one in the original Volkov-Akulov action.

The partial wave amplitudes are defined by

\[ M_{\lambda\lambda}^\ell = \frac{1}{64\pi} \int_{-1}^{1} d(\cos \theta) P_\ell(\cos \theta) M_{\lambda\lambda}, \]  
(65)

where \( P_\ell \) denotes the Legendre Polynomials: \( P_0(z) = 1, P_1(z) = z, P_2(z) = \frac{1}{2}(3z^2 - 1), \) etc. Since the particles in the final state are identical, we compensate the integral over all of phase space by multiplying by a factor of 1/2. Substituting the expressions in Eqs. (62–64), we obtain the following \( s \)-wave amplitude for the \( \lambda\lambda \) scattering.

\[ M_{\lambda\lambda}^{\ell=0} = \frac{1}{16\pi} \frac{s^2}{f^4} - \frac{5}{32\pi} \frac{m_P^2 m^2 s^2}{f^8} - \frac{1}{16\pi} \frac{m_P^2 m^4 s}{f^8} \left[ 3 - \left( 4 + \frac{3m^2}{s} \right) \ln \left( 1 + \frac{s}{m^2} \right) \right]. \]  
(66)

The first term in the RHS comes from \( M_{\lambda\lambda}^{(\det A)} \), while the remaining terms come from \( M_{\lambda\lambda}^{(HH)} \). There is no parameter to control the relative sign of the two contributions.
Figure 1: The magnitude of the s-wave amplitude as a function of $\sqrt{s}/f$. Upper and lower curves represent contributions from $(-f^4/2)\det A$ and $(H_{\mu\nu}H^{\mu\nu} - H_{\mu}^{\mu}H_{\nu}^{\nu})$ terms, and the middle curve represents the total amplitude. Here, $m_P = 0.7f$ and $m = 1.2f$ are used as an example.

One can see that two $O(s^2)$ terms in Eq. (66) always have an opposite sign, and thus the graviton contribution partially cancels the growth of the amplitude. The magnitude of the s-wave amplitude is plotted in Fig. 1 as a function of $\sqrt{s}/f$. The upper and lower curves represent contributions from $(-f^4/2)\det A$ and $(H_{\mu\nu}H^{\mu\nu} - H_{\mu}^{\mu}H_{\nu}^{\nu})$ interactions (i.e., the first and second row of the RHS of Eq. (66), respectively). The middle curve represents the total amplitude. The parameters $m_P = 0.7f$ and $m = 1.2f$ are chosen for illustration.

We define the perturbative-unitarity-violation scale $E_s$, by the energy where the tree level s-wave amplitude of $\lambda\lambda$ scattering reaches the value 0.5. In the case of the example depicted in Fig. 1, the pure Goldstino amplitude (i.e., the upper curve) gives $E_s \sim 2.2f$. The contribution of the spin-two particle to this amplitude has the opposite sign (lower curve). This contribution partially cancels the pure Goldstino amplitude, delaying the onset of the strong coupling regime. For the parameter values used in Fig. 1, one finds $E_s \sim 3.4f$.

Perturbativity imposes additional constraints, since at high energies both the Goldstino and the graviton become strongly coupled. These are of three kinds. First, the interactions in the Volkov-Akulov action modify $\lambda\lambda \rightarrow \lambda\lambda$ scattering at one-loop. Compared to the tree-level amplitude, the one-loop amplitude gives a relative correction of $O(s^2/((4\pi)^2 f^4))$, which suggests that the natural cut-off scale is $O(\sqrt{4\pi}f)$. At
Figure 2: The contour of $E_\ast = m$ and $\Lambda_\ast^{(5)} = \sqrt{2}m$ in the parameter space of $m/f$ and $m_P/f$. In the region to the left of the thick lines, both $E_\ast > m$ and $\Lambda_\ast^{(5)} > \sqrt{2}m$ are satisfied.

$\sqrt{s} \sim m \sim f$ these corrections are $O(1/(4\pi)^2)$ and the expansion parameter is small.

Next, the interactions of the massive graviton provide additional and stronger constraints. The “Einstein gravity” interactions grow with energy and become strong at energies of order $E \sim 4\pi m_P$. With $m_P \gtrsim f/\sqrt{4\pi}$ this is of order $\Lambda$ or larger. There is however a stronger constraint, since the spin-two field is massive. The coupling of the longitudinal component becomes strong at a lower scale, proportional to $(m_P m^4)^{1/5}$ \[20\]. This estimate is obtained from using the equivalence theorem in the limit $E \gg m$. Factors of $4\pi$ can be estimated using naive dimensional analysis \[38\]. For example, the one-loop contribution of the Goldstino to the vacuum polarization of the spin-two state becomes comparable to the tree-level propagator at roughly an energy scale

$$\Lambda_\ast^{(5)} = (4\pi m_P m^4)^{1/5}.$$ \[67\]

One then expects higher dimension operators involving the spin-two field to be suppressed by this scale. We will use this scale as a crude estimate of the energy at which the low-energy effective theory is no longer calculable. Note that for $m = m_P$ the theory becomes strongly coupled at $E \sim 1.7m$, not far above threshold.

The parameter region in which the spin-two resonance can be consistently incorpo-
rated in the effective theory is then approximately bounded by these considerations. For illustration, in Fig. 2 we plot the contour of \( E^* = m \) and \( \Lambda^{(5)*} = \sqrt{2}m \) in the parameter space of \( m \) and \( m_P \). In the region to the left of the thick lines, both \( E^* > m \) and \( \Lambda^{(5)*} > \sqrt{2}m \) are satisfied, and we expect that the production of the spin-two resonance in the single-graviton channel can be treated in perturbation theory, that is, for \( E \sim m < \min[E^*, \Lambda^{(5)*}] \).

At higher energies though, the theory becomes strongly coupled. As we have seen, for generic values of the parameters the scale of strong coupling is not far above the mass of the spin-two particle. Since pair production of the spin-two resonances or scattering of spin-two resonances requires \( E \gtrsim 2m \), these processes are generically not calculable. Thus we have a situation where processes describing the production and decay of a single on-shell spin-two resonance are plausibly perturbative, but soon becomes strongly coupled above the single-particle threshold.

### 4.2 Phenomenological Signatures

In the effective theory the leading order interactions between the spin-two and Higgs boson are given by

\[
\mathcal{K}_{\text{kin.}} = \delta_H \delta^4(\theta - \lambda) H^{\mu\nu} \left[ A_\mu^a A_\nu^b (D_a \phi)^\dagger e^{-2gV} D_b \phi \right] \\
+ \delta'_H \delta^4(\theta - \lambda) \text{Tr} H \left[ \eta^{ab} (D_a \phi)^\dagger e^{-2gV} D_b \phi \right] 
\]  

and

\[
\mathcal{K}_{\text{pot.}} = \delta''_H \delta^4(\theta - \lambda) \text{Tr} H \left[ -\delta m^2 \phi^\dagger e^{-2gV} \phi - \frac{\delta k}{4} \left( \phi^\dagger e^{-2gV} \phi \right)^2 \right].
\]

where \( \delta_H \), \( \delta'_H \) and \( \delta''_H \) are parameters assumed to be of \( O(1) \). These interactions begin at \( O(h_{\mu\nu}h h) \) or \( O(h_{\mu\nu}VV) \) (\( V = W, Z \)). Other interactions are possible, such as replacing \( H_{\mu\nu} \) with \( g_{\mu\nu} \). Such interactions are equivalent to those above, since the difference can be adsorbed in the normalization of the kinetic and potential terms for the Higgs. One may also consider the above interactions multiplied by \( \det A^{-1} \sqrt{-g} \); to zeroth order in \( \lambda \partial \lambda \) these are the same. The interaction terms proportional to \( \text{Tr} H \) do not contribute to any tree amplitude when the spin-two field is on-shell.

A general feature of the spin-two field that distinguishes it from massive gravitons from extra dimensions (i.e., from a metric), is that it does not have a minimal coupling. That is, it does not couple universally to the total stress-energy tensor of matter, or
even non-universally to the stress-energy tensor of each particle. This is explicitly evident in its couplings to the Higgs boson, Eqs. (68) and (69), since the parameters $\delta_H$, $\delta'_H$ and $\delta''_H$ are unrelated. Only for a specific ratio of these parameters does the spin-two field couple to the stress-energy tensor of the Higgs boson. However in the effective theory there is no symmetry principle which would enforce such a condition.

At one-loop these interactions will generically modify the Fierz-Pauli form of the graviton mass term. This introduces a ghost at high energy. For $\delta_H \sim \delta'_H \sim O(1)$, the ghost pole is above the cut-off scale provided

$$\Lambda \lesssim (4\pi m^2 m_P)^{1/3}. \quad (70)$$

For $m \lesssim 4\pi m_P$ the RHS is always larger than the cutoff $\Lambda^{(5)}_*$ above which the graviton is strongly coupled. The spin-two interactions with the Higgs boson therefore do not introduce a ghost below $\Lambda^{(5)}_*$ provided this condition is satisfied and the couplings in Eqs. (68) and (69) are no larger than $O(1)$. It is therefore not surprising that (70) is also parametrically the same as the largest possible cutoff of an interacting massive graviton [56] (i.e., the scale $\Lambda^{(3)}$ in the notation of that reference). It is perhaps more of a coincidence that (70) is also the maximum cutoff of an electromagnetically coupled spin-two particle of charge $e$ [57] with the correspondence $e \rightarrow \delta_H m/m_P$.

With the assumption that of the Standard Model particle content only the Higgs boson couples directly to the strong dynamics breaking SUSY, the dominant decay modes of the spin-two particle are then

$$h_{\mu\nu} \rightarrow \lambda\bar{\lambda}, \; hh, \; WW, \; ZZ. \quad (71)$$

One obtains

$$\Gamma(h_{\mu\nu} \rightarrow \lambda\bar{\lambda}) = \frac{1}{16\pi} \frac{1}{5} \frac{m_P^2 m^7}{f^8} \quad (72)$$

for the (invisible) Goldstino final state, and

$$\Gamma(h_{\mu\nu} \rightarrow hh) = \frac{1}{16\pi} \frac{1}{5} \frac{\delta^2_H}{4} \frac{m^3}{3m_P^2} \left(1 - \frac{4m^2_h}{m^2}\right)^{5/2} \quad (73)$$

for the Higgs boson final state. The powers of velocity appearing here may be understood by noting that its rest frame, the spin-two particle couples to the velocity of the Higgs boson, leading to two powers in the amplitude, or five in the rate. In the limit $m \gg m_W, m_h$ the Equivalence Theorem applies, giving $\Gamma(h_{\mu\nu} \rightarrow hh) = \Gamma(h_{\mu\nu} \rightarrow$
\( ZZ \) = \( \Gamma(h_{\mu\nu} \rightarrow WW)/2 \). Note the spin-two resonance is narrow provided the three mass scales \( m, f, m_P \) are all comparable and \( \delta_H^2 \) is \( O(1) \).

We next define a field strength
\[
F_{\mu\nu} \equiv \frac{i}{g}[A_{\mu}^a D_a, A_{\nu}^b D_b],
\] (74)
by using the “covariant” derivative in Eq. (44). A graviton-gluon interaction term can then be written as
\[
K_{\text{glue}} = -\frac{\delta g}{4k_g} \delta^4(\theta - \lambda) H^{\mu\nu} G^{\nu\sigma} \text{Tr} (F_{\mu\nu} F_{\rho\sigma}).
\] (75)
The normalization factor \( k_g \) is defined by \( \text{Tr}(T_i T_j) = k_g \delta_{ij} \). These interactions give
\[
h_{\mu\nu} \rightarrow gg.
\] (76)
By assumption the interaction term is small – \( \delta g \ll 1 \) – and therefore this decay is suppressed compared to other channels.

This interaction however leads to the production of spin-two particles through the collisions of gluons. In the narrow-width approximation the leading-order differential production cross-section to \( hh \) at the parton level is
\[
\frac{d\hat{\sigma}}{d\cos \theta}(gg \rightarrow h_{\mu\nu} \rightarrow hh) = \frac{1}{16\pi \hat{s}} \frac{\delta^2 \delta_H^2}{2048 m_P^4} \left( 1 - \frac{4m_h^2}{m^2} \right)^{5/2} \frac{\pi}{m \Gamma_{tot}} \delta(\hat{s} - m^2) \sin^4 \theta.
\] (77)
The phase space is \( 0 \leq \theta \leq \pi/2 \) because of the identical particles in the final state. For \( m \gg m_h \) the two Higgs bosons are highly boosted. The observation of the \( \sin^4 \theta \) dependence of the cross section will be an interesting confirmation of the spin-two resonance.

The total cross section at the LHC is then given by
\[
\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \int d\hat{s} \delta(\hat{s} - x_1 x_2 s) f_g(x_1, m^2) f_g(x_2, m^2) \tilde{\sigma}(\hat{s}) s \delta(\hat{s} - m^2),
\] (78)
where
\[
d\tilde{\sigma} = d\sigma \cdot \delta(\hat{s} - m^2)
\] (79)
and \( s \) is the proton-proton center-of-mass energy. The expression for the cross-section reduces to
\[
\sigma = \frac{dL(\tau)}{d\tau} \tilde{\sigma}(m^2),
\] (80)
where the luminosity function $dL(\tau)/d\tau$ is defined by
\begin{equation}
\frac{dL(\tau)}{d\tau} \equiv \int_{\tau}^{1} dx \frac{1}{x} f_g(x, \tau s) f_g(\tau/x, \tau s), \quad \left( \tau \equiv \frac{m^2_s}{s} \right), \tag{81}
\end{equation}
and the $\bar{\sigma}$ factor is
\begin{equation}
\bar{\sigma}(m^2) = \frac{\pi}{64s} \frac{m^2}{m_P^2} \cdot \delta_g^2 \cdot B(h_{\mu\nu} \to hh). \tag{82}
\end{equation}

The cross section (81) for $pp \to h_{\mu\nu} \to hh$ are shown in Fig. 3 for a choice of parameters, using the CTEQ6M parton distribution functions [58]. The spin-two couplings to the Higgs boson and the gluon are chosen to be $\delta_H = 1$ and $\delta_g = 0.1$. The production rate depends on the Higgs boson mass only through phase space, and is therefore significant only for $2m_h$ comparable to $m$; in Fig. 3 we have set $m_h = 114$ GeV. We have varied $m$ while holding the relations $f = m_P = m$ fixed. The reason for this is to satisfy the requirements of weak coupling, as discussed previously. For this choice of parameters, the cross-section is larger than 1 fb at 14 TeV (10 TeV) for spin-two masses $m \lesssim 1.75 \ (1.5)$ TeV. We note that the production cross-section is sensitive to the spin-two coupling $\delta_g$ to gluons, so that larger rates are possible.

We conclude this section with comments on other phenomenological signatures of the spin-two field.
When all the mass scales are comparable and $\delta_H = O(1)$ no one decay mode dominates over any other. For example, with $\delta_H = 1$ and $f = m = m_P \gg m_h$ one has
\[
\frac{\Gamma(h_{\mu\nu} \to \lambda \bar{\lambda})}{\Gamma(h_{\mu\nu} \to hh) + \Gamma(h_{\mu\nu} \to ZZ) + \Gamma(h_{\mu\nu} \to WW)} = 3.
\] (83)

Since producing the graviton in association with a gluon jet is also proportional to $\delta_g^2$, searching for the invisible decay in this channel may be promising and important to validate this scenario. Note monojets are also a generic signature of large extra dimensions [59].

Couplings of the spin-two field to electroweak and hypercharge field strengths may also occur. These operators are analogous to its interactions with gluons (75). Although by assumption they too are suppressed, the rare decay
\[
h_{\mu\nu} \to \gamma\gamma
\] (84)

is of obvious experimental interest. The rapidity distribution of the photons depends on the spin of the resonance, which in principle may be used to distinguish the graviton from a scalar.

Finally, spin-two particles can also be produced through vector boson fusion $qq' \to qq'h_{\mu\nu}$, which has been recently studied in [60]. In our model the production rate is proportional to $\delta_H^2$ and therefore under our assumptions cannot be made arbitrarily small, unlike the production through gluon-gluon fusion which is suppressed by the small parameter ($\delta_g$). Compared to Higgs production from vector boson fusion, the production rate of the spin-two particle is suppressed by a factor of $v^2/m_P^2$. This is simply because in unitary gauge the amplitude for producing the spin-two field involves two Higgs vev insertions, whereas the same amplitude for producing the Higgs boson has only a single insertion. The Higgs and graviton production cross-sections through vector boson fusion are not trivially related however, since they scale differently with mass due to the dependence on spin. The rate in this channel could be of experimental interest if the spin-two mass is low and the scale $m_P$ not too large.

Experimentally discovering the spin-two field in different production channels and measuring the branching ratios to the invisible and all visible decay channels will obviously help discriminate between different models of composite or Kaluza-Klein spin-two fields.
5 Summary

If SUSY is broken near the TeV scale by strong dynamics then there may be composites that can be accessed at the LHC. It is desirable to have a formalism for writing the effective theory describing the interactions between the matter or gauge fields and the composites, especially in the situation where the matter and gauge fields are not participants of these dynamics. The challenge then in this case is that the matter and gauge fields appear in linearly realized multiplets, whereas the composites do not. We have presented a formulation of non-linearly realized global SUSY in which this can be done.

As an application, we consider two scenarios. In both, the Higgs boson is such a composite. No Higgsinos are present in the low-energy theory. We show that it is possible to write down SUSY invariant Yukawa couplings and $A$ terms, despite the presence of only one Higgs boson.

Next, we further suppose that the composites include a light spin-two field in addition to the Higgs boson. The construction of the SUSY invariant action is in analogy to the Hidden Local Symmetry of chiral dynamics, where the $\rho$ meson is a massive vector boson of a hidden local $SU(2)_V$ symmetry. Here though the hidden symmetry is local Poincaré. Thus we find that a massive graviton can naturally be incorporated in a theory with an enlarged spacetime symmetry, which in this case is global SUSY.

Some phenomenological signatures are discussed. Unlike a generic Kaluza-Klein graviton, the spin-two particle does not couple to the stress-energy tensor. Instead its interactions with matter and gauge fields are constrained only by gauge, Lorentz and non-linear SUSY invariance. Its dominant decay mode is to Goldstinos (invisible), electroweak gauge bosons and the Higgs boson. Search strategies to find boosted Higgs bosons are particularly interesting for this scenario. Vector boson fusion producing the spin-two particle occurs and may be of experimental interest. Rare decays to diphotons also occur but the rate is more model-dependent. Search strategies to find the Standard Model Higgs boson are therefore simultaneously sensitive to finding the spin-two particle.

This scenario has the usual low-energy SUSY experimental signatures (without Higgsinos), while in addition possessing signatures of both large $\kappa$ and warped extra
dimensions [61] - monojets (ADD) and a spin-two resonance (RS) - even though there is no extra dimension. The discovery of SUSY particles and a single spin-two resonance is not sufficient to claim discovery of an extra (supersymmetric) dimension. It may just be due to four-dimensional strong SUSY breaking dynamics.

6 Acknowledgements

The work of M.G. and M.K. is supported by the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396. Additional support for M.K. is provided by a LANL Director’s Fellowship. The work of R.K. is supported in part by the Grant-in-Aid for Scientific Research (No. 18071001) from the Japan Ministry of Education, Culture, Sports, Science and Technology.

References

[1] S. Weinberg, Phys. Rev. D 13, 974 (1976); Phys. Rev. D 19, 1277 (1979); L. Susskind, Phys. Rev. D 20, 2619 (1979).

[2] See for reviews, e.g., E. Farhi and L. Susskind, Phys. Rept. 74, 277 (1981); C. T. Hill and E. H. Simmons, Phys. Rept. 381, 235 (2003) [Erratum-ibid. 390, 553 (2004)] and references therein.

[3] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990); Phys. Rev. D 46, 381 (1992); G. Altarelli and R. Barbieri, Phys. Lett. B 253, 161 (1991); G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. B 369, 3 (1992) [Erratum-ibid. B 376, 444 (1992)].

[4] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.

[5] R. Kitano and Y. Nomura, Phys. Rev. D 73, 095004 (2006) [arXiv:hep-ph/0602096].

[6] G. F. Giudice and R. Rattazzi, Nucl. Phys. B 757, 19 (2006) [arXiv:hep-ph/0606105].

[7] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B 189, 575 (1981).

[8] S. Dimopoulos and S. Raby, Nucl. Phys. B 192, 353 (1981).
[9] M. A. Luty, J. Terning and A. K. Grant, Phys. Rev. D 63, 075001 (2001) [arXiv:hep-ph/0006224].

[10] H. Murayama, arXiv:hep-ph/0307293.

[11] R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama, Phys. Rev. D 70, 015002 (2004) [arXiv:hep-ph/0311349].

[12] E. Witten, Nucl. Phys. B 188, 513 (1981).

[13] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706, 045 (2007) [arXiv:hep-ph/0703164].

[14] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. D 71, 063534 (2005) [arXiv:astro-ph/0501562]; A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M. Viel, JCAP 0905, 012 (2009) [arXiv:0812.0010 [astro-ph]].

[15] A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, Nucl. Phys. B 666, 105 (2003) [arXiv:hep-ph/0301121]; M. Dine, N. Seiberg and S. Thomas, Phys. Rev. D 76, 095004 (2007) [arXiv:0707.0005 [hep-ph]].

[16] R. Barbieri and A. Strumia, Phys. Lett. B 462, 144 (1999) [arXiv:hep-ph/9905281]; R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B 703, 127 (2004) [arXiv:hep-ph/0405040].

[17] M. Roček, Phys. Rev. Lett. 41, 451 (1978).

[18] E. A. Ivanov and A. A. Kapustnikov, J. Phys. G 8 (1982) 167.

[19] S. Samuel and J. Wess, Nucl. Phys. B 221, 153 (1983).

[20] T. Gherghetta and A. Pomarol, Phys. Rev. D 67, 085018 (2003) [arXiv:hep-ph/0302001].

[21] T. Gherghetta and A. Pomarol, Nucl. Phys. B 586, 141 (2000) [arXiv:hep-ph/0003129].

[22] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985).

[23] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. B 259, 493 (1985).
[24] M. Bando, T. Fujiwara and K. Yamawaki, Prog. Theor. Phys. 79, 1140 (1988).

[25] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988).

[26] M. Harada and K. Yamawaki, Phys. Rept. 381, 1 (2003) [arXiv:hep-ph/0302103].

[27] Z. Komargodski and N. Seiberg, arXiv:0907.2441 [hep-th].

[28] E. A. Ivanov and A. A. Kapustnikov, JINR-E2-10765, (1977); E. A. Ivanov and A. A. Kapustnikov, J. Phys. A 11, 2375 (1978).

[29] E. A. Ivanov and A. A. Kapustnikov, Phys. Lett. B 143, 379 (1984); E. A. Ivanov and A. A. Kapustnikov, Nucl. Phys. B 333, 439 (1990).

[30] A. Salam and J. A. Strathdee, Fortsch. Phys. 26, 57 (1978).

[31] D. V. Volkov and V. P. Akulov, JETP Lett. 16, 438 (1972) [Pisma Zh. Eksp. Teor. Fiz. 16, 621 (1972)].

[32] T. E. Clark and S. T. Love, Phys. Rev. D 70, 105011 (2004) [arXiv:hep-th/0404162].

[33] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2239 (1969); C. G. Callan, S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2247 (1969).

[34] T. Uematsu and C. K. Zachos, Nucl. Phys. B 201, 250 (1982).

[35] S. Samuel and J. Wess, Nucl. Phys. B 233, 488 (1984).

[36] M. A. Luty and E. Ponton, Phys. Rev. D 57, 4167 (1998) [arXiv:hep-ph/9706268].

[37] I. Antoniadis and M. Tuckmantel, Nucl. Phys. B 697, 3 (2004) [arXiv:hep-th/0406010].

[38] A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).

[39] D. Bailin and A. Love, Bristol, UK: IOP (1994) 322 p. (Graduate student series in physics)

[40] C. J. Isham, A. Salam and J. A. Strathdee, Phys. Rev. D 3, 867 (1971).

[41] A. H. Chamseddine, A. Salam and J. A. Strathdee, Nucl. Phys. B 136 (1978) 248.
[42] J. Lukierski, Phys. Lett. B 121, 135 (1983).

[43] W. Siegel, Phys. Rev. D 49, 4144 (1994) [arXiv:hep-th/9312117].

[44] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A 173, 211 (1939).

[45] H. van Dam and M. J. G. Veltman, Nucl. Phys. B 22, 397 (1970).

[46] P. van Nieuwenhuizen, Nucl. Phys. B 60, 478 (1973).

[47] R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, Phys. Lett. B 216, 325 (1989) [Erratum-ibid. B 229, 439 (1989)].

[48] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, Phys. Rev. D 69, 055006 (2004) [arXiv:hep-ph/0305237].

[49] C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. 92, 101802 (2004) [arXiv:hep-ph/0308038].

[50] R. S. Chivukula, D. A. Dicus and H. J. He, Phys. Lett. B 525, 175 (2002) [arXiv:hep-ph/0111016]; R. S. Chivukula and H. J. He, Phys. Lett. B 532, 121 (2002) [arXiv:hep-ph/0201164]; R. S. Chivukula, D. A. Dicus, H. J. He and S. Nandi, Phys. Lett. B 562, 109 (2003) [arXiv:hep-ph/0302263].

[51] R. Foadi, S. Gopalakrishna and C. Schmidt, JHEP 0403, 042 (2004) [arXiv:hep-ph/0312324]; J. Hirn and J. Stern, Eur. Phys. J. C 34, 447 (2004) [arXiv:hep-ph/0401032]; R. Casalbuoni, S. De Curtis and D. Dominici, Phys. Rev. D 70, 055010 (2004) [arXiv:hep-ph/0405188]; M. Perelstein, JHEP 0410, 010 (2004) [arXiv:hep-ph/0408072]; H. Georgi, Phys. Rev. D 71, 015016 (2005) [arXiv:hep-ph/0408067].

[52] R. S. Chivukula, E. H. Simmons, H. J. He, M. Kurachi and M. Tanabashi, Phys. Rev. D 70, 075008 (2004) [arXiv:hep-ph/0406077]; Phys. Lett. B 603, 210 (2004) [arXiv:hep-ph/0408262]; Phys. Rev. D 71, 035007 (2005) [arXiv:hep-ph/0410154]; Phys. Rev. D 71, 115001 (2005) [arXiv:hep-ph/0502162]; Phys. Rev. D 72, 015008 (2005) [arXiv:hep-ph/0504114]; Phys. Rev. D 75, 035005 (2007) [arXiv:hep-ph/0612070]; Phys. Rev. D 78, 095003 (2008) [arXiv:0808.1682 [hep-ph]].
[53] R. S. Chivukula, B. Coleppa, S. Di Chiara, E. H. Simmons, H. J. He, M. Kurachi and M. Tanabashi, Phys. Rev. D 74, 075011 (2006) [arXiv:hep-ph/0607124].

[54] See also for earlier works on the application of the Hidden Local Symmetry to the extended electroweak gauge sector: R. Casalbuoni, S. De Curtis, D. Dominici and R. Gatto, Phys. Lett. B 155, 95 (1985); R. Casalbuoni, A. Deandrea, S. De Curtis, D. Dominici, R. Gatto and M. Grazzini, Phys. Rev. D 53, 5201 (1996) [arXiv:hep-ph/9510431].

[55] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001) [arXiv:hep-th/0104005]; C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. D 64, 105005 (2001) [arXiv:hep-th/0104035].

[56] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. 305, 96 (2003) [arXiv:hep-th/0210184].

[57] M. Porrati and R. Rahman, Nucl. Phys. B 814, 370 (2009) [arXiv:0812.4254 [hep-th]].

[58] S. Kretzer, H. L. Lai, F. I. Olness and W. K. Tung, Phys. Rev. D 69, 114005 (2004) [arXiv:hep-ph/0307022].

[59] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436, 257 (1998) [arXiv:hep-ph/9804309]; N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59, 086004 (1999) [arXiv:hep-ph/9807344]; G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544, 3 (1999) [arXiv:hep-ph/9811291].

[60] K. Hagiwara, J. Kanzaki, Q. Li and K. Mawatari, Eur. Phys. J. C 56, 435 (2008) [arXiv:0805.2554 [hep-ph]]; K. Hagiwara, Q. Li and K. Mawatari, arXiv:0905.4314 [hep-ph].

[61] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].