Generalized Second Law of Thermodynamics in Quintom Dominated Universe

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Abstract

In this paper we will investigate the validity of the Generalized Second Law of thermodynamics for the Quintom model of dark energy. Reviewing briefly the quintom scenario of dark energy, we will study the conditions of validity of the generalized second law of thermodynamics in three cases: quintessence dominated, phantom dominated and transition from quintessence to phantom will be discussed.

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1 Introduction

One of the most important problems of cosmology, is the problem of so-called dark energy (DE). The type Ia supernova observations suggests that the universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [1, 2, 3]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on dynamics of a scalar or multi-scalar fields (e.g quintessence [4, 5] and quintom model of dark energy, respectively). Primary scalar field candidate for dark energy was quintessence scenario, a fluid with the parameter of the equation of state lying in the range, $-1 < w < \frac{-1}{3}$. While the most model independent analysis suggest that the acceleration of the universe to be below the de Sitter value [6], it is certainly true that the body of observational data allows for a wide parameter space compatible with an acceleration larger than the de Sitter’s [9, 8]. If eventually this proves to be the case, the fluid driving the expansion would violate not only the strong energy condition $\rho + 3P > 0$, but the dominate energy condition $\rho + P > 0$, as well. Fluids of such characteristic dubbed phantom fluid [9]. In spite of the fact that the field theory of phantom fields encounter the problem of stability which one could try to bypass by assuming them to be effective fields [10, 11], it is nevertheless interesting to study their cosmological implication. Recently there are many relevant studies on phantom energy [12]. The analysis of the properties of dark energy from recent observations mildly favor models with $w$ crossing -1 in the near past. But, neither quintessence nor phantom can fulfill this transition. In the quintessence model, the equation of state $w = p/\rho$ is always in the range $-1 \leq w \leq 1$ for $V(\phi) > 0$. Meanwhile for the phantom which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has $w \leq -1$. Neither the quintessence nor the phantom alone can fulfill the transition from $w > -1$ to $w < -1$ and vice versa. Although for k-essence[13] one can have both $w \geq -1$ and $w < -1$, it has been lately considered by Ref[14, 15] that it is very difficult for k-essence to get $w$ across $-1$ during evolving. But one can show [16, 17] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with $w \geq -1$. In the other term the quintom model of dark energy represents a transition of dark energy equation of state from $w > -1$ to $w < -1$, or vice versa, namely from $w < -1$ to $w > -1$ is also one realization of quintom, as can be seen clearly in [18]. We must mention that there are another possibilities in model building regarding quintom, see [19] for a dark energy model which includes higher derivative operators in the Lagrangian with a single scalar field which gives rise to an equation of state larger than $-1$ in the past and less than $-1$ at the present time. One another is the scalar field model with non-minimal coupling to the gravity [20]. Another model is a single scalar field minimally coupled to the gravity but with a non-minimal kinetic term, in this model a dimensionless function of temperature $f(T)$ is in front of the kinetic term. During the evolution of the universe when $f(T)$ change sign from positive to negative, the possibility of quintessence-phantom transition appears [16].

In 1973, Bekenstein [21] assumed that there is a relation between the event of horizon and the thermodynamics of a black hole, so that the event of horizon of the black hole is a measure of the entropy of it. This idea has been generalized to horizons of cosmological models, so that each horizon corresponds to an entropy. Thus the second
law of thermodynamics was modified in the way that in generalized form, the sum of all time derivative of entropies related to horizons plus time derivative of normal entropy must be positive i.e. the sum of entropies must be increasing function of time. In [23], the validity of Generalized Second Law (GSL) for the cosmological models which departs slightly from de Sitter space is investigated. However, it is only natural to associate an entropy to the horizon area as it measures our lack of knowledge about what is going on beyond it. In this paper we show that the sum of normal entropy and the horizon entropy in phantom dominated universe is non-decreasing function of time. Also, the transition from quintessence to Phantom dominated universe is considered and the conditions of the validity of GSL in transition is studied. Also for quintom model of dark energy [16], we study the GSL in quintom dominated universe and conclude the same results when we consider two scalar fields with no coupling potential term. In our calculations we use 

\[ c = 8\pi G_N = 1. \]

### 2 The quintom model of dark energy

The quintom model of dark energy [16] is of new models proposed to explain the new astrophysical data, due to transition from \( w > -1 \) to \( w < -1 \), i.e. transition from quintessence dominated universe to phantom dominated universe. Here we consider the spatially flat Friedman-Robertson-Walker universe, where has following space-time metric

\[ ds^2 = -dt^2 + a(t)^2(dr^2 + r^2d\Omega^2) \]

Containing the normal scalar field \( \sigma \) and negative kinetic scalar field \( \phi \), the action which describes the quintom model is expressed as the following form

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{2} - \frac{1}{2} g^\mu\nu \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^\mu\nu \partial_\mu \sigma \partial_\nu \sigma + V(\phi, \sigma) \right), \]

where we have not considered the lagrangian density of matter field. In the spatially flat Friedman-Robertson-Walker (FRW) universe, the effective energy density, \( \rho \), and the effective pressure, \( P \), of the scalar fields can be described by

\[ \rho = -\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 + V(\phi, \sigma), \]

\[ P = -\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 - V(\phi, \sigma). \]

So, the equation of state can be written as

\[ w = \frac{-\dot{\phi}^2 + \dot{\sigma}^2 - 2V(\phi, \sigma)}{-\dot{\phi}^2 + \dot{\sigma}^2 + 2V(\phi, \sigma)}. \]

From the equation of state , it is seen that for \( \dot{\sigma} > \dot{\phi}, w \geq -1 \) and for \( \dot{\sigma} < \dot{\phi}, \) we will have, \( w < -1 \). Alike [17], we consider a potential with no direct coupling between two scalar fields

\[ V(\phi, \sigma) = V_\phi(\phi) + V_\sigma(\sigma) = V_{\phi 0} e^{-\lambda_\phi \phi} + V_{\sigma 0} e^{-\lambda_\sigma \sigma}. \]
Where the $\lambda_\phi$ and $\lambda_\sigma$, are two dimensionless positive numbers characterizing the slope of the potential for $\phi$ and $\sigma$ respectively. So, the evolution equation for two scalar fields in FRW model will have the following form

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV_\phi(\phi)}{d\phi} = 0, \quad (7)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV_\sigma(\sigma)}{d\sigma} = 0, \quad (8)$$

where, $H$ is the Hubble parameter.

### 3 Generalized second law and quintom model of dark energy

To study the GSL through the universe which is dominated by quintom scenario, we deduce the expression for normal entropy using the first law of thermodynamics.

$$TdS = dE + PdV = (P + \rho)dV + Vd\rho \quad (9)$$

From the equations (3),(4) we have

$$P + \rho = -\dot{\phi}^2 + \dot{\sigma}^2 \quad (10)$$

and the Friedman constraint equation will be

$$H^2 = \frac{1}{3}(-\frac{\dot{\phi}^2}{2} + V_\phi + \frac{\dot{\sigma}^2}{2} + V_\sigma). \quad (11)$$

So, using relations (7) and (8), it is seen that

$$\dot{H} = \frac{1}{2}(\dot{\phi}^2 - \dot{\sigma}^2) = -\frac{1}{2}(P + \rho) \quad (12)$$

Thus, if $\dot{\phi}^2 < \dot{\sigma}^2$ then $\dot{H} < 0$, i.e. for the quintessence dominated universe and if $\dot{\phi}^2 > \dot{\sigma}^2$ then $\dot{H} > 0$, for the phantom dominated universe. Rewriting the first law of thermodynamics with respect to relations above and using $V = \frac{4}{3}\pi R_h^3$, in which the $R_h$ is the event of horizon, one can obtain

$$TdS = -2\dot{H}dV + Vd\rho = -8\pi R_h^2\dot{H}dR_h + 8\pi R_h^3HdH \quad (13)$$

where $T$ is the temperature of the quintom fluid. Therefore, the time derivative of normal entropy will have the following form

$$\dot{S} = \frac{8\pi \dot{H}R_h^2}{T}(HR_h - \dot{R}_h) \quad (14)$$

As we know, the quintom is the combination of normal scalar filed, i.e, quintessence and phantom scalar field. From the definition of event of horizon

$$R_h = a(t) \int_t^{t_0} \frac{dt'}{a(t')} \quad \text{and} \quad \int_t^{t_0} \frac{dt'}{a(t')} < \infty. \quad (15)$$
where for different space times $t_s$ has different values, e.g for de Sitter space time $t_s = \infty$, $R_h$ satisfies the following equation which is true for both scalar fields individually

$$\dot{R}_h = HR_h - 1$$  \hfill (16)$$

where $\dot{R}_h \leq 0$ for phantom dominated universe [22] and $\dot{R}_h \geq 0$ for quintessence dominated universe [23]. As the final form, we write the time derivative of normal entropy of the quintom fluid using relation (13)

$$\dot{S} = \frac{8\pi \dot{H} R_h^2}{T}$$  \hfill (17)$$

As it is seen from relation (14), it is shown that, the sign of $\dot{S}$ depends on the sign of $\dot{H}$, hence for quintessence dominated universe $\dot{S} < 0$ and for phantom dominated universe $\dot{S} > 0$.

The entropy of a black hole is proportional to the area of its event horizon is well understood, it has deep physical meaning. The status of an entropy associated to a cosmological event horizon is not well established. In some cases like the case a de Sitter horizon this seems plausible, with some caveats, but in general this is a topic of current research; see [24]. If, the horizon entropy is taken to be $S_h = \pi R_h^2$, the generalized second law stated that

$$\dot{S} + \dot{S}_h \geq 0$$  \hfill (18)$$

Thus, we will have

$$\dot{S} + \dot{S}_h = \frac{8\pi \dot{H} R_h^2}{T} + 2\pi R_h \dot{R}_h \geq 0$$  \hfill (19)$$

To investigate the validity of equation (19), we will consider three different cases, the first case we dominate the phantom fluid, the second, the quintessence will be dominated, and the third, the transition from quintessence to phantom.

a) Phantom dominated:
In this case $\dot{R}_h \leq 0$ and $\dot{H} > 0$, then $\dot{S}_h < 0$. If the phantom fluid temperature $T > 0$ the condition for validity of GSL is as

$$\dot{H} \geq \frac{|T \dot{R}_h|}{2R_h}$$  \hfill (20)$$

If the temperature is assumed to be proportional to the de Sitter temperature [23]

$$T = \frac{bH}{2\pi}$$  \hfill (21)$$

where $b$ is a parameter, the GSL hold when:

$$b \leq \frac{4\pi \dot{H} |R_h|}{H |\dot{R}_h|}$$  \hfill (22)$$

in de Sitter spacetime case $R_h = \frac{1}{H^2}$, then $b \leq 1$. In phantom model case which is small perturbed around de Sitter space, one can expect $T \leq \frac{H}{2\pi}$, which is the condition that the phantom fluid be cooler than the horizon temperature.
b) Quintessence dominated:

In this case $\dot{R}_h \geq 0$, and $\dot{H} < 0$ so the sum of normal entropy and horizon entropy could be positive, if $T > 0$ then the condition for validity of GSL is

$$|\dot{H}| \leq \frac{T \dot{R}_h}{2R_h} \quad (23)$$

using eq.(21) this condition has following form

$$b \geq \frac{4\pi |\dot{H}|R_h}{H R_h} \quad (24)$$

c) Phase transition from quintessence to phantom:

As $\dot{R}_h \geq 0$ in quintessence model and $\dot{R}_h \leq 0$ in phantom model, and assuming that $R_h$ variates continually one can expect that in transition from quintessence to phantom $\dot{R}_h = 0$. So the horizon entropy in transition time will be zero, also in transition time $\dot{H} = 0$, using eq.(17), we obtain $\dot{S} = 0$. Therefore in the transition time the total entropy is differentiable and continuous.

4 Conclusion

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have. Investigating the principles of thermodynamics and specially the second law- as global accepted principle in the universe - in different models of DE, as one of these possibilities, has been widely studied in the literature, since this investigation can constrain some of parameters in studied models, say, P. C. Davies [23] studied the change in event horizon area in cosmological models that depart slightly from de Sitter space and showed that for this models the GSL is respected for the normal scalar field, provided the fluid to be viscous.

In the present paper we have considered total entropy as the entropy of a cosmological event horizon plus the entropy of a normal scalar field $\sigma$ and ghost scalar field $\phi$. In the quintom model of dark energy $\dot{H}$ is given by eq.(12), for the phantom dominated case $\dot{H} > 0$, in this case $\dot{R}_h \leq 0$, then the horizon entropy is constant or decreases with time, i.e $\dot{S}_h \leq 0$, therefore the phantom entropy must increases with expansion so long as $T > 0$. In fact the phantom fluids possess negative entropy and equals to minus the entropy of black hole of radius $R_h$. In contrast with the previous case in the quintessence dominated case, $\dot{H} < 0$ and $\dot{R}_h \geq 0$, then $\dot{S}_h \geq 0$. By considering the influence of the transition from the quintessence to phantom dominated universe on the GSL , one can obtain that the time derivative of the future event horizon and the entropy must be zero at the transition time. In the summary, we have examined the quintessence and phantom dominated universe, and we have shown that by satisfying the conditions (20), (23) the total entropy is non-decreasing function of time. Otherwise the second law of thermodynamics break down. Note that in [25] these calculations have been done
for the case of interacting holographic dark energy with dark matter, the authors have shown, in contrast to the case of the apparent horizon, both the first and second law of thermodynamics break down if one consider the universe to be enveloped by the event horizon with the usual definitions of entropy and temperature.

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