Recent Advances on Intersection Graphs of Hypergraphs: A Survey

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Abstract: In this survey, we have attempted to show some developmental milestones on the characterizations of intersection graphs of hypergraphs. The theory of intersection graphs of hypergraphs has been a classical topic in the theory of special graphs. To conclude, at the end, we have listed some open problems posed by various authors whose work has contributed to this survey and also the new trends coming out of intersection graphs.

Keywords: Hypergraphs, Intersection graphs, Line graphs, Representative graphs, Derived graphs, Algorithms (ALG), Forbidden induced subgraphs (FIS), Krausz partitions, Eigenvalues.

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1. Introduction.

We will follow the terminology of Harary, F., in [11] and Berge, C., in [2] and [3]. A graph or a hypergraph is a pair \((V, E)\), where \(V\) (the vertex set) is a finite nonempty set and \(E\) the edge set is a finite family of nonempty subsets of \(V\). Two edges of a hypergraph are \(l\)-intersecting if they share at least \(l\) common vertices. This concept was studied in [4] and [13] by Bermond, Heydemann and Sotteau. A hypergraph \(H\) is called a \(k\)-uniform hypergraph if its edges have \(k\) number of vertices. A hypergraph is linear if any two edges have at most one common vertex. A 2-uniform linear hyper graph is called a graph.

Let \(H\) be a hypergraph without isolated vertices. The intersection graph \(L(H)\) is the graph whose vertex set is the edge set of \(H\), and such that two vertices are adjacent in \(L(H)\) if and only if the corresponding edges are adjacent or intersecting edges in \(H\). Among the earliest works on intersection graphs is seen in Whitney’s work [35]
in 1934, Krausz’s work [19] in 1943 and Hoffman’s work [15] in 1964. They called intersection graphs as line graphs in their work. Harary in [11] noticed that the concept of intersection graph is so appealing that it has been introduced under different names by many authors. Berge called these as representative graphs in his famous book on hypergraphs [3]. The intersection graph of a hypergraph, as defined by Berge [3], is a $l$-intersection graph with $l=1$, it doesn’t matter whether the adjacent edges have one or more common vertices. For various other definitions and names related to intersection graphs, see [11].

We call $l$ the multiplicity of a hypergraph $H$ if any two edges of $H$ intersect in at most in $l$ vertices, $1 \leq l \leq k$. We will denote by $I_l(k)$, the set of all graphs which are intersection graphs of some $k$-uniform hypergraphs with multiplicity $l$. Thus $I_1(k)$ is the set of graphs which are intersection graphs of some $k$-uniform linear hypergraphs. We will write $I(k)$ for the set $U_{l \geq 0} I_l(k)$.

A family of graphs $M$ is said to be hereditary if $G \in M$ implies all the vertex induced subgraphs of $G$ are also in $M$. The families $I(k)$’s are clearly hereditary families of graphs. Now let $M$ be a hereditary family of graphs. If $G$ does not belong to $M$, then clearly $G$ is not an induced subgraph of any graph in $M$. A graph $G$ does not belong to $M$ is said to be a minimal forbidden graph for $M$ if all vertex induced subgraphs of $G$ are in $M$. Let $F(M)$ denote the family of all minimal forbidden graphs for $M$. Clearly, then $M$ can be characterized by saying that $G \in M$ if and only if any graph of $F(M)$ is not an induced subgraph of $G$. The well-known result of Beineke [1] stated below shows that $F(I_1(2))$ consists of the nine graphs.

2. The characterization of intersection graphs of 2-uniform linear hypergraphs (graphs).

The following theorem on edge isomorphism is from Whitney.

Theorem 2.1 [35]: If $G$ and $H$ are connected graphs and $L(G) \cong L(H)$, then $G \cong H$ unless one is $K_3$ and the other is $K_{1,3}$. Further, if the orders of $G$ and $H$ are greater than 4, then for any isomorphism $f : L(G) \to L(H)$, there exists a unique isomorphism between $G$ and $H$ inducing $f$. 
The first characterization of intersection graphs given below in theorem 2.2 comes from Krausz which gives a global characterization of line graphs. In fact, this powerful result is instrumental in the development of characterizations of intersection graphs of hypergraphs.

Theorem 2.2 [19]: A graph G is an intersection graph if the edges of G can be partitioned into complete subgraphs in such a way that no vertex lies in more than two of the complete graphs (cliques).

Rooij and Wilf were able to describe in a structural criterion for a graph to be an intersection graph. They introduced odd triangles to make their case. A triangle T ($K_3$) of a graph is called odd if there is a vertex of G adjacent to an odd number of its vertices of T, and is even otherwise. Their characterization is:

Theorem 2.3 [30]: A graph G is an intersection graph if G does not have $K_{1,3}$ as an induced subgraph, and if two odd triangles have a common edge, then the subgraph induced by their vertices is $K_4$.

A classical result of Beineke and Robertson N. (unpublished) listed exactly 9 forbidden induced subgraphs (FIS) which cannot occur in intersection graphs aka derived graphs called in Beineke’s paper [1].

Theorem 2.4 [1]: A graph G is an intersection graph if and only if none of the nine graphs given below from [38] is an induced subgraph of G.

![Graphs](image-url)
Likewise, the FIS characterization of an intersection graph of a multigraph was studied by Bermond and Meyer in 1973 in terms of the seven forbidden subgraphs, see in [3].

Roussopoulos in [31] and Lehot in [20] gave a max $\{|E|, |V|\}$ — time recognition algorithm for intersection graphs. In [31], the algorithm is based on the characterizations of intersection graphs due to Krausz whereas the algorithm in [20] is based on the characterizations of intersection graphs due to Rooij and Wilf. In [7], the algorithm from Degiorgi et al. is based on the local attributes. In [27], Naor et al. proposed a parallel algorithm for line-to-root graph construction based on a divide-and-conquer scheme.

3. Intersection graphs of $k$-uniform linear hypergraphs, $k = 3$.

The situation changes radically if one takes $k = 3$ instead of $k = 2$. For $k \geq 3$, the problem of characterizing $I(k)$’s becomes more complicated in terms of $F(I(k))$’s. A global characterization of representative graphs of $k$-uniform hypergraphs and that of linear $k$-uniform hypergraphs for an arbitrary $k$ are given by Berge in [3]. In [22], Lovasz stated the problem of characterizing the class $I(3)$ and noted that it cannot be characterized in terms of FIS. In [4], Bermond et al., Gardner [9], Germa and Nickel in [3] showed that this class cannot be characterized by a finite list of forbidden induced subgraphs (FIS).

The difficulty in finding a characterization for $I(3)$ is due to the fact that there are infinitely many forbidden induced subgraphs shown as below. For an integer, $m > 0$, consider a chain of $(m + 2)$ diamond graphs such that the consecutive diamonds share vertices of degree two and add two pendant edges at every vertex of degree 2 to get one of the families of minimal forbidden subgraphs of Naik et al. in [29] and [3] as shown below from [40]. Call this graph as $G_1(m)$. Now let, $G_3 = \{G_1(m), m > 0\}$. It can be verified that $G_3 \in F(I(3))$. Additional examples can be found in [9].
This does not rule out either the existence of polynomial time recognition (ALG) or the possibility of a forbidden induced subgraph (FIS) characterizations of k-uniform linear hypergraphs similar to Beineke's list of forbidden subgraphs. However, in [3], and [29], it is shown that $I_1(3)$ can be characterized by a finite list of forbidden induced subgraphs in the class of graphs whose vertex degrees are at least 69.

In [29], authors also gave a global characterization of the members of the family $I_1(k)$, which is a generalization of a criterion of the intersection graphs due to Krausz. It is as follows.

Theorem 3.1 [29]: If G is a graph, then $G \in I_1(k)$ if and only if in G there exists a set $C = \{c_1, \ldots, c_m\}$ of cliques of sizes $> 1$ such that the following two conditions hold:

(i) Every edge of $G$ is in a unique click of $C$, and
(ii) Every vertex of $G$ is in at most $k$ cliques of $C$.

The following theorem 3.2 is a characterization of the family $I_1(k)$. For $k = 2$, it is almost similar to aforesaid Rooij and Wilf theorem 2.3.

Theorem 3.2 [29]: If G is a graph, then $G \in I_1(k)$ if and only if G has a set $T$ of triangles satisfying the following two conditions:

(i) If abc, and abd are in $T$ with $c \neq d$, then cd $\in E(G)$ and acd, bcd are also in $T$;
(ii) Given any $(k + 1)$ distinct edges of $G$, all having a vertex in common, at least two of these edges are in a triangle of $G$ which is in $T$.

The theorems 3.1 and 3.2 are used in proving the theorems 3.3 and 4.2 of section 4. The proofs of theorem 3.1 and 3.2 are similar to the proofs for $k = 2$. 
A set of cliques of G is a linear r-covering of G if G is the union of these cliques and any two cliques from the set have at most one common vertex and each vertex of G belongs to at most r cliques. A clique of size $\geq k^2 - k + 2$ is called k-large in $I_1(k)$. In [20], [21], [23], [24], [25], [26], [28], [29], [36], [37] and [38], it is well noted that if G has a linear k-covering, then the covering must contain the set of all subgraphs of G induced by maximal k-large cliques.

Theorem 3.3 [29]: There is a finite family F of forbidden graphs such that any graph with minimum degree at least 69 belongs to $I_1(3)$ if and only if G has no induced subgraph isomorphic to a member of F.

In [23], Metelsky et al. reduced the minimum degree bound from 69 to 19. Their proof is based on the theorems of Krausz, Beineke, and the Krausz characterization for the class $I_1(3)$ in [29] and the properties of graph cliques from this class obtained by Levin et al. in [21]. They formed a finite family F of forbidden graphs called as “A” different from the family F of the Theorem 3.2. Their theorem states as follows.

Theorem 3.4 [23]: For a graph G with minimum degree $\geq 19$, the following two statements are equivalent

(i) $G \in I_1(3)$,
(ii) None of the graphs from the list A is an induced subgraph of G.

The list A differs from the list of forbidden subgraphs obtained in Theorem 3.3. In [23], the same authors gave the following polynomial algorithm (ALG) for $I_1(3)$.

Theorem 3.5 [23]: There is a polynomial recognition algorithm to decide whether $G \in I_1(3)$, for G with minimum degree $\geq 19$.

To prove this, as noticed in [29], the process is to construct a linear 3-covering for the edges E of G.

In [18], Jacobson M. S. et al. also gave a polynomial recognition algorithm (ALG) for theorem 3.5. The existence of the polynomial recognition algorithm follows from a simpler recursive characterization of graphs in $I_1(3)$ and relies on the fact that there is a polynomial time recognition algorithm for the members of $I_1(2)$. 
They did not provide the finite forbidden subgraph characterization (FIS) for $I_1(3)$ for minimum degree $\geq 19$.

Denote by $\delta_{alg}$ the minimal integer such that the problem "$G \in I_1(3)$" is polynomially solvable in the class of graphs $G$ with minimal degree $\geq \delta_{alg}$ and by $\delta_{fis}$ the minimal integer such that $I_1(3)$ can be characterized by a finite list of forbidden induced subgraphs in the class of graphs $G$ with minimal degree $\geq \delta_{fis}$.

In [25], a polynomial algorithm solving the recognition problem for the graphs with bound on $\delta_{alg} \geq 13$ is given.

In [32], Skums et al. improved the bound on the minimum degree conditions on theorem 3.5. Their work deals mainly with the structural properties of the graphs from the class $I_1(3)$ connected with the geometry of the cliques. Their theorems are:

Theorem 3.6 [32]: There exists an algorithm with complexity $O(nm)$ solving the recognition problem “$G \in I_1(3)$” in the class of graphs $G$ with minimum degree $\geq 10$.

Theorem 3.7 [32]: If a graph $G$ with minimum degree $\geq 16$ contains no graph from the set $B$ as an induced subgraph, then $G \in L_1(3)$.

The set $B$ of forbidden subgraphs (FIS) is obviously different from the sets obtained previously on the previous minimum degree conditions.

In [24], Matelesky et al. proposed $6 \leq \delta_{alg}$ and $6 \leq \delta_{fis}$.

4. The Improvements on Beineke’s 9 forbidden graphs by Metelsky et al.

In [23], it is shown $G \in I_1(2)$ can be characterized in terms of the following 6 graphs out of the 9 graphs from the theorem 2.3 provided the minimum degree of
G ≥ 5. The new list [38] is:

5-graph 31 6-graph 127 claw graph Johnson solid skeleton 12

6-wheel graph

(2,3)-king graph

5. Intersection graphs of k-uniform linear hypergraphs, k > 3.

Define inductively a family $G_k, k \geq 3$, as follows. $G_3$ is already defined above. $G_k = \{G | G$ is obtained by adding a pendent edge at every vertex of degree k in $G_1$ where $G_1 \in G_k\}. Clearly, G_k is in $F(I_1(k))$. Additional infinite families of forbidden graphs for $I_1(k)$ or $I(k)$ can be found in [4], [13], [28] and [29]. However, all these graphs have minimum degrees. Naik et al. raised an interesting question in [28], and [29] whether the theorem 3.1 can be generalized for $k > 3$ or not. In the same paper of 1980 the authors conjectured that the theorem 3.1 cannot be generalized and later in 1997 this conjecture was proved by Metelsky et al. The proof of the conjecture is theorem 4.1 stated below.

Theorem 4.1 [23]: For, $k > 3$, and an arbitrary constant c, the set of all graphs G in $I_1(k)$ with minimum degree $\geq c$ cannot be characterized by a finite list of forbidden induced subgraphs.

Metelsky et al. exhibited a new graph by taking the graph $G_1$ in [28], [29] as described above and pasting $(k - 3)$ pairwise disjoint copies of the clique of size, $s = \max \{k^2 - k + 2, c\}$ to each vertex of $G_1$. Infinitely, many such graphs can be constructed. So, if $k > 3$, no such finite list characterizations exists for k-uniform linear hypergraphs, $k > 3$, no matter what lower bound is placed on degrees?

To characterize $I_1(k), k > 3$, Naik, Rao, Shrikhande and Singhi developed a machinery in [29] based on “edge degree” of G. Edge degree of an edge e in G is
the number of triangles in $G$ containing the edge $e$ and the minimum edge degree of $G$ is the minimum over all the edges of $G$. Their characterization of $I_1(k), k > 3$, based on the edge degree is as follows:

Theorem 4.2 [29], [3]: For a polynomial $f(k) = k^3 - 2k^2 + 1$, there exists a finite family $F(k)$ of forbidden graphs such that any graph $G$ with minimum edge degree $\geq f(k)$ belongs to $I_1(k)$ if and only if $G$ has no induced subgraph isomorphic to a member of $F(k)$.

Jacobson et al. [18] sharpened the bound on the polynomial $f(k)$ to $2k^2 - 3k + 1$ by exhibiting a polynomial recognition algorithm (ALG). Zverovich, I., in [36] gave a forbidden graph characterization (FIS) for $f(k) = 2k^2 - 3k + 1$, their theorems are as follows.

Theorem 4.3 [18]: There is a polynomial algorithm (ALG) to decide whether $G \in I_1(k), k \geq 3$ with edge-degree $\geq 2k^2 - 3k + 1$.

Theorem 4.4 [36]: For $k \geq 3$, there is a finite set $F(k)$ of graphs such that a graph $G$ with minimum edge-degree $\geq 2k^2 - 3k + 1$ belongs to $I_1(k)$ if and only if $G$ has no induced subgraph isomorphic to a member of $F(k)$.

The complexity of recognizing intersection graphs of linear $k$-uniform hypergraphs without any constraint on minimum vertex degree (or minimum edge-degree) is not known.

6. Some open Problems and perspectives:

6.1 Metelsky et al. proposed to find the exact values for $\delta_{alg}$ and $\delta_{fis}$, see in [24]. Authors also proposed whether $\delta_{alg} = \delta_{fis}$ or not. Furthermore, they made an interesting conjecture that $\delta_{alg} = 9$.

6.2 Zverovich, I. [36] proposed the following question on Theorem 4.4. Is it possible to improve the bound $2k^2 - 3k + 1$ on edge-degree? The guess is that the bound cannot be improved.

6.3 Naik et al. [28] have the following problem connecting class of graphs $I_1(k)$ to the Eigen Values of graphs.
Eigen values of a graph are the Eigen values of its (0, 1) adjacency matrix [11]. We will denote by $\alpha(G)$ the minimum Eigen value of G. For an arbitrary real number $\alpha$, we define, $E_{\alpha}\{G \mid G \text{ is a graph with } \alpha(G) > \alpha\}$. It is evident that $E_{\alpha}$ is a hereditary family and that $I_1(k) \leq E_{\alpha}$ for all $\alpha < -k \leq -2$.

They proposed a problem to describe the family $F(E_{\alpha})$ for all real number $\alpha$.

Since $E_{\alpha}$ for $\alpha < -k$ is a larger family than $I_1(k)$, the set $F(E_{\alpha})$ may have simpler structures than that of $F(I_1(k))$. This is also suggested by Hoffman’s theorem [15-17]. Hoffman’s theorem, in fact, describes the families $E_{\alpha}$ in terms of families of $I_1(k)$.

6.4 Zverovich, I. proved that the class of graphs $C(k, l), k \geq 0, l \geq 0$ sprung out of Krausz partitions has FIS characterizations. Author exhibited 14 FISs for $C(3, 1)$, $C(k, l)$ is the class of all graphs having an $l$-bounded and $k$-colorable, see [38].

6.5 Tyshkevich et al. studied Line Hypergraphs in [33]. The work is based on Krausz’s global characterization of line graphs and Whitney’s theorem on edge isomorphism.

6.6 In [10] and [14], the work on Krausz dimensions, computational complexities and other related topics branched out from the intersections graphs of k-uniform linear hypergraphs are discussed.

6.7 In [25], Matelsky et. al. studied the class $I_2(3)$ the intersection graphs of k-uniform hypergraphs, $k \leq 3$ with multiplicity at most 2 (non-linear) and characterized them by a means of finite list of forbidden induced subgraphs in the class of threshold graphs [6],[8]. O(n) - time algorithm is given.

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