Orbit: Probabilistic Forecast with Exponential Smoothing

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Abstract

Time series forecasting is an active research topic in academia as well as industry. Although we see an increasing amount of adoptions of machine learning methods in solving some of those forecasting challenges, statistical methods remain powerful while dealing with low granularity data. This paper introduces a refined Bayesian exponential smoothing model with the help of probabilistic programming languages including Stan. Our model refinements include additional global trend, transformation for multiplicative form, noise distribution and choice of priors. A benchmark study is conducted on a rich set of time-series data sets for our models along with other well-known time series models.

1. Introduction

Time series forecasting is one of the most popular and yet the most challenging tasks, faced by researchers and practitioners. It has played an important role in a wide range of areas, such as statistics, machine learning, artificial intelligence, and econometrics. At Uber, time series forecasting has its applications from demand/supply prediction in the marketplace to the Ads budget optimization in the marketing data science.

In recent years, machine learning and deep learning methods (Gers et al., 1999; Huang et al., 2015; Selvin et al., 2017) have gained increasing attention in the time series forecasting, due to their great ability in capturing the non-linear trend and complex interactions within multivariate time series. However, the theories for deep learning are still in the active research and development progress and not well established yet in the forecasting literature, especially in the case of univariate time series. At the same time, the black-box nature of machine learning/deep learning models causes difficulties in interpretability and explainability (Gunning, 2017; Gilpin et al., 2018). In a benchmark study with different time series data (Hewamalage et al., 2019), the authors show mixed performances of various deep learning models when compared against traditional statistical models.

On the other hand, traditional statistical parametric models have a well-established theoretical foundation, such as the popular autoregressive integrated moving average (ARIMA) (Box & Jenkins, 1968) and exponential smoothing (Gardner Jr, 1985). Recently, researchers from Facebook developed Prophet (Taylor & Letham, 2018), which is based on an additive model where non-linear trends are fit with seasonality and holidays.

In this paper we propose a family of refined Bayesian exponential smoothing models, with great flexibility on the choice of priors, model type specifications, as well as noise distribution. Our model introduces a novel global trend term, which works well for short term time series. Most importantly, our models come with a well crafted compute software/package in Python, called Orbit (Object-oriented Bayesian Time Series). Our package leverages the probabilistic programming languages, Stan (Carpenter et al., 2017) and Pyro (Bingham et al., 2019), for the underlying MCMC sampling process and optimization. Pyro, developed by researchers at Uber, is a universal probabilistic programming language (PPL) written in Python and supported by PyTorch and JAX on the backend. Orbit currently has a subset of the available prediction and sampling methods available for estimation using Pyro.

The remainder of this paper is organized as follows. In section 2, a review is given on some popular statistical parametric time series models. In section 3, we give our refined model equations, and Orbit package design review. In section 4, an extensive benchmark study is conducted to evaluate the proposed models performance and compare with other time series models discussed in the paper. section 5 is about the conclusion and future work. We focus on the univariate time series in this work.

2. Review

Let \( y = \{y_1, \ldots, y_t\} \) denote a univariate time series. Forecasting denotes the process of estimating the future values of \( y, y_{t+h} \), where \( h \) denotes the forecasting horizon.
2.1. ARIMA

ARMA model is one of the most commonly used methods to model univariate time series. ARMA(p,q) combines two components: AR(p), and MA(q). In AR(p) model, the value of a given time series, \( y_t \), can be estimated using a linear combination of the \( p \) past observations, together with an error term \( \epsilon_t \) and a constant term \( c \):

\[
y_t = c + \sum_{i=1}^{p} \psi_i y_{t-i} + \epsilon_t
\]

where \( \psi_i, \forall i \in \{1, \cdots, p\} \) denote the model parameters, and \( p \) represents the order of the model. Similarly, the MA(q) model uses past errors as explanatory variables:

\[
y_t = \mu + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t
\]

where \( \mu \) denotes the mean of the observations, \( \theta_i, \forall i \in \{1, \cdots, q\} \) represents the parameters of the models and \( q \) denotes the order of the model. Essentially, the method MA(q) models the time series according to the random errors that occurred in the past \( q \) lags.

The model ARMA(p,q) can be constructed by combining the AR(p) model with the MA(q) model, i.e.,

\[
y_t = c + \sum_{i=1}^{p} \psi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t
\]

The ARMA(p,q) model is defined for stationary data. However, many realistic time series in practice exhibit a non-stationary structure, e.g., time series with trend and seasonality. The ARIMA(p,d,q) (or SARIMA) overcomes this limitation by including an integration parameter of order \( d \). In principle, ARIMA works by applying \( d \) differencing transformations to the time series until it becomes stationary before applying ARMA(p,q).

2.2. Exponential Smoothing

The exponential smoothing model is similar to the AR(p) model in the sense that it models the future values of time series using a linear combination of its past observations. However, exponential smoothing models use weighted averages of the past values, where the weights decay exponentially as the observations go older. For example, in a simple exponential smoothing method, the prediction for \( y_{t+1} \) can be defined as follows:

\[
y_{t+1} = \beta_0 y_t + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots
\]

where \( \{\beta_i\} \) represent the weights of past observations. There are many variations of exponential smoothing methods. Hyndman et al. 2008 give a complete review on the literature.

2.3. Bayesian Structural Time Series

Bayesian structural time series, one type of state-space models, is another important approach for the time series forecasting task. In general, a structural time series model is defined by two equations, the observation equation and the transition equation. The observation equation establishes the relationship between observations \( y_t \) and a vector of latent variables \( \alpha_t \) known as the state:

\[
y_t = Z_t' \alpha_t + \epsilon_t
\]

The transition equation defines how the latent states evolves over time

\[
\alpha_{t+1} = T_t \alpha_t + R_t \eta_t
\]

Here, the error terms \( \epsilon_t \) and \( \eta_t \) are Gaussian and independent. The model matrices \( Z_t, T_t, R_t \) typically contain a mix of known values and unknown parameters. A model that can be formulated by the two equations above is said to be in the state space form. A very large class of models can be expressed in the state space form, including the ARIMA and VARMA models.

One big advantage of BSTS model is that they are modular, in the sense that independent state components can be combined by concatenating their observation vectors \( Z_t \) and arranging the other model matrices as elements in a block diagonal matrix. This provides considerable flexibility for modeling trend, seasonality, regressors, and potentially other state components that may be necessary in practice.

2.4. Prophet

Researchers at Facebook (Taylor & Letham, 2018) use a decomposable time series model with three main components: trend, seasonality, and holidays. They can be combined in the following equation:

\[
y_t = g_t + s_t + h_t + \epsilon_t
\]

where \( g_t \) is the piecewise linear or logistic growth curve to model the non-periodic changes in the time series, \( s_t \) is the seasonality term, \( h_t \) is the holiday effect with irregular schedules, and \( \epsilon_t \) is the error term. On a high level, Prophet is framing the forecasting problem as a curve-fitting exercise rather than looking explicitly at the time based dependence of each observation within a time series. As a computational tool/software, moreover, Prophet allows users to manually supply change points in fitting the trend term and set the boundaries for saturation growth, which gives great flexibility in business applications.

3. Our Models

3.1. Refined Models

Our proposed SLGT (Seasonal Local and Global Trend) model is based on models from Rlgt (Smyl et al., 2019) with a re-parametrization on the multiplicative form through log transformation. For the prediction equation, it is

\[
y_t' = ln(y_t)
\]
\[ y_t' = l_{t-1} + \theta b_{t-1} + \xi l_{t-1}^{\lambda} + s_{t-m} + r_t + \epsilon_t \]
\[ \epsilon_t \sim \text{Student}(\nu, 0, \sigma) \]
\[ \sigma \sim \text{HalfCauchy}(0, \gamma) \]

where \( l_{t-1}, \theta b_{t-1}, \xi l_{t-1}^{\lambda}, s_{t-m}, r_t \) are level, local trend, global trend, seasonality, and regression components, respectively.

The advantages over Rlgt with such form are two-fold. First, with the help of the transformation, NUTS (No-U-Turn Sampler) is more effective on the additive form. Second, \( \sigma \) in Rlgt is parameterized with \( y_t \) which introduces dependency in noise generation. Hence, \( \sigma \) is refined as an independent noise in SLGT and hence significantly reduces computation cost to generate forecast.

In the update process, it follows a triple exponential smoothing additive form (ETS-AAA) from Rob Hynd (De Livera et al., 2011; Hyndman & Athanasopoulos, 2018):

\[
\begin{align*}
  l_t &= \rho_l(y_t - y_{t-1} - r_t) + (1 - \rho_l)l_{t-1} \\
  b_t &= \rho_b(l_t - l_{t-1}) + (1 - \rho_b)b_{t-1} \\
  s_t &= \rho_s(y_t - l_t - r_t) + (1 - \rho_s)s_{t-m} \\
  r_t &= \Sigma_j \beta_j x_{jt}
\end{align*}
\]

where \( \rho_l, \rho_b, \rho_s \) are the smoothing parameters and \( \theta, \xi, \lambda \) are the trend control parameters, \( \beta_j \) is the regression coefficient for j-th regressor.

As a Bayesian time series model, SLGT provides great flexibility to choose priors for different parameters,

\[
\beta_j \sim \text{Gauss}(\mu_j, \sigma_j) \]
\[ \rho_l \sim \text{Uniform}(0,1) \]
\[ \rho_b \sim \text{Uniform}(0,1) \]
\[ \rho_s \sim \text{Uniform}(0,1) \]

It also allows hierarchical prior on the standard deviation of the error term,

\[ \sigma_j \sim \text{HalfCauchy}(0, \text{scale}) \]

The scale here is a hyperparameter and can be derived from the response size.

Note that in the prediction formula, the model assumes \( y_t > 1 \) which means \( y_t > 1, \forall t \) due to log-transformation.

Furthermore, our algorithm has the following practical advantages:

- Readily extended to other model formulations, such as damped local trend model (DLT) which modifies the prediction equation as such:

\[ y_t' = \ln(y_t) \]

and update process as such:

\[
\begin{align*}
  g_t &= g_{t-1} + \delta \\
  l_t &= \rho_l(y_t' - g_t - s_{t-m} - r_t) + (1 - \rho_l)l_{t-1} \\
  b_t &= \rho_b(l_t - l_{t-1}) + (1 - \rho_b)b_{t-1} \\
  s_t &= \rho_s(y_t' - l_t - r_t) + (1 - \rho_s)s_{t-m} \\
  r_t &= \Sigma_j \beta_j x_{jt}
\end{align*}
\]

where \( g_t \) and \( \theta \) can be interpreted as deterministic global trend and damped factor of the local trend. Unlike SLGT, this model allows any non-negative values of \( y_t \).

- Great flexibility in the choice of different priors.
- Fitting and prediction are very fast, with the help of PyStan probabilistic programming language and PyTorch.
- Great model transparency and interpretation ability.
- Allows for multiplicative format.

### 3.2. Package Orbit Design

Orbit is our Python package to implement the refined models discussed above. This package is written and designed from a strict object oriented perspective with the goals of reusability, ease of maintenance, and high efficiency.

The base Estimator class contains generic logic to handle interaction with the underlying inference engine (e.g PyStan, Pyro) along with utilities to load and save Orbit models, and the specifics of the model are implemented in each model class. Figure 1 is the overall workflow of Orbit package.

![Figure 1. Overall Design of Orbit Package.](image-url)
4. Model Benchmark

4.1. Data

We performed a comprehensive benchmark study on four datasets:

- US and Canada cities rider weekly first-trips with Uber (52 series)
- US and Canada cities driver weekly first-trips with Uber (52 series)
- M3 monthly series (1428 series)
- M4 weekly series (359 series)

where M3/M4 time series are well-known in the forecast community (Makridakis et al., 2018).

4.2. Performance Metric

We use symmetric mean absolute percentage error (SMAPE), a widely adopted forecast metric, as our performance benchmark metric

\[
SMAPE = \frac{1}{h} \sum_{t=1}^{h} \frac{|F_t - A_t|}{(|F_t| + |A_t|)/2}
\]

where \(X_t\) represents the value measured at time \(t\) and \(h\) is the forecast horizon.

Note that in our case, \(h\) is simply the number of holdouts in each series. As the original competitions suggested, we use 13 steps and 18 steps respectively for M4 weekly and M3 monthly series. For the dataset provided by Uber, we use two splits with 13 incremental steps and 26 forecast steps to get a more robust result. The calculation is done with the help of our backtest utilities built in the Orbit package.

4.3. Results

We compared our proposed models, SLGT and DLT, to other popular time series models such as SARIMA and Facebook Prophet. Both Prophet and Orbit models use Maximum A Posteriori (MAP) estimates and they are configured as similar as possible in terms of optimization and seasonality settings. For SARIMA, we fit the (1,1,1) x (0, 0, 0, S) structure by maximum likelihood estimation (MLE) where \(S\) represents the choice of seasonality.

Within a dataset, the models in consideration were run on each time series separately, and then we report the aggregated metrics. Table 1 gives the average and the standard deviation (with parentheses) of SMAPE across different models and dataset. Table 2 is a comparison between LGT and DLT with MCMC sampling. Figure 2 depicts the bar charts with error bars for Ubers data.

It shows that our models consistently deliver better accuracy than other candidate time series models. Orbit is also relatively efficient computationally. For example, the average compute time per series with full MCMC sampling and prediction from a subset of M4 weekly data is about 2.5 minutes and 16 ms. The run time for the same series in Prophet is about 10 minutes for sampling and 2.4 s for prediction. That’s a 4x speed up in training, and orders of magnitude difference in prediction.

Code and M3/M4 data used in this benchmark study are available upon request.
5. Conclusion

We have shown that our approach outperforms the baseline time series models consistently in terms of SMAPE metrics. Furthermore, we also identified compute cost improvements when using the Orbit package. For our future work, we will continue to actively maintain the package, incorporate new model types, and provide new features or enhancements (to support dual seasonality, fully Pyro integration, etc).

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