Classical transitions with the topological number changing in the early Universe

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ABSTRACT: We consider classical dynamics of two real scalar fields within a model with the potential having a saddle point. The solitons of such model are field configurations that have the form of closed loops in the field space. We study the formation and evolution of these solitons, in particular, the conditions at which they could be formed even when the model potential has only one minimum in the field space. These non-trivial field configurations represent domain walls in the three-dimensional physical space. The set of these configurations can be split into disjoint equivalence classes. We provide a simple expression for the winding number of an arbitrary closed loop in the field space and discuss the transitions that change the winding number. It is also shown that non-trivial field configurations could be responsible for a variety of phenomena observed in the early Universe. This fact should be taken into account when studying inflation with a complex form of the potential.

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1 Introduction

Inflationary models with several fields have recently become the subject of a growing
interest [1–3]. Most of them are based on complicated potentials, which arise in the brane
approach [4, 5], various supersymmetric models [6], and the landscape paradigm, see, e.g.,
[7–11].

Multi-field inflation has been thoroughly elaborated, with its influence on CMB fluc-
tuations analyzed in, e.g., [12–15]. An important feature of multi-field inflationary models
is a potential with many minima [16]. It is well-known that a field-theoretic system with
such a potential can admit non-trivial configurations of the scalar field — solitons [17].
In the course of inflation such configurations, in particular, can collapse into black holes
[18–20] or form domains with extra energy density. Remarkably, solitons could be formed
even if the model potential has only one minimum [17]. This work considers the conditions
at which this can occur.

As shown in [21], the number of saddle points of the multi-field potential can be much
greater than the number of its minima. Therefore, if the horizon of our universe was formed
near an appropriate minimum of the potential [22, 23], there is a substantial probability to
have its saddle point(s) which has to be taken into account. This provides additional mo-
tivation to study the saddle point influence on the inflationary dynamics. Of the plethora
of the inflationary models, those with potentials having saddle points have important im-
lications. They include the models of hybrid inflation [24] and their modifications, and
other modern models. For example, Aligned Natural Inflation was recently shown to have
higher altitude inflationary trajectories passing through the saddle points of the two-field
potential [25]; the influence of a nearby saddle point on the inflationary process was also
studied in field spaces of higher dimensions [26].
Our paper considers an inflationary system with two real scalar fields, denoted as $\varphi$ and $\chi$. We show that the multi-field inflation allows the formation of a domain wall even if the potential has only one minimum. We demonstrate that non-trivial field configurations can occur when the fields reach the same minimum at different spatial asymptotics. The necessary condition for the realisation of this phenomenon is the existence of a saddle point of the potential. We perform the topological classification of such configurations, which allows us to divide them into disjoint homotopic classes. We also investigate the conditions at which the configurations can move from one homotopic class to another. Note that such transitions are forbidden if the fields reach different minima of the potential at different spatial asymptotics; in this case only quantum transitions due to tunneling are allowed [27].

Qualitatively, the arguments regarding the field dynamics and formation of the initial conditions can be set out as follows. During the inflation, the fields $\varphi$ and $\chi$ undergo both quantum and classical evolution. At the former stage, the quantum fluctuations of $\varphi$ and $\chi$ are large, of the order of $\delta \varphi \sim \delta \chi \sim H_I/2\pi$, where $H_I$ is the Hubble parameter during the inflation [28]. These large fluctuations lead to a highly inhomogeneous fields distribution soon after the beginning of the inflation. Let us select a space region with the fields distributed around a saddle point of the potential. The potential, in turn, determines the steepest descent line $l_{sd}$ passing through the saddle point $(\varphi_s, \chi_s)$. The space region under consideration consists of two subregions — one of them, $B$, contains the fields on the one side of the saddle point while the fields in another region $U$ are located on the other side. The space boundary $\partial B$ of the domain $B$ consists of points where $\varphi = \varphi_s$, $\chi = \chi_s$, and therefore the potential is maximal at those points of the line $l_{sd}$, which belong to the border $\partial B$.

In the process of further evolution the fields in $B$ and $U$ will roll down into the potential minimum $(\varphi_{\text{min}}, \chi_{\text{min}})$, but their trajectories will lie on the different sides of the saddle point. As a result, after a long period of evolution the fields arrive to a configuration that encircles a local maximum of the potential. The values of the fields at spatial points $x$ far from the boundary $\partial B$ are close to $(\varphi_{\text{min}}, \chi_{\text{min}})$. The line $l_{sd}$ contains the saddle point $(\varphi_s, \chi_s)$ with both its ends arriving at the same minimum $(\varphi_{\text{min}}, \chi_{\text{min}})$. Therefore, starting at one end of $l_{sd}$ and moving to the other, the fields’ values have to pass through $(\varphi_s, \chi_s)$ that defines the border $\partial B$, which thus becomes a domain wall. Our numerical simulations, discussed in detail in what follows, confirm this qualitative picture and reveal the conditions at which these arguments hold.

Although our initial setup considers the fields evolving in the (3+1)-dimensional space-time, we will show that effectively the problem can be reduced to a (1+1)-dimensional one. Multi-field inflationary models could thus potentially benefit from connections to (1+1)-dimensional field-theoretical models with potentials with one or more minima, where many interesting and important results have been obtained recently [29–42], in particular related to planar domain walls [43–45], see also review [46] and books [17, 47].

Our paper is organized as follows. In section 2 we describe the model and the approximations employed by us, and introduce the homotopic classification of the field configurations. In section 3 we show the results of our numerical simulation of the evolution of
selected initial configurations. In section 4 we discuss the change of the winding number during the classical field evolution, considering selected initial configurations in a model with a tilted “Mexican hat” potential. Section 5 presents a discussion of the non-trivial field configurations within modern inflationary models. In section 6 we conclude with a brief discussion of the results and the prospects for future work.

2 Nontrivial field configurations and their classification

We consider a model with two real scalar fields $\varphi$ and $\chi$ in the (3+1)-dimensional space-time. The dynamics of the system is determined by the Lagrangian

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi + \partial_\mu \chi \partial_\nu \chi) - V(\varphi, \chi),$$

where the metric tensor $g_{\mu\nu}$ for the Friedmann-Robertson-Walker universe is

$$g_{\mu\nu} = \text{diag} \left( 1, -a^2(t), -a^2(t) r^2, -a^2(t) r^2 \sin^2 \theta \right).$$

The equations of motion following from the Lagrangian (2.1) are

$$\Box \varphi = - \frac{\partial V}{\partial \varphi}, \quad \Box \chi = - \frac{\partial V}{\partial \chi},$$

where $\Box = \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \right)$ is the d’Alembert operator. Taking into account the metric (2.2), we obtain:

$$\begin{cases} 
\varphi_{tt} + 3H \varphi_t - a^{-2}(t) \varphi_{rr} - \frac{2}{r} a^{-2}(t) \varphi_r = - \frac{\partial V}{\partial \varphi}, \\
\chi_{tt} + 3H \chi_t - a^{-2}(t) \chi_{rr} - \frac{2}{r} a^{-2}(t) \chi_r = - \frac{\partial V}{\partial \chi},
\end{cases}$$

where the Hubble parameter $H = \dot{a}/a$ is small after the end of the inflation. The friction term is not necessary, however, it provides the dumping of long-period oscillations during the reheating stage. We are looking for a domain $\mathcal{B}$, which is surrounded by the border $\partial \mathcal{B}$ with the fields’ values at the saddle point $(\varphi_s, \chi_s)$. As we show below, a nontrivial field configuration can be formed even if the vacuum state is the same at both sides of the border.

During the inflation, the size of the domain $\mathcal{B}$ significantly increases. Hence, for an observer near the border $\partial \mathcal{B}$ the latter is almost flat, and one can restrict the problem to the case of flat solutions $\varphi(x, t), \chi(x, t)$ depending on one spatial coordinate $x$. The equations of motion (2.3) can also be transformed in this case in such a way that they only depend on $t$ and $x$. The terms $\frac{2}{r} a^{-2}(t) \varphi_r$ and $\frac{2}{r} a^{-2}(t) \chi_r$ can also be omitted as it is usually done within the thin wall approximation. In the following we use the physical distances $R = a(t) r$ and assume that the cosmological expansion is small. In this approximation equations (2.4) become

$$\begin{cases} 
\varphi_{tt} + 3H \varphi_t - \varphi_{xx} = - \frac{\partial V}{\partial \varphi}, \\
\chi_{tt} + 3H \chi_t - \chi_{xx} = - \frac{\partial V}{\partial \chi},
\end{cases}$$

(2.5)
The potential $V(\varphi, \chi)$ with a saddle point is chosen in the form

$$V(\varphi, \chi) = d(\varphi^2 + \chi^2) + a \exp \left[-b (\varphi - \varphi_0)^2 - c (\chi - \chi_0)^2\right],$$

(2.6)

where $a > 0, b > 0, c > 0, d > 0$ and $\varphi_0, \chi_0$ are the physical parameters of the model. The parameters $\varphi_0$ and $\chi_0$ fix the position of the maximum of the potential, while $a$ defines the height of the maximum. The constants $b$ and $c$ describe the shape of the maximum. The typical shape of the potential (2.6) is shown in figure 1. With exponentially small errors,

![Figure 1. Left panel: one of the possible initial field configurations. Right panel: initial field configuration (3.2) which is used in our calculations. The units on all the axes are the Planck ones.](image-url)

the minimum of the potential (2.6) is at the point $(0, 0)$. Note that we will employ the Planck units in what follows — as argued above, the value of $H_I$ gives the natural scale of the initial field configurations.

Field configuration at any moment of time is a pair of smooth functions $\varphi(x), \chi(x)$, which map the physical space $x$ to the field space $(\varphi, \chi)$. We can imagine that the point $(\varphi, \chi)$ is on the potential surface (2.6). Thus the field configuration at any given time moment can be visualized as a curve on the potential surface (2.6).

As already mentioned in the Introduction, the initial field configurations are the results of quantum fluctuations during the inflation. In the post-inflationary period, the initial values of the fields in causally disconnected domains substantially differ. After the inflation the horizon starts to grow, and many such domains become causally connected. The initial field configuration in any space domain is unique and can correspond to an arbitrary smooth curve in the $(\varphi, \chi)$ plane, see, e.g., figure 1 (left panel). The initial curve chosen there as an example is not closed. Nevertheless, both ends of this curve tend to evolve to the same point — the potential minimum, resulting in a final configuration represented by a closed curve.

The set of all closed curves in $(\varphi, \chi)$ plane can be split into homotopic classes (equivalence classes). The equivalent curves are those having the same winding number $N$ — the number of turns of the curve around the point $(\varphi_0, \chi_0)$. An explicit formula for $N$ can be obtained using the residue theorem: the line integral of a function $f(\zeta)$ of the complex variable $\zeta = \varphi + i\chi$ around the curve is equal to $2\pi i$ times the sum of residues of $f(\zeta)$ at isolated singular points, each counted as many times as the curve winds around the point.
We use the following simple function:

\[ f(\zeta) = \frac{1}{\zeta - \zeta_0}, \quad (2.7) \]

which has a simple pole at \( \zeta_0 = \varphi_0 + i\chi_0 \), and the residue of \( f \) at this point is equal to 1. This results in

\[ N = \frac{1}{2\pi i} \oint f(\zeta)d\zeta = \frac{1}{2\pi i} \oint \frac{d\zeta}{\zeta - \zeta_0}, \quad (2.8) \]

where the contour integration is performed along the closed curve, which winds \( N \) times around the pole. Substituting \( \zeta = \varphi + i\chi, \zeta_0 = \varphi_0 + i\chi_0 \) and simplifying the expression, we obtain

\[ N[\varphi,\chi] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{(\varphi - \varphi_0)\chi_x - (\chi - \chi_0)\varphi_x}{(\varphi - \varphi_0)^2 + (\chi - \chi_0)^2} \, dx. \quad (2.9) \]

We have thus separated the set of all closed curves in the \((\varphi,\chi)\) plane into homotopic classes, depending on the number of windings of the curve around the potential maximum \((\varphi_0,\chi_0)\). Depending on the shape of the potential, one of the following three scenarios can be realized:

1. The height of the maximum of the potential at the point \((\varphi_0,\chi_0)\) is negligible, and any trajectory contracts to the point \((\varphi_{\text{min}},\chi_{\text{min}})\) as the result of the classical evolution. The evolution of any configuration thus turns it into the trivial configuration, i.e. \( \varphi(x,t \to +\infty) \to \varphi_{\text{min}}, \chi(x,t \to +\infty) \to \chi_{\text{min}} \). In this case the splitting into the homotopic classes does not make sense.

2. The height of the maximum of the potential at the point \((\varphi_0,\chi_0)\) is infinite, and the winding number \( N \) is conserved during the classical dynamics of any closed trajectory.

3. The maximum of the potential has a finite height. As we demonstrate below, in this case the winding number of any configuration can be conserved or can decrease, depending on the initial conditions and the parameters of the model.

Below we discuss the third scenario, which appears to be the most interesting one, and to our best knowledge has not been studied before.

3 The winding number of the configurations. Numerical simulation

We will now consider the third scenario identified above. Studying the classical evolution of the fields, we show that their dynamics can vary substantially, depending on the initial configuration and the values of the parameters of the model.

We start from an initial field configuration in the form of a closed loop in the \((\varphi,\chi)\) plane, so that the boundary conditions are the following:

\[
\begin{align*}
\varphi(-\infty,t) &= \varphi(+\infty,t), \\
\varphi_x(-\infty,t) &= \varphi_x(+\infty,t), \\
\chi(-\infty,t) &= \chi(+\infty,t), \\
\chi_x(-\infty,t) &= \chi_x(+\infty,t).
\end{align*}
\quad (3.1)
\]
(a) The case of \( N = 1 \).

(b) The case of \( N = 2, 3 \). The number of turns of the curve around the local maximum is equal to \( N \).

**Figure 2.** Steady stationary configuration of the fields \( \varphi, \chi \) on the potential surface (2.6) at various winding numbers \( N \).

Furthermore, we specifically consider the following initial configuration which encircles the local maximum of the potential (2.6):

\[
\begin{align*}
\varphi(x) &= \varphi_0 + R \cos \theta(x), \\
\chi(x) &= \chi_0 + R \sin \theta(x),
\end{align*}
\]  

(3.2)

where the dependence \( \theta(x) \) is given by

\[
\theta(x) = N \pi (1 + \tanh(x)).
\]  

(3.3)

This initial trajectory is characterized by the parameter \( R \) — the radius of the circle.

It is easy to see that \( 0 \leq \theta(x) \leq 2\pi \) at \( -\infty \leq x \leq +\infty \). Note also that this initial trajectory fulfills the boundary conditions (3.1). We show this trajectory in figure 1 (right panel). Substituting (3.2) and (3.3) in (2.9) gives the correct answer for the winding number, equal to the coefficient \( N \) in (3.3).

Equations (2.5), (3.1), (3.2), and (3.3) form a well-posed problem. Equations (2.5) are inhomogeneous hyperbolic equations of the second order with the periodic boundary conditions (3.1), which can be solved by the standard methods. We employed an implicit version of the finite difference method, complemented with the modified tridiagonal matrix algorithm, see, e.g., [48]. The following values of the parameters of the potential (2.6) were used: \( a = 5 \cdot 10^2, b = 1, c = 1, d = 1, \varphi_0 = -5, \chi_0 = 0 \). Furthermore, we set \( R = \sqrt{\varphi_0^2 + \chi_0^2} \), see figure 1 (right panel).

We performed a numerical simulation of the evolution of the initial configuration (3.2), (3.3) for \( N = 1, 2, 3 \). Our results are shown in figures 2 and 3. At all considered winding numbers we observed the tightening of the loop around the maximum of the potential. This means that a space trajectory connecting a point of the domain \( B \) with a point of \( \mathcal{U} \) necessarily goes through a domain wall (at \( N = 1 \)) or a series of domain walls (at \( N \geq 2 \)).
The energy density of the field configurations can be written as the sum of the kinetic and the potential term,

$$\varepsilon(x) = \varepsilon_k(x) + \varepsilon_p(x),$$

where

$$\varepsilon_k(x) \equiv \frac{1}{2} (\varphi^2 + \chi^2), \quad \varepsilon_p(x) \equiv \frac{1}{2} (\varphi_x^2 + \chi_x^2) + V(\varphi, \chi).$$
Figure 4. Steady stationary state of the fields $\varphi, \chi$ on the potential surface (4.1) in the case when the winding number is conserved.

Figure 5. Left panel: Steady stationary spatial distribution of the fields $\varphi, \chi$ for the potential (4.1). Right panel: the energy density $\varepsilon(x)$ of the soliton.

The distributions of $\varepsilon$ depending on $x$ for various winding numbers $N$ are shown in figure 3. One can see that the evolution of the fields results in the formation of soliton-like configurations at $t \rightarrow +\infty$.

4 Change of the winding number during the classical field evolution

Moving on to discussion of the conditions under which the change of the winding number becomes possible during the classical evolution of the fields, we consider the well-known tilted Mexican hat potential [49]. For two real scalar fields $\varphi$ and $\chi$ this potential has the form:

$$V(\varphi, \chi) = \lambda \left( \varphi^2 + \chi^2 - \frac{g^2}{2} \right)^2 + \Lambda^4 \left( 1 - \frac{\varphi}{\sqrt{\varphi^2 + \chi^2}} \right).$$

In our calculations we used $g = 2$ and $\Lambda = 10^{-4}$ in the Planck units, see figure 4.

Solving the equations of motion (2.5) with the boundary conditions (3.1) and the initial conditions (3.2) with $\varphi_0 = \chi_0 = 0$ and $R = 1.1g/\sqrt{2}$ for the potential (4.1), we observed
the formation of a soliton analogous to those studied above, see figure 5(a). The energy density of this soliton, in turn, is shown in figure 5(b).

We have found a critical value $\lambda_{cr}$ of the parameter $\lambda$ (keeping the values of the other parameters fixed) such that at $\lambda < \lambda_{cr}$ the winding number decreases in the course of the classical evolution. As a consequence, the soliton decays, as illustrated in figure 6. The fields $\varphi$ and $\chi$ at the spatial infinity take the values $\varphi_{\text{min}}$ and $\chi_{\text{min}}$, i.e. they roll down to the minimum of the potential.

The initial configuration of the fields is the result of the chaotic fluctuations during the inflationary stage. It can be shown that the critical value $\lambda_{cr}$ depends on the particular initial configuration. There are many ways to change the initial field configuration; one of the simplest is to change the parameter $R$ in (3.2), which results in two different critical curves, see figure 6.

As a check of our numerical results, we also calculated the total energy of the field configuration,

$$E[\varphi, \chi] = \int_{-\infty}^{+\infty} \varepsilon(x) \, dx.$$  \hspace{1cm} (4.2)

As expected in the situation when a soliton decays, it monotonically decreases with time, while the potential energy density oscillates, see figure 7.

Thus we conclude that the initial field configuration (3.2) evolves differently depending on the shape of the potential. If the height of the potential maximum is small, the formation of the configurations with non-zero winding number is not observed. On the other hand, if the height of the maximum is bigger than some critical value, the configurations with non-zero winding numbers can be formed. As we show in the next section, the formation of the topologically non-trivial configurations substantially affects the process of the formation of the Universe.
Figure 7. Typical time-dependence of the total energy $E$ and the potential energy $E_p$ for the potential (4.1) at $\lambda = 1.5$, $\Lambda = 0.1$, $g = 2$ and initial conditions (3.2) with $R = 1.1g/\sqrt{2}$. The parameters are chosen in such a way that the soliton overcomes the maximum of the potential and then decays.

5 Inflationary models and the formation of topologically non-trivial configurations

First of all we mention the Natural Inflation model \[50, 51\]. Within this model the potential can be described by eq. (4.1). Apparently observations do not confirm this model. Nevertheless, it is worth discussing because it contains all necessary ingredients and rather simple. In our simulations we used the model parameters $\Lambda = 10^{-4}$ and $g = 2$ in the Planck units, which match those used in \[50\], where it was shown that these parameters provide necessary period of the slow-rolling during the inflationary stage and do not contradict observations.

We have shown that, depending on the value of the parameter $\lambda$, there are two different scenarios of inflation. The first one is being implemented if the height of the potential maximum is small (at small $\lambda$), and is considered in \[50, 51\]. It these papers it is implied that the winding number of configurations is equal to zero.

The second scenario is being realized at large values of $\lambda$ — configurations with non-zero winding numbers can be formed. As it has been shown by the authors of refs. \[19, 20\], appearance of such configurations leads to strong inhomogeneities in the form of closed domain walls. These walls, in turn, can shrink and form black holes. Figure 8, which is taken from ref. \[20\], presents distribution of these primordial black holes (PBHs), which does not contradict observations.

In our numerical simulations we observed that the critical value of the parameter $\lambda$ depends on $R$ and $\Lambda$ (at fixed $g$), see figure 6. If $\lambda$ is bigger than the critical value, then domain walls and subsequently PBHs can be formed. Notice that existence of the saddle point of the potential (4.1) is a necessary condition for the formation of topologically non-trivial configurations.

The next application, which we would like to mention here, is the inflation in random Gaussian landscape \[8\] and multi-stream inflation \[14\] mentioned in the Introduction.
Figure 8. Mass spectrum of primordial black holes. This figure is taken from ref. [20].

Figure 9. Potential (2.6) with parameters $a = 2, b = c = 5, d = 0$ (in $m_{\text{Pl}}$ units). Red line shows one of the possible initial field configurations.

These inflationary scenarios require at least several hills and wells in the potential landscape. Parameters of the potential have to lead to slow-rolling during the inflation and should not contradict observable temperature fluctuations. The slow classical motion is given by trajectories that flow around the potential hills. We investigated the fields dynamics in a vicinity of a hill, which was modeled by eq. (2.6), see figure 9. The main constraint is imposed on the parameter $d$, which is proportional to the inflaton mass squared. For the quadratic gravity $d \sim 10^{-12}$ in the Planck units.

At small hill’s height no inhomogeneities appear (nevertheless, substantial deformation of the CMB spectrum has been predicted in [15]). The case of a small hill in our simulations corresponds to $a < 2$. In the case of a high hill ($a \gtrsim 2$, see figure 9) the field dynamics leads to formation of closed domain walls and finally to the clusters of the primordial black holes as was discussed above.
6 Conclusion

We have considered an inflationary model with two real scalar fields with the potential having one minimum, local maximum(s), and saddle point(s). The evolution of non-trivial field configurations that have the form of closed loops in the \((\varphi, \chi)\) plane has been studied. The set of these configurations can be split into disjoint equivalence classes that are identified by the winding number of the field configurations. We have provided simple expressions for analytical and numerical calculation of the winding number of arbitrary field configurations.

Evidently, in the case of an infinite height of the maximum of the potential the winding number can not change during the classical evolution. On the other hand, a finite height of the maximum allows the change of the homotopic class of a given field configuration, along with the stepwise decrease of the winding number. The realisation of such changes depends on the parameters of the potential and on those of the initial field configuration. We have quantitatively confirmed these qualitative features of the transitions that change the winding number, considering an example of a model with a tilted Mexican hat potential and the multi-stream inflationary model.

Field configurations similar to those considered in this work could be formed during the inflationary stage, and lead to strong inhomogeneities in the cosmic microwave background radiation temperature fluctuations. Similar mechanism of PBH formation and its consequences were discussed in, e.g., [19, 20, 57, 60, 65]. It was shown that these PBHs could be responsible for the large structure formation, reionization and existence of early quasars. This phenomenon could develop in a substantial number of the inflationary models with the potential having a saddle point, as we mentioned in the Introduction. The new type of solitons studied here may be also useful as a mechanism of production of black holes with complex mass spectra [18–20, 52–55] and could be used as a part of the solution of a lot of cosmological (high-z quasars and supermassive black holes within galactic nuclei [18–20, 52, 54, 56–59], dark matter [53, 54, 60–62]) and astrophysical (unidentified sources of gamma-radiation [53, 63, 64], reionization [53, 65–67], the absence of intermediate-mass black holes [59, 68]) problems.

In conclusion, we emphasize that our study opens wide prospects for further research. For instance, our consideration is limited to planar domain walls in the three-dimensional space, whereas the formation and evolution of string-type configurations will be the subject of a future study.

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