What can we learn about the lepton CP phase in the next 10 years?

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ABSTRACT: We discuss how the lepton CP phase can be constrained by accelerator and reactor measurements in an era without dedicated experiments for CP violation search. To characterize globally the sensitivity to the CP phase $\delta_{\text{CP}}$, we use the CP exclusion fraction, which quantifies what fraction of the $\delta_{\text{CP}}$ space can be excluded at given input values of $\theta_{23}$ and $\delta_{\text{CP}}$. Using the measure we study the CP sensitivity which may be possessed by the accelerator experiments T2K and NO$\nu$A. We show that, if the mass hierarchy is known, T2K and NO$\nu$A alone may exclude, respectively, about 50% – 60% and 40% – 50% of the $\delta_{\text{CP}}$ space at 90\% CL by 10 years running, provided that a considerable fraction of beam time is devoted to the antineutrino run. The synergy between T2K and NO$\nu$A is remarkable, leading to the determination of the mass hierarchy through CP sensitivity at the same CL.

KEYWORDS: Neutrino Physics, CP violation
1 Introduction  

After accumulating hints and indications, the elusive lepton mixing angle $\theta_{13}$ was finally discovered to be non-zero and measured with high precision [1–9]. Thus, we are left with the CP violating phase $\delta_{CP}$, the unique unknown parameter in the lepton flavor mixing matrix [10], which could remain a mystery for sometime together with the problem of determining the neutrino mass hierarchy. Lepton CP violation due to $\delta_{CP}$, in association with the one by the possible Majorana phases, may hide the secret behind the baryon number asymmetry in our universe [11]. However, because of the smallness of the effects of $\delta_{CP}$, being suppressed by the small ratio of two $\Delta m^2$ and products of mixing angles, its measurement will require dedicated facilities such as Hyper-Kamiokande [12] and LBNE [13] as well as intense neutrino beams.
Here, the potential problem is that it will take a long time, \(\sim 10\) years, to construct and operate such facilities. Therefore, it may be worthwhile to ask the question, “What can be done in the next 10 years toward the observation of lepton CP violation?” To sharpen up our concern we may ask a more scrutinizing question: “How can an experiment that is not actually capable of observing CP violation due to \(\delta_{CP}\) help us to pave the way to the final discovery?” It is the purpose of this paper to give a partial answer to these questions.

To reveal the sensitivity to CP violation at a particular time, one can think of two different approaches: bring all available data together to collect every tiny piece of information on \(\delta_{CP}\) in them to enhance CP sensitivity, the spirit of the so called global fits [14–16], or focus on a few measurements which have relatively higher sensitivities to CP. In this paper, following our previous analysis [17], we take the latter strategy with implementing the precision reactor measurement of \(\theta_{13}\) [18]. There are pros and cons in each approach. In a global fit the sensitivity is higher, but it is achieved at the price of combining many experiments with different systematic errors. In our approach that drawback is somewhat cured though it may not reveal the best possible sensitivity to CP violation. We believe that it is important to proceed in both ways as they are complementary to each other.

In addressing CP violation, one of the relevant issues is how the CP sensitivity achievable by a particular experimental setting can be quantified and displayed. Though it might sound a bit technical, this is an important point because getting a robust hint, although it could still be only a slight indication, is an important step for the successful completion of the long-term race toward detecting and measuring the lepton CP violating phase \(\delta_{CP}\), the marathon in neutrino physics. To quantify the maximum sensitivity possessed by a particular experiment, or by a set of experiments to measure \(\delta_{CP}\), we analyze the fraction of values of \(\delta_{CP}\) that can be excluded for a given set of input parameters, which we call the “CP exclusion fraction”. Notice that it is essentially the same measure as the “CP coverage” which was introduced in [19] and extensively used in the analyses in [20]. See Sec. 2 and Appendix A for more about the relationship between the measures for CP sensitivity.

We argue that one of the most important goals related to lepton CP violation that may be reached by the ongoing and the upcoming experiments is to exclude a significant fraction of the \(\delta_{CP}\) space. The CP exclusion fraction will serve for the discussion of this point. We will use the global measure to investigate the CP exclusion potential of T2K and NO\(\nu\)A within a 10 years perspective. It is interesting and timely to discuss the following questions: What is the impact of running T2K also in the antineutrino mode on the determination of \(\delta_{CP}\)? What would be the optimal time sharing between neutrino and antineutrino beams in order that T2K can say something meaningful on \(\delta_{CP}\)? How T2K and NO\(\nu\)A compare with each other in \(\delta_{CP}\) sensitivity? Can the combination of equal-time running of T2K and NO\(\nu\)A say more on \(\delta_{CP}\) than each one of these experiments with doubled running time? Or, rephrasing, is there a synergy between them?

Our results demonstrates that CP sensitivities which may be achievable by 10 years running of T2K and NO\(\nu\)A are not so low, even after admitting the fact that these experiments are not originally designed to discover CP violation. We have found that running T2K in the antineutrino mode makes the experiment, in general, much more powerful in ex-
cluding regions of $\delta_{\text{CP}}$ in a way independent of the neutrino mass hierarchy, the $\theta_{23}$ octant, and of the sign of $\sin \delta_{\text{CP}}$. Our study shows that the optimal setting would be to run about half the time in neutrino and the other half in antineutrino mode. If we compare these two experiments, it appears that T2K has better sensitivity for CP, but NO$\nu$A can make an unique contribution by its higher sensitivity to the matter effects. As a consequence the synergy between these two experiments is quite visible. See, for example, [21, 22] for related works.

2 CP exclusion fraction; A measure of CP sensitivity for non-conclusive experiments

In this paper, we investigate the experimental sensitivity to $\delta_{\text{CP}}$ of T2K and NO$\nu$A and quantify it by using the fraction of $\delta_{\text{CP}}$ values which can be disfavored by these experiments for a given set of input parameters. We call this fraction the “CP exclusion fraction” $f_{\text{CPX}}$. As explained in more detail in Appendix A.1, $f_{\text{CPX}}$ is calculated as the fraction of $\delta_{\text{CP}} \in [-\pi, \pi]$ values which can be excluded by the experiment at a given confidence level for each input point of the parameter space ($\sin^2 \theta_{23}, \delta_{\text{CP}}$). It is thus a global measure which covers the entire input parameter space. It should be mentioned that the CP exclusion fraction is related to the “CP coverage” defined and extensively used in [19, 20] as $f_{\text{CPX}} = 1 - \text{CP coverage}/360^\circ$. Instead of using the CP coverage we choose to work with CP exclusion fraction for an appeal to intuition to facilitate understanding the plots. In this work, we use the standard parameterization for the neutrino mixing angles as well as the CP phase $\delta_{\text{CP}}$ found in Ref. [23].

While expert readers can go directly to Sec. 3 for the analysis results, a comparative discussion of CP exclusion fraction $f_{\text{CPX}}$ with CP violation (CPV) fraction may be illuminating for a wide range of non-expert readers to reveal the nature of the two measures for CP sensitivity and their difference. The latter gives us the fraction of $\delta_{\text{CP}}$ values for which CPV can be established as a function of the input parameter values, usually as a function of $\sin^2 \theta_{13}$.

We first note that in the general context of revealing CP sensitivity, they are complementary to each other. Then, what are the differences?

The CP exclusion fraction plot is a particularly useful tool to reveal the potential for exploring the CP phase effects by a “non-conclusive experiment” which is not designed as a dedicated CP violation discoverer. Suppose that there are two experiments each of which alone can not discover (establish) CPV at a given CL. In this case the CPV fraction vanishes for both experiments, and it does not provide us with any useful informations. But, with the use of $f_{\text{CPX}}$ we are able to reveal the CP sensitivity of each experiment and can tell which one has higher capability of restricting the allowed range of $\delta_{\text{CP}}$. In this way, the CP exclusion fraction serves as a viable way of quantifying the experimental CP sensitivity for non-conclusive experiments, and provides a better chance for a fruitful discussion of synergy.

The merit of using the CPV fraction is that it conveys a clear cut message by focusing on “yes or no” to CP violation. Because of the definition, however, it suffers from the “bias”

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1 The CPV fraction is used in many papers including, for example, Refs. [12, 21].
of choosing $\delta_{CP}$ equal 0 or $\pi$ as a reference point to measure the capability of detecting CP violation. That is, the CPV fraction plot neither tells us whether the experiment is able to exclude for example, $\delta_{CP} = \pi/2$ or $-\pi/2$, nor allows us to extract the precision on $\delta_{CP}$ determination, e.g., at around these points. We emphasize that the exclusion of the region around $\delta_{CP} = \pm \pi/2$, depending upon the mass hierarchy, is likely to be the initial footprint of the near future experiments which first step into probing the CP phase.

3 Sensitivity to CP phase expected by T2K

In this and the following sections we discuss the results of our analyses, the sensitivities to CP phase determination or exclusion to be expected by the T2K and NO\(\nu\)A experiments, respectively, assuming accurate measurement of $\theta_{13}$ by the reactor experiments. Details of our analysis method are described in Appendix B. An intuitive understanding of some of the salient features of the analysis results will be offered in Appendix C.

Considering the nature of the experiments as the initial phase of CP measurement we will use, throughout this section, the CP exclusion fraction in $\delta_{CP} - \sin^2 \theta_{23}$ space defined at 90% CL to display the sensitivity to CP phase $\delta_{CP}$.\(^2\) We note that while 90% CL may not guarantee high enough confidence for exclusion, the criterion is often used to place useful constraints on physics parameters in the literatures, for example, in the reports from Bugey \cite{24}, Chooz \cite{25}, and T2K \cite{1} experiments. While we show only the results corresponding to 90% CL in this paper, we have also performed the computations to obtain the contours at 95% CL ($\approx 2\sigma$ CL). Very roughly speaking, the change of CP exclusion fraction when we use 95% CL is that the contours of equal $f_{CPX}$ at 90% CL are to be interpreted as $f_{CPX} - (0.1 - 0.15)$ at 95% CL, the precise values of $f_{CPX}$ reduction depend on $\delta_{CP}$ and $\sin^2 \theta_{23}$.

We focus our discussion primarily on the possibility of a total of 10 years of data taking. The reason being, as we will see shortly, that after a total of 5 running years T2K will only be able to exclude 50% of $\delta_{CP}$ values in a very limited parameter space in the $\delta_{CP} - \sin^2 \theta_{23}$ plane, even if we assume that the mass hierarchy is known. We would like to explore the possibility of increasing the CP sensitivity of the experiment in a longer time span. As we mentioned in Sec. 1, most probably, the construction of a dedicated CP explorer needs longer than 10 years from now, so that it is not an unrealistic scenario to examine.

The inverted mass hierarchy has been favored by some experimental analyses \cite{26, 27}, however feebly. Hence, the choice of the hierarchy to be displayed in our figures is basically arbitrary, and we opt for the inverted one. Our treatment will not be completely equal for T2K and NO\(\nu\)A, because our analysis of NO\(\nu\)A can not be as mature as that of T2K for which we can profit from the informations of the experiment in operation.

3.1 Total of 5 running years ($5 \times 10^{21}$ POT)

In Fig. 1, the contours of equal CP exclusion fraction are plotted in the space spanned by the true values of $\delta_{CP}$ and $\sin^2 \theta_{23}$. A total running time of 5 years is assumed with

\(^2\) Of course, since $\theta_{13}$ has been measured rather accurately it is now more appropriate to discuss the sensitivity to $\delta_{CP}$ in the $\delta_{CP}$ vs $\sin^2 \theta_{23}$ space as $\theta_{23}$ is now the least known angle.
the nominal design luminosity, and the results for the $\nu + \bar{\nu}$ beam time sharing of $5 + 0$, $3 + 2$, and $2 + 3$ years are shown (panels from left to right). Intermediate runnings, like $4 + 1$ years, lie between the results shown. In the upper panels (lower panels) of Fig. 1 the inverted (normal) hierarchy is assumed as the input true mass hierarchy. It is quite likely that the mass hierarchy will not be determined with high confidence level when T2K completes its running period of 5 years. Therefore, we present here only the case where we fit for an unknown mass hierarchy, obtained by marginalizing over both cases.

![Figure 1](image)

**Figure 1.** CP exclusion fraction isolines plotted on the $\delta_{\text{CP}} \sim \sin^2 \theta_{23}$ plane at 90% CL, for T2K running in $\nu + \bar{\nu}$ mode for $5 + 0$ (left), $3 + 2$ (center) and $2 + 3$ (right) years. The top (bottom) panels are for the case of inverted (normal) input mass hierarchy. The fit marginalizes over both hierarchies.

The numbers on the isolines correspond to the CP exclusion fraction that can be achieved at 90% CL. By comparing the CP exclusion fractions of the three cases of $\nu + \bar{\nu}$ running periods of $5 + 0$, $3 + 2$, and $2 + 3$ years in Fig. 1, it is evident that running in antineutrino mode helps to improve the CP sensitivity. It is notable that the performance of $3 + 2$ and $2 + 3$ years of runnings are roughly comparable to each other.

We note some characteristic features of the exclusion fraction iso-contour lines we can see in Fig. 1:

- Overall, the regions of relatively high sensitivity to CP are centered around $\delta_{\text{CP}} \simeq \pm \pi/2$.
- In the $5 + 0$ years running option the CP sensitive region is restricted mostly to two regions centered at $(\delta_{\text{CP}} \simeq \pi/2, \text{low } \sin^2 \theta_{23})$ and $(\delta_{\text{CP}} \simeq -\pi/2, \text{high } \sin^2 \theta_{23})$, whereas
in 3 + 2 and 2 + 3 years running options (center and right panels) the dependence on \( \sin^2 \theta_{23} \) is weakened, particularly at around \( \delta_{\text{CP}} \simeq \pi/2 \) and \( \delta_{\text{CP}} \simeq -\pi/2 \) for the inverted and the normal hierarchies, respectively.

From the probability point of view, one naively expects that the highest sensitivity to CP would be at around \( \delta_{\text{CP}} \simeq \pm \pi/2 \), in agreement with the first feature mentioned above. However, as statistics increases these most favorable values become less favorable than \( \delta_{\text{CP}} = 0 \), depending on the \( \theta_{23} \) value and our knowledge on the mass hierarchy, as will be shown in Figs. 2-4. An attempt to explain such a change in behavior in terms of the bi-probability plot can be found in Appendix C (see Fig. 8 and its description). The second feature explained above regarding the dependence on \( \theta_{23} \) can also be understood qualitatively in terms of the bi-probability plot, see Fig. 7 and the related discussions in the Appendix C. Associated questions on the effect of the uncertainty of \( \theta_{23} \) on \( \delta_{\text{CP}} \) determination in the precision era has been addressed in [28].

### 3.2 Total of 10 running years (10^{22} \text{ POT})

In Fig. 2 we present similar contours of equal CP exclusion fraction for a total of 10 running years with \( \nu + \bar{\nu} \) beam time sharing of 10 + 0, 7 + 3, and 5 + 5 years (panels from left to right), assuming the nominal design luminosity. The results for 3 + 7 running years (not shown) are similar to the latter two cases, which represent the best sensitivities among the studied cases of a total of 10 running years. The results presented in the top panels were obtained by marginalizing over the mass hierarchies (black contours). The middle and bottom panels are for cases of a fit assuming the normal (blue contours) and the inverted (red contours) mass hierarchies, respectively. In Fig. 2, only the case for inverted mass hierarchy as input is shown.

The main features of the CP exclusion fraction contours for the normal mass hierarchy as input may be obtained, in the zeroth order approximation, by doing the re-parameterization \( \delta_{\text{CP}} \rightarrow \pi - \delta_{\text{CP}} \) in Fig. 2. This approximation is valid because of the small matter effect in the T2K setting. The particular case of T2K 5 + 5 running years with the normal hierarchy as input is shown in the next section, see Fig. 3.

It should be emphasized first that as in the case of 5 years of data taking, the inclusion of antineutrino running time significantly improves the sensitivity to CP phase. Some of the distinctive features of running T2K for 10 years, shown in Fig. 2, compared to the results in 5 years running shown in Fig. 1, are:

- With marginalization over the mass hierarchies (top panels) the null sensitivity regions become significantly smaller, in particular, if we compare the last two top panels of each figure.

- The 7 + 3 and 5 + 5 years running results, when fitted assuming the inverted mass hierarchy (the correct one in this case), can exclude 50\% (or higher) values of \( \delta_{\text{CP}} \) in almost the entire \( \delta_{\text{CP}} - \sin^2 \theta_{23} \) plane allowed by the current oscillation data. This can be seen in the bottom center and right panels.
Figure 2. CP exclusion fraction isolines plotted on the $\delta_{\text{CP}} - \sin^2 \theta_{23}$ plane at 90% CL, for T2K running in $\nu+\bar{\nu}$ mode for 10 + 0 (left), 7 + 3 (center) and 5 + 5 (right) years. The input mass hierarchy is the inverted one. The top panels are for a fit marginalizing over the hierarchies, while the middle (bottom) panels are for a fit imposing the normal (inverted) hierarchy.

- The 7+3 and 5+5 years running results, when fitted using the normal mass hierarchy, can exclude a fraction of $\delta_{\text{CP}}$ values up to 80%-90% for $\delta_{\text{CP}} > 0$. The higher exclusion power is due to the assumption of the wrong mass hierarchy. But for $\delta_{\text{CP}} < 0$, specially when $\theta_{23}$ is in the second octant, the exclusion fraction tends to be much less than the one for the right hierarchy.

What is the meaning of doing a fit assuming the wrong mass hierarchy? If the hierarchy is known with high confidence level, of course, there is no physics sense of attempting a fit assuming the wrong mass hierarchy. The real question is: What does it mean at the time in which the mass hierarchy is not established? We argue that it is an alternative and useful way of probing the mass hierarchy sensitivity in terms of the CP exclusion fraction.
Since this point will become clearer in the discussion of NOνA results we will come back to it in the next section.

4 Sensitivity to CP phase expected by NOνA and by Its Combination with T2K

4.1 10 running years: NOνA ($6 \times 10^{21}$ POT)

In Fig. 3 the contours of equal CP exclusion fraction are plotted for a total of 10 running years of the NOνA experiment with $\nu + \bar{\nu}$ beam time sharing of $5 + 5$ years. The left and middle panels are for the case of inverted and normal mass hierarchies, respectively. The results for $7 + 3$ years running (not shown) are similar to the ones in Fig. 3. To make a comparison with T2K sensitivity to CP phase easier we place on the right panels of Fig. 3 the contours of equal CP exclusion fraction obtained by T2K $5+5$ years running in the case of normal input mass hierarchy. (For similar contours with the inverted mass hierarchy see Fig. 2.) As in Fig. 2 the upper panels are for cases marginalized over the mass hierarchies (black contours). The middle and bottom panels are for cases of a fit assuming the normal (blue contours) and the inverted (red contours) mass hierarchies, respectively.

We notice the following two significant features of NOνA’s CP sensitivity in comparison to that of T2K:

- The sensitivity of NOνA to CP phase is worse than that of T2K when marginalized over the mass hierarchies (top panels), almost losing the sensitivity in the negative (positive) half plane of $\delta_{CP}$ for the input inverted (normal) mass hierarchy.

- Similarly, T2K is slightly better than NOνA in the CP sensitivity assuming the right mass hierarchy (middle panels of the second and third columns), having 60% contours of CP exclusion in both half planes of $\delta_{CP}$. On the other hand, in the wrong mass hierarchy fit the NOνA CP sensitivity is overwhelming, making almost a complete exclusion at 90% CL of one of the half planes possible.

Let us understand these characteristics. It appears that the relatively low NOνA CP sensitivity compared to that of T2K comes partly from the relatively low statistics. Although the number of events depends on the input parameters, it appears that in general T2K is able to accumulate 20–30% more statistics than NOνA. In addition to this, as discussed in Appendix C, the fact that the major axis of the CP ellipse for NOνA is shorter than that for T2K (see Fig. 7) makes the CP sensitivity of NOνA worse than that of T2K, even for similar statistics.

On the other hand, the powerfulness of excluding almost half the space (positive $\delta_{CP}$ region for the inverted, and negative $\delta_{CP}$ region for the normal mass hierarchies) in the wrong hierarchy fit is due to the larger matter effect thanks to the longer baseline of NOνA.

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3 One should keep in mind that we are comparing between the sensitivities expected by these two experiments by taking the particular official values of neutrino fluxes, $10^{21}$ and $6 \times 10^{20}$ POT a year for T2K and NOνA, respectively.
Using this property the CP exclusion fraction may be used as a powerful indicator of the mass hierarchy though in a particular region of $\delta_{\text{CP}}$. Therefore, it appears to us that these two experiments complement each other quite nicely.

4.2 Combination of NO$\nu$A with T2K and the synergy

One of the most intriguing questions would be how high is the sensitivity to the CP phase when T2K and NO$\nu$A are combined, and to what extent a synergy can be expected. To answer these questions, we present in Fig. 4 the contours of CP exclusion fraction obtained by combining 5+5 years running of T2K and NO$\nu$A (a total of 10 years each) for the input
Figure 4. CP exclusion fraction isolines plotted on the $\delta_{CP} - \sin^2 \theta_{23}$ plane at 90% CL, for NOvA and T2K running in $\nu + \bar{\nu}$ mode for 5 + 5 years (each) combined as well as T2K running for 10 + 10 years. The left and center panels are for the combination using the inverted and normal mass hierarchy, respectively, as input. The right panels are for T2K with inverted mass hierarchy as input. The top panels are for a fit marginalizing over the mass hierarchies, while the middle (bottom) panels are for a fit imposing the normal (inverted) mass hierarchy.

normal (left panels) and inverted (middle panels) mass hierarchies. To extract the effect of the synergy we place in the right panel of Fig. 4 the contours obtained by a hypothetical 10 + 10 years running of T2K (a total 20 years). Although we do not consider it a realistic option, we show it for the sake of revealing the synergy.

The distinctive features of Fig. 4 are as follows:

- One of the most significant features in Fig. 4 is that the wrong mass hierarchy is excluded at 90% CL in almost the entire allowed region in $\delta_{CP} - \sin^2 \theta_{23}$ space. In the settings discussed in this paper it can only be achieved by combining T2K and
NOνA.

- It is quite remarkable in the left and center set of panels that in the entire $\delta_{\text{CP}} - \sin^2 \theta_{23}$ space is covered by 60% or higher exclusion fraction region, even with marginalization over the mass hierarchies, or in the right mass hierarchy fit.

- For the case of T2K and NOνA combined or the T2K only but for the known mass hierarchy, the region of the highest sensitivity tends to exist at $\delta_{\text{CP}} \sim 0$ or $\pm \pi$, which is different from the cases of lower statistics where the highest sensitivity likely to occur at $\delta_{\text{CP}} \sim \pm \pi/2$. A qualitative explanation of this feature based on the bi-probability plots is found in Appendix C.

- Another salient feature in Fig. 4 is that the effect of the synergy is evident when T2K and NOνA combination (each for a total of 10 years, left and center panels) is compared to T2K running for 20 years (right panels). This is so, in particular, in the case with marginalization over the mass hierarchies, or for the wrong mass hierarchy fit.

5 The interplay between $\delta_{\text{CP}}$ and $\theta_{23}$ octant for the experimental strategy

Until now, we have focused on the sensitivity to CP phase and discussed some strategy to optimize it. Actually, T2K and NOνA can endeavor to measure another very important unknown: the octant of $\theta_{23}$. That said, we raise the straightforward question “How the strategies for determining $\delta_{\text{CP}}$ and the $\theta_{23}$ octant are related?”

To answer this question, let us first recollect some relevant features of the $\theta_{23}$ octant measurement. Due to high statistics of the disappearance channels $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$, $\sin^2 2\theta_{23}$ can be measured with high precision, but they are insensitive to the $\theta_{23}$ octant. On the other hand, because of its relatively low statistics, the appearance channels $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ have a capability of breaking the octant degeneracy only if the determinations of $\sin^2 2\theta_{13}$ and $\sin^2 2\theta_{23}$ are precise enough. For concreteness, let us focus on T2K. After 10 years of running, we expect that the determination of $\theta_{23}$ by the disappearance channels is dominated by systematic errors. Hence its sensitivity to $\sin^2 2\theta_{23}$ would be approximately independent of the running configuration.

Now, we discuss how to proceed with the $\nu_e$ and $\bar{\nu}_e$ appearance channels. If T2K runs solely in the neutrino mode, the octant degeneracy becomes virtually unsolvable even if we take into account energy spectrum as we will see below. First, let us consider only the total rates. From Fig. 7, we can see that by only using the neutrino mode, even if we know the true mass hierarchy and the precise value of the oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$, unless we know rather well the value of $\delta_{\text{CP}}$, $\theta_{23}$ different octants can be confused. This is in general true apart from the case where $\theta_{23}$ lies in the 1st (2nd) octant and $\delta_{\text{CP}}$ is close to $\pi/2$ ($-\pi/2$).

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4 Note, however, that solving the $\theta_{23}$ octant degeneracy is not the whole story, as stressed in [28].
Figure 5. Appearance probabilities $P(\nu_\mu \rightarrow \nu_e)$ for neutrino (left panel) and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ for antineutrino (right panel) as a function of the neutrino energy, for $\delta_{\mathrm{CP}} = 0, \pm \pi/2$ for the case where $0.95 < \sin^2 2\theta_{23} < 0.97$.

We can try to see if the energy spectrum information will help in resolving this degeneracy. Let us look at Fig. 5 which shows the appearance probabilities $P(\nu_\mu \rightarrow \nu_e)$ for neutrino (left panel) and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ for antineutrino (right panel) as a function of the neutrino energy for the case where $0.95 < \sin^2 2\theta_{23} < 0.97$ for various different values of $\delta_{\mathrm{CP}}$. We can see from the left panel of Fig. 5 that the two cases of $\delta_{\mathrm{CP}} = -\pi/2$ with $\theta_{23}$ in the 1st octant (filled band by red color) and $\delta_{\mathrm{CP}} = 0$ with $\theta_{23}$ in the 2nd octant (the band delimited by the dashed blue curves) are easily confused even if we take into account the energy spectrum. However, these two cases give very different probabilities in the antineutrino modes, as we can see from the right panel of Fig. 5, which means that combining with antineutrino data will certainly help in resolving the octant degeneracy. The importance of the antineutrino run in resolving the octant degeneracy was inherent in the analysis in Ref. [34], and some of the related points are discussed recently in Ref. [29, 30].

When the antineutrino running is incorporated in T2K, the comparison between the event rates as well as the energy spectra of the $\nu + \bar{\nu}$ modes challenges the degeneracy toward its resolution in a more robust way. In order to understand to what extent the mechanism works, we present in Fig. 6 the regions of resolution of the octant degeneracy in $\delta_{\mathrm{CP}} - \sin^2 2\theta_{23}$ space, calculated by imposing a Gaussian uncertainty in $\sin^2 2\theta_{23}$ of 0.02 at 68% CL. The regions colored in blue, green and red represent the region on the plane of the true values of $\delta_{\mathrm{CP}}$ and $\sin^2 2\theta_{23}$ in which the octant of $\theta_{23}$ can be distinguished at 1$\sigma$, 2$\sigma$, and 3$\sigma$ CL, respectively. Around maximal $\theta_{23}$, no identification of the $\theta_{23}$ octant is possible even at 1$\sigma$ CL. In the panels from left to right are shown $10 + 0$, $7 + 3$, and $5 + 5$ years running. It is also worth mentioning that for $3 + 7$ years running almost the same sensitivity as for running $7 + 3$ or $5 + 5$ years is obtained.

As can be seen, the inclusion of the antineutrino run significantly improves the sensitivity to the octant determination of $\theta_{23}$ because it can break the octant degeneracy as discussed above. We also notice that once the fraction of time allocated for antineutrino
Figure 6. Regions in which the $\theta_{23}$ octant degeneracy is resolved are plotted on the $\delta - \sin^2 \theta_{23}$ plane for 10+0 (left panel), 7+3 (middle panel), 5+5 (right panel) years of $\nu + \bar{\nu}$ running of T2K. In the upper and lower panels the cases of inverted and normal mass hierarchies, respectively, are shown.

running reaches 30% or so of the total running time the sensitivity to the octant of $\theta_{23}$ is remarkably stable against variations of this fraction. As we saw, the impact of the $\nu + \bar{\nu}$ beam time sharing on the octant determination is much less important than on the CP sensitivity, at least for the experiments we are interested in. It seems to us that the optimal proportion of antineutrino to neutrino could be mainly dictated by the sensitivity for the CP phase without losing essentially that for the octant $\theta_{23}$ determination.

6 Conclusion

In the near future, 5 to 10 years from now, we do not expect to be able to measure the lepton CP phase, since we will not yet dispose of neutrino experiments designed to discover CP violation due to non-zero $\sin \delta_{\text{CP}}$. However, the accelerator based neutrino oscillation experiments, T2K and NO$\nu$A, after the precise measurement of $\sin^2 \theta_{13}$ by the reactor experiments, will have some sensitivity to $\delta_{\text{CP}}$. This sensitivity will depend on the true values of $\delta_{\text{CP}}$, $\sin^2 \theta_{23}$, the neutrino mass hierarchy as well as the amount of data taking in neutrino and antineutrino modes.

To study the maximal sensitivity to $\delta_{\text{CP}}$ attainable by a single or a set of experiments we employed the CP exclusion fraction, which quantifies the range of $\delta_{\text{CP}}$ that can be excluded, at a certain confidence level (we adopted 90% in this paper), by a set of experimental observables. We expect that the CP exclusion fraction is particularly useful to examine the potential of exploring CP phase possessed by the near future experiments which may be
the unique sources of information on the CP phase in an era without dedicated apparatus designed for the discovery of CP violation.

By using the CP exclusion fraction we have analyzed the CP sensitivity of T2K and NOνA experiments. We have shown that it is important to run T2K in the antineutrino mode in order to significantly enhance the CP sensitivity of this experiment. The optimal situation seems to be to share the time equally between neutrino and antineutrino beams. If one could run T2K for 10 years one would be able to exclude 50% or more of the $\delta_{\text{CP}}$ values in almost the entire half plane of $\delta_{\text{CP}} > 0$ ($\delta_{\text{CP}} < 0$) for the inverted (normal) mass hierarchy and $\sin^2 \theta_{23} \in [0.35, 0.65]$, even not knowing the neutrino mass hierarchy. If the neutrino mass hierarchy is known by that time, one could exclude 50% (or more) of the $\delta_{\text{CP}}$ values on almost the entire $\delta_{\text{CP}} - \sin^2 \theta_{23}$ plane.

We have shown that NOνA is less powerful than T2K for the CP sensitivity as measured with the CP exclusion fraction. An intuitive understanding of this feature is offered by using the bi-probability plot and by noting the difference in statistics in both experiments. By combining both experiments, however, one could exclude 60% or more of $\delta_{\text{CP}}$ values in the $\delta_{\text{CP}} - \sin^2 \theta_{23}$ plane. It should be noticed that the synergy between these two experiments is quite visible, allowing the exclusion of the wrong mass hierarchy at 90% CL in almost the entire $\delta_{\text{CP}} - \sin^2 \theta_{23}$ space.

We have also examined T2K sensitivity to the $\theta_{23}$ octant, showing that adding antineutrino run also helps the experimental sensitivity to $\sin^2 \theta_{23}$. The determination of this parameter will further help constraining $\delta_{\text{CP}}$, as it will exclude part of the currently allowed region of $\delta_{\text{CP}}$ and $\sin^2 \theta_{23}$.

We emphasize that the 10% uncertainty we adopt in our analyses, for both experiments, may be a very conservative choice, in particular for the analysis of 10 years running. This is because T2K already achieved the uncertainty of $\simeq 10\%$ for running in neutrino mode, and it is conceivable that this will be improved in the future. A caution is, however, that so far little experimental information is accumulated in the antineutrino mode.

The results of our analysis in this paper underlines the necessity of dedicated experiments specially designed to access the lepton CP violating phase $\delta_{\text{CP}}$. Examples for such apparatus include Hyper-Kamiokande or LBNE. Nonetheless, we emphasize the importance of getting as much information as we can on $\delta_{\text{CP}}$ before that day of dedicated machines arrives. It will certainly help us to lay the foundations for winning perhaps the long-term hardest job of hunting the lepton CP phase, the marathon in neutrino physics.

A Definition of CP exclusion fraction and its properties

A.1 CP exclusion fraction; Definition

We follow the conventional $\chi^2$ method to calculate the likelihood, at a given confidence level, of rejecting points in the parameter space ($\sin^2 \theta_{23}, \delta_{\text{CP}}$) for a given input value of the parameters ($\sin^2 \theta_{23}^{\text{in}}, \delta_{\text{CP}}^{\text{in}}$). Toward the goal, we compute the expected number of events $T_i$ in the $i$-th energy bin as a function of the input parameters, $T_i(\theta_{23}^{\text{in}}, \delta_{\text{CP}}^{\text{in}}, h^{\text{in}})$, where $h^{\text{in}}$ is the input neutrino mass hierarchy. We also compute the expected number of events $F_i$ in the $i$-th energy bin for a given set of fit and nuisance parameters $\{\alpha\}$,
\( F_{i}(\theta_{13}^{fit}, \theta_{23}^{fit}, \delta_{CP}^{fit}, h^{fit}, \{\alpha\}) \). These numbers include neutrino and antineutrino events, according to the assumed exposure. With these we can build the likelihood function

\[
-2 \ln L(\theta_{23}^{in}, \delta_{CP}^{in}, h^{in}, \delta_{CP}^{fit}) = \min_{\{\theta_{13}^{in}, \theta_{23}^{in}, h^{in}, \delta_{CP}^{fit}\}} \left\{ \sum_{i=1}^{n_{h}} 2 \left( F_{i} - T_{i} + T_{i} \ln \frac{T_{i}}{F_{i}} \right) + \sum_{j} \left( \frac{\alpha_{j}}{\sigma_{j}} \right)^{2} \right. \\
+ \left. \left( \sin^{2} 2\theta_{13}^{in} - \sin^{2} 2\theta_{13}^{fit} \right)^{2} \sigma_{13}^{2} \right\}, \\
\text{(A.1)}
\]

where we set \( \sin^{2} 2\theta_{13}^{in} = 0.089 \). The expected number of events includes the contribution from signal and background so that schematically \( F_{i} = \alpha_{j} F_{i}^{\text{signal}} + \alpha_{j+1} F_{i}^{\text{bck}} \). The likelihood (A.1) will be used to calculate, at a given confidence level, the fraction of values of \( \delta_{CP} \) that are not compatible with the assumed input values. For T2K, we use 23 energy bins of 50 MeV and for NO\( \nu \)A 20 bins of 150 MeV. In both cases we assume \( \sigma_{13} = 0.005 \), and all \( \sigma_{j} = 0.1 \).

### A.2 Relationship between CP exclusion fraction and the uncertainty on \( \delta_{CP} \)

An another global measure to display the CP sensitivity used in the literature is simply to evaluate the uncertainty on the determination of \( \delta_{CP} \), at a certain CL, as a function of the true value of \( \delta_{CP} \). This measure has been used recently, e.g., in Ref. [32]. We note here that our CP exclusion fraction has intimate relationship with the uncertainty on \( \delta_{CP} \) determination. Since \( 1 - f_{CPX} \) is equal to the fraction of the allowed range of \( \delta_{CP} \), at least naively, \( (1 - f_{CPX}) / 2 \) would imply the uncertainty associated with \( \delta_{CP} / 2\pi \). Unfortunately this is not quite true, if the allowed range of \( \delta_{CP} \) is disconnected or there are multiple fake solutions,\(^5\) in which case the interpretation can be misleading. In fact, in T2K and NO\( \nu \)A with \( \sim 10 \) years perspective we expect, mainly due to the lack of statistics, that the determination of \( \delta_{CP} \) will be plagued by large uncertainties and degeneracies, which entails severe non-Gaussian features of the \( \chi^{2} \) distribution.

On the other hand, in precision measurement era in which \( \delta_{CP} \) can be measured with high precision, the \( \chi^{2} \) will become locally Gaussian. Because of the above mentioned properties, we expect that \( (1 - f_{CPX}) / 2 \) will turn smoothly to be the uncertainty on \( \delta_{CP} / 2\pi \). Therefore, while we prefer to work with the CP exclusion fraction for the time being because it is more tolerant to degeneracies, the two measures will become more closely related to each other in the era of dedicated CP experiments.

### B Analysis Method

In order to simulate T2K \( \nu_{\mu} \rightarrow \nu_{e} \) and \( \bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e} \) events, we used a similar machinery as the one developed in Ref. [17]. We took the fluxes for the neutrino and antineutrino modes, as well as the backgrounds, from the Hyper-Kamiokande letter of intent [12], normalizing the numbers to the T2K experimental parameters. We used the cross sections from Ref. [34]. The migration of events were taken into account as below, in a similar way as done in [17].

\(^{5}\) A related detailed discussion about this point can be found in [20].
We considered four systematic uncertainties, that is, the signal and background absolute normalizations for both neutrino and antineutrino modes. We took all of them to be 10%, as described in the previous section. In view of the fact that T2K comes already very close to 10% level systematic errors, it is a conservative choice for the neutrino mode, but may be a reasonable choice for the antineutrino mode.

To mimic the T2K neutrino energy reconstruction, we built migration matrices for quasi-elastic (QE) and non-quasi-elastic (nQE) events for both neutrino and antineutrino modes. For simplicity, we used a Gaussian smearing for all cases. For each migration matrix, we set a width of the Gaussian and an energy shift at 0.55 GeV and 0.75 GeV, and we inter/extrapolated for all energies of interest. The precise values we used are shown in Table 1. The efficiencies were taken to be almost constant for QE events, around 80%, and slightly decreasing for nQE, around 25% and 45% for the neutrino and the antineutrino channels, respectively. We simulate T2K disappearance modes according to Ref. [34], obtaining a sensitivity to $\sin^2 2\theta_{23}$ around 0.02 (0.013) at 90% CL for a 5 (10) years running only in the neutrino mode.

As a small variation of the value $|\Delta m^2_{31}|$ within the current uncertainty (which will be further reduced by T2K/NOνA) has little impact on the appearance channel we do not let it vary in our simulations. We fix it to $2.47 \times 10^{-3}$ eV$^2$ ($2.43 \times 10^{-3}$ eV$^2$) for the normal (inverted) hierarchy [16]. For the same reason, we fix the solar neutrino oscillation parameters to $\sin^2 \theta_{12} = 0.31$ and $\Delta m^2_{21} = 7.54 \times 10^{-5}$ eV$^2$.

Regarding NOνA simulation, we have based it on the simulation done in [21, 35], considering both the appearance and disappearance channels for the neutrino and antineutrino modes, using the latest experimental configuration [36, 37]. We used the fluxes available from [38] and take the cross sections from Refs. [39, 40]. In this case, we do not put a prior on $\sin^2 2\theta_{23}$, but let it be determined by NOνA itself.

Implementing the precisely measured value of $\theta_{13}$ is an indispensable ingredient in our method of detecting CP violation by ongoing and near future accelerator experiments [18]. To incorporate the precision reactor measurement of $\theta_{13}$, we assume the final sensitivity to match Daya Bay’s current systematic uncertainty of $\sin^2 2\theta_{13}$, that is, $\delta(\sin^2 2\theta_{13}) = 0.005$ [7, 8].

To study the maximum capacity of the experiment to contribute to our knowledge on the true value of $\delta_{\text{CP}}$ we calculate the fraction of $\delta_{\text{CP}}$ values that can be ruled out with a certain confidence level by the experimental data as a function of the input parameters $(\sin^2 \theta_{23}^{\text{min}}, \delta_{\text{CP}}^{\text{in}})$, either by assuming a known neutrino mass hierarchy or by marginalizing over

|                | 0.55 GeV |                | 0.75 GeV |
|----------------|----------|----------------|----------|
|                | width (MeV) | shift (MeV) | width (MeV) | shift (MeV) |
| $\nu$ QE      | 85       | -10           | 98       | -15       |
| $\nu$ nQE     | 70       | -325          | 110      | -390      |
| $\bar{\nu}$ QE| 57       | -20           | 60       | -20       |
| $\bar{\nu}$ nQE| 100      | -270          | 120      | -310      |

**Table 1.** T2K energy reconstruction parameters used in this paper.
it. We have done this for different number of running years in neutrino and antineutrino modes, as we will describe in what follows. We assume that 1 year running of T2K and NOνA corresponds, respectively, to delivery of $10^{21}$ and $6 \times 10^{20}$ protons on target (POT). The fiducial mass of Super-Kamiokande is taken as 22.5 kt and NOνA detector as 14 kt.

C Qualitative discussions based on Bi-probability plots

In this section, we present a simple way to understand some of the notable features in the analysis results presented in Secs. 3 and 4 by using the bi-probability plots introduced in Ref. [41]. As shown in Fig. 7 it is a simultaneous presentation of the appearance probabilities, $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, calculated by continuously varying $\delta_{CP}$ from $-\pi$ to $\pi$ while the other oscillation parameters are fixed. We show in the left and right panels of Fig. 7, the bi-probability plots which correspond roughly to the T2K ($L = 295$ km and $E = 0.6$ GeV) and the NOνA ($L = 810$ km and $E = 2.0$ GeV) setups, respectively, for $\sin^2 \theta_{23} = 0.4, 0.5,$ and 0.6 for both mass hierarchies.

In order to have some idea about the expected precision of the measurements in terms of the probabilities, we also show in Fig. 7 the expected uncertainty ellipses for the case where the mass hierarchy is normal, $\delta_{CP} = -\pi/2$ and $\sin^2 \theta_{23} = 0.5$, based only on the statistical uncertainty on the number of signal events assuming the data taking of 3 years each for neutrinos and antineutrino runs for both NOνA and T2K. Note that in reality, the precise evaluation of the uncertainties is much more complicated as one should take into account several factors such as energy dependence, backgrounds, systematic uncertainties and their correlations.

By looking into Fig. 7, one can notice the following features of the bi-probability plots for T2K and NOνA setup, which are very important to understand the results of our analysis shown in this paper.

(i) For a given set of oscillation parameters, the CP ellipses for T2K are thinner and their major axis, which are proportional to the $\sin \delta_{CP}$ term in the probability, are longer than that for NOνA. These properties follow because the neutrino energy taken for T2K is closer to the first oscillation maximum, $|\Delta m^2_{32}|L/(4E) = \pi/2$.

(ii) For a given set of oscillation parameters, the two CP ellipses for the normal and the inverted mass hierarchies are more separated for NOνA than for T2K due to a stronger matter effect in the former setup.

From these observations one can naively expect that the feature described in (i) would make T2K more sensitive than NOνA to $\delta_{CP}$ determination, assuming that the numbers of events for these two experiments are similar. While the one in (ii) is the feature familiar to us, it would make NOνA more sensitive than T2K to the mass hierarchy, which potentially could help also in increasing the sensitivity to $\delta_{CP}$ by reducing the degeneracy related to the unknown mass hierarchy.

Let us discuss the importance of exploiting the observation of both the neutrino and the antineutrino modes. For the purpose of illustration, let us look at the bi-probability
The three ellipses for the both mass hierarchies are for $\sin^2 \theta_{13} = 0.089$, $\delta_{CP} = 295$ km, and $E = 0.6$ GeV (left panel) and the case where the mass hierarchy is normal, $\delta_{CP} = 0$ GeV (right panel) of Fig. 7. Bi-probability plots, or the simultaneous presentation of the appearance probabilities, $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, by continuously varying $\delta_{CP}$ from $-\pi$ to $\pi$ for the T2K set up of $L = 295$ km and $E = 0.6$ GeV (left panel) and the NOνA one of $L = 810$ km and $E = 2.0$ GeV (right panel). The three ellipses for the both mass hierarchies are for $\sin^2 \theta_{23} = 0.4, 0.5$, and 0.6. In order to have some idea about the precision of the measurements, the expected uncertainties for T2K and NOνA are indicated by the solid (1$\sigma$) and dashed (2$\sigma$) black curves by taking into account only statistical uncertainties for 3 + 3 years of exposure for neutrino plus antineutrino modes for the case where the mass hierarchy is normal, $\delta_{CP} = -\pi/2$ and $\sin^2 \theta_{23} = 0.5$.

plot for the T2K experiment. Let us consider the case of the normal mass hierarchy. Suppose that $\sin^2 2\theta_{23} = 0.96$ (corresponds to $\sin^2 \theta_{23} = 0.4$ or 0.6), and that only the appearance probability in the neutrino mode is observed with the result $P(\nu_\mu \to \nu_e) = 5\%$. Then, we can not distinguish the cases between $\sin^2 \theta_{23} = 0.4$ with $-3\pi/4 \lesssim \delta_{CP} \lesssim -\pi/4$, and $\sin^2 \theta_{23} = 0.6$ with $\pi/4 \lesssim \delta_{CP} \lesssim 3\pi/4$ (see the left panel of Fig. 7). However, if the antineutrino appearance probability is also measured, these two cases can be distinguished since their probabilities are rather different, $P(\bar{\nu}_\mu \to \bar{\nu}_e) \sim 2\%$ for $\sin^2 \theta_{23} = 0.4$ against $P(\bar{\nu}_\mu \to \bar{\nu}_e) \sim 6\%$ for $\sin^2 \theta_{23} = 0.6$, and consequently, one can constrain better also the allowed range of $\delta_{CP}$. Therefore, the additional running of the $\bar{\nu}_\mu \to \bar{\nu}_e$ mode clearly helps. This is true also for the NOνA experiment, as the qualitative behavior of the CP ellipses for NOνA is similar to that for T2K, see the right panel of Fig. 7.

Let us also comment about some expectations on CP sensitivity to establish CP violation. In Fig. 7, we assume that we know the true mass hierarchy and consider only the statistical uncertainties. Despite the optimistic assumption it is clear that we can not establish CP violation at 3$\sigma$ CL by either one of these experiments (T2K or NOνA), or by both, after 3 + 3 years of $\nu$ and $\bar{\nu}$ running. Therefore, the presence or absence of CP violation may not be a useful measure for these experiments. However, even in the case without capability of establishing CP violation, depending on the true value of $\delta_{CP}$, it may be possible to exclude a certain range of $\delta_{CP}$ values at some CL. Therefore, it can be useful to quantitatively measure the sensitivity of experiments to measure $\delta_{CP}$, by using the CP exclusion.
fraction, as discussed in this paper. From Fig. 7, one expects that T2K can exclude larger ranges of $\delta_{\text{CP}}$ than NO$\nu$A, for the same exposure. This expectation was confirmed by the actual calculation in in Sec. 4.

![Figure 8](image_url)

**Figure 8.** Bi-probability plot for T2K for $\sin^2 \theta_{23} = 0.4$ and the normal mass hierarchy with the error ellipse for $\delta_{\text{CP}} = 0$ and $-\pi/2$, which explains the impact of the increase of statistics on the CP exclusion fraction.

What would be the values of $\delta_{\text{CP}}$ which give larger or smaller CP exclusion fractions? The answer depends on the statistics as well as our knowledge on the mass hierarchy. Let us look at Fig. 8 in which the bi-probability plot with the normal mass hierarchy is shown for $\sin^2 \theta_{23} = 0.4$. Suppose that the true value of $\delta_{\text{CP}}$ is zero and the statistics is so low that the error ellipse cover the whole range of $\delta_{\text{CP}}$ as the red solid curve in Fig. 8 does. In this case, no region of $\delta_{\text{CP}}$ space would be excluded. Even in this case, however, it is possible to exclude roughly half of the range of $\delta_{\text{CP}}$ if the true value of $\delta_{\text{CP}}$ is equal to $\pm \pi/2$, as we can see from the solid blue curve ($\delta_{\text{CP}} = -\pi/2$) in Fig. 8. Therefore, $\delta_{\text{CP}} = \pm \pi/2$ is the most favorable value for highest CP exclusion fraction whereas $\delta_{\text{CP}} = 0$ is the least.

The situation would change significantly as the statistics increases. If we compare the cases of $\delta_{\text{CP}} = 0$, the red dashed curve, and $\delta_{\text{CP}} = -\pi/2$, the blue dashed curve in Fig. 8 we observe that the exclusion fraction of $\delta_{\text{CP}}$ region of these two cases become comparable due to a Jacobian effect, and is approximately equal to 2/3. If the statistics increases further, the exclusion fraction for $\delta_{\text{CP}} = 0$ is expected to be larger than that for $\delta_{\text{CP}} = -\pi/2$. Thus, the favorable and unfavorable values of $\delta_{\text{CP}}$ for the CP exclusion fraction will be interchanged as the statistics increases. This can be confirmed by our results shown in Secs. 3 and 4, by comparing, for e.g., the right top panel of Fig. 1 for T2K 2+3 running and the middle right panel of Fig. 4 for T2K 10 + 10 running where the mass hierarchy was assumed to be known. Even if the hierarchy is unknown, the same feature can be seen when T2K and NO$\nu$A are combined as the hierarchy information comes from the result of
the fit in this case.

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