Theoretical and Computational Analysis on the Melt Flow Behavior of Polylactic Acid in Material Extrusion Additive Manufacturing under Vibration Field

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Abstract: Material extrusion (ME), an extrusion-based rapid prototyping technique, has been extensively studied to manufacture final functional products, whose forming quality is significantly influenced by the melt flow behavior (MFB) inside the extrusion liquefier. Applied vibration has a great potential to improve the MFB, and thereby promote the forming quality of the built product. To reveal the mechanism, a dynamic model of the melt flow behavior (DMMFB) is established based on fluid dynamics, Tanner nonlinear constitutive equation and Newton’s power law equation. The MFB, i.e., pressure drop, shear stress and apparent viscosity, is investigated without and with different vibration applied. The corresponding finite element analysis (FEA) is then carried out. From the comparison between DMMFB and FEA results, it is concluded that the proposed model is reliable. When vibration is applied onto the extrusion liquefier, the time-domain MFB will change periodically. Its effective value decreases significantly, and further decreases with the increase of vibration frequency or amplitude. This paper provides the theoretical basis to improve the MFB by applied vibration, and thereby to enhance the forming quality of ME products.

Keywords material extrusion; melt flow behavior; applied vibration; dynamic model; finite element analysis; forming quality

1. Introduction

Material extrusion (ME), used by ISO/ASTM 52900:2015 to include FDM (fused deposition molding)/FFF (fused filament fabrication) technologies, is one of the most promising rapid prototyping techniques [1,2] due to the advantages of ease of operation, low cost, broad resources of raw materials, etc. In this technique, the raw material filament is first fed into an extrusion liquefier, where it is melted and flowed. The solid portion of the filament is acting as a piston to push the melt out of the liquefier through a nozzle. The extrudate then cools, solidifies and bonds with the extruded material layer beneath it. A three-dimensional model or part entity is finally completed by repeating this procedure through to the last layer [3,4]. Although the products built via ME have been increasingly widely used for electronic, automotive and aerospace applications [5], widespread
工业采用这一技术的限制在于最终部分性能的限制，且仍存在改进其成型质量的大量空间 [6,7]。

由于熔融流动行为 (MFB) 是影响 ME 产品成型质量的一个关键因素 [8,9]，因此提高其质量至关重要。Ramanath 等人 [10,11] 建立了一种数学模型来研究在 ME 萃取器中温度和流动行为随输入条件变化。有限元分析 (FEA) 也用于验证该模型。发现压力损失随数学模型和 FEA 保持在可接受的范围内。压力损失和熔融流动速率随流道参数而变化。所提出的模型可用于反映质量的子产品。Peng 等人 [12] 提出了一种实验方法 (颜料分布), 用于说明温度和流动历史在挤出过程中。结果表明，在中心处发生了更小的剪切速率，导致了更钝的流速剖面。挤出过程具有显著非-等温性，尤其是在高挤出速度时。Pandey 和 Pradhan [13] 理论上研究了 MFB 内 ME 的最佳给料速率。熔融流动速率、压力损失、剪切应力和温度可以通过动量平衡方程和能量平衡方程预测。Stewart 等人 [14] 分析了 MFB 内聚乳酸 (PLA) 的行为，以促进产品质量、过程控制和效率。

他们实验上分析了电阻式热源的输出功率，理论地研究了热-转移系数并验证了液化器温度。结果表明温度在不同区域中是不同的。研究提供了更好的挤出机设计和材料选项。Zhang 等人 [15] 研究了喷嘴直径、填料尺寸、树脂粘度和 MFB 内的体积含量对挤出机的影响。结果表明喷嘴直径和体积含量对熔融流动性能有显著影响。阻塞更可能发生在材料粘度更高、材料颗粒更大和喷嘴直径更小时。Coogan 和 Kazmer [16] 实验性地研究了 ME 中熔融流动的混炼过程。MFB 现代化了设计的在-线 rheometer。挤出机的 rheometer 被修改以匹配压力转接器和 thermocouple。结果表明在线 rheometer 提供了准确的粘度测量，并应用了适当的校正。Mackay [17] 深入研究了 MFB 内 ME 的 rheological 行为，主要包括热-转移、MFB、填充、剪切、粘性、流动特征、粘度、温度等。他强调了对 MFB 和制造过程的交互作用的分析是必要的，以证明 ME 产品质量可以改进。

Liu 等人 [18-20] 研究了熔融流动的振荡在 ME 中的振动率和速度的效应。熔融流动行为在 ME 中得到了充分的利用，以凸显其重要性。主要包括热-转移、MFB, 填充、粘性、流动特征、粘度、温度等。他强调了对 MFB 和制造过程的交互作用的分析是必要的，以证明 ME 产品质量可以改进。

同样地，将振动应用到 ME 过程中具有很大的潜力，以提高 MFB，并且可以增强 ME 产品的成型质量。然而，很少有学者提出了这一概念在现有 [22,23] 中，并且没有足够的实验数据 (理论或实验数据) 可用。为覆盖这一缺口，本文建立了动态 MFB (DMMFB) 的模型来分析压力损失、剪切和表观粘度的粘度随振动断的不同而变化。对应的结果是分别根据模型结果进行模拟比较。模型结果表明了振动对 MFB 的影响。提出的模型提供了一种理论工具来分析 MFB 过程并提出理论基础来改进 ME 产品的成型质量。
technical reference to improve the forming quality of similar manufacturing processes (like injection molding) in industry.

2. The DMMFB

The MFB within ME, i.e., pressure drop, shear stress and apparent viscosity, is related to the geometry and size of the internal passage of the extrusion liquefier, as well as the viscoelastic behavior of the melt. The interior of the liquefier can be mainly divided into three geometric regions, as shown in Figure 1. Region I is the upper zone of the liquefier (cylindrical in shape), where the material filament enters and melts into a molten state, Region II is the transition zone for the molten material from Region I to III in the liquefier, and the shape is a truncated cone and Region III is the lower zone of the liquefier (also cylindrical in shape but smaller), from which the molten material is extruded out.

![Figure 1. Schematic diagram of the internal geometric region of the extrusion liquefier.](image1)

Since the modeling principle is similar among the three regions, details are provided for Region I, with its geometry and coordinates shown in Figure 2. The length and radius of the cylindrical passage are $L$ and $R$, respectively. For ease of analysis about Region I, a cylindrical coordinate system is used to describe the coordinate points. Among them, $Z$ is the direction of the melt flow, $R$ is the direction of the velocity gradient and $\theta$ is the neutral direction.

![Figure 2. Cylindrical area coordinates.](image2)

To further facilitate the analysis, the following assumptions are proposed:

1. The melt flow is laminar, $Re$ (Reynolds number) $< 5$.
2. The temperature is constant inside the whole liquefier.
3. The melt is incompressible.
4. The effect of gravity force is negligible.
5. There is no external force, like magnetic, etc.
6. The velocity near the wall surface is zero, and it does not change in the neutral direction.
7. No flow occurs in the \( r \) direction.
   The modeling details are provided in Appendix A.

3. FEA

FEA was carried out to simulate the MFB inside the extrusion liquefier without and with different types of sinusoidal vibration applied. The Fluent module with fluid analysis function in ANSYS workbench (commercial software) was used for simulation analysis. There were three steps, including pre-processing, solving and post-processing. Firstly, the three-dimensional (3D) geometric model of the internal flow channel of the liquefier was set up according to Table 1. It was imported into the pre-processing sub-module to define the entry, exit and boundary conditions. Since the geometric model was simple and axisymmetric, an unstructured grid automatically generated by the sweeping method was used to divide mesh of the internal flow channel wall of the liquefier, as shown in Figure 3.

| Entrance Radius | Exit Radius | Upper End Length | Lower End Length | Exit Angle |
|-----------------|-------------|------------------|------------------|------------|
| \( r_1 \) (mm) | \( r_2 \) (mm) | \( L_1 \) (mm) | \( L_2 \) (mm) | \( \alpha \) (rad) |
| 0.9             | 0.2         | 10               | 1                | \( 2\pi/3 \) |

Figure 3. The meshed internal flow passage.

In the solving sub-module, the analysis type was transient analysis, and the model entrance and exit were set to velocity-inlet and pressure-outlet, respectively. According to Assumptions 1 and 3, the simulation environment was set to be good at solving the pressure correction algorithm of incompressible flow, and the semi implicit original algorithm (suitable for laminar flow calculation) was adopted. For each vibration cycle, there were 50 time sub-steps divided. The maximum number of iterations in each sub step is 70, with the iteration accuracy set to 0.00001. In addition, the velocity of the melt at the internal wall was zero due to Assumption 6. The inlet velocity boundary conditions were set as in Equation (A4). The inlet temperature of the melt was \( T = 333K \), the temperature of the internal wall was \( T_w = 433K \) and the reference temperature was \( T_A = 423.15K \). Besides, PLA, the working material in this research, was a type of biological renewable and biodegradable polymer with good workability and forming stability [24]. The basic physical properties of PLA are shown in Table 2. After the solution was completed, the corresponding information in terms of MFB could be obtained by entering the post-processing sub-module.
Table 2. Physical properties of polylactic acid (PLA) [25].

| Item                        | Value | Unit       |
|-----------------------------|-------|------------|
| Power law index $n$         | 0.232 |            |
| Relaxation time $\lambda$  | 1.1   | s          |
| Density in molten state $\rho$ | 1073  | kg/m$^3$   |
| Activation energy $E$       | 67.526| kJ/mol     |
| Material wire feed speed $v_0$ | 0.006 | m/s        |
| Specific heat capacity $C$  | 1800  | J/kg·k     |
| Thermal conductivity       | 0.13  | W/m·K      |

4. Results and Discussion

In this section, the MFB results within ME are compared between DMMFB and FEA, and the influencing rule of different applied vibration is then provided. According to fractional factorial design [26], each section shows the detailed effect of only one single variable on the MFB, and gives corresponding discussions.

4.1. Effect of Vibration on the Pressure Drop

4.1.1. 0.25 g Vibration at Different Frequency

Figure 4 shows the time-domain internal pressure drop of the extrusion liquefier obtained by DMMFB and FEA without and with different-frequency vibration applied, of which the amplitude is 0.25 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. When vibration is applied, the time-domain pressure drop changes periodically as the inlet velocity of the extrusion liquefier changes from a constant to a periodic variation. From the comparison, it can be seen that the analytical pressure drop (obtained by DMMFB) is in good agreement with the simulated one (obtained by FEA), both in value and trend.
Figure 4. The internal pressure drop of the extrusion liquefier without and with 0.25 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 3 summarizes the effective value of the pressure drop for each condition. It can be seen that the value of the pressure drop decreases with the application of vibration and further decreases with the increase of vibration frequency. For example, when no vibration is applied, the effective value of the pressure drop predicted by DMMFB is 16.1 MPa. When the frequency of the applied vibration is 200 Hz, the value is reduced down to 11.7 MPa, decreased by 27.3%. When the frequency is 500 Hz, the value is further reduced to 8.5 MPa, decreased by 47.2%. This can be explained by the fact that increasing vibration frequency will decrease the amplitude of inlet velocity and speed up the changes according to Equation (A4), the amplitude of the pressure drop is then reduced and the changes are fast, as referred to by Equation (A20). Therefore, the larger the introduced vibration frequency is, the smaller the magnitude in pressure drop will be.

Table 3. Effective value of the pressure drop without and with 0.25 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (MPa) | Drop (%) | FEA (MPa) | Drop (%) |
|---------------|-------------|----------|-----------|----------|
| 0 (No vibration) | 16.1 | - | 15.7 | - |
| 100           | 13.7 | 14.9 | 13.3 | 15.3 |
| 200           | 11.7 | 27.3 | 11.4 | 27.4 |
| 300           | 10.3 | 36.0 | 10.0 | 36.3 |
| 400           | 9.4 | 41.6 | 9.1 | 42.0 |
| 500           | 8.5 | 47.2 | 8.3 | 47.1 |

*DMMFB means the dynamic model of the melt flow behavior.

4.1.2. 0.3 g Vibration at Different Frequency

Figure 5 shows the comparison of pressure drop between DMMFB and FEA. The amplitude of the applied vibration is 0.3 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. From the comparison, it can be seen that the results predicted by DMMFB agree well with those obtained by FEA, no matter without or with vibration applied.
Figure 5. The internal pressure drop of the extrusion liquefier without and with 0.3 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

The effective value of the pressure drop for each condition is detailed in Table 4. The results show the same trend as above, that is, the pressure drop is decreased with the application of vibration, and it is further decreased with the increase of applied vibration frequency. The reason for this phenomenon is the same as discussed above.

Table 4. Effective value of the pressure drop without and with 0.3 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (MPa) | Drop (%) | FEA (MPa) | Drop (%) |
|---------------|-------------|----------|-----------|----------|
| 0 (No vibration) | 16.1        | -        | 15.7      | -        |
| 100           | 11.9        | 26.1     | 11.6      | 26.1     |
| 200           | 10.0        | 37.9     | 9.8       | 37.6     |
| 300           | 9.0         | 44.1     | 8.8       | 44.0     |
| 400           | 8.3         | 48.4     | 8.1       | 48.4     |
| 500           | 7.7         | 52.2     | 7.5       | 52.2     |

4.1.3. 0.35 g Vibration at Different Frequency

The comparison of the pressure drop within ME obtained respectively by DMMFB and FEA is shown in Figure 6. No matter without or with vibration applied (the amplitude is 0.35 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz), the analytical and simulated pressure drop have a good agreement with each other.
Figure 6. The internal pressure drop of the extrusion liquefier without and with 0.35 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 5 details the effective value of the above pressure drop for each condition. Again, the value decreases with vibration applied, and further decreases with the increase of vibration frequency.

From the above, applying vibration onto the ME extrusion liquefier can reduce the internal pressure drop, and it will be further reduced with the increasing frequency of applied vibration. This can help improve the continuity and uniformity of the extrudate, and thereby enhance the forming quality of ME products.

Table 5. Effective value of the pressure drop without and with 0.35 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (MPa) | FEA (MPa) | Drop (%) | FEA Drop (%) |
|----------------|-------------|-----------|----------|--------------|
| 0 (No vibration) | 16.1 | - | 15.7 | - |
| 100 | 10.9 | 32.3 | 10.7 | 31.8 |
| 200 | 9.5 | 41.0 | 9.2 | 41.4 |
| 300 | 8.3 | 48.4 | 8.2 | 47.8 |
| 400 | 7.6 | 52.8 | 7.5 | 52.2 |
| 500 | 6.9 | 57.1 | 6.8 | 56.7 |

4.1.4. Effect of Different Vibration Amplitude

Figure 7 compares the analytical and simulated results of the effective value of the pressure drop without and with different vibration applied, whose amplitude is, separately, 0.25, 0.3 and 0.35
Generally, the predictions agree well with the simulations. When vibration is applied, the pressure drop is reduced. The value is further reduced when the amplitude of applied vibration is growing. For example, when the amplitude of applied vibration is 0.25 and 0.35 g at 300 Hz, the effective value of the analytical pressure drop is 10.3 and 8.3 MPa, respectively. Compared with that under the no-vibration field (16.1 MPa), the pressure drop is decreased separately by 36.0% and 48.4%. This can be explained by the fact that increasing vibration amplitude will decrease the inlet velocity according to Equation (A4), and the pressure drop is then reduced as referred to in Equation (A20). Therefore, increasing the amplitude of applied vibration can reduce the pressure drop inside the extrusion liquefier of ME, which helps enhance the forming quality of ME products.

Figure 7. Effective value of the internal pressure drop inside the extrusion liquefier without and with different-amplitude vibration applied.

4.2. Effect of Vibration on the Shear Stress

4.2.1. 0.25 g Vibration at Different Frequency

Figure 8 shows the time-domain shear stress at the internal wall of the extrusion liquefier obtained by DMMFB and FEA without and with different-frequency vibration applied, of which the amplitude is 0.25 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. With vibration applied, the shear stress in time domain changes periodically as the pressure drop changes to a periodic variation. From the comparison, it can be seen that both the values and trend of the analytical shear stress are in good agreement with the simulated ones.
Figure 8. The shear stress at the internal wall of the extrusion liquefier without and with 0.25 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 6 summarizes the above effective value of the shear stress for each condition. The shear stress is decreased with the application of vibration and further decreased with the increase of vibration frequency. For example, when no vibration is applied, the effective value of the shear stress predicted by DMMFB is 0.89 MPa, when the frequency of the applied vibration is 100 Hz, the value is reduced down to 0.75 MPa, decreased by 15.7%, and when the frequency is 400 Hz, the value is further reduced to 0.44 MPa, decreased by 50.6%. This is because the shear stress is directly influenced by the pressure drop, see Equation (A5). Its changing situation and trend are similar to those of the pressure drop, as discussed previously.

Table 6. Effective value of the shear stress without and with 0.25 g vibration applied at different frequencies.

| Frequency (Hz) | Effective Value | Drop (%) | Effective Value | Drop (%) |
|----------------|-----------------|----------|-----------------|----------|
|                | DMMFB (MPa)     |          | FEA (MPa)       |          |
| 0 (No vibration) | 0.89            | -        | 0.88            | -        |
| 100            | 0.75            | 15.7     | 0.74            | 15.9     |
| 200            | 0.63            | 29.2     | 0.62            | 29.5     |
| 300            | 0.52            | 41.6     | 0.51            | 42.0     |
| 400            | 0.44            | 50.6     | 0.43            | 51.1     |
| 500            | 0.38            | 57.3     | 0.37            | 58.0     |

4.2.2. 0.3 g Vibration at Different Frequency

Figure 9 shows the comparison of the shear stress at the internal wall of the extrusion liquefier between DMMFB and FEA. The amplitude of the applied vibration is 0.3 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. From the comparison, it can be seen that the shear stress predicted by DMMFB agrees well with that simulated by FEA, no matter without or with vibration applied.
Figure 9. The shear stress at the internal wall of the extrusion liquefier without and with 0.3 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

The effective value of the shear stress for each condition is detailed in Table 7. The results show the same trend as above, i.e., the pressure drop is decreased with the application of vibration, and it is further decreased with the increase of applied vibration frequency. The reason for this phenomenon is the same as that discussed in Section 4.2.1.

Table 7. Effective value of the shear stress without and with 0.3 g vibration applied at different frequencies.

| Frequency (Hz) | Effective Value |           |
|---------------|-----------------|-----------|
|               | DMMFB (MPa)     | FEA (MPa) |
|               | Drop (%)        | Drop (%)  |
| 0             | 0.89            | 0.88      |
| (No vibration)| -               | -         |
| 100           | 0.59            | 0.58      | 34.1 |
| 200           | 0.51            | 0.50      | 43.2 |
| 300           | 0.44            | 0.43      | 51.1 |
| 400           | 0.39            | 0.37      | 58.0 |
| 500           | 0.35            | 0.34      | 61.4 |

4.2.3. 0.35 g Vibration at Different Frequency

The comparison of the shear stress at the internal wall of the extrusion liquefier within ME obtained respectively by DMMFB and FEA is provided in Figure 10. No matter without or with vibration applied (the amplitude is 0.35 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz), the analytical and simulated shear stress have a good agreement.
Figure 10. The shear stress at the internal wall of the extrusion liquefier without and with 0.35 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 8 details the effective value of the above shear stress for each condition. Again, the value decreases with vibration applied, and further decreases with the increase of vibration frequency.

Table 8. Effective value of the shear stress without and with 0.35 g vibration applied at different frequencies.

| Frequency (Hz) | Effective Value |
|---------------|-----------------|
|               | DMMFB (MPa)     | FEA (MPa) | Drop (%) | Drop (%) |
| 0 (No vibration) | 0.89            | 0.88      | -        | -        |
| 100           | 0.55            | 0.54      | 38.2     | 38.6     |
| 200           | 0.48            | 0.46      | 46.1     | 47.7     |
| 300           | 0.40            | 0.39      | 55.1     | 55.7     |
| 400           | 0.35            | 0.34      | 60.7     | 61.4     |
| 500           | 0.31            | 0.30      | 65.2     | 65.9     |

To sum up, introducing vibration into the extrusion process can reduce the shear stress at the internal wall of the liquefier, and increasing the vibration frequency will further reduce the shear stress, which helps improve the forming quality of ME products.

4.2.4. Effect of Different Vibration Amplitude

Figure 11 compares the analytical and simulated effective value of the shear stress without and with different vibration applied, whose amplitude is, separately, 0.25, 0.3 and 0.35 g. It is shown that the predictions agree well with the simulations. When vibration is applied, the shear stress is
reduced. The value is further reduced when the amplitude of applied vibration is growing. For example, when the amplitude of applied vibration is 0.3 and 0.35 g at 200 Hz, the effective value of the analytical shear stress is 0.51 and 0.48 MPa, respectively. Compared with that under the no-vibration field (0.89 MPa), the shear stress is decreased separately by 42.7% and 46.1%. The reason for this is that increasing vibration amplitude will decrease the pressure drop of the melt inside the extrusion liquefier, leading to the reduction in shear stress according to Equation (A5). Therefore, increasing the amplitude of applied vibration can further reduce the shear stress inside the extrusion liquefier of ME, helping to improve the continuity and uniformity of the extrudate, and thereby enhance the forming quality of ME products.

Figure 11. Effective value of the shear stress without and with different-amplitude vibration applied.

4.3. Effect of Vibration on the Apparent Viscosity

4.3.1. 0.25 g Vibration at Different Frequency

Figure 12 shows the time-domain apparent viscosity of the melt inside the extrusion liquefier obtained by DMMFB and FEA without and with different-frequency vibration applied, of which the amplitude is 0.25 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. It can be seen that when vibration is applied, the apparent viscosity changes periodically with time as the shear stress changes from a constant to a periodic variation, according to Equation (A8). Generally, both values and trend of the analytical apparent viscosity agree well with the simulated ones.
**Figure 12.** The apparent viscosity of the melt inside the extrusion liquefier without and with 0.25 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 9 gives the effective apparent viscosity of the melt inside the extruder liquefier for each condition. The viscosity is decreased with the application of vibration and further decreased with the increase of vibration frequency. For example, when no vibration is applied, the effective value of the viscosity predicted by DMMFB is 3601 Pa·s, when the frequency of the applied vibration is 200 Hz, the value is reduced down to 2220 Pa·s, decreased by 38.4%, and when the frequency is 300 Hz, the value is further reduced to 1760 Pa·s, decreased by 51.1%. This is because the apparent viscosity of the melt is reduced with vibration applied, and it is further reduced when the vibration frequency is increased [27,28]. In addition, due to the close relationship between apparent viscosity and shear stress according to Equation (A8), the changing situation and trend are similar to those of the shear stress, as discussed previously.

**Table 9.** Effective value of the apparent viscosity without and with 0.25 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (Pa·s) | Drop (%) | FEA (Pa·s) | Drop (%) |
|---------------|--------------|----------|------------|----------|
| 0 (No vibration) | 3601 | - | 3438 | - |
| 100 | 2852 | 20.8 | 2673 | 22.3 |
| 200 | 2220 | 38.4 | 2061 | 40.1 |
| 300 | 1760 | 51.1 | 1619 | 52.9 |
| 400 | 1368 | 62.0 | 1302 | 62.1 |
| 500 | 1134 | 68.5 | 1090 | 68.3 |

4.3.2. 0.3 g Vibration at Different Frequency

Figure 13 shows the comparison of apparent viscosity between DMMFB and FEA. The amplitude of the applied vibration is 0.3 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz. From the comparison, it can be seen that the results predicted by DMMFB agree well with those obtained by FEA whenever vibration is applied or not.
Figure 13. The apparent viscosity without and with 0.3 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 10 shows the effective value of the apparent viscosity for each condition. The results show the same trend as above, that is, the apparent viscosity is decreased with the application of vibration, and it is further decreased with the increase of applied vibration frequency. The reason for this phenomenon is the same as that discussed above.

Table 10. Effective value of the apparent viscosity without and with 0.3 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (Pa·s) | Drop (%) | FEA (Pa·s) | Drop (%) |
|---------------|--------------|----------|------------|----------|
| 0 (No vibration) | 3601         | -        | 3438       | -        |
| 100           | 2431         | 32.5     | 2241       | 34.8     |
| 200           | 1712         | 52.5     | 1578       | 54.1     |
| 300           | 1306         | 63.7     | 1216       | 64.6     |
| 400           | 1090         | 69.7     | 1015       | 70.5     |
| 500           | 962          | 73.3     | 895        | 74.0     |

4.3.3. 0.35 g Vibration at Different Frequency

The comparison of the apparent viscosity within ME obtained respectively by DMMFB and FEA is shown in Figure 14. No matter without or with vibration applied (the amplitude is 0.35 g and the frequency is, separately, 100, 200, 300, 400 and 500 Hz), the analytical and simulated apparent viscosity have a good agreement with each other.
Figure 14. The apparent viscosity without and with 0.35 g vibration applied at different frequencies. (a) No vibration applied, (b) 100 Hz, (c) 200 Hz, (d) 300 Hz, (e) 400 Hz, (f) 500 Hz.

Table 11 provides the effective value of the above apparent viscosity for each condition. Again, the value decreases with vibration applied, and further decreases with the increase of vibration frequency.

Table 11. Effective value of the apparent viscosity without and with 0.35 g vibration applied at different frequencies.

| Frequency (Hz) | DMMFB (Pa·s) | Drop (%) | FEA (Pa·s) | Drop (%) |
|---------------|--------------|----------|------------|----------|
| 0 (No vibration) | 3601 | - | 3438 | - |
| 100 | 2088 | 42.0 | 1978 | 42.5 |
| 200 | 1460 | 59.5 | 1358 | 60.5 |
| 300 | 1050 | 70.8 | 993 | 71.1 |
| 400 | 831 | 76.9 | 788 | 77.1 |
| 500 | 697 | 80.6 | 662 | 80.7 |

In summary, applying vibration onto the extrusion liquefier can reduce the apparent viscosity inside the liquefier, which will be further reduced with the increasing vibration frequency. This could improve the continuity and uniformity of the extrudate, and thus enhance the forming quality of ME products.

4.3.4. Effect of Different Vibration Amplitude

Figure 15 compares the analytical and simulated effective value of the apparent viscosity without and with different vibration applied, whose amplitude is, separately, 0.25, 0.3 and 0.35 g. It is shown that the predictions agree well with the simulations. When vibration is applied, the apparent viscosity is reduced. The value is further reduced when the amplitude of applied vibration is growing. For example, when the amplitude of applied vibration is 0.3 and 0.35 g at 500 Hz, the effective value of the analytical apparent viscosity is 962 and 697 Pa·s, respectively. Compared with that under the no-vibration field (3601 Pa·s), the apparent viscosity is decreased separately by 73.3% and 80.6%. The reason is that the apparent viscosity is reduced with vibration applied, and it is further reduced when the vibration amplitude is increased [27,28]. This can also be seen from the relationship between apparent viscosity and shear stress in Equation (A8). Therefore, increasing the amplitude of applied vibration can reduce the apparent viscosity inside the extrusion liquefier of ME, which helps promote the forming quality of ME products.
5. Conclusions

In this paper, the DMMFB inside the extrusion liquefier within ME is established based on fluid dynamics, Tanner nonlinear constitutive equation and Newton’s power law equation. The influencing rule of applied vibration on the MFB (i.e., pressure drop, shear stress and apparent viscosity) is illustrated. FEA is then performed accordingly. The conclusions are drawn as follows:

(1) The time-domain pressure drop, shear stress and apparent viscosity inside the extrusion liquefier change periodically when vibration is applied.

(2) With vibration applied on the liquefier, the pressure drop, shear stress and apparent viscosity decrease significantly, and they will further decrease with the increase of frequency or amplitude of the applied vibration.

(3) The DMMFB results of MFB are in good agreement with those obtained by FEA both in value and trend.

(4) When vibration is applied, the MFB within ME will be reduced, which helps enhance the continuity and uniformity of the extrudate, and thus improves the forming quality of ME products.

(5) This research provides a theoretical tool for MFB process monitoring within ME and gives technical reference for fabrication improvement of similar manufacturing processes in industry.

In the near future, experiments are recommended to validate the DMMFB and FEA in terms of MFB to thoroughly clarify the corresponding mechanism.

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Appendix A

The kinematic equation of the melt flow in the direction can be expressed as follows:
\[
\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{r\theta} \right) + \frac{1}{r} \frac{\partial \tau_{\theta \theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} \right] + \rho g_z \tag{A1}
\]

where, \( \rho \) is the density of the polymer melt, \( v_z \) is the velocity vector of \( z \) direction, \( t \) is time, \( p \) is the pressure, \( \tau_{r\theta} \) and \( \tau_{zz} \) are stress tensors and \( g_z \) is the gravitational acceleration vector of \( z \) direction. According to the assumption mentioned above, \( v_r, \frac{\partial v_z}{\partial \theta}, \frac{\partial v_z}{\partial z}, \tau_{r\theta}, \tau_{zz} \) and \( g_z \) can be regarded as 0, thus Equation (A1) becomes

\[
\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau) \tag{A2}
\]

Integrating both sides of Equation (A2), the shear stress can be obtained by

\[
\tau = \frac{r}{2} \frac{\partial p}{\partial z} + \frac{1}{2} \frac{\partial}{\partial r} \left[ \frac{v(t)}{r} \right] \tag{A3}
\]

Applying vertical vibration to the extrusion liquefier, the velocity variation inside the liquefier is

\[
v(t) = v_0 + v_1 = v_0 - A \cos \omega t \tag{A4}
\]

where, \( v_0 \) is the material wire feed speed, \( v_1 \) is the speed of applied vibration, \( A = A_0 \cdot 0.1 \lambda g \), \( A_0 \) is the vibration amplitude, \( g \) is the gravitational acceleration and \( \omega \) is the angular frequency of vibration.

Substituting Equation (A4) into (A3), the internal shear stress of the melt in the channel is

\[
\tau = \frac{r}{2} \frac{\partial p}{\partial z} + \frac{1}{2} \frac{\partial}{\partial r} \left[ \frac{v(t)}{r} \right] A \omega \sin \omega t \tag{A5}
\]

Taking into account the material properties, the melt is regarded as Newtonian fluid and its flow behavior follows the power law [29], which is expressed by

\[
\mu = K \cdot \dot{\gamma}^{n-1} \tag{A6}
\]

where, \( \mu \) is the apparent viscosity, \( K \) is the consistency coefficient, \( n \) is the power law index and \( \dot{\gamma} \) is the shear rate.

According to Newton’s power law, the shear rate and shear stress have the following relationship:

\[
\dot{\gamma} = K \cdot \tau^{\frac{1}{n}} \cdot \tau^{\frac{1}{n}} \tag{A7}
\]

Substituting Equation (A7) into (A6), the relationship between apparent viscosity and shear stress can be obtained by

\[
\mu = K \cdot \tau^{\frac{1}{n}} \cdot \tau^{\frac{1}{n}} \tag{A8}
\]

Considering the influence of vibration field applied, the Tanner nonlinear constitutive equation modified from Maxwell linear constitutive equation is used to accurately reflect the time-varying and viscoelastic behavior of the melt [20]. The expression is
\[
\tau + \lambda \frac{\partial \tau}{\partial t} = \eta \left( \dot{\gamma} \right) = K \left( \frac{\partial v}{\partial r} \right)^n
\]
(A9)

where, \( \lambda \) is the relaxation time of the polymer.

Based on Assumption 1 and 3, the pressure gradient can be expressed by

\[
\frac{\partial p}{\partial z} = \frac{\Delta p}{L'}
\]
(A10)

where, \( L' \) is the length of the fully developed flow region, \( \Delta p \) is the pressure difference of two extremities of the fully developed flow region. Since Region I (shown in Figure 2) is the upper end of the liquefier, the material cannot completely melt into liquid and the melt does not fully flow here, thus the pressure gradient is corrected by Bagley correction method. The corrected pressure gradient is given by

\[
\frac{\partial p}{\partial z} = \frac{\Delta p}{L + N^B \cdot R}
\]
(A11)

where, \( N^B \) is the revised factor of pressure gradient under the vibration field and \( R \) is the pipe radius of Region I.

Substituting Equations (A5) and (A11) into Equation (A9) yields

\[
\frac{r}{2} \frac{\Delta p}{L + N^B \cdot R} \left( \frac{\rho r}{2} A \omega \sin \omega t \right) + \frac{\rho r}{2} A \omega^2 \cos \omega t = K \left( -\frac{\partial v}{\partial r} \right)^n
\]
(A12)

The velocity gradient can be obtained by

\[
\left\{ \frac{1}{K} \left[ \frac{1}{2} \frac{\Delta p}{L + N^B \cdot R} + \frac{\rho r}{2} A \omega \sin \omega t + \frac{\rho r}{2} A \omega^2 \cos \omega t \right] \right\} \frac{1}{n} \cdot \frac{r}{n} \cdot \frac{\partial v}{\partial r} = -\frac{\partial v}{\partial r}
\]
(A13)

According to Assumption 6, \( v(R) = 0 \), substituting \( r = R \) into Equation (A13), the velocity can be obtained by

\[
v(r) = \left\{ \frac{1}{K} \left[ \frac{1}{2} \frac{\Delta p}{L + N^B \cdot R} + \frac{\rho r}{2} A \omega \sin \omega t + \frac{\rho r}{2} A \omega^2 \cos \omega t \right] \right\} \frac{1}{n} \cdot \frac{n}{1 + n} \cdot \frac{R^{n+1}}{n+1} \left( \frac{r}{R} \right)^n
\]
(A14)

The relationship between flow rate and velocity can be obtained by

\[
dq = 2\pi rdv(r)
\]
(A15)

Substituting Equation (A14) into (A15) and integrating Equation (A15), the flow expression can be obtained by

\[
Q = \pi \left\{ \frac{1}{K} \left[ \frac{1}{2} \frac{\Delta p}{L + N^B \cdot R} + \frac{\rho r}{2} A \omega \sin \omega t + \frac{\rho r}{2} A \omega^2 \cos \omega t \right] \right\} \frac{1}{n} \cdot \frac{n}{1 + n} \cdot \frac{R^{n+1}}{n+1} \left( \frac{r}{R} \right)^n
\]
(A16)

According to Equations (A4) and (A16), the velocity can be given by
\[
\frac{Q}{\pi R^2} = v_0 - A \cos \omega t
\]

Rearranging Equation (A17), the pressure drop can be obtained by

\[
\Delta p_1 = \left[ K(v_0 - A \cos \omega t)^n \cdot n \cdot \sec^n \left( \frac{a+1}{n} \right) \cdot \frac{1}{r_1^n} - \frac{\rho}{2} A \omega \sin \omega t - \frac{\rho}{2} A \omega^2 \cos \omega t \right] \cdot 2(L_4 + N_B \cdot R)
\]

Although PLA melt is a pseudoplastic fluid, it can be considered as a Newtonian fluid at zero shear rate since the thermal effect can continuously regenerate the polymer chain [30]. As a Newtonian fluid, the relationship between viscosity and temperature can be expressed by the Arrhenius equation, which is shown below

\[
H(T) = \exp \left( E \left( \frac{1}{T} - \frac{1}{T_A} \right) \right)
\]

where, \( E \) is the activation energy, \( T \) is the working temperature and \( T_A \) is the reference temperature. Combining Equations (A18) and (A19), the final expression of the pressure drop becomes

\[
\Delta p_1 = \left[ K(v_0 - A \cos \omega t)^n \cdot n \cdot \sec^n \left( \frac{a+1}{n} \right) \cdot \frac{1}{r_1^n} - \frac{\rho}{2} A \omega \sin \omega t - \frac{\rho}{2} A \omega^2 \cos \omega t \right] \cdot 2(L_4 + N_B \cdot R) \cdot \exp \left( E \left( \frac{1}{T} - \frac{1}{T_A} \right) \right)
\]

The pressure drop for Region II can be derived by discretizing the tapered regions into differential cylindrical regions and summing the differential cylindrical regions. For Region III, its expression of pressure drop is similar with that of Region I. Therefore, \( \Delta p_2 \) and \( \Delta p_3 \) are given by the following equations:

\[
\Delta p_2 = \frac{2}{\tan \left( \theta / 2 \right)} \left[ K(v_0 - A \cos \omega t)^n \cdot r_2^n \cdot \left( \frac{3n+1}{n} \right)^n \cdot \frac{1}{3n} \left( \frac{1}{r_2^n} - \frac{1}{r_1^n} \right) \cdot \frac{\rho}{2} A \omega \sin \omega t \right] \cdot \exp \left( E \left( \frac{1}{T} - \frac{1}{T_A} \right) \right)
\]

\[
\Delta p_3 = \left[ K(v_0 - A \cos \omega t)^n \cdot r_2^n \right] \cdot \left( \frac{3n+1}{n} \right)^n \cdot \frac{1}{2} A \omega \sin \omega t \sim \frac{\rho}{2} A \omega^2 \cos \omega t \cdot 2L_3 \cdot \exp \left( E \left( \frac{1}{T} - \frac{1}{T_A} \right) \right)
\]

Therefore, the total pressure drop inside the extrusion liquefier under vibration field is given by

\[
\Delta P = \Delta p_1 + \Delta p_2 + \Delta p_3
\]

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