AMLSA Algorithm for Hybrid Precoding in Millimeter Wave MIMO Systems

Fulai Liu1,a, Zhenxing Sun2,3,b*, Ruiyan Du1,c and Xiaoyu Bai2,d
1Engineering Optimization & Smart Antenna Institute, Northeastern University at Qinhuangdao, Qinhuangdao, China
2School of Computer Science and Engineering, Northeastern University, Shenyang, China
3Department of Electrical and Information Engineering, Northeast Petroleum University at Qinhuangdao, Qinhuangdao, China
Email: afulailiu@126.com, bsunzx-0@163.com, cruiyandu@126.com, dbxyu.good@163.com

Abstract. In this paper, an effective algorithm will be proposed for hybrid precoding in mmWave MIMO systems, referred to as alternating minimization algorithm with the least squares amendment (AMLSA algorithm). To be specific, for the fully-connected structure, the presented algorithm is exploited to minimize the classical objective function and obtain the hybrid precoding matrix. It introduces an orthogonal constraint to the digital precoding matrix which is amended subsequently by the least squares after obtaining its alternating minimization iterative result. Simulation results confirm that the achievable spectral efficiency of our proposed algorithm is better to some extent than that of the existing algorithm without the least squares amendment. Furthermore, the number of iterations is reduced slightly via improving the initialization procedure.

1. Introduction
With the development of wireless communication technology, the capacity of wireless networks have to drastically grow to meet the explosive demands for high-data-rate multimedia access. To be specific, the upcoming 5G networks will be expected to achieve 1000X increase in capacity by 2020 [1]. Millimeter-wave (mmWave) communication is a burgeoning technology for future 5G cellular networks [2]. It has the underlying ability to provide gigabit-per-second data rates by developing the large bandwidth available at mmWave frequency range from 30 GHz to 300 GHz [3]. Millimeter-wave MIMO precoding can use large-scale antennas at transceivers to provide prominent beamforming gains to combat the path loss and to synthesize highly directional beams, since the small wavelength of mmWave signals.

For a classical MIMO communication system, fully digital processing is made possible by connecting to every antenna dedicated baseband and radio-frequency (RF) hardware. In a mmWave MIMO communication system making use of large arrays, this approach consumes a lot of power due to components such as analog-to-digital converters (ADCs) and RF devices [4]. Hybrid MIMO architectures [5] can overcome this limitation by splitting the MIMO processing between the analog and digital domains. The analog pre-coding stage processes the received signals at the different antenna elements feeding a number of radio-frequency (RF) chains much lower than the array size. A mmWave criterion with a hybrid beamforming architecture has already been designed and tested in [6].
Hybrid precoding is a newly-emerged technique in mmWave MIMO systems [7]. So far the main efforts are on the fully-connected structure [8]. In [9], an optimal hybrid precoder design in a special case was identified, i.e., when the number of RF chains is at least twice that of the data streams. VGA-enabled hybrid precoding is investigated according to different design criteria[10]. By removing VGAs from the RF domain, low-power analog precoders with phase shifters were also considered, whose phases are heuristically extracted from those of the VGA-enabled solution.

On the other hand, much less attention has been paid on the partially-connected structure. In [11], [12], codebook-based design of hybrid precoders was presented for narrow-band and orthogonal frequency division multiplexing (OFDM) systems, respectively. Although the codebook-based design enjoys a low complexity, there will be certain performance loss, and it is not clear how much performance gain can be further obtained. By utilizing the idea of successive interference cancellation (SIC), an iterative hybrid pre-coding algorithm for the partially-connected structure was proposed in [13]. The algorithm is established based on the assumption that the digital precoding matrix is diagonal, which means that the digital precoder only allocates power to different data streams, and the number of RF chains should be equal to that of the data streams.

The remainder of this paper is organized as follows. We shall introduce the system model and channel model, followed by the problem formulation in Section II. An alternating minimization algorithm with the least squares amendment for the fully-connected structure is depicted in Section III. In Section IV, simulation results will be presented. Finally, we will conclude this paper in Section V.

Notations: the notations below are used throughout this paper. \( \mathbf{a} \) and \( \mathbf{A} \) stand for a column vector and a matrix, respectively; \( a_{ij} \) is the entry on the \( i \) th row and \( j \) th column of \( \mathbf{A} \); The conjugate, transpose and conjugate transpose of \( \mathbf{A} \) are represented by \( \mathbf{A}^* \), \( \mathbf{A}^T \) and \( \mathbf{A}^{\dagger} \); \( \det(\mathbf{A}) \) and \( \|\mathbf{A}\| \) denote the determinant and Frobenius norm of \( \mathbf{A} \); \( \mathbf{A}^+ \) is the Moore-Penrose pseudo inverse of \( \mathbf{A} \); \( \text{Tr}(\mathbf{A}) \) and \( \text{vec}(\mathbf{A}) \) indicate the trace and vectorization; Expectation and the real part of a complex variable is noted by \( \mathbb{E}[\cdot] \) and \( \mathbb{R}[\cdot] \).

2. System Model and Problem Formulation

Firstly, the system model and channel model of the considered mmWave MIMO system will be described in this section, subsequently the hybrid precoding problem will be formulated to introduce the objective function and constraint condition.

2.1 System Model

Consider a single-user mmWave MIMO system as shown in Figure 1, where \( N_d \) data streams are sent and collected by \( N_r \) transmit antennas and \( N_r \) receive antennas. The numbers of RF chains at the transmitter and receiver are respectively denoted as \( N_{RF}^t \) and \( N_{RF}^r \), which are subject to constraints \( N_d \leq N_{RF}^t \leq N_r \) and \( N_d \leq N_{RF}^r \leq N_r \).

The hybrid precoders consist of an \( N_{RF}^t \times N_d \) digital baseband precoder \( \mathbf{f}_{ab} \) and an \( N_r \times N_{RF}^r \) analog RF precoder \( \mathbf{f}_{sr} \). The normalized transmit power constraint is given by \( \|\mathbf{f}_{ab}\|_F^2 = N_d \). Thus, the received signal after decoding processing is given as

\[
\mathbf{y} = \sqrt{\rho} \mathbf{w}_{ab}^H \mathbf{w}_{sr}^H \mathbf{H} \mathbf{f}_{sr} \mathbf{s} + \mathbf{w}_{sr}^H \mathbf{w}_{ab}^H \mathbf{n}
\]

where \( \rho \) stands for the average received power, \( \mathbf{h} \) is the channel matrix, \( \mathbf{w}_{ab} \) is the \( N_d \times N_{RF}^t \) digital baseband decoder, \( \mathbf{w}_{sr} \) is the \( N_r \times N_{RF}^r \) analog RF decoder at the receiver, \( \mathbf{s} \) is the \( N_d \times 1 \) symbol vector such that \( \mathbb{E} [\mathbf{s}\mathbf{s}^H] = \frac{1}{N_d} \mathbf{I}_{N_d} \), and \( \mathbf{n} \) is the noise vector of independent and identically distributed (i.i.d.) \( \mathcal{CN}(0, \sigma_n^2) \) elements. In this paper, we assume that perfect channel state information (CSI) is known at both the transmitter and receiver. The achievable spectral efficiency when transmitted symbols follow a Gaussian distribution can be expressed as
Furthermore, the analog precoders are implemented with phase shifters, which can only adjust the phases of the signals. Thus, all the nonzero entries of $F_{an}$ and $W_{an}$ should satisfy the unit modulus constraints, namely $\| F_{an} \|, \| W_{an} \| = 1$ for nonzero elements.

According to different signal mapping strategies from RF chains to antennas, the transceiver architecture can be categorized into the fully-connected (as illustrated in Figure 1) and partially-connected hybrid precoding structures. For the fully-connected structure, which this paper concentrates on researching, the output signal of each RF chain is sent to all the antennas through phase shifters.

Subsequently, we consider the problem of the channel model. Due to high free-space path loss, the mmWave propagation environment is well characterized by a clustered channel model, namely the Saleh-Valenzuela model [14]. This model depicts the mmWave channel matrix as

$$H = \frac{N_r N_t}{N_s N_{ray}} \sum_{i=1}^{N_s} \sum_{l=1}^{N_{ray}} \alpha_i a_i(\phi_i^r, \theta_i^r) a_i(\phi_i^t, \theta_i^t)^H$$

(3)

where $N_s$ and $N_{ray}$ represent the number of clusters and the number of rays in each cluster, and $\alpha_i$ denotes the gain of the $i$th ray in the $i$th propagation cluster. Furthermore, $a_i(\phi_i^r, \theta_i^r)$ and $a_i(\phi_i^t, \theta_i^t)$ represent the receive and transmit array response vectors, where $\phi_i^r(\phi_i^t)$ and $\theta_i^r(\theta_i^t)$ stand for azimuth and elevation angles of arrival and departure (AoAs and AoDs), respectively. In this paper, we consider the uniform square planar array (USPA) with $N \times N$ antenna elements. Therefore, the array response vector corresponding to the $i$th ray in the $i$th cluster can be written as

$$a_i(\phi_i^r, \theta_i^r) = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{j \frac{2 \pi}{\lambda} (\rho \sin \phi_i + \sin \phi_i \sin \theta_i + \cos \theta_i)} \\ \vdots \\ e^{j \frac{2 \pi}{\lambda} ((\sqrt{N} - 1) \sin \phi_i + \sin \phi_i \sin \theta_i + (\sqrt{N} - 1) \cos \theta_i)} \end{bmatrix}$$

(4)

where $\lambda$ and $\lambda$ are the antenna spacing and the signal wavelength, $0 \leq \rho < \sqrt{N}$ and $0 \leq q < \sqrt{N}$ are the antenna indices in the 2D plane. While this channel model will be used in simulations, our precoder design is applicable to more general models.

**Figure 1.** The transmitter-receiver single user mmWave system architecture
2.2 Problem Formulation

The design of precoders and decoders can be separated into two subproblems, i.e., the precoding and decoding problems. They have similar mathematical formulations except that there is an extra power constraint in the former.

Therefore, we will mainly focus on the precoder design in the remaining part of this paper and the algorithms proposed in this paper can be equally applied for the decoder. The corresponding problem formulation is given by

$$
\begin{align*}
\text{minimize} & \quad \left\| F_{op} - F_{sa}F_{bb} \right\|_F \\
\text{subject to} & \quad \left\| F_{sa}F_{bb} \right\|_F = N_s, \\
& \quad \left\| F_{sa} \right\|_F = 1.
\end{align*}
$$

(5)

where $F_{op}$ stands for the optimal fully digital precoder, while $F_{sa}$ and $F_{bb}$ are the analog and digital precoders to be optimized. It has been shown in [17] that minimizing the objective function in (5) approximately leads to the maximization of the spectral efficiency. It is also intuitively true that the optimal hybrid precoders should be sufficiently “close” to the unconstrained optimal digital precoder. In addition, the unconstrained optimal precoder and decoder are comprised of the first $N_s$ columns of $\mathbf{v}$ and $\mathbf{u}$ respectively, which are unitary matrices derived from singular value decomposition (SVD) of the channel, i.e., $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$.

In this paper, by decoupling the analog and digital precoders, alternating minimization stands out as an efficient method to obtain an effective solution. With the principle of alternating minimization, we will alternately solve for $F_{sa}$ and $F_{bb}$ while fixing the other, which will be the essential idea of this paper.

3. AMLSA Algorithm for the Fully-Connected Structure

For the fully-connected structure, literature [9] has shown that the Frobenius norm in (5) can be made exactly zero under the condition that $N_s^2 \geq 2N_s$. This means that the hybrid precoders can achieve the performance of the fully digital precoder in this special case, and the optimal hybrid precoders were obtained in [9]. Therefore, we will focus on the range of $N_s \leq N_s' \leq 2N_s$ in this paper.

Inspired by [15], this paper will propose an alternating minimization algorithm with the least squares amendment for the fully-connected structure, which is used to minimize the objective function in (5) and obtain the hybrid precoding matrix. The proposed algorithm introduces an orthogonal constraint to the digital precoding matrix $F_{bb}$, which is amended subsequently by the least squares after obtaining its iterative result. So that the result here is regarded as the final result of $F_{sa}$. It is shown in simulation results that the achievable spectral efficiency of our proposed algorithm is better to some extent than the existing algorithm without the least squares amendment. Furthermore, the number of iterations is reduced slightly via improving the initialization procedure.

3.1 Hybrid Precoder Design

It is well known that the columns of the unconstrained optimal precoding matrix $F_{op}$ are mutually orthogonal in order to cut down the interference between the multiplexed streams. On the basis of this structure of the unconstrained precoding solution, a similar constraint is introduced that the columns of the digital precoding matrix should be mutually orthogonal, i.e.,

$$
F_{bb}^T F_{bb} = I_{N_s}.
$$

(6)

where $I_{N_s}$ is an identity matrix that is of size $N_s \times N_s$. Therefore, the objective function in (5) can be rewritten as
In terms of [15] and [16], the objective function can be further recast as

\[
\begin{align*}
\|F_{opt} - F_{as}F_{ub}\|_2 &= \text{Tr}(F_{as}^H F_{as}) - \text{Tr}(F_{as}^H F_{as}F_{ub}) - \text{Tr}(F_{as}^H F_{as}F_{as}^H F_{opt}) \\
&\quad + \text{Tr}(F_{as}^H F_{as}^H F_{as}F_{ub}) \\
&= \|F_{as}\|_F^2 - 2\text{Tr}(F_{as}^H F_{as}F_{as}^H F_{opt}) + \|F_{as}^H F_{as}\|_F^2.
\end{align*}
\] (8)

Subsequently, the third item of (8) is focused and has the following upper bound

\[
\begin{align*}
\|F_{as}\|_F^2 &= \text{Tr}(F_{as}^H F_{as}F_{as}) \\
&= \text{Tr}\left(\begin{bmatrix} I_N & 0 \\ 0 & K_{N_{as}}^H F_{as}K_{N_{as}} \end{bmatrix}\right) \\
&\leq \text{Tr}\left(K_{N_{as}}^H F_{as}K_{N_{as}}\right) \\
&= \|F_{as}\|_F^2.
\end{align*}
\] (9)

Where \( F_{as}^H F_{as} = K \begin{bmatrix} I_N & 0 \\ 0 & K_{N_{as}}^H F_{as}K_{N_{as}} \end{bmatrix} \) is the SVD of \( K_{as} \) and the equality holds when \( N_{as} = N_j \). In terms of the mathematical expression above, the expression (8) can be further recast as

\[
\begin{align*}
\|F_{opt}\|_2 - 2\text{Tr}(F_{as}^H F_{as}F_{as}^H F_{opt}) + \|F_{as}\|_F^2 &= \text{Tr}(F_{as}^H F_{as}) - 2\text{Tr}(F_{as}^H F_{as}F_{as}^H F_{opt}) \\
&\quad + \text{Tr}(F_{as}^H F_{as}^H F_{as}^H F_{opt}) \\
&= \|F_{as}^H F_{as} - F_{as}\|_F^2.
\end{align*}
\] (10)

Because of the product form between digital precoding matrix and the analog precoding matrix, the existing algorithm that directly optimizes the objective function (8) will still bring in high complexity. In this context, we plan on adopting the upper bound (10) as the objective function rather than the original one. Thus, by adopting (10) as the objective function, the hybrid precoder design problem is given as

\[
\begin{align*}
\min_{s_{as},s_{ub}} \quad & \|F_{as}^H F_{as} - F_{as}\|_F^2 \\
\text{subject to} \quad & \|F_{as}\|_F^2 = N_j, \\
& \left|F_{as}^H F_{as}\right|_{i,j} = 1, \\
& F_{as}^H F_{as} = I_{N_j}.
\end{align*}
\] (11)

Confronted with the expression above, we propose to solve (11) by an alternating minimization procedure. To be specific, since the analog precoding matrix removes the product form with digital precoding matrix, it has a closed-form solution as shown in [15].

\[
\begin{align*}
\min_{s_{as},s_{ub}} \quad & \|F_{as}^H F_{as} - F_{as}\|_F^2 \\
\text{subject to} \quad & \|F_{as}\|_F^2 = N_j, \\
& \left|F_{as}^H F_{as}\right|_{i,j} = 1, \\
& F_{as}^H F_{as} = I_{N_j}.
\end{align*}
\] (11)
\[
\arg (F_{\text{ub}}) = \arg (F_{\text{op}} F_{\text{ub}}^H)
\]  

(12)

where \( \arg (F) \) generates a matrix whose entries are the phases of the corresponding entries of matrix \( F \).

Since \( F_{\text{ub}} \) is obtained, it can be regarded as fixed, we intend subsequently to solve a digital precoder problem. In terms of [16], OPP tries to minimize \( \| \mathbf{A} \Omega - \mathbf{B} \|_F \), where the optimization variable \( \Omega \) is a square unitary matrix. The mathematical model of OPP is similar to the objective function in (11) if the constraint of \( \Omega \) is relaxed to only columns orthogonality which has the same form of \( F_{\text{ub}} \). The solution to this problem, in terms of [16], is given by

\[
F_{\text{ub}} = V_{\text{s}}, U^H
\]  

(13)

where \( F_{\text{op}} F_{\text{ub}} = U \Sigma V^H = U S V_{\text{s}}^H \), which is the SVD of \( F_{\text{op}} F_{\text{ub}} \), \( S \) is a diagonal matrix whose elements are the first \( N_s \) nonzero singular values, and \( V_{\text{s}} \) is the first \( N_s \) columns of right singular vector \( V \).

As stated before, the proposed algorithm can alternately optimize the digital precoder and the analog precoder resort to expression (12) and (13), respectively. For better performance, the least squares is used to update \( F_{\text{ub}} \) after the last iteration of the algorithm, which is given as

\[
F_{\text{ub}} = (F_{\text{op}} F_{\text{ub}})^{-1} F_{\text{op}}^H F_{\text{ub}}
\]  

(14)

So that the result here is regarded as the final result of \( F_{\text{ub}} \). It is shown in simulation result that the achievable spectral efficiency of our proposed algorithm is better to some extent than the existing algorithm without the least squares amendment. Furthermore, in some existing algorithms, such as [15], the initialization of \( F_{\text{ub}} \) has been achieved with random phases, which can increase the number of iterations. The proposed method reduces slightly the number of iterations, which is depicted as follows: first, the initial digital precoding matrix \( F_{\text{ub}} \) is generated randomly which is orthogonalised subsequently to guarantee its columns orthogonality. Second, initialize analog precoding matrix \( F_{\text{ub}} \) in terms of the expression (12). It is shown in simulation result that the number of iterations is reduced slightly via improving the initialization procedure.

### 3.2 Algorithm Procedure

As stated before, the algorithm procedure is described as follows:

---

**AMLSA Algorithm**

**Input:** \( F_{\text{ub}} \in N_s \times N_s \)

1: Generate \( F_{\text{ub}} \) randomly and orthogonalize it subsequently to guarantee its columns orthogonality;

2: Initialize \( F_{\text{ub}} \) in terms of the expression

\[
\arg (F_{\text{ub}}) = \arg (F_{\text{op}} F_{\text{ub}}^H);
\]

3: Repeat

4: Fix \( F_{\text{ub}} \), compute \( F_{\text{op}} F_{\text{ub}} = U \Sigma V^H = U S V_{\text{s}}^H \);

5: \( F_{\text{ub}} = V_{\text{s}}, U^H \);

6: Fix \( F_{\text{ub}} \), compute \( \arg (F_{\text{ub}}) = \arg (F_{\text{op}} F_{\text{ub}}^H) \);

7: Until a stopping criterion triggers;

8: Update by least squares \( F_{\text{ub}} = (F_{\text{op}} F_{\text{ub}})^{-1} F_{\text{op}}^H F_{\text{ub}} \);

9: For the digital precoder at the transmit end, normalize

\[
\hat{F}_{\text{ub}} = \sqrt{N_s} \frac{F_{\text{ub}}}{\| F_{\text{ub}} \|_F}
\]

---
4. Simulation Results

This section will numerically evaluate the performance of our proposed algorithms. Suppose that data streams are transmitted from a transmitter with $N_t = 144$ to a receiver with $N_r = 36$ antennas, while both are equipped in the shape of USPA. The channel parameters are set as $N_{cl} = 5$ clusters, $N_{ray} = 10$ rays and the average power of each cluster is $\sigma_i^2 = 1$. The azimuth and elevation angles of AoDs and AoAs follow the Laplacian distribution with uniformly distributed mean angles over $[0, 2\pi)$ and angular spread of 10 degrees. The antenna elements in the USPA are separated by a half wavelength distance and all simulation results are averaged over 1000 channel realizations. The initial phases of the analog precoder matrix comply with the rule which is depicted in the algorithm procedure. The PE-AltMin algorithm is regarded as the opponent in contrast to our proposed algorithm.

4.1 Spectral Efficiency Evaluation

The spectral efficiency is evaluated when the number of RF chains is complied with the rule $N_s \leq N_{\text{RF}} < 2N_s$. Figure 2 compares the spectral efficiency of different precoding schemes for different $N_{\text{RF}}$. In this case, as shown in Figure 2, for the fully-connected structure, the proposed algorithm achieves higher spectral efficiency in some extent than the existing PE-AltMin algorithm, which benefits from the least squares amendment. Meanwhile, it also has been shown that when compliance with the rule $N_{\text{RF}} = N_{\text{ss}} = N_s$, it is the worst case since the number of RF chains cannot be smaller than $N_s$ as previously mentioned.

4.2 The Number of Iterations Evaluation

The number of iterations is evaluated when $N_s = 6$ and $N_{\text{RF}} = N_{\text{ss}} = 9$. Figure 3 compares the number of iterations of different precoding schemes for different accuracy. In this case, as shown in Figure 3, for the fully-connected structure, the proposed algorithm experiences less number of iterations slightly than the existing PE-AltMin algorithm, which benefits from reasonable initialization of the analog precoding matrix.

As stated before, the performances including spectral efficiency and the number of iterations, which belong to the proposed algorithm, have been both improved to some extent in contrast to the existing algorithm without the least squares amendment and improved initialization procedure.

![Figure 2](image-url)  
**Figure 2.** Spectral efficiency of different precoding schemes for different $N_{\text{RF}}$ when $N_s \leq N_{\text{RF}} < 2N_s$. 

4.3 Conclusions
5. Conclusions
In this paper, based on the principle of alternating minimization, an AMLSA algorithm is proposed for hybrid precoding in mmWave MIMO systems. The proposed method can increase the spectral efficiency to some extent, as well as reduce slightly the number of iterations. Simulation results confirm the effectiveness of the proposed algorithm, i.e., the spectral efficiency is better to some extent than the existing algorithm without the least squares amendment, moreover, the number of iterations is reduced slightly via improving the initialization procedure.

6. Acknowledgment
This work was supported by the Program for New Century Excellent Talents in University (NCET-13-0105), and by the Support Program for Hundreds of Outstanding Innovative Talents in Higher Education Institutions of Hebei Province, under Grant No.BR2-259, and by Natural Science Foundation of Hebei Province (No. F2016501139), and by the Specialized Research Fund for the Doctoral Program of Higher Education of China (No.20130042110003), and by the Fundamental Research Funds for the Central Universities under Grant No. N142302001, and by the Fundamental Research Funds for the Central Universities under Grant No.N162304002.

7. References
[1] J. G. Andrews et al., “What will 5G be?”, IEEE J. Sel. Areas Commun., vol. 32, no. 6, 2014, pp. 1065–1082.
[2] A. Ghosh et al., “Millimeter-wave enhanced local area systems: A highdata-rate approach for future wireless networks,” IEEE J. Sel. Areas Commun., vol. 32, no. 6, 2014, pp. 1152–1163.
[3] M. R. Akdeniz et al., “Millimeter wave channel modeling and cellular capacity evaluation”, IEEE J. Sel. Areas Commun., vol. 32, no. 6, 2014, pp. 1164–1179.
[4] A. Alkhateeb, J. Mo, N. González-Prelcic, and R. W. Heath, Jr., “MIMO precoding and combining solutions for millimeter-wave systems”, IEEE Commun. Mag., vol. 52, no. 12, 2014, pp. 122–131.
[5] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, “Spatially sparse precoding in millimeter wave MIMO systems”, IEEE Trans. Wireless Commun., vol. 13, no. 3, 2014, pp. 1499–1513.
[6] W. Roh et al., “Millimeter-wave beamforming as an enabling technology for 5G cellular communications: Theoretical feasibility and prototype results”, IEEE Commun. Mag., vol. 52, no. 2, 2014, pp. 106–113.
[7] P. Wang, Y. Li, L. Song, and B. Vucetic, “Multi-gigabit millimeter wave wireless communications for 5G: From fixed access to cellular networks”, IEEE Commun. Mag., vol. 53, no. 1, 2015, pp. 168–178.

[8] M. Kim and Y. Lee, “MSE-based hybrid RF/baseband processing for millimeter wave communication systems in MIMO interference channels,” IEEE Trans. Veh. Technol., vol. 64, no. 6, 2015, pp. 2714–2720.

[9] E. Zhang, C. Huang, “On achieving optimal rate of digital precoder by RF-baseband codesign for MIMO systems,” in Proc. 80th IEEE Veh. Technol. Conf. (VTC-Fall), Vancouver, BC, Canada, Sep. 2014, pp. 1–5.

[10] G. Wang and G. Ascheid, “Joint pre/post-processing design for large millimeter wave hybrid spatial processing systems,” in Proc. 20th Eur. Wireless Conf., Barcelona, Spain, May 2014, pp. 1–6.

[11] J. Singh and S. Ramakrishna, “On the feasibility of codebook-based beamforming in millimeter wave systems with multiple antenna arrays,” IEEE Trans. Wireless Commun., vol. 14, no. 5, 2015, pp. 2670–2683.

[12] C. Kim, T. Kim, and J.-Y. Seol, “Multi-beam transmission diversity with hybrid beamforming for MIMO-OFDM systems,” in Proc. IEEE Global Commun. Conf. Workshops (GLOBECOM), Atlanta, GA, USA, Dec. 2013, pp. 61–65.

[13] L. Dai, X. Gao, J. Quan, S. Han, and C.-L. I, “Near-optimal hybrid analog and digital precoding for downlink mmwave massive MIMO systems,” in Proc. IEEE Int. Conf. Commun. (ICC), London, U.K., Jun. 2015, pp. 1334–1339.

[14] T. S. Rappaport, R. W. Heath, R. C. Daniels, and J. N. Murdock, “Millimeter Wave Wireless Communications”, New York, NY, USA: Pearson Education, 2014.

[15] X. Yu, J. C. Shen, J. Zhang, K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems”, IEEE Journal Selected Topics in Signal Processing, vol. 10, no. 3, April 2016, pp. 485-500.

[16] P.H. Schönemann, “A generalized solution of the orthogonal Procrustes problem”, Psychometrika, vol. 31, no. 1, 1966, pp. 1–10.