MAGNETIC DOMAINS IN MAGNETAR MATTER AS AN ENGINE FOR SOFT GAMMA-RAY REPEATERS AND ANOMALOUS X-RAY PULSARS

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ABSTRACT

Magnetars have been suggested as the most promising site for the origin of observed soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs). In this work, we investigate the possibility that SGRs and AXPs might be observational evidence for a magnetic phase separation in magnetars. We study magnetic domain formation as a new mechanism for SGRs and AXPs in which magnetar matter separates into phases containing different flux densities. We identify the parameter space in matter density and magnetic field strength at which there is an instability for magnetic domain formation. We conclude that such instabilities will likely occur in the deep outer crust for the magnetic Baym, Pethick, and Sutherland model and in the inner crust and core for magnetars described in the relativistic Hartree theory. Moreover, we estimate that the energy released by the onset of this instability is comparable with the energy emitted by SGRs.

Key words: gamma rays: stars – instabilities – stars: interiors – stars: magnetic field – stars: neutron – X-rays: stars

Online-only material: color figures

1. INTRODUCTION

Soft gamma-ray repeaters (SGRs) are compact objects undergoing episodic instabilities that produce super-Eddington X-ray outbursts. Up to now, six SGRs (four confirmed and two candidates) have been observed.3 They are believed to be a new class of γ-ray transients that are different from the source of ordinary gamma-ray bursts. Observations of the spin-down timescale (Kouveliotou et al. 1998) have confirmed the fact that these SGRs are newly born neutron stars with a very large surface magnetic field ($B \sim 10^{15}$ G). Such stars have been named magnetars (Duncan & Thompson 1992; Kouveliotou et al. 2003). About 10 anomalous X-ray pulsars (AXPs) are also categorized as magnetars (Kaspi 2007; van Paradijs et al. 1995). Even though the magnetar model is generally accepted as the paradigm for both SGRs and AXPs, it is not easy to explain both objects simultaneously by a consistent set of parameters.

Woods et al. (2001) have reported evidence for a sudden reconfiguration of the magnetic field in SGR 1900+14 during the giant flare of 1998 August 27. This scenario requires a reorganization of the magnetic field both inside and outside the star. Sharp field gradients are postulated to create a fracture in the rigid outer crust of the neutron star. Cheng et al. (1995) have shown that SGR events and earthquakes share four distinctive statistical properties: (1) power-law energy distributions, (2) log-symmetric waiting time distributions, (3) strong positive correlations between waiting times of successive events, and (4) weak or no correlation between intensities and waiting times. These statistical similarities, together with the fact that the crustal energy liberated by starquakes is sufficient in principle to fuel the soft gamma-ray flashes, suggest that SGRs are indeed powered by starquakes. Moreover, there is also a strong correlation between the magnitude and waiting times both for active earthquake regions and for SGR 1806-20. Thus, the statistical similarities between earthquakes and SGR events argue for physically similar origins. The quasi-periodic oscillations (QPOs) observed at late times of giant flares of SGR 1806-20, SGR 1900+14, and SGR 0525-66 (Mareghetti 2008; Watts & Strohmayer 2007) constitute another piece of observational evidence in favor of starquakes. These QPOs are most likely due to seismic oscillations induced by the large crustal fractures occurring in extremely energetic events similar to what happens after earthquakes. Such oscillations could be limited to the crust or involve the entire neutron star.

Crackling noise (Sethna et al. 2001) arises when a system responds to changing external conditions through discrete, impulse events spanning a broad range of sizes. Bak et al. (1987) introduced a connection between dynamical critical phenomena and crackling noise. They emphasized how systems may end up naturally at the critical point through a process of self-organized criticality. Based upon this idea of crackling noise, Kondratyev (2002) studied the statistics of magnetic noise in neutron star crusts and compared its intensity and statistical properties to the burst activity of SGRs. He then argued that the noise could originate from magnetic avalanches. However, because of the required inhomogeneous crust structure, he postulated the existence of magnetic domains within the neutron star crust for an interior magnetic field strength in the range of $10^{16}$–$10^{17}$ G. Using the randomly jumping interacting moment model, it was shown that the burst intensity and waiting time distributions are not only in good agreement with observations, but also are analogous with the statistical properties of SGRs.

Whether or not magnetars are the source of SGRs and AXPs, as relics of stellar interiors, the study of the magnetic fields in and around degenerate stars should give important information on the role such fields play in star formation and stellar evolution (Suh & Mathews 2001a). The scalar virial theorem (Chandrasekhar & Fermi 1953) implies an allowed internal field strength of $B \lesssim 2 \times 10^{5} (M/M_{\odot})(R/R_{\odot})^{-2}$ G for a star of size $R$ and mass $M$. For a typical neutron star, the maximum interior field strength could thus reach $B \sim 10^{18}$ G. Since strong interior magnetic fields modify the nuclear equation of state for degenerate stars, their structure will also be changed (Cardall et al. 2001).

3 http://www.physics.mcgill.ca/~pulsar/magnetar/main.html
Even though SGRs appear to be observable consequences of starquakes or surface fractures, their detailed mechanism is not known. Moreover, it is unlikely that AXPs could only be explained by starquakes in strong magnetic fields. Therefore, in this work we suggest a magnetic domain model to correlate smoothly between the statistics of starquakes and magnetic avalanches in magnetar crusts. In this work, magnetic properties of magnetar matter such as the magnetization and the susceptibility are calculated in the framework of three different representative equations of state. We consider an ideal npe gas, the relativistic Hartree mean field theory, and the magnetic Baym, Pethick, and Sutherland (BPS) model (Lai & Shapiro 1991). It has been shown (Broderick et al. 2000) that the magnetization of magnetar matter undergoes large de Haas–van Alphen oscillations. The magnetic susceptibility can then lead to a region unstable to the formation of magnetic domains. It has not yet been demonstrated, however, that magnetic domains actually form in magnetar matter. Here, we show that it is indeed possible to form such magnetic domains in magnetars, and that these could affect the surface properties and structure of magnetars, possibly leading to observable consequences such as starquakes, glitches, and X- or γ-ray emission.

2. THE DIFFERENTIAL SUSCEPTIBILITIES FOR A MAGNETIC npe GAS

The magnetic equation of state for magnetar matter has been described in Suh & Mathews (2001a) for an ideal npe gas. The magnetization of simple magnetar-matter material can be derived from the thermodynamic potential (Blandford & Hernquist 1982). Broderick et al. (2000) have generalized the formalism for a multicomponent system including interacting nucleons. Hereafter, we introduce the following notation for magnetic fields. For material in a uniform magnetic field, the magnetic field $H$ is related to the flux density $B$ by the relation (Pippard 1980):

$B = H + 4\pi (1 - D) \mathcal{M}(B), \tag{1}$

where $D$ is the demagnetization coefficient that is fixed by the geometry of the system. For example, $D \approx 0$ for a neutron star crust permeated by an approximately vertical magnetic field. In chemical equilibrium, the total magnetization $\mathcal{M}$ is given by a simple sum of the constituent magnetizations:

$\mathcal{M} = \sum_{j=n,p,e} \mathcal{M}_j. \tag{2}$

The magnetic susceptibilities are then given by $\chi = \mathcal{M}/H$, and the differential susceptibility $\eta$ is defined (Blandford & Hernquist 1982) by

$\eta_j = \left( \frac{\partial \mathcal{M}_j}{\partial B} \right)_{\mu,T,V}, \tag{3}$

where $\mu$ is the chemical potential, $T$ is the temperature, and $V$ is the volume of the system. The total differential susceptibility for magnetar matter above the neutron drip density is then given by

$\eta = \sum_j \eta_j, \text{ where } j = n, p, e. \tag{4}$

For a cold ideal npe gas, we can obtain simple expressions for the differential susceptibilities of the various components. For the electron differential susceptibility $\eta_e$, we have

$\eta_e = \frac{\alpha}{4\pi^2} \sum_{x',n} \left[ 4n_p^f \ln \left( \frac{\epsilon_e + \sqrt{\epsilon_e^2 - m_e^2}}{m_e} \right) - 2\gamma_e \left( \frac{n_p^f}{m_e} \right)^2 \left( \frac{\epsilon_e}{\sqrt{\epsilon_e^2 - m_e^2}} \right) \right]. \tag{5}$

where $m_e = \sqrt{1 + 2\gamma_e n_p^f}$. For protons,

$\eta_p = \frac{\alpha}{4\pi^2} \sum_{x',n} \left[ \left( 4A_p - 2\gamma_p \frac{n_p^f}{(1 + 2\gamma_p n_p^f)^{3/2}} \right) + \gamma_p A_p \left( \ln \left( \frac{\epsilon_p + \sqrt{\epsilon_p^2 - m_p^2}}{m_p} \right) - \frac{\epsilon_p}{\sqrt{\epsilon_p^2 - m_p^2}} \right) \right]. \tag{6}$

where $A_p = n_p^f/\sqrt{1 + 2\gamma_p n_p^f} - s_p^f \kappa_p$, and $m_p = \sqrt{1 + 2\gamma_p n_p^f}$. In Equations (5) and (6), $n_p^f = n + 1/2 = s_p^f p$, where $n = 1, 2, 3, \ldots$ denotes the Landau levels and $s_p^f p$ is the electron or proton spin projection on the magnetic field direction; $\gamma_e,p = B/B_{c1}^e,p$, where $B_{c1}^e,p = e\hbar/m_{e,p} c^3$ are the quantum critical field for electrons and protons; and $\epsilon_e,p = E_F^e,p/m_{e,p} c^2$ with the electron and proton Fermi energy $E_F^e,p$, respectively (Suh & Mathews 2001a).

Finally, for neutrons, we obtain

$\eta_n = \frac{\alpha}{2\pi^2} \sum_{x',n} \left( s_n^f \kappa_n \right)^2 \left[ n_n^0 + s_n^f \kappa_n \gamma_n \eta_n \right]. \tag{7}$

where

$\eta_n^0 = -\frac{1}{2} \left\{ \epsilon_n \sqrt{\epsilon_n^2 - m_n^2} + \tilde{m}_n \ln \left( \frac{\epsilon_n + \sqrt{\epsilon_n^2 - m_n^2}}{\tilde{m}_n} \right) \right\}, \tag{8}$

and

$\eta_n^s = \tilde{m}_n \ln \left( \frac{\epsilon_n + \sqrt{\epsilon_n^2 - m_n^2}}{\tilde{m}_n} \right) \tag{9}$

with $\tilde{m}_n = 1 + 2s_n^f \kappa_n \gamma_n$ and $\gamma_n = B/B_{c1}^n$, $B_{c1}^n = e\hbar/m_n^2 c^3$. In Equations (5)–(7), $\alpha = e^2/\hbar c$ is the fine structure constant, $s_n^f p$ is the neutron spin projection in the magnetic field direction, and $\kappa_p, \kappa_n$ are the anomalous magnetic moments for protons and neutrons, respectively, as given in Equation (10).

3. THE DIFFERENTIAL SUSCEPTIBILITIES IN THE RELATIVISTIC HARTREE THEORY

For a system of strongly interacting baryons (neutrons and protons), the relativistic mean field (Hartree) theory should be a reasonable approximation for the description of the equation of state for magnetar matter at high density (Broderick et al. 2000; Chakrabarty et al. 1997) through the exchange of $\sigma$ and vector $\omega$ and $\rho$ mesons in a strong magnetic field. In the baryon Lagrangian for the relativistic Hartree theory, the anomalous magnetic moments are included through the coupling of the baryons to the electromagnetic field tensor with $\sigma_{\mu\nu} \equiv \frac{i}{2} [\gamma_{\mu}, \gamma_5]$ and the strengths $\kappa_\rho$ and $\kappa_n$ are given by

$\kappa_\rho = \frac{e}{2m_\rho c} \left( \frac{g_\rho}{2} - 1 \right) \quad \text{and} \quad \kappa_n = \frac{e}{2m_n c} \frac{g_n}{2}, \tag{10}$

where $g_\rho$ and $g_n$ are the anomalous magnetic moments for protons and neutrons, respectively.
where \( g_p = 5.58 \) and \( g_n = -3.82 \) are the Lande \( g \)-factors for protons and neutrons, respectively. In this work, we can ignore the possible self-interactions among the scalar \( \sigma \), the vector \( \omega \), and iso-vector \( \rho \) mesons. Therefore, although the electromagnetic field is included in the total Lagrangian, it is assumed to be externally generated (and thus has no associated field equation) and only frozen-field configurations will be considered. The effective baryon mass \( m_{b=p,n} \) is then given by the coupling to the \( \sigma \) meson,

\[
m^*_b = m_b - (g_\sigma/m_\sigma)^2 (n^*_p + n^*_n),
\]

where \( g_\sigma \) and \( m_\sigma \) are the \( \sigma \) meson coupling constant and mass, respectively. In Equation (11), \( n^*_p \) is the scalar number density for protons,

\[
n^*_p = \frac{1}{2\pi^2} \sum_{s_p, n} \frac{m^*_p c}{\hbar} \left( \frac{\epsilon^*_p + \sqrt{\epsilon^*_p^2 - m^*_p^2}}{\epsilon^*_p + \sqrt{\epsilon^*_p^2 - m^*_p^2}} \right),
\]

while the scalar number density for neutrons is

\[
n^*_n = \frac{1}{4\pi^2} \sum_{s_n} \frac{m^*_n c}{\hbar} \left[ \frac{\epsilon^*_n + \sqrt{\epsilon^*_n^2 - m^*_n^2}}{\epsilon^*_n + \sqrt{\epsilon^*_n^2 - m^*_n^2}} \right],
\]

where \( m^*_p = 1 + s^*_{p} \kappa_{n} \gamma_{p} \) and \( \gamma_{p} = (m_{e}/m_{n}^{*}) \gamma_{e} \). For simplicity, as in Chakrabarty et al. (1997), the nucleon rest mass is taken as \( m = m_{n} = m_{p} \) in the numerical calculations.

Assuming a mixture of neutrons, protons, and electrons in chemical equilibrium, the chemical potentials are related by

\[
\mu_n = \mu_p + \mu_e,
\]

while the condition of charge neutrality gives

\[
\rho_p (\epsilon_p, \gamma_p) = \rho_e (\epsilon_e, \gamma_e).
\]

Given the nucleon-meson coupling constant and the coefficients in the scalar self-interactions, the field equations can be solved self-consistently for the chemical potentials, \( \mu_j \) (\( j = n, p, e \)), and the meson field strengths in a uniform magnetic field \( B \) along the \( z \)-axis corresponding to the choice of the gauge for the vector potential \( A^\mu \) (Broderick et al. 2000). In this work, we adopt the following coupling constants and meson masses: \( g_{\sigma}^2 (m_N/m_\sigma)^2 = 357.47 \), \( g_{\rho}^2 (m_N/m_\rho)^2 = 273.78 \), and \( g_{\omega}^2 (m_N/m_\omega)^2 = 97.0 \) (Horowitz & Serot 1981).

Figure 1 shows the effective baryon mass \( m^{*}/m \) as a function of baryon density \( \rho \) for magnetic field strengths of \( \gamma_e = 0.01 \) (solid line) and \( 10^2 \) (dashed line) calculated in the model of Horowitz & Serot (1981). For a magnetic field strength less than \( \sim 10^{18} \) G, this figure shows that the effective nucleon mass is not significantly affected by the magnetic field strength. Broderick et al. (2000) and Chakrabarty et al. (1997) have obtained similar results. This effective baryon mass modifies the baryon dispersion relation in dense magnetar matter.

4. THE DIFFERENTIAL SUSCEPTIBILITY IN THE MAGNETIC BPS MODEL

For matter in thermodynamical equilibrium below the neutron drip density (\( \rho_{\text{drip}} \approx 4.3 \times 10^{14} \) g cm\(^{-3}\)), we adopt the magnetic BPS model (Lai & Shapiro 1991) and use the semi-empirical mass formula (Shapiro & Teukolsky 1983). For simplicity, we only consider \( ^{56} \)Fe nuclei in the numerical calculation. Then, the magnetization and the differential susceptibility for the magnetic BPS equation of state are given by

\[
M_{\text{BPS}} = M_e + \frac{1}{3} M_L \quad \text{and} \quad \eta_{\text{BPS}} = \eta_e + \frac{1}{3} \eta_L,
\]

where \( M_e \) is the magnetization of the electron gas and \( \eta_e \) is given in Equation (5). In Equation (16), \( M_L \) is the magnetization for the \( bcc \) Coulomb lattice energy. Then, we derive here the lattice differential susceptibility \( \eta_L \) to be

\[
\eta_L = -1.444 \left( \frac{1}{2\pi^2} \right)^{4/3} Z^{2/3} \alpha^2 \sum_{\chi_{p,n}} \frac{1}{(\epsilon^2 - m^2)^{2/3}} \gamma_e^{7/3} \\
\times \left[ (\epsilon^2 - m^2)^{-1/3} - 2 \gamma_e n_f^2 (\epsilon^2 - m^2)^{4/3} - 2 (\gamma_e n_f^2)^2 \right],
\]

where \( Z \) is the average atomic number of the nuclei.

5. MAGNETIC DOMAIN FORMATION

In general, the magnetization of a system is small compared with the external magnetic field \( H \). However, when the system is sufficiently cool so that its thermal energy is smaller than the spacing of the Landau levels, the magnetization can undergo large de Haas–van Alphen oscillations (Lifshitz & Pitaevskii 1980) with either changing magnetic fields or a changing Fermi energy. Under these conditions, it
The dash line indicates the nuclear saturation density values for which conditions are unstable for the formation of magnetic domains. \( \eta > \) obeys Hernquist (1982), that is, when the differential susceptibility gas, in certain regions \( n_{\text{pe}} \) becomes energetically favorable for the system to separate into two phases containing different flux densities. This is the so-called Schoenberg effect (Pippard 1980). This means that although \( 4\pi \eta \) is less than unity for a magnetized npe gas, in certain regions \( 4\pi \eta \) can exceed unity, which implies the possible existence of magnetic domains (Blandford & Hernquist 1982), that is, when the differential susceptibility obeys \( \eta > 1/4\pi \) and \( \partial H/\partial B < 0 \). Then, magnetar matter in thermodynamic equilibrium becomes unstable to the formation of magnetic domains of alternating magnetization. For the case of a vanishing demagnification coefficient, \( D = 0 \), the material will separate into two phases corresponding to different magnetization.

For magnetar matter above the neutron drip density \( \rho_{\text{drip}} \), we considered an ideal pure non-interacting cold npe gas as well as the relativistic Hartree model. However, in the density region between \( \rho_{\text{drip}} \) and \( \rho_{\text{nuc}} \approx 2.8 \times 10^{14} \text{ g cm}^{-3} \), neutron star matter is composed of electrons, nuclei, and free neutron gas so that we cannot directly apply the ideal npe gas model in this density regime. For example, for non-magnetic neutron star matter in this intermediate density regime, we can employ the Baym, Bethe, and Pethick (BBP) equation of state (Baym et al. 1971). This BBP model is based upon a compressed liquid drip model of nuclei. It gives some corrections to the ideal npe equation of state. Therefore, if we adopt the magnetic BBP model, the region below the dash line in Figures 2 and 3 will be shifted to the left by the BBP equation of state corrections (see Shapiro & Teukolsky 1983). This means that for a fixed magnetic field strength the density region in which magnetic domains can be formed increases. However, since there is no other physical model for the intermediate density regime with a strong magnetic field, we can still gain qualitative understanding by describing this regime using an ideal npe gas model.

Figure 2 shows the regions of matter density and magnetic flux density for an ideal npe gas above the \( \rho_{\text{drip}} \) in which magnetar matter is unstable to phase separation into magnetic domains. For magnetic field strengths of \( \sim 10^{14} - 10^{15} \text{ G} \), magnetic domains cannot be formed in the lower density region below \( \sim 10^{13} \text{ g cm}^{-3} \). However, it is possible for magnetic domains to form in the density region higher than \( \sim 10^{13} \text{ g cm}^{-3} \).

Figure 3 shows the unstable region for a relativistic Hartree mean field model in which the baryon effective mass is taken into account. When we consider the effective baryon mass within the relativistic Hartree theory, domain formation can significantly occur above a density of \( \rho \sim 10^{14} \text{ g cm}^{-3} \) and a field strength \( B \sim 10^{16} \text{ G} \). The effective baryon mass lowers the density at which magnetic domain formation occurs in the core of a magnetar, in which strong magnetic fields are expected. We also find that magnetic domain formation could not be formed for a magnetic field strength of \( \lesssim 10^{15} \text{ G} \).

Finally, in Figure 4, the magnetic domain instability regions obtained by using the magnetic BPS model are depicted in the outer crust of magnetars. Unlike the other two cases, in this case the magnetization is dominated by electrons. Therefore, the conditions for the onset of the magnetic domain instability are affected by the occupation of Landau levels. This causes the instability conditions to vary as a function of field strength for fixed density. In Figure 4, we see that magnetic domains cannot be formed if the density is less than \( \rho \sim 5 \times 10^{10} \text{ g cm}^{-3} \) for a magnetar having a typical surface magnetic field of \( B \sim 10^{14} - 10^{15} \text{ G} \). However, in the density region around the neutron drip density, magnetic domains can be formed. We can also see magnetic domain formation in regions at low density \( \rho \lesssim 10^{11} \text{ g cm}^{-3} \) and at a relatively low magnetic field strength \( B \lesssim 10^{13} \text{ G} \). This is the case considered by Blandford & Hernquist (1982). Adam (1986) also obtained a similar result for a magnetized electron gas in magnetic white dwarfs.

We thus find that possible unstable regions for magnetic domain formation are within the deep outer crust, deeper part of the inner crust, and core of magnetars. This means that the outer...
Figure 4. Magnetic domain formation instability for $^{56}$Fe in the magnetic BPS model. The shaded regions denote the parameter regions for which conditions are unstable for the formation of magnetic domains. (A color version of this figure is available in the online journal.)

shell at low density $\rho \lesssim 10^{10}$ g cm$^{-3}$ is stable against magnetic domain formation and should consist of strongly magnetized material without magnetic domains.

6. SGR AND AXP MECHANISMS

This magnetic domain formation might be an important clue to explain SGRs and AXPs in the magnetar model (Suh & Mathews 2001b). As the density increases, the number of the maximum occupied Landau orbital also increases. Therefore, similar to Blandford & Hernquist (1982), we can calculate the scale height $\Delta z$ of a domain of fixed magnetization. We start with the equation of hydrostatic equilibrium, for which $(dP/dz) = -g \rho$, where $g$ is the surface gravity. For a crust supported by electron degeneracy pressure, we have $dP \propto \epsilon_{\gamma e}^{3/2} d\epsilon_{\gamma e}$, while $\rho \propto \mu_e \epsilon_{\gamma e}$, where $\epsilon_{\gamma e} = E_{\gamma e} / m_e c^2$ is the Fermi energy of the electron and $\mu_e \sim 2-3$ is the mean molecular weight per electron. This gives $\Delta z \propto \Delta \epsilon_{\gamma e} / (g \mu_e)$. Now, we wish to find the range of $\Delta z$ over which the electrons remain in a single Landau level. The number of the maximum Landau is given by $n = (\epsilon_{\gamma e}^2 - 1) / 2 \gamma_e$ (Suh & Mathews 2001a), where $\gamma_e = B / B_e$. Hence, for a fixed $B$, $\Delta n \propto \epsilon_{\gamma e} \Delta \epsilon_{\gamma e} / B$. Setting $\Delta n = 1$, we can find the change of Fermi energy $\Delta \epsilon_{\gamma e} \propto B / \epsilon_{\gamma e}$ corresponding to the length scale associated with a single maximum Landau level. Collecting the appropriate constants, we then deduce the following scaling:

$$\Delta z \sim 100 \left( \frac{g}{10^{11} \text{ cm s}^{-2}} \right)^{-1} \left( \frac{\mu_e}{2} \right)^{-1} \left( \frac{\epsilon_{\gamma e}}{5} \right)^{-1} \left( \frac{B}{10^{15} \text{ G}} \right)^{1/2} \text{ m.}$$

(18)

Under the conditions for domain formation, the spacing $\Delta z$ would be the average vertical scale between domain interfaces and also the horizontal scale of the domains if they are in local pressure equilibrium. (However, the actual size and shape of the domains are difficult to determine.)

For a fixed Fermi energy, we can also estimate the variation in the magnetic field associated with the occupation of a single Landau level. Directly from $n = (\epsilon_{\gamma e}^2 - 1) / 2 \gamma_e$, the difference in the magnetic field $\Delta B$ over which $\Delta n = 1$ can be written as

$$\frac{\Delta B}{B} \sim 50 \left( \frac{B}{10^{15} \text{ G}} \right) \frac{1}{\epsilon_{\gamma e}^2 - 1}. \quad (19)$$

This gives the scale of the different magnetic flux densities and magnetic pressure in regions where magnetic domains can form. However, in the outer crust at lower density, the magnetic fields are homogeneously distributed and tightly pinned to the matter.

According to the magnetic domain theory (Pippard 1980), the magnetic domain walls move or grow, and the magnetic domains rotate within the material. Therefore, the movement and rotation of magnetic domains can cause a physical dimensional change and produces the maximum possible strain on the crust material. This process is very similar to that of the internal structure of the Earth in which a sudden collapse or strain of the mantle below the Earth’s crust sometimes occurs.

In regions where magnetic field distortion increases, magnetic domains could be formed. At the boundary, there will also exist regions around each wall where the magnetic field is distorted. The formation or adjustment of domain structure involves magnetic field fluctuations of a few percent amplitude which have an anisotropic magnetostrictive stress $2\pi M^2$ associated with the magnetization. Any sudden readjustment of the domain structure will cause a local departure from isostasy which will be relieved on an ohmic dissipation timescale ($\tau_D \sim \sigma A / 4\pi c^2 \sim 10^4$ yr; Blandford & Hernquist 1982). These anisotropic magnetostrictive stresses may be large enough to crack the outer crust (Blaes et al. 1989).

Then, we can estimate the physical length variation of the magnetic domains. The bulk modulus ($K_B$) is defined as the pressure increase needed to effect a given relative decrease in volume. For a gas, the adiabatic bulk modulus $K_S$ is approximately given by $K_S = \gamma P$, where $\Gamma$ is the adiabatic index. Then,

$$\frac{\delta V}{V} \approx \frac{\lambda}{K_S} \gamma e \sim 0.1\lambda. \quad (20)$$

for $\Gamma \simeq 1.8$ and $\Gamma \simeq 1$ at the neutron drip density $(4.4 \times 10^{11}$ g cm$^{-3}$ in the magnetic BPS model; Lai & Shapiro 1991).

Therefore, with $\lambda \approx 0.6$ for Fe (Stewart 1954), the length change due to magnetic domain formation is finally given by

$$\frac{\delta l}{l} = \frac{1.5V}{3V} \approx 0.02. \quad (21)$$

This means that there is a 2 m length change for a 100 m characteristic domain size when a magnetic domain is formed in the deep outer crust of a magnetar. We can also estimate the cracking timescale in the outer crust to be $\tau_c \sim \Delta z / v_c \approx 0.1 \text{ ms}$, where $v_c = \sqrt{Y/\rho}$ (with $Y$ being the shear modulus) is the shear velocity (Blaes et al. 1989). Kondratyev (2002) has analogized this cracking timescale as the avalanche spanning time, which is consistent with the rise time for SGR giant bursts (Hurley 2000; Kouveliotou et al. 2003; Mareghetti 2008).

Finally, we can estimate the elastic energy released, $\Delta E_D \sim \Delta z / \sigma R_D^2 \sigma$, where $R_D \sim \Delta z$ is the characteristic horizontal size of the domain, using the magnetic stress energy $\sigma \sim \chi^2 B^2$. Then, we obtain the released elastic energy

$$\Delta E_D \sim 6 \times 10^{42} \chi^2 \theta_m \left( \frac{R_D}{10^6 \text{ cm}} \right)^2 \left( \frac{B}{10^{15} \text{ G}} \right)^2 \text{ erg.} \quad (22)$$
where $\theta_m$ the maximum allowed strain angle and $\chi = M/H$ is the magnetic susceptibility. This is also the typical energy released in SGRs. Hence, this cracking of the crust by magnetostrictive stress could be the mechanism of the observed SGRs. We suggest that magnetic domain formation and any sudden readjustment of the domains can produce an energy source for soft gamma rays in SGRs and X-rays in AXPs.

However, there is evidence that AXPs have stronger magnetic fields than SGRs (Mareghetti 2008). With stronger magnetic fields, it would be hard for the magnetic domains to be formed in the outer crust of magnetars. This means that there would be little cracking of the outer crust by the magnetostrictive stress. The possibility remains, however, that AXPs could produce a giant blast like SGRs in this magnetic domain model.

7. SUMMARY

In this work, we have studied magnetic domain formation as a new mechanism for SGRs and AXPs. In this paradigm, magnetar matter separates into phases containing different flux densities. We have identified the parameter space in matter density and magnetic field strength at which there is an instability for magnetic domain formation and have shown that such instabilities are likely to occur in the deep outer crust for the magnetic BPS model and in the deeper part of the inner crust and core for magnetars described by the relativistic Hartree theory. Moreover, we have estimated the strain on the outer crust induced by the formation of such domains and found that the anticipated energy release is comparable with the energy emitted by typical SGRs. Hence, we propose that the magnetic domain formation scenario described here represents a new possible mechanism that drives the giant flares of SGRs as well as X-ray outbursts and the quiescent phase of AXPs. At the very least, this proposal warrants further investigation. Moreover, since the physical length variation caused by the magnetic domain formation might lead to solid crustal deformation and catastrophic cracking, SGRs might be sources of gravitational waves (GWs; Abbott et al. 2008) even though there is no evidence yet of GWs associated with observed SGR bursts. However, if GWs were ever detected from SGRs, then that may be a way to verify this magnetic domain model in magnetars. Clearly, the next step is to undertake detailed dynamical numerical studies of the formation and evolution of such magnetic domains in neutron star crusts. Efforts along this line are currently underway (I.-S. Suh et al. 2010, in preparation).

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