CP and T violation in neutrino oscillations\textsuperscript{1}

J. Bernabéu \textsuperscript{a}, M.C. Bañuls \textsuperscript{b}

\textsuperscript{a} Dep. de Física Teórica, Universidad de Valencia
\textsuperscript{b} IFIC, Centro Mixto Univ. Valencia - CSIC

Abstract

The conditions to induce appreciable CP-and T-odd effects in neutrino oscillations are discussed. The propagation in matter leads to fake CP-and CPT-odd asymmetries, besides a Bohm-Aharonov type modification of the interference pattern. We study the separation of fake and genuine CP violation by means of energy and distance dependence.

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Complex neutrino mixing for three neutrino families originates CP and T violation in neutrino oscillations. The requirement of CPT invariance leads to the condition
\[ A(\alpha \rightarrow \beta; t) = A^*(\beta \rightarrow \alpha; -t) \]
for the probability amplitude between flavour states, so that CP or T violation effects can take place in Appearance Experiments only.

CPT invariance and the Unitarity of the Mixing Matrix imply that the CP-odd probability
\[ D_{\alpha\beta} = |A(\alpha \rightarrow \beta; t)|^2 - |A(\beta \rightarrow \alpha; t)|^2 \]
is unique for three flavours: \( D_{e\mu} = D_{\mu\tau} = D_{\tau\nu} \). The T-odd probabilities
\[ T_{\alpha\beta} = |A(\alpha \rightarrow \beta; t)|^2 - |A(\beta \rightarrow \alpha; t)|^2 \]
are odd functions of time by virtue of the hermitian character of the evolution Hamiltonian. The last property does not apply to the effective Hamiltonian of the \( K^0 - \bar{K}^0 \) system.

The oscillation terms are controlled by the phases \( \Delta_{ij} \equiv \Delta m^2_{ij} L/4E \), where \( L = t \) is the distance between source and detector. In order to generate a non-vanishing CP-odd probability, the three families have to participate actively: in the limit \( \Delta_{12} \ll 1 \), where (1, 2) refer to the lowest mass eigenvalues, the effect tends to vanish linearly with \( \Delta_{12} \). In addition, all mixings and the CP-phase have to be non-vanishing. Contrary to the CP-violating \( D_{\alpha\beta} \), the CP-conserving probabilities are not unique. If \( \Delta_{12} \ll 1 \), the flavour transitions can be classified by the mixings leading to a contribution from the main oscillatory phase \( \Delta_{23} \approx \Delta_{13} \). The strategy would be the selection of a “forbidden” transition, i.e., an appearance channel with very low CP-conserving probability, in order to enhance the CP-asymmetry. This scenario appears plausible and perhaps favoured by present indications on neutrino masses and mixings from atmospheric and solar neutrino experiments. The use of the atmospheric results for \( |\Delta m^2_{23}| \) leads to the energy dependent
\[ |\Delta_{23}| \approx 4 \times 10^{-3} \left( \frac{L}{Km} \right) \left( \frac{GeV}{E} \right) \]
which is of the order of unity for long-base-line experiments. This oscillation phase generates an “allowed” transition \( \nu_\mu \rightarrow \nu_\tau \) proportional to \( s_{23}^2 \) and a “forbidden” transition \( \nu_\mu \rightarrow \nu_e \) proportional to \( s_{13}^2 s_{23}^2 \) for terrestrial or atmospheric neutrinos. In that case, the CP-even probability is independent of \( \Delta_{12} \). The CP-odd probability is, on the contrary, linear in \( s_{13} \) and \( \Delta_{12} \). We conclude

We disregard the alternative solution where the solar neutrino oscillations are associated to an almost degenerate \( \Delta m^2_{23} \).
that the ratio \( \Delta_{12}/s_{13} \) is the crucial parameter to induce an appreciable CP-odd asymmetry in the forbidden \( \nu_\mu \rightarrow \nu_e \) transition.

The large-mixing-angle MSW solution \([4]\) to the solar neutrino data provides \( \Delta m_{12}^2 \) of the order of few \( \times 10^{-5} \) eV\(^2\) and a mixing \( \sin^2 2\theta_{12} \simeq 0.7 \). For reactor neutrinos, the survival probability proceeds through \( \Delta_{13} \) (neglecting \( \Delta_{12} \ll 1 \)) with the result

\[
P_{\nu_e \rightarrow \nu_e} = c_{13}^2 + s_{13}^2 + 2 c_{13}^2 s_{13}^2 \cos(2\Delta_{13}) 
\simeq 1 - 4 s_{13}^2 \sin^2 \Delta_{23} \tag{4}
\]

The CHOOZ limit \([3]\) gives \( s_{13}^2 \lesssim 0.05 \). Under these circumstances, one can reach \([3]\) CP-odd asymmetries for the forbidden channel \( \nu_\mu \rightarrow \nu_e \) of the order 10-20 %.

Table 1

| Vacuum | Matter |
|--------|--------|
| \( \Delta_{23} \) | \( \Delta_{23} \) |
| \( \Delta_{13} \) | \( \Delta_{13} - \frac{aL}{4E} \) |
| \( \Delta_{12} \) | \( \frac{aL}{4E} \) |

| Oscillation Lengths | \( s_{23} \) | \( s_{23} \) |
|---------------------|-----------------|-----------------|
| \( s_{13} \) | \( s_{13} \) |
| \( s_{12} \) | \( \frac{\Delta_{12}}{4} \) | \( s_{12}c_{12} \) |

Long-base-line experiments have large matter effects. They are described in the flavour basis by the effective hamiltonian

\[
H_\nu = \frac{1}{2E} \left\{ \begin{array}{c|c} m_1^2 & 0 \\ \hline m_2^2 & \text{U} \\ m_3^2 & 0 \end{array} \right\} U^+ + \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}
\]

\[
H_{\bar{\nu}} = \frac{1}{2E} \left\{ \begin{array}{c|c} m_1^2 & 0 \\ \hline m_2^2 & \text{U}^T \end{array} \right\} U - \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \tag{5}
\]

where \( U \) is the neutrino mixing matrix and \( a \) is the effective potential of electron-neutrinos with electrons. The mismatch of this charged current electron-flavour interaction induces a relative phase \([3]\) among the electron- and the other neutrinos which is energy independent

\[
\frac{aL}{2E} \simeq 0.58 \times 10^{-3} \left( \frac{L}{Km} \right) ; \quad a = G\sqrt{\Sigma}N_c 2E \tag{6}
\]

with \( N_c \) the number density of electrons in the Earth. We have then the hierarchy \( \Delta m_{23}^2 \lesssim a \gg \Delta m_{12}^2 \). When we diagonalize \( H_\nu \) and \( H_{\bar{\nu}} \) in the \( \Delta_{12} = 0 \) limit, we observe that matter effects break the degeneracy and there is a resonance energy obtained from the condition \( a = \Delta m_{23}^2 \equiv a_R \). The effective mixing \( s_{13} \) in matter is affected by the resonance amplitude for neutrino beams. Although the vacuum mixing \( s_{12} \) is irrelevant in the \( \Delta_{12} = 0 \) limit, the effective mixing matrix in matter becomes determined, with \( U_{e2} = 0 \). This transmutation of the vanishing \( \Delta_{12} \) in vacuum to the vanishing \( U_{e2} \) in matter forbids, in both cases, genuine CP violating effects. Contrary to the “allowed” \( \nu_\mu \rightarrow \nu_\tau \) transition, which is little affected by matter effects, the forbidden transition \( \nu_\mu \rightarrow \nu_e \) in matter has a CP-even probability

\[
P_{\nu_\mu \rightarrow \nu_e} = 4 \left( \frac{s_{13}}{1 - \frac{a}{\Delta m_{23}^2}} \right)^2 s_{23}^2 \sin^2 \left( \Delta_{23} - \frac{aL}{4E} \right) \tag{7}
\]

This result shows both the enhanced probability for neutrinos (suppressed for antineutrinos, for which \( a \rightarrow -a \)), and a modification of the interference pattern with an energy independent phase-shift induced by matter. This quantum-mechanical interference provides a Bohm-Aharonov type experiment able to detect a potential difference between the two “arms” of an interferometer. The interferometer is represented here by the Mixing Matrix, the optical path difference by \( \Delta_{23} \) and the potential by the energy-independent term \( \frac{aL}{2E} \).

Although there are no genuine CP-violating effects in the limit \( \Delta_{12} = 0 \), the medium induces fake CP-odd effects. Even with fundamental CPT invariance, the survival probability of electron-neutrinos in matter gets modified when going to antineutrinos: \( P_{\nu_e \rightarrow \bar{\nu}_e} \neq P_{\bar{\nu}_e \rightarrow \nu_e} \). The corresponding asymmetry is, for this background effect, an even function of \( L \). In order to generate genuine CP-odd effects in matter, one has to allow a non vanishing \( \Delta_{12} \). The results in perturbation theory are given in Table \([3]\) where
$\Delta_{12}$ is only maintained when needed to avoid a zero.

The CP-asymmetry in matter contains then two different terms: one fake component induced by matter asymmetry, which is an even function of $L$, and one genuine component, odd function of $L$, which is a true signal of CP violation (modified by matter effects). Again the CP-odd asymmetry associated with the “forbidden” transition $\nu_\mu \to \nu_e$ in long-base-line experiments is much more promising. The separation of fake and genuine components is possible by using the energy and distance dependence of the asymmetry.

An alternative to the CP-asymmetry is provided by T-odd effects [8]. As matter is, in good approximation, T-symmetric, the T-odd asymmetry does not suffer from fake effects. However, its implementation will need the construction of neutrino factories from muon-storage-rings, able to provide both $\nu_\mu$ and $\nu_e$ beams. Under the conditions discussed above, the corresponding T-odd asymmetry for the “forbidden” process $\nu_\mu \to \nu_e$ is given by

$$A_T \sim -\Delta_{12} \left( 1 + \frac{a}{s_{13} s_{\text{R}}} \right) \frac{\sin \left( \frac{a L}{2E} \right)}{\frac{a L}{2E}}$$

(8)

which can reach again appreciable values. As in vacuum, the asymmetry varies linearly with the crucial parameter $\Delta_{12}/s_{13}$.

If neutrinos were Majorana particles, nothing would change in the discussion of this paper, as long as we discuss only flavour oscillations. The additional Majorana phases [9] do not enter into the relevant Green function $\langle 0 | T \{ \psi(x) \bar{\psi}(0) \} | 0 \rangle$, neither for vacuum oscillations nor for matter [10]. One would need a Majorana propagator $\langle 0 | T \{ \psi(x) \bar{\psi}(0) \} | 0 \rangle$ to be sensitive to these new ingredients. Such a situation would affect the so-called “neutrino-antineutrino oscillations”.

We conclude with the comment that CP and T violation in neutrino oscillations, although possible in appearance experiments involving three neutrino families, will be difficult to observe. With a hierarchical spectrum to explain atmospheric and solar neutrino data, better prospects appear for the forbidden transition $\nu_\mu \to \nu_e$ in long-base-line experiments and the large mixing angle MSW solution to the solar neutrino observation. The CP-odd asymmetry becomes linear in $\Delta_{12}/s_{13}$, the parameter associated with “forbiddeness”. Although matter effects break the degeneracy in (1, 2), the CP-odd asymmetry is still linear in $\Delta_{12}/s_{13}$ as induced by the effective mixings in matter (see Table 1).

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