Enhanced collectivity in neutron-deficient Sn isotopes in energy functional based collective Hamiltonian

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Abstract

The low-lying collective states in Sn isotopes are studied by a five-dimensional collective Hamiltonian with parameters determined from the triaxial relativistic mean-field calculations using the PC-PK1 energy density functional. The systematics for both the excitation energies of \( \gamma^2 \) states and \( B(E2; 0_1^+ \rightarrow \gamma^2_1) \) values are reproduced rather well, in particular, the enhanced \( E2 \) transitions in the neutron-deficient Sn isotopes with \( N < 66 \). We show that the gradual degeneracy of neutron levels \( 1g_{7/2} \) and \( 2d_{5/2} \) around the Fermi surface leads to the increase of level density and consequently the enhanced paring correlations from \( N = 66 \) to 58. It provokes a large quadrupole shape fluctuation around the spherical shape, and leads to an enhanced collectivity in the isotopes around \( N = 58 \).

Keywords: covariant energy density functional, collective Hamiltonian, low-lying states, electromagnetic transition, Sn isotopes

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The evolution of shell structure and nuclear collectivity along either isotopic or isotonic chains, inferred from the systematics of the nuclear deformation energy surface \( \Omega \), the nucleon separation energy \( \Delta \), the excitation energy of low-lying state and the electric quadrupole transition strength \( Q_{\text{JM}} \), becomes of great interest in nuclear physics. In recent years, a number of experiments have measured the \( B(E2; 0_1^+ \rightarrow \gamma^2_1) \) transition strengths in neutron-deficient Sn isotopes with \( N < 66 \). The deduced \( B(E2) \) values increase when going from \( N = 66 \) to \( N = 64 \), and then remains roughly constant within the experimental uncertainties when decreasing the neutron number down to \( N = 56 \). This picture deviates from the parabolic trend, with a peak at mid-shell, predicted by the single \( j \)-shell seniority model \( \text{[13]} \). Furthermore, the \( B(E2) \) values deduced from the measurement of the lifetimes of the first excited \( 2^+ \) states in \( ^{112,114,116}\text{Sn} \) at the UNILAC accelerator of the Gesellschaft für Schwerionenforschung (GSI) are overall smaller than the previously reported values and present a shallow minimum at \( N = 66 \). These findings have generated a renewal interest in the study of low-lying states in Sn isotopes.

The theoretical description of the enhanced \( B(E2; 0_1^+ \rightarrow \gamma^2_1) \) transition strength in the neutron-deficient Sn isotopes is not straightforward. Similarly as the seniority scheme, the seniority truncated large-scale shell model calculations predicted a nearly parabolic behavior of \( B(E2) \) values with a maximum at \( N = 66 \). By using new interactions and including core polarization terms up to both the third order and \( 5\hbar \omega \) core excitations in the shell model calculations, the predicted \( B(E2) \) values exhibit a shallow maximum around \( N = 68 - 70 \), but are still far away from the experimental data \( \text{[10]} \). Shell model calculations in the \( sd\text{gh} \) major shell using CD-Bonn and Nijmegen I two-body effective nucleon-nucleon (NN) interactions were also carried out for the mid-heavy even-even Sn isotopes in Refs. \( \text{[13, 16]} \). An overall good agreement with the observed energy spectra of Sn isotopes is found. However, the \( E2 \) transitions are not presented. Recently, a quasiparticle-phonon model (QPM) was adopted to investigate the evolution of the \( \gamma^2_1 \) state in Sn isotopes \( \text{[17]} \). The calculation reproduced the trend of the energies and, partly, the observed deviations of the \( E2 \) strengths from the parabolic behavior. It was shown that such an asymmetric trend was the consequence of several factors: single-particle energies, polarization of the \( N = Z = 50 \) core, interplay between pairing plus quadrupole, and quadrupole pairing interactions \( \text{[17]} \), all of which, however, are quite model-dependent.

In the framework of nuclear energy density functional theory or self-consistent Hartree-Fock-Bogoliubov (HFB) approach, due to the robust \( Z = 50 \) shell gap, the ground states of all the Sn isotopes are dominated by the spherical configuration. For the \( \gamma^2_1 \) states, which are expected to be a one-phonon state caused by surface vibration around the spherical shape, the theoretical studies are obviously beyond the mean-field approximation. For this purpose, one can either introduce random-phase approximation (RPA) or generator coordinate method (GCM), where the polarization of \( N = Z = 50 \) core in the shell model can be taken into account by particle-hole excitation in RPA or configuration mixing of different shapes in GCM. The implementations of the quasiparticle RPA on top of the nonrelativistic and relativistic mean-field calculations were carried out to
investigate the evolution of low-lying states along the Sn isotopic chain\cite{18, 19, 20, 21}. In Refs.\cite{19, 20}, the systematics of $B(E2; 0^+ \rightarrow 2^+_1)$ values can be roughly reproduced. However, these studies were focused on the substantial rise of the $B(E2)$ values at neutron magic numbers 50 and 82, and did not provide any interpretation for the enhanced collectivity in the neutron-deficient Sn isotopes.

In recent years, the GCM has been introduced into the self-consistent mean-field approaches by combing with projection techniques in the modern energy density functional calculations\cite{22, 23, 24, 25, 26, 27, 28, 29}. The dynamic correlation effects related to the symmetry restoration and quadrupole fluctuation (along both $\beta$ and $\gamma$ directions) around the mean-field minimum could be taken into account. These methods restricted to axial case have been applied to study the low-lying collective excitation spectra of the neutron-deficient Pb isotopes\cite{30, 31}. However, the application of these methods with triaxial degree-of-freedom for systematic study is still much time-consuming. Up to now, such kind of study is mostly restricted to light nuclei\cite{32, 33} and some specific medium heavy nuclei\cite{34, 35}. As the Gaussian overlap approximation of GCM, collective Hamiltonian with parameters determined by self-consistent mean-field calculations is much simple in numerical calculations, and has achieved great success in description of nuclear low-lying states for deformed and transitional nuclei\cite{36, 37, 38, 39, 40, 41, 42}. In this work, we are going to apply this approach to study the low-lying collective states, for the first time, in the semi-magic Sn isotopes. Our aim is to provide a microscopic mechanism for the enhanced $E2$ transitions in the neutron-deficient Sn isotopes.

In the calculation, we use a recent parameterized relativistic functional PC-PK1\cite{50} for the particle-hole channel, and a separable force\cite{51, 52} for the particle-particle channel. The solution of the equation of motion for the nucleons is accomplished by an expansion of the Dirac spinors in a set of three-dimensional harmonic oscillator basis functions in Cartesian coordinates with 12 major shells. More details about the calculations can be found in Ref.\cite{27, 28, 53}. The intrinsic triaxially deformed states are obtained as solutions of the self-consistent RMF+BCS equations constrained on the mass quadrupole moments related to the Bohr parameters ($\beta, \gamma$) varying $\beta \in [0.0, 0.6]$ and $\gamma \in [0^\circ, 60^\circ]$ with step size $\Delta \beta = 0.05$ and $\Delta \gamma = 10^\circ$.

The nuclear collective excitations associated with three-dimensional rotation and quadrupole fluctuations are described with a five-dimensional collective Hamiltonian (5DCH). The dynamics of the 5DCH is governed by the seven functions of the intrinsic deformations $\beta$ and $\gamma$: the collective potential $V_{coh}$, the three mass parameters: $B_{\beta\beta}, B_{\gamma\gamma}, B_{\beta\gamma}$, and the three moments of inertia $I_\beta$. These functions are determined using cranking approximation formula based on the intrinsic triaxially deformed states. The diagonalization of the Hamiltonian yields the excitation energies and collective wave functions that are used to calculate observables\cite{58}.

The upper panel of Fig.\ref{fig:1} displays the excitation energies of $2^+_1$ states for Sn isotopes from our 5DCH calculations, in comparison with those by the shell model with 100\textsuperscript{Sn} core (dashed line), RQRPA (dotted line), and 5DCH based on PC-PK1 density functional (solid line), as well as that by the recent shell model calculations\cite{15, 16}. Although the realization of these two models for excitation states are quite different. Compared with the shell model calculation, the discrepancy between our 5DCH result and the data is relatively larger for $^{120-130}$Sn, from ~ 200 keV to ~ 1 MeV. However, it is worthwhile to note that the 5DCH is fully self-consistent and without any free parameter. In addition, we find that, beside $2^+_1$ state, the excitation energies of other low-lying states for $^{108-116}$Sn are described as well as that by the recent shell model calculations\cite{15, 16}. Taking $^{116}$Sn as an example one finds that the rms deviation for seven low-lying excitation states (the first two $0^+$, three $2^+$, and two $4^+$) is: $\sigma = 0.279$ MeV for the 5DCH, which is in compari-
son with the corresponding value 0.429 MeV obtained by using Nijmegen I effective interaction in Ref. [15], and 0.137 MeV in the calculation of Ref. [16].

In the lower panel of Fig. 1, we present the \( B(E2; 0^+_1 \rightarrow 2^+_1) \) values from 5DCH calculations, in comparison with the results by shell model and RQRPA, as well as the recently measured data. It is shown that both the 5DCH and RQRPA calculations reproduce rather well the systematics of \( B(E2; 0^+_1 \rightarrow 2^+_1) \) values in the whole isotopic chain, in particular the increase of \( E2 \) strength from \(^{116}\text{Sn}\) down to \(^{108}\text{Sn}\), which cannot be described by the shell model. Quantitatively, the RQRPA slightly underestimates those \( E2 \) strengths around \(^{116}\text{Sn}\). While the 5DCH overestimates the \( E2 \) strength \( \sim 0.1 \varepsilon^2 b^2 \) when approaching to the neutron shell closure, where the low-lying states are mainly caused by the excitation of few quasiparticles. Concerning our interest and the power of our model, in the following we will mainly focus on the isotopes around middle shell, where the excitation is dominated by the collective vibration and enhanced \( E2 \) transition is observed.

To understand the enhanced \( E2 \) transitions around \( N = 108 \), we plot the deformation energy curves of the even-even \(^{108-116}\text{Sn}\) isotopes as functions of the axial deformation parameter \( \beta \). Energies are normalized with respect to the binding energy of the corresponding spherical state.

Figure 2: (Color online) Self-consistent RMF+BCS binding energy curves for the even-even \(^{108-116}\text{Sn}\) isotopes as functions of the axial deformation parameter \( \beta \). Energies are normalized with respect to the binding energy of the corresponding spherical state.

Figure 3: (Color online) The predicted curvature of the PEC (panel a) and average neutron pairing gaps (panel b) at spherical point in Sn isotopic chain calculated by PC-PK1 density functional. For comparison, the odd-even mass differences (denoted by solid squares) extracted by the five-point formula are also plotted in panel b.

Figure 4 displays the neutron single-particle energies at spherical shape along Sn isotopic chain. The balls indicate the position of Fermi surface. It is seen that the levels \( 1g_{7/2} \) and \( 2d_{5/2} \) move closer as the neutron number decreases from \( N = 82 \) to \( N = 50 \) shell. Meanwhile, the Fermi surface comes toward to these two levels for the isotopes with \( A < 116 \). This leads to the increasing level density around Fermi surface and consequently enhanced paring correlations. Moreover, it is found that the Fermi surface crosses the \( 3s_{1/2} \) orbit in between \( N = 64 \) and \( N = 70 \). The considerable occupation probability of \( 3s_{1/2} \) orbit hinders the collectivity of these nuclei. Meanwhile, the obstructive effect is further enhanced by the low density of single-particle states around the Fermi surface, which reduces the par correlations (c.f. in Fig. 3). It provides a qual-
In summary, the low-lying collective states in Sn isotopes from $N = 50$ to $N = 82$ have been studied by solving a five-dimensional collective Hamiltonian with parameters determined from the triaxial relativistic mean-field calculations. The calculations reproduce the systematics of both excitation energies for $E2$ transitions from $N = 66$ to $N = 58$ has been found to be the consequence of increasing quadrupole shape fluctuation around spherical shape, induced by the strengthening neutron pairing correlations. This conclusion is consistent with that given in Ref. [14].

In particular, the enhancement of $E2$ transitions from $N = 66$ to $N = 58$ has been found to be the consequence of increasing quadrupole shape fluctuation around spherical shape, induced by the strengthening neutron pairing correlations. This conclusion is consistent with that given in Ref. [14].

Acknowledgments

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