keV Warm Dark Matter via the Supersymmetric Higgs Portal

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Warm dark matter (WDM) may resolve the possible conflict between observed galaxy halos and the halos produced in cold dark matter (CDM) simulations. Here we present an extension of MSSM to include WDM by adding a gauge singlet fermion, $\chi$, with a portal-like coupling to the MSSM Higgs doublets. This model has the property that the dark matter is necessarily warm. In the case where $M_\chi$ is mainly due to electroweak symmetry breaking, the $\chi$ mass is completely determined by its relic density and the reheating temperature, $T_R$. For $10^2 \text{ GeV} \lesssim T_R \lesssim 10^3 \text{ GeV}$, the range allowed by $\chi$ production via thermal Higgs annihilation, the $\chi$ mass is in the range 0.3−4 keV, precisely the range required for WDM. The primordial phase-space density, $Q$, can directly account for that observed in dwarf spheroidal galaxies, $Q \approx 5 \times 10^6 \text{eV/\text{cm}^3}/(\text{km/s})^3$, when the reheating temperature is in the range $T_R \approx 10 – 100 \text{ TeV}$, in which case $M_\chi \approx 0.45 \text{ keV}$. The free-streaming length is in the range 0.3-4 Mpc, which can be small enough to alleviate the problems of overproduction of galaxy substructure and low angular momentum of CDM simulations.

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Introduction: Cold dark matter (CDM) with a cosmological constant (Lambda CDM) is remarkably successful in explaining the large scale structure of the observed universe. Numerical simulations based on the ACDM model predict cusped central densities. The observational situation is less clear. It has been suggested that observed galaxy halos have cores, implying that CDM has a cusp problem. However, the observational evidence has also been interpreted to support cusped halos. Here we will consider the former possibility. One way to solve the cusp problem, should it exist, is to have dark matter with a sufficiently large velocity dispersion, known as warm dark matter (WDM). In addition, CDM simulations result in overproduction of galactic substructure and low angular momentum. Several candidates have been suggested to account for WDM, notably sterile neutrinos in a minimal extension of the Standard Model (MSM) and superWIMPs. These models may also account for other phenomena, such as pulsar velocities and baryogenesis in the case of sterile neutrinos, and can have distinctive collider phenomenology, as in the case of a gravitino superWIMP, which requires large masses $\gtrsim 500 \text{ GeV}$ for the NLSP of the MSSM. The mass of the dark matter candidate in these models is not fixed by the model, in which case the dark matter is not necessarily warm. The unique feature of the model we will present here is that the dark matter candidate is fixed by its relic density to be necessarily warm.

In the case of the MSSM, the only possible WDM candidate is a stable gravitino. (Sterile neutrinos could also play this role in the MSSM extended to include neutrino masses.) However, this is not compatible with the MSSM in the case of gravity-mediated SUSY breaking as the gravitino is too heavy. The form of SUSY breaking may be determined at the LHC from the pattern of SUSY particle masses. Here we present a new WDM candidate, the $Z_2$-singlino, in a portal-like extension of the minimal supersymmetric (SUSY) standard model (MSSM) in which the stability of the WDM particle is ensured by a $Z_2$-parity. The $Z_2$-singlino was previously introduced to provide a stable dark matter candidate in R-parity violating SUSY models. Here we will show that, in a different region of its parameter space, the model provides a dark matter particle which is necessarily warm and with the right properties to explain non-singular galaxy halos and to alleviate the galaxy substructure and low angular momentum problems of CDM models.

The Model: We extend the MSSM by adding a chiral superfield $\chi$ and messenger field $S$ of mass $M_S$. We also impose an additional $Z_2$ symmetry under which $\chi$ is odd, while all other fields are even. The effective superpotential after integrating out $S$ is given by

$$W = W_{\text{MSSM}} + \frac{f \mu H_u H_d}{M_S} + \frac{M_\chi \chi^2}{2}.$$  

Since $\chi$ is odd under $Z_2$, its lightest component cannot decay to any of the MSSM fields. Therefore the fermionic component of $\chi$, the $Z_2$-singlino, is a good candidate for DM. In Eqn. we have included a SUSY mass $M_\chi$ for $\chi$. Since in general the $\chi$ mass must be small relative to the weak scale in order to account for WDM, a particularly interesting case is where the $\chi$ mass is entirely due to the Higgs expectation values, with $M_\chi$, zero or negligibly small. The small $\chi$ mass can then be understood in terms of a large value for $M_S$ compared with the Higgs expectation values. The absence of a $\chi$ mass term in Eqn. guarantees if there is an unbroken R-symmetry which allows the $\mu H_u H_d$ term of the MSSM, since in this case the R-charge of $\chi$ must be zero.

$Z_2$-singlino dark matter is interesting as a SUSY implementation of gauge singlet dark matter. Gauge singlet scalar dark...
matter interacting via the Higgs portal [14] was first proposed in [15] for the case of complex scalars and in [16] for real scalars, and was further discussed in [17, 18, 19, 20, 21]. Couplings to hidden sector particles are currently of considerable interest [22, 23, 24, 25, 26, 27, 28, 29]. The simplicity of the terms in Eqn. (1) should allow them to easily form part of a hidden sector dark matter model.

In the following we will calculate the relic abundance of \( \chi \) as a function of its mass and reheating temperature, \( T_R \). We will show that in the case where the \( \chi \) mass is mostly due to the Higgs expectation value, \( \chi \) is necessarily warm with a mass in the keV range when it accounts for the observed dark matter density. For reasonable values of \( T_R \) the primordial phase-space density can then account for that observed in dwarf spheroidal galaxies (dSphs), while the free-streaming length can damp the density perturbation on small scales and so reduce galaxy substructure formation and angular momentum loss.

**Relic Abundance of \( \chi \):** Production of \( \chi \) will occur mainly through thermal Higgs annihilations\(^2\). We will see that most of the \( \chi \) are produced at temperatures close to the reheating temperature, \( T_R \). In this case we can consider the Higgs expectation values to be zero and calculate with the weak eigenstate Higgs doublets. \( \chi \) production via thermal Higgs boson pair annihilation occurs due to the contact interaction in the Lagrangian \( f \chi \chi H_u H_d / M_S \). The total rate of \( \chi \) production per Higgs pair annihilation is then

\[
\frac{dn_\chi}{dt} + 3 H n_\chi = \Gamma n_H ; \quad \Gamma = 8 n_H \sigma_H v_{\text{rel}},
\]

(2)

In this \( n_H = 2.4 T^3 / \pi^2 \) is the number density of a complex Higgs scalar, \( \sigma_H = (f / M_S)^2 / 16 \pi \) is the cross-section for \( h_u^+ h_d^- / \chi \chi \) (with the same for \( h_u^- h_d^+ / \chi \chi \)) and \( v_{\text{rel}} = 2 \) is the relative Higgs velocity. There is an overall factor of 2 in \( \Gamma \) since each Higgs pair annihilation produces two \( \chi \) particles. In addition, there will be \( \chi \) production via Higgsino-Higgs boson annihilation to \( \chi \chi \). This will increase the \( \chi \) production rate by approximately a factor of 2, which we have included in \( \Gamma \). In terms of \( T \) we obtain

\[
\Gamma_T = \frac{2.4 M_{\chi_{sb}}^2 T^3}{\pi^2 v^4 \sin^2 2\beta},
\]

(3)

where \( \tan \beta = v_u / v_d \) with \( v = \sqrt{v_u^2 + v_d^2} = 174 \text{ GeV} \). \( M_{\chi_{sb}} \) is the contribution to the \( \chi \) mass from electroweak symmetry breaking, \( M_{\chi_{sb}} = f v^2 \sin 2\beta / M_S \). This relates the symmetry breaking contribution to the \( \chi \) mass to the strength of the \( \chi \) interaction with the MSSM Higgs, \( f / M_S \). As a result, the \( \chi \) relic density fixes the \( \chi \) mass in the case where it is mostly due to symmetry breaking. The total \( \chi \) mass is then \( M_\chi = M_\chi^{\text{obs}} + M_{\chi_{sb}} \). Since \( \Gamma_\chi \propto T^3 \) while \( H \propto T^2 \) during radiation-domination, the production of \( \chi \) will occur mostly at the highest temperature during radiation-domination, which is the reheating temperature, \( T_R \). (The temperature can be higher during the inflaton-dominated era before reheating, but since \( H \propto T^4 \) during this era, its contribution to \( \chi \) production is small compared with production at reheating.) We define the thermalization temperature \( T_{th} \) by the condition \( \Gamma_T = H \equiv k T^2 / M_{Pl} \), where \( k = \pi^2 g(T) / 45 \) and \( g(T) \) is the number of relativistic degrees of freedom. Therefore

\[
T_{th} = \frac{k T^3}{2.4 M_{\chi_{sb}}^2 M_{Pl}}.
\]

(4)

Assuming \( g(T) \) is constant, Eqn. (2) can be written as

\[
\frac{d}{dT} \left( \frac{n_\chi}{T^3} \right) = -\frac{n_H}{T_{th}} \frac{1}{T^3},
\]

(5)

where we have used the relation \( \Gamma_T / HT = 1 / T_{th} \), which is generally true during radiation-domination. Integrating this from \( T_R \) to \( T \), and noting that \( n_H / T^3 = 2.4 / \pi^2 \) is a constant, we obtain

\[
\frac{n_\chi(T)}{T^3} = \frac{2.4(T_R - T)}{\pi^2 T_{th}}.
\]

(6)

Clearly most of the production of the comoving \( \chi \) density occurs at \( T \approx T_R \). Including an entropy dilution factor for the change of \( g(T) \) from \( T_R \) to the present CMB temperature, \( T_s = 2.4 \times 10^{-10} \text{ GeV} \), we find for the present \( \chi \) number density,

\[
n_\chi(T_s) = \frac{2.4 T_s^3}{\pi^2} \frac{g(T_s)}{g(T_R)} \frac{T_R}{T_{th}}.
\]

(7)

Using Eqn. (4) to eliminate \( T_{th} \), the relic abundance of \( \chi \) from Higgs annihilations is then given by

\[
\Omega_\chi = \frac{4(1.2)^2}{\pi^3} \left( \frac{M_{\chi_{sb}}}{M_\chi} \right)^2 \frac{g(T_s)}{g(T_R)} \frac{T_R^3}{k T_s v^4 \sin^2 2\beta} \frac{M_{Pl}^2}{\rho_c},
\]

(8)

where \( \rho_c = 8.1 \times 10^{-47} h^2 \text{ GeV}^4 \) is the critical density. Therefore

\[
M_\chi = 1.92 \left( \frac{M_{\chi_{sb}}}{M_\chi} \right)^{2/3} \left( \frac{\Omega_\chi h^2}{0.113} \right)^{1/3} \times \left( \frac{1 \text{ TeV}}{T_R} \right)^{1/3} \frac{1}{\sin^2 2\beta} \frac{\text{keV}}{M_{Pl}}
\]

(9)

where we have used \( g(T_s) = 2 \) and \( g(T_R) = 228.75 \), corresponding to the MSSM degrees of freedom, and we have expressed \( \Omega_\chi \) relative the observed dark matter abundance, \( \Omega_{DM} h^2 = 0.113 \pm 0.0034 \).\(^3\)

From Eqn. (9) we see that in the case where the \( \chi \) mass is due to symmetry breaking, \( M_\chi \) is automatically close to a keV. As a result, the dark matter in this model is necessarily warm.

The relationship between \( M_\chi \) and \( T_R \) for different values of \( \tan \beta \) is shown in Fig. 1 for the case \( M_{\chi_{sb}} = M_\chi \). From

\(^2\) \( \chi \) can also be produced by decay of thermal Higgs particles. However, we find that this is negligible compared with Higgs annihilation except for \( T_R \lesssim 300 \text{ GeV} \) where both processes give a similar contribution to \( n_\chi \). We will therefore neglect the contribution of Higgs decays in the following.

\(^3\) From Eqn. (9) we see that in the case where the \( \chi \) mass is due to symmetry breaking, \( M_\chi \) is automatically close to a keV. As a result, the dark matter in this model is necessarily warm.
this we find that the range of $\Omega$ mass is tightly constrained. $T_R \lesssim 10^4 \text{GeV}$ is necessary in order to avoid thermal gravitino overproduction (for the case with hadronic gravitino decay modes) [31] while $T_R \gtrsim 10^2 \text{ GeV}$ in order to have the thermal Higgs necessary to produce the $\Omega$ density. In the case where the $\Omega$ mass is due to spontaneous symmetry breaking, $M_{\Omega} = M_{\Omega, sb}$, the $\Omega$ mass is in the range

$$0.19 \sin^2\beta \, \text{ keV} \lesssim M_{\Omega} \lesssim 4.1 \sin^2\beta \, \text{ keV}.$$  

(10)

Moreover, $T_R < T_{\text{th}}$ is necessary in order to keep $\Omega$ out of thermal equilibrium, which is assumed in Eqn. (2). This gives $M_{\Omega} \gtrsim 0.3 \text{ keV}$. Thus for reasonable values of $\sin \beta$ we expect $0.3 \text{ keV} \lesssim M_{\Omega} \lesssim 4 \text{ keV}$. The corresponding range of reheating temperature is $10^2 \text{ GeV} \lesssim T_R \lesssim 10^3 \text{ GeV}$.

In this we have used the observed abundance of dark matter as an input, which then determines the mass of the dark matter particle to be of the order of a keV. In doing so, we have implicitly tuned the mass of the dark matter particle for a given $T_R$. To put this tuning in perspective, we can compare the present model with the most popular dark matter scenario, that of thermal relic weakly interacting dark matter. In this case, the dark matter density is broadly of the correct order of magnitude when the dark matter particle mass and its interaction strength are determined by the weak scale. However, although phenomenologically encouraging, theoretically this amounts to an implicit tuning of the physics of thermal freeze-out against the completely unrelated physics of baryogenesis in order to obtain abundances of baryons and dark matter which are within a factor of 6. In other words, the interaction strength, which is determined by its mass scale, must be tuned in order to obtain the correct dark matter abundance relative to the baryon density. In this case the output is a mass for the dark matter particle of the order of a keV. Therefore, from a theoretical point of view, the two models are not dissimilar in their need for tuning. Such tuning is a generic problem for any dark matter model which does not directly address the baryon-to-dark matter ratio.

The Phase-Space Density of $\Omega$: Dark matter with a finite phase-space density may be able to explain the finite density of galaxy cores [32]. The coarse-grained phase-space density is defined by $Q \equiv \langle \rho \chi \rangle / \Sigma_{\Omega \chi}$, where the 1-D velocity dispersion is $\sigma_{\Omega \chi} = (1/3) \langle p^2 \rangle / M_{\Omega \chi}^2$ [32]. If $Q$ is finite then there is a limit on how large $\rho$ can be for a given velocity dispersion, which prevents the formation of singular galaxy cores. $Q$ can be expressed in terms of the distribution of $\Omega$ produced by thermal Higgs annihilation [33]

$$Q \equiv \frac{3^{3/2} M_{\Omega}^2 \rho_{\chi}}{(p_{\chi}^3 / 3)^{1/2}} = \frac{3^{3/2} M_{\Omega}^2 \rho_{\chi} \rho_{\chi} / \sigma_{\Omega \chi}^3}{T_d^3},$$  

(11)

where $T_d = (g(T_d) / g(T_R))^{1/3} T_R$ for a decoupled density created at $T_R$ and $J = \int_0^\infty \chi^2 f(y) dy / (\int_0^\infty \chi^4 f(y) dy)$. Here $f(y)$ is the distribution function for production of $\Omega$ from thermal Higgs annihilation, where $y = (p_{\chi}^2 / T_d)$ is the comoving momentum. Therefore

$$Q = 5.6 \times 10^7 \left( \frac{\Omega_{\Omega \chi} h^2}{0.113} \right) \left( \frac{M_{\Omega}}{1 \text{ keV}} \right)^3 \left( \frac{J}{0.1} \right)^{3/2} \text{ eV/cm}^3 (\text{km/s})^3.$$  

(12)

The distribution function due to thermal Higgs annihilation is not known at present. However, $J$ may be estimated by assuming that the momentum of the $\Omega$ at $T_R$ is equal to the mean momentum of the thermal Higgs particles, $p \approx 3T$, in which case $f(y) = \delta(y - 3)$. This gives $J = 1 / y^2 = 1 / 3^2 = 0.11$. This is in good agreement with the value obtained in the case of thermal Higgs decay, $J = 0.12$ [34]. $Q$ is conserved for an adiabatically contracting collisionless gravitational system [32]. More generally, $Q$ can only decrease from its primordial value during relaxation in collisionless systems. Simulations of structure formation have shown that the phase-space density can decrease by a factor $10^2$ to $10^3$ [35].

If the distribution of dark matter in dwarf spheroidal galaxies (dSphs) is cored, then a synthesis of recent photometric and kinematic dSph data [3], which indicates a mean density $\sim 5 \text{GeV/cm}^3$ and central velocity dispersion $\sim 10 \text{km/s}$, implies that [33]

$$Q_{\text{dSph}} \approx 5 \times 10^6 \text{ eV/cm}^3 (\text{km/s})^3.$$  

(13)

Eqn. (12) then provides an upper limit on $M_{\Omega}$ in the case with an explicit SUSY $\Omega$ mass. If we assume a dynamical suppression of $Q$ by at most a factor $10^3$, then the largest value of $Q$ consistent with observations is $\approx 10^6 \text{eV/cm}^3 / (\text{km/s})^3$. From Eqn. (12) the largest mass compatible with this is $M_{\Omega} \approx 6 \text{ keV}$. This is not much larger than the mass range 0.3-4 keV implied by the relic density in the case with $M_{\Omega} \approx M_{\Omega, sb}$. Since there is no reason to expect a SUSY mass for $\Omega$ in the keV range.
range, it is more natural to assume that $M_{\chi}$ is zero or negligible, with the small $\chi$ mass then generated by electroweak symmetry breaking together with $M_S \gg v$.

Using Eqn. (9) to eliminate $M_{\chi}$ from Eqn.(12), the phase space density of $\chi$ as a function of $T_R$ is given by

$$Q = 4.0 \times 10^8 \sin^2 2\beta \left( \frac{M_{\chi}}{M_{\chi sb}} \right)^2 \times \left( \frac{\Omega_{\text{H}} h^2}{0.113} \right)^2 \left( \frac{1 \text{ TeV}}{T_R} \right) \left( \frac{J}{0.1} \right)^{3/2} \text{eV/cm}^3 \left( \frac{\text{km/s}}{\text{cm}} \right)^3.$$  \hspace{1cm} (14)

Comparing with Eqn. (13), we see that the value of $Q$ can be equal to or larger than $Q_{\text{dSph}}$ for reasonable values of $T_R$. From Eqn. (14) the required reheating temperature is

$$T_R \approx 80 \sin^2 2\beta \left( \frac{Q_{\text{dSph}}}{Q} \right) \left( \frac{M_{\chi}}{M_{\chi sb}} \right)^2 \text{TeV}. \hspace{1cm} (15)$$

Thus in the case where the $\chi$ mass is mostly from the Higgs expectation value and the phase-space density of dSph corresponds to the primordial phase-space density without suppression, 10 TeV $\lesssim T_R \lesssim 100$ TeV is typically required, depending on $\sin 2\beta$. From Eqn. (9) the corresponding $\chi$ mass is $M_{\chi} \approx 0.45$ keV. If the primordial phase space density is suppressed during formation of dSphs, then larger $Q$ and lower $T_R$ are required. In general, for $\sin 2\beta \gtrsim 0.1$ and $10^2$ GeV $\lesssim T_R \lesssim 10^3$ GeV, $Q$ due to $\chi$ dark matter is in the range $10^6 - 10^{10}$ eV/cm$^3$/km/s$^3$.

**Free-streaming length of $\chi$:** The free-streaming length, $\lambda_{fs}$, below which primordial perturbations are suppressed, is roughly equal to the horizon when the $\chi$ particles become non-relativistic. In general, for a distribution of relativistic decoupled particles, $\lambda_{fs}$ is given by $\left[36\right]$

$$\lambda_{fs} \approx 1.2\text{ Mpc} \left( \frac{1 \text{ keV}}{M_{\chi}} \right) \left( \frac{10.75}{g(T_R)} \right)^{1/3} \left( \frac{\langle p/T \rangle}{3.15} \right). \hspace{1cm} (16)$$

where $\langle p/T \rangle$ is the mean momentum over $T$ of the initial relativistic $\chi$ distribution. Since the $\chi$ are produced by annihilation of thermal Higgs at $T \approx T_R$, we expect that their mean momentum will be approximately equal that of the thermal Higgs at $T_R$, such that $\langle p/T \rangle \approx 3$. Thus with $g(T_R) = 228.75$ and $0.3$ keV $\lesssim M_{\chi} \lesssim 4$ keV, $\lambda_{fs}$ is in the range $0.3 - 4$ Mpc. (Smaller values are possible if the mean momentum of the $\chi$ distribution is less than thermal.) The lower end of this range is comparable with the scale of galaxies and so may alleviate the problems of overproduction of substructure and low angular momentum observed in CDM simulations $\left[6,7\right]$.

**Lyman-$\alpha$ constraints:** Observation of Lyman-$\alpha$ absorption spectra constrains the matter power spectrum on small-scales and so provides a lower bound on the WDM particle mass given its momentum distribution $\left[37\right]$. Lower bounds on the mass of sterile neutrino WDM were obtained in $\left[38,39\right]$. Lower bounds were obtained for WDM momentum distribution functions which are generalizations of the Fermi-Dirac distribution. In the case of thermal relics which decoupled while relativistic, a lower bound of 1.7 keV (95% c.l.) was obtained, while for non-resonantly produced sterile neutrinos (less-than-equilibrium density but with a thermal momentum distribution) the corresponding lower bound was 9.5 keV $\left[41\right]$.

Since the distribution function from thermal Higgs annihilation is not expected to be of Fermi-Dirac form, we cannot directly apply the results of existing Lyman-$\alpha$ analyses$^3$. However, as the $\chi$ density from thermal Higgs annihilation is less than the thermal equilibrium density but the mean $\chi$ momentum is of the order of the thermal Higgs momentum at $T \approx m_{\chi}$, we expect the Lyman-$\alpha$ lower bound on the $\chi$ relic density to lie between the lower bounds of $\left[41\right]$, which may allow a window below the 4 keV upper bound from the $\chi$ relic density.

In addition, Lyman-$\alpha$ observations can easily be consistent with WDM in the case where there is a significant ($\gtrsim 40\%$) component of CDM $\left[41\right]$. Since the neutralino is a CDM candidate in our model in the case where there is an unbroken R-parity, mixed dark matter is a possibility. This does not diminish the advantage of a dark matter candidate which is necessarily warm.

**Discussion and Conclusions:** We have shown that the $Z_2$-singlino can account for the observed phase-space density of dwarf spheroidal galaxies, which may be evidence of non-singular cores. In the case where the $\chi$ mass comes entirely from the Higgs expectation value, $M_{\chi}$ is fixed by the $\chi$ relic density. The observed abundance of dark matter implies that $M_{\chi}$ is in the range $0.3 - 4$ keV, which coincides exactly with the range required for $\chi$ to act as WDM. Therefore dark matter is necessarily warm in this model. The model can directly account for the phase-space density of dwarf spheroidal galaxies when $T_R \approx 10 - 100$ TeV, while dynamical suppression of the primordial phase-space density allows smaller values of $T_R$ to be consistent with dSphs. The free-streaming length is in the range $0.3 - 4$ Mpc, the lower end of which may reduce the overproduction of satellites and loss of angular momentum observed in CDM simulations of galaxy formation. The small mass of $\chi$ can be understood in terms of a large messenger mass, $M_S \approx 10^{10}$ GeV. Such a heavy $S$ field might be identified with the messenger sector of gauge mediated SUSY breaking models. We will return to this possibility in a future study $\left[43\right]$.

Depending on $T_R$ and $\sin 2\beta$, a range of primordial phase-space densities can be generated, with $Q \approx 10^5 - 10^{10}$ eV/cm$^3$/km/s$^3$ when $\sin 2\beta \gtrsim 0.1$ and $10^2$ GeV $\lesssim T_R \lesssim 10^3$ GeV. This may allow the wide range of observed values of $Q$, ranging from order $10^6$ eV/cm$^3$/km/s$^3$ in dSphs to $10^4$ eV/cm$^3$/km/s$^3$ or less in normal spiral galaxies $\left[44\right]$, to be understood, for example by having the values of $Q$ in dSphs close to their primordial values and the values in normal spirals suppressed by non-adiabatic evolution during structure formation. It has

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$^3$ Similarly, constraints on the WDM particle mass from globular cluster-based observations of the phase space density of the Fornax dwarf galaxy $\left[42\right]$ cannot be directly applied to the present model.
been suggested that too many dSphs may be generated when the mean primordial $Q$ is equal to that in dSphs $11,45$. In that case a possible solution might be to have a mean primordial $Q$ much smaller than that observed in dSphs, with dSphs then forming from a high phase-space fraction of the $\chi$ particles in the low momentum tail of the distribution $11,45$.

In addition, strong constraints on $\chi$ dark matter may be expected from Lyman-\alpha observations. However, existing constraints on WDM masses cannot be directly applied as the $\chi$ momentum distribution function due to thermal Higgs annihilation differs from those considered in existing studies. We will return to these issues in a future study $43$.

If $R$-parity is unbroken in the MSSM then the model can be extended to a mixed dark matter model, with the $R$-stabilized MSSM LSP providing CDM in addition to the $Z_2$-stabilized $\chi$ WDM. In this case WDM should easily be compatible with Lyman-\alpha constraints.

Testing the model at colliders will be challenging due to the small effective coupling of $\chi$ to the Higgs bosons, with the $\chi\chi$ coupling being of the order of $v/M_q \approx 10^{-8}$. However, the simple form of the superpotential may allow the model to form part of a more complete model which could have distinctive collider signatures.

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[37] V. K. Narayanan, D. N. Spergel, R. Dave and C. P. Ma, arXiv:astro-ph/0005095.

[38] K. Abazajian, Phys. Rev. D 73, 063513 (2006) [arXiv:astro-ph/0512631].

[39] U. Seljak, A. Makarov, P. McDonald and H. Trac, Phys. Rev. Lett. 97, 191303 (2006) [arXiv:astro-ph/0602430].

[40] A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M. Viel, arXiv:0812.3256 [hep-ph]. M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. Lett. 97, 071301 (2006) [arXiv:astro-ph/0605706]. M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. D 71, 063534 (2005) [arXiv:astro-ph/0501562].

[41] A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M. Viel, arXiv:0812.0010 [astro-ph].

[42] L. E. Strigari, J. S. Bullock, M. Kaplinghat, A. V. Kravtsov, O. Y. Gnedin, K. Abazajian and A. A. Klypin, Astrophys. J. 652, 306 (2006) [arXiv:astro-ph/0603775].

[43] J. McDonald and N. Sahu, in preparation.

[44] J. J. Dalcanton and C. J. Hogan, Astrophys. J. 561, 35 (2001) arXiv:astro-ph/0004381.

[45] J. Madsen, Phys. Rev. D 64, 027301 (2001) arXiv:astro-ph/0006074.