Study on Excitation-triggered Damage Mechanism in Perilous Rock

Hongkai Chen1,* and Shengjuan Wang
1Institute of Geotechnical Engineering, Chongqing Jiaotong University, Chongqing 400074, China
E-mail: chkxf@163.com.Tel:023-62982694

Abstract: Chain collapse is easy to happen for perilous rock aggregate locating on steep high slope, and one of the key scientific problems is the damage mechanism of perilous rock under excitation action at perilous rock rupture. This paper studies excitation-triggered damage mechanism in perilous rock by wave mechanics, which gives three conclusions. Firstly, when only the normal incidence attenuation spread of excitation wave is considered, while the energy loss is ignored for excitation wave to spread in perilous rock aggregate, the paper establishes one method to calculate peak velocity when excitation wave passes through boundary between any two perilous rock blocks in perilous rock aggregate. Secondly, following by Sweden and Canmet criteria, the paper provides one wave velocity criterion for excitation-triggered damage in the aggregate. Thirdly, assuming double parameters of volume strain of cracks or fissures in rock meet the Weibull distribution, one method to estimate micro-fissure in excitation-triggered damage zone in perilous rock aggregate is established. The studies solve the mechanical description problem for excitation-triggered damage in perilous rock, which is valuable in studies on profoundly rupture mechanism.

1. Introduction
As one of global far-ranging geological disasters, collapse seriously threatens highways and railways in mountainous areas, mines, urbans and residents. Hitherto, foreign scholars have studied collapse after perilous rock rupture, such as Strom and Korup considered that both perilous rock rupture and rockfall movement are the kinetic process for rock slope evolution [1]. At present, researches gradually focus on rupture of perilous rock at source of collapse, for example, Xuqiang et al. discovered there was key block for Jiweishan avalanche in Chongqing [2]. Based on grey theory and catastrophe theory, Tian Qingyan and Fu Helin established a new model to predict collapse time of rock slope [3]. In view of geostress and rockcell depth, development of perilous rock was discussed by Zhang Yongxing et al [4]. A catastrophe model to determine landslide was established in elastic mechanics by Wang Zhiqiang et al [5]. Frayssines and Hantz discussed failure of limestone slope [6]. One method to determine Poisson’s ratio and elastic modulus of rock at end of dominant fissure in perilous rock had been established by Chen et al [7]. Especially, excitation effect at perilous rock failure is discovered by Chen et al [8], and the effect is significant in degrading stability of perilous rock. This paper is focus on excitation-triggered damage mechanism in perilous rock, which must be facilitation to mechanical rupture of perilous rock.

2. Method to Identify Peak Vibration Velocity Excitation-triggered at Rupture of Perilous Rock
Cases identify that perilous rocks exist in cliff as perilous rock aggregate. In an instant at rupture of any one perilous rock block in the aggregate, excitation wave is produced and passed to surrounding
perilous rock block certainly, and this process belongs to vibration process, but also a energy consumption process. Surfaces among perilous rock blocks are likened to mass rock joint, and perilous rock aggregation can be likened to joint rock mass. Joints belong to interface of wave impedance. When excitation wave passes to interface the wave will produce reflection and transmission of wave (Fig.1), whose transmission path alter and arouse reassignment of wave field energy. Only the situation of excitation wave’s normal incidence is considered in this paper, and takes energy loss of excitation wave in the interface into account.

**Figure1.** Transmission and reflection model for excitation wave to pass through boundary between perilous rock blocks in perilous rock aggregate

Based on equation Zoeppritz, wave will produce four kinds of wave, reflection P-wave, reflection S-wave, transmission P-wave, transmission S-wave to pass the interface (Fig.1). where, SR1, PR1, ST1, PT1 are reflection P-wave, reflection S-wave, transmission P-wave, transmission S-wave, SR12, PR12, ST12, PT12 are reflection P-wave, reflection S-wave, transmission P-wave produced by ST1, SR22, PR22, ST22, PT22 are new reflection P-wave, reflection S-wave, transmission P-wave, transmission S-wave produced by PT1, respectively. This equation expresses the relation between incidence P-wave and S-wave, transmission coefficient and angle of incidence, density and velocity, and determined energy distribution of excitation wave. The equation is as follows,

\[
\begin{bmatrix}
\sin \alpha_1 & \cos \beta_1 & -\sin \alpha_2 & \cos \beta_2 \\
-\cos \alpha_1 & \sin \beta_1 & -\cos \alpha_2 & -\sin \beta_2 \\
\sin 2\alpha_1 & \frac{v_{p1} \cos 2\beta_1}{v_{s1}} & \cos 2\beta_1 & \frac{v_{s1} \sin 2\alpha_2}{v_{p1}} \\
\cos 2\beta_1 & -\frac{v_{s1} \sin 2\alpha_1}{v_{p1}} & -\cos 2\beta_2 & \frac{v_{p1} \cos 2\beta_1}{v_{s1}} \\
\end{bmatrix}
\begin{bmatrix}
R_p \\
R_s \\
T_p \\
T_s \\
\end{bmatrix}
= \begin{bmatrix}
-\sin \alpha_1 \\
-\cos \alpha_1 \\
\sin 2\alpha_1 \\
-\cos 2\beta_1 \\
\end{bmatrix}
\] (1)

in which, \(\alpha_1, \beta_1, \alpha_2, \beta_2\) are normal angles of reflection P-wave, reflection S-wave, transmission P-wave, transmission S-wave on joint plane; \(R_p\) and \(R_s\) are respectively reflection coefficient of P-wave and S-wave; \(T_p\) and \(T_s\) respectively are transmission coefficient of P-wave and S-wave; \(\rho_1\) and \(\rho_2\) are reflection plane density on two sides of medium, kg/m\(^3\); \(v_{p1}\) and \(v_{s1}\) are velocity of incidence P-wave and S-wave, m/s; \(v_{p2}\) and \(v_{s2}\) are velocity of reflection P-wave and S-wave, m/s. In the paper, only normal incidence transmission problem of excitation wave is considered, namely \(\alpha_1=0\), and normal incidence wave coefficient is following by Equation (1),
\[ T_p = \frac{2v_{p1}p_1}{v_{p2}p_2 + v_{p1}p_1} \]  

Excitation wave energy depends on its square of the amplitude by wave theory. Due to the P-wave in vertical incidence doesn’t produce reflection S-wave and transmission S-wave, so parameters are zeros such as reflection P-wave and transmission P-wave’s reflection and transmission angle. For the study of energy distribution characteristics of excitation wave transmission in the interface, transmission coefficient \( E_k \) was defined by according to equal interface normal energy flux density energy principles.

\[ E_k = \frac{Z_2A_2^2}{Z_1A_1^2} \]  

Where, \( Z_2 = V_{p2}p_2; A_2 \) is amplitude of transmission wave; \( Z_1 = V_{p1}p_1; A_1 \) is amplitude of incidence wave.

Excitation wave transmission coefficient can express to the ratio of transmission wave and incidence wave, so

\[ \frac{A_2}{A_1} = \frac{2v_{p1}p_1}{v_{p2}p_2 + v_{p1}p_1} \]  

Put Eq.(4) into Eq.(3), excitation wave energy transmission coefficient expression is follows as

\[ E_k = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2} \]  

For example, \( E_k \) of marble is 30.8281 by Ju Yang’s studies, and \( E_k \) of granite is 30.7088, elastic strain energy expression \( \overline{W} \) produced sudden failure of perilous rock, i.e.,

\[ \overline{W} = \int_0^h \frac{M^2(x)}{2EI} dx + \frac{\gamma^2 l^2 l_1 bh}{2G} + \frac{\sigma^2 ebl_2}{2E} \]  

Where, \( h \) and \( b \) are height and width of perilous rock respectively, (m); \( A = bh, \) (m\(^2\)); \( E \) is elastic modulus, (kPa); \( I \) is inertia moment of cross section for perilous rock, (m\(^4\)); \( M \) is bending moment, (kN-m); \( l_1 \) is length for perilous rock 1#, (m); \( l_2 \) is length for perilous rock 2#, (m); \( e \) is full water depth of dominant fissure, (m); \( \gamma \) is fissure water of dominant fissure, (kPa); \( G \) is shear modulus, (kPa); \( \gamma \) is unit weight, (kN/m\(^3\)).

After excitation wave transmit the interface, formula \( W_f \) for transmitted wave energy is as followsings.

\[ W_f = E_k \overline{W} = E_k \left[ \int_0^h \frac{M^2(x)}{2EI} dx + \frac{\gamma^2 l^2 l_1 bh}{2G} + \frac{\sigma^2 ebl_2}{2E} \right] \]  

Where, units of \( \overline{W} \) and \( W_f \) are KJ; the others of the variable physical meaning is the same as above.

Further peak vibration velocity formula was deduced by rest energy from excitation wave transmitted the interfaces.

\[ V_f = \sqrt{\frac{2vE_k}{EA}} \left[ \int_0^h \frac{M^2(x)}{2EI} dx + \frac{\gamma^2 l^2 l_1 bh}{2G} + \frac{\sigma^2 ebl_2}{2E} \right] \]  

where, \( V_f \) is peak vibration velocity of excitation wave to cross the interface, (m/s); \( v \) is propagation velocity of excitation wave, (m/s); The others is the same as above.

Similarly, peak vibration velocity for perilous rock triggered by excitation wave to cross interface between anyone of perilous rock blocks, and with the damage criterion excitation damage scope from failure of perilous rock can be determined preliminarily.
3. Damage Criterion for Perilous Rock under Excitation Effect

Excitation effect from rupture of perilous rock is likened to vibration load produced by miniature blasting in structural plane of jointed rock mass. Peak vibration velocity is available to confirm damage scope to adjacent perilous rock under excitation effect. For acquiring damage threshold of perilous rock on the basis of particle peak vibration velocity, this paper used two kinds of damage criterion\[9\], Sweden criterion and Canmet criterion, which respectively confirmed maximum and minimum peak vibration velocity for perilous rock.

Sweden criterion adopted upper limit threshold under the condition of rock stretch to determine maximum peak vibration velocity that rock can bear:

\[ V_{\text{max}} = \frac{\sigma_T v}{E} \quad (9) \]

in which, \( V_{\text{max}} \) is the maximum peak vibration velocity that rock can bear before rock broke, m/s. \( \sigma_T \) is uniaxial tension strength, kPa. The rest of the variable meaning is the same as above.

Canmet criterion obtained minimum peak vibration velocity that rock can bear on the basis of lower limit threshold value of damage degree under weak blasting, and its expression is as follows,

\[ V_{\text{min}} = \frac{0.021 \sigma_T}{v \rho} \quad (10) \]

in which, \( V_{\text{min}} \) is the minimum peak vibration velocity that protogenesis crack of perilous rock expanded, m/s. \( \sigma_T \) is uniaxial tension strength, kPa. The rest of the variable meaning is the same as above.

It is thus clear that damage criterion of unstable under excitation effect is,

For \( V_T < V_{\text{min}} \), excitation effect from rupture of perilous rock to damage and break effect to adjacent perilous rock can be neglected;

For \( V_{\text{min}} \leq V_T < V_{\text{max}} \), excitation effect from rupture of perilous rock has damage effect to adjacent perilous rock;

For \( V_{\text{max}} \leq V_T \), excitation effect from rupture of perilous rock has destructive effect to adjacent perilous rock.

4. Microcosmic Crack Estimation Method in the Damage Scope by Excitation Effect from Rupture of Perilous Rock

Perilous rock is formed by brittle rock blocks, such as sandstone, marlstone, limestone, which develops a lot of microcrack and microfissure. Excitation wave produced by abrupt failure of perilous rock inevitably arouses microcrack and microfissure to be nucleus, grow up and expand. Futher, mechanical property decayed, even perilous rock block broke. According to studies of Grady and Kipp, crack density is the ratio of rock volume for influenced crack scope and total rock volume, and numbers of fissure and crack activate obey two-parameter Weibull distribution of volume tensile strain:

\[ C_d = \beta N a^3 \quad (11) \]

where, \( C_d \) is crack density; \( \beta \) is coefficient, approximately equals to 1; \( N \) is the number of microfissure or microcrack excited; \( a \) is characteristic length of microfissure or microcrack under action of excitation, can be calculated by the following formula,

\[ a = \frac{1}{2} \left[ \frac{\sqrt{20 K_{IC}}}{\rho v \theta_{\text{max}}} \right]^{2/3} \quad (12) \]

where, \( K_{IC} \) is fracture toughness for intact rock of perilous rock, (kN.m\(^{3/2}\)). \( \rho \) is rock density, (kg/m\(^3\)), \( v \) is velocity of excitation wave, (m/s), \( \theta_{\text{max}} \) is maximum volume tension strain rate, (s\(^{-1}\)).

Damage variable \( D \) was defined by elastic modulus of medium as followings:
where, £ is initial elastic modulus for rock, (MPa), £ is effective elastic modulus after perilous rock damaged, (MPa), Yu Shouwen and Feng Siqiao [14] suggested the following formula to calculate the parameter.

\[
\hat{E} = \frac{E}{1 + \frac{16(1 - \mu^2)(10 - 3\mu)}{45}}
\]

(14)

in which, \(\mu\) is poisson ratio of perilous rock.

Combining Eq. (12), Eq. (13) and Eq. (14), and \(\beta=1\), expression of damage variable \(D\) becomes,

\[
D = 1 - \left[1 + \frac{8(1 - \mu^2)(10 - 3\mu)}{9} - \frac{K_{IC}}{\rho \theta_{max}}\right]^{-\frac{1}{2}}
\]

(15)

In addition, based on \(V_{max}\) that rock can bear before rock broke and \(V_{min}\) expanded by original fissure damage variable expression was defined as follows:

\[
D = \frac{V_T - V_{min}}{V_{max} - V_{min}}
\]

(16)

Vibration velocity equation \([9]\) of perilous rock under excitation was obtained as formula (17).

\[
C = \sqrt{\frac{2\nu}{EAT}} \left[ \int_{0}^{l} \frac{M^2(x)}{2EI} dx + \frac{\gamma^2 l^2 bh}{2G} + \frac{\sigma^2 ebl_2}{2E} \right]
\]

(17)

Where, \(C\) is vibration velocity of perilous rock under excitation effect: the rest of physical meaning of variable is the same as above.

Combining Eq. (15), Eq. (16) and Eq. (17),

\[
D = \frac{\sigma_T \nu}{E} - \frac{21\sigma_T}{1000\nu\rho}
\]

(18)

Numbers of microfissure of left perilous rocks likely excited under excitation can be estimated by formula (19).

\[
N = \frac{8\sigma_T \nu}{9} \sqrt{\frac{2\nu}{EAT}} \left[ \int_{0}^{l} \frac{M^2(x)}{2EI} dx + \frac{\gamma^2 l^2 bh}{2G} + \frac{\sigma^2 ebl_2}{2E} \right] \left[1 - \frac{K_{IC}}{\rho \theta_{max}}\right]^{2}
\]

(19)

5. Case Analysis

Cliff of YangCha River is located in south of QiJiang in Chongqing of Three Gorges Reservoir, lies in core of anticline, is composed of limestone and mudstone, and presents remarkable geomorphology of footstep state. Dispersion of cliff is between 34 m and 36 m. Stratum is composed of interactive limestone and mudstone, and thickness of lower mudstone is about 20 m with 4.3 m limestone placing in between. About 5 m high and 3 m deep rock cell develops in Mudstone outcropped place above the limestone, which is a relieved eroded platform with about 25 m wide, and SongZao mining area
railway across the platform. Bottom of perilous rock zone is weak foundation bed composed of mudstone. Due to mudstone’s weak resistance to weather, concave rock cell usually develops in cliff. Upper perilous rock is mainly close vertical limestone mass, and its physics and mechanical parameters are listed in table 1. Geometric dimensions of perilous rocks W10 is 5m(height)×5m(width)×4m(length), and length of dominant fissure is 2.5 m.

\[ V_{\text{min}} \] for Yangcha River perilous rock is 2.9 mm/s, and \[ V_{\text{max}} \] is 68.1 mm/s combining with Eq. (9) and Eq. (10). Peak vibration velocity of perilous rock deduced by rupture of perilous rock is 27.7 mm/s, and it is greater than \[ V_{\text{min}} \], which clearly states that excitation wave from rupture of perilous rock could damage adjacent perilous rock. With the fractured rock mass physical and mechanical parameters in the joint surface rather hard to obtain, this paper takes energy transmission coefficient 0.6 crossing jointed rock mass referring to SHPB test conducted by Ju Yang etc, and put 0.6 into Eq. (8) peak vibration velocity of perilous rock W11 is 2.9 mm/s; Microcrack numbers of each perilous rock can be obtained by Eq. (19) in the damage scope. For example, excitation wave excited 38 microcracks with excitation vibration source crossing the interface between perilous rock W10 and W11.

| Table 1. Physical and mechanical parameters of perilous rocks in YangCha River |
|---|
| category | Unit weight (kN/m³) | Elastic modulus (10⁴ MPa) | Tension strength of rock (MPa) | Tension strength of rock mass (MPa) | Maximum volume tension strain rate (l/s) | Poisson ratio | Fracture toughness (kPa√m) |
| --- | --- | --- | --- | --- | --- | --- | --- |
| limestone | dry | 24.8 | 25.2 | 0.52 | 2.60 | 0.52 | 0.03 | 0.21 | 1600 |
| mudstone | natural | 25 | 25.4 | 0.31 | / | / | / | 0.32 | 1300 |

![Figure 2. The W10, W11 and W12 perilous rock in the middle reaches of Yangcha river basin](image)

6. Conclusion
Firstly, pay attention to the truth of excitation effect at rupture of perilous rock in the aggregate, the authors establish one method to calculate peak excitation velocity when excitation wave pass interface between perilous rock blocks by wave mechanics.

Secondly, based on the rule of Sweden and the rule of Canmet, wave velocity damage criteria for perilous rock excitation effect was proposed.

Thirdly, Abide by the assumption of bi-parameters Weibull distribution function for volume tension strain of cracks in rock, crack estimation method in the damage scope by excitation effect from rupture of perilous rock is established.

7. Acknowledgements
This paper was funded by the National Natural Science Foundation of China (51678097, 51378521), the innovative team building programs of Chongqing University in 2016 (CXTDG201602012), and
the Par-Eu Scholars Program of Chongqing(201309).

8. Reference
[1] Strom, A.L. and Korup, O. Extremely large rockslides and rock avalanches in the Tien Shan mountains 2006. Kyrgyzstan. Landslides, 3(2):125-136.
[2] XU Qiang, Huang Runqiu, Yin Yueping, et al. The Jiweishan landslide of June 5, 2009 in Wulong Chongqing: characteristics and failure mechanism 2009. Journal of engineering geology, 17(4):433-444.
[3] Tian Qinyan, Fu Helin. Failure time prediction of slope collapse of block rockmass based on gray and catastrophic theories 2009. Journal of south china university of technology (natural science edition), 37(12):122-126.
[4] Zhang Yongxing, Lu Li, Zhang Siping, et al. Development and failure principle of differential weathering overhanging rock 2010. Journal of Civil, Architectural & Environmental engineering, 32(2):1-6.
[5] Wang Zhiqiang, Wu Minying, Pan Yue. Fold catastrophe model of slope destabilization and its starting velocity 2009. Journal of China University of Mining & Technology, 38 (2):175-181.
[6] Frayssines M. and Hantz, D. Modelling and back-analysing failures in steep limestone cliffs 2009. International Journal of Rock Mechanics and Mining Sciences, 46(7):1115-1123.
[7] Chen Hongkai, Tang Hongmei, Ye Siqiao. Damage model of control fissure in perilous rock 2006. Applied Mathematics and Mechanics, 27 (7):967-974.
[8] Chen Hongkai, Tang Hongmei, Wang Zhi, et al. Study on frequency domain characteristics of excitation signals at rupture of perilous rock 2014. Journal of Vibration and shock, 33(19):64-68.