Phase shift in an atom interferometer induced by the additional laser lines of a Raman laser generated by modulation

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The use of Raman laser generated by modulation for light-pulse atom interferometer allows to have a laser system more compact and robust. However, the additional laser frequencies generated can perturb the atom interferometer. In this article, we present a precise calculation of the phase shift induced by the additional laser frequencies. The model is validated by comparison with experimental measurements on an atom gravimeter. The uncertainty of the phase shift determination limits the accuracy of our compact gravimeter at $8 \times 10^{-8}$ m/s$^2$. We show that it is possible to reduce considerably this inaccuracy with a better control of experimental parameters or with particular interferometer configurations.

I. INTRODUCTION

Light-pulse atom interferometry \cite{1} is a promising technology to obtain highly sensitive and accurate inertial sensors. Laboratory experiments have already demonstrated state of the art performances for gravimeter \cite{2,3}, gradiometer \cite{4} and gyroscope \cite{5,6}. Inertial sensors start to be tested in mobile platforms such as a plane \cite{7} or a truck \cite{8}. An important research effort has still to be done in order to have instruments and particularly the laser system more compact and robust for practical applications like navigation \cite{9}, space mission \cite{10–13}, gravity mapping and monitoring \cite{14}, subsurface detection \cite{15}.

The key point of light-pulse atom interferometer is the stimulated Raman transition between two stable states of the atom \cite{16}. This transition is driven with two counter-propagating lasers with a frequency difference corresponding to the energy difference between the stable states. This transition allows to make the equivalent of beam splitters and mirrors for matter waves. To generate this Raman laser, two phase locked lasers are generally used \cite{17}. Another solution consists in a laser which is intensity or phase modulated at the frequency difference between the ground states. This technology gives a laser system more compact (only one laser is needed) and more robust (no phase lock). Moreover, the frequency noise of the laser is not reported into the Raman phase noise \cite{18}. The drawback of this technique is the presence of additional laser frequencies which can perturb the atom interferometer. The realization of an atom interferometer using a modulated Raman laser has been demonstrated experimentally \cite{17,19} but with an additional phase shift which have not been quantified yet.

In this article, the perturbations induced on an atom interferometer by the additional frequencies present in a modulated Raman laser are studied in detail. In the first part, we show a precise calculation of a stimulated Raman transition with a modulated Raman laser. In the second part, we calculate for a Mach-Zehnder interferometer the phase shift induced by the additional laser frequencies. In a third part, we compare the results of our calculation to experimental measurements of the phase shift. Finally, we present the limitation in accuracy of an atom interferometer using a modulated Raman laser. We also show a configuration where the inaccuracy induced by the additional laser frequencies is considerably reduced.

II. STIMULATED RAMAN TRANSITION WITH A MODULATED LASER

The atom corresponds to a Λ-type three-level system with two lower levels $|a\rangle$ and $|b\rangle$ separated by an energy $\hbar G$ and an excited state $|i\rangle$ separated from the level $|b\rangle$ by $\hbar \omega_0$ (see Fig. 1). The atom interacts with a laser retro-reflected by a mirror at the position $z_M$. The spectrum of the laser is composed of lines separated in frequency by $\Delta \omega$. This kind of spectrum is obtained with a laser modulated in amplitude or in phase with a frequency $\Delta \omega$. The electric field seen by the atom can be written as:

$$E = \sum_{n=-\infty}^{\infty} E_n \cos ((\omega_L + n \Delta \omega) (t - z/c) + \varphi_n)$$

$$+ E_n \cos ((\omega_L + n \Delta \omega) (t + (z - 2z_M)/c) + \varphi_n)$$

(1)

When the frequency of modulation $\Delta \omega$ is close to the frequency difference of the two lower levels $G$, each couple of laser lines separated by $\Delta \omega \simeq G$ drives stimulated Raman transitions between the two lower states (see Fig. 1). In this article, one considers only two-photon Raman transition with counter-propagating beams with the higher frequency beam propagating downward (see Fig. 2). If the velocity of the atoms is big enough, the other kinds of Raman transition (co-propagating and counter-propagating with opposite direction) are out of resonance thanks to the Doppler effect and can be neglected. The couple of counter-propagating beams with frequencies $\omega_L + (n + 1) G$ and $\omega_L + n G$ is coupling the state $|a, p\rangle$ to the state $|b, p + \hbar k_{\text{eff}} + n \hbar \Delta k\rangle$ where $k_{\text{eff}} = (2 \omega_L + \Delta \omega)/c$ and $\Delta k = 2 \Delta \omega/c$. The effective Rabi frequency $\Omega_n$ as-

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associated with this transition is equal to:
\[ \Omega_n = \frac{\Omega_{n+1} a_i \Omega_{n+1} b_i}{2(\Delta + n \Delta \omega)} = \frac{E_{n+1} E_n}{E_1 E_0} \frac{\Delta}{\Delta + n \Delta \omega} \Omega_0 \]
where \( \Delta = \omega_L - \omega_0 \) and \( h \Omega_{nxi} = -\langle i | d_{xi} E_n | x \rangle \) with \( x = a \) or \( b \).

![FIG. 1. Atomic level system and Raman transitions.](image1)

![FIG. 2. Raman transition with a retro-reflected modulated laser.](image2)

The calculation of the transition matrix will be done within the following approximations. The kinetic energy difference induced by the term \( \Delta k \) will be neglected. In typical atom interferometer (\( G/2\pi \lesssim 10 \text{ GHz} \), \( v \lesssim 1 \text{ m/s} \)), the Doppler effect associated with \( \Delta k \) is below 67 Hz and can be neglected for typical interaction time of 10 \( \mu \text{s} \). We will therefore consider that the states \( |a, n\rangle \) together with the states \( |b, n\rangle \) are degenerated in energy. We also suppose that the Raman laser is in resonance i.e. the frequency of modulation \( \Delta \omega \) is equal to the frequency difference between the states \( |a, n\rangle \) and the states \( |b, n\rangle \) eventually affected by the light shift. Within this approximation and by adiabatically eliminating the excited state and using the rotating wave approximation, the system can be described by the following effective Hamiltonian:

\[ H = \frac{\hbar}{2} \sum_n \sum_m \Omega_m e^{i \phi_m} |b, n + m\rangle \langle a, n| + \Omega_m e^{-i \phi_m} |a, n - m\rangle \langle b, n| \]

where \( \phi_m \) is the phase of the Raman laser associated with the couple \( m \) of Raman laser:

\[ \phi_m = \varphi_m - \varphi_{m+1} - (k_{\text{eff}} + m \Delta k)z_M + \frac{\Delta k}{2}z_M \]

If a free fall frame is used, one should replace in this expression \( z_M \) by \( z_M - \frac{1}{2}g t^2 \). To compensate the Doppler shift induced by gravity, one generally applies a frequency chirp \( \alpha \) to the Raman frequency. In this case, one should add to the phase \( \phi_m \) the term \(-\alpha t^2/2\). The terms \( \Delta k z_M \) and \( k_{\text{eff}} z_M \) are constant and can be ignored in the following. If the intensity or phase modulated laser propagates through a dispersive medium, \( \varphi_{m+1} - \varphi_m \) is not null and is proportional to \( m \). This term is equivalent to a modification of the mirror distance \( z_M \) and can be ignored. Finally, the expression of \( \phi_m \) can be written into a term independent of \( m \) and a term proportional to \( m \):

\[ \phi_m = A + mB \]

with

\[ A = \frac{1}{2} (k_{\text{eff}} g - \alpha) t^2 \]

\[ B = \Delta k \left( \frac{1}{2} g t^2 - z_M \right) \]
The evolution operator of the atom interacting with the Raman laser is given by:

$$U = e^{-i \frac{B}{\hbar} t}$$  (7)

One can show that the elements of the evolution operator can be written as:

$$\langle a, n + m | U | a, n \rangle = t_m e^{i m B}$$
$$\langle b, n + m | U | b, n \rangle = t_{-m} e^{i m B}$$
$$\langle b, n + m | U | a, n \rangle = -i r_m e^{i A} e^{i m B}$$
$$\langle a, n + m | U | b, n \rangle = -i r_{-m} e^{-i A} e^{i m B}$$  (8)

The coefficient $r_m$ and $t_m$ are real and can be calculated numerically by truncating the number of states coupled and the number of laser lines. Typically, we perform the calculation with 22 quantum states and 5 laser lines. The calculation of the transition amplitude is done for the interaction times: $\tau = \frac{\pi}{2} \times t_m$ and $\tau = \frac{\pi}{2} \times t_m$ corresponding to $\pi/2$ and $\pi$ pulses without additional laser lines (beam splitter and mirror). The result of the calculation is presented in Table I for the case of stimulated Raman transition on Rubidium 87.

We can notice that the probability transition into parasite states ($n \neq 0$) is small but not negligible. We should therefore take into account the additional paths engendered by these parasite transitions in order to determine precisely the phase of an atom interferometer.

| $n$ | $t_n^+$ | $r_n^+$ | $t_n^-$ | $r_n^-$ |
|-----|---------|---------|---------|---------|
| -2  | $\sqrt{1.1 \times 10^{-3}}$ | $-\sqrt{8.3 \times 10^{-3}}$ | $\sqrt{1.3 \times 10^{-4}}$ | $-\sqrt{1.5 \times 10^{-4}}$ |
| -1  | $\sqrt{1.2 \times 10^{-4}}$ | $-\sqrt{3.2 \times 10^{-4}}$ | $\sqrt{9.2 \times 10^{-4}}$ | $-\sqrt{1.1 \times 10^{-4}}$ |
| 0   | $0.0497$ | $0.0497$ | $-0.30 \times 10^{-6}$ | $0.979$ |
| 1   | $\sqrt{1.2 \times 10^{-3}}$ | $-\sqrt{1.5 \times 10^{-4}}$ | $\sqrt{9.2 \times 10^{-4}}$ | $9.1 \times 10^{-4}$ |
| 2   | $\sqrt{1.1 \times 10^{-3}}$ | $-\sqrt{1.8 \times 10^{-4}}$ | $\sqrt{1.3 \times 10^{-4}}$ | $1.5 \times 10^{-4}$ |

TABLE I. Transition amplitude with a phase modulated Raman laser with $G/2\pi = 6.8$ GHz, $\Delta/2\pi = 0.7$ GHz, $E_n = J_n(1.25)$ where $J_n$ is the Bessel function of the first kind of order $n$.

III. PHASE SHIFT IN A MACH ZEHNDER INTERFEROMETER INDUCED BY THE ADDITIONAL LASER LINES OF A MODULATED RAMAN LASER

In this section, we present the calculation of a Chu-Bordé interferometer using a modulated laser for stimulated Raman transition. This interferometer consists of three Raman laser pulses of duration $\tau$, $2\tau$ and $\tau$ ($\Omega_0 \tau = \pi/2$) separated in time by $T$. This interferometer is equivalent to an optical Mach Zehnder interferometer where the first and last pulses act as beam splitters and the second pulse acts as a mirror. The evolution operator of this interferometer is given by:

$$U = U_3 \cdot U_L \cdot U_2 \cdot U_L \cdot U_1$$  (9)

where $U_n$ is the evolution matrix for the $n$th laser pulse which has been calculated in the previous section and $U_L$ is the free evolution during the time $T$. The free falling frame will be used for the calculation. The gravitational potential is therefore not included in the free evolution and one has only the internal energy and the kinetic energy:

$$U_L = \sum_n e^{-i \omega_n T} |a, n\rangle \langle a, n| + e^{-i \omega_n T} |b, n\rangle \langle b, n|$$  (10)

with:

$$\omega_{an} = \frac{(p + n \hbar \Delta k)^2}{2\hbar M}$$
$$\omega_{bn} = \frac{(p + n \hbar \Delta k + \hbar k_{eff})^2}{2\hbar M} + G$$  (11)

where $M$ is the mass of the atom.

We assume that the atom is initially in the internal state $|a\rangle$ and has a momentum probability amplitude $\varphi(p)$:

$$|\Psi_0\rangle = \int \varphi(p) |a, p\rangle dp$$  (12)

The momentum probability amplitude $\varphi(p)$ will be modeled by a gaussian wave packet with a mean momentum $p_0$, a width given by the temperature of the atoms $T_a$ and centered at the position $z = 0$.

$$\varphi(p) = \frac{1}{(2\pi \hbar)k_B T_a}^{1/4} \exp \left(-\frac{(p - p_0)^2}{4 \hbar k_B T_a}\right)$$  (13)

After the interferometer sequence, the probability to be in the state $|a\rangle$ is given by:

$$P_a = \int |\langle a, p | U | \Psi_0\rangle|^2 dp$$  (14)

By inserting equation [12] into the expression of $P_a$ and using the family of quantum states coupled by $U$ (see Fig. 3), one obtains:

$$P_a = \int \left| \sum_n \varphi(p + n \hbar \Delta k) \langle a, p | U | a, p + n \hbar \Delta k \rangle \right|^2 dp$$  (15)

We assume now that the atomic coherence length $l_c = \hbar/\sqrt{\hbar} k_B T_a$ is small compared to the microwave wavelength $2\pi/\Delta k$. This approximation is completely valid for atoms at a temperature of $\sim 1 \mu K$ ($l_c \sim 0.5 \mu m$) and with $G/2\pi \lesssim 10$ GHz ($2\pi/\Delta k \gtrsim 1.5$ cm). Within this approximation, one can write $\varphi(p + n \hbar \Delta k) \approx \varphi(p)$ and one obtains:

$$P_a = \int \left| \sum_n \langle a, p | U | a, p + n \hbar \Delta k \rangle \right|^2 |\varphi(p)|^2 dp$$  (16)
By decomposing $U$ in free evolution and laser interaction (Eq. (9)), one obtains:

$$P_a = \int |\varphi(p)|^2 |C_{ab}|^2 + |C_{ba}|^2 + |C_{aa}|^2 + |C_{bb}|^2 \, dp$$

(17)

with:

$$C_{ij} = \sum_{n',n,m} e^{-i(\omega_{j,n} + \omega_{i,m})} T \langle a, 0 | U_3 | j, n' \rangle \langle j, n' | U_2 | i, n \rangle$$

$$\langle i, n' | U_3 | a, n \rangle$$

(18)

In this expression, $C_{ij}$ represents the probability amplitude of an atom to follow the path $a \rightarrow i \rightarrow j \rightarrow a$ where $i,j = a$ or $b$. We will consider only the interference between the term $C_{ab}$ and $C_{ba}$. The interferences with the terms $C_{aa}$ and $C_{bb}$ vanish if the path separation $D = h k_{\text{eff}} T / M$ is much bigger than the coherence length of the atoms. This assumption is perfectly verified in practical situation with $T > 1$ ms ($D > 100 \mu$m) and atoms at a temperature $T_a \sim 1 \mu K$ ($\ell_c \sim 0.5 \mu$m). We will consider therefore only the interferences between the term $C_{ab}$ and $C_{ba}$ and the expression of $P_a$ becomes:

$$P_a = \int |\varphi(p)|^2 (|C_{aa}|^2 + |C_{bb}|^2 + |C_{ab}|^2 + |C_{ba}|^2 + C_{ab} \cdot C_{aa}^* + C_{ab}^* \cdot C_{ba}) \, dp$$

(19)

By inserting the equations (18), (11) and (8) in the previous expression and neglecting the phase terms proportional to $h k_{\text{eff}} / 2 M T$ ($\sim 10^{-6}$ for $^{87}$Rb and $T = 48$ ms), we obtain:

$$P_a = P_0 + \frac{C}{2} \cos((k_{\text{eff}} g - \alpha) T^2 + \Delta \phi)$$

(20)

where $P_0$ is the mean value of $P_a$, $C$ is the contrast and $\Delta \phi$ is the phase shift induced by the additional laser lines.

$$P_0 = \sum_{m,m',m''} \frac{h k_{\text{eff}}^2}{2 M} \sum_{m,m',m''} T_{m,m'} T_{m''} e^{-i(\omega_{m} + \omega_{m''})^2} e^{i \Delta k(m z_A + m' z_B + m'' z_C + m'' z_D)}$$

(21)

$$+ \sum_{m,m',m''} T_{m,m'} T_{m''} e^{-i(\omega_{m} + \omega_{m''})^2} e^{i \Delta k(m z_A + m' z_B + m'' z_D)}$$

$$+ \sum_{m,m',m''} T_{m,m'} T_{m''} e^{-i(\omega_{m} + \omega_{m''})^2} e^{i \Delta k(m z_A + m' z_C + m'' z_E)}$$

$$+ \sum_{m,m',m''} T_{m,m'} T_{m''} e^{-i(\omega_{m} + \omega_{m''})^2} e^{i \Delta k(m z_A + m' z_B + m'' z_E)}$$

In this expression, we constat that the additional laser lines are responsible for a decrease of the interferometer contrast. For practical case, this contrast diminution is small ($\sim 3\%$) and does not affect the performance of the inertial sensor. We find also that the phase of the interferometer is modified. Without additional laser lines, we obtain the classical result $(k_{\text{eff}} g - \alpha) T^2$. The presence of the additional lines gives an additional phase shift $\Delta \phi$ which induces if not corrected an error on the acceleration measurement. This phase shift depends on the parameters of the atom interferometer: $T$, $p_0$, $T_a$, $E_a$, $z_m$. In the expression (21), one can see that the phase shift has a periodic dependence with the mirror position $z_m$ and the initial velocity $p_0 / m$ with respectively a period of $2 \pi / \Delta k$ and $2 \pi / (\Delta \phi / T)$.

With the expression (21), it is possible to numerically calculate the phase shift induced by the additional laser lines. For a $^{87}$Rb atom interferometer with the parameters $T = 48$ ms and $\Delta / 2 \pi = 0.7$ GHz, the phase shift is between $\pm 160$ mrad depending on the mirror position $z_m$. This phase shift corresponds to an acceleration of $\pm 4 \times 10^{-6} \text{ m/s}^2$ has to be evaluated precisely in order to reach the state of the art accuracy of $\sim 10^{-8} \text{ m/s}^2$.

And consequently, the parameters needed in the calculation of the phase shift have to be known precisely.

IV. SIMPLIFIED EXPRESSION OF THE PHASE SHIFT IN THE LIMIT OF LOW TEMPERATURE

It is possible to simplify the expression of the additional phase $\Delta \phi$ by assuming that the spatial separation
between parasite paths $\hbar \Delta k T / m$ is small compared to the atomic coherence length $l_c$:

$$\theta = \frac{\Delta k^2 T^2 k_B T_a}{2 M} \ll 1 \quad (22)$$

For typical Rubidium atom interferometer parameters ($T = 50 \text{ ms}$, $T_a = 2 \mu \text{K}$, $G/2\pi = 7 \text{ GHz}$), $\theta = 0.02$. Within this approximation, the equation (21) can be simplified and one obtains:

$$\Delta \varphi = \varphi_p(z_A) + \varphi_p(z_E) - \varphi_p(z_B) - \varphi_p(z_C) \quad (23)$$

where

$$\varphi_p(z) = \arg \left( \sum_m \Omega_m e^{im\Delta k z} \right) \quad (24)$$

In the next section, we will validate our model by comparing the calculated phase shift with experimental measurements.

V. COMPARISON WITH AN EXPERIMENT

Our experimental apparatus consists in a compact atom gravimeter using $^{87}\text{Rb}$. For this atom, the two ground states used for the Raman transition are $5^2 S_{1/2}$ $F=1$ and $F=2$ separated in frequency by $6.8 \text{ GHz}$. The laser used in our atomic interferometer is similar to the one described in [10]. The laser consists in a DFB laser at 1560 nm frequency doubled in a PPLN crystal. A fiber phase modulator at 1560 nm is used to generate sidebands at 6.8 GHz for the Raman transition. The source of cold atoms is a magneto-optical trap of $^{87}\text{Rb}$. After a stage of sub-Doppler cooling and a Zeeman selection, the atoms are in the state $F = 1, m_F = 0$ at a temperature of $1.75 \mu \text{K}$. The atoms interact with a vertical Raman laser retro-reflected on a mirror placed on a passive vibration isolation table. The detuning of the Raman laser compared to the excited state $5^2 P_{3/2}$ $F'=2$ is equal to $-0.7 \text{ GHz}$. The interferometer sequence starts 9.5 ms after the beginning of the atoms fall and consists in three pulses of duration 10, 20 and 10 $\mu$s separated by a time $T$ which is chosen between 5 and 48 ms. During the interferometer a frequency chirp $\alpha$ is applied to the Raman frequency in order to compensate the Doppler effect coming from the gravitational acceleration. After the interferometer sequence, the relative atomic populations in the states $F=2$ and $F=1$ are detected by fluorescence. Interference fringes can be measured by varying the chirp $\alpha$ applied to the Raman frequency. The gravity acceleration is obtained by measuring the relative atomic population on each side of the central fringe ($\alpha_0 = k_{\text{eff}} g$) [17].

In order to validate our theoretical model described in the previous section, we measured the gravity for different times $T$ and for two different distances of the mirror compared to the atoms (see Fig. 4). Each measurement of gravity corresponds to an averaging over 1000 atoms drops. Systematic effects on the gravity measurement depending on $T$ can perturb the comparison theory/experiment of the phase shift due to additional laser lines. The systematic effects depending on $k_{\text{eff}}$ (first order light shift, magnetic field) are canceled by alternating the sign of $k_{\text{eff}}$ [20]. The only important systematic effect which does not depend on the sign of $k_{\text{eff}}$ and has a dependence in $T$ is the two photons light shift [21, 22].

In the result presented, we take into account this effect by subtracting to the data the predicted error induced by the two-photon light shift.

The different parameters used in the calculation of the phase shift induced by the additional lines have been evaluated. The relative intensities of the laser lines of the Raman laser $I_n$ have been measured with an optical Fabry-Perot interferometer at 780 nm. The relative phase of the laser lines are determined by the fact that the laser is phase modulated: $E_n = \sqrt{T_n}$ for $n > 0$ and $E_n = (-1)^n \sqrt{T_n}$ for $n < 0$. By taking into account the two excited states coupled by the laser with two different Clebsch-Gordan coefficients, we obtain for the effective Rabi frequencies:

$$\frac{\Omega_n}{\Omega_0} = \frac{E_{n+1}E_n}{E_1 E_0} \frac{1}{\Delta_2 + n \Delta_1} + \frac{1/3}{\Delta_2 + 1/3 \Delta_1} \quad (25)$$

where $\Delta_2$ (resp. $\Delta_1$) is the Raman detuning compared to the excited state $F' = 2$ (resp. 1). The initial velocity of the atoms is deduced from the gravity and from the delay between the beginning of the atomic fall and the first Raman pulse. The temperature of the atoms has been measured with Raman spectroscopy and is equal to $1.75 \mu \text{K}$. In our experiment, we can not evaluate precisely the distance between the atoms and the mirror ($z_M$). This parameter will be adjustable in our calculation in order to obtain the best fit of our experimental data. For the sec-
that minimized the uncertainty of the phase shift. In our case, the optimum distance is obtained for $z_M \simeq z_0$ (first set of measurement).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$X$ & $\delta X$ & $\delta \Delta \varphi$ & $\delta \Delta \varphi$ \\
& & ($z_M = z_0$) & ($z_M = z_0 - 7.5\, \text{mm}$) \\
\hline
$z_M$ & 0.3 mm & 2.4 mrad & 2.9 mrad \\
$p_0/m$ & 5 mm s$^{-1}$ & 1.6 mrad & 7.7 mrad \\
$T_\alpha$ & 0.5 $\mu$K & 0.96 mrad & 0.97 mrad \\
$I_{1}/I_0$ & 0.02 & 0.18 mrad & 3.6 mrad \\
$I_{-1}/I_0$ & 0.02 & 0.18 mrad & 3.7 mrad \\
$I_2/I_0$ & 0.004 & 0.05 mrad & 1.6 mrad \\
$I_{-2}/I_0$ & 0.004 & 0.13 mrad & 1.2 mrad \\
$I_3/I_0$ & 0.0015 & 0.26 mrad & 2.2 mrad \\
$I_{-3}/I_0$ & 0.0015 & 0.16 mrad & 0.20 mrad \\
\hline
\end{tabular}
\caption{Experimental uncertainties on the parameters of the atom interferometer and resulted uncertainties on the estimation of the phase shift due to additional laser lines ($G/2\pi = 6.8\, \text{GHz}, \Delta_2/2\pi = -0.7\, \text{GHz}, T = 48\, \text{ms}, p_0/M = 9.3\, \text{mm/s}, T_\alpha = 1.75\, \mu\text{K}, I_{-3,-2,-1,1,2,3}/I_0 = 0, 0.066, 0.614, 0.636, 0.062, 0)$.}
\end{table}

On Fig. 5 we compare the measured phase shift on our atom interferometer with the calculated phase shift induced by the additional laser lines. We obtain an excellent agreement between theory and experiment. The differences are compatible with the error bars. We validate our model of phase shift calculation up to an accuracy of few mrad. For a more precise validation, one needs a better estimation of the atom interferometer parameters and better gravity measurements. For our compact gravimeter, the uncertainty of the phase shift induced by the additional laser lines limits the accuracy of our acceleration measurement at $8 \times 10^{-8}\, \text{m/s}^2$. This level of accuracy is close to the state of the art of $10^{-8}\, \text{m/s}^2$ and satisfies most of the applications in gravimetry. In the next part, we will show some methods which reduce the inaccuracy coming from the use of a modulated Raman.

\section{VI. Methods for Reducing the Inaccuracy Induced by Additional Laser Lines}

If one wants to improve the accuracy limitation induced by the additional laser lines, a first possibility is to have a better control of the interferometer parameters. On table III one can see that for our gravimeter, the inaccuracy is limited by the uncertainty of the mirror position and of the atoms velocity. The mirror position is not well controlled in our experiment because the mirror is on an anti-vibration table which can freely move vertically. If the entire experiment is placed on a vibration isolation table, the mirror position is fixed and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Measurement of the phase shift versus $T$ for two different mirror distances and comparison with the theory. The points are the experimental measurements. The solid line is the result of the calculation of the phase shift due to the additional laser lines. The only adjustable parameter is the mirror distance $z_M$ for the first graph. The dashed lines correspond to the error of the calculated phase shift due to experimental uncertainties of the interferometer parameters (see Table III).}
\end{figure}
the fluctuation of the mirror distance is only given by the magneto-optical trap position fluctuations which are in the order of 20 μm\[23\]. The control of the atoms velocity can be improved with a Raman velocity selection\[24\]. A control of the mean velocity and the velocity width under 0.1 mm/s can be easily achieved. With these two improvements of the parameters control, one obtains for our compact gravimeter (T = 48 ms) a phase shift uncertainty of 0.45 mrad corresponding to an acceleration accuracy of 1.2 × 10⁻⁸ m/s².

Another solution to improve the accuracy limitation is to find a configuration of the interferometer where the phase shift is reduced. On the approximated expression of the phase shift (23), one can see that the phase shift is equal to zero when zA − 2zC and zB − zE are multiple of the microwave wavelength 2π/∆k. As zA, zB, zC, zE are the position of the atoms at different points of the interferometer, this condition is equivalent to say that the distances between the position of the atoms at the moment of the three Raman pulses is multiple of 2π/∆k.

The parameters of the atom interferometer which satisfy this condition are:

\[ T = \sqrt{n^2 \frac{2\pi}{\Delta k g}} \]

\[ v_0 = \left( n - n' \right) \frac{2\pi g}{\sqrt{\Delta k}} \]

where \( n, n' \) are integers.

If this condition is satisfied, the exact phase shift should be minimized. We perform thus the calculation of this phase shift. An excellent agreement between experimental measurements and calculation has validated our model. For our compact gravimeter, the uncertainty of the parameters needed for the phase shift calculation limits the accuracy at the level of 8 × 10⁻⁸ m/s². However, with a better control of the interferometer parameters or with particular configurations, the inaccuracy is reduced below 10⁻⁸ m/s² and does not prevent to reach the state of the art in gravity accuracy.

The calculation presented here can be easily extrapolated to other inertial sensors like gyroscopes or gravimeters. In conclusion, the use of a modulated Raman laser allows to have a laser system more compact and robust and does not prevent to reach an accuracy at a level of 10⁻⁸ m/s².

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VII. CONCLUSION

In this article, we have showed that the additional laser lines present in a Raman laser generated by modulation induce in an atom interferometer a supplementary phase shift. We have presented a model which allows a precise calculation of this phase shift. An excellent agreement between experimental measurements and calculation has validated our model. For our compact gravimeter, the uncertainty of the parameters needed for the phase shift calculation limits the accuracy at the level of 8 × 10⁻⁸ m/s². However, with a better control of the interferometer parameters or with particular configurations, the inaccuracy is reduced below 10⁻⁸ m/s² and does not prevent to reach the state of the art in gravity accuracy.

The calculation presented here can be easily extrapolated to other inertial sensors like gyroscopes or gravimeters. In conclusion, the use of a modulated Raman laser allows to have a laser system more compact and robust and does not prevent to reach an accuracy at a level of 10⁻⁸ m/s².

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