Research Article

Vibration and Acoustic Radiation of Stiffened Plates Subjected to In-Plane Forces

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This study is devoted to vibration and acoustic radiation of stiffened plates in the presence of in-plane normal and shear loads using the finite element method. In structural modelling, the plate and stiffeners are treated as separate elements and the strain and kinetic energies of the stiffened plate with an elastic boundary are introduced. The results show good agreement with those obtained using other methods. Parametric studies show that in-plane normal forces have obvious influences on the acoustic radiation efficiency and the sound power level of the structure. Furthermore, the position of in-plane normal forces warrants attention; e.g., the farther the boundary in-plane normal forces from the boundary constraint are, the greater the effect on the acoustic performance is. However, in-plane edge shear loading has little influence on the acoustic performance of structures.

1. Introduction

Stiffened plates are one of the most common forms of thin walled structures in the aerospace, automobile, and shipbuilding industries. These thin structures often undergo in-plane stresses during their service life, which in turn influence the dynamics and acoustic radiation properties of these structures.

Dickinson [1] considered the free lateral vibrations of simply supported rectangular plates under the stress of both direct and shear in-plane forces. Leissa and Ayoub [2] presented a Ritz solution for a cantilever plate subjected to combined stresses due to a general in-plane acceleration force, which can extend to plates with other boundary conditions. Matsunaga [3] analysed the natural frequencies and buckling loads of a simply supported thick plate subjected to in-plane initial tensile and/or compressive forces by using the power series expansion of displacement components. Lal and Saini [4] employed the differential quadrature method to obtain the buckling and vibration behaviour of nonhomogeneous rectangular plates of uniform thickness on the basis of classical plate theory when the two opposite edges are simply supported and are subjected to a linearly varying in-plane force. Takahashi et al. [5] applied the Rayleigh-Ritz method to the problem of vibration, buckling, and dynamic stability of a cantilever rectangular plate subjected to an in-plane sinusoidally varying load applied along the free end. These scholars first showed the buckling properties and natural frequencies of the plate, then presented unstable regions for various loading conditions, and obtained simple parametric resonances and combination resonances with a sum type for various loading conditions, static load, and damping. Gutierrez and Laura [6] proposed the differential quadrature method to allow the straightforward solution of transverse vibrations of a thin rectangular plate subjected to nonuniform, in-plane loading. The results revealed that this method may be used to one’s advantage by first solving the plane stress problem and then tackling the transverse vibrations problem if the solution of the plane stress problem is not known in advance. Sato [7] presented a study on the vibration and buckling of a clamped elliptical plate supported by a well-known Winkler-type foundation and subjected to a uniform in-plane force along the plate edge according to ordinary thin plate theory. It was shown that letting the natural frequency be zero in the frequency equations derived by the analytical method yields buckling conditions. Huang [8] proposed an approach to analyse the influences of classical boundary conditions, cutouts, and in-plane loading on the vibration of a
rectangular plate with a circular hole. Zeng et al. [9] utilized the moving least squares (MLS) method to analyse the vibrations and stability of a side-cracked rectangular plate under uniform loading at its two opposite edges. Two boundary condition combinations are considered. One is simply supported along all the edges, and the other is simply supported along the two loaded edges and free along the other edges. Schmidt and Frampton [10] made the first attempt to quantify the sound power radiated from a rectangular simply supported plate subjected to a convected fluid on one surface and a vacuum on the other under various stress conditions, both compressive and tensile. Gunasekaran et al. [11] carried out analytical studies on the vibroacoustic response behaviour of an isotropic plate under nonuniform edge loads subjected to steady-state mechanical excitation, and the results reveal that the effect of the applied nonuniform edge load is significant only in the stiffness region; hence, a remarkable change in sound transmission loss is observed only in the stiffness region compared to the damping and mass regions.

Since analytical or semianalytical methods are not effective for practical structures with complex geometries or boundary constraints, numerical calculations are playing an increasingly important role in engineering fields (such as isogeometric analysis and the finite element method). Based on isogeometric analysis, level set, and simple first-order shear deformation theory, Yu et al. [12] proposed a new efficient method for calculating the vibration and buckling characteristics of composite laminates with different classical boundary conditions. Hu et al. [13] applied isogeometric analysis to solve static and dynamic calculations of laminated curved microbeams with three types of boundary conditions. Zhang et al. [14] used Mindlin–Reissner plate theory to develop a computational approach in terms of the extended isogeometric analysis for free vibration and buckling behaviours of cracked functionally graded plates. Srivastava [15] studied the buckling and vibration characteristics of a stiffened plate subjected to in-plane partial edge loading using the finite element method and analysed the stresses all over the region for different kinds of loading and edge conditions. Based on a review of the literature, the effect of in-plane forces on the radiated sound radiation properties from stiffened plates has seldom been presented. Therefore, the goal of this paper is to use the separation method to solve the above problem.

This paper is arranged in four sections. Following this introduction, Section 2 provides the theoretical background for in-plane forces and elastic boundaries, and the corresponding matrix representation is given. In Section 3, the effects of tensile, compressive in-plane loads, and shear in-plane loads on the vibration and acoustic radiation performance of stiffened plates are discussed. The final section summarizes the main results of the paper.

2. Strain Energy $U$, Kinetic Energy $T$, and Potential Energy $W$

To obtain a better representation, this section describes the theory and formulation of plates and stiffeners, which are modelled as separate elements using the finite element method. A detailed explanation of the above content is presented below. Notably, the current investigations are based on the following assumptions: (I) The plate bending deformation conforms to the Mindlin hypothesis, namely, a line that is straight and normal to the midsurface before loading is assumed to remain straight but not necessarily normal to the midsurface after loading. Thus, transverse shear deformation is allowed. (II) The lateral deflection is much less than the midplane size. (III) The derivative of $w$ with respect to $z$ is negligible.

2.1. Modelling of a Plate. Consider a plate with a total thickness $t$ in the thickness direction, and set the $x-y$ plane to coincide with the middle plane of the plate with the $z$-axis oriented. Let the displacement of any point $(x, y, z)$ in the element be of the form [16]

$$
\begin{align*}
U(x, y, z) &= u(x, y) + z\theta_x, \\
V(x, y, z) &= v(x, y) + z\theta_y, \\
W(x, y, z) &= w(x, y),
\end{align*}
$$

where $u$, $v$, and $w$ are the displacements in the $x$, $y$, and $z$ directions of the midplane of the corresponding point. $\theta_x$ and $\theta_y$ are rotations of the midplane around the $x$-axis and $y$-axis, respectively. By invoking the Mindlin plate bending theory and the von Karman large deflection assumptions, the strain–displacement relations for the midplane can be expressed as [17]

$$
\begin{align*}
\{\varepsilon\} &= \{\varepsilon_0\} + \{\varepsilon^p\},
\end{align*}
$$

where $\{\varepsilon_0\}$ and $\{\varepsilon^p\}$ are the linear strain and nonlinear strain components of the plate, respectively.

$$
\begin{align*}
\{\varepsilon_0\} &= \left\{u_{,x}v_{,y} + v_{,x}u_{,y} + r_{,x}t_{,y} + r_{,y}t_{,x}, \right. \\
\{\varepsilon^p\} &= \left. \frac{1}{2}w_{,x}^2 + \frac{1}{2}w_{,y}^2 + \frac{1}{2}w_{,x}w_{,y} \right\}.
\end{align*}
$$

The strain energy $U_p$ for the plate is

$$
U_p = \frac{1}{2} \iiint_V \{\varepsilon^p\}^T \{D_p\} \{\varepsilon^p\} dV,
$$

where $\{D_p\}$ is the elastic matrix.

The potential energy of the plate associated with the membrane forces can be expressed as [18]

$$
W_p = -\iint \left( \frac{1}{2}N_x w_x^2 + \frac{1}{2}N_y w_y^2 + N_{xy} w_x w_y \right) dA,
$$

where $N_x$ and $N_y$ are the membrane normal forces in the $x$ and $y$ directions, respectively, and $N_{xy}$ is the membrane shear force in the $x-y$ direction.

2.2. Modelling of the Stiffener. In this section, it is assumed that the beam element is arranged along the directions of $x$ and $y$. The middle plane of the plate is the reference plane of the eccentric stiffener. Based on the von Karman hypothesis, the generalized strain–displacement relation of $x$-directional stiffeners can be expressed as [19]
\{ \varepsilon_i \} = \{ \varepsilon_0' + \varepsilon_i' \}, \quad \text{(6)}

where \{ \varepsilon_0' \} and \{ \varepsilon_i' \} are the linear strain and nonlinear strain components of the stiffeners, respectively.

\[
\{ \varepsilon_0 \} = \{ u_x \theta_{x,x} + \theta_{x,y} + w_x \},
\]
\[
\{ \varepsilon_i' \} = \frac{1}{2} \{ w_i^0 + 000 \}^T .
\]

The energy function of the stiffener is obtained as follows:

\[
U_s = \frac{1}{2} \iint_V \{ \varepsilon_0 \}^T \{ \sigma \} dV,
\]  
\[
\text{where} \{ \varepsilon_0 \} \text{ and } \{ \sigma \} \text{ are the strain and stress vectors of the stiffener, respectively.}
\]

The potential energy of the stiffeners caused by the initial stress is given by

\[
W_s = -\iint_V \{ \varepsilon_i' \}^T \{ \sigma \} dV,
\]  
\[
\text{where } \{ \sigma \} \text{ is the initial stress vector, which is calculated in the prebuckling stage.}
\]

### 2.3. Modelling of the Elastic Boundary and Elastic Foundation

The strain energy of the elastic boundary and elastic foundation in the plate element, as shown in Figure 1, can be given by [20]

\[
 V_{b_f} = \frac{1}{2} \left[ \int_0^b \left[ k_{x,0} u_x^2 + k_{y,0} u_y^2 \right] d x + \frac{1}{2} \int_0^b \left[ k_{x,1} u_x^2 + k_{y,1} u_y^2 \right] d x + \frac{1}{2} \int_0^b \left[ k_{x,2} u_x^2 + k_{y,2} u_y^2 \right] d x \right] d y + \frac{1}{2} \int_0^a \left[ k_f u_x^2 \right] d x d y,
\]  
\[
\text{where} \ k_{x,0} \text{ and } k_{x,1} \text{ (} k_{y,0} \text{ and } k_{y,1} \text{) are the linear spring constants, and } k_{x,2} \text{ (} k_{y,2} \text{ and } k_{y,3} \text{) are the rotational spring constants at } x = 0 \text{ and } x = a \ (y = 0 \text{ and } y = b), \text{ respectively, and } k_f \text{ is the elastic coefficient of the foundation.}
\]

### 2.4. Finite Element Model of a Stiffened Plate with Elastic Boundary Conditions

A finite element formulation based on the four-node isoparametric shell element is used here, in which the displacement field is phrased in terms of the nodal variables [17].

\[
\{ U \} = \{ U^0 \} + \{ U^1 \},
\]  
\[
\text{where } \{ U^0 \} \text{ is the bilinear coordination function formed by natural coordinates, which can be represented by node degrees of freedom, and } \{ U^1 \} \text{ is a higher-order displacement field represented by in-plane rotational degrees of freedom.}
\]

**Figure 1: The stiffened plate subjected to an in-plane force.**

\[
\{ U^0 \} = \{ u^0 v^0 \theta_{x,y} \theta_{y,y} \}^T = \sum_{i=1}^4 N_i^0 \{ u_i v_i \theta_{x,i} \theta_{y,i} \}^T,
\]  
\[
\{ U^1 \} = \{ u^1 v^1 000 \}^T = \sum_{i=1}^4 \{ N_{a,b}^0 N_{a,b}^0 \}^T \theta_{x,i}.
\]  
\[
\text{Substituting equations (12) and (13) into (11), equation (11) can be rewritten as}
\]

\[
\{ U \} = [N] \{ d \}^T,
\]
\[
[N] = [N_1 N_2 N_3 N_4],
\]  
\[
\text{where}
\]

\[
[N_i^0] = \begin{bmatrix}
N_i^0 & 0 & 0 & 0 & N_{i1} \\
0 & N_i^0 & 0 & 0 & N_{i2} \\
0 & 0 & N_i^0 & 0 & 0 \\
0 & 0 & 0 & N_i^0 & 0 \\
\end{bmatrix} (i = 1, 2, 3, 4).
\]  
\[
\text{The stiffeners are placed in the plate along the } x- \text{ and } y- \text{axes. Similar to isoparametric plate elements, the stiffeners are modelled by isoparametric 2-noded beam elements. Taking, for example, the displacement field of the beam elements for the } x \text{ directional stiffener is expressed as}
\]

\[
\{ U_x \} = \{ u_1 u_1 \theta_1 u_2 u_2 \theta_2 \}.
\]  
\[
\text{The element displacement function can be expressed in detail as}
\]

\[
u_s = u_1 (1 - \xi) + u_2 \xi
\]
\[
w_s = \begin{bmatrix} N_1 N_2 N_3 N_4 \end{bmatrix} \begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix}. \quad \text{(17)}
\]  
\[
\text{The expressions of the shape functions } N_j (j = 1, 2, 3, 4) \text{ along } \xi \text{ for the Euler Bernoulli beam are as follows:}
\]
The equation of vibration of stiffened plates takes the form
\[
\begin{align*}
N_1 &= 1 + 2\xi^2 - 3\xi^2, \\
N_2 &= x(\xi - 1)^2, \\
N_3 &= 3\xi^2 - 2\xi^3, \\
N_4 &= x(\xi^2 - \xi), \quad (18)
\end{align*}
\]

where \([Z]\) is the acoustic impedance matrix, which can be obtained by the surface Rayleigh integral of a baffled plate. Additionally, \([v_n]\) is the vector of the normal velocity, which can be written as
\[
[v_n] = [G]^T[v] = i\omega [G]^T[U]. \quad (23)
\]

Combining equations (21)–(23), the unknown displacement vector can be determined using
\[
[U] = [-\omega^2 [M] + i\omega [C] + i\omega [G][A][Z][G]^T + [K_s]]^{-1}[F]. \quad (24)
\]

Once \([U]\) is obtained, the surface pressure \([P]\) and normal velocity \([v_n]\) can then be solved.

The acoustic power of the structure can be defined as
\[
W = \frac{1}{2} \int \text{Re}(p v_n^*) ds. \quad (25)
\]

The asterisk denotes the complex conjugate and \(P\) is the surface pressure of stiffened plate.

The mean square velocity of the stiffened plate can be calculated from the following formulas [23]:
\[
\nu_n^2 = \frac{1}{s} \int_s v_n^2 ds, \quad (26)
\]
\[
\sigma = \frac{W}{1/2 \rho_a c_a ab \langle v_n^2 \rangle},
\]
where \(S\) is the plate surface, \(\rho_a\) is the fluid density, \(c_a\) is the sound velocity, \(a\) is the plate length, and \(b\) is the plate width.

### 3. Numerical Results and Discussion

#### 3.1. Model Validation

In this paper, Fortran language is used to calculate the vibration and acoustic radiation of stiffened plates. First, for finite element analysis, a convergence study is needed to estimate the order of mesh size to obtain more accurate numerical solutions. The support stiffness and rotation stiffness are taken to be infinite to represent the rigid fixed boundary. Similarly, the support stiffness and rotation stiffness are taken to be small values to represent the free boundary. As shown in Table 1, the numerical calculation of the vibration frequency of the stiffened plate shows good convergence. Furthermore, the frequencies are compared with those of Patel et al. [24] (with the following geometric dimensions and physical parameters: \(a = 0.6096\) m; \(b = 0.4046\) m; dimensions of stiffener I: \(0.0254 \times 0.0127\) mm, \(E = 2.11 \times 10^{11}\) N/m², \(\rho = 7842.72\) kg/m³, \(v = 0.3\), and the results are in good agreement; this consistency verifies the accuracy of the numerical calculation method in this paper. A mesh size of \(20 \times 20\) is found to be sufficient to attain convergence, and this size is used for all the subsequent analyses. Second, to verify the accuracy of the calculation model, the natural frequency of the simply supported plate with four sides [25] is calculated when subjected to in-plane forces. The length \(a = 2\) m, width \(b = 1\) m, and thickness \(t = 0.5 m m\) of the plate are calculated. The long side is subjected to in-plane tension, and the short side is subjected to in-plane tension.
The natural frequencies of the plates obtained are shown in Table 2, and the results are basically equivalent to those in the literature. Finally, the sound pressure levels of a flat plate are calculated. The geometric dimensions and physical parameters are as follows: \(a = 1 \text{ m}, b = 1 \text{ m},\) and \(t = 0.025 \text{ m}.\) A unit harmonic force is applied at the midpoint of the plate. The sound pressure levels of a plate under CFFF boundary conditions are shown in Figure 2, and the computed results are in good agreement with results in [26].

### Table 1: Convergent behaviour of the natural frequency of the simply supported stiffened plate.

| Mesh size | Present | Patel [24] |
|-----------|---------|------------|
| 4 × 4     | 261.13  | 265.63     |
| 8 × 8     | 265.63  | 265.92     |
| 12 × 12   | 265.92  | 266.04     |
| 16 × 16   | 266.04  | 266.1      |
| 20 × 20   | 266.1   | 262.702    |

### Table 2: Natural frequencies of the plate subjected to in-plane forces.

| Order modal | Present | Abdullat [25] |
|-------------|---------|---------------|
| 1           | 2.3066  | 2.2915        |
| 2           | 3.6042  | 3.5697        |
| 3           | 5.4108  | 5.3826        |
| 4           | 6.0353  | 5.9567        |
| 5           | 7.1537  | 7.0454        |

3.2. Modal Frequencies. Figure 1 shows a CAD illustration of a stiffened plate, which is a common structure in ocean engineering, in which the coordinate system is shown. The dimensions of the stiffened plates are as follows: \(a = 0.6 \text{ m}, b = 1 \text{ m},\) and \(t = 0.003 \text{ m}.\) The dimensions of stiffener \(T\) are \((3 \times 20/3 \times 30) \text{mm},\) and the dimensions of stiffener \(I\) are \(20 \times 3 \text{ mm}.\) Both the plate and the stiffeners are made of the same isotropic material with a Young’s modulus of \(E = 6.9 \times 10^{10} \text{N/m}^2,\) a Poisson’s ratio of \(\nu = 0.347,\) and a material density of \(\rho = 2700 \text{ kg/m}^3.\) In all cases, the structure is immersed in air. The density \(\rho_a = 1.21 \text{ kg/m}^3,\) and the sound speed \(c_a = 343 \text{ (m/s).}\) The reference sound power value is \(10^{-12} \text{W};\) the excitation is a transverse point force applied at \((x = 0.225 \text{m}, y = 0.3125 \text{m})\) from a corner of the plate. The elastic foundation stiffness, translational stiffness, and rotational stiffness are \(k_f = 1 \times 10^9 \text{N/m},\) \(k_l = 1 \times 10^{11} \text{N/m},\) and \(k_r = 1 \times 10^4 \text{ (N/rad)},\) respectively. For the sake of discussion, \(\chi = (F/F_{cr})\) is the ratio of distributed in-plane edge loading \((F)\) to the buckling load \((F_{cr}).\)

The frequency curves of stiffened plate structures subjected to uniform compressive or tensile forces are shown in Figure 3. Two distinct features can be seen in the figure. First, the natural frequency of the stiffened plate increases with increasing in-plane tensile stress or decreasing in-plane compressive stress because the overall stiffness of the stiffened plate structure increases when the in-plane tensile stress increases or the in-plane compressive stress decreases. The frequency of the stiffened plate structure changes to 0 when the compressive stress is equal to the buckling stress, and the frequency of the second mode changes the most when the boundary of the stiffened plate is subjected to in-plane tensile stress. Second, the modal order is likely to change with increasing in-plane tensile stress or decreasing in-plane compressive stress. For example, the first and second orders of the modes are reversed when the compressive stress is approximately 0.2 times the buckling load, and the third and fourth orders of the modes are reversed when the tensile stress is approximately 4 times the buckling load.

3.3. Sound Radiation. Figure 4 shows the mean square value curve and radiated sound power curve of the stiffened plate with distributed in-plane edge normal loading at \(y = b\) (the following is the same without special instructions). The compressive stress makes the mean square velocity curve of the stiffened plate structure move to low frequency, which causes the low-frequency mean square velocity to increase. The boundary tensile stress makes the mean square velocity curve of the stiffened plate structure move to a high frequency, and the low-frequency mean square velocity decreases. However, in the middle and high-frequency stages, the mean square velocity of the stiffened plate structure does not change much with either distributed compressive or tensile in-plane edge loading on the boundary.

The sound radiation efficiency is greatly influenced by in-plane normal loading; that is, the in-plane compressive loading reduces the sound radiation efficiency of the stiffened plate, and the in-plane tensile loading increases the sound radiation efficiency of the stiffened plate. In particular, the sound radiation efficiency of the stiffened plate structure increases significantly in the middle- and high-frequency stages with the presence of in-plane tensile loading. The radiated sound power curve of the stiffened plate structure moves to high frequency with increasing in-plane force. In the low-frequency stage, the radiated sound

![Figure 2: Comparison of the radiated sound power of the rectangular plate with that from [26].](image-url)
power of the stiffened plate structure decreases with increasing in-plane loading, and in the high-frequency stage, the radiated sound power of the stiffened plate increases with increasing in-plane loading.

As shown in Figure 5, \( d \) denotes the width of in-plane loading, and \( c \) denotes the distance of uniform in-plane loading relative to the bottom edge of the stiffened plate. In this section, \( d = 0.5a, c = 0.25a \), and \( \chi = 4 \). Schematic diagrams of the internal forces in the middle position and near the boundary are shown in Figures 5(a) and 5(b). The curves of the mean square velocity and radiated sound power of the stiffened plate at different positions of in-plane tension are shown in Figure 6. It can be seen from the figure that the sound radiation efficiency of the stiffened plate when there is in-plane tensile loading in the middle of the stiffened plate boundary is significantly higher than that of the stiffened plate when there is in-plane tensile loading near the stiffened plate boundary; as a result, more vibration energy is converted into sound energy. Therefore, the sound power radiated by the former is obviously greater than that of the latter. For example, the radiated sound power of the stiffened plate structure increases by approximately 12 dB at the first resonance frequency, and the resonant frequencies also increase significantly in the higher frequency band. This is because the increase of the overall stiffness of the stiffened plate is relatively small when in-plane tensile loading exists at both ends of the stiffened plate boundary due to the limitation of the boundary condition. However, the overall stiffness of the stiffened plate clearly increases when in-plane tensile loading exists in the middle of the stiffened plate boundary due to the small influence of the boundary condition.

Figure 7 shows the mean square value curve, radiated sound power, and sound radiation efficiency curve of the stiffened plate with distributed in-plane edge shear loading. It can be seen from the figure that in-plane edge shear

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**Figure 3:** Effects of uniform boundary pressure on the modal frequencies of the stiffened plate. (a) Tensile forces. (b) Compressive forces.

**Figure 4:** Effects of in-plane forces on the stiffened plates: (a) the mean square velocity, (b) the sound radiation efficiency, and (c) the sound power level.
loading has little influence on acoustic radiation power, sound radiation efficiency, and mean square velocity. This is because the tensile force of one part of the stiffened plate increases its stiffness, while the compressive force of the other part decreases its stiffness correspondingly.

4. Conclusions

Investigations of the vibration and acoustic radiation of stiffened plates with elastic boundary conditions subjected to in-plane forces have been carried out using finite element analysis. Conclusions based on the results and discussions are summarized as follows:

1. The increase in tension in the plane causes the vibration frequency to increase, while the increase in compression force causes the vibration frequency to decrease until the buckling state; at the same time, the modal order of the structure may change.

2. The existence of boundary in-plane normal forces has a great effect on acoustic radiation. In-plane normal compressive loading reduces the sound radiation efficiency of the stiffened plate, and in-plane normal tensile loading increases the sound radiation efficiency of the stiffened plate. In particular, the sound radiation efficiency of the stiffened plate structure increases significantly in the middle-
high-frequency stages with the presence of in-plane tensile loading.

(3) The position of the in-plane normal forces also has an obvious influence on the acoustic radiation efficiency of the structure. The farther the boundary in-plane forces from the boundary constraint, the greater the effect on the acoustic performance.

(4) In-plane edge shear loading has little influence on acoustic radiation power, sound radiation efficiency, and mean square velocity. Therefore, the effect of the boundary shear force on the structural acoustic performance can be ignored in actual acoustic calculations.

Appendix

A. Stiffness Matrix of the Stiffened Plate

The stiffness matrix of the plate element is given by [17]

$$\left[ K_p \right] = \int \left[ B_p \right]^T [D] [B_p] dA,$$  \hspace{1cm} (A1)

where $[B_p]$ can be expressed as

$$[B_p] = [B_1, B_2, B_3, B_4],$$

where $l$ is the length of the beam, $E$ is the elastic modulus, and $A$ is the cross-sectional area of the beam.

B. Geometric Stiffness of the Stiffened Plate

The geometric stiffness of the plate element due to in-plane forces is rewritten in the following form:

$$\left[ K_{pg} \right] = \int \left[ G_p \right]^T [R][G_p] dA,$$ \hspace{1cm} (A3)

where $[G]$ is the membrane force matrix of the plate.

The geometric stiffness of the stiffener element due to in-plane forces can be obtained as follows [27]:

$$[K_s] = \begin{bmatrix}
\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\
0 & \frac{12EI}{l^3} & 6EI & 0 & \frac{12EI}{l^3} & 6EI \\
\frac{12EI}{l^3} & 6EI & \frac{2EI}{l^2} & 0 & \frac{2EI}{l^2} & 6EI \\
\frac{4EI}{l} & 0 & \frac{6EI}{l^2} & 0 & \frac{4EI}{l} & \frac{2EI}{l^2} \\
\end{bmatrix},$$
\[ K_{ae} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 36 & 3l & 0 & -36 & 3l \\ 4l^2 & 0 & -3l & -l^2 \\ Symmetry & 0 & 0 \\ 36 & -3l \\ 4l^2 \end{bmatrix} \]  

where \( P \) is the axial force of the beam.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The author declares that there are no conflicts of interest regarding the publication of this paper.

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