Coordination of Decisions in a Spatial Agent Model

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Abstract

For a binary choice problem, the spatial coordination of decisions in an agent community is investigated both analytically and by means of stochastic computer simulations. The individual decisions are based on different local information generated by the agents with a finite lifetime and disseminated in the system with a finite velocity. We derive critical parameters for the emergence of minorities and majorities of agents making opposite decisions and investigate their spatial organization. We find that dependent on two essential parameters describing the local impact and the spatial dissemination of information, either a definite stable minority/majority relation (single-attractor regime) or a broad range of possible values (multi-attractor regime) occurs. In the latter case, the outcome of the decision process becomes rather diverse and hard to predict, both with respect to the share of the majority and their spatial distribution. We further investigate how a dissemination of information on different time scales affects the outcome of the decision process. We find that a more “efficient” information exchange within a subpopulation provides a suitable way to stabilize their majority status and to reduce “diversity” and uncertainty in the decision process.

Key words: multi-agent system, spatial structures, collective phenomena, communication, decision processes, phase separation
PACS: 05.40.+j, 82.40.-g

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Preprint submitted to Physica A 6 September 2001
1 Introduction

Decision making is one of the fundamental processes in economy but also in social systems. If these systems consist of many interacting elements – which we call agents here – the system dynamics may be described on two different levels: the microscopic level, where the decisions of the individual agents occur and the macroscopic level where a certain collective behavior can be observed. To find a link between these two levels remains one of the challenges of complex systems theory not only with respect to socio-economic systems [1, 9, 32, 37, 44].

Among the various factors that may influence the decision of agents we mention the information available on a particular subject – such as the price or the quality of a particular product, in an economic context, or the benefits and harms that might result from the decision, in a social context, but also information about the decisions of others. A somehow extreme example is given by the rational agent model, one of the standard paradigms of neoclassical economic theory. It is based on the assumption of the agent’s complete knowledge of all possible actions and their outcomes or a known probability distribution over outcomes, and the common knowledge assumption, i.e. that the agent knows that all other agents know exactly what he/she knows and are equally rational [39].

This implicitly requires an infinitely fast, loss-free and error-free dissemination of information in the whole system. A more realistic assumption would be based on the bounded rationality of agents, where decisions are not taken upon complete a priori information, but on incomplete, limited knowledge distributed with finite velocity. This however would require to model the information flow between the agents explicitly. A possible approach to this problem is given by the spatio-temporal communication field [34, 38], that is also used in this paper (cf. Sect. 2).

Based on incomplete information, how does an agent make her decision on a particular subject? The “rational” agent usually calculates her private utility and tries to maximize it. Besides the methodological complications involved e.g. in the definitions of the utility functions (that sometimes may anticipate the results observed in computer simulations), it turns out in a world of uncertainty that the maximization of private utilities can be only achieved by some supplemented strategies.

In order to reduce the risk of making the wrong decision, it often seems to be appropriate just to copy the decisions of others. Such an imitation strategy is widely found in biology, but also in cultural evolution. Different species including humans imitate the behavior of others of their species to become
successful or just to adapt to an existing community \[\text{3}]. A physically visible example of such an imitation strategy is the formation of trails commonly created among humans, hoofed animals or social insects. Very similar to the model discussed in the present paper, this can be simulated in an agent model based on a non-linear positive feedback between the agents, where sometimes different kind of information is involved \[\text{12, 35}\].

But imitation strategies are also most powerful in economic systems, where late entrants quite often size markets from pioneers \[\text{31}\]. While the latter ones are forced to spend heavily on both product and market development, imitators can often profit from an already existing market, while avoiding the risk of making costly mistakes. In the case, where agents can observe the payoffs generated by other agents, information contagion \[\text{2}\] has been presented as an explanation for particular patterns of macrobehavior in economic systems, for example path-dependence and lock-in-effects \[\text{21, 43}\]. This has been also simulated within the concept of social percolation \[\text{41, 45}\], which may explain the occurrence of extreme market shares, based on positive feedback processes between adaptive economic agents.

Information contagion however involves the transmission of two different information, the decision made by an agent and the payoff received. The situation becomes different when agents only observe the choices of other agents and tend to imitate them, without complete information about the possible consequences of their choices. This is commonly denoted as herding behavior which plays a considerable role in economic systems \[\text{3, 19}\], in particular in financial markets \[\text{25}\], but also in human and biological systems where panic can be observed \[\text{11}\].

In social systems, herding behavior may result from the many (internal or external) interdependencies of an agent community that push or pull the individual decision into a certain direction, such as peer pressure or external influences. The social impact theory \[\text{22, 28}\] that intends to describe the transition from “private attitude to public opinion” has covered some of these collective effects in a way that can be also formalized within a physical approach \[\text{15, 24, 27}\].

One of the modelling impacts of the social impact theory was the “rediscovering of physical space” in sociology, i.e. distance matters for social influence \[\text{13, 23, 27}\]. Hence, instead of mean-field approaches where all actions of agents are coupled via a mean field, spatial models become of increasing interest. In addition to the question of how individual decisions of agents may affect the macrobehavior of the system, now the question becomes important how these decisions may organize themselves in space, i.e. what kind of spatial patterns may be observed on the global level.
The most commonly “spatial” model used is based on the cellular automaton approach, i.e. each agent occupies a lattice site. Often the lattice is also fully covered by agents, i.e. there is a homogeneous distribution in “space”. As a third assumption mostly used, only nearest-neighbor interactions are included, however the social impact theory has also provided an ansatz, where the interaction between each two agents can be considered in a weighted manner (different from the mean field).

Within the framework of cellular automata different models of collective opinion formation and group decision processes have been introduced, such as ising-like models [7, 8, 42], or models based on a modified social impact theory [14, 17, 18] that have been also applied to the coordination of individual economic decisions [26]. Also sociological problems such as the formation of support networks [10] have been simulated within an cellular automaton approach. The role of local information exchange has been further considered within the framework of the Minority Game [4], where the emergence of rich and poor spatial domains could be shown [40]. Here, however, only nearest-neighbor interactions have been taken into account (more precisely, only interactions with the left-hand neighbor in a linear chain model).

Taking into account the different perspectives on decision processes outlined above, we may conclude the following requirements for a modeling approach: It should be (i) an agent-based approach that allows to simulate individual decisions, (ii) a spatial model that takes into account physical distances between agents, but (iii) is not restricted to nearest-neighbor or mean-field interaction. Further it should allow (iv) a heterogeneous spatial distribution of both agents and information and (v) an explicit modeling of information exchange. Regarding the decision process, the model should take into account (vi) the influence of information locally available to the agent (instead of a common knowledge assumption), (vii) strategic elements such as imitation that go beyond the calculation of a private utility.

In the next section, we want to propose an agent model, that is novel in the sense that it meets all of the requirements listed above – at least to some extent – and is further simple enough to serve as a toy model for investigating spatial effects in decision processes both analytically and by means of computer simulations.

As we will show, this model describes the emergence of a majority and a minority of agents making the same decision (Sect. 3). But besides the existence of a global majority, there are regions that are dominated by the minority, hence a spatial coordination of decisions among the agents occurs (Sect. 4).

The main result of this paper, however, is not just the observation of the clustering of opinions, which is a pervasive feature of a wide range of models
Although in economics this phenomenon is very often described only on the aggregated (macro) level, but not on the (micro) level of the agents. In this paper, we derive conditions (Sect. 4) under which – for the same given parameters – a large variety of possible spatial decision patterns can be found, and the outcome of the decision process becomes certainly unpredictable, both with respect to the share of the majority and to the spatial distribution. We further show how the exchange of information affects the possibility of a broad range of spatial decision patterns (Sect. 5). In particular, we discuss how “efficient” information exchange provides a suitable way to stabilize the majority status of a particular subpopulation – or to avoid “diversity” and uncertainty in the decision process (Sect. 5).

2 Toy Model of Communicating Agents

Let us consider a 2-dimensional spatial system with the total area \( A \), where a community of \( N \) individuals exists. In general, \( N \) can be changed by birth and death processes but \( A \) is assumed fixed. In a more abstract sense, each individual \( i \) shall be treated as an agent, i.e. a rather autonomous entity which is assigned two individual parameters: its position in space, \( r_i \), which should be a continuous variable, and its current “opinion”, \( \theta_i \) (with respect to a definite aspect or problem). The latter one is a discrete valued parameter representing an internal degree of freedom (which is a rather general view of “opinion”).

To be more specific, let us discuss for instance the separate disposal of recyclable material. Each agent in the system needs to decide whether she will cooperate in the recycling campaign or defect. Then, there are only two (opposite) opinions, i.e. \( \theta_i \in \{+1, -1\} \), or \( \{+, -\} \) to be short. \( \{+\} \) shall indicate the cooperating agent, and \( \{-\} \) the defecting agent.

From the classical economic perspective, the agents’ decision about her opinion may depend on an estimate of her utility, i.e. what she may gain compared to her own effort, if she decides to cooperate or not. Here, we neglect any question of utility and may simply assume that the agent will more likely do what others do with respect to the specific problem, i.e. she will decide to cooperate in the recycling campaign if most of her neighbors will do so, and defect if most of their neighbors have the same opinion in this case. This type of herding behavior in decision processes – a special kind of the imitation strategy – is well known from different fields, as discussed in Sect. 1.

This example raises the question about the interaction between agents at different locations, i.e. how is agent \( i \) at position \( r_i \) affected by the decisions of other agents at closer or far distant locations? In a checkerboard world, commonly denoted as cellular automaton, a common assumption is to consider
only the influence of agents, which are at the (four or eight) nearest neighbor sites or also at the second-nearest neighbor sites, etc. Contrary, in a mean-field approximation, all agents are considered as influential via a mean field, which affects each agent at the same time in the same manner.

Our approach will be different from these ones in that we will consider a continuous space and a gradual, time delayed interaction between all agents. We assume that agent $i$ at position $r_i$ is not directly affected by the decisions of other agents, but only receives information about their decisions via a communication field generated by the agents with the different opinions. This field is assumed a scalar multi-component spatio-temporal field $h_\theta(r, t)$, which obeys the following equation:

$$\frac{\partial}{\partial t} h_\theta(r, t) = \sum_{i=1}^{N} s_i \delta_{\theta, \theta_i} \delta(r - r_i) - k_\theta h_\theta(r, t) + D_\theta \Delta h_\theta(r, t). \quad (1)$$

Every agent contributes permanently to this field with her personal “strength” or influence, $s_i$. Here, $\delta_{\theta, \theta_i}$ is the Kronecker Delta indicating that the agents contribute only to the field component which matches their opinion $\theta_i$. $\delta(r - r_i)$ means Dirac’s Delta function used for continuous variables, which indicates that the agents contribute to the field only at their current position, $r_i$.

The information generated this way has a certain life time $1/k_\theta$, further it can spread throughout the system in a diffusion-like process, where $D_\theta$ represents the diffusion constant for information exchange. We have to take into account that there are two different opinions in the system, hence the communication field should also consist of two components, $\theta = \{+1, -1\}$, each representing one opinion. Note, that the parameters describing the communication field, $s_i$, $k_\theta$, $D_\theta$ do not necessarily have to be the same for the two opinions.

The spatio-temporal communication field $h_\theta(r, t)$ is used to reflect some important features of communication in social systems:

(i) the existence of a memory, which reflects the past experience. In our model, this memory exist as an external memory, the lifetime of which is determined by the decay rate of the field, $k_\theta$.

(ii) an exchange of information in the community with a finite velocity. It means that the information will eventually reach each agent in the whole system, but of course at different times.

(iii) the influence of spatial distances between agents. Thus, the information generated by a specific agent at position $r_i$ will affect agents at a closer spatial distance earlier and thus with larger weight, compared to far distant agents.

The communication field $h_\theta(r, t)$ influences the agent’s decisions as follows: At a certain location $r_i$ agent $i$ with e.g. opinion $\theta_i = +1$ is affected by two kinds of
information: the information $h_{\theta=+1}(r_i, t)$ resulting from agents who share her opinion, and the information $h_{\theta=-1}(r_i, t)$ resulting from the opponents. The diffusion constants $D_\theta$ determine how fast she will receive any information, and the decay rate $k_\theta$ determines, how long a generated information will exist. Dependent on the information received locally, the agent has two opportunities to act: she can change her opinion or she can keep it. A possible ansatz for the transition rate to change the opinion reads [34]:

$$w(-\theta_i|\theta_i) = \eta \exp\left\{ \frac{h_\theta(r_i, t) - h_{-\theta}(r_i, t)}{T} \right\}; \quad w(\theta_i|\theta_i) = 0 \quad (2)$$

The probability to change opinion $\theta_i$ is rather small, if the local field $h_\theta(r_i, t)$, which is related to the support of opinion $\theta_i$, overcomes the local influence of the opposite opinion. Here, $\eta$ defines the time scale of the transitions. $T$ is a parameter which represents the erratic circumstances of the opinion change, based on an incomplete or incorrect transmission of information. Note, that $T$ is measured in units of the communication field. In the limit $T \to 0$ the opinion change rests only on the difference $\Delta h(r_i, t) = h_\theta(r_i, t) - h_{-\theta}(r_i, t)$, leading to “rational” decisions (cf. also [35]), i.e. decisions that are totally determined by the external information. In the limit $T \to \infty$, on the other hand, the influence of the information received is attenuated, leading to “random” decisions. We note that $T$ can be also interpreted in terms of a “social temperature” \cite{17,18}, i.e. it is a measure for the randomness in social interaction.

For $N = \text{const.}$, the community of agents may be described by the time-dependent canonical $N$-particle distribution function

$$P(\bar{\theta}, \bar{r}, t) = P(\theta_1, \ldots, \theta_N, \bar{r}, t), \quad (3)$$

which gives the probability to find the $N$ agents with the opinions $\theta_1, \ldots, \theta_N$ in the vicinity of $r_1, \ldots, r_N$ on the surface $A$ at time $t$. The time dependent change of $P(\bar{\theta}, \bar{r}, t)$ is then given by the following master equation [44]:

$$\frac{\partial}{\partial t} P(\bar{\theta}, \bar{r}, t) = \sum_{\bar{\theta}'} \left[ w(\bar{\theta}, \bar{\theta}') P(\bar{\theta}', \bar{r}, t) - w(\bar{\theta}' | \bar{\theta}) P(\bar{\theta}, \bar{r}, t) \right] \quad (4)$$

Eq. (4) describes the “gain” and “loss” of agents with the coordinates $r_1, \ldots, r_N$ due to opinion changes, where $w(\bar{\theta}, \bar{\theta}')$ means any possible transition within the opinion distribution $\bar{\theta}'$ which leads to the assumed distribution $\bar{\theta}$. Eq. (4) together with eqs. (1), (2) forms a complete description of our system, which depends on the parameters describing the agent density, i.e. $N, A$, and the components of the communication field, $s_i, k_\theta, D_\theta$. In order to find possible solutions of the master equation, we will use computer simulations, and in particular apply the stochastic simulation technique \cite{6,33}. But before investigating the spatially distributed system, we first will discuss a mean-field
approximation, in order to get some insight into the complex dynamics of the agent system.

3 Mean-Field Approach

In this section, we will neglect any spatial effects of the agents distribution and the communication field. This case, which has been discussed in more detail also in [34], may have some practical relevance for communities existing in small systems with short distances between different agents. In particular, in such small communities a very fast exchange of information may hold, i.e. spatial inhomogeneities in the communication field are equalized immediately. Thus, in this section, the discussion can be restricted to subpopulations with a certain opinion rather than to agents at particular locations.

Let us define the share \( x_\theta \) of a subpopulation \( \theta \) and the respective mean density \( \bar{n}_\theta \) in a system of size \( A \) consisting of \( N \) agents:

\[
x_\theta(t) = \frac{N_\theta(t)}{N} ; \quad \bar{n}_\theta(t) = \frac{N_\theta(t)}{A}
\]  

(5)

where the total number of agents sharing opinion \( \theta \) at time \( t \) fulfils the condition

\[
\sum_\theta N_\theta(t) = N_+(t) + N_-(t) = N = \text{const.} ; \quad x_+(t) = 1 - x_-(t)
\]  

(6)

In the mean-field approach, the communication field \( h_\theta(r, t) \) can be approximated by a mean value \( \bar{h}_\theta(t) \) which obeys the following dynamic equation:

\[
\frac{\partial \bar{h}_\theta(t)}{\partial t} = -k_\theta \bar{h}_\theta(t) + s_\theta \bar{n}_\theta
\]  

(7)

Here, we have assumed that agents with the same opinion \( \theta \) will have the same influence \( s_i \rightarrow s_\theta \). We note that the case of a “strong leader”, where one agent has a personal strength \( s_i \) much larger than the usual strength \( s_i \) has been discussed in [16, 18, 34].

The dynamic equation for the size of subpopulation \( \theta \) can be derived from eq. (4) in the mean–field approximation as follows [34]:

\[
\dot{x}_\theta = (1 - x_\theta) \eta \exp(a) - x_\theta \eta \exp(-a) ; \quad a = \left[ \bar{h}_\theta(t) - \bar{h}_{-\theta}(t) \right] / T
\]  

(8)

Via \( \Delta \bar{h}(t) = \bar{h}_\theta(t) - \bar{h}_{-\theta}(t) \), this equation is coupled to eq. (7).
Let us now investigate the possible existence of stationary states, \( \dot{x}_\theta = 0, \dot{h}_\theta = 0 \). For the two field components we find from eq. (7) with \( \bar{n} = N/A \):

\[
\bar{h}^{stat}_+ = \frac{s_+}{k_+} \bar{n} x_+ ; \quad \bar{h}^{stat}_- = \frac{s_-}{k_-} \bar{n} (1 - x_+) \quad (9)
\]

Let us for the moment assume that the parameters of both field components are identical, i.e. \( s_+ = s_- \equiv s, k_+ = k_- \equiv k \), a more complex case will be discussed in Sect. 5. Then, the value \( a \), eq. (8), which depends on the difference between the field components, can in the stationary limit be expressed as:

\[
a = \kappa \left( x_+ - \frac{1}{2} \right) \quad \text{with} \quad \kappa = \frac{2s \bar{n}}{k T} \quad (10)
\]

The parameter \( \kappa \) plays the role of a bifurcation parameter that includes the specific internal conditions within the community, such as the population density, the individual strength of the opinions, the life time of the information generated or the randomness \( T \). Combining eqs. (8), (10), the condition for stationary solutions can be expressed by means of a function \( \mathcal{L}(x, \lambda) \) and \( \kappa = \lambda/2 \) as

\[
\mathcal{L}(x_+, 2\kappa) = 0 \quad (11)
\]

where \( x \in (0, 1) \) and \( \mathcal{L}(x, \lambda) \) is defined as:

\[
\mathcal{L}(x, \lambda) := \ln \frac{x}{1-x} - \lambda \left( x - \frac{1}{2} \right) \quad (12)
\]

Fig. 1. Stationary solutions for \( x_+ \) (eq. 11) for different values of \( \kappa \). The bifurcation at the critical value \( \kappa^c = 2 \) is clearly visible. For \( \kappa = 2.66 \) that is used for some of the computer simulations, we find in the mean field limit the stationary values \( x_+ = 0.885 \) and \( x_+ = 0.115 \) for the majority and the minority status, respectively.

In [34] we found that, depending on \( \kappa \), different stationary values for the fraction of the subpopulations exist (cf. also Fig. 1). For \( \kappa < 2, x_+ = 0.5 \)
is the only stationary solution, which means a stable community where both opposite opinions have the same influence. However, for $\kappa > 2$, the equal distribution of opinions becomes unstable, and a separation process towards a preferred opinion is obtained, where $x_\pm = 0.5$ plays the role of a separatrix. Then, two stable solutions are found where both opinions coexist with different shares in the community, as shown in Fig. 1. Hence, each subpopulation can exist either as a majority or as a minority within the community. Which of these two possible situations is realized, depends in a deterministic approach on the initial fraction of the subpopulation. For initial values of $x_+$ below the separatrix, $0.5$, the minority status will be most likely the stable situation, as shown in [34].

The share of the majority or minority for a given value of $\kappa > 2$ can be implicitly calculated from eq. (11). The critical value for $\kappa$ where the bifurcation occurs is determined by eq. (11) together with:

$$\frac{\partial L(x_\pm, 2\kappa)}{\partial x_+}\bigg|_{x_+ = 1/2} = 4 - 2\kappa := 0$$

which results in $\kappa^c = 2$. From this condition we can derive by means of eq. (10) a critical population size,

$$N^c = k AT/s,$$

where for larger populations an equal fraction of opposite opinions is certainly unstable. We note that this critical value has been derived based on a mean field analysis and therefore does not consider finite size or discrete effects.

If we consider e.g. a growing community with fast communication, then both contradicting opinions are balanced, as long as the population number is small. However, for $N > N^c$, i.e. after a certain population growth, the community tends towards one of these opinions, thus necessarily separating into a majority and a minority. Note that eq. (14) for the critical population size can be also interpreted in terms of a critical social temperature, which leads to an opinion separation in the community. This has been discussed in more detail in [34].

From Fig. 1, we see further, that the stable coexistence between majority and minority breaks down at a certain value of $\kappa$, where almost the whole community shares the same opinion. From eq. (11) it is easy to find that e.g. $\kappa \approx 4.7$ yields $x_\theta \approx \{0.01; 0.99\}$, which means that about 99% of the community share either opinion $+1$ or $-1$. For smaller values of $\kappa$, for instance for $\kappa = 2.66$, we find $x_+ = 0.885$ for the majority status, and $x_+ = 0.115$ for the minority status respectively. Of course, due to the symmetry between both opinions, the opposite relation, $x_+ = 0.115$ and $x_- = 0.885$ is possible with the same probability. Which one is realized may depend on the fluctuation during the early stage of the evolution of the agent system. Fig. 2 shows a
particular realization obtained from computer simulation of 400 agents who at $t = 0$ are randomly assigned one of the opinions $\{+1, -1\}$.

Fig. 2. Computer simulation of the relative subpopulation sizes $x_+ (\bigodot)$ and $x_- (\bigcirc)$ vs. time $t$ for a community of $N = 400$ agents. Parameters: $A = 400$, $s = 0.1$, $k = 0.1$, $T = 0.75$, i.e. $\kappa = 2.66$. Initially, each agent has been randomly assigned opinion $+1$ or $-1$. The dashed lines indicate the initial equal distribution ($x_\theta = 0.5$) and the minority and majority sizes ($x_\theta = \{0.115; 0.885\}$) which follow from eq. (11).

Fig. 3. Components $h_+ (\bigodot)$ and $h_- (\bigcirc)$ of the mean communication field vs. time $t$ for the simulation shown in Fig. 2. After the initial time lag $t^* \sim 5/k = 50$, both field components evolve differently.

As indicated in Fig. 2, there is a latent period in the beginning before the minority/majority relation emerges, i.e. during this period it is not clear which one of the two subpopulations, the cooperators or the defectors will gain the majority status. This initial time lag $t^*$ can be roughly estimated as $t^* \sim 5/k = 50$ [30], where $k$ is the decay constant of the field. $t^*$ is needed to establish the communication field, as shown in Fig. 3. With the same set of parameters, both components of the communication field at first evolve in
the same manner. However after the initial time lag, a selection in favor of one of the components occurs which then breaks the symmetry between both opinions. Hence, the communication field plays the role of an order parameter as known from synergetics \[9\]. Consequently, for \( t \geq t^* \), a transition from the unstable equal distribution between both opinions toward a majority/minority relation is clearly visible in Fig. 2. The time period to eventually establish this relation is then rather short, since the case discussed in this section is related to a very fast exchange of information.

Eventually, we want to point out that the symmetry between the two opinions can be broken due to external influences on the agents. In \[34\] we have considered two similar cases: (i) the existence of a strong leader in the community, who possesses a strength \( s_l \) which is much larger than the usual strength \( s \) of the other individuals, (ii) the existence of an external field, which may result from government policy, mass media, etc. which support a certain opinion with a strength \( s_m \). The additional influence \( s_{ext} := \{s_l/A, s_m/A\} \) mainly effects the mean communication field, eq. (7), due to an extra contribution, normalized by the system size \( A \). We found within the mean-field approach that at a critical value of \( s_{ext} \), the possibility of a minority status completely vanishes. Hence, for a certain supercritical external support, the supported subpopulation will grow towards a majority, regardless of its initial population size, with no chance for the opposite opinion to be established. This situation is quite often realized in communities with one strong political or religious leader (“fundamentalistic dictatorships”), or in communities driven by external forces, such as financial or military power (“banana republics”).

4 Spatial Information Dissemination

4.1 Results of Computer Simulations

The previous section has shown within a mean-field approach the emergence of a minority/majority relation in the agents community. With respect to the example of the recycling campaign addressed in the beginning, it means that either most of the agents decide to cooperate or most of them defect.

The question remains how the cooperators and the defectors organize themselves in space by means of a spatial dissemination of information. In order to consider the spatial dimension of the system explicitly, let us start with the previous example of \( N = 400 \) agents randomly distributed in a system of size \( A \) (cf. Fig. 4), with random initial opinions. They get information about the opinions of other agents by means of the two-component communication field \( h_\theta(r,t) \), eq. (1), which now explicitly considers space and therefore “diffu-
sion” of information. The two-dimensional system is here treated as a torus, i.e. we assume periodic boundary conditions.

In this section, we assume again that the parameters describing the communication field, are the same for both components, i.e.

\[ s_+ = s_- \equiv s; \quad k_+ = k_- \equiv k; \quad D_+ = D_- \equiv D \]  

(15)

a different case will be investigated in Sect. 5. Fig. 4 shows three snapshots of the spatial distribution of the cooperators and the defectors, while Fig. 5 shows the respective evolution of the subpopulation shares. Evidently, we find again the emergence of a majority/minority relation – this time however, on a larger time scale compared to Fig. 2, which is basically determined by the information diffusion, expressed in terms of \( D \). But the initial latent time lag for the emergence of the majority/minority relation is about the same, which is needed again to establish the communication field.

As the different snapshots of Fig. 4 show, the minority and majority organize themselves in space in such a way that they are separated. Thus, besides the existence of a global majority, we find regions in the system which are dominated by the minority. From this we can conclude a spatial coordination of decisions, i.e. agents which share the same opinion are spatially concentrated in particular regions. With respect to the example of the recycling campaign this means that those agents who defect to cooperate (or cooperate in the opposite case), are mostly found in a spatial domain of a like-minded neighborhood. This result might remind on the famous simulations of segregation in social systems \[10, 29, 30\] – however, we would like to note that in our case the agents do not migrate toward supportive places; they rather adapt to the opinion of their neighborhood based on the information received.

The spatial distribution of the majority and the minority is also reflected in the different components of the communication field, as shown in Fig. 6. We find that the maxima of both components are of about equal value, however, the information generated by the majority, is roughly spread over the whole system, whereas the information generated by the minority eventually concentrates only in specific regions dominated by them.

Running the simulations of the agent system different times with the same set of parameters but just different initial random seeds, reveals an interesting effect that can be observed in Fig. 7, in comparison to Fig. 5. We see that instead of a single fixed majority/minority relation for the spatially extended system a large range of such relations exist, which even in the presence of fluctuations are stable over a very long time. This means on the other hand that under certain conditions the global outcome of the decision process becomes hard to predict.
The different global majority/minority relations further correspond to different spatial coordination patterns. Fig. 8 gives a snapshot of the spatial distribution of collaborators and defectors of a much larger system, but with the same parameters as in Fig. 4 (in particular with the same average density of agents). We see that in this case the majority and the minority are of about the same size. Further, the minority no longer exists rather isolated (cf. Fig. 4c), but in a spatially extended domain. We note that besides some stochastic
Fig. 5. Relative subpopulation sizes $x_+$ (◇) and $x_-$ (○) vs. time $t$ for the computer simulation shown in Fig. 4.

Fig. 6. Spatial distribution of the two-component communication field: (top) $h_+(r, t)$, (bottom) $h_-(r, t)$ at time $t = 10^4$, which refers to the spatial agent distribution of Fig. 4c.

fluctuations the observed coordination pattern remains stable also in the long run ($t = 5 \cdot 10^4$).

Thus, we may conclude from these simulations that the spatially extended system – under certain conditions – possesses multiple attractors for the collective
Fig. 7. Relative subpopulation sizes $x_\theta$ vs. time $t$ from 20 computer simulations with the same set of parameters as in Fig. 4, but different initial random seeds.

Fig. 8. Snapshot of the spatial distribution of cooperators (◊ left) and defectors (○ right) at $t = 5 \cdot 10^4$. System size $A = 1600$, total number of agents $N = 1600$, for the other parameters see Fig. 4. In this particular realization, the frequency of collaborators is $x_+ = 0.543$ and the frequency of defectors is $x_- = 0.456$, respectively.

dynamics that makes the outcome of the decision process hard to predict. This holds not only for the global minority/majority ratio, but also for the possible spatial patterns that correspond to the different attractors. This shall be discussed in more detail in the following section.
4.2 Analytical Investigation of the Two-Box Case

In order to get more insight into the attractor structure of the spatially extended system, we want to investigate analytically the most simple spatial case, consisting of only two boxes. The frequency to find agents with opinion +1 in box \( i \in \{1, 2\} \) shall be denoted by \( y_{i+} \), whereas the frequency in the total system is still given by \( x_+ = N_+/N \), eq. (5). With a total system area \( A \) and an assumed homogeneous distribution of agents with an average density \( \bar{n} = N/A \), it yields:

\[
y_{i+} = \frac{N_{i+}}{N/2} = 1 - y_{i-}; \quad i = 1, 2
\]

\[
x_+ = \frac{1}{2} (y_{1+} + y_{2+})
\]

Fig. 9 gives an overview of the relevant variables of the two-box system. The dynamics is described by six coupled equations, which read explicitly:

\[
\dot{y}_{i+} = (1 - y_{i+}) \eta \exp \left[ \frac{h_{i+} - h_{i-}}{T} \right] + y_{i+} \eta \exp \left[ - \frac{h_{i+} - h_{i-}}{T} \right]; \quad i = 1, 2
\]

\[
\dot{h}_{i+} = -k_+ h_{i+} + s_+ \bar{n} y_{i+} + D_+ \frac{8(h_{j+} - h_{i+})}{A}; \quad i, j \in \{1, 2\}; \quad i \neq j
\]

\[
\dot{h}_{i-} = -k_- h_{i-} + s_- \bar{n}(1 - y_{i+}) + D_- \frac{8(h_{j-} - h_{i-})}{A}
\]

The stationary solutions of the set of equations follow from \( \dot{y}_{i+} = 0; \dot{h}_{i+} = \dot{h}_{i-} = 0 \). Combining the six equations of eq. (18) in the stationary case, we
arrive after some transformations at the following two conditions:

\[
\begin{align*}
\ln \left( \frac{y_{1+}}{1 - y_{1+}} \right) + \ln \left( \frac{y_{2+}}{1 - y_{2+}} \right) &= 2 \left( y_{1+} + y_{2+} \right) \left[ \frac{s_+ \bar{n}}{k_+ T} + \frac{s_- \bar{n}}{k_- T} \right] - 4 \frac{s_- \bar{n}}{k_- T} \\
\ln \left( \frac{y_{1+}}{1 - y_{1+}} \right) - \ln \left( \frac{y_{2+}}{1 - y_{2+}} \right) &= 2 \left( y_{1+} - y_{2+} \right) \times \\
&\quad \times \left[ \frac{s_+ \bar{n}}{k_+ T} \left( 1 + \frac{16 D_+}{A k_+} \right)^{-1} + \frac{s_- \bar{n}}{k_- T} \left( 1 + \frac{16 D_-}{A k_-} \right)^{-1} \right]
\end{align*}
\]  

(19)

The coupled equations of eq. (19) define the possible stationary frequencies \( y_{1+}, y_{2+} \) of agents in the two coupled boxes and therefore serve as a starting point for a subsequent bifurcation analysis. It will be convenient to assume first the parameters of the different field components as equal again, cf. eq. (15). In this case, we can recover the bifurcation parameter \( \kappa \), eq. (10) of the mean field limit as follows:

\[
\kappa = \frac{s_+ \bar{n}}{k_+ T} + \frac{s_- \bar{n}}{k_- T} = \frac{2 \bar{s} \bar{n}}{k T}
\]  

(20)

By means of the previously defined function \( \mathcal{L}(x, \lambda) \), eq. (12) we are now able to express the conditions for stationary solutions, eq. (19) in the following compact form:

\[
\begin{align*}
\mathcal{F}_{1+} := \kappa (1 - \bar{D}) (y_{2+} - y_{1+}) + \mathcal{L}(y_{2+}, 2\kappa) &= 0 \\
\mathcal{F}_{2+} := \kappa (1 - \bar{D}) (y_{1+} - y_{2+}) + \mathcal{L}(y_{1+}, 2\kappa) &= 0
\end{align*}
\]  

(21)

where

\[
\bar{D} = \left( 1 + \frac{16 D}{A k} \right)^{-1}
\]  

(22)

If we compare eq. (21) with eq. (11) for the mean-field case, we find that each solution \( x_+ \) of the mean-field case is also a solution of eq. (21) by setting \( y_{1+} = y_{2+} = x_+ \). Moreover, if \( D = 0 \), i.e. no exchange between the two boxes, then \( \bar{D} = 1 \), and eq. (21) reduces to eq. (11).

The coupled equations (21) have been solved numerically, further we have conducted a stability analysis. The corresponding bifurcation diagram is shown in Fig. 10 for a single box (top) and the total system (bottom). If we compare the latter one with the bifurcation diagram of the mean field limit, Fig. 1, we find that the bifurcation at \( \kappa^c = 2 \) is obtained again, cf. curve (a). That means, for \( \kappa < \kappa^c \), the equal distribution of both opinions is the stable state also for the spatially heterogeneous system, while for \( \kappa > \kappa^c \) a majority and a minority emerges, which organizes itself in space in the way shown in Fig. 4.

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Fig. 10. Stationary solutions for $y_\theta$ (top: single box) and $x_\theta$ (bottom: total system) resulting from eq. (21) for different values of $\kappa$. Parameters: $A = 400$, $D/k = 6.0$. The solid lines indicate stable solutions, the dashed lines instable ones.

With increasing $\kappa$, we find a second bifurcation, curve (b) at the critical value $\kappa^c_2$. For consistency, the first critical value shall be denoted as $\kappa^c_1 \equiv \kappa^c = 2$. As long as $\kappa^c_1 < \kappa < \kappa^c_2$, the minority/majority ratio is clearly defined by just one possible value, cf. Fig. 10(bottom). For $\kappa^c_2 < \kappa$, however there are different stable majority/minority relations possible in the system. This can be also clearly observed in Fig. 11 that shows the stable and instable stationary states of the 2-box system for three different values of $\kappa$.

The appearence of the second bifurcation curve (b) in Fig. 10(bottom) can be understood when looking at the bifurcation diagram of a single box at the top part of Fig. 10. To avoid confusion, the critical bifurcation values of the single box are indicated by $'$ instead of $c$. In the single box, the first bifurcation is found at $\kappa^c_1 \equiv \kappa^c_1 = 2$ as in the mean-field case, and a second bifurcation at $\kappa^c_2 > \kappa^c_1$. The resulting new stationary states, however still lead to an average frequency $x_\theta = 0.5$ in the total system, because of $y_{1+} = \ldots$
Fig. 11. Stationary solutions for \( y_{1+} \) vs. \( y_{2+} \) resulting from eq. (21). (left) \( \kappa'_1 < \kappa = 2.1 < \kappa'_2 \), (middle) \( \kappa'_2 < \kappa = 2.6 < \kappa'_3 \), (right) \( \kappa'_3 < \kappa = 3.0 \). Other parameters see Fig. 10. The filled circles mark stable states, the open squares unstable ones. The dashed lines are used to indicate possible separatrices.

Fig. 12. Stationary solutions for \( y_{1+} \) vs. \( y_{2+} \) resulting from eq. (21) for \( \kappa = 3.0 \) and \( D/k = 0 \), i.e. \( \bar{D} = 1 \). This figure should be compared to Fig. 11(right), where \( D/k = 6.0 \).

\[ 1 - y_{2+} \] in this particular case. Only after a third bifurcation in the single box that occurs at the critical value \( \kappa'_3 = \kappa^c_2 \), we find stationary frequencies in each box that do not automatically sum up to 1, and therefore allow for different frequencies for the total system. That means, in order to know about the possible appearance of different majority/minority relations in the 2-box system, we have to concentrate on the appearance of the third bifurcation for the single box.

Eventually, we also investigate how the different critical bifurcation values \( \kappa'_i \) shift with the scaled diffusion constant \( D/k \). The numerical results are shown in Fig. 13. While \( \kappa'_1 \) remains constant, \( \kappa'_2 \) increases linearly with \( D/k \), and \( \kappa'_3 \) increases in nearly linear manner.

At least for the linear dependence, the shift of \( \kappa' \) with \( D/k \) can be derived analytically. As Fig. 10(top) shows, the first two bifurcations occur for \( y_{i+} = 1/2 \). Looking e.g. at box \( i = 1 \), we find for \( \mathcal{F}_{1+} \), eq. (21): \( \mathcal{F}_{1+} (y_{1+} = 1/2) = 0 \). The dependence \( \kappa' (\bar{D}) \) results then from the condition:

\[
\left. \frac{\partial \mathcal{F}_{1+}}{\partial y_{1+}} \right|_{y_{1+}=1/2} = 0
\]  

(23)
This eventually leads to a quadratic equation for $\kappa'_i (\bar{D})$:

$$\bar{D} \kappa'^2 - 2(1 + \bar{D}) \kappa' + 4 = 0$$

with the two solutions

$$\kappa'_1 = 2 ; \quad \kappa'_2 = \frac{2}{\bar{D}} = 2 \left(1 + \frac{16\bar{D}}{A k}\right)$$

which agree with the results of Fig. 13.

We note that all three bifurcations keep existing also in the limit $D/k \to 0$, until the bifurcation values $\kappa'_i$ coincide at $D/k = 0$. The diffusion between the two boxes of course affects the values of the $\kappa'_i$, as shown in Fig. 13, but the values of the stable states are not changed much. This can be seen by comparing Fig. 11(right) with Fig. 12 valid for $D/k = 0$, that clearly indicates how the size of the attractor regions around the stable states is influenced by diffusion. In the limit of $D/k \to \infty$ the situation of the mean-field limit is recovered again.

In conclusion, the investigation of the two-box case has shown that above a certain critical value $\kappa'_2 = \kappa'_3$ the possibility of different stable majority/minority ratios exist. For a given supercritical population density, cf. eq. (14), the velocity of information exchange, expressed in terms of the diffusion constant $D$ and the lifetime of information $k$ determine whether these multiple attractors can exist.

These results can be also adapted to the spatially extended “multi-box” case as an approximation of the continuous space. If for the single box multiple attractors exist, we can observe in the multi-box case a larger variety of spatial coordination patterns, with different patches of likeminded agents coexisting.
(cf. Fig. 8). Regarding the possible values of the minority/majority ratio for the total system, we note that in the spatially extended system almost every value can be realized. These multiple stable states for the total system can be envisioned as a combination of the different possible stable states of the single boxes. An increase of the diffusion constant of the information field, on the other hand, restricts the stability of multiple attractors of the single boxes and this way limitates the variety of possible spatial coordination patterns.

5 Information Dissemination on Different Time Scales

From the investigations in the previous section, we found the emergence of different majority/minority relations in a spatially extended agent system. So far, however, fluctuations during the initial time lag observed may decide whether the cooperators or the defectors will appear as the majority. If we start from an unbiased initial distribution, i.e. an equal distribution between both opinions, then there is no easy way to break the symmetry towards e.g. cooperation, except, an external bias is taken into account.

In this section, we will investigate a possibility to break the symmetry by means of different information dissemination, i.e. we may exploit the different properties of the information exchange in the system, as expressed in terms of the parameters $s_\theta$, $k_\theta$, $D_\theta$ of the communication field. For instance, we may assume that the information generated by of one of the subpopulations is distributed faster in the system than the information generated by the other one. Alternatively, we may also consider different life times of the different components of the communication field. However, in order to model a faster exchange of information, it is not sufficient to simply increase the value of $D_\theta$, we need to consider its effect on the local values of the communication field in more detail.

Eq. (1) is known as a reaction-diffusion equation. In order to elucidate the influence of the different parameters, we will assume for the moment a one-dimensional system with the spatial coordinate $x$. Further, there shall be only one group of $N$ agents, $\theta = 0$, which are located at the same place, $x = 0$ and have the same personal strength, $s_0$. Then, eq. (1) can be simplified to:

$$\frac{\partial}{\partial t} h_0(x,t) = s_0 \frac{N}{A} \delta(x-0) - k_0 h_0(x,t) + D_0 \Delta h_0(x,t)$$  \hspace{1cm} (26)

Initially, $h(x,0) = 0$ yields. After a certain time which depends in particular on $D_0$, the communication field $h_0(x,t)$ generated by the agents will reach a
stationary distribution. With \( \dot{h}(x,t) = 0 \) and \( n = N/A \) we find from eq. (26):

\[
h_{0}^{\text{stat}}(x) = \frac{s_0 n}{2 \sqrt{k_0 D_0}} \exp \left\{ -\sqrt{\frac{k_0}{D_0}} |x| \right\}
\]  

(27)

Fig. 14. Stationary distribution \( h_{0}^{\text{stat}}(x) \), eq. (27), of the communication field, assumed that there is only one group of \( N \) agents located at the same place, \( x = 0 \).  
\( \longrightarrow D_0 = 0.06, \ \longleftrightarrow D_0 = 0.12 \), other parameters: \( N = 10, A = 1, s_0 = 0.1, k_0 = 0.1 \).

Eq. (27) is plotted in Fig. 14 for two different parameters of the diffusion constant \( D_0 \). Obviously, in addition to the time scale of information dissemination, an increase of \( D_0 \) mainly affects the stationary value of the communication field \( h_{0}^{\text{stat}}(x) \) at the position of the agents, \( x = 0 \). I.e. a faster communication in the system via a faster diffusion of the generated information, also lowers the information available at the agent’s position. This might be considered as a drawback in modeling information exchange by means of reaction-diffusion equations. Obviously, the field \( h_\theta(r,t) \) obeys certain boundary conditions and conservation laws, which do not hold for “information per se”. In particular, the local value of available information is not lowered if this information spreads out faster, but the local value of the “communication field” obeying eq. (1) does.

In order to compensate the unwanted effect of a local decrease of \( h_\theta(r,t) \), the production rate \( s_\theta \) can be increased accordingly, which however effects again the stationary values. To be consistent, we have to choose that both the ratios

\[
\frac{k_\theta}{s_\theta} = \beta; \quad \frac{D_\theta}{s_\theta} = \gamma
\]  

(28)

need to be constant for both components \( \theta = \{+1, -1\} \). In this case, eq. (1) for the dynamics of the multi-component communication field can be rewritten
as:

\[
\frac{\partial}{\partial \tau} h_\theta(r, \tau) = \sum_{i=1}^{N} \delta_{\theta, \theta_i} \delta(r - r_i) - \beta h_\theta(r, \tau) + \gamma \Delta h_\theta(r, \tau). \tag{29}
\]

where the time scale \( \tau \) is now defined as \( \tau = t (D_\theta / \gamma) \). If both parameters \( \beta \) and \( \gamma \) are kept constant, eq. (29) means that the dynamics of the respective component of the communication field occurs on a different time scale \( \tau \), dependent on the value of \( D_\theta \). An increase in the diffusion constant \( D_\theta \) then models indeed the information exchange on a faster time scale as expected, without affecting the stationary distribution resulting from eq. (29).

The effect of the different diffusion constants can be understood by means of computer simulations, where the parameters \( \beta \) and \( \gamma \), eq. (28) are kept constant and only the ratio

\[
d = \frac{D_+}{D_-} \tag{30}
\]

is varied. Fig. 15 shows the evolution of the subpopulations in time for \( d = 1/3 \) and \( d = 1 \) for comparison. Two features can be noticed from these two par-

![Fig. 15. Relative subpopulation sizes \( x_+ (\diamond) \) and \( x_- (\circ) \) vs. time \( t \). Parameters: \( A = 400, N = 400, \beta = 1, \gamma = 0.6, (a) D_+ = 0.02, D_- = 0.06, (b) D_+ = D_- = 0.06 \) (cf. Fig. 5).](image)

icular runs: (i) For \( d = 1/3 \), the initial time lag when the decision which subpopulation becomes the majority is yet pending, has vanished. I.e. compared to \( d = 1 \), there is a considerably reduced period of time for early fluctuations to break the symmetry toward one of the subpopulations. The time lag has been related before to the establishment of the communication field needed to provide the coupling between the agents. But here we see that the symmetry is broken very fast, even without a fully established communication field. (ii) For \( d = 1/3 \), the subpopulation with the faster (more “efficient”) communication
has become the majority, while for $d = 1$ both subpopulations have an equal chance to become the majority in the system. These conclusions however need some substantiation, therefore we have conducted more extensive computer simulations presented in the following.

Fig. 16 shows the total fraction of agents of subpopulation $\{-1\}$ over time averaged over 20 runs, for different values of $d$. This mean value gives an estimate of the chance that subpopulation $\{-1\}$ becomes the minority or majority in the system. We find that for $d = 1$ this chance is about 50 percent, i.e. only random events decide about its status, as long as the information dissemination of both subpopulations occurs on the same time scale. With increasing $d$ (i.e. with an increasing information diffusion of the other subpopulation), the trend towards a minority status clearly increases for subpopulation $\{-1\}$, as shown in the average fraction $\langle x_- \rangle$. Already for $d = 1.5$, i.e. for example $D_+ = 0.06$, $D_- = 0.04$, the average subpopulation fraction reaches a constant minimum value in the stationary limit, which means that deviations in size have considerably decreased.

![Figure 16](image-url)

**Fig. 16.** Relative subpopulation size $\langle x_- \rangle$ averaged over 20 runs. The different numbers give the value of $d = D_+/D_-$. Other parameters see Fig. 15.

A closer inspection is given in the series of Fig. 17, which shows the mean values of $\langle x_- \rangle$ together with the minimum and maximum values of the 20 runs to indicate the scattering.

We find that for $d \in \{1.1; 1.2\}$, inspite of the clear tendency towards the minority status, there are still possibilities that the subpopulation $\{-1\}$ ends up as the majority in the system – even with a slower communication. With increasing $d$, these possibilities vanish, as Fig. 17 (c,d) indicate. However, for a range of $d \in \{1.2; 1.4\}$ we find that the size of the minority population still shows a large range of possible values. Only for $d > 1.5$, these deviations
become small enough to allow only one stable size of the minority subpopulation.

Considering the results for possible attractors in the spatially extended system obtained in the previous section, we can discuss the outcome of the above simulations also from a different perspective. They allow to distinguish between two different stationary regimes: (i) a multi-attractor regime, where different values for a stable minority/majority ratio are possible, and (ii) a single-attractor regime, where only one stable minority/majority ratio exists. In the considered case, the crossover between these two regimes can be obtained by an increase/decrease of the ratio \( d = D_+ / D_- \), provided a subcritical population density and constant values of \( \beta \) and \( \gamma \), eq. (28) are fulfilled. The effect of an increasing diffusion coefficient of one subpopulation – or a more efficient communication, respectively, can be understood similar to the two-box case, Sect. 4.2. There we have shown that larger values of \( D \) decrease the
attractor basins of some stable states for the single boxes, and thus reduce the possibility of multiple stable states for the total system.

6 Conclusions

In this paper, we have investigated the coordination of decisions in a spatially distributed agent community. The interaction between the agents is described by a scalar, multi-component communication field that stores the information about the agents’ decisions. This spatio-temporal field is assumed to obey a reaction-diffusion equation. This way, memory effects are modeled by means of a limited lifetime of the information, whereas the exchange of information is modeled by means of a finite diffusion coefficient. Dependent on the different information received at her particular position, each agent makes a decision in a binary choice problem. I.e. she chooses either \{+\} or \{-\} after comparing the local information available.

From analytical investigations of the mean-field case, we found that the subpopulations of agents making a particular choice coexist at different shares within the community. They can be found either as a majority or a minority, provided some internal conditions (such as a supercritical population density) are fulfilled.

In this paper, we are mainly interested in how the majority and the minority of agents emerge in a spatially heterogeneous system and how they organize themselves in space. We observed that the formation of minority/majority subpopulations goes along with a spatial separation process, i.e. besides the existence of a global majority, there are regions that are dominated by the minority. Hence, a spatial coordination of decisions among the agents occurs.

Further, we found that – different from the mean-field case – a large range of possible global minority/majority relations can be observed that refer to different spatial coordination patterns. We have investigated analytically and by means of computer simulations, under which conditions these multiple steady states occur and stable exist. The results can be concluded by looking at the two influential parameters of the model, \( \kappa \) and \( D \).

\( \kappa \) includes the specific internal conditions within the agent community, namely the population density \( \bar{n} \), the production rate of information per agent \( s \), the lifetime of information \( k \) and the randomness \( T \) that can be envisioned as a measure of the incompleteness or incorrect transformation of information. Defining \( \nu = s\bar{n}/k \) as the net information density, \( \kappa = 2\nu/T \) describes the relation between the mean information \( \nu \) available at any location and its efficiency \( \sim 1/T \) – in other words, the impact of the information produced. We
recall that the limit $T \to 0$ means a large impact of the available information leading to “rational” decisions, whereas in the limit $T \to \infty$ the influence of the information is attenuated, leading to “random” decisions.

In order to gain at least some impact of the available information, a supercritical value of $\kappa > \kappa^c = 2$ is needed. In this case the emergence of a minority and a majority within the agent community can be observed, whereas for $\kappa < \kappa^c$ only random decisions occur. To allow multiple steady states in the spatially extended system instead of just one fixed minority/majority relation, $\kappa$ has to be $\kappa^c < \kappa_2^c < \kappa$, where $\kappa_2^c$ itself is increasing with the diffusion constant $D$. Then a variety of possible spatial decision patterns can be found, and the outcome of the decision process becomes certainly unpredictable, both with respect to the share of the majority and to the spatial distribution.

This means that the spatial couplings, expressed in terms of $D$, are not large enough to globally organize the system. Since $\kappa$ characterizes the average local situation in a spatially extended system in terms of a net information density, this can be also interpreted in a way that the impact resulting from the information exchange does not overcome the impact resulting from the local information production.

For values of $\kappa$ below $\kappa_2^c(D)$, however, these local effects become smaller, and the spatial couplings are able to organize the whole system. Thus only one minority/majority relation occurs on the global level, which relates to randomly different, but very similar spatial patterns.

If we put these results in the context of a social system, we could conclude that strong local influences, expressed in a high information impact, can prevent the global system from being equalized and “globalized” by some ruling information. While such a diversity might be among the wanted effects, we note again that this on the other hand makes the system difficult to predict.

Eventually, we have addressed in this paper the influence of information exchange on different time scales. In particular, we have assumed that one of the two subpopulations communicates faster – or more efficient – than the other one, and have investigated how this affects the global outcome of the decision process. Dependent on the ratio $d = D_+/D_-$ of the two diffusion constants we found (i) a tendency that the subpopulation with the faster communication more likely becomes the majority, and (ii) that the possibility of multiple steady states tends to vanish with an increasing/decreasing $d$. In conclusion, “efficient” information exchange provides a suitable way to stabilize the majority status of a particular subpopulation – or to avoid “diversity” and uncertainty in the decision process.

Finally, we want to add that the toy model of communicating agents investigated in this paper may be easily modified or extended to describe other
processes. Without giving up the whole framework, we may consider e.g. other types of information dissemination in the system, i.e. eq. (1) for the communication field may be replaced – for example by a more network-type communication among the agents. Another possible modification is regarding the decision process described in this paper by means of eq. (2). Here, we may envision various dependences on the information received from likeminded or opponent agents.

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