Revisit assignments of the new excited Ω_c states with QCD sum rules

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Abstract

In this article, we distinguish the contributions of the positive parity and negative parity Ω_c states, study the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ using the QCD sum rules in a consistent way, and revisit the assignments of the new narrow excited Ω_c states. The predictions support assigning the Ω_c(3000) to be the 1P Ω_c state with $J^P = \frac{1}{2}^-$, assigning the Ω_c(3090) to be the 1P Ω_c state with $J^P = \frac{3}{2}^-$ or the 2S Ω_c state with $J^P = \frac{1}{2}^+$, and assigning Ω_c(3119) to be the 2S Ω_c state with $J^P = \frac{3}{2}^+$. 

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1 Introduction

Recently, the LHCb collaboration studied the Ξ_b^-K^- mass spectrum and observed five new narrow excited Ω_c states, Ω_c(3000), Ω_c(3050), Ω_c(3066), Ω_c(3090), Ω_c(3119) [1]. The measured masses and widths are

Ω_c(3000) : $M = 3000.4 \pm 0.2 \pm 0.1$ MeV, $\Gamma = 4.5 \pm 0.6 \pm 0.3$ MeV,
Ω_c(3050) : $M = 3050.2 \pm 0.1 \pm 0.1$ MeV, $\Gamma = 0.8 \pm 0.2 \pm 0.1$ MeV,
Ω_c(3066) : $M = 3065.6 \pm 0.1 \pm 0.3$ MeV, $\Gamma = 3.5 \pm 0.4 \pm 0.2$ MeV,
Ω_c(3090) : $M = 3090.2 \pm 0.3 \pm 0.5$ MeV, $\Gamma = 8.7 \pm 1.0 \pm 0.8$ MeV,
Ω_c(3119) : $M = 3119.1 \pm 0.3 \pm 0.9$ MeV, $\Gamma = 1.1 \pm 0.8 \pm 0.4$ MeV.

There have been several assignments for those new Ω_c states, such as the 2S Ω_c states with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ [2] [3] [4] [5], the P-wave Ω_c states with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ or $\frac{5}{2}^-$ [3] [4] [5] [6] [7] [8] [9] [10] [11] [12], the pentaquark states or molecular pentaquark states with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ or $\frac{5}{2}^-$ [13] [14] [15], or the D-wave Ω_c states [16].

In Refs [2] [5], Agaev, Azizi and Sundu construct the interpolating currents without introducing the relative P-wave to study the Ω_c states by taking into account the 1S, 1P, 2S states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions in the QCD sum rules. They use the 1S state plus continuum model to obtain the masses and pole residues of the 1S states firstly, then take them as input parameters and use the 1S state plus 1P state plus continuum model to obtain the masses and pole residues of the 1P states, finally use the 1S state plus 1P state plus 2S state plus continuum model to obtain the masses and pole residues of the 2S states. In Ref. [12], Aliev, Bilmis and Savci use the same interpolating currents to study the Ω_c states by taking into account the 1S and 1P states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions in the QCD sum rules. The potential quark models predict
that the 1P and 2S $\Omega_c$ states have the masses about $3.0 - 3.2$ GeV \cite{17, 18}. If the 1P and 2S $\Omega_c$ states lie in the same energy region, it is difficult to distinguish their contributions in the QCD sum rules \cite{2, 5, 12}.

In Refs.\cite{19, 20, 21, 22}, we construct the interpolating currents without introducing the relative P-wave to study the $J^P = \frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ heavy, doubly-heavy and triply-heavy baryon states with the QCD sum rules in a systematic way by subtracting the contributions from the corresponding $J^P = \frac{1}{2}^\mp$ and $\frac{3}{2}^\mp$ heavy, doubly-heavy and triply-heavy baryon states, and obtain satisfactory results. In Ref.\cite{8}, we study the new excited $\Omega_c$ states with the QCD sum rules by introducing an explicit P-wave involving the two $s$ quarks, the predictions support assigning the $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$ to be the P-wave baryon states with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{3}{2}^-\text{ and } \frac{5}{2}^-$, respectively.

In this article, we distinguish the contributions of the S-wave (positive parity) and P-wave (negative parity) $\Omega_c$ states, study the masses and pole residues of the 1S, 1P, 2S and 2P $\Omega_c$ states with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ using the QCD sum rules in details, and revisit the assignments of the new narrow excited $\Omega_c^0$ states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the S-wave and P-wave $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ $\Omega_c$ states in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

## 2 QCD sum rules for the $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ $\Omega_c$ states

Firstly, we write down the two-point correlation functions $\Pi(p)$ and $\Pi_{\alpha\beta}(p)$ in the QCD sum rules,

\[ \Pi(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ \eta(x)\bar{\eta}(0) \} | 0 \rangle, \]
\[ \Pi_{\alpha\beta}(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ \eta_{\alpha}(x)\bar{\eta}_{\beta}(0) \} | 0 \rangle, \]  

where

\[ \eta(x) = \varepsilon^{ijk} s_i^T(x) C \gamma_\alpha s_j(x) \gamma_5 \gamma^\alpha c_k(x), \]
\[ \eta_{\alpha}(x) = \varepsilon^{ijk} s_i^T(x) C \gamma_\alpha s_j(x) c_k(x), \]  

the $i$, $j$ and $k$ are color indexes, and the $C$ is the charge conjugation matrix. In this article, we choose the simple Ioffe type interpolating currents.

At the hadron side, we insert a complete set of intermediate $\Omega_c$ states with the same quantum numbers as the current operators $\eta(x)$, $i\gamma_5\eta(x)$, $\eta_{\alpha}(x)$ and $i\gamma_5\eta_{\alpha}(x)$ into the correlation functions $\Pi(p)$ and $\Pi_{\alpha\beta}(p)$ to obtain the hadronic representation \cite{23, 24}. We isolate the pole terms of the lowest 1S, 1P, 2S and 2P $\Omega_c$ states ($\Omega_c$ and $\Omega_c'$), and obtain the results:

\[ \Pi(p) = \lambda_{\frac{1}{2}^\pm}^+ \frac{p^\pm + M_+^2}{M_+^2 - p^2} + \lambda_{\frac{1}{2}^\pm}^- \frac{p^\pm - M_-^2}{M_-^2 - p^2} + \lambda_{\frac{3}{2}^\pm}^+ \frac{p^\pm + M_+^2'}{M_+^2' - p^2} + \lambda_{\frac{3}{2}^\pm}^- \frac{p^\pm - M_-^2'}{M_-^2' - p^2} + \cdots, \]
\[ = \Pi_{\frac{1}{2}}(p^2) + \cdots, \]
\[
\Pi_{\alpha\beta}(p) = \left( \lambda_{\frac{3}{2}}^{+} \frac{p + M_{p}}{M_{p}^{2} - p^2} + \lambda_{\frac{3}{2}}^{-} \frac{p - M_{p}}{M_{p}^{2} - p^2} + \lambda_{\frac{1}{2}}^{+} \frac{p + M_{s}^{0}}{M_{s}^{0}^{2} - p^2} + \lambda_{\frac{1}{2}}^{-} \frac{p - M_{s}^{0}}{M_{s}^{0}^{2} - p^2} \right)
\]
\[
- g_{\alpha\beta} + \frac{\gamma_{\alpha} \gamma_{\beta}}{3} + \frac{2 p_{\alpha} p_{\beta}}{3 p^2} - \frac{p_{\alpha} \gamma_{\beta} - p_{\beta} \gamma_{\alpha}}{3 \sqrt{p^2}} + \cdots,
\]
\[
= \Pi_{\frac{3}{2}}(p^2) (-g_{\alpha\beta}) + \cdots. \tag{5}
\]

The currents \(\eta(0)\) and \(\eta_{\alpha}(0)\) couple potentially to the spin-parity \(J^P = \frac{1}{2}^\pm\) and \(\frac{3}{2}^\pm\) \(\Omega_c\) states \(\Omega_{\frac{1}{2}}^{(\pm)}\) and \(\Omega_{\frac{3}{2}}^{(\pm)}\), respectively\ [22, 25, 26, 27],

\[
\langle 0 | \eta(0) | \Omega_{\frac{1}{2}}^{(\pm)}(p) \rangle = \lambda_{\frac{1}{2}}^{(\pm)} U_{\frac{1}{2}}^{(\pm)}(p, s),
\]
\[
\langle 0 | \eta_{\alpha}(0) | \Omega_{\frac{1}{2}}^{(\pm)}(p) \rangle = \lambda_{\frac{1}{2}}^{(\pm)} U_{\alpha}^{(\pm)}(p, s), \tag{6}
\]
\[
\langle 0 | \eta(0) | \Omega_{\frac{3}{2}}^{(\pm)}(p) \rangle = \lambda_{\frac{3}{2}}^{(\pm)} i\gamma_5 U_{\frac{3}{2}}^{(\pm)}(p, s),
\]
\[
\langle 0 | \eta_{\alpha}(0) | \Omega_{\frac{3}{2}}^{(\pm)}(p) \rangle = \lambda_{\frac{3}{2}}^{(\pm)} i\gamma_5 U_{\alpha}^{(\pm)}(p, s), \tag{7}
\]

where the \(\lambda_{\frac{1}{2}}^{(\pm)}\) and \(\lambda_{\frac{3}{2}}^{(\pm)}\) are the pole residues or the current-baryon couplings, the spinors \(U_{\frac{1}{2}}^{(\pm)}(p, s)\) and \(U_{\alpha}^{(\pm)}(p, s)\) satisfy the relations,

\[
\sum_{s} U(p, s) \overline{U}(p, s) = \not{p} + M_{\frac{1}{2}}^{(\pm)},
\]
\[
\sum_{s} U_{\alpha}(p, s) \overline{U}_{\beta}(p, s) = \left( \not{p} + M_{\frac{3}{2}}^{(\pm)} \right) \left(-g_{\alpha\beta} + \frac{\gamma_{\alpha} \gamma_{\beta}}{3} + \frac{2 p_{\alpha} p_{\beta}}{3 p^2} - \frac{p_{\alpha} \gamma_{\beta} - p_{\beta} \gamma_{\alpha}}{3 \sqrt{p^2}} \right), \tag{8}
\]

and \(p^2 = M_{\frac{1}{2}}^{(\pm)}^2\) on mass-shell, the \(s\) are the polarizations or spin indexes of the spinors, and should be distinguished from the \(s\) quark or the energy \(s\).

We obtain the hadronic spectral densities at hadron side through dispersion relation,

\[
\frac{\text{Im} \Pi_j(s)}{\pi} = \not{p} \left[ \lambda_{j}^{+} \delta(s - M_{+}^2) + \lambda_{j}^{-} \delta(s - M_{-}^2) + \lambda_{j}^{+} \delta(s - M_{+}^{02}) + \lambda_{j}^{-} \delta(s - M_{-}^{02}) \right]
\]
\[
+ \left[ M_{+} \lambda_{j}^{+} \delta(s - M_{+}^2) - M_{-} \lambda_{j}^{-} \delta(s - M_{-}^2) + M_{+} \lambda_{j}^{+} \delta(s - M_{+}^{02}) - M_{-} \lambda_{j}^{-} \delta(s - M_{-}^{02}) \right]
\]
\[
= \not{p} \rho_{j,H}^{1}(s) + \rho_{j,H}^{0}(s), \tag{9}
\]

where \(j = \frac{1}{2}, \frac{3}{2}\), the subscript \(H\) denotes the hadron side, then we introduce the weight
function \( \exp\left(-\frac{s}{T^2}\right) \) to obtain the QCD sum rules at the hadron side,

\[
\int_{m_0^2}^{s_0} ds \left[ \sqrt{s}\rho^1_{j,H}(s) + \rho^0_{j,H}(s) \right] \exp\left(-\frac{s}{T^2}\right) = 2M_+\lambda^2_j \exp\left(-\frac{M^2_+}{T^2}\right) + 2M'_+\lambda^+_{j\p} \exp\left(-\frac{M^2'_+}{T^2}\right), \tag{10}
\]

\[
\int_{m_0^2}^{s_0} ds \left[ \sqrt{s}\rho^1_{j,H}(s) - \rho^0_{j,H}(s) \right] \exp\left(-\frac{s}{T^2}\right) = 2M_-\lambda^-_{j\p} \exp\left(-\frac{M^2_-}{T^2}\right) + 2M'_-\lambda^-_{j\p} \exp\left(-\frac{M^2'_-}{T^2}\right), \tag{11}
\]

where the \( s_0 \) are the continuum thresholds and the \( T^2 \) are the Borel parameters \[27\]. We distinguish the contributions of the positive parity and negative parity \( \Omega_c \) states unambiguously according to Eqs.\( (9-11) \).

At the QCD side, we calculate the light quark parts of the correlation functions \( \Pi(p) \) and \( \Pi_{\alpha\beta}(p) \) with the full light quark propagators \( S_{ij}(x) \) in the coordinate space \[28\],

\[
S_{ij}(x) = \frac{i\delta_{ij}}{2\pi^2x^4} - \frac{\delta_{ij}m_s}{4\pi^2x^2} - \frac{\delta_{ij}\langle ss \rangle}{12} + \frac{i\delta_{ij}\langle m_s \rangle}{48} - \frac{\delta_{ij}x^2\langle \bar{s}g_s\sigma_Gs \rangle}{192} - \frac{i\delta_{ij}x^2\langle m_s \rangle}{1152} - \frac{i\bar{s}g_sG^n_{\alpha\beta}x_{ij}\langle \bar{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\bar{x} \rangle}{32\pi^2x^2} - \frac{1}{8}\langle \bar{s}j\sigma^{\mu\nu}\sigma_{ij} \rangle + \cdots,
\]

and take the full \( c \)-quark propagator \( C_{ij}(x) \) in the momentum space \[24\],

\[
C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \frac{\delta_{ij}}{k - m_c} - \frac{g_sG^n_{\alpha\beta}x_{ij}\sigma^{\alpha\beta}(k + m_c) + (k + m_c)\sigma^{\alpha\beta}}{4(k^2 - m^2_c)^2} - \frac{g^2_s(x_{ij})G_{\alpha\beta}G_{\mu\nu}^{a}(f^{a\beta\mu\nu} + f^{a\mu\beta\nu} + f^{a\nu\mu\beta})}{4(k^2 - m^2_c)^5} + \cdots \right\}, \tag{13}
\]

\[
f^{\alpha\beta\mu\nu} = (k + m_c)\gamma^\alpha(k + m_c)\gamma^\beta(k + m_c)\gamma^\mu(k + m_c)\gamma^\nu(k + m_c),
\]

\( q = u, d, s, l^n = \frac{\lambda^n}{2} \), the \( \lambda^n \) is the Gell-Mann matrix. In Eq.\( (12) \), we add the term \( \langle \bar{s}_j\sigma^{\mu\nu}s_i \rangle \) originates from the Fierz re-ordering of the \( \langle s_is_j \rangle \) to absorb the gluons emitted from other quark lines to form \( \langle \bar{s}_jg_sG_{\alpha\beta}G_{\mu\nu}^{a}(\sigma^{\alpha\beta}\mu\nu + \sigma^{\alpha\beta}\mu\nu + \sigma^{\alpha\beta}\mu\nu) \rangle \) to extract the mixed condensate \( \langle \bar{s}g_s\sigma_Gs \rangle \). The term \( -\frac{1}{8}\langle \bar{s}_j\sigma^{\mu\nu}s_i \rangle \) was introduced in Ref.\[29\]. We compute the integrals both in the coordinate space and momentum space to obtain the correlation functions \( \Pi_j(p^2) \), then obtain the QCD spectral densities through dispersion relation,

\[
\frac{\text{Im}\Pi_j(s)}{\pi} = \frac{1}{2} \rho^1_{j,QCD}(s) + \rho^0_{j,QCD}(s), \tag{15}
\]

where \( j = \frac{1}{2}, \frac{3}{2} \), the explicit expressions of the QCD spectral densities \( \rho^1_{j,QCD}(s) \) and \( \rho^0_{j,QCD}(s) \) can be rewritten in a concise form after multiplying the weight function \( \exp\left(-\frac{s}{T^2}\right) \) to obtain the integrals \( \int_{m_0^2}^{s_0} ds \sqrt{s}\rho^1_{j,QCD}(s) \exp\left(-\frac{s}{T^2}\right) \) and \( \int_{m_0^2}^{s_0} ds \rho^0_{j,QCD}(s) \exp\left(-\frac{s}{T^2}\right) \).
We take the quark-hadron duality, introduce the continuum thresholds $s_0$ and the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules:

$$
\int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_j(s) + \rho^0_j(s) \right] \exp\left(-\frac{s}{T^2}\right) = \int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_j(s) + \rho^0_j(s) \right] \exp\left(-\frac{s}{T^2}\right),
$$

where $j = \frac{1}{2}, \frac{3}{2}$,

$$
\rho^0_j(s) = m_c \rho^0_j(s),
$$

$$
\rho^1_j(s) = \rho^1_j(s),
$$

$$
\rho^{\frac{1}{2}}_j(s) = \frac{3}{32\pi^2} \int_{x_i}^{1} dx \left( 1 - x \right)^2 \left( s - m_c^2 \right)^2 - \frac{3m_s \langle \bar{s}s \rangle}{2\pi^2} \int_{x_i}^{1} dx
$$

$$
+ \frac{1}{2} \left( \frac{\alpha_s G_F}{\pi} \right) \int_{x_i}^{1} dx \left( 1 - x \right)^2 \left[ 1 - \frac{s}{3} \delta(s - \tilde{m}_c^2) \right]
$$

$$
+ \frac{1}{2} \left( \frac{\alpha_s G_F}{\pi} \right) \int_{x_i}^{1} dx \frac{2 - 3x}{x}
$$

$$
- \frac{m_s \langle \bar{s}g_s \sigma G_s \rangle}{4\pi^2} \int_{x_i}^{1} dx \frac{1}{x} \delta(s - \tilde{m}_c^2)
$$

$$
+ \frac{5m_s \langle \bar{s}g_s \sigma G_s \rangle}{12\pi^2} \delta(s - m_c^2) + \frac{4\langle \bar{s}s \rangle^2}{3}\delta(s - m_c^2)
$$

$$
+ \langle \bar{s}s \rangle \left( \langle \bar{s}g_s \sigma G_s \rangle \right) \left( 1 - \frac{2m_c^2}{T^2} \right) \delta(s - m_c^2)
$$

$$
- \frac{\langle \bar{s}g_s \sigma G_s \rangle^2}{12T^4} \left( 1 + \frac{3m_c^2}{T^2} \right) \delta(s - m_c^2),
$$

(19)
\[ \rho_{\frac{1}{2}}^1 (s) = \frac{1}{16\pi^4} \int_{x_i}^1 dx x (1 - x)^3 (5s - 3\bar{m}_c^2)(s - \bar{m}_c^2) - \frac{m_s \langle \bar{s}s \rangle}{\pi^2} \int_{x_i}^1 dx \]

\[ + \frac{m_s \langle \bar{s}s \rangle}{\pi^2} \int_{x_i}^1 dx x (1 - x) [3 + s \delta (s - \bar{m}_c^2)] \]

\[ + \frac{1}{48\pi^2} \left( \frac{\alpha_s G_G}{\pi} \right) \int_{x_i}^1 dx \left\{ (1 - x) [3 + s \delta (s - \bar{m}_c^2)] + (1 - 2x) \right\} \]

\[ - \frac{m^2}{72\pi^2} \left( \frac{\alpha_s G_G}{\pi} \right) \int_{x_i}^1 dx \left\{ (1 - x)^3 \left( 1 + \frac{s}{2T^2} \right) \delta (s - \bar{m}_c^2) \right\} \]

\[ - \frac{m_s \langle \bar{s}g_s G_s \rangle}{3\pi^2} \int_{x_i}^1 dx x \left( 1 + \frac{s}{2T^2} \right) \delta (s - \bar{m}_c^2) \]

\[ - \frac{m_s \langle \bar{s}g_s G_s \rangle}{8\pi^2} \int_{x_i}^1 dx \delta (s - \bar{m}_c^2) \]

\[ + \frac{m_s \langle \bar{s}g_s G_s \rangle}{4\pi^2} \delta (s - m_c^2) + \frac{2(\bar{s}s)^2}{3} \delta (s - m_c^2) \]

\[ - \frac{\langle \bar{s}s \rangle \langle \bar{s}g_s G_s \rangle}{6T^2} \left( 1 + \frac{2m_c^2}{T^2} \right) \delta (s - m_c^2) \]

\[ - \frac{m^2 \langle \bar{s}g_s G_s \rangle^2}{24T^6} \left( 1 - \frac{m_c^2}{T^2} \right) \delta (s - m_c^2), \] (20)

\[ \rho_{\frac{1}{2}}^0 (s) = \frac{1}{64\pi^4} \int_{x_i}^1 dx (x + 2)(1 - x)^2 (s - \bar{m}_c^2)^2 - \frac{m_s \langle \bar{s}s \rangle}{4\pi^2} \int_{x_i}^1 dx (2 - x) \]

\[ - \frac{m^2}{576\pi^2} \left( \frac{\alpha_s G_G}{\pi} \right) \int_{x_i}^1 dx \frac{(x + 2)(1 - x)^2}{x^3} \delta (s - \bar{m}_c^2) \]

\[ + \frac{1}{192\pi^2} \left( \frac{\alpha_s G_G}{\pi} \right) \int_{x_i}^1 dx \left[ \frac{(x + 2)(1 - x)^2}{x^2} - (2 - x) \right] \]

\[ + \frac{m_s \langle \bar{s}g_s G_s \rangle}{24\pi^2} \int_{x_i}^1 dx \delta (s - \bar{m}_c^2) + \frac{m_s \langle \bar{s}g_s G_s \rangle}{12\pi^2} \delta (s - m_c^2) \]

\[ + \frac{\langle \bar{s}s \rangle^2}{3} \delta (s - m_c^2) - \frac{m^2 \langle \bar{s}s \rangle \langle \bar{s}g_s G_s \rangle}{6T^4} \delta (s - m_c^2) \]

\[ - \frac{m^2 \langle \bar{s}g_s G_s \rangle^2}{24T^6} \left( 1 - \frac{m_c^2}{2T^2} \right) \delta (s - m_c^2), \] (21)
\[
\rho_j^1(s) = \frac{1}{64\pi^4} \int_{x_i}^1 dx x(x+2)(1-x)^2(s-m_c^2)^2 - \frac{m_s(\bar{s}s)}{4\pi^2} \int_{x_i}^1 dx x(2-x) \\
- \frac{m_c^2}{576\pi} \langle \frac{\alpha_sGG}{\pi} \rangle \int_{x_i}^1 dx \frac{(x+2)(1-x)^2}{x^2} \delta(s-m_c^2) \\
- \frac{1}{192\pi^2} \langle \frac{\alpha_sGG}{\pi} \rangle \int_{x_i}^1 dx x(2-x) \\
+ \frac{m_s(\bar{s}sG\sigma Gs)}{24\pi^2} \int_{x_i}^1 dxx(s-m_c^2) + \frac{m_s(\bar{s}sG\sigma Gs)}{12\pi^2} \delta(s-m_c^2) \\
+ \frac{(\bar{s}s)^2}{3} \delta(s-m_c^2) - \frac{(\bar{s}s)(\bar{s}sG\sigma Gs)}{6T^2} \left(1 + \frac{m_c^2}{T^2}\right) \delta(s-m_c^2) \\
+ \frac{m_s^4(\bar{s}s\sigma Gs)^2}{48T^8} \delta(s-m_c^2),
\]

\(\bar{m}_c^2 = \frac{m_c^2}{s}, \ x_i = \frac{m_c^2}{s}\).

The QCD sum rules can be written more explicitly,

\[
2M_+ \lambda_j^+ \exp\left(-\frac{M^2}{T^2}\right) + 2M_+ \lambda_j^+ \exp\left(-\frac{M^2}{T^2}\right) = \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{ij}^{1,\text{QCD}}(s) + \rho_{ij}^{0,\text{QCD}}(s)\right] \exp\left(-\frac{s}{T^2}\right), \tag{23}
\]

\[
2M_- \lambda_j^- \exp\left(-\frac{M^2}{T^2}\right) + 2M_- \lambda_j^- \exp\left(-\frac{M^2}{T^2}\right) = \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{ij}^{1,\text{QCD}}(s) - \rho_{ij}^{0,\text{QCD}}(s)\right] \exp\left(-\frac{s}{T^2}\right). \tag{24}
\]

The contributions of the positive parity and negative parity \(\Omega_c\) states are separated explicitly.

Firstly, we choose low continuum threshold parameters \(s_0\) so as not to include the contributions of the 2S and 2P \(\Omega_c\) states (\(\Omega_c^\prime\)), and obtain the QCD sum rules for the masses of the 1S and 1P \(\Omega_c\) states,

\[
M_{+}^2 = \frac{d}{\pi(1/T^2)} \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{ij}^{1,\text{QCD}}(s) + \rho_{ij}^{0,\text{QCD}}(s)\right] \exp\left(-\frac{s}{T^2}\right), \tag{25}
\]

\[
M_{-}^2 = \frac{d}{\pi(1/T^2)} \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{ij}^{1,\text{QCD}}(s) - \rho_{ij}^{0,\text{QCD}}(s)\right] \exp\left(-\frac{s}{T^2}\right), \tag{26}
\]

then obtain the pole residues \(\lambda_j^+\) and \(\lambda_j^-\).

Now we take the masses and pole residues of the 1S and 1P \(\Omega_c\) states as input parameters, and postpone the continuum threshold parameters \(s_0\) to larger values to include the contributions of the 2S and 2P \(\Omega_c\) states, and obtain the QCD sum rules for the masses.
of the 2S and 2P $\Omega_c$ states,

\[
M^+ = \frac{\int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{J,\text{QCD}}(s) + \rho^0_{J,\text{QCD}}(s) \right] \exp \left( -\frac{s}{T^2} \right) - 2M_+ \lambda^j_+ \exp \left( -\frac{M^2}{T^2} \right)}{\int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{J,\text{QCD}}(s) + \rho^0_{J,\text{QCD}}(s) \right] \exp \left( -\frac{s}{T^2} \right) - 2M_+ \lambda^j_+ \exp \left( -\frac{M^2}{T^2} \right)},
\]

\[
M^- = \frac{\int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{J,\text{QCD}}(s) - \rho^0_{J,\text{QCD}}(s) \right] \exp \left( -\frac{s}{T^2} \right) - 2M_- \lambda^j_- \exp \left( -\frac{M^2}{T^2} \right)}{\int_{m_c^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{J,\text{QCD}}(s) - \rho^0_{J,\text{QCD}}(s) \right] \exp \left( -\frac{s}{T^2} \right) - 2M_- \lambda^j_- \exp \left( -\frac{M^2}{T^2} \right)},
\]

(27)

(28)

then obtain the pole residues $\lambda^j_+$ and $\lambda^j_-$.

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{GeV}^2$, $\langle \bar{s}G G s \rangle = (0.33 \text{GeV})^4$ at the energy scale $\mu = 1 \text{GeV}$ \cite{23,24,30,31}, $m_c(m_c) = (1.275 \pm 0.025) \text{GeV}$ and $m_s(\mu = 2 \text{GeV}) = (0.095 \pm 0.005) \text{GeV}$ from the Particle Data Group \cite{32}. The updated values from the Particle Data Group in version 2016 \cite{32} are slightly different from the corresponding ones in version 2014, we take the old values to make consistent predictions with the same parameters and criteria chosen in previous works. If we choose the updated values $m_c(m_c) = (1.28 \pm 0.03) \text{GeV}$ and $m_s(\mu = 2 \text{GeV}) = 0.096^{+0.008}_{-0.007} \text{GeV}$ \cite{32}, the central value of the predicted mass of the $\Omega_c(1S)$ is 2.6991 GeV rather than 2.6983 GeV, the predicted mass presented in Table 2 survives, so the old values are OK. The values of the $m_0^2$, $\langle \bar{s}s \rangle/\langle \bar{q}q \rangle$ and $\langle \bar{s}g_s\sigma Gs \rangle/\langle \bar{q}g_s\sigma Gq \rangle$ vary in rather large ranges from different theoretical determinations, for example, in Ref.\cite{33}, $\langle \bar{s}g_s\sigma Gs \rangle/\langle \bar{q}g_s\sigma Gq \rangle = 0.95 \pm 0.15$, which differs from the standard value $\langle \bar{s}g_s\sigma Gs \rangle/\langle \bar{q}g_s\sigma Gq \rangle = \langle \bar{s}s \rangle/\langle \bar{q}q \rangle = 0.8 \pm 0.1$ remarkably \cite{30}. In this article, we take the standard values or the old values still accepted now \cite{30,31}.

We take into account the energy-scale dependence of the input parameters from the renormalization group equation,

\[
\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^\frac{1}{3},
\]

\[
\langle \bar{s}g_s\sigma Gs \rangle(\mu) = \langle \bar{s}g_s\sigma Gs \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^\frac{2}{3},
\]

\[
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^\frac{1}{3},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^\frac{12}{25},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2^2 (\log^2 t - \log t - 1)}{b_0^2 t^2} + b_0 b_2 \right],
\]

(29)
where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2n_f}{12\pi^2} \), \( b_1 = \frac{153-19n_f}{24\pi^2} \), \( b_2 = \frac{2857-2823n_f+137n_f^2}{128\pi^4} \), \( \Lambda = 213 \) MeV, 296 MeV and 339 MeV for the flavors \( n_f = 5 \), 4 and 3, respectively \([32]\), and evolve all the input parameters to the optimal energy scales \( \mu \) to extract the masses of the \( \Omega \) states. The energy scale dependence of the quark masses and quark condensates is known beyond the leading order, the energy scale dependence of the mixed quark condensates is only known in the leading order \([34, 35]\). In this article, we take the leading order approximation in a consistent way, and take the energy scale dependence of the mixed condensates presented in Refs.\([33, 36]\), this may be our next work. For the heavy degrees of freedom, we take the favors \( n_f = 4 \), the power in the \( m_c(\mu) \) is \( \frac{22}{25} \). For the light degrees of freedom, we take the flavors \( n_f = 3 \), the powers in the \( \langle \bar{s}s\rangle(\mu) \), \( \langle \bar{g}_s\sigma Gs\rangle(\mu) \) and \( m_s(\mu) \) are \( \frac{1}{2} \) (or \( \frac{12}{27} \)), \( \frac{2}{27} \) and \( \frac{4}{27} \), respectively. If we take the favors \( n_f = 4 \), the powers in the \( \langle \bar{s}s\rangle(\mu) \), \( \langle \bar{g}_s\sigma Gs\rangle(\mu) \) and \( m_s(\mu) \) are \( \frac{2}{27} \), \( \frac{2}{27} \) and \( \frac{12}{27} \), respectively, in fact, the induced tiny difference in numerical calculations can be neglected. As far as the fine constant \( \alpha_s(\mu) \) is concerned, we choose the next-to-next-to-leading order approximation, which is consistent with the values determined experimentally \([32]\).

In Fig.1, we plot the correlation functions \( \Pi_{j,+} \) and \( \Pi_{j,-} \) with variations of the energy scales \( \mu \) and the Borel parameters \( T^2 \),

\[
\Pi_{j,+} = \int_{m_c^2}^{\infty} ds \left[ \sqrt{5} \rho^1_{j,QCD}(s) + \rho^0_{j,QCD}(s) \right] \exp \left( -\frac{s}{T^2} \right), \quad (30)
\]

\[
\Pi_{j,-} = \int_{m_c^2}^{\infty} ds \left[ \sqrt{5} \rho^1_{j,QCD}(s) - \rho^0_{j,QCD}(s) \right] \exp \left( -\frac{s}{T^2} \right). \quad (31)
\]

From the figure, we can see that the \( \Pi_{j,+} \) and \( \Pi_{j,-} \) increase remarkably with increase of the energy scale \( \mu \) at the region \( T^2 > 4.0 \) GeV\(^2\), while at the region \( T^2 < 3.0 \) GeV\(^2\), the \( \Pi_{j,+} \) and \( \Pi_{j,-} \) increase slowly with increase of the energy scale \( \mu \). All in all, we cannot obtain energy scale independent QCD sum rules, some constraints are needed to determine the energy scales of the QCD spectral densities in a consistent way.

Now we take a short digression to discuss how to choose the optimal energy scales. In the heavy quark limit, the heavy quark \( Q \) serves as a static well potential and combines with a light quark \( q \) to form a heavy diquark in color antitriplet, or combines with a light diquark in color antitriplet to form a heavy baryon in color singlet. The heavy antiquark \( \bar{Q} \) serves as another static well potential and combines with a light antiquark \( \bar{q} \) to form a heavy antidiquark in color triplet, or combines with a light antidiquark in color triplet to form a heavy antibaryon in color singlet. Then the heavy diquark and heavy antidiquark combine together to form a hidden-charm or hidden-bottom tetraquark state. The heavy baryons \( B \) and tetraquark states \( X/Y/Z \) are characterized by the effective heavy quark masses \( M_Q \) (or constituent quark masses) and the virtuality \( V = \sqrt{M_B^2 - M_Q^2} \), \( \sqrt{M_{X/Y/Z}^2 - (2M_Q)^2} \) (or bound energy not as robust). The diquark-antidiquark type baryon states and diquark-antidiquark type tetraquark states are expected to have the same effective \( Q \)-quark masses \( M_Q \), which embody the net effects of the complex dynamics \([29, 37]\). In Refs.\([29, 38]\), we study the acceptable energy scales of the QCD spectral densities for the hidden-charm (hidden-bottom) tetraquark states and molecular states.
in the QCD sum rules in details for the first time, and suggest an energy scale formula 
\[ \mu = \sqrt{\frac{M_X^{2}}{g\phi}} - (2M_m \Omega)^2 \] by setting \( \mu = V \) to determine the optimal energy scales with 
the effective heavy quark masses \( M_m \).

We fit the effective \( c \)-quark mass \( M_c \) to reproduce the experimental value of the mass of 
the \( Z_c^+(3900) \) in the scenario of tetraquark state \( [29] \). In this article, we use the empirical 
energy scale formula \( \mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi}} - (2M_m M_c)^2 \) to determine the optimal energy scales of the QCD 
spectral densities, and take the updated value of the effective \( c \)-quark mass \( M_c = 1.82 \text{ GeV} \) 
\( [39] \). For detailed discussions about the energy scale formula 
\[ \mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi}} - (2M_m M_c)^2 \], one can consult Ref.\( [37] \). According to the energy scale formula 
\[ \mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi}} - (2M_m M_c)^2 \], we extract 
the masses of the ground states (see Eqs.(25-26)) and the first radial excited states (see 
Eqs.(27-28)) at different energy scales.

In Fig.2, we plot the masses and pole residues of the \( \Omega_c(1S, \frac{1}{2}) \), \( \Omega_c(1S, \frac{3}{2}) \), \( \Omega_c(1P, \frac{1}{2}) \) and 
\( \Omega_c(1P, \frac{3}{2}) \) with variations of the energy scale \( \mu \) for the central values of the Borel 
parameters and threshold parameters shown in Table 1. From the figure, we can see 
that the predicted masses decrease monotonously but mildly with increase of the energy 
scale \( \mu \), the constraint \( \mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi}} - (2M_m M_c)^2 \) is not difficult to satisfy. On the other hand, 
the pole residues increase monotonously and mildly with increase of the energy scale \( \mu \), 
which is consistent with Fig.1, as the Borel parameters are chosen as \( T^2 < 3.0 \text{ GeV}^2 \). At 
the vicinities of the energy scales presented in Table 1, the uncertainties induced by the 
uncertainties of the energy scales are tiny.

For the \( Z_c(3900) \), the uncertainty of the energy scale of the QCD spectral density is 
about \( \delta \mu = 0.1 \text{ GeV} \), the uncertainty of the effective \( c \)-quark mass \( M_c \) can be estimated 
as \( \delta M_c = \frac{M_c}{\mu_0} \delta \mu = 0.02 \text{ GeV} \) from the equation,

\[
\mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi} - (2M_m M_c)^2} = \sqrt{\frac{M_{\Omega_c}^2}{g\phi} - 4M_c^2} \sqrt{1 + \frac{8M_c^2 \delta M_c}{M_{\Omega_c}^2}} = \mu_0 \pm \frac{M_c}{\mu_0} \delta \mu ,
\]

where the \( \mu_0 \) is the central value. The uncertainties \( \delta \mu \) in this article can be estimated as 
\( \delta \mu = \frac{M_c}{\mu_0} \delta M_c < 0.02 \text{ GeV} \) from the equation,

\[
\mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi} - (2M_m M_c)^2} = \mu_0 \pm \frac{M_c}{\mu_0} \delta M_c .
\]

The predicted masses and pole residues are not sensitive to variations of the energy scales, 
the small uncertainty \( \delta M_c = 0.02 \text{ GeV} \) or \( \delta \mu < 0.02 \text{ GeV} \) can be neglected safely.

We search for the ideal Borel parameters \( T^2 \) and continuum threshold parameters \( s_0 \) 
according to the four criteria:

1. Pole dominance at the hadron side, the pole contributions are about \( (40 - 70)\% \);  
2. Convergence of the operator product expansion, the dominant contributions come 
from the perturbative terms;  
3. Appearance of the Borel platforms;  
4. Satisfying the energy scale formula \( \mu = \sqrt{\frac{M_{\Omega_c}^2}{g\phi}} \), 
by try and error, and present the optimal energy scales \( \mu \), ideal Borel parameters \( T^2 \),
Figure 1: The correlation functions with variations of the energy scales $\mu$ and Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ correspond to the $\Pi_{\frac{1}{2}+}$, $\Pi_{\frac{3}{2}+}$, $\Pi_{\frac{1}{2}-}$ and $\Pi_{\frac{3}{2}-}$, respectively.
continuum threshold parameters $s_0$, pole contributions and perturbative contributions in Table 1. From Table 1, we can see that the criteria 1 and 2 can be satisfied, the two basic criteria of the QCD sum rules can be satisfied, and we expect to make reliable predictions.

We take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the 1S, 1P, 2S and 2P $\Omega_c$ states, which are shown explicitly in Table 2. From Table 2, we can see that the criterion 4 can be satisfied. In Figs.3-4, we plot the masses and pole residues of the 1S, 1P, 2S and 2P $\Omega_c$ states with variations of the Borel parameters $T^2$ at much larger intervals than the Borel windows shown in Table 1. In the Borel windows, the uncertainties originate from the Borel parameters $T^2$ are very small, the Borel platforms exist, the criterion 3 can be satisfied. Now the four criteria are all satisfied, and we expect to make reliable predictions. In the Borel windows, the uncertainties of the predicted masses are about $(3-5)\%$, as we obtain the masses from a ratio, see Eqs.(25-28), the uncertainties originate from a special parameter in the numerator and denominator cancel out with each other, so the net uncertainties are very small. On the other hand, the uncertainties of the pole residues are about $(10-16)\%$, which are much larger. The uncertainties $\delta\lambda_{\Omega_c}$ are compatible with the uncertainties of the decay constants $f_\pi = 127 \pm 15$ MeV and $f_\rho = 213 \pm 20$ MeV from the QCD sum rules [31].

In Table 2, we also present the experimental values [1, 32]. The present predictions support assigning the $\Omega_c(3000)$ to be the 1P $\Omega_c$ state with $J^P = \frac{1}{2}^-$, assigning the $\Omega_c(3090)$ to be the 1P $\Omega_c$ state with $J^P = \frac{3}{2}^-$ or the 2S $\Omega_c$ state with $J^P = \frac{3}{2}^+$, and assigning the $\Omega_c(3119)$ to be the 2S $\Omega_c$ state with $J^P = \frac{3}{2}^+$ (or the 1P $\Omega_c$ state with $J^P = \frac{5}{2}^-$ [8]). The present predictions indicate that the 1P $\Omega_c$ state with $J^P = \frac{3}{2}^-$ and the 2S $\Omega_c$ state with $J^P = \frac{1}{2}^+$ have degenerate masses, it is difficult to distinguish them by the masses alone, we have to study their strong decays. Other predictions can be confronted to the experimental data in the future.

In Refs.[2, 5], Agaev, Azizi and Sundu study the $\Omega_c$ states by taking into account the 1S, 1P, 2S states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions, and assign the $\Omega_c(3000)$,
|       | $J^P$ | $\mu$(GeV) | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole          | perturbative |
|-------|-------|------------|----------------|------------------|---------------|--------------|
| $\Omega_c(1S)$ | $\frac{1}{2}^+$ | 2.0 | 2.3 – 2.9 | 3.30 ± 0.10 | (41 – 69)% | (86 – 90)% |
| $\Omega_c(1S)$ | $\frac{3}{2}^+$ | 2.1 | 2.4 – 3.0 | 3.40 ± 0.10 | (46 – 72)% | (87 – 91)% |
| $\Omega_c(1P)$ | $\frac{1}{2}^+$ | 2.4 | 2.2 – 2.8 | 3.40 ± 0.10 | (40 – 68)% | (117 – 130)% |
| $\Omega_c(1P)$ | $\frac{3}{2}^-$ | 2.5 | 2.2 – 2.8 | 3.50 ± 0.10 | (39 – 67)% | (106 – 114)% |
| $\Omega_c(2S)$ | $\frac{1}{2}^+$ | 2.5 | 2.6 – 3.2 | 3.45 ± 0.10 | (43 – 68)% | (90 – 93)% |
| $\Omega_c(2S)$ | $\frac{3}{2}^+$ | 2.5 | 2.7 – 3.3 | 3.50 ± 0.10 | (45 – 69)% | (90 – 93)% |
| $\Omega_c(2P)$ | $\frac{1}{2}^-$ | 2.9 | 2.4 – 3.0 | 3.70 ± 0.10 | (53 – 78)% | (111 – 118)% |
| $\Omega_c(2P)$ | $\frac{3}{2}^-$ | 2.9 | 2.4 – 3.0 | 3.75 ± 0.10 | (49 – 75)% | (104 – 108)% |

Table 1: The optimal energy scales $\mu$, Borel parameters $T^2$, continuum threshold parameters $s_0$, pole contributions (pole) and perturbative contributions (perturbative) for the $\Omega_c$ states.

|       | $J^P$ | $M$(GeV) | $\lambda$(GeV$^3$) | (expt) (MeV) |
|-------|-------|----------|---------------------|-------------|
| $\Omega_c(1S)$ | $\frac{1}{2}^+$ | 2.70$^{+0.11}_{-0.13}$ | 1.09$^{+0.17}_{-0.15}$ $\times 10^{-1}$ | 2695.2 |
| $\Omega_c(1S)$ | $\frac{3}{2}^+$ | 2.76$^{+0.11}_{-0.12}$ | 0.64$^{+0.09}_{-0.08}$ $\times 10^{-1}$ | 2765.9 |
| $\Omega_c(1P)$ | $\frac{1}{2}^+$ | 3.02$^{+0.12}_{-0.12}$ | 0.90$^{+0.13}_{-0.10}$ $\times 10^{-1}$ | ? 3000.4 |
| $\Omega_c(1P)$ | $\frac{3}{2}^-$ | 3.09$^{+0.08}_{-0.06}$ | 0.29$^{+0.04}_{-0.04}$ $\times 10^{-1}$ | ? 3090.2 |
| $\Omega_c(2S)$ | $\frac{1}{2}^+$ | 3.09$^{+0.11}_{-0.12}$ | 0.82$^{+0.09}_{-0.09}$ $\times 10^{-1}$ | ? 3090.2 |
| $\Omega_c(2S)$ | $\frac{3}{2}^-$ | 3.12$^{+0.12}_{-0.12}$ | 0.37$^{+0.03}_{-0.04}$ $\times 10^{-1}$ | ? 3119.1 |
| $\Omega_c(2P)$ | $\frac{1}{2}^+$ | 3.40$^{+0.10}_{-0.10}$ | 0.91$^{+0.09}_{-0.09}$ $\times 10^{-1}$ | |
| $\Omega_c(2P)$ | $\frac{3}{2}^-$ | 3.46$^{+0.10}_{-0.11}$ | 0.27$^{+0.04}_{-0.03}$ $\times 10^{-1}$ | |
| $\Omega_c(1P)$ | $\frac{5}{2}^-$ | 3.11$^{+0.10}_{-0.10}$ | 1.07$^{+0.17}_{-0.17}$ $\times 10^{-1}$GeV | ? 3119.1 |

Table 2: The masses and pole residues of the $\Omega_c$ states, the masses are compared with the experimental data, the values of the $\Omega_c(1P)$ with $J^P = \frac{5}{2}^-$ are taken from Ref.[8].
\(\Omega_c(3050), \Omega_c(3066)\) and \(\Omega_c(3119)\) to be the \((1P, \frac{1}{2}^-), (1P, \frac{3}{2}^-), (2S, \frac{1}{2}^+)\) and \((2S, \frac{3}{2}^+)\) states, respectively. In Ref.\[12\], Aliiev, Bilmis and Savci use the same interpolating currents to study the \(\Omega_c\) states by taking into account the 1S and 1P states with \(J = \frac{1}{2}\) and \(\frac{3}{2}\) in the pole contributions, and assign the \(\Omega_c(3000)\) and \(\Omega_c(3066)\) to be the \((1P, \frac{1}{2}^-)\) and \((1P, \frac{3}{2}^-)\) states, respectively. In Refs.\[2, 5\]\[12\], the contributions of the \(\Omega_c\) states with positive parity and negative parity are not separated explicitly, there are some contaminations from the 2S or 1P states. In Ref.\[8\], we separate the contributions of the positive parity and negative parity \(\Omega_c\) states explicitly, and study the new excited \(\Omega_c\) states with the QCD sum rules by introducing an explicit P-wave involving the two s quarks. The predictions support assigning the \(\Omega_c(3050), \Omega_c(3066), \Omega_c(3090)\) and \(\Omega_c(3119)\) to be the P-wave \(\Omega_c\) states with \(J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^-\) and \(\frac{5}{2}^-\), respectively. Compared with Refs.\[2, 5\]\[12\], the methods used in the present work and Ref.\[8\] have the advantage that the contributions of the \(\Omega_c\) states with positive parity and negative parity are separated explicitly, there are no contaminations from the 2S or 1P states.

In the diquark-quark models for the heavy baryon states, the angular momentum between the two light quarks is denoted by \(L_\rho\), while the angular momentum between the light diquark and the heavy quark is denoted by \(L_\lambda\). In Refs.\[2, 5\]\[12\] and present work, the currents with \(L_\rho = L_\lambda = 0\) are chosen to explore the P-wave \(\Omega_c\) states, although the currents couple potentially to the P-wave \(\Omega_c\) states, we are unable to know the substructures of the P-wave \(\Omega_c\) states, and cannot distinguish whether they have \(L_\lambda = 1\) or \(L_\rho = 1\). In Ref.\[8\], we choose the currents with \(L_\lambda = 1\) to interpolate the \(\Omega_c\) states, and obtain the predicted masses \((3.06 \pm 0.11)\) GeV and \((3.06 \pm 0.10)\) GeV for the \(J^P = \frac{3}{2}^-\) \(\Omega_c\) states with slightly different substructures, which support assigning the \(\Omega_c(3066)\) and \(\Omega_c(3090)\) to be the P-wave \(\Omega_c\) states with \(J^P = \frac{3}{2}^-\) and \(L_\lambda = 1\). While in the present work, we obtain the mass \(3.09^{+0.08}_{-0.06}\) GeV for the \(J^P = \frac{3}{2}^-\) \(\Omega_c\) state. If we take the central values of the predicted masses as references, the \(\Omega_c(3066)\) and \(\Omega_c(3090)\) can be tentatively assigned to be the \(\frac{3}{2}^-\) \(\Omega_c\) states with \(L_\lambda = 1\) and \(L_\rho = 1\), respectively. However, the assignment \(\Omega_c(3090) = (2S, \frac{1}{2}^+)\) is also possible according to the predicted mass \(3.09^{+0.11}_{-0.12}\) GeV for the \((2S, \frac{1}{2}^+)\) state.

Now we summarize the assignments based on the QCD sum rules in Table 3. From Table 3, we can see that all the calculations based on the QCD sum rules support assigning the \(\Omega_c(3000)\) to be the 1P \(\frac{1}{2}^-\) state, while the assignments of the other \(\Omega_c\) states are under debate. We have to study the decay widths to make the assignments on more solid foundation. In Ref.\[5\], Agaev, Azizi and Sundu study the decays of the \(\Omega_c\) states to the \(\Xi^+_c K^-\) by calculating the hadronic coupling constants \(g_{\Omega_c \Xi K}\) with the light-cone QCD sum rules, however, they use an over simplified hadronic representation and neglect the contributions of the excited \(\Xi_c\) states.

Experimentally, we can search for those new excited \(\Omega_c\) states through strong decays and electromagnetic decays to the final states \(\Xi^+_c K^-\), \(\Xi^0 c\bar{K}^0\), \(\Xi^+ c\bar{K}^-\), \(\Xi^0 c\bar{K}^0\), \(\Xi^+_c c\bar{K}^-\), \(\Xi^0 c\bar{K}^0\), \(\Xi^- D^+\), \(\Xi^0 D^0\), \(\Omega_c(2695)\gamma\), \(\Omega_c(2770)\gamma\), and measure the branching fractions precisely, which can shed light on the nature of those \(\Omega_c\) states. More theoretical works on the partial decay widths based on the QCD sum rules are still needed.
Figure 3: The masses of the $\Omega_c$ states with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$ correspond to the $\Omega_c$ states with the quantum numbers $(1S, \frac{1}{2}^+)$, $(1S, \frac{3}{2}^+)$, $(1P, \frac{1}{2}^-)$, $(1P, \frac{3}{2}^-)$, $(2S, \frac{1}{2}^+)$, $(2S, \frac{3}{2}^+)$, $(2P, \frac{1}{2}^-)$ and $(2P, \frac{3}{2}^-)$, respectively.
Figure 4: The pole residues of the $\Omega_c$ states with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$ correspond to the $\Omega_c$ states with the quantum numbers $\left(1S, \frac{1}{2}^+\right)$, $\left(1S, \frac{3}{2}^+\right)$, $\left(1P, \frac{1}{2}^+\right)$, $\left(1P, \frac{3}{2}^-\right)$, $\left(2S, \frac{1}{2}^+\right)$, $\left(2S, \frac{3}{2}^-\right)$, $\left(2P, \frac{1}{2}^+\right)$ and $\left(2P, \frac{3}{2}^-\right)$, respectively.
| $J^P$ | $nL$ | References |
|-------|-------|------------|
| $\Omega_c(3000)$ | $\frac{1}{2}^-$ | 1P | [2, 5, 8, 12, 21] and This Work |
| $\Omega_c(3050)$ | $\frac{3}{2}^-$ | 1P | [2, 5] |
| $\Omega_c(3050)$ | $\frac{1}{2}^+$ | 1P | [8] |
| $\Omega_c(3066)$ | $\frac{1}{2}^+$ | 1P | [8, 12] |
| $\Omega_c(3066)$ | $\frac{1}{2}^+$ | 2S | [2, 5] |
| $\Omega_c(3090)$ | $\frac{1}{2}^+$ | 1P | [8] and This Work |
| $\Omega_c(3090)$ | $\frac{1}{2}^+$ | 2S | This Work |
| $\Omega_c(3119)$ | $\frac{3}{2}^+$ | 2S | [2, 5] and This Work |
| $\Omega_c(3119)$ | $\frac{5}{2}^-$ | 1P | [8] |

Table 3: The possible assignments of the new $\Omega_c$ states based on the QCD sum rules.

4 Conclusion

In this article, we distinguish the contributions of the S-wave and P-wave $\Omega_c$ states unambiguously, study the masses and pole residues of the 1S, 1P, 2S and 2P $\Omega_c$ states with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ using the QCD sum rules in a consistent way, and revisit the assignments of the new narrow excited $\Omega_c$ states. The present predictions support assigning the $\Omega_c(3000)$ to be the 1P $\Omega_c$ state with $J^P = \frac{1}{2}^-$, assigning the $\Omega_c(3090)$ to be the 1P $\Omega_c$ state with $J^P = \frac{3}{2}^-$ or the 2S $\Omega_c$ state with $J^P = \frac{1}{2}^+$, and assigning the $\Omega_c(3119)$ to be the 2S $\Omega_c$ state with $J^P = \frac{3}{2}^+$. The present predictions indicate that the 1P $\Omega_c$ state with $J^P = \frac{3}{2}^-$ and the 2S $\Omega_c$ state with $J^P = \frac{1}{2}^+$ have degenerate masses, it is difficult to distinguish them by the masses alone, we have to study their strong decays. Other predictions can be confronted to the experimental data in the future.

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