Branes and Six Dimensional Fixed Points

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Abstract

We analyze brane configurations corresponding to non-trivial six dimensional fixed points. Several results previously obtained from a pure field theoretical analysis are rederived in the brane language.
1. Introduction

Recently, the world volume of branes was used to study field theories in various dimensions with different amount of supersymmetry [5,2,3,4,7,12,15,17]. The original construction in [5] was invented to study field theories in 2+1 dimensions with $N = 4$ supersymmetry. The main idea used in that paper is to stretch $N_c$ parallel 3 branes of IIB string theory between NS 5 branes and to study the world volume theory of the 3 branes, which is an $SU(N_c)$ gauge theory. Matter is included by introducing additional D5 branes, which give rise to fundamental fields when they intersect the 3 branes. This setup was modified to study 4 dimensional $N = 1$ theories in [3]. In that paper Seiberg’s duality was interpreted in the brane setup. The introduction of orientifolds into the setup of [3] enabled the authors of [15] to study also $SO$ and $Sp$ groups. In [7] many more 4d, $N = 1$ field theory phenomena such as product gauge groups and superpotentials were given an interpretation in a brane picture. $d = 4$ theories with $N = 2$ are studied in [6]. Here also quantum effects are interpreted in the brane setup by considering the bending of the higher dimensional branes when the lower dimensional branes end on them. In [2] the construction of 5-dimensional $N = 1$ theories via branes is discussed. The authors construct new 5-dimensional superconformal field theories. Their paper contains also results on $d = 3$, $N = 2$ theories, which were also studied in [4]. In this paper, we will study 6-dimensional $N = 1$ field theories in the brane setup. In section 2 several aspects of the brane constructions in various dimensions are reviewed. The 5-dimensional theories of [2] are discussed in some more detail in section 3. Here, we also check the consistency of the construction in [2] using orientifolds. Section 4 contains the 6-dimensional analysis. We show how 6-dimensional $SU$, $SO$ and $Sp$ gauge theories with a tensor multiplet can be described in the brane picture. We compare our results to the field theory results of [10] and [8].

2. Hanany-Witten in various dimensions

The Hanany-Witten [5] setup yields an $N = 4$ supersymmetric theory in three dimensions. Its basic ingredients are IIB NS 5 branes, D5 branes and D3 branes. The worldvolumes of the various branes occupy the following directions:

|       | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| NS 5  | x     | x     | x     | x     | x     | x     | o     | o     | o     | o     |
| D 5   | x     | x     | x     | o     | o     | o     | o     | x     | x     | x     |
| D 3   | x     | x     | x     | o     | o     | o     | x     | o     | o     | o     |

In the $x^6$ direction the 3 branes are suspended between 5 branes. The effective low energy theory is the 2+1 dimensional field theory living on the extended 3 brane directions. Fluctuations of the larger branes are considered much heavier than those of the 3 branes. Moving around the higher dimensional branes hence corresponds to changing parameters of the low energy theory, moving around the 3 branes corresponds to changing moduli of the
theory. The idea behind this approach can be summarized as the statement, that the low energy dynamic is determined by the lowest dimensional brane in the setup.

The simplest realization of the Hanany-Witten setup is a configuration of $N_c$ 3 branes suspended between two NS 5 branes. An analysis of the boundary conditions at the intersection of the branes shows that this configuration describes a pure $N = 4$ supersymmetric $SU(N_c)$ gauge theory in three dimensions. There are two ways to incorporate flavors into this theory. One is to add D5 branes. Whenever a D5 brane approaches the stretched D3 branes we obtain a massless flavor hypermultiplet from strings stretching between the D5 and the D3 brane. The other approach is to add semi-infinite 3 branes stretching to the right of the right or to the left of the left NS brane [6]. The flavor multiplets arise from 3-3 strings stretching from a semi-infinite 3 brane to the finite 3 branes. Since the worldvolume of these semi-infinite branes has 4 infinite directions they do not participate in the low energy dynamics according to the general philosophy that only the lowest dimensional objects determine the low energy effective theory. Their motion again corresponds to parameters of the theory. Latter approach has the advantage that it is in general easier to deal with those configurations since we avoid introducing the third kind of brane. But we have to pay a price. Using semi-infinite 3 branes makes the Higgs branch of the theory very hard to see. ¹

There are two ways to generalize this setup. One is to rotate the 5 branes [3,7,12]. This reduces the supersymmetry from $N = 4$ to $N = 2$. The other is to apply T-duality to a direction in the NS 5 brane worldvolume ² and transverse to the D3 brane [6]. The resulting configuration is a IIA D4 brane stretched between the two NS branes. Flavors can again be added by incorporating D6 branes or semi-infinite D4 branes. The low energy theory is a four dimensional $N = 2$ theory. A combination of these two approaches has been pioneered in [3] to yield a $d = 4$, $N = 1$ theory.

It is natural to T dualize once more to get $d = 5$, $N = 1$ from IIB D5 branes still stretched between the two NS branes. Note that this is not possible in the rotated (hence $N = 1$) theory since we would T dualize in a direction transverse to the rotated NS brane. This is consistent with the fact that $N = 1$ is the minimal supersymmetry in 5 dimensions. This step has been taken by [2]. We will review the parts of their construction relevant for our analysis in the next section.

One can T dualize once more to obtain IIA 6 branes suspended between the two NS 5 branes. This will be the subject of this paper.

In all the configurations obtained by T-dualizing the original Hanany-Witten setup, there is an unbroken $SO(3)$ subgroup of the 10d Lorentz group corresponding to rotations in the 789 plane. This corresponds to the $SU(2)_R$ symmetry present in all theories with the equivalent of $N = 2$ in $d = 4$.

¹We can change a flavor from a D5 brane to a flavor from a semi-infinite 3 brane by moving the D5 brane off to infinity. Higgs branches are associated to motion of D3 branes connecting the flavor giving D5 branes with the various other branes. Moving off the flavor giving D5 brane to infinity also moves the Higgs branches off to infinity as well.

²A NS 5 brane only stays a NS 5 brane under T-dualities in one of its worldvolume directions.
2.1. Bending of branes

From what we have said so far it is clear that the configurations we are dealing with are “color” D d branes suspended between two NS 5 branes of IIA(B) theory for even(odd) d, giving rise to a d-1 dimensional low energy field theory. Even though we always considered the 5 branes as being flat, this classical picture is not the full story, since the d brane ending on the 5 brane creates a dimple. The form of this deformation depends crucially on d.

The end of the d brane looks like a \( d-1 \) brane in the NS brane worldvolume or as a charged particle in the \( 6 - d \) dimensional subspace of the NS worldvolume transverse to the d brane worldvolume. Generalizing the description of this phenomenon in [6] one would like to describe the bending of the NS brane as an equation

\[ x^6 = x^6(y) \]

where \( y \) denotes a vector in the 6-d dimensional transverse NS space. Since in \( y \) space the end of the d brane looks like a charged particle we expect \( x^6 \) to satisfy the following equation:

\[ \Delta x^6 = 0 \]

away from the ends of the d branes. The behaviour in various dimensions can be analyzed by looking at the Green’s function of the Laplacian in the corresponding \( 6 - d \) dimensional transverse NS space.

The net charge is proportional to the number of d branes ending on the left of a given NS brane minus the number of d branes ending on the right, since they contribute charges of opposite sign.

- \( d = 3 \): the Green’s function in the 3 dimensional transverse NS space goes like \( x^6 = \frac{1}{|y|} + \text{constant} \). Far away from the disturbance the position of the NS brane approaches a definite value. It makes sense to talk about the position of the NS brane. The inverse coupling constant is given by the distance between these brane positions.

- \( d = 4 \): the Green’s function in 2 dimensions yields a logarithm. This is the case discussed in [6]. The distance between the two NS branes does not reach a finite value but diverges logarithmically for large \( |y| \). Witten interprets this as the sign of asymptotic freedom in the brane picture. The distance in \( y \) sets the energy scales one is probing. Asymptotic freedom is manifest through the logarithmic divergence of the inverse gauge coupling. The coefficient in front of the logarithm is given by the difference of the net charges of the two NS branes. This coincides with the one-loop beta function of the corresponding field theory.

- \( d = 5 \): in 1 dimension we have a linear Green’s function yielding piecewise straight branes. The gauge coupling is piecewise linear as expected from the field theory analysis. We will say more about this case in the next section.

- \( d = 6 \): we can not have a net charge in 0 dimensions. The brane describes a consistent theory only if the net charge on every 5 brane vanishes.
3. The 5d analysis

3.1. The building blocks

Let us briefly review the analysis of [2] of the 5 dimensional setup. We are dealing with NS 5 branes still along the 012345 coordinates and D5 branes long 012346, stretched between the NS branes in the 6 direction. (This convention differs slightly from the one used in [2], where they take the NS brane to live in 012389 and the D5 in 01789. Our notation is the one which should be obvious if one constructs this setup by applying T-duality to the original Hanany-Witten setup.) Flavors will be incorporated by semi-infinite 5 branes. In 5d the alternative would be including D7 branes in the 01234789 direction. The introduction of such objects would lead to new complications, since these objects affect the asymptotic space-time geometry. So we avoid them by using the semi-infinite 5 branes to add the flavors. As a trade back, we loose the brane realization of the Higgs branches, as mentioned above.

Next let’s discuss the bending of the branes in more detail. We already saw in the last section, that we are always dealing with straight branes. They change their orientation, when they hit another 5 brane. So one has to deal with branes living in the full 01234 space and describing straight lines in the 56 plane. An NS brane only lives in the 5 and a D brane only in the 6 direction. We can classify them by their charge under the 2-form gauge potentials present in the IIB setup. The NS brane is usually called a (1,0) brane and the D5 brane is the (0,1) brane. There are also bound states of D and NS branes, (p,q) branes for arbitrary p and q relatively prime. They behave exactly like ordinary 5 branes, since they are related by a $SL(2,\mathbb{Z})$ duality transformation. $(np,nq)$ branes can be viewed as n copies of a (p,q) brane lying on top of each other. Such a (p,q) brane also lives in the full 01234 space. The line it describes in the 56 plane is given by the equation

$$px^6 + qx^5 = \text{constant}.$$ 

Charge conservation tells us that whenever n (0,1) branes end on the left of m (1,0) branes, the m (1,0) branes will be “bended” to a (m,n) brane (for m,n not relatively prime, this again has to be interpreted as several branes lying on top of each other). This agrees with the solution of the Laplace equation with a source term of strength $\frac{n}{m}$. An example of this effect is shown in figure 1, which we borrowed from [2].
3.2. The low energy field theory

According to the general philosophy behind the field theoretic analysis one should now look for the lowest dimensional objects in this setup, since they determine the low energy field theory description. In all the lower dimensional examples these objects were the color d branes suspended between the NS branes. Here the situation becomes more complicated: all finite 5 brane pieces should appear on an equal footing. Only the semi-infinite 5 branes are heavy and do not participate in the low energy dynamics. Hence in this setup brane motions that leave the asymptotic form of the 5 branes invariant should be viewed as moduli of the theory while a change in the asymptotic behaviour of the semi-infinite 5 branes corresponds to changing the parameters of the theory.

To analyze this in more detail, let us start as in [2] with the setup that naively would correspond to a pure $SU(2)$ gauge theory: Two D5 branes suspended between two NS branes. Taking into account the linear bending, the situation is summarized in figure 2, which we borrowed from [2]. Even though we expect that we get contributions from the finite (1,0) pieces as well as from the finite (0,1) pieces, it is easy to see that there is only one deformation of this configuration which does not change the asymptotic forms of the branes, corresponding to shrinking the rectangle homogeneously in the 5 and 6 direction and thus going from configuration (a) to configuration (b). This one parameter deformation can still be interpreted as the Coulomb branch of an $SU(2)$ gauge theory.
Figure 2: Pure $SU(2)$ gauge theory in five dimensions. Horizontal lines represent D5 branes, vertical lines represent NS 5 branes, and diagonal lines at an angle $\theta$ such that $\tan(\theta) = p/q$ represent $(p, q)$ 5 branes. Figure (a) shows a generic point on the Coulomb branch, figure (b) shows a point near the origin of moduli space, and figure (c) corresponds to the strong coupling fixed point. Figure borrowed from [2]

When the two D5 branes meet, that is the rectangle degenerates to a line, we expect the full $SU(2)$ symmetry to be restored. The distance between the two vertices, where the semi-infinite 5 branes meet corresponds to the inverse bare gauge coupling $1/g_0^2$. For finite $g_0^2$ this corresponds to a trivial IR fixed point. For infinite $g_0^2$ this theory was argued to have a non-trivial fixed point in [9]. This fixed point corresponds to figure (c). This situation can be generalized to the claim of [2] that every configuration of $n$ semi-infinite half 5 branes emanating from the same point correspond to a five dimensional IR fixed point. Some of these fixed points may be trivial (for example for $n=2$). In the rest of this paper we will assume that this is not the case for $n>2$. It is also an open question, if all fixed points generated this way are actually distinct.

3.3. The fixed point theories

Aharony and Hanany studied the possible deformations of these fixed points. Of special interest are deformations that correspond to Coulomb branches coming out of the fixed point, like in the $SU(2)$ example reviewed above. If after going to finite coupling these Coulomb branches include only areas bounded in the 5 direction by $N_c$ parallel D5 branes, these branches have a natural interpretation as Coulomb branches of an $SU(N_c)$ gauge theory. In addition there are several fixed points which have Coulomb branches which can not be associated to Coulomb branches of an gauge theory in any obvious way and fixed points which do not have any Coulomb branches at all. There are no deformations leaving the heavy branes untouched corresponding to the Higgs branches of the gauge theories. This is not too surprising, since we produced our flavors by semi-infinite branes. This moved the Higgs branches off to infinity, as discussed earlier. They are associated to motion of semi-infinite branes which appear as parameters of the theory, not as moduli.

Given this brane realization of non-trivial IR fixed points, an interesting question to ask is: given a field theory with certain gauge group and matter content, does it allow a non-trivial fixed point when we take the coupling to infinity. This question was analyzed
by Intriligator, Seiberg and Morrison from the field theory point of view. They found that the answer is yes if the number of flavors satisfies an upper bound. Aharony and Hanany reproduced this answer for SU groups in the brane picture. There is a slight discrepancy for SU(2), which we will discuss later. The point is that one has to avoid configurations like in figure 3, which we again borrowed from [2].

![Figure 3](image)

**Figure 3:** An attempt to construct the SU(2) theory with \(N_f = 5\). Figure borrowed from [2]

Even though we started out with two parallel D5 branes, the resulting gauge theory is no longer SU(2) since we get additional degrees of freedom from the finite branes in the triangle in the lower part of the picture. To avoid configurations like this, the branes extending to infinity in the positive or negative 5 direction should tend away from each other to avoid additional intersection points like in the example discussed above. The limiting case is if the branes come in parallel from positive infinity and leave parallel to negative infinity. Any additional flavor (a semi-infinite brane stretching purely in the 6 direction) would bend them towards each other and enforce an additional intersection point. It is easy to see, that this limit of parallel branes corresponds to having \(N_c\) flavors from the left and \(N_c\) flavors from the right. Taking the coupling to zero (i.e take the parallel branes to be on top of each other) in these configurations we get an interacting fixed point. Hence we get the condition \(N_f \leq 2N_c\) for the existence of a brane configuration corresponding to a non-trivial fixed point for a given SU(\(N_c\)) gauge group. This corresponds exactly to the condition found in [1] for \(N_c \geq 3\). The \(N_c = 2\) case is exceptional and allows also fixed points for \(N_f = 5, 6, 7, 8\). They have to correspond to configurations that do not correspond to an SU(2) gauge theory in the obvious manner described so far. Note that it would be worse if we would have found a fixed point that corresponded to a gauge theory which is not allowed to have a fixed point from gauge
theory considerations. While the problem encountered in the $N_c = 2$ case is just a matter of completeness (not all gauge theory fixed points can be realized as a brane configuration in an obvious way) the latter is a matter of consistency. It would be in contradiction to the assumption that all configurations of this type lead to non-trivial fixed points.

3.4. $SO$ and $Sp$ groups

As a consistency check of the construction of [2] we have reviewed so far we will generalize the setup to $SO$ and $Sp$ groups. We again want to analyze the conditions on the matter content of a given gauge group that has to be satisfied in order to have a brane construction that corresponds to this gauge theory in the obvious way described above and develops a non-trivial fixed point at infinite coupling. We again find an upper bound on the number of flavors which corresponds exactly to the one found in the field theory analysis of [1].

As discussed by various authors [13,14,15,4] $Sp$ and $SO$ groups can be realized by including an orientifold plane along the suspended color d branes, hence in our case an O5 plane. The charge of the O5 is $\pm 1$ the charge of a physical D brane, depending on the sign of $\Omega^2$ [16]. For $\Omega^2 = 1$ the gauge group associated to $N_c/2$ physical D branes on top of the O5 is $SO(N_c)$ and the charge of the O5 is $-1$. For $\Omega^2 = -1$ we get a $Sp(N_c)$ gauge theory and the O5 charge is $+1$. Whenever the O5 crosses an NS brane $\Omega^2$ changes sign and hence also the charge [15]. This way the orientifold contributes to the bending of branes. Without this sign flip it would just pull equally from both sides and leave the orientation of the NS brane unchanged. But taking into account the sign flip, it pushes from one side and pulls from the other side, effectively increasing the number of physical branes by one on the one side and decreasing it by one on the other side.

Consider the case where $\Omega^2 = +1(-1)$ between the NS branes 4 For $N_c/2$ color D5 branes we get a $SO(N_c)$ ($Sp(N_c)$) gauge theory from strings stretching between the branes and their mirrors. Each flavor brane (semi-infinite 5 brane to the left or right) gives us the equivalence of 2 chiral multiplets in $d = 4$, $N = 1$, that is one hypermultiplet. The charge of the O5 brane is the opposite (the same) as the charge of a physical brane, that is a brane-mirror pair between the NS branes, in the interior, and the same (the opposite) left of the left and right of the right NS brane, in the exterior. The limiting case discussed in the last section (parallel branes extending to positive and negative infinity in the 5 direction) is achieved, if for every NS brane the total interior charge (D5 branes and O5) is the same as the total exterior charge. If $n_l$ denotes the number of flavor branes from the left and $n_r$ the number of flavor branes from the right, this tells us:

$$n_l + 1 = n_r + 1 = N_c/2 - 1$$

$$n_l - 1 = n_r - 1 = N_c/2 + 1$$

3To be able to treat $SO$ and $Sp$ groups on an equal footing and not to introduce an additional factor of 2 we use the convention, that $Sp(N_c)$ denotes the group with the $N_c$ dimensional fundamental representation, $SU(2)$ is $Sp(2)$. Hence the $Sp$ groups make only sense for even $N_c$ while the $SO$ groups are also defined for odd $N_c$. The reader should keep this in mind, since we won’t mention it anymore from now on.

4We have to choose our configuration in such a way, that at the point where the O5 crosses the part of the 5 brane polymer extended in the 5 direction, the 5 brane is actually $(1,0)$, i.e a NS brane. Otherwise the polymer would not have a mirror symmetry with the O5 as a symmetry axis.
From this we obtain the condition that \( N_f = n_l + n_r \), the total number of hypermultiplets, has to satisfy the relation \( N_f \leq N_c - 4 \) (\( N_f \leq N_c + 4 \)) for \( SO(N_c) \) (\( Sp(N_c) \)) gauge group in order to get a corresponding brane construction of a non-trivial fixed point. These coincide precisely with the conditions found from pure field theoretic arguments in [1].

4. The 6d analysis

4.1. The building blocks

There is still one more T-duality we can apply without destroying our NS 5 branes, namely T-dualizing the 5 direction. This takes us once more to type IIA and the following setup of branes: two NS 5 branes as usual along the 012345 coordinates. Stretched between them \( N_c \) D6 branes with a worldvolume along the 0123456 coordinates. In addition there are \( n_l \) semi-infinite 6 branes ending on the left NS brane and \( n_r \) semi-infinite 6 branes ending on the right. We will again call them the \( N_f = n_l + n_r \) flavor branes. The configuration is shown schematically in figure 4.

![Figure 4](image)

Figure 4: The brane configuration under consideration, giving rise to a 6 dimensional field theory. Horizontal lines represent D6 branes, the crosses represent NS 5 branes.

As discussed earlier, there is no way the 5 branes can support a nonvanishing charge from the ends of the 6 branes, since the charged objects, the ends of the 6 branes, are again 5 branes filling the whole worldvolume. Therefore we must have \( n_l = n_r = N_c \) and hence \( N_f = 2N_c \). This condition should arise in the low energy field theory as an anomaly requirement. This is precisely what happens, as we will see later. We can do the same analysis for \( SO \) (\( Sp \)) groups. Since the case we are interested in is again the case, where the total charge on the left of an given NS brane is the same as on the right, we can almost copy the calculation from the end of the last section. The only difference is that we are now dealing with O6 orientifolds instead of O5 orientifolds. Taking into account that the charge of the O6 is twice that of the O5, we obtain the conditions \( N_f = N_c - 8 \) (\( N_f = N_c + 8 \)) for \( SO(N_c) \) (\( Sp(N_c) \)) gauge group respectively.
4.2. The low energy field theory

To get the low energy field theory corresponding to this brane setup we should again look for the lowest dimensional objects. These are as usual the finite color branes, but this time we get an equally important contribution from the full NS branes. Since the configuration is a T-dual of the Hanany-Witten setup, we know that we are dealing with a $N = 1$ supersymmetric gauge theory in six dimensions. Let us analyze the contributions from the various branes. The IIA NS brane can be viewed as a M-theory 5 brane. We know that the field theory living on its worldvolume is that of a chiral $N = 2$ supersymmetric tensor multiplet. By introducing the 6 branes we break $N = 2$ to $N = 1$. The $N = 2$ tensor multiplet decomposes into an $N = 1$ tensor and a hypermultiplet. Scalar fields correspond to fluctuations of the branes. The $N = 2$ tensor has 5 scalars which are associated to motions of the brane in the 6, 7, 8, 9, 10 directions. Four of these scalars are part of the $N = 1$ hyper, the fifth scalar is part of the $N = 1$ tensor. In our setup the worldvolume of the NS brane coincides with the infinite directions of the worldvolume of the sixbrane, so there is no room for fluctuations and the hypermultiplet does not contribute to the massless spectrum of the theory. There is a tensor living on both of the NS branes. Effectively, one tensor decouples from the theory because the scalar in one tensor multiplet parametrizes the center of mass motion of the two fivebranes. The scalar in the remaining tensor is related to the distance between the two NS branes. If the two NS branes come together, we arrive at a point where a non-trivial fixed point is possible. The tension of strings arising from membranes ending on the 5 branes is proportional to the expectation value of the scalar in the remaining tensor multiplet and thus vanishes at the fixed point. So we are looking at a theory with one tensor multiplet, vector and hypermultiplets. In the previous section we have seen that the brane setup allows only a limited amount of matter. From a field theory point of view these restrictions can be reproduced by an analysis of the anomaly [10]. The anomaly arising from vector and hypermultiplets is

$$I = \alpha \text{tr} F^4 + c (\text{tr} F^2)^2\tag{1}$$

where tr is the trace in the fundamental representation. In the case $\alpha = 0, c > 0$ the anomaly can be cancelled by introducing a tensor multiplet. So these are the field theories which we can compare with the theory arising from our brane configuration. If the gauge group of the field theory has an independent fourth order Casimir element (as is the case for $SU(N)$ with $N \geq 4$ and $SO(N)$ with $N \geq 5$ and $Sp(N)$ with $N \geq 4$) the two conditions $\alpha = 0$ and $c > 0$ have to be solved independently. The result is [10] that for the $SU$ groups only the matter content $N_f = 2N_c$ leads to a consistent theory. For $SO$ the only possibility is $N_f = N_c - 8$ and for $Sp$ we can only have $N_f = N + 8$. This is in agreement with the brane analysis. However, if the gauge group does not have an independent fourth order Casimir element ($SU(2)$, $SU(3)$) we only have to satisfy the condition $c > 0$. The anomaly analysis gives only an upper bound on the matter content. Considering also the global anomaly [11] for $SU(2)$ only the cases $N_f = 4, 10, 16$ and for $SU(3)$ only the cases $N_f = 0, 6, 12$ are allowed. We cannot see the additional possibilities in our brane picture. This is however not surprising because also in 5 dimensions the results obtained from the brane picture and the field theory analysis did not agree in the case of $SU(2)$.
We can also find the distance between the branes in the field theory picture. We have already mentioned that it is related to the expectation value of the scalar in the tensor multiplet. The interactions of this scalar to the gauge field is described by the following terms in a Lagrangian:

\[ \frac{1}{g^2} F^2_{\mu\nu} + (\partial \Phi)^2 + \sqrt{c} \Phi F^2_{\mu\nu}. \]

As remarked in [8] it is possible to absorb the coupling \( \frac{1}{g^2} \) in \( \Phi \). The bosonic terms are

\[ (\partial \Phi)^2 + \sqrt{c} \Phi F^2_{\mu\nu}. \]

We obtain an effective coupling

\[ \frac{1}{g_{eff}^2} = \sqrt{c} \Phi. \]

It is this product which we see in the brane picture. Note that this is different in 5 dimensions, where one can also include a bare coupling:

\[ \frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + c\Phi. \]

In the brane picture the bare coupling can be identified with the distance between the NS 5 branes at the point in moduli space, where the D 5 branes coincide. The constant \( c \) determines the tension of the magnetic BPS strings and hence is associated with the area of the rectangle between the branes at infinite bare coupling\(^5\).

5. Conclusion

We showed that by applying T-duality to the Hanany-Witten setup one can get a brane description of field theories in up to 6 dimensions. We constructed a brane realization of various six dimensional field theories yielding non-trivial infrared fixed points. Various results already known from field theory can be reproduced in the brane language. Consistency of the brane configuration gets mapped to anomaly cancellation in the field theory analysis.

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