Vacuum structure in 5D $SO(10)$ GUT on $S^1/Z_2$

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Abstract

We study the vacuum structure in 5D $SO(10)$ grand unified theory (GUT) compactified on $S^1/Z_2$ orbifold, where $SO(10)$ is broken into $SU(3) \times SU(2) \times U(1)^2$ through the boundary conditions. Although a lot of people extended to 6D to avoid massless colored particle, we can show they obtain finite masses by the radiative corrections. In a supersymmetric case, the fermionic partner of the zero-mode can also acquire non-vanishing mass through the SUSY breaking effects, and the gauge coupling unification can be recovered by use of brane localized kinetic terms.
1 Introduction

Grand unified theories (GUTs) are very attractive models in which the three gauge groups are unified at a high energy scale. However, one of the most serious problems to construct a model of GUTs is how to realize the mass splitting between the triplet and the doublet Higgs particles in the Higgs sector. This problem is so-called triplet-doublet (TD) splitting problem. A new idea for solving the TD splitting problem has been suggested in higher dimensional GUTs where the extra dimensional coordinates are compactified on orbifolds[1]-[6]. In these scenarios, Higgs and gauge fields are propagating in extra dimensions, and the orbifolding realizes the gauge group reduction and the TD splitting since the doublet (triplet) Higgs fields have (not) Kaluza-Klein (KK) zero-modes. A lot of attempts and progresses have been done in the extra dimensional GUTs on orbifolds[8]-[9]. Among them, the reduction of \(SO(10)\) gauge symmetry and the TD splitting solution are first considered in 6D models in Refs.[10] and [11], where \(SO(10)\) is broken into \(SU(3) \times SU(2) \times U(1)^2\) through the boundary conditions. 5D \(SO(10)\) models have been also considered, for example, in Refs.[12] and [13]. However, in Ref.[12], \(SO(10)\) is reduced into \(SU(3) \times SU(2) \times U(1)^2\), but some colored fields are remaining as the zero-modes of the extra dimensional components of the gauge fields. The introduction of a pair of bulk \(16\) multiplets and a bulk singlet that develop non-vanishing vacuum expectation values (VEVs) can make the zero-modes massive, the model became a little complicated. On the other hand, in Ref.[13], \(SO(10)\) is broken to only the Pati-Salam \(SU(4) \times SU(2) \times SU(2)\) symmetry[14] by the boundary condition, where all the extra dimensional components of the gauge fields acquire masses of the order compactification scale. This Pati-Salam gauge symmetry should be broken down to the standard gauge symmetry by the usual Higgs mechanism.

In this paper we reanalyze the 5D theory on a \(S^1/Z_2\) orbifold where \(SO(10)\) is broken into \(SU(3) \times SU(2) \times U(1)^2\) by the boundary conditions. Colored fields of the Wilson line degrees of freedom are surely remaining as the zero-modes in the tree level. However, we will show they obtain finite masses of order the compactification scale by the radiative corrections. In the supersymmetric (SUSY) case, the fermionic partners of the zero-modes can also acquire non-vanishing masses through the SUSY breaking effects. Although the scale of the masses is the SUSY breaking scale and the gauge coupling unification seems to be disturbed, the suitable boundary localized kinetic terms may conserve the gauge coupling unification.

2 \(SO(10)\) GUT on orbifold

We consider a 5D \(SO(10)\) GUT in which the gauge and Higgs fields propagate in the bulk. The 5th dimensional coordinate \((y)\) is assumed to be compactified on an \(S^1/Z_2\) orbifold. Under the parity transformation of \(Z_2\) which transforms \(y \rightarrow -y\), the gauge field \(A_M(x^\mu, y)\) \((M = \mu(=0-3), 5)\) in 5D transforms as

\[
A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P^\dagger, \quad (1)
\]

\[
A_5(x^\mu, -y) = -PA_5(x^\mu, y)P^\dagger, \quad (2)
\]
where $P$ is the operator of $Z_2$ transformation. Two walls at $y = 0$ and $\pi R$ are fixed points under $Z_2$ transformation. The physical space can be taken to $0 \leq y \leq \pi R$. Considering the $S^1$ boundary condition, $A_M(x^\mu, y + 2\pi R) = TA_M(x^\mu, y)T^\dagger$, the reflection around $y = \pi R$, $Z'_2$, is given by $P' = TP$. The gauge field $A_M(x^\mu, y)$ transforms

$$A_{\mu}(x^\mu, \pi R - y) = P'A_{\mu}(x^\mu, \pi R + y)P'^\dagger,$$

$$A_5(x^\mu, \pi R - y) = -P'A_5(x^\mu, \pi R + y)P'^\dagger,$$

under the parity transformation of $Z'_2$. It should be noticed that the signs of parities of $A_5$ are opposite to those of $A_{\mu}$. On the other hand, two bulk Higgs fields, $H_i$ ($i = u, d$), which are $10$ representation of $SO(10)$, transform as

$$H_i(x, -y) = PH_i(x, y), \quad H_i(x, \pi R - y) = P'H_i(x, \pi R + y),$$

under the parities.

The boundary conditions are taken as $P = \sigma_2 \otimes I_5$ and $P' = I_2 \otimes \text{diag}(1, 1, 1, -1, -1)$ in the base of $SO(10)$, which commute with the generators of the Georgi-Glashow $SU(5) \times U(1)$, and the Pati-Salam $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ groups, respectively. Seeing the zero modes of gauge and Higgs fields in the bulk, we can show that the $SO(10)$ gauge group is broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$, and the TD splitting is realized. Besides, there appear two zero modes, $(3, 2)$ and $(\bar{3}, 2)$, from $A_5$ as the Wilson line degrees of freedom. They are unwanted massless scalar fields in the low energy. This is the reason why many people have considered $SO(10)$ GUT in not 5D but 6D, where the above unwanted massless scalar fields obtain KK masses and do not appear in the low energy. However, radiative correction can induce the mass of the Wilson line phases in general. So we need to calculate the one loop effective potential of the Wilson line phases to study the vacuum structure and estimate the radiatively induced mass of the Wilson line phases in 5D theory. The 5D theory has a merit of a simple anomaly cancellation comparing to the 6D theory.

### 3 The effective potential of $SO(10)$

At first we show the effective potential induced from the gauge and ghost contributions. The adjoint representation, $45$, is divided by $P = \sigma_2 \otimes I_5$ and $P' = I_2 \otimes \text{diag}(1, 1, 1, -1, -1)$. $P$ and $P'$ break $SO(10)$ to $SU(5) \times U(1)_X$ and $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, respectively. The adjoint representation is decomposed as

$$45 = 24_0 + 1_0 + 10_4 + \bar{10}_{-4},$$

$$45 = (15, 1, 1) + (1, 3, 1) + (1, 1, 3) + (6, 2, 2),$$

under $P$ and $P'$, respectively.

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*In Ref. [10][11], they take $T^2/(Z_2 \times Z_2)$ orbifold with the boundary condition as $P_5 = I_{10}, T_5 = \sigma_2 \otimes I_5$ for the 5th coordinate and $P_6 = I_{10}, T_6 = I_2 \otimes \text{diag}(1, 1, 1, -1, -1)$ for the 6th coordinate. This boundary condition can avoid massless zero modes of $(3, 2)$ and $(\bar{3}, 2)$, from $A_5$ and $A_6$. The anomaly cancellation is not trivial in such a 6D theory, for examples, there should be accompanied two $10$ hypermultiplets with a $SO(10)$ gauge multiplet, and a $10$ hypermultiplets with a $16$ (or $\overline{16}$) in $N = 1$ SUSY ($(1,0$-SUSY)).*
The Wilson line degrees of freedom are \((3, 2)\) and \((\overline{3}, 2)\) components of \(A_5\) in the base of \(SU(3)_c \times SU(2)_L\). In the base of flipped \(SU(5)_F \times U(1)_X\), to which \(SO(10)\) is broken by the product of parities \(PP'\), the VEVs of them can be set as

\[
\langle A_5 \rangle = \frac{1}{2gR} \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & b \\
0 & 0 & 0 \\
a & 0 & 0 \\
b & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}_0
\]

(8)

by the residual global symmetry.

Since the adjoint representation is decomposed as \(45 = 24 + 1 + 10 + \overline{10}\) in terms of the flipped \(SU(5)_F\), the eigenvalues of \(D_y(A_5)^2\) for a adjoint field \(B\) depending on VEVs are calculated as

\[
\frac{(n \pm a)^2}{R^2}, \quad \frac{(n \pm b)^2}{R^2}, \quad 2 \times \frac{(n \pm a/2)^2}{R^2}, \quad 2 \times \frac{(n \pm b/2)^2}{R^2},
\]

\[
2 \times \frac{(n \pm (a-b)/2)^2}{R^2}, \quad 2 \times \frac{(n \pm (a+b)/2)^2}{R^2}, \quad 2 \times \frac{(n \pm (a-1)/2)^2}{R^2},
\]

\[
2 \times \frac{(n \pm (b-1)/2)^2}{R^2}, \quad 2 \times \frac{(n \pm (a-b-1)/2)^2}{R^2}, \quad 2 \times \frac{(n \pm (a+b-1)/2)^2}{R^2},
\]

(9)

where the eigenfunctions of \(B\) are expanded by \(\cos \frac{ny}{R}\) and \(\sin \frac{ny}{R}\) \((\cos \frac{(n+1/2)y}{R}\) and \(\sin \frac{(n+1/2)y}{R}\)) for \(PP' = +(PP' = -)\) components. According to the calculational method proposed in Ref.17, we find the effective potential induced from the gauge sector is given by

\[
V_{\text{eff}}^{\text{gauge}} = -\frac{3}{2} C \sum_{n=1}^{\infty} \frac{1}{n^5} \left[ \cos(2\pi na) + \cos(2\pi nb) + 2 \cos(\pi na) + 2 \cos(\pi nb)
\right.
\]

\[
+ 2 \cos(\pi n(a-b)) + 2 \cos(\pi n(a+b)) + 2 \cos(\pi n(a-1))
\]

\[
+ 2 \cos(\pi n(b-1)) + 2 \cos(\pi n(a+b-1)) + 2 \cos(\pi n(a-b-1)) \right].
\]

(10)

where \(C \equiv 3/(64\pi^7 R^5)\).

Next let us show the contributions from Higgs fields in the bulk. The \(10\) representation is divided as

\[
10 = 5_2 + \overline{5}_2,
\]

(11)

\[
10 = (6, 1, 1) + (1, 2, 2),
\]

(12)

under \(P\) and \(P'\), and

\[
10 = 5 + \overline{5}
\]

(13)

under \(PP'\). The eigenvalues of \(D_y(A_5)^2\) for a \(10\) representation field \(B\) depending on VEVs are

\[
\frac{(n \pm a/2)^2}{R^2}, \quad \frac{(n \pm b/2)^2}{R^2}, \quad \frac{(n \pm (a-1)/2)^2}{R^2}, \quad \frac{(n \pm (b-1)/2)^2}{R^2},
\]

(14)

where the eigenfunctions of \(B\) are expanded by \(\cos \frac{ny}{R}\), \(\sin \frac{ny}{R}\), \(\cos \frac{(n+1/2)y}{R}\) and \(\sin \frac{(n+1/2)y}{R}\) depending on the parities, \(P\) and \(P'\). Thus, the effective potential induced from the Higgs
sector is given by

\[ V_{\text{eff}}^{Higgs} = -C \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(\pi na) + \cos(\pi nb) + \cos(\pi n(a-1)) + \cos(\pi n(b-1)), \] (15)

for one complex 10 Higgs scalar. When there are \( N_{v,s} \) 10 Higgs scalars in the bulk, the effective potential from the Higgs contributions is given by \( N_{v,s} \times \text{Eq. (15)}. \)

When there is a fermion multiplet (\( \psi \)) or a spinor Higgs, 16, in the bulk, they contribute to the effective potential. Under the parities, they transform as

\[ \psi(x, -y) = \eta P \gamma^5 \psi(x, y), \quad \psi(x, \pi R - y) = \eta' P' \gamma^5 \psi(x, \pi R + y), \] (16)

where \( \eta, \eta' \) = ±, and the effective potential induced from these bulk fields depends on the sign of the product, \( \eta \eta' \).

The eigenvalues of \( D_y(A_5)^2 \) depending on VEVs for a 16 representation field \( B \) with \( \eta' = \pm \) are

\[
\begin{align*}
&\left( \frac{n \pm a}{2} \right)^2 \frac{R}{2}, \quad \left( \frac{n \pm b}{2} \right)^2 \frac{R}{2}, \quad \left( \frac{n \pm (a-1)}{2} \right)^2 \frac{R}{2}, \quad \left( \frac{n \pm (b-1)}{2} \right)^2 \frac{R}{2}, \\
&\left( \frac{n \pm (a+b)}{2} \right)^2 \frac{R}{2}, \quad \left( \frac{n \pm (a-b)}{2} \right)^2 \frac{R}{2},
\end{align*}
\]

(17)

where the eigenfunctions of \( B \) are expanded by \( \cos \frac{n\eta}{R}, \sin \frac{n\eta}{R}, \cos \left( \frac{n+1/2}{R} \right) \eta \) and \( \sin \left( \frac{n+1/2}{R} \right) \eta \) depending on the parities, \( P \) and \( P' \). Thus the effective potential induced from the 16 representation fields is given by

\[ V_{\text{eff}}^{16(\pm)} = \frac{d^{(\pm)}}{2} C \sum_{n=1}^{\infty} \frac{1}{n^5} \left[ \cos(\pi na) + \cos(\pi nb) + \cos(\pi n(a-1)) + \cos(\pi n(b-1)) + \cos(\pi n(a + b)) + \cos(\pi n(a - b)) \right], \] (18)

where \( d \) denotes the number of degree of freedom, for examples, +4 for one Dirac fermion and -2 for one complex scalar. When there are \( N_{s,s} \) 16 complex scalars and \( N_{s,f}^{(\pm)} \) 16 fermions with \( \eta' = \pm \) in the bulk, the contribution to the effective potential from the 16 fields is given as \( 4N_{s,f}^{(\pm)} - 2N_{s,s}^{(\pm)} \times V_{\text{eff}}^{16(\pm)}. \)

The radiatively induced mass of the Wilson line phases at the symmetric point, \( a = b = 0 \), is easily calculated by using the Riemann’s zeta function, \( \zeta_R(d) = \sum_{n=1}^{\infty} \frac{1}{n^d} \). The mass is given as

\[
m_{A_5}^2 = (gR)^2 \left. \frac{\partial^2 V_{\text{eff}}}{\partial a^2} \right|_{a=b=0} = 2\pi g_4^2 R^3 C \pi^2 \left[ \frac{3}{2} \left( 10 - 6 \times \frac{3}{4} \right) + \left( 1 - 3 \times \frac{3}{4} \right) N_{v,s} + \left( 1 - 3 \times \frac{3}{4} \right) (-2N_{s,f}^{(+)} + N_{s,s}^{(+)}) + \left( 3 - 3 \times \frac{3}{4} \right) (-2N_{s,f}^{(-)} + N_{s,s}^{(-)}) \right] \zeta_R(3)
\]

\[
= \frac{3\zeta_R(3)g_4^2}{128\pi^4} \left[ 33 + N_{v,s} + 5 \left( 2N_{s,f}^{(+)} - N_{s,s}^{(+)} \right) - 9 \left( 2N_{s,f}^{(-)} - N_{s,s}^{(-)} \right) \right] \left( \frac{1}{R} \right)^2. \] (19)

where the 4D gauge coupling constant \( g_4 \), which is defined as \( g_4 = g/\sqrt{2\pi R} \), is assumed to be \( \mathcal{O}(1) \). So the zero modes, (3, 2) and (3, 2), components of \( A_5 \) in the base of \( SU(3)_c \times SU(2)_L \).
obtain masses of $\mathcal{O}(1/R)$ from the radiative corrections, even for the minimal bulk content, that is, only the gauge multiplet propagates in the bulk. When the compactification scale is around the GUT energy scale, the Wilson line phases obtain the heavy masses radiatively, and do not survive in the low energy.

This symmetric point, $a = b = 0$, is the global minimum degenerated with the point $a = b = 1$ in the wide region of the parameter space, $N_{v,s}$, $N_{s,f}^{(\pm)}$, $N_{s,s}^{(\pm)}$. Unless the value of $2N_{s,f}^{(-)} - N_{s,s}^{(-)}$ is larger than that of $2N_{s,f}^{(+)} - N_{s,s}^{(+)}$ and/or $m_{A_5}^2 < 0$, where $(a, b) = (1, 0)$ and $(0, 1)$ points become the global minimum, the symmetric point is remaining as the global minimum. Even when the symmetric point is a local minimum, the tunnel transition from the symmetric point to the $(a, b) = (1, 0)$ or $(0, 1)$ points is negligible as long as $m_{A_5}^2 \gg 0$. We can easily see the tunnel transition between $a = b = 0$ and $a = b = 1$ is also negligibly small. So once the vacuum exists at $a = 0$ in the early universe, we can consider this color conserving vacuum is stable.

The SUSY version of the effective potentials are easily obtained from those of the non-SUSY version in the case of the Scherk-Schwarz (SS) SUSY breaking. They are obtained by adding a factor $(1 - \cos(2\pi n\beta))$ in the summations and modifying the coefficients depending on the number of the degree of freedoms as in Eq. (18). The $\beta$ parameterizes the SS SUSY breaking, which should be taken as to induce the soft mass of order $\beta/R$. Then the SUSY version of the effective potentials are given by

$$V_{\text{gauge}}^{\text{eff}} = -2C \sum_{n=1}^{\infty} \frac{1}{n^5} (1 - \cos(2\pi n\beta))[\cos(2\pi na) + \cos(2\pi nb) + 2 \cos(\pi na) + 2 \cos(\pi nb) + 2 \cos(\pi n(a - b)) + 2 \cos(\pi n(a + b - 1)) + 2 \cos(\pi n(a - b - 1))],$$

$$V_{\text{Higgs}}^{\text{eff}} = 2C \sum_{n=1}^{\infty} \frac{1}{n^5} (1 - \cos(2\pi n\beta)) \times [\cos(\pi na) + \cos(\pi nb) + \cos(\pi n(a - 1)) + \cos(\pi n(b - 1))],$$

$$V_{\text{16}}^{(\pm)}_{\text{eff}} = 2C \sum_{n=1}^{\infty} \frac{1}{n^5} (1 - \cos(2\pi n\beta))[\cos(\pi na) + \cos(\pi nb) + \cos(\pi n(a - 1)) + \cos(\pi n(b - 1)) + \cos(\pi n(a + b - 1)) + \cos(\pi n(a - b - 1)) + \cos(\pi n(a + b - 1))],$$

for the gauge multiplet, one 10 hypermultiplet and one 16 hypermultiplet with $\eta_\eta' = \pm$, respectively.

As in the non-SUSY case, two points, $a = b = 0$ and $a = b = 1$, are degenerated and become the global minimum when the number of 16 hypermultiplet with $\eta_\eta' = -$ is not so large, and $m_{A_5}^2 > 0$. The tunnel transition between them is also negligible. The masses of the Wilson line phases at the symmetric point are given, by using the approximation formula,

$$\sum_{n=1}^{\infty} \frac{\cos(n\xi)}{n^3} \simeq \zeta(3) + \frac{\xi^2}{2} \ln \xi - \frac{3}{4} \xi^2,$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\xi)}{n^3} (\xi^n) \simeq -\frac{3}{4} \zeta(3) + \frac{\xi^2}{2} \ln 2,$$
for a small (positive) $\xi$, as

$$m_{A_5}^2 = (gR)^2 \frac{\partial^2 V_{\text{eff}}}{\partial a^2} \bigg|_{a=b=0} = 2\pi g_4^2 R^2 \left[ 2 \left( 10 \times (-2\pi^2 \beta^2 \ln(2\pi \beta) + 3\pi^2 \beta^2) - 6 \times 2\pi^2 \beta^2 \ln 2 \right) - 2 \left( (-2\pi^2 \beta^2 \ln(2\pi \beta) + 3\pi^2 \beta^2) - 2\pi^2 \beta^2 \ln 2 \right) N_v \right. \\
\left. - 2 \left( (-2\pi^2 \beta^2 \ln(2\pi \beta) + 3\pi^2 \beta^2) - 3 \times 2\pi^2 \beta^2 \ln 2 \right) N_s^{(+)} \right] \\
- 2 \left( 3 \times (-2\pi^2 \beta^2 \ln(2\pi \beta) + 3\pi^2 \beta^2) - 2\pi^2 \beta^2 \ln 2 \right) N_s^{(-)} \right]$$

$$= \frac{3g_4^2}{16\pi^2} \left[ (-20 + 2N_v + 2N_s^{(+)} + 6N_s^{(-)}) \ln(2\pi \beta) \right. \\
\left. + (30 - 3N_v - 3N_s^{(+)} - 9N_s^{(-)}) \right] \times \left( \frac{\beta}{R} \right)^2,$$

where $N_s$ and $N_s^{(\pm)}$ denote the number of the $10$ hypermultiplets and that of the $10$ hypermultiplets with $\eta\eta' = \pm$ in the bulk, respectively. Thus, the scalar components of the zero modes, $(3, 2)$ and $(\bar{3}, 2)$, of $A_5$ obtain masses of $O(\beta/R)$ from the radiative corrections. As for the fermion components of $(3, 2)$ and $(\bar{3}, 2)$, they also obtain masses of order $\beta/R$ via the SS mechanism as the $\mu$-term generation in the gauge-Higgs unification scenario. It is because they are a part of a $SU(2)_R$ doublet. So that all the component of the Wilson line phases become massive, although they might spoil the success of the gauge coupling unification, due to their small masses of $O(\beta/R)$.

However, there is the possibility that boundary localized kinetic terms can recover the gauge coupling unification. In general, boundary localized kinetic terms can exist independently of bulk kinetic terms, and affect the gauge coupling unification in orbifold GUT scenarios. Although such contribution is often assumed to be negligible, even order one boundary localized gauge couplings may give the same order contribution from one pair of $(3, 2)$ and $(\bar{3}, 2)$ chiral multiplet with a mass of order SUSY-breaking. For example, $g_2 = 1$ and $g_3 = 3/2$ on a boundary induce the difference of the fine structure constants as

$$\Delta\alpha^{-1} = 4\pi \left( \frac{1}{g_2^2} - \frac{1}{g_3^2} \right) = \frac{20}{9}\pi \sim 7,$$

while the pair contributes as

$$\Delta\alpha^{-1} \sim \frac{1}{2\pi} (b_2 - b_3) \ln \left( \frac{1/R}{\beta/R} \right) \sim 5,$$

where $b_a$ denotes the contribution to the renormalization coefficient from the pair, $b_2 = 3, b_3 = 2$, with $\beta \sim 10^{-13}$. So here we assume the gauge coupling unification is recovered by the contribution from boundary localized kinetic terms.

## 4 Summary and discussion

In a 5D orbifold GUT, if $SO(10)$ is broken into $SU(3) \times SU(2) \times U(1)^2$ by the boundary conditions, colored components of the 5th dimensional component of the gauge field become
massless at the tree level. In order to give them mass, people have extended the model to the 6D, or introduced the additional bulk hypermultiplets with the field developing non-vanishing VEVs by hand. However, in this paper, we have shown the radiative corrections make the colored zero-modes acquire masses. In SUSY case, SS SUSY breaking makes the fermionic partner of the zero-modes be massive, too. The boundary localized kinetic terms can recover the gauge coupling unification.

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†In a warped geometry, colored light modes may not spoil the success of the gauge coupling unification in the minimal SUSY SU(5) model. And a SO(10) model is also considered in warped background.
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