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Research of Fuzzy-Systems of Automatic Frequency and Phase Control on Basis of MATLAB

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1. Introduction

Systems of phase auto fine tuning (phase locked loop – PLL-system) are one of the most widespread functional knots of various electronic systems. A phase-locked loop (PLL) is a closed-loop feedback control system that generates and outputs a signal in relation to frequency and phase of an input “reference” signal. They are used in digital networks for synchronization of some exact setting generators on the reference generator, in frequency synthesizers for multichannel communication systems with frequency consolidation where the grid of carrier frequencies for division of channels is required, for example, in broadcasting and TV, in schemes of restoration bearing and clock frequencies in synchronous communication systems, as discriminators for demodulation is frequency- and phase-modulated bearing, as voice-frequency decoders in telephone switchboards for decoding of figures from accepted voice-frequency combinations. In the given chapter fuzzy-systems of frequency and phase auto fine tuning (PLL-system with the digital fuzzy controllers working on the basis of fuzzy logic) are investigated. Research of fuzzy-systems PLL is spent by mathematical modeling with use of the interactive environment for scientific and engineering calculations MATLAB and a powerful tool of modelling and research of control systems with feedback Simulink.

2.1 Fuzzy-systems of Phase Locked Loop – PLL

Let’s consider widely used system of the PLL frequency control which function diagram is resulted on fig. 2.1.

Fig. 2.1. System of the PLL frequency control.
A open-ended circuit of system is consists of serial connection of frequency discriminator FD, an amplifier A, an engine E with a reducer R, a controlling unit CU, a voltage-controlled oscillator VCO, a mixer M and intermediate – frequency amplifier IFA. System operation detail is described in [1]. A frequency discriminator can be presented by the series connection of a device of comparing, nonlinearity \( K_\omega \) and the low-pass filter LPF. An elementary low-pass filter is described by transfer function \( G_F(s) = K_F(s + 1)^{-1} = k / (s + b) \), where \( k = K_F / T_F \), \( b = 1 / T_F \), \( K_F \) - gain factor \( T_F \) - a constant of time of the filter on the output of discriminator. We accept \( \omega_{\text{hp}} = \text{const} \) and \( \omega_e = \text{const} \). Inertial properties of intensive elements can be neglected. An engine is used as an integrating element for giving to system astaticism the first order that allows to reduce a dynamic error. It is possible to use the electronic integrator instead of the electromechanical engine. Therefore transfer function of control object may be written down in a type \( G_a(s) = a(s + a)^{-1} \), where \( a = 1 / T_C \), \( T_C \) - constant of time of the generator with a controlling unit.

The block diagram of system of the PLL frequency control in interactive system MATLAB we will present in a kind fig. 2.2. For maintenance of demanded dynamics of system it is used Fuzzy controller or PID-controller.

A curve \( K(\omega) \) is called the static discrimination characteristic. The fluctuation component on an output of discriminator is described by spectral density \( S_\omega \) and dependence \( \omega S_\omega(\omega) \) is called of fluctuation characteristic of discriminator. At simulation fluctuation component can considered as voltage \( V(t) \) - random disturbance applied to the output of the discriminator. A mathematical model of nonlinearity \( K(\omega) \) we are describing expression

\[
K(\omega) = K_d \omega \exp\left(-\frac{\omega^2}{\Delta^2}\right)
\]

where \( K_d \) - a conversion coefficient of the discriminator, and \( \Delta \) - a half-width of the discrimination characteristic, determines resolution capacity of the discriminator, \( \omega = \Delta \omega_{\text{hp}} \) - detuning concerning nominal intermediate frequency \( \omega_{\text{hp}} = \text{const} \). Graphically a nonlinearity \( K(\omega) \) at \( \omega = \epsilon \) is presented at the fig. 2.3

![Fig. 2.2. The block diagram of system of the PLL frequency control.](www.intechopen.com)
Fig. 2.3. A nonlinearity $K(\omega)$ at $\omega = e$

It should be emphasized, that on an output of the unit of nonlinearity on the block diagram
the mismatch error is defined as $K[\omega(t)] = K[e(t)]$. The mismatch error on the input a
frequency discriminator is defined as $\omega(t) = e(t)$.

At the description of the discrimination characteristic represented on fig. 2.3, usually used
approximation in a type

$$K(e) = K_e e^{\exp(-ae^2)}. \quad (2.2)$$

For determine the parameter $a$ at the given half-width of the discrimination characteristic
$\Delta$ area, bounded by a branch of function (2.2), which is defined as

$$K_a \int_{0}^{\infty} e^{\exp(-ae^2)} de = \frac{K_e}{2a},$$

replace an area of a right triangle (fig. 2.3) with length of legs $\Delta$ and $K_a \Delta$ which is defined
as $K_a \Delta^2 / 2$.

From here we have $a = 1 / \Delta^2$ also expression (2.2) at $\omega = e$ it will be transformed to
expression (2.1).

A frequency discriminator FD (see fig. 2.1) in a figure 2.2 is presented by the comparing
circuit from the adder and characteristic of discrimination $\text{Discrim} - K(\omega)$. A low-pass
filter LPF is described by link TransferFcn1 with transfer function $G(s)$.

The object of control - an integrator and the clock oscillator with a controlling unit - with transfer function
$G(s)$ is presented by links Integrator and Transfer Fcn2.

The digital fuzzy controller (Fuzzy controller on fig. 2.2) is fulfilled under the block diagram
with the identical triangular membership function erected in degree and consists of the
block of the shaper of sizes $A(t)$ and $B(t)$, the block of comparing of sizes $A$ and $B$ and
calculation $u$, and the block of normalization variable [2]. The mismatch error $\theta(t)$ from an
output of a low-pass filter arrives on a analog-digital converter (AD converter) (Zero-Order
Hold), included at the input of a fuzzy controller. A quantization step of AD converter $h$
$=0.01s$. At the output of a fuzzy controller is included the digital to analog converter (Zero-
Order Hold1).

For simplification of normalization (recalculation of values of signals in values of elements
of universal set) ranges of change input and output signals (parameters of a fuzzy
controller) we is accepting by symmetric:
\[
\theta_{\text{max}} = -\theta_{\text{min}}; \quad \dot{\theta}_{\text{max}} = -\dot{\theta}_{\text{min}}; \quad \ddot{\theta}_{\text{max}} = -\ddot{\theta}_{\text{min}}; \quad m_{\text{max}} = -m_{\text{min}}.
\]

Then recalculation of values of signals in values of elements of universal set perform according to the formulas

\[
\begin{align*}
t_1^* &= (\theta^* + A_m) / (2A_m); \\
t_2^* &= (\dot{\theta}^* + B_m) / (2B_m); \\
t_3^* &= (\ddot{\theta}^* + C_m) / (2C_m).
\end{align*}
\]

Values of ranges \( (A_m = \theta_{\text{max}} = -\theta_{\text{min}}; \quad B_m = \dot{\theta}_{\text{max}} = -\dot{\theta}_{\text{min}}; \quad C_m = \ddot{\theta}_{\text{max}} = -\ddot{\theta}_{\text{min}}; \quad D_m = m_{\text{max}} = -m_{\text{min}}) \) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task.

If studying of system (see fig. 2.2) the method of mathematical simulation selection following parameters:

\[
k = 1; a = 10 c^{-1}; b = 12,5 c^{-1}; c = 1; \quad a = a = 15; \quad \Delta = 1; \quad K_d = 1.
\]

The system was studied when exposed on input an equivalent harmonic signal

\[
u(t) = 1 + 0,5\sin 2\pi Ft,
\]

with carrier frequency \( F = 0,1Hz \).

The adjustment of a fuzzy controller is carried out by criterion of a minimum of a dynamic error. Is received following optimal parameters of a fuzzy controller:

\[
A_m = \theta_{\text{max}} = 0,05; \quad B_m = \dot{\theta}_{\text{max}} = 0,4; \quad C_m = \ddot{\theta}_{\text{max}} = 10; \quad D_m = m_{\text{max}} = 150.
\]

Processes in system of the PLL frequency control (see fig. 2.2) with a fuzzy controller are shown on fig 2.4 where \( u(t) \) - an input action, \( x(t) \) - an output a system (see fig. 2.4,a), \( e(t) = \text{Err} \) - a mismatch error on a discriminator input (see fig. 2.4, b), \( m(t) \) - an output of a fuzzy controller (see fig. 2.4, c).

The maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed 0,7 % from amplitude of a sinusoid.

At the fig. 2.4, d transition is presented process of system of the PLL frequency control - response to unit step action. The system fulfills input influence in time, not exceeding 0,16 s, without overshoot.

Thus, the digital fuzzy controller provides not only greater accuracy castings input influence, but and high system performance by an step action.

It should be noted that the research system with the above parameters, without the regulator shows that the system lacks stability.

The system (see fig. 2.2) with a digital fuzzy controller it is researched also in the presence of a stationary white noise (Band-Limited White Noise) on a frequency discriminator output. Implementation of a white noise \( v(t) \) is shown at the fig. 2.5, a. A mismatch error on an input of the slope detector \( e(t) \) and an output of a fuzzy controller \( m(t) \) thus are casual processes (see fig. 2.5, b and c).
Fig. 2.4. Processes in system of PLL frequency control.

Fig. 2.5. Processes in system in the presence of a stationary white noise.
Researches of system in the presence of a stationary white noise show that noise of such intensity (see fig. 2.5 a,) slightly influences a system output $x(t)$ (see fig. 2.5,d), however error of system increases (we compare fig. 2.4, b and 2.5b), therefore at small input influences (at small frequency deviations of the generator from set) noise on a frequency detector output will make the considerable impact on a dynamic error of system. System (see fig.2.2) was also studied by using a more simple fuzzy controller. Parameter $a$ in the fuzzy controller is chosen to be 0.2, and ranges of variation of input and output variables left unchanged: $A_m = 0.05; \ B_m = 0.4; \ C_m = 10; \ D_m = 150.$ Processes in system (see fig. 2.2) with a fuzzy controller where $u(t)$ – an input , $x(t)$ – an output of system (fig.2.6,a), $e(t) = Err$ – a mismatch error on a discriminator input (fig.2.6,b). $x(t)$ and $m(t)$ - response to a unit jump of the input signal $u(t)$ respectively, at the system output and the output of fuzzy controller (see fig.2.6,c,d).

Fig. 2.6. Processes in system with a fuzzy controller

The maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed 0.64 % from amplitude of a sinusoid. The system fulfills input influence in time, not exceeding 0.13s, without overshoot.

Of interest to examine the processes in the system (see Fig.2.2), by using instead of a fuzzy controller of the traditional PID controller. The block diagram of digital PID controller shown in fig.2.2. The transfer function of the regulator

$$W(z) = G_1 + G_2 \frac{z + 1}{z - 1} + G_3 \frac{z - 1}{z},$$
where \( G_1 = K_c \), \( G_2 = \frac{K_i h_0}{2} \), \( G_3 = \frac{K_d}{h_0} \), \( h_0 \) – digitization step.

As a result of tuning of PID-controller at the indicated higher parameters of the system and input an equivalent harmonic signal \( u(t) = 1 + 0.5 \sin 2\pi F t \), with carrier frequency \( F = 0.1 \text{Hz} \), is received following optimal parameters of a PID-controller, at the \( h_0 = 0.01 \text{s} \): \( G_1 = 171.2; \ G_2 = 0.48; \ G_3 = 1800 \).

Processes in system (see fig. 2.2) with a PID-controller are shown at the fig. 2.7 where \( u(t) \) - an input action, \( x(t) \) - an output a system (see fig. 2.7, a), \( e(t) = \text{Err} \) – a mismatch error on a discriminator input (see fig. 2.7, b), \( m(t) \) - an output of a fuzzy controller (see fig. 2.7, c). The maximum dynamic error on system of the of the PLL frequency control with a PID-controller (except for initial burst at the moment of signal capture) does not exceed 2.4% from amplitude of a sinusoid. The maximum dynamic error on system of the PLL frequency control with a PID-controller in 3.5 times more than the maximum dynamic error in system of the PLL frequency control with a fuzzy controller and in 3.75 times with a fuzzy controller.

![Fig. 2.7. Processes in system with a PID-controller](image-url)

At the fig. 2.4, d transition is presented process of system with a PID-controller - response to unit step action. The transitional process - the oscillating with the overshoot of more than 20%. The system fulfills input influence in time, exceeding 0.4 s. The regulation time in system of the PLL frequency control with the PID-controller approximately in 2.5 times is more than regulation time in system with a fuzzy controller and in 3 times with an simple fuzzy controller.
Thus, the fuzzy controllers provide accuracy fulfills of input influences and high-speed performance of system of the PLL frequency control it is much better, then the PID-controller.

The system (see fig. 2.2) with a digital PID-controller it is researched also in the presence of a stationary white noise (Band-Limited White Noise) on a frequency discriminator output. Implementation of a white noise \( v(t) \) is shown at the fig. 2.8.a.

Researches of system in the presence of a stationary white noise show that noise of such intensity (see fig. 2.8 a,) slightly influences a system output \( x(t) \) (see fig. 2.5,d) however error of system increases (we compare fig. 2.7, b and 2.8b) therefore at small input influences (at small frequency deviations of the generator from set) noise on a frequency detector output will make the considerable impact on a dynamic error of system.

![Fig. 2.8. Processes in system in the presence of a stationary white noise](image)

Let’s consider a mathematical model (block diagram) of system of the PLL frequency control in interactive system MATLAB (see fig. 2.9), with the low-pass filter of the second order which has transfer function in a numerical type

\[
G(s) = \frac{k}{a_2s^2 + a_1s + 1} = \frac{0.008}{0.008s^2 + 0.18s + 1}.
\]

A frequency discriminator FD at the fig. 2.9 is presented by the comparator circuit on the adder and the discrimination characteristic (Discrim)

\[
K_e = K_ee^{\exp\left(-\frac{\epsilon^2}{\Delta}\right)} = e^{\exp\left(-\frac{\epsilon^2}{0.64}\right)}.
\]
where conversion coefficient of the discriminator $K_d = 1$, and the half-width of the discrimination characteristic defining resolution capability of the discriminator, $\Delta = 0.8$.

The object of control - the integrator and the without inertia clock generator - with transfer function $G_0(s) = \frac{1}{s}$, is presented by link Transfer Fcn1.

The mismatch error $\theta(t)$ from an output of a low-pass filter arrives on a analog-digital converter (AD converter) (Zero-Order Hold), included at the input of a fuzzy controller. A quantization step of AD converter $h = 0.01s$. On an output of a fuzzy controller is included the digital/analog converter DAC (Zero-OrderHold1).

For simplification of normalization (recalculation of values of signals in values of elements of universal set) ranges of change input and output signals (parameters of a fuzzy controller) we is accepting by symmetric:

$$\theta_{max} = -\theta_{min}; \; \theta_{max} = -\theta_{min}; \; \theta_{max} = -\theta_{min}; \; m_{max} = -m_{min}.$$

Then recalculation of values of signals in values of elements of universal set perform according to the formulas

$$u_1^* = \left(\theta^* + A_m\right) / \left(2A_m\right);$$
$$u_2^* = \left(\theta^* + B_m\right) / \left(2B_m\right);$$
$$u_3^* = \left(\theta^* + C_m\right) / \left(2C_m\right);$$

Values of ranges (Am = $\theta_{max} = -\theta_{min}$$; \; Bm = \theta_{max} = -\theta_{min}; \; Cm = \theta_{max} = -\theta_{min}; \; Dm = m_{max} = -m_{min}$) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task.

Fig. 2.9. The block diagram of system of the PLL frequency control

In system is used the elementary digital fuzzy controller (Fuzzy controller at the fig. 2.9) is fulfilled under the block diagram. The controller consists of the block of the shaper of sizes A (t) and B (t), the block of comparing of sizes A and B and calculation $u_c$ and the block of normalization output variable[2].

The mismatch error $\theta(t)$ from an output of a low-pass filter arrives on a analog-digital converter (AD converter) (Zero-Order Hold), included at the input of a fuzzy controller. A quantization step of AD converter $h = 0.01s$. On an output of a fuzzy controller is included the digital/analog converter DAC (Zero-OrderHold1).

For simplification of normalization (recalculation of values of signals in values of elements of universal set) ranges of change input and output signals (parameters of a fuzzy controller) we is accepting by symmetric:

$$\theta_{max} = -\theta_{min}; \; \theta_{max} = -\theta_{min}; \; \theta_{max} = -\theta_{min}; \; m_{max} = -m_{min}.$$

Then recalculation of values of signals in values of elements of universal set perform according to the formulas

$$u_1^* = \left(\theta^* + A_m\right) / \left(2A_m\right);$$
$$u_2^* = \left(\theta^* + B_m\right) / \left(2B_m\right);$$
$$u_3^* = \left(\theta^* + C_m\right) / \left(2C_m\right);$$

Values of ranges (Am = $\theta_{max} = -\theta_{min}$$; \; Bm = \theta_{max} = -\theta_{min}; \; Cm = \theta_{max} = -\theta_{min}; \; Dm = m_{max} = -m_{min}$) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task.

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The system was studied when exposed on input an equivalent harmonic signal
\[ u(t) = 1 + 0.5 \sin 2\pi F t, \]
with carrier frequency \( F = 0.1 \text{Hz} \).
The adjustment of a fuzzy controller is carried out by criterion of a minimum of a dynamic error. Is received following optimal parameters of a fuzzy controller:
\[ \text{Am} = \theta_{\text{max}} = 0.05; \quad \text{Bm} = \theta_{\text{max}} = 0.4; \quad \text{Cm} = \theta_{\text{max}} = 10; \quad \text{Dm} = m_{\text{max}} = 150. \]
Processes in system of the PLL frequency control (see fig. 2.9) with a fuzzy controller are shown on fig. 2.10 where \( u(t) \) - an input action, \( x(t) \) - an output a system (see fig. 2.10 a), \( e(t) = \text{Err} \) - a mismatch error on a discriminator input (see fig. 2.10,b). \( m(t) \) - an output of a fuzzy controller (see fig. 2.10,c).

Fig. 2.10. Processes in system
The maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed 0.4% from amplitude of a sinusoid. At fig. 2.10,d,e are presented reactions to unit step of input signal \( u(t) \) respectively, at the output of system \( x(t) \) and at the output of a fuzzy controller \( m(t) \). The transition process – the oscillation, which the overshoot of more than 20%. The system fulfills input influence in time, not exceeding 0.25 s.

At usage in system (fig. 2.9 see) a digital PID-controller it is received following results. Tuning of PID-controller at the indicated higher parameters of the system and input an equivalent harmonic signal \( u(t) = 1 + 0.5\sin 2\pi F t \), with carrier frequency \( F = 0.1Hz \), is received following earlier optimal parameters of a PID-controller, at the \( h_0 = 0.01s \):

\[
G_1 = 171.2; \quad G_2 = 0.48; \quad G_3 = 1800.
\]

Therefore processes in system (see fig. 2.9) with a digital PID-controller same what are shown at the fig. 2.7.

The maximum dynamic error in system of the PLL frequency control with the PID-controller (except for initial burst at the moment of signal capture) does not exceed 2.4% from amplitude of a sinusoid. Transfer process - oscillatory, with overshoot more than 20%. The system fulfills input influence for time exceeding 0.4 with. The maximum dynamic error in system of the of the PLL frequency control (fig. 2.9) with the PID-controller in 6 times more the maximum dynamic error in system of the of the PLL frequency control with a fuzzy controller see, and regulation time in system of the of the PLL frequency control with the PID-controller approximately in 1.6 times is more than regulation time in system with a fuzzy controller.

Thus, the fuzzy controller is provided with accuracy of working off of input influences and high-speed performance of system of the frequency self-tuning it is much better, than the PID-controller.

### 2.2 Fuzzy systems of clock synchronization

Systems of clock synchronization CS found a wide circulation in various areas of technique, in particular on digital networks and transmission systems of information DTSI. The modern development of technique demands requires the CS to meet the new higher quality of indicators.

At the fig. 2.11 shows a typical function diagram of CS DTSI. Diagram includes leading and slave clock oscillator - LCO and SCO accordingly; the input device of regenerator - IDR; the extractor of clock synchro signal (ECS); a solver - S; a phase detector - PD; a low-pass filter LPF; a manager -M; a communication line - a CL.

![Fig. 2.11. System of clock synchronization](www.intechopen.com)
Clock synchronization of digital signals of telecommunication DST is a process of establishment and maintaining the required phase relations between the significant moments of a digital signal of telecommunication DST and a clock synchro signal. Thus, into the decision of the task of clock synchronization include: 1) formation of the clock synchro signal solver of the receiver or a regenerator; 2) “binding” of a phase of the specified synchro signal to a phase of the significant moments DST, accepted from a communication line; 3) monitoring of the given phase relationship between the timing signal accepted from a communication line and a synchro signal submitted on solver of the receiver; 4) framing of controlling influence in the presence of an error at violation of the specified phase relationship; 5) base service of a phase error according to the worked out control action.

The solution of these problem is possible, first of all, creation of system of automatic control of a phase of a clock synchronize of the receiver or regenerator DST. Clearly that in-phase operation of the transmitter and receiver DST is inconceivable without presence synchro information (upsetting control of system of automatic control). Clock synchro signal are worked out by clock oscillator CO. Consequently, it is necessary to carry out generating of a clock synchro signal on transmission and on reception that in turn causes of presence TG on transmission and on reception. Thus, the first stage of solving the problem of clock synchronization in practice is implemented by presence TG in a regenerator or a receiving part of the equipment DST, and clock synchronization can be shown to synchronization CO, i.e. process of establishment and maintenance of demanded phase relationships between clock synchro signal two or several CO. Thus CO of transmitter is leading, he works out setting action; and CO the receiver or a regenerator - slave, the phase of his synchro signal is a measured variable of automatic system of PLL of a phase. This circumstance is an important feature of the synchronization system, which becomes a telemechanical system.

Setting action - a phase of a clock synchro signal of leader CO in the general case, due to random distortions when moving the synchro signal from the transmitter to the receiver, instability of generating of pulses by leader CO, and other is unknown function of time, i.e. \( \varphi_{TC}(t) = \text{var} \). This position gives the grounds to classify system of clock synchronization as follow-up systems of automatics, and at \( \varphi_{TC}(t) = \text{const} \) - as stabilizing systems.

Basic element of system of clock synchronization is the system PLL which is selected by a dotted line for fig. 2.11

In phase-locked loop systems the coordination of phases of two oscillations is carried out - the current phase of controlled oscillations is set up under a current phase of setting oscillations so that ideally the phase difference of these oscillations saves constant value. As persistence of a phase difference is possible only at equality of oscillation frequencies in PLL systems frequency of controlled oscillations ideally is installed to equal frequency of setting oscillations. This property of PLL systems defines their double assignment - as systems of self-locked of frequency and as systems of self-locked of a phase.

In the first case the immediate aim of application of system is establishment of demanded correspondence between frequencies of two oscillations, and in the second - establishment of demanded correspondence between phases of two oscillations.

The phase-locked systems which immediate aim is establishment of demanded correspondence between frequencies of two oscillations, received a title of phase-locked loop systems of frequency control (PLL frequency control).

If the immediate purpose of functioning of system is the coordination of phases of two oscillations it name the same as also all class of considered systems - the phase-locked loop system (PLL).
Distinction of titles reflects only distinction in immediate mission and does not mean distinctions in principles of actions. Principles of action of systems PLL OF and PLL are identical.

System PLL defines accuracy and high-speed performance of all system of clock synchronization. Object of control in system PLL is the clock VCO which taking into its inertial can be described transfer function

\[ G(s) = \alpha [s(s + \alpha)]^{-1} \]

(If an output variable of the generator is the phase of oscillations), or transfer function

\[ G(s) = \alpha / (s + \alpha) \]

(If an output variable of the generator is oscillation frequency),

where \( \alpha = K_C / T, \ \alpha = 1 / T, \ K_C \) - transmission factor, \( rad / (s \cdot V) \), \( T \) - time constant VCO. Assuming, that the VCO is without inertial element of system PLL, its transfer function can be defined as

\[ G(s) = \alpha / s \]

(If an output variable of the generator is the phase of oscillations), or the transfer function

\[ G(s) = \alpha \]

(If an output variable of the generator is oscillation frequency), where \( \alpha = K_C, \ K_C \) - transmission factor VCO, \( rad / (s \cdot V) \).

The mathematical model of the phase detector can be present series connection of the device of comparing, non linearity \( K(\varphi) \) and low-pass filter LPF. The simplest LPF is described by the transfer function

\[ G_F(s) = K_F / (T_F s + 1) = k / (s + b), \]

where \( k = K_F / T_F, \ b = 1 / T_F, \ K_F \) - an amplification factor, \( T_F \) - the constant of the filter phase detector.

Curve \( K(\varphi) \) is called the static discrimination characteristic. Mathematical model of the nonlinearity of the type “discrimination feature” can be described by

\[ K(\varphi) = K_d \sin \varphi, \]

where \( K_d \) - gain of phase detector, \( \varphi(t) = \phi(t) \) - a mismatch error on an input of phase detector, or expression at the “linear discrimination characteristic

\[ K(\varphi) = \begin{cases} K_d \varphi \text{ at } -\Delta \leq \varphi \leq \Delta, \\ 0 \text{ at } |\varphi| > \Delta. \end{cases} \]

Usually the half-width \( \Delta \) “linear” discrimination characteristic of phase detector is equal \( \pi / 2, \ \pi \) or \( 2\pi \).

Discrimination characteristic of phase-frequency detector can be described by the expression
Let’s consider a block diagram (a mathematical model) of system of the PLL presented in interactive system MATLAB, in fig. 2.12 [3,4]. To ensure the dynamics of the system using fuzzy and PID-controllers (Fuzzy controller and PID at the fig.2.12).

Fig. 2.12. System of the PLL.

Phase detector FD (see fig. 2.11) in fig.2.12 is presented by the compare circuit on the adder and discrimination characteristic Discrim - \( K(\phi) = K_{f} \sin \phi \). The low-pas filter LPF is described by link Transfer Fcn1 with transfer function \( G_{f}(s) \). The clock generator with transfer function \( G(s) \) is described by links Integrator and Transfer Fcn2.

The digital fuzzy controller (fuzzy controller at the fig. 2.12) is fulfilled under the block diagram, with the identical triangular membership function erected in degree and consists of the block of the shaper of sizes A (t) and B (t), the block of comparing of sizes A and B and calculation \( u_{c} \) and the block of normalization output variable[2]. The mismatch error \( \theta(t) \) from an output of a low-pass filter arrives on an analog-digital converter (AD converter) (Zero-Order Hold), included at the input of a fuzzy controller. A quantization step of AD converter \( h = 0,01s \). On an output of a fuzzy controller is included the digital/analog converter (Zero-Order Hold1).

For simplification of normalization (recalculation of values of signals in values of elements of universal set) ranges of change input and output signals (parameters of a fuzzy controller) we is accepting by symmetric:

\[
\begin{align*}
\theta_{\text{max}} &= -\theta_{\text{min}}, & \dot{\theta}_{\text{max}} &= -\dot{\theta}_{\text{min}}, & \ddot{\theta}_{\text{max}} &= -\ddot{\theta}_{\text{min}}, & m_{\text{max}} &= -m_{\text{min}}.
\end{align*}
\]

Then recalculation of values of signals in values of elements of universal set perform according to the formulas.
Values of ranges \( (A_m = \theta_{\text{max}} - \theta_{\text{min}}; B_m = \hat{\theta}'_{\text{max}} - \hat{\theta}'_{\text{min}}; C_m = \hat{\theta}'_{\text{max}} - \hat{\theta}'_{\text{min}}; D_m = m'_{\text{max}} = -m'_{\text{min}}) \) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task.

Let’s notice that mathematical models of systems at the fig. 2.12 and fig. 2.2 differ only discrimination characteristics, therefore at identical parameters of other elements it is necessary to expect identical or close results at research of these systems. If studying of system (see fig. 2.12) the method of mathematical simulation selection following parameters: \( k = 1; a = 10 \, c^{-1}; b = 12.5 \, c^{-1}; c = 1; a = \alpha = 15 \).

The system was studied when exposed on input an equivalent harmonic signal \( u(t) = 1 + 0.5 \sin 2\pi Ft \) with carrier frequency \( F = 0.1 \, Hz \).

The adjustment of a fuzzy controller is carried out by criterion of a minimum of a dynamic error. Is received following optimal parameters of a fuzzy controller: \( A_m = 0.05; \, B_m = 0.4; \, C_m = 10; \, D_m = 150 \).

Processes in system (see fig. 2.12) are shown at the fig. 2.13 where \( u(t) - \) an input action, \( x(t) - \) an output a system (see fig. 2.13,a), \( e(t) = \text{Err} - \) a mismatch error on a discriminator input (see fig. 2.13). \( m(t) - \) an output of a fuzzy controller (see fig. 2.13c). The maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed 0.7\% from amplitude of a sinusoid. At the fig. 2.13,d transition is presented process of system - response to unit step action. The system fulfills input influence in time, not exceeding 0.22s, with overshoot not more than 10\%.

Thus, the fuzzy controllers provide accuracy fulfills of input influences, but and high-speed performance of system at the jump action. It should be noted that the study of the system without the controller shows, that the system does not have the stability.

The system (see fig. 2.12) with a fuzzy controller it is researched also in the presence of a stationary white noise (Band-Limited White Noise) on an output of frequency discriminator [5].

Implementation of a white noise \( v(t) \) is shown at the fig. 2.14,a. A mismatch error on an input of the slope detector \( e(t) \) and an output of a fuzzy controller \( m(t) \) thus are casual processes (see fig. 2.14, b and c ).

Researches of system in the presence of a stationary white noise show that noise of such intensity slightly influences a system output \( x(t) \) (see fig. 2.14 d) however error of system increases (we compare fig. 2.4b and 2.5b) therefore at small input influences (at small frequency deviations of the generator from set) noise on a detector output will make the considerable impact on a dynamic error of system.

It is of interest to consider the processes in the system of PLL (see fig.2.12)if using instead of the fuzzy controller the PID-controller [6]. The transfer function of the digital PID (PID on fig.2.12) controller

\[
W(z) = \frac{G_1 + G_2 \frac{z + 1}{z - 1} + G_3 \frac{z - 1}{z}}{z - 1},
\]
Fig. 2.13. Processes in system

Fig. 2.14. Processes in system with a fuzzy controller in the presence of a stationary white noise
where \( G_1 = K_1 \), \( G_2 = \frac{K_1 h_0}{2} \), \( G_3 = \frac{K_d}{h_0} \), \( h_0 \) - a digitization step.

As a result of tuning a controller at the indicated higher parameters of the system and input an equivalent harmonic signal is received following optimal parameters of a PID-controller, at the \( h_0 = 0.01 \). \( G_1 = 130; \ G_2 = 0.48; \ G_3 = 2000 \). Processes in system (see fig. 2.12) with a PID-controller are shown on fig. 2.15 where \( u(t) \) - an input action, \( x(t) \) - an output a system (see fig. 2.15,a), \( e(t) = Err \) - a mismatch error on a discriminator input (see fig. 2.15,b), \( m(t) \) - an output of a fuzzy controller (see fig. 2.15,c).

![Fig. 2.15. Processes in system with a PID-controller](image)

The maximum dynamic error in system of PLL with a PID-controller (except for initial burst at the moment of signal capture) attains of 2.8% from amplitude of a sinusoid. The maximum dynamic error in system of PLL with a PID-controller in 4 times more than the maximum dynamic error in system of PLL with a fuzzy controller.

On fig. 2.15,d transition is presented process of system with a PID-controller - response to unit step action. The transitional process - the oscillating with the overshoot of more than 20%. The system fulfills input influence in time, exceeding 0.31 s. The overshoot in system of PLL with the PID-controller in 2 times, and the regulation time in 1.4 times more than the appropriate parameters in system of PLL with a fuzzy controller.

When reducing the dynamic error in the PLL with PID-controller increases the overshoot and control time, and decrease in these parameters of the transition process increases dynamic error.

Thus, the fuzzy controllers provide accuracy fulfills of input influences, and high-speed performance of system PLL much better, then PID-controller.
The system (see fig. 2.12) with a PID-controller is researched also in the presence of a stationary white noise (Band-Limited White Noise) on an output of discriminator. Implementation of a white noise \( v(t) \) is shown at the fig. 2.16,a. A mismatch error on an input of the detector \( e(t) \) and an output of a fuzzy controller \( m(t) \) thus are casual processes (see fig. 2.16,b and c). Researches of system in the presence of a stationary white noise show that noise of such intensity slightly influences a system output \( x(t) \) (see fig. 2.16,d) however error of system increases (we compare fig. 2.15,b and 2.16,c) therefore at small input influences (at small frequency deviations of the generator from set) noise on a frequency detector output will make the considerable impact on a dynamic error of system.

![Fig. 2.16. Processes in system in the presence of a stationary white noise](image)

Let’s consider the system of the PLL of oscillator with a digital fuzzy controller. One of the main elements of the PLL oscillator is an analog phase detector, the static discrimination characteristic which can be written down in a kind

\[
U_{FD} = K_{FD} \cos \varphi
\]

where \( K_{FD} \) - the constant coefficient equal to the maximum value of voltage on an output of the detector, \( \varphi \) - difference in phase fluctuation (one of which is the reference) of the same frequency, field by to the first and second inputs of the detector (when equality the frequencies of two oscillation, the phase difference of these oscillations is constant).

When you change the frequency of input signal, the phase difference become a function of time:

\[
\varphi(t) = \varphi_0 + 2\pi \int \Delta f(t) dt
\]
where $\phi_0$ - starting value of phase difference at the moment $t=0$, when $\Delta f = f_1 - f_2 = 0$.

Taking into account expressions (2.3) and (2.4) block diagram of a phase detector with variable frequency $f_1$ and $f_2$ input signal will have the form shown at the fig.2.17 [7].

![Block diagram of a phase detector](image)

Fig. 2.17. Block diagram of a phase detector

Based on the structural scheme of the phase detector, taking into account the inertia of the filter at the output of the detector and the control element at the input of the oscillator (the filter at the output of the detector and the control element is usually described aperiodic links) can make a mathematical model of system of the PLL frequency control.

The mathematical model system of PLL with a digital fuzzy controller, compiled using the interactive system MATLAB, presented at the fig.2.18. Filter at the output of the digital phase detector and an controlling element of the generator describe the transfer functions:

$$G_1(s) = k / (s + b) = 10 / (s + 12.5), \quad G_2(s) = a f / (s + a) = 3 / (s + 20).$$

The digital fuzzy controller (Fuzzy controller at the fig. 2.18) is fulfilled under the block diagram, with the identical triangular membership function erected in degree and consists of the block of the shaper of sizes $A(t)$ and $B(t)$ (block 1 collected by the diagram), the block of comparing of sizes $A$ and $B$ and calculation $u_c$ (block 2 collected by the diagram) and the block of normalization output variable (block 3 collected by the diagram)[2].

![System of the PLL frequency control](image)

Fig. 2.18. System of the PLL frequency control.
The digitization step is chosen 0.001s. Values of ranges \( (\theta_{\text{max}} = \theta_{\text{min}}; \ Bm = \theta_{\text{max}} = -\theta_{\text{min}}; \ Cm = \theta_{\text{max}} = -\theta_{\text{min}}; \ Dm = m_{\text{max}} = -m_{\text{min}}) \) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task. At research of the system we will accept, that the difference of frequencies of two fluctuations changes under the sinusoidal law: \( \Delta f = 0.2 \sin(\pi / 5) \) or \( \Delta f = 0.2 \sin(\pi / 10) \) (i.e. the maximum deviation of frequency of the generator operated pressure, from the set reaches \( \pm 20\% \)). System of PLL should compensate a deviation of frequency of the generator, therefore an fuzzy controller it is necessary to adjust on the minimum current error of a mismatch in system. As a result of adjustment it is received following optimum parameters of an fuzzy controller:

\[
Am=0.03; \ Bm=0.5; \ Cm=10; \ Dm=20; \ c=1.
\]

Processes in system (see fig.2.18) at setting action \( 0.2 \sin(\pi / 5) \) are presented at the fig.2.19, at setting action \( 0.2 \sin(\pi / 10) \) at the fig. 2.20. In figure \( e(t) \) - the error of a mismatch on frequency on an input of the phase detector, \( \theta(t) \) – the error of a mismatch on frequency on an output of the phase detector, \( m(t) \) - operating pressure on an output of an fuzzy controller, \( u(t) \) and \( x(t) \) - an input and output of the system accordingly.

Independently of the frequency of input signal the transition process in the system finished for 3 s. Maximum dynamic mismatch error of frequency at the input of the phase detector at the input action \( 0.2 \sin(\pi / 5) \) does not exceed \( 2.8 \times 10^{-3} \) (1.4% of the amplitude of the input action), and for input action \( 0.2 \sin(\pi / 10) \) is approximately \( 1.4 \times 10^{-3} \) (0.7% of the amplitude of the input).

![Fig. 2.19. Processes in system at setting action 0.2sin(π/5)](www.intechopen.com)
As shown by the researches of the system (see fig. 2.18), a fuzzy controller allows to increase the accuracy of the PLL system practically by two orders of magnitude compared to the system without a controller.

Research of the PLL system, accuracy of tracking which defines a synthesized fuzzy controller, on the basis of the mathematical apparatus of the interactive system MATLAB allows to define simply enough one of the system key parameters - a strip of dynamic tracking. The strip of dynamic tracking of the PLL system is the greatest offset of the frequency of the input basic signal concerning the nominal frequency of oscillations of the voltage-controlled oscillator VCO. In this strip, the slave device of synchronization should remain in a mode of tracking irrespective of the speed of change of the input frequency in all ranges of frequencies. In other words, in a strip of dynamic tracking the PLL system can track arbitrarily fast changes of the input frequency, including spasmodic changes of frequency at the input of the system.

The strip of dynamic tracking of the PLL system at the research of the mathematical model of the PLL system in the interactive system MATLAB is defined simply enough, namely, submitting step signals on the input of the PLL system it is necessary to find the value of the jump of the input signal at which there is a tracking failure (i.e. at which the system ceases to fulfill the jump of the input signal). For researched system transients action (responses to jumps of the input signal) are shown at fig. 2.21. Disruption of tracking occurs when the amplitude of the jump $A=0.67$.

It is necessary to mark that at sinusoidal input action (at the set speed of change of input frequency) tracking failure occurs at certain amplitude of the sinusoidal effect, and the amplitude at which there is a tracking failure in this case will be more than at spasmodic change of input signal. For researched system processes (responses to input action...
$A \sin(\pi / 10)$ at various amplitudes $A$ an input signal) are shown at the fig. 2.22. Disruption of tracking occurs when the amplitude $A=0.78$.

![Image](image1.png)

Fig. 2.21. Disruption of tracking occurs when the amplitude of jump $A=0.67$.

![Image](image2.png)

Fig. 2.22. Responses to input action $A \sin(\pi / 10)$ at various amplitudes $A$ an input signal.
Disruption of tracking occurs when the amplitude $A=0.78$.

Consider the fuzzy system PLL generator with a digital phase detector [8]. Version execution a diagram of a phase detector is shown in fig. 2.23 a.

![Diagram of a phase detector](image)

**Fig. 2.23. Execution a diagram and principle of operation of a phase detector**

The principle of operation of digital phase detector can be explained as follows [7]. At the input of detector come the reference $u_o$ and $u$ input signals, that are previously transformed into meander shape (see. Fig. 2.23, b). The positive half-cycle - logical unit, negative - logical zero. We suppose that the periods of both signals are identical and equal $T_o$, and the phase difference is equal $\phi$. The counting pulses $u_{GCP}$ from the generator counting pulses arrive on an input of 1 reverse counter RC only at coincidence in time of input reference signal and for an input 2 in the presence of reference and absence input signals. At the end of each period, the next number at the output of down reversible counter is equal to the difference between the numbers $q_1$ and $q_2$ (see fig.2.23,b) is remembered and the reverse counter is nullified by the device of reset DR.

$$K_{FD} = \frac{T_o}{2T_c} \cdot$$ a constant rate, is equal to the maximum number on the output of the reversible counter of digital phase detector, $T_o$ - period of reference signal, $T_c$ - period of counting pulses, $\phi$ - difference in phase fluctuation (one of which is the reference) of the same frequency, field by to the first and second inputs of the detector (when equality the frequencies of two oscillation, the phase difference of these oscillations is constant).

Sampling interval at phase sample and hold devise is defined as $\delta \phi = 2\pi \frac{T_c}{T_o}$. A phase shift $\phi$ corresponds to a time shift $\Delta t = \frac{\phi}{2\pi} T_o$. 

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The static discrimination characteristic of the digital phase detector (see fig. 2.24,a) can be represented as the shaper of function, give at on specific intervals

\[
Q_{FD}(\phi) = \begin{cases} 
\frac{2}{\pi} K_{FD}(\frac{\pi}{2} - \phi) & \text{at } 0 \leq \phi \leq \pi; \\
\frac{2}{\pi} K_{FD}(\phi - \frac{3\pi}{2}) & \text{at } \pi \leq \phi \leq 2\pi.
\end{cases}
\] (2.5)

And continued periodically on intervals \(2k\pi \leq \phi \leq 2(k+1)\pi, \ k = 1,2,3,...\), and device of sampling-storage DSS (clamp).

Using the Fourier series expansion is extended periodically at the intervals \(2k\pi \leq \phi \leq 2(k+1)\pi, \ k = 1,2,3,...\), of the functions \(Q_{FD}(\phi)\) we obtain expression in the form

\[
Q_{FD}(\phi) = \frac{8}{\pi^2} K_{FD}[\cos \phi + \frac{\cos 3\phi}{9} + \frac{\cos 5\phi}{25} + \frac{\cos 7\phi}{49} + ...].
\] (2.6)

As practice shows, for determination of a mathematical model of the static discrimination characteristic of the digital phase detector is sufficient to the first three terms of Fourier series.

When you change the frequencies of input signals, the phase difference becomes a function of time (see the formula (2.4)). Taking into account expressions (2.4), (2.6) and the clamp the block diagram of a digital phase detector under varying frequencies \(f_1 \ i \ f_2\) of input signals will have the form, shown at the fig. 2.24,b.
At the output of clamp can be activated a filter for smooth the pulsations, for example, a simple analog filter with a transfer function of an aperiodic link \( \frac{k}{s+b} \).

On the basis of the block diagram of the numeral phase detector taking into account iner tance of the filter on the output of a detector and a controlling unit on the input of oscillator (a controlling unit is usually also described as an aperiodic link) can be make a mathematical model of system of the PLL frequency control.

The mathematical model of system of PLL with the digital fuzzy controller and digital phase detector, compiled using of interactive system MATLAB, is presented on fig. 2.25.

At research of the system we will accept, that the difference of frequencies of two fluctuations changes under the sinusoidal law: \( \Delta f = 0.2 \sin(\pi/5) \) (i.e. the maximum deviation of frequency of the generator from the set reaches \( \pm 20\% \)). System of PLL should compensate a deviation of frequency of the generator, therefore a fuzzy controller it is necessary to adjust on the minimum current error of a mismatch in system. As a result of adjustment it is received following optimum parameters of a fuzzy controller, \( c=1; A_m=0,08; B_m=2.5; C_m=50; D_m=100 \).

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Processes in system (see fig. 2.25) at setting action $0.2 \sin(\pi / 5)$ are presented at the fig. 2.26. In the figures $e(t)$ - a mismatch error in frequency at the input of digital phase detector (a) and its steady form in the dynamic mode (b), $\theta(t)$ - error at the output of phase detector - input of fuzzy controller (c) and its steady form in the dynamic mode (d), $m(t)$ - an operating pressure on an output of an fuzzy controller (e), $u(t)$ and $x(t)$ - an input and output of the system (see fig. 2.26) accordingly (f).

Transition process in the system ends in a time not exceeding 0.6 sec. The maximum dynamic error of a mismatch on frequency on an input of the phase detector on setting action $0.2 \sin(\pi / 5)$ does not exceed $4 \cdot 10^{-3}$. Research shows the system (see fig. 2.25), the fuzzy controller can increase the accuracy of the PLL system is almost two orders of magnitude compared to the system without controller.

![Processes in system at setting action 0.2 sin(\pi / 5)](image_url)
Application of fuzzy controllers in PLL appropriate. Since the fuzzy controller is a digital correction device, it can be successfully applied in the digital PLL.

2.3 Combined fuzzy-system of PLL

The mathematical model of combined system of PLL with the digital fuzzy controller, made with use of interactive system MATLAB, is presented on fig.2.27.

Fig. 2.27. The mathematical model of combined system of PLL with the digital fuzzy controller

Automatic control system name combined (or system with the combined control) when in the law of control except an error, derivatives and integrals from an error, additional communications on setting action or/and on revolting action are entered.

The basic system in detail considered earlier [fig. 2.18], consists of the phase detector which block diagram is resulted on fig. 2.17 and is presented by the comparison device, an integrating link with factor $2\pi$ (Transfer Fcn), the discrimination characteristic $\cos \phi$ (Discrim) and the low-pass filter on an output of the detector with transfer function $G_1(s) = k / (s + b)$ (Transfer Fcn1), an fuzzy controller (Fuzzy controller), the generator operated pressure, with transfer function $G_2(s) = a / (s + a)$ (Transfer Fcn2), and an individual negative feedback.

Filter at the output of the detector and an operating element of the generator have the transfer functions in a numerical kind: $G_3(s) = k / (s + b) = 10 / (s + 12,5)$; $G_4(s) = a / (s + a) = 3 / (s + 20)$.

The digital fuzzy controller with the identical triangular membership functions erected in degree (Fuzzy controller at the fig.2.27) is executed under the basic scheme. The fuzzy controller consists of the block of the shaper of sizes $A(t)$ and $B(t)$, the block of comparison of sizes $A$ (t)and $B$ (t) and calculation $u_c$ and the block of normalization output variable[2].

The digitization step is chosen 0,001s. Values of ranges ($A_{min}$; $A_{max}$; $C_{min}$; $C_{max}$; $D_{min}$; $D_{max}$) get out at controller adjustment. Additional coupling on setting action includes following elements: the frequency discriminator with the discrimination characteristic (Discrim1).
where \( K_d = 1 \) and \( \Delta = 1 \), and the filter of the bottom frequencies with transfer function 
\[ G_3(s) = k / (s + b) \] (Transfer Fcn3), and the amplifier (Gain) with changeable factor of strengthening.

From consideration of mathematical model of combined system of PLL with a digital fuzzy controller it is visible, that it is impossible to carry out the transfer function of additional coupling equal to return transfer function of generator 
\[ 1 / G_2(s) = (s + a) / (s + 20) / 3 \], i.e. it is impossible to carry out absolute invariancy of system from setting action, but it is possible to reduce a dynamic error of system by selection of transmission factor of amplifier Gain (to reach \( \varepsilon \)-invariancy).

At research of the combined system we will accept, that the difference of frequencies of two fluctuations changes under the sinusoidal law: 
\[ \Delta f = 0,2[1 + \sin(\pi / 5)] \] (i.e. the maximum deviation of frequency of the generator operated pressure, from the set reaches 40%).

System of PLL should compensate a deviation of frequency of the generator, therefore an fuzzy controller it is necessary to adjust on the minimum current error of a mismatch in system. As a result of adjustment it is received following optimum parameters of fuzzy controller:

\[ Am=0,03; Bm=0.5; Cm=10; Dm=20; c=1. \]

Optimum factor of strengthening of amplifier Gain \( K=7,3 \).
Processes in system (see fig. 2.27) at setting action \(0.2[1 + \sin(\pi / 5)]\) are presented at the fig. 2.28 (at the disconnected coupling on setting action) and at the fig. 2.29 (at the included coupling on setting action). In drawings: a) and b) \(e(t)\) - a dynamic error of a mismatch on frequency on an input of the phase detector in the transitive and established modes, c) \(m(t)\) - operating pressure on an output of an fuzzy controller, d) \(u(t)\) and \(x(t)\) - an input and output of the system accordingly.

The maximum dynamic error of a mismatch on frequency on an input of the phase detector in system of PLL at the disconnected coupling on setting action \(0.2[1 + \sin(\pi / 5)]\) in the established dynamic mode makes size \(4.9, 8 \times 10^{-4}\), and the maximum dynamic error of a mismatch on frequency on an input of the phase detector in system of PLL at the included coupling on setting action \(0.2[1 + \sin(\pi / 5)]\) in the established dynamic mode makes size \(3.6 \times 10^{-4}\). Transient time (time of capture of an input signal) in system at the disconnected coupling on setting action makes 4.3 seconds, and in system at the included coupling on setting action 2.5 seconds.

![Graphs](image)

Fig. 2.29. Processes in system at setting action \(0.2[1 + \sin(\pi / 5)]\) (at the included coupling on setting action)

Research combined (with additional coupling on setting action) system of PLL with a digital fuzzy controller by method of mathematical modeling has shown, that additional coupling on setting action reduces the established dynamic error in 2.7 times, and time of capture of an input signal in 1.7 times, in comparison with these indicators in system without coupling on setting action.
Ever more reduce the steady dynamic error can be achieved if in additional coupling on setting action to enter ideal forcing link with transfer function \( G(T_s + 1) \), where \( T = 1/a \), and in the factor \( G \) should enter \( k \) factor and additional adjusting factor \( K \).

The combined system of PLL with the digital fuzzy controller and forcing link shown in Fig. 2.30.

\[
U_{out}(n) = k[U_{in}(n) + T[U_{in}(n) - U_{in}(n-1)]]/h.
\]

The scheme of a digital ideal forcing link is resulted on fig. 2.31 (only this link the system on fig. 2.30 differs from the system shown on fig. 2.27). At research of the combined system we will accept, that the difference of frequencies of two fluctuations changes under the sinusoidal law: \( \Delta f = 0.2[1 + \sin(\pi/5)] \) (i.e. the maximum deviation of frequency of the generator, from the set reaches 40%).

Fig. 2.31. The scheme of a digital ideal forcing link
System of PLL should compensate a deviation of frequency of the generator, therefore a fuzzy controller it is necessary to adjust on the minimum current error of a mismatch in system. As a result of adjustment it is received the same optimum parameters of an fuzzy controller, as in the system shown on fig.2.27: Am=0.03; Bm=0.5; Cm=10; Dm=20; c=1. But the optimum amplification factor of amplifier Gain will be another: $K=0.82$.

Processes in system (see fig.2.30) at setting action $0,2[1 + \sin(\pi / 5)]$ are presented on fig.2.32 (at the disconnected coupling on setting action) and on fig. 2.33 (at the included coupling on setting action). In drawings: a) and b) $e(t)$ - a dynamic error of a mismatch on frequency on an input of the phase detector in the transitive and established modes, c) $m(t)$ - operating pressure on an output of an fuzzy controller, d) $u(t)$ and $x(t)$ - an input and output of the system accordingly.

The maximum dynamic error of a mismatch on frequency on an input of the phase detector in system of PLL at the disconnected coupling on setting action $0,2[1 + \sin(\pi / 5)]$ in the established dynamic mode makes size $9.8 \times 10^{-4}$, and the maximum dynamic error of a mismatch on frequency on an input of the phase detector in system of PLL at the included coupling on setting action $0,2[1 + \sin(\pi / 5)]$ in the established dynamic mode makes size $2.5 \times 10^{-4}$.

![Fig. 2.32. Processes in system at setting action $0,2[1 + \sin(\pi / 5)]$ (at the disconnected coupling on setting action)](image-url)
Fig. 2.33. Processes in system at setting action $0,2[1+\sin(\pi/5)]$ (at the included coupling on setting action)

Transient time (time of capture of an input signal) in system at the disconnected coupling on setting action makes 4.3 seconds, and in system at the included coupling on setting action 2.4 seconds.

Thus, in system of PLL with the digital fuzzy controller and a digital ideal forcing link additional communication on setting action reduces the established dynamic error in 3.9 times, and time of capture of an input signal in 1.79 times, in comparison with these indicators in system without communication on setting influence.

2.4 System of PLL with the phase-frequency discriminators
The phase-frequency discriminator has the specific characteristic which can be presented a non-linear element of type "saturation" (see fig. 2.34, a,b) that allows to work in broad range of frequencies.

Fig. 2.34. The characteristic of the phase-frequency discriminator
Discrimination characteristic phase-frequency discriminator (see Fig. 2.34, a) can be described by

\[
K(\phi) = \begin{cases} 
2\pi K_d & \text{at } \phi < -2\pi, \\
-2\pi K_d & \text{at } -2\pi \leq \phi \leq 2\pi, \\
-K_d \phi & \text{at } \phi > 2\pi.
\end{cases}
\]

In interactive system MATLAB the diagram of discriminator with the characteristic presented at the fig. 2.34, a it is possible to present as the connections of three block (see fig. 2.35): the switch (Switch), the inverter (Gain) and the saturation block (Saturation). At the positive signal of an error on a discriminator input (in block Switch parameter Threshold \(=0.000001\)) closed a upper contact of switch and the is formed a plot of characteristic \(\phi \geq 0\), at the negative signal of an error on input discriminator is closed bottom contact and formed a plot of characteristic \(\phi \leq 0\). Levels of restrictions in a block of saturation (Saturation) are \(\pm 2\pi K_d\).

![Fig. 2.35. Diagram of discriminator with the characteristic presented at the fig. 2.34, a](image1)

Discrimination characteristic phase-frequency discriminator (see Fig. 2.34, b) can be described by

![Fig. 2.36. Operating principle of phase-frequency discriminator](image2)
In an interactive system MATLAB the diagram of discriminator with the characteristic presented on fig 2.34,b can present only a block saturation (Saturation).

Phase-frequency discriminator with a characteristic shown in fig.2.34, b, is a machine that has five states (-2, -1, 0, +1, +2), and switchable on a leading edge of the reference signal and feedback signal. Reference signal adds to the state machine unit and the feedback signal on the contrary, subtracts. Operating principle of phase-frequency discriminator is shown at the fig.2.36 (darkened color is shown zero state machine).

At fig.2.37 shows the model on which to explore the two systems having the same phase-frequency discriminator (Discrim1), made under the scheme fig.2.35, and the same voltage-controlled oscillator (TransferFcn1), with a transfer function \( G(s) = \alpha f / s \). Transfer coefficient VCO \( \alpha f = K_G = 104 \text{rad} / (s \cdot V) \).

Fig. 2.37. System of PLL with phase-frequency discriminator

With the closure of a switch ManualSwitch1 in the down position simulated the system of PLL. The block diagram of this system, except for blocks Discrim1 and TransferFcn1 (simulating the discriminator and VCO), have two additional blocks: Gain (transfer factor of detector \( K_{fd} = 3,7/(4\pi) \text{V/ rad} \)) and block Correction circuit (an active filter with an operational amplifier). The transfer function of filter

\[
 F(s) \equiv \frac{r_s + 1}{r_s (r_s + 1)} = \frac{3330(s + 4,55)}{s(s + 100)}. 
\]

System with active filter on the operational amplifier was studied when exposed to on input of an equivalent harmonic signal \( u(t) = 10 + 5 \sin 2\pi Ft \) with carrier frequency \( F = 0,1 \text{Hz} \).
The processes in the system of the PLL frequency control with active filter Correction circuit (see fig.2.37) are shown in fig.2.38 where \( u(t) \) - an input action, \( x(t) \) - an output system (see fig.2.38.a), \( e(t) = \text{Err} \) - a mismatch error on the input of the discriminator (see fig.2.38, b), \( m(t) \) - a filter output (see fig.2.38,c). At fig.2.38, shows reaction system on step input \( u(t) = 10 \).

The results show that the system of the PLL frequency control with active filter Correction circuit has a very high exactness working off entrance influence: a dynamic mismatch error for a given harmonic signal is \( 4,2 \cdot 10^{-4} \) or 0.082% of the amplitude of input sine impact. System performance is also very high. Settling time is approximately equal to 0.06, but the transient process is oscillatory, with overshoot almost 60%.

![Fig. 2.38. The processes in the system of the PLL frequency control with active filter](image)

With the closure of a switch ManualSwitch1 in the upper position simulated the system of PLL with a low-pass filter LPF pass second and digital fuzzy controller (Fuzzy controller at the fig.2.37) which is executed by the block diagram. The controller consists of the block of the shaper of sizes \( A(t) \) and \( B(t) \), the block of comparing of sizes \( A \) and \( B \) and calculation \( u \), and the block of normalization output variable (block 3 collected by the diagram)[2]. The filter has the transfer function

\[
F(s) = \frac{k}{a_0 s^2 + a_1 s + 1} = \frac{0.002}{0.008 s^2 + 0.18 s + 1},
\]

which can also be written as
The mismatch error \( \theta(t) \) from an output of a low-pass filter arrives on a analog-digital converter (AD converter) (Zero-Order Hold), included in the input of a fuzzy controller. A quantization step of AD converter \( h = 0,01 \) s. On an output of a fuzzy controller is included the digital/analog converter (Zero-Order Hold 1).

For simplification of normalization (recalculation of values of signals in values of elements of universal set) ranges of change input and output signals (parameters of a fuzzy controller) we is accepting by symmetric:

\[
\hat{\theta}_{\min} = -\theta_{\max}; \quad \hat{\dot{\theta}}_{\min} = -\hat{\theta}_{\max}; \quad m_{\max} = -m_{\min}.
\]

Then recalculation of values of signals in values of elements of universal set perform according to the formulas [2]

\[
\begin{align*}
\hat{u}_1^* &= (\theta^* + A_m) / (2A_m); \\
\hat{u}_2^* &= (\dot{\theta}^* + B_m) / (2B_m); \\
\hat{u}_3^* &= (\ddot{\theta}^* + C_m) / (2C_m).
\end{align*}
\]

Values of ranges \( (Am = \theta_{\max} = -\theta_{\min}; \quad Bm = \dot{\theta}_{\max} = -\dot{\theta}_{\min}; \quad Cm = \ddot{\theta}_{\max} = -\ddot{\theta}_{\min}; \quad Dm = m_{\max} = -m_{\min}) \) at adjustment of a fuzzy controller steal up either manually, or automatically by the decision of the optimization task. The system of PLL with a low pass filter LPF and a digital fuzzy controller was investigated when exposed at input an equivalent harmonic signal \( u(t) = 10 + 5 \sin 2\pi F t \) with carrier frequency \( F = 0,1 \) Hz.

The adjustment of a fuzzy controller is carried out by criterion of a minimum of a dynamic error. Is received following optimal parameters of fuzzy controller:

\[
\begin{align*}
Am &= 0,0174; \quad Bm = 0,2458; \quad Cm = 4,6607; \quad Dm = 249,89.
\end{align*}
\]

When setting up a fuzzy controller in an interactive system MATLAB we using a block NCD (Nonlinear Control Design), which implements the method of dynamic optimization for the design of control systems. This tool is designed for use with Simulink, automatically adjusts a system parameters (in the system fig.2.37 adjusted parameters of the fuzzy controller, Am, Bm, Cm, Dm), based on certain constraints on the temporal characteristics (time of regulating and overshoot for reaction to a step action and limits for the dynamic mismatch error).

The processes in the system of the PLL frequency control with low pass filter LPF and the digital fuzzy controller (see fig.2.37) are shown in Fig.2.39 where \( u(t) \) - a input action, \( x(t) \) - a system output (see fig.2.39, a) \( e(t) = \text{Err} \) - a mismatch error at the input of the discriminator (see fig.2.39, b), \( m(t) \) - an output of fuzzy controller (see fig.2.39, c).

The maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed \( 4,2 \cdot 10^{-3} \) or \( 0,82\% \) from amplitude of a sinusoid.

At fig.2.39, d shows the system response to a step action \( u(t) = 10 \) on the output of system \( x(t) \) and the output a fuzzy controller \( m(t) \). Transient process have overshoot 3,4\%. The system fulfills input action in a time not exceeding 0,16 sec.
Fig. 2.39. The processes in the system of the PLL frequency control with low pass filter LPF and the fuzzy controller

We remark that the PLL with a low pass filter LPF, but without the digital fuzzy controller, has no stability.

Thus, can conclude that the PLL with a low pass filter LPF and the digital fuzzy controller has a bad quality indicators than the PLL with an active filter on the operational amplifier, but has transient process (overshoot).

Now consider the operation of these systems in the presence of internal noise. In this case, the switch Manual Switch on Fig.2.37 is closed in the upper position, which corresponds to the filing of the noise signal - a stationary white noise (Band-Limited White Noise) on the output phase-frequency detector (Discrim1). Noise signal \( n(t) \) represented on Fig.2.40. Amplitude of the noise emissions of approximately 1% of the input signal.

Fig. 2.40. Noise signal on the output phase-frequency detector
Fig. 2.41. Mismatch errors on the input of phase-frequency discriminator

At fig. 2.41 are presented mismatch errors on the input of phase-frequency discriminator at the presence of stationary white noise at the output of the discriminator in the system of a phase-frequency locked loop with low pass filter LPF and the digital fuzzy controller Fuzzy controller (see fig. 2.41 a) and in the system of the PLL frequency control with active filter Correction circuit (see Fig. 2.41, b).

As seen from the oscillograms the system of system of the PLL frequency control with a low pass filter LPF and a digital fuzzy controller Fuzzy controller has less mismatch error at the input phase-frequency discriminator in the presence of stationary white noise than the system phase-frequency locked loop with active filter Correction circuit.

2.5 Servomechanism of synchronous and in-phase rotation shaft part of drive from the alternating reference voltage

The mathematical model of servomechanism in an interactive system MATLAB is presented in fig. 2.42.

The mathematical model of the phase detector is represented serial connection of comparison device, the nonlinearity (Discrim) \( K(\phi) = K_d \sin \phi \), where \( K_d \) - coefficient of conversion of the phase detector, and \( \phi(t) = \phi(t) \) - a mismatch error on a phase detector input, and the low-pass filter LPF (Transfer Fcn).

Transfer function of a low-pass filter LPF (Transfer Fcn)[2]

\[
G(s) = \frac{1}{\tau s + 1}, \quad a = 1/\tau, \quad \tau = RC = 0.08 \text{ s.}
\]

Transfer function of the amplifier of power-modulator P-M (Transfer Fcn1)[2]

\[
G_1(s) = \frac{12.5}{s + 12.5}, \quad a = 1/\tau, \quad \tau = RC = 0.08 \text{ s.}
\]

Transfer function of engine EE (Zero-Pole)[2]

\[
G_2(s) = \frac{500}{s + 100}, \quad a_1 = 1/T_y, \quad T_y = 0.01 \text{ s.}
\]

Transfer function of engine EE (Zero-Pole)[2]

\[
G_3(s) = \frac{\alpha}{s(s + b)} = \frac{90}{s(s + 10)},
\]

where \( b = 1/T_{db}; \quad T_{db} = 0.1 \text{ c}; \quad \alpha = Kdb / T_{db}; \quad Kdb = 9 \text{ rad} / (V \cdot s) \)
Fig. 2.42. The mathematical model of servomechanism

Transfer function of the tacho-generator (Gane + Derivation)[2]

\[ G_s(s) = \frac{k}{s} \cdot \frac{0.03}{V \cdot s \cdot \text{rad}} \]

At adjustment of a fuzzy controller in interactive system MATLAB we use unit NCD (Nonlinear Control Design) which implements a method of dynamic optimization for designing of management systems.

The system is researched at typical influences on an input: input: 1. single jump, 2. the equivalent of a harmonic signal \( u(t) = 0.5 \sin 2\pi F t \) with carrying frequency \( F = 0.1 \text{Hz} \) and 3. a linear signal Ramp \( u(t) = t \).

Adjustment of a fuzzy controller is carried out by criterion of a minimum of a dynamic error at the equivalent harmonic signal without influence of noise on a phase detector output (contacts of switch are closed in the lower position). Receive the following optimal parameters of fuzzy controller: \( A_m = 0.072; B_m = 0.168; C_m = 1.579; D_m = 61.798 \).

Processes in servomechanism (fig. 2.42 see) with a fuzzy controller without influence of noise on a phase detector output (contacts of switch Manual Switch are closed in the lower position) are shown on fig. 2.43 at an single jump of an input signal, on fig. 2.44 at influence of the equivalent harmonic signal \( u(t) = 0.5 \sin 2\pi F t \), with carrying frequency \( F = 0.1 \text{Hz} \) and on fig. 2.45 at of the linear signal \( u(t) = t \). In pictures \( u(t) \) - input influence, \( x(t) \) - a system output, \( e(t) = \text{Err} \) - a mismatch error on a discriminator input, \( m(t) \) - a signal on an engine input.

At an single jump of an input signal transient phenomenon has overshoot of 13 % and ends at time 2.8 (see fig. 2.43). The installed dynamic error is equal to zero. At action of a harmonic signal the maximum dynamic error (except for the release initial at the moment of
signal capture) does not exceed 0.5% from amplitude of a sinusoid (see fig. 2.44). At action of the linear signal \( u(t) = t \) transient phenomenon comes to an end in time, not exceeding 2c, and the installed dynamic error is close to zero (approximately \( 2 \cdot 10^{-4} \)). Therefore the signal output practically repeats input action (fig. 2.45 see).

![Fig. 2.43. Processes in servomechanism at a single jump](image1)

![Fig. 2.44. Processes in servomechanism at the equivalent harmonic signal](image2)
Adjustment of a fuzzy controller by criterion of a minimum of a dynamic error at action of noise on a phase detector output (contacts of switch Manual Switch are closed in the upper position) is enough difficult. The noise signal $n(t)$ - a stationary white noise (Band-Limited White Noise) on a phase detector output (Discrim) is presented on fig. 2.46,a. Current value of a root from root-mean-square of a noise signal (root mean squared value) calculated in unit CRMS is shown at the fig. 2.46b.

Instead of fuzzy controller FC (Fuzzy Controller), is executed under the basic scheme, with identical triangular functions of an accessory in the presence of a noise signal on an output of the phase detector for obtaining of a smaller dynamic error in system it is expedient to use fuzzy controller (Fuzzy Controller), fulfilled on the schematic circuit with identical exponential membership functions. For the purpose of an exception (or the considerable decrease) a dynamic error at tracking controlling action, on an output of a fuzzy controller two integrators for giving to a closed circuit of follow-up system astaticism the second order are included.

At system adjustment, except selection of values of ranges $A_m, B_m, C_m, D_m$ and parameter $c$ of exponential membership functions in a fuzzy controller, it was required to optimize coefficient $k$ of the tacho-generator of an alternating current. At adjustment of a fuzzy controller in interactive system MATLAB is used unit NCD (Nonlinear Control Design) and received the following optimal parameters of system:

![Fig. 2.45. Processes in servomechanism at the linear action](image-url)
Fig. 2.46. Noise signal

Fig. 2.47. Processes in system at the unit step

Fig. 2.48. Processes in system at the equivalent harmonic signal
Processes in servomechanism (fig. 2.42 see) with a fuzzy controller at influence of noise on a phase detector output (contacts of switch Manual Switch are closed in the upper position) are shown on fig. 2.47 at an unit step of an input signal, on fig. 2.48 at action of the equivalent harmonic signal $u(t) = 0.5 \sin 2\pi F$, with carrying frequency $F = 0.1 \text{Hz}$ and on fig. 2.49 at action of the linear signal $u(t) = t$. In figure $u(t)$ – an input action, $x(t)$ – a system output, $e(t) = \text{Err}$ – a mismatch error on input of a discriminator, $m(t)$ - a signal on an input of engine.

At an single jump of an input signal transient phenomenon has overshoot of 16% and comes to an end during 0.93 s (fig. 2.47 see). The installed dynamic error is equal to zero. At action of a harmonic signal the maximum dynamic error (except for initial burst at the moment of signal capture) does not exceed 1.5% from amplitude of a sinusoid (fig. 2.48 see).

At action of the linear signal $u(t) = t$ transient phenomenon ends at time, not exceeding 1.2 s, and the installed dynamic error does not exceed $8 \cdot 10^{-3}$. The signal output practically repeats input action (fig. 2.49 see).

Fig. 2.49. Processes in system at the linear action

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MATLAB - A Ubiquitous Tool for the Practical Engineer
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A well-known statement says that the PID controller is the “bread and butter” of the control engineer. This is indeed true, from a scientific standpoint. However, nowadays, in the era of computer science, when the paper and pencil have been replaced by the keyboard and the display of computers, one may equally say that MATLAB is the “bread” in the above statement. MATLAB has become a de facto tool for the modern system engineer. This book is written for both engineering students, as well as for practicing engineers. The wide range of applications in which MATLAB is the working framework, shows that it is a powerful, comprehensive and easy-to-use environment for performing technical computations. The book includes various excellent applications in which MATLAB is employed: from pure algebraic computations to data acquisition in real-life experiments, from control strategies to image processing algorithms, from graphical user interface design for educational purposes to Simulink embedded systems.

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