Thermo-compositional diabatic convection in the atmospheres of brown dwarfs and in Earth’s atmosphere and oceans

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- Brown dwarfs spectral sequence:

Clouds?
Brown dwarfs spectral sequence:

or reduced T gradient?
Brown dwarfs spectral sequence:

Convection linked to CO/CH4 transition?

or reduced T gradient?

Why?
What is in common between:
Convective systems but not adiabatic, they are all subject to:

- Energy exchange (latent heat, thermal diffusion, radiative transfer)
- and/or compositional source terms (chemical reactions, condensation/evaporation, compositional diffusion)
- Thermohaline or fingering convection

\[
\left( \nabla_T - \nabla_{\text{ad}} \right) \kappa_\mu - \nabla_T \kappa_T > 0
\]

Stern 1960

Ulrich 1972
Steam/liquid or moist convection

\[ \nabla T - \nabla_{\text{ad}} \left( 1 + \frac{X_{eq} L}{R_d T_0} \right) > 0 \]

Moist « pseudo-adiabat »

von Bezold 1893
What is adiabatic convection?

Thermal adiabatic case

\[ \frac{\partial \ln \theta}{\partial t} + \mathbf{u} \nabla (\ln \theta) = 0 \]

\[ \theta = T \left( \frac{P_{\text{ref}}}{P} \right)^{\frac{\gamma-1}{\gamma}} \]

\[ P = \rho k_b T / \mu \]
What is adiabatic convection?

**Thermal adiabatic case**

\[ \frac{\partial \ln \theta}{\partial t} + \vec{u} \cdot \nabla (\ln \theta) = 0 \]

\[ \theta = T(P_{\text{ref}}/P)^{(\gamma-1)/\gamma} \]

\[ P = \rho k_b T/\mu \]

- **Unstable if:** \[ \frac{\partial \ln \theta_0}{\partial z} < 0 \]

- **Schwarzschild criterion (1906)**

\[ \nabla_T - \nabla_{\text{ad}} > 0, \quad \nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0} \]

\[ \frac{\partial T_0}{\partial z} < \frac{g}{C_p} \]
What is adiabatic convection?

Thermo-compositional adiabatic case

\[
\frac{\partial \ln \theta}{\partial t} + \vec{u} \nabla (\ln \theta) = 0
\]

\[
\frac{\partial X}{\partial t} + \vec{u} \nabla (X) = 0
\]

\[
\theta = T \left( \frac{P_{\text{ref}}}{P} \right)^{(\gamma - 1)/\gamma}
\]

\[
P = \rho k_b T / \mu(X)
\]
- **What is adiabatic convection?**

- **Thermo-compositional adiabatic case**

\[
\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u} \nabla (\ln \theta) = 0 \quad \theta = T(P_{\text{ref}}/P)^{(\gamma-1)/\gamma}
\]

\[
\frac{\partial X}{\partial t} + \overrightarrow{u} \nabla (X) = 0 \quad P = \frac{\rho k_b T}{\mu(X)}
\]

- **Unstable if:**

\[
\nabla T - \nabla_{\text{ad}} - \nabla_{\mu} > 0
\]

\[
\nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0}, \quad \nabla_{\mu} = \frac{\partial \ln \mu_0}{\partial \ln P_0}
\]

- **Ledoux criterion**

(1947)
- What is adiabatic convection?
- Thermo-compositional diabatic case

\[
\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u} \nabla (\ln \theta) = \frac{H(X, T)}{T} \quad \theta = T \left( \frac{P_{\text{ref}}}{P} \right)^{(\gamma-1)/\gamma}
\]
\[
\frac{\partial X}{\partial t} + \overrightarrow{u} \nabla (X) = R(X, T) \quad P = \rho k_b T / \mu(X)
\]
- What is adiabatic convection?

- Thermo-compositional diabatic case

\[
\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u} \nabla \left( \ln \theta \right) = \frac{H(X, T)}{T} \quad \theta = T \left( \frac{P_{\text{ref}}}{P} \right)^{\frac{\gamma-1}{\gamma}}
\]

\[
\frac{\partial X}{\partial t} + \overrightarrow{u} \nabla (X) = R(X, T)
\]

- **Unstable if:** \( \nabla_T - \nabla_{\text{ad}} - \nabla_\mu > 0 \)

or

\[
(\nabla_T - \nabla_{\text{ad}}) \omega_X' - \nabla_\mu \omega_T' < 0
\]

with \( \omega_X' = R_X + R_T \left( T_0 \partial \ln \mu_0 / \partial X \right) \)

and \( \omega_T' = H_T + H_X \left( T_0 \partial \ln \mu_0 / \partial X \right)^{-1} \)
Thermohaline or fingering convection

\[
(\nabla_T - \nabla_{ad})\omega'_X - \nabla_{\mu}\omega'_T < 0
\]

\[
(\nabla_T - \nabla_{ad})\kappa_\mu - \nabla_{\mu}\kappa_T > 0
\]

Stern 1960
Ulrich 1972
Steam/liquid or moist convection

\[(\nabla_T - \nabla_{ad}) \omega_X' - \nabla_\mu \omega_T' < 0\]

\[X = X_{eq}(P, T)\]

Moist « pseudo-adiabat »

von Bezold 1893
- Thermo-compostional diabatic convection

\[
(\nabla_T - \nabla_{\text{ad}})\omega'_X - \nabla_\mu \omega'_T < 0
\]

with  \( \omega'_X = R_X + R_T(T_0 \partial \ln \mu_0 / \partial X) \)

and  \( \omega'_T = H_T + H_X(T_0 \partial \ln \mu_0 / \partial X)^{-1} \)

and probably many more….
- CO/CH4 radiative convection

\[(\nabla_T - \nabla_{ad})\omega'_{X} - \nabla_{\mu}\omega'_{T} < 0\]

with \[R = - (X - X_{eq})/\tau_{\text{chem}}\]

and \[H = 4\pi k/c_p (J - \sigma T^4)\]

Brown dwarfs and giant exoplanets

Moist convection

CO/CH4 radiative convection
Generalisation of mixing length theory

adiabatic: \( \ln \theta \) is conserved

or

diabatic: \( \ln \theta' = \ln \theta - X \frac{\partial \ln \mu_0}{\partial X} \frac{\omega_T'}{\omega'_X} \) is conserved
- Generalisation of mixing length theory

adiabatic: \( \ln \theta \) is conserved

Can define an adiabatic convective flux:

\[
F_{ad} = \rho c_p w_{ad} T_0 (\nabla_T - \nabla_{ad})
\]

or

Can define a diabatic convective flux:

\[
\ln \theta' = \ln \theta - X \frac{\partial \ln \mu_0}{\partial X} \frac{\omega'_T}{\omega'_X}
\]
is conserved

Can define a diabatic convective flux:

\[
F_d = \rho c_p w_d T_0 (\nabla_T - \nabla_{ad} - \nabla \mu \omega'_T / \omega'_X)
\]
Generalisation of mixing length theory

Can define a diabatic convective flux:

\[ F_d = ρc_p w_d T_0(∇T - ∇_{ad} - ∇μω'_T/ω'_X) \]

similar to mass/flux convection parametrizations used for moist convection (review: Arakawa & Jung 2011)

\[ \ln θ' = \ln θ - XL/c_p T_0 \]
- Generalisation of mixing length theory

Can define a diabatic convective flux:

\[ F_d = \rho c_p w_d T_0 (\nabla_T - \nabla_{\text{ad}} - \nabla_\mu \omega'_T / \omega'_X) \]
- Generalisation of mixing length theory

adiabatic: \( \ln \theta \) is conserved

Can define an adiabatic convective flux:

\[
F_{\text{ad}} = \rho c_p w_{\text{ad}} T_0 (\nabla_T - \nabla_{\text{ad}})
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or

diabatic: \( \ln \theta' = \ln \theta - X \frac{\partial \ln \mu_0}{\partial X} \frac{\omega'_T}{\omega'_X} \) is conserved

Can define a diabatic convective flux:

\[
F_d = \rho c_p w_d T_0 (\nabla_T - \nabla_{\text{ad}} - \nabla_{\mu} \omega'_T/\omega'_X)
\]
- Bifurcation between adiabatic and diabatic convection
- Boiling crisis in steam/liquid convection
- Bifurcation between adiabatic and diabatic convection
- Boiling crisis in steam/liquid convection

Could provide a natural explanation of the boiling crisis?
- Bifurcation between adiabatic and diabatic convection
- L/T transition in brown-dwarf spectra?

\[ F_d = \rho c_p w_d T_0 (\nabla_T - \nabla_{ad} - \nabla_{\mu_T}/\omega_X) \]

\[ F_{ad} = \rho c_p w_{ad} T_0 (\nabla_T - \nabla_{ad}) \]

Dupuy & Liu 2012
- Bifurcation between adiabatic and diabatic convection
- L/T transition in brown-dwarf spectra?

A giant cooling crisis?
Idealized case study: a possible CO/CO₂ transition in hot (irradiated or young) rocky exoplanets

Expected dominant species in an terrestrial planet atmospheres (abiotic)

Forget and Leconte (2013), « Possible climate on terrestrial exoplanets » Phil. Trans. Royal Society A. (2014) (arXiv:1311.3101)

Daley-Yates et al. in prep
- Idealized case study: a possible CO/CO$_2$ transition in hot (irradiated or young) rocky exoplanets

Daley-Yates et al. in prep

5000$^3$ 3D run on 1000 GPUs
- You are skeptical about this? Good we all should be!

BUT

- You should also be skeptical about the role of clouds in brown dwarfs

- Clouds are present for sure in some objects, e.g. the silicate 10 um feature but they are not necessarily responsible for the NIR reddening:
  - Why don’t we see strong polarization in BDs?
  - Start of a consensus about the need of a reduced temperature gradient to explain T/Y dwarfs spectra: Leggett et al. 2019
  - Why cloud models are failing on low-gravity objects where they should work best?
  - Why only one L/T transition when crossing 10’s of condensation curves?