Optimisation of variable helix end millings tools by minimising self excited vibration

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Abstract. During machining, self-excited vibrations known as regenerative chatter can occur. This instability can be avoided by modifying the tool geometry, thereby influencing the time delay terms that arise in the governing equations. The present study uses Differential Evolution (DE) to optimise the tool helix geometry, so as to avoid chatter. The results are compared to those from Sequential Quadratic Programming (SQP). It is shown that the DE approach can significantly increase the chatter stability, and substantially out-performs the SQP algorithm. The performance of the DE approach is due to its ability to perform global optimisation in the presence of significant nonlinearities.

1. Introduction
Aerospace, automotive, mold/die and general manufacturing industries face pressure to ensure lower cost, greater productivity and improved quality in order to encourage the economic growth of the machine tool industry. However, machining productivity using a high material removal rate is inhibited by the dynamic deflection of tool and workpiece. This causes sudden large amplitude vibrations when energy input exceeds the energy dissipated from the system, producing chatter. Chatter is a self excited vibration that occurs in metal cutting if the chip width is too large with respect to the dynamic stiffness of the system, especially when machining with a high material removal rate. It produces a poor surface finish and high tool wear and can even damage machine tools. The boundary of stability (between stable or unstable machining) is a function of depth of cut and spindle speed.

Researchers have created various types of chatter prediction models such as time domain simulation [1], analytical delay differential equations with time periodic coefficients [2], time finite element analysis [3], and semi discretisation methods [4]. To suppress chatter, passive methods such as the use of vibration absorbers or damping treatments have been applied. Active approaches, such as spindle speed modulation or active vibration control have also been considered, but passive methods are often preferred due to their simplicity.

This paper investigates a passive approach to chatter suppression that disrupts the chatter vibration using variable helix tools. Figure 1 illustrates uniform helical end milling tools compared to non-traditional variable helix and variable pitch designs. Variable pitch tools were proposed by Slavicek [5]. Later, Opitz et al. [6] studied irregular tooth pitches that produce a higher stable depth of cut. Only Stone [7] applied an irregular helix in the early period before 1970s. Variable pitch tools were re-considered by Altintas et al. [2] who utilised an invariant time constant and a non uniform multiple regeneration time delay to optimise pitch geometry. Meanwhile, Budak [8] modelled and optimised a
non constant pitch cutter using an analytical stability model. Recently, Olgac and Sipahi [9]
theoretically maximised the material removal rate by applying an irregular pitch cutter that optimised
with the Cluster treatment of Characteristic roots (CTCR) as a mathematical objective function.

This brief overview has demonstrated that current research on passive chatter suppression methods
only takes into account variable pitch tools, and has tended to overlook variable helix tools. Besides
reducing chatter, the helical angle can also break chip formation and change the line of contact
between the tool and workpiece. For the present study, a semi discretisation method (SDM) has been
combined with Differential Evolution (DE) to optimise variable helix end milling tools. The target is
to reduce chatter, and Sequential Quadratic Programming (SQP) is used to benchmark the optimisation
performance.

The paper is presented as follows. First, a summary of SDM applied to the current problem is
briefly described. The DE optimisation algorithm and application into SDM are then presented. An
optimisation study is then presented for a three flute variable helix milling tool. SQP optimisation is
finally carried out to validate the performance of DE.

2. Semi Discretisation Method
The Semi Discretisation Method (SDM) is a well known technique in the Finite Element Method and
Computer Fluid Dynamics. SDM analyses stability of linear retarded dynamical systems based up on a
discretisation technique. The effects of time delays and time periodicity are considered to produce a
high dimensional linear discrete system. SDM is more effective in time and accuracy than fully
discrete methods due to the delayed states and time dependent coefficient. Details of SDM are given in
ref.[4]. Recently, Sims et al. [10] applied SDM to model irregular helix and pitch tools.

The regenerative chatter in milling can be represented as shown in Figure 2. It comprises structural
dynamics, regenerative effect and dynamic cutting force coefficients that interact to form a potentially
unstable feedback path. The regenerative system includes a time delay term that arises due to the
angular spacing between the teeth on the tool. This delay term can be modified by using variable helix
or variable pitch tools.

The stability of the feedback path shown in Figure 2 can be determined from the characteristic
multipliers ($CM$) or eigenvalues that emerge from the semi-discretisation method. Characteristic
Multipliers are obtained for each combination of spindle speed and depth of cut. When the maximum
value of $CM$ is greater than unity then the system is unstable. The type of instability can characterised
by the location where the $CM$ crosses the unit circle (cyclic fold bifurcation, secondary hopf
bifurcation and period doubling bifurcation). Consequently, the results of the method can be
summarised graphically by plotting stability (maximum $CM$ greater than unity) on a plot of spindle
speed versus depth of cut. Meanwhile, all of the corresponding $CM$’s can be plotted on the complex
plane (along with the unit circle) to indicate the relative stability of the system.

3. Differential Evolution
Evolution Algorithms (EA) such as Genetic Algorithm (GA), Evolutionary Programming (EP) and
Evolution Strategy (ES) have been researched for several decades. Differential Evolution (DE) was
introduced by Price et al. [11], and can be considered to be an improved Genetic Algorithm (GA)
version with different strategies for faster optimisation. It is similar to other evolution algorithms in
which mutation plays the key role, with real valued parameters that directly search for the global
optimum. A basic idea in DE is that of adapting the search during the evolution process. Compared to
other algorithms, DE has the advantages of simple structure, ease of use, speed and robustness. In
machining applications, Saikumar and Shunmugan [12] applied DE to select the best cutting speed,
feedrate and depth of cut to achieve optimum surface finish while Krishna [13] applied DE in a
grinding process to search for suitable process grinding in minimising surface grinding.

DE can solve objective functions that are non differentiable, non linear, noisy, flat, multi
dimension, and with multiple local minima. Such functions are difficult to solve analytically, and the
variable helix optimisation problem fits within this scope. DE begins using initial samples at multiple
random chosen initial points. With simple algorithms, DE can search for the optimal condition very quickly with minimal control parameters such as mutation, crossover, selection and population. The concept is evolved from GA’s with layer population and special evolutionary strategy of self adaptive mutation. However, instead of a binary-coded populations (as in a GA), differential evolution deals with real-coded populations, and uses its own processes of mutation and crossover. The mutation process is created randomly from the selection of three individual vector differences. In the crossover process, any individual population member has equal opportunity to survive in the next generation based on its fitness value. The process of evolving mutation, recombination and selection through generations or new populations is repeated until the optimum solution is achieved. In the present work, the DE source code by Markus Buehren [14] was used. The code is based on the DE algorithm of Price et al. [11].

4. Implementation

By combining with the analytical method for chatter stability prediction, the process of DE optimisation can be used to optimise tool/helix geometry. Figure 3 shows the sequence of operations, namely: optimisation setting process, SDM process, objective function evaluation process and DE optimisation process. DE parameters are first set to create an initial population in the optimisation process. To search for the optimum values, DE requires the predicted values from the SDM stability analysis with consideration of the input variables (i.e. helix angles $\beta_i$ and pitch angles $\phi_i$, cutting parameters, etc.). The fitness of each population member is evaluated in terms of the chatter stability: consequently the DE process will strive to obtain tool geometry (helix and pitch angles) that maximize the chatter stability. This process continues until the termination criteria are met.

The values DE parameters used in the present work are given in Table 1. Crossover rate, $CR$ is 0.7, scaling factor, $F$ is 0.7 and number of population, $NP$ is 10 times the dimensions ($10^\text{D}$) and 70 generations are employed in DE optimisation. A ‘strategy-7’ (DE/rand/1/bin) methodology [13, 12] was implemented owing to its wide application in the literature. This methodology involves random perturbation of a population vector (‘/rand’), perturbation of a difference vector for the mutation process (‘/1’), and binomial crossover (‘/bin’).

During the optimisation process the candidate values of tooth helix $\beta_i$ and pitch $\phi_i$, may result in milling cutters whose flutes intersect each other. This is clearly inadmissible from a practical viewpoint. To prevent helical angles intersecting, the helical height different ($\Delta h_\beta$) is introduced as a constraint in the DE optimisation. The helical angle difference calculations for variable helix and variable helix with variable pitch are as follows:

$$\Delta h_\beta = \frac{d_c - d_c (2 N_c)^{-1}}{\tan \beta_i} - \frac{d_c - 2 d_c (N_c)^{-1}}{\tan \beta_{i+1}} \quad \text{(variable helix)}$$

$$\Delta h_\beta = \frac{d_c - d_c (2 N_c)^{-1}}{\tan \beta_i} - \left(\frac{d_c - 2 d_c (N_c)^{-1}}{\tan \beta_{i+1}}\sin \left(2^{-1} \Delta \phi \right)\right) \quad \text{(variable helix with variable pitch)}$$

Here, Eqn (1) relates to optimization problems where the free end of the milling cutter has teeth that are uniformly spaced, but the variable helix of each flute means that the teeth are irregularly spaced along the rest of the cutter’s axial length. Eqn (2) relates to cutters with both variable helix and a variable pitch at the free end. The $\Delta h_\beta$ value is based on the parameters illustrated in Figure 1c, along with the number of cutter teeth, $N_c$ and cutter diameter, $d_c$, and pitch difference, $\Delta \phi$. The final optimisation problem can be specified as follows:

Objective function: Minimise the maximum value of $f_{CM}(\beta_i, \phi_i, n) = CM$ for all values of $n$ and $b$

Subject to constraints:
Spindle speed \( n_{\text{min}} \leq n \leq n_{\text{max}} \) (r/min)

Depth of cut \( b_{\text{min}} \leq b \leq b_{\text{max}} \) (mm)

Helical Angle \( 25 \leq \beta_i \leq 55 \) \( ^{\circ} \)

Pitch Angle \( \phi + 22.5 \leq \phi_i \leq \phi + 22.5 \) \( ^{\circ} \)

Helical height different \( \Delta h_{\beta} \geq 5 \) (mm)

The constraint on \( \beta_i \) (25\(^{\circ}\) to 55\(^{\circ}\)) is to ensure good chip evacuation [8, 9]. The DE needs to search for the suitable value of \( \beta_i \) that produces minimum chatter across the chosen spindle speed and depth of cut range (along with the additional constraints).

In order to benchmark the DE optimisation method, a traditional gradient based optimisation method (Sequential Quadratic Programming (SQP)) was combined with the Semi Discrete Method to minimise chatter and optimise variable helix geometry. The SQP approach is widely used in manufacturing applications: In machining, Kurdi et al.[15] compared SQP with Particle Swam Optimisation (PSO) for optimising the surface location error and material removal rate at same time as minimising chatter. The main idea of SQP is to obtain a search direction from a quadratic program solution together with its constraints. The general method to solve constrained optimisation is stated in [16]. In this research, the SQP in Matlab Optimization Toolbox was used by employing the constrained minimisation function [17].

5. Results

The results from a three flute variable helix from Sims et al. [10] is initially presented. The parameters from [10] are presented in Table 2. The milling cutter helix geometry is then optimised based on variable helix and variable pitch modifications to reduce chatter. Stability lobes and \( CM \) results of optimum cutting tools are illustrated and compared with the original design that had been chosen arbitrarily.

The original tool geometry consisted of a three flute variable helix (25,30,35) cutter with variable pitch (120,100,140) at its free end. In Figure 4, a large unstable region can be observed at high depth of cut, giving eigenvalues outside the unit circle. For this low radial immersion scenario three instability conditions are seen: cyclic fold bifurcation, secondary hopf bifurcation and period doubling bifurcation. The optimisation routines were used to adjust the tool helix so as to obtain the most stable chatter performance across the illustrated spindle speed range. Two scenarios were considered: a variable helix with a uniform pitch at the tool’s free end, and a variable helix with a variable pitch at the tool’s free end.

The performance of the DE and SQP algorithms is summarised in Figure 5. Note that the objective function is a maximum value for a \( CM \), so a value less than unity represents complete stability over the chosen spindle speeds and depths of cut.

The DE algorithm achieved complete stability (maximum \( CM = 0.8965 \)) by the second generation. The corresponding variable helix (52,52,41) with variable pitch (107,169,90) stability diagram is shown in Figure 6. Since the maximum \( CM \) is less than unity, no chatter is observed. In comparison, the SQP algorithm achieved a maximum \( CM \) of 0.952 after 25 iterations. The corresponding best-case stability diagram is shown in Figure 7. Although this system is also completely stable, the \( CM \)s are considerably greater than those obtained with the DE algorithm. This indicates a lower margin of stability.

For the scenario where the tool’s free-end pitch angle was uniform, Figure 5 shows that a poorer performance was obtained for both optimisation methods. The DE algorithm converged at 50 generations with 1.005 evaluation value, with a variable helix of (53,27,54). The resulting unstable conditions of stability prediction and eigenvalues outside the unit circle are given in Figure 8.
Meanwhile, for the SQP result, the maximum CM value higher (1.071) and converges at 5 iterations as shown in Figure 5. The corresponding stability prediction is shown in Figure 9, indicating a very large unstable island (eigenvalues plotted outside the unit circle) in contrast to the DE (Figure 8) and the original (Figure 4) results.

6. Discussions
To recap, the DE algorithm was able to design a three flute variable helix tool (with variable pitch at its free end) that showed a great improvement in chatter stability. However, the importance of also allowing a variable pitch at the free end of the tool has been highlighted. Without this variable pitch at the free end, the variable helix geometry is much less effective because there is less potential to ‘disrupt’ the time-delay parameters in the governing stability equations.

Both the DE and SQP algorithms were able to improve the chatter stability. However, by referring to the maximum CM value, the optimisation results for the DE algorithm are consistently better than for the SQP algorithm. For the variable helix tool with a uniform helix at the free end, the SQP algorithm is clearly trapped in a local optimum. Furthermore, the performance strongly depends on the chosen initial value because it uses gradient based optimisation [15].

7. Conclusion
This article has illustrated how variable helix milling tools can effectively suppress the chatter vibration of a workpiece during milling processes, using an analytical/numerical study. A differential evolution algorithm was shown to perform much better than a traditional optimisation routine (SQP). The poorer performance of SQP can be attributed to its local nature, in contrast the global optimisation achieved with the DE method.

Although the validation with experimental results has not been presented, the present study suggests that the proposed algorithm can have good potential to optimise variable helix geometries. Further work is needed to validate in the underlying stability algorithms, in order to ensure the capability to increase productivity in practical machining problems.

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**Figure 1.** Typical cutting helical end milling tools a) 4 flute tool, b)Uniform helix, c) Variable helix, d) Uniform pitch and e) Variable pitch

| Parameter                      | Value |
|--------------------------------|-------|
| Number of Generation           | 70    |
| Population, NP                 | 10*D  |
| Crossover rare, CR             | 0.7   |
| Scaling factor, F              | 0.8   |

**Table 1.** Typical DE parameters settings

| Tool and cutting parameters    |
|--------------------------------|
| Tool diameter, mm              | 19.05 |
| Radial immersion, mm           | 1.00  |
| Kt, MPa                        | 550   |
| Kn, MPa                        | 200   |

| Modal property in x-direction mode |
|------------------------------------|
| Natural frequency, f (Hz)          | 169.3 |
| Modal effective mass, m (kg)       | 6.5363 |
| Damping Ratio, ξ                    | 0.0056 |

**Table 2.** Cutting, modal and tool parameters for optimization

**Figure 2.** Schematic block diagram in state space form for regenerative chatter
Figure 3. The sequence operation of between Semi Discretisation Method and optimization of Differential Evolution

Figure 4. Original stability prediction for variable helix (25,30,35) with variable pitch (120,100,140) [10] (——) stability region, (■) secondary hopf bifurcation, (ο) cyclic fold bifurcation and (∇) period doubling bifurcation

Figure 5. Performance of DE and SQP. (——) DE, variable helix, variable pitch at end (⋯⋯) SQP, variable helix, variable pitch at end (-----) DE, variable helix, uniform pitch at end and (---) SQP, variable helix, uniform pitch at end.
Figure 6. DE Optimised stability prediction for variable helix (52,52,41) with variable pitch (107,163,90)

Figure 8. DE optimised stability prediction for variable helix (53,27,54) (—) stability region, (—) stability region, (■) secondary hopf bifurcation and (V) period doubling bifurcation

Figure 7. SQP optimised stability prediction for variable helix (25,38,55) with variable pitch (120,105,135)

Figure 9. SQP optimised stability prediction for variable helix (28,32,37) (—) stability region, (■) secondary hopf bifurcation and (o) cyclic fold bifurcation