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The challenge of measuring poverty and inequality:  
a comparative analysis of the main indicators

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Abstract. This paper presents a review of the main available indicators to measure poverty and income inequality, examining their properties and suitability for different types of economic analyses, and providing real-world data to illustrate how they work. Although some of these metrics, such as the Gini coefficient, are most frequently used for this purpose, it is crucially important for researchers and policy-makers to take into account alternative methods that can offer complementary information in order to better understand these issues at all levels.

Keywords. Income inequality, indicators, inequality metrics, poverty metrics

JEL classification. D31, D63.

Introduction

Poverty and inequality have long been topics of interest in the economic literature, because of the concerns about an equitable distribution of the fruits of economic growth. However, before tackling the analysis of the causes and potential consequences of such phenomena, we have to face the issue of which is the best way to measure them.

Although they are inseparably connected, we should first distinguish between poverty and income inequality. While inequality is a much broader concept, since it focuses on the way income, or wealth or consumption, is distributed in an entire population, poverty focuses on the living conditions of the individuals placed in the lowest end of income distribution, below a threshold called ‘poverty line’.

But for the measure of these two variables to be useful, it is desirable that they fulfil certain conditions. In this respect, on the one hand, regarding poverty measures, Morduch (2006) mentions the following properties: scale invariance (also known as population size independence), which means that, if the number of individuals in the population is multiplied by a constant for all income levels, the results of the measurement should not change; focus, which implies that the indicator should only be focused on individuals living below a certain level of
income, called the ‘poverty line’, so that an improvement or deterioration in the living conditions of those above this level of income should not change the results of the measurement; monotonicity, which means that if an individual living below the poverty line loses income, the results of the measurement should worsen, or at least not improve; transfer sensitivity, also known as Pigou-Dalton condition, proposed by Pigou (1912) and Dalton (1920), which implies that if there is a transfer of income from a richer household to a poorer one without changing their relative positions within the income distribution or their average income, the poverty measure must fall, and vice versa; and finally, an additional desirable property is that poverty measures can be decomposed according to different criteria, so that we can analyse the poverty level of different subgroups, being the sum of the subgroup indicators equal to the poverty level of the entire population. A key feature for a measure to be decomposable is that the sub-groups should not overlap and that together they should encompass the entire population.

On the other hand, as for inequality measures, Haughton and Khandker (2009) point out that it is desirable they have as many of the following properties as possible: scale invariance and transfer sensitivity (both explained above); mean independence, which means that multiplying the income of all individuals in the population by a constant should not change the results of the measurement; symmetry or anonymity, which implies that if two individuals of the population exchange places within the distribution, the results of the measurement should not be altered; and, as additional properties, we could also mention decomposability, equivalent to the aforementioned property; fixed range, so that the measurement of inequality is performed on a scale varying between two fixed values, ideally 0 and 1; and statistical testability, which means that the researcher should be able to test for the significance of changes in the indicator over time.

Thus, splitting the set of indicators in poverty and inequality measures for reasons explained above, this paper is organized as follows: in Section 1, we review the main poverty measures; in Section 2, we study the most important inequality measures, and finally, in Section 3, we present our concluding remarks.

1. Poverty indicators

1.1 Foster-Greer-Thorbecke class of measures

Foster, Greer, and Thorbecke (1984) developed a group of indicators in order to assess the living standards of individuals that are below the so-called “poverty line”. This threshold can be set by the researchers as a share of the mean or median income of a population, which would be a measure of “relative poverty”, e.g. Eurostat sets it at 60% of the median equivalized disposable income after social transfers, or as an arbitrarily selected value, that is, a measure of “absolute poverty”, e.g. the World Bank currently sets the “international poverty line” in 1.90 American dollars a day valued at 2011 purchasing power parity.

The general expression of the FGT measures can be written as:
\[ P_a = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{G_i}{z} \right)^a, \quad (a \geq 0), \]

where \( N \) is the number of individuals in the population, \( G_i \) is the difference between the poverty line and the actual income of an individual (being \( G_i = 0 \) for those above the poverty line), \( z \) is the poverty line, and \( a \) is a constant that represents the indicator sensitivity to poverty, i.e., it can take values from 0 to infinity, and by giving it higher and higher values to \( a \), we can gradually increase the sensitivity of the indicator to poverty.

There are three cases of the FGT measures that are so widely used that researchers designate them with specific names: \( P_0 \) is called ‘headcount ratio’ or ‘at-risk-of-poverty rate’; \( P_1 \) is known as ‘poverty gap index’; and \( P_2 \) is referred to as ‘squared poverty gap index’ or ‘severity index’.

The simplest and most popular way to assess the poverty level of a population is the headcount ratio, also known as at-risk-of-poverty rate, which measures the share of people with an equivalized disposable income below the poverty line. It can be calculated using the following formula:

\[ P_0 = \frac{N_p}{N}, \]

where \( N \) is the number of individuals in the population, and \( N_p \) is the number of them below the poverty line.

Using the headcount ratio, the highest levels of poverty in the EU-28 can be seen in Romania, Latvia, Lithuania, Spain and Bulgaria, where poor households represent more than 20% of the population. At the other end of the scale, we have the Czech Republic, the Netherlands, Denmark, Slovakia and Finland, where this indicator lies somewhere around 12% (the ISO codes corresponding to each EU country are in Table 1 of the Annex). It is also worth mentioning that only nine member states managed to reduce their poverty levels in the last decade (Figure 1).

Although the headcount ratio offers an easy-to-interpret first glance to poverty measurement, it is a simple ratio that does not allow us to quantify the extent to which individuals fall below the poverty line. Thus, it does not change if the living conditions of the poor improve or deteriorate as long as they remain below the poverty line.

In order to address these flaws, the poverty gap index allows us to measure how far poor individuals fall below the poverty line, and it can be calculated as:

\[ P_1 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{G_i}{z} \right), \]

where \( N \) is the number of individuals in the population, \( G_i \) is the difference between the poverty line and the actual income of an individual (being \( G_i = 0 \) for those above the poverty line), and \( z \)
is the poverty line.

![Figure 1. Headcount ratio in the EU-28 countries (2006, 2015 and period peak). Source: Own elaboration based on statistics from Eurostat. Notes: (i) Countries appear ranked according to the highest at-risk-poverty rate in 2015. (ii) Croatia's data correspond to 2010 instead of 2006; Romania's data correspond to 2007 instead of 2006.](image)

Also, if we want to increase the “sensitivity to poverty” of our indicator, we can use the squared PGI, also known as severity index, which is calculated by averaging the square of the distance to the poverty line of every individual below this line:

\[ P_2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{G_i}{z} \right)^2, \]

where all the elements are just the same as in the case of \( P_1 \).

By squaring the \( G_i/z \) component of the poverty gap formula, we make the indicator more sensitive to changes in income of individuals far below the poverty line, i.e. distributionally-sensitive.

**1.2 Sen and Sen-Shorrocks-Thon indices**

In order to assess more dimensions of poverty with the same indicator, Sen (1976) proposed a new metric that combines the relative number of poor people, their income level, and the income distribution within the group, which may arguably be considered his main contribution in the measurement of poverty.

It can be calculated as:
\[
P_{SEN} = P_0 \left[ 1 - (1 - GINI_P) \frac{x_p}{z} \right],
\]

where \(P_0\) is the headcount ratio of the population, \(GINI_P\) is the Gini coefficient (see Section 2.4) among the poor, \(\bar{x}_p\) is the average income of the poor, and \(z\) is the poverty line. Shorrock's (1995) presented a modified version of the Sen index, currently known as Sen-Shorrocks-Thon index, which introduces in the calculation the poverty gap index and the Gini coefficient of the poverty gap ratios for the entire population to better gauge poverty intensity.

It can be expressed as:

\[
P_{SST} = P_0 P^P (1 - \tilde{G}_P)
\]

where \(P_0\) is the headcount ratio of the population, \(P^P\) is the poverty gap applied only to those below the poverty line, and \(\tilde{G}_P\) is the Gini coefficient of the poverty gaps of the poor.

These variables allow researchers to track the source of the changes in poverty levels measured by the SST index in three basic dimensions: number of poor, the depth of their poverty, and income distribution among the poor.

**1.3 Watts index**

This last indicator, proposed by Watts (1968), was the first distribution-sensitive poverty indicator. It can be calculated using the following formula:

\[
W = \frac{1}{N} \sum_{i=1}^{q} [\ln(z) - \ln(x_i)],
\]

where \(N\) is the number of individuals in the population, \(q\) is the number of individuals below the poverty line, \(z\) is the poverty line, and \(x_i\) is the income level of a certain individual below the poverty line.

By introducing logarithms, Watts makes the indicator more sensitive to changes in the lowest end of the income distribution. This way, the indicator will improve the most when poorer individuals improve their living conditions.

**2. Inequality indicators**

The measurement of inequality can be made using the whole income distribution, which includes the income of every individual in the population or sample under analysis, or using quantiles (deciles, percentiles, etc.). The latter is not as informative as the former since they only consider specific segments of the distribution, while ignoring the rest. For this reason, it is important to clearly identify which type of measure is best suited to the needs of the researcher and to maintain consistency throughout the study.
2.1 Income shares

The simplest way to assess how income is distributed in a given population is dividing the observations of our sample in quantiles, e.g., quartiles, quintiles, deciles, percentiles, etc., and analyzing the evolution of the income share corresponding to each quantile over time.

If the main focus of our analysis is the lowest end of the income distribution, we may choose to evaluate how the income share of the first decile or quintile has evolved over a certain period of time.

For the EU-28 in 2015, the countries where households in the lowest end of the distribution had a greater share of equivalized disposable income were the Czech Republic, Finland, Slovenia, the Netherlands and Slovakia, where the income of the poorest 20% almost reached the 10% of the total income. While in the opposite situation, we can find Romania, Spain, Latvia, Greece and Belgium, where the bottom 20% only amounted to about 6% of the total income. It also should be noted that only in nine out of twenty-eight countries, the income share of the poorest households has increased over the last decade.

Figure 2. Income share by the first two deciles in the EU-28 countries (2006, 2015, and period peak).
Source: Own elaboration based on statistics from Eurostat. Notes: (i) Countries appear ranked according to the highest income share of the poorest 20% in 2015. (ii) Croatia's data correspond to 2010 instead of 2006; Romania's data correspond to 2007 instead of 2006.

Furthermore, in order to analyse the accumulation of income by the households at the highest end of the distribution, we may also use the income shares to quantify the share of total income in the hands of these households.

The most widely used segments are the tenth decile and the hundredth decile, but to assess
the increasing importance of the “super rich”, we may want to focus on increasingly small segments of the top of the distribution, e.g., top 0.5%, top 0.1%, top 0.01%.

As we can see in Figure 3, the EU-28 countries where the top 1% has the larger share of the total income are Romania, Cyprus, Lithuania, Bulgaria and the United Kingdom, while Slovakia, Slovenia, Sweden, Malta and Finland are in the other end of the scale. There is no general trend in this regard among the member states, since sharp falls of the income share suffered by the top 1% in Slovakia, Ireland and Hungary coincided in time with strong increases in countries such as Romania, Cyprus and Latvia.

**Figure 3.** Income share by the hundredth percentile in the EU-28 countries (2006, 2015, and period peak).

*Source:* Own elaboration based on statistics from Eurostat. *Notes:* (i) Countries appear ranked according to the highest income share by the richest 1% in 2015. (ii) Croatia's data correspond to 2010 instead of 2006; Romania's data correspond to 2007 instead of 2006.

### 2.2 Quantile ratios

With the purpose of assessing together the aforementioned segments of the income distribution, it may be useful quantifying the gap between the poorest and the richest households. For this purpose, we have at our disposal several ratios that are easy to construct and interpret. Nevertheless, we should note that, even though these ratios are widely used, they do not measure inequality properly since they are calculated without taking into account the central segment of the distribution, so they can be considered income polarization indicators.

The two most well-known ratios are: (i) the S80/S20 ratio, defined as the ratio of the richest 20% of the population's share in gross total income, divided by the poorest 20% of the
population’s share, and used by the United Nations Development Programme Human Development Indicators; and (ii) the Palma ratio, developed by the Chilean economist Gabriel Palma, and defined as the ratio of the richest 10% of the population’s share in gross total income, divided by the poorest 40% of the population’s share. Palma (2011) proposed using these two particular segments since there is evidence that in most countries the central segment of the income distribution amounts to about 50% of total income while the other 50% is distributed between the top 10% and the bottom 40%. Considering that the way this half of the total income is distributed between these two segments varies greatly among countries and over time, this ratio can be extremely useful to track changes in income polarization over time, or to compare income distribution among countries or regions.

As we can see in Figures 4 and 5, both ratios show similar outcomes since their correlation coefficient for this period is 96.40%. The member states with the highest income polarization are Lithuania, Romania and Bulgaria for both indicators, while Slovakia, Slovenia, Czech Republic, Finland and Sweden are the least polarized countries in terms of income.

These results could be complemented with the data presented in Figures 2 and 3 to try to establish whether the source of the changes experienced by these ratios is in the highest or lowest end of the income distribution, or both.

![Figure 4. Palma ratio in the EU-28 countries (2006, 2015, and period peak). Source: Own elaboration based on statistics from Eurostat. Notes: (i) Countries appear ranked according to the highest Palma ratio in 2015. (ii) Croatia’s data correspond to 2010 instead of 2006; Romania’s data correspond to 2007 instead of 2006.](image-url)
Figure 5. S80/S20 ratio in the EU-28 countries (2006, 2015, and period peak). Source: Own elaboration based on statistics from Eurostat. Notes: (i) Countries appear ranked according to the highest S80/S20 ratio in 2015. (ii) Croatia’s data correspond to 2010 instead of 2006; Romania’s data correspond to 2007 instead of 2006.

However, there are many other ratios, such as P90/P10, P90/P50 and P50/P10, than can be used to assess the gap between certain segments of a given population. The P ratios are calculated by dividing the incomes at the respective percentiles, rather than the share of income of all those higher or lower than that percentile as in the S ratios. The P ratios therefore have the advantage of being easier to calculate and are quite insensitive to the data-missingness that is more common in the tails of the income distribution.

For instance, the P90/P10 that, similarly to the ratios commented before, measures the gap between the highest and lowest ends of the distribution, and, needless to say, it will give results highly correlated with those ratios. Moreover, the P90/P50 ratio is used to appraise the gap between the highest income individuals and the median income of the population, whereas the P50/P10 ratio is employed to gauge the divergence of the poorest households from the median income of their population.

2.3 Measures of statistical dispersion: squared coefficient of variation (SCV) and relative mean deviation (RMD)

The following measures are not designed to analyse the level of inequality in a distribution of income; they are indicators used to assess the variability of any set of observations with regard to their average.

That is the reason why these general statistics, despite not having been designed
specifically to analyse the degree of inequality in income distribution, can be used to quantify the dispersion of an income distribution, so that a higher level of dispersion would also mean higher inequality.

First, we have the squared coefficient of variation (SCV), which is a variant of the coefficient of variation, a measure of dispersion that can be used for any data set. It fulfills all the requirements explained in the introduction, except the additive decomposability and the fixed range, it can take values from 0 to infinity. As will be explained in Section 2.6, this indicator is more sensitive to changes at the highest end of the income distribution. For its calculation, the formula is the following:

\[ SCV = \left( \frac{\sigma}{\mu} \right)^2, \]

where \( \sigma \) is the standard deviation and \( \mu \) is the arithmetic mean.

As a second measure of dispersion, we can use the relative mean deviation (RMD), which was developed by Schutz (1951), and represents the percentage of income that should be transferred from those with higher-than-average income to those with lower-than-average income, so that both groups have exactly the same average income (Kakwani, 1980). It can be calculated according to the following formula:

\[ RMD = \frac{1}{N} \sum_{i=1}^{N} \left| x_i - \bar{x} \right| / \left| \bar{x} \right|, \]

where \( N \) is the number of individuals in the population, \( x_i \) is the income level of a given individual, and \( \bar{x} \) is the average income of the population.

The main problem with this measure is its insensitivity to transfers between individuals in the same side of the average income.

### 2.4 The Lorenz curve and the Gini coefficient

The Gini coefficient is based on the Lorenz curve, which is a graphical representation of a cumulative distribution function, and is mathematically defined as the cumulative share of total income assumed by cumulative shares of the population.

The Lorenz curve is always represented paired with the line of egalitarian income distribution, that is the 45-degree line, and represents an ideal situation where every individual in the population has the same income level. This way, we can easily compare how far the Lorenz curve is from this line of absolute equality.

So, graphically, the Gini coefficient is defined as the ratio of the area between the line of complete equality and a given Lorenz curve, and the total area under the line of perfect equality. In Figure 6, it can be calculated as \( A/(A+B) \).

Due to its comparability between regions and through time, regardless of population sizes, exchange rates, price levels, etc., and its easy-to-interpret results, which always range between 0 ("perfect equality") and 1 ("perfect inequality"), the Gini coefficient is the most widely used indicator to measure inequality.
Assuming that the Lorenz curve is a finite discrete function, the area between the perfect equality line and the Lorenz curve can be approximated as a frequency polygon using the following formula (Abounoori and McCloughan, 2003):

\[ G = 1 - \sum_{i=1}^{N}(x_i - x_{i-1})(y_i - y_{i-1}), \]

where \( N \) is the number of intervals into which the population is divided, \( x_i \) is the cumulative share of income, and \( y_i \) is the cumulative share of population.

Besides, given that the Lorenz curve is a twice-differentiable, monotonic increasing and convex function \( L(x) \) where \( x \) is the cumulative share of income, we can calculate the Gini coefficient this way:

\[ G = 1 - 2 \int_{0}^{1} L(x)dx \]

Figure 6. Lorenz curve for the EU-28 (2015). Source: Own elaboration based on statistics from Eurostat.

According to the data presented in Figure 7, the most unequal EU-28 countries are, once again, Lithuania, Romania, Bulgaria and Latvia, whereas the most egalitarian are Slovakia, Slovenia, the Czech Republic and Sweden. These results are extremely correlated to the Palma and S80/S20 ratios, with coefficients of correlation of 99.19% and 96.78%, respectively. Nonetheless, the relative increases and decreases (i.e., in terms of percentages) are far less pronounced in the Gini coefficient than in the ratios.

This behaviour may be related to the fact that the Gini coefficient is more sensitive to changes at the center of the distribution, while the ratios focus exclusively on what happens at its ends. This situation may lead to the researchers more interested in income polarization to use the aforementioned ratios rather than the Gini coefficient as inequality indicators,
considering their almost perfect correlation.

Additionally, the Gini coefficient is unable to differentiate between two populations where the area under the Lorenz curve is the same, but the shape of the curve is different, i.e. they have different inequality patterns, and it is completely unresponsive to structural demographic changes.

Finally, the Gini coefficient is not easily decomposable as the sum of the Gini indices of different subgroups. Nonetheless, many techniques for its decomposition have been proposed over the years (Pyatt, 1976; Lerman and Yitzaki, 1985; Silber, 1989).

Figure 7. Gini coefficient of equivalized disposable income in the EU-28 countries (2006, 2015, and period peak). Source: Own elaboration based on statistics from Eurostat. Notes: (i) Countries appear ranked according to the highest Gini coefficient in 2015. (ii) Croatia’s data correspond to 2010 instead of 2006; Romania’s data correspond to 2007 instead of 2006.

2.5 The Hoover Index

This indicator is closely associated to the Gini coefficient because the Lorenz curve is also used for its calculation. Originally created as a measure of industrial localization (Hoover, 1936), it represents the share of income that should be redistributed to attain a hypothetical situation of complete equality, that is why it is also commonly referred as the “Robin Hood index”.

It can be graphically represented as the maximum vertical distance between a given Lorenz curve and the 45-degree line of perfect equality (Figure 8), and for its calculation we have to use the following formula after dividing the income distribution into quantiles:

\[
H = \frac{1}{2} \sum_{i=1}^{N} \left[ \frac{F_i}{F_{total}} - \frac{A_i}{A_{total}} \right],
\]

...
where \( N \) is the number of quantiles, \( A \) is the width of said quantiles, \( E_i \) is the income level of a given quantile, \( A_i \) is the number of individuals in the quantiles.

Although it provides little information about how income is distributed in a population, it can be used to illustrate how far a population is from the egalitarian distribution.

Figure 8. Lorenz curve and Hoover index for the EU-28 (2015). Source: Own elaboration based on statistics from Eurostat.

### 2.6 Generalized Entropy measures: Theil index and Mean Log Deviation (MLD)

The Theil index and the Mean Log Deviation (MLD) are special cases for the Generalized Entropy index (GE), an indicator originated in information theory, and developed by Henry Theil (1967).

The GE can be calculated using the following formula (Haughton and Khandker, 2009):

\[
GE(\alpha) = \frac{1}{N\alpha(\alpha-1)} \sum_{i=1}^{N} \left[ \left( \frac{x_i}{\bar{x}} \right)^{\alpha} - 1 \right], (\alpha \geq 0),
\]

where \( N \) is the number of individuals in the population, \( x_i \) is the income level of a given individual, \( \bar{x} \) is the average income of the population, and \( \alpha \) is the weight for distances between incomes in different parts of the income distribution. This last parameter allows the researcher to adjust the index sensitivity to their preferences, since for values below (above) 1 GE is more sensitive to changes in the lowest (highest) end of the distribution. For \( \alpha = 1 \), it applies equal weights across the income distribution.

As the poverty indices presented in Section 1, there are three cases of the GE that are so
widely used that researchers designate them with specific names: when $\alpha = 0$, the generalized entropy index is the mean log deviation; when $\alpha = 1$, it is the Theil index; and when $\alpha = 2$, it is half the squared coefficient of variation (see Section 2.3).

Although these indicators can take values from zero to infinity, and therefore, they do not fulfill the fixed range requirement, they are easily decomposable, allowing both the segmentation of the income distribution according to different criteria, and the disaggregation of total inequality in between and within group components.

The decomposability of these indicators allows us to analyse the evolution of inequality patterns over the reference period using many segmentation criteria provided by the household finances surveys.

The Theil index, also known as Theil’s $T$, can be calculated using the following formula:

$$GE(1) = T = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x_i}{\bar{x}} \ln \frac{x_i}{\bar{x}} \right),$$

where $N$ is the number of individuals in the population, $x_i$ is the income level of a given individual, and $\bar{x}$ is the average income of the population.

It can be decomposed as the following sum (Haughton and Khandker, 2009):

$$T = \sum_{i=1}^{m} s_i T_i + \sum_{i=1}^{m} s_i \ln \frac{\bar{x}_i}{\bar{x}},$$

where $m$ is the number of subgroups, $s_i$ is the share of total income of each subgroup, $T_i$ is the Theil index of each subgroup, $\bar{x}_i$ is the average income of each subgroup, and $\bar{x}$ is the average income of the population.

The first term of the expression above is the weighted sum of the Theil indices calculated for the different subgroups, where the weights are given by each subgroup’s share on total income. This term represents the component of inequality attributed to income differences within the same group.

The second term is the Theil index corresponding to a distribution in which each individual receives the average income of their subgroup. This component then represents the income inequality between subgroups of the population.

On the other hand, the mean logarithmic deviation (MLD), also known as Theil’s $L$, is the percentage of income difference between a randomly selected individual or household of a certain population and the average income of the population.

We can calculate it in this way:

$$GE(0) = MLD = \frac{1}{N} \sum_{i=1}^{N} \ln \frac{\bar{x}}{x_i},$$

where $N$ is the number of individuals in the population, $x_i$ is the income level of a given individual, and $\bar{x}$ is the average income of the population.
It can be decomposed as the following sum:

\[
L = \sum_{i=1}^{m} \frac{n_i}{N} L_i + \frac{n_i}{N} \sum_{i=1}^{m} l_i \ln \left( \frac{\bar{x}_i}{\bar{x}} \right),
\]

where \( N \) is the number of individuals in the population, \( m \) is the number of subgroups, \( n_i \) is the number of individuals in each subgroup, \( L_i \) is the MLD of each subgroup, \( \bar{x}_i \) is the average income of each subgroup, and \( \bar{x} \) is the average income of the population.

Similarly to the Theil index, the first term represents the inequality within subgroups, while the second represents the inequality between subgroups.

Using the Theil index, i.e. GE(1), the most unequal EU-28 countries were Belgium, Latvia, Greece, Lithuania and Portugal. The results are almost identical if we focus on the changes in the lowest end of the income distribution, using the MLD, i.e. GE(0). Nonetheless, using the SCV, i.e. twice the GE(2), that focus on the highest end, we can perceive bigger changes: countries like France or Cyprus appear for the first time among the most unequal member states.

![Figure 9](image)

**Figure 9.** Generalized entropy measures for the EU-28 countries, 2012. *Source:* Own elaboration based on statistics from the European Commission. *Note:* Countries appear ranked according to the highest Theil index.

All three measures in Figure 9 show high levels of correlation. But, as one might expect, MLD and SCV are the ones less correlated (76.27%), whilst the correlation coefficients of the other two pairs are above 90%, Theil and MLD, 94.92%; Theil and SCV, 91.99% (see Table 2 in Annex).
2.7 Atkinson class of measures

Similar to the Generalized Entropy measures, this group of statistics developed by Anthony Atkinson (1970) allows the researcher to calibrate the indicator’s sensitivity to inequality by giving values to a theoretical constant $\varepsilon$ that measures the "inequality aversion level".

It can be calculated using the following formula (Litchfield, 1999):

$$A(\varepsilon) = 1 - \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x_i}{\bar{x}} \right)^{\frac{1}{1-\varepsilon}} \right]^{\frac{1}{1-\varepsilon}} (\varepsilon \geq 0),$$

where $N$ is the number of individuals in the population, $x_i$ is the income level of a given individual, $\bar{x}$ is the average income of the population, and $\varepsilon$ is the inequality aversion level, also known as Atkinson constant.

For increasingly higher values of the constant, the Atkinson index becomes more and more sensitive to changes in the lowest end of the distribution. By doing so, if we calculate the Atkinson index for different levels of inequality aversion, we can determine if the changes in income inequality in a certain population are being driven more by changes at the top or at the bottom of the distribution.

To understand how it works, we can compare the different results of the index for some values of the constant. As we can see in Figure 10, for a low level inequality aversion, $\varepsilon = 0.5$, the most unequal EU countries are Bulgaria, Latvia, Greece, Portugal and Lithuania, while the most egalitarian are Slovenia, Slovakia, the Czech Republic, the Netherlands and Sweden. If we move towards a neutral level of inequality aversion, $\varepsilon = 1$, Lithuania is replaced by Romania as one of the countries with higher inequality, while Finland does the same with Sweden in the other group. Finally, for a high level of inequality aversion, $\varepsilon = 2$, we can see major changes in the first group: Austria, Spain and Italy enter the "most unequal group", which is also made up of Greece and Romania. At the other end, nevertheless, the Czech Republic, Slovenia, the Netherlands and Finland remain as the most egalitarian member states, the entry of Denmark is the only significant change in this group.

These divergences become even more evident if we calculate the correlation coefficient of the Atkinson index for the three $\varepsilon$ values chosen. $A(0.5)$ and $A(1)$ have a 98.64% correlation, whilst $A(1)$ and $A(2)$ show a correlation of 73.32%, and $A(0.5)$ is only 64.94% correlated with $A(2)$ (see Table 2 in Annex).
Finally, it should be noted that this indicator is the only measure analysed in this study that fulfills all the conditions presented in the introduction.

3. Concluding remarks

The purpose of this study is summarizing and reviewing the most widely used poverty and inequality indicators, weighing up their advantages and disadvantages. As we explain in the introduction, it is a critical issue for researchers and policy-makers to know and use these indicators in order to target, analyse and correct both poverty and inequality.

Since every indicator or group of indicators, presented in this paper provides complementary information, they should be used in conjunction with others for the purpose of having the best possible overall picture of the circumstances in a certain population.

Researchers should choose the indicators they will use considering their needs and the information that each one can provide. For instance, if we want to focus on the living conditions of the poor, we should choose the FGT class of measures to quantify the number of households below the poverty line and their distance to such threshold. But if we want to have a better understanding of the income distribution among the poor, only one FGT measure would be useful: the severity index, which measures the distribution of income among individuals below the poverty line using the square of the coefficient of variation. A second measure that could be useful for this purpose is the SST index, which takes into account this dimension of the problem by including the Gini index as one of its components.

Moreover, if we want to measure the gap between the rich and the poor, or the distance between these groups and the median household, we should use the quantile ratios since they...
are the most suitable indicators for measuring income polarization. Additionally, considering their greater variability and their high correlation with the Gini coefficient, we could choose them as proxies of income inequality.

Conversely, if our focus is on income inequality for an entire population, we should use many of the aforementioned measures, but always taking into account the problems they have: although the Gini coefficient is the most widely used indicator to this end due to its simple interpretation and its comparability over time and across countries, we must also bear in mind that it is relatively insensitive to changes in the ends of the distribution, it cannot distinguish inequality patterns and it cannot be decomposed; the Hoover index provides little information about the way income distributes in a population and should be only used as a first glance for this issue; the Generalized Entropy measures allow to adjust its sensibility to poverty and are decomposable, which makes them the ideal choice for unraveling the patterns of inequality according to several criteria, but they are not easily comparable since they can theoretically take values from zero to infinity; finally, the Atkinson measures suffer none of the drawback listed above, but nevertheless they are relatively little used, so there are scarce data available of them.

Regarding data availability on these issues, it should be noted that there are several databases where we can easily download normalized macro data for research purposes, Eurostat, the World Bank, the United Nations University World Institute for Development Economics Research, the OECD database, the Luxembourg Income Survey, the World Wealth and Income Database, or the Standardized World Income Inequality Database (Solt, 2009). However, we should also note that these sources only offer time series for selected variables: headcount index, income shares and quantile ratios, and Gini coefficient, leaving aside the rest of indicators commented in this paper.

Lastly, we must bear in mind that the data available on these subjects have many limitations owing to their sources. Almost every data source on income distribution comes from household surveys that involve issues such as an ever-growing unit and item non-response rate, and an increasingly large measurement error due to less accurate responses provided by the respondents (Meyer et al., 2015).

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Appendix 1

Table A.1. ISO codes for each EU country.

| Code | Country          |
|------|-----------------|
| AT   | Austria         |
| BE   | Belgium         |
| BG   | Bulgaria        |
| CY   | Cyprus          |
| CZ   | Czech Republic  |
| DE   | Germany         |
| DK   | Denmark         |
| EE   | Estonia         |
| EL   | Greece          |
| ES   | Spain           |
| FI   | Finland         |
| FR   | France          |
| HR   | Croatia         |
| HU   | Hungary         |
| IE   | Ireland         |
| IT   | Italy           |
| LT   | Lithuania       |
| LU   | Luxembourg      |
| LV   | Latvia          |
| MT   | Malta           |
| NL   | Netherlands     |
| PL   | Poland          |
| PT   | Portugal        |
| RO   | Romania         |
| SE   | Sweden          |
| SI   | Slovenia        |
| SK   | Slovakia        |
| UK   | United Kingdom  |

Table 2. Correlation coefficients of several inequality measures for the EU-28 countries in 2012. Source: Own elaboration based on statistics from the European Commission.

|        | GINI | MLD   | SCV   | THEIL | AT0.5 | AT1   | AT2   |
|--------|------|-------|-------|-------|-------|-------|-------|
| GINI   | 10.00| 0.9761| 0.8364| 0.9821| 0.9952| 0.9772| 0.6213|
| MLD    | 0.9761| 10.000| 0.7627| 0.9493| 0.9860| 0.9999| 0.7422|
| SCV    | 0.8364| 0.7627| 10.000| 0.9199| 0.8477| 0.7644| 0.3576|
| THEIL  | 0.9821| 0.9492| 0.9199| 10.000| 0.9870| 0.9501| 0.5755|
| A(0.5) | 0.9952| 0.9860| 0.8477| 0.9870| 10.000| 0.9864| 0.6494|
| A      | 0.9772| 0.9999| 0.7644| 0.9500| 0.9864| 10.000| 0.7432|
| AT2    | 0.6213| 0.7422| 0.3576| 0.5755| 0.6494| 0.7431| 10.000|