Gamma-ray Bursts as Cosmological Tools

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\textbf{ABSTRACT}

In recent years there has been considerable activity in using gamma-ray bursts as cosmological probes for determining global cosmological parameters complementing results from type Ia supernovae and other methods. This requires a characteristics of the source to be a standard candle. We show that contrary to earlier indications the accumulated data speak against this possibility. Another method would be to use correlation between a distance dependent and a distance independent variable to measure distance and determine cosmological parameters as is done using Cepheid variables and to some extent Type Ia supernovae. Many papers have dealt with the use of so called Amati relation, first predicted by Lloyd, Petrosian and Mallozzi, or the Ghirlanda relation for this purpose. We have argued that these procedure involve many unjustified assumptions which if not true could invalidate the results. In particular, we point out that many evolutionary effects can affect the final outcome. In particular, we demonstrate that the existing data from \textit{Swift} and other earlier satellites show that the gamma-ray burst may have undergone luminosity evolution. Similar evolution may be present for other variables such as the peak photon energy of the total radiated energy. Another out come of our analysis is determination of the luminosity function and the comoving rate evolution of gamma-ray bursts which does not seem to agree with the cosmic star formation rate. We caution however, that the above result are preliminary and includes primarily the effect of detection threshold. Other selection effects, perhaps less important than this, are also known to be present and must be accounted for. We intend to address these issues in future publications.

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1. INTRODUCTION

The change in our understanding of gamma-ray bursts (GRBs) in less than a decade has been unprecedented. We have gone from groping for ways to determine their distances (from solar system to cosmological scales) to attempts to use them as cosmological probes. Observations by instruments on board a series of satellites starting with BeppoSAX and continuing with HETE, INTEGRAL and Swift, have been the primary source of this change. The higher spatial resolution of these instruments has allowed the measurement of redshifts of many well-localized GRBs, which has in turn led to several attempts to discover some emission characteristics which appears to be a “standard candle” (SC for short), or shows a well defined correlation (with a small dispersion) with another distance independent measurable characteristic. One can use such relations to determine the distances to GRBs in a manner analogous to the use of the Cepheid variables. Example of this are the lag-luminosity and variability-luminosity relations (Norris et al 2000, Norris 2002, Fenimore & Ramirez-Ruiz 2000, Reichart et al 2001) which were exploited for determining some cosmological aspects of these sources (Lloyd, Fryer & Ramirez-Ruiz 2002, Kocevski & Liang 2006) using the methods developed by Efron & Petrosian (1992, 1994, 1999). More recently there has been a flurry of activity dealing with the observed relation between the peak energy $E_p$ of the $\nu F_\nu$ spectrum and the total (isotropic) gamma-ray energy output $E_{iso}$ ($E_p \propto E_{iso}^\eta$, $\eta \sim 0.6$) predicted by Lloyd et al. (2000, LPM00) and established to be the case by Amati et al. (2002) (see also Lamb et al. 2004, Attiea et al 2004). Similarly Ghirlanda et al. (2004a) has shown a correlation with smaller dispersion between $E_p$ and the beaming corrected energy $E_\gamma$ ($E_p \propto E_{\gamma}^{\eta'}$, $\eta' \sim 0.7$) where:

$$\frac{E_{\gamma}}{E_{iso}} = \frac{(1 - \cos(\theta_{\text{jet}}))}{2}$$

and $\theta_{\text{jet}}$ is the half width of the jet.

These are followed by many attempt to use these relations for determining cosmological parameters (Dai et al. 2004, Ghirlanda et al. 2004b, 2005 [Gea05], Friedman & Bloom 2005, [FB05]). There are, however, many uncertainties associated with the claimed relations and even more with the suggested cosmological tests. The purpose of this paper is to investigate the utility of GRBs as cosmological tools either as SCs or via some correlation. In the next section we review the past and current status of the first possibility and in §3 we discuss the energy-spectrum correlation and whether it can be used for cosmological model parameter determination. Finally in §4, we address the question of cosmological luminosity and rate density evolution of GRB based on the existing sample with known redshifts.

2. Standard Candle?

The simplest method of determining cosmological parameters is through SCs. Type Ia supernovae (SNIa) are a good example of this. But currently their observations are limited to relatively nearby universe (redshift $z < 2$). Galaxies and active galactic nuclei (or AGNs), on the other hand, can be seen to much higher redshifts ($z > 6$) but are not good SCs. GRBs
are observed to similar redshifts and can be detected to even higher redshifts by current instruments. So that if there were SCs they can complement the SNIa results. In general GRBs show considerable dispersion in their intrinsic characteristics. The first indication that GRBs might be SCs came from Frail et al. (2001) observation showing that for a sample of 17 GRBs the dispersion of the distribution of $E_\gamma$ is significantly smaller than that of $E_{iso}$. The determination of $E_\gamma$ requires a well defined light curve with a distinct steepening. The jet opening $\theta_{jet}$ depends primarily on the time of the steepening and the bulk Lorentz factor, but its exact value is model dependent and depends also weakly on $E_{iso}$ and the density of the background medium

$$\theta_{jet} = 0.101 \ \text{radian} \times \left( \frac{t_{\text{break}}}{1 \ \text{day}} \right)^{3/8} \left( \frac{\eta}{0.2} \right)^{1/8} \left( \frac{n}{10 \ \text{cm}^{-3}} \right)^{1/8} \left( \frac{1 + z}{2} \right)^{-3/8} \left( \frac{E_{\text{iso}}}{10^{53} \ \text{ergs}} \right)^{-1/8},$$

In Figure 11 we show the distribution of $E_{iso}$ and $E_\gamma$ for 25 pre-Swift GRBs (mostly compiled in FB05). As evident, there is little difference between the two distributions (except for their mean values) and neither characteristic is anywhere close to being a SC.

We have calculated the jet angle $\theta_{jet}$ (from equation 2) for 25 pre-Swift GRBs with relatively well defined $t_{\text{break}}$. Assuming a gamma-ray efficiency $\eta = 0.2$ and a value of circumburst density estimated from broadband modeling of the lightcurve when available (otherwise we use the default value of $n = 10 \ \text{cm}^{-3}$).

Unfortunately fewer than expected Swift GRBs have optical light curves and their X-ray light curves show considerable structure (several breaks and flaring activity) with several GRBs showing no sign of jet-break or beaming (Nousek et al. 2006). This has brought the whole idea of jet breaks and calculating $E_\gamma$ into question. The upshot of this is that Swift $E_\gamma$, like $E_{iso}$ also has a broad distribution extending over two decades. Thus, any cosmological use of GRBs must include the effects of the breadth of the distributions.

3. Correlations

When addressing the correlation between any two variables, one should distinguish between a one-to-one relation and a statistical correlation. In general the correlation between two variables (say, $E_p$ and $E_{iso}$) can be described by a bi-variate distribution $\psi(E_p, E_{iso})$. If this is a separable function, $\psi(E_p, E_{iso}) = \phi(E_{iso})\zeta(E_p)$, then the two variables are said to be uncorrelated. A correlation is present if some characteristic (say the mean value) of one variable depends on the other: e.g. $\langle E_p \rangle = g(E_{iso})$. Only in the absence of dispersion there

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1It should be also noted that there is an observational bias in favor of detecting smaller jet angles (i.e. earlier jet-breaks), so that the population as a whole (including those with very late jet-breaks) will have even a broader distribution. Note also that a SC $E_\gamma$ means that the Ghirlanda relation would have an index $\eta' \sim 0$
Fig. 1.— Distribution of $E_{\text{iso}}$ and $E_{\gamma}$ for 25 pre-Swift GRBs with evidence for a jet-break and beaming. $E_{\gamma}$ distribution is shifted by about two orders of magnitudes compared to $E_{\text{iso}}$ distribution due to the beaming factor correction. However the dispersion of the two distribution are very similar ($\sigma_{E_{\text{iso}}} = 0.68$, $\sigma_{E_{\gamma}} = 0.52$) and broad indicating that GRBs cannot be assumed to be SCs.
will be a one-to-one relation; \( \zeta(E_p) = \delta(E_p - g(E_{iso})) \).

In general, the determination of the exact nature of the correlation is complicated by the fact that the extant data suffers from many observational selection biases and truncations. An obvious bias is that most sample are limited to GRBs with peak fluxes above some threshold. There are also biases in the determination of \( E_{p,\text{obs}} \) (see e.g. Lloyd & Petrosian 1999: LP99). The methods devised by Efron & Petrosian (1992, 1994) are particularly suitable for determination of correlations in such complexly truncated data.

The first indication of a correlation between the energetics and spectrum of GRBs came from Mallozzi et al (1995), who reported a correlation between observed peak flux \( f_p \) and \( E_{p,\text{iso}} \). A more comprehensive analysis by LPM00, using the above mentioned methods, showed that a similar correlation also exist between the observed total energy fluence \( F_{\text{tot}} \) and \( E_{p,\text{obs}} \). Both these quantities depend on the redshift \( z = Z - 1 \);

\[
F_{\text{tot}} = \frac{E_{\text{iso}}}{4\pi d_m^2 Z}, \quad \text{and} \quad E_{p,\text{obs}} = \frac{E_p}{Z},
\]

Here \( d_m \) is the metric distance, and for a flat universe

\[
d_m(Z) = \frac{c}{H_0} \int_1^Z dZ' (\Omega(Z'))^{-1/2}, \quad \text{with} \quad \Omega(Z) = \frac{\rho(Z)}{\rho_0},
\]

describing the evolution of the total energy density \( \rho(z) \) of all substance (visible and dark matter, radiation, dark energy or the cosmological constant). LPM00 also showed that the correlation expected from these interrelationships is not sufficient to account for the observed correlation, and that there must be an intrinsic correlation between \( E_p \) and \( E_{\text{iso}} \). Without knowledge of redshifts LPM00 predicted the relation \( E_{\text{iso}} \propto E_p^{0.5} \) which is very close 0.5, which is very similar to the so-called Amati relation obtained for GRBs with known redshifts. However, it should be emphasized that the LPM00 result implies a statistical correlation and not a one-to-one relation needed for using GRBs as a reliable distance indicator. Nakar & Piran (2004, 2005) and Band & Preece (2005) have shown convincingly that the claimed tight one-to-one relations cannot be valid for all GRBs. We believe that the small dispersion seen in GRBs with known redshifts is due to selection effects arising in the localization and redshift determination processes: e.g., these GRBs may represent the upper envelope of the distribution. A recent analysis by Ghirlanda et al. (2005) using pseudo-redshift shows a much broader dispersion (as in LPM00). The claimed tighter Ghirlanda relation, could be due to additional correlation between the jet opening angle \( \theta_{\text{jet}} \) and \( E_{\text{iso}} \), \( E_p \), or both. However, as mentioned above the picture of jet break and measurements of \( \theta_{\text{jet}} \) and \( E_{\gamma} \) is a confusing state in view of Swift observation.

We have reanalyzed the existing data and determined the parameters of the Amati and Ghirlanda relations. In Figure 2 we show the the \( E_{\text{iso}} \) (and \( E_{\gamma} \) excluding some outliers) vs \( E_p \) for all GRBs with known redshifts (and \( \theta_{\text{jet}} \)). We compute best power law fit for both these correlations and we describe the dispersion around it by the standard deviation. For the \( E_p - E_{\text{iso}} \) we find: \( E_p \propto E_{\text{iso}}^{\eta} \), \( \eta \sim 0.328 \pm 0.036 \) and \( \sigma_{\text{iso}} = 0.286 \). For \( E_p - E_{\gamma} \) correlation we find: \( E_p \propto E_{\gamma}^{\eta'} \), \( \eta' \sim 0.555 \pm 0.089 \) and \( \sigma_{\gamma} = 0.209 \). We find that additional data has reduced the significance of the correlations or has increased the dispersions (compare our values of \( \sigma_{\text{iso}} \sim .... \) and \( \sigma_{\gamma} \sim .... \) obtained by Amati et al. and Ghirlanda et al.). This is contrary to
what one would expect for a sample with a true correlation. From this we conclude that, as predicted by LPM00, there is a strong correlation between $E_p$ and $E_{iso}$ (or $E_\gamma$), but for the population GRBs as a whole both variables have a broad distribution and most GRBs do not obey the tight relations claimed earlier.

3.1. Correlations and Cosmology

Attempts to use observations of extragalactic sources for cosmological studies have shown us that extreme care is required. All observational biases must be accounted for and theoretical ideas tested self-consistently, avoiding circular arguments. This is especially true for GRBs at this stage of our ignorance about the basic processes involved in their creation, energizing, particle acceleration and radiation production. Here we outline some of the difficulties and how one may address and possibly overcome them.

Let us assume that there exists a one-to-one but unknown relation between $E_{iso}$ and $E_p$, $E_{iso} = E_0 f(E_p/E_0)$, and that we have a measure of $F_{tot}$ and $z$. Here $E_0$ and $E_p$ are some constants, and for convenience we have defined $f(x)$ which is the inverse of the function $g$ introduced above. The

From equations (3) and (4) we can write

$$\int_1^Z dZ'\left[\Omega(Z')\right]^{-1/2} = \left(\frac{f(E_{iso}Z)/E_0}{ZF_{tot}/F_0}\right)^{1/2}$$

with $F_0 = \frac{E_0}{4\pi(c/H_0)^2}$. 

For general equations of state $P = w_i \rho$, $\Omega(Z) = \sum_i \Omega_i Z^{3(1+w_i)}$. The aim of any cosmological test is to determine the values of different $\Omega_i$ and their evolutions (e.g. changes in $w_i$). If we make the somewhat questionable assumption of complete absence of cosmological evolutions of $E_{iso}$, $E_p$ and the function $f(x)$, then this equation involves two unknown functions $\Omega(z)$ and $f(x)$. In principle, if the forms of these functions are known, then one can rely on some kind of minimum $\chi^2$ method to determine the parameters of both functions, assuming that there is sufficient data to overcome the degeneracies inherent in dealing with large number of parameters. By now the parametrization of $\Omega(Z)$ has become standard. However, the form and parameter values of $f(x)$ is based on poorly understood data and theory, and currently requires an assumed cosmological model. Using the form (e.g. the power law used by Geo05) derived based on an assumed cosmological model to carry out such a test is strictly speaking circular. (It is even more circular to fix the value of parameters, in this case the index $\eta$, obtained in one cosmological model to test others as done by Dai et al. 2004). Even though different models yield results with small differences, this does not justify the use of circular logic. The differences sought in the final test using equation 5 will be of the same order. The situation is even more difficult because as stressed above the correlation is not a simple one-to-one relation but is a statistical one. Finally, the most important unknown which plagues all cosmological tests using discrete sources is the possibility of the existence of an a priory unknown evolution in one or all of the relevant characteristics. For example, the intrinsic luminosity $L_{iso}$ might suffer large evolution which we refer to as luminosity evolution. The value of $E_p$ can also be subject to selection effects, or the correlation function $f(x)$ may evolve...
Fig. 2.— $E_p - \mathcal{E}_{iso}$ and $E_p - \mathcal{E}_\gamma$ correlations. The 43 gray circles are all the bursts from our sample that had good enough spectral observations to find the energy peak of the $\nu F_\nu$ spectrum, and the 21 black diamonds are a subset of those bursts with a jet break found in their optical lightcurve. Solid lines are the best fit we find for the two correlations ($E_p \propto \mathcal{E}_{iso}^\eta$, $\eta \sim 0.328 \pm 0.036$ and $E_p \propto \mathcal{E}_\gamma^{\eta'}$, $\eta' \sim 0.555 \pm 0.089$) and dashed lines are the best fit found by Amati et al. 2003 with 20 bursts ($E_p \propto \mathcal{E}_{iso}^{0.35}$) and Ghirlanda et al. 2004 with 16 bursts ($E_p \propto \mathcal{E}_\gamma^{0.70}$).
with redshift, i.e. $\eta = \eta(z)$. For such a general case we are dealing with 4 unknown $???$ of the two above. Moreover the rate function of GRBs most likely is not a constant and can influence the results with a broad distribution. We address some of these questions now.

4. GRB Evolutions

For a better understanding of GRBs themselves and the possibility of their use for cosmological tests we need to know whether characteristics such as $E_{\text{iso}}$, $\theta_{\text{jet}}$, $E_p$, the correlation function $f(x)$ and the occurrence rate $\rho_{\text{GRB}}$ (number of GRBs per unit co-moving volume and time) change with time or $Z$. For example to use the $E_p - E_{\text{iso}}$ correlation for cosmological purposes, one need to first establish the existence of the correlation and determine its form locally (low redshift). One then has to rely on a theory or non-circular observations to show that either this relation does not evolve or if it does how it evolves. The existing GRB data is not sufficient for such a test. In fact there seems to be some evidence that there is evolution. Lie??? has shown by subdividing the data into 4 $z$-bins, he obtained different index $\eta$ which change significantly, rendering previous use of this relation for cosmological test invalid. This emphasizes the need for a solid understanding of the evolution of all GRB characteristics. Two of the most important characteristics are the energy generation $E_{\text{iso}}$ and the rate of GRBs. These are also two characteristics which can be determined more readily and $???$ with higher uncertainty. In what follows we address these two questions. We will use all GRBs with known $Z$ irrespective of whether we know the jet angle $\theta_{\text{jet}}$ because this gives us a larger sample and because in view of new Swift observations (Nousek et al. 2006) the determination of the latter does not seem to be straightforward. Also since it is often easier to determine the peak flux $f_p$ rather than the fluence threshold, in what follows we will use the peak bolometric luminosity $L_p = 4\pi d_m^2 f_p$ instead of $E_{\text{iso}} = \int L(t) dt$.

4.1. Evolution with Pseudoreddshifts

Before considering GRBs with known redshifts we briefly mention that there has been two indications of strong evolutionary trends from use of pseudo redshifts based on the so-called luminosity-variability and lag-luminosity correlations (Lloyd et al. 2002, Kocevski & Liang 2006) using the methods developed by Efron & Petrosian (1992, 1994). These works show existence of a relatively strong luminosity evolution $L(z) = L_0 Z^\alpha$ ($\alpha = 1.4 \pm 0.5, 1.7 \pm 0.3$) from which one can determine a GRB formation rate which also varies with redshifts and can be compared with other cosmological rates such as the star formation rate.
4.2. Evolution with Measured Redshifts

4.2.1. Description of the Data

We have compiled the most complete list of GRBs with known redshift. Since the launch of the Swift satellite, this list has become significantly larger. We include only bursts with good redshift determination meaning that GRBs with only upper or lower limits on their redshift are not in our sample. On total, our sample contains 86 bursts, triggered by 4 different instruments: BATSE on board CGRO (7 bursts), BeppoSAX (14), HETE-2 (13), and Swift (52). For each burst we collected fluence and peak flux in the energy bandpass of the triggering instrument, as well as the duration of the burst. When available we have also collected spectral information namely the parameters that define the Band function; the energy peak ($E_{\text{peak}}^\text{obs}$) as well as the low ($\alpha$) and high ($\beta$) energy indexes of the $\nu F_\nu$ spectrum. When a good spectral analysis was not available, we took as default values the mean of the BATSE distributions based on large number of bursts: $< \alpha > = -1.0$, $< \beta > = -2.3$ and $< E_{\text{peak}}^\text{obs} > = 250$ keV. For non-Swift bursts, all this information was mostly extracted from FB05 data set to which we added results from recent spectral analysis released in GCN. For Swift GRBs, redshift, duration, fluence and peak flux were compiled from the Swift Information webpage (http://swift.gsfc.nasa.gov/docs/swift/archive) and spectral information have been retrieved from GCN releases and we have also looked at spectral analysis ourselves for some of them. We assumed the following cosmological model:

$$\Omega_M = 0.3, \Omega_\Lambda = 0.7, H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

in order to determine the intrinsic properties (e.g. $E_{\text{iso}}$ see eq. [3]). $E_{\text{iso}}$ is here calculated for a rest-frame bandpass [20,2000] keV. Note that K-correction due to the shift of the photons into the instrument bandpass has been properly taken into account for the $E_{\text{iso}}$ calculation (see Bloom et al. 2001). From this, we calculate the average isotropic-equivalent luminosity as:

$$L_{\text{iso}} = \frac{E_{\text{iso}}}{T_{90}}$$

when $T_{90}$ is the duration of the burst that includes 90% of the total counts.

Because different instruments have been used to collect this information, the sample is very heterogeneous and suffers from various selection and truncation effects that vary from burst to burst. The most simple of these truncation effects is due to the limiting sensitivity of the instruments. A GRB trigger will occur when the peak flux of the burst exceeds the average background variation by a few sigmas (depending on the setting of the instrument). In an attempt to carefully take into account this effect into our study, we used the analysis carried out by Lamb et al. 2005 for pre-Swift instruments. In this analysis, they computed the sensitivity for each instrument depending on the spectral parameters of the bursts. Therefore, for each specific burst of our data set, from its spectral parameters it is possible to determine the limiting photon flux of the instrument (for specific GRB with its
Fig. 3.— Isotropic average luminosity versus redshift for all bursts in our sample (86). Different symbols represent bursts observed by different instruments: BATSE(7), Beppo-SAX (14), HETE-2 (13), Swift (52). For all non-Swift burst, a vertical line is plotted representing the range of isotropic luminosity in which the burst would still have been observable by the instrument keeping all its others parameters fixed. Using the work of Lamb & al. 2005, the limiting luminosity is taken to be only dependent on the energy peak $E_p$ of the bursts. For Swift bursts, a conservative threshold flux of 0.8 ergs s$^{-1}$ cm$^{-2}$ has been chosen. This limit is shown as a dashed line.
specific spectral parameters). From these, we can easily compute the limiting peak flux $f_{p,\text{lim}}$ for our burst (assuming the Band function for our spectrum). Finally, we can determine the detection threshold of the observed energy fluence $F_{\text{obs}}$. This lower limit $F_{\text{obs,lim}}$ is obtained via the simple relationship (Lee & Petrosian 1996):

$$\frac{F_{\text{obs}}}{F_{\text{obs,lim}}} = \frac{f_p}{f_{p,\text{lim}}}$$

Using the same reasoning we can obtain the limiting values for the intrinsic quantities meaning the intrinsic values that a given burst needs to have in order to be detected:

$$\frac{\mathcal{E}_{\text{iso}}}{\mathcal{E}_{\text{iso,lim}}} = \frac{L_{\text{iso}}}{L_{\text{iso,lim}}} = \frac{f_p}{f_{p,\text{lim}}}$$

Those limiting average luminosities for each bursts of our sample are represented in Figure 3. This analysis was not carried out for BAT instrument on board Swift therefore we used a conservative threshold of 0.8 erg s$^{-1}$ cm$^{-2}$ for all of Swift bursts.

4.2.2. Analysis and results

We now describe our determination of luminosity and density rate evolution of the parent population of our GRB sample. Our analysis is based on the work done by Efron & Petrosian (1992, 1994). We refer the reader to these two papers for details. We will here simply describe the most important steps of the analysis and what it allows us to infer on our data sample. This method has been developed in order to take into account effects of data truncation and selection bias on a heterogeneous sample from different instruments with different sensitivities as described above and shown in Figure 3. The method corrects for this bias by applying a proper rankings to different subset of our sample. The first step is to compute the degree of correlation between the isotropic luminosity and redshift. For that we use the specialized version of Kendall’s $\tau$ statistics. The parameter $\tau$ represents the degree of correlation found for the entire sample with proper accounting for the data truncation. $\tau = 0$ means no correlation is found between the two parameters being inspected (luminosity and redshift in our case). Any other specific value $\tau_0$ implies presence of a correlation with a significance of $\tau_0 \sigma$. With this statistic method in place, we can calculate the parametrization that best describe the luminosity evolution. To establish a functional form of the luminosity-redshift correlation, we assume a power law luminosity evolution: $L(z) = L_0 Z^\alpha$. We then remove this dependency from the observed luminosity: $L' \rightarrow L_{\text{observed}}/(1+z)^\alpha$ and calculate the Kendall’s $\tau$ statistics as a function of $\alpha$. Figure 4 shows the variation of $\tau$ with $\alpha$.

Once the parametric form for the luminosity evolution have been determined, this non-parametric maximum likelihood techniques can be used to determine the cumulative distribution for luminosity and redshift (see Efron & Petrosian 1994) say $\Phi(L)$ and $\sigma(z)$, which gives the relative number of bursts under a certain redshift $z$. From this last function, we can
easily draw the comoving rate density \( \dot{n}(z) \), which is the number of GRB per unit comoving volume and unit time:

\[
\dot{n}(Z) = \frac{d\sigma(Z)}{dZ} \frac{Z}{dV/dZ}
\]  

(6)

where the factor \( Z \) is to take the time dilatation into account. Note that this method do not provide any constrain on the normalization of any of the quantity mentioned above. Normalization will therefore be set arbitrarily on all our figures representing these functions.

We find a 3.68 \( \sigma \) evidence for luminosity evolution (see the \( \tau \) value at the onset of Figure 4 when \( \alpha = 0 \)). From this figure, we can also infer that \( \alpha = 2.21 \) for value obtained when \( \tau = 0 \) gives the best description of the luminosity evolution for the assumed form has a one sigma range of \([1.75, 2.74]\). Constrain on the \( \alpha \) parameter is not very tight off course since the size of our sample is still limited. While current satellites accumulate more data, we will be able to increase our data set and further constrain this parameter in the future.

Fig. 4.— Variation of the \( \tau \) parameter with the power law index \( \alpha \) of the luminosity evolution. \( \tau = 0 \) means no correlation which gives the best value of \( \alpha = 2.21 \) for the assumed power law form with a one sigma range of 1.75 to 2.74.
The cumulative functions are both shown in Figure 5 and the estimated comoving density rate is shown by the jagged curve in Figure 6. Most of the high frequency variation is not real and is due to taking the derivative of a noisy curve ($\sigma(z)$). The dashed line was obtained by fitting the cumulative density distribution by the following parametrized function:

$$\sigma(Z) \propto \frac{(Z/Z_0)^{p_1}}{(1 + Z/Z_0)^{p_1 - p_2}}$$

with the following values for the parameters: $Z_0 = 1.8$, $p_1 = 7.1$, and $p_2 = 0.95$

These results are still very preliminary as more data become available, accuracy of the density function will increase constraining further the evolution rate of long bursts. By tackling this problem for the first time we hope to set the ground for further analysis in the future.

The behavior of the comoving density rate for our sample of long bursts is quite peculiar with a significant rate increase happening at low redshifts. This effect might be due to some selection effects that we have not included in our analysis. For instance, it might be a consequence of the fact that instruments detect more easily low-redshifts host galaxies and therefore create a bias toward low redshifts GRBs. Another interesting feature is the steady increase we obtain in the GRB rate at high redshifts ($z > 3$). Figure 7 compares the estimated comoving rate evolution with different models of Star Formation Rates (SFRs I, II, III).

For comparison with Star Formation history, we used three different models taken from the literature:
Fig. 6.— The comoving rate density $\dot{n}(z)$. The dashed line was obtained by fitting the cumulative density distribution by the parametrized smooth function of equation [7]. We also show comparison of the density rate (from Figure 5) with three different SFR scenarios taken from literature. No SFR scenario seems to match the density rate deduced from our analysis.
Fig. 7.— Comparison of the comoving density rate evolution of the total sample with that of several sub-sample where we impose three different luminosity thresholds: $L_{iso} > 10^{49}$, $> 10^{50}$, and $> 8 \times 10^{50}$ ergs.s$^{-1}$. We also looked at a low luminosity population where we imposed of maximal luminosity of $8 \times 10^{50}$ ergs.s$^{-1}$. Each sub-sample is subject to the same analysis and has provided different luminosity evolution as evident from the different values of $\alpha$. As expected, the rate at low redshift decreases with increasing values of threshold.
- Steidel et al. 1999:

\[ \dot{n} = 0.16 h_{70}^2 \frac{e^{3.4z}}{e^{3.4z} + 22} M_\odot \text{yr}^{-1} \text{Mpc}^{-3} \]  

(8)

- Porciani & Madau 2000:

\[ \dot{n} = 0.22 h_{70}^2 \frac{e^{3.05z - 0.4}}{e^{2.93z} + 15} M_\odot \text{yr}^{-1} \text{Mpc}^{-3} \]  

(9)

- Cole et al. 2001:

\[ \dot{n} = \frac{(a + bz) h_{70}}{1 + (z/c)^d} M_\odot \text{yr}^{-1} \text{Mpc}^{-3} \]  

(10)

with \((a, b, c, d) = (0.0166, 0.1848, 1.9474, 2.6316)\)

As evident, no SFR scenario seems to match the density rate evolution deduced from our analysis, specially at low redshift. How much of this difference is real and how much is due to other selection effects that we have not quantified is unclear. Because of increasing difficulty of identifying the host galaxy with increasing redshift one would expect some bias against detection of high redshift bursts. But the largest densities for SFR seems to be in the intermediate redshift range.

An other possibility is that there may exist subclasses of GRBs such as low or high luminosity classes. In order to test this eventuality we have defined several subsets of our total GRB sample carried out the above analysis for each subsamples, determining a new luminosity evolution (a new \(\alpha\)) and then proceeding to obtain \(\dot{n}(z)\) from the smooth function fitting \(\sigma(z)\). We impose different luminosity thresholds for the different subsamples. Three different threshold have been chosen: \(L_{\text{iso}} > 10^{49}\) ergs.s\(^{-1}\), \(L_{\text{iso}} > 10^{50}\) ergs.s\(^{-1}\), and \(L_{\text{iso}} > 8 \times 10^{50}\) ergs.s\(^{-1}\). We also looked at a low luminosity population where we imposed of maximal luminosity of \(10^{50}\) ergs.s\(^{-1}\). Figures 6 and 7 compare the new rates with that of the total sample. As expected high luminosity samples contribute less to the rate at low redshifts. But the general trend and the differences with SFR are essentially still present. However the method and framework we presented would be a very valuable tool when enough data has been accumulated.

5. Summary and Conclusion

We have considered GRBs as cosmological tools. We find that GRBs are not SCs and the correlations found so far are statistical in nature and too broad to be very useful for cosmological model parameter determination. In addition we have shown that there is strong evidence for evolution of the peak luminosity of GRBs. This indicate a very likely possibility that \(E_{\text{iso}}\) and \(E_{\gamma}\) may also have undergone comparable changes. There may also be evolution
of $E_p$, $\theta_{\text{jet}}$ or other relevant characteristics. This makes use of GRBs as cosmological tools more difficult.

We may therefore ask is this process hopeless? Strictly speaking the answer is no. Some broad brush conclusion can already be reached. For example, one can test the relative merits of different forms for $f(x)$. As shown by FB05 the SC assumption ($\phi(E_{\text{iso}}) \rightarrow \delta(E_{\text{iso}} - E_0)$) gives unacceptable fit to essentially all cosmological models, but the use of the power-law form agrees with Lemaitre type model (FB05, Gea05) with a relatively long quasi-static phase (referred to as a loitering model). Such models, which were in vogue some time ago (see Petrosian 1974), are currently unacceptable because of their low (baryonic plus dark) matter density and large curvature. This indicates that the form of the correlation and/or other assumptions (e.g. no evolution) are not correct. In this paper, we have shown that there is strong evolution of luminosity and $E_{\text{iso}}$. Lei et al. have also shown that the form of the $E_p - E_{\text{iso}}$ correlation may evolve. These are tentative results and more data are required to determine these evolution trend and their meaning.

The relevant variables in addition to redshift are $E_{p,\text{obs}} = E_p/(1+z)$ and $F_{\text{tot}}$, determination of which requires a good description of the total spectrum. We need to know the observational selection biases for all the variables and their parameters, and use accurate statistical methods to account for the biases and data truncations. From these one can learn about the distributions of $E_{\text{iso}}$ and $E_p$ and their correlations.

Obviously Swift observations will be extremely helpful and eventually may provide the data required for this complex task. In the near future, however, from analysis of the incoming and archived data we will (and need to) first learn more about the nature of the GRBs than cosmological models. Eventually we may have enough information to construct a well defined “SC”, which can be used for global cosmological tests as is done using type Ia supernovae. The immediate situation may be more analogous to galaxies where the cosmological tests are rendered complicated because of the multivariate situation and broad distributions of the relevant variables. Consequently, over the years the focus of activity has shifted from the determination of the few global cosmological parameters to the investigation of structure formation, the building process of the black holes and the star formation rate (SFR). Similarly, we expect that from investigation of GRBs we will learn about the evolution, distributions and correlations of their intrinsic characteristics, and the relationship of these with the evolutionary rates of other cosmological sources and the formation rates of stars, supernovae and black holes.

In summary, on the long run cosmological test with GRBs may be possible, either carried out with clever statistical methods, or by identification of a subclass of “SCs”. On a shorter time scale, we need to learn more about the intrinsic characteristics of GRBs, and provide a reasonable theoretical interpretation for them and their cosmological evolution. As an example, as shown here, one outcome of our analysis is the determination of the evolution rate of GRBs. We have shown that for all GRBs with known redshifts this rate appears to be different form the SFR. These differences seems to be present with different subdivision of the sample and may be consequences of other selection biases not included in our analysis. But the difference is not what one would expect from some possible observational selection...??
6. REFERENCES

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