NORMALIZING THE TEMPERATURE FUNCTION OF CLUSTERS OF GALAXIES

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ABSTRACT

We reexamine the constraints that can be robustly obtained from the observed temperature function of X-ray studies of cluster of galaxies. The cluster mass function has been thoroughly and analytically studied in simulations, but a direct simulation of the temperature function is presented here for the first time. Adaptive hydrodynamic simulations using the cosmological Moving Mesh Hydro code are used to calibrate the temperature function for different popular cosmologies. Applying the new normalizations to the present-day cluster abundances, we find \( \sigma_8 = 0.53 \pm 0.05 \Omega_0^{-0.45} \) for a hyperbolic universe and \( \sigma_8 = 0.53 \pm 0.05 \Omega_0^{-0.53} \) for a spatially flat universe with a cosmological constant. The simulations followed the gravitational shock heating of the gas and dark matter and used a crude model for energy injection by supernova heating. The error bars are dominated by uncertainties in the heating/cooling models. We present fitting formulae for the mass-temperature conversions and cluster abundances based on these simulations.

Subject headings: cosmology: theory — galaxies: clusters: general — hydrodynamics — methods: numerical

1. INTRODUCTION

X-ray studies of clusters of galaxies have provided a host of quantitative data for the study of cosmology. They are ideally suited for the normalization of structure formation, of quantitative data for the study of cosmology. They are

rely on the cluster temperature function and the number robust measurements of the cluster abundance currently

to P-S calculations calibrated by

\[ \text{N}(>kT) \]

expressed in units of \( h^3 \text{ Mpc}^{-3} \). Henry & Arnaud (1991, hereafter HA) and Edge et al. (1990) have analyzed the

HEAO-1 A2 all-sky X-ray survey to obtain temperatures for clusters at a flux limit of \( F_{2\text{keV}} - F_{\text{10keV}} > 3 \times 10^{-11} \)

ergs \( \text{cm}^{-2} \text{s}^{-1} \). The sample contains the 25 X-ray brightest clusters and is over 90% complete at Galactic latitude \( |b| > 20^\circ \). By virtue of the brightness of the clusters, their temperatures are easily and well determined. In this paper we will make use of this excellent objective sample.

From the theoretical standpoint, the cluster temperature function is a clean measurement, since the normalization of fluctuations can be established independently of the baryon fraction \( \Omega_0 \) or the Hubble constant \( H_0 \equiv 100 h \text{ km s}^{-1} \text{ Mpc}^{-1} \). Clusters are close to isothermal, both in observations and in simulations, which makes their temperature determination robust and insensitive to numerical resolution or telescope angular resolution. The temperature of a cluster depends primarily on the depth of the potential well of the dark matter and the state of equilibrium of the gas. This is in contrast to the cluster luminosity which depends strongly on small-scale parameters like clumping and core radius. While heating or cooling processes could slightly change the temperatures, the gravitational potential provided by the dark matter buffers the system and causes the temperatures in hydrostatic equilibrium to be weakly affected by energy injection or loss. We will verify this effect through direct simulations of heat injection. Furthermore, measurements of galaxy velocity dispersions appear to be in good agreement with the temperatures and with the ratio of gas temperature to velocity dispersion \( \beta_{\text{fit}} \equiv kT/\mu m_p \sigma^2 \approx 0.95 \pm 0.05 \) (Bahcall & Lubin 1994). This suggests that non-thermal pressures are negligible.

Recent work by Eke, Cole, & Frenk (1995, hereafter ECF) and Viana & Liddle (1996, hereafter VL) used the cluster temperature function to normalize the amplitude of fluctuations in spheres of radius \( 8h^{-1} \text{ Mpc} \), denoted \( \sigma_8 \). They compared the observed cluster temperature function of HA to P-S calculations calibrated by N-body simulations. These studies did not include any gasdynamic effects, which left questions about the statistical properties of cluster equilibria. They concluded that the present-day normalization in a flat standard cold dark matter (CDM) is \( \sigma_8 = 0.5 \). Modeling the predicted cluster abundance in a subcritical density universe is more difficult. VL proposed \( \sigma_8 \propto \Omega_0^{0.5} \), while ECF have a smaller error budget in the exponent. ECF only measured the mass function in simulations. In order to convert from a mass function to a temperature function, these models assumed that all clusters are perfect singular isothermal spheres that formed at some prescribed redshift. While the errors in the normalization \( \sigma_8 \) from the Poissonian scatter are very small, any error in mass-temperature conversion translates into a similar-sized error in the normalization. That issue is the point of largest uncertainty in these semianalytic calculations. In order to address this problem, we extend their work by performing full hydrodynamic simulations. We directly measure the X-ray emission weighted temperature
in the simulation. The simulations are used to measure directly the mass-temperature relation, which are then fed into the P-S calculations. This approach allows us to explore a large dynamic range in volumes and parameters.

In this paper we will not use the term “open universe.” Instead we will call universes with negative spatial curvature “hyperbolic universe.” This circumvents the misleading suggestion that hyperbolic universes should be spatially infinite (Pen & Spergel 1995). We will proceed in \( \S \) 2 to describe the simulations. In \( \S \) 3 we summarize the observed cluster temperature function. Our new results are presented in \( \S \) 4. We discuss the cosmological implication of these results in \( \S \) 5.

2. SIMULATIONS

We use the Moving Mesh Hydrodynamic (MMH) code (Pen 1998); it implements a Total Variation Diminishing (TVD, Xin & Jin 1995) high-resolution shock capturing hydrodynamics scheme on an adaptively deforming mesh. The gravitational potential is solved by using a multigrid iteration (Pen 1995). Dark matter is modeled by using a particle-mesh algorithm on the same moving grid. The grid is continuously adjusted to maintain an approximately constant mass per cell. This is achieved through a pure potential flow grid velocity field. The full Euler fluid equations are solved on this moving mesh, and the fluid is allowed to develop vorticities. By following the mass, the MMH code has improved spatial resolution in dense regions such as clusters of galaxies. Since their cores are \( 10^3-10^4 \) times overdense, any mass-based method will have a 10–20 fold better length resolution in these high-density regions. A recent study has compared this code to several other cosmological hydrodynamic codes for a cluster formation scenario (Frenk et al. 1998), and general agreement exists between this code and the others compared. Excellent agreement was found for the temperature properties of simulated clusters. Similar agreement was obtained in a comparison to a suite of existing codes (Pen 1998; Kang et al. 1994). All simulations were run on an SGI power challenge at the National Center for Supercomputing Applications. They all used \( 128^3 \) grid cells and \( 256^3 \) particles. The initial power spectrum was taken from Bardeen et al. hereafterBBKS). The simulations were started at a redshift \( z = 100 \).

Table 1 summarizes the cosmological parameters used in each simulation. Cooling has not been incorporated in these simulations and will be addressed in a future paper (Cen et al. 1998).

For the CDM model, we used the best-fit values suggested by ECF with \( \Omega_0 = 1, \Omega_k = 0.05, h = 0.5, \sigma_8 = 0.5, \) and \( n = 1 \) on a box of side length \( 80 \) \( h^{-1} \) Mpc. \( n \) denotes the unprocessed power spectrum (BBKS), whose Harrison-Zeldovich-Peebles (HZP) value is \( n = 1 \). The simulations were repeated 2 times with different random seeds to improve the statistical measures. These comprise models CDMD1–3. We note that the finite box sizes truncate the long wave modes, which has the effect of lowering the fluctuation amplitude in the numerical realization by 15% (Gelb & Bertshinger 1994; Pen 1997a). To check its significance and to test the resolution dependence, we ran a simulation in a larger box of side length \( 200 \) \( h^{-1} \) Mpc, which we call CDM4. At such small values of \( \Omega_0 \), the corrections to the power spectrum are negligible compared to other sources of error (Holtzman 1989). As long as the gravitational potential is dominated by the dark matter, we can easily rescale the result to any value of \( \Omega_0 \). We then ran a model with heat input from early star formation, which we call PREHEAT. At a redshift \( z = 1 \), we injected \( 1 \) keV of energy per nucleon into the plasma. Details are described in \( \S \) 4. We next ran a cosmological constant model in a box of side length \( 120 \) \( h^{-1} \) Mpc. We repeated the simulation with identical initial conditions for a hyperbolic universe \( \Omega_0 = 0.37 \). These are called LCDM and OCDM, respectively.

Since clusters are rare objects, their identification is a relatively simple matter. We use the gas densities to identify clusters. We search the volume for all density peaks in the unsmeared gas field, which are overdense by at least 200 over the mean cosmic density, and separated by at least 2 \( h^{-1} \) Mpc. If two peaks are closer than that distance, the less dense one is discarded. From this sample, we computed the total emission-weighted temperature for each cluster in a \( 1 \) \( h^{-1} \) Mpc radius, weighting the temperature in each cell by \( \rho^2 T^{1/2} \), and sorted these clusters by temperature to find the cumulative temperature function.

3. OBSERVED TEMPERATURE FUNCTION

We use the cluster sample from HA. The sample is based on the objects from the HEAO-1 A2 survey, which identified all objects at a flux limit \( F_x > 3 \times 10^{-12} \) ergs cm\(^{-2}\) s\(^{-1}\) in the 2–10 keV band at Galactic latitude \( |b| > 20° \). The 2–10 keV window in which HEAO is sensitive is well matched to the temperatures of rich clusters. HA identified 25 clusters with that sample. They range in temperature from 2.5 to 9.5 keV, with luminosities in the range \( 10^{43} \) to \( 10^{45} \) \( h^{-2} \) ergs s\(^{-1}\). The furthest cluster is at redshift \( z = 0.09 \), which is sufficiently low that evolutionary corrections are small. This sample has excellent completeness properties, and most clusters have multiply-measured temperatures. The typical temperature error is 10%. Following ECF, we measure the cumulative cluster density

\[
N(>kT) = \sum_{kT > kT}^{} \frac{1}{V_{\text{max},i}},
\]

where \( V_{\text{max},i} \) is the maximal volume to which each cluster could have been seen at the flux limit. The formal Poisson error on the estimate is given by

\[
\sigma(>kT)^2 = \sum_{kT > kT}^{} \frac{1}{V_{\text{max},i}^2}.
\]

This tends to underestimate the true error for several reasons. Since we are measuring a cumulative abundance, errors at different temperatures are correlated. Clusters tend to cluster strongly with each other with correlation lengths of about 20 \( h^{-1} \) Mpc (Bahcall 1996). This boosts the error.
in equation (2) for the low-luminosity clusters, where $V_{\text{max}}$ is comparable to the correlation volume. The errors are Poissonian with respect to the true underlying distribution, not with respect to the estimated density. Using the estimated density will again systematically underestimate the errors. Instead of attempting to model these uncertain errors directly, we will use a heuristic fit based on the estimates of ECF. They found that the abundance is best represented near 5 keV, which lies in the center of the temperature window. This value is close to the temperature formed by the collapse of an 8 h$^{-1}$ Mpc radius sphere in a $\Omega = 1$ model, and empirically appears to be the pivot point of the distribution as one varies parameters. Normalizing abundances at that temperature, ECF found statistical errors resulting in variations of $\sigma_8$ of only about 2%. As we will see below, the errors arising from thermal history uncertainties and numerical limitations are significantly larger, so we will neglect the Poissonian errors hereafter. The cluster abundance and two past fits are shown in Figure 1, renormalized using the results of this paper (see eqs. [8], [9], and [11] below).

### 4. MASS-TEMPERATURE RELATION

To obtain an analytic estimate of the temperature function, we will use the P-S Ansatz. The fraction of mass in bound objects is

$$f(>M) = \sqrt{\frac{2}{\pi}} \int_{\delta_c(>M) \Delta}^{\infty} e^{-u^2/2} du . \quad (3)$$

\(\sigma(M)\) is defined as the rms density fluctuations in tophat spheres of mass \(M\). The distribution function has been multiplied by 2, such that all the mass is accounted for when \(\sigma \to \infty\). \(\delta_c = 1.686\) is the linearly extrapolated overdensity at which an object virializes. Formally, it is only exact for \(\Omega_0 = 1\), but it varies only by a few percent to \(\Omega_0 = 0.3\), and we will consider it to be constant. We will numerically normalize the mass-temperature relation, which will absorb any changes in \(\delta_c\) and simplifies calculations. We define the dimensionless mass \(m \equiv M/M_8\), where \(M_8 \equiv 4\pi \bar{n}(8 \text{ h}^{-1} \text{ Mpc})^3/3\) is the mass contained in an 8 h$^{-1}$ Mpc sphere. \(\bar{n} \equiv 3\Omega_0 H_0^2/8\pi G\) is the mean density of the universe today in terms of the Hubble constant \(H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}\). Differentiating equation (3), we obtain the differential number density of objects \(dn/dm = (\bar{n}/M_8) df/dm\) as

$$\frac{dn}{dm} = \sqrt{\frac{2}{\pi}} \frac{\bar{n} \delta_c}{M_8 \alpha m^2} \left| \frac{d\ln \sigma}{d\ln m} \right| \exp \left( -\frac{\delta_c^2}{2\alpha^2} \right) . \quad (4)$$

The observed abundance (eq. [1]) is a cumulative statistic, so it is desirable to integrate the differential density function (eq. [4]). To simplify the algebra, we will assume a pure power-law dependence \(\sigma = \sigma_8 m^{-\alpha}\). \(\alpha\) is related to the effective power spectrum \(P(k) = k^{-3\alpha}\). For CDM-like power spectra, the BBKS fit to the power spectrum at \(r = 8 \text{ h}^{-1} \text{ Mpc}\) has

$$\alpha = 0.222 + 0.2495 \Gamma - 0.0232 \Omega_0 , \quad (5)$$

where \(\Gamma \equiv \Omega_0 \bar{n} h \exp \left[ -\Omega_0 - \Omega_0/\Omega_8 \right] (\text{VL})\). Equation (5) is defined by requiring \(\sigma_8/\sigma_g\) to be exact, where \(X\) is the radius which forms a 5 keV cluster. We can then integrate equation (4);

$$n(>m) = \frac{3}{2048\pi^{3/2}} \left( \frac{2\alpha^2}{2\sigma_8} \right)^{1/2} \times \Gamma \left( \alpha - 1, \frac{m^{2\alpha} \delta_c^2}{2\sigma_8^2} \right) h^3 \text{ Mpc}^{-3} , \quad (6)$$

where \(\Gamma[a, x]\) is the incomplete Gamma function. The cumulative number abundance (eq. [6]) has units of clusters per Mpc$^3$ $h^{-3}$. The asymptotic semiconvergent sequence for \(\Gamma\) (Arfken 1985) allows us to expand equation (6) to

$$n(>m) \approx 0.063 \times 10^{-4} \left( \mu^{1/2} v \right) \exp \left( -v \right) \left( 1 - \frac{p+1}{v^2} + \frac{p(p+1)}{v^3} + \cdots \right) h^3 \text{ Mpc}^{-3} , \quad (7)$$

where we have abbreviated \(\mu = \delta_c^2/2\sigma_8^2\), \(v = \mu m^{2\alpha}\), and \(p = (\alpha + 1)/2\alpha\). The terms in equation (7) initially converge for large \(v\), but at some point diverge again. Each term has an alternating sign, and each approximation brackets the true solution. For our purposes, we can truncate the sum and modify the first two terms as

$$n(>m) \approx 0.063 \times 10^{-4} \left( \mu^{1/2} v \right) \exp \left( -v \right) \left( 1 - \frac{p}{2v} \right) h^3 \text{ Mpc}^{-3} . \quad (8)$$

The accuracy of equation (8) is shown in Figure 2, where we have used the mass-temperature relation derived below. Three sets of lines are shown, which correspond to an integral of equation (4) using the BBKS power spectrum (solid lines), the abundance using the power-law fit with equations (5)–(6) (dashed lines), and the two-term asymptotic expansion (eq. [7]) (dotted lines). For this illustration, we picked one model with \(\Omega = 1\) and one cosmological constant model with \(\Omega_\Lambda = 0.65\). We see that the approximations are accurate to about 3% in temperature over the temperature range 3 to 10 keV.

For the next step, we need to convert the cumulative mass function into a temperature function. Various approaches (see, e.g., ECF) assumed all objects to be perfect
isothermal spheres forming at $z = 0$. In the chain of calculations, this step is the least certain link, and it is the purpose of this paper to quantify it. We will continue to rely on the virial relation $m \propto T^{3/2}$, which follows if the formation redshift and state of equilibrium are uncorrelated with cluster mass. Bryan & Norman (1997) recently demonstrated this relationship to hold to a good approximation in hydrodynamic simulations. We then only need to know the relation coefficient $T_8$:

$$m = \left[ \frac{kT}{T_8(1+z)} \right]^{3/2},$$

where $T_8$ is the effective temperature of a cluster that forms during the collapse of an $8\ h^{-1}\ Mpc$ radius sphere. In a low-density universe, the same sphere contains much less mass, and consequently $T_8$ depends on $\Omega_0$. While the overdensity parameter $\delta_c$ in the top-hat model has a weak $\Omega_0$ dependence, we will fix its value at $\delta_c = 1.686$ and absorb all modeling into $T_8$.

To solve for $T_8$, we identified clusters in each $\Omega_0 = 1$ simulation, resulting in the numerical cumulative distribution $n > T$, shown as the step lines in Figure 3. We then applied equation (8) to the linearly extrapolated density field of the simulation’s initial conditions $\delta\rho(z = 0) = \delta\rho(z) \times (1+z)$. This density field is then smoothed on tophat spheres of varying radii, from which we obtain the predicted P-S abundance of the collapsed mass fraction shown as the diagonal dashed lines in Figure 3. We then solve for $T_8$, such that equation (8) agrees with the predicted abundance in the simulation volume for the five hottest clusters. This approach compensates for the finite box size effect that results in a loss of power on large scales (Pen 1997a). The infinite volume average is the solid diagonal curve in Figure 3. We see that the smaller boxes systematically underestimate the cluster abundance because of the suppression of $\sigma_8$ from the truncation of the power spectrum in the finite box.

The resulting best fit is

$$T_8 = 4.9 \pm 0.2\ keV.$$  \hfill (10)

This value should be compared to $T_8 = 5.5\ keV$ for the perfect isothermal sphere model (ECF). We would expect the ECF model to overestimate $T_8$ for several reasons. The gas is almost certainly not in perfect hydrostatic equilibrium, which would lower the temperature. A scatter in the mass-temperature relation also creates more hot clusters due to Malmquist bias. Cluster profiles may have departures from isothermality, with slight temperature gradients throughout the cluster. The X-ray emission weighted temperature can be slightly different from the mean mass-weighted virial temperature. ECF provided for a fudge factor $\beta$ that describes the ratio between the temperature they used and the actual statistical temperature of clusters. Note that the mean mass-temperature relation for an ensemble of clusters is not sufficient to substitute correctly into the P-S relation (eq. [8]). Since clusters are very rare, the mass function is steep, and any scatter in the mass-temperature relation will introduce a bias in the abundance. What we really want is to normalize equation (8) directly to the temperature function measured in simulations. One can rescale the ECF result by choosing $\beta = 1.1$ in their temperature conversions. Concurrent work by Bryan & Norman (1997) produced results very similar to this study.

They defined a parameter $f_T$ as the ratio of the actual temperature of a cluster compared to that obtained from the tophat model with some assumed radial density profile. In our notation, a rough correspondence would be $f_T = T_8/5.5$,
for which we would obtain $f_p = 0.89 \pm 0.03$, while they found values in the range $0.75 \leq f_p \leq 0.92$.

The error interval is the 1-$\sigma$ standard deviation which compares the different simulations and represents departures from a deterministic P-S theory. To first order, we have compensated for the numerical cosmic variance and also the loss of large-scale power from the finite box size. The simulations only have a limited sampling volume and do not directly simulate the rarest, richest clusters. By using the P-S formalism, we can extrapolate the simulation normalization to larger volumes by assuming the mass-temperature relation scales as one would expect from virial equilibrium. Each simulated model still had at least one cluster above the pivot temperature of 5 keV. In this analysis, P-S allows us to reduce the error bars by simultaneously fitting a range in cluster temperatures.

The first nongravitational effect that needs to be incorporated is the effect of heat injection from stars. Direct observations of the iron line emission suggests that the intracluster medium (ICM) is not pristine and has passed through an earlier generation of stars. This may have raised the initial entropy of the gas. The present-day metallicity of the ICM is near one-third solar, from which we may infer up to 1 keV of energy per nucleon to have been injected (Loewenstein & Mushotzky 1996). While it is not known when this might have happened, clusters at redshift 0.3–1 appear to have similar metallicities as nearby clusters (Mushotzky & Loewenstein 1997; Hattori et al. 1997). Early enrichment would have a smaller effect since the adiabatic expansion of the universe cools the preheated gas. In model PREHEAT we inject gas consistent with the observed lack of evolution to $z = 1$. The most extreme model postulates 1 keV of energy injection at $z = 1$. We simulated such a model by evolving a simulation to $z = 1$, raising the thermal energy everywhere by 1 keV and continuing the evolution to $z = 0$. We then measured the present cluster temperature function. We find that clusters are slightly hotter with $T_8 = 5.3$ keV. We should consider this an upper limit on the plausible effect of heating. Because the gas remains in hydrostatic equilibrium with the dark matter potential, the injection of 1 keV only raised the mean temperatures by 0.4 keV. For our choice of flat-universe parameters, this lowers the normalization to $\sigma_8 = 0.50$, relative to the adiabatic value of 0.53 (see below). Future work will also address the effects of cooling (Cen et al. 1998). Fabian (1994) estimates that up to 20% of the X-ray luminosity of a cluster may arise from a cooling flow, which might affect the emission-weighted temperature by a similar amount as the heating. The latter depends sensitively on the smallest scale inhomogeneities and poses a larger computational challenge. For now we will assume temperature errors to be symmetric.

Once we have solved for $T_8$ by using the $\Omega_0 = 1$ simulations, we proceed to repeat the procedure for the $\Lambda$ and hyperbolic cosmologies. By using the same random seed, we expect the differences between the models to be modeled more accurately than for each model individually. The $\Omega$ dependence is incorporated into the mass-temperature relation as

$$T_8 = 4.9 \pm 0.2\Omega_0^{2/3}\Omega(z)^{1/2} \text{ keV}. \quad (11)$$

The two-third scaling accounts for the smaller virial mass enclosed in the 8 $h^{-1}$ Mpc spheres as the density is lowered.

$\gamma$ is used to parameterize remaining corrections, such as the change in formation redshift and in virial radius and collapse density. Solving for $\gamma$ from the $\Lambda$ simulation yields $\gamma = 0.283$ for a $\Lambda$ universe and $\gamma = 0.133$ in the hyperbolic scenario. The temperature function fits for the simulated low-density parameters are shown in Figure 4.

We now have all the required relations to solve for $\sigma_8$, given the cluster abundance. For a given $\Omega_0$, we have the left-hand side of equation (8) from HA at any given temperature, from which we follow ECF and use $kT = 5$ keV. Equations (9) and (11) allow us to convert the temperature into $m$. The only remaining unknown variable is $\sigma_8$ which...
determines $\mu$ and $v$. We then obtain the result $\sigma_8 = 0.53 \pm 0.05 \Omega_{b}^{-0.53}$ in the $\Lambda$ model, and $\sigma_8 = 0.53 \pm 0.05 \Omega_{\gamma}^{-0.45}$ in the hyperbolic universe. The error bars are obtained by linearly adding the uncertainty in the mass-temperature relation to the effects of supernova heat injection. Figure 5 summarizes the results from of the numerical normalizations. In the hyperbolic model, matter domination ends at a relatively higher redshift, $z \sim (1/\Omega) - 1$, than in a cosmological constant model. Clusters of the same mass will thus have a smaller radius in the hyperbolic case and a higher virial temperature. This accounts for the smaller value of $\gamma$ and $\sigma_8$ in hyperbolic models. Our modeling of the temperature function (eq. [11]) implicitly accounts for all these effects since it is normalized by simulations.

5. COSMOLOGICAL IMPLICATIONS

In the context of structure formation, the simplest adiabatic models, which could arise for example from inflation, have only a single free parameter to normalize the spectrum of fluctuations for a HZP spectrum. For a COBE normalized flat universe, this implies $\sigma_8 = 1.2$, which is at great odds with the observed cluster abundance. If we wish to retain a flat universe, we can try to change one of several parameters. The first would be the baryon fraction $\Omega_b$ (White et al. 1995) which suppresses fluctuations due to acoustic oscillations and Silk damping. COBE normalized CDM agrees with the cluster abundance for $\Omega_m = 0.45$, which requires a dramatic revision of big bang nucleosynthesis (Walker et al. 1991). A different parameter that can lower the COBE normalized value of $\sigma_8$ is a tilted power spectrum, i.e., deviations from HZP. Leaving only this one parameter free, we find a satisfactory fit for $n = 0.63$ when we also allow for tensor modes. This violates the limits on the slope allowed by the 4 yr COBE data (Wright et al. 1996). A combination of these two parameters violates each of these constraints more weakly, and one could envision combinations such as $n = 0.7$ and $\Omega_b = 0.15$, which would also be consistent with the cluster gas fractions (Pen 1997b, White et al. 1993; White & Fabian 1995). One can also take more radical departures and lower the Hubble constant to $h \sim 0.3$ (Bartlett et al. 1995). We conclude that no single parameter modification of $\Omega = 1$ inflationary cosmology is even marginally consistent with observations.

For low $\Omega_b$ models, the opposite problem arises. COBE normalized fluctuations result in too low values of $\sigma_8$. For the hyperbolic universe with parameters $\Omega_b = 0.37$ and $h = 0.7$, we obtain $\sigma_8 = 0.57$ which is significantly lower than the 0.83 suggested by the cluster abundance. To raise it to match the cluster abundance, we can either raise $\Omega_b$ to 0.43, raise the Hubble constant to $h = 1.1$, or introduce a tilt $n = 1.17$, or any combination thereof. The cosmological constant models also have their share of free parameters. By lowering $\Omega_\Lambda$, we lower the normalization, but it increases with larger Hubble constant $h$. For the choices under discussion, $\Omega_b = 0.37$ and $h = 0.7$, we have COBE normalized $\sigma_8 = 1.0$, just slightly higher than what the cluster abundance suggests. This can easily be addressed by lowering $h$ to 0.63 or by using the slight tilt $n = 0.95$ as suggested by Ostriker & Steinhardt (1995).

A third alternative is to consider nonadiabatic initial conditions, for example, from topological defects (Pen, Seljak, & Turok 1997). In these models the P-S abundance must be modified to account for the non-Gaussianity (Chiu, Ostriker, & Strauss 1998). These models have not been studied directly with hydrodynamical simulations of the cluster temperature function, but the preliminary results indicate a cluster abundance consistent with a COBE normalized HZP spectrum. We note that these models have problems with other observations, including the galaxy power spectrum and small-scale microwave background anisotropies.

The HA sample has the great advantage that it has well-established completeness criteria which allows us to accurately measure cluster abundances. Many more clusters have measured temperatures, and one can ask how those might affect our estimates. Most importantly, we are not limited by Poissonian statistics but rather by systematic errors. It has been proposed that the evolution of the temperature function is a strong test of cosmologies. At a fixed temperature, the difference in cluster abundance at $z = 0.5$ is over an order of magnitude between flat and hyperbolic models (ECF). Again, we are not limited by statistics. Instead, we must ask what the expected difference in cluster temperature is at fixed abundance. Presently, the hottest cluster in a $300 h^{-1}$ Mpc radius is about 8 keV (see Fig. 1). At a redshift $z = 0.3$, the hottest cluster in the same sample volume would be essentially the same temperature in a hyperbolic universe and about 6 keV in a flat universe. One must make sure that systematic temperatures measurement errors are less than 25% at these redshifts. Since the metallicities have not evolved significantly, it appears that heating will not contribute significantly to the cluster temperature evolution. The presence of hot clusters such as A2163 and MG2016 (Hattori et al. 1997) may well pose problems for flat cosmologies (Pen, David, & Tucker 1997). Recently, Carlberg et al. (1997) studied the abundance of galaxy clusters at intermediate redshift using galaxy velocity dispersions (see also Fan, Bahcall, & Chen 1997). The difficulty here is converting local cluster temperatures into velocity dispersions (Bird et al. 1995; Bahcall & Lubin 1995) since the expected change is small. Any scatter in the velocity-temperature relation will introduce systematic biases which need to be understood. A systematic overestimate of 10% in the line-of-sight velocity dispersion $\sigma_v$ in the CNOC sample relative of the converted $\sigma_v$ from the HA temperature data is sufficient to offset the decrease in the cluster abundance predicted in an $\Omega = 1$ cosmological model. It is essential in these comparisons that homogeneous samples are used at both low and high redshift, which are checked directly against simulations.

Clusters of galaxies provide, coincidentally, a similar constraint on $\Omega_\Lambda$, as velocity field measurements. Measurements of peculiar velocities constrain $\beta_v \equiv \Omega_\Lambda^{0.6}/b$, where the bias $b$ is the ratio of fluctuation in galaxies relative to the dark matter (Strassler & Willick 1995). Typical values for $\beta_v$ are in the range of 0.3 to 0.8. Cluster abundances from the temperature function constrain a very similar function $\sigma_8 \Omega_\Lambda^{-0.6} \sim 0.5$, where $\sigma_8 = 1/b$ for optical galaxies. While velocity fields measure fluctuations in the linear regime, the cluster abundances are determined for highly nonlinear bound objects. It is reassuring that the values obtained from the two very different methods are consistent with each other. The downside is that we cannot determine $\Omega_\Lambda$ and $b$ independently by using present-day measurements alone.

6. CONCLUSIONS

The most robust cosmological constraints using clusters of galaxies come from the cluster temperature function
which primarily depends on the gravitational potential wells of the dark matter. We used new gas-dynamic simulations to test the \( N \)-body and P-S estimates by ECF and VL. We found good general agreement in the normalization to the observed cluster temperature function, with \( \sigma_8 = 0.53 \pm 0.05 \Omega_m^{0.45} \) for a hyperbolic universe and \( \sigma_8 = 0.53 \pm 0.05 \Omega_m^{0.53} \) for a spatially flat universe with a cosmological constant. This result is only weakly sensitive to models of the thermal history of the intracluster medium, which we have modeled with preheating. We have presented improved P-S fits to predict more accurately the mass-

temperature relation scalings (eq. [11]) for a range in values of \( \Omega_m \). Applications to high redshift X-ray clusters are in progress (Pen, David, & Tucker 1997).

\[ \text{COBE} \] normalization for \( \Omega = 1 \) with a Harrison-Zeldovich-Peebles spectrum overpredicts cluster abundances by many orders of magnitude, but this can be addressed if the spectrum is strongly tilted and a large baryon fraction is invoked. The \( \Omega_m = 0.35 \) hyperbolic or cosmological constant models lie closer to observations on all measures, including the age of the universe, the slope of the baryon fraction, and the slope of the galaxy power spectrum, just to mention a few. While no single measurement is at a very high significance, the combination does appear to carry a heavy vote. Lensing statistics (Kochanek 1996) and deceleration parameter measurements (Perlmutter et al. 1997; Pen 1997b) would favor a hyperbolic universe over one with a cosmological constant, and alternative scenarios (e.g., the string-dominated model, Spergel & Pen 1997) are also viable. \[ \text{COBE} \] normalized topological defect models fare reasonably well on the cluster abundance (Chiu et al. 1997).

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