Transverse vibrations of an underground cylindrical structure

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Abstract. The transverse vibrations of an extended cylindrical underground structure under the action of concentrated forces changing according to a harmonic law are considered. The problem of transverse vibrations of a cylindrical rod interacting with an elastic homogeneous medium was solved by the Fourier method. Approximate solutions are obtained using the method of compensating loads. The influence of soil conditions and geometrical dimensions of the structure, the location of active forces on the amplitude-frequency characteristics of the object are analyzed.

1. Introduction

Investigation of the state of extended cylindrical structures is relevant, since most hydraulic and transport structures have this shape. The location of such structures in a seismically active zone requires the study of their condition under the action of dynamic influences. It is important to study both separately the structure [1, 2, 3, 4], soil [5, 6, 7], and their interactions [8, 9, 10, 11, 12, 13, 14, 15, 16]. The assessment of the dynamic behavior and the study of the dynamic characteristics of earthen dams of earthen dams were carried out in [1, 2, 3], and the vibrations of multilayer viscoelastic composite toroidal pipes are considered in [4]. Accounting for the foundations in assessing the dynamics of the earth's structure is considered in [5]. Wave processes in determining the mechanical characteristics of soils were studied in [6], in [7] the attenuation of longitudinal waves in a nonlinearly viscoelastic medium was studied.

In [8], axisymmetric natural torsional vibrations of an infinitely long cylindrical shell in an elastic inertial medium are considered. The dependences of the first vibration frequency of the shell in an elastic medium on the dimensionless wavelength are obtained.

In [9], natural vibrations of thin cylindrical shells (with a ratio of thickness to radius - h / R ≠ 0.25) in an infinite elastic medium are considered. At the contact of the shells with the medium, the condition of rigid or sliding contact is satisfied. The frequency equation is obtained, which is solved by the numerical method of Muller. The local dependence on the length of natural frequencies and the forms of plane transverse vibrations of a thin inhomogeneous rod in an elastic medium with a variable stiffness coefficient and arbitrary boundary conditions of elastic fastening is studied in [10]. In [11], the longitudinal, torsional and bending vibrations of a pipeline laid in the ground were investigated, formulas for calculating the frequencies of natural vibrations were obtained, and graphs of the dependence of frequencies on the parameters of the soil and structures for the corresponding mode of vibration were given. Longitudinal vibrations of a cylindrical rod interacting with an elastic medium
surrounding it were analyzed in [12], the solution was obtained in rows, the coefficients of which include Hankel functions, graphs are given that characterize the change in the nature of the frequency response of the structure depending on the physical and mechanical characteristics of the soil and the structure, the ratio of sizes structures. The dynamic interaction of an elastic medium and a cylindrical structure is considered in [13, 14, 15, 16]. It is important to study the possibility of reducing the vibrations of underground structures from the impact of dynamic forces, as in [17, 18, 19] this possibility was investigated using dynamic vibration dampers.

2. Method
When solving the problem, special attention is paid to the interaction of the structure with the environment. In this case, the structure is considered as a source of waves radiated into space and satisfying the conditions of healing at infinity
The differential equation of motion of a cylindrical rod located in an elastic medium has the form

\[ EJ \frac{d^4 W^*}{dz^4} + \rho_0 F_0 \frac{d^2 W^*}{dt^2} + q = P(z, t), \]

here, \( W^* \) - rod movement
\[ W^*(r, \theta, z, t) = W(r, \theta, z)e^{-i\omega t}, \]
\( E, \rho_0, r_0, F_0 \) - respectively bending stiffness, specific gravity, radius and cross-sectional area of the bar;
\( q \) - the resistance force of the medium to the movement of the rod, having the form
\[ q = r_0^2 \alpha \sigma \cos \theta r_0 d\theta + \frac{d}{dz} \left[ 2r_0 \int_{-\pi/2}^{\pi/2} r \sigma \cos \theta r_0 d\theta \right], \]
\( \sigma, \tau_{rz} \) - normal and shear stress in the medium

![Figure 1. Design scheme](image)

The load acting on the rod is presented as
\[ P(z, t) = \frac{P_0 \alpha_m}{zL^{2m-1}} \sin \alpha_m z e^{-i\omega t}, \alpha_m = \frac{\pi 2m-1}{2L}. \]

Components of displacement of points of an elastic medium
\[ u_r(r, \theta, z, t) = u_r(r, \theta, z)e^{-i\omega t}, \]
\[ u_\theta(r, \theta, z, t) = u_\theta(r, \theta, z)e^{-i\omega t}, \]
\[ u_z(r, \theta, z, t) = u_z(r, \theta, z)e^{-i\omega t}, \]
express in terms of elastic potentials
\[ u_r = \frac{d\phi}{dr} + \frac{1}{r} \frac{d\psi_1}{d\theta} + \frac{1}{r} \frac{d^2 \psi_2}{d\theta dz}, \]
\[ u_\theta = \frac{1}{r} \frac{d\phi}{d\theta} - \frac{d\psi_1}{dr} + \frac{1}{r} \frac{d^2 \psi_2}{dr dz}, \]
\[ u_z = \frac{d\phi}{dz} + k^2 \psi_2 + \frac{d^2 \psi_2}{dz^2}. \]

We represent elastic potentials in the form
\( \phi = \sum_{m=1}^{\infty} A_m H_1^1(k_1r) \cos \theta \sin \alpha m z, \)
\( \Psi_1 = \sum_{m=1}^{\infty} B_m H_1^1(k_2r) \sin \theta \sin \alpha m z, \)
\( \Psi_2 = \sum_{m=1}^{\infty} C_m H_1^1(k_2r) \cos \theta \cos \alpha m z. \)  
(7)

Here, \( A_m, B_m, C_m \) - unknown coefficients determined from the boundary conditions;
\( H_1^1(z) \) - Hankel function of 1-st kind 1-st order;
\( k_1^2 = k_1^2 - \alpha_m^2, k_1^2 = k_2^2 - \alpha_m^2, k_2^2 = k_2^2 - \alpha_m^2 = \frac{\omega_1}{c_1} \)
(8)

The displacements of the elastic medium, taking into account expressions (7), can be written in the following form
\[
\begin{align*}
    u_r &= \sum_{m=1}^{\infty} \left\{ \frac{1}{k_1 r} A_m H_1^1(k_1 r) + B_m H_1^1(k_2 r) + \frac{1}{k_1 r} C_m \alpha m H_1^1(k_2 r) \right\} \cos \theta \sin \alpha m z, \\
    u_\theta &= \sum_{m=1}^{\infty} \left\{ \frac{1}{k_1 r} A_m H_1^1(k_1 r) + B_m H_1^1(k_2 r) + \frac{1}{k_1 r} C_m \alpha m H_1^1(k_2 r) \right\} \sin \theta \sin \alpha m z, \\
    u_z &= \sum_{m=1}^{\infty} \{ A_m H_1^1(k_1 r) + C_m (k_2^2 - \alpha_m^2) H_1^1(k_2 r) \} \cos \theta \cos \alpha m z, 
\end{align*}
\]
(9)

The prime in the expressions for the Hankel function means differentiation with respect to \( r \).

Stresses in an elastic medium are determined by the formulas of the theory of elasticity
\[
\begin{align*}
    \sigma_r &= 2\mu \frac{du_r}{dr} + \lambda \left( \frac{du_r}{dr} + \frac{du_\theta}{dr} + \frac{du_z}{r \, d\theta} + \frac{u_z}{r} \right), \\
    \tau_r \theta &= \mu \left( \frac{1}{r} \frac{du_\theta}{dr} + \frac{du_r}{dr} - \frac{u_\theta}{r} \right), \\
    \tau_r z &= \mu \left( \frac{du_z}{dr} + \frac{du_\theta}{dr} \right). 
\end{align*}
\]
(10)

The unknown coefficients \( A_m, B_m, C_m \) are determined from the following boundary conditions.

The first boundary condition is the equality to zero of one of the components of the medium displacement at the contact boundary with the rod, i.e. at \( r = r_0 \).
\[
u_z = (r_0, \theta, z) = 0. 
\]
(11)

The second boundary condition follows from the assumption that the shear stress is zero, that is, at \( r = r_0 \).
\[
\tau_r \theta = (r_0, \theta, z) = 0. 
\]
(12)

From conditions (11) and (12) the coefficients \( A_m, B_m, C_m \) are determined, and to determine the coefficient \( C_m \) we use the equation of motion of the rod (1).

Assuming the cross-section of the bar to be non-deformable, the expression for the transverse displacements of the bar is represented in the following form
\[
W = \frac{1}{2} \left( \frac{u_r}{\cos \theta} - \frac{u_\theta}{\sin \theta} \right). 
\]
(13)

Taking into account expressions (9) and carrying out some transformations, we obtain an expression for the transverse displacements of the bar in the form
\[
W = \frac{r_0}{2\pi \mu (1 - \delta^2)} \sum_{m=1}^{\infty} \Delta_1 \sin \alpha m z / \left[ \frac{(\eta^2 \pi^4) (1 + \delta^2)(2m - 1)^4 - 1}{\eta^2 \pi^4} \right] + \frac{r_0^2 \delta^2 (2m - 1)^2 \pi^2 \eta^2}{\pi^2 \delta^2 (1 - \delta^2)}, 
\]
(14)

Here, \( \Delta_1, \Delta_2, \Delta_3 \) - constants obtained from the boundary conditions and determined in terms of the Hankel functions;
\[
\alpha = kr_0, \quad R_C = \frac{k_2}{k_1}, \quad R_v = \frac{k}{k_1}, \quad \delta = \frac{r_0}{r_1}. 
\]
3. Results and Discussions

The expression for the transverse displacements of the rod (14) includes the constants $\eta, R_v, \Omega$, which are the physical and mechanical properties of the soil-structure system. To analyze the effect of these constants on the amplitude-frequency characteristics (AFC) of the system, the expression of the transverse displacements of the rod (14) is presented as a function of the dimensionless frequency parameter $\alpha$ and in the range $0 \leq \alpha \leq 8$, we will calculate the frequency response. The arguments of the Hankel function, depending on the values of $\alpha, R_v, \eta$, can take imaginary values. In this case, the Hankel function $H_n^1(iz)$ goes over the Macdonald function and the following dependence is used

$$K_\vartheta(z) = \frac{\pi i}{2} e^{\frac{\vartheta \pi i}{2}} H^1_\vartheta \left( z e^{\frac{\pi i}{2}} \right).$$

To determine the frequencies of natural vibrations of the structure, we use the following frequency equation

$$\left[ \frac{\tau (2m-1)^4 \Omega}{\alpha^2 + (2m-1)^2 \pi^2 \eta^2} \right] \Delta_2 + \frac{4}{R_v^2} \left[ \frac{(\pi \varrho^2-2)\Delta_2 - \Delta_3 (2m-1)^2 \pi^2 \eta^2}{4 \alpha^2 + (2m-1)^2 \pi^2 \eta^2} \right] = 0.$$  

When determining the frequencies at which infinite resonant displacements are obtained, it is necessary to consider the cases when (16) will have real roots. In other cases, the structure vibrates with a small amplitude over the entire range of the impact frequency. This is observed under the condition $R_v > 1$.

4. Discussion

The frequency response was calculated for the following values: Poisson’s ratio - 0.25, relative pipeline thickness - $\delta = 0.95$. The calculations were carried out for typical combinations of quantities: $\Omega = 5$, $\eta = 0.01; 0.05; 0.10$; $R_v = 0.5; 1; 3$, which reflect the effect of changes in the properties of the “soil-structure” system on the oscillation process. Frequency response graphs are shown in Fig. 2-6. As the AFC plots show, the nature of the structure's oscillations depends on the parameter $\eta$, which characterizes the flexibility of the pipeline. In this case, it turns out that for flexible structures ($\eta = 0.01$) strong resonant oscillations occur only in the region of low frequencies (Fig. 2a), therefore, one can restrict oneself to considering the first resonance. At $\eta = 0.05$, the spectrum is denser (Fig. 2b). In all cases, the most dangerous is the first (main) resonance zone, since it has a fairly wide range in the frequency domain and the resonance peak is quite high. The amplitudes of oscillations outside the resonance zone are small, and, as a rule, are less dangerous for the structure.

Figure 2. AFC of the rod. $R_v = 0.5$; a - $\eta = 0.01$; b - $\eta = 0.05$. 
Figure 3. AFC of the rod. $R_v = 0.5; \eta = 0.1$.

Figure 4. AFC of the rod. $R_v = 1; \eta = 0.01$

Figure 5. AFC of the rod. $R_v = 1; \alpha - \eta = 0.1; \eta = 0.05$. 
Figure 6. AFC of the rod. $R_v = 3; a - \eta = 0,01; b - \eta = 0,1.$

The nature of pipeline vibrations at $\eta = 0.10$ differs from the previous cases in that the frequency spectrum is rather rarefied, but the displacement amplitudes are insignificant (Figure 3). With an increase in the rigidity of the soil $R_v$, the resonant frequencies shift towards high frequencies (Figure 2 and Figure 4), and resonances occur in a wider frequency range.

The above studies of the lateral vibrations of an extended pipeline showed that the most unfavorable state for the construction is the first (main) mode of vibration. In this case, the first resonant zone has a fairly wide range in the frequency domain and the resonant amplitude is quite high compared to the peaks in other regions. The oscillation amplitudes outside the resonance zone are relatively small and, as a rule, are less dangerous for the structure. Changes in soil characteristics greatly affect the nature of vibrations. An increase in the rigidity of the environment leads to a shift of the resonant frequencies to the high frequency zone and leads to an expansion of the resonance region.

5. Conclusions
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