Abstract

All known string theory models may be obtained as partial fermionization, projection and background Ansätze from the original, purely bosonic string theory. The latter theory in turn has been recently shown to describe a chirally and non-commutatively doubled and manifestly T-dual target spacetime. We show herein that this, so-called metastring theory automatically includes mirror symmetry.

Introduction:
A closer look at the underlying structures in string theory shows that the familiar and (almost) independent left- and right-moving degrees of freedom span a chirally doubled phase space-like non-commutative and modular target spacetime $\mathcal{M}$ [1,2]. This so-called Born geometry is specified by (i) the symplectic structure $\omega$, (ii) the bi-orthogonal metric $\eta$, and (iii) the doubly Lorentzian metric $H$. More precisely, this metastring target spacetime, $\mathcal{M}$, is an almost symplectic and para-Hermitian manifold $\mathcal{M}$ that has a compatible foliation [3]: Locally at every point, $p$, $\mathcal{M}$ is of the form $\mathcal{M}_p = M_p \times \tilde{M}_p$, with local coordinates $(x^a, \tilde{x}_b)_p \in M_p \times \tilde{M}_p$, where $x^a := x^a_L + x^a_R$ and $\tilde{x}^a := x^a_L - x^a_R$ are combinations of the zero-modes of the corresponding worldsheet scalar fields. For local diffeomorphisms (implemented by the Dorfman generalization of the Lie derivative) to be integrable to finite translations, the so-called “section condition” is imposed, which halves the spacetime akin to the quantum-mechanical restriction of the classical phase space, $(p, q)$, in the coordinate or momentum representation — or indeed any other $\pi := (\alpha p + \beta q)$-polarization, as familiar from Geometric Quantization program [4,5].

Mirror Doubling:
A hallmark of the above structure is that $\pi[T_{\mathcal{M}}] = (T \oplus T^*)_{\pi[\mathcal{M}]}$: a polarization of (the total space of) the tangent bundle on $\mathcal{M}$ is the generalized/doubled tangent bundle on that polarization of $\mathcal{M}$. In local coordinates of the $\pi_x$-polarization, $\pi_x[\mathcal{M}] = M_x$, elements of $T_x\mathcal{M}$ are given as $v^a(x, \tilde{x}) \partial_a + w_a(x, \tilde{x}) dx^a$, as $\tilde{x}$ are locally constant ($\tilde{\partial}^a, d\tilde{x}_a \rightarrow 0$) [3]. In turn, the same element is $v^a(x, \tilde{x}) d\tilde{x}_a + w_a(x, \tilde{x}) \tilde{\partial}^a$ in the $\pi_{\tilde{x}}$-polarization, $\pi_{\tilde{x}}[\mathcal{M}] = \tilde{M}_{\tilde{x}}$, where $x$ are locally constant and $\partial_a, dx^a \rightarrow 0$. Therefore, swapping the polarization $M_x \leftrightarrow \tilde{M}_{\tilde{x}}$ explicitly identifies $T_{M_x} = T^*_{\tilde{M}_{\tilde{x}}}$ and $T_{\tilde{M}_{\tilde{x}}} = T_{M_x}$ — which is the underlying premise of mirror symmetry. Displacing $M$ and $\tilde{M}$ for clarity, these relationships are sketched

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1 Such spaces admit a signature-$(d,d)$ metric $\eta$ and a symplectic structure $\omega$ such that $K := \eta \omega$ is an almost product structure, $K^2 = +1$. This $\eta$ need not be flat, and almost means that $\omega$ need not be a closed
Simple pos. roots of $\tilde{M}$ and $T_{\tilde{M}}$, and both depicted together in Figure 1, and corroborate the argument presented in [6, \S 5.1]. The Born geometry of $\mathcal{M}$ explicitly includes mirror symmetry by virtue of having to polarize on one half of $\mathcal{M}$, such that polarizing on the complementary half swaps the corresponding tangent and cotangent spaces. This provides a non-commutative version of the generalized Calabi-Yau space framework [7] for the entire spacetime, resonating with the recent proposal [8].

**Born Heterosis:** The metasting intrinsic mirror symmetry applies also to an adaptation of the heterotic Ansatz [9], which relies on a partial fermionization and supersymmetry: On the worldsheet, each chiral boson maps non-locally and nonlinearly but exactly 1–1 to a (chiral) fermion [10, 11]; also [12–15]. A suitable superconformal action for the resulting collection of worldsheet scalar and fermion fields then induces supersymmetry in the target space, leading to the familiar result that all the various superstring models may be obtained from the bosonic string [16], all of which eliminate the tachyonic instability of the purely bosonic string; see also [17].

Re-bosonizing and adapting the original heterotic Ansatz [9,16], we partition the 26+26 chiral bosons of the bosonic (meta)string as follows, by identifying $x^0, x^1, \tilde{x}^0, \tilde{x}^1$ in the light-cone gauge with the worldsheet (chirally doubled) coordinates:

\[
\begin{array}{c|c|c|c}
\text{WS} & \text{light-cone } x\text{-coordinates on } M & \text{light-cone } T_M = T_M^*\text{-fiber} & \text{Simple pos. roots of } E_8 \\
\hline
x^0, x^1 & x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9 & x^{10}, x^{11}, x^{12}, x^{13}, x^{14}, x^{15}, x^{16}, x^{17} & x^{18}, x^{19}, x^{20}, x^{21}, x^{22}, x^{23}, x^{24}, x^{25} \\
\hline
\tilde{x}^0, \tilde{x}^1 & \tilde{x}^2, \tilde{x}^3, \tilde{x}^4, \tilde{x}^5, \tilde{x}^6, \tilde{x}^7, \tilde{x}^8, \tilde{x}^9 & \tilde{x}^{10}, \tilde{x}^{11}, \tilde{x}^{12}, \tilde{x}^{13}, \tilde{x}^{14}, \tilde{x}^{15}, \tilde{x}^{16}, \tilde{x}^{17} & \tilde{x}^{18}, \tilde{x}^{19}, \tilde{x}^{20}, \tilde{x}^{21}, \tilde{x}^{22}, \tilde{x}^{23}, \tilde{x}^{24}, \tilde{x}^{25} \\
\end{array}
\]

For $a = 10, \ldots, 17$, the chiral bosons $x^a, \tilde{x}^a$ are the preimages of 8+8 fermions, $\psi^a, \tilde{\psi}^a$, that are induced by supersymmetry to span the indicated (co)tangent bundles. The worldsheet action for these $x^a, \tilde{x}^a$ must reflect the re-bosonization of the supersymmetric structure of the action involving $\psi^a, \tilde{\psi}^a$. In turn, for $a = 18, \ldots, 25$, $x^a, \tilde{x}^a$ are compactified on two copies of the $\mathbb{R}^8/L(E_8)$ torus. Their dynamics is the (nonlocal!) re-bosonization of the structure specified for their fermionized counterparts, $x^a \mapsto \lambda^a$ and $\tilde{x}^a \mapsto \tilde{\lambda}^a$, adapting the standard description [9,16]. The worldsheet actions are indeed much simpler (and local!) in the fermionized picture of $x^a, \tilde{x}^a$ for $a > 9$; the re-bosonized specification given here however highlights the implications of the metasting Born geometry.

The non-commutative metastring structure of $\mathcal{M}$ then has the following immediate implications: 

2-form. Owing to the almost symplectic structure, the existence of the corresponding flat (Bott) connection guarantees that a foliated space is everywhere locally a product of two half-dimensional affine spaces.
1. The metastring metric, $\eta_{AB}|_{10\cdots 17}$, reproduces the canonical fiber-wise pairing of the two standard bundles: $\langle T_M, T_M^*\rangle = (T_8^*, T_8) = 1$.

2. The metastring metric, $\eta_{AB}|_{18\cdots 25}$, implies a dual pairing between the $E_8$ ’s simple positive roots and those of $\tilde{E}_8$. This implies that the roots of $E_8$ are $\eta$-canonically reciprocal to those of $E_8$ — which identifies $E_8$ as the Langlands dual, $E_8'$ [18], i.e., the electromagnetic dual [19, 20]! Being simply laced, $E_8' \approx E_8$, and they are physically indistinguishable. The $E_8$-group elements are exponential functionals of the $\{x^{18}, \cdots, x^{25}\}$, which can be arranged to commute [1, 2] with the $\tilde{E}_8$-group exponential functionals of the $\{\tilde{x}^{18}, \cdots, \tilde{x}^{25}\}$, realizing the $E_8 \times \tilde{E}_8$ Yang-Mills gauge group as expected.

3. For $a = 2, \cdots, 9$, the diagram (1) identifies $x^a, \tilde{x}^a$ with the light-cone base coordinates, $x^{a+8} \mapsto dx^a = \delta^a$ and $\tilde{x}^{a+8} \mapsto \partial_a = d\tilde{x}_a$ with the light-cone fibre coordinates (as befits the preimages of fermions typically spanning the tangent and cotangent bundles), and $(x^{a+16}, \tilde{x}^{a+16}) \mapsto (\rho_a, \rho_a)$ with the simple positive roots. Then:

   (a) All target spacetime fields are a priori bi-local: they depend on both $x^\mu$ and $\tilde{x}_\mu$.
   (b) They are valued as follows: $A_\mu^a$ in $T_M^* \times E_8$ and $A^{a\alpha}$ in $T_M \times \tilde{E}_8$, the latter of which define $A_\mu^\alpha := H_{\mu\nu} A^{\nu\alpha}$ in $T_M^b \times \tilde{E}_8$ using the metastring $H_{AB}$-metric.

   These originally bi-local fields then give rise to both an $E_8$-gauge field plus the dual (“dark”) field valued in the same $E_8$ algebra. With the analogous for the $\tilde{E}_8$-valued gauge fields, we have both $A_\mu^a(x), A_\mu^\alpha(x)$ and their duals, $\tilde{A}_\mu^a(\tilde{x}), \tilde{A}_\mu^\alpha(\tilde{x})$.

4. The metastring non-commutativity implies the correlation, $\frac{x^2}{L^2} \int_x \mathcal{F}^{\mu\nu}[A_\mu^a(x), A_\nu^\beta(\tilde{x})] \eta_{a\beta}$, between the visible $E_8$-fields and the dark $\tilde{E}_8$-fields, as well as the analogous correlation term with $E_8 \leftrightarrow \tilde{E}_8$ swapped [21].

The above statements in 3 insures that our Ansatz (1) reproduces the standard gauge field content of the heterotic string. The statements 1 and 2 however show that the Born geometry is not only perfectly aligned with the standard geometry and dynamics in the heterotic string theory, but additionally links the two copies of $E_8$ as each other’s Langlands dual. Finally the correlation 4 is a straightforward but novel result [21].

**Outlook:** In conclusion, we comment on various ramifications of this new view of mirror symmetry. Our discussion of mirror symmetry and its relation to Born geometry of an intrinsically non-commutative and T-duality covariant formulation of string theory can be naturally related to the old observation that T-duality is deeply related to mirror symmetry [22]. In that case, as in our discussion, one should pay close attention to the issues of the local versus global formulation of mirror symmetry as T-duality. In particular, our phase space treatment of string theory, and mirror symmetry, as encapsulated in Born geometry, naturally relates to homological mirror symmetry involving the (derived) Fukaya categories (symplectic structure) on one side and the (derived) category of coherent sheaves (complex structure) [23]. Also, the world-sheet formulation of metastring theory should be related to the topological A and B model projections [24], so that these appear as gauge fixed
versions of the metastring formulation that explicitly incorporates Born geometry. Finally, we would like to understand non-perturbative aspects of this approach in the context of a purely bosonic but non-commutative (and, in principle, non-associative) matrix-model like formulation; see [21].

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