Hall magnetoresistivity response under Microwave excitation revisited

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(Dated: February 1, 2008)

We theoretically analyzed the microwave-induced modification of the Hall magnetoresistivity in high mobility two-dimensional electron systems. These systems present diagonal magnetoresistivity oscillations and zero-resistance states when are subjected to microwave radiation. The most surprising modification of the Hall magnetoresistivity is a periodic reduction which correlates with a periodic increase in the diagonal resistivity. We present a model that explains the experimental results considering that radiation affects directly only the diagonal resistivity and the observed Hall resistivity changes are coming from the tensor relationship between both of them.

PACS numbers:
Microwave-Induced Resistivity Oscillations (MIRO)\textsuperscript{1,2} and Zero Resistance States (ZRS)\textsuperscript{3,4} are some of the most striking physical phenomena recently discovered in the field of condensed matter physics. MIRO and ZRS are produced in the diagonal resistivity ($\rho_{xx}$) of a two-dimensional electrons system (2DES) when is subjected simultaneously to a static and moderate magnetic field ($B$) and Microwave (MW) radiation. A very intense activity is being developed on this topic and recent experimental\textsuperscript{2,3,4,5,6,7,8,9,10,11}, and theoretical\textsuperscript{12,13,14,15,16,17,18,19,20,21} contributions are being published in a continuous basis. Some theoretical contributions have been presented giving explanation for some of the emerging experimental outcomes\textsuperscript{22,23,24,25,26,27,28}. However no much attention has been paid to the study of the influence of MW radiation on the Hall magnetoresistivity ($\rho_{xy}$). On the one hand, experimental results has been obtained\textsuperscript{29,30} showing that $\rho_{xy}$ indeed present some remarkable features as, unexpected oscillations which are in anti-phase with $\rho_{xx}$ oscillations, the experimental curve presents an average negative slope vs $B$, and finally, the MW-induced correction to $\rho_{xy}$ tends to vanish in ZRS regions. On the other hand, almost no theoretical effort, with some exception\textsuperscript{31}, has been presented to date to explain this striking results on $\rho_{xy}$.

In this letter we propose a theoretical explanation on the $\rho_{xy}$ response obtained under MW excitation. We consider that radiation affects directly just $\rho_{xx}$ and the observed $\rho_{xy}$ changes are coming only from the tensor relationship between both of them. Thus, following our approach, MW-induced $\rho_{xx}$ response needs first to be studied. In a recently presented model by the author\textsuperscript{19,28}, it was demonstrated that when a 2DES is subjected to a perpendicular $B$ and MW radiation, the electronic Larmor orbit centers oscillate back and forth in the $x$ direction (current direction) with the same frequency as the MW field: \textit{MW driven Larmor orbits}. An important extension of this model\textsuperscript{19,28} is presented here which allows to consider linear polarized MW radiation with the electric field vector oriented in any direction of the $x - y$ two-dimensional (2D) plane. For that purpose, symmetric gauge has been introduced to represent the vector potential of $B$: $\vec{A}_B = -\frac{1}{2} \vec{r} \times \vec{B}$. We first obtain the \textit{exact} expression of the electronic wave vector for a 2DES.
in a perpendicular $B$ and MW radiation$^{19,28}$:

$$\Psi(x, y, t) = \phi_N [(x - X - a(t)), (y - b(t)), t]$$

$$\times \exp \frac{i}{\hbar} \left[ m^* \left( \frac{da(t)}{dt} x + \frac{db(t)}{dt} y \right) + \frac{m^* w_c [b(t)x - a(t)y]}{2} - \int_0^t L dt' \right] \sum_{p=-\infty}^\infty J_p(A_N) e^{ip\omega t}$$  \tag{1}$$

where $\phi_N$ are analytical solutions for the Schrödinger equation with a 2D parabolic confinement, known as Fock-Darwin states$^{32}$. Fock-Darwin states converge to a Landau level spectrum when $B$ is large enough or it is the only source of confinement (present case). $X$ is the center of the orbit for the electron spiral motion, $w$ is the frequency of the MW field and $w_c$ the cyclotron frequency. $L$ is the classical lagrangian, and $J_p$ are Bessel functions$^{19,28}$. $a(t)$ for the x-coordinate and $b(t)$ for the y-coordinate, are the classical solutions for a driven 2D harmonic oscillator whose expressions depend on the direction of the linearly polarized MW field. If the polarization is aligned with the current (x-direction), i.e., the harmonic driving force is acting only in the x-direction, then the expressions are:

$$a(t) = \frac{e E_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos wt = A_1 \cos wt$$

$$b(t) = \frac{e E_o w_c}{m^* \sqrt{(w_c^2 - w^2)^2 + w_c^2 \gamma^4}} \sin wt = A_2 \sin wt$$  \tag{2}$$

$\gamma$ is a material and sample dependent damping parameter which affects dramatically the MW-driven electronic orbits movement and which has been introduced phenomenologically. Along with this movement there occur interactions between electrons and lattice ions yielding acoustic phonons and producing a damping effect in the electronic motion. In ref [21], we introduced a microscopical model to calculate $\gamma$ obtaining a numerical value of $\gamma \approx 10^{12} s^{-1}$ for GaAs. $E_o$ is the amplitude of the MW field. In the case of a MW field aligned with the y-direction (the harmonic driving force is acting in the y-direction): $a(t) = A_2 \cos wt$ and $b(t) = A_1 \sin wt$.

The first important result is that, apart from phase factors, the exact wave function, $\Psi(x, y, t)$, is the same as a Fock-Darwin state$^{32}$ where the center of the orbits performs and elliptical motion given by $\frac{a^2}{A_1^2} + \frac{b^2}{A_2^2} = 1$ for x-linearly polarized MW. In the case of MW polarized in the y direction, the elliptical motion is according to $\frac{a^2}{A_2^2} + \frac{b^2}{A_1^2} = 1$. When
$w_c = w$ the motion becomes circular for both cases. Another important outcome from our model is that irrespective of the linear polarization direction, the elliptical motion for the orbit center is reflected in the current direction as oscillatory with the same frequency as the MW field. This MW-induced oscillatory motion will affect dramatically the way in which electrons in their orbits interacts with scatterers compared to the dark case. Thus, we introduce the scattering suffered by the electrons due to charged impurities randomly distributed in the sample. Following the model described in [19], firstly we calculate the electron-charged impurity scattering rate $1/\tau$ (being $\tau$ the scattering time). Secondly we find the average effective distance advanced by the electron in every scattering jump, $\Delta X^{MW} = \Delta X^0 + A_{1,2} \cos w\tau$, where $\Delta X^0$ is the effective distance advanced when there is no MW field present. The magnitude $A_{1,2}$ is the amplitude of the oscillatory motion in the current direction, depending on the orientation of the MW linear polarization: subindex 1 corresponds to $x$ and 2 to $y$. Finally the diagonal or longitudinal conductivity $\sigma_{xx}^{MW}$ can be calculated: $\sigma_{xx}^{MW} \propto \int dE \frac{\Delta X^{MW}}{\tau} (f_i - f_f)$, being $f_i$ and $f_f$ the corresponding electron distribution functions for the initial and final states respectively and $E$ energy. To obtain $\rho_{xx}$ we use the well-known tensor relation $\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$, where $\sigma_{xy} \approx \frac{n_i e B}{2}$, being $n_i$ the impurity density.

In Fig. 1, we represent in two panels calculated $\rho_{xx}$, $\rho_{xy}$ and $\Delta \rho_{xy} = \rho_{xy}^{MW} - \rho_{xy}^{dark}$ vs $B$ for $w = 50\,GH\,z$. In the top panel, $\rho_{xx}$ with and without MW on the left x-axis and $\rho_{xy}$ on the right one. It can be observed clearly the typical MIRO and ZRS in $\rho_{xx}$ and the linear dependence of $\rho_{xy}$ with $B$. In the bottom panel we present $\rho_{xx}$ with MW on the left x-axis and $\Delta \rho_{xy}$ on the right x-axis. For all cases x-linearly polarized MW has been used. There appears to be an oscillatory variation in $\Delta \rho_{xy}$ where a reduction in magnitude correlates with an increase in $\rho_{xx}$. It is remarkably also that the calculated $\Delta \rho_{xy}$ curve presents an average negative slope vs $B$. Finally it is demonstrated that the MW-induced correction to the Hall resistivity, disappears as $\rho_{xx} \to 0$. In other words, the plot illustrates similar features as in experiments.

In Fig. 2, we represent the same as in Fig.1, with the exception of $\rho_{xy}$ vs $B$ which is not represented, and for $w = 100\,GH\,z$. A similar behavior is obtained. This striking
behavior for $\Delta \rho_{xy}$ can be readily explain observing carefully its developed expression:

$$
\Delta \rho_{xy} = \rho_{xy}^{MW} - \rho_{xy}^{dark} = \frac{\sigma_{xy}}{(\sigma_{xx}^{MW})^2 + \sigma_{xy}^2} - \frac{\sigma_{xy}}{(\sigma_{xx}^{dark})^2 + \sigma_{xy}^2}
$$

$$
\simeq \left[ \frac{B}{n_i e} \right]^3 \left[ (\sigma_{xx}^{dark})^2 - (\sigma_{xx}^{MW})^2 \right]
$$

(3)

where we have taken into account that $(\sigma_{xx}^{dark}, \sigma_{xx}^{MW}) \ll \sigma_{xy}$. Following our model, we have considered for MW and dark cases the same expression for $\sigma_{xy}$: $\sigma_{xy} \simeq \frac{n_i e}{B}$. Thus, we propose that the full MW effect on $\rho_{xy}$ is coming only from $\sigma_{xx}^{MW}$ through the tensor relationship and that $\sigma_{xy}$ is unaffected by the MW field. Considering that, in the range of moderate $B$ we are working with, $\sigma_{xx}^{dark}$ is practically constant (see, for instance, top panel of Fig. 1), the important features of $\Delta \rho_{xy}$ are going to depend mainly on the term $[-(\sigma_{xx}^{MW})^2]$. This would explain that the corresponding oscillations of $\sigma_{xx}^{MW}$ would be reflected as anti-phase oscillations of $\Delta \rho_{xy}$. MW-induced increases (decreases) in $\sigma_{xx}^{MW}$ will produce decreases (increases) in $\Delta \rho_{xy}$. In the same way, $\Delta \rho_{xy}$ behaves as an oscillating curve around an average straight line with negative slope as a function of $B$. Remember that for moderate values of $B$, $-B^3 \rightarrow -B$, i.e., a straight line of negative slope. This can be clearly observed in Figs. 1 and 2, (see dashed-dotted line in bottom panels, blue color on line). Finally, when $\sigma_{xx}^{MW} \rightarrow 0$, (ZRS region) we will obtain $\Delta \rho_{xy} \rightarrow \left[ \frac{B}{n_i e} \right]^3 \left[ (\sigma_{xx}^{dark})^2 \right]$. We have calculated, using experimental parameters, that $\sigma_{xx}^{dark}$ has an average value of $\sigma_{xx}^{dark} \simeq 5 \times 10^{-6} \Omega^{-1}$. Then for an average $B$, we can estimate that $\Delta \rho_{xy} \simeq 0.02 \Omega$, which is very small. Similar behavior for all $B$ range has been found. Therefore $\Delta \rho_{xy}$ obtains a very small value in ZRS regions. Thus, it appears as if $\Delta \rho_{xy}$ would tend to zero. Again this is in good agreement with experiments.

This work has been supported by the MCYT (Spain) under grant MAT2005-06444, by the Ramón y Cajal program and by the EU Human Potential Programme: HPRN-CT-2000-00144.
M.A. Zudov, R.R. Du, J.A. Simmons and J.L. Reno, Phys. Rev. B 64, 201311 (2001).

S.A. Studenikin, M. Potemski, A. Sachrajda, M. Hilke, L.N. Pfeiffer, K.W. West, Phys. Rev. B, 71, 245313, (2005); S.A. Studenikin, M. Potemski, P.T. Coleridge, A. Sachrajda, Z.R. Wasilewski, Solid State Comm 129, 341 (2004).

R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson, V. Umansky, Nature 420 646 (2002).

M.A. Zudov, R.R. Du, N. Pfeiffer, K.W. West, Phys. Rev. Lett. 90 046807 (2003).

R.L. Willett, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 93 026804 (2004).

R.G. Mani, Appl. Phys. Lett. 85, 4962, (2004); R.G. Mani, Physica E, 22, 1, (2004).

M.A. Zudov, R.R. Du, N. Pfeiffer, K.W. West, Phys. B 9073 041303 (2006).

C.L. Yang, R.R. Du, L.N. Pfeiffer and K.W. West, 74, 045315, (2006).

R.G. Mani, Phys. Rev. B, 72 075327, (2005)

M.A. Zudov, R.R. Du, N. Pfeiffer, K.W. West, Phys. Rev. Lett. 96, 236804 (2006).

J.H.Smet, B. Gorshunov, C.Jiang, L.Pfeiffer, K.West, V. Umansky, M. Dressel, R. Dressel, R. Meisels, F.Kuchar, and K.von Klitzing, Phys. Rev. Lett. 95, 116804 (2005).

A.C. Durst, S. Sachdev, N. Read, S.M. Girvin, Phys. Rev. Lett.91 086803 (2003)

C.Joas, J.Dietel and F. von Oppen, Phys. Rev. B 72, 165323, (2005).

X.L. Lei, S.Y. Liu, Phys. Rev. Lett.91, 226805 (2003);

V. Ryzhii and V. Vyurkov, Phys. Rev. B 68 165406 (2003); V. Ryzhii, Phys. Rev. B 68 193402 (2003).

P.H. Rivera and P.A. Schulz, Phys. Rev. B 70 075314 (2004)

Junren Shi and X.C. Xie, Phys. Rev. Lett. 91, 086801 (2003).

A.V. Andreev, I.L. Aleiner and A.J. Millis, Phys. Rev. Lett. 91, 056803 (2003)

J. Íñarrea and G. Platero, Phys. Rev. Lett. 94 016806, (2005)

I.A. Dimitriev, M.G. Vavilov, I.L. Aleiner, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B, 71, 115316, (2005)

Assa Auerbach and G. Venketeswara, cond-mat/0612469
22 J. Iñarrea and G. Platero, Phys. Rev. B 72, 193414 (2005).
23 X.L. Lei, S.Y. Liu, Phys. Rev. B 72, 075345 (2005).
24 J. Iñarrea and G. Platero, Appl. Phys. Lett. 89, 052109, (2006).
25 Kang-Hun Ahn, J. Korean Phys. Soc., 47 (4), 666-672, (2005).
26 J. Iñarrea and G. Platero, Appl. Phys. Lett. 89, 172114, (2006).
27 X.L. Lei, Phys. Rev. B 73, 235322, (2006)
28 J. Iñarrea and G. Platero, cond-mat/0612429
29 R.G. Mani, V. Narayanamurti, K. von Klitzing, J.H. Smet, W.B. Johnson, V. Umansky, Phys. Rev. B 69, 161306(R) (2004)
30 S.A. Studeniking, M. Potemski, P.T. Coleridge, A. Sachrajda and Z.R. Wasilewski, Solid. State. Comm. 129, 341, (2004).
31 V. Ryzhii J. Phys. Soc. Jpn. 73, 1539, (2004).
32 V. Fock, Z. Phys. 47, 466 (1928); C.G. Darwin, Proc. Cambridge Philos. Soc. 27, 86, (1930).
FIG. 1:

Figure 1 caption: Top panel: on the left x-axis $\rho_{xx}$ with MW (single line, black color online) and without MW (dotted line, blue color online), and on the right x-axis $\rho_{xy}$ with MW (dashed line, red color online). Bottom panel: $\rho_{xx}$ with MW (single line, black color on line) on the left x-axis and $\Delta \rho_{xy} = \rho_{xy}(MW) - \rho_{xy}(dark)$, (dotted line, red color online) on the right x-axis. All as a function of $B$ and for $w = 50GHz$. MW is x-linearly polarized for all cases. $T=1K$.

Figure 2 caption: Same as in Fig.1, (except $\rho_{xy}$ vs $B$) and for a MW frequency of $100GHz$. We obtain a similar qualitatively behavior of the different magnitudes versus $B$.
FIG. 2: