Determination of the scalar glueball mass in QCD sum rules

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Abstract

The $0^{++}$ glueball mass is analyzed in the QCD sum rules. We show that in order to determine the $0^{++}$ glueball mass by using the QCD sum rules method, it is necessary to clarify the following three ingredients: (1) to choose the appropriate moment with acceptable parameters which satisfy all of the criteria; (2) to take account into the radiative corrections; (3) to estimate an additional contribution to the glueball mass from the lowest lying $\bar{q}q$ resonance. We conclude that it is the key point to choose suitable moments to determine the $0^{++}$ glueball mass, the radiative corrections do not affect it sensitively and the composite resonance have a little effect on it.

1 Introduction

The self-interaction among gluons is a distinctive feature in the QCD theory. It may lead to build bound gluon states, glueballs. Thus discovering of the glueball will be a direct test to the QCD theory. Although there are several glueball candidates experimentally, there is no conclusive evidence on them. People recently pay particular attention to two scalar states: $f_0(1500)(J=0)$ [1] and $f_J(1710)$ $(J=0)$ [2], they seem like glueballs. However, the explicit analyses [3] on them reveal that neither of them appears to be a pure meson or a pure glueball. Most probably they are mixtures of glueball and $\bar{q}q$ meson.

The property of the glueball has been investigated in the lattice gauge theory and in many models based on the QCD theory. Even in the lattice gauge calculation, there are different predictions for the $0^{++}$ glueball [4][5][6]. Some years ago, the mass of the $0^{++}$ glueball was predicted around $700 - 900$ MeV. Recently, IBM group [4] predicts the lightest $0^{++}$ glueball mass: $(1710 \pm 63)$ MeV, and UK QCD group [5] gives the estimated mass: $(1625 \pm 92)$ MeV respectively. The improvement of determination of the $0^{++}$ glueball mass originates from the more accuracy of the lattice technique, however, at present the uncertainty still exists .

V. A. Novikov et al [7] first tried to estimate the scalar glueball mass by using QCD sum rules [8], but they only took the mass to be $700$ MeV by hand because of uncontrolled instanton contributions. Since then, P. Pascual and R. Tarrach [9], S. Narison [10] and J. Bordes et al [11] presented their calculation on the scalar glueball mass in the framework of QCD sum rules. They all got a lower mass prediction around $700 - 900$ MeV when they used
the moments $R_{-1}$ or $R_0$ and neglected the radiative corrections in their calculation of the correlators. E. Bagan and T. Steele [12] first took account of the radiative corrections in the correlator calculation. Choosing appropriate moments($R_0$ and $R_1$) for their calculation, they got a higher glueball mass prediction around 1.7 GeV. It seems that the radiative corrections make a big difference on the prediction of the scalar glueball mass. Obviously, there are some uncertainties in the determination of the scalar glueball mass, in order to give the reliable values in the QCD sum rules reasonably, an analysis of these uncertainties is necessary.

In this paper, we first give the criteria to choose the moments, which are obtained by the Borel transformation of the correlator weighted by different powers of $q^2$, according to application of QCD sum rules. It is important to choose suitable moments to determine the glueball mass[13]. From the criteria follows that different moment has different result, but not all of them are reliable. By choosing appropriate moment, we get the glueball mass without radiative corrections: 1.7 GeV. When the radiative corrections are included in, glueball mass shifts a little: $\sim 1.65$ GeV.

Secondly, we consider the effect of mixing between lowest-lying $0^{++}$ glueball and $\bar{q}q$ meson, i.e., the gluonic currents and quark currents couple both to glueball states and $\bar{q}q$ states. Therefore, there are some exotic form factors to be determined. By using the low-energy theorem, we can construct a sum rule for the mixing correlation function (one gluonic current and one quark current). Through these relationship and based on the assumption of two states (lowest-lying states of glueball and $\bar{q}q$ meson) dominance, we find the mass for $0^{++}$ glueball is around: 1.9 GeV, which is a little higher than the pure resonance prediction while the mass for $0^{++}$ meson is around: 1.0 GeV, which is a little lower than the pure resonance prediction.

The paper is organized as follows. In Sect. 2 a brief review about the calculation of the mass of physical state from QCD sum rules is given. In Sect. 3 we discuss the criteria of choosing the moments and the effect of the radiative corrections. The mixing effect of the glueball with the meson state is studied in Sect. 4. Finally, the last section is reserved for a summary.

## 2 QCD sum rules and moments

Let us consider the correlator

$$\Pi(q^2) = i \int e^{iqx} \langle 0 | T \{ j(x), j(0) \} | 0 \rangle dx, \quad (1)$$

where $j(x)$ is the current with definite quantum numbers.

In the deep Euclidean domain($-q^2 \rightarrow \infty$), it is suitable to carry out operator product expansion (OPE)

$$\Pi(q^2) = \sum_n C_n(q^2) O_n, \quad (2)$$

where the $C_n(q^2)$ are Wilson coefficients. Then, the correlator can be expressed in term of vacuum expectation values of the local operators $O_n$.

On the other hand, the imaginary part of $\Pi(q^2)$ in the Minkovski domain(at positive values of $q^2$), which is called the spectral density, is relevant with the physical observables.
Therefore, we can extract some information of the hadrons from QCD calculation by using the dispersion relation

\[
\Pi(q^2) = \frac{(q^2)^n}{\pi} \int \frac{Im \Pi(s)}{s^n(s-q^2)} ds + \sum_{k=0}^{n-1} a_k(q^2)^k,
\]

where \(a_k\) are some subtraction constants originated from the facial divergence of \(\Pi(q^2)\). In order to keep control of the convergence of the OPE series and enhance the contribution of the lowest lying resonance to the spectral density, the standard Borel transformation is used. However, in practice, it may be more convenient to use the moments \(R_k\) instead, which is defined by

\[
R_k(\tau, s_0) = \frac{1}{\pi} \hat{L}[(q^2)^k \{\Pi(Q^2) - \Pi(0)\}] - \frac{1}{\pi} \int_0^{+\infty} s^k e^{-s\tau} Im \Pi^{(pert)}(s) ds
\]

where \(\hat{L}\) is the Borel transformation and \(\tau\) is the Borel transformation parameter, \(s_0\) is the starting point of the continuum threshold. Using the higher rank moments, one can enhance the perturbative contribution and suppress resonance contribution. In the following, we will see the role of \(R_k\) in our analysis.

### 3 Criteria of choosing the moments

In this paper, the \(0^{++}\) gluonic current is defined as

\[
j(x) = \alpha_s G^a_{\mu\nu} G^a_{\mu\nu}(x),
\]

where \(G^a_{\mu\nu}\) in Eq.(3) stands for the gluon field strength tensor and \(\alpha_s\) is the quark-gluon coupling constant. The current \(j(x)\) is the gauge-invariant and non-renormalization(to two loops order) in pure QCD.

Through operator product expansion, the correlator without radiative corrections becomes

\[
\Pi(q^2) = a_0(Q^2)^2 \ln(Q^2/\nu^2) + b_0\langle \alpha_s G^2 \rangle + c_0 \langle gG^3 \rangle/Q^2 + d_0 \langle \alpha_s^2 G^4 \rangle/(Q^2)^2,
\]

with \(Q^2 = -q^2 > 0\), and

\[
a_0 = -2(\frac{\alpha_s}{\pi})^2, \quad b_0 = 4\alpha_s, \\
c_0 = 8\alpha_s^2, \quad d_0 = 8\pi\alpha_s.
\]

For the non-perturbative condensates the following notations and estimates are used

\[
\langle \alpha_s G^2 \rangle = \langle \alpha_s C^a_{\mu\nu} C^a_{\mu\nu} \rangle, \\
\langle gG^3 \rangle = \langle g f_{abc} C^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \rangle, \\
\langle \alpha_s^2 G^4 \rangle = 14\langle (\alpha_s f_{abc} C^a_{\mu\nu} G^b_{\rho\lambda} G^c_{\rho\mu})^2 \rangle - \langle (\alpha_s f_{abc} C^a_{\mu\nu} G^b_{\rho\lambda} G^c_{\rho\mu})^2 \rangle. 
\]
Now, we can apply the standard dispersion representation for the correlator
\[ \Pi(Q^2) = \Pi(0) - \Pi'(0) + \frac{(Q^2)^2}{\pi} \int_0^{+\infty} \frac{Im\Pi(s)}{s^2(s + Q^2)} ds \]  
(7)
to connect the QCD calculation with the resonance physics. From the low energy theorem \[7\] follows that
\[ \Pi(0) = \frac{32\pi}{11} \langle \alpha_s G^2 \rangle. \]  
(8)

For the physical spectral density \( Im\Pi(s) \), one can divide it into two parts: low energy part and high energy part. Its high-energy behavior is known as trivial,
\[ Im\Pi(s) \longrightarrow \frac{2}{\pi} s^2 \alpha_s^2(s), \]  
(9)
while at low energy region, \( Im\Pi(s) \) can be expressed in the single narrow width approximation. The single resonance model for \( Im\Pi(s) \) leads
\[ Im\Pi(s) = \pi f^2 M^4 \delta(s - M^2), \]  
(10)
where \( M, f \) are the glueball mass and coupling of the gluon current to the glueball. Thus we can proceed the following calculation.

To construct the sum rules, we use the moments \( R_k \) defined above, then the standard dispersion relation is transformed into
\[ R_k(\tau, s_0) = \frac{1}{\pi} \int_0^{s_0} s^k e^{-s\tau} Im\Pi(s) ds, \]  
(11)
and from Eq.(11) we have (for \( k \geq -1 \))
\[ R_k(\tau, s_0) = (-\frac{\partial}{\partial \tau})^{k+1} R_{-1}(\tau, s_0). \]  
(12)

Renormalization-group improvement of the sum rules amounts to the substitution:
\[ \nu^2 \rightarrow \frac{1}{\tau}, \]
\[ \langle gG^3 \rangle \rightarrow \left[ \frac{\alpha_s}{\alpha_s(\nu^2)} \right]^{7/11} \langle gG^3 \rangle. \]

\( R_{-1}(\tau, s_0) \) without radiative corrections can be obtained from Eq. (11).

If we had a complete knowledge of resonances and QCD, we would be able to fix the glueball mass, then different moments \( R_k \) would give the same result definitely, but we are far from this goal. In practice, we cannot calculate the infinite terms in OPE. Therefore, the result will depend on the choice of the moments. There should be a criteria to choose some suitable moments at appropriate \( s_0 \). As shown in Ref. [12], the \( R_{-1} \) sum rule leads to a much smaller mass scale due to the anomalously large contribution of the low-energy part \( \Pi(0) \) of the sum rule and it violates asymptotic freedom at large energy region. They claimed that \( R_{-1} \) was not reliable to predict the \( 0^{++} \) glueball mass and employed the \( R_0 \) and \( R_1 \) moments to predict the \( 0^{++} \) glueball mass by fitting the stability criteria with the radiative
corrections considered. Their approach showed that the $R_0$ and $R_1$ sum rules with the radiative corrections result in a higher mass scale compared to previous mass determination. They didn’t analyze how reliable these moments $R_k$ are for determining the glueball mass. After analyzing the different moment with the criteria of QCD sum rules, one can find that $R_0$ is not reliable too for the calculation of $0^{++}$ glueball in the single narrow width resonance approximation. In order to determine which moment is the more suitable and give a reliable mass prediction, we re-examine the $R_k$ sum rules.

To improve the convergence of the asymptotic series, we study the ratio $\frac{R_{k+1}}{R_k}$, such as $\frac{R_0}{R_{-1}}$ and $\frac{R_1}{R_0}$. In the narrow width approximation, we have

$$M^{2k+4} f^2 \exp(-\tau M^2) = R_k(\tau, s_0),$$

and (with $k \geq -1$)

$$M^2(\tau, s_0) = \frac{R_{k+1}(\tau, s_0)}{R_k}.$$  \hspace{1cm} (13)

To proceed calculation, we choose the following parameters

$$\langle \alpha_s G^2 \rangle = 0.06 \text{GeV}^4,$$

$$\langle gG^3 \rangle = (0.27 \text{GeV}^2) \langle \alpha_s G^2 \rangle,$$

$$\langle \alpha_s^2 G^4 \rangle = \frac{9}{16} (\langle \alpha_s G^2 \rangle)^2,$$

$$\Lambda_{\overline{MS}} = 200 \text{MeV},$$

$$\alpha_s = \frac{-4\pi}{11 \ln(\tau \Lambda_{\overline{MS}}^2)}.$$  \hspace{1cm} (12)

$M^2$ and $f^2$ are the functions of $s_0$ which is the starting point of the continuum threshold, $s_0 > M^2$. Since the glueball mass $M$ in Eq. (13) depends on $\tau$ and $s_0$, we take the stationary point of $M^2$ versus $\tau$ at an appropriate $s_0$ as the square of the glueball mass.

To determine the suitable moment and the appropriate $s_0$, the following criteria are employed: (1), The moments should be chosen to have a balance between the perturbative and the lowest lying resonance contribution to the sum rule, which means that both the perturbative contribution and the lowest resonance contribution to the sum rule are dominant in the sum rules; (2), $s_0$ should be a little higher than the physical mass and approaches it as near as possible due to the continuum threshold hypothesis and the narrow width approximation; (3), The choice of moments and a suitable $s_0$ should lead to not only a widest flat portions of the plots of $M^2$ versus $\tau$ but also an appropriate parameter region of $\tau$ with the parameter region compatible to the value of the glueball mass. According to these criteria, the acceptable region of $s_0$ is chosen from $s_0 = 3.0 \text{ GeV}^2$ to $s_0 = 4.3 \text{ GeV}^2$.

Let’s begin our analysis through the $R_k$ sum rules without radiative corrections. It is known that different moment has different suppression to the nonperturbative contribution and the lowest resonance contribution, moments with higher rank enhance the perturbative contribution and suppress the lowest resonance contribution to the sum rules.

In the sum rule of the moments $R_{-1}$ and $R_0$, although there is a platform for mass prediction (see Fig. 1), the perturbative contribution is less than 30%, which is not fit the criteria (1), so it is not acceptable.
Using the moment \( R_0 \) and \( R_1 \), one can obtain a balance between the perturbative and the lowest resonance contribution to the sum rules, however there is no platform for mass prediction (see Fig. 2). It doesn’t satisfy the criteria (3), so this moment is not suitable for the mass prediction either. All the previous calculations without radiative corrections were based on either moment \( R_{-1} \) and \( R_0 \) or moment \( R_0 \) and \( R_1 \), so the results are not very reliable.

The ratio \( \frac{R_2}{R_1} \) in Fig. 3 gives an excellent platform, and we can find a balance between the perturbative and the lowest resonance contribution to the sum rules, which keep the perturbative contribution and the lowest resonance contribution dominant in the sum rules, the moment \( R_1 \) satisfies all of the criteria and is reliable for the glueball mass determination. The curve shows that the \( 0^{++} \) glueball mass is 1710 MeV. In the acceptable region of \( s_0 \), the \( 0^{++} \) glueball mass is 1710 ± 80 GeV.

The moments with higher rank can’t stress the lowest resonance contribution in the sum rule, because the higher dimension condensates will not be negligible (we have little knowledge about higher dimension condensates at present). Therefore, we have no way to proceed our prediction from \( R_k \) with \( k > 2 \).

After taking into account radiative corrections, the correlator is

\[
\Pi(q^2) = \left( a_0 + a_1 \ln(Q^2/\nu^2) \right)(Q^2)^2 \ln(Q^2/\nu^2) + b_0 + b_1 \ln(Q^2/\nu^2) \right) (\alpha_s G^2) + (c_0 + c_1 \ln(Q^2/\nu^2)) \left( \frac{gG_5}{Q^2} + d_0 \alpha_s^2 G^4 \right). \tag{14}
\]

where

- \( a_0 = -2\left( \frac{\alpha_s}{\pi} \right)^2 (1 + \frac{51}{4} \frac{\alpha_s}{\pi}) \)
- \( b_0 = 4\alpha_s (1 + \frac{49}{12} \frac{\alpha_s}{\pi}) \)
- \( c_0 = 8\alpha_s^2 \)
- \( d_0 = 8\pi \alpha_s \)
- \( a_1 = \frac{11}{2} \left( \frac{\alpha_s}{\pi} \right)^3 \)
- \( b_1 = -11\frac{\alpha_s^2}{\pi} \)
- \( c_1 = -58\alpha_s^3 \)

The predicted mass from ratio \( \frac{R_2}{R_1} \) is \( \sim 1.65 \text{ GeV} \) (see Fig. 4). The value is a little lower than the one without radiative corrections.

In this section, we show how the predicted glueball mass depends on the choice of the moment. We give the criteria on choosing suitable moments and \( s_0 \) to calculate the glueball mass in QCD sum rules. From the criteria, only \( R_1 \) and \( R_2 \) are reliable for determination of the \( 0^{++} \) glueball mass and the result is 1.7 GeV. The radiative corrections do not affect the mass determination sensitively, they shift the glueball mass a little lower: 1.65 GeV.

4 Low energy theorem to the mixing picture

Now we proceed to discuss the mixing effect to determination of \( 0^{++} \) glueball mass. Let’s consider the \( 0^{++} \) quark current with isospin \( I = 0 \)

\[
j_2(x) = \frac{1}{\sqrt{2}} (\bar{u}u(x) + \bar{d}d(x)). \tag{15}
\]
Through operator product expansion, the correlator of the $j_2(x)$ is given by\cite{14}

$$\Pi_2(q^2) = \langle a_0'(Q^2)^2 \ln(Q^2/\nu^2) + \frac{3}{Q^2} \langle m\bar{q}q \rangle + \frac{1}{8\pi Q^2} \langle \alpha_s G^2 \rangle + \frac{b'_0}{(Q^2)^2} \langle \bar{q}q \rangle^2, \tag{16}$$

where $Q^2 = -q^2 > 0$, and

$$a'_0 = \frac{3}{8\pi^2} \left(1 + \frac{13\alpha_s}{3\pi}\right), \quad b'_0 = -\frac{176}{27\pi}\alpha_s.$$

The correlator of the $j_1(x)$ without radiative corrections is not changed.

In order to estimate the vacuum expectation values of higher dimension operators, the vacuum intermediate states dominance approximation\cite{8} has been employed

$$\langle \bar{q}\sigma_{\mu\nu}\lambda^a q\bar{q}\sigma_{\mu\nu}\lambda^a q \rangle = -\frac{16}{3} \langle \bar{q}q \rangle^2,$$

$$\langle \bar{q}\gamma_{\mu}\lambda^a q\bar{q}\gamma_{\mu}\lambda^a q \rangle = -\frac{16}{9} \langle \bar{q}q \rangle^2.$$

To proceed the numerical calculation, in addition to the parameters we have chosen above, the following parameters are taken

$$\langle \bar{q}q \rangle = -(0.25 GeV)^3,$$

$$\langle m\bar{q}q \rangle = -(0.1 GeV)^4,$$

$$\alpha_s = 0.28,$$

where the scale of the running coupling is set at the glueball mass.

Through the $R_k$ defined above, we can get the corresponding moments $R_k$ and $R'_k$ for $\Pi(q^2)$ and $\Pi_2(q^2)$

$$R_0(\tau, s_0) = -\frac{2a_0}{\tau^3} [1 - \rho_2(s_0 \tau)] + c_0 \langle gG^3 \rangle + d_0 \langle \alpha_s G^4 \rangle \tau, \tag{17}$$

$$R_1(\tau, s_0) = -\frac{6a_0}{\tau^4} [1 - \rho_3(s_0 \tau)] - d_0 \langle \alpha_s G^4 \rangle, \tag{18}$$

$$R_2(\tau, s_0) = -\frac{24a_0}{\tau^5} [1 - \rho_4(s_0 \tau)], \tag{19}$$

$$R'_0(\tau, s_0) = \frac{a'_0}{\tau^2} [1 - \rho_1(s_0 \tau)] + 3 \langle m\bar{q}q \rangle + \frac{1}{8\pi} \langle \alpha_s G^2 \rangle + b'_0 \tau \langle \bar{q}q \rangle^2, \tag{20}$$

$$R'_1(\tau, s_0) = \frac{2a'_0}{\tau^3} [1 - \rho_2(s_0 \tau)] - b'_0 \langle \bar{q}q \rangle^2, \tag{21}$$

where

$$\rho_k(x) \equiv e^{-x} \sum_{j=0}^{k} \frac{x^j}{j!}.$$ 

By using the Low-energy theorem \cite{15}, we can construct another correlator for the quark current with the gluonic current

$$\lim_{q \to 0} i \int dx e^{i qx} \langle 0 | T \left[ \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d), \alpha_s G^2 \right] | 0 \rangle = \frac{72\sqrt{2\pi}}{29} \langle \bar{u}u \rangle.$$ \tag{23}
In order to factorize the spectral density, we define the couplings of the currents to the physical states in the following way

\begin{align}
\langle 0 | j_1 | Q \rangle &= f_{12}m_2, \quad \langle 0 | j_1 | G \rangle = f_{11}m_1, \quad (24) \\
\langle 0 | j_2 | Q \rangle &= f_{22}m_2, \quad \langle 0 | j_2 | G \rangle = f_{21}m_1,
\end{align}

where \( m_1 \) and \( m_2 \) refer to the glueball (including few part of quark component) mass and the \( \bar{q}q \) meson (including few part of gluon component) mass, \( |Q\rangle \) and \( |G\rangle \) refer to the \( \bar{q}q \) meson state and the glueball state respectively.

We indicate that the gluon current couples to both the glueball and quark states, so does the quark current. In the real physical world, the physical state is not pure glueball state or quark state, the mixing effect should not be omitted without any reasonable argument. After choosing the two resonances plus continuum state approximation, the spectral density of the currents of \( j_1(x) \) and \( j_2(x) \) read in following respectively

\begin{align}
Im\Pi_1(s) &= m_2^2 f_{12}^2 \delta(s-m_2^2) + m_1^2 f_{11}^2 \delta(s-m_1^2) + \frac{2}{\pi} s^2 \alpha_s^2 \theta(s-s_0), \\
Im\Pi_2(s) &= m_2^2 f_{22}^2 \delta(s-m_2^2) + m_1^2 f_{21}^2 \delta(s-m_1^2) + a_0' s \theta(s-s_0). \quad (26)
\end{align}

Then it is straightforward to get the moments

\begin{align}
R_0 &= \frac{1}{\pi} \{m_2^2 e^{-m_2^2 r} f_{12}^2 + m_1^2 e^{-m_1^2 r} f_{11}^2\}, \quad (27) \\
R_1 &= \frac{1}{\pi} \{m_2^4 e^{-m_2^2 r} f_{12}^2 + m_1^4 e^{-m_1^2 r} f_{11}^2\}, \quad (28) \\
R_2 &= \frac{1}{\pi} \{m_2^6 e^{-m_2^2 r} f_{12}^2 + m_1^6 e^{-m_1^2 r} f_{11}^2\}, \quad (29) \\
R'_0 &= \frac{1}{\pi} \{m_2^2 e^{-m_2^2 r} f_{22}^2 + m_1^2 e^{-m_1^2 r} f_{21}^2\}, \quad (30) \\
R'_1 &= \frac{1}{\pi} \{m_2^4 e^{-m_2^2 r} f_{22}^2 + m_1^4 e^{-m_1^2 r} f_{21}^2\}. \quad (31)
\end{align}

In the meanwhile, assuming the states \( |G\rangle \) and \( |Q\rangle \) saturate the l.h.s of Eq. (23), we can obtain

\[ \lim_{q \to 0} i \int dx e^{iqx} \langle 0 | T \left[ \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \alpha_s G^2 \right] | 0 \rangle = f_{22} f_{12} + f_{21} f_{11}. \quad (32) \]

The next step is to equate the QCD side with the hadron side one by one, and we get a set of equations. Giving various of reasonable parameters \( s_0 \) and \( \tau \) and through solving this series of equations, we can get a series of the two states’ masses. We illustrate our result in Fig.5. In this figure, the solid line corresponding to the glueball and the doted line corresponding to the meson, the points of the plateau compatible to the parameters are regarded as the mass prediction points. We find that \( s_0 = 3.7 \text{ GeV}^2 \) is the best favorable value for \( s_0 \). There is no platform for \( \tau \) above 0.6 GeV\(^{-2}\), we can read the masses prediction: glueball with mass around 1.9 GeV and meson with mass around 1.0 GeV. We find that the glueball mass a little higher than the pure glueball state while the quark state mass is a little lower than the pure quark state.
5 Summary

In this paper, we analyze the determination of the scalar glueball mass based on the duality among resonance physics and QCD. The modified Borel transformation has been employed, it makes the calculation more convenient and reasonable.

We first conclude that it is important to choose suitable moments for the determination of $0^{++}$ glueball mass. To stress the contribution of the lowest resonance and make the perturbative contribution dominant in sum rules, the criteria on the choice of the moment and continuum threshold are given. These criteria make it reliable to choose a suitable moment for the calculation of the glueball mass. We find moments $R_{-1}$, $R_0$ and $R_k$ with higher rank $k > 2$ aren’t suitable for the mass determination in the single narrow width resonance approximation. The ratio of moments $R_2 / R_1$ is the most preferable for the determination of $0^{++}$ glueball mass. The numerical calculation shows that the mass is around 1.7 GeV without radiative corrections.

When the radiative correction is take into account, it shifts to 1.65 GeV.

Secondly, we consider the physical states as composite resonances, which include both gluon component and quark component, so we saturate the spectral density with two physical resonances, in this way we consider not only the couplings of gluonic current to both glueball state and quark state, but also the couplings of quark current to quark state and glueball state. Employing the Low-energy theorem and different moments, we predict the masses of glueball and normal meson from a set of coupled equations: glueball mass is around 1.9 GeV, which is a little higher than the one without mixing($\sim 1.7 GeV$), while mass of the quark state is around 1.0GeV. a little lower than the pure quark state($\sim 1.1Gev$). We conclude that the mixing between the glueball and the quark state is not large.

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Figure caption

Figure 1: $\frac{R_0}{R_{-1}}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ without radiative corrections.

Figure 2: $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ without radiative corrections.

Figure 3: $\frac{R_2}{R_1}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ without radiative corrections.

Figure 4: $\frac{R_2}{R_1}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ with radiative corrections.

Figure 5: $M$ versus $\tau$ at $s_0 = 3.7$ GeV$^2$. 
Fig. 1
Fig. 2

$S_0 = 3.6$

$M^2 (\text{GeV}^2)$ vs. $\tau (\text{GeV}^2)$
Fig. 3
Fig. 4

$M^2 (GeV^2)$ vs $\tau (GeV^2)$
Fig. 5

$S_0 = 3.7$