Cooperative Collision Avoidance in Mobile Robots using Dynamic Vortex Potential Fields

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Abstract—In this paper, the collision avoidance problem for non-holonomic robots moving at constant linear speeds in the 2-D plane is considered. The maneuvers to avoid collisions are designed using dynamic vortex potential fields (PFs) that are functions of relative velocities in polar coordinates. Introduction of vorticity in the calculation of the gradients leads to a cooperative collision avoidance behaviour between the robots and also ensures the absence of local minima. Such a repulsive field is activated by a robot only when it is on a collision path with other mobile robots or stationary obstacles. By analysing the kinematics-based engagement dynamics in polar coordinates, the PF parameters are identified that ensure collision avoidance with stationary and moving robots, as well as those actively seeking to collide with it. Experimental results acquired using a mobile robot platform that support the theoretical contributions are presented.

Index Terms—Vortex Potential Fields, Mobile Robots, Collision Avoidance, Reciprocity

I. INTRODUCTION

Collision avoidance (CA) is a crucial element in many applications involving a swarm of mobile robots, for example, in area exploration, [1]–[3]. A commonly applied technique to avoid collisions in such dynamic environments is based on artificial potential fields (APFs), where, an attractive PF is defined at the goal location and repulsive fields are defined around obstacles. The robot inputs that steer it to its goal, while avoiding obstacles, are calculated based on the gradients of the PF functions. Harmonic PF functions and other alternatives, that avoid this issue, are proposed in [4] and [5]–[7], respectively.

Another variant is the use of the so-called curl-free vector field, also the vortex field, where the field defined around an obstacle rotates around the obstacle. The direction of rotation can be selected by the appropriate choice of signs of the gradients. Such fields have been used in [3], [8]–[11] to design repulsive PFs for CA. In our work, we select such a rotating repulsive field to ensure that a pair of robots on a collision path turn in the same direction and avoid collisions; this formulation holds even for multiple robots. As described in [9], fixing the direction of rotation of the vortex repulsive field corresponds to fixing the “rule of the road”. Repulsive PFs akin to the vortex PF we use are designed in [12], [13], but, these designs do not explicitly consider the relative positions or the velocities of the robots.

We use the dynamic vortex field function proposed in [14] - designed for robotic manipulators - to design the repulsive field, where, the field is a function of the position and velocity of a robot relative to another robot. By assigning signs to the gradients to calculate the robot inputs, the direction of rotation of the field becomes fixed, which in turn, ensures that robots turn in the same direction, thus leading to an emergent cooperative behaviour. Indeed, the idea of requiring robots to turn in the same direction is the essence of the Hybrid Reciprocal Velocity Obstacle (HRVO) algorithm, [15], as well as the Optimal Reciprocal CA algorithm, [16], [17]. In the HRVO algorithm, a robot computes a new velocity that lies outside the velocity obstacle induced by other robots and which is also closest to its preferred velocity. What we propose does not require the computation of velocity obstacles and reciprocity appears as a natural consequence of assigning signs to the gradients of the repulsive field.

The main contributions of this paper are the application of a dynamic vortex potential field for CA in robots

1) that naturally introduces cooperation in robots; there is no need to state rules that define the directions of turn;

2) against stationary or mobile robots that do not apply this repulsive PF or those that are actively seeking to collide with a robot applying the repulsive PF; and

3) that consider the size of the robots - assumed to be a circle of known radius.

Proofs of CA are presented based on kinematics-based engagement dynamics between robots moving at constant linear speeds. The proofs\(^1\) are based on necessary and sufficient conditions that hold, for collision in 2-D space, as identified in [18]. We present experimental results for all our theoretical contributions in an indoor robotics platform that consists of several differential-drive robots. The paper is organised as follows: The kinematic model of the robot as well as the

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\(^{1}\)See [https://github.com/waynepaulmartis/Preprint-Cooperative-Collision-Avoidance-in-Mobile-Robots-using-Dynamic-Vortex-Potential-Fields](https://github.com/waynepaulmartis/Preprint-Cooperative-Collision-Avoidance-in-Mobile-Robots-using-Dynamic-Vortex-Potential-Fields) for all detailed proofs.
engagement dynamics between two robots on a collision path are presented in Sec. II. The attractive and dynamic vortex repulsive fields that are used to design the robot maneuvers are described in Sec. III; the proofs of CA are also briefly sketched here. Experimental results on the application of the proposed approach on differential drive robots are presented in Sec. IV, followed by concluding remarks in Sec. V.

II. PRELIMINARIES

A swarm of \( N \geq 2 \) homogeneous robots that move at constant linear speeds, \( V \), is considered. Each robot is assumed to be a point-mass and non-holonomic, moving in a Cartesian plane \((X – Y)\) according to

\[
\begin{align*}
\dot{x}_{Ri} &= V \cos (\phi_{Ri}), \quad \dot{y}_{Ri} = V \sin (\phi_{Ri}), \quad \dot{\phi}_{Ri} = \omega_{Ri},
\end{align*}
\]

where \((x, y)_{Ri}\) are the coordinates of robots \(R_i\), \(i = 1, \ldots, N\), in Cartesian space with a known origin; \(\phi_{Ri}\) is the heading angle, which is measured positive counter-clockwise about the \(X\)-axis; and \(\omega_{Ri}\) is the angular velocity, which can be varied. This variation leads to the accelerations \(\ddot{x}_{Ri} = -V \sin (\phi_{Ri}) \dot{\phi}_{Ri}, \ddot{y}_{Ri} = V \cos (\phi_{Ri}) \dot{\phi}_{Ri} = F_y_{Ri}\), where \(F_x_{Ri}, F_y_{Ri}\) can be interpreted as forces acting on the robot that steer the robot to known goal locations \((x, y)_{RiG}\).

The conditions that lead to collision between robots are identified in terms of relative velocities in polar coordinates. Consider 2 robots, \(R_1\) and \(R_2\), as shown in Fig. 1. From this figure, by computing the separation distance, \(r_1 = r_2 = r\), between the two robots, and the line-of-sight (LOS) angles, \(\theta_2 = \pi + \theta_1\), the relative velocities and accelerations, respectively, are

\[
\begin{align*}
\dot{r} &= V_r = V \cos (\phi_{R2} - \theta) - V \cos (\phi_{R1} - \theta), \\
\dot{\theta} &= V_\theta = V \sin (\phi_{R2} - \theta) - V \sin (\phi_{R1} - \theta), \quad \theta_1 = \theta,
\end{align*}
\]

and

\[
\begin{bmatrix}
\dot{V}_r \\
\dot{V}_\theta
\end{bmatrix}
= \begin{bmatrix}
\frac{V_r^2}{r} \\
- \frac{V_r V_\theta}{r}
\end{bmatrix} + \begin{bmatrix}
\cos \theta & \sin \theta \\
- \sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
F_x_{R2} - F_x_{R1} \\
F_y_{R2} - F_y_{R1}
\end{bmatrix},
\]

(2c)

Fig. 1. A pair of robots on a collision path

As proved in [18, Lemma 2], a pair of robots moving at a constant velocity are on a collision path if \(V_r < 0\) and \(V_\theta = 0\);

these are both necessary and sufficient conditions for collision. Note that the linear speeds of the robots can be different. Thus, if the forces acting on each robot are selected such that these conditions are violated, then, the robots avoid colliding with each other. These forces are chosen based on the gradients of the dynamic vortex potential field—which forms the repulsive field around each robot considered as an obstacle—and an attractive field defined at the goal location.

III. MAIN RESULTS

Attractive PF: Given the known and stationary goal locations, \((x, y)_{RiG}\), for each robot, the attractive PF function is selected as the scaled distance between the current location of the robot and the goal, given by

\[
U_{Att} = \kappa_r G, \quad r_G = \sqrt{(x_{Ri} - x_{RiG})^2 + (y_{Ri} - y_{RiG})^2}, \quad \kappa > 0.
\]

The robot inputs, along \(X\) and \(Y\) directions, are selected as the negative gradients

\[
\begin{align*}
F_{xRiAtt} &= -\frac{\partial U_{Att}}{\partial (x_{Ri} - x_{RiG})} = +\kappa \cos (\theta_{RiG}) \quad (4a) \\
F_{yRiAtt} &= -\frac{\partial U_{Att}}{\partial (y_{Ri} - y_{RiG})} = +\kappa \sin (\theta_{RiG}), \quad (4b)
\end{align*}
\]

where \(\theta_{RiG}\) is the LOS angle between the robot’s location and its goal. Based on these inputs, the accelerations of the robot relative to the goal become \(\dot{V}_r = (V^2_{RiG}/r_{G}) - \kappa, \quad V_\theta = -V r_{G} V_{G}/r_{G}\). Now, the robot is attracted towards its goal, that is \(r_G = 0\), if \(V_{rG} < 0\) and \(V_{\theta G} = 0\). It can be shown by analysing the properties of the equilibrium point \((r_G, V_{rG} = 0, V_{\theta G} = -V)\) using the Lyapunov function \(V_{LAtt} = \kappa_r G + 0.5 (V^2_{RiG} + (V_{rG} + V)^2)\) and calculating its derivatives along trajectories similar to (2), that the robot is attracted to the goal and that it actually becomes “trapped” at this location.

Dynamic Vortex Repulsive PF: Consider two robots, \(R_i\) and \(R_j\) moving in the plane to their goal locations. Let their trajectories be such that they will collide, that is, the collision conditions \(V_r < 0\) and \(V_\theta = 0\), from (2a) and (2b), are satisfied. For the robots to avoid a collision, their respective inputs are designed according to the repulsive PF function, as proposed in [14] and given by

\[
U_{Repi} = \begin{cases}
\lambda (\cos \gamma_i)^2 \frac{\|V_{ij}\|}{r} & \text{if } \frac{\pi}{2} < \gamma \leq \pi \quad , \lambda > 0, \quad (5a) \\
0 & \text{otherwise}
\end{cases}
\]

where

\[
\begin{align*}
\cos \gamma_i &= \frac{V^T_{ij} x_{ij}}{\|x_{ij}\| r}, \quad x_{ij} = \begin{bmatrix} (x_{Rj} - x_{Ri}) \\ (y_{Rj} - y_{Ri}) \end{bmatrix} \\
V_{ij} &= \begin{bmatrix} V \cos (\phi_{Rj}) - V \cos (\phi_{Ri}) \\ V \sin (\phi_{Rj}) - V \sin (\phi_{Ri}) \end{bmatrix}
\end{align*}
\]

The term \(\cos \gamma_i\), (5b), is a function of the relative velocities and positions of the two robots in Cartesian space. Expressing these states in polar coordinates results in

\[
\cos \gamma_i = \frac{V_r}{V_{rel}}, \quad V_{rel} = V \sqrt{2 (1 - \cos (\phi_{Ri} - \phi_{Rj}))}.
\]

61
If the two robots are on a collision course, then, the relative velocity \(-2V \leq V_r < 0\), while the relative speed \(2V \geq V_{rel} > 0\). Thus, \(\gamma_i = \frac{\pi}{2} < \gamma_i \leq \pi\) - this is the inequality used to define the repulsive PF, (5a).

Thus, both robots trigger the repulsive PF only if they are on a collision path. In (6), the relative speed condition \(V_{rel} = 0\) holds when the heading angles of the two robots satisfy \(\phi_{Ri} = \phi_{Rj}\). This condition occurs when they either are moving in parallel or are one behind another; in the latter case, since they are moving at the same linear speed, they also cannot overtake each other. In both these conditions, the repulsive PF is not triggered.

When the repulsive PF is triggered, to avoid collisions, robot \(R_i\) applies its inputs according to

\[
F_{xRiRep} = -\frac{\partial U_{Rep}}{\partial (y_{Rj} - y_{Ri})}, ~ F_{yRiRep} = -\frac{\partial U_{Rep}}{\partial (x_{Rj} - x_{Ri})},
\]  

(7a)

where

\[
\frac{\partial U_{Rep}}{\partial (x_{Rj} - x_{Ri})} = -\frac{\lambda V_r}{V_{rel} r^2} (2V_\theta \sin \theta_i + V_r \cos \theta_i),
\]

(7b)

\[
\frac{\partial U_{Rep}}{\partial (y_{Rj} - y_{Ri})} = +\frac{\lambda V_r}{V_{rel} r^2} (2V_\theta \cos \theta_i - V_r \sin \theta_i).
\]

(7c)

The vortex nature of the application of the repulsive PF appears from the choosing the inputs according to (7a). Further, by fixing the signs of these inputs as described, the PF “rotates” in the same direction for all robots, see Fig. 2; flipping the signs reverses the direction of rotation. As has been proved in [19], based on the expression of the repulsive field function and selecting the gradients according to (7), the field becomes curl-free. As the repulsive field is set to zero when the robots are moving away from each other, that is, when \(V_r \geq 0\), there are no arrows in the space “behind” each robot.

![Fig. 2. Gradients of the repulsive dynamic vortex PF when two robots are on a head-on collision course.](image)

**Proof of Cooperative CA:** This aspect is proved for the case when each of the two robots that are on a collision course applies its inputs given by (7); the robots are assumed to be initially separated by a non-zero distance, \(r_0 > 0\). CA is proved by analysing the closed-loop dynamics

\[
\dot{V}_r = V_r, ~ \dot{V}_\theta = \frac{V_r^2}{r} + 4 \left(\frac{\lambda}{V_{rel} r^2}\right) V_r V_\theta,
\]

(8a)

\[
\dot{V}_\theta = -\frac{V_r V_\theta}{r} + 2 \left(\frac{\lambda}{V_{rel} r^2}\right) V_r^2.
\]

(8b)

derived with the substitution of the robots’ inputs given in (7) along with the equality \(\theta_j = \pi + \theta_i\). As can be observed, the unique equilibrium point of these dynamics is \(V_r = V_\theta = 0\) - this condition can hold only when \(r = 0\), that is, when the robots collide. To show that this is an unstable equilibrium point, the Lyapunov function \(V_{LRep} = r + 0.5(V_r^2 + V_\theta^2)\) and its derivative \(\dot{V}_{LRep} = V_r (1 + 6\lambda V_r V_\theta/(V_{rel} r^2))\) are analysed. It can be shown that, for any \(\lambda, r_0 > 0\), the ratio \(0 < (\dot{V}_{LRep}/V_r) < 1\), i.e. the product \(\dot{V}_{LRep} V_r < 0\). Using these cases, it can be shown that the sign of \(\dot{V}_{LRep}\) changes from negative to positive, and that at this time instant, the separation distance is non-zero. Thus collision never occurs. A similar analysis can be performed to show that if it is only one of the robots that applies the inputs in (7), it can avoid colliding with either a stationary robot or one moving at a constant velocity.

To highlight the difference caused by the vortex PF, let the robots’ inputs be chosen according to

\[
F_{xRiRep} = -\frac{\partial U_{Rep}}{\partial (x_{Rj} - x_{Ri})}, ~ F_{yRiRep} = -\frac{\partial U_{Rep}}{\partial (y_{Rj} - y_{Ri})},
\]

(9)

which is the conventional approach when PFs are used. In this case, the closed-loop relative accelerations become

\[
\dot{V}_r = \frac{V_\theta^2}{r} - 2 \left(\frac{\lambda}{V_{rel} r^2}\right) V_r^2 \]  

(10a)

\[
\dot{V}_\theta = -\frac{V_r V_\theta}{r} + 4 \left(\frac{\lambda}{V_{rel} r^2}\right) V_r V_\theta.
\]

(10b)

For these dynamics, the equilibrium conditions are the pairs \(V_r = V_\theta = 0\) and \(V_r = -2V, V_\theta = 0\). The latter is an equilibrium condition since the relative velocities satisfy \(V_r^2 + V_\theta^2 = V_{rel}^2\), thus, even if the magnitude of \(\dot{V}_r\) is high, the relative velocity \(|V_r| \leq 2V\). Now, by choosing the Lyapunov function \(V_{LNV} = r + 0.5(V_\theta^2 + (V_r + 2V)^2)\) and analysing its derivative, it can be shown that the conditions for collision form an asymptotically stable equilibrium point, implying that the robots collide with each other starting from the initial conditions \(r \neq 0, V_r < 0, V_\theta = 0\).

The choice of vortex PF ensures that the robots turn away from each other, since \(\dot{V}_\theta \neq 0\). In the non-vortex case, from (10b), it can be seen that \(V_\theta = 0 \iff t \geq 0\), implying at the robots do not turn away from each other and eventually collide.

**Implementation:** Consider robots that are defined by circles of radius \(R_{Rob} > 0\) and have limits on their accelerations, \(F_{lim}\). By analysing the case when a pair of robots are moving at speed \(V\), initially separated by distance \(r_0\), and are on a head-on collision course, it can be shown that if \(F_{lim} \geq \frac{2R_{Rob} V^2}{r_0^2 - R_{Rob}^2}\), then the robots avoid grazing each other while performing the CA maneuvers. Thus, if the difference \((r_0^2 - R_{Rob}^2)\) is small, then, the acceleration limit has to be proportionately large for the robots to turn away from each other to avoid a collision.

The proposed PF-based CA algorithms can be implemented by controlling \(\phi_{Ri}\), the heading angle of robot \(R_i\), to the
desired value
\[ \phi_{Ri\text{Des}} = \tan^{-1} \left( \frac{F_{yRi\text{An}} + \sum_j F_{yRij\text{Rep}}}{F_{xRi\text{An}} + \sum_j F_{xRij\text{Rep}}} \right), \]  
where, \( F_{xRi\text{An}} \) and \( F_{yRi\text{An}} \) are given by (4), while \( F_{xRi\text{Rep}} \) and \( F_{yRi\text{Rep}} \) are calculated using (7); this approach is motivated by that suggested in [8]. Now, to ensure \( \phi_{Ri} \rightarrow \phi_{Ri\text{Des}}, \) the angular speed, \( \omega_{Ri} \) in (1), can be designed as the output of the controller \( \omega_{Ri} = K_p (\phi_{Ri\text{Des}} - \phi_{Ri}) \), where \( K_p > 0 \) is the control gain. This control law can also be chosen as a Proportional-Integral controller so that bounded disturbances acting on the robot can be suppressed.

**IV. Experimental Results**

The dynamic vortex PF algorithm is implemented on the QBOT 2E mobile robot platform by Quanser\(^2\). All experiments are conducted in the Autonomous Vehicles Research Studio\(^3\), also from Quanser.

The results of two robots, \( R_{1,2} \), each of which traverses to the other’s initial locations, while avoiding collisions, is shown in Fig. 3. The repulsive and attractive PF parameters are set at \( \lambda = 10 \) and \( \kappa = 10 \), respectively. The reciprocal and cooperative CA maneuvers is evident. Each robot turns towards its right in order to avoid collisions; this direction of turn is decided by the choice of the signs of the gradients that define the inputs. The trajectory in the \( V_r - V_{\theta} \) space reveals that there are no stable equilibria in the \( \{V_r < 0 \text{ and } V_{\theta} = 0\} \) domain, thus ensuring that the conditions for collision are never satisfied. In addition, the robots do not display any oscillatory behaviour as well.

![Fig. 3. Two robots moving to the other’s initial location. As time progresses, the shade of the circles becomes lighter; this visualisation idea is borrowed from [16].](image)

The motions of 3 robots are shown in Fig. 4. In this case, the robots follow the “symmetric roundabout” behaviour, described in [9], as they trace a circle in the clockwise direction leading to this behaviour. The proof of CA for the multiple robot case is similar to that for a pair of robots.

![Fig. 4. Collision avoidance by three robots, all of which are initially on a collision course.](image)

The results of a case, where one robot, say \( R_2 \), is actively seeking to collide with robot \( R_1 \), are shown in Fig. 5. \( R_1 \) applies the vortex repulsive PF to avoid \( R_2 \) while moving to it goal, while \( R_2 \) applies its inputs given by the Attractive PF, (4), considering \( R_1 \) as its goal. The inputs of \( R_2 \) become \( F_{x2} = -\kappa \cos (\theta_1) \), \( F_{y2} = -\kappa \sin (\theta_2) \), where, \( \theta_i \) is the LOS angle with reference to \( R_1 \). Now, \( R_1 \) “projects” the dynamic vortex PF around robot \( R_2 \) and applies its inputs to avoid a collision. The dynamics of the relative velocities become

\[ \dot{V}_r = \frac{V_{\theta}^2}{r} + 2 \left( \frac{\lambda}{V_{rel}V_{rel}} \right) V_r V_{\theta} - \kappa \]  
\[ \dot{V}_{\theta} = -V_r V_{\theta} + \left( \frac{\lambda}{V_{rel}V_{rel}} \right) V_r^2. \]  

It can be shown that if the initial separation distance, \( r_0 \), between \( R_1 \) and \( R_2 \) satisfies \( r_0 \geq \sqrt{3\lambda V} \), then, \( R_1 \) is able to avoid colliding with \( R_2 \). This result emerges from analysing the Lyapunov function \( V_{L\text{Atck}} = kr + 0.5(V_{\theta}^2 + V_r^2) \) and its derivative, and showing that the equilibrium point \( r = V_r = V_{\theta} \) is unstable.

![Fig. 5. A robot applying the vortex repulsive PF to avoiding collision with another robot applying the attractive PF to collide with the former.](image)

For the results in Fig. 5, the robots are initially on a head-on collision course. The PF parameters are selected such the conditions for \( R_1 \) to avoid colliding with \( R_2 \) hold. As can be seen, \( R_2 \) maneuvers and begins to “chase” \( R_1 \), while \( R_1 \) has already changed its heading angle in order to avoid colliding.
with \( R_2 \). In this experiment, once \( R_1 \) has reached the vicinity of its goal, it stops moving and only then does \( R_2 \) collide with it. During experimentation, it was observed that a lower value of the repulsive PF parameter, \( \lambda \), resulted in \( R_1 \) making several (> 2) circular turns before it reached its goal; however, at no instant was \( R_2 \) able to collide with \( R_1 \). This behaviour is equivalent to the oscillatory behaviour recognised in the analysis of \( V_{Latck} \).

Simulation results of application of the vortex and non-vortex PFs for two robots on a head-on collision course are shown in Fig. 6. As is evident, without the vortex field, the robots do not turn and eventually collide with each other, thus supporting the theoretical analysis presented in Sec. III.

![Diagram of trajectories](image)

Fig. 6. Trajectories of the cooperative robots in \( X-Y \) space with (left) and without (right) the vortex repulsive PF.

V. CONCLUSIONS

In this article, we presented a dynamic vortex PF algorithm for CA in planar mobile robots. The nature of the vortex PF guarantees that robots turn in the same direction while avoiding collisions with one another, thus lending the robots a cooperative behaviour. The algorithm can be extended to robots navigating spaces such as corridors or paths where the robot maybe forced to turn in one direction. In such cases, the problem of how to select the direction of rotation of the robot would need to be addressed.

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