Teleportation on a quantum dot array

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We present a model of quantum teleportation protocol based on a double quantum dot array. The unknown qubit is encoded using a pair of quantum dots, with one excess electron, coupled by tunneling. It is shown how to create a maximally entangled state using an adiabatically increasing Coulomb repulsion between different dot pairs. This entangled state is exploited to perform teleportation again using an adiabatic coupling between itself and the incoming unknown state. Finally, a sudden separation of Bob’s qubit allows a time evolution of Alice’s which amounts to a modified version of standard Bell measurement. A transmission over a long distance could be obtained by considering the entangled state of a chain of coupled double quantum dots.

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I. INTRODUCTION

Building a quantum computer is a challenge of modern physics. Required ingredients are entanglement, quantum channels with high fidelity for information transfer, and two (unconditional and conditional) qubit gates. During last years many efforts have been done in different fields to design a feasible quantum computer. A very efficient protocol to transfer a state from one place to another is represented by quantum teleportation.

Experimental realizations have been performed with optical systems and NMR techniques, with spatially localized qubits since the degree of freedom of codification were represented by polarization for photons and spin for molecules.

In a solid state approach teleportation schemes have been proposed using spin and vacuum single exciton qubit on semiconducting quantum dots. The first one of these schemes considers the use of a circuit of three normal and two superconducting QDs; the teleportation is achieved by measuring the spin-polarized current through the dot array, and the maximum distance of information transport limited by the coherence length of the superconducting device.

Ground state-single exciton qubits in quantum dots have been also proposed for quantum computation architecture using non adiabatic processes. Recently the realization of number state qubits had interesting developments. In this framework a teleportation protocol has been successfully implemented with photons. Here we propose a solid state implementation based on a double quantum dot (DQD) array that permits to create entanglement and to perform quantum teleportation. Our qubit is represented by a pair of spatially separated quantum dots coupled by tunneling. Interactions between different qubits are driven by means of Coulomb repulsion.

These charge (or number) states are characterized by one excess electron occupying the first or the second dot, creating an isolated well defined quantum two level system.

A similar configuration can be adopted in an ion trap quantum computing device localizing a neutral atom in one member of a pair of trapping potentials. Despite decoherence obstacles, principally due to cotunneling effects, electron-phonon interactions and background charge fluctuations, coherent oscillations in DQD systems have been recently experimentally observed, and therefore it becomes more and more interesting to analyze the potentialities of these schemes. In order to obtain the desired operations, both adiabatic and nonadiabatic changes in the interaction parameters are requested. The charge transport through dots using adiabatic variations of pairing parameters has been extensively described, while in a different scenario (NMR) quantum adiabatic gates have been experimentally achieved.

We preliminary show how to create Alice’s general unknown state starting from an initial one by tunneling. Next we show how to generate the entangled state of two qubits, which represents the support for teleportation by adiabatic switching on Coulomb interaction. The unknown state is then entangled with the support state again by adiabatic switching on of Coulomb interaction. Bob’s qubits is obtained by detachment of last pair. From now on the system evolves for a suitable time at which the measurement of a number state of one of Alice’s dots ends the teleportation process. By a classical communication of Alice’s measurement to Bob the unknown state can be reconstructed. This simple scheme can be easily extended to an N pairs teleportation support and is presumably robust with respect to electron-phonon interaction. Decoherence effects will be mainly due to coupling with external leads.
II. ENTANGLED STATES

Let us consider a system composed by a DQD with just one excess electron with respect to the ground state. This system represents a charge (or number) qubit with basis elements \( |0_1, 1_2 \rangle \) and \( |1_1, 0_2 \rangle \). If dots are coupled by tunneling, in the presence of a vector potential \( \mathbf{A} \) directed from dot 1 to dot 2, the system is described by the following Hamiltonian:

\[
H_{12} = -(w e^{i \varphi} a_1^\dagger a_2 + w e^{-i \varphi} a_1^\dagger a_4 + \epsilon (a_1^\dagger a_2 + a_1^\dagger a_4)) \tag{1}
\]

where \( a_i (a_i^\dagger) \) represents the annihilation (creation) fermionic operator on the site \( i \) and \( \varphi = \frac{2 \pi}{N} \). For sake of simplicity and without loss of generality we shall assume \( \epsilon = 0 \). This Hamiltonian has eigenvalues \( E = \pm w \) associated to the eigenvectors \( |E\rangle = \frac{1}{\sqrt{2}} [e^{i \varphi} |0_1, 1_2 \rangle \mp |1_1, 0_2 \rangle] \).

If, assuming \( \hbar = 1 \), we suppose that the system is in a particular state at \( t = 0 \) (e.g. \( |\phi(0)\rangle = |0_1, 1_2\rangle \)), the time evolution creates a coherent superposition:

\[
|\phi(t)\rangle = \cos wt |0_1, 1_2\rangle + i \sin wt e^{-i \varphi} |1_1, 0_2\rangle \tag{2}
\]

Thus, by instantaneously switching off the tunneling at a suitable time \( t \), we can encode a qubit \( |\phi(t)\rangle = |\chi\rangle = \alpha |0_1, 1_2\rangle + \beta |1_1, 0_2\rangle \).

The entangled support for teleportation is an array of four QDs labelled with subscripts 3, 4, 5, 6 disposed as indicated in figure 1. The Hamiltonian

\[
H_{3456} = -w \left( a_3^\dagger a_4 + a_5^\dagger a_6 + h.c. \right) + U(\theta) \left( n_3 n_6 + n_4 n_5 \right) \tag{3}
\]

takes into account both of tunneling interaction along vertical lines and of Coulomb repulsion along horizontal lines. Here \( n_i = a_i^\dagger a_i \) is the occupation number operator on the site \( i \). Double occupation on a single dot, as well as double occupation on a DQD will be completely neglected. Lateral interdot tunneling interdiction has been considered also in [12]. Starting from \( U(0) = 0 \) the Hamiltonian is separable: \( H_{3456} = H_{34} + H_{56} \). For convenience we shall assume that the system is prepared in its ground state:

\[
|\psi(0)\rangle = \frac{1}{2} \left[ |0_3, 1_4 \rangle + |1_3, 0_4 \rangle \right] \left[ |0_5, 1_6 \rangle + |1_5, 0_6 \rangle \right] \tag{4}
\]

An adiabatic growth of Coulomb repulsion between dot localized on the same plane will create a near maximally entangled state. Here adiabatic means slow with respect to the lower frequency of the system. Due to the adiabatic theorem [13], the overall system will remain in its instantaneous ground state. The asymptotic behavior is a good approximation of a maximally entangled state in the limit of \( w/U \rightarrow 0 \):

\[
\begin{align*}
|\psi(t \rightarrow \infty)\rangle &= \frac{1}{\mathcal{N}} \left[ |0_3, 1_4, 0_5, 1_6 \rangle + |1_3, 0_4, 1_5, 0_6 \rangle \right] + \\
&- \frac{1}{4w} \left( -U + \sqrt{U^2 + 16w^2} \right) |1_3, 0_4, 0_5, 1_6 \rangle + \\
&- \frac{1}{4w} \left( -U - \sqrt{U^2 + 16w^2} \right) |0_3, 1_4, 1_5, 0_6 \rangle
\end{align*} \tag{5}
\]

where \( \mathcal{N} \) is a normalization coefficient and \( U = U(t \rightarrow \infty) \); in the limit \( w/U \rightarrow 0, \mathcal{N} \sim 1/\sqrt{2} \). It can be shown that a maximally entangled state is a good approximation even in the case of an array with \( N > 2 \) dot pairs. Bearing in mind the limit of approximation we consider as starting point for the following manipulation the state

\[
|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left[ |0_3, 1_4, 0_5, 1_6 \rangle + |1_3, 0_4, 1_5, 0_6 \rangle \right] \tag{6}
\]

The same entangled state can be obtained by quantum annealing procedure in the presence of a time independent interaction.

III. THE TELEPORTATION

The creation of these entangled states enables us to implement a quantum teleportation protocol allowing the transfer of the information associated to an unknown incoming state from one place to another. For sake of comparison with our proposal we recall briefly the standard teleportation protocol: Alice has an unknown qubit \( |\chi\rangle_U = \alpha |0\rangle_U + \beta |1\rangle_U \) and she wants to send it to Bob (in order to avoid confusion this general description \( |0\rangle \) and \( |1\rangle \) are merely label indicating the qubit states, without referring to charge occupation). They need to share a maximally entangled two qubits state (e.g.\( |0_A, 1_B\rangle + |1_A, 0_B\rangle \)) where subscripts A and B denote respectively Alice and Bob subspaces (normalization
factors will be ignored). The whole state can now be rewritten in terms of Bell basis \(|\phi^\pm\rangle = (|0,0\rangle \pm |1,1\rangle)\):

\[
|\Psi\rangle_{UAB} = \alpha |0,0,1\rangle_{UAB} + \alpha |0,1,0\rangle_{UAB} + \beta |1,0,1\rangle_{UAB} + \beta |1,1,0\rangle_{UAB} = \]

\[|\phi^+\rangle_{UA} \sigma_x |\alpha\rangle_B - i |\phi^-\rangle_{UA} \sigma_y |\alpha\rangle_B + |\psi^+\rangle_{UA} |\alpha\rangle_B + |\phi^-\rangle_{UA} \sigma_z |\alpha\rangle_B \quad (7)
\]

having indicated with \(\sigma\) standard Pauli matrices. Making a projective measure on \(U\) and \(A\) (Alice’s qubits) and transmitting via a classical channel the result, we are able to recover the original unknown state in the Bob site without need of measurements in the destination place. The Bell measurement process can be performed (as suggested by G. Brassard and coworkers) in two sequential steps: first, Bell states are rotated in the computational basis \(|\langle 0,0|,|0,1|,|1,0|,|1,1\rangle\rangle\), then the projective measure is performed in this latter basis.

Here we propose a slightly modified procedure wherein the Bell states involved are two instead of four; furthermore, we exploit temporal evolution to perform the first step of Brassard method, making simple the final one.

Our protocol exploits an adiabatic switching on of Coulomb interaction between the qubit we want to teleport and the entangled state. Now we deal with a system composed by three DQDs (see figure 2), one of them is used to encode the unknown qubit and the other two as entangled support. The Hamiltonian is

\[
H_{123456} = -w (a_1^\dagger a_4 + a_4^\dagger a_1) - w' (t) (a_1^\dagger a_6 + a_6^\dagger a_1) + \]

\[- w'' (t) (a_1^\dagger a_2 + a_2^\dagger a_1) + U (t) (n_3 n_6 + n_4 n_5) + \]

\[+ U' (t) (n_1 n_4 + n_2 n_3) \quad (8)
\]

where \(U'(0)\) and \(w''(0)\) vanishes, while \(w'(0) = w\).

Making use of encoding technique and entanglement generation above described, the incoming overall state is

\[
|\Psi\rangle_{123456} = \frac{1}{\sqrt{2}} (\alpha |01,12\rangle + \beta |11,02\rangle) \times \]

\[(|03,14,05,16\rangle + |13,04,15,06\rangle) \quad (9)
\]

If \(U'(t)\) is adiabatically increased, the state evolves and reaches its new ground state

\[
|\Psi\rangle_{123456} = \frac{1}{\sqrt{2}} (\alpha |01,12,03,14,05,16\rangle + \beta |11,02,13,04,15,06\rangle) \quad (10)
\]

a GHZ-like three qubits nonmaximally entangled state.

So far we have described the coupling between unknown qubit and entangled state. Next step represents the analogous of Bell measurement. To prepare it we need to detach Bob QDs (5 and 6) from the others and to start a temporal evolution of the state which involves dots from 1 to 4. By instantaneously turning on the tunneling \(w'(t)\), and turning off the tunneling \(w''(t)\) and the Coulomb interaction \(U(t)\) (from now on the time will be measured starting from the switching instant), the system is forced to belong to a state in which dots from 1 to 4 evolve following the Hamiltonian of Eqn. 8 (with appropriate indices), while Bob’s dots are frozen. Neglecting terms of the order of \(w/U, |01,12,03,14\rangle\) evolves in \(|\cos \omega t |01,12,03,14\rangle + i \sin \omega t |11,02,13,04\rangle\) and \(|11,02,13,04\rangle\) evolves in \(|\cos \omega t |11,02,13,04\rangle + i \sin \omega t |01,12,03,14\rangle\) where \(\omega = 2w^2/U\). Thus, the whole state becomes

\[
|\Psi(t)\rangle_{123456} = \frac{1}{\sqrt{2}} (|01,12,03,14\rangle |\chi^+\rangle_{56} + \]

\[i |11,02,13,04\rangle |\chi^-\rangle_{56}) \quad (11)
\]

having introduced \(|\chi^\pm (t)\rangle_{56} = |\langle 05,16| \pm i |\langle 15,06|\rangle \rangle\rangle\). Waiting a suitable time (\(\omega t = \pi/4\) we obtain, associated with two orthogonal computational states on the four Alice’s dots, \(\alpha |05,16\rangle + i \beta |15,06\rangle\) and \(\alpha |05,16\rangle - i \beta |15,06\rangle\). Measuring the charge on a dot (e.g. the number 1), Alice transmits the result as classical bit to Bob, that can choose the correct unitary rotation to perform in order to completely recover \(|\chi\rangle\) on its site. Note that due to the nonlinearity of interactions involved in this model, there are no conceptual obstacles for which Bell measurements cannot reach a 100% of success probability.

The teleportation protocol described allows an information transfer over typical interdot distances, but it’s straightforward to extent this model on a array of \(N\) DQDs aligned and coupled one-to-one by Coulomb interaction. In effect, the state of last \(N-1\) DQDs can be arranged, by means of adiabatic couplings between them, in a maximally entangled state

\[
\frac{1}{\sqrt{2}} (|0,0,1,0,1,0,\ldots,0,1\rangle + |1,0,1,0,1,0,\ldots,1,0\rangle)
\]

(12)
The coupling with the first, adjunctive, DQD used to encode a qubit, like in the foregoing discussion, will drive the system in the generalization of Eqn. 10 allowing the final step of teleportation. Thus, we deal with a quantum channel with high fidelity: losses are only due to contributes to ground state coming from the time evolution such as in Eqn. 5 and can be strongly reduced with a suitable choice of coupling parameters.

IV. CONCLUSIONS

To summarize, we have proposed a solid state implementation of a quantum teleportation protocol. Qubits are represented by DQDs coupled by tunneling with one excess electron staying coherently in the first or in the second dot. Coherent oscillations in these structures, experimentally observed, show how it’s possible to conceive a network of DQDs. In this paper we have illustrated a scheme to generate entanglement between adjacent qubits using adiabatic changes in the system parameters. Extending this technique also to nonadiabatic variations we have formulated a proposal for optimal quantum teleportation. Furthermore, this technique can be generalized to an array of $N$ DQDs embedded in a chain, allowing the transmission with high fidelity over relatively long distances of the unknown qubit.

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