RESEARCH ARTICLE

Optimization Method on the Tuning, Sound Quality, and Ergonomics of the Ancient Guitar Using a DSP-FEM Simulation

SPYROS POLYCHRONOPOULOS1, KONSTANTINOS BAKOGIANNIS1,2, DIMITRA MARINI1, AND GEORGIOS TH. KOUROUPETROGLOU1, (Member, IEEE)

1Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, 15784 Athens, Greece
2Department of Music Studies, National and Kapodistrian University of Athens, 15784 Athens, Greece

Corresponding author: Georgios Th. Kouroupetroglou (koupe@di.uoa.gr)

This work was supported by the European Regional Development Fund of the European Union and Greek National Funds through the Operational Program Competitiveness, Entrepreneurship, and Innovation (under the call Research-Create-Innovate Project MNESIAS: “Augmentation and Enrichment of Cultural Exhibits via Digital Interactive Sound Reconstitution of Ancient Greek Musical Instruments”) under Grant T1EDK-02823/MIS 5031683.

ABSTRACT The key parameters musicians consider when they evaluate the quality of a musical instrument are its tuning (reproducing a musical scale), sound quality, and ergonomics. Musical instrument makers, even up to this day, primarily base their practice on empirical knowledge and costly physical experimentation. A computational model that combines all the aforementioned key parameters is presented to predict the building details of an instrument. The simulation of the instrument is introduced to a multi-objective optimizer to calculate its optimal set of geometrical and material features, considering the importance of the three key parameters. For a string musical instrument, this approach is based on a hybrid model of Digital Signal Processing simulating the vibrating string and of Finite Elements Method simulating the effect of the body. The simulation technique has been validated by building an ancient guitar and comparing its recordings with its analogous digital model. The proposed method can be put into practice to investigate the building details of any instrument by introducing the relevant simulation and the objective’s function parameters. This work is expected to provide a powerful tool for the musical instrument makers towards a more efficient design of a bespoke instrument.

INDEX TERMS Ancient guitar, digital signal processing, finite elements method, musical acoustics, optimization, simulation.

I. INTRODUCTION Musical instruments have always been an essential element of every human civilization. Hence, they always attract the interest of scientists in various fields. The most popular studies focus on the physics governing their sound production mechanism [1] the musicians’ impact on the generated sound [2] and their interaction with the instrument [3], the enrichment of cultural exhibits [4], and the digital simulation of existing [1] or conceptual ones [5]. The musical instruments’ accurate simulation, even up to this day, is not trivial. Firstly, because of the complicated physical phenomena governing the sound generation and, as a result, secondly, because of the computationally expensive algorithms required for their description. Early attempts at the beginning of the 1990s opened the field productively for discussion [6]. Nevertheless, the models could not accurately reproduce the sound of the physical instrument. Over the past decades, a better understanding of their mechanisms, as well as the development of computational power, brought about more promising techniques based on Digital Signal Processing (DSP), Finite Differences Methods (FDM) [7], [8], [9], [10], and machine learning [11], [12]. The field attracted both academic and industry researchers, leading to systems and...
new applications that go beyond the accurate simulation of a musical instrument [13], [14].

Depending on the way an instrument creates sound can be practically classified into four categories: idiophones, membranophones, aerophones (i.e., wind instruments) and chordophones (i.e., string instruments). In this work, the case study instrument is the ancient guitar which can be classified as a chordophone. Chordophones can be studied in two parts: the excitation mechanism (i.e., the string) and the resonator (i.e., the body). The excitation mechanism has been thoroughly studied, and various simulation techniques are available [15], [16], [17]. However, because of the complicated shape, structure, and material properties of the body the string is attached to, the simulation of the instrument is not trivial. The first attempts to study the resonances of a violin using magnetic pickups attached to the bridge [18] or spatially averaged measurements of direct radiation from a violin excited by force-hammer impact at the bridge using a rotating semicircular [19] require the instrument in its physical form. More recent works depend on Finite Elements Method (FEM) to model a classical guitar [20] where the string oscillation is transferred through the bridge to the body, which then interacts with the enclosed and surrounding air to radiate the instrument’s tone. Nevertheless, the coupling of the string-body is not trivial, and it is still an active field of research [21], [22], [23].

Exploring the simulation potentials, researchers created user interfaces of the virtual instruments [24], that can simulate their sound in real spaces [25], [26], [27], or even tried to better understand the performance of state-of-the-art instruments (e.g., Stradivarius violins using FEM [28]). Furthermore, the simulation of clarinet, using mathematical models based on impedance, was introduced in a numerical optimizer to propose a geometry that fulfills the requirements regarding the generated frequencies [29]. However, this method is limited to a single objective goal, which is the set of fundamental frequencies and their first harmonics, overlooking their relevant amplitude (significant for the instrument’s timbre) and the ergonomics of the instrument. Using FEM to simulate musical instruments can be more accurate than analytical equations allowing the study of instrument’s parts with precision [30], nevertheless, it is not a computationally cheap method that can run several iterations fast in an optimizer.

Using a simulation model based on FEM, the shape of an acoustic horn was optimized as a minimization problem of an objective using a Broyden–Fletcher–Goldfarb–Shann (BFGS) quasi-Newton algorithm concerning its ability to provide impedance matching to the surrounding air [31]. Another study obtained a target set of modal frequencies for vibraphone or marimba-type bars [32]. This study was based on FEM and eigenanalysis coupled with optimization procedures. However, none of the above two works consider all three key parameters to provide an optimal set of building details.

This paper introduces a hybrid simulation (DSP and FEM) method of a string instrument along with a multi-optimization technique based on which the user first rates the importance of the instrument’s key parameters (i.e., tuning, sound quality, and ergonomics), then sets the limits for the modifiable building details (i.e., material and geometrical features), and the algorithm returns an optimal set of building parameters that fulfill the chosen criteria. The case study instrument is the ancient guitar (also known in ancient Greece as Phorminx) which is a wooden string musical instrument. A set of nine material properties was used (see Table 1) to correspond the proposed material properties to commonly used material for the body, as well as to reduce the runtime of the simulation which is introduced to the multi-objective optimizer.

Until this day, the study of old instruments is a pole of attraction. Recently, researchers formulated a model of the instrument [33] based on FEM to investigate the acoustic efficiency of the thickness of the top plate of the early viola da gamba. In this paper the case of the ancient guitar is shown. Homer, when referring to the ancient Greek guitar, commonly used the word Phorminx. It is the instrument of the singers of the Homeric period but also of the muses in the ancient and classical times. It has two symmetrical arms inserted into a hollow, horseshoe shape, and generally wooden resonator. The resonator’s upper inner part has a transverse semicircular section shown in Fig. 1 and Fig. 2a. The front flat surface of the resonator ensures the perfect fit of the resonator with the wooden vibrating surface of the instrument (the soundboard). A metal tailpiece was used to secure the sheep-gut strings to the resonator. The instrument was played either with a bone, ivory, or metal pick or with the fingers of the left and the right hand by a usually centrally positioned player who could sit, march or dance [34].

In this work a method to simulate string instruments and optimize their geometry according to the importance of three key parameters is being proposed. The simulation method described in this work covers the instruments that fall under the chordophone class, showing challenges in the simulation method [35]. However, the proposed multi-optimization technique can be applicable, considering the relevant simulation and the objective’s function parameters, for the rest of the three classes: idiophones [36], [37], membranophones [38], [39], and aerophones [6], [10], [40].

The structure of the current work is as follows: first, the simulation of the ancient guitar and the numerical optimizer are described in detail in the methods section. Then, in the results section the simulation verification and the optimization results for four cases are presented. Finally, the conclusion section summarizes the main findings of this work, and provides a discussion for future works.

II. METHODS

A. INSTRUMENT’S SIMULATION

String instruments can be divided in two functional parts: the string and the body. Each one of them plays its role in the process of sound production. The instrument’s body (in this work assumed to be a linear mechanical-acoustic system)
transforms the forces imposed by the string’s vibration, through the bridge, into sound pressure waves that propagate in the air. Each element has been simulated independently, and then the two generated signals have been combined to obtain the final signal of the instrument. The vibrating string is simulated using Digital Waveguides, and the impulse response of the body is calculated using FEM. Next, the final signal \( S(t) \) is calculated by exciting the string model with the body’s impulse response \( h(t) \) [41]. This is the main idea of commuted synthesis which is computationally cheaper since the body filter can be replaced by an inexpensive lookup table [17] of precalculated impulse responses (in this case, generated using FEM models). In future work, creating a less computationally expensive FEM model (or reduce the runtime using parallel computing) will allow it to be part of the optimizer and run iteratively. In this case, the model will be able to propose modifications for the geometrical features of the body of the instrument as well.

The string is oscillated by three triggering mechanisms: 1) via percussion [42] (e.g., piano), 2) via pulling, either with fingers [15] or with a pick [43] (e.g., guitar), and 3) via a bow [44] (e.g., cello). The string is the primary vibrating element of the instrument as it determines the frequency response of the produced sound. However, the sound energy produced is weak and needs amplification to meet the performance needs of the instrument. This amplification can be achieved by attaching the string to a resonator (i.e., the instrument’s body). The string’s vibration is transferred to the soundboard (the front surface of the resonator on which the bridge is mounted). For instruments without an opening, through which the air enclosed in the resonator communicates directly with the surrounding air (e.g., piano), the string’s vibration is transmitted approximately only to the surface. On the other hand, for instruments with an opening (i.e., acoustic guitar), the resonator can be seen as a coupled system of a vibrating surface with a Helmholtz resonator. Thus, in this kind of musical instruments, the coupling with the surrounding air must be included in the study of the vibration of the soundboard [45]. It should be noted that the selected case study instrument’s (ancient guitar) body does not have an opening. Furthermore, as the key parameters (tuning, sound quality, and ergonomics) of the instrument are not affected by the sympathetic string vibrations (when one string is excited, some others are also excited via the body), the phenomenon was not taken into account in the simulation. Using the proposed hybrid (DSP & FEM) model the instrument’s sound production mechanism is simulated.

The simulation of the ancient guitar was based on a hybrid technique. The string was simulated using a DSP model of digital waveguides [46] (see DSP box, Fig. 1) and the body using a 3D model, designed using Autodesk 3ds Max 2020, introduced in COMSOL Multiphysics 5.5 Acoustics Module for FEM analysis (see FEM box, Fig. 1). Figure 1 illustrates three seconds of the final simulated instrument’s signal expressing the displacement of the bridge \( S(t) \). It is calculated by exciting the string model with the body’s impulse response, calculated by the FEM model, expressed as displacement in the time domain \( h(t) \), Fig. 1. In order to validate the proposed simulation model, a replica of the ancient guitar with a 43.5 cm strings’ length was physically built (Fig. 1, red box and Fig. 2a) at the premises of the Speech and Accessibility Laboratory, Department of Informatics and Telecommunications, National and Kapodistrian University of Athens Greece based on the description and the images considering the relevant literature [47]. For more details regarding the validation of the proposed model, see second paragraph section III.

1) STRING

The vibration of an ideal flexible string, with both ends fixed, can be expressed, as transverse waves in a uniform chord [1], by (1)

\[
\frac{\partial^2 y}{\partial t^2} = \frac{T}{\epsilon} \frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial x^2},
\]

where \( x \) is the position measured along the string, \( c = \sqrt{T/\epsilon} \) is the velocity of a transverse wave on the string which is stretched by a tension \( T \) and has a linear density \( \epsilon \), and \( y \) is the transverse displacement of the string. By considering a string that is fixed at one end and the other end moves up and down, a pulse will be created which will move at speed \( c \) and maintain its shape as it moves along the string. When this pulse reaches the end of the string it will be reflected backwards creating a standing wave. In a typical string instrument, the nut/fingerboard end of the string is commonly assumed that is an ideal termination [1]. The bridge abuts on the instrument’s body and moves along with the body. Thus, this end of the string cannot be considered motionless. Further, this is the point where, for the vibrating string, a slight energy loss is introduced. Other losses are related to the movement of the
string such as the internal damping due to friction and the viscous losses as the string cuts the air [41]. The plucking position of the string plays a vital role regarding the overtones [1]. For example, plucking the string at a quarter of the distance from the end eliminates the fourth partial. In the model of this work, this parameter has a logical constant value: a distance of 4.5 cm from the bridge.

In this work, the string is modeled in 1D modeling only one of the transverse string waves, using digital waveguides (comprising digital delay lines). This is a well-established technique for audio synthesis and a commonly used one, especially in physical modeling [46], [48], [49]. The length of the digital delay line is \( N \) samples corresponding to the physical length of the string, \( N = \left\lfloor \frac{L}{T} \right\rfloor \), where \( L \) is the string’s length, \( T \), the sampling period, and \( \lfloor \cdot \rfloor \) denotes the rounding to the nearest integer. The frequency calculation error introduced by the discretization of the physical length of the string, is minimized by using fractional delay filtering techniques [50]. The technique used here is a third-order Lagrange interpolation implemented as a filter \((L(z))\), see Fig. 1). The damping of the string is frequency depended and it is simulated using a loop filter \((H_L(z))\), see Fig. 1). For the filter to be realistic and reflect the case of an ancient guitar its parameters were extracted using a recorded signal from the replica [51]. This resulted in a first-order IIR filter \( H_L(z) = \frac{0.8817 + 0.0513 z^{-1}}{1 - 0.0465 z^{-1}} \). The nodal effect of the plucking position is simulated by a comb filter consisting of a \( M \)-samples delay line (see Fig. 1), where the delay corresponds to the time it takes for the excitation to travel from the plucking position to the nut [17].

2) BODY

The string is the primary vibrating element of a string instrument as it determines the frequency content of the produced sound. However, the sound energy produced by a vibrating string is weak and needs amplification to serve the instrument’s purpose. This amplification is achieved by attaching the string to a solid element, the instrument’s body. The string’s vibration is transferred through the bridge to the rest of the body, which produces the instrument’s sound. Beyond the distinct study of the string and the body, a complete study of the produced sound should consider the role of the bridge [52]. In previous works, the instrument’s body was studied by hammer-excitation [19] or attaching magnetic pickups on the bridge [18]. In this work, as the building details of the body are not known in advance and vary at every iteration of the optimizer, techniques that require the instrument on its physical form are not relevant.

The impulse response of the body of the instrument was studied using FEM in COMSOL Multiphysics 5.5 Acoustics Module. A picture of the replica showing the dimensions of the instrument, its 3D model (using Autodesk 3ds Max 2020), the mesh, and an instance of the body’s response (displacement in cm) after exciting the bridge with a short Gaussian pulse of 0.01 seconds to simulate a Dirac delta excitation, are shown in Fig. 2a, b, c, and d, respectively. The FEM model is solved in the time domain, and its inputs are the building details of the body (i.e., geometrical features and material properties). The model outputs the impulse response at the bridge expressed as a displacement in the time domain. The mesh was carefully chosen considering the building details of the instrument and the expected maximum frequency (at least six elements along a wavelength) of interest (2 kHz), resulting in 46136 elements and 250194 degrees of freedom. The model’s runtime to output an impulse of 0.3 s at a sample rate of 44100 Hz was approximately one hour using a Dell desktop computer with an i-5 CPU at 3.1 GHz and 8 GB of RAM.

The material of the body is wood which is a complex natural material. However, for simulation purposes, in this work, it is modeled as a homogeneous orthotropic elastic material (simulating the relative damping by an orthotropic loss factor). The properties of nine different, commonly used by instrument makers, types of wood [53] considered in this work are shown in Table 1. Their impulse responses are precalculated and introduced in the optimizer as a lookup table.

The ten lowest modes of the instrument, using the nine materials shown in Table 1, were calculated using FEM eigenfrequency analysis. The model shares the same simulation parameters as described above and takes less than 1 min per material to calculate the natural frequencies (Table 2). As can be concluded from Table 2 the greater the material’s stiffness the lower the pitch of the natural frequencies. The first ten vibration modes, using the first material (Black Walnut wood), are illustrated in Fig. 3. The patterns shown are governed by the presence of fixed (motionless) elements (e.g., the neck above the soundboard) and the presence of the bridge. The position of the bridge on the soundboard determines the antinodal zones.

3) STRING – BODY COUPLING

The bridge couples the string’s motion to the body. The body can be modeled as a radiation filter expressed via its impulse response. The FEM model, as shown above, outputs the body’s impulse response expressed as displacement in the time domain \((h(t))\). Given that the model is linear and time-invariant, commuted synthesis can be used [17] and excite the string model with the body’s impulse response [41] (see
TABLE 1. Young’s Modulus, Shear Modulus, Poisson’s Ratio, and density of nine commonly used materials by the instrument makers used in this work for the ancient guitar’s body: 1: Black Walnut, 2: African Mahogany, 3: Honduras Mahogany, 4: Sitka Spruce, 5: Engelmann Spruce, 6: Northern White Cedar, 7: Western Red Cedar, 8: Redwood, and 9: Douglas-fir.

| Material | Density [kg/m^3] | Young’s Modulus [GPa] | Shear Modulus [GPa] | Poisson’s Ratio |
|-----------------|-----------------|-----------------|-----------------|----------------|
|                 | E_x             | E_y             | E_z             | G_xz           | G_yx           | G_zy           | ν_xz           | ν_yx           | ν_zy           |
| 1               | 610             | 12.76           | 0.715           | 1.353          | 1.085          | 0.791          | 0.268          | 0.495          | 0.632          | 0.718          |
| 2               | 640             | 10.67           | 0.533           | 1.184          | 0.939          | 0.630          | 0.224          | 0.224          | 0.641          | 0.604          |
| 3               | 590             | 11.33           | 0.725           | 1.212          | 0.748          | 0.974          | 0.317          | 0.314          | 0.533          | 0.600          |
| 4               | 425             | 10.89           | 0.468           | 0.849          | 0.697          | 0.664          | 0.033          | 0.372          | 0.467          | 0.435          |
| 5               | 385             | 9.79            | 0.578           | 1.253          | 1.214          | 1.175          | 0.098          | 0.422          | 0.462          | 0.530          |
| 6               | 350             | 6.05            | 0.490           | 1.107          | 1.270          | 1.131          | 0.091          | 0.337          | 0.340          | 0.458          |
| 7               | 370             | 8.47            | 0.466           | 0.686          | 0.737          | 0.728          | 0.042          | 0.378          | 0.296          | 0.484          |
| 8               | 415             | 10.12           | 0.901           | 0.880          | 0.668          | 0.779          | 0.111          | 0.360          | 0.346          | 0.373          |
| 9               | 510             | 14.74           | 0.737           | 1.002          | 0.943          | 1.150          | 0.103          | 0.292          | 0.449          | 0.390          |

TABLE 2. Ten lowest vibration modes of the body materials calculated using eigenfrequency analysis in FEM where 1: Black Walnut, 2: African Mahogany, 3: Honduras Mahogany, 4: Sitka Spruce, 5: Engelmann Spruce, 6: Northern White Cedar, 7: Western Red Cedar, 8: Redwood, and 9: Douglas-fir.

| Material | Natural Frequencies [Hz] |
|----------|--------------------------|
| 1        | 249 400 566 627          |
| 2        | 280 466 626 741          |
| 3        | 295 462 683 708          |
| 4        | 329 546 741 854          |
| 5        | 297 516 655 820          |
| 6        | 296 480 673 749          |
| 7        | 321 549 707 890          |
| 8        | 327 526 747 817          |
| 9        | 282 460 639 726          |

Fig. 1). The proposed simulation method was found to be valid, for the purpose of this project, as the comparison of the signals between the physical instrument vs the virtual instrument show good compliance (for more details see subsection III-A).

B. OPTIMIZER

The optimizer is a controlled elitist genetic algorithm running on MATLAB R2021b using the Optimization Toolbox to try a new set of variables ($V_j$) at every iteration ($j$) to minimize the sub-objectives ($G_{11}$, $G_{21}$, $G_{31}$ and $G_{32}$). When the optimizer finishes the calculations, the sub-objectives are combined and normalized between 0 and 1 to form the key parameters $G_1$, $G_2$, and $G_3$. Then, considering the significance of each parameter (weighting factors), the multi-objective function is calculated by (7), see Fig. 4. A multi-objective function has been used firstly because it is more flexible to independently consider the significance of each sub-objective, and secondly, because the values of the sub-objectives need to be comparable thus, were normalized. Forming a single-objective function for the optimizer [54] would not fulfill the above two points.

Sub-objective $G_{11}$ (tuning) examines the tuning accuracy by calculating the absolute deviation between the simulated fundamental frequencies ($f_s^{0}$) and the corresponding fundamental frequencies of the goal musical scale ($f_{ms}^{0}$). The final value of this sub-objective is the average absolute deviation of all the strings. Minimizing $G_{11}$ results in better tuning.

$$G_{11} = \frac{\sum_{k=1}^{K} |1200 \log_2 \frac{f_s^{0}}{f_{ms}^{0}}|}{K}, \quad (2)$$

where $k$ is the string’s index, $K$ is the number of strings (in this work $K=7$), and $|$ $|$ denotes the absolute value.

Sub-objective $G_{21}$ (sound quality) examines whether the relative resonant frequencies between two signals similarly affect the timbre and is calculated by considering the resonant frequencies (fundamentals and overtones) amplitudes. It is the average (mean) correlation between a reference and a generated matrix of all the stings ($K$). The matrices are comprised of the resonance numbers arranged considering their amplitudes in descending order. In this work, regarding the reference matrix ($I_{ref}$), as lower the number of the partial is, the higher significance respectively it holds, the order will be: fundamental, first overtone, second overtone, etc., resulting in the matrix $[0, 1, 2, \ldots, M]$. Maximizing the correlation (or minimizing the minus correlation ($G_{21}$) of these matrices results in timbres of which the resonant frequencies share a
similar amount of contribution compared to an ideal contribution of resonant frequencies, considering their amplitude.

\[ G_{21} = \text{mean}_{K}(\text{-correlation}(I_{\text{ref}}, I_k)), \]  

(3)

where \( I_{\text{ref}} = [0, 1, 2, \ldots, M] \) is the reference matrix and \( I_k \) is the generated matrix from the signal of the simulated instrument. It should be noted here, that since sound absorption in the air increases with frequency [55] and considering that in the reference matrix partials’ significance decreases with frequency, sound radiation will only modify the amplitude of the resonances and not their relative order.

Sub-objective \( G_{31} \) is the first part of \( G_3 \) (ergonomics). It computes the absolute difference between the strings’ length for the simulated instrument and the reference (optimal) length, in meters. Minimizing the absolute difference of the strings’ length from a reference ideal length results in a proposed length as close as possible to the reference one and hence, better ergonomics.

\[ G_{31} = |L^s - L_{\text{ref}}|, \]  

(4)

where \( L^s \) is the simulated instrument’s strings’ length, and \( L_{\text{ref}} \) is the reference instrument’s strings’ length, in meters.

Sub-objective \( G_{32} \) is the second part of \( G_3 \) (ergonomics). It computes the standard deviation of the strings’ tension. By minimizing the standard deviation of the strings’ tension, results with highly balanced tensions are derived.

\[ G_{32} = \sqrt{\frac{K}{\sum_{k=1}^{K} (T_k^s - \mu)^2}}, \]  

(5)

where \( T_k^s \) is the string’s tension of the simulated instrument, \( \mu = \frac{\sum_{k=1}^{K} T_k^s}{M} \).

The objectives \( G_1 \) (tuning), \( G_2 \) (sound quality), and \( G_3 \) (ergonomics) are calculated by:

\[ G_1 = \hat{G}_{11}, \quad G_2 = \hat{G}_{21} \] and \[ G_3 = \frac{G_{31} + G_{32}}{2}, \]  

(6)

where \( \hat{\text{mean}} \) denotes the normalization between 0 and 1.

III. RESULTS AND DISCUSSION

A. INSTRUMENT’S SIMULATION

The ancient guitar with a musician exciting each of the seven strings was recorded, using his fingers at a distance of 4.5 cm from the bridge. The microphone was placed approximately 1 m away from the instrument, and the musician played at piano (soft) volume level every note once. The recordings took place at the studio of the Laboratory of Music Acoustics and Technology, Department of Music Studies, National and Kapodistrian University of Athens, on March 4th, 2021. The equipment used: microphone preamplifier Millennia HV-3D, A/D converter RME ADI A8 DS, and microphone Neumann KM 184.

For this work, the simulation’s primary goal is to accurately output the ancient guitar’s resonant frequencies (fundamental and overtones) and their relative amplitudes. Figure 1 illustrates the comparison of the signals in the frequency domain of the simulated and the physical instrument show good agreement. In more detail, the signals of the seven notes of the simulated and the physical instrument (replica) were compared. The absolute deviation in cents (\( \psi \)) of the first partials’ frequencies (fundamental frequencies) shows an average deviation of 0 \( \psi \) and a standard deviation of 0 \( \psi \). The second partials (first overtones) show an average deviation of 4 \( \psi \) and a standard deviation of 7 \( \psi \); the third partials.
(second overtones) show an average deviation of 2 \( \varsigma \) and a standard deviation of 4 \( \varsigma \); the fourth partials (third overtones) show an average deviation of 7 \( \varsigma \) and a standard deviation of 8 \( \varsigma \); the fifth partials (fourth overtones) show an average deviation of 5 \( \varsigma \) and a standard deviation of 8 \( \varsigma \). All the above frequency mismatches can be considered perceptually insignificant as they are well below the Just Noticeable Difference (JND) [56], [57]. Further, the relative amplitudes of the resonant frequencies (fundamentals and overtones) are in descending order from the fundamental (highest amplitude) to the fifth partial (lowest amplitude) in both the recording and the simulated instrument’s signals. Considering the above the proposed simulation is accurate and serves the purpose of this project.

**B. OPTIMIZER**

The proposed simulation of the ancient guitar is added to a multi-objective optimizer to predict an optimal set of building details by considering the three key parameters (tuning, sound quality and ergonomics). In this work, the varying parameters are the strings’ tension, length, linear density and the body’s material. The case study musical instrument has seven strings. Therefore, there are fourteen variables for the strings’ tension and the linear density, one variable for the strings’ length (same length for all strings), and one variable for the body’s material. It should be noted here, that in order for the proposed material to reflect a commonly used one, as well as to reduce the runtime of the simulation, a set of nine materials was used (for more details, see the Body subsection of the Methods section).

The multi-objective function includes three sub-objectives (see (7)). \( G_1 \) is responsible for the successful reproduction of the instrument’s tuning (i.e., the notes of a predetermined musical scale). It is defined here as the deviation between the calculated (via simulation) and the goal fundamental frequencies. Although the musician can adjust the tuning by altering the strings’ tension, the ability of the instrument to reproduce a specific musical scale depends primarily on instrumental characteristics. In this case these characteristics are the length of the phorminx arm determining the length of the strings and given this length, the choice of the set of strings (expressed as strings’ linear density). In each iteration the optimizer calculates the deviation (\( G_{11} \), in cents) of the fundamental for each string of the signal generated by the simulated instrument and the set of frequencies corresponding to the preferable tuning. The selected tuning of the seven strings of the ancient guitar (named in antiquity as hypatē, parhypatē, lichanos, mesē, triē, paranē, nētē) corresponds to a natural minor hexatomic scale starting from the musical note E3 (i.e., E3, F♯3, G3, A3, B3, D4, E4) which is consistent with the standard tuning process of the ancient Greek music system (i.e., hypatē and mesē forming a perfect fourth interval, while mesē and nētē forming a perfect fifth interval) [58]. \( G_2 \) is responsible for the sound quality, and it is defined as the order of the resonant frequencies (\( G_{21} \) (fundamentals and overtones) considering their amplitude. The goal for the partials’ amplitude levels is be in descending order from the fundamental to the fourth overtone. \( G_3 \) is responsible for the ergonomics considering the comfort of the musician playing the instrument and it is defined here in two parts as a) \( G_{31} \): the difference between the string’s length and an ideal one, and b) \( G_{32} \): the string’s tension standard deviation. Based on the relevant literature for the ancient stringed instruments [59] the ideal string length was set to 43.5 cm. Moreover, as a balanced string’s tension is essential for a natural and balanced feel when playing the instrument, one more goal for the optimizer was set to minimize the differences in the strings’ tensions. Note that the sub-objectives \( G_{11}, G_{21}, G_{31}, \) and \( G_{32} \) are normalized between 0 and 1 and contribute equally to calculate \( G_1, G_2, \) and \( G_3 \). The importance of each of the three parameters, \( G_1 \): tuning, \( G_2 \): sound quality and \( G_3 \): ergonomics, is expressed using weighting factors \( w_{(1-3)} \) (see (7)). For more details about the multi-objective function (\( O \)) please see the Optimizer subsection of the Methods section.

\[
O = w_1 G_1 + w_2 G_2 + w_3 G_3, \tag{7}
\]

The multi-objective function was introduced in a controlled elitist genetic algorithm running on MATLAB R2021b using Optimization Toolbox. To minimize the sub-objectives: \( G_{11}, G_{21}, G_{31}, \) and \( G_{32} \) the optimizer was created until the termination of 104 generations, consisting of 200 individuals, resulting in a total population of 20800 individuals. The runtime for a single generation was approximately 10 minutes on a desktop computer with i5-8600 CPU at 3.10 GHz, 8 GB of RAM, and an MS-Windows 10 Operating System. When the optimizer calculates all the generations, the values of the sub-objectives’ values are normalized between 0 and 1 (see Fig. 5) and form the main objective (see (7)). Then considering the significance of each of the key parameters (using the weighting factors), the optimal solution is selected, and the corresponding building details (\( V \): optimal set of variables) is the output (see Fig. 5).

Figure 5 illustrates the proposed method to modify a set of variables (musical instrument’s building details) of an ancient guitar for optimal \( G_1 \): tuning, \( G_2 \): sound quality and \( G_3 \): ergonomics, considering the significance (using weighting factors) of each of the three key parameters. The inputs in Fig. 5 include: the weighting factors, the musical scale that the instrument should reproduce, the optimal strings’ length (based on the ergonomics), the lower and upper bound for the strings’ length, the lower and upper bounds for the strings’ tension and linear density (seven strings), and a set of materials for the body of the instrument. In the calculation section the sub-objectives (\( G_{11}, G_{21}, G_{31}, \) and \( G_{32} \)), the whole population of all generations (grey dots) of the multi-objective optimizer plotted in a 3D plot where the x axis is \( G_1 \), y axis is \( G_2 \) and z axis is \( G_3 \), four cases of different weighting factors (significance of the key parameters) and the relevant sub-objectives’ values are shown. In this work four cases of a different set of weighting factors (case A: only the tuning is important, case B: only the sound quality is important, case C: only the ergonomics is important, and case D: 25%
importance for tuning, 45% importance for sound quality and 30% importance for ergonomics) with their relevant sub-objectives ($G_{11}$, $G_{31}$, and $G_{32}$) and $G_1$, $G_2$, and $G_3$ are shown. In case A (blue), where $G_1$ is the only relevant sub-objective, $G_{11} = 3.4 \times 10^{-3}$ a tuning mismatch lower than JND and in all the rest of the cases where it is non-significant (case B and case C) or less significant (case D) $G_{11}$ values are higher. In case B (red), where $G_2$ is the only relevant sub-objective, $G_{21} = -1$ i.e., optimal order of the resonant frequencies. In all the rest of the cases where it is non-significant (case A and case C) or less significant (case D) $G_{21}$ values are higher. In case C (green), where $G_3$ is the only relevant sub-objective, $G_{31} = 0.02$ mm and $G_{32} = 0$ N i.e., optimal string’s length and a low strings’ tension variance. In all the rest of the cases where it is non-significant (case A and case B) or less significant (case D) $G_{31}$ and $G_{32}$ values are higher. In case D (black), a random set of weighting factors are shown illustrating a more realistic resulting in $G_{11} = 10.8 \times 10^{-3}$, $G_{21} = -1$, $G_{31} = 0.02$ mm, and $G_{32} = 0.005$ N. The output of the method is a set of instrument’s building details (optimal variables $V$), that meet the selected criteria, one for each case of weighting factors. The optimal building details ($V$) for each case are illustrated in Fig. 5, where the set of values are: one value for strings’ length, seven values for strings’ tension (the values in the first parenthesis), seven values for strings’ linear density (the values in the second parenthesis), and one value for body’s material as per Table 1.

**IV. CONCLUSION**

This study proposes a method to simulate stringed musical instruments and determine their building details to optimize their key parameters (tuning, sound quality and ergonomics). The simulation of any string musical instrument becomes trivial as the proposed method requires just a 3D model of the body (which can be either designed using a 3D software or a 3D scanner to obtain the geometrical features of a physical instrument) and the strings’ parameters. An ancient guitar has been used as a case study instrument. The simulation was validated by building the instrument and comparing its recordings with the signal generated by the proposed physical model. However, the method is not limited to string instruments. It can be put into practice to investigate the building details for any instrument by introducing the relevant simulation and modifying the multi-objective’s parameters to correspond to the specific instrument.

In this work the proposed method was put into practice for four cases, where the importance of the key parameters varies, to obtain the relative building details for the optimized ancient guitar. As a future work, subjective tests could be conducted, including building this work’s four optimized instruments and evaluating their performance according to the goals of each case with musicians.

We anticipate that this work will help musical instrument makers to more efficiently design a new bespoke instrument that could be introduced to other models to simulate the sound of the instrument in a real space. In order to optimize the key parameters of a new instrument, the method described here does not require a physical musical instrument, which is relatively expensive and time consuming to build. Moreover, the building details of an instrument can be customized considering the musician’s specific needs. This work’s method can propose building details for a new musical instrument that...
will be able to reproduce a goal 1) musical scale, 2) unique timbre, and 3) specific ergonomics for the musician (e.g., considering the impairment of a musician). Archaeomusicologists can also use the proposed method for excavated (or conceptual) musical instruments. As most of the excavated instruments are not in one piece (or they have missing parts), this work can reveal the missing information by considering details such as the possible musical scale, the human anatomy, and the available materials.

REFERENCES

[1] T. Fletcher and N. Rossing, *The Physics of Musical Instruments*, 2nd ed. New York, NY, USA: Springer, 1998.

[2] K. Bakogiani, S. Polychronopoulos, D. Marini, C. Terzis, and G. T. Kouroupetrogrou, “ENTROTUNER: A computational method adopting the musician’s interaction with the instrument to estimate its tuning,” *IEEE Access*, vol. 8, pp. 53185–53195, 2020, doi: 10.1109/ACCESS.2020.2981007.

[3] S. Papetti and C. Saitis, *Musical Haptics*. Cham, Switzerland: Springer, 2018.

[4] G. T. Kouroupetrogrou, S. Polychronopoulos, and K. Bakogiani, “Augmentation and enrichment of cultural exhibits via digital interactive sound reconstruction of ancient Greek musical instruments,” *Archeol. e Calcolatori*, vol. 32, no. 1, pp. 423–438, 2021, doi: 10.19282/ac.32.1.2021.23.

[5] M. V. Mathews, “The digital computer as a musical instrument,” *Science*, vol. 142, no. 3592, pp. 553–557, Nov. 1963, doi: 10.1126/science.142.3592.553.

[6] J. O. Smith III, “Physical modeling synthesis update,” *Comput. Music J.* vol. 20, no. 2, pp. 44–56, 1996, doi: 10.1121/1.1913511.

[7] J. O. Smith III, “Physical modeling synthesis update,” *Comput. Music J.* vol. 29, no. 6, pp. 556–571, 2008, doi: 10.1121/10.0001416.

[8] J. A. Torres, C. A. Soto, and D. Torres-Torres, “Exploring design variations of the Titian Stradivari violin using a finite element model,” *J. Acoust. Soc. Amer.* vol. 148, no. 3, pp. 1496–1506, Sep. 2020, doi: 10.1121/10.0001952.

[9] D. Noreland, J. Kergomard, F. Laloé, C. Vergez, P. Guillemain, and A. Guilloteau, “Exploring design variations of the Titian Stradivari violin using a finite element model,” *Acta Acustica Unaita Acustica*, vol. 99, no. 3, pp. 615–628, 2013.

[10] A. Lefebvre and G. P. Scavone, “Characterization of woodwind instrument toneholes with the finite element method,” *J. Acoust. Soc. Amer.*, vol. 131, no. 4, pp. 3153–3163, Apr. 2012, doi: 10.1121/1.3658481.

[11] E. Bangtsson, D. Noreland, and M. Berggren, “Shape optimization of an acoustic horn,” *J. Acoust. Soc. Amer.*, vol. 192, pp. 1533–1571, Mar. 2021, doi: 10.1121/0004-7825/2021/00656-4.

[12] L. L. Henriques and J. Antunes, “Optimal design and physical modelling of mallet percussion instruments,” *Acta Acustica Unaita Acustica*, vol. 89, no. 6, pp. 948–963, 2003.

[13] V. Chatziioannou and A. Georgaki, “Physical modeling of the ancient Greek wind musical instrument aulos: A double-reed exciter linked to an acoustic resonator,” *IEEE Access*, vol. 9, pp. 8915–89160, 2021, doi: 10.1109/ACCESS.2021.3095720.

[14] H. Hawley, V. Chatziioannou, and A. Morrisonad, “Synthesis of musical instrument sounds: Physics-based modeling or machine learning?” *Phys. Today*, vol. 16, no. 1, pp. 20–28, 2020, doi: 10.1063/1.5071610.2020.

[15] S. Gonzalez, D. Salvi, D. Baeza, F. Antonacci, and A. Sarti, “A data-driven approach to violin making,” *Sci. Rep.*, vol. 11, no. 1, pp. 1–9, May 2021, doi: 10.1038/s41598-021-88931-z.

[16] N. Giordano and V. Chatziioannou, “Status and future of modeling of musical instruments: Introduction to the JASA special issue,” *J. Acoust. Soc. Amer.*, vol. 150, no. 3, pp. 2294–2301, Sep. 2021, doi: 10.1121/10.0006439.

[17] M. Karjalainen, V. Välimäki, and T. Tolonen, “Plucked-string models: The fretboard,” in *Proc. DAFX*, Erlangen, Germany, 2014, pp. 137–144.

[18] A. Lefebvre and G. P. Scavone, “Characterization of woodwind instrument toneholes with the finite element method,” *J. Acoust. Soc. Amer.*, vol. 101, no. 5, pp. 2307–2316, May 2002, doi: 10.1121/10.0005062.

[19] P. Bestle, P. Eberhard, and M. Hanss, “Musical instruments—Sound synthesis of virtual idophones,” *J. Sound Vib.*, vol. 395, pp. 187–200, May 2017, doi: 10.1016/j.jsv.2017.02.010.

[20] L. Trautmann, S. Petrausch, and R. Rabenstein, “Physical modeling of drums by transfer function methods,” in *Proc. ICASSP*, Salt Lake City, UT, USA, 2001, pp. 3385–3388, doi: 10.1109/ICASSP.2001.940385.
[40] R. Tabata, R. Matsuda, T. Koiwaya, S. Iwagami, H. Midoriwaka, T. Kobayashi, and K. Takahashi, “Three-dimensional numerical analysis of acoustic energy absorption and generation in an air-jet instrument based on Howe’s energy corollary,” J. Acoust. Soc. Amer., vol. 149, no. 6, pp. 4000–4012, Jun. 2021, doi: 10.1121/10.0005133.

[41] P. R. Cook, “Computer music,” in Springer Handbook of Acoustics. Berlin, Germany: Springer-Verlag, 2014, pp. 747–778.

[42] J. Chabassier, A. Chaigne, and P. Joly, “Modelling and simulation of a grand piano,” J. Acoust. Soc. Amer., vol. 134, no. 1, pp. 648–665, Jul. 2013, doi: 10.1121/1.4809649.

[43] G. Evangelista, “Physically inspired playable models of guitar, a tutorial,” in Proc. ISCSIP. Limassol, Cyprus, 2010, pp. 1–4, doi: 10.1109/ISCSIP.2010.5463365.

[44] H. Mansour, J. Woodhouse, and G. P. Scavone, “Time-domain simulation of the bowed cello string: Dual-polarization effect,” J. Acoust. Soc. Amer., vol. 133, no. 5, p. 3270, May 2013, doi: 10.1121/1.4805313.

[45] N. Giordano, “Some observations on the physics of stringed instruments,” in Springer Handbook of Systematic Musicology. Berlin, Germany: Springer, 2010, doi: 10.1007/978-1-4419-7110-4.

[46] J. O. Smith III, “Physical modeling using digital waveguides,” Comput. Music J., vol. 16, no. 4, pp. 74–91, 1992, doi: 10.2307/680470.

[47] M. Maas, “Phorminx in classical Greece,” J. Amer. Music. Instrum. Soc., vol. 2, pp. 34–55, Jan. 1976.

[48] L. Gabrielli, V. Välimäki, H. Penttinen, S. Squarini, and S. Bilbao, “A digital waveguide-based approach for clavinet modeling and synthesis,” EURASIP J. Adv. Signal Process., vol. 2013, no. 1, pp. 1–14, Dec. 2013, doi: 10.1186/1687-6180-2013-103.

[49] V. Välimäki, M. Laurson, and C. Erkut, “Commuted waveguide synthesis of the clavichord,” Comput. Music J., vol. 27, no. 1, pp. 71–82, 2003.

[50] L. Savioja and V. Välimäki, “Reducing the dispersion error in the digital waveguide mesh using interpolation and frequency-warping techniques,” IEEE Trans. Speech Audio Process., vol. 8, no. 2, pp. 184–194, Mar. 2000, doi: 10.1109/89.824704.

[51] M. Karjalainen, V. Välimäki, and Z. Jánosy, “Towards high-quality sound synthesis of the guitar and string instruments,” in Proc. ICMI, Tokyo, Japan, 1993, pp. 56–63.

[52] T. D. Rossing, The Science of String Instruments. New York, NY, USA: Springer, 2010, doi: 10.1007/978-1-4419-7110-4.

[53] R. J. Ross, Wood Handbook: Wood as an Engineering Material, Madison, WI, USA: United States Department of Agriculture, 2010, doi: 10.23773/FPL-GTR-190.

[54] S. Polychronopoulos and G. Memoli, “Acoustic levitation with optimized reflective metamaterials,” Sci. Rep., vol. 10, no. 1, pp. 1–10, Mar. 2020, doi: 10.1038/s41598-020-60978-4.

[55] J. E. Piercy, T. F. W. Embleton, and L. C. Sutherland, “Review of noise propagation in the atmosphere,” J. Acoust. Soc. Amer., vol. 61, no. 6, pp. 1403–1418, Jun. 1977, doi: 10.1121/1.381455.

[56] B. Kollmeier, T. Brand, and B. Meyer, “Perception of speech and sound,” in Springer Handbook of Speech Processing, J. Benesty, M. M. Sondhi, and Y. A. Huang, Eds. Berlin, Germany: Springer, 2008, pp. 61–82, doi: 10.1007/978-3-540-49127-9_4.

[57] M. Long, Architectural Acoustics. Amsterdam, The Netherlands: Elsevier, 2005.

[58] C. Terzis, “Musical instruments of Greek and Roman antiquity,” in A Companion to Ancient Greek and Roman Music, T. A. C. Lynch and E. Rocconi, Eds. Hoboken, NJ, USA: Wiley, 2020, ch. 16, pp. 213–228, doi: 10.1002/9781119275510.ch16.

[59] S. Hagel, Ancient Greek Music: A New Technical History. Cambridge, U.K.: Cambridge Univ. Press, 2009.

**SPYROS POLYCHRONOPOULOS** received the B.Sc. degree in physics and the Ph.D. degree in acoustics. He is currently a Research Fellow and a Fixed-Time Lecturer with the Department of Informatics and Telecommunications, National and Kapodistrian University of Athens. Since 1997, he has been composing music and he has released 18 albums. He has created his own algorithms to compose and distribute his music. His research interests include acoustics, ultrasound, audio digital processing, musical acoustics, and room acoustics.

**DIMITRA MARINI** was born in Athens, Greece, in 1994. She received the B.Sc. degree from the Department of Informatics and Telecommunications (DIT), National and Kapodistrian University of Athens (NKUA), Greece, in 2018. Since 2018, she has been a Research Assistant with the DIT, NKUA. Her research interest includes the physical modeling of musical instruments.

**GEORGIOS TH. KOUROUPETROGIOLOU** (Member, IEEE) received the B.Sc. degree in physics and the Ph.D. degree in communications and signal processing from the National and Kapodistrian University of Athens (NKUA), Greece. He is currently a Professor with the Department of Informatics, the President of the Association for the Advancement of Assistive Technology in Europe (AAATE), a member of the European Academy of Sciences and Arts, the Founder of the “Accessibility Unit,” NKUA, and the Founder of the Speech and Accessibility Research Laboratory. He worked as the Chairperson of the Department of Informatics and Telecommunications, the Director of Postgraduate Studies and Ph.D. Studies, the Director of the “Communication and Signal Processing Division,” and the Director of the M.Sc. Program in “Language Technology.” His research interests include computer accessibility and voice user interfaces, as a part of the major domain of human–computer interaction. He is a member of the Editorial Board of the journals Universal Access in the Information Society and Technology and Disability.

**KONSTANTINOS BAKOGIANNIS** received the M.Eng. degree in electrical and computer engineering from the National Technical University of Athens (NTUA), Greece, the M.A. degree in musicology from the National and Kapodistrian University of Athens (NKUA), Greece, and the Ph.D. degree in the field of music informatics from the School of Electrical and Computer Engineering, NTUA. He is currently a Research Fellow with the Department of Music Studies, NKUA, and a Fixed-Term Lecturer with the Department of Digital Arts and Cinema, NKUA. He is also a researcher in the field of sound and music computing. He is a classical-trained musician, holding degrees in piano performance, classical music theory, and composition issued by the Greek Ministry of Culture. His research interests include computational musicology, computer music, interactive dance, and physical modeling of musical instruments. Besides research, he is an active musician and composer.