High-accuracy Three-dimensional Measurement by Improving the Asymmetry of Dithered Patterns

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Abstract. The previously proposed dithering defocusing technology performs well for three-dimensional (3D) measurement when stripes are relatively wide, yet suffers if stripes are narrow. This paper finds two asymmetries in dithered patterns generated by the Sierra Lite dithering algorithm and verifies the longitudinal fringes are more advantageous for phase-shifting technique over transverse fringes. Furthermore, this paper proposes an algorithm with a meandering scan. In each pattern, the pixels of odd lines are scanned from left to right while the even lines are scanned from right to left. The proposed method avoids the quantization errors propagating in a specific direction and greatly improves the symmetry of longitudinal fringes. Both simulation and experimental results have shown this method can effectively improve accuracy of 3D measurement especially for narrow stripes.

1. Introduction
Past researches have shown that digital fringe projection technology is becoming one of the important methods of three-dimensional (3D) shape measurement due to its high precision, high resolution and high speed [1]. However, because of the nonlinearity of the projector, it will cause phase unwrapping errors [2]. Therefore, the 3D measurement method, which is not affected by non-linearity, has become a difficulty. The most representative method to solve this obstacle is the binary defocusing technology [3, 4]. This method has the characteristics of no need for nonlinearity correction. For the binary defocusing technology, the binary fringe patterns are projected onto the surface of an object after properly defocusing by the projector. Meanwhile, the measurement speed is faster than the direct projection of the sinusoidal fringe pattern [4].

Previous studies have shown that high quality sinusoidal fringe patterns can be obtained using defocusing dithering [5] consist of random dithering[6], ordered dithering[7] and error-diffusion dithering[8] when stripes are wide, but suffer if stripes are narrow. Many improved algorithms have been proposed. Zhang proposed improved dithering technique from intensity [9] and Dai would like to optimize the technique from phase [10].But the results are still unsatisfactory. We find there are asymmetries [11] in dithered fringe patterns generated by these dithering algorithm which will increase the phase error because fringe patterns for phase-shifting method should ensure the symmetry in a single cycle [12]. The Sierra Lite dithering algorithm [13] is a simple and fast dithering algorithm and can generate higher quality sinusoidal fringes than most dithering algorithms but still encounter this problem.
In this paper, the Sierra Lite dithering algorithm is used to generate binary defocusing patterns. The asymmetry of dithering patterns generated by Sierra Lite dithering arithmetic comes from two aspects: (1) the proportion of error diffusion in horizontal direction and vertical direction is asymmetric; (2) the proportion of error diffusion to the right and left in horizontal direction is asymmetric. We firstly studied the effects of the transverse and longitudinal fringes on the phase errors under different amounts of defocusing, different stripe widths and different phase-shifting step algorithms, so as to obtain a more advantageous stripe direction. Then to reduce the asymmetry of fringe patterns, a Sierra Lite dithering algorithm with a meandering scan was proposed. The proposed method greatly improves the symmetry of longitudinal fringes and effectively improve fringe quality especially for narrow fringes.

Section 2 introduces the N-step phase-shifting algorithm we will use to obtain the phase. Section 3 explains the principles of the proposed improved dithering algorithm. Section 4 and section 5 present simulation and experimental results respectively. And section 6 summarizes the paper.

2. N-step phase-shifting algorithm
The phase-shifting method has a wide range of applications in the fields of optical measurement and optical metrology. The method has the characteristics of simplicity, high precision and rapidity. For the most common N-step phase-shifting algorithm with equal phase shift, the fringe patterns can be described as:

\[ I_{n}(x, y) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + 2n\pi / N] \]  \hspace{1cm} (1)

Where \( n = 1, 2, \ldots, N \), \( I(x,y) \) is the average intensity, \( I'(x,y) \) is the intensity modulation, and \( \phi(x,y) \) is the phase to be solved for using the following equation:

\[ \phi(x, y) = -\tan^{-1}\left[ \frac{\sum_{n=1}^{N} I_{n}(x, y) \sin(2n\pi / N)}{\sum_{n=1}^{N} I_{n}(x, y) \cos(2n\pi / N)} \right] \]  \hspace{1cm} (2)

Since the phase value \( \phi(x, y) \) is obtained by solving the inverse tangent function, it will result a value ranging \([-\pi, +\pi]\) with \(2\pi\) jump. If encoding is performed using a sinusoidal pattern larger than one cycle, the final continuous phase value can be calculated by the temporal or spatial phase unwrapping algorithm [14,15]. Due to the accuracy of temporal phase unwrapping algorithm, we employed this method and then reconstructed the 3D shape from the unwrapped phase in this paper.

3. Sierra Lite dithering algorithm
The Sierra Lite dithering algorithm [13] is a kind of dithering algorithm based on error diffusion that is simple in form and fast in operation. It quantizes the current image pixel to two gray levels of 0 or 255 according to the threshold, and then spreads the quantization error residuals of a single pixel to a plurality of adjacent unprocessed pixels according to a certain ratio, thereby realizing the image color reduction. The principle of error propagation can be expressed as follows:

\[ I'(i, j) = I(i, j) + \sum_{k,l} w(k,l)e(i-k, j-l) \]  \hspace{1cm} (3)

Where \( I(i,j) \) is the original image, and \( I'(i,j) \) is the gray value of the original image after adding the quantization error of the surrounding pixel diffusion at \( I(i,j) \). The quantization error \( e(i,j) \) is diffused to the neighboring pixels by a two-dimensional weighting matrix \( w(k,l) \), which is also called diffusion kernel. The diffusion kernel of the Sierra Lite dithering algorithm is:

\[ w(k,l) = \frac{1}{4} \begin{bmatrix} x & 2 \\ 1 & 1 & 0 \end{bmatrix} \]  \hspace{1cm} (4)
Here, the values of 2, 1, 1, 0 in the matrix represent the error diffusion ratio, \( x \) represents the pixel in processing, and - represents the pixel that has been processed. The algorithm quantizes a single pixel each time, and achieves the quantization process of the entire image by scanning the image line by line. After a pixel has been processed, the image gray value of that pixel will not change.

It can be seen from the diffusion kernel of the Sierra Lite dithering algorithm that the ratio of error diffusion in the horizontal and vertical directions is asymmetrical, and the ratio of error diffusion to the right and left in the horizontal direction is also asymmetric.

Thus, if the original symmetric sinusoidal fringe images are binarized using the Sierra Lite dithering algorithm, the resulting binarized images will be out of symmetry. Since the error diffusion ratio of Sierra Lite dithering algorithm in horizontal and vertical directions is asymmetric, the results may differ when it treats the transverse and longitudinal sinusoidal fringes. At the same time, due to the asymmetric diffusion ratio of the Sierra Lite dithering algorithm to the right and left in the horizontal direction, the binarization will cause the accumulation of diffusion error in a certain direction.

3.1. Asymmetry in horizontal and vertical directions
Figure 1 shows the relationship between the error diffusion direction of the Sierra Lite dithering algorithm and the sinusoidal stripe direction. For figure 1(a), the scanning direction is parallel to the stripe direction, while for figure 1(b), the scanning direction and stripe direction are vertical to each other. Since the diffusion kernel \( w(k,l) \) has different error diffusion ratios in the vertical and horizontal directions, the two methods have different effects on 3D reconstruction of the digital projection. In this paper, both simulations and experiments will be conducted to demonstrate effects of these two methods on phase errors under different amounts of defocusing, different stripe widths and different phase-shifting step algorithms, so as to obtain a more advantageous stripe direction.

![Figure 1](image1.png)

(a) Parallel scanning. (b) Vertical scanning

3.2. Asymmetry of the left and right in horizontal direction
It can be seen from the error diffusion ratio of the diffusion kernel \( w(k,l) \) of the Sierra Lite dithering algorithm that the error diffusion direction has asymmetry of the left and right in the horizontal direction. The conventional Sierra Lite dithering algorithm quantizes the pixels line by line from left to right, as shown in figure 2(a), which causes a superposition effect when errors propagate and further affect the quality of the dithered images.

In this paper, for the longitudinal fringes, we propose a Sierra Lite dithering algorithm with a meandering scan as shown in figure 2(b) and 2(c). In each pattern, the pixels of odd lines are scanned from left to right, while the even lines are scanned from right to left. The meandering scanning method avoids the quantization errors propagating in a specific direction and thus reduces the accumulation of errors. To some extent, the asymmetric texture in the dithered image is improved, thus the generated
The dithered image has better symmetry and higher quality than the conventional algorithm. In the paper, we will verify the performance of this method through simulations and experiments.

![Image](image_url)

**Figure 2.** The Sierra Lite dithering algorithm with a meandering scan. (a) Conventional method. (b) Proposed method. (c) Error diffusion.

### 4. Simulation results

#### 4.1. Influencing factors of phase errors

We firstly analyzed the asymmetry of the error diffusion ratio of horizontal and vertical directions of dithered patterns. In this paper, the Sierra Lite dithering algorithm is utilized to binarize the transverse and longitudinal sinusoidal fringe patterns respectively, and the effects of the two methods on the phase under different amounts of defocusing, different stripe widths and different phase-shifting step algorithms were studied.

The simulations can be divided into the following steps:

- Generating Sierra Lite dithering fringe patterns from sinusoidal fringe patterns;
- Generating smoothed dithered patterns with different defocusing quantities blurred by Gaussian filters;
- Using the phase-shifting algorithm with different steps to calculate wrapped phases, then utilizing temporal phase unwrapping algorithm to calculate the corresponding absolute phases;
- Comparing the absolute phases obtained by defocusing dithering fringe patterns with the absolute phases obtained by sinusoidal fringe patterns. The absolute values of the difference between the two are defined as the phase errors.

In the simulations, the stripe directions of the sinusoidal fringe patterns were respectively set as transverse fringes and longitudinal fringes, and the stripe widths were set to 30, 60, 90, ..., 180 pixels respectively. The defocusing effect of the projection system was simulated by Gaussian filtering method. The amounts of defocusing were realized by the Gaussian filters with the five different sizes of 7, 9, 11, 13, and 15 pixels respectively. And this paper analyzes three-step, four-step, five-step and six-step phase-shifting methods one by one, and uses the thirty-step phase-shifting method as the maximum number of steps and standard result.

Figure 3 and figure 4 show the phase errors mean results for the transverse fringes and longitudinal fringes. In the each figure, the subgraphs from left to right are results of three-step, four-step, five-step, six-step and thirty-step phase-shifting methods respectively. Comparing the phase errors of transverse fringes and corresponding longitudinal fringes, we can see that the Sierra Lite dithering algorithm has fringe direction selectivity and the phase mean errors of longitudinal fringes tends to be smaller, which is more advantageous for 3D reconstruction. For each figure, we can see that (1) As the stripe widths increase, the phase errors in both transverse and longitudinal fringe patterns decrease and tend to be stable and when the stripe widths are narrow, the phase errors are relatively large (approximately 0.5 for the width of 30 pixels); (2) In the case of small steps, as the amounts of defocusing increase, the phase mean errors decrease. However, the amounts of defocusing are closer to each other in the case of large step; (3) The phase average errors obtained by the Sierra Lite dithering algorithm under the five-step phase-shifting algorithm has been relatively low.
Figure 3. Simulation results of phase errors by parallel direction scanning.
(a) Three-step. (b) Four-step. (c) Five-step. (e) Six-step. (f) Thirty-step.

Figure 4. Simulation results of phase errors by vertical direction scanning.
(a) Three-step. (b) Four-step. (c) Five-step. (e) Six-step. (f) Thirty-step.

4.2. The improved Sierra Lite dithering algorithm
It can be seen from aforementioned results that phase average errors of longitudinal fringe patterns tend to be smaller, so only the longitudinal fringe patterns are discussed here. Since the five-step phase-shifting algorithm can satisfy both the system speed and accuracy, the five-step phase-shifting algorithm was employed for simulation.
Figure 5. Asymmetric of the Sierra Lite dithering algorithm. (a) Longitudinal dithered patterns by proposed method. (b) Longitudinal dithered patterns by conventional method. (c) Transverse dithered patterns by conventional method. (d) Cross Section of (a). (e) Cross Section of (b). (f) Cross Section of (e). (g) Difference between the defocusing dithered patterns and the ideal sinusoidal patterns.

In order to compare the proposed Sierra Lite dithering algorithm with the traditional one, we respectively generated the vertical stripe patterns (stripe width is 60 pixels) by the improved algorithm and the horizontal and vertical patterns by the traditional algorithm. For better comparison, we rotated the horizontal stripes by 90 degrees. Figure 5(a) shows longitudinal fringes generated by the modified Sierra Lite algorithm. The stripes are relatively uniform with no significant texture and distortion. Figure 5(b) and (c) show the vertical and horizontal stripes generated by the traditional Sierra Lite algorithm separately. The asymmetric textures of the fringes are severe and there are a lot of details lost, which seriously reduces the fringe quality. In addition, asymmetric textures of horizontal stripes are more severe than vertical stripes to some extent. To quantitatively describe the sinusoidality of fringe patterns generated by the Sierra Lite dithering algorithm, we used a Gaussian filter of 15 pixels to defocus the dithered patterns of figure 5(a), figure 5(b) and figure 5(c). Figure 5(d), 5(e) and 5(f)
are cross sections of the corresponding defocusing patterns, and figure 5(g) shows the difference between the defocusing dithered patterns and the ideal sinusoidal patterns in several cycles. It can be seen that the intensity curve of the improved algorithm and the standard sinusoid curve are basically the same while the traditional algorithm generates poor sinusoidal patterns and the difference of longitudinal stripes are lower than that of the horizontal stripes.

For verifying the performance of the proposed Sierra Lite dithering algorithm, we also designed simulations to study the phase errors under different amounts of defocusing and different stripe widths.

![Figure 6](image)

**Figure 6.** Simulation results of phase errors by proposed method for five-step phase-shifting technology.

As we can see from figure 6 that phase errors of longitudinal fringes using the algorithm proposed. Compared with figure 3, it can be seen that:(1) The Sierra Lite dithering algorithm with a meandering scan has significantly improved phase errors compared to the conventional algorithm under different amounts of defocusing and different stripe widths;(2)When the stripes width is narrow, the phase errors are still small, even only 0.0045 under the maximal amount of defocusing for 30 pixels. And as the stripe width increases, the phase errors decrease and tend to be stable;(3) As the amount of defocusing increases, the phase mean errors decrease. These results mean that the proposed method performs better in almost all cases compared to the conventional Sierra Lite dithering algorithm especially for narrow fringes.

5. Experimental results

In order to confirm the significance of the proposed method, we established a 3D measurement experimental system. In the experiments, we employed an LCoS projector (Esonic HD-720P) and a digital camera (MER-130-30UM). The camera is attached with a 8 mm focal length megapixel lens (Computar M0814-MP2).

Phase errors caused by the asymmetry in horizontal and vertical directions was experimented first under different defocusing quantities, different stripe widths and different phase-shifting step algorithms. In the experiment, we firstly measured a white plate with the same fringe patterns used in simulations and rotated lens focus ring manually to achieve five different amounts of defocusing. Figure 7 and figure 8 show the results. It can be seen that longitudinal fringes perform better than transverse fringes whatever the amounts of defocusing, stripe widths and phase-shifting step algorithms are. Apparently, the experimental results are consistent with simulation results, and thus strongly argue that in 3D shape measurement the stripe direction parallel to scanning direction of the Sierra Lite dithering algorithm should be selected preferentially.

We also carried out an experiment to study phase errors under different amounts of defocusing and different stripe widths. Figure 9 shows the proposed method has significantly improved phase errors...
compared to the conventional algorithm under different amounts of defocusing and different stripe widths and the phase errors are all small (less than 0.013 even low to 0.003). The experimental results are consistent with the simulation results.

**Figure 7.** Experimental results of phase errors by parallel direction scanning.
(a) Three-step. (b) Four-step. (c) Five-step. (e) Six-step. (f) Thirty-step.

**Figure 8.** Experimental results of phase errors by vertical direction scanning.
(a) Three-step. (b) Four-step. (c) Five-step. (e) Six-step. (f) Thirty-step.
Figure 9. Experimental results of phase errors by proposed method for five-step phase-shifting technology.

Figure 10. Experimental results of measuring a 3D object.
(a) Face model. (b) Five-step phase-shifting image. (c) 3D result of proposed method for longitudinal fringes. (d) Depth map of proposed method for longitudinal fringes. (e) Depth map result of conventional method for longitudinal fringes. (f) Depth map of conventional method for transverse fringes.

In order to evaluate the 3D shape measurement quality of the proposed method, a complex 3D object-face model was measured, as shown in figure 10(a). The width of the sinusoidal fringes are 60 pixels, and the direction of the fringes are vertical. Figure 10(b) shows one of the five-step phase-shifting images, and figure 10(c) shows 3D shape of the face model. Figure 10(d) is the depth map of
the surface of the face model. To compare it with traditional methods, we also measured the face model based on the dithered fringe patterns binarized by the conventional Sierra Lite dithering algorithm with the longitudinal and transverse fringes separately. Figures 10(e) and 10(f) show the corresponding results and can be seen that the longitudinal fringes can achieve higher measurement quality than the transverse fringes. At the same time, the results of the proposed method is better than the results of conventional method whether the stripes are horizontal or vertical and more details are restored.

6. Conclusion
Starting from the diffusion kernel of the Sierra Lite dithering algorithm, this paper finds two asymmetries of the dithered fringe patterns and verified that the Sierra Lite dithering algorithm is selective to the stripe direction and the longitudinal fringes are more advantageous for phase-shifting technique from simulation and experiments. Furthermore, a Sierra Lite dithering algorithm with a meandering scan was proposed. The proposed method greatly improves the symmetry of longitudinal fringes, and both simulation and experimental results have demonstrated this method can effectively improve accuracy of 3D measurement especially for fringe quality especially for narrow stripes.

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