Small-scale statistics in direct numerical simulation of turbulent channel flow at high-Reynolds number

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Abstract. Small-scale statistics in turbulent channel flow are studied by high resolution direct numerical simulations with the friction Reynolds number up to $Re = 2560$. The second order velocity statistics in the inertial subrange of the log-low region are found to be not far from those predicted by Kolmogorov’s universal equilibrium theory, although the agreement is not perfect. The shear parameter $S^+$ decreases with the distance $y^+$ from the wall and $S^+ \lesssim 0.1$ in $y^+ > 400$ for large $Re$, where $S^+$ is the ratio of the Kolmogorov time scale to the mean shear time scale. For larger $Re$, the $y^+$-dependence of $S^+$ is closer to a simple theoretical estimate in the log-low region.

1. Introduction

According to Kolmogorov’s theory (Kolmogorov, 1941) (referred as K41 below), there is a certain kind of universality in the small-scale statistics of fully developed turbulence at high Reynolds number ($Re$) and far from flow boundaries. As regards two-point second order velocity correlations, or equivalently the velocity correlation spectra, the idea of K41 has been well supported by field observations, laboratory experiments and direct numerical simulations (DNSs) of nearly isotropic turbulence or turbulence far from flow boundaries.

One may then ask whether this is true also in wall-bounded turbulence (WBT). For example, one may ask ”Are the local isotropy and the energy spectra predicted by K41 realized in WBT?”. Supports by experiments for the prediction have been accumulated (see e.g., Saddoughi & Veeravalli, 1994; Tsuji, 2003; McKeon & Morrison, 2007).

In this study, we address the question by using high resolution DNS of turbulent channel flow (TCF), which is one of the most canonical WBTs. DNS has the advantage of being free from experimental uncertainties. Since the universality in the sense of K41 can be expected only if $Re$ is sufficiently high, it is of crucial importance here to simulate high $Re$ turbulence. In order to achieve $Re$ as high as possible within given computational facilities, we have focused on TCF in a so-called small-domain (see Table 1 below, for the detail). Underlying the use of such a small-domain is the idea that although the large scale statistics may be sensitive to the domain size, the small scale statistics with which we are concerned in this study is not so sensitive. This is in accordance with the idea of K41. We have simulated TCFs with the friction Reynolds number $Re_f$ (see below for the definition) up to 2560, on the Earth Simulator (ES2). To the
authors' knowledge, \(Re_\tau = 2560\) is the highest \(Re_\tau\) so far achieved in DNS of TCF. A particular attention is paid to the statistics in the so-called log-law region, as well as their dependence on \(Re_\tau\) and the distance from the wall.

2. Run conditions and Numerical methods

We have simulated TCFs of an incompressible fluid obeying the Navier-Stokes (NS) equations, under periodic boundary conditions with fundamental domain size \(L_x\) and \(L_z\) in the streamwise \((x)\) and spanwise \((z)\) directions respectively, and the non-slip boundary conditions on the walls at \(y = \pm h\), where \(h\) is the channel half-width. The DNS is based on the Fourier-spectral method in \(x\)- and \(z\)-directions, and the Chebyshev-tau method in the wall-normal \(y\)-direction. The alias errors are removed by the 3/2 rule. The NS equations are expressed in terms of the wall-normal vorticity components and the Laplacian of wall-normal velocity, as in Kim \textit{et al.} (1987). Time advancement is accomplished by a third-order Runge-Kutta method for the convection term, and the first-order implicit Euler method for the viscous terms, as in Jiménez \textit{et al.} (2001).

The parameters in the DNSs are given in Table 1. The superscript + denotes non-dimensional quantity scaled by the wall unit \(u_\tau\) and \(\nu\), where \(u_\tau\) and \(\nu\) are the wall friction velocity and the kinematic viscosity, respectively. The friction Reynolds number \(Re_\tau\) is defined by \(Re_\tau \equiv u_\tau h/\nu\). \(Re_\tau\) and the numbers of grid points \(N_x\), \(N_y\), and \(N_z\) are so chosen that the relations \(\Delta x^+ \sim 8, \Delta y^+ \sim 8, \Delta z^+ \sim 4\) are satisfied (Jiménez, 2003), where \(\Delta x^+\) and \(\Delta z^+\) are the \(x\)- and \(z\)-resolutions, respectively, and \(\Delta y^+\) is the \(y\)-resolution at the center of channel. The statistics shown below are the averages over the time interval at least \(20T\) where \(T = L_x/U\).

3. Results

Figure 1 shows the profiles of the mean streamwise velocity \(U^+\) for Cases 1, 2, 3, and 4. It is confirmed that there is a flow region, (for example in Case 4, \(40 < y^+ < 600\)), in which the mean streamwise velocity fits well to the so-called log-law

\[
U^+ = \frac{1}{\kappa} \log y^+ + A,
\]

where \(\kappa\) is the von Kármán constant and \(A\) is a constant. This region is called here the log-law region.

Figure 2 shows the \(y^+\) dependence of Taylor microscale Reynolds number

\[
R_{\lambda_{uu}}^{(x)}(y) = \frac{u'(y)\lambda_{uu}^{(x)}(y)}{\nu},
\]

where \(u'(y) = \langle u(x, y, z)^2 \rangle^{1/2}\) is the root-mean-square of streamwise velocity fluctuation (here and below the brackets \(\langle \rangle\) denote the average over the \(x\)-\(z\) plane), and \(\lambda_{uu}^{(x)}(y)\) is the Taylor
**Figure 1.** Mean velocity profiles. Solid and dashed lines indicate the log-law $U^+ = (1/0.4) \log y^+ + 5.0$ and the wall-law $U^+ = y^+$, respectively.

$\frac{\partial^2 R_{uu}^{(x)}(y, r)}{\partial r^2} \bigg|_{r=0} = -\frac{1}{(\lambda_{uu}^{(x)}(y))^2}, \quad (3)$

in which $R_{uu}^{(x)}(y, r)$ is the longitudinal two-point velocity correlation in the streamwise direction defined by

$R_{uu}^{(x)}(y, r) = \frac{\langle u(x + r, y, z)u(x, y, z) \rangle}{u'^2(y)}. \quad (4)$

It is seen in figure 2 that $R_{uu}^{(x)}$ increases with $y^+$ in the log-law region and reaches a maximum value ($\sim 280$) at the end of the log-law region in Case 4.
Figure 3. Comparison between compensated longitudinal spectra $E_{11}^{(x)}(k_x)$ and $E_{33}^{(z)}(k_z)$, at $y^+ = 100, 200, 400$ and $600$ in Case 4, where $k_x = k_z(\equiv k)$. $\eta$ is Kolmogorov’s length scale. Solid line is $k^{5/3}E(k)/\epsilon^{2/3} = C_1$ with $C_1 = 0.5$.

Figure 4. Comparison between compensated transverse spectra (a) $E_{22}^{(x)}(k_x)$ and $E_{22}^{(z)}(k_z)$, (b) $E_{33}^{(x)}(k_x)$ and $E_{11}^{(z)}(k_z)$, at $y^+ = 100, 200, 400$ and $600$ in Case 4, where $k_x = k_z(\equiv k)$. $\eta$ is Kolmogorov’s length scale. Solid line is $k^{5/3}E(k)/\epsilon^{2/3} = C'_1$ with $C'_1 = (4/3)0.5$.

According to K41, the one-dimensional longitudinal energy spectra $E_{11}^{(x)}(k_x)$ and $E_{33}^{(z)}(k_z)$ and the transverse energy spectra $E_{22}^{(x)}(k_x)$, $E_{33}^{(z)}(k_x)$, $E_{33}^{(z)}(k_z)$, and $E_{11}^{(z)}(k_z)$ are given by

$$E_{11}^{(x)}(k) = E_{33}^{(z)}(k) = C_1\epsilon^{2/3}k^{-5/3},$$

$$E_{22}^{(x)}(k) = E_{33}^{(z)}(k) = E_{11}^{(z)}(k) = C'_1\epsilon^{2/3}k^{-5/3},$$

in the inertial subrange, where $k_x = k_z(\equiv k)$, $C_1$ is a non-dimensional universal constant, $C'_1 = (4/3)C_1$, $\epsilon$ is the mean energy dissipation rate per unit mass averaged over $x$-$z$ plane, the subscripts 1, 2, and 3 respectively refer the velocity components in the $x$, $y$, and $z$ directions, and the superscripts $x$, $y$, and $z$ respectively refer the directions of the wave vector $k = (k_x, k_y, k_z)$.

Figure 3 shows the comparison between compensated longitudinal spectra $E_{11}^{(x)}(k_x)$ and
Figure 5. $y^+$-dependence of shear parameter $S^*$. The inset shows log-log plots of the same data. Solid line is $S^* = (\kappa y^+)^{-1/2}$ with $\kappa = 0.4$.

$E^{(z)}_{33}(k_z)$ in Case 4 at fixed $y^+$, where $k_x = k_z (\equiv k)$. The classical value for the Kolmogorov constant $C_1 = 0.5$ (see e.g., Sreenivasan, 1995) is also shown in figure 3. It is seen that there is a wavenumber range near $k\eta \sim 0.5$, in which each spectrum is not far from the K41 spectra given by Eq. (5). The difference between $E^{(x)}_{11}(k_x)$ and $E^{(z)}_{33}(k_z)$ is seen in figure 3 to decrease with the increase of the Taylor microscale Reynolds number $R_{\lambda uu}^{(x)}$. The similar is also the case for the transverse spectra shown in figure 4.

Figure 5 shows the $y^+$-dependence of the shear parameter $S^*$ for Cases 1, 2, 3, and 4, where

$$S^* \equiv (\nu/\epsilon)^{1/2} S,$$

and $S = dU/dy$ is the mean velocity gradient. $S^*$ is a representative measure characterizing the importance of the shear effect at the small scale $\sim$ Kolmogorov’s length scale $\eta$. In figure 5, it is seen that

(i) the $y^+$-dependence of $S^*$ is weak near the wall,
(ii) $S^*$ is smaller for larger $Re_\tau$,
(iii) $S^*$ rapidly decreases with $y^+$ at $y^+ \sim 20$ and $S^* \lesssim 0.1$ in $y^+ > 400$ for large $Re_\tau$.

It is also seen in the inset that the data for larger $Re_\tau$ are closer to a simple theoretical estimate

$$S^* \sim (\kappa y^+)^{-1/2},$$

in the log-law region.

4. Conclusions

We have performed high resolution DNSs of turbulent channel flows with friction Reynolds numbers up to $Re_\tau = 2560$ and analyzed the small-scale statistics in the log-law region. The one-dimensional energy spectra in the inertial subrange of the log-low region are not far from those predicted by Kolmogorov’s theory, although the agreement is not perfect. The dependence of the shear parameter $S^*$ on the distance $y^+$ from the wall is closer to a simple theoretical estimate $S^* \sim (\kappa y^+)^{-1/2}$ in the log-low region for larger $Re_\tau$, and $S^* \lesssim 0.1$ in $y^+ > 400$ for large $Re_\tau$. 
Acknowledgments

The computations were carried out on the Earth Simulator at Japan Agency for Marine-Earth Science and Technology and on the FX1 system at the Information Technology Center of Nagoya University. This work was partially supported by Grant-in-Aids for Scientific Research (C)23560194 and (S)20224013 from the Japan Society for the Promotion of Science.

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