AB INITIO PULSAR MAGNETOSPHERE: THREE-DIMENSIONAL PARTICLE-IN-CELL SIMULATIONS OF AXISYMMETRIC PULSARS

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ABSTRACT

We perform “first-principles” relativistic particle-in-cell simulations of aligned pulsar magnetosphere. We allow free escape of particles from the surface of a neutron star and continuously populate the magnetosphere with neutral pair plasma to imitate pair production. As pair plasma supply increases, we observe the transition from a charge-separated “electrosphere” solution with trapped plasma and no spin-down to a solution close to the ideal force-free magnetosphere with electromagnetically dominated pulsar wind. We calculate the magnetospheric structure, current distribution, and spin-down power of the neutron star. We also discuss particle acceleration in the equatorial current sheet.

Key words: plasmas – pulsars: general – stars: magnetic field – stars: rotation

Online-only material: color figures

1. INTRODUCTION

More than 40 years after their discovery, many fundamental questions about pulsars are still with us. Both the radio and gamma-ray emission lack a comprehensive theory, mainly because plasma conditions in various parts of the magnetosphere are not well understood. Production and acceleration of plasma have yet to be modeled in a self-consistent way as part of the global magnetospheric solution. In recent years, considerable progress has been made in constructing global models of pulsar magnetospheres under the assumption of abundant plasma supply. For example, self-consistent force-free numerical solutions of axisymmetric (Contopoulos et al. 1999; Gruzinov 2005; McKinney 2006; Timokhin 2006) and oblique (Spitkovsky 2006; Kalapotharakos & Contopoulos 2009; P´etri 2012; Li et al. 2012; Kalapotharakos et al. 2012) pulsar magnetospheres were developed. These studies were extended to the full magnetohydrodynamic (MHD) regime (Komissarov 2006; Tchekhovskoy et al. 2013) that take plasma pressure and inertia into account. All such solutions generically find thin return current layers at the boundary of closed and open field lines and a strong current sheet beyond the light cylinder. Strong current regions have been identified as potential sites of particle acceleration and gamma-ray emission, mainly based on geometrical considerations of light-curve fitting (e.g., Bai & Spitkovsky 2010; Kalapotharakos et al. 2013), supported by plausible estimates of characteristic emission from current sheet reconnection (Arka & Dubus 2013; Uzdensky & Spitkovsky 2014). However, plasma instabilities and particle acceleration in the magnetosphere that are required for ultimate emission modeling cannot be fully addressed within the MHD or force-free approach, necessitating a kinetic treatment. The current sheet also cannot be easily studied in isolation from the rest of the magnetosphere as it is causally connected to the global solution.

The goal of this Letter, therefore, is to lay the foundation for global kinetic simulations of pulsar magnetospheres. We do this by formulating a novel boundary condition for conducting rotating stars and use it to construct a first-principles relativistic particle-in-cell (PIC) model of the aligned pulsar magnetosphere. We consider two extreme limits of plasma supply: charge-separated emission from the stellar surface in the absence of pair production, and abundant neutral plasma injection approximating efficient pair formation. In Section 2, we describe our numerical method and test problem setup, and in Section 3 we present our results on the three-dimensional (3D) magnetospheric structure and discuss particle acceleration in the current sheet.

2. NUMERICAL METHOD AND SETUP

To simulate the pulsar magnetosphere we use the 3D electromagnetic PIC code TRISTAN-MP (Spitkovsky 2005). To a good accuracy, the star can be described as a rotating magnetized conductor. Unipolar induction generates an electric field corresponding to a quadrupolar surface charge. In the limit of zero work function, these charges can be pulled from the surface by the electric field, thus populating the magnetosphere with plasma. This space-charge limited outflow from a spherical star is difficult to simulate without encountering virtual cathode oscillations and stair-stepping artifacts on a Cartesian grid (Spitkovsky & Arons 2002; hereafter, SA02). Instead of prescribing corotating electric fields inside a stair-stepped star and injecting a fraction of the local surface charge at every time step, we tried to develop a more physical boundary condition for simulating unipolar induction. We model the star as a dense magnetized plasma ball with particles that are pushed into rotation by an external drag force. This force emulates the “lattice” force acting inside a moving solid:

\[ \mathbf{f}_{\text{drag}} = -m(V - V_0)/\tau, \]  

(1)

where \( m \) is the particle mass, \( V \) is the particle velocity, \( V_0 \) is the particle velocity in the steady state, and \( \tau \) is the equilibration time. As they are pushed, particles will move across the background magnetic field \( B_0 \) with the drift velocity

\[ V_{\text{drift}} = c \frac{\mathbf{f}_{\text{drag}} \times B_0}{q B_0^2}, \]  

(2)

where \( q \) is the particle charge. Since the drift velocity depends on particle charge, the drag force will create charge separation and the corresponding electric field inside the plasma conductor:

\[ \mathbf{E} = -V_0 \times B_0/c. \]  

(3)
The exterior electric fields will then develop field-aligned components which can extract the surface charges near the edge of the sphere. We find that with this setup the simulation of space-charge limited emission from our plasma conductor does not require special control of the injection rate at the surface. Stair-stepping is also avoided because we do not set boundary fields directly. When particles fall back on the conductor, they are slowed down and forced to move with the star by the lattice force.

In the next section, we test our boundary condition by launching an Alfvén wave from the surface of a moving conductor. This process is essential for emitting and returning poloidal currents in the rotating magnetosphere that will be described in Section 3.

2.1. Test Problem: Two-dimensional Flux Tube

We consider a two-dimensional magnetized conducting plate of finite thickness that is immersed in background plasma (“flux tube”; see Figure 1(a) for a description of the simulation geometry), with the following velocity profile inside the conductor:

$$v_z(y) = \frac{V_0}{L} \left( y - y_0 \right),$$

where $y_0$ is the position of the conductor’s center and $L$ is its length. The electric field inside the conductor is then

$$E_y(y) = -V_0 B_0 \frac{y - y_0}{cL/2}.$$  

Since electric field parallel to the boundary should be continuous, the same field is established behind the outgoing Alfvén wave. When the magnetization is large (Alfvén velocity $\approx c$), the toroidal field equals the electric field:

$$B_z(y) = E_y(y) = -V_0 B_0 \frac{y - y_0}{cL/2}. $$ 

The charge and current necessary to support this outflow are

$$\rho = \frac{\nabla \cdot E}{4\pi} = -\frac{V_0 B_0}{2\pi c L},$$

and

$$j = \frac{c \nabla \times B}{4\pi} = -\frac{V_0 B_0}{2\pi L} = \rho c,$$

which are just GJ charge and current densities (Goldreich & Julian 1969).

With these simple analytical results we can check the results of numerical simulations. The current flow structure is shown in Figure 1(b). In the bulk outflow, the Alfvén wave establishes the GJ current density. We find that the necessary current is launched if the plasma density inside the conductor exceeds the background density $\rho$ by a factor of 40 for external magnetization $\sigma = B_0^2/(4\pi n m_e c^2) = 20$ and particle velocity $V_0 = 0.1 c$ at the edges of the plasma plate. The current closure happens in thin return current layers within several plasma skin depths at the edges of the outflow. The out-of-plane component of the magnetic field, $B_z$, and the electric field component parallel to the conductor surface, $E_y$, are shown in Figures 1(c) and (d). Their profiles agree with analytical expectation (6). The “plasma conductor” boundary condition provides good current closure, and supports the formation of thin current sheets. Despite the current being equal to $pc$, the particles in the bulk flow move much slower than $c$ (at most 0.2$c$) because the quasineutral external plasma supports counterstreaming.

3. PULSAR MAGNETOSPHERE

We now apply the new boundary condition to the case of a 3D magnetized rotating sphere. We start from a dense pair plasma sphere with radius $R_*/=50$ cells, located in the center of a uniform Cartesian grid with 800$^3$ cells. Inside the sphere there are 100 particles per cell, and the local skin depth is only
We discuss two different limits of plasma supply in the magnetosphere. In the first one, the only source of plasma particles is the extraction of charges from the central star; in the second, in addition to the plasma from the star, neutral pair plasma is injected throughout the magnetosphere. These limits should capture the transition from “dead pulsars” with no pair formation to “active pulsars” with abundant pair formation. The initial evolution is similar for the two cases. The unipolar in-

no pair formation, the solution reaches a disk–dome configuration, shown in Figure 2 (Krause-Polstorff & Michel 1985; Smith et al. 2001; SA02). The dome particles are electrostatically trapped in the vicinity of the star and are rotating with it. The equatorial disk is not corotating with the star and is unstable to the diocotron instability (SA02; Pétri et al. 2002; see Figure 3 for an equatorial slice of the 3D simulation). The instability leads to nonaxisymmetric charge modulations and radial charge transport in the disk. At the end of our simulation, while some equatorial charges reach the light cylinder (note the extended spiral structures in Figure 3), the current flow is insufficient to appreciably modify the dipole magnetic field structure or cause pulsar spin-down. While $E \cdot B = 0$ on the star in this solution, large accelerating gaps exist in the vacuum regions between the domes and the torus. However, $E < B$ everywhere.

In the second plasma supply scheme, initially there is no plasma outside the star, but at each time step we inject neutral pair plasma at a fixed rate in every cell where the magnetization exceeds a certain limit, $\sigma > 400(R_*/r)^3$. This limiter helps prevent overloading of the magnetosphere with plasma, particularly in the closed zone, while keeping the flow well-magnetized. The rate of injection over the whole space is $5\, \text{GJ}$ charges per rotation, where we define GJ charge as the integral of the GJ density over a dipolar field inside the light cylinder, $0.8\Omega B_0 R_*^2 \ln(c/\Omega R_*)/ce$. Before the limiter is engaged, the injected plasma is distributed uniformly over the box, but in steady state the effective injection falls off as $1/r^3$, and is mostly important within the light cylinder. The rate of injection in steady state is $4\, n_{\text{GJ}}$ per rotation near the pole, and $2\, n_{\text{GJ}}$ per rotation at the light cylinder, where $n_{\text{GJ}}$ is the local GJ density. In the steady state the number of particles outside the star is constant, and the final magnetization of our solution is $\sigma \approx 400$ near the pole (around 15 particles per cell, local skin depth is 1.3 cells) and $\sigma \approx 20$ at the light cylinder (around 1 particle per cell, local skin depth is 5 cells). In steady state the number of simulation particles is of order $10^8$. Magnetization remains nearly constant.

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1 We assume that positive charges are positrons and ignore potential complications of extracting positive charges from the surface of a star.
in the wind zone, so the solution is close to force-free conditions everywhere. When the magnetosphere is being filled with neutral plasma, the charge-separated dome–torus stage is only transient. Two torsional Alfvén waves are emitted from the surface of the star and modify the magnetospheric structure. Near the star the waves launched from different hemispheres cancel, creating a region of zero poloidal current. At larger distances along the equator the current sheet starts to form. While in the beginning the sheet is charge-separated, the additional neutral plasma inflowing into the sheet quickly makes it quasineutral, though with a net negative charge (consistent with $\mathbf{\Omega}_\star \cdot \mathbf{B} < 0$). The physical thickness of the sheet decreases with additional plasma density, remaining at several local skin depths.

The magnetic field in a slice through the 3D magnetosphere is shown in Figure 4(a), where the color represents the out-of-plane magnetic field component $B_y$. The solution is remarkably similar to force-free or MHD solutions. It consists of the region of the closed field lines that carries no poloidal current and the region of open field lines with asymptotically radial poloidal field lines. The Y-point is located approximately at the light cylinder. A small fraction of field lines close through the current sheet, suggesting that reconnection occurs there (3D structure of the field lines is shown in Figure 5). The poloidal current flow structure is shown with color in Figure 4(b). The current launched from the polar cap region returns to the star through the current sheet and the separatrix current layer, which are resolved in our simulation.

The spin-down energy loss of the PIC solution, measured as the Poynting flux integrated over a sphere with $r = R_{LC}$, is $L = (1 \pm 0.1) \mu^2 \Omega^2 / c^3$, consistent with previous ideal force-free and MHD work. Figure 4(c) shows the angular distribution of the Poynting flux, consistent with the two-peak structure first found by Timokhin (2006). The radial dependence of the Poynting flux is shown in Figure 4(d). We find that less than 15% of the Poynting flux is dissipated near the light cylinder. The discrepancy is likely due to the assumption of null current flowing in the current sheet that was made in Contopoulos et al. (2013). Plasma in the current sheet beyond the light cylinder supports counterstreaming and produces space-like current, in agreement with the ideal force-free solution (Bai & Spitkovsky 2010).

Our kinetic solution has multiplicity $n/n_{GJ} \approx 10$ near the pole, so we do not observe significant gap regions where parallel electric fields can accelerate particles. The bulk of the polar outflow is thus subrelativistic inside the light cylinder.
Nevertheless, particle acceleration is possible in the equatorial current sheet. The spatial distribution of mean particle Lorentz factors is shown in Figure 5. Energetic particles exist only in the sheet, reaching maximum energy of $\gamma \approx 15$ for the solution with magnetization about 20 at the light cylinder. We note that solutions with smaller magnetization have accelerated particles with lower energy, $\gamma_{\text{max}} \propto \sigma$. The average energy increases with distance along the sheet, and the spectrum is consistent with a drifting Maxwellian. Particles are confined in the sheet and have similar drift velocities in the azimuthal and radial directions. The energy gain is likely due to heating from reconnection. Further acceleration may be possible from the growth of the tearing mode in the sheet and energization at X-points. Our present runtime and resolution (two cells per skin depth in the sheet) are not sufficient to see this effect yet. The skin depth becomes poorly resolved with time because of the density increase from plasma inflow into the sheet. In future work we will study the acceleration mechanism and particle spectrum in more detail. At five stellar rotational periods we observe the growth of drift-kink instability in the current sheet. It causes oscillations of the sheet in the direction perpendicular to the current, with an amplitude of about several local plasma skin depths (see Figure 4(b)). This oscillation widens the current sheet but does not seem to significantly affect the global magnetospheric structure by the end of our run.

4. DISCUSSION

We performed first-principles relativistic PIC simulations of an aligned pulsar magnetosphere by allowing free escape of particles from the stellar surface and feeding the magnetosphere with neutral plasma. We confirm that given sufficient plasma supply the magnetosphere reaches a solution close to the ideal force-free state. The particle energization in our solution happens only inside the equatorial current sheet, which is obtained self-consistently as part of the global magnetosphere structure. The availability of self-consistent three-dimensional kinetic simulations of magnetospheres of magnetized conductors will help the development of quantitative models of observed radiation from pulsars and other magnetized objects.

Admittedly, our kinetic solution is highly idealized. For simplicity we assumed volume production of plasma, so in the future we will need to relax this assumption and study how realistic pair creation prescriptions may affect the magnetospheric structure. The reemergence of accelerating regions may be particularly interesting for high energy emission modeling. In addition, we will need to extend this study to the case of oblique rotators. Runs with higher resolution will allow us to study the current sheet physics and counterstreaming instabilities more reliably. However, even at this stage our kinetic solution is likely a good approximation to the magnetospheric structure of young pulsars with vigorous pair formation. More work can also be done on the numerical side. Our boundary condition currently cannot handle non-uniform fields inside the star, necessitating a switch to a constant interior field. Although we do not see any adverse effects of this, it will be improved on in future work.

Previously, several groups have attempted particle simulations of pulsar magnetospheres with pair production. In SA02, PIC simulations with volume injection of plasma led to effective reduction of accelerating fields and formation of a wind-like outflow for oblique rotators. However, this failed to work for aligned rotators, due to inconsistent boundary conditions on the star. In the present work, we rectified this deficiency with our new plasma conductor boundary condition. Aligned rotators were also simulated by Wada & Shibata (2011) using an electrostatic PIC scheme, and by Gruzinov (2013a) using PIC with modified particle equations of motion. These authors find solutions with large vacuum gaps and regions with $E > B$, which accelerate charges to a large fraction of the available potential. The accelerated particles then effectively decouple from the field beyond the light cylinder. We believe such solutions are an artifact of low pair multiplicity in the magnetosphere. Realistic pulsars are expected to effectively produce quasineutral plasma with densities well in excess of local GJ density. Our kinetic solution shows that for pulsars that have large plasma multiplicity there are no pathologies at the light cylinder and any accelerating fields are likely confined to small volumes in the magnetosphere. Also, we find little evidence for dramatic dissipation of Poynting flux beyond the light cylinder (unlike Gruzinov 2012 and Contopoulos et al. 2013), a conclusion that is likely to only strengthen with improved current sheet resolution. It remains possible that pulsars with weak pair formation form a different class of magnetospheres (Gruzinov 2013b); however, in the limit of vanishing pair formation these solutions must be bounded by the completely charge-separated solutions that we find here. We conjecture that solutions with large vacuum gaps and pair formation confined to near the star eventually evolve to quasi-neutral configurations, although the time it takes to approach this state may scale inversely with the pair production rate. We will check this conjecture with future simulations.

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