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Work-hardening behaviour of sheet steels in large strain regions and its simple approximation

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Abstract. The equivalent plastic strains in stretch flanging and bending frequently exceed the strain equivalent to the uniform elongation in uniaxial tension. However, the uniaxial tensile test does not directly provide the true-stress and true-strain relationship in this strain region. In this study, the simple shear tests and the hydraulic bulge tests of an interstitial free steel, and 590 and 980 MPa grade dual-phase high-strength steels were carried out to investigate the work-hardening behaviour, i.e., the relationship between equivalent stress and equivalent plastic strain as well as the evolution of instantaneous $n$-value, in such a large strain region. A parameter $\kappa$ that correlates stress and strain for a simple shear (or equi-biaxial stretching) to an uniaxial tension leads to a good agreement of stress-strain curves for these testing modes. It was also found that $\kappa$ is practically independent of strain region in the present study. The work-hardening behaviour obtained by a simple shear or bulge test well agrees with that by a uniaxial tensile test. On the other hands, work-hardening equations proposed by previous studies do not reproduce the evolution of instantaneous $n$-value, i.e., a strain dependency of the $n$-value, for the present steel grades. Thus, the authors propose a new work-hardening equation that can reproduce the strain dependency of the $n$-value for any steel grade in this study.

1. Introduction

Computer-aided engineering (CAE) is widely used industrially to reduce development times and costs by omitting production trials. In sheet metal forming, the work-hardening behaviour of sheet metals is among the most influential factor affecting the formability, stiffness, and crushability of formed parts. Therefore, constitutive models capable of accurately reproducing the work-hardening behaviour are necessary.

The true-stress and true-strain relationship (S-S curve), used to determine the work-hardening properties of sheet metals in CAE, is usually measured by uniaxial tensile testing. However, ordinary uniaxial tensile testing does not directly provide the S-S curve in large-strain regions exceeding the limit of uniform elongation, because diffuse necking occurs. In contrast, the equivalent plastic strains in stretch flanging and bending frequently reach the large-strain region. The traditional method used to address this problem has been the extrapolation of the S-S curve in the large-strain region by using a work-hardening equation. However, this risks erroneous CAE results.

To obtain the S-S curve in the large-strain region, material testing methods have been developed by many researchers [1]-[5]. The simple shear test is considered especially effective because it permits...
stable measurement of stress and strain from the yield point to the large-strain region. However, such a procedure requires an appropriate yield locus for the material to determine the equivalent stress and equivalent strain relationship (equivalent S-S curve).

In this study, the basis of a convenient and high-precision procedure to obtain the equivalent S-S curve of steel sheet in the large-strain region using the simple shear test is established. Then, a new work-hardening equation that accurately reproduces the equivalent S-S curves of a variety of steel sheets is proposed.

2. Measurement of the equivalent S-S curve of steel sheets using the simple shear test

2.1. Specimens and experimental procedure

The test materials used in this study were 1.6-mm-thick as-received cold-rolled interstitial-free steel sheet (IF steel) and 590-MPa and 980-MPa grade dual-phase high-strength steel sheets (590DP and 980 DP). The mechanical properties obtained from uniaxial tensile testing in the rolling direction (RD) of the materials are listed in Table 1. In the uniaxial tensile tests, a JIS 5 type specimen and a 50-mm gauge length (GL) extensometer were used.

| Materials | \( t_0 \) (mm) | YS\(^*\) (MPa) | TS\(^*\) (MPa) | \( u\)-EL\(^*\) (%) | EL\(^*\) (%) |
|-----------|----------------|----------------|----------------|---------------------|--------|
| IF steel  | 1.6            | 150            | 286            | 29                  | 54     |
| 590DP     | 1.6            | 407            | 626            | 19                  | 31     |
| 980DP     | 1.6            | 730            | 1051           | 8                   | 14     |

\( YP \): yield stress, \( TS \): tensile strength, \( u\)-EL: uniform elongation, \( EL \): total elongation.

* Measured in RD.

A schematic of the simple shear test used in this study is shown in Figure 1. The details of the testing apparatus can be found in [5]. The shear strain \( \varepsilon_s \) and the shear stress \( \sigma_s \) were calculated by the following equations:

\[
\varepsilon_s = \tan \theta
\]

\[
\sigma_s = \frac{F}{A_0}
\]

where \( \theta \) is the slope of the line drawn on the specimen surface along the short-side direction, \( F \) is the shear force applied to the specimen, and \( A_0 \) is the initial cross-sectional area of the specimen along the longer side.

A hydraulic bulge test was also performed to evaluate the validity of the equivalent S-S curves obtained from the shear tests (see section 2.2). The bulge die with a hole diameter of 200 mm was used. The strain and curvature at the apex of the bulged specimen were measured using a digital image...
correlation method with a grid of 1 mm × 1 mm drawn on the specimen surface. The equi-biaxial stress $\sigma_b$ was determined as:

$$\sigma_b = \frac{P \cdot \rho}{2 \pi \cdot e_b \cdot \exp(-2 \epsilon_b)}$$

(3)

where $P$ is the internal pressure, $t_0$ is the initial thickness of the specimen, and $\epsilon_b$ and $\rho$ are the strain and the curvature, respectively, at the apex of the specimen.

The equivalent strain rate applied to the test materials was approximately $10^{-3} \text{s}^{-1}$ in all material testing.

2.2. Transformation method for the equivalent S-S curve

Denoting the tensile true stress and logarithmic plastic strain measured in the uniaxial tensile test along the RD as $\sigma_u$ and $\epsilon_u^P$, respectively, the stress and strain equivalent to $\sigma_u$ and $\epsilon_u^P$ are defined as the equivalent stress $\bar{\sigma}$ and equivalent plastic strain $\bar{\epsilon}^P$. To transform the shear S-S curve to the $\sigma_u$-$\epsilon_u^P$ curve, the parameters $\kappa_\sigma$ and $\kappa_\epsilon$ are introduced as:

$$\bar{\sigma} = \kappa_\sigma \cdot \sigma_s$$

(4)

$$d\bar{\epsilon}^P = \kappa_\epsilon \cdot d\epsilon_s^P$$

(5)

as proposed in [6].

Assuming that the yield stress of a material is determined as a function of the plastic work per unit volume, $W^p$, consumed by the material throughout all deformation processes, the following equation is obtained:

$$dW^p = \bar{\sigma} \cdot d\bar{\epsilon}^P = \sigma_s \cdot d\epsilon_s^P$$

(6)

From Eqs. (4), (5) and (6), we obtain

$$d\bar{\epsilon}^P = \sigma_s \cdot d\epsilon_s^P = \frac{1}{\kappa_\sigma} \cdot d\epsilon_s^P \quad \therefore \quad \kappa_\epsilon = \frac{1}{\kappa_\sigma}$$

(7)

Moreover, $\kappa_\sigma$ (equivalently, $\kappa_\epsilon$) is constant when the material work-hardens isotropically. Thus, the $\bar{\sigma}$-$\bar{\epsilon}^P$ curve can be determined from the following equations using $\kappa_s$ as

$$\bar{\sigma} = \kappa_s \sigma_s, \quad d\bar{\epsilon}^P = \frac{1}{\kappa_s} \cdot d\epsilon_s^P, \quad \kappa_s = \text{Const.}$$

(8)

In this study, the value of $\kappa_s$ was determined by the least-squares method of fitting the S-S curve calculated from Eq. (8) to the uniaxial curve.

Parameter $\kappa_b$, which transforms the equi-biaxial S-S curve to the $\bar{\sigma}$-$\bar{\epsilon}^P$ curve, was also determined by replacing $\sigma_s$ and $d\epsilon_s^P$ in Eq. (8) with $\sigma_b$ and $d\epsilon_b^P$, where $d\epsilon_b^P$ is defined using the increment of thickness strain, $d\epsilon_z^P$, as

$$d\epsilon_b^P = -d\epsilon_z^P$$

(9)

2.3. Validity of the transformation method for the equivalent S-S curve

Figure 2 shows $\kappa_\sigma$ and $1/\kappa_\epsilon$ determined for identical plastic work in each material test. $\kappa_{u/s}$ and $1/\kappa_{u/s}$ are those for the uniaxial tensile and simple shear tests, and $\kappa_{u/b}$ and $1/\kappa_{u/b}$ are those for the hydraulic bulge and simple shear tests. The constant parameters $\kappa_s$ and $\kappa_b$ are also shown in Figure 2. For the IF steel, $\kappa_{u/s}$ does not coincide with $1/\kappa_{u/s}$. The values of $\kappa_{u/s}$ and $1/\kappa_{u/b}$ are smaller than $\kappa_s$ near initial yielding, and are increased with increases in plastic work, finally becoming equal to $\kappa_s$ in the strain range of $\bar{\epsilon}^P > 0.1$. For the 590DP and 980DP, $\kappa_\sigma$ and $1/\kappa_\epsilon$ of each of the material tests agreed with each of the constant parameters in the measurable strain region.

The difference of $\kappa_\sigma$ and $1/\kappa_\epsilon$ observed in the IF steel means that its yield stress is not determined only by a function of $W^p$. The changes in $\kappa_\sigma$ and $1/\kappa_\epsilon$ indicate anisotropic hardening behaviour. It
may be necessary to consider such behaviour for the transformation method, if the degree of anisotropic hardening is significant. Practically, however, $\kappa^\sigma$ and $1/\kappa^\varepsilon$ in the IF steel can be assumed as equal and constant. The equivalent S-S curve obtained by simple shear or bulge testing agrees well with that by uniaxial tensile testing, as shown in Figure 3.

**Figure 2.** Comparison of the stress and strain ratio at equal plastic work values for different material tests; (a) IF steel, (b) 590DP, and (c) 980DP. Dashed and dotted lines are calculated by the proposed method.

**Figure 3.** $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves of specimens obtained from the material tests; (a) IF steel, (b) 590DP, and (c) 980DP.

### 3. Reproducibility of existing work-hardening equations

The reproducibility of the $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves in the following work-hardening equations was investigated using the curves obtained from the simple shear test.

**Swift’s equation** [7]:

$$\bar{\sigma} = C (\bar{\varepsilon}^p + \bar{\varepsilon}_0)^n$$

**Ludwigson’s equation** [8]:

$$\bar{\sigma} = K_1 (\bar{\varepsilon}^p)^{n_1} + \exp(K_2 + n_2 \cdot \bar{\varepsilon}^p)$$

**Voce’s equation** [9]:

$$\bar{\sigma} = \sigma_s - (\sigma_s - \sigma_l) \cdot \exp(n_V \cdot \bar{\varepsilon}^p)$$

where $C$, $\varepsilon_0$, and $n^*$ in Eq. (10), $K_1$, $n_1$, $K_2$ and $n_2$ in Eq. (11), $\sigma_s$, $\sigma_l$ and $n_V$ in Eq. (12) are curve-fitting parameters.
The experimental data of the $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves obtained from the simple shear test have equal strain increments of $4 \times 10^{-4}$. The material parameters of Eqs. (10), (11), and (12) were determined by the least-squares method by fitting the $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves calculated from these equations to the experimental one. Additionally, the instantaneous $n$-value ($n'$-value) was calculated from the $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves. The $n'$-value was defined as the instantaneous slope of the $\bar{\sigma}$-$\bar{\varepsilon}^p$ curve plotted on a double logarithmic chart for the incremental strain of 0.025.

Figures 4 and 5 compare the calculated $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves and $n'$-values with the experimental ones. The reproducibility of Voce’s equation in the IF steel, that of Swift’s equation in the 590DP, and that of Ludwigson’s equation in the 980DP were the highest. As shown in Figure 5, the behaviour of the $n'$-value differs according to the materials. Therefore, a work-hardening equation must have a flexibility to reproduce the strain dependency of the $n$-value for any steel grade.

**Figure 4.** $\bar{\sigma}$-$\bar{\varepsilon}^p$ curves and those approximated using selected work-hardening laws; (a) IF steel, (b) 590DP, and (c) 980DP.

**Figure 5.** Measured values of $n'$ and those obtained from selected work-hardening laws; (a) IF steel, (b) 590DP, and (c) 980DP.

4. Development of new work-hardening equation

From the investigation of the $n'$-value for the steel sheets used in this study, a novel work-hardening equation to accurately reproduce the strain dependency of the $n$-value is proposed as:

$$\bar{\sigma} = K(\bar{\varepsilon}^p + a)^{n + b/\bar{\varepsilon}^p + c}$$  \hspace{1cm} (13)
where $K$, $a$, $\bar{a}$, $b$, and $c$ are curve-fitting parameters. This equation was developed based on Swift’s equation; the $n'$-value asymptotically approaches the value of $\bar{a}$. A change in the $n'$-value is described by the fitting parameters $b$ and $c$. For $b = 0$, the equation is identical to Swift’s equation. Figure 6 compares the $\bar{\sigma}$-\(\bar{\varepsilon}_p\) curves and the $n'$-value between the experimental data and approximated results using Eq. (13). The proposed work-hardening equation reproduces the strain dependencies of the $n$-values for the three specimens.

By investigating a variety of steel sheets, the proposed equation with $a = c$ can practically reproduce the work-hardening behaviour of steel with satisfactory precision. Figure 7 shows the errors of the flow stresses of the approximated results using Eq. (13) for a variety of steel sheets. The proposed work-hardening equation can reproduce the $\bar{\sigma}$-\(\bar{\varepsilon}_p\) curve for any steel grade with an error of less than 0.5%.

Figure 6. Comparison between experimental results and approximated results using proposed work-hardening equation; (a) $\bar{\sigma}$-\(\bar{\varepsilon}_p\) curves and (b) the $n'$-value.

Figure 7. Residual error of flow stresses of approximated results using the proposed work-hardening equation, Eq. (13), with $a = c$.

5. Conclusions
1. The simple shear test is an effective material testing method for accurately measuring the $\bar{\sigma}$-\(\bar{\varepsilon}_p\) curve of steel sheets from initial yielding to the large-strain region.
2. The proposed work-hardening equation capably reproduces the equivalent S-S curve and thus the strain dependency of the $n$-value for any steel grade.

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