BUNDLING AND PRICING DECISIONS FOR
BRICKS-AND-CLICKS FIRMS WITH CONSIDERATION OF
NETWORK EXTERNALITY

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Abstract. The development of the Internet has dramatically changed firms’ business models. Companies can now use both virtual and physical channels to enhance their competitiveness and profitability. In addition, bundling is a commonly used promotion strategy, although managers should consider the characteristics of the candidate bundled products. This study proposes a two-stage game theoretic model, in which a manufacturer may start an online channel along with an existing physical one which is operated by a dealer, i.e., a bricks-and-clicks approach, to examine the bundling and pricing strategy when selling two products with different network externalities. In the first stage, the manufacturer offers the products to the dealer, who may sell the two products individually or in a bundle to customers. In the second stage, and with the aim of expanding market share, the manufacturer may consider starting an online channel to integrate with the existing physical channel. We consider four cases, in which the manufacturer and dealer may sell the two products either individually or bundled in the two channels, in order to obtain the corresponding optimal pricing strategies with the aim of maximizing their profits. We also perform a numerical analysis to investigate the effects that network externality has on the bundling strategies and profits of the two channels. The results indicate that the bricks-and-clicks business model benefits both the manufacturer and dealer, and their profits would increase as network externality increases. In particular, when the network externalities of the two products are both high, a mixed strategy, which sells the two products in a bundle in the online channel and individually in the physical channel, should be adopted.

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1. **Introduction.** Firms often sell products through integrated channels, which include both physical and online stores, forming what is known as the bricks-and-clicks business model (Bendoly et al., 2007 [7]; Bernstein et al., 2008 [8]; Hsiao and Chen, 2014 [24]). For example, Apple, Walmart, and Gap used to sell products only in conventional retailing stores, but the rise of the Internet enabled them to start their own online channels to enhance their market shares (Chiang et al., 2003 [16]; Liu et al., 2006 [37]; Zhang, 2009 [56]). However, the relationship between the conventional physical channel and new online channel is not only complementary, but also competitive (Chen et al., 2008 [13]; Dumrongsi et al., 2008 [20]). Consumers perceive the two channels differently, and the perceived convenience, value, and risk of the two channels influence the selection of either one (Jeffers and Nault, 2011 [28]; Xiong et al., 2012 [52]). While many firms have shifted their business models from brick and mortar stores to a bricks-and-clicks approach, this move has raised potential problems of channel conflicts and the selection of the appropriate promotion strategies (Yao and Liu, 2005 [54]). The emergence of virtual channels offers consumers more choices, but also complicates purchasing behaviors (Viswanathan, 2005 [49]; Jeffers and Nault, 2011 [28]).

Bundling is a commonly used marketing strategy which puts two or more products or services together for sale at a price which is often lower than the total price of the individual items (Stremersch and Tellis, 2002 [43]; Balachander et al., 2010 [4]; Alexandrov and Bedre-Defolie, 2014 [1]). Bundling goods may bring clustering effects, which can enhance the perceived value of the bundled items, and thus encourage consumers, who were originally interested in only one product, to pay more for the whole bundle. However, firms may also lose those consumers who do not want one of the products in the bundle (Ibragimov and Walden, 2010 [27]; Hui et al., 2012 [26]). In addition, product characteristics may also affect the decision whether or not to purchase bundling products (Hitt and Chen, 2005 [23]; Wu et al., 2008 [50]; Jiang et al., 2011 [29]).

Changes in network and digital technologies have increased the mobility of information dissemination, thus changing certain long-standing economic rules, with firms no longer dealing with single consumers but instead with groups of them, and, as a result, there is a need to focus on economies of scale on the demand-side (Stremersch et al., 2007 [44]; Cheng and Tang, 2010 [15]). The success of many businesses now depends on the network externality of their products, which rises when the perceived value of a product increases with the number of users it has (Katz and Shapiro, 1985 [31]; Chen and Xie, 2007 [14]). The network externalities associated with high-technology and Internet-related products can often lead to a rapid growth in the number of users, due to the widespread use of the Internet, and can then increase the value perceived by potential users (Sun et al., 2004 [45]; Parker and van Alstyne, 2005 [41]; Kornish, 2006 [32]). Network externalities are thus an important factor which would affect both consumer preferences for purchasing bundled goods and firms’ decisions about pricing and sale channels (Basu et al., 2003 [6]; Chien and Chu, 2008 [17]; Katona et al., 2011 [30]; Derdenger and Kumar, 2013 [19]).

Based on the aforementioned discussion, while the use of the bricks-and-clicks business model is growing, it is also complicated. If a bricks-and-clicks firm can use bundling strategies appropriately with consideration of the network externality of the related products, then the optimal pricing can be determined in order to attract more consumers to make purchases, so increasing the firm’s competitive advantage.
and profitability. Bricks-and-clicks firms can either physically or virtually contact consumers to satisfy those with different requirements, and consumers can select their shopping channels based on their preferences and the products or services needed. Bundling can enhance clustering effects and enable firms to raise the perceived values of bundled products, but bundling products with different network externalities and characteristics may require different pricing strategies. However, to date there is relatively little research on the bricks-and-clicks business model with the use of bundling strategies, and channel conflicts are rarely considered in this context. In addition, previous research often investigated the effects that different product types have on the profitability of bundling strategies, without considering other related factors. To address this gap in the literature, this study considers a manufacturer, who originally sells his products through the existing physical channel of a dealer, then starts an online channel to adopt the bricks-and-clicks business model, and the product characteristics and network externalities are considered in this context when using the bundling strategy in either channel. We investigate the effects that network externality may have on the firm’s decision to sell products bundled or individually, as well as consumers’ selection of a shopping channel, and derive the optimal pricing for the manufacturer with the aim of maximizing profits.

The determination of an appropriate pricing strategy to enhance profitability is a critical issue for bricks-and-clicks firms. Cattani et al. (2006) [11] considered a competitive context in which a manufacturer starts an online channel to sell the focal product at a wholesale price, which is the same as the retail price, to compete with an in-store retailer in order mitigate channel conflict, and found that this approach can maximize the manufacturer’s profit, and satisfy both the retailer and consumers. Ofek et al. (2011) [40] proposed a game-theoretic model to investigate the effects that the choice of operating online and in-store channels for two competing retailers has on pricing and profitability, with consideration of product returns due to customer dissatisfaction, and found that an asymmetric equilibrium solution would occur when one retailer opens an online channel which earns less profit than that seen for the in-store channel only. Hua et al. (2010) [25] noted that previous studies indicate that the service quality has a greater impact on consumer acceptance of online channels than product price, and used a game-theoretic model to investigate the optimal delivery lead time and pricing strategies for centralized and decentralized dual channels. Li et al. (2015) [34] proposed a cost-sharing model for a warehouse-retailer network design game. Liu et al. (2017) [36] proposed a dynamic pricing model in which two firms sell products to consumers with bounded rationality with consideration of network effects. Lai et al. (2018) [33] investigated the optimal pricing and production decisions in a dual-channel supply chain under symmetric and asymmetric information sharing conditions. They considered that information asymmetry is beneficial to the reseller, but is inefficient to the manufacturer and the entire supply chain. Yang et al. (2018) [53] studied advertising games on national brand and store brand products in a dual-channel supply chain. They concluded that the dual-channel supply chain can earn more profits if the member who has greater marginal profits from cooperative advertising is the dominant player. Sun et al. (2019) [46] investigated the competition in a duopolistic dual-channel supply chain, and concluded that, in general, the supplier can earn more profits in most cases by raising the retail price or holding down the self-price elasticity.
With regard to bundling, both the types and characteristics of products need to be considered when forming bundles. McCardle et al. (2007) [39] considered the bundling and pricing problem of two types of retail products, commodities and fashion goods, in which the former often have longer life cycles and more predictable demand, and then investigated the optimal bundling strategy. They found that the profitability from bundling depends on the demand for individual products, as well as the relationship among the demands for the bundled products. Banciu et al. (2010) [5] considered the optimal bundling strategy of a seller who offers two vertically differentiated products, with consideration of space constraints, and found that the availabilities and sub-additivity of the two products would greatly impact the optimal solution when adopting the bundling strategy, and that the mixed and pure bundling approaches would not be optimal for the condition of super-additivity. Geng et al. (2005) [22] stated that the average value perceived by consumers for information goods would decrease with the number of such products consumed, and found that when the consumers’ perceived value decreases slowly in this content, then the bundling strategy is more or less optimal. Bakos and Brynjolfsson (2000) [3] stated that bundling can create benefits from aggregation effects for information goods with low marginal costs, and found that the profit from selling bundles is higher than that from selling separate products, and consumers would find that buying bundles makes more economic sense than buying separate products at the optimal price. Li et al. (2018) [35] investigated the optimal bundling supply chain and concluded that the profits of members in the supply chain can be enhanced using the bundling strategy. Xie et al. (2018) [51] investigated the impact that the stochastic demand and manufacturers’ decisions have on the bundling strategy, and suggested that manufacturers should cooperate with retailers to maximize their profits. Zhang et al. (2018) [57] stated that more and more e-commerce companies are willing to bundle their products with services for sale on Internet. They concluded that the bundling discount plays a different role according to the degree of service uncertainty for e-business. Taleizade et al. (2019) [47] considered a supply chain in which two complementary fashion products are manufactured and sold as a bundle, and such a bundle has a short selling season and a stochastic price dependent on demand.

The appropriate bundling strategies are different for complementary and unrelated products. Derdenger and Kumar (2013) [19] stated that the profitability of bundling depends on the degree of homogeneity between consumer perceived value and the bundled products. They found that bundling can be effective in classifying consumers, and that consumers would perceive bundled goods as independent products when the degree of homogeneity is low. Bhargava (2013) [9] stated that mixed bundling problems are often too complicated to solve analytically, and developed a simplified mixed bundling model for two independently valued products. Bitran and Ferrer (2007) [10] considered the problem of how to determine the composition and price of a bundle with the aim of maximizing the profit, and stated that the components in bundles have to meet certain technical constraints. Researchers have also examined bundling products with network externalities. Chao and Derdenger (2013) [12] developed a game-theoretic model for mixed bundling in a two-sided market, in which the purchase of a product may increase the potential demand for another complementary product, and investigated the effects that mixed bundling has on prices and the benefits to both the firm and consumers.
Prasad et al. (2010) [42] incorporated the concept of network externality into traditional bundling to consider a monopolist who sells two products in the market, and found that when both products have low marginal costs or high network externalities then pure bundling is more profitable, and when the two products are different in their costs and network externalities, then pure components and mixed bundling are more profitable. Luo et al. (2017) [38] developed a mixed bundling strategy for information products with network externality. They concluded that the significance of the impact of network externality on optimal bundling depends on the releasing time of the bundle. Aoyagi (2018) [2] developed a Bertrand competition model for explaining network externalities, and the analytical results can be applied to platform competition in a two-sided market. Yi and Yang (2017) [55] investigated the dynamics between the strategic marketing choice of the retailers and the wholesale pricing of the manufacturer in a market with network externality. They concluded that increasing network externality cannot generate more profits if the market position of retailers is inferior, but a large network externality can indeed partially compensate the profit losses of the manufacturer which caused by the improved bargaining power of retailers.

In sum, based on a review of the literature, the objectives of this study are as follows: (1) to investigate whether the manufacturer should start an online channel; (2) to determine the optimal bundling and pricing strategy for the manufacturer and dealer; (3) to investigate the effects that network externality has on the pricing strategy, channel expansion strategy, bundling strategy, and consumers’ selection of shopping channel. The remainder of this study is organized as follows: Section 2 states the research problem, framework, and assumptions. Section 3 describes the process of the model construction, which is then used to derive the optimal bundling and pricing strategy with consideration of the network. Section 4 presents an analytical discussion of the results. Finally, Section 5 gives the concluding remarks and some suggestions for future research.

2. The bricks-and-clicks business model. We propose a two-stage game theoretic model from the perspective of the manufacturer. At the first stage, the manufacturer offers two products, products 1 and 2, to a dealer, who can determine whether to sell the two products in a bundle or individually. At the second stage, the manufacturer may start an online channel to integrate with the existing physical channel, i.e., a bricks-and-clicks business model, to expand market share and satisfy consumers' various requirements. Both the manufacturer and dealer can determine their own bundling strategies and selling prices to enhance consumers’ willingness to make a purchasing and thus increase their sale volumes. The manufacturer and dealer may sell the two products in different ways: (1) pure components: products 1 and 2 are sold separately, (2) bundling: products 1 and 2 are sold as a bundled product $B$ at a discounted price, and (3) mixed bundling: the two products are sold individually and in a bundle.

At the first stage, the manufacturer offers two products with different network externalities through a dealer to the market. The unit cost of product $j$ is $c_j$, where $j = 1, 2$, $B$ indicates products 1, 2, and the bundle $B$. The manufacturer offers product $j$ at the wholesale price $w_j$ to the dealer. Suppose that the manufacturer would offer a discounted price, $w_B$, where $w_B \leq w_1 + w_2$, for product $B$, if the dealer orders products 1 and 2 as a bundle. We assume that the dealer sells product $j$ in a
physical store with the market potential, \(D_{rj}\) which is affected by the network externality of product \(j\), \(\delta_j\). The dealer can determine the selling price of product \(j\), \(p_{rj}\), and consumers would pay \(p_{rj}\) for product \(j\). Note that the price of bundled product \(B\), \(p_{rB}\), is lower than the total price of the two products, i.e., \(p_{rB} \leq p_{r1} + p_{r2}\). Otherwise, no consumer would choose to purchase bundled products. Consumers select products and shopping channels according to their preferences and shopping experiences. Suppose that consumers are uniformly located on a Hostelling line, that is, each consumer is located at \(x\), which has a uniform distribution, i.e. \(x \sim U[0,1]\). We assume that the consumers’ perceived value with regard to product \(j\) is \(v_j\), and this value is independent of the perceived value of other products. Accordingly, the consumers’ perceived value for bundle \(B\) is the sum of the consumers’ perceived value for products 1 and 2. When consumers decide to purchase product \(j\) in the physical channel, a consumer at \(x\) would be responsible for the transport cost \(t_x\), which is related to the unit distance cost, \(t\). The consumers’ utility from purchasing product \(j\) in the physical channel is thus given by \(v_j + \delta_j D_{rj} - p_{rj} - t_x\). In order to ensure that the utility function must be greater than zero, the condition \(D_{rj} > (1/\delta_j) (p_{rj}+t_x - v_j)\) must be satisfied.

At the second stage, in order to expand the market, the manufacturer may start an online channel to directly sell the product to consumers. In doing so, the manufacturer would incur a fixed setup cost \(F\). Without sharing profit with intermediaries, the manufacturer can offer product \(j\) at a lower price, \(p_{dj}\), to consumers. However, when consumers look for products in online channels they can only learn about the items of interest through photos and text descriptions, and thus cannot directly examine their quality, which may result in a deficit between the imagined state of the focal item and its actual condition. In addition, consumers have to wait for the ordered product to be delivered. Therefore, we assume that the costs that consumers’ perceived inconvenience, uncertainty, and risk in relation to the online channel, along with the shipping fee required for delivery, would be set as \(s\), and the market potential of the online channel is \(D_{dj}\). The consumers’ utility from purchasing product \(j\) in the online channel is thus given by \(v_j + \delta_j D_{dj} - p_{dj} - s\). The positive utility means that the customers are willing to purchase product \(j\) in the online channel, and therefore it implies that the condition \(D_{dj} > (1/\delta_j) (p_{dj}+s - v_j)\) is satisfied.

The assumptions of this study are as follows.

1. The physical and online channels may compete with each other in the same market. When consumer demand is not satisfied by one channel, consumers would turn to purchase the product from the other channel.
2. Products 1 and 2 are assumed to have different network externalities, so they may be bundled for sale to increase the overall visibility and reduce consumers’ searching cost.
3. The consumers’ perceived value of bundled product \(B\) is the sum of the consumers’ perceived value of products 1 and 2, and the selling price of bundle \(B\) is less than the total price of products 1 and 2.

The notations used in this study are as follows.

- \(i\) The selling channel, where \(i = d\) indicates the online channel and \(i = r\) indicates the physical channel.
- \(j\) The product type and \(j = 1, 2, B\).
- \(D_{ij}\) The demand for product \(j\) in selling channel \(i\).
- \(p_{ij}\) The selling price of product \(j\) in selling channel \(i\).
3. The bundling and pricing strategy. This study investigates the dynamics between the selling prices of the two products in the two shopping channels and consumer demand with consideration of consumer preferences toward online and in-store shopping to derive the optimal bundling and pricing strategy for the two products with different network externalities.

3.1. The first stage. In the first stage, the dealer may sell the two products either individually or in a bundle. The profits for the dealer and manufacturer for different bundling strategies are considered, and the corresponding optimal pricing strategies are thus derived.

Suppose that the dealer determines to use the pure component strategy and sell products 1 and 2 individually to the market. According to Chun and Kim (2005), a consumer located at \( x \) who purchases product \( j \) in the physical channel would have the utility as given by

\[
u_{ij} = v_j + D_{rj} \delta_j - p_{rj} - tx.
\]

Suppose that the total market potential is standardized as 1, and consumers would make their purchasing decisions based on the maximization of their surplus. We can derive the demand for the two products by solving the equation \( v_j + D_{rj} \delta_j - p_{rj} - tx = 0 \) for \( x \). Suppose that each consumer is located at \( x \), which has a uniform distribution, i.e. \( x \sim U[0, 1] \).

Since the market potential is increased to \( D_{r1} (1 + \delta_1) \), due to the effects of network externality on demand, the market demand for product 1 is thus given by

\[
D_{r1} = \frac{p_{r1} - v_1}{1 + \delta_1}.
\]

Likewise, the demand for product 2 can also be obtained. In considering the pure component strategy, in which products 1 and 2 are sold individually, the profit of the dealer is the sum of the profits gained from products 1 and 2, which is given by

\[
\pi_c^e = (p_{r1} - w_1) \left( \frac{(p_{r1} - v_1) (1 + \delta_1)}{-t + \delta_1 + \delta_1^2} \right) + (p_{r2} - w_2) \left( \frac{(p_{r2} - v_2) (1 + \delta_2)}{-t + \delta_2 + \delta_2^2} \right),
\]

and the profit for the manufacturer is thus given by

\[
\pi_m = (w_1 - c_1) \left( \frac{(p_{r1} - v_1) (1 + \delta_1)}{-t + \delta_1 + \delta_1^2} \right) + (w_2 - c_2) \left( \frac{(p_{r2} - v_2) (1 + \delta_2)}{-t + \delta_2 + \delta_2^2} \right).
\]

**Theorem 3.1.** When the dealer sells products 1 and 2 individually, the optimal selling prices for product \( j \) and the maximum profits are given by

\[
p_{rj}^* = \frac{1}{2} (v_j + w_j),
\]
respec- respectively, and the profit for the manufacturer is given by
\[
\pi_c = \frac{1}{4} \left( \frac{(v_1 - w_1)^2 (1 + \delta_1)}{-t + \delta_1 + \delta_1^2} - \frac{(v_2 - w_2)^2 (1 + \delta_2)}{-t + \delta_2 + \delta_2^2} \right),
\]
(4)
respectively, and the profit for the manufacturer is given by
\[
\pi_c^m = \frac{1}{2} \left( \frac{(v_1 - w_1)(-c_1 + w_1)(1 + \delta_1)}{-t - \delta_1 (1 + \delta_1)} + \frac{(v_2 - w_2)(-c_2 + w_2)(1 + \delta_2)}{t - \delta_2 (1 + \delta_2)} \right).
\]
(5)
Proof. Since the second order derivative of the profit function of the dealer with respect to \( p_{r1} \) and \( p_{r2} \) are both less than zero, i.e., \( \frac{\partial^2 \pi^c}{\partial p_{r1}^2} = \frac{2(1+\delta_1)}{t+\delta_1+\delta_1^2} < 0 \) and \( \frac{\partial^2 \pi^c}{\partial p_{r2}^2} = \frac{2(1+\delta_2)}{t+\delta_2+\delta_2^2} < 0 \), when \( t > \delta_1(1 + \delta_1) \), the profit function is convex and has a global maximum value. By setting the first order derivative of the profit function of the in-store retailer with respect to \( p_{r1} \) and \( p_{r2} \), respectively, we have
\[
\frac{\partial \pi^c}{\partial p_{r1}} = \frac{2(p_{r1} - v_1 - w_1)(1 + \delta_1)}{-t + \delta_1 + \delta_1^2} = 0 \quad \text{and} \quad \frac{\partial \pi^c}{\partial p_{r2}} = \frac{2(p_{r2} - v_2 - w_2)(1 + \delta_2)}{-t + \delta_2 + \delta_2^2} = 0.
\]
We can thus obtain the optimal selling price for product \( j \) by solving the simultaneous equations. Moreover, the manufacturer would assess his own demand and profit based on the dealer’s optimal selling strategy to maximize his own profit.

Theorem 3.1 verifies that network externality would not affect the determination of the optimal selling price for the dealer. However, both the profits of the manufacturer and dealer are positively correlated to network externality. The manufacturer and dealer can thus enhance product network externality by using promotion activities and increasing product visibility to gain more demand and earn more profits.

On the other hand, suppose the dealer would use the pure bundling strategy to put products 1 and 2 together as bundle \( B \), which indicates that only one product exists in the market. In this case consumers who are originally interested in one product may pay more to purchase the whole bundle. However, some consumers may want to purchase only one product, and thus there may be a potential loss in this situation. According to the aforementioned assumptions, the perceived value of bundle \( B \) is the total perceived value of products 1 and 2. Therefore, we can obtain the consumer utility for bundle \( B \) in the physical channel, which is given by \( u_{rB} = v_1 + v_2 - p_{rB} - tx + (\delta_1 + \delta_2)D_{rB} \). The indifference point is derived as \( x_0 = \frac{p_{rB} + v_1 + v_2 + D_{rB}(\delta_1 + \delta_2)}{t} \), which can be used to classify consumers who would either purchase bundle \( B \) or no product. In considering network externality, the market potential of bundle \( B \) is increasing as \( (1 + \delta_1 + \delta_2)D_{rB} \), we can thus obtain the demand for bundle \( B \), which is given by \( D_{rB} = \Pr \left( x < \frac{p_{rB} + v_1 + v_2 + D_{rB}(\delta_1 + \delta_2)}{t} \right) (1 + \delta_1 + \delta_2) = \frac{(p_{rB} - v_1 - v_2)(1 + \delta_1 + \delta_2)}{-t + \delta_1^2 + \delta_2 + \delta_1 (1 + 2\delta_2)}. \)
Therefore, the profit of the dealer, who sells only bundle \( B \), can be obtained as
\[
\pi^e_B = (p_{rB} - w_B) \left( \frac{(p_{rB} - v_1 - v_2)(1 + \delta_1 + \delta_2)}{-t + \delta_1^2 + \delta_2 + \delta_1 (1 + 2\delta_2)} \right),
\]
(6)
and the profit of the manufacturer is given by
\[
\pi^m_B = (w_B - (c_1 + c_2))(\frac{p_{rB} - v_1 - v_2)(1 + \delta_1 + \delta_2)}{-t + \delta_1^2 + \delta_2 + \delta_1 (1 + 2\delta_2)}).
\]
(7)

Theorem 3.2. When the dealer determines to sell only bundle \( B \), the optimal selling prices and maximum profits are given by
\[
p^*_r = \frac{1}{2} (v_1 + v_2 + w_B),
\]
(8)
and

$$\pi_B = -\frac{(v_1 + v_2 - w_B)^2}{4(-t + \delta_1^2 + \delta_2 + \delta_2^2 + \delta_1 (1 + 2\delta_2))}. \quad (9)$$

respectively, and the profit for the manufacturer is given by

$$\pi_B^m = -\frac{(c_1 + c_2 - w_B)(-v_1 - v_2 + w_B)(1 + \delta_1 + \delta_2)\quad (10)}{2(-t + \delta_1^2 + \delta_2 + \delta_2^2 + \delta_1 (1 + 2\delta_2))}.$$  

Proof. Since $0 \leq \delta_j \leq 1$, the second order derivative of the profit function with respect to $p_{rB}$,

$$\frac{\partial^2 \pi_B}{\partial p_{rB}^2} = \frac{2(1+\delta_1+\delta_2)}{-t+\delta_1^2+\delta_2^2+\delta_1(1+2\delta_2)}$$

would be negative when $t > \delta_2(1+\delta_2) + \delta_1 (1 + \delta_1 + 2\delta_2)$. That is, the profit function is convex and has a maximum value. By letting the first order derivative be zero, i.e., $\frac{\partial \pi_B}{\partial p_{rB}} = \frac{(2p_{rB} - v_1 - v_2 - w_B)(1+\delta_1+\delta_2)}{-t+\delta_1^2+\delta_2^2+\delta_1(1+2\delta_2)} = 0$, we can thus obtain the optimal selling price of bundle B, and so do the maximal profits for the manufacturer and dealer. \qed

The optimal selling price of bundle B for the dealer is only affected by the consumers’ perceived value and the manufacturer’s wholesale price, and network externality has great impacts on the manufacturer’s and dealer’s profitability. In addition, the manufacturer has to consider the product cost, which affects the determination of the wholesale price and thus influences the dealer’s profit and optimal selling pricing.

### 3.2. The second stage.

At the second stage, in order to expand the market and meet different consumer demands, the manufacturer considers starting an online channel along with the existing physical channel to attract different types of customers by offering more services. In such a case, both the online and physical channels offer homogenous products, which include products 1 and 2 and bundle B, and would determine their own bundling and pricing strategies with consideration of consumer shopping preferences, the advantages and characteristics of different selling channels, and the synergistic effects of network externality. Table 1 shows the four possible combination strategies that the manufacturer and dealer may use.

| The Bricks-and-Clicks Operation |
|---------------------------------|
| **The Online Channel**          |
| Pure component                  |
| Pure Bundling                   |
| **The Physical Channel**        |
| Pure component                  |
| I                               |
| II                              |
| Pure Bundling                   |
| IV                              |
| Case I: Pure Component Strategy in both Online and Physical Channels |

When the manufacturer starts an online channel in the market, and the both channels use the pure component strategy to individually sell product j at their own selling prices to attract customers, consumers select a shopping channel based on their preferences and shopping experiences. According to Liu et al. (2006), the consumers’ utility for product j in the online channel can be given by $u_{dj} = v_j + \delta_j D_{dj} - p_{dj} - s$, where s denotes the inconvenience cost for consumers in this context. The consumers’ utility on product j in the physical channel is the same as stated in Section 3.1. When $u_{r1} > u_{d1}$, consumers would purchase product 1 in the physical channel. Otherwise, they would purchase product 1 in the online channel. Therefore, when $x < \frac{c_1 + c_2 - w_B}{t + \delta_1 + \delta_2 + \delta_1 (1 + 2\delta_2)}$, consumers who are near
the physical store would purchase product 1 in the physical channel. Otherwise, consumers would turn to purchase product 1 in the online channel. Since network externality would increase demand to \( D_{ij} (1 + \delta_j) \), the demands for product \( j \) in the physical and online channels are given by

\[
D_{r1} = \frac{(1+\delta_1)(s-t+p_{d1}+p_{r1}+\delta_1+\delta_j^2)}{t+2\delta_1(1+\delta)}
\]

and

\[
D_{d1} = \frac{(1+\delta_1)(s-t+p_{d1}+p_{r1}+\delta_1+\delta_j^2)}{t+2\delta_1(1+\delta)}
\]

respectively. Accordingly, the profits of the physical and online channels are derived as

\[
\pi^p_i = \sum_{j=1}^{2} \left( p_{rj} - w_j \right) \left( \frac{(1+\delta_j)(-s-p_{d1}+p_{rj}+\delta_j+\delta_j^2)}{-t+2\delta_j(1+\delta_j)} \right)
\]

and

\[
\pi^m_i = \sum_{j=1}^{2} \left( w_j - c_j \right) \left( \frac{(1+\delta_1)(-s-p_{d1}+p_{r1}+\delta_1+\delta_1^2)}{-t+2\delta_1(1+\delta_1)} \right)
\]

respectively.

**Theorem 3.3.** When both the manufacturer and dealer use the pure component strategy, the optimal selling prices of product \( j \) and maximum profits for the dealer and manufacturer would be given by

\[
p^*_{rj} = \frac{1}{2} \left( -s + 2t + 2w_j - 3\delta_j - 3\delta_j^2 \right),
\]

\[
p^*_j = \frac{1}{4} \left( s + 2t + 4w_j - 5\delta_j - 5\delta_j^2 \right),
\]

\[
\pi^m_i = \sum_{j=1}^{2} \left( 1+\delta_j \right) \left( \frac{8(t-2(\delta_j+\delta_j^2))}{s-2t+3(\delta_j+\delta_j^2)^2} \right) \left( \frac{8(t-2(\delta_j+\delta_j^2))}{s-2t+3(\delta_j+\delta_j^2)^2} \right) - F,
\]

and

\[
\pi^m_i = \sum_{j=1}^{2} \left( 1+\delta_j \right) \left( \frac{8(t-2(\delta_j+\delta_j^2))}{s-2t+3(\delta_j+\delta_j^2)^2} \right) \left( \frac{8(t-2(\delta_j+\delta_j^2))}{s-2t+3(\delta_j+\delta_j^2)^2} \right) - F,
\]

**Proof.** This study assumes that the manufacturer and dealer are the leader and follower, respectively. We use backward induction to derive the response selling price of the dealer. Since the second order derivative of the profit for the dealer with respect to \( p_{r1} \), i.e., \( \frac{\partial^2 \pi^m_i}{\partial p_{r1}} = -\frac{2(1+\delta_1)}{t-2\delta_1(1+\delta)} \), would be negative when \( t > 2\delta_1(1+\delta) \), the profit function has a global maximum value. By letting the first order derivative of the profit with respect to \( p_{r1} \) be zero, i.e., \( \frac{\partial \pi^m_i}{\partial p_{r1}} = \frac{(1+\delta_1)(s+p_{d1}+w_1-\delta_1(1+\delta_1))}{t-2\delta_1(1+\delta)} = 0 \), we have the optimal selling price of the dealer in response to the manufacturer’s selling strategy, which is given by \( p_{r1} = \frac{1}{2} (s+p_{d1}+w_1-\delta_1(1+\delta_1)) \). By substituting the obtained selling pricing of the dealer, into the demand functions of the two selling channels, i.e., equations (15) and (16), we can obtain the profit of product 1 for the manufacturer. Moreover, since the second order derivative of the profit of the manufacturer with respect to \( p_{d1} \), i.e., \( \frac{\partial^2 \pi^m_i}{\partial p_{d1}} = -\frac{1+\delta_1}{t-2\delta_1(1+\delta)} \), is less than zero under the same condition stated above, we have a global maximum value. By letting the first order derivative of the profit with respect to \( p_{d1} \) be zero,
i.e., \( \frac{\partial \pi_n^m}{\partial p_{dB}} = -\frac{(1+\delta_1)(s-2t+2p_{dB}-2\delta_1+3\delta_1(1+\delta_1))}{2t-2\delta_1(1+\delta_1)} \) = 0, we can obtain the optimal selling price of the manufacturer. By substituting the obtained optimal selling price of the manufacturer into the response selling price of the dealer, we can obtain the optimal selling price of the dealer. Likewise, the optimal selling price of product 2 can be obtained.

Theorem 3.3 indicates that the optimal selling prices for the dealer and manufacturer are influenced by the same factors. The network externality is negatively correlated with the optimal selling prices. Consumers’ perceived costs for both channels, which include those related to transport and inconvenience, should be considered to determine the optimal selling prices. In addition, since the transport and inconvenience costs have fewer effects on the manufacturer than on the dealer, the dealer has to set a higher selling price than the manufacturer does in response to the same increase in the transport cost.

**Case II: Pure Bundling Strategy in both Online and Physical Channels**

Both the manufacturer and dealer may use the pure bundling strategy to draw consumers’ attention to purchase the two products together, and thus increase their sales volumes. In such a case, suppose the online and physical channels sell bundle \( B \) at prices \( p_{dB} \) and \( p_{rB} \), respectively. The consumer’s utility for bundle \( B \) in the online channel is given by \( u_{dB} = v_1 + v_2 + (\delta_1 + \delta_2)D_{dB} - p_{dB} - s \), and the utility in the physical channel is the same as that in at the first stage for the pure bundling strategy. Likewise, when \( x < \frac{s + p_{dB} - p_{rB} - (D_{dB} - D_{dB})}{\delta_1 + \delta_2} \) consumers are willing to purchase bundle \( B \) in the physical channel. Otherwise, consumers would turn to purchase bundle \( B \) in the online channel. The demands for consumers to purchase bundle \( B \) in the physical and online channels are thus derived as \( D_{rB} = \frac{(1+\delta_1+\delta_2)(-s-p_{dB}+p_{rB}+\delta_1+\delta_2+2\delta_1\delta_2)}{-t+2\delta_1^2+2\delta_2+2\delta_1^2+\delta_1(2+4\delta_2)} \) and \( D_{dB} = \frac{(1+\delta_1+\delta_2)(s-t+p_{dB}+p_{rB}+\delta_1+\delta_2+2\delta_1\delta_2)}{-t+2\delta_1^2+2\delta_2+2\delta_1^2+\delta_1(2+4\delta_2)} \), respectively. Therefore, the profits for the dealer and manufacturer are given by

\[
\pi_{II}^d = (p_{dB} - w_B)(1 + \delta_1 + \delta_2)(-s - p_{dB} + p_{rB} + \delta_1 + \delta_2 + 2\delta_1\delta_2 + \delta_2^2)
- t + 2\delta_1^2 + 2\delta_2 + 2\delta_1^2 + \delta_1(2 + 4\delta_2)
\]

(17)

and

\[
\pi_{II}^m = (w_B - (c_1 + c_2))(1 + \delta_1 + \delta_2)(-s - p_{dB} + p_{rB} + \delta_1 + \delta_2 + 2\delta_1\delta_2 + \delta_2^2)
- t + 2\delta_1^2 + 2\delta_2 + 2\delta_1^2 + \delta_1(2 + 4\delta_2)
+ (p_{dB} - (c_1 + c_2))(1 + \delta_1 + \delta_2)(s - t + p_{dB} - p_{rB} + \delta_1 + \delta_2 + 2\delta_1\delta_2 + \delta_2^2)
- t + 2\delta_1^2 + 2\delta_2 + 2\delta_1^2 + \delta_1(2 + 4\delta_2)
- F.
\]

Theorem 3.4. When both the manufacturer and the dealer use the pure bundling strategy, the optimal selling prices of bundle \( B \) would be given by

\[
p_{rB}^* = \frac{1}{4}(s + 2t + 4w_B - 5\delta_1 - 5\delta_2 - 10\delta_1\delta_2 - 5\delta_2^2),
\]

(19)

and

\[
p_{dB}^* = \frac{1}{2}(-s + 2t + 2w_B - 3\delta_2 - 3(\delta_1^2 + \delta_2^2 + \delta_1(1 + 2\delta_2))
\]

(20)

respectively.
Proof. Since the second order derivative of the profit of the in-store retailer with respect to \( p_B \), i.e., \( \frac{\partial^2 \pi_{I1}^r}{\partial p_B^2} = \frac{2(1+\delta_1+\delta_2)}{-t+2\delta_1^2+2\delta_2^2(1+\delta_2)+\delta_1(2+4\delta_2)} \), would be negative when \( t > 2\delta_1^2 + 2\delta_2 (1 + \delta_2) + \delta_1 (2 + 4\delta_2) \), the profit is convex and has a global maximum value. By letting the first order derivative of the profit with respect to \( p_B \) be zero, i.e., \( \frac{\partial \pi_{I1}^r}{\partial p_B} = \frac{(1+\delta_1+\delta_2)(-s-p_B+2p_B-w_B+\delta_1+\delta_2+2\delta_1\delta_2+\delta_1^2)}{-t+2\delta_1^2+2\delta_2^2(1+\delta_2)+\delta_1(2+4\delta_2)} = 0 \), we can have the optimal selling price of the dealer in response to the manufacturer’s selling strategy, which is given by \( p_{RB} = \frac{1}{2}(s + p_{dB} + w_B - \delta_1^2 - \delta_2 (1 + \delta_2) - \delta_1 (1 + 2\delta_2)) \). By substituting the obtained optimal selling price of the in-store retailer into the demand functions of the physical and online channels, i.e., equations (17) and (18), we can have the profit for the manufacturer. Likewise, the second and first order derivatives of the profit for the manufacturer with respect to \( p_{dB} \), are given as \( \frac{\partial^2 \pi_{II}^m}{\partial p_{dB}^2} = \frac{1+\delta_1+\delta_2}{-t+2\delta_1^2+2\delta_2^2(1+\delta_2)+\delta_1(2+4\delta_2)} \) and \( \frac{\partial \pi_{II}^m}{\partial p_{dB}} = \frac{(1+\delta_1+\delta_2)(s-2t+2p_{dB}+w_B-\delta_1^2-\delta_2 (1 + \delta_2) - \delta_1 (1 + 2\delta_2))}{2(t-2\delta_1^2-2(\delta_1^2+\delta_2(1+2\delta_2)))} \). Since the second order derivative is negative under the above condition, by letting the first order derivative be zero, we can have the optimal selling price of the manufacturer into the response selling price of the dealer, we can have the optimal selling price of the dealer.

Theorem 3.4 indicates that the effects that the transport and inconvenience costs have on the optimal selling price are similar as in Case I, but the effects that network externality has on the optimal selling price is greater than that in Case I. Note that the dynamics between the network externalities of the two products are also involved.

Case III: Pure Bundling Strategy in the Online Channel and Pure Component Strategy in the Physical Channel

In this case the manufacturer uses the pure bundling strategy in the online channel and sells bundle \( B \) at \( p_{dB} \). Suppose that for the sake of increasing sales of bundle \( B \), the dealer can only sell product 1 at \( p_{r1} \) in the market. That is, consumers can purchase either bundle \( B \) in the online channel or product 1 in the physical channel. When \( u_{r1} > u_{dB} \), consumers would purchase the product in the physical channel, and thus we can know that when \( x < \frac{s+p_{dB}+p_{r1}-v_2+D_{r1}+D_{dB}(\delta_1+\delta_2)}{s-\delta_1-\delta_2} \), consumers would purchase product 1 in the physical channel. Therefore, the consumer demands for the physical and online channels are given by

\[
D_{r1} = \frac{(1+\delta_1)(-s-p_{dB}+p_{r1}+v_2+\delta_1+\delta_2+2\delta_1\delta_2+\delta_1^2)}{-t+2\delta_1^2+\delta_2^2+2\delta_1(1+\delta_2)}
\]

and

\[
D_{dB} = \frac{(s-\delta_1-\delta_2)(-s+\delta_1+\delta_2+\delta_1^2)}{-t+2\delta_1^2+\delta_2^2+2\delta_1(1+\delta_2)},
\]

respectively. Accordingly, the profits for the dealer and manufacturer can be derived as

\[
\pi_{II}^r = (p_{r1} - w_1)(\frac{(1+\delta_1)(-s-p_{dB}+p_{r1}+v_2+\delta_1+\delta_2+2\delta_1\delta_2+\delta_1^2)}{-t+2\delta_1^2+\delta_2^2+2\delta_1(1+\delta_2)})
\]

and

\[
\pi_{II}^m = (w_1 - c_1)(\frac{(1+\delta_1)(-s-p_{dB}+p_{r1}+v_2+\delta_1+\delta_2+2\delta_1\delta_2+\delta_1^2)}{-t+2\delta_1^2+\delta_2^2+2\delta_1(1+\delta_2)}) + (p_{dB} - c_1)(\frac{(s-\delta_1-\delta_2)(-s-\delta_1-\delta_2)(\delta_1+\delta_2)}{-t+2\delta_1^2+\delta_2^2+2\delta_1(1+\delta_2)}) - F,
\]

respectively.
Theorem 3.5. When the manufacturer uses the pure bundling strategy and the dealer uses the pure component strategy, the optimal selling prices for product 1 and bundle B are given by

\[ p_{r1}^* = \frac{(w_1 + \delta_1 + c_1 \delta_2)}{4(1 + \delta_1 + \delta_2)} + \frac{1}{4} \left( s + 2t + c_2 - v_2 + 3w_1 - 3\delta_2 (1 + 2\delta_1 + \delta_2) - 5\delta_1 (1 - \delta_1) \right), \]

and

\[ p_{dB}^* = \frac{(w_1 + \delta_1 + c_1 \delta_2)}{2(1 + \delta_1 + \delta_2)} + \frac{1}{2} \left( -s + 2t + c_2 + v_2 + w_1 - \delta_2 (1 + \delta_2) - \delta_1 (3 + 2\delta_2 - 3\delta) \right), \]

respectively.

Theorem 3.5 indicates that the optimal selling prices of bundle B and product 1 should incorporate the consumers’ perceived values and costs. Since product 2 is only available in the online channel, consumers, who are willing to purchase product 2, have to turn to purchase bundle B in the online channel, which would enable the manufacturer to earn more profits.

Proof. Since \( 0 < \delta_1 < 1 \), the second and first order derivatives of the profit of the in-store retailer with respect to \( p_{r1} \), i.e., \( \frac{\partial^2 \pi_{r1}}{\partial p_{r1}^2} = \frac{2(1 + \delta_1)}{t + 2\delta_1^2 + \delta_2^2 + 2\delta_1 (1 + \delta_2)} \), would be negative when \( t > 2\delta_1^2 + \delta_2^2 + 2\delta_1 (1 + \delta_2) \), the profit is convex and has a global maximum value. By letting the first order derivative of the profit for the retailer with respect to \( p_{r1} \) be zero, i.e., \( \frac{\partial \pi_{r1}}{\partial p_{r1}} = \frac{(1 + \delta_1) (-s - p_{dB} + 2p_{r1} + v_2 - w_1 + \delta_1 + \delta_2 + 2\delta_1 \delta_2 + \delta_2^2)}{t + 2\delta_1^2 + \delta_2^2 + 2\delta_1 (1 + \delta_2)} \), we can have the response selling price of product 1 for the dealer, which is given by

\[ p_{r1} = \frac{1}{2} \left( s + p_{dB} - v_2 + w_1 - \delta_1 (3 + 2\delta_2) \right). \]

By substituting the obtained optimal selling price of the manufacturer into the demands of the physical and online channels, we can then have the profit for the manufacturer. Likewise, the second and first order derivatives of the profit for the manufacturer with respect to \( p_{dB} \) are

\[ \frac{\partial^2 \pi_{m}}{\partial p_{dB}^2} = \frac{1 + \delta_1 + \delta_2}{t + 2\delta_1^2 + \delta_2^2 + 2\delta_1 (1 + \delta_2)} \quad \text{and} \quad \frac{\partial \pi_{m}}{\partial p_{dB}} = (c_1 - w_1) \left( 1 + \delta_1 \right) - (c_1 + c_2 - p_{dB}) \left( 1 + \delta_1 + \delta_2 \right) + (1 + \delta_1 + \delta_2) \left( s - 2t + p_{dB} - v_2 - w_1 + 3\delta_1 (1 + \delta_1) + \delta_2 + 2\delta_1 \delta_2 + \delta_2^2 \right) \]

\[ \left[ (2 (s + p_{dB} - v_2 - w_1 + 3\delta_1 (1 + \delta_1) + 2\delta_1 \delta_2 + \delta_2^2)) \right]^{-1}, \]

respectively. Since the obtained second order derivative of the selling price of the dealer is less than zero under the same condition, by letting the first order derivative of the profit for the manufacturer with respect to \( p_{dB} \) be zero, we can have the optimal selling price of the manufacturer, \( p_{dB}^* \). By substituting the obtained optimal selling price of the manufacturer into the response selling price of product 1 for the dealer, we can have the optimal selling price \( p_{r1}^* \). Then, by substituting the obtained optimal selling prices into the demand and profit, we can have the optimal selling prices for the manufacturer and dealer.

Case IV: Pure Component Strategy in the Online Channel and Pure Bundling Strategy in the Physical Channel

In this case, the manufacturer uses the pure component strategy and the dealer uses the pure bundling strategy. For the sake of increasing sales of bundle B, the manufacturer decides to sell only product 1 at \( p_{d1} \) in the market, and the dealer sells bundle B at \( p_{rB} \). When \( u_{rB} > u_{d1} \), i.e., when \( x < \frac{s + p_{dB} - v_2 - D_{dB} \delta_1 + D_{d1} (\delta_1 + \delta_2)}{t} \), consumers would purchase bundle B in the physical channel. Therefore, the demands for the physical and online channels are given by
The profits for the dealer and manufacturer are derived as

$$D_{r_B} = -\frac{\left(s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)\right)(1 + \delta_1 + \delta_2)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}$$

and

$$D_{d1} = \frac{-\left(s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)\right)(1 + \delta_1 + \delta_2)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)},$$

respectively. Accordingly, the profits for the dealer and manufacturer are derived as

$$\pi_{IV}^d = [p_{r_B} - w_B] \left(-\frac{s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}\right) \quad (25)$$

and

$$\pi_{IV}^m = (w_B - (c_1 + c_2)) \left(-\frac{s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}\right) + (p_{d1} - c_1) \left(-\frac{s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}\right) - F, \quad (26)$$

respectively.

**Theorem 3.6.** When the manufacturer uses the pure component strategy and the dealer uses the pure bundling strategy, the optimal selling prices for product 1 and bundle B are given by

$$p^*_d = -\frac{(2 + c_1 + c_2 - w_B + 6\delta_1 + 4\delta_2^2)\delta_2 + 2 (1 + \delta_1) \delta_2^2}{2 (1 + \delta_1)}$$

and

$$p^*_B = \frac{1}{4} \left(s + 2t - c_2 + v_2 + 4w_B - 5\delta_1 - 5\delta_1^2 - 2\delta_2^2\right) - \frac{(2 + c_1 + c_2 - w_B + 6\delta_1 + 4\delta_2^2)\delta_2}{4 (1 + \delta_1)} \quad (28)$$

respectively.

**Proof.** The second and first order derivatives of the profit of the dealer with respect to $p_{r_B}$ are

$$\frac{\partial^2 \pi_{IV}^d}{\partial p_{r_B}^2} = \frac{2(1 + \delta_1 + \delta_2)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}$$

and

$$\frac{\partial \pi_{IV}^d}{\partial p_{r_B}} = \frac{\left(s + p_{d1} - p_{r_B} + v_2 - \delta_1 (1 + \delta_1)\right)(1 + \delta_1 + \delta_2)}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)} \quad ,$$

respectively. Since $0 < \delta_1 < 1$, the second order derivative would be less than zero, when $2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)$, the profit is convex and has a global maximum value. By letting the first order derivative of the profit of the dealer with respect to $p_{r_B}$ be zero, we can have the response selling price of the product 1 for the dealer, which is given by $p_{r_B} = \frac{s + p_{d1} + v_2 + w_B - \delta_1 (1 + \delta_1)}{2}$. By substituting the obtained selling price of the dealer into the demands of the physical and online channels, the profit for the manufacturer can be obtained. Likewise, the second and first order derivatives of the profit for the manufacturer with respect to $p_{d1}$ are given by

$$\frac{\partial^2 \pi_{IV}^m}{\partial p_{d1}^2} = \frac{1 + \delta_1}{t + 2\delta_1^2 + \delta_2 + 2\delta_1 (1 + \delta_2)}$$

and

$$\frac{\partial \pi_{IV}^m}{\partial p_{d1}} = \frac{\left[2 (t - 2\delta_1^2 - 2\delta_1 (1 + \delta_2) - \delta_2 (1 + \delta_2))\right]^{-1} \left[-(1 + \delta_1) (s - 2t + c_2 + 2p_{d1} + v_2 - 2w_B + 3\delta_1 (1 + \delta_1)) - (2 + c_1 + c_2 - w_B + 6\delta_1 + 4\delta_2^2)\delta_2 + 2 (1 + \delta_1) \delta_2^2\right]}{\partial p_{d1}} \quad ,$$

respectively. Since the second order derivative is less than zero under the same condition, by letting the first order of the profit with respect to $p_{d1}$ be zero we can have the optimal selling price of the manufacturer. The optimal selling prices of the products can be obtained by solving the simultaneous equations. 

□
Since product 2 is only offered in the physical channel, the consumers’ perceived value for product 2, which would affect the profits for the dealer and manufacturer, should be considered before the determination of the selling strategies.

4. Property analysis and numerical application. A number of related factors would impact the selection of appropriate strategies for the manufacture and dealer, and managers may manipulate these to maximize the profits.

4.1. Property analysis. Primary cost for the dealer to consider is the wholesale price, which is also critical to the manufacturer. Both the manufacturer and dealer can adjust their bundling and pricing strategies according to the wholesale price to gain more profits.

Proposition 1: At the first stage, when only the dealer sells the two products in the market:

1) If the network externalities of the two products are the same, i.e., \( \delta_1 = \delta_2 = \delta \), adopting the pure bundling strategy would be better when

\[
 w_B < (v_1 + v_2) - \sqrt{\frac{(1 + \delta)(t - 2\delta + 4\delta^2)\left((v_1 - w_1)^2 + (v_2 - w_2)^2\right)}{(1 + 2\delta)(t - \delta - \delta^2)}}. \tag{29}
\]

2) Moreover, if the two products do not have network externality, i.e., \( \delta_1 = \delta_2 = 0 \), adopting the pure bundling strategy would be better when

\[
 w_B < v_1 + v_2 - \sqrt{(v_1 - w_1)^2 + (v_2 - w_2)^2}. \tag{30}
\]

Proof. Let the profits of the pure bundling and pure component strategies be the same, i.e., \( \pi_c^* = \pi_B^* \), and we can thus obtain the above relationships.

From Proposition 1, when the manufacturer offers a relatively large discount for the bundle, i.e., a lower wholesale bundle price, adopting the pure bundling strategy would be more profitable for the dealer. This is because bundling may encourage consumers who are originally interested in purchasing only one product to purchase them in a bundle based on the perceived added value and a promising discounted price, thus increasing the demand for both products. When the manufacturer determines a high wholesale price, which is greater than the threshold, the dealer should use the pure component strategy, and further determine his own optimal pricing strategy.

Proposition 2: At the first stage, when the dealer uses the pure bundling strategy to sell the two products, the profit would increase \( \frac{t(1 + \delta_1 + \delta_2)}{t - \delta_2(1 + \delta_2) - \delta_1(1 + \delta_1 + 2\delta_2)} \) times because of the network externalities of \( \delta_1 \) and \( \delta_2 \).

Proof. By dividing the profit using the pure bundling strategy with consideration of network externalities by the profit using the pure bundling strategy when network externalities are set to zero, we can have

\[
 \frac{\pi_c^*}{\pi_c^*|\delta_1 = \delta_2 = 0} = \frac{(v_1 + v_2 - w_B)^2(1 + \delta_1 + \delta_2)}{4(-t + \delta_1 + \delta_2 + \delta_2(1 + 2\delta_2))} = \frac{t(1 + \delta_1 + \delta_2)}{-t + \delta_1^2 + \delta_2 + \delta_2^2 + \delta_1(1 + 2\delta_2)}. \]

\(\square\)
Proposition 2 indicates that when using the pure bundling strategy, the profit gained from the products with network externalities would be greater than those without these. When the dealer sells two products with network externalities, whose perceived value would increase along with the number of users, higher network externalities would also cause consumers to become more sensitive to the perceived value of the products and the number of users, and thus increasing the demand for bundles. Therefore, the dealer can observe the relationship between the number of users and network externality to indirectly assess consumer perceived value for the two products, and then determine the optimal bundling and pricing strategy.

In considering the bundling strategy, the manufacturer and dealer also have to assess the effects that various related factors have on their profits.

Proposition 3: At the second stage, when both the manufacturer and dealer can sell the two products in the market, then if the two products have no network externality, i.e., \( \delta_1 = \delta_2 = 0 \), Case I is the best bricks-and-clicks approach when \( t > \frac{1}{4} (-2s - c_2 + v_2) \).

Proof. By comparing the profits obtained for the manufacturer and dealer, we have

\[
\pi_m^I - \pi_m^{IV} = \frac{(c_2 - v_2)(2s + 4t - c_2 + v_2)}{4t} > 0,
\]

and

\[
\pi_m^m - \pi_m^{mIII} = \frac{(s + 2t)(c_2 - v_2)}{4t} > 0.
\]

The optimal strategies can thus be derived.

Proposition 3 indicates that reducing the transport cost by locating a physical store near an area with high product demand, and reducing the inconvenience cost by offering free shipping and online coupons, can increase the profits of both the manufacturer and dealer. However, the manufacturer has to obtain sufficient information before selecting a strategy. Note that the profits in Cases I and IV are always greater than those in Cases II and III. When the transport cost is high, the use of a mixed bricks-and-clicks operation strategy, i.e., Cases III and IV, is not suggested.

Proposition 4: At the second stage, if the two products have the same network externality, i.e., \( \delta_1 = \delta_2 = \delta \),

(1) when the pure component strategy is adopted, the manufacturer would gain more profits by starting an online channel than selling the two products only through the dealer if

\[
F < \frac{(1+\delta)(s-2t+3\delta^2)}{4t-8\delta-8\delta^2},
\]
and the condition would then become
\[
F < \frac{(s - 2t)^2 - 1}{t} \sum_{i=1}^{2} [(c_i + 2w_i) t + c_i v_i + w_i (w_i - c_i - v_i)],
\]
(32)

when the two products have no network externality, i.e., \( \delta_1 = \delta_2 = 0 \).

(2) when the pure bundling strategy is adopted, the manufacturer would gain more profits by starting an online channel than selling the two products only through the dealer if
\[
F < \frac{(s - 2t)^2}{8 t} \sum_{i=1}^{2} [(c_i + 2w_i) t + c_i v_i + w_i (w_i - c_i - v_i)],
\]
(33)

and the condition would then become
\[
F < \frac{(s - 2t)^2 - 4 (c_1 + c_2 - w_B) (2t - v_1 - v_2 + w_B)}{8 t},
\]
(34)

when the two products have no network externality, i.e., \( \delta_1 = \delta_2 = 0 \).

**Proof.** The profit for the manufacturer is \( \pi^{m}_m \) when using the pure component strategy through the dealer at the first stage, and is \( \pi^{m}_m \) when using the pure component strategy for both the physical and online channels. If the manufacturer starts an online channel then this would result in the additional setup cost, \( F \), and by letting the profits of the cases be the same we can derive the threshold of \( F \).

As noted above, starting an online channel would result in an additional setup cost, and Proposition 4 provides the manufacturer with the threshold which can be used to determine whether it is profitable to start such a channel, along with the observed market conditions.

**Proposition 5:** At the second stage, when both the manufacturer and dealer can sell the two products in the market.

(1) In Case II, when \( w_B > \delta_2 (1 + 2\delta_1 + \delta_2) \), the optimal bundling price would be lower than that of the mixed bundling strategy in Cases III and IV.

(2) When \( s > \frac{1}{3} \), the optimal selling prices for the dealer would be higher than those for the manufacturer in Cases I and II.

(3) In Case II, the discount of the bundle for the dealer should be \( t + w_1 + w_2 - w_B + 3\delta_1\delta_2 - s/2 \).

**Proof.** (1) By subtracting the bundling price in Case II from that in Case III, we have \( p^{II}_{dB} - p^{II}_{dI} = \frac{1}{2} \left( c_2 + v_2 + w_1 + \frac{w_1 (1 + \delta_1) + c_1 \delta_2}{1 + \delta_1 + \delta_2} + 2 (-w_B + \delta_2 (1 + 2\delta_1 + \delta_2)) \right) \). It is observed that when \( w_B > \delta_2 (1 + 2\delta_1 + \delta_2) \), \( p^{II}_{dB} - p^{II}_{dI} > 0 \) and \( p^{II}_{dB} > p^{II}_{dI} \). Moreover, since \( p^{IV}_{dB} - p^{IV}_{dI} = \frac{(v_2 - v_2) (1 + \delta_1) + \delta_2 (-c_2 - c_2 + w_B + 3(1 + \delta_1) (1 + 2\delta_1 + \delta_2))}{4 (1 + \delta_1)} \), we would have \( p^{IV}_{dB} > p^{IV}_{dI} \).

(2) By subtracting the selling price of the in-store retailer from the selling prices for the manufacturer in Cases I and II, we have \( p^{I}_{rj} - p^{I}_{d1} = \frac{3s - 2t + \delta_1 + \delta_2 + \delta_2 + \delta_1 (1 + 2\delta_2)}{4} \) and \( p^{II}_{rj} - p^{II}_{d1} = \frac{1}{2} (3s - 2t + \delta_1 + \delta_2 + \delta_2 + \delta_1 (1 + 2\delta_2)) \). It is then observed that when \( s > \frac{1}{3} \), \( p^{I}_{rj} > p^{II}_{d1} \) and \( p^{II}_{rj} > p^{II}_{d1} \).

(3) By subtracting the total of the selling prices of products 1 and 2 in Case I from the bundling price of the in-store retailer in Case II, we have \( p^{II}_{rB} - (p^{I}_{r1} + p^{I}_{r2}) = -s + 2((w_1 + w_2 - w_B) + 3\delta_1\delta_2) \).
Proposition 5 provides guidelines for the determination of the selling prices and amount of discount for the dealer and manufacturer for different cases. The selling price would affect consumers’ purchasing decisions, and using appropriate promotion activities can enhance consumers’ willingness to purchase, which can benefit both the manufacturer and dealer.

4.2. Numerical application. According to the parameter settings of the numerical example in Xiao and Cheng (2014), in addition, we also suppose that the two products are perceived values as \( v_1 = 34 \) and \( v_2 = 20 \), respectively, and the network externalities of products 1 and 2 are \( \delta_1 = 0.5 \) and \( \delta_2 = 0.3 \), respectively, which indicates that when the number of users increases then perceived value of product 1 would increase more than that of product 2. At stage 1, the manufacturer sells products 1 and 2 at the wholesale prices \( w_1 = 20 \) and \( w_2 = 12 \), respectively. If the dealer purchases products 1 and 2 together as a bundle B, the manufacturer would offer a discounted wholesale price, \( w_B = 29 \). The production costs for products 1 and 2 are \( c_1 = 5 \) and \( c_2 = 3 \), respectively, for the manufacturer. The manufacturer would require an additional setup cost, \( F = 1,200 \), to start an online channel. The inconvenience cost is \( s = 16 \) for online shopping, and the unit transport cost is \( t = 20 \) for the physical store. The parameter settings are summarized in Table 2.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( \delta_1 \) | 0.5 | \( \delta_2 \) | 0.3 |
| \( v_1 \) | 34 | \( v_2 \) | 20 |
| \( c_1 \) | 5 | \( c_2 \) | 3 |
| \( w_1 \) | 20 | \( w_2 \) | 12 |
| \( w_B \) | 29 | \( s \) | 16 |
| \( t \) | 20 | \( F \) | 1,200 |

By substituting the parameters into the equilibrium solutions in the previous section, we can obtain the optimal selling prices and profits for the manufacturer and dealer, as shown in Table 3.

| Strategy | Selling Price | Profit ($000) | Total Profit ($000) |
|----------|---------------|---------------|---------------------|
| The Physical Channel (The First Stage) | | | |
| Pure Component | \( p_{r1} = 27 \) | \( \pi_R^C = 4.8789 \) | 15.4472 |
| | \( p_{r2} = 16 \) | \( \pi_R^C = 10.5684 \) |
| Pure Bundling | \( p_{rB} = 41.5 \) | \( \pi_{RB} = 15.1536 \) | 40.6116 |
| | | \( \pi_{RB} = 25.4580 \) |
| Bricks-and-Clicks (The Second Stage) | | | |
| Case I | \( p_{d1} = 30.875 \) | \( \pi_I^R = 26.1846 \) | 57.5858 |
| | \( p_{d2} = 23.415 \) |
| | \( p_{r1} = 33.0625 \) | \( \pi_I^R = 31.4012 \) |
| | \( p_{r2} = 25.5125 \) |
| Case II | \( p_{rB} = 41.2 \) | \( \pi_{RB} = 15.6491 \) | 46.5392 |
| | \( p_{d1} = 38.84 \) | \( \pi_{RB} = 30.8901 \) |
| Case III | \( p_{r1} = 27.67 \) | \( \pi_{RB} = 4.9547 \) | 36.2380 |
| | \( p_{d1} = 40.78 \) | \( \pi_{RB} = 31.2833 \) |
| Case IV | \( p_{rB} = 47.0175 \) | \( \pi_{RB} = 32.8094 \) | 58.6167 |
| | \( p_{d1} = 29.785 \) | \( \pi_{RB} = 25.8073 \) |
As can be seen in Table 3, the profits for the manufacturer do not vary much for the different scenarios, but are notably low under the condition in which the dealer, as the only channel, uses the pure component strategy. The profit for the dealer is only greater than that for the manufacturer for Case IV. It is observed that when the dealer, who operates as the only channel, adopts the pure bundling strategy then this would benefit both the manufacturer and dealer. When the bricks-and-clicks business model is adopted, the dealer should use the pure bundling strategy and the manufacturer should use the pure component strategy. In addition, since product 1 has greater consumer perceived value and network externality than product 2, the optimal selling price of product 1 would be greater than that of product 2 in the case of using the pure component strategy. With regard to the profits of the entire supply chain, they are not very different in Cases I and IV, but both of these are greater than those in Cases II and III. Therefore, if the manufacturer and dealer can integrate their selling channels and use complementary strategies, they would earn more profits under coordination.

The setup and inconvenience costs would affect the decision to start an online channel for the manufacturer, and thus we first investigate the difference in profit, \( \pi_m I - \pi_m C \) and \( \pi_{mI} - \pi_m B \), for the manufacturer when using the pure component and bundling strategies with and without the bricks-and-clicks operation, and the results are shown in Figures 1 and 2.

![Figure 1. The Decision Zone for the Pure Component Strategy](image)

As can be seen in Figures 1 and 2, when the manufacturer uses the pure component strategy and when the inconvenience and setup costs are relatively high, then selling the two products in only the physical channel would gain more profits. However, when using the pure bundling strategy, the two costs would be less influential. Since an increase in the setup and inconvenience costs would result in a decrease in the manufacturer’s profit, then only when the profit from selling the two products can offset the setup cost can the manufacturer benefit. In such a case, the manufacturer should not start an online channel but instead retain the current condition. Moreover, the effects of the two costs on the pure bundling strategy seem greater than those on the pure component strategy. Therefore, adopting the pure component strategy would produce more profits than the pure bundling strategy with these parameter settings.
At the second stage, when the manufacturer starts an online channel then both he and the dealer may use either the pure component or pure bundling strategies. We investigate the effects that network externality has on the profits in the different cases, and the results are shown in Figure 3.

Since the profits in Case III are always lower than those in the other three cases with the same parameter settings, we show the comparison of the profits for Cases I, II and IV in Figure 3. The areas occupied by the three cases are similar, and the area occupied by Case IV is a slightly larger than those occupied by Cases I and II. When the two products have relatively high network externalities then Case IV would earn more profits due to the greater effects of network externality on bundling. This can be explained by the fact that consumers can purchase the products individually or in a bundle based on their preferences in this case. The manufacturer should use the pure bundling strategy in both channels when products 1 and 2 have low and high network externalities, respectively, since an increase of the sales volume of products with high network externality would increase the sales.
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volume of the products. When the two products have the relatively low network externalities, the pure component strategy should be adopted in both channels.

At the first stage, adopting the pure bundling strategy would earn more profits than adopting the pure component strategy for both the manufacturer and dealer. We investigate the effects that network externality and consumers' perceived values have on the profit for the manufacturer, and the results are shown in Figure 4.

As can be seen in Figure 4, network externality and consumers' perceived value are both positively correlated with the manufacturer's profits. The perceived value of product 1 has greater effects than the other related factors. The perceived values of the products would affect the selling prices and demand for these items, and thus profitability. Managers should therefore perform a market survey to assess consumers' product preferences before determining an appropriate selling price. Since network externality would increase both demand and perceived value, the products with high network externality would bring more profits for both the manufacturer and dealer. Therefore, the manufacturer can promote product 2 or enhance the network externality of product 2 to increase his profits. Since the network externality and perceived value of product 1 are slightly higher than those of product 2, the effects that network externality and perceived value have on product 1 would be greater than those on product 2.

We then investigate the effects that the related costs have on the manufacturer's profit when the dealer uses the pure bundling strategy, and the results are shown in Figure 5.

As can be seen in Figure 5, the costs of products 1 and 2 do not have much effect on the manufacturer's profits, and the transport cost to the physical store has the greatest impact. However, the manufacturer can effectively reduce the transportation cost by offering coupons which can only be redeemed in the physical store, and thus earn more profits.

Since the transport cost seems to have a great impact on the manufacturer's profit, and would also significantly affect the consumers' selection of a shopping channel, we investigate the effects that the transport cost has on the manufacturer's
Figure 5. Effects of the Related Costs on the Manufacturer’s Profits and dealer’s profits when using different strategies, and the results are shown as Figure 6.

As can be seen in Figure 6, the transport cost has a great impact on the manufacturer’s profit. This may be explained by the fact that when consumers who are originally interested in purchasing only product 1 decide to purchase bundle B when both the manufacturer and dealer use the bundling strategy, then this would increase both demand and profits. When considering the cases for the pure bundling and component strategies, the manufacturer’s profits are all greater than those of the dealer. In addition, when the transport cost increases it would eventually exceed the cost that consumers are willing to pay, which would thus reduce their willingness to make a purchase, and so decrease profits.
At the second stage the manufacturer uses the bricks-and-clicks business model, and thus we also investigate the effects that the wholesale prices and network externalities of products 1 and 2 have on the manufacturer’s profit in Case I, with the results shown in Figure 7.

![Figure 7. The Effects of the Wholesale Price and Network Externality on the Profits](image)

As can be seen in Figure 7, the effects of the wholesale price are greater than those of network externality, and thus the former can lead to a greater increase in profit for the manufacturer. The influences of network externality are similar to those in the physical channel at the first stage. Since the wholesale price and network externality of product 1 are both greater than those of product 2, the sales of product 1 would have greater impacts on the profit for the manufacturer. When the wholesale price increases, the manufacturer can gain more profit, but that for the dealer would decrease. As a result, the dealer has to set a relatively high selling price in response to the increasing wholesale price, which would result in a fall in demand and thus the manufacturer’s profit. Therefore, the manufacturer and dealer may coordinate to obtain an optimal wholesale price to benefit both parties.

We also investigate the effects that the related costs have on the profits of the manufacturer in Case I, with the results shown in Figure 8.

As can be seen in Figure 8, when the manufacturer starts an online channel then this will incur an additional setup cost. The manufacturer would thus only start an online channel when the expected increase in profit is greater than the setup cost. The impact of the inconvenience cost is slightly greater than that of the setup cost. However, the relative impact of the inconvenience cost would gradually decrease and eventually be similar to that of the setup cost as they both increase. This may be explained by the fact that when the inconvenience cost of the online channel is greater than the cost of transport to the physical store, consumers would turn to purchase the products in the physical store, which would decrease demand in the online channel. However, the increasing demand for the dealer would also affect the manufacturer’s profit. Figure 9 shows how the inconvenience cost affects the profits for the manufacturer and dealer in Case I.
Since the inconvenience cost is the most sensitive factor among the related costs, we investigate the effects that the inconvenience cost has on the profits in the four cases, with the results shown in Figure 10.

As can be seen in Figure 10, the impacts on the profits are similar for the first three cases, but different for Case IV. When the inconvenience cost decreases, Case III is the optimal one as the mixed bundling strategy can satisfy consumers with different preferences. Since the transport cost to the physical store is greater than the inconvenience cost of the online channel, consumers who are located at a significant distance from the physical store, and are originally interested in purchasing only a single product, would purchase bundles from the online channel, which gives the manufacturer opportunities to earn more profits. When the inconvenience cost increases and exceeds the transport cost, the optimal case would become Case II,
i.e., consumers would decide to purchase the products in the physical channel, since the manufacturer uses the pure bundling strategy to enhance his profitability, and the pure bundling strategy can add more value to the product, which indicates that consumers can obtain the other product only at a relatively low price. In addition, since the network externality of bundles is greater than that of individual products, the manufacturer can use this characteristic to attract more consumers.

Since the network externality of product 1 has a greater impact than that of product 2 in Figure 8, we investigate the effects that the network externality of product 1 has on the profits for the manufacturer in the four cases, and Figure 11 shows the results.

Figure 10. The Effects of the Inconvenience Cost on the Profits in the Four Cases

Figure 11. The Effects of the Network Externality on the Profits in the Four Cases
As can be seen in Figure 11, when the network externality of product 1 increases, the profits in the four cases would all increase. The effects that the network externality has on the profits in the first three cases do not vary a lot, and the profit in Case III is slightly greater than that in the cases. However, the effects that the network externality has on the profit in Case I are relatively weak. This may be explained by the fact that the network externality of bundles is greater than that of individual products. The products are all sold individually in Case I, but sometimes bundled in other cases. The optimal selling strategy shifts from the pure component approach to pure bundling as the network externality increases. In addition, Case IV is the dominant strategy. Therefore, when the network externality increases the profit for the dealer can be maximized in Case III.

5. Conclusion. Although the use of multiple selling channels can expand a market, conflicts would also occur due to competition among channels. Firms also have to consider other related factors, such as the level of market demand, types of consumers, and consumer preferences, before entering an online market. This study investigates whether the manufacturer should start an online channel and use a bricks-and-clicks approach to earn more profits. Channel coordination is an important way for the manufacturer to eliminate channel conflicts when an online channel is started. This study constructs a model in which a manufacturer may consider starting an online channel for use along with an existing physical channel which is operated by a dealer, i.e., a bricks-and-clicks approach, to offer two products with network externalities to the market. The optimal selling strategies for the manufacturer and dealer are determined. We also investigate how the use of the bundling strategy can enhance the related synergistic effects, and how the use of pure component strategy can be profitable by selling individual products to consumers with the highest perceived value. With regard to product characteristics, this study also investigates effects that network externality has on the bundling and pricing strategies for the manufacturer and dealer. The effects that the related factors have on the profitability for the manufacturer and dealer in the different cases are also investigated, and the results of this work can thus be used to provide managers with a number of guidelines in this context.

Future research may be extended to consider bundling more products, such as when, for example, consumers are allowed to choose the number of products in a bundle. Future research can also consider stochastic demand for the products. The proposed model can also be extended to consider a two-stage multi-channel supply chain, and then used to investigate the interaction and competition among multiple channels in the market. Finally, future research could incorporate related inventory problems into the model, such as by considering possible shortages of products.

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