Analysis of publications about the methods of static friction definition for bulk material

O O Erokhina

Peter the Great St. Petersburg Polytechnic University, 29, Politekhnicheskaya Street, St. Petersburg, 195251, Russia

e-mail: o.o.erokhina@gmail.com

Abstract. Static friction is one of the most important factors affecting the parameters of technological processes in the mining and metallurgical industries. To obtain reliable results of mathematical and computer simulations, there is a need to estimate a number of physical parameters of materials. With respect to the metallurgical and mining industries, the static friction parameter is of particular importance because of its applicability to relatively common process steps. Evaluation of static friction for bulk materials causes particular difficulties, which is justified by the variability of the particle size distribution, shape and density of particles. This article provides an overview of the existing methods for assessing static friction of bulk substances, their applicability, as well as the development trends of this scientific field.

1. Introduction

In the metallurgical industry there are a number of technological processes involving the use of bulk materials, including sintering and drying in tubular rotary kilns, roasting in a fluidized-bed roaster, sinter roasting, and others [1,2]. In addition, there are several side operations, for example, the supply of bulk materials from bunkers. Mining and mineral processing industries, in turn, mainly implement technological processes using bulk materials [3].

At the moment there are difficulties associated with the design and automation of pyrometallurgical and electrometallurgical processes that occur at elevated temperatures. They can be explained by such factors as: the complexity of studying, controlling and mathematical description of high temperature processes with existing measuring instruments; the difficulty of determining the occurring chemical transformations in a multicomponent system and others [4,5].

From the point of view of the mining industry, enrichment methods are one of the most energy-intensive processing areas, as a result of which there is a need for high-precision methods for estimating the intensity of wear and its profile for various units [6,7]. The above affects the complexity of studying technological processes with the aim of their further optimization and modernization.

The use of standard measurement tools is limited by both the cost of automation equipment and the complexity of the automatic process control system. To fully describe the process, it is necessary to use a large number of measuring instruments, which is often not suitable [8]. As a result, complex technological processes have a number of assumptions with relation to automation and control.

A variety of computer modeling methods are used to evaluate the processes occurring in the aggregates. The used software allows predicting the temperature distribution, acceleration, pressure and other parameters characterizing liquid, solid and gaseous media [9-11]. However, there are difficulties in assessing the behavior of bulk materials, which is caused by the variability of the particles properties, therefore there is no possibility to take the jet as a homogeneous body [12]. The above causes both a
low number of software products that implement modeling for bulk materials, and the duration of further calculations. 

Regarding the metallurgical and mining industries, computer modeling of bulk materials makes it possible to assess the possibility and locations for the occurrence of stagnant zones and zones of increased wear, to select the optimal technological parameters and to correct the existing ones [13-15]. Such optimization of technological processes can significantly reduce the costs associated with the current production process and possible equipment failure [16-17].

Regarding computer simulation of bulk materials, the most widely used method is DEM-modeling, which allows to consider each particle as a discrete element, as well as the interaction between these discrete particles [18]. Software that implements DEM-modeling (including EDEM, Rocky and others) bases calculations on Newtonian physics. And to evaluate the behavior of bulk material, it is necessary to set a number of parameters for the bulk materials under consideration, including dynamic and static friction, recovery coefficient, distribution particle sizes and others [19,20].

There are difficulties associated with the selection of these parameters, quite often the parameters are set on the basis of reference materials, which, however, may not be sufficiently accurate relative to the bulk material under consideration. Alternatively, it is possible to conduct laboratory experiments and further iterative selection of DEM parameters to achieve convergence of the results of laboratory and computer simulations [12].

The most relevant method for increasing the convergence of computer simulation results is to estimate the parameters of the materials and media under consideration using calibration techniques, which allows us to differentiate computer simulation results from laboratory models [21].

Methods of determining static friction will be considered in this article, since this parameter is particularly important for the metallurgy and mining industries [22,23]. Static friction is a measure of the immobility of bodies under various external influences and makes it possible to estimate the necessary thrust force for the beginning of the particle motion.

Static friction can be considered not only within the framework of modeling, but also when designing tubular vehicles, the charge of which is carried out when the device is at an angle. Improving the accuracy of measuring static friction will simplify the process of selecting and controlling the inclination angle of devices [24].

Also, this parameter allows one to determine the occurrence of stagnant zones by assessing the required inclination of surfaces for further free fall. A number of articles identify a significant correlation between the angle of repose and static friction, which makes it possible to apply static friction not only within technological units [25-27].

There are static friction between particles and static friction between the particle and the wall, which affects the variety of methods for estimating this parameter. This article will discuss methods for determining both types of static friction.

It should be noted that despite the similarity of the considered techniques, all dependencies are different and sometimes contradict each other, which is explained by the applicability of each installation for a specific material or conditions.

2. Methods for determining static friction between a particle and a surface

2.1 Inclined plane method

Mathematically static friction between a particle and a wall (µ) is determined by the tangent of the angle of inclination (α) at which the particle begins to move along a plane (without rolling), which is shown in Figure 1.

![Figure 1. The scheme of measuring static friction.](image-url)
Thus, the dependence of static friction and the inclination angle of the platform can be described by the following formula:

$$\mu = \tan \alpha.$$  \hspace{1cm} (1)

There are a number of devices that involve the determination of the friction angle by a similar technique [28,29]. Changing the inclination angle of the shelf is realized when using traction devices containing moving blocks with a thread thrown over (Figure 2). The change of the inclination angle of the shelf 1 is realized by the thread, thrown between two moving blocks 6. The measurement of the inclination angle of the platform is assessed on a scale 3. Subsequently static friction is calculated by the formula (1).

![Figure 2](image)

**Figure 2.** Device for determining the coefficient of static friction force where: 1 – platform, 2 – bedplate, 3 – scale, 4 – box without bottom, 5 – viscoplastic material, 6 – traction equipment, 7 – support contour, 8 – gaskets, 9 – leading roller, 10 – slab material, 11 – bulk material particles, 12 – shock absorber, 13 – restrictive leash [28].

Differences in these installations are often associated with the placement of bulk material, which is presented below:

- In the invention [28], the bulk material is immersed in a pre-heated viscoplastic material to form adhesive bonds between the particles.
- In the invention [29] a plate with longitudinal grooves is used, the radius of which is greater than the maximum curvature size of the cross-sectional contour of particles of the bulk material, but with a smaller diameter of their length.

The installations obtained are applicable for the evaluation of static friction of bulk materials, however, they have a number of drawbacks, which include unstable discreteness of the inclination angle of the platform, pulling the threads and their further sagging, difficulties in determining the inclination angle on a scale.

2.2 **The method of shifting the form of bulk material on a horizontal surface**

To estimate the static friction between the particle and the wall, it is possible to use the method presented in Figure 3. In this case, the movement of the tank, filled with bulk material, is realized due to the traction device with suspended loads. The calculation of the static friction coefficient is realized by the formula (2):

$$\mu = \frac{(m_w - m_c)(1 - W \sqrt{2})}{m_b + m_{tm} + m_{pp} + m_{lw}},$$  \hspace{1cm} (2)

where $m_w$ – the mass of weights in the cargo cup, kg; $m_c$ – weight of the cargo cup, kg; $W$ – block rotation resistance factor; $m_b$ – weight of the box without the bottom, kg; $m_{tm}$ – mass of the test medium in the box without the bottom, kg; $m_{pp}$ – weight of the push platform, kg; $m_{lw}$ – load weight on the push platform, kg.

A similar method is presented in [31], but large metal balls of the same size are used as the material under consideration. For this type of material, it is possible to replace the material of the plane along which the capacitance with the sample under consideration begins to roll onto the sample material, which will make it possible to estimate the static friction that occurs between two particles.
Figure 3. General view of the device for determining the coefficient of static friction force where: 1 – platform, 2 – box without bottom, 3 – cargo cup, 4 – cord, 5 – block, 6 – push platform, 7 – load weight, 8 – screw supports [30]

Significant disadvantages of this method are also low automation of the process and the complexity of the implementation of the design, and the influence of the human factor on the results obtained.

3. Methods for determining static friction arising between particles

Methods for determining static friction between particles are often reduced to displacement of layers of bulk material relative to each other [32, 33]; installation, by definition, static friction between particles is shown in Figure 4.

Figure 4. A plant by definition of static friction between particles [33].

Measurement in this case is reduced to the displacement of the layers under the action of the load, the necessary thrust force for displacing the layers relative to each other is measured by the potentiometers used. The change in the position of the layers during the experiment is shown in Figure 5.

Figure 5. Computer model of the shear method for estimating static friction arising between particles [32].

These installations are characterized by high complexity of implementation [34,35]; however, they allow one to obtain the most accurate results than those using other methods for estimating the static friction parameter between two particles [33].
4. Integrated parameter detection methods

Currently, DEM calibration methods are widely used, the parameters are implemented using video cameras that record indirect parameters. These parameters make it possible to estimate a number of parameters, including the aggregate of static friction both between particles and between a particle and a plane, using a neural network.

These methods are often quite simple both in the conduct of the experiment and in the implementation of the installations and, at the same time, they imply the exclusion of the human factor in the measurement process.

At the moment, the definition of the parameters of DEM-modeling by the angle of repose is widespread. There are both vertical installations with retractable side faces [36], and installations similar to the bunker device [37,38] as shown in Figure 6.

![Figure 6. A plant of the bunker type to determine the parameters of DEM-modeling, where 1 – upper damper, 2 – lower dumper [38].](image_url)

Such methods are characterized by high accuracy of results, the lack of mechanisms that are driven by engines [39], and a high degree of automation.

Currently, various stands are being developed for estimating the parameters of DEM-modeling of bulk materials, including static friction, which can be integrated into the general parameter calibration system, which corresponds to the main world trends, including the transition to Industry 4.0, The Fourth Industrial Revolution. Further studies are underway to simplify the design of measuring stands, as well as to eliminate mechanization in the development of stands, which will allow one to achieve high reproducibility of results that are not dependent on external factors.

5. Conclusion

At present there is a fairly large number of different methods for assessing static friction. In the laboratory, methods are used to estimate static friction between a particle and the surface, while the development of installations for assessing static friction between particles of the material under consideration has begun relatively recently. Existing methods are distinguished by a high degree of mechanization, inaccurate and difficult to reproducible, and also contradictory about each other, which is associated with the binding of methods to various types of bulk materials.

The main trend is the development of installations that allow you to calibrate a set of parameters of DEM-modeling, including both types of static friction. The accuracy of the results obtained using these methods should not depend on the bulk material in question. The values obtained are reproducible and applicable both in computer and mathematical modeling, as well as in the design of units and their control in real time.

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