Supplementary information for:

The enrichment paradox in adaptive radiations: emergence of predators hinders diversification in resource rich environments

This supplementary note includes:

- Supplementary text:
  1. Detailed description of model analysis
  2. Derivation of the fitness gradient and curvature of the fitness landscape
  3. Analytical derivation of conditions that enable diversification
  4. Effects of different separation in the trait axis between the optima to feed in the different food resources and of an uneven partitioning of productivity among the preexisting resources
  5. Evolutionary oscillations in the intraclade consumer population
  6. Effects of separation in the trait axis between the consumer niche and the ancestral population
  7. Diversification in sexually reproducing populations with cannibalistic interactions

- Table S1
- Figures S1 to S7

Other supplementary materials for this manuscript includes:

- MATLAB code of the model for numerical simulations and the individual-based model.
1. Detailed description of model analysis

Analysis of conditions that enable initial diversification

We investigate the conditions that enable initial diversification, by analytically studying the behavior of the system in the simplest case: when one population colonizes an environment with two different food resources. To do so, we use the adaptive dynamics framework, which assumes that evolution occurs via small, infrequent mutation steps, and thus that the system reaches the ecological equilibrium when a mutant arises. This separation of the ecological and evolutionary timescales allows us to derive analytical expressions for resource density, $R_1(\eta^*)$ and $R_2(\eta^*)$, and use these expressions to calculate the fitness gradient of the colonizing population $\partial W(\eta^*, R_1, R_2) / \partial \eta^*$. According to Rueffler et al\textsuperscript{1}, diversification can occur when a population evolves towards a phenotype corresponding to a local fitness minimum, in other words, when this phenotype is an attractor of the evolutionary dynamics. We therefore investigate how the model parameters affect diversification by evaluating the conditions that enable an equilibrium to be an attractor of the evolutionary dynamics and at the same time a local fitness minimum. To do so, we examine the stability of the evolutionary equilibrium (trait value at which the fitness gradient is zero), as well as the curvature of the fitness landscape. The stability analysis informs us whether the equilibrium corresponds to an attractor of the evolutionary dynamics; whereas the curvature of the fitness landscape informs us whether this phenotype corresponds to a local fitness minimum. The derivation of the results of our analytical investigation can be found in the section 3 of supporting information.

Numerical simulations of the eco-evolutionary dynamics of adaptive radiations

The analytical investigation of the previous section provides us with the conditions for diversification in the first stage of an adaptive radiation, but it tells us little about eco-evolutionary dynamics of adaptive radiations and evolution of intraclade consumers. We use simulations to study adaptive radiations when ecological and evolutionary processes occur at comparable timescales. Specifically, we simulate adaptive radiations when intraclade consumers cannot evolve and when they can evolve. In the first case, the consumer niche $\theta_c$ is too far from any optimal trait value $\theta_1$ to consume preexisting resources along the trait axis (i.e. $\theta_c$ equals -10), preventing individuals of any of the radiation members to consume other members of the radiation. In the alternative case, the consumer niche $\theta_c$ equals 0; therefore, the consumer niche $\theta_c$ is close to the optimal trait value $\theta_1$ to consume the first preexisting resource. We use as a reference the radiations in which intraclade
consumers cannot evolve and compare these reference radiations to radiations in which they can evolve, all other conditions being equal.

Because we are interested in understanding how environmental factors (i.e. productivity and diversity of preexisting resources) constrain or promote the evolution of intraclade consumers in adaptive radiations, we exclude the possibility that other factors affect the evolution of intraclade consumers. In particular, we exclude the possibility that the consumer niche does not become occupied due to large separation in the trait space between this and the occupied preexisting resource niches. To do so, we seed the adaptive radiation with a population with a trait value that allows the occupancy of the preexisting resource niche that is closest to the consumer niche (i.e. near $\theta_1$) in the first stage of the radiation (we nonetheless explore the eco-evolutionary dynamics of adaptive radiations when this assumption is relaxed in the section 5 of supporting information).

Specifically, we introduce a population with a trait value equal to 2 (that is between $\theta_c$ and $\theta_1$), and compute the ecological equilibrium. We use the population size and density of resources in the equilibrium to initialize the simulations of the eco-evolutionary dynamics of the adaptive radiation. Then, we simulate the evolutionary dynamics of the ecomorph populations under directional selection using the canonical equation of adaptive dynamics (eq. 3). We halt the simulation when trait change is smaller than 0.0001 per unit of evolutionary time. At this point, we calculate the curvature of the fitness landscape at the trait value of the populations using eq. S2.3 (in supporting information 2). If the trait value $\eta_j$ of the $j$th ecomorph corresponds to a local fitness minimum (i.e. the curvature of the fitness landscape is positive), we assume this ecomorph population to be split into two ecomorph populations with a trait value 0.1% larger and smaller than $\eta_j$, and abundance equal to $N_j/2$. We iteratively simulate the evolutionary dynamics of directional selection (using eq. 3) and evaluate the sign of the curvature of the fitness landscape (using eq. S2.3) until this evaluation reveals that the trait of all ecomorph populations present in the system correspond to a fitness maximum (i.e. diversification halts). The evolution of intraclade consumers results in evolutionary oscillations that repeatedly cause the extinction of one consumer subpopulation and the persistence of the other (see supporting information 5). These oscillations occur indefinitely in the system; therefore, we allow the simulation to iteratively run several of these evolutionary cycles, specifically 20, finishing the run with the dynamics of directional selection (when directional selection ceases). At the end of the run, we check that all ecomorphs with a trait value equal or larger than $\theta_1$ cannot further diversify (i.e. their phenotype corresponds to a maximum of the fitness landscape).
Throughout the simulation we follow the number of ecomorphs, their trait value and their abundances as well as the densities of resources, which we use to calculate the fitness landscape. We subsequently count the different ecomorphs with positive density to calculate the radiation richness. To calculate the density of intraclade consumers, we add the densities of all ecomorphs with trait value smaller than $\theta_1/2$, i.e. a trait value that enables a higher attack rate on other members of the clade than on any of the preexisting resources. We use standard MATLAB functions to numerically solve the differential equations of the system for the parameter values in table S1 (supporting information); the code is available in supporting information.

Implementation of the individual-based model

We implement an individual-based model with sexual reproduction based on the ecological dynamics of the model described above. Individuals are assigned two diploid genotypes: one set of loci determines the feeding niche trait $\eta$ and the other the mating trait $m$. $\eta$ is given by the sum of the values of the first set of alleles, whereas the $m$ is determined by the average value of the second set of alleles. Alleles associated to the niche trait can take any value, and those associated with the mating trait can take values of -1 and 1. Assortative mating depends on a mate-choice function according to Dieckmann and Doebeli. Individuals carrying an intermediate mating trait mate randomly ($m = 0$). Individuals with negative $m$ mate disassortatively. Individuals carrying a positive $m$ mate assortatively. We assume offspring to inherit maternal and paternal alleles at each locus independently (free recombination). With probability $\mu_a = 0.005$, a mutation occurs in the inherited alleles and change their value (in the first set the mutation can cause changes of +0.1 and -0.1, and in the second it reverses the sign of the allele value). Simulations start with a population of 40 individuals (20 females and 20 males) with identical niche trait ($\eta = 2$) and random mating ($m = 0$). Because the resource use of each individual depends on its own trait, cannibalism or rather predation on individuals with similar niche trait is possible. Furthermore, demographic stochasticity and, thus stochastic extinction is also enabled.

We present the temporal dynamics of a simulation when the predator cannot evolve ($\theta_c = -10$; figure SSA) and when the predator can evolve ($\theta_c = 0$; figure SSB). In addition, we show the niche trait distribution and the probability of gene flow calculated following the method of Sobel and Chen for the final clade resulting from 10 independent simulations.
2. **Derivation of the fitness gradient and curvature of the fitness landscape in the equilibria**

The relative fitness of a rare mutant population with trait value \( \eta_j' \) depends on the environment set by ecomorph populations with trait values \( \eta = (\eta_1, ..., \eta_m) \). Therefore, we evaluate the relative fitness of a rare mutant \( \eta_j' \) at ecological equilibrium when resident ecomorphs have traits \( \eta \):

\[
W(\eta_j', \eta) = \varepsilon \left( \sum_{i=1}^{n} a_i(\eta_j') R_i(\eta) + \sum_{k\neq j}^{m} a_c(\eta_j') \lambda N_k(\eta) \right) - \sum_{k\neq j}^{m} a_c(\eta_k) \lambda N_k(\eta) - \delta.
\]

(S2.1)

The fitness gradient is thus given by:

\[
\frac{\partial W(\eta_j', \eta)}{\partial \eta_j} \bigg|_{\eta_j' = \eta_j} = \varepsilon \left( \sum_{i=1}^{n} A \frac{(\theta_i - \eta_j')^2}{\tau^2} \exp \left[ -\frac{(\theta_i - \eta_j')^2}{2\tau^2} \right] R_i(\eta) + \sum_{k\neq j}^{m} A \frac{(\theta_c - \eta_j')^2}{\tau^2} \exp \left[ -\frac{(\theta_c - \eta_j')^2}{2\tau^2} \right] \lambda N_k(\eta) \right)
\]

(S2.2)

Directional selection halts where the selection gradient vanishes, in other words when eq S2.2 equals 0. At this point, selection can be stabilizing if the trait value at the equilibrium corresponds to a fitness maximum, or disruptive if it corresponds to a fitness minimum. The curvature of the fitness function therefore determines whether evolution halts or an ecomorph splits into two different ecomorphs. Hence, the second derivative of the fitness function with respect to \( \eta_j' \) evaluated at the ecological equilibrium when ecomorphs have traits \( \eta \) allows us to determine whether a diversification event occurs:

\[
\frac{\partial^2 W(\eta_j', \eta)}{\partial \eta_j^2} \bigg|_{\eta_j' = \eta_j} = \varepsilon \left( \sum_{i=1}^{n} A \frac{(\theta_i - \eta_j')^2 - \tau^2}{\tau^4} \exp \left[ -\frac{(\theta_i - \eta_j')^2}{2\tau^2} \right] R_i(\eta) \right)
\]

\[
+ \sum_{k\neq j}^{m} A \frac{(\theta_c - \eta_j')^2 - \tau^2}{\tau^4} \exp \left[ -\frac{(\theta_c - \eta_j')^2}{2\tau^2} \right] \lambda N_k(\eta) \right)
\]

(S2.3)
3. Analytical derivation of conditions that enable diversification

Here, we derive the conditions that enable diversification when one population with density \( N \) colonizes an environment with two different food resources, \( R_1 \) and \( R_1 \). We restrict our analytical investigation to the simplest case, in which \( R_{1\text{max}} = R_{2\text{max}} = P/2 \). The optimal niche trait values to feed on resource 1 and resource 2 are \( \theta_1 \) and \( \theta_2 \), respectively, and their distance in the trait axis equals \( D \). Following the adaptive dynamics framework, we assume that evolution occurs much more slowly than the ecological dynamics. This separation of the ecological and evolutionary timescales enables us to approximate the ecological dynamics by assuming that they are always in a quasi-steady state determined by the ecomorph population with trait value \( \eta \). We can thus derive analytical expressions for the population and resource density at ecological equilibrium. Subsequently, we investigate the fitness landscape of a rare mutant in the environment set by the population with a trait value \( \eta \).

**Ecological equilibrium** Without a loss of generality, we assume that \( \theta_1 = 0 \) and \( \theta_2 = D \). At ecological equilibrium, the population density and resources satisfy the following equations

\[
0 = \varepsilon (a_1(\eta)R_1 + a_2(\eta)R_2) - \delta \tag{1}
\]
\[
0 = \rho (R_{\text{max}} - R_1) - a_1(\eta)R_1N \tag{2}
\]
\[
0 = \rho (R_{\text{max}} - R_2) - a_2(\eta)R_2N \tag{3}
\]

Equations (2) and (3) give us

\[
R_1 = \frac{\rho R_{\text{max}}}{\rho + a_1(\eta)N} \quad \text{and} \quad R_2 = \frac{\rho R_{\text{max}}}{\rho + a_2(\eta)N} \tag{4}
\]

By substituting \( R_1 \) and \( R_2 \) in equation (1), we obtain an equation for \( N \):

\[
0 = \varepsilon (a_1(\eta)\frac{\rho R_{\text{max}}}{\rho + a_1(\eta)N} + a_2(\eta)\frac{\rho R_{\text{max}}}{\rho + a_2(\eta)N}) - \delta
\]

which is equivalent to the quadratic equation

\[
0 = \alpha N^2 + \beta N + \gamma \tag{5}
\]

where

\[
\alpha = \delta a_1(\eta)a_2(\eta) \tag{6}
\]
\[
\beta = \rho \delta a_1(\eta) + \rho \delta a_2(\eta) - 2\varepsilon \rho R_{\text{max}} a_1(\eta)a_2(\eta) \tag{7}
\]
\[
\gamma = \rho^2 \delta - \varepsilon \rho^2 R_{\text{max}}(a_1(\eta) + a_2(\eta)) \tag{8}
\]

The two solutions of equation (5) are

\[
N^*(\eta) = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha}
\]

where \( \Delta = \beta^2 - 4\alpha \gamma = \delta^2 \rho^2 (a_1(\eta) - a_2(\eta))^2 + 4\rho^2 \varepsilon^2 R_{\text{max}}^2 a_1(\eta)^2 a_2(\eta)^2 \).
Next, we investigate how the population density and the resources at equilibrium change with the trait by calculating their derivatives with respect to $\eta$.

\[
\frac{\partial N^*}{\partial \eta} = \frac{2\alpha \left( -\frac{\partial \beta}{\partial \eta} + \frac{\partial \gamma}{\partial \eta} (\sqrt{\Delta}) \right) - \left( -\beta + \sqrt{\Delta} \right)}{4\alpha^2} \tag{9}
\]

where

\[
\frac{\partial \alpha}{\partial \eta} = \frac{\delta}{\tau^2} a_1(\eta)a_2(\eta)(D - 2\eta) \tag{10}
\]

\[
\frac{\partial \beta}{\partial \eta} = \frac{-\rho \delta \eta}{\tau^2} a_1(\eta) - \frac{2\varepsilon \rho R_{\text{max}}}{\tau^2} a_1(\eta)a_2(\eta)(D - 2\eta) \tag{11}
\]

\[
\frac{\partial \gamma}{\partial \eta} = -\rho \delta \frac{\eta - D}{\tau^2} a_2(\eta) \tag{12}
\]

\[
\frac{\partial \sqrt{\Delta}}{\partial \eta} = \frac{1}{2\sqrt{\Delta}} \left( 2\delta^2 \rho \left( a_1(\eta) - a_2(\eta) \right) \frac{\partial a_1}{\partial \eta} - \frac{\partial a_2}{\partial \eta} + 8\rho^2 \varepsilon^2 R_{\text{max}}^2 a_1(\eta)a_2(\eta) \left( a_2 a_2 \frac{\partial a_1}{\partial \eta} + a_1(\eta) a_1 a_2 \frac{\partial a_2}{\partial \eta} \right) \right) \tag{13}
\]

The derivative of $R_2$ with respect to $\eta$ is

\[
\frac{\partial R_2}{\partial \eta} = \frac{-\rho R_{\text{max}} \left( \frac{\partial a_2}{\partial \eta} N + a_2 \frac{\partial N}{\partial \eta} \right)}{(a_2 N + \rho)^2} = \frac{\partial a_2}{\partial \eta} N + a_2 \frac{\partial N}{\partial \eta} \frac{a_2 N + \rho}{a_2 N + \rho} \tag{14}
\]

Finally, we evaluate the population density and the resources and their derivatives at equilibrium when $\eta^* = \frac{D}{2}$. For this value of $\eta$, the only positive solution of equation (5) is

\[
N^*(\frac{D}{2}) = \frac{-2\rho \delta a_1(\frac{D}{2}) + 2\varepsilon \rho R_{\text{max}} a_1(\frac{D}{2})^2 + 2\rho \varepsilon R_{\text{max}} a_1(\frac{D}{2})^2}{2\delta a_1(\frac{D}{2})^2} = \frac{-\rho \delta + 2\varepsilon \rho R_{\text{max}} a_1(\frac{D}{2})}{\delta a_1(\frac{D}{2})} \tag{15}
\]

Evaluating equations (10) to (13) at $\eta = \frac{D}{2}$ and substituting them in equation (9) yields

\[
\frac{\partial N^*}{\partial \eta} \left( \frac{D}{2} \right) = 0 \tag{15}
\]

Finally,

\[
\frac{\partial R_2}{\partial \eta} \left( \frac{D}{2} \right) = \frac{-\delta \frac{D}{2 \rho} a_2(\frac{D}{2}) N^*(\frac{D}{2})}{2\varepsilon a_2(\frac{D}{2}) a_2(\frac{D}{2}) N^*(\frac{D}{2}) + \rho} + \frac{1}{a_2(\frac{D}{2})^2} \tag{16}
\]

\[
= \frac{-D\delta}{4\varepsilon^2} a_2(\frac{D}{2}) N^*(\frac{D}{2}) + \rho \tag{17}
\]

\[
= \frac{-D\delta}{4\varepsilon^2} a_2(\frac{D}{2}) N^*(\frac{D}{2}) + \rho \tag{18}
\]

A population always evolves to the trait range $[\theta_1, \theta_2]$, here $[0, D]$. The fitness experienced by a mutant is given by:

\[
\tilde{W}(\zeta, \eta) = \varepsilon (a_1(\zeta) R_1(\eta) + a_2(\zeta) R_2(\eta)) - \delta. \tag{19}
\]
The trait value increases when the fitness gradient is positive, i.e. \( \frac{\partial W}{\partial \zeta}(\eta, \eta) > 0 \) and it decreases when the fitness gradient is negative, i.e. \( \frac{\partial W}{\partial \zeta}(\eta, \eta) < 0 \). We show that the fitness gradient is positive for any trait value smaller than 0 and it is negative for any trait value larger than \( D \). We first need to calculate \( \frac{\partial W}{\partial \zeta}(\eta, \eta) \):

\[
\frac{\partial W}{\partial \zeta}(\eta, \eta) = \varepsilon \left( \frac{\partial a_1}{\partial \zeta} R_1(\eta) + \frac{\partial a_2}{\partial \zeta} R_2(\eta) \right)
= -\frac{\varepsilon}{\tau^2} \eta(a_1(\eta) R_1(\eta) + a_2(\eta) R_2(\eta)) + \frac{\varepsilon D}{\tau^2} a_2(\eta) R_2(\eta)
= -\frac{\delta}{\tau^2} \eta + \frac{\varepsilon D}{\tau^2} a_2(\eta) R_2(\eta)
\tag{20}
\]

Let us assume that \( \eta < 0 \). Because \( a_2 \) and \( R_2 \) are always positive, it is clear from equation (20) that \( \frac{\partial W}{\partial \zeta}(\eta, \eta) > 0 \). Let us assume that \( \eta = D + \alpha \), for \( \alpha > 0 \). Following equation (20) we have

\[
\frac{\partial W}{\partial \zeta}(\eta, \eta) = -\frac{\delta}{\tau^2} D - \frac{\delta}{\tau^2} \alpha + \frac{\varepsilon D}{\tau^2} a_2(\eta) R_2(\eta)
= \frac{D}{\tau^2} (\varepsilon a_2(\eta) R_2(\eta) - \delta) - \frac{\delta}{\tau^2} \alpha
= -\frac{D}{\tau^2} \varepsilon a_1(\eta) R_1(\eta) - \frac{\delta}{\tau^2} \alpha
< 0
\tag{21}
\]

The step between equations (22) and (23) is a direct consequence of equation (2).

**When does a population experience disruptive selection?** One of the necessary conditions for diversification is that a population experiences disruptive selection. This occurs when the population is in an evolutionary equilibrium that corresponds to a fitness minimum, hence the curvature of the fitness landscape around its trait value is positive. To determine when this condition is satisfied, we investigate the fitness landscape of a rare mutant with niche trait value, \( \zeta \), in the environment set by the population with a trait value, \( \eta \). The fitness experienced by a mutant is given by equation (19). A population with a trait value \( \eta^* \) is in an evolutionary equilibrium if, at this point, its selection gradient equals zero, i.e. \( \frac{\partial W}{\partial \zeta}(\eta^*, \eta^*) = 0 \). This equilibrium corresponds to a fitness minimum if \( \frac{\partial^2 W}{\partial \zeta^2}(\eta^*, \eta^*) > 0 \). To evaluate these conditions when \( \eta^* = \frac{D}{\tau} \), we first need to calculate \( \frac{\partial^2 W}{\partial \zeta^2}(\eta, \eta) \). Following equation (20), we have

\[
\frac{\partial^2 W}{\partial \zeta^2}(\eta, \eta) = \varepsilon \left( \frac{\partial^2 a_1}{\partial \zeta^2} R_1(\eta) + \frac{\partial^2 a_2}{\partial \zeta^2} R_2(\eta) \right)
= \varepsilon \left( \frac{1}{\tau^2} a_1(\eta) \left( \frac{\eta^2}{\tau^2} - 1 \right) R_1(\eta) + \frac{1}{\tau^2} a_2(\eta) \left( \frac{(\eta - D)^2}{\tau^2} - 1 \right) R_2(\eta) \right)
= \varepsilon \left( \frac{\eta^2}{\tau^2} - 1 \right) a_1(\eta) R_1(\eta) + a_2(\eta) R_2(\eta) + \frac{D}{\tau^2} a_2(\eta) R_2(\eta) (D - 2\eta)
\tag{25}
\]

When \( \eta^* = \frac{D}{\tau} \), \( R_1(\frac{D}{\tau}) = R_2(\frac{D}{\tau}) \), hence

\[
2\varepsilon a_2(\frac{D}{\tau}) R_2(\frac{D}{\tau}) = \delta
\tag{26}
\]

because in any ecological equilibrium the per capita growth rate equals 0 (i.e. \( W(\eta, \eta) = 0 \)). Using equations (20) and (26), we can show that \( \eta^* = \frac{D}{\tau} \) is an evolutionary equilibrium for any
$D > 0.$

\[
\begin{align*}
\frac{\partial W}{\partial \eta}(\frac{D}{2}, \frac{D}{2}) &= -\frac{\delta}{\tau^2} D + \frac{\varepsilon D}{\tau^2} a_2(\frac{D}{2}) R_2(\frac{D}{2}) \\
&= D \frac{\delta}{\tau^2} \left( -\frac{\delta}{2} + \varepsilon a_2(\frac{D}{2}) R_2(\frac{D}{2}) \right) \\
&= 0
\end{align*}
\]

Finally, we evaluate the curvature of the fitness landscape at $\eta^* = \frac{D}{2}$ using equations (25) and (26)

\[
\frac{\partial^2 W}{\partial \eta^2}(\frac{D}{2}, \frac{D}{2}) = \frac{\delta}{\tau^2} \left( \frac{D^2}{4\tau^2} - 1 \right)
\]

The curvature of the fitness landscape for $\eta^* = \frac{D}{2}$ is positive, i.e. $\frac{\partial^2 W}{\partial \eta^2}(\frac{D}{2}, \frac{D}{2}) > 0$, and thus selection is disruptive in the evolutionary equilibrium $\eta^* = \frac{D}{2}$ when

\[
D > 2\tau
\]

**When does a population evolve toward the evolutionary equilibrium where selection is disruptive?** The second necessary condition for diversification is that a population evolves toward the evolutionary equilibrium where selection is disruptive. This occurs when this equilibrium is an evolutionary attractor. To determine for which values of $D$, the equilibrium $\eta^* = \frac{D}{2}$ is an attractor, we investigate its stability.

An evolutionary equilibrium $\eta^*$ is stable if $\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\eta, \eta) \right) < 0$. Hence, we first derivate equation (20) with respect to $\eta$.

\[
\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\eta, \eta) \right) = -\frac{\delta}{\tau^2} - \frac{\varepsilon D}{\tau^2} a_2(\eta) R_2(\eta) + \frac{\varepsilon D}{\tau^2} a_2(\eta) \frac{\partial R_2}{\partial \eta}(\eta)
\]

Next, we evaluate $\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\eta, \eta) \right)$ at $\eta^* = \frac{D}{2}$.

\[
\begin{align*}
\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\eta, \eta) \right) (\frac{D}{2}) &= -\frac{\delta}{\tau^2} - \frac{\varepsilon D}{\tau^2} a_2(\frac{D}{2}) \frac{D}{2} - \frac{D}{2} \frac{\partial R_2}{\partial \eta}(\frac{D}{2}) + \frac{\varepsilon D}{\tau^2} a_2(\frac{D}{2}) \frac{\partial R_2}{\partial \eta}(\frac{D}{2}) \\
&= -\frac{\delta}{\tau^2} + \frac{\varepsilon D^2}{4\tau^4} a_2(\frac{D}{2}) \left( \frac{D}{2} \frac{\partial R_2}{\partial \eta}(\frac{D}{2}) - \frac{\partial R_2}{\partial \eta}(\frac{D}{2}) \right) \\
&= -\frac{\delta}{\tau^2} + \frac{8\varepsilon R_{max} \tau^4 a_2(\frac{D}{2})}{8\varepsilon R_{max} \tau^4 a_2(\frac{D}{2})} \\
&= -\frac{\delta}{\tau^2} + \frac{8\varepsilon R_{max} \tau^4 a_2(\frac{D}{2})}{8\varepsilon R_{max} \tau^4 a_2(\frac{D}{2})}
\end{align*}
\]

In the step between equations (32) and (33) we use equation (18).

Because $\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\eta, \eta) \right) (\frac{D}{2})$ is an increasing function of $D$, there exists $\bar{D}$ such that $\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\frac{D}{2}, \frac{D}{2}) \right) < 0$ for any $D < \bar{D}$ and $\frac{\partial}{\partial \eta} \left( \frac{\partial W}{\partial \eta}(\frac{D}{2}, \frac{D}{2}) \right) > 0$ for any $D > \bar{D}$. Thus, $\bar{D}$ is the unique solution of
the equation \( \frac{\partial}{\partial \eta} \left( \frac{\partial \mathcal{W}}{\partial \xi} (\eta, \eta) \right) \left( \frac{D}{2} \right) = 0 \) which we rewrite using equation (35):

\[
0 = \frac{-\delta}{\tau^2} + \frac{\delta^2 D^2}{8\varepsilon R_{\text{max}} \tau^4 A} \frac{\rho^2}{e^{\frac{8\rho^2}{\tau^2}}} \\
= \frac{-1}{\tau^2} + \frac{\delta D^2}{8\varepsilon R_{\text{max}} \tau^4 A} \frac{\rho^2}{e^{\frac{8\rho^2}{\tau^2}}} \tag{36}
\]

Equation (37) is equivalent to

\[
\frac{D^2}{8\tau^2} \frac{\rho^2}{e^{\frac{8\rho^2}{\tau^2}}} = \frac{\varepsilon R_{\text{max}} A}{\delta} \tag{37}
\]

Let us define \( u = \frac{D^2}{8\tau^2} \) and rewrite equation (38) as follows.

\[
u e^u = \frac{\varepsilon R_{\text{max}} A}{\delta} \tag{39}\]

The solution of equation (39) is given by the principal branch \( W_0 \) of the Lambert function. That is \( u = W_0(\frac{\varepsilon R_{\text{max}} A}{\delta}) \). Hence, we have

\[
D = 2\sqrt{2\tau^2 W_0(\frac{\varepsilon R_{\text{max}} A}{\delta})} \tag{40}\]

Therefore, for any \( D \) smaller than the right-hand side of equation (40), the evolutionary equilibrium \( \eta^* = \frac{D}{2} \) is stable and thus an evolutionary attractor.

In conclusion, diversification occurs at the evolutionary equilibrium \( \eta^* = \frac{D}{2} \) when

\[
2\tau < D < 2\sqrt{2\tau^2 W_0(\frac{\varepsilon R_{\text{max}} A}{\delta})}.
\]
4. **Effects of different separation in the trait axis between the optima to feed in the different food resources and of an uneven partitioning of productivity among the preexisting resources**

Our aim is to investigate the role of total productivity and diversity of preexisting resources in the habitat of the radiating clade. In the main text, we focus on the case in which the optimal trait to consume each preexisting resource $\theta_i$ and the optimal trait to feed upon other members of the radiating clade $\theta_c$ are equally distant from one another by a distance $D$ along the feeding niche trait axis. Likewise, in the main text we investigate the case in which $R_{i_{\text{max}}}$ is equal for all preexisting resources, i.e. $R_{i_{\text{max}}} = P/n$; where $P$ is the total productivity of the system and $n$ is the number of preexisting resources.

In this section, we relax these assumptions and explore the effect of varying distances between the optimal trait values as well as varying $R_{i_{\text{max}}}$. In the first case, we randomly draw each optimal trait value $\theta_i$ from a uniform distribution with a minimum and a maximum value $(-D - \varphi_\theta)$ and $(D + \varphi_\theta)$, respectively. The optimal trait to feed upon other members of the radiating clade $\theta_c$ is drawn from a uniform distribution with a minimum and a maximum value $-\varphi_\theta$ and $\varphi_\theta$ when the intraclade consumer can evolve, and from a uniform distribution with a minimum and a maximum value $(-10 - \varphi_\theta)$ and $(-10 + \varphi_\theta)$ when the intraclade consumer cannot evolve. For each combination of productivity and diversity of preexisting resources, we simulate 50 different scenarios of randomly generated $\theta$ values and investigate the eco-evolutionary dynamics of adaptive radiations using numerical simulations (see Numerical simulations of the eco-evolutionary dynamics of adaptive radiations in supporting information 1). We present the average values resulting from the simulations of these 50 scenarios for $\varphi_\theta = 0.3$ and $\varphi_\theta = 0.75$ in figure S6. These $\varphi_\theta$ values result in uniform distributions whose intervals span 20% and 50% of the average distance $D$ between optimal traits.

In the second case, we consider a fixed productivity that is unevenly partitioned among the preexisting resources. Specifically, we generate a vector that contains $n$ random values $R_{i_{\text{max}}}$ lying in the interval with a minimum and a maximum value $P(1 + \varphi_{R_{\text{max}}})/n$ and $P(1 - \varphi_{R_{\text{max}}})/n$, respectively, and subject to the condition that their sum be equal to $P$. Therefore, $\varphi_{R_{\text{max}}}$ determines the degree of deviation from $R_{i_{\text{max}}} = P/n$. We specifically use the MATLAB function randfixedsum.m to generate this vector of random $R_{i_{\text{max}}}$ values. For each combination of productivity and diversity of preexisting resources, we simulate 50 different scenarios of randomly generated $R_{i_{\text{max}}}$ values and investigate the eco-evolutionary dynamics of adaptive radiations using numerical simulations (see Numerical simulations of the eco-evolutionary dynamics of adaptive radiations in supporting information 1). We present the average values resulting from the simulations of these 50 scenarios for $\varphi_{R_{\text{max}}} = 0.2$ and $\varphi_{R_{\text{max}}} = 0.5$ in figure S7.

Similar to the results presented in the main text, when the diversity of preexisting resources is high the evolution of intraclade consumers reduces the clade richness compared to the expected richness in the absence of consumers. High variation, however, weakens the magnitude of the negative effect of the intraclade consumer on species richness. Specifically, the difference in the resulting diversity in the absence and presence of intraclade predators is smaller when variation in the distances between the optimal traits and in $R_{i_{\text{max}}}$ is large (blue region in the bottom panel is smaller when $\varphi_\theta = 0.75$ compared to $\varphi_\theta = 0.3$ and when $\varphi_{R_{\text{max}}} = 0.5$ compared to $\varphi_{R_{\text{max}}} = 0.2$). Nonetheless, the negative effect persists, and as observed in figure 4,
the intraclade consumers adds to the diversity of the clade only when the number of preexisting resources is low.
5. **Evolutionary oscillations in the intraclade consumer population**

Small evolutionary cycles occur in the intraclade consumer population after its emergence. In figure 1B, after time 9000, these cycles produce changes in the trait of the consumer between values of 0.68 and 0.85. Interestingly, directional selection drives the trait to a value of 0.7. At this point, selection is disruptive and the population is split into two subpopulations. One that experiences a positive selection gradient that stabilizes at a trait value of 0.85. The other one experiences a negative selection gradient. As its trait value decreases, the population undergoes extinction at a value of 0.68. After the extinction of one of the populations, the other population (that with a trait value of 0.85) experiences directional selection towards the trait value where selection is disruptive. The population once again split into two subpopulations, of which one will result again in extinction. This process causes the continuous addition and removal of a subpopulation of consumer. This type of oscillations, known as branching-extinction evolutionary cycles, has been observed in a variety of systems including polymorphic populations with emerging trophic interactions⁴, similar to the ones described in our model. The maximal difference of traits between the subpopulations is 0.17, whereas the trait distance to the closest ecomorph is 5.32. Due to the high similarity between subpopulations of consumers and the high dissimilarity with any other ecomorph, these cycles do not have a negative effect on the evolution of reproductive isolation between the intraclade consumer and other ecomorphs. In fact, our individual based-model simulations show that the evolution of assortative mating in the intraclade consumer population is possible (figure S5).
6. Effects of separation in the trait axis between the consumer niche and the ancestral population

Our main interest is to examine how environmental factors (i.e. productivity and abundance of available niches) may limit the evolution of intraclade consumers in adaptive radiations. Therefore, until now we exclude the possibility that other factors limit the evolution of intraclade consumers. In this section, we investigate how the separation in the trait space between the niche consumer trait and the trait of the ancestral population affect the eco-evolutionary dynamics of adaptive radiations. We study these dynamics by seeding the radiation with a population with different trait values.

Similar to the case when the ancestral population has a trait of 2 (figure 1B and figure S4A), the evolution of intraclade consumers causes reduced diversity also when the clade is seeded with a population with a trait value equal to 6 and 12 (figure S4 B and C). Specifically, four niches ($\theta_1$, $\theta_7$, $\theta_9$ and $\theta_{10}$), that in the absence of intraclade consumers would have been occupied by the clade, remain unoccupied. This means that pre-existing resources $R_1$, $R_7$, $R_9$ and $R_{10}$ are not used by the clade. When the ancestral population has a larger trait value (18, 24 and 30 in figure S4 D, E and F, respectively), niches corresponding to trait values $\theta_7$, $\theta_9$ and $\theta_{10}$ are filled, however $\theta_1$ remains unfilled.

A delayed evolution of the intraclade consumer due to large separation in the trait space between the niche consumer trait and the trait of the ancestral population can thus reduce the negative effect on the final diversity of the clade. However, this occurs when the ancestral population has a trait value separated by several niches from the consumer niche, and even, when the consumer niche is the last to be filled by the clade, at least one niche remains unfilled. As a consequence, in the better case, a radiation in which an intraclade consumer can evolve has the same diversity than a radiation in which the consumer cannot evolve.
7. Diversification in sexually reproducing populations with cannibalistic interactions

Up to now, the model results presented in the main text are based on the assumption of asexual reproduction. For speciation to occur in sexual populations, assortative mating needs to evolve to limit gene flow. We evaluate whether assortative mating can evolve in the model system using an individual-based model (IBM) incorporating sexual reproduction. The IBM additionally allows us to check whether our results hold under demographic stochasticity and cannibalistic interactions. Our simulations show that assortative mating indeed evolves to reduce gene flow between populations (figure S5). Furthermore, we find that the clade does not diversify to occupy all available niches when the predator can evolve, similar than in the asexual deterministic model (figure 1). However, the trajectory followed by the radiating clade is not the same when reproduction is sexual than when it is asexual. In contrast to the asexual case, diversification is not sequential in sexual populations. Instead, sexual reproduction causes the initial phase of the radiation to be a hybrid swarm. Initially, this hybrid swarm expands over the trait space without differentiation of populations. Once, biomass has been built up in the clade, the first ecomorph that differentiates is the intraclade consumer. After its emergence, the expansion of the clade is quickly halted (expansion front is near $\theta_7$ in figure S5B), and short after, other ecomorphs emerge from the hybrid swarm (ecomorphs using resources 1 to 6; individuals with trait near $\theta_7$ got extinct in figure S5B). The final diversity of the clade is seven different ecomorphs (one intraclade consumer ecomorph and 6 ecomorphs feeding on preexisting resources). We run 10 different simulations with the same parameters and find that in all of them the intraclade consumer evolved. In addition, in 8 of them the final diversity was 7, in 1 it was 9, and in 1 it was 5 (figure S5C). Extinctions occur in half of them. We furthermore calculate the probability of gene flow using the method of Sobel and Chen\(^1\) and found that the evolution of assortative mating reduces gene flow to probabilities below 0.1, and in many cases, to probability 0 (figure S5D).
Table S1. Parameter values

| Parameter                                                                 | Symbol | Value | Units                        |
|---------------------------------------------------------------------------|--------|-------|------------------------------|
| Renewal rate of preexisting resources†                                   | 𝜌     | 0.01  | (unit of time)⁻¹             |
| Total productivity of the habitat                                        | 𝑃     | varied| g L⁻¹                        |
| Number of preexisting resources                                          | 𝑛     | varied| -                            |
| Maximum attack rate†                                                     | 𝐴     | 0.2   | g (unit of time)⁻¹           |
| Optimal trait value to feed upon other members of the radiating clade    | 𝜃ₜ    | 0*    | g (unit of time)⁻¹           |
| Optimal trait value to feed upon other members of the radiating clade    | 𝜃ₜ    | -10** | -                            |
| Distance between optimal trait values to feed on the preexisting         | 𝐷     | 3***  | -                            |
| resources and on other members of the clade                              |        |       |                              |
| Width of the Gaussian curve describing the degree of                     | 𝜏     | 1     | -                            |
| specialization to successfully attack a food resource                     |        |       |                              |
| Feeding niche trait                                                     | 𝜂     | Evolving trait | -                         |
| Assimilation efficiency                                                 | 𝜖     | 0.6   | -                            |
| Fraction of biomass available for an intraclade consumer                 | 𝜆     | 0.1   | -                            |
| Background mortality†                                                   | 𝜁     | 0.01  | (unit of time)⁻¹             |
| Mutation rate per birth event                                           | 𝜇     | 0.001 | -                            |

†Rates have an arbitrary unit of time (e.g. day)

*When the consumer can evolve.

**When the consumer cannot evolve.

***This value is selected based on the conditions for diversification that we analytically derived (expressions in (4) and (5) in Results section).
References

1. Rueffler, C., Van Dooren, T. J. M., Leimar, O. & Abrams, P. A. Disruptive selection and then what? *Trends Ecol. Evol.* **21**, 238–245 (2006).

2. Dieckmann, U. & Doebeli, M. On the origin of species by sympatric speciation. *Nature* **400**, 354–357 (1999).

3. Sobel, J. M. & Chen, G. F. Unification of methods for estimating the strength of reproductive isolation. *Evolution (N. Y.)* **68**, 1511–1522 (2014).

4. Dercole, F. Remarks on branching-extinction evolutionary cycles. *J. Math. Biol.* **47**, 569–580 (2003).