The Perturbations Estimation in Two Gas Plants

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ABSTRACT The perturbations are the unwanted and unknown inlets in nonlinear plants which can affect the outlets. In this article, an estimator is studied for the variables and perturbations estimation in nonlinear plants. The saturation map is used in our estimator instead of the signum map to decrease the chattering, and we ensure the estimator convergence by the Lyapunov analysis. The conditions required by our estimator gains are found to reach the variables error convergence, and these gains are used for the perturbations estimation. An algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator. The studied estimator is applied for the variables and perturbations estimation in the gas turbine and gasification plants.

INDEX TERMS Variables, perturbations, estimation, gas turbine, gasification, convergence.

I. INTRODUCTION

The perturbations are the unwanted and unknown inlets in nonlinear plants which can affect the outlets. This issue has occurred in many nonlinear plants. Since perturbations can affect the sensors, actuators, or plants yielding additional costs, and since most nonlinear plants regulators require the knowledge of perturbations; an approach for the perturbations estimation is welcome.

There are some studies about regulators for perturbed plants. In [1] and [2], the active strategy for the perturbations attenuation is mentioned. In [3] and [4], the singular perturbations approach for the perturbations attenuation is considered. The variables and perturbations estimation in plants is focused in [5]–[7], and [8]. In [9]–[12], and [13], the robust analysis for the perturbed plants stabilization is focused. The perturbations estimation with fuzzy regulators is mentioned in [14] and [15]. In [16]–[18], and [19], the authors use the neuro-fuzzy approximations for the perturbed plants regulation. The adaptive laws for the perturbed plants regulation are focused in [20], [21], and [22]. In [23] and [24], the structure theory for the perturbations attenuation is mentioned. From the above studies, in [1], [2], [5], [6], [7]–[11], [12], [13], [17], and [19], the authors use approaches for the variables or perturbations estimation in nonlinear plants; then it would be welcome to be focused on this issue. The novelty of this article is that each nonlinear plant has a different structure; consequently, a special estimator with the structure of the gas plants must be discussed.

There are various estimators who use the plant outlets for the variables estimation [1], [2], [5], [6], but there are not many estimators who use the plant outlets for the perturbations estimation. In [7], [8], [11], [13], [17], [19], previous studies of estimators are focused for the perturbations estimation, but with the two below differences: in the previous studies estimators with adaptive or feedback outlets are used, while in this article an estimator with sliding modes outlets is used, in the plants of previous studies the noise is not used, while in the plants of this article the noise is used.

In this article, the sliding mode approach is utilized in our estimator for the variables and perturbations estimation in nonlinear plants. Since the sliding mode approach uses the signum map, it can yield the unwanted chattering [3], [12], [14], [15].

The first contribution of this article is that an estimator is designed, it is described by the following characteristics: 1) the saturation map is used in our estimator instead of the signum map to decrease the chattering; and later, we ensure...
the estimator convergence by the Lyapunov analysis, 2) find the conditions required by our estimator gains that allow it to reach the variables error convergence; it yields an acceptable performance in the variables and perturbations estimation.

The second contribution of this article is that an algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator, it is described as follows: 1) we choose a value for the gain 1, 2) we obtain the estimator matrix, 3) we obtain the eigenvalues of the estimator matrix, if the real parts of all the eigenvalues of the estimator matrix are negative, then the gain 1 is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose the matrix 1 of the Lyapunov equation, 5) we substitute matrix 1 and estimator matrix into the Lyapunov equation, and we find the matrix 2 of the Lyapunov equation, if matrix 1 and matrix 2 of the Lyapunov equation are positive definite, then matrix 1 is correctly chosen and we can go to the step 6, otherwise, we must return to step 1, 6) we choose the gain 2, gain 3, and gain 4 to solve the estimator, if the estimator reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant, then the algorithm finishes, otherwise, we must return to step 6.

Our estimator is applied for the variables and perturbations estimation in the gas turbine and gasification plants. The gas turbine plant is used for the electrical energy generation from the gas [25], while the gasification plant is used for the gas generation from biomass [26].

The rest of the article is described below. Section II presents the estimator design containing the variables error convergence, and the perturbations estimation in nonlinear plants, later, an algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator. Sections III and IV estimators are studied for the variables and perturbations estimation in the gas turbine and gasification plants. Section V express the conclusion and future work.

II. THE ESTIMATOR FOR THE VARIABLES AND PERTURBATIONS ESTIMATION

In this section, a variables estimator, and a perturbations estimator, which are termed as estimator will be studied for the variables and perturbations estimation in nonlinear plants.

In this article, a special nonlinear plant will be used in which the outlets have a linear combination with the variables, the variables have a nonlinear combination with the variables, and the perturbations are entered additively. The nonlinear plant is [25], [26]:

\[
\begin{align*}
\dot{h} &= Ah + f(h, v) + Bu + Bx \\
y &= Ch
\end{align*}
\]  

(1)

\(h \in \mathbb{R}^n\) as plant variables, \(v \in \mathbb{R}^n\) as the plant inlets, \(y \in \mathbb{R}^n\) as the plant outlets, \(u \in \mathbb{R}^n\) as the perturbations, \(f(h, v) \in \mathbb{R}^n\) as a nonlinear map, \(x \in \mathbb{R}^n\) as the noise, \(A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times 1}\), \(C \in \mathbb{R}^{n \times n}\) as matrices.

A. THE VARIABLES ESTIMATOR

The goal of the variables estimator is that using the inlets and outputs, the variables of the variables estimator should estimate the nonlinear plant variables.

The estimator error \(\hat{y}\) is:

\[
\hat{y} = y - \tilde{y} = C\tilde{h}
\]  

(2)

\(\tilde{y}\) as the variables estimator outlet, \(\tilde{h} = h - \hat{h}\) as the variables error, \(\hat{h}\) as the estimator variables. The variables estimator is:

\[
\begin{align*}
\dot{\hat{h}} &= A\hat{h} + f(\hat{h}, v) + K\tilde{y} + Esat(M\tilde{y}) \\
\tilde{y} &= C\hat{h}
\end{align*}
\]  

(3)

\[
sat(M\tilde{y}) = \begin{cases} 
\frac{c}{|c|}cc1 & M\tilde{y} > 1 \\
M\tilde{y} & |M\tilde{y}| \leq 1 \\
-1 & M\tilde{y} < -1,
\end{cases}
\]

\(\hat{h}\) as the estimator variables, \(\tilde{y}\) as the estimator outlets, sat(\(\cdot\)) as the saturation map, \(M\) as a matrix where \(MC \in \mathbb{R}^{n \times n}\) is a positive semi-definite constant, \(K \in \mathbb{R}^{n \times 1}\) and \(E \in \mathbb{R}^{n \times 1}\).

B. THE CONVERGENCE ANALYSIS OF THE VARIABLES ESTIMATOR

In this sub-section, the convergence of the variables estimator applied to nonlinear plants is analyzed based on the Lyapunov approach [3], [4].

The closed loop model of the variables estimator is the subtraction of (3) to (1) and using the estimator error (2) as:

\[
\begin{align*}
\dot{\tilde{h}} &= A\tilde{h} + f(\tilde{h}, v) + Bu + Bx - f(\hat{h}, v) \\
&\quad - K\tilde{y} - Esat(M\tilde{y}) \\
\Rightarrow \tilde{h} &= A\tilde{h} + \tilde{f} + Bu + Bx - KC\tilde{h} - Esat(MC\tilde{h}) \\
\Rightarrow \hat{h} &= A\hat{h} + \hat{f} + Bu + Bx - Esat(MC\hat{h})
\end{align*}
\]  

(4)

\(\tilde{f} = f(h, v) - f(\hat{h}, v), A = A - KC\). The nonlinear map \(\tilde{f}\) is bounded as \(|\tilde{f}| \leq \bar{f}, \bar{f}\) as the absolute value.

The below theorem analyzes the variables estimator convergence.

Theorem 1: The variables error of the variables estimator (2)-(3) applied to estimate the nonlinear plant variables in (1) is convergent, \(\gamma = \lambda_{\min}(Q, P^{-1})\), \(|\tilde{f} + Bu + Bx| \leq \bar{u}, \bar{u} \leq E, |||\cdot|||\) as the Euclidean norm in \(\mathbb{R}^n\), \(|\cdot|\) as the absolute value, \(P_s \in \mathbb{R}^{n \times n}\) and \(Q_s \in \mathbb{R}^{n \times n}\) are positive definite matrices which meet:

\[
A_s^T P_s + P_s A_s = -Q_s
\]  

(5)

\(A_s\) as is expressed in (4).

Proof: The Lyapunov candidate map is:

\[
L = \tilde{h}^T P_s \tilde{h}
\]  

(6)

The derivative of (4) is:

\[
\dot{L} = \tilde{h}^T \dot{\tilde{h}} + \tilde{h}^T P_s \tilde{h}
\]  

(7)
Using the second term of (7), \( sat(M\tilde{C}\tilde{h}) = sat(\tilde{h}) \), and 
\[ |f + Bu + Bx| \leq \bar{u}, \]
is:
\[ 2\tilde{h}^T P_s [f + Bu - Esat(M\tilde{C}\tilde{h})] = 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]  
(8)

Using (8) in (7) is:
\[ \dot{L} = \tilde{h}^T \left( A_s^T P_s + P_s A_s \right) \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]  
(9)

Using \( A_s^T P_s + P_s A_s + \mu P_s = -Q_s \) of (5), is: The equation of (9) can be expressed as:
\[ \dot{L} = -\tilde{h}^T Q_s \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]  
(10)

Since (5), \( f + Bu + Bx \leq \bar{u}, \bar{u} \leq E \), and since (3), (7),
\[ sat(\tilde{h}) = \begin{cases} \frac{1}{\tilde{h}} & \tilde{h} > 1 \\ \hat{h} & \tilde{h} \leq 1 \end{cases} \]
, we notice that there are three cases of the saturation map. 1) If \( \tilde{h} > 1 \), then \( sat(\tilde{h}) = 1 \) and \( \tilde{h} = \tilde{h} \) [T], we replace in (10) as:
\[ \dot{L} = -\tilde{h}^T Q_s \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]
[11]

2) If \( \tilde{h} \leq 1 \), then \( sat(\tilde{h}) = \tilde{h} \) and \( \tilde{h}^T \tilde{h} = [\tilde{h}]^T [\tilde{h}] \), we replace in (10) as:
\[ \dot{L} = -\tilde{h}^T Q_s \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]
[12]

\[ \dot{L} = -\tilde{h}^T Q_s \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]
(11)

3) If \( \tilde{h} < 1 \), then \( sat(\tilde{h}) = -1 \) and \( \tilde{h} = -\tilde{h} \), we replace in (10) as:
\[ \dot{L} = -\tilde{h}^T Q_s \tilde{h} + 2\tilde{h}^T P_s [f + Bu + Bx] - 2\tilde{h}^T P_s Esat(\tilde{h}) \]
(12)

Using \( \gamma = \lambda_{\min}(Q_s P_s^{-1}) \) (14) becomes to:
\[ \dot{L} \leq -\gamma L \]  
(15)

Since (15), it concludes that the variables error of the variables estimator applied to estimate the nonlinear plant variables is convergent.

\[ C. \text{THE PERTURBATIONS ESTIMATOR} \]

The goal of the perturbations estimator is that using the outlet, the perturbations of the perturbations estimator should estimate the nonlinear plant perturbations.

Since the Theorem 1, it is:
\[ \lim_{T \to \infty} \tilde{h} \equiv 0 \]  
(16)

Using, \( \lim_{T \to \infty} \tilde{h} - \lim_{T \to \infty} Bx \leq \lim_{T \to \infty} \tilde{h} - \lim_{T \to \infty} \tilde{f} \equiv 0 \), \( \tilde{h} \) as the upper bound of \( \tilde{h} \) in the first equality of (4) is:
\[ \lim_{T \to \infty} A\tilde{h} + \lim_{T \to \infty} \tilde{f} + \lim_{T \to \infty} Bu - \lim_{T \to \infty} K\bar{y} \]
(17)

\[ B^{-1} \text{as the pseudo-inverse of } B. \text{ Since all the terms of (17) are bounded independently of } T, \text{ it becomes to:} \]
\[ \tilde{u} \equiv B^{-1} \left[ K\bar{y} + Esat(M\bar{y}) + \bar{h} \right] \]  
(18)

\( \tilde{u} \) as the perturbations of the perturbations estimator, \( u \) as the nonlinear plant perturbations, \( \bar{h} \) as the upper bound of \( \tilde{h} - Bx \).

Remark 1: Note that the usage of the map \( sat(\cdot) \) yields that the derivative \( \dot{\tilde{h}} \) does not tend to zero even the map \( \tilde{h} \) tends to zero, but it could be said that \( \tilde{h} \) is bounded, this fact is used to express \( \tilde{h} \) as the estimated perturbations of \( u \).

Remark 2: Note that in (2), (18), \( \tilde{u} \) only allows to estimate perturbations \( u \), and since in this article the map \( sat(\cdot) \) is used, it can reduce the unwanted chattering. The possibility of changing the non-continuous map \( sat(\cdot) \) for a softer one also could reduce the chattering.

The Figure 1 shows the variables estimator and perturbations estimator for the variables and perturbations estimation. The variables estimator of (2), (3) and perturbations estimator of (2), (18) are termed as the estimator of (2), (3), (18).

Remark 3: Note it is not required that the nonlinear plant (1) must be convergent to achieve a satisfactory performance in our estimator:

Remark 4: For the satisfactory operation of the estimator (2), (3), (18), the theory and application conditions mentioned below must be met: a) propose a gain \( K \) that meets the theory condition (5) such that the variables \( \tilde{h} \) of the variables estimator (2), (3) must reach as soon as possible the nonlinear plant variables \( h \) of (1), b) propose the gain
we must return to step 1, 6) we choose the gains $E$, $M$, and $\tilde{h}$ to solve the proposed estimator as $\hat{h} = Ah + f(\tilde{h}, v) + K\tilde{y} + Esat(M\tilde{y})$, $\tilde{y} = Ch$, $\tilde{u} \equiv B^{-1} [K\tilde{y} + Esat(M\tilde{y}) + \tilde{h}]$, if the proposed estimator reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant $h = Ah + f(h, v) + Bu + Bz$, $y = Ch$, then the algorithm finishes, otherwise, we must return to step 6.

In the below sections, the root mean squared error (MSE) is used for comparisons, it is:

$$J = \left( \frac{1}{T} \int_{0}^{T} h^2 dt \right)^\frac{1}{2} \quad (19)$$

$J = J_h$, $h^2 = \tilde{h}^2$ for variables, $J = J_y$, $h^2 = \tilde{y}^2$ for outlets, $J = J_u$, $h^2 = \tilde{u}^2 = (u - \tilde{u})^2$ for perturbations.

### III. THE GAS TURBINE PLANT

The Figure 3 shows the gas turbine plant.

The gas turbine plant consists of a compressor, a combustion chamber, a turbine, and a power turbine. $T_3$, $T_4$, $T_5$, $T_6$ as the temperatures in the different stages, $M_3$, $M_4$, $M_5$, $M_6$ as the amounts of mass. $m_f$ as the flow rate of the fuel. The inlets are $v_1 = m_3$, $v_2 = v_3 = P_2$, the variables are $h_1 = m_3$, $h_2 = m_4$, $h_3 = m_5$, $h_4 = m_6$, $h_5 = T_3$, $h_6 = T_4$, $h_7 = T_5$, $h_8 = T_6$, $h_9 = P_3$, $h_{10} = P_4$, $h_{11} = P_5$, $h_{12} = P_6$, and the outlet is $y = P_6$. Table 1 shows the gas turbine plant constants.

The gas turbine plant is represented in the form (1) with $A = \text{diag}[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}]$, $a_1 = -1$, $a_2 = -1$, $a_3 = -1$, $a_4 = -1$, $a_5 = -\frac{R}{C_1}$, $a_6 = -\frac{R}{C_2}$, $a_7 = -\frac{R}{C_1}$, $a_8 = -\frac{R}{C_2}$, $a_9 = -\frac{R}{\nu}$, $a_{10} = -DP$, $a_{11} = -\frac{R}{\nu}$,
TABLE 1. The gas turbine plant constants.

| Constant          | Value       |
|-------------------|-------------|
| MFDP = 1 × 10^{-2} kg/s | m_a = 70 kg/s |
| m_{3cor} = 7.731 kg/s | \( R_{2ref} = 288.15 \) K |
| C_p = 1005 (J/kg)K | \( \eta_c = 1.46966 \) |
| C_v = 718 (J/kg)K | \( \gamma = 0.8 \text{ m}^3 \) |
| \( R = 287 \) (J/kg)K | \( \eta_b = 0.99 \) |
| LHV = 44124 (J/kg)K | \( \eta_p = 1 \times 10^{-4} \) |
| \( P_{2ref} = 103.125 \) Pa | \( DP = 0.99 \) Pa |
| m_y = 32.94229 kg/s | \( \eta_{pr} = 1 \times 10^{-4} \) |
| \( P_{4ref} = 680.125 \) Pa | \( T_{4ref} = 1400 \) K |
| \( C_{v_g} = 863 \) (J/kg)K | \( \gamma = 1.4 \) |
| \( P_{5ref} = 416.0 \) Pa | \( T_{5ref} = 1000 \) K |
| m_{5cor} = 2.7184 kg/s | \( \sigma = 1 \times 10^{-6} \) |
| m_{6cor} = 0.3912 kg/s | \( \gamma_g = 1.33 \) |

\[ f_{12} = -\frac{8}{T} f(h, v) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}]^T, \quad C = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1], \quad h = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}, h_{12}]^T, \quad v = [v_1, v_2, v_3]^T, \quad B = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \]

\[ a_{12} = -\frac{8}{T} f(h, v) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}]^T, \quad C = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1], \quad h = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}, h_{12}]^T, \quad v = [v_1, v_2, v_3]^T, \quad B = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \]

A. RESULTS

In this sub-section, the estimator of this research called Estimator 1 is compared against the estimator of [7], [8], called Estimator 2. The initial conditions of the plant are \( h_0 = [75.9, 75.9, 75.9, 75.9, 691.7694, 1.6164 \times 10^3, 1.2708 \times 10^3, 932.4205, 2.0561 \times 10^6, 2.0355 \times 10^5, 5.7420 \times 10^5, 1.2679 \times 10^6] \). The goal of the estimators is that the variables \( \hat{h} \) of the variables estimator have to reach the plant variables \( h \) and that the perturbations \( \hat{u} \) of the perturbations estimator have to reach the plant perturbations \( u \). The plant inlets are \( v_1 = 1.6 + 0.1 \sin(2T) \) kg/s, \( v_2 = 288 + 1 \sin(2T) \) K, and \( v_3 = 1.013 \times 10^5 + 1 \sin(2T) \) Pa from 0 s to 20 s. The perturbation is \( u = 7.6923 \times 10^{-3} \sin(2T) \) from 0 s to 20 s. The noise is \( x = 0.1 \) rand from 0 s to 20 s. RAND is random values between 0 and 1.

Estimator 2 is expressed as [7], [8] with initial conditions as \( h_0 = [76, 76, 76, 76, 691, 1.616 \times 10^3, 1.27 \times 10^3, 932, 2.05 \times 10^6, 2.03 \times 10^6, 5.74 \times 10^5, 1.24 \times 10^5] \) and gains as \( L = [4 \times 10^{-8}, 4 \times 10^{-8}, 2 \times 10^{-8}, 2 \times 10^{-8}, 2 \times 10^{-8}, 4 \times 10^{-8}, 4 \times 10^{-8}, 4 \times 10^{-8}, 6 \times 10^{-6}, 6 \times 10^{-6}, 6 \times 10^{-6}] \).
\[ \hat{f}_8 = \frac{1}{\hat{h}_4 C_v} \left\{ \frac{MFDPm_{\text{m,cor}}}{(\hat{h}_{11}/P_{\text{ref}})^{0.5}} \right\} \times \left( C_p \hat{h}_7 \left( 1 + \eta_{pt} \left[ 1 - \left( \frac{\hat{h}_{12}}{\hat{h}_{11}} \right)^{0.5} \right] \right) - C_v \hat{h}_8 \right) \]

\[ \hat{f}_9 = \frac{R}{V} \left( \hat{h}_9 + \hat{h}_4 \hat{h}_9 + \hat{h}_5 \hat{h}_1 \right) \]

\[ \hat{f}_{10} = DP \left( \hat{h}_{10} + \hat{h}_9 \right) \]

\[ \hat{f}_{11} = \frac{R}{V} \left( \hat{h}_{11} + \hat{h}_5 \hat{h}_1 + \hat{h}_7 \hat{h}_3 \right) \]

\[ \hat{f}_{12} = \frac{R}{V} \left( \hat{h}_{12} + \hat{h}_4 \hat{h}_8 + \hat{h}_8 \hat{h}_4 \right) \]

The initial conditions are \( \hat{h}_0 = [76, 76, 76, 691, 1.616 \times 10^3, 1.27 \times 10^3, 932, 2.05 \times 10^6, 2.03 \times 10^8, 5.74 \times 10^5, 1.24 \times 10^8]^T \).

The gains \( K \in \mathbb{R}^{12}, E \in \mathbb{R}^{12}, M \in \mathbb{R}^{12}, \) and \( \overline{h} \in \mathbb{R} \) for the Estimator 1 are chosen using the proposed algorithm of Figure 2 detailed as follows: 1) we choose a value for \( K = [4 \times 10^{-9}, 4 \times 10^{-9}, 2 \times 10^{-9}, 2 \times 10^{-9}, 4 \times 10^{-9}, 4 \times 10^{-9}, 4 \times 10^{-9}, 6 \times 10^{-7}, 6 \times 10^{-7}, 6 \times 10^{-7}, 6 \times 10^{-7}]^T \), 2) we obtain \( A_2 = A - KC = [a_{ij}] \in \mathbb{R}^{12 \times 12}, a_{11} = -1, a_{22} = -1, a_{44} = -1, a_{55} = -287, a_{66} = -287, a_{77} = -287, a_{88} = -287, a_{99} = -358.75, a_{1010} = -0.99, a_{1111} = -358.75, a_{1212} = -358.75, a_{4112} = -1, a_{4212} = -1, a_{4312} = -1, a_{5112} = -1, a_{5212} = -1, a_{5312} = -1, a_{5412} = -1, a_{5512} = -1, a_{5612} = -1, a_{5712} = -1, a_{5812} = -1, a_{5912} = -1, a_{101112} = -1, a_{111112} = -1, a_{121112} = -1, \) the other terms of \( A_i \) have a value of 0, 3) we obtain the eigenvalues of \( A_{ss} = s I - A_i as -0.42028 + 0.11615i, -0.42028 - 0.11615i, -347.89, -1.3013, -18.240, -364.18 - 9.7741i, -364.18 + 9.7741i, -0.31231 - 4.1895 \times 10^{-2}i, -0.31231 + 4.1895 \times 10^{-2}i, -0.75333 + 0.25701i, -0.75333 - 0.25701i, -1.124, \) if the real parts of all the eigenvalues of \( A_{ss} \) are negative, then the gain \( K \in \mathbb{R}^{12} \) is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose \( P_2 = [p_{ij}] \in \mathbb{R}^{12 \times 12}, p_{11} = 0.5, p_{22} = 0.5, p_{33} = 0.5, p_{44} = 0.5, p_{55} = 1.2509, p_{66} = 1.5035, p_{77} = 1.2509, p_{88} = 1.2509, p_{99} = 1.3937 \times 10^{-3}, p_{1010} = 0.50505, p_{1111} = 1.3937 \times 10^{-3}, p_{1212} = 1.3937 \times 10^{-3}, \) the other terms of \( P_2 \) have a value of 0, 5) we substitute \( P_1 = [p_{ij}] \in \mathbb{R}^{12 \times 12} \) and \( A_2 = A - KC \) into the Lyapunov equation \( A_r^T P_2 + P_2 A_2 = -Q_2, \) and we find \( Q_2 = [q_{ij}] \in \mathbb{R}^{12 \times 12}, q_{11} = 1.0, q_{22} = 1.0, q_{33} = 1.0, q_{44} = 1.0, q_{55} = 1.0, q_{66} = 1.0, q_{77} = 1.0, q_{88} = 1.0, q_{99} = 0.99998, q_{1010} = 1.0, q_{1111} = 0.99998, q_{1212} = 0.99998, \) the other terms of \( Q_2 \) have a value of 0, if \( P_1 = [p_{ij}] \in \mathbb{R}^{12 \times 12} \) and \( Q_2 = [q_{ij}] \in \mathbb{R}^{12 \times 12} \) are positive definite, then \( P_1 = [p_{ij}] \in \mathbb{R}^{12 \times 12} \) is correctly chosen, the Theorem 1 is met, and we can go to the step 6, otherwise, we must return to step 6, we choose the gains \( E = [1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}]^T, M = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]^T, \) and \( \overline{h} = 2.14 \).

The Figures 4 and 5 show the variables and perturbations estimation of the Estimator 1 and Estimator 2 applied to the gas turbine plant. The Table 2 shows the MSE of (19).

In the Figures 4 and 5, since the Estimator 1 reaches better variables and perturbations estimation of the plant than the Estimator 2, it is seen that the Estimator 1 reaches better performance. In addition, the Figures 4 and 5 show that in the variables and perturbations estimation of Estimator 1, the nonlinear plant \( h = Ah + f(h, v) + Bu + Bx, y = C_1 h, \) then the algorithm finishes, otherwise, we must return to step 6.

![Figure 4. The variables estimation in the gas turbine plant.](image)

![Figure 5. The perturbations estimation in the gas turbine plant.](image)
TABLE 2. The gas turbine plant results.

|          | Estimator 2 | Estimator 1 |
|----------|-------------|-------------|
| $J_h$    | 4.084       | 0.0255      |
| $J_y$    | 3.2915      | 0.0248      |
| $J_u$    | 0.1134      | 0.0387      |

the unwanted chattering is not presented. In the Table 2, since the MSE is smaller for the Estimator 1 than for the Estimator 2, it is seen that the Estimator 1 reaches better exactness for the variables and perturbations estimation.

IV. THE GASIFICATION PLANT

The Figure 6 shows the gasification plant. The gasification plant model consists of drying, pyrolysis, oxidation or combustion, and reduction. $C_{CO}$, $C_{CO_2}$, $C_H$, $C_{H_2}$, $C_{H_2O}$, $C_{O_2}$, $C_{N_2}$ as concentrations of gases $CO$, $CO_2$, $H_2$, $H_4$, Tar, $H_2O$, $O_2$, $N_2$ in mol/cm$^3$, $\rho_{coal}$, $\rho_{char}$ as the coal solids densities, char in g/cm$^3$, $T$ as the gases temperature in K, $Ts$ as the solids temperature in K, $u$ as the injected gases flow in mols/cm$^2$/s, $\delta$ as the steam flow in mols/cm$^2$/s, $R_1$, $R_2$, $R_3$ as the chemical reactions, $m_{O_2}$, $m_{H_2O}$ as the internal molar fractions of $O_2$ and $H_2O$. The variables are $h_1 = \rho_{coal}$, $h_2 = \rho_{char}$, $h_3 = Ts$, $h_4 = C_{CO}$, $h_5 = C_{CO_2}$, $h_6 = C_{H_2}$, $h_7 = C_{H_4}$, $h_8 = C_{Tar}$, $h_9 = C_{H_2O}$, $h_{10} = C_{O_2}$, $h_{11} = C_{N_2}$, the inlet is $v = u$, the perturbation is $\delta$, and the outlet is $y = C_{H_2O}$. Table 3 shows the gasification plant constants.

The gasification plant has the form of (1) with $h = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{11}]^T$, $v = u$, $A = -\beta diag(1) \in R^{11 \times 11}$, $diag(1)$ as a diagonal matrix with a number 1 in the main diagonal, $C = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$, $B = [0, 0, 0, 0, 0, 0, 1, 0, 0]$, $f(h, v) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}]^T$, the terms of $f(h, v)$ are [26]:

$$f_1 = -M_1 R_1$$
$$f_2 = M_2[pr_{char} R_1 - R_2 - R_3]$$
$$f_3 = \frac{1}{C_s}[h_1(T - h_3) - \Delta q_2 R_2 - \Delta q_3 R_3]$$
$$f_4 = p_{R_1CO} R_1 + R_3$$
$$f_5 = p_{R_1CO_2} R_1 + R_2$$
$$f_6 = p_{R_1H_2} R_1 + R_3$$
$$f_7 = p_{R_1H_2O} R_1$$
$$f_8 = p_{R_1Tar} R_1$$
$$f_9 = p_{R_1H_2O} R_1 + p_{R_2H_2O} R_2 - p_{R_3H_2O} R_3 + \frac{\Pi v}{L}$$
$$f_{10} = -p_{R_2O_2} R_2 + \frac{\Pi v}{L}$$
$$f_{11} = \frac{\Psi}{L} v$$
$$R_1 = 5 \frac{h_1}{M_1} \exp \left( \frac{-6039}{h_3} \right)$$
$$R_{m2} = \frac{1}{10} h_1 m_{O_2}$$
$$R_{c2} = \frac{1}{M_2} \left[ 0.55 \times 10^8 h_2 m_{O_2} P \exp \left( \frac{-22142}{h_3} \right) h_3^{-0.5} \right]$$
$$R_2 = \frac{1}{\frac{1}{\frac{R_{c2}}{R_{m2}} + 1}}$$
$$R_{m3} = \frac{1}{10} h_1 m_{H_2O}$$
$$R_{c3} = \frac{h_2 m_{H_2O}^2 P^2 \exp \left( \frac{5.052 - 12908}{h_3} \right) h_3^{0.5}}{M_2 \left[ m_{H_2O}^2 + \exp \left( -22.216 + \frac{24880}{h_3} \right) \right]^2}$$

TABLE 3. The gasification plant constants.

| Constant | Constant |
|----------|----------|
| $p_{R_1char}$ = 0.766 | $C_s = 7.3920$ cal/g/K |
| $p_{R_1CO}$ = 0.008 | $h_1 = 0.001$ cal/s/K/cm$^3$ |
| $p_{R_1CO_2}$ = 0.058 | $\beta = 7 \times 10^{-6}$ mol/cm$^2$/s |
| $p_{R_1H_2}$ = 0.083 | $\Delta q_2 = -93929$ cal/mol |
| $p_{R_1CH_4}$ = 0.044 | $\Delta q_3 = 31309.7$ cal/mol |
| $p_{R_1Tar}$ = 0.0138 | $\Xi = 0.77$ |
| $p_{R_1H_2O}$ = 0.055 | $\Psi = 7.6$ |
| $p_{R_2H_2O}$ = 0.075 | $M_1 = 54.9916$ g/mol |
| $p_{R_2O_2}$ = 1.02 | $M_2 = 52.9666$ g/mol |
| $P = 4.83$ | $T = 430$ K |
| $L = 100$ cm |
Estimator 2. The plant initial conditions are

\[ m_{O_2} = \frac{h_5}{C_T + h_9} \]

\[ m_{H_2O} = \frac{h_9}{C_T + h_9} \]

\[ C_T = h_4 + h_5 + h_6 + h_7 + h_8 + h_{10} + h_{11} \]

**A. RESULTS**

In this sub-section, the estimator of this research called Estimator 1 is compared against the estimator of [7, 8], called Estimator 2. The plant initial conditions are \( h_0 = [0.48, 0.48, 0.0, 0.0, 0.0, 0.0, 4.2 \times 10^{-4}, 1.6 \times 10^{-6}]^T \). The goal of the estimators is that the variables \( \hat{h} \) of the variables estimator have to reach the plant variables \( h \) and that the perturbations \( \hat{a} \) of the perturbations estimator have to reach the plant perturbations \( a \). The plant inlet is \( v = 0.2 \times 10^{-3} \text{sin}(2T) \) moles/cm²/s from 0 s to 20 s. The perturbation is \( u = 1.5 \times 10^{-3} \text{sin}(2T) \) moles/cm²/s from 0 s to 20 s. The noise is \( \sigma = 1 \times 10^{-5} \) rand from 0 s to 20 s. rand are random values between 0 and 1.

Estimator 2 is expressed as [7, 8] with initial conditions as \( \hat{h}_0 = [0.48, 6 \times 10^0, 480, 1 \times 10^{-9}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-9}, 1 \times 10^{-7}, 4.2 \times 10^0, 1.6 \times 10^{-6}]^T \) and gains as \( L = [1 \times 10^{-1}, 1 \times 10^1, 1 \times 10^{-1}, 1 \times 10^{-1}, 1 \times 10^{-2}, 2 \times 10^{-2}, 1 \times 10^{-2}, 4 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{-3}, 1 \times 10^{-3}]^T \).

Estimator 1 is expressed as (2), (3), (18) with \( \hat{h} = [\hat{h}_1, \hat{h}_2, \hat{h}_3, \hat{h}_4, \hat{h}_5, \hat{h}_6, \hat{h}_7, \hat{h}_8, \hat{h}_9, \hat{h}_{10}, \hat{h}_{11}]^T \). \( f(\hat{h}, v) = \{ \hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4, \hat{f}_5, \hat{f}_6, \hat{f}_7, \hat{f}_8, \hat{f}_9, \hat{f}_{10}, \hat{f}_{11} \}^T \), the terms of \( f(\hat{h}, v) \) are:

\[
\hat{f}_1 = -M_1 \hat{R}_1 \\
\hat{f}_2 = M_2 [p_{R_1, char} \hat{R}_1 - \hat{R}_2 - \hat{R}_3] \\
\hat{f}_3 = \frac{1}{C_s} [h_1 (T - \hat{h}_3) - \Delta q_2 \hat{R}_2 - \Delta q_3 \hat{R}_3] \\
\hat{f}_4 = pr_{R_2, CO} \hat{R}_1 + \hat{R}_3 \\
\hat{f}_5 = pr_{R_2, CO} \hat{R}_1 + \hat{R}_3 \\
\hat{f}_6 = pr_{R_3, H_2O} \hat{R}_1 + \hat{R}_3 \\
\hat{f}_7 = pr_{R_4, CH_4} \hat{R}_1 \\
\hat{f}_8 = pr_{R_5, Ta} \hat{R}_1 \\
\hat{f}_9 = pr_{R_5, H_2O} \hat{R}_1 + pr_{R_2, H_2O} \hat{R}_2 - pr_{R_3, H_2O} \hat{R}_3 + \frac{\Pi}{L} v \\
\hat{f}_{10} = -pr_{R_2, O_2} \hat{R}_2 + \frac{\psi}{L} v \\
\hat{f}_{11} = \frac{5 h_1}{M_1} \exp \left( \frac{-6039}{h_3} \right) \\
\hat{R}_1 = \frac{1}{10} h_1 m_{O_2} \\
\hat{R}_{m_2} = \frac{1}{10} h_1 m_{H_2O} \\
\hat{R}_{c_2} = \frac{1}{M_2} \left[ 9.55 \times 10^8 h_2 m_{O_2} P \exp \left( \frac{-22142}{h_3} \right) \right] \hat{R}_{c_2}^{0.5} \\
\hat{R}_3 = \frac{1}{R_3 + \frac{1}{R_{n_2}}} \]

the initial conditions as \( \hat{h}_0 = [0.48, 1 \times 10^{-5}, 480, 1 \times 10^{-9}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-9}, 1 \times 10^{-7}, 4.2 \times 10^4, 1.6 \times 10^{-6}]^T \).
In this article, an estimator was studied for the variables and perturbations estimation in nonlinear plants. The variables error convergence was analyzed with the Lyapunov approach. Our estimator was compared to a previous estimator in the gas turbine and gasification plants concluding that our estimator reached a better performance than the previous estimator for the variables and perturbations estimation. In addition, in our estimator, the unwanted chattering is not presented. Our estimator can be applied to many types of nonlinear plants such as electric, mechanical, hydraulic or thermal. In the future work, we will seek to use another alternative map that allows us to reduce the unwanted chattering for this type of estimators, we will explore other types of strategies for the perturbations estimation, for the perturbations attenuation, or for trajectories reaching in perturbed plants.

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