The disinformation problem for black holes

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No escape?

1966: “event horizon . . . is the boundary of the region from which particles or photons can escape to infinity . . . a black hole is a region . . . from which particles or photons cannot escape”

1976: “Because part of the information about the state of the system is lost down the hole, the final situation is represented by a density matrix rather than a pure quantum state”

1997: “Whereas Stephen Hawking and Kip Thorne firmly believe that information swallowed by a black hole is forever hidden from the outside universe, and can never be revealed even as the black hole evaporates and completely disappears . . . ”

2004: “Thus the total path integral is unitary and information is not lost in the formation and evaporation of black holes. The way the information gets out seems to be that a true event horizon never forms, just an apparent horizon” [Hawking quotes]

How should black holes be defined? Event and apparent horizons are too indirectly defined.
Assume spherical symmetry for simplicity (everything generalizes).
Area $A$, null coordinates $x^\pm$: $g^{-1}(dx^\pm, dx^\pm) = 0$, future-pointing, unique up to $x^\pm \mapsto \tilde{x}^\pm(x^\pm)$.
Null expansions $\theta_\pm = \partial_\pm A/A$, $\theta_\pm = \partial/\partial x^\pm$.
Normally, outgoing light rays diverge, $\theta_+ > 0$;
ingoing light rays converge, $\theta_- < 0$;
outgoing wavefront expands;
ingoing wavefront contracts:

But if there is a massive source inside the surface,
itself gravitational field has an attractive or converging effect.
Close enough to a massive enough source,
the outgoing light rays may also be converging, $\theta_+ < 0$; a trapped surface:

Everything inside is trapped within a shrinking area.
Nothing can escape, not even light.
In between, there will be a marginal surface (marginally trapped surface), $\theta_+ = 0$,
where the outgoing light rays are instantaneously parallel:

This is a black hole: its surface is located by the marginal surface, where outgoing light rays are instantaneously parallel, $\theta_+ = 0$, ingoing light rays are converging, $\theta_- < 0$, and outgoing light rays are diverging just outside and converging just inside, $\partial_- \theta_+ < 0$. As time develops, the marginal surfaces generate a hypersurface in space-time.
Gravity traps

A surface is

- untrapped
- marginal
- trapped

if \( \theta_+ \theta_- \leq 0 \), \( \theta_+ > 0 \), \( \theta_- < 0 \), respectively.

Trapping horizon: a hypersurface foliated by marginal surfaces,

- future trapped if \( \theta_+ \theta_- < 0 \)
- future marginal if \( \theta_+ \theta_- > 0 \)

A future trapped horizon can be taken as the local definition of a black hole.

e.g. Schwarzschild and Reissner-Nordström black holes:

Signature law:

NEC (null energy condition) \( \Rightarrow \) trapping horizons are spatial or null.

and null if and only if the ingoing energy density vanishes.

Proof. Take the case \( \theta_+ \geq 0 \) (evaluation on the horizon),
write the horizon-generating vector \( \xi = \xi^+ \partial_+ + \xi^- \partial_- \),
then \( 0 \equiv \xi \cdot d\theta_+ = \xi^+ \partial_+ \theta_+ + \xi^- \partial_- \theta_+ \) relates three signs:

- spatial
- null
- temporal

for an \( \xi^+ > 0, \xi^- > 0 \) trapping horizon;

\( \partial_- \theta_+ \) for an \( \partial_- \theta_+ > 0 \) for an \( \partial_- \theta_+ < 0 \) trapping horizon;

\( \partial_+ \theta_+ \) relates three signs:

\( \partial_+ \theta_+ \equiv -8\pi GT_{++} \) (null focusing equation, from Einstein equation),
\( T_{\pm \pm} = T(\partial_\pm , \partial_\pm ) \) (energy tensor), NEC \( \Rightarrow T_{\pm \pm} \geq 0 \).

\( \Rightarrow \) Black hole horizons are one-way traversable;
one can fall into a black hole but not escape:
Area law: NEC ⇒
\[
\begin{align*}
\{ \text{future outer or past inner} \} & \text{ trapping horizons have } \{ \text{non-decreasing area} \}, \\
\{ \text{past outer or future inner} \} & \text{ trapping horizons have } \{ \text{non-increasing area} \}, \\
\end{align*}
\]
instantaneously constant \((A' = 0)\) if and only if the horizon is null.

Proof. \(A' = \xi \cdot dA = A(\xi^+ \theta_+ + \xi^- \theta_-) \approx A\xi^- \theta_-\). Fixing orientation \(\xi^+ > 0\),
\[
\xi^- \begin{cases} 
\leq 0 & \text{for outer horizons} \\
\geq 0 & \text{for inner horizons} \\
0 & \text{for null horizons} 
\end{cases}
\]
as above; \(\theta_- \begin{cases} 
< 0 & \text{for future horizons} \\
> 0 & \text{for past horizons} 
\end{cases}\)

⇒ Black holes grow if they absorb any matter, and otherwise remain the same size.
(Actually general results, not restricted to spherical symmetry).

Evaporating black holes: Hawking radiation is based on pair production of
positive-energy outgoing radiation, \(T_{--} > 0\),
and negative-energy ingoing radiation, \(T_{++} < 0\).
⇒ area and signature properties reverse:

\[
\begin{align*}
\text{ingoing energy density negative} & \Rightarrow \{ \text{outer trapping horizons are } \text{temporal} \}, \\
\{ \text{future outer or past inner} \} & \text{ trapping horizons have } \{ \text{decreasing area} \}, \\
\{ \text{future inner or past outer} \} & \text{ trapping horizons have } \{ \text{increasing area} \}, \\
\end{align*}
\]

The black-hole horizon is shrinking and two-way traversable:
Matter and information clearly will escape.

\(\text{Losing it?}\)
No reason to expect a purely thermal spectrum for an evaporating black hole;
Hawking’s thermal spectrum holds only for a stationary black hole,
ignoring the back-reaction of the radiation on the black hole, which is not evaporating.

Endpoint of evaporation? Usual picture:

PO: \(T_{++} > 0\), outer ⇒ spatial, growing.
ON: \(T_{++} < 0\), outer ⇒ temporal, shrinking.
The horizon shrinks to zero size, intersecting the singularity.
Some of the matter (and information) escapes, some is lost in the singularity.
However, this is a semi-classical approximation, valid only away from the singularity.
In full quantum gravity, is the singularity resolved?
The great escape
Assume that the centre never becomes singular.
Regular centre ⇒ ∃ untrapped neighbourhood
⇒ ∃ inner (future) trapping horizon.
Inner horizon and centre approach, become almost null,
matter becomes almost pure outgoing radiation:
(∃ exact Einstein-Klein-Gordon solution).
During black-hole formation:
PO: $T_{++} > 0$, outer ⇒ spatial, growing.
PI: $T_{++} > 0$, inner ⇒ temporal, shrinking.
During black-hole evaporation:
ON: $T_{++} < 0$, outer ⇒ temporal, shrinking.
IN: $T_{++} < 0$, inner ⇒ spatial, growing.
The outer and inner sections eventually meet,
marking the endpoint of evaporation.
Metric $C^2$ ⇒ inner and outer sections join smoothly.
⇒ ∃ single trapping horizon of $S^2 \times S^1$ topology,
enclosing a compact region of trapped surfaces.
The black hole has evaporated completely,
with all its contents re-emerging, albeit mangled.

No information loss; no event horizon. Obtained by (at least) three independent methods:
• Euclidean quantum gravity [Hawking, unpublished];
• loop quantum gravity [Ashtekar & Bojowald, unpublished];
• this qualitative classical analysis with minimal assumptions about “correct” quantum gravity,
  that a singularity never forms
  and that the ingoing radiation has negative energy density during the entire evaporation phase.

Orthodox paradox
• Logically there are no true paradoxes, just misunderstandings, usually simple but fundamental.
  The disinformation problem for black holes: defining them by event horizons as regions of no escape.
• Understanding black holes locally by trapping horizons, the supposed paradox disappears.