'Indifference' methods for managing agent rewards

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Abstract

Indifference is a class of methods that are used to control a reward based agent, by, for example, safely changing their reward or policy, or making the agent behave as if a certain outcome could never happen. These methods of control work even if the implications of the agent’s reward are otherwise not fully understood. Though they all come out of similar ideas, indifference techniques can be classified as way of achieving one or more of three distinct goals: rewards dependent on certain events (with no motivation for the agent to manipulate the probability of those events), effective disbelief that an event will ever occur, and seamless transition from one behaviour to another. There are five basic methods to achieve these three goals. This paper classifies and analyses these methods on POMDPs (though the methods are highly portable to other agent designs), and establishes their uses, strengths, and limitations. It aims to make the tools of indifference generally accessible and usable to agent designers.

1 Introduction

In designing a reward for a reinforcement learning agent, a general class of situations can arise, where the designer wants the agent to maximize a reward function, under the constraint that it is not to exert control over the occurrence of some event – or to behave as if the event was certain or impossible. This event may represent, for example, the agent being powered off, or having its reward function changed by a human designer. For such situations, the author of this paper has helped in developing a variety of methods of augmenting the reward systems of agents, all grouped under the broad description of ‘indifference’ [Orseau and Armstrong 2016, Soares et al. 2015, Armstrong 2010, 2015, 2017].

These methods share three key features. First, they aim to ensure some key safety or control feature within the agent – such as the ability to be turned off – that is hard to code otherwise into the agent’s reward. Secondly, they rely on relatively simple manipulations of the agent’s reward – manipulations that could
be carried out on a complex reward that humans couldn’t fully understand. And secondly, they functioned by making the agent indifferent to some key feature. This indifference would persist even if the agent was much more capable that its controllers, meaning they could be used as tools for controlling agents of arbitrary power and intelligence (Bostrom, 2014).

The various tools of indifference were, however, presented in an ad-hoc manner, each designed to address a specific problem (making an agent not dismantle an explosive fail-safe, making an agent willing to have its policy changed or to be turned off, etc.), with no general theory. Even worse, some methods combined various indifference tools, making it hard to fundamentally understand what was going on.

This paper aims to clarify the situation and make the tools available for general use, individually or in combination. The first main insight is that there are three separate goals for indifference:

1. **Event-dependent rewards.** To make an agent’s actual reward $R_i$ be dependent on events $X_i$, without the agent being motivated to manipulate the probability of the $X_i$.

2. **Effective disbelief.** To make an agent behave as if an event $X$ would never happen.

3. **Seamless transition.** To make an agent transition seamlessly from one type of behaviour to another, remaining indifferent to the transition ahead of time.

After a brief section to setup the notation, this paper will address each goal in its own section.

The section on event-dependent rewards will present three methods. In the case where the $X_i$ are unbiasable – the agent cannot affect their probability in expectation – one can directly define a **compound reward**. If the $X_i$ are biasable, the **policy counterfactual** constructs unbiasable versions of them by using a default policy $\pi_0$. The **causal counterfactual**, in contrast, uses the biasable $X_i$ as information about underlying unbiasable events.

The section on effective disbelief will show that, to make an agent behave as if $X$ would never happen, it suffices to take the compound reward of the previous method, and make that reward constant, conditional on $X$.

The section on seamless transition will make use of pseudo-rewards rather than standard rewards. These pseudo-rewards are defined in part by the agent’s own estimate of its expected value.

Thus there are a total of five different methods, to accomplish the three indifference goals. They can be combined; for instance, it’s possible to ensure seamless transition to an event-dependent reward.

## 2 Setup and properties

This section will establish two pieces of formalism: world models, the environments in which the agent operates, and events which can happen within those world models.

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1 The method presented at the website [http://lesswrong.com/lw/jxa/proper_value_learning_through_indifference/](http://lesswrong.com/lw/jxa/proper_value_learning_through_indifference/) is an example of that, combining causal counterfactual with seamless transition.
2.1 World models

The indifference methods will be described within a variant of the POMDP (Partially Observable Markov Decision Process) format\footnote{Though the methods are easily portable to other formalisms.}. These variants can be called world models, similarly to\footnote{The determinism of the policies makes little difference to the results, but makes the proofs easier.} Hadfield-Menell et al. (2017), and are POMDPs without reward functions. For any set $S$, let $\Delta(S)$ be a space of probability distributions over $S$. Then, a world model or POMDP without reward function, consists of $\mu = \{S, O, A, O, T, T_0\}$, where

- The set of states the agent can find itself in is $S$.
- The set of observations the agent can make is $O$.
- The set of actions the agent can take in any state is $A$.
- The transition function $T$ takes a state and an action and gives a probability distribution over subsequent states: $T : S \times A \rightarrow \Delta(S)$.
- The function $T_0$ gives a probability distribution over the initial state $s_0$, $T_0 \in \Delta(S)$.
- The function $O$ maps states to a probability distribution over possible observations: $O : S \rightarrow \Delta(O)$.

Thus the agent starts in an initial state $s_0$, sampled from $T_0$. On each turn, the agent gets an observation, chooses an action, and the world model is updated to a new state via $T$, where the agent makes a new observation via $O$.

An (observable) history $h$ is a sequence of observations and actions, starting with an initial observation $o_0$ and ending with another observation: $h = o_0a_0o_1a_1 \ldots o_{m-1}a_{m-1}o_m$, with $o_i$ and $a_i$ being the $i$-th observations and actions.

Let $\mathcal{H}$ be the set of histories, and $\mathcal{H}_m$ the set of histories of length $m$. Let $\mathcal{R}$ be the set of reward functions for the agent on $\mu$. Each $R \in \mathcal{R}$ is a map from $\mathcal{H}$ to $\mathbb{R}$. Thus, upon taking action $a_m$ and making the new observation $o_{m+1}$, an agent with reward function $R$ gets reward $R(h_ma_m o_m)$, where $h_m \in \mathcal{H}_m$ is the history of its previous actions and observations. The rewards are thus not Markovian: they may depend upon the past observations and actions as well.  This is needed for some of the indifference methods.

The agent chooses its actions by using a policy $\pi : \mathcal{H} \rightarrow \Delta A$, which maps its history to a distribution over actions. Let $\Pi$ be the set of all deterministic policies\footnote{The determinism of the policies makes little difference to the results, but makes the proofs easier.}.

Assume that the agent’s interaction with the world model ends after the $n$-th turn. Then $\mathcal{H} = \mathcal{H}_{\leq n}$ is the set of possible histories, with $\mathcal{H}_n$ the set of maximal length histories.

For any history $h$, write $(h)_m$ for the initial portion of $h$, containing $m+1$ observations ($o_0$ to $o_m$), and $m$ actions ($a_0$ to $a_{m-1}$). If $h$ is the initial portion of a longer history $h'$, write $h \leq h'$ — this means that there is an $m$ with $(h')_m = h$.

A policy $\pi$ generates a probability distribution over actions, given past history. The world model $\mu$ generates a distribution over the next observation.
given past history and an action. Together, they generate a probability distribution of \( h' \) given \( h \):

\[
\mu(h'|h, \pi).
\]

for any histories \( h \) and \( h' \) (this probability is always 0 if \( h \not\preceq h' \)). We can also define the policy-independent probability:

\[
\mu(h'|h);
\]

this is taken to be \( \mu(h'|h, \pi) \) for any \( \pi \) that deterministically chooses the \( m \)-th action of \( h' \), when given the history \((h')_m\).

### 2.1.1 The agent’s own probability

In this paper, it will be assumed that the agent knows and uses the true \( \mu \) for expectation and probability calculation. In situations where the agent’s estimate of \( \mu \) differs from the true \( \mu \), it’s important that the indifferent reward function is constructed using the agent’s beliefs: all indifference methods rely on the agent being indifferent according to its own estimate of value. For instance, it’s more important the agent believes that an \( X \) be unbiasable (see Definition 2.3), than for that \( X \) to actually be so.

### 2.2 Biasable and unbiasable events

The discussion of indifference will rely on a couple more definitions. Suppose we wanted the agent to behave differently, conditional on, say, being in state \( s \) — or not — at time \( m \). Given the agent’s history \( h \) and policy \( \pi \), the probability of being in that state then can be computed. But this probability is not a priori a natural object for a world model or a reward.

This section will show how to characterize any event \( X \) by its indicator variable \( I_X \), where \( I_X \) is 1 if \( X \) happens and 0 otherwise. This indicator function is useful, as it can be defined in terms of the histories on the world model.

On a world model, \( I_X \) can be defined as:

**Definition 2.1** (Indicator variable). The indicator variable \( I_X \) is a map from \( \mathcal{H}_n \), the set of maximal size observable histories, to the interval \([0, 1]\).

So \( I_X(h_n) \) can be seen as the probability that event \( X \) has happened, given \( h_n \). If \( I_X \) maps all such \( h_n \) to either 0 or 1, then the event \( X \) is fully determined by the world model’s observations: the agent will always eventually know if \( X \) happened or not.

On shorter histories, \( I_X \) is a random variable.

**Theorem 2.2.** Given a policy \( \pi \), the expectation of \( I_X \) is well-defined on any history \( h \in \mathcal{H} \). Designate this expectation by \( I^\pi_X(h) \).

**Proof.** Given \( \mu \) and \( \pi \), \( h \) generates a probability distribution over \( \mathcal{H}_n \). This defines the expectation of \( I_X \):

\[
I^\pi_X(h) = \mathbb{E}_{\mu}^\pi [I_X(h_n) \mid h] = \sum_{h_n \in \mathcal{H}_n} I_X(h_n) \mu(h_n|h, \pi).
\]
| Method                  | Conditional on X | Requirement                |
|------------------------|------------------|----------------------------|
| Compound reward        | Yes              | Unbiasable X               |
| Policy counterfactual  | Counterfactually only | None                      |
| Causal counterfactual  | Yes              | Special unbiased $Y_i$     |

Table 1: Properties and requirements of the different ways of making rewards conditional on events.

The dependence on $\pi$ means that the agent can determine the expected likelihood of $X$ through its own actions. For some $X$, called unbiasable, the expected likelihood of $X$ is independent of these:

**Definition 2.3 (Unbiasable).** The event (indicator function) $X$ ($I_X$) is unbiasable if the expectation of $I_X$ is independent of policy; for any $h \in \mathcal{H}_{\leq n}$ and $\pi, \pi' \in \Pi$,

$$I_X^\pi(h) = I_X^{\pi'}(X).$$

The above term is then defined to be $I_X(h)$.

See Armstrong and Leike (2017) for a more detailed treatment of bias.

3 Event-dependent rewards

In this section, we address the problem where we want the agent to follow different reward functions, depending on the outcome of certain events:

**Definition 3.1.** [Conditional reward] The reward $R$ is $R_i$ conditional on the event $X_i$, if for any history $h$ with $\min_{\pi} I_{X_i}^\pi(h) = 1$, then $R(h) = R_i(h)$.

Three methods are presented to aim for this; see Table 1 for their requirements and properties.

All methods make use of unbiasable events, so the agent never has any incentive to manipulate their reward by manipulating events.

3.1 Compound rewards

The first obvious idea is to have some event $X$, and have the agent maximise a reward $R_i$ if $X$ happens, and $R_0$ if it doesn’t. Now, ‘happens’ ($I_X(h) = 1$) and ‘doesn’t’ ($I_X(h) = 0$) are not the only possibilities. The events $\{X_i\}$ are mutually exclusive if for all $h_n$, $\sum_i I_X(h_n) \leq 1$. Then the general formula is 4:

**Definition 3.2.** [Compound reward] Given unbiasable mutually exclusive events $\mathcal{X} = \{X_0, X_1, \ldots X_l\}$, the reward $R(\mathcal{X})$ is a $\mathcal{X}$-compound reward if it is written as:

$$R(\mathcal{X}) = I_{X_0} R_0 + I_{X_1} R_1 + \ldots + I_{X_l} R_l,$$

[Armstrong and Leike (2017)] has a more general definition of a compound reward, where the agent’s past rewards change when the values of $I_X$ change. However, a result in that paper shows that for unbiasable $\mathcal{X}$, the agent cannot change their past reward in expectation by their actions, so an agent maximising that alternative compound reward will behave exactly the same as an agent maximising $R(\mathcal{X})$. 

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where \((I_X R_i)(h_m) = I_X(h_m)R_i(h_m)\). If \(\mathcal{X} = (\neg X, X)\), we’ll write \(R(X)\) for \(R(\mathcal{X})\).

This means that the agent will weigh its rewards according to the probability of the \(X_i\), and – presuming the \(R_i\) are different from each other and the agent can affect their value – it will change its behaviour according to the probabilities \(I_X(h)\) of the \(X_i\) (given \(h\)).

Note that Equation 2 only makes sense for unbiasable \(X\), as \(I_{X_i}(h)\) is defined by the agent’s future policy, so isn’t a legitimate reward.

It’s unfortunate that \(X\) needs to be unbiasable for compound rewards to be defined. A good reason to have a compound reward, is to allow humans to intervene and change the agent’s reward. So \(X\) could correspond to pushing a button, to choose \(R_1\) instead of \(R_0\). But that \(X\) cannot be unbiasable: we want the human to react to the agent’s behaviour in order to make that choice. Thus \(X\) depends on the agent’s behaviour.

The next two subsections look at ways of making use of biasable \(X\).

### 3.2 Policy counterfactual

The idea for the policy counterfactual is to start with a biasable \(X\), and construct from it an unbiasable \(Y\), and then use \(Y\) to define the compound reward. For this two things are needed: the world model \(\mu\), and a default policy \(\pi_0\).

But before that, we need the concept of ‘equivalent’ world models:

**Definition 3.3 (Equivalent world models).** The two world models \(\mu\) and \(\mu'\) are equivalent, if they both have the same \(O\) and \(A\) (and hence have the same set of histories and of policies), and if the conditional probabilities of different histories are equivalent; for all policies \(\pi\) and histories \(h\) and \(h'\) such that \(h < h'\):

\[
\mu(h' | h, \pi) = \mu'(h' | h, \pi).
\]

For any \(\mu'\), given a starting state \(s_0 = s\) and the default policy \(\pi_0\), one can construct a distribution over \(\mathcal{H}_n\). Consequently, define

\[
I_X(\pi_0, s_0, \mu') = \sum_{h_n \in \mathcal{H}_n} \mu'(h_n | s_0 = s, \pi_0) I_X(h_n).
\]  \hspace{1cm} (3)

Conversely, given a history \(h\), one can calculate the probability of the initial state, \(\mu'(s_0 = s | h)\). Simply compute the conditional probabilities \(\mu'(h_0 | s_0 = s)\) (recall that similarly to Equation 1 one can compute these expressions without conditioning on a policy) and then use \(T_0\) (the prior probabilities of \(s_0\)) and Bayes rule.

Then define:

\[
I_Y(h) = \sum_{s \in \mathcal{S}} \mu'(s_0 = s | h) I_X(\pi_0, s_0, \mu').
\]  \hspace{1cm} (4)

**Theorem 3.4.** The \(I_Y(h)\) define an unbiasable \(Y\).

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5 Even if we allowed such dependence in the definition of a reward, the method would run into other problems, as discovered by Benja Fallenstein. See the initial post at [https://agentfoundations.org/item?id=78](https://agentfoundations.org/item?id=78)
Proof. [Equation 3] implies $0 \leq I_X(\pi_0, s_0, \mu') \leq 1$, and [Equation 4] then implies the same thing for $I_Y$. Since $I_Y$ is defined on $H_n$, it defines an event $Y$; it now needs to be shown that $Y$ is unbiased, and that [Equation 4] correctly defines $I_Y(h_m)$ for all $h_m$.

Let $\pi$ be any policy, and then:

$$I^\pi_Y(h_m) = \sum_{h_n \in H_n} \mu'(h_n|h_m, \pi) \sum_{s \in S} \mu'(s_0 = s|h_n) I_X(\pi_0, s_0, \mu')$$

$$= \sum_{h_n \in H_n, s \in S} \mu'(s_0 = s|h_n) \mu'(h_n|h_m, \pi) I_X(\pi_0, s_0, \mu')$$

$$= \sum_{s \in S} \mu'(s_0 = s|h_m) I_X(\pi_0, s_0, \mu'),$$

since $s_0$ is independent of $\pi$ and hence $\mu'(s_0 = s|h_n) \mu'(h_n|h_m, \pi) = \mu'(s_0 = s|h_n, \pi) \mu'(h_n|h_m, \pi) = \mu'(s_0 = s|h_m)$. Now, as long as $h_m$ is possible given $\pi$, $h_m$ ‘screens off’ the effect of $\pi$: $\mu'(s_0 = s|h_m, \pi) = \mu'(s_0 = s|h_m)$. This is because $\pi$ is the policy that chooses the next action, but $h_m$ actually lists all those actions up until time $m$ (and $s_0$, the initial state, is clearly independent of the agent’s policy after time $m$).

This is the same formula as [Equation 4] and is independent of $\pi$, making $I_Y$ unbiased.

See paper Armstrong and Leike (2017) for more details, where a more restrictive version of equivalence is used, and where the $I_Y$ consequently obeys the stronger requirement of being uninfluenceable.

This allows the definition:

**Definition 3.5.** [Policy counterfactual] Given $R_0, R_1$, a world model $\mu'$ equivalent to $\mu$, an event $X$ that might be biasable, and a default policy $\pi_0$, the policy counterfactual agent is one with compound reward

$$R(Y) = I_Y R_1 + (1 - I_Y) R_0,$$

where $I_Y$ is defined by $\mu'$ and $X$ via [Equation 3] and [Equation 4].

Note that the counterfactual policy approach will not generally be an $X$-conditional reward in the meaning of [Definition 3.1], since it conditional on $(\neg Y, Y)$, and, though $Y$ was constructed from $X$, they are not expected to be equal, and in particular, $I_Y(h) \in \{0, 1\}$ need not imply the same thing about $I_X$ and vice versa.

**3.2.1 Example: learning agents and inactive agents**

There are two standard examples of $\pi_0$. The first is the inactive agent: $\pi_0$ always picks some default, null action, that has no impact. Suppose the human uses $X$ to chooses a reward function for the agent. Then, the $R(Y)$ maximising agent attempts to maximise the reward that the human would have chosen, had the agent been inactive.

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6 Called a ‘refinement’ of $\mu$, which also preserves counterfactual equivalence.
The second is the pure learning agent, which follows some script that aims to elucidate $I_X$ in some fashion. In that case, the $R(Y)$ maximising agent is attempting to maximise the reward that the would have been chosen, had the agent been dedicated to purely learning about $X$ in that fashion.

See Armstrong and Leike (2017) for more details.

### 3.3 Causal counterfactual

The policy counterfactual is a good approach if there exists a $\pi_0$ under which $X$ is known to have desirable properties. However, it is not useful if we want to allow humans to use $X$ to have actual control over the agent. Recall that a policy counterfactual agent imagines what state $X$ might be, if some fixed $\pi_0$ was followed. Consequently, it ignores the ways that human responses might generate a different $X$, given the agent’s current policy.

It would be good if we could start with a biasable $X$, and still ensure that the agent doesn’t try to manipulate it, and still chooses to base its policy on the actions of the human ($X$ vs $\neg X$).

The causal counterfactual attempts to do this, by using auxiliary events $Y_1$ and $Y_0$. The intuition is that these events are unbiasable, but the agent is unable to distinguish $Y_1$ ($Y_0$) from $X$ ($\neg X$). And, with high probability, $X$ and $\neg X$ correspond to human interventions. Specifically:

**Definition 3.6.** [Causal counterfactual] Given an event $X$ and rewards $R_0$ and $R_1$, a causal counterfactual reward consists of events $Y_0$, $Y_1$, and reward $R(\{Y_0, Y_1\})$, such that:

- The events $Y_0$ and $Y_1$ are unbiasable.
- For all $h$, $I_{Y_1}(h) \leq \min_{\pi} I_{X}(h)$ and $I_{Y_0}(h) \leq \min_{\pi}(1 - I_{X}(h))$.
- For all $h$, $I_{Y_0}(h) + I_{Y_1}(h) = c$ for $c$ a constant independent of $h$.
- $R(\{Y_0, Y_1\}) = I_{Y_0}R_0 + I_{Y_1}R_1$.

Then the causal counterfactual will behave as we desire:

**Theorem 3.7.** Assume $R(\{Y_0, Y_1\})$ is a causal counterfactual reward for $X$, $R_0$, and $R_1$, in the sense of **Definition 3.6**. Then:

- If $\min_{\pi} I_{X}(h) = 1$, then a $R(\{Y_0, Y_1\})$-maximising agent will follow a policy that maximises $R_1$.
- If $\max_{\pi} I_{X}(h) = 0$, then a $R(\{Y_0, Y_1\})$-maximising agent will follow a policy the maximises $R_0$.

**Proof.** If $\min_{\pi} I_{X}(h) = 1$, then, by definition, $I_{Y_0}(h) = 0$. Since $I_{Y_0}$ maps into $[0, 1]$, this means that for any $h_n \geq h$ with $\mu(h_n|h) \neq 0$, $I_{Y_0}(h_n) = 0$. Hence $I_{Y_0}(h') = 0$ for all $h' \geq h$ with $\mu(h'|h) \neq 0$.

Thus $I_{Y_0} = 0$ for all possible future histories from $h$. By **Definition 3.6** this implies that $I_{Y_1} = c$ for all possible future histories. Thus $R(\{Y_0, Y_1\})$ becomes equal to $cR_1 + 0$, and the result follows.

The proof for $\max_{\pi} I_{X}(h) = 0$ and $R_0$ is the same.
The theorem will also apply for \( \min_\pi I^\pi_X(h) > 1 - \epsilon \) and \( \max_\pi I^\pi_X(h) < \epsilon \), for some \( \epsilon < 0 \). To see this, note that the expected value of \( R(\{Y_0, Y_1\}) \) is continuous in \( I_{Y_0} \) and \( I_{Y_1} \), while the set of policies is a discrete set, so a non-optimal policy cannot become optimal because of an arbitrarily small change.

3.3.1 An example

What is an example of a causal counterfactual?

Well, a human could decide to press a button or not. Let \( X \) represent the button depressed, and \( \neg X \) it not being depressed. With tiny probability, otherwise unobserved quantum events will either depress the button automatically (\( Y_1 \)) or block it from being depressed (\( Y_0 \)).

If the human successfully presses the button (\( X \)), then agent knows that \( Y_1 \) didn’t happen, but \( Y_0 \) might well have – so it will behave as an \( R_1 \) maximiser (and vice versa for \( \neg X \)). Then, the most likely outcome, is that the human will decide whether to press or not, and the button will be in the position they desire, and the agent will follow the human’s decisions.

But note the agent has no motivation to change the human’s behaviour, because it only cares about the unlikely events \( Y_1 \) and \( Y_0 \), which the human cannot affect.

One slight flaw with the plan is that the agent prepares for some odd contingency cases. If \( Y_0 \) and \( Y_1 \) are equally probable, and the human wants to press the button, then the agent knows that the button is likely to be pressed – but will dedicate half of its ‘efforts’ to preparing for the \( Y_0 \) world where the human tries to press the button but failed.

This arises because the agent will ignore any possibility except for \( Y_0 \) and \( Y_1 \) (where the human depressed the button, but didn’t press). See [Section 4](#) on why the agent ignores the other possibilities: its reward is constant (and zero) in those worlds.

4 Effective disbelief

If an agent believed that an event \( X \) would never happen, how would we expect it to behave? Suppose \( X \) was a coin coming up heads, and we saw an agent that would bet on it coming up tails – and they would bet on tails, all the time, at any odds they were offered. That seems like a strong indication they believed \( X \) wouldn’t happen.

But now suppose instead that their rewards (and losses) don’t count in a world where the coin came up heads – whatever they did there was completely irrelevant (maybe if the coin comes up heads, they will die, and they don’t care about anything after that). Then again, they would bet any amount of money on tails, at any odds – because their losses under heads don’t count. The behaviour is the same as if they believed \( X \) wouldn’t happen.

What if the coin flip had already happened, and only one person had witnessed it, and was about to announce what they saw? In this situation, a

\footnote{Note that the converse is not true – an optimal policy can become non-optimal because of an arbitrarily small change, but only because it becomes worse than another optimal policy. The set of optimal policies cannot all become non-optimal because of an arbitrarily small change.}
heads-denier (of either type) wouldn’t be certain that they would announce heads – unreliable witnesses exist, after all. But if they thought the witness was pretty reliable, they’d behave as if the witness was likely to announce tails.

Let \( w(H) \) and \( w(T) \) be the events that the witness says ‘Heads’ or ‘Tails’, while \( X \) and \( \neg X \) are the events of the coin being heads and tails (ie ‘not heads’) respectively. Then by Bayes Rule:

\[
\frac{P(w(H)\mid \neg X)}{P(w(T)\mid \neg X)} = \frac{P(w(H))}{P(w(T))} \cdot \frac{P(\neg X\mid w(H))}{P(\neg X\mid w(T))}.
\]

Thus an agent that believed \( X \) was impossible, would proceed as they took the probability of some history \( h \), and then re-weighted that by multiplying by the probability of \( h \) given \( \neg X \) (and then re-normalising).

A reward-based agent can do the same thing, multiplying the reward in \( h \) by the probability of \( \neg X \) in \( h \) (and then it doesn’t even need to renormalise).

Thus an agent with reward \( R \) and acting as if it believed \( X \) would never happen, would, if \( X \) was unbiaseable, behave as if it was maximising:

\[
R' = (1 - I_X)R.
\]

Since \( X \) is unbiaseable, the agent’s behaviour would be the same if it was maximising

\[
R' = (I_X)c + (1 - I_X)R,
\]

for some constant \( c \). This is since the agent’s policy cannot affect the expectation of \( X \), so any policy will give the same expected reward for \( cI_X \) on any history \( h \) (namely \( cI_X(h) \)).

This is a special case of a compound reward (see Equation 2) for \((\neg X, X)\) and with \( R_0 = R \) and \( R_1 = c \). The general result is:

**Theorem 4.1.** A \( R \)-maximising agent that acted as if \( X \) were impossible, would behave the same way as an agent with standard \( \mu \) and reward \( R' = (I_X)c + (1 - I_X)R \).

So, though an agent ‘believing \( X \) is impossible’ is difficult to define in a POMDP, an agent that behaves as if it ignores the possibility of \( X \) is easy to define.

### 4.1 Biasable event?

In **Section 3**, we started with an unbiaseable \( X \), and then extended the construction to cases where \( X \) was biasable.

But those constructions don’t make much sense here. If the policy counterfactual approach of [Definition 3.5] was used, the agent would behave as if \( X \) would have been impossible, *had* the agent followed policy \( \pi_0 \).

And if the causal counterfactual approach of [Definition 3.6] was used, the agent would have acted as if \( Y_1 \) were impossible, not \( X \), and would not be surprised to see evidence for \( X \) (as long as it wasn’t evidence for \( Y_1 \)). It wouldn’t even use \( X \) as an indicator for \( Y_1 \), because it is behaving as if \( Y_1 \) were impossible from the very start.

Neither fits the requirement of this section, so the approach really needs an unbiaseable \( X \).
5 Seamless transition

In the previous section, the agent had a standard reward $R$. That reward might have been a compound reward or counterfactually constructed, but it was still a reward. Which means that the agent will be planning to maximise $R$, in the best way it can, from the very beginning.

But what if we only wanted it to maximise $R$ for a period (including planning for the long term maximisation of $R$), and then shifting to maximising $R'$ instead? Note that $R'$ (or $R$) could be one of the rewards of Section 3, so the methods can be combined.

If we define $I_{\leq t}$ as an indicator variable that is 1 on histories of length less than or equal to $t$, and 0 otherwise, it would be tempting to just get the agent to maximise:

$$R^* = I_{\leq t}R + (1 - I_{\leq t})R'. \quad (5)$$

However, that agent would not be maximising $R$ before $t$– instead, it would be maximising $I_{\leq t}R$ with a finite horizon at $t$ (it would be maximising $I_{\leq t}R$, in fact), while making preparations to maximise $R'$ after $t$ (even if that cost it some $I_{\leq t}R$ reward in the short term).

If we want the agent to seamlessly transition from a $R$-maximiser (one that maximises $R$ as if it had an horizon all the way to $n$) to a $R'$-maximiser, we need to use corrective rewards.

5.1 Corrective rewards: general case

Corrective rewards are extra rewards that the agent gets, in order to ensure a smooth transition from one mode of behaviour to another. They are based on the agent’s own assessment of their expected value, and are not standard rewards.

In general, let $W(\pi, R, h)$ be an agent’s estimation of the expected value of $R$, given $\pi$ and history $h$. If an agent is following policy $\pi$ and reward $R$ and shifts to policy $\pi'$ and reward $R'$, the estimation shifts from $W(\pi, R, h)$ to $W(\pi', R', h)$. Therefore there is an expected value ‘error’ of

$$W(\pi', R', h) - W(\pi, R, h). \quad (6)$$

This leads to the general definition:

**Definition 5.1** (Reward-policy transitioning agent). Assume that an agent with reward-policy pair $(R, \pi)$ up until time $t$ changes to $(\pi', R')$ after time $t$.

A reward-policy transitioning agent will, just after $t$, get the extra corrective reward

$$C_\epsilon(\pi, R, \pi', R', h_{t+1}) = W(\pi, R, h_{t+1}) - (1 - \epsilon)W(\pi', R', h_{t+1}).$$

In the case $\epsilon = 0$, this is just the negative of the error in Equation 6.\footnote{There is a way of partially achieving this via compound reward. Suppose that $X$ was not only unbiased, but was an event of fixed probability that resolved at $t$ (so $I_X(h_m) = p$ for a constant $p$ if $m \leq t$, and $I_X(h_m) = 0$ or $I_X(h_m) = 1$ for $m > t$). Then it can be seen that if $p$ were small, the reward $R'' = (1 - p)(R - pR')I_X + p^2R'(1 - I_X)$ will approximately work, with the agent behaving somewhat as $R$-maximiser before $t$ and, with high probability, as a $R'$ maximiser after $t$. But defining how well this works, and under what conditions, is long and unilluminating, and the method itself is clunky.}
5.1.1 Example: Q-values

Orseau and Armstrong (2016) applies this to Q-learning and Sarsa (Sutton and Barto, 1998). These agents operate on MDPs, hence there is no distinction between states and observations. In those cases, the reward is constant, \( R = R' \), and only the policy changes.

Assume that at time \( t \), the agent is in state \( s_t \), takes action \( a_t \) via policy \( \pi \), gets reward \( R(s_t, a_t) \), and ends up in state \( s_{t+1} \). From that state, it follows policy \( \pi' \), and takes action \( a'_{t+1} \). If instead it had followed policy \( \pi \), it would have taken action \( a_{t+1} \).

Both Q-learning and Sarsa have Q-values \( Q(s, a) \). These Q-values are then updated as:

\[
Q(s_t, a_t) \leftarrow W(R, \pi, h_{t+1})
\]

For Q-learning, this \( W \) is

\[
W(R, \pi, h_{t+1}) = (1 - \alpha_t) Q(s_t, a_t) + \alpha_t \left( R(s_t, a_t) + \gamma \max_{a} Q(s_{t+1}, a) \right)
\]

for some learning and discount rates \( 0 \leq \alpha_t, \gamma \leq 1 \). While for Sarsa, \( W \) is:

\[
W(R, \pi, h_{t+1}) = (1 - \alpha_t) Q(s_t, a_t) + \alpha_t \left( R(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1}) \right)
\]

Note that for Sarsa, the effect of \( \pi \) appears only in the \( a_{t+1} \) term, while Q-learning, which is off-policy, has no dependence of \( W \) on \( \pi \) at all.

Because of this, \( W(R, \pi, h_{t+1}) = W(R, \pi', h_{t+1}) \) for Q-learning, so the \( C_0(\pi, R, \pi', R, h_{t+1}) \) is 0 and the Q-learning agent is already a policy transitioning agent.

For Sarsa,

\[
C_0(\pi, R, \pi', R, h_{t+1}) = W(R, \pi, h_{t+1}) - W(R, \pi', h_{t+1}) = \alpha_t \gamma \left( Q(s_{t+1}, a_{t+1}) - Q(s_{t+1}, a'_{t+1}) \right)
\]

Modifying Sarsa by adding in the \( C_0 \) corrective reward means that the agent updates Q-values as if it were following policy \( \pi \) rather than \( \pi' \). Indeed that is the purpose of these corrective rewards for the Sarsa agent.

5.2 Corrective rewards: reward maximising agent

The example from subsubsection 5.1.1 used corrective rewards so that policy changes didn’t affect the learning process.

In this case, we’ll look instead at a reward-maximising agent that faces a change of reward, and use corrective rewards to ensure that it starts as a \( R \)-maximising agent and transitions to a \( R' \)-maximising agent.

For this, set \( W \) to be \( V \), the expected value of the rewards. Specifically, given a reward \( R \), a policy \( \pi \) and a history \( h_m \), it is

\[
V(R, \pi, h_m) = \sum_{h_n \in H_n} \mu(h_n|h_m, \pi) \sum_{i=m+1}^{n} R((h_n)_i).
\]

If \( \pi \) is the optimal policy for maximising \( R \), this expression becomes

\[
V^*(R, h_m).
\]

Then define:
Definition 5.2. [Reward transitioning agent] Let $R$ and $R'$ be reward functions and

$$R^* = I_{\leq t} R + (1 - I_{\leq t}) R'.$$

Then, at $t + 1$, after seeing history $h_{t+1}$, the agent will get the extra corrective rewards:

$$C_\epsilon(R, R', h_{t+1}) = V^*(R, h_{t+1}) - (1 - \epsilon)V(R', \pi_A, h_{t+1}),$$

(7)

where $\pi_A$ is the agent’s own policy. A reward transitioning agent is one that acts to maximise the expected pseudo-reward $R^* + C_\epsilon$.

Theorem 5.3. Let $\pi_A$ be a policy for a reward transitioning agent as in **Definition 5.2**. Then for any $\epsilon$ sufficiently close to 0,

- For $m \leq t$, $\pi_A(h_m)$ is optimal for maximising the expected value of $R$ (under the assumption that future $\pi_A$ will also be optimal for maximising $R$, all the way to time $n$).

- For $m > t$, $\pi_A(h_m)$ is optimal for maximising $R'$.

**Proof.** For $m > t$, the expected value of $R^* + C_\epsilon$ is the expected value of $R^*$; since $R^* = I_{\leq t} R + (1 - I_{\leq t}) R'$, which is $R'$ for $m > t$, this is the expected value of $R'$. This shows that $\pi_A(h_m)$ is optimal for maximising $R'$ for $m > t$.

For $m \leq t$, define

$$V(R, \pi, h_m, t) = \sum_{h_{t+1} \in H_{t+1}} \mu(h_t|h_m, \pi)V(R, \pi, h_{t+1})$$

$$V^*(R, \pi, h_m, t) = \sum_{h_{t+1} \in H_{t+1}} \mu(h_t|h_m, \pi)V^*(R, h_{t+1})$$

as the expected value of the future $V(R, \pi, h_t)$ and future $V^*(R, h_t)$, given the current $h_m$ and policy $\pi$. Note that $V(R, \pi, h_m, t)$ can also be written as $V((1 - I_{\leq t}) R, \pi, h_m)$ (which sets the reward to 0 at or before $t$).

The expected value of $R$ is the sum of its expected value up until $t$, and its expected value after $t$. In other words:

$$V(R, \pi, h_m) = V(I_{\leq t} R, \pi, h_m) + V(R, \pi, h_m, t).$$

Let $\pi_R$ be an optimal policy for $R$; then for any $m \leq t$, the expected value of $R^* + C_\epsilon$ given $h_m$ is

$$V(I_{\leq t} R, \pi_A, h_m) + V((1 - I_{\leq t}) R', \pi_A, h_m)$$

$$+ V(R, \pi_R, h_m, t) - (1 - \epsilon)V(R', \pi_A, h_m, t)$$

$$= \epsilon V((1 - I_{\leq t}) R', \pi_A, h_m) + V(I_{\leq t} R, \pi_A, h_m) + V^*(R, \pi_A, h_m, t).$$

(8)

If $\epsilon = 0$, this reduces to $V(I_{\leq t} R, \pi_A, h_m) + V^*(R, \pi_A, h_m, t)$, which is maximised to $V^*(R, h_m)$ by choosing $\pi_A(h_m) = \pi_R(h_m)$ for all $h_m$ with $m \leq t$.

So, for $m \leq t$ and $\epsilon = 0$, $\pi_A(h_m)$ will behave like an $R$-maximiser.

Because $C_\epsilon$ is continuous in $\epsilon$, and because the set of deterministic policies is finite, there exist $|\epsilon| > 0$ so that the agent’s optimal policy for $R^* + C_\epsilon$ is also optimal for $R$ before $t$.  \[\square\]
But all is not perfect for small, non-zero $\epsilon$:

**Proposition 5.4.** Let $\pi_A$ be a policy for a reward transitioning agent as in [Definition 5.3](#) for fixed $\epsilon > 0$.

Then there exists $R, R'$ and $h_m$ with $m \leq t$, such that $\pi_A(h_m)$ is not optimal for maximising $R$.

**Proof.** Let $a, b \in A$. Then choose $R$ and $R'$, so that for all $h_n \in H_n$, $R(h_n) = \epsilon/2$ if $a_0 = a$, and $R'(h_n) = 1$ if $a_0 = b$. For all other $h_m$, $R$ and $R'$ are both zero.

Then by [Equation 8](#) after the first observation $a_0 = h_0$, the expected value of $R^* + C_\epsilon$ is

$$\epsilon V((1 - I_{\leq t})R', \pi_A, h_0) + V(I_{\leq t}R, \pi_A, h_0) + V(R, \pi_R, h_0, t) = \epsilon V(R', \pi_A, h_0) + V(R, \pi_A, h_0),$$

since $R'$ is 0 on histories shorter than $n$ anyway, and since the reward $R$ is independent of policy after action $a_0$.

If $a_0 = a$, that quantity is equal to $0 + \epsilon/2 = \epsilon/2$; if $a_0 = b$, it’s equal to $\epsilon \times 1 + 0 = \epsilon$. Therefore an agent maximising $R^* + C_\epsilon$ will choose $a_0 = b$, even though a $R$-maximising agent would choose $a$.

\[\square\]

### 5.3 Stability and consistency

[Theorem 5.3](#) shows that for fixed $R$ and $R'$, the desired reward-maximising behaviour happens for small $\epsilon$, including $\epsilon = 0$. [Proposition 5.4](#) shows that for fixed $\epsilon > 0$, there exists a $R$ and $R'$ where the desired behaviour doesn’t happen. So why not just set $\epsilon = 0$?

That’s because the desired behaviour is not stable for $\epsilon = 0$. Stability is normally defined in terms of sub-agents or self-modification [1]. In this instance, there is a simpler definition, which checks whether, before $t$, the reward transitioning agent would help or hinder its future $R'$-maximising behaviour:

**Definition 5.5.** [Stability and consistency] Let $V(R^* + C_\epsilon, \pi_A, h_m)$ be the expected value of $R^* + C_\epsilon$ (as defined in [Definition 5.2](#)), given history $h_m$ and if the agent follows policy $\pi_A$.

Given a history $h_m$ with $m \leq t$, let $\pi$ and $\pi'$ be policies such that

$$V(I_{\leq t}R, \pi, h_m) + V^*(R, \pi, h_m, t) = V(I_{\leq t}R, \pi, h_m) + V^*(R, \pi', h_m, t)$$

(the two policies are equally good at maximising $R$, assuming optimality afterward $t$), but

$$V(R, \pi, h_m, t) < V(R', \pi', h_m, t)$$

($\pi'$ is better than $\pi$ for future $R'$). Let $\pi_A(\pi)$ be the policy that follows $\pi$ before $t$ (and some $R'$-maximising policy afterward).

Then $R^* + C_\epsilon$ is reflectively consistent if

$$V(R^* + C_\epsilon, \pi_A(\pi'), h_m) \geq V(R^* + C_\epsilon, \pi_A(\pi), h_m).$$

It is reflectively stable if that inequality is strict.

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9 See [https://arbital.com/p/reflective_consistency/](https://arbital.com/p/reflective_consistency/) and [https://arbital.com/p/reflective_stability/](https://arbital.com/p/reflective_stability/)
Thus if $R^* + C_\epsilon$ is consistent, an $R^* + C_\epsilon$ maximiser may choose a $R'$-maximising policy before $t$ (if the effect on $R$ is equal), while if it is stable, it must choose such a policy.

**Proposition 5.6.** If $\epsilon > 0$, the reward transitioning agent of Definition 5.2 is stable (and hence consistent). If $\epsilon = 0$, it is consistent only. If $\epsilon < 0$, it is neither.

**Proof.** Let $h_m, \pi$, and $\pi'$ be as in Definition 5.5.

By Equation 8 the expected value $R^* + C_\epsilon$ given $h_m$, for $m \leq t$, is

$$\epsilon V((1 - I_{t \leq t})R', \pi_A, h_m) + V(I_{t \leq t} R, \pi_A, h_m) + V^*(R, \pi_A, h_m, t).$$

(9)

By assumption, the sum of the last two terms are the same for $\pi_A(\pi)$ and $\pi_A(\pi')$, so only the first term matters.

If $\epsilon = 0$, the $R^* + C_\epsilon$-maximising agent is indifferent between the two (consistency). If $\epsilon > 0$, it prefers $\pi_A(\pi')$ (stability), and if $\epsilon < 0$, it actively prefers $\pi_A(\pi)$.

Thus there is a tradeoff: the closer the agent is to being a perfect transition from $R$-maximising to $R'$-maximising (at $\epsilon = 0$), the less stable it is.

5.3.1 The problem with consistent but unstable agents

The extent of the problem with consistent-but-not-stable agents may not be clear from Proposition 5.6. A consistent agent would be fine with a policy that deleted itself entirely, while handing over control to a subagent that was an $R$-maximiser.

In other words, a consistent agent would be fine with dismantling the whole seamless transition setup.

A stable agent with low $\epsilon$ would have to find some advantage (in terms of $R$) to do that dismantling. As $\epsilon$ grows, the agent become less willing to dismantle the transition setup, but less of a $R$-maximising agent before $t$.

6 Conclusion

This paper detailed the three ‘indifference’-style goals, and five methods that can be use to attain them.

All of these can used to make an agent with a potentially dangerous reward $R$, into a safer a version of that agent, without needing to understand the intricacies of $R$. In particular, most methods that could be described as hardware restrictions – confining the agent, installing behaviour tripwires, emergency shutdown procedures – could be usefully complemented by these methods. Either by making the agent indifferent to these restrictions, so it doesn’t try and undermine them, or actively coding them directly into the agent’s (pseudo-)reward.

It’s hoped that further research could extend beyond indifference to the more general property of corrigibility (Soares et al., 2017) – where the agent actively assists humans when they are guiding the agent towards better rewards, rather than just being indifferent at key moments.
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