Economic Viability of Data Trading with Rollover

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Abstract—Mobile Network Operators (MNOs) are providing more flexible wireless data services to attract subscribers and increase revenues. For example, the data trading market enables user-flexibility by allowing users to sell leftover data to or buy extra data from each other. The rollover mechanism enables time-flexibility by allowing a user to utilize his own leftover data from the previous month in the current month. In this paper, we investigate the economic viability of offering the data trading market together with the rollover mechanism, to gain a deeper understanding of the interrelationship between the user-flexibility and the time-flexibility. We formulate the interactions between the MNO and mobile users as a multi-slot dynamic game. Specifically, in each time slot (e.g., every day), the MNO first determines the selling and buying prices with the goal of revenue maximization, then each user decides his trading action (by solving a dynamic programming problem) to maximize his long-term payoff. Due to the availability of monthly data rollover, a user’s daily trading decision corresponds to a dynamic programming problem with two time scales (i.e., day-to-day and month-to-month). Our analysis reveals an optimal trading policy with a target interval structure, specified by a buy-up-to threshold and a sell-down-to threshold in each time slot. Moreover, we show that the rollover mechanism makes users sell less and buy more data given the same trading prices, hence it increases the total demand while decreasing the total supply in the data trading market. Finally, numerical results based on real-world data unveil that the time-flexible rollover mechanism plays a positive role in the user-flexible data trading market, increasing the MNO’s revenue by 25% and all users’ payoff by 17% on average.

I. INTRODUCTION

A. Background and Motivation

Mobile Network Operators (MNOs) typically offer various wireless data plans. The most widely used data plan is a three-part tariff defined by a data cap, a monthly one-time subscription fee, and a linear price for any usage exceeding the data cap [1]. Due to the stochastic nature of users’ data demand, the fixed monthly data cap occasionally ends up with a data leftover (waste) in low-demand months or an average usage (additional fee) in high-demand months, both of which users hope to avoid. To attract more subscribers (and further increase revenue), many MNOs have been exploring various innovations on the three-part tariff data plans to offer more flexibility and reduce users’ data consumption uncertainty over time. Two successful examples are the rollover mechanism and the data trading market.

The rollover mechanism offers the time-flexibility by allowing a user’s unused data from the previous month to be used in the current month. Many MNOs (e.g., AT&T [2], China Unicom Hong Kong [3], and China Mobile [4]) have adopted the rollover data plan since 2015. Our previous study in [5] has shown that the time-flexible rollover mechanism leads to a win-win situation. That’s, it enables users to hold the leftover data in low-demand months to compensate the heavy demand in the future. It also allows the MNO to extract more consumer surplus through a higher subscription fee. As a result, the rollover mechanism increases the total social welfare.

Different from the rollover mechanism, the data trading market promotes the user-flexibility. In 2014, China Mobile Hong Kong (CMHK) launched the first data trading market, called the 2nd exChange Market (2CM) [6]. It allows subscribers to sell leftover data to or buy extra data from others. The transaction prices may change over time, depending on the supply and demand relationship in the data trading market. In addition, CMHK acts as the middleman and benefits from the difference between the buying price and selling price. It is shown in [7] that the trading market is beneficial to the MNO, since its revenue gain from the trading market is larger than the revenue loss from overage fee.

Although the two mechanisms are increasingly popular, the economic viability of the data trading market with a rollover mechanism has not been clearly demonstrated, which may impede the MNO’s joint adoption in the telecom market.

First, the rollover mechanism offers users a conservative way of managing their leftover data, i.e., holding for future use, while the data trading encourages a more radical attitude towards data manipulation by taking advantage of the time-variant trading prices. It is easy to imagine that the rollover
mechanism will remarkably change users’ data trading behaviors, which motivates us to ask the first question:

**Question 1.** How will users adjust their selling and buying decisions under the rollover mechanism?

Second, although the joint adoption of the rollover mechanism and the data trading market offers more flexibility to mobile users, it is not yet clear whether the joint adoption is also beneficial for the MNO, comparing with adopting them separately. This motivates us to ask the second question:

**Question 2.** Will rollover mechanism increase or reduce the MNO’s benefit from data trading market?

In this paper, we will study the economic viability of the data trading market with a rollover mechanism (as in Fig. 1) and address the above two key questions. As far as we know, the interrelationship between the time-flexibility and the user-flexibility has not been studied before. We hope that this paper could lead to a much better understanding about them.

**B. Key Results and Contributions**

Our main results and key contributions are as follows:

- **Viability Analysis on the Data Trading Market with a Rollover Mechanism:** We formulate the interactions between the MNO and users as a multi-slot dynamic game. In each time slot (e.g., every day), the MNO first determines the selling price and buying price with the goal of revenue maximization, then each user determines his trading action to maximize his long-term payoff. Due to the availability of monthly data rollover, a user’s optimal trading decision corresponds to a dynamic programming problem with two time scales (e.g., day-to-day and month-to-month). Despite the complexity of the model, we are able to explicitly characterize the optimal decisions for the MNO and users.

- **Optimal Trading Decision:** We analyze users’ trading problem through backward induction and unveil a target interval trading policy specified by a buy-up-to threshold and a sell-down-to threshold in each time slot. Such a target interval policy allows simple implementation for users in practice and it recovers the optimal trading decision without the rollover mechanism as a special case.

- **Impact of Rollover Mechanism:** By comparing the thresholds of the target interval policy, we find that the rollover mechanism makes users sell less and buy more data under the same trading prices, which allows users to hold more data to avoid the potential overage fee and wait for higher selling prices. Hence the rollover mechanism increases the total trading market demand and reduces the total supply.

- **Performance Evaluation based on Real-world Data:** We use a group of mobile users’ six-month daily data usage traces to evaluate the performance improvement brought by the rollover mechanism. The numerical results show that offering rollover service can increase the MNO’s average revenue by 25% and users’ average payoff by 17%, which demonstrate that the rollover mechanism can be very beneficial to the data trading market.

**C. Related Literature**

This work focuses on the interrelationship of two data pricing mechanisms, i.e., the data trading market and the rollover mechanism. For clarity we list in Table I the related literature in terms of the mechanism and main methodologies. There are many excellent works on the traditional mobile data market without considering the two new mechanisms. Some focused on the optimization of pricing (e.g., [8], [9]) and data cap (e.g., [10]) from the MNO’s perspective. Other studies (e.g., [11], [12]) examined a single user’s data consumption dynamics using multi-period models.

Data trading market has been studied in [13]–[15]. Zheng et al. in [13] considered the auction-based trading platform. Yu et al. in [14] analyzed the users’ behavior with prospect theory. Andrews et al. in [15] developed a multi-period model to study a single user’s trading actions. However, these papers did not take into account the rollover mechanism.

The rollover mechanisms have been studied in [16]–[20]. Zheng et al. in [16] found that moderately price-sensitive users can benefit from subscribing to the rollover data plan. Wei et al. in [17] focused on the impact of expiration time. In our previous works, we studied the consumption priority of the rollover data [18], the MNOs’ market competition [19], and the multi-cap design problem [20]. However, none of above studies considered users’ optimal data trading decisions in the data trading market.

Our paper differs from the aforementioned works in the following key aspects: First, our model generalizes the data trading market and the rollover mechanism into a unified framework with two-dimensional flexibilities (i.e., user and time). Second, we explicitly characterize a user’s optimal trading policy in each time slot considering the availability of data rollover. Third, we examine the viability of the data trading market and the rollover mechanism, which has never been studied before.

The remainder of this paper is organized as follows. Section II introduces the system model. In Section III we study users’ trading policy. In Section IV we analyze the MNO’s pricing problem. Section V presents the numerical results and Section VI concludes this paper.

### II. System Model

We consider a telecom market where a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of mobile users subscribe to a Mobile Network Operator.
(MNO). The MNO offers a three-part tariff data plan together with the rollover and data trading services. The time is slotted with the index \( t \in \{1, 2, 3, \ldots \} \). In each slot \( t \) (e.g., every day), the MNO determines the trading prices and each user \( n \in \mathcal{N} \) takes a trading action (how much to buy or sell). The data rollover happens at the end of each billing cycle (i.e., every month), as a result of the user’s data consumption and data trading decisions in the month.

Next we first introduce the wireless data services in Section [II-A]. Then we formulate users’ trading problem and MNO’s pricing problem in Section [II-B] and Section [II-C] respectively.

### A. Wireless Data Services

We introduce the MNO’s wireless data services from the following four aspects.

1) **Mobile Data Plan**: We characterize a mobile data plan by a tuple \( T = \{Q, \Pi, \pi\} \). The user pays a monthly subscription fee \( \Pi \) for the data consumption up to the data cap \( Q \). For unit data consumption exceeding the data cap, the user pays the overage fee \( \pi \). Such a tuple \( T \) includes both the pure usage-based data plan (i.e., \( Q = 0 \) and \( \Pi = 0 \)) and the unlimited data plan (i.e., \( Q = +\infty \)) as special cases.

In practice, mobile users usually sign a one-year or two-year contract with the MNO for a particular data plan. We consider a time horizon consisting of \( M \) months and suppose that there are a total of \( K \) time slots in each month (e.g., 30 days). Denote \( m \in \{1, 2, \ldots, M\} \) as the \( m \)-th month and \( k \in \{1, 2, \ldots, K\} \) as the \( k \)-th time slot in a particular month. Hence the time slot \((m,k)\) corresponds to \( t = K(m - 1) + k \). For the sake of presentation, we use \((m,k)\) and \( t \) interchangeably.

2) **Rollover Mechanism**: The rollover mechanism allows a user’s leftover data from the previous month to be used in the current month. Different rollover mechanisms can be classified based on the consumption priority between the rollover data and the current monthly data cap, the impact of which has been extensively studied in [18]. In this work, we focus on one of the most common implementations by MNOs (including China Mobile and China Unicom HK): the rollover data from the previous month is consumed prior to the current monthly data cap and expires at the end of the current month.

3) **Data Trading Market**: Mobile users can sell their leftover data or buy extra data in the data trading market. The MNO determines the selling price \( p^t_1 \) and the buying price \( p^t_0 \) based on the total trading market supply (from sellers) and the total trading market demand (from buyers) in each time slot \( t \). In this work, we consider the MNO’s revenue-maximizing pricing strategy, i.e., the MNO decides the trading prices \( \{p^t_1, p^t_0\} \) to maximize its revenue in slot \( t \). Our later analysis in Section [IV] shows that the revenue-maximizing pricing clears the data trading market, i.e., the trading market demand equals to the trading market supply. Other objectives (e.g., maximizing the long-term revenue) are also possible but may not clear the market efficiently, which will be discussed in Section [V].

4) **Implementation of Data Rollover & Trading**: There will be some new considerations for the MNO when offering both rollover and trading services. Under the rollover mechanism, users have two types of data caps that differ in terms of the expiration time, which we call the short-term data cap and the long-term data cap.

- The short-term data cap can be consumed in the current month, and expires at the end of the current month.
- The long-term data cap not only can be consumed in the current month, but also can rollover to the next month.

Data trading behavior affects the short-term and long-term data caps. When a user buys some extra data from other users, most MNOs (e.g., AT&T and China Mobile) require that the purchased data cannot rollover (i.e., it is short-term) and is consumed with the top priority. When a user wants to sell some extra data to other users, we assume that the short-term data is sold prior to the long-term data, which will lead to the maximum flexibility to the user.

To sum up, the short-term data includes two parts: the rollover data from the previous month and the purchased data from the trading market in the current month. The long-term data only corresponds to the current monthly data cap \( Q \). Moreover, the short-term data is consumed and sold prior to the long-term one. Fig. 2 provides an illustration of the MNO’s policy of data rollover and trading.

### B. Mobile Users’ Decisions

We introduce how to model users’ decisions in five aspects.

1) **Data Volume**: We use \( \{Q^n, \Pi^n, \pi^n\} \) to represent the data plan of user \( n \). Note that the per-unit fee \( \pi \) is usually the same among various data plans from the same MNO \( \{\} \). In time slot \( t \), we denote \( e^n_t \) and \( q^n_t \) as the short-term data volume and the long-term data volume, respectively. Since \( Q^n \) is the potential maximal long-term data, \( q^n_t \leq Q^n \). We further denote \( e^n_t = \{e^n_{tn}, n \in \mathcal{N}\} \) and \( q^n_t = \{q^n_{tn}, n \in \mathcal{N}\} \) as all users’ short-term data vector and long-term data vector, respectively.

The sequence of events in each time slot \( t \) is as follows (also see Fig. 3):

- **MNO Pricing**: The MNO decides the prices \( p^t_1 = \{p^t_1, p^t_0\} \).
- **Users Review**: Each user \( n \in \mathcal{N} \) reviews his leftover data volume \( (e^n_t, q^n_t) \) and the trading prices \( p^t_1 = \{p^t_1, p^t_0\} \).
- **Users Trade**: Each user \( n \) takes a trading action (i.e., buy, sell, or no trading) and his leftover data becomes \( (\hat{e}^n_t, \hat{q}^n_t) \).
- **Users Consume**: After user \( n \) consumes data, his leftover data volume decreases to \( (\hat{e}^n_t, \hat{q}^n_t) \) at the end of slot \( t \).
Next we explain users’ trading actions and data consumptions. We adopt the notations of \((\cdot)^+ = \max\{\cdot, 0\}\) and \((\cdot)^- = \min\{\cdot, 0\}\) for brevity.

2) Trading Action: We denote \(a^n_t\) as the data trading action of user \(n\) in time slot \(t\). It covers the following three cases.

- **Buy data** \((a^n_t > 0)\): User \(n\) buys \(a^n_t\) amount of data with a unit-price \(p^n_t\). His short-term data volume increases, i.e., \(\hat{e}^{n}_t = e^{n}_t + a^n_t\), while the long-term data volume does not change, i.e., \(\hat{q}^{n}_t = q^n_t\).

- **Sell data** \((a^n_t < 0)\): User \(n\) sells \(|a^n_t|\) amount of data with a unit-price \(p^n_t\). His short-term data is sold first, i.e., \(\hat{e}^{n}_t = (e^{n}_t + a^n_t)^+\). If that is not enough to, then he will sell the long-term data, i.e., \(\hat{q}^{n}_t = q^n_t + (e^{n}_t + a^n_t)^-\).

- **No trading** \((a^n_t = 0)\): User \(n\) does not sell or buy data. His data volume remains the same, i.e., \(\hat{e}^{n}_t = e^{n}_t\) and \(\hat{q}^{n}_t = q^n_t\).

To sum it up, user \(n\)’s leftover short-term and long-term data volumes after data trading can be represented as:

\[
\begin{cases}
\hat{e}^{n}_t = (e^{n}_t + a^n_t)^+,
\hat{q}^{n}_t = q^n_t + (e^{n}_t + a^n_t)^-,
\end{cases}
\]

where \(a^n_t \geq -(e^n_t + q^n_t)\), since user \(n\) can sell at most \(e^n_t + q^n_t\) units of data. For notation simplicity, we denote \(z^n_t\) as the total data volume of user \(n\) after his trading action \(a^n_t\), given by

\[
z^n_t = \hat{e}^{n}_t + \hat{q}^{n}_t = e^{n}_t + q^n_t + a^n_t.
\]

3) Data Consumption: As each user \(n\)’s data consumption in time slot \(t\) is stochastically random (i.e., not known by the user beforehand), we model it as a random variable \(x^n_t\) with a PDF \(f_n(\cdot)\). The distribution information is available from the historical data consumption record. In Section [V] we will use real-world data to estimate the distribution \(f_n(\cdot)\) for each user.

According to the MNO’s policy in Section [II-A4], user \(n\) consumes his short-term data first, i.e., \(\hat{e}^{n}_t = (e^{n}_t - x^n_t)^+\). If the short-term data is not enough, then he further consumes the long-term data. After the long-term data decreases to zero, further data consumption will lead to an overage charge with a unit price \(\pi\). Therefore, the leftover short-term and long-term data volumes after data consumption are

\[
\begin{cases}
\hat{e}^{n}_{t+1} = (\hat{e}^{n}_t - x^n_t)^+,
\hat{q}^{n}_{t+1} = (\hat{q}^{n}_t + (e^{n}_t - x^n_t)^-)^+.
\end{cases}
\]

4) One-slot User Payoff: A user’s one-slot payoff depends on the one-slot utility from consuming data, overage charge, and the trading income or cost. First, we use a general utility function \(u_n(x)\) to represent user \(n\)’s one-slot utility of consuming \(x\) units data. Function \(u_n(x)\) is assumed to be an increasing and concave function in \(x\). Second, user \(n\) has to pay the overage charge \(\pi(x^n_t - e^n_t - q^n_t - a^n_t)^+\) if his data demand exceeds the leftover total data volume after trading. Third, the data trading decision \(a^n_t\) brings a monetary income \(p^n_t \cdot (-a^n_t)^+\) or a monetary cost \(p^n_t \cdot (a^n_t)^+\). Therefore, the one-slot payoff of user \(n\) is

\[
v^n_t(e^n_t, q^n_t, p_t, a^n_t, x^n_t) = u_n(x^n_t) - \pi(x^n_t - e^n_t - q^n_t - a^n_t)^+ + p^n_t \cdot (-a^n_t)^+ + p^n_t \cdot (a^n_t)^+.
\]

We take the expectation over the random data consumption \(x^n_t\) to derive user \(n\)’s one-slot expected payoff:

\[
\hat{v}^n_t(e^n_t, q^n_t, p_t, a^n_t) = \int_0^{+\infty} v^n_t(e^n_t, q^n_t, p_t, a^n_t, x) f_n(x)dx,
\]

(5)

To illustrate the key insights of the optimal trading policy (in Section [III]), we treat \(z^n_t\) (defined in (2)) as the trading decision variable of user \(n\), instead of using \(a^n_t\). Hence we rewrite the user’s one-slot expected payoff as follows:

\[
\hat{v}^n_t(e^n_t, q^n_t, p_t, z^n_t) = W(z^n_t) + J(e^n_t + q^n_t - z^n_t, p_t),
\]

(6)

where \(W_u(z)\) and \(J_z(p_t)\) are given by

\[W(z) = \int_0^{+\infty} f_n(x)dx,\]

(7)

\[J_z(p_t) = p^n_t \cdot (z)^+ - p^n_t \cdot (-z)^+.
\]

(8)

5) Multi-slot Data Trading & Rollover: Based on user \(n\)’s one-slot expected payoff in (6), we will further formulate a user’s multi-slot data trading problem. Before that, we first introduce the transition between consecutive time slots, which contains the following two cases:

- **If the current time slot \(t\) is not the end of a month, then the user’s data volume at the beginning of the next time slot equals to that at the end of the current time slot, i.e.,**

\[
\begin{cases}
e^{n}_{t+1} = \hat{e}^{n}_t,
q^{n}_{t+1} = \hat{q}^{n}_t.
\end{cases}
\]

(9)

- **If the current time slot \(t\) is the end of a month, then the \(\hat{e}^{n}_t\) units of short-term data expires, while the \(\hat{q}^{n}_t\) units of long-term data will rollover and becomes the short-term data of the next month. Moreover, the monthly data cap \(Q^n\) becomes available to user \(n\) again. Therefore, the data volume in the next time slot is**

\[
\begin{cases}
e^{n}_{t+1} = \hat{q}^{n}_t,
q^{n}_{t+1} = Q^n.
\end{cases}
\]

(10)

We denote \(V^n_t(e^n_t, q^n_t, p_t)\) as user \(n\)’s maximal expected total discounted payoff from slot \(t\) to his contract end, given his current data volume \((e^n_t, q^n_t)\) and the trading prices \(p_t\). We
also refer to \( V^n_t(e^n_t, q^n_t, p_t) \) as user \( n \)'s value function in time slot \( t \). Accordingly, we can formulate user \( n \)'s multi-slot data trading as the following dynamic programming problem.

**Problem 1** (User \( n \)'s Multi-Slot Data Trading Problem). For user \( n \) in the \( k \)-th time slot of the \( m \)-th month, i.e., \( t = K(m - 1) + k \), his value function has three cases:

1. If \( m = M \) and \( k = K \), the value function is
   \[
   V^n_t(e^n_t, q^n_t, p_t) = \max_{z_{t+1} \geq 0} \left\{ J(e^n_t + q^n_t - z^n_{t+1}, p_t) + W(z^n_{t+1}) \right\}.
   \]  
   \[ (11) \]

2. If \( m < M \) and \( k = K \), the value function is
   \[
   V^n_t(e^n_t, q^n_t, p_t) = \max_{z_{t+1} \geq 0} \left\{ J(e^n_t + q^n_t - z^n_{t+1}, p_t) + W(z^n_{t+1}) \right\} + \delta \cdot \mathbb{E}_t \left[ V^n_{t+1}(e^n_{t+1}, q^n_{t+1}, p_{t+1}) \right].
   \]  
   \[ (12) \]

3. If \( k < K \), the value function is
   \[
   V^n_t(e^n_t, q^n_t, p_t) = \max_{z_{t+1} \geq 0} \left\{ J(e^n_t + q^n_t - z^n_{t+1}, p_t) + W(z^n_{t+1}) \right\} + \delta \cdot \mathbb{E}_t \left[ V^n_{t+1}(e^n_{t+1}, q^n_{t+1}, p_{t+1}) \right].
   \]  
   \[ (13) \]

Case 1 corresponds to the very last time slot (contract ending day). On the RHS, the terms inside the brackets is the one-slot expected payoff.

Case 2 corresponds to the last time slot of each month (excluding the contract-ending month). The term \( \delta \cdot \mathbb{E}_t \left[ V^n_{t+1}(e^n_{t+1}, q^n_{t+1}, p_{t+1}) \right] \) is the expected maximal discounted payoff from slot \( t+1 \) to the end of the contract. Here \( \delta \in (0, 1) \) is the discount factor. We use \( \mathbb{E}_t[\cdot] \) to denote \( \mathbb{E}_{Z_{t+1}} [\mathbb{E}_{Z_{t+2}} [\cdot]] \) for brevity.

Moreover, we have substituted \( (10) \) in \( (12) \) here.

Case 3 corresponds to the time slots that are not the last slot of any month. We have substituted \( (11) \) in the third term of the RHS of \( (13) \), i.e., \( \delta \cdot \mathbb{E}_t \left[ V^n_{t+1}(e^n_{t+1}, q^n_{t+1}, p_{t+1}) \right] \).

In each time slot \( t \), user \( n \) needs to make his optimal data trading decision \( z^n_{t+1} \) based on the trading prices \( p_t \) and his leftover data volume \( e^n_t, q^n_t \), while taking into account his random data demand \( \bar{a}^n_t \). We will derive the users’ optimal trading policy in Section III.

**C. MNO’s Decision**

We formulate MNO’s pricing problem based on the above users’ model. Recall that user \( n \) makes his trading decision \( a^n_t \) based on his leftover data and the trading prices \( p_t = \{p_1^t, p^K_t\} \).

The user might become a seller (i.e., \( a^n_t < 0 \)) or a buyer (i.e., \( a^n_t > 0 \)), or choose not trade at all (i.e., \( a^n_t = 0 \)). Therefore, the total trading market demand (from all buyers) is

\[
D_t(p_t) = \sum_{n \in N} (a^n_t)^+, \quad (14)
\]

and the total trading market supply (from all sellers) is

\[
S_t(p_t) = \sum_{n \in N} (-a^n_t)^+. \quad (15)
\]

In (14) and (15), all users’ trading decision vector \( a_t = \{a^n_t, n \in N\} \) depends on the prices \( p_t \). We will derive the corresponding more detailed expression in Section IV after analyzing users’ optimal data trading policy in Section III.

Given the total demand \( D_t(p_t) \) and the total supply \( S_t(p_t) \), the total transaction quantity in the market becomes

\[
\min \left\{ D_t(p_t), S_t(p_t) \right\}.
\]

The MNO obtains \( p^b_t - p^s_t \) revenue from each unit of transaction data. Therefore, we formulate the MNO’s revenue-maximizing pricing problem as follows:

**Problem 2** (MNO’s Pricing Problem).

\[
\max_{p_t \geq 0} (p^b_t - p^s_t) \cdot \min \left\{ D_t(p_t), S_t(p_t) \right\}.
\]  
   \[ (16) \]

Now we have introduced the full model. Next we first study users’ optimal data trading policy in Section III and then look at MNO’s revenue-maximizing pricing in Section IV.

**III. USERS’ TRADING POLICY**

Next we will study the user’s optimal data trading policy under two different scenarios:

- **Plain trading**: A user decides his trading action to maximize the total discounted payoff in the current month, if no rollover happens at the end of the current month.
- **Rollover-involved trading**: A user decides his trading action to maximize the total discounted payoff in the future, if rollover happens at the end of the current month.

Users will be in the plain trading case if the MNO does not offer the rollover mechanism. If the MNO offers, users will face the plain trading case in the last month of the contract period and the rollover-involved trading case in other months.

Next we study the trading policy under the two scenarios and then compare the key differences between them. Since our analysis focuses on a generic user’s optimal decision, we will suppress the superscript \( n \) unless it is not clear. Due to space limit, proofs are provided in the on-line technical report [22].

**A. Plain Trading**

Now we study the plain trading case. Specifically, we will analyze the optimal trading policy of each time slots in the contract-ending (\( M \)-th) month, i.e., \( t = K(M - 1) + k \) and \( k \in \{1, 2, ..., K\} \). Theorem 1 summarizes the result. Due to page limit, we provide proof sketches for Theorems 1 and 2 and omit the proofs of other results.

**Theorem 1** (Plain Trading Policy). For any \( t = K(M - 1) + k \) and \( k \in \{1, 2, ..., K\} \), given the leftover data volume \((e, q)\) and the trading prices \( p = \{p^s, p^b\} \), there exists a pair of thresholds \( \{L_{k,M}^\text{Plain}(p^s), U_{k,M}^\text{Plain}(p^s)\} \) with \( L_{k,M}^\text{Plain}(p^b) \leq U_{k,M}^\text{Plain}(p^s) \), such that the optimal trading action is

\[
z_t^k = \begin{cases} 
L_{k,M}^\text{Plain}(p^b), & \text{if } e + q < L_{k,M}^\text{Plain}(p^s), \\
e + q, & \text{if } L_{k,M}^\text{Plain}(p^s) \leq e + q \leq U_{k,M}^\text{Plain}(p^s), \\
U_{k,M}^\text{Plain}(p^s), & \text{if } e + q > U_{k,M}^\text{Plain}(p^s).
\end{cases}
\]  
   \[ (17) \]

Theorem 1 shows that the optimal plain trading policy is a target interval policy specified by the buy-up-to threshold \( L_{k,M}^\text{Plain}(p^b) \) and the sell-down-to threshold \( U_{k,M}^\text{Plain}(p^s) \). More
specifically, if the user’s total leftover data volume \(e + q\) is less than \(L_{k,m}^{\text{Plain}}(p^b)\), then the user needs to buy extra data and increase the leftover data volume to \(L_{k,m}^{\text{Plain}}(p^b)\). If the leftover data \(e + q\) is higher than \(U_{k,m}^{\text{Plain}}(p^s)\), then he needs to sell some data and reduce the leftover data volume to \(U_{k,m}^{\text{Plain}}(p^s)\). If the leftover data \(e + q\) is already between these two thresholds, the user should choose not to trade.

Fig. 4(a) illustrates the optimal plain trading policy in the \(k\)-th time slot of the \(M\)-th month. The horizontal and vertical axes correspond to the user’s long-term data \(q\) and the short-term data \(e\). Each point in the plane specifies the data volume \((e, q)\). The blue region between the selling line and the buying line represents those states where the user do not need to trade, i.e., \(L_{k,m}^{\text{Plain}}(p^b) \leq e + q \leq U_{k,m}^{\text{Plain}}(p^s)\). The red squares represent the data volume before trading, i.e., \((e_t, q_t)\). The blue squares represent the data volume after trading, i.e., \((e_b, q_b)\). We also represent the optimal trading action with the blue arrows.

Next we present the key properties of the threshold values in Corollary 1

**Corollary 1.** The two thresholds \(\{L_{k,m}^{\text{Plain}}(p^b), U_{k,m}^{\text{Plain}}(p^s)\}\) in Theorem 1 have the following properties.

1. The buy-up-to threshold \(L_{k,m}^{\text{Plain}}(p^b)\) decreases in \(p^b\).
2. The sell-down-to threshold \(U_{k,m}^{\text{Plain}}(p^s)\) decreases in \(p^s\).
3. Given the trading price \(p\), we have
   \[
   \begin{align*}
   L_{1,1}^{\text{Plain}}(p^b) &\geq L_{2,1}^{\text{Plain}}(p^b) \geq \cdots \geq L_{K,1}^{\text{Plain}}(p^b), \\
   U_{1,1}^{\text{Plain}}(p^s) &\geq U_{2,1}^{\text{Plain}}(p^s) \geq \cdots \geq U_{K,1}^{\text{Plain}}(p^s).
   \end{align*}
   \] (18)

In Corollary 1, the first property indicates that a higher buying price \(p^b\) leads to a lower buy-up-to threshold \(L_{k,m}^{\text{Plain}}(p^b)\), hence the user tends to buy less data. The second property indicates that a higher selling price \(p^s\) leads to a lower sell-down-to threshold \(U_{k,m}^{\text{Plain}}(p^s)\), hence the user will sell more data. The third property shows that the thresholds decrease in time. This is because that the need to maintain a high data inventory decreases over time.

**B. Rollover-involved Trading**

Now we study the rollover-involved trading and analyze the optimal trading policy in each time slot before the last month, i.e., \(t = K(m - 1) + k\) where \(k \leq K\) and \(m < M\). We first present the optimal rollover-involved trading policy in Theorem 2 and then elaborate it in details.

**Theorem 2 (Rollover-involved Trading Policy).** For all \(t = K(m - 1) + k\) and \(m < M\), given the data volume \((e, q)\) and the trading prices \(p = \{p^s, p^b\}\), there exists a pair of thresholds \(\{L_{k,m}^{\text{Roll}}(p^b, q), U_{k,m}^{\text{Roll}}(p^s, q)\}\) with \(L_{k,m}^{\text{Roll}}(p^s, q) \leq U_{k,m}^{\text{Roll}}(p^s, q)\), such that the optimal trading action is

\[
\bar{z}_t = \begin{cases} 
L_{k,m}^{\text{Roll}}(p^b, q), & \text{if } e + q < L_{k,m}^{\text{Roll}}(p^b, q), \\
\bar{e} + q, & \text{if } L_{k,m}^{\text{Roll}}(p^b, q) \leq e + q \leq U_{k,m}^{\text{Roll}}(p^s, q), \\
U_{k,m}^{\text{Roll}}(p^s, q), & \text{if } e + q > U_{k,m}^{\text{Roll}}(p^s, q).
\end{cases}
\] (19)

Theorem 2 shows that the optimal rollover-involved trading policy is still a target interval policy, which is similar to that in the plain trading case. However, the buy-up-to threshold \(L_{k,m}^{\text{Roll}}(p^b, q)\) and the sell-down-to threshold \(U_{k,m}^{\text{Roll}}(p^s, q)\) not only depends on the trading price \(p\), but also the leftover long-term data volume \(q\). This is because that the long-term data \(q\) also plays a role in the next month. Before we illustrate the rollover-involved trading policy, let us first introduce some key properties in Corollary 2

**Corollary 2.** For all \(t = K(m - 1) + k\) and \(m < M\), the thresholds \(\{L_{k,m}^{\text{Roll}}(p^b, q), U_{k,m}^{\text{Roll}}(p^s, q)\}\) in Theorem 2 have the following properties.

1. The buy-up-to threshold \(L_{k,m}^{\text{Roll}}(p^b, q)\) decreases in \(p^b\).
2. The sell-down-to threshold \(U_{k,m}^{\text{Roll}}(p^s, q)\) decreases in \(p^s\).
3. Given the trading prices \(p\), we have
   \[
   \begin{align*}
   L_{1,1}^{\text{Roll}}(p^b, q) &\geq L_{2,1}^{\text{Roll}}(p^b, q) \geq \cdots \geq L_{K,1}^{\text{Roll}}(p^b, q), \\
   U_{1,1}^{\text{Roll}}(p^s, q) &\geq U_{2,1}^{\text{Roll}}(p^s, q) \geq \cdots \geq U_{K,1}^{\text{Roll}}(p^s, q).
   \end{align*}
   \] (20)
4. If \(q = 0\), then we have
   \[
   \begin{align*}
   L_{k,m}^{\text{Roll}}(p^b, 0) &= L_{k,m}^{\text{Plain}}(p^b), \\
   U_{k,m}^{\text{Roll}}(p^s, 0) &= U_{k,m}^{\text{Plain}}(p^s).
   \end{align*}
   \] (21)

The first three properties of Corollary 2 indicate similar intuitions as those in Corollary 1. The fourth property of Corollary 2 further shows that the two thresholds of the rollover-involved case degenerate into those of plain trading if there is no long-term data, i.e., \(q = 0\). That is, the plain trading policy is a special case of the rollover-involved trading policy.

Fig. 4(b) illustrates the rollover-involved trading policy in the \(k\)-th time slot of the \(m\)-th month, where \(m < M\). Here
the blue region between the selling curve and the buying curve represents those states where the user does not need to trade, i.e., \( L_{k,n}^{\text{Roll}}(p^b, q^b) \leq e + q \leq U_{k,n}^{\text{Roll}}(p^n, q^n) \). By comparing Fig. 4(b) with Fig. 4(a) we note that the rollover mechanism changes the straight selling and buying lines (as in Fig. 4(a)) into nonlinear curves (as in Fig. 4(b)). In the following, we discuss more insights on the trading thresholds and examine the impact of rollover mechanism.

### C. Impact of Rollover Mechanism

Based on the analysis of the optimal trading policy of the two cases, we investigate the impact of rollover mechanism on users’ trading behaviors in Corollary 3.

**Corollary 3.** Considering the same \( k \)-th time slot of different months, given the trading prices \( p = \{p^s, p^b\} \), the corresponding thresholds satisfy

\[
\begin{align*}
L_{k,1}^{\text{Roll}}(p^b, q^b) &\geq L_{k,2}^{\text{Roll}}(p^b, q^b) \geq \ldots \geq p_{k,M}^{\text{Plain}}(p^b), \\
U_{k,1}^{\text{Roll}}(p^n, q^n) &\geq U_{k,2}^{\text{Roll}}(p^n, q^n) \geq \ldots \geq U_{k,M}^{\text{Plain}}(p^n).
\end{align*}
\]

(22)

We illustrate Corollary 3 by combining Fig. 4(a) and Fig. 4(b) together in Fig. 4(c). We note that there are two regions (i.e., Regions I and II) indicating different trading actions:

- **Region I:** The optimal policy is buying data based on the dash curves (with rollover), but no-trading based on the solid lines (plain trading). That’s, the rollover mechanism makes users buy more data.
- **Region II:** The optimal policy is no-trading based on the dash curves (with rollover), but selling data based on the solid lines (plain trading). That’s, the rollover mechanism makes users sell less data.

To sum up, we find that the rollover mechanism makes users hold more data by shifting the buying curve and selling curve upwards. A high data inventory not only helps reduce the potential overage charge, but also increases the potential selling income (as the user can wait for higher selling prices).

### IV. MNO’S PRICING

We study the MNO’s revenue-maximizing pricing problem, considering users’ optimal data trading policy derived in Theorems 1 and 2. In this section, we will recover the superscript \( n \) for each user.

Given all users’ leftover data volume \( e_t \) and \( q_t \) in time slot \( t \), under the trading prices \( p_t = \{p_t^s, p_t^b\} \), the total trading market demand (from buyers) is given by

\[
D_t(p_t^b) = \sum_{n \in N} \left( L_t^n(p_t^b, q_t^n) - e_t^n - q_t^n \right)^+,
\]

(23)

where \( L_t^n(p_t^b, q_t^n) \) is the buy-up-to threshold of user \( n \) in time slot \( t \).

Similarly, the total trading market supply (from sellers) is

\[
S_t(p_t^s) = \sum_{n \in N} \left( e_t^n + q_t^n - U_t^n(p_t^s, q_t^n) \right)^+,
\]

(24)

where \( U_t^n(p_t^s, q_t^n) \) is the sell-down-to threshold of user \( n \) in time slot \( t \).

Fig. 5: An illustration of the market clear pricing.

**Fig. 5** illustrates the different scenarios of demand and supply. In each sub-figure, the vertical and horizontal axes correspond to the price (labeled by \( p \)) and the quantity, respectively. The two curves are the demand curve \( D_t(p) \) and the supply curve \( S_t(p) \).

- **Fig. 5(a)** If the total demand is larger than the total supply, i.e., \( S_t(p_t^s) < D_t(p_t^b) \), then the area of gray region is the MNO’s total revenue, i.e., \( \{p_t^b - p_t^s\} \cdot S_t(p_t^s) \). It is obvious that the MNO can increase its revenue by raising the buying price from \( p_t^b \) to \( p_t^b \). Accordingly, the area of green region is the MNO’s revenue increment.

- **Fig. 5(b)** If the total demand is smaller than the total supply, i.e., \( S_t(p_t^s) > D_t(p_t^b) \), then it is obvious that the MNO can increase its revenue by decreasing the selling price from \( p_t^s \) to \( p_t^s \). Similarly, the area of green region is the revenue increment for the MNO.

Based on the above insights, we can conclude that the MNO should set the price such that the demand equals to the supply in the data trading market. Next we characterize the MNO’s optimal prices in Theorem 3. For notation simplicity, we first define \( P_{t,D}(\theta) \) and \( P_{t,S}(\theta) \) as the inverse functions of \( D_t(p) \) and \( S_t(p) \), respectively. Here \( \theta \) is the quantity of total market demand or supply.

**Theorem 3.** In time slot \( t \), the MNO’s revenue-maximizing prices, denoted by \( \hat{p}_t^s, \hat{p}_t^b \), are given by

\[
\begin{align*}
\hat{p}_t^s &= P_{t,S}(\theta^*), \\
\hat{p}_t^b &= P_{t,D}(\theta^*),
\end{align*}
\]

(25)

where \( \theta^* \) is the optimal transaction quantity, given by

\[
P_{t,D}(\theta^*) + \theta^* \cdot P'_{t,D}(\theta^*) = P_{t,S}(\theta^*) + \theta^* \cdot P'_{t,S}(\theta^*).
\]

(26)

In Theorem 3 we characterize the optimal selling price \( \hat{p}_t^s \) and buying price \( \hat{p}_t^b \) through the optimal transaction quantity \( \theta^* \) based on the demand and supply curves. In practice, the MNO can estimate the demand and supply curve based on all users’ leftover data volumes \( e_t \) and \( q_t \). In Section V we will further quantitatively evaluate the MNO’s revenue based on the real-world data.

### V. NUMERICAL RESULTS

We apply our analysis to a real-world usage trace. We collect a group of mobile users’ data consumption records in China from December 2017 to June 2018. For each user in our
dataset, we have the information of the data consumption and the corresponding time duration for each Internet connection. We first use the empirical data to estimate users’ demand distributions, then compare the buy-up-to and sell-down-to thresholds based on the data. Finally, we evaluate the impact of rollover mechanism on MNO revenue and user payoffs.

A. Empirical Results

Mobile users’ data consumption highly depends on their daily activity and mobility. Many statistical studies (e.g., [23]) have shown a clear periodic nature for Internet connections and the period is twenty-four hours. Therefore, we follow the previous studies in [12] by viewing one day as the minimum time slot and estimate users’ daily demand distribution by time-variant prices in Section V-C.

We first use the empirical data to estimate users’ demand thresholds based on the data. Finally, we evaluate the impact of rollover mechanism on MNO revenue and user payoffs.

B. Optimal Trading Policy

Next we simulate the trading thresholds for each user based on the estimated data consumption distribution. To make reasonable comparison, here we fix the trading prices as \( p^u = 10 \text{HKD/GB} \) and \( p^b = 15 \text{HKD/GB} \). We will examine the time-variant prices in Section V-C.

1) Plain Trading: Fig. 6 shows the trading thresholds of User 1 and User 2 in the plain trading case. In each subfigure, the horizontal axis represents the \( k \)-th day of the month. For the illustration purpose, we only show the results of the second half month, i.e., \( k = 16, 17, \ldots, 30 \). Moreover, we investigate the impact of time preference by considering three time discount values, i.e., \( \delta \in \{0.92, 0.95, 0.98\} \).

Overall, both of the buy-up-to threshold and sell-down-to threshold decrease in \( k \). However, the value of buy-up-to threshold \( L_{k, M}^\text{Plain} \) is less sensitive to \( k \) and always remain small. This is because that users do not need to buy a lot of extra data in advance. Instead, they prefer to maintaining a small amount of leftover data to avoid overage fee in the current day, then only buy more data when the data consumption reaches a significant level in the current month.

We observe from Fig. 7 that a larger time discount leads to the increase of both threshold values. That is, the user tends to sell less and buy more data as \( \delta \) increases. This is because that a larger discount \( \delta \) corresponds to a better joint consideration for the current and the future, which is twofold:

- The user is willing to be more patient to sell data for immediate income.
- The user is less sensitive to incur immediate cost from buying data.

2) Rollover-involved Trading: Now we look at the rollover-involved trading scenario. Recall that the buy-up-to threshold \( L_{k, m}^\text{Roll} \) and the sell-down-to threshold \( U_{k, m}^\text{Roll} \) are related to the long-term data volume \( q \). Fig. 8 shows how the long-term data volume \( q \) affects the two thresholds with \( \delta = 0.98 \). Similarly, the horizontal axis represents the \( k \)-th day in a month. The two black circle curves represent the two thresholds with zero long-term data, i.e., \( L_{k, m}^\text{Roll}(p^b, 0) \) and \( U_{k, m}^\text{Roll}(p^u, 0) \). The two red square curves are the thresholds with the maximal long-
term data, i.e., $I_{k,m}^\text{Roll}(p^b, Q)$ and $U_{k,m}^\text{Roll}(p^b, Q)$. Therefore, the buy-up-to threshold $I_{k,m}^\text{Roll}(p^b, q)$ will appear in the red region and sell-down-to threshold $U_{k,m}^\text{Roll}(p^b, q)$ will appear in the blue region. On average, the long-term data volume $q$ leads to 20% and 10% increase of the buy-up-to and sell-down-to threshold values, respectively.

C. Performance Evaluation

To quantitatively evaluate the effect of rollover mechanism, we consider $Q = 1$GB data cap with $\Pi = 100$HKD monthly subscription fee. The per-unit fee is $\pi = 30$HKD/GB. Based on the estimated daily data consumption distribution, we randomly generate six-month daily data usage for 500 mobile users. We consider two cases with and without the rollover mechanism. For each case, we first determine whether the user will subscribe to this data plan based on his value function, then we simulate the MNO’s pricing, users’ trading and data consumption process for six months.

Fig. 9(a) plots the average daily trading prices in a month. The two red curves (marked by triangles) represent the trading prices without rollover mechanism, while the two blue curves (marked by circles) represent the case with rollover mechanism. We note that the rollover mechanism drives both the buying and selling prices higher. This is because that the time-flexibility enables users to buy more but sell less data, leading to a seller’s market with higher trading prices. Fig. 9(b) and Fig. 9(c) plot users’ average monthly payoff and the MNO’s average monthly revenue, respectively. We observe from the black curves (marked by stars) that the rollover mechanism can increase users’ monthly payoff by 17% on average and the MNO’s monthly revenue by 25% on average, compared with the case without rollover mechanism. These improvements are quite substantial.

VI. CONCLUSION

In this paper, we studied the economic viability of the data trading market with the rollover mechanism and investigated the interrelationship between the time-flexibility and user-flexibility. We found that the time-flexible rollover mechanism benefits the user-flexible data trading market, in the sense that it can substantially increases both users’ expected payoff and the MNO’s expected revenue.

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