Ground State Wavefunctions of General Filling Factors in the Lowest Landau Level

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We present a set of explicit trial wavefunctions for the filling factors $\nu = n/(2n \pm 1)$ and $\nu = 1/2$ in the symmetric gauge. We show that the zeroes of the wavefunction, except those dictated by the Fermi statistics, are detached from the particles. The evolution of zeroes as the filling factor is varied is examined. We show that the wavefunction at half-filling exhibits a $2k_F$-like oscillation in its occupation number profile. The center-of-mass motion of the ground state droplet is described in terms of the intra-Landau-level excitations of composite fermions.

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The remarkable discovery of the fractional quantum Hall effect has led to a rich variety of beautiful new concepts in the condensed matter physics of low-dimensional systems \cite{1}. The essential ingredient in the description of the fractional Hall systems is the Jastrow correlation, with which one can write down the eigenstates in the systems \cite{1}. They are based on the "composite fermion (CF) wavefunction", obtained from the corresponding electronic state. The sum $\sum_i N_i$ adds up to the total particle number $N$. In this notation, the $1/3$ Laughlin state is given by $[N]$; only the LLL is filled. The filled electronic Landau level $\nu = 1$ is given by $[1,1,...,1]$. No specific prescription was given for other general filling factors.

In physical terms, the Jastrow factor $J_i$ implies that each particle carries with it a correlation hole with respect to all the other particles. The projection $P_{L3}$ is tantamount to replacing the anti-holomorphic variable $\bar{z}$ with $2(\partial/\partial z)$. Some of the correlation zeroes, both in the compressible and incompressible states, must be displaced since $\partial/\partial z$ acting on the Jastrow factor will in general displace its zeroes. It is believed that the resulting particle-hole pair makes a dipole-like entity. The field theory of these dipole structures is a subject of active study in recent years \cite{7,8}. The Laughlin state contains no anti-holomorphic part, hence all the zeroes are tightly bound on the particles \cite{7}. For other fractions, the interplay between zeroes and the particle coordinates in the system is not as clearly understood.

We propose a class of ground state wavefunctions with the filling factor $\nu = n/(2n \pm 1)$, including $\nu = 1/2$, in the symmetric gauge using Eqs. (3) and (3). They are valid for a symmetric confining potential which preserves the angular momentum $M_{\text{tot}}$. The filling factor is defined by $\nu = N(N-1)/2M_{\text{tot}}$, where $N$ can be small (quantum dot) or a macroscopically large number. Conceptually our wavefunctions are based on the flux attachment scheme that binds two flux quanta to each electron, but differ from the conventional CF picture in the way the residual flux is treated. For example, at $\nu = 2/5$, one is left with $1/2$ flux quanta per particle. The standard CF picture is obtained by assuming this residual flux to form
a $\nu^* = 2$ state. However, one can also regard the residual flux as three integer flux quanta plus an extra one pointing in the opposite direction, shared between four particles. As a result, three-quarters of particles that bind to upward flux lie in the holomorphic LL while the other one-quarter lies in the anti-holomorphic LL [14]. The anti-holomorphic wavefunctions become derivatives acting on $J_i$ with the usual substitution $\bar{z}_i^m \rightarrow \partial_i^m$. According to this picture $\nu = 2/5$ is given by $[3N/4, 1, \ldots , 1]$, which is the case as shown below. The same interpretation applied to other fractions indicate that the ground state is given by $[(\nu - 1 - 1)N/2, 1, \ldots , 1]$. At $\nu = 1/2$, once the initial flux attachment is made, half the particles bind to an additional flux in the “up” direction while the other half bind to “down” flux quanta, leading to a symmetric population of $z_i^n$ and $\partial_i^n$ $= \bar{z}_i^n$ in the wavefunction.

Following these ideas, we explicit construct the ground state wavefunctions at $\nu = 2/5, 3/7, 3/5, 1/2$, and $2/3$ and list the results in Table I. The states at half-filling, $M_{tot} = N(N - 1)$, were discussed previously [18]. The total angular momentum of $\nu = 2/5$ state, for example, is $M_{tot} = 5N(N - 1)/4$. In order to have an integer-valued $M_{tot}$, $N$ is either a multiple of 4, 4k, or 4k + 1. One can check that the desired angular momentum is obtained for

$$N = 4k + 1 : \{3k + 1, 1, \ldots , 1\},$$

$$N = 4k : \{3k, 2, 1, \ldots , 1\}. \quad (4)$$

For $N \leq 7$, our constructions give the correct ground states at the given angular momentum $M_{tot}$ [18].

We can make general observations about the wavefunction. First, the states are given by $N_0/N = (\nu - 1 - 1)/2$ for large $N$, $N_1 = 1$ or 2 depending on the parity of the particle number, and $N_i = 1 (i \geq 2)$, largely in agreement with the argument presented above. There exists an interesting dual relation between states $\nu = n/(2n + 1)$ and $\nu = n/(2n - 1)$ for given $n$. Starting from one state, we can obtain the other by interchanging $z_i^m$ with $\partial_i^m$ in the determinant, Eq. (3). This is not the same as the particle-hole symmetry which relates two states the sum of whose filling factors is one [2]. One has $1/n$ residual flux per particle pointing either upward or down at $\nu = n/(2n \pm 1)$. Our argument for the assignment of the residual flux goes through in exactly the same manner for both states, with the role of the holomorphic and the anti-holomorphic parts reversed. In the projected space, this leads to the duality under the interchange of $z_i^m$ with $\partial_i^m$.

The low-energy excitation spectrum of the states is easily classified within the CF picture. One can distinguish between intra-Landau-level excitations which preserve $n$, and the inter-Landau-level excitations which change the LL index. An example of the intra-Landau-level, $m \rightarrow m + 1$ excitation is shown in Fig. 1. For $\delta M_{tot} = 1$, we have $l$ distinct intra-Landau-level excitations if the ground state consists of $l$ different LLs, $[N_0, \ldots , N_{l - 1}]$. Such a plethora of low-energy states is difficult to understand for incompressible fractions, where the low-energy modes are known to be one-dimensional edge excitations [19]. We shall return to the resolution of this puzzle later after we have explained the presence of the “occupation tail” in our wavefunctions.

$$\psi = [3, 1, 1]$$

$$\psi^* = [3^*, 1, 1]$$

FIG. 1. A schematic picture of the ground state $[3, 1, 1]$ and the excited state $[3^*, 1, 1]$ in terms of CF levels.

The center-of-mass (CM) motion of a droplet is described by multiplying the droplet wavefunction with powers of the CM coordinate $Z = \sum_{i=1}^{\infty} z_i/N$ [13, 17]. In the absence of the external potential, the displaced state is degenerate in energy with the original state because the interaction Hamiltonian, projected to the LLL, commutes with $Z$ [13]:

$$\sum_{k,i<j} V(k) e^{ik(z_i-z_j)/2} e^{ik(\partial_i-\partial_j)}, Z = 0. \quad (5)$$

We have found that $\psi^* = Z\psi_{g.s.}$ has a remarkably simple interpretation in terms of the CF picture. Since $\psi^*$ is degenerate with $\psi_{g.s.}$ itself, it is natural to expect $\psi^*$ to be expressed as a linear combination of the intra-Landau-level excitations of CFs. We denote $[N_0, \ldots , N_i, \ldots ]$ as the state where the highest angular momentum of the $n = i$ LL has increased by unity (See Fig. 1). We have explicitly confirmed for $\nu = 2/5 (N = 4), \nu = 1/2 (N = 3, 4, 5)$ and $\nu = 2/3 (N = 4, 5)$ that the following relation holds:

$$\psi^* = [N_0^*, \ldots ] + [N_0, N_1^*, \ldots ] + \cdots + [\ldots , N_{l-1}^*] \quad (6)$$

Note that each bracketed state is not normalized but rather is a Slater determinant of Eq. (3) with the overall constant equal to one. Equation (4) obviously holds in the special limit of $\psi_{g.s.} = [N]$ [16].

We can gain a lot of insight about the nature of the ground states by plotting the zeroes of the wavefunction, $\psi(z_1, z_2, \ldots , z_N) = 0$, with respect to $z_1$ while the other coordinates $z_2$ through $z_N$ are held fixed. At least one zero occurs at each fixed position itself due to the exclusion principle, hence our task lies in finding out where the extra zeroes (those not dictated by Fermi statistics) are. We have derivatives of $J_i$ entering the determinant, whose zeroes do not in general occur at $z_1 = z_i$, but somewhat displaced from it. We expect that the full many-body state will also share this feature. Figure 3(a)-(f)
shows a plot of zeroes for two different distribution of $z_i$, $i = 2, \ldots, N$ ($N = 17$), for several filling factors. The total number of zeroes that occur is the same as the degree of $\psi(z_1, \ldots, z_N)$ treated as a polynomial in $z_1$. We denote this quantity as $m_{\text{eff}}$ and show its value in Table I. One sees that $m_{\text{eff}}/N = (3 + \nu - 1)/2 (\nu < 1)$ for large systems.

Some qualitative observations can be made about the evolution of zeroes shown in Fig. 2. As we move from $\nu = 1/3$ to $2/5$, two out of three zeroes detach themselves and form a (two vortices + one particle) composite. Since there are fewer zeroes per particle, some of the composites have to share their zeroes. As we increase the density, some of the zeroes move away from the particle cluster while the density of zeroes in the interior goes down. At $\nu = 2/3$, we have a very distinct “halo” of zeroes surrounding the particles. At $\nu = 1$ both the halo and the interior zeroes disappear completely. We have also observed that the number of extra zeroes which appear inside the particle cluster is roughly given by $N_0$, the number of CFs in the LLL. For example, one-quarter of the total particles reside in the LLL for $\nu = 2/3$, and indeed we have $16 \times (1/4) = 4$ zeroes in the cluster’s interior in the plots shown in Fig. 2.

The exception is the $\nu = 1/3$ state, where the outer zeroes merge with the particles. A naive definition of the filling factor as the number of zeroes per particle will give a lower $\nu$ than what we obtain for $\nu$ in terms of $M_{\text{tot}}$. In other words, our trial wavefunction carries more zeroes than expected in the thermodynamic limit without the boundary.

The over-abundance of zeroes and their tendency to be located outside the cluster can be related to another feature of the trial wavefunction, the long tail in the occupation number [15]. One can verify that $m_{\text{eff}}$ is the maximum angular momentum state for which $n_m = \langle c^\dagger_m c_m \rangle$ is nonzero. A compact droplet would have its last occupied state occur at $m_{\text{ed}} \approx \nu^{-1} N$. A short examination of Table I shows that $m_{\text{eff}} > m_{\text{ed}}$ and the difference $W = m_{\text{eff}} - m_{\text{ed}}$ is precisely the number of CFs inhabiting the higher LLs. For example, $m_{\text{eff}} - m_{\text{ed}} = N/2$ for half-filled case while $N - N_0$ is also $N/2$. At half filling, we have demonstrated that the actual occupation numbers in the tail region $m_{\text{ed}} < m < m_{\text{eff}}$ is quite small [15].

A few cases we have examined for other filling fractions also show similar behavior. This is consistent with our plot of zeroes since we expect the electron probabilities to be small in regions with high density of zeroes.

The tail region $W$ is non-uniform in density, and there is no reason to expect incompressibility in this region. It is appropriate to regard the tail as a 2D metallic state which, like the ordinary 2D Fermi surface, supports a macroscopic number of particle-hole excitations. The abundance of low-energy modes in our wavefunctions can thus be understood as primarily excitations in the tail.

We have found evidence of a Fermi liquid behavior in the occupation profile of a half-filled droplet.
In conclusion, we proposed here a class of ground state wavefunctions for the droplet in the LLL, written entirely in terms of the holomorphic coordinates. A long and small-density occupation tail is a generic feature of the states, which is also reflected in the distribution of the zeroes in the wavefunction. The zeroes of the wavefunction for the droplet in the LLL, written entirely in terms of the holormophic coordinates. A long

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| $\nu$ | $N$ | $\psi$ | $m_{eff}$ |
|------|-----|-------|---------|
| 2/5  | 4k  | [k,2,1,...,1] | 11k - 3 |
|      | 4k + 1 | [k + 1,1,...,1] | 11k |
| 3/7  | 6k  | [k,2,1,...,1] | 16k - 3 |
|      | 6k + 1 | [k + 1,1,...,1] | 16k |
|      | 6k + 3 | [k + 2,2,1,...,1] | 16k + 5 |
|      | 6k + 4 | [k + 3,1,...,1] | 16k + 8 |
| 1/2  | 2k  | [k,2,1,...,1] | 5k - 3 |
|      | 2k + 1 | [k + 1,1,...,1] | 5k |
| 3/5  | 6k  | [k,2,1,...,1] | 14k - 3 |
|      | 6k + 1 | [k + 1,1,...,1] | 14k |
|      | 6k + 3 | [k + 2,2,1,...,1] | 14k + 4 |
|      | 6k + 4 | [k + 2,1,...,1] | 14k + 7 |
| 2/3  | 4k  | [k,2,1,...,1] | 9k - 3 |
|      | 4k + 1 | [k + 1,1,...,1] | 9k |