Horizontal Branch stars as AmFm/HgMn stars

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Abstract. Recent observations and models for horizontal branch stars are briefly described and compared to models for AmFm stars. The limitations of those models are emphasized by a comparison to observations and models for HgMn stars.

Key words: Stars: abundances – Stars: diffusion – Stars: Population II – Galaxy: globular clusters – Stars: radiative accelerations – Stars: Horizontal Branch

1. Introduction

At around 16000 K, the $T_{\text{eff}}$, where the horizontal branch (HB) crosses the main–sequence, HB stars have the same $T_{\text{eff}}$ and log $g$ as HgMn stars. Both have very shallow surface convection zones and they appear to have some abundance anomalies in common (Sargent & Searle, 1967, 1968). At slightly cooler $T_{\text{eff}}$, HB stars have a smaller log $g$ and their convection zones become as deep or even deeper than those of AmFm stars. On the main–sequence, stellar evolution models assuming a mixed outer zone have been relatively successful at explaining AmFm stars (Richer et al., 2000; Michaud et al., 2005). Detailed models for HgMn stars are more complex and have involved NLTE calculations of individual species (see Michaud 1981 for a review but also Michaud et al. 1974 and Proffitt et al. 1999).

We first very briefly ($\S$ 2) describe stellar evolution calculations done with all effects of atomic diffusion including radiative accelerations, $g_{\text{rad}}$. We then show applications to Pop II stars ($\S$ 3). We will briefly describe recent observations of abundance anomalies in HB stars and what fraction of them can be explained by stellar evolution models with a mixed outer zone ($\S$ 3.2.2), a model similar to that used for AmFm stars. After a few examples of results for AmFm and HgMn stars ($\S$ 4), the role of chemical differentiation in the atmosphere of HB stars is discussed in $\S$ 5 as well as the potential role of rotation in reducing anomalies, thus linking HB and HgMn stars.

2. Stellar evolution with radiative accelerations

The particle transport equations are introduced into a stellar evolution code which was described in VandenBerg (1985), Proffitt & Michaud (1991) and Proffitt (1994). For
each species one adds a force equation (eq. [18.1] of Burgers 1969) and a heat equation (eq. [18.2] of Burgers). There are consequently two coupled differential equations for each of the 28 included species. Similar equations are written for electrons. It is generally assumed that each atomic species can be treated locally as being in an average state of ionization. One needs to know $Z_i$, an appropriate mean of the number of lost electrons.

The dominant term for each species contains $g_{\text{rad}}(A) - g$ as a factor, where $g_{\text{rad}}(A)$ is an appropriate average of the radiative acceleration over the states of ionization of element $A$. Over most of the stellar interior and for most of the evolution, it dominates transport even though the electric field, “diffusion” and thermal diffusion terms are also included in the calculations and are important for part of the evolution in some stars.

Rosseland opacities, mean ionic charges, and mean radiative forces are calculated using the same interpolation method, based on the principle of corresponding states described in § 2.2 of Rogers & Iglesias (1992); see in particular their equations (4) to (6). Interpolation weights are determined for a subset of the data grid, and used to interpolate locally all these variables.

In first approximation, evaluating $g_{\text{rad}}(A)$ amounts to calculating the fraction of the momentum flux that each element absorbs from the photon flux. In stellar interiors, it takes the form:

$$g_{\text{rad}}(A) = \frac{L_{\text{rad}}}{4\pi^2c} \frac{k_R}{X_A} \int u \kappa_u(A) \frac{\mathcal{P}(u)}{\kappa_u(\text{total})} du$$  \hspace{1cm} (1)

where most symbols have their usual meaning. The quantities $\kappa_u(\text{total})$ and $\kappa_u(A)$ are respectively the total opacity and the contribution of element $A$ to the total opacity at frequency $u$, with $u$ and $\mathcal{P}(u)$ given by:

$$u = \frac{h\nu}{kT}$$  \hspace{1cm} (2)

and

$$\mathcal{P}(u) = \frac{15}{4\pi^3} u^4 \frac{e^n}{(e^n - 1)^2}.$$  \hspace{1cm} (3)

The calculations of $g_{\text{rad}}(A)$ involve carrying out the integration over the $10^4 u$ values for each atomic species, $A$. It implies using a 1.5 Gigabyte spectrum data base from OPAL. The integrations must be repeated for each atomic species, at each mesh point (typically 2000 in our models) and at each time step (typically $10^4$ up to the giant branch). The Rosseland average opacity is also continuously recalculated making these calculations fully self consistent with all composition changes.

3. Pop II and HB stars

Evolutionary calculations have been carried out for a large number of masses and metallicities (Richard et al., 2002a,b). These have in particular been used to determine the age of M92, one of the oldest globular clusters, to be 13.5 Gyr, and to constrain the Li abundance to be expected from stellar evolution in the oldest stars. Both were compared to observations of WMAP (Richard et al., 2005; VandenBerg et al., 2002).
3.1. From the Main Sequence to the tip of the Giant Branch

Here, we follow a 0.8 $M_\odot$ model with $Z = 10^{-4}$ from the zero age main–sequence to the middle of the HB. Its metallicity allows a comparison to observations in the globular cluster M15. The evolution is carried out without the adjustment of any parameter except in so far as the mixing length was determined to fit the solar model (Turcotte et al., 1998).

During main–sequence evolution, concentration variations develop throughout the star. At turnoff, a 0.8 $M_\odot$ star has $T_{\text{eff}} \sim 6500$ K and an age $\sim 11.5$ Gyr (Richard et al., 2002b). The concentration variations caused by atomic diffusion extend over 30% of the radius at the level of a factor of 1.5 in contrast and over 60% of the radius at the level 1.2 in contrast. There are overabundances of all atomic species from Na to Ni in the atmospheric regions but overabundance factors vary among the species. Some of those abundance anomalies may have been seen in M92 by King et al. (1998) but at the limit of detection so that this result requires confirmation since the signal to noise ratio was not quite satisfactory. Metals pushed from the deeper interior also accumulate within the star where $g_{\text{rad}} = g \sim 0$. This occurs progressively deeper in, as one considers species with larger atomic numbers from S to Ni, since a given electronic configuration occurs progressively deeper in the star as the atomic number increases.

As the evolution proceeds on the subgiant and giant branches, dredge up occurs and mixes some 58 % of the star. The abundance anomalies are largely but not completely eliminated. At the He flash, there remains a 0.04 dex difference in metal abundance concentration between the surface and the core and a 0.003 $M_\odot$ difference in helium core mass between a model calculated with and one without diffusion.

3.2. Horizontal Branch Stars

The transition to the HB was carried out as described in Michaud et al. (2007): a fraction of the envelope mass was removed and the star was reconverged on the HB following approximately the procedure used by Sweigart (1987). On the HB there are effects of diffusion in three regions of the star: just outside the He burning core, just below the H burning shell and to produce surface abundance anomalies. The first two are discussed in § 3.2.1 and the last one in § 3.2.2.

3.2.1. Structural effects of diffusion

In canonical stellar evolution it has been standard practice since the suggestion of Paczyński (1970) to assume that the He burning core is extended by overshooting or penetration in order to maintain convective neutrality at the boundary. Since the opacity of carbon rich material is larger than that of He rich material, the radiative gradient increases in the core as He burning transforms He into C. If the core boundary is stationary, strong superadiabaticity develops at the boundary. It has been customary to assume that this causes overshooting. This process is however poorly understood (see Sweigart 1994) and its efficiency is essentially unknown. In stellar evolution with atomic diffusion, this superadiabaticity does not develop. Instead atomic diffusion, even in the
Figure 1. Concentration variations for all calculated species, *left panel*, in a 0.61 $M_\odot$ model after 30 Myr on the HB ($T_{\text{eff}} \sim 12400$ K) and, *right panel*, in a 2.0 $M_\odot$ Pop I star ($T_{\text{eff}} \sim 8005$ K) after 616 Myr on the main–sequence as appropriate for a Hyades star. The gray scale is adjusted so that an overabundance by a factor of 1.5 or more appears black in both cases while an underabundance by a factor of 0.8 or less appears white. The radial scale is linear in $r$. Horizontal lines indicate the mass of the spherical shell outside a certain radius ($\Delta m$) labeled by $\log(\Delta m/ M_\ast)$. The HB model is more concentrated since there is about $\log(\Delta m/ M_\ast) = -2$ outside of the fractional radius where there is $\log(\Delta m/ M_\ast) = -1$ in the main–sequence model. This leads to larger effects of diffusion on the HB. The outer 50% by radius is affected by diffusion in the HB model ($\log(\Delta m/ M_\ast) = -3$) while it is the outer 25% by radius in the Pop I model ($\log(\Delta m/ M_\ast) = -3$) even if 20 times longer was available to the Pop I model.

absence of overshooting, is sufficient to cause core expansion and maintain convective neutrality at the boundary (see Fig. [10] of Michaud et al. 2007). This does not prove that overshooting does not exist, but it is not necessary to ascribe to it a significant mixing efficiency since the transport it was assumed to carry can be done by atomic diffusion when it is properly included.

The second structural effect of atomic diffusion is related to H burning. Atomic diffusion of the H which is burning in the shell causes an extension inward of this shell. Hydrogen makes contact with C that was synthesized during the He flash. It leads to an increase in H burning and so of luminosity but only of $\sim 0.01$ dex.

Those structural effects of diffusion are caused by atomic diffusion. They are not influenced by the turbulent transport that was introduced in the outer envelope as described in §3.2.2.

3.2.2. Surface abundance anomalies

Since HB stars are Pop II stars that just left the giant branch of globular clusters, they are all expected to have the same concentration of metals, at least of those heavier than Al (Gratton et al., 2004). The concentration of CNO and other relatively light
species might show small variations but Fe is not expected to be affected. Michaud et al. (1983b) however suggested that $g_{\text{rad}}$ should lead to overabundances of at least some metals in those stars where settling causes underabundances of He. Glaspey et al. (1989) have confirmed the overabundance of Fe in one star of one cluster but at the limit of detection and this observation required confirmation. This prediction has now been confirmed in many clusters (Behr et al., 1999; Moehler et al., 2000; Fabbian et al., 2005; Pace et al., 2006) but in particular by Behr (2003) for M15. Overabundances of Fe by factors of 50–100 are seen in nearly all HB stars with $T_{\text{eff}} > 11500$ K while the cooler ones have the same Fe abundance as cluster’s giants (see Fig. 2).

In stellar evolution calculations, the surface concentrations can be affected by the exterior boundary conditions. In the calculations of Michaud et al. (2008), the simplest assumption was made, that of a mixed outer zone without any mass loss. The concentration variations within a 0.61 $M_\odot$ HB star, 30 Myr after ZAHB, are illustrated in the left panel of Fig. 1. One notes that, in this HB model, all species more massive than O, except S, are overabundant in the external region. There is a second abundance peak of metals that is progressively deeper in the star from S (at $\log(\Delta m/ M_\ast) \sim -5$) to Fe (at $\log(\Delta m/ M_\ast) \sim -3$). This is caused by $g_{\text{rad}}$ being linked to electronic shells and heavier elements getting into a given electronic shell at higher $T$.

The density dependence of the turbulent diffusion coefficient was adjusted to reproduce approximately the observations of Fe in one of the stars observed by Behr (2003)
in M15. In practice, the outer \((\log(\Delta m/M) \sim -7\) corresponding to the envelope above \(T \sim 10^5\) K) was mixed. The same model reproduced reasonably well the observations in other high \(T_{\text{eff}}\) stars of that cluster as may be seen in Fig. 2. Furthermore, as may be seen in Fig. 11 and 12 of Michaud et al. (2008) the other anomalies are also reasonably well reproduced. This is a striking confirmation of the role of \(g_{\text{rad}}\)s in HB stars.

Some hints as to possible additional separation mechanisms may be suggested by the anomalies which are not as well reproduced. In particular as may be seen from the right panel of Fig. 2, He is expected to be underabundant with the turbulence model determined by the Fe abundance and indeed it is observed to be underabundant. But the observed underabundance is larger than expected from the calculations. If one assumes that the mixed zone only comprises the region above \(T \sim 30000\) K, expected He underabundances are increased by many orders of magnitude. Given the size of the observed error bars of the He abundance, it may be premature to conclude that observations require separation within the outer \(\log(\Delta m/M) \sim -7\). The right panel of Fig. 2 is however suggestive of the potential role of additional separation in atmospheric regions.

### 4. Pop I, AmFm and HgMn stars

The evolution code used above for HB stars had been used before by Richer et al. (2000) to model AmFm stars. In their model, the separation occurs relatively deep in the star, at \(\log(\Delta m/M) \sim -5\) where Ca is in the Ne configuration. The concentration variations within a typical AmFm star may be seen in the right panel of Fig. 1. Results of the calculations were compared to observations for a number of stars. For instance in Fig. 19 of Richer et al. (2000) it is shown that there is reasonable agreement for 10 of the 11 chemical elements common to the observations and calculations for the Hyades star 68 Tau. In Fig. 18 of the same paper reasonable agreement is found for 12 out of 16 elements for Sirius.

While an alternate model for AmFm stars, assuming separation just below the H convection zone, is a possible alternative (Watson, 1971; Smith, 1973; Michaud et al., 1983a; Alecian, 1996), a strong argument for the Richer et al. (2000) model is offered by the observations of the system \(\alpha\) Leo whose primary and secondary are both observed to be AmFm stars. The \(M, L\) and \(T_{\text{eff}}\) are relatively well constrained by the observations. The A component is observed to be unusually cool for an AmFm star. As may be seen in the upper left hand panel of Fig. 3, the A component is just about to become a red giant and is in a very rapid evolutionary stage. The presence in a binary system confirms its position in the HR diagram and so the rapid evolutionary stage it is in. In the lower left hand panel, it is seen that the Fe concentration marks the AmFm character for both components even though the primary is in an advanced subgiant state. It is just about to lose this Fe overabundance however. A model with only superficial abundance anomalies on the main–sequence would not maintain them to the evolutionary stage of \(\alpha\) Leo A. That the outer \(10^{-5}\) of the stellar mass have modified
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Figure 3. **Left panel,** in the upper part are shown, in the HR diagram, the observed positions indicated by crosses of the primary o Leo A and the secondary o Leo B as determined by Griffin (2002). One model is shown for the A component: 2.24 \( M_\odot \) (solid line) and one for the B component: 1.97 \( M_\odot \) (dashed line). When the computed primary has the observed \( T_{\text{eff}} \) (square), the 1.97 \( M_\odot \) secondary has the \( T_{\text{eff}} \) and \( L \) indicated by a circle, which is within the observed error bars. In the bottom part is shown the surface Fe abundance divided by the original value. **Right panel,** schematic description of an upper atmosphere assumed stable enough for separation to be important, from Michaud et al. 1974. In this example, Hg is pushed from the stellar interior by \( g_{\text{rad}}(\text{Hg}) \) to the point where \( g_{\text{rad}} \approx g \). The reduction of the \( g_{\text{rad}} \) as \( \tau \) decreases is caused by its progressive ionization into Hg III where it has fewer lines in the outgoing radiation flux than Hg II. Mercury accumulates there and forms a cloud.

Concentrations in the Richer et al. (2000) model explains that such a subgiant can keep the AmFm characteristics.

In the right hand panel of Fig. [3] is found a schematic description of a HgMn atmosphere which is expected to have no outer convection zone. Clouds of various species are expected to form and can modify the concentration variations that come from the interior. This relatively simple model was found to explain many abundance anomalies observed on HgMn stars. For instance, Borsenberger et al. (1979) reproduced reasonably well the Heacox (1979) observations of Sr. Jomaron et al. (1999) measured Mn surface abundances in agreement with the Alecian & Michaud (1981) calculations.

The most careful calculation yet done of \( g_{\text{rad}} \) in HgMn stars is probably that of Profitt et al. (1999) for Hg. It included careful evaluation of momentum sharing between
ions. It was however carried for a homogeneous Hg abundance in the atmosphere. They
obtain that an overabundance by a factor of $10^4$ of Hg can be supported by $g_{\text{rad}}(\text{Hg})$
instead of the observed $10^5$ overabundance. This is not that bad an agreement taking into
account that it is most likely that a cloud of Hg actually forms in the atmosphere and
that this would modify the Hg supported in the atmospheric region. Their assumption
of a completely mixed outer atmosphere would certainly not explain the Hg isotope
anomalies observed on HgMn stars (Preston, 1971). Hg isotope anomalies seem to
require separation in the outer atmosphere (Michaud et al., 1974). The presence of iso-
tope anomalies is probably the strongest argument in favor of separation going on in
the atmosphere in addition to the bottom of the mixed zone, both in HgMn stars and
the probably related HB stars.

5. Conclusion

Assuming the outer $\sim 10^{-7}$ of the mass of HB stars to be mixed one obtains, with
complete evolutionary models from the zero age main–sequence, abundance anomalies
corresponding to those observed on M15 (§3.2.2). Similar success has been obtained
for AmFm stars (§4). However such an assumption is almost certainly an oversimpli-
fication. There are some indications of atmospheric effects in the underabundances of
He. If the values quoted by Behr (2003) are not too affected by NLTE effects, larger
underabundances of He are observed than expected in our model. Our model leads
to underabundances of He but not nearly as large as the underabundances claimed by
Behr. However as mentioned in §4, there are additional separations going on in the
atmospheric regions of HgMn stars to which HB stars are very closely related as original-
ly suggested by Sargent & Searle (1967, 1968). The similarity in $T_{\text{eff}}$ and $\log g$ with
HgMn stars suggests that separation in the atmosphere may play a role in HB stars also.
Note from the right hand panel of Fig. 2 that the model with the thinnest mixed zone
develops much smaller He abundances than the one with complete mixing of the outer
$\sim 10^{-7}$ of the mass. Extensive $g_{\text{rad}}$ calculations in HB star atmospheres such as those
of Hui-Bon-Hoa et al. (2000) would be required in order to couple surface and interior
concentration variations more precisely.

The calculations described in §3.2.2 explain most of the $T_{\text{eff}}$ dependence of [Fe/H]
seen in Fig. 2, but not all of it. A potential link between the observed anomalies and
rotation in HB stars has been analyzed in Quievy et al. (2007). From Figure 1 of that
paper, there seems to be little doubt that rotation plays a role; furthermore according
to their calculations involving a parameter free meridional circulation model, rotation
explains why stars with $T_{\text{eff}} < 11000$ K have normal Fe abundances and why one of
the hotter stars in M15 (the one indicated by an arrow in the right hand panel of Fig.
2) does not have abundance anomalies. While atomic diffusion driven by $g_{\text{rad}}$ appears
to play the major role in explaining the anomalies, the slow rotation of the hotter HB
stars appears to be important also. It remains to explain why they rotate so slowly. Simi-
larly it is not understood why the main–sequence equivalent, the HgMn stars, rotate
as slowly as they do. In this case one may assume a small original rotation rate but it
is not clear that one may extend that assumption to HB stars. The slow rotation of the HB stars with $T_{\text{eff}} > 11\,500$ K remains unexplained.

Finally mass loss may also play a role. As shown by Vick and Michaud (C. Vick, these proceedings), including mass loss in models for AmFm stars leads to approximately as good agreement with observations as obtained in models assuming turbulent mixing. Observational tests are suggested to distinguish between the two processes but current observations are not accurate enough to allow the elimination of one or the other model. It would be interesting to determine if mass loss could be as successful as turbulent mixing for HB stars also.

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