Anomalous Global Strings and Primordial Magnetic Fields

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We propose a new mechanism for the generation of primordial magnetic fields, making use of the magnetic fields which are induced by anomalous global strings which couple to electromagnetism via Wess-Zumino type interactions. This mechanism can be realized in QCD by utilizing pion strings, global vortices which appear in the linear sigma model which describes physics below the QCD confinement scale. During the chiral symmetry breaking phase transition, pion strings can be produced, thus leading to primordial magnetic fields. We calculate the magnitude and coherence length of these fields. They are seen to depend on the string dynamics. With optimistic assumptions, both the magnitude and coherence scale of the induced magnetic fields can be large enough to explain the seed magnetic fields of greater than $10^{-23}$ Gauss necessary to produce the observed galactic magnetic fields via the galactic dynamo mechanism.

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I. INTRODUCTION

The origin of galactic magnetic fields is still a mystery. Observations (see e.g. [1] for a recent review) indicate that magnetic fields in galaxies have strengths of order $10^{-6}$ Gauss which are coherent on scales of several kpc. Such fields can be produced by the galactic dynamo mechanism (see e.g. [2] for a recent study) provided seed magnetic fields of strength of between $10^{-23}$ and $10^{-19}$ Gauss are present initially.

There have been several proposals to generate the required seed magnetic fields (see e.g. [3,4] for recent reviews). A breaking of electromagnetic gauge invariance in the early Universe [5] or a change in the gravitational coupling [6] are two radical proposals which require changes in basic physical principles. In string cosmology, these effects may naturally arise [7]. Phase transitions in the early Universe leads to another class of mechanisms for generating primordial magnetic fields. The random orientation of the fields may produce magnetic fields by the Kibble mechanism [8]. Although the initial magnitude of the fields can be large on small scales in this scenario, it is hard to generate the required amplitude of the coherent seed fields on large scales [9]. Magnetic field also can be generated in the collision of bubbles formed at a first order phase transition [10], or by dynamical charge separation during the phase transition [11]. Once again, it is hard to obtain coherence on large enough scales. Superconducting cosmic strings may lead to magnetic fields via turbulence effects which take place in the sting wakes [12]. Astrophysical mechanisms have also been proposed (see e.g. [13-15])

In this paper, we propose a new mechanism for the generation of primordial magnetic fields. It is based on the realization [16] that anomalous global strings couple to electricity and magnetism via an induced $F_{\mu\nu}F^{\mu\nu}$ term in the low energy effective lagrangian and induce magnetic fields (note that such a term also is a crucial feature in magnetic field generation mechanisms [17,18] based on a pseudoscalar coupling to electricity and magnetism). The major advantage of this mechanism is that the coherence scale of the magnetic fields induced by these global vortex lines is set by the length scale $\xi(t)$ of the strings (the typical curvature radius of the strings). In models which admit strings, strings are inevitably produced during the symmetry breaking phase transition in the early Universe [19]. Immediately after the phase transition, the string length scale is microscopic. However, the string network rapidly approaches a scaling solution (see e.g. [20,21] for recent reviews on cosmic strings) during which $\xi(t)$ is proportional to time $t$. In this way, there is a natural way in which large-scale coherent fields can be produced.

A concrete realization of our mechanism exists in QCD. As was shown in [22], there exists a class of string-like classical solutions of the effective theory (linear sigma model) which describes strong interaction physics below the confinement scale. These solutions are called pion strings. During the chiral symmetry breaking phase transition at a temperature $T_c \sim 100 - 200 \text{MeV}$, a network of pion strings forms. Pion strings are not topologically stable, and they will hence eventually decay at a temperature $T_d \sim 1 \text{MeV}$.

We calculate the magnitude and coherence scale of the primordial magnetic fields induced by the anomalous global strings as a function of $T_c$ and $T_d$ and demonstrate that it is possible (but requires stretching of some parameters) to use pion strings to generate the required seed magnetic fields of greater than $10^{-23}$ Gauss coherent on comoving scales of a few kpc.

In the following section, we give a brief review of pion
strings and discuss the mechanism by which they generate primordial magnetic fields. In Section 3 we calculate the magnitude and coherence length of the primordial magnetic fields induced by anomalous global vortex lines as a function of \( T_c \) and \( T_d \). The calculations apply in the general case. We then specialize to the case of pion strings. We conclude the paper with a discussion of further applications of and open issues concerning our general mechanism for magnetic field generation. For most of the paper, we work in natural units.

II. PION STRINGS AND INDUCED MAGNETIC FIELDS

In this section we will review the work of [27] in which it was shown that below the chiral symmetry breaking scale, the effective lagrangian of strong interaction physics admits global vortex line solutions, the pion strings. We then use the results of [20] to demonstrate that these vortex lines generate primordial magnetic fields.

We consider an idealization of QCD with two species of massless quarks \( u \) and \( d \). The lagrangian of strong interaction physics is invariant under \( SU_L(2) \times SU_R(2) \) chiral transformations

\[
\Psi_{L,R} \rightarrow \exp(-i\vec{\sigma} \cdot \vec{\tau})\Psi_{L,R},
\]

where \( \Psi_{L,R} = (u, d)_{L,R} \). However this chiral symmetry does not appear in the low energy particle spectrum since it is spontaneously broken to the diagonal subgroup \( SU_{L+R}(2) \) by the vacuum of QCD via quark condensate formation. Consequently, three Goldstone bosons, the pions, appear and the (constituent) quarks become massive. At low energy, the spontaneous breaking of chiral symmetry can be described by an effective theory, the linear sigma model, which involves the massless pions \( \vec{\pi} \) and a massive \( \sigma \) particle.

As usual, we introduce the field

\[
\Phi = \sigma \frac{\sigma^0}{2} + i \frac{\vec{\tau}}{2} \Phi^+, \]

where \( \sigma^0 \) is unity matrix and \( \vec{\tau} \) are the Pauli matrices with the normalization condition \( Tr(\tau^a \tau^b) = 2\delta^{ab} \). Under \( SU_L(2) \times SU_R(2) \) chiral transformations, \( \Phi \) transforms as

\[
\Phi \rightarrow L^+ \Phi R.
\]

The renormalizable effective lagrangian of the linear sigma model is given by

\[
L = L_\Phi + L_q,
\]

where

\[
L_\Phi = Tr[(\partial_\mu \Phi)^+ \partial^\mu \Phi] - \lambda [Tr(\Phi^+ \Phi) - \frac{f_\sigma^2}{2}],
\]

and

\[
L_q = \bar{\Psi} L i \gamma^\mu \partial_\mu \Psi_L + \bar{\Psi} R i \gamma^\mu \partial_\mu \Psi_R - 2g \bar{\Psi}_L \Phi \Psi_R + h.c.
\]

During chiral symmetry breaking, the field \( \sigma \) takes on a nonvanishing vacuum expectation value, which breaks \( SU_L(2) \times SU_R(2) \) down to \( SU_{L+R}(2) \). This results in a massive sigma \( \Phi^+ \) and three massless Goldstone bosons \( \vec{\pi} \), as well as giving a mass \( m_q = g f_\pi \) to the constituent quarks. Numerically, \( f_\pi \sim 94 \text{ MeV} \) and \( m_q \sim 300 \text{ MeV} \).

In an earlier paper, we studied the classical solutions of this model and discovered a class of vortex-like configurations which we refer to as pion string [2]. Like the Z string [34] of the standard electroweak model, the pion string is not topologically stable. Nevertheless, as demonstrated recently in numerical simulations in the case of semilocal strings [3] (which are also not topologically stable) pion strings are expected to be produced during the QCD phase transition in the early Universe (and also in heavy-ion collisions). The strings will subsequently decay.

The pion string is a static configuration of the lagrangian \( L_\Phi \) of Eq. (1). To construct these solutions, we define new fields (following the notation of Ref. [27])

\[
\phi = \frac{\sigma + i \vec{\pi}^0}{\sqrt{2}},
\]

\[
\pi^\pm = \frac{\pi^1 \pm i \pi^2}{\sqrt{2}}.
\]

The lagrangian \( L_\Phi \) now can be rewritten as

\[
L = (\partial_\mu \phi)^* \partial^\mu \phi + 3 \phi \pi^+ \partial^\mu \pi^+ - 3 \lambda (\pi^+ \pi^- + \phi^* \phi - \frac{f_\sigma^2}{2}).
\]

For static configurations, the energy functional corresponding to the above lagrangian is given by

\[
E = \int d^3x \left[ \nabla^2 \phi^* \nabla \phi + \nabla \pi^+ \nabla \pi^- + \lambda (\pi^+ \pi^- + \phi^* \phi - \frac{f_\sigma^2}{2}) \right].
\]

The time independent equations of motion are:

\[
\nabla^2 \phi = 2\lambda (\pi^+ \pi^- + \phi^* \phi - \frac{f_\sigma^2}{2}) \phi, \quad (11)
\]

\[
\nabla^2 \pi^+ = 2\lambda (\pi^+ \pi^- + \phi^* \phi - \frac{f_\sigma^2}{2}) \pi^+. \quad (12)
\]

The pion string solution extremising the energy functional of Eq. (10) is given by
\[ \phi = \frac{f_\pi}{\sqrt{2}} \rho(r)e^{i\phi}, \]  
(13)

\[ \pi^\pm = 0, \]  
(14)

where the coordinates \( r \) and \( \theta \) are polar coordinates in the \( x-y \) plane (the string is assumed to lie along the \( z \) axis), and \( \rho(r) \) satisfies the following boundary conditions

\[ r \to 0, \rho(r) \to 0; \]  
(15)

\[ r \to \infty, \rho(r) \to 1. \]  
(16)

To study primordial magnetic field generation, we turn on the \( U_{em}(1) \) electromagnetic interaction by replacing the derivatives in eqs. (5) and (6) by covariant derivatives. Since a pion string is made of \( \sigma \) and \( \pi^0 \) fields, it is neutral under the \( U_{em}(1) \) symmetry. However, the \( \pi^0 \) will couple to photons via the Adler-Bell-Jackiw anomaly. In the linear sigma model, the effective couplings are generated during the phase transition.

At the time of the phase transition. Thus, magnetic fields coherent with the strings are automatically set up by the analog of the Kibble mechanism. Integrating out the heavy particles, the sigma and \( \pi^0 \) fields are annihilated by the leading anomaly. In the notation of Ref. [20], this solution is

\[ \eta(t) = \kappa(t) \]  
(21)

\[ \kappa_L = -i\gamma^1 u_L. \]  
(22)

This zero mode corresponds to a fermion travelling in the \( \pm \) direction at the speed of light and generates a zero mode current

\[ j_{zero-mode} = \frac{Q_u n}{2\pi} f(r)(1, 0, 0, 1), \]  
(23)

where \( n \) is the number of quarks per unit length along the pion string, \( Q_u = 2e/3 \) and

\[ f(r) = \frac{\exp[-2gf_\pi \int_0^r \rho(r')dr']}{\int_0^\infty \exp[-2gf_\pi \int_0^r \rho(r')dr']} \]  
(24)

which measures the (normalized) transverse spread of the zero mode current and falls off exponentially as \( e^{-2m_\pi r} \). Down quarks couple to \( \phi^* \sim \sigma - i\pi^0 \) instead of to \( \phi \). Hence, for down quarks the pion string looks like an anti-vortex \( \Phi \). Consequently, the zero mode fermion travels in the opposite direction. Since \( Q_d = -\frac{2}{3} e \) and \( Q_\pi = -\frac{1}{3} e \), the net zero mode current flows in the \( -\hat{z} \) direction,

\[ j^\mu = N_c \frac{e}{2\pi} f(r)(1, 0, 0, 1). \]  
(25)

This allows the determination \( \Phi \) of the coefficients \( C_+ \) and \( C_- \), with the result \( C_+ = 0 \) and \( C_- = N_c \frac{e}{2\pi} r_0^{-\alpha/\pi} \), where that \( r_0 \geq (2m_\pi)^{-1} \). Thus we have

\[ E_r = -B_\theta = N_c \frac{e}{2\pi} (2m_\pi)^{\alpha/\pi} r_0^{-1+\alpha/\pi}. \]  
(26)

III. SEED MAGNETIC FIELD FROM GLOBAL STRINGS

In this section we estimate the magnitude of the magnetic field generated by global anomalous strings which are coherent on galactic scales. The two important scales are temperature \( T_c \) when the symmetry breaking phase transition which gives rise to the strings occurs, and the
temperature $T_d$ when the string network decays (when the strings become unstable). Plasma effects will, for the moment, be neglected. We will briefly discuss these effects in the discussion section.

Generalizing the discussion of the previous section of magnetic fields from pion strings to the case of general anomalous global strings, we conclude that the coherent magnetic fields induced by the strings at a distance $r$ from the string have the magnitude

$$B = N_c \frac{en}{2\pi r_0} \frac{r}{r_0}^{\alpha/\pi} \frac{1}{r},$$

where $n$ is the number density of charge carriers on the string, $r_0$ is the width of the string and $\alpha \ll 1$. By dimensional analysis, at the time $t_c$ when the strings form, their width will be $r_0 \sim T_c^{-1}$, and the number density of charge carriers is $n \sim T_c$. For our general study, we take $\alpha$ to be an effective anomalous coupling. In our specific model based on the chiral phase transition of QCD, it is to be an effective anomalous coupling constant. However, it could be larger than that for a model with a big coefficient of the Wess-Zumino term.

The evolution of the string network in the time interval during which the Universe cools from $T_c$ to $T_d$ is complicated. Initially, the strings have a typical curvature radius and separation of $\xi(T_c)$ (“correlation length”) which then increases rapidly and eventually approaches a scaling solution in which $\xi(t) \sim t$. During this evolution, the charge density is diluted as the strings stretch (by total charge conservation), but the mergers of small string segments into larger ones leads to a buildup of charge which can be modelled as a random walk superposition of the charges of the individual segments. Hence, the charge carrier density $n(t_d)$ of a string at time $t_d$ when the correlation length is $\xi(t_d)$ is

$$n(t_d) \sim \frac{\xi(t_c)}{\xi(t_d)} \frac{\xi(t_d)}{\xi(t_c)}^{1/2} n(t_c),$$

where the first factor on the r.h.s. comes from the stretching, and the second from the random walk superposition.

If the initial separation of strings is microscopic, then their evolution at early times will be friction-dominated and

$$\xi(t) \sim t^p$$

where $p = 5/4$ or $p = 3/2$ until $\xi(t)$ becomes comparable to the Hubble radius $t$. At late times, the correlation length scales as $t$ (i.e. $p = 1$ in (28)). We assume that the strings decay during the radiation-dominated epoch. In this case it follows from (28) that

$$n(t_d) \sim \left( \frac{T_d}{T_c} \right)^p n(t_c).$$

Combining the above results, it follows that

$$B(T_d) \sim N_c \frac{e}{2\pi r} \left( \frac{T_d}{T_c} \right)^p \left( \frac{r_Tc}{\alpha} \right)^{\alpha/\pi}$$

$$= N_c \frac{e}{2\pi} \frac{T_d}{T_c} r_m^{-1} \left( \frac{T_d}{T_c} \right)^p \left( \frac{r_Tc}{\alpha} \right)^{\alpha/\pi} \frac{\text{GeV}}{m},$$

where $r_m$ is the distance in meters. Converting from natural to MKSA units (and remembering to insert the factors of $c$ and $\mu_0$) this gives

$$B(T_d) \sim 10^{7.5} \frac{T_d}{T_c} r_k^{-1} \left( \frac{T_d}{T_c} \right)^p \left( \frac{r_Tc}{\alpha} \right)^{\alpha/\pi} \text{Gauss},$$

where $T_0$ is the present time (temperature), and $r_k$ is the present distance from the original comoving location of the string expressed in kpc. Note that $r$ is the physical distance at $T_d$.

In the case of the pion string, we make the assumption that $\xi(t) \sim t$ for all times. In contrast, topologically stable strings formed at the QCD scale would be formed with a microscopic $\xi(t_c)$ and would remain in the friction-dominated period with $\xi(t) \ll t$ until the present time. However, since pion strings are not topologically stable, their formation probability will be smaller and it is not unreasonable to take the causality bound $t$ as an estimate for the string separation. Given this assumption, we can estimate the resulting magnetic field strength by setting $T_c \sim 1\text{GeV}$, $T_d \sim 1\text{MeV}$ and $p = 1$ in (31), thus obtaining

$$B(t_0) \sim 10^{-26} \left( r_Tc \right)^{\alpha/\pi} \text{Gauss}.$$
where \( \beta \) is a constant which for scaling strings is of the order 1. On scales larger than \( \xi(t_d) \), the fields add up incoherently as a random walk, yielding for the average field

\[
\bar{B} = \left( \frac{1}{N} \right)^{1/2} B(t_0),
\]

where

\[
N = \left( \frac{d}{\xi(t_d)} \right)^2 = d[kpc]^2 \beta^{-2} 10^4 T_d [MeV]^2,
\]

where \( d \) is the scale on which we want to calculate the coherent magnetic field. For the values of the pion string chosen above, \( \bar{B} \) is reduced compared to (35) by about two orders of magnitude. Note, however, that the magnetic field lines may actually not be frozen in comoving coordinates \( \xi \), in which case the suppression factor associated with the \( N \) of (38) would be reduced.

Thus, we conclude that although the global string mechanism discussed in this paper can easily explain how QCD-scale strings can generate primordial magnetic fields which are coherent on the required scales, one requires either a large value of the anomalous coupling \( \alpha \) in order that the amplitude is sufficient for seeding a galactic dynamo, or else one needs to invoke nonlinear hydrodynamical effects which may increase the magnitude of the magnetic fields by several orders of magnitude.

### IV. DISCUSSION

In this paper we have discussed the cosmological magnetic fields induced by global anomalous strings. Due to the existence of charged zero modes on the string, coherent magnetic fields are generated when the strings form at a temperature \( T_c \) and evolve thereafter. We have pointed out that the stretching of the string network provides a natural mechanism of increasing the coherence length of the magnetic fields to scales which could be important for cosmology.

We have focused our attention on a particular example, pion strings which form at the QCD phase transition. Pion strings are unstable and hence the network of strings will decay at a temperature \( T_d \ll T_c \). We have calculated the amplitude and coherence length of the magnetic fields as a function of \( T_c, T_d \) and of the effective coupling \( \alpha \) to the anomaly. This calculation is applicable to general theories with global anomalous strings, in particular to axion strings.

The typical coherence scale of the induced magnetic fields is determined by the decay temperature \( T_d \). Provided that \( T_d < 10^{-2} \text{MeV} \) then the coherence scale of the fields induced by a single string at \( T_d \) is sufficiently large to explain the length scale of coherent galactic magnetic fields (assuming that the string network is scaling at \( T_d \)).

The amplitude of the induced magnetic fields depends sensitively on the history of the string network. We obtain the largest amplitude if we assume that the strings were scaling throughout their history (i.e. \( p = 1 \) in (29)). In this case, the amplitude is independent of \( T_d \) and depends on \( T_c \) only via factor \((rT_c)^{3/2}\) which comes from the crucial fact that the fields fall off less fast as a function of the distance from the string than would be expected from classical considerations. Neglecting this factor, we find that the amplitude is too small by a few orders of magnitude to explain the required seed fields for the galactic dynamo. However, by taking this factor into account, it may be possible to generate sufficiently large fields. If \( T_c \) is the QCD scale and \( \alpha \) the electromagnetic fine structure constant, the enhancement factor is too small (only order unity). However, if the effective coupling to the anomaly is large or if \( T_c \) is very large, such as for axion strings, then the fields generated by the mechanism discussed in this paper will be sufficiently large to explain the seed fields for the galactic dynamo. Even without the enhancement factor coming from a large value of \( \alpha \), the magnetic fields may be sufficiently large if nonlinear hydrodynamical effects are taken into account.

Plasma effects have been neglected in this paper. However, it is known that a magnetized plasma will resist the expansion of magnetic fields in comoving coordinates (see e.g. (44) and references therein). The time scale \( \tau(l) \) required for magnetic fields to diffuse a physical distance \( l \) in a plasma of temperature \( T \) is

\[
\tau = \frac{\sigma l^2}{l},
\]

where \( \sigma \) is the diffusion constant. By dimensional analysis (see also (44)), we expect \( \sigma \sim T \). Hence, it follows that the time scale for a magnetic field to diffuse a distance \( \xi(t_d) \sim t_d \) at a plasma temperature \( T_d \) is much larger than the Hubble time at the corresponding time \( t_d \), which implies that before recombination, the magnetic fields produced by the strings will not look like the vacuum configurations on a scale of \( t_d \) However, after recombination the Universe will be electrically neutral, and the resistance to the expansion of the magnetic fields will disappear. Provided that the plasma effects do not destroy the magnetic fields set up at the time \( t_d \) by the strings, the fields will rapidly approach their vacuum distribution calculated in the previous section.

Since there are generically charged zero modes for superconducting cosmic strings, the mechanism for magnetic field generation discussed here may also apply to such strings (a very similar mechanism has been proposed in (47)). However, one must be careful to consider the cosmological constraints on the amplitude of zero modes on superconducting cosmic strings (44).

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