Multivariate Control Charts based on Bayesian State Space Models

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Abstract

This paper develops a new multivariate control charting method for vector autocorrelated and serially correlated processes. The main idea is to propose a Bayesian multivariate local level model, which is a generalization of the Shewhart-Deming model for autocorrelated processes, in order to provide the predictive error distribution of the process and then to apply a univariate modified EWMA control chart to the logarithm of the Bayes' factors of the predictive error density versus the target error density. The resulting chart is proposed as capable to deal with both the non-normality and the autocorrelation structure of the log Bayes' factors. The new control charting scheme is general in application and it has the advantage to control simultaneously not only the process mean vector and the dispersion covariance matrix, but also the entire target distribution of the process. Two examples of London metal exchange data and of production time series data illustrate the capabilities of the new control chart.

Some key words: time series, SPC, multivariate control chart, state space model, EWMA.

1 Introduction

In the last decades multivariate Statistical Process Control (SPC) has received considerable attention, since in practice many processes are observed in a vector form (Montgomery). Univariate control charts have been extensively discussed in the literature (Montgomery, Box and Luceño, del Castillo) and many efforts have been devoted to upgrading the control charts for: (a) cases of correlated univariate processes; and (b) cases of multivariate uncorrelated processes.

Multivariate control charting has been discussed in many studies, e.g. Tracy et al., Liu, Kourti and MacGregor, Mason et al., Vargas, Ye et al. and Pan among many others. Review papers on multivariate control charts include Lowry and Montgomery, Sullivan and Woodall, Montgomery and Woodall, Bersimis et al. and Yeh et al. Most of the current research has been focused on the Hotelling’s $T^2$ control chart and the multivariate EWMA control chart for controlling the process mean. Yeh et al., Surtihadi et al., Cheng and Thaga and Costa and Rahim propose and study multivariate EWMA and CUSUM control charts to control the dispersion of a multivariate process. As stated before univariate control charts for autocorrelated processes have been discussed in the literature.

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(Montgomery\textsuperscript{1}, Box and Luceño\textsuperscript{2}), however, for multivariate processes the general focus has been placed on uncorrelated processes. Dyer \textit{et al.}\textsuperscript{20}, Jiang\textsuperscript{21}, Kalgonda and Kulkarni\textsuperscript{22} and Noorossana and Vaghefi\textsuperscript{23} consider multivariate control charting for autocorrelated processes based on autoregressive-moving-average (ARMA) time series models and the $T^2$ and multivariate CUSUM control charts are illustrated. Pan and Jarrett\textsuperscript{24} build a multivariate $T^2$ control chart for the forecast errors of the process. They consider a state-space approach for modelling the underlying process and they point out that the problem of monitoring multivariate processes is a problem of multivariate time series forecasting as well as a problem of control charting. Some forms of Bayesian control charts, known also as adaptive or dynamic control charts, are discussed in Tagaras\textsuperscript{25}, Tagaras and Nikolaidis\textsuperscript{26}, de Magalhães \textit{et al.}\textsuperscript{27} and in references therein. Adaptive control charts offer the flexibility and versatility to dynamically change the sampling size and the sampling interval of a Shewhart control chart, but they are disadvantaged in that the complexity is increased and usually the modeller has to resort to Monte Carlo simulation.

Our aim in this paper is to construct a multivariate control chart for autocorrelated processes in such a way that the scheme will be capable to monitor the process mean vector only, the process dispersion covariance matrix only, or both the process mean vector and the process dispersion covariance matrix. We propose a new control chart based on the theory of sequential Bayes’ factors (West and Harrison\textsuperscript{28}). First we fit a local level model to the multivariate process and then we apply a univariate modified EWMA control chart to the logarithm of the Bayes’ factor to monitor the dispersion of the predictive distribution of the data from the target distribution. Our model makes use of a generalization of the Shewhart-Deming model for multivariate autocorrelated processes (Deming\textsuperscript{29}, del Castillo\textsuperscript{3}, Triantafyllopoulos \textit{et al.}\textsuperscript{30}).

Section 2 gives the necessary time series background. The proposed control chart is discussed in detail in Section 3. In Sections 4 and 5 two examples, consisting of data from the London metal exchange and from a production of a plastic mould, illustrate the methodology and give light to the design and implementation of the new control chart. Concluding comments are given in Section 6 and the appendix details a proof of an argument in Section 3.

## 2 Background

The conventional control charts are based on the Shewhart-Deming model, e.g. for a $p \times 1$ process vector $y_t$ this model sets

$$y_t = \mu + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}_p(0, \Sigma),$$

where $\mu$ is the process mean vector and $\Sigma$ is the process dispersion covariance matrix, known also as the measurement covariance matrix. Here $\mathcal{N}_p(0, \Sigma)$ indicates the $p$-dimensional normal distribution with mean vector zero and covariance matrix $\Sigma$. The measurement drift sequence $\{\epsilon_t\}$ is assumed uncorrelated and this makes the generating process $\{y_t\}$ an uncorrelated sequence too. In this paper we extend the above model by considering equation (1), but now $\mu$ is replaced by a time-dependent $\mu_t$, which follows a multivariate random walk model, known also as local level model (Durbin and Koopman\textsuperscript{31}).

Discount Weighted Regression (DWR), which originated in the path-breaking work of Brown\textsuperscript{32}, is a method for forecasting autocorrelated time series. Considering univariate time
series Ameen and Harrison\textsuperscript{33} developed further DWR for more complex time series. The reviews of Ameen\textsuperscript{34} and Goodwin\textsuperscript{35} suggest that DWR can model efficiently time series in a wide range of situations. Triantafyllopoulos and Pikoulas\textsuperscript{36} developed a multivariate version of DWR and these authors focused on the estimation of the measurement covariance matrix.

In this paper we consider the DWR method of Triantafyllopoulos\textsuperscript{37} for multivariate local level models defined by

\[ y_t = \mu_t + \epsilon_t \quad \text{and} \quad \mu_t = \mu_{t-1} + \omega_t, \tag{2} \]

where \( \epsilon_t \sim \mathcal{N}_p(0, \Sigma) \) and \( \omega_t \sim \mathcal{N}_p(0, \Omega_t \Sigma) \). The scalar \( \Omega_t \) is specified with the aid of a discount factor \( \delta \) and the sequences \( \{\epsilon_t\} \) and \( \{\omega_t\} \) are mutually and individually uncorrelated, e.g. \( \mathbb{E}(\epsilon_i \epsilon_j') = \mathbb{E}(\omega_i \omega_j') = \mathbb{E}(\epsilon_i \omega_j') = 0 \), for all \( i \neq j, k \neq \ell \) and for all \( r, s \). Here \( \mathbb{E}(\cdot) \) denotes expectation and \( \epsilon_j' \) denotes the row vector of \( \epsilon_j \). The model definition is complete by specifying a prior distribution \( p(\mu_0|\Sigma) \), which is usually the \( p \)-dimensional normal distribution, e.g. \( \mu_0|\Sigma \sim \mathcal{N}_p(m_0, P_0 \Sigma) \), for some known prior mean vector \( m_0 \) and a positive scalar \( P_0 > 0 \).

It is further assumed that \( \mu_0 \) is uncorrelated of all \( \omega_t \). For some positive integer \( N > 0 \), let \( y^t = (y_1, y_2, \ldots, y_t) \) be the information set comprising data up to and including time \( t \), for \( t = 1, 2, \ldots, N \).

With the prior \( \mu_0|\Sigma \sim \mathcal{N}_p(m_0, P_0 \Sigma) \), the posterior density of \( \mu_t|\Sigma, y^t \) is \( \mu_t|\Sigma, y^t \sim \mathcal{N}_p(m_t, P_t \Sigma) \), where \( m_t \) and \( P_t \) are updated by

\[ m_t = m_{t-1} + \frac{P_{t-1}}{\delta + P_{t-1}} e_t = \frac{\delta m_{t-1} + P_{t-1} y_t}{\delta + P_{t-1}} \quad \text{and} \quad P_t = \frac{1}{\delta + P_{t-1}}, \tag{3} \]

with \( e_t = y_t - \mathbb{E}(y_t|y^{t-1}) = y_t - m_t \) being the one-step forecast error vector at time \( t - 1 \). Define the residual error vector \( r_t = \mathbb{E}(\epsilon_t|y^t) = y_t - m_t \). For each time \( t \) the estimator \( S_t \) of \( \Sigma \) is achieved by least squares estimation as

\[ S_t = \frac{1}{t} \sum_{i=1}^{t} r_i e_i' = \frac{1}{t} \sum_{i=1}^{t} \frac{\delta e_i e_i'}{\delta + P_{t-1}}, \tag{4} \]

after observing that

\[ r_t = y_t - m_t = y_t - m_{t-1} - \frac{P_{t-1}}{\delta + P_{t-1}} e_t = e_t - \frac{P_{t-1} e_t}{\delta + P_{t-1}} = \frac{\delta e_t}{\delta + P_{t-1}}. \]

Details of the derivations of \( m_t \), \( P_t \) and \( S_t \) appear in Triantafyllopoulos and Pikoulas\textsuperscript{36} and Triantafyllopoulos\textsuperscript{37}.

From the above it follows that the one-step forecast density is

\[ y_{t+1}|\Sigma = S_t, y^t \sim \mathcal{N}_p \left\{ m_t, \frac{(\delta + P_t)S_t}{\delta} \right\} \]

and the corresponding one-step forecast error density is

\[ e_{t+1}|\Sigma = S_t, y^t \sim \mathcal{N}_p \left\{ 0, \frac{(\delta + P_t)S_t}{\delta} \right\}, \tag{5} \]

where \( e_{t+1} = y_{t+1} - \mathbb{E}(y_{t+1}|y^t) = y_{t+1} - m_t \).

The adequacy of the model is evaluated via the mean of squared standard one-step forecast error vector (MSSE), the mean of absolute percentage one-step forecast error vector (MAPE)
and the mean of absolute one-step forecast error vector (MAE). These statistics are discussed in Chatfield\textsuperscript{38} and for data \(y_1, y_2, \ldots, y_N\) they are defined by

\[
MSSE = \frac{1}{N} \sum_{t=1}^{N} \left[ (e_{1t}^*)^2 (e_{2t}^*)^2 \cdots (e_{pt}^*)^2 \right], \quad e_t^* = \left( \frac{(\delta + P_{t-1})S_{t-1}}{\delta} \right)^{-1/2} e_t,
\]

\[
MAPE = \frac{1}{N} \sum_{t=1}^{N} \left[ \frac{|e_{1t}|}{y_{1t}} \frac{|e_{2t}|}{y_{2t}} \cdots \frac{|e_{pt}|}{y_{pt}} \right], \quad MAE = \frac{1}{N} \sum_{t=1}^{N} [|e_{1t}| |e_{2t}| \cdots |e_{pt}|],
\]

where \(e_t^*\) is the standard one-step forecast error, \(y_t = [y_{1t} \ y_{2t} \cdots y_{pt}]\), \(e_t = [e_{1t} \ e_{2t} \cdots e_{pt}]\) and \(\left( \delta^{-1}(\delta + P_{t-1})S_{t-1} \right)^{-1/2}\) denotes the inverse of the symmetric square root of the matrix \(\delta^{-1}(\delta + P_{t-1})S_{t-1}\) based on the spectral decomposition of symmetric matrices (Gupta and Nagar\textsuperscript{39}; pages 6-7). If the model fit is good the MSSE should be close to the vector \([1 \ 1 \ \cdots \ 1]\), while MAPE and MAE should be as small as possible in absolute value. Note that the MAPE, as a percentage statistic, makes sense only for a positive valued process \(y_t\), for all \(t\). If this is not the case, then MAPE can not have a meaningful interpretation and it should be excluded from the statistical analysis (Chatfield\textsuperscript{38}).

### 3 The Bayesian Control Chart

#### 3.1 The Main Idea

Bayes’ factors have been extensively discussed in the statistics literature and recently they have been applied sequentially for time series, see e.g. West and Harrison\textsuperscript{28} (Chapter 11). Salvador and Gargallo\textsuperscript{40} propose a monitoring scheme, based on Bayes’ factors, for multivariate time series, but this approach is not suitable for control charting, because it is applied in a model selection problem. In addition to this, most of the Bayesian time series monitoring (including the work of Salvador and Gargallo\textsuperscript{40}) relies upon simulated based methods and in particular Monte Carlo simulation. In this paper we favour non-iterative techniques, because they are faster, more flexible and easier to apply.

Once we have the distribution \(\text{[5]}\) we can construct a target distribution for the dispersion of \(y_t\) from the target mean and then compare these two distributions. It is well known (see e.g. Pan and Jarrett\textsuperscript{24}) that the forecast errors \(e_i\) and \(e_j\) \((i \neq j)\) are approximately uncorrelated and the approximation is so good as \(S_t\) is closer to \(\Sigma\). Suppose now that the target mean of \(\{y_t\}\) is denoted by \(\mu\) and the process dispersion covariance matrix is denoted by \(V\). This notation is consistent with the Shewhart-Deming model as in equation \(\text{[1]}\), with \(V = \Sigma\) so that \(E(y_t) = \mu\) and \(\text{Var}(y_t) = \Sigma\), where \(\text{Var}(y_t)\) denotes the covariance matrix of \(y_t\). Is is assumed that \(\mu\) is a generally unknown vector, but not stochastic. In our model of equation \(\text{[2]}\) we have \(E(y_t|\mu_t) = \mu_t\) and \(\text{Var}(y_t|\mu_t) = \Sigma\), but now \(\mu_t\) is stochastic and it also changes with time according to the random walk model of \(\text{[2]}\). We postulate that, if the process is in control, the one step forecast mean of \(y_t\) will be close to the target mean vector \(\mu\) and the forecast covariance matrix of \(y_t\) will be close to the target dispersion covariance matrix \(V\). Thus we can define the target error distribution by \(\varepsilon_t \sim N_p(0, V)\), where \(\varepsilon_t = y_t - \mu\) is the process error, also known in the process adjustment literature (del Castillo\textsuperscript{3}) as disturbance drift. Here we assume that \(V\) is positive definite matrix, although the proposed approach can be modified when \(V\) is positive semi-definite. According to the above postulate, if model \(\text{[1]}\) describes well the in-control process, density \(\text{[5]}\) should be close to the above target distribution. In
order to find out “how close” it is, we form the Bayes’ factor at time $t$:

$$BF(t) = \frac{f_{e}(t)}{f_{\varepsilon}(t)} = \frac{f_{e}(\varepsilon_{t}|\Sigma = S_{t-1}, y^{t-1})}{f_{\varepsilon}(\varepsilon_{t})}, \quad t = 1, 2, \ldots, N,$$

where $f_{e}(t)$ and $f_{\varepsilon}(t)$ denote the probability density functions of $e_{t}$ and $\varepsilon_{t}$, respectively.

For consistency in the above equation we need to make the convention $y^{0} = \emptyset$ (the null or empty set). Since both densities $f_{e}(t)$ and $f_{\varepsilon}(t)$ are normal we have

$$BF(t) = \sqrt{\frac{\delta^{p} \det (V)}{(\delta + p \delta + P_{t-1})^{p} \det (S_{t-1})}} \exp \left\{ (y_{t} - \mu)/V^{-1}(y_{t} - \mu)/2 - \delta(y_{t} - m_{t-1})/S_{t-1}^{-1}(y_{t} - m_{t-1})/(2\delta + 2P_{t-1}) \right\}, \quad (6)$$

where $\det(\cdot)$ denotes the determinant of a square matrix. The Bayes’ factor $BF(t)$ takes values from 0 to $+\infty$. We will say that the process $y_{t}$ is in control at time $t$, if $f_{e}(t) = f_{\varepsilon}(t)$, or if $BF(t) = 1$; otherwise the process will be out of control, at this time point. An out of control signal might be caused because of a mean shift (e.g. when $E(y_{t}|y^{t-1}) = m_{t-1}$ is significantly different than $\mu$) or because of a dispersion shift (e.g. $\text{Var}(y_{t}|\Sigma = S_{t-1}, y^{t-1}) = (\delta + P_{t-1})S_{t-1}/\delta$ is significantly different than $V$).

### 3.2 The Modified EWMA Control Chart for Correlated Data

A control chart for the Bayes’ factor $BF(t)$ can conclude whether $BF(t)$ is close to 1 and thus whether the process is in control or not. Since $BF(t)$ is positive valued, it is more convenient to work with the logarithm of the Bayes’ factor

$$LBF(t) = \log BF(t) = p \log \delta/2 + \{\log \det (V)\}/2 - p\{\log(\delta + P_{t-1})\}/2 - \{\log (S_{t-1})\}/2 + (y_{t} - \mu)/V^{-1}(y_{t} - \mu)/2 - \delta(y_{t} - m_{t-1})/S_{t-1}^{-1}(y_{t} - m_{t-1})/(2\delta + 2P_{t-1}) \quad (7)$$

and so we can construct an appropriate univariate control chart for $LBF(t)$. In order to propose such a chart we need to deal with two issues: (a) the values of $LBF(t)$ will be serially correlated and (b) the distribution of $LBF(t)$ might not be normal.

Considering (a), in our development it is clear that, from the definition of the $BF(t)$, either the original data $y_{t}$ are i.i.d. or auto-correlated, the resulting data $BF(t)$ (or $LBF(t)$) will be correlated and hence, if the Shewhart or any other control chart is to be used successfully, they should be modified appropriately to accommodate for correlated observations. Many authors have demonstrated that the Shewhart control charts need to be modified in order to cater for serially correlated observations (Vasilopoulos and Stamboulis⁴¹; Schmid⁴²). Similarly, the EWMA needs also to be modified and the resulting modified EWMA control chart has been discussed in many articles including Schmid⁴³ and VanBrackle and Reynolds⁴⁴. According to Harris and Ross⁴⁵ ignoring serial correlation has a stronger effect in EWMA than in the Shewhart control chart, but as we will see later the EWMA control chart is preferable to Shewhart, because it is more robust to the assumption of normality. One could also consider the modified CUSUM chart for correlated observations, but we will not further discuss this in the present paper.

Proceeding with (b) one needs to check the assumption of normality, before applying a modified EWMA (or Shewhart or CUSUM) control chart. Borror et al.⁴⁶ studied the ARL
performance of the EWMA and they suggested that the EWMA with a smoothing parameter equal to 0.05 is very effective, even in the presence of non-normality of the observations. This result agrees with Montgomery who states for the EWMA “It is almost a perfectly non-parametric (distribution free) procedure”. Maravelakis et al. study the robustness to normality of the EWMA by tabulating characteristics of the run length distributions (e.g. ARL) for observations generated by several gamma distributions. These results conclude that, for relatively low values of the damping parameter of the EWMA and for shifts in the mean the EWMA control chart can be used, even in the absence of normality. Moreover, if the process is in-control following a symmetrical, but not normal, distribution, then the EWMA can be applied successfully. To the following we look at the empirical distribution of $LBF(t)$ when the process is in control and when it is out of control.

We generate 1000 vectors from a bivariate normal distribution $N_2(\mu, V)$ with

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

and we generate 1000 vectors for three out of control scenarios. In scenario 1 we simulate data from $N_2(\mu_d, V)$ (deviations from the mean $\mu$); in scenario 2 we simulate data from $N_2(\mu, V_d)$ (deviations from the covariance matrix $V$); in scenario 3 we simulate data from $N_2(\mu_d, V_d)$ (deviations from both $\mu$ and $V$), where

$$\mu_d = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \quad \text{and} \quad V_d = \begin{bmatrix} 1 & 2.5 \\ 2.5 & 8 \end{bmatrix}.$$
Figure 1: Histograms of the log Bayes’ factor $LBF(t)$ for an in-control process (panel (a)) and out-of-control processes (panels (b)-(d)). The out of control scenarios considered are deviations from the mean vector (panel (b)), deviations from the covariance matrix (panel (c)) and deviations from both the mean vector and the covariance matrix (panel (d)).

**Phase II:** We fit the DWR model with the model components from Phase I (e.g. $\delta = \delta_{opt}$, $m_t = m_{opt}$, $\Sigma = S_{opt}$) and we apply a modified EWMA control chart at observations $LBF(t)$ with the control limits identified at Phase I, for $t = N^* + 1, N^* + 2, \ldots, N$.

In order to apply the modified EWMA control chart we first calculate the series $z_t$ with observations $x_t = LBF(t)$ as

$$z_t = \lambda x_t + (1 - \lambda)z_{t-1}, \quad 0 < \lambda \leq 1.$$  

(8)

The parameter $\lambda$ is the EWMA smoothing parameter and as it is mentioned above, for $\lambda = 0.05$ or $\lambda = 0.1$ the control chart is robust to normality. Then, the control limits of the modified EWMA control chart are

$$\mu_z \pm c\sigma_z,$$  

(9)

where $\mu_z = E(z_t)$, $\sigma_z^2 = \lim_{t \to \infty} \text{Var}(z_t)$ (asymptotic variance of $z_t$) and $c > 0$ is determined according to the required ARL. For AR(1) dependence $x_t = \phi x_{t-1} + \nu_t$ and for large $t$, the
asymptotic variance $\sigma_z^2$ is

$$
\sigma_z^2 = \frac{\sigma^2 \lambda \{1 + \phi(1 - \lambda)\}}{(1 - \phi^2)(2 - \lambda)(1 - \phi(1 - \lambda))},
$$

where $\nu_t \sim \mathcal{N}(0, \sigma^2)$ and $\sigma^2, \phi$ are assumed known. In practice these parameters are estimated at Phase I. According to Schmid\(^{43}\) the asymptotic variance $\sigma_z^2$ performs better than the exact variance of $z_t$, which is given in Schmid\(^{43}\) and which produces time-dependent control limits. Most of the literature on this topic focuses on deriving the variance $\sigma_z^2$ assuming simple time series models for $x_t$, e.g. as in the above AR(1) or as in the ARMA(1,1) model considered in VanBrackle and Reynolds\(^{44}\).

Algorithm 1 can be simplified, if at Phase I, the quantities $P_t$ and $S_t$ converge to stable values and these values are determined in Phase I for both phases. This brings up a well known problem, which has received considerable attention in the time series literature (see e.g. Durbin and Koopman\(^{31}\)). However, for the DWR and similar multivariate models limiting results for $P_t$ and $S_t$ have not been yet established. The next theorem (which proof is in the appendix) states that $P_t$ and $S_t$ converge to stable limiting values.

**Theorem 1.** In the DWR model (2) the estimator $S_t$ of the measurement covariance matrix $\Sigma$ converges in probability to $\Sigma$ and the non-stochastic scalar parameter $P_t$ converges to the limit $P = (\sqrt{\delta^2 + 4} - \delta)/2$, i.e. $S_t \xrightarrow{p} \Sigma$ and $P_t \longrightarrow P$.

From Theorem 1 the estimator $S_t$ is consistent and from the proof of this theorem (given in the appendix), $S_t$ is also unbiased estimator. Theorem 1 suggests that $P_{t-1}$ in the calculation of $\text{LBF}(t)$ of equation (7) can be replaced by its limit $P$. From equation (3) and Theorem 1 the forecast of $y_t$, $m_{t-1}$ can be approximated by

$$
    m_{t-1} = m_0 + \frac{P}{\delta + P} \sum_{i=1}^{t-1} e_i = m_0 + \frac{\sqrt{\delta^2 + 4} - \delta}{\sqrt{\delta^2 + 4} + \delta} \sum_{i=1}^{t-1} e_i,
$$

where $P_{t-1}$ of equation (7) is replaced by $P$. Figure 2 shows how fast $\{P_t\}$ converges to its limit $P$, for a prior $P_0 = 1/1000$ and three values of $\delta$. This figure points out that $P_t$ is bounded above by 1, but for $\delta = 0.2$, this bound is only achieved after $t > 13$ (solid line in Figure 2), while for $\delta = 0.9$, this bound is achieved for any $t > 1$ (dotted line in Figure 2). This gives an empirical indication of the speed of convergence of $\{P_t\}$, for several values of $\delta$.

The limit $P$ is known before the algorithm starts (e.g. $P$ depends only on $\delta$) and, given enough data in Phase I, the limit $\Sigma$ can be approximated by $\Sigma \approx S_{N^*}$, in the end of Phase I. This can have an additional benefit on computational savings, but more importantly it gives a theoretical justification that the DWR produces a good copy of the process $\{y_t\}$ and therefore this model is appropriate for the monitoring part at Phase II of Algorithm 1. For example, if $P_t$ and $S_t$ were not converging to stable values, no matter how many data we collected at Phase I, the covariance matrix of $y_t$ and thus its uncertainty would change over time resulting in an unstable time series model. False alarms are probable in the framework of such unstable models, which should be avoided.

In the design and application of the control chart it is important to suggest values of $m_0$, $P_0$, $\delta$ and $S_0$ and to study their sensitivity and influence to the performance of the proposed control chart. Since these suggestions are related to forecasting as in equation (5), results on the sensitivity of such prior parameters follow from Triantafyllopoulos and Pikoulas\(^{36}\) and
The solid line plots \( \{ P_t \} \) for \( \delta = 0.2 \), the dashed line plots \( \{ P_t \} \) for \( \delta = 0.5 \), the dotted line plots \( \{ P_t \} \) for \( \delta = 0.9 \) and the dashed/dotted line is the critical bound of 1.

Figure 2: Rate of convergence for the sequence \( \{ P_t \} \) of Theorem 1; the solid line plots \( \{ P_t \} \) for \( \delta = 0.2 \), the dashed line plots \( \{ P_t \} \) for \( \delta = 0.5 \), the dotted line plots \( \{ P_t \} \) for \( \delta = 0.9 \) and the dashed/dotted line is the critical bound of 1.

Triantafyllopoulos\textsuperscript{37}. It is worthwhile noting that, given enough data in Phase I, the values of \( m_0 \), \( P_0 \) and \( S_0 \) are not critical to the forecast performance, as in time series modelling prior information is deflated over time. This is indicated in Theorem 1 from the fact that \( P \) does not depend on \( P_0 \). The value of \( \delta \) can be critical in forecasting and a general recommendation is that several values of \( \delta \) (in the range of \((0, 1)\)) are applied in Phase I and according to the forecast performance (see Section 2) a value of \( \delta \) is decided. One should note that high values of \( \delta \) (e.g. \( \delta = 0.9 \)) yield smooth forecasts with low forecast variances, but these forecasts are sometimes unable to forecast abrupt changes in the data; low values of \( \delta \) (e.g. \( \delta = 0.1 \)) yield more precise forecasts in the presence of “wild data”, but these forecasts come with increased forecast variances.

Our proposal for the modified EWMA control chart for the \( LBF(t) \) process is motivated from the fact that the observations \( LBF(t) \) possess autocorrelation and non-normality. The approach is model-based, and so a comparison with traditionally used multivariate control charts, such as the Hotelling’s \( T^2 \) and the M-EWMA (which are both data-based control charts), is difficult and in many occasions it can not give justice. Within the model-based control charting methods, it appears that our approach can be compared with the residual chart (Pan and Jarrett\textsuperscript{24}), but again the comparisons need to make sure that model uncertainty (whether for example the DWR is a good model or an alternative time series model performs better) should be ideally removed before any comparison is attempted. For example a miss-specification of a time series model might result to a false result in the comparison
of the competing control charts. From our experience the DWR works generally well (since it is a generalization of the Shewhart-Deming model), but this might not be the case for every multivariate process. We believe that such a comparison should deserve the length and the detail of a whole paper and thus here we do not pursue this project. Next we give two examples illustrating the design and application of the proposed control chart.

To the above we have assumed that given a process \( \{y_t\} \) the interest is in building a control chart for monitoring simultaneously the process mean and the dispersion covariance matrix. However, in some cases the interest is placed on monitoring the dispersion covariance matrix only. In this case we can modify the control scheme by considering a modified EWMA control chart of the log-Bayes’ factors of the first order difference process \( z_t = y_t - y_{t-1} \), which from equation (2) has zero mean. Control charts based on \( \{z_t\} \) will be more robust as compared to those for \( \{y_t\} \), since the uncertainty of monitoring the process mean of \( \{y_t\} \) has been removed.

4 London Metal Exchange Data

London metal exchange (LME) is the world’s premier non-ferrous metals market trading currently aluminium, copper, lead and zinc, among other non-ferrous metals. Information on the LME and its functions can be found in its web site: [http://www.lme.co.uk](http://www.lme.co.uk). The review of Watkins and McAleer explores the recently growing literature on the LME market and Triantafyllopoulos discusses the correlation of spot and future contract prices of aluminium based on the DWR model of Section 2. In this paper we discuss data of spot prices for the four metals aluminium (variable \( \{y_{1t}\} \)), copper (variable \( \{y_{2t}\} \)), lead (variable \( \{y_{3t}\} \)) and zinc (variable \( \{y_{4t}\} \)).

The data are collected from January 2005 until October 2005 for every trading day excluding weekends and bank holidays; Figure 5 plots the data. We form the observation vector \( y_t = [y_{1t} \ y_{2t} \ y_{3t} \ y_{4t}]' \) and we are interested in knowing whether volatility is apparent, for \( t = 151 \) until \( t = 220 \). In other words we want to know whether from \( t \) to \( t+1 \), the variability of the observations \( y_t \) and \( y_{t+1} \) has changed. This is a major concern to econometricians, because if there is evidence for volatility, this means there is uncertainty in investments and ideally the volatility should be understood and explained. In order to answer this important question we form the first order difference of the series \( \{y_t\} \), defined by \( x_t = y_t - y_{t-1} \), for \( t > 1 \) (Figure 4). Adopting the usual forecasting strategy of commodity forecasting, given data up to time \( t-1 \), the forecast mean of \( y_t \) at time \( t \) is just the value of \( y_{t-1} \) and so we can write \( E(y_t|y_{t-1}) = y_{t-1} \). We note that the true mean of \( x_t \) may not be zero (unless in model (2) it is \( \mu_t = \mu + \omega_t \)), but it is true that conditionally on \( y_{t-1} \) or \( y_{t-1} \) we have \( E(x_t|y_{t-1}) = E(y_t) - y_{t-1} = 0 \), since \( E(y_t|y_{t-1}) = y_{t-1} \). From Figure 4 we observe that the series \( \{x_t\} \) fluctuates around zero and volatility can be detected as significant deviations from the zero target; such deviations can be detected with the aid of a control chart of Section 3.

First we need to make sure that the DWR model fits the differenced series \( \{x_t\} \) well. We take \( t = 1 - 150 \) as Phase I, in which the adequacy of the DWR model is evaluated. The performance statistics of Section 2 are: \( MSE = [0.993 \ 1.486 \ 0.866 \ 1.323]' \) and \( MAE = [18.932 \ 45.187 \ 14.569 \ 19.082]' \), suggesting an acceptable fit. Of course the MAPE is not available, since \( \{x_t\} \) is not a positive valued process (Section 2).

We have designed a modified EWMA control chart for the \( LBF(t) \) of the process \( \{x_t\} \) according to the discussion of Section 3. Figure 5 shows four control charts corresponding to
Figure 3: LME data $y_t = [y_{1t} \, y_{2t} \, y_{3t} \, y_{4t}]$, consisting of aluminium ($\{y_{1t}\}$), copper ($\{y_{2t}\}$), lead ($\{y_{3t}\}$) and zinc ($\{y_{4t}\}$) spot prices (in US dollars per tonne of each metal).

four values of the EWMA smoothing parameter $\lambda$. Typically the control chart is robust to normality for small values of $\lambda$, but for these values the control chart is only detecting very small drifts in the mean this might not be desirable. As $\lambda$ increases the modified EWMA control chart is losing its robustness over normality, but for symmetric process distributions, such as the empirical distribution of the $LBF(t)$ shown in Figure 1, the EWMA control chart might still be used for $\lambda = 0.5$. The correlation of the $LBF(t)$ is accounted by the autoregressive model of Section 3 and an analysis involving the data at Phase I shows that an autoregressive parameter $\phi = 0.1$ is adequate to capture the autocorrelation of $LBF(t)$. According to Tables for the $ARL$ of the modified EWMA control chart (see e.g. Shiau and Hsu49) we choose the value of $c$ in equation (9) so that $ARL = 370.4$, e.g. for $\lambda = 0.05$ and $\phi = 0.1$ we have $c = 2.469$. The remainder of the control limits are calculated as in equation (9).

Figure 5 shows that the process in Phase II appears to be in control, for $\lambda = 0.05$ and $\lambda = 0.1$, while for $\lambda = 0.2$ and $\lambda = 0.5$ the control chart returns an out of control point at $t = 172$ (with values $z_{172} = -1.852$ and $z_{172} = -2.999$, respectively). The mean of the EWMA $z_t$ is slightly lower than zero, which indicates that, for the entire process $\{x_t\}$, there will be some deviation of the predictive density $f_e$ from the target density $f_\varepsilon$. It is up to the modeller to decide whether such deviation from the target distribution is worth of declaring the process out of control. In search of a more automatic approach, one can lift up the whole control chart so that in Phase I the mean of $z_t$ is exactly zero. This can be performed automatically, in the end of Phase I, and this will declare the process in control in Phase II,
London metal exchange differenced process

Figure 4: LME differenced process $x_t = [x_{1t} \ x_{2t} \ x_{3t} \ x_{4t}]'$, consisting of aluminium ($\{x_{1t}\}$), copper ($\{x_{2t}\}$), lead ($\{x_{3t}\}$) and zinc ($\{x_{4t}\}$). The horizontal lines, placed at zero, indicate no volatility.

for $\lambda = 0.05, 0.1$, while for $\lambda = 0.2, \lambda = 0.5$ there is an out of control point at $t = 172$. In Figure 5 the value of $\lambda = 0.5$ is rather high to ensuring correct control limits of the modified EWMA chart (see the relevant discussion in page 4); here the chart with $\lambda = 0.5$ is mainly shown for comparison purposes with the charts with lower values of $\lambda$, but in practice we suggest that $\lambda$ does not exceed 0.2, unless there is strong evidence to support the assumption of normality for the distribution of $LBF(t)$. It is worth pointing out that the concentration of consecutive EWMA values under the mean in Phase II is causing warning, which is apparent in all charts. The phenomenon is more apparent in the charts for $\lambda = 0.05$ and $\lambda = 0.1$ and it can suggest the out of control state of the process at $t = 172$, which is apparent in the charts with $\lambda = 0.2$ and $\lambda = 0.5$. The interpretation of the out of control signal at $t = 172$ can not be done just by looking at Figure 4 and more dedicated methods of out of control variable identification need to be employed, see e.g. Bersimis et al.\textsuperscript{14}.

5 Production Time Series Data

In an experiment of production of a plastic mould the quality is centered on the control of temperature and its variation. For this purpose five measurements of the temperature of the mould have been taken, for 276 time points. The experiment is fully described in Pan and Jarrett\textsuperscript{24} and these authors show that this 5-dimensional production process $\{y_t\}$ is both autocorrelated and serially correlated including both vector autoregressive and moving aver-
Figure 5: Modified EWMA control chart for the log Bayes’ factor of the LME differenced process. Plots (a)-(d) show the modified control chart for different values of the smoothing parameter $\lambda$. In each plot of the panel, the solid horizontal line indicates the mean of the EWMA and the dotted horizontal lines indicate the control limits; the vertical line separates Phase I ($t = 1 - 150$) and Phase II ($t = 151 - 210$).

These authors use a vector state space charting approach based on the Hotelling control chart resulting on 12 out of control signals at Phase II (time points from $t = 181$ to $t = 220$) and hence concluding that the process falls badly out of control at Phase II.

We have used the data at Phase I (time points $t = 1 - 180$) in order to estimate the target mean vector $\mu = [208.245 \ 153.638 \ 53.063 - 22.742 \ 16.126]'$ (as the average of each $y_{it}: t = 1 - 180$) and the dispersion covariance matrix

$$V = \begin{bmatrix} 0.168 & -0.001 & 0.633 & -0.438 & 0.015 \\ -0.001 & 0.023 & -0.017 & 0.006 & -0.002 \\ 0.633 & -0.017 & 25.621 & -15.658 & 0.453 \\ -0.438 & 0.006 & -15.658 & 14.181 & -0.596 \\ 0.015 & -0.002 & 0.453 & -0.596 & 0.951 \end{bmatrix}$$

(as the sample covariance matrix of each $y_{it}: t = 1 : 180$), where $y_{it} = [y_{i1t}, y_{i2t}, y_{i3t}, y_{i4t}, y_{i5t}]'$. The DWR fits well with $MSSE = [0.855 \ 0.950 \ 0.992 \ 1.161 \ 0.996]'$, which is close to $[1 \ 1 \ 1 \ 1 \ 1]$. The other two performance statistics are $MAE = [1.378 \ 0.899 \ 4.450 \ 3.316 \ 0.945]'$ and $MAPE = [0.007 \ 0.006 \ 0.089 - 0.059]'$, where for $\{y_{it}\}$ the “–” indicates that the MAPE is not available, since this variable is not positive valued (see the relevant discussion for MAPE in Section 2). The above performance statistics suggest that the model fit is good and therefore we can proceed with control charting at Phase II ($t = 181 - 279$).

The first thing to do is to find a suitable AR(1) model for the process $LBF(t)$. A suitable model is the AR(1): $LBF(t) = -4.624 + 0.062LBF(t-1) + \nu_t$. According to the discussion
Figure 6: Modified EWMA control chart for the log Bayes’ factor of the Production process. Plots (a)-(b) show two charts for values of the smoothing parameter $\lambda = 0.05$ and $\lambda = 0.1$. For both plots, the solid horizontal line indicates the target mean 0 and the dotted horizontal lines indicate the control limits; the solid vertical line separates Phase I (for $t = 1 - 180$) and Phase II (for $t = 181 - 276$).

above, we remove the intercept $-4.624$ so that we can obtain a in-control process in Phase I. Thus we design the modified EWMA control chart for $LBF(t) + 4.624$. Again we use tables for the modified EWMA control chart and for $\lambda = 0.05$ the resulting control chart is given in Figure 6. This figure agrees with the residual chart of Pan and Jarrett\textsuperscript{24}, that finds the process in Phase II out of control for most of the data points. In Phase I chart of panel (b) of Figure 6 gives one out of control point, which is in agreement with Pan and Jarrett\textsuperscript{24}, but in panel (a) of Figure 6 the control chart detects more out of control points in Phase I. The EWMA control chart is robust to non-normality for the low values of $\lambda = 0.05$ and $\lambda = 0.1$, but for $\lambda = 0.05$ the chart is more sensitive to small shifts in the mean of $LBF(t)$, resulting to the detection of out of control points in Phase I. Any out of control points in Phase I should be immediately investigated and usual SPC procedures of removing influence of these points in the calculation of the control limits should be applied (Montgomery\textsuperscript{1}).

6 Conclusions

This paper develops a new multivariate control chart based on Bayes’ factors. This control chart is specifically aimed at multivariate autocorrelated and serially correlated processes. The general idea is to form a target distribution, to construct a predictive density with good forecast ability and then to apply a univariate control chart for the logarithm of the Bayes’ factor of the predictive error density against the target error density. Although in this
paper, for simplicity, we have considered normal distributions for the target and the predictive densities, in general application the proposed control charts can be applied considering other densities too as long as they are available in analytic form.

We have restricted our discussion to the modified EWMA control chart, but other control charts such as the modified CUSUM and non-parametric control charts can be applied. A major advantage of our approach as compared to other multivariate control charts is that once we have obtained the log Bayes’ factors we can apply any appropriate univariate control chart. A difficulty appears to be that the resulting Bayes’ factors process is both autocorrelated and non-normal, but we believe the design of the proposed chart is a challenge that can attract and motivate further research in this so important area of statistical process control.

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Appendix

Proof of Theorem 1. First we prove \( S_t \stackrel{P}{\rightarrow} \Sigma \). It suffices to prove that \( S_t \) is unbiased estimator and that its covariance matrix converges to zero. From equations (4) and (5) we obtain

\[
\mathbb{E}(S_t) = \frac{1}{t} \sum_{i=1}^{t} \frac{\delta \mathbb{E}(e_i e_i^\prime)}{\delta + P_{i-1}} = \frac{1}{t} \sum_{i=1}^{t} \frac{\delta (\delta + P_{i-1}) \Sigma}{(\delta + P_{i-1}) \delta} = \frac{1}{t} (t \Sigma) = \Sigma
\]

and so \( S_t \) is unbiased for \( \Sigma \). For the convergence, let \( \text{vech}(\cdot) \) denote the column stacking operator of a lower portion of a covariance matrix and let \( \| \cdot \| \) denote a matrix norm defined in a suitable linear space. From equation (5) we have

\[
\text{Var}\{\text{vech}(S_t)\} = \frac{1}{t^2} \sum_{i=1}^{t} \left( \frac{\delta}{\delta + P_{i-1}} \right)^2 \text{Var}\{\text{vech}(e_i e_i^\prime)\}.
\]

From equation (5) \( e_i \) follows a \( p \)-variate normal distribution and so by writing \( e_i = [e_{i1} e_{i2} \cdots e_{ip}]' \), we have that \( \text{Cov}(e_{ij}, e_{ik}) = \mathbb{E}(e_{ij} e_{ik}) \) are bounded, since these expectations are expressed as moments of the multivariate normal distribution (Triantafyllopoulos50). Hence \( \text{Var}\{\text{vech}(e_i e_i^\prime)\} \) has finite elements and so we can write \( \| \text{Var}\{\text{vech}(e_i e_i^\prime)\} \| < M \), for some \( M > 0 \). For any \( \epsilon > 0 \) define \( t_0 = \lfloor \epsilon M \rfloor \) (the integral part of \( \epsilon M \)). From \( P_{i-1} > 0 \) we have that \( \delta/(\delta + P_{i-1}) < 1 \), for all \( i = 1, 2, \ldots t \). Then

\[
\| \text{Var}\{\text{vech}(S_t)\} \| = \frac{1}{t^2} \left\| \sum_{i=1}^{t} \left( \frac{\delta}{\delta + P_{i-1}} \right)^2 \text{Var}\{\text{vech}(e_i e_i^\prime)\} \right\| 
\leq \frac{M}{t^2} \left\| \sum_{i=1}^{t} \left( \frac{\delta}{\delta + P_{i-1}} \right)^2 \right\|
\leq \frac{tM}{t^2} = \frac{M}{t} < \epsilon,
\]

for any \( t > t_0 \). This shows that \( \lim_{t \to \infty} \text{Var}\{\text{vech}(S_t)\} = 0 \) and so \( S_t \stackrel{P}{\rightarrow} \Sigma \).
Proceeding now with \( \{P_t\} \) we show that \( \{P_t\} \) is a Cauchy sequence in the real line and hence \( \lim_{t \to \infty} P_t = P \) exists. To prove that \( \{P_t\} \) is a Cauchy sequence, it suffices to prove that \( \lim_{t \to \infty} |P_t - P_{t-1}| = 0 \), where \( |\cdot| \) denotes absolute value. First we show that exists positive integer \( t_0 \) such that for all \( t > t_0 \) it is \( P_t < 1 \). The proof of this is by contradiction. Suppose that for all \( t_0 \) exists \( t > t_0 \) such that \( P_t \geq 1 \). Without loss in generality take \( t_0 = t^* \) and \( P_{t^*} = 1 \). Then we see that \( P_{t^*+1} = 1/(\delta + P_{t^*}) = 1/(\delta + 1) < 1 \), \( P_{t^*+2} = 1/(\delta + P_{t^*+1}) = (\delta + 1)/(\delta^2 + \delta + 1) < 1 \) and likewise \( P_{t+k} < 1 \), for all \( k \geq 1 \). So we can pick \( t_0 = t^* + 1 \) so that we can not find any \( t > t_0 \) with \( P_t \geq 1 \), which contradicts the hypothesis. Thus exists \( t_0 > 0 \) so that for all \( t > t_0 \) it is \( P_t < 1 \). This in turn implies that

\[
\delta + P_{t-1} > 1, \quad \forall t > t_0.
\]

(A-2)

From the definition of \( P_t \) of equation 3, we obtain

\[
P_t - P_{t-1} = \frac{1}{\delta + P_{t-1}} - \frac{1}{\delta + P_t} = -\frac{P_{t-1} - P_t}{(\delta + P_{t-1})(\delta + P_t)} = \cdots = \frac{(-1)^t(P_t - P_0)}{\prod_{i=1}^{t-1}(\delta + P_{t-i})}.
\]

Now pick \( t_0 \) as in (A-2) and define \( M = \min\{\delta + P_{t-1}, (\delta + P_{t-2})^2, \ldots, (\delta + P_{t_0+1})^2\} \) so that \( M > 1 \). Then

\[
|P_t - P_{t-1}| = \frac{|1 - \delta P_0 - P_0^2|}{\prod_{i=0}^{t_0}(\delta + P_i)^2} \prod_{i=1}^{t_0-2}(\delta + P_{t-i})(\delta + P_{t-i-1}) < \frac{|1 - \delta P_0 - P_0^2|}{\prod_{i=0}^{t_0}(\delta + P_i)^2 M^{t-t_0-1}} \to 0,
\]

since \( \lim_{t \to \infty} M^{t-t_0-1} = +\infty \). This proves that \( \lim_{t \to \infty} |P_t - P_{t-1}| = 0 \) and so \( \{P_t\} \) is a Cauchy sequence. Thus \( \lim_{t \to \infty} P_t = P \) exists and from equation 3 we have \( P = 1/(\delta + P) \), for which we derive \( P = (\sqrt{\delta^2 + 4} - \delta)/2 \), after rejecting the negative root \( P = (-\sqrt{\delta^2 + 4} - \delta)/2 \).

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