D-World: Decay Small-World for Optimizing Swarm Knowledge Synchronization

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ABSTRACT Knowledge synchronization in robot swarm systems is a challenging task under communication constraints. Swarm robots cannot maintain the complete communication structure with classic fixed communication network, which leads to the decline of motion decision effects. It is expected to improve the performance of swarm robot motion decisions under communication constraints with more researches on dynamic complex networks. This study established a feedback search control model for robot swarm systems for target search tasks in a 3D environment. Inspired by the WS model, which showed good synchronization performance in complex systems owing to its small average shortest path length, a novel dynamic small-world network model called decay small-world was proposed for negative impact of communication constraints. Decay small-world model reconnects robot communication links by controlling the rewired probability based on the half-life formula. It realizes dynamic network topology by decentralized computing. This new model maintains a small-world pattern over time without degenerating it into a random network. Simulations show that good knowledge synchronization of swarm robots can be realized using the decay small-world model. And the results also show that target search performances are promoted by this method.

INDEX TERMS Average consensus, knowledge synchronization, small-world, robot swarms.

I. INTRODUCTION
For individual of robot swarm systems (RSSs) with limited capabilities, decentralized control offers significant advantages in real-world applications [1], [2]. The key to decentralized control is the realization of knowledge-sharing among neighboring robots in the network using specific interaction rules, such as average consensus [3]. By integrating decentralized knowledge, agents effectively cooperate with others and achieve higher robustness, adaptability, and scalability.

Real-time knowledge of the positions of objects is conducive to improving the overall performance of robot swarms [4]. In this study, the influence of communication and no communication on swarm motion decision is compared, and it is proved that no communication will significantly reduce the swarm’s accurate perception of target position. At the same time, this perceptual inaccuracy of swarm will seriously affect the motion decision performance. Nevertheless, real-time knowledge-sharing is difficult to realize owing to the limitations of communication bandwidth and computing resources [5]. Broadcasting is the most straightforward way to solve the knowledge-sharing problem among decentralized schemes. However, this results in a broadcast storm problem that breaks the network over time [6]. In addition, too many communication connections lead to an information overload in a large-scale swarm [7].

To avoid broadcast storms and reduce information overload, each robot must communicate with only a few neighbors, filter the input information to integrate into its own opinions, and only send what is helpful to its neighbors. A practical approach is to integrate and “diffuse” knowledge in a network of mobile robots utilizing average consensus with Laplace matrix [8]. This consensus algorithm and its decentralized form exist widely in natural systems, such as bees selecting new nests [9] and flying towards them [10], migrating locusts [11], birds, and fish swarm...
movements [12]. Most of them have similar characteristics to the consensus problem of RSSs.

However, fast and accurate knowledge synchronization may not always be achieved with average consensus in a spontaneous network topology because of its high average shortest path length. The communication links between robots and their neighbors form a network topology with statistical characteristics belonging to complex networks in RSSs. Well-known mathematical models of complex networks mainly include the random network model [13], Watts and Strogatz model (WS model) [14], and scale-free network models [15]. The scale-free network model is not suitable for RSSs because the links of the nodes increase exponentially the robot has a limited number of communication links [16]. For RSSs, the shorter the link, the more stable the communication. However, the random network does not prioritize the formation of short links, resulting in instability of the network system.

Watts and Strogatz [14] studied the characteristics of a small-world network and found that it has a high average clustering coefficient and low average path length, which can be employed to improve the convergence speed of the average consensus. WS model was also used for swarm optimization in PSO [17]–[21]. Compared with the regular network (RE) model, the WS model can effectively solve the problem of inefficient knowledge diffusion caused by the average clustering factor and path length. However, it is not easy to spontaneously form effective small-world networks in natural RSSs. Furthermore, the WS model has only been used to solve a system with a fixed topology, and there has been no research on the characteristics of dynamic RSSs with small-world networks [22], [23]. And other works have mainly focused on some simplified network cases such as leader-based multi-agents or second-order networks of leader-free multi-agents [24]–[28].

To study the swarm robot search problem under communication constraints, a decentralized feedback control model is built, as shown in Fig. 1. In this model, the motion decision part is composed of the swarm control algorithm, which makes the motion decision and sends a control command to the robot actuator. The artificial fish swarm algorithm (AFSA) [29] was selected and modified to adapt to RSSs with limited communication. The status part is updated according to the status of the swarm and sensor information, and synchronizes the status knowledge to other robots through the swarm network. To solve the problem of fast knowledge synchronization in this controller model, a novel small-world network model called decay small-world (D-world) is proposed based on the half-life formula for dynamical systems.

In addition, the impacts of the D-world model on knowledge synchronization and swarm search algorithms are discussed. Finally, a hybrid algorithm based on D-world, average consensus, and AFSA is proposed for target search tasks under communication and perception constraints. The main contributions of this paper can be listed as follows.

- A hybrid control algorithm of robot swarms with average consensus and AFSA is proposed.
- For the first time, the small-world network is applied to the swarm search problem, and its effectiveness in network knowledge synchronization is verified.
- A dynamic small-world network, D-world, is proposed for communication constrained networks.
- The above problems are verified by designing 3D underwater robot simulations with limited communication and perception.

The D-world model with dynamic characteristics can not only be widely applied to various RSSs control systems with dynamic communication networks, but also has potential help to the research of social networks.

The remainder of this paper is structured as follows: The second section provides the preliminaries and problem formation of this paper, including graph theory, average consensus, and small-world networks. The third section proposes a feedback swarm search controller with the D-world model and an improved AFSA suitable for average consensus. The fourth section presents the simulations and discussion. Finally, in the fifth section, we present our conclusions.

II. PREIMINARIES AND PROBLEM FORMATION
A. RELATED WORK

In this subsection we review the relevant studies in swarm systems, complex networks, and control systems of robots. We focus on how swarm robots achieve state consensus by communication protocols and network changes. When swarm robots cooperate to complete a task, they have to deal with the problems of communication effectiveness in order to coordinate each other’s work. For classic RSSs control algorithm studies, the main focus is on swarm behavior research, including: 1) the spatial organization behavior such as aggregation [38], pattern formation [39], self-organization [40], object collection [41]. 2) Navigation behavior, such as swarm exploring [42], cooperative movement [43]. 3) Collective decisions: consensus achievement [44], task allocation [45].

However, the real robots are limited by actuator saturation, different errors, communication capacity limitations and other constraints, so that the algorithms of motion decision
TABLE 1. Summarizing the results of the search for a systematic literature review.

| Authors          | Year | Conference journal                  | Network                        | Approach                        |
|------------------|------|------------------------------------|--------------------------------|---------------------------------|
| Abdessamour et al. [30] | 2015 | Automatical                        | Second-Order network           | Small-gain framework            |
| Dong et al. [31]   | 2016 | Control Eng Practice               | Switching network              | Finite-time bipartite consensus  |
| Myeong, Jin et al. [23] | 2017 | IEEE Transactions on Cybernetics   | Second-Order network           | Betweenness Centrality-Based Consensus Protocol |
| Cai et al. [32]    | 2017 | Automatical                        | Second-Order network           | Adaptive distributed observer approach |
| Du et al. [21]     | 2018 | Complexity                        | Second-Order network           | Genetic Simulated annealing algorithm |
| Yang et al. [33]   | 2018 | International Journal of Systems Science | Second-Order network          | K filter and disturbance observer based control |
| Cheng et al. [28]  | 2019 | International Journal of Control, Automation and Systems | Second-Order network         | Velocity-free consensus protocols |
| Wu et al. [34]     | 2020 | Neurocomputing                     | Bipartite network              | Decentralized bipartite tracking consensus |
| Nicola et al. [35] | 2020 | Energies                           | Small-world network            | Small-world theory              |
| Jianquan et al. [36] | 2021 | IEEE Transactions on Systems       | Fixed network with 5 nodes     | Distributed adaptive finite-time protocol |
| Peiming et al. [37] | 2022 | Neurocomputing                     |                                |                                 |

are not efficient when directly applied to the physical robots. In distributed computing, RSSs need to face the problem of information exchange between individual robots. The average consensus and its control algorithms were used to distributed robots information exchange [3]. For the RSSs, the consensus of state is the basis of other tasks, and the swarms need to maintain the common state, such as cooperative positioning, tracking [46]. If the swarm system is a homogeneous integrator with a fixed network topology, consensus can be achieved by linear feedback of relative states [47].

The average consensus of the fixed topology is the main focus of the previous research due to the simplification of the problem. For example, Abdessamour et al. [30] designs a consensus protocol with communication time delay and indirect information for second-order multi-robot systems. He et al. [32] proposes a distributed observer approach for cooperative output regulation for nonlinear multi-agent systems. Yang et al. [33] designed a k-filter and disturbance observer controller for consensus tracking of second-order nonlinear systems. The method used only the relative position information of agents. Cheng et al. [28] proposed a velocity-free consensus protocols, which could ensure the relative positions of followers and leader without velocity measurement. Wu et al. [34] designed a decentralized bipartite tracking consensus controller to deal with bipartite tracking consensus problems. Peiming et al. [37] proposes a distributed adaptive finite-time protocol, which is established based on the recursion algorithm and neural networks for distributed adaptive finite-time control problem.

Generally, RSSs is more complex than simple fixed network system. Swarm robots form complex network system [48] through communication connections and information sharing. Complex networks are often composed of large numbers of individuals interacting in relatively simple ways, and the overall outcome of these interactions appears chaotic and unpredictable [49]. Complex network theory has also been used to study the communication problems of multi-robots, such as Lizihi et al. [50] used complex network theory to model the control system of UAV group. As one of the most classic complex network models, the concept of “small-world effect” was first proposed by Stanley Milgram, a psychologist at Harvard University, in 1967 in a social experiment [51], which aimed to find out whether any pair of individuals in a social network can be interconnected through a short connection. Small-world network model [14] is the mathematical form of the small-world effect proposed by Watts and Strogatz in 1998. It is a model between regular network and random network constructed by changing the connection mode of edges in regular network with a small probability. It has been proven to be existed in many systems. At present, small-world research has made achievements in many research fields [52]. Small-world network can improve the information communication of complex network system, so it is widely studied in many fields, such as social network, brain network [53]. For multi-robot systems, small-world networks are also studied. Du et al. [21] propose a genetic simulated annealing (GSA) algorithm to improve the efficiency of transforming other kinds of networks into small-world networks by adding edges. Nicola et al. [35] adopts the small-world theory control a swarm of moving individual vehicles to improve synchronization and robustness of information sharing. To sum up, current researches mainly focus on the consensus of multi-agents with simple fixed topology. There is little attention to the consensus of complex networks, but in-depth research is lacking, as shown in table.1. Meanwhile, these studies did not discuss the defect that fixed topology cannot adapt to change under communication constraints, which seriously affects the movement-decision ability of agents, which discussed in section 4.2 of this study.

B. PREIMINARIES

1) GRAPH THEORY

Denote a time-varying digraph \( G = (V, E(t), A(t)) \) as the knowledge exchange model among a group of homogeneous agents, where \( V = \{v_i \mid i \in [1, N]\} \) is the set of agents. \( E(t) = \{e_{ij} : i \text{ and } j \in [1,2,\ldots,N]\} \subset V \times V \) and \( A(t) \) are edges set and adjacency matrix which are respect to the time instant \( t \) respectively. For the matrix \( A(t) = [a_{ij}(t)] \in \mathbb{R}^{N \times N} \), \( a_{ij} = 1 \) holds if and only if \( e_{ij} \in E(t) \), otherwise \( a_{ij} = 0 \).

\[
a_{ij} = \begin{cases} 1, & e_{ij} \in E(t) \\ 0, & \text{otherwise} \end{cases}
\]

2) AVERAGE CONSENSUS

an algorithm model used to describe the process of continuous knowledge flowing and reaching consensus in a system
where \( L \) is the Laplacian matrix which is defined as
\[
L = D - A
\]
where \( D = \text{diag}(d_1, \ldots, d_n) \) is the degree matrix of G.

3) SMALL-WORLD NETWORK

Together with the six-degree separation theory \([51]\), small-world theory was first proposed by Stanley Milgram in 1967 and its mathematical model was formally defined by Watts and Strogatz in 1998 \([14]\). In \([14]\), it was shown that a small-world network is a form between a regular network and a random network and has the characteristics of a lower average shortest path length than a regular network, high knowledge propagation speed, and strong synchronization, as shown in Fig.2.

C. PROBLEM FORMATION

This study investigated the optimization of the average consensus in swarm search with communication and perception constraints. In the case of limited perception, \( k, k = 0, 1, \ldots, N \) robots perceive the target at time \( t \). The average consensus algorithm integrates the knowledge of targets and robots received from the network and sensors into new knowledge published in the network.

Suppose that one or more targets, \( T_k, k = 1, 2, \ldots, M \), are randomly distributed in the area. The target’s current position is \( P_K(t) \) and moves at a constant velocity \( v \) towards a random destination \( P_D \) or remains in \( P_S \). Subsequently, the target \( T_k \) movement follows the following rules:

\[
P_D = \text{random}(\text{range}),
\quad P_K(t) = \begin{cases} P_D & \text{if } P_K(t) = P_D \\ P_K(t) + v \cdot \Delta t \cdot (P_D - P_K(t)) & \text{else}, \end{cases},
\]

(4)

A SRS containing \( N \) robots with dynamics is considered.

\[
p_i^k(t) = Af(t) + Bu_i(t) \quad i = 1, 2, \ldots, N,
\]

(5)

\( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times p} \) are the constraint matrices, \( f(t) \) denotes the average consensus function, \( p_i \in \mathbb{R}^n \) denotes the position estimation of robot \( i \) to target \( k \), and \( u_i \in \mathbb{R}^p \) is the system control input calculated by the improved AFSA.

Three types of ways are defined for robots to obtain knowledge:

1) **Global communication (GC):** A centralized information unit exists to collect and share target knowledge. Each robot can obtain target position knowledge from this unit, as shown in Fig.3(a).

2) **Local communication (LC):** Limited by the perception and communication devices, each robot perceives the target within a finite range and shares knowledge with other robots within the communication range via interaction, as shown in Fig.3(b).

3) **No communication (NC):** Only equipped with a perception device, each robot perceives the target within a limited range without a communication device. As shown in Fig.3(c), they cannot share knowledge with others via communication.

In this study, experiments were conducted on the performance of robot swarms in three different communication modes. The target’s position is the knowledge transmitted between the robots through interaction or a centralized information unit. AFSA and feedback control methods were employed to make the robot swarms converge towards the targets.

III. MAIN RESULT

The experimental robot was assumed to be an autonomous underwater vehicle (AUV) with limited perception and communication. A motion simulation program was developed for underwater robot simulation based on python3 and Matplotlib. The simulation program is a simplified underwater robot system used to study the movement decisions of robot swarms under the condition of limited communication and perception in a 3D space. To simplify the problem, this study did not consider the problems of robot kinetic control and obstacle avoidance. The platform can realize three modes: 1) global communication with global perception, 2) local communication with local perception, and 3) no communication with local perception, corresponding to the three forms of global communication (GC), local communication (LC), and no communication (NC) defined in this study.
Algorithm 1 Information Processing Algorithm

Require: pre_info, processed_info, sense_info
1: for object_shared ∈ processed_info do
2:   count ← 0
3:   varPosInfo ← 0
4:   for info_record ∈ pre_info do
5:     if object_shared.id ∈ info_record then
6:       count ++
7:       varPosInfo ← varPosInfo + info_record[object_shared.id].pos − processed_info[object_shared.id].pos
8:     end if
9: end for
10: processed_info[object.id].pos ← processed_info[object.id].pos + varPosInfo/count
11: end for
12: for object_sensed ∈ sense_info do
13:   processed_info[object_sensed.id].pos ← object_sensed.pos
14: end for
15: pre_info ← Empty
16: sense_info ← Empty

A. FEEDBACK CONTROLLER WITH D-WORLD MODEL

This section proposes a dynamic small-world network model, the D-world model for a feedback swarm control system (5) with limited communication. The robots were separated into two parts in the LC network. One is the robots that directly observe the targets to acquire the exact position such that \( f(t) = \varepsilon_i \), and the other robots acquire the knowledge using the communication and consensus algorithm. \( \varepsilon_i \) denotes the position of the target. The method used to obtain the positions of the other robots in the network is similar to that of the target. The robot is the most direct observer of its coordinates and is shared over the network.

We designed a controller that obeys the feedback control (5), for communication constrained swarm robot, as shown in Fig.4. At each controller update, the four stages of 1) perception, 2) information processing, 3) motion decision, and 4) communication establishment are executed in sequence. It is assumed that the controller maintains three lists: 1) pre-info, in which communication information shared by adjacent robots is stored; 2) sense-info, in which the sensed object information is stored; and 3) processed-info, in which the processed information is stored. These three lists jointly comprise the state part of the controller.

For Perception, it is assumed that the robots can accurately identify each object with the corresponding ID and locate it in position \( P = (x, y, z) \). The object recognition algorithm can execute this procedure in the real world to assign each object a corresponding ID label. When executing the sensing stage, the controller stores the ID and coordinates \( P \) corresponding to the perceived target in the sense-info list.

In the information processing stage, when the robot observes the target, the value is replaced by practical knowledge. In other cases, the controller uses the knowledge in these three lists to perform an average consensus policy, as shown in (6) to update the controller state.

\[
 f(t) = K
 \nabla
 \varphi(p(t)) = K \sum_{j \in N_i} a_{ij}(p_j(t) - p_i(t)), \tag{6}
\]

where \( K \) denotes the weight parameter of knowledge diffusion, and its physical meaning is the diffusion coefficient, which is the knowledge of diffusion velocity. At this stage, the pre-info and sense-info lists are utilized to update the processed-info list, as shown in Alg.1.

After information processing, it steps into the motion decision stage. The motion decision followed the AFSA modified in this study, as described in Section III.B.

In the communication establishment stage, the robot designates connections with \( k \) neighboring robots if the controller adopts the RE model. In contrast, if the WS model is adopted according to the pre-generated network topology, the controller attempts to designate a communication link with the connected robot. The WS model implements the rewiring procedure of the connection based on the probability \( P_{	ext{rewire}} \) to yield a fixed network topology. However, RSSIs cannot be applied to a system with a limited communication range owing to their preset network topology, as shown in Fig.5. The connections will fail for the preset network topology when the communication range is overreached, which leads to the robot within the range not being effectively utilized in the communication connection. To introduce the small-world network topology and adapt to the dynamic trend of RSSIs, a dynamic topology, namely D-world, is defined based on a half-life period equation. An Alg.2 (pseudo-code representation) is executed for each controller to update with D-world.

First, it is assumed that the maximum communication links of a single robot are \( \zeta \) and the communication distance is \( R_p \). At the same time, the general communication links number is set to \( K \). In LC, a kd-tree [54] is used to establish the spatial relationship of the robots in the controller with a processed-info list. The position list \( \varphi \) of robots within distance \( R_p \) is queried. It is assumed that the current robot communication link list is represented as \( \text{Index}_\varphi = \{ \text{index}_1, \text{index}_2, \ldots, \text{index}_\kappa, \kappa < \zeta, \text{index}_\kappa \in \varphi \} \), where \( \text{index}_\kappa \) denotes the robot ID.

\[
 \varphi = \text{kd_tree}.\text{query}(P, R_p) \tag{7}
\]

where \( P \) denotes the position of the current robot. The controller first executes pre-communication to detect whether the robots in the \( \text{Index}_\varphi \) list exceed the communication range. Then, the communication link of overreaching ones will be removed from the \( \text{Index}_\varphi \). If \( \kappa < k \), the \( k - \kappa \) of neighboring robots are randomly picked from \( \varphi \) into \( \text{Index}_\varphi \) to designate the communication links. If \( \kappa < \zeta \) and \( \kappa > k \), a neighbor is randomly selected from \( \varphi \) with QueryPosibility probability into \( \text{Index}_\varphi \). It is considered that the longer the connection time, the more likely the communication link is to alter (decay). Simultaneously, only one communication link changes in one update of the controller iteration for local link stability. When
Algorithm 2 D-World Algorithm

Require: QueryNum, QueryPosibility, k, LinksMax, LinksMin, T1, di, epsilon, alpha
1: availableRobots ← PreCommunication()
2: RemoveRobotsNotInRange(availableRobot)
3: if getNumOfComNeighbors() < k then
4:   if Random() < QueryPosibility or getNumOfComNeighbors() < LinksMin then
5:     for i = 0 → QueryNum do
6:       if Len(availableRobots) > 0 then
7:         newRobotPartner ← Sample(availableRobots)
8:         isEstablish ← EstablishComLinkWith(newRobotPartner)
9:       if isEstablish then
10:          break
11:     end if
12:   end if
13: end for
14: end if
15: end if
16: ComRobotList ← GetComRobots()
17: for robotID ∈ ComRobotList do
18:   communicationTime[robotID] ← communicationTime[robotID] + epsilon
19:   t ← communicationTime[robotID]
20:   selfPos ← GetPosition(selfID)
21:   decayRobotPos ← GetPosition(robotID)
22:   p1p2Dist ← Distance(selfPos, decayRobotPos)
23:   T ← T1/(1 + (p1p2Dist * di))
24:   communicationSigma[robotID] ← 0.5(t/T)
25:   if Random() < alpha * (1 − communicationSigma[robotID]) * epsilon then
26:     for time ∈ communicationTime do
27:       time ← 0
28:     end for
29:     EndComLinkWith(robotID)
30: end if
31: end for

FIGURE 4. A feedback search control model.

a change occurs, the other communication links recover their initial stability, σ(t) = a₀. The instability factor σ(t) denotes the unchanging probability of communication link per second. In each controller update iteration, the period interval
is $\epsilon$, and the changing likelihood of the communication link in each iteration is $\alpha(1 - \sigma(t))\epsilon$. According to the half-life equation, $m = m_0(\frac{1}{2})^T$, (8) is proposed to estimate the value of $\sigma(t)$.

$$\frac{d\sigma(t)}{dt} = \frac{\sigma_0}{T}(\frac{1}{2})^T \ln\frac{1}{2}$$  \hspace{1cm} (8)

$$T = T_1/(1 + (distance(p_1, p_2)))$$  \hspace{1cm} (9)

$T$ denotes the time required for $\sigma(t)$ to decay to half its initial value. The further the node $p_1$ is from $p_2$, the shorter the time for $\sigma(t)$ to reach half of the initial value, as shown in (9). $\sigma_0$ denotes the initial stability and $\sigma_0 = 1$ in this study. When decay occurs, the link is dismissed from Index$_i$ and communication is interrupted. And all of the robot’s $\sigma(t) = \sigma_0$. The robot sends the processed information to each neighbor and stores it in the pre-info list as an object-shared message.

**B. DECENTRALIZED AFSA FOR AVERAGE CONSENSUS**

AFSA is an optimization algorithm based on swarm intelligence theory, which mimics the social behavior of fish schools and the motion mode. Through diverse individual behaviors, the fish sustain the aggregation of the school and are more likely to reach food in the water. AFSA has become one of the best optimization algorithms, owing to its sligher parameter reliance, higher convergence speed, flexibility, fault tolerance, and increased accuracy [55]. Based on the AFSA, a decentralized AFSA (dAFSA) was designed for average-consensus-based control. The flowchart is shown in Fig.7. For dAFSA, fish corresponds to the robot, fish school to the robot swarms, and food to the target to be searched.

First, a fitness function was defined to compute the value of the target distribution in the area. Equation (10) is as follows.

$$F_{sig} = Sig(F_{fitness}, \theta, \eta)$$  \hspace{1cm} (10)

$Sig$ is the sigmoid function, as shown in (13), where $\theta = 0.5$, $\eta = 0.5$. $F_{fitness}$ denotes fitness in the area, as shown in (11). $F_{sig}$ denotes the fitness after normalization using the sigmoid function.

$$F_{fitness} = argmax_k(1 - f_s(x, y, z))$$  \hspace{1cm} (11)

$$f_s(x, y, z) = \sqrt{(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2}$$  \hspace{1cm} (12)

$x, y, z$ denote the coordinates in space and $x_k, y_k, z_k, k = 1, 2, \ldots, M$ are the target coordinates. SenseDistance denotes the perceiving distance of the current robot. According to (11), the closer the spatial position is to any targets, the more eminent its fitness is.

$$Sig(X, \theta, \eta) = \frac{1}{1 + e^{-\eta(X-\theta)}}$$ \hspace{1cm} (13)

At the same time, a crowding factor $\delta$ is specified to estimate the crowding degree of fishes, avoid undeserved concentration of fishes, and enhance the searching capacity.

$$\delta = -\frac{F_{sig}(x_c, y_c, z_c)}{count_{in\_range}(R_c) \times F_{sig}(x_i, y_i, z_i)}$$  \hspace{1cm} (14)

$count_{in\_range}(R_c)$ denotes the number of robots within radius $R_c$.

The algorithm described four behaviors: preying, following, clustering, and leap. The fitness of the behaviors are estimated for determining which behavior pattern to adopt. The four behaviors are described as follows. 1) Swarm behavior: AFSA mimics the basic natural behavior of fish. For example, in the deep sea, fish can visually sense the concentration of food in water and then decide how to act. Fish schools avoid predators or dangers as much as possible by swarming together to save themselves. When applied to the RSSs algorithm, swarm behavior can be described as follows:

- $x_i, y_i, z_i$ is the current position of the fish.
- Equation (15) calculates the center position of the swarm $x_c, y_c, z_c$.
- Equation (10) estimates the fitness of the corresponding position.

If the fitness of the swarm center is greater than that of the current position and the crowding factor $\delta$ is less than the maximum crowding factor $\delta_{Max}$, as shown in (14). The robot then moves toward the swarm center, as shown in (16).

$$x_c = \frac{1}{N} \sum_{j=1}^{N} x_j$$  \hspace{1cm} (15)

$$y_c = \frac{1}{N} \sum_{j=1}^{N} y_j$$

$$z_c = \frac{1}{N} \sum_{j=1}^{N} z_j$$

In (15), $N$ denotes the total number of robots, and the average positions of all robots that can be perceived in the swarm are calculated as $x_c, y_c, z_c$.

$$\hat{P} = \frac{v\Delta t(P_c - P_i)}{\|P_c - P_i\|}$$  \hspace{1cm} (16)

$P_i$ denotes the current robot position, $X_i, Y_i, Z_i, P_c$ denotes the center of the robots, and $v$ denotes the average velocity of the robots.

2) Following behavior: In movement, some fishes may perceive food earlier and thus have higher fitness, so others may find food faster by following it. The RSSs face the same problem in robots, and the robots that follow the neighbor, which hold higher fitness, contribute to the rapid convergence.
of the swarm to the targets. This behavior can be described as follows: \(x_i, y_i, z_i\) denotes the robot’s current position, and it obtains the surrounding neighbors \(X_j, y_j, z_j\). If it is a GC, the real-time positions of all robots are obtained directly. The neighbors’ positions are obtained by communication with the average consensus for LC. There are only available neighbors’ positions within the visual range \(d_{ij} < \text{Visual}\) for no communication. Finally, the neighbor with the maximum \(Y_j\) value is selected. If the neighbor’s fitness is greater than the current fitness and \(\delta < \delta_{\text{max}}\), the neighbor’s position \(x_j, y_j, z_j\) is closer to the targets and less crowded, and then moves one step towards the neighbor \(X_j\), as shown in (17).

\[
\dot{P} = \frac{v \Delta t(P_j - P_i)}{\|P_j - P_i\|}
\]  

(17)

\(P_j\) denotes the position of neighbor \(x_j, y_j, z_j\). If none of them are satisfied, the prey behavior will be executed.

3) Prey behavior: An essential task for fish survival in water is to search for food. When performing the prey behavior of robot swarms, the robot moves one step to the position with the highest fitness in the range of perception, as depicted in (19).

\[
P_m = P_j + \text{Sense.rand}() \tag{18}
\]

\[
\dot{P} = \frac{v \Delta t(P_j - P_m)}{\|P_j - P_m\|} \tag{19}
\]

4) Leap behavior: With the increase in time, the behaviors of fishes will gradually become invariant, which may cause them to fall into the position of local extreme value. In robot swarms, particularly in LC, the swarm may converge to a “false target” in advance. The leap behavior allows the swarm to jump out of the local extreme value to obtain a better solution. In the algorithm, in an iteration, when more worthy fitness cannot be obtained by multiple (larger than Max Prey

FIGURE 6. Impact of communication on search. (a) Target searching with no communication (NC). (b) Target searching with local communication (LC). (c) Target searching with Global communication (GC).

FIGURE 7. dASFA flowchart.
IV. EXPERIMENTAL STUDIES

In this section, the numerical simulations and motion simulations are conducted. The primary characteristics of small-world and RE networks were explored in the search task of targets in numerical simulations. Furthermore, the impact on the sharing performance of the target knowledge was investigated using numerical simulations. The impact of each network model on the search algorithm is discussed along with the communication constraints in the motion simulation.

A. EXPERIMENTAL SETUP

The robot swarms, and the targets are considered as one-dimensional particles in the numerical simulations. The targets can move or remain in a one-dimensional area, an appointed robot (or a random robot) can sense them, and the knowledge of the targets is spread and synchronized in the swarm. The robot’s estimation of the initial target position was randomly distributed in the [0, 100] interval, and its standard deviation and mean deviation from the target position were calculated. The standard deviation $k_1 = 1.5$ and $k_2 = 1.0$, mean deviation $k_3 = 3$, and $k_4 = 1$ divide the knowledge synchronization into five stages. It is argued that before $K_1$, the agreeing stage between $K_1$ and $K_2$, the agreed stage after $K_2$, the converging stage between $K_3$ and $K_4$, and the converged stage after $K_4$. The experimental environment was extended to a 3D area for the motion simulation, and the simulation environment was $2000 \times 2000 \times 2000$, as shown in Fig.1. In the simulations, multiple red robots worked together to search for green-wedge targets. Robots that enter the communication range can establish communication links, as represented by green dotted lines. The robot’s visual field of perception is represented by a red dotted line. The dotted blue line shows the destination of the robot and the solid light blue line shows the trajectory of the robot. The fitness and crowded degree maps represent the current global target fitness and crowded degree distribution gradient of the AFSA, respectively. It is assumed that the robots are underwater robots with a limited communication range, perception range, and motion velocity. Moreover, its perception and communication model is shown in fig.9. The number of robots is $N = 100$. Its communication modes are set to three types: GC, LC, and NC, as shown in Fig.3. For LC, the knowledge of the swarm and targets is conveyed by the communication network. It is assumed that GC has the best search efficiency, and LC is between NC and GC, as proven by the simulations in the next section.

The simulation experiment mainly discusses the following issues.

1) How do the RE model, WS model, and D-world influence knowledge synchronization in the network with the average consensus rule?

2) How do the communication modes influence the swarm search efficiency with AFSA?

3) In the case of limited communication and perception, does the D-world model perform better than the RE network and WS model?

B. IMPACT OF COMMUNICATION STATUS ON NETWORK PERFORMANCE

This section explores the impact of the GC (Fig.3(a)), LC (Fig.3(a)), and NC (Fig.3(b)) Target perception and search.
First, numerical simulations of the network performance are discussed in the one-dimensional area. For GC, because each robot acquires the precise position of the targets at any time, all the robots’ knowledge of the targets is consistent with the actual position. For LC and NC, the perception is very different. To diminish the differences caused by network topology, it is assumed that only one robot senses the target simultaneously, and a new robot is randomly selected to sense the target every ten iterations of time. The robot swarm perception results for the target are shown in Fig.8(a) and Fig.8(b), while the target is immobile at \( x = 30 \). Fig.8(a) shows the robot’s perception of the target in NC mode, similar to Fig.8(c) (the target moves from position 0 with a speed of 0.3 per time iteration). The swarm robot’s knowledge of the target cannot converge to the target’s actual position. Fig.8(b) and Fig.8(d) are LC, whose topology is the RE network (Fig.10(a)). For Fig.8(b) with an immobile target, the robot swarms’ knowledge of the target can reach a consensus and converge to the actual value after several iterations. When the target is moving, the knowledge of the target cannot reach an effective consensus, and its convergence trend lags, as shown in Fig.8(d). The simulation results show that the robots have a significant difference in the perception of the target without communication, compared with the local communication results. In the absence of communication, only a few robots can accurately estimate the target position by directly observing the target, but most of the robots’ estimation of the target is seriously lagging, or even wrong.

Simultaneously, motion simulations with a single immobile target for the consensus search algorithm were conducted, and the impact of communication on search performance is shown in Fig.6. In the simulations, the robot’s angle of perception range with the three communication modes was \( \pi/2 \), and the perceptive distance was 800. Each robot in GC mode can immediately acquire knowledge of the swarm and target. The robots can only share knowledge through the RE network in the LC mode, but the communication distance is not limited in this section. There is no information to transmit in NC mode. Robots can only use their perception to locate their neighbors and targets. Consistent with our hypothesis, the search speed of GC is faster than that of LC, and LC is faster than that of NC. The experimental results show that no communication will seriously affect the swarm’s perception of the target, and the inaccuracy of the perception will affect the effectiveness of the robots’ motion decision.

### C. Impact of Network Topology on Knowledge Synchronization Without Communication Constraints

The previous section confirmed that communication can affect the sharing of target knowledge in the swarm and the
search convergence of the dAFSA for the target. This section will compare the impact of each network topology on knowledge synchronization. First, the change in network structure with time in the D-world model was tested, as shown in Fig. 11. The 100 nodes were arranged in a ring, with RE model connections starting between them. The setting of D-world is as follows. \( \text{QueryNum} = 10, \text{QueryPosibility} = 0.01, k = 6, \text{LinksMax} = 10, \text{LinksMin} = 2, T_1 = 100, d_i = 10, \epsilon = 0.03, \alpha = 0.1 \). The color depth of the nodes is related to their degree. The darker the color of the nodes, the greater the number of edges. The color of the edge is related to its stability. If it is close to light blue, the edge is
difficult to rewire, and if it is close to red, the probability of rewiring increases. It can be seen from the results that nodes tend to connect with neighboring nodes first in the network ruled by the D-world model. The farther apart the nodes are, the faster their links decay. Over time, D-world has held a small-world network pattern, as shown in Fig.11.

It is assumed that all robots follow the communication mode shown in Fig.3(b), namely LC. Meanwhile, its information processing and decision making follow the rules of (6). Then, three different network topologies are considered, namely, the RE network in Fig.10(a), WS model of Fig.10(b), and the D-world model in Fig.11. Through numerical simulations, the impact of each network topology on knowledge synchronization is shown in Fig.12. A target is immobile at the position of \( x = 30 \). Among the 100 robots, the same robot continuously sensed the target’s actual position to form new knowledge shared with the networks. After 500 iterations, each robot estimated its target position, as shown in Fig.12. At the same time, we also tested the result of repeating the random one robot perception target every ten iterations, as shown in Fig.13. Repeated statistical simulations were conducted because of the randomness of the generation of the network topology structure and the initial robot positions. Mode 1 was fixed robot perception, and Mode 2 was iterative random robot perception every ten iterations, as shown in the Table.2.

It can be seen from the results that the D-world is superior to the WS model in terms of consensus and convergence speed, while the WS model is superior to the RE network.

D. IMPACT OF D-WORLD ON SWARM SEARCH WITH COMMUNICATION CONSTRAINT

The above experiments reached the premises of the problem in this section. 1) LC can enhance the performance of swarm search, but there is still room for improvement. 2) The D-world model is superior to the WS model, and the WS
model is superior to the RE model in knowledge synchronization performance optimization.

Motion simulations were conducted on three network models, as shown in Fig.1. The number of robots was 100, the angle of perception range was $\pi/2$, the perception distance was 800, the communication distance was 1000, and the robot’s velocity was 30 per iteration. An iteration represents the minimum time interval of the controller’s updating cycles.
The number of robot connections in the RE and WS models was set to $K = 6$, the rewire probability of the WS model was set to $p_{\text{rewire}} = 0.1$, and the setting of D-world was as follows: $\text{QueryNum} = 10$, $\text{QueryPossibility} = 0.01$, $k = 6$, $\text{LinksMax} = 10$, $\text{LinksMin} = 2$, $T_1 = 100$, $d_i = 10$, $\epsilon = 0.03$, $\alpha = 0.1$. The impact of each network topology on the search performance was tested, and the simulation results are shown in Fig.14, Fig.15, and Fig.16. The results show that the robots can converge to the target position in a limited time with the three communication.
network topologies; however, the robots can find the target faster with the D-World model. It is assumed that robots within the target radius of 100 are used to find the target. For a single simulation, the number of robots that find the target at different times was counted, and the results are shown in Fig. 17. It can be seen from the results that the D-world model of robots can find the target faster. At the same time, the experiment was repeated 30 times. Three time points by

**FIGURE 16. Impact of D-world model on search algorithm.**
confirmed that D-world could hold the dynamic small-world network topology over time. The communication decay algorithm in the D-world model can maintain a few connections between the robot and its distant neighbors, and the stability of the local connections is also guaranteed in the network.

The characteristics of fixed topology of WS model make it advantageous in the application of network system with long-term constant structures, such as brain network, computer communication network, power transmission network, et cetera. But, the dynamic D-world extends the application advantages of small-world network to dynamic network systems, such as mobile robots communication network and social network. It makes small-world network have a broader application prospect. Unlike the classic WS model, the D-world is more adaptive to RSSs owing to its dynamic and decentralized characteristics. However, the WS model can more easily alter the aggregation factor, which is not controllable in D-world, and this needs to be improved in future studies.

V. CONCLUSION

Based on the classic small-world network model, this study proposes D-world model for the problem of ineffective motion decision caused by communication and perception constraints. Different from the fixed network structure of the WS model, this model can dynamically change the topology of the communication network to adapt to the limitation of the current communication range by a distributed method, without the failures of the fixed network structure. Then, the effect of knowledge synchronization and the ability of motion decision are enhanced. And a feedback control model for a swarm search task was designed for targets searching under communication constraints. The simulations show that, the swarms without communication will seriously affect the accurate perception of the target, compared with local communication. At the same time, these perceptual errors will further affect the effectiveness of swarm motion decision. It proves that communication can enhance swarm motion decision effectiveness. And there is a performance gap between the GC and LC modes based on the RE model. The D-world model achieves the best target search performance under communication constraints to minimize the gap. It shows a better knowledge synchronization performance than the WS and RE models through numerical simulation. Meanwhile, it was confirmed that D-world could hold the dynamic small-world network topology over time. The communication decay algorithm in the D-world model can maintain a few connections between the robot and its distant neighbors, and the stability of the local connections is also guaranteed in the network.

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