A study of the dynamical fluctuation property of jets is carried out using the Monte Carlo method. The results suggest that, the anisotropy of dynamical fluctuations in the hadronic system inside jets, changes abruptly with the variation of the cut parameter $y_{\text{cut}}$. A transition point exists, where these fluctuations behave like those in soft hadronic collisions, i.e. being circular in the transverse plane with respect to dynamical fluctuations.

The presently most promising theory of strong interaction — Quantum Chromo-Dynamics ($\text{QCD}$) has the special property of both asymptotic freedom and colour confinement. For this reason, in any process, even though the energy scale, $Q^2$, is large enough for perturbative $\text{QCD}$ ($\text{pQCD}$) to be applicable, there must be a non-perturbative hadronization phase before the final state particles can be observed. Therefore, the transition or interplay between hard and soft processes is a very important problem.

An ideal “laboratory” for studying this problem is hadron production in moderate-energy $e^+e^-$ collisions, e.g. at c.m. energy about in the range [10, 100] GeV. The initial condition in these processes is simple and clear. It can safely be considered as a quark-antiquark pair, moving back to back with high momenta. On the contrary, in other processes, e.g. in hadron-hadron collisions, the initial condition is complicated with the problem of hadron structure involved.

Theoretically, the transition between perturbative and non-perturbative $\text{QCD}$ is at a scale $Q_0 \sim 1–2$ GeV. Experimentally, the transition between hard and soft processes is determined by the identification of jets through some jet-finding process, e.g. the Durham algorithm. In these processes, there is a parameter — $y_{\text{cut}}$, which, in the case of the Durham algorithm, is essentially the relative transverse momentum $k_t$ squared, $k_t = \sqrt{y_{\text{cut}} \cdot \sqrt{s}}$. From the experimental point of view, $k_t$ can be taken as the transition scale between hard and soft. Its value depends on the definition of “jet”.

Historically, the discovery in 1975 of a two-jet structure in $e^+e^-$ annihilation at c.m. energies $\geq 6$ GeV has been taken as an experimental con-
firmation of the parton model, and the observation in 1979 of a third jet in $e^+e^-$ collisions at 17–30 GeV has been recognised as the first experimental evidence of the gluon. These jets, being directly observable in experiments as “jets of particles”, will be called “visible jets”. Our aim is to find the scale corresponding to these visible jets and to discuss its meaning.

For this purpose, let us recall that the qualitative difference between the typically soft process — moderate energy hadron-hadron collision — and the typically hard process — high energy $e^+e^-$ collision — can be observed most clearly in the property of dynamical fluctuations therein. The latter can be characterized as usually by the anomalous scaling of normalized factorial moments ($nfm$):

$$F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m (n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \propto (M)^{\phi_q} \quad (M \to \infty),$$

where a region $\Delta$ in 1-, 2- or 3-dimensional phase space is divided into $M$ cells, $n_m$ is the multiplicity in the $m$th cell, and $\langle \cdots \rangle$ denotes vertically averaging over the event sample. Note that when the fluctuations exist in higher-dimensional (2-D or 3-D) space, the projection effect will cause the second-order 1-D $nfm$ to go to saturation according to the rule:

$$F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a},$$

where $a = 1, 2, 3$ denotes the different 1-D variables. The parameter $\gamma_a$ describes the rate of approach to saturation of the $nfm$ in direction $a$ and is the most important characteristic for the higher-dimensional dynamical fluctuations. If $\gamma_a = \gamma_b$, the fluctuations are isotropic in the $a, b$ plane. If $\gamma_a \neq \gamma_b$, the fluctuations are anisotropic in this plane. The degree of anisotropy is characterized by the Hurst exponent $H_{ab}$, which can be obtained from the values of $\gamma_a$ and $\gamma_b$ as $H_{ab} = (1 + \gamma_b)/(1 + \gamma_a)$. The dynamical fluctuations are isotropic when $H_{ab} = 1$, and anisotropic when $H_{ab} \neq 1$.

For the 250 GeV/c $\pi(K)$-p collisions from NA22, the Hurst exponents are found to be $H_{p_t\phi} = 0.99 \pm 0.01$, $H_{pp_t} = 0.48 \pm 0.06$, $H_{p\phi} = 0.47 \pm 0.06$, which means that the dynamical fluctuations in this moderate-energy hadron-hadron collisions are isotropic in the transverse plane and anisotropic in the longitudinal-transverse planes. This is what should be expected, because

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\(^a\) In order to eliminate the influence of momentum conservation, the first few points ($M = 1, 2$ or 3) should be omitted when fitting the data to Eq.(2).
there are almost no hard collisions at this energy and the direction of motion of the incident hadrons (longitudinal direction) should be privileged.

In high energy $e^+e^-$ collisions, the longitudinal direction is chosen along the thrust axis, which is the direction of motion of the primary quark-antiquark pair. Since this pair of quark and antiquark moves back to back with very high momenta, the magnitude of the average momentum of final state hadrons is also anisotropic due to momentum conservation. However, the dynamical fluctuations in this case come from the QCD branching of partons, which is isotropic in nature. Therefore, in this case the dynamical fluctuations should be isotropic in 3-D phase space. A Monte Carlo study for $e^+e^-$ collisions at 91.2 GeV confirms this assertion. Also the presently available experimental data on $e^+e^-$ collisions at 91.2 GeV show isotropic dynamical fluctuations in 3-D.

Now we apply this technique to the “2-jet” sub-sample of $e^+e^-$ collisions obtained from a certain, e.g. Durham, jet-algorithm with some definite value of $y_{\text{cut}}$. Doing the analysis for different values of $y_{\text{cut}}$, the dependence of dynamical-fluctuation property of the “2-jet” sample on the value of $y_{\text{cut}}$ can be investigated. Two event samples are constructed from the Jetset7.4 and Herwig5.9 generators, each consisting of 400 000 $e^+e^-$ collision events at c.m. energy 91.2 GeV. The variation of $\gamma$’s of the 2-jet sample with $y_{\text{cut}}$ ($k_t$) are shown in Fig’s 1(a) and (b), respectively. It shows an interesting pattern. When $y_{\text{cut}}$ ($k_t$) is very small, the three $\gamma$’s are separate. As $y_{\text{cut}}$ ($k_t$) increases, $\gamma_p$ and $\gamma_\varphi$ approach each other and cross over sharply at a certain point. After that, the three $\gamma$’s approach a common value. The latter is due to the fact that when $y_{\text{cut}}$ is very large, the “2-jet” sample coincides with the full sample and the dynamical fluctuations in the latter are isotropic.

We will call the point where $\gamma_p$ crosses $\gamma_\varphi$ the transition point. It has

Fig.1 The variation of the parameter $\gamma$ with $y_{\text{cut}}$ ($k_t$)
the unique property $\gamma_p = \gamma_c \neq \gamma_y$, i.e., the jets at this point are circular in the transverse plane with respect to dynamical fluctuations. These jets will, therefore, be called circular jets.

The above-mentioned results are qualitatively the same for the two event generators, but the $y_{\text{cut}}$ ($k_t$) values at the transition point are somewhat different. The cut parameters $y_{\text{cut}}$, the values of the $\gamma$, the corresponding Hurst exponents $H$ and the relative transverse momenta $k_t$ at the transition point are listed in Table I.

| $\sqrt{s}$ (GeV) | $y_{\text{cut}}$ | $k_t$ (GeV/c) |
|------------------|------------------|--------------|
| 50               | 0.0186 $\pm$ 0.0012 | 6.82 $\pm$ 0.03 |
| 30               | 0.059 $\pm$ 0.002  | 7.28 $\pm$ 0.03 |

It is natural to ask the question: Is there any relation between the circular jets determined by the condition $\gamma_p = \gamma_c \neq \gamma_y$ and the visible jets directly observable in experiments as “jets of particles”? In order to answer this question, we plot in Fig. 2 the ratios $R_2$ and $R_3$ of “2-jet” and “3-jet” events as functions of the relative transverse momentum $k_t$ at different c.m. energies.

Let us consider the point where a third jet starts to appear. Historically, a third jet was firstly observed in $e^+e^-$ collisions at c.m. energy 17 GeV. It can be seen from Fig.2 that, for $\sqrt{s} = 17$ GeV, $R_3$ starts to appear at around $k_t = 8–10$ GeV/c, cf. the dashed vertical lines in Fig. 2. This value of $k_t$ is consistent with the $k_t$ value (4.3–6.3 GeV/c) of a circular jet within a factor of 2, cf. Table I. Thus we see that the circular jet, defined as a kind of jet circular in the transverse plane with respect to dynamical fluctuations, and the visible jet, defined as a kind of jet directly observable in experiments as a “jet of particles”, have about the same scale — $k_t \sim 5–10$ GeV/c.

In order to check how sensitively the magnitude of this scale depends on the c.m. energy of $e^+e^-$ collisions, a similar analysis is carried out for $\sqrt{s} = 50$ and 30 GeV using Jetset7.4, cf. Fig’s.1 (c, d). It can be seen that although

Table I $\gamma$, $H$, $y_{\text{cut}}$ (GeV/c) and $k_t$ (GeV/c) at the transition point

|            | $y_{\text{cut}}$ | $\gamma_y$ | $\gamma_p$ | $\gamma_c$ | $H_{y_{Pt}}$ | $H_{y_{Pt}}$ | $H_{y_{Pt}}$ | $k_t$ |
|------------|------------------|------------|------------|------------|--------------|--------------|--------------|-------|
| Jetset     | 0.0007 $\pm$ 0.0007 | 0.514      | 0.461      | 0.73       | 0.70         | 0.96         | 6.32         |
| Herwig     | 0.0022 $\pm$ 0.0008 | 1.237      | 0.633      | 0.637      | 0.73         | 1.00         | 4.28         |
the shape of $\gamma_i$ versus $y_{\text{cut}}$ ($k_t$) ($i = y, p_t, \phi$) changes considerably with energy the qualitative trend is the same for these energies. In particular, the transition point where $\gamma_{p_t}$ crosses $\gamma_{\phi}$ exists in all cases. The values of $y_{\text{cut}}$ and $k_t$ at the transition point are listed in Table II. It can be seen that the $k_t$ values are also in the range 5–10 GeV/c. This shows that the scale $k_t \sim 5–10$ GeV/c for the circular jet is universal, at least for moderate energy $e^+e^-$ collisions.

This scale is to be compared with the scale $k_t \sim 1–2$ GeV/c, which is the scale for the transition between the perturbative and non-perturbative domains. It is interesting also to see what happens in the results of jet-algorithm at this scale. It can be seen from Fig.2a (Jetset7.4) that, at this scale ($k_t \sim 1–2$ GeV/c) the ratio $R_2$ of “2-jet” events tends to vanish almost independently of energy, provided the latter is not too low. This can be explained as follows. Consider, for example, an event with only two hard partons, having no perturbative branching at all. Even in this case, the two partons will still undergo non-perturbative hadronization to produce final-state particles. If the $k_t$ is chosen to be less than $1–2$ GeV/c, then the non-perturbative hadronization with small transverse momentum will also be considered as the production of new “jets” and this “should-be” 2-jet event will be taken as a “multi-jet” event too. This means that, when $k_t < 1–2$ GeV/c, events with small transverse momentum will also become “multi-jet” ones, and $R_2$ vanishes. However, even when $k_t < 1–2$ GeV/c, a few 2-jet events may still survive if the hadronization is almost collinear. This effect becomes observable when the energy is very low, see, e.g., the $R_2$ curve for $\sqrt{s} = 6$ GeV in Fig.2a. A similar picture holds also for the results from Herwig5.9, cf. Fig.2b, but the almost-collinear hadronization appears earlier.

Let us give some comments on the physical picture behind the above-mentioned two scales. A circular (or visible) jet is originated from a hard parton. The production of this parton is a hard process. Its evolution
into final state particles includes a perturbative branching and subsequent hadronization. The hadronization is a soft process. The perturbative branching (sometimes called parton shower) between the hard production and soft hadronization connects these two processes. This perturbative branching inside a circular jet is certainly not soft, but is also not so hard. This kind of processes is sometimes given the name semi-hard in the literature. The isotropic property of dynamical fluctuations provides a criterion for the discrimination of the hard production of circular jets and the (semi-hard) parton shower inside these jets.

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