On the Deconfinement Phase Transition in Hot Gauge Theories with Dynamical Matter Fields

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Abstract

The phase structure of hot gauge theories with dynamical matter fields is reexamined in the canonical ensemble with respect to triality. Since this ensemble implies a projection to the zero triality sector of the theory we introduce a proper quantity which is able to reveal a critical behaviour of the theory with fundamental quarks. We discuss the properties of both the chromoelectric and chromomagnetic sectors of the theory and show while electric charges carrying a unit of $Z(N_c)$ charge are screened at high temperatures by dynamical matter loops, this is not the case for the $Z(N_c)$ magnetic flux. An order parameter is constructed to probe the realization of local discrete $Z(N_c)$ symmetry in the magnetic sector. We argue it can be used to detect a deconfinement phase being defined in terms of the screening mechanism as a phase of unscreened $Z(N_c)$ flux. It may be detectable at long range via the Aharonov-Bohm effect. We discuss the possible phase structure of QCD in this approach.

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1 Introduction

A new round of considerable interest in discrete global and local gauge symmetries started when it was conjectured that the discrete center $Z(N_c)$ of an underlying gauge group $G$ can be of crucial importance for quark confinement [1, 2]. Another milestone appeared with understanding the deeply rooted relation between the spontaneous breaking of the global $Z(N_c)$ symmetry and the deconfinement phase transition in pure gauge models on the lattice and in continuum [3]. This paper continues the previous studies of hot gauge theories with dynamical matter fields and emphasizes the importance of the magnetic sector in the investigation of their phase structure reliably. We propose a new order parameter to test the realization of the discrete $Z(N_c)$ symmetry at finite temperature and to measure screening effects in different regions of temperature and couplings. The result seems equally applicable to any $Z(N_c)$ and $SU(N_c)$ gauge models coupled to Higgs and/or fermion fields carrying $Z(N_c)$ charge, i.e. which are nontrivial on the center group.

A brief overview of commonly recognized results of finite temperature QCD analysis occurs quite pertinent for an introduction to the subject. So then, gluon fields are strictly periodic in time, while quarks are antiperiodic with a period given by the inverse temperature. Pure gauge theory has an exact $Z(N_c)$ global symmetry. The gauge invariant operator, the Polyakov loop (PL),

$$L_{\vec{x}} = \frac{1}{N_c} Tr \prod_{t=1}^{N_t} U_0(\vec{x}, t),$$

transforms under $Z(N_c)$ global transformations as

$$L_{\vec{x}} \rightarrow ZL_{\vec{x}}, \quad Z = \exp\left[\frac{2\pi i}{N_c} n\right], \quad n = 0, ..., N_c - 1.$$  \hspace{1cm} (1)

The PL can be used as an order parameter to test $Z(N_c)$ symmetry in pure gauge theory. The expectation value of the PL is interpreted as the free energy of a probe quark $F_q$ immersed in a pure gluonic bath

$$\langle L_{\vec{x}} \rangle = \exp\left(-\frac{1}{T} F_q\right).$$  \hspace{1cm} (2)

Unbroken $Z(N_c)$ symmetry implies $\langle L \rangle = 0$ and $F_q = \infty$. When the global $Z(N_c)$ symmetry is spontaneously broken, $\langle L \rangle$ develops a non-zero value giving a finite magnitude to $F_q$, i.e. it costs a finite energy only to create a single quark in the gluonic bath. However, in [4] it was discussed that in a system with a finite ultraviolet (UV) cutoff the free energy should not diverge. Usually, Monte Carlo (MC) and analytical calculations are also performed with periodic boundary conditions (PBC) in space directions. It has been shown, however, that the Gauss’ law and this PBC in space are inconsistent unless the sum of quark and gluon colour charges vanishes, what looks controversial since they have different triality. In fact,
it was concluded that for space PBC the expectation value of the PL is not the free energy of heavy quark.

The second trouble appears if we realize that in the spontaneously broken phase $<L>$ may pick up $N_c$ different values corresponding to $N_c$ equivalent minima of the free energy. Thus, $L$ can be negative or even complex and Eq.(2) tells us that the free energy could be some complex number. This gives rise to doubts that Eq.(2) has the proper physical interpretation. When dynamical quarks are included the picture becomes more complicated and new troubles appear. The fermion determinant generates loops winding around the lattice a number of times which is not a multiple of $N_c$. Such loops present a propagation of single quarks and transform non trivially under $Z(N_c)$. This means that dynamical quarks break $Z(N_c)$ symmetry explicitly and screen sources of heavy quarks at any temperature. The expectation value of the PL prefers the phase with $\arg L = 0$ which provides the minimum of the free energy. Other $Z(N_c)$ phases with $\arg L = \frac{2\pi k}{N_c}$, $k = 1, \ldots, N_c - 1$ become metastable. They possess, however, such unphysical properties as a complex free energy or entropy [5]. Recently, it has been discovered that chiral symmetry is not restored in $Z(N_c)$ phases [6], what led to another reexamination of degenerate $Z(N_c)$ phases and interfaces between them in pure gauge theory [7] and one of the conclusions is that all $Z(N_c)$ phases correspond to the same physical state and the interfaces are unphysical. The operator, which we introduce here may test directly the $Z(N_c)$ phases and hence can be used to clarify this important problem in the description of finite temperature QCD.

The main motivation of our study comes, however, from the following. It became almost a dogma to consider that the deconfinement phase transition is related to the appearance of nonzero triality above the critical point. A number of ‘order parameters’ were constructed aiming to detect a phase transition and nonzero triality states [8, 9]. To our knowledge all of them failed to display a phase transition and to test any nontrivial triality state. The most conclusive results here have been obtained in $Z(N_c)$ gauge theories with Higgs fields [10]. Since it had been known that at zero temperature this system has two phases, a confining/screening phase and a deconfining phase separated by a critical line and since such a critical line was not found at finite temperature it was concluded that presumably at finite temperature the critical behaviour is not present at all. Moreover, it has been claimed that there is no smooth relation of finite and zero temperature theory because no phase has been found at arbitrary small (but finite) temperature which could correspond to a deconfining phase at zero temperature in the same interval of bare coupling values. By now it became almost usual to define deconfinement as a ‘crossover’ but not a genuine phase transition in full theory and to study the ‘phase transition’ related to chiral symmetry restoration at light quark masses [11]. The question how this
The phase transition is related to the deconfining crossover at larger quark masses has not been answered positively yet.

Two points are to be discussed here. First of all, the deconfined phase was identified with the phase where nonzero triality should exist and order parameters have been constructed to detect such ‘free triality states’. The correctness of such a conjecture is not obvious a priori. In the context of this model, we demonstrate here that there might be a phase transition which, however, is not related to the liberation of triality but rather to different screening mechanisms of triality and introduce an operator which tests these different mechanisms. Second, the previous emphasizes have been put on the possible realization of the $Z(N_c)$ global symmetry in the electric sector of the theory as explained above. The magnetic sector has been untouched from this point of view. We intend to overcome this deficiency in the present paper putting the problem on a rigorous basis and suggesting to study local $Z(N_c)$ symmetry in this sector.

Certainly, we need an appropriate framework to study the problem. The grand canonical ensemble (GCE) with respect to triality is obviously not a proper tool since the $Z(N_c)$ symmetry is explicitly broken in this ensemble. In our previous papers we introduced an ensemble canonical with respect to triality [12] (see also [13]) to analyze the QCD phase structure [14, 15]. Since the $Z(N_c)$ symmetry is not explicitly broken in the canonical ensemble (CE) it is more suitable to study the QCD phase structure, especially the chromoelectric sector, therefore we shall use this formulation in the present paper as well. Of course, in general we expect in the thermodynamic limit the same phase structure both in the GCE and in the CE.

The paper is organized as follows.

In Section 2 we introduce the $Z(N_c)$ invariant quantities, $A_t$ and $A_s$, which can probe a unit of $Z(N_c)$ charge or flux and discuss its properties. These quantities give an opportunity to treat the chromoelectric and chromomagnetic sectors of the theory on the same footing applying them for the investigation of the ground state. In Section 3 we discuss in some details the general phase structure of gauge theories at zero and finite temperatures and outline a realization of local $Z(N_c)$ symmetry in the chromomagnetic sector of the theory different from the usually discussed chromoelectric sector. We define the deconfinement phase by analogy with the zero temperature deconfinement phase, i.e. as a phase where the kinetic screening due to the gluon-gluon interaction is stronger than the dynamic screening due to dynamical matter loops and propose an order parameter to test the corresponding phase transition. Further, we explain how to investigate the problem of domain walls in QCD and suggest a possible resolution. To check our predictions we study in the Section 4 a model of $Z(N_c)$ gauge fields coupled to discrete Higgs fields. We calculate the order parameters $A_t$ and $A_s$ in different sectors and for various
regimes of coupling constants and temperature. Finally we summarize our results and outline perspectives for the application of our results for $Z(N_c)$ and $SU(N_c)$ models with dynamical quarks.

2 An order parameter for $Z(N_c)$ charges and domain walls

In this section we construct an order parameter at finite temperature which may probe a nontrivial charge and/or flux in the Debye screened phase and can be represented by a $Z(N_c)$ invariant operator in a finite volume. Such a quantity was introduced some years ago in the context of gauge theories with nontrivial discrete abelian center at zero temperature \cite{16}. We will discuss that this quantity is able to detect a nontrivial phase of some state in a region enclosed by a surface $\Sigma$

$$A(\Sigma, C) = \frac{< F(\Sigma) W(C) >}{< F(\Sigma) > < W(C) >} ,$$

with the limits $\Sigma, C \to \infty$ taken. $W(C)$ is a space-time Wilson loop nontrivial on the $Z(N_c)$ subgroup. $\Sigma$ may be thought as a set of plaquettes in a given time-slice whose dual plaquettes $\Sigma^*$ form a closed two-dimensional surface for this time slice. Further, $\Sigma$ has to be chosen in such a way that it encloses at the considered slice a time-like line of the loop $C$. As will be discussed below, the operator $F(\Sigma)$ frustrates the plaquettes of $\Sigma$ by the so-called “singular gauge transformations” with a non-trivial element of $Z(N)$.

The action of this order parameter is based on the Aharonov-Bohm effect: despite the absence of electric fields a singular potential influences particles at arbitrary long distances giving them a nontrivial phase during winding around the solenoid. In our case a $Z(N_c)$ charge may be viewed as a kind of such a singular solenoid placed at some space position. To get this configuration one should take the Wilson loop $W(C)$ and consider the limit $C \to \infty$. Since the Wilson loop is a source of $Z(N_c)$ charge one can probe it via the “device” $F(\Sigma)$. A nontrivial $Z(N_c)$ charges may be detected only in the case when the fermionic screening is suppressed relatively to the Debye screening

$$\lim_{\Sigma, C \to \infty} A(\Sigma, C) = \exp \left[ \frac{2\pi i}{N_c} K(\Sigma, C) \right] ,$$

where $K(\Sigma, C)$ is the linking number of the surface $\Sigma$ and the loop $C$. If the triality charge is totally screened by fermions one gets $\lim_{\Sigma, C \to \infty} A(\Sigma, C) = 1$. 

5
Let us discuss the implementation of this order parameter for gauge theories at finite temperature. The following discussion is equally applicable for any gauge theory if it admits an introduction of the PL which transforms nontrivially under the discrete center of the gauge group, i.e. SU($N_c$) or $Z(N_c)$. We consider the lattice gauge model with a standard Wilson action given by

$$S_W = \lambda \sum_p Tr U_p ,$$

where $U_p$ is a product of gauge link matrices around a plaquette $p$. Then, since the fundamental PL is a source of $Z(N_c)$ charge at finite temperature one can insert the correlation function of PLs into (3) instead of a Wilson loop to get

$$A_t(\Sigma, R) = \frac{\langle F(\Sigma)L_0L_R \rangle}{\langle F(\Sigma) \rangle},$$

The operator $F$ which frustrates the plaquettes of $\Sigma$ is defined by

$$F(\Sigma) = \exp \left[ \lambda \sum_{p \in \Sigma} (Z-1)Tr U_p \right],$$

where $Z$ is a non-trivial element of $Z(N_c)$. One can then define the “frustrated action” in the following way

$$S_W \to S_F = \lambda \sum_p Tr U_p + \lambda \sum_{p \in \Sigma} Z Tr U_p .$$

A schematic diagram of the surface $\Sigma$ in space is shown in Fig.1 for the case of a (2 + 1)-dimensional theory. The two Polyakov loops $L_0$ and $L_R$ are closed in time direction. The frustrated plaquettes $\Sigma$ are drawn, they enclose the loop $L_R$. The links dual to the plaquettes of $\Sigma$ form a closed line $\Sigma^*$ surrounding $L_R$. They are shown by a dotted line. The frustration of the plaquettes of $\Sigma$ can also be achieved by multiplying the time-like links enclosed by $\Sigma$ with $Z$. Bold lines represent these links.

The canonical partition function reads

$$Z = \frac{1}{N_c} \sum_{k=1}^{N_c} \int dU_l \prod_{x,i} d\bar{\Psi}_x d\Psi_x e^{-S_W - S_q(k)} .$$

$S_q$ is the standard quark action, where we have to substitute $U_0 \to \exp[\frac{2\pi i k}{N_c}]U_0$ for one time slice in the CE [12]. Then, $\sum_k$ projects to zero triality states. We define a new partition function in the CE with the action $S_F$ (3) as

$$Z_F = \frac{1}{N_c} \sum_{k=1}^{N_c} \int dU_l \prod_{x,i} d\bar{\Psi}_x d\Psi_x e^{-S_F - S_q(k)} .$$
In what follows we refer to the expectation values $< \cdots >_F$ calculated in the ensemble defined by (10) as those obtained in the $F$-ensemble to distinguish them from the corresponding values $< \cdots >_0$ in the standard ensemble. It is easily seen that with such a definition $A_t$ in (6) reads

$$A_t(\Sigma, R) = \frac{< L_0 L_R >_F}{< L_0 L_R >_0}.$$

(11)

Figure 1: Correlation function of Polyakov loops in the $F$-ensemble for the case of a (2+1)-dimensional theory. The two Polyakov loops $L_0$ and $L_R$ extend in time direction. The frustrated plaquettes are explicitly depicted. Their dual links form a closed (dotted) line around $L_R$. Frustration can be achieved by multiplying of the bold links with a nontrivial element $Z \in Z(N_c)$.

To probe a $Z(N_c)$ charge in space one has to take the limit $R \to \infty$ where correlations of the ratio (11) reduce to one PL. Thus, one can probe a unit of $Z(N_c)$ charge enclosed by the surface $\Sigma$.

A similar $F$-ensemble can be constructed for the spatial Wilson loop. In this case the order parameter $A$ is essentially the same as in zero temperature theory, i.e. it is defined in formula (3) where we should insert a pure space-like Wilson loop instead of a time-like one. In what follows we use the notation $A_s$ to distinguish it from $A_t$ introduced above. The meaning of the operator in this case is however different as we shall discuss in the next section.
Let us investigate now some simple properties of $A_t$ and $A_s$. In a pure gauge system the surface $\Sigma$ may be completely gauged away by a change of variables $U_0(x) \to Z^* U_0(x)$ on the time-like links in the three dimensional volume enclosed by the surface $\Sigma$. It follows that $A = Z \neq 1$ for the pure gauge system since the fundamental PL in the origin changes sign, precisely as in the theory at zero temperature [16]. When dynamical fermion (Higgs) fields are added, the surface $\Sigma$ cannot be gauged away since the corresponding time-like fields $U_0$ will be flipped in the fermionic (Higgs) part of the action, so that

$$S = \lambda \sum_p Tr U_p + \alpha \sum_{l \notin \Omega(\Sigma)} Q_l(U_l) + \alpha \sum_{l \in \Omega(\Sigma)} Q_l(Z^* U_l),$$

where $Q_l$ means the density of either the fermion or the Higgs action and $\Omega(\Sigma)$ is a volume enclosed by the surface $\Sigma$. $\sum_l$ is the sum over links. The same obviously holds for $A_s$ with the corresponding substitution of space-like links instead of time-like ones.

An important interpretation for the influence of $F(\Sigma)$ comes from the following observation. $F(\Sigma)$ tries to implement a phase change at the spatial surface $\Sigma$. If the global $Z(N_c)$ symmetry is spontaneously broken $F(\Sigma)$ produces a stable interface between the volume enclosed by $\Sigma$ and the surrounding vacuum. The different phases can be detected by the PL and the order parameter $A_t(\Sigma, C)$. If the symmetry is unbroken $F(\Sigma)$ cannot produce a stable interface and the value of the PL is not influenced by $F(\Sigma)$ which is far apart.

The most essential property of $A_t$ and $A_s$ is, however, that they measure screening effects of dynamical fermion (Higgs) loops (dynamical screening) together with screening effects of the kinetic energy due to the pure gluonic interaction and show which screening is stronger at given conditions. The 'Aharonov-Bohm effect' works only when screening due to fermion (Higgs) loops is weaker so that it is possible to detect a charge (flux) at spatial infinity.

The simplest example where the quantity $A$ can be calculated exactly is the Gaussian model for a scalar field coupled to an external field. We want now to consider this example which may be quite suggestive since it illustrates how the operator works; in this case it does not correspond to any discrete charge but rather shows the competition between two kinds of screening and measures which one is stronger. Let us define the partition function of the massive Gaussian model in an external field $h_x$ as

$$Z_0 = \int_{-\infty}^{\infty} \prod_x d\sigma_x \exp \left( -\frac{1}{2} \sum_{x,n} (\sigma_x - \sigma_{x+n})^2 - m^2 \sum_x \sigma_x^2 + \sum_x h_x \sigma_x \right).$$

In this case $\Sigma$ consist of links dual to a $(D-1)$-dimensional surface $\Sigma^*$. The operator
\( F(\Sigma) \) is introduced on the stack of links \( \Sigma \) and has the form

\[
F(\Sigma) = \exp \left[ -\frac{1}{2} \sum_{l \in \Sigma} (\sigma_x + \sigma_{x+n})^2 + \frac{1}{2} \sum_{l \in \Sigma} (\sigma_x - \sigma_{x+n})^2 \right] = \exp \left[ -2 \sum_{l \in \Sigma} \sigma_x \sigma_{x+n} \right].
\]

In what follows we consider a \( D \)-dimensional periodic lattice with period \( L \) and number of sites \( V = L^D \). The region \( \Omega \) enclosed by \( \Sigma \) we choose to be a \( D-1 \) torus winding around the lattice. Performing now the substitution \( \sigma \to -\sigma \) in the interior \( \Omega \), we can write down the partition function in the \( F \)-ensemble as

\[
Z_F = \int_{-\infty}^{\infty} \prod_x d\sigma_x \exp \left[ -\frac{1}{2} \sum_{x,n} (\sigma_x - \sigma_{x+n})^2 - m^2 \sum_x \sigma_x^2 + \left( \sum_{x \in \Omega(\Sigma)} - \sum_{x \in \Omega(\Sigma)} \right) h_x \sigma_x \right].
\]

(15)

Figure 2: Correlation function of spins in the \( F \)-ensemble for the case of a 2\(D\) theory. The frustrated links \( \Sigma \) are explicitly depicted. Their dual links form two closed lines around \( \sigma_R \) and are chosen to wind around the whole lattice.

After this transformation we have

\[
A = -\frac{< \sigma_0 \sigma_R >_F}{< \sigma_0 \sigma_R >_0},
\]

(16)

where \( < \sigma_0 \sigma_R >_0 \) and \( < \sigma_0 \sigma_R >_F \) are the correlation functions calculated in the usual and the \( F \)-ensemble, correspondingly. A diagram of the correlation function in the \( F \)-ensemble is plotted in Fig.2. On a finite lattice \( < \sigma_0 \sigma_R >_0 \) is known to be

\[
< \sigma_0 \sigma_R >_0 = \frac{1}{2V} \sum_{k=0}^{L-1} M_k^{-1} \exp \left[ \frac{2\pi Rk}{L} \right]
+ \frac{1}{4V} \left( \sum_{k=0}^{L-1} M_k^{-1} h_k \exp \left[ \frac{2\pi Rk}{L} \right] \right) \left( \sum_{k=0}^{L-1} M_k^{-1} \right),
\]

(17)
where
\[ h_k = \frac{1}{\sqrt{V}} L \sum_{k=0}^{L-1} h_x \exp\left[i \frac{2\pi x k}{L}\right] \] (18)
and the propagator \( M_k^{-1} \) in the momentum space has the well known form
\[ M_k^{-1} = (D + m^2 - \sum_n \cos \frac{2\pi k_n}{L})^{-1}. \] (19)

For \( <\sigma_0\sigma_R>_F \) we should change signs for \( h_x \) in the Fourier transform (18) at corresponding sites. Let us consider now the simple case of a constant external field. We have from (18) \( h_k = \sqrt{V} \delta_{k,0} h \). In the thermodynamic limit \( V \rightarrow \infty \) we get the following expression
\[
A = -\frac{G(R) + h^2/2 \left(M_0^{-1} - 2f(R)\right) \left(M_0^{-1} - 2f(0)\right)}{G(R) + h^2/2(M_0^{-1})^2},
\] (20)
where we used
\[
G(R) = \int_0^{2\pi} \left(\frac{d\phi}{2\pi}\right)^D \frac{e^{i\phi R}}{D + m^2 - \sum_{n=1}^D \cos \phi_n},
\] (21)
\[
f(R) = \sum_{n=0}^{L_F-1} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{e^{i\phi(R+n)\cos \phi}}{1 + m^2 - \cos \phi}
\] (22)
and \( M_0 = m^2 \). \( L_F \) in the last formula denotes the number of sites with negative sign of \( h \). In any dimension \( G(R) \) goes to zero exponentially as \( R \) increases. In the limit \( R \) and \( L_F \rightarrow \infty \) one can get the final expression for \( A \)
\[
A = \frac{m}{\sqrt{2 + m^2}}.
\] (23)

One can see from the last formula that for a constant external field \( A \) is always positive. It is interesting to note that the limit \( m \rightarrow 0 \) exists even for the one- and two-dimensional cases and gives \( A = 0 \). The limit \( m \rightarrow \infty \) corresponds to the value \( A = +1 \). This is the only mass when \( A = 1 \) since the scalar field \( \sigma \) does not carry a unit of any discrete charge. As we discussed earlier in Eq. (4), for fields carrying a discrete charge we would find only discrete values of \( A \).

The interpretation of these results is rather transparent. There is a competition in the correlation function between the term \( G(R) \) and the second term of (17). \( G(R) \) comes from the kinetic energy of the system, decays exponentially and describes the corresponding Debye screening. The second term comes from the external field and describes “screening” due to this field. We think this example shows that \( A \) is able...
to determine which screening mechanism is stronger. In fact, it is not difficult to find a nonconstant external field such that in the limit $R \rightarrow \infty$ the quantity $G(R)$ goes to zero slower than the second term of the correlation function and hence $A = -1$. A special example of this type we will mention later.

3 Screening at zero and finite temperatures and the deconfinement phase transition

We are now ready to proceed to study the phase structure of gauge models with a nontrivial center. We would like to explore the above mentioned properties of $A_t$ and $A_s$ to reveal some features of these models. Let us start with a discussion of pure gauge theory at zero temperature. First, we are interested in $Z(N_c)$ gauge models. These models are known to exhibit confinement at strong coupling and deconfinement behaviour at weak coupling. This means that the Wilson loop obeys an area law in the first case and a perimeter law in the second. The interpretation of such a behaviour in terms of the quantity $A$ has been given in [16] and amounts essentially to: $A = -1$ and the absence of dynamical screening of a $Z(N_c)$ charge at any coupling. The Wilson loop in the $F$-ensemble always changes its sign, though in different ways, in strong and weak coupling regimes. At weak coupling the ground state of the system consists of gauge spins flipped inside a volume bounded by $\Sigma$ relatively to gauge spins outside. A perimeter law for the Wilson loop arises in this phase due to the kinetic energy of gluons as can be seen most easily in the Hamiltonian formulation (see [13] for a comprehensive discussion of this topic). If we add Higgs fields to this system dynamical screening appears, causing a competition with the screening coming from the gluon kinetic energy. Before we proceed further we need to give more precise definitions of dynamical screening and of screening coming from the gluon kinetic energy.

Let $W(C)$ be a Wilson loop in $Z(N_c)$ lattice gauge theory either at zero or at finite temperature. In the weak coupling phase of pure gauge theory one may write

$$< W(C) > \sim \exp(-\gamma_{gl} P),$$

where $P$ means the perimeter of the loop $C$. We refer to nonzero $\gamma_{gl}$ in the deconfinement phase as coming from the kinetic energy of the pure gluonic interaction and attribute the corresponding kinetic screening to this interaction. Let us imagine now this system coupled to dynamical matter fields, either to Higgs or to fermion ones. Among other contributions to the Wilson loop the dynamical fields generate a perimeter decay of the loop at any coupling and/or temperature. In the $C \rightarrow \infty$
limit one always finds the following behaviour of the loop in the strong coupling limit, i.e. \( \lambda = 0 \) in (9)

\[
<W(C) \sim \exp(-\gamma_{dyn} P). \tag{25}
\]

We refer to \( \gamma_{dyn} \) as coming from the dynamical screening due to the dynamical Higgs or fermion fields.

Let us suppose now that the system is in a region of coupling constants where the pure gauge interaction produces only area law decay for the Wilson loop, i.e. \( \lambda \ll 1 \). One has the following formal expansion

\[
<W(C) \sim K_1 \exp(-\gamma_{dyn} P) + K_2 \exp(-\alpha_{gl} S) + ... , \tag{26}
\]

where \( \alpha_{gl} \) is a string tension of the pure gauge theory and \( S \) is the minimal area enclosed by the loop \( C \). Dots mean all the (perimeter and area) terms which vanish faster in the thermodynamic and \( C \to \infty \) limits than the corresponding written terms. From the properties of \( A \) described in the previous section it is clear that in the \( F \)-ensemble the term with area law decay should change its sign. The expansion might look like

\[
<W(C) \sim K_1 \exp(-\gamma_{dyn} P) - K_2 \exp(-\alpha_{gl} S) + ... . \tag{27}
\]

Substituting these expansions into (28) and taking the limit \( C \to \infty \) we find \( A = 1 \) since the area term coming from the pure gluon interaction vanishes clearly faster

\[
A(\Sigma, C) = \frac{< W(C) >_F}{< W(C) >_0} \to_{C \to \infty} 1. \tag{28}
\]

Thus, \( A \) takes a trivial value and we interpret this behaviour as reflecting the following fact: in the confinement region the dynamical screening is the only one screening present in the system. Actually, in this region there is not even a real competition. However, in the pure gauge system there is a critical point which of course is not a critical point of a full system. Above that one has instead of (28)

\[
<W(C) \sim K_3 \exp(-\gamma_{dyn} P) + K_4 \exp(-\gamma_{gl} P) + ... . \tag{29}
\]

In the \( F \)-ensemble one has to change the sign of \( K_4 \). There is a competition between the kinetic screening and the dynamical screening. One gets

\[
A(\Sigma, C) = \begin{cases} 1, & \gamma_{dyn} < \gamma_{gl}, \\ -1, & \gamma_{dyn} > \gamma_{gl}. \end{cases} \tag{30}
\]

For the second possibility the kinetic screening is stronger and a \( Z(N_c) \) charge can be detected. We interpret this as a permanent feature of the deconfinement phase. Simple and maybe well known conclusions are:
1) deconfinement is the phase at a small gauge coupling constant;
2) deconfinement may take place even in the presence of dynamical screening;
3) $A$ acts equally good for time and space-like Wilson loops since there is no actual difference between them in zero temperature theory. It distinguishes between confined/screened phase and deconfined phase.

The first observation is rather important especially in the context of finite temperature theory. Usually, the deconfinement phase is associated with the high temperature region identifying the latter with the region of small couplings. In fact, at finite temperature the original formulation involves two free and completely independent parameters: the temperature and the bare coupling. The region of small bare coupling needs a priori not to be the phase of high temperature. In fact, rigorous results on the deconfinement transition in pure gauge models \cite{18} state that as soon as the temperature is turned on there exists such a coupling below that the system is in the deconfined phase.

It is also interesting to mention that the equation \eqref{30} predicts an exact equation for the critical line in the theory, namely
\begin{equation}
\gamma_{\text{dyn}}(\alpha) = \gamma_{\text{gl}}(\lambda),
\end{equation}
where $\lambda$ and $\alpha$ are the gauge and Higgs (fermion) couplings, respectively.

The main question arising from these observations which we would like to put here concerns special features of the deconfined phase at finite temperature relatively to the one at zero $T$? We argue that in fact there are no special features, breakdown of global and/or local $Z(N)$ symmetry at high temperature may be misleading in this respect if one deals with the full theory. Triality is always a conserved quantity and to reveal the critical behaviour one has to look at the screening mechanisms of triality in different regions of coupling. We therefore suggest to define the deconfinement phase in the theory with dynamical matter fields as the weak coupling phase where the kinetic screening due to the gluon interaction is stronger than dynamical screening. A similar proposition was discussed also in \cite{13}. Technically however the situation at finite temperature may differ from the picture discussed above.

Let us discuss now in detail what we expect for finite temperature theory. We continue considering $Z(N_c)$ gauge models in the Euclidean path integral formulation. There is no symmetry anymore between electric and magnetic sectors at finite temperature, in particular, one can find a drastic difference in the behaviour of the corresponding Wilson loops, i.e. the spatial Wilson loop and the thermal (Polyakov) loop. The spatial Wilson loop behaves essentially as in the zero temperature case. Thus, the first observation which can be easily deduced from this fact is that the quantity $A_s$ should be as good an order parameter at finite temperature as it is at zero temperature because all formulae above are valid in this case if we substitute
$W(C)$ by the spatial Wilson loop and $A$ by $A_s$. A nontrivial value of $A_s$ should imply that a unit of $Z(N_c)$ flux is not screened dynamically and can be detected at long-range. Therefore, if there is a critical behaviour at zero temperature it should also be present at finite temperature. We would like to stress that this observation does not depend on the ensemble (CE or GCE) used to study the finite temperature system (see also next section).

On the other hand, the behaviour of order parameter $A_t$ could be qualitatively described as follows. In the strong coupling phase the correlation function of PLs calculated in the pure gluonic sector goes down exponentially with increasing distance. The fermionic sector, however generates terms which screen heavy quarks and lead thus to a constant value of the correlator even at spatial infinity. This implies $A_t = 1$. In the weak coupling region the pure gluonic sector also gives finite values for the correlation at spatial infinity leading in such a manner to the competition with dynamic screening. In such a situation the direct use of $A_t$ as an indicator of a phase transition is not possible because both contributions stay finite in the $R \to \infty$ limit. One may argue, however that $A_t = 1$. To understand qualitatively what one should expect in such a situation one can study some simpler models with similar properties. Such studies were done by us in [17] on the example of the $3D$ Ising model where we were able to define and to calculate an analogue of the quantity $A$ for spin systems in the presence of an external magnetic field. It was done both analytically and in MC simulations. Applying that consideration to the $Z(2)$ gauge model we conjecture that in the weak coupling phase the value $A_t = 1$ has the following meaning: The operator $F$ introduces a stable interface in the pure gauge system. In the weak coupling region the main state is the one where all the PLs are flipped inside the volume enclosed by the surface $\Sigma$ relatively to the loops outside of the volume. $A_t = -1$, and this means that the interfaces are stable. Dynamical matter loops try to destroy such interfaces. At sufficiently weak coupling the main state has all loops aligned in one of the $Z(N_c)$ direction (in the canonical ensemble). The state with the loops flipped inside of $\Sigma$, which could potentially lead to a nontrivial value appears to be suppressed by a volume term whereas the contribution of the state with aligned loops is suppressed with a surface term. There is a necessary mathematical condition for $A_t$ to take a nontrivial value: the expansion for large masses or small Higgs coupling has to converge in the electric sector and in the thermodynamic limit. The consideration in [4] shows that such an expansion is presumably not convergent at any finite temperature. It means in turn that it is impossible to expand the correlation of the PLs similarly to the Wilson loop at zero temperatures, e.g. (26) and (29). Indeed, if one wants to make such an expansion one would find at the first nontrivial order in small Higgs or fermion and
large inverse gauge couplings the equations for $A_t$ (see next section for details)

$$A_t(\Sigma, R) = \frac{< F(\Sigma)L_0L_R >}{< F(\Sigma) >> L_0L_R >} = \pm V C e^{-4D\lambda},$$  \hspace{1cm} (32)

where $\lambda$ is gauge coupling, $V$ is the volume and the constant $C$ is either proportional to the Higgs or to the fermion coupling. A similar expansion one can find for the correlator in the $F$-ensemble. One sees from here that the thermodynamic limit does not exist already at the first nontrivial order, even for the correlation of the PLs. One concludes that in this situation $A_t$ cannot determine which screening is stronger. However, it gives an interesting information on the domain wall structure of the high temperature phase. The value $A_t = 1$ corresponds as explained above to an instability of the interfaces of pure gauge theory. We argue thus that the order parameter $A_t$ can be used to resolve a long standing problem of high temperature QCD, namely the problem of the stability of domain walls. In the CE all $Z(N_c)$ directions are degenerate but in the thermodynamic limit, as the behaviour of $A_t$ shows there is no stable interfaces between regions with different orientations of the PL. In the next section we mention however a possibility where the interfaces could be stable and $A_t = -1$.

Let us shortly summarize. In the confined phase a $Z(N_c)$ charge does not exist in the sense that quarks are bound in triality zero states, i.e. mesons and baryons. Whatever small it is, dynamical screening is finite at any long range. Thus, a ‘free’ $Z(N_c)$ electric charge cannot be found and $A = 1$. In the weak coupling phase, quarks may become free and kinetic screening appears to play an important role. The correlation function is finite because of this screening which is defined as coming from the pure gluonic part. It is however rather impossible to distinguish between this and dynamical screening because both of them lead to a finite value of the correlation of the PLs at spatial infinity. The attempt to separate the different types of screening leads to volume divergences. This fact does not imply that the critical behaviour is lost, it concerns only purely time-like loops. One can use the quantity $A_s$ to measure screening in the magnetic sector, i.e. the presence of a unit of the $Z(N_c)$ flux. Such a treatment should reveal the critical behaviour if it is present at zero temperature. Indeed, the screening mechanism in the magnetic sector is not much affected by dynamical matter loops winding around the lattice in time direction and should be essentially the same (in the thermodynamic limit) as at zero temperature.

In the next section we show how these ideas work in the theory of $Z(2)$ gauge spins coupled to Higgs fields.
4 Application: $A$ in $Z(2)$ gauge theory

As an application of the proposed approach we would like to examine here the model of $Z(2)$ gauge spins coupled to discrete Higgs fields at finite temperature. The canonical partition function of the $Z(2)$ gauge model is given by the path integral

$$Z = \frac{1}{2} \sum_{\sigma = \pm 1} \sum_{s_t = \pm 1} \sum_{z_x = \pm 1} e^{S_W + S_H}.$$  \hspace{1cm} (33)

The sum over $\sigma$ is the sum over two n-ality sectors. We have denoted

$$S_W = \sum_{p_0} \lambda_0 U_{p_0} + \sum_{p_n} \lambda_n U_{p_n},$$  \hspace{1cm} (34)

where $U_{p_n} (U_{p_0})$ is a product of $Z(2)$ gauge link variables $s_\mu(x)$ around a space(time) plaquette. The Higgs action at finite temperature we write down as

$$S_H = S_H^{sp} + S_H^{cl} = \sum_{x,n} h_n z_x s_n(x) z_{x+n} + \sum_x h_0 z_x \sigma s_0(x) z_{x+0},$$  \hspace{1cm} (35)

where both gauge $s_\mu(x)$ and Higgs $z_x$ fields obey a periodicity condition. The phase diagram of this theory at zero temperature was well established long ago [10, 19] and was discussed in the previous section. The phase diagram at finite temperature is rather unclear. It has been recently claimed that pure $Z(N_c)$ gauge model may well possess 4 distinct phases in terms of bare coupling depending on actual values of $\lambda_0$, $\lambda_n$ and $N_t$ [20]. Ref. [9] claims that the coupling of gauge spins to the Higgs field leads to a trivial phase structure, the system is in a confining/screening phase at all couplings and temperatures. The $U(1)$ abelian Higgs model at nonzero temperature was studied earlier in [21]. It has been argued that there is no phase transition in temperature, neither increasing the temperature from the confining/screening phase nor from the deconfined phase. However, there should be a phase transition at finite temperature in the coupling constant from the confining/Higgs phase to the deconfined phase. It was speculated that this phase boundary may disappear above some finite temperature but the issue remains unsettled. We expect a similar scenario to be realized in the $Z(2)$ Higgs model, i.e. the deconfinement phase transition is a transition in the coupling constant either at zero or at some finite temperature.

At zero temperature the order parameter $A$ was calculated for the case of $Z(2)$ gauge spins coupled to Higgs fields in Ref. [13]. It was shown that $A$ indeed is an order parameter changing abruptly from $A = 1$ (confining/dynamical screening phase) to $A = -1$ (deconfining phase).
Let us study first $A_t$ in the finite temperature Higgs model. Shifting the surface $\Sigma$ via $Z(2)$ singular gauge transformations to the Higgs part of the action we get

$$A_t(\Sigma, R) = \frac{< L_0 L_R >_F}{< L_0 L_R >_0},$$

where the PL is given by

$$L_0 = \prod_{t=1}^{N_t} s_0(0, t).$$

The time-like part of the Higgs action reads

$$S^t_H = \sum_x h_0(x) z_x \sigma s_0(x) z_{x+0},$$

where

$$h_0 \rightarrow h_0(x) = \begin{cases} h_0(x \notin \Omega), \\ -h_0(x \in \Omega) \end{cases}$$

and $\Omega$ is as in (12). We would like now to analyze two regions of the gauge couplings $\lambda_0$ and $\lambda_n$, the weak and the strong coupling regions, at arbitrary $N_t$. The corresponding expansions are known to be convergent, precisely as in the zero temperature theory [10, 22].

I. $A_t$ in the strong coupling region, $\lambda_0$ and $\lambda_n << 1$.

We want to get an expression for the partition and the correlation functions which could be used in both ensembles and both for $A_t$ and $A_s$. For that purpose we introduce a space-time dependence in the Higgs couplings and use the general notation $h_{\mu}$. Following the standard scheme of the strong coupling expansion we get (up to the third order), e.g. for the partition function the expression

$$Z = C(\lambda) \prod_l \cosh h_{\mu}(x)[1 + \sum_p \tanh \lambda_{\mu} \prod_{l \in p} \tanh h_{\mu}(x) + \sum_{p \neq p'} \tanh \lambda_{\mu} \tanh \lambda_{\mu'} \prod_{l \in p,p'} \tanh h_{\mu}(x) + \sum_{p \neq p'} \tanh \lambda_{\mu} \tanh \lambda_{\mu'} \prod_{l \in p,p'} \tanh h_{\mu}(x)]$$

and we introduced the obvious notation $\lambda_{\mu}$. $\sum_p$ means sum both over time-like and space-like plaquettes, $\sum_{p}$ means that one has to omit plaquettes which have a link in common and $\sum_{p}$ means sum over plaquettes which have one link in common. $\prod_{l \in p,p'}$ means that one has to omit that link from the product which is common for both plaquettes. At last,

$$C(\lambda) = (\cosh \lambda_0)^{N_{p_0}} (\cosh \lambda_n)^{N_{p_n}},$$
where \( N_{p_0}, N_{p_a} \) is the number of time(space)-like plaquettes, correspondingly. The general expansion for the correlation of the PL is

\[
< L_0 L_R > = \frac{1}{Z} C(\lambda) \sum_{s_1=\pm 1} \sum_{z_2=\pm 1} L_0 L_R e^{S_H} \left[ 1 + \sum_{p} \tanh \lambda_p S_p + \sum_{p \neq p'} \tanh \lambda_{p'} S_{p'} + O(\lambda^3) \right]. \tag{41}
\]

Now it is straightforward to calculate correlations of the PL in both the standard and the \( F \)-ensemble. Let us suppose that the frustrated links are in the time slice \( t = 1 \). Since in this case space couplings \( h_n(x) \) are not affected by a singular transformation one can omit the space-time dependence. It gives up to second order

\[
< L_0 L_R > = \prod_{t=1}^{N_t} \tanh h_0(0, t) \tanh h_0(R, t) [1 + 2D N_t \tanh \lambda_0 \tanh^2 h_n(1 - \tanh^2 h_0)], \tag{42}
\]

where \( D \) is the space dimension. Since the linking number of the PL in the origin and the surface \( \Sigma \) is 1, it gives the result

\[
A_t = 1 + O(\lambda^2).
\]

In fact, it is obvious that the expression in the square brackets of (41) is an even function of \( h_0(t) \) up to the order \( (\tanh \lambda_0)^{L_\Sigma} \), where \( L_\Sigma \) is the radius of the region \( \Omega \). Only on the boundary the corresponding plaquettes will change signs. Thus,

\[
A_t = 1 - N_\Sigma C_1 (\tanh \lambda_0)^{L_\Sigma}, \tag{43}
\]

where \( N_\Sigma \) is the number of frustrated plaquettes and \( C_1 \) is independent of \( \Sigma \). It leads in the \( \Sigma \to \infty \) limit to the expected result \( A_t = 1 \).

II. \( A_t \) in the weak coupling region \( \lambda_0, \lambda_n \gg 1 \).

We fix a static gauge where all gauge spins in time direction \( s_0(x) \) are set to 1 except for one time-slice \( t = 1 \) which includes the frustrated plaquettes on \( \Sigma \). The Debye screening comes from the pure gluonic action and produces the formula for the correlation of PLs

\[
\ln < L_0 L_R > \propto \exp[-M_D R], \tag{44}
\]

where \( M_D \) is the Debye mass. Though it may seem at the first glance that the fermionic contribution to the correlation function is suppressed as \( h_0^{2N_t} \) and could be neglected relatively to the Debye screening, this is misleading. The crucial point is that in order to skip this contribution one should isolate the contribution from the fermionic sector. To do that, one needs to make an expansion of the correlation
function in small $h_0$. But such an expansion is known to be divergent \[9\]. Thus, there is no direct way to separate the Debye screening from the fermionic screening in the electric sector. In this situation to calculate $A_t$ we should construct a weak coupling expansion not supposing a small value for the Higgs coupling, take the thermodynamic limit and only then we are allowed to make an expansion at small $h_0$. This procedure can be easily accomplished at least in the main orders of the large $\lambda$ expansion if one knows the main state of the gauge system for time-like links which has to be perturbated. In the usual ensemble this is a state with all the links up or down depending on which $n$-ality sector we consider in the CE. In the $F$-ensemble there is a competition between one of these states and the state where all the time-like spins are flipped inside a volume bounded by $\Sigma$ relatively to the links outside. The classical action on these two configurations is, correspondingly

$$S^I = \lambda_0 N_{p_0} + h_0 (V - 2\Omega(\Sigma)), \quad (45)$$

$$S^{II} = \lambda_0 (N_{p_0} - 2N_\Sigma) + h_0 V. \quad (46)$$

Therefore,

$$S^{II} - S^I = -2\lambda_0 N_\Sigma + 2h_0 \Omega(\Sigma). \quad (47)$$

On any finite lattice there always exists such a large $\lambda_0$ that the surface term wins and thus the first configuration will represent the main state. Since the PL is not flipped in this case in the corresponding triality sector of the CE one finds $A_t = -1$. Since, however we have to consider the limit $\Sigma \to \infty$, which is equivalent to $\Omega \to \infty$ and $N_\Sigma \to \infty$, we get easily convinced that this state gets metastable when the volume term suppresses the surface term in this limit. Hence, it is the second state \[16\] which gives the dominating contribution to the thermodynamic limit. Since the PL in the origin flips the sign one finds $A_t = 1$.

These qualitative arguments can be made more precise if we consider an effective model for the PL which can be obtained in the region $\lambda_0 \gg \lambda_n$, $h_0 \gg h_n$. Summing over space gauge fields and over Higgs fields we come to the standard effective model for the PL which in this case reads (we omit all irrelevant constants)

$$Z_{eff} = \sum_{L_x=\pm1} \exp \left[ \gamma \sum_{x,n} L_x L_{x+n} + \sum_x \alpha_x L_x \right], \quad (48)$$

where $x$ denotes now the sites of the $D$ dimensional lattice. This is the familiar Ising model in the external magnetic field with effective couplings defined as

$$\tanh \gamma = (\tanh \lambda_0)^{N_t}, \quad \tanh \alpha = (\tanh h_0)^{N_t}.$$

In the usual ensemble $\alpha_x = \alpha$, while in the $F$-ensemble one has to change sign in $\alpha$ if $x \in \Omega(\Sigma)$. We calculated the correlation function in the $F$-ensemble and the
quantity $A_t$ in the Ising model both analytically and in MC simulations \[17\]. The
results found support completely the qualitative arguments we presented above. The
interpretation of such behaviour we gave in the previous section.

It is interesting to mention that there is, at least, one obvious possibility which
leads to a nontrivial value $A_t = -1$ even in the thermodynamic limit. This is
the case of an inhomogeneous magnetic field in the effective Ising model \[48\]. For
instance, one can take a slowly varying field of the form

$$\alpha_x = \alpha_0 \cos \frac{2\pi}{L} x, \quad (49)$$

where $L$ is the linear size of the system. In such a field one expects that the ground
state is again a state with spins flipped inside a volume enclosed by an appropriate $\Sigma$
and, thus $A = -1$ at large values of $\gamma$. At small $\gamma$ $A = 1$ as can be easily shown via
the high temperature expansion. This implies that in the model with Ising spins in
the external field \[48\] there could be a phase transition. With respect to the Higgs
coupling it means that $h(x,t)$ may be a constant everywhere except one time-slice
where it has to be of the form \[49\]. Whether such a field could have any physical
significance in the context of gauge models remains however unclear.

We turn now to the order parameter $A_s$. Let $W_s(C)$ be a space-like Wilson
loop in the finite temperature gauge-Higgs model defined in \[33\] and $\Sigma_s$ be a two
dimensional surface on a dual lattice, $\Omega_s$ – the corresponding volume. Following the
above procedure we have

$$A_s(\Sigma_s, C) = - \frac{< W_s(C) >_F}{< W_s(C) >_0}. \quad (50)$$

The temporal part of the Higgs action is not affected by the singular $Z(2)$ gauge
transformations whereas for the spatial part we get in the $F$-ensemble

$$S^{sp}_H = \sum_{x,n} h_n(x) z_x s_n(x) z_{x+n}, \quad (51)$$

where $h_n(x)$ is defined similarly to \[39\]. Again, we consider the strong and weak
coupling phases in the gauge coupling $\lambda$.

I. $A_s$ in the strong coupling region $\lambda_n \ll 1$.

Using the general expansion \[14\] one gets up to second order

$$< W(C) > = \prod_{l \in C} \tanh h_l [1 + 2DP_C \tanh \lambda \tanh^2 h(1 - \tanh^2 h)], \quad (52)$$

where we consider the symmetric case $\lambda_0 = \lambda_n$ and $h_0 = h_n$, for simplicity. $P_C$ is
the perimeter of the loop $C$. It gives

$$A_s = 1 - O(\lambda^2).$$
As in the previous case for $A_t$, it is straightforward to see that

$$A_s = 1 - N_{\Sigma_s} C_2 (\tanh \lambda)^{L_{\Sigma_s}},$$

which gives $A_s = 1$ in the $\Sigma_s \to \infty$ limit.

II. $A_s$ in the weak coupling region $\lambda_n > 1$.

The gauge part gives the following contribution to the Wilson loop for the space dimension $D = 3$

$$< W(C) > \propto \exp \left[ -2 P_C (e^{-2\lambda})^6 \right].$$

We again analyze the symmetric case as above. The fermionic screening is now suppressed as $h_n P_C$, i.e.

$$< W(C) > \propto (\tanh h_n)^{P_C}.$$  

The crucial point distinguishing this case from the similar expansion in the electric sector is that now we are allowed to expand in small $h_n$ since there are no loops going around the lattice in space direction which could destroy the convergence. It is straightforward to calculate, e.g.

$$< F(\Sigma_s) > \propto \exp \left[ -\delta N_{\Sigma_s} + O(h^6) \right],$$

where $\delta \approx 2h^4 \tanh \lambda$. If we remove the surface $\Sigma_s$ back to the pure gauge action, then the main contribution in the $F$-ensemble comes from configurations of the gauge fields $s_n(x)$ flipped in the volume $\Omega(\Sigma_s)$ relatively to $s_n(x)$ outside of $\Omega(\Sigma_s)$. Since the linking number of the surface $\Sigma_s$ and the Wilson loop $C$ is 1, the Wilson loop changes sign in the $F$-ensemble. The corresponding expansions in large $\lambda$ and in small $h_n$ are converging. Therefore, in the limit of infinite $\Sigma_s$ and $C$ all the corrections go to zero and we find

$$A_s(\Sigma_s, C) = -1.$$  

This indicates perhaps a deconfinement phase transition in the gauge coupling to a phase where the kinetic screening dominates the dynamical screening. Above the critical point one can detect a unit of $Z(2)$ flux by the Aharonov-Bohm effect. In the main order the critical line is determined as

$$\exp \left[ -2(e^{-2\lambda_c})^6 \right] = \tanh h_n^c.$$  

Our last comment concerns the usage of the CE. One can easily check that $A_s$ exhibits essentially the same behaviour in both ensembles, at least in the main order of the Higgs coupling which is not affected by the temporal loops. Thus, both ensembles should lead to the same phase structure. The use of the CE is more relevant, as we believe, for the investigation of the $Z(N_c)$ symmetry in the electric sector, in particular to give a proper description of the metastable states.
5 Summary

In this paper we proposed a method which distinguishes between different screening mechanisms in gauge theories at finite temperature. This method relies on the action of the order parameter $A$ used earlier in the context of zero temperature theory \[16\] and is based on the Aharonov-Bohm effect. Various phenomena can be studied using this approach. The most notable application is a possibility for describing the deconfinement phase transition in terms of screening mechanisms. We suggested a general line for such an application and illustrated it on the example of $Z(2)$ gauge spins coupled to Higgs fields. Among other applications it is worth to mention the following:

1) The quantity $A_t$ can be used to study the problem of domain walls and of the stability of the corresponding interfaces in finite temperature theories with dynamical matter fields. That this order parameter indeed may show the instability of the interface in the thermodynamic limit we showed in [17] on the example of the 3D Ising model.

2) A very important and long standing problem in the dynamics of gauge systems (not only for finite temperature ones) is the difference between the Higgs and the fermionic screening. In particular, it is needed to explain the non-confining character of the weak interaction. Since the quantity $A$ provides a detailed description of the screening in both systems this problem can be formulated rigorously in the present approach.

3) We conjecture here that a similar behaviour of $A$ can also be found in the model of $Z(N_c)$ gauge spins coupled to fundamental fermions. The general phase structure of the latter model may however differ from the Higgs model studied here. Two essential points distinguish the fermion model from the Higgs model. First of all, in the limit of large $\lambda$ one can get a model of a free fermion gas which does not possess a phase transition. On the other hand, in the limit of strong coupling one can find a phase transition in the fermion sector which is related to chiral symmetry restoration. It is an interesting opportunity to explore the properties of $A$ in both electric and magnetic sectors in order to investigate a phase structure of this theory and to get a deeper insight into the connection (if any) between screening mechanisms of triality and chiral symmetry restoration.

4) The application of all previous suggestions and results to $SU(N_c)$ gauge models at finite temperature is, in principle straightforward though it is technically much more involved. Summarizing we would like to stress that although a final answer to the issue of deconfinement in QCD is not available yet, certainly the order parameter $A$ should be able to check the scenario proposed here also in the context of full QCD.

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