Abstract

Wide-angle sonar mapping of the environment by mobile robot is nontrivial due to several sources of uncertainty: dropouts due to "specular" reflections, obstacle location uncertainty due to the wide beam, and distance measurement error. Earlier papers address the latter problems, but dropouts remain a problem in many environments. We present an approach that lifts the overoptimistic independence assumption used in earlier work, and use Bayes nets to represent the dependencies between objects of the model. Objects of the model consist of readings, and of regions in which "quasi location invariance" of the (possible) obstacles exists, with respect to the readings. Simulation supports the method's feasibility. The model is readily extensible to allow for prior distributions, as well as other types of sensing operations.

1 INTRODUCTION

Sonar mapping of the environment by mobile robot is nontrivial due to several sources of uncertainty. The first impediment to mapping is that most sonars used in mobile robots are wide-beam, in order to allow for sufficient coverage for obstacle avoidance. The wide angle causes a large uncertainty of location of the detected obstacle(s), within the arc at the detected distance from the sonar (such as region $A_1$ in figure 1). Second, readings are inexact due to noise and pulse-width problems (thickness of region $A_1$). These problems have been largely addressed in work by Elfes using a Bayesian model and an occupancy grid [3].

Third, surfaces of objects slanted far from the perpendicular to the direction of the beam may reflect almost no acoustic energy in the direction of the sensor (which is also the emitter - the sonar(s) mounted on the robot). We call such a false negative a dropout, although sometimes, rather than generating no reading at all, the beam, after one or more secondary reflections, is detected by the sonar (a spurious reading) and results in a phantom obstacle. This problem becomes acute if there are many flat surfaces, which are acoustically "specular", i.e. reflect almost all acoustic energy in a narrow cone around the angle of incidence, resulting in frequent dropouts, as has been encountered by various researchers and implementers [12]. The "specularity" of a surface (determined by material, surface roughness, etc.) is usually represented as a "critical angle" from the perpendicular, beyond which the reading is likely to be bogus (in fact the probability that the reading is bogus may depend on other factors, such as signal strength, area, etc., but we would like to abstract away from these issues in this paper).

Most schemes that perform sonar mapping consider information from numerous sonar readings in order to overcome the errors. In a mapping architecture developed by Elfes [3], (2D) space is partitioned into a grid, where each grid point can either be occupied or free. Each grid point is seen as a random variable, denoting probability of occupancy. Information is accumulated, for each grid point, from multiple sonar readings and robot locations (the latter appear as circles in figure 5), resulting in an occupancy probability map as in figure 7 (where black stands for high occupancy probability). An error model of the sonar reading, that handles mostly range error due to noise and pulse width, is used. In general, Elfes makes the following independence assumption: all grid points are independent random variables. Updating the occupancy probability $P(O)$ of each grid point is done according to the sensor reading model $p(r|O), p(r|\neg O)$, independently from updates to other grid points, as follows:

$$P(O_{\text{new}}) = \frac{p(r|O)P(O_{\text{old}})}{p(r|O)P(O_{\text{old}}) + p(r|\neg O)(1 - P(O_{\text{old}}))}$$  \hspace{1cm} (1)

Another system [12] also uses a grid-based scheme, but with Dempster-Shafer probabilities [2]. Each grid point (cell) has the values $h_o$ and $h_f$, denoting belief that the cell is occupied, and belief that the cell is free, respectively. Update of the model is, again, indepen-
dent in each cell, as follows \(^2\): 

\[
\begin{align*}
    h_{f}^{\text{new}} &= \frac{1}{\delta} (h_{f}^{\text{old}} h_{f}^{\text{obs}} + h_{f}^{\text{old}} h_{u}^{\text{obs}} + h_{u}^{\text{old}} h_{f}^{\text{obs}}) \\
    h_{o}^{\text{new}} &= \frac{1}{\delta} (h_{o}^{\text{old}} h_{o}^{\text{obs}} + h_{o}^{\text{old}} h_{u}^{\text{obs}} + h_{u}^{\text{old}} h_{o}^{\text{obs}})
\end{align*}
\]

with \( h_{u} = 1 - h_{o} - h_{f} \), and the superscripts \( \text{old}, \text{new}, \) \( \text{obs} \) denoting old belief values, new belief values, and observed values (by the current sensing operation), respectively. \( \delta \) is a "normalizer", used to account for the probability mass of contradictory evidence, as follows:

\[
\delta = 1 - (h_{o}^{\text{old}} h_{f}^{\text{obs}} + h_{f}^{\text{old}} h_{o}^{\text{obs}})
\]

Dropouts (false negatives) are handled by direct filtering on successive readings during travel, and the problem of uncertainty due to wide beam is alleviated by the facts that: a) The sonar beam width of only about \( 10^\circ \), compared to the roughly \( 25^\circ \) beam width of the standard Polaroid sonars. b) Triangulation between readings from different robot locations (also facilitated by the narrower beam). Remaining errors are handled by the updating mechanism above. One disadvantage of the scheme is obviously the larger number of readings required due to the narrower beam. The paper [12] does not give a clear semantics to the prior belief values for the cells. It is also unclear whether the method is applicable to a more cluttered environment than the rather simple one in the experiment, or to objects with a more specular reflectance pattern.

In essence, both schemes assume that grid points are independent variables, both a-priori and given the sensing operations. There are several problems in assuming independence of grid points and of not keeping track of which readings originated the information. Consider, for example, the spatial configuration of readings in figure 1. In the figure, the sectors are regions where, had there been an obstacle, the readings would have been other than observed (assuming no false negatives). The thickened arcs are regions where some object is likely to be (as a cause for the reading value). Using grid-point independence, reading \( R_2 \) has no effect on the occupancy probability in region \( A_1 \), whereas the fact that a possible obstacle in \( A_1 S_2 \) is contra-indicated by \( R_2 \), and a possible obstacle in \( A_1 A_2 \) is confirmed by \( R_2 \), should change the probability that region \( A_1 \) is occupied.

Increased probability of occupancy in region \( A_1 A_2 \), and decreased probability of occupancy in region \( S_1 S_2 \) occur both in Elfes' model and our own. Extending the example, consider a reading \( R_3 \) that might be caused by an object in region \( A_2 \), which partially overlaps \( S_1 \), as in figure 2. The probability that \( R_3 \) is a dropouts increases, and this should decrease the probability of occupancy for region \( A_1 \). This kind of inference cannot be handled by a scheme that assumes independence of grid points, and does independent updates.

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\(^2\)In the cited paper, \( h \) values are actually in the range 0 to 100, but that is immaterial.
2 THE SENSING MODEL

We first present and justify our single sonar reading sensor model assumptions, and then discuss the issue of independence assumptions we have remaining in the multi-reading sensing model.

2.1 Assumptions and Definitions

Our single reading model assumes the following:

1. The probability that an obstacle is detectable by a reading depends only on whether it is (at least in part) in a sector in front of the sonar, defined by the beam-width. Probability of detecting an object outside the sector is 0.

2. Given a true reading r, the probability of occupancy of each unit area in the thickened (thickness ε, the reading inaccuracy) arc about distance r from the sensor is uniform. The probability that there is an object in the beam at a distance d < r − ε is 0.

3. Spurious readings are uniformly random.

4. The probability of a reading given an object in the thickened (detection) arc is independent of its location within the arc.\(^3\)

All the above assumptions are approximations to a more realistic sensor model, made so that we can partition space into equi-distributed regions, to facilitate our main task of correcting dropouts. Assumption 2, uniform distribution (in contrast with the realistic model, of a normal distribution along the distance axis), is reasonable if the error of a true reading is small compared to other errors (such as arc length). In many sonar systems, where distance error is on the order of 1 inch, the above condition holds. Assumption 3 is very difficult to overcome without further spatial data, which is unavailable in an unmapped environment, but we believe that the results are not very sensitive to its violation.

In order to define a probability distribution over a set of readings, several independence assumptions are used, as follows:

1. Readings are independent measurements.
2. Occupancy of regions are (a-priori) independent.

While the first assumption is commonly used, the second is more problematic. Prior work (e.g. [3]) makes this assumption w.r.t. individual occupancy grid cells. Doing so w.r.t. regions should present no difficulty. In fact, it should be advantageous, since one can consider the size of a region, as well as its shape, when determining prior probabilities of occupancy. However, since regions are arbitrary, and modified dynamically, this is equivalent to assuming independence of grid points. Later on, we adjust this assumption. The main difference with prior work is that region occupancies are not independent given the readings, contrary to Elfes’s model.

The resulting probabilistic model for multiple readings is as follows. Conditional probabilities for each reading are dependent only on the occupancy of regions within their sector or thickened arc. Let reading r be a random variable with range \([R_{\text{min}}, R_{\text{max}}]\), \(A_r\) be its arc region, and \(S_r\) be its sector. Let \(\text{free}(X)\) be a predicate that denotes that region X is completely free (i.e. has no obstacles)\(^4\). Then:

\[
\begin{align*}
p(r & \text{free}(S_r)) = p_r(r) \\
p(r | \text{free}(S_r) \land \text{free}(A_r)) = p_0(r) \\
p(r | \text{free}(S_r) \land \neg \text{free}(A_r)) = p_r(r)
\end{align*}
\]

where the three cases stand for a dropout in region \(S_r\), a spurious reading due to noise where no objects exist (such as crosstalk, sensor error, etc.), and a true reading, respectively. The probability of a reading is independent of occupancy of any regions outside \(S_r\) and \(A_r\). We now define the conditional distributions \(p_r(r), p_0(r), p_r(r)\) in the regions of \(R_{\text{min}} \leq r \leq R_{\text{max}}\). Outside this range, the distributions are defined arbitrarily, to make them normalized.

Under our assumptions, the conditional distribution \(p_r(r)\) is piecewise uniform, with a high value \(p_r^h\) at roughly the distance of the region \(A_r\), from the source, and low value \(p_r^l\) elsewhere inside the sonar range. The exact ratio depends on the prior probability \(P_r\) that the primary reflection from objects in \(A_r\) is detected: \(P_r = 2cp_r^h\). The distribution \(p_0(r)\), for dropouts in region \(S_r\), is uniform, under the assumptions. As we have no good model for other sources of errors, and have not encountered them in our system, we use a piecewise uniform \(p_0(r)\), with a low value for all possible readings (i.e. within the sensor range). In fact, since the reading in our system is discrete, we use a discrete probability distribution instead of densities.

For each region \(X\), the probability of \(\text{free}(X)\) is dependent on area, shape, and location (if there is any prior information). The set of regions used in the model is the Cartesian product of the distinct regions of all n readings. That is, let \(\{A_{r_1}, S_{r_1}, E_{r_1}\}\) be the arc, sector, and the remaining space for reading \(r_1\), respectively. Then our set of regions, for \(n\) readings, is:

\[A = \prod_{i=1}^{n}\{A_{r_i}, S_{r_i}, E_{r_i}\}\]

Note that although the size of \(A\) seems to be exponential in \(n\), there are two mitigating factors: a) In

\(^3\)Strictly speaking, taken together with assumption 2, this entails that readings \(r\) and distances of objects from the sonar \(d\) are equi-distributed, which involves complicated spatial assumptions. However, if \(\epsilon\) is reasonably small this effect is negligible, and we thus ignore this constraint henceforth.

\(^4\)We denote the random variables standing for the occupancy status of an region by its name \(X\), and the state that it is completely free by the predicate \(\text{free}(X)\).
practice, most of the regions are spatially empty re­
gions, and are thus ignored. If all thickened arcs and
sectors were convex, the actual maximum number of
non-empty regions would be low-order polynomial in
the number of readings. The arcs are not convex, but
are nearly so, and we would thus expect the number
of regions to be comparatively small. b) In the actual
practice, most of the regions are spatially empty re­
gion occupancies, from a
Bayesian model.

The probabilistic reasoning issue we need to tackle is
to find the distribution of region occupancies given the
sonar readings. We actually solve a somewhat simpler
problem - finding the (marginal) probability that each
region is occupied given the readings. Representing
the distribution as a Bayesian network allows us to use
existing tools to solve the problem.

Equation 3 is a conditional distribution where reading
probability is dependent only on a set of region vari­
ables, and region variables are independent. It thus
exhibits a 2-level dependency structure, and can thus
be implemented as a 2-level Bayesian network, one root
node for each region in A, and one sink node for each
reading. However, since the in-degree is essentially
unbounded, and a reading node is not a type of node
for which specialized implementations exist in the lit­
erature (as is the case for, e.g. noisy-or), we prefer to
implement the model as a 3-level network, by adding,
for each reading-node r, two intermediate (binary val­
ed) parents: "pro" and "con", which are or-nodes,
defined as follows:

\[ P("\text{pro} \mid r_i\, \text{true}) = \begin{cases} 1 & \exists A \subseteq A_r, \neg\text{free}(A) \\ 0 & \text{otherwise} \end{cases} \]

\[ P("\text{con} \mid r_i\, \text{true}) = \begin{cases} 1 & \exists A \subseteq S_r, \neg\text{free}(A) \\ 0 & \text{otherwise} \end{cases} \]

Additionally, since in the implementation, a reading
node is always set to the observed reading, it is suffi­
cient that r be a binary-valued node, with one value,
"obs", standing for the observed value, and another
standing for all other possible readings.

\[ P(r_i = \text{obs} \mid \text{pro}_{r_i}, \text{con}_{r_i}) = \begin{cases} P_r & \text{pro}_{r_i} \\ P_{\neg r} & \text{con}_{r_i} \land \neg\text{pro}_{r_i} \\ P_{\text{con}_{r_i} \land \neg\text{pro}_{r_i}} & \text{otherwise} \end{cases} \]

Note that readings are assumed independent given the
evironment, but are dependent in an unknown environ­
ment. Likewise, regions are (approximately) independent
a-priori, but must become dependent given the readings.
Hence, the Bayes network must be constructed in the
causal direction: regions preceding readings in the DAG.
We use a constant low $P_{0r} = P_0$ (that is, assume few spurious readings that are not dropouts), and $P_r$, a constant $P_r$, a reasonably high probability for initial experiments. In fact, this number should be determined by the spatial configuration, but we do not as yet have a good model for determining it. $P_{0r} = P_0$ is determined by the prior probability that an object in the sector will cause no detectable primary reflection (i.e. that we will get a dropout). Each region $A$ is represented by a binary random variable, denoting $\text{free}(A)$, with prior probability as discussed above. The resulting network can now be implemented with existing Bayes network software that handles or-nodes as special cases (see implementation section on using systems with explicit distribution representation). Such networks are similar to BNO2 networks [1], except for the negative-cause con nodes. For example, the Bayes network for the configuration of Figure 2 is depicted in figure 3.

### 3 IMPLEMENTATION

The experimental system has two main components: the readings analyzer, and the probabilistic reasoner (Figure 4). The former communicates with the Nomadic Software Development package (simulator) for the Nomad 200 mobile robot (which can also communicate in turn with the physical Nomad 200 robot).

The readings analyzer is a C++ program consisting of four modules: The robot activation module, the analysis module, the output module and the graphics module. The analyzer uses a data structure with two partitions: dynamic structure for the sonar readings and regions, and dynamic grid/pixels relations with the above structure. Each sonar reading produces two or more new regions. While it is possible to use a purely geometrical representation in our model, the grid was found to be easier and possibly a more efficient way to implement the system in practice.

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**Figure 3: Bayes Net for 3 Sonar Readings**

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**Figure 4: Architecture of the experimental system**

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**Figure 5: Scenario 1: Corridor**

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The probabilistic reasoner constructs the Bayes net dynamically (using IDEAL [11] in LISP), according to topological data from the readings analyzer, sets up the readings as evidence, and performs belief updating. Dynamic construction of the network is by performing actions for each new input item, as follows:

1. Add-reading(name, distance): create new reading node, con node and pro node. Connect them and set conditional probabilities.
2. Add-region(name, area): create a new region node, and set up its prior probability according to its size.
3. Add-cause(region, reading, polarity): add region node as a parent of pro or con node (according to polarity) of reading. If an or-node (pro or con node) has more than 2 parents, add an intermediate or-node.

The intermediate or-nodes in add-cause are a pure implementation issue: our version of IDEAL represents all nodes as distribution arrays, which may become too large if we allowed arbitrary in-degree. Additionally, some of the algorithms we use for belief updating work better with a small in-degree.

### 4 SIMULATION RESULTS

Results presented are for a simulated environment. The maximal reflection angle (critical angle) was set at 60° for the simulation. We compare the results of our mapping to results that might be obtained by assuming grid-point independence (the Elfes model), in several spatial configurations defined with the simulator's polygon mode editing. The complexity of the network
generated for a large number of sonar readings forced us, in the current implementation, to select a particular mix of readings that does not generate too many regions, but with some dropouts, in order to show that our model can discount them. Hence, our experimental setting is somewhat contrived. The configurations selected for the paper are 1) a turn in a corridor (figure 5), and 2) an irregular polygon (figure 6) where the simulated robot’s path shown as a sequence of circles.

Results are presented as an array of gray-level pixels, with white representing a probability of 0 (for an obstacle), and black a probability of 1, on a linear scale. The relative grid point scale is 1 inch per grid point. Sonar arc thickness (error) width was 3 pixels.

In displaying results for independent updates (Figures 7,8) we show both the sonar readings, and the occupancy probability computed with the grid-point independence assumption, using equation 1, with initial probability being $p_{\min} = 0.3$. If the grid point is in the detection arc of the reading, we used $p(r \mid O) = 0.9$, and $p(r \mid -O) = 0.3$. If the grid point is in the “free” sector of a reading, we used $p(r \mid O) = 0.1$ and $p(r \mid -O) = 0.9$. Parameters for our model were $P_e = 0.9$ (the probability of true positive), prior probability of a dropout, $P_d = 0.1$, $P_0 = 0.05$, and $p_{\max}$ was set at 0.8. Grid points traversed by the robot body have $P(\text{free}) = 1$.

For comparison, region occupancy probabilities in our model (which stand for probability that there is some obstacle anywhere within the region) must be translated into a probability that a single grid point be occupied. For any region $A$, let $P(\neg\text{free}(A))$ be its prior probability of occupancy (from equation 4), and $P_e(\neg\text{free}(A))$ be its posterior probability of occupancy given the evidence (the readings). Probability of occupancy for a grid point $g$ in region $A$ is given by:

$$P(\text{occupied}(g)) = p_{\min} \frac{P_e(\neg\text{free}(A))}{P(\neg\text{free}(A))}$$

The above equation is only for comparative display of the results. We do not intend to argue that it represents the actual probability that the (region of space covered by) a grid point is occupied given the evidence.

For scenario 1 (Figure 5), we have 26 readings, resulting in 73 regions excluding robot body, with 69 positive area regions. The network had 104 support arcs. Results are depicted in figures 9, and 10 (thresholds on figure 9, set at $P(\text{occupied}(g)) = 0.25$ and 0.35).

For scenario 2 (Figure 6), we have 20 readings. This resulted in 83 regions, excluding regions covered by the robot body, and 65 regions after removing regions including no grid points. The generated Bayes network had 139 support arcs. Results are depicted in figures 11, and 12.

With the independence assumption, several good read-
ings are essentially disregarded (occupancy within the
detection arc less than the global prior, $p_{\text{min}}$) due to
one or more dropouts. The system would thus tend to
ignore some nearby obstacles. Our model overcomes
dropouts in both scenarios: probability of region occu­
pancy higher than prior, despite the dropouts. The
latter are much discounted, as shown by applying of
thresholds to the results. This property occurs due to
the fact that detection arcs of different readings (even
disjoint ones), occurring wholly or partially within a
"free" sector (of a dropout), tend to support each other
in the model, and to decrease the probability that the
"free" sector of the dropout is, indeed, free.

Evaluation time for the above Bayes networks was
about 1 CPU-minute on a Sparc ELC, with the Jensen
(junction-tree) algorithm in IDEAL. However, with
more readings, or readings with a larger tendency to
intersect, computation time grows rapidly. A configu­
ration (not shown) with 25 readings and 115 regions,
and took 4 hours CPU time. Larger configurations
could not be evaluated at all. This problem is par­
tially addressed below.

5 DISCUSSION

It is possible to add prior information, or information
from other sensors, into this system. One way to do
this is by using different priors for regions (and other
parameters than just size to decide priors). Another
way is to add a prior dependency model on the regions.
In order to incorporate prior knowledge of objects into
the map (e.g. a wall known to be at a certain posi­
tion, with some given probability), one could further
partition regions to accommodate such known objects.

One weakness of our system is in coping with regis­
tration or location errors: if the location of a known
object is incorrect, incorporating that is somewhat dif­
ficult. That is because the system cannot tell where
region partitions should be added. Although one pos­
sible solution would be to add a fine partition, cat­
erg for all possible placements of the object, this would
clearly be inefficient. A similar problem is encountered
if the location of the sensors (w.r.t. earlier placement
of the sensors) is known imprecisely. In both cases,
the problem is not severe if the error is small. Using
localization schemes available in the literature one can
solve the latter problem. Handling the location errors
for other objects is still an open problem.

Currently, the system handles only the "gross" uncer­
tainty resulting from sonar readings, caused by the
beam-width and secondary reflections. It would be
desirable to incorporate handling of the "small" errors
resulting from inexact readings, which are handled by
various sensor fusion schemes, such as Kalman filter­
ing. In principle, this may be done after the "gross"
sources of error have been eliminated.

Another obvious problem is that the Bayes network
quickly becomes too large for evaluation by general,
extact methods. We are considering several directions
for overcoming the problem, as follows:

1. Using a scheme similar to that used in [1] for the
   somewhat similar BNO2 networks.
2. Make decisions about regions $A$ that have ex­
treme probabilities $P(\text{free}(A))$, after a few read­
ings, thus partitioning, as well as decreasing the size of the network.

3. Using order of magnitude probabilities as an approximation.

4. Using sampling algorithms (e.g. [6, 10]). Initial experiments in [10] show some measure of success with this method.

Numerous other papers have time-dependent or dynamic Bayes network for handling problems of uncertainty in sensing and results of actions. They all, however, deal with different kinds of sensors and environments. For example, [8] has a discrete spatial representation in terms of "rooms" where light beam sensors are placed at the doors, to detect crossing objects. Discrete locations of moving objects are then estimated by dynamically constructing a Bayes network and evaluating it. Sensors may either fail to detect objects, or detect "ghosts". The spatial configuration in that paper is essentially reduced to a topology, whereas in our paper, the spatial geometry is important. Note that in our paper, the grid is used just as a convenient computational aid, and in fact we could have used a continuous geometrical representation of regions. The latter is not possible in [8].

Other examples are in traffic scene analysis [7], which uses Bayes networks to track discrete objects (cars), and in the BATmobile autonomous vehicle simulation [4] which uses Bayes networks to decide the position of obstacles (e.g. other cars). In both papers, the inherent assumption is that we can separate out the individual objects, which is indeed possible due to e.g. relative motion. That is not possible in our domain, and hence we need to use a different sensor model.

6 SUMMARY

This paper presents a probabilistic model for "gross" sonar sensor fusion, that relaxes several independence assumptions made in related work. Such assumptions may be over-optimistic for various environments. The dependencies we introduced between readings are represented by the topology of a (dynamically constructed) Bayes network, which also represents the conditional distributions between elements of the model (readings and spatial regions). Initial experiments indicate that the model can overcome spurious sonar reflections, by accumulating evidence from a number of readings, and thus greatly discount the impact of the bogus readings. This evidence accumulation is done automatically by the probability updating mechanism for the probability model (as implemented by the Bayes network). We believe that the method is reasonably easy to extend to incorporating prior information, both certain and uncertain, as well as information from other types of sensors.

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