Distribution Locational Marginal Pricing Under Uncertainty Considering Coordination of Distribution and Wholesale Markets

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Abstract—A rapidly growing amount of small-scale distributed energy resources (DERs) integrated into distribution systems call for an effective distribution electricity market to manage the uncertainty of DERs and remove barriers to the participation of DERs in wholesale electricity markets. To this end, this paper proposes an uncertainty-aware distribution locational marginal pricing (DLMP) mechanism based on robust optimization for day-ahead distribution markets within a transmission-distribution coordinated framework. The transmission-level model clears the wholesale market and forms transmission locational marginal prices (LMPs) to price energy, reserve, and uncertainty. At the distribution level, a robust optimization-based DLMP mechanism is proposed that internalizes uncertainties and coordinates with the wholesale market. Besides active and reactive power DLMPs, the uncertainty DLMP is introduced to reward reserve and charge uncertainties. The novel DLMP mechanism provides transparent and comprehensive price signals for managing voltage, congestion, loss, especially uncertainty. The coordinated model is solved in a decentralized manner by heterogeneous decomposition algorithm. Distribution and wholesale markets are integrated through the coordinated mechanism to fully utilize generation resources. Accordingly, DMLPs are highly correlated and consistent with transmission LMPs, and thus allow DERs to participate in wholesale markets. The effectiveness of the proposed method is verified via numerous case studies.

Index Terms—Coordination, distribution electricity market, distribution locational marginal pricing (DLMP), robust optimization, uncertainty, wholesale electricity market.

Nomenclature

Indices and Sets

| Symbol | Description |
|--------|-------------|
| $t$    | Index for time intervals |

Variables and Parameters

| Symbol | Description |
|--------|-------------|
| $C_G(i)$ | Energy cost of thermal generator $i$ |
| $C_R(i)$ | Reserve cost of thermal generator $i$ |
| $\pi_{i,j}^L, \pi_{i,j}^U$ | LMP and uncertainty LMP (ULMP) at bus $i$ in the transmission system |
| $\pi_{i,j}^Q$ | Reactive power price at the substation |
| $\chi_{i,j}, \chi_{i,j}', \chi_{i,j}''$ | Active power, reactive power, and uncertainty DLMPs at node $i$ |
| $c_{i,j}^T, c_{i,j}^Q$ | Active and reactive power bid prices of DER $i$ |
| $P_{DS}^i, R_{DS}^i$ | Energy and reserve demands of the distribution system at bus $i$ purchased from wholesale markets |
| $Q_{DS}^i$ | Reactive power demand of a distribution system |
| $P_{i,j}^a, Q_{i,j}^a$ | Active and reactive power of DERs |
| $P_{i,j}^d, Q_{i,j}^d$ | Active and reactive power of load demands |
| $\Delta P_{i,j}^{set}$ | Reserve capacity provided by microturbine $i$ |
| $\epsilon_{i,j}^{rdg}$ | Forecast deviation (uncertainty) of load demand and wind power in transmission systems |
| $u_{i,j}^{D, W, rdg}$ | Forecast deviation of renewable DERs |

Indices

| Symbol | Description |
|--------|-------------|
| $\mathcal{N}_T$ | Set of buses in transmission systems |
| $\mathcal{D}ER$ | Set of distributed energy resources (DERs) |
| $\mathcal{R}DG$ | Set of renewable DERs |
| $\mathcal{M}T$ | Set of microturbines |
| $\mathcal{ESS}$ | Set of energy storage systems (ESSs) |
| $\mathcal{N}, \mathcal{L}$ | Set of nodes and lines in distribution systems |
| $pr(i), cr(i)$ | Set of parent and children of node $i$ |

I. INTRODUCTION

Driven by the goal of clean and low-carbon energy, the power industry is undergoing significant transformations. The rapid growth of distributed energy resources (DERs) accelerates the transition of the traditional passive distribution system to an active, bottom-up, and localized control paradigm. Establishing a distribution electricity market can provide an effective solution for managing large amounts of small-scale DERs in distribution systems [1].
However, the establishment of such distribution markets faces significant challenges, two of which lie in the uncertainty of renewable DERs and the coordination with the wholesale electricity market in the distribution market clearing and pricing. On one hand, the uncertainty of renewable DERs affects the normal operation of distribution systems, causing problems such as overvoltage and network congestion [2]. A novel market mechanism is required to internalize the uncertainty in price formation for effective uncertainty management in distribution systems. On the other hand, new regulations, such as the FERC Order 2222 in the U.S [3] and the guideline on constructing a unified electricity market in China [4], promote the participation of DERs in wholesale markets. Thus, the distribution system operators (DSOs) should account for the coordination and interaction with the transmission system operator (TSO). The distribution market clearing and pricing need to be consistent with the wholesale market. As a result, the wholesale and distribution markets are integrated as a unified electricity market, so as to provide a theoretical foundation for DERs to trade energy and flexibility with the bulk power grid.

Operation policies applied in transmission systems can be employed in distribution systems with high penetration of DERs [5]. At the transmission level, the locational marginal pricing (LMP) mechanism is the dominant methodology in deregulated electricity markets over the world [6], [7]. By extending the LMP concept from the wholesale market to distribution markets, the distribution LMP (DLMP) has been developed and attracted considerable attention. In [1], DLMPs are derived based on the proposed market clearing model to reflect the temporal-spatial values of the services provided by DERs, and motivate DERs to alleviate network congestion and provide voltage support in distribution systems. In [8], a social welfare optimization problem is solved to obtain DLMPs in order to alleviate network congestion induced by electric vehicles. In [9], a new DLMP method based on quadratic programming is presented for congestion management in distribution systems with high penetration of flexible demands. The authors in [10] introduce a robust virtual battery model to describe aggregated prosumer flexibilities and employ iterative DLMPs to schedule prosumer resources with the purpose of preventing network congestion. In [11], a new DLMP model is proposed based on a linearized variant of the global energy balance formulation, which can achieve an efficient flexibility resource allocation in local distribution markets. The studies in [1] and [8], [9], [10], [11] demonstrate the effectiveness of DLMPs in managing DERs in distribution markets. However, the uncertainty of renewable DERs is not taken into account in DLMP derivation, which may hinder its application in practice and lead to market inefficiencies and economic losses [12].

In order to internalize system uncertainties in nodal pricing, the uncertainty-aware LMP methods have been first proposed and developed in wholesale markets to price the uncertainties of generation and load demand. In [13], a new market-clearing mechanism based on a direct current (DC) optimal power flow (OPF) model is proposed to introduce uncertainty components in LMPs. In [14], a stochastic wholesale market mechanism is proposed to internalize the uncertainty of renewable energy sources and risk tolerance of the TSO in price formation process. In [15], a day-ahead wholesale market is cleared to derive pool energy prices and balancing prices, which are introduced to price energy and deployed reserve, respectively. In [16], a stochastic market clearing model considering uncertainties is developed to yield LMPs for pricing energy through a linearized lossless DC representation of the network. In [17], a novel day-ahead market clearing mechanism is proposed to credit the generation and reserve and to charge the load and uncertainty. The uncertainty components in the above LMP methods provide price signals to reflect the system costs as a result of generation and demand uncertainty at different locations, incentivizing the uncertainty management in wholesale markets.

Naturally, the uncertainty-aware LMP methods [13], [14], [15], [16], [17] in wholesale markets are expected to be employed in distribution markets to internalize uncertainties into DLMP derivation. However, the widely-used DCOPF model in transmission systems is not suitable for DLMP calculation due to its incapability of addressing losses and reactive power as well as the high R/X ratio of distribution networks [18]. For instance, an improved DLMP method in day-ahead market is proposed in [19] to handle the uncertainty of flexible demand based on the DCOPF model, but the network losses and reactive power are neglected.

To fully preserve the features of distribution systems, the latest studies [20], [21], [22] have proposed different uncertainty-aware DLMP methods in distribution markets. In [20], a novel stochastic market model considering the uncertainties of load demands and renewable sources is presented based on a joint active and reactive OPF model. Although this paper models and assesses the impact of uncertainties on active and reactive DLMPs, it does not provide an approach for uncertainty pricing to produce effective price signals for uncertainty management. In addition, the stochastic optimization approach is computationally challenging due to a large number of scenarios. Another issue of stochastic approach is that it only provides probabilistic guarantees to the system reliability and cannot ensure the feasibility for all uncertainties [23]. In [21], a new market pricing mechanism is proposed by means of a chance-constrained ACOPF model to internalize uncertainties of DERs in DLMP computations. The DLMPs for active power, reactive power, and balancing regulation are derived to price distribution-level energy and reserve products. The balancing regulation DLMP may not reflect the difference of uncertainty level at different nodes as it is identical for all nodes in a distribution system. Also, the chance-constrained approach requires the probabilistic information of uncertainty parameters, which may be inaccessible in practice [24]. Different from stochastic and chance-constrained approaches, the robust optimization approach has the minimum data requirement and significantly reduced computation burden [17], [25]. Moreover, the robust optimization method constructs a solution of the best performance with respect to worst cases within an uncertainty set. As a result, its solution is more reliable toward uncertainties, which is consistent with the risk-averse fashion in which the power system is operated [24], [25]. Therefore, the robust
optimization approach has been widely adopted to address engineering and management problems. In [22], a distribution market clearing mechanism is proposed based on a robust economic dispatch model, through which DLMPs are derived to price active power and uncertainty. However, network losses and congestion are neglected in the economic dispatch model.

It could be even more challenging to price the uncertainties in distribution markets and wholesale markets in a coordinated and coherent framework. The large-scale integration of DERs change the aggregate demand curve of distribution systems, thus impact transmission system operation and LMPs. In turn, LMPs also impact the scheduling schemes of DERs and distribution systems as well as the DLMP calculation. However, the boundary LMPs at the power supply points connected to the transmission system is usually assumed to be constant in DLMP calculation and not impacted by the DSO behaviors. The separate market clearing and pricing without considering transmission-distribution coordination may present some challenges [26]: a) Generation resources may be not fully utilized, which impairs the overall benefits of the transmission and distribution systems. b) Network congestion may be more serious, leading to the increase of electricity prices. c) Power mismatch may occur at the power supply point and affect system operation.

Even though there is a massive amount of existing work on distribution and transmission coordination [27], [28], [29], [30], [31], few research on DLMPs has accounted for coordination with transmission systems and wholesale markets. In [32], a bi-level model is presented to clear the distribution markets and calculate DLMPs through the equilibrium problem with equilibrium constraints approach. In [33], an iterative approach is proposed based on a lossy DCOPF model to derive DLMPs. References [32] and [33] integrate the wholesale and distribution markets to schedule the generation and load resources across the entire grid. However, the deterministic method adopted in [32] and [33] makes it challenging to introduce uncertainty components in DLMPs.

In order to fulfill the above research gaps of existing DLMP methods, an uncertainty-aware DLMP mechanism based on robust optimization is proposed for day-ahead distribution market-clearing and pricing within a transmission-distribution coordinated framework. The transmission-level model clears the wholesale market through a robust economic dispatch model and forms LMPs to price energy, reserve, and uncertainty. At the distribution level, a robust optimization-based DLMP mechanism is proposed that internalizes uncertainties and coordinates with the wholesale market. A heterogeneous decomposition (HGD) algorithm is utilized to solve the coordinated model with limited information interaction in a decentralized manner. The main contributions of this paper are highlighted as follows:

1) A novel uncertainty-aware DLMP mechanism is proposed. Besides active and reactive power DLMPs, the uncertainty DLMP is introduced to charge the renewable DERs for their uncertainties and reward the generators who provide reserve capacity to mitigate uncertainties. Each DLMP can be decomposed into the marginal costs of energy, loss, congestion, and voltage through the robust optimization paradigm, which is consistent with the existing DLMP mechanism. Thus, the proposed DLMP mechanism can provide transparent and comprehensive signals for managing loss, congestion, and voltage in distribution systems. Furthermore, the uncertainty of DERs is internalized in the price formation of each DLMP. Accordingly, the novel pricing mechanism can incentivize the uncertainty management in distribution markets.

2) A transmission-distribution coordination mechanism is presented in DLMP calculation. The influence of large amounts of DERs on wholesale markets, as well as the influence of LMPs on distribution market-clearing and DLMP formation are all considered. The coordination method integrates distribution and wholesale markets to fully utilize the generation resources in the whole system. Moreover, LMPs and DLMPs are highly correlated through the coordination mechanism. They have similar decomposition forms and physical meanings, and both internalize uncertainties based on the robust optimization. The above characteristics determine the consistency of distribution markets with wholesale markets. In a sense, the wholesale and distribution markets can be regarded as a unified electricity market in terms of market clearing and pricing. Accordingly, any generation resources can be settled by the locational LMP or DLMP according to the nodal pricing concept, and the cleared generation and flexibility can be scheduled across the entire system. Therefore, the proposed coordination mechanism contributes to the participation of DERs in wholesale markets based on DLMPs.

The remainder of this paper is organized as follows: Section II proposes the transmission-distribution coordinated market framework. Section III and Section IV present the clearing and pricing models in wholesale markets and distribution markets, respectively. Section V elaborates the solution methodology. Section VI conducts simulation studies and Section VII draws main conclusions for this paper.

**II. TRANSMISSION-DISTRIBUTION COORDINATED MARKET FRAMEWORK**

The day-ahead wholesale and distribution markets typically include 24 hours with time resolution of 1 hour. It is assumed that all participants bid at their marginal costs. The proposed transmission-distribution coordinated market framework is depicted in Fig. 1.

At the transmission level, the TSO operates the wholesale market which includes participants such as thermal generators, wind farms, load serving entities, and distribution systems. Firstly, wind farms and load serving entities submit day-ahead
reactive power prices at the power supply points are assumed day-ahead market-clearing mechanism for reactive power in making of TSO and DSOs. It should be noted that there is no the decentralized method guarantees the independent decision-making of DSOs; with limited information exchange, DSO as well as LMPs and ULMPs need to be exchanged at the power supply points. Only the boundary active power and reserve demands of each DSO are determined and sent to the TSO. Finally, the DLMPs for active power, reactive power, and uncertainty are derived based on the linear programming model.

The transmission-distribution coordinated model is solved iteratively by the HGD algorithm in a decentralized manner. Only the boundary active power and reserve demands of each DSO as well as LMPs and the unit commitment of TSO and DSOs. It should be noted that there is no day-ahead market-clearing mechanism for reactive power in existing wholesale markets [1], [11]. Therefore, the boundary reactive power prices at the power supply points are assumed to be constants, and the reactive power demands of DSOs are not sent to TSO and do not affect the wholesale market clearing and pricing.

III. DAY-AHEAD WHOLESALE MARKET CLEARING AND PRICING

A day-ahead wholesale market clearing model is presented in [17] within a robust optimization framework. The robust unit commitment determines the unit commitment status and worst-case uncertainty realization. Then the robust economic dispatch is utilized to derive the uncertainty-aware LMPs. The active power and reserve demands of distribution systems are taken into account in wholesale markets in this paper. Given the unit commitment and worst-case scenarios, the modified robust economic dispatch model can be formulated as:

\[
\min \quad \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left( C_i^G \left( p_{i,t}^G \right) + C_i^R \left( \Delta p_{i,t}^G \right) \right)
\]

s.t.

\[
\sum_{i \in \mathcal{G}} p_{i,t}^G = \sum_{i \in \mathcal{G}} p_{i,t}^D + \sum_{i \in \mathcal{D} \mathcal{G}} p_{i,t}^D - \sum_{i \in \mathcal{W} \mathcal{G}} p_{i,t}^W, \quad \forall t \in \mathcal{T}, \mathcal{G}
\]

\[
I_{t, i} p_{i, t, \min}^G \leq p_{i, t}^G \leq I_{t, i} p_{i, t, \max}^G, \quad \forall i \in \mathcal{G}, t \in \mathcal{T}
\]

\[
-r_{i,t}^{RD} (1 - v_{i,t}) - r_{i,t}^{SD} v_{i,t} \leq p_{i,t}^G - p_{i,t-1}^G \leq r_{i,t}^{RU} (1 - u_{i,t}) + r_{i,t}^{SU} u_{i,t}, \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, \mathcal{G}
\]

\[
-F_t \leq \sum_{i \in \mathcal{G}} GSF_{i,t} \left( \sum_{j \in \mathcal{G} \mathcal{T}} \sum_{j \in \mathcal{D} \mathcal{G} \mathcal{T}} \left( p_{j,t}^G + \Delta p_{j,t}^G \right) - \sum_{j \in \mathcal{W} \mathcal{G} \mathcal{T}} p_{j,t}^W \right) \leq F_t, \forall t \in \mathcal{T}
\]

\[
r_{i,t}^{RD} (1 - v_{i,t}) - r_{i,t}^{SD} v_{i,t} \leq \left( p_{i,t}^G + \Delta p_{i,t}^G \right) \leq r_{i,t}^{RU} (1 - u_{i,t}) + r_{i,t}^{SU} u_{i,t}, \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, \mathcal{G}
\]

\[
I_{t, i} p_{i, t, \min}^G \leq p_{i,t}^G + \Delta p_{i,t}^G \leq I_{t, i} p_{i, t, \max}^G, \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, \mathcal{G}
\]

\[
\sum_{i \in \mathcal{G}} \Delta p_{i,t}^G = \sum_{i \in \mathcal{G}} \delta_{i,t}^G + \sum_{i \in \mathcal{D} \mathcal{G}} \Delta p_{i,t}^D - \sum_{i \in \mathcal{W} \mathcal{G}} \delta_{i,t}^W W_{i,t} \left( \Delta t_{i,t} \right)
\]

\[
I_{t, i} p_{i, t, \min}^G \leq p_{i,t}^G + \Delta p_{i,t}^G \leq I_{t, i} p_{i, t, \max}^G, \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, \mathcal{G}
\]

\[
-F_t \leq \sum_{i \in \mathcal{G}} GSF_{i,t} \left( \sum_{j \in \mathcal{G} \mathcal{T}} \sum_{j \in \mathcal{D} \mathcal{G} \mathcal{T}} \left( p_{j,t}^G + \Delta p_{j,t}^G \right) - \sum_{j \in \mathcal{W} \mathcal{G} \mathcal{T}} p_{j,t}^W \right) \leq F_t, \forall t \in \mathcal{T}
\]

where \( \mathcal{G} \), \( \mathcal{D} \mathcal{G} \), and \( \mathcal{W} \mathcal{G} \) denote the sets of thermal generators, wind farms, load serving entities, and distribution systems connected to bus \( i \); \( I_{t, i} \) denotes the line index; \( F_t \) is the capacity of line \( i \); \( GSF_{i,t} \) is the generation shift factor of bus \( i \) to line \( i \); \( p_{i,t}^D \) and \( p_{i,t}^W \) are the forecast values of load demand and wind power. For thermal generator \( i \), \( p_{i,t}^G \) denotes the energy provision to meet the forecast value of net load, including load demand, wind power, and energy demand of DSOs; \( \Delta p_{i,t}^G \) is the reserve provision to cope with the forecast deviation of load demand and wind power as well as the reserve demand of DSOs; \( \Delta p_{i,t}^D \) and \( \Delta p_{i,t}^W \) denote the lower and upper output limits, respectively; \( r_{i,t}^{RU}, r_{i,t}^{RD}, r_{i,t}^{SU}, \) and \( r_{i,t}^{SD} \) denote the maximum ramp-up/down and startup/shutdown ramp rate, respectively; \( u_{i,t} \) and \( v_{i,t} \) indicate the startup status (1 for startup, 0 otherwise) and shutdown status (1 for shutdown, 0 otherwise) respectively, \( I_{t, i} \) is the unit status (1 for online, 0 otherwise). The dual variables are in brackets. In (1)–(3), the unit commitment status \( u_{i,t} \) and \( v_{i,t} \), and \( I_{i,t} \) are assumed to be known. The forecast deviation of load demand and wind power is bounded within a box set [34], which can be expressed in (4). In robust optimization method, the upper and lower bounds of box sets are taken as the worst-case scenario for load demand and wind power, respectively [35].
gram model [36]. Accordingly, the proposed robust scheduling model is formulated to clear the distribution market and derive initial system operating status. Then a linear programming formulation is performed to solve the robust scheduling model. According to the definition, the LMP and ULMP at bus \( i \) are active and reactive power flow on line segments.

\[
\begin{align*}
\pi_{i,t}^P & = \lambda_i^P + \sum_l GSF_{i,l} \left( \eta_{i,t}^P - \eta_{i,t}^{P+} + \eta_{i,t}^{P-} \right) \\
\pi_{i,t}^U & = \lambda_i^U + \sum_l GSF_{i,l} \left( \eta_{i,t}^U - \eta_{i,t}^{U+} + \eta_{i,t}^{U-} \right)
\end{align*}
\]

IV. DAY-AHEAD DISTRIBUTION MARKET CLEARING AND PRICING

A robust scheduling model is first presented to obtain the worst-case uncertainty realization of renewable DERs and the initial system operating status. Then a linear programming formulation is performed to clear the distribution market and derive the uncertainty-aware DLMPs. The market clearing includes dispatch and redispatch processes, where the dispatch process schedules controllable DERs to support the active and reactive power demands, and the redispatch process optimizes the reserve of microturbines to cope with the uncertainty (forecast deviation) of renewable DERs in distribution systems.

A. Robust Scheduling Model for Distribution Systems

A day-ahead distribution-level scheduling model that fully captures the features of distribution systems and network losses can be generally formulated as a second-order cone program model [36]. Accordingly, the proposed robust scheduling model for a distribution system at bus \( m \) in the transmission system is formulated as follows.

\[
\begin{align*}
\max_{\pi} & \min_{x} \sum_{i \in \mathcal{T}} \left( \pi_{i,m}^P P_{i,m}^D + \pi_{i,m}^Q Q_{i,m}^D + \pi_{i,m}^U R_{i,m}^D \right) \\
& + \sum_{i \in \mathcal{DER}} \left( \epsilon_i^P P_{i,t}^D + \epsilon_i^Q Q_{i,t}^D \right) + \sum_{i \in \mathcal{MT}} c_i^P P_{i,t}^D + c_i^Q Q_{i,t}^D \\
\text{s.t.} & \quad \left( I_{i,j,t}^P ight)^2 + w_{i,j,t} \left( I_{i,j,t}^P \right) - w_{i,j,t} \left( I_{i,j,t}^P \right)^2 = 0, \forall i \in \mathcal{N}, \forall (i,j) \in \mathcal{L}, \forall t \\
& \quad \sum_{k \in \mathcal{PR}(i)} P_{i,j,t}^P - \sum_{j \in \mathcal{CR}(i)} P_{i,j,t}^P = P_{i,t}^D - P_{i,t}^R, \forall i \in \mathcal{N}, \forall t \\
& \quad \sum_{k \in \mathcal{PR}(i)} \left( Q_{i,j,t}^P - w_{i,j,t} P_{i,j,t}^Q \right) - \sum_{j \in \mathcal{CR}(i)} Q_{i,j,t}^P = Q_{i,t}^D - Q_{i,t}^Q, \forall i \in \mathcal{N}, \forall t \\
& \quad u_{i,t}^P - 2 \left( r_{ij} P_{i,j,t}^P + x_{ij} Q_{i,j,t}^P \right) + \left( w_{i,j,t}^2 \right)^2 = u_{i,j,t}^P, \forall i,j \in \mathcal{CR}(i), \forall t \\
& \quad 0 \leq w_{i,j,t}^2 \leq I_{ij,t}^2, \forall (i,j) \in \mathcal{L}, \forall t \\
& \quad \left( P_{i,j,t}^P \right)^2 + \left( Q_{i,j,t}^P \right)^2 \leq \left( S_{ij,t}^P \right)^2, \forall (i,j) \in \mathcal{L}, \forall t \\
& \quad \left( P_{i,t}^P \right)^2 + \left( Q_{i,t}^P \right)^2 \leq \left( P_{i,t}^D \right)^2, \forall i \in \mathcal{N}, \forall t
\end{align*}
\]

where \( \mathcal{T} \) is the set of nodes, \( \mathcal{N} \) is the set of buses, \( \mathcal{DER} \) is the set of DERs, and \( \mathcal{MT} \) is the set of microturbines. The objective is to minimize the total energy and reserve costs. The constraints (2a) and (3a) represent the power balance constraints. (2b) and (3b) are the generation limits. (2c) and (3c) are the ramping constraints. (2d) and (3d) denote the line capacity limits.

According to the definition, the LMP and ULMP at bus \( i \) can be obtained in (5) and (6), respectively [17]. The detailed derivation process is given in Appendix A.
the active power, reactive power and capacity. For ESS \(i\), \(P_{\text{ess},c}^{i,t}\) and \(P_{\text{ess},d}^{i,t}\) denote the charge and discharge active power; \(Q_{\text{ess}}^{i,t}\) denote the reactive power; \(\eta_{s}^{i}\) and \(\eta_{d}^{i}\) are the charge and discharge efficiency; \(SOC_{\text{min}}^{i}\) and \(SOC_{\text{max}}^{i}\) are the lower and upper limits of state of charge; \(E_{i,c}^{i}, E_{i,d}^{i} \text{and} S_{\text{ess}}^{i,max}\) denote the remaining energy, rated energy capacity, and inverter capacity, respectively. \(x\) denotes the optimization variables.

\[
x = \left\{ P_{\text{ess},c}^{i,t}, Q_{\text{ess}}^{i,t}, \Delta P_{\text{DS}}^{m,t}, P_{\text{DS}}^{m,t}, Q_{\text{DS}}^{m,t} \right\} \tag{14}
\]

Eq. (7) minimizes the total costs of purchasing active energy, reactive energy, and reserve from the outside, and scheduling active and reactive power of DERs and reserve of microturbines locally, respectively. Eq. (8) and (9) are constraints for dispatch and redispatch processes, respectively. Eq. (8b)–(8c) and (9b)–(9c) are nodal power balance constraints. Eq. (8d) and (9d) denote voltage equations. Eq. (8e)–(8f) and (9e)–(9f) indicate voltage and current limits. Eq. (8g) and (9g) are line power flow limits. Eq. (8h) and (9h) is a second-order-cone form for the relationship between current, voltage, and line power flow. For renewable DERs, (10a) and (10b) indicate the reactive power constraints in dispatch and redispatch processes, respectively; (10c) is the uncertainty set. For microturbines, (11a) denotes the capacity limit; (11b) and (11c) denote ramping limits. For ESSs, (12a) denote charge and discharge power constraints; (12b) reveals the remaining energy; (12c) denotes the state of charge limit; (12d) indicates that the stored energy at the beginning is equal to that at the end of a day; (12e) is the inverter capacity limit. Eq. (13) linearizes the reactive power of DERs [1].

In the distribution system, the surplus or deficit of reactive power is balanced at the root node, i.e., the boundary substation connected to the transmission system. The reactive power price at the root node is determined by the marginal cost of supplying reactive power from the transmission system [11]. Whether the marginal cost is made available and how the reactive power should be priced are still a topic of ongoing discussion in existing wholesale markets [37], [38]. Thus, the reactive power price \(\pi_{Q}^{\text{r}}\) is kept in objective function (7) for flexibility, and assumed to be constants in this paper.

The scheduling model (7)–(13) can provide the accurate results of the worst-case realization of renewable DERs and the initial system operating status, which will be employed in the linear programming model. In addition, although the scheduling model does not include discrete variables, it can be extended easily to take into account discrete voltage regulation devices, such as on load tap changers and capacitor banks.

B. Linearized Power Flow Model

Eq. (7)–(13) is a convex optimization problem which can be utilized to derive DLMPs. However, the convex DLMP formulation is not intuitive [11]. There are obstacles for DSOs to interpret DLMPs and translate them into their financial settlements with market participants. To this end, a linear programming model considering uncertainties is formulated to derive the uncertainty-aware DLMPs in a more general form. The novel DLMP mechanism allows individual components of DLMPs to be analyzed in detail, which can help in interpreting DLMPs for DSOs in a manner similar to the uncertainty-aware LMP formulation at the transmission level.

The linear programming model can be established based on the following linearized power flow model 1)-3).

1) Polygonal Inner-Approximation Method: A polygonal inner-approximation method is utilized to linearize the quadratic capacity constraints (8g), (9g), (11a), and (12e), which can be formulated as follows [39]:

\[
\alpha_{c,0} P + \alpha_{c,1} Q + \alpha_{c,2} S \leq 0, \forall c \in \{1, 2, \ldots, 12\} \tag{15}
\]

where \(\alpha_{c,0}, \alpha_{c,1}\), and \(\alpha_{c,2}\) are the linearized coefficients.

2) Branch Flow and Voltage Sensitivity Factors: Branch flow and voltage can be expressed as [1]:

\[
P' = LSF \cdot P_{\text{inj}}, Q' = LSF \cdot Q_{\text{inj}} \tag{16}
\]

\[
\Delta V = RP' + XQ' = R \cdot LSF \cdot P_{\text{inj}} + X \cdot LSF \cdot Q_{\text{inj}} \tag{17}
\]

where \(P_{\text{inj}}\) and \(Q_{\text{inj}}\) are nodal active power and reactive power injections; \(\Delta V\) is the voltage change relative to the root node; \(R\) and \(X\) are the resistance and reactance matrices between any two nodes on the path; \(LSF\) is the load shift factor indicating the ratio of line loading change with respect to the injected nodal load.

The sensitivity factors are derived from (16) and (17).

\[
SF_{vp} = \frac{\partial \Delta V}{\partial P_{\text{inj}}} = R \cdot LSF, SF_{vq} = \frac{\partial \Delta V}{\partial Q_{\text{inj}}} = X \cdot LSF \tag{18}
\]

\[
SF_{lp} = \frac{\partial P'}{\partial P_{\text{inj}}} = LSF, SF_{lq} = \frac{\partial Q'}{\partial Q_{\text{inj}}} = LSF \tag{19}
\]

where \(SF_{vp}\) and \(SF_{vq}\) are the sensitivity factors of nodal voltage change with respect to nodal active power and reactive power injections; \(SF_{lp}\) and \(SF_{lq}\) are the sensitivity factors of line flows with respect to nodal active power and reactive power injections.

3) Delivery Factor and Fictitious Nodal Demand: To account for network losses in distribution systems, the delivery factor and fictitious nodal demand are utilized in the linearized power flow model. The delivery factor is a ratio representing the part of injected power that can be actually transmitted in the network considering losses. Based on the distribution-level scheduling results, the delivery factor for active and reactive power at node \(i\) can be expressed as [1]:

\[
DF_{p,1}^{f} = 1 - \sum_{i \in \mathcal{L}} 2P_{1,i}^{*,p} \sum_{i \in \mathcal{N}} SF_{lp,1-i} \tag{20}
\]

\[
DF_{q,1}^{f} = 1 - \sum_{i \in \mathcal{L}} 2Q_{1,i}^{*,q} \sum_{i \in \mathcal{N}} SF_{lq,1-i} \tag{21}
\]

where \(P_{1,i}^{*,p}\) and \(Q_{1,i}^{*,q}\) denote the active and reactive power flow of line \(l\). Note that all the variables with * represent the fixed values obtained in the scheduling model (7)–(13).

The fictitious nodal demand is a virtual nodal demand to stand for network losses which assigns the power loss of a line to its nodes at both sides equally. The fictitious nodal demand for active power and reactive power can be expressed as:

\[
F_{p,1}^{f} = \frac{1}{2} \sum_{j \in \mathcal{N}(i)} r_{ij} \left( P_{ij,t}^{*,p} \right)^{2}, F_{q,1}^{f} = \frac{1}{2} \sum_{j \in \mathcal{N}(i)} x_{ij} \left( Q_{ij,t}^{*,q} \right)^{2} \tag{22}
\]
C. Distribution Market Clearing and Pricing Mechanism

The linear programming model can be formulated as:

\[ \min C(x) \]  
\[ \text{s.t.} \quad \sum_i DF_{p,t}^q p_{e,t}^i - \sum_i DF_{p,t}^q p_{d,t}^i - p_{\text{loss},t}^i = 0, \quad \forall i \in \mathcal{N}, t \quad (23a) \]
\[ \sum_i DF_{q,t}^i Q_{e,t}^i - \sum_i DF_{q,t}^i Q_{d,t}^i - Q_{\text{loss},t}^i = 0, \quad \forall i \in \mathcal{N}, t \quad (23b) \]
\[ P_{i,t}^g \leq P_{i,t}^g \leq P_{i,t}^g \max, \quad \forall i \in \mathcal{N}, t \quad (23c) \]
\[ Q_{i,t}^g \leq Q_{i,t}^g \leq Q_{i,t}^g \max, \quad \forall i \in \mathcal{N}, t \quad (23d) \]
\[ V_{\min} \leq \sum_j SF_{vq, i-j,t} \left( p_{d,t}^j - p_{i,t}^j + p_{f,b}^j \right) \leq V_{\max}, \ \forall i \in \mathcal{N}, t \quad (23e) \]
\[ \alpha_{c,0} \sum_i SF_{lp, i-l,t} \left( p_{d,t}^i - p_{l,t}^i + p_{f,b}^i \right) + \alpha_{c,1} \sum_i SF_{lq, i-l,t} \left( Q_{d,t}^i - Q_{i,t}^i + F_{q,b}^i \right) + \alpha_{c,2} S_{l,t,\text{max}} \leq 0, \quad \forall i \in \mathcal{N}, l \in \mathcal{L}, t \quad (24f) \]
\[ \sum_i DF_{p,t}^q \left( \Delta P_{p,t}^g + \epsilon_{d,t}^g \right) - \left( p_{\text{loss},t}^i - p_{\text{loss},t}^i \right) = 0, \quad \forall i \in \mathcal{N}, t \quad (25a) \]
\[ V_{\min} \leq \sum_j SF_{vq, i-j,t} \left( p_{d,t}^j - p_{i,t}^j + p_{f,b}^j \right) \leq V_{\max}, \ \forall i \in \mathcal{N}, t \quad (25b) \]
\[ \alpha_{c,0} \sum_i SF_{lp, i-l,t} \left( p_{d,t}^i - p_{l,t}^i + p_{f,b}^i \right) + \alpha_{c,1} \sum_i SF_{lq, i-l,t} \left( Q_{d,t}^i - Q_{i,t}^i + F_{q,b}^i \right) + \alpha_{c,2} S_{l,t,\text{max}} \leq 0, \quad \forall i \in \mathcal{N}, l \in \mathcal{L}, t \quad (25c) \]

where \( C(x) \) corresponds to the objective function (7); \( P_{\text{loss},t}^i \) and \( Q_{\text{loss},t}^i \) denote the total active and reactive power losses; the dual variables are in brackets.

Eq. (23) is the objective function. Eq. (24a) and (25a) are constraints for dispatch and redispatch processes, respectively. Eq. (24a), (24b), and (25a) are power balance constraints. Eq. (24c), (24d), and (25b) denote the generation limits. (24e) and (25c) represent voltage limits. Eq. (24f) and (25d) denote the line capacity constraints.

The Lagrange function of the linear programming model (23)–(25) can be formulated as:

\[ L = C(x) - \sum_t \lambda_{p,t}^i \left( \sum_i DF_{p,t}^q p_{e,t}^i - \sum_i DF_{p,t}^q p_{d,t}^i - p_{\text{loss},t}^i \right) \]
\[ - \sum_t \lambda_{q,t}^i \left( \sum_i DF_{q,t}^i Q_{e,t}^i - \sum_i DF_{q,t}^i Q_{d,t}^i - Q_{\text{loss},t}^i \right) \]
\[ + \sum_i \sum_j \rho_{p,t}^i \left( p_{d,t}^i - p_{f,b}^i \right) + \sum_i \sum_j \rho_{q,t}^i \left( Q_{d,t}^i - Q_{f,b}^i \right) \]
\[ + \sum_i \sum_j \mu_{\text{loss},t}^i \left( V_{\text{min}} - V_{\text{max}} \right) \]
\[ + \sum \sum \mu_{\text{loss},t}^i \left( V_{\text{min}} - V_{\text{max}} \right) \]
\[ + \sum \sum \eta_{\text{loss},t}^i \left( V_{\text{min}} - V_{\text{max}} \right) \]
\[ + \sum_i \sum \eta_{\text{loss},t}^i \left( V_{\text{min}} - V_{\text{max}} \right) \]

Similar to the uncertainty-aware LMPs in wholesale markets, the active and reactive power DLMPs at node \( i \) in distribution systems are defined as the partial derivative of the Lagrange function with respect to the forecasted active and reactive load demands. The uncertainty DLMP is defined as the partial derivative of the Lagrange function with respect to the forecast deviation of net load at that node. The DLMPs for
active power, reactive power, and uncertainty can be derived as follows.

\[ x_{i,t}^p = \frac{\partial L/\partial P_{i,t}}{\lambda_{p,t}} = \lambda_{p,t}^b + \sum_j S F_{i,p,j,t} (\mu_{i,t}^r - \mu_{i,t}^r - \mu_{i,t}^r + \mu_{i,t}^r) \]

\[ + \sum_c a_c S F_{i,p,j,t} (\eta_{i,c,t}^r + \eta_{c,t}^r + \lambda_{p,t}^b (DF_{i,t}^{p,b} - 1) \]

(27)

\[ x_{i,t}^q = \frac{\partial L/\partial Q_{i,t}}{\lambda_{q,t}} = \lambda_{q,t}^b + \sum_j S F_{i,q,j,t} (\mu_{i,t}^r - \mu_{i,t}^r - \mu_{i,t}^r + \mu_{i,t}^r) \]

\[ + \sum_c a_c S F_{i,q,j,t} (\eta_{i,c,t}^r + \eta_{c,t}^r + \lambda_{q,t}^b (DF_{i,t}^{q,b} - 1) \]

(28)

\[ x_{i,t}^u = \frac{\partial L/\partial U_{i,t}}{\lambda_{u,t}} = \lambda_{u,t}^b + \sum_j S F_{i,u,j,t} (\mu_{i,t}^r - \mu_{i,t}^r - \mu_{i,t}^r + \mu_{i,t}^r) \]

\[ + \sum_c a_c S F_{i,u,j,t} (\eta_{i,c,t}^r + \eta_{c,t}^r + \lambda_{u,t}^b (DF_{i,t}^{u,b} - 1) \]

(29)

The uncertainty DLMP is introduced in this paper to reward the reserve provision and charge the uncertainty in distribution markets. Each type of DLMPs in (27)–(29) includes energy, voltage, congestion, and loss components sequentially, which is consistent with the existing DLMP mechanism to ensure the applicability. Moreover, besides uncertainty DLMP, the active and reactive power DLMPs both include the dual variables \( (\mu_{i,t}^r, \eta_{i,c,t}^r, \lambda_{p,t}^b, \lambda_{q,t}^b, \lambda_{u,t}^b) \) which correspond to the voltage constraint (25c) and congestion constraint (25d) in redispatch process. Thus, each type of DLMPs can reflect the impact of uncertainties on DLMP formation.

The above characteristics reveals that the proposed DLMP mechanism allows individual components of DLMPs to be analyzed in detail, which contributes to interpreting the DLMPs in a clear and intuitive way. The novel DLMP method can provide effective price signals for managing voltage, congestion, and loss in distribution systems. More importantly, it internalizes the system uncertainties, so as to extend the application scope of existing DLMP methods to incentivize the uncertainty management in distribution markets.

It can be found from (27)–(29) that any type of the three DLMPs has the same energy component for all nodes in the distribution system, which is in line with the concept of nodal pricing. The diversity for each type of DLMPs in a distribution system depends on the difference in voltage, congestion, and loss components for different nodes. According to the definition, three types of DLMPs in (27)–(29) are utilized to price active energy, reactive energy and uncertainty/reserve, respectively. The active and reactive power generation, reserve provision, and uncertainty (forecast deviation) of a participant \( i \) at node \( m \) and time \( t \) in distribution systems are assumed as \( P_{i,t}^{ds}, Q_{i,t}^{ds}, R_{i,t}^{ds} \) and \( U_{i,t}^{ds} \), respectively. The cleared revenue can be expressed as

\[ \text{rev}_{i,t}^{ds} = \sum_m \xi_{P_{i,t}} P_{i,t}^{ds} + \sum_m \xi_{Q_{i,t}} Q_{i,t}^{ds} + \sum_m \xi_{R_{i,t}} R_{i,t}^{ds} - \sum_m \xi_{U_{i,t}} U_{i,t}^{ds} \]

(30)

The revenue of a participant can be decomposed into terms not only for active power, reactive power, reserve capacity, and uncertainty, but also for energy, voltage, congestion, and loss. The proposed DLMP mechanism accounts for uncertainties and provides different feasible decomposition ideas for the revenues of participants, which can guide the distribution system management clearly and efficiently.

On the other hand, it should be noted that the proposed DLMP mechanism accounts for the transmission-distribution coordination, which is implemented by solving the transmission-distribution coordinated model with the iterative solution method demonstrated in Section V. Accordingly, DLMPs in (27)–(29) have a similar decomposition form with LMP and ULMP in (5)–(6) except voltage and loss components which are not considered in wholesale markets. The similar decomposition strengthens the consistency and interaction between wholesale and distribution markets. The coordination method makes it possible for DERs to participate in wholesale markets. The generation and demand of DERs are settled based on the proposed DLMPs, and the cleared resources can be scheduled across the entire power grids.

V. SOLUTION METHODOLOGY

The robust optimization problem (7)–(13) can be effectively solved by the column-and-constraint generation algorithm [40]. The HGD algorithm [41] is adopted to solve the transmission-distribution coordinated model.

The compact form of the transmission-level problem (1)–(3) can be written as follows.

\[ \min_{P_T, \Delta P_T} c_T (P_T, \Delta P_T) \]

s.t.

\[ A_{P_T} P_T + A_{\Delta P_T} \Delta P_T + A_{PB_T} P_B + A_{\Delta PB_T} \Delta P_B = a_T (\lambda_{a_T}) \]

\[ B_{P_T} P_T + B_{\Delta P_T} \Delta P_T + B_{PB_T} P_B + B_{\Delta PB_T} \Delta P_B \geq b_T (\eta_{b_T}) \]

\[ E_{P_T} P_T + E_{\Delta P_T} \Delta P_T \geq e_T (\mu_{e_T}) \]

(31)

where (31a) is the objective function (1), (31b) includes (2a) and (3a), (31c) includes (2d) and (3d), (31d) includes (2b), (2c), (3b), and (3c); \( P_T \) and \( \Delta P_T \) denote \( P_{G_t} \) and \( \Delta P_{G_t} \), respectively; \( P_B \) and \( \Delta P_B \) denote \( P_{DS_t} \) and \( R_{DS_t} \), respectively; \( A_{P_T}, A_{\Delta P_T}, A_{PB_T}, A_{\Delta PB_T}, B_{P_T}, B_{\Delta P_T}, B_{PB_T}, B_{\Delta PB_T}, E_{P_T}, \) and \( E_{\Delta P_T} \) denote the coefficient matrices of the corresponding constraints in (2) and (3); \( a_T, b_T, \) and \( \mu_{e_T} \) are the constant column vectors obtained from (2) and (3); \( \lambda_{a_T}, \eta_{b_T}, \) and \( \mu_{e_T} \) denote the dual variables.

According to the derivation process in Appendix A, LMP and ULMP in wholesale markets can be derived from (31), which are shown in (32a) and (32b), respectively.

\[ \xi_{P_B} = -A_{PB_T}^T \lambda_{a_T} - B_{PB_T}^T \eta_{b_T} \]

(32a)

\[ \xi_{\Delta P_B} = -A_{\Delta PB_T}^T \lambda_{a_T} - B_{\Delta PB_T}^T \eta_{b_T} \]

(32b)

The compact form of the distribution-level problem (23)–(25) can be written as follows.

\[ \min_{P_D, Q_D, \Delta P_D} c_D (P_D, Q_D, \Delta P_D) + \xi_{P_B} P_B + \xi_{\Delta P_B} \Delta P_B \]

(33a)
The solution steps of solving the transmission-distribution coordinated model (31) and (33) through the HGD algorithm are as follows:

1) Initialize $P_B$ and $\Delta P_B$. Set the iteration number $k = 1$;
2) Solve the transmission-level problem (31) and publish the boundary LMPs $\xi_{P_B,k}$ and ULMPs $\tilde{\xi}_{P_B,k}$ to DSOs;
3) Solve the distribution-level problem (33) and send active energy and reserve demands (i.e., $P_{B,k}$ and $\Delta P_{B,k}$) to the TSO;
4) Set a convergence tolerance $\varepsilon$. If the convergence criterion (34) is satisfied, exit the iteration process. Otherwise, update $k = k + 1$ and go to step 2).

$$\max |P_{B,k} - P_{B,k-1}| \leq \varepsilon, \quad \max |\Delta P_{B,k} - \Delta P_{B,k-1}| \leq \varepsilon$$

The solution process to solve the transmission-distribution coordinated model (31) and (33) is the same as that in solving (31) and (33). It should be noted that the coefficient $1/2$ in (35a) guarantees the optimality of the solution [41]. The proofs for the optimality and convergence of the HGD algorithm are shown in Appendix B.

VI. CASE STUDIES

The effectiveness of the proposed DLMP mechanism is verified by the TSD33 and T118D69 systems in this section. All simulations are performed in MATLAB 2020a with YALMIP interface and MOSEK 9.2 on a computer with Intel Core i7-10700F CPU and 16 GB RAM.

A. Simulation Settings

The TSD33 system is comprised of a modified PJM 5-bus transmission system depicted in Fig. 2 and 100 identical IEEE 33-node distribution systems, in which one half are at bus C and the other half are at bus D in the transmission system. Fig. 2 shows the capacity and bid prices of thermal generators. The other parameters are given in [42]. The ratio of the forecasted base load of load serving entities at buses B, C, and D is 3:3:4. The maximum forecast deviation of the load serving entities at buses B, C, and D is 10%, 5%, and 0% to their forecasts. A 200MW wind farm is located at bus E.

The IEEE 33-node distribution system is shown in Fig. 3, and the load demand data is given in [1]. The line flow limit is set to 4 MVA. Each distribution system includes eight PVs with capacity of $\{0.6, 0.6, 0.5, 0.6, 0.5, 0.8, 0.6, 0.8\}$ MW at nodes $\{4, 7, 11, 15, 18, 25, 28, 32\}$, two WTs with capacity of $\{0.2, 0.2\}$ MW at nodes $\{13, 20\}$, two microturbines with capacity of $\{1, 1\}$ MW and ramp up/down rate limit of $\{0.5, 0.5\}$ MW/h at nodes $\{17, 32\}$, two 3MWx2h ESSs at nodes $\{3, 29\}$. The voltage magnitude of the root node is set to 1.0 p.u. The voltage limits are $[0.95, 1.05]$ p.u.

The wind farm in the transmission system as well as the PVs and WTs in distribution systems follow the forecast profiles considering uncertainty in [43]. The microturbines and ESSs bid at 15 $/MWh$ and 20 $/MWh$ respectively for active power and 3 $/MVArh$ for reactive power. The thermal generators and
microturbines provide reserve at half of their active power bid prices [44]. The optimality gap of HGD algorithm is set to 1%.

B. Market Clearing and Pricing

In the wholesale market, LMPs and ULMPs are shown in Fig. 4. The prices are different at different buses in each period of hours 1-2, 8, and 17-24 due to the network congestion. The generation and demands of market participants affect the price curves, which are the highest at bus D and followed by bus C.

The prices in the wholesale market directly affect DLMPs. The active power and uncertainty DLMPs in the distribution systems at buses C and D are illustrated in Fig. 5 and Fig. 6, where each curve represents a node. The DLMPs in the distribution system at bus C are different from those at bus D due to the different location in the transmission system.

Fig. 4–Fig. 6 indicate that DLMPs are similar for the nodes in a distribution system and are closely related to the boundary LMP and ULMP. The reason is that boundary LMP and ULMP determine the energy components of the corresponding type of DLMPs (LMP versus active power DLMP, ULMP versus uncertainty DLMP) when DSOs trade with the TSO and the traded energy and reserve is not congested by substations. From the DSO perspective, the transmission system is a virtual source and can be regarded as the marginal unit in distribution markets whose marginal cost is at LMP and ULMP. Thus, the boundary LMP and ULMP are equal to the energy components of corresponding DLMPs, which is shown in Table I.

The diversity of each type of DLMPs in a distribution system shown in Fig. 5 and Fig. 6 depends on the difference in voltage, congestion, and loss components for different nodes. The distribution system at bus D is selected for discussion.

Fig. 7 shows the voltage component in active power DLMP and its decomposition in dispatch process ($\sum_{j} SF_{vp,i-j,t}(\mu_{b,i,t} - \mu_{b,j,t})$) and redispatch process ($\sum_{j} SF_{vp,i-j,t}(\mu_{r,i,t} - \mu_{r,j,t})$). The voltage component reflects voltage condition and implements voltage support considering uncertainties. Taking node 18 in hour 7 as an example, the voltage component and its decomposition in dispatch and redispatch processes are $-2.45$, $-0.36$, and $-2.09$ respectively, which have been marked in Fig. 7.

According to the strong duality theory, $\mu_{b,i,t} + \mu_{r,j,t}$ and $\mu_{b,i,t} - \mu_{r,j,t}$ are both nonnegative. Thus, “$-0.36$” and “$-2.09$” indicate that $\mu_{b,i,t} + \mu_{r,j,t} > 0$ and $\mu_{b,i,t} - \mu_{r,j,t} = 0$. Accordingly, the related voltage constraints (24e) and (25c) are binding respectively, and the voltage reaches upper limit in dispatch and redispatch processes, which is caused by excessive power generation and uncertainty. Overall, the negative value “$-2.45$” motivates

| Hour | ULMP ($$/MW) | E.COM. ($$/MW) | ULMP ($$/MW) | E.COM. ($$/MW) |
|------|--------------|--------------|--------------|--------------|
| 1    | 20.00        | 17.71        | 26.11        | 23.85        |
| 2    | 20.00        | 17.70        | 26.11        | 23.83        |
| 3    | 12.00        | 9.43         | 12.00        | 9.38         |
| 4    | 12.00        | 9.23         | 12.00        | 9.15         |
| 5    | 5.00         | 2.42         | 5.00         | 3.54         |
| 6    | 12.00        | 9.61         | 12.00        | 9.65         |
| 7    | 12.50        | 10.24        | 12.50        | 10.34        |
| 9    | 12.00        | 9.58         | 12.00        | 9.53         |
| 10   | 12.00        | 9.41         | 12.00        | 9.35         |
ESSs and microturbines to reduce active power generation to alleviate the voltage problem.

Fig. 8 illustrates the congestion component in active power DLMP in dispatch and redispatch processes (i.e., $\sum_l \sum_c a_{c,0} SF_{l,p,\text{LP},l}\eta_{l,c,t}^h$ and $\sum_l \sum_c a_{c,0} SF_{l,p,\text{RP},l}\eta_{l,c,t}^r$). Each curve represents one hour. The other hours except hours 5, 6, and 11 are all zero and not shown in Fig. 8. The congestion component can reflect congestion condition and contribute to congestion management while considering the uncertainty. For instance, the congestion component in hour 11 is negative in dispatch process for all nodes except nodes 1–2 and 19–22. According to the line capacity constraint (24f), it reveals that congestion occurs on line (2,3) in dispatch process. The power flow direction is from node 3 to 2, which is caused by the active power reverse from large amounts of PVs at noon. Based on (30), microturbines and ESSs are motivated to reduce active power generation for more revenue. The congestion can be alleviated accordingly. In hour 6, the component is positive for all nodes except node 1 in dispatch process due to the uncertainty of WTs. It indicates that the congestion occurs on line (1,2) and the power flow is from node 1 to 2. DERs are stimulated to increase active power to alleviate congestion.

Fig. 9 shows the loss component ($\chi_{l,p}^h (DF_{l,i,l}^{p,h} - 1)$) in active power DLMP. The positive values motivate DERs to increase active power generation, and vice versa. The network loss are reduced accordingly.

Fig. 7–Fig. 9 demonstrate the management ability of active power DLMP on active power generation of DERs in terms of voltage, congestion, and loss, while considering uncertainties. The reactive power DLMP in (28) has the same decomposition form with active power DLMP in (27). Thus, the analysis on operation management by reactive power DLMP is similar to that by active power DLMP.

In terms of the uncertainty DLMP, (27) and (29) indicate that its voltage and congestion components are identical to those in active power DLMP in dispatch process ($\sum_l \sum_c a_{c,0} SF_{l,p,\text{LP},l}\eta_{l,c,t}^h$ and $\sum_l \sum_c a_{c,0} SF_{l,p,\text{RP},l}\eta_{l,c,t}^r$). Therefore, Fig. 7(c) and Fig. 8(b) are also the voltage and congestion components of uncertainty DLMP. In Fig. 7(c), the negative values indicate that microturbines should pay the TSO for their reserve provision in terms of voltage management, which in the face of conventional wisdom. The reason is that reserve is scheduled to cope with uncertainties. When reserve is fully deployed in real-life situation, the excess active power generation in distribution systems may aggravates the existing overvoltage problem. Thus, the negative voltage component stimulates microturbines to reduce reserve in dispatch process to mitigate the voltage risk. In Fig. 8(b), the values are positive except at node 1, and thus motivates microturbines to increase reserve and stimulates PVs and WTs to improve forecast accuracy based on (30). Then the congestion can be alleviated accordingly.

Based on the above discussion, individual components can be analyzed in detail in the proposed DLMP mechanism, which is interpreted to market participants in a clear and intuitive way. The pricing mechanism can fully explain the system operation, and provide transparent price signals for managing voltage, congestion, and loss as well as implementing the uncertainty management in distribution systems.

DLMPs affect the output of participants. The active power and reserve of microturbines in the distribution system at bus D are shown in Fig. 10. It can be seen that the microturbines give priority to generating active power for more revenue when active power DLMPs are high. When the active power DLMPs decreases, microturbines utilize the residual capacity to provide reserve. Two microturbines have different output due to the different DLMPs at different nodes.

LMPs and ULMPs affect the distribution market clearing and pricing. The overall demands of distribution systems change accordingly. Fig. 11 illustrates the active power and
reserve demands of the distribution systems at bus C and D. From Fig. 4, Fig. 10, and Fig. 11, it can be concluded that DSOs schedule controllable DERs to generate more energy and reserve to reduce the demand when boundary LMPs and ULMPs are high, and vice versa. The overall demands of the distribution systems at buses C and D are different due to their different location.

C. Sensitivity Analysis of Uncertainty Level

The box set (10c) utilizes upper/lower bounds to describe uncertainty. In order to verify the impact of uncertainty level on market clearing and pricing, $\Lambda$ is introduced to adjust the uncertainty interval of (10c) for WTs and PVs in distribution systems [17], which can be expressed as

$$-\Lambda \cdot u_{rdg}^{i,t} \leq \epsilon_{rdg}^{i,t} \leq \Lambda \cdot u_{rdg}^{i,t} \quad \forall i \in RDG, \forall t$$ (37)

A bigger $\Lambda$ increases the width of box sets and means strong uncertainty. In Sections VI-A and VI-B, $\Lambda = 1$ (denoted as Case 1). The sensitivity analysis of uncertainty level on market clearing and pricing are performed by adding two cases (Case 2 and Case 3), where $\Lambda = 1.2$ and $\Lambda = 1.4$, respectively. The other parameters in Cases 2 and 3 are the same with those in Case 1. The market clearing and pricing results are demonstrated in Table II for the distribution system at bus D and in Table III for the wholesale market.

It can be found that high uncertainty increases not only the uncertainty DLMP which is directly related to uncertainty, but also active and reactive power DLMPs. The reason is that the market clearing model co-optimizes the dispatch and redispatch processes, resulting in the mutual influence of energy and reserve schemes. Accordingly, active and reactive power DLMPs both reflect the influence of uncertainty.

On the other hand, high uncertainty of renewable DERs increases their payment for reserve and reduces their total profit. Thus, the proposed DLMP mechanism can stimulate uncertainty sources to improve forecast accuracy. Moreover, the DLMPs motivate microturbines to provide reserve capacity to mitigate uncertainties, increasing their profit based on DLMPs. In a word, the proposed DLMP mechanism provides effective price signals to incentivize the uncertainty management.

High uncertainty of renewable DERs increases the operation costs of distribution systems as well as the active energy and reserve demands purchased from TSO. The increased demands of DSOs lead to the shortage of energy and reserve resources in transmission systems, and thus affects the transmission market clearing and pricing. Table III indicates that high uncertainty in distribution systems increases LMPs and ULMPs, as well as the transmission-level operation costs. In turn, the increased LMPs and ULMPs affects the distribution market clearing and pricing. Based on the above discussion, the DLMP method considers the influence of large amounts of DERs on wholesale markets, and the influence of LMPs and ULMPs on distribution markets. The coordination mechanism integrates distribution and wholesale markets to fully utilize resources in the whole system.

### Table II

| Comparison items | Case 1 | Case 2 | Case 3 |
|-----------------|-------|-------|-------|
| Average DLMPs  | Active power ($/MWh$) | 29.79 | 29.98 | 30.61 |
|                 | Reactive power ($/MVArh$) | 2.16 | 2.21 | 2.33 |
|                 | Uncertainty ($/MWh$) | 16.68 | 16.9 | 17.53 |
| Revenue of WTs  | Active Power | 222.41 | 224.55 | 225.86 |
|                 | Reserve | -5.09 | -60.81 | -73.17 |
|                 | Profit | 174.37 | 166.5 | 154.92 |
| Revenue of microturbines | Active Power | 1214.33 | 1242.39 | 1269.77 |
|                 | Reserve | 12.37 | 4.74 | 1.96 |
|                 | Profit | 89.36 | 101.66 | 123.35 |
| Revenue of ESSs  | Active Power | 208.69 | 211.48 | 257.46 |
|                 | Reserve | 93.45 | 111.64 | 97.57 |
|                 | Profit | 63.54 | 82.26 | 110.65 |
| Purchase from TSO | Active Power (MWh) | 5.95 | 6.41 | 6.96 |
|                 | Reserve (MW) | 4.58 | 6.49 | 7.33 |
| Operation Cost of DSO | Active Power | 691.2 | 723.31 | 743.48 |
|                 | Reserve | 12.15 | 11.94 | 16.16 |
|                 | Total | 140.17 | 157.69 | 212.41 |
| $\Delta$        | 843.52 | 892.94 | 972.05 |

### Table III

| Comparison items | Case 1 | Case 2 | Case 3 |
|-----------------|-------|-------|-------|
| Average Price   | LMP | 21.52 | 21.81 | 21.97 |
|                 | ULMP | 12.88 | 13.17 | 13.35 |
| Operation Cost of the Transmission Reserve | Energy | 160745.1 | 160950.05 | 161255.49 |
|                 | Reserve | 23405.63 | 24753.96 | 25562.7 |
| System Total    | 184150.73 | 185704.01 | 186798.19 |
TABLE IV
PERFORMANCE OF THE HGD ALGORITHM

| System   | Comparison item | HGD          | Centralized |
|----------|-----------------|--------------|-------------|
| T5D33    | Iteration       | 6            | 267451.01   |
|          | Objective       |              | 267451.01   |
| T118D69  | Iteration       | 10           | 2314927.58  |
|          | Objective       |              | 2314927.58  |

1) The uncertainty is internalized in the derivation of active and reactive power DLMPs, which can provide intuitive price signals in scheduling active and reactive power for managing voltage, congestion, and loss while considering the uncertainty.

2) The proposed uncertainty DLMP charges renewable DERs for their uncertainties and rewards reserve capacity to mitigate uncertainties. Accordingly, the uncertainty management is incentivized in distribution markets.

3) The coordination mechanism ensures the consistency between the wholesale market and distribution markets in terms of market clearing and price formation.

APPENDIX A
DERIVATION PROCESS OF LMP AND ULMP

The Lagrange function of the economic dispatch model (1)–(3) can be formulated as:

\[
L = \sum_{i \in T} \sum_{t \in \mathcal{J}_i} \left( C_i^C \left( P_{i,t}^C \right) + C_i^P \left( \Delta P_{i,t}^C \right) \right) + \sum_{i \in T} \sum_{t \in \mathcal{J}_i} \left( \sum_{j \in \mathcal{N}_i} \left( \left[ \sum_{j \in \mathcal{N}_i} \sum_{j \in \mathcal{W}_j} \sum_{j \in \mathcal{D}_j} \sum_{j \in \mathcal{G}_j} \right] \right) \right)
\]

The T5D33 system requires 6 iterations and 97.8 seconds, and the T118D69 system requires 10 iterations and 857.5 seconds to converge to the optimal solution. Taking the distribution system at bus C of the T5D33 system in hour 3 as an example, the convergence performance of HGD algorithm is shown in Fig. 12. It can be found that high LMPs lead to the reduction of active power demand of distribution systems, and vice versa. The coordination and interaction between TSO and DSOs are revealed accordingly. In addition, the convergence is accelerated by the introduced sensitivity after the third iteration. \( P_{R,6} = P_{R,5} = 0.33 \), thus the HGD algorithm converges at the sixth iteration based on the convergence criterion (34), which verify the convergency of HGD algorithm.

VII. CONCLUSION

This paper proposes a novel DLMP mechanism for day-ahead distribution market clearing and pricing that accounts for the uncertainty of renewable DERs and the coordination with wholesale markets. The effectiveness of the proposed method has been verified by case studies. The impact of uncertainties and transmission-distribution interactions on market pricing and system operation can be evaluated and analyzed through the proposed DLMP mechanism. Some conclusions can be drawn:

Fig. 12. Convergency performance.
According to the definition, the LMP at bus $i$ is defined as the partial derivative of the Lagrange function with respect to the forecast value of net load considering the active energy demand of distribution systems.

$$\pi_i = \frac{\partial L}{\partial (\sum_{j \in D \gamma(i)} p_{\gamma(i)}^D - \sum_{j \in W \gamma(i)} p_{\gamma(i)}^W)} = \lambda_i + \sum_l GSF_{l,i} \left( \eta_{l,i} - \eta_{l,i}^R \right)$$  

(A-3)

The ULMP at bus $i$ is defined as the partial derivative of the Lagrange function with respect to the forecast deviation of net load considering the reserve demand of distribution systems.

$$\pi_i^R = \frac{\partial L}{\partial (\sum_{j \in D \gamma(i)} \delta_{j}^D - \sum_{j \in W \gamma(i)} \delta_{j}^W)} = \lambda_i^R + \sum_l GSF_{l,i} \left( \eta_{l,i} - \eta_{l,i}^R \right)$$  

(A-2)

APPENDIX B

PROOFS FOR OPTIMALITY AND CONVERGENCY OF HGD ALGORITHM

A. Proof for Optimality

The transmission-distribution coordinated model (31) and (35) can be integrated into a centralized transmission and distribution coordinated model, which is a convex model and can be formulated in (B-1).

$$\min_{P_T, \Delta P_T, P_D, Q_D, \Delta P_D} c_T(P_T, \Delta P_T) + c_D(P_D, Q_D, \Delta P_D)$$  

(B-1a)

$$A_{tT} P_T + A_{tT} \Delta P_T + A_{P_{B_T}} P_B + A_{DB_{PB_T}} \Delta P_B = a_T (\lambda_{tT})$$  

(B-1b)

$$B_{tT} P_T + B_{tT} \Delta P_T + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B \geq b_T (\eta_{tT})$$  

(B-1c)

$$E_{tT} P_T + E_{tT} \Delta P_T \geq e_T (\mu_{tT})$$  

(B-1d)

$$A_{P_{P_T}} P_T + A_{DB_{P_T}} \Delta P_T + A_{Q_{P_D}} Q_D + A_{P_{PB_T}} P_B + A_{DB_{P_{PB_T}}} \Delta P_B = a_D$$  

(B-1e)

$$B_{P_{P_T}} P_T + B_{DB_{P_T}} \Delta P_T + B_{Q_{P_D}} Q_D + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B \geq b_D$$  

(B-1f)

$$E_{P_{P_T}} P_T + E_{DB_{P_T}} \Delta P_T + E_{Q_{P_D}} Q_D \geq e_D (\mu_{tT})$$  

(B-1g)

The Lagrange function of the centralized model (B-1) is

$$L = c_T(P_T, \Delta P_T) + c_D(P_D, Q_D, \Delta P_D)$$

$$- \lambda_{tT} (A_{tT} P_T + A_{tT} \Delta P_T + A_{P_{B_T}} P_B + A_{DB_{P_{B_T}}} \Delta P_B - a_T)$$

$$- \eta_{tT} (B_{tT} P_T + B_{tT} \Delta P_T + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B - b_T)$$

$$- \mu_{tT} (E_{tT} P_T + E_{tT} \Delta P_T + E_{P_{PB_T}} P_B + E_{DB_{P_{PB_T}}} \Delta P_B - e_T)$$

Therefore, the KKT conditions of the centralized model can be obtained as follows:

1) The partial derivative of the Lagrange function (B-2) with respect to the optimal variables is equal to zero.

$$\frac{\partial L}{\partial P_T} = \frac{\partial c_T(P_T, \Delta P_T)}{\partial P_T} - A_{tT}^T \lambda_{tT} - B_{tT}^T \eta_{tT} - E_{tT}^T \mu_{tT} = 0$$  

(B-3a)

2) The complementarity constraints are

$$\eta_{tT} (B_{tT} P_T + B_{tT} \Delta P_T + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B - b_T) = 0$$  

(B-3b)

3) The original constraints of the centralized model are

$$A_{P_{P_T}} P_T + A_{DB_{P_T}} \Delta P_T + A_{Q_{P_D}} Q_D + A_{P_{PB_T}} P_B + A_{DB_{P_{PB_T}}} \Delta P_B = a_D$$  

(B-3c)

$$B_{P_{P_T}} P_T + B_{DB_{P_T}} \Delta P_T + B_{Q_{P_D}} Q_D + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B \geq b_D$$  

The optimality of HGD algorithm can be proved when its converged solution satisfies the KKT conditions (B-3) for the convex centralized model (B-1). In the HGD algorithm, the transmission-level model is

$$\min_{P_T, \Delta P_T} c_T(P_T, \Delta P_T)$$

$$A_{P_{P_T}} P_T + A_{DB_{P_T}} \Delta P_T + A_{P_{PB_T}} P_B + A_{DB_{P_{PB_T}}} \Delta P_B = a_T$$

(B-4a)

$$B_{P_{P_T}} P_T + B_{DB_{P_T}} \Delta P_T + B_{P_{PB_T}} P_B + B_{DB_{P_{PB_T}}} \Delta P_B \geq b_T$$

(B-4b)

$$E_{P_{P_T}} P_T + E_{DB_{P_T}} \Delta P_T + E_{P_{PB_T}} P_B + E_{DB_{P_{PB_T}}} \Delta P_B \geq e_T$$

(B-4c)
where $P_B^{up}$ and $\Delta P_B^{up}$ are sent from DSOSs to the TSO in iterative calculation and assumed to be constants in (B-4).

The distribution-level model is shown in (35). Assume that the HGD algorithm converges at the $k$th iteration. Similar to the centralized model, the KKT conditions of HGD algorithm are as follows:

\[
\begin{align*}
\frac{\partial T(P_T, \Delta P_T)}{\partial P_T} & = -A^T_{P_T} \lambda_{AT} - B^T_{P_T} \eta_{BT} - E^T_{P_T} \mu_{ET} = 0 \\
\frac{\partial T(P_T, \Delta P_T)}{\partial \Delta P_T} & = -A^T_{\Delta P_T} \lambda_{AT} - B^T_{\Delta P_T} \eta_{BT} - E^T_{\Delta P_T} \mu_{ET} = 0 \\
\eta_{P_T}(B_{P_T} P_T + B_{\Delta P_T} \Delta P_T + B_{PB} P_{PB}^B + B_{\Delta PB} \Delta P_B^B - b_T) & = 0 \\
\mu_{ET}(E_{P_T} P_T + E_{\Delta P_T} \Delta P_T - e_P) & = 0 \\
A_{P_T} P_T + A_{\Delta P_T} \Delta P_T + A_{PB} P_{PB} + A_{\Delta PB} \Delta P_B & = a_T \\
B_{P_T} P_T + B_{\Delta P_T} \Delta P_T + B_{PB} P_{PB} + B_{\Delta PB} \Delta P_B & = b_T \\
E_{P_T} P_T + E_{\Delta P_T} \Delta P_T + E_{Q_D} Q_D & = e_D \\
\eta_{Q_D} & = 0, \mu_{ED} = 0
\end{align*}
\]  

(B-5)

The KKT conditions of the distribution model (35) are as follows:

\[
\begin{align*}
\frac{\partial C(D_P, Q_D, \Delta D_P)}{\partial D_P} & = -A^T_{D_P} \lambda_{AD} - B^T_{D_P} \eta_{BD} - E^T_{D_P} \mu_{CD} = 0 \\
\frac{\partial C(D_P, Q_D, \Delta D_P)}{\partial Q_D} & = -A^T_{Q_D} \lambda_{AD} - B^T_{Q_D} \eta_{BD} - E^T_{Q_D} \mu_{CD} = 0 \\
\frac{\partial C(D_P, Q_D, \Delta D_P)}{\partial \Delta D_P} & = -A^T_{\Delta D_P} \lambda_{AD} - B^T_{\Delta D_P} \eta_{BD} - E^T_{\Delta D_P} \mu_{CD} = 0 \\
\lambda_{AD} - S_{\Delta D_P} \Delta D_P & = 0 \\
\eta_{BD} & = 0, \mu_{CD} = 0
\end{align*}
\]  

(B-6)

When the algorithm converges, $P_{B,k} = P_{B,k-1}, \Delta P_{B,k} = \Delta P_{B,k-1}, P_{P,B}^{up} = P_{B,k},$ and $\Delta P_B^{up} = \Delta P_{B,k} S_{\xi,P,B,k} \eta_{Dr,b} S_{\xi,P,B,k}$ are eliminated in the fourth and fifth equations in (B-6). Thus, the introduction of sensitivity in (35) does not affect the solution optimality of HGD algorithm. Then bring $\xi_{P,B} = -A^T_{PB} \lambda_{AT} - B^T_{PB} \eta_{BT}$ and $\xi_{\Delta P_B} = -A^T_{\Delta PB} \lambda_{AT} - B^T_{\Delta PB} \eta_{BT}$ shown in (32) into (B-6) and compare (B5)-(B6) with (B3). It can be found that the KKT conditions of HGD algorithm are the same with those for the centralized model. Therefore, the HGD algorithm guarantees the global optimality.

B. Proof for Convergence

The HGD algorithm has a linear convergence in the neighbor of the optimum value of the centralized model (B-1) with some assumptions [41]: a) the optimal solutions of the transmission-level problem (31) and distribution-level problem (35) are regular points; b) the optimal solutions satisfy the second-order sufficient optimality conditions and the strict complementarity slackness condition. When these assumptions are satisfied, the following mapping relationships hold at the $k$th iteration [45].

\[
\begin{align*}
\xi_{P_B,k} & = h_{TB,P_{B}}(P_{B,k}^{up}) \\
P_{B,k+1}^{up} & = h_{BD,P_B}^{-1}(\xi_{P_B,k}) \\
\xi_{\Delta P_B,k} & = h_{TB,\Delta P_B}(\Delta P_{B,k}^{up}) \\
\Delta P_{B,k+1}^{up} & = h_{BD,\Delta P_B}^{-1}(\xi_{\Delta P_B,k})
\end{align*}
\]  

(B-7)

where $h_{TB,P_{B}}, h_{BD,P_B}, h_{TB,\Delta P_B},$ and $h_{BD,\Delta P_B}$ denote the mapping functions, which are unique once continuously differentiable based on the assumptions.

Then the composite mapping $\Phi_{PB}$ and $\Phi_{\Delta PB}$ can be obtained as follows:

\[
\begin{align*}
P_{B,k+1}^{up} & = h_{BD,P_B}^{-1} \circ h_{TB,P_{B}}(P_{B,k}^{up}) \Phi_{PB} \\
\Delta P_{B,k+1}^{up} & = h_{BD,\Delta P_B}^{-1} \circ h_{TB,\Delta P_B}(\Delta P_{B,k}^{up}) \Phi_{\Delta PB}
\end{align*}
\]  

(B-8)

In (B-9), the derivative of $\Phi_{PB}$ is calculated by multiplying the derivatives of $h_{BD,P_B}$ and $h_{TB,P_{B}}$. Likewise, the derivative of $\Phi_{\Delta PB}$ is calculated by multiplying the derivatives of $h_{BD,\Delta P_B}$ and $h_{TB,\Delta P_B}$, which can be calculated based on the work in [47].

In the T5D33 system and T118D69 system in Section VI-D, the norm of the derivatives of $\Phi_{PB}$ and $\Phi_{\Delta PB}$ are both less than 1. Thus, the convergence of HGD algorithm is guaranteed.

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