Relating the ElectroWeak scale to an extra dimension: 
constraints from ElectroWeak Precision Tests

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Abstract

In models with supersymmetry breaking by boundary conditions on an extra dimension it is possible to relate in a quantitative natural way the Fermi scale to the size of the extra dimension. We analyze in detail the compatibility of such models with ElectroWeak Precision Tests.
1 Introduction

In the last decade the Standard Model (SM) has received impressive experimental confirmations by the ElectroWeak Precision Tests (EWPT). However, despite its experimental success, the core of the SM, i.e. the physical mechanism of ElectroWeak Symmetry Breaking (EWSB), remains unexplained. The SM, through the Higgs mechanism, probably catches the essence of EWSB, without providing a physical theory for it. The Higgs boson mass has a quadratic sensitivity to the ultraviolet (UV) physics. This fact led to the expectation of discovering New Physics around the TeV scale. Today, thanks to EWPT, the problem of the stabilization of the Higgs potential assumes a new, entirely low energy, aspect. The 1-loop correction to the Higgs boson mass due to the top quark is given by

$$\delta m^2_H = \frac{3}{\sqrt{2} \pi^2} G_F m^2_t \Lambda^2 \simeq (120 \text{ GeV})^2 \left( \frac{\Lambda}{430 \text{ GeV}} \right)^2$$

Equation (1)

The EWPT suggest an indirect evidence for a light Higgs boson ($m_H < 237 \text{ GeV}$ at 95% of C.L. [1]). Without invoking a very finely-tuned cancellation between a counterterm and the loop correction (1), one requires the latter not being much larger than the physical Higgs mass. One then needs a physical mechanism, such as the appearance of some new particle, which cuts off $\delta m^2_H$ not too far from the ElectroWeak scale. The fact that the EWPT showed no signals of physics beyond the SM makes the gap between the Higgs mass and a typical scale of new physics problematic from the naturalness point of view, even if this scale is in the $5 \div 10 \text{ TeV}$ range. This is the so-called “Little Hierarchy Problem”.

These kind of arguments led many to the expectation of seeing some supersymmetric particle at LEP. But this did not happen. In Supersymmetry it is the appearance of a stop quark with mass $m_{\tilde{t}}$ which cuts off eq. (1) rendering logarithmic the quadratic divergence. However, the fact that neither the Higgs boson nor any supersymmetric particle has been experimentally discovered implies, in the context of supersymmetric models, that a certain amount of fine-tuning has necessarily to be accepted.

All this motivates further thoughts on the problem of EWSB. Precisely, one should look for alternative models which i) are possibly predictive, ii) naturally foresee the existence of a light (not fine-tuned) Higgs boson and iii) are not in conflict with EWPT. We want to focus on a recent proposal of supersymmetry breaking obtained by imposing boundary conditions along a compact extra dimension [2, 3, 4]. This approach allows to connect the ElectroWeak scale to the radius of compactification of an extra dimension. The implementations of this idea reach the goals i) and ii). In fact the models which have been built are extremely predictive in terms of a minimal number of free parameters and the connection between the Higgs mass and the radius of compactification can be established with very high precision [5]. The “Little
Hierarchy problem" is solved since the Higgs mass is UV-insensitive. In this paper we want to analyze in detail the point iii): the compatibility of these models with EWPT.

The paper is organized as follows. Mostly for the ease of the reader, we recall in Sec. 2 the main features of the idea to connect the ElectroWeak scale to the radius of compactification of an extra dimension and describe the two possible implementations of this setup (Subsec. 2.2 and 2.3). In Sec. 3 we analyze in detail the impact of EWPT on the two implementations. The conclusions are drawn in Sec. 4.

2 The models

2.1 The setup

Let us consider a $SU(3) \times SU(2) \times U(1)$ invariant theory with a compact spatial extra-dimension, seen in the following as a segment of length $\pi R/2$. We require that the 5D theory is supersymmetric. Given a generic field $f$, supersymmetry implies the existence of its superpartner $\tilde{f}$. Furthermore the existence of the conjugated fields $f^c, \tilde{f}^c$ follows from 5D Poincaré invariance. They have the same chirality as $f, \tilde{f}$ but opposite quantum numbers. Up to now every field has a zero mode. It is possible, in a unique way, to get rid of the unwanted states and remain with only the SM zero modes. This can be achieved by imposing suitable boundary conditions to the fields of the theory. In fact imposing Dirichlet (+) or Neumann (−) conditions at the two boundaries, only $(+, +)$ fields have a zero mode: $(+, -)$ and $(−, +)$ lightest modes are at $1/R$, while the $(−, −)$ are at $2/R$. The assignment of the boundary conditions to the various fields is shown in Fig. 1.

Now the mass of every particle is shifted of $1/R$ with respect to its superpartner: the boundary conditions have non-locally broken 5D supersymmetry. This mechanism is known
Table 1: Continuous $R$ charges for gauge, Higgs and matter components. Here, $m$ represents $q, u, d, l, e$.

as supersymmetry breaking à la Scherk-Schwarz [6, 7]. However there is a residual local supersymmetry. In fact the initial $N = 1$ 5D supersymmetry can be seen, from the 4D point of view, as two $N = 1$, 4D supersymmetries $\xi_1, \xi_2$ linked by an $SU(2)_R$ symmetry. After imposing the boundary conditions the theory is still supersymmetric under $(\xi_1^{(+,-)}, \xi_2^{(-,+)})$. The two supersymmetric transformation parameters have thus to satisfy proper boundary conditions. Then at the boundaries there are two different $N = 1$, 4D supersymmetries: at $y = 0$ one has $(\xi_1 \neq 0, \xi_2 = 0)$; at $y = \pi R/2$ one has $(\xi_1 = 0, \xi_2 \neq 0)$. This residual supersymmetry is crucial for the calculability of the model: it renders insensitive to the UV physics several observables.

Furthermore, after imposing the boundary conditions, there are two other residual symmetries.

- A continuous R-symmetry with R-charges given in Table 1 intact even after EWSB. The absence of any A-terms or Majorana gaugino masses can be traced back to this symmetry.

- A local $y$–parity $P_5$ under which any field transforms as

$$\varphi(y) \rightarrow \eta \varphi(\pi R/2 - y)$$

where $\eta$ is the parity assignment at any one of the two boundaries. This symmetry forbids local mass terms for the hypermultiplets.

The most general Lagrangian compatible with the symmetries of the theory is the following

$$\mathcal{L} = \mathcal{L}_5 + \delta(y) \mathcal{L}_4 + \delta(y - \frac{\pi R}{2}) \mathcal{L}_4'$$

where $\mathcal{L}_5$ is supersymmetric under the full $N = 1$, 5D supersymmetry, while $\mathcal{L}_4$ and $\mathcal{L}_4'$ respect the two different $N = 1$, 4D supersymmetries at the boundaries. The Yukawa interactions, necessary to give a mass to the SM fermions, are forbidden by 5D supersymmetry and thus have to be localized at the boundaries. We take the top quark Yukawa coupling,
crucial for EWSB, to be localized at $y = 0$:

$$\delta(y) \int d^2 \theta \lambda_U QUH + \text{h.c.}$$

(4)

while the bottom Yukawa coupling is necessarily localized at the opposite boundary. The tree-level Higgs potential is constrained from supersymmetry to have the form

$$V_{H,0} = \frac{g^2 + g'^2}{8} |\phi_H|^4$$

(5)

where $g$ and $g'$ are the 4D $SU(2)$ and $U(1)$ gauge couplings.

There are two possible implementations of this setup. The first one is the so-called “Constrained Standard Model” (CSM) \[2, 3\] and the second one is the so-called “Quasi-Localized Top Model” (QLTM) \[4, 5\]. We now describe them in more detail.

2.2 The CSM

If we implement the setup previously exposed with no modification, we are in the position to compute the 1-loop Higgs boson mass due to the top Yukawa coupling \[4\] and, more in general, the Coleman-Weinberg potential in order to analyze EWSB. The 1-loop Higgs mass is given by

$$\delta m_H^2 = -\frac{63}{\sqrt{2\pi^4}} \frac{G_F m_t^2}{R^2} \simeq -0.19 \frac{1}{R^2}$$

(6)

The finiteness of (6) is a consequence of local supersymmetry conservation in 5D, as previously discussed. Thus EWSB is triggered radiatively by the top Yukawa interaction.

One can then compute the full 1-loop potential due to the top multiplet. It is a function of the top Yukawa coupling (determined by the top quark mass) and of $1/R$: there is one parameter less than in the SM, hence the name of the model. Requiring that the Higgs potential has its minimum at the Higgs vev $v$, determined by the Fermi constant $G_F$, one can compute the value of $1/R$ which turns out to be in the 400 GeV range \[2\]. With this value of $1/R$, taking the second derivative of the potential evaluated at its minimum, one finds the Higgs mass, predicted to be about 130 GeV \[2\].

In the CSM with a single Higgs supermultiplet a quadratically divergent Fayet-Iliopoulos (FI) term arises for the hypercharge vector supermultiplet \[8\]. However, given the value of the cut-off, determined below in Sect. \[2, 3\] the FI is numerically negligible \[9\]. Nevertheless, in order to ensure the quantum consistency of the theory, one has to give up the local parity \[2\]. Then it is necessary to consider possible mass parameters for all the hypermultiplets. If they are taken to be small with respect to $1/R$ this consists of a small modification of the theory \[3\]. If they are $\gtrsim 1/R$ one has a qualitatively different model, described in the next section.
Figure 2: The Higgs function as a function of $1/R$ for $M_U = M_Q = M_D$ and $M_H = 0$.

### 2.3 The QLTM

Giving up the local parity one can consider the effect of a mass term for every hypermultiplet. For the hypermultiplet of components $(\psi, \psi^c, \varphi, \varphi^c)$, the 5D mass term is

$$
\mathcal{L}_m = - (\psi m(y)\psi^c + \text{h.c.}) - M^2 (|\varphi|^2 + |\varphi^c|^2) \\
- 2M (\delta(y) + \delta(y - \pi R/2)) (|\varphi|^2 - |\varphi^c|^2)
$$

The mass term $m(y)$ must respect $(-, -)$ boundary conditions. We consider a constant mass term inside the segment $(0, \pi R/2)$. For a matter hypermultiplet all the KK modes acquire a mass which goes as $|M|$ for $|MR| \gg 1$. The fermionic zero mode remains massless, while one of the two lightest scalars has mass which goes to zero as $|MR| \to \infty$. The wave function of the fermion zero mode gets exponentially localized at on boundary depending on the sign of $M$

$$
\psi_0(y) \propto |M|^{1/2} \exp [-My + (M - |M|)\pi R/2]
$$

The same thing happens for the scalar particle which becomes massless. In the limit $MR \to \infty$ a supersymmetric spectrum is recovered.

We are now interested in the following model. We introduce a second Higgs hypermultiplet $H_d$ in order to exactly cancel the 1-loop generated FI term. This additional Higgs doublet does not get a vev and is irrelevant for EWSB. The only mass terms which are relevant for EWSB are those for the third quark generation and for the Higgs. We choose to localize the third generation of quarks towards the $y = 0$ boundary through a common mass term $M_U = M_Q = M_D \equiv M$, setting for the time being $M_H = 0$. With this configuration the FI
term vanishes. Now one can compute the 1-loop Higgs potential which now depends on two parameters: $1/R$ and $MR$. Thus the Fermi constant allows to connect $MR$ with $1/R$. Then the Higgs mass is a function of $1/R$. The connection between the Higgs mass and $1/R$ can be established with a few percent accuracy, calculating the relevant 2-loop contribution to the Higgs potential [5], even if the relation between $1/R$ and $MR$ is known with significantly less precision. We are interested in typical values $MR \simeq 1.5 \div 2.5$: for too large values of $MR$ EWSB does not occur successfully, while for too small values of $MR$ one needs a mass term for the Higgs in order to stabilize the potential. The result is shown in Fig. 2. The typical value of $1/R$ has increased of about one order of magnitude with respect to CSM. This is due to the near cancellation between the 1-loop electroweak correction and the top 2-loop correction to the Higgs mass.

If one allows an equal mass term $M_H$ for the two Higgses the FI still vanishes. This affects EWSB giving a tree level mass term to the Higgs potential that stabilizes the Higgs potential, allowing lower values of $MR$. In Fig. 3 we show the Higgs mass as a function of $1/R$ leaving $M_H$ free to vary. However $M_H$ is limited from above by the request of a maximum cancellation of 10% in the slope of the potential and from below by the stability of the potential. In the upper range of $1/R$, $M_H$ is irrelevant and we recover the previous results. The calculation of the potential done in [5] is reliable either for $1/R \lesssim 1$ TeV (where the 1-loop top contribution dominates) or for $1/R \gtrsim 2$ TeV (where a quasi-localized approximation can be used). For intermediate values of $1/R$ a more accurate calculation of the 2-loop contribution could be necessary.

Figure 3: The Higgs function as a function of $1/R$ for $M_U = M_Q = M_D$ and $M_H \neq 0$. 

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2.4 The UV cutoff

The previous models are formulated in 5D and are non-renormalizable. Indeed they have to be regarded as effective field theories, valid up to an energy scale $\Lambda$ where they will be completed by some kind of a more fundamental theory.

Since perturbative calculations in an effective field theory are organized as an expansion series in $(E/\Lambda)$, the range of their validity is controlled by the size of the cutoff $\Lambda$. It is then necessary to correctly estimate its value.

A lower bound on the cutoff can be given by estimating the scale at which one of the interactions of the model ceases to be perturbative. Then the theory (or at least one of its sectors) should be completed by UV unknown physics at energies of the order of $\Lambda$.

As already shown in [2, 4] the first interactions that become strong are the Yukawa couplings of the 3rd generation.

This is due to the fact that Yukawa couplings are localized at the boundaries and break discrete 5th-momentum conservation. The number of open channels in a process mediated by a Yukawa interaction grows more than linearly with the energy, bringing 5D Yukawa couplings to become strong before any gauge coupling of the same 4D strength.

Here, we recall the estimation of $\Lambda$ from top and bottom Yukawa interactions. We consider also bottom interactions because in presence of hypermultiplet masses the hierarchy $m_b/m_t$ can be generated from localization effects alone and the size of $\lambda_t$ and $\lambda_b$ in 5D can be comparable.

Let us focus first on the CSM case. Here only $\lambda_t$ matters since all the hypermultiplets masses are zero. $\lambda_t(\Lambda)$ can be estimated either by Naive Dimensional Analysis (NDA) [10] properly adapted to 5D [11] or by 4D NDA, taking into account the number $N$ of KK modes below the cutoff. In the first case we get

$$\tilde{\lambda}_t(\Lambda) \simeq \frac{1}{16 \pi^2} \left( \frac{24 \pi^3}{2 \pi \Lambda R} \right)^{3/2} \simeq 8.2 (\Lambda R)^{-3/2}$$

while in the second

$$\tilde{\lambda}_t(\Lambda) \simeq 4 \pi \left( \frac{1}{\Lambda R} \right)^{3/2} \simeq 8.9 (\Lambda R)^{-3/2}$$

where $\tilde{\lambda}_t = \lambda_t/(2\pi R)^{3/2} \simeq m_t/v$, so that $\Lambda R \sim 5$.

In the QLTM model since the 3rd generation is localized towards $y = 0$ the Yukawa coupling of both the top and the bottom are relevant for perturbativity and are indeed equal at $MR \sim 4/\pi$. The relation between $\tilde{\lambda}_{t,b}$ and $\Lambda$ now depends on the localization parameter $MR$. This dependence comes from two sources: the most important one is the relation between the 5D and the 4D couplings, the other one is the fact that the KK spectrum is shifted in presence of non-zero masses. Both sources are important for big masses and the second one is
Figure 4: Cutoff estimation from top and bottom Yukawa coupling perturbativity as a function of $MR$. The thick point represents the case of the CSM.

particularly relevant for low cutoffs. The impact of these two effects of perturbativity can be easily estimated by means of 4D NDA, remembering that the masses of the excited states go like $m^2 = (2n/R)^2 + M^2$ for large $MR$ and that the relation between the 5D and 4D couplings reads

$$\hat{\lambda}_{t,b}(\Lambda_{t,b}) \simeq \frac{m_{t,b}}{v} \frac{2}{\pi MR (\coth(MR\pi/2) \pm 1)}$$

with the sign $+$ for the top and the $-$ for the bottom.

The estimated cutoff is shown in Fig. 4. For values of $MR \lesssim 1.4$ the cutoff is determined by the top Yukawa coupling and increases with $MR$, while for bigger values $\Lambda R$ is controlled by the bottom and decreases exponentially with the localization parameter. In particular for the QLTM one has successful EWSB for $MR \lesssim 2.3$; at that maximum value of $MR$ one has $\Lambda R \simeq 3 \div 4$. If the model is not UV completed before $\Lambda$, a strong interacting sector appears at one of the boundaries. For $\Lambda R \gg 1$ we can think of the theory in this regime as composed by a bulk sector with perturbative gauge couplings coupled to a strong interacting sector localized at one of the boundary. This motivates the possibility, when considering the effects of higher order SUSY operators on EWPT, to focus on those localized at the boundaries.

3 EWPT constraints

The effects on EWPT arise from various sources. First of all there are calculable loop effects generated by non supersymmetric operators. In fact, as we shown in Sect. 2 supersymmetry is globally broken: the mass splitting between particles belonging to the same supersymmetric multiplet is $1/R$. However the residual supersymmetry of the theory restricts the form of the
possible counterterms. Observables corresponding to operators which have no counterterm allowed by the residual supersymmetry are thus necessarily finite. This is the case for the Higgs boson mass [2], but also for several other observables like $b \to s\gamma$ [12], the muon $g - 2$ [13] and the Higgs boson decay into two gluons [14]. The calculations for $b \to s\gamma$ and $g_{\mu} - 2$ show that for $1/R \gtrsim 400$ GeV these effects are below the current experimental sensitivity.

Beyond these calculable effects one should also consider the effect of supersymmetric operators, mainly those localized at the boundaries. Indeed one has to look for operators respecting the $N = 1$ supersymmetries present at the boundaries and which affect EWPT. The fact that they are allowed by supersymmetry makes 1-loop computations divergent. These operators, whose coefficients have negative dimension in mass, are generated by radiative corrections at the scale where perturbation theory breaks down and are weighted by powers of $1/\Lambda R$. In order to build a fully reliable and predictive theory one has to consider the impact of such operators on EWPT. We analyze the localized operators assuming that their coefficients saturate perturbation theory at the scale $\Lambda$, estimating them by 5D NDA.

### 3.1 Universal effects

We make an analysis in terms of the form factors $\hat{S}, \hat{T}, W, Y$ introduced in Ref. [15]. Such an analysis is valid for a wide class of “universal theories” in which the only gauge interaction (except QCD) of all the light fermions of the SM is

$$\mathcal{L}_{\text{int}} = \bar{\Psi} \gamma^\mu \left( T^a \bar{W}_\mu^a + Y \bar{B}_\mu \right) \Psi$$  \hspace{1cm} (12)$$

where $\bar{W}, \bar{B}$ are not necessary the physical $W, B$. In our case, if no localization parameter is present, it follows from momentum conservation in the 5th dimension that, at tree level, the “interpolating” fields $\bar{W}, \bar{B}$ are exactly the zero modes of the 5D gauge bosons. We shall come back to the localization effects later on.

Upon use of the equations of motion and neglecting terms vanishing with the fermion masses, a complete set of dimension-6 operators which contribute to $\hat{S}, \hat{T}, W, Y$ are [16]:

$$\mathcal{O}_{W B} = \frac{1}{gg'} (H^\dagger \tau^a H) W_\mu^a B_{\mu \nu}$$  \hspace{1cm} (13a)$$

$$\mathcal{O}_H = |H^\dagger D_\mu H|^2$$  \hspace{1cm} (13b)$$

$$\mathcal{O}_{BB} = \frac{1}{2g'^2} (\partial_\mu B_{\mu \nu})^2$$  \hspace{1cm} (13c)$$

$$\mathcal{O}_{WW} = \frac{1}{2g^2} (D_\rho W_\mu^a)^2$$  \hspace{1cm} (13d)$$

If they appear in a 4D lagrangian as

$$\delta \mathcal{L} = \frac{1}{v^2} (c_{WB}\mathcal{O}_{WB} + c_H\mathcal{O}_H + c_{WW}\mathcal{O}_{WW} + c_{BB}\mathcal{O}_{BB})$$  \hspace{1cm} (14)$$
where $v = \langle H \rangle = 174$ GeV is the Higgs vev, they give the following contributions to the EW form factors

$$
\begin{align*}
\hat{S} &= 2 \frac{c_W}{s_W} c_{WB}, \\
\hat{T} &= -c_H, \\
W &= -g^2 c_{WW}, \\
Y &= -g^2 c_{BB}
\end{align*}
$$

(15)

Since at the boundaries there are $N = 1$ supersymmetries, we have to find the supersymmetric completion of the operators $[13]$. This can be easily accomplished using supersymmetric gauge covariant derivatives (see for example [17]). Their properties are briefly recalled in appendix A. A simple power counting shows that all the SUSY and gauge invariant operators up to dimension-6 only, involving Higgs and vector superfields are

$$
\begin{align*}
\int d^4\theta (\hat{H}^\dagger e^{gV} \hat{H})^2 \\
\int d^2\theta \, \text{Tr} \left( \nabla^\mu \hat{W}^\alpha \nabla_\mu \hat{W}_\alpha \right) + \text{h.c.} \\
\int d^2\theta \, \text{Tr} \left( C_{\alpha\hat{\beta}} \nabla^{\alpha\hat{\beta}} \hat{W}_\alpha \nabla^{\beta\hat{\beta}} \hat{W}_\beta \right) + \text{h.c.} \\
\int d^4\theta \hat{H}^\dagger e^{gV} \hat{W}_\alpha e^{-gV} \nabla_\alpha e^{gV} \hat{H} + \text{h.c.} \\
\int d^4\theta \hat{H}^\dagger e^{gV} \nabla^\mu \nabla_\mu \hat{H} \\
\int d^4\theta \hat{H}^\dagger e^{gV} \hat{H} \\
\int d^2\theta \, \text{Tr} \left( \hat{W}^\alpha \hat{W}_\alpha \right) + \text{h.c.}
\end{align*}
$$

(16a-g)

where $\hat{H}$ is the Higgs chiral superfield while $V$ is a general vector superfield and $\hat{W}_\alpha$ its chiral supersymmetric field strenght. They refers both to the $W$ and to the $B$ vector fields. The symbol $C$ is the usual antisymmetric matrix used in raising and lowering spinorial indices. In our notation $C_{\alpha\beta} = \sigma_2$.

Using (anti)commutation rules the other unlisted operators are shown to be equivalent to suitable combinations of the previous ones. Expanding them in component fields, one can see that all the operators in [13] are originated, up to terms that vanish by the equations of motion and/or in the limit of massless fermions. Therefore the basis selected in [13] can be supersymmetrized to $N = 1$ in 4D. In particular the first operator originates $O_H$, the second and the third originate $O_{WW}$ and $O_{BB}$ while the fourth contributes to $O_{WB}$. On the contrary the localized kinetic terms [13] only contribute through the mixing of the zero modes with the Kaluza Klein states. The related effects, also of “universal” nature, are however

\footnote{The normalization of the vector fields is such that $\mathcal{L}_{\text{kin}} = -\frac{1}{4g^2} W_\mu W^\mu - \frac{1}{4g^2} B_\mu B^\mu$.}
subdominant in the parameter region of interest, with respect to the direct contributions from dimension 6 operators and we shall neglect them in the following.

To estimate the coefficients of the operators $\mathcal{O}_i$ we use 5D NDA. Notice that the existence of the supersymmetric operators $\mathcal{O}_i$ makes the 1 loop corrections to these coefficients divergent. For example, the Yukawa contributions to $c_H$, or $\hat{T}$, has a quadratic sensitivity to the UV, while the dependence of $\hat{S}$ is logarithmic. This can be easily found by dimensional analysis. In fact the coefficient $c_H$ of the operator $\mathcal{O}_H$ localized at one of the boundaries and written in terms of the 5D fields has dimension of $(\text{mass})^{-4}$. 1-loop Yukawa corrections will be proportional to the fourth power of the 5D Yukawa coupling. Then the quadratic dependence on $\Lambda$ comes out immediately remembering that the dimension of a 5D Yukawa coupling is $(\text{mass})^{-3/2}$. In the same way $c_{WB}$ for a localized contribution has dimension of $(\text{mass})^{-3}$ and the Yukawa corrections to $\mathcal{O}_{WB}$ are proportional to the square of the Yukawa coupling giving a logarithmic dependence on $\Lambda$.

Let us now suppose that the operators $\mathcal{O}_i$ contribute to a localized term $\delta \mathcal{L}_4$ in eq. (3), where the fields $H, W^a$, and $B_\mu$ are 5D fields localized at a boundary. The dominant contribution to the EWPT from the operators of dimensions 6 in eq. (16a-e) comes from the zero modes of the various fields, i.e. the standard gauge and Higgs bosons. The localized operators we are interested in are then the following

$$\delta \mathcal{L}_4 = C_H |\bar{H}^\dagger D_\mu \bar{H}|^2 + \frac{C_{WB}}{gg'} (\bar{H}^\dagger \tau^a \bar{H}) W^a_\mu B_\mu + \frac{C_{BB}}{2g'^2} (\partial_\mu B_\mu)^2 + \frac{C_{WW}}{2g^2} (D_\mu W^a_\mu)^2$$

(17)

where $\bar{H}$ is the zero mode of the Higgs boson with canonical 5D normalization, whereas the vector bosons are already normalized to 4D (and $g, g'$ are the standard 4D gauge couplings). Using naive dimensional one finds

$$C_H = \frac{(24\pi^3)^2}{16\pi^2} \frac{1}{\Lambda^4}, \quad C_{WB} = gg' \frac{(24\pi^3)^2}{16\pi^2} \frac{1}{\Lambda^3}, \quad C_{BB} = C_{WW} = \frac{1}{16\pi^2} \frac{1}{\Lambda^2}$$

(18)

To connect the 5 dimensional coefficients of (18) to the 4 dimensional coefficients of eq. (14) one has to rescale the 5D Higgs field in terms of the 4D zero mode: $\bar{H} = H/\sqrt{2\pi R}$. Using (14) and (15) one gets

$$\hat{S}(\mathcal{O}_{WB}) \sim \frac{3}{2} g^2 \frac{(vR)^2}{(\Lambda R)^3} \simeq 1.6 \cdot 10^{-4} \frac{(R \cdot \text{TeV})^2}{(\Lambda R/5)^3}$$

(19a)

$$\hat{T}(\mathcal{O}_H) \sim 9\pi^2 \frac{(vR)^2}{(\Lambda R)^4} \simeq 4.3 \cdot 10^{-3} \frac{(R \cdot \text{TeV})^2}{(\Lambda R/5)^4}$$

(19b)

$$W(\mathcal{O}_{WW}) \sim Y(\mathcal{O}_{BB}) \sim \frac{g^2}{16\pi^2} \frac{(vR)^2}{(\Lambda R)^2} \simeq 3.3 \cdot 10^{-6} \frac{(R \cdot \text{TeV})^2}{(\Lambda R/5)^2}$$

(19c)

One can notice that the dominant contributions come from $C_H$. $C_{WW}, C_{BB}, C_{WB}$ become comparable to $C_H$ only for values of the cut-off $\Lambda R \simeq 100$ which can never be attained (see
Figure 5: Bounds on the CSM and the QLTM models from EWPT. The shaded region is excluded at 99 % of C.L.

Fig. 4). Comparing the contributions (19) to the experimental values [15]

\[
\hat{S} = (-0.7 \pm 1.3) \cdot 10^{-3}, \quad \hat{T} = (-0.5 \pm 0.9) \cdot 10^{-3}, \quad W = (0.2 \pm 0.6) \cdot 10^{-3}, \quad Y = (0.0 \pm 0.6) \cdot 10^{-3},
\]

(20)

we can obtain the bound shown in Fig. 5. The shaded region is excluded at 99 % of C.L. The sign of \( C_H \) is irrelevant. The black point represents the CSM described in Sec. 2.2, the red continuous line represents the QLTM model described in Sec. 2.3 for \( M_U = M_Q = M_D \) and \( M_H = 0 \), while the region inside the green lines corresponds to the QLTM model varying \( M_H \) as described in Sec. 2.3. This region consists in two disconnected areas since we excluded the region of the parameter space where the Higgs is too light and already excluded by direct searches. The red continuous line is stopped at \( 1/R \approx 3.2 \) TeV because the Higgs mass drops below the experimental limit (see Fig. 2) and at about 7 TeV when a fine-tuning of about 10 % in the potential occurs. One has to remember that the connection between \( MR \) and \( 1/R \) is not as precise as the one between the Higgs mass and \( 1/R \). Thus the red continuous line can move horizontally of about \( \Delta(AR) \approx 3 \) due to this uncertainty. From these universal effects it seems problematic to reconcile the CSM with the EWPT. One should not forget, on the other hand, that eqs. (19) are a naive estimate, dependent on a high power of \( \Lambda R \).
Furthermore, an unknown contribution coming from the UV completion of the theory could provide a fine-tuned cancellation in order to make the size of the contribution to $\hat{T}$ in Eq. (19b) sufficiently small. Conversely, in the QLTM model both for $M_H = 0$ and for $M_H \neq 0$, the typical values of $1/R$ are large enough not to create conflicts with EWPT.

3.2 Non universal effects

We now come back to the non universal effects involving the bottom quark.

In the CSM a localized operator

$$\int d^4\theta \, \hat{H}^\dagger \hat{H} \, Q_3^\dagger e^{\theta V} Q_3$$

is generated, with a similar coefficient to $C_H$ in eq. (18). It is a correction to the $Zb\bar{b}$ vertex and thus one obtains a similar bound to the one discussed in the previous Subsection.

In the QLTM and for $M_H = 0$ it is the bottom Yukawa coupling which becomes non perturbative first, hence one expects that the most important contribution comes from the operator (21) localized at the $y = \pi R/2$ boundary. But since the bottom gets quasi-localized mostly at the $y = 0$ boundary, a wave function suppression is present making this effect negligible. If we turn $M_H$ on, then successful EWSB occurs for lower values of $MR$ where it is the top Yukawa coupling that determines the cut-off. Then one should consider the operator at $y = 0$. However one obtains a bound that does not differ significantly from the one obtained above from $\hat{T}$, which is weaker than the one coming from flavor violation effects, analyzed in the following.

In addition to this kind of effect, if we turn on a mass term for the third generation of quarks, the interpolating field defined by eq. (12) is no longer the same for all the light fermions since, for the bottom, it is a superposition of all the KK modes. Then for the third generation an additional interaction to (12) is present at the tree level which gives non universal effects mainly concerning the bottom quark. These effects produce 4-fermion interactions involving the bottom quark and modify the $Zb\bar{b}$ vertex only in presence of localized kinetic terms for the gauge or Higgs multiplets. The size of this effect is comparable to the one produced by the operator (21).

The last source of effects comes from flavor violation. If one localizes only the third quark generation then it is the different coupling of the KK gluons to the first two generations with respect to the third one which gives an effect. First of all we have to assume that the Yukawa matrices and the 5D mass matrices are diagonal in the same basis. Assuming mixing angles and phases of the down-quark Yukawa coupling matrix comparable to those of the Cabibbo-Kobayashi-Maskawa matrix, then the strongest bound comes from the $\epsilon$-parameter in $K$ physics: one needs $1/R \gtrsim 1.5 \text{ TeV}$ [18]. This bound applies to the QLTM model. In
the region where $1/R \lesssim 2$ TeV (and $M_H \neq 0$) there is a weaker bound due to the weaker localization. Even if this bound is stronger than the one coming from EWPT when $\Lambda R \gtrsim 10$, the region inside the green lines showed in Fig. 3 is entirely allowed.

We can then conclude that while in the CSM it appears problematic to avoid a conflict with EWPT without invoking a fine-tuning mechanism, the QLTM model, which foresees natural values of $1/R \gtrsim 2$ TeV, is perfectly compatible with EWPT.

4 Conclusions

We have analyzed in detail the constraints coming from EWPT to the possible implementations of the idea to relate the ElectroWeak scale to the radius of a compact extra dimension. We have recalled how, insisting on supersymmetry in 5D and breaking it through boundary conditions à la Scherk-Schwarz, it is possible to build essentially two kind of models. In the first one, the CSM, the radius of the extra dimension turns out to be in the 400 GeV range, while the Higgs mass is about 130 GeV. In the other possible implementation of the general setup, the QLTM model, we considered a theory where the third generation of quarks is localized close to the boundary $y = 0$ through a mass term which is of the order of $1/R$. In this theory the typical value of $1/R$ is increased by almost one order of magnitude with respect to the CSM due to a quasi cancellation between the electroweak and the top corrections to the Higgs mass. We analyzed both the case of vanishing Higgs mass term and the case where $M_H \neq 0$.

The possible effects come from:

1. Calculable loop effects generated by non supersymmetric operators.
2. Supersymmetric operators localized at the boundaries.
3. Flavor physics.

We have shown that the strongest constraints come from 2 and 3. In particular we have shown that it is difficult to reconcile the CSM with 2 without invoking some fine-tuning mechanism which suppresses the effects of the higher order operators to the $\rho$ parameter ($\hat{T}$) or to the $Zb\bar{b}$ vertex.

On the contrary we have shown that the QLTM model, both with $M_H = 0$ and $M_H \neq 0$, have no problem with EWPT. The bound 2 is stronger for low values of the cut-off: for $\Lambda R \gtrsim 10$ the bound 3 dominates. However the regions corresponding to the QLTM are always allowed.

We can then conclude that the idea of supersymmetry breaking by boundary conditions along an extra dimension successfully addresses the Little Hierarchy problem. It allows to build very predictive models with a naturally light Higgs boson, insensitive to the UV. The
phenomenology of such models is very rich and testable at the Tevatron and the LHC. We have shown that at least one of the possible implementations of this idea is perfectly compatible with all the possible constraints coming from EWPT.

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A Supersymmetric gauge covariant derivatives

The supersymmetric gauge covariant derivatives in the chiral representation are defined as

\[ \nabla_\alpha = e^{-gV}D_\alpha e^{gV} \quad \bar{\nabla}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \quad \nabla_{\alpha\dot{\alpha}} = -i\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\} \]  \tag{22}

where \( D_\alpha, D_{\dot{\alpha}} \) are ordinary SUSY covariant chiral derivatives. Under a gauge transformation they transform as

\[ \nabla_A \rightarrow e^A \nabla_A e^{-A} \]  \tag{23}

where \( A = \alpha, \dot{\alpha}, \mu \). In this basis, chiral and antichiral superfields transform as

\[ \Phi \rightarrow e^A \Phi \quad \Phi e^{gV} \rightarrow \bar{\Phi} e^{-gV} e^{-A} \]  \tag{24}

under a gauge transformation.

References

[1] The LEP Electroweak Working Group, \url{http://lepewwg.web.cern.ch/LEPEWWG/}.
[2] R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D63, 105007 (2001), \url{hep-ph/0011311}.
[3] R. Barbieri, G. Marandella and M. Papucci, Phys. Rev. D66, 095003 (2002), \url{hep-ph/0205280}.
[4] R. Barbieri et al., Nucl. Phys. B663, 141 (2003), \url{hep-ph/0208153}.
[5] R. Barbieri, G. Marandella and M. Papucci, Nucl. Phys. B668, 273 (2003), \url{hep-ph/0305044}.
[6] J. Scherk and J. H. Schwarz, Phys. Lett. B82, 60 (1979).
[7] J. Scherk and J. H. Schwarz, Nucl. Phys. B153, 61 (1979).
[8] D. M. Ghilencea, S. Groot Nibbelink and H. P. Nilles, Nucl. Phys. B619, 385 (2001), \texttt{hep-th/0108184}.

[9] R. Barbieri, L. J. Hall and Y. Nomura, \texttt{hep-ph/0110102}.

[10] A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984).

[11] Z. Chacko, M. A. Luty and E. Ponton, JHEP 07, 036 (2000), \texttt{hep-ph/9909248}.

[12] R. Barbieri, G. Cacciapaglia and A. Romito, Nucl. Phys. B627, 95 (2002), \texttt{hep-ph/0107148}.

[13] G. Cacciapaglia, M. Cirelli and G. Cristadoro, Nucl. Phys. B634, 230 (2002), \texttt{hep-ph/0111288}.

[14] G. Cacciapaglia, M. Cirelli and G. Cristadoro, Phys. Lett. B531, 105 (2002), \texttt{hep-ph/0111287}.

[15] R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, \texttt{hep-ph/0405040}.

[16] B. Grinstein and M. B. Wise, Phys. Lett. B265, 326 (1991).

[17] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, Front. Phys. 58, 1 (1983), \texttt{hep-th/0108200}.

[18] A. Delgado, A. Pomarol and M. Quiros, JHEP 01, 030 (2000), \texttt{hep-ph/9911252}.