Neutron Matter from Low-Momentum Interactions

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We present a perturbative calculation of the neutron matter equation of state based on low-momentum two- and three-nucleon interactions. Our results are compared to the model-independent virial equation of state and to variational calculations, and we provide theoretical error estimates by varying the cutoff used to regulate nuclear interactions. In addition, we study the dependence of the BCS \(^1S_0\) superfluid pairing gap on nuclear interactions and on the cutoff. The resulting gaps are well constrained by the nucleon-nucleon scattering phase shifts, and the cutoff dependence is very weak for sharp or sufficiently narrow smooth regulators with cutoffs \(\Lambda > 1.6 \text{ fm}^{-1}\).

§1. Introduction

The determination of a reliable equation of state of nucleonic matter plays a central role for the physics of neutron stars\(^1\) and core-collapse supernovae.\(^2, 3\) Furthermore the superfluidity and superconductivity of neutrons and protons is an important phenomenon in nuclear many-body systems,\(^4, 5\) in particular for the cooling of neutron stars.\(^6\) In this contribution, we present calculations of the neutron matter equation of state at finite temperature and of the \(^1S_0\) superfluid gap in the BCS approximation based on low-momentum interactions.

Renormalization group methods coupled with effective field theory (EFT) offer the possibility for a systematic approach to the equation of state. By evolving nuclear forces to low-momentum interactions \(V_{\text{low}} k\)\(^7, 8\) with cutoffs around \(2 \text{ fm}^{-1}\), the model-dependent short-range repulsion is integrated out and the resulting low-momentum interactions are well constrained by the nucleon-nucleon (NN) scattering data. Furthermore, the corresponding leading-order three-nucleon (3N) interactions (based on chiral EFT) become perturbative in light nuclei for \(\Lambda \lesssim 2 \text{ fm}^{-1}\).\(^10\)

With increasing density, Pauli blocking eliminates the shallow two-nucleon bound and nearly-bound states, and the contribution of the particle-particle channel to bulk properties becomes perturbative in nuclear matter.\(^8\) The Hartree-Fock approximation is then a good starting point for many-body calculations with low-momentum NN and 3N interactions, and perturbation theory (in the sense of a loop expansion) around the Hartree-Fock energy converges at moderate densities. This can be understood quantitatively based on the behavior of the Weinberg eigenvalues as a function of the cutoff and density.\(^8, 9\)

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Some uncertainty remained concerning a possible dependence of the $^1S_0$ pairing gap on the input NN interaction in low-density neutron matter ($k_F < 1.6\text{ fm}^{-1}$). We address this point and explore the dependence of $^1S_0$ superfluidity on nuclear interactions at the BCS level in detail. We find that the BCS gap is well constrained by the NN phase shifts. Therefore, any uncertainties are due to polarization (induced interaction), dispersion and three-nucleon interaction effects.

§2. Equation of State of Neutron Matter

Using the Kohn-Luttinger-Ward theorem\textsuperscript{(11),12)}, the perturbative expansion of the free energy (at finite temperature) can be formulated as a loop expansion around the Hartree-Fock (HF) energy. In this work, we include the first-order NN and 3N contributions, as well as normal and anomalous second-order NN diagrams. Other thermodynamic quantities are computed using standard thermodynamic relations.

![Fig. 1. Energy per particle $E/N$ as a function of the density $\rho$ at first order (left panel) and including second-order NN contributions (right panel).\textsuperscript{13}]

The resulting energy per particle $E/N$ as a function of the density $\rho$ is shown in Fig. 1 for a cutoff $\Lambda = 2.1\text{ fm}^{-1}$ and temperatures $T = 3$, 6 and $10\text{ MeV}.\textsuperscript{13}$ The results presented in the left panel are the first-order NN and 3N contributions, and those in the right panel includes all second-order diagrams with NN interactions. For $T = 6\text{ MeV}$, we also give a band spanned between $\Lambda = 1.9\text{ fm}^{-1}$ (lower line) and $\Lambda = 2.5\text{ fm}^{-1}$ (upper line). The inclusion of second-order contributions significantly reduces the cutoff dependence of the results. The model-independent virial equation of state\textsuperscript{14)} and the variational calculations of Friedman and Pandharipande (FP)\textsuperscript{15)} are displayed for comparison.

The inclusion of second-order correlations lowers the energy below the variational
results for densities $\rho \lesssim 0.05 \text{fm}^{-3}$, and we observe a good agreement for $E/N$ with the $T = 10 \text{MeV}$ virial result when the second-order contributions are included. In the virial equation of state these contributions are included via the second-order virial coefficient, while in the variational calculation the state dependence of such correlations is only partly accounted for.\textsuperscript{16} Furthermore, the generic enhancement of the effective mass at the Fermi surface leads to an enhancement of the entropy at low temperatures above the variational and HF results.\textsuperscript{13,16,17}

§3. BCS gap in the $^1S_0$ channel

We solve the BCS gap equation in the $^1S_0$ channel

$$\Delta(k) = -\frac{1}{\pi} \int dp \, p^2 \frac{V_{\text{low}}(k, p) \Delta(p)}{\sqrt{\xi^2(p) + \Delta^2(p)}}, \quad (3.1)$$

with the (free-space) low-momentum NN interaction $V_{\text{low}}(k, k')$. Here $\xi(p) \equiv \varepsilon(p) - \mu$, $\varepsilon(p) = p^2/2$ and $\mu = k_F^2/2$ ($c = h = m = 1$).

We find that the neutron-neutron BCS gap is practically independent of the NN interaction.\textsuperscript{18} Consequently, $^1S_0$ superfluidity is strongly constrained by the NN scattering phase shifts. The maximal gap at the BCS level is $\Delta \approx 2.9 - 3.0 \text{MeV}$ for $k_F \approx 0.8 - 0.9 \text{fm}^{-1}$. For the neutron-proton $^1S_0$ case, we find somewhat larger gaps, reflecting the charge dependence of realistic nuclear interactions.\textsuperscript{18}

In Fig. 2 we show the dependence of the neutron-neutron $^1S_0$ superfluid pairing gap on the cutoff starting from the N$^3$LO chiral potential of Ref.\textsuperscript{19} for three representative densities.\textsuperscript{18} We employed different smooth exponential regulators $f(k) = \exp[-(k^2/\Lambda^2)^n]$, as well as a sharp cutoff. As long as the cutoff is large com-
pared to the dominant momentum components of the bound state ($\Lambda > 1.2k_F$), the gap depends very weakly on the cutoff. This shows that the $^1S_0$ superfluid pairing gap probes low-momentum physics. Below this scale, which depends on the density and the smoothness of the regulator, the gap decreases, since the relevant momentum components of the Cooper pair are then partly integrated out.

§4. Conclusions

In summary, we have studied the equation of state at finite temperature including many-body contributions in a systematic approach. We have found good agreement with the virial equation of state in the low-density–high-temperature regime. Analyzing the cutoff dependence of our results provides lower bounds for the theoretical uncertainties. The possibility of estimating theoretical errors plays an important role for reliable extrapolations to the extreme conditions reached in astrophysics.

In addition, we have shown that the $^1S_0$ superfluid pairing gap in the BCS approximation is practically independent of the choice of NN interaction, and therefore well constrained by the NN scattering data. This includes a very weak cutoff dependence with low-momentum interactions $V_{\text{low } k}$ for sharp or sufficiently narrow smooth regulators with $\Lambda > 1.6 \text{fm}^{-1}$. At lower densities, it is possible to lower the cutoff further to $\Lambda > 1.2k_F$. Furthermore, the pairing gap clearly reflects the charge dependence of nuclear interactions. The weak cutoff dependence indicates that, in the $^1S_0$ channel, the contribution of 3N interactions is small at the BCS level.

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