Geometric nonlinear dynamic model of rubber bearing based on the stress-strain analysis of micro-unit

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Geometric nonlinear dynamic model of rubber bearing based on the stress-strain analysis of micro-unit

Z D Lin and H Shi
City Institute, Dalian University of Technology, Dalian, 116600, China
zhidanlin@sina.com

Abstract. In this paper, the partial differential equation of geometric nonlinear dynamic response of laminated rubber bearing is established. Based on the assumption of homogenous column and the nonlinear first-order shear deformation beam theory, the governing equations and boundary conditions of nonlinear dynamic response of laminated rubber bearings are deduced by the stress-strain analysis of micro-unit. The governing equation is a third-coupled geometric nonlinear partial differential equation.

1. Introduction
Rubber bearings are made of rubber and steel layer by layer composited as a column, its detailed analysis is more difficult, so it need to be appropriate simplification based on the characteristics of actual bearing formation. In addition, from the overall perspective considering the laminated rubber bearing, it is a heterogeneous short column structure with good stability, high compressive capacity and shear deformation capacity. In order to establish the dynamic model of a laminated rubber bearing, it is assumed that the bearing is approximately equivalent to a homogeneous cylinder. Assuming that its cross section remains flat but not necessarily perpendicular to the central axis after the deformation, and the effect of rotational inertia is taken into account. The above assumptions was first carried out by Gent[1] according to Haringx's theory of buckling rubber[2]. The detailed derivation process has been described in the document[3-4].

The research[5-6] are basically on the characteristic analyses of the rubber bearings based on static at home and abroad. The rubber bearings show extremely strong nonlinear characteristic and large deformation in earthquake excitation, especially under the great earthquake. Therefore, learning from the beam theory of nonlinear large deformation in solid mechanics theory, it can help to improve the understanding of the mechanical properties of rubber bearings. In order to explore its dynamic characteristics and random response, to reveal its nonlinear phenomena and explain its mechanism, the most crucial issue is to establish the correct and reasonable mathematical model of laminated rubber bearings under dynamic system.

The laminated rubber bearing is taken as the research object under certain basic assumptions, Based on the previous researches, the rubber bearing mathematical model of nonlinear dynamic analysis is established based on the nonlinear first-order shear deformation beam theory, by the stress-strain analysis of micro-unit.
2. Geometric nonlinear dynamic model of rubber bearing

2.1. Nonlinear geometric equation
The cross-section of column is assumed that it only rotates of the neutral axis in the deformation. After the deformation, the axis of the column is still in the \((x, y)\) plane. Considering the effect of moment of inertia and shear deformation about the homogeneous column, the deformation of the column micro-section is shown in figure 2, where \(P\) is the constant axial force, \(M\) is the section bending moment, \(Q\) is the shear force and \(\varphi\) is the bending moment caused by the cross-section corner, \(\gamma\) is the shearing angle caused by shear, \(\partial x/\partial y\) is the actual rotation of the column axis caused by the shear force and bending moment.

\[
\begin{align*}
\frac{\partial v}{\partial y} + \frac{\partial M}{\partial y} dy + Q + \frac{\partial Q}{\partial y} dy = 0 \\
M + \frac{\partial M}{\partial y} dy = 0 \\
Q + \frac{\partial Q}{\partial y} dy = 0
\end{align*}
\]

a) Rubber bearing schematic diagram  
b) Deformation of rubber bearing

**Figure 1.** Rubber bearing

Displacement field:
The center line of the homogeneous column is assumed by the \(y\) axis and the axis of symmetry of the cross section is the \(x\) axis, according to the first-order shear deformation beam theory, there is a displacement field:

\[
\begin{align*}
\frac{\partial v}{\partial y} + \frac{\partial M}{\partial y} dy + Q + \frac{\partial Q}{\partial y} dy = 0 \\
M + \frac{\partial M}{\partial y} dy = 0 \\
Q + \frac{\partial Q}{\partial y} dy = 0
\end{align*}
\]
\[ U_y = u(y,t) - x\varphi(y,t) \\
U_z = v(y,t) \]  

(1)

Where \( u \) and \( v \) are the axial and lateral displacements on the center line of the column, respectively, and \( \varphi \) denotes the normalized corner of the cross-section.

Based on the displacement field, the nonlinear geometric equation can be obtained by the finite deformation:

\[
\begin{align*}
\varepsilon_x &= \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 - \frac{x}{2} \frac{\partial \varphi}{\partial y} \\
\gamma_{xy} &= \frac{\partial v}{\partial y} - \varphi
\end{align*}
\]

(2)

2.2. Equilibrium equation

By the balance of micro-unit, it is not difficult to get three equilibrium equations:

(1) \( \sum F_y = 0 \), projecting all the forces onto the \( y \)-axis, neglecting higher order minima:

\[
\frac{\partial N}{\partial y} + p = \rho A u \ddot{u}
\]

(3)

(2) \( \sum F_x = 0 \), projecting all the forces onto the \( x \)-axis, neglecting higher order minima:

\[
\frac{\partial Q}{\partial y} + \frac{\partial}{\partial y} \left( P \frac{\partial v}{\partial y} \right) + q = \rho A v \ddot{v}
\]

(4)

(3) \( \sum M_z = 0 \), taking the moment of the cross section center by all the forces in the \( x \)-\( y \) plane and neglecting higher order minima:

\[
Q + P \frac{\partial v}{\partial y} - \frac{\partial M}{\partial y} = \rho I, \ddot{\phi}
\]

(5)

where "\( \dddot{\cdot} \)" represents the second derivative of the time variable,

\( q_y = p - \rho A u \ddot{u}, \quad q_x = q - \rho A v \ddot{v}, \quad q_\varphi = \rho I, \dddot{\phi}, \)

\( p, \quad q \) is vertical and horizontal force respectively.

2.3. Physical equation

From the theory of the beam, for the homogeneous column, assuming a linear and isotropic elastic material, its constitutive relation:

\[
\sigma_y = E \varepsilon_y, \quad \tau_{xy} = G \gamma_{xy}, \quad G = \frac{E_r}{2(1 + \mu)}
\]

(6)

Where \( E_r \) is the modified bending elastic modulus of the laminated rubber bearing, \( G_r \) is shear modulus, \( \mu \) is Poisson's ratio.

From the integral available axial force \( N(y) \), bending moment \( M(y) \), respectively:

\[
N = \int_A \sigma dA = \int_A E_r \left( \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 - \frac{x}{2} \frac{\partial \varphi}{\partial y} \right) dA = E_r A \left[ \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 \right]
\]

\[
M = \int_A \sigma y dA = \int_A E_r \left( \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 - \frac{x}{2} \frac{\partial \varphi}{\partial y} \right) y dA = -E_r I, \frac{\partial \varphi}{\partial y}
\]

(7)
The shear component caused by axial force is supposing proportional to the actual rotation angle of the beam axis\[^{[7-8]}\], that is:

\[ Q = \kappa G A \left( \frac{\partial v}{\partial y} - \varphi \right) - P \frac{\partial v}{\partial y} \]  

(8)

In the formula, \( p \) and \( q \) are the forces along the coordinate axis respectively. When the magnitude of \( N \frac{\partial v}{\partial y} \) is relatively small, and the change of \( N \) is relatively slow, or the rotation angle is relatively small, the formula can be written as

\[
\begin{align*}
\frac{\partial N}{\partial y} + p &= \rho A \ddot{u} \\
\frac{\partial Q}{\partial y} + P \frac{\partial^3 v}{\partial y^3} + q &= \rho A \ddot{v} \\
Q + P \frac{\partial v}{\partial y} \frac{\partial M}{\partial y} &= \rho I \ddot{\varphi}
\end{align*}
\]  

(9)

Substituting equations (6) ~ (8) into equation (9):

\[
\begin{align*}
\rho A \ddot{u} - E A \frac{\partial^2 u}{\partial y^2} - E A \frac{\partial^2 v}{\partial y^2} - p &= 0 \\
\rho A \frac{\partial^2 v}{\partial t^2} + P \frac{\partial^2 v}{\partial y^3} - E A \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} - E A \frac{\partial^2 u}{\partial y^2} - \frac{3}{2} E A \left( \frac{\partial v}{\partial y} \right)^2 \frac{\partial^2 v}{\partial y^3} &= 0
\end{align*}
\]  

(10)

\[ -\kappa G A \left( \frac{\partial^2 v}{\partial y^2} - \frac{\partial \varphi}{\partial y} \right) - q = 0 \]

\[ \rho I \frac{\partial^2 \varphi}{\partial t^2} - E I \frac{\partial^2 \varphi}{\partial y^2} - \kappa G A \left( \frac{\partial v}{\partial y} - \varphi \right) = 0 \]  

(12)

2.4. Boundary conditions and initial conditions

Boundary conditions:

\[
\begin{align*}
u &= 0, v = 0, \varphi = 0, \frac{\partial v}{\partial y} = 0 \\
E A \left[ \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 \right] &= P \\
\frac{1}{2} E A \left[ 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} + \left( \frac{\partial v}{\partial y} \right)^3 \right] + \kappa G A \left( \frac{\partial v}{\partial y} - \varphi \right) - P \frac{\partial v}{\partial y} &= Q \\
E I \frac{\partial \varphi}{\partial y} &= M_i \\
\varphi &= 0, \frac{\partial v}{\partial y} = 0
\end{align*}
\]  

(13)

Initial conditions:

The homogeneous column at \( t < 0 \) is in the natural state, when \( t \geq 0 \) is meeting the following initial conditions:
\[ u|_{t=0} = 0, \quad \dot{u}|_{t=0} = 0; \quad v|_{t=0} = 0, \quad \dot{v}|_{t=0} = 0; \quad \varphi|_{t=0} = 0, \quad \dot{\varphi}|_{t=0} = 0; \]  

(14)

3. Conclusion

Based on the first-order shear deformation beam theory, the nonlinear governing equations of homogeneous column are presented under the assumption of elastic homogeneous column, which include three generalized displacements and two displacement, considering the geometrical nonlinearity of the series isolation system, using the stress-strain analysis of micro-unit. The geometrical nonlinear dynamic response equations and boundary conditions of the rubber bearing is established. In the process of model derivation, the material is linear and uniform, and its deformation is geometric nonlinear.

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