Summary: In this paper, we address the minimum-cost node-capacitated multiflow problem in undirected networks. For this problem, Babenko and Karzanov (JCO 24: 202–228, 2012) showed strong polynomial-time solvability via the ellipsoid method. Our result is the first combinatorial polynomial-time algorithm for this problem. Our algorithm finds a half-integral minimum-cost maximum multiflow in $O(m \log(nCD)SF(n', m', \eta))$ time, where $n$ is the number of nodes, $m$ is the number of edges, $k$ is the number of terminals, $C$ is the maximum node capacity, $D$ is the maximum edge cost, and $SF(n', m', \eta)$ is the time complexity of solving the submodular flow problem in a network of $n'$ nodes, $m'$ edges, and a submodular function with $\eta$-time-computable exchange capacity. Our algorithm is built on discrete convex analysis on graph structures and the concept of reducible bisubmodular flows.

MSC:
90C27 Combinatorial optimization
05C21 Flows in graphs

Keywords:
minimum-cost node-capacitated multiflow; discrete convex analysis; cost-scaling method; submodular flow; reducible bisubmodular flow

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