A review of modern numerical and analytical models of heat transfer in a dielectric layer during melting due to microwave radiation

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Abstract. In this work, numerical and analytical solutions of heat transfer in a dielectric layer during melting in the microwave field were considered. We considered solutions, where the source term was obtained based on the solution of Maxwell equation, as well as using the Lambert law. The conditions applicable for analytical solutions, allowing the parametric analysis, are determined. The areas of application of the technology of microwave melting of dielectrics, in particular with melting ice on water, defrosting products, etc., were also considered.

1. Introduction
The effect of microwaves on the melting process of dielectrics has been studied by plenty researchers. An example of the need to use microwaves for melting is ice-covered waters in cold regions, which is the main driver of cost growth in the maritime transport industry [1]. Icebreaking ships are used to cross the ice-covered water. Due to the strong resistance of ice, a large amount of fuel is required, and the launch of an icebreaker in a small lake is uneconomical. There are a number of projects exploring the use of microwaves to melt ice around stations, ships, etc.

Microwaves (with wavelengths in the range of 1 mm - 1 m) have the ability to heat solid, liquid and gaseous bodies [2,3]. The effect of microwaves on different molecular structures is not the same due to different absorption coefficients. Analysis using numerical solution of the effect of microwaves on water was carried out at various pressures and temperatures [4-5]. According to the research results, the most productive and efficient microwave frequency is 2.45 GHz. This work presents an overview of modern mathematical models of varying degrees of complexity for describing the main processes in the melting of a dielectric due to microwave radiation.

2. Numerical simulation
One of such works is [6]. This article proposes a new method of using microwaves to melt ice around floating equipment in freezing conditions in cold regions. The scheme of the problem is shown in Figure 1.
Figure 1. Scheme of the effect of microwaves on the layer of ice.

At the same time, in this work, a number of simplifications were used, associated primarily with the complexity of the process of interaction of microwave radiation with ice during the phase transition. These simplifications include: microwaves propagate only across the surface of the ice layer; the Boussinesq approximation is adopted and the microwave power does not change.

The basic equations of the phenomenon of ice melting with the use of microwaves, including equations for electromagnetic field, mass and heat transfer, are given. The energy balance for thermal energy stored is based on heat flux by conduction and absorbed microwave energy. The volumetric power generated inside the dielectric can be found as follows:

\[ P = 2\pi f \varepsilon_0 \varepsilon_r \tan \delta E^2 \]

The moving boundary condition between the lowest layer of ice and the water is (Figure 1):

\[ \frac{\partial R}{\partial t} = \frac{1}{\rho_1 L} \left( \lambda_1 \frac{\partial T_1}{\partial x} \bigg|_{x=R} - \lambda_1 \frac{\partial T_1}{\partial x} \bigg|_{x=R} \right) \]

Other boundary conditions are as follows:

\[ t \geq 0, \ x = 0 : \frac{\partial T}{\partial x} = 0, P = P_{in} \]

\[ t \geq 0, \ x \to \infty : \frac{\partial T}{\partial x} = 0 \]

\[ t \leq 0, \ T = T_0 \]

The dependences of the main parameters for water and ice used in the calculations were also presented (absorption coefficient of microwave, heat capacity, thermal conductivity, density, etc.). The process of melting by microwave radiation of ice is associated with a large number of difficulties; therefore, when choosing a scheme for the numerical solution, the choice was made in favor of an
implicit scheme. This solution implies taking into account the energy balance equation and heat transfer equations.

As compared to the calculations of other researchers, the computational errors in various numerical tests are less than 3%, which is a very acceptable result.

At the same time, a number of the following points were noted:
The absorption of microwave energy in the lower layer of liquid has an inverse ratio with an increase in the thickness of the water phase in the upper layer;
The minimum temperature is reached in the solid phase (ice), while the maximum temperature is observed in water. This is primarily due to the difference in the electrophysical parameters of phases;
The use of microwaves to melt ice around floating equipment is being introduced as a potential anti-icing method for the following reasons: no mechanical wear, low energy dissipation, no mechanical stress due to the interaction of a floating body with ice, and, finally, acceptable speed of the process of defrosting small lakes.

There are many works related to microwave defrosting of products. The work [7] is of particular interest, since it covers the comparison of two approaches to modeling microwave exposure:
The first one consists in solving the Maxwell equations and the source term is deduced from the electric field. The second method is known as the Lambert law. This method consists in calculating a volumetric heat source based on the depth of penetration of microwaves into the dielectric. This method does not require the electric field computation inside the heated materials, which gives a certain simplicity in calculations.

This work is devoted to the study of microwave melting of a piece of tylose. Simulation was carried out in the software package COMSOL Multiphysics, where the approaches based on electromagnetic field calculation and Lambert law were compared. The sample itself (86mm×43mm×50mm) was placed in a rectangular waveguide (Figure 2).

Microwave radiation is presented as monochromatic wave (TE\textsubscript{10}), with a frequency of 2.45 GHz, a power of 1000 W. A number of assumptions were also made related to the homogeneity and isotropy of the sample, as well as the constancy of temperature over the layer and the absence of changes in the electromagnetic field along the Y axis.

Heat transfer equation:
\[ \rho C_{p_{\text{app}}} \frac{\partial T}{\partial t} = \text{div}(k \nabla T) + Q \]

Where Q goes for the internal heat generation and is calculated based on approach. Thermophysical properties and their temperature dependences were also taken into account. Experiments were also carried out to validate the obtained calculations.

**Figure 3.** Comparison of experimental results and temperature calculations based on Lambert law
- experimental measurements – numerical simulations. T1 - temperature near the surface (maximum temperature), T2 - temperature in the center of the layer, T3 - lowest temperature in the layer.

**Figure 4.** Comparison of experimental results and temperature calculations based on solving Maxwell equations
- experimental measurements – numerical simulations. T1 - temperature near the surface (maximum temperature), T2 - temperature in the center of the layer, T3 - lowest temperature in the layer.

The figures (Figure 3, Figure 4) above demonstrate comparisons of temperatures at three different points for two approaches with experiments. Moreover, as it can be seen for the Lambert approach, the coincidence with experiments is very accurate. Small differences may be related to inaccuracy of
dielectric properties taken from the literature. For the approach based on Maxwell equations, the calculations are in very good agreement with the experiments in the frozen zone, however, in the unfrozen zone, the differences are very large. This is because the numerical model based on Maxwell equations is very sensitive to dielectric properties, which can vary greatly during melting. This leads to significant changes in the electric field strength. In this model, dielectric properties of tylose are considered to be constant in the defrosted zone. Regarding differences between model and experimental data, this assumption seems insufficient for this model.

There are also a number of other works that differ in their formulation, but at the same time retain the general approaches to modeling the phase transition under microwave action.

3. **Analytical simulation**

One of the very first works on this topic can be considered the work of Lame-Clapeyron for Stefan problems. As it is stated earlier, in the literature there are extensive numerical and experimental studies on microwave melting and heating, but just a few of studies use analytic solutions and techniques to analyze the fundamentals of microwave processing. In [8], the authors present a closed form analytic solution for temperature distribution in a three-dimensional rectangular block under microwave processing. Knowing the distribution of the electric field from the Maxwell equation, a three-dimensional, non-stationary, inhomogeneous energy equation containing a microwave radiation source is solved by the method of integral transformation. At the same time, to obtain an analytical solution, the following assumptions were used: (a) uniform plane (TEM) microwave impingement; (b) electroneutrality of the system is taken for granted; (c) electrophysical and thermal properties are temperature independent; (d) magnetic permeability can be approximated from its value in free space. The results showed that with correctly selected parameters: frequency, sample thickness, processing time, etc., it is possible to achieve uniform temperature distribution and, therefore, uniform defrosting of the object.

In the case of uneven temperature distribution during the phase transition, it is possible to use approximate analytical solutions, as it was done in one of the authors' works [9]: at the stage of microwave melting, the problem can be solved approximately analytically. This process is considered as a nonlinear Stefan problem with a liquid-vapor phase transition. The quasi-stationary approximation allowed determination of the dynamics of the phase interface motion, which ultimately allows calculation of such parameters as the process time, energy consumption, etc. This approximation is based on the fact that the speed of the phase transition boundary is much less than the speed of heat pulse propagation. Therefore, in this approximation, the time-limiting temperature dependence can be used to determine the motion of the front of the phase transition boundary. As it was shown by the authors of this work, analytical solutions under certain conditions (the absence of a strong dependence of the electrophysical and thermophysical properties on temperature) can give very high accuracy in calculating the distribution of temperature and phase transition time.

4. **Conclusion**

In this work, we compared various models of melting of a dielectric object under the impact of microwave energy. Areas of application were shown for approaches based on the solution to the Maxwell equation, as well as using the Lambert law. It is shown that, despite the abundance of numerical solutions, analytical solutions are also present, allowing, under certain assumptions, the parametric analysis in order to find the optimal processing conditions for a dielectric sample. The wide field of application of the technology of microwave melting of dielectrics is also shown.

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