Research on the influence mechanism of friction factor of gear based on mathematical statistics principle

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Abstract. This paper first introduces the development status of gear friction factor research. Secondly, the advantage of the experimental method based on mathematical statistics is discussed. In addition, the mathematical statistics principle is used to design the orthogonal experiment to verify the existing formula of gear friction factor calculation. The correlation of each parameter was studied according to the experimental data. The deficiency of calculation formula is analysed and the prospect of future research is put forward.

1. Research status of gear friction factor

Gear mechanism is the most widely used transmission mechanism. Scientists have never stopped studying the gear mechanism. Because of its complicated mechanism, tooth surface friction is always one of the hot and difficult points in the study. The main effects of gear surface friction on gear transmission include: reduction of gear transmission efficiency, aggravation of gear tooth failure (tooth surface wear, tooth surface pitting, tooth surface cementation, etc.), system vibration and noise [1][2]. Therefore, the study of tooth surface friction has great economic benefits and is of great significance to improve the performance of gear transmission system.

In theory, involute gear transmission does not generate friction, because the two gears are pure rolling at the nodes, and the gears are rigid. However, in actual generation, the gear is not rigid, and there is inevitable elastic deformation. The elastic deformation makes the gear meshing no longer pure rolling in theory, and the actual machining accuracy and installation precision cannot achieve the theoretical accuracy, which also makes the actual meshing different from the theory, so there will be gear friction. Since Martin first proposed the use of Reynolds equation to analyse gear friction in 1916. At present, the formula of gear friction is based on elastohydrodynamic lubrication theory. For example, the formula for calculating the friction factor of the numerical solution of linear contact isothermal total membrane elastomeric presented by Dowson and Higginson is as follows [3]:

\[
fs = \frac{u_2 - u_1}{F_{n\min}} \int_{-\vartheta}^{+\vartheta} \eta dx
\]

Calculation formula of friction factor of gear for thermal elastohydrodynamic lubrication of non-newtonian fluid proposed by Xu [4]:

\[
\mu = e^f P_{h}^{b_2} \left| SR \right|^{b_3} V_{e}^{b_4} V_{0}^{b_5} R^{b_6}
\]

It should be pointed out that the current calculation formula is mainly established for steady state, and the parameters do not change with time, which is different from the actual motion process of gear.

2. Orthogonal experimental design based on mathematical statistics theory

Orthogonal experiment is one of the experimental design methods. In brief, the experimental design is to study how to arrange experiments scientifically and obtain more comprehensive information with less consumption. The experiment is well designed, with half the effort. On the contrary, it will not only
fail to achieve the expected effect, but also consume a lot of manpower, material resources and time. Therefore, how to carry out test design is an important part of successful test.

Orthogonal experimental design is a scientific and reasonable arrangement of experiments based on the principle of mathematical statistics. From the results, the optimal conditions of the process can be found quickly, and it has the advantages of judging which factors are the main factors and the mutual influence between the influencing factors. Orthogonal experimental design is a common technique for experimental optimization. The so-called experimental optimization refers to an optimization method for optimal design under the guidance of the optimization thought. For example, if there are 3 influencing factors in a test and each influencing factor has 3 levels, then there are $3^3 = 27$ permutations in the comprehensive test on 3 factors and 3 levels. Orthogonal experimental design is to select some representative test points from the comprehensive test points to carry out experiments. It can achieve the experimental effect and reduce the number of experiments. 3 factors 3 level test using orthogonal test design only requires 9 tests.

China began to study this subject in 1950s and gradually applied it to industrial and agricultural production. It is proved that orthogonal experimental design is an important method of experimental design technology and brings great economic benefits. Orthogonal experimental design has become a necessary technology for engineers and researchers.

3. The formula of gear friction factor is verified by orthogonal experiment

As mentioned above, XU proposed the calculation formula of friction factor of non-newtonian thermal elastohydrodynamic lubrication gear:

$$
\mu = e^f P_h^{b_2} |SR|^{b_3} V_e^{b_6} V_0 R^{b_8}
$$

$$
f = b_1 + b_4 |SR| P_h \log_{10}(v_0) + b_5 e^{-|SR|P_h \log_{10}(v_0)} + b_9 e^{x}
$$

Where $\mu$ is friction coefficient; SR is the sliding velocities($V_S$) and entraining velocity($V_e$) of tooth surface meshing point, called Slide-to-roll ratio, $SR = V_S/V_e$; R is radius of curvature; $P_h$ is max. Hertzian pressure; $V_0$ is the dynamic viscosity of the lubricant; $S$ is surface roughness; $b_i$ is a constant related to gear oil, using a common gear oil (75W90), $b_i$ ($i = 1, 2, ..., 9) = (-8.92, 1.03, 1.04, -0.35, 2.81, -0.10, 0.75, -0.39, 0.62)$.

According to the formula analysis, it is considered that SR, radius of curvature, max. Hertzian pressure, surface roughness and entraining velocity have great influence on the friction factor of gear. So take 5 levels for each of these five factors, listed in table 1.

| Table 1 | Factors and levels |
|---------|-------------------|
| Radius of curvature (R), m | Entraining velocity (Ve), m/s | Slide-to-roll ratio (SR) | Surface roughness (S), m | Max. Hertzian pressure (Ph), GPa |
| 1 | 0.01 | 1.00 | 0.01 | 0.05 | 0.50 |
| 2 | 0.02 | 5.00 | 0.05 | 0.10 | 1.00 |
| 3 | 0.03 | 10.00 | 0.10 | 0.20 | 1.50 |
| 4 | 0.04 | 15.00 | 0.15 | 0.30 | 2.00 |
| 5 | 0.08 | 20.00 | 0.20 | 0.40 | 2.50 |

This experiment is a 5 factor and 5 level test, each factor occupies one column, without considering the interaction, so there are a total of 5 columns. Therefore, orthogonal table $L_{25}(5^5)$ was selected. According to the orthogonal table, the parameters with multiple levels were filled in successively, and the formula was used for calculation. The experimental results and the analysis were included in table 2 together. Complete orthogonal table in appendix. Details of the test can be found in [5].
### Table 2. Test results and intuitive analysis

|     | R       | Ve      | SR     | S       | Ph       |
|-----|---------|---------|--------|---------|----------|
| **K1** | 0.054606 | 0.040223 | 0.004612 | 0.032484 | 0.016061 |
| **K2** | 0.039714 | 0.043447 | 0.021807 | 0.029857 | 0.024697 |
| **K3** | 0.032482 | 0.036307 | 0.047862 | 0.028842 | 0.037468 |
| **K4** | 0.030071 | 0.034220 | 0.054392 | 0.043491 | 0.052797 |
| **K5** | 0.027115 | 0.029792 | 0.055315 | 0.049313 | 0.052966 |
| **Range** | 0.027491 | 0.013655 | 0.050703 | 0.020471 | 0.036905 |

Principal→SubordinateSR>P_h>SR>V_e

The intuitive analysis of the test results, that is, range analysis, generally speaking, the range of each column is not equal. This indicates that the change of the level of each factor has different effects on the test results. The greater the difference, the greater the change of the value of this factor within the test range, which will lead to greater changes of the test index in the value. Therefore, the column with the maximum range value is the factor whose level has the greatest influence on the test results, which is the most important factor. For this experiment, it can be seen that the range is from large to small: SR>P_h>SR>V_e. In other words, SR is the biggest influence on the friction factor of gear, and V_e is the smallest.

### Table 3. Analysis of variance table

|     | sums of squared deviations(SSj) | degree of freedom (df) | SSj/df | F       | significance test |
|-----|---------------------------------|------------------------|--------|---------|-------------------|
| **R** | 9.67E-05                         | 4                      | 2.416E-05 | 4.319  | un-significance   |
| **Ve** | 2.24E-05                         | 4                      | 5.595E-06 |        |                   |
| **SR** | 4.07E-04                         | 4                      | 1.018E-04 | 18.190 | significance      |
| **S** | 6.63E-05                         | 4                      | 1.658E-05 | 2.962  | un-significance   |
| **Ph** | 2.19E-04                         | 4                      | 5.471E-05 | 9.778  | significance      |

Analysis of variance is a very practical and effective statistical test method, which can be used to test the significance of the influence of related factors on the test results. For example, a typical analysis of variance is how to find out whether different catalysts have significant effects on the rate of a product when reflecting time, temperature, pressure and other conditions.

As shown in table 3, F_{0.05}(4,4) = 6.39 was obtained from the F-distribution table. F_{SR}>F_{α}, F_{Ph}>F_{α}, so the slide-to-roll ratio SR and Max. Hertzian pressures Ph have significant influence on the friction factor of gear, and the slide-to-roll ratio SR has the biggest influence on the friction factor. Similarly, F_{R}<F_{α}, F_{Ve}<F_{α}, F_{S}<F_{α}, so the radius of curvature R, entraining velocity Ve and surface roughness S have no significant influence on the friction factor of gear, and entraining velocity Ve has the least influence.

### 4. Conclusions

Through the test and analysis of the formula, it can be seen that the slide-to-roll ratio SR and the Max. Hertzian pressures Ph have a great influence on the friction factor of gear, and the radius of curvature R, entraining velocity Ve and the surface roughness S have little influence. According to the practical experience, Max. Hertzian pressures Ph has a great influence on the friction factors. However, surface roughness S has little effect on the friction factor, but it does not accord with it. In addition, the entraining velocity Ve has the least influence on the friction factor, and the slide-to-roll ratio SR has
the greatest influence on the friction factor. But the slide-to-roll ratio $SR = \frac{V_s}{V_e}$ ratio shows that the sliding speed $V_s$ has a great influence on the friction factor. To sum up, this formula is basically in line with the law of gear friction factor, but there are some deficiencies, which need further correction and supplement. In this paper, the principle of mathematical statistics is used to design the orthogonal experiment, and the formula is verified. This process is a qualitative analysis of each parameter. It is helpful to understand the influencing factors and grasp the research direction. It provides new ideas for future experimental research.

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Appendix

Complete orthogonal table

| Text NO | R  | Ve  | SR  | s   | Ph  | μ   |
|---------|----|-----|-----|-----|-----|-----|
| 1       | 0.01 | 1.00 | 0.01 | 0.05 | 0.50 | 0.0006 |
| 2       | 0.01 | 5.00 | 0.05 | 0.10 | 1.00 | 0.0052 |
| 3       | 0.01 | 10.00 | 0.10 | 0.20 | 1.50 | 0.0121 |
| 4       | 0.01 | 15.00 | 0.15 | 0.30 | 2.00 | 0.0172 |
| 5       | 0.01 | 20.00 | 0.20 | 0.40 | 2.50 | 0.0195 |
| 6       | 0.02 | 1.00 | 0.05 | 0.20 | 2.00 | 0.0088 |
| 7       | 0.02 | 5.00 | 0.10 | 0.30 | 2.50 | 0.0137 |
| 8       | 0.02 | 10.00 | 0.15 | 0.40 | 0.50 | 0.0067 |
| 9       | 0.02 | 15.00 | 0.20 | 0.05 | 1.00 | 0.0094 |
| 10      | 0.02 | 20.00 | 0.01 | 0.10 | 1.50 | 0.0011 |
| 11      | 0.03 | 1.00 | 0.10 | 0.40 | 1.00 | 0.0089 |
| 12      | 0.03 | 5.00 | 0.15 | 0.05 | 1.50 | 0.0094 |
| 13      | 0.03 | 10.00 | 0.20 | 0.10 | 2.00 | 0.0106 |
| 14      | 0.03 | 15.00 | 0.01 | 0.20 | 2.50 | 0.0017 |
| 15      | 0.03 | 20.00 | 0.05 | 0.30 | 0.50 | 0.0018 |
| 16      | 0.04 | 1.00 | 0.15 | 0.10 | 2.50 | 0.0118 |
|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 17 | 0.04 | 5.00 | 0.20 | 0.20 | 0.50 | 0.0058 |
| 18 | 0.04 | 10.00 | 0.01 | 0.30 | 1.00 | 0.0007 |
| 19 | 0.04 | 15.00 | 0.05 | 0.40 | 1.50 | 0.0049 |
| 20 | 0.04 | 20.00 | 0.10 | 0.05 | 2.00 | 0.0069 |
| 21 | 0.08 | 1.00 | 0.20 | 0.30 | 1.50 | 0.0100 |
| 22 | 0.08 | 5.00 | 0.15 | 0.40 | 2.00 | 0.0093 |
| 23 | 0.08 | 10.00 | 0.10 | 0.05 | 2.50 | 0.0062 |
| 24 | 0.08 | 15.00 | 0.05 | 0.10 | 0.50 | 0.0011 |
| 25 | 0.08 | 20.00 | 0.01 | 0.20 | 1.00 | 0.0005 |
| K1 | 0.054606 | 0.040223 | 0.004612 | 0.032484 | 0.016061 |
| K2 | 0.039714 | 0.043447 | 0.021807 | 0.029857 | 0.024697 |
| K3 | 0.032482 | 0.036307 | 0.047862 | 0.028842 | 0.037468 |
| K4 | 0.030071 | 0.034220 | 0.054392 | 0.043491 | 0.052797 |
| K5 | 0.027115 | 0.029792 | 0.055315 | 0.049313 | 0.052966 |
| Range | 0.027491 | 0.013655 | 0.050703 | 0.020471 | 0.036905 |

Principal → Subordinate: SR > Ph > R > S > Ve