Probabilistic Risk Metrics for Navigating Occluded Intersections

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Abstract—Among traffic accidents in the USA, 23% of fatal and 32% of non-fatal incidents occurred at intersections. For driver assistance systems, intersection navigation remains a difficult problem that is critically important to increasing driver safety. In this letter, we examine how to navigate an unsignalized intersection safely under occlusions and faulty perception. We propose a real-time, probabilistic risk assessment for parallel autonomy control applications for occluded intersection scenarios. The algorithms are implemented on real hardware and are deployed in a variety of turning and merging topologies. We show phenomena that establish go/no-go decisions, augment acceleration through an intersection and encourage nudging behaviors toward intersections.

Index Terms—Intelligent Transportation Systems, Human Factors and Human-in-the-Loop, Autonomous Vehicle Navigation.

I. INTRODUCTION

Intersections present one of the most challenging driving scenarios because a vehicle must interact with others to navigate safely. In 2016, 23% of fatal and 32% of non-fatal traffic incidents in the U.S. occurred at intersections.\(^1\) Recent advances in robotic perception and control routines promise to enhance the safety of passengers on the road through fully autonomous cars or by augmenting the human with advanced driver assistance systems (ADAS). However, reasoning about intersections remains a major challenge.

Perception systems may fail to detect other vehicles as a consequence of occluded views. Causes of these occlusions may stem from cross-traffic behavior, buildings or road geometries that cause poor visibility \(^2\). As a consequence, tracking and detecting other road objects may fail in unpredictable ways. New models must account for the resulting uncertainty and risk while ensuring robustness to imperfect perception and remaining computationally efficient.

With respect to human drivers, collisions at intersections often occur due to inattention or misjudgment of the other cars’ dynamics \(^3\). This remains an open problem for autonomous vehicles, which can struggle to navigate intersections without incident \(^4\) or to interact naturally with cars driven by humans \(^5\). In this letter, we aim to address this challenge. Our goals include modeling the risk of collision at junctions and deploying both full and shared autonomous systems to enhance vehicle safety. Our driver-assistance framework embodies a parallel autonomy system, where the human maintains primary control of the vehicle, but the autonomous system can intervene for safety.

We propose a risk model that reasons about several crucial aspects of road interactions. It accounts for cross traffic, occlusions, sensor errors and driver attentiveness. This work focuses on intersections that do not contain a stop light to regulate the traffic flow, and where the ego-vehicle must yield to all cross traffic. Figure 1 provides an example of such an occluded, unsignalized intersection that presents difficulty for human-operated and autonomous systems alike. In this scenario, the ego-vehicle attempts a left turn maneuver into crossing traffic. Houses abutting the intersection obstruct the views of the ego-vehicle and crossing vehicles, creating uncertainty in the estimation of each others’ poses.

The proposed algorithm estimates the risk for the ego-vehicle and we demonstrate the use of this estimate in a parallel autonomy framework \(^6\) to increase driver safety. Generalizing beyond the left turn scenario, the approach we present extends to various junction topologies, including roundabouts and merge lanes. The main contributions of the paper are summarized as follows:

1) A model and an algorithm to estimate the risk at intersections in the presence of occlusions and uncertain sensor measurements;
2) Integration of our estimator into an online, shared control method for negotiating intersections; and
3) Demonstration on a physical hardware platform in both fully autonomous and parallel autonomy modes.

Related Work: Human drivers negotiate intersections through arrival times of oncoming traffic \(^7\). Outlined by \(^8\), usual intersection behavior consists of identifying gaps, assessing risk
Fig. 1. An unsignalized intersection with an occluded view, shown above, presents a risky scenario for performing unprotected turns across traffic. Below, vertical bars show the risk per discretized road segment, while horizontal bars show the occupancy estimation belief, described in Section II.

and mitigating threats. Gap estimation [9] is a crucial aspect in making go/no-go decisions at junctions, providing a common metric for making turns. In assessing risk, the critical times to cross an intersection also play a vital role in decision making [10]. This risk aware decision making process varies based on the attention that human drivers give to the situation [11] and these differences have a major influence on how active safety systems interact with human drivers [8], [12] to reduce threats. Notably, current gap estimation methods lack real time applicability [13].

Successful ADAS deployment requires accurate assessment of the intersection topology and turning prediction. Increased fidelity in lane-level maps can improve turning predictions [14], and studies on human drivers have identified typical distances for turning decisions [15]. Predicting when to warn the driver serves as a key feature of ADAS systems. In [16], the authors combine SVM-based intention prediction with an RRT-based threat assessment to evaluate potential risks and warn the driver. Augmented reality systems may prove useful in warning and assisting the driver [12], [17].

We use techniques from fully-autonomous control to improve our parallel autonomy system. Autonomous vehicle systems often utilize a model-based cost function in reasoning about lane changes and merges [18], [19]. Recent work shows promise in using deep reinforcement learning for intersection navigation [20]. In addition, if cars communicate via a vehicle-to-vehicle (V2V) network, higher-level intersection management techniques can mediate this traffic flow from reservations or auction systems [21]. Our presented work relies on local information and existing topology maps, instead of V2V communication.

Fig. 2. The ego-car traverses from its original lane (blue lines), over crossed lanes (green lines), to its new lane (dashed green and blue). Red circles mark lane origins, with thin red lines showing the ego-car’s path from the original lane to the new lane. Exemplary road distances, marked with $d_i$, and an occlusion, $\Omega_1$, are visualized. For each discretized road segment, shown in (b), we maintain the belief state $P(C_i)$ of other cars and the conditional risk $P(E_i|C_i)$.

Similar to previous work [22], [23], we address the problem of occluded traffic in the environment. Our method accommodates sensor and motion uncertainty, as opposed to over-approximation of predicted occupancies and possible states in the set based verification method of [22]. Additionally, our model can estimate risk due to unlikely distracted or highly speeding traffic agents. Like [23], we provide a simple control mechanism for accelerating and decelerating in the presence of risk before and after an intersection, while approaches in similar occluded scenarios use only braking [24].

The remainder of the paper is organized as follows: in Section II, we describe our model of intersections and probabilistic approaches to traffic arrival, sensor observations and driver inattention. We quantify risk by using a probabilistic graphical model and considering the likelihoods at all potential cross traffic locations. Section III details our control strategy for leveraging the estimation of risk at an intersection. In Section IV, we describe our experimental design, including algorithm and discretization choices. We show risk estimation results for both simulated and real world scenarios. By testing on one-tenth model scale cars, we show the performance of the model for both fully autonomous and driver assistance (parallel autonomy) operating modes. Finally, we present our conclusions in Section V.

II. RISK ASSESSMENT MODEL

In this section, we describe our model for assessing risk at unsignalized intersections. We define unsignalized intersections as road junctions that do not have lights or signs mediating traffic. This includes an unprotected left turn across traffic, as well as merges and roundabouts. We represent the intersection as a junction node with lanes of traffic entering and exiting the node, and Figure 2 illustrates various topological representations. We compute our risk assessment by incorporating (i) traffic density, (ii) sensor noise and physical occlusions that hinder observations of other vehicles and (iii) attention limitations of other drivers. We use this risk assessment in determining a “go” or “no-go” decision at an intersection.
One common approach to intersection risk assessment is to use gap estimation, where the car uses a pre-selected critical gap size to determine its go and no-go decisions. This gap is defined by the spacing between two vehicles in the desired traffic lane, with the critical gap size being the minimum tolerated spacing for safe maneuvers [25]. When the car sees a critical gap opening, go and no-go decisions are equally likely [26]. Gap estimation algorithms perform well in intersections without occlusions and under the assumption of perfect observability. By incorporating occlusions and noisy observations, in Section IV we show our model outperforms a baseline gap estimation algorithm.

A. Modeling Risk

Consider an ego-car moving across an intersection. During its maneuver, it must move from an origin lane to a new lane, passing through a junction, while avoiding incidents with other vehicles. We define an incident to include collisions, near-miss braking incidents, traffic conflicts [27], small gap spacing [15], or the threat of accident from other vehicles [13]. The risk to the ego-car at time $t$, denoted $r_t$, is computed as the expected number of incidents that will occur if the ego-car enters the intersection at time $t$.

For an intersection with $n_l$ lanes, we discretize each lane $l \in \{1, \ldots, n_l\}$ into $n_s$ segments of length $\delta$ meters. For a segment $i$ in lane $l$, the distance from the segment to the intersection is denoted $d_i$. We assume that traffic in each lane, $l$, has some average velocity $\nu_l$, such that for sampling intervals $\Delta t$, $\delta = \nu_l \Delta t$. For the ego-car, we denote its velocity as $\nu_e$ and its distance to the intersection as $d_e$.

By construction, we compute the overall risk by summing over all segments in all lanes. Thus, we formulate the properties of incident probability, occupancy and observation noise per road segment. This formulation is Eulerian, computing risk properties for traversible space, unlike Lagrangian techniques [28], that reasons about vehicle tracks [29]. As the identity of the individual objects is unimportant, this Eulerian model provides computational efficiency [30].

We denote the event of an incident within segment $i$ of lane $l$ at time $t$ as $E_{i,l}^t \in \{0, 1\}$. We condition $E_{i,l}^t$ on the occupancy of a segment, with the occupancy denoted $C_{i,l}^t \in \{0, 1\}$. We denote $O_{i,l}^t \in \{0, 1\}$ as the observation of another vehicle in segment $i$ of lane $l$ at time $t$. For clarity, when the target lane and time are fixed, we use the shorthand notation $E_i$ and $C_i$. Figure 2 illustrates the lane topologies and maintained probabilities within an intersection.

The overall risk $r_t$ is the sum over all segments and lanes,

$$r_t = \sum_{i=1}^{n_l} \sum_{l=1}^{n_s} E_i E_{i,l}^t (O^{t-} - 1),$$

where $O^{-t}$ denotes the observations seen until time $t$ over all lanes in $n_l$. Note that in this model, we consider the expected number of incidents, $E_i E_{i,l}^t (O^{t-} - 1)$, in separate lanes and segments. In Section III, we use this risk to determine when it is safe for the vehicle to move through the intersection.

Fig. 3. A graphical model describing the belief about cars per road segment over time. Shaded nodes represent observations of vehicles, and a node represents the event of a vehicle occupying road segment $i$ at time $t$.

Our formulation of approximate risk serves as a union bound on risky incidents, since the event of at least one risky incident is no more likely than the sum of all risky events. Thus, $r_t$ is an upper bound on expected incidents. It becomes accurate in the case of low risk levels, where correct estimation of the risk is important. For high-risk situations, the ego-car should avoid the intersection, determined as risk $r_t$ greater than a safety threshold, $r_{go}$.

B. Occupancy Estimation

Here, we discuss how the probability of occupancy depends on the velocity and traffic. We use a dynamic Bayesian network [31], which allows us to reason about probability and model the likelihood of occupied segments. This network is depicted in Figure 3. We initialize the probability of occupancy at the lane origin, $P(C_i^t | C_{i-1}^t)$, with emission rate $\lambda_i$ since theoretical and real world studies show that the Poisson model captures vehicle arrivals at uncontrolled intersections [32], [33]. With $\lambda_i = 1$, road segments are marked occupied until a free space observation is made.

We define occupancy belief update as $P(C_i^{t+1} | C_i^t)$, based on $P(C_i^{t+1} | C_i^t) P(C_i^t)$, where $C_i^t$ denotes the occupancy over all surrounding segments, in the following manner. Lane velocities at each road segment are normally distributed, such that $\nu_i \sim \mathcal{N}(\nu, \sigma)$. With $K_m$ as a discrete Gaussian convolution kernel with variance $\sigma_l$ and window size, $m$, we compute $P(C_i^{t+1} | C_i^t) = \sum_{m} P(C_i^{t+1} \cdot K_m \cdot C_{i-m}^t)$. Thus, the estimate of occupancy, $P(C_i^{t+1})$, is conditioned on the occupancy belief of surrounding segments at the previous timestep.

C. Driver Attention

When the ego-car enters the intersection, other vehicles may not notice due to limited attention. Failure to react to the ego-car may result in an incident. Here, we present how our model of the conditional risk of an incident, $P(E_i | C_i)$, incorporates driver attention. Consider a vehicle at distance $d_s$ from the intersection. Let $t_{go}$ denote the time it takes the ego-car to clear the intersection, which is a function of the path length through the intersection and velocity $\nu_e$. 

\[ t_{go} = \frac{d_s}{\nu_e} \]
We define the conditional risk as
\[
P(E_i|C_i) = \begin{cases} 
0 & d_i > v_t \cdot t_c \\
1 & d_i < d_s \\
\exp(-\lambda_a (d_i - d_s)) & \text{otherwise}
\end{cases},
\]
where \(\lambda_a > 0\) is a parameter modeling the attention of the other vehicle and \(d_s\) is defined as the comfortable stopping distance. This exponential decay is motivated by [34]. If the other vehicle will take longer than \(t_c\) to arrive at the intersection, \(P(E_i|C_i) = 0\). Note that, as \(t_c\) increases, so does the number of road segments needed to model the lane. Conversely, if the other vehicle is within its comfortable stopping distance \(d_s\), then \(P(E_i|C_i) = 1\). This does not imply a collision, but possible cut-offs, tailgating and other unsafe situations included in our definition of incidents. The conditional risk for an ego vehicle crossing a lane of traffic is shown in Figure 4.

We also define \(d_k^s\) as the required forward distance between the ego-car and other vehicles in the target lane. For incidents with other vehicles in the new lane, the conditional risk is an indicator function, \(P(E_i|C_i) = 1(d_i < d_k^s)\). This implies the ego-car must also maintain a comfortable stopping distance behind the car it is following.

D. Occlusions and Noisy Observations

The occupancy probability depends on both noisy observations as well as line-of-sight occlusions that may prevent the ego vehicle from seeing the road segment. Road segments beyond the vehicle’s sensor range are considered occluded. We compute the likelihood of an occlusion conditioned on the presence of a vehicle at segment \(i\) with probability \(P(O_i|C_i)\) by incorporating both occlusions and a model of noisy observations. For an environment with \(n_o\) occlusions, let \(\Omega_k\) for \(k = \{1, \ldots, n_o\}\) represent an occlusion. We define the occlusion as blocking some segments \(a\) to \(b\) from the ego car’s view, \(\Omega_k = [d_k^a, d_k^b]\). We refer to the set of all occlusions as \(\Omega\), and use the notation \(i \in \Omega\) to refer to a segment \(i\) that is occluded from view. For occluded segments, \(P(O_i|C_i), i \in \Omega = 0\).

For unoccluded segments, we model standard perception and tracking pipelines with noisy observations. Observed cars are associated with a single segment in the lane, and a belief update is computed for unoccluded \(i \notin \Omega\) as
\[
P(C_i = 1|O_i, i \notin \Omega) = \frac{P(O_i|C_i = 1)P(C_i = 1)}{\sum_{j \in [0,1]} P(O_i|C_i = j)P(C_i = j)},
\]
where \(P(C_i = 1)\) is computed based on the distribution of \(v_t\), and the belief from the previous time step. In our experiments presented in Section IV, we set \(P(O_i = 1|C_i = 1) = 0.85\) and \(P(O_i = 1|C_i = 0) = 0.05\).

Using the probability of an event of an incident in each lane, we act based on the expected number of incidents. The expected risk from lane \(l\) is the inner product of the conditional risk and vehicle occupancy estimate,
\[
\mathbb{E}_t[E_i|O^{-t}] = \sum_i P(E_i|C_i)P(C_i|O^{-t}),
\]
where \(P(C_i|O^{-t})\) denotes the belief of a car in segment \(i\) given observations until time \(t\) at all segments observed by the ego-car.

E. Combined Risk Algorithm

From (4), we sum the expected number of incidents over the new and crossed lanes to find the overall risk, \(r_l\) in (1), \(r_l = \sum_{i=1}^{n_i} \mathbb{E}_t[E_i|O^{-t}]\). Algorithm 1 outlines our process for updating the conditional risk using driver attention, occupancy, occlusions and noisy observations. Observations \(O_i\) of a car in segment \(i\) are made for a discrete time \(t\). Our risk model is evaluated every \(\Delta t\) seconds, and in both our simulations and experiments, \(\Delta t = 0.1\). An example of our risk calculation in simulation is shown in Figure 6.

Key variables in our model include vehicle velocities and environment visibility. Here, we discuss how our model behaves as these variables are taken to their limits.

1) No Occlusions: With no occluded regions, \(\Omega = \emptyset\), the problem is akin to gap estimation. Here, our model also includes the uncertainty estimates from \(P(C_i|O_i), v_t ~ N(v_t^0, \sigma_t)\) and \(\lambda_a\).

2) Fully-Occluded View: For \(\Omega = \{(-\infty, 0]\},\) the other vehicles and ego-car cannot observe each other until the ego-car enters the intersection. The model then estimates \(P(C_i) = \lambda_i\) for all points \(i\) since \(P(O_i) = 1 = 0\) when \(i\) is occluded, and the

![Conditional Likelihood as a function of distance](image-url)
lane is initialized with likelihood $\lambda_t$ for a Poisson emission rate. Thus, $r_t = \sum_j \lambda_t \sum_p P(E_t | C_t)$.

3) High-Speed Traffic: When other vehicles travel at high velocity, $v_l \to \infty$, the model yields the same expected risk as in the no observability case, since the ego car can never clear the intersection in time with $d_e > t_c \cdot v_l$.

4) High-Speed Ego Car: As $t_c \to 0$, the risk integration is performed over a single road segment per lane. Per (1), if the accumulated risk over intersecting lanes $l$ becomes less than $r_{go}$, the car can occupy that road segment on the lane.

5) Low-Speed Ego Car: With $t_c \to \infty$, the model integrates risk over an infinite number of road segments. Thus, any other cars in view of the ego car, regardless of position, would prevent the ego car from entering the intersection.

III. SHARED CONTROL APPROACHES

We leverage our risk model for control policies before and after entering an intersection. Before the intersection, the ego-car uses the risk to determine if it can proceed, or if it must to nudge toward the intersection to reduce uncertainty. Once the ego-car enters the intersection, we use the risk to adjust its velocity along the path through the intersection. We summarize the overall control algorithm in Algorithm 2.

A. Nudging Into Intersections

Figure 5 shows an ego-car approaching an intersection with occlusions on either side of its origin lane. The risk decreases as the car approaches the intersection, due to the increase in visible road segments. The risk is dependent on the time it takes the ego-car to clear the intersection, $t_c$. Figure 5 plots several values of $t_c$ and the corresponding risks.

In the presence of occlusions or noisy observations, the ego-car should spend time in high visibility positions to reduce uncertainty in occupancy estimation. This desire to reduce uncertainty and to prevent risky go/no-go decisions motivates our nudging control policy,

$$v'_e = \begin{cases} d_{\text{nudge}} v_e & (r_t > r_{go}) \text{ and } (d_e < d_{\text{nudge}}) \\ v_e & \text{otherwise} \end{cases}, \quad (5)$$

where $v'_e$ is the regulated velocity, bounded by the original desired velocity $v_e$, and $d_{\text{nudge}}$ is the location where the vehicle begins to nudge towards the intersection. The value $r_{go}$ provides a threshold for how much risk is tolerated before going into an intersection. When $d_e > d_{\text{nudge}}$, or the risk is low, the regulated velocity matches the commanded velocity, $v'_e = v_e$. Otherwise, the vehicle slows to a stop before the intersection, while gathering more observations.

B. Clearing the Intersection

Once the risk $r_t < r_{go}$, the ego-car enters the intersection and follows a pre-computed trajectory to clear the intersection. Let $L_p$ represent the length of the remaining path, $p$, from the original lane to the new lane through the intersection. We assume the ego-car cannot move backwards along its path, thus $L_p$ monotonically decreases over time. The minimum time it takes the ego-car to cross the intersection is given by $t_{c \text{min}} = \frac{L_p}{v_{\text{max}}}$. The maximum time to clear the intersection, while keeping risk under the threshold $r_{go}$, is $t_{c \text{max}}$, and depends on other traffic approaching the intersection. We impose an upper bound on $t_{c \text{max}}$ to require the ego-car to complete its turn and not to stop within the intersection. Thus, the regulated velocity after the ego-car enters the intersection is given as

$$v_e' = \max \left( \frac{L_p}{t_{c \text{max}}}, v_e \right). \quad (6)$$

We combine the nudging policy and clearing policy together in Algorithm 2 for the ego-car’s overall control policy through the intersection. In Algorithm 2, we assume the path through the intersection $p$ is pre-defined, and a Pure Pursuit algorithm [35] is used to control the steering commands $\theta_e$.

While the above methods establish low level control mechanisms, opportunities exist to utilize high level planners that utilize risk during planning e.g., to maximize $t_c$ for comfort, as in [23].

IV. EXPERIMENTAL RESULTS AND VALIDATION

To evaluate this model, we perform both simulations and physical experiments on the one-tenth scale MIT racecar platform [36]. We show that full and parallel autonomy control

\begin{algorithm}
\caption{Ego-Car Control.}
\begin{algorithmic}
\Input{Turning path, $p$, desired velocity, $v_e$}
\Output{Steering, $\theta_e$, and modified velocity, $v'_e$, commands}
\While{$L_p > 0$}
\State Update the risk according to Algorithm 1
\If{$r_t < r_{go}$}
\State Bound velocity $v_e$ to $v'_e$ according to (5)
\Else
\State Bound velocity $v_e$ to $v'_e$ according to (6)
\EndIf
\State Find pure pursuit steering, $\theta_e$, based on $p$
\State Command vehicle according to $v'_e$ and $\theta_e$
\EndWhile
\end{algorithmic}
\end{algorithm}
systems, when leveraging our model's integrated risk, can avoid accidents at occluded junctions.

A. Simulations

We simulate topologically distinct environments to prototype our methods. To verify risk calculations, we vary cross traffic densities ($\lambda_l$), velocities ($v_l$) and environment occlusions ($\Omega$), while generating test trajectories for the ego-cars and other cars. At an unsignalized, occluded left turn junction, the ego car approaches an intersection with two lanes of cross traffic.

Topological adaptions, such as those shown in Figure 7, demonstrate the flexibility of our theory and how the risk model extends to other motions and occlusions. Mimicking blind merges exiting tunnels or on-ramps, the merging scenario requires the ego-car to enter a lane of traffic with occlusions blocking the view until near the merge. For the roundabout, the ego-car must enter the roundabout, but features within the roundabout occlude the car from knowing the positions of the other cars. We refer the reader to our video supplement for animations of risk, akin to Figure 6, for these topologies.

B. Experiments

To benchmark our system, we consider five separate scenarios, including two full autonomy scenarios, two parallel autonomy scenarios and a fully manual scenario. In the parallel autonomy and manual control cases, the expert human operator utilizes a first-person view from cameras mounted on the vehicle. Each autonomy scenario is evaluated with our risk-based controller and a baseline gap-acceptance controller. All scenarios include a pre-planned path, with the computer and human supplying velocity commands in time.

- **Human Expert:** An expert fully controls the vehicle velocity.
- **Full Autonomy (Ours):** The car uses Algorithm 2, setting the desired velocity $v_e$ to $v_l$ of the new lane.
- **Full Autonomy (Baseline):** The car uses the baseline gap-acceptance algorithm, similar to the warning system of [37], where other vehicles need to stop only within a time gap. In place of a warning, the vehicle stops. The time gap is chosen based on lane speed and car length.
- **Parallel Autonomy (Ours):** An expert sets the desired velocity, $v_e$, which the risk assessment system may override, per Algorithm 2.
- **Parallel Autonomy (Baseline):** An expert controls velocity, but the gap-acceptance system may override to stop the vehicle.

Tables I and II summarize the run time and success rates, where a success indicates that a vehicle completes a turn without incurring contact with another vehicle.

C. Full Autonomy

To validate our model on a fully autonomous system, we first implement a motion controller that executes turns at junctions. Given the task of moving across traffic into a new lane, the control policy executes a pure pursuit controller [35] to follow a path, with no ability to stop for cross traffic. Pure pursuit sets
the steering angle, $\theta_c$, but not velocity $v_c$, which is set to $v_l$ in the case of full autonomy. It is set by the human in the case of parallel autonomy and manual control.

We then ran two turning aids for making a go/no-go decision: a gap estimator and our risk model. We used $d_s = 0.2$ m as the gap size in the baseline estimator and set $d^F_s = 0.6$ m to accommodate the wheel-base length between vehicle pose and its front bumper. To model arrivals and attention, we set $\lambda_l = 0.05$ and $\lambda_s = 0.5$. We evaluated six $l_c$ values between 2 and 4.5 seconds, with a risk tolerance of $T_{go} = 0.02$, such that the ego-vehicle will not wait indefinitely at the intersection. By applying (5) and (6), the controller will slow down or speed up the ego-car along the trajectory to minimize risk. For the gap estimator, if a gap in traffic exists, then the car will attempt to turn; otherwise, the ego-velocity is set to zero.

We tested making left turns into a crowded two lane roadway with an unsignalized intersection, shown in Figure 2b, and applied our control approach. To simulate a stream of traffic, we used six other vehicles driving in a loop at $v_l = 0.75$ m/s under ground truth information from a motion tracking system, shown in Figure 1. Virtual occlusions were added to preclude observations. A computer program chose the start time for the ego-car to approach the intersection uniformly at random, with no advance information about the positions of the looping cars. For every example, the ego car started approximately 75 cm before the junction, with its view occluded. The baseline gap estimation system completed 70% of left turns, while the risk based model successfully completed 90% of trials.

Slowing $v_l$ to 0.5 m/s, the risk based model performance dropped to 70%. However, the gap estimation system could not complete a single trial without collision when other cars were in proximity. This is because the gap estimation system attempts a turn as soon as it sees a gap without regard for the risk of occluded vehicles. To test this, we increased $d_s$ to 0.6 m, which effectively causes the gap estimation system to require a larger gap in traffic. This resulted in the gap estimation system scoring 80%, while our risk estimation system maintained a 70% success rate.

### V. Conclusions

In this letter, we have presented a probabilistic model for assessing risk at occluded intersections with uncertain perception data. Whereas previous approaches typically adopt a Lagrangian perspective, based on the trajectories of perceived vehicles, our Eulerian approach reasons about the risk for road segments. This enables us to handle occluded junctions efficiently with limited perceptual data. We have deployed the risk assessment algorithm on scale model cars, testing for both autonomous control and remote human interaction (parallel autonomy). When a vehicle enters an occluded intersection, the risk assessment algorithm augments the commanded inputs to improve the safety of maneuvers, while also reducing the amount of time the vehicle waits before crossing traffic.

For future work, we will explore integration of our risk metrics into complete planning approaches, including more diverse

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**TABLE II**  
**COMPARING THE RISK MODEL AND THE GAP BASELINE**

|                | Autonomy | Trials | Success | Avg Time | Std Dev |
|----------------|----------|--------|---------|----------|---------|
| Ours           | Full     | 30     | 77%     | 5.89s    | 1.29s   |
| Baseline       | Full     | 20     | 75%     | 3.79s    | 1.03s   |
| Ours           | Parallel | 20     | 85%     | 9.68s    | 3.36s   |
| Baseline       | Parallel | 20     | 65%     | 8.93s    | 3.43s   |
| Ours           | Full & Parallel | 50 | 80% | 7.41s | 2.12s |
| Baseline       | Full & Parallel | 40 | 70% | 6.36s | 2.23s |

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Fig. 8. The ego-vehicle viewpoint from three cameras corresponds to the configuration shown in Figure 1. Note there is a vehicle to the left completely occluded by the blue house.
vehicle platforms. Additionally, we plan to explore ways to adapt our algorithms with respect to additional road topologies and naturalistic data, as well as learning optimal model parameters.

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