Numerical modeling of unsteady MHD flow of Casson fluid in a vertical surface with chemical reaction and Hall current

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Abstract
The non-Newtonian fluid flow plays an important role in industrial and manufacturing processes such as painting, printing, coating, and biomechanical. In the present paper, the steady two-dimensional MHD free convective flow of Casson fluid over a vertical surface is elucidated numerically with the impact of thermal radiation and chemical reaction are offered. The interaction of transverse magnetic field, viscous dissipation, and Hall current are taken into the account. A nonlinear flow system is developed to decompose the heat and mass transmission effects of this liquid, which is done by taking an appropriate extreme boundary condition. Non-dimensional nonlinear ordinary differential equations (ODEs) are diminished from governing partial differential equations (PDEs) via fitting transformations. The fundamental model of nonlinear constitutive flow laws is tackled numerically through the ND-solve technique in a Mathematica 11.0 environment. The computational upshot is also described explicitly to examine the consequences pertinent parameters. The attained sundry parameters are analyzed expediently, which include magnetic factor, radiation, chemical reaction, Prandtl number, heat source, modified Grashof number, and Schmidt number. The features of the governing relevant parameters on the flow frameworks are analyzed correctly via plots and tables. Additionally, the drag force, heat, and mass transference coefficients outlines are examined.

Keywords
Numerical technique, Magneto-hydrodynamic (MHD), Casson fluid, Hall current, heat source, chemical reaction

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Introduction
The synthetic industry, oil and gas, atomic energy, electrical energy, etc are the popular heat transfer phenomena nowadays. The section of particles from various temperatures is viewed as the exchange of hotness. Taking everything into account, heat move is the progression of hotness across the gadget limit because of the distinction in temperature between the framework and the climate. The hotness move in the framework can likewise be performed at various focuses inside the framework as a result of temperature contrasts. The

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presence of nanoparticles in the liquid furthermore impacts the hotness move of the liquid. The mixture of solid nanoparticles with a base liquid is known as nano-fluid that was proposed by Choi.1 There are several groups for the nanoparticles; chemically stable metals (gold, copper), metal oxide (alumina, silica), metal carbides (silicon carbide), metal nitrides (aluminum nitride, silicon nitride), and carbon in the various form (diamond, graphite) reported by Sarkar et al.2

Khan and Pop3 initiated the thermophoresis together with the Brownian motion effects on the boundary layer flow of a nanofluid past a steady stretching sheet numerically. Yirga and Shankar4 claimed that the wall shear stress increases when the presence of nanoparticle volume fraction on MHD flows across a steady porous sheet. An analogous issue of MHD flow with the combination of thermal radiation and buoyancy effect has been reported by Pourmehran et al.5 Samrat et al.6 considered the effect of three kinds of nanoparticles respectively; silver, ferrous, and copper with water as a base fluid. The creator used the perturbation strategy to tackle and dissect the issue and has reasoned that the silver-water nanofluid is a decent coolant contrasted with copper and ferrous in water nanofluid. Rawi et al.7 studied the Casson fluid with copper as nanoparticles that dissolved in a base fluid, carboxymethyl cellulose solution (CMC). The nanoparticle volume fraction in Casson fluid flow has enhanced the temperature profile, as illustrated by Rawi et al.7 The Keller Box method has been used by Rawi et al.7 and discovered that it is a productive technique to fix the issue in the presence of copper-carboxymethyl cellulose-based nanofluid past a stretching sheet.

Hybrid nanofluids got the reaction of industry, particularly designing to investigate profoundly because of its broad specialized, modern, and logical applications like transportation, microfluidics, clinical assembling, and so on. The ideal mix of different nanoparticle properties has exhibited an exceptional heat transfer coefficient improvement with a low-pressure decline limit.8–12 According to Waskeen et al.,9 the blend of balanced-out nanofluids increment the warm conductivity rate with a minor measure of nanoparticle fixations in the liquid. Moreover, the proper hybridization of the better thermal conductivity of the nanocomposites has enlarged the heat transfer of the fluid and has reduced the disadvantage of increment of viscosity by Minea and Moldoveanu.13 Normally, stable hybrid nanoparticles can be classified into three classes; metal, ceramic, and polymer.14–16

Several researchers have studied the heat transfer of metal nanomaterial alumina/copper because the hybrid nanoparticles can intensify the thermal conductivity and result in increasing the convective heat transfer.17–20 The existence of nanoparticles Al2O3 in a fluid has improved the Nusselt number coefficient, which characterizes the fluid’s heat transfer compared to other metal oxides by Mikkola.21 It is obvious because the thermal conductivity of Al2O3 is higher as contrasted to other metal oxides. Furthermore, Al2O3 likewise shows a few brilliant properties like generally excellent dependability and synthetic latency by Suresh et al.17 As stated in Mikkola,21 the copper nanoparticle’s heat capacity is higher than the silver nanoparticle. Genuinely, heat limit is the measure of hotness expected to expand the temperature of a substance by 1°C. The fluid takes a long time to gain heat and a long time to cool. Therefore, the incorporation of copper and alumina in a liquid can altogether work on the thermal characters of the liquid just as steady hybrid nanoparticles.

Magneto-hydrodynamic (MHD) along with thermal radiation impacts on the wire coatings analysis for numerous categories of fluids are considered by several investigators.22–29 All these researches have so far presented numerical as well as analytical techniques for the solution of wire coating problems subjected to different types of non-Newtonian fluids. Marmeni30 then studied various features of mass transfer effect to liquid flow with constant heat flux as another flow issue. Narahari and Raghavan31 examined the true computational outcomes for natural convective flow of Casson fluid subject to radiation effect and rising wall temperature impact. The diffusion thermal effect was later explored by Jha and Ajibade.32 The Dufor effect raised the temperature of the fluid, implying that it can diminish the time it takes for it to reach steady state. The flow of unsteady hydro magnetic convective flow through a rotating framework in permeable channel along with Hall effects was analyzed by Seth et al.33 The model of fractal-fractional unsteady convection flow of Newtonian liquid has been analyzed by Khan et al.34 None of them did any analytical work on non-Newtonian fluid flow in a channel. Casson fluid flow is a new study on non-Newtonian fluid flow in vertical channels that was just published by a group of researchers. The effect of thermal radiation on Casson fluid flow via a vertical conduit was investigated by Khan et al.35 They used a fixed vertical plate and an oscillating vertical plate to tackle the problem. The unsteady flow of Casson fluid through a fixed channel was examined by Ahmad Qushairi et al.36 Mallikarjuna et al.37 used numerical methods to solve the problem of pulsatile Casson fluid flow. Bukhari et al.,38 in addition to slip velocity, included the impacts of porosity and heat radiation. Divya et al.39 described hydro magnetic...
flow Casson liquid with heat and mass transfer effects. Ahmad Sheikh et al.\textsuperscript{40} studied non-Newtonian liquid flow in a channel with an external magnetic field and thermal radiation effect.

To author’s best idea, the subsequent specific objectives of this numerical study have remained undone, and the objectives are:

- To investigate Hall current and viscous dissipation on MHD free convective flow of Casson fluid past a vertical plate in presence of thermal radiation with heat source/sink and chemical reaction.
- The mathematical modeling equations of the non-linear partial differential equations are transformed into ordinary differential equations and then solved numerically.
- The effect of various pertinent parameters on velocity, temperature, and concentration are demonstrated.
- Also the skin friction, Nusselt, and Shrewood numbers are calculated numerically and given in the tabulated form.

**Mathematical formulation**

In this section, we describe the physical and mathematical description of the proposed model.

Physical description: we consider a steady state 2D hydrodynamic convective flow of Casson liquid by a vertical sheet having boundary layers, subjected to thermal and concentration buoyancy forces. By fixing the sheet in \( x \) – axis and \( y \) – axis is taken normal to the surface. The physical sketched and coordinate system is revealed in Figure 1. The wall is kept at fixed temperature \( T_w \) and, concentration \( C_w \), higher than the ambient temperature \( T_\infty \) and concentration \( C_\infty \). A transverse and magnetic field having strength \( B_0 \) is considered, which is sufficiently strong to produce the Hall current in the flow field. Furthermore, we assuming 2D flow, so that the physical constraints are independent of \( x \), because of the length of the plate are large enough. The effects of hall current, viscous dissipation, chemical reaction; thermal radiation, and heat source/sink parameters are present.

The basic equation for the Casson fluid can be expressed as\textsuperscript{1}:

\[
\tau_{pq} = \begin{cases} 
2\left(\mu_B + \frac{\tau_B}{\sqrt{2\pi}}\right)e_{pq}, & \pi > \pi_c \\
2\left(\mu_B + \frac{\tau_B}{2\pi}\right)e_{pq}, & \pi < \pi_c.
\end{cases}
\]  

\( \tau_{ij} \) denote the yield stress of the fluid, \( \mu_B \) signify the plastic dynamic viscosity of the non–Newtonian fluid; \( \pi = e_{pq}e_{pq} \) is the product of the component of deformation rate with itself, and \( e_{pq} \) are the \((p,q)\)th component of the deformation rate; \( \pi_c \) is a critical value of \( \pi \) based on non-Newtonian Casson fluid.

**Mathematical description**

The governing flow equations are given in equations (2)–(5):

\[
\frac{\partial \tau}{\partial \bar{y}} = 0 \tag{2}
\]

\[
\rho\frac{\partial \bar{u}}{\partial \bar{y}} = \mu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2 \sin^2 \chi}{(1 + m)} \bar{n} + B_T \rho g (T - T_\infty) + B_c \rho g (C - C_\infty) \tag{3}
\]

\[
\bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \left( \frac{k}{\rho C_p} \right) \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \left( \frac{Q_0}{\rho C_p} \right) (T - T_\infty) - \frac{1}{\rho C_p} \frac{\partial \bar{T}}{\partial \bar{y}} \left( \frac{\beta B_0^2 \sigma}{\rho C_p} \right) (n)^2 \tag{4}
\]

\[
\bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - R(C - C_\infty) \tag{5}
\]

Subject to boundary conditions:

\[
\bar{n} = 0, \quad T = T_w, \quad \bar{C} = \bar{C}_w \text{ for } \bar{y} = 0 \quad \bar{n} \rightarrow 0, \quad T \rightarrow \bar{T}_w, \quad \bar{C} \rightarrow \bar{C}_w \text{ for } \bar{y} \rightarrow \infty \tag{6}
\]

Here, \( \bar{x}, \bar{y} \) designate the directions of the dimensional components along and normal to the sheet surface. Whereas \( \bar{n}, \bar{v} \) represent velocities components along \( \bar{x}, \bar{y} \) directions; \( g \) denote acceleration due to gravity; \( \bar{T} \) dimensional temperature of the fluid near the plate; \( \bar{T}_w \) free stream dimensional temperature; \( \bar{C} \) dimensional concentration; \( \bar{C}_w \) free stream concentration; \( \beta \) Casson parameter; \( \nu = \mu/\rho \) kinematic viscosity; \( k \) thermal
conductivity nature of the fluid; $\rho$ fluid density; $m$ Hall current; $\sigma$ electrical conductivity; $C_p$ specific heat; $B_T$ and $B_C$ denotes thermal and concentration expansion coefficient; $Q_0$ heat absorption coefficient; $D$ mass diffusivity.

The radiative heat flux $q_r$ is given by

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I^*$$  (7)

$I^* = \int_0^\infty K_{\lambda w}(\frac{a_{\lambda w}}{c_T})d\lambda$, $e_{\lambda w}$ is Planck’s function and, $K_{\lambda w}$ denote the absorption coefficient at the wall.

Introducing equation (8)

$$\eta = \frac{y}{v_0}; F = \frac{y}{v_0}; T = \frac{y}{T_w - T_\infty}; C = \frac{C - T_\infty}{T_w - T_\infty}$$  (8)

Equation (2) is justified routinely whereas equations (3)–(5) are reduced into nondimensional form by using equations (7) and (8):

$$\left(1 + \frac{1}{\beta}\right) \frac{d^2F}{d\eta^2} + \frac{dF}{d\eta} - P_1 F + GrT + GmC = 0$$  (9)

$$\frac{d^2T}{d\eta^2} + Pr \left(\frac{dT}{d\eta} + (S - Rd)T + Ec \left(\frac{dF}{d\eta}\right)^2 + MEcF^2\right) = 0$$  (10)

$$\frac{d^2C}{d\eta^2} + Sc \left(\frac{dC}{d\eta} + \gamma C\right) = 0$$  (11)

Where $M_1 = \frac{M}{1 + m^2}, P_1 = M_1 \sin^2 \chi$

with the following extreme conditions

$$F = 0, T = 1, C = 1 \text{ at } \eta = 0$$

$$F \to 0, T \to 0, C \to 0 \text{ at } \eta \to \infty$$  (12)

The variables appearing in equations (9)–(11) are defined as follows:

$$M = \frac{B_0^2 \sigma v}{\rho v_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad S = \frac{Q_0 v}{\rho C_p v_0}, \quad Ec = \frac{v_0^2}{C_p(T_w - T_\infty)}, \quad Sc = \frac{v}{D}, \quad Rd = \frac{4F v}{\mu C_p v_0},$$

$$\gamma = \frac{R^* v}{v_0^2}, \quad Gr = \frac{pg \beta F v^2(T_w - T_\infty)}{\mu v_0^3}, \quad Gm = \frac{pg \beta C v^2(C_w - C_\infty)}{\mu v_0^3}$$  (13)

Defines as magnetic factor; Prandtl number; Heat source; Eckert number; Schmidt number; Radiation parameter; Chemical reaction parameter; Grashof and Modified Grashof numbers.

Physical quantities of interest

The important relationships for the engineering concern in the present exploration are $C_f$, $Nu_x$, and $Sh_x$, with expressions defined below:

**Skin friction**

The dimensionless form of the $C_f$ coefficient for the velocity profile is given by

$$C_f = \frac{dF}{d\eta}|_{\eta = 0}$$  (14)

**Nusselt number**

The heat flow rate coefficient $Nu_x$ in dimensionless form is given by

$$Nu_x = -\frac{dT}{d\eta}|_{\eta = 0}$$  (15)

**Sherwood number**

The mass flow rate coefficient $Sh_x$ in the dimensionless form given by

$$Sh_x = -\frac{dC}{d\eta}|_{\eta = 0}$$  (16)

Results and discussion

This section is devoted to understanding clearly the physical insight and graphical interpretation. The nonlinear flow equations (9)–(11) subjected boundary postulates, in equation (12), are tackled numerically through ND-solve technique. While keeping $\beta = 0.5; Rd = 0.3; S = 0.5; Pr = 2.0; m = 1.0; Sc = 0.3; Gr = 2.0; Gm = 2.0; Ec = 0.2; M = 0.5; \chi = \pi/4$. These values considered constant unless they are cited. The following remarks have been made:

The variations in drag force coefficient $C_f$ under the effect different physical constraints are recorded numerically from Table 1. As noticed the primary skin-friction enhances with larger estimation of $\beta$, $P_1$, and $Gr$. Table 2 shows the numerical outcomes for the rate of heat transference coefficient $Nu_x$. It can be seen that heat transfer rate rises for increasing data of $Pr$ while diminishes the same profile with $Ec$ and $S$. The mass flow rate enhances under the influence of $Sc$ and $\gamma$ which can be found from Table 3.

Figure 2, explains velocity curves reduce with increase of magnetic influence $M$ when the magnetic strength develops a resistive force is invented known as Lorentz force which obstructs the fluid motion as result velocity curves diminish. Since the buoyancy force is
much higher than the viscous force with upsurge in $Gm$ values. Consequently, $Gm$ augments the fluid movement which leads to boosts flow field as well as thickness of momentum boundary layer is revealed in Figure 3. A similar result may be illustrated for the behavior of Grashof number $Gr$ on velocity profiles. As the thermal buoyancy force becomes improve and the momentum boundary layer is enlarging. The momentum boundary layer outlines increase subject to larger estimation of $Gr$ parameter see Figure 4. Figure 5 illustrate the action of variation in $\beta$ on velocity. The higher values of $\beta$ develop a low resistance to the yield stresses, which decreases the flow field. In consequence, velocity profiles rise. Hall current is produced due to potential difference. Here, larger estimation of Hall parameter $m$ diminishes the effective conductivity, which reduces the magnetic damping force. In consequence, velocity outlines develop and evaluated through Figure 6. Figure 7 describes heat source parameter $S$ on temperature field.

This plot reveals temperature field enhancement for higher $S$. One can perceived that $S$ added extra heat energy to the stretching surface due to which thermal field rises. Figure 8 display the results of the fluid

| $\beta$ | $P_1$ | $Gr$ | $C_f$ |
|---------|-------|------|-------|
| 1.0     | 0.10  | 0.3  | 0.447549 |
| 1.2     |       |      | 0.503993 |
| 1.4     |       |      | 0.552956 |
| 1.6     |       |      | 0.595738 |
| 1.0     | 0.10  |      | 0.447549 |
| 0.15    |       |      | 0.462060 |
| 0.20    |       |      | 0.490699 |
| 0.25    |       |      | 0.600547 |
| 0.10    | 0.3   |      | 0.447549 |
| 0.4     |       |      | 0.598866 |
| 0.5     |       |      | 0.752418 |
| 0.6     |       |      | 0.909507 |

| $Pr$ | $Ec$ | $S$ | $Nu_x$ |
|------|------|-----|-------|
| 0.3  | 0.4  | 0.1 | 0.256853 |
| 0.5  | 0.4  | 0.1 | 0.444604 |
| 0.7  | 0.4  | 0.1 | 0.634459 |
| 0.9  | 0.4  | 0.1 | 0.820368 |
| 0.3  | 0.1  | 0.1 | 0.35758 |
| 0.3  | 0.2  | 0.1 | 0.328279 |
| 0.3  | 0.3  | 0.1 | 0.295122 |
| 0.3  | 0.4  | 0.1 | 0.256853 |
| 0.1  | 0.10 |    | 0.35758 |
| 0.15 |    |    | 0.321915 |
| 0.20 |    |    | 0.280667 |
| 0.25 |    |    | 0.231317 |

Table 1. Variations in drag force coefficient under the influence of pertinent parameters.

Table 2. Variations in heat transmission coefficient under the impact of pertinent constraints.

| $Sc$ | $\gamma$ | $Sh_x$ |
|------|----------|--------|
| 0.2  | 0.1      | 0.284398 |
| 0.4  | 0.1      | 0.484826 |
| 0.6  | 0.1      | 0.687633 |
| 0.8  | 0.1      | 0.889952 |
| 0.2  | 0.3      | 0.367252 |
| 0.5  | 0.5      | 0.432536 |
| 0.7  | 0.7      | 0.487633 |

Table 3. Variations in mass flow coefficient under the influence of pertinent parameters.
temperature with discrete values of the radiation factor $Rd$. The thermal field lines upsurges with an increment in $Rd$ factor. The upshot of conduction rises via higher value of radiation parameter, and the fluid temperature increases at every point away from the surface, which increases the thermal outlines. Figure 9 illustrate the nature of thermal field curve against the changed values of Pr constraint. The Pr is defined as the ratio of kinematic viscosity to thermal diffusivity. An upsurge in Pr diminish fluid flow which decrease thermal conductivity and improve fluid viscosity. Furthermore, the thermal boundary layer thickness declines, as fluid thermal conductivity reducing with higher estimation of Pr which leads to decrease fluid temperature. Attributes of Eckert number $Ec$ on thermal field curves are interpreted in Figure 10. From this plot it can be detected that the fluid thermal behavior development happens as heat energy is deposited in the liquid because of frictional heating. Figure 11 describes $Sc$ result against concentration field. Physically, $Sc$ and $D$ are related inversely. Therefore, an upsurge in $Sc$ reasons a decay in the fluid concentration profiles. In reality, $D$ declines when $Sc$ augments. The outcomes of chemical reaction parameter $\gamma$ are exposed in Figure 12. From this graph a decline in the concentration field subject to larger estimation of $\gamma$. Such situation is perceived as higher $\gamma$ implies destructive chemically species rate which
dissolves fluid specie efficiently. In consequence, fluid concentration reduces. Figures 13 to 15 elaborate the nature of pertinent constraints on the quantities of physical importance for the engineering consequences. Figure 13 expositions the nature of drag force versus $\beta$ and $Gr$. As the values of theses constrains improved, the surface drag coefficient is increased. Figure 14 reveals the heat transfer nature against $Pr$ and $Ec$. An increment in Prandtl, Eckert numbers improves; the heat flow rate is diminished. The mass flow rate coefficient for diverse values of $\gamma$ and $Sc$ are observed
through Figure 15. As noticed mass transfer rate developed for higher estimation of these parameters.

Closing points

A numerical approach, the ND-Solve technique is employed to investigate Hall current and chemical reaction effects on the flow of Casson fluid over a vertically stretching sheet. The strategic remarks of current analysis are as follows:

- The growing data of $Gr$, $Gm$, $\beta$, and $m$ help to increase the velocity outlines although the pertinent constraint $M$ have diminishing propensity on the same outlines.
- It has been perceived that the fluid temperature develops with an augmentation in $S$, $Rd$, and $Ec$ whereas the pertinent parameter like Pr, diminishing the thermal field curves.
- The fluid concentration profiles are noticed to have deteriorated with augmentation in $Sc$ and $\gamma$ parameters.
- However, the surface drag force coefficient shown increasing attitude for the rising data of $\beta$ and $Gr$ parameters.
- Heat flow rate coefficient has shown the plunging behavior with rising values of $Pr$, $Ec$, and $S$ parameters.
- The mass flow rate coefficient has been noticed as a growing function of $Sc$ and $\gamma$ parameters.

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**Appendix**

**Notation**

$x, y$ Dimensionsal distance along and perpendicular to the plate

$g$ Gravitational Acceleration

$T_x$ Free stream dimensional Temperature

$C_v$ Free stream dimensional Concentration

$\nu$ Kinematic viscosity

$Gm$ Modified Grashof number

$k$ Thermal conductivity of the fluid

$\sigma$ Electrical Conductivity
| Symbol | Description                        | Symbol | Description                        |
|--------|-----------------------------------|--------|-----------------------------------|
| $B_T$  | Thermal expansion coefficient     | $Gr$   | Grashof number                     |
| $Q_0$  | Heat absorption coefficient       | $\overline{q}_r$ | Radiative Heat flux               |
| $D$    | Mass Diffusivity                  | $\rho$ | Density of the fluid               |
| $m$    | Hall effect parameter             | $c_p$  | Specific heat capacity at constant pressure |
| $C_{fx}$ | Skin fraction coefficient        | $B_C$  | Concentration expansion coefficient |
| $Sh_x$ | Sherwood number                   | $Rd$   | Radiation parameter                |
| $\overline{u}, \overline{v}$ | Dimensional velocities         | $\gamma$ | Mass Diffusivity                   |
| $T$    | Dimensional Temperature           | $Sc$   | Schmidt number                     |
| $C$    | Dimensional Concentration         | $Nu_x$ | Nusselt number                     |
| $\beta$ | Casson parameter                 | $B_0$  | Magnetic field                     |