Model building in SUSY QCD

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In $N = 1$ Supersymmetric gauge theories, we can calculate non-perturbative effects in the superpotential. It is interesting to apply this effect to the phenomenology. We show two models here. One is the low energy supersymmetry (SUSY) breaking scenario without messenger sector, and the second is the composite model generating Yukawa interaction dynamically.

1 Introduction

Recently, we can calculate the non-perturbative effects in the $N = 1$ SUSY gauge theories. Many people are trying to apply this effect to the phenomenology. We will show two models among them. One is the low energy SUSY breaking scenario without messenger sector, and the second is the composite model generating Yukawa couplings dynamically.

We consider $SU(N_c)$ gauge group and $N_f$ (ant-)fundamental representations $Q, \overline{Q}$ for the matter. The non-perturbative effects, which show vacuum structures, largely depend on the number of $N_f$. We will show the model building in $N_f = N_c - 1$ and $N_f = N_c + 1$. Each case induces the non-perturbative superpotential.

$$W_{\text{dyn}} = \frac{\Lambda^b_0}{\det M} \quad (N_f = N_c - 1), \quad (1)$$

$$W_{\text{dyn}} = \frac{1}{\Lambda^0_0} (BM \overline{B} - \det M) \quad (N_f = N_c + 1). \quad (2)$$

2 A model of gauge mediation

2.1 Why we need SUSY breaking theory?

Before showing a model let us consider briefly why we need SUSY breaking theory. SUSY guarantees the smallness of Higgs mass through the chiral symmetry of fermions. It is the main motivation of introducing SUSY from the viewpoint of phenomenology. The standard model (SM) can be extended to the supersymmetric standard model (SSM), and the minimal extension is so-called minimal supersymmetric standard model (MSSM). If SUSY is exact, the

\[^{a}\text{Here } M \equiv qQ, B \equiv \epsilon Q \cdot \cdots \cdot \overline{Q}, \overline{B} \equiv \epsilon \overline{Q} \cdot \cdots \cdot \overline{Q}. \text{ Since the superfields are boson, the gauge invariant operator } B, \overline{B} \text{ exist only in the region } N_c \leq N_f.\]
parameter number 19(1) in the SM does not increase in the MSSM. However, since SUSY is broken in our real world, the parameter number increases expansionary by the SUSY breaking parameters. The total parameter number becomes 125(44) in the MSSM. Most part comes from SUSY breaking parameters and especially from their flavor indices. Therefore, if these SUSY breaking parameters are arbitrary, MSSM induces too large flavor changing neutral current (FCNC) in $K^0 - \bar{K}^0$ and $\mu \rightarrow e\gamma$, and too large electric dipole moment (EDM) of neutron and electron. Thus, if there exists SUSY in our world, there must exist underlying theory of SUSY breaking, which should naturally derive highly degenerated squark/slepton masses $\delta\tilde{m}^2/\bar{m}^2 < O(10^{-3})$ for the supersymmetric version of GIM cancellation to work, and small $CP$ phases (less than $O(10^{-2})$) in SUSY breaking parameters.

2.2 How is SUSY breaking messaged to our world?

The underlying theory of SUSY breaking might be the model of spontaneous SUSY breaking. However, tree level supertrace formula $\text{Str} M^2 = 0$ shows that the squark/slepton which are lighter than quark/lepton appear when SUSY is spontaneously broken. Break through of this formula is possible by three ways. Correspondingly there are three ways to mediate SUSY breaking to our world, that is, (a): Gravity mediate scenario (Hidden sector scenario), (b): Anomalous $U(1)$ gauge scenario, (c): Gauge mediate scenario (Visible sector scenario).

The 125(44) parameters in the MSSM decrease in each scenario. They are (a): 22(3), (b): 34(3), and (c): 22(3). Each model has both merits and demerits. Here we concentrate on the scenario (c), which has the merits of calculability and naturally derives small FCNC since gauge interaction is flavor blind.

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| The number in ( ) shows the number of $CP$ phase. |
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| $^c$Gauge couplings 3; quark/lepton masses 9; KM matrix 4(1); Higgs mass 1; Higgs self-coupling 1; $\theta$ 1. |
| $^d$Gauge couplings 3; quark/lepton masses 9; KM matrix 4(1); $\theta$ 1; $\mu$ 2(1); $B$ 2(1); $m_{H_u}^2$ 1; $m_{H_d}^2$ 1; $\xi$ 1; gaugino mass $M_\lambda$ 2(1) × 3; squark/slepton mass $\tilde{m}^2$ 9(3) × 5; A 18(9) × 3; minus 4 phases (field redefinition), which is easily obtained by counting the reduction of the global symmetry $U(1)_\mu U(1)_B U(1)_{PQ} U(1)_R \rightarrow U(1)_B U(1)_L$ by introducing $\mu$ and SUSY breaking parameters. |
| $^e$A few TeV superparticle mass can also avoid large EDM. |
| $^f$Here we count anomalous $U(1)$ charges for the parameters. |
| $^g$Here we count $\mu$ 2(1); $B$ 2(1). |
2.3 A SUSY breaking model without messenger sector

The dynamical SUSY breaking can explain the large hierarchy between the Planck scale and the weak scale. Eq. (1) shows that in the case of \( N_f = N_c - 1 \) the vacuum can be lifted dynamically. SUSY is broken in general when there are no no-flat directions and the continuous global symmetry is spontaneously broken. The 3-2 model is just the case whose superpotential is

\[
W_{3-2} = \lambda X_1 + \frac{2\Lambda^7}{X_3}. \tag{3}
\]

The second term is induced from the non-perturbative effects as Eq. (1). \( X_i \)s (\( i = 1 \sim 3 \)) are moduli fields. Since we analyze the potential in the Higgs picture for the calculability, where gauge coupling is weak enough to use the canonical Kähler potential, \( \lambda \) should be small of \( \mathcal{O}(10^{-7}) \). Such small parameter can be avoided by building modified 3-2 model as

\[
W'_{3-2} = \lambda' \frac{X_1 X_3}{M_{\text{Planck}}} + \frac{2\Lambda^7}{X_3}. \tag{4}
\]

Now let us introduce the vector-like fields \( q, \bar{q} \) and \( l, \bar{l} \), which are the components of \( 5, \bar{5} \) of \( SU(5) \). By the suitable \( R \) charge assignment of vector-like fields, we can build a simple model of gauge mediation (c) without messenger sector whose superpotential is

\[
W = W'_{3-2} + h_1 \frac{X_3}{M_{\text{Planck}}^3} q \bar{q} + h_2 \frac{X_3}{M_{\text{Planck}}^3} l \bar{l}. \tag{5}
\]

The flavor blind squark/slepton masses and gaugino mass are generated by loop diagram of gauge interactions. This is one of the simplest models of gauge mediation (c), which naturally avoid large FCNC since \( \delta \tilde{m}^2 / \tilde{m}^2 \simeq \tilde{m}_{\text{gravity}}^2 / \tilde{m}_{\text{loop}}^2 \simeq \mathcal{O}(10^{-4}) \). The demerits of this model are that \( \mu \) and \( B \) are arbitrary parameters and that the color conserving minimum might not be the true vacuum.

\( ^k \)It is because the Goldstone boson can not have the supersymmetric partner.\( ^i \)The 3-2 model is the SM without the right-handed electron.\( ^j \)Three moduli fields remains in the Higgs phase: 14 (degrees of freedom of fields) – 8 (\( SU(3) \) gauge boson) – 3 (\( SU(2) \) gauge boson).\( ^k \)It might be not so unnatural to introduce the scale \( M_{\text{Planck}} \) in the superpotential. It is because there exists supergravity theory behind. The massless Goldstone fermion can not be absorbed unless we consider supergravity theory.\( ^l W'_{3-2} \) is obtained by changing the \( R \) charge assignment of \( X_{1,2} \). In this case \( \lambda' \simeq \mathcal{O}(1) \).\( ^m \)We can easily show that there is no SUSY vacuum in this model.
3 A composite model

Next we would like to consider the application of the case $N_f = N_c + 1$. This is the case of confinement where only $M, B,$ and $\overline{B}$ are massless fields below the strong dynamics scale. Here we consider enough high energy scale to neglect the SUSY breaking effects. People build composite models by using Eq. (2). Here we show a model which generates Yukawa coupling dynamically. Our motivation is very simple. It is that when we build the preon model whose “meson” $M$ is Higgs and “(ant-)baryon” $B (\overline{B})$ is quark/lepton, the first term of Eq. (2) becomes the Yukawa coupling.

We introduce $SU(7)_H$ gauge group and $N_f = 8$ preons $P_i, \overline{P}_i$, which also have SM gauge quantum numbers as shown in Table 1. When the $SU(7)_H$ gauge group becomes strong, confinement states $M, B$ and $\overline{B}$ appear as Table 2 and Table 3.

$$W_{\text{dyn}} = y_u \bar{U}(\Phi_u + \overline{\phi}_u)Q + y_d \bar{D}(\Phi_d + \overline{\phi}_d)Q + y_{3} \bar{N}(\Phi_u + \overline{\phi}_u)\ell + y_e \bar{E}(\Phi_d + \overline{\phi}_d)\ell + \alpha_u \bar{U}G_u Q + \alpha_d \bar{D}G_d Q + \beta_u \bar{U}Y \ell + \beta_d \bar{D}X \ell + \beta_n \bar{N}XQ + \beta_e \bar{E}YQ - \text{[Det]}.$$  (6)

This model is the modified version of Ref.4.
It shows that the Yukawa couplings are really generated dynamically. And this model predicts 4 Higgs doublets. Unfortunately, the color octet Higgs $G$ and lepto-quark $X,Y$ are also generated as “meson” fields, and $W_{\text{dyn}}$ contains their Yukawa couplings. Since they are all massless, we must introduce

$$W_{\text{tree}} = g_G \frac{P_Q P_U \bar{P}_D}{M} + g_\mu \frac{P_{\ell} P_{\ell} \bar{P}_{E} \bar{P}_N}{M} + g_x \frac{P_Q P_{\ell} \bar{P}_D \bar{P}_N}{M} + g_y \frac{P_Q P_{\ell} \bar{P}_U \bar{P}_E}{M}$$

by hand in order to give them masses. Eq.(7) is the most general gauge invariant 4 point interactions, which also generate $\mu$ terms ($\mu \simeq \Lambda^2/M$).

This model can be extended to the 3 generation model. Considerable extensions are (1): $SU(7)_H \rightarrow SU(7)_H^3$, (2): $SU(7)_H \rightarrow SU(23)_H$$^p$, (3): The first and second generations are elementary and the third generation and Higgs are composite$$^q$.

### 4 Summary

We have shown two models of phenomenological application of the non-perturbative effects in SUSY QCD. One is the low energy SUSY breaking scenario without messenger sector, and the second is the composite model generating Yukawa interaction dynamically. It is interesting to try to apply the non-perturbative effects in SUSY QCD to other phenomenologies.

$^a$The anomalies in this model are the same in the SM, $SU(2)^2U(1)_{B+L}.U(1)^2_{B+L}$.

$^p$It corresponds to the extension of global symmetry $U(1)_B U(1)_{L} \rightarrow SU(3)_F U(1)_B U(1)_L$.

$^q$The idea (3) comes from the fact that preons (Table 1) are just like “mirror fields”. Mirror fields are conjugated states of ordinal fields, which appear in string theory.
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