Axially deformed relativistic Hartree Bogoliubov with separable pairing force

Yuan Tian\textsuperscript{1,2,3}, Zhong-yu Ma\textsuperscript{1,2,4}, P. Ring\textsuperscript{2,3}

(1) China Institute of Atomic Energy, Beijing 102413, P.R.of China
(2) Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China
(3) Physikdepartment, Technische Universität München, D-85748, Garching, Germany and
(4) Centre of Theoretical Nuclear Physics, National Laboratory of Heavy Collsion, Lanzhou 730000, P.R.of China

A separable form of pairing interaction in the $^1S_0$ channel has been introduced and successfully applied in the description of both static and dynamic properties of superfluid nuclei. By adjusting the parameters to reproduce the pairing properties of the Gogny force in nuclear matter, this separable pairing force is successful in depicting the pairing properties of ground states and vibrational excitations of spherical nuclei on almost the same footing as the original Gogny force. In this article, we extend these investigations for Relativistic Hartree Bogoliubov theory in deformed nuclei with axial symmetry (RHBZ) using the same separable pairing interaction. In order to preserve translational invariance we construct one- and two-dimensional Talmi-Moshinsky brackets for the cylindrical harmonic oscillator basis. We show that the matrix elements of this force can then be expanded in a series of separable terms. The convergence of this expansion is investigated for various deformations. We observe a relatively fast convergence. This allows for a considerable reduction in computing time as compared to RHBZ-calculations with the full Gogny force in the pairing channel. As an example we solve the RHBZ equations with this separable pairing force for the ground states of the chain of Sm-isotopes. Good agreement with the experimental data as well as with other theoretical results is achieved.

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I. INTRODUCTION

Covariant density functional theory (DFT) based on Relativistic Mean Field (RMF) theory provides a microscopically consistent description of the nuclear many-body problem \cite{1}. Conventional DFT without particle-particle (pp) correlations can be applied only for a few doubly closed shell nuclei. For the vast majority of nuclei and in particular those far away from the $\beta$-stability line, that play an important role in astrophysical applications, the inclusion of particle-particle correlations is essential for a quantitative description of many phenomena in nuclear structure. In the framework of DFT pairing correlations are taken into account in the form of Hartree-Bogoliubov theory \cite{2,3,4,5,6} for the ground states. Because of the numerical complexity of the deformed Hartree-Bogoliubov equations, monopole pairing or density dependent $\delta$-pairing interactions have been widely used in the literature for deformed mean field calculations \cite{5}. But the results are affected by a cutoff parameter which has to be introduced in a rather arbitrary way. Recent investigations show clearly that many results depend on this cut-off \cite{3}. Therefore, in order to avoid the complicated problem of a pairing cutoff, the finite range Gogny force \cite{8} has been applied in many relativistic applications \cite{5}. The parameters of this force have been adjusted very carefully in a semi-phenomenological way to characteristic properties of the microscopic effective interactions and to experimental data \cite{10,11}. It has been shown in several applications that the RHB model with Gogny pairing provides an excellent tool for the description of ground state properties in finite nuclei. Such investigations have been first devoted to spherical applications, such as to halo phenomenon in light nuclei \cite{12,13}, to the properties of nuclei near the neutron drip line \cite{14}, to the reduction of the spin-orbit potential in nuclei with extreme isospin values \cite{15}, to ground-state properties of Ni and Sn isotopes \cite{16}, etc. RHB theory with Gogny pairing has also been applied to investigate deformed \cite{17,18,19,20} and rotating \cite{20} nuclei, but such calculations are limited due to their numerical complexity.

Recently we have introduced a new separable form of the pairing force for RHB calculations in spherical nuclei \cite{21,22}. The parameters of this separable force are adjusted to reproduce the pairing properties of the Gogny force in nuclear matter. It preserves translational invariance and has finite range. A similar ansatz has been used in the pairing channel of non-relativistic Skyrme calculations in Refs. \cite{23,24}. This pairing interaction is separable in momentum space. In $r$-space the translational invariance leads to a $\delta$-force in the center of mass coordinates and therefore, at a first glance, translational invariance forbids exact separability. However using well known techniques of Talmi and Moshinsky \cite{25,26,27} it has been shown in Ref. \cite{21} that this force can be represented by a sum of separable terms which converges quickly. This avoids the complicated problem of a cutoff at large momenta or energies inherent in zero range pairing forces. As we discussed in Fig. 6 of Ref. \cite{21}, we found that although the $\delta$-force can give the same average gap as the Gogny force D1S if the size of the strength is adjusted properly, the individual matrix elements of the forces and the matrix elements of the pairing field $\Delta$ are very different from each other. The $\delta$-force behaves in some sense very much like a constant pairing. In contract our separable force has a very similar behavior to
This simple separable force can reproduce the pairing properties of the ground-state for spherical nuclei on almost the same footing as the original Gogny pairing interaction. Recently it has also been applied for studying dynamic properties of spherical nuclei. The relativistic quasiparticle random phase approximation (RQRPA) based on the same separable pairing interaction in the pairing channel was used for calculations of low-lying 2+ and 3− excited states in a chain of Sn-isotopes, which are very sensitive to the pairing channel. In comparison with experimental data and with the results of the original Gogny force it was shown that this simple separable pairing interaction is also very successful in describing the dynamical pairing properties of vibrational excitations.

So far, the new separable pairing force has been used only in spherical nuclei. In this article we apply this separable pairing force for axially deformed relativistic Hartree-Bogoliubov (RHBZ) calculations. For axially symmetric shapes the densities are invariant with respect to a rotation around the symmetry axis, which is taken to be the z-axis. Therefore it is convenient to work in cylindrical coordinates. The Talmi and Moshtinsky techniques used in Ref. are restricted to spherical coordinates. Therefore we had to develop similar techniques for cylindrical coordinates working in an anisotropic oscillator basis. Again the matrix elements of this pairing force in this basis are no longer fully separable. However they can be expanded, as in the spherical case, in a series of separable terms. Obviously the convergence of this expansion is not as fast as in the spherical case, it is still quick enough to save considerable numerical effort as compared to the full Gogny calculations. Finally we investigate for a chain of Sn-isotopes the ground-state properties such as binding energies and deformations using the RHBZ-program with this new separable pairing force. As it is known, the ground-state properties of the deformed nuclei are highly affected by the pairing gap. Good agreement is found, when comparing with experimental data and with theoretical calculations using RMF + BCS theory with a constant pairing force, or using non-relativistic Hartree-Fock Bogoliubov (HFB) theory with the full Gogny force D1S.

The paper is arranged as follows. The theoretical formalism of RHBZ with the separable form of the pairing interaction is presented in Sec. II. The convergence of the expansion of the pairing force elements in the cylindrical harmonic oscillation basis is investigated in Sec. III. In Sec. IV the ground-state properties of a chain of Sn-isotopes are calculated in the RHBZ approach. They are discussed in Sec. V. Finally we give a brief summary in Sec. V.

II. THEORETICAL FORMALISM

We start our investigations in symmetric nuclear matter with various densities. The gap equation in the 1S0 channel has the form,

\[ \Delta(k) = -\int_0^\infty \frac{k'^2 dk'}{2\pi^2} \langle k|V^1S0|k'\rangle \Delta(k') \ , \]

where a separable form of the pairing force is introduced,

\[ \langle k|V^1S0|k'\rangle = -Gp(k)p(k') \ . \]

A simple Gaussian ansatz \( p(k) = e^{-a k^2} \) is assumed. In Ref. the two parameters \( G \) and \( a \) have been fitted to the density dependence of the gap at the Fermi surface \( \Delta(k_F) \). Comparing with the Gogny force, we found two sets of parameters \( G = 738 \text{ MeV} \cdot \text{fm}^3 \) and \( a = 0.636 \text{ fm} \) for the parameter set D1 [8] and \( G = 728 \text{ MeV} \cdot \text{fm}^3 \) and \( a = 0.644 \text{ fm} \) for the set D1S [11].

In the Hartree approximation for a consistent mean field, the RHB equations read

\[
\begin{pmatrix}
\hat{h}_D - \lambda \\
-\Delta^* - \hat{h}_D + \lambda
\end{pmatrix}
\begin{pmatrix}
U_k(r) \\
V_k(r)
\end{pmatrix}
= E_k
\begin{pmatrix}
U_k(r) \\
V_k(r)
\end{pmatrix}
, \]

where \( \hat{h}_D \) is the single nucleon Dirac Hamiltonian,

\[
\hat{h}_D = -i\alpha \cdot \nabla + \beta (m + g_\sigma \sigma(r)) + g_\omega \tau_3 \omega^0(r) + g_\rho \rho^0(r) + e(1 - \tau_3) A^0(r) - m.
\]

The Dirac Hamiltonian contains the mean-field potentials of the isoscalar scalar \( \sigma \)-meson, the isoscalar vector \( \omega \)-meson, the isovector vector \( \rho \)-meson, as well as the photon. \( m \) is the nucleon mass and the term \( -m \) subtracts the rest-mass and normalizes the energy scale to the continuum limit. The chemical potential is to be determined by the subsidiary particle number condition, where the expectation value of the particle number operator in the ground state equals the number of nucleons. The column vectors are the quasiparticle spinors and \( E_k \) are the quasiparticle energies. \( \Delta \) is the pairing fields, which is an integral operator with the kernel

\[
\Delta_{ab}(r, r') = \frac{1}{2} \sum_{c,d} V_{abcd}(r, r') \rho_{cd}(r, r') \ ,
\]

where \( a, b, c, d \) denote the quantum numbers that specify the Dirac indices of the spinor. They run over the two spin orientations and the large and small components. \( V_{abcd}(r, r') \) are matrix elements of two-body pairing interaction. In general this should be a relativistic force and involve large and small components. However, since pairing correlations in nuclei are a purely non-relativistic effect it has been shown in Ref. [29] that we can neglect the pairing matrix elements between large and small
components as well as the effect of the pairing matrix elements between small components and we consider only the upper part of the pairing field $\Delta$ in Eq. (3). The pairing tensor is defined as

$$\kappa_{\alpha\beta}(\mathbf{r},\mathbf{r}') = \sum_{E_\perp > 0} U_{\alpha\beta}(\mathbf{r})^* V_{\alpha\beta}(\mathbf{r}').$$

(6)

For the axially symmetric deformed shape rotational symmetry is broken and therefore the total angular momentum $J$ is no longer a good quantum number. However, the densities are still invariant with respect to a rotation around the symmetry axis, which is taken to be the z-axis. It then turns out to be useful to work with the cylindrical coordinates

$$\mathbf{r} = (r_\perp \cos \varphi, r_\perp \sin \varphi, z).$$

(7)

In these coordinates the Dirac equation can be reduced to a coupled set of partial differential equations in the two variables $z$ and $r_\perp$ that are solved by an expansion in an anisotropic harmonic oscillator basis [30].

Since the interaction in the particle-hole (ph)-channel is identical to earlier calculations, here we only discuss the derivation of the matrix elements of the pairing interaction for the separable form of Eq. (2) in the pp-channel. First, we transform the separable force in Eq. (2) from momentum space to coordinate space and obtain

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = -G\delta(\mathbf{R} - \mathbf{R}')\frac{1}{2}(1 - P_T)P(r)P(r')$$

(8)

where $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ are the center of mass and relative coordinates of two paired particles, respectively. And $P(r)$ is obtained from the Fourier transform of $p(k)$,

$$P(r) = P(z, r_\perp) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\frac{z^2 + r_\perp^2}{4a^2}}.$$

(9)

The term $\delta(\mathbf{R} - \mathbf{R}')$ in Eq. (8) insures the translational invariance. It also shows that this force is not completely separable in coordinate space.

We start from the eigenfunctions of the deformed harmonic oscillator:

$$|\alpha\rangle = |n_z, n_r, m_l, m_s\rangle$$

$$= \frac{1}{\sqrt{b_\perp b_z}} \phi\phi_{m_l}(\frac{r_\perp}{b_\perp}) \frac{1}{\sqrt{2\pi}} e^{im_l\varphi} X_{m_s}(s),$$

(10)

with

$$\phi\phi_{m_l}(x) = N_{n_z} H_{n_z}(x) e^{-\frac{x^2}{4}},$$

$$\phi_{m_l}(x) = N_{n_m} \sqrt{2} e^{im_l x} l_{m_l}(x^2) e^{-\frac{x^2}{4}}.$$

(11)

The quantity $b_z = \sqrt{\hbar/\mu z}$ and $b_\perp = \sqrt{\hbar/\mu \omega_\perp}$ are the harmonic oscillator length. The polynomials $H_n(x)$ and $L_n^\nu(\eta)$ are Hermite polynomials and associated Laguerre polynomials as defined in Ref. [31]. The normalization constants are given by

$$N_{n_z} = \frac{1}{\sqrt{\sqrt{\pi} 2^{n_z} n_z!}}, \quad N_{n_m} = \sqrt{\frac{n_m!}{(n_z + |m_l|)!}},$$

(12)

where the $m_l$ and $m_s$ are the components of the orbital angular momentum and of the spin along the symmetry axis. The eigenvalue of $j_z$, which is a conserved quantity in these calculations, is $\Omega = m_l + m_s$, the parity is given by $\pi = (-)^{n_z + m_l}$. The conventional deformation parameter $\beta$ is obtained from the calculated quadrupole moments through

$$Q = Q_n + Q_p = \frac{16\pi}{5} \frac{3}{4\pi} A R_0^2 \beta,$$

(13)

with $R_0 = 1.2 A^{1/3} \text{ fm}$. Where The quadrupole $Q_{n,p}$ moments for neutrons and protons are calculated using the expressions

$$Q_{n,p} = \langle 2r^2 P_2(\cos \theta) \rangle_{n,p} = \langle 2z^2 - x^2 - y^2 \rangle_{n,p}.$$

(14)

In the pairing channel, although the total angular momentum $J$ is no longer a good quantum number, we still have $\Omega_1 + \Omega_2 = 0$. Together with the projector $\frac{1}{2}(1 - P)$, the two-particle wave function can be written as:

$$|\alpha_1, \alpha_2\rangle = |n_z, n_{z_2}, m_{l_1}, m_{s_1}, n_{r_2} - m_{l_1} - m_{s_1}\rangle.$$

(15)

As it is known, the separable force in Eq. (8) is expressed in the center of mass frame, the two-particle wave function has to be transformed into the same frame. So we use the fact that the product of two oscillator functions in the coordinates of the two particles can be expanded in terms of products of oscillator functions in the relative and in the center of mass coordinates. This fact holds for the one-dimensional oscillators in z-direction

$$|n_{z_1}, n_{z_2}\rangle = \sum_{N_{z_1} N_{z_2}} M_{N_{z_1} N_{z_2}}^n |N_{z_1} n_{z_2}\rangle,$$

as well as for the two-dimensional oscillators in $r_\perp$-direction

$$|n_{r_1} m_{l_1} n_{r_2} m_{l_2}\rangle = \sum_{N_{r_1} N_{r_2}} \sum_{N_{m_1} N_{m_2}} M_{N_{r_1} N_{r_2} N_{m_1} N_{m_2}}^n |N_{r_1} N_{r_2} m_{l_1} m_{l_2} m_{s_1} m_{s_2}\rangle.$$

Using the multinomial coefficients defined as

$$\binom{n}{m_1 \ldots m_\nu} = \frac{n!}{m_1! m_2! \ldots m_\nu!},$$

(16)

with $n = m_1 + m_2 + \cdots + m_\nu$, for $\nu = 2$ (binomial coefficients, which is expressed as $\binom{n}{m_1}$ and $\nu = 4$, the one- and two-dimension Talmi-Moshinsky transform brackets can be written as [32]:
with pairing field $\Delta$ in Eq. (5) has the form separable terms in Eq. (19). Using these expressions the used in the RHB equation can be evaluated by a sum of

\[
M_{N_z}^{n_1 n_2} = \frac{1}{\sqrt{2N_z + n_1}} \frac{1}{\sqrt{N_z + n_2}} \sum_{n_z = 0}^{n_z + n_2} (-)^s \left( \begin{array}{c} N_z \\ n_z - n_z + s \end{array} \right) \left( \begin{array}{c} n_z \\ s \end{array} \right),
\]

(17)

\[
M_{N_p M_p n_p m_p}^{n_1 m_1 n_2 m_2} = \frac{(-)^{N_p + n_p - n_1 - n_2}}{\sqrt{2N_p + 2n_p + |M_p| + |m_p|}} \sum_{n_1, n_2} \left( \begin{array}{c} (n_1) ![n_1 + |m_1|] ![n_2 + |m_2|] \right) \left( \begin{array}{c} (N_p) ![N_p + |M_p|] ![n_p + m_p] \right) !
\times \delta_{2n_1 + |m_1| + 2n_2 + |m_2|, 2N_p + |M_p| + 2n_p + |m_p|} \delta_{m_1 + m_2, M_p + m_p}
\times \sum_{Q, R, S = 0}^{N_p} \sum_{t = 0}^{M_p} \sum_{r, s = 0}^{n_p} \sum_{t = 0}^{n_p} (-)^{r + s + t} \left( \begin{array}{c} N_p - Q - R - S \\ Q \quad R \quad S \end{array} \right) \left( \begin{array}{c} M_p \\ T \end{array} \right) \left( \begin{array}{c} n_p - q - r - s \\ q \quad r \quad s \end{array} \right) \left( \begin{array}{c} m_p \\ t \end{array} \right).
\]

(18)

Therefore the matrix element of the separable force in axially deformed oscillator basis can be written as:

\[
\langle 12 | V | 1'2' \rangle = \langle n_z, n_r m_1, n_z n_r m_2 | V | n_z', n_r m_1', n_z' n_r m_2' \rangle = -G \sum_{N_z}^{N_p} \sum_{N_p}^{N_n} W_{12}^{N_z N_p} W_{1'2'}^{N_z N_p},
\]

(19)

where

\[
W_{12}^{N_z N_p} = \frac{1}{b_z^{1/2} b_{1/2}} \frac{1}{8\pi^{1/2}} V_{12}^{N_z N_p},
\]

(20)

with

\[
V_{12}^{N_z} = M_{N_z, N_z}^{n_z + n_z} \int_{-\infty}^{\infty} \phi_{n_z}(x) e^{-\frac{x^2}{\alpha_z^2}} dx,
\]

(21)

\[
V_{12}^{N_p} = M_{N_p, 0, n_p, 0}^{n_1 m_1 n_2 m_2} \int_{0}^{\infty} \phi_{0}(x) e^{-\frac{x^2}{\alpha_p^2}} dx,
\]

(22)

with the definition of

\[
n_z = n_z + n_z - N_z,
n_p = n_r + n_r - |m_1| + N_p.
\]

and $\alpha_z = a/b_z$, $\alpha_p = a/b_{1\perp}$. Thus we find that the pairing matrix elements for the separable pairing interactions used in the RHB equation can be evaluated by a sum of separable terms in Eq. (19). Using this expressions the pairing field $\Delta$ in Eq. (5) has the form

\[
\Delta_{12} = -G \sum_{N_z}^{N_0} \sum_{N_p}^{N_0} W_{12}^{N_z N_p} P_{N_z N_p}
\]

(23)

with

\[
P_{N_z N_p} = \frac{1}{2} \sum_{12} W_{12}^{N_z N_p} \kappa_{12}
\]

(24)

The results of the RHBZ model will depend on the choice of the effective RMF Lagrangian in the ph-channel, as well as on the treatment of pairing correlations. In this work the effective interaction NL3 [33] is adopted for the RMF Lagrangian and in the pairing channel we use the separable form of the pairing force in Eq. (8) adjusted to the pairing part of the Gogny D1S force in Ref. [21].

### III. STUDY OF CONVERGENCE

In the following investigations we solve the RHBZ equation (Eq. 3) with this separable pairing force. As in Ref. [34] the Dirac spinors are expanded in an axially deformed oscillator basis with $N_F = 20$ major oscillator shells. As we see from the Eq. (19) the separable pairing interaction is not fully separable in the axially deformed harmonic oscillator basis. We have a sum over the quantum number $N_z$ and $N_p$ characterizing the major shells of the deformed harmonic oscillator in the center of mass coordinate. For the self-consistent calculation of the RHBZ equation, the matrix elements of the $V_{12}^{N_z}$ and $V_{12}^{N_p}$ are calculated and stored in memory before the iteration. Therefore the time spend on the later calculations of the pairing matrix elements is really negligible as compared to the total time, while its take a large percentage of computing time for the RHBZ model with the fully Gogny force in the pairing channel. Due to this big advantage much computer time can be saved in RHBZ calculations for axially deformed nuclei.

For $N_F = 20$ the maximum number of $N_z$ and $N_p$ for the expansion of the pairing matrix elements in Eq. (19) is $N_0 = 40$ and $N_0 = 20$ respectively, which means a large number of 8000 separable terms in the pairing channel. As we mentioned in last section, therefore a large memory space is required for this series. To reduce the storage we study the convergence of the expansion with
Due to the prolate shape of the ground state of $^{164}$Er, the deformation parameter $\beta$ is already large enough. On the other side, for the normal oblate deformation, $N^0_\beta = 10$ is large enough, and we need at least $N^0_\beta = 7$ to reach the full convergence. The details of these results are given in the Tab. I.

In practical applications for normal deformed prolate nuclei it turns out that the expansion of the pairing matrix elements in Eq. (19) can be restricted to finite values $N_\beta \leq N^0_\beta = 14$ and $N_p \leq N^0_p = 5$ for obtain sufficient accuracy. For specific cases, where higher precision is required or for very large deformations, e.g. for superdeformed configurations larger values for $N^0_\beta$ or $N^0_p$ are required and convergence has to be checked. The matrix elements $V^{N_\beta}_{12}$ and $V^{N_p}_{12}$ are calculated and stored before starting the iterations. As compared to the calculations with the full expansion of the separable force in the pairing channel this corresponds to a considerable reduction in the memory and computing time.

the number of separable terms $N^0_\beta$ and $N^0_p$ and take the nucleus $^{164}$Er as example. For the self-consistent solution of the RHBZ equations with the full expansion of the separable pairing matrix elements the deformation of the ground state of $^{164}$Er is found to be prolate with $\beta = 0.325$. Varying the numbers $N^0_\beta$ and $N^0_p$ of the expansion we plot in Fig. 1 the neutron and proton pairing energies of $^{164}$Er with respect to the values of $N^0_\beta$ and $N^0_p$. In the main panel of Figs. 1(a) and 1(b), the contour lines correspond to lines of constant neutron and proton pairing energy. The value of these quantities increases with darkness. It is clearly seen that the neutron pairing energy converges for $N^0_\beta = 14$ and $N^0_p = 5$. The top and right panels of Fig. 1(a) show how the neutron pairing energy converges as a function of $N^0_\beta$ and $N^0_p$ for fixed values of $N^0_\beta = 5$ and $N^0_p = 14$, respectively. The same is shown in Fig. 1(b) for the proton pairing energy. Due to the prolate shape of $^{164}$Er the convergence in $N^0_\beta$ is slower than that in $N^0_p$.

in Fig. 2(a) the potential energy surface (PES) of the heavy nucleus $^{240}$Pu is plotted as a function of the deformation parameter $\beta$. The full pairing matrix elements Eq. (19) with $N^0_\beta = 40$ and $N^0_p = 20$ are adopted in these calculations. The corresponding pairing energies of protons and neutrons are given in Fig. 2(b). It is found that the ground state of $^{240}$Pu has a normal prolate deformation (NpD) at $\beta_{\text{NpD}} = 0.278$. Further two minima in the PES are observed with a normal oblate deformation (NoD) at $\beta_{\text{NoD}} = -0.277$, and a super prolate deformation (SD) at $\beta_{\text{SD}} = 0.865$. As discussed above, a reduced number of separable terms with $N^0_\beta = 5$ and $N^0_p = 14$ in Eq. (19) is large enough to obtain convergence for the pairing matrix elements at the normal prolate deformation. We also investigate the convergence with the number $N^0_\beta$ and $N^0_p$ of separable terms for the other two minima in the PES. We find that for the super deformation a larger value of $N^0_\beta = 18$ in the direction of the symmetry axis is needed to obtain convergence for the global properties while in the perpendicular direction $N^0_p = 5$ is already large enough. On the other side, for the normal oblate deformation, $N^0_\beta = 10$ is large enough, and we need at least $N^0_p = 7$ to reach the full convergence. The details of these results are given in the Tab. I.
\[ \beta_{ND} = -0.277 \]

| \( \beta_{SD} = 0.865 \) | \( E_{\text{full}} \) [MeV] | \( E_{\text{norm}} \) [MeV] | \( E_{\text{super}} \) [MeV] |
|-----------------|----------------|----------------|----------------|
| full            | -1807.984      | -14.025        | -9.639         |
| \( N_e = 10, N_p = 5 \) | -1807.123      | -8.870         | -8.718         |
| \( N_e = 10, N_p = 6 \) | -1807.819      | -13.022        | -9.609         |
| \( N_e = 10, N_p = 7 \) | -1807.972      | -13.975        | -9.606         |

**TABLE I**: The total energy \( E \) and the pairing energies \( E_n \) and \( E_p \) for protons and neutrons for the nucleus \( ^{240}\text{Pu} \) at normal and at super deformation for various values of \( N_e^0 \) and \( N_p^0 \).

FIG. 3: (Color online) Dependence of the binding energy (BE) and discrepancy between calculated binding energies and the available experimental data \( \Delta \text{BE} \) on the number of neutrons for a chain of Sm(Z=62) isotopes.

IV. SM-ISOTOPES

We also perform RHBZ calculations with the separable pairing interaction for a chain of Sm (Z=62) isotopes in the rare-earth region. Several shape transitions are expected along this isotope chain. The calculated ground state properties of Sm-isotopes, especially the deformations, are shown in Figs. 3, 4 and 5.

In the top panel of Fig. 3 we plot the total binding energy for Sm-isotopes as a function of neutron number for \( 66 \leq N \leq 102 \). In comparison, we also show results obtained in two other theoretical models: (i) the RMF model with the parameter set NL3 [35] in the \( s \)-channel and the pairing correlations included by the Bardeen-Cooper-Schrieffer (BCS) formalism with constant pairing gaps obtained from the prescription of Ref. [36], and (ii) non-relativistic HFB calculations [37] with the Gogny force D1S [11]. In the lower panel of Fig. 3 we display the discrepancy of the binding energies obtained in these three models from the experimental data [38]. All the calculated binding energies are in good agreement with the experimental data. We observe that our results obtained from RHBZ-calculations with the new separable pairing interaction deviate from the experimental binding energies by less than 0.2% and that they are for many cases in slightly better agreement with experiment than the other models.

The deformations and shapes of nuclei play a crucial role in defining the properties such as nuclear sizes and isotopes shifts. They are strongly affected by the pairing correlations. In the Fig. 4 we show the quadrupole deformation parameter \( \beta \) derived from Eq. (13) for the Sm isotopes between \( N=66 \) and \( N=126 \). We find that the deformations obtained from RHBZ-calculations with the new separable pairing force are very close to the results from non-relativistic HFB-calculations with the Gogny force D1S and in good agreement with the experimental data [39]. Although simple RMF + BCS theory provides a reasonable description for the binding energies, the deformation parameters of neutron-rich nuclei calculated in this model deviate slightly from the experimental data in comparison with those obtained for RHBZ and HFB theory.

Finally we show in Fig. 5 that there is excellent agreement between the calculations with the separable force presented in this paper and the original Gogny force in the pairing channel. The resulting deformation parameters shown in the upper panel are identical and for the pairing energies in the lower panel the differences are negligible.

V. SUMMARY

We have presented first results obtained by axially symmetric relativistic Hartree-Bogoliubov calculations using the new separable pairing force introduced in Ref. [21]. This separable force is translational invariant and has finite range. It contains two parameters which are adjusted to reproduce the bell shape curve of the pairing gap at the Fermi surface obtained from the
Gogny force in nuclear matter. In these RHBZ calculations for finite nuclei the two-body matrix elements of this force are not exactly separable because of translational invariance. However, using one and two dimensional Talmi-Moshinsky brackets, they can be evaluated in an anisotropic axially symmetric harmonic oscillator basis as a sum of separable terms. We investigate the convergence properties of this series for various deformations, in particular for normal prolate, for normal oblate and for super deformed cases. We find that the expansion of the pairing matrix elements converges relatively well and that an appropriate truncation provides an excellent approximation. This allows a considerable reduction of computing recourses such as time and memory in practical applications of the RHBZ theory. In particular we study the ground state properties of a chain of well deformed Sm-isotopes within this model. We find excellent agreement of our results with those obtained by using the non-relativistic HFB theory with the Gogny force D1S, the RMF + BCS theory based on the constant pairing gap approximation, and with the available experimental data.

Results obtained by RHB-theory with the original Gogny force D1S and with the separable force derived from it are basically identical. Therefore we can conclude that this simple pairing interaction can be applied in future applications of the RHBZ approach in nuclei far from stability instead of the complicated Gogny force.

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