Optimisation of regularisation methods for differentiation of measurement data in monitoring of human movements

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Abstract. The research reported in this paper is related to the regularised differentiation of measurement data from impulse-radar sensors and infrared depth sensors applied in systems for unobtrusive monitoring of elderly persons. Four strategies for selection of regularisation parameters are compared in terms of their potential of decreasing the uncertainty of walking velocity estimation. The comparison is based on synthetic and real-world data. The best results are obtained by means of a strategy based on the so-called Stein’s unbiased risk estimator.

1. Introduction

The number of people with functional deficiencies or ill health, including persons suffering from dementia, has been growing in Europe since the last decade of the XXth century; on the other hand, the recruitment of the healthcare personnel has not been increasing proportionally [1]. Hence the growing importance of research on new technologies that could be employed in monitoring systems supporting care services for elderly persons. Those systems are expected to detect and predict dangerous events, such as person’s falls [2]. It has been shown recently that the analysis of gait may contribute to the fall prevention [3].

Healthcare-oriented analysis of human gait may comprise the estimation of different gait-related parameters [4]. This paper is devoted to the most important among them – the monitored persons’ walking velocity. Even though many factors may affect the value of the walking velocity, healthcare practitioners consider it to be a good indicator of the general health; experimental results suggest that it is correlated with the risk of falling [5].

Several techniques for estimation of the walking velocity of elderly persons in their home environment, based on different types of sensors, have been proposed; some examples can be found in [6–8]. This paper is focused on systems for estimation of walking velocity, based on impulse-radar sensors and depth sensors, which provide data representative of the monitored person’s two-dimensional position trajectory [9]. The numerical differentiation of those data – necessary for the estimation of the walking velocity – is an ill-posed problem: small errors in the data may contribute to large errors corrupting the derivative estimate. Therefore, regularised differentiation methods should be used. The results of the authors’ previous studies indicate that a method based on the Tikhonov regularisation may yield sufficiently accurate estimates of velocity. However, the performance of this method depends strongly on the value of a regularisation parameter which can be selected according to different strategies (see, e.g., [10] for their exhaustive list). In the experiments described here, four such strategies are compared empirically in terms of their applicability in the above-mentioned monitoring systems.
2. Formulation of research problem
In the study to be reported here, the spatial dimensions have been treated independently; so – for the sake of simplicity – only one-dimensional data are considered in the following mathematical formulæ.

The estimates of the monitored person’s velocity \( \hat{x}^{(i)} \) of order 3; \( \hat{x} \equiv [\hat{x}_0 ... \hat{x}_N]^\top \) are the data representative of the monitored person’s position trajectory, corrupted with measurement errors whose standard deviation \( \sigma \) is assumed to be known \( \text{a priori} \); \( 1 \equiv [1 ... 1]^\top \). Four strategies for the selection of \( \alpha \) are described in the following section. The methodology and results of their comparison are presented in sections 4 and 5.

3. Compared strategies for parameter selection
The strategy, labelled here with the acronym DP, is based on the discrepancy principle which states that “the quality of the results of a computation must be comparable to the quality of the input data” [12]; it has been implemented by computing the parameter value in the following way:

\[
\hat{\alpha}_{\text{DP}} = \arg_{\alpha} \min \left\{ \frac{1}{N-1} \left\| \mathbf{A} \hat{x}^{(i)}(\alpha) - (\hat{x} - \hat{x}_0) \right\|^2 - \sigma^2 \right\}
\]

(2)

The strategy, labelled here with the acronym RR, is based on the so-called Regińska’s parameter choice rule, and consists in computing the parameter value in the following way [13]:

\[
\hat{\alpha}_{\text{RR}} = \arg_{\alpha} \min \left\{ \left\| \mathbf{A} \hat{x}^{(i)}(\alpha) - (\hat{x} - \hat{x}_0) \right\| \left\| \hat{x}^{(i)}(\alpha) \right\|^2 \right\}
\]

(3)

The strategy, labelled here with the acronym GCV, is based on the generalised cross-validation procedure and consists in computing the parameter value in the following way [14]:

\[
\hat{\alpha}_{\text{GCV}} = \arg_{\alpha} \min \left\{ \left\| \left( I - \mathbf{A} \left( \mathbf{A}^\top \mathbf{A} + \alpha \mathbf{D}^\top \mathbf{D} \right)^{-1} \mathbf{A}^\top \right) \hat{x}^{(i)}(\alpha) \right\|^2 \left\| \left( \mathbf{I} - \mathbf{A} \left( \mathbf{A}^\top \mathbf{A} + \alpha \mathbf{D}^\top \mathbf{D} \right)^{-1} \mathbf{A}^\top \right) \right\|^2 \right\}
\]

(4)

The strategy, labelled here with the acronym SURE, consists in selecting the parameter value which minimises the so-called Stein’s unbiased risk estimator, defined in the following way [15]:

\[
\hat{\alpha}_{\text{SURE}} = \arg_{\alpha} \min \frac{1}{N} \left\| \mathbf{A} \hat{x}^{(i)}(\alpha) - (\hat{x} - \hat{x}_0) \right\|^2 - \sigma^2 + \frac{2\sigma^2}{N} \text{tr} \left\{ \mathbf{A} \left( \mathbf{A}^\top \mathbf{A} + \alpha \mathbf{D}^\top \mathbf{D} \right)^{-1} \mathbf{A}^\top \right\}
\]

(5)

4. Methodology of experimentation

4.1. Experiments based on synthetic data
The synthetic data, used for experimentation, have been obtained using a function defined in the following way:

\[
f(t) = \left[ \tanh(t - \frac{1}{2}) \right]^9 \quad \text{and} \quad f^{(i)}(t) = 9 \left[ \tanh(t - \frac{1}{2}) \right]^8 - 9 \left[ \tanh(t - \frac{1}{2}) \right]^{10} \quad \text{for} \quad t \in [0,5] \]

(6)

Those data have been disturbed additively:

\[
\hat{x}_n = f(t_n) + \Delta \hat{x}_n \quad \text{for} \quad n = 0, ..., N
\]

(7)

where \( t_n = 5n/N \), \( N = 100 \), and \( \Delta \hat{x}_n \) are pseudorandom numbers following a zero-mean normal distribution. The level of disturbances in the data has been characterised by the signal-to-noise ratio, defined as:

\[
SNR_\beta = 10 \log_{10} \left\{ \frac{\sum_{n=1}^{N} (f(t_n))^2}{\sum_{n=1}^{N} (\hat{x}_n - f(t_n))^2} \right\}^{-1}
\]

(8)
For 15 different values of $SNR_\alpha \in [15, 40]$, $R = 500$ data sets have been generated using different pseudorandom sequences $\{\Delta \tilde{x}_n\}$. For each data set, the values of $\alpha$ have been computed using the strategies described in section 3, and those values have been applied for estimation of the first derivative according to equation (1). The signal-to-noise ratio in the estimates $\hat{x}_n^{(1)}$ has been determined according to the formula:

$$SNR_\alpha (\alpha) = 10 \log_{10} \left\{ \left[ \sum_{n=1}^{N} \left( f^{(1)} \left( t_n \right) \right)^2 \right] \left[ \sum_{n=1}^{N} \left( \hat{x}_n^{(1)} \left( \alpha \right) - f^{(1)} \left( t_n \right) \right)^2 \right] \right\}^{-1} \right\} \quad (9)$$

For each data set, a reference value of $SNR_\alpha$, defined as $SNR_{\alpha_{\text{ref}}} = \sup \{ SNR_\alpha (\alpha) | \alpha \in [10^{-10}, 10^0] \}$, and a reference parameter value $\alpha_{\text{ref}}$, such that $SNR_\alpha (\alpha_{\text{ref}}) = SNR_{\alpha_{\text{ref}}}$, have been determined. The parameter selection strategies have been compared in terms of two criteria: the ratio $SNR_\alpha / SNR_\alpha$ and the discrepancy between the selected and reference parameter value, defined as $\delta \hat{\alpha} = \log \hat{\alpha} - \log \alpha_{\text{ref}}$.

4.2. Experiments based on real-world data

The real-world data, used for experimentation, have been acquired by means of a pair of impulse-radar sensors and a single depth sensor. Walking with the velocity of a predefined value (called reference value hereinafter) has been assured by making half-meter steps in equal time intervals signalled by a metronome set to an appropriate tempo. In the experiments, a person has walked $R = 40$ times along straight-line trajectories with six reference values of velocity $v_{\text{ref}} = 0.5, 0.6, \ldots, 1.0$ m/s. The estimated trajectories have been differentiated according to equation (1), using the values of $\alpha$ selected by means of the strategies described in section 3. The following indicators of the bias and the standard deviation of errors have been computed on the basis of the obtained sequences of estimates of velocity $\{\hat{v}_n\}$:

$$\bar{b} = \frac{1}{R} \sum_{r=1}^{R} b(r) \quad \text{with} \quad b(r) = \frac{1}{N+1} \sum_{n=0}^{N} \hat{v}_n(r) - v_{\text{ref}}$$

and

$$\bar{\sigma} = \left\{ \frac{1}{RN} \sum_{r=1}^{R} \sum_{n=0}^{N} \left[ \hat{v}_n(r) - v_{\text{ref}} - b(r) \right]^2 \right\}^{1/2} \quad (10)$$

The expanded uncertainty of velocity estimation, defined as $u = |\bar{b}| + 3\bar{\sigma}$, has been used as the criterion for comparing the strategies of $\alpha$ selection.

5. Results and conclusions

The results of experiments based on synthetic data are presented in figures 1 and 2, and of those based on real-world data – in figures 3 and 4.

The novelty of the research results, presented in this paper, consists in a systematic comparison of four strategies for selection of the regularisation parameter in a numerical differentiation method based on the Tikhonov regularisation – a comparison aimed at evaluation of their applicability for estimation of walking velocity in monitoring systems based on impulse-radar sensors or depth sensors. The results of the reported experiments indicate that the strategy based on the minimisation of the so-called Stein’s unbiased risk estimator can be recommended. Only in the case of synthetic data corrupted with large errors, better results have been obtained by means of other strategies.

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Figure 1. Dependence of $\text{SNR}/\text{SNR}_0$ on $\text{SNR}_0$.

Figure 2. Dependence of $\hat{a}$ on $\text{SNR}_0$.

Figure 3. Dependence of $u$ on $v_{\text{ref}}$ for velocity estimates obtained by means of impulse-radar sensors.

Figure 4. Dependence of $u$ on $v_{\text{ref}}$ for velocity estimates obtained by means of depth sensor.

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