Robust and optimal laser cooling of trapped ions

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We present a robust and fast laser cooling scheme suitable for trapped atoms and ions. Based on quantum interference, generated by a special laser configuration, it is able to rapidly cool the system such that the final phonon occupation vanishes to zeroth order in the Lamb-Dicke parameter in contrast to existing cooling schemes. Furthermore, it is robust under conditions of fluctuating laser intensity and frequency, thus making it a viable candidate for experimental applications.

Introduction — Laser cooling is a crucial ingredient for probing quantum properties of matter \(^1\), \(^2\), \(^3\). It is a key factor in a wide variety of experiments ranging from Bose-Einstein condensates and quantum computing to quantum simulation with atoms and ions. Variants of cooling schemes range from Doppler cooling for free particles \(^4\), \(^5\), \(^6\), and its partner, side band cooling for bound particles \(^7\), \(^8\), to dark state cooling schemes for free \(^9\), \(^10\) and bound particles \(^11\) relying on quantum interference which arises thanks to their non-trivial internal electronic structures.

At present, sideband cooling is the method of choice for trapped ions. It is a necessary requirement for efficient cooling that motional sidebands with frequency \(\nu\) can be resolved, i.e., the linewidth of the optical transition \(\gamma \ll \nu\). Cooling is then achieved by the red sideband transition which excites the atom while at the same time annihilating a phonon to ensure energy conservation. This transition rate must be higher than that on the carrier and blue sideband transitions that heat the system either through recoil after spontaneous decay (carrier) or coherent generation of a phonon (blue sideband). The selection of the red transition is ensured by the rotating wave approximation, i.e. energy conservation, as long as the Rabi frequency \(\Omega\) of the laser satisfies \(\Omega \ll \nu\). One method to suppress the carrier and blue sideband transitions employs destructive interference exhibited for example in dark states \(^8\). For this reason, Electromagnetically Induced Transparency (EIT) \(^3\) has become an inspiration for a variety of proposed laser cooling schemes for trapped ions such as EIT cooling \(^10\) and Stark-shift cooling \(^11\). In EIT cooling, interference eliminates the carrier transition to improve cooling performance while in the Stark-shift scheme this is achieved in a rotated basis in a suitable interaction picture \(^12\).

In EIT cooling \(^10\) the existence of a dark state of a three level system allows final temperatures below \(kT/k_B\). This is achieved in a three level scheme subject to Raman lasers with strong blue single-photon detuning that couple both the ground state and a meta-stable state to an excited dissipative state. Among the dressed states of the system there is one dark state that cancels the carrier transition. Well chosen parameters, can center the red sideband transition under a peak of the Fano-like absorption spectrum, while constraining the blue sideband to a region with negligible absorption, thus achieving low final temperatures. The final state of the system is then

\[
\rho^{(EIT)} = |dark\rangle\langle dark| \otimes \sum_n a_n |n\rangle\langle n| + o(\eta^2),
\]

where \(|dark\rangle\) is the internal level steady state and \(|n\rangle\) are the number states of the external degrees of freedom (dof), with a final mean phonon number of order \((\gamma/4|\Delta|)^2\) \(^10\).

In Stark-shift cooling \(^11\) on the other hand one laser drives transitions between the ground and a meta-stable state and another two resonant Raman lasers couple a superposition of both to the excited state. This first laser generates Rabi oscillations between the dark state and the orthogonal bright state. If the coupling is correctly tuned, the oscillations will also involve neighboring mechanical levels so that states \(|\text{dark}\rangle|n\rangle\) and \(|\text{bright}\rangle|n-1\rangle\) are coupled while carrier transitions are eliminated. As the EIT scheme, Stark-shift cooling achieves a final temperature that is in leading order independent of the Lamb-Dicke parameter \(\eta\).

It is now natural to ask whether these schemes can be combined to reach vanishing temperature in leading order in the Lamb-Dicke parameter. As this work demonstrates, this is indeed possible. We present here a novel cooling scheme, a judicious combination of EIT and Stark-shift cooling, which share the same dark state. This in turn allows for both the carrier and the blue sideband transitions, and hence heating, to be suppressed by interference. As a result the final temperature, to zeroth order in the Lamb-Dicke parameter, vanishes. To the best of our knowledge this is the first scheme that is able to achieve this for trapped particles.

That the combination of EIT and Stark-shift cooling is successful, may seem surprising. A classical analysis would lead one to expect that the efficiency of the combined scheme to be the arithmetical average of the individual ones. It is the quantum nature of the interaction however that makes them interfere constructively in such a way as to outperform its constituent schemes. The quantum interference can be designed to completely cancel the two leading heating processes in the system.
The quantum theory of laser cooling of trapped particles was developed in [13, 14, 15] and we will follow the notation in [15]. Expanding the Hamiltonian up to first order in the Lamb-Dicke parameter and the interaction of the laser beams with the ion due to the Stark-shift part:

$$H_{SSH} = \Omega_B \sigma_x^{g_1,g_2} \eta_A + \eta_B \Omega_B \sigma_y^{g_1,g_2} (b + b^\dagger) \quad (5)$$

with $\sigma_x^{g_1,g_2} = (|m\rangle\langle n| + h.c.)$ and $\sigma_y^{g_1,g_2} = (i|m\rangle\langle n| + h.c.)$.

The physics of the scheme can best be understood by extending the analysis of the steady state Eq. (3) to the next order in the Lamb-Dicke parameter. By inspection of the Hamiltonian one can consider the following non-normalized pure state:

$$|\Psi\rangle = |g_1 - g_2\rangle |0\rangle - i\eta_B |g_1 + g_2\rangle |1\rangle + o(\eta^2) \quad (6)$$

Under the effect of the EIT couplings (Eq. 4) both terms of the superposition interfere destructively and as a result $|\Psi\rangle$ remains invariant. However $|\Psi\rangle$ is not invariant under the free Hamiltonian (Eq. 3) which introduces a phase shift between the two components. For a suitable tuned parameters, the Stark shift coupling part (Eq. 4) can cancel the effect of the free Hamiltonian and as a consequence $|\Psi\rangle$ will be a dark state of the Liouvillian as it does not suffer any spontaneous emission losses either. This is achieved when

$$\frac{\eta_B}{\eta_A} = \left( \frac{\nu}{\Omega_B} + 2 \right). \quad (7)$$

From an experimental point of view, it is worth noting that this resonance condition is characterized by the quotient of the Lamb-Dicke parameters. These can be set up at the beginning of the experiment to a high precision.

Losses from the excited level are incorporated in the master equation:

$$\frac{d\rho}{dt} = -i\hbar [H_0 + H_{EIT} + H_{SSH}, \rho] + \mathcal{L}^d \rho \quad (8)$$

where $\mathcal{L}^d$ contains the spontaneous emission of the excited level:

$$\mathcal{L}^d = \sum_{i=g_1,g_2} \gamma_{e,i} 2\sigma_{i,e,\rho} \sigma_{e,i} - \rho \sigma_{e,e} - \sigma_{e,e} \rho \quad (9)$$

where $\sigma_{j,k} = |j\rangle \langle k|$ and $\mathcal{L} = \frac{1}{\hbar} \int_0^1 dx e^{ikx} \rho e^{-ikx}$.

After expansion of the rest of the terms up to second order in the Lamb-Dicke parameter and adiabatic elimination of the internal dof we find the rate equation:

$$\frac{d\rho_{n,n}^{ext}}{dt} = ((n + 1)(A_{-\rho_{ext}^{n+1,n+1}} - A_{+\rho_{ext}^{n,n}}) + (n + 1)(A_{+\rho_{ext}^{n,n-1}} - A_{-\rho_{ext}^{n,n}}) \quad (10)$$

In the spirit of [15] the rates $A_{\pm}$ can be expressed

$$A_{\pm} = 2 Re [D + S(\mp \nu)] \quad (11)$$

and

where $D$ is the diffusion coefficient due to spontaneous emission from the excited atomic states. Here $D = 0$ as
the population of the excited states vanishes due to the dark state nature of the final state. $S(\nu)$ is the fluctuation spectrum of Heisenberg operator $F(t)$:

$$S(\nu) = \frac{1}{2M\nu} \int_0^\infty dt e^{i\nu t} \langle F(t)F(0) \rangle$$  \hspace{1cm} (12)

where $F = F_{EIT} + F_{SSH}$ and $F_{EIT} = \sigma^{g_1}_y - \sigma^{g_2}_y$ and $F_{SSH} = \eta_B \Omega_B \sigma^{g_1}_y \sigma^{g_2}_y$ are the part in the interaction Hamiltonian Eq. 4 and Eq. 5 that multiply $b + b^\dagger$. The average can be calculated using the quantum regression theorem. We may decompose the operator $F$ into components of EIT and Stark-shift so that the overall heating rate can then be split into three parts: the EIT part $A^{EIT}_+$, the Stark-shift part $A^{SSH}_+$ and interaction between EIT and the Stark-shift part $A^{int}_+$ for the remaining cases. Their results are: $A^{EIT}_+ = (\eta_A (\nu + 2\Omega_B))^2/D$, $A^{SSH}_+ = (\eta_B \Omega_B)^2/D$ and $A^{int}_+ = -2\eta_A\eta_B(\nu + 2\Omega_B)\Omega_B/D$ where $\frac{1}{D} = \frac{2\nu^2}{\Gamma^2[(\nu + 2\Omega_A \Omega_B)(\nu + \Omega_B)]}$.

Then

$$A_+ = \frac{\eta_A(\nu + 2\Omega_B)}{D} - \eta_B \Omega_B^2 \frac{2\nu^2}{\Gamma^2[(\nu + 2\Omega_A \Omega_B)(\nu + \Omega_B)]}$$  \hspace{1cm} (13)

which will be identically zero for the condition in Eq. 7. This emphasizes the role of the interaction between the EIT and Stark-shift cooling in order to assure the ground state is populated with probability 1. The mean occupation number $n$ is given by $A_+ - A_-$, thus yielding $\langle n \rangle = 0$. Fig. 2 shows the numerical results of the approach to the $\langle n \rangle = 0$ point as the Lamb-Dicke parameter goes to zero.

**Robustness** — The constructive interference between EIT and Stark-shift contribution is also crucial for understanding the robustness of the scheme under fluctuating parameters. If the Rabi-frequencies deviate from Eq. 7, the final populations as

$$\langle n \rangle \propto (\Delta \Omega_A)^4 (\Delta \Omega_B)^2$$  \hspace{1cm} (14)

instead of second order as is usually the case. As is exemplified in fig. 3 under a given value of the fluctuations of the laser intensities, the final mean occupation decreases abruptly as one moves away of the Stark-shift only or EIT only regimes. This guarantees promising performance under real experimental conditions, overcoming the main drawback of the previous dark-state cooling schemes.

**Rate** — The interference structure of this scheme is also essential for the high cooling rate $W = A_- - A_+$. For the resonance condition eq. 7 we find

$$W = \frac{8\Gamma \eta_B^2 \nu^2 \Omega_A^2}{(2\Omega_A^2 + (\nu - 2\Omega_B)(\Delta - \nu + \Omega_B))^2 + \Gamma^2(\nu - 2\Omega_B)^2}$$  \hspace{1cm} (15)

$\Omega_B$ is the only Rabi frequency involved in the condition Eq. 7. As a function of $\Omega_B$, Eq. 15 takes the approximate shape of a squared Lorentzian, with a peak close to $\Omega_B = \nu/2$ at which point the cooling rate expression reduces to

$$W = \frac{\Gamma \eta_B^2 \nu^2}{8\Omega_A^2}.$$  \hspace{1cm} (16)

This is also an optimal point for Stark-shift cooling but it should be noted that the cooling rate of Robust cooling is slightly higher than that of Stark-shift cooling. Since the internal dynamics should be much faster than the external one for the perturbation approach to work the analytic result is not valid for $\Omega_A < \eta_A \nu$, which means that the cooling can be as fast as 1 order of magnitude less than the trap frequency. Since the final state (eq. 6) is a pure state up to second order in the Lamb-Dicke parameter there is a simple unitary rotation such that the final state has vanishing number of phonons to fourth order in the Lamb-Dicke parameter.

**Implementation** — The way the effective $\Omega_B$ and $\eta_B$ couplings can be physically implemented is not unique.
One way is to use two lasers to create the EIT cooling part and to use magnetic gradients [16] for the Stark-shift part. In this system the magnetic gradients create a coupling of the following type: \( \lambda \sigma_z (b + b^\dagger) \), where \( \lambda \) is proportional to the magnetic gradients and the TLS is driven using a microwave: \( \Omega_d \sigma_x \cos \omega_d t \), where \( \Omega_d \) corresponds to the Rabi frequency of the driving and \( \omega_d \) to the driving frequency. After a polaron transformation the resulting Hamiltonian is exactly as in Eq. 5 when the Rabi frequency is replaced by \( \Omega_d \) and the Lamb Dicke parameter is replaced by \( \frac{\Delta}{\nu} \).

This scheme can be especially useful to cool nano scale resonators, by using the setup described in [17]. In this setup an NV center is coupled to a diamond cantilever, the coupling is achieved by magnetic gradients resulting in the same Hamiltonian as described above. In cantilevers the speed of cooling is very important due to the finite \( Q \) value, which is an important factor limiting the achievable final temperatures at present. The high cooling rate achieved by the described scheme will result in lower final temperatures bringing us closer to the goal of achieving the quantum regime in cantilever systems.

Alternatively, one can also use Raman beams with large single-photon detuning to couple levels \( |g_1\rangle \) and \( |g_2\rangle \). By adiabatically eliminating the upper level the relationships between our effective parameters \( \Omega_B \) and \( \eta_B \) and the physical values \( \Omega_p \) and \( \eta_p \) are found to be \( \Omega_B = \Omega_p^2 \left[ \frac{1}{2} - \frac{\Delta^2 - \nu^2}{2 \Delta^2} (2n + 1) \right] \) and \( \Omega_B \eta_B = \Omega_p^2 \eta_p \frac{2 \Delta^2 - \nu^2}{\Delta^2} \). Neglecting the \( \eta^2 \) correction in the first expression we find \( \eta_B = \eta_p \frac{2 \Delta^2 - \nu^2}{\Delta^2} \) which yields \( \Delta = 2 \Delta_p \) for sufficiently large detunings.

If we chose the optimal point for the cooling rate and fluctuations \( \Omega_B = \frac{\Delta}{\nu} \), the condition becomes \( \eta_p/\Delta = 4 \). This can be achieved for a layout where beam B is collinear to the trap axis and the beam A is 60° away from the axis. Even if the optimal situation is for an angle separation of 60°, the robustness of the scheme ensures excellent performance for any geometrical configuration. Proposed experimental layouts [20] use vacuum chambers with windows at 22.5° and/or 45° from the trap axis, which generally allow for an angle range of about ±10°. Taking 45° as an operating value, the Lamb-Dicke parameter quotient becomes \( \eta_B/\Delta = 2 \sqrt{2} \). Different optimal set of parameters can be obtained for this situation depending on whether the cooling rate or the final temperature want to be optimized. For the latter, condition Eq. 6 has to be observed, \( \Delta = 0.6 \nu \) and \( \Delta \approx 0 \). This will assure an extremely stable cooling rate which is generally 2 orders of magnitude smaller than EIT cooling but still better than sideband cooling. This final result can be improved depending on the particular values of the transition linewidth \( \Gamma \). On the contrary, if the cooling rate is to be enhanced, condition Eq. 6 won't be satisfied. In particular, for \( \Delta = 0.4 \nu \), \( \Omega_B = 0.45 \nu \) with 5% fluctuation and \( \Delta \approx -2 \nu \) the population can still be as low as \( 10^{-3} \) while having a cooling rate several orders of magnitude over that of EIT cooling. Taking into account the fact that angles up to 55° are accessible the cooling rate can still be improved by up to two orders of magnitude.

**Multi mode cooling** — Finally, the cooling scheme has also been tested for an ion chain using Monte Carlo simulation [18, 19]. The robustness of the scheme implies a wide range of operational Rabi frequencies or, in a different perspective, a wide range of trap frequencies for a given Rabi frequency. Thus, a particular central mode frequency can be addressed so that also the neighboring modes benefit from the cooling. In a multi-mode environment with up to 3 ions promising results have been obtained and a detailed study will be presented elsewhere.

**Conclusion** — To conclude, we have introduced a cooling scheme that cools to zero temperature without any corrections in zeroth order in the Lamb Dicke parameter. Beyond the academic interest of proving the existence of such a scheme, its robustness makes it extremely attractive for experimental realizations.

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