Scalar and Pseudo-Scalar Higgs Production at Hadron Colliders*

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The evaluation of the NNLO QCD corrections to the production of a scalar and a pseudo-scalar Higgs boson is described.

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1. Introduction and Motivation

The experimental discovery of a Higgs boson could be one of the next big milestones in particle physics. If a Higgs boson exists, it will most certainly be found at the Large Hadron Collider (LHC), or even before, at the Tevatron.

The Higgs sector is the least explored part of the Standard Model. In particular, it is unclear if really the minimal model with a single Higgs doublet is realized in Nature. Extended models, like the Minimal Supersymmetric Standard Model (MSSM), predict a larger variety of Higgs bosons which differ among each other for example by their mass, charge, CP-parity, and couplings.

Hints towards physics beyond the Standard Model could be obtained from measuring these properties of a Higgs boson, once discovered. In this respect, it is essential to understand the theoretical values of these quantities in the framework of the Standard Model as precisely as possible.

A clear understanding of the Higgs properties is often based on the precise knowledge of the production rates. One can distinguish four main production mechanisms at hadron colliders: Gluon fusion, associated production with weak bosons, weak boson fusion, and associated production of the Higgs boson with a top–anti-top pair. The theoretical progress in each one of these modes has been enormous over the past few years, both for the signal and the background processes (see, e.g., Ref. [1] for a review and the corresponding references). In the following we will only focus on the signal cross section in the gluon fusion channel.

2. Gluon fusion

The gluon fusion mode has been shown to be the dominant production mechanism for Higgs bosons at hadron colliders a long time ago [2]. The coupling of the Higgs boson to gluons is mediated by a top quark triangle, so that the leading order process is described by the one-loop diagram shown in Fig. 1.

The fact that the LO process already requires a one-loop calculation makes the evaluation of higher orders even more challenging. It turns out, however, that there is an approximation to the problem that simplifies the task of evaluating higher orders significantly, without much loss in theoretical accuracy. The relevant limit is given by taking the top quark very heavy as compared to the Higgs boson. Keeping the full top mass dependence at LO and evaluating the higher orders in the heavy top limit leads to a very good approximation to the exact result. This was explicitly demonstrated at NLO [3], where the full top mass dependence is known.

In the heavy top limit, the original diagrams factorize into a massive component with vanishing external momenta, and a massless component with the physical momenta of the in- and outgoing particles. The massive component represents an “effective coupling constant” $C(\alpha_s)$ which multiplies the $ggH$ interaction vertex. $C(\alpha_s)$ can be evaluated perturbatively and is known up to N$^3$LO [4].
3. NNLO calculation

The sub-processes to be evaluated for the NNLO rate are

- $gg \rightarrow H$ up to two-loop level
- $gg \rightarrow Hg$, $qq \rightarrow Hq$, $q\bar{q} \rightarrow Hg$ up to one-loop level
- $gg \rightarrow Hgg$, $gg \rightarrow Hq\bar{q}$, $qg \rightarrow Hqg$, $q\bar{q} \rightarrow Hq\bar{q}$, $qq \rightarrow Hqq$ at tree level

All the one-loop processes can be evaluated analytically using known loop and phase space integrals. On the other hand, the evaluation of the two-loop virtual diagrams as well as the phase space integration of the double real emission contributions required new techniques, even though a very similar process, the Drell-Yan production of lepton pairs, had been evaluated up to NNLO about ten years before [5].

The initial impulse was provided in a paper by Baikov and Smirnov [6].\(^1\) It contains a recipe to evaluate the three-point functions at two loops in complete analogy to massless three-loop propagator diagrams. The evaluation of the latter is standard in the field of multi-loop calculations. It can be done with the help of the FORM program MINCER [7]. Using Ref. [6], one can evaluate general massless two-loop vertex diagrams with two massless legs in a similar fashion. The corresponding integration routine can easily be connected to the diagram generator QGRAF [8] in the framework of the package GEFICOM [9] (see also Ref. [10]), so that the evaluation of the NNLO virtual contributions to $gg \rightarrow H$ is done fully automatically [11].

Let us now describe the evaluation of the phase space integrals for the emission of two gluons or quarks. In a first step, they were evaluated in the soft limit, i.e., when the momenta of the final state gluons (or quarks) tend to zero. This limit suffices to cancel the infra-red divergences of the virtual and soft single-emission contributions, so that after mass factorization one obtains a finite result. The result was then combined with the previously known collinear terms

\[ \propto \ln^3(1 - x) \] \(^{12}\) \((x \equiv M_H^2/s)\) to arrive at the so-called “soft+sl” \(^{13}\) or “SVC” \(^{14}\) approximation. Numerical differences between the final results of Ref. [13] and [14] could be attributed partly to the different treatment of formally subleading terms $\propto \ln^n(1 - x)$, $n = 0, 1, 2$. The remaining difference was due to the different sets of parton distribution functions: No published set of NNLO parton distributions was available at that time, so that Ref. [13] used NLO sets for the NNLO curves, while Ref. [14] used unpublished NNLO sets.

The soft limit was used as a starting point for the method to evaluate the complete NNLO result [15]. This means that a systematic expansion of the partonic cross section around the soft limit $x \rightarrow 1$ was constructed, where $s$ is the partonic c.m.s. energy. This leads to a series in $(1 - x)^n$, whose coefficients depend logarithmically on $(1 - x)$, up to $\ln^3(1 - x)$ at NNLO. The series was evaluated analytically up to $n = 16$ [15]. The resulting hadronic cross section was obtained from this by convolution with the proper parton distributions. For $n > 5$, its prediction is almost independent on $n$, indicating that small values of $x$ have no significant influence on the final result. The physical hadronic rate is thus perfectly described in this approach.

The series in $(1 - x)$ as described above, taken up to $n = \infty$, is nothing but a representation of a sum of tri-, di-, and simple logarithms,\(^2\) multiplied by powers of of $x$, $1/x$, and $1/(1 - x)$, just like, e.g.,

\[ \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{(1 - x)^n}{n} \left( \frac{1}{n} - \ln(1 - x) \right) \] \(^{1}\) can be viewed as a representation of $Li_2(x)$. Thus, making an ansatz for the resummed result with unknown coefficients for these functions and expanding it in terms of $(1 - x)$, one can actually determine the coefficients from the truncated series, by solving a system of linear equations. This requires the knowledge of a sufficient number of terms in the expansion, of the order of 100 in our case. The required efforts for the evaluation and manipulation of the intermediate expressions

\(^{1}\)R.H. is indebted to A. Czarnecki for initially pointing out the relevance of this paper.

\(^{2}\)This can be deduced from the known NNLO result for the Drell-Yan process [5,15].
are quite remarkable. The resummation has been achieved in Ref. [16], and the result is identical to the expression of Ref. [17], obtained through a completely different method (see also Ref. [18]). For the final numerical results presented below, however, it is irrelevant if we use the truncated series of Ref. [15], or the closed form of Ref. [17].

In order to arrive at a consistent NNLO result, it is not sufficient to evaluate the partonic cross section up to NNLO. One also needs to account for the parton evolution up to the same order. So far, the exact evolution kernels are not known. Thus, until the exact NNLO evolution becomes available, we use the approximate NNLO parton set of Ref. [19] which is based on moments of the structure functions [20].

4. Results

Let us now present the results for the production of a Standard Model Higgs boson at the Tevatron and the LHC [15,17]. Fig. 2 shows the total cross section for Higgs production at (a) the Tevatron, and (b) the LHC. One observes a nicely converging perturbative series, together with a clear reduction of the scale dependence in both cases. Soft gluon resummation increases the NNLO curves by about 10%, confirming the stability of the perturbative result [21].

To investigate the scale dependence in more detail, we show the variation of the cross section with respect to the renormalization and the factorization scale $\mu_R$ and $\mu_F$ for a fixed Higgs mass of $M_H = 115$ GeV at the Tevatron and the LHC in Fig. 3 and 4, respectively. In the left panel of each figure, $\mu_R$ and $\mu_F$ are identified and varied simultaneously within the rather conservative range of $M_H/4 < \mu < 4M_H$. In the center panels, the renormalization scale $\mu_R$ is identified with $M_H$, while $\mu_F$ is varied, and in the right panels, $\mu_F$ is fixed, and $\mu_R$ is varied. Using the variation between $M_H/2$ and $2M_H$ as an indication of the theoretical uncertainty, one arrives at the conclusion that the cross section at the Tevatron is known to about $\pm 15\%$, at the LHC to about $\pm 10\%$.

5. Pseudo-scalar Higgs production

The method described in Sect. 3 can also be applied to evaluate the production rate of a pseudo-scalar Higgs boson, as it is predicted in extended models like the MSSM. One should keep in mind, however, that the couplings of the Higgs bosons may be altered in such theories. In the MSSM, for example, the coupling to bottom quarks might be enhanced by large values of $\tan \beta$. In this case, bottom loop contributions cannot be neglected even at higher orders in QCD.

In Ref. [22], the top loop contribution to the total rate for pseudo-scalar Higgs production was evaluated up to NNLO QCD. The corrections were found to be very similar to the scalar Higgs case, while the leading order result determines the different overall normalization. In consequence, within the limit where bottom quark contributions can be neglected, the production rate for pseudo-scalar Higgs bosons is known to the same level of accuracy as for scalar Higgs bosons.

Conclusions. The production rate for scalar and pseudo-scalar Higgs bosons has been shown to be described by a well-behaved perturbative series up to NNLO. The newly developed calculational methods of Ref. [15] and Ref. [17] should prove useful also in many other applications.

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Figure 2. Total cross section for Higgs production in gluon fusion at (a) Tevatron Run II, and (b) the LHC at leading (dotted), next-to-leading (dashed), and next-to-next-to leading order (solid). The upper (lower) curve of each pair corresponds to a choice of the renormalization and factorization scale of $\mu_R = \mu_F = M_H/2 \ (\mu_R = \mu_F = 2 M_H)$.

Figure 3. Scale dependence of the cross section for $M_H = 115$ GeV at the Tevatron Run II ($\sqrt{s} = 2$ TeV). Left: variation of $\mu \equiv \mu_R = \mu_F$; center: variation of $\mu_F$ with $\mu_R = M_H$; right: variation of $\mu_R$ with $\mu_F = M_H$. 
Figure 4. Same as Fig. 3, but for the LHC.

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