Charmed nuclei within a microscopic many-body approach

I. Vidaña$^1$, A. Ramos$^2$ and C. E. Jiménez-Tejero$^3$

$^1$Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Dipartimento di Fisica “Ettore Majorana”, Università di Catania, Via Santa Sofia 64, I-95123 Catania, Italia
$^2$Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos (ICCUB), Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain and
$^3$Barcelona Center for Subsurface Imaging, Institute of Marine Sciences, Spanish Research Council, Passeig Marítim de la Barceloneta 37, E-08003 Barcelona, Spain

Single-particle energies of the $\Lambda_c$ charm baryon are obtained in several nuclei from the relevant self-energy constructed within the framework of a perturbative many-body approach. Results are presented for a charm baryon-nucleon ($Y_cN$) potential based on a SU(4) extension of the meson-exchange hyperon-nucleon potential $\tilde{A}$ of the Jülich group. Three different models (A, B and C) of this interaction, that differ only on the values of the couplings of the scalar $\sigma$ meson with the charmed baryons, are considered. Phase shifts, scattering lengths and effective ranges are computed for the three models and compared with those predicted by the $Y_cN$ interaction derived in Eur. Phys. A 54, 199 (2018) from the extrapolation to the physical pion mass of recent results of the HAL QCD Collaboration. Qualitative agreement is found for two of the models (B and C) considered. Our results for $\Lambda_c$-nuclei are compatible with those obtained by other authors based on different models and methods. We find a small spin-orbit splitting of the $p-, d-$ and $f-$wave states as in the case of single $\Lambda$-hypernuclei. The level spacing of $\Lambda_c$ single-particle energies is found to be smaller than that of the corresponding one for hypernuclei. The role of the Coulomb potential and the effect of the coupling of the $\Lambda_c$ and $\Sigma_c$ channels on the single-particle properties of $\Lambda_c$-nuclei are also analyzed. Our results show that, despite the Coulomb repulsion between the $\Lambda_c$ and the protons, even the less attractive one of our $Y_cN$ models (model C) is able to bind the $\Lambda_c$ in all the nuclei considered. The effect of the $\Lambda_cN - \Sigma_cN$ coupling is found to be almost negligible due to the large mass difference of the $\Lambda_c$ and $\Sigma_c$ baryons.

I. INTRODUCTION

Soon after the discovery of charmed hadrons [1–6], the possible existence of charmed nuclei (bound systems composed of nucleons and charmed baryons) was proposed in analogy with hypernuclei (see e.g., Refs. [7–11]). This possibility motivated several authors to study the properties of these systems within different theoretical approaches, predicting a rich spectrum and a wide range of atomic numbers [12–18]. Production mechanisms of charmed nuclei by means of charm exchange or associate charm production reactions, analogous to the ones widely used in hypernuclear physics, were also proposed [12–20]. However, the experimental production of charmed nuclei is difficult and, up to now, only threeambiguous candidates have been reported by an emulsion experiment carried out in Dubna in the mid-1970s [21, 22]. Experimental difficulties arise mainly from (i) the kinematics of the production reactions: charmed particles are formed with large momentum making their capture by a target-nucleus improbable; and (ii) the short lifetimes of $D$-meson beams, which makes necessary to place the target as close as possible to the $D$-meson production point. Such difficulties will be hopefully overcome at the future GSI-FAIR (Gesellschaft für Schwerionenforschung–Facility for Antiproton and Ion Research) and J-PARC (Japan Proton Accelerator Research Complex) facilities [20, 27]. The production of charmed particles in these facilities will be sufficiently large to make the study of charmed nuclei possible. Studies of $\bar{p}$ reactions in nuclei at the conditions of the PANDA experiment predict forward differential cross sections for the formation of $\Lambda_c$ hypernuclei in the range of a few mb/sr [28]. These future prospects have injected a renewed interest in this line of research [29]. In the last few years, theoretical estimations of the charmed baryons properties in nuclear matter and finite nuclei have been revisited using the quark-meson coupling model [30–33], a relativistic mean field approach [34], effective Lagrangians satisfying the heavy quark, chiral and hidden local symmetries [35], the quark cluster model [36], or a single-folding potential employing a Lattice QCD (LQCD) simulation of the $\Lambda_cN$ interaction [37]. An extrapolation to the physical pion mass of the former LQCD $\Lambda_cN$ interaction has recently become available [38].

In this work we study the single-particle properties of the $\Lambda_c$ charm baryon in several nuclei using a microscopic many-body approach. Our starting point is a nuclear matter $G$-matrix derived from a bare charmed baryon-nucleon ($Y_cN, Y_c = \Lambda_c, \Sigma_c$) potential based on a SU(4) extension of the hyperon-nucleon ($Y'N$) potential $\tilde{A}$ of the Jülich group [39]. This $G$-matrix is used to calculate the self-energy of the $\Lambda_c$ in the finite nucleus including corrections up to the second order. Solving the Schrödinger equation with this self-energy we are able to determine the single-particle energies and the wave function of the bound $\Lambda_c$. Our approach also provides the real and imaginary parts of the $\Lambda_c$ optical potential at positive energies, and therefore, allows one to study the $\Lambda_c$-nucleus scattering properties. This method was al-
II. THE YcN INTERACTION

Our model for the YcN interaction is based on a generalization of the meson exchange YN potential \( A \) of the J"ulich group \( [39] \). In analogy with that model, we describe the three different channels, \( \Lambda cN \to \Lambda cN \), \( \Sigma cN \to \Sigma cN \) and \( \Lambda cN \leftrightarrow \Sigma cN \), only as the sum of single scalar, pseudoscalar and vector meson exchange potentials shown in Fig. I. As in the YN J"ulich potential, the exchange of the effective scalar \( \sigma \) meson parametrizes the contribution of correlated \( 2\pi \)-exchange. The basic input of our model are the baryon-baryon-pseudoscalar (BBP) and the baryon-baryon-vector (BBV) vertices described, respectively, by the Lagrangian densities

\[
\mathcal{L}_{BBP} = g_{NN\sigma}(N^+ \bar{p}N) \cdot \bar{\pi} \\
+ g_{\Lambda c, \Sigma c, \pi}[\bar{\Sigma}^+_c \cdot \bar{\pi} \Lambda c + \Lambda^+_c \bar{\Sigma}^-_c \cdot \bar{\pi}] \\
- i g_{\Sigma c, \pi}[\bar{\Sigma}^+_c \bar{\Sigma}^-_c \cdot \bar{\pi}] \\
+ g_{N\Lambda c, D}[N^+ D \Lambda c + \Lambda^+_c (D^+ N)] \\
+ g_{N\Sigma c, D}[N^+ \bar{\Sigma}^-_c + \Sigma^+_c (D^+ \bar{\pi} N)]
\]

and

\[
\mathcal{L}_{BBV} = g_{NN\rho}(N^+ \bar{\rho}N) \cdot \bar{\rho} \\
+ g_{NN\rho, \rho}[\bar{\Sigma}^+_c \cdot \bar{\rho} \Lambda c + \Lambda^+_c \bar{\Sigma}^-_c \cdot \bar{\rho}] \\
- i g_{NN\rho, \rho}[\bar{\Sigma}^+_c \bar{\Sigma}^-_c \cdot \bar{\rho}] \\
+ g_{N\Lambda c, D^*}[N^+ D^* \Lambda c + \Lambda^+_c (D^{*+} N)] \\
+ g_{N\Sigma c, D^*}[N^+ \bar{\Sigma}^-_c + \Sigma^+_c (D^{*+} \bar{\pi} N)] \\
+ g_{NN\Lambda c, \omega} N^+ \Lambda c \omega \\
+ g_{NN\Sigma c, \omega} \Lambda^+_c \bar{\Sigma}^-_c \omega \\
+ g_{\Sigma c, \omega} \Sigma^+_c \Sigma^-_c \omega.
\]

We note that the isospin structure of these vertices is the same as that of their analogous strange ones. Similarly to the J"ulich YN interaction, which is itself based on the Bonn NN one, the YcN model presented here also neglects the contribution of the \( \eta \) and \( \eta' \) mesons.

We use the SU(4) symmetry to derive the relations between the different coupling constants. Note, however, that this symmetry is strongly broken due to the use of the different physical masses of the various baryons and mesons, and that we employ it rather as a mathematical tool to get a handle on the various couplings of our model.

In particular, we are dealing with \( J^P = \frac{1}{2}^+ \) baryons and \( J^P = 0^- \) mesons belonging to \( 20' \)- and \( 15' \)-plet irreducible representations of SU(4), respectively. Since the baryon current can be reduced according to

\[
20' \otimes 20' = 1 \oplus 15_1 \oplus 15_2 \oplus 20'' \oplus 45 \oplus 45 \oplus 84 \oplus 175 ,
\]

there are two ways to obtain an SU(4)-scalar for the coupling \( 20' \otimes 20' \). Because the baryon current contains two distinct 15'-plet representations, \( 15_1 \) and \( 15_2 \). They couple to the meson 15'-plets with strength \( g_{15_1} \) and \( g_{15_2} \), respectively. It is quite straightforward to relate these two couplings to the couplings \( g_\rho \) and \( g_F \) of the usual symmetric ("D-coupling") and antisymmetric ("F-coupling") octet representations of the baryon current in SU(3). They read

\[
g_{15_1} = \frac{1}{4}(7g_D + \sqrt{3}g_F) = \sqrt{\frac{10}{3}}g_8(7 - 4\alpha) \\
g_{15_2} = \frac{\sqrt{3}}{20}(5g_D - 5g_F) = 4\sqrt{10}g_8(1 - 4\alpha)
\]

where in the last step we have written \( g_D \) and \( g_F \) in terms of the conventional SU(3) octet strenght coupling \( g_8 \), and the so-called \( F/(F + D) \) ratio \( \alpha \)

\[
g_D = \frac{40}{\sqrt{30}}g_8(1 - \alpha) , \quad g_F = 4\sqrt{6}g_8\alpha.
\]

Let us consider first the coupling of the baryon current to the pseudoscalar mesons. The relations between all the relevant BBP coupling constants can be easily obtained...
by using SU(4) Clebsch–Gordan coefficients [46] and the above relations. They read

\[
\begin{align*}
\frac{g_{\Lambda_{c} \Sigma_{c} \pi}}{\sqrt{3}} &= \frac{2}{\sqrt{3}} g_{NN\pi}(1 - \alpha_{p}) \\
\frac{g_{\Sigma_{c} \Sigma_{c} \pi}}{\sqrt{3}} &= \frac{2}{\sqrt{3}} g_{NN\pi}(1 + \alpha_{p}) \\
\frac{g_{NN, D}}{\sqrt{3}} &= -\frac{1}{\sqrt{3}} g_{NN\pi}(1 + 2\alpha_{p}) \\
\frac{g_{NN, D}}{\sqrt{3}} &= \frac{g_{NN\pi}(1 - 2\alpha_{p})}{\sqrt{3}},
\end{align*}
\]

where we have added the subindex \( p \) to the ratio \( \alpha \) to specify that this is the ratio for the coupling of baryons with the pseudoscalar mesons and distinguish it from that for the vector ones used below.

Similarly, the corresponding relations for the BBV couplings can be obtained by simply making the replacements \( \pi \rightarrow \rho, D \rightarrow D^*, \alpha_{p} \rightarrow \alpha_{v} \) in the above expressions. In addition, the couplings to the \( \omega \) meson are

\[
\begin{align*}
g_{NN\omega} &= g_{NN\rho}(4\alpha_{v} - 1) \\
g_{\Lambda_{c} \Lambda_{c} \omega} &= \frac{g_{NN\rho}(6\alpha_{v} + 3)}{9} \\
g_{\Sigma_{c} \Sigma_{c} \omega} &= g_{NN\rho}(2\alpha_{v} - 1),
\end{align*}
\]

where we have assumed that the physical \( \omega \) meson results from the ideal mixing of the mathematical members of the 15-plet \( \omega_{8} \) and \( \omega_{1} \).

The relations for the tensor coupling constants \( f_{BBM} \) can be obtained by applying the corresponding SU(4) relations to the “magnetic” coupling \( G_{BBM} = g_{BBM} + f_{BBM} \). Thus, in the above relations \( g_{c} \) has to be replaced simply by \( G_{c} \) and \( \alpha_{v} \) by \( \alpha_{t} \).

To determine the couplings of the scalar \( \sigma \) meson with the charmed baryons, we should remind that this meson is not a member of any SU(4) multiplet and, therefore, it is not possible to obtain these couplings by invoking the SU(4) symmetry as we did for the couplings with the pseudoscalar and vector mesons. This leaves us certain freedom to chose the values of the couplings \( g_{\Lambda_{c} \Lambda_{c} \sigma} \) and \( g_{\Sigma_{c} \Sigma_{c} \sigma} \). To explore the sensitivity of our results to these couplings, in this work we consider three different sets of values for them that, together with those for the pseudoscalar and vector meson couplings, define three models for the \( Y_{c}N \) interaction. From now on we will refer to these models simply as A, B and C. In model A the couplings of the \( \sigma \) meson with the charmed baryons are assumed to be equal to its couplings with the \( \Lambda \) and \( \Sigma \) hyperons, and their values are taken from the original \( Y_{c}N \) potential \( A \) of the Jülich group [39]. In models B and C these couplings are reduced by 15\% and 20\%, respectively, with respect to model A. The coupling \( g_{NN\sigma} \) have been taken, for the three models, equal to that of the Jülich \( A \) \( Y_{c}N \) potential.

Taking the values \( \alpha_{p} = 0.4 \), \( \alpha_{v} = 1 \) and \( \alpha_{t} = 0.4 \) employed in [39], we obtain the couplings reported in Table I where we also show the cutoff masses \( \Lambda_{BBM} \) of the monopole form factors of the different vertices. We note that, to describe the nucleon-nucleon data quantitatively, the coupling \( g_{NN\omega} \) in the Jülich \( A \) \( Y_{c}N \) model was increased by a factor 1.626 with respect to its SU(3) value, \( g_{NN\omega} = 3g_{NN\rho} \), thereby accounting for missing short-range correlations in an effective way. In the present work, we apply the same increasing factor to the \( g_{\Lambda_{c} \Lambda_{c} \omega} \) and \( g_{\Sigma_{c} \Sigma_{c} \omega} \) coupling constants of Eq. (4). We note also that the relation of these coupling constants to \( g_{NN\rho} \) is a factor of two smaller than that obtained in the SU(3) sector, while the relations in Eq. (4), involving charmed baryons and the \( \pi, \rho, D \) and \( D^* \) mesons, are the same as those involving their counterparts in the strange sector.

The three \( Y_{c}N \) interaction models have then been used to solve the coupled-channel \( (\Lambda_{c}N, \Sigma_{c}N) \) Lipmann–Schwinger equation to obtain several scattering observables from the corresponding scattering amplitudes. The \( \Lambda_{c}N \) phase shifts in the \( 1^{1/2}_{0} \) and \( 3^{1/2}_{1} \) partial waves are shown as a function of the center-of-mass kinetic energy.

\[\text{Table I: Baryon–baryon–meson coupling constants } g_{BBM}, f_{BBM} \text{ and cutoff masses } \Lambda_{BBM} \text{ for the models A, B and C of the } Y_{c}N \text{ interaction constructed and used in this work.}\]

| Model | Vertex | \( g_{BBM}/\sqrt{4\pi} \) | \( f_{BBM}/\sqrt{4\pi} \) | \( \Lambda_{BBM} \) (GeV) |
|-------|--------|-----------------|-----------------|------------------|
| A,B,C | \( NN\pi \) | 3.795 | – | 1.3 |
| A,B,C | \( \Lambda_{c} \Sigma_{c} \pi \) | 3.067 | – | 1.4 |
| A,B,C | \( \Sigma_{c} \Sigma_{c} \pi \) | 2.277 | – | 1.2 |
| A,B,C | \( \Lambda_{c} \Lambda_{c} \pi \) | –3.506 | – | 2.5 |
| A,B,C | \( \Sigma_{c} \Sigma_{c} \pi \) | 1.518 | – | 2.5 |
| A,B,C | \( NN\rho \) | 0.917 | 5.591 | 1.4 |
| A,B,C | \( \Lambda_{c} \Lambda_{c} \rho \) | 0.000 | 4.509 | 1.16 |
| A,B,C | \( \Sigma_{c} \Sigma_{c} \rho \) | 1.834 | 3.372 | 1.41 |
| A,B,C | \( NN\omega \) | 4.472 | 0.000 | 1.5 |
| A,B,C | \( \Lambda_{c} \Lambda_{c} \omega \) | 1.490 | 2.758 | 2.0 |
| A,B,C | \( \Sigma_{c} \Sigma_{c} \omega \) | 1.490 | –2.907 | 2.0 |
| A,B,C | \( NN\sigma \) | 2.385 | – | 1.7 |
| A | \( \Lambda_{c} \Lambda_{c} \sigma \) | 2.138 | – | 1.0 |
| A | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 1/2) | 3.061 | – | 1.0 |
| A | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 3/2) | 3.102 | – | 1.12 |
| B | \( \Lambda_{c} \Lambda_{c} \sigma \) | 1.817 | – | 1.0 |
| B | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 1/2) | 2.601 | – | 1.0 |
| B | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 3/2) | 2.636 | – | 1.12 |
| C | \( \Lambda_{c} \Lambda_{c} \sigma \) | 1.710 | – | 1.0 |
| C | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 1/2) | 2.448 | – | 1.0 |
| C | \( \Sigma_{c} \Sigma_{c} \sigma \) (I = 3/2) | 2.481 | – | 1.12 |
The physical pion mass of the recent results of the HAL QCD Collaboration is shown by the green band for comparison. One can clearly see from the phase shifts that model A predicts a more attractive $\Lambda_c N$ interaction in the $^1S_0$ and $^3S_1$ partial waves than the one derived in Ref. [38]. The reduction of the $g_{\Lambda_c,\Lambda_c,\sigma}$ and $g_{\Sigma_c,\Sigma_c,\sigma}$ couplings in models B and C leads to a reduction of attraction in these two partial waves which translates into a qualitatively better agreement between the phase shifts predicted by these two models and those obtained from the interaction of Ref. [38], particularly in the low energy region. Note that the interaction derived in [38] predicts similar phase shifts for both partial waves since the corresponding $^1S_0$ and $^3S_1$ potentials are almost identical, a feature already noted by the HAL QCD Collaboration at different values of the pion mass (see Ref. [37]) that seems to persist when extrapolating the lattice results to the physical point. This, however, is not the case of our models A, B and C which predict more overall attraction in the $^3S_1$ partial wave as it can be seen for example in Fig. 3 where we show the diagonal $^1S_0$ and $^3S_1$ matrix element in momentum space of the $\Lambda_c N \to \Lambda_c N$ channel.

For completeness we report in Table II the singlet and triplet $\Lambda_c N$ scattering length and effective range predicted by the three models. The results obtained by Haidenbauer and Krein in Ref. [38] are shown for comparison in the last column of the table. There is a good agreement between model C and the result of [38] for both scattering lengths. However, it is pointed out in Ref. [38] that the scattering lengths at the physical pion mass could in fact be as larger as $-1.3$ fm if the uncertainty of $\pm 0.2$ fm, given by the HAL QCD Collaboration for their result at $m_\pi = 410$ MeV, is combined with the observation that variations in the scattering lengths of $\pm 0.05$ fm at this value of the pion mass amount to differences of about $\pm 0.1$ fm at $m_\pi = 138$ MeV. In this case, the prediction of model B would be in better agreement with the result of Haidenbauer and Krein than model C. Model A predicts a single effective range compatible with that obtained in Ref. [38] although a smaller triplet one. On the other hand, models B and C give a singlet effective range larger than that of [38] but their agreement is qualitatively better for the triplet one.

### Table II

| Model | $a_s$ (fm) | $r_s$ (fm) | $a_t$ (fm) | $r_t$ (fm) |
|-------|-----------|-----------|-----------|-----------|
| Model A | $-2.60$ | $2.86$ | $-15.87$ | $1.64$ |
| Model B | $-1.11$ | $4.40$ | $-1.52$ | $2.79$ |
| Model C | $-0.84$ | $5.38$ | $-0.99$ | $3.63$ |
| Ref. [38] | $-0.85$ | $2.88$ | $-0.81$ | $3.50$ |

**III. $\Lambda_c$ SINGLE-PARTICLE PROPERTIES IN FINITE NUCLEI**

Here we briefly describe a method to obtain the $\Lambda_c$ single-particle energies in a finite nucleus using an effective in-medium $Y_c N$ interaction derived from the bare $Y_c N$ potential presented in the previous section. The starting point of this method is the calculation of all the $Y_c N$ G-matrices, which describe the interaction between a charmed baryon ($Y_c = \Lambda_c, \Sigma_c$) and a nucleon in infinite nuclear matter. The G-matrices are obtained by solving the coupled-channel Bethe–Goldstone equation, written...
schematically as
\[
G_{Y,N \rightarrow Y'_N} (\omega) = V_{Y,N \rightarrow Y'_N} + \sum_{Y''_N} V_{Y,N \rightarrow Y''_N} \\
\times \frac{Q_{Y''_N}}{\omega - \epsilon_{Y''} - \epsilon_{N''} + i\eta} G_{Y''_N \rightarrow Y'_N} (\omega), \tag{6}
\]
where \(V\) is the bare \(Y, N\) potential derived in the previous section, \(Q\) is the Pauli operator, that prevents the nucleon in the intermediate state \(Y'_N\) from being scattered below the Fermi momentum \(k_{FN}\), and \(\omega\) is the nuclear matter starting energy which corresponds to the sum of the masses and the non-relativistic energies of the interacting charmed baryon and nucleon. We note that the Bethe–Goldstone equation has been solved in momentum space including partial waves up to a maximum value of the total angular momentum \(J = 4\). We note also here that the so-called discontinuous prescription has been adopted, \(i.e.,\) the single-particle energies \(\epsilon_{Y'}\) and \(\epsilon_{N''}\) in the denominator of Eq. \(6\) are simply taken as the sum of the non-relativistic kinetic energy plus the mass of the corresponding baryon.

The finite nucleus \(Y,N\) G-matrix, \(G_{FN}\), can be obtained, in principle, by solving the Bethe–Goldstone equation directly in the finite nucleus \([47, 48]\). The Bethe–Goldstone equation in finite nucleus is formally identical to Eq. \(6\), the only difference being the intermediate particle-particle propagator (\(i.e.,\) Pauli operator and energy denominator), which corresponds to that in the finite nucleus. Alternatively, one can find the appropriate \(G_{FN}\) by relating it to the nuclear matter \(Y,N\) G-matrix already obtained. Eliminating the bare interaction \(V\) in both finite nucleus and nuclear matter Bethe–Goldstone equations it is not difficult to write \(G_{FN}\) in terms of \(G\) through the following integral equation:

\[
G_{FN} = G + G \left[ \frac{Q}{E} \right] G_{FN} \\
= G + G \left[ \frac{Q}{E} \right] G \\
+ G \left[ \frac{Q}{E} \right] G \left[ \frac{Q}{E} \right] G + \cdots, \tag{7}
\]

which involves the nuclear matter G-matrix and the difference between the finite nucleus and the nuclear matter propagators, written schematically as \((Q/E)_{FN} - (Q/E)\). This difference, which accounts for the relevant intermediate particle-particle states has been shown to be quite small (see Refs. \([40, 45]\)) and, therefore, in all practical calculations \(G_{FN}\) can be well approximated by truncating the expansion \(7\) up second order in the nuclear matter G-matrix. Therefore, we have

\[
G_{FN} \approx G + G \left[ \frac{Q}{E} \right] G. \tag{8}
\]
The finite nucleus \(\Lambda_c\) self-energy can be obtained in the so-called Brueckner–Hartree–Fock approximation using the \(G_{FN}\) as an effective \(Y,N\) interaction, as it is shown in diagram (a) of Fig. 4. According to Eq. \(8\) it can be split into the sum of the diagram (b), which represents the first-order term on the right-hand side of Eq. \(8\), and the diagram (c), which stands for the so-called two-particle-one-hole (2p1h) correction, where the intermediate particle-particle propagator has to be viewed as the difference of propagators appearing in Eq. \(8\). Schematically, it reads

\[
\Sigma_{BHF} = \sum_N \langle \Lambda_c N | G_{FN} | \Lambda_c N \rangle \approx \sum_N \langle \Lambda_c N | G | \Lambda_c N \rangle \\
+ \sum_{Y,N} \langle \Lambda_c N | G | Y_c N \rangle \left[ \frac{Q}{E} \right] \langle Y_c N | G | \Lambda_c N \rangle.
\]

Detailed expressions for the first-order and the 2p1h contributions to \(\Sigma_{BHF}\) can be derived in close analogy to those for the finite nucleus \(\Lambda\) self-energy given in Refs. \([42, 43]\) being, in fact, formally identical. The interested reader is referred to these works for details on the derivation and specific expressions of both contributions.

Finally, the self-energy can then be used as an effective \(\Lambda_c\)–nucleus mean field potential in a Schrödinger equation in order to obtain the energies and wave functions of the bound states of the \(\Lambda_c\) in a finite nucleus. The Schrödinger equation is solved by diagonalizing the corresponding single-particle Hamiltonian in a complete basis within a spherical box following the procedure outlined in detail in Refs. \([40, 45]\). Note that the Hamiltonian includes also the Coulomb potential since the \(\Lambda_c\) is a positively charged baryon.

IV. RESULTS

The energy of \(\Lambda_c\) single-particle bound states in \(_5\Lambda_c\) He, \(_{13}\Lambda_c\) C, \(_{17}\Lambda_c\) O, \(_{41}\Lambda_c\) Ca, \(_{91}^{197}\) Zr and \(_{89}^{209}\) Pb are shown in Table \(\ref{table1}\) for the three models considered. For comparison the
|          | \(^5\Lambda\)He | \(^3\Lambda\)He | \(^{13}\Lambda\)C | \(^{15}\Lambda\)C | \(^{17}\Lambda\)O | \(^{19}\Lambda\)O |
|----------|----------------|---------------|----------------|----------------|----------------|----------------|
| Model A  | Model B Model C | JÅ             | Model A Model B Model C | JÅ             | Model A Model B Model C | JÅ             |
| 1s\(_{1/2}\) | -13.58 -3.24 -1.05 | -1.49         | -27.26 -10.20 -5.47 | -7.84         | -31.76 -12.47 -6.96 | -10.04         |
| 1p\(_{3/2}\) | -1.74   |  | -14.91 -2.13 |  | -19.99 -4.32 -0.51 | -0.33          |
| 1p\(_{1/2}\) | -0.39   | - | -13.42 -1.03 | - | -18.79 -3.22 | -0.35          |
| 1d\(_{5/2}\) | -     | - | -4.10   | - | -9.02 | -               |
| 1d\(_{3/2}\) | -     | - | -2.13   | - | -6.96 | -               |
| 2s\(_{1/2}\) | -     | - | -3.59   | - | -7.13 | -               |

|          | \(^{41}\Lambda\)Ca | \(^{41}\Lambda\)Ca | \(^{91}\Lambda\)Zr | \(^{91}\Lambda\)Zr | \(^{208}\Lambda\)Pb | \(^{208}\Lambda\)Pb |
|----------|----------------|---------------|----------------|----------------|----------------|----------------|
| Model A  | Model B Model C | JÅ             | Model A Model B Model C | JÅ             | Model A Model B Model C | JÅ             |
| 1s\(_{1/2}\) | -41.09 -16.89 -9.60 | -17.33         | -44.76 -18.46 -10.51 | -24.61         | -52.52 -20.33 -10.32 | -31.41         |
| 1p\(_{3/2}\) | -32.39 -10.41 -4.13 | -7.67          | -39.60 -14.27 -6.75 | -17.66         | -49.06 -18.28 -8.82 | -27.59         |
| 1p\(_{1/2}\) | -31.60 -9.67 -3.42 | -7.78          | -39.24 -14.00 -6.49 | -17.58         | -48.84 -18.10 -8.64 | -27.58         |
| 1d\(_{5/2}\) | -23.10 -3.91   | -            | -33.74 -9.63 -2.57 | -9.12          | -42.37 -12.94 -4.25 | -19.24         |
| 1d\(_{3/2}\) | -21.84 -2.74   | -            | -33.17 -9.01 -1.95 | -8.91          | -41.97 -12.58 -3.88 | -19.20         |
| 1f\(_{7/2}\) | -13.54     | -            | -27.06 -4.65     | -1.35          | -37.47 -9.11 -0.59 | -10.51         |
| 1f\(_{5/2}\) | -11.82     | -            | -26.29 -3.80     | -1.13          | -37.07 -8.65 -0.10 | -10.41         |
| 2s\(_{1/2}\) | -20.47 -2.74 | -            | -31.13 -8.05 -1.29 | -6.60          | -40.53 -10.20 -1.13 | -17.43         |
| 2p\(_{3/2}\) | -10.20     | -            | -22.81 -2.23     | -0.39          | -39.21 -9.28 -0.03 | -7.68          |
| 2p\(_{1/2}\) | -9.24      | -            | -22.24 -1.45     | -0.38          | -38.95 -9.06     | -7.60          |
| 2d\(_{5/2}\) | -2.04      | -            | -14.62          | -             | -30.28 -5.36     | -4.85          |
| 2d\(_{3/2}\) | -0.95      | -            | -14.03          | -             | -29.83 -4.75     | -4.79          |
| 2f\(_{7/2}\) | -         | -            | -7.90           | -             | -22.57          | -              |
| 2f\(_{5/2}\) | -         | -            | -6.81           | -             | -22.10          | -              |
| 3s\(_{1/2}\) | -1.15      | -            | -13.41          | -             | -23.80 -1.51     | -3.59          |
| 3p\(_{3/2}\) | -         | -            | -5.65           | -             | -22.32          | -              |
| 3p\(_{1/2}\) | -         | -            | -5.61           | -             | -21.95          | -              |
| 3d\(_{5/2}\) | -         | -            | -               | -             | -19.05          | -              |
| 3d\(_{3/2}\) | -         | -            | -               | -             | -18.33          | -              |
| 3f\(_{7/2}\) | -         | -            | -               | -             | -5.58           | -              |
| 3f\(_{5/2}\) | -         | -            | -               | -             | -5.02           | -              |
| 4s\(_{1/2}\) | -         | -            | -               | -             | -14.31          | -              |
| 4p\(_{3/2}\) | -         | -            | -               | -             | -1.19           | -              |
| 4p\(_{1/2}\) | -         | -            | -               | -             | -0.78           | -              |
| 4d\(_{5/2}\) | -         | -            | -               | -             | -0.68           | -              |
| 5s\(_{1/2}\) | -         | -            | -               | -             | -0.52           | -              |

TABLE III: Energy of \(\Lambda\) single-particle bound states of several charm nuclei from \(^{5}\Lambda\)He to \(^{208}\Lambda\)Pb obtained for the three models considered. Results for the single-particle bound states of the \(\Lambda\) hyperon in the corresponding hypernuclei predicted by the original Jülich \(\Lambda YN\) interaction are also shown for comparison. Units are given in MeV.

energy of the single-particle bound states of the \(\Lambda\) hyperon in the corresponding hypernuclei, obtained with the original Jülich \(\Lambda YN\) interaction using the method described in the previous section, are also reported in the
models are probably more realistic than model A. The Coulomb contribution increases because the Λ interaction is also in Refs. [31–34] that give rise to different predictions for the Λ single-particle energies. In addition, we also note that, since the Λ is heavier than the Λ the level spacing of the Λ single-particle energies is, for three models, always smaller than the corresponding one for the hypernuclei.

These observations are in agreement with the results previously obtained by in Tsushima and Khanna in Refs. [31, 32] using the quark-meson coupling model and, later, by Tan and Ning in Ref. [34] within a relativistic mean field approach. Although there exist formal differences between our calculation and those of Refs. [31, 34] that give rise to different predictions for the Λ single-particle bound states in finite nuclei, our results (particularly those for models B and C) are in general compatible with those of these works (see e.g., tables I and II of Ref. [31] and table I of Ref. [34]).

It has been pointed in Refs. [31, 32] and, more recently, also in Ref. [37] that the Coulomb interaction plays a non-negligible role in Λc nuclei, and that their existence is only possible if their binding energy is larger than the Coulomb repulsion between the Λc and the protons. To understand better the effect of the Coulomb force in our calculation, in Fig. 6 we explicitly show the contributions of the kinetic energy, the ΛγN interaction and the Coulomb potential to the energy of the Λc single-particle bound state 1s1/2 as a function of the mass number of the Λc nuclei.

Looking now back at the table we observe (as in the case of single Λ—hypernuclei) a small spin-orbit splitting of the p, d— and f—wave states in all Λc—nuclei, specially in the case of the heavier ones where it is of the order of few tenths of MeV. In addition, we also note that, since the Λc is heavier than the Λ, the level spacing of the Λc single-particle energies is, for the three models, always smaller than the corresponding one for the hypernuclei. These observations are in agreement with the results previously obtained by in Tsushima and Khanna in Refs. [31, 32] using the quark-meson coupling model and, later, by Tan and Ning in Ref. [34] within a relativistic mean field approach. Although there exist formal differences between our calculation and those of Refs. [31, 34] that give rise to different predictions for the Λc single-particle bound states in finite nuclei, our results (particularly those for models B and C) are in general compatible with those of these works (see e.g., tables I and II of Ref. [31] and table I of Ref. [34]).

It has been pointed in Refs. [31, 32] and, more recently, also in Ref. [37] that the Coulomb interaction plays a non-negligible role in Λc nuclei, and that their existence is only possible if their binding energy is larger than the Coulomb repulsion between the Λc and the protons. To understand better the effect of the Coulomb force in our calculation, in Fig. 6 we explicitly show the contributions of the kinetic energy, the ΛγN interaction and the Coulomb potential to the energy of the Λc single-particle bound state 1s1/2 as a function of the mass number (A = N + Z + 1, with N the neutron number and Z the atomic number) of the Λc nuclei considered. Note that, while the Coulomb contribution increases because of the increase of the number of protons with the atomic number, those of the kinetic energy and the ΛγN interaction decrease when going from light to heavy Λc nuclei.

The kinetic energy contribution decreases with the mass number because the wave function of the 1s1/2 state (see Fig. 5) becomes more and more spread due to the larger extension of the nuclear density over which the Λc wants to be distributed. The increase of the nuclear density with A leads to a more attractive Λc self-energy (see e.g., figures 2 and 3 of Ref. [45] for a detail discussion in the

| Model   | 1s1/2 | 1p3/2 | 1p1/2 | 1d5/2 | 1d3/2 | 2s1/2 | JΛ |
|---------|-------|-------|-------|-------|-------|-------|----|
| Model A | -31.54 | -12.57 | -7.11 | -8.78 |
| Model B | -19.69 | -4.37 | -0.58 | -     | -     | -     | -  |
| Model C | -18.45 | -3.24 | -     | -     | -     | -     | -  |

TABLE IV: Energy of Λc single-particle bound states of 17O when the coupling of the ΛcN and the ΣcN channels is switched off. Results for the Λ hyperon in 17O obtained with the original Jülich ΛYN interaction are also shown for comparison. Units are given in MeV.
case of single Λ—hypernuclei) that translates into a more negative contribution of the Y_cN interaction. Note that, when adding the three contributions, the energy of the 1s_{1/2} bound state decreases by several MeV in the low mass number region and then it tends to saturate (being almost constant for model C) for heavier nuclei. This is due to a compensation between the attraction of the Y_c interaction and the repulsion of the Coulomb force. We note that this compensation leads, particularly in the case of model B, to values of the Λ_c single-particle bound state energies similar to those obtained for single Λ-hypernuclei with the Jülich AYN potential (see Table IV). We want to point out that even the less attractive one of our Y_cN interactions (model C), despite the Coulomb repulsion, is able to bind the Λ_c in all the nuclei considered. This is in contrast with the recent results of the HAL QCD Collaboration [37] which suggest that only light or medium-mass Λ_c—nuclei could really exist. However, we note that this conclusion is based on results obtained for a value of the pion mass of 410 MeV which give rise to a Y_cN interaction much less attractive than ours and the one derived in Ref. [38] when these lattice results are extrapolated to the physical pion mass (see figures 1 and 2 of Ref. [38]).

Now we would like to focus the attention of the reader for a while on the effect of the coupling of the Λ_cN and Σ_cN channels. These two channels are located at approximately 3224 and 3394 MeV, respectively. Being separated by about 170 MeV it is expected, as it was already pointed out by Tsushima and Khanna (see e.g., [31, 32]), that the effect of their coupling on charmed nuclei will be less important than that of the ΔN and ΣN channels (separated only by ~ 80 MeV) on hypernuclei. This is illustrated in Table IV where we show as an example the energy of the Λ_c (Λ) single-particle states bound states of 17^1_Xe (17^1_O) when the Λ_cN — Σ_cN (ΔN — ΣN) coupling is switched off. Note that the differences between the levels obtained with the complete coupled-channel calculation for 17^1_O (see Table III) and without the Λ_cN — Σ_cN coupling are almost negligible, being of the order of few tenths of MeV or less, whereas those for 17^1_Xe are slightly larger than 1 MeV. Note also that the elimination of the coupling between the Λ_cN and Σ_cN channels leads, in the case of models B and C, to a bit more of attraction, contrary to what happens in the hypernuclei case where the Λ bound states become less bound when the ΔN — ΣN coupling is eliminated.

We finish this section by showing in Fig. 6 for the three models, the probability density distribution (i.e., the square of the radial wave function) of the Λ_c in the 1s_{1/2} state for the six Λ_c—nuclei considered. The result when the Coulomb interaction is artificially switched off is also shown for comparison (dashed lines). Note that, due to the increase of the nuclear density, when moving from light to heavy nuclei the probability density of finding the Λ_c close to the center of the nucleus decreases, and it becomes more and more distributed over the whole nucleus. Note also that, as expected, due to the Coulomb repulsion the Λ_c is pushed away from the center of the nuclei. A similar discussion can be done for the probability densities of the other Λ_c single-particle bound states.
V. SUMMARY AND CONCLUSIONS

In this work we have determined the single-particle energies of the $\Lambda_c$ charmed baryon in several nuclei. To such end, we have developed a charmed baryon-nucleon interaction based on a SU(4) extension of the meson-exchange hyperon-nucleon potential $\tilde{A}$ of the Jülich group. We have considered three different models (A, B and C) of this interaction that differ only on the energies of the couplings of the scalar $\sigma$ meson with the charmed baryons. Several scattering observables have been computed with the three models and compared with those predicted by the $Y_cN$ interaction derived by Haidenbauer and Krein [38] from the extrapolation to the physical pion mass of the recent results of the HAL QCD Collaboration [37]. Qualitative agreement has been found between the predictions of our models B and C and those of the model by Haidenbauer and Krein [38].

The three models have then been used to obtain the self-energy of the $\Lambda_c$ in finite nuclei by using a many-body approach that started with the construction of a nuclear matter $Y_cN$ $G$-matrix from which a finite nucleus one was derived through a perturbative expansion. Using the resulting $\Lambda_c$ self-energy as an effective $\Lambda_c$–nucleus mean field potential in a Schrödinger equation we have finally obtained the energies and wave functions of the bound states of the $\Lambda_c$ in the different nuclei.

Our results (particularly those for models B and C) are compatible with those obtained by Teshushima and Khanna [31][33] and Tan and Ning [34], despite the formal differences between our calculation and those of these works based, respectively, on the quark-meson coupling model and the relativistic mean field approach. A small spin-orbit splitting of the $p$, $d$– and $f$–wave states has been found as in the case of single $\Lambda$-hypernuclei. It has been also observed that level spacing of the $\Lambda_c$ single-particle energies is smaller than the corresponding one for hypernuclei.

We have analyzed the role played by the Coulomb potential on the energies of the $\Lambda_c$ single-particle bound states. This analysis has shown that the compensation between the $Y_cN$ interaction and the repulsion of the Coulomb force leads, particularly in the case of model B, to values of the $\Lambda_c$ single-particle bound state energies similar to those obtained for the single $\Lambda$-hypernuclei with the original Jülich $\tilde{A}$ $Y_cN$ potential. The analysis has also shown that, despite the Coulomb repulsion, even the less attractive one of our $Y_cN$ interactions (model C) is able to bind the $\Lambda_c$ in all the nuclei considered. This is in contrast with the recent results of the HAL QCD Collaboration [37] which suggest that only light or medium-mass $\Lambda_c$–nuclei could really exists. However, the conclusion of this work is based on results obtained for a value of the pion mass of 410 MeV which give rise to a $Y_cN$ interaction much less attractive than ours and the one derived in Ref. [28] when these lattice results are extrapolated to the physical pion mass.

Finally, we have shown that the effect of the coupling of the $\Lambda_cN$ and $\Sigma_cN$ channels on the single-particle properties of charmed nuclei is much less important (being in fact almost negligible) than that of the $\Lambda N$ and $\Sigma N$ channels on the corresponding properties of single $\Lambda$-hypernuclei, due to the large mass difference of the $\Lambda_c$ and $\Sigma_c$ baryons of $\sim$ 170 MeV.

Acknowledgments

The authors are very grateful to Johann Haidenbauer for his useful comments. This work has been partly supported by the COST Action CA16214 and by the Spanish Ministerio de Economía y Competitividad (MINECO) under the project MDM-2014-0369 of ICCUB (Unidad de Excelencia “María de Maeztu”), and, with additional European FEDER funds, under the project FIS2017-87534-P.

[1] C. L. Aubert et al., Phys. Rev. Lett. 33, 1404 (1974).
[2] J. Augustin et al., Phys. Rev. Lett. 33, 1406 (1974).
[3] E. G. Cazzoli et al., Phys. Rev. Lett. 34, 1125 (1975).
[4] G. Goldhaber et al., Phys. Rev. Lett. 37, 255 (1976).
[5] I. Peruzzi et al., Phys. Rev. Lett. 37, 569 (1976).
[6] B. Knapp et al., Phys. Rev. Lett. 37, 882 (1976).
[7] A. A. Tyapkin, Yad. Fiz. 22, 181 (1975); Sov. J. Nucl. Phys. 22, 89 (1976).
[8] C. B. Dover and S. H. Kahana, Phys. Rev. Lett. 39, 1506 (1977).
[9] C. B. Dover, S. H. Kahana and T. L. Trueman, Phys. Rev. D. 16, 799 (1977).
[10] S. Iwao, Lett. Nuovo Cimento 19, 647 (1977).
[11] R. Gatto and F. Paccanoni, Nuovo Cimento 46A, 313 (1978).
[12] G. Bhamathi, Phys. Rev. C 24, 1816 (1981).
[13] N. N. Kolesnikov et al., Sov. J. Nucl. Phys. 34, 957 (1981).
[14] H. Bandi and M. Bando, Phys. Lett. B 109, 164 (1982).
[15] H. Bandi and S. Nagata, Prog. Theor. Phys. 69, 557 (1983).
[16] H. Bandi, Prog. Theor. Phys. Suppl. 81, 197 (1985).
[17] B. F. Gibson, C. B. Dover, G. Bhamathi, D. R. Lehman, Phys. Rev. C 27, 2085 (1983).
[18] G. Bhamathi, Nuovo Cimento A 102, 607 (1989).
[19] T. Bressani and F. Iazzi, Nuovo Cimento A 102, 597 (1989).
[20] S. A. Bunyatov, V. V. Lyukov, N. I. Starkov and V. A. Tsarev, Nuovo Cimento A 104, 1361 (1991).
[21] Y. A. Batusov et al., JINR Preprint E1-10069 (Dubna 1976).
[22] Y. A. Batusov et al., Pis’ma Zh. Éksp. Teor. Fiz. 33, 56 (1981).
[23] Y. A. Batusov et al., JETP Lett. 33, 56 (1981).
[24] Y. A. Batusov et al., JINR Communication P1-85-495 (Dubna 1985);
[25] V. V. Lyukov, Nuovo Cimento A 102, 583 (1989);
[26] J. Riedl, A. Schäfer, M. Stratmann, Eur. Phys. J. C 52, 987 (2007);
[27] E. Tomasi-Gustafsson [PANDA Collaboration], Hyperfine Interact. 239, no. 1, 30 (2018);
[28] R. Shyam and K. Tsushima, Phys. Lett. B 770, 236 (2017);
[29] G. Krein, A. W. Thomas and K. Tsushima, Prog. Part. Nucl. Phys. 100, 161 (2018);
[30] K. Tsushima and F. C. Khanna, Phys. Lett. B 552, 138 (2003);
[31] K. Tsushima and F. C. Khanna, Prog. Theor. Phys. Suppl. 149, 160 (2003);
[32] K. Tsushima and F. C. Khanna, Phys. Rev. C 67, 015211 (2003);
[33] K. Tsushima and F. C. Khanna, J. Phys. G 30, 1765 (2004);
[34] Y.-H. Tan and P.-Z. Ning, Europhys. Lett. 67, 355 (2004);
[35] Y.-R. Liu and M. Oka, Phys. Rev. D 85, 014015 (2012);
[36] S. Maeda, M. Oka, A. Yokota, E. Hiyama and Y. R. Liu, PTEP 2016, no. 2, 023D02 (2016);
[37] T. Miyamoto et al., Nucl. Phys. A 971, 113 (2018);
[38] J. Haidenbauer and G. Krein, Eur. Phys. J. A 54, 199 (2018);
[39] A. Reuber, K. Holinde, and J. Speth, Nucl. Phys. A 570, 543 (1994);
[40] M. Borromeo, D. Bonatsos, H. Mütter and A. Polls, Nucl. Phys. A 539, 189 (1992);
[41] M. Hjorth–Jensen, H. Mütter and A. Polls, Phys. Rev. C 50, 501 (1994);
[42] M. Hjorth–Jensen, A. Polls, A. Ramos and H. Mütter, Nucl. Phys. A 605, 458 (1996);
[43] I. Vidaña, A. Polls, A. Ramos and M. Hjorth–Jensen, Nucl. Phys. A 644, 201 (1998);
[44] I. Vidaña, Ph.D. Thesis, University of Barcelona (2001). http://www.tesisenxarxa.net/TDX-0709102-133910/;
[45] I. Vidaña, Nucl. Phys. A 958, 48 (2017);
[46] E. M. Haacke, J. W. Moffat, and P. Savaria, J. Math. Phys. 17, 2041 (1976);
[47] J. Hao, T. T. S. Kuo, A. Reuber, K. Holinde, J. Speth and D. J. Millener, Phys. Rev. Lett. 71, 1498 (1993);
[48] D. Halderson, Phys. Rev. C 48, 581 (1993).