Kerman, Ron; Rawat, Rama; Singh, Rajesh K.

Dilation-commuting operators on power-weighted Orlicz classes. (English) Zbl 1419.42013
Math. Inequal. Appl. 22, No. 2, 463-486 (2019).

Summary: Let \( \Phi \) be a nondecreasing function from \( \mathbb{R}_+ = (0, \infty) \) onto itself. Fix \( \gamma \in \mathbb{R} = (-\infty, \infty) \) and let \( L_{\Phi, t^\gamma}(\mathbb{R}_+) \) be the set of all Lebesgue-measurable functions \( f \) from \( \mathbb{R}_+ \) to \( \mathbb{R} \) for which

\[
\int_{\mathbb{R}_+} \Phi(k|f(t)|)t^\gamma \, dt < \infty
\]

for some \( k > 0 \). Define the gauge \( \rho_{\Phi, t^\gamma} \) at \( f \in L_{\Phi, t^\gamma}(\mathbb{R}_+) \) by

\[
\rho_{\Phi, t^\gamma}(f) = \inf \left\{ \lambda > 0 : \int_{\mathbb{R}_+} \Phi\left(\frac{|f(t)|}{\lambda}\right) t^\gamma \, dt \leq 1 \right\}.
\]

Our principal goal in this paper is to find conditions on the nondecreasing functions \( \Phi_1 \) and \( \Phi_2, \gamma \in \mathbb{R} \) and an operator \( T \) so that the assertions

\[
\rho_{\Phi_1, t^\gamma}(Tf) \leq C \rho_{\Phi_2, t^\gamma}(f) \tag{G}
\]

and

\[
\int_{\mathbb{R}_+} \Phi_1(|Tf|)(t) t^\gamma \, dt \leq K \int_{\mathbb{R}_+} \Phi_2(K|f|)(s) s^\gamma \, ds \tag{M}
\]

concerning \( f \in S(\mathbb{R}_+) \), the class of simple functions supported in \( \mathbb{R}_+ \), are equivalent and then to find necessary and sufficient conditions in order that \( (M) \) holds. In addition, we investigate the connection between \( (G) \) and the assertion that

\[
T : \hat{L}_{\Phi_2, t^\gamma}(\mathbb{R}_+) \rightarrow L_{\Phi_1, t^\gamma}(\mathbb{R}_+)
\]

where \( \hat{L}_{\Phi_2, t^\gamma}(\mathbb{R}_+) \) is the closure of \( S(\mathbb{R}_+) \) in \( L_{\Phi_2, t^\gamma}(\mathbb{R}_+) \).

MSC:

42B25 Maximal functions, Littlewood-Paley theory
26D15 Inequalities for sums, series and integrals
28A25 Integration with respect to measures and other set functions

Keywords:

dilation-commuting operators; Orlicz space; norm inequalities; modular inequalities; Hardy operator; maximal function; Hilbert transform

Full Text: DOI arXiv

References:

[1] S. BLOOM ANDR. KERMAN, \textit{Weighted L}^p \textit{integral inequalities for operators of Hardy type}, Studia Math. 110, 1 (1994), 35-52.
[2] S. BLOOM ANDR. KERMAN, \textit{Weighted Orlicz space integral inequalities for the Hardy-Littlewood} \textit{maximal operator}, Studia Math. 110, 2 (1994), 149-167. · Zbl 0813.42014
[3] C. BENNETT ANDR. SHARPLEY, \textit{Interpolation of operators}, Pure and Applied Mathematics, vol. 129, Academic Press Inc., Boston, MA, 1988.
[4] A. CIANCHI, \textit{Hardy inequalities in Orlicz spaces}, Trans. Amer. Math. Soc. 351, 6 (1999), 2459-2478. · Zbl 0920.46021
[5] R. COIFMAN ANDC. FEFFERMAN, \textit{Weighted norm inequalities for maximal functions and singular} \textit{integrals}, Studia Math. 51, 3 (1974), 241-250.
[6] M. FABIAN, P. HABALA, P. H ´AJEK, V. MONTESINOS ANDV. ZIZLER, \textit{Banach space theory}, CMS Books in
[7] R. KERMAN AND L. PICK, *Explicit formulas for optimal rearrangement-invariant norms in Sobolev inequalities*, Studia Math. 206, 2 (2011), 97-119.

[8] R. KERMAN AND A. TORCHINSKY, *Integral inequalities with weights for the Hardy maximal function*, Studia Math. 71, 3 (1982), 277-284.

[9] L. MALIGRANDA, *Orlicz spaces and interpolation*, Sem. Math. 5, Dep. Mat., Univ. Estadual de Campinas, Campinas SP, Brazil (1989).

[10] W. MATUSZEWSKA AND W. ORLICZ, *On certain properties of ϕ-functions*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 8 (1960), 439-443.

[11] S. MAZUR AND W. ORLICZ, *On some classes of linear spaces*, Studia Math. 17 (1958), 97-119.

[12] W. ORLICZ, *A note on modular spaces. I*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 9 (1961), 157-162. · Zbl 0109.33404

[13] A. TORCHINSKY, *Real-variable methods in harmonic analysis*, Pure and Applied Mathematics, vol. 123, Academic Press, Inc., Orlando, FL, 1986. · Zbl 0621.42001

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.