Enhancing manipulation of algebraic equation through Balance Method

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Abstract. This action research study measured the effectiveness of the Balance Method as an intervention in teaching and learning of manipulation of the algebraic equation. The lesson intervention involved twenty-five students from two Year 9 classes in one all-girls secondary school in Brunei. Data collection involved pre-test and post-test attempted by all students, followed by the interview of three selected students. Three lessons designed explicitly as the intervention in the learning of manipulation of algebraic equation, which transition from the use of manipulatives to written work, making use of Balance Method concept. Any improvement or beneficial outcome of the previous lesson was used to adapt the following experience as part of the Design Research process. A paired sample t-test revealed that there was a significant improvement in students’ performance on the topic after the intervention lesson using Balance Method. More answers that are correct were obtained when students used the Balance Method, compared to their previous failed attempts or incorrect attempts when using another method. Whereas interview revealed that whether students prefer Balance Method or others, they correctly attempted manipulation of the algebraic equation through Balance Method, particularly with equations that involved the combination of operations. It provides evidence that Balance Method should be introduced and taught as the correct concept of manipulating algebraic equation at students’ first encounter of the topic.

1. Introduction

Algebra is an important topic in mathematics curriculum and agreed by teachers as one that is difficult to teach and make students understand. This topic brings about an abstract idea, which is difficult for the learners to grasp at first [1-2]. Students find it difficult to learn algebra as they fail to relate why they should learn this particular topic and how it relates to real world [1].

In Brunei, algebra is introduced to students in Year 6 where they are taught basic concepts. These basic concepts involve simple algebraic terms compared to the secondary level syllabus, such as adding and subtracting like terms, substituting values and solving equation of one unknown. Students who encountered problems in algebra at primary level will face difficulties at higher level, which require deeper understanding and mastery of the concept and skills [3-4].

The National Education System for the 21st Century, SPN21, was first implemented in Brunei in 2008 and proposed to raise the students’ achievements in core subjects including mathematics as one of the rationale for the changes in the education system [5]. The new curriculum listed the expected
learning outcomes of students for each learning area in which they need to accomplish the specific objectives identified by subject content. Students are expected to develop learning skills, conceptual understanding and attitudes to handle mathematics in everyday life, intended to achieve learning outcomes that includes developing the ability to interpret and communicate mathematical ideas both orally and written, stimulate students’ interest in learning mathematics and nurture good learning habits. It is important that students improve their understanding on the mathematics concept by displaying good mathematical thinking and reasoning in solving mathematical problem [5]. As part of the objectives of SPN21, equipping students with necessary skills, such as collaboration, creative thinking, problem solving and critical reasoning and ICT skills is crucial for future encounter in university and career [5]. Thus, this intervention lesson aimed to improvise students’ way of presenting their procedures to solutions both verbally and written as this could reflect their conceptual understanding and thinking process. This may also provide assurance whether students has fully understand the concept of equal sign in an equations. The research question used to guide this study: How is the balancing equation method affecting students’ performance?

Mathematics is considered as an essential and fascinating subject because it can be learnt without complications if only the students are manage to produce understanding and fluency of what they need to learn [6]. Mathematical concepts may be difficult to understand or grasp at the beginning of lesson due to unfamiliarity of the concept especially algebra. Most of mathematical concepts can only be understood fully if procedural skills of the concept are acquired by the learners’ themselves [7]. Students are still struggling with mathematical concepts even at their upper secondary level of education and that misconceptions learnt by them appeared to be the cause of their poor achievements in test and examinations [8].

In algebra, some of the errors and misconceptions that can be found include errors in forming an equation based on a given word problem or statement [9]. Students often misinterpret the statement or directly translated the words into an equation, making errors in expansion and factorization of algebraic expressions or equation and errors in rearranging or manipulating algebraic equations [10, 11]. Other factors that may result in poor performance in mathematics algebra include low English proficiency in interpreting word problems. When a mathematics question becomes difficult to understand, students will most likely not attempt the question or use inappropriate approach in solving the problem [10].

Algebraic manipulation involves the rearrangement of variables in an equation to obtain the desired outcome. For this section of algebra, these basic skills in manipulating an equation are important and foundation to other higher level concepts algebra. Manipulation of algebraic equation is also applicable in other topics such as function, geometry, and trigonometry. Because of students’ inability to manipulate algebraic equation properly, it has led them to have difficulties in solve algebraic problems such as formulating word problem [11].

Students were found to encounter difficulties when solving problems that involve manipulation of algebraic equation [12]. Therefore, enriching and enhancing students’ understanding in manipulating equations might increase their capability to solve other problems not only in algebra but also in other topics in mathematics. With the students having better understanding of mathematical concept, it may realize Brunei’s aims to increase the students’ performance in mathematics.

Balance Method Model takes into consideration that in a mathematics equation, a state of equilibrium achieve in any variables or constants in the left hand side are equal to those in the right hand side. In Balance Method approach, you aim to leave the unknown variable to one side by applying any operation on both side of the equation, while still maintain equilibrium or balance. If we imagine this as seesaw at a balance state, adding or subtracting the same amount from both side will maintain balance, although there is changes value in both sides of the balance. Similarly, multiplying or dividing is the same amount will result in similar outcome. This concept of manipulating of quadratic equation through the Balance Method was successful in several studies [13-15]. Balance Method does have limitation where negative numbers cannot visually be represented [15-17].
2. Method

The initial participants involved in this quantitative study consisted of convenient sampling of thirty-four female students of Year 9 International General Certificate of Secondary Education (IGCSE) students, from one of the all-girl secondary schools in Brunei-Muara District. The abilities of these students ranged from average to low since these students were from two of the IGCSE classes in the school. Since there were some absentees during the pre-test and post-test, only results from twenty-five students were analysed for this study.

The test instruments consist of pre-test and post-test. Both the tests contained the same ten items with questions designed to bring about possible errors and misconceptions that students might commit. Items 1 to 4 involved individual addition, subtraction, multiplication and division operation, followed by two items containing square and square root operation and the remaining four items involving equations that contained combination of any of the four operations with square and fraction (Table 1). Correct attempts in question 1-4 each carried one mark, while the rest carried two marks each. The questions for the test were adapted from past year papers of IGCSE mathematics and Mathematics IGCSE textbook for Year 9. Students were expected to complete the test within 30 minutes and the use of calculators was permitted.

Table 1. List of items included in the test.

| Item number | Item question |
|-------------|---------------|
| 1           | Given that \( x = y - 1 \). Express \( y \) in terms of \( x \). |
| 2           | Given that \( p = m + n \). Express \( m \) in terms of \( p \) and \( n \). |
| 3           | Given that \( C = 2\pi r \). Express \( r \) in terms of \( C \). |
| 4           | Given that \( Q = \frac{p}{3} \). Express \( P \) in terms of \( Q \). |
| 5           | Given that \( y = a + bc \), make \( c \) as the subject. |
| 6           | Given that \( v = \frac{w-7}{x} \). Express \( w \) in terms of \( x \) and \( y \). |
| 7           | Given that \( t = m^2 \), make \( m \) as the subject. |
| 8           | Given that \( T = 2\sqrt{n} \). Express \( n \) in terms of \( T \). |
| 9           | Given that \( C = \frac{5}{F} - 3 \). Express \( F \) in terms of \( C \). |
| 10          | Given that \( A = \frac{5\pi r^2}{2} \), make \( r \) as the subject. |

Prior to the actual study, the test instrument was administered to a group of year 9 IGCSE students of the same school not involved in the main study to check for the reliability of the test questions. A test-retest approach of one week apart under same conditions produced high positive correlation coefficient, indicating reliable instrument. Moreover, students were able to complete the test within 30 minutes during the first and second time taking the test. Three teachers validated the test itself with at least five years’ experience teaching mathematics at this level.

Pre-test was carried out at the beginning of the research where students were not informed in advance about the test to anticipate their current understanding of algebraic manipulation. Post-test was given to students after conducting all the intervention lessons as to measure the impact of the intervention lessons. A paired sample t-test comparing the means of pre-test and post-test was done using Statistical Package for Social Sciences or SPSS to determine if there is the significant difference; hence determine whether the intervention lessons have any impact towards students’ performance in the topic.

Interviews were conducted after the post-test, which aimed to gain more feedback from the students regarding their opinions on the method introduced during the intervention. Three randomly selected students were interviewed with the designed questions related to the research question. The selection...
for the interview was also based on the availability of the students at the time of the interview. The interview was audio recorded and later transcribed for analysis.

For the intervention, three lessons of one hour each were conducted separately in both classes. The three lessons were planned to ensure that each part in rearranging algebraic equation or formula would be covered with sufficient work examples and exercises for each and combined operation. The lessons were designed in a sequence of increasing difficulties and hence students were expected to enhance their understanding in each operation so that they will be able to proceed to the next part of the intervention. (Table 2). In addition, any improvement or useful outcome of the prior lesson was added or adapted to the following lesson as part of the Design Research process.

Table 2. The intervention lessons conducted in this research study

| Lesson | Focus                                                                 | Materials used                               | Duration (hour) |
|--------|----------------------------------------------------------------------|----------------------------------------------|-----------------|
| 1      | Introduction of balance concept and use this concept, with the aid of manipulatives, to manipulate equations involving addition, subtraction, multiplication and division. | Manipulatives, activity sheets, exercise sheet | 1               |
| 2      | Applying balance concept to continue with manipulation of equation, expanding to square, square roots, fractions and combination of operations. | Manipulatives, Activity sheets, exercise sheets | 1               |
| 3      | Reinforcement questions on of manipulation of equation using combinations of operations | Work sheet                                   | 1               |

The first lesson introduced the concept of balance to resemble the equation and any addition, deletion, manipulation of number or variable in the equation needed to be done on both side. Students also used Manipulative to support the learning concepts (Figure 1).

Figure 1. Boards and cards used during activities involving equation manipulation using balancing method.

Four operations were focused in the first lesson, which correspond to specific actions on manipulatives (Table 3).

Table 3. The intervention lessons conducted in this research study

| Operation | Action with manipulative in Lesson 1 activity                                                                 |
|-----------|----------------------------------------------------------------------------------------------------------------|
| Addition  | Adding the corresponding number or unknown on both side                                                      |
| Subtraction | Removing the corresponding number or unknown on both side                                                |
| Multiplication | Repeated addition of the corresponding number and unknown on both side                                 |
| Division  | Regrouping into divisor set on both side, follow by removal/leaving only one set.                        |

The second lesson expanded the manipulation of equation to include operation of square, square root and fractions, in addition to reinforcement of the four operations previously covered (single and
combined). Initially, the manipulative was only planned in the first lesson, but was kept in the following two lessons so that teacher could use as quick reminder to students the concept related to Balance Method to help and reinforce students visualise the concept of actual balanced equation. Students could also use manipulatives to self-check and clarify any confusion. The lesson moves on to more manipulation via written work. The final intervention lesson stressed is on the reinforcement of all aspects in previous lessons.

3. Result and discussion
The descriptive statistics (Table 4) of the pre-test results indicated that the mean score of the pre-test was 2.68 with standard deviation (S.D.) of 1.82. The lowest score obtained was 0 and the highest score was 8. Surprisingly, only one student out of the total participants passed the test (scored 8). In the post-test, the mean score was 10.40 (S.D. = 2.65). The lowest score gained by students in the post-test was 6 while the highest score obtained was 16, with twenty-one students passed the test (scored 8 and above).

| Total students | Score |   |   |   |
|----------------|-------|---|---|---|
| Pre-test       | 25    | 16| 2.68| 1.82| 0| 8|
| Post-test      | 25    | 16| 10.40| 2.65| 6| 16|

Prior to conducting the paired sample t-test, the conditions for suitability for t-test were determined if fulfilled. The dependent variables are continuous (marks of 0 to 20), the independent variable consists of related groups, no significant outliers marks, and the distribution of the differences in the pre-test and post-test is approximately normally distributed. Thus, paired-sample t-test was carried out with the pre-test and post-test results.

| Mean difference | Standard Deviation | t   | df | Sig (2 tailed) |
|-----------------|--------------------|-----|----|----------------|
| Post-test – Pre-test | 7.720            | 2.072| 18.629| 24 | 0.000 |

A paired sample t-test (Table 5) produced mean score increase of 7.720, with pre-test having mean of 2.68 (SD = 1.82) and the post-test reaching mean of 10.40 (SD = 2.65), with t = 18.629, df = 24 and p = 0.000 (p < 0.05 indicates significance changes). Since the p-value was less than 0.05, analysis indicated that there was significant improvement of the post-test compared to pre-test, thus giving evidence that intervention lessons based on Balance Method has improved students’ performance in that topic.

| Item number | Item number |
|-------------|-------------|
| 1 | 1 |
| 2 | 2 |
| 3 | 1 |
| 4 | 2 |
| 5 | 1 |
| 6 | 1 |
| 7 | 0 |
| 8 | 0 |

Table 6. Number of students got correct responses for each question in the both tests

From Table 6, in the pre-test, Question 1, 2 and 4 had high correct responses with 15, 16 and 17 students respectively. This coincided with questions involving addition, subtraction and division. Low correct attempts in pre-test for question 3 indicated students are having difficulty in successfully attempting multiplication question. Similarly, the rest of the questions indicated extremely low correct attempt (no students correctly responded question 9 and 10 at all), proving that participants are unable
to attempt combination and complex questions of manipulating equations. Comparing to the post-test responses, the number of correct responses of every questions increased; with question 2 and 7 had correct response from all students. Question 7 had the most increase in the number of correct responses with only one correct response in the pre-test to 25 correct responses in the post-test. This question is on rearranging equation that involve square in which that Balance Method greatly facilitated correct attempts on such question.

Question 3 which involved multiplication, also increased by large amount of correct responses in the post-test from 3 to 24 correct responses. For the small increase in the correct responses, question 6 only increased by four correct responses in the post-test. This small increase might due to possibly students having the lack of practice in questions involving mixed of operations as well as fraction.

From the interview with the selected students, two of them revealed that they prefer to use balancing equation method in manipulating algebraic equation, which evidently was reflected in their big increase in test’s performance. The other student who still preferred ‘move to the other side’ method also performed better in the post-test by using balancing equation method. It was observed that students were able to manipulate equation correctly using Balance Method with consistence use of the method and more practices, particularly with equations that involved combination of operations.

The significant results from paired sample t-test proved that there was a significant improvement in students’ performance on the topic after the intervention of Balance Method. Students attempting questions using Balance Method in post-test produce more correct answers compared to ‘move to the other side’ method majority of students applied during the pre-test. Students committed fewer errors in the post-test using Balance Method, even for those complex and combination equations [18].

These findings showed that teaching the right mathematical concept did enhanced students’ performance on the topic, and hopefully, understanding of the topic. With these, it is important for teachers to develop both conceptual and procedural understanding for the students in learning mathematics, which should be implied when designing a lesson [19-21].

4. Conclusion
Manipulating algebraic equation is introduced with Balance Method in most textbooks, yet teachers tend to use to keep using alternative or short-cut method such as the ‘move to the other side’ method, which consistently show misconceptions and fail attempts by students. Some students do not even know ‘move to the other side’ method itself is a consequence of Balance Method and on its own such concept does not exist. Thus, teachers should apply the right mathematical concept and procedure to deliver such topic to the students to ensure not only strengthens students’ understanding of this basic concept but also as to avoid teaching accidental misconception to begin with.

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