Axion field induces exact symmetry

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Abstract
While no regularisation is consistent with the anomalous chiral symmetry which occurs for massless fermions, the artificial axion-induced symmetry for massive fermions is shown here to be consistent with a standard regularisation, even in curved spacetime, so that it can be said to have no anomaly in gauge or gravitational fields. Implications for $\theta$ terms are pointed out.

1. Introduction
Chiral symmetry, which is exact for massless fermions at the classical level and approximate for light quarks, has been very useful in particle physics. In analogy with this chiral symmetry, an artificial chiral-like symmetry [1] was introduced some time back for the strong interactions with massive quarks. It implies the occurrence of a light pseudoscalar particle, the axion [2], which has however not been detected in spite of elaborate searches [3]. This symmetry has been suspected to be fraught with an anomaly because of the involvement of a chiral transformation. However, this is not obvious and has to be examined by careful regularisation of the theory. This is important because of implications for the symmetry of the theory.

While chiral symmetry of the action, corresponding to the transformation
\[ \psi \rightarrow e^{i\alpha \gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{-i\alpha \gamma_5}, \] (1)
is broken by a non-vanishing quark mass term $m\bar{\psi}\psi$, with mass $m$, the artificial chiral symmetry for massive fermions works by letting a new field $\varphi$ absorb the chiral transformation. The mass term is replaced by
\[ \bar{\psi}m e^{i\alpha \gamma_5} \psi, \] (2)
which is classically invariant under the transformation
\[ \psi \rightarrow e^{i\alpha \gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{i\alpha \gamma_5}, \quad \varphi \rightarrow \varphi - 2\alpha, \] (3)
which is also a symmetry of the other terms of the action.

The original interaction introduced by Peccei and Quinn [1] was of the form
\[ \bar{\psi} \left[ \frac{\Phi^{1 + \gamma_5}}{2} + \Phi^{1 - \gamma_5} \right] \psi, \] (4)
where $\Phi$ is a complex scalar field with a symmetry breaking potential. The artificial chiral symmetry transformation here is
\[ \Phi \rightarrow e^{-2i\alpha} \Phi, \quad \psi \rightarrow e^{i\alpha \gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{i\alpha \gamma_5}. \] (5)
$\Phi$ may be taken to be of the form $\rho e^{i\varphi}$. The amplitude $\rho$ of the scalar field acquires a vacuum expectation value because of symmetry breaking, which provides a massive boson. The phase $\varphi$ is the zero mode of the potential and provides a Goldstone boson. This is the axion, which should acquire a mass because of the quark masses, but does not appear to exist. The kinetic term of $\Phi$ yields $\frac{1}{2} \rho \frac{\partial^2}{\partial \varphi^2} \varphi - \rho \frac{\partial \varphi}{\partial \varphi}$ as the kinetic term for $\varphi$, with $\rho_0$ the vacuum expectation value of $\rho$. The mass $m$ comes from $\rho_0$ and the coupling constant of $\Phi$.

The anomaly, which occurs for the usual chiral symmetry and makes the divergence of the axial current $\bar{\psi} \gamma_5 \gamma_5 \psi$ nonvanishing, appears when the singularities in the theory are handled by regularisation. Even the measure picture of anomalies requires a regularisation for actual calculations. We shall therefore use an explicit
regularisation for studying chiral transformations of fermions in the presence of the axion coupling. This will tell us whether the continuous symmetry (3) introduced classically for massive quarks is anomalous or not.

The new symmetry is expected to be broken spontaneously by the vacuum because of the translation of the spinless field by a $c$-number. The field must have a definite value in a fixed vacuum: let it be denoted by $\varphi_0$. Then

$$\varphi' = \varphi - \varphi_0$$

(6)
is the shifted field with vanishing vacuum expectation value. This is a pseudoscalar field, so that $\bar{\psi}\varphi'\gamma_5\psi$ and $(\varphi' \text{ tr } F^{\mu\nu}F_{\mu\nu})$ are scalars. On the other hand, $(\varphi_0 \text{ tr } F^{\mu\nu}F_{\mu\nu})$ is a pseudoscalar, like the $\theta$ term $(\theta \text{ tr } F^{\mu\nu}F_{\mu\nu})$. If a $(\varphi_0 \text{ tr } F^{\mu\nu}F_{\mu\nu})$ term is generated when the fermion is integrated out, it can modify the $\theta$ term. The possibility of this occurrence is related to the question of an anomaly in the symmetry (3).

If the symmetry (3) is anomalous, no regularisation of the fermions will be consistent with it. On the other hand, if any regularisation is found to be consistent with it, the symmetry has no anomaly in this quantisation and such symmetric regularisations are the ones to be preferred. Pauli–Villars regularisation will be studied in flat and in curved spacetime.

2. Pauli–Villars regularisation

We recall first the simple form of the Pauli–Villars regularisation. For a fermion with mass $m$, the Lagrangian density

$$\bar{\psi}[i\not{D} - m]\psi + \bar{\chi}[i\not{D} - M]\chi$$

(7)

involves regulator spinor fields $\chi$, $\bar{\chi}$ which however are assigned Bose statistics. The regulator mass $M$ is ultimately taken to infinity when the regulator fields decouple. The regulator fields couple to the gauge fields in the standard way. The regulator mass term breaks the chiral symmetry and yields the chiral anomaly in the $M \to \infty$ limit. This is the original form of the Pauli–Villars regularisation. A more general form with several species of regulator fields is also available [4]:

$$\bar{\psi}[i\not{D} - m]\psi + \sum_j \sum_{k=1}^{[c_j]} \bar{\chi}_j[i\not{D} - M_j]\chi_{jk}.$$  

(8)

Here $c_j$ are integers whose signs are related to the statistics assigned to $\chi_{jk}$. They have to satisfy some conditions to ensure regularisation of the divergences [4]:

$$1 + \sum_j c_j = 0, \quad m^2 + \sum_j c_j M_j^2 = 0.$$  

(9)

Chiral transformations work on $\chi$ as well as $\psi$. Hence the axion coupling has to be introduced for both $\psi$, $\chi$, like the gauge coupling. One has

$$\bar{\psi}[i\not{D} - me^{i\xi\gamma_5}]\psi + \bar{\chi}[i\not{D} - Me^{i\xi\gamma_5}]\chi.$$  

(10)

This regularisation (10) is invariant under the combined transformation

$$\psi \to e^{i\psi}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\chi}, \quad \chi \to e^{i\chi}\chi, \quad \bar{\chi} \to \bar{\chi}e^{i\chi}, \quad \varphi \to \varphi - 2\alpha,$$  

(11)

which is the Pauli–Villars extension of (3). This means the symmetry survives when the regularisation is removed by taking the limit $M \to \infty$ and hence is not anomalous. Unlike the regularised axial current, which has a pseudoscalar divergence arising from the masses, the Noether current for the extended symmetry, namely

$$\bar{\psi}\gamma_\mu\gamma_5\psi + \bar{\chi}\gamma_\mu\gamma_5\chi + 2\alpha\partial_\mu\partial_\mu\varphi,$$  

(12)
is conserved. The non-vanishing divergence of the axial current is cancelled by that of the $\varphi$ piece by virtue of the axion equation of motion.

It is interesting to note that the $\varphi$ phases in (10) can be removed by a joint chiral transformation of both the physical fermions and the regulator spinors:

$$\psi \to e^{-i\psi}\psi, \quad \bar{\psi} \to \bar{\psi}e^{-i\chi}, \quad \chi \to e^{-i\chi}\chi, \quad \bar{\chi} \to \bar{\chi}e^{-i\chi},$$  

(13)
The derivative operators in the action produce derivatives of $\varphi$, which appear after the transformation. Now the transformation has a trivial Jacobian because the contribution of the fermion field is cancelled by that of the bosonic regulator field as in the latter case the determinant arising from functional integration comes in the denominator. The argument for the generalised Pauli–Villars regularisation, with (10) replaced by
\[ \psi^c [i \mathcal{J} - m e^{i \gamma_5}] \psi + \sum_{j} \sum_{k=1}^{\ell} \chi_{j} \chi_{j}^{\dagger} \psi \quad \text{(14)} \]

involves a Jacobian with the factor \(1 + \sum_{j} c_{j} = 0\) in the exponent [5]. Consequently the effective action does not contain any \(\varphi\) \(\text{tr} \tilde{F}^{\mu \nu} F_{\mu \nu}\) in this regularisation. The conserved Noether current here is

\[ \bar{\psi} \gamma_{\mu} \gamma_{5} \psi \quad + \sum_{j} \sum_{k=1}^{\ell} \chi_{j} \chi_{j}^{\dagger} \psi \quad + 2 \rho_{0}^{2} \partial_{\mu} \varphi, \quad \text{(15)} \]

Note that the argument may be easily extended to curved spacetime, where the Dirac operator

\[ i \mathcal{J} = i \gamma^\mu \left( \partial_{\mu} - i A_{\mu} - \frac{i}{2} A^{\eta \mu}_{\nu} \sigma_{\eta \nu} \right) \quad \text{(16)} \]

comes with a tetrad \(e^{\mu}_{i}\) and a spin connection \(A^{\eta \mu}_{\nu}\) in addition to the gauge field, but it continues to anticommute with \(\gamma^5\). The regularised action is again invariant under the Pauli–Villars extension of (3). This means that there is not even an \(RR\) anomaly [6] in this symmetry. Similar results have also been found by others [7].

3. Conclusion

We have examined the chiral symmetry of the fermion action including \(\varphi\). This classical symmetry is preserved by the explicit regularisation (10). When an acceptable regularisation preserves a symmetry, one can conclude that the quantum theory defined by the limit of that regularisation will also satisfy it. Hence, after the fermion is integrated out, the effective action must be invariant under translations of \(\varphi_{0}\), i.e., independent of this variable. This is indeed true of this regularisation, as seen above, and there is no \(\varphi\) \(\text{tr} \tilde{F}^{\mu \nu} F_{\mu \nu}\) term. This makes the modification of a \(\theta\) term by a contribution from \(\varphi_{0}\) impossible. The field \(\varphi\) occurs only in the form of derivatives. The extension of the argument to curved spacetime means that an existing \(RR\) term [5] too cannot be modified.

There is another way of seeing this. The exact classical chiral symmetry could be expected to play a rôle like the chiral symmetry that holds for massless fermions. With massless fermions, the \(\text{tr} \tilde{F}^{\mu \nu} F_{\mu \nu}\) term is not anomalous, as we have seen. So \(\theta\) cannot be modified in that way.

It should be pointed out here that any effect of \(\theta\) terms can be removed by setting \(\theta = 0\) [5] or by making it explicitly dynamical, which forces the topological charge to vanish. A dynamical \(\theta\)-like term is essentially a Peccei–Quinn \((\varphi_{0} \text{tr} \tilde{F}^{\mu \nu} F_{\mu \nu})\) term without an axion particle. However, as only fields are dynamical in field theory, the field \(\varphi\) was used in [1] and a direct \((\varphi \text{tr} \tilde{F}^{\mu \nu} F_{\mu \nu})\) term can remove \(\theta\).

One may wonder how much freedom one has in choosing regularisations. Different regularisations have been used in the past and it is known that all do not lead to the same result. A key point is that symmetries of the action are generally sought to be preserved by the regularisation. Thus one is always looking for Lorentz invariant and gauge invariant regularisations for actions having such symmetries. Regularisations which maintain all symmetries are technically natural. We have seen that the Pauli–Villars regularisation conveniently respects the symmetry (3) of the fermion action. Non-symmetric regularisations are possible, but are not preferable in any way. Regularisations that break an existing classical symmetry like (3) may to that extent be termed technically unnatural. The artificial chiral symmetry here is not anomalous as a regularisation consistent with it has been demonstrated to exist. In these circumstances, working with regularisations inconsistent with it would be a needless and avoidable violation.

We end with the hope that the exact chiral symmetry in the presence of axions will be useful also in other calculations and even in curved spacetime.

References

[1] Peccei R and Quinn H 1977 Phys. Rev. Lett. 38 1440
[2] Weinberg S 1978 Phys. Rev. Lett. 40 223
[3] Włodarcik P 1978 Phys. Rev. Lett. 40 279
[4] Patrignani C et al 2016 Review of particle physics Chin. Phys. C 40 100001
[5] Dvali G and Funcke L 2016 arXiv:1608.08969
[6] Deser S, Duff M and Isham C 1980 Phys. Lett. B 93 419
[7] Kimura T 1969 Prog. Theor. Phys. 42 1191