The ‘Proton Spin’ Effect - Theoretical Status ’97

G.M. Shore

Department of Physics, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, U.K.

The theoretical status of the ‘proton spin’ effect is reviewed. The conventional QCD parton model analysis of polarised DIS is compared with a complementary approach, the composite operator propagator-vertex (CPV) method, each of which provides its own insight into the origin of the observed suppression in the first moment of $g_1^p$. The current status of both experiment and non-perturbative calculations is summarised. The future role of semi-inclusive DIS experiments, in both the current and target fragmentation regions, is described.

1. Introduction

It is now nearly a decade since the EMC collaboration announced measurements of the first moment of the polarised proton structure function $g_1^p$ which in a simple parton picture could be interpreted as showing that the quarks carry only a small fraction of the spin of the proton. This ‘proton spin’ problem has been the focus ever since of an extraordinary experimental and theoretical effort to extend and improve the data and provide a thorough understanding of the phenomenon in terms of QCD.

Perhaps the 1998 Montpellier conference will mark the tenth anniversary of the EMC paper with a comprehensive review of all this work. In this ninth anniversary review, I will instead take the opportunity to present a more individual look at what the ‘proton spin’ effect means, how it can be understood in non-perturbative QCD, and what future experiments it suggests.

The central theme of this review is the comparison of two complementary approaches to the description of deep inelastic scattering (DIS) - the parton model and the composite operator propagator-vertex (CPV) method. We will show how each provides its own insight into the ‘proton spin’ effect - the first from a quark-gluon constituent point of view and the second combining a holistic description of the proton with non-perturbative, target-independent, QCD physics.

The status of non-perturbative calculations using QCD spectral sum rules (QSSR) and lattice methods is then briefly reviewed, followed by a discussion of the current experimental situation for $g_1^p(x, Q^2)$ and the important unresolved questions concerning the small $x$ region. Finally, the present and future role of semi-inclusive DIS experiments, both in the current and target fragmentation regions, is considered. A recent proposal for testing the target-independent suppression mechanism suggested by the CPV approach is described and predictions for cross section moment ratios for $eN \rightarrow ehX$ are given.

2. The sum rule for $\Gamma_1^p$ and the OZI rule

The polarised structure functions are measured in polarised DIS (see Fig. 1), either $\mu p \rightarrow \mu X$ (EMC, SMC) or $ep \rightarrow eX$ (SLAC, HERMES).

\[ x \frac{d\Delta\sigma}{dx dy} = \frac{16\pi^2\alpha^2}{s} g_1^p(x, Q^2) + \ldots \] (1)
where $Q^2 = -q^2$ and $x = Q^2/2p_2q$ are the Bjorken variables, $y = Q^2/xs$ and $Y_P = (2-y)/y$. (The dots denote terms of $O(M^2 x^2/Q^2)$, including the second polarised structure function $g_2^p$, for which interesting new measurements are becoming available.)

On the theoretical side, $g_1^p$ is determined by the proton matrix element of two electromagnetic currents carrying large spacelike momentum. The sum rule for the first moment $\Gamma_1^p$ is derived using the twist 2, spin 1 terms in the operator product expansion (OPE) for the currents:

$$J^p(q)J^p(-q) \sim \frac{2e^{\rho_x\mu_q}}{Q^2} \times \left[ C_1^{NS}(\alpha_s)A_{\mu}^3 + C_1^{S}(\alpha_s)A_{\mu}^0 \right]$$

where the Wilson coefficients $C_1^{NS}(\alpha_s)$ and $C_1^{S}(\alpha_s)$ are now both known to $O(\alpha_s^3)$. It reads:

$$\Gamma_1^p(Q^2) = \int_0^1 dx \ g_1(x, Q^2) = \frac{1}{12} C_1^{NS}(a^3 + 1/a^8) + \frac{1}{9} C_1^{S} a^0(Q^2)$$

(3)

Here, $a^3$, $a^8$ and $a^0(Q^2)$ are the form factors in the forward proton matrix elements of the renormalised axial current, i.e.

$$\langle p, s|A_{\mu}^3|p, s \rangle = s_{\mu} \frac{1}{2} a^3$$

$$\langle p, s|A_{\mu}^8|p, s \rangle = s_{\mu} \frac{1}{2\sqrt{3}} a^8$$

$$\langle p, s|A_{\mu}^0|p, s \rangle = s_{\mu} a^0(Q^2)$$

(4)

where $p_\mu$ and $s_\mu$ are the momentum and polarisation vector of the proton.

Because of the chiral $U_A(1)$ anomaly, $\partial^{\mu} A_{\mu}^0 - 2n_f Q \sim 0$, the flavour singlet current $A_{\mu}^0$ is not conserved. It is renormalised and mixes with the topological density. Defining the bare operators $A_{\mu B}^0 = \sum \bar{q}_\mu \gamma_5 q$ and $Q_B = \frac{3}{2} e^{\mu\nu\rho\sigma} \text{tr}G_{\mu\nu}G_{\rho\sigma}$, we have (for $n_f$ flavours)

$$A_{\mu}^0 = Z A_{\mu B}^0$$

$$Q = Q_B - \frac{1}{2n_f}(1-Z)\partial^{\mu} A_{\mu B}^0$$

(5)

where $Z$ is a divergent renormalisation constant.

The associated anomalous dimension $\gamma$ is known to 3 loops. Matrix elements of $A_{\mu}^0$ therefore have a non-trivial renormalisation group (RG) scale dependence governed by $\gamma$. In particular,

$$\frac{dt}{dt} a^0 = \gamma a^0$$

(6)

where $t = \ln Q^2/\Lambda^2$, so that the singlet axial charge $a^0(Q^2)$ is scale dependent. This is crucial to understanding the ‘proton spin’ effect in QCD.

The axial charges $a^3$ and $a^8$ are known in terms of the $F$ and $D$ constants ($\alpha^3 = F + D$, $\alpha^8 = 3F - D$) found from neutron and hyperon beta decays. The interest of the sum rule centres on the flavour singlet axial charge $a^0(Q^2)$. In the absence of an alternative experimental derivation of $a^0(Q^2)$, the simplest ansatz is to assume that it obeys the OZI (Zweig) rule, i.e. $a^0(Q^2) = a^8$. This gives a theoretical prediction for $\Gamma_1^p$ which is known as the Ellis-Jaffe sum rule. It is now known to be violated (see sect. 6), with $a^0(Q^2)$ strongly suppressed relative to $a^8$. This is what is known as the ‘proton spin’ problem.

In fact, it is not at all surprising that the OZI rule should fail in this case. The first clue is the anomaly-induced scale dependence of $a^0(Q^2)$. If the OZI rule were to hold, at what scale should it be applied? For the same reason, it is immediately clear that $a^0(Q^2)$ cannot really measure spin. Moreover, it is known that the pseudovector and pseudoscalar channels are linked through the Goldberger-Treiman relations. Since large anomaly-induced OZI violations are known to be present in the pseudoscalar channel ($U_A(1)$ problem, $\eta'$ mass, etc.) it is natural to find them also for $a^0(Q^2)$ in the pseudovector channel.

While this immediately resolves the ‘proton spin’ problem, clearly we want to understand the origin of the suppression in $a^0(Q^2)$ much more deeply. The following two sections describe two complementary approaches to this question – the
conventional QCD parton model and the CPV method developed in refs. [2–4].

3. The QCD parton model

In the most simple parton model, where the proton structure for large $Q^2$ DIS is described by parton distributions corresponding to free quarks only, the polarised structure function is given by

$$g_1^p(x) = \frac{1}{2} \sum_{i=1}^{n_f} e_i^2 \Delta q_i(x)$$

(7)

where $\Delta q_i(x) = q_i^+(x) + q_i^-(x) - q_i^+(x) - \bar{q}_i(x)$ is the difference of the distributions of quarks with helicities parallel and antiparallel to the nucleon spin. It is convenient to work with the flavour non-singlet and singlet combinations:

$$\Delta q^{NS}(x) = \sum_{i=1}^{n_f} \left( e_i^2 < e_i^2 > - 1 \right) \Delta q_i(x)$$

$$\Delta q^S(x) = \sum_{i=1}^{n_f} \Delta q_i(x)$$

(8)

In this model, the first moment of the flavour singlet quark distribution $\Delta q^S = \int_0^1 dx \Delta q^S(x)$ can indeed be identified as the sum of the helicities of the quarks. Interpreting the structure function data in this model then leads to the conclusion that the quarks carry only a small fraction of the spin of the proton – the ‘proton spin’ problem.

However, this simple model leaves out many important features of QCD – gluons, RG scale dependence, the chiral $U_A(1)$ anomaly, etc. When these effects are included, in the QCD parton model, the naive identification of $\Delta q^S$ with spin no longer holds and the experimental results for $g_1^p$ are readily accommodated.

The QCD parton model picture of DIS is shown in Fig. 2. The polarised structure function is written in terms of both quark and gluon distributions as follows:

$$g_1^p(x, Q^2) = \frac{1}{9} \int_x^1 \frac{dy}{y} \left( C^{NS} \left( \frac{x}{y} \right) \Delta q^{NS}(y, t) + C^S \left( \frac{x}{y} \right) \Delta q^S(y, t) \right) + C^g \left( \frac{x}{y} \right) \Delta g(y, t)$$

(9)

where $C^S$, $C^g$ and $C^{NS}$ are perturbatively calculable functions related to the Wilson coefficients in sect. 2 and the quark and gluon distributions have a priori a $t = \ln Q^2/\Lambda^2$ dependence.

The RG evolution (DGLAP) equations for these polarised distributions are:

$$\frac{d}{dt} \Delta q^{NS}(x, t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P^{NS} \left( \frac{x}{y} \right) \Delta q^{NS}(y, t)$$

(10)

and, abbreviating the notation of (10),

$$\frac{d}{dt} \left( \Delta q^S \right) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left( P^q \frac{P_{qq}}{P_{gg}} \right) \left( \Delta q^S \right)$$

(11)

showing the mixing between the singlet quark and the gluon distributions. The splitting functions $P$ are also calculable in perturbative QCD, their moments being related to the anomalous dimensions of the series of increasing spin operators appearing in the OPE (2).

In this language, the first moment sum rule for $g_1^p$ reads:

$$\Gamma^p(Q^2) = \frac{1}{9} \left[ C^{NS} \Delta q^{NS} + C^S \Delta q^S + C^g \Delta g \right]$$

(12)

where $\Delta q^{NS}$, $\Delta q^S$ and $\Delta g$ are the first moments of the above distributions. Comparing with (3), we see that the axial charge $a^0(Q^2)$ is identified with a linear combination of the first moments of the singlet quark and gluon distributions. It is often, though not always, the case that the moments of parton distributions can be identified in one-to-one correspondence with the matrix elements of local operators. The polarised first moments are special in that two parton distributions correspond to the same local operator. This adds an extra subtlety to the identification.

The RG equations for the first moments of the parton distributions follow immediately from
(10,11) and depend on the matrix of anomalous dimensions for the lowest spin, twist 2 operators. This introduces a renormalisation scheme ambiguity. The issue of scheme dependence has been studied thoroughly by Ball, Forte and Ridolfi [12] and an excellent summary can be found in ref. [13]. It is shown there that it is possible to choose a scheme known as the Adler-Bardeen or AB scheme (strictly, a class of schemes [12, 13]) for which the parton distributions satisfy the following RG equations:

\[
\frac{d}{dt} \Delta q^S = 0 \quad \frac{d}{dt} \Delta q^S = 0
\]

\[
\frac{d}{dt} \alpha_s \Delta g(t) = \gamma \left( \frac{\alpha_s}{2\pi} \Delta g(t) - \frac{1}{n_f} \Delta q^S \right)
\]

with the implication \(C_2^g = -n_f \frac{\alpha_s}{2\pi} C_2^S\). It is then possible to make the following identifications with the axial charges:

\[
a_g^3 = \Delta u - \Delta d
\]

\[
a_g^8 = \Delta u + \Delta d - 2\Delta s
\]

\[
a_g^0(Q^2) = \Delta q^S - n_f \frac{\alpha_s}{2\pi} \Delta g(Q^2)
\]

where \(\Delta u = \int_0^1 dx (\Delta u(x,t) + \Delta \bar{u}(x,t))\) etc. Notice that in the AB scheme, the singlet quark distribution \(\Delta q^S\) (which is often written as \(\Delta \Sigma\)) is scale independent. All the scale dependence of the axial charge \(a_g^0(Q^2)\) is assigned to the gluon distribution \(\Delta g(Q^2)\).

This was the identification originally introduced for the first moments by Altarelli and Ross [14]. We emphasise that (13) is true only in a particular renormalisation scheme (the AB scheme) and that it is only in this scheme that the identifications (14) hold.

In this picture, the Ellis-Jaffe sum rule follows from the assumption that in the proton both \(\Delta s\) and \(\Delta g(Q^2)\) are zero. This is equivalent to the naive OZI approximation \(a_g^0(Q^2) = a_g^8\) described above. Clearly, given the RG scale dependence of \(a_g^0(Q^2)\), this assumption is in contradiction with QCD where the anomaly requires \(a_g^0(Q^2)\) to scale with the anomalous dimension \(\gamma\).

Since neither \(\Delta \Sigma\) nor \(\Delta g(Q^2)\) are currently measurable in other processes, the parton model is unable to make a quantitative prediction for the first moment \(\Gamma_1\). While the model can accommodate the observed suppression, it cannot predict it.

An interesting conjecture, proposed in the original paper of Altarelli and Ross [14], is that the observed suppression in \(a_g^0(Q^2)\) is due overwhelmingly to the gluon distribution \(\Delta g(Q^2)\). If so, the strange quark distribution \(\Delta s \simeq 0\) in the proton and so \(\Delta \Sigma \simeq a_g^8\). This is entirely plausible because it is the anomaly (which is due to the gluons and is responsible for OZI violations) which is responsible for the scale dependence in \(a_g^0(Q^2)\) and \(\Delta g(Q^2)\) whereas (in the AB scheme) \(\Delta \Sigma\) is scale invariant. The essence of this conjecture will reappear in the next section where we describe the CPV method.

To test this conjecture, we need to find a way to measure \(\Delta g(Q^2)\) itself, rather than the combination \(a_g^0(Q^2)\). One possibility (see also sect. 6) is to exploit the different scaling behaviours of \(\Delta q^S(x)\) and \(\Delta g(x,Q^2)\) to distinguish their contributions in measurements of \(g_1^p(x,Q^2)\) at different values of \(Q^2\). A second is to extract \(\Delta g(x,Q^2)\) from processes such as open charm production, \(\gamma^* g \rightarrow c\bar{c}\), which will be studied in various forthcoming experiments at COMPASS, RHIC, etc.

4. The CPV method for DIS

This approach to sum rules in DIS was developed in collaboration with S. Narison and G. Veneziano in a series of papers on the ‘proton spin’ effect. The starting point, as described above, is the use of the OPE in the proton matrix element of two currents. This gives the standard form for a generic structure function moment:

\[
\int_0^1 dx x^{n-1} F(x;Q^2) = \sum_i C_i^n(Q^2) \langle p|O_i^n(0)|p\rangle (15)
\]

where \(O_i^n\) are the set of lowest twist, spin \(n\) operators in the OPE and \(C_i^n(Q^2)\) the corresponding Wilson coefficients. In the CPV approach, we now factorise the matrix element into the product of composite operator propagators and vertex functions, as illustrated in Fig. 3.
perturbative, target-dependent vertex functions then takes the form

\[ \sum_i \sum_j C_j^{(n)}(Q^2) \langle 0 | T \bar{\mathcal{O}}_j^{(n)} \mathcal{O}_i | 0 \rangle \Gamma_{\bar{\mathcal{O}},pp} \]

As emphasised in refs. [3, 4], it is important to recognise that this decomposition of the matrix function into three pieces — first, the Wilson coefficients \( C_j^{(n)}(Q^2) \) which control the \( Q^2 \) dependence and can be calculated in perturbative QCD; second, non-perturbative but target-independent QCD correlation functions (composite operator propagators) \( \langle 0 | T \bar{\mathcal{O}}_j^{(n)} \mathcal{O}_i | 0 \rangle \); and third, non-perturbative, target-dependent vertex functions \( \Gamma_{\bar{\mathcal{O}},pp} \) describing the coupling of the target proton to the composite operators of interest. The vertex functions cannot be calculated directly from first principles. They encode the information on the nature of the proton state and play an analogous role to the parton distributions in the more conventional parton picture.

As emphasised in refs. [3, 4], it is important to recognise that this decomposition of the matrix elements into products of propagators and proper vertices is exact, independent of the choice of the set of operators \( \mathcal{O}_i \). In particular, it is not necessary for \( \mathcal{O}_i \) to be in any sense a complete set. All that happens if a different choice is made is that the vertices \( \Gamma_{\bar{\mathcal{O}},pp} \) themselves change, becoming ‘1PI’ with respect to a different set of composite fields. Of course, while any set of \( \mathcal{O}_i \) may be chosen, some will be more convenient than others. Clearly, the set of operators should be as small as possible while still capturing the essential physics (i.e. they should encompass the relevant degrees of freedom) and indeed a good choice can result in vertices \( \Gamma_{\bar{\mathcal{O}},pp} \) which are both RG invariant and closely related to low energy physical couplings, such as \( g_{\pi NN} \) or \( g_{\pi\gamma\gamma} \). In this case, (16) provides a rigorous relation between high \( Q^2 \) DIS and low-energy meson-nucleon scattering.

For the first moment sum rule for \( g_i^p \), it is most convenient to use the chiral anomaly immediately to re-express \( a^0(Q^2) \) in terms of the forward matrix element of the topological density \( Q \), i.e.

\[ a^0(Q^2) = \frac{1}{2M} 2n_f \langle p| Q | p \rangle \]

\[ (17) \]

Our set of operators \( \mathcal{O}_i \) is then chosen to be the renormalised flavour singlet pseudoscalars \( Q \) and \( \Phi_5 \) where, up to a crucial normalisation factor, the corresponding bare operator is \( \Phi_{5B} = \sum g_{\gamma\gamma} q \). This normalisation factor is chosen such that in the absence of the anomaly, or more precisely in the OZI limit of QCD (see footnote 3), \( \Phi_5 \) would have the correct normalisation to couple with unit decay constant to the \( U(1) \) Goldstone boson which would exist in this limit. This also ensures that the vertex is RG scale independent [3]. We then have

\[ \Gamma^p_{1 \, \text{sing}} = \frac{1}{9} \frac{1}{2M} 2n_f C_1^5(\alpha_s) \times \]

\[ \left[ \langle 0 | T \bar{Q} Q | 0 \rangle \Gamma_{Qpp} + \langle 0 | T \bar{Q} \Phi_5 | 0 \rangle \Gamma_{\Phi_5 pp} \right] \]

\[ (18) \]

where the propagators are at zero momentum and the vertices are 1PI wrt \( Q \) and \( \Phi_5 \) only. This is illustrated in Fig. 4.
The composite operator propagator in the first term is the zero-momentum limit of the QCD topological susceptibility $\chi(k^2)$, viz.

$$\chi(k^2) = \int dx e^{i k \cdot x} i\langle 0 | T Q(x) Q(0) | 0 \rangle$$

(19)

The anomalous chiral Ward identities show that $\chi(0)$ vanishes for QCD with massless quarks, in contrast to pure Yang-Mills theory where $\chi(0)$ is non-zero. Furthermore, it can be shown\(^4\) that the propagator $\langle 0 | T Q \Phi_2 | 0 \rangle$ at zero momentum is simply the square root of the first moment of the topological susceptibility. We therefore find:

$$\Gamma_{1\;\text{sing}}^p = \frac{1}{9} \frac{1}{2 M^2} 2 n_f \, C^S(\alpha_s) \, \sqrt{\chi(0)} \, \Gamma_{\Phi_{pp}}$$

(20)

The quantity $\sqrt{\chi'(0)}$ is not RG invariant and scales with the anomalous dimension $\gamma$, i.e.

$$\frac{d}{dt} \sqrt{\chi'(0)} = \gamma \sqrt{\chi'(0)}$$

(21)

On the other hand, the proper vertex has been chosen specifically so as to be RG invariant. The renormalisation group properties of this decomposition are crucial to the resolution proposed in ref.\(^3\) of the ‘proton spin’ problem.

The proposal of ref.\(^2\)\(^3\) is that we should expect the source of OZI violations to lie in the RG non-invariant, and therefore anomaly-sensitive, terms, i.e. in $\chi'(0)$. Notice that we are using RG non-invariance, i.e. dependence on the anomalous dimension $\gamma$, merely as an indicator of which quantities are sensitive to the anomaly and therefore likely to show OZI violations. Since the anomalous suppression in $\Gamma_{1\;\text{sing}}^p$ is assigned to the composite operator propagator rather than the proper vertex, the suppression is a target independent property of QCD related to the chiral anomaly, not a special property of the proton structure\(^5\).

To convert this into a quantitative prediction we use the OZI approximation for the vertex $\Gamma_{\Phi_{pp}}$ and a QCD spectral sum rule estimate of the first moment of the topological susceptibility. We find\(^2\), for $n_f = 3$,

$$\sqrt{\chi(0)}_{Q^2=10\text{GeV}^2} = 23.2 \pm 2.4 \text{ MeV}$$

(22)

This is a suppression of approximately a factor 0.6 relative to the OZI value $f_x/\sqrt{6}$.

Our final result, in the chiral limit, is then

$$a^0(Q^2 = 10\text{GeV}^2) = 0.35 \pm 0.05$$

(23)

from which we deduce

$$\Gamma_{1\;\text{sing}}^p \big|_{Q^2=10\text{GeV}^2} = 0.143 \pm 0.005$$

(24)

This is to be compared with the Ellis-Jaffe (OZI) prediction of $a^0 = 0.58$.

The complementary nature of the QCD parton model and CPV methods is now clear. Both involve at present incalculable non-perturbative functions describing the proton state – the quark and gluon distributions in the parton picture and the 1PI vertices in the CPV method. Both exhibit a degree of universality – the same parton distributions may be used in different QCD processes such as DIS or hadron-hadron collisions, while the vertices (when they can be identified with low-energy couplings) also provide a link between high $Q^2$ DIS and soft meson-nucleon interactions.

One of the main advantages of the CPV method is that some non-perturbative information which is generic to QCD, i.e. independent of the target, is factored off into the composite operator propagator. This allows us to distinguish between non-perturbative mechanisms which are generic to all QCD processes and those which are specific to a particular target. As explained above, our contention is that the anomalous suppression in the first moment of $g_1^p$ is of the first, target-independent, type. This target-independence conjecture can in principle be tested by DIS with non-nucleon targets. This may effectively be realised in semi-inclusive DIS (see sect. 7 and ref.\(^\text{[H]}\)).

Both the parton and CPV methods allow a natural conjecture in which the origin of this suppression is attributed to ‘glue’ – either through
a large polarised gluon distribution $\Delta g(Q^2)$ in the parton description or due to an anomalous suppression of the first moment of the topological susceptibility $\sqrt{\chi(0)}$ in the CPV description. These conjectures are based on assumptions that the appropriate RG invariant quantities, $\Delta \Sigma$ or $\Gamma_{\Phi_{3,pp}}$, obey the OZI rule. The motivation for this is particularly strong in the CPV case, where it is supported by a range of evidence from low-energy meson phenomenology in the $U_A(1)$ channel.

The two approaches therefore provide related, but complementary, insights into the nature of the ‘proton spin’ effect. Perhaps the most important message of this review is that both insights are needed and both methods have a full part to play in understanding this intriguing and subtle phenomenon.

5. Non-perturbative results – QSSR and lattice

The challenge to non-perturbative calculational methods in QCD is therefore to evaluate $a^0(Q^2)$, either directly through the matrix element $\langle p|A^0_\mu|p \rangle$ or, using the anomaly, from $\langle p|Q|p \rangle$. Alternatively, if we accept the conjecture proposed above, we can deduce $a^0(Q^2)$ from a calculation of $\chi'(0)$. In any case, the topological susceptibility is a fundamental correlation function in QCD of great importance in a variety of contexts and deserves to be studied in its own right.

These non-perturbative calculations can be performed either using lattice gauge theory or the method of QCD spectral sum rules (QSSR). It must be stressed that to be meaningful, these calculations must be performed in full QCD, or at the very least in an approximation that incorporates chiral $U_A(1)$ breaking by the anomaly. The occurrence of massless singularities is quite different in truncations such as large $N_c$ or the quenched approximation (see footnote 3) where the anomaly is not properly included, due to the erroneous appearance of a $U_A(1)$ Goldstone boson.

Lattice calculations of $a^0(Q^2)$ from $\langle p|A^0_\mu|p \rangle$ have been performed by several authors and are reviewed in ref.\[17\]. Since only quenched configurations have been used, the results cannot give the true $a^0(Q^2)$. However, a major step towards the true answer can be made by including explicitly OZI-violating ‘disconnected’ diagrams with purely gluonic intermediate states as well as the OZI respecting ‘connected’ diagrams (see Fig. 5).

Using this method, Dong, Lagae and Liu\[18, 19\] find $a^0 = 0.25 \pm 0.12$, and confirm that the contribution from the connected diagrams ($\alpha_{\text{conn}} = 0.62 \pm 0.09$) reproduces the OZI expectation. It is not clear, however, how much this result will change if non-quenched gauge configurations including the effects of dynamical fermions are used.

For $\langle p|Q|p \rangle$, so far only quenched calculations have been performed\[17\], although the Pisa group\[20\] are currently running simulations in full QCD. Existing calculations of proton matrix elements using QSSR have also not fully incorporated OZI breaking.

The first moment of the topological susceptibility, $\chi'(0)$, has been evaluated using QSSR in ref.\[3\] and the result is quoted above in (22). This shows $\sqrt{\chi'(0)}$ to be suppressed to around 0.6 of the OZI value. The method and calculational details are described in full in\[3\].

The application of QSSR to the $U_A(1)$ sector of QCD has been criticised repeatedly by Ioffe (see e.g.\[21\]) on two grounds: (i) that there are important neglected contributions from ‘instantons’, i.e. higher dimensional condensates, and (ii) that when the strange quark mass is included, QSSR results for current-current correlators show quite unrealistic $SU(3)$ breaking. Neither criticism is valid, and both will be refuted in detail in a forthcoming paper\[22\]. In fact, the stabilisation scale in the calculation of ref.\[3\] viz. $\tau^{-1} \sim 2 \text{GeV}^2$, is sufficiently big for higher
dimensional condensates to be suppressed, and indeed the calculation does display the hierarchy of gluonium to light meson hadronic scales anticipated by ref.\[23\]. Ioffe’s second criticism is based on a calculation\[21\] where radiative corrections are not properly implemented, giving a false result. The correct results are given in ref.\[22\], where we extend our previous analysis systematically beyond the chiral limit using a new set of generalised Goldberger-Treiman relations.

$\chi'(0)$ is a particularly difficult correlation function to calculate on the lattice, requiring algorithms that implement topologically non-trivial configurations in a sufficiently fast and efficient way. However, very preliminary results from the Pisa group\[20\] of calculations in full QCD with dynamical quarks indicate a value of $\sqrt{\chi'(0)} \simeq 16 \pm 3$MeV. Given the preliminary nature of the lattice simulations, the agreement with the QSSR result (22) is encouraging and further results from the lattice are eagerly awaited.

6. Experiment and the small $x$ region

The most recent published results from the SMC collaboration on $g_1^p$ are given in ref.\[24\]. For the first moment, SMC quote:

$$\Gamma_1^p\big|_{Q^2=10\text{GeV}^2} = 0.136 \pm 0.013 \pm 0.009 \pm 0.005$$

(25)

where the last error is a theoretical one, related to the evolution of the measured data to a standard $Q^2$. This implies

$$a^0(Q^2 = 10\text{GeV}^2) = 0.28 \pm 0.16$$

(26)

This is to be compared with the prediction (23,24) obtained in sect. 4.

Although this agreement is very promising and suggests strongly (see also below) that the explanation of the ‘proton spin’ effect in terms of a suppression in the topological susceptibility is correct, there is an important uncertainty in the presentation of this data. In fact, SMC only take measurements in the region $x > 0.003$. The contribution of the small $x$ region to $\Gamma_1^p$ is estimated in ref.\[24\] by a dubious Regge extrapolation, which gives only a very small addition of $0.0042 \pm 0.0016$. Ball, Forte and Ridolfi\[12\] (see also \[24\]) have proposed an alternative small $x$ extrapolation, using the data to fit the parton densities at small $x$ at a low $Q^2$ scale, then using the perturbative QCD evolution equations to deduce the form of $g_1^p(x, Q^2 = 10\text{GeV}^2)$. Examples of their fits are shown in Fig. 6.

The important feature of the evolution equations is that while $\Delta \Sigma(x, Q^2)$ falls with increasing $Q^2$, the polarised gluon distribution $\Delta g(x, Q^2)$ rises. The net effect is that $g_1^p(x, Q^2)$ is driven strongly negative at $Q^2 = 10\text{GeV}^2$ for sufficiently small $x$. This gives a potentially large negative contribution to the first moment $\Gamma_1^p$, but with relatively large errors.

This summer, SMC\[26,27\] have released new, still preliminary, data on $g_1^p$ based on the 1996 run. See Fig. 7.
It shows two significant features compared with the older data – the smallest \( x \) point has dropped appreciably, while there is a small rise in the medium \( x \) data points.

In presenting these results (see Fig. 8), SMC have given fits to the small \( x \) region using both the ‘Regge’ and ‘QCD’ extrapolations. They quote the following results. For the measured range (including an uncontroversial \( x > 0.7 \) extrapolation):

\[
\int_{0.003}^{1} dx \ g_1^p(x; Q^2 = 10\text{GeV}^2) = 0.146 \pm 0.006 \pm 0.009 \pm 0.005 \quad (27)
\]

while with small \( x \) extrapolations:

\[
\Gamma_1^{\text{Regge}}_{Q^2=10\text{GeV}^2} = 0.149 \pm 0.006 \pm 0.009 \pm 0.005(28)
\]

\[
\alpha^0(Q^2 = 10\text{GeV}^2)^{\text{Regge}} = 0.41 \pm 0.11 \quad (29)
\]

and

\[
\Gamma_1^{QCD}_{Q^2=10\text{GeV}^2} = 0.135 \pm 0.006 \pm 0.009 \pm 0.011(30)
\]

\[
\alpha^0(Q^2 = 10\text{GeV}^2)^{QCD} = 0.27 \pm 0.15 \quad (31)
\]

Notice that these results are for \( g_1^p \) taken from proton data alone. The values of \( \Gamma_1^p \) and \( \alpha^0 \) for \( g_1^d \) taken from combined proton and deuteron data are systematically lower [24,25].

The small \( x \) region is therefore an important challenge to experimentalists. Resolving the above uncertainty in \( \Gamma_1^p \) and \( \alpha^0 \) will enable a rigorous test of the CPV conjecture and the link with the topological susceptibility. In parton terms, measuring the small \( x \) region at different \( Q^2 \) will, through analysis of the RG scaling behaviour, enable the gluon distribution \( \Delta g(x, Q^2) \) to be isolated [22,28] accurately. These will be important tasks for future experiments at HERA with a polarised p beam.

7. Semi-inclusive DIS

As well as pushing further into the small \( x \) frontier, an increasingly important role in the study of polarisation and the proton structure will be played in future by semi-inclusive processes. We have already mentioned the importance of open charm production in isolating the gluon distribution \( \Delta g(x, Q^2) \). In this section, we describe what may be learnt from semi-inclusive DIS, both in the current and target fragmentation regions.
events are described by parton fragmentation functions \( D_i^h(\frac{x_1}{1-x}, Q^2) \), where \( i \) denotes the parton, while the target fragmentation events are described by fracture functions\[^{[29]}\] \( M_i^{hN}(x, \bar{z}, Q^2) \) representing the joint probability distribution for producing a parton with momentum fraction \( x \) and a detected hadron \( h \) (with momentum \( p_h' \)) carrying energy fraction \( \bar{z} \) from a nucleon \( N \).

The lowest order cross section for polarised semi-inclusive DIS is:

\[
x \frac{d\Delta\sigma}{dx dy d\bar{z}} = \frac{Y_p}{2} \frac{4\pi\alpha^2}{s} \sum_i c_i^2 \left[ \Delta M_i^{hN}(x, \bar{z}, Q^2) + \Gamma_i \right] D_i^h \left( \frac{\bar{z}}{1-x}, Q^2 \right)
\]

(32)

where \( \bar{z} = E_h/E_N \)\[^{[31]}\]. The notation is slightly different from sect. 3. Here, \( \Delta q_i(x) \) refers to quarks and antiquarks separately and a sum over both is implied. \( \Delta M_i^{hN}(x, \bar{z}, Q^2) \) is the polarisation asymmetry of the fracture function. The NLO corrections to (32) are given in ref.\[^{[31]}\].

Quite different physics emerges from the two regions. A programme of semi-inclusive DIS in the current fragmentation region has already been carried out by SMC\[^{[31]}\] and will be pursued by HERMES and COMPASS. The particular interest is that it allows the distributions for quarks and antiquarks to be separated, giving information on the polarised 'valence' and 'sea' distributions defined as \( \Delta q_i^v(x) = \Delta q_i(x) - \bar{\Delta} \bar{q}_i(x) \) and \( \Delta q_i^{sea}(x) = 2\Delta \bar{q}_i(x) \). These are found by comparing cross section asymmetries for positive and negative charged hadrons \( h \), with the assumption that the fragmentation functions \( D_i^h \) in (32) satisfy isospin and charge conjugation symmetry and are helicity independent\[^{[12]}\]. The first results for the moments \( \Delta u^v \) and \( \Delta d^v \) are given in ref.\[^{[31]}\].

Recently, a new proposal to exploit semi-inclusive DIS in the target fragmentation region to elucidate the 'proton spin' effect has been presented\[^{[3]}\]. The idea is to test the 'target independence' conjecture suggested by the CPV method by using semi-inclusive DIS in effect to make measurements of the polarised structure functions of other hadronic targets besides the proton and neutron.

The basic conjecture of ref.\[^{[3]}\] is that for any hadron, the singlet axial charge in (3) can be substituted by its OZI value multiplied by a universal (target-independent) suppression factor \( a(Q^2) \) determined, up to radiative corrections, by the anomalous suppression of the first moment of the topological susceptibility \( \chi(0) \). For example, for a hadron containing only \( u \) and \( d \) quarks, the OZI relation is simply \( a_0 = a^8 \), so we predict:

\[
\Gamma_1 = \frac{1}{12} C_1^NS \left( a^4 + \frac{1}{3} (1 + 4s) a^8 \right)
\]

(33)

where

\[
s(Q^2) = \frac{C_1^S(\alpha_s)}{C_1^NS(\alpha_s)} a^{0}(Q^2)
\]

(34)

Since \( s \) is target independent, we can use the value measured for the proton to deduce \( \Gamma_1 \) for any other hadron simply from the flavour non-singlet axial charges. From our spectral sum rule estimate of \( \chi(0) \), we find \( s \sim 0.66 \) at \( Q^2 = 10 \text{GeV}^2 \), while the central value of the SMC result (26) gives \( s \sim 0.55 \).

The non-singlet axial charges for a hadron \( B \) are given by the matrix elements of the flavour octet axial currents, so can be factorised into products of \( SU(3) \) Clebsch-Gordon coefficients times reduced matrix elements. Together with the target independence conjecture, this allows predictions to be made for ratios of the first moments of the polarised structure functions \( g_1^B \) for different \( B \). Some of the most intriguing are:

\[
\Gamma_1^o/\Gamma_1^t = \frac{2s - 1 - 3(2s + 1) F/D}{2s + 6s F/D}
\]

(35)

\[
\Gamma_1^{\Delta^{++}}/\Gamma_1^{-} = \frac{\Sigma_{c}^{++}/\Sigma_{c}^{0}}{\Gamma_1^{\Delta^{--}}/\Gamma_1^{\Delta^{-}}} = \frac{2s + 2}{2s - 1}
\]

(36)

where \( \Sigma_{c}^{++} \) (\( \Sigma_{c}^{0} \)) is the state with valence quarks \( uuc \) (\( ddc \)). The results for \( \Delta \) and \( \Sigma_c \) are particularly striking because of the \( 2s - 1 \) denominator factor, which is very small for the range of \( s \) favoured by experiment. These examples therefore show spectacular deviations from the valence quark counting (OZI) expectations, which would give the ratio 4.

The proposal of ref.\[^{[3]}\] is that these ratios can be realised in semi-inclusive DIS in a kinematical region where the detected hadron \( h \) (a pion or D meson in these examples) carries a large target.
energy fraction, i.e. \( \tilde{z} \) approaching 1. While this process is most rigorously described in terms of fracture functions, it can be pictured as the single Regge exchange shown in Fig. 11. The fracture function description, which utilises the recently introduced extended fracture functions of ref. [33, 34], may be found in ref. [3].

The moment of the polarised structure function \( g_1^B \) of the Reggeon \( B \) can be extracted from the polarisation asymmetry of the differential cross section moment in the limit \( z \to 1 \):

\[
\frac{1}{2} \int_0^{1-z} dx \frac{d\Delta \sigma_{\text{target}}}{dx dy dz dt} = \frac{Y_B}{4\pi \alpha^2} \Delta f(z, t) \int_0^1 dx_B \frac{g_1^B(x_B, t; Q^2)}{s} \Delta f(0, t) \Delta x (10)
\]

where \( z = p_2 \cdot q / p_2 \cdot q, x_B = Q^2 / 2k \cdot q, 1 - z = x / x_B, \) and \( t = -k^2 \ll O(Q^2) \) so that \( z \approx \tilde{z} \). The emission factor \( \Delta f(z, t) \) cancels in the ratios.

Since the predictions (36) depend only on the \( SU(3) \) properties of \( B \), together with target independence, they will hold equally well when \( B \) is interpreted as a Reggeon rather than a pure hadron state. The ratios (36) can therefore be found (see e.g. Fig. 12) by considering the processes \( e p \to e\pi^- (D^-) X \) and \( en \to e\pi^+ (D^0) X \).

Of course, the ratios (36) are only obtained in the limit as \( z \) approaches 1, where the reaction \( eN \to ehX \) is dominated by the process in which most of the target energy is carried through into the final state \( h \) by a single quark. At the opposite extreme, for \( z \) approaching 0, the detected hadron carries only a small fraction of the target nucleon energy and has no special status compared to the other inclusive hadrons \( X \). In this limit, the ratio of cross section moments for \( ep \to e\pi^- (D^-) X \) and \( en \to e\pi^+ (D^0) X \) is simply the ratio of the structure function moments for the proton and neutron (35).

Interpolating between these limits, we expect the ratios of \( \int_0^{1-z} dx \frac{d\Delta \sigma_{\text{target}}}{dx dy dz dt} \) in the range \( 0 < z < 1 \) for \( en \to e\pi^+ (D^0) X \) over \( ep \to e\pi^- (D^-) X \) to look like the sketch in Fig. 13.

Fig.11 Single Reggeon exchange model of \( ep \to ehX \).

Fig.12 Quark diagram for the \( NhB \) vertex in the reaction \( ep \to e\pi^- X \) where \( B \) has the quantum nos. of \( \Delta^{++} \).

![Fig.11](image1.png)

![Fig.12](image2.png)

![Fig.13](image3.png)
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