Contact force analysis in a planar mechanism with translational clearance joint considering complex contact modes

M B Qian¹,², Z Qin², S Z Yan¹,³ and L Zhang¹

¹State Key Laboratory of Tribology, Department of Mechanical Engineering, Tsinghua University, Beijing, 100084, P R China
²Department of Mechanical Engineering, Zhejiang A & F University, Hangzhou, 311300, China
E-mail: yansz@mail.tsinghua.edu.cn

Abstract. A translational joint model with clearance was established. A contact mode identification method was derived on the basis of the classification of contact modes. The positional relationships were classified into three different contact modes. The identification method was applied to a planar mechanism with translational joint. The influences of clearance size and velocity on dynamic characteristics between them were studied. The mean square error value of contact force can be used to evaluate the influence of the clearance on the dynamic characteristics of the mechanism.

1. Introduction
Clearance is an important factor affecting the dynamic performance of kinematic pairs because its existence is a necessary condition for relative motion between the connected components thereof [1-3]. A translational joint is a common motion pair: collisions between the translational joint sub-elements during the process of movement cause changes in dynamic characteristics due to the clearances therein [4-7].

At present, much research has been carried out on the subject of the effect of joint clearances on the dynamic characteristics of mechanical systems [8-11]. Translational joints with clearance are the research focus. By using a planar mechanism as an example, Jia et al [12,13] studied the influence of the clearance of the tripod ball sliding joint on dynamic characteristics of the mechanism, and a force constraint equation was established when there is a clearance in the kinematic pair. Song et al [14] proposed a dynamic analysis method for planar linkages, which considers the impact of translational joint with clearances. A virtual prototype model of a mechanism with translational joint with clearance was established based on the study of contact force and Coulomb friction force of the non-linear spring damper by Tong et al [15]. This model can reflect the collision and friction phenomenon between the sub-elements of translational joint with clearance.

To sum up, there is little research available on the motion state of translational joints and the contact mode between two elements when the clearance is considered. There is no systematic study of the motion characteristics of the two elements of the translational joint with clearance: however, the research in this field is important in any theoretical study and practical design of the dynamic behaviour of the mechanism. Here, we established a translational joint model with clearance and
derived a contact-mode identification method on the basis of the classification of contact modes. To study the non-linear dynamic behaviour of mechanism with clearance, numerical calculation and simulation of a planar mechanism were conducted.

2. Translational joint with clearance model

Figure 1 shows a simplified model of a translational joint with clearance. The slider and the slide way are represented by \( i \) and \( j \) respectively. The local coordinate systems attached to \( i \) and \( j \) are \( O_i-x_iy_i \) and \( O_j-x_jy_j \), respectively. Points \( P, Q, R \) and \( S \) on the surface of the inner hole of the slideway refers to the geometric contact limits for the slider [16].

![Figure 1. Two different contact conditions of translational joint with clearance (a) Free movement; (b) contact deformation.](image)

Two different contact conditions of the slider and the slide way are shown in figure 1. Provided that the point \( A_j \) on the slide way is the potential contact point of the point \( A_i \) on the slider, \( A_j \) can be represented in the global coordinate system as [17]

\[
r_{ij}^{A} = r_{ij}^P + t^T (r_{ij}^A - r_{ij}^P) t
\]

\[
\delta = r_{ij}^A - r_{ij}^A
\]

When the slider and slideway are not in contact, vector \( \delta \) represents aligned with vector \( n \), otherwise it is opposite it: \( c \) is applied to represent the contact condition between \( i \) and \( j \):

\[
c = \begin{cases} 
1, & \delta^T n > 0 \\
0, & \delta^T n \leq 0 
\end{cases}
\]

where \( \delta \) denotes the piercing depth and its size is expressed as

\[
\delta = \sqrt{\delta^T \delta}
\]

The collision velocity \( \dot{\delta} \) can be given as

\[
\dot{\delta} = r_{ij} + \dot{N}_i s_{ij}^A - r_{ij} - \dot{N}_i s_{ij}^A
\]

2.1. The tangential friction

The contact force between the sub-elements is mainly composed of normal contact force and
tangential friction force in the contact region. The modified Coulomb friction model is applied to describe the tangential friction [18]:

$$F_T = -\mu(v_t)F_N\frac{v_t}{|v_t|}$$  \hspace{1cm} (6)

Where $\mu(v_t)$ represents the effective dynamic friction coefficient, $v_t$ is the relative sliding speed of the two bodies in the tangential direction.

The normal contact force can be analysed according to the different contact modes of the sub-elements (figure 2).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Three contact modes between $i$ and $j$. (a) Surface contact; (b) line contact; (c) point contact.}
\end{figure}

2.2. The normal contact force

- When the slider is in surface contact with the slide way (figure 2(a)), a linear model is used to calculate the normal contact force on the contact surface [19]:

$$F_N = K\delta$$  \hspace{1cm} (7)

where $K$ is the generalised stiffness coefficient, which can be expressed as:

$$K = \frac{a}{0.475(\sigma_i + \sigma_j)}$$  \hspace{1cm} (8)

$$\sigma_k = \frac{1-\mu_k^2}{E_k} \cdot (k = i, j)$$  \hspace{1cm} (9)

where $a$ represents half of the perimeter of the contact surface, $\sigma_i$ and $\sigma_j$ are the material coefficients of the slider and the slide way, respectively, $\mu_k$ and $E_k$ are the Poisson’s ratio and elastic modulus of the material, respectively.

- When the slider is in line contact with the slide way (figure 2(b)), the contact is considered to be a cylinder-plane contact and the contact region can be regarded as a narrow rectangle, and a damping model is applied to calculate the normal contact force on the contact surface [20,21]:

$$F_N = Pl + C\hat{\delta}\hat{\delta}$$  \hspace{1cm} (10)

$$\delta = \frac{P\left(1-v^2\right)}{\pi E} \left(2\ln\frac{2r}{a} - \frac{v}{1-v} \right)$$  \hspace{1cm} (11)
\[ a = \left( \frac{4PR}{\pi E^*} \right)^{\frac{1}{2}} \]  

where \( C \) represents the damping coefficient, and is generally 0.1% ~ 1% of the equivalent contact stiffness \( K \). \( P \) denotes the pressure per unit length, \( 2a \) is the contact width, \( E \) is the elastic modulus of the material, \( r \) is the radius of the cylinder, and \( E^* \) and \( R \) are defined as

\[ \frac{1}{E^*} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \]  

\[ R = \frac{r_1 r_2}{r_2 - r_1} \] 

- When the slider is in point contact with the slide way (figure 2(c)), the contact region can be idealised as an arc of radius \( R_c \). The non-linear spring damping model is used to solve the normal contact force in this case [22]:

\[ F_N = K \delta^e + C \dot{\delta} \]  

where \( e \) is the Hertz contact force index (generally taken as between 1.3 and 1.5 for metals).

3. Case study

In this section, a planar mechanism with translational clearance joint was studied (figure 3), and the geometric and inertial properties of the mechanism are listed in table 1. The difference in size between the slider and the slide way defines the size of the clearance. The crank is the driving link of the mechanism and rotates at a constant angular velocity of \( \pi \) rad/s. The step-size and time of simulation are 0.01 and 2 s respectively in the case study.

![Figure 3. The mechanism with translational clearance joint.](image)

**Table 1. Geometric and inertial properties of the mechanism**

| Rods | Length (mm) | Width (mm) | Depth (mm) | Density (kg/m³) | Elastic modulus (N/m²) | Poisson’s ratio |
|------|-------------|------------|------------|-----------------|------------------------|----------------|
| 1    | 50          | 5          | 2.5        | 7801.0          | 2.07x10¹¹              | 0.29           |
| 2    | 142.2       | 14.2       | 7.1        | 7801.0          | 2.07x10¹¹              | 0.29           |
| 3    | 40          | 20         | 30         | 7801.0          | 2.07x10¹¹              | 0.29           |
| 4    | 250         | 20         | 30         | 7801.0          | 2.07x10¹¹              | 0.29           |

Based on the division of the contact modes of the sub-elements of the translational joint, a user
A sub-routine that can be dynamically linked with MSC.ADAMS was written in FORTRAN. Accordingly, the contact modes can be divided into three different types: point contact, line contact, and surface contact.

The influences of clearance size and velocity on the contact force between sub-elements were studied by changing the size of the inner hole of the slider while the width and depth of the slide way are fixed at 2 mm and 3 mm respectively. The parameters of the inner hole of the slider used in different cases are listed in table 2. The peak fluctuation, when the direction of contact force is positive, is taken as the research object, and the results are shown in figure 4.

### Table 2. Parameters of the inner hole of the slider used in the four cases.

| Cases | Length (mm) | Width (mm) | Depth (mm) | Clearance (mm) | Operation speed (rad/s) |
|-------|-------------|------------|------------|----------------|-------------------------|
| 1     | 40          | 2.2        | 3.2        | 0.1mm          | π                       |
| 2     | 40          | 2.1        | 3.1        | 0.5mm          | π                       |
| 3     | 40          | 2.2        | 3.2        | 0.1mm          | 1.22π                   |
| 4     | 40          | 2.2        | 3.2        | 0.1mm          | 2π                      |

![Figure 4. Contact force in different cases.](image1)

As can be seen from the figure 4, with the clearance size decreases or the speed increases, the contact force between sub-elements would increase. Meanwhile, the contact force was calculated according to different contact types, and we could assess the contact force under different contact modes. The result of the contact force in different contact modes in Case 1 is shown in figure 5. The simulation results are taken from different simulation intervals within one cycle.

The simulation results of the mechanism with 0.1 mm clearance size and π rad/s operation speed are divided according to different contact modes, as shown in figure 5. The simulation step-size and time of simulation are the same as the case study in figure 4. It is shown from figure 5 that, when the sub-elements are under different contact modes, the maximum contact force ($F_{\text{max}}$) in different modes during the simulation is different. This is because the magnitude of contact force is related to the positional relationship between sub-elements. We can also find that the contact force under different contact modes show irregular changes at the beginning of each simulation: however, the contact force continues to decrease after 0.5 s, which is consistent with the variation in contact force found previously.

This work can be employed for design optimization for translational joint design. The fluctuation amplitude and frequency of the contact force curve would change due to the existence of clearance. Therefore, the mean-square error value of contact force can be used to evaluate the influence of the
clearance on the dynamic characteristics of the mechanism.

4. Results and discussion
We established a translational joint model with clearance and derived a contact mode identification method on the basis of the classification thereof. The positional relationships of the sub-elements were classified into three different contact modes: surface contact, line contact, and point contact. The contact force between sub-elements was calculated separately under each of the three different contact modes.

A contact-mode identification method was applied to a planar mechanism with translational joint. The influences of clearance size and velocity on the contact force between sub-elements were studied. When the sub-elements are under different modes of contact, the maximum contact force ($F_{max}$) in different modes during the simulation is different.

Acknowledgments
This work is supported by National Science Foundation of China under Contract Nos. 11872033, 51875531, and 51505432, and Natural Science Foundation of Beijing under Contract No. 3132017.

References
[1] Flores P, Ambrósio J, Claro J C P, Lankarani H M and Koshy C S 2006 A study on dynamics of mechanical systems including joints with clearance and lubrication Mech. Mach. Theory 41 247-61
[2] Li J L, Huang H Z, Yan S Z and Yang Y 2017 Kinematic accuracy and dynamic performance of a simple planar space deployable mechanism with joint clearance considering parameter uncertainty Acta. Astronaut. 136 34-45
[3] Xiang W K, Yan, Wu J N and Robert X G 2014 Complexity evaluation of nonlinear dynamic behavior of mechanisms with clearance joints by using the fractal method Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci. 228 3482-3495
[4] Schwab, MeijJaard J P and Meijers P 2002 A comparison of revolute joint clearance models in the dynamic analysis of rigid and elastic mechanical systems Mech. Mach. Theory 37 895-913
[5] Xiang W K, Yan S Z and Wu J N 2015 A comprehensive method for joint wear prediction in planar mechanical systems with clearances considering complex contact conditions Sci. China Tech. Sci. 58 86-96
[6] Farahanchi F and Shaw S W 1994 Chaotic and periodic dynamics of a slider-crank mechanism with slider clearance J. Sound Vib. 177 307-24
[7] Olyaei and Ghazavi M R 2012 Stabilizing slider-crank mechanism with clearance joints Mech. Mach. Theory 53 17-29
[8] Mukras S, Kim N H and Mauntler N A 2010 Analysis of planar multi-body systems with revolute joint wear Wear 268 643-52
[9] Bauchau A and Rodriguez J 2002 Modeling of joints with clearance in flexible multi-body systems Int. J. Solids Struct. 39 41-63
[10] Flores P, Koshy C S, Lankarani H M, Ambrósio J and Claro J C P 2011 Numerical and experimental investigation on multi-body systems with revolute clearance joints Nonlinear Dyn. 65 383-98
[11] Flores P and Ambrósio J 2004 Revolute joints with clearance in multi-body systems Comput. Struct. 82 1359-69
[12] Jia X H, Ji L H, Jin D W and Zhang J C 2000 Modeling of tripod-ball sliding joint with clearance J. Tsinghua Univ. 40 125-8
[13] Jia X H, Ji L H, Jin D W and Zhang J C 2000 Dynamic analysis of the crank slider mechanism including tripod-ball sliding joint with clearance J. Mech. Sci. Technol. 19 698-700
[14] Song L and Yang J 2001 A method of planar linkage dynamic analysis with slider joint
impact in Mach. Des. Res. 17 36-9
[15] Tong L and Wang H J 2007 Dynamic simulation of a crank-slider mechanism including sliding joint with clearance based on virtual prototyping J. Chongqing Univ. Sci. Tech. 9 25-7
[16] Flores P, Ambrósio J and Claro J C 2008 Translational joints with clearance in rigid multibody systems J. Comput. Nonlinear Dyn. 3 011007
[17] Flores P, Leine R and Glocker C 2010 Modeling and analysis of planar rigid multibody systems with translational clearance joints based on the non-smooth dynamics approach Multibody Sys. Dyn. 23 165-90
[18] Flores P 2009 Modeling and simulation of wear in revolute clearance joints in multibody systems Mech. Mach. Theory 44 1211-22
[19] Tian Q, Liu C, Machado M and Flores P 2011 A new model for dry and lubricated cylindrical joints with clearance in spatial flexible multibody systems Nonlinear Dyn. 64 25-47
[20] Bai Z F and Zhao Y 2013 A hybrid contact force model of revolute joint with clearance for planar mechanical systems Int. J. Nonlinear Mech. 48 15-36
[21] Lankarani H M and Nikravesh P E 1990 A contact force model with hysteresis damping for impact analysis of multibody systems J. Mech. Des. 112 369-76
[22] Yigit A S, Ulsoy A G and Scott R A 1990 Spring-dashpot models for the dynamics of a radially rotating beam with impact J. Sound Vib. 142 515-25