A Genetic Algorithm Based Form-finding of Tensegrity Structures with Multiple Self-stress States

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Abstract
A form-finding method of tensegrity systems is a process of finding an equilibrium configuration and a key step in the design of tensegrity. Over the past few years, several studies have been made on the form-finding methods of tensegrity systems, however, these methods are limited in the tensegrity systems with multiple self-stress states. In this study, a numerical method is presented for form-finding of tensegrity structures with multiple states of self-stress by using a force density method combined with a genetic algorithm. The proposed method can design the desired tensegrity shape through a genetic algorithm with appropriate constraints. The design variable can be uniquely defined in the case of multiple states of self-stress using only the constraint of the member types. An eigenvalue decomposition of the force density matrix and a singular value decomposition of the equilibrium matrix are performed repeatedly in order to determine a feasible solution for nodal coordinates and force densities. A genetic algorithm is then adopted to uniquely define a single integral feasible set of force densities. Several numerical examples are presented to prove efficiency in searching for self-equilibrium configurations of tensegrity structures. In all cases, the single integral feasible self-stress states can be obtained.

Keywords: tensegrity structure; force density method; form-finding; genetic algorithm; self-stress state

1. Introduction
Tensegrities are spatial, reticulated and lightweight structures that consist of a set of discontinuous compressive components inside a set of continuous tensile components (Ali et al., 2010). The design of tensegrities is divided into three distinct steps: form-finding, structural stability and load analysis (Schenk, 2005). A key step in the design of tensegrity structures is the determination of their equilibrium configuration, known as form-finding. Recently Tibert and Pellegrino (2011) announced a review of available form-finding methods, offering a review and classification of seven form-finding methods for tensegrity structures.

As pioneering work in form-finding, the force density method was first proposed by Schek (1974) for cable structures. The concept of the force density method is based on the force-length ratio (or force densities). Estrada et al. (2006) presented a multi-parameter form-finding procedure for tensegrity structures using the force-density method. Masic et al. (2005) extended the force-density method by explicitly incorporating shape constraints for general and symmetric tensegrity structures. Zhang and Ohsaki (2006) presented the adaptive force density method for the form-finding problem of tensegrity structures. Most recently, Tran and Lee (2010) proposed a numerical method for tensegrity structures based only on the given topology and member types. They also presented an approach for a form-finding method of tensegrity structures with multiple states of self-stress (Tran and Lee, 2011).

Most form-finding methods assume a given topology and try to find equilibrium configurations using some given constraints. A different approach is to find the topology with a genetic algorithm. A genetic algorithm provides up-to-the-minute search techniques by adapting mechanisms found in genetics (Goldberg, 1989). Recently, several studies have researched the form-finding methods of tensegrity structures using genetic algorithms for searching self-equilibrium topology. Paul et al. (2005) used genetic algorithms to develop from an initial arbitrary topology into a stable one. Xu and Luo (2010) presented a form-finding method of irregular tensegrities based on the genetic algorithm. Yamamoto et al. (2011) proposed a genetic algorithm based form-finding method to obtain
tensegrity structures with fewer design variables. Koohestani (2012) provided an efficient form-finding method using a genetic algorithm that is used as an optimization technique. However, most of these studies used symmetric constraints in searching suitable forms of the tensegirities.

In this paper, the work by Tran and Lee (2011) is extended to obtain unique feasible sets of force densities by using the force density method combined with a genetic algorithm. The proposed method can determine multiple shapes of a tensegrity through a genetic algorithm with appropriate constraints. That is, the desired shape of the tensegrity can be obtained by adjusting suitable constraints. The procedure only requires the topology and the types of members (i.e., either compression or tension). First, the eigenvalue decomposition (EVD) of the force density matrix is implemented to determine the nodal coordinates that compose the equilibrium matrix. Second, the multiple states of force densities are obtained through the singular value decomposition (SVD) of the equilibrium matrix. A genetic algorithm is then performed to uniquely define a single integral feasible set of force densities. Finally, this process is iteratively performed to determine the range of feasible sets of the nodal coordinates and the force densities until the required rank deficiencies of the force density and equilibrium matrices for the case of multiple states of self-stress are met. Several numerical examples of tensegrity structures with multiple states of self-stress are presented to demonstrate the efficiency and accuracy of the proposed method.

2. Formulation of Self-equilibrium Equations

2.1 Basic Assumptions

In this study, the basic assumptions regarding tensegrity structures are stated as follows:

- The topology of the structure in terms of nodal connectivity is known.
- Members are connected by pin-joints.
- No external load is applied and the self-weight of the structures is neglected during the form-finding procedure.
- There are no dissipative forces acting on the systems.
- Neither local nor global buckling are considered.

2.2 Self-equilibrium Equations

A topology of $d$-dimensional ($d = 2$ or 3) tensegrity structure with $b$ members and $n$ free nodes can be expressed by a connectivity matrix $C$ ($\in R^{b\times n}$) as discussed in (Tran and Lee, 2010). Since a tensegrity system does not require any support (fixed node), in this study, only the free node is considered. According to Fig. 1, if member $k$ connects nodes $i$ and $j$ ($i < j$), the $i^{th}$ and $j^{th}$ elements of the $k^{th}$ row of the $C$ matrix are then set to 1 and -1, respectively, as follows:

$$C_{(k,p)} = \begin{cases} 1 & \text{for } p = i \\ -1 & \text{for } p = j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

When the external load and self-weight are ignored, the equilibrium equations in each direction of a general tensegrity structure given by (Schek, 1974) can be stated as

$$C^T Q C x = 0 \quad (2a)$$
$$C^T Q C y = 0 \quad (2b)$$
$$C^T Q C z = 0 \quad (2c)$$

Refer to the Cartesian coordinate system $(O-xyz)$, the nodal coordinate vector $x$, $y$ and $z$ are used. The notation $Q$ ($\in R^{3\times3}$) is diagonal square matrix, calculated by

$$Q = diag(q) \quad (3)$$

where $q$ ($\in R^{3\times3}$) suggested by Schek (1974) is the force density vector. Each component of this vector is the member force $f_i$ to length of element $l_i$ ($k = 1, 2, 3...b$) ratio ($q_{ik} = f_i/l_i$). The equilibrium equations (Eq. (2)) can be rewritten as follows:

$$\begin{bmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

where $D$ ($\in R^{3\times3}$) is the force density matrix (Tiber and Pellegrino, 2011, Estrada et al., 2006), or stress matrix (Connelly, 1982).

Eq. (2) can be reorganized as

$$A q = 0 \quad (5)$$

where $A$ ($\in R^{3\times3}$) is known as the equilibrium matrix, defined by

$$A = \begin{pmatrix} C^T \text{diag}(Cx) \\ C^T \text{diag}(Cy) \\ C^T \text{diag}(Cz) \end{pmatrix} \quad (6)$$
Eq. (4) shows the relationship between force density matrix $\mathbf{D}$ and nodal coordinates, and Eq. (5) illustrates the relationship between the equilibrium matrix $\mathbf{A}$ and force densities. Generally, both mechanical and geometrical parameters are obtained as a result of the form-finding process.

### 2.3 Rank Deficiency Conditions

In this form-finding procedure for tensegrity structures, two rank deficiency conditions (rank deficiency conditions of force density and equilibrium matrices) are required. For a $d$-dimensional tensegrity structure with $n$ free nodes, a dimension of rank deficiency of $\mathbf{D}$ (null space) can be expressed as

$$ n_D = n - r_D $$

where $r_D = \text{rank} (\mathbf{D})$. The rank deficiency of $\mathbf{D}$ ($n_D$) has at least one state of self-stress, since the sum of the elements of row or column of force density matrix is always equal to zero, and $n_D$ has at least $d$ useful particular solutions (Tran and Lee, 2010). Therefore, the rank deficiency condition is defined as

$$ n_D \geq d + 1 $$

The second rank deficiency condition is related to a dimension of null space of the equilibrium matrix $\mathbf{A}$ as follows:

$$ n_A = b - r_A $$

where $r_A = \text{rank} (\mathbf{A})$. The dimension of null space of the equilibrium matrix $\mathbf{A}$ is identical to "$s$" known as the number of independent states of self-stress. This study is limited to a consideration of tensegrity structures with multiple states of self-stress, which ensures the existence of at least two states of self-stress and can be stated as

$$ s = n_A \geq 2 $$

### 3. Algorithm

#### 3.1 Force Density Method

In the proposed method, the dimension, the connectivity of nodes, and the type of each member are only required for a form-finding procedure. Based on the type of each member, the initial force density coefficients of cables and struts are automatically assigned as $+1$ and $-1$, respectively.

Firstly, the connectivity matrix $\mathbf{C}$ is composed by Eq. (1) for the given of the connectivity of nodes. The force density matrix is then calculated from the initial force density vector and the nodal coordinates are adopted from the eigenvalue decomposition of the force density matrix $\mathbf{D}$.

The square force density matrix $\mathbf{D}$ can be factorized as follows by using the eigenvalue decomposition (Meyer, 2000):

$$ \mathbf{D} = \Phi \Lambda \Phi^T $$

where $\Phi (\in \mathbb{R}^{n \times n})$ is the orthogonal matrix whose $i^{th}$ column is the eigenvector basis $\phi_i (\in \mathbb{R}^n)$ of $\mathbf{D}$. $\Lambda (\in \mathbb{R}^{n \times n})$ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues, i.e., $\Lambda_{ii} = \lambda_i$. The eigenvector $\phi_i$ of $\Phi$ corresponds to eigenvalue $\lambda_i$ of $\Lambda$. The eigenvalues are in increasing order as

$$ \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n $$

It is clear that the number of zero eigenvalues of $\mathbf{D}$ is equal to the dimension of its null space. The first $d+1$ eigenvectors of $\Phi$, corresponding to the first $d+1$ smallest eigenvalues, respectively, are chosen as nodal coordinates $[x, y, z]$ for $d$-dimensional tensegrity structure. Subsequently, these nodal coordinates are substituted into Eq. (5) to select the candidates for a set of force densities by the singular value decomposition of the matrix $\mathbf{A}$.

$$ \mathbf{A} = \mathbf{U} \mathbf{W}^T $$

where $\mathbf{U}$ and $\mathbf{W}$ matrices are orthogonal matrices. $\mathbf{V}$ is a diagonal matrix with non-negative singular values of $\mathbf{A}$. The matrices $\mathbf{W}$ from Eq. (13) can be expressed as (Pellegrino, 1993).

$$ \mathbf{W} = [w_1, w_2, \cdots, w_d, q_1, q_2, \cdots, q_s] $$

As a result, the general solution $\bar{\mathbf{q}}$ of Eq. (5) that lies in the null space of $\mathbf{A}$ is formulated as

$$ \bar{\mathbf{q}} = \sum_{i=1}^{s} c_i \mathbf{q}_i $$

where the coefficients $c_i$ are arbitrary values and $\mathbf{q}_i (\in \mathbb{R}^n$, $i = 1, 2, \ldots, s$) are the particular solutions of Eq. (5).

A genetic algorithm is then used to obtain the coefficients $c_i$ related to a set of force densities that satisfy Eq. (4). Finally, the process is iteratively calculated to search for a feasible set of the nodal coordinates and force densities until Eq. (8) is satisfied. Additionally, the vector of unbalanced forces $\epsilon_i (\in \mathbb{R}^m)$ defined as follows can be used to evaluate the accuracy of the results:

$$ \epsilon_i = A \bar{\mathbf{q}} $$

The Euclidean norm of $\epsilon_i$ is used to define the design error $\epsilon$ as

$$ \epsilon = \sqrt{(\epsilon_i)^T \epsilon_i} $$

### 3.2 Genetic Algorithm

As pointed out in the previous section, a genetic algorithm is used to obtain a set of the coefficients $c_i$ and force densities $\bar{\mathbf{q}}$ in Eq. (15). These problems are formulated as constrained optimization problems that draw feasible solutions from form-finding. The key point in developing a form-finding method for a tensegrity structure using genetic algorithms is how to define fitness functions and constraints. In this study, the fitness function combined with coefficients $c_i$ is constructed to minimize it using a genetic algorithm; the magnitude of $c_i$ is assumed to be a unit. Also, the
fitness function is provided with penalty functions to significantly improve accuracy. The process of determining a unique feasible set \( \bar{q} \) is formulated as a constrained optimization problem as

\[
\text{Minimize : } \left| 1 - \sum_{i=1}^{s} \epsilon_i^2 \right| \tag{17}
\]

Towards a more effective algorithm, three constraints are added into the objective function shown as follows:

- **Constraint 1**
  \[
  \begin{align*}
  &\begin{cases} 
  0 < q_c \leq 1 \ (q_c \in \Gamma_{\text{cable}}) \\
  -1 \leq q_s < 0 \ (q_s \in \Gamma_{\text{strut}})
  \end{cases} \tag{18a}
  \end{align*}
  \]

- **Constraint 2**
  \[
  \frac{(q_i - q_j)}{q_i} \leq \epsilon_0 \ (q_i, q_j \in \Gamma^k) \tag{18b}
  \]

- **Constraint 3**
  \[
  \frac{(q_m - q_d)}{q_m} \leq \epsilon_0 \ (q_m, q_n \in \Gamma) \tag{18c}
  \]

in which \( \Gamma \) denotes the total set of the force density, \( \Gamma_{\text{cable}} \) is the set of the force density for cable members, and \( \Gamma_{\text{strut}} \) is the set of the force density for strut members. In Eq. (18a), \( q_c \) and \( q_s \) are force densities that are allowed to take a value from 0 to +1 for cables and from -1 to 0 for struts, respectively. In Eq. (18b), subscript \( i \) and \( j \) denote element numbers in the equal \( k \) group. \( \epsilon_0 \) is then used to define the tolerance. Eq. (18a) indicates the unilateral condition for tensegrity, which is necessary in order to have a unique value for force density. On the other hand, Eq. (18b) and Eq. (18c) are optional constraints for drawing the desired shape of the tensegrity from the optimization problem. Eq. (18b) denotes grouping constraints indicating that the force density values of the members are identical if they are in the same group \( k \). Eq. (18c) is optional constraints for two distinct members that are in a linear relation to each other. In the previous paper (Tran and Lee, 2011), the members are required to be grouped based on the symmetry of the structure in order to find the single integral feasible self-stress state.

These can be time-consuming and it is often difficult to perform suitable groupings. However, in this study, the force density can be uniquely defined for the tensegrity with multiple states of self-stress even for grouping constraints. If any regular form of a tensegrity structure is necessary, the other optional constraints can be applied to the fitness function. In other words, the specific solutions can be derived in a versatile manner using the appropriate constraints, and user-defined shapes can be achieved as well.

### 3.3 Solution Procedure

An outline of the form-finding process is shown in Fig.2. The feasible sets of the nodal coordinates and the single integral force densities of the tensegrity structures can be simultaneously defined by the proposed form-finding method through the following procedure.

- **Initialization:**
  1. Specify the dimension size, the connectivity of nodes, and the type of each member (cable or strut). An initial force density vector set is needed to determine the first generation. Assign the initial force density coefficients of cables and struts as +1 and -1, respectively.
  2. Define the connectivity matrix \( C \) by Eq. (1).

- **Iterations:**
  3. By the number of \( s \), perform EVD (Eq. (11)) and SVD (Eq. (13)) to define nodal coordinates \( [x,y,z] \) and force densities \( q \), respectively.
  4. Perform the genetic algorithm (Eq. (17)) to define the coefficients \( c_i \) through Eq. (15). And then evaluate accuracy of the results \( \bar{q} \) through Eq. (16).
  5. If the design error \( \epsilon_i \) is greater than stopping criteria, the results are substituted into matrix \( D \) of Eq. (4) and go to Step 4.

- **Termination:**
  6. The process is terminated until Eq. (16) has been checked. The final coordinates and force density vector are the solutions.

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**Fig.2. Outline of the Proposed Form-finding Procedure**

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4. Numerical Examples

In this section, numerical examples of tensegrity structures with multiple states of self-stress are presented to demonstrate the efficiency of the proposed method. Based on the algorithm developed, both the connectivity matrix and the force density vector are simultaneously defined with the nodal connectivity and the type of each member. Note that all of the force densities given in tables were normalized with respect to the force density coefficient of Element 1.

Each run of the genetic algorithm is conducted for 200 maximum generations, using a population size of 200. The convergence of all results was verified using a stopping criteria $\varepsilon_f = 0.01$, and the tolerance $\varepsilon_o$ of the constraints is set to 0.01.

Since the 2D hexagonal tensegrity structure has two states of self-stress ($s=2$), two force density sets of the elements could be obtained as follows:

Table 1. The Force Density Sets and Coefficients for Two State of Self-stress ($s=2$), 2D Hexagonal Tensegrity Structure

| Elem. No. | $q_1$ | $q_2$ | $c_1$ | $c_2$ | $\bar{q}$ |
|-----------|-------|-------|-------|-------|----------|
| 1         | -0.316 | -0.301 | 0.392 | -0.316 | 0.153    |
| 2         | -0.316 | -0.301 | 0.153 |        |          |
| 3         | -0.316 | -0.301 | 0.153 |        |          |
| 4         | -0.316 | -0.301 | 0.153 |        |          |
| 5         | 0.440  | -0.145 | 0.306 |        |          |
| 6         | 0.440  | -0.145 | 0.306 |        |          |
| 7         | 0.123  | -0.446 | 0.459 |        |          |
| 8         | 0.123  | -0.446 | 0.459 |        |          |
| 9         | -0.281 | 0.296  | -0.382|        |          |
| 10        | -0.281 | 0.296  | -0.382|        |          |
| 11        | 0.158  | 0.150  | -0.076|        |          |

Finally, the general force density solution set $\bar{q}$ of Eq. (5) is obtained as shown in Table 1. Table 2. shows that the general force density solution sets are in good agreement with those of previous studies. All of the force densities given in Table 2. were normalized with respect to the force density coefficient of Element 1.

Table 2. Comparison of the Force Densities Obtained by the Proposed Method with the Previous Study

| Group | Element | Tran and Lee (2011) | Present |
|-------|---------|---------------------|---------|
| 1     | 1-4     | 1.0                 | 1.0     |
| 2     | 5-6     | 2.0                 | 2.0     |
| 3     | 7-8     | 3.0                 | 3.0     |
| 4     | 9-10    | -2.5                | -2.5    |
| 5     | 11      | -0.5                | -0.5    |

Table 3. Comparison of the Force Densities Obtained by the Proposed Method with the Previous Study. 3D Three-strut Octahedral Cell

| Elem. No. | Constraint | Present | Tran and Lee (2011) |
|-----------|------------|---------|---------------------|
| 1         | 1          | 1.0     | 1.0                 |
| 2         | 1          | 1.0     | 1.0                 |
| 3         | 1          | 1.0     | 1.0                 |
| 4         | 1          | 1.0     | 1.0                 |
| 5         | 0.7       | 0.5     | 0.5                 |
| 6         | 0.8       | 0.5     | 0.5                 |
| 7         | 0.7       | 0.5     | 0.5                 |
| 8         | 0.8       | 0.5     | 0.5                 |
| 9         | 0.7       | 0.5     | 0.5                 |
| 10        | 0.8       | 0.5     | 0.5                 |
| 11        | 0.7       | 0.5     | 0.5                 |
| 12        | 0.8       | 0.5     | 0.5                 |
| 13        | -1.3      | -1.5    | -1.5                |
| 14        | -1.8      | -2.4    | -2.4                |
| 15        | -1.5      | -2.9    | -1.0                |

Fig.3. The Initial Topology of the 2D Hexagonal Tensegrity Structure with Eight Cables. The Struts and Cables are Depicted by Thick and Thin Lines, Respectively

Fig.4. The Initial Topology of the 3D Three-strut Octahedral Cell
4.2 3D Three-strut Octahedral Cell

The 3D three-strut octahedral cell is a tensegrity with three struts and 12 cables as shown in Fig.4. The input parameters for this example are \( n = 6 \) and \( b = 15 \) and \( d = 3 \). After investigating the rank deficiency of the force density matrix, the tensegrity is formed to have three self-stress states (\( s = 3 \)).

In the previous study (Tran and Lee, 2011), in order to find the single integral feasible self-stress state, the symmetry properties of the structure are required. The method requires appropriate grouping in order to obtain a single feasible pre-stressed mode. Accordingly, a trial and error scheme should be employed to find an appropriate grouping. In this study, three cases of this example are performed. First, only the constraint for the member type (Eq. (18a)) is applied (Constraint [1]). As a second case, the grouping constraint (Eq. (18b)) is added to the member type constraint (Constraint [1, 2]). The members are grouped according to the geometry and symmetry. The final case (Constraint [1, 2, 3]) is performed using all three constraints (member type (Eq. (18a)), grouping (Eq. (18b)), and force density ratio among the members (Eq. (18c))).

In the first and the second cases, the obtained force densities differ from those of the previous study as in Table 3. To obtain identical results, the specific force density ratio constraint of element 1 and 5 (\( q_1 = 2q_5 \)) is additionally imposed to the fitness function. As a result, the same force density values are obtained, as presented in Table 3. Fig.5 shows the case of Constraint [1], Constraint [1, 2], and Constraint [1, 2, 3], respectively. As shown in Fig.5., the different shape of the tensegrity is achieved by applying different constraints. Even for a tensegrity with a multiple self-stress status, form-finding can be achieved by simply imposing a member type constraint. If the obtained shape of the tensegrity is not satisfactory, an alternative shape can be obtained by adjusting grouping or ratio constraints only. The desired shape of the tensegrity can be obtained by imposing appropriate constraints.

4.3 3D Six-strut Tensegrity

A 3D six-strut tensegrity has six struts and 24 cables, and the initial topology has 12 nodes and 30 elements (Fig.6.). After investigating rank deficiency, the structure obtained two states of self-stress (\( s = 2 \)). This example is also performed using two constraint cases. Firstly, only the constraint for the member type (Eq. (18a)) is applied. As the second scenario, two constraints, member type (Eq. (18a)) and grouping (Eq. (18b)), are provided. The second case used in this study is the same as that used in the condition of the previous study (Tran and Lee, 2011). Fig.7.(a) and 7.(b) show a comparison between the final topologies obtained by two cases. Each case achieved a single integral feasible self-stress state. Table 4. shows the obtained force density coefficients. According to Table 4., the results of the second scenario agreed well with those of the previous study.

For the purpose of designing an arbitrary tensegrity shape, a new constraint condition is provided for fitness function. A linear force density ratio between Element 28 and 30 is additionally imposed \( q_{28} = q_{30} \). In this example, since using all three constraints is a strict restriction for obtaining a single feasible self-stress state, the grouping constraint is not provided. Fig.7.(c) shows a newly-designed 3D six-strut tensegrity shape. This new tensegrity shape differs from other shapes in Fig.7.; the results of force densities differ significantly.
5. Conclusions

In this study, a numerical method using a force density method combined with a genetic algorithm has been proposed as a form-finding process for tensegrity structures with multiple states of self-stress. The proposed method comprises equilibrium equations using a force density method.

Both the eigenvalue decomposition of the force density matrix and the singular value decomposition of the equilibrium matrix are iteratively executed to find the range of feasible sets of the nodal coordinates and the force densities. Then a genetic algorithm is used to find a unique feasible set of force densities. The method could be adapted simply to determine the uniquely defined force density. The feature of the proposed method is that a grouping or symmetric constraint is not required to find the single integral feasible self-stress state. The desired tensegrity shapes can be designed through a genetic algorithm with appropriate constraints. Three examples of tensegrity structures with multiple states of self-stress are performed. A very good performance of the proposed method has been shown in the numerical examples; they clearly show that the specific solutions can be derived with versatility using the appropriate constraints.

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