Search for CP Violation with a Neutrino Factory

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ABSTRACT

We have discussed the search of the genuine $CP$ violation in a neutrino factory, in which well known neutrino beams are provided by a high intensity muon storage ring. Both $\Delta P(\nu_\alpha \rightarrow \nu_\beta)$ and $CP$ odd asymmetry $A_{CP}$ in neutrino oscillations are investigated by taking account of the atmospheric neutrino data and the solar neutrino data. If the large mixing angle MSW solution is taken, the magnitude of $\Delta P(\nu_e \rightarrow \nu_\mu)$ could be 1%. If $s_{13}$ is lower than 0.05 with the maximal $CP$ violating phase, the matter effect is negligible in $\Delta P(\nu_e \rightarrow \nu_\mu)$ and $A_{CP}$. We have proposed how to extract the genuine $CP$ violation effect in the three family scheme from the neutrino oscillation data $\Delta P(\nu_e \rightarrow \nu_\mu)$, $\Delta P(\nu_\mu \rightarrow \nu_e)$ and $\Delta P(\nu_e \rightarrow \nu_\tau)$.

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Neutrino flavor oscillations provide information of the fundamental property of neutrinos such as masses, flavor mixings and the $CP$ violating phase. Recent experimental data of neutrinos make big impact on these property. Most exciting one is the results at Super-Kamiokande on the atmospheric neutrinos, which indicate the large neutrino flavor oscillation of $\nu_\mu \rightarrow \nu_x$ [1]. Solar neutrino data also provide the evidence of the neutrino oscillation, however this problem is still uncertain [2]. Now, a new stage is represented by the long baseline (LBL) neutrino oscillation experiments. The LBL accelerator experiment K2K [3] begins taking data in this year (1999), whereas the MINOS [4] and a CERN to Gran Sasso project [5] will start in the first year of the next century.

Some authors [6, 7, 8, 9, 10] have already discussed possibilities of observing $CP$ violation in LBL experiments by measuring the difference of transition probabilities between $CP$-conjugate channels [11, 12], which originates from the phase of the neutrino mixing matrix [13], such as $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_e$. However, the direct measurement is very difficult in the planned LBL experiments since the magnitude of its difference is usually expected below 0.01 and the difference of energy distributions of neutrino beams $\nu_\mu$ and $\nu_\mu$ disturbs this measurement in the order of $O(0.01)$. Moreover, the matter effect due to the earth makes difficult to extract the genuine $CP$ violating effect from the neutrino oscillation data.

On the other hand, a neutrino factory [14, 15], in which excellent neutrino beams are provided by a high intensity muon storage ring, is planned. These experiments may make possible to search for $CP$ violation in neutrino oscillations because neutrinos from muon decays are well known as to flavors and their energy distributions when the muon polarization is known [15]. For instance, the averaged energy of $\nu_e$ is 7 (12)GeV in the decay of the muon with 10 (20)GeV. Based on this set up with $L = 732$Km, the observability of $CP$ violation was studied in some mixing angles in ref.[15].

In this letter, we estimate the magnitude of $CP$ violation in the neutrino factory by using relevant parameters, in which the $CP$ violating effect is sizable, and then discuss how
to extract the genuine $CP$ violation from neutrino oscillation data, which include the matter effect. In order to estimate the $CP$ violating effect in the standard model, we consider the three family model without sterile neutrinos. Our starting point as to the neutrino mixing is the large $\nu_\mu \to \nu_\tau$ oscillation of atmospheric neutrinos with $\Delta m^2_{\text{atm}} = (2 \sim 6) \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta_{\text{atm}} \geq 0.84$, which are derived from the recent data of the atmospheric neutrino deficit at Super-Kamiokande [1]. In the solar neutrino problem [2], we consider three solutions: the small mixing angle MSW solution, the large mixing angle MSW solution and the vacuum oscillation solution. These mass difference scales are $\Delta m^2_{\odot} = 10^{-10} \sim 10^{-4} \text{eV}^2$, which are much smaller than the atmospheric one. We put $\Delta m^2_{\text{atm}} = \Delta m^2_{32}$ and $\Delta m^2_{\odot} = \Delta m^2_{21}$, and so disregard the LSND data [10].

If neutrinos are Majorana particles, one finds three $CP$ violating phases. However, the effect of extra Majorana phases is suppressed by the factor $(m_\nu/E)^2$ [17]. Therefore, $CP$ violation in the neutrino flavor oscillations relates directly to the $CP$ violating phase $\phi$ in the following mixing matrix $U$ for massive neutrinos:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13} \end{pmatrix},$$ (1)

where $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$ are vacuum mixings. The amplitude of $\nu_\alpha \to \nu_\beta$ transition with the neutrino energy $E$ after traversing the distance $L$ can be written as

$$A(\nu_\alpha \to \nu_\beta) = e^{-iEL} \left\{ \delta_{\alpha\beta} + \sum_{k=2}^{3} U_{\alpha k}U^*_{\beta k} \left[ \exp \left( -i \frac{\Delta m^2_{jk} L}{2E} \right) - 1 \right] \right\},$$ (2)

where $U_{\alpha i}$ is a neutrino mixing matrix element in eq.(1) [13], in which $\alpha$ and $i$ refer to the flavor eigenstate and the mass eigenstate, respectively. The amplitude $A(\nu_\alpha \to \bar{\nu}_\beta)$ is given by replacing $U$ with $U^*$ in the right hand side in eq.(2). Direct measurements of $CP$ violation originated from the phase $\phi$ are differences of transition probabilities between $CP$-conjugate channels [11, 12]:

$$\Delta P_{CP} \equiv P(\nu_\mu \to \nu_\tau) - P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\tau) - P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$$
\[ P(\nu_e \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_e) = 4J_{\nu CP}^\nu f_{\nu CP}, \]  

(3)

where \( f_{\nu CP} \) is the sum of oscillatory terms as

\[ f_{\nu CP} \equiv \sin \Delta_{12} + \sin \Delta_{23} + \sin \Delta_{31}, \]  

(4)

with

\[ \Delta_{ij} = \Delta m^2_{ij} \frac{L}{2E}, \]  

(5)

and the rephasing invariant quantity \( J_{\nu CP}^\nu \) is given as \[^{[18]}\]

\[ J_{\nu CP}^\nu = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \phi. \]  

(6)

Since oscillatory terms are periodic in \( L/E \) and \( \Delta_{12} + \Delta_{23} + \Delta_{31} = 0 \) is satisfied, these terms tend toward cancellation among them.

How large is \( J_{\nu CP}^\nu f_{\nu CP} \)? The magnitude \( f_{\nu CP} \) depends on \( \Delta m^2_{\text{atm}} \) and \( \Delta m^2_{\odot} \). The detail behavior was discussed in ref.[7]. We show the neutrino energy dependence of \( f_{\nu CP} \) in the case \( \Delta m^2_{\text{atm}} = 3 \times 10^{-3}, \ 6 \times 10^{-3} \text{eV}^2 \) and \( \Delta m^2_{\odot} = 10^{-5}, \ 10^{-4} \text{eV}^2 \) with \( L = 732 \text{Km} \) in fig.1. The expected magnitude of \( |f_{\nu CP}| \) is at most 0.06 in \( E \geq 5 \text{GeV} \). The magnitude \( f_{\nu CP} \) is suppressed as \( \Delta m^2_{\text{atm}} \) and \( \Delta m^2_{\odot} \) decrease.

On the other hand, \( J_{\nu CP}^\nu \) depends on mixing angles. The mixings \( s_{23}, s_{12} \) and \( s_{13} \) are constrained by atmospheric neutrinos, solar neutrinos and CHOOZ experiments \[^{[19]}\], respectively. We take \( s_{23} = 1/\sqrt{2} \), which is the typical mixing angle from the atmospheric neutrino data, and \( s_{13} \leq 0.2 \) from the CHOOZ data. Since larger mixings give larger \( J_{\nu CP}^\nu \), the large mixing angle solution of the solar neutrino is favored to search for \( CP \) violation. Putting \( s_{12} \simeq 0.5 \), which is a typical large mixing angle MSW solution, we get \( |J_{\nu CP}^\nu| \leq 0.04 \sin \phi \).

The magnitude of \( \Delta P \) can reach \( 10^{-3} \sim 10^{-2} \) in the case of \( \Delta m^2_{\odot} \simeq 10^{-4} \text{eV}^2 \) \[^{[7]}\]. The vacuum oscillation solution is unfavor for observing \( CP \) violation since \( f_{\nu CP} \) is considerably suppressed due to \( \Delta m^2_{\odot} \simeq 10^{-10} \text{eV}^2 \). Then, \( \Delta P \) is at most \( 10^{-8} \). If we use the small angle MSW solution, \( \Delta P \) is at most \( 10^{-4} \) due to the small mixing angle \( s_{12} \sim 0.04 \). In conclusion,
one can expect to observe the $CP$ violating effect in neutrino oscillations when the large mixing angle MSW solution is the true solution in the solar neutrino problem. Therefore, we show numerical calculations by taking this solution in this letter.

In practice, a realistic observable may be the $CP$ odd asymmetry $A_{CP}$ \cite{11, 15}:

$$A_{CP} \equiv \frac{P(\nu_{\alpha} \to \nu_{\beta}) - P(\bar{\nu}_{\alpha} \to \bar{\nu}_{\beta})}{P(\nu_{\alpha} \to \nu_{\beta}) + P(\bar{\nu}_{\alpha} \to \bar{\nu}_{\beta})}.$$  \hspace{1cm} (7)

Even if $\Delta P_{CP}$ is smaller than $10^{-3}$, the asymmetry could be $O(1)$. We also calculate $A_{CP}$ as well as $\Delta P_{CP}$ in this letter.

Even if the distance travelled by neutrinos is less than 1000Km in LBL experiments, those data include the background matter effect which is not $CP$ invariant. The matter effect should be carefully analyzed since the effect depends strongly on the mass hierarchy, mixings and the incident energy of the neutrino as shown in previous works \cite{20, 21, 22}. The effective mass squared in the matter $M_{m}^{2}$ for the neutrino energy $E$ in weak basis is

$$M_{m}^{2} = U \begin{pmatrix} m_{1}^{2} & 0 & 0 \\ 0 & m_{2}^{2} & 0 \\ 0 & 0 & m_{3}^{2} \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  \hspace{1cm} (8)

where $a \equiv 2\sqrt{2}G_F n_e E$. For antineutrinos, the effective mass squared is given by replacing $a \to -a$ and $U \to U^\ast$. Taking the constant matter density $\rho = 2.8g/cm^3$, the effective mixing angles and the phase are given in terms of vacuum mixings and the effective neutrino masses, which are eigenvalues in eq.(8) \cite{22}. It may be important to note that the constant matter density, assumed in our calculations, is not always quantitatively accurate and that a real earth model must eventually be used in the calculation of the matter effects. Actually, Koike and Sato have discussed the matter effect in the K2K experiment by using the real earth model \cite{23}.

In the neutrino factory, one has a $\nu_{\mu} + \bar{\nu}_e (\bar{\nu}_{\mu} + \nu_e)$ beam in the decay of $\mu^- (\mu^+)$. The search for the $CP$ violation effect is possible in different four oscillation channels. Although the magnitude of the genuine $CP$ violation is expected to be same in all channels as seen
in eq. (3), matter effects are different in general. The matter effect may enhance \( P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \) in the one channel, but suppress it in the other one. Therefore, it is important to search for \( P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \) in different channels.

At first, we show the result of \( \Delta P(\nu_e \to \nu_\mu) \) in the matter as well as in vacuum in fig.2 by taking reference set-up \( L = 732\text{Km} \) and \( E = 7\text{GeV} \), which leads to \( a = 1.5 \times 10^{-3}\text{eV}^2 \). The large mixing angle MSW solution is taken for relevant parameters, \( s_{12} = 0.5, s_{23} = 1/\sqrt{2} \) and \( \phi = -90^\circ, \Delta m_{32}^2 = 3 \times 10^{-3}\text{eV}^2 \) and \( \Delta m_{21}^2 = 10^{-4}\text{eV}^2 \). It is remarked that the matter effect increases as \( s_{13} \) increases and the magnitude of \( \Delta P(\nu_e \to \nu_\mu) \) is at most 0.25\% as seen in fig 2. However, it could reach to 1\% if we take \( \Delta m_{32}^2 = 6 \times 10^{-3}\text{eV}^2 \), which is the upper bound of the experimental data. The matter effect becomes negligible if \( s_{13} \) is lower than 0.05 with \( \phi = -90^\circ \).

On the other hand, the CP odd asymmetry \( A_{CP} \) of the same process is large as seen in fig.3, in which parameters are taken to be same ones as in fig.2. The matter effect is unimportant if \( s_{13} \leq 0.05 \). The larger \( A_{CP} \) corresponds to the smaller absolute value of the oscillation probability. We show these absolute values as well as the \( A_{CP} \) in the matter and vacuum in table 1. If \( A_{CP} \) is larger than 20\%, the magnitude of \( P(\nu_e \to \nu_\mu) \) is smaller than \( 10^{-3} \), which means that the observation of signals of CP violation is difficult.

Although we have calculated \( \Delta P \) numerically, the approximate formula is useful to investigate the qualitative structure of the matter effect. The formulae have been given in the lowest order approximation by Arafune, Koike and Sato [7]. In the case of \( aL/2E \ll 1 \) and \( \Delta m_{21}^2 L/2E \ll 1 \) with \( a, \Delta m_{21}^2 \ll \Delta m_{32}^2 \), we show two differences of transition probabilities between CP-conjugate channels as follows:

\[
\Delta P(\nu_e \to \nu_\mu) \equiv P(\nu_e \to \nu_\mu) - P(\overline{\nu}_e \to \overline{\nu}_\mu) \\
\simeq P_m s_{23}^2 + \Delta P_{CP},
\]

\[
\Delta P(\nu_\mu \to \nu_e) \equiv P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e) \\
\simeq P_m s_{23}^2 - \Delta P_{CP},
\]
\[ \Delta P(\nu_e \rightarrow \nu_\tau) \equiv P(\nu_e \rightarrow \nu_\tau) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) \]
\[ \simeq P_m c_{23}^2 - \Delta P_{CP}, \quad (9) \]

where
\[ P_m = (1 - 2 s_{13}^2) c_{13} s_{13} \left( \frac{16 a}{\Delta m_{31}^2} \sin^2 \frac{\Delta m_{31}^2 L}{4E} - \frac{4 a L}{2E} \sin \frac{\Delta m_{31}^2 L}{2E} \right), \quad (10) \]

and \( \Delta P_{CP} \) is the genuine CP violating effect \( 4J^e_{CP} f_{CP} \). Since the matter effect in \( \Delta P(\nu_\mu \rightarrow \nu_\tau) \) is different from \( P_m \), we do not discuss this process. Approximate equations in eqs.(9) suggest how to extract the genuine CP violating effect from the experimental data. By use of these equations, we get
\[ \Delta P_{CP} = \frac{1}{2} \left[ \Delta P(\nu_e \rightarrow \nu_\mu) - \Delta P(\nu_\mu \rightarrow \nu_e) \right], \quad (11) \]
\[ \Delta P_{CP} = \frac{1}{2} \left[ (\cos 2 \theta_{23} + 1) \Delta P(\nu_e \rightarrow \nu_\mu) + (\cos 2 \theta_{23} - 1) \Delta P(\nu_e \rightarrow \nu_\tau) \right]. \quad (12) \]

These formulae are very useful to exclude the matter effect. If \( s_{23} \) will be determined precisely in LBL experiments, these formulae become a test of the CP violating phase in the three family model. The eq.(11) is guaranteed exactly by the T violation relation as far as we use constant matter density. Since eq.(12) is an approximate formula with \( a \ll \Delta m_{31}^2 \) in principle, we have to test that these are well satisfied in relevant parameter regions without any approximation. In fig.4, we show the expected \( \Delta P(\nu_e \rightarrow \nu_\mu) \) and \( \Delta P(\nu_e \rightarrow \nu_\tau) \) as well as \( 4J^e_{CP} f_{CP} \) in the case of \( s_{12} = 0.5, \ s_{23} = 1/\sqrt{2} \) and \( \phi = -90^\circ \). The difference of \( |\Delta P(\nu_e \rightarrow \nu_\mu)| \) and \( |\Delta P(\nu_e \rightarrow \nu_\tau)| \) is due to the matter effect because \( \Delta P(\nu_e \rightarrow \nu_\mu) = -\Delta P(\nu_e \rightarrow \nu_\tau) \) is expected in vacuum. This difference is advantage to getting the genuine CP violation. In fig.5, we show \( \Delta P_{CP} \) and the \( 4J^e_{CP} f_{CP} \), in which \( \Delta P_{CP} \) is estimated by using eq.(12). The \( \Delta P(\nu_e \rightarrow \nu_\mu) \) and \( \Delta P(\nu_e \rightarrow \nu_\tau) \) are calculated numerically without any approximation. As seen in fig.5, the calculated \( \Delta P_{CP} \) agrees to \( 4J^e_{CP} f_{CP} \) within 1%.

Thus, eqs.(11) and (12) provide a good test for CP violation in the three family model. If we find the disagreement between \( \Delta P_{CP} \)'s obtained from both equations, we should consider
it as an evidence of new physics. For example, our formulae are modified if the sterile neutrino mixes with active neutrinos. The detail discussions will be presented elsewhere.

We have studied the possibility to observe the genuine CP violation in a neutrino factory. Both $\Delta P(\nu_\alpha \rightarrow \nu_\beta)$ and CP odd asymmetry $A_{CP}$ have been estimated by taking account of the large mixing angle MSW solution. The magnitude of $\Delta P(\nu_e \rightarrow \nu_\mu)$ could be 1%. If $s_{13}$ is lower than 0.05, the matter effect is negligible in $\Delta P(\nu_e \rightarrow \nu_\mu)$. If $A_{CP}$ is larger than 20%, the observation of $P(\nu_e \rightarrow \nu_\mu)$, which is smaller than $10^{-3}$, is difficult. We have proposed how to extract the genuine CP violation effect from the neutrino oscillation data $\Delta P(\nu_e \rightarrow \nu_\mu)$, $\Delta P(\nu_\mu \rightarrow \nu_e)$ and $\Delta P(\nu_e \rightarrow \nu_\tau)$. The proposed method is expected to be advantage to observing CP violation in the neutrino factory.

Acknowledgments

I would like to thank E. Akhmedov for the useful comment on the T violation. I also thank J. Maalampi for useful discussions and his hospitality in University of Helsinki. This research is supported by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan(No.10640274 ).
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Table 1: Expected probabilities and asymmetries for the $\nu_e \rightarrow \nu_\mu$ oscillation in the case $s_{12} = 0.5$, $s_{23}^2 = 0.5$, $\phi = -90^\circ$, $\Delta m_{21}^2 = 10^{-4}\text{eV}^2$, $\Delta m_{32}^2 = 3 \times 10^{-3}\text{eV}^2$ with $E = 7\text{GeV}$ and $L = 732\text{Km}$. $\overline{P}$ denotes the $CP$-conjugate process. [vac] and [mat] mean “in the vacuum” and “in the matter”, respectively.

| $s_{13}$ | $P_{\text{vac}}$ | $\overline{P}_{\text{vac}}$ | $\Delta P_{\text{vac}}$ | $P_{\text{mat}}$ | $\overline{P}_{\text{mat}}$ | $\Delta P_{\text{mat}}$ | $A_{CP_{\text{vac}}}$ | $A_{CP_{\text{mat}}}$ |
|----------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 0.10     | 0.00352          | 0.00282          | 0.00070          | 0.00365          | 0.00263          | 0.00102          | 0.11             | 0.16             |
| 0.05     | 0.00102          | 0.00067          | 0.00035          | 0.00106          | 0.00063          | 0.00043          | 0.21             | 0.26             |
| 0.01     | 0.00013          | 0.00006          | 0.000071         | 0.00013          | 0.00006          | 0.000074         | 0.36             | 0.38             |
Figure 1: Neutrino energy dependence of $f_{CP}$ in the cases of a: $\Delta m^2_{32} = 6 \times 10^{-3}eV^2$, $\Delta m^2_{21} = 10^{-4}eV^2$, b: $\Delta m^2_{32} = 3 \times 10^{-3}eV^2$, $\Delta m^2_{21} = 10^{-4}eV^2$, and c: $\Delta m^2_{32} = 3 \times 10^{-3}eV^2$, $\Delta m^2_{21} = 10^{-5}eV^2$. 
Figure 2: The $s_{13}$ dependence of $\Delta P(\nu_e \rightarrow \nu_\mu)$. The dashed-curve denotes the vacuum oscillation.
Figure 3: The asymmetry of $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$. The dashed-curve denotes the vacuum oscillation.
Figure 4: The $s_{13}$ dependences of $\Delta P(\nu_e \rightarrow \nu_\mu)$ and $\Delta P(\nu_e \rightarrow \nu_\tau)$ including the matter effect. Parameters are fixed as $s_{12} = 0.5$, $s_{23}^2 = 0.5$, $\Delta m_{21}^2 = 10^{-4}\text{eV}^2$, $\Delta m_{32}^2 = 3 \times 10^{-3}\text{eV}^2$ with $E = 7\text{GeV}$ and $L = 732\text{Km}$. The dashed-curve denotes the vacuum oscillation.
Figure 5: The estimated genuine $CP$ violation, which is given by subtracting the matter effect from the data by using eq.(12).