Hydrodynamic outcomes of planet scattering in transitional discs

Nickolas Moeckel\textsuperscript{1} and Philip J. Armitage\textsuperscript{2,3}

\textsuperscript{1}Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA
\textsuperscript{2}JILA, 440 UCB, University of Colorado, Boulder, CO 80309-0440, USA
\textsuperscript{3}Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder, CO 80309, USA

ABSTRACT
A significant fraction of unstable multiple planet systems are likely to scatter during the transitional disc phase as gas damping becomes ineffectual. Using a large ensemble of \textsc{fargo} hydrodynamic simulations and \textsc{mercury} \textit{N}-body integrations, we directly follow the dynamics of planet–disc and planet–planet interactions through the clearing phase and through 50 Myr of planetary system evolution. Disc clearing is assumed to occur as a result of X-ray-driven photoevaporation. We find that the hydrodynamic evolution of individual scattering systems is complex, and can involve phases in which massive planets orbit within eccentric gaps, or accrete directly from the disc without a gap. Comparing the results to a reference gas-free model, we find that the \textit{N}-body dynamics and hydrodynamics of scattering into one- and two-planet final states are almost identical. The eccentricity distributions in these channels are almost unaltered by the presence of gas. The hydrodynamic simulations, however, also form a population of low-eccentricity three-planet systems in long-term stable configurations, which are not found in \textit{N}-body runs. The admixture of these systems results in modestly lower eccentricities in hydrodynamic as opposed to gas-free simulations. The precise incidence of these three-planet systems is likely a function of the initial conditions; different planet setups (number or spacing) may change the quantitative character of this result. We analyse the properties of surviving multiple planet systems, and show that only a small fraction (a few per cent) enter mean motion resonances after scattering, while a larger fraction form stable resonant chains and avoid scattering entirely. Our results remain consistent with the hypothesis that exoplanet eccentricity results from scattering, though the detailed agreement between observations and gas-free simulation results is likely coincidental. We discuss the prospects for further tests of scattering models by observing planets or non-axisymmetric gas structure in transitional discs.

Key words: hydrodynamics – scattering – planets and satellites: dynamical evolution and stability – planet–disc interactions – protoplanetary discs – planetary systems.

1 INTRODUCTION
Prior to gap opening, gravitational interactions between planets and their surrounding gas discs act to efficiently damp planetary eccentricity. This damping implies that the formation of giant planets via core accretion leads to initially circular orbits, and that postformation dynamical effects must be sought to explain the observed eccentricity distribution of massive extrasolar planets (Butler et al. 2006). Dynamical instability in multiplanet systems, leading to orbit crossing and the ejection or merger of some of the planets, is one way to achieve this (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997). Extensive \textit{N}-body experiments have shown that with realistic mass functions, the dynamics of unstable two-planet (Ford & Rasio 2008), three-planet (Chatterjee et al. 2008) or richer systems (Jurić & Tremaine 2008) can successfully reproduce the observed eccentricity distribution.

The good agreement between \textit{N}-body dynamical experiments and observations is somewhat surprising, as it is hard to envisage a consistent scattering scenario in which gas disc interactions are not important. The simplest model is one in which the typical planetary system forms with just two massive planets. In this case, the only unstable systems have planets that are separated by less than the threshold for Hill stability (Marchal & Bozis 1982; Gladman 1993), and most of the unstable subset scatter on a time-scale that is much shorter than the gas disc dispersal time ($\sim 10^5$ yr; Wolff...
Scattering is therefore likely to occur as soon as gas disc damping declines below some threshold level, and will be synchronized with the epoch of disc dispersal. Related arguments apply, less robustly, to richer planetary systems. For more than two planets there is no analytic criterion for stability, and relatively loosely packed initial configurations can delay the onset of scattering for tens or hundreds of Myr (Chatterjee et al. 2008). Absent fine-tuning, however, it is not possible to construct initial conditions in which all of the instabilities occur on >Myr time-scales, without overproducing stable low-eccentricity systems that are not seen observationally. As the gas dissipates, there should therefore be a significant number of systems primed to go unstable, scattering in the presence of at least some remnant gas.

The large computational cost of hydrodynamics relative to gravity has necessarily motivated approximate approaches that marry disc physics to N-body simulations of scattering. One method is to include an analytic description of planet–disc interactions as a force within an N-body calculation. This has been used to study the migration of planets into resonance (e.g. Snellgrove, Papaloizou & Nelson 2001; Lee & Peale 2002), and through resonance into scattering (Adams & Laughlin 2003; Moorhead & Adams 2005; Lee, Thommes & Rasio 2009). Thommes, Matsumura & Rasio (2008) followed the growth of planetary systems from planetesimals in a gas disc, with the disc modelled in 1D and torque on the bodies calculated from that density profile. This technique was used by Chambers et al. (2008) and Matsumura et al. (2010) to study scattering of more than two planets in an embedding disc. There have also been preliminary studies that include a full hydrodynamic treatment of the gas disc during active periods of planet scattering. Moekel, Raymond & Armitage (2008) performed 3D simulations starting from very unstable equal-mass, two-planet systems with a remnant gas disc exterior to the planets’ orbits. Marzari, Baruteau & Scholl (2010) performed 2D simulations studying migration and resonant trapping in three-planet systems, some of which led naturally to scattering when resonance was broken. While both of these studies hinted at possibly significant dynamical effects mediated by the gas, they were too limited in scope to attempt statistical comparison to gas-free results.

Our goal in this work is to directly quantify the impact of residual gas discs on the outcome of planet scattering. We assume that the interesting dynamics is coincident with the epoch of disc dispersal, and follow the planet–planet and planet–disc interactions through this transition phase using a 2D hydrodynamic code. We consider three-planet systems, set up almost identically to previous gas-free calculations (Chatterjee et al. 2008; Raymond, Armitage & Gorelick 2010), and model disc dispersal using a physically motivated photoevaporation model (Owen, Ercolano & Clarke 2011). The hydrodynamics are run long enough to take the systems through the transition phase into the pure N-body regime, after which we switch to a standard integrator for a further 50 Myr of gravitational interactions.

2 THE SIMULATIONS

2.1 Initial conditions

To facilitate direct comparison with previous N-body results, we consider three-planet systems around stars with mass $M_\star = 1 M_\odot$, with the mass $m$ of each planet drawn randomly from a power-law distribution $f(m) \alpha m^{-1.1}$ in the range of 0.3–5 $M_\oplus$. The innermost planet is placed with a semimajor axis $a_1 = 3$ au, and the spacing of the other planets is determined following the approach of Chambers, Wetherill & Boss (1996), with each adjacent planetary pair separated by a fixed number of mutual Hill radii $R_{\text{Hill}}$. That is, with

$$R_{\text{Hill}} = \left( \frac{m_i + m_j}{3M_\star} \right)^{1/3} \frac{a_i + a_j}{2},$$

the middle planet is placed at $a_2 = a_1 + K R_{\text{Hill}}$, and the outer planet at $a_3 = a_2 + K R_{\text{Hill}}$. The constant $K$ largely determines the time-scale on which gravitational instability occurs (Chambers et al. 1996; Marzari & Weidenschilling 2002; Chatterjee et al. 2008). We set $K$ at approximately the maximum value that does not over-produce stable systems, whose low final eccentricity would be inconsistent with exoplanet observations. With our mass spectrum, experimentation showed that $K = 4.0$ yielded a distribution of instability times with a median $t_i \sim 4 \times 10^5$ yr, and $\sim 10$ per cent of the set-ups unscattered after 50 Myr.\(^1\)

The planets are initially embedded in a gas disc which we model between 1 and 40 au. The surface density initially follows a radial profile $\Sigma(r) = 200(r/1 \text{ au})^{-1}$ g cm$^{-2}$. We include photoevaporation of the disc using the mass-loss profiles of Owen et al. (2011), which are based on the coupled hydrodynamic and radiative transfer models introduced by Owen et al. (2010). In these models, X-ray luminosity drives photoevaporative disc dispersal. Our discs initially contain just over 5 $M_\oplus$ of gas, which is both the typical total mass of the three planets and the mass at which the photoevaporative mass-loss begins to alter the disc structure on a fast time-scale. We pragmatically set the X-ray luminosity of our star at $1.4 \times 10^{30}$ erg s$^{-1}$, which yields a disc clearing time-scale of just over $10^5$ years depending on the details of the planet–disc interactions (see Fig. 1). This time-scale, coupled with our chosen planet spacing, means that about half our systems might be expected to scatter when there is still a significant amount of disc material left.

We evolve 100 random realizations of the planetary initial conditions within the fixed initial conditions for the disc. We use direct hydrodynamic simulations to model the coupled planetary and disc dynamics while the disc mass is high enough to still influence the

\(^1\)This instability time-scale is about an order of magnitude larger than the result for $K = 4.0$ with equal-mass planets found by Chatterjee et al. (2008), where for a given $K$ each planetary pair is at a fixed separation relative to the major mean motion resonances. With our mass spectrum and $K = 4$, each adjacent pair is somewhere between roughly the 3:2 and 2:1 mean motion resonances.
planets’ evolution, followed by a longer N-body run once the disc has been cleared. For the hydrodynamic portion of each run we use a modified version of the 2D code FARGO (Masset 2000), and for the N-body follow-on we use MERCURY (Chambers 1999).

FARGO is a 2D Eulerian hydrodynamic code tailored to planet–disc interactions (Masset 2000). We have modified the code somewhat to make it more suitable for this work (similar modifications were made by Marzari et al. 2010). The N-body integrator was changed from a fifth-order Runge–Kutta to a (7,8)-order embedded pair (Prince & Dormand 1981). We include stepsize control in the following way: the results of the seventh- and eighth-order estimate of each component of each planet’s velocity and position are compared at the end of each hydrodynamic time-step. If the fractional difference between the two exceeds 10^{-2}, the planets are integrated through the hydrodynamic step with a smaller N-body time-step, repeating this process until our accuracy criterion is met.

We have also implemented the removal of escaping planets if they are on a hyperbolic orbit and more than 400 au from the star, as well as collisions between planets and between planets and the star. Planet–planet mergers are mass and momentum conserving and take place whenever two planets’ surfaces are detected to touch, with all planets assumed to have radii of 1.3 R_J. Planets merge with the star if they approach within 5 R_J.

2.2 Numerical details

Photoevaporation is included using the X-ray-driven mass-loss profile of Owen et al. (2011), scaled to our 1 M⊙ star and chosen X-ray luminosity of 1.4 × 10^{30} erg s^{-1}. The azimuthally constant but radially dependent mass-loss rate from these profiles is imposed across the numerical grid as an externally specified sink term in the continuity equation. In these models, photoevaporation opens a gap near 2 au, and when the gas interior to the gap is gone the directly illuminated outer disc erodes from the inside out (see fig. 9 of Owen et al. 2011). We implement this transition to direct illumination somewhat approximately, as we do not follow the disc inwards of 1 au and set a lower surface density limit of 2 × 10^{-3} g cm^{-2} below which we do not photoevaporate any material for numerical reasons. As the simulation progresses, we track the azimuthally averaged surface density of each ring. When the innermost radius has an average surface density below 8 × 10^{-3} g cm^{-2}, we rescale the inner radius of the photoevaporation profile to that radius.

We allow accretion on to the planets using an approach similar to the unmodified version of FARGO, based on the prescription used by Kley (1999). The planet’s Roche lobe (approximated by a constant radius R_J) is drained on some time-scale, with material inside 0.45 R_J, drained at twice the rate of that between 0.45 and 0.75 R_J. Material outside 0.75 R_J is left alone. The time-scale on which the Roche lobe is drained is set to twice the planet’s orbital period. With our chosen parameters, the disc is typically dispersed via a combination of planetary dynamics and photoevaporation (with a small contribution from accretion) between 1 and 1.5 × 10^5 yr, with a few instances stretching to 0.8–1.9 × 10^5 yr. In Fig. 1 we show the mass evolution of all the discs in our runs.

We use open boundaries at the inner and outer edges of the grid. We use a fixed disc aspect ratio of H/r = 0.04, and an alpha-viscosity with α = 10^{-5}. The grid is set up with n_θ = 512 azimuthal cells and n_r = 207 rings. The radial spacing is logarithmic, with approximately square cells between 2.5 and 10 au, but coarser radial spacing between 1–2.5 and 10–40 au so that those cells are rectangular with a 2:1 aspect ratio. Our highest resolution is thus concentrated in the region of highest interest, where the majority of the planets spend most of the time.

We tested the adequacy of the grid resolution by setting up a planet with m = 1 M_J, and eccentricity e = 0.5, in an initially unperturbed disc. We considered two initial disc masses, Σ(1 au) = 200 and 50 g cm^{-2}, and three resolutions, n_θ = 256, 512 and 1024. The results are shown in Fig. 2. The more massive disc results are shown in blue, and the low-mass disc in red. The lightest shade is n_θ = 256, and the darkest shade is n_θ = 1024. The low-mass disc is well converged in both the orbital evolution and planet mass. The high-mass case is only moderately converged. While the orbit of the planet ends up at approximately the same place for all three resolutions, the mass evolution of the planet is quite different. The differences in the orbital evolution are most pronounced at the time when the planet is accreting the most mass, between 1000 and 2000 years. Visual inspection of the surface density of these runs shows that gas is accreting on to the planet in streams that are not well resolved at low n_θ. We conclude that for studying the orbital evolution of the planets – the prime focus of this work – n_θ = 512 is sufficient. Higher resolution, along with a more physical treatment of mass accretion within the Roche lobe, would likely be needed to model mass evolution in detail.

Figure 2. Resolution tests in which a 1 M_J planet is placed on an eccentric orbit in an initially unperturbed disc. The series of blue lines shows a disc with Σ(1 au) = 200 g cm^{-2}, and the red lines have Σ(1 au) = 50 g cm^{-2}. In each series, the lightest shade is run with n_θ = 256, the medium shade with n_θ = 512 and the dark shade with n_θ = 1024.
The results of the resolution test suggest that a massive planet scattering into an undisturbed disc will accrete material at a resolution-dependent rate until a gap is opened. This manifests itself as an offset in the mass evolution of the disc and the planet (see the similar slopes of the $M_{\text{planet}}$ and $M_{\text{disc}}$ curves in Fig. 2 after $\sim 1500$ years). Runs that scatter very early, when a large amount of gas is still in the disc, are susceptible to this numerical effect, which may alter the disc dissipation time-scale. Offsets of the order of $0.1 M_\odot$ will not move the disc mass evolution out of the main band seen in Fig. 1, though, and are certainly small compared to the expected physical dispersion in dissipation history that results from varying stellar X-ray luminosity. Hence, we do not consider this to be of great importance.

Each set of initial conditions is integrated with FARGO until the disc is photoevaporated down to our density floor, at which point the run is terminated. The planetary system is then evolved further using the FARGO integrator with no disc until $2 \times 10^5$ yr. At this point, the system is turned over to the Burlisch–Stoer integrator of MERCURY (Chambers 1999) and integrated to 50 Myr. As in the FARGO portion of the integration, planetary mergers and ejections are allowed with the same parameters. These hydrodynamic runs are then compared to an ensemble of 500 purely N-body simulations, set up using identical initial conditions but integrated using MERCURY throughout.

3 RESULTS

The hydrodynamic runs are characterized by several phases: an initial clearing of gaps and relaxation of the disc, the onset (or not) of dynamical instability and planet scattering, renewed relaxation of the disc and eccentricity damping, and clearing of the disc by photoevaporation. These phases are illustrated for two example runs in Figs 3 and 4. In each of these examples, scattering occurs, and the once-prominent gaps are filled in. As the disc comes to terms with

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**Figure 3.** The evolution of the planetary system and the gas disc for an example run. The orbits of the planets are plotted on top of the surface density plots in the same colours as the plots of the orbital elements and masses. The surface density plot on the left shows the system before scattering, when the red, green and blue planets have partially cleared a mutual gap. During the scattering event, the green and blue planets collide to form the orange planet on an orbit with $e \sim 0.6$. The aftermath of the scattering is shown in the middle density plot. Over the next $\sim 2.5 \times 10^5$ yr, disc interactions damp the eccentricity of the outer planet; after disc clearing, the two planets are stable. The right-hand density plot shows the system near the end of the photoevaporative clearing, with each planet orbiting in a gap.
3.1 Instability time-scale and outcome

In Fig. 5 we compare the distribution of instability times between the hydrodynamic and N-body control runs. During the FARGO runs, the time of scattering is identified as the first time the semimajor axis of any planet changes by 10 per cent between two output times. Visual inspection of the results confirms that this is a reasonable estimate of the onset of instability. During the MERCURY runs, an encounter between two planets within three Hill radii is the criterion. The bulk of the distribution is similar for both sets of simulations; the main difference is at the long-time tail of the distribution, where the FARGO runs have roughly 50 per cent more runs that never become unstable over the course of the simulations.

In the right-hand panel of the figure we plot the percentage of unstable runs ending in a given planetary configuration. The main difference between the simulation sets is the number of unscattered systems and the presence, in the hydrodynamic runs only, of a population of scattered systems that retain all three planets. There are also a handful of double systems that are not provably Hill stable in the MERCURY runs, but not enough to alter any conclusions. We discuss the difference in the percentage of unscattered triple systems further in Sections 3.3 and 3.4.

3.2 Eccentricity distributions

With realistic planet masses, scattering from nearly (but not identically) coplanar and circular initial conditions is quite successful at reproducing the observed eccentricity distribution of giant extrasolar planets. Good agreement can be obtained irrespective of the initial number of planets (Chatterjee et al. 2008; Ford & Rasio 2008; Juric & Tremaine 2008). These authors compare their results...
to the observed sample.\(^3\) In our study, a subtlety arises from the fact that \textsc{fargo} is a 2D code, and thus we simulate the planetary dynamics strictly in the plane. This has some consequences for the eccentricity distribution obtained by purely gravitational studies; namely, the distribution is shifted towards lower eccentricities, as clearly seen in Fig. 6. We address this point further in Appendix A. When comparing our \textsc{fargo} runs to a discless \(N\)-body result, we thus compare to a coplanar suite of experiments run with \textsc{mercury} rather than directly to the observed sample. With non-zero inclinations, these initial conditions yield a distribution quite close to the observed exoplanets in a pure \(N\)-body suite of experiments, which we verify by running the same initial conditions with random inclinations up to 1\(^\circ\). For the observed sample we exclude exoplanets with \(a < 0.1\) au to take tidal circularization out of consideration.

To statistically compare the sets of simulations, we perform two-sample Kolmogorov–Smirnov (KS) tests and two-sample Anderson–Darling (AD) tests on the cumulative distributions. The latter test is more sensitive to deviations at the tails of the distribution. We reject the null hypothesis that the two samples are drawn from the same distribution when \(p\)-values are less than 0.05. In Fig. 6 we show the eccentricity distributions of the disc and discless cases. The top panels show the results including only those systems that are provably stable, i.e. single-planet and Hill-stable double-planet systems. The null hypothesis is not ruled out by the KS (\(p\)-value = 0.10), though the AD test rejects it (\(p\)-value = 0.037). The greater weight the AD test gives to the distributions’ tails validates the visual impression that the main difference is at the high eccentricity end. For comparison, we also plot the observed eccentricity distribution, as well as the result of our simulations with inclinations up to 1\(^\circ\). While the eccentricity distributions alone are quite similar for stable systems, the combined eccentricity–semimajor axis distribution is marginally different. Fig. 7 shows scatter plots of these distributions. A 2D KS test (using the method of Fasano & Franceschini 1987) rejects the null hypothesis, with a \(p\)-value of 0.041.

The bottom panels of Fig. 6 show the same comparison between the hydrodynamic and \(N\)-body integrations, but now including all systems that scattered prior to 50 Myr, regardless of provable stabil-

\(^3\) Available from http://exoplanet.eu.

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\(^4\) For these integrations we used the hybrid integrator of \textsc{mercury}, with a 20-d stepsize.
Since the disc is the driver of any difference that might arise between the distributions, we split the sample into those in which instability begins when disc material is still present, and later scatterers. These distributions are shown in Fig. 9. Whether or not we include the unstable systems, the distribution of early and late scatterers is significantly different, with KS and AD \textit{p}-values of 0.0016 and $3.2 \times 10^{-4}$. While the early scatterers are likewise convincingly statistically distinct from the discless runs (\textit{p}-values 0.0069 and $6.6 \times 10^{-4}$), the two tests also ambiguously reject the null hypothesis for the discless runs and the late scatterers (\textit{p}-values 0.067 and 0.033). We would not necessarily expect any difference here, as there is no disc to alter the dynamics, and Chatterjee et al. (2008) showed that the time of instability is unimportant in determining the final outcome of a scattering experiment. Note that we suffer somewhat from low numbers of runs in this late scattering category, due to the large number of runs that never scatter over the length of our integrations.

### 3.3 Mean motion resonances: two planets

Convergent migration in a disc can lead to trapping into resonance (e.g. Snellgrove et al. 2001; Lee & Peale 2002; Kley, Peitz & Bryden 2004; Moorhead & Adams 2005), and is seen with high frequency in the previous work most relevant to this one, the coupled \textit{N}-body–1D disc models of Matsumura et al. (2010). We first searched for mean motion resonances (MMRs) among the 39 stable double systems by observing the apsidal alignment $\sigma_2 - \sigma_1$, and the resonance angles for a $(p + q):q$ MMR given by (e.g. Murray & Dermott 1999)

$$\varphi_{1,2} = (p + q) \lambda_2 - p \lambda_1 - q \sigma_{1,2}.$$  \hspace{1cm} (2)

Here, subscripts 1 and 2 refer to the inner and outer planets, $\lambda$ is the mean longitude, and $\sigma$ is the longitude of pericentre. We searched for resonances up to eighth order by monitoring these angles for libration around fixed values.

We find just two double systems in 3:1 MMRs with libration semi-amplitudes of $\sim 150^\circ$ and $\sim 80^\circ$. The case with the larger libration amplitude did not scatter until the disc had been photoevaporated. The low frequency of two planet resonances, and the large libration angles, are symptomatic of random entry into resonance due to purely gravitational effects (Raymond et al. 2008), rather than systematic driving into resonance due to gas disc migration (Matsumura et al. 2010). We observe a dramatically lower resonant fraction than the $\sim 75$ per cent seen by Matsumura et al. (2010). The difference is probably due to the shorter duration of the discs in this study compared to the Myr dispersal timescales assumed by Matsumura et al. (2010), which allows less time for post-scattering migration to move planets into resonance.

As noted in Section 3.1, the disc runs have a higher fraction that remain stable for the entire simulation than the \textit{N}-body only runs. Because our initial conditions naturally place the planets near to several MMRs, there is the possibility that two adjacent planets may get trapped into resonance, a situation that can enhance the stability of a three-planet system (Fabrycky & Murray-Clay 2010). We also search each adjacent pair of planets in the systems that never scattered for MMRs. Out of 27 such cases, we find three cases in which the inner two planets are in resonance (two 3:2 and one 2:1), and three cases in which the outer two planets are in a 2:1 resonance. These resonant systems are potentially part of the reason for the large number of unscattered systems.
3.4 Mean motion resonances: three-planet resonant chains

Further investigating the unscattered runs, we discovered that 12 of the 27 unscattered runs are in three-body resonance. In eight cases, the inner–middle and middle–outer pairs move into resonance with librating resonance angles (equation 2), and in three of these the inner–outer pair is likewise in a resonance. The chains we find are 9:6:4 (four cases), 6:3:2, 3:2:1 (two cases) and 4:2:1. In each case a Laplace angle of the form

$$\psi_L = m\lambda_1 - (m + n)\lambda_2 + n\lambda_3$$

(3)
is identified (Aksnes 1988), which librates around a fixed value for small integer values of $m$ and $n$, with subscripts 1, 2 and 3 referring to the inner, middle and outer planets, respectively.

The other four of the 12 triple resonances are less deeply in resonance; all three exhibit libration of the $m = 1$, $n = 3$ Laplace angle but the adjacent planet pairs are not in the 2:1 resonances. In Fig. 10, we plot the resonant behaviour of three example chains; a representative of the 9:6:4 cases (i.e. a double 3:2 resonance), the 6:3:2 case and the Laplace resonance case, the former two of which exhibit asymmetrical resonance in some of the angles. In the 9:6:4 run, each adjacent pair of planets is trapped into a 3:2 resonance at ~4000 years, the same time when the Laplace angle begins librating around $\psi_L = \pi$. The inner–outer pair moves into apsidal alignment, but the resonant angles continue to circulate.

The 6:3:2 case moves into triple resonance at ~1000 years, roughly the same time when the inner–middle pair locks into a 3:2 resonance, but somewhat before the other two pairings fully enter resonance at ~10^4 years. The initial inner–middle resonance and the Laplace angle all appear to be settling towards resonant angles of 0 or $\pi$, but when middle–outer and inner–outer resonances begin, most of the angles move into asymmetrical resonances, including the Laplace angle which shifts to librate around a value (very) slightly more than $\pi$.

The double 2:1 Laplace resonance shows libration of $\psi_L$ around an angle completely incommensurate with $\pi$. In this case, the inner–middle 2:1 pair immediately enters asymmetric resonance, along with the Laplace angle. The middle–outer 2:1 pair is in a symmetric resonance, and the inner–outer pair is flirting with asymmetric resonance, though $\phi_1$ continues to circulate. We note that this asymmetric behaviour in the Laplace resonance is a preferred outcome of simulations of migration into double resonance (Beaugé et al. 2008), although this particular case did not undergo very much migration. It would appear that the high incidence of triple resonances, as well as the perhaps enhanced stability due to a single adjacent pair migrating into resonance, is the underlying reason behind the large number of unscattered systems in our hydrodynamic simulations compared to the N-body runs.

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Figure 10. The evolution of the planetary system and the gas disc for an example run. The orbits of the planets are plotted on top of the surface density plots in the same colours as the plots of the orbital elements and masses. The surface density plot on the left shows the system before scattering, when the red, green and blue planets have partially cleared a mutual gap. During the scattering event, the green and blue planets collide to form the yellow planet on an orbit with $e \sim 0.6$. The aftermath of the scattering is shown in the middle density plot. Over the next ~2.5 × 10^4 years, disc interactions damp the eccentricity of the outer planet; after disc clearing, the two planets are stable. The right-hand density plot shows the system near the end of the photoevaporative clearing, with each planet orbiting in a gap.
3.5 Periods of observability

Disc–planet interactions may be able to excite eccentricity to modest values, $e \sim 0.1$–0.2 (D’Angelo, Lubow & Bate 2006), but planetary dynamics are probably necessary to reach higher values. Some of the more interesting features in the disc surface density in our simulations occur when eccentricities are high, during or just after scattering (see Figs 3 and 4). These include non-axisymmetric features, new gaps being opened and eccentric gaps. If scattering (or at least its signature in the eccentricity of the planets) is reasonably common in transitional discs, then these features may be observable. We estimate the likelihood that these features might be observed by calculating the amount of time that systems spend with at least one planet’s eccentricity above a threshold value, while also having at least 1 $M_J$ of gas left. This distribution is shown in Fig. 11 for three values of the threshold eccentricity, $e_{\text{min}} = 0.1$, 0.25 and 0.5.

For the higher and more interesting thresholds, most of the runs do not have eccentric enough planets for any reasonable amount of time. If we require that interesting features are present for something like 25 per cent of our typical disc lifetime, $t_{\text{obs}} \sim 3 \times 10^4$ yr, a threshold of $e_{\text{min}} = 0.25$ means that 10–15 per cent of the runs will have planets in eccentric enough orbits to be noticeable; thus, a few per cent of transition discs might be expected to harbour planets caught in the act or the aftermath of scattering.

4 CONCLUSIONS

If planets exit the gas disc phase in non-resonant configurations, a significant fraction of unstable systems are likely to scatter roughly contemporaneously with the transition between gas-rich and gas-poor conditions. We have studied whether the presence of small masses of gas, in photoevaporating transition discs, significantly modifies the outcome of scattering. Previous work that has addressed this question has been ambiguous. Detailed statistical studies, using approximate hydrodynamic treatments of the disc, have suggested that gas-free initial conditions are a reasonable approximation to the more physically complete case that includes gas (Chatterjee et al. 2008; Matsumura et al. 2010). On the other hand, earlier hydrodynamical work suggested a possibly significant effect (Moeckel et al. 2008; Marzari et al. 2010), but was hampered by very limited statistics. The immediate aftermath of a scattering event, when multiple gravitationally interacting planets can have high-eccentricity orbits that take them into previously undisturbed regions of the disc, is clearly the most challenging situation to model reliably with simple prescriptions. As shown, for example, in Fig. 3, these phases can be important in shaping the long-term evolution of scattered systems, though such individual examples do not mean that the statistical results obtained from a damping prescription might not be ‘good enough. Here, we have presented results from a direct hydrodynamic simulation of a moderately large ensemble of systems, which we have used to extract statistical results for the eccentricity distribution and for the abundance of resonant planetary configuration. We find some differences between our hydrodynamic results and previously published results derived using approximate methods, though we defer for future work the question of whether other analytic prescriptions for planet–disc interactions would yield better agreement.

Our main result is that scattering of random three-planet systems, from non-resonant initial conditions, yields significantly lower eccentricities (as compared to $N$-body runs) when the effects of hydrodynamics are included. This difference does not, however, arise because of any major change in the dynamics of scattering events that either eject planets or result in planetary mergers. In these – dominant – channels, scattering in the hydrodynamic and $N$-body simulations leads to statistically very similar eccentricity distributions, differing (if at all) only in the fraction of highly eccentric planets. Rather, the lower eccentricities found in the hydrodynamic case occur because a modest number of scattering events evolve into three-planet systems that appear to be stable over an observationally interesting time-scale (at least a Gyr). These systems, which tend to have lower than average eccentricities, are simply not present in runs without gas, whose end state after scattering is uniformly either one- or two-planet systems.

Despite the lower eccentricities, we interpret our results as supporting the model of planetary scattering as the dominant physical mechanism giving rise to observed exoplanet eccentricity. Although gas does impact the outcome of scattering, it does not lead to large-scale circularization of planetary orbits. The differences between the hydrodynamic and gas-free simulations, in fact, are comparable to the variations seen in scattering simulations that start with different planetary mass functions (Raymond et al. 2010). It would almost certainly be possible to find some set of plausible initial conditions that would evolve hydrodynamically, under scattering, to match the observed distribution of massive planet eccentricity, given the freedom to fine-tune the initial planetary spacing or disc dispersal time. Alternatively, and perhaps more naturally, the fraction of stable triple planet systems that are responsible for the lower eccentricity could likely be suppressed by starting with $n_p > 3$. Given the wealth of such possibilities, our results do not constitute an argument against the physical importance of scattering, though they do imply that the relatively precise agreement between $N$-body calculations and observations is likely fortuitous.

The frequency of resonant systems of multiple planets is potentially a much more sensitive probe than the eccentricity distribution of the physics predating scattering. Under purely gravitational evolution, the occupation fraction of resonant systems mirrors the available phase space, and MMRs are uncommon (a few per cent; Raymond et al. 2008). In a laminar, viscous disc, conversely, almost arbitrarily high-resonant fractions can be achieved (Matsumura et al. 2010). Our results fall somewhere in between these extremes. The frequency of two-planet resonances that we find is comparable to...
that found in pure $N$-body simulations. This is consistent with the conclusion, noted above, that the hydrodynamics and dynamics of scattering are very similar for those channels that result in planetary ejection or merger. However, we also observe a high frequency of resonant chains of three-planet systems. Out of 100 runs, 12 evolved into libration of the Laplace angle $\phi_L$. This resonance trapping stabilized the systems against instability, pushing the number of unscattered systems in our runs to nearly 30 per cent. Similar effects were seen in the $N$-body simulations of Raymond et al. (2010), where the dissipative role that is here played by gas was instead furnished by planetesimals. Observationally, some resonant systems are seen, but it is reasonably clear that their number is lower than theoretical expectations based on an extended period of evolution in a viscous disc. One possibility is that survival in resonance is frustrated at early times, when the disc is massive and turbulent fluctuations lead to stochastic torques on planets (Adams, Laughlin & Bloch 2008), such that resonance capture only becomes possible in a limited window just prior to disc dispersal. The quantitative details of such a scenario are, however, poorly known, and it is likely that our limited understanding of the mechanisms that can disrupt resonances is the dominant uncertainty in current models of early planetary system evolution.

Finally we have explored the potential of future imaging to directly observe the disc structures formed during the epoch of planetary scattering, which include eccentric gaps and massive planets directly accreting from the disc gas. The predicted structures are distinctive, but the limited synchronization between the onset of scattering and the dispersal of the disc means that they should only rarely be present in a sample of transition discs. We estimate an abundance that might be of the order of a few per cent. A blind search of transition discs is thus unlikely to uncover any systems with ongoing scattering, though in principle it may be possible to exploit the large-scale asymmetries generated during scattering to identify possible candidates.

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APPENDIX A: THE EFFECT OF ZERO-INCLINATION INITIAL CONDITIONS

The FARGO and MERCURY scattering calculations reported in this paper employ exactly planar planetary initial conditions. This is done for reasons of computational economy: 3D hydrodynamic simulations with the same in-plane resolution as the 2D runs would require around two orders of magnitude greater resources. For many purposes, moreover, assuming planarity is not a bad approximation. Prior $N$-body scattering calculations, run using the same mass distribution as ours, show that the median inclination of surviving planets is only $\approx 10^\circ$ (Raymond et al. 2010). This means that the initial phase of scattering typically involves inclinations $i \sim \dot{h}/r$, or smaller, and accordingly we expect that any hydrodynamic interaction with the disc will be dominated by planar effects (eccentricity damping and accretion) rather than fully 3D ones (such as warp excitation). As noted in Section 3, however, the assumption of exact coplanarity does introduce a technical subtlety, since the final $f(e)$ that results from scattering in planar systems is not the same as that which results from otherwise identical initial conditions with a small (of the order of 1$^\circ$) non-zero inclination. Because of this subtlety, we compare our hydrodynamic simulations not to the same $N$-body runs that match the observed eccentricity distribution, but rather to their zero-inclination analogues.

We emphasize that the point of this paper is not to argue that the result of the planar calculation is physically realistic. Rather, from the fact that in 2D the effect of hydrodynamics on the final
Figure A1. The cumulative eccentricity distributions, following scattering, obtained from purely \( N \)-body simulations of three-planet systems initialized with different inclinations. The highest mean eccentricities result from initial conditions in which all three planets had inclinations chosen randomly and uniformly in the interval \( i = [0^\circ, 1^\circ] \) (blue curve). Lower eccentricities result from choosing \( i = [0^\circ, 0.1^\circ] \), \( i = [0^\circ, 0.01^\circ] \) and \( i = 0^\circ \). The black curve shows results for a strictly planar geometry. In all cases, only surviving planets from those systems that went unstable during the integration are plotted.

As the eccentricity distribution is small, we infer that in 3D a hydrodynamic calculation would yield results similar to the observed distribution. That said, a physical effect is in principle possible if the gas disc were able to damp mutual inclinations (Lubow & Ogilvie 2001; Marzari & Nelson 2009) to the point where the final system was effectively planar. To quantify this possibility, we have run a set of \( N \)-body \textsc{mercury} experiments to identify how small the initial inclination must be for the system to scatter as if it were exactly planar. The set-up is identical to that used earlier. We start with three planets on circular orbits, separated by \( K = 4.0 \) mutual Hill radii (equation 1), with the inner planet at 3 au. The mass spectrum is \( f(m) \propto m^{-1.1} \) between 0.3 \( M_J \) and 5 \( M_J \), and the planetary radius is 1.3 \( R_J \). We run four ensembles of 500 simulations, with the planetary inclinations chosen randomly and uniformly in the range \( i = [0^\circ, 1^\circ] \), \( i = [0^\circ, 0.1^\circ] \), \( i = [0^\circ, 0.01^\circ] \) and \( i = 0^\circ \). The systems are integrated for 15 Myr, after which we plot the eccentricity distribution of the surviving planets in the unstable systems (at this epoch, this is about 80 per cent of the total). As before, we discard double systems that are not Hill stable, which removes only \( \sim 10 \) runs from each set.

The results for the eccentricity distribution are shown in Fig. A1. We find that the ensembles initialized with \( i = [0^\circ, 0.1^\circ] \) and \( i = [0^\circ, 0.01^\circ] \) are similar to, but statistically distinct from, the zero-inclination limit (KS \( p \)-values 0.049 and 0.023), whereas inclinations of the order of 1\(^\circ\) are sufficient to result in markedly larger final eccentricities, distinct from all other distributions. To an order of magnitude, the condition for scattering behaviour that is distinct from the 3D limit in these three planet systems is thus that the initial inclinations should satisfy \( i < 0.1^\circ \).

Could real planetary systems be flat enough that their scattering dynamics are effectively 2D? It seems unlikely. Although a gas disc damps the inclination of a single planet on a short time-scale, it will damp the orbit until it is coincident with the local angular momentum vector of the gas, which will not be exactly constant with radius. A warp whose magnitude (over several au) exceeds the threshold \( \Delta i = 0.1^\circ \) seems almost inevitable (Terquem & Papaloizou 2000; Lubow & Pringle 2010). Even if the gas disc were perfectly flat, any temporary resonances between the planets would likely excite their mutual inclination (Thommes & Lissauer 2003) to the level at which subsequent scattering would occur in the 3D limit.

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