Effects of losses on the sensitivity of an actively correlated Mach-Zehnder interferometer

Qiang Wang1, Gao-Feng Jiao1, Zhifei Yu1, L. Q. Chen1,2,3, Weiping Zhang2,3,4, and Chun-Hua Yuan†

1State Key Laboratory of Precision Spectroscopy, Quantum Institute for Light and Atoms, Department of Physics, East China Normal University, Shanghai 200062, China
2School of Physics and Astronomy, and Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China
3Shanghai Research Center for Quantum Sciences, Shanghai 201315, China and
4Collaborative Innovation Center of Extreme Optics, Shanzi University, Taiyuan, Shanxi 030006, China

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We theoretically studied the quantum Cramér-Rao bound of an actively correlated Mach-Zehnder interferometer (ACMZI), where the quantum Fisher information obtained by the phase-averaging method can give the proper phase-sensing limit without any external phase reference. We numerically calculate the phase sensitivities with the method of homodyne detection and intensity detection in the presence of losses. Under lossless and very low loss conditions, the ACMZI is operated in a balanced case to beat the standard quantum limit (SQL). As the loss increases, the reduction in sensitivity increases. However within a certain range, we can adjust the gain parameters of the beam recombination process to reduce the reduction in sensitivity and realize the sensitivity can continue to beat the SQL in an unbalanced situation. Our scheme provides an optimization method of phase estimation in the presence of losses.

I. INTRODUCTION

The fundamental setup of a metrology device is a Mach-Zehnder interferometer (MZI), which has been used in many fields such as phase estimation and gravitational wave detection [1, 2]. Sensitivity is one of the most important indexes of an interferometer. However in a generic interferometric measurement using an MZI and classical light sources the precision of estimating the relative phase delay $\phi$ inside the interferometer is bounded by $\Delta^2 \phi \geq 1/n^2$, where $n$ is the average photon number inside interferometer. This bound is referred to as the shot-noise limit or standard quantum limit (SQL) [3].

In order to improve the sensitivity, researchers have proposed a number of schemes. The first way is to use quantum states to beat SQL such as squeezed states [4], N00N states [5], twin Fock states [6], and two-mode squeezed states [7]. The other way of beating SQL is to use active elements in an interferometer. SU(1,1) interferometer was proposed to one of them by Yurke in 1986 [7]. For SU(1,1) interferometer, the beam splitters in the MZI are replaced by nonlinear beam splitter such as an optical parametric amplifier (PA) or a four-wave mixers, which are mathematically characterized by the group SU(1,1). Because the sensitivities of these interferometers can achieve Heisenberg limit, this type of interferometers have received extensive attention both experimentally [8, 17] and theoretically [18, 23].

Combination of the quantum state input and active elements in interferometers, a new variant of MZI, actively correlated Mach-Zehnder interferometer (ACMZI) recently was proposed [30]. Compared to the traditional MZI, in addition to a coherent state in one input port, one mode of a two-mode squeezed-vacuum state is in the other input port and the output is detected with the active elements. That is the final interference output of the MZI is detected with the method of active correlation output readout.

In this paper, we theoretically derive the quantum Cramér-Rao bound (QCRB) of ACMZI according to the quantum Fisher information (QFI) obtained by the phase-averaging method, which can give the proper phase-sensing limit without any external phase reference. To approach the QCRB, we calculate the phase sensitivities with the method of homodyne detection (HD) and intensity detection (ID) in the presence of losses. Under lossless and very low loss conditions, the ACMZI is operated in a balanced case to beat the SQL. As the loss increases, the reduction in sensitivity increases. We can adjust the gain parameters of the beam recombination process to reduce the reduction in sensitivity and realize the sensitivity can continue to beat the SQL within a certain range. The corresponding optimization conditions are given.

Our paper is organized as follows. In Sec. II, the QCRB of ACMZI is derived according to the phase-averaging method. In Sec. III, the phase sensitivity is studied with the method of HD and ID in the presence of losses. Due to losses the sensitivity is reduced in a balanced case, and we describe that the sensitivity can continue to beat the SQL in an unbalanced situation by optimizing the gain ratio. Finally, our results are summarized.
In this section, we review the input-output relation of ACMZI. As is shown in Fig. 1, one input port of MZI is injected with a strong pump field, and another input port is sent with one mode of a two-mode entanglement state generated by PA1. The unknown phase shift is embedded in one arm of MZI as the estimator. One of the output fields of the MZI and the other mode of the two-mode squeezed state are combined by with PA2 to realize enhanced readout. The four modes in the ACMZI are described by the annihilation operators $\hat{a}_1$, $\hat{b}_1$, $\hat{c}_1$ and $\hat{d}_i$ ($i = 0, 1, 2, 3$). The phase shift is described as $U(\phi) = e^{i\phi n_{a1}}$. In the Heisenberg picture, the output ports $\hat{a}_2$ and $\hat{b}_3$ described by

$$\hat{a}_2 = T_1 \hat{a}_0 + T_2 \hat{b}_1 + T_3 \hat{c}_1, \quad \hat{b}_3 = M_1^* \hat{a}_0 + M_2^* \hat{b}_0 + M_3^* \hat{c}_0,$$

with

$$T_1 = G_2 G_1 + g_1 g_2 e^{i(\theta_2 - \theta_1)} (T - R e^{-i\phi}),$$
$$T_2 = G_2 g_1 e^{i\theta_1} + G_1 g_2 e^{i\theta_2} (T - R e^{-i\phi}),$$
$$T_3 = \sqrt{TR} g_2 e^{i\theta_2} (1 + e^{-i\phi}),$$
$$M_1^* = G_1 g_2 e^{i\theta_2} + G_2 g_1 e^{i\theta_1} (T - R e^{i\phi}),$$
$$M_2^* = g_1 g_2 e^{i(\theta_2 - \theta_1)} + G_2 G_1 (T - R e^{i\phi}),$$
$$M_3^* = \sqrt{TR} G_2 (1 + e^{i\phi}),$$

where $R$ and $T$ are the reflectivity and transmissivity of the two BSs, $G_1$ and $G_2$ are the gain factors of PA1 and PA2, for wave splitting and recombination with $G_i^2 - g_i^2 = 1$ ($i = 1, 2$), $\theta_1$ and $\theta_2$ describe the phase shifts of the PAs for wave splitting and recombination, respectively.

II. QCRB OF ACMZI

A. Input-output relation of ACMZI

In this section, we review the input-output relation of ACMZI. As is shown in Fig. 1, one input port of MZI is injected with a strong pump field, and another input port is sent with one mode of a two-mode entanglement state generated by PA1. The unknown phase shift is embedded in one arm of MZI as the estimator. One of the output fields of the MZI and the other mode of the two-mode squeezed state are combined by with PA2 to realize enhanced readout.

The four modes in the ACMZI are described by the annihilation operators $\hat{a}_1$, $\hat{b}_1$, $\hat{c}_1$ and $\hat{d}_i$ ($i = 0, 1, 2, 3$). The phase shift is described as $U(\phi) = e^{i\phi n_{a1}}$. In the Heisenberg picture, the output ports $\hat{a}_2$ and $\hat{b}_3$ described by

$$\hat{a}_2 = T_1 \hat{a}_0 + T_2 \hat{b}_1 + T_3 \hat{c}_1,$$
$$\hat{b}_3 = M_1^* \hat{a}_0 + M_2^* \hat{b}_0 + M_3^* \hat{c}_0,$$

B. Phase averaged QFI

In this section, the QFI and phase averaged QFI of ACMZI are given, respectively. The QFI is the intrinsic information about the quantum state and is not related to the actual measurement procedure. The phase estimation bound can be dealt with the method of QFI \cite{31, 32}. We consider one input $\hat{c}_0$ is a coherent state $|\alpha\rangle$, and the other two input $\hat{a}_0$ and $\hat{b}_0$ is a vacuum state. As shown in Fig. 1, the probe state is a pure state $|\psi_0\rangle$, and after injecting into the PA1 and BS1 the state transform into the correlated probe state $|\psi_1\rangle$, and the probe state $|\psi_1\rangle$ is modified as $|\psi_0\rangle$ after phase shifted. For the symmetric logarithmic derivatives (SLD) operator, the QFI is $F = 4(\partial_\psi |\psi_0\rangle \langle \psi_0| - |\partial_\psi |\psi_0\rangle |\psi_0\rangle)^2$, where $|\partial_\psi |\psi_0\rangle = \partial |\psi_0\rangle / \partial \phi$ \cite{33, 34}. After the calculation, the QFI is given by

$$F = N_c \left[ 4T^2 + 4TR (G_1^2 + g_1^2) \right] + g_1^2 \left[ 4R^2 G_1^2 + 4TR \right],$$

where $N_c = \langle \alpha | \partial_\alpha |\alpha\rangle$. When $N_c \gg G_1^2$, $F \approx N_c \left[ 4T^2 + 4TR (G_1^2 + g_1^2) \right]$.

As pointed by Jarzyna et al. \cite{35}, the QFI-only approach may have the pitfall that the optimal POVM attaining the QCRB might contain huge amounts of hidden resources. The remedy is to exclude any external resources that might provide some phase information to the measuring device when implementing the best POVM. Such a "rule-out" protocol was introduced by Jarzyna \cite{35}, where the issue is resolved by introducing the phase-averaging of the three-mode input state via a common phase shift. The QFI obtained by the phase-averaging method can give the proper phase-sensing limit without any external phase reference \cite{36, 37}.

Let us expand the input state $|\psi_0\rangle = \sum_n c_n |n\rangle \langle m| \otimes |0\rangle_B \otimes |0\rangle_A$.  

$$|\psi_0\rangle = \sum_{nm} c_n c_m |n\rangle \langle m| \otimes |0\rangle_B \otimes |0\rangle_A.$$
where \(|n⟩\) is the photon number state. The reference framed between the inputs and the measurement is removed by phase averaging the input state as

$$|ψ_0⟩⟨ψ_0|_{ave} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{dθ}{dφ} e^{iθ(n-m)} |0n⟩⟨0m| = \sum_n \sum_m P_n |n⟩⟨n| \otimes |0⟩_B \otimes |0⟩_A$$

(6)

where \(V^A = e^{iθa^† a}, V^B = e^{iθb^† b}, V^C = e^{iθc^† c}\), \(P_n = c_n c^*_m\), and \(\sum_n P_n = 1\). The output state is given by

$$|ψ_φ⟩⟨ψ_φ|_{ave} = T^{a,φ}_{T,\phi} (|ψ_0⟩⟨ψ_0|_{ave}) T^{T,φ}_{T,\phi} = \sum_n P_n |ψ_n⟩⟨ψ_n|,$$

(7)

where \(|ψ_n⟩ = T^{a,φ}_{T,}\phi |0n⟩ = e^{iφn} B_{T,\phi} |0n⟩\), \(B_{T,\phi}\) represents the combined action of PA1 and BS1.

Due to the convexity of the QFI \([38]\), we can obtain the total QFI of the above density matrix, which is given by

$$F_{ave} = \sum_n P_n F (|ψ_n⟩)$$

$$= 4R^2 G_1^2 g_1^2 + 4TR [N_c G_1^2 + g_1^2 (N_c + 1)]$$

$$= 4N_c TR (G_1^2 + g_1^2) [4R^2 G_1^2 + 4TR].$$

(8)

Because of \(N_c \gg G_1^2\), we rewrite the above equation as \(F_{ave} \approx 4N_c TR (G_1^2 + g_1^2)\). Comparing \(F\) with \(F_{ave}\), we can simply find that the fluctuation terms of the coherent light \((4TR)\) are eliminated. The \(F_{ave}\) value of phase averaging is more accurate, then we use \(F_{ave}\) as our bound. When \(T = R = 0.5\), the value of \(F_{ave}\) is the maximum value:

$$\max F_{ave} = N_c (G_1^2 + g_1^2).$$

(9)

The QCRB states that whatever the measurement chosen, the following bound on the estimation uncertainty holds

$$\Delta^2 φ \geq \frac{1}{F_{ave}}.$$  

(10)

III. PHASE SENSITIVITIES

In this section, we study the ultimate precision limit of ACMZI in noisy metrology and the effects of losses on phase sensitivities \(Δφ\) and optimal gain ratio \((G_2/G_1)_{opt}\). The QFI represents the maximum amount of information that can be extracted from quantum experiments. However, saturating the limit obtained by QFI is an important issue. Here, we consider two detection methods:

![Image 320x571 to 573x750](image)

FIG. 2. Phase sensitivity as a function of \(φ\) (a) balanced case and (b) unbalanced case with \(G_2^2 = 20\). Parameters: \(N_C = 1000\), \(G_1^2 = 5\), and \(T = R = 0.5\).

ID and HD. Through an error propagation analysis the sensitivity is given by \([39]\)

$$\Delta^2 φ = \frac{D^2 (O)}{|d(O)/dφ|^2},$$

(11)

where \(D^2 (O) = \langle O^2 \rangle - \langle O \rangle^2\), \(\langle Δ^2 O \rangle\) denotes the noise of observable \(O\), and \(d(O)/dφ\) is the slope with respect to the corresponding phase shift.

By adding the fictitious beam splitters, the arms of the MZI have different transmission rates, where \(η_a, η_b\) are the external transmission rates of the MZI, and \(η_c, η_d\) are the internal transmission rates of the MZI, as shown in Fig. 1. Then the transforms of the fields is given by

$$\hat{a}'_1 = \sqrt{η_a} \hat{a}_1 + \sqrt{1 - η_a} \hat{v}_a, \hat{b}'_2 = \sqrt{η_b} \hat{b}_2 + \sqrt{1 - η_b} \hat{v}_b,$$

$$\hat{c}'_2 = \sqrt{η_c} \hat{c}_2 + \sqrt{1 - η_c} \hat{v}_c, \hat{d}'_1 = \sqrt{η_d} \hat{d}_1 + \sqrt{1 - η_d} \hat{v}_d.$$

(12)

where the \(\hat{v}_i\) \((i = a, b, c, d)\) represent the vacuum. There-
fore, the input-output relation is written as
\begin{align}
\hat{a}_{2l} & = T_{1l} \hat{a}_0 + T_{2l}^\dagger \hat{b}_0^\dagger + T_{3l} \hat{c}_0 + T_4 \hat{v} + T_5 \hat{v}_c^\dagger + T_7 \hat{v}_d^\dagger, \\
\hat{b}_{3l} & = M_{4l}^* \hat{a}_0^\dagger + M_{4l}^* \hat{b}_0 + M_{3l}^* \hat{c}_0 + M_4^* \hat{v} + M_5^* \hat{v}_c + M_7^* \hat{v}_d,
\end{align}
(13)
(14)
where
\begin{align}
T_{1l} & = G_1 G_2 \sqrt{\eta_a + \eta_b e^{i(\theta_2 - \theta_1)}} \sqrt{\eta_b (T \sqrt{\eta_d} - R \sqrt{\eta_c} e^{-i\phi})}, \\
T_{2l} & = G_2 g_1 \sqrt{\eta_b e^{i\theta_1}} + G_1 g_2 e^{i\theta_2} \sqrt{\eta_b (T \sqrt{\eta_d} - R \sqrt{\eta_c} e^{-i\phi})}, \\
T_{3l} & = \sqrt{T R} g_2 e^{i\theta_2} \sqrt{\eta_b (1 - \eta_c)} e^{i\theta_2}, \\
T_4 & = g_2 \sqrt{1 - \eta_b} e^{i\theta_2}, \\
T_5 & = g_2 \sqrt{1 - \eta_b} e^{i\theta_2}, \\
M_{4l}^* & = G_2 g_1 \sqrt{\eta_b e^{i\theta_2}} + G_2 g_2 e^{i\theta_1} \sqrt{\eta_b (T \sqrt{\eta_d} - R \sqrt{\eta_c} e^{-i\phi})}, \\
M_{3l}^* & = g_1 g_2 \sqrt{\eta_b e^{i(\theta_2 - \theta_1)}} + G_2 g_1 \sqrt{\eta_b (T \sqrt{\eta_d} - R \sqrt{\eta_c} e^{-i\phi})}, \\
M_{4l}^* & = G_2 \sqrt{T R} \sqrt{\eta_b (1 - \eta_c)}, \\
M_{5l}^* & = G_2 \sqrt{1 - \eta_b}, \\
M_{7l}^* & = \sqrt{T G_2} \sqrt{\eta_b (1 - \eta_d)}.
\end{align}
(15)

A. Homodyne detection

For the HD, the measurement operator is $\hat{Y} = i(\hat{b}_3^\dagger - \hat{b}_3)$. Under the condition of $\theta_1 = 0$ and $\theta_2 = \pi$, the slope and variance are given by
\begin{equation}
\frac{\partial \langle \hat{Y} \rangle}{\partial \phi} = 2 \sqrt{N C \sqrt{T R} G_2} \sqrt{\eta_b \eta_c} \cos \phi,
\end{equation}
(16)
and
\begin{equation}
\langle \Delta^2 \hat{Y} \rangle = 2 \eta_a G_2 g_2^2 + 2 T^2 G_2^2 \eta_b \eta_d + 2 R^2 G_1^2 \eta_b \eta_c + 4 (G_1 \sqrt{\eta_a g_2} - T G_2 g_2 \sqrt{\eta_b} \sqrt{\eta_d}) \times (R G_2 g_1 \sqrt{\eta_b} \sqrt{\eta_c}) \cos \phi + 2 g_2^2 (1 - \eta_a) - 4 T G_1 G_2 g_2 \sqrt{\eta_b} \sqrt{\eta_c} \sqrt{\eta_d} + 1.
\end{equation}
(17)

From Eq. (17), we can obtain the phase sensitivity $\Delta \phi^{HD}$ in the presence of losses. From the phase sensitivity $\Delta \phi^{HD}$, the optimal gain $G_2$ should satisfy the following condition
\begin{equation}
\left( \frac{g_2}{G_2} \right)_{opt} = \frac{G_1 g_2 \sqrt{\eta_b} \sqrt{\eta_c} (T \sqrt{\eta_d} + R \sqrt{\eta_c})}{\eta_b G_1^2 + (1 - \eta_a) - 1/2}.
\end{equation}
(18)
As the loss increases, the sensitivity reduction increases. However, we adjust the gain ratio $G_2/G_1$ to reduce the reduction in sensitivity. Due to the SQL is independent of $G_2$, we adjust the gain parameter $G_2$ of the beam recombination process to reduce the reduction in sensitivity and realize that the sensitivity can continue to beat SQL within a certain range.
sensitivity at \( \phi = \pi \). For balanced case \((G_1 = G_2)\), the optimal phase sensitivity \( \Delta^2 \phi^{HD}_{\text{opt}} \) is written as

\[
\Delta^2 \phi^{HD}_{\text{opt}} = \frac{1}{4TRG^2N_c}.
\]  

(21)

In Fig. 2(a), the sensitivity \( \Delta \phi^{HD}_{\text{opt}} \) can beat the SQL. It can approach but cannot reach the QCRB. For the unbalanced case \((G_1 \neq G_2)\), the optimal \( \Delta^2 \phi^{HD}_{\text{opt}} \) is written as

\[
\Delta^2 \phi^{HD}_{\text{opt}} = \frac{2(G_2g_1 - G_1g_2)^2 + 1}{4NG^2TR}.
\]  

(22)

When the gain ratio \( G_2/G_1 \geq 2 \) and increases, the sensitivity \( \Delta \phi^{HD}_{\text{opt}} \) always reach the QCRB, as shown in Fig. 2(b) and Fig. 3. For a given \( G_1 \), to obtain the optimal sensitivity the value of \( G_2 \) should satisfy the following conditions

\[
\left( \frac{g_2}{G_2} \right)_{\text{opt}} = \frac{2G_1g_1}{G_1^2 - 1}.
\]  

(23)

In the presence of losses, from Eq. (11) and Eq. (17) we can get the optimal \( \Delta \phi^{HD}_{\text{opt}} \) when \( \phi = \pi \). On balanced case, the phase sensitivity \( \Delta \phi^{HD}_{\text{opt}} \) as a function of transmission rates of two arms can be obtained, where the SQL denoted as SQL0 in the contour figure. After optimizing the \( G_2 \), the phase sensitivity \( \Delta \phi^{HD}_{\text{opt}} \) as a function of transmission rates of two arms can also be obtained, where the SQL denoted as SQL1 in the contour figure. For given \( G_1 \) the value of SQL0 and SQL1 is equal, the position of SQL in the contour figure of the sensitivity versus transmission rates has changed.

When only considering the external loss \( \eta_a \) and \( \eta_b \) \((\eta_a = \eta_b = 1)\) or internal loss \( \eta_c \) and \( \eta_d \) \((\eta_a = \eta_b = 1)\), the SQL0 (solid line), SQL1 (dashed line) and the contour line of optimized \( G_2/G_1 \) as a function of \( \eta_a \) and \( \eta_b \) or \( \eta_c \) and \( \eta_d \) are shown in Fig. 4(a) and Fig. 4(b), respectively. The phase sensitivities in the area of upper right corner and within the SQL lines in Fig. 4(a) and Fig. 4(b) can beat the SQL. From these two figures, it is demonstrated that for a given \( G_1 \) and transmission rates of two arms by optimizing \( G_2/G_1 \), the small area between the SQL0 and SQL1 can still beat the SQL.

By Comparison, in Fig. 4(b) when only considering the internal loss \( \eta_c \) and \( \eta_d \), the added area that can beat SQL between the SQL0 and SQL1 is larger. We obtain that the phase sensitivity can tolerate the internal loss \( \eta_c \) and \( \eta_d \), and it can still beat SQL with about 20% of the photon loss of \( \eta_c \) even if \( \eta_d = 0 \). The reason is that the internal loss only affects the output field of MZI. However, the external loss affects not only the output field of MZI, but also the quantum field of another combined beam.

B. Intensity detection

As shown in Fig 1, we use the \( \hat{n}_{32} = \hat{a}_3^\dagger \hat{a}_2 + \hat{b}_3^\dagger \hat{b}_2 \) as the detection variable. We analyze the phase sensitivity at \( \theta_1 = 0 \) and \( \theta_2 = \pi \). The slope is

\[
\frac{\partial \langle \hat{n}_{32} \rangle}{\partial \phi} = [2TR\eta_b \sqrt{\eta_c} (G_2^2 + g_2^2) (N_c - g_1^2) + 4RG_2G_1g_2 \eta_b \eta_c \sin \phi]
\]  

(24)

the fluctuation of the quadrature \( \Delta \hat{n}_{32} \) is

\[
\langle \Delta^2 \hat{n}_{32} \rangle = \frac{N_c [h_3 h_3^* + h_3 h_3^* + h_3 h_3^* + h_3 h_3^*]}{2G_2 g_2 (N_c - g_1^2) + 4RG_1G_2 g_2 \eta_b \eta_c \sin \phi}
\]  

(25)

where \( h_i h_j^* = T_i T_j^* + M_i M_j^* \) (i, j = 1, 2, 3, 4, 5, 6, 7).

Under the condition of lossless, the phase sensitivity is given by

\[
\Delta \phi^{ID}_{\text{opt}} = \left[ \frac{K + J^2 + L (\cos \phi + 1)^2}{I \sin^2 \phi} \right]^{1/2},
\]  

(26)

where subscript ID indicates the intensity detection and

\[
I = [2TR (G_2^2 + g_2^2) (N_c - g_1^2) + 4RG_1G_2 g_2 R_1^2],
\]

\[
J = G_1 G_2 (G_2^2 + g_2^2) [2TR - 2 + 2TR \cos \phi] + 4RG_2 G_1 g_2 (G_1^2 + g_1^2) (T - R \cos \phi),
\]

\[
K = [2G_2 G_1 g_2 - g_1 (G_2^2 + g_2^2)]^2 TR(N_c + 1) + [(G_2^2 + g_2^2) G_1 - 2G_2 g_2 R_1]^2 TR(N_c + 1) + [4G_2 g_2 R_1]^2 TR(N_c + 1) + [4G_2 g_2 R_1]^2 TR(N_c + 1) + 4TR^2 G_2^2 g_2^2 N_c.
\]  

(27)

Next, we analyze the results of balanced \((G_1 = G_2)\) and unbalanced cases \((G_1 \neq G_2)\). For balanced case, when \( \phi \approx \pi \), we can simply come to the following conclusion \( J = 0 \) and \( \frac{\cos \phi + 1}{\sin^2 \phi} \approx 0 \). Therefore the optimal sensitivity of \( \Delta \phi^{ID}_{\text{opt}} \) is

\[
\Delta \phi^{ID}_{\text{opt}} \approx \frac{K}{T} \approx \frac{1}{4TR (G_2^2 + g_2^2) N_c}. \]  

(28)

Compared with the bound obtained from \( F_{\text{ave}} \), the optimal sensitivity using the ID can approach the corresponding QCRB, as the solid line shown in Fig. 2(a). By comparing \( \Delta \phi^{HD}_{\text{opt}} \) and \( \Delta \phi^{ID}_{\text{opt}} \) we can obtain that the sensitivity of ID is superior than that of HD in the balanced case, i.e. \( \Delta \phi^{ID}_{\text{opt}} < \Delta \phi^{HD}_{\text{opt}} \). The sensitivities of two approaches can beat the SQL and the sensitivity of ID can reach the the QCRB.

For unbalanced case, when \( \phi \) tends to \( \pi \), the sensitivity of ID will diverge and the optimal phase point is very near \( \pi \) and dependent on the values of \( G_1 \) and \( G_2 \), as shown in Fig. 2(b). The optimal value \( \phi_{\text{opt}} \) as a function
FIG. 5. Results of ID. SQL0 (solid line), SQL1 (dashed line) and the contour line of optimized $G_2/G_1$ as a function of (a) $\eta_a$ and $\eta_b$ with $\eta_c = \eta_d = 1$, and (b) $\eta_c$ and $\eta_d$ with $\eta_a = \eta_b = 1$. Parameters: $N_c = 1000, T = R = 0.5, G_1^2 = 5$. The phase sensitivities in the area of upper right corner and within the line SQL0 or SQL1 can beat the SQL.

of $G_2/G_1$ for ID is shown in Fig. 5. For lossless case, when $G_1 = G_2$ the detection results of ID is the best. When the gain ratio $G_2/G_1$ is greater than 1, the phase sensitivity $\Delta \phi^{ID}$ always approaches but does not reach the QCRB.

Next, we research the effect of loss on the phase sensitivity $\Delta \phi^{ID}$ and corresponding optimal gain ratio $(G_2/G_1)^{opt}$. When only considering the external loss $\eta_a$ and $\eta_b$ ($\eta_c = \eta_d = 1$) or internal loss $\eta_c$ and $\eta_d$ ($\eta_a = \eta_b = 1$), the SQL0 (solid line), SQL1 (dashed line) and the contour line of optimized $G_2$ as a function of $\eta_a$ and $\eta_b$ or $\eta_c$ and $\eta_d$ are shown in Fig. 5(a) and Fig. 5(b), respectively. Different from HD method, by optimizing $G_2$ the ID method can only increase a little area that can beat SQL. Comparing the two detection methods, it is found that the method HD is more tolerant of photon losses.

IV. CONCLUSION

In conclusion, we theoretically have derived the QCRB of ACMZI according to the QFI obtained by the phase-averaging method. To approach the QCRB, we have calculated the phase sensitivities with the method of ID and HD. Under the condition of lossless, the phase sensitivity with the method of ID can approach the QCRB in balanced case. In the presence of losses, the phase sensitivity is reduced and associated optimal condition is also changed. Within a certain loss range, we can adjust the gain parameters of the beam recombination process to reduce the reduction in sensitivity and realize the sensitivity can continue to beat SQL in an unbalanced situation. In the presence of loss, comparing the results obtained by two different detection methods, we found that the HD is more tolerant of internal loss.

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