Geometrically nonlinear transient vibrations of actively damped anti-symmetric angle ply laminated composite shallow shell using active fibre composite (AFC) actuators

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Abstract. In present work, the thin laminated composite shallow shell as smart structure with AFC material’s ACLD treatment is analyzed for geometrically nonlinear transient vibrations. The AFC material is used to make the constraining layer of the ACLD treatment. Golla-Hughes-McTavish (GHM) is used to model the constrained viscoelastic layer of the ACLD treatment in time domain. Along with a simple first-order shear deformation theory the Von Kármán type non-linear strain displacement relations are used for deriving this electromechanical coupled problem. A 3-dimensional finite element model of smart composite panels integrated with the ACLD treated patches has been modelled to reveal the performance of ACLD treated patches on improving the damping properties of slender anti-symmetric angle-ply laminated shallow shell, in controlling the transient vibrations which are geometrically nonlinear. The mathematical results explain that the ACLD treated patches considerably enhance the damping properties of anti-symmetric angle-ply panels undergoing geometrically nonlinear transient vibrations.

1. Introduction
In the advancement of design of lightweight flexible structures necessitated the use of laminated composite structures in the form of beams, plates and panels. These light weight structures are vulnerable to outsized vibrations with extensive decay time because of small internal damping. To circumvent these problems appropriate active control system which comprises of actuators and/or sensors is integrated on the host structure making them as smart structure [1]. The use of distributed actuators and/or sensors made of piezoelectric material is predominant and enticed the attention of several researchers from past several years [2-6]. “These lightweight flexible structures when coupled with the layer/patches of piezoelectric materials acting as distributed sensors and/or actuators are customarily known as smart structures”. It has been observed that PZT piezoceramics used as sensor/actuator in research works related to smart structures has drawn attention towards increasing the performance of structural control. The PZT piezoceramics has disadvantages such as brittleness, low control authority due to their low stress/strain constants, non-conformability to curved surface and relatively higher density compared to conventional composite structural materials, etc. In an attempt to
get the better performance from the existing monolithic piezoelectric materials, Hau and Fung [7] focused on the effect of active constrained layer damping (ACLD) treatment on analysis of a flexible beam. The unwanted vibrations of the substrate structure can be minimized by adding ACLD treatment to it. Since, ACLD treatment gives the characteristics of both active and passive damping [8] occurring simultaneously and has been expansively used for skilful and consistent active damping of flexible structures. An analysis of ACLD of geometrically nonlinear transient vibrations of thin composite plates using horizontally reinforced piezo-fiber composite material is carried out by R. Suresh Kumar and M C Ray [9]. Ray and co-workers [10] conducted the active damping of nonlinear vibrations of functionally graded plates and vertically reinforced 1-3 PZC as the constraining layer ACLD treatment material for active control of geometrically nonlinear vibrations of composite structures [11]. Much of the work is not reported in open literature to address the ACLD treatment of geometrically nonlinear vibrations of laminated composite shallow shells using AFC material so that in-plane actuation could be used for active control.

2. Finite element modelling
Figure 1 represents the shallow shell composite panel consist of \( N \) orthotropic layers. The letters \( h, s, \phi, a, \) and \( R \) are used to denote thickness, circumferential width, shallowness angle, length and average radius of the panel respectively. The AFC patches are applied on the top surface of the panel through ACLD Technique as shown in Figure 1. ACLD treated viscoelastic constrained layer is having a thickness of \( h_v \) and the electrode spacing in inter-digitated electrode pattern for actuating AFC patch material is \( h_p \). The overall deformation kinematics of the entire panel consists of laminated panel and ACLD treated AFC patches is studied using first order shear deformation theory (FSDT).

Letters \( u, v, \) and \( w \) are used to represent the generalized displacements along \( x, y, \) and \( z \) directions at any point in any coat of the entire panel respectively and can be written as:

\[
\{d\} = \{d_r\} + [Z] \{d_f\}
\]

where \( \{d\} = [u \quad v \quad w]^T \), \( \{d_r\} = [u_r \quad v_r \quad w_r]^T \), \( \{d_f\} = [\theta_x \quad \theta_y \quad \phi_x \quad \phi_y \quad \gamma_x \quad \gamma_y]^T \),

![Figure 1. Laminated shallow shell with patches of ACLD treatment.](image)
but thin structures are subjected to shear locking effect and to avoid this, selective integration rule is employed by selecting the state of the strains as transverse shear strains \( \{ \varepsilon_t \} \) and in-plane strains \( \{ \varepsilon_b \} \) respectively, where:
\[
\{ \varepsilon_s \} = [ \varepsilon_{xz} \ varepsilon_{yz} ]^T \quad \text{and} \quad \{ \varepsilon_b \} = [ \varepsilon_x \ varepsilon_y \ varepsilon_{xy} ]^T
\]
(2)

In the entire panel, the in-plane and transverse shear stresses state at any point can be written as:
\[
\{ \sigma_b \} = [ \sigma_x \ \sigma_y \ \sigma_{xy} ]^T \quad \text{and} \quad \{ \sigma_s \} = [ \sigma_{xz} \ \sigma_{yz} ]^T
\]
(3)

The host panel’s constitutive relations for the orthotropic layers are given by
\[
\{ \sigma^b_k \} = [ \tilde{C}^b_k ] \{ \varepsilon^k_b \} \quad \text{and} \quad \{ \sigma^s_k \} = [ \tilde{C}^s_k ] \{ \varepsilon^k_s \}, \quad k = 1, 2, 3, \ldots N
\]
(4)

The AFC layer applied on the panel will undergo the effect of applied electric field \( (E_x) \) acting along its length (i.e. in x-direction). Therefore the smart structure’s coupled constitutive relations can be given as:
\[
\{ \sigma^k_b \} = [ \tilde{C}^k_b ] \{ \varepsilon^k_b \} - \{ \sigma^k_b \} \{ \varepsilon^k_b \} \{ \varepsilon^k_s \} = [ \tilde{C}^k_s ] \{ \varepsilon^k_s \} \quad \text{and} \quad D_x = [ \tilde{e}^k_b ]^T \{ \varepsilon^k_b \} + \tilde{e}_{11} E_x, \quad k = N + 2
\]
(5)

It is assumed that material used for the viscoelastic layer is linearly viscoelastic and isotropic material. Here, to analyze and model the viscoelastic material of the ACLD treated smart structure in time domain the Golla-Hughes-McTavish (GHM) method is used. Using Stieltjes integral [12] form, the viscoelastic material’s constitutive equation can be expressed as:
\[
\{ \sigma_s \} = \frac{1}{G(t - \tau)} \frac{\partial (\varepsilon_s)}{\partial \tau} \, d\tau
\]
(6)

where \( G(t) \) is the relaxation function for viscoelastic material.

The two types of energies of the smart structure with ACLD treated patches made of AFC material i.e. total potential energy \( T_p \) and kinetic energy \( T_k \) can be expressed as:
\[
T_p = \frac{1}{2} \sum_{k=1}^{N+2} \left( \{ \varepsilon^k_b \}^T \{ \sigma^k_b \} + \{ \varepsilon^k_s \}^T \{ \sigma^k_s \} \right) d\Omega - \int \frac{E_x \, D_x \, d\Omega}{A} - \int \{ f \}^T \{ \varphi \} \, dA
\]
(7)

and
\[
T_k = \frac{1}{2} \sum_{k=1}^{N+2} \left[ \frac{\rho}{kN} \left( \dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) \right] \, d\Omega
\]
(8)

where \( f \) is the externally applied surface traction vector acting over a surface area \( A \) and \( \Omega \) is the volume of the concerned domain. \( \rho \) is the mass density of \( k \)th layer. Using eight nodded isoparametric quadrilateral elements for discretization of the overall panel the total potential energy \( T_p \) of a typical element with AFC treatment made of AFC material can be quantified as:
\[
T_p = \frac{1}{2} \int \left[ \{ \varepsilon^e \}^T \{ \varepsilon^e \} \right] + \{ \varepsilon^e \}^T \{ \sigma^e \} + \{ \sigma^e \}^T \{ \varepsilon^e \} \, d\Omega + \sum_{k=1}^{N+2} \frac{\rho}{kN} \left( \dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) \, d\Omega
\]
(9)

Using GHM method to model viscoelastic layer [8] and to find the global equations of equilibrium, the elemental governing equations are assembled into the global space represented as:
\[
[M][\ddot{X}] + [K_t][X_t] + [K_{tr}][X_{tr}] + [K_{fsv}][\dot{X}_t] [G(t-r) \frac{\partial}{\partial r} \{X_t\} \dot{\tau} + [K_{frsv}][G(t-r) \frac{\partial}{\partial r} \{X_{tr}\} d\tau = \{F\} + ([F_{tm}] + [F_{tp}])V
\]

(10)

A simple velocity feedback control law is used to find the control voltage needed to apply for activating the patches. The control voltage of each patch is given in the form of derivatives of the global nodal degrees of freedom as shown in equation 11.

\[
V^j = -K^j_d \ddot{w} = -K^j_d[U^j]\{\dot{X}\}
\]

(11)

Where \( K^j_d \) is the control gain for the \( j\)th patch and unit vector \([U^j]\) is a defining the location of sensing the velocity signal which would be fed back to this patch. Using equation 11 the motion equations governing the closed loop dynamics of the substrate plates activated by the patches of ACLD treatments can be obtained as follows:

\[
[M^*][\ddot{X}] + [C^*_d]\{\dddot{X}\} + [K^*]\{X\} = \{F^*\}
\]

(12)

where an active damping matrix is given by \([C^*_d] = [C^*] + \sum_{j=1}^{M} K^j_d[F^*_p][U^j]\]

3. Numerical results

The numerical results are obtained using the FE model derived in the earlier section for evaluating the performance of the ACLD patches on controlling the geometrically nonlinear vibrations of laminated composite shallow shells. Anti-symmetric angle-ply thin circular shell having the square plan form \( a \times a \) and integrated with two rectangular patches of ACLD treatment are used for evaluating the numerical results.

Material properties of the orthotropic layers of the substrate panel, AFC layer and viscoelastic layer are taken as follows [13]:

\[
E_t=172.9\text{GPa}, E_r/E_s=25, G_{ts}=0.5E_t, G_{tr}=0.2E_t, v_{ts}=v_{tr}=0.25, \rho = 1600\text{kg/m}^3
\]

with the symbols indicating the common meaning. The PZT5H and epoxy are used to make the piezoelectric fibres and the matrix of the active patches respectively. For AFC material the effective elastic with 40% fibre volume fraction are taken as [11]

\[
C_{11} = 131.8\text{GPa}; C_{12} = 71.15\text{GPa}; C_{22} = 148.9\text{GPa}; C_{44} = 32.35\text{GPa};
C_{66} = 39.14\text{GPa}; \rho = 3640\text{kg/m}^3,
\]

while the piezoelectric coefficients of the AFC material are given by

\[
e_{12} = e_{13} = 3.34\text{C/m}^2; e_{16} = 0
\]

The different thicknesses of the AFC patch, viscoelastic patch and the substrate panel are considered as 250\,\mu m, 50.8\,\mu m and 3\,mm, respectively. The values of the axial length \((a)\), the shallowness angle \((\phi)\) of the panel and the piezoelectric fiber orientation angle \((\psi)\) of the ACLD patches are taken as 0.5m, 20° and 0°, respectively unless otherwise mentioned. Also, the thicknesses of the orthotropic layers of the substrate panel are equal. The mechanical load \( P \) acting upward is supposed to be uniformly distributed. The SS boundary conditions at the edges of the overall panels taken for evaluating the numerical results are specified by

\[
v_0 = w = \theta_y = \phi_y = \gamma_y = 0 \text{ at } x = 0, a \quad \text{and} \quad u_0 = w = \theta_y = \phi_y = \gamma_y = 0 \text{ at } \gamma = \Omega_0, s
\]

Now to validate the current FE model with [8] for the ACLD patches made of AFC material and with identical conditions for SS symmetric cross-ply \((0º/90º/0º)\) substrate panel with shallowness angle
\( \phi = 20^\circ \). The results shown in figures 2 and 3, indicate that the results obtained by the current order agree in an excellent manner with the existing one [8] in passive and active mode respectively.

**Figure 2.** Comparison of present model with [11] for responses of a SS1 symmetric cross-ply \((0^\circ/90^\circ/0^\circ)\) substrate panel with ACLD \((\phi = 20^\circ, Q = 750)\) using AFC under passive mode

**Figure 3.** Comparison of present model with [8] for responses of a SS symmetric cross-ply \((0^\circ/90^\circ/0^\circ)\) substrate panel with ACLD \((\phi = 20^\circ, Q = 750)\) using AFC under active mode

Figure 4 illustrates nonlinear dynamic responses of a SS anti-symmetric angle-ply \((-45^\circ/45^\circ/-45^\circ/45^\circ)\) substrate panel undergoing ACLD. In this case the AFC patch significantly damps
out the vibration of the shallow shell. The control voltage needed to achieve this control response shown in figure 4 is illustrated in figure 5 in which the values of the voltages are quite nominal and the corresponding phase plot is shown in figure 6.

**Figure 4.** Nonlinear dynamic responses of a simply supported anti-symmetric angle-ply (−45°/90°/−45°/90°) substrate panel undergoing ACLD (ϕ = 20°, Q=750) using AFC

**Figure 5.** Control voltages needed for the ACLD of nonlinear transient vibrations of an anti-symmetric angle-ply (−45°/90°/−45°/90°) substrate panel (ϕ = 20°, Q=750) using AFC
4. Conclusions

In this article, a 3D FE analysis has been conducted to examine the performance of the ACLD treated patches for controlling geometrically nonlinear transient vibrations of thin anti-symmetric angle ply shallow shell composite panels. AFC material is considered for the constraining layer of the ACLD patches.

The results display the effective use of AFC material patches used in ACLD treatment in achieving the active control to suppress the geometrically nonlinear transient vibrations of anti-symmetric angle ply laminated composite shallow shell.

5. References

[1] Rogers C A 1994, Sci. Mach. J. 46 977-983
[2] Mallik N, Ray M C 2005, Int J of Mech and Mat in Design, 2 81-97
[3] Yang S M, Lee Y J 1994, J. Sound and Vibration, 176 289-300
[4] Maurini C, Porfiri M, Pouget J 2006, J. Sound and Vibration, 298 918-933
[5] Maxwell N D, Asokanathan S F 2004, J. Sound and Vibration, 269 19-31
[6] Kuscuoglu Z K, Fallahi B, Royston T J 2004, J. Sound and Vibration 276 27-44
[7] Hau L C, Fung E H K 2004, J. Sound and Vibration, 269 549-567
[8] Shivakumar J, Ashok M H, Ray M C 2013, Acta Mechanica, 224 1-15
[9] Suresh Kumar R, Ray M C 2016, J. Mech of Adv Materials and Structures, 23 652-669
[10] Panda S, Ray M C 2008, Smart Materials and Structures, 17 1-15
[11] Sarangi S, Ray M C 2010, Smart Materials and Structures, 19 875-890
[12] McTavish D J, Hughes P C 1993, 115 103-113
[13] Shivakumar J, Ray M C 2008, J. Reinforced Plastics and Composite, 28 525-541