MAGNETOHYDRODYNAMIC TURBULENT MIXING LAYERS: EQUILIBRIUM COOLING MODELS

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ABSTRACT

We present models of turbulent mixing at the boundaries between hot \((T \sim 10^6 - 10^7 \text{ K})\) and warm material \((T \sim 10^4 \text{ K})\) in the interstellar medium, using a three-dimensional magnetohydrodynamic code, with radiative cooling. The source of turbulence in our simulations is a Kelvin-Helmholtz instability, produced by shear between the two media. We find that because the growth rate of the large-scale modes in the instability is rather slow, it takes a significant amount of time \((\sim 1 \text{ Myr})\) for turbulence to produce effective mixing. We also find that the total column densities of the highly ionized species \((\text{C}^4, \text{N}^6, \text{O}^6)\) per interface (assuming ionization equilibrium) are similar to previous steady state nonequilibrium ionization models but grow slowly from \(\log N \sim 10^{11}\) to a few \(10^{12} \text{ cm}^{-2}\) as the interface evolves. However, the column density ratios can differ significantly from previous estimates, with an order of magnitude variation in \(N(\text{C}^4)/N(\text{O}^6)\) as the mixing develops.

Subject headings: ISM: general — ISM: structure — MHD — turbulence

1. INTRODUCTION

The interstellar medium (ISM) is a very complex entity. It is extremely rich in structure, highly turbulent, and embedded in a dynamically important magnetic field. Although the concept of “phases” is up for debate (Cox 2005), we can safely say that the ISM shows regions of distinct physical conditions, which range from cold and molecular to hot and ionized. A factor that controls the structure of the ISM to a large extent is the balance between heating and cooling processes. In this work, we are mainly concerned with the interfaces between the hot \((T \sim 10^6 \text{ K})\) and warm \((T \sim 10^4 \text{ K})\) media, as well as the transfer of heat between them. At temperatures between the hot and warm media the efficiency of radiative cooling is maximal (see Sutherland & Dopita 1993). Material at such temperatures cools very quickly and therefore should be rarely observed. However, absorption-line studies in the far-ultraviolet (FUV) have found otherwise (e.g., Savage et al. 2000). Enhancement of thermal diffusivity in magnetized plasmas due to turbulent motions is discussed in Cho et al. (2003).

In this work, we study the formation and evolution of turbulent mixing layers by means of the K-H instability. We use a magnetohydrodynamic (MHD) code, which includes radiative cooling. The layout of the paper is as follows: in § 2 we provide a brief review of the theory behind turbulent mixing layers, and in § 3 we describe the code and our numerical setup. The results, including estimates of column densities and line ratios of highly ionized species, can be found in § 4, followed by a summary in § 5.

2. TURBULENT MIXING LAYERS

The idea of a turbulent mixing layer was proposed in BF90 as an important heat transfer mechanism between two media at different temperatures. The basic picture proposed is that turbulence at the boundary of the two fluids will provide a constant input to the intermediate-temperature mixture. If energy is conserved, this mixing layer would grow indefinitely, and eventually all of the material would be at such an intermediate temperature. To prevent this they proposed a steady state in which the energy lost by radiation (radiative cooling is most efficient precisely at such
intermediate temperatures) is balanced by a turbulent heat flux into the mixing layer. As it is usual, most of the energy in the turbulence is on the largest scales and cascaded down to a dissipative level. Thus, the model of BF90 is basically a three-phase steady state fluid in which the losses due to radiation are balanced with energy entrained by turbulence into the intermediate-temperature zone. In their model, the temperature of the mixing layer is determined by mass flux balance:

$$\bar{T} = \frac{\bar{m}_\text{hot} T_\text{hot} + \bar{m}_\text{cold} T_\text{cold}}{\bar{m}_\text{hot} + \bar{m}_\text{cold}},$$  \hspace{1cm} (1)

where $\bar{m}_\text{hot}$ and $\bar{m}_\text{cold}$ are the mass flux rates from the hot and from the cold phase into the layer, and $T_\text{hot}$ and $T_\text{cold}$ are the temperatures of the hot and cold phases, respectively. Since neither the heat transfer nor the mixing will be perfect, BF90 introduced two efficiency factors: $\eta_\text{hot}$, the fraction of mass and energy that is deposited by the hot medium to the mixing layer, and $\eta_\text{cold}$, an efficiency for the hydrodynamic mixing. With these factors the mass flux rates become

$$\bar{m}_\text{hot} = \eta_\text{hot} \rho_\text{hot} v, \hspace{1cm} (2)$$

$$\bar{m}_\text{cold} = \eta_\text{cold} (\rho_\text{hot} / \rho_\text{cold})^{1/2} v, \hspace{1cm} (3)$$

and the resulting intermediate temperature will be

$$\bar{T} \approx \left( \frac{\eta_\text{hot} + \eta_\text{cold} (T_\text{cold} / T_\text{hot})^{1/2}}{\eta_\text{cold} + \eta_\text{hot} (T_\text{cold} / T_\text{hot})^{1/2}} \right) (T_\text{cold} T_\text{hot})^{1/2} \equiv \xi (T_\text{cold} T_\text{hot})^{1/2}. \hspace{1cm} (4)$$

The definition of the two different efficiencies reveals what the authors had in mind as the mechanism that provides the mixing. The efficiency associated with the entrainment of hot gas simply corresponds to an enthalpy flux $\dot{e} \eta_\text{hot} \rho v$, where $\rho$ is the pressure and $v$ the turbulent velocity. This yields equation (2), where $\rho_\text{hot}$ is the mass density of the hot medium and the contribution of turbulent kinetic energy has been neglected. The cold gas is then pulled into the hot medium by turbulent eddies at a rate $\eta_\text{cold} \rho_\text{cold} v$ due to turbulence. Turbulent eddies are formed on scales $l_\text{cold} < l_\text{turb}$, where $l_\text{turb}$ is the total thickness of the layer, and $l(T_\text{cold})$ can be thought of as an eddy turnover time for eddies of size $l(T_\text{cold})$. BF90 used the Kelvin-Helmholtz growth rate $\eta_\text{K-H}(l(T_\text{cold})) \sim (\rho_\text{cold} / \rho_\text{hot})^{1/2} l(T_\text{cold})^{1/2}$, where $\rho_\text{cold}$ is the mass density of the cold medium, leading to equation (3). However, the form of the timescale $\sim (\rho_\text{cold} / \rho_\text{hot})^{1/2} l(T_\text{cold})^{1/2}$ is not exclusive for the K-H instability but suitable for fully developed turbulence in general. The main uncertainty in the BF90 model lies in these efficiency factors. Their model is also derived assuming fully developed turbulence (i.e., rapid mixing) and therefore does not include the effects of cooling in the dynamical development of turbulence.

Later, SSB93 expanded on the ideas of BF90 and ran a grid of models based on one-dimensional, instantaneous, steady state mixing, characterized principally by $T$ and $v_\text{turb}$. These included the effects of nonequilibrium ionization and self-photoionization of the gas in the mixing layer, but adopted a somewhat ad hoc value for $\xi$ and the efficiencies ($\eta$).

In this work we focus on the dynamical formation and development of the mixing layer. We do not include effects of nonequilibrium or self-photoionization, but we measure the actual value of $\bar{T}$, which is the result of a continuous distribution of temperatures that range from $T_\text{hot}$ to $T_\text{cold}$, \textsuperscript{6} instead of fixing any efficiency factor.

3. OUR MODEL

3.1. The Code

We solve the following system of equations:

$$\frac{\partial p}{\partial t} + \nabla \cdot (p v) = 0, \hspace{1cm} (5)$$

$$\frac{\partial p}{\partial t} + v \cdot \nabla v + \frac{1}{\rho} \nabla p - \frac{\nabla \cdot (B \times B)}{4\pi \rho} = 0, \hspace{1cm} (6)$$

$$\frac{\partial p}{\partial t} + v \cdot \nabla p + \nu \nabla \cdot (v \times B) = 0, \hspace{1cm} (7)$$

$$\frac{\partial B}{\partial t} - \nabla \times (v \times B) = 0. \hspace{1cm} (8)$$

with $\nabla \cdot B = 0$. Here $p$ is the mass density, $v$ is the velocity, $\rho$ is the pressure, and $B$ is the magnetic field. We use an ideal gas equation of state $p = (\gamma - 1)\rho u$, where $\gamma$ is the ratio of the specific heats ($\gamma = c_p / c_v$) and $u$ is the specific internal energy. We solve equations (5)–(8) using a monotonic upstream-centered scheme for conservation laws (MUSCL; see van Leer 1979) with HLL fluxes (Harten et al. 1983) and monotonized central limiter (see Kurganov et al. 2001), on a regular Cartesian grid. We use the flux-interpolated constrained transport (or “flux-CT”) scheme (Toth 2000) to enforce the divergence-free condition of the magnetic field. A similar version of the code was used by Gammie et al. (2003) for relativistic MHD calculations. Our code gives satisfactory results for standard shock tube tests (see, e.g., Brio & Wu 1988; Ryu & Jones 1995). When compared with our earlier hybrid ENO (essentially nonoscillatory) code (see an isothermal version in Cho & Lazarian 2002), our current code runs faster and yet gives consistent results. The overall scheme is second-order accurate. After updating the system of equations along $x_1$-direction, we repeat similar procedures for the $x_2$ and $x_3$-directions, with the appropriate rotation of indexes. We use a two-stage Runge-Kutta method for time integration. The cooling at a given temperature is assumed to be that of plasma in collisional ionization equilibrium, with solar metallicity, as given in Benjamin et al. (2001). We consider $\gamma = 5/3$ and solve the time evolution of specific internal energy. For the runs with cooling, this is applied before updating equations (5)–(8), using an implicit scheme (i.e., we simply subtract the internal energy that will be lost in the time step $\delta t$). We do not explicitly include heating due to absorption of radiation or thermal conduction. The time step for the MHD part is determined by the standard “Courant” condition. When we include cooling, we monitor the minimal cooling time in the computational box and compare it with that of the MHD part. If the minimum cooling time is shorter, it replaces the time step. However, this was very rarely the case, and typically the dynamical time was shorter than the cooling time (by 1 or 2 orders of magnitude in most cases).

We do not include thermal conduction explicitly. The diffusion of heat at small scales is determined in our simulations by the numerical diffusion, whose properties may be different from those of diffusion in the actual interstellar gas. However, this does not greatly affect our final results if the heat transport is determined by turbulence. A discussion about turbulent and thermal dispersion is beyond the scope of this paper.

\textsuperscript{6} Instead of $T_\text{cold}$ we call it $T_\text{warm}$ because we choose the parameters to coincide with the warm phase of the ISM (at $T \sim 10^5$ K).
conduction in a magnetized medium can be found in Lazarian (2006). Turbulence provides an effective diffusion coefficient \( \sim v_{\text{turb}} L_{\text{inj}} \), where \( v_{\text{turb}} \) is the turbulent velocity and \( L_{\text{inj}} \) is the scale of energy injection, corresponding to the size of the largest eddies (Cho et al. 2003). We show below that in our models the injection scale is utterly determined by the size of our computational box. If the turbulent diffusion coefficient is much larger than the thermal diffusion coefficient, the heat transfer is dominated by turbulence. An important property of turbulent heat transfer, as well as of the other transport processes, is that they do not depend on the microscopic diffusivity. Indeed, provided that the gas is turbulent, the mixing and the heat transport are happening approximately over one large eddy turnover time. If the diffusivity on the atomic level decreases, the turbulent cascade goes to yet smaller scales ensuring heat transport that still scales as above. For typical parameters of turbulent mixing layers in our models (i.e., \( T \sim 10^5 \) K, \( n \sim 10^{-2} \) cm\(^{-3}\), \( L_{\text{inj}} \sim 10 \) pc, and \( v_{\text{turb}} \sim 20 \) km s\(^{-1}\)), one obtains a Spitzer thermal diffusion coefficient of \( \kappa_{\text{Sp}} \sim 10^{24} \) cm\(^2\) s\(^{-1}\) and a turbulent diffusion coefficient of \( \kappa_{\text{turb}} \sim 10^{25} \) cm\(^2\) s\(^{-1}\). The difference is modest, but the presence of a magnetic field will further suppress thermal conduction. Moreover, the magnetic field does not have to be dynamically dominant to produce an important effect on the thermal conductivity. And in some sense, our unmagnetized models are equivalent to models with a very weak magnetic field (ubiquitous in the ISM), which is sufficient to suppress thermal conductivity significantly. At the beginning of the simulations turbulence is not fully developed and the turbulent diffusivity is small, but at the same time the magnetic field is aligned with the interface, dramatically decreasing electron conduction as well. At more evolved stages, however, turbulence will develop and thermal conduction will be less suppressed as the magnetic field becomes entangled. Estimates by Narayan & Medvedev (2001) suggest that for fully developed turbulence the thermal conductivity is decreased by a factor of \( \sim 5 \) from the Spitzer value. Their model is, however, rather restrictive as only turbulence with Alfvén Mach number \( (M_A) \) equal to unity is discussed. For \( M_A \) both much larger and much smaller than unity the electron thermal conductivity is less (see Lazarian 2006).

3.2. The Numerical Setup

We start with warm gas, at a temperature of \( T_{\text{warm}} = 10^4 \) K, with a relatively high hydrogen number density \( n_{\text{warm}} = 0.1 \) cm\(^{-3}\) and no mean motion, at the right side of the computational box. At the opposite side we have a low-density \( (n_{\text{hot}}) \), hot medium

\[
\begin{array}{llllll}
\text{Model} & \log T_{\text{hot}} & n_{\text{hot}} & v_{\text{hot}} & B_{\text{warm}} & \text{Grid size} \\
144-\text{Th}6-\text{V}200-\text{B}0 & 6 & 1 \times 10^{-3} & 50 & 0 & 144^3 \\
144-\text{Th}6-\text{V}200-\text{B}0 & 6 & 1 \times 10^{-3} & 200 & 0 & 144^3 \\
256-\text{Th}6-\text{V}200-\text{B}0 & 6 & 1 \times 10^{-3} & 200 & 0 & 256^3 \\
\text{LX-7} & 1 & 1 \times 10^{-4} & 50 & 0 & 144^3 \\
\text{LX-7} & 1 & 1 \times 10^{-4} & 20 & 0 & 144^3 \\
\text{144-\text{Th}6-\text{V}200-\text{B}1} & 6 & 1 \times 10^{-3} & 50 & \sim 2 & 144^3 \\
\text{144-\text{Th}6-\text{V}50-\text{B}1} & 6 & 1 \times 10^{-3} & 200 & \sim 2 & 144^3 \\
\text{144-\text{Th}7-\text{V}50-\text{B}0} & 7 & 2 \times 10^{-4} & 50 & \sim 2 & 144^3 \\
\text{144-\text{Th}7-\text{V}200-\text{B}1} & 7 & 2 \times 10^{-4} & 200 & \sim 2 & 144^3 \\
\text{144-\text{Th}7-\text{V}200-\text{B}1} & 7 & 2 \times 10^{-4} & 200 & \sim 2 & 144^3 \\
\end{array}
\]

Notes.—All the runs start with a temperature on the warm side of \( T_{\text{warm}} = 10^4 \) K and a hydrogen density of \( n_{\text{warm}} = 0.1 \) cm\(^{-3}\). The box size in the vertical direction corresponds to 10 pc.

\* All models were run either without cooling or with collisional ionization equilibrium cooling (see text for details). An additional prefix “NC” (no cooling) or “EC” (equilibrium cooling) to the run name is added.

\( T_{\text{hot}} \), moving upward (\( z \)-direction) with a velocity \( v_{\text{hot}} \). In some of our simulations we include a magnetic field \( B_{\text{warm}} \) threading the warm gas aligned in the vertical direction, with a magnitude corresponding to \( B_{\text{warm}} \sim \frac{\rho_{\text{gas}}}{\rho_{\text{mag}}} \sim 1 \) km. A sketch of the computational domain is presented in Figure 1.

The two media are initially in total pressure equilibrium. The transition region (between hot and warm media) follows a hyperbolic tangent profile in the \( x \)-direction, initially occupying \( 1/10 \) of the box size. The division line for the runs in a \( 144^3 \) grid was set at the middle of the box. The vertical direction in all cases corresponds to a physical scale of \( L_z = 10 \) pc. In the cases without cooling, the simulations can be rescaled arbitrarily, but when cooling is included the scale lengths are fixed by the value of the cooling rate used. The boundary conditions are periodic in the \( z \) and \( y \)-directions. For the \( x \)-direction in the warm side of the box (right) we have a reflective boundary, and on the hot side (left) we have a source boundary condition. The latter reinforces the initial condition after each time step, acting as a reservoir of hot material, which helps to balance the energy lost through radiation. We initialized the computational cube with sinusoidal perturbations (3 harmonics, with random phases) in the component of the velocity normal to the interface of the two media \( (v_z) \). The shear at the boundary will excite a K-H instability, which will eventually lead to the development of turbulence. A summary of the parameters we use is presented in Table 1.

4. RESULTS

4.1. Evolution of the Mixing Layer

Our models, as described in the previous section, are dynamical and include a continuous transition between the hot and the warm media. In order to compare the result of our calculations with previous models of turbulent mixing layers (SSB93) we have to define what a region of “intermediate temperature” is, in our computational box. We adopted a threshold of 20% departure from any of the nominal temperatures for the hot and warm media. That is, we consider in the transition region all material above \( 1.2 \times 10^4 \) K and below \( 8 \times 10^4 \) K for the runs in which \( T_{\text{hot}} = 1 \times 10^6 \) K, or below \( 8 \times 10^4 \) K for the runs with \( T_{\text{hot}} = 1 \times 10^5 \) K. We evolve the initial conditions for \( \sim 20,000 \) time steps for all the cases with \( 144^3 \) resolution. In Figures 2 and 3 we
show the time evolution of all the purely hydrodynamic and magnetized runs, respectively.

We find in general that the volume-average temperature within the mixing layer depends strongly on the threshold used to define the intermediate-temperature region. The density-weighted temperature is more robust against the particular choice of threshold. And since the emission observable is also density weighted, it is a better measure of the properties of mixing layers, and we use it to interpret our results. Keeping in mind that the temperature in BF90 and SSB93 with $\xi = 1$ [dotted horizontal lines]; see eq. [1]), (b) Hydrogen column density in the mixing layer. (Both panels are averages over different lines of sight normal to the layer along the $x$-axis.)

Fig. 2.— Time evolution of the mixing layers, for the purely hydrodynamic runs. (a) Density-weighted average (thick lines) and volume-average (thin lines) temperature of the mixing layers (material with a departure of 20% from the nominal hot and warm temperatures). For reference we show the harmonic mean of the hot and warm temperatures (which corresponds to $\bar{T}$ in BF90 and SSB93 with $\xi = 1$ [dotted horizontal lines]; see eq. [1]). (b) Hydrogen column density in the mixing layer. (Both panels are averages over different lines of sight normal to the layer along the $x$-axis.)

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Fig. 3.— Same as Fig. 2, but for the cases with magnetic field ($\beta \sim 1$) on the warm side.

be measured from the simulations; however, it would require the choice of an arbitrary temperature threshold to obtain a mean (volume average) temperature, which turns out to be very sensitive to such a threshold. Thus, it is impractical to compare the models through estimates of the efficiencies

4.1.1. Formation of the Mixing Layer: Early Stages

Using a density-weighted average, our calculations show a relatively cold boundary layer (of a few $\times10^4$ K) at the early stages of formation. Because the mass is initially on the warm side, this indicates a somewhat inefficient mixing at such early times.

Figures 2b and 3b show how the mixing layer develops. When we do not include cooling, the thickness of the boundary layer increases monotonically. The growth rate is faster when the shear velocity is larger, as expected for the K-H instability. For the cases in which cooling is included, we see a very different evolution, with the boundary layer shrinking with time at the beginning and after some time starting to grow again. The smallest
perturbations in the K-H instability have the fastest growth rate but do not have the required energy to pull (enough) material into the mixing interface. Since the cooling is very effective, the transition layer is rather sharp. At this point, we mostly have warm gas being condensed at the interface, but mixing is not very effective yet. This is demonstrated in Figure 4, where we show cuts perpendicular to the mixing layer ($x$-$z$ plane) in two of our simulations after $t/C_{24}/0:7$ Myr. The cases with magnetic field do not show significant differences in the formation of the mixing layer when compared with the unmagnetized runs at early times ($t < 0.7$ Myr).

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4.1.2. Evolution of the Mixing Layers at Later Times

Until now, we have seen the formation of turbulent mixing layers by means of a K-H instability. However, we have not seen evidence of reaching a steady state, which is the original idea of the whole process. Since the largest modes in the K-H instability grow rather slowly, it is not practical to follow the evolution of all our models until steady state is achieved. Moreover, in a qualitative way, we have observed similar behavior for the runs that

We found marginal dependence on numerical resolution for the formation of the mixing layers. We show, however, that the mixing at later times is utterly dominated by the largest scales, when the modes that correspond to the longest wavelengths excited by the instability become apparent.

**Fig. 4.**—Selected snapshots of cuts in the $x$-$z$ plane from our simulations, showing density and temperature maps, along with the velocity field. The largest arrow in the velocity representation corresponds to a magnitude of $\sim 200$ km s$^{-1}$ $(a, b)$ Initial conditions for $v_{\text{hot}} = 200$ km s$^{-1}$ and $T_{\text{hot}} = 10^6$ K. $(c, d)$ Evolution after $t < 0.7$ Myr, without including cooling. $(e, f)$ Result after approximately the same time, but with the cooling included. We can see how the thickness of the mixing layer grows from the initial conditions in which no cooling is present. At the same time, we can see the dramatic effect of cooling, showing a rather sharp transition zone, and also how the largest wavelength modes of the K-H instability start to develop.

**Fig. 5.**—We ran a couple of cases in a $256^3$ grid. The results are shown in Figure 5.
started with a larger temperature ($T_{\text{hot}} = 10^6$ K) but with the additional difficulty that we require finer time stepping for such cases, making it very difficult to cover a large span in time. To study the long-term evolution of the layer, we followed the fastest evolving model, i.e., the unmagnetized, $T_{\text{hot}} = 10^6$ K, and $v_{\text{hot}} = 200$ km s$^{-1}$ model. A related problem in our 1443 resolution runs was that by the time the mixing layer was reaching its asymptotic state (after $\gtrsim 1.5$ Myr), it was getting very close to the boundary of the computational box. To overcome this problem, we extended the dimensions of the box to 256 cells in the $x$-direction, and we placed the division between hot and warm media off-center. The new initial conditions (model LX-Th6-V200-B0 in Table 1) have only one-fourth of the volume filled with warm gas and the remainder with hot gas. This is because we have much more heat capacity in the warm gas (due to the density contrast) and because our simulations did not have enough hot gas to provide a steady state. Figure 6 illustrates the evolution of this extended model, which shows how the mixing layer broadens after the first signs of large modes of the instability appear (see Figs. 4e and 4f).

We also show snapshots of cuts perpendicular to the layer, after 2.3 Myr in Figure 7. This figure highlights how a fully operational K-H instability effectively mixes the two media and provides a much larger interface zone, compared with the times when only small-scale modes are present. Even after $\sim 2.5$ Myr, in the longer box (256 cells in the $x$-direction), the simulations have not reached a steady state. Nonetheless, our results provide important insights on how these mixing layers will develop and change over time.

4.2. Observational Diagnostics for Mixing Layers

Notably, the time evolution we observe for these mixing layers is associated with a significant evolution of spectral line diagnostics,
most importantly the column densities and ratios of highly ionized species. These measurements depend not only on the metallicity, density contrast, and relative velocities of the mixing zones but also on the evolutionary state of the layer. As a first step toward examining the evolution, we have calculated the column densities of C$^{+}$, N$^{+}$, and O$^{+}$ ions through our simulated layers as a function of time, using the temperature and density profiles along synthetic lines of sight (LOSs) through the data cube, chosen perpendicular to the boundary layer (in the direction of the x-axis). These column densities were computed under the assumptions of solar metallicity and collisional ionization equilibrium (Benjamin et al. 2001). Figures 8 and 9 show the range of values of the C$^{+}$/O$^{+}$ and N$^{+}$/O$^{+}$ ratios along with other models of high ion production in the literature (compiled by Indebetouw & Shull 2004a, 2004b). The central points are the mean C$^{+}$/O$^{+}$ and N$^{+}$/O$^{+}$ values averaged over all the possible LOSs, at a given point in time. The error bars were obtained considering the variability with the different LOSs.

The line ratios and column densities from our simulations should not be interpreted too literally but rather as a guide to obtain insight into the evolution of mixing layers. In particular, the assumption of collisional ion fraction equilibrium limits the accuracy of line ratios. As pointed out by SSB93, mixing layers can be quite far from equilibrium. The overall dynamics and time evolution of mixing layers is not likely to change dramatically by nonequilibrium effects. However, observational diagnostics such as line ratios can be affected significantly. A study of nonequilibrium cooling models with more emphasis on the observational implications is necessary, and we plan to provide it in a future paper.
It is interesting that a clear difference is present between the runs that include cooling and those that do not. However, note that except for the model LX-Th6-V200 runs, the mixing layer is still in a very early stage of formation. It is remarkable that the models without cooling show similarity to the results of SSB93. Certainly their models did not account for the dynamical effect of the cooling: their model provides the intermediate state.

With new models, observations may be able to provide insights about the time dependence of the mixing. Another thing to note is the larger scatter for the cases that have been run for longer times. This scatter can be explained noting that for longer timescales we have a more dynamical (turbulent) picture, with longer times. This scatter can be explained noting that for longer times the models also assumed fully developed turbulence, and our set of runs have not achieved the corresponding stationary state.

With new models, observations may be able to provide insights about the time dependence of the mixing. Another thing to note is the larger scatter for the cases that have been run for longer times. This scatter can be explained noting that for longer timescales we have a more dynamical (turbulent) picture, with many structures moving in and out of a particular LOS. Actually, the values of the \( C_{iv} / O_{vi} \) and \( N_{v} / O_{vi} \) ratios (and the rather large scatter) are somewhat consistent with the observations presented by Indebetouw & Shull (2004b). It is also evident that in the magnetized cases the K-H instability (and therefore the development of turbulence at the boundary) is delayed. This can be seen from the smaller scatter of points in Figure 9 compared to those in Figure 8. However, magnetic reconnection should not allow the magnetic field to form knots and prevent turbulent mixing motions (see Lazarian & Vishniac 1999).

It remains difficult to reconcile the models with high ion column densities. It has been pointed out (e.g., SSB93; Savage et al. 2003; Indebetouw & Shull 2004a, 2004b) that to explain the typical column densities observed, the LOS must pass through several mixing layers (sometimes as many as a hundred). Indeed, several interfaces are likely to be blended for long lines of sight (as in the case of observations in the halo of the Galaxy). However, in many cases fairly smooth and symmetrical line profiles, with little centroid dispersions, are observed. This evidence indicates that merely summing over a large number of interfaces may not explain the observed column densities. In Table 2 we present the column densities of an arbitrary artificial LOS after 0.7 Myr for all the models with \( T_{\text{hot}} = 10^6 \, \text{K} \). For comparison we include the values that correspond to the initial conditions.

In Figure 10 we show the mean column densities of \( C_{iv} \), \( N_{v} \), and \( O_{vi} \) (for synthetic LOSs normal to the interface), as a function of time for the more evolved runs.

Average values of column densities in the halo of the Galaxy observed with the Far Ultraviolet Spectroscopic Explorer (FUSE) are \( \log N(C_{iv}) \sim 14.3 \), \( \log N(N_{v}) \sim 13.7 \), and \( \log N(O_{vi}) \sim 14.5 \) (Savage et al. 2003; Indebetouw & Shull 2004b). A survey with the Copernicus satellite in the Galactic plane by Jenkins (1978a, 1978b) suggested that individual \( O_{vi} \) components have column densities closer to \( \log N(O_{vi}) \sim 13 \), which is similar to observations with FUSE of very nearby stars (Oegerle et al. 2005). The column densities obtained in the model that ran for the longest time (at \( t \sim 2.5 \, \text{Myr} \)) are slightly larger than those predicted by SSB93 but would still require too many interfaces to explain observations. However, as we have discussed above, the simulations were stopped before fully reaching a stationary state, and the thickness of the intermediate-temperature zone was still increasing with time (see, e.g., Fig. 6b). The results presented here suggest that not only the number of mixing layers but also the time available for turbulence to develop are factors to consider for the proper interpretation of observations.

5. SUMMARY

Turbulent mixing layers at the interfaces of the hot \( (T \sim 10^6-10^7 \, \text{K}) \) and warm \( (T \sim 10^4 \, \text{K}) \) media in the ISM can be produced by high shear, via a Kelvin-Helmholtz instability. We
use a MHD code with radiative cooling to model the formation and evolution of turbulent mixing layers through this instability.

The introduction of cooling into the dynamical evolution was found to produce dramatic differences compared with the same models run without cooling. For instance, the deposition of momenta from hot gas condensing onto the mixing layer modifies the growth rate of the K-H instability, making it develop faster than the same case run without cooling.

The low density of the hot medium caused the growth rate of the large-scale modes of the instability to be slow. At early stages of the formation of the mixing layer (\(t \approx 1 \text{ Myr}\)) the rapid cooling of material at intermediate temperatures \((T \approx 10^5 \text{ K})\) makes for a sharp transition between the hot and warm media.

At later times, the instability excites large-scale fluctuations that are powerful enough to provide more efficient mixing, and the transition zone broadens. We see evidence that for typical ISM conditions \((T_{\text{hot}} \approx 10^4 \text{ K} \text{ and } T_{\text{warm}} \approx 10^4 \text{ K}, \text{ with a shear velocity of } 200 \text{ km s}^{-1}\)), the mixing layer is still growing after 2.5 Myr.

We included a dynamically important magnetic field \((\sim 2 \mu G)\) in the warm phase for some runs, and its effect was found to be minimal at earlier times, \(\lesssim 0.7 \text{ Myr}\). At later times the magnetic field inhibits/delays the development of the K-H instability and thus the turbulent mixing.

Assuming solar metallicities and collisional ionization equilibrium fractions, we estimated column densities and line ratios of highly ionized species (\(\text{C iv}, \text{ N v}, \text{ and O vi}\)) from our simulations. We compared our results with previous models (SSB93) of turbulent mixing layers and found similar column densities. The \(\text{C iv}/\text{O vi}\) and \(\text{N v}/\text{O vi}\) mean line ratios are slightly different, with a lower \(\text{C iv}/\text{O vi}\) \(\text{vi}\). We also found a significant scatter between different lines of sight, in particular for the models evolved to longer times.

We were not able to fully reach a stationary state in our simulations, but our results suggest that given more time (\(\geq 2.5 \text{ Myr}\) for typical conditions), the column density of high ions should be significantly larger than in previous models, partially alleviating the problem of number of layers required to explain observations.

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