A volume structure based ECS-PER algorithm for the hybrid electromagnetic field-circuit simulation

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ABSTRACT
A new algorithm for the hybrid electromagnetic (EM) field-circuit simulation is proposed. It employs the equivalent current source with parallel equivalent resistance (ECS-PER) to simulate the effect of an EM subsystem on a circuit subsystem and adopts the volume structure to map the lumped circuit into the finite-difference time-domain cells. By including an additional current term in Maxwell-Ampere’s Law to represent the current density of the lumped circuit and using the finite difference scheme, the formulas of the proposed volume structure based ECS-PER algorithm is deduced for the first time. Two numerical examples of microwave circuits radiated under external EM incidence are simulated, and the results verify the accuracy and versatility of the proposed algorithm.

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I. INTRODUCTION

Hybrid electromagnetic (EM) field-circuit simulation is very important in modern science and engineering. For example, when circuits are radiated by EM wave energy, due to the coupling effect, the analysis must be a combination of the EM simulation and the circuit simulation. Another example is that when circuits work at high frequencies where the wavelength becomes comparable to the circuits’ physical dimensions, the parasitic and coupling effects among electrical devices also require the hybrid EM field-circuit simulation. Various methods for this simulation have been proposed. Many are based on the finite-difference time domain (FDTD) method, a popular full-wave algorithm. It solves Maxwell equations in time domain directly on the space grids and is intuitive and convenient to work with transient simulators of circuits (such as PSPICE) to achieve the hybrid EM field-circuit simulation. For example, the lumped-element (LE)-FDTD distributes lumped circuits into FDTD grids and updates EM fields explicitly with analytical expressions. However, a limitation exists, i.e., it cannot easily and accurately solve two-terminal circuits consisting of the arbitrary connection of multiple lumped elements. The lumped-network (LN)-FDTD method is an extension of the LE-FDTD, which integrates the computation of a high-order linear lumped network into FDTD iteration equations explicitly, but involves complex domain transformations.

Besides the above methods, the equivalent source approach has also been widely investigated. It divides a hybrid system into an EM subsystem and a circuit subsystem and simulates them by the EM simulation such as the FDTD and the circuit simulation, respectively. The effect of EM subsystem on circuit subsystem is described by the equivalent source, which can be a Maxwell-Ampere law based equivalent current source (ECS) or a Maxwell-Faraday law based equivalent voltage source (EVS).

So far, this approach includes three types, i.e., ECS with parallel equivalent capacitance (ECS-PEC), ECS with parallel equivalent resistance (ECS-PER), and EVS with serial equivalent inductance (EVS-SEI).

In these approaches, a critical step is to map lumped circuits into FDTD cells. Three techniques have been proposed which map lumped circuits into FDTD cells as wire, surface, or volume structures. Accordingly, the lumped circuit occupies one-, two-, or three-dimension FDTD space, respectively.

In comparison with the volume structure, the wire and surface structures are relatively simple and easy to implement, and...
The lumped circuit must be mapped into FDTD cells. This strategy is thus adopted by most hybrid EM field-circuit simulations. Some researchers\textsuperscript{11–13} pointed out that the wire or surface structure has some defects. For example, they do not take the actual dimensions of circuit devices into account and are usually associated with numerical parasitic elements that can dramatically spoil the simulation results. Besides, when the frequency rises or electrical devices become miniature with the development of the semiconductor technology, lumped circuits’ physical dimensions may be close to the wavelength. Therefore, their volume can no longer be ignored. However, few literature studies have researched the volume structure. The ECS-PEC and EVS-SEI approaches adopting the volume structure have been derived, and two examples are given to prove the effectiveness of the approach.\textsuperscript{13} However, so far, no literature on ECS-PER adopting the volume structure (ECS-PER-VS) has been reported.

This work investigates the ECS-PER-VS, including analyzing the approach’s principle and deducing the mathematical formulas and validating it by two numerical examples.

II. PRINCIPLE AND FORMULAS OF THE ECS-PER APPROACH ADOPTING THE VOLUME STRUCTURE

The proposed ECS-PER-VS approach divides a hybrid EM field-circuit system into an EM subsystem and a lumped circuit subsystem. It uses an equivalent current source $I_{eq}$ with a parallel equivalent resistance $R_{eq}$ to simulate the effect of EM subsystem on circuit subsystem. As shown in Fig. 1, the equivalent source is calculated in the EM subsystem and transferred to the circuit subsystem, then the total current $I_d$ and total voltage $V_d$ across the lumped circuit are obtained in the circuit subsystem and sent back to the EM subsystem. As a result, the hybrid EM field-circuit simulation can be achieved.

In this work, the EM simulation adopts the FDTD method. First, the lumped circuit must be mapped into FDTD cells. As illustrated in Fig. 2, for generality, the lumped circuit is assumed to be loaded in the y direction, which occupies cells $i = i_1, i_2, ..., i_M$, $j = j_1, j_2, ..., j_N$, and $k = k_1, k_2, ..., k_P$ in the FDTD’s Yee lattices. It means that there are $M \times P$ columns in the loading direction, with $N$ cells in each of them. It is obvious that the lumped circuit occupies three-dimension FDTD space, so this mapping technique is called the volume structure.

After the lumped circuit is mapped into the FDTD cells, an additional current term is added to Maxwell-Ampere’s Law, which represents the effect of the lumped circuit,

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{I}_d, \quad (1)$$

where $\vec{I}_d$ is the current density of the lumped circuit, $\vec{H}$ is the magnetic field intensity, $\vec{E}$ is the electric field intensity, $\sigma$ is the conductivity, and $\varepsilon$ is the permittivity.

Equation (1) can be expanded as

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_y}{\partial x} = \frac{\sigma E_y}{\varepsilon} \frac{\partial E_y}{\partial t} + \varepsilon E_y + \vec{J}_d, \quad (2)$$

where the subscript $y$ represents the lumped circuit’s loading direction.

Based on the finite difference method, (2) is discretized in space and time. The iterative equation of the electric field along the $y$ direction can be approximately written as

$$E_y^{n+1/2}(i, j + 1/2, k) = A \times E_y^{n-1/2}(i, j + 1/2, k) + B \times \left( \frac{H_x^{n+1/2}(i, j + 1/2, k + 1/2) - H_x^{n+1/2}(i, j + 1/2, k - 1/2)}{\Delta z} \right) + \frac{\Delta z}{\Delta x} \left( \frac{H_z^{n+1/2}(i + 1/2, j + 1/2, k) - H_z^{n+1/2}(i - 1/2, j + 1/2, k)}{\Delta x} \right) - B \times J_{dy}^{n+1/2}, \quad (3)$$

where $n, i, j, k$ represents the time step and space step in the $x, y$, and $z$ directions, respectively. It should be noted that the following equations are approximate ones when it involve difference forms. For magnetic field, the iterative equation can be obtained from the Maxwell-Faraday

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**FIG. 1.** The simulation model for the ECS-PER-VS approach.

**FIG. 2.** Lumped circuit mapped into FDTD cells.
law and the form is similar. A and B are, respectively, expressed as
\[
A = \frac{2\varepsilon(i,j + 1/2,k) - \Delta t \times \sigma(i,j + 1/2,k)}{2\varepsilon(i,j + 1/2,k) + \Delta t \times \sigma(i,j + 1/2,k)}
\]
and
\[
B = \frac{2\Delta t}{2\varepsilon(i,j + 1/2,k) + \Delta t \times \sigma(i,j + 1/2,k)}.
\]
According to the relationship between the electric field intensity and the voltage, the approximate expression can be obtained as follows:
\[
V_d^n = \sum_{j=1}^{b_n} E_y^n(i,j + 1/2,k) \Delta y.
\] (4)
Substituting (3) into (4), the approximate relationship between the circuit and the electromagnetic field is established as
\[
V_d^n(i,j + 1/2,k) = -I_d^n \left[ \sum_{j=n}^{b_n} \left( B \times \frac{1}{\Delta x \times \Delta z} \right) \Delta y \right] + \sum_{j=n}^{b_n} \frac{A \times E_y^{n+1} (i,j + 1/2,k)}{-B} + \sum_{j=n}^{b_n} \left\{ \frac{H_x^{n+1/2} (i,j + 1/2,k+1/2) - H_x^{n+1/2} (i,j + 1/2,k-1/2)}{\Delta z} + \frac{H_x^{n+1/2} (i,j + 1/2,k+1/2) - H_x^{n+1/2} (i,j + 1/2,k-1/2)}{\Delta x} \right\} \times \Delta y.
\] (5)
The units on both sides of (5) are volts, so (5) can be rewritten in the following form:
\[
i_{ik}^{n+1/2} = \frac{V_d^n}{R_{ik}} + I_{ik}^n,
\] (6)
where
\[
R_{ik} = \sum_{j=n}^{b_n} \left( B \times \frac{\Delta y}{\Delta x \times \Delta z} \right)
\] (7)
and
\[
i_{ik}^{n-1/2} = \frac{\Delta y}{R_{ik}} \times \sum_{j=n}^{b_n} \left[ A \times E_y^{n-1} (i,j + 1/2,k) + \frac{H_x^{n+1/2} (i,j + 1/2,k+1/2) - H_x^{n+1/2} (i,j + 1/2,k-1/2)}{\Delta z} + \frac{H_x^{n+1/2} (i,j + 1/2,k+1/2) - H_x^{n+1/2} (i,j + 1/2,k-1/2)}{\Delta x} \right].
\] (8)
Equations (7) and (8) are the equivalent resistance (ER) and equivalent current (EC), respectively, corresponding to each column in the $M \times P$ columns. Figure 3(a) shows how the $M \times P$ equivalent sources are connected in shunt. Figure 3(b) is the simplified circuit, in which
\[
I_{eq} = \sum_{i=1}^{b_n} \sum_{k=1}^{b_n} i_{ik}^{n-1/2}
\] (9)
\[FIG. 3. (a) Equivalent circuit representation of the ECS with volume structure and (b) a simplified diagram.\]
FIG. 4. Simulation flow chart of the ECS-PER-VS algorithm.

and

$$R_{eq} = \left( \sum_{i=1}^{M} \sum_{k=1}^{P} \frac{1}{R_{ik}} \right)^{-1}. \quad (10)$$

The voltage $V_d$ and current $I_d$ of the lumped circuit can be obtained by the transient simulators of circuits (such as PSPICE). According to the expression (11), the current density of lumped circuit is obtained. Finally, Eq. (11) is substituted into iterative Eq. (3) to achieve the update of the electromagnetic system,

$$j_{max} = \frac{E^n_{max} - V^n}{M \Delta x \times P \Delta z}. \quad (11)$$

The wire structure and surface structure can be obtained from the volume structure. When either $M$ or $P$ equals to 1, (9) and (10) are the formulas for the ECS with surface structure. When both $M$ and $P$ are equal to 1, these formulas are for the ECS with wire structure, which are consistent with Ref. 6.

The specific iteration process is shown in Fig. 4, where $E^n$ is the electric field and $H^{n+1/2}$ is the magnetic field. At first, the $I_{eq}$ and $R_{eq}$ are obtained by the EM subsystem. Then they are transferred into the circuit subsystem to obtain the voltage $V_d$ and current $I_d$. The current density of lumped circuit is substituted into field iterative equation to achieve the update of the EM subsystem. To conclude the description above, the EM simulation for the EM subsystem and the circuit simulation for the lumped circuit are executed alternately in the iterative simulation procedure of the approach through the exchange of data ($I_{eq}$, $R_{eq}$) and ($I_d$, $V_d$).

III. NUMERICAL EXAMPLES

In this section, two numerical examples are presented to demonstrate the ECS-PER-VS algorithm. One is an active patch antenna, and another is a frequency selective surface (FSS) loaded with Schottky diodes. Both are radiated under microwave power, and thus, the hybrid EM field-circuit simulation is quite necessary for their analysis.

A. An active patch antenna

As illustrated in Fig. 5, an active patch antenna resides on a dielectric substrate with the thickness of 0.789 mm and relative permittivity of 2.33, and is embedded with two Gunn diodes. As a typical hybrid EM field-circuit example, this active patch antenna has been presented in Refs. 14–16 and has been analyzed by using various methods, including theoretical analysis such as the modal analysis,14 and hybrid EM field-circuit simulation such as the EM field-circuit the time domain integral equation (TDIE)15 and finite-element time-domain (FETD).16

Here, the ECS-PER-VS algorithm is employed for simulating this active antenna. In the algorithm, the whole system is meshed by FDTD cells where the lumped circuit (the Gunn diode) is mapped. The spatial dimensions of the cells are $\Delta x = 0.148$ mm, $\Delta y = 0.231$ mm, and $\Delta z = 0.1315$ mm, and convolutional perfectly...
TABLE I. Result comparison for the simulated fundamental frequency (GHz).

| Methods          | Fundamental frequency | Reference |
|------------------|-----------------------|-----------|
| Modal analysis   | 12.2                  | 14        |
| TDIE             | 12                    | 15        |
| FETD             | 11.8                  | 16        |
| ECS-PER-VS       | 12.1                  | This paper|

matched layers (CPML) as the absorption boundary is used to truncate the computing space. In the simulation, the diode occupies a cell in the lateral direction and 6 cells in the longitudinal direction.

The active patch antenna is excited by a plane wave source with a cosine-modulated gaussian pulse, which will cause the two antenna elements to operate in an inverting mode and produce a steady-state inverting oscillating voltage across the two diodes, and thereby results in a resonant fundamental frequency for the diode terminal voltage.

The simulation of the ECS-PER-VS algorithm follows the procedure illustrated in Fig. 4. In each time step, first the EM field distribution is calculated, then the equivalent source parameters $I_{eq}$ and $R_{eq}$ are obtained by making use of (9) and (10), and finally the voltage across the diode can be obtained. After all iterative steps have been finished, the time-varying voltage at both ends of the diode can be obtained, and thereby the frequency spectrum of the voltage across one diode is got by using a discrete Fourier transform.

It can be clearly seen from Fig. 6, the fundamental frequency of the oscillations computed by the ECS-PER-VS algorithm is 12.1 GHz. Table I compares this result to those obtained by making use of other methods. One can observes that they are in good agreement, and hence it demonstrates the feasibility of the ECS-PER-VS algorithm as well as the correctness of the formulas deduced in this work.

B. FSS loaded Schottky diodes

In our previous work, an FSS loaded with Schottky diodes is proposed. It is a $4 \times 4$ array fabricated on a printed circuit board (PCB) with a substrate thickness of 1.0 mm and a relative dielectric constant of 2.55. As shown in Fig. 7, its unit cell consists of a square patch loaded in a square loop, and 4 diode pairs are mounted across the square patch and the loop. Each diode pair includes two antiparallel Schottky diodes HSMS-286C with SOT-323 package from Avago.

This nonlinear FSS is radiated under external microwave power and was simulated by utilizing the ECS-PEC algorithm in Ref. 17. Here, the ECS-PER-VS algorithm is used to analyze its transmission characteristics for the external microwave radiation.

The whole structure of the FSS is meshed by FDTD cells where the lumped circuit (the Schottky diode pair) is mapped. The spatial dimensions of the FDTD cells are $\Delta x = 0.25 \text{ mm}$, $\Delta y = 0.25 \text{ mm}$, and $\Delta z = 0.2 \text{ mm}$. According to the package size of the diodes, the diodes occupy $4 \times 8 \times 4$ cells in the simulation.

Following the process shown in Fig. 4, the transmission coefficient of the FSS over a frequency band from 1 GHz to 4.5 GHz is simulated by the ECS-PER-VS algorithm. As illustrated in Fig. 8, results are compared with the simulated and measured ones in Ref. 17. One can observed that the transmission coefficients simulated by the ECS-PER-VS algorithm agree well with the simulated ones by the ECS-PEC algorithm in Ref. 17 and are considerably consistent.
with the measured one. It should be noted that our laboratory equipment cannot distinguish overlapping incident and reflected waves, so there is no comparison result of reflection coefficient.

IV. CONCLUSION

For the hybrid EM-circuit systems, the equivalent source approach is an effective and popular simulation method. As one subclass of the equivalent source approach, the ECS-PER-VS algorithm, which uses an equivalent current source with a parallel equivalent resistance to simulate the effect of EM subsystem on circuit subsystem and maps lumped circuits into FDTD cells as volume structure, has not been investigated in previous literature. In this work, by including an additional current term in Maxwell-Ampere’s Law, the formulas of the ECS-PER-VS algorithm are deduced for the first time.

To validate the ECS-PER-VS algorithm and the deduced formulas, two typical hybrid EM field-circuit examples, an active patch antenna and an FSS loaded with Schottky diodes, are presented. Their performances under external EM incidence are analyzed by the ECS-PER-VS algorithm. The results are compared to other simulations by using other algorithms as well as measurements used in the references. A good agreement can be observed, and it proves the correctness of the deduced formulas and demonstrates the capability of the ECS-PER-VS algorithm in analyzing the hybrid EM field-circuit system.

ACKNOWLEDGMENTS

This work was supported by Fund of Ministry of Education of China under Grant No. 6141A0202504.

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