Mean Field Effect on $J/\psi$ Production in Heavy Ion Collisions

Baoyi Chen, Kai Zhou, and Pengfei Zhuang
Physics Department, Tsinghua University, Beijing 100084, China
(Dated: February 17, 2012)

PACS numbers: 25.75.-q, 12.38.Mh, 24.85.+p

$J/\psi$ is a tightly bound state of heavy quarks $c$ and $\bar{c}$, its dissociation temperature $T_d$ in hot medium is much higher than the critical temperature $T_c$ of the deconfinement phase transition. Therefore, the measured $J/\psi$s in nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) carry the information of the hot medium created in the early stage of the collisions and has long been considered as a signature of the new state of matter, the so-called quark-gluon plasma (QGP) [2].

The hot nuclear matter effect on $J/\psi$ production includes elastic and inelastic interactions. The latter, for instance the gluon dissociation process $g + J/\psi \rightarrow c + \bar{c}$ and its inverse process, leads to a $J/\psi$ suppression and regeneration [12,17] and is widely considered in the models for $J/\psi$ production in heavy ion collisions. The former, which changes the particle properties like mass, is weak for a heavy quark system at low temperature and therefore neglected in almost all the models. However, from the calculations with quantum chromodynamics (QCD) sum rules [18] and QCD second-order Stark effect [19], there is a sudden change in the mass of $J/\psi$ around $T_c$. For instance, at temperature $T/T_c = 1.05$ the mass shift $\delta m_{J/\psi} = m_{J/\psi}(T) - m_{J/\psi}(0)$ can reach $(-100 - 200)$ MeV, which is already comparable with the mass change for light hadrons and should have remarkable consequence in $J/\psi$ production in heavy ion collisions at RHIC and LHC where the fireball temperature is believed to be much higher than $T_c$.

Let’s first qualitatively estimate the mean field effect on the $J/\psi$ distribution. The decreased mass reduces the threshold value for the $J/\psi$ production in QGP, and should result in an enhancement for the $J/\psi$ yield. Secondly, the attractive force between the inhomogeneous medium and $J/\psi$, $F(x,p) = -\frac{m_{J/\psi}}{E_{J/\psi}} \nabla m_{J/\psi} = -\frac{m_{J/\psi}}{E_{J/\psi}} \frac{\partial m_{J/\psi}}{\partial T} \nabla T$

(1)

with $J/\psi$ energy $E_{J/\psi} = \sqrt{m^2_{J/\psi} + p^2}$, pulls $J/\psi$ to the center of the fireball and kicks $J/\psi$ to a lower momentum region. This will lead to an enhancement at low momentum and a reduction at high momentum. In comparison with the reduced threshold by the temperature dependent mass $m_{J/\psi}(T)$, the mean field force induced by the inhomogeneous fireball $T(x)$ is a second order correction. For a homogeneous fireball, the threshold is reduced, but the force disappears.

Since charmonia are so heavy and difficult to be thermalized in hot medium, we use a detailed transport approach [20] to describe their distribution functions $f_\Psi(p,x,\tau|b)$ in the phase space at fixed impact parameter $b$ of a nucleus-nucleus collision,

$$\frac{\partial f_\Psi}{\partial \tau} + \frac{p}{E_\Psi} \cdot \nabla x f_\Psi + F \cdot \nabla p f_\Psi = -\alpha f_\Psi + \beta f_\Psi,$$

(2)

where $\nabla_x$ and $\nabla_p$ indicate three dimensional derivatives in coordinate and momentum spaces. Considering the fact that in proton-proton collisions the contribution from the decay of the excited charmonium states to the $J/\psi$ yield is about 40% [21], we must take all the transport equations for $\Psi = J/\psi, \psi'$ and $\chi_c$ in the study of $J/\psi$.

The collision terms on the right hand side are from the inelastic interaction with the medium. The lose term yield is about 40% [21], we must take all the transport equations for $\Psi = J/\psi, \psi'$ and $\chi_c$ in the study of $J/\psi$.

The local temperature $T(x,\tau)$ appeared in the elastic and inelastic terms are obtained from the hydrodynamic equations [10] for the space-time evolution of the QGP. The charm quark mass $m_c$ is determined by the charmonium mass $m_\Psi$ and its binding energy $\epsilon_\Psi$ [24], $m_c = (m_\Psi + \epsilon_\Psi)/2$. The cold nuclear matter effects on $J/\psi$ production, such as nuclear absorption [27], Cronin effect [28,29] and shadowing effect [30] can be reflected in the initial charmonium distributions $f_\psi(x,\tau_0|b)$ with $\tau_0$ being the formation time of the hot medium. To simplify the numerical calculations, we neglect in the following the nuclear absorption and shadowing effect at RHIC and LHC energy and consider only the Cronin effect with a Gaussian smearing treatment [17,31].
The transport equation with the mean field force can not be solved analytically. Supposing that the very small elastic cross section for charmonia at hadron level is still true at parton level, the mean field term can be considered as a perturbation, and the transport equation can be solved perturbatively. Fully neglecting the mass shift, the zeroth-order transport equation can be strictly solved with the solution [20]

\[ f^{(0)}_{\psi}(p, x, \tau | b) = f_{\psi}(p, x_0, \tau_0 | b) e^{-\int_{\tau_0}^{\tau} d\tau_1 \alpha_\psi(p, x_1, \tau_1 | b)} + \int_{\tau_0}^{\tau} d\tau_1 \beta_\psi(p, x_1, \tau_1 | b) e^{-\int_{\tau_1}^{\tau} d\tau_2 \alpha_\psi(p, x_2, \tau_2 | b)} \]  

(3)

with the coordinate shift \( x_n = x - p/E_\psi(\tau - \tau_n) \) which reflects the leakage effect [1, 32] during the time period \( \tau - \tau_n \). The first term on the right hand side is the contribution from the initial production, it suffers from the gluon dissociation during the whole evolution of QGP. The second term arises from the regeneration and suffers also the suppression from the production time \( \tau_1 \) to the end of QGP.

Taking into account the fact that the dissociation temperatures for the excited states are around \( T_c \) [24], we do not consider their mass shifts in QGP. Therefore, for the transport equations for \( \psi' \) and \( \chi_c \), their masses are temperature independent and the mean field forces disappear, the zeroth-order solutions become the full distributions, \( f_{\psi'} = f^{(0)}_{\psi} \) and \( f_{\chi_c} = f^{(0)}_{\psi} \).

We now consider the mean field effect on the \( J/\psi \) distribution \( f_{J/\psi} \). We extract the mass shift from the calculation using QCD second-order Stark effect [19],

\[ \delta m_{J/\psi}(T) = m_{J/\psi}(T) - m_{J/\psi}(0) = -\frac{7\pi^2 a^2}{18} e \left[ \frac{2}{11} M_0(T) + \frac{3}{4} \frac{\alpha_s(T)}{\pi} M_2(T) \right] \]  

(4)

with constants \( a = 0.271 \) fm and \( e = 640 \) MeV, the temperature dependent coupling constant \( \alpha_s \), and the functions \( M_0 = e - 3P \) and \( M_2 = e + P \) determined by the energy density \( e \) and pressure \( P \) of the medium and extracted from lattice QCD simulations [32]. Leaving out first the mean field force induced by the inhomogeneous property of the fireball but keeping the temperature dependence of the mass, the solution of the transport equation has the same form with the zeroth-order distribution, the only difference is the replacement of \( m_{J/\psi}(0) \) \( \rightarrow m_{J/\psi}(T) \),

\[ f^{(1)}_{J/\psi} = \left| m_{J/\psi}(0) \rightarrow m_{J/\psi}(T) \right| f_{J/\psi} \]  

(5)

If the effect of the mean field force is not very strong, we may infer that it is not such a bad approximation to replace \( f_{J/\psi} \) in the mean field term by the known \( f^{(1)}_{J/\psi} \), the approximate transport equation

\[ \frac{\partial f_{J/\psi}}{\partial \tau} + \frac{p}{E_{J/\psi}} \cdot \nabla_x f_{J/\psi} = -\alpha_{J/\psi} f_{J/\psi} + \beta_{J/\psi} - F \cdot \nabla_p f^{(1)}_{J/\psi} \]  

(6)

is then similar to the equation for \( f^{(1)}_{J/\psi} \) but with a replacement for the regeneration

\[ \beta_{J/\psi} \rightarrow \beta_{J/\psi} - F \cdot \nabla_p f^{(1)}_{J/\psi}. \]  

(7)

This means that the mean field force can be considered as an effective regeneration, which is not from the coalescence of heavy quarks but due to the \( J/\psi \) mass shift. The approximate solution so obtained is known as the second-order \( J/\psi \) distribution,

\[ f^{(2)}_{J/\psi}(p, x, \tau | b) = f^{(1)}_{J/\psi}(p, x, \tau | b) - \int_{\tau_0}^{\tau} d\tau_1 \left[ F(x_1) \cdot \nabla_p f^{(1)}_{J/\psi}(p, x_1, \tau_1 | b) \right] e^{-\int_{\tau_1}^{\tau} d\tau_2 \alpha_\psi(p, x_2, \tau_2 | b)}. \]  

(8)

Substituting the obtained \( f^{(2)}_{J/\psi} \) into the mean field term, and solving the transport equation again, we can derive the third-order distribution function \( f^{(3)}_{J/\psi} \). With the similar way, the distribution to the \( n \)-th order can be generally expressed as a series of the mean field force,

\[ f^{(n)}_{\psi}(p, x, \tau | b) = f^{(1)}_{\psi}(p, x, \tau | b) - \sum_{m=1}^{n-1} (-1)^{m-1} \int_{\tau_0}^{\tau_1} \int_{\tau_0}^{\tau_{m+1}} \cdots \int_{\tau_0}^{\tau_{n-1}} d\tau_1 \cdots d\tau_{m} \times \left[ \prod_{j=1}^{m} F(x_j) \cdot \nabla_p f^{(1)}_{\psi}(p, x_m, \tau_m | b) \right] e^{-\int_{\tau_m}^{\tau} d\tau' \alpha_\psi(p, x', \tau' | b)}. \]  

(9)

Note again that the mass shift affects the \( J/\psi \) production in two aspects. The reduced threshold, which is considered already in the first-order distribution \( f^{(1)}_{J/\psi} \), makes the production more easily, and the attractive force...
between $J/\psi$ and the matter, which is included only in the higher-order distributions $f_{J/\psi}^{(n)}$ $(n = 2, 3, \cdots)$, makes a momentum shift and leads to a low $p_t$ enhancement and a corresponding high $p_t$ suppression.

By integrating the charmonium distribution function $f_{\Psi}(p, x, \tau | b)$ over the hyper surface of hadronization determined by $T(x, \tau) = T_c$, and considering the decay of the excited states to the ground state, we can calculate the number $N_{AA}$ of survived $J/\psi$s in a heavy ion collision. Let's first examine the differential nuclear modification factor $R_{AA}(p_t) = N_{AA}(p_t) / (N_c N_{pp}(p_t))$ as a function of transverse momentum $p_t$ in mid rapidity region, where $N_{pp}$ is the number of $J/\psi$s in a nucleon-nucleon collision and $N_c$ the number of nucleon-nucleon collisions. To see the largest mean field effect, we calculate first $R_{AA}(p_t)$ in central collisions with $b = 0$ where the formed fireball is hot and large and the surviving time is long. Fig. 1 shows our numerical results for Au+Au collisions at top RHIC energy $\sqrt{s_{NN}} = 200$ GeV. The thick and thin lines indicate our results with and without considering the $J/\psi$ mass shift, by taking $f_{J/\psi}$ and $f_{J/\psi}^{(0)}$, respectively. The dotted, dashed and solid lines are the calculations with initial production only, regeneration only and the total.

From our numerical results, the series $f_{J/\psi}^{(n)}$ $(n = 0, 1, 2, \cdots)$ converges rapidly, there is $\left| R_{AA}^{(2)} - R_{AA}^{(1)} / R_{AA}^{(4)} \right| < 3\%$ for any of the three cases. This confirms our qualitative estimation that the mean field effect is mainly reflected in the change in the threshold, the attractive force is a second order effect and the higher order corrections with $n > 2$ can be safely neglected. In all the numerical calculations in this paper we take $f_{J/\psi}^{(2)} / f_{J/\psi}^{(0)}$. Since the initial production has ceased before the formation of the hot medium, the change in the threshold does not affect it, and the correction is only from the small mean field force. That is the reason why the initial contribution shown in Fig. 1 is almost the same for the two cases with and without considering the mass shift. However, the regeneration happens in the hot medium, it is affected by both the reduced threshold and the mean field force, the overall correction should be much larger in comparison with the correction to the initial production. Considering the fact that the regenerated $J/\psi$s in the early stage of the QGP will be eaten up by the hot medium and only the $J/\psi$s regenerated in the later stage of the QGP at lower temperature can survive, the enhanced regeneration is mainly in the low $p_t$ region, as shown in Fig. 1. At $p_t = 0$, the total $R_{AA}$ goes up from 0.46 to 0.58, the enhancement is 26%. Since some $J/\psi$s are kicked to low $p_t$ region by the mean field force, there is a little reduction of $R_{AA}$ in the mid $p_t$ ($2 - 3$ GeV) region. At high enough $p_t$, there is almost no effect of the mass shift, and the $J/\psi$ distribution is dominated by the perturbative QCD in the initial production. In comparison with our previous calculations [34, 35], the Gaussian smoothing treatment [17, 31] for the Cronin effect used here reduces the $R_{AA}$ at high $p_t$.

To compare with the experimental data, we show in Fig. 2 the RHIC data at centrality 10% [36, 37] and our calculation at $b = 0$. With decreasing centrality, the fireball temperature drops down and the mean field effect is gradually reduced. However, from $b = 0$ to 4.2 fm, the change in the mean field is small, and the model calculations are almost the same. We show in Fig. 2 only the total calculation. Since the current data are still with large uncertainty, both the calculations with and without $J/\psi$ mass shift seem in reasonable agreement with the data. Especially, the dip at about $p_t = 2.5$ GeV coming from the competition [34] between the initial production and regeneration is slightly amplified by the attractive mean field force. To confirm the mean field effect, we need more precise data at low $p_t$. At high $p_t$ where the mean field effect disappears, the higher dissociation temperature for fast moving charmonia [35] will increase the $J/\psi$ surviving probability.

We calculated also the $R_{AA}(p_t)$ in central Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, as shown in Fig. 3. Due to the higher temperature, larger size and longer life time of the formed fireball, most of the initially produced $J/\psi$s at LHC are eaten up by the medium, leading to a very small contribution to the $R_{AA}$ at any $p_t$. On the other hand, the regenerated $J/\psi$s in QGP at very high temperature $T > T_q$ are immediately dissociated in the
medium, and only the regeneration at temperature $T < T_d$ contributes to the finally observed $J/\psi$'s. Therefore, the regeneration at LHC is not sensitive to the fireball temperature, and the mean field effect on $J/\psi$ production is similar to that at RHIC, see Fig. 3.

While the mean field effect changes significantly the differential nuclear modification factor at low transverse momentum, it does not affect the global yield remarkably. From Figs. 1 and 2 the most important mean field effect is at $p_t = 0$ and the effective region is around $p_t = 0.5$ GeV which is much less than the averaged $J/\psi$ transverse momentum $\langle p_t \rangle \approx 2 - 3$ GeV at RHIC and LHC energies. Therefore, the $p_t$-integrated nuclear modification factor $R_{AA}(N_p)$ as a function of the number $N_p$ of participant nucleons becomes not sensitive to the mean field effect. From our numerical calculations, the mass shift induced enhancement of $R_{AA}(N_p)$ is very small in peripheral and mid-central collisions and less than 10% even in most central collisions. Different from $R_{AA}$ which is a summation of the initial production and regeneration, the averaged transverse momentum square $\langle p_t^2 \rangle$ is governed by the ratio of regeneration to initial production [34, 35]. We found that the enhanced $J/\psi$ yield at low $p_t$ leads to a less than 10% suppression of $\langle p_t^2 \rangle$ in most central collisions.

In summary, we developed a perturbative expansion to study the mean field effect on $J/\psi$ production in heavy ion collisions. Taking the mean field term as a perturbation, we analytically solved the $J/\psi$ transport equation with elastic and inelastic terms to any order and found a rapid convergence of the perturbative expansion. The initial $J/\psi$ production, which happens before the formation of QGP, is not sensitive to the mean field force, but the continuous regeneration, which happens in the evolution of QGP, is significantly affected by the mean field. As a result, the differential nuclear modification factor of $J/\psi$ is strongly enhanced at low $p_t$ in heavy ion collisions at RHIC and LHC.

**Acknowledgement:** We thank useful discussions with Yunpeng Liu, Zhen Qu and Nu Xu. The work is...
supported by the NSFC (Grant Nos. 10975084 and 11079024) and RFDP (Grant No.20100002110080).

[1] T.Matsui and H.Satz, Phys. Lett. B178, 416(1986).
[2] For instance, see Quark-Gluon Plasma, edited by R.C. Hwa (World Scientific, Singapore, 1990).
[3] M.C. Abreu et al. [NA50 Collaboration], Nucl. Phys. A610, 404c(1996).
[4] J.-P. Blaizot and J.-Y. Ollitrault, Phys. Rev. Lett. 77, 1703(1996).
[5] A. Capella, A. Kaidalov, A. Kouider Akil and C.Gerschel, Phys. Lett. B393, 431(1997).
[6] J. Hufner and P. Zhuang, Phys. Lett. B559, 193(2003).
[7] A. Polleri, T. Renk, R. Schneider and W.Weise, Phys. Rev. C70 044906(2004).
[8] E. Bratkovskaya, W. Cassing, H. Stöcker and N. Xu, Phys. Rev. C71 044901(2005).
[9] P.Zhuang and X.Zhu, Phys. Rev. C67, 067901(2003).
[10] X.Zhu, P.Zhuang and N.Xu, Phys. Lett. B607, 107(2005).
[11] C.Wong, Phys. Rev. C72, 034906(2005).
[12] P.Braun-Munzinger and J. Stachel, Phys. Lett. B490, 196(2000).
[13] M.I.Gorenstein, A.P.Kostyuk, H.Stöcker and W. Greiner, Phys. Lett. B509, 277(2001).
[14] R.L.Thews, M.Schroedter and J.Rafelski, Phys. Rev. C63, 054905 (2001).
[15] L.Grandchamp, R.Rapp and G.E.Brown, Phys. Rev. Lett. 92, 212301(2004).
[16] L.R.Thews and M.L.Mangano, Phys. Rev. C73, 044904(2006).
[17] X.Zhao and R.Rapp, Phys.Lett.B664,253(2008).
[18] E.Megias, E.Ruiz Arriola, and L.L.Salcedo, Phys. Lett. B563, 173(2003).
[19] S.H.Lee and K.Morita, Phys. Rev. D79, 011501(R)(2009).
[20] L.Yan, P.Zhuang, N.Xu, Phys.Rev.Lett97,232301(2006).
[21] A. Zoccoli et. al. [HERA-B Collaboration], Eur. Phys. J. C43, 179(2005).
[22] M.E.Peskin, Nucl. Phys. B156, 365(1979).
[23] G.Bhanot and M.E.Peskin, Nucl. Phys. B156, 391(1979).
[24] H.Satz,J. Phys. G32, R25(2006).
[25] F.Karsch and E.Laermann, [arXiv:hep-lat/0305025]
[26] E.Shuryak and I.Zahed, Phys. Rev. D70, 054507(2004).
[27] C.Gerschel and J.Hufner, Annu. Rev. Nucl. Part. Sci. 49, 255(1999).
[28] S.Gavin and M.Gyulassy, Phys. Lett. B214, 241(1988).
[29] J.Hufner, Y.Kurihara and H.J.Pirner, Phys. Lett. B215, 218(1988).
[30] R.Vogt, Phys. Rev. C81 044903(2010).
[31] Y.Liu, B.Chen, N.Xu and P.Zhuang, Phys. Lett. B697, 32(2011).
[32] J.Hufner and P.Zhuang, Phys. Lett. B515, 115(2001).
[33] G.Boyd, J.Engels, F.Karsch, E.Laermann, C.Legeland, M.Lüttgemeier, and B.Petersson, Nucl. Phys. B469, 419(1996).
[34] Y.Liu, Z.Qu, N.Xu, and P.Zhuang, Phys. Lett. B678, 72(2009).
[35] K.Zhou, N.Xu, and P.Zhuang, Nucl. Phys. A834, 249c(2010).
[36] A.Adare et al.[PHENIX Collaboration], Phys. Rev. Lett. 98, 232301(2007).
[37] Z.Tang [STAR Collaboration], J. Phys. G38, 124107(2011).
[38] Y.Liu, N.Xu, and P.Zhuang, dissociation temperature of moving quarkonia in deconfined matter, in progress.