Prediction of Water Consumption for Per Ten Thousand Yuan of Ningxia’s GDP Based on Combinatorial Grey Forecasting Model

Jing ZHANG\(^1\), Lian TANG\(^{1,2,3,*}\), Tian-yan HUANG\(^1\) and Xiao-juan CHENG\(^{1,2}\)

\(^1\)School of Civil & Hydraulic Engineering, Ningxia University, Yinchuan 750021, China
\(^2\)Engineering Research Center of Ministry of Education for Efficient Utilization of Modern Agriculture Water Resources in Arid Areas, Ningxia University, Yinchuan 750021, China
\(^3\)Research Team of Ministry of Education for “Efficient Utilization of Modern Agriculture Water Resources in Arid Areas”, Ningxia University, Yinchuan 750021, China

*Corresponding author

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Abstract. This paper, based on grey system theory, establishes a grey forecasting model combining grey GM(1.1) forecasting model and Logistic equation grey forecasting model to predict water consumption per 10,000 yuan of GDP in the region. It by building models with data collected between 2006-2013 of Ningxia Hui Autonomous Region and utilizing the current computer programs, analyzes and predicts the water consumption per 10,000 yuan of GDP of the region in the future. The findings show that all the three forecasting models meet the requirement for precision, but the combinatorial grey forecasting model conforms more to the real situation.

Introduction

China’s water scarcity is recognized almost across the world. Presently, how to reduce water waste and improve the efficiency of water utility draws close attention in the academia. Water consumption per 10,000 yuan of GDP, as an important macro quota indicator for water resources and an internationally-used comparative quota indicator for the utility efficiency of water resource, can measure the economic water consumption and its efficiency of a country, region or industry. Significant reduction in water consumption per 10,000 yuan of GDP in 2020 has already been set as a major goal for China’s development\(^[1]\). Therefore, it’s of great importance to predict water consumption per 10,000 yuan of GDP and know how it develops in a country, a region or an industry.

This paper, taking Ningxia Hui Autonomous Region as an example, chooses three forecasting methods—grey GM(1.1) model, Logistic equation grey forecasting model and a combinatorial forecasting model, to predict the water consumption per 10,000 yuan GDP in the hope of providing some reference for improving water utility efficiency and water-conservation planning.

Construction of the Forecasting Model

Grey GM(1.1) Model

Based on grey system theory, Grey GM(1.1) forecasting model establishes an differential equation that extends from the past to the future according to the system’s available or uncertain information of the past or present. It then seeks system changing patterns which can be used to determine the system’s future development\(^[2]\). This model is characterized with less data, simple calculation and high credibility\(^[3,4]\). The differential equation is:

\[
\frac{dx^{(1)}}{dt} + \lambda x^{(1)} = u
\]  

(1)
Logistic Equation Grey Forecasting Model

Logistic Equation[5,6]:

\[ y = \frac{L}{1 + ae^{bt}} \]  

(2)

In the equation,  
\( y \) — is the growing index of the water consumption per 10,000 yuan of GDP to be measured;  
\( L \) — is the upper limit of \( y \)'s growth;  
\( b \) — is the free growth rate of the water consumption per 10,000 yuan of GDP to be measured;  
\( t \) — is the growth time of water consumption per 10,000 yuan of GDP;  

Logistic equation grey forecasting model is based on grey GM(1.1) forecasting. In the equation, make \( x = 1/y; B = 1/L \), then we have:

\[ x = B(1 + ae^{-tb}) \]  

(3)

Compute the derivative of both sides of equation (3), and then we have:

\[ \frac{dx^{(1)}}{dt} + bx^{(1)} = Bb \]  

(4)

If we make \( \lambda = b \), \( u = Bb \), then the Logistic equation can be written in the same way as grey GM(1.1) forecasting model. That’s equation (1). Then according to the calculation steps of grey GM(1.1) forecasting model, we are to solve \( \lambda \) and \( u \), where \( x \) is the reciprocal of the index \( y \) to be measured. The steps are as follows:

\[ (\lambda \quad u) = \left(X^TX\right)^{-1}X^TX_N \]  

(5)

\[
X = \begin{bmatrix}
-\frac{1}{2}(x_1 \pm x_2) & 1 \\
-\frac{1}{2}(x_2 \pm x_3) & 1 \\
\vdots & \vdots \\
-\frac{1}{2}(x_{n-1} \pm x_n) & 1
\end{bmatrix}
\]

(6)

\[
x_N = \left(\frac{x_2 - x_1}{t_2 - t_1}, \frac{x_3 - x_2}{t_3 - t_2}, \ldots, \frac{x_n - x_{n-1}}{t_n - t_{n-1}}\right)^T
\]

(7)

From equation (5), (6) and (7), we know that in the Logistic equation, \( b = \lambda, \quad L = u, \quad a = (L/y_0)e^{b0} \), and then we can establish the Logistic equation forecasting model of water consumption per 10,000 yuan of GDP.

Combinatorial Grey Forecasting Model

The combinatorial grey forecasting model integrates grey GM(1.1) forecasting model and Logistic equation grey forecasting model, so it is more comprehensive than one single forecasting model and can improve the precision of prediction as it balances the errors of two forecasting models. The advantage of this model is reflected exactly in the combination of the prediction results, i.e. the average of the forecasting results of the two models. It can be formulated as follows:

\[ x = \frac{1}{2}(\hat{X}^{(0)} + y) \]  

(8)
In the equation,
\( \hat{X}^{(n)} \) means the predicted value of grey GM(1.1) for water consumption per 10,000 yuan of GDP;
y means the predicted value of Logistic equation grey forecasting model for water consumption per 10,000 yuan of GDP.

**Case Application**

**Calculation of Ningxia’s Water Consumption per 10,000 Yuan of GDP by Three Models**

The paper is to establish models with data of Ningxia’s water consumption per 10,000 yuan of GDP from 2006 to 2013 and predict its water consumption from 2014 to 2020 with the three different models. The available data of Ningxia’s water consumption per 10,000 yuan of GDP can be seen in Table 1.

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
|------|------|------|------|------|------|------|------|------|
| Water consumption per 10,000 yuan of GDP (m³/10,000 yuan) | 1098 | 851 | 675 | 541 | 440 | 357 | 298 | 281 |
| 1/y | 0.0009 | 0.0012 | 0.0015 | 0.0018 | 0.0023 | 0.0028 | 0.0034 | 0.0036 |

According to the above methods, the calculations of the three models are as follows:

**Grey GM(1.1) Forecasting Model**

By calculating, we have:

\[(\lambda, u) = (0.2040, 1144.23)\]

The model is:

\[ \hat{X}^{(1)}_{k+1} = \left( \frac{1144.23}{0.2040} \right) \left( e^{-0.2040k} \right) + \frac{1144.23}{0.2040} \]

By one subtracting, we can get the simulated values of Ningxia’s water consumption per 10,000 yuan of GDP from 2006 to 2013. See Table 2.

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | Averag e |
|------|------|------|------|------|------|------|------|------|----------|
| Actual Value /100 million m³ | 1098 | 851 | 675 | 541 | 440 | 357 | 298 | 281 | - |
| Simulated Value | 1098 | 832 | 679 | 554 | 451 | 368 | 300 | 245 | - |
| Relative Error (%) | - | -2.19 | 0.56 | 2.31 | 2.59 | 3.10 | 0.73 | -12.89 | 3.48 |

From the simulated results of grey GM(1.1) forecasting model in Table 2, the relative error is between -12.89~3.10 and the average relative error is 3.48%. The simulated values and the actual values of this model are close and its result is of high precision.

**Logistic Equation Grey Forecasting Model**

Through calculating, we have:

\[(\lambda, u) = (-0.0369, 0.00003), \text{i.e.} b=\lambda=-0.0369, L=-123.5358, a=-1.0723.\]
The model is:

\[ y = \frac{-123.5358}{1 - 1.0723e^{0.0369t}} \]

The simulated results can be seen in Table 3. It is clear that the relative error is between -7.39~6.88 and the average relative error is 4.51, which meets the required precision.

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | Average |
|------|------|------|------|------|------|------|------|------|---------|
| Actual value / 100 million m³ | 1098 | 851 | 675 | 541 | 440 | 357 | 298 | 281 | - |
| Simulated Data | 1098 | 801 | 625 | 509 | 427 | 366 | 318 | 281 | - |
| Relative Error (%) | -5.91 | -7.39 | -5.87 | -2.93 | 2.48 | 6.88 | -0.08 | 4.51 |

With the stimulated results from grey GM(1.1) forecasting model and Logistic equation grey forecasting model, we get the stimulated results of a combinatorial grey forecasting model. They are the average of the above results that can been seen in Table 4.

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | Average |
|------|------|------|------|------|------|------|------|------|---------|
| Actual value / 100 million m³ | 1098 | 817 | 652 | 531 | 439 | 367 | 309 | 263 | - |
| Stimulated Data | 1098 | 817 | 652 | 531 | 439 | 367 | 309 | 263 | - |
| Relative Error (%) | -4.05 | -3.42 | -1.78 | -0.17 | 2.79 | 3.80 | -6.49 | 3.21 |

The stimulated results in Table 4 show that the relative error is between -6.49~3.8 and the average error is 3.21. The model is of high precision.

**Prediction of Ningxia’s Water Consumption per 10,000 Yuan of GDP**

By calculating, it is found that the simulated values of the three forecasting models are all close to the actual values. These models meet the required precision and can be used to predict water consumption per 10,000 yuan of GDP in Ningxia. The simulated values and predicted values of other models are in Figure 1 and the predicted results are in Table 5.
Table 5. The Predicted Results of Water Consumption per 10,000 Yuan of GDP of Ningxia between 2014-2020.

| Year | Grey GM(1.1) Forecasting Model | Logistic Equation Grey Forecasting Model | Combinatorial Grey Forecasting Model |
|------|---------------------------------|----------------------------------------|--------------------------------------|
| 2014 | 200                             | 250                                    | 225                                  |
| 2015 | 163                             | 225                                    | 194                                  |
| 2016 | 133                             | 203                                    | 168                                  |
| 2017 | 108                             | 185                                    | 146                                  |
| 2018 | 88                              | 169                                    | 129                                  |
| 2019 | 72                              | 155                                    | 114                                  |
| 2020 | 59                              | 143                                    | 101                                  |

Case Application

Based on grey system theory, Grey GM(1.1) forecasting model, logistic equation grey forecasting model and combinatorial grey forecasting model need less data, simple calculation and the predicted results are accurate.

From the predicted results, it is found that the three forecasting models all meet the requirement of precision, among which the relative error of grey GM(1.1) forecasting model is 3.48, Logistic equation grey forecasting model is 4.51 and the combinatorial model is 3.21. The precision of the combinatorial grey forecasting model is highest among these three models and its simulated values are closest to the actual values.

The predicted results indicate that, with the establishment of a water-saving society and the setting of the target values of the most strict water resource management system ever in Ningxia’s history, the region’s water consumption per 10,000 yuan of GDP is constantly declining and has successfully reached the target set in the Twelfth Five-Year Plan. Besides, the water consumption per 10,000 yuan of GDP should be put into the performance evaluation system and provide support for the sustainable socioeconomic development and the rational planning of water resources of Ningxia.

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