Reducing peak to average power ratio (PAPR) of OFDM systems using complex BIFORE precoding transform

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Abstract. Due to its valuable features, orthogonal frequency division multiplexing (OFDM) is utilized extensively with the modern communication systems. In despite, such employment is impacted with some drawbacks that has been received a great interest in the recent studies. One of the negative ramifications owing to the supersession of transmitted signal through inverse fast Fourier transform (IFFT) is the large values of peak to average power ratio (PAPR). Consequently, reducing PAPR has received a good effort in the current studies. Owing to an efficient data rate and low computational complexity further to reduce PAPR without distortion, precoding OFDM system is employed in the recent information transmission techniques. This paper proposed the complex BIFORE transform (CBT) as a precoding scheme for OFDM systems and compared with Walsh Hadamard transform (WHT). In the sequel, the CBT achieved more PAPR reduction than WHT when employing as a precoding transform with the OFDM systems because of reducing superposition of the transmitted signals. As clearly shown in the simulation results, the CBT can be considered as an efficient precoding transform with OFDM systems for reducing PAPR.

Keywords: OFDM, complex BIFORE transform (CBT), precoding techniques, peak to average power ratio (PAPR), discrete Fourier transform (DFT), Walsh-Hadamard transforms (WHTs).

1. Introduction
Over the last decades, OFDM system can be used efficiently to deliver high rate of data used for wired and wireless communications. it is a multicarrier non-overlapping orthogonal digital communication structure where the whole available bandwidth is divided into many narrow bands of low data rates and then modulated at the same time by multiple carriers. The widespread adoption of OFDM in wireless communications can be explained by its capacity to compensate for impairments of the radio environment due to the multi-path propagation that produces selective frequency fading, shadow fading and interference by other users \cite{1, 2}. Basically, the modulation (Mapping) system generally calculates the required carrier amplitude and phase of the carrier using different techniques such as M-PSK or M-QAM. The schematic of main blocks of the conventional (uncoded) OFDM system is depicted in figure 1.
In addition, the inverse fast Fourier transform (IFFT) is utilized at the transmitter side to ensure carrier orthogonality where the signal is changed from frequency-domain to time-domain. Whereas, fast Fourier transform (FFT) is applied at the receiver side to reverse the operation and recover the original samples [3, 4]. In wireless communications, the impact of fading is spread over multiple samples by using the OFDM system. Therefore, in high data rate applications, few bits in adjacent places might be demolished by the deep fading further to each bit that slightly affected by the fading. Moreover, as given in equation 1, the discrete Fourier transform (DFT) is employed to subdivide the wideband signal into many orthogonal narrowband subcarriers [4, 5]:

\[ x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \exp\left(\frac{j2\pi kn}{N}\right) \]  

(1)

Where, \( X_n = [X_0, X_1, ..., X_{N-1}] \) is the transmitted samples and \( N \) the number of subcarriers. As shown in Figure 2, the cyclic prefix (CP) has been utilised as a guard interval to eliminate the inter symbol interference (ISI). Such a guard interval should be equal or larger than to the maximum delay of the path for the channel [6].

Due to the superposition among samples transmitted through the modulator (IFFT) at the transmitter side of the OFDM system, the peak to average power ratio (PAPR) is increased. The peak power is generated by \( N \) carrier signals that may contain equal phase, that is \( N \) times the average power. The large values of the PAPR influence results in a non-linear distortion of the power amplifier and the clipping noise [2].

In recent research, precoding techniques were highlighted as the most prominent solution to reduce the high PAPR problem. Authors in [7] applied the Walsh Hadamard transform (WHT) as a precoder to
DFT modulator and it has proven its effectiveness in PAPR reducing. Also Hsu C-Y and Liao H-C [8] studied a generalized precoding method that uses a generalized precoding method to provide a flexible scheme to generate precoded data with low PAPR and complexity.

The paper is organized in six sections as follows: section 2 presents the basic definition of PAPR and its effects in OFDM systems. Section 3 deals with the proposed precoding techniques used in this paper follows by OFDM systems based on WHT and CBT transforms presented in section 4. Comparison of different OFDM systems are evaluated by conducting the simulation results of proposed WHT/CBT systems and the conventional OFDM system is given in section 5. Finally, section 6 concludes the paper.

2. Peak to Average Power Ratio (PAPR)

Every multi-carrier signal is the summation of several distinct or orthogonal signals that may produce high PAPR [8]. For this signal, the PAPR can be defined as:

$$\text{PAPR}(x_k) = \frac{\max_{0 \leq k \leq N-1} |x_k|^2}{E[|x_k|^2]}$$  \hspace{1cm} (2)

Where $E[\cdot]$ denotes the expected value, $E[|x_k|^2]$ indicates the signal average power and $\max_{0 \leq k \leq N-1}$ indicates the maximum instantaneous power ratio.

The typical method for estimating PAPR is by the Complementary Cumulative Distribution Function (CCDF) graph. The CCDF shows the risk for the block of data, PAPR exceeding a given threshold, which can be defined as:

$$F(z) = 1 - \exp(-z)$$  \hspace{1cm} (3)

As a result, the probability of PAPR can be defined as:

$$P(\text{PAPR} > z) = 1 - (F(z))^N$$

$$= 1 - (1 - \exp(-z))^N$$  \hspace{1cm} (4)

Highest values of PAPR leads the transmitter power amplifier (PA) to work in the nonlinear section that distorts the transmitted signals and causes the in-band and out-band distortions [9, 10].

3. Precoding Techniques

During the growth of OFDM technologies, significant strategies for reducing PAPR have been introduced. The key purpose of this procedure for the OFDM signal is to minimize the PAPR to an appropriate level prior transmitting the OFDM signal. There are many techniques for reducing PAPR such as selected mapping technique (SLM), amplitude clipping and filtering, partial transmit sequence (PTS) technique and precoding Technique [9-11]. In this paper, we study the precoding technique along with their effect on the PAPR at the transmitter of the OFDM system. As shown in the literature, precoding transforms are employed efficiently to suppress the peak power of the transmitted signals. Consequently, decreasing PAPR can compensate for the non-linearity of high power amplifier (HPA), which decreases the bit error rate (BER). The key properties of precoding schemes are including preservation of bandwidth, lower peak power, no data rate loss, and reducing BER without distortion. Some related precoded transforms are addressed below.

3.1. Walsh-Hadamard Transform (WHT)

Walsh-Hadamard Transform (WHT) is a linear, real, non-sinusoidal orthogonal transform. This means that the implementation of WHT does not involve a substantial increase in complexity of the system. WHT matrices are indexed with values of (+1,-1) [12]. Thus, WHT matrices could be written as:

$$[H(k)] = \begin{bmatrix} [H(k-1)] & [H(k-1)] \\ [H(k-1)] & -[H(k-2)] \end{bmatrix}, \text{ } k=1,2,3...$$  \hspace{1cm} (5)
\[ [H(1)] = [1] \]
\[ [H(2)] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]
\[ [H(4)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \] (6)

It is worth mentioning that both the WHT and the inverse WHT (IWHT) are symmetric and identical. Furthermore, merging WHT with the IFFT at the transmitter side of OFDM system [13] achieves a significant BER improvement [14]. In this paper, we will review the WHT and compare it with the Complex BIFORE transform by simulation results, because the two transfers belong to the same family.

3.2. Complex BIFORE Transform (CBT)

Complex binary Fourier representation (BIFORE) Transform (CBT) [15], is a progressed case from WHT, linear orthogonal transform, but non-real complex transform. This means that the implementation of CBT would not require a substantial improvement in system complexity. CBT is a sequence of orthogonal functions with elements (±1, ±i). Furthermore, if we equate CBT with DFT it is much smoother and quicker to calculate.

The presented complex BIFORE transform structure will minimize the existence of large peaks compared to the conventional OFDM method. CBT can be utilized efficiently to reduce the autocorrelation among the transmitted symbols to reduce the peak power of the modulated signals. Consequently, CBT matrix can be expressed as:

\[ H_c(N) = \begin{bmatrix} [H(k - 1)] & [H(k - 1)] \\ P(1) \otimes [H(k - 2)] & -P(1) \otimes [H(k - 2)] \end{bmatrix} \] (8)

Where \( k = \log_2 N \) and \( \otimes \) denotes Kronecker product [16].

\[ [H_c(0)] = [1], \]
\[ [H_c(1)] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]
\[ [H_c(2)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & i \\ 1 & i & -1 & -i \end{bmatrix} \] (9)

\[ P(1) = \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \]

The CBT can be used in numerous applications, such as image recognition, voice recognition, filtering, and the study of the power spectrum. Moreover, fast CBT (FCBT) can be used efficiently with lower complexity requirements [17].

4. Proposed Systems

Figure 3 shows the proposed WHT and CBT precoding based OFDM system.
In this system, the precoding matrix $P$ is applied to the modulated (M-QAM or M-PSK) data samples in frequency-domain $X_n = [X_0, X_1, \ldots, X_{N-1}]^T$ to generate a new vector $Y_m = [Y_0, Y_1, \ldots, Y_{N-1}]^T$ which can be expressed as:

$$Y_m = \sum_{n=0}^{N-1} P_{m,n} X_n , \quad m=0, 1, \ldots, N-1$$

where $P_{m,n}$ is a matrix of size $N \times N$ represents the precoder.

In consequence, the time-domain OFDM samples can be calculated:

$$X_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} Y_m \exp \left( j \frac{2\pi nm}{N} \right) , \quad m=0, 1, \ldots, N-1$$

Substituting (12) into (13), we get the output of the proposed OFDM system as

$$Y_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} P_{m,n} Y_m \exp \left( j \frac{2\pi nm}{N} \right) \quad n,m=0, 1, \ldots, N-1$$

### 5. Simulation Results

To demonstrate the performance outcomes of the presented system, M-PSK and M-QAM modulation schemes are used to calculate the PAPR. To demonstrate the overall effect of WHT and CBT transforms for reducing PAPR over the OFDM system and without loss of generality, a randomly generated data are modulated into M-array PSK and QAM with the number of subcarriers $N=256$ and modulation index $M=16$ as shown in figures 4 and 5 respectively.
Figure 4. PAPR of the CBT-OFDM, WHT-OFDM and traditional OFDM with 16-PSK.

Figure 4 illustrates the ability of CBT scheme to reduce PAPR comparing with WHT scheme and conventional OFDM signal with 16-PSK modulation. As obviously shown, the CBT scheme reduces the PAPR by 1.8 dB comparing with the conventional OFDM system.

Figure 5. PAPR of the CBT-OFDM, WHT-OFDM and traditional OFDM with 16-QAM.

Furthermore, figure 5 shows the CCDF utilized systems with 16-QAM modulation. As clearly shown, the CBT scheme reduces the PAPR by 1.5 dB over the conventional OFDM system. Such a PAPR reduction consequences from the reduction of the superposition among the transmitted samples.
On the other hand, for any scheme that may proposed for PAPR reduction its effects on the BER performance should be taken into account. Thus, as shown in Figure 6, similar to other distortionless precoded transforms that are used in frequency-domain, the proposed CBT-OFDM does not has any negative ramifications on the BER performance. In addition, precoded systems do not need for any side information that may effect on the data rate of the transmitted signals. In the sequel, the CBT can be used efficiently with OFDM systems by achieving PAPR reduction with distortionless effect.

6. Conclusion

In this paper, WHT and CBT precoding techniques are presented as an ideal solution to reduce the PAPR in OFDM. Obviously, CBT-OFDM system is achieved better PAPR reduction than WHT-OFDM and conventional OFDM systems. Such PAPR reduction is achieved owing to the lower superposition among the transmitted samples and without bit-rate losses, bandwidth expansion, and performance distortion. The proposed CBT-OFDM system takes advantage of changes in frequency and therefore can achieve satisfactory BER performance in fading of multipath channels compared to the conventional OFDM system.

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