The structure of the manipulators of industrial robots

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Abstract. The paper discusses the specifics of using the technology of applying the force action of servo-drives to manipulator components in accordance with the control signals from the robot control device. Consideration is given to solving the inverse kinematics of manipulators of the robot manipulator in continuous path control. Solving the inverse problem description of dependencies is formed in an analytical and tabular form.

1. Introduction
A robot is a complex technical device created on the basis of the mechatronic method, consisting of a manipulator and a control device and intended for the transport of working tools in a three-dimensional space. In robotic manipulators, single rotational or rectilinear sliding kinematic pairs of the fifth class are used [1].

The power effects of the servos on the components of the manipulator are formed in accordance with the control signals coming from the robot control device which are formed according to a predetermined capture motion as well as the position of the robot and the surrounding environment. Under the influence of the servo, the robot arm and the gripper will make very definite movement in space.

2. Materials and methods
In a robot, to perform the motor function, the manipulator is used which is a series of kinematic links coupled together by kinematic pairs (Figure 1).
One of the links is stable and fixed, while others may make controlled movements due to the influence of drives.

The robot manipulator can be formed according to the principle of the open kinematic chain (Figure 2) with consecutive links (anthropomorphic humanoid robots) or on the basis of a closed kinematic chain (Figure 3) with a series-parallel connection (the robot with parallel kinematics, specifically hexapods on Stewart's platform) [2].

The manipulator is a mechanism that itself is a closed or open kinematic chain that is used to obtain the desired grip in the space.

Transformation laws depend on the nature of kinematic pairs and their relative positions in the manipulator.

The kinematic pairs allow some movement of the constituting parts relative to each other. This certainty is due to constraints of the pairs depending of the geometry of their elements.

Pairs are classified into classes, the number of which equals the number of constraints (the number of communication conditions) imposed by this pair:
- 1 limitation (1 connection condition) - a pair of class I;
- 2 limitations (2 connection conditions) - a pair of classes II;
- 3 limitations (3 connection conditions) - a pair of classes III;
- 4 limitations (4 connection conditions) - a pair of classes IV;
5 limitations (5 connection conditions) - the pair of classes V (pair of 5-th class can be translational and rectilinear sliding) [3].

One of the important characteristics of the manipulator is the number of degrees of mobility, the number of degrees of freedom.

In an arbitrary kinematic chain the formula of Luba-Malyshev can be applied: \( \Sigma \)

\[
W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1 = 6n - \sum_{i=1}^{5} i p_i, \tag{1}
\]

where \( p_i \) - the number of kinematic pairs of the i-th class.

For robotic manipulators with kinematic pairs of the 5-th class [4]:

\[
W = 6n - 5p_5. \tag{2}
\]

Solution of the inverse kinematics problem of manipulators is one of the main tasks of the kinematic and dynamic analysis and synthesis of manipulators.

The inverse problem is solved by a continuous path control of the robot. In case where the gripper must move along a path defined in time and space, we need to find values of the generalized coordinates of the manipulator in accordance with a predetermined position of the gripper.

Given there is a law of gripper motion, i.e. the rules of changing its coordinates and orientation angles:

\[
\begin{align*}
\begin{cases}
    x_{0n} = x_{0n}(t); & y_{0n} = y_{0n}(t); & z_{0n} = z_{0n}(t) \\
    \bar{x}_0\bar{y}_n = f_1(t); & \bar{y}_0\bar{z}_n = f_2(t); & \bar{x}_0\bar{z}_n = f_3(t),
\end{cases}
\end{align*}
\tag{3}
\]

Consequently, we can define a matrix, setting positions and orientations of the gripper:

\[
T_{0n}^{II} = \begin{bmatrix}
    a_{11}^{II}(t) & a_{12}^{II}(t) & a_{13}^{II}(t) & a_{14}^{II}(t) \\
    a_{21}^{II}(t) & a_{22}^{II}(t) & a_{23}^{II}(t) & a_{24}^{II}(t) \\
    a_{31}^{II}(t) & a_{32}^{II}(t) & a_{33}^{II}(t) & a_{34}^{II}(t) \\
    0 & 0 & 0 & 1
\end{bmatrix},
\tag{4}
\]

where

\[
a_{12}^{II}(t) = \cos[\bar{x}_0\bar{y}_n]; & a_{13}^{II}(t) = \cos[\bar{x}_0\bar{z}_n]; & a_{23}^{II}(t) = \cos[\bar{y}_0\bar{z}_n];
\tag{5}
\]

\[
a_{14}^{II}(t) = x_{0n}(t); & a_{24}^{II}(t) = y_{0n}(t); & a_{34}^{II} = z_{0n}(t). \tag{6}
\]

We suppose that at some k-th point of time \( t_k \) (\( k = 0, \ldots, K \)), the predetermined gripper position: \( a_{12}^{II} (t_k), a_{13}^{II} (t_k), \ldots, a_{34}^{II} (t_k) \) coincides with the actual position supported by the current values \( q_i^k \) of generalized coordinates [5].

Then:

\[
T_{0n}^{II}(t_k) - T_{0n}^{Q,k}(q_1^k, \ldots, q_n^k; t_k) = 0,
\tag{7}
\]

whence:

\[
a_{12}^{II}(t) - a_{12}^{II} = 0, \ a_{13}^{II}(t) - a_{13}^{II} = 0, \ldots, a_{34}^{II}(t) - a_{34}^{II} = 0. \tag{8}
\]

Generalized coordinates, at which the equivalence of the matrices \( T_{0n}^{II} \) and \( T_{0n}^{Q,k} \) is achieved, are taken as the value \( q_i^{k+1} = t_k + 1 \), corresponding to time \( t_{k+1} \).
Thereafter position \((k + 1)\) is taken as the k-th and the next new position of the manipulator, i.e. a mismatch again introduced into the resulting equation, which should be removed after the determination of the following values of the generalized coordinates. This continues until the moment when the values are defined for each of generalized coordinates required by positions K of the manipulator at a predetermined trajectory.

Let us define the criterion function. Since the aim of the solution is to find such generalized coordinate values, for which the difference between this position of gripper and its actual position is equal to zero, the parameter reflecting this difference should be taken as a criterion.

In this regard, for the automatic execution of the probable range of changes of generalized coordinates, the penalty function is introduced in the form of the of the following restrictions on the values \(q_i\):

\[
F_i^1 = \begin{cases} 
0, & \text{if } q_i - q_i^{\text{min}} \geq 0; \\
W_i^1(q_i^{\text{min}} - q_i), & \text{if } q_i - q_i^{\text{min}} < 0, 
\end{cases} 
\]

\[
F_i^2 = \begin{cases} 
0, & \text{if } q_i^{\text{max}} - q_i \geq 0; \\
W_i^2(q_i - q_i^{\text{max}}), & \text{if } q_i^{\text{max}} - q_i < 0, 
\end{cases} 
\]

where \(W_i^1, W_i^2\) - the weight of the penalty function, which can adjust its slope [6].

In addition to these, there can be other limitations.

In the role of the objective function, one can use a function that is the sum of test functions and penalty function:

\[
K(q_1, ..., q_n) = \Delta + \sum_{i=1}^{i=n} F_i^1 + \sum_{i=1}^{i=n} F_i^2. \tag{11}
\]

We will use as a method of finding new values of generalized coordinates, the q method of the gradient which is expressed by the following relation:

\[
q_i^{m+1} = q_i^m - h_i \cdot \frac{\Delta(q_1^m, ..., q_i^{m+\Delta q_i}, ..., q_n^m) - \Delta(q_1^m, ..., q_i^m, ..., q_n^m)}{\Delta q_i}, \tag{12}
\]

where \(h_i\) - a step for the i-th generalized coordinate; \(\Delta q_i\) - a small increment of the i-th generalized coordinate, used for finding the partial derivative of \(q_i\).

It should be noted that the old value \(q_i^k\) is used as a first approximation \(q_i^m(m = 1)\), i.e. at the initial moment \(q_i^1 = q_i^k\) is supposed.

The computational process of approaching the values of \(q_i^{k+1}\) may be interrupted by any attribute used in the nonlinear programming. In particular, it can be supplemented here by condition \(\left| q_i^{m+1} - q_i^m \right| \leq \delta_i\), where \(\delta_i\) is the value of the deviation of the i-th generalized coordinate permitted by tolerance accuracy of the trajectory.

Given there is the nature of the function (11), the final value should be taken as:

\[
q_i^{k+1} = 0.5 \cdot (q_i^{m+1} + q_i^m). \tag{13}
\]

Let us note that the \((m + 1)\) th step to the point \((k + 1)\) in the calculation process may be performed using various algorithms typical of nonlinear programming techniques. Using a method of the gradient step \((m + 1)\) is performed simultaneously on all the coordinates after calculating the orientation anti-gradient movement.

To do this, we will use the central subtraction:
\[
\begin{align*}
\dot{q}_i^k &= \frac{q_i^{k+1} - q_i^{k-1}}{2\Delta t};
\ddot{q}_i^k &= \frac{q_i^{k+1} - 2q_i^k + q_i^{k-1}}{\Delta t^2}.
\end{align*}
\]

(14)

Thus, functions \(\dot{q}_i^k\) and \(\ddot{q}_i^k\) will also be obtained in a tabular form. To form the control action as continuous functions, rather than a table, it is important to bring the table values of the generalized coordinates, velocities and accelerations. To do this we can use, for example, Lagrange interpolation equation:

\[
q_i(t) = q_i^0 \cdot \frac{(t - t_1) \cdot (t - t_2) \cdot \ldots \cdot (t - t_K)}{(t_0 - t_1) \cdot (t_0 - t_2) \cdot \ldots \cdot (t_0 - t_K)} + q_i^1 \cdot \frac{(t - t_0) \cdot (t - t_2) \cdot \ldots \cdot (t - t_K)}{(t_1 - t_0) \cdot (t_1 - t_2) \cdot \ldots \cdot (t_1 - t_K)} + \ldots + q_i^K \cdot \frac{(t - t_0) \cdot (t - t_2) \cdot \ldots \cdot (t - t_K)}{(t_K - t_0) \cdot (t_K - t_1) \cdot \ldots \cdot (t_K - t_{K-1})}. 
\]

(15)

3. Conclusion
As a result, a continuous function \(q_i(t)\) was found, which will pass through the points \(q_i^k\) at fixed moments in time \(k\).

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