Dicke super-radiance as non-destructive probe for super-fluidity in optical lattices

Nicolai ten Brinke and Ralf Schützhold

Fakultät für Physik, Universität Duisburg-Essen, Lotharstrasse 1, 47057 Duisburg, Germany

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We study Dicke super-radiance as collective and coherent absorption and emission of photons from an ensemble of ultra-cold atoms in an optical lattice. Since this process depends on the coherence properties of the atoms (e.g., super-fluidity), it can be used as a probe for their quantum state. This detection method is less invasive than time-of-flight experiments or direct (projective) measurements of the atom number (or parity) per lattice site, which both destroy properties of the quantum state such as phase coherence.

I. INTRODUCTION

Ultra-cold atoms in optical lattices are very nice tools for investigating quantum many-body physics since they can be well isolated from the environment and cooled down to very low temperatures. Furthermore, it is possible to control these systems and to measure their properties to a degree which cannot be reached in many other scenarios. For example, the quantum phase transition \( J \) between the highly correlated Mott insulator state and the super-fluid phase in the Bose-Hubbard model \([2,3]\) has been observed in \([4]\). This observation was accomplished by time-of-flight experiments where the optical lattice trapping the atoms is switched off and their positions are measured after a waiting time. As another option for detecting the state of the atoms, the direct \textit{in situ} measurement of the number of atoms per lattice site (or more precisely, the parity) has been achieved recently, see, e.g., \([3,6]\). However, both methods are quite invasive since they destroy properties of the quantum state such as phase coherence. In the following, we study an alternative detection method based on Dicke super-radiance, i.e., the collective and coherent absorption and emission of photons from an ensemble of ultra-cold atoms, see, e.g., \([2,11]\).

II. BASIC FORMALISM

Under appropriate conditions, bosonic ultra-cold atoms in optical lattices are approximately described by the Bose-Hubbard Hamiltonian \([1]\) \((\hbar = 1)\)

\[
\hat{H}_{BH} = -\frac{J}{2} \sum_{\mu \nu} T_{\mu \nu} \hat{b}_\mu^\dagger \hat{b}_\nu + \frac{U}{4} \sum_{\mu} \hat{n}_\mu^{(b)} (\hat{n}_\mu^{(b)} - 1),
\]

(1)

with the hopping rate \( J \), the interaction strength \( U \), the adjacency matrix \( T_{\mu \nu} \), and the coordination number \( Z \). Here, we assume a quadratic lattice with \( Z = 4 \). Furthermore, \( \hat{b}_\mu^\dagger \) and \( \hat{b}_\mu \) denote the creation and annihilation operators of the atoms (in their ground state) at lattice sites \( \mu \) and \( \nu \), respectively, and \( \hat{n}_\mu^{(b)} = \hat{b}_\mu^\dagger \hat{b}_\mu \) is the number operator. Assuming unit filling \( \langle \hat{n}_\mu^{(b)} \rangle = 1 \), this model displays a quantum phase transition \([1]\) between the super-fluid phase where \( J \) dominates and the Mott insulator state where \( U \) dominates. In the extremal limits \( J \gg U \) and \( U \gg J \), the ground states simply read

\[
|\Psi\rangle_{\text{Mott}}^J = \prod_\mu |\hat{b}_\mu^\dagger| 0 \rangle = \bigotimes_\mu |1\rangle_\mu,
\]

(2)

for the Mott insulator state and

\[
|\Psi\rangle_{\text{superfluid}}^U = \frac{1}{\sqrt{N!}} (\hat{b}_0^\dagger)^N |0\rangle \propto \left( \sum_\mu \hat{b}_\mu^\dagger \right)^N |0\rangle,
\]

(3)

for the super-fluid phase, where \( N \) is the total number of lattice sites (which equals the number of particles).

Now, we consider the interaction of these atoms with infra-red photons. Assuming that the wavelength of these infra-red photons is much larger than the lattice spacing of the optical lattice, we have a large number of atoms within one (infra-red) photon wavelength. In addition, the atomic recoil due to the absorption or emission of an infra-red photon is negligible, which is another basic requirement for Dicke super-radiance. The interaction between atoms and photons is then described by

\[
\hat{H}_{\text{int}} = \int d^3 k \ g_k(t) \hat{a}_k \sum_\mu \hat{c}_\mu^\dagger \hat{b}_\mu \exp (i k \cdot r_\mu) + \text{H.c.},
\]

(4)

where \( \hat{a}_k \) is the annihilation operator of a photon with wave-number \( k \) and \( r_\mu \) is the position of the atom at the lattice site \( \mu \). The excited atoms at lattice sites \( \mu \) and \( \nu \) are described by the creation and annihilation operators \( \hat{c}_\mu^\dagger \) and \( \hat{c}_\nu \) and thus one has to extend the Bose-Hubbard Hamiltonian \([1]\) accordingly.

Furthermore, since the typical life-time of an excited atom is very short compared to the characteristic time scales of the lattice dynamics (determined by \( J \) and \( U \)), we envisage the following sequence: After the absorption of the infra-red photon by one of the atoms, a Raman pulse is applied equally to all the atoms such that the excited atom is transferred into a meta-stable state while all the non-excited atoms remain unchanged. Then, after a waiting period \( \Delta t \) during which the atoms have time to tunnel (\( J \)) and to interact (\( U \)), another Raman pulse is applied to all the atoms which transfers the atom which
absorbed the photon back into the original excited state such that it can decay to the ground state by emitting an infra-red photon. This switching process is encoded in the time-dependence of the coupling strength $g_k(t)$.

III. EMISSION PROBABILITY

In lowest-order perturbation theory, the probability (density) to first absorb a photon with wave-number $\kappa_\text{in}$ and later (after the waiting time $\Delta t$) emit a photon with wave-number $\kappa_\text{out}$ is given by

$$P = \int dt_1 dt_2 dt_3 dt_4 g_{\text{out}}(t_4)g_{\text{out}}(t_2)g_{\text{in}}(t_3)g_{\text{in}}(t_1)$$
$$\times \exp\left\{i(\omega_\text{in} t_3 - \omega_\text{out} t_4) - i(\omega_\text{in} t_1 - \omega_\text{out} t_2)\right\}$$
$$\times D(t_1, t_2, t_3, t_4),$$

(5)

with the operatorial part containing the lattice dynamics

$$D(t_1, t_2, t_3, t_4) = \sum_{\mu\nu\rho\eta} \exp\left\{i(\kappa_\text{out} \cdot \tau_\mu - \kappa_\text{in} \cdot \tau_\eta)\right\}$$
$$\times \exp\left\{-i(\kappa_\text{out} \cdot \tau_\mu - \kappa_\text{in} \cdot \tau_\nu)\right\}$$
$$\times \langle \hat{b}_\mu^\dagger(t_3) \hat{c}_\eta(t_3) \hat{c}_\rho(t_4) \hat{b}_\nu(t_4) \hat{b}_\mu(t_2) \hat{c}_\nu(t_2) \hat{c}_\rho(t_1) \hat{b}_\nu(t_1) \rangle. \quad (6)$$

The expectation value in the last line should be taken in the initial state, which could be a pure state, such as the super-fluid state (3) or the Mott state (2), or a mixed state such as a thermal density matrix.

As a result, this probability (density) depends on the above four-times eight-point function, which contains information about the underlying state. Unfortunately, since the Bose-Hubbard model is (as far as we know) not integrable, we do not have an explicit solution for this eight-point function apart from some limiting cases. For $J = 0$, the atoms are pinned to their lattice sites and we get the usual Dicke super-radiance. In the other limiting case $U = 0$, we may simplify this eight-point function further. Assuming that there are no excited atoms initially $\langle \hat{c}_\mu^\dagger \hat{c}_\nu \rangle = 0$, the eight-point function can be reduced to a four-point function in terms of the operators $\hat{b}_\mu^\dagger$ and $\hat{b}_\nu$. After a Fourier transform, this four-point function $\langle \hat{b}_{\mu}\hat{c}_{\nu}\hat{b}_{\mu}^\dagger\hat{b}_{\nu}\rangle$ depends on two wave-numbers $p$ and $q$ (assuming translational invariance). If we have a Gaussian state (for $U = 0$), such as a thermal state or (to a very good approximation) the super-fluid state (3), it can be expanded into a sum of products of two-point functions via the Wick theorem. Finally, if the initial state is diagonal in the $k$-basis – which is also the case for the super-fluid state (3) and thermal states – these two-point functions just give the spectrum $N_k$, i.e., the number of particles per mode. For example, the expectation value $\langle \hat{b}_{\mu}\hat{c}_{\nu}\hat{b}_{\mu}^\dagger\hat{b}_{\nu}\rangle$ becomes $N_q - \kappa_{\text{in}}\delta(\kappa_{\text{in}}, \kappa_{\text{out}})$.

IV. SUPER-RADIANCE

As an example for the general considerations above, let us consider the super-fluid state $\kappa$ with $U = 0$ as initial state. In this case, the probability (density) in Eq. $(5)$ is independent of the waiting time $\Delta t$ and yields

$$P = N^2 \delta(\kappa_{\text{in}}, \kappa_{\text{out}}) P_{\text{single}},$$

(7)

to leading order, where $P_{\text{single}}$ is the corresponding expression for a single atom. As a result, we obtain the same Dicke super-radiance as in the case of immovable atoms. Note that one factor of $N$ originates from the simple fact that $N$ atoms absorb the incident photon more likely than one atom – whereas the other factor of $N$ corresponds to the coherent enhancement of the collective decay probability (i.e., Dicke super-radiance).

As the next example, let us consider a state where $N_1$ atoms are in the super-fluid state while the other $N_2$ atoms are equally distributed over all $k$-modes. This can be considered as a simple toy model for a thermal state with partial condensation, for example. In this situation, the probability (density) in Eq. $(5)$ does depend on the waiting time $\Delta t$ and behaves as

$$P = \left|N_1 e^{\varphi(\Delta t)} + N_2 J(\Delta t)\right|^2 \delta(\kappa_{\text{in}}, \kappa_{\text{out}}) P_{\text{single}}.$$ 

(8)

The phase $\varphi(\Delta t)$ can lead to interference effects between the two terms and is given by $\varphi(\Delta t) = J(T_k - 1)\Delta t$, where we have abbreviated $\kappa = \kappa_{\text{in}} = \kappa_{\text{out}}$ and $T_k$ denotes the Fourier transform of the adjacency matrix $T_{\mu \nu}$. For a quadratic lattice with lattice spacing $\ell$, it reads $T_{k} = [\cos(k_x \ell) + \cos(k_y \ell)]/2$. The remaining function $J(\Delta t)$ describes the reduction of super-radiance due to the hopping of the excited atoms during the waiting time

$$J(\Delta t) = \frac{1}{N} \sum_k \exp\left\{iJ(T_k - T_{k-\Delta t})\Delta t\right\} \leq 1.$$ 

(9)

For small wave-numbers $|k|\ell \ll 1$ and large enough waiting times $\Delta t$ such that $J\Delta t|k|\ell = O(1)$, it can be approximated by Bessel functions

$$J(\Delta t) \approx J_0\left(\frac{J\Delta t}{2} k_x \ell\right) J_0\left(\frac{J\Delta t}{2} k_y \ell\right). \quad (10)$$

As a result, the peak in forward direction decays with time $\Delta t$ unless the photon was incident in orthogonal direction $k_x = k_y = 0$. This can be explained by the fact that the (excited) atoms tunnel during the waiting time $\Delta t$ and thus the initial and final phases $\exp(i k \cdot r_{\mu})$ do not match anymore.

V. PHASE TRANSITION

Now, after having discussed the two cases $J = 0$ and $U = 0$ separately, let us consider a phase transition between the two regimes. After the initial Mott state $\kappa$
has absorbed the incident photon with wave-number $\kappa_{\text{in}}$, we have the following excited state

$$|\Psi_{\text{excited}}^{\text{Mott}}\rangle = \frac{1}{\sqrt{N}} \sum_{\mu} \exp \left( i \kappa_{\text{in}} \cdot r_\mu \right) \hat{c}_\mu^\dagger \prod_{\nu \neq \mu} \hat{b}_\nu^\dagger |0\rangle . \quad (11)$$

Actually, for $U \gg J$, this is an approximate eigenstate of the Bose-Hubbard Hamiltonian (1) in the sub-space where one atom is excited and the $N-1$ others are not. Now, assuming that $N$ is large but finite, we could envisage an adiabatic transition from the initial Mott regime $U \gg J$ to the super-fluid phase where $J \gg U$. Due to the adiabatic theorem, an initial eigenstate such as the state (11) stays an eigenstate during that evolution and thus we end up with the state (for $N \gg 1$)

$$|\Psi_{\text{excited}}^{\text{superfluid}}\rangle = \frac{1}{\sqrt{(N-1)!}} (\hat{b}_{k=0}^\dagger)^{N-1} c_{\kappa_{\text{in}}}^\dagger |0\rangle , \quad (12)$$

where the excited atom possesses the initial wave-number $\kappa_{\text{in}}$ of the absorbed photon and the $N-1$ other atoms are in the super-fluid state (9). Calculating the emission probability from this state, we find that it shows precisely the same characteristic features of Dicke super-radiance and thus photons are emitted predominantly in $\kappa_{\text{in}}$-direction (as one would expect).

Now, after having discussed an adiabatic passage from the Mott state to the super-fluid phase, let us study the other extremal case and consider a sudden switching procedure. Again starting in the state (11), we now envisage an abrupt change from $J = 0$ (and $U > 0$) to $U = 0$ (and $J > 0$). After this sudden switch, the state (11) is no longer an eigenstate of the Bose-Hubbard Hamiltonian (1) but a mixture of excited states. Calculating the emission probability from this state, we find that it coincides with Eq. (9) for $N_2 = N$ and $N_1 = 0$. Ergo, the initial Mott state – after the sudden switch – behaves as a state where all momenta are equally populated. This is a quite intuitive result, but one should keep in mind that the state (11) is not a Gaussian state such that some care is required by applying the results from Sec. IV.

VI. CONCLUSIONS

We studied Dicke super-radiance from an ensemble of ultra-cold atoms in an optical lattice described by the Bose-Hubbard Hamiltonian (1) and found that the character of the emission probability can be employed to obtain some information about the quantum state of the atoms. In the non-interacting case $U = 0$, for example, the temporal decay of the emission peak in forward direction (9) can be used to infer the number $N_1$ of condensed atoms. Comparing the adiabatic passage from the Mott state to the super-fluid phase with a sudden transition, we found that these two cases can also be distinguished via the temporal behavior of the emission probability. Note that the above method is complementary to other techniques since it yields information about the coherence properties of the atoms without destroying them.

Acknowledgments

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