A cooperative game model with bankruptcy theory for Water Allocation: A case study in China Tarim River Basin

Jiahe Tian  
China Agricultural University College of Water Resources and Civil Engineering

Yang Yu  
Tsinghua University

Tongshu Li  
China Agricultural University College of Water Resources and Civil Engineering

Yi Zhou  
China Agricultural University College of Water Resources and Civil Engineering

Jingjun Li  
Tsinghua University

Xingpeng Wang  
Tarim University

Yu Han (yhan@cau.edu.cn)  
China Agricultural University

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Abstract

China Tarim River Basin is located in an arid area, whose rapid socioeconomic development intensifies the current water resources shortage. To allocate water resources reasonably, this paper introduces the bankruptcy theory into the cooperative game model to contract a linear function describing the degree of satisfaction of each region's declared water demand. Bankruptcy theory solves the problem of insufficient information about players in the cooperative game. From the perspective of the cooperative game's stability, the bankruptcy allocation stability index (BASI) is used to evaluate and compare water resource allocation results in the Tarim River Basin in 2025 and 2030 under different scenarios. Moreover, this paper uses the improved TOPSIS model to build the harmony index of water-economy-environment (HWEE) to evaluate the harmony of water resources, economy and environment in each region. The results show that the model is more suitable for the actual water allocation game and has a good application value than the classical bankruptcy theory. Moreover, the stability index and HWEE proposed in this paper also have better applicability, and the allocation scheme with the same game weight in each region is more stable.

1 Introduction

Water resource has become a pledge for sustainable development which is the foundation of natural resources and strategic economic resource (Li et al., 2019). The rapid economic and social growth will help enhance a country or region's economic strength and promote social prosperity and stability. Still, at the same time, socioeconomic demands can exert pressure on water resources and lead to a decline in ecosystem services by adversely affecting water distribution. An integrated response to this problem becomes even more challenging when water demands are competitive and local stakeholder priorities diverge (Kapetas et al., 2019). In the China Tarim River Basin - the arid inland area, precipitation is scarce, evaporation is too large, and production flow is little. The allocation of water resources in different regions restricts regional economic development. Therefore, allocating water resources effectively and fairly and adapting to regional economic growth has become scholars' research focus in recent years.

At present, scholars have made specific achievements in the study of water resource allocation. From the linear decision-making of water resources system, network flow algorithm to the current macro-economy-based water resources optimal allocation theory, from linear programming to multi-level, multi-objective, group decision-making methods, their underlying logic is to use mathematical analysis models to maximize the objective function to perform calculations and comprehensive evaluations. Cohon et al. (1975) evaluated the current situation of the multi-objective solution technology used in water resources analysis and believed that selecting the evaluation target's metric should consider environmental issues and the mode of interaction between physical and social systems. To remedy the environmental impact of land salinization in the Lower Arkansas River Basin in Colorado, Rohmat et al. (2021) introduced geographic information systems into river basin management models to explore the best. Quinn et al. (2004) integrated hydrological climate models, surface water and groundwater hydrological models,
economic and environmental impact models, and social impact assessment techniques to cope with the allocation of water resources under future climate change conditions.

However, conventional optimization methods can not deal with the strategic interaction and conflicting behaviour characteristics between the parties well (Shou-ke et al., 2009). The quasi-public product attributes of water resources and the externality existing in the water-utilization make it necessary to introduce the game theory into the allocation to consider cross-administrative water competition and cooperation. Eleftheriadou et al. (2008) applied the game theory to analyze the negotiation problem of cross-administrative river conflicts between Greece and Bulgaria, and found a compromise solution that both parties may accept, and use interconnected game methods to expand the options of stakeholders. Wei et al. (2009) used a large amount of actual data to simulate and analyze the conflicts of interest in the South-to-North Water Transfer Project's water resources management based on non-cooperative games and cooperative games. He concluded that when some of the net benefits of cooperation were transferred from the winner to the loser, all parties in the game can benefit, and the basic theory of "Prisoner's Dilemma" can solve such problems. The compromise planning method as an effective method to solve the multi-objective decision-making problem has been applied in water resources planning and management. Fattahi et al. (2010) applies the compromise planning method to the multi-objective optimization problem of integrated water resources management. Bender et al. (2000) used the fuzzy compromise planning method to study water resources system planning under uncertain conditions, and the results showed that the compromise planning method provided support for group decision-making. Abrishamchi et al. (2005) took the city of Zahedan, Iran, as an example and applied the compromise planning method to selecting a possible plan in the urban water supply system. The results show that decision-makers can use this method for urban integrated water resources management.

Although game theory has a good reference for solving the actual conflict in allocating water resources across administrative regions, the game theory needs to be based on mastering enough relevant stakeholders' information, such as risk preference, utility function, and possible decision-making. Complex realities challenge the accuracy and accessibility of this information. Bankruptcy theory is applied to the distribution of residual value among creditors after bankruptcy. The sum of due value declared by creditors is greater than the distributed residual value, which leads to the conflict between related stakeholders, which is similar to the conflict of water resources allocation across administrative regions (Mianabadi et al., 2014). The total amount of water allocated by each administrative area of a river basin is often greater than the total amount of water resources available for allocation. Therefore, bankruptcy theory can be used to study conflicts in allocating river water resources across administrative areas (Madani et al., 2014).

Based on the above research, under the background of water shortage in the Tarim River Basin, this paper introduces the bankruptcy theory into the cooperative game model to simulate water resources allocation across five regions. It considers the space-time constraint rules of water resources across regions to explore the Nash Equilibrium in allocating water resources in the Tarim River Basin.
2 Overview Of The Research Area

China Tarim River Basin is located in the inland arid area, living in 50% of Xinjiang's population, has an important economic status. It is far away from the sea and surrounded by high mountains. The basin's average annual precipitation is less than 50 mm, while the average annual evaporation is as high as 2300–3000 mm. Tarim River flows through Bayingolin Mongolian Autonomous Prefecture, Aksu region, Kizilsu Kirgiz Autonomous Prefecture, Kashgar region and Hotan Region, and only has a surface hydraulic connection with Aksu River, Yeerqiang River and Hotan River. The agricultural economy dominates the Tarim River Basin's economy. In recent years, with the gradual increase of agricultural irrigation area and the continuous growth of population, water resources are overexploited, the stock of water resources is decreasing, and the natural ecosystem is seriously degraded. Under the background of the continuous growth of water demand in the future, water shortage will continue. Therefore, water allocation should adapt to the basin's geographical relationship to achieve water efficiency and fairness among the five regions. How to reasonably design water resources allocation in the Tarim River Basin is very important for the Basin's overall development and even Xinjiang.

This paper will use the cooperative game model with bankruptcy theory to establish a new mechanism of water resource allocation across regions in the Tarim River Basin, comprehensively considering the water use efficiency and fairness of the different hydrological years. Figure 1 is the Tarim River basin system's general schematic diagram, which has three main tributaries, the mainstream of the Tarim River and five water consumption areas. When the declared water demand of each water-use region exceeds the total amount of water resources for allocation, the water resources will go bankrupt. Therefore, the bankruptcy theory can be applied to allocate available water resource according to each region's declared water demand in the basin.

The water demand index of this paper adopts the historical water quantity data from 2015 to 2020 in the "General Control Scheme of Water Use in Xinjiang" and the prediction results of relevant planning in 2025 and 2030 (Table 1). In order to better evaluate the applicability of the model allocation results in the future economic development of the Tarim River Basin, this paper applies the existing historical water supply scenarios to the future water demand background to explore the internal law of water resources allocation in the Tarim River Basin as an inland arid area.

Table 1

Annual water demand of Tarim River Basin ($10^8$ m$^3$).
3 Model

3.1 Classical bankruptcy theory model

The classical bankruptcy theory to solve the conflict of water resources allocation across administrative regions is mainly Proportional (P) rule and adjusted proportional rule (AP) (Madani et al., 2014). In period $t$, the declared water consumption of the region $i$ ($i = 1, 2, \cdots m$) in the set composed of different regions is $c_i$, and the total amount of river water resources is $C$. The sum of declared water consumption of each region is shown in expression (1).

$$C = \sum_{i=1}^{m} c_i$$

(1) P rule and AP rule can be defined as follows:

(1) P rule: P rule only considers the declared water demand of each region, the water allocated to each region is the same proportion (Eq.2) of its declared water demand, and the water allocated to the region $i$ is $x_i$

$$p = \frac{E}{C} \ (p \leq 1)$$

$$x_i = c_i \times p$$
4

(2) AP rule: according to the AP rule, if all regions except region \( i \) are satisfied with their water allocation, then the initial water allocation obtained by region \( i \) is the surplus after allocating other regions. To determine the initial water \( v_i \) allocated to region \( i \), compare the sum of declared water demand for all other regions with the available water resource. The initial water allocation of region \( i \) is equal to the remaining water when there is still a surplus after the water allocation meets the declared water demand of other regions. Otherwise, the initial allocation of region \( i \) is set to 0. It is assumed that all regions agree on the initial water allocation calculated by this method. Moreover, after the initial water \( v_i \) is determined, the declared water demand will be modified. The modified declared water demand of a given region is made to be the minimum of the available water resource and the initially declared water demand of the region. Then the P rule is applied to the remaining available water resource, and the modified declared water demand.

The whole allocation process is divided into two steps: first, define the initial water \( v_i \) of each region (Eq. 5)). That is, there is a minimum allocation of water quantity for any region. When the sum of declared water demand \( R_j \) in other regions is greater than the available water resource, the value of \( v_j \) is 0. When the sum of declared water demand in other regions is less than the available water resource, the value of \( v_j \) is \( E-R_j \). In the second step, according to Eq. (6=), the final allocation of water resource is completed, in which the modified declared water demand

\[ c_i^E \]

is as follows Eq. (7):

\[ v_i = \max(0, E - R_i) \]

5

\[ x_i = v_i + (E - \sum_{j \in M} v_j) \frac{(c_i^E - v_i)}{\sum_{j \in M} (c_i^E - v_j)} \]

6

\[ c_i^E = \min(c_i, E) \]

7
3.2 A cooperative game model with bankruptcy theory

The classical bankruptcy theory assumes that different agents have the same access to homogeneous resources, suitable for one-time allocation problem. Therefore, they may not be suitable for water resource allocation, which is temporal and spatial differences in accessibility. Due to the change of water flow with time and space, especially in a river system with multiple tributaries, the accessibility of water at a specific time or location may be limited. Therefore, the classical bankruptcy theory may produce an infeasible river system allocation scheme.

In this study, the cooperative game method is used to solve the Nash equilibrium of water resources allocation in the Tarim River Basin by introducing the AP rule into the cooperative game model. The model simplifies the physical characteristics of the natural river system (shape, number of tributaries and number of water users) and can also cover the water diversion or water conveyance infrastructure, which has a strong ability to adapt to complex water resource allocation problems.

The generalized Nash equilibrium refers to the equilibrium solution obtained by players after many games. Each player has two strategies: cooperation or non-cooperation, which needs to be constructed through the minimum utility, utility function and other elements, to seek the Pareto optimality of collective utility and individual fairness in the cooperative game. This model uses the bankruptcy theory to obtain each region's minimum utility - the minimum game water right. When the water supply is less than the minimum utility point, the players refuse to cooperate; when the water supply is greater than the minimum utility point, they choose to cooperate.

The cooperative game model maximizes the utility of water resources composed of five regions through the Nash product. The minimum utility point obtained by ruin theory provides a mathematical basis for the subsequent solution. The utility maximization of water resource in each region can be understood as the smallest difference between the actual allocation result of water resource and its declared water demand, and the largest difference to its minimum utility point. Therefore, the utility function of each region is defined as a linear interpolation function (Wang et al., 2013, and the specific expressions are as follows:

\[
 f_i(x_i) = \frac{x_i - v_i}{c_i - v_i}
\]

8

\[
 d_i = f_i(v_i)
\]

9

where \(F_i(x_i)\) is the annual utility function of water use in the region \(i\), and \(d_i\) is the annual minimum utility of the region \(i\).
Based on bankruptcy theory and the utility functions, the Nash equilibrium in the cooperative game is solved (Harsanyi, 1961):

\[ \text{Max} \{ Z = \prod_{i=1}^{m} (f_i(x_i) - d_i) w_i \} \]

\[ \sum_{i=1}^{m} w_i = 1 \]

In Eq. (10), Z is the Nash product of the total annual water use utility of the whole region of water resource allocation. In Eq. (11) \( w_i \) is the weight of the region i's game ability.

Also, the model is subject to conventional water resource allocation constraints, including guaranteed water use constraints and water consumption constraints:

\[ v_i \leq x_i \leq c_i \]

\[ \sum_{i=1}^{m} x_i = E \]

### 3.3 Model comparison

To compare the performance of the classical bankruptcy theory model and the improved model in the actual allocation of water resource, this paper brings the declared water demand data into the AP model (Model 1) and the cooperative game model (Model 2) to solve the problem. By comparing the model allocation results with the actual allocation data in the same year (Eq. (14)), the two model allocation results' average deviation degree relative to the actual allocation data is obtained, as shown in Fig. 2. Generally, in the series of data from 2015 to 2020, the average deviation degree of Model 2 is less than that of Model 1.

\[ d = \left| \frac{x_i - t_i}{t_i} \right| \times 100\% \]

To further compare the performance of the two models in the historical data of water resource allocation in the Tarim River Basin, the t-test (Fan et al., 2012) is conducted for the average deviation degree.
between the allocation results of the two models relative to the actual allocation data from 2015 to 2020. The t-test uses t-distribution theory to infer the probability of difference so as to compare whether the difference between two means is significant. The test statistic $t$ is calculated as follows:

$$
t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}
$$

Where $\bar{X}_1$ and $\bar{X}_2$ are the samples' mean, $S_1^2$ and $S_2^2$ are the samples' variance, $n_1$ and $n_2$ are the samples' size.

T-test results are shown in Table 2. At the significance level of 1%, the average deviation degree of model 2 is less than that of model 1. That is to say, there is more than 99% possibility to reject the hypothesis that the average deviation degrees of model 1 and model 2 is the same. Therefore, from the perspective of the whole basin, the application of model 2 is better than that of model 1.

|                   | Model 1 | Model 2 | difference | $t$     |
|-------------------|---------|---------|------------|---------|
| T-test result     | 0.069258| 0.066054| 0.003004***| (3.954) |

Notes: * $p<0.1$, ** $p<0.05$, *** $p<0.01$

3.4 Game weight

In the cooperative game, the bargaining power $w_i$ of each region also affects the final result. Because the bargaining power involves risk aversion, fairness, externality and other factors, it is not easy to analyze quantitatively at present. Based on the actual situation of water use in the Tarim River Basin, this paper adopts three kinds of game weight distribution schemes under the ideal state.

(1) Equal weight scheme: From the perspective of regional equity, it fully recognizes the equal status of each game subject in water use so that the game weights of all regions are equal.

(2) Water demand weight scheme: From the perspective of regional actual water demand, it assumes that the greater the water demand weight of the region, the greater the game weight, and the model will increase the allocation of water resources in the region as much as possible. Because its political economy is more developed than others, and its water use status is relatively high, the water consumption needs to be appropriately tilted.

(3) CRITIC weight scheme (Yin et al., 2017): From the perspective of regional historical water consumption, the weight is determined according to the variability and conflict of water consumption in a
long series. Variability refers to the value difference under the same factor, which is reflected by the standard deviation. The larger the standard deviation is, the larger the gap is, and it also shows that the amount of information reflected by the sample is also larger; the correlation coefficient reflects conflict, whose size, positive and negative determine the weight component. The more significant the negative correlation between water consumption in different regions, the greater the conflict, indicating that the more pronounced the information difference of water consumption in different regions, the greater the game weight, and vice versa. The specific solving steps are as follows.

a) Standardize water consumption:

\[
x^*_ij = \frac{x_{ij} - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}
\]

b) Solve the coefficient of variation:

\[
v_j = \frac{s_j}{x_j}, \ j = 1, 2, \cdots, n
\]

where \( \overline{x}_j \) and \( s_j \) represent the average and mean square deviation of historical water consumption in the region \( i \) respectively.

c) Solve the independence coefficient:

\[
\eta_j = \sum_{k=1}^{n} (1 - r_{kj}), \ j = 1, 2, \cdots, n
\]

where \( r_{kj} \) is the correlation coefficient of historical water consumption of the region \( k \) and the region \( i \).

d) Calculate of comprehensive coefficient:

\[
C_j = v_j \eta_j, \ j = 1, 2, \cdots, n
\]

e) Calculate CRITIC weight:
To sum up, this paper uses the water demand data from 2015 to 2030 in “General Control Scheme of Water Use in Xinjiang” to set the weights as follows (Table 3).

Table 3

Weight setting.

| Regions      | Bayingolin | Aksu | Kizilsu Kirgiz | Kashgar | Hotan |
|--------------|------------|------|----------------|---------|-------|
| 2025         | Equal weight | 0.200 | 0.200          | 0.200   | 0.200 |

\[
w_j = C_j / \sum_{j=1}^{n} C_j
\]

4 Stability Analysis Of Different Scenarios In The Future

4.1 Scenario setting

The "General Control Scheme of Water Use in Xinjiang" proposes a phased water consumption control plan, which requires that Xinjiang's total water consumption should be controlled within 52.674 billion m³ by 2030. According to the plan, this paper sets six scenarios composed of two years' expected water supply scenarios with three different game weight schemes (Table 4), taking the control indicators of 2025 and 2030 as the declared water demand.

Table 4

Scenario setting.

| Water demand weight | 0.157 | 0.330 | 0.036 | 0.339 | 0.138 |
| CRITIC weight       | 0.104 | 0.421 | 0.178 | 0.173 | 0.124 |
| Equal weight        | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| 2030 Water demand weight | 0.162 | 0.324 | 0.037 | 0.333 | 0.144 |
| CRITIC weight       | 0.104 | 0.421 | 0.178 | 0.173 | 0.124 |
In this study, LINGO (linear interactive and general optimizer) is used to compile the model method, and the neural network algorithm is used to solve the water resource allocation results of the Tarim River Basin under different scenarios. The input data comes from the "General Control Scheme of Water Use in Xinjiang".

### 4.2 Allocation results

Based on the model comparison in Sect. 3.3, this paper further analyzes water resource allocation results under Model 2. Firstly, the allocation proportions of water resource in different scenarios of the Tarim River Basin are obtained (Table 5).

| Regions       | Scenario 1a | Scenario 1b | Scenario 1c | Scenario 2a | Scenario 2b | Scenario 2c |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Bayingolin    | 14.69%      | 13.52%      | 12.15%      | 15.23%      | 14.10%      | 12.68%      |
| Aksu          | 33.90%      | 36.68%      | 36.68%      | 33.25%      | 36.04%      | 36.04%      |
| Kizilsu Kirgiz| 3.98%       | 1.63%       | 3.98%       | 4.15%       | 1.62%       | 4.15%       |
| Kashgar       | 34.84%      | 37.61%      | 36.09%      | 34.18%      | 36.98%      | 35.42%      |
| Hotan         | 12.59%      | 10.55%      | 11.10%      | 13.20%      | 11.25%      | 11.71%      |

According to the allocation results of water resources, the water proportion obtained in Tarim River Basin regions is different under different scenarios. However, the degree of change is relatively small, and the overall water allocation is relatively stable. Taking Kashgar with the largest water demand as an example, the water allocation proportion of the water demand weight scheme is greater than that of the equal weight scheme, which indicates that the model will tilt more water to the regions with larger water demand when considering the water demand weight. Taking Aksu with the largest weight of variability and conflict as an example, the water allocation proportion of the CRITIC weight scheme is greater than that of equal weight scheme, which shows that the model will tilt more water to the regions with greater variability and conflict when considering the variability and conflict of regional water use. Both scenario 1 and scenario 2 have added water consumption constraints, while in scenario 2, with the relatively small...
water supply, the proportion of water obtained in Kashgar is significantly less than that in scenario 1, which indicates that the game model will reduce the water supply in regions with large water consumption in the case of water shortage when balancing efficiency and fairness, so that the water quantity obtained by other regions can be better guaranteed.

The results of water demand satisfaction of each region are shown in Fig. 3. It can be seen from Fig. 3 that under different water supply scenarios, the water satisfaction degree of most areas can reach more than 60%. However, to achieve the future water saving demand in "General Control Scheme of Water Use in Xinjiang", there is still significant pressure on the water resources of the Tarim River. Besides, the overall satisfaction of scenario 1 and scenario 2 are similar, which indicates that the model can adapt to different hydrological conditions in the case of water saving in the future. To further explore the law of water use, in addition to analyzing the results of water allocation, it is also necessary to evaluate the stability of cooperation after the output of the model.

4.3 Stability evaluation

In the conflict problem, the possibility that the solution is accepted by the main body of the conflict and remains stable is very important. The conflict solution's stability ensures the effective implementation of the conflict solution, and the more stable conflict solution is easier to be accepted and implemented by all parties. This study's allocation model maximizes the efficiency of water resource utilization based on the cooperative game. According to different definitions of fairness, different weights will be set. In negotiations with multiple-decision makers, parties who base decisions on individual rationality may find the social planner solution unfair, because they think they can get more water resources under another allocation scheme (Madani, 2011). Therefore, after obtaining the results, we need to evaluate the allocation results' equilibrium stability (Read et al., 2014) and then evaluate the allocation results' realizability and game law.

This paper adopts the bankruptcy allocation stability index (BASI) (Madani et al., 2014) to compare the allocation results under different scenarios from the cooperative game's stability evaluation. The specific expressions are as follows:

\[
BPI_i = \frac{x_i - v_i}{\sum_{j \in M} (x_j - v_j)}
\]

\[
BASI = \frac{\sigma_{BPI}}{BPI}
\]
Figure 4 shows the stability index of the allocation results of different scenarios. The smaller the value is, the more balanced and stable the cooperative game system is. Overall, each scenario's allocation stability index is about 0.34, which can be considered that the inter-regional water resources game under this model has reached a certain cooperative system equilibrium (Madani et al., 2014).

From the perspective of the water supply situation, scenario 2 with relatively less water supply is more stable than scenario 1, which shows that after the implementation of the water-saving plan, under the condition of water supply reduction, the cooperation of water resources in various regions of Tarim River Basin is more stable according to the model in this paper.

From the perspective of the game weight scheme, the cooperation system of scenario a with equal weight is more stable than that of scenario B with water demand weight and scenario C with CRITIC weight. It shows that the cooperative game model treats all regions' water consumption equally and ignores the relative size, conflict, and variability of their declared water consumption in the basin system, minimising the hydraulic hegemony (Zeitoun and Allan, 2008) in the system and making the system more stable. The water allocation scheme for each region is relatively stable. Many regions with a small proportion of water demand are more willing to participate in the overall cooperation of water resources, which is also more in line with water users' game law in the actual management of water resources.

5 Discussion

Water resources have externality and non-competitiveness, so it is necessary to analyze the harmony between water resources allocation results and the local economy and environment. As shown in Table 4, this paper uses the TOPSIS model with entropy method (Gao et al., 2021), using five indexes in "General Control Scheme of Water Use in Xinjiang" and water resource satisfaction of allocation results of the cooperative game model, to construct three indexes to evaluate water resource endowment, economic development and environmental friendliness respectively in each region (Table 6). Then use the TOPSIS evaluation method to construct the harmony index of water-economy-environment (HWEE) to analyze the coordination degree of the water resources allocation results in this paper with each region's economy and environment.

Table 6

Integrated indicator system of HWEE.
| Primary indicators                  | Secondary indicators                                                                 |
|------------------------------------|----------------------------------------------------------------------------------------|
| Water resource endowment           | Water resource satisfaction                                                          |
|                                    | Water utilization coefficient of agricultural irrigation                                |
| Economic development               | Decline rate of water consumption per 10000 yuan of industrial added value             |
|                                    | Decrease rate of water consumption per 10000 yuan GDP                                   |
| Environmental friendliness         | Reduced irrigation area                                                                 |
|                                    | Development area of high efficiency water saving irrigation                            |

### 5.1 Entropy Method

The main steps of the entropy method are as follows:

Firstly, the information entropy of index $j$ is $e_j$:

$$
e_j = -\frac{1}{\ln(n)} \sum_{i=1}^{n} (x_{ij} \ln x_{ij})$$

where $x_{ij}$ is the original evaluation matrix value after normalization and standardization.

Secondly, the information utility value of index $j$ is $g_j$:

$$
g_j = 1 - e_j$$

Finally, calculate the information weight $w_j$ of the index $j$ and the weight set $w$:

$$
w_j = \frac{g_j}{\sum_{j=1}^{n} g_j}$$

$$
w = [w_1, w_2, \ldots, w_n]$$  \hspace{1cm} (26)

### 5.2 Improved TOPSIS model
The main steps of TOPSIS improved by entropy method are as follows:

Firstly, calculate the weighted normalized decision matrix $Z$:

$$Z = (z_{ij})_{m,n}, \quad z_{ij} = w_j \times x_{ij} \quad (i = 1, K, m; \quad j = 1, K, n) \quad (27)$$

Secondly, determine the ideal optimal state $Z^+$ and the worst state $Z^-$:

$$Z^+ = (z_{1}^+, z_{2}^+, K, z_{j}^+, K, z_{n}^+) \quad, \quad z_{j}^+ = \max_{i} (x_{ij}) \quad (28)$$

$$Z^- = (z_{1}^-, z_{2}^-, K, z_{j}^-, K, z_{n}^-) \quad, \quad z_{j}^- = \min_{i} (x_{ij}) \quad (29)$$

Thirdly, calculate the Euclidean distance between the evaluation object $i$ and the optimal state ($d_i^+$) and the worst state ($d_i^-$) respectively:

$$d_i^+ = \sqrt[n]{\sum_{j=1}^{n} (z_{ij} - z_{j}^+)^2}$$

Finally, calculate the evaluation score $S_i$ of the evaluation object $i$:

$$S_i = \frac{d_i^-}{d_i^+ + d_i^-}$$

### 5.3 HWEE analysis

In this paper, three secondary evaluation indexes are considered to calculate HWEE, and the results of water resources allocation in scenario 1a are analyzed with the harmony degree of local economy and environment, as shown in Fig. 5. There are significant differences in HWEE scores among different regions. Aksu and Kashgar regions have higher scores, indicating that the results of water resources allocation are in good harmony with the local economy and environment. On the one hand, Aksu and Kashgar get more water resources in this allocation model; on the other hand, in the improved TOPSIS
model, these two regions get a better evaluation of economic development and environmental friendliness. It is mainly due to decline rate of water consumption per 10000 yuan of industrial added value, decline rate of water consumption per 10000 yuan GDP, reduced irrigation area, and development area of high efficiency water saving irrigation. Therefore, under the comprehensive effects of little gap in water resources allocation, high water use efficiency, economic and social development, ecological and environmental protection, the water resources, economy, and environment of Aksu and Kashgar are in good harmony.

In general, this paper’s cooperative game model provides a new idea for the water resource allocation of the Tarim River Basin in 2030, which has certain feasibility. Under the different scenarios of balancing efficiency and fairness, the model has different emphasis on the water supply of different water demand proportion regions, resulting in the difference in cooperation stability. In the future research, more scenarios can be considered, such as annual hydrological changes, water demand changes, socio-economic changes, and the actual supply capacity constraints of water supply plants can also be added. It can then be integrated into the policy recommendations for the unified management and scheduling of water resources in the Tarim River Basin, which has significant reference value for the practice of water resources management in inland arid areas.

6 Conclusion

This paper introduces the AP rule in bankruptcy theory to construct each region's utility function and establishes a cooperative game model of water resources allocation across regions. By analyzing the water consumption forecast data of various regions in 2025 and 2030 in the "General Control Scheme of Water Use in Xinjiang", this article provides a plan for allocating water resources in the Tarim River Basin under water-saving conditions in the future. Conclusion are drawn as below:

Firstly, compared with the AP criterion of the classical bankruptcy theory, applying the cooperative game model with the bankruptcy theory in the allocation of water resources has less error than the actual allocation results from 2015 to 2020.

Secondly, in the case of water conservation in the future, the model in this paper can adapt to different water supply scenarios and hydrological conditions, and the water supply satisfaction degree in most regions can reach more than 60%.

Thirdly, this paper uses the BASI to assess the stability of different allocation schemes between regions, hoping to maximize the Tarim River Basin's overall welfare. The improved TOPSIS model is used to construct HWEE indicators to evaluate the degree of coordination between the results of water resources allocation and the economy and environment of different regions. The results show that the same game weight has better stability. And the water resources, economy, and environment of Aksu and Kashgar are in good harmony.

In addition to providing solutions to cross-regional water resource allocation, the model in this paper is also applicable to other resource allocation problems with temporal and spatial availability constraints.
Examples of such problems include aid relief during and after disasters and the distribution of utilities (natural gas, electricity, water) during supply shocks. Besides, if the data permits, the time step can be further reduced to optimize the model, and the sustainability analysis of the cooperation can be introduced to further consider the robustness of the cooperation.

**Declarations**

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**Author contribution**

**Jiahe Tian**: Conceptualization, Methodology, Software, Formal analysis, Writing - Original Draft

**Yang Yu**: Conceptualization, Methodology

**Tongshu Li**: Investigation, Data Curation, Writing - Review & Editing

**Yi Zhou**: Investigation, Data Curation

**Jingjun Li**: Visualization

**Xingpeng Wang**: Resources

**Yu Han**: Conceptualization, Validation, Writing - Review & Editing, Supervision, Funding acquisition

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**Declarations**

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**Competing interests** The authors declare no competing interests

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**Figures**
Figure 1

Tarim River basin system's general schematic diagram. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.
**Figure 2**

Comparison of average deviation degree between two models.

![Comparison of average deviation degree between two models.](image)

**Figure 3**

Water demand satisfaction of each region. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

![Water demand satisfaction of each region.](image)

**Figure 4**

BASI of the allocation results in different scenarios.

![BASI of the allocation results in different scenarios.](image)
Figure 5

HWEE calculation results of five regions.