Chapter 5

Universality class of the nonequilibrium phase transition in two dimensional Ising ferromagnet driven by propagating magnetic field wave.
5.1 Introduction:

In the previous chapters, I have discussed about dynamical phase transition in different kinds of kinetic Ising ferromagnet driven by magnetic waves (mainly propagating and standing magnetic waves) and tried to emphasize on their various dynamical states, dependence of transition temperature on various parameters of magnetic waves and system parameters (such as strength of anisotropy $D$, in the case of BC model, magnitude $S$ of general spin-$S$ Ising model) for a fixed lattice size or dimension. However, in these studies the detailed finite size analyses were not yet done to know the universality class of these nonequilibrium phase transitions observed in Ising ferromagnet driven by magnetic field wave. Finite size analyses and critical exponents of ferromagnetic transitions in the Ising model were studied \[12\]-\[15\] under various occasions, but the effect of spatio-temporal variation of magnetic field was not considered in the earlier studies. Now I will address this issue in this chapter. In the following sections I will discuss on the nonequilibrium behavior, the finite size analysis and the critical aspects of the dynamic phase transition near the dynamic transition temperature in a square type spin-$\frac{1}{2}$ Ising ferromagnet driven by propagating magnetic wave. The journal reference of these studies is as follows:

Ajay Halder and Muktish Acharyya Applied Mathematics 10 (2019) 568-577.

5.2 Dynamical Phase Transition in spin-$\frac{1}{2}$ Ising ferromagnet driven by propagating magnetic waves:

Dynamical phase transition (DPT) in an Ising spin-$\frac{1}{2}$ ferromagnet under the influence of propagating magnetic field wave has been discussed in details in the section 2.2.1. The magnetic perturbation in the form of propagating magnetic wave keeps the Ising ferromagnet away from equilibrium. Hence, the phase transition discussed earlier is a nonequilibrium phase transition. Depending on the strength of the magnetic field wave and the temperature of the system, two dynamical phases have been observed to form. These phases are quite different from the well known equilibrium ferro-para transition. Naturally, we may ask, whether this kind of phase transition belongs to any known universality class. If so, which universality class does such dynamical phase transition
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

belong to? I will address these questions here.

5.2.1 Response to propagating magnetic wave:

We write the time dependent Hamiltonian of a two dimensional Ising spin-$\frac{1}{2}$ ferromagnet having uniform nearest neighbor interaction, in presence of a magnetic field wave as-

$$
H(t) = -J\sum_{\prime} s^z(x, y, t) s^z(x', y', t) - \Sigma h^z(x, y, t) s^z(x, y, t),
$$

(5.1)

where, $s^z(x, y, t)$ represents the $z$ component of the spin. It takes any of the two values +1 or −1. The first term in the Hamiltonian represents the spin-spin cooperative energy, whereas the second term is the spin-magnetic field interaction energy. The cooperative interaction strength $J$ is considered to be uniform throughout the system. "\(\Sigma'\)" indicates that summation is extended up to nearest neighbors only. The magnetic field $h^z(x, y, t)$ at site $(x, y)$ at time $t$, has the following form of propagating wave:

$$
h^z(x, y, t) = h_0 \cos(2\pi ft - 2\pi \frac{x}{\lambda}).
$$

Here, $h_0$, $f$ and $\lambda$ are the field amplitude, the frequency of magnetic field oscillation and the wavelength of the propagating magnetic field wave respectively.

Monte-Carlo simulation:

Consider square lattices of different sizes $L \times L$ over which 2-state (spin-$\frac{1}{2}$) Ising spins are arranged. Periodic boundary conditions, applied to both directions preserve the translational invariance in lattice. The dynamics of spins are imposed by the standard Monte-Carlo method. At high temperatures these spins are uniformly distributed (with equal probability) over the two states $s^z = \pm 1$. Starting from this initial (paramagnetic) configuration the system is now cooled down slowly in small steps of temperature and any dynamical steady state is thus reached. Various dynamical quantities are measured in nonequilibrium steady state, which is achieved by keeping the system in a particular heat bath of steady temperature for a sufficiently long time. The spins are updated at the metropolis rate given by equation 2.3:

$$
W(s^z_i \rightarrow s^z_f) = \text{Min}[\exp(-\Delta E/kT), 1].
$$

Here $s^z_i$ and $s^z_f$ represent the initial and final values of any spin state at a site $(x, y)$, before and after updating, respectively. $\Delta E$ represents the change in energy due to spin
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

flip from initial state to final state; \( k \) is the Boltzmann constant. The field amplitude \( h_0 \) is measured in the unit of \( J \) whereas, the temperature \( T \) is measured in the unit of \( \frac{J}{k} \). The unit of time \( MCSS \) is the time required to update \( L^2 \) number of spins once in an \( L \times L \) square lattice.

Finite-size analysis and scaling hypothesis:

To understand the scaling behavior of an Ising \((S = \frac{1}{2})\) ferromagnet driven by propagating magnetic field wave, the method of finite size scaling analysis is adopted. From this analysis the values of critical exponents for the dynamic phase transition may be determined to know the universality class. The usual method is to express the measured dynamical quantities, describing the dynamical phase transition, as a function of the system size \( L \). Normally, in any ferromagnetic system, order parameter \( Q \) and susceptibility \( \chi^Q \) play the important role in describing the phase transition. Hence, we may assume the following scaling forms for the order parameter \( Q \) and susceptibility \( \chi^Q \) at the critical temperature:

\[
\langle Q \rangle_L \propto L^{-\beta/\nu} \quad (5.2)
\]

\[
\chi^Q_L \propto L^{\gamma/\nu}. \quad (5.3)
\]

Though the above scaling forms are used to determine the critical exponents for ferro-para equilibrium phase transition [4], the detailed investigations ([5]) carried out previously, suggest that these forms are also applicable to classify the universality classes of the driven magnetic systems, exhibiting nonequilibrium phase transition.

Results:

Monte-Carlo (MC) technique with parallel updating rule [3] is employed to simulate the dynamics of a square Ising ferromagnet (of size \( L \times L \)) through which a linearly polarized propagating wave is passing. In steady state two different phases, namely: high temperature symmetric phase and low temperature symmetry-broken phase, have been identified, depending upon the temperature \((T)\) and magnetic field amplitude \((h_0)\). At high temperature the ferromagnetic spins are symmetrically distributed over its two possible states \( s^z = \pm 1 \). Because of higher thermal agitation at higher temperatures, the spins are easily flipped by the externally applied magnetic perturbation and alternate
bands of spins are observed to move coherently along with the propagating magnetic wave. Hence, this phase has been called as the \textit{propagating} phase. But, at temperatures below the transition temperature, the mutual spin-spin interaction holds the spins together along a particular direction giving rise to the \textit{pinned} phase. The order parameter for such a transition is defined, as the time average of magnetization per lattice site over a full period of magnetic field oscillation of propagating magnetic wave, given by the \textit{equation}:

\[
Q = f \times \frac{1}{T} \sum_{x,y,t} \langle s_i^z(x, y, t) \rangle_{t}
\]

where \( s_i^z \) is the value of magnetization per site at a particular time. For an asymmetric distribution of spins in the pinned phase, the value of magnetization per site \( M(t) \) and hence the value of order parameter \( Q \) is large, whereas due to symmetric distribution of spins in the propagating phase \( M(t) \) and \( Q \) have small (nearly zero) values. As the system is cooled down below the transition temperature it undergoes dynamical phase transition continuously from the propagating phase to the pinned phase. Lattice morphologies of the two dynamical phases and the variation of magnetization per site \( M(t) \) with time in both the phases are shown in the figure 5.1.

The dynamical transition temperature is determined from the peaks in the temperature variation of the \textit{dynamic susceptibility} \( \chi_L^Q \), which is defined as:

\[
\chi_L^Q = L^2 \chi^Q = L^2 \left[ \langle Q^2 \rangle - \langle Q \rangle^2 \right]
\]

\( \chi^Q \) is the variance of the order parameter \( Q \). It is known from the behavior of the kinetic Ising model that the scaled variance \( \chi^Q \) of the dynamical order parameter may be regarded as the ”dynamic susceptibility” of the system. The correlation between the spins in any dynamical phase may be understood from the finite size analysis of the phase transition near the transition temperature. It is observed that the correlation grows with the increase in lattice size. The correct scaling form is found from the precise values for different dynamical quantities obtained at the transition temperature. The dynamical transition temperature may be found, more precisely, from the common point of intersection in the temperature variation of the fourth order Binder Cumulant \( U_L(T) \) [[9]-[11]] for different lattice sizes \( L \). \( U_L(T) \) is defined by the following \textit{equation} as:

\[
U_L(T) = 1 - \frac{\langle Q^4 \rangle_L}{3\langle Q^2 \rangle_L^2}
\]

\textit{Figures 5.2} show respectively the temperature variation of \( Q, U_L \) and \( \chi_L^Q \) for four
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

Figure 5.1: (Color online) fig.5.1(a) & fig.5.1(b) show the lattice morphology of the pinned and propagating phase respectively at time $t = 39937$ MCSS for $L = 64$. fig.5.1(c) & fig.5.1(d) show the dynamical symmetry breaking (change in the value of magnetization per site from non-zero to nearly zero value). fig.5.1(a) & fig.5.1(c) are at temperature $T = 1.8$ whereas fig.5.1(b) & fig.5.1(d) are at temperature $T = 2.5$.

different lattice sizes ($L = 16$, 32, 64 and 128). The values of $Q$, $U_L$ and $\chi^Q_L$, used in these graphs are the steady state values of the respective dynamical variables. These typical data set are obtained from the square lattices of four different sizes $L$. A propagating wave of wavelength $\lambda = 16 lu$, frequency $f = 0.01$ (MCSS)$^{-1}$ and amplitude $h_0 = 0.3$ is allowed to pass through the lattices. Hence, 100 MCSS time is required for one complete cycle of magnetic field oscillation. Any dynamical state corresponding to each lattice size $L$ is reached by cooling the system down from high temperature, in small steps of temperature ($\Delta T = 0.005 \frac{\mu B}{K}$). To reach the dynamical steady state the system is kept at any heat bath of fixed temperature for a sufficiently long time; 12000 (for $L = 128$) to 32000 (for $L = 16$) cycles of magnetic oscillations. Data for the initial (or transient) 1000 cycles are discarded and then average of dynamical quantities are calculated over
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

![Graphs showing temperature variation of different quantities for different values of linear system size $L$.](image)

Figure 5.2: (Color online) Temperature variation of different quantities for different values of linear system size $L$: (a) Order parameter $Q$, (b) Binder cumulant $U_L$ and (c) scaled variance of order parameter $\text{var}_Q$ or susceptibility $\chi^Q_L$.

the remaining cycles. It has been verified that the system has reached the dynamical steady state during this 1000 cycles or 100000 $MCSS$ time interval. It is observed here in the figure 5.2 that the order parameter $Q$ increases steadily and continuously and takes nonzero values below the transition temperature. This indicates the dynamic
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-\(\frac{1}{2}\) ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

Phase transition between the symmetric phase and the symmetry-broken phase in a spin-\(\frac{1}{2}\) Ising lattice driven by propagating magnetic field wave. The precise value of dynamical transition temperature, corresponding to this typical set of \(\lambda, f\) and \(h_0\) of the propagating wave, is \(T_d = 2.011 J/k\). This value is obtained from the common intersection point in the graphs of \(U_L\) vs \(T\) for different lattice size \(L\). This is a Standard technique to determine the transition temperature and is widely used in the cases of equilibrium phase transition [9, 1, 2]. The peaks in the \(\chi^Q_L\) vs \(T\) curves near the transition temperature indicate the tendency of divergence near \(T_d\) as \(L\) increases. These higher peaks for greater lattice size \(L\) show that the correlation grows with the lattice size or dimension.

From the scaling ansatz of dynamic order parameter \((Q_L \sim L^{-\beta/\nu})\) (Eqn.5.2) and the dynamic susceptibility \((\chi^Q_L \sim L^{7/4})\) (Eqn.5.3), the values of critical exponents may be estimated. The slopes of the straight line curves of \(Q\) vs \(L\) or \(\chi^Q_L\) vs \(L\), drawn in double logarithmic scale, give estimates of the critical exponents [6, 5, 7]. Figures 5.3 show the log-log plot of dynamic order parameter \(Q\) and dynamic susceptibility (or scaled variance) \(\chi^Q_L\) as the system size \(L\) respectively. These values of \(Q\) and \(\chi^Q_L\) are obtained at the critical temperature for different \(L\). It may be noted that since the critical temperature is found from the \(U_L\) vs \(T\) curves for different \(L\), the values of \(Q\) and \(\chi^Q_L\) have been read out from their respective temperature variations, at this particular critical temperature. Hence, these values are not the simulated values, rather, they represent the average values at that temperature. The values of the critical exponents found from these Monte-Carlo studies are \(\beta/\nu = 0.146 \pm 0.025\) and \(\gamma/\nu = 1.869 \pm 0.135\) (measured from the data read at the critical temperature obtained from Binder cumulant), and \(\gamma/\nu = 1.746 \pm 0.017\) (measured from the peak positions of dynamic susceptibility) respectively for \(Q\) and \(\chi^Q_L\).

We may compare the estimated values of critical exponents with those of the two dimensional Ising ferromagnet showing equilibrium ferro-para phase transition. The values, reported from Onsager exact solution [26], are respectively \(\beta/\nu = 1/8\) and \(\gamma/\nu = 7/4\). Thus, these findings suggest that the estimated values of the critical exponents, near the dynamic transition temperature, are very close, within the limits of statistical errors, to those for the two dimensional equilibrium Ising ferromagnet. It may be concluded that such a kind of nonequilibrium phase transition in a 2D- Ising ferromagnetic system driven by propagating magnetic wave belongs to the same universality class of the two-
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

Dimensional equilibrium Ising ferro-para phase transition. Recently, Manoj Kumar and Chandan Dasgupta (IISc, Bangalore) has studied the nonequilibrium phase transition in the kinetic Ising model. In this study they have followed the violation of principle of detailed balance. They also estimated the value of critical exponents in close agreement with the present observations. It is worthy here to mention that recently, similar study of universality class has been done also in the BC model and triangular lattice under an oscillating magnetic field [5, 6].

Figure 5.3: (Color online) Log-log plot of (a) order parameter $Q$ and (b) scaled variance $\chi_Q$ or susceptibility $\chi_Q$ as a function of linear system size $L$. In (b) red dots represent the value of susceptibility at $T_d$ whereas blue triangles represent the same at peak positions.
5.2. DYNAMICAL PHASE TRANSITION IN SPIN-$\frac{1}{2}$ ISING FERROMAGNET DRIVEN BY PROPAGATING MAGNETIC WAVES:

Bibliography:

1. S. W. Sides, P. A. Rikvold, M. A. Novotny, Phys. Rev. Lett. 81 (1998) 834.
2. S. W. Sides, P. A. Rikvold, M. A. Novotny, Phys. Rev. E 59 (1999) 2710.
3. K. Binder and D. W. Heermann, Monte-Carlo Simulation in Statistical Physics, Springer Series in Solid State Sciences, Springer, New York, 1997.
4. K. Huang, Onsager solution (Chapter 15), Statistical Mechanics, Second Edition, John Wiley & sons Inc, Wiley India edition 2010.
5. E. Vatansever and N. Fytas, Phys. Rev. E. 97 (2018) 012122.
6. E. Vatansever, arXiv:1706.03351 [cond-mat.stat-mech].
7. E. Vatansever and N. Fytas, Phys. Rev. E. 97 (2018) 062146.
8. S. H. Tsai, S. R. Salinas, Braz. J. Phys., 28 (1998) 58.
9. K. Binder, Z. Phys. B: Condens. Matter, 43 (1981) 119; Phys. Rev. Lett., 47 (1981) 693.
10. W. Selke, Eur. Phys. J. B, 51 (2006) 223-228.
11. W. Selke and L.N. Shchur, Phys. Rev. E, 80 (2009) 042104.
12. A B Harris J. Phys. C: Solid State Phys, 7 (1974) 1671.
13. J. Villain Phys. Rev. Lett., 52 (1984) 1543.
14. K. G. Wilson and M. E. Fisher Phys. Rev. Lett. 28 (1972) 240.
15. D. V. Boulatov and V. A. Kazakov Physics Letters B 186 (1987) 379-384.