I. INTRODUCTION

Quantum state discrimination (QSD) is the problem of determining the state of a quantum system among a set of possible candidates. It constitutes a fundamental primitive in quantum information processing, with applications ranging from long-distance communication [1–9], cryptography [10–17], and, recently, quantum machine learning [18–26].

In the past few years, the use of machine learning methods to deepen the understanding of fundamental physics has become a standard technique [27–36]. Machine learning approaches have been traditionally classified into three categories: supervised learning, unsupervised learning, and reinforcement learning (RL). In particular, RL studies the behavior of an agent interacting with an environment via observations, actions, and rewards. The goal is to optimize such interactions in order to maximize a suitable figure of merit, e.g., the expected reward over time. Combinations of these three machine-learning classes have recently led to outperforming the best human GO player, discovering strategies never played before [37]. Recently, RL techniques have also been proved successful in quantum information, e.g., in the design of novel quantum experiments [32], quantum error-correction codes [33], quantum communication protocols [34], and optimal control of quantum systems [35,36].

In the present work we consider the discrimination of two coherent states with passive linear optics, photodetectors, and classical adaptive control. This is a prototypical problem in quantum information theory [1,38,39], of great technological significance for long-distance communication [4–8,40]: The optimal measurement to discriminate two coherent states is known [1,38] but its implementation is demanding at the state of the art, i.e., via the so-called Dolinar receiver [40–50] that requires asymptotically many control rounds. Moreover, its extension to multiple states is not fully understood [48,51,52], although it may bring us a step closer to achieving the Holevo communication capacity of real-world channels.

We propose an innovative and experimentally appealing approach to the problem: The search for optimal discrimination strategies is cast as a testbed for RL by studying how well an agent can perform in calibrating a receiver by means of model-free methods. The nature of our approach is particularly appealing for scenarios where an accurate description of the system is not possible, e.g., due to intrinsic complexity, experimental constraints or imperfections, untrusted devices, or simply lack of knowledge. This is precisely the case for applications of coherent-state discrimination in communication scenarios, where discriminating multiple hypotheses may require tuning long sequences of gate parameters [53–56], the detectors may be affected by losses and dark counts [43,52], the actual communication channel may add different kinds of noise depending on the physical implementation [4,5,57,58].
and device-independent security may be additionally required [13,17].

In this paper we show that a RL agent can achieve near-optimal control of a coherent-state receiver when it has zero prior knowledge of: (i) the energy of the coherent states themselves; (ii) the actual operations that the detector performs; (iii) the underlying quantum-mechanical laws governing the system. By trial and error, the RL agent has to sequentially press buttons and select actions according to previous measurement outcomes and at a final stage guess for one of the possible hypotheses. A nonzero reward is given only if the guess is correct. By repeating the procedure over several episodes (or runs), the agent earns experience and learns a near-optimal discrimination protocol and guessing rule with the resources at its disposal.

Our approach differs from previous applications of RL in quantum information [32–36] at least in three crucial aspects: (i) our agents can simultaneously learn and be tested in a completely model-free setting; (ii) each reward is obtained directly from a single-shot experiment and not indirectly inferred from a known model or from several runs of the experiment as in the case were the reward is, say, a target fidelity or a success probability; (iii) we will not only be concerned about finding near-optimal detectors but also, importantly, on the actual online success rate of the agents as measured by the cumulative reward, i.e., on the whole learning curve of the agent.

We tackle the problem in three stages of increasing complexity: First, in the model-aware setting, where the outcome probability function of the receiver is known, we find the optimal action sequence by solving the Bellman equation via dynamic programming [43,59]; second, in the model-free setting, where the receiver is completely unknown, we apply Watkins’ Q learning [60,61], a standard RL method whose update rule approximates the optimal Bellman equation; third, exploiting potential optimal strategies and exploring new ones, the model-aware setting, where the outcome probability function for a RL agent controlling the receiver. In Sec.III we describe the implementation of these methods and analyze their performance in terms of the cumulative reward. The bandit problem is introduced here as a basic framework to study, quantify, and optimize the real-time performance of agents over sequential learning strategies. We analyze and compare the performance of standard and bandit-inspired learning strategies in a variety of experimentally relevant settings. We conclude in Sec. V by mentioning possible extensions of our work.

II. PRELIMINARIES

We consider the discrimination of two electromagnetic signals with opposite phases, described by two coherent states of the field, |±α⟩, whose energy is proportional to |α|^2. When the energy of the signals approaches zero, i.e., |α|^2 ≪ 1, quantum effects become evident and it becomes impossible to discriminate between them perfectly.

Any binary discrimination protocol is described compactly by a quantum positive-operator-valued measurement (POVM), ℳ = {ℳ₀, ℳ₁} with ℳ₁,₂ ≥ 0 and ℳ₁ + ℳ₂ = 1. Defining the kth hypothesis as α^(k) = (−1)^kα, with prior probability p_k, the probability of obtaining outcome k given that hypothesis k was true is p(ˆk|α^(k)) = ⟨α^(k)|ℳ_k|α^(k)⟩ and the best guess is given by the most likely hypothesis given that outcome. Thus, the average success probability over all outcomes is given by

\[ P_s(α, ℳ) = \sum_{k=0,1} \max_{k=0,1} p(α^(k), ℳ_k) \]

\[ = \sum_{k=0,1} \max_{k=0,1} p(ˆk|α^(k))p_k. \] (1)

For nonorthogonal quantum states, this quantity is bounded below by 1 so-called Helstrom bound [1], which in our case reads

\[ P_s^{\text{hel}}(α) = \max_ℳ P_s(α, ℳ) = \frac{1}{2}(1 + \sqrt{1 - e^{-4|α|^2}}), \] (2)

where the optimization is carried out over all two-outcome POVMs; note that the Helstrom probability tends to 1/2 for |α| → 0, i.e., the states become indistinguishable at very low energies. The optimal Helstrom measurement that attains Eq. (2) is a difficult projection on a superposition of |±α⟩, i.e., a Schrödinger-cat-like state, which cannot be realized with simple linear-optical operations [44]. Quite surprisingly, Dolinar [41] showed that Eq. (2) can be asymptotically attained by continuous-time control of a displacement operator—his receiver has since then been extended to the discrete-time scenario by Takeoka et al. [44]. Nevertheless, the practical implementation of these receivers still proves demanding at present [46,51], due to various experimental limitations. Moreover, in a general communication scenario, the states will be transferred through a noisy channel and could be subject to various kinds of noise, e.g., loss, thermal noise, and phase diffusion [52,58].

Based on these premises, we aim to construct a model-free RL agent that, without any knowledge of the problem at hand nor of the receiver setup, learns to tune the receiver’s parameters in order to maximize its success probability. In this
way, when placed in a real-life situation, the agent will be able to train and optimize the receiver for the specific experimental conditions it encounters in real time. The receiver we consider comprises passive linear optics, photodetectors, and classical feed-forward, structured into successive processing layers \( \ell = 0, \ldots, L \), as depicted in Fig. 1; this receiver is known to attain the Helstrom probability in the limit \( L \to \infty \) [44]. For each layer \( \ell < L \), the following operations are applied:

1. The input signal \( |\alpha\rangle \) is split on a beamsplitter (BS) of transmissivity \( \theta \), effectively extracting a fraction \( 1 - \theta \) of the energy for detection. The BS transforms the input signal and vacuum states as

\[
|\alpha\rangle|0\rangle \mapsto |\alpha\sqrt{\theta}\rangle|\alpha\sqrt{1-\theta}\rangle_{\text{ref}},
\]

where the added phase of the second mode has been corrected via a proper phase shift, not shown in the figure.

2. The reflected part of the signal undergoes a displacement operation \( D(\beta) \), realizable via interference with a strong coherent signal on a small reflectivity BS, not shown in the figure. The resulting state is \( |\bar{\alpha}(\beta, \theta)\rangle = |\alpha\sqrt{1-\theta} + \beta\rangle \).

3. The displaced signal is measured via an on/off photodetector, which detects no photon, i.e., outcome \( o_{\ell+1} = 0 \), with conditional probability

\[
p(o_{\ell+1} = 0|\alpha, (\beta, \theta)) = |\langle 0|\bar{\alpha}(\beta, \theta)|\rangle|^2 = e^{-|\bar{\alpha}(\beta, \theta)|^2},
\]

and detects one or more photons, i.e., \( o_{\ell+1} = 1 \), with probability \( 1 - p(o_{\ell+1} = 0|\alpha, (\beta, \theta)) \).

4. The transmitted part of the signal enters layer \( \ell + 1 \).

Finally, the last processing layer \( \ell = L \) consists in elaborating a guess \( \hat{k} \) of the true hypothesis \( k \), based on previous measurement outcomes and parameter choices.

For an initial coherent state \( |\alpha\rangle \), the input state at the \( \ell \)th layer is \( |\alpha\rangle = |\alpha\sqrt{\theta_0} \ldots \sqrt{\theta_{\ell-1}}\rangle \). Since the experimenter can use all the past history \( h_t = (a_0, o_1, \ldots, a_{\ell-1}, o_{\ell}) \), with \( h_0 = \emptyset \), to decide the next value of \( (\beta, \theta) \) and the final guess, the total set of parameters over all possible histories is of exponential size in \( L \). We label them compactly as \( a_\ell(h_t) = (\beta_t, h_t) \) and \( a_t(h_t) = \hat{k} \), omitting the label \( \ell \) or the dependence on \( h_t \) when it is clear from the context. Hence, the average success probability of this strategy over all possible outcomes’ sequences \( o_{1:L} = (o_1, \ldots, o_L) \) can be written as

\[
P_\tau(\alpha, (a_L)) = \sum_{o_{1:L}} \prod_{t=1}^L p(o_t|a^{(k)}, a(h_{t-1})) p_k|k=\alpha(a_t)|,
\]

where \( \{o_t\} \) is the total set of actions over all histories and we have written the conditional probability of the sequence of outcomes \( o_{1:L} \) factors as a product of single-layer conditional probabilities (4).

In the model-aware setting, this expression can be optimized using dynamic programming, as we show in Sec. IV A, finding the set of optimal parameters \( \{a^*_\ell\} \) for any given \( \alpha \) and \( L \):

\[
\{a^*_\ell\} = \arg \max_{\{a\}} P_\tau(\alpha, (a_L)).
\]

As a shorthand we denote the optimal success probability (over the available actions) as

\[
P^*_\tau(\alpha) = \max_{\{a\}} P_\tau(\alpha, (a_L))
\]

and omit the label \( L \) when it is clear from the context. In the model-free setting instead, the agent has no knowledge of Eqs. (4) and (5), so it must resort to exploring the set of possible parameters and sample from the probability of (5) during several runs of the experiment to discover an optimal choice of parameters and guess rule by trial and error.

### III. SEQUENTIAL DECISION MAKING

The framework of RL is based on the interaction between an agent and an environment during several episodes [61]. At each time step \( t = 0, \ldots, L \) of each episode \( \tau = 1, \ldots, T \), the agent observes the environment in a state \( s^{(t)}_\ell \in S \) and chooses an action \( a^{(t)}_\ell \in A \); as a consequence, the agent enjoys a reward \( r^{(t)}_{\ell+1} \in R \) and observes a new state of the environment, \( s^{(t)}_{\ell+1} \in S \), where \( S, A, \) and \( R \) stand for the sets of states, actions, and rewards the agent may experience.

The environment is usually modeled to be Markovian: its dynamics is completely determined by the last time step via the transition function \( \tau(s', r|s, a) \), i.e., the conditional probability of ending up in a state \( s' \) and receiving a reward \( r \) given that the previous state was \( s \) and the agent took an action \( a \); the next future states accessible from \( s \) are thus restricted to \( S(s) = \{s' : \tau(s'|s, a) \neq 0\} \subseteq S \). The agent does not have control of nor access to the transition function, but it will influence the dynamics of the environment by choosing actions according to an interaction policy \( \pi(a|s) \), i.e., the conditional probability of performing an action \( a \) when the observed environment’s state is \( s \); hence the available actions at a given state may be restricted to a subset \( A(s) \subseteq A \). This setting is usually known as a Markov decision process (MDP).

Informally, the agent’s objective is to interact with the environment through an optimal policy \( \pi^* \), such that the total reward acquired during an episode is as high as possible. To achieve this goal, a value function is assigned to each state and optimized over all possible policies, as further explained below in Sec. III A.

The Markov assumption is justified whenever the agent’s observations provide a complete description of the state of the environment \( s_t \). However, in general this is not the case, and the agent has only access to partial observations \( o_t \in O \) at each time step. Such observations would not allow us to determine the dynamics even if \( r \) was known, and they are generated from the current state and the previous action. In RL literature this is called a partially-observable MDP (POMDP) and developing methods to solve it efficiently constitutes an active area of research [66–70]; usually, the problem is tackled by first reducing it to an effective MDP. The most...
straightforward approach is to define an effective state that contains all the past history of observations and actions up to a given time step, i.e., \( h_t = (a_0, o_1, \ldots, a_{t-1}, o_t) \). In this way, the dynamics observed by the agent can always be described by an effective MDP with transition function \( \tau(h', r|h, a) \), which is unknown to the agent and determined by the underlying environmental transition function. Clearly, this approach makes the problem intractable for large time steps, since the number of states increases exponentially in \( L \). In the model-aware setting, one can condense the history in a belief distribution over the states, \( b_o(s') = p(s'|o, a, b_o) \), i.e., the probability that the environment is in state \( s' \) given the current observation \( o' \), the previous action \( a \) and the belief at the previous time step \( b_o \). The belief has an initial value \( b(s) \) equal to the prior distribution over the initial states and at each time step it is updated using Bayes’ rule. In the following parts of this section we will introduce several tools for MDPs, which can be immediately adapted to POMDPs by exchanging the unknown state with the history \( h \) or the belief \( b_o(s) \).

A. Value functions and the Bellman equation

The agent’s objective is to acquire as much reward as possible during an episode. As a matter of fact, this strongly depends on the agent’s policy. At the end of episode \( t \), in which a sequence of \( L \) tuples \( \{ (s_t, a_t, r_{t+1}) \}_t=0 \) has been experienced (with \( s_{t+1} \) a terminal state and \( L \) generally varying among different episodes), the agent’s performance at each time step \( \ell \) can be evaluated using the so-called return,

\[
G^{(t)}_\ell = \sum_{j=0}^{L-\ell} \gamma^j r^{(t)}_{\ell+j+1},
\]

i.e., the weighted sum of rewards obtained at all future time steps, with a discount factor \( \gamma \in (0, 1] \), which weighs more the rewards that are closer in the future. Note that for infinite-horizon MDPs, i.e., \( L \to \infty \), it must hold \( \gamma < 1 \) to ensure that \( G_t \) remains finite.

By introducing the return, it is straightforward to assign a value to a state \( s \) for a given interaction policy \( \pi \), via the so-called value function:

\[
v_\pi(s) = E_\pi[G_t|s_t = s],
\]

which is the expected return over all possible trajectories that start from state \( s \), take actions according to policy \( \pi \), and whose dynamics is governed by \( \tau \). In other words, the value function measures how convenient it is to visit state \( s \) when policy \( \pi \) is being followed. Note that this quantity is completely determined by the future trajectories accessible from \( s \) and hence its dependence on the time step \( \ell \) can have at most the effect of restricting the set of states on which \( v_\pi(s) \) is supported at that time. We keep this dependence implicit unless otherwise stated. By writing explicitly the expected value for the first future time step in Eq. (9) and then applying the definition of \( v \) recursively, it is easy to show that the state-value function satisfies, for any policy, the following Bellman equation [59]:

\[
v_{\pi}(s) = \sum_{a \in A, s' \in S, r \in R} \tau(s', r|s, a)\pi(a|s)(r + \gamma v_{\pi}(s')).
\]

This equation relates the value of a state \( s \) with that of its nearest neighbors \( s' \), which can be reached with a single action from \( s \) and with the corresponding reward obtained by performing such action.

The problem can then be solved by finding an optimal policy \( \pi^* \), namely one that maximizes the state-value function for each \( s \) (also called optimal value function) and thus satisfies the optimal Bellman equation:

\[
v^*(s) := v_{\pi^*}(s) = \max_\pi v_\pi(s) = \max_{a \in A} \sum_{s' \in S, r \in R} \tau(s', r|s, a)(r + \gamma v^*(s')).
\]

Similarly, one can define the state-action value function (or \( Q \) function) as the expected return when starting from state \( s \) and performing action \( a \):

\[
Q_\pi(s, a) = E_\pi[G_t|s_t = s, a_t = a],
\]

which is related to the state-value function by \( v_\pi(s) = \sum_{a \in A} \pi(a|s)Q_\pi(s, a) \). Thus, the optimal policy \( \pi^* \) can also be obtained by maximizing the \( Q \) function, with a corresponding optimal Bellman equation

\[
Q^*(s, a) := Q_{\pi^*}(s, a) = \max_\pi Q_\pi(s, a) = \sum_{s' \in S, r \in R} \tau(s', r|s, a)(r + \gamma \max_{a' \in A} Q^*(s', a')).
\]

B. Model-aware learning

In the model-aware setting, where the transition function is known, an optimal policy can be efficiently found offline by optimizing the corresponding state-value function. This problem, known as planning [61], can be solved for finite-horizon MDPs via dynamic programming methods. We follow the method introduced by Bellman [59], which makes use of the recursive relation of Eq. (11) to find the optimal policy step by step; for this we assume that every episode deterministically ends at a fixed time step \( L \) and denote by \( v_\pi^L(s) \) the optimal value function of state \( s \) at time step \( \ell \) (problems in which \( L \) is not fixed can be solved, for instance, via value iteration [61]).

Since the optimal policy consists in taking the best possible action from any given state, it can be constructed by concatenation of the optimal policies at each time step: We start by solving Eq. (11) at the last time step,

\[
v_\pi^L(s) = \max_{a \in A(n)} \sum_{r \in R} \tau(r|s, a)r,
\]

where we have omitted the terminal state \( s_{L+1} \) and used the fact that \( v_{\pi^L}(s) = 0 \). The solution to Eq. (14) provides the optimal action at step \( L - 1 \) for each \( s \) and the optimal value function \( v_\pi^L(s) \). Then we plug the latter into the optimal Bellman equation for the previous time step, which in turn can be solved to obtain the optimal action and value function \( v_\pi^{L-1}(s) \). By repeating this procedure iteratively for each time step \( \ell = L, \ldots, 0 \), we can obtain the optimal sequence of actions and value functions for any state at any time step.
C. $Q$ learning

In the model-free setting, the agent not only has to find an optimal policy by exploiting valuable actions but also needs to characterize the environment in the first place by exploring possibly advantageous configurations. This is known as the exploration-exploitation tradeoff and lies at the core of RL problems [61]. In this setting, the $Q$ function is quite helpful since it associates a value to the transitions determined by taking action $a$ from state $s$ and following policy $\pi$ thereafter.

$Q$ learning was first proposed by Watkins [60], and it is often used as a basis for more advanced RL algorithms [67,71]. It is based on the observation that any Bellman operator, i.e., the operator describing the evolution of a value function as in Eqs. (10), (11), and (13), is contractive [72]. This implies that, under repeated applications of a Bellman operator, i.e., the operator describing the evolution of a value function converges to a fixed point, which by construction satisfies the corresponding Bellman equation. Thus, in order to find $Q^*(s,a)$, $Q$ learning turns the optimal Bellman equation for $Q$, Eq. (13), into an update rule for $\hat{Q}(s_t, a_t)$, i.e., the $Q$-function’s estimate available to the agent at a given time step $\ell$ of any episode $t = 1, \ldots, T$.

After an interaction step $s_{t} \rightarrow a_{t} \rightarrow s_{t+1}$ is experienced, the update rule for the $Q$ estimate is

$$
\hat{Q}(s_t, a_t) \leftarrow (1 - \lambda_t(s_t, a_t))\hat{Q}(s_t, a_t) + \lambda_t(s_t, a_t)(r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1}, a'))
$$

(15)

where $\lambda_t(s, a)$ is the learning rate, which depends on the number of times the state-action pair $(s_t, a_t)$ has been visited. Note that in order to do the update at each time step $\ell$, it is only necessary to enjoy the next immediate reward $r_{t+1}$ and observe the next state $s_{t+1}$; this method thereby allows an online learning of the MDP. A pseudocode of the algorithm is given in Algorithm 1.

---

**Algorithm 1.** Q-learning pseudocode.

**input:** $\hat{Q}(s, a)$ arbitrarily initialized for all $s \in \mathcal{S}$, $\forall a \in \mathcal{A}(s)$; learning rates $\lambda_t(s_t, a_t) \in (0, 1]$

**output:** $\hat{Q}(s, a)$

for $t$ in $1..T$ do
  initialize $s_0$
  for step $\ell$ in episode $t$ do
    if $t \leq T$ do
      initialize $s_0$
      for step $\ell$ in episode $t$ do
        take action $a_t$ according to $\pi$ (e.g., $\epsilon$-greedy)
        observe reward $r_{t+1}$ and next state $s_{t+1}$
        update $\hat{Q}(s_t, a_t)$ according to:
        $\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \lambda_t(s_t, a_t)(r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1}, a'))$
        if $s_{t+1}$ is terminal state then break
      else
        $s_t \leftarrow s_{t+1}$
  end
end

---

After a large number $n$ of iterations of the update rule Eq. (15) for all state-action couples, the convergence of the $Q$ estimate to the optimal $Q$ function is guaranteed by two general conditions on the learning rate (also known as Robinson conditions) [60,61]:

$$
\hat{Q}(s, a) \rightarrow Q^*(s, a) \forall s \in \mathcal{S}, a \in \mathcal{A}(s)
$$

iff $\sum_{t \in \mathbb{N}} \lambda_t(s, a) = \infty, \sum_{t \in \mathbb{N}} \lambda_t(s, a)^2 < \infty$,

(16)

where the sums are taken over all interactions at which a given state-action couple is visited. Once the optimal $Q$ function is obtained, an optimal deterministic policy can be constructed by “going greedy” with respect to it, i.e., $\pi^*(a|s) = \delta(a, \arg\max_{a \in \mathcal{A}} Q^*(s, a))$ for all $s \in \mathcal{S}$, where $\delta(x, y)$ is a Kronecker delta.

In RL literature, $Q$ learning is classified as an off-policy method [61], meaning that it learns the state—action values of a target policy—in this case the optimal policy—by taking actions according to an interaction policy, generally differing from the first one. The standard $Q$-learning method commits to an $\epsilon$-greedy interaction policy, where with probability $\epsilon$ the agent chooses a random action and otherwise it chooses the greedy action that maximizes the current $Q$ estimate. However, as we will see below, more general strategies can be considered.

---

IV. MODEL-FREE REINFORCEMENT LEARNING OF DISCRIMINATION STRATEGIES

In the following we frame the optimization of the receiver described in Sec. II into a RL context, in which an agent has to attain optimal reward-per-episode rate (success rate) by departing from a situation of complete ignorance of the experiment. For simplicity, we assume that the sender and receiver have a shared reference frame, so that we can take the states and displacements to be real, $a, \beta \in \mathbb{R}$, without loss of generality.

We can establish a correspondence between the notions introduced in Sec. II related to our discrimination setup to RL concepts of Sec. III:

(i) Each episode $t$ corresponds to an independent discrimination experiment, with a new default state $s_0 = a^{(k)}$ sampled from $p_k$, $k \in [0, 1]$; we set $\gamma = 1$ (the return (8) is well defined since the process has finite horizon).

(ii) Each episode consists of $L + 1$ time step $\ell = 0, \ldots, L$, corresponding to the $L$ detection layers followed by the final guessing stage.

(iii) The possible states of the environment at time steps $\ell$ are $s_{\ell} = a^{(k)}$, i.e., the transmitted part of $s_0$ at that layer.

(iv) The agent is not aware of the state $s_{\ell}$, in particular it does not know which hypothesis is true, but it can observe the measurement outcome $o_{\ell}, 0 < \ell \leq L$.

(v) The actions $a_{\ell}$ available at time-step $0 \leq \ell < L$ are the displacements $\beta_{\ell}$ and BS parameters $\beta_{\ell}$ available at that layer, conditioned on the history of observations $h_{\ell}$, while at the last step they constitute the guess, $a(h_L) = k \in [0, 1]$.

(vi) The reward $r \in [0, 1]$ is nonzero only at the end of the episode and provided that the guess is correct, hence the
transition function for the environment is
\[
\tau(a_{\ell+1}^{(k)}|a_{\ell}^{(k)}, a_\ell) = \delta(k', k) \quad \forall \ell \leq L, \tag{17}
\]
\[
\tau(r_{\ell+1}|a_{\ell}^{(k)}, a_\ell) = \delta(r_{\ell+1}, 1)\delta(a_\ell, k), \tag{18}
\]
where we omitted the trivial reward for \( \ell \leq L \).

A. Benchmarking the success probability via dynamic programming

In order to benchmark the performance of our RL agent, we start by considering a model-aware POMDP where the agent knows the amplitude \(|\alpha|\) of the optical signals and the transition probabilities and its task is to optimize the success probability of Eq. (5). In this case, we define \( b_\ell(k) = p(a_\ell^{(k)}|o_\ell, a_{\ell-1}, b_{\ell-1}) \) to be the belief distribution over the states \( a_\ell \), after performing action \( a_{\ell-1} \) and observing \( o_\ell \), with prior belief \( b_{\ell-1}(k) \). The initial value of the prior is \( b_0(k) = p_k \) and its update rule follows Bayes’ theorem:
\[
b_\ell(k) = \frac{p(a_\ell^{(k)}|o_\ell, a_{\ell-1}) b_{\ell-1}(k)}{\sum_k p(a_\ell^{(k)}|o_\ell, a_{\ell-1}) b_{\ell-1}(k)}. \tag{19}
\]

The optimal Bellman equation, Eq. (14), for the state-value function of this POMDP at step \( \ell \) reads
\[
v_\ell^\star(b_\ell) = \max_k b_\ell(k), \tag{20}
\]
which means that at the last step, if the final belief distribution over the states is known, the best guess is the hypothesis with maximum likelihood. The optimal Bellman equation at step \( \ell < L \) instead reads
\[
v_\ell^\star(b_\ell) = \max_{a_\ell \in \mathcal{A}(b_\ell)} \sum_{a_{\ell+1} \in \mathcal{O}} \sum_k p(o_{\ell+1}|a_\ell^{(k)}, a_\ell)b_\ell(k)v_{\ell+1}^\star(b_{\ell+1}). \tag{21}
\]

These equations can be solved iteratively by inserting the solution \( v_{\ell+1}^\star(b_{\ell+1}) \) into the equation for \( v_\ell^\star(b_\ell) \), starting with \( \ell = L - 1 \) and \( v_L^\star(b_L) \) found in Eq. (20). Note that, since \( v_{\ell+1}^\star(b_{\ell+1}) \) is computed for a discrete set of values of the belief distribution, these cannot always coincide with the values, determined by Eq. (19), needed to solve Eq. (21) and hence we use interpolation methods to obtain them. The maximum success probability attainable with the receiver is equal to the optimal value function at step \( \ell = 0 \), which is the latter corresponds to the expected reward starting from the initial belief distribution:
\[
v_0^\star(b) = \mathbb{E}_{\mathcal{S}}[r_{L+1}|b_0(k)] = p_0^{(b)}(\alpha) \tag{22}
\]
as can be seen by repeated applications of Eqs. (19) and (21) and detailed in Appendix A.

In Fig. 2 we show the optimal success probability obtained with this method as a function of \(|\alpha|^2\) and for up to \( L = 8 \) layers. We also show the results at fixed \( \theta_t \) such that the input state of each layer has equal amplitude \( a_\ell^{(k)} = \frac{\alpha}{\sqrt{L-1}} \) for all \( \ell \) (dashed lines). We observe that for all \( L \geq 2 \) there is an energy threshold above which allowing adaptive optimization of the attenuations gives a better success probability than adding one layer with fixed attenuations.
where \( r_{L+1} = \{1, 0\} \) stands for the correctness of the guess made at episode \( i \) and (ii) the success probability of the best actions according to the agent, at the current episode,

\[
P_t = P_a(\alpha, \{a_i^{(\alpha)}\}),
\]

where the best actions \( \{a_i^{(\alpha)}\} \) at episode \( t \) are obtained by going greedy with respect to the current \( Q \) estimate, i.e.,

\[
a_0^{(\alpha)}(h_0) = \arg \max_{a \in A(h_0)} Q(h_0, a) \rightarrow h_1^* = (o_1, a_0^{(\alpha)})
\]

\[
a_1^{(\alpha)}(h_1^*) = \arg \max_{a \in A(h_1^*)} Q(h_1^*, a) \rightarrow h_2^* = (a_0^{(\alpha)}, o_1, a_1^{(\alpha)}(h_1^*), o_2)
\]

\[
...\]

\[
a_L^{(\alpha)}(h_L^*) = \arg \max_{a \in A(h_L^*)} Q(h_L^*, a).
\]

The first figure of merit, \( R_t \), is usually employed to describe the learning process in RL and it evaluates the success rate obtained by the agent so far. On the other hand, the second figure of merit, \( P_t \), is standard in QSD and in our context it evaluates the best strategy discovered by the agent so far.

As \( t \rightarrow \infty \), for a good learner it is expected that \( R_t \rightarrow P_t \), i.e., with enough learning time the average reward should tend to the success probability for the best actions found by the agent, which in turn should converge to the optimal success probability \( P_t^{(L)} \). Therefore, the learner is not only expected to find a good discrimination strategy but to also follow it: The interaction policy should tend to the optimal policy. This feature is captured by the evolution of \( R_t \) over different episodes: A good learner is asked to obtain as much reward as possible during the learning process.

Here and in the rest of the paper, we restrict to \( L = 2 \) interaction layers and fix the attenuation coefficients to give equal amplitude at each layer, since the difference in success probability is small compared as shown in Sec. IV A. We choose a resolution of 21 points for each displacement, each one ranging from \(-1\) to \(1\) with step 0.1, leading to a fairly large state-action space: The agent has 3528 possible \( Q \) values to learn from (including the last guess). We note that each discretized displacement is an independent action or “button” in the eyes of the agent—the agent is dispossessed of any notion of closeness between buttons corresponding to similar values of \( \beta \). As the behavior of the RL agent strongly depends on the actions chosen at early episodes, we averaged the learning curves over 24 agents. Our results are compared with: (i) the maximum success probability achievable with this number of layers and discretization of displacements, Eq. (7), and (ii) the success probability attainable via a standard homodyne measurement, which is optimal among Gaussian receivers [40].

In Fig. 3 we plot these two figures of merit for \( Q \)-learning agents with three different \( \epsilon \)-greedy interaction policies: (i) a completely random one, i.e., \( \epsilon = 1 \), (ii) a 0.3-greedy one, i.e., \( \epsilon = 0.3 \), and (iii) a dynamic one (exp-greedy) that becomes exponentially greedier as time passes, i.e., \( \epsilon(t) = \max[e^{-\tau t}, \epsilon_0] \); this standard choice assures that at initial episodes the agent favors exploration, whereas at \( t = \tau \log \frac{1}{\epsilon_0} \) the agent’s behavior collapses to an \( \epsilon_0 \)-greedy policy.

In the first place we note in Fig. 3 that a fully random search over the action space (1-greedy policy) leads to the extremely poor cumulative reward per episode of \( R_t \approx 1/2 \), even for long times, which is expected because a random guess (last action) leads to \( P_t(\alpha, a_i) = 1/2 \). Instead, since all the actions will be sampled enough times for the agent to learn the optimal policy, \( P_t \) will converge to optimal value at large enough episode number. Nevertheless, if the action space is large, the fully random strategy will require a large number of episodes to explore each action a significant number of times, and for moderate times a \( \epsilon \)-greedy strategy might reach a better strategy. Indeed, Fig. 3 shows that the 0.3-greedy policy has at all episodes a higher \( P_t \) than the 1-greedy one, being 99\% the optimal success probability \( P_t^{(L=2)} \) at episode \( t = 10^5 \). Of course, for 0.3-greedy policy the agent collects many more rewards (actual correct guesses) than for the 1-greedy but it is still limited to \( R_t \approx 0.7P_t^{(L=2)} \). In order to reach a better exploration-exploitation tradeoff, it is customary to consider an episode-dependent \( \epsilon \), e.g., \( \epsilon(t) = \max[e^{-\tau t}, \epsilon_0] \). Figure 3 shows the results for this tunable interaction policy with \( \tau = 2 \times 10^2 \) and \( \epsilon_0 = 0.01 \). This allows the agent’s \( R_t \) to surpass the homodyne limit at about episode ~5 \times 10^5 (which is comparable with the size of the action space), while at later times the performance converges to that of the 0.01-greedy policy. Finally, notice that 0.3-greedy discovers a strategy whose \( P_t \) surpasses the homodyne limit at episode ~3 \times 10^5.

In Fig. 4 we study the guessing rule discovered by the 0.3-greedy agent at episode \( t = 5 \times 10^5 \). For each sequence of outcomes \( o_1, o_2 \), we plot the difference between the \( Q \) values of guessing for \( | - \alpha \), i.e., \( a_1 = 1 \), and \( | + \alpha \), i.e., \( a_1 = 0 \), as a function of the past actions:

\[
\hat{Q}(a_0, o_1, a_1(h_1), o_2, 1) - \hat{Q}(a_0, o_1, a_1(h_1), o_2, 0).
\]  

Note that the sign of Eq. (27) corresponds to the agent’s best guess for the true hypothesis, since the latter is obtained by going greedy towards \( Q(h_{\alpha}, a_{i}) \), as explained in Appendix A. We compare these results with the optimal guessing rule in the model-aware setting, plotting a shaded region when the maximum-likelihood guess is \( | \pm \alpha \). The plot shows that the
agent perfectly learn the guessing rule at the given resolution. Moreover, the difference between the two $Q$ values is more pronounced in the surroundings of the optimal $\beta$ values, meaning that the agents are more confident about their guess in these regions.

Our numerical results show that standard $Q$ learning successfully trains agents that surpass the homodyne limit of optical detection and discover strategies whose error rate is comparable with that of the optimal receiver. This is remarkable, especially taking into consideration that the agents are not initially trained for this task and run in a model-free setting entirely based on the feedback they get (correct/incorrect) on their guess. As mentioned above, although many RL schemes focus on extracting the optimal policy from the agent (as measured, e.g., by $R^*$), our central figure of merit, $R$, captures the real performance of the agent and can actually be assessed by the agent itself. It is hence important to design strategies that not only aim at finding the optimal policy within an episode but also maximize the cumulative reward per episode, reaching $R\rightarrow R^*$ as fast as possible. We have seen above an example of such strategy (the exponential greedy) and in the next sections we will study more advanced ones. For this purpose we will first study a simplified setting, called the multiarmed bandit problem, where the intraepisode dynamics is trivial, and the main focus is drawn on how to optimize the interepisode learning strategy. In passing, we introduce some theoretical tools in order to study the bandits’ learning curves, which are a cornerstone to tackle more challenging situations such as learning optimal policies over a MDP.

C. The multiarmed bandit problem

Multiarmed bandit problems are MDPs with a single default state at which the agent faces a fixed set of actions $a \in A$, each one leading to a reward $r \in R$ with an unknown probability $\tau(r|a)$. After action $a$ is performed, the reward $r$ is enjoyed and the episode finishes. The return defined in Eq. (8) becomes $r$. The situation models a gambler at a row of slot machines that has to decide which arms to pull, how many times to pull each one and in which order, with the aim of maximizing the earned rewards.

The bandit problem is an ideal framework to highlight the aforementioned crucial difference between learning strategies that accomplish the main goal of identifying the optimal policy after a given number of episodes and more refined strategies that also procure high de facto cumulative returns during the learning process. This is why bandit problems are very relevant in real-life applications where the final success probability is not the only figure of merit, as for example in clinical trials [73] where one needs to find the right compromise between advancing in the search of the best treatment while effectively treating current patients.

The general traits of the cumulative return per episode $R$, in Fig. 3 could inspire several ways of quantifying the performance of the learning agent, e.g., (a) the onset episode at which $R_\star$ starts exceeding the random policy; (b) the transient episode at which $R$ reaches a given fraction of the optimal success probability; (c) the learning speed as quantified by the slope of $R_\star$ after the onset episode; (d) the learning speed at which $R_\star$ converges to the optimal success probability. Unfortunately, very little is known about these or alternative ways to characterize the learning curves. Nonetheless, bandit theory provides us with a framework were some of these notions can be defined and rigorously studied. In particular, bandit theory defines the so-called cumulative regret:

$$L_t = \mathbb{E} \sum_{k=1}^t (Q(a^*) - Q(a^{(k)})) = t (Q(a^*) - \mathbb{E} R_\star), \quad (28)$$

where $\mathbb{E}$ indicates the expected value with respect to different agents following the same strategy, and $a^{(k)}$ is the action actually taken by an agent at episode $t$. The cumulative regret is closely related to the (expected) cumulative return per episode $R$, and quantifies the price to pay, or loss, for taking actions different from the optimal one ($a^*$). In other words, it quantifies the difference in earnings of the agent with respect to those of a model-aware superagent. One of the most fundamental results in bandit theory is the Lai-Robbins bound [74] for the asymptotic expected cumulative regret:

$$L_t \geq \log t \left( \sum_{a \in A} \frac{\Delta_a}{\text{KL}(\pi|\star)} + o(1) \right) = C_L R \log t \quad (29)$$

with $\Delta_a = Q(a^*) - Q(a)$ and $\text{KL}(\pi|\star)$ the Kullback-Leibler divergence between the reward distributions $\tau(r|a)$ and...
\[ \tau(r|a^\ast). \] Recalling the definition in Eq. (28) we note that the Lai-Robbins bound characterizes the learning curve in the asymptotic regime, indeed the average return over agents, \( \mathbb{E} \mathbf{R} \), can approach the optimal value not faster than \( \mathbb{E} \mathbf{R} \lesssim Q(a^\ast) - C_{LR} \log \frac{t}{1} \). Let us now briefly present some possible bandit strategies in light of this ultimate performance bound.

The most straightforward policy to use is the \( \epsilon \) greedy (already introduced in Sec. III C), as described in Algorithm 2.

\begin{algorithm}
\caption{\( \epsilon \) greedy for bandit problems.}
\begin{algorithmic}
\State \textbf{input} : \( \hat{Q}(a) \) arbitrarily initialized and learning rates \( \lambda_t(a) \in (0, 1) \) \forall a \in \mathcal{A}, \epsilon \in (0, 1] \)
\For {\( t \) in \( 1, \ldots, T \)}
\State generate a random number \( j \)
\If {\( j \leq \epsilon \)}
\State choose \( a \) at random
\Else
\State choose \( a = \arg \max_{a \in \mathcal{A}} \hat{Q}(a) \)
\EndIf
\State observe \( r \)
\State update \( \hat{Q} \):
\State \( \hat{Q}(a) \leftarrow \hat{Q}(a) + \lambda_t(a) [r - \hat{Q}(a)] \)
\EndFor
\end{algorithmic}
\end{algorithm}

Note that by choosing the learning rates \( \lambda_t(a) \) to be the inverse of the number of times action \( a \) was visited up to time \( t \), then
\[ \hat{Q}(a) \to_{t \to \infty} \sum_{r \in \mathbf{R}} r \tau(r|a) = Q(a) \ \forall a \in \mathcal{A}. \]

As stressed in Sec. IV B, the \( \epsilon \)-greedy policy never attains the optimal success rate, \( \mathbb{E} \mathbf{R} \to Q(a^\ast) \) for all \( t \), since at every episode there is a finite probability \( \epsilon \) that the agent performs a suboptimal action. For this reason, the expected cumulative regret grows linearly with \( t \). It is then clear that there is room for improvement before reaching the logarithmic scaling of the ultimate limit of Eq. (29). We will present two strategies, one based on upper confidence bounds (UCB) [63,74,75] and the other called Thompson sampling (TS) [64,73,76,77], which substantially improve the performance of \( \epsilon \) greedy and may even attain the Lai-Robbins ultimate bound [63,78].

In UCB, the agent keeps a record of the number of times each action \( a \) was selected up to episode \( t \), which we denote as \( N_t(a) \). Hoeffding’s inequality bounds the probability that the \( \hat{Q}(a) \) underestimates the true value of \( Q(a) \) by more than \( \epsilon(t) > 0 \), as
\[ \Pr[\hat{Q}(a) < Q(a) - \epsilon(t)] \lesssim e^{-2N_t(a)\epsilon(t)^2} =: \mathcal{P}(t), \]
where here and in the rest of this section we assume that \( r \in [0, 1] \). Then, for a given \( N_t(a) > 0 \), the upper confidence bound, defined as
\[ \text{ucb}_t(a) := \hat{Q}(a) + \epsilon(t) = \hat{Q}(a) + \sqrt{\frac{-\log \mathcal{P}(t)}{2N_t(a)}}, \]
represents an upper bound to the true value \( Q(a) \) with (high) probability \( 1 - \mathcal{P}(t) \). This value is used to compare and choose among the different actions, i.e.,
\[ a = \arg \max_{a \in \mathcal{A}} \text{ucb}_t(a), \]
and responds to the motto “optimism under the face of uncertainty:” actions that have not been visited enough are assigned an “optimistic” value of the return and hence more chances of being picked; in addition, actions whose \( Q \) estimate is accurate but suboptimal will have little chances to be picked again. The functional form of \( \mathcal{P}(t) \) can be tuned to balance exploration and exploitation. In particular, for the standard choice \( \mathcal{P}(t) = t^{-1} \) (here called UCB-1) it can be easily seen that the expected cumulative regret follows the logarithmic scaling [62,63]. In Appendix B we also discuss a different choice, which is known to saturate the Lai-Robbins bound, but performs worse than UCB-1 for our setting and the moderately small number of episodes that we consider.

\begin{algorithm}
\caption{UCB for bandit problems.}
\begin{algorithmic}
\State \textbf{input} : \( \mathcal{P}(t) \), initialize \( \hat{Q}(a), N(a) \) to zero \forall a \in \mathcal{A}.
\For {\( t \) in \( 1, \ldots, T \)}
\If {\( t \leq |\mathcal{A}| \)}
\State (choose each action once) \( a = t \)
\Else
\State choose \( a = \arg \max_{a \in \mathcal{A}} \text{ucb}_t(a) \) (Eq. (32))
\State observe reward \( r \)
\State record visit:
\State \( N_t(a) \leftarrow N_t(a) + 1 \)
\State update Q-value:
\State \( \hat{Q}(a) \leftarrow \hat{Q}(a) + \frac{[\hat{Q}(a)]}{N_t(a)} \)
\EndIf
\EndFor
\end{algorithmic}
\end{algorithm}

Thompson sampling (TS) departs from the standard \( Q \)-learning paradigm, which is based on keeping track and updating the \( Q \) table. Instead, TS follows a Bayesian approach and at every episode assigns a full prior distribution (not just an expectation value) to the expected reward \( \tilde{r} \) of every arm \( a \), \( f_t(\tilde{r}|a) \forall a \in \mathcal{A} \). This distribution characterizes the knowledge the agent has about the expected earnings of each arm, \( Q(a) \), and at the first episode can be taken to be flat over the whole interval \([0,1]\). The policy then consists in sampling an expected reward \( \tilde{r} \sim f_{t-1}(\tilde{r}|a) \) for each possible action \( a \) and choosing the action with the largest sample \( \tilde{r} = \arg \max_{a \in \mathcal{A}} (\tilde{r} \sim f_{t-1}(\tilde{r}|a)) \). Finally, the distribution for the chosen action is updated according to the true reward \( r \) obtained, using Bayes’ theorem.

In order to avoid computationally expensive Bayesian updates, families of distributions that are closed under the update rule are used. In the case of Bernoulli bandits, beta-distributions are employed since those are precisely their conjugate priors. That is, given
\[ f_t(\tilde{r}|a) = \text{Beta}(\mu_t(a), \nu_t(a)) \propto \tilde{r}^{\mu_t(a)-1} (1 - \tilde{r})^{\nu_t(a)-1}, \]
upon obtaining a reward \( r \) the prior is updated to another beta distribution with parameters \( \mu_{t+1}(a) = \mu_t(a) + r, \nu_{t+1}(a) = \nu_t(a) + 1 - r \), where at the first episode it is \( \mu_1(a) = \nu_1(a) = 1 \forall a \in \mathcal{A} \) (flat prior). By mimicking the underlying distributions, TS gauges exploration according to the information acquired so far: If a certain action has not been sampled enough at episode \( t \), its reward distribution will still be broad and, when sampled, can easily return a higher value of \( \tilde{r} \) than that obtained from other (more peaked) distributions;
curves are averaged over 10³ agents. Furthermore, we compare the asymptotic behavior of TS, studying different strategies. In this case, With this, each possible displacement with binary outcome, we have assumed a fixed decision rule. considering each action conditioned on the past history as a bandit problem. The displacements considered are $\beta \in [0, -\alpha, \beta^\star]$, for $\alpha = 0.4$ and $\beta^\star = -0.74$. All curves are averaged over 10³ agents. Furthermore, we compare the asymptotic behavior of TS, studying bandit problem 2, with $\beta \in \{-\alpha, \beta^\star, -1.5\alpha\}$.

thereby TS will favor to explore such action. At the same time, if a suboptimal action has been sampled enough episodes, it will be very unlikely that it is sampled again, since the corresponding prior will be highly peaked at low values. The pseudocode of TS for Bernoulli bandits is described in Algorithm 4.

Algorithm 4. TS for Bernoulli bandit problems.

```
input : $\mu_1(a)$, $v_1(a)$ initialized to one $\forall a \in A$
for $t$ in 1, ..., $T$ do
  for $a$ in $A$ do
    draw $r_a$ according to Beta($\mu_1(a)$, $v_1(a)$)
    choose $a = \arg \max_a r_a$
    observe reward $r$
    update Beta distribution:
      $\mu_{i+1}(a) = \mu_i(a) + r$
      $v_{i+1}(a) = v_i(a) + 1 - r$
```

Figure 5 shows the performance of different strategies in a three-armed bandit problem. For this purpose, we have considered a simple optical receiver as described in Sec. II with a single layer $L = 1$. Since there is only a single detector with binary outcome, we have assumed a fixed decision rule. With this, each possible displacement $\beta$ constitutes an action $a_0$ (recalling the notation used in last section) of a bandit problem. The figure shows that the cumulative regret scales linearly with time for the $\epsilon$-greedy strategy, while it has a logarithmic scaling for the UCB and TS strategies. The inset shows the cumulative regret as a function of $\log t$ together with the ultimate bound given by Lai-Robbins bound. The achievability of this bound is hard to observe in simulations because the convergence to the asymptotic results is very slow [79], i.e., subleading constants and terms of order $\log(\log t)$ might be important. Nevertheless, in the setting of Fig. 5 we see that the leading term captured by the slope of the curve is consistent with the ultimate bound.

Let us conclude this overview of bandit theory by introducing the simple regret, another widely used figure of merit that, as $P_\ell$, quantifies how well has the agent learned so far, regardless of his actual performance:

$$\Lambda_\ell = \mathbb{E}(Q(a^* \mid \mathcal{F}_t) - Q(a_t \mid \mathcal{F}_t)), \quad (34)$$

where $a_t^{\star\star}$ is the agent’s recommendation of which the optimal action is at episode $t$, which, e.g., in $Q$ learning would be given by $a_t^{\star\star} = \arg \max_a \hat{Q}(a)$. Strategies designed to minimize $\Lambda_\ell$ will prioritize to learn what the optimal arm to pull is, and the probability that the agent confuses the optimal arm by a suboptimal one will be exponentially small, hence $\Lambda_\ell$ will converge to zero exponentially fast. Recent results [80] show that the exploitation-exploration tradeoff manifests itself in the asymptotic scaling of the simple and cumulative regret in the sense that one imposes lower and upper bounds on the other, and therefore optimizing one usually affects the performance of the other.

D. Enhancing the agent via UCB and TS

In this section we consider two enhanced RL strategies, inspired by the advanced bandit methods introduced in Sec. IV C and adapted to our MDP problem. The first strategy employs the standard $Q$-learning update rule for the estimate $\hat{Q}$, as described in Eq. (15), but it employs UCB to determine the interaction policy at each episode, as described in Sec. IV C, with $P(t) = t^{-\delta}$ [see Appendix B for a comparison of different choices on $P(t)$]. This is implemented by keeping a count of the number of visits of each history-action couple up to the current episode $t$, i.e., $N_h(t, a_t)$, which is then used to compute an upper confidence bound, $ucb(h_t, a_t)$ as in Eq. (32), for each action $a_t$ and history $h_t$. Finally, at time step $\ell$ the agent chooses the greedy action w.r.t. the UCB, i.e., $a_t^{\star\star} = \arg \max_a ucb(h_t, a_t)$.

The second strategy is instead based entirely on TS, considering each action conditioned on the past history as a bandit problem and rewarding each sequence of actions that led to a successful experiment. In detail, the agent keeps a beta-distribution, Eq. (33), of the mean reward obtainable at each time step $\ell$ from each action $a_t$ given each possible history $h_c$, i.e., $f_{h_c}(\hat{r} \mid h_c, a_t)$. In order to choose a new action at time step $\ell$ given history $h_c$, the agent samples an expected reward $\hat{r} \sim f_{h_c}(\hat{r} \mid h_c, a_t)$ for each $a_t$ and selects the action with the largest sample $\hat{r}$. At the end of the episode a reward is obtained as usual, and $f_{h_c}(\hat{r} \mid h_c, a_t)$ is updated in a Bayesian way for all the history-action couples visited at the episode. In this case, when computing $P_\ell$, the best actions are chosen by going greedy w.r.t. their mean reward distribution $f_{h_c}(\hat{r} \mid h_c, a_t)$ [64].

In Fig. 6 we plot the two figures of merit $R_\ell$, $P_\ell$, for agents trained using these two enhanced strategies, as well as
for those based on the exp-greedy and 0.3-greedy strategies, considered in Sec. IV B, which had, respectively, the largest
final $R$, and $P$, out of all the analyzed strategies. We observe that UCB performs a thorough exploration of the action space
and indeed it is able to attain a value of $P$ close to that of 0.3-greedy. This result comes at the price of a small $R$ value, which nevertheless shows that UCB has better exploitation
properties than 0.3-greedy; in particular it has a strikingly larger slope than the latter at long times. As for TS, we observe that this strategy attains the best $R$, values, surpassing exp-greedy at intermediate times. Moreover, TS also radically improves the values of $P$, w.r.t. exp-greedy and it is even able to attain the performance of the other two strategies that favor exploration. Overall, it appears that for our problem
TS provides the most profitable balance of exploration and exploitation; we expect this strategy to perform worse in scenarios in which the underlying distribution probability does not belong to the family of distributions used by TS.

Finally, we show that RL agent’s performance is independent of the coherent states’ energy. For this we evaluate, for a range of different amplitudes $|\alpha|$, the values of $R$, and $P$, attained by different agents at episode $t = 5 \times 10^5$, comparing them with the optimal success probability $\mu(\alpha = 2)$, as can be seen in Fig. 7. In the following we turn to test model-free methods in realistic experimental scenarios, where the ultimate success probability is affected by the presence of noise.

E. Noise robustness

In the previous subsections we have shown that our RL agents are able to learn near-optimal discrimination strategies and—most importantly—exploit them in real time, employing exclusively the detectors’ outcomes and the rewards at the end of each episode. Here we show that these results do not sensibly change in the presence of noise, i.e., that the same
agents are able to attain near-optimal performance even when unknown errors affect the experiment and hence the learning process.

First, we consider a common experimental imperfection known as dark counts: Due to the presence of background noise, each photodetector in the receiver has a nonzero probability $p_{dc}$ of detecting a photon even when it receives a vacuum signal. Accordingly, the conditional probability of obtaining an outcome 0 given an input state $|\alpha\rangle$, Eq. (4), is modified by a multiplicative factor (1 − $p_{dc}$).

In Fig. 8 we plot $R$, and $P$, at time $t = 5 \times 10^5$ for several RL strategies as a function of $p_{dc} \in [0, 1]$, along with the maximum success probability attainable by the corresponding receiver. We see that the final values of $P$, are near optimal for all values of $p_{dc}$, while $R$, seems to be slightly affected in an intermediate region of values of $p_{dc}$. We notice that the agents are be able to learn optimal strategies in the long term, as shown by the high values of $P$, attained. However, since a dark count effectively increases the chance of (not) obtaining a reward for a (correct) wrong action, the time it takes to learn a near-optimal strategy and to start exploiting is also increased, as reflected by the behavior of $R$. Note that for $p_{dc} \sim 1$ the best guess is the random one and thus easier to learn.
Next, we consider the case where the phase of the incoming signal is flipped before arriving to the receiver, with probability $p_f$. In this scenario, if the agent guesses for the correct received phase, the corresponding reward will be zero since the phase initially sent was opposite than the received one. In particular, the probability of observing a string of outcomes $p(o_{1:L} | \alpha, \{a(h_{L-1})\})$ in Eq. (4) is modified such that

$$
p(o_{1:L} | \alpha, \{a(h_{L-1})\}) \rightarrow (1 - p_f)p(o_{1:L} | \alpha, \{a(h_{L-1})\}) + p_f p(o_{1:L} | -\alpha, \{a(h_{L-1})\}).
$$

In Fig. 9 we depict the values of $R_t$ and $P_t$ attained by several agents at episode $t = 5 \times 10^5$, as a function of $p_f \in [0.5, 1]$, along with the maximum success probability attainable by the corresponding receiver. As in the case of dark counts, we see that for all values of $p_f$, the agents are able to converge to near-optimal $P_t$ values and they exhibit very small variations in $R_t$ as $p_f$ increases.

V. DISCUSSION AND CONCLUSIONS

In this paper we provided an in-depth study of RL methods for the online optimization of coherent-state receivers based on current technology. Such receivers are crucial for the deployment of high-data-rate long-distance communications in free space or optical fiber and they are based on the interplay of several simple quantum gates and measurements, combined to create a complex structure. The RL methods that we analyzed enable to optimize such structure based on the actual experimental conditions and limitations of the communication channel and of the receiver. Thus, they possess a high potential for increasing the flexibility and effectiveness of current receivers and provide a useful addition to the current experimental toolbox. This is even more so the case if we consider that the methods we studied are relatively simple and rely only on "shallow" RL techniques, i.e., they are not based on the use of neural networks, which would allow to consider larger state-action spaces, possibly at the cost of a longer training time and computational overheads. We expect that the use of such "deep" RL methods could allow to control receivers of multiple and/or multimode coherent states, whose best performance is still to be determined at present. On the other hand, the characterization of the cumulative or simple regret for intermediate episodes through general nonasymptotic upper and lower bounds, as well as its extension from bandits to more general MDPs is still an open and active field of research. Quantum technologies can benefit from this progress, and nontrivial quantum features might appear in more general quantum learning scenarios.

Let us now briefly comment on the time needed to perform the search. The proposed scheme is an adaptive learning protocol in two distinct ways, and thereby we may distinguish between two different timescales. During a single experiment, the architecture performs a weak measurement of the (displaced) signal and feeds this result forward to perform the next control operation; this defines what we may call an "internal timescale." The "external timescale" concerns instead the adaptation of the agent’s strategy from one experiment to another, based solely on the correctness of the guess. The technical requirements for the internal timescales are within reach of current technology [46,50]. Regarding the external timescales instead, the computations done between experiments are not particularly demanding for the problem at hand (in particular because of the “shallow” RL methods employed, as discussed above).

Finally, we would like to stress that the RL problem induced by real-time state discrimination is characterized by intrinsically noisy and stochastic rewards. As such, it stands out from other instances where RL has been applied to quantum physics. Indeed, even when performing a good set of actions and guessing rule, an agent might still not be rewarded. This is due to two crucial factors: (i) quantum states are intrinsically indistinguishable, i.e., even the best receiver has a nonzero probability of discrimination error; (ii) our methods can be applied in real time to the experiment, hence the binary reward received for a given set of actions is stochastic and thereby not sufficient to estimate the success probability of the corresponding receiver. Still, the best among our agents is able to reach good configurations and start exploiting them in a number of experiments which is roughly sufficient to try each set of actions only once. This is a key signature of the agents’ intelligent behavior, showing that they make the most out of each reward rather than blindly trying actions at random. Hence, we believe that the discrimination problem provides an interesting, rich, and flexible sandbox for testing RL in quantum-physics-inspired scenarios and will constitute an interesting line of work at the intersection between these two fields.

The code developed to obtain the numerical results of this research can be found at [81]. Any suggestions, comments, and even collaborations are welcome.

ACKNOWLEDGMENTS

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant
APPENDIX A: OPTIMAL STATE-ACTION VALUES

In this section we verify that—by construction—the optimal policy leads to the maximum success probability $P^L(\alpha)$. It is assumed that $Q$ is always associated with the optimal policy $\pi^*$; we simplify notation by $Q = Q_{\pi^*}$.

At step $L$, given any history $h_L$, the actions available to the agent are $\ell = a_L$, i.e., guessing for one of the possible phases of the coherent state. The $Q$ values at this time step read as

$$Q(h_L, a_L) = E[G_L|h_L, a_L] = \sum_{i_{L+1}} r_{L+1} p(r_{L+1}|h_L, a_L),$$

with $o_{1:L} = \{o_1, o_2, \ldots, o_L\}$ the observations obtained up to the $L$th photon detector, and $a_{0:L} = \{a_0, a_1, \ldots, a_L\}$ the actions done up to step $L$. Recalling that the optimal action, given $h_L$, is obtained from $Q$ as $\pi^*(h_L) = \arg\max_{a_L} Q(h_L, a_L)$, the optimal guess $a_L^*$ is the one of maximum likelihood:

$$a_L^* = \arg\max_{a_L} p(a^{(k)}|o_{1:L}; a_{0:(L-1)})|k=a_L.$$

By definition of the optimal policy and because optimal Bellman equation Eq. (13) holds, the optimal action to take given history $h_{L-1}$ at step $L - 1$ is

$$a_L^{*} = \arg\max_{a_L} Q(h_{L-1}, a_L)$$

$$= \arg\max_{a_L} \sum_{a_{L-1}} p(a_{L-1}|o_{1:(L-1)}; a_{0(L-1)}) \max_{a_0} Q(h_1, a_1)$$

$$= \arg\max_{a_L} \sum_{a_{L-1}} p(a_{L-1}|o_{1:(L-1)}; a_{0(L-1)}) \max_{k} p(a^{(k)}|o_{1:L}; a_{0(L-1)})$$

$$\times p(o^{(k)}|o_{1:L}; a_{0(L-1)})$$

$$= \arg\max_{a_L} \sum_{a_{L-1}} \max_{k} p(a^{(k)}|o_{1:L}; a_{0(L-1)})p_k,$$

where in the last line we have used Bayes following. This line of reasoning, we can obtain the optimal actions $a^*_L$ at any time step. In particular, for $t = 0$, by recursively applying the optimal Bellman equation [Eq. (13)] we have

$$Q(h_0, a_0) = \sum_{a_1} p(o_1|a_0) \max_{a_1} Q(h_1, a_1)$$

$$= \sum_{a_1} p(o_1|a_0) \max_{a_1} \sum_{a_2} p(o_2|a_1) \max_{a_2} Q(h_2, a_2)$$

$$= \sum_{a_1} p(o_1|a_0) \max_{a_1} \sum_{a_2} p(o_2|a_1) \max_{a_2} \sum_{a_3} (\ldots)$$

$$\times \left( \sum_{a_0} p(o_{L-1}|o_{1:(L-1)}; a_{1:(L-1)}) \max_{a_L} Q(h_L, a_L) \right)$$

$$= \sum_{a_1} \sum_{a_2} \sum_{a_3} (\ldots)$$

$$\times \left( \sum_{a_0} p(o_{L-1}|o_{1:(L-1)}; a_{1:(L-1)}) \max_{a_L} Q(h_L, a_L) \right) \cdot p_k.$$ (A3)

Therefore, by taking the optimal action $a^*_0 = \arg\max_{a_0} Q(h_0, a_0)$, we obtain

$$\max_{a_0} Q(h_0, a_0) = P^L(\alpha).$$ (A4)

As pointed out in the main text, the value and action-value functions are related via $v_\pi(s) = \sum_{a} \pi(a|s)Q_\pi(s, a)$. Therefore, the optimal value function for the initial state is the optimal success probability:

$$v_\pi(h_0) = \sum_{a} \delta(a, \arg\max_{a} Q(h_0, a)) = Q(h_0, a^*_0) = P^L(\alpha).$$ (A5)

In Fig. 10 we show several sections of the estimates $\hat{Q}$, using 1-greedy as the interaction policy and each update made according to Algorithm 1, at episode $\epsilon = 10^8$.

APPENDIX B: COMPARISON OF DIFFERENT UCB STRATEGIES

In this section we show numerical results on how the choice of $\mathcal{P}(t)$ for the UCB strategy leads to different learning curves behavior, for $L = 2$. As explained in Sec. IV C, the probability to overestimate the state-action value can be
that the exploration-exploitation tradeoff is an intrinsic feature of our problem. bounded by Hoeffding’s inequality. This probability can be forced to depend on the episode. In Fig. 11 we show the performance of three different choices of \( \mathcal{P}(t) \) for the same receiver considered in Sec. IV B. First we consider UCB-1, which is the standard choice of \( \mathcal{P}(t) = t^{-4} \), which for a bandit problem with \( K = |\mathcal{A}| \) arms can be easily proven to have an asymptotic cumulative regret upper bounded by [62]

\[
\mathcal{L}_t \leq 8 \sum_{a \in \mathcal{A}(\mathcal{A})} \log t + K \pi^2 / 3
\]  

which together with the Lai-Robbins bound of Eq. (29) implies that \( \mathcal{L}_t = O(\log(t)) \). Secondly, we consider UCB-2, with a choice of \( \mathcal{P}(t) \) proved to be asymptotically optimal in bandit problems [62]. Lastly, an heuristic and instance dependent variation of \( \mathcal{P}(t) \), UCB-3, leads to better \( \mathcal{R} \), only in the short term, as exploration is damped too fast (which is also reflected in suboptimal \( \mathcal{P} \), even in the long term).

![Algorithm's performance comparison](image)

**FIG. 11.** We compare two different variants of UCB showing that the exploration-exploitation tradeoff is an intrinsic feature of our problem.

[1] C. W. Helstrom, Quantum detection and estimation theory, J. Stat. Phys. 1, 231 (1976).
[2] A. S. Holevo, Quantum Systems, Channels, Information (De Gruyter, Berlin, Boston, 2012).
[3] M. Takeoka and S. Guha, Capacity of optical communication in loss and noise with general quantum Gaussian receivers, Phys. Rev. A 89, 042309 (2014).
[4] A. Waseda, M. Takeoka, M. Sasaki, M. Fujiwara, and H. Tanaka, Quantum detection of wavelength-division-multiplexing optical coherent signals, J. Opt. Soc. Am. B 27, 259 (2010).
[5] A. Waseda, M. Sasaki, M. Takeoka, M. Fujiwara, M. Toyoshima, and A. Assalini, Numerical evaluation of PPM for deep-space links, J. Opt. Commun. Netw. 3, 514 (2011).
[6] S. Guha, Structured Optical Receivers to Attain Superadditive Capacity and the Holevo Limit, Phys. Rev. Lett. 106, 240502 (2011).
[7] H. Krovi, S. Guha, Z. Dutton, and M. P. Da Silva, Optimal measurements for symmetric quantum states with applications to optical communication, Phys. Rev. A 92, 062333 (2015).
[8] M. Rosati, A. Mari, and V. Giovannetti, Multiphase Hadamard receivers for classical communication on lossy bosonic channels, Phys. Rev. A 94, 062325 (2016).
[9] W. Zwoliński, M. Jarzyka, and K. Banaszek, Range dependence of an optical pulse position modulation link in the presence of background noise, Opt. Express 26, 25827 (2018).
[10] B. Huttner, N. Imoto, N. Gisin, and T. Mor, Quantum cryptography with coherent states, Phys. Rev. A 51, 1863 (1995).
[11] M. Dušek, M. Jahma, and N. Lütkenhaus, Unambiguous state discrimination in quantum cryptography with weak coherent states, Phys. Rev. A 62, 022306 (2000).
[12] F. Grosshans and P. Grangier, Continuous Variable Quantum Cryptography Using Coherent States, Phys. Rev. Lett. 88, 057902 (2002).
[13] N. Gisin, G. Ribordy, W. Titel, and H. Zbinden, Quantum cryptography, review of modern physics, Rev. Mod. Phys. 74, 145 (2002).
[14] J. A. Bergou, Quantum state discrimination and selected applications, J. Phys. Conf. Ser. 84, 012001 (2007).
[15] C. H. Bennett and G. Brassard, Quantum cryptography: Public key distribution and coin tossing, Theor. Comput. Sci. 560, 7 (2014).
[16] U. Chabaud, T. Douce, F. Grosshans, E. Kashefi, and D. Markham, Building trust for continuous variable quantum states, Schloss Dagstuhl–Leibniz-Zentrum für Informatik 158, 3:1 (2019).
[17] S. Pirandola, U. L. Andersen, L. Banchi, M. Berta, D. Bunandar, R. Colbeck, D. Englund, T. Gehring, C. Lupo, C. Ottaviani, J. Pereira, M. Razavi, J. S. Shaari, M. Tomamichel, V. C. Usenko, G. Vallone, P. Villoresi, and P. Wallden, Advances in quantum cryptography, arXiv:1906.01645 (2019).
[18] M. Schuld and F. Petruccione, Supervised Learning with Quantum Computers (Springer, Cham, 2018).
[19] G. Sentís, E. Bagan, J. Calsamiglia, and R. Muñoz-Tapia, Multicopy programmable discrimination of general qubit states, Phys. Rev. A 82, 042312 (2010).
[20] G. Sentís, M. Guţă, and G. Adesso, Quantum learning of coherent states, EPI Quantum Technol. 2, 17 (2015).
[21] M. Guţă and W. Kotłowski, Quantum learning: Asymptotically optimal classification of qubit states, New J. Phys. 12, 123032 (2010).
[22] S. Lloyd and C. Weedbrook, Quantum Generative Adversarial Learning, Phys. Rev. Lett. 121, 040502 (2018).
[23] M. Fanizza, A. Mari, and V. Giovannetti, Optimal universal learning machines for quantum state discrimination, IEEE Trans. Inf. Theory 65, 5931 (2019).

[24] C. Blank, D. K. Park, J.-K. Kevin Rhee, and F. Petruccione, Quantum classifier with tailored quantum kernel, npj Quantum Inf. 6, 41 (2020).

[25] G. Sergioli, R. Giuntini, and H. Freytes, A new quantum approach to binary classification, PLoS One 14, e0216224 (2019).

[26] M. Benedetti, E. Grant, L. Wossnig, and S. Severini, Adversarial quantum circuit learning for pure state approximation, New J. Phys. 21, 043023 (2019).

[27] G. Carleo and M. Troyer, Solving the quantum many-body problem with artificial neural networks, Science 355, 602 (2016).

[28] G. Torlai and R. G. Melko, Neural Decoder for Topological Codes, Phys. Rev. Lett. 119, 030501 (2017).

[29] E. P. L. Van Nieuwenburg, Ye Hua Liu, and S. D. Huber, Learning phase transitions by confusion, Nat. Phys. 13, 435 (2017).

[30] J. Carrasquilla and R. G. Melko, Machine learning phases of matter, Nat. Phys. 13, 431 (2017).

[31] G. Torlai, G. Mazzola, J. Carrasquilla, M. Troyer, R. Melko, and G. Carleo, Neural-network quantum state tomography, Nat. Phys. 14, 447 (2018).

[32] A. A. Melnikov, H. Poulsen Nautrup, M. Krenn, V. Dunjko, M. Tiersch, A. Zeilinger, and H. J. Briegel, Active learning machine learns to create new quantum experiments, Proc. Natl. Acad. Sci. USA 115, 1221 (2018).

[33] T. Fösel, P. Tighineanu, T. Weiss, and F. Marquardt, Reinforcement Learning with Neural Networks for Quantum Feedback, Phys. Rev. X 8, 031084 (2018).

[34] J. Wallnöfer, A. A. Melnikov, W. Dür, and H. J. Briegel, Machine learning for long-distance quantum communication, arXiv:1904.10797 (2019).

[35] M. Bukov, A. G. R. Day, D. Sels, P. Weinberg, A. Polkovnikov, and P. Mehta, Reinforcement Learning in Different Phases of Quantum Control, Phys. Rev. X 8, 031086 (2018).

[36] M. Y. Niu, S. Boixo, V. Smelyanskiy, and H. Neven, Universal quantum control through deep reinforcement learning, in AIAA Scitech 2019 Forum, Vol. 5 (Nature Publishing Group, Reston, Virginia, 2019), p. 33.

[37] D. Silver, A. Huang, C. J. Maddison, A. Guez, L. Sifre, G. van den Driessche, J. Schrittwieser, I. Antonoglou, V. Panneershelvam, M. Lanctot, S. Dieleman, D. Grewe, J. Nham, N. Kalchbrenner, I. Sutskever, T. Lillicrap, M. Leach, K. Kavukcuoglu, T. Graepel, and D. Hassabis, Mastering the game of Go with deep neural networks and tree search, Nature (London) 529, 484 (2016).

[38] M. Osaki, M. Ban, and O. Hirota, Derivation and physical interpretation of the optimum detection operators for coherent-state signals, Phys. Rev. A 54, 1691 (1996).

[39] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. 84, 621 (2012).

[40] M. Takeoka and M. Sasaki, Discrimination of the binary coherent signal: Gaussian-operation limit and simple non-Gaussian near-optimal receivers, Phys. Rev. A 78, 022320 (2008).

[41] S. J. Dolinar, Communication and sciences engineering, Q. Prog. Rep. (Research Lab. Electron.) 111, 115 (1973).

[42] R. S. Kennedy, Near-optimum receiver for the binary coherent state quantum channel, MIT Res. Lab. Electron. Q. Prog. Rep. 108, 219 (1973).

[43] J. M. Geremia, Distinguishing between optical coherent states with imperfect detection, Phys. Rev. A 70, 062303 (2004).

[44] M. Takeoka, M. Sasaki, P. Van Loock, and N. Lütkenhaus, Implementation of projective measurements with linear optics and continuous photon counting, Phys. Rev. A 71, 022318 (2005).

[45] M. Takeoka, M. Sasaki, and N. Lütkenhaus, Binary Projective Measurement via Linear Optics and Photon Counting, Phys. Rev. Lett. 97, 040502 (2006).

[46] R. L. Cook, P. J. Martin, J. M. Geremia, B. A. Chase, and J. M. Geremia, Optical coherent state discrimination using a closed-loop quantum measurement, Nature (London) 446, 774 (2007).

[47] M. P. Da Silva, S. Guha, and Z. Dutton, Achieving minimum-error discrimination of an arbitrary set of laser-light pulses, Phys. Rev. A 87, 052320 (2013).

[48] R. Nair, S. Guha, and S. H. Tan, Realizable receivers for discriminating coherent and multicyclop quantum states near the quantum limit, Phys. Rev. A 89, 032318 (2014).

[49] M. Rosati, A. Mari, and V. Giovannetti, Coherent-state discrimination via nonheralded probabilistic amplification, Phys. Rev. A 93, 062315 (2016).

[50] M. T. Dimario and F. E. Becerra, Robust Measurement for the Discrimination of Binary Coherent States, Phys. Rev. Lett. 121, 023603 (2018).

[51] F. E. Becerra, J. Fan, G. Baumgartner, J. Goldhar, J. T. Kosloski, and A. Migdall, Experimental demonstration of a receiver beating the standard quantum limit for multiple nonorthogonal state discrimination, Nat. Photonics 7, 147 (2013).

[52] C. R. Müller and Ch. Marquardt, A robust quantum receiver for phase shift keyed signals, New J. Phys. 17, 032003 (2015).

[53] S. Guha and M. M. Wilde, Polar codes to achieve the Holevo capacity of a pure-loss optical channel, Proceedings of IEEE International Symposium on Information Theory (IEEE, Piscataway, NJ, 2012), p. 546.

[54] M. M. Wilde and S. Guha, Polar codes for classical-quantum channels, IEEE Trans. Inf. Theory 59, 1175 (2013).

[55] M. Rosati and V. Giovannetti, Achieving the Holevo bound via a bisection decoding protocol, J. Math. Phys. 57, 062204 (2016).

[56] M. Rosati, A. Mari, and V. Giovannetti, Capacity of coherent-state adaptive decoders with interferometry and single-mode detectors, Phys. Rev. A 96, 012317 (2017).

[57] Ze Liang Xiang, M. Zhang, L. Jiang, and P. Rabl, Intracry Quantum Communication Via Thermal Microwave Networks, Phys. Rev. X 7, 011035 (2017).

[58] M. T. DiMario, L. Kunz, K. Banaszek, and F. E. Becerra, Optimized communication strategies with binary coherent states over phase noise channels,npj Quantum Inf. 5, 65 (2019).

[59] R. Bellman, Dynamic Programming (Reprinted version) (Dover Publications, 2003).

[60] C. J. C. H. Watkins and P. Dayan, Q-learning, Mach. Learn. 8, 279 (1992).

[61] R. Sutton and A. G. Barto, Reinforcement Learning Sutton (MIT Press, Cambridge, MA, 2018).

[62] T. Lattimore and C. Szepesvari, Bandit Algorithms (Cambridge University Press, 2018), p. 542.

[63] P. Auer, N. Cesa-Bianchi, and P. Fischer, Finite-time analysis of the multiarmed bandit problem, Mach. Learn. 47, 235 (2002).
[64] D. J. Russo, B. Van Roy, A. Kazerouni, I. Oshband, and Z. Wen, A tutorial on Thompson sampling, Found. Trends Mach. Learn. 11, 1 (2018).

[65] C. Jin, Z. Allen-Zhu, S. Bubeck, and M. I. Jordan, Is $Q$-Learning Provably Efficient? (Curran Associates, Inc., 2018), pp. 4863–4873.

[66] S. P. Singh, T. Jaakkola, and M. I. Jordan, Learning without state-estimation in partially observable markovian decision processes, in Proceedings of Machine Learning 1994 (Morgan Kaufmann, 1994), pp. 284–292.

[67] V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. Riedmiller, Playing atari with deep reinforcement learning, arXiv:1312.5602 (2013).

[68] G. Shani, J. Pineau, and R. Kaplow, A survey of point-based POMDP solvers, Auton. Agent. Multi. Agent. Syst. 27, 1 (2013).

[69] M. Egorov, Deep reinforcement learning with pomdps, Technical report, Stanford University, 2015, http://cs229.stanford.edu/proj2015/363_report.pdf.

[70] P. Zhu, Xin Li, P. Poupart, and G. Miao, On improving deep reinforcement learning for POMDPs, arXiv:1704.07978 (2018).

[71] D. Silver, G. Lever, N. Heess, T. Degris, D. Wierstra, and M. Riedmiller, Deterministic policy gradient algorithms, In Proceedings of the 31st International Conference on International Conference on Machine Learning - Volume 32 (JMLR, 2014) ICML-14, p. I-387-I-395.

[72] Csaba Szepesvri, Algorithms for Reinforcement Learning (Morgan and Claypool Publishers, 2010).

[73] W. R. Thompson, On the likelihood that one unknown probability exceeds another in view of the evidence of two samples, Biometrika 25, 285 (1933).

[74] T. L. Lai and H. Robbins, Asymptotically efficient adaptive allocation rules, Adv. Appl. Math. 6, 4 (1985).

[75] R. Agrawal, Sample mean based index policies by $O (\log n)$ regret for the multi-armed bandit problem, Adv. Appl. Probab. 27, 1054 (1995).

[76] W. R. Thompson, On the theory of apportionment, Am. J. Math. 57, 450 (1935).

[77] S. L. Scott, A modern Bayesian look at the multi-armed bandit, Appl. Stoch. Model. Bus. Ind. 26, 639 (2010).

[78] E. Kaufmann, N. Korda, and R. Munos, Thompson sampling: An asymptotically optimal finite-time analysis, in Algorithmic Learning Theory, edited by Nader H. Bshouty, Gilles Stoltz, Nicolas Vayatis, and Thomas Zeugmann (Springer Berlin Heidelberg, Berlin, Heidelberg, 2012), pp. 199–213.

[79] A. Garivier, P. Ménard, and G. Stoltz, Explore first, exploit next: The true shape of regret in bandit problems, Math. Oper. Res. 44, 377 (2019).

[80] S. Bubeck, R. Munos, and G. Stoltz, Pure exploration in finitely-armed and continuous-armed bandits, Theor. Comput. Sci. 412, 1832 (2011).

[81] github.com/matibilkis/marek.git.