Can one measure $C$-odd asymmetry in $e^+e^- \to \pi^+\pi^-$? *

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C-odd asymmetry can be studied from an accurate measurement of the angular distribution due to the interference between the $S$- and $P$-waves in $e^+e^- \to \pi^+\pi^-$ at order $\alpha^3$. Though the integrated total cross-section is zero as expected from the Furry’s theorem, the asymmetry is dominated by the pion rescattering diagram which is enhanced by the presence of the $\ln s/m^2_e$ and is quite large ($\approx 11\%$ at $\theta = 30^0$ and $\sqrt{s} < M_{\pi\pi}$) compared to $\alpha/\pi \simeq 0.3\%$. This process can also be used for alternatively measuring the size of the rescattering term and the phase of the $S$-wave amplitude, but does not help to solve the present discrepancy between the hadronic spectral functions from $e^+e^-$ and $\tau$-decay data.

Introduction

• At present, experiments on colliding electron-positron beams with high-luminosity and high-statistics are carried out intensively. Low-energy region below 1 GeV, in the vicinity of the $J^{PC} = 1^{--}$, is measured with increasing precisions by the new generations of $e^+e^-$ experiments [1].

• On one hand, these precision measurements are motivated by the important contribution of this region to the anomalous magnetic moment of the muon where improved measurement is planned in the future BNL $g-2$ experiments [2] for further tests of the Standard Model (SM) and for eventually detecting new physics beyond the SM. At present, this project is, unfortunately, obscured by the present discrepancies between the $\pi^+\pi^-$ spectral function from $e^+e^-$ [13] and $\tau$-decay data [4], which affects the evaluation of the hadronic contribution to the muon anomaly [3678], and which are expected to be clarified by the future accurate data in this low-energy region.

• On the other, they are motivated by the improved measurements of the light meson parameters which are necessary for a better understanding of the structure and of the dynamics of these mesons, which are important for testing different QCD non-perturbative methods including lattice calculations and QCD spectral sum rules predictions [9].

• In this paper, we shall discuss the possibility of studying properties of scalar resonances (direct coupling to $\gamma\gamma$), and of related processes with positive $C$–parities ($\pi\pi$ non-resonant and rescattering S-wave amplitudes), in the low-energy region, which remains a long standing puzzle in non-perturbative QCD [10111213], and which may eventually help for solving the $e^+e^-$ and $\tau$-decay data discrepancy.

• Contrary to the negative $C$–parities $\rho$, $\omega$ and $\phi$ mesons, which are easily observed in the processes via a one-photon exchange (Fig. 1), the observation of processes with positive $C$–parities only occurs via two-photon intermediate state (Table I), which are a priori difficult to observe due to the additional QED coupling $\alpha^2$ suppression compared to the one in Fig. 1. However, one may (a priori) expect that the interference between the Born amplitude in Fig. 1 and the one in Table I can be reached at the present experimental accuracy because of the less power of $\alpha$ in the interference term relative to the diagonal one.

• In the following, we plan to study this process, from the $C$-odd asymmetry angular distribution:

$$A_{PS} = \frac{d\sigma(\theta)}{d\Omega} - \frac{d\sigma(\pi - \theta)}{d\Omega} / \frac{d\sigma(\theta)}{d\Omega}_{\text{Born}},$$

(1)

which can differentiate the $C$-even and $C$-odd parities (the Born cross-section refers to the process in Fig. 1). Unlike the $C$-even process, this observable requires the detection of the charge of the final state particle, which, in the present case, is the pion. An analogous observable has been discussed in the pure QED process $e^+e^- \to \mu^+\mu^-$ [14], and in $e^+e^- \to f_2 \to \pi^+\pi^-$ [14] Here, we consider, instead, the interference between the usual one-photon exchange amplitude involving the $P$-wave $\pi^+\pi^-$ shown in Fig. 1

$$e^+e^- \to \gamma \to \pi^+\pi^-,$$

(2)

with the $S$-wave amplitude shown in Table I

$$e^+e^- \to \gamma \gamma \to \pi^+\pi^-,$$

(3)

which dominates over the $D$-wave one below $M_{f_2}=1.27$ GeV [3]. The measurement of the $\pi^+\pi^-$ process is certainly less accurate than the pure QED final state $\mu^+\mu^-$, though one expects that it can be reached with a good accuracy at the present and forthcoming high-statistic

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4 We plan to reconsider this process in a future work.

5 This feature has been explicitly checked in $\gamma\gamma$ scattering processes [1213].
and the pion coupling to $\gamma\gamma$ are introduced through the interaction terms:

\[ \mathcal{L}_{S\gamma\gamma} = g_{S\gamma\gamma} S F^{(1)}_{\mu\nu} F^{\mu\nu}_{\nu}, \quad \mathcal{L}_{S\pi\pi} = g_{S\pi\pi} S \pi^+ \pi^-, \quad \text{(5)} \]

and:

\[ \mathcal{L}_{\pi\gamma\gamma} = \frac{1}{2} g_{\pi\gamma\gamma} \pi^0 \epsilon^{\mu\nu\rho\sigma} F^{(1)}_{\mu\nu} F^{\rho\sigma}_{\nu}, \quad \text{(6)} \]

where $F^{(i)}_{\mu\nu}$ is the photon field strength. The model dependence enters into the size of the couplings, which are normalized as in [11] and fixed from the data. In the range of energy where we are working, we use a vector meson dominance model (VDM) by replacing, with a good approximation, the virtual photon propagator by the ones of vector mesons. This good VDM approximation being confirmed by the observation of the $\rho$-dominance of the $e^+e^- \rightarrow \gamma \rightarrow \pi^+\pi^-$ cross-section below 1.27 GeV. Using VDM, the Born s-channel amplitude in Eq. (2) is shown in Fig. 1 and reads:

\[ M_{\text{Born}} = e^2 v(l') \left( \frac{p' - \bar{p}}{q^2 + i\epsilon} \right) u(l) F_{\pi}(q^2), \quad \text{(7)} \]

where: $(l + l')^2 = (p + p')^2 = q^2 \equiv s$, and

\[ |F_{\pi}(s)|^2 \simeq \frac{M_\rho^4 (1 + \Gamma_\rho^2/M_\rho^2)^2}{|s - M_\rho^2 - iM_\rho\Gamma_\rho|^2}, \quad \text{(8)} \]

is the square of pion form factor normalized as $|F_{\pi}(s = 0)|^2 = 1$. This expression leads to the well-known angular distribution:

\[ \frac{d\sigma}{d\Omega} \bigg|_{\text{Born}} \simeq \alpha^2 \frac{|\bar{p}|^3}{s^2 \sqrt{s}} \sin^2 \theta |F_{\pi}(s)|^2, \quad \text{(9)} \]

where $\theta$ is the polar angle between the electron beam and the outgoing charged negative pion.

### Evaluation of the interference amplitudes

- For this purpose, we use the chiral Lagrangian in Eq. (4), and the couplings given previously in Eqs. (5) and (6).
- We compute directly the interference term between the lowest-order Born amplitude in Eq. (7) and shown in Fig. 1 with the $\gamma\gamma$ diagrams given in Table 1. The Feynman rules for deriving the amplitude are standard. Using a Vector Meson Dominance Model (VDM), the photon line is replaced by the $\rho$-meson form factor.
- Due to the complexity of the calculation, we evaluate the trace of Dirac matrices with the 	extit{FeynCalc} program [19] linked to 	extit{Mathematica} for expressing the results in terms of the Passarino-Veltman–t Hooft (PAVE) integrals [20]. The analytic expressions are lengthy, which are not appropriate to present in this letter. However, they can be send by demand.
- We obtain the final results by computing numerically, either with 	extit{Fortran} or with 	extit{Mathematica}, these different PAVE integrals using the 	extit{LoopTools} program [21].
- To the order we are working, we find that the contributions of the contact term and of the scalar-meson

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Table 1

| Diagrams contributing to the process in Eq. (3) |
|------------------------------------------------|
| 1. Contact term and Scalar-meson s-channel exchange |
| 2. One $\pi$- or $V$-meson exchange in the t channel |

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8Footnote: This is the case of the $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \mu^+\mu^-\gamma$ processes calculated in Ref. [1], where both contributions (real and virtual photon processes) should be added for cancelling IR divergences. We should note that the process which we shall calculate is very similar to the one: $\gamma\gamma \rightarrow \pi^+\pi^-$ which comes from a similar box diagram. The $\gamma\gamma$ process does not also need the inclusion of real photon contribution for being meaningful.

9Footnote: We shall work to leading order of these couplings.

10Footnote: Inclusion of resonances into chiral lagrangian has been discussed in the literature (see e.g. [12,18]).
exchange in Table I are zero. These null contributions are due to the Lorentz structure of the vertex couplings and are model-independent, where the process is proportional to \( l^2 = l'^2 = m^2 \) as expected from symmetry arguments of the \( \gamma \gamma \) amplitude.

- The contributions of the last two diagrams in Table I with one pion or one vector meson exchange (within VDM) in the t-channel is UV and IR (photon mass taken to zero) finite, which is a remarkable property. The vanishing of the UV divergence is obvious for each box diagram due to the form of numbers of propagators and of the algebraic form of the numerators. We have checked this UV convergence by checking the absence of the \( 2/e \) pole or equivalently by introducing a scale \( \mu_{\text{new}} \equiv e^{2\gamma_{\text{Euler}}} \mu_{\text{old}} \). The result is invariant when changing \( \mu \) in a large range. The vanishing of the IR singularities is less trivial, which is due to a fine reorganization of the different PAVE loop integrals, and is expected in the absence of virtual photons in the t-channel [21]. We have checked the absence of the IR divergence by giving a mass \( \lambda \) to the photon and by varying it in a large range. As expected, the result is independent of \( \lambda \). The previous successful UV and IR numerical tests following the recommendation in the LoopTools user’s manual [21], are a good indication on the reliability of our results, which (indirectly)indicate that, in the process which we have considered, we have not missed some other diagrams to this order.

- Using the experimental value of the dominant \( \omega \pi \gamma \) coupling for the t-channel vector-meson (V) exchange, one also finds that this contribution is negligible \( (10^{-3}) \) of the one of pion exchange). It also indicates that the \( \pi^0 \pi^0 \) production dominated by this contribution is unobservable.

### Angular distribution, and C-odd Asymmetry

- We show in Fig. 2a) the angular distribution including radiative corrections and compared with the corresponding Born term given in Eq. (9) at the value of \( \sqrt{s} = 0.5 \) GeV. The C-odd asymmetry \( A_{PS} \) defined in Eq. (1) versus \( \theta \) in degree.

![](image)

**Figure 2.** a) Angular distribution \( d\sigma/d\Omega \) in units of nanobarn versus \( \theta \) (polar angle between \( e^- \) and \( \pi^- \)) in degree at \( \sqrt{s}=0.5 \) GeV; the continuous line is the Born contribution; the dashed line is the one including radiative corrections; b) The asymmetry \( A_{PS} \) defined in Eq. (1) versus \( \theta \) in degree.

in Eq. (1) is given in Fig. 2) using the previous value \( \sqrt{s} = 0.5 \) GeV.

- The radiative correction is asymmetric for \( \theta \) and \( \pi - \theta \), while it is maximal at small and large angles.

- At \( \theta = 30^0 \) and \( \sqrt{s}=0.5 \) GeV, the correction to the angular distribution is about 5.5%, i.e., 11% for \( A_{PS} \), which is relatively large compared with the naive counting \( (\alpha/\pi) \approx 0.3\% \) and may be observed with improved accurate data.

- We check that the \( \ln^2 (s/m^2) \) contribution is zero. We fit numerically the coefficient of the \( \ln (s/m^2) \) term expected from the electron exchange in the t-channel [21]. We found the functional dependence at \( \sqrt{s} = 0.5 \) GeV and \( \theta = 30^0 \):

\[
\frac{d\sigma}{d\Omega} \approx \frac{d\sigma}{d\Omega} \bigg|_{\text{Born}} \left\{ 1 + \left( \frac{\alpha}{\pi} \right) \left[ 2.0 \ln \frac{s}{m^2} - 1.5 \right] \right\}, \quad (10)
\]

where the numerical coefficients contain \( m^2 \) and \( M^2 \) terms.

- Eq. (10) shows the huge contribution from the \( \ln (s/m^2) \)-term. The presence of the \( \ln (s/m^2) \) term seems to be a general feature of a QED calculation with a large external momentum \( s \) and a virtual light particle (electron) appearing in a loop. A classical example is the QED calculation of the 2nd order correction due to electron loop for the muon \( g-2 \), where a log \( (m_{\mu}/m_e) \) appears (\( m_{\mu} \) is here the external momentum). Fixing again \( \theta = 30^0 \), we study the relative strength of the radiative corrections for \( \sqrt{s} < M_{T_2} = 1.27 \) GeV, where it is expected that VDM provides a good approximate description of the data. We notice that \( A_{PS} \) is almost unaffected by the change of the \( s \)-values in this range of energy.

### Isospin symmetry, \( \tau \)-decay and the \( g-2 \) of leptons

The important rôle of the \( \ln (s/m^2) \) term present in the expression of \( A_{PS} \) [Eq. (10)] can indicate that the effect of the C-odd asymmetry is more pronounced in \( e^+e^- \rightarrow \pi^+\pi^- \) differential cross-section than in the \( \tau^- \rightarrow \nu_\tau \pi^0\pi^- \) differential decay rate. For checking this result, we evaluate the similar process for \( \tau \)-decay. We found that the box diagram with internal \( \tau, W \) and \( \gamma \) lines behaves like \( \ln (s/M^2) \), while the one with internal \( \nu_\tau, W \) and \( Z \) lines vanishes in the chiral limit \( m^2 \rightarrow 0 \). The two contributions are relatively negligible compared to the electron case due to the \( W, Z \) propagator suppressions in the box diagram calculation. The difference between the strength of the two processes indicates a violation of the isospin symmetry rotation for the asymmetry \( A_{PS} \). However, the effects of the interference term vanish in the integrated total cross-section, which is expected from the Furry theorem [10]. Though this result cross-checks the validity of the results obtained in this paper, it (unfortunately) does not help to explain the present discrepancy between the hadronic spectral functions extracted from \( e^+e^- \rightarrow \pi^+\pi^- \) hadron total cross-section and the one from \( \tau^- \rightarrow \nu_\tau \pi^0\pi^- \) total decay rate, which are expected to be equal in the \( SU(2) \) isospin symmetry limit [14]. In fact, these spectral functions play a crucial rôle in the present evaluation of the lepton anomalous magnetic \( a_l \equiv 1/2(g_l - 2) \) [5–8].

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10As mentioned earlier, the contact term and the s-channel contributions shown in Table I also vanish.

11Some effects of I=0 scalar mesons which only contribute to \( e^+e^- \) but not to \( \tau \)-decay have been also analyzed in [7], where the contributions are tiny and do not solve the present discrepancy between the two data.
Conclusions

• We begin this paper by wondering if one can measure the C-odd asymmetry \( A_{PS} \) in \( e^+e^- \rightarrow \pi^+\pi^- \). This project may be realized at enough small polar angle between the electron beam and outgoing \( \pi^- \) in high-statistic and high-precision present and future experiments with a good pion identification.

• The dominance of the pion rescattering contribution indicates that contrary to the \( \gamma\gamma \) and \( \pi\pi \) scattering processes, it is possible to disentangle, for this process, the pion rescattering contribution from the scalar and vector meson exchanges. This feature being relevant in e.g. the analytic K-matrix model discussed in [12,13], where one can separate the direct coupling of the scalar resonance to \( \gamma\gamma \) from the rescattering contribution. The null contribution of the \( \sigma \) exchange in the s-channel, i.e. of the \( I = 0 \) part of the S-wave amplitude, may indicate that the non-zero contributions from the box diagram are only due to the \( I = 2 \) part of the S-wave amplitude.

• \( A_{PS} \) can also serve for alternatively measuring the \( S \)-wave phase which can be compared with the one obtained from elastic \( \pi\pi \) scattering via Watson theorem.

• Due to the important rôle of the \( \ln(s/m_\pi^2) \) term, \( A_{PS} \) due to the rescattering process may not be observed in the reaction involving heavy leptons such as \( \tau \rightarrow \nu_\tau\pi^0\pi^- \), where \( m_\tau \) is (naively) replaced by \( M_\tau \).

• Finally, the vanishing of the interference term in the total cross-section, as expected from the Furry theorem, cross-checks the validity and reliability of our results. Unfortunately, this feature does not help to explain the present discrepancy between the hadronic spectral functions extracted from \( e^+e^- \rightarrow \pi^+\pi^- \) total cross-section and the one from \( \tau^- \rightarrow \nu_\tau\pi^0\pi^- \) total decay rate which govern the hadronic contribution to the lepton anomalous magnetic moment \( a_\ell \).

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