The Variance of QSO Counts in Cells

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Abstract

From three quasar samples with a total of 1038 objects in the redshift range $1.0 \div 2.2$ we measure the variance $\sigma^2$ of counts in cells of volume $V_u$. By a maximum likelihood analysis applied separately on these samples we obtain estimates of $\sigma^2(\ell)$, with $\ell \equiv V_u^{1/3}$. The analysis from a single catalog for $\ell = 40 \, h^{-1} \, \text{Mpc}$ and from a suitable average over the three catalogs for $\ell = 60$, $80$ and $100 \, h^{-1} \, \text{Mpc}$, gives $\sigma^2(\ell) = 0.46^{+0.27}_{-0.27}$, $0.18^{+0.14}_{-0.15}$, $0.05^{+0.14}_{-0.05}$ and $0.12^{+0.13}_{-0.12}$, respectively, where the 70% confidence ranges account for both sampling errors and statistical fluctuations in the counts. This allows a comparison of QSO clustering on large scales with analogous data recently obtained both for optical and IRAS galaxies: QSOs seem to be more clustered than these galaxies by a biasing factor $b_{\text{QSO}}/b_{\text{gal}} \sim 1.4 - 2.3$.

*Subject headings:* galaxies: clustering — quasars: general, surveys — large-scale structure of the universe
1. Introduction

Only in recent years the rapid growth of quasar surveys has made possible the analysis of their clustering properties. The availability of faint quasar samples, with their high surface density and size, has allowed a detailed study at scales \( r \leq 150 \, h^{-1} \) Mpc (e.g. Shanks et al. 1987; Anderson, Kunth, & Sargent 1988; Iovino & Shaver 1988; Andreani, Cristiani, & La Franca 1991). There is now substantial agreement on the results of the quasar two-point correlation function \( \xi(r) \). This function is larger than unity at scales \( r < 10 \, h^{-1} \) Mpc, but the issue of its evolution with redshift is still matter of debate (Iovino, Shaver, & Cristiani 1991; Boyle et al. 1991; Andreani & Cristiani 1992).

In this work we analyze QSO clustering by means of the variance of counts in cells. The advantage of this method is to provide information on clustering at various scales (i.e. various cell sizes), even when the volume covered by the catalog does not form a connected region; this is particularly useful for the available QSO samples. Statistics of counts in cells have been recently considered by various authors (e.g. Efstathiou et al. 1990; Saunders et al. 1991; Loveday et al. 1992; Gaztañaga 1992; Bouchet et al. 1993), to obtain reliable constraints on the amplitude of galaxy clustering on different scales, through the variance, and on its possible deviations from a Gaussian behavior, through higher order moments such as the skewness. On the other hand, it is relatively easier, within a model for structure formation, to obtain theoretical predictions for the moments of counts in cells at various scales. Moreover, this kind of analysis, combined with similar studies performed for optical and IRAS galaxies, allows a direct determination of the biasing factor relating the clustering of QSOs with that of these classes of objects.

After shot–noise subtraction, the variance of the continuous density fluctuation field, smoothed over the cell size \( \ell \), is related to the spatial correlation function \( \xi(r) \) by the integral

\[
\sigma^2(\ell) = \int_0^\infty dr \, r^2 \xi(r) \mathcal{F}_\ell(r),
\]

where the window function \( \mathcal{F}_\ell(r) \) takes into account the details of the cell geometry. For spherical cells of radius \( R \), one finds

\[
\mathcal{F}_R(r) = \frac{18}{\pi R^3} \int_0^\infty dx \, j_1^2(x) \, j_0(xr/R) \approx \frac{3}{R^3} \vartheta_H(R - r),
\]

where \( j_\ell \) are spherical Bessel functions of order \( \ell \) and \( \vartheta_H(x) \) is the Heaviside function (which is zero for \( x < 0 \) and one for \( x > 0 \)). These relations allow to connect the results
of this work with previous data on the quasar–quasar correlation function. Actually, the two methods are complementary: the variance yields a more compact information on the clustering amplitude at the scale of the cell–size, while the correlation function gives a more detailed geometrical information. Being a volume average of the correlation function, the variance is characterized by a higher signal–to–noise ratio.

2. Data Samples and Statistical Analysis

Table I lists our database, which consists of eight different surveys already published. Table I reports the sample name (column 1), the effective covered area (column 2), the limiting magnitude (column 3), the number of objects with \( M_B \leq -23 \) (column 4), within the assumed redshift range (column 5), and the number of objects between redshift 1 and 2.2 (column 6).

The samples contain objects selected with different techniques: UV–excess, variability and slitless spectroscopy. Attention has been paid to use only complete catalogues, in order to minimize systematic biases. The optimal redshift range for our statistical study is 1–2.2: this is because the highest QSO number density is in this redshift range and the catalog completeness decreases beyond \( z = 2.2 \).

In spite of the different catalog selection criteria, the high completeness in the considered redshift range allows to subdivide our database in three groups (named Sample A, B and C, in the following) on the only basis of their limiting magnitude; each of these samples will then be characterized by its own selection function. Sample A (510 objects): APM; Sample B (332 objects): Boyle et al. (1990), \( m_J \leq 21 \) sample (hereafter HVI) from Hawkins & Véron (1993), Zitelli et al. (1992) and Osmer & Hewett (1991), all cut at the limiting magnitude \( m_J = 20.85 \), which leads to a 2.5% decrease in the number of objects; Sample C (122 objects): La Franca et al. (1992), \( m_J \leq 19.5 \) sample (hereafter HVII) from Hawkins & Véron (1993) and Crampton et al. (1989), cut at \( m_J = 19.5 \), with a 34% decrease in the number. The actual limiting magnitude has been chosen slightly different for each sample, to take into account the different galactic extinction. The \( B \) magnitudes of La Franca et al. (1992) have been converted to \( J \) magnitudes, according to the relation \( m_J = m_B - 0.05 \).

To compute the moments of QSO counts in cells we first divide our three samples

\[ \text{The absolute } B \text{ magnitude is calculated assuming Hubble constant } H_0 = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}^{-1}, \] with \( h = 0.5 \) in a flat universe, with vanishing cosmological constant; \( k \)-corrections are as in Cristiani & Vio (1990) and galactic extinction as in Burstein & Heiles (1982).
in shells with mean radii \( r_a \) centered on the observer, further divided in \( M_a \) cells of volume \( V_u \). Let \( N_j \) be the number of objects in the \( j \)-th cell \((j = 1, \ldots, M_a)\) of a given shell and \( V_j \leq V_u \) the cell volume actually included in the sample boundary, estimated by means of a standard Monte Carlo technique. Cells with \( V_j < 0.5V_u \) have not been used.

In calculating the variance of counts in cells we had to account for the volume incompleteness of our samples. Following Efstathiou et al. (1990) we write

\[
\sigma_a^2 \equiv \Sigma_a^2 = \frac{\sum_j (N_j - V_j \bar{N}_a/V_u)^2 - (1 - \sum_k V_k^2/\sum_k V_k^2) \sum_j N_j}{(\bar{N}_a/V_u)^2 \left[ \sum_k V_k^2 - 2 \sum_k V_k^3/\sum_k V_k + (\sum_k V_k^2)^2/(\sum_k V_k)^2 \right]},
\]

where \( \Sigma_a^2 = (\Delta N/\bar{N}_a)^2 \) and \( \bar{N}_a = V_u \sum_j N_j/\sum_j V_j \) is the expected number of objects in a cell of volume \( V_u \) belonging to the \( a \)-th shell. The shot–noise subtraction in Eq.(3) may result in negative values for the estimates of \( \sigma_a^2 \) (see, e.g., Figure 1 in Efstathiou et al. 1990): this is because the Poisson model only approximately describes discreteness effects. In this sense one can safely state that Eq.(3) represents an estimate of the excess variance above the Poisson level; this also assumes that the expected variance is independent of the cell–volume, which is the case if the missing volume in incomplete cells is small, given that \( \sigma_a^2 \) is likely to be a weak function of cell size.

When using different catalogs grouped within a sample (as in Samples B and C), even if the selection method is the same, the effects of systematic errors (e.g. in the zero point of the magnitude calibration) have to be considered. We have therefore normalized the different catalogs of Samples B and C by selecting as a reference catalog the one with the highest surface density in the sample and reducing the effective cell volumes of the remaining ones by the ratio of their surface density (derived from Table I) to that of the reference catalog. In this way, we expect to have removed the above mentioned systematic effects, leaving a bias, if any, in the sense of underestimating the variance.

Errors in the estimate of the variance, \( \text{Var}(\Sigma^2) \), are computed by the quadratic sum of two terms: a first one, \( \text{Var}_{\text{samp}}(\Sigma^2) \), accounting for the sampling errors inherent in our data, and a second one, \( \text{Var}_{\text{stat}}(\Sigma^2) \), corresponding to the statistical uncertainty. In order to quantify the sampling errors in our data we used a bootstrap resampling technique (e.g. Barrow, Bhavsar, & Sonoda 1984) in each separate sub–sample, accounting for the different densities. The second contribution to the variance of \( \Sigma_a^2 \) can be estimated under the simplifying assumptions that the cells are independent; making
then use of the Central Limit Theorem one can approximate the underlying distribution by a Gaussian with variance \( \sigma_a^2 = \Sigma_a^2 \) (see Efstathiou et al. 1990). This results in

\[
\text{Var}_{\text{stat}}(\Sigma_a^2) = \frac{2 (1 + \sigma_a^2) + 4 N_a \sigma_a^2 + 2 (N_a)^2 \sigma_a^4}{M_a (N_a)^2}.
\]

(4)

This method of calculating \( \text{Var}(\Sigma^2) \) allows to deal with catalogs characterized by both reduced number of cells (such as Sample C) and dilution effects (Sample A): in the former case the larger contribution comes from the theoretical variance, in the latter one from the bootstrap error. Note, however, that this method leads to a more conservative estimate of error bars, which result typically higher than in previous analyses of the variance of counts in cells.

The final variance, \( \sigma^2 \), for the cell counts of QSOs at a given scale, separately for samples A, B and C, is obtained by maximizing the likelihood function:

\[
\mathcal{L}(\sigma^2) = \prod_a \frac{1}{[2\pi \text{Var}(\Sigma_a^2)]^{1/2}} \exp\left[-\frac{(\sigma_a^2 - \sigma^2)^2}{2 \text{Var}(\Sigma_a^2)}\right],
\]

(5)

where the product extends over all shells.

3. Results and Discussion

We report the results for the variance of counts in cells of sizes \( \ell \equiv V_{\ell}^{1/3} = 60, 80 \) and \( 100 \ h^{-1} \) Mpc. For sample B, which is the one of highest density, we can also consider cells of size \( \ell = 40 \ h^{-1} \) Mpc. All these cells are obtained with parallelepiped–shaped geometry, with line–of–sight dimension larger than the transversal ones by a factor of 1.55, in order to better follow the geometry of the catalogs.

Figures 1, 2 and 3 show, for the considered cell–sizes, the variance of QSO counts in cells for Sample A, B and C respectively, obtained from Eq.(3), with error bars given by \( \text{Var}(\Sigma^2) \). The maximum likelihood estimates of the variance as a function of the cell–size for the three samples separately are reported in Table II; the 70% errors are obtained by computing the values of the variance where the likelihood in Eq.(5) drops by a factor of 1.71 from its maximum. When the lowest value becomes negative we consistently replace it with zero. Table II also shows the \( \chi^2 \) values and the number of radial shells \( N_s \) for each determination.

Having consistently computed three independent estimates of \( \sigma^2 \) at various scales, the maximum likelihood method [Eq.(5)] can now be used to estimate the overall variance, considering all samples together, at 60, 80 and \( 100 \ h^{-1} \) Mpc: we find
\( \sigma^2 = 0.18^{+0.14}_{-0.15}, 0.05^{+0.14}_{-0.05} \) and \( 0.12^{+0.13}_{-0.12} \), (70% confidence range), respectively. We can also compare these data with the estimate of the variance resulting from the QSO correlation function, \( \xi(r) \), obtained from Sample A, B and C, separately, according to the methods described in Andreani, Cristiani, & La Franca (1991). We fit a spline to \( \xi(r) \) and numerically integrate Eq.(1) and Eq.(2), with a spherical top-hat filter of equivalent volume. The results and the errors, obtained by bootstrap resampling, are shown in Figure 4 together with the maximum likelihood estimates of \( \sigma^2 \) from the counts. Within the (70%) error bars, the two methods provide compatible results, although the values of \( \sigma^2 \) derived from the counts in Sample B are systematically higher.

In order to evaluate the possible redshift dependence of the clustering amplitude we calculated the variance for the two separated redshift ranges \( 1 \leq z \leq 1.6 \) and \( 1.6 \leq z \leq 2.2 \). We found that the results are compatible with a constant comoving clustering amplitude within the error bars (e.g. Andreani & Cristiani 1992), although a slight tendency to have larger variance in the nearest strip occurs in Sample B and C.

Given the size of the error bars, which in some cases make the results compatible with no clustering, we decided to check the presence of real clustering in our data by performing a Kolmogorov–Smirnov test against the null hypothesis that our counts are drawn from a Poisson parent distribution. To this aim we generated 10,000 mock Poisson catalogs with the same density, selection function and volume coverage, separately for Sample A, B, and C. We then compared the resulting histograms of the cell counts with the real ones. They are shown in Figure 5, where the dotted lines correspond to histograms of the counts in cell for the Poissonian case and the solid lines to those of the real data for the three samples and different cell sizes. The Kolmogorov–Smirnov test indicates that the Poisson hypothesis can be rejected at a high confidence level \( 10^{-20} \) at 40 \( h^{-1} \) Mpc and down to \( 10^{-2} \) at 100 \( h^{-1} \) Mpc, with the only exception of Sample A, for which the Poissonian hypothesis cannot be rejected; in this case indeed we found that the bootstrap errors dominate the overall variance of \( \Sigma^2 \).

Our results can be compared with those for the variance of IRAS galaxies in the QDOT sample, analyzed by Efstathiou et al. (1990); they find \( \sigma^2 = 0.21^{+0.11}_{-0.07} \) and \( 0.05^{+0.06}_{-0.03} \), for cubic cells of size 40 and 60 \( h^{-1} \) Mpc respectively. Loveday et al. (1992), who performed a similar analysis in the Stromlo–APM redshift survey of optical galaxies, obtain \( \sigma^2 = 0.14^{+0.08}_{-0.05}, 0.05^{+0.06}_{-0.03} \) and \( 0.02^{+0.05}_{-0.01} \), for cubic cells of size 40, 60 and 75 \( h^{-1} \) Mpc, respectively. A recent estimate is given by Bouchet et al. (1993) for the 1.2 Jy
IRAS Galaxy Redshift Survey; they get the best-fit \(\log \sigma^2(R) = (1.17 \pm 0.05) - (1.59 \pm 0.06) \log R\), for spherical cells of radius \(R\). This corresponds to \(\sigma^2 \approx 0.09\) and 0.05, for \(\ell = 40\) and 60 \(h^{-1}\) Mpc respectively (having accounted for the different geometry of the cells). Note that the (95\%) confidence ranges quoted by Efstathiou et al. (1990) and Loveday et al. (1992) are obtained by considering only the theoretical part of the error, i.e. neglecting sampling fluctuations, whilst we made the more conservative choice of summing up the two uncertainties.

Our data are compatible, within the errors, with all results above. Nevertheless, it could be argued that QSOs are biased over both IRAS and optical galaxies; we find \(b_{QSO}/b_{gal} = \sigma_{QSO}/\sigma_{gal}\) in the range 1.4 – 2.3. This effect is indeed predicted by hierarchical theories of quasar formation within massive haloes (Efstathiou & Rees 1988; Cole & Kaiser 1989; Haehnelt & Rees 1993), although the amplitude of such a bias strongly depends on the specific model of structure formation. This issue clearly deserves more realistic modeling of quasar origin, also taking into account the recent observational constraints from large-scale structures, such as the normalization implied by COBE data (Smoot et al. 1992). On the other hand, our statistical analysis shows that more stringent constraints on quasar clustering will only be obtained when new catalogs will be constructed with homogeneous selection criteria and over wider and deeper regions of the sky: a goal which can be reached within few years, thanks to the availability of multiobject spectrographs.

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Table I

Quasar surveys

| survey                  | surface\(^*\) sq. deg. | limiting magnitude | # objects\(^†\) | z range  | # object in 1 ≤ z ≤ 2.2 |
|-------------------------|--------------------------|--------------------|----------------|----------|--------------------------|
| APM\(^*\)              | 516                      | \(m_J \leq 18.5\)  | 1006           | 0.2–3.1  | 510                      |
| Boyle et al. (1990)     | 10.15\(^†\)             | \(m_J \leq 20.9\)  | 320            | 0.2–2.2  | 236                      |
| Crampton et al. (1989)  | 4.8                      | \(m_J \leq 20.5\)  | 135            | 0.2–3.1  | 87                       |
| HVI\(^\diamond\)       | 2                        | \(m_J \leq 21.0\)  | 29             | 0.3–2.2  | 24                       |
| HVII\(^\diamond\)      | 19                       | \(m_J \leq 19.5\)  | 66             | 0.3–2.2  | 40                       |
| La Franca et al. (1992) | 10                       | \(m_B \leq 19.9\)  | 95             | 0.35–2.2 | 63                       |
| Osmer & Hewett (1991)   | 6.1                      | \(m_J \leq 21.7\)  | 113            | 0.2–3.1  | 66                       |
| Zitelli et al. (1992)   | 0.69                     | \(m_J \leq 20.85\) | 21             | 0.6–2.8  | 12                       |

\(^*\) claimed effective area  
\(^†\) number of objects with \(M_B \leq -23\) in a \(h = 0.5\) and \(\Omega_0 = 1\) universe.  
\(^\diamond\) Foltz et al. (1987); Foltz et al. (1989); Hewett et al. (1991); Chaffee et al. (1991); Morris et al. (1991)  
\(^\dagger\) only 5 out of 8 fields have been used in this work  
\(^\diamond\) Hawkins & Véron (1993)
Table II

Variance from counts in cells

| Sample | $\ell$ ($h^{-1}$ Mpc) | $\sigma^2$ | $\sigma^2$ (70%) | $\chi^2/N_s$ |
|--------|-----------------------|------------|-------------------|---------------|
| A      | 60                    | 0.22       | 0.00 – 0.62       | 0.88/11       |
|        | 80                    | 0.00       | 0.00 – 0.32       | 0.32/8        |
|        | 100                   | 0.00       | 0.00 – 0.24       | 0.64/6        |
| B      | 40                    | 0.46       | 0.19 – 0.73       | 6.90/16       |
|        | 60                    | 0.27       | 0.07 – 0.47       | 2.72/11       |
|        | 80                    | 0.13       | 0.00 – 0.31       | 2.10/8        |
|        | 100                   | 0.20       | 0.00 – 0.40       | 2.09/6        |
| C      | 60                    | 0.00       | 0.00 – 0.25       | 4.39/11       |
|        | 80                    | 0.00       | 0.00 – 0.22       | 1.09/8        |
|        | 100                   | 0.10       | 0.00 – 0.37       | 0.38/6        |
Figure captions

**Figure 1.** The variance $\sigma^2$ in cells of size $\ell = 60$, 80 and 100 $h^{-1}$ Mpc as a function of redshift, for sample A. The error bars are one standard deviation, Var($\Sigma^2$). The solid line represents the maximum likelihood estimate of the variance; the dotted lines correspond to the 70\% confidence range.

**Figure 2.** The variance $\sigma^2$ in cells of size $\ell = 40$, 60, 80 and 100 $h^{-1}$ Mpc as a function of redshift, for sample B. Error bars and lines as in Figure 1.

**Figure 3.** The variance $\sigma^2$ in cells of size $\ell = 60$, 80 and 100 $h^{-1}$ Mpc as a function of redshift, for sample C. Error bars and lines as in Figure 1.

**Figure 4.** Comparison of two estimates of the QSO variance $\sigma^2$ for the three samples: filled squares refer to the estimate obtained from the counts in cells, open squares to the integral of the correlation function. Error bars give the 70\% confidence range. For clarity, the two different estimates are shown with a small horizontal shift.

**Figure 5.** Histograms of the counts in cells for the three separated samples A, B and C and for different cell sizes (from 40 to 100 $h^{-1}$ Mpc). The dotted lines correspond to the Poissonian case, while the solid ones to the real data. The hypothesis that the distribution of objects in cells is compatible with a Poissonian one can be rejected at a very high confidence level for samples B and C, but not for sample A.