A New Method for Deformation of B-spline Surfaces

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Abstract. This paper presents an efficient method for deforming B-spline surfaces, based on the surface energy minimization. Firstly, using an analogy between the B-spline surface patch and the thin-plate element of the finite element method, and applying external forces on the surface with some given geometric constraints, the forces can locate on part of the surface or the surface. Then, the energy of the B-spline surface can change with the change of the forces. Finally, a new B-spline surface is generated by solving an optimization problem of change of the energy. The forces can be a single force, a distributed force and set of isolated force. The method can accomplish easily local deformation and total deformation of the B-spline surface.

Introduction

B-spline surface is one of the most wisely used parametric surfaces in computer aided geometric design (CAD) and computer graphics (CG), such as ship hull design, automobile body design, etc. After creating B-spline surfaces, we often need to modify them to satisfy the design’s requirements. Many efforts have been made towards developing more convenient and effective methods for shape deformation of B-spline surfaces. Piegl[1,2] proposed two methods to modify the shape of B-spline surfaces from their mathematical representation: control-point-based modification and knot-based modification. Celniker and Gossard[3] presented a method to modify the B-spline surfaces interactively based on the energy optimization thinking, in this method energy of the surface is regarded as the objective function, all kinds of characteristic line are regarded as the geometric constraint, and external load is also considered. Celniker and Welch[4] investigated a method for constrained deformation of B-spline surfaces by using linear constraints and global energy function minimizing. Tzvetomir[5] proposed shape modification of non-uniform B-spline surfaces based on minimizing an energy function. Cheng siyuan[6] proposed a method to modify the shape of B-spline surface with geometric constraints.

In this paper, a new method for deforming the shape of B-spline surface with the physically based method. The designers can pick points that locate on a surface and move them to new locations by external forces that are associated with given geometric constraints.

Deformable model of a B-spline surface

A B-spline surface is defined by

\[ S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)P_{i,j} \]

Where \( P_{i,j} \) are the control points, \((n+1)\) and \((m+1)\) are the number of control points in \( u \) and \( v \) directions, \( N_{i,p}(u) \) and \( N_{j,q}(v) \) are the normalized B-spline basis function of order \( p \).
and \( q \) in the \( u \) and \( v \) direction, respectively. The basis functions are defined recursively by Cox-deBoor algorithm. For example, \( N_{i,p}(u) \) is defined by

\[
N_{i,p}(u) = \begin{cases} 
1 & \text{if } u_i < u < u_{i+1} \\
0 & \text{otherwise}
\end{cases}
\]

\[
N_{i,p}(u) = \frac{u-u_i}{u_{i+1}-u_i}N_{i,p-1}(u) + \frac{u_{i+p}-u}{u_{i+1}-u_i}N_{i+1,p-1}(u).
\]  

(2)

B-spline surface has nice properties that make it suitable representation form in computer aided design, such as convex hull, affine invariance, and local control property.

A B-spline surface is modeled as a metal plate in the physically based method and a deformable model cloud be obtained by applying geometric constraints on the surface. A B-spline surface deforms in a natural way when it is applied external forces and constraints. Celniker and Gossard\cite{2} suggested the following energy functional for B-spline surfaces

\[
E_s = \int \int [\alpha_{uu} (\partial^2 S/\partial u^2) + \alpha_{vv} (\partial^2 S/\partial v^2) + \beta_{uu} (\partial^2 S/\partial u^2)^2 + \beta_{vv} (\partial^2 S/\partial v^2)^2 + \beta_{uv} (\partial^2 S/\partial u \partial v)^2 - 2FS] dudv.
\]

(3)

Where \( S \) represents a B-spline surface, \( \alpha_{uu} \) and \( \alpha_{vv} \) are stretching stiffness in \( u \) and \( v \) direction, \( \beta_{uu} \) and \( \beta_{vv} \) are bending stiffness in \( u \) and \( v \) direction, respectively, while \( \beta_{uv} \) is cross bending stiffness in \( u \) and \( v \) directions, \( f \) is the applied external forces on the surface. The energy \( E_s \) in Eq.3 can be subdivided into two parts \( E_s = E_{int} - E_{ext} \):

\[
E_{int} = \int \int [\alpha_{uu} (\partial^2 S/\partial u^2) + \alpha_{vv} (\partial^2 S/\partial v^2) + \beta_{uu} (\partial^2 S/\partial u^2)^2 + \beta_{vv} (\partial^2 S/\partial v^2)^2 + \beta_{uv} (\partial^2 S/\partial u \partial v)^2] dudv.
\]

(4)

\[
E_{ext} = 2 \int \int FS dudv.
\]

(5)

Where \( E_{int} \) represents the energy of the surface itself and \( E_{ext} \) the energy due to the applied forces.

Substituting Eq.1 into Eq.4, the energy of the surface itself \( E_{int} \) is given by

\[
E_{int} = \int \int [\alpha_{uu} P^T (\partial^2 N/\partial u^2) P + \alpha_{vv} P^T (\partial^2 N/\partial v^2) P + \beta_{uu} P^T (\partial^2 N/\partial u^2)^2 + \beta_{vv} P^T (\partial^2 N/\partial v^2)^2 + \beta_{uv} P^T (\partial^2 N/\partial u \partial v)^2] dudv.
\]

(6)

\( P \) is independent of parameters \( u \) and \( v \), Eq.6 can be expressed as

\[
E_{int} = P^T K P.
\]

(7)

Where \( K \) is stiffness matrix and that can be expressed as

\[
K = \int \int [\alpha_{uu} (\partial^2 N/\partial u^2)^T (\partial^2 N/\partial u^2) + \alpha_{vv} (\partial^2 N/\partial v^2)^T (\partial^2 N/\partial v^2) + \beta_{uu} (\partial^2 N/\partial u^2)^T (\partial^2 N/\partial u^2)^2 + \beta_{vv} (\partial^2 N/\partial v^2)^T (\partial^2 N/\partial v^2)^2 + \beta_{uv} (\partial^2 N/\partial u \partial v)^T (\partial^2 N/\partial u \partial v)^2] dudv.
\]

(8)
Eq. 3 is now simplified to

\[ E_s = P^T K P - 2 \int\int_{\Omega} f S d u d v. \] \hspace{1cm} (9)

Minimizing \( E_s \) will ensure the smoothness and the fairness of the result surface. Based on the principle of stationary potential energy, the minimum can be expressed as

\[ KP = \int\int_{\Omega} f (N)^T d u d v. \] \hspace{1cm} (10)

The Eq. 10 represents a deformable B-spline surface model and could be calculated by finite element method. Finite element model of a deformable B-spline surface is obtained by meshing a B-spline surface into small patches (element). So each B-spline surface patch is considered as an element. A typical B-spline surface patch and its corresponding finite element are shown in Fig. 1.

![Fig. 1 A B-spline surface patch](image)

The stiffness matrix \( K \) and the integral \( \int\int_{\Omega} f (N)^T d u d v \) in Eq. 10 can be calculated numerically using Gaussian quadrature[7]. The Eq. 10 can be solved with standard linear equation solver. Once the matrix \( P \) in Eq. 10 is determined that is the control points of the resulting surface, which satisfies the designer’s requirements.

**Practical Examples**

Several examples are presented in this section to verify the effects of the proposed method. The boundaries of the surface are fixed and the parameters of the initial B-spline surface (Fig. 2) are as follows:

\[
\begin{align*}
p &= 4, \quad q = 4, \quad m = 10, \quad n = 10 \\
U &= [0.0, 0.0, 0.0, 0.0, 0.0, 0.1, 0.3, 0.4, 0.6, 0.8, 0.9, 1.0, 1.0, 1.0, 1.0] \\
V &= [0.0, 0.0, 0.0, 0.0, 0.0, 0.1, 0.3, 0.4, 0.6, 0.8, 0.9, 1.0, 1.0, 1.0, 1.0].
\end{align*}
\] \hspace{1cm} (11)

![Fig. 2 The original B-spline surface](image)

The first example is that a B-spline surface is deformed based on external applied forces solely. There are two types for this problem. One type is to apply a single point force on the surface; the other is to apply a distributed force on the surface. Fig. 3 (a) shows a single point force to deform the surface, the parametric position location of the force is (0.4, 0.6), the value of the force is 1000N. Fig. 3 (b) shows a distributed force, the value of the force is 5000N.
The other example is that a B-spline surface is deformed based on external applied forces with an area constraint. The constraint divides the surface into the fixed zone and the moving zone. This can easily realize local deformation of the B-spline surface. Fig. 4 shows a distributed force with a square area constraint $0.4 \leq u, v \leq 0.6$ to deform the surface, the area is free to move and fixed show as (a) and (b), respectively.

Conclusions
This paper presents a new method for the deformation of the B-spline surface based on the surface energy minimization. The user applies external forces on the surface to accomplish its local deformation and total deformation with some given geometric constraints.

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