Some further validations and comparison of the Bearing Area Model (BAM) for adhesion of rough surfaces

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Abstract

In the present short note, we attempt further validations and comparisons of a recent simple model for the estimate for adhesion between elastic (hard) rough solids with Gaussian multiple scales of roughness, BAM (Bearing Area Model) belonging to a DMT class of models. In one case, we use the GJP (Generalized Johnson Parameter) model, which is an empirical fit validated on the same (and so far most extensive) set of data on which BAM was validated, namely that of Pastewka and Robbins. In the second case, we compare with another approximate DMT theory, that of Persson and Scaraggi, which turns out extremely close to the BAM model, despite much more complex: GJP however can lead to significant discrepancies.

Keywords:
Roughness, Adhesion, BAM model, GJP model, DMT model

1. Introduction

The science of adhesion starts with adhesion of ”hard” solids, neglecting any elasticity, Bradley found in 1932 (Bradley, 1932) the first pull-off force between a spherical particle and a flat surface. The second solution only came some 40 years later, with JKR (from Johnson et al., 1971), and that solution differed only by a small prefactor from the Bradley solution, indicating in that case (of quadratic surfaces), almost no influence of the elastic behaviour, except that the elastic behaviour manifested itself in instabilities for pull-in and pull-off which provoke hysteresis (the hysteresis of the JKR solution was even very recently further investigated by Ciavarella et al (2017)). Since then,
many more aspects have been studied and there has been an explosion of interest in adhesion, motivated also by the quest for small scale engineering, bioengineering and bio-inspired engineering. We know today that for “soft” bodies, adhesion shows instabilities like in the simple case of a single sinusoid in the so called JKR regime (Johnson 1995) which leads to a very strong hysteretic behavior, with wear adhesion resulting in some cases and strong adhesion in others, depending on a single parameter, $\alpha_1$, defined in terms of surface energy $w$

$$\alpha_1 = \sqrt{\frac{2}{\pi^2}} \frac{w\lambda}{E^* h^2}$$

(1)

for a single sinusoid (this is why we use the subscript ”1”) of wavelength and amplitude $\lambda, h$, and where $\alpha$ represents the square of the ratio of the surface energy in one wavelength to the elastic strain energy when the wave is flattened. Here, $E^*$ is elastic modulus in plane strain, where $1/E^* = 1/E^*_1 + 1/E^*_2$ and $E^*_i = E_i/(1-\nu^2_i)$ where $E, \nu$ are Young’s modulus and Poisson’s ratio of each elastic body. It turns out that for $\alpha > 0.57$, there is a spontaneous snap into full contact, and from this state, detachment should occur only at values of stress close to theoretical strength. We shall return to this parameter, as we have recently generalized it for a multiscale surface — in what we called the ”Generalized Johnson Parameter” (GJP) model (Ciavarella & Papangelo, 2017a). Turning back to historical developments, JKR was found to be the correct limit for high Tabor parameters in the case of a sphere of radius $R$ (Tabor, 1977)

$$\mu = \left(\frac{Rw^2}{E^*^2 \Delta r^3}\right)^{1/3} = \left(\frac{Rl_a^2}{\Delta r^3}\right)^{1/3}$$

(2)

where $l_a = w/E^*$ is an alternative way to measure adhesion as a length scale, and $\Delta r$ is the range of attraction, which for a crystal is of the order of atomic spacing $a_0$. The DMT approximations originally developed for the spherical problem (Derjaguin et al., 1975), instead holds for $\mu \to 0$. In DMT, the contact is assumed to be split into ”repulsive” contact areas and ”attractive” contact areas, and no effect of tensile tractions occurs within the repulsive contact area. This opens the possibility to apply sophisticated solutions using the DMT idea to Persson’s detailed approximate solution of the repulsive problem (Persson & Scaraggi, 2014), and we shall return on this model to make some comparisons and validations. DMT leads to possibly serious errors for cylinders and spheres, unless the Tabor parameter is really
low, and it is still quite unclear what is the degree of approximation in general rough contact (Ciavarella, 2017).

If the simple problem of the sphere has taken so long to be completely understood, it is not surprising that the case with roughness, starting from the ideal case of two nominally flat surfaces, is still quite remote from being solved. The effect of roughness is in fact quite less obvious than it seems. For very soft and large bodies (JKR regime), and special types of roughness, the effect of roughness could be even to enhance adhesion instead of reducing it as it is common (Guduru, 2007), but this remains a rather special condition. However, at the other extreme, like at nanoscales and for hard solids, very simple equations like Rumpf-Rabinowicz (Rumpf, 1990, Rabinovich et al., 2000) assuming no elasticity at all, work very well for the spherical geometry, and show large reduction with very small amplitude of roughness. These findings are for example confirmed by extensive AFM experiments by Jacobs et al (2013) where from atomic corrugation up to a few nm, the measured work of adhesion was found to decrease by more than an order of magnitude. This naturally also raise the very delicate issue of what is “work of adhesion” in experimental measurements of adhesion forces.

This significant dependence on rms amplitude of roughness is relatively in agreement with classical theories which attempt to consider roughness with ”asperities” and seemed to have been confirmed in simple experiments with low modulus materials like smooth rubber lenses against roughened surfaces (Fuller and Tabor, 1975). However, classical asperity models like those of Fuller and Tabor are questionable for the modern view of ”random fractal” surfaces, and more in general it is a problem that no reliable estimates can be made of quantities like real contact area, mean slope or mean curvature of surfaces (see Ciavarella and Papangelo, 2017b): the problem of sensitiveness to ”small scale” truncation creates a big effect in asperity models, as their adhesion stickiness parameter includes the radius of asperities, which clearly would go to zero in the fractal limit, making stickiness impossible for any fractal dimension (in the fractal limit), a result which is contradicted also in the models we are about to discuss.

Unfortunately, with the advent of fractal surfaces, the emphasis shifted from the well defined rms amplitude of asperities heights $h_{rms}$, which is the most easily measured quantity, and in most common situations reflects the reduction of stickiness, to much more sophisticated quantities, and no theory at present is able to clarify the general behaviour of rough contacts under
adhesive conditions, not even in terms of orders of magnitude estimate, which justifies the attempts the two simple models we are about to discuss, BAM (Bearing Area Model, Ciavarella, 2017a), and GJP (Ciavarella & Papangelo, 2017a), which at least permit a full exploration of the parameters of the problem to be done analytically with simple equations.

Numerical investigations are also extremely demanding and still not very many in the literature, with the notable exception of Pastewka & Robbins (2014) who consider self-affine rough surfaces with 3 orders of magnitude in wavelengths from nano/atomic scale wavelengths to microscale: however, Pastewka & Robbins (2014, PR in the following) seemed to reach quite strikingly strange conclusion: that stickiness (as indicated by the area-load slope) should be independent on rms amplitude of asperities heights $h_{rms}$, and only depend on a parameter which combines the rms slopes and rms curvatures. This conclusion was examined by the present authors in a number of papers (Ciavarella, 2016, 2017a,b, Ciavarella and Papangelo, 2017c, Ciavarella, Papangelo and Afferrante, 2017), but perhaps the problem was just that they looked at loading curves, instead of the much more significant unloading curves, especially since their data on pull-off in the Supplementary Material part of the paper do not seem to indicate the same dependence of ”stickiness” on rms quantities. Indeed, we have used mostly the data on pull-off in subsequent modelling. Although it is true that, as remarked by PR, their pull-off data decay (Fig. S3) do not correlate well with classical Fuller-Tabor asperity model predictions by various orders of magnitude, it is also true that they do not correlate well with the PR parameter (actually worse), and this motivated the implementation of the BAM and GJP models.

The present paper therefore has three purposes:-

1) to review and further validate BAM and GJP with a new set of data, namely with Persson and Scaraggi (2014) paper

2) to compare BAM and GJP also outside the range of conditions for which they were both validated.

3) to suggest possible further research by making a number of observations, for example about force-separation laws

Notice that the GJP (Generalized Johnson Parameter, Ciavarella & Papangelo, 2017a), is not a proper ”model”, but simply postulates that pull-off should depend on a single parameter, by analogy with a single sinusoid case as we have discussed above. The actual dependence on this generalized $\alpha$ was simply fitted to the Pastewka-Robbins data, and found to be exponential, but even in these original data, the approximation was not excellent
(despite better than any other single parameter with which we compared). Therefore, we expect some significant error is possible, whereas the BAM model, being based on a reasonable DMT theory, should give results which are comparable to Persson and Scaraggi (2014) paper, at a much reduced cost, and the possible errors would require in general a very sophisticated numerical investigation, similar or better than Pastewka & Robbins (2014, PR), which is way beyond the scope of the present paper, and in general quite demanding.

2. A short review of BAM and GJP models

2.1. BAM

In a recent note, the BAM (Bearing Area Model) model was introduced by Ciavarella (2017a), a single-line equation estimate for adhesion between elastic (hard) rough solids with Gaussian multiple scales of roughness. BAM starts from the observation that the entire DMT solution for “hard” spheres (Tabor parameter tending to zero) assuming the Maugis law of attraction, is very easily obtained using the Hertzian load-indentation law and estimating the area of attraction as the increase of the bearing area geometrical intersection when the indentation is increased by the Maugis range of attraction (see Fig.1).
BAM shows that adhesion, as already well known, for hard solids at macroscopic scale is destroyed quite easily and the problem remains that contact area is a ill-defined “magnification” dependent quantity (Ciavarella and Papangelo, 2017b). When elastic modulus decreases sufficiently, observable adhesion may be possible, although then the assumptions of BAM may become questionable, in particular those of the DMT behaviour, which for a sphere would be low Tabor parameter — whereas Tabor parameter for a multiscale roughness can only be estimated, perhaps at the smallest scale, although this itself is not a solid and unique definition.

The BAM model, summing up repulsive (coming from Persson’s (2007) solution, corrected in a prefactor as in Papangelo, Hoffmann and Ciavarella (2017)) and attractive contributions which is estimated purely from geometrical considerations, in the case of low fractal dimension (which is where
Persson’s solution is simplest and shows no dependence on the small scale details, but is also the case of most practical interest, see Persson et al., 2005), gives
\[
\frac{\sigma(u)}{\sigma_{th}} \simeq \frac{3}{8\gamma} q_0 h_{rms} E^* \exp \left( -\frac{u}{\gamma h_{rms}} \right) - \frac{1}{2k} \times \left[ \text{Erfc} \left( \frac{u - \Delta r}{\sqrt{2} h_{rms}} \right) - \text{Erfc} \left( \frac{u}{\sqrt{2} h_{rms}} \right) \right]
\]
where \( \frac{\sigma(u)}{\sigma_{th}} \) is the ratio between the actual stress between the surfaces (compressive as positive) with respect to theoretical stress; \( q_0 \) is the short wavevector for a power-law fractal self-affine profile (a more general definition of the BAM model would require the use of the entire Power Spectrum Density of a surface, but this is not a problem).

This closed form result for the entire curve of pressure vs mean separation obviously results in a pull off if we find numerically the minimum as a function of \( u \). The equation depends only on \( h_{rms}, q_0 \) and no other aspect of Power Spectrum, so the pull-off depends only on these two quantities, and not on small scale details. In particular, notice that using the constant \( k \simeq 2 \) which is imposed to attempt to fit the complicated shape of adhesive zones in a rough contact which are rather elongated (see Pastewka-Robbins, 2014) and also to cover intermediate Tabor parameters, comes at the expense of modelling the very low \( h_{rms} \) as obviously the limit becomes \( \frac{\sigma(u)}{\sigma_{th}} \simeq -\frac{1}{k} \) and not \(-1\). The comparison with Pastewka-Robbins, 2014 data was rather satisfactory, as we shall see again when we make further comparisons below.

2.2. GJP

In another note (Ciavarella & Papangelo, 2017a), we introduced a ”generalized Johnson’s parameter”, which is
\[
\alpha(\zeta) = \frac{w}{U_{el}(\zeta)} = \frac{l_a}{\frac{\pi}{2} \int_{q_0}^{q_1} q^2 C(q) \, dq} = \frac{l_a}{l(\zeta)} \tag{4}
\]
where is indeed the generalized Johnson parameter for a multiscale surface, since the elastic strain energy to flatten the surface, \( U_{el}(\zeta) \), depends on the entire Power Spectrum Density (PSD) of the (isotropic) rough surface \( C(q) \), where the surface is considered up to the magnification \( \zeta = q_1/q_0 \)

\[\text{1}\text{The important point is that this ”truncation” which in mathematical terms is perfectly fine, but in practise is quite arbitrary assumption typical of many ”modern” fractal theories}\]
$q_0, q_1$ are the low cutoff and high wavevector cut-off of the ideal power-law spectrum (more general spectra require no difficult generalization). Notice that we have introduced an effective length of adhesion $l(\zeta)$, and see the derivation of eqt.8-9 of Persson (2002) for $U_{el}(\zeta)$.

Introducing this parameter, does not solve the problem. The novelty of the GJP note was the postulate that, as in sinusoidal case the pull-off value depends mainly on $\alpha$ (at intermediate range of Tabor parameters), the multiscale problem will depend mostly on a generalized Johnson $\alpha$ parameter. A comparison with other known single parameters (that of Fuller and Tabor, that of Pastewka and Robbins, and that of the $h_{rms}$ alone), proved that indeed this postulate was the best. We did not make a full comparison however with the BAM model. Indeed, for power law PSD and the usual case of $H > 0.5$ (low $D$) the integral converges quickly (see the original paper for details), and takes the limit value

$$l(\infty)_{lowD} = \frac{\pi h_0^2}{2\lambda_0} \frac{H}{2H-1}$$

which gives a very gentle dependence on Hurst exponent: the energy is mainly stored in the long wavelength components. Notice that there is no true threshold below which surfaces are "absolutely sticky" (probably because of energy barriers), whereas in practical terms we can define a threshold for them to be "absolutely unsticky".

3. Comparisons

For the typical fractal dimension $D = 2.2$ ($H = 0.8$), GJP becomes

$$\alpha = \frac{3}{4} \frac{2w/E}{q_0 h_{rms}^2}$$

and from the Ciavarella & Papangelo (2017a) fit,

$$\log \left| \frac{\sigma_{min}}{\sigma_{th}} \right| = -1.62 - \frac{2.14}{\alpha} = -1.62 - \frac{2.14}{0.6} \frac{q_0 h_{rms}^2}{2w/E}$$

including those of Persson for which we can look at a problem at different "magnifications", will not be always needed, as in the most important practical class of problems, those of low fractal dimension, this parameter converges.
which in particular for the majority of PR results has \( \frac{w}{E} = l_a = 0.05a_0 \), whereas \( q_0 = \frac{2\pi}{(2048a_0)} \)

\[
\log \left| \frac{\sigma_{\text{min}}}{\sigma_{\text{th}}} \right| = -1.62 - 0.088 \left( \frac{h_{\text{rms}}}{a_0} \right)^2
\]

whereas for \( \frac{w}{E} = l_a = 0.005a_0 \),

\[
\log \left| \frac{\sigma_{\text{min}}}{\sigma_{\text{th}}} \right| = -1.62 - 0.88 \left( \frac{h_{\text{rms}}}{a_0} \right)^2
\]

and BAM and GJP are compared in Fig.2: solid lines are GJP predictions, and cross symbols are for BAM (red for \( l_a = 0.05a_0 \), blue for \( l_a = 0.005a_0 \)), and notice that we have included also data for larger fractal dimensions (different symbols circles, squares and triangles indicate the fractal dimension), as well as open symbols which indicate larger rms slope \( h'_{\text{rms}} = 0.3 \) (closed symbols are for \( h'_{\text{rms}} = 0.1 \)). The fit of both BAM and GJP is reasonable, despite there seems to be a certain deviation for the low adhesion case for \( l_a = 0.005a_0 \) and the effect of rms slopes is not entirely clear.

Notice that both GJP and BAM would have a different prediction for larger fractal dimension, and while GJP has been tested also to include this effect (and has shown again reasonable prediction, within the limited number and possible accuracy of data), BAM would require the implementation of the fuller Persson’s solution and this has not been done, also because it is of limited practical interest.
Scales.

the fractal dimension in Fig. S3 does not appear in correct order, as a slow inversion of the data for higher rms amplitude, whereas the opposite trend should occur. Probably there is an

3.1. Extrapolations

We remain with the "Lennard-Jones" estimate of the potential, \( w/E = l_a/0.05a_0 \), but we now vary the small wavevector \( q_0 = 2\pi/(2048xa_0) \) where \( x \) is a variable to vary the PR case. In the GJP model, we obtain using the same fit of the PR data, and

\[
\log \left| \frac{\sigma_{\text{min}}}{\sigma_{\text{th}}} \right| \approx -1.62 - \frac{179}{2048x} \left( \frac{h_{\text{rms}}}{a_0} \right)^2
\]

which is easy to follow in the diagram\(^2\).

\(^2\)Surprisingly, in Fig.S3 there are some blue closed symbols \( (h_{\text{rms}}' = 0.1 \) and \( l_a/a_0 = 0.005) \), which appear curious, as they appear as non-sticky in Fig. 4 of the paper. Also, the fractal dimension in Fig. S3 does not appear in correct order, as low \( H \) seem to have higher rms amplitude, whereas the opposite trend should occur. Probably there is an inversion of the data for \( H = 0.3 \) and \( H = 0.8 \), which is however irrelevant for the present scopes.

Fig.2. Pull-off value decay with parameter. Data are shown in PR paper with the same symbols as they will be shown here, so \( h_{\text{rms}}' = 0.1, 0.3 \) (closed, open symbols), and for \( l_a/a_0 = 0.05, 0.005 \) (red, blue) — we omit the change in size of the symbols since \( q_0/a_0 \) increasing from 4 to 64 also corresponds to an increase of \( h_{\text{rms}} \) which is easy to follow in the diagram\(^2\).
scale), stickiness increases, as it is intuitive. Further, given the shape of
the curve, we can assume that stickiness is exhausted when \( \frac{\sigma_{\text{min}}}{\sigma_{\text{th}}} = 10^{-5} \) or
log \( 10^{-5} = -11.5 \), giving

\[
\left( \frac{h_{\text{rms}}}{a_0} \right)_{\text{thresh}} \simeq 10.61 \times 1^{1/2}
\]

which can be compared with the estimate made with the BAM model, which
instead requires a numerical routine to find the minimum of the BAM equa-
tion.

The comparison is shown in Fig.3, where we see that BAM (solid black
curve) and GJP (solid red line) compare reasonably well in terms of threshold,
but differ considerably at intermediate values. The disagreement at very low
rms amplitudes also comes from having assumed a simple exponential fit and
was judged reasonable given that amplitudes below the lattice spacing \( a_0 \) do
not make any sense.

![Fig.3. Pull-off value decay with different wavelength \( \lambda_0 \) (black solid lines are BAM model, red solid lines GJP) While the threshold for stickiness is almost perfectly coincident for the two models, significant deviations occur for the prediction of actual pull-off in the sticky region](image)

4. Further validations and comparisons
In the BAM model, we have assumed a Maugis type of potential, because that becomes very convenient for the extension to the random rough surfaces. In the classical spherical problem, the exact details of the force law are not important, and this was explained in very nice details in Barber (2013) due to similarity considerations in quadratic profiles for the rigid body, but it translates to a very good approximation also in the case of elastic spheres (but we maintain homogeneous halfspace). However, some caution should be exercised when modelling non-quadratic profiles. For a perfectly flat surface, i.e. in the limit of negligible roughness, indeed, the force law gives exactly the force for a given separation, and the details of the force give the actual minimum (theoretical strength), for a given surface energy (which is after all simply a measure of the integral of force-law from zero to infinity).

This should also be borne in mind in future simulations, where the details of the potential will matter. Pastewka & Robbins (2014) use a truncated spline representation of their force-law, which we have attempted to model with a Maugis law in the BAM model.

Persson & Scaraggi (2014) introduce a pure DMT solution, which makes use of the elaborate Persson’s solution for repulsive (adhesionless) contact, which includes the full distribution of separations (with multiple recursive integrals have to be done, and then further convoluted with the force separation law, i.e. integrated, to estimate the attractive force), and not just the macroscopic force-separation law as we have used in BAM. They use a purely adhesive potential (since in DMT the repulsion is taken care by Signorini boundary condition with zero separation in the contact zone) which imitates the Lennard-Jones potential

\[
p_a^+ (u) = B^+ \left[ \left( \frac{\Delta r}{u + \Delta r} \right)^n - \beta \left( \frac{\Delta r}{u + \Delta r} \right)^m \right]
\]  

and most of their results are for \( \beta = 0 \), where \( B^+ \) is adjusted to make the integral equal to the given surface energy. Here, \( n = 3 \) resembles van der Waals attractive law. Obviously for \( \beta = 0 \), \( p_a^+ (0) = B^+ \) is by construction the theoretical strength, and since \( B^+ = \frac{w}{\Delta r} (n - 1) \) there is a very significant (linear) dependence on the value of \( n \). Notice that for Maugis as we use in BAM, \( \sigma_{th} \approx \frac{w}{\Delta r} \) and this would seem to correspond to \( B^+ \) when \( n = 2 \) — however, this is obviously only the correspondence of the peak values, but when roughness will be present, details may matter. But we shall see that this coincidence seems indeed to correspond to much better agreement of the full Persson-Scaraggi DMT model with the simple BAM.
For their calculations, Persson & Scaraggi use $\Delta r = 4 \times 10^{-10} m$, and $E^* = \frac{4}{3} TPa$, so notice that $\frac{w}{\Delta r} = w / (E^* \Delta r) = 0.1 / (\frac{4}{3} 10^{12} \times 4 \times 10^{-10}) = 1.88 \times 10^{-4}$ which is 2 orders of magnitude smaller the value expected for Lennard Jones of 0.05. This law in the case of $\beta = 0$ introduces a spike at ”zero separation” (theoretical strength $\sigma_{th}$) which would tend to increase the adhesive force, given in a state of repulsive contact there are significant regions at near zero separations and indeed the probability distribution is singular there (see Figure 9 of Almqvist et al., 2011). Their comparison with Persson’s own JKR theory seems to indicate a very significant difference (and the JKR theory seems limited to positive loads anyway, so there is no comparison for the most important part of an adhesive solution) – part of this difference is intrinsic in the assumptions: in particular, Fig. 6 shows contact area results, where for DMT the area is purely repulsive, and for JKR it is both repulsive and attractive. However, in some cases this is not sufficient to explain why repulsive DMT areas are sometimes higher than the total JKR areas. A full check of DMT solution is missing in their paper because the full numerical solution is limited to very small bandwidths, presumably as computational cost for a true multiscale surface solved with a non-linear adhesive BEM code is still too demanding for present computers, and indeed Pastewka & Robbins (2014), are probably the most advanced still today. Unfortunately, Persson and Scaraggi’s method is not trivial to implement as is BAM, nor the code is available to the public. However, we can make some qualitative comparisons using the calculations they report in a few cases. Their surfaces have roll-off and this would require using more sophisticated version of Persson’s repulsive solution. Indeed, the most important contribution to roll-off is to make surfaces more Gaussian, and it does not contribute significantly to the rms amplitude nor to the repulsive solution, so we can neglect the roll-off region and use the pure power-law solution, for the sake of simplicity. Hence, low wavevector is $q_0 = 10^6$, rms amplitude is $h_{rms} = 6 \times 10^{-10} m$. With the GJP model, we obtain

$$\log \left| \frac{\sigma_{min}}{\sigma_{th}} \right| = -1.62 - \frac{2.14}{0.6} \frac{10^6 (0.6 \times 10^{-9})^2}{2} \frac{4}{3} \frac{10^{12}}{w} = -1.62 - 0.684/w$$

(10)

whereas BAM requires to find numerically the minimum of the force-separation.

A comparison of the predictions with the Persson and Scaraggi’s results is in Tab.1, where we see that GJP prediction is actually rather poor (there are 1-2 orders of magnitude differences with higher pull-off in the Persson-Scaraggi’s
estimate) whereas BAM is rather close and very reasonable, considering the much simpler implementation of a single closed form equation. Notice we have removed the factor $k = 2$ of the original BAM model, which results in a more realistic trend towards the very low amplitudes of roughness, tending to theoretical strength. However, notice that the fit of Persson-Scaraggi’s data would improve for some data, and not for others.

![Graph](image)

Fig.4. Pull-off value decay for results from Persson & Scaraggi (2014) Fig.7 for different surface energies $w$ (solid filled circles), and comparison with BAM (black solid lines) with reasonable agreement. Instead, comparison with GJP (empty circles) shows less good agreement.

| $w$ [J/m$^2$] | $\sigma_{th}$ [GPa] | $\sigma_{min}/\sigma_{th}$ (PS) | $\sigma_{min}/\sigma_{th}$ (GJP) | $\sigma_{min}/\sigma_{th}$ (BAM) |
|-------|--------|----------------|----------------|----------------|
| 0.1   | 0.5    | 0.04           | $2.12 \times 10^{-4}$ | 0.08           |
| 0.2   | 1      | 0.12           | $6.5 \times 10^{-3}$ | 0.132          |
| 0.3   | 1.5    | 0.18           | $2.02 \times 10^{-2}$ | 0.157          |
| 0.4   | 2      | 0.225          | $3.6 \times 10^{-2}$ | 0.173          |

Tab.1 - Some results from Persson & Scaraggi (2014) Fig.7, and comparison with GJP and BAM
Tab.2 - Some results from Persson & Scaraggi (2014) Fig.8, $w = 0.3 J/m^2$.

For comparison with GJP and BAM see Tab.1

\[ |\sigma_{\min}/\sigma_{\text{th}}| (GJP) = 2.02 \times 10^{-2} \text{ and } |\sigma_{\min}/\sigma_{\text{th}}| (BAM) = 0.157 \]

Tab.2 shows that the choice of the power law in the adhesive force-separation law is not indifferent in the results obtained by Persson and Scaraggi, although its effect is not dramatic and the prediction of BAM extremely close to the case $n = 2$, which incidentally we suspected was the closest to a Maugis potential in the beginning of the paragraph.

5. Conclusion

We have compared two simple models for pull-off of hard elastic solids for low Tabor parameters, one based on a new geometrical variant of the DMT solution for the sphere, and another based on the postulate that pull-off should depend only on the ratio of surface energy and elastic strain energy to flatten the surface. We have seen that in the range of data from Pastewka-Robbins for which the data were calibrated, there is obviously very similar predictive capability. However, outside this range, the difference may be larger, despite the threshold of stickiness seems to be given by reasonably very close results.

Indeed, validating the models for an independent set of data, those obtained with the Persson and Scaraggi’s DMT model, we found that the BAM model seems very similar to the much more complex Persson and Scaraggi’s DMT model, and therefore is most promising. GJP instead, which was an empirical fit after all, shows worryingly large differences in the case of Persson and Scaraggi’s DMT paper, with respect to both BAM and Persson and Scaraggi.

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