Spectral scrambling in Coulomb-blockade quantum dots

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Abstract

We study the fluctuations of an energy level as a function of the number of electrons $m$ added to a Coulomb-blockade quantum dot. A microscopic calculation in the limit of Koopmans’ theorem predicts that the standard deviation of these fluctuations behaves as $\sqrt{m}$ in the absence of surface charge but is linear in $m$ when the effect of a surface charge in a finite geometry is included. The microscopic results are compared to a parametric random-matrix approach. We estimate the number of electrons it takes to scramble the spectrum completely in terms of the interaction strength, the dimensionless Thouless conductance, and the symmetry class.

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The simplest model for describing a quantum dot in the Coulomb-blockade regime is the constant interaction (CI) model [1], in which the electrons occupy single-particle levels and the Coulomb interaction is taken as an average electrostatic energy that depends only on the number of electrons. When the single-particle dynamics in the dot are chaotic (or diffusive), one can apply random matrix theory (RMT) to describe the statistical properties of the single-particle wave functions and energies within an energy window whose width is the Thouless energy. RMT was successful in describing the conductance peak height distributions and their sensitivity to time-reversal symmetry [2,3]. However, other measured statistics, most notably the peak spacing distribution [4], indicated that it is necessary to include interaction effects beyond the simple CI model [5]. The best way to take into account interactions while retaining a single-particle framework is the Hartree-Fock (HF) approximation, which has been used to explain some of the observed features of the peak spacing statistics [5].

The HF single-particle wave functions and energies are calculated self-consistently and therefore can change as electrons are added to the dot. In the statistical regime (i.e., in a chaotic or diffusive dot), this phenomenon is called scrambling. Scrambling was observed in the decay of correlations between the $m$-th excited state in the dot and the ground-state of a dot with an additional $m$ electrons [7], and indirectly through the saturation of the peak-to-peak correlations as a function of temperature [8,9]. A phenomenological way to describe scrambling is to consider a discrete set of Hamiltonians (corresponding to the different number of electrons in the dot) that are random but have the correct symmetries. Such a set is known as a discrete Gaussian process (GP), and can be embedded in a continuous GP, i.e., random matrices that depend on a continuous parameter [10]. This parametric approach leads to a nearly Gaussian peak spacing distribution [11], and explains the saturation of the number of correlated peaks versus temperature [12]. While the parametric approach is appealing in its simplicity, it is not clear how well it describes features obtained in a microscopic approach. For example, in the parametric approach, the fluctuation standard deviation (FSD) of a typical level upon the addition of $m$ electrons into the dot is propor-
tional to $m$. One can also evaluate the FSD of a level in the microscopic approach through the fluctuations of the diagonal interaction matrix elements (in the limit where the single-particle wave functions do not change with the addition of electrons, hence the off-diagonal matrix elements can be neglected). When only the bulk screened Coulomb interaction is accounted, we find that the associated FSD behaves like $\sqrt{m}$, unlike the standard parametric approach. However, in the finite geometry of a quantum dot, the level fluctuation may be dominated by the variation of the mean-field potential arising from surface charge. We shall show that in this case the FSD is linear in $m$ (for $1 \ll m \ll g$, where $g$ is the dimensionless Thouless conductance), in overall agreement with parametric RMT. The main results of the microscopic approach are summarized in Eqs. (8), (12) and (13) in the absence of surface charge and Eqs. (16), (18) and (19) in the presence of surface charge. Extrapolating our results to larger values of $m$, we also estimate the dependence of the number of added electrons $m_c$ required for complete scrambling on $g$ and the interaction strength both in the absence and presence of surface charge (Eqs. (14) and (21), respectively).

Scrambling implies variation of both eigenstates and energy levels. However, in this work we approach the problem from the limit where the wave functions do not change (Koopmans’ theorem [12]), and we only estimate the FSD of energy levels. In the parametric approach this limit is valid when the change in the parameter upon the addition of $m$ electrons is small compared to the mean parametric distance between avoided crossings. Complete scrambling occurs when the energy fluctuations become comparable to $\Delta$ (the mean-level spacing in the middle of the band).

We first discuss the parametric approach. The variation of the single-particle energies and eigenfunctions (e.g., in a mean-field approximation) with the addition of electrons into the dot is described by a parametric variation of the Hamiltonian [13]. We denote by $H(x_N)$ the effective single-particle Hamiltonian of the dot with $N$ electrons. We assume that the dot is either diffusive or ballistic with chaotic dynamics, and that the statistical properties of $H(x_N)$ are not modified by the interactions. Restricting ourselves to the universal regime, i.e., to $\sim g$ levels in the vicinity of the Fermi energy [14], we assume that $H(x_N)$ belongs to
one of the Gaussian ensembles of random matrices whose symmetry class \( \beta \) is independent of \( x_N \). The sequence of Hamiltonians \( H(x_N) \) forms a discrete GP that can be embedded in a continuous GP \( H(x) \) \[^{[18]}\]. A simple GP is given by \[^{[10,19]}\] \[ H(x) = \cos xH_1 + \sin xH_2, \]

where \( H_1 \) and \( H_2 \) are \( N \times N \) uncorrelated random matrices chosen from the Gaussian ensemble of symmetry class \( \beta \): \[ P(H) \propto e^{-\beta a^2 \text{Tr}H^2}. \]

We choose \( a = (2/N)^{1/2} \zeta \) so that the average level density (a semicircle) has a constant band width of \( 2a\sqrt{2N} = 4\zeta \) and the mean-level spacing in the middle of the spectrum is \( \Delta = \pi a/\sqrt{2N} = \pi \zeta/N \). The average distance between avoided crossings is given by the inverse of the rms level velocity \( \delta x_c = \Delta [(\partial \epsilon_\alpha/\partial x)^2]^{-1/2} = \pi (\beta/2N)^{1/2} \), where \( \epsilon_\alpha \) is an energy level. This distance is larger for the GUE by a factor of \( \sqrt{2} \) compared with the GOE. Two-point parametric correlators become universal when the parameter \( x \) is measured in units of \( \delta x_c \); i.e., as a function of a scaled parameter \( \bar{x} \equiv x/\delta x_c \).

The energy levels \( \epsilon_\alpha \) scramble as the parameter \( \bar{x} \) changes: \[ \delta \epsilon_\alpha = \epsilon_\alpha(\bar{x} + \delta \bar{x}) - \epsilon_\alpha(\bar{x}). \]

The variance of \( \delta \epsilon_\alpha \) is estimated in the limit \( \delta \bar{x} \ll 1 \) to be \[ \sigma^2(\delta \epsilon_\alpha) = \Delta^2 (\delta \bar{x})^2 \] using first order perturbation theory in \( \delta \bar{x} \) (i.e., ignoring the change of the single-particle wave function as \( \bar{x} \to \bar{x} + \delta \bar{x} \)).

In the parametric approach it is assumed that \( \bar{x} \) changes by \( \delta \bar{x}_1 \) upon the addition of one electron into the dot (independently of the number of electrons \( N \)). Thus for the addition of \( m \) electrons \( \delta \bar{x}_m = m\delta \bar{x}_1 \), and as long as \( \delta \bar{x}_m \ll 1 \), we can still use first order perturbation theory to find

\[ \sigma(\delta \epsilon_\alpha^{(m)}) = \Delta m \delta \bar{x}_1 = m\sigma(\delta \epsilon_\alpha^{(1)}) , \]

(1)

where \( \delta \epsilon_\alpha^{(m)} \equiv \epsilon_\alpha^{(N+m)} - \epsilon_\alpha^{(N)} \) (\( \epsilon_\alpha^{(N)} \) is the energy of level \( \alpha \) in a dot with \( N \) electrons).

To relate the parametric approach to a microscopic mean-field approach, we describe the effect that adding electrons has on a particular single-particle HF level. The Hamiltonian of the dot is

\[ H = \sum_{ij} h_{ij}^{(0)} a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl} v_{ijkl} a_i^\dagger a_i^\dagger a_j a_k \] ,

(2)
where $h^{(0)}$ is the one-body part (accounting for the disorder or the chaotic confining potential) and $v_{ij;kl}^A \equiv \langle ij|v|kl \rangle - \langle ij|v|lk \rangle$ are the antisymmetrized matrix elements of the two-body interaction. The HF Hamiltonian in the self-consistent basis for $N$ electrons is given by

$$h^{(N)}_{\alpha\gamma} = \epsilon_{\alpha\gamma}^{(0)} + \sum_{\delta=1}^{N} v_{\alpha\delta;\gamma\delta}^A \quad \text{(where the sum is over the lowest $N$ occupied levels)},$$

and the eigenvalues of $h^{(N)}_{\alpha\gamma}$ are the HF single-particle energies $\epsilon_{\alpha}^{(N)}$. The diagonal part of the Hamiltonian (2) in the HF basis is

$$H_{\text{diagonal}} = \sum_{\alpha} \epsilon_{\alpha\alpha}^{(0)} \hat{n}_\alpha + \frac{1}{2} \sum_{\alpha,\gamma} v_{\alpha\gamma}^A \hat{n}_\alpha \hat{n}_\gamma \quad \text{(3)},$$

where $\hat{n}_\alpha$ is the number operator for single-particle state $\alpha$ and $v_{\alpha\gamma}^A \equiv v_{\alpha\gamma;\alpha\gamma}$.

In Koopmans’ limit, the single-particle wave functions do not change when an electron is added to the dot. Consequently, the change in the single-particle energy $\epsilon_{\alpha}$ when an electron is added to the $N+1$-st level is given by a diagonal interaction matrix element

$$\delta \epsilon_{\alpha}^{(1)} \equiv \epsilon_{\alpha}^{(N+1)} - \epsilon_{\alpha}^{(N)} \approx v_{\alpha N+1}^A \quad \text{(4)}.$$

Assuming the HF wave functions satisfy similar statistics as in the non-interacting case, we can estimate the variance of the matrix element in (4). The two-body interaction used in the HF approximation is often an effective interaction, e.g., an RPA screened interaction [16]. Alternatively, one can employ a short-range dressed interaction [17] $v(r-r') = \lambda \Delta V \delta(r-r')$, where $\lambda$ is the interaction strength and $V$ is the system’s volume. A diagonal interaction matrix element is then given by $v_{\alpha\gamma} = \lambda \Delta V \int dr \psi_{\alpha}^*(r) \psi_{\gamma}^*(r) \psi_{\alpha}(r) \psi_{\gamma}(r)$, and its variance

$$\sigma^2(v_{\alpha\gamma}) = \lambda^2 \Delta^2 V^2 \int dr_1 \int dr_2 \left[ \langle \psi_{\alpha}^*(r_1) \psi_{\gamma}^*(r_1) \psi_{\alpha}(r_1) \psi_{\gamma}(r_1) \psi_{\alpha}(r_2) \psi_{\gamma}(r_2) \psi_{\alpha}^*(r_2) \psi_{\gamma}^*(r_2) \rangle - \langle \psi_{\alpha}^*(r_1) \psi_{\gamma}^*(r_1) \psi_{\alpha}(r_1) \psi_{\gamma}(r_1) \rangle \langle \psi_{\alpha}(r_2) \psi_{\gamma}(r_2) \psi_{\alpha}^*(r_2) \psi_{\gamma}^*(r_2) \rangle \right] \quad \text{(5)}.$$

Only the connected part of the ensemble average over the product of eight wave functions contributes to the r.h.s. of (5). In the following we restrict our calculations to the GUE symmetry. All possible pairwise and quadruplet-wise contractions should be taken into account, and they can be regrouped to give
\[
\sigma^2(v_{\alpha \gamma}) = \frac{\lambda^2 \Delta^2 V^2}{V} \int dr_1 \int dr_2 \left\{ \left[ \langle \psi^*_{\alpha}(r_1) \psi_\alpha(r_1) \psi_\alpha^*(r_2) \psi^*_{\alpha}(r_2) \rangle - \frac{1}{V^4} \right]
+ \left[ \langle \psi^*_{\alpha}(r_1) \psi_\alpha(r_1) \psi_\gamma(r_2) \psi^*_{\alpha}(r_2) \rangle - \frac{1}{V^4} \right]
+ \left[ \langle \psi^*_{\alpha}(r_1) \psi_\alpha(r_2) \psi_\gamma(r_2) \psi^*_{\alpha}(r_2) \rangle \right] \right\},
\]

(6)

where the 1/V^4 factors are subtracted to avoid double counting of pairwise contractions (here \langle \ldots \rangle denotes an ensemble average and not just the connected part).

Wave-function correlations appearing in Eq. (5) were calculated in Ref. [20,21]. Including contributions of order 1/g^2 and assuming that the distance between levels \alpha and \gamma is less than the Thouless energy, we have

\[
\sigma^2(v_{\alpha \gamma}) = \frac{\lambda^2 \Delta^2}{V^2} \int dr_1 \int dr_2 \left\{ [1 + k_d(r)(1 + \Pi(r_1, r_2))] + \Pi(r_1, r_2) + b(r_1, r_2)]^2 - 1
+ [1 + k_d(r)] \Pi(r_1, r_2) + c(r_1, r_2)]^2 - 1 + [k_d(r) + \Pi(r_1, r_2)]^2 \right\}.
\]

(7)

The function \( k_d(r) \) \( (r = r_1 - r_2) \) is given by \( k_d(r) \equiv (\pi \nu)^2 \langle \text{Im} G(r) \rangle^2 \) (where \( G^R \) is the retarded Green function and \( \nu \) the electron density of states per unit volume) and describes the short-range correlations. In 2D \( k_d(r) = \exp(-r/\ell)J_0^2(k_F r) \) (where \( \ell \) is the mean free path and \( J_0 \) is a Bessel function), while in 3D \( k_d(r) = \exp(-r/\ell)(\sin k_F r/k_F r)^2 \). \( \Pi(r_1, r_2) = (\pi \nu)^{-1} \sum_Q \phi_Q(r_1) \phi_Q(r_2)/DQ^2 \) is the long-range diffuson propagator where \( \phi_Q \) is the eigenfunction of the diffusion operator corresponding to the eigenvalue \( DQ^2 \) (\( D \) is the diffusion constant). The functions \( b, c \) in Eq. (7) are of order 1/g^2 (\( c \approx \Pi^2(r_1, r_2)/2 \) for \( \ell \ll r \ll L \) [21] where \( L \) is the linear size of the dot). The main contributions to (6) arise from the long-range terms \( \Pi^2 \) \( (\int dr_1 \Pi(r_1, r_2) = 0 \) and does not contribute\). Using the normalization conditions \( \int dr_1 \int dr_2 \langle \psi^*_{\alpha}(r_1) \psi_\alpha(r_1) \psi_\alpha^*(r_2) \psi^*_{\alpha}(r_2) \rangle = 1 \) and \( \int dr_1 \int dr_2 \langle \psi^*_{\alpha}(r_1) \psi_\alpha(r_1) \psi_\gamma(r_2) \psi^*_{\alpha}(r_2) \rangle = 1 \), it follows that \( k_d(r)(1 + \Pi(r_1, r_2)) + b(r_1, r_2)] = 0 \) and \( \int dr_1 \int dr_2 [1 + k_d(r)\Pi(r_1, r_2) + c(r_1, r_2)] = 0 \), respectively. Thus the functions \( b \) and \( c \) do not contribute to order 1/g^2 and

\[
\sigma^2(v_{\alpha \gamma}) = 2\kappa (\lambda \Delta/g)^2,
\]

(8)
where $g = 2\pi \nu DL^{d-2}$ is the dimensionless Thouless conductance. For a cube in $d$ dimensions,

$$\kappa = \frac{4}{\pi} \sum_{n \neq 0} 1/|n|^4,$$

where the summation is over $n = (n_1, \ldots, n_d)$ with $n_i$ non-negative integers $[n \neq (0, \ldots, 0)]$.

We next consider the addition of $m$ electrons into the dot. Extending Koopmans’ limit to $m$ electrons, i.e., assuming the single-particle wave functions do not change with the addition of $m$ electrons, we can express the change in a single-particle level $\alpha$ as

$$\delta \epsilon^{(m)}_\alpha \equiv \epsilon^{(N+m)}_\alpha - \epsilon^{(N)}_\alpha \approx \sum_{i=1}^{m} v^A_{\alpha N+i}.$$ (9)

For $m < g$, the variance of (9) is given by

$$\sigma^2(\delta \epsilon^{(m)}_\alpha) = m\sigma^2(\epsilon_{\alpha \gamma}) + m(m-1)(v_{\alpha \gamma} v_{\alpha \delta} - v_{\alpha \gamma} v_{\alpha \delta})$$

(since the variances and covariances are independent of the particular wave functions). In analogy to Eq. (6), the covariance of $v_{\alpha \gamma}$ and $v_{\alpha \delta}$ is given by

$$\frac{v_{\alpha \gamma} v_{\alpha \delta} - v_{\alpha \gamma} v_{\alpha \delta}}{\lambda^2 \Delta^2 V^2} \int dr_1 \int dr_2 \left\{ \left[ \psi^*_\alpha(r_1) \psi_\alpha(r_1) \psi_\delta(r_2) \psi^*_\delta(r_2) \right] - \frac{1}{V^4} \right\} ,$$ (10)

and using the known wave-function correlations

$$\frac{v_{\alpha \gamma} v_{\alpha \delta} - v_{\alpha \gamma} v_{\alpha \delta}}{\lambda^2 \Delta^2 V^2} \int dr_1 \int dr_2 \left\{ [1 + k_d(r)(1 + \Pi(r_1, r_2)) + \Pi(r_1, r_2) + b(r_1, r_2)] \right.$$ 

$$\times \left[ 1 + k_d(r)\Pi(r_1, r_2) + c(r_1, r_2) \right] - 1 + [1 + k_d(r)\Pi(r_1, r_2) + c(r_1, r_2)]^2 - 1 \right\} .$$ (11)

Terms such as $k_d^2\Pi$, $k_d\Pi^2$ and $k_d c$ in Eq. (11) contribute $\sim (\ell/L)^2 \lambda^2 \Delta^2 / g^3$ and behave like $\sim \lambda^2 \Delta^2 / g^3$ in the ballistic limit $\ell \sim L$. Other terms contribute $\sim 1/g^4$. Thus to leading order in $1/g^2$

$$\frac{v_{\alpha \gamma} v_{\alpha \delta} - v_{\alpha \gamma} v_{\alpha \delta}}{\lambda^2 \Delta^2 V^2} \approx 0 .$$ (12)

Using Eqs. (9), (8) and (12) we find \cite{23}

$$\sigma^2(\delta \epsilon^{(m)}_\alpha) = m\sigma^2(\delta \epsilon^{(1)}_\alpha) = 2m\kappa(\lambda \Delta / g)^2 .$$ (13)
We conclude that the statistical fluctuations accumulated by sequentially adding electrons into the dot are added independently, namely, the FSD of an energy level behaves like $\sqrt{n}$, contrary to the parametric approach where it is linear in $m$ (see Eq. (I)).

A complete scrambling of the spectrum corresponds to a number of electrons $m_c$ for which $\sigma(\delta \epsilon^{(m_c)}) \sim \Delta$. Using (13) we find

$$m_c \sim \left( \frac{g}{\lambda} \right)^2 .$$

We note that since we have neglected wave-function scrambling due to off-diagonal interaction matrix elements, our estimate in Eq. (14) should only be regarded as an upper bound.

Next we consider the effect due to excess negative charge on the boundaries of the dot. The effective interaction due to such charge is $v(r, r') = V_\kappa(r) + V_\kappa(r')$ where $V_\kappa(r)$ describes the variation of the mean-field potential upon the addition of one electron. For a disk of radius $R$ in 2D we have $V_\kappa(r) = -\left( e^2 / 2\kappa R \right)(R^2 - r^2)^{-1/2}$ where $\kappa$ is the inverse screening length. The related interaction matrix element is now $v_{\alpha \gamma} = \int dr |\psi_\alpha(r)|^2 V_\kappa(r) + \int dr |\psi_\gamma(r)|^2 V_\kappa(r)$. Using the wavefunction correlations to order $1/g^2$, its variance is

$$\sigma^2(v_{\alpha \gamma}) = \frac{2}{V^2} \int dr_1 \int dr_2 \left[ (V^2 |\psi_\alpha(r_1)|^2 |\psi_\alpha(r_2)|^2) - 1 \right] + \left( V^2 |\psi_\alpha(r_1)|^2 |\psi_\beta(r_2)|^2 - 1 \right)$$

$$\times V_\kappa(r_1) V_\kappa(r_2)$$

$$\frac{4}{\beta V^2} \int dr_1 \int dr_2 \left[ k_d(r) [1 + (\beta + 2)] \Pi(r_1, r_2) / \beta \right]$$

$$+ \Pi(r_1, r_2) + b(r_1, r_2) + c(r_1, r_2) \} V_\kappa(r_1) V_\kappa(r_2) .$$

(15)

The leading order contribution to (15) comes from $\Pi(r_1, r_2)$. Using the expansion of $\Pi(r_1, r_2)$ in terms of the diffusion modes $\phi_Q$, the respective contribution is

$$\sigma^2(v_{\alpha \gamma}) = \frac{4}{\beta V^2} \frac{1}{\pi \nu} \sum_{Q \neq 0} \frac{[\int dr \phi_Q(r) V_\kappa(r)]^2}{DQ^2} = \frac{\lambda^2 \Delta^2}{\beta g} .$$

(16)

For a disk in 2D the coefficient $a = (2\pi)^{-1} \sum \phi_q \left[ \int dr \phi_q(r)(1 - r^2)^{-1/2} \right]^2 / q^2$, where $\phi_q$ are the diffusion modes in a disk of radius $R = 1$ ($Q = R^{-1}q$). The short-range term $k_d$ in (15) contributes a term $\sim (\ell / L)^2 (\Delta^2 / \beta g)$ that for a diffusive dot is parametrically small compared with (16), but in the ballistic limit its contribution be-
comes comparable to (16). The surface charge contribution to the variance (16) dominates the bulk contribution (8). The bulk-surface cross-correlation term is given by 

\[ (2\lambda \Delta /V^2) \int dr_1 \int dr_2 [V^2 \langle |\psi_\alpha(r_1)|^2 |\psi_\alpha(r_2)|^2 \rangle - 1] V_\alpha(r_2). \]

The dominating contribution due to \( k_d (\sim (\ell /L)^2 \Delta^2 /g) \) is suppressed in a diffusive dot but is comparable to (16) in the ballistic limit.

Similarly, the covariance of \( v_{\alpha \gamma} \) and \( v_{\alpha \delta} \) (due to surface charge) is evaluated to be

\[
\begin{align*}
\sigma^2 (\delta \epsilon^{(m)}) &= \frac{1}{2} m(m + 1) \sigma^2 (\delta \epsilon^{(1)}) = \frac{1}{2} m(m + 1) a \frac{\Delta^2}{\beta g} .
\end{align*}
\]

Thus, for \( m \gg 1 \), the FSD of a level is linear in the number of added electrons \( m \), in agreement with the result (1) of the parametric approach. The scrambling parameter \( \delta \bar{x}_1 \) can be determined by comparing the variance (11) in the parametric approach with the corresponding variance (19) in the microscopic approach

\[
\delta \bar{x}_1 = \left( \frac{a}{2\beta g} \right)^{1/2} .
\]

A complete scrambling of the spectrum corresponds to a parametric change of one avoided crossing, i.e., \( \delta \bar{x}_m \sim 1 \). Using (20), we find

\[
m_c \sim \frac{1}{\delta \bar{x}_1} \sim (\beta g)^{1/2} .
\]
When an interaction scale $\lambda$ is introduced (e.g., screening due to external gates), we find $m_c \sim (\beta g)^{1/2}/\lambda$.

In a recent experiment [24] the standard deviation $\sigma_p(m) \equiv \sigma(V_{N+m} - V_N)$, where $V_N$ is the position in gate voltage for the $N$-th peak, was measured as a function of the separation $m$ in peak number. In Koopmans’ limit $V_{N+m} - V_N \approx \epsilon^{(N+m)}_{N+m} - \epsilon^{(N)}_N = \Delta_m^{(N+m)} + \delta \epsilon^{(m)}_N$, where $\Delta_m^{(N+m)} \equiv \epsilon^{(N+m)}_{N+m} - \epsilon^{(N+m)}_N$ is the distance between levels separated by $m$ consecutive spacings in a dot with a fixed number of electrons ($N+m$). The latter quantity is unrelated to scrambling and its fluctuation for $m < g$ is completely determined from RMT: $\sigma(\Delta_m^{(N+m)}) \propto \Delta \ln m$. In principle one can extract $\sigma(\delta \epsilon^{(m)}_N)$ from the measured $\sigma_p(m)$. In practice this is difficult to do with the available experimental statistics since for $m < m_c$, $\sigma(\delta \epsilon^{(m)}_N)$ is smaller than $\sigma(\Delta_m^{(N+m)})$, and $\sigma_p(m)$ is dominated by the $\ln m$ term of RMT. Nevertheless, $\sigma_p(m)$, when plotted versus $\ln m$, appears to follow a straight-line behavior with a small concave deviation [25], in qualitative agreement with an additional contribution due to scrambling.

In conclusion, we have studied spectral scrambling when several electrons are added to a Coulomb-blockade quantum dot. In the absence of surface charge the fluctuation standard deviation of a level upon the addition of $m$ electrons behaves like $\sqrt{m}$, but when surface charge is present this standard deviation is linear in $m$. The latter result is the one obtained in the conventional parametric random matrix approach. Our analysis was carried out within the framework of the HF-Koopmans picture and was limited to energy fluctuations. A microscopic description of wave functions scrambling is an outstanding problem.

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