Research on Conventional Beamforming Based on Compressive Sensing

J. Shi, H.Y. Song, B.S. Liu, C.Y. Yang and M. Diao

ABSTRACT

Compressive sensing, or compressive sampling (for short, CS) is a novel sensing/sampling paradigm. With the rapid development of the theory and algorithms for sparse recovery in finite dimensions, compressive sensing has already inspired some notable investigation in the context of Direction Of Arrival (DOA) estimation. In this paper, we show how CS can be applied in the DOA estimation and be solved by the well-established toolbox, CVX. In order to ensure a high spatial resolution, the conventional compressive beamforming formulation is further extended to a virtual array case. Numerical simulations illustrate the effectiveness of the DOA estimation algorithm based on CS. In addition, numerical tests also show that under some challenging scenarios such as low SNR, coherent arrivals and few snapshots, compressive beamforming based on virtually expanded array (for convenience, called V-CS) can distinguish closely spaced sources and has higher resolving probability than conventional compressive beamforming.

KEYWORD: Compressive sensing; Direction Of Arrival; CVX; sparse recovery

INTRODUCTION

The topic of Direction Of Arrival (DOA) estimation has become an important area of the array signal processing research, which is widely used in sonar, radar, medical ultrasound imaging, wireless communication and other areas (Krim & Viberg 1996). During the last half century, many important milestones of DOA estimation have been reached, such as Conventional Beamforming (CBF),

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Maximum Entropy method (ME), the Maximum Likelihood method (ML), Multiple Signal Classification method (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and so on (Shi & Yang 2012). However, the super-resolution approaches developed to overcome the Rayleigh resolution limit of conventional beamforming suffer from performance degradation severely under noisy conditions, coherent sources and few snapshots (Shi & Yang 2011).

Recently, a novel data acquisition technique called Compressive Sensing (CS) which goes against the conventional sampling rate (the so-called Nyquist rate) has been proposed (Candes & Wakin 2008). By turning to the compressive sensing framework, we are able to exploit the inherent sparsity of the underlying signal in space domains to achieve super-resolution even in a noisy and coherent environment with few snapshots (Malioutov & Çetin 2005, Angeliki & Peter 2014). Gorodnitsky et al. view direction-of-arrival (DOA) estimation as an underdetermined problem and use a recursive weighted minimum-norm algorithm termed the FOCal Underdetermined System Solver (FOCUSS) to find its sparse solutions (Gorodnitsky & Rao 1997). Ma et al. adopt three algorithms, including diagonal loading least squares, regularization, and Orthogonal Matching Pursuit (OMP) to solve the high-resolution DOA estimation problem (Li & Ma 2013).

In this paper, we show how CS can be applied in the DOA domain and be solved by the well-established interior point method, CVX. In order to ensure a high spatial resolution, we also discuss how to extend the conventional compressive beamforming formulation to a virtual array case. Numerical simulations illustrate the effectiveness of the DOA estimation algorithm based on CS and also show that under some challenging scenarios such as low SNR, coherent arrivals and few snapshots, compressive beamforming based on virtually expanded array (for convenience, called V-CS) can distinguish closely spaced sources and has higher resolving probability than conventional compressive beamforming.

PRELIMINARY KNOWLEDGE ABOUT COMPRESSIVE SENSING

The Shannon/Nyquist sampling theorem specifies that to avoid losing information when capturing a signal, the sampling rate must be at least twice the maximum frequency present in the signal (the so-called Nyquist rate). In many engineering applications, the Nyquist rate is so high that too many samples result. Compressive sensing is a novel sensing/sampling paradigm that captures and represents compressible signals at a rate significantly below the Nyquist rate (Yonina & Gitta 2012).

Generally speaking, many natural signals can be expressed in a convenient orthonormal basis. For example, any signal $x \in \mathbb{R}^N$ can be expanded in an orthonormal basis $\Phi = [\psi_1, \psi_2, \cdots, \psi_s]$ as follows:

$$x = \sum_{i=1}^{s} s_i \psi_i \quad \text{or} \quad x = \Psi s$$  \hspace{1cm} (1)

The signal $x$ is compressible in the sense that Eq.(1) has just a few large coefficients and many small coefficients.

Consider a general linear measurement process:

$$y = \Phi x$$  \hspace{1cm} (2)
Substitute Eq.(1) for $x$ in Eq.(2), and $y$ can be written as

$$y = \Phi x = \Phi \Psi s = \Theta s$$  \hspace{1cm} (3)

Eq.(3) is called the compressive sensing problem (Donoho 2006).

**COMPRESSIVE SENSING FOR DOA ESTIMATION**

**Sparse Array Signal Representation and Compressive Beamforming**

The DOA problem can be formulated as a sparse representation problem. Let \( \{\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n\} \) \( (\hat{\theta}_i \in [-90^\circ, 90^\circ]) \) be a sampling grid of all directions of interest, and we construct the sensing matrix \( \tilde{A} \) formed by steering vectors corresponding to each potential source directions as its columns

$$\tilde{A} = [a(\hat{\theta}_1), a(\hat{\theta}_2), \ldots, a(\hat{\theta}_N)]$$ \hspace{1cm} (4)

At the same time, the signal field \( S(t) \) can be represented by a new \( \tilde{N} \times 1 \) vector \( \tilde{S}(t) \). Then, the DOA problem is shown as a sparse representation problem in Fig.1

$$X(t) = \tilde{A} \tilde{S}(t) + N(t)$$ \hspace{1cm} (5)

![Figure 1. Sparse array signal model.](image)

Generally speaking, the actual number of sources is small compared with all possible source directions of interest, so the underlying spatial spectrum is sparse, and we can solve Eq.(5) by \( l_1 \)-norm methodology. In the presence of the noise field \( N(t) \), Eq.(5) can be solved as (Angeliki & Peter 2014)

$$\min \| \tilde{S}(t) \|_1 \quad \text{s.t.} \quad \| \tilde{A} \tilde{S}(t) - X(t) \|_2 \leq \varepsilon$$ \hspace{1cm} (6)

where \( \varepsilon \) is the upper bound for the noise energy (\( l_2 \)-norm). For convenience, we call Eq.(12) conventional compressive sensing beamforming. Eq.(6) is called compressive sensing beamforming, which is a convex optimization problem and can be readily handled CVX toolbox (Grant & Boyd 2015).

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**Compressive Beamforming Based on Virtually Expanded Array**

The proposed method is by virtually expanding a real array into virtual arrays and then making utilize of compressive beamforming to achieve superior DOA resolutions in comparison with the standard algorithm.
According to the linear prediction theory, the real array can be virtually expanded by either forward prediction or backward prediction as shown in Fig.2.

The virtual signal from the \( N + 1 \)th sensor can be represented by forward prediction

\[
x_{N+1}(t) = \sum_{i=1}^{N} c_i^f x_i(t)
\]  

(7)

At the same time, the signal from the \(-1\)th sensor can be represented by backward prediction

\[
x_{-1}(t) = \sum_{i=1}^{N} c_i^b x_i(t)
\]

(8)

where \( x_i(t) (i = 1, \cdots, N) \) represents the real signal from the \( i \)th sensor, and \( c_i^f \) and \( c_i^b \) ( \( i = 1, \cdots, N \) ) respectively represent the forward prediction coefficient and the backward prediction coefficient corresponding to the \( i \)th sensor. Furthermore, the signals from other virtual sensors can also be obtained in the same way.

On the basis of the virtually expanded array, the compressive beamforming algorithms can be applied. For convenience, we call this technique virtually compressive sensing beamforming, for short, V-CS.

SIMULATION ANALYSIS

Consider a uniform linear array with 8 omni-directional sensors and half-wavelength sensor spacing. Two coherent signals in the far-field impinge on the array from distinct DOAs, \( 10^\circ \) and \( 16^\circ \) respectively. The total number of snapshots is 60, SNR is 20dB.

The numerical results from single snapshot are shown in Fig.3 and Fig.4. The original signal amplitude at different time can be recovered from each snapshot by CS and V-CS respectively, and superimposed together or stacked over time, as shown in Fig.3. Then, the spatial spectrum corresponding to each snapshot can be obtained and shown in Fig.4 (in the same way as in Fig.3).

Fig.4 compares the spatial spectra obtained by CS, V-CS, and CBF with single snapshot, as we already noted, our proposed technique V-CS is still able to resolve the two sources, whereas CS and CBF can no longer distinguish the two distinct DOAs. In Fig.4 (a), there is only one large peak and accordingly in Fig.4 (c), there is only one striation indicating the false estimation. In Fig.4(b) and (d), two narrow striations clearly indicate the two arrivals at \( 10^\circ \) and \( 16^\circ \), moreover, the striation obtained by V-CS are less fluctuant with time. In Fig.4(e), the broad striation which denotes the main-lobe of CBF merges the distinct DOAs, and the side-lobe of CBF disturbs the
background of the whole map. So under the challenging scenario of more closely spaced sources, V-CS has higher resolution than CS and CBF.

Figure 3. Amplitude recovery results from single snapshot for two coherent sources at 10° and 16°.
(a) Superimpose the result from CS with each snapshot
(b) Superimpose the result from V-CS with each snapshot
(c) Stack the result from CS with each snapshot over time
(d) Stack the result from V-CS with each snapshot over time
(e) Stack the result from CBF with each snapshot over time

Figure 4. Spatial spectrum results from single snapshot for two coherent sources at 10° and 16°.

CONCLUSIONS

This paper converts the DOA problem into the CS framework, which is actually a convex problem and can be solved by the CVX toolbox. In order to achieve higher resolution, we have extended compressive beamforming to a virtual array case, called V-CS. The simulation results show that under some challenging scenarios such as low SNR, coherent arrivals and few snapshots, compressive beamforming based on virtually expanded array (for convenience, called V-CS) can distinguish closely
spaced sources and has higher resolving probability than conventional compressive beamforming.

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