Steady State Analysis and Optimization for Autonomous Underwater Vehicle

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Abstract. Autonomous Underwater Vehicle (AUV) is widely used in ocean engineering field. Steady state analysis of AUV is considered as a necessary part in the design process. Motivated by improving the stability, a downwash filter based feedback optimization methodology for AUV is presented. First, combined with AUV structural parameters, the AUV four-degree of freedom motion space mathematical model is calculated to provide a mathematical basis for rock-bottom steady state control system. Second, the transfer function of the filter is derived according to the zero-pole and trajectory map of the AUV ontic system. Finally, the AUV optimized control system is established by series downwash filter. The simulation analysis result shows that the stability of the system is greatly increased after the whole system is connected to the filter.

1. Introduction

In contemporary society, land resources are becoming increasingly in shortage. Countries around the world are turning their attention to the sea. The development and utilization of the ocean are the future trend. Therefore, AUVs have been developed rapidly [1]. From the 1970s and 1980s, subsea surveys have emerged. AUVs, which is safe and efficient, can be widely used in all aspects of marine-related work, such as marine environmental survey and marine geological exploration. The United States, Germany, Russia, the United Kingdom, France, Italy, Japan, China and other countries have been developing a variety of AUVs [2]. AUVs are considered as the swords for countries to develop and use the ocean. With the rapid development and advancement of correlative technology, various AUVs will be applied in marine-related research work.

To copy with extremely complex subsea environment, the anti-environmental interference ability, rapid response ability and strong robustness ability should be involved in the consideration of design [3]. Therefore, the steady state design and analysis of AUV are considered to be significant. In this paper, AUV stability design is carried out by two aspects [4]: AUV's ontological structure steady state analysis and internal control system optimization. The ontological structure is the natural property of AUV, and its stability is directly reflected by the AUV's anti-rollover ability and self-return capability [5]. Stability analysis of the AUV mechanical structure is used as the standard for the design of the ontological structure. In order to further improve the stability and robust control of AUV, the downwash filter is designed, and the control model is analysed by the impulse response function to verify the AUV control effect [6-7].

The reminder of this paper is organized as follows. In Section 2, the kinematics model of AUV is established. In Section 3, the hydrodynamic model of AUV is established. Section 4 shows steady state analysis and optimization for AUV. Section 5 summarizes the paper.
2. Kinematic model of AUV

The object of analysis is the self-developed detection type AUV, which is a non-open frame structure, as shown in Figure 1, and the following assumptions are usually made in studying its equation of motion:

1. AUV is seen as a rigid body;
2. Considering the acceleration component of the center of gravity after ignoring the influence of the Earth's rotation;
3. The force acting on the AUV ontological structure is only considered as the effects of gravity, hydrostatic force, and hydrodynamic force;
4. Fluid dynamic coefficient or parameter is constant.

![Figure 1. The self-developed AUV](image)

The ground coordinate system \( E - \xi \eta \zeta \) is used as the inertial coordinate system, \( E - \xi \) points to the center of the earth, \( E - \eta \) points to the geographic east, and \( E - \zeta \) points to the geographic south.

As shown in Figure 2, the vehicle coordinate system \( O - x y z \) is established, which is also known as the moving coordinate system. \( O - x \) is consistent with the AUV main symmetry axis, \( O - z \) is parallel to the front thruster pointing to the AUV bottom surface, and \( O - y \) conforms to the right hand rectangular coordinate system.

![Figure 2. Coordinate system definition](image)

The spatial position of the AUV is described as the three components of the moving coordinate system origin \( O \) in the static coordinate system, i.e. \( \{ \xi O , \eta O , \zeta O \} \), and the three attitude angles of the dynamic coordinate system for the static coordinate system: horizontal inclination angle \( \varphi \), pitch angle \( \theta \), and yaw angle \( \psi \). In the ground coordinate system \( \{ E \} \), the speed of the AUV is recorded as \( V \), and the angular velocity is recorded as \( \Omega \). The projections of \( V \) and \( \Omega \) on the three coordinate axes of the moving coordinate system \( \{ O \} \) are recorded as \( u , v , w \) and \( p , q , r \), respectively. The complex motion state of the AUV is decomposed into motions in two planes, as shown in Figure 2 (b) and (c), namely horizontal plane motion and vertical plane motion, respectively. In the low-speed navigation case, the coupling effects of motion in two planes can be ignored.

3. Hydrodynamic model of AUV
3.1. AUV external force
The calculation method of the external force \( F \) and external torque \( M \) using [8],

\[
F = F_0 + B + P + \sum_{i=1}^{n} T_i
\]

\[
M = M_0 + M_B + M_P + \sum_{i=1}^{n} M_{t_i}
\]

Where \( F_0 \) is the hydrodynamic force of AUV, \( B \) is buoyancy of AUV, \( P \) is gravity of AUV, \( \sum_{i=1}^{n} T_i \) is thrust of AUV, and \( M_0, M_B, M_P, \sum_{i=1}^{n} M_{t_i} \) represent their corresponding torque.

3.2. AUV space motion equation
Only the four-degree of freedom space motion equation is considered. Based on the analysis in the above part, the space motion equation of AUV can be obtained using [9-10].

\[
m(u - vr + wq) = T_x - (P - B) + X_u u + X_w u^2
\]

\[
m(w - uq + vp) = T_y - (P - B) \cos \theta \cos \varphi + Z_w w + Z_q q
\]

\[
I_x q + (I_y - I_z)rp = M_y - z_c B \sin \theta + M_q q + M_q q + M_q q
\]

\[
I_z r + (I_y - I_z)pq = M_z + N_v v + N_r r + N_v v + N_r r
\]

Where \( m \) is the quality of the AUV. The \( X_u, X_w, Z_w, Z_q, M_q, M_q, M_w, M_q, N_v, N_r, N_v, N_r \) represent hydrodynamic coefficient, which are set as empirical values. The \( I_x, I_z, I_y \) represent the corresponding axis’s rotational inertia of AUV rotation.

4. Steady state analysis and optimization for AUV
4.1. AUV motion state space model
The AUV spatial equation of state can be generated by using Eq. (1)-(6) [11].

\[
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t)
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t)
\end{bmatrix}
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22} \\
b_{31} & b_{32} \\
b_{41} & b_{42}
\end{bmatrix}
\begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_1(t) \\
y_2(t)
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{21} & c_{13} & c_{14} \\
c_{21} & c_{22} & c_{23} & c_{24}
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t)
\end{bmatrix}
\]

Where \( x_1(t) \) is swing angle (rad); \( x_2(t) \) is swing angular velocity, (rad/s); \( x_3(t) \) is pitch angular velocity, (rad/s); \( x_4(t) \) is pitch angle, (rad); \( u_1(t) \) is thrust difference of rear thruster,
(N); \( u_2(t) \) is thrust difference of front thruster, (N); \( y_1(t) \) is output swing angle (rad); \( y_2(t) \) is output swing angular velocity (rad/s).

Assume that the AUV underwater navigation speed is 2 knots/s, and the AUV ontic design parameters set \{A\} and \{B\} are calculated by using self-developed AUV parameters.

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} \\
    a_{21} & a_{22} & a_{23} & a_{24} \\
    a_{31} & a_{32} & a_{33} & a_{34} \\
    a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix} = \begin{bmatrix}
    -0.0058 & -0.0968 & 0.0082 & 0.0015 \\
    0.0080 & -0.0150 & -0.0018 & 0.0000 \\
    -0.0250 & 0.0880 & -0.0050 & 0.0000 \\
    0.0000 & 0.0805 & 1.0000 & 0.0000
\end{bmatrix} \tag{9}
\]

\[
B = \begin{bmatrix}
    b_{11} & b_{12} \\
    b_{12} & b_{22} \\
    b_{13} & b_{32} \\
    b_{14} & b_{42}
\end{bmatrix} = \begin{bmatrix}
    0.00002 & 0.00000 \\
    -0.00500 & 0.00075 \\
    0.05200 & 0.01300 \\
    0.00000 & 0.00000
\end{bmatrix} \tag{10}
\]

4.2. AUV ontic stability analysis

4.2.1. Analysis of system Pole-Zero

The zero-pole map of the AUV ontological structure can be calculated by using the Eq. (7)-(8) \cite{12}, as shown in Figure.3.

The map shows that there is a pair of conjugate poles in the AUV state space model, indicating the system is progressively stable and comply with design requirements. However, since the pole is close to the imaginary axis, the system damping is relatively small, and the robust performance is low. Once the damping of the system is improved, the AUV’s ability will be improved to cope with complex sea conditions.

4.2.2. Unit impulse response analysis of AUV system

The stable performance of the AUV system under interference conditions is analyzed by loading the unit impulse. The unit impulse response of the system is shown in Figure 4.
The adaptive ability of the previous time after the interference is considered extremely important in the actual voyage of the AUV. Figure 4 (a) illustrates the system has certain robustness. The whole system tends to stabilize after being disturbed by the unit impulse for about 150 seconds, indicating that the pitch angle and the swing angle keep oscillating at first, and then the oscillation decreases and stabilizes gradually. It can also be seen that AUV has a weak anti-interference ability in a short time, considering the first 20 seconds of the system unit impulse response, as shown in Figure 4 (b). In order to improve the stability of the AUV system, the system is to be corrected by designing a feedback closed-loop.

4.3. Downwash filter optimization design

It is necessary to ensure that the root trajectory cannot move further to the left half plane in the Zero-pole map, and the downwash filter is designed to improve the stability of the AUV system [13]. The transfer function by using,

\[ G(s) = \frac{s}{s + \alpha} \]

The zero point of the system is fixed at the origin, therefore, the root trace of the downwash filter is limited to the near origin. Set the \( \alpha = 0.2 \), and the system is desired to start to converge when the impulse response is 5 seconds so that the time constant \( s \) is 5. The downwash filter-based closed-loop model is connected in series to the AUV system to form a new system, which zero-pole root trajectory map is shown in Figure 5.

![Figure 4. Unit impulse response of AUV](image1)

(a) 300s  
(b) 20s

![Figure 5. Closed-loop model zero-pole root trajectory map](image2)
The trajectory map indicates that the series downwash filter system is also stable. According to the general requirements of the stability system, when the system damping ratio $\xi = 0.3$, the system gain is 1.74, as shown in Figure 6.

![Root Locus](image)

**Figure 6.** $G(s) = 1.74$

The closed loop system with a series of downwash filter is designed to obtain a steady-state curve of the thrust difference input and the swing angle output, as shown in Figure 7.

![Response Curves](image)

(a) Swing angle stability response curve  
(b) Parameter stability response curve

**Figure 7.** Response curve

It can be seen from Figure 7 (a), the system starts to converge in about 5 seconds, then tends to be stable. Besides, the system oscillation is reduced, the stability is improved, and the robustness is improved. The stability curves of other parameters are shown in Figure 7 (b). It proves that the feedback system of the series downwash filter has strong robustness and stability, which can adapt to the complex underwater environment and ensures the safe operation of the robot system.

5. **Conclusions**

In this paper, the AUV kinematics and hydrodynamics are analyzed. The AUV spatial motion is decomposed into horizontal and vertical plane motions, and the AUV kinematics equation is derived from two planes. Combined with AUV structural parameters as well as gravity buoyancy and characteristics, the AUV external force analysis is carried out, and the AUV four-degree of freedom
motion space mathematical model is further proposed providing a mathematical basis for the AUV underlying steady state control. The AUV ontology space motion equation is derived, and the state space model is established. Then the unit impulse interference signal is applied to the ontology structure model to verify that the AUV mechanical structure has certain stability control ability. After receiving external interference for about 150 seconds, the whole system is stabilised, indicating that the structural design meets the stability criteria.

In order to further improve the stability of the AUV, the internal control system is optimized. The zero-pole and trajectory map of the AUV system is established, which is used as a guide to backward determine the key optimization parameters of the system stability. The AUV downwash filter is designed, and the stability control is realized through the intervention of the rock-bottom control program. The simulation analysis shows that the stability of the system is significantly improved after the whole system was connected to the filter.

6. References

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