Vanishing Zeeman energy in a two-dimensional hole gas

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Quantum confined holes exhibit highly desirable properties for emerging quantum technologies exploiting spins and topological states. The study of spin physics, however, has concentrated on electron systems due to the challenge in developing low-disorder materials for holes. Herein, we demonstrate that a high-mobility two-dimensional hole gas in strained germanium quantum well is a unique material platform to uncover and harness hole spin-related phenomena due to its simple band structure. A clear signature of Zeeman split states crossing in Landau fan diagram is observed and the underlying mechanisms are discussed based on a perturbative model yielding a closed formula for the critical magnetic fields. The latter depend strongly on the energy difference between the top-most and the neighboring valence bands and are sensitive to the quantum well thickness, strain, and spin-orbit-interaction. This framework quantifies straightforwardly the key parameters of hole-states from simple measurements, thus paving the way for its widespread use in design and modelling of hole-based quantum devices.

The inherent large and tunable spin-orbit interaction (SOI) energies of holes and their reduced hyperfine coupling with nuclear spins are behind the surging interest in implementing hole spin qubits with fast all-electrical control11–5. In addition to the large SOI, holes can also host superconducting pairing correlations, a key ingredient for the emergence of Majorana zero modes6–10 for topological quantum computing. Because of its attractive properties1,11–22, strained Ge low-dimensional system has been proposed as an effective building block to develop this new classes of quantum devices. Interestingly, the simplicity of this system also makes it a textbook model to uncover and elucidate subtle hole spin-related phenomena leading, for instance, to the recent observation of pure cubic Rashba spin-orbit coupling23.

Measuring Zeeman splitting (ZS) of hole states under an external magnetic field has been central in probing hole spin properties, as it is directly related to the hole g-factor, which is itself strongly influenced by the underlying SOI, strain, symmetry, and quantum confinement24,25. In III-V semiconductors24,26–38, it was demonstrated that hole spin splitting depends nonlinearly on the out-of-plane magnetic field strength B, causing Landau level crossings/anti-crossings28 and Zeeman crossings/anti-crossings32,39,40. The nonlinearity is usually modeled by a quadratic-in-field contribution to ZS24, which owes its existence to valence band mixing. Depending on the sign of the splitting, Zeeman energy can even vanish at some finite critical field, Bc. Theoretical studies attribute these nonlinearities to the mixing of heavy-hole (HH) and light-hole (LH) bands at finite energy26. Alongside with valence band mixing, Rashba and Dresselhaus spin-orbit coupling were also shown to have an influence on the crossing field, due to the inversion asymmetry of the underlying crystal lattice and of the confining potential.

Notwithstanding earlier theoretical and experimental investigations including the few on group IV systems23,40,41, detailed mechanisms of ZS of hole states are yet to be unravelled and understood. Moreover, ZS treatments for zinc-blende or diamond crystals that explicitly consider strain and SOI strength remain conspicuously missing in literature. Note that Winkler et al.40 reported calculations of Landau levels in Ge/SiGe QW to interpret cyclotron resonance experiments in Ref.42. Crossing of spin split states within the first HH subband were present in their calculations, and the corresponding field position was found to be sensitive to the strength of spin-orbit coupling. In that work, the authors insisted on the importance of including explicitly the split-off hole band, which was required to achieve a good agreement with experiments. It is also noteworthy that studies that included both strain and SOI were diagonalizing numerically the full k · p matrix29,40. However, this mathematical rigor comes at the expense of identifying the physics governing the non-linearities in ZS.

To solve these outstanding limitations and elucidate the underlying mechanisms of ZS, herein we present clear signature of ZS crossings in Ge high-mobility two-dimensional hole gas (2DHG). We also derive a theoretical framework describing the crossing of Zeeman split states that includes explicitly the SOI strength and strain. A closed formula for the crossing fields is obtained and compared to measurements confirming the agreement between theory and experiment. In addition to establishing the key parameters in Zeeman crossings, this analysis also provides a toolkit for a direct quantification from a straightforward magnetotransport measurement of important physical quantities including the HH g-factor, the HH-LH splitting, and the cubic Rashba
The investigated 2DHG is obtained in a Ge/SiGe heterostructure grown by reduced-pressure chemical vapor deposition (see Methods). The heterostructure consists of a strain-relaxed Si$_{0.2}$Ge$_{0.8}$ buffer setting the overall lattice parameter, a compressively-strained Ge quantum well (QW), and a Si$_{0.2}$Ge$_{0.8}$ barrier separating the QW from a sacrificial Si cap layer. Hall bar shaped heterostructure field effect transistors (H-FETs) are fabricated and operated with a negatively biased gate to accumulate a 2D hole gas into the QW and tune the carrier density. Fig. 1a shows an optical micrograph of the H-FET and a cross-section schematic of the active layers of the heterostructure and of the gate stack. An optimized barrier thickness of 17 nm was chosen, which is thin enough to allow for a large saturation carrier density in the QW (up to $7.5 \times 10^{14} \text{ cm}^{-2}$), and at the same time provides sufficient separation to reduce scattering of carriers in the QW from remote impurities, leading to large hole mobility ($2.6 \times 10^5 \text{ cm}^2/\text{V} \cdot \text{s}$). Large density range and high mobility are key ingredients that enable the measurements of Landau level fan diagrams in magnetotransport with the quality required to reveal subtle spin-related features such as level crossings and anti-crossings.

The fan diagram in Fig. 1b shows the normalized magnetoresistance oscillation amplitude $\Delta \rho_{xx}/\rho_0 = (\rho_{xx} - \rho_0)/\rho_0$ as a function of energy and out-of-plane external magnetic field $B$ aligned along the growth direction $\hat{z}$ and perpendicular to the 2DHG plane, where $\rho_0$ is the $\rho_{xx}$ value at $B = 0$. The Zeeman split energy gap, corresponding to odd integer filling factors $\nu$, deviates from its linear dependence on $B$, vanishes when the magnetic field reaches a critical value $B_c$, and then reopens at higher $B$ values. We clearly observe the associated crossing of Zeeman split states for odd integers $\nu = 3, 5, 7, \text{ and } 9$. Partial signatures of Zeeman crossings occurring at similar magnetic fields were observed in earlier studies, albeit the fan diagram measurements were limited in density range or affected by thermal broadening. These observations point to an underlying mechanism that is independent of the QW position with respect to the surface gate.

To identify the mechanisms behind the non-linearities in ZS and the key parameters affecting the crossing field, we developed a perturbative model to describe the hole dispersion in quasi-2D layers as a function of the out-of-plane magnetic field. The model assumes an abrupt and infinite band offset between the QW and its barriers and is based on a 6-band $k \cdot p$ Hamiltonian for HH, LH and split-off (SO) bands. We start the perturbative approach by first dividing the total Hamiltonian $H$ for the hole dispersion in two terms: $H = H_0(e_B; k_z) + H'(n, B; k_z)$, where $n$ is the Landau level index. $H_0$ depends on neither $n$ nor $B$ and describes the hole spectrum at $B = 0$. The term $H'$ introduces the magnetic field and will be treated as a perturbation, with $H'(n, 0; k_z) = 0$. The idea is to eliminate $H'$ to second order perturbation and to write an effective Hamiltonian for the 2-fold $l$th HH subband. The explicit expression of $H_0$ and $H'$ is outlined in Supplementary Information (SI, Appendix A). The diagonalization of $H_0$ results in either pure HH states of energy $E_{l,0}^{\text{HH}}$ or states that consist of a superposition of LH and SO holes of energy $E_{l,0}^{\eta}$, where $\eta = \{+, -\}$ is a generic label to distinguish the two orthogonal LH-SO states. Energies and eigenstates of $H_0$ are detailed further in SI (Appendix B). The Hamiltonian $H$ is then projected onto the eigenbasis of $H_0$, and $H'$ is eliminated to second order by a Schrieffer-Wolff transformation. Remarkably, the resulting effective $2 \times 2$ Hamiltonian for the
The $l$th HH subband does not couple spin-up (+) and spin-down states (−), which remain good quantum numbers to second order perturbation. The HH dispersion as a function of $B$ is thus simply the diagonal entries of the effective matrix:

\[
E_{+,l,n}^{(2)}(B) = E_{l}^{\text{HH}} - [(2n - 1)(\gamma_1 + \gamma_2) + 3\kappa - 6nF_l] \mu_B B \\
+ 3n(n + 1)(\gamma_2 + \gamma_3)^2 (\mu_B B)^2 \sum_{\eta=\pm} \left( \frac{l_0^3 + \sqrt{2}s_0^3}{E_{l}^{\text{HH}} - E_{\eta}^l} \right)^2.
\]

\[
E_{-,l,n}^{(2)}(B) = E_{l}^{\text{HH}} - [(2n + 5)(\gamma_1 + \gamma_2) - 3\kappa - 6(n + 2)F_l] \mu_B B \\
+ 3(n + 1)(n + 2)(\gamma_2 + \gamma_3)^2 (\mu_B B)^2 \sum_{\eta=\pm} \left( \frac{l_0^3 + \sqrt{2}s_0^3}{E_{l}^{\text{HH}} - E_{\eta}^l} \right)^2.
\]

where the $\gamma$s and $\kappa$ are the Luttinger parameters, $\mu_B$ is the Bohr magneton, $l_0^3$ and $s_0^3$ are LH and SO contributions (see SI, Appendix B) and $n \geq 1$ ($n \geq -2$) for spin-up (spin-down). $F_l$ indicates the strength of the interaction of the $l$th HH subband with neighboring $\eta$-states:

\[
F_l = \frac{32\alpha_0\gamma_3}{L^2} \sum_{j=1}^{\infty} \frac{[1 - (-1)^{j+1}] I_j^2 j^2}{(l_0^2 - j^2)^2} \sum_{\eta=\pm} \left( \frac{l_0^3 - s_0^3 + \sqrt{2}}{E_{l}^{\text{HH}} - E_{\eta}^l} \right)^2,
\]

Eq. (1) can also be evaluated at $B = B_c^{(2)}$ to reveal the energy difference that separates the HH band edge from the energy at a Zeeman crossing position. When $n \to \infty$ (or equivalently $\nu \to \infty$) this energy difference is independent of $n$:

\[
\Delta E \equiv E_{l}^{\text{HH}} - \lim_{n \to \infty} E_{+,l,n}^{(2)}(B_{c}^{(2)}(l,n)) = \left[ \gamma_1 + \gamma_2 - \frac{3}{4} (\kappa + 3F_l) \right] \mu_B B_c^*.
\]

Eqs. (1)-(3) and $B_c^*$ can be simplified if SOI is large enough, that is if $\Delta$ satisfies criterion (B1) (see SI Appendix B for details). We thus obtain:

\[
B_c^* \approx \frac{\kappa - F_l}{\mu_B (\gamma_2 + \gamma_3)^3} \left( E_{l}^{\text{HH}} - E_{l-n}^0 \right)^{-1}
\]

and

\[
F_l \approx \frac{32\alpha_0\gamma_3}{L^2} \sum_{j=1}^{\infty} \frac{1}{E_{l}^{\text{HH}} - E_{\eta}^l} \frac{[1 - (-1)^{j+1}] I_j^2 j^2}{(l_0^2 - j^2)^2}.
\]

Let us now test the accuracy of the second order dispersion (Eq. (1)) compared to the dispersion given by solving numerically $H$. We take Ge as the QW material with width $L$ and strain $\epsilon_3$ as free parameters. Since Ge has a rather high spin-orbit energy $\Delta = 260 \text{meV}^{43}$, it is worthwhile to look also at the behavior of Eq. (1) when $\Delta \to \infty$. We also focus on relaxed or compressively
strained wells, which always result in a HH-like valence band edge. The calculated fan diagram of the ground HH subband is displayed in Fig. 2a for a 16 nm-thick well with $\varepsilon_\parallel = -0.6\%$, similar to the system analyzed in Fig. 1. Assuming finite $\Delta$, Eq. (1) reproduces perfectly well the experimental fan diagram up to $\sim 2$ T, which implies that $6(\kappa - F_l)$ is a very accurate approximation for the HH g-factor at low fields. As the magnetic field increases, quadratic terms in $B$ become more important and the dispersions eventually cross. The dispersion of a state with spin-up projection in a given spin-split Landau pair always has a bigger curvature than the spin-down one, which can be straightforwardly inferred from the coefficients $n(n+1)$ and $(n+1)(n+2)$ in Eq. (1). For that reason, a Zeeman crossing cannot occur, at least to second order, if the spin-up state lies closer to the band gap than the spin-down one. Crossing fields are indicated in Fig. 2a for filling factors $\nu = 3$ and $\nu = 5$. The numerical solution of $H$ gives a crossing field $B_c = 7.27$ T for $\nu = 3$, whereas the second order formula (Eq. (5)) gives $B_c^{(2)} = 5.04$ T. Here the second order approximation underestimates $B_c$ as it diverges from the numerical dispersion before the crossing. When assuming $\Delta \to \infty$, however, the dispersion diverges less dramatically than its finite SOI counterpart and instead overestimates the crossing field. Assuming an infinite SOI for this particular system turns out to be a good approximation, because the right-hand side of (B1) equals 21.2 meV, which is much smaller than spin-orbit gap in Ge.

DISCUSSION

Fig. 2b depicts the behavior of the crossing field as a function of the well thickness and strain, with and without the assumption of an infinite SOI. The crossing field $B_c$ is well approximated by $B_c^{(2)}$ for a well thickness $> 10$ nm with reduced strain levels, as in our experiments. For narrower and highly strained wells, third or higher perturbative terms become more important. These could be included in the model, but at the cost of extremely cumbersome equations, even with infinite SOI. On the other hand, for $\Delta \to \infty$, $B_c^{(2)}$ misses completely the increase of the crossing field for thin wells, which highlights the explicit role of the SOI strength. This is consistent with criterion (B1) : thin wells increase the right-hand side in (B1) as $1/L^2$, thus requiring $\Delta$ to be even larger for this criterion to be satisfied.

From the present model, we see that Zeeman crossings still occur under the assumption of an infinite QW (no barrier effects), an infinite band gap (6-band $k \cdot p$), and even an infinite spin-orbit gap (4-band $k \cdot p$ for HH and LH). Consequently, LH-HH mixing plays a crucial role in the crossing of spin-split states. Our assumptions
and the parameter $F_l$. For ground subbands, compressive strain typically increases $E_i^{\text{HH}} - E_i^\eta$, which explains the increase of $B_c$ at higher compressive strain. SOI also increases $E_i^{\text{HH}} - E_i^\eta$, mainly through the spin-orbit energy $\Delta$ for $\eta = +$ or through the out-of-plane effective mass for $\eta = -$. At $\Delta = 0$ and any strain, the HH subbands share the same spectrum as the $\eta = +$ or $\eta = -$ states. Eq. (6) then gives $B^* = 0$ hence no Zeeman crossing occurs. SOI lifts this degeneracy and thus allows the existence of Zeeman crossings.

The experimental observation of Zeeman crossings are further highlighted by plotting portions of the fan diagram from Fig. 1b as a function of energy and filling factor (Fig. 3a-d). The upper part of each panel shows the $\rho_{xx}$ as a function of the energy $E$ at odd-integer values of filling factors from $\nu = 3$ to 9. Fingerprints of Zeeman crossing are observed for filling factors up to $\nu = 17$. In addition to describing the crossings in Zeeman split states, the theoretical framework described above also allows a straightforward evaluation of several parameters. Since only the first HH subband is involved in the measurements, we assume $l = 1$ and drop the subscripts $l$ for simplicity. First, we fit the crossing fields extracted from Fig. 3a ($\nu = 3, 5, ..., 17$) with Eq. (5) using $B^*$ as the sole fitting parameter. This yields $B^* = 25.258$ T and the crossing fields obtained from Eq. (5) match the experimental values with a relative error < 4% for $\nu = 3, 5, 7, 9, 11$ and < 10% for $\nu = 13, 15, 17$ (Fig. 3b). Zeeman crossings also approach a fixed energy value as $\nu$ increases, as demonstrated in Eq. (7). From Fig. 1(b), we have $\Delta E \approx 17$ meV. Knowing $B^*$ and $\Delta E$ gives the value of $F$, which in turn can be used to determine the HH effective mass and weak-field g-factor. A rearrangement of Eq. (7) gives:

$$F = \frac{4}{9} \left( \gamma_1 + \gamma_2 - \frac{\Delta E}{\mu_B B^*} \right) - \frac{\kappa}{3} \approx 1.52. \quad (10)$$

From Eqs. (4) and (10), we extract $g^* = 11.35$, which is close to the g-factor value of 12.9 obtained by solving $H$ numerically. An expression for the subband-edge HH in-plane effective mass $m^*$ involving the parameter $F$ can also be derived by inserting Eq. (5) from Ref. 44 into Eq. (4) : $m^*/m_0 = (\gamma_1 + \gamma_2 - 3F)^{-1} \approx 0.077$. This value is also close to those reported in literature39,45. A close relation exists between the crossing fields, the HH g-factor and the HH-\eta splitting (Eqs. (6) and (8)). Knowing two of these quantities is enough to obtain the third. For the system described in Fig.1, the criterion (B1) is also satisfied, thus the HH-LH splitting is found directly from Eq. (8):

$$E_i^{\text{HH}} - E_i^{\text{LH}} = \frac{6 (\gamma_2 + \gamma_3)^2 \mu_B B^*}{g^*} \approx 76.0 \text{ meV}. \quad (11)$$
A numerical solution of $H$ yields a HH-LH splitting of 62.8 meV. This value does not change significantly when an effective out-of-plane electric field is introduced in $H$. This is expected from square QWs whose HH-LH splitting is dominated by strain and quantum confinement. For that reason, we assume that the HH-LH splitting does not change with hole concentration, or applied gate voltages. From the HH-LH splitting energy (Eq. (11)), one can finally estimate the cubic Rashba coefficient $\alpha_3$:

$$\alpha_3 = \frac{e\alpha_2\gamma_3}{12(\gamma_2 + \gamma_3)^3} \left( \frac{g^*}{\mu B} \right)^2 \approx 4.25 \times 10^5 e \text{Å}^4,$$

where $e$ is the elementary charge. $\alpha_3$ appears in the cubic Rashba SOI Hamiltonian of HH states $H_3 = \beta_3 i(k^z \sigma_z - k^r \sigma_r)$, where $k^z = k_x \pm ik_y$ and $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ with $\sigma_{x,y}$ the Pauli spin matrices, and $\beta_3 = \alpha_3 E_z$, with $E_z = e\varphi/e$ the effective out-of-plane electric field in the accumulation mode 2DHG, $p$ the hole density and $\epsilon$ the Ge dielectric constant. The obtained $\alpha_3$ is almost twice as large as the one obtained for Ge QW in Ref. $^{23}$, which had a bigger HH-LH splitting of 110 meV. As mentioned above, we expect $\alpha_3$ to be independent of the gate voltage or hole concentration, since it depends mostly on the HH-LH splitting. The Zeeman crossings appear at a density $p \approx 6.1 \times 10^{11} \text{cm}^{-2}$, corresponding to $E_z \approx 6.8 \times 10^{-4} \text{V Å}^{-1}$ (by taking $\epsilon = 16.2\epsilon_0$ for Ge), which yields $\beta_3 \approx 290 \text{eV Å}^3$. Note that $\alpha_3$ or $\beta_3$ are hitherto hard to measure in these high mobility structures with established methodologies: weak anti-localization measurements are impractical due to the small characteristic transport field $B_L$ associated with μm-scale mean free paths; Shubnikov-de Haas oscillations lack sufficient spectral resolution before onset of ZS to resolve the beatings associated with spin-split subbands. The analysis outlined above provides a straightforward framework to evaluate these quantities and other fingerprints of the hole states from simple magnetotransport measurements. Crucially, the detailed knowledge of the physical parameters of the underlying experimental material platform will provide the necessary input to further advance design and modelling of hole spin qubits and other hole-based quantum devices.

**METHODS**

**Heterostructure Growth:** The undoped Ge/SiGe heterostructure is grown in an Epsilon 2000 (ASM) reduced pressure chemical vapor deposition reactor on a 100 nm n-type Si(001) substrate. The growth sequence starts with the deposition of a Si$_{0.2}$Ge$_{0.8}$ virtual substrate. This virtual substrate is obtained by growing a 1.6 μm strain-relaxed Ge buffer layer, a 1.6 μm reverse-graded Si$_{1-x}$Ge$_x$ layer with final Ge composition $x = 0.8$, and a 500 nm strain-relaxed Si$_{0.2}$Ge$_{0.8}$ buffer layer. A 16 nm compressively-strained Ge quantum well is then grown on top of the Si$_{0.2}$Ge$_{0.8}$ virtual substrate, followed by a strain-relaxed 17 nm-thick Si$_{0.2}$Ge$_{0.8}$ barrier. An in-plane compressive strain $\epsilon_{||} = -0.63\%$ is found in the QW via X-ray diffraction measurements. A thin (2 nm) sacrificial Si cap completes the heterostructure. This cap is readily oxidized upon exposure to the clean room environment after unloading of the Ge/SiGe heterostructure from the growth reactor.

**H-FET Fabrication:** A 170 nm deep trench mesa is dry-etched around the Hall-bar shaped H-FET in order to isolate the bonding pads from the device. The sample is dipped in HF to remove the native oxide from the ohmic contact areas prior to a 60 nm Pt layer deposition via e-beam evaporation. Ohmic contacts are obtained by diffusion of Pt into the quantum well occurring during the atomic layer deposition of a 30 nm Al$_2$O$_3$ dielectric layer at a temperature of 300°C. Finally, a 10/200 nm-thick Ti/Au gate layer is deposited.

**Magnetotransport Characterization:** We measure the longitudinal and transversal ($\rho_{xx}$ and $\rho_{yy}$) component of the 2DHG resistivity tensor via a standard four-probe low-frequency lock-in technique. The measurements are recorded at a temperature of $T = 260 \text{mK}$, measured at the cold finger of a 3He dilution refrigerator. A source-drain voltage bias $V_{sd} = 0.1 \text{mV}$ is applied at a frequency of 7.7 Hz. The magnetoresistance characterization of the device reported in Fig. 1b is performed by sweeping the voltage gate $V_y$ and stepping $B$ with a resolution of 15 mV and 25 mT, respectively. The energy $E$ is obtained using the relation $E = p\pi h^2/m^*$, where we obtain the carrier density $p$ by Hall effect measurements at low $B$ and we use the effective mass $m^*$ measured as a function of density in similar heterostructures. The $\rho_{xx}$ energy profiles in the upper panels of Fig. 3a have been smoothed for clarity by using a Matlab routine based on Savitzky-Golay filtering method.

**Theoretical calculations:** The model is based on a 6-band $k \cdot p$ Hamiltonian for HH, LH and SO bands. The total Hamiltonian $H$ for the hole state is written as $^{18}$: $H = H_k + H_\epsilon + H_{SO} + H_B + V$ where $H_k$ is a function of the wavevector operator $k = (k_x, k_y, k_z)$, $H_\epsilon$ is the Bir-Pikus Hamiltonian and depends on the strain tensor components $\epsilon_{ij}$, $H_{SO}$ is the spin-orbit term proportional to the spin-orbit energy $\Delta$ and $H_B$ includes the interaction of the free electron spin with the magnetic field. $V$ is the infinite well potential for a square well of width $L$. We consider QWs grown along [001] direction and subjected to biaxial bi-isotropic strain. Thus, $\epsilon_{ij} = 0$ if $i \neq j$, $\epsilon_{yy} = \epsilon_{zz} \equiv \epsilon_3$ and $\epsilon_{xz} = 0$, where $D_{001}$ is the Poisson ratio. $H$ was numerically diagonalized by projecting it into the position basis via the substitution $k_z \rightarrow -i\partial/\partial z$, in which the $z$-derivative was implemented by finite differences over the simulation domain. A constant mesh grid size of 0.01 nm was used for every diagonalization.
The Matlab eigs() routine was used to retrieve the desired subset of eigenvalues. The Ge Luttinger parameters $\gamma_1, \gamma_2, \gamma_3$ and deformation potentials were taken from Ref.49, while the parameter $\kappa$ was taken from Ref.50.

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AUTHORS CONTRIBUTION STATEMENT

A. S. provided the Ge heterostructures designed by M. L. and G. S. M. L. fabricated the devices and performed the electrical characterisation. G. S. supervised the material growth, device fabrication, and electrical characterisation. P. D. V developed the theory and carried out the calculations under the supervision of O. M. P. D. V, M. L., G. S., and O. M. wrote the manuscript. G. S. and O. M. conceived and led the project. Correspondence and requests for materials should be addressed to: G. S. (material and experimental work) O. M. (theory).

Data availability

Datasets supporting the findings of this study are available at 10.4121/uuid:c64b0509-2247-4d51-ade0-90c361b928a4

REFERENCES

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1. N. W. Hendrickx, D. P. Franke, A. Sammak, G. Scappucci, and M. Veldhorst, Nature 577, 487 (2020), arXiv:1904.11443.
2. R. Maurand, X. Jehl, D. Kotekar-Patil, A. Corna, H. Bohuslavskyi, R. Laviéville, L. Hutin, S. Barraud, M. Vinet, M. Sanquer, and S. De Franceschi, Nature Communications 7, 13575 (2016), arXiv:1605.07599.
23. R. Moriya, K. Sawano, Y. Hoshi, S. Masubuchi, Y. Shiraki, A. Wild, C. Neumann, G. Abstreiter, D. Bougeard, T. Koga, and T. Machida, Physical Review Letters 113, 086601 (2014).

24. R. Kotlyar, T. L. Reinecke, M. Bayer, and A. Forchel, Physical Review B 63, 85340 (2001).

25. R. Winkler, Spin Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems, edited by Springer (Springer, Berlin, 2003) p. 228.

26. N. Traynor and R. Warburton, Physical Review B 55, 15701 (1997).

27. M. J. Lawless, R. J. Warburton, R. J. Nicholas, N. J. Pulford, J. K. Moore, G. Duggan, and K. Woodbridge, Physical Review B 45, 4266 (1992).

28. R. J. Warburton, R. J. Nicholas, S. Sasaki, N. Mirua, and K. Woodbridge, Physical Review B 48, 12323 (1993).

29. V. Jovanov, T. Eissfeller, S. Kapfinger, E. C. Clark, F. Klotz, M. Bichler, J. G. Keizer, P. M. Koenraad, M. S. Brandt, G. Abstreiter, and J. J. Finley, Physical Review B 85, 165433 (2012).

30. R. Danneau, O. Klochan, W. R. Clarke, L. H. Ho, A. P. Micolich, M. Y. Simmons, A. R. Hamilton, M. Pepper, D. A. Ritchie, and U. Zülicke, Physical Review Letters 97, 026403 (2006), arXiv:0607355 [cond-mat].

31. M. Kubis, K. Ryczko, J. Jadczak, L. Bryja, J. Misiewicz, and M. Potemski, Acta Physica Polonica A 119, 609 (2011).

32. P. S. Grigoryev, O. A. Yugov, S. A. Eliseev, Y. P. Efimov, V. A. Lovtciev, V. V. Petrov, V. F. Sapega, and I. V. Ignatiev, Physical Review B 93, 205425 (2016), arXiv:1608.04774.

33. F. Fischer, R. Winkler, D. Schuh, M. Bichler, and M. Grayson, Physical Review B 75, 073303 (2007).

34. P. E. Faria Junior, D. Tedeschi, M. De Luca, B. Scharf, A. Polimeni, and J. Fabian, Physical Review B 99, 195205 (2019).

35. D. Tedeschi, M. De Luca, P. E. Faria Junior, A. Granados Del Águila, Q. Gao, H. H. Tan, B. Scharf, P. C. Christiaenen, C. Jagadish, J. Fabian, and A. Polimeni, Physical Review B 99, 161204(R) (2019), arXiv:1811.04922.

36. W. Bardyszewski and S. P. Lepkowski, Physical Review B 90, 075302 (2014).

37. D. A. Broido and L. J. Sham, Physical Review B 31, 885 (1985).

38. U. Ekenberg and M. Altarelli, Physical Review B 32, 3712 (1985).

39. M. Lodari, A. Tosato, D. Sabbagh, M. A. Schubert, G. Capellini, A. Sammak, M. Veldhorst, and G. Scappucci, Physical Review B 100, 041304(R) (2019).

40. R. Winkler, M. Merkler, T. Darnhofer, and U. Rössler, Physical Review B 53, 10858 (1996).

41. R. Moriya, Y. Hoshi, K. Sawano, Y. Shiraki, N. Usami, S. Masubuchi, and T. Machida, IEEE : Extended abstract , 786 (2013).

42. C. M. Engelhardt, D. Többen, M. Aschauer, F. Schäffler, G. Abstreiter, and E. Gornik, Solid State Electronics 37, 949 (1994).

43. M. P. Polak, P. Scharoch, and R. Kudrawiec, Journal of Physics D : Applied Physics 50, 195103 (2017).

44. I. L. Drichko, A. A. Dmitriev, V. A. Malyshev, I. Y. Smirnov, H. Von Känel, M. Kummer, D. Chrastina, and G. Isella, Journal of Applied Physics 123, 165703 (2018), arXiv:1804.03876.

45. L. A. Terrazos, E. Marcellina, S. N. Coppersmith, M. Friesen, A. R. Hamilton, X. Hu, B. Köller, A. L. Saraiva, D. Culcer, and R. B. Capaz, arXiv , :1803.10320 (2018), arXiv:1803.10320.

46. S. Hikami, A. I. Larkin, and Y. Nagaoka, Progress of Theoretical Physics 63, 707 (1980).

47. S. V. Iordanskii, Y. B. Lyanda-Geller, and G. E. Pikus, ZhETF Pis ma Redaktsiiu 60, 199 (1994).

48. T. Eissfeller and P. Vogl, Physical Review B 84, 195122 (2011).

49. D. J. Paul, Journal of Applied Physics 120, 043103 (2016).

50. P. Lawaetz, Physical Review B 4, 3460 (1971).
Supplementary Information to:
Vanishing Zeeman energy in a two-dimensional hole gas

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Appendix A

The matrix representation of $H$ is presented in the following $|j,m\rangle$ angular momentum basis:

$$\left\{ \begin{array}{c|c|c|c|c}
\frac{3}{2},\frac{3}{2} & \frac{3}{2},\frac{1}{2} & \frac{3}{2},\frac{1}{2} & \frac{1}{2},\frac{3}{2} & \frac{1}{2},\frac{1}{2}
\end{array} \right\}$$

The magnetic field-free Hamiltonian $H_0$ is:

$$H_0 = -\begin{pmatrix}
\alpha_0 (\gamma_1 - 2\gamma_2) k_x^2 & 0 & 0 & 0 & 0 \\
0 & \alpha_0 (\gamma_1 + 2\gamma_2) k_x^2 & 0 & 0 & -\sqrt{8}\gamma_2 k_z^2 \\
0 & 0 & 0 & 0 & \sqrt{8}\gamma_2 k_z^2 \\
0 & -\sqrt{8}\gamma_2 k_z^2 & 0 & \alpha_0 (\gamma_1 - 2\gamma_2) k_x^2 & 0 \\
\alpha_0 (\gamma_1 + 2\gamma_2) k_x^2 & 0 & \sqrt{8}\gamma_2 k_z^2 & 0 & \alpha_0 \gamma_1 k_z^2 + \Delta \\
\end{pmatrix} + (2 - D_{001}) a_v \epsilon_{||} 6 \times 6 +
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -\sqrt{2} \\
-1 & 0 & 0 & -\sqrt{2} & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
(1 + D_{001}) b e_{||},$$

where $\alpha_0 = \hbar^2/(2m_0)$, $m_0$ being the free electron mass, and $a_v$ and $b$ are deformation potentials. Its eigenstates and eigenvalues are described in Appendix B. For perpendicular-to-plane magnetic fields it is convenient to write $k_x$ and $k_y$ in terms of the ladder operator $a$:

$$k_x = \frac{1}{\sqrt{2\lambda}}(a + a^\dagger) \quad k_y = \frac{i}{\sqrt{2\lambda}}(a - a^\dagger),$$

where the magnetic length $\lambda = \sqrt{\hbar/eB}$, $e$ being the elementary charge. Also, $[a, a^\dagger] = 1$, $a |n\rangle = \sqrt{n} |n - 1\rangle$ and $a^\dagger a |n\rangle = |n\rangle$, where $n$ is the Landau number. In the axial approximation the vector

$$\begin{pmatrix}
|n-1\rangle |l\rangle_{3/2,3/2} \\
|n\rangle |l\rangle_{3/2,1/2} \\
|n+1\rangle |l\rangle_{3/2,-1/2} \\
|n+2\rangle |l\rangle_{3/2,-3/2} \\
|n\rangle |l\rangle_{1/2,1/2} \\
|n+1\rangle |l\rangle_{1/2,-1/2}
\end{pmatrix}$$

is an eigenstate of $H$, where $|z |l\rangle_{j,m}$ is the spatial envelope function of the hole component with angular momentum $|j,m\rangle$ and subband index $l \geq 1$. This ansatz allows to write $H$ as a function of the quantum numbers $n$ and to eliminate the ladder operators $a$. The perturbation $H'$ takes the form (with $g_0 = 2$):

$$\left\{ \begin{array}{c|c|c|c|c}
\frac{3}{2},\frac{3}{2} & \frac{3}{2},\frac{1}{2} & \frac{3}{2},\frac{1}{2} & \frac{1}{2},\frac{3}{2} & \frac{1}{2},\frac{1}{2}
\end{array} \right\}$$
where we defined $\gamma_{\pm} = \gamma_1 \pm \gamma_2$ and $\tilde{\gamma} = \gamma_2 + \gamma_3$. Note that $\alpha_0/\lambda^2 = \mu_B B$.

Appendix B

If $B = 0$ (and $k_x = k_y = 0$) the subbands are either pure HH states or pure spin-1/2 states (LH-SO superposition). The subband eigenstates are

$$|HH, \sigma, l\rangle = \left| \frac{3}{2}, \frac{3\sigma}{2} \right| \left| l \right\rangle$$

$$|\eta, \sigma, l\rangle = \left( l^\eta \left| \frac{3}{2}, \frac{3\sigma}{2} \right\rangle + \sigma s^\eta \left| \frac{1}{2}, \frac{\sigma}{2} \right\rangle \right) \left| l \right\rangle,$$

where $l \geq 1$ is the subband index, $\sigma = \{+, -\}$ is the pseudo-spin index (spin-up -down respectively) and $\eta = \{+, -\}$ labels two orthogonal spin-1/2 states. We have

$$l^\eta = \frac{\xi_- + \delta_\eta}{\sqrt{(\xi_- + \delta_\eta)^2 + 8\xi_-^2}}$$

$$s^\eta = \frac{-\sqrt{8}\xi_-}{\sqrt{(\xi_- + \delta_\eta)^2 + 8\xi_-^2}}$$

$$\delta_\pm = \frac{\Delta}{2} \pm \sqrt{(\xi_- + \Delta/2)^2 + 8\xi_-^2}$$

$$\xi_\pm = -\alpha_0 \left[ \left( \frac{1 \pm \frac{1}{2}}{2} \right) \gamma_1 + \gamma_2 \right] \left( \frac{L}{l\pi} \right)^2 + \left[ \left( \frac{1 \pm \frac{1}{2}}{2} \right) (2 - D_{001})a_v - (1 + D_{001})b/2 \right] \epsilon_\parallel.$$

The spatial part is

$$\langle z \left| l \right\rangle = \sqrt{\frac{2}{L}} \sin \left( \frac{l\pi z}{L} \right).$$

The energy spectrum for the HH and \eta-states is

$$E^\text{HH}_l = -\alpha_0 \left( \gamma_1 - 2\gamma_2 \right) \left( \frac{l\pi}{L} \right)^2 + \left[ (2 - D_{001})a_v + (1 + D_{001})b \right] \epsilon_\parallel$$

$$E^\eta_l = \xi_+ - \delta_\eta.$$

Infinite SOI regime is reached when $\Delta \gg |\xi_-|$. Under compressive strain this expands to
\[
\Delta \gg \alpha_0 \gamma_2 \left( \frac{l \pi}{L} \right)^2 + \frac{(-b)}{2} (1 + D_{001}) |\epsilon|_\parallel.
\]

(1)

Assuming \( \Delta \to \infty \) is a good approximation only if \( \Delta \) satisfies criterion (1). The square root in \( \delta_\pm \) can then be eliminated by a Taylor expansion, and the following results immediately follow:

\[
\begin{align*}
\delta_- &= -\xi_- , \\
l_i^- &= 1 , \\
l_i^+ &= 0 ,
\end{align*}
\]

\[
\begin{align*}
\delta_+ &= \Delta + \xi_- , \\
s_i^- &= 0 , \\
s_i^+ &= 1 .
\end{align*}
\]

Consequently,

\[
\begin{align*}
| -, \sigma, l \rangle \to | \text{LH}, \sigma, l \rangle &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} \sigma \\ 2 \end{pmatrix} | l \rangle \\
| +, \sigma, l \rangle \to | \text{SO}, \sigma, l \rangle &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} \sigma \\ 2 \end{pmatrix} | l \rangle
\end{align*}
\]

\[
\begin{align*}
E_i^- \to E_i^{\text{LH}} &= -\alpha_0 (\gamma_1 + 2 \gamma_2) \left( \frac{l \pi}{L} \right)^2 + [(2 - D_{001})a_v - (1 + D_{001})b] |\epsilon|_\parallel \\
E_i^+ \to E_i^{\text{SO}} &= -\alpha_0 \gamma_1 \left( \frac{l \pi}{L} \right)^2 - \Delta + (2 - D_{001})a_v |\epsilon|_\parallel,
\end{align*}
\]

corresponding to a pure LH and pure SO spectrum.

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