A New Class of LRS Bianchi Type-I Cosmological Models in Lyra’s Manifold

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Abstract

LRS Bianchi type-I models have been studied in the cosmological theory based on Lyra’s geometry. A new class of exact solutions has been obtained by considering a time dependent displacement field for variable deceleration parameter models of the universe. We have compared our models with those of Einstein’s field theory with the cosmological term Λ. Our frame of reference is restricted to the recent Ia observations of supernovae. Some physical behaviour of the models is also examined in the presence of perfect fluids.

Key Words : Cosmology; L R S Bianchi type-I Models; Lyra Geometry
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1 Introduction

In 1917 Einstein introduced the cosmological constant into his field equations in order to obtain a static cosmological model since, as is well known, without the cosmological term his field equations admit only non-static solutions. After the discovery of the redshift of galaxies and its explanation as being due to the expansion of the universe, Einstein regretted his introduction of the cosmological constant. Recently, there has been much interest in the cosmological term in context of quantum field theories, quantum gravity, super-gravity theories, Kaluza-Klein theories and the inflationary-universe scenario. Shortly after Einstein’s general theory of relativity Weyl, in 1918, suggested the first so-called unified field theory based on a generalization of Riemannian geometry. In retrospect, it would seem more appropriate to call Weyl’s theory a geometrized theory of gravitation and electromagnetism (just as the general theory was a geometrized theory of gravitation only), rather than a unified field theory. It is not quite clear to what extent the two fields have been unified, even though they acquire (different) geometrical significances in the same geometry. The theory was never taken seriously because it was based on the concept of non
integrability of length transfer, and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl’s geometry provides an interesting example of non-Riemannian connections, and recently Folland [11] has given a global formulation of Weyl manifolds thereby clarifying considerably many of Weyl’s basic ideas.

In 1951 Lyra [2] proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl’s geometry. But in Lyra’s geometry, unlike Weyl’s, the connection is metric preserving as in Riemannian; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the “normal” gauge the curvature scalar is identical to that of Weyl. In consecutive investigations Sen [3], Sen and Dunn [4] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra’s geometry. It is thus possible [3] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl’s “unified” field theory without, however, the inconvenience of non-integrability length transfer.

Halford [5] has pointed out that the constant vector displacement field \( \phi_i \) in Lyra’s geometry plays the role of cosmological constant \( \Lambda \) in the normal general relativistic treatment. It is shown by Halford [6] that the scalar-tensor treatment based on Lyra’s geometry predicts the same effects, within observational limits, as the Einstein’s theory. Several authors Sen and Vanstone [7], Bhamra [8], Karade and Borikar [9], Kalyanshetti and Wagmode [10], Reddy and Innahia [11], Beesham [12], Reddy and Venkateswarlu [13], Soleng [14], have studied cosmological models based on Lyra’s manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one of convenience and there is no a priori reason for it. Beesham [15] considered FRW models with time dependent displacement field. He has shown that by assuming the energy density of the universe to be equal to its critical value, the models have the \( k = -1 \) geometry. Singh and Singh [16], Singh and Desikan [17] have studied Bianchi-type I, III, Kantowski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein’s theory with cosmological term and in the cosmological theory based on Lyra’s geometry. Soleng [14] has pointed out that the cosmologies based on Lyra’s manifold with constant gauge vector \( \phi \) will either include a creation field and be equal to Hoyle’s creation field cosmology [18] or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.

The Einstein’s field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or that space-time
admits killing vector symmetries \[19\]. Solutions to the field equations may also be generated by applying a law of variation for Hubble’s parameter which was proposed by Berman \[20\]. In simplest case the Hubble law yields a constant value for the deceleration parameter. It is worth observing that most of the well-known models of Einstein’s theory and Brans-Deke theory with curvature parameter \( k = 0 \), including inflationary models, are models with constant deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by several authors \[21\]. But redshift magnitude test has had a chequered history. During the 1960s and the 1970s, it was used to draw very categorical conclusions. The deceleration parameter \( q_0 \) was then claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. Today’s situation, we feel, is hardly different. Observations (Knop et al. \[22\]; Riess et al., \[23\]) of Type Ia Supernovae (SNe) allow to probe the expansion history of the universe. The main conclusion of these observations is that the expansion of the universe is accelerating. So we can consider the cosmological models with variable cosmological term and deceleration parameter. The readers are advised to see the papers by Vishwakarma and Narlikar \[24\] and Virey et al. \[25\] and references therein for a review on the determination of the deceleration parameter from Supernovae data.

Recently Pradhan and Otarod \[26\] have studied the universe with time dependent deceleration parameter and \( \Lambda \)-term in presence of perfect fluid. Motivated with the situation discussed above, in this paper, we study a new class of LRS Bianchi-I cosmological models in Lyra geometry by considering a time dependent deceleration parameter in an expanding universe. This paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3 we deal with a general solution. The Sections 4, 5, and 6 contain the three different cases for the solutions in exponential, polynomial and sinusoidal forms respectively. Finally in Section 7 concluding remarks will be given.

2 The Metric and Field Equations

We consider LRS Bianchi type I space-time

\[
\mathrm{d}s^2 = \mathrm{d}t^2 - A^2 \mathrm{d}x^2 - B^2 (\mathrm{d}y^2 + \mathrm{d}z^2)
\]  

(1)

where \( A = A(x,t), B = B(x,t) \). We take a perfect fluid form for the energy momentum tensor

\[
T_{ij} = (p + \rho)u_i u_j - pg_{ij}
\]  

(2)

together with co-moving coordinates \( u^i u_i = 1 \), where \( u_i = (0,0,0,1) \).
The field equations in normal gauge for Lyra’s manifold, as obtained by Sen \[3\] are

\[
R_{ij} - \frac{1}{2}g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -8\pi G T_{ij}
\]  

(3)

where \( \phi \) is a time-like displacement field vector defined by \( \phi_i = (0,0,0,\beta(t)) \) and other symbols have their usual meaning as in Riemannian geometry. Here we
want to mention the fact that the ansatz choosing the coordinate system with matter require the vector field happens to be in the required form exactly in the matter comoving coordinates. The essential difference between the cosmological theories based on Lyra geometry and the Riemannian geometry lies in the fact that constant vector displacement field $\beta$ arises naturally from the concept of gauge in Lyra geometry where as cosmological constant $\Lambda$ was introduced in adhoc fashion in the usual treatment.

Now the field equations can be set up and one obtains

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B''}{A^2B^2} + \frac{3}{4} \dot{\beta}^2 = -\chi p$$  \hspace{1cm} (4)

$$\frac{\ddot{B}'}{B'} - \frac{\dot{B}'\dot{A}}{A} = 0$$  \hspace{1cm} (5)

$$\frac{\dddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2B} + \frac{A'B'}{A^3B} + \frac{3}{4} \beta^2 = -\chi p$$  \hspace{1cm} (6)

$$\frac{2B''}{A^2B} - \frac{2A'B'}{A^3B} + \frac{B'^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} + \frac{3}{4} \beta^2 = \chi p.$$  \hspace{1cm} (7)

The energy conservation equation is

$$\chi \dot{\rho} + 3 \frac{\dot{\beta}}{2} + \left[\chi (p + \rho) + \frac{3}{2} \beta^2\right] \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0$$  \hspace{1cm} (8)

where $\chi = 8\pi G$. Here and in what follows a prime and a dot indicate partial differentiation with respect to $x$ and $t$ respectively. Equations (4)-(7) are four equations in five unknowns $A, B, \beta, p$ and $\rho$. For complete determinacy of the system one extra condition is needed. One way is to use an equation of state. The other alternative is a mathematical assumption on the space-time and then to discuss the physical nature of the universe. In this paper we confine ourselves to assume an equation of state

$$p = \gamma \rho, \ 0 \leq \gamma \leq 1$$  \hspace{1cm} (9)

### 3 Solutions of the field equations

Equation (10), after integration, yields

$$A = \frac{B'}{l}$$  \hspace{1cm} (10)

where $l$ is an arbitrary function of $x$. Equations (4) and (6), with the use of equation (10), reduces to

$$\frac{B}{B'} \left(\frac{\ddot{B}}{\dot{B}}\right)' + \dot{B} \frac{d}{dt} \left(\frac{B'}{B}\right) + \frac{l^2}{B^2} \left(1 - \frac{Bl'}{B'l}\right) = 0$$  \hspace{1cm} (11)
Setting
\[ \frac{B'}{B} = \text{functions of } x \]  
(12)

Equation (11) yields on integration
\[ B = lS(t), \]  
(13)

where \( S(t) \) is an arbitrary function of \( t \). With the help of equation (13), equation (10) becomes
\[ A = \frac{l'}{l} S \]  
(14)

Now the metric (1) takes the form
\[ ds^2 = dt^2 - S^2(t)[dX^2 + e^{2X}(dy^2 + dz^2)], \]  
(15)

where \( X = \ln l \). Equations (4) and (4) give
\[ \chi_p = \frac{1}{S^2} - 2 \frac{\dot{S}}{S} - \frac{3}{4} \beta^2 \]  
(16)

\[ \chi_p = \frac{3\dot{S}^2}{S^2} - \frac{3}{S^2} - \frac{3}{4} \beta^2 \]  
(17)

Using equation (4) and eliminating \( \rho(t) \) from equations (16) and (17) we have
\[ 2 \ddot{S} + (1 + 3\gamma) \frac{\dot{S}^2}{S^2} - (1 + 3\gamma) \frac{1}{S^2} + \frac{3}{4} (1 - \gamma) \beta^2 = 0 \]  
(18)

Now the expressions for the energy density and the pressure are given by
\[ \chi_p = \chi_{\gamma \rho} = \frac{4\gamma}{(1 - \gamma)} \left[ \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} + \frac{\ddot{S}}{2S} \right] \]  
(19)

The function \( S(t) \) remains undetermined. To obtain its explicit dependence on \( t \), one may have to introduce additional assumptions. Accordingly we assume the deceleration parameter to be variable and set
\[ q = - \frac{S \ddot{S}}{S^2} = - \left( \frac{\dot{H} + H^2}{H^2} \right) = b(\text{variable}), \]  
(20)

where \( H = \dot{S}/S \) is the Hubble parameter. The above equation may be rewritten as
\[ \frac{\dddot{S}}{S} + b \frac{\dot{S}^2}{S^2} = 0. \]  
(21)

The general solution of Eq. (21) is given by
\[ \int e^{\frac{b}{S}\dot{S}} dS = t + k, \]  
(22)
where $k$ is an integrating constant.

In order to solve the problem completely, we have to choose $\int \frac{b}{S} dS$ in such a manner that Eq. (22) be integrable.

Without loss of generality, we consider

$$\int \frac{b}{S} dS = \ln L(S), \quad (23)$$

which does not affect the nature of generality of solution. Hence from Eqs. (22) and (23), we obtain

$$\int L(S) dS = t + k. \quad (24)$$

Of course the choice of $L(S)$, in Eq. (24), is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider the following case:

### 4 Solution in the Exponential Form

Let us consider $L(S) = \frac{1}{k_1 S}$, where $k_1$ is arbitrary constant.

In this case on integration of Eq. (24) gives the exact solution

$$S(t) = k_2 e^{k_1 t}, \quad (25)$$

where $k_2$ is an arbitrary constant. Using Eqs. (4) and (5) in (15) and (16), we obtain expressions for displacement field $\beta$, pressure $p$ and energy density $\rho$ as

$$\beta^2 = \frac{4(1 + 3\gamma)}{3(1 - \gamma)k_2^2 e^{2k_1 t}} - \frac{4(1 + \gamma)k_1^2}{(1 - \gamma)} \quad (26)$$

$$\chi p = \chi \gamma \rho = \frac{4}{(1 - \gamma)} \left[ \frac{3}{2} k_2^2 - \frac{1}{k_2^2 e^{2k_1 t}} \right]. \quad (27)$$

From Eq. (26), since scale factor can not be negative, we find $S(t)$ is positive if $k_2 > 0$. From Figure 1, it can be seen that in the early stages of the universe, i.e., near $t = 0$, the scale factor of the universe had been approximately constant and had increased very slowly. At specific time the universe had exploded suddenly and expanded to large scale. This is consistent with Big Bang scenario.

From Eq. (26), it is observed that $\beta^2$ is a decreasing function of time. As mentioned earlier the constant vector displacement field $\phi_i$ in Lyra’s geometry plays the role of cosmological constant $\Lambda$ in the normal general relativistic treatment and the scalar-tensor treatment based on Lyra’s geometry predicts the same effects, within observational limits, as the Einstein’s theory. Recent cosmological observations (Garnavich et al. [27], Perlmutter et al. [28], Riess et al. [29], Schmidt et al. [30]) suggest the existence of a positive cosmological constant $\Lambda$ with the magnitude $\Lambda(G\tilde{h}/c^3 \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an
accelerating one with induced cosmological density through the cosmological \( \Lambda \)-term. In our model, it is seen that \( \beta \) plays the same role as cosmological constant and preserves the same character as \( \Lambda \)-term, in turn with recent observations.

The expressions for \( \beta^2 \) and \( \rho \) cannot be determined for the empty universe \( (p = \rho = 0) \) and stiff matter \( (p = \rho) \) models. In this case, the Ricci scalar is obtained as

\[
R = \frac{6(k_1^2 k_2 e^{k_1 t} - 1)}{k_2^2 e^{2k_1}} + 6k_1^2. \tag{28}
\]

From above equation, we see that the Ricci scalar remains positive for

\[
k_1 > \frac{1}{\sqrt{k_2(1 + k_2)}}.
\]

The expansion and shear scalar are given by

\[
\theta = 3k_1, \quad \sigma = 0. \tag{29}
\]

The model represents uniform expansion as can be seen from Eq. (29). The flow of the fluid is geodetic with the acceleration vector \( f_i = (0, 0, 0, 0) \).
5 Solution in the Polynomial Form

Let \( L(S) = \frac{1}{2k_3 \sqrt{S + k_4}} \), where \( k_3 \) and \( k_4 \) are constants.

In this case, on integrating, Eq. (24) gives the exact solution

\[
S(t) = \alpha_1 t^2 + \alpha_2 t + \alpha_3, \tag{30}
\]

where \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are arbitrary constants. Using Eqs. (9) and (30) in (18) and (19), we obtain the expressions for displacement field \( \beta \), pressure \( p \) and energy density \( \rho \) as

\[
\beta^2 = \frac{4(1 + 3\gamma)\{8\alpha_1(\alpha_1 t + \alpha_2)t + 4\alpha_1\alpha_3 + \alpha_2^2 - 1\}}{(1 - \gamma)(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^2}, \tag{31}
\]

\[
\chi p = \chi \gamma \rho = \frac{4[5\alpha_1(\alpha_1 t + \alpha_2)t + \alpha_1\alpha_3 + \alpha_3^2 - 1]}{(1 - \gamma)(\alpha_1 t^2 + \alpha_2 t + \alpha_3)^2}. \tag{32}
\]

Figure 2: The plot of scale factor \( S(t) \) vs time with parameters \( \alpha_1 = 1.00, \alpha_2 = 4.00, \alpha_3 = 1.00 \) and \( \gamma = 0.5 \)

From Eq. (30), we note that \( S(t) > 0 \) for \( 0 \leq t < \infty \) if \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are positive constants. Figure 2 shows that the scale factor is a decreasing function of time, implying that our universe is expanding.

Eq. (31) shows that \( \beta^2 < 0 \) for all times as \( \gamma - 1 < 0 \) and is a decreasing function of time, characteristically similar to \( \Lambda \) in Einstein’s theory of gravitation. In this model, \( \beta \) plays the role as cosmological constant and it preserves the same
character as $\Lambda$-term. This is consistent with recent observations (Garnavich et al. [27], Perlmutter et al. [28], Riess et al. [29], Schmidt et al. [30]). A negative cosmological constant adds to the attractive gravity of matter; therefore, universe with a negative cosmological constant is invariably doomed to re-collapse. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most of the time, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expand exponentially.

The expressions for $\beta^2$ and $\rho$ cannot be determined for the empty universe ($p = \rho = 0$) and stiff matter ($p = \rho$) models. In this case, the Ricci scalar is obtained as

$$R = \frac{6[(2\alpha_1 t + \alpha_2)^2 + 2\alpha_1 - 1]}{(\alpha_1^2 t + \alpha_2 t + \alpha_3)^2}.$$  \hfill (33)

From Eq. (33) we observe that Ricci scalar remains positive if $\alpha_2^2 > \frac{1 - 2\alpha_1}{6}$. This condition also implies that $\alpha_1 < \frac{1}{2}$. The scalar of expansion is given by

$$\theta = \frac{3(2\alpha_1 t + \alpha_2)}{(\alpha_1^2 t + \alpha_2 t + \alpha_3)}.$$ \hfill (34)

It is of the interest to note that all physical parameters in our models are defined at $t = 0$ and we do not have any singularity.

### 6 Solution in the Sinusoidal Form

Let $L(S) = \frac{1}{\sqrt{1 - S^2}}$, where $\beta$ is constant. In this case, on integrating, Eq. (24) gives the exact solution

$$S = M \sin(\beta t) + N \cos(\beta t) + \beta_1,$$ \hfill (35)

where $M$, $N$ and $\beta_1$ are constants. Using Eqs. (9) and (35) in (18) and (19), we obtain the expressions for displacement field $\beta$, pressure $p$ and energy density $\rho$ as

$$\beta^2 = \frac{4 [(1 + 3\gamma)(1 - \beta^2(M \cos \beta t - N \sin \beta t)^2) + 2\beta^2 Q(Q - \beta_1)]}{3(1 - \gamma)Q^2},$$ \hfill (36)

$$\chi p = \chi \gamma \rho = \frac{2 \beta^2 \{P - 6MN \sin \beta t \cos \beta t - \beta_1(Q - \beta_1)\} - 2}{(1 - \gamma)Q^2},$$ \hfill (37)

where

$$Q = M \sin \beta t + N \cos \beta t + \beta_1,$$

$$P = (2M^2 - N^2) \cos \beta t^2 + (2N^2 - M^2) \sin \beta t^2.$$

In this case the Ricci scalar and the scalar of expansion are obtained as

$$R = \frac{6 [(M \beta \cos(\beta t) - N \beta \sin(\beta t))^2 - \beta^2 (M \sin(\beta t) + N \cos(\beta t))^2 - 1]}{(M \sin(\beta t) + N \cos(\beta t) + \beta_1)^2},$$ \hfill (38)
Figure 3: The plot of scale factor $S(t)$ vs time with parameters $M = 2.00$, $N = 1.00$, $\beta = 10.00$, $\beta_1 = 0.2$, and $\gamma = 0.5$

$$\theta = \frac{3\beta[M \cos \beta t - N \sin \beta t]}{M \sin \beta t + N \cos \beta t + \beta - 1}. \quad (39)$$

From the Figure 3, we note that at early stage of the universe, the scale of the universe increases gently and then decreases sharply, and afterwards it will oscillate for ever. We must mention here that the oscillation takes place in positive quadrant this has physical meaning. Since, in this case, we have many alternatives for choosing values of $M$, $N$, $\beta$, $\beta_1$, it is just enough to look for suitable values of these parameters, such that the physical initial and boundary conditions are satisfied.

### 7 Conclusions

In this paper we have obtained exact solutions of Sen equations in Lyra geometry for time dependent deceleration parameter. The nature of the displacement field $\beta(t)$ and the energy density $\rho(t)$ have been examined for three cases (i) exponential form (ii) polynomial form and (iii) sinusoidal form. The solutions obtained in Sections 4, 5 and 6 are to our knowledge quite new. Here the displacement field $\beta$ plays the role of a variable cosmological term $\Lambda$.

Recently there is an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology. Therefore the study of cosmological models in Lyra geometry are relevant for
inflationary models. Further the space dependence of the displacement field $\beta$ is important for inhomogeneous models in the early stage of the evolution of the universe. Besides, the implication of Lyra’s geometry for bodies of astrophysical interest is still an open question. The problem of equations of motion and gravitational radiation need investigation.

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