Syncyclons
or
Solitonic Signals from Extra Dimensions

C.S. Aulakh
Institute of Physics, Sachivalya Marg
Bhubhaneshwar, 751005, India

ABSTRACT

In theories where spacetime is a direct product of Minkowski space ($M^4$) and a d-dimensional compact space ($K^d$), there can exist topological solitons that simultaneously wind around $R^3$ (or $R^2$ or $R^1$) in $M^4$ and the compact dimensions. A paradigmatic non-gravitational example of such “co-winding” solitons is furnished by Yang-Mills theory defined on $M^4 \times S^1$. Pointlike, stringlike and sheetlike solitons can be identified by transcribing and generalizing the procedure used to construct the periodic instanton (caloron) solutions. Asymptotically the classical pointlike objects have non-Abelian magnetic dipole fields together with a non-Abelian scalar potential while the “color” electric charge is zero. However quantization of collective coordinates associated with zeromodes and coupling to fermions can radically change these quantum numbers due to fermion number fractionalization and its non-Abelian generalization. Interpreting the YM group as color (or the Electroweak $SU(2)$ group) and assuming that an extra circular dimension exists thus implies the existence of topologically stable solitonic objects which carry baryon(lepton) number and a mass $O(1/g^2 R)$, where $R$ is the radius of the compact dimension.

PACS Nos: 12.10.-g, 04.20.Jb, 04.60.+n, 14.80.Hv

The speculation that space-time enjoys an extension from Minkowski space $M^4$ to $M^4 \times K^d$ where $K$ is some d-dimensional compact space is a common ingredient of the Kaluza-Klein and Superstring scenarios. However, so far, generic signals
of the presence of extra dimensions are lamentably absent. The reasons for this sorry divorce between speculation and its experimental verifiability are most easily explained in terms of field theoretic framework \(^1\) developed for Kaluza-Klein theory. Modulo intrinsically stringy effects, what follows holds also for the low-energy behaviour of string theory.

In Kaluza-Klein theory physics is taken to be governed by an action which is a functional of fields \(\phi_i(x, y)\) defined on \(M^4 \times K^d\). The compact space \(K\) is characterized by some radii \(R_1, R_2, \ldots R_d\). Physics at energy scales \(<< R_i^{-1}\) is extracted by expanding the fluctuation field around an appropriate background \(\bar{\phi}_i\) in harmonics \(\{f_n^i(y)\}\) on the compact space \(K\):

\[
\phi_i(x, y) = \bar{\phi}_i + \sum_n \phi_n^i(x) f_n^i(y) \tag{1}
\]

The harmonics \(\{f_n^i(y)\}\) are eigenfunctions of the appropriate laplacian on \(K\):

\[
- \nabla_i^2 f_n^i(y) = \left(\frac{n^2}{R^2}\right) f_n^i(y) \tag{2}
\]

Here \(\left(\frac{n^2}{R^2}\right)\) is the appropriate “Casimir” and \(R\) the appropriate combination of length scales associated with \(K\). On integrating the part of the action quadratic in the small fluctuations over the compact coordinates \(\{y^a\}\) one gets a spectrum of small fluctuations which consists of massless modes \(\{\phi_i^{(0)}\}\) (“\(n^2 = 0\)”) and massive modes \(\phi_n^i(x)\) (“\(n^2 \neq 0\)”) whose masses are \(\sqrt{\left(\frac{n^2}{R^2}\right)}\). A rough estimate of the upper limit on \(R\) based on the 100 GeV scale of current experiments and their 1% accuracy yields \(R^{-1} \geq 1 TeV\) and this is confirmed by more detailed comparison\(^2,3\) of the generic predictions of such models with experiment. The relevant quantum numbers in such theories are defined by the symmetries of the massless sector and, barring pathologies of individual models, the enormous phase space available ensures that any heavy modes initially present will have long decayed. Conversely their high mass ensures that such modes are very difficult to excite in the laboratory.
Thus the only possibly observable effects of the heavy excitations come from their participation in virtual processes and these are also severely suppressed by their large masses.

Topological solitons (monopoles, vortices, strings, etc.) are accepted members of the speculative particle physics zoo. Generically the static field configurations \( \{\phi_i(x, y)\} \) provide a map from \( R^3 \times K^d \) into some configuration space \( \mathcal{M} \). The distinct homotopy classes of these maps will then generically include extremae of the energy and these soliton solutions of the field equations will manifest as stable particles (modulo quantum effects) or (if gravity is involved) topologically nontrivial space-time structures. Such solitons divide naturally into three classes. The special cases

\[
\{\phi_i(x)\} : R^3 + \{\infty\} \longrightarrow \mathcal{M} \tag{3a}
\]

\[
\{\phi_i(y)\} : K^d \longrightarrow \mathcal{M} \tag{3b}
\]

correspond respectively to the usual monopoles vortices etc. of four dimensional physics and the configurations used to provide stress energy to curve the compactified space \( K^d \) in Kaluza-Klein type scenarios. The third category

\[
\{\phi_i(x, y)\} : \{R^3 + \{\infty\}\} \times K^d \longrightarrow \mathcal{M} \tag{3c}
\]

consists of field configurations in which neither the \( x \) nor the \( y \) dependence of the fields can be contracted away i.e in which the fields co-wind (wind around both spaces simultaneously). Due to this co-winding attribute we call such configurations syncyclons (from the greek \( \sigmaνυ \) (with) \( κυκλος \) (turning)). From the point of view of 4-dimensional low-energy physics the solitons of type (3.c) will appear at distances \( \gg R \) as stable particles with masses \( \sim R^{-1} \) which may possibly have dressed themselves by binding, in an early epoch, with lighter particles.
Although the Gross-Perry-Sorkin monopole can be considered as an example of type (3.c), since it involves the extra assumption that the massless $U(1)$ field to which the monopole couples comes from the higher dimensional graviton multiplet, its claim to providing a generic signature of extra dimensions is much weakened for there are many models with gauge fields from other sources. Moreover the mass of such monopoles is necessarily of the order of the Planck mass since the radius of the compact space is fixed to be $O(L_p)$ once the gauge coupling is taken to be $O(10^{-2} - 10^{-1})$. Nevertheless, inasmuch as the TAUB-NUT solutions of the Euclidean Einstein equations on $R^3 \times S^1$ correspond to gravitational periodic instantons, they are closely related to the example we shall discuss below which is based on the periodic instanton or caloron solutions of YM theory.

The crucial point is that the topology of maps from a product of spaces into a compact space is in general much more complex than that of maps from either space individually. For example although $\pi_1(S^3) = \pi_2(S^3) = 0$, $S^3$ is well known to be an $S^1$ bundle over $S^2$ (the Hopf-fibration). Another well known example is furnished by $S^3, S^4$ and $S^7$.

Yang-Mills theory in 5-dimensions with signature $(-+++)$ is a simple theoretical studio wherein the basic features of the syncyclonic scenario come into focus. As we shall see the properties of such solitons are quite rich and one seems to be able to go quite far in predicting the kind of signals that the non-Abelian sector of the standard model would give if there was an extra circular dimension.

Thus consider the $SU(N)$ Yang-Mills action on $M^4 \times S^1$:

$$S = -\left(\frac{1}{g^2 R}\right) \int d^4 x \int_0^R dx^5 \frac{1}{4} F_{MN}^a \cdot F_{MN}^a$$

(4)

where $F_{MN}^a = \partial_M A_N^a + f^{abc} A_M^b A_N^c$, $(M, N = 0, 1, 2, 3, 5)$. The fields $A_M$ have dimension 1 and the normalization ($g$ is the 4-dimensional gauge coupling) is is fixed by that of the zero mode sector. $R$ is the perimeter of the extra compact
dimension. If we make the static ansatz \( \partial_0 = A_0 = 0 \) the field equations become
\[
0 = D_\mu F_{\mu\nu} \quad (\mu, \nu = 1, 2, 3, 5).
\]
The field \( A_\mu(\bar{x}, x^5 = y^1) \) is independent of time and periodic in \( y \). The energy density is \((i, j = 1, 2, 3)\)
\[
T_{00} = \frac{1}{g^2 R} \left( \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} F_{ij5}^a F_{ij5}^a \right)
\]
We wish to find topologically stable finite energy solutions of the 4-dimensional Euclidean YM equations on \( R^3 \times S^1 \). The solutions that we are looking for are thus nothing but the periodic instantons or calorons\(^5\) studied in the context of YM theories at finite temperature \(!\). We can therefore simply carry over the results summarized in Ref. 6 and reinterpret them in the present context.

The topological data is coded \(^6\) in the so-called Polyakov operator
\[
\Omega(\bar{x}) = P \exp \int_0^R dx^5 A_5(\bar{x}, x^5)
\]
where \( P \) denotes path-ordering with respect to \( x^5 \). The boundary conditions on the fields are those demanded by finite energy \((F_{\mu\nu}^2 \longrightarrow r^{-(3+\epsilon)} \text{ as } r = |\bar{x}| \longrightarrow \infty)\) and periodicity in \( x^5 \). Under periodic gauge transformations \( U(\bar{x}, x^5+R) = U(\bar{x}, x^5) \)
\[
\Omega(\bar{x}) \longrightarrow U(\bar{x}, 0) \Omega(\bar{x}) U^{-1}(\bar{x}, 0)
\]
The eigenvalues \( \{\lambda(\bar{x})\} \) of \( \Omega(\bar{x}) \) are therefore gauge invariant observables. The finite energy boundary conditions imply that \( \{\lambda(\bar{x})\} \) approaches a direction independent limit \( \{\lambda^\infty\} \) as \( r \longrightarrow \infty \). Thus \( \Omega^\infty(\bar{x}) = V(\bar{x}) \lambda^\infty V(\bar{x})^{-1} \) provides, via \( V(\bar{x}) \), a map from provides a map from \( S^2_\infty \) into \( G/H_{\{\lambda^\infty\}} \) where \( H_{\{\lambda^\infty\}} \) is the subgroup of \( G \) which commutes with \( \{\lambda^\infty\} \) i.e \( U(1)^k \bigotimes_{\alpha=0}^k SU(m_\alpha) \) where \( m_\alpha \) are the degeneracies of the distinct eigenvalues \( \mu_\alpha(\alpha = 0, 1 \ldots k) \). The topology of \( \Omega(\bar{x}) \) for \( r \longrightarrow \infty \) is thus classified by \( \pi_2(G/H) = \pi_2(H) = \mathbb{Z}^k \). The \( k \) integers are nothing but the quantized magnetic charges in the distinct eigenvalue sectors labelled by \( \alpha \):
where $P_\alpha$ projects $\vec{B}$ onto the $\alpha$th eigensubspace of $\Omega(\vec{x})$. The remaining topology is labelled by a Pontryagin index $\nu$ which arises as follows: $\Omega(\vec{x})$ can be rewritten as $V(\vec{x})\omega(\vec{x})V(\vec{x})^{-1}$ where $\omega(\vec{x})$ is continuous and well defined throughout $R^3$ while $V(\vec{x})$ is continuous on $R^3/L$ where $L$ is a set of line singularities corresponding to the magnetic charges $\{q_\alpha\}$. Then

$$Q = \nu(\omega^{-1}\bar{\partial}\omega) + \sum_\alpha \frac{\ln \mu_\alpha}{2\pi i} q_\alpha$$

$$= \nu(\omega^{-1}\bar{\partial}\omega) + \frac{1}{8\pi^2} \int d^2\vec{S} \cdot tr(ln(\Omega(\vec{x})\vec{B}))$$

$$\nu(\vec{J}) = [Q] = \frac{1}{48\pi^2} \int d^3\vec{x} \quad \vec{J} \cdot (\vec{J} \times \vec{J})$$

Here $[Q]$ is the integer part of the total topological charge $Q$. The total topological data $\mathcal{T} = (\{\mu_\alpha, q_\alpha, m_\alpha\}, \nu)$ is invariant under smooth deformations of fields and in the general case is considerably richer than that for YM theory on $R^4$. This topological richness is generic to the syncyclonic scenario. For each $\mathcal{T}$ one expects to find a solution to the static field equations since the energy obeys

$$\mathcal{E} = \frac{1}{g^2 R} \int \left( \frac{1}{4} F_{a}^{a} \right)^2 \geq \frac{1}{g^2 R} \left| \int \frac{1}{4} (F_{a}^{a} \tilde{F}_{a}^{a}) \right| = \frac{8\pi^2}{g^2 R} |Q|$$

For the case $\nu = 1, q_\alpha = 0$ explicit solutions\(^5\) can be constructed by using the ’t Hooft’s multi-instanton solution\(^7\) on $R^4$. We specialize the group to SU(2) which is a subgroup of every SU(N) group. In the ’t Hooft solution one sets $A_{\mu}^{a} = -\bar{\eta}_{a\mu\nu} \partial_{\nu} \ln \Pi$ where $\bar{\eta}_{a\mu\nu}$ are the anti-selfdual ’t Hooft symbols and $\Pi$ is a scalar function called the superpotential. The self-duality conditions $F_{\mu\nu}^{a} = \tilde{F}_{\mu\nu}^{a}$ then reduce to the linear equation $\Pi^{-1} \partial^2 \Pi = 0$ whose solution
\[ \Pi(x^\mu) = 1 + \sum_{n=1}^{K} \frac{\rho_n^2}{(x - z_n)^2} \quad (11) \]

describes \( K \) instantons with positions \( \{ z_n \} \) and scales \( \{ \rho_n \} \) in a singular gauge. To obtain the periodic instanton one sets \( \rho_n = \rho, z_n = nR \) \( \tilde{x}^5, n \in \mathbb{Z} \) and sums over \( n \), thus effectively compactifying \( x^5 \), and gets

\[ \Pi(\vec{x}, x^5) = 1 + \frac{\pi \rho^2}{rR} \frac{\sinh \tilde{r}}{\cosh \tilde{r} - \cos \tilde{x}^5} \quad (12) \]

\[ \tilde{r} = \frac{2\pi r}{R} \quad \tilde{x}^5 = \frac{2\pi x^5}{R} \]

The gauge potential has a singularity at \( r = x^5 = 0 \) which, however, can be removed by a periodic gauge transformation for instance

\[ U = (\sin^2 x^5 + r^2 \cos^2 x^5)^{-\frac{1}{2}} (\sin x^5 + ix^j \cos x^5 \sigma^j) \quad (13) \]

The winding number for such configurations (in the physical strip \( x^5 \in [0, R] \) ) is 1 and labels the homotopy class of the maps “\( S^2_\infty \times S^1 \rightarrow S^3 \)” furnished by them. We have written “\( S^2_\infty \)” since in a singular gauge the winding is actually around an infinitesimal sphere centered at \( r = x^5 = 0 \). Since \( F_{\mu 0}^a = 0 \) they have zero electric field and thus zero nonabelian charge. Their energy (mass) is \( 8\pi^2 / (g^2 R) \). The scale parameter \( \rho \) is not fixed by the compactification scale \( R \) and remains arbitrary. Multi-centered solutions can be obtained trivially by summing over several distinct chains separately to obtain:

\[ \Pi(\vec{x}, x^5) = 1 + \sum_{k=1}^{N} \frac{\pi \rho_k^2}{r_k R} \frac{\sinh \tilde{r}_k}{\cosh \tilde{r}_k - \cos \tilde{x}^5_k} \quad (14) \]

\[ \tilde{r}_k = \frac{2\pi r_k}{R} \quad \tilde{x}_k^5 = \frac{2\pi (x^5 - x_k^5)}{R} \]

\[ r_k = \sqrt{(\vec{x} - \vec{x}_k)^2} \]
and \( \{ \vec{x}_k, x_k^5 \} \) are the positions of the \( N \) non-interacting solitons with total energy \( 8\pi^2 N/(g^2 R) \).

In the Kaluza-Klein limit \( r >> R \) the fields become

\[
A_5^a \rightarrow \frac{-x^a}{r^2(1 + \frac{rR}{\pi \rho^2})} \tag{15}
\]

\[
B_i^a = \frac{1}{2} \epsilon_{ijk} F_{jk}^a \rightarrow -\frac{1}{r^2(1 + \frac{rR}{\pi \rho^2})^2} \left[ \frac{x^i x^a}{r^2} - \frac{rR}{\pi \rho^2} (\delta^{ia} - \frac{3x^i x^a}{r^2}) \right]
\]

If \( r >> \frac{\rho^2}{R} \) then one has the fields of a non-Abelian magnetic dipole (the magnetic moment is \( \mu_i^a = (\pi \rho^2 / R) \delta_i^a \)) together with a scalar potential:

\[
B_i^a \rightarrow \frac{\pi \rho^2}{R} (\delta^{ia} - \frac{3x^i x^a}{r^2}) \frac{1}{r^3}
\]

\[
A_5^a \rightarrow -\frac{\pi \rho^2 x^a}{R r^3} \tag{16}
\]

while in the intermediate regime \( \frac{\rho^2}{R} >> r >> R \) the fields are

\[
A_5^a \rightarrow -\frac{x^a}{r^2} \tag{17}
\]

\[
B_i^a \rightarrow -\frac{x^i x^a}{r^4}
\]

The energy density \( (1/4) F_{\mu\nu}^a \) is confined to the smallest scale available i.e \( \rho \) if \( \rho \approx R \) or \( \rho << R \) and \( R \) if \( \rho >> R \). Thus classically such solitons will appear as pointlike particles with masses \( \mathcal{O}(\frac{1}{g^2 R}) \) and dipolar asymptotic fields which become monopolar at short distances (>> \( R \)).

The gauge field possesses zero modes associated with translations (translation = rotation on the internal circle !) dilatations and global gauge rotations. Collective coordinates must be introduced for each of these and quantized by the standard
semiclassical methods to find the quantum states of the soliton. Moreover since we expect the YM fields to be coupled to light fermions in any realistic example fermionic zero modes should also be taken into account. This leads to fermion number fractionalization and the induction of non-Abelian charges on the soliton. Barring pathologies, the quantization of the collective coordinates associated with global gauge rotations implies that the states of the soliton occupy representations of the YM group. It is interesting to note that since the asymptotic falloff of the fields in the present case is dipolar the “breakdown of color” that takes place when the corresponding collective coordinates of non-Abelian (grand unified) ’t Hooft-Polyakov monopoles are quantized should be evaded since this breakdown has been shown to be connected with the $r^{-2}$ falloff of the color magnetic fields and is absent in the case of a monopole-antimonopole system with $r^{-3}$ asymptotics. These questions require detailed analysis presently in progress in collaboration with V.Soni and will be reported on subsequently.

A preliminary sketch of the situation is as follows. In keeping with our basic aim of trying to deduce model independent consequences of the existence of extra compact dimensions let us take the YM group to be SU(3) color in which the SU(2) group associated with the soliton is embedded trivially. In that case it is reasonable to assume that the scale parameter $\rho$ is smaller than $1\text{GeV}^{-1}$ since at such scales QCD becomes confining and classical considerations are largely irrelevant. On scales $R << r << 1\text{Gev}^{-1}$ the QCD coupling is weak and semi-classical heuristics are applicable. The fermion masses of the standard model with $N_f$ flavors are essentially all negligible compared to $\frac{1}{g^2 R} > 1\text{TeV}$ and can at most lead to small splittings of the degenerate ground states that arise due to fermion number fractionalization. $N_f$ fermionic zeromodes develop and for antiperiodic boundary conditions around $S^1$ their wave functions are exponentially localized ($e^{-r/R}$) around the soliton. A degeneracy of ground states associated with the zeromode being occupied or not
develops. A baryon number

\[ \langle B \rangle = \frac{1}{3} \left( \int d^4x \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i \right) \]
\[ = -\frac{N_f}{192\pi^2} \int d^4x \text{ tr}(F \tilde{F}) \]
\[ = -\frac{N_f}{6} Q \tag{18} \]

is induced. Similarly one gets \( B/2 \) for the electric charge. Using the result of Ref.12 for the induced color charge one finds

\[ \langle Y^A \rangle = \left( \int d^4x \sum_{i=1}^{N_f} \bar{\psi}_i Y^A \psi_i \right) \]
\[ = -\frac{N_f}{64\pi^2} \int d^4x \text{ tr}(Y^A F \tilde{F}) \]
\[ = -\frac{N_f}{2} Q\delta^{A8} \tag{19} \]

\((A,B = 1,2...8)\) and the quark hypercharges have been normalized to \((1,1,-2)\). The effective action \( \Gamma_{ind} \) from which this expectation value may be deduced as \( \int \frac{\delta \Gamma_{ind}}{\delta A_0} \) is the Chern-Simons term in 5 dimensions \(^{12}\). An analogy with the Skyrmeon problem \(^{13}\) with the roles of flavor and color interchanged is evident and a six-dimensional representation (i.e \( \int_{M^6} tr F^3 \) with \( \partial M^6 = M^4 XS^1 \)) of the induced action is available to describe the topologically non-trivial sectors of such theories.

Since the ground states pick up a color charge the soliton should now have a nonzero color electric field and this will result in the formation of a bag at scales \( O(1GeV^{-1}) \) with quarks trapped within it to neutralize the color charge of the semiclassical states discussed above. Similar scenarios may be spelt out for the Salam-Weinberg model at scales \( r << (10^2 GeV)^{-1} \) where the v.e.v of the scalar field is negligible and will result in lepton number carrying objects with large masses.

It is interesting to note that the map provided by the Standard Model doublets from \( S^2_{\infty} XS^1 \) to \( S^3 \) can be noncontractible in contrast to the usual situation.
Let us now briefly sketch how stringlike and sheetlike solitonic structures could arise. Consider a YM theory in 6 dimensions of which two are circular (Self-Dual Euclidean solutions on $R^2$ could also be used). Let $(x_0, x_1, x_2, x_3)$ be coordinates for $M^4$ and $(x^5 = y^1, x^6 = y^2)$ those for the extra dimensions of radii $(R_1, R_2)$. Then for field configurations $A_\mu(x_0, x_1, x_2, x_3), (\mu = 1, 2, 5, 6)$ independent of time and $x^3$ the field equations are as before except that now one must solve them on $R^2 \times S^1 \times S^1$. The same procedure used for going from solutions on $R^4$ to solution on $R^3 \times S^1$ may now be used to go from $R^3 \times S^1$ to $R^2 \times S^1 \times S^1$ since the multisoliton solutions (14) are available for arbitrary locations of arbitrary numbers of solitons. Thus the superpotential for our stringlike objects is

$$
\Pi(\vec{x}, y_1, y_2) = 1 + \sum_{(n_1, n_2) \in \mathbb{Z}^2} \frac{\rho^2}{\vec{x}^2 + (y_1 - n_1 R_1)^2 + (y_2 - n_2 R_2)^2} \\
= 1 + \sum_{n \in \mathbb{Z}} \frac{\pi \rho^2}{r_n R_1} \frac{\sinh \tilde{r}_n}{\cosh \tilde{r}_n - \cos \tilde{y}_1}
$$

$$
\tilde{r}_n = \frac{2\pi}{R_1} \sqrt{x_1^2 + x_2^2 + (y_2 - n R_2)^2} = \frac{2\pi}{R_1} r_n
$$

$$
\tilde{y}_1 = \frac{2\pi}{R_1} y_1
$$

(20)

It is obviously periodic in $(y^1, y^2)$. Actually since the series above has a divergence the above formula needs a subtraction proportional to $\zeta(1)$, after which it displays the logarithmic asymptotics in $\rho = \sqrt{x_1^2 + x_2^2}$ expected for solutions of the equation for the superpotential in two non-compact dimensions. The winding number per unit length in the physical strip $y^i \in [0, R_i)$ is obviously again one and is associated with the homotopy of maps $S^1_\infty \times S^1 \times S^1$. Thus the energy per unit length is $O(\frac{1}{g^2 R})$. Once again coupling to fermions will induce fermion number, charge and non-Abelian charge on the string. A cylindrical bag may be expected to form around QCD strings of this type at distances from the core of order $1(\text{GeV})^{-1}$. The cosmological properties of such structures may provide a
window of allowed values of the compactification scale.

The same procedure can be repeated to include sheetlike defects. The relevant homotopy is that of maps $S^1XS^1XS^1XZ_2 \rightarrow S^3$, where the $Z_2$ is associated with the two sides of the sheet.

To conclude we have utilized the known properties of an available solution$^5,6$ of the Euclidean YM field equations in 4 dimensions to illustrate the complexity and richness of the generic scenario of co-winding solitons that may well be a common and relatively model independent necessary implication of the plethora of higher dimensional models proposed over the last few years. This model independence and the topological stability of such solitons combined with the fact that the known gauge group of the standard model will be sensitive to the extra dimensions through the cowinding channel singles out syncyclons as one of the few possible distinct signatures of higher dimensional scenarios.

Acknowledgements: I am grateful to G.Senjanovic, S.Ouvry, J.McCabe, A.Comtet, Prof. I. Todorov, S. Phatak, and A.Khare for helpful discussions, to J.P.Técourt for etymological advice, to V.Soni for instructive collaboration and to the Theory Group at IPN, Orsay, where this work was largely done, for their warm and pressure free hospitality.

Bibliographical Note: After this idea was reported at a seminar at ICTP, Trieste we were informed by S.Randjbar-Daemi that Strominger$^{14}$ had made some remarks along the same lines. The “five-branes” of Ref.14 are also based on instanton solutions. Their behaviour under compactification of some of the coordinates is interesting.

References
1) A.Salam and J.Strathdee, Ann. of Physics 141 (1982) 316.
2) V.A.Kostelecký and S.Samuel, Phys. Lett. B270 (1991)21.
3) R.Sorkin, Phys. Rev. Lett. 51 (1983) 87.
4) D.Gross and M.Perry, Nucl. Phys. B226 (1983) 29.
5) B.J.Harrington and H.K.Shepard, Phys. Rev. D17 (1978), 2122.
6) D.Gross, R.Pisarski, L.Yaffe, Rev. Mod. Phys. 53 (1981) 43.
7) G.’t Hooft (unpublished);Phys. Rev. D14(1976)3432.
   E.Corrigan and D.Fairlie, Phys. Lett. B67 (1977), 69.
   R.Jackiw, C.Nohl. and C.Rebbi, Phys. Rev. D15, (1977)1642.
8) R.Rajaraman, *Solitons and Instantons*, North Holland, 1982 and references therein.
9) A.Abouelsaood, Nucl. Phys. B226 (1983) 309.
   P.Nelson, Phys. Rev. Lett. 50 (1983), 939.
   P.Nelson and A.Manohar, Phys. Rev. Lett. 50 (1983) 943.
   C.P.Dokos and T.N.Tomaras, Phys. Rev. D 21 (1980), 2940.
10) S.Soleman and P.Nelson, Nucl. Phys. B237 (1984)1.
11) R.Jackiw and C.Rebbi, Phy. Rev. D13 (1976) 3398.
    B.Grossman, Phys. Lett. A61 (1977), 86.
12) A.Niemi and G.W.Semenoff, Physics Reports 135 (1986) 99;
    Phys. Rev. Lett. 51 (1983) 2077.
13) E.Witten, Nucl. Phys. B223 (1983) 422.
    S.Jain and S.R.Wadia , Nucl. Phys. B258(1985) 713.
14) A.Strominger Nucl. Phys. B 343 (1990) 167.