SCALING VIOLATIONS IN THE $Q^2$ LOGARITHMIC DERIVATIVE OF $F_2$

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Abstract

We examine the latest HERA experimental measurements of the $Q^2$ logarithmic derivative of the proton structure function. We analyze the characteristics of DGLAP with and without screening, as well as a Regge type model, and compare their predictions to all available data on $\frac{\partial F_2}{\partial \ln Q^2}$ including those for fixed $W$. Our results show that the present data can be described in pQCD taking into account shadowing corrections. However, such a description cannot be considered a conclusive signal of gluon saturation since the experimental data on $\frac{\partial F_2}{\partial \ln Q^2}$ do not allow one to discriminate between the various approaches, in spite of their very different construction.
1 Introduction

The physics of small $Q^2$ and small $x$ is associated with the search for the scale where gluon saturation [1], implied by s-channel unitarity [2], sets in. Gluon saturation signals the transition from the perturbative (hard) to the non perturbative (soft) QCD regime. We expect this transition to be preceded by signatures of screening corrections (SC) which should be experimentally visible even though the relevant scattering amplitude has not yet reached the unitarity (black disk) limit. Although, the general theoretical framework for saturation is reasonably well understood [1, 2], the specific kinematic domain where we expect to see evidence of the role it plays, is as yet not determined. Consequently, this QCD component depends on the relevant DIS experimental data and the associated phenomenology. While the global analysis of the proton structure function $F_2$ (or $\sigma^{\gamma*\gamma*}(W,Q^2)$) data shows no significant deviations from DGLAP [3], there are dedicated HERA measurements of the $Q^2$ logarithmic derivative of the proton structure function $F_2$ [4, 5, 6], which add to our knowledge as they provide detailed information on the rate of change of the logarithmic $Q^2$ dependence of $F_2$, and, hence, they magnify possible deviations from the expected DGLAP behaviour at small $x$ and small $Q^2$.

The interest in $(\frac{\partial F_2}{\partial \ln Q^2})$ stems from the observation that:

1. In leading order DGLAP evolution the $Q^2$ logarithmic slope of $F_2$ at low $x$, is directly proportional to the gluon structure function [7], since

$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{2\alpha_s}{9\pi} xG^{DGLAP}(x, Q^2),$$

where $xG^{DGLAP}(x, Q^2)$ denotes the DGLAP gluon distribution of the proton.

2. $\frac{\partial F_2}{\partial \ln Q^2}$ is also related to the dipole cross section [8]

$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^3} \sigma_{dipole}(r_\perp^2, x) = \frac{Q^2}{3\pi^2} \int db^2 \text{Im} a_{el}(r_\perp^2, x; b);$$

where $a_{el}(r_\perp^2, x)$ is the elastic scattering amplitude at fixed impact parameter of a dipole of size $r_\perp = 2/Q$ with energy $x$. For the amplitude $a_{el}(r_\perp^2, x)$ we have a non-linear equation in the region of high density QCD (see Refs.[2, 9]) as well as the unitarity constraint $a_{el}(r_\perp^2, x; b) \leq 1$. Using Eq. (2) one can extract the dipole - target amplitude directly from the experimental data.

In our study we will incorporate the following initial observations and constraints:
Figure 1: The $Q^2$ behaviour of $\frac{\partial F_2}{\partial \ln Q^2}$ at $x = 10^{-5}$ ($Q^2$ in GeV$^2$).

1. Recent measurements of $F_2$ [13], extending to very low $Q^2$, find that it has a monotonic behaviour down to the photo production limit. These data suggest a smooth transition from the perturbative (DGLAP, high $Q^2$) to the non-perturbative (Regge or classical gluon field, exceedingly small $Q^2$) regime.

2. Preliminary HERA data [5, 6] on $\frac{\partial F_2}{\partial \ln Q^2}$ at small $Q^2$ and small $x$ do not show any turnover when plotted at fixed $x$ or fixed $Q^2$.

3. An approach which provides a way of successfully describing $\frac{\partial F_2}{\partial \ln Q^2}$ should also be able to reproduce data in other channels which are sensitive to $xG(x, Q^2)$ such as $F_2^c$, $F_L$ and the photo and DIS production of heavy vector mesons.

The main goal of this letter is to investigate and compare:

1. The ability of conventional DGLAP, using the pdfs [10, 11, 12], to reproduce the available data on $\frac{\partial F_2}{\partial \ln Q^2}$.

2. The role of screening corrections (SC). In this approach [14, 15, 16], DGLAP is modified by SC due to the $q\bar{q}$ percolation through the target, as well as by modifications due to screening in the gluon distribution $xG(x, Q^2)$.

3. Within the framework of pQCD no allowance is made for explicit soft contributions, both in the non-screened and screened formulations of DGLAP. The latter provides a smooth interpolation between the high and low $Q^2$ regimes. In the present investigation we have checked the validity of our methods at low values of $Q^2$, i.e. just above the DGLAP $Q^2_0$. 


threshold (defined for each pdf input). For the case with SC, we have also considered an extrapolation for \( Q^2 \leq Q^2_0 \). Our aim is to determine the values of \( Q^2 \) that one can successfully describe without the help of a soft component. To this end we compare with the Donnachie and Landshoff (DL) two Pomeron model [17], which explicitly uses the sum of a hard and a soft Pomeron.

4. We conclude with a critical discussion and caution regarding attempts that have been made to determine the gluon saturation scale using data with highly correlated variables such as in the Caldwell and fixed W plots [4, 6, 18]. We also comment on the general problems associated with gluon saturation and screening corrections relating to other models which include SC [18, 19, 20, 21].

2 The small \( Q^2 \) and small \( x \) behavior of \( \frac{\partial F_2}{\partial \ln Q^2} \) in various models

As is well known, a global DGLAP analysis of the data with the recent pdfs [10, 11, 12] is in good agreement with the experimental data. A study [22] comparing the screened and non-screened DGLAP calculations of \( F_2(x,Q^2) \) showed only a small difference due to SC even in the small \( Q^2 \) and \( x \) region attained by present HERA measurements. A significant deviation of the data from Eq. (1), where \( xG^{DGLAP} \) is obtained from the global \( F_2 \) analysis, may serve as an experimental signature indicating the growing importance of unitarity corrections. This was recently suggested by Caldwell [4], showing a rather complicated plot of \( \frac{\partial F_2}{\partial \ln Q^2} \) in which each measured point had different correlated values of \( Q^2 \) and \( x \). The Caldwell plot suggested a dramatic turnover of \( \frac{\partial F_2}{\partial \ln Q^2} \) corresponding to \( Q^2 \) of about 3 \( GeV^2 \) and \( x < 5 \times 10^{-3} \), in contrast to the behavior expected from GRV94 [23] at sufficiently small \( Q^2 \) and \( x \). The problem with this presentation is that, as suggestive as it may seem, it does not discriminate between different dynamical interpretations [14, 15, 17, 18, 19, 21]. It is, actually compatible with an overall data generator [24] as well as the latest pdfs which were adjusted to account for this observation.

Better discrimination is achieved if we carefully study the small \( Q^2 \) and \( x \) dependences of \( \frac{\partial F_2}{\partial \ln Q^2} \) at either fixed \( Q^2 \) or fixed \( x \) values, so as to be free of the kinematic correlation between \( Q^2 \) and \( x \) that is endemic in the data displayed in the Caldwell plot. Such preliminary HERA data have recently become available [5, 6]. Note that the method that has been used to specify the logarithmic derivative of the structure function \( F_2 \) in [5, 6] is by fitting the \( F_2 \) data at fixed \( x \), to the expression

\[
F_2(x,Q^2) = a(x) + b(x) \ln Q^2 + c(x)[\ln Q^2]^2
\]

and then make the identification

\[
\frac{\partial F_2}{\partial \ln Q^2} = b(x) + 2c(x) \ln Q^2
\]

H1 [5] compared the results using Eq. (4) with the results obtained by taking the derivative of their QCD parameterization (H1 QCD fit). They find that for \( x \geq 0.0002 \), the L.H. and R.H. sides of Eq. (4) match very well, however, for \( x < 2 \times 10^{-4} \) there is a discrepancy. We note that the experimental data of [5, 6] are correlated, such that the lower values of \( x \) are associated with the lower values of \( Q^2 \).

At the initial stage of our investigation we examined whether DGLAP (using the pdfs [10, 11, 12] in the NLO approximation) reproduce the data. The results are shown in Fig. 2. We note that:

1. GRV98 overestimates the low \( Q^2 \leq 5 GeV^2 \) data. For higher \( Q^2 \), the reproduction of the data is good.
2. MRS99 underestimates the data at $Q^2 = 1.9$ and $40 \text{ GeV}^2$. It is above the data in the $3 \leq Q^2 \leq 10 \text{ GeV}^2$ range and does well for $Q^2 = 12$ and $30 \text{ GeV}^2$.

3. CTEQ5 (we have used CTEQ5HQ as it reproduces the energy dependence of $J/\psi$ photoproduction [25]) provides a good reproduction of the experimental data over the entire range $1.9 \leq Q^2 \leq 40 \text{ GeV}^2$. A few words of explanation are called for regarding the numerical NLO calculation of $F_2$ from the CTEQ pdf (we discuss only CTEQ in this context since both GRV98 and MRS99 supply a code for calculating the structure function, from which the parton densities have been parameterized). When calculating $F_2$ in NLO one should take care when inserting the threshold for charm production, so that $F_2$ remains a smooth function of $Q^2$. The effect of the threshold is small if one looks at the structure function, but it can be rather large when examining it’s $Q^2$ derivative. We found that by following the consistent treatment of charm in [26], we obtain a smooth behaviour of both $F_2$ and $\frac{\partial F_2}{\partial \ln Q^2}$.

In order to study the role of SC in our calculations, we follow the eikonal SC formalism presented in Ref.[15], where screening is calculated in DLA in both the quark sector, to account for the percolation of a $q\bar{q}$ through the target, and the gluon sector, to account for the screening of $xG(x, Q^2)$. The factorizable result that we obtain is

$$\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln Q^2} = D_q(x, Q^2)D_g(x, Q^2)\frac{\partial F_2^{DGLAP}(x, Q^2)}{\partial \ln Q^2}. \quad (5)$$

SC in the quark sector are given by

$$D_q(x, Q^2)\frac{\partial F_2^{DGLAP}(x, Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} \int db^2 \left( 1 - e^{-\kappa_q(x, Q^2; b^2)} \right), \quad (6)$$

$$\kappa_q = \frac{2\pi\alpha_S}{3Q^2} xG^{DGLAP}(x, Q^2) \Gamma(b^2). \quad (7)$$

The calculation is simplified if we assume a Gaussian parameterization for the two gluon non perturbative form factor,

$$\Gamma(b^2) = \frac{1}{R^2} e^{-b^2/R^2}. \quad (8)$$

SC in the gluon sector are given by

$$xG^{SC}(x, Q^2) = D_g(x, Q^2)xG^{DGLAP}(x, Q^2), \quad (9)$$

where

$$xG^{SC}(x, Q^2) = \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_0^{Q^2} dQ'^2 \int db^2 \left( 1 - e^{-\kappa_g(x', Q'^2; b^2)} \right), \quad (10)$$

and $\kappa_g = \frac{9}{4}\kappa_q$.

An obvious difficulty in the above calculation of $xG^{SC}$ stems from the fact that the $Q'^2$ integration spans not only the short (pQCD), but also the long (npQCD) distances. To overcome this difficulty we assume that

$$xG \left( x, Q^2 < \mu^2 \right) = \frac{Q^2}{\mu^2} xG \left( x, \mu^2 \right). \quad (11)$$

\[1\] For theoretical and phenomenological status of this approach see Ref. [16].
Our choice of the above extrapolation is motivated by the gauge invariance requirement that $xG \propto Q^2$ when $Q^2 \to 0$, and is supported by recent ZEUS measurements of $F_2$ at exceedingly small $Q^2$ [13].

The SC calculation described above can be applied to any given input pdf where the only adjusted parameters are $R^2$ and $\mu^2$. For the radius $R^2$, see Eq. (8), we use the value $8.5 \text{GeV}^{-2}$ which is determined directly from the forward slope of $J/\Psi$ photo production [26]. $\mu^2$ is conveniently fixed at $Q^2_0$, the lowest $Q^2$ value accessible for the input pdf we use. Once we have chosen our pdf, our SC calculation is essentially parameter free. We have checked that our output results are not sensitive to the fine tuning of these fixed parameters.

Before presenting our results we would like to recall two important features of SC:

1. They can only dampen (reduce) the results obtained with the unscreened input pdfs.

2. They are negligible for large enough $Q^2$ and/or $x$. This can be easily deduced from the functional dependence of $\kappa_q$ and $\kappa_g$ on $Q^2$ and $x$ [15].

Throughout the SC calculation we have used as input the $\overline{\text{MS}}$ version of NLO GRV98. The results obtained using GRV98 DIS are very similar.

Our fixed $Q^2$, and fixed $x$ results are displayed in Fig. 3 and Fig. 4, respectively. As can be seen in the limit of small $Q^2$ and $x$ there is a significant difference between the screened and non screened values of $\frac{\partial F_2}{\partial \ln Q^2}$. As expected the SC results are smaller and softer than the non screened input. All in all, our reproduction of the experimental data for $Q^2 > 2 \text{GeV}^2$ is very good. The ZEUS fixed $Q^2 = 1.9 \text{GeV}^2$ data are somewhat softer than our predictions. This feature is conspicuous when we compare with the very small $Q^2 = 0.3$ and $0.75 \text{GeV}^2$ data, where we have made use of the extrapolation given in Eq.(11). In Fig. 4 we see that, using this extrapolation, screened GRV’98 is also successful in describing the data at values of $Q^2 \leq 1 \text{GeV}^2$, where in the DL parameterization (see Fig. 6) the soft component dominates.

To further examine the role of the soft (non perturbative) component in $\frac{\partial F_2}{\partial \ln Q^2}$, we compare the above DGLAP results with the DL two Pomeron model [17]. The DL parameterization is based on the Regge formalism, and consists of the coherent sum of contributions from a hard and soft Pomeron, a normal Reggeon and an additional contribution from the charmed sector which is proportional to the hard Pomeron. Each of these fixed j-poles are multiplied by a fitted $Q^2$ form factor. The hard Pomeron with a fixed trajectory has an intercept of 1.43 at $t=0$, while the soft Pomeron has an intercept of 1.08 at $t=0$. As can be seen in Fig. 3, the DL model reproduces the data at all values of $Q^2$.

We conclude that one can obtain a good description of $\frac{\partial F_2}{\partial \ln Q^2}$ data with $Q^2 \geq 1.9 \text{GeV}^2$ using any of the following options:

1. Conventional DGLAP evolution using CTEQ5 pdfs as input.

2. DGLAP evolution using GRV98 pdfs as input, provided SC are included.

3. The two Pomeron (DL) model combining hard and soft components.

For smaller $Q^2 (< Q^2_0)$, a soft contribution is called for, for which there are several parameterizations available, but not yet a precise theory.

Recently, Golec-Biernat and Wuesthoff [18] have suggested studying the $Q^2$ and $x$ behaviour of $\frac{\partial F_2}{\partial \ln Q^2}$ at fixed $W$ as a method to determine the gluon saturation scale from the anticipated turnover in these plots. The new ZEUS low $Q^2$ presentations [6] of these plots show, indeed, the anticipated turnover structure in these figures, suggesting that gluon saturation is attained
Figure 2: $x$ dependence of HERA data for the logarithmic slope at fixed $Q^2$ (in GeV$^2$) compared with calculations for the unscreened pdfs.
Figure 3: $x$ dependence of HERA data for logarithmic slope at fixed $Q^2$ (in GeV$^2$) compared with our calculations for screened GRV98 and the DL model.
at $Q^2 \simeq \text{a few GeV}^2$. We advocate using the invariant variables $x$ and $Q^2$ when describing $F_2$. Introducing other variables such as $W$ to study the structure function gives rise to spurious effects which are predominantly kinematic. In the specific procedure suggested by Golec-Biernat and Wusthoff, the combination of the kinematic relation between $x, Q^2$ and $W$ with the very general behaviour of $xG(x, Q^2)$ is sufficient to produce a turnover. Its exact location depends on the details of the numerical input. Consequently, the suggested fixed $W$ plots do not convey any new dynamical information on the saturation problem even if such information is contained in the analyzed data. We note that a fixed $W$ plot is natural and informative for Regge type models. It appears that any parameterization for $F_2$ (or $xG$) which has factorizable $Q^2$ and $x$ dependences, and provides a reasonable description of the data such as the Buchmueller- Haidt (BH) model [27], gives rise to the fixed $W$ turnover effect.

To illustrate this point we consider the fixed $W$ behaviour in three parameterizations which have completely different dynamics:

1. DGLAP with CTEQ5 pdf. As noted, the results of this parameterization are close to the experimental data for $\frac{\partial F_2}{\partial \ln Q^2}$ at fixed $Q^2$ and at fixed $x$ values over the kinematic range of interest. This parameterization does not include a specific soft component.

2. The GLMN model [15], which is a pure pQCD dipole model with SC. As such, the model relates indirectly to gluon saturation, even though it is constructed so as to include unitarity corrections below actual saturation. This parameterization has no explicit soft component.

3. The DL two Pomeron parameterization [17].

All three models, as well as the BH parameterization, follow the experimental behaviour of $\frac{\partial F_2}{\partial \ln Q^2}$ at fixed $W$ rather well, including the observed turnover. We demonstrate this in Fig. 5 by comparing the ZEUS data [6] with the results of our SC model. In Fig. 6 we plot the DL model predictions versus the data at fixed $W$. We also display in this figure the contribution of the hard DL Pomeron.

We conclude with three general comments:

1. Gluon saturation is not unique in producing a turnover at fixed $W$.

2. The gluon saturation scale may be estimated theoretically from the contours produced at the boundary of $\kappa_g = 1$, as discussed in our papers [14, 15]. This theoretical analysis suggests that gluon saturation occurs at $Q^2 \approx 1 \text{ GeV}^2$. Note that $\kappa_g$ depends on $xG(x, Q^2)$, which is determined from a DGLAP pdf fit to the global $F_2$ data.

3. We note that in the BH and DL parameterizations the soft contributions are concentrated at a rather large typical scale $\geq 2 \text{ GeV}^2$. This observation supports the idea that the soft Pomeron originates from rather short distances [28].

3 Discussion and Conclusions

The main experimental observation regarding $\frac{\partial F_2}{\partial \ln Q^2}$ is that at fixed $x$ it is a monotonic decreasing function of $Q^2$, and for fixed $Q^2$ it increases as $x$ becomes smaller. This is the observed pattern even for the lowest measured value of $Q^2 = 0.3 \text{ GeV}^2$ [6]. The main objective of our
Figure 4: $Q^2$ (in GeV$^2$) dependence of $H1$ and ZEUS logarithmic slope data at fixed $x$ compared with our calculations for screened GRV98.
Figure 5: ZEUS logarithmic slope data at fixed $W$ (in GeV) compared with our SC calculation with GRV98 input ($Q^2$ in GeV$^2$).
Figure 6: ZEUS logarithmic slope data at fixed $W$ (in GeV) compared with DL two Pomeron model ($Q^2$ in GeV$^2$).
Figure 7: Predictions for the different parameterizations at fixed low values of $x$ ($Q^2$ in GeV$^2$).
investigation was to check whether the preliminary H1 [5] and ZEUS [6] data contains a decisive experimental signature indicating signs of gluon saturation. This effect is associated with the transition from the relatively well understood pQCD hard domain to the more complex, and less understood soft domain. We have shown in this paper that the gluon saturation is capable of describing all HERA data on $\frac{\partial F}{\partial \ln Q^2}$, and therefore, the data do not contradict the idea that HERA has reached a new QCD regime: high parton density QCD.

However, the final conclusion of our analysis is that the present data can be described by a number of models, which are very different in their basic construction. Specifically, we show that all available data with $Q^2 > Q^2_0$, of $\frac{\partial F}{\partial \ln Q^2}$ can be described by DGLAP using the CTEQ5 pdf as input, as well as GRV98 pdf which have been corrected for screening. The DL model which is Regge motivated and includes a significant soft component, also provides a good description of the data.

We have shown that the turnover seen in $\frac{\partial F}{\partial \ln Q^2}$ at fixed $W$, as a function of either fixed $Q^2$ or $x$, is not necessarily connected with saturation effects and cannot be used to discriminate between models. There has also been a successful reproduction of the Caldwell plot by Kaidalov et al. [19] within the framework of the CKMT model [20], and by Forshaw et al. [21] using a colour dipole model formalism. We are of the opinion that fitting the Caldwell as well as the fixed $W$ plots is a necessary first step, but a comparison with all the detailed measurements is essential to test the adequacy of a given model.

In Fig. 7 we attempt to clarify the similarities and differences between the non screened CTEQ5 and the screened GRV98 DGLAP calculations whose results are very similar, (see Figs. 2 and 3). To this end we display the calculated distributions of $\frac{\partial F}{\partial \ln Q^2}$ and $\alpha_s xG(x,Q^2)$ at fixed values $x = 10^{-3}, 10^{-4}, 10^{-5}$ and $10^{-6}$. As can be seen the results using CTEQ5 (non screened) and GRV98 (screened) in the limit of very small $x$ are very similar. It is therefore suggestive, that CTEQ5 contains significant screening effects, that are absent in the boundary conditions used in GRV98. Note that CTEQ5 is not defined for $x < 10^{-5}$. Our estimates of SC for the CTEQ5 parameterization (based on Eq. (5) - Eq. (10)) is about 10%, which should be considered as the uncertainty in this parameterization. We also show the DL predictions for $\frac{\partial F}{\partial \ln Q^2}$, which differ from the hard partonic DGLAP at small enough $x$.

The ZEUS data at exceedingly low $Q^2$ [13] are of particular interest when investigating the transition from the DGLAP dominated region to the non perturbative (low $Q^2$) region. This transition was expected to be observed experimentally since $F_2 \approx Q^2$ as $Q^2 \rightarrow 0$ due to gauge invariance requirements, and for large $Q^2$, $F_2 \approx (Q^2)^\gamma$ from DGLAP evolution, where $\gamma$ is the anomalous dimension. This transition has been seen by ZEUS [13] in their measurements of $F_2(x,Q^2)$ at small values of $x$ and $Q^2$, where the transition appears to be at $Q^2 \approx 1 \text{ GeV}^2$, and is compatible with our early theoretical estimates [14, 15]. This provides an indirect indication that this is the scale for the onset of gluon saturation, but we still lack a decisive signature for this effect.

Based on our present investigation we conclude that the behaviour of $\frac{\partial F}{\partial \ln Q^2}$ as measured in the kinematic region presently accessible at HERA, does not display unambiguous signs of saturation. This is compatible with the information displayed in Fig. 1 where we see that the anticipated gluon saturation scale is close to $Q^2_0$, and to the scale at which the soft contribution becomes significant.

We are extending our search for possible effects of gluon saturation to the channel of photo and DIS production of $J/\Psi$ [25], where the cross section should be very sensitive to this signal, as it is proportional to $|xG(Q^2,x)|^2$.

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