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Modulation-assisted tunneling in laser-fabricated photonic Wannier–Stark ladders

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Abstract

We observe Wannier–Stark (W–S) localization in curved photonic lattices, realized using arrays of evanescently coupled optical waveguides. By correctly tuning the strength of inter-site coupling in the lattice, we observe that W–S states become increasingly localized, and eventually fully localized to one site, as the curvature of the lattice is increased. We then demonstrate that tunneling can be successfully restored in the lattice by applying a resonant sinusoidal modulation to the lattice position, an effect that is a direct analogue of photon-assisted tunneling. This precise tuning of the tunneling matrix elements, through resonant modulation-assisted tunneling, opens a novel route for the creation of gauge fields in laser-fabricated photonic lattices.

1. Introduction

Quantum matter or light propagating in engineered lattices offer versatile platforms for the quantum simulation of new states of matter, such as topological phases\cite{1–5}. This approach relies on novel technologies allowing for the tuning of microscopic parameters that characterize lattice models of interest, such as on-site interactions and tunneling matrix elements. In fact, generating complex-valued tunneling matrix elements can potentially induce non-trivial gauge structures in the lattice, e.g. artificial magnetic fields and non-Abelian gauge potentials\cite{1, 5, 6}, offering an interesting route for quantum simulation\cite{7}. In this context, photon-assisted tunneling, a powerful method by which tunneling can be controlled in lattice systems, has been recently exploited in cold gases\cite{8–14} and ion traps\cite{15, 16}; this led to the experimental realization of the Hofstadter model\cite{17, 18} and to the detection of the topological Chern number\cite{19} with cold atomic gases.

The photon-assisted-tunneling method relies on two main ingredients\cite{10, 13–15}: (a) an artificial electric field generating a large energy offset \(\Delta\) between neighboring sites, hence inhibiting the bare hopping, and (b) a time-modulation of the on-site energy, whose frequency is resonant with respect to the static offset \(\omega = \Delta / \hbar\); this restores the tunneling in an efficient and tunable manner. In this paper, we experimentally demonstrate the realization of modulation-assisted tunneling, an analogue of photon-assisted-tunneling, in arrays of coupled optical waveguides.

The transport of light in a system of coupled optical waveguides, a photonic lattice, can be described by a Schrödinger-like equation. As a result, photonic lattices can be used to observe phenomena known from solid state physics. In recent years photonic lattices have been used to study fundamental solid state phenomena including Bloch oscillations\cite{20, 21}, dynamic localization\cite{22–24}, Bloch–Zener dynamics\cite{25}, and Landau–Zener dynamics\cite{26}. These phenomena are each related to the manner in which a charged particle behaves in a periodic potential and external electric field. In such a system, a static electric field destroys the translational symmetry of the lattice, and the delocalized Bloch states become localized in space. These states are known as Wannier–Stark (W–S) states, originally predicted by Wannier in 1960\cite{27}.
In the absence of an external electric field, the eigenstates of an electron in a periodic potential are the Bloch states. A static electric field \( \mathcal{E}_{dc} \) destroys the degeneracy of these spatially delocalized Bloch states. In this situation, the eigenstates and energies are \( [28, 29] \)

\[
\phi_m = \sum_n J_{n-m}(2\kappa/e\mathcal{E}_{dc,0})|n\rangle, \\
E_m = (e\mathcal{E}_{dc,0})m,
\]

where \( J_n(\cdot) \) is the Bessel function of order \( \nu, a \) is the lattice spacing, \( 4\kappa \) is the bandwidth, \( e \) is the electronic charge and \( |n\rangle \) are the WannIER states. The span of the first Brillouin zone is \( -\pi/a \leq k \leq \pi/a \). In the limit \( 2\kappa/e\mathcal{E}_{dc,0} \to 0 \) there is only one term in equation (1), i.e. the eigenstates exactly correspond to the localized WannIER states \( |m\rangle \). In fact, in a strong external electric field and weak inter-site interaction i.e. \( 2\kappa/e\mathcal{E}_{dc,0} \ll 1 \), the spatial width of W–S state is less than the inter-site separation, \( a \). In this limit, the W–S states will be localized to a single lattice site, indicating that the energy offset \( \Delta = e\mathcal{E}_{dc,0}a \gg \kappa \) generated by the static electric field inhibits the bare hopping between neighboring sites. Importantly, when driving the system, the strongly localized electronic states on individual lattice sites can interact through photons and tunnel to the nearest lattice sites. This type of tunneling with discrete energy exchange is known as photon-assisted tunneling; it has been observed in superconducting diodes [30], semiconductor superlattices [31], quantum dots [32], and also with cold atoms trapped in optical lattices [33].

In this paper, we use photonic lattices, fabricated using the technique of ultrafast laser inscription, as a powerful platform to investigate the dynamics of W–S states in a strong static electric field and weak inter-site interaction. We demonstrate strong localization of the W–S state, seen for the first time using curved photonic lattices, where the curvature is analogous to the inverse of a static electric field in the electronic case. When the electric field exceeds a threshold value, we observe that the W–S state becomes localized to a single lattice site. Importantly, we then also demonstrate that a strongly localized W–S state becomes delocalized when an appropriate (specific frequencies and amplitudes) sinusoidal modulation is applied to the lattice. The latter result constitutes the first photonic-crystal analogue of photon-assisted tunneling, based on fabricated sinusoidal modulations, offering a promising method for the generation of gauge fields in photonic lattices.

2. The photonic lattice

The propagation of the electric field envelope \( \Phi \) in the material is governed by

\[
i\lambda \frac{\partial\Phi}{\partial z} = \left[ -\frac{\lambda^2}{2n_0} \nabla_\perp^2 + V(x' - x_0(z), y) \right] \Phi,
\]

where \( \Phi \) depends on \( x', y \) and \( z \), \( \lambda = 2\pi\lambda \) is the free-space wavelength and \( \nabla_\perp^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y^2} \). \( V(x', y) = \sum_n V_0(x' - x_n, y - y_m) \) describes the refractive index modulation in the transverse cross section where \( V_0(x', y) \) is the refractive index profile of a single waveguide at position \( x_n, y_m \). The function \( x_0(z) \) determines the transverse shift of the whole lattice depending on the propagation distance \( z \). By making a change of reference frame \( x = x' - x_0(z) \) equation (3) can be rewritten as

\[
i\lambda \frac{\partial\Phi}{\partial z} = \left[ -\frac{\lambda^2}{2n_0} \nabla_\perp^2 + V(x, y) - Fx \right] \Phi
\]

with \( F = -n_0\frac{\partial^2}{\partial x^2}x_0(z) \) and where \( \Phi(x, y, z) \) is the consequently transformed state [23, 34]. The transport of light in a circularly curved one-dimensional (1D) photonic lattice, with sinusoidal modulation (figure 1(c)) is then governed by the paraxial equation [23, 34]

\[
i\lambda \frac{\partial\Phi}{\partial z} = \left[ -\frac{\lambda^2}{2n_0} \frac{\partial^2}{\partial x^2} - \Delta n(x) - \frac{n_0}{R}x - n_0A\omega_0^2 \sin(\omega_0z) \right] \Phi,
\]

where the lattice is bending along the \( x \) direction with a radius of curvature \( R \). The amplitude and frequency of the \( z \)-dependent ‘ac’ modulation are \( A \) and \( \omega_0 \) respectively, \( n_0 \) is the refractive index of the substrate material and \( \Delta n(x, y) \) is the transverse refractive index profile. Equation (5) is analogous to the Schrödinger equation of a particle with effective mass \( n_0 \) and charge \( e \) moving in a 1D periodic potential \( V(x) = -\Delta n(x) \), with an external (artificial) electric field \( \mathcal{E} = \mathcal{E}_{dc} + \mathcal{E}_{ac} \), where \( e\mathcal{E} = n_0/R + n_0A\omega_0^2 \sin(\omega_0z) \). Here, \( z \) plays the role of time, \( e\mathcal{E}_{dc} = n_0/R \), and \( e\mathcal{E}_{ac} = n_0A\omega_0^2 \sin(\omega_0z) \) (see equations (1) and (2)).

For well-confined single-mode waveguides, equation (5) can be solved using the tight-binding approximation. For a 1D photonic lattice, and supposing that only the lowest band is excited, equation (5) gives the coupled-mode equations.
This semi-classical picture generalizes to

\[ \Phi_{nA} = \frac{K}{\omega_0 \epsilon} \]

where the oscillations in quasi-momentum space are small, and the wave packet essentially remains localized around

\[ \nu \in \mathbb{Z}, \]

is satisfied. Hence, for a photonic lattice satisfying \( n_0 a / \Lambda \omega_0 = \nu \in \mathbb{Z} \), tunneling between neighboring sites is restored. This leads to an effective coupling constant, \( \kappa_{\text{eff}} \), whose amplitude is given by (see [10, 14] and section 5)

\[ \left| \frac{\kappa_{\text{eff}}}{\kappa} \right| = \left| J_0 \left( \frac{K}{\omega_0} \right) \right|, \]

where \( J_0 \) is the Bessel function of order \( \nu \); see also figure 6. Let us point out that the real part of the effective coupling constant \( \Re (\kappa_{\text{eff}}) = \kappa J_0 (K / \omega_0) \) is allowed to take positive, but also negative values. This effect has a simple semi-classical interpretation based on the micro-motion associated with the ac field, and which generates oscillations in quasi-momentum space, see [14, 35]. Let us consider the case \( \nu = 0 \) for simplicity, and suppose that a wave packet is prepared in the vicinity of the band’s ground-state at \( k = 0 \). For weak modulations \( K \ll \omega_0 \), the oscillations in quasi-momentum space are small, and the wave packet essentially remains localized around \( k = 0 \); the effective tunneling rate \( \kappa_{\text{eff}} \approx \kappa \) is only slightly affected. For strong modulations, the micro-motion oscillations become comparable to the size of the Brillouin zone, and the wave packet often visits the high-energy regions of the band at \( k = \pm \pi / a \). This situation is consistent with a change of sign of the tunneling rate, i.e. \( \text{sign}(\kappa_{\text{eff}}) = -\text{sign}(\kappa) \), as described by the Bessel function \( J_0 \). This semi-classical picture generalizes to arbitrary \( \nu \neq 0 \).

3. W–S localization

To investigate W–S localization, fifteen 1D lattices (lattice constant \( a = 16 \mu m \)) were fabricated using ultrafast laser inscription, figure 1(d). In these lattices, no modulation was created (\( A = 0 \)), however the radius of curvature of the lattice was varied between 1.5 and 0.1 m (\( R = 1.5, 1.4, \ldots, 0.1 \) m), see figure 1(b). An additional straight lattice was also fabricated (\( R = \infty \)), see figure 1(a). The white-light transmission micrograph of the facet of a lattice is shown in figure 1(e). The refractive index profile of each waveguide was controlled using the ‘slit-beam’ shaping method [36]. Each waveguide was inscribed by translating the 30 mm long glass sample (Corning Eagle2000), at a translation speed of 8 mm s\(^{-1}\), once through the focus of a 500 kHz train of 1030 nm femtosecond laser pulses. The laser inscription parameters were optimized to produce waveguides that were single-mode and well confined at 780 nm. For a more detailed description of the
waveguide fabrication procedure, see [37]. It should be highlighted that although the propagation loss of a waveguide depends on its curvature, we do not expect this to be important in our discussion since all waveguides in each lattice have the same bend radius at a given value of $z$. In other words, there is no site dependent loss for a given lattice.

The nearest-neighbor coupling, $\kappa$, was measured to be $0.072 \text{ mm}^{-1}$. The next-nearest neighbor coupling strength, $\kappa_{NN}$, was insignificant for the 30 mm long lattices with 16 $\mu$m lattice constant. Figure 2 shows the output intensity distribution measured for the lattices with radii of curvature $R = \infty$ (i.e. the straight lattice), 1, 1.2, 0.5, 0.3 and 0.2 m. It is clear from figure 2 that the light becomes increasingly localized as the radius of curvature is reduced, as would be expected from equation (1). To investigate this phenomenon further, we fabricated 10 mm long 2D lattices with a lattice constant $a = 15 \mu$m along both $x$ and $y$ axes, where each lattice curves only along the $x$ direction. For these lattices, the measured coupling strengths along the $x$ and $y$ axes were $0.085 \text{ mm}^{-1}(\kappa_x)$ and $0.095 \text{ mm}^{-1}(\kappa_y)$ respectively. From simulations, the estimated value of next-nearest-neighbor coupling was $0.019 \text{ mm}^{-1}(\kappa_{NN})$. As can be seen from figure 3, localization occurs only along the $x$ axis, the direction of the artificial electric field. To quantify localization along the two axes, the inverse participation ratio (IPR) was calculated. The IPR is a measure of localization and is defined as the inverse of the absolute value of the average of the fourth power of the wave function. For our purpose, the IPR for the $x$ axis was obtained by summing all the intensity values in each column to obtain a vector of values along the $x$ axis. The IPR was then calculated using this vector. The IPR along the $y$ axis was calculated using the same procedure, but by summing
the rows rather than columns. For a localized state, the IPR is equal to 1. As can be seen from figure 4, there is no effect of electric field along the y axis, as would be expected, but complete localization is observed along the x axis once the artificial electric field exceeds a threshold value.

The localization phenomenon can also be explained using the theory of waveguide optics. It can be shown [34], using a conformal transformation, that a 1D array of circularly curved waveguides with periodic transverse refractive index profile is equivalent to an array of straight waveguides with a new refractive index profile. In the limit $a/R \ll 1$, the new refractive index profile is the superposition of the original periodic index profile and a linear ramp of refractive index, and the radius of curvature controls this ramp. In other words, the mode supported by each waveguide in the curved array has a different propagation constant compared to a straight array. In the limit of high curvature, the mode is equivalent to an array of straight waveguides with a new refractive index profile.

4. Analog photon-assisted tunneling

To observe the effect analogous to photon-assisted tunneling, three sets of 30 mm long lattices were fabricated with a sinusoidal modulation (figure 1(c)). For sets 1, 2 and 3, the periods $z_0 = 2\pi/\omega_0$ were set to 3.9 mm, 7.8 mm and 11.7 mm respectively, corresponding to $\nu = 1, 2$ and 3 in equation (7). For all sets, the radius of curvature and inter-site separation were set to $R = 120$ mm and $a = 16$ $\mu$m respectively. For each set, 15 lattices were fabricated and the amplitude of oscillation, $A$, was varied; $0.5$ $\mu$m $\leq A \leq 14.5$ $\mu$m (for set 1), $5.0$ $\mu$m $\leq A \leq 40.0$ $\mu$m (for set 2) and $14$ $\mu$m $\leq A \leq 70$ $\mu$m (for set 3). The measured output intensity distributions are shown in figure 5. The effective coupling of a modulated lattice, for a given value of $A$, $z_0$ and $R$, was evaluated by simulating a 30 mm long straight photonic lattice, and varying the coupling strength to optimally fit the observed output intensity distributions. The normalized effective coupling strength $|K_{\text{eff}}|/|K|$ is plotted graphically as a function of $K_{\text{eff}} = K/\omega_0$ in figure 6, where it can be seen that the normalized effective coupling strength has a characteristic (Bessel-function) dependency on $K_{\text{eff}}$ as predicted by equation (8). This is clear evidence that the tunneling has been partially restored through an analogue of photon-assisted tunneling. As a final note, it should be stressed that simulations performed using the experimentally evaluated parameters indicate that significant tunneling is absent when $z_0$ was not an integer multiple of 3.9 mm.

5. Application: artificial magnetic fluxes

Photon-assisted tunneling constitutes a powerful method to generate artificial gauge structures in lattice systems [13–15]. To illustrate this concept, let us start with a simplified version of the system described by equations (6), (7). Consider a two-level system $\{|0\}, \{|1\}\}$, i.e. two sites of a lattice treated in a single-band tight-binding approximation, with energy offset $\alpha = \omega_0 \nu$ and resonant ‘time’ modulation with frequency $\omega_0$; here $\nu \in \mathbb{Z}$. The Hamiltonian is taken to be of the form
which is indeed strictly equivalent to restricting the Schrödinger equation in equations (6), (7) to two lattice sites \( s = 0, 1 \). Note that we have introduced the phase of the modulation \( \phi \), and we note that \( K \approx \omega_0 \) in the strong-driving regime. We tackle the Schrödinger equation by first performing a unitary transformation

\[
\hat{R}(z) = \exp \left\{ -i \left[ \nu_0 z - K_0 \cos \left( \omega_0 z + \phi \right) \right] \right\},
\]

which results in the modified z-dependent Hamiltonian

\[
\hat{H}(z) = -\kappa \left( |0\rangle \langle 1| + |1\rangle \langle 0| \right) - |1\rangle \langle 1| \left[ \nu_0 \nu_0 + K_0 \omega_0 \sin \left( \omega_0 z + \phi \right) \right],
\]

where we used the Jacobi–Anger expansion, \( \exp \left( i x \cos y \right) = \sum_{n=-\infty}^{\infty} i^n J_n(x) e^{i n y} \). To lowest order in \( \kappa / \omega_0 \ll 1 \), the effective Hamiltonian \([14, 35]\) is well approximated by the time-average over one period (i.e. \( j = 0 \) in equation (9)), which yields

\[
\hat{H}(z) = \frac{1}{2} \left( |0\rangle \langle 1| + |1\rangle \langle 0| \right) - |1\rangle \langle 1| \left[ \nu_0 \nu_0 + K_0 \omega_0 \sin \left( \omega_0 z + \phi \right) \right].
\]
\[ \hat{H}_{\text{eff}} \approx -\kappa_{\text{eff}} |0\rangle \langle 1| + \text{h.c.}, \quad \kappa_{\text{eff}} = \kappa \mathcal{J} (K_0) e^{-i\nu + \pi / 2}, \]  

(10)

Importantly, the effective coupling elements \( \kappa_{\text{eff}} \) are now complex-valued for \( \nu \neq 0 \), with a phase factor \( \exp(\pm i\nu) \) that explicitly depends on the phase of the modulation \( \phi \). The two-site result in equation (10) directly generalizes to the full modulated W-S ladder (equations (6), (7)), which is then well described by the effective Hamiltonian

\[ \hat{H}_{\text{eff}} \approx -\kappa_{\text{eff}} \sum_s |s - 1\rangle \langle s| + \text{h.c.}, \quad \kappa_{\text{eff}} = \kappa \mathcal{J} (K_0) e^{-i\nu + \pi / 2}, \]  

(11)

as already announced in equation (8) for \( \phi = 0 \).

While the phase factor \( \exp(\pm i\nu) \) in equation (11) is irrelevant in the 1D geometries considered in this work (i.e. the phase is associated with a constant gauge potential; see [38-40] for physical consequences), it can potentially lead to dramatic effects in 2D geometries if it is made explicitly space-dependent [13-15]. Indeed, let us consider a modulated W-S ladder aligned along the \( x \) direction, with sites labeled by the index \( s_x \), and let us assume that the phase of the modulation \( \phi = \phi(x, y) \) can be controlled in a space-dependent manner. If an additional lattice is aligned along the \( y \) direction, with lattice sites labeled by \( s_y \), then the 2D effective Hamiltonian will be of the form [13-15, 41, 42]

\[ \hat{H}_{\text{eff}} \approx - \sum_{s_x, s_y} \kappa_{\text{eff}}^{x,y} e^{-i\phi(s_{x,y})} \left[ s_x + 1, s_y \right] \left[ s_x, s_y + 1 \right] + \text{h.c.}, \]  

(12)

where \( \kappa_{\text{eff}}^{x,y} \) are real-valued tunneling rates along the \( x \) and \( y \) directions, respectively, and where we considered the case \( \nu = 1 \) for simplicity. If the phase is designed to be in the form \( \phi(s_{x,y}) = 2\pi \phi_{x,y} s_x s_y \), then the system realizes the Harper–Hofstadter Hamiltonian [41-43]: a 2D lattice penetrated by a uniform magnetic flux \( 2\pi \phi_{x,y} \) per plaquette. Indeed, the wavefunction of a particle circulating around any plaquette of the lattice acquires an Aharonov-Bohm phase associated with the flux \( 2\pi \phi_{x,y} \). Hence, engineering the spatial dependence of the phase of the modulation \( \phi \), which still constitutes an experimental challenge, would allow for the creation of synthetic fluxes in photonic lattice systems, opening an interesting route for photonic quantum-Hall (topological) physics.

6. Conclusion

In this paper, we have demonstrated that an appropriately designed array of evanescently coupled curved optical waveguides can be used to observe a W–S state that is fully localized on a single lattice site. From the perspective of solid state physics, the localization is due to an analogue of a strong external dc electric field that breaks the degeneracy of the Bloch states and results in a W–S ladder. We also demonstrate that tunneling in such photonic lattices can be restored by applying an analogue of an ac electric field, and that the strength of this tunneling obeys a characteristic dependency on the frequency and amplitude of the ac modulation, which is in excellent agreement with the existing theory of photon-assisted tunneling. By further tuning the spatial dependence of the laser-fabricated modulation, this method could be used to produce effective magnetic fluxes [13–15, 44] in 2D photonic lattices; this goes beyond the realization of artificial electric fields, as presented and exploited in this work. The interplay between such artificial fields and the presence of nonlinearities opens a promising route for the study of interacting particles in large magnetic fields and topological phenomena.

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References

[1] Carusotto I and Ciuti C 2013 Rev. Mod. Phys. 85 299
[2] Rechtsman M C, Zeuner J M, Pлотник Y, Lumer Y, Podolsky D, Dreisow F, Nolte S, Segev M and Šantner A 2013 Nature 496 196
[3] Hafezi M, Demler E A, Lukin M D and Taylor J M 2011 Nat. Phys. 7 907
[4] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. 80 885
[5] Goldman N, Juzeliūnas G, Öhberg P and Spielman I B 2014 Rep. Prog. Phys. 77 126401
[6] Dalibard J, Gerbier F, Juzeliūnas G and Öhberg P 2011 Rev. Mod. Phys. 83 1523
[7] Bloch I, Dalibard J and Nascimbene S 2012 Nat. Phys. 8 267
[8] Ruostekoski J, Dunne G V and Javanainen J 2002 Phys. Rev. Lett. 88 180401
[9] Jaksh D and Zoller P 2003 New J. Phys. 5 56
[10] Eckardt A, Jinasundera T, Weiss C and Holthaus M 2005 Phys. Rev. Lett. 95 200401
[11] Eckardt A and Holthaus M 2007 Europhys. Lett. 80 50004
[12] Gerbier F and Dalibard J 2010 New J. Phys. 12 30007
[13] Kolovsky A R 2011 Europhys. Lett. 93 20003
[14] Goldman N, Dalibard J, Aidelsburger M and Cooper N R 2015 Phys. Rev. A 91 033632
[15] Bermudez A, Schaetz T and Porras D 2011 Phys. Rev. Lett. 107 150501
[16] Bermudez A, Schaetz T and Porras D 2012 New J. Phys. 14 053049
[17] Aidelsburger M, Atala M, Lohse M, Barreiro J T, Paredes B and Bloch I 2013 Phys. Rev. Lett. 111 85301
[18] Miyake H, Siviloglou G A, Kennedy C J, Burton W C and Ketterle W 2013 Phys. Rev. Lett. 111 185302
[19] Aidelsburger M, Lohse M, Schweizer C, Atala M, Barreiro J T, Nascimbène S, Cooper N R, Bloch I and Goldman N 2015 Nat. Phys. 11 162
[20] Bloch F 1928 Z. Phys. 52 555
[21] Pertsch T, Dannberg P, Ellein W, Bräuer A and Lederer F 1999 Phys. Rev. Lett. 83 4752
[22] Dunlap D H and Kenkre V M 1986 Phys. Rev. B 34 3625
[23] Longhi S, Marangoni M, Lobino M, Ramponi R, Laporta P, Cianci E and Foglietti V 2006 Phys. Rev. Lett. 96 243901
[24] Szameit A, Garanovich I L, Heinrich M, Sukhorukov A A, Dreisow F, Pertsch T, Nolte S, Tünnermann A, Longhi S and Kivshar Y S 2010 Phys. Rev. Lett. 104 235903
[25] Dreisow F, Szameit A, Heinrich M, Pertsch T, Nolte S, Tünnermann A and Longhi S 2009 Phys. Rev. Lett. 102 076802
[26] Dreisow F, Szameit A, Heinrich M, Nolte S, Tünnermann A, Ornigotti M and Longhi S 2009 Phys. Rev. A 79 055802
[27] Wannier G H 1960 Phys. Rev. 117 432
[28] Fukuyama H, Bari R A and Fedoghy H C 1973 Phys. Rev. B 8 5579
[29] Emin D and Hart C F 1987 Phys. Rev. B 36 7353
[30] Tien P and Gordon J P 1963 Phys. Rev. 129 647
[31] Guimaraes P S S, Keay B J, Kaminski J P, Allen S J, Hopkins P F, Gossard A C, Florez I T and Harbison J P 1993 Phys. Rev. Lett. 70 3792
[32] Kouwenhoven L P, Jauhar S, Orenstein J, McEuen P L, Nagamune Y, Motohisa J and Sakaki H 1994 Phys. Rev. Lett. 73 3443
[33] Stas C, Lignier H, Singh Y P, Zenesini A, Ciampini D, Morsch O and Arimondo E 2008 Phys. Rev. Lett. 100 040404
[34] Heiblum M and Harris J H 1975 IEEE JQE 11 75
[35] Goldman N and Dalibard J 2014 Phys. Rev. X 4 031027
[36] Ams M, Marshall G, Spence D and Withford M 2005 Opt. Express 13 5676
[37] Mukherjee S, Spracklen A, Choudhury D, Goldman N, Öhberg P, Andersson E and Thomson R R 2015 Phys. Rev. Lett. 114 245504
[38] Creffield C E and Sols F 2011 Phys. Rev. A 84 023630
[39] Jiménez-Garcia K, LeBlanc L J, Williams R A, Beeler M C, Perry A R and Spielman I B 2012 Phys. Rev. Lett. 108 225303
[40] Struck J, Olschläger C, Weinberg M, Hauke P, Simonet J, Eckardt A, Lewenstein M, Sengstock K and Windpassinger P 2012 Phys. Rev. Lett. 108 225304
[41] Creffield C E and Sols F 2014 Phys. Rev. A 90 023636
[42] Creffield C E and Sols F 2013 Europhys. Lett. 101 40001
[43] Hofstetter D R 1976 Phys. Rev. B 14 2239
[44] Dubcek T, Lelas K, Jukić D, Pezer R, Soljacic M and Buljan H 2015 arXiv:1505.06690