Turbulence Spectra in the Stable Atmospheric Boundary Layer

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Stratification can cause turbulence spectra to deviate from Kolmogorov’s isotropic -5/3 power-law scaling in the universal equilibrium range at high Reynolds numbers. However, a consensus has not been reached with regard to the exact shape of the spectra. Here we propose a theoretically-derived shape of the turbulent kinetic energy (TKE) and temperature spectra in horizontal wavenumber that consists of three regimes at small Froude number: the buoyancy subrange, a transition region and isotropic inertial subrange through derivation based on previous research. These regimes are confirmed by various observations in the atmospheric boundary layer. We also show that DNS may not apply in the study of very stable atmospheric boundary layers at very high Reynolds numbers as they cannot correctly represent the observed spectral regimes because of the lack of scale separation limited by current computational capacity. In addition, the spectrum in the transition regime explains why Monin-Obukhov similarity theory cannot entirely describe the behavior of the stable atmospheric boundary.

1. Introduction

Isotropic turbulence spectra follow a -5/3 power-law scaling in the inertial subrange \cite{Kolmogorov1941}. However, stratification \cite{Lilly1983} is widely observed in geophysical turbulence, for example, in nocturnal atmospheric boundary layer \cite{Mahrt1998}, in most part of troposphere \cite{Fiedler1970, Nastrom1985, Tulloch2006}, above the tropopause \cite{Nastrom1985, Tulloch2006, Smith2006} and below ocean mixed layer \cite{Baker1987} and generates anisotropy. Because of this stratification \cite{Bolgiano1959, Dougherty1961, Lumley1964, Phillips1965, Weinstock1978}, the spectra deviate from the -5/3 scaling of isotropic turbulence. At very large, synoptic scales, theory and observation have shown that horizontal wavenumber spectra of turbulence kinetic energy (TKE) exhibit a \textsuperscript{-3} power-law scaling \cite{Charney1971, Nastrom1985}. In the mesoscale regime, a scaling close to -5/3 is still observed \cite{Nastrom1985}. Recently, a direct energy cascade hypothesis \cite{Lindborg2006} was supported by direct numerical simulation (DNS) studies \cite{Brethouwer2007, Kimura2012, Riley2003, Waite2004} to explain the -5/3 scaling of anisotropic turbulence.

In the atmospheric surface layer \footnote{Email address for correspondence: yc2965@columbia.edu} (at lower part of the boundary layer), and thus at high
Reynolds numbers, it is widely accepted that Monin-Obukhov similarity theory (MOST) (Monin & Obukhov 1954) works well in neutral or weakly stable cases but does not apply in very stable cases (Mahrt 1998, 2014). Several hypotheses have been proposed to account for the failure of MOST, such as upside-down boundary layers (Mahrt 1985; Ohya et al. 1997), mesoscale motions (Mahrt 1999; Smeets et al. 1998), surface heterogeneity (Derbyshire 1995) and Kelvin-Helmholtz instability with discontinuous and intermittence turbulence (Cheng et al. 2005). When applying MOST to calculate velocity and temperature variance, the -5/3 spectra scaling in the equilibrium range is always assumed (Kaimal et al. 1972).

Weinstock (1978) derived the stratified turbulence spectra by assuming “approximately isotropic” fluid motions. For eddies below “energy containing” scales in the atmospheric boundary layer, we relax the “approximately isotropic” hypothesis for horizontal wavenumber spectra, which consist of buoyancy subrange, transition region and isotropic inertial subrange. We will show that the transition region is related to three scales: the buoyancy length scale (Billant & Chomaz 2001), the Dougherty-Ozmidov scale (Dougherty 1961; Ozmidov 1965) and the distance from wall (Townsend 1976; Katul et al. 2014) in horizontal wavenumber spectra of both TKE and temperature in the equilibrium range at high Reynolds numbers. We further suggest that the deviation from the -5/3 power-law scaling in the transition region leads to the failure in applying MOST to very stable atmospheric boundary layers (ABL).

2. Derivation of spectra scaling

2.1. Scale definition

The strength of the stratification is assessed using the horizontal Froude number \( Fr_h = \frac{U}{N L_h} \), where \( U \) is root mean square of the horizontal velocity, \( N \) is the Brunt-Väisälä frequency and \( L_h \) is a horizontal length scale. The horizontal length scale \( L_h = \frac{U^3}{\epsilon} \) is determined by invoking Taylor’s frozen turbulence hypothesis (Taylor 1938), where \( \epsilon \) is TKE dissipation rate. The Reynolds number is \( Re = \frac{UL_h}{\nu} \), where \( \nu \) is kinetic viscosity and the buoyancy Reynolds number is \( Re_b = Re F r_h^2 \) (Brethouwer et al. 2007). Buoyancy length scale is \( L_b = 2\pi L_h \) (Billant & Chomaz 2001) and the corresponding wavenumber is

\[
k_b = \frac{2\pi}{L_b}.
\]

The Dougherty-Ozmidov length scale is \( L_O = 2\pi \left( \frac{\nu}{N} \right)^{1/2} \) (Dougherty 1961; Ozmidov 1965), where the stratification effect becomes important. The corresponding wavenumber to Dougherty-Ozmidov length scale is

\[
k_O = \frac{2\pi}{L_O}.
\]

As Dougherty and Ozmidov independently defined the length scale, we call it Dougherty-Ozmidov length scale following Grachev et al. (2015). We thus assume that isotropic turbulence applies only for length scales below \( L_O \) when there is no wall. The ratio of the buoyancy scale to Dougherty-Ozmidov scale is related to the horizontal Froude number as \( \frac{L_b}{L_O} = Fr_h^{-1/2} \). Therefore, small \( Fr_h (Fr_h \ll 1) \) is required for a scale separation between \( L_b \) and \( L_O \). The Kolmogorov length scale (Kolmogorov 1941) is \( \eta = \left( \frac{\nu^3}{\epsilon} \right)^{1/4} \) and the ratio of Dougherty-Ozmidov to Kolmogorov length scale is \( \frac{L_O}{\eta} = 2\pi \frac{Re_b^{3/4}}{\eta} \). So large \( Re_b (Re_b \gg 1) \) (Brethouwer et al. 2007) is required for the separation between \( L_O \)
Turbulence Spectra in the Stable Atmospheric Boundary Layer

and \( \eta \). In the rest of this manuscript we thus assume that both \( Fr_h \ll 1 \) and \( Re_b \gg 1 \) conditions are met.

2.2. Spectra shape

We start from the spectral TKE balance equation (Lumley & Panofsky 1964)

\[
\frac{\partial E(k)}{\partial t} + \frac{\partial Q(k)}{\partial z} = S(k)\frac{\partial U_0}{\partial z} - \frac{\partial \epsilon(k)}{\partial k} + B(k) - 2\nu k^2 E(k),
\]

where \( E(k) \) is spectral kinetic energy density, \( \frac{\partial Q(k)}{\partial z} \) is the vertical transfer of turbulent energy in physical space, \( S(k) \) is spectrum of the Reynolds stress \( -\overline{uw} \), \( u \) and \( w \) are streamwise and vertical velocity fluctuation respectively, \( U_0 \) is mean streamwise velocity, \( \epsilon(k) \) is net rate of spectral energy transfer, \( B(k) \) is spectrum of buoyancy flux \( -\frac{\partial}{\partial z}\overline{wT} \) and \( 2\nu k^2 E(k) \) is rate of energy dissipation by molecular viscosity \( \nu \). Assuming steady state, neglecting \( \frac{\partial Q(k)}{\partial z} \) for eddies smaller than the energy containing scales and neglecting molecular viscosity (i.e. for eddies much larger than Kolmogorov scale), Weinstock (1978) obtained \( \frac{\partial \epsilon(k)}{\partial k} = S(k)\frac{\partial U_0}{\partial z} + B(k) \) for universal equilibrium range. The buoyancy flux spectrum can also be written as \( B(k) = -R_f S(k)\frac{\partial U_0}{\partial z} \) (Lumley 1965), where \( R_f = \frac{\overline{wT}}{U_0 \overline{uw} \frac{\partial U_0}{\partial z}} \) is flux Richardson number and \( T' \) is fluctuation from mean temperature \( \overline{T} \). Substituting \( B(k) \) into spectral balance equation, Weinstock (1978) obtained \( \frac{\partial \epsilon(k)}{\partial k} = B(k)\left(1 - \frac{1}{R_f}\right) \), where \( R_f \) is larger than 0 for stratified turbulence.

Weinstock did not realize that there is an upper bound (Townsend 1958; Nieuwstadt 1984; Schumann & Gerz 1995; Zilitinkevich et al. 2013) for \( R_f \) under the steady state assumption. Here we will take the upper bound of \( R_f \) to be 0.25 (Nieuwstadt 1984; Zilitinkevich et al. 2013; Katul et al. 2014) for the existence of continuous turbulence with Richardson-Kolmogorov cascade, which is discussed in detail in Grachev et al. (2013). Assuming approximately isotropic turbulence, Weinstock (1978) obtained

\[
B(k) = -\alpha a N^2 \epsilon(k)^{\frac{5}{3}} \frac{v_m k^{-2/3}}{0.8 N^2 + k^2 v_m^2},
\]

where \( \alpha = 1.5 \) is the Kolmogorov constant (Sreenivasan 1995; Pope 2000), \( a \) is an anisotropic factor (we will have detailed discussion for \( a \) given as 1 for isotropic turbulence and 0.5 for “approximately isotropic” turbulence. \( v_m \) is the root mean square of fluctuating velocity for eddies smaller than energy containing range given as \( v_m^2 = \frac{2}{3} \int_{k_m}^{\infty} E(k)dk \), where \( k_m \) is the smallest wavenumber in the universal equilibrium range. When \( 0.8 N^2 \gg k^2 v_m^2 \), the effect of buoyancy denoted by \( N \) is large and \( B(k) \) is in the buoyancy subrange; when \( 0.8 N^2 \ll k^2 v_m^2 \), the effect of buoyancy is small and \( B(k) \) is in the inertial subrange. The transition wavenumber was defined (Weinstock 1978) as \( k_{BW} = \frac{0.81^{1/2} N}{v_m} \), whose corresponding length scale is \( L_{BW} = \frac{2\pi}{k_{BW}} \). Note that the above analysis by Weinstock assumes a similar spectra shape between horizontal wavenumber and vertical wavenumber spectra, i.e., “approximately isotropic”. However, that assumption does not apply in our hypothesis as vertical wavenumber spectra are not considered in the atmospheric boundary layer. We will relax the “approximately isotropic” turbulence hypothesis below by only considering horizontal wavenumber spectra, e.g., equation (2.4) will be used only for horizontal wavenumber.

We choose the buoyancy scale \( L_b \) to be the minimum scale of buoyancy subrange, which requires \( 1.118 \frac{4}{U_0} \ll 1 \). If the Dougherty-Ozmidov length scale \( L_O \) is the maximum scale of isotropic inertial subrange for turbulence without walls, then the wavenumber
$k_O = \frac{2\pi}{L_O}$ will have to satisfy $0.8N^2 \ll k_O^2 v_m^2$, which requires $1.118 \frac{v_m}{U} \gg Fr_h^{\frac{1}{2}}$. In order to observe the transition region between the Dougherty-Ozmidov scale $L_O$ and buoyancy scale $L_b$, we thus need to satisfy the condition:

$$Fr_h^{\frac{1}{2}} \ll 1.118 \frac{v_m}{U} \ll 1.$$  \hspace{1cm} (2.5)

In the equilibrium range of horizontal wavenumber in stably stratified turbulence without walls, scales above $L_b$ will thus be in the buoyancy subrange, scales below $L_O$ will be in the isotropic inertial subrange, and scales in between will be in a transition region.

However, like Weinstock (1978), the above analysis does not consider the effects of wall on turbulence in the atmospheric boundary layer. Equation (19a') in Weinstock (1978) assumes a similar spectra shape between $w$ and $u$ in horizontal wavenumber as the focus was “approximately isotropic” turbulence without considering wall effects. In the atmospheric boundary layer, the existence of wall will cause $w$ spectrum to deviate from $u$ spectrum at low horizontal wavenumbers. Field observation (Grachev et al. 2013) and models (Katul et al. 2014) suggest a shallower slope than -5/3 at low horizontal wavenumbers in $w$ spectrum. Following Katul et al. (2014), we consider the wavenumber $k_a$ that satisfies

$$k_a z = 1,$$  \hspace{1cm} (2.6)

where $z$ is height above ground. If $k_a \leq k_O$, we assume the aforementioned anisotropic factor $a$ to be as follows

$$a = \begin{cases} 
1, & k \geq k_O, \\
\left(\frac{k}{k_O}\right)^{2/3}, & k < k_O.
\end{cases}$$  \hspace{1cm} (2.7)

If $k_a > k_O$, we assume the anisotropic factor $a$ to be as follows

$$a = \begin{cases} 
1, & k \geq k_a, \\
\left(\frac{k}{k_a}\right)^{2/3}, & k < k_a.
\end{cases}$$  \hspace{1cm} (2.8)

Under this assumption, the $w$ spectrum will be different from $u$ spectrum when $k < \max(k_O, k_a)$ in horizontal wavenumber and $w$ variance will be smaller than $u$ variance as $k$ decreases. However, we can still assume a -5/3 scaling for horizontal wavenumber TKE spectrum as the variance of horizontal velocity largely dominates at low wavenumbers. Note that we only consider horizontal wavenumber spectrum in the atmospheric boundary layer as turbulence in the vertical direction is not homogeneous (Fiedler & Panofsky 1970). Contrary to Weinstock’s theory, we do not assume a similar shape between vertical wavenumber spectra and horizontal wavenumber spectra because we only consider horizontal wavenumber spectra in the ABL. Calculations for buoyancy flux and energy spectrum will then differ from Weinstock (1978) when $k < \max(k_O, k_a)$. The energy spectrum in horizontal wavenumber can be obtained for the universal equilibrium range as Weinstock (1978)

$$E(k) = \alpha \epsilon_0^2 \left[1 - \frac{5\alpha}{12} \frac{v_m}{v_0} \left(1 - \frac{R_f}{R_f}C\left(\frac{k}{k_{BW}}\right)\right)\right]^2 k^{-\frac{5}{3}},$$  \hspace{1cm} (2.9)

where $v_0^2 = \frac{2}{3} \int_{k_{BW}}^{\infty} E(k)dk$, $\epsilon_0$ is viscous dissipation rate and $C\left(\frac{k}{k_{BW}}\right)$ in Weinstock.
\( C(\frac{k}{k_{BW}}) = \begin{cases} 
\frac{3-3\left(\frac{k}{k_{BW}}\right)^{\frac{1}{3}}}{1+\frac{1}{3}\left(\frac{k}{k_{BW}}\right)^{\frac{1}{3}}} + 0.5, & k < k_{BW}, \\
\frac{3}{3} - \left(\frac{k}{k_{BW}}\right)^{\frac{1}{3}}, & k \geq k_{BW}. 
\end{cases} \tag{2.10} \)

Details of the functional forms of \( C(\frac{k}{k_{BW}}) \) under different cases in the ABL are discussed as follows.

2.2.1. Case \( k_a \leq k_O \)

If \( k_a \leq k_O \), similarly to Weinstock (1978), we obtain

\[ C\left(\frac{k}{k_{BW}}\right) = \begin{cases} 
\frac{\alpha}{\sqrt{3}} \left(\frac{k}{k_{BW}}\right)^{-\frac{5}{3}} + \frac{n}{2} \left[\arctan\left(\frac{k}{k_{BW}}\right) - \arctan\left(\frac{k_O}{k_{BW}}\right)\right], & k < k_O, \\
\frac{\alpha}{\sqrt{3}} \left(\frac{k}{k_{BW}}\right)^{-\frac{5}{3}}, & k \geq k_O. 
\end{cases} \tag{2.11} \]

When \( k_b < k < k_O \), letting \( A = \frac{5\alpha}{12} \frac{v_0}{v_w} \left(1 - \frac{R_f}{R_L}\right) \) we obtain:

\[ \left[1 - AC\left(\frac{k}{k_{BW}}\right)\right]^2 = \sigma_0^2 - 2An\sigma_0 k \frac{k}{k_{BW}} + A^2n^2\left(\frac{k}{k_{BW}}\right)^2 + \frac{2}{3}An\sigma_0\left(\frac{k}{k_{BW}}\right)^3 + O\left(\left(\frac{k}{k_{BW}}\right)^4\right) > 0, \tag{2.12} \]

where \( \sigma_0 = A\left(m + n\arctan\left(\frac{k_O}{k_{BW}}\right)\right) - 1, m = \frac{\alpha}{3} \left(\frac{k_{BW}}{\frac{k}{k_{BW}}}\right)^{-\frac{5}{3}}, n = k_{BW}^{-\frac{4}{3}}k_O^{-\frac{2}{3}}. \) The spectra slope may vary at different combinations of variables and we will refer to observations. When \( k > k_O \), the condition \( Fr_h^{\frac{1}{3}} \ll 1.118\frac{v_0}{U} \ll 1 \) ensures \( k \gg \frac{k}{k_{BW}} \) so \( E(k) = \alpha\epsilon_0^2 k^{-\frac{5}{3}} \) is always a good approximation and thus the spectrum becomes isotropic. For the largest eddies, \( k < k_b \), the condition \( Fr_h^{\frac{1}{3}} \ll 1.118\frac{v_0}{U} \ll 1 \) ensures that \( k \gg \frac{k}{k_{BW}} \), so \( E(k) = \alpha\epsilon_0^2 k^{-\frac{5}{3}} \) is again a good approximation and thus we would expect the spectrum to exhibit a shape similar to the isotropic one but for different physical reasons.

Therefore, we expect \( E(k) \) spectra in horizontal wavenumber to exhibit a \(-5/3\) power-law scaling both below \( k_b \) and above \( k_O \), and having a scaling that may be different from \(-5/3\) in the transition region between \( k_b \) and \( k_O \).
2.2.2. Case $k_a > k_O$

If $k_a > k_O$, similarly to Weinstock (1978), we have

$$C\left(\frac{k}{k_{BW}}\right) = \begin{cases} \frac{4}{5} \left(\frac{k}{k_{BW}}\right)^{-\frac{2}{5}} & + \left(\frac{k_{BW}}{k_a}\right)^{\frac{2}{5}} \left[\arctan\left(\frac{k_a}{k_{BW}}\right) - \arctan\left(\frac{k}{k_{BW}}\right)\right], \ k < k_a, \\ \frac{2}{5} \left(\frac{k}{k_{BW}}\right)^{\frac{2}{5}} \left[\arctan\left(\frac{k_a}{k_{BW}}\right) - \arctan\left(\frac{k}{k_{BW}}\right)\right], \ k < k_a. \\ 1 + \frac{1}{5} \left(\frac{k}{k_{BW}}\right)^{\frac{2}{5}}, \ k \geq k_a. \end{cases}$$ (2.13)

When $k_b < k < k_a$, letting $A = \frac{5\alpha_0^2}{12} \frac{v_m}{v_0} \left(\frac{1-R_f}{R_f}\right)$ we obtain:

$$\left[1 - AC\left(\frac{k}{k_{BW}}\right)\right]^2 = \sigma_1^2 - 2AN\sigma_1 \frac{k}{k_{BW}} + A^2 n^2 \left(\frac{k}{k_{BW}}\right)^2 + \frac{2}{3} AN\sigma_1 \left(\frac{k}{k_{BW}}\right)^3 + O\left(\left(\frac{k}{k_{BW}}\right)^4\right) > 0,$$ (2.14)

where $\sigma_1 = A \left(m + n \arctan\left(\frac{k_a}{k_{BW}}\right)\right) - 1$, $m = \frac{4}{5} \left(\frac{k}{k_{BW}}\right)^{-\frac{2}{5}}$, $n = k_{BW}^{\frac{2}{5}} k_a^{-\frac{2}{5}}$. The spectrum slope in this case is uncertain and we will illustrate this scenario with observational results. When $k > k_a$, the condition $Fr_h^2 \ll 1.118 \frac{v_m}{U} \ll 1$ ensures $k_{BW} \gg 1$ so $E(k) = \alpha_0^2 k^{-\frac{5}{3}}$ is always a good approximation and thus the spectrum becomes isotropic. For the largest eddies, $k < k_b$, the condition $Fr_h^2 \ll 1.118 \frac{v_m}{U} \ll 1$ ensures that $k_{BW} \ll 1$, so $E(k) = \alpha_0^2 k^{-\frac{5}{3}}$ is again a good approximation.

To summarize, we expect the following three regions of turbulence spectra in horizontal wavenumber of the equilibrium range in the stable ABL: the isotropic inertial subrange at $k > \max(k_O, k_a)$; the transition region at $k_b < k < \max(k_O, k_a)$; the buoyancy subrange with the -5/3 power-law scaling at $k < k_b$. Also note that we do not assume vertical wavenumber spectra are comparable to horizontal wavenumber spectra as only the latter is considered in the ABL. Assuming that the temperature spectrum $E_\theta(k)$ in horizontal wavenumber is proportional to $E(k)$ (Weinstock 1983), we have $E_\theta(k) = F(N, \epsilon_0)E(k)$, where $F(N, \epsilon_0)$ is a proportionality faction related to $N$ and $\epsilon_0$ for stratified turbulence and thus we would expect overall similar spectrum behavior for the temperature spectrum. To confirm our theoretical expectation, we now turn to results from observations and DNS.

3. Experiment and numerical results

3.1. Observations of the stable atmospheric boundary layer

High frequency (20 Hz) velocity and temperature were recorded at 4 different heights (1.66 m, 2.31 m, 2.96 m and 3.61 m above water level) with eddy-covariance (EC) systems in the stable ABL over Lake Geneva during August-October, 2006 (Bou-Zeid et al. 2008). Wind velocity measurements had errors on the order of 0.02 m s$^{-1}$ with a maximum of 0.054 m s$^{-1}$ under zero wind conditions (Vercauteren et al. 2008). Wind velocity had a standard derivation of 0.001 m s$^{-1}$ from instrument. Mean of temperature measurements
were corrected by the relative mean offsets of instruments. Temperature measurements had a standard deviation of 0.002°C. The Kolmogorov scales in the 9 representative periods were 0.0019 m, 0.0017 m, 0.0015 m, 0.0013 m, 0.0011 m, 0.0009 m, 0.0011 m, 0.0011 m and 0.0007 m respectively. The smallest scales resolved using Taylor’s frozen turbulence hypothesis (Taylor 1938) in the 9 periods were 0.20 m, 0.23 m, 0.25 m, 0.28 m, 0.37 m, 0.50 m, 0.28 m, 0.30 m and 0.65 m respectively. Mean streamwise wind were 1.90 m s$^{-1}$, 2.27 m s$^{-1}$, 2.39 m s$^{-1}$, 2.70 m s$^{-1}$, 3.62 m s$^{-1}$, 4.83 m s$^{-1}$, 2.72 m s$^{-1}$, 2.92 m s$^{-1}$ and 6.30 m s$^{-1}$ respectively. The standard deviation of wind speed were 0.183 m s$^{-1}$, 0.279 m s$^{-1}$, 0.369 m s$^{-1}$, 0.547 m s$^{-1}$, 0.645 m s$^{-1}$, 0.648 m s$^{-1}$, 0.342 m s$^{-1}$, 0.343 m s$^{-1}$ and 1.058 m s$^{-1}$ respectively. Details about the experiment setup and data can be found in Bou-Zeid et al (2008), Vercauteren et al (2008), Li & Bou-Zeid (2011), and Li et al (2018).

High frequency (10 Hz) velocity and temperature were recorded at 3.5 m above ground with EC in Dome C, Antarctica (Vignon et al 2017a,b). The temperature gradient in the stable boundary layer was obtained from balloon sounding measurements (Petenko et al in press). Four 30-minute periods around 8 PM January 9th, 2015 were selected. Wind speed measurements had an accuracy of 0.05 m s$^{-1}$ and temperature had an accuracy of 0.01°C (Vignon et al 2017a). The Kolmogorov scales in the 4 representative periods were 0.0022 m, 0.0015 m, 0.0023 m and 0.0017 m respectively. The smallest scales resolved using Taylor’s frozen turbulence hypothesis (Taylor 1938) in the 4 periods were 0.45 m, 0.37 m, 0.43 m and 0.45 m respectively. Mean streamwise wind were 2.17 m s$^{-1}$, 1.81 m s$^{-1}$, 2.10 m s$^{-1}$ and 2.16 m s$^{-1}$ respectively. The standard deviation of wind speed were 0.261 m s$^{-1}$, 0.182 m s$^{-1}$, 0.156 m s$^{-1}$ and 0.215 m s$^{-1}$ respectively. Details of the EC setup and the site can be found in Vignon et al (2017a).

Fiber optics were set up at 4 heights (1.00 m, 1.25 m, 1.50 m, and 1.75 m above ground) along a 233-meter long transect at Oklahoma State University Range Research Station from 20 May to 15 July 2016. Details of the Distributed Temperature Sensing (DTS) (Selker et al 2006, Tyler et al 2009) experiment can be found in Cheng et al (2017). Temperature data were collected every 0.127 meters along the fiber and every 1.5 seconds. Mean wind velocity $U$ and scaling temperature $T_s = -\frac{w^2}{u^*}$ ($u^*$ is friction velocity) were obtained from a nearby EC tower. Two representative 30-minute periods in stable nocturnal boundary layer were analyzed. Temperature resolution were 0.16°C and 0.22°C before and after the fiber optics transect respectively. The effective spatial resolution was 0.56 m and temporal resolution was 3 s. The smallest scales resolved using Taylor’s frozen turbulence hypothesis (Taylor 1938) in the 2 representative periods were both 4.00 m, while the smallest scales resolved by spatial data were 1.52 m in both periods. Mean streamwise wind were both 1.28 m s$^{-1}$. The standard deviation of temporal temperature in the 2 periods were 0.43°C and 0.47°C respectively. The standard deviation of spatial temperature of the 2 periods were 0.45°C and 0.51°C respectively.

White noise in instruments will typically cause a flat spectrum, so high wavenumber spectrum will be influenced more. In the 2 subplots of the TKE spectra (figure 1) and 1 subplot of temperature spectra (figure 3) in the Lake EC data, a shallower slope than $-5/3$ can be seen at the highest wavenumbers, which indicates the influence of white noise. In the spectra plots of the Dome C data, the influence of white noise is not obvious. In temporal spectra of DTS data, temperature variation inside the constant temperature cooler box is regarded as white noise, which is then removed from the raw temperature spectra (Schilperoort 2017, personal communication).
3.2. DNS of stably stratified Ekman layer

The incompressible Navier-Stokes equations with Boussinesq approximation and the temperature equation are numerically integrated in time. The flow is driven by a steady pressure gradient, assuming a geostrophic balance in regions far above the surface, where the Coriolis force balances the large scale pressure gradient. Numerical details of the code can be found in Shah & Bou-Zeid (2014). A neutrally stratified turbulent Ekman layer flow (Coleman et al. 1992) over a smooth surface is first simulated for $ft \approx 3$ with $Re_D = U_g D/\nu$, where $f$ is the Coriolis parameter, $t$ is time, $U_g$ is the geostrophic wind speed, $D$ = $\sqrt{2\nu/f}$ is the laminar Ekman-layer depth and $\nu$ is the viscosity of air. A cooling surface buoyancy flux, $B_0$, constant in time (i.e. a Neumann boundary condition), is then applied, similarly to Gohari & Sarkar (2017).

A stably stratified Ekman flow is often used to represent an idealized stable planetary boundary layer (Ansorge & Mellado 2014). The bulk Richardson number $R_i$ evolves with time, where $\delta_*$ is the turbulent Ekman layer length scale given by $\delta_*=u_*/f$, $T_{ref}$ is the reference temperature at far distance and $T_0$ is the surface temperature which changes with time as a result of the imposed cooling buoyancy flux. We can use the Obukhov length scale $L$ (Obukhov 1946) to measure the near-surface stability, which is given by $L^+$ when scaled with the inner variables $L^+ = -\frac{u^3}{\kappa g w^T} u^*$, where $\kappa$ is the von Kármán constant and $u_*$ is friction velocity. Frequency spectra were transformed into horizontal wavenumber spectra invoking Taylor’s frozen turbulence hypothesis (Taylor 1938). Wavelet spectra (Torrence & Compo 1998) of temporal series (the wavelet software was provided by C. Torrence and G. Compo, and is available at URL: http://paos.colorado.edu/research/wavelets/) were calculated.

Similarly to Kaimal et al. (1972) both energy spectra in horizontal wavenumber and frequency are normalized in log-log plots. In TKE spectra of Lake EC (figure 1) and Dome C data (figure 2a), there are three regions corresponding to the buoyancy subrange, a transition region and the isotropic inertial subrange. At $k > \max(k_O, k_a)$, the spectra match the -5/3 power-law scaling of isotropic turbulence (Kolmogorov 1941; Dougherty 1961; Ozmidov 1965). The observation also shows that Dougherty-Ozmidov scale is a better demarcation of the isotropic inertial subrange compared to $L_{BW}$ defined in Weinstock (1978). In the transition region at $k_b < k < \max(k_O, k_a)$, the slope of spectra is not universal but less steep than -5/3. Katul et al. (2012) has reported a -1 slope at low wavenumbers in the TKE spectra of the stable atmospheric boundary layer. It appears to be a specific case of the transition region described here, with a slope not as steep as -5/3. Above the buoyancy scale, observations show another -5/3 slope, which agrees with spectra shown in figure 7 of Muschinski et al. (2004) and is consistent with the theory presented in section 2.

In the temperature spectra of the Dome C (figure 2b) and Lake data (figure 3), three regimes similar to the TKE spectrum can be observed. The observed -1 slope
Table 1. Details of DNS setup. L_x, L_y and L_z are the dimensions of computational domain in x (streamwise), y (spanwise) and z (vertical) directions respectively; Δx(y)⁺, Δz⁺ are the resolutions in x(y) and z directions; N(x,y,z) denotes the number of points for computation.

| Re_D | L⁺ (t = 0) | L_x, L_y, L_z | Δx(y)⁺, Δz⁺ | N_x × N_y × N_z |
|------|------------|---------------|-------------|-----------------|
| 1000 | 1600       | 90D, 90D, 30D | 3.90, 0.353  | 1024 × 1024 × 3840 |

Figure 1. Normalized temporal spectra of TKE in 9 representative 15-minute periods of Lake Geneva EC data. E_{ωω} is wavelet spectrum in frequency, U is mean streamwise wind velocity, z is measurement height above lake, u*_f is friction velocity, ω = 2πf is angular frequency, Var is variance of wind velocity, L is Obukhov length, Fr is horizontal Froude number and Rf is flux Richardson number. "H1", "H2", "H3" and "H4" denotes observation heights 1.66 m, 2.31 m, 2.96 m and 3.61 m above the lake respectively. "Ozmidov", "buoyancy", "L_{BW}" and "k_a" denotes Dougherty-Ozmidov scale, buoyancy scale, L_{BW} and k_a respectively.

Katul et al. (2016), Li et al. (2015) at low wavenumber in the temperature spectra of stable atmospheric boundary layer is actually a specific scaling in the transition region. In the temperature spectra of DTS data (figure 2c and 2d), only the buoyancy subrange and transition region are observed due to limited temporal and spatial resolution and temporal averaging (Cheng et al. 2017). The spatial spectra were used to check if temporal spectra that invoke Taylor’s frozen turbulence hypothesis are a good approximation, but those spatial spectra are more limited in terms of the number of decades they cover (≈ 3). More detailed discussion on Taylor’s hypothesis can be found in Cheng et al. (2017). Both temporal spectra and spatial spectra exhibit an approximately -5/3 slope at low wavenumber and a shallower slope at high wavenumber. The minimum length resolved in the temporal and spatial spectra were 4.00 and 1.52 meters respectively, which are close to the Dougherty-Ozmidov scale typically observed in the atmospheric boundary layer (Jiménez & Cuxart 2005, Li et al. 2016) so that we cannot observe the smaller scale isotopic behavior. About one decade to the left of the Dougherty-Ozmidov scale, a -5/3 scaling can be observed, corresponding to the buoyancy subrange previously defined.
Figure 2. (a) Normalized temporal spectra of TKE in Dome C data. $T_\ast$ is scaling temperature and other variables have the same meaning as those in figure 1. “spectrum t1”, “spectrum t2”, “spectrum t3” and “spectrum t4” denotes the spectra of 4 different 30-minute stable periods in January 9, 2015 respectively. The 4 length scales correspond to the time period of “spectrum t1”. (c) and (d) Spatial spectra and spatial mean of temporal spectra of temperature along fiber optics in 2 representative 30-minute periods of DTS data. Only the buoyancy subrange and transition region are resolved due to limited temporal and spatial resolution and temporal averaging. Variables have the same meaning as those in figure 1. “spatial” denotes the spatial spectra of temperature at height 1.75 m. “H1 mean”, “H2 mean”, “H3 mean” and “H4 mean” denotes the spatial mean of temporal spectra at 4 fiber optics measurement heights respectively.

The transition region in the DTS data resembles a flat white noise spectrum compared to other observations. One possible explanation is that spatial averaging is applied to the temporal spectrum of a single spatial point.

The Kolmogorov scale (see introduction in section 3.1) in Lake EC data is about 4 decades away from the Dougherty-Ozmidov scale, suggesting that turbulence is still well defined rather than being intermittent as shown in DNS results [Flores & Riley 2011; Ansorge & Mellado 2016]. The spectral bump (figure 4) as $Ri_B$ changes from 0.21, 0.65 to 0.98 in the DNS is not as clear as those shown in the atmospheric data. Indeed, in the DNS the Dougherty-Ozmidov scale is nearly similar to Kolmogorov scale (Waite 2014). It may be argued that around the $k_x z$ indicated by the scale of $1/L_b$, the spectral bump appears consistent with the scaling analysis above. However, DNS can only achieve a narrow range of inertial subrange compared to the observational data in the atmospheric boundary layer, which is at a much higher Reynolds number and thus DNS do not appear to have sufficient scale separation [Kunkel & Marusic 2006] to correctly represent the three regimes highlighted above due to the limitation of current computational capacity. As stratification increases, the Dougherty-Ozmidov length scale further decreases (and the wavenumber $k_\ast$ increases), the highest wavenumbers become closer to the $-5/3$ scaling and are less impacted by dissipative effects. However, a larger separation of scales between the inertial subrange and the dissipation range is required and not met with currently achievable DNS Reynolds numbers. Although DNS has been useful
in reproducing quantitatively and qualitatively some surface layer similarity relations according to MOST (Chung & Matheou 2012), we have to question the applicability of DNS for the study of very stable boundary layers at very high Reynolds numbers as they cannot correctly represent the observed spectral regimes seen in actual atmospheric boundary layers because of the lack of scale separation due to the limitation of current computational capacity. LES will obviously face similar issues in addition to the subgrid scale modeling that will have to be applied at scales well below the Dougherty-Ozmidov length scale to be applicable. In LES, spectra below the Dougherty-Ozmidov scale are thus not present (Beare et al. 2006; Waite 2011; Khani & Waite 2014).

A schematic of the TKE and temperature spectra in horizontal wavenumber in the stable atmospheric boundary layer is shown (figure 5a). In the universal equilibrium range, the $-5/3$ power-law scaling at scales larger than buoyancy scale is due to stratification effects (Weinstock 1978), while the $-5/3$ scaling at $k > \max(k_O, k_a)$ is due to isotropic turbulence (Dougherty 1961; Kolmogorov 1941; Ozmidov 1965). In the intermediate transition region, there is not a universal power-law scaling for spectra although the slope is less steep than $-5/3$ and even sometimes close to white noise. Our focus is the equilibrium range in the atmospheric boundary layer so spectra slope at the energy containing range is not shown.

The Lake EC data also show (figure 5b) that $Fr_h$ and $z/L$ are consistent in describing stability of atmospheric boundary layer, i.e., the higher $z/L$, the lower $Fr_h$ and the more stable the atmosphere is. When $k_O > k_a$, the width of the transition region denoted by the ratio of buoyancy scale to Dougherty-Ozmidov scale increases with $z/L$ (figure 5c). Therefore, the transition region will play a more important role in more stable conditions. Due to the increased impact of the transition region, the typically assumed $-5/3$ spectra scaling (Kaimal et al. 1972) has to be revised to better represent TKE, temperature variance or other fluxes while applying MOST. In the limit of extremely stable boundary layer, the transition region would occupy a very large fraction of the

**Figure 3.** Temporal spectra of temperature in 9 representative 15-minute periods of Lake EC data. $T_*$ is scaling temperature and other variables have the same meaning as those in figure 1.
3.4. Discussion

In Kaimal (1973), a $-5/3$ power-law scaling was shown in the spectra of $u$, $v$ and $w$ at the high-frequency horizontal wavenumbers and was approximated by an empirical formula. The shallower slope of spectra in the “transition region” between the buoyancy scale and Dougherty-Ozmidov scale was not reported. However, figure 1 in Caughey
Turbulence Spectra in the Stable Atmospheric Boundary Layer showed a shallower slope in $u$ and $T$ spectra at lower frequency compared to the isotropic $-5/3$ scaling frequency range at the height of 8 meters. Such a shallower slope of spectra was not as obvious at heights of 46 meters and 91 meters above the surface, at which the atmosphere was not as stable as at 8 meters. So figure 1 in Caughey (1977) actually showed the existence of a transition region to the left hand side of the isotropic $-5/3$ scaling in log-log plots. Figure 2 in Caughey (1977) showed the slope of the transition region was shallower than $-5/3$. The Dougherty-Ozmidov scale and buoyancy scale were not shown in figures of Caughey (1977) so the importance of the two scales under stratification might have been overlooked at that time. Grachev et al. (2015) proposed a similarity theory based on the Dougherty-Ozmidov scale under stable conditions, which provided the evidence that Dougherty-Ozmidov characterizes the small scale turbulence well. Li et al. (2016) showed Dougherty-Ozmidov scale was the limitation of momentum transporting eddies as stability increased. These results were consistent with our finding that Dougherty-Ozmidov scale decreases and is the limit of isotropic turbulence as stratification increases. Smyth & Moum (2000) suggested that the buoyancy Reynolds number is limited in current DNS studies due to the computational limitation and that turbulence decays when bulk Richardson number exceeds 1/4, which agrees with the condition $R_f < 0.25$ applied here for continuous Richardson-Kolmogorov cascade. Riley & Lindborg (2008) showed that some geophysical turbulence spectra under stratification at scales larger than the Dougherty-Ozmidov scale should not be explained by Kolmogorov’s isotropic turbulence hypothesis. We further show that stratification influences the turbulence spectra at scales larger than the Dougherty-Ozmidov scale by observation and derivation. The transition region that is highlighted in our work was not emphasized in previous research such as in Riley & Lindborg (2008) or Lindborg (2006).

The original theory of Weinstock (1978) applied the “locally inertial” relation $E(k) = \alpha \epsilon \left(k^{2/3}k^{-5/3}\right)$ for “approximately isotropic” turbulence in the equilibrium range (scales smaller than energy containing range) on condition that $\left|\frac{k \partial \epsilon}{\epsilon \partial k}\right| \ll 1$. The condition $\left|\frac{k \partial \epsilon}{\epsilon \partial k}\right| \ll 1$ is shown to be satisfied in most parts (see pages 644-645 of Weinstock (1978)), so the “locally inertial” assumption is approximately valid in that region when stratification is weak. The deviation of the TKE spectra from isotropy is not large, which was supported by observation (Reiter & Burns 1965) that vertical velocity spectrum is comparable to horizontal spectrum. So the theory is self-consistent in assuming “local inertial” relation in the equilibrium range when vertical wavenumber spectra are similar to horizontal wavenumber spectra and wall effects can be neglected. Indeed, Weinstock’s theory is inherently isotropic and does not apply to strongly stratified turbulence which is “anisotropic” as horizontal length scale will be much larger than vertical length scale under strong stratification (Billant & Chomaz 2001). For strongly stratified turbulence, people may assume that Lindborg’s stratified turbulence hypothesis (Lindborg 2006) rather than Weinstock’s theory describes turbulence above Dougherty-Ozmidov scale. However, in the case of homogeneous stratified turbulence in DNS studies, the suggested vertical wavenumber $k_v^{-3}$ scaling by Lindborg (2006) is not always reproduced. Waite & Bartello (2004) showed $k_v^{-5/3}$ spectrum in the vertical direction under weak stratification, Waite & Bartello (2006) reported shallower spectrum than $k_v^{-3}$. Bartello & Tobias (2013) suggested no evidence of vertical spectrum $k_v^{-3}$ expected between the buoyancy scale and Dougherty-Ozmidov scale, which is due to limited stratification. Maffioli & Davidson (2010) also showed the vertical spectra are closer to $k_v^{-5/3}$ rather than $k_v^{-3}$ owing to insufficiently low Froude number. Almalkie & de Bruyn Kops (2012) suggested $k_v^{-3}$ is not apparent in most cases of stratified turbulence simulations.
Yu Cheng, Qi Li, Stefania Argentini, Chadi Sayde, Pierre Gentine (2017) used a scale decomposition and shows the $k_v^{-3}$ scaling in large scales. An approximate balance (Maffioli 2017) between buoyancy and inertia has been proposed for scales between Dougherty-Ozmidov scale and buoyancy scale. These studies suggest Lindborg’s $k_v^{-3}$ hypothesis cannot entirely describe vertical wavenumber spectrum around the buoyancy scale and that vertical wavenumber spectra is not that different from horizontal wavenumber spectrum in weakly stratified conditions where horizontal length scale is close to vertical length scale. Therefore, the approximately isotropic hypothesis may still partly apply under certain conditions in weakly stratified turbulence.

Different from Weinstock (1978), we do not assume that vertical wavenumber spectra are comparable to horizontal wavenumber spectra, i.e., isotropic hypothesis is not applied here. For horizontal wavenumbers, we do not assume a similar spectra shape between $w$ and $u$ in the ABL. Indeed, in the atmospheric boundary layer, turbulence in the vertical direction (perpendicular to land surface) is not homogeneous (Fiedler & Panofsky 1970) but turbulence in the horizontal directions can be considered homogeneous. Generally vertical wavenumber spectrum is not calculated or considered in the atmospheric boundary layer. For turbulence spectrum at horizontal wavenumber, we relax the isotropic hypothesis and assume $w$ spectrum is different from $u$ spectrum at low wavenumbers. One may still argue that the $-5/3$ scaling around the buoyancy scale in our observation should be explained by Lindborg’s anisotropic $k_h^{-5/3}$ cascade. Compared to free atmosphere motion, atmospheric boundary layer turbulence is microscale motion and is essentially three-dimensional (Fiedler & Panofsky 1970; Monin & Yaglom 1975) as boundary layer scale is much smaller than the density scale height (about 10 km). Waite (2011) suggested that both buoyancy scale and Dougherty-Ozmidov scale are much smaller than energy containing horizontal scale in stratified turbulence. That is to say, scales around the buoyancy scale and Dougherty-Ozmidov scale are in the equilibrium range in the stable atmospheric boundary layer, which supports our analysis for horizontal wavenumber spectrum. Waite (2011) also pointed out that the $k_h^{-5/3}$ cascade is driven by anisotropic eddies with horizontal scales much larger than buoyancy scale, i.e., Lindborg’s stratified turbulence cascade mainly describes turbulence in the energy containing range. Therefore, horizontal turbulence in the energy containing range is described by Lindborg’s $k_h^{-5/3}$ cascade, while our analysis is below the energy containing range. Besides, a lot of numerical studies (Augier et al. 2015; Brethouwer et al. 2007; Maffioli & Davidson 2016; Waite 2011) showed a spectral bump around the buoyancy scale, which is neither predicted nor explained by Lindborg’s $k_h^{-5/3}$ spectra of anisotropic turbulence hypothesis. These results also suggest that buoyancy scale is not in the energy containing range in the horizontal direction and support our equilibrium range analysis.

Here some hypotheses on the spectra shape are discussed. Townsend (1958) assumed that fluid motion may consist of gravity waves when buoyancy forces are dominant. Bolgiani (1959) noted that TKE is converted to potential energy in the case of buoyancy stratification. Lumley (1964) suggested that wave-like behavior exists at low wavenumbers. Weinstock (1978) showed that gravity waves are weakly damped at low wavenumbers but are heavily damped at high wavenumbers in the isotropic inertial subrange. Zilitinkevich et al. (2007) showed energy transfer between TKE and turbulent potential energy from budget equations for both of them. Similarly to the derivation in Weinstock (1978), the $-5/3$ spectra slope at scales larger than the buoyancy scale is caused by constant dissipation rate, which is smaller than $\epsilon_0$ in isotropic turbulence below the Dougherty-Ozmidov scale. However, the variation of dissipation rate in wavenumber space leads to different spectra slopes in the transition region. Therefore, the different spectra characteristics between buoyancy subrange and transition region correspond to
different interaction strengths between gravity waves and turbulence. The spectra shallower than -5/3 in the transition region indicate extra energy between buoyancy scale and Dougherty-Ozmidov scale. The extra energy might possibly come from potential energy that is converted to kinetic energy through counter-gradient heat flux (Holt et al. 1992; Komori & Nagata 1996; Schumann 1987; Zilitinkevich et al. 2007) in stratified turbulence. Another possible explanation of the shallower spectra might be Kelvin-Helmholtz instabilities (Brethouwer et al. 2007; Waite 2011). In fact, Brethouwer et al. (2007) found counter-gradient fluxes appearing nearly simultaneously with Kelvin-Helmholtz-type instabilities. Therefore, further research are needed to explain the relation between shallower spectra, counter-gradient fluxes and Kelvin-Helmholtz instabilities. However, it is unlikely that an inverse energy cascade causes shallower spectra in transition region because direct transfer of energy was shown (Augier et al. 2015; Waite 2011) from energy-containing scale to buoyancy scale.

4. Conclusion

In the stable atmospheric boundary layer, we showed for the first time that buoyancy subrange, transition region and isotropic inertial subrange are separated by $k_b$ and $\text{max}(k_0, k_a)$ in TKE and temperature spectra of horizontal wavenumber in the equilibrium range. The transition region between buoyancy scale and Dougherty-Ozmidov scale will be observed when $Fr_{h}^{1/2} \ll 1.118 \frac{\nu}{U} \ll 1$ and $R_f < 0.25$ (for the existence of continuous turbulence with Richardson-Kolmogorov cascade) are satisfied. To represent the full spectra in horizontal wavenumber in the stably stratified atmospheric boundary layer, LES needs to resolve scales as small as the Dougherty-Ozmidov scale and use suitable subgrid scale model. To correctly represent those regimes, DNS would have to be run at much higher Reynolds number to obtain larger scale separation between the Dougherty-Ozmidov and Kolmogorov scale, as well as in the energy containing range. The impact of the variation of the spectra in the transition region should generate departure from MOST in very stable atmospheric boundary layer as this region expands when stratification increases.

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