Neutrino masses: evidences and implications

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Abstract.
I give an overview of the evidences for neutrino masses and mixing, the associated neutrino mass generation schemes, as well as the resulting implications in particle physics experiments and cosmology.

1. Introduction
Ever since the first historic hints for neutrino oscillations the evidences for neutrino masses and mixing have mounted, and have by now become overwhelming. The bulk of the current evidence fits nicely into a three-neutrino paradigm which we will tacitly adopt in this talk. The basic tool to describe oscillations is the lepton mixing matrix, characterized by three angles, one phase which affects oscillations, and two other phases intrinsic to the Majorana nature of neutrinos [1]. The latter are important in the description of lepton-number-violating processes such as neutrinoless double beta decay, better described in terms of the original [1, 2] symmetric parametrization over its equivalent presentation adopted by the PDG 1.

Neutrino oscillation data come from a variety of solar (Homestake, SAGE, GALLEX/GNO, Super-K, SNO and Borexino), atmospheric (mainly Super-K), reactor (in particular the recent Double-Chooz, Daya-Bay and RENO experiments) and accelerator experiments (mainly MINOS and the recent T2K) [4]. To describe them one assumes the simplest unitary approximation for the lepton mixing matrix and uses state-of-the-art solar and atmospheric flux calculations. Here we summarize the results of the global analysis of neutrino oscillation parameters (for details see the review [5] and references therein).

The results obtained for $\sin^2 \theta_{13}$ and $\delta$ are summarized in Fig. 1. In the upper panels we show the $\Delta \chi^2$ profile as a function of $\sin^2 \theta_{13}$ for normal (left panel) and inverted (right panel) neutrino mass hierarchies. The solid blue/dark line corresponds to the result obtained from the combination of all the data samples while the others correspond to the individual reactor data samples, as indicated. One sees from the constraints on $\sin^2 \theta_{13}$ coming from each of the new reactor experiments separately 2 that, as expected, the global constraint on $\theta_{13}$ is dominated by the recent reactor measurements. One finds overwhelming $10.2\sigma$ exclusion of $\theta_{13} = 0$ for both neutrino mass hierarchies. In contrast, one sees that there is no “preferred region” for the CP phase $\delta$ at $\Delta \chi^2 = 1$ (note that, given the approximations adopted in the atmospheric

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1 In seesaw-type schemes the lepton mixing has a rectangular form, which leads to lepton-flavour violation involving charged leptons [3] and non-standard neutrino interactions (talk by Miranda).
2 Here we have fixed all the other oscillation parameters to their best fit values.
Figure 1. Upper panels: Determination of $\sin^2\theta_{13}$ from the analysis of the total event rate in Daya Bay (solid magenta/light line), RENO (dotted line) and Double Chooz (dashed line) as well as from the global neutrino data analysis (solid blue/dark line). Lower panels: contours of $\Delta\chi^2 = 1, 4, 9$ in the $\sin^2\theta_{13} - \delta$ plane from the global oscillation fit. Left (right) panels correspond to normal (inverted) neutrino mass hierarchy.

Besides $\theta_{13}$ and $\delta$, the global neutrino data analysis yields best fit values and allowed ranges for all the other neutrino oscillation parameters, as summarized in Fig. 2.

Figure 2. Global determination of the neutrino oscillation parameters $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, $\Delta m^2_{31}$, $\Delta m^2_{12}$ and $\delta$, from [7]. The solid (dashed) lines in the central and right panels correspond to normal (inverted) mass hierarchy. The $\Delta\chi^2$ profile for the CP phase $\delta$ is nearly flat.

Note that at $3\sigma$ we find approximately a 17% accuracy in the determination of $\Delta m^2_{31}$, an
improvement that follows from the new long-baseline neutrino oscillation data. Also the best fit values for the atmospheric mass splitting parameter $\Delta m^2_{31}$ have been shifted to somewhat larger values mainly due to the new MINOS disappearance data.

Turning to the atmospheric mixing angle we note a slight rejection for maximal $\theta_{23}$ values. In particular, our global fit shows a preference for the second octant. This preference is very weak for the normal mass hierarchy case (see the symmetric solid $\Delta \chi^2$ profile in the middle-top panel of Fig. 2). However, for inverse hierarchy, the profile is asymmetric (see details in [7]). Note that maximal mixing, i.e. $\theta_{23} = \pi/4$ is disfavored at $\sim 90\%$ C.L. for both mass hierarchies. As discussed in Ref [7] the preference for non-maximal values of the atmospheric mixing angle comes directly from the new MINOS data, while the choice of a particular octant comes from the interplay of long-baseline, reactor and atmospheric neutrino data.

2. Origin of neutrino mass

Now we turn to the big question: What is the origin of neutrino mass? The answer remains as elusive as ever. The simplest way to induce neutrino masses makes use of Weinberg’s dimension-five operator, illustrated in Fig. 3 (left panel).

![Figure 3. Weinberg operator and type-1 and type-2 seesaw schemes.](image)

2.1. Seesaw mechanism

As illustrated by the other panels in Fig. 3, the dimension-five operator may be induced by the exchange of “messenger” states, either fermions or Higgs scalars. Effective neutrino masses arise when the electroweak symmetry breaks through a non-zero vacuum expectation value (vev) of the Higgs doublet. Their smallness relative to the charged standard model fermion masses would arise from the messengers being super-heavy. This is the idea behind the simplest seesaw mechanism [1] [8, 9, 10, 11, 12, 13, 14]. In type-1 seesaw the messenger states are $\text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1)$ singlet “right”-handed neutrinos, while in type-2 a heavy triplet scalar boson is the messenger particle, as indicated in the right panel of Fig. 3. The associated large scale characterizing the violation of lepton number symmetry will depend on the precise model realization considered.

As a result of the arbitrary number of gauge singlets that may be added in the seesaw [1] one can realize the type-1 version also at low-scale, the smallness of the lepton-number-violating parameter being natural in ’t Hooft sense. Moreover such small L-violating parameter may arise dynamically through radiative corrections [15]. The inverse [16] and linear seesaw schemes [17, 18, 19] provide examples of these constructions. In the presence of supersymmetry such low-scale seesaw schemes allow for a sneutrino-like state to be the lightest supersymmetric particle and play the role of cold dark matter, instead of the standard neutralino of minimal supergravity [20, 21] (talk by De Romeri).

If the breaking of lepton number takes place spontaneously there are additional states associated to neutrino mass generation. For instance, if lepton number is ungauged, there is a remnant physical Goldstone boson, generically called Majoron. In contrast to the case of
high-scale seesaw [12, 22], in low-scale versions [23] the Majoron may show up as Higgs boson decay products [24].

On the other hand in such extended schemes with gauged lepton number \(^3\) [17, 18, 19] there is (at least) an additional neutral gauge boson coupled to neutrinos. For example one can show that in SO(10) the B-L breaking scale need not be the one that determines the lightness of neutrinos [19]. As a result it can lie low enough to be within reach of current and future experiments. The associated \(Z'\) may be searched for either directly through Drell-Yan production at high-energy hadron colliders [26], such as the LHC, and also through low-energy electroweak precision measurements [27] (talk by Garces).

2.2. Supersymmetry as origin of neutrino mass: probing neutrinos at the LHC

There are also neutrino mass generation schemes based on weak-scale physics. An important example is “low-energy” supersymmetry without a conserved R-parity [28]. If the breaking of R-parity occurs spontaneously \(^4\) [29, 30, 31, 32] the associated scale is of the same order as the supersymmetry breaking scale. Below that scale one is left with a very interesting scheme called bilinear R-parity violation (BRpV) [33].

In this case one finds that the neutralino decays, typically inside collider detectors, such as those at the Large Hadron Collider (LHC) (see Fig. 4), decreasing the missing energy expected in the unbroken R-parity supersymmetry.

![Neutralino decays](image)

**Figure 4.** Neutralino decays typically produce displaced vertices inside the detector in the mSUGRA BRpV model. For illustration we show the decay length with \(A_0=100\) GeV, \(\text{sign}(\mu)=+1\), \(\tan \beta=10\). Left: in the \(m_{1/2}, M_0\) plane. Right: as a function of the LSP mass, for \(200 \leq M_0 \leq 1000\) GeV and \(240 \leq m_{1/2} \leq 1000\) GeV.

The neutrino mass spectrum is expected to be normal hierarchy with the atmospheric neutrino mass scale generated by tree seesaw-like diagrams such as the one in the left plot in Fig. 5 involving the exchange of supersymmetric fermions, while the solar scale is induced radiatively (right). This provides a rationale for the smallness of the solar squared mass splitting with respect to the atmospheric. The model provides a successful phenomenological scheme for neutrino masses and mixings [34, 35].

The generic CP conserving form of this model is characterized by six free parameters, which correspond to the three mixing angles and three neutrino masses. As a result it does not predict the neutrino mixing angles \(^5\). However, the same parameters that determine neutrino masses and mixing also induce LSP decays, leading to tight correlations between both, whose details depend on the nature of the LSP [37]. An important consequence of this is that in this model...

\(^3\) For an early paper see, e.g. [25].

\(^4\) In contrast to the seesaw Majoron which has \(|L|=2\), in this case the Majoron carries \(|L|=1\).

\(^5\) This changes if one adds flavor symmetries. For a predictive “flavored” BRpV scheme see Ref. [36].
neutrino properties can be directly probed in high energy hadron colliders [38, 39, 40, 41, 42, 43], illustrating the complementarity of accelerator and non-accelerator approaches in elementary particle physics. As an example, in the minimal supergravity BRpV model the semi-leptonic neutralino decay branching ratios correlate with the neutrino mixing angles describing oscillations, as illustrated in Fig. 6.

Finally we note that in this case the neutralino can not be dark matter, as it decays with a short lifetime. However one can show that the gravitino can play the role of dark matter [44] while keeping the above phenomenological tests.

3. Flavour symmetries and neutrino mixing angles

Non-Abelian continuous and discrete flavour symmetries have been extensively used to account for the observed pattern of neutrino mixing [45, 46, 47]. The tri-bimaximal (TBM) mixing ansatz [48], with an effective bimaximal mixing of $\nu_\mu$ and $\nu_\tau$ at the atmospheric scale and trimaximal mixing of $\nu_e$ with $\nu_\mu$ and $\nu_\tau$ at the solar scale, was proposed in 2002 by Harrison, Perkins and Scott as a good approximation for the observed neutrino mixing angles,

$$U = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}.$$  \hspace{1cm} (1)
TBM *ansatz* as a good first approximation to the neutrino mixing pattern. However in concrete models there can be large corrections to the TBM pattern, so that we can still take it as a useful reference *ansatz*.

Assigning the three known generations of leptons to non trivial irreducible representations (*irreps*) of a non-Abelian discrete flavour symmetry group one can make predictions for masses and mixings in the lepton sector. In general the number of free parameters in models based on Abelian flavour symmetries is typically larger than the corresponding number of free parameters needed to describe non-Abelian flavour symmetry theories. Moreover there are non-Abelian discrete groups with triplet irreps, exactly as the number of the standard model generations.

The smallest group that contains triplet irreps is $A_4$, the group of the even permutations of four objects, isomorphic to the group of the symmetries of the tetrahedron $T^6$. A realistic $A_4$ model was proposed in Ref. [49] predicting maximal atmospheric mixing and vanishing $\theta_{13}$ to first approximation. Although expected to be sizeable, the solar mixing angle was not predicted and neutrino masses were (quasi)degenerate.

However one can obtain the full tri-bimaximal pattern in Eq. (1) from a non-Abelian flavor symmetry. Consider $A_4$ as an example. $A_4$ contains two Abelian subgroups, namely $Z_2$ and $Z_3$. When broken into $Z_3$ in the charged sector, and into $Z_2$ in the neutrino sector, $A_4$ leads to a lepton mixing matrix of tri-bimaximal form, Eq. (1).

In order to yield the tri-bimaximal pattern in Eq. (1), the neutrino mass matrix must be

$$M_\nu = \begin{pmatrix} y & x & x \\ x & y + z & x - z \\ x & x - z & x + z \end{pmatrix},$$  

(2)

where $x, y, z$ are free parameters. The above matrix has two properties:

- it is $\mu - \tau$ invariant giving maximal atmospheric and zero reactor angles;
- it satisfies the relation $(M_\nu)^{11} + (M_\nu)^{12} = (M_\nu)^{22} + (M_\nu)^{23}$ giving trimaximal solar angle.

Note that Eq. (2) is diagonalized by the TBM mixing matrix in Eq. (1) independently of the mass eigenvalues. The derivation of TBM mixing from a flavour symmetry was first given in Refs. [50, 51, 52]. Note that $A_4$ is totally broken, therefore deviations to TBM are expected. In general one can not align $A_4$ in the $Z_3$ and $Z_2$ directions in the charged and neutral lepton sectors respectively, this is known as the *alignment* problem. This may be circumvented by using extra dimensions and/or supersymmetry [50, 51] or by assuming a suitably chosen soft breaking sector. Alternatively one may use a large discrete group, such as $Z_3^2 \times U_L(1)^3 \times S_3$ [53].

4. Implications of a “large” reactor angle

At the Neutrino 2012 conference in Kyoto, reactor and accelerator experiments have presented important results. In particular

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}) \quad \text{at 5.2}\sigma \quad \text{(DayaBay [54])}$$ (3)

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}) \quad \text{at 4.9}\sigma \quad \text{(RENO [55])}$$ (4)

indicating that $\sin \theta_{13} \sim \lambda_C^2$ where $\lambda_C \approx 0.2$ is the Cabibbo angle, and casting doubts on the validity of the TBM *ansatz*. As we saw, in order to have the TBM mixing pattern we need to break separately our flavour group (for instance in $A_4$) into $Z_3$ in the charged sector and into $Z_2$ in the neutrino sector. Therefore the flavour group is completely broken. Since the flavour symmetry leading to the TBM *ansatz* is in general broken we expect deviations from TBM.

For a classification of the irreps of different non Abelian discrete groups see for instance Ref. [45].
which, in particular, could generate a nonzero reactor angle. As a result TBM can still be taken as a good first order approximation.

However many models with TBM at leading order are ruled out because they give corrections of the same order $\delta \theta$ to the three angles $\theta_{13}$, $\theta_{12}$ and $\theta_{23}$. If $\delta \theta \sim \lambda_C$ we have

$$\sin^2 2\theta_{13} = 0.087 \quad \text{for } \delta \theta = 0.15,$$

$$\sin^2 2\theta_{13} = 0.152 \quad \text{for } \delta \theta = 0.20$$

close to the best fits. However the deviations of the solar mixing from its trimaximal values $\sin^2 \theta_{12}^{TBM} \equiv 1/3$ will be too large if we take $\delta \theta = 0.15 \sim \lambda_C$, namely

$$\sin^2 (\theta_{12}^{TBM} + \delta \theta) = 0.48 \ (0.38 \ @3\sigma),$$

$$\sin^2 (\theta_{12}^{TBM} - \delta \theta) = 0.20 \ (0.27 \ @3\sigma).$$

This in principle rules out many TBM schemes. Indeed, most extensions of TBM models which allow for a large reactor angle also predict a deviations of the atmospheric and solar mixing angle from their TBM values [56, 57]. Therefore a burning question is to evaluate the extent to which solar and atmospheric mixing angles deviate from their TBM values.

However, not all TBM models proposed in the past are excluded, for example in the model of Ref. [58], based on $A_4$, large reactor angle $\theta_{13} \sim \lambda_C$ has been obtained with deviation of $\theta_{12}^{TBM}$ of order of $\lambda_C^2$, in agreement with data. There are other examples in the literature of models where such deviations are small, despite the relatively large reactor angle value, see for instance [59, 60, 61, 62, 63]. Needless to say that this poses no problem for the model in [49], as it does not predict the solar angle.

Finally we note that alternative ansatze have been suggested to circumvent this problem. An interesting possibility is that the leading order neutrino mass matrix is not diagonalized by the TBM ansatz, but rather by the bi-maximal one (where both solar and atmospheric mixing angles are maximal from the start) [64] or simply bi-large [65], or by the golden ratio [63, 66]. Clearly the “large” reactor angle will not only open a new world of CP violation in the lepton sector, but may also shed light into the flavour problem, one of the most challenging puzzles in particle physics.

5. Neutrinoless double beta decay and neutrino properties

Given that neutrinos have mass one expects $0\nu\beta\beta$ to receive a contribution from tree-level neutrino exchange if they are, as expected, Majorana fermions. The amplitude is proportional to an effective mass parameter combination $\langle |m_{ee}| \rangle$ illustrated in Fig. 7. In contrast to neutrino oscillations, this is sensitive also to the absolute scale of neutrino masses, which is independently tested also in searches for tritium beta decay [70] and cosmology [71, 72]. For example, the model in [49] implies a lower bound on the absolute neutrino mass $m_\nu \gtrsim 0.4 \text{ eV}$ and therefore will be tested fairly soon. In addition, this amplitude can bring complementary information on the underlying structure of flavour in the lepton sector, as we now discuss.

Many of the models based on non–Abelian discrete flavour symmetries are characterized by a specific (complex) relation between neutrino mass eigenvalues, leading to mass sum rules, e. g.

\begin{align*}
A) \quad & \chi m_2^\nu + \xi m_3^\nu = m_1^\nu, \\ 
B) \quad & \frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}, \\ 
C) \quad & \chi \sqrt{m_2^2} + \xi \sqrt{m_3^2} = \sqrt{m_1^2} \\ 
D) \quad & \frac{\chi}{\sqrt{m_2^2}} + \frac{\xi}{\sqrt{m_3^2}} = \frac{1}{\sqrt{m_1^2}}. 
\end{align*}
Figure 7. The broad upper and lower branches correspond to the flavor-generic inverse (yellow) and normal (gray) hierarchy neutrino spectra, respectively. Flavor model predictions are indicated by the green and red (darker-shaded) regions. Only these sub-bands are allowed by the corresponding mass sum rule. The left panel is taken from Ref. [67] while the right plot is from Ref. [68]. We also give the current limit and future sensitivities on $\langle |m_{ee}| \rangle$ [69] and $m_\nu$ [70].

Here $m_\nu^i$ denote neutrino mass eigenvalues, up to a Majorana phase factor, while $\chi$ and $\xi$ are free parameters characterizing the model, taken as positive without loss of generality.

The effective neutrino mass parameter $|m_{ee}|$ determining the $0\nu\beta\beta$ decay amplitude in a flavour-generic scheme is given in Fig. 7, as a function of the lightest neutrino mass. Allowing the neutrino oscillation parameters to vary in their allowed ranges one obtains two regions in the $(|m_{ee}|, m_{\nu\text{light}})$ plane corresponding to normal and inverse hierarchy spectra, as shown by the broad bands in Fig. 7. There is a lower bound for the parameter $|m_{ee}|$ only in the case of inverse mass hierarchy.

Let us turn to the case where relations like $(A),(B),(C)$ and $(D)$ hold, such as in the tri-bimaximal mixing pattern. In Fig 7 we show the prediction for $|m_{ee}|$ as function of $m_{\nu\text{light}}$ for the indicated models 7. Note that when deriving a lower bound on the effective $0\nu\beta\beta$ decay amplitude parameter $|m_{ee}|$, we include explicitly the effects of non-vanishing $\theta_{13}$ as indicated by recent experiments [73, 74] as well as global neutrino oscillation fits [7] by taking the 3 $\sigma$ oscillation parameter ranges 8. Note that within specific flavor models a lower bound on $|m_{ee}|$ can be set even for the case of normal neutrino mass hierarchy, since a destructive interference among the light neutrinos [75, 76, 77] is prevented in this case by the flavor symmetry. Of course, all inverse hierarchy schemes corresponding to various choices of $(\chi, \xi)$ within sum-rules A-D have a lower bound for the parameter $|m_{ee}|$. However, the numerical value obtained depends on the flavor scheme.

Before closing let me stress that the importance of $0\nu\beta\beta$ lies in that it so far provides the only feasible way to test the nature of neutrinos. Indeed, the black box theorem [78, 79] states that the observation of the $0\nu\beta\beta$ would point towards the Majorana nature of neutrinos irrespective of the mechanism generating their mass and irrespective of the mechanism inducing the decay, as illustrated in Fig. 8 9.

6. Lepton flavour violating phenomena

The phenomenon of lepton flavour violation required to account for neutrino oscillation data may show up in processes involving charged leptons. A remarkable fact is that their associated

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7 A compilation is summarized in Tab. I of Ref. [67]. See talks by Dorame and Rojas.
8 Corrections leading to $\theta_{13} \neq 0$ may arise from higher dimensional operators or the charged lepton sector.
9 In principle CP and electromagnetic properties of neutrinos are also sensitive to their nature [76, 80, 81, 82].
Figure 8. The observation of $0\nu\beta\beta$ implies the Majorana nature of a neutrino [78].

strength need not be suppressed by the smallness of neutrino masses with similar results for leptonic CP violation [83, 84, 85, 86]. For example, in the presence of supersymmetry, high-scale seesaw schemes may induce sizeable lepton flavour violation decays such as $\mu^-\rightarrow e^-\gamma$ and flavour violating tau decays (Fig. 9) as well as nuclear $\mu^- - e^-\gamma$ conversion (Fig. 10) as a result of the exchange of supersymmetric leptons. The existence of such loop effects, illustrated in Fig. 9, has been known for a while [87, 88]. The resulting rates will be accessible to the upcoming

Figure 9. Supersymmetric Feynman diagrams for $l_i^- \rightarrow l_j^-\gamma$.

Figure 10. Long-distance (a) and short-distance (b) contributions to nuclear $\mu^- - e^-\gamma$ conversion.

generation of experiments [89, 90], the pattern of lepton flavour violation processes depending on the model details.

6.1. Heavy neutrino exchange in low-scale seesaw

Low-scale seesaw schemes generate neutrino masses from fermion messengers ("right"-handed neutrinos) with mass at the TeV scale. As interesting examples we have the inverse and the linear seesaw schemes [16, 17, 18, 19]. These are potentially accessible to the LHC, especially in the presence of a new gauge boson "portal" associated, for example, to left-right symmetry [91, 92].

Within low-scale seesaw mechanisms lepton flavour violation and/or CP violating effects arise at the one-loop level. In these models the $9 \times 9$ Majorana neutrino mass matrix is diagonalized


by a unitary matrix $U_{\alpha\beta}$, $\alpha, \beta = 1, \ldots, 9$, leading to the 3 light eigenstates $\nu_i$ with $i = 1, 2, 3$ and 6 heavy ones $N_j$ with $j = 4, \ldots, 9$. The leptonic mixing matrix is given as a rectangular matrix \[ L_{CC} = \frac{g}{\sqrt{2}} K_{i\alpha} t_i^{\gamma_5} (1 + \gamma_5) N_\alpha W^\mu, \] (13)

where $i = 1, 2, 3$ label the left-handed charged leptons and $\alpha$ the neutrals. The well-known one-loop contribution to this branching ratio is given by

$$ Br(l_i \rightarrow l_j\gamma) = \frac{\alpha^2 s_W^2 m_i^5}{256 \pi^2 M_W^4} \frac{1}{\Gamma_i} |G_{ij}|^2, $$

(14)

where $G_{ij}$ is the appropriate loop function [93, 94]. We note that the resulting branching ratio can be sizeable, since it need not be suppressed by the smallness of neutrino masses [83, 84, 85, 86, 95] Similar results hold other lepton flavour violation processes, including nuclear muon-electron conversion [96] whose expected rates correlate with those of $\mu \rightarrow e\gamma$, see Fig. 11. In contrast to the general case considered in [96, 97], in the presence of a flavor symmetry [95] it is easier to display the dependence of $B(\mu \rightarrow e\gamma)$ upon the parameters $\mu, v_L$ characterizing the low-scale violation of lepton number, and the heavy neutrino mass $M$, fixed as $M = 100, 200, 1000$ GeV (continuous, dashed and dot-dashed lines) in Fig. 11. Note also that, in a given flavor model, there may be predictions for ratios of lepton flavour violation branching ratios such as

$$ \frac{Br(\tau \rightarrow e\gamma)}{Br(\mu \rightarrow e\gamma)} = \left( \frac{m_\tau}{m_\mu} \right)^5 \frac{\Gamma_{\mu}}{\Gamma_{\tau}} \approx 0.18. $$

(15)

Figure 11. Left: $Br(\mu \rightarrow e\gamma)$ correlates with muon-electron conversion in nuclei. From Ref. [96]. Right: $Br(\mu \rightarrow e\gamma)$ versus the lepton number violation scale characterizing the inverse (red), and the linear seesaw (blue) of Ref. [95].

6.2. Lepton flavour violation in supersymmetric low-scale seesaw

We now consider lepton flavour violation rates in the framework of a generic supersymmetric inverse seesaw model [97]. The left panel in Fig. 12 displays the dependence of the branching ratios for $\mu^- \rightarrow e^-\gamma$ conversion in Au, Ti and Al (left) and $\mu^- \rightarrow e^-\gamma$ (right) with the small neutrino mixing angle $\theta_{13}$, for different values of $\theta_{12}$. The inverse seesaw parameters are given by: $M = 1$ TeV and $\mu = 30$ eV. The light neutrino parameters used are from [5], except for $\theta_{13}$ which is varied as shown in the plots. The vertical lines indicate the sin$^2 \theta_{13}$ values indicated by recent experiments, [7]. These rates may be testable in the upcoming lepton flavour violation experiments [89, 90]. A novel feature present in low-scale ($M$ in the TeV range) supersymmetric seesaw models is the interplay between heavy neutrino and supersymmetric

\[ ^{10} \text{The situation for very large } M \text{ ("double seesaw" limit) has been recently considered in Ref. [98] and they show how the Z-penguin diagram can enhance the } l_i \rightarrow 3l_j \text{ decays, in contrast to naive estimates.} \]
contributions to $Br(\mu \rightarrow e\gamma)$ and other lepton flavour violation processes such as nuclear $\mu^- - e^-$ conversion, depicted in Fig. 10, which fall within the sensitivity of future experiments [89]. Note that large lepton flavour violation rates are possible even in the massless neutrino limit, hence the allowed lepton flavour and CP violation rates are unsuppressed by the smallness of neutrino masses [83, 84, 85, 86]. Finally, for low enough $M$ the corresponding heavy neutrinos can be searched directly at particle accelerators [99, 100]. In the presence of new gauge bosons, such as right-handed gauge bosons these heavy neutrinos can be searched at the LHC [91, 92].

6.3. Supersymmetric high-scale $A_4$-based seesaw

The exchange of supersymmetric leptons induces flavour violating processes, as illustrated in Fig. 10 and discussed in [101, 102, 103, 104, 105]. Instead of a generic “flavour-blind” supersymmetric high-scale seesaw scheme we now consider, as an example, the $A_4$-based model introduced in Ref. [49]. The allowed parameter space is determined by scanning the multidimensional parameter space, keeping all supersymmetric masses real and in the range 100 GeV to 1000 GeV in Ref. [106].

The spectra fall into two different groups. The normal hierarchy having two low mass sleptons ($\sim 150$ GeV) and one heavy (above $\sim 500$ GeV), and the inverted hierarchy case having two heavy sleptons and one light. In both cases at least one slepton mass lies below about 200 GeV, detectable at the LHC. Most points fall into the case of normal hierarchy, corresponding to a normal hierarchy for the neutrinos as well, with one small and two large mixing angles.

One finds a prediction for the charged lepton decays $\ell_i \rightarrow \ell_j \gamma$ seen in Fig. 13, with a lower bound of $10^{-9}$ for $BR(\tau \rightarrow \mu\gamma)$, which lies within reach of B-factories. Similarly, $BR(\mu \rightarrow e\gamma)$ is found to be larger than about $10^{-15}$ which may be observed in the future [90].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{LFV_branching-ratios.png}
\caption{LFV branching ratios in the supersymmetric inverse seesaw model (see text).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Expected_branching-ratios.png}
\caption{Expected branching ratios for the processes $\ell_i \rightarrow \ell_j \gamma$ as a function of $\tan(\beta)$.}
\end{figure}
7. Neutrino mass and dark matter

Dark matter and neutrino masses are two sectors that clearly require physics beyond the Standard Model. These may, in fact, be closely related [107, 108]. I now consider some “neutrino-motivated” dark matter candidates and discuss their direct/indirect detection prospects. While they are all cold insofar as the properties of the Cosmic microwave background (CMB) are concerned, some, like the Majoron, behave as warm dark matter regarding structure formation.

7.1. Sneutrino-like dark matter in low-scale seesaw

If supersymmetrized, the simplest SU(3) \( \otimes \) SU(2) \( \otimes \) U(1) inverse seesaw model of neutrino masses [16, 23] may lead to a sneutrino-like state as the lightest super-particle, instead of the lightest neutralino [20], as illustrated in the left plot in Fig. 14. This happens even in the constrained Minimal Supersymmetric Standard Model (CMSSM) picture inspired by minimal supergravity [20]. As discussed in Ref. [15] the low-scale breaking of lepton number may arise dynamically.

Indeed, in the shaded (red) and light (yellow) areas in Fig. 14 (plotted for a given choice for \( \tan \beta = 35, A_0 = 0 \) and \( \mu > 0 \)) one finds that, for a suitable choice of parameters associated to the inverse seesaw scheme a sneutrino-like state is the LSP [20]. Note also that the light (yellow) region includes all the region where the \( \tilde{\tau} \) is the LSP in the standard mSUGRA case. On the other hand the dark (blue) region is excluded by experimental and theoretical constraints. This region is now substantially larger due to the recent LHC results of ATLAS and CMS [109], as illustrated by the extended dark region in Fig. 14 (left).

As shown in the mid panel in Fig. 14, the model can reproduce the correct relic sneutrino-like dark matter abundance \( \Omega h^2 \), when the supersymmetric parameters lie within the ranges \( 100 \text{ GeV} < m_0 < 3 \text{ TeV}, 100 \text{ GeV} < m_{1/2} < 3 \text{ TeV}, A_0 = 0, 3 < \tan \beta < 50 \) (see Ref. [20] for details). Finally, the right panel in Fig. 14 illustrates how one can obtain an accessible direct detection rates in nuclear recoil experiments.

![Figure 14](image)

**Figure 14.** Left: regions of mSUGRA parameters where the “sneutrino” is the LSP, see [20] for details. Middle: Blue points give the LSP relic abundance \( \Omega h^2 \) versus the LSP mass. Right: “sneutrino”–nucleon scattering cross section vs. its relic abundance, for the same scan of parameters. The horizontal (light blue) band denotes the current sensitivity of direct detection experiments. The yellow bands delimit the WMAP cold dark matter 3\( \sigma \) range [110].

7.2. Dark matter stabilized by flavour symmetry

The observed pattern of mixing angles radically differs from that which characterizes the Cabibbo-Kobayashi-Maskawa angles [111, 112]. Attempts to understand this in terms of basic flavour symmetries have brought in the idea that perhaps the symmetry explaining flavour may also account for the required dark matter stability. A non-Abelian flavour symmetry model where dark matter is stabilized through a discrete unbroken \( Z_2 \) subgroup has been recently proposed in Ref. [113]. The model is based on \( A_4 \) and extends the Higgs sector of
the Standard Model with three scalar doublets. After electroweak symmetry breaking two of
the scalars of the model acquire vacuum expectation values (vevs), leaving a residual parity
symmetry. The lightest neutral odd scalar must be stable and is a viable dark matter candidate.
The phenomenology of this model has been studied in [114]. The dark matter particle is
thermally produced via a Higgs portal (Fig. 15, left) and obeys all constraints from current
laboratory experiments and astrophysical observations. The right panel in Fig. 15 gives the spin-
independent dark matter scattering cross section off-protons as a function of the dark matter
mass. Direct detection prospects are promising for the near future as many collaborations are
closing in on the low mass region of the WIMP parameter space. The orange regions show the
DAMA/LIBRA annual modulation regions including (neglecting) the channeling effect (dashed
and solid, respectively) [115]. The green region corresponds to the COGENT data [116]. Dashed
and dotted lines correspond to the upper bound from CDMS (respectively from [117] and [118]).
XENON100 bounds [119] are indicated by a solid black line. Coming to indirect detection [114]
one finds that the sensitivity of current Fermi-LAT observations are starting to probe the model
for low-intermediate dark matter masses (see e.g. [120, 121]). Predictions for the neutrino sector
are discussed in [114]. This scheme is just an example of a class which may possibly realize a
deeper connection between neutrino and dark matter physics (see talk by Peinado).

Figure 15. Left: Higgs portal. Right: Spin-independent dark matter scattering cross section
off-protons versus the dark matter mass. Current experimental sensitivities indicated.

7.3. Majoron as decaying dark matter

A wide class of neutrino mass models is based upon the spontaneous breaking of ungauged
lepton number [22, 12]. The associated Goldstone boson - the Majoron - will pick up a mass
as a result of quantum gravity effects [122]. The massive Majoron will necessarily decay to a
pair of neutrinos [123]. The lifetime and mass are constrained by cosmic microwave background
observations as indicated in Fig. 16 [124]. The scenario fits nicely in models where neutrinos get
mass a la seesaw, and may lead to other possible cosmological implications associated to structure
formation. Indeed, although the Majoron in this mass range behaves as cold dark matter as far
as the CMB is concerned, its small thermal velocities might play a role in structure formation.

In a number of schemes the decaying Majoron has also a sub-leading decay mode into two
photons see, e.g. Refs. [108] and [125, 126]. There are upper limits on the decay rate arising
from NGC3227 (red), the Milky Way halo observed with a prototype cryogenic spectrometer
(salmon), XMM observations of the Milky Way (sand), Chandra observations of the Bullet
Cluster and M31 (orange), HEAO-1 observations of the diffuse X-ray background (aquamarine),
INTEGRAL SPI line search in the Milky Way halo (blue). These exclude the various shaded
regions, which have been compiled and extended in Ref. [125] where further details and references
can be found. Improved sensitivities are expected by the proposed XENIA mission [127].
Figure 16. Left panel: (Color online) Distortion of CMB anisotropy spectrum arising from decaying dark matter Majoron, see Ref. [124] for details. Right panel: (Color online) Contours of the 68% (green/dark) and 95% (yellow/light) CL allowed mass-decay-rate regions.

Figure 17. Majoron decay rate to photons versus Majoron mass, for different values of the triplet vev, $v_3$. We assume the invisible decay bound to be saturated and a (quasi)degenerate neutrino mass spectrum. Shaded regions are excluded by observations [125] and vertical lines characterize the cosmological Majoron production mechanism [124].

7.4. Decaying gravitino as dark matter in BRPV

Weak-scale supersymmetry with broken R-parity [28] provides an attractive way to induce neutrino masses and mixing in an intrinsically supersymmetric way, as indicated in Fig. 5. In the presence of $SU(3) \otimes SU(2) \otimes U(1)$ singlet “right-handed” neutrino superfields one can indeed induce the spontaneous breaking of R-parity in agreement with LEP measurements of the invisible Z width [29, 30]. This leads to bilinear R-parity violation (BRpV) as an effective model [33], the simplest extension of the minimal supersymmetric standard model (MSSM) with lepton number violation that can account for the present neutrino oscillation data [34, 35].

One of its main features is that the lightest neutralino can not be dark matter, as it decays typically inside collider detectors, as illustrated in Fig. 4. A natural possibility is to assume that the LSP is the gravitino [44]. In this picture the same lepton number violating superpotential which generate neutrino masses and mixing, also induce R-parity breaking gravitino decays, into a photon plus a neutrino. The latter is doubly suppressed, (i) by the smallness of the R-parity violating couplings, and (ii) by its gravitational origin. The allowed gravitino mass-lifetime region consistent with neutrino oscillation data and astrophysical bounds from negative searches for gamma-ray lines from dark matter decay is indicated in Fig. 18 (left). The yellow region is excluded by gamma-ray line searches at Fermi and EGRET. The lower and upper black lines correspond to $m_{1/2}=240$ and 3000 GeV respectively. The right plot shows the parameters in terms of reheat temperature and gravitino mass [44]. In the orange area a consistent gravitino relic abundance would require gluino masses already excluded by present collider searches, while the blue area corresponds to very large gluino masses, $M_3 >6000$ GeV. The yellow region is excluded by astrophysical gamma-ray line searches.
Note that the BRpV gravitino dark matter model can be probed at future collider experiments, like the LHC or the Next Linear Collider, since the decay pattern of the next-to-lightest supersymmetric particle provides a direct connection with the lepton mixing angles determined by neutrino experiments [38, 39, 40, 41, 42], as illustrated in Fig. 4.

Figure 18. Left: Allowed gravitino mass-lifetime region (grey color) consistent with neutrino oscillation data and astrophysical bounds on gamma-ray lines from dark matter decay. Right: The allowed parameters (white band) in terms of reheat temperature and gravitino mass [44].

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