FORMATION AND STRUCTURE OF HALOS IN A WARM DARK MATTER COSMOLOGY

VLADIMIR AVILA-REESE AND PEDRO COLIN
Instituto de Astronomía, UNAM, A.P. 70-264, 04510, México, D.F., México

OCTAVIO VALENZUELA
Astronomy Department, New Mexico State University, Box 30001, Department 4500, Las Cruces, NM 88003-0001

ELENA D’ONGHIA
Università degli Studi di Milano, via Celoria 16, 20100 Milano, Italy

AND

CLAUDIO FIRMANI
Instituto de Astronomía, UNAM, A.P. 70-264, 04510, México, D.F., México

Received 2000 October 26; accepted 2001 June 6

ABSTRACT

Using high-resolution cosmological N-body simulations, we study how the density profiles of dark matter halos are affected by the filtering of the density power spectrum below a given scale length and by the introduction of a thermal velocity dispersion. In the warm dark matter (WDM) scenario, both the free-streaming scale, $R_f$, and the velocity dispersion, $v_w$, are determined by the mass, $m_w$, of the WDM particle. We found that $v_w$ is too small to affect the density profiles of WDM halos. Down to the resolution attained in our simulations ($\sim 0.01$ virial radii), there is not any significant difference in the density profiles and concentrations of halos obtained in simulations with and without the inclusion of $v_w$. Resolved soft cores appear only when we artificially increase the thermal velocity dispersion to a value that is much higher than $v_w$. We show that the size of soft cores in a monolithic collapse is related to the tangential velocity dispersion. The density profiles of the studied halos with masses down to $\sim 0.01$ the filtering mass $M_f$ can be described by the Navarro-Frenk-White shape; soft cores are not formed. Nevertheless, the concentrations of these halos are lower than those of the CDM counterparts and are approximately independent of mass. The cosmogony of halos with masses $\lesssim M_f$ is not hierarchical: they form through monolithic collapse and by fragmentation of larger structures. The formation epoch of these halos is slightly later than that of halos with masses $\approx M_f$. The lower concentrations of WDM halos with respect to their CDM counterparts can be accounted for by their late formation epoch. Overall, our results point to a series of advantages of a WDM model over the CDM one. In addition to solving the substructure problem, a WDM model with $R_f \sim 0.16$ Mpc ($m_w \approx 0.75$ keV; flat cosmology with $\Omega_\Lambda = h = 0.7$) also predicts concentrations, a Tully-Fisher relation, and formation epochs for small halos, which seems to be in better agreement with observations than CDM predictions.

Subject headings: dark matter — galaxies: halos

1. INTRODUCTION

The damping of small-scale modes in the power spectrum of density fluctuations has been proposed to overcome potential observational difficulties at small scales of the hierarchical cold dark matter (CDM) scenario for structure formation (e.g., Avila-Reese, Firmani, & Hernández 1998; Moore et al. 1999b; Sommer-Larsen & Dolgov 2001; Hogan 1999; Kamionkowski & Liddle 1999; Hogan & Dalcanton 2000; White & Croft 2000; Colin, Avila-Reese, & Valenzuela 2000). The damping in the power spectrum may be produced either during its primordial generation (in the biased scalar inflationary models, for example) or from Landau and free-streaming damping, which erase galactic or subgalactic density fluctuations when the dark matter particles are warm (e.g., Blumenthal, Pagels, & Primack 1982), although the free-streaming damping could also be present in CDM models if CDM particles are nonthermally produced (Lin et al. 2001). In this paper we study warm dark matter (WDM) models; however, it is important to remark that our results are also valid for other models in which the power spectrum is damped at small scales.

A potential shortcoming of the CDM scenario is the large amount of substructure (satellites) that is predicted for Milky Way–like halos with respect to observations under the assumption that at each small halo a dwarf galaxy is formed (Kauffmann, White, & Guideroni 1993; Klypin et al. 1999; Moore et al. 1999a; but see, e.g., Bullock, Kravtsov, & Weinberg 2000 for an alternative solution). The cosmological N-body experiments of Colín et al. (2000, hereafter Paper I) have shown that the observed satellite circular velocity function of Milky Way and Andromeda can be reproduced if the power spectrum is exponentially damped (filtered) at scales $0.1$–$0.2$ Mpc (a flat cosmological model with $\Omega_\Lambda = 0.7$ and $h = 0.7$ was used). A free-streaming scale $R_f$ of $0.1$–$0.2$ Mpc is attained for particle masses $m_w$ of $0.6$–$1$ keV (Sommer-Larsen & Dolgov 2001). Interestingly, Narayanan et al. (2000) derived a lower limit for $m_w$ of $750$ eV from the restriction that the predicted power spectrum should reproduce the observed properties of the Ly$\alpha$ forest in quasar spectra.

In Paper I it was also found that the concentrations of satellite WDM halos decreases as $R_f$ increases, these concentrations being lower than those of the corresponding CDM halos. Unfortunately, these small halos were poorly resolved and we could not explore in detail the inner density profiles of halos with masses below $M_f$. Studying the density profiles of WDM halos with high resolution is necessary, in particular because another potential problem
of the CDM scenario is related to the inner density profile and concentration of small halos. It seems to be a discrepancy between numerical simulation results and observations of dwarf and low surface brightness (LSB) galaxies (e.g., Moore 1994; Flores & Primack 1994; Burkert 1995; de Blok & McGaugh 1997; de Blok et al. 2001), although other observational studies have challenged this result (van den Bosch et al. 2000; Swaters, Madore, & Trehwellia 2000; van den Bosch & Swaters 2001).

In the present paper the density profiles of WDM halos with masses close to or up to ~100 times smaller than $M_f$ are studied with a mass resolution 2–3 orders of magnitude larger than in Paper I. Our conclusions will be based partly on simulations with a power spectrum corresponding to $m_W \sim 0.6$ keV. However, in order to study with high resolution the halos with masses below $M_f$, we need to increase $M_f$. Thus, we also use simulations for $m_W \sim 0.13$ keV ($M_f \sim 10^{14} h^{-1} M_\odot$) and assume that the structure of halos should depend on the halo mass-to-$M_f$ ratio rather than on the specific value of $M_f$. The cosmogony of halos with masses less than $M_f$ is not hierarchical. Halos with masses close to or larger than $M_f$ by factors of 2–5 assemble a large fraction of their present-day mass by a coherent and nearly monolithic collapse, whereas halos with masses smaller than $M_f$ form from the gravitational fragmentation of larger pancake-like structures, as in the top-down scenario (Zeldovich 1970). Firmani et al. (2001) emphasized that the density profiles of dark halos formed through a monolithic collapse depend significantly on the velocity dispersion of the collapsing particles (see also Aguilar & Merritt 1990); this velocity dispersion may be primordial (thermal relict) or can be acquired during an inhomogeneous gravitational collapse (dynamical).

The nonzero thermal velocities of warm particles (warmons) should produce minimal shallow halo cores because of the phase packing limit (e.g., Tremaine & Gunn 1979). However, this thermal velocity dispersion, $v_{th}$, for warmons with masses of ~1 keV seems to be too small to produce a noticeable core (Hogan & Dalcanton 2000). In Paper I this velocity was neglected, leaving uncertain the effect it could have on the structure of simulated halos (Sellwood 2000). Here we include different thermal velocity values in the simulations in order to explore their influence on the halo density profile. An analytical (dynamical) approach will be also used to study the “hot” monolithic collapse and clarify the conditions required to form shallow cores.

After the completion of this paper, a similar paper by Bode, Ostriker, & Turok (2001) appeared in the preprint list of Los Alamos National Laboratory. Bode et al. study the cosmogony and structure of halos for models with damped power spectra in a cosmological box. They also find that halos with masses below the filtering mass are less concentrated and that these halos form later than the CDM ones. In addition, they also describe the spatial distribution and the mass function of these small halos; in this sense, their study complement ours.

In § 2 we present the WDM models to be explored in this paper. In § 3 we briefly describe the numerical simulations. In § 4 we present the results from our experiments aimed at studying the density profiles and concentrations of halos with masses below the mass corresponding to the filtering scale. The influence of the velocity dispersion over the inner density profile of halos formed by monolithic collapse is explored in § 5.1. In § 5.2 we present results from our WDM cosmological simulations, including the thermal velocity dispersion of warmons. In § 6.1 we describe and discuss how structure formation proceeds in simulations with a power spectrum suppressed at small scales. The viability of the WDM scenario is discussed in § 6.2. Finally, our conclusions are given in § 7.

### 2. COSMOLOGICAL MODELS

As in Paper I, here we also use the currently favored flat low-density universe with $\Omega_m = 0.7$, $h = 0.7$, and $\sigma_8 = 1$, and instead of CDM we introduce WDM. Two characteristics distinguish WDM from CDM: the damping of the small-scale density fluctuations and the fact that thermal velocity dispersion might not be negligible at the time of structure formation. The free-streaming length $R_f$ is related to the mass of the warmon through the following equation (e.g., Sommer-Larsen & Dolgov 2001):

$$R_f = 0.2(\Omega_{\text{WDM}} h^2)^{1/3} \left( \frac{m_W}{1 \text{ keV}} \right)^{-4/3} \frac{1}{M_\odot} \text{ Mpc},$$

where $\Omega_{\text{WDM}}$ is the contribution to the energy density of the universe from WDM. Modifications to equation (1) exist; for example, a larger coefficient of proportionality is given by Pierpaoli et al. (1998). Using their coefficient, for a given $R_f$, $m_W$ should be roughly 2 times larger than the values used here.

We follow Sommer-Larsen & Dolgov and define a characteristic free-streaming wavenumber $k_f$, as the $k$ for which the WDM transfer function (eq. [4]) is one-half; that is, $k_f = 0.46 R_f$. Once $R_f$ has been fixed (this is the “true” input parameter in our simulations), a characteristic filtering mass $M_f$ can be defined conditionally. Unfortunately, this has not been done in an unique way in the literature, introducing some confusion; its value can vary by about 3 orders of magnitude from one definition to another. For instance, defining $M_f$ simply as $4\pi/3\rho_m R_f^3$ gives a value 318 times smaller than the value obtained with the definition given by Sommer-Larsen & Dolgov (2001), which we use in this paper:

$$M_f \equiv \frac{4\pi}{3} \frac{\rho_m}{\de\lambda_f} \frac{\lambda_f}{2}^3,$$

where $\lambda_f \equiv 2\pi/k_f = 13.6 R_f$. This definition of $M_f$ is justified by the numerical experiments of Tittley & Couchman (1999). They found that the initial density distribution produced by a sharp cut in the power spectrum at $k_c = 2\pi/r_s$ (the exponential drop in the power spectrum in our case is at $k_c = 2\pi/\lambda_f$) is similar to that produced when the initial density field is convolved with a top-hat window of smoothing radius $\sim r_s/2$. The analysis of the halo properties below shows that the mass at which the WDM halos begin to diverge from the CDM ones is just around the mass given by equation (2).

We use the WDM power spectrum given in Bardeen et al. (1986):

$$P(k) = T_W(k)p_{\text{CDM}}(k),$$

where the WDM transfer function is approximated by

$$T_W(k) = \exp \left[ -\frac{kR_f}{2} - \frac{(kR_f)^2}{2} \right]$$

In § 6.2, we compare our numerical results with these analytical approximations.
and $P_{\text{CDM}}$ is the CDM power spectrum given by Klypin & Holtzman (1997) (see Paper I).

3. THE NUMERICAL SIMULATIONS

The simulations were performed using the multiple-mass scheme variant of the adaptive refinement tree (ART) $N$-body code (Kravtsov, Klypin, & Khokhlov 1997). The ART code achieves high spatial resolution by refining the base uniform grid in all high-density regions with an automated refinement algorithm. The multiple-mass scheme, described in detail elsewhere (Klypin et al. 2001; see also Paper I), is used to increase the mass and spatial resolution in few selected halos, which hereafter we call “hosts,” as in Paper I. However, while in Paper I the host halos were structures much larger than $M_f$, and selected to be relatively isolated, here, in simulations with large $R_f$, they are smaller than $M_f$, can be embedded within filaments, and can belong to groups at the intersection of filaments. On the other hand, in both CDM and WDM, we call “guest halos” the bound structures surviving within the virial radii of the host halos. These definitions are more technical than conceptual; note that in the simulations mentioned above, the host halos contain guest halos, but on their own they may be “guests” of other larger systems.

We start our simulations by making a low mass resolution run with $32^3$ or $64^3$ particles in a grid with $256^3$ cells in which all particles have the same mass. We use these runs to select (host) halos to be resimulated with higher mass and force resolution. We then identify all particles within $\sim 2$ virial radii and trace them back to get their Lagrangian positions at $z = 40$. For the cosmology used in this paper, the virial radius $r_v$ is defined as the radius within which the average halo density is 334 times the background density, according to the spherical collapse model. Next, the initial distribution with particles with different masses is generated, and the models are evolved to the present time with the multiple-mass variant of ART. Models with $R_f = 0.2$ Mpc have four mass levels (particles with masses 1, 8, 64, and $256 \times m_h$), whereas models with $R_f = 1.7$ Mpc have three levels. The mass resolution on the finest mass level corresponds to a box of $256^3$ particles in both series of simulations.

As in Paper I, the bound density maxima (BDM) group-finding algorithm was used to locate the host halos in the low mass resolution run. The BDM algorithm finds the positions of local maxima in the density field smoothed at the scale of interest and applies physically motivated criteria to test whether a group of particles is a gravitationally bound system. The BDM is also used to find the host and the guest halos in the multiple-mass high-resolution simulations.

In Table 1 we present an overview of all the simulations used in this paper. The formal force resolution shown in column (6) is the size of a cell in the finest refinement grid, and the mass per particle (col. (4)) is the mass for those particles that belong to the finest level of mass resolution. The halo density profile can be studied with high confidence only for radii larger than $\sim 4$ times the formal force resolution (Klypin et al. 2001) and containing within them more than 50–100 particles.

The first set of runs in Table 1 was aimed at studying in detail the density profiles and concentrations of halos with virial masses close to or below the corresponding filtering mass $M_f$. Model AWDM is the same simulation presented in Paper I for $R_f = 0.2$ Mpc but run in a smaller box, $L_{\text{box}} = 7.5$ h$^{-1}$ Mpc, in order to attain more mass resolution. The host halo studied in this run is 1.7 times less massive than the same halo in the run with $L_{\text{box}} = 15$ h$^{-1}$ Mpc. Unfortunately, in the simulation with the box size reduced, the most massive guest halos turned out to be small, containing fewer than 1000 particles.

Since it is not easy to know the masses of the guest halos a priori, and since our aim is just to explore the density profiles of halos with masses smaller than $M_f$, a better strategy is to study halos in simulations where we select them a priori (hosts), with the desired masses below $M_f$. Although from the technical point of view of our approach these halos are hosts, they may be well embedded within larger halos, i.e., they could be satellites (guests). We set $R_f = 1.7$ Mpc, which corresponds to $M_f = 1.7 \times 10^{14}$ h$^{-1}$ M$_\odot$, and resimulate with high resolution smaller and smaller host halos in runs with decreasing box sizes. For this model ($m_p = 125$ eV), the structure formation process is close to the hot dark matter regime. Nevertheless, in the assumption that the formation and structure of halos depend on $M_\odot/M_f$.

\begin{table}[h]
\centering
\caption{Models and Simulations Parameters}
\begin{tabular}{llllllll}
\hline
Run & $R_f$ & $M_f$ & $v_{\text{th}}$ & $m_p$ & Box & Resolution & $N_{\text{host}}$
\hline
AWDM & 0.2 & $2.9 \times 10^{13}$ & 0.0 & $2.1 \times 10^6$ & 7.5 & 0.1 & 1
AWDM & 1.7 & $1.7 \times 10^{14}$ & 0.0 & $2.1 \times 10^6$ & 7.5 & 0.4 & 1
AWDM & 1.7 & $1.7 \times 10^{14}$ & 0.0 & $1.7 \times 10^7$ & 15.0 & 0.4 & 1
AWDM & 1.7 & $1.7 \times 10^{14}$ & 0.0 & $1.3 \times 10^4$ & 30.0 & 1.8 & 4
AWDM & 0.2 & $2.9 \times 10^{13}$ & 2 & $2.1 \times 10^6$ & 7.5 & 0.1 & 1
AWDM & 1.7 & $1.7 \times 10^{14}$ & 2 & $1.1 \times 10^6$ & 60.0 & 1.8 & 4
AWDM & 1.7 & $1.7 \times 10^{14}$ & 16 & $1.1 \times 10^6$ & 60.0 & 1.8 & 4
\hline
\end{tabular}
\end{table}

*This is in fact what we find in all of our WDM experiments: the mass of a given halo becomes smaller as $L_{\text{box}}$ is reduced. However, this mass reduction is not dramatic, and we do not expect that it will change our conclusions about concentrations and density profiles of the halos, because neither are typically very dependent on mass.

4 Thermal velocity dispersion added to the particles at $z = 40$, in units of the relict thermal velocity corresponding to the warmon mass used in the simulation, $v_{\text{th}}$. 
rather than the specific value of \( M_f \), this model will give guidance to the physics on smaller scales in a model with a more realistic warmon mass. We think this assumption is plausible. In any case, general trends can be compared with the (low-resolution) results from simulations with \( m_w = 605 \, \text{eV} \).

Runs \( \Lambda \text{WDM}_{15}, \Lambda \text{WDM}_{30}, \) and \( \Lambda \text{WDM}_{60} \) (Table 1), are for \( L_{\text{box}} = 7.5, 15, 30, \) and \( 60 \, h^{-1} \, \text{Mpc} \), respectively. For comparison, we also run a CDM simulation \( (R_f = 0) \) with the same initial condition as \( \Lambda \text{WDM}_{60} \). In model \( \Lambda \text{WDM}_{60} \), we resimulated four host halos with masses 2–4 times smaller than \( M_f \). They were resolved with \( \sim 4–7 \times 10^4 \) particles. In order to obtain halos with masses even \( \sim 10 \) times smaller but with the same mass resolution, we reduced the box size by a factor of 2. This is run \( \Lambda \text{WDM}_{30} \), where four halos with masses 22–33 times smaller than \( M_f \) were resimulated. In run \( \Lambda \text{WDM}_{15} \), the only host halo resimulated is already 45 times smaller than \( M_f \) and has \( 3.54 \times 10^5 \) particles. The guest halos (those contained within the virial radii of hosts) in all of these simulations have masses smaller than \( \sim 0.01 M_f \) and are resolved with \( \leq 7000 \) particles. Finally, in run \( \Lambda \text{WDM}_{7.5} \) the selected halo is 300 times less massive than \( M_f \) and it is resolved with \( 2.73 \times 10^5 \) particles.

At this point, it is important to note the effect that a reduction of the box size has on the results from WDM simulations. In CDM simulations we do not expect that a reduction in the box size affects the internal structure of halos with radii much smaller than \( L_{\text{box}} \) (e.g., Frenk et al. 1988). However, in WDM simulations the loss of long wavelengths may critically affect the formation and the structure of halos if the fundamental wavelength \((= L_{\text{box}})\) is close to or smaller than the characteristic filtering wavelength, \( \lambda_f \). When dealing with WDM power spectra, one should keep in mind that structures of scales smaller than \( \sim \lambda_f \) arise from a transfer of power from large to small

---

**Fig. 1.** Distribution of dark matter particles in run \( \Lambda \text{WDM}_{60} \) inside a cube of \( 40 \, h^{-1} \, \text{Mpc} \) comoving on a side at different epochs: \( a = 0.3, 0.5, 0.75, \) and 1.0, where \( a \) is the scale factor. All particles that are within a sphere of radius \( 1.5 \, h^{-1} \, \text{Mpc} \) centered on a host halo at \( a = 1 \) were traced back to the chosen epoch, and their center of mass was computed. Cubes were then centered on these centers of masses. We have color-coded particles on a gray scale according to the logarithm of their local density (a \texttt{pgplot} program kindly provided by A. Kravtsov). The local density at the particle positions, on the other hand, was computed using SMOOTH, a publicly available code developed by the HPCC group in the University of Washington Department of Astronomy.
scales (see § 6.1). Hence, if the large-scale modes are omitted, especially those around the peak, then the formation of small structures becomes affected. For CDM this is not a problem because small scales have ever more power than the large ones, and the structure formation process is dominated by the smallest scales.

Since the filtering wavelength for $R_f = 1.7$ Mpc is $\lambda_f = 16 \, h^{-1}$ Mpc (see § 2), run $\Lambda WDM_{15}$ is at the limit of confidence, while for run $\Lambda WDM_{7.5}$ the mode corresponding to the peak of the WDM power spectrum is absent (the fundamental mode lies beyond the peak). The density profile of the halo resimulated in this peculiar model (a narrow and exponentially damped power spectrum) indeed deviates from profiles of halos obtained in other WDM simulations with larger box sizes, as will be seen in § 4.

The second set of simulations shown in Table 1 was aimed at studying the effect that a warm nonzero thermal velocity, $v_{th}$, has on the internal structure of simulated halos. From this study we conclude that $v_{th}$ does not affect the inner density profile of WDM halos, at least down to the scale at which we are confident of the resolution of our simulations ($\sim 0.01r_e$). Run $\Lambda WDMt2$ is the same as $\Lambda WDM (R_f = 0.2$ Mpc), but introducing a thermal velocity component twice as large as $v_{th}$. Runs $\Lambda WDM_{60}t2$, $\Lambda WDM_{60}t4$, and $\Lambda WDM_{60}t16$ are the same as $\Lambda WDM_{60}$ ($R_f = 1.7$ Mpc), but introducing thermal velocities with amplitudes 2, 4, and 16 times larger than the corresponding $v_{th}$, respectively.

In order to attain a visual impression of how structure formation does proceed for models with a damped power spectrum, we present in Figure 1 snapshots at different epochs of the run $\Lambda WDM_{60}$. The figure snapshots show the distribution of particles inside a cube of $40 \, h^{-1}$ Mpc comoving on a side. The center of each cube is the center of mass of the particles that were traced back to the selected epoch from a sphere of radius $1.5 \, h^{-1}$ Mpc centered in one of the host halos at $z = 0$ (Fig. 3, see below). In Figure 2 we show the same as in Figure 1, but for the corresponding CDM model, run $\Lambda CDM_{60}$. The different formation histories of halos with masses $\approx M_f$ or below in models with and without damped power spectra are highlighted in these two

![Fig. 2.—Same as Fig. 1, but for CDM, run $\Lambda CDM_{60}$](image-url)
figures; in particular, we note that (1) these halos form later in model $\Lambda WDM_{60}$ than in $\Lambda CDM_{60}$ (the amount of substructure at $z \sim 2$ in model $\Lambda WDM_{60}$ is reduced to a solitary sharp filament), and (2) they are only located within filaments in the WDM case, while in the CDM one they can also be found outside the filaments (see also Bode et al. 2001). In Figure 3 we show a zoom of the central region of Figure 1 at two times, $z = 1$ and $z = 0$. In the right panel all particles inside a sphere of $1.5 \, h^{-1}$ Mpc centered on a host halo at $z = 0$ are plotted. The mass of this halo is about half the filtering mass ($M_f = 1.7 \times 10^{14} \, h^{-1} \, M_\odot$), and it was one of the first structures to collapse in the simulation. In the left panel, the particles plotted in the right panel were traced back to $z = 1$. Note that the virialized halos seem to emerge from the filamentation and fragmentation of pancakes. In any case, the detailed study of halo formation in WDM-like models is beyond the scope of this paper.

An effect that is becoming typical of cosmological high-resolution simulations with damped power spectra is the finding of relatively small halos regularly spaced in filaments. An example of this is shown in Figure 1 at $a = 0.5$; see those knots that lie inside the filament that points from right to left to the center of the cube. Several of these knots were indeed selected by our halo-finding algorithm as self-gravitating structures. The space regularity, also seen in other of our simulations, makes us suspect that the origin of these very small halos with respect to $M_f$ may be a numerical artifact attributed perhaps to finite grid effects. A detailed study of this effect is highly desirable, since it might produce a steepening of the circular velocity function distribution of satellites at the small velocity end.

4. DENSITY PROFILES OF HALOS WITH MASSES NEAR OR BELOW THE FILTERING MASS

In Paper I host halos of a few $10^{12} \, h^{-1} \, M_\odot$ were resimulated using the multiple-mass scheme in a box size of $15 \, h^{-1}$ Mpc for a $\Lambda CDM$ model with $R_f = 0.2$ Mpc. The mass per particle in the finest level of resolution corresponded to $1.66 \times 10^7 \, h^{-1} \, M_\odot$. Thus, the guest halos reported in Paper I (with scales below the filtering scale) were poorly resolved; they only had a few $10^2$ particles, a number not high enough to study in detail their inner density profiles. As mentioned in § 3, we also run the same simulation of Paper I, but with the box size reduced by a factor of 2 (run $\Lambda WDM$). We find that the most massive guest halos are smaller than the most massive ones from the $15 \, h^{-1}$ Mpc run, in such a way that they were resolved with not many more particles than in Paper I. Figure 4 (top) shows the density profiles of the host and guest halos with more than 1000 particles (the latter were shifted vertically in order to avoid too much overlapping). The radius of the innermost point is at least 4 times the formal force resolution and contains more than 50 particles within it (see § 3; this criteria applies for all density profiles shown in this paper). For the host halo, the innermost radius is $4 \times 0.1 \, h^{-1}$ kpc, and for the guest halos is $\sim 1.2 \, h^{-1}$ kpc. As one sees from Figure 4, the density profiles of guest halos—whose masses are $\sim 0.01$ times smaller than the filtering mass $M_f = 2.8 \times 10^{11} \, h^{-1} \, M_\odot$—are well described by the Navarro, Frenk, & White (1997, hereafter NFW) profile (dashed lines), from $\sim 0.04$ to $1 \, r_v$.

As described in § 3, for a series of $\Lambda WDM$ simulations with $R_f = 1.7$ Mpc and different box sizes, we resimulated with high resolution selected halos (hosts) with masses smaller than the corresponding $M_f = 1.7 \times 10^{14} \, h^{-1} \, M_\odot$, obtaining in this way halos with a large number of particles.

In Figure 5 we show the density profiles of the host halos with masses 2–4 times smaller than $M_f$ from run $\Lambda WDM_{60}$ (filled circles) and those obtained in the same simulation but with CDM instead of WDM (run $\Lambda CDM_{60}$; open circles). Although both simulations started with identical phases (seeds) and large-scale normalization, the CDM halos are slightly more massive than the corresponding WDM ones (by factors of $\sim 1.1$–1.4). The dashed lines show the NFW shape describes rather well the density
profiles of both the WDM and CDM halos, for radius ranging from ~0.01 to 1 $r_v$, although in the innermost parts the slope tends to be steeper than $r^{-1}$, in particular for the CDM halos (Moore et al. 1999a). The accuracy of the fit can be estimated with the parameter $D \equiv D/N$, where $D \equiv \Sigma \left( \log \left( \rho_i/\rho_{\text{med}} \right) \right)^2 \rho_i \rho_{\text{med}}$ is the quantity used for minimization of the fitting, and $N$ is the number of radial bins (points to fit). For the host halos in runs $\Lambda$WDM$_{60}$ and $\Lambda$CDM$_{60}$, on average $D \approx 6.5\%$ and $5.5\%$, respectively.

In order to explore the structure of halos with masses much smaller than $M_f$, we have run simulations with the same $R_f = 1.7$ Mpc but with box sizes of 30, 15, and 7.5 $h^{-1}$ Mpc (runs $\Lambda$WDM$_{30}$, $\Lambda$WDM$_{15}$, and $\Lambda$WDM$_{7.5}$, respectively); in these simulations we resimulate host halos with masses of $\approx 6-95 \times 10^{11} h^{-1} M_\odot$. Note that these halos may be embedded within larger halos, i.e., they could be satellites (see §3 for the technical definitions of host and guest halos).

Figure 6 shows the density profiles of the four host halos obtained in the run $\Lambda$WDM$_{30}$ (top panel), and of the two host halos obtained in runs $\Lambda$WDM$_{15}$ and $\Lambda$WDM$_{7.5}$ (bottom panel). For comparison, the density profile of a CDM halo of $2.5 \times 10^{12} h^{-1} M_\odot$ presented in Paper I is also plotted (open circles). Although now the masses of the halos are up to ~50 times smaller than $M_f$, the NFW fit (dashed lines) continues to be a good description for the density profiles of WDM halos explored with accuracy down to ~0.01 $r_v$. The deviation parameter $D$ is on average ~6\% for host halos from run $\Lambda$WDM$_{30}$ and ~3.5\% for the only host halo from run $\Lambda$WDM$_{15}$. Nonetheless, the WDM halos are less concentrated and shallower in the center than their CDM counterparts.

The density profile of the host halo from simulation $\Lambda$WDM$_{1.5}$ shows a prominent shallow core and a disturbed outer density profile. Since this run has a box size already smaller than the filtering wavelength $\lambda_f$, it samples only the high-frequency drop of the WDM power spectrum; we thus expect the inner structure of the formed halo to be significantly affected (see §3).

The results obtained here confirm our previous result: the WDM halos with masses near or below the filtering mass are less concentrated than the corresponding CDM halos (Paper I). In Figure 7 (top) we plot the $c_{1/5}$ concentration parameter$^2$ versus virial mass $M_v$ for the host halos from the different runs with $R_f = 1.7$ Mpc. We also include in this plot guest halos with more than 1000 particles. They are only a few, and their inner density profiles are resolved only down to ~0.03 $r_v$. The concentrations and inner

$^2$ The concentration $c_{1/5}$ is defined as the ratio between the virial radius $r_v$ and the radius at which 1/5 of the total halo mass is contained (Avila-Reese et al. 1999). This definition of the concentration parameter is independent of the particular fitting applied to the halo density profile.
The inner points and the normalization of the radius are as in Fig. 4.

son, the profile of a CDM halo from Paper I is also plotted (open circles). The inner points and the normalization of the radius are as in Fig. 4.

Bottom: Same as the top panel, but for the two host halos from runs \( \text{AWDM}_{15} \) (crosses) and \( \text{AWDM}_{15} \) (squares), respectively. As one expects, the density profile of the halo from run \( \text{AWDM}_{15} \) is affected by the small size of the box (see § 3).

Density profiles of surviving guest halos are not expected to be significantly affected by the fact that they are within larger systems. In fact, some of the host halos in the different runs are also within larger halos. Results for the WDM model with \( R_f = 0.2 \) Mpc are also shown in Figure 7 (top): crosses are from Paper I and three-point stars correspond to the \( \text{AWDM} \) run presented here. Halos with less than 90 particles were excluded.

In order to compare the concentrations of WDM and CDM halos, we also plot in Figure 7 (top) the results from our \( \Lambda \text{CDM} \) simulation of size 60 h\(^{-1}\) Mpc (open circles), as well as a linear fitting for thousands of isolated and clustered halos found in a \( \Lambda \text{CDM} \) simulation (Avila-Reese et al. 1999). The trend in Figure 7 (top) is clear: as the halo mass becomes smaller than \( M_f \), the \( c_{1/5} \) concentration departs more and more from the corresponding CDM concentration. Although the dispersion in the concentration is large (see Fig. 8 in Avila-Reese et al. 1999 and Fig. 11 below), the trend seen in Figure 7 (top) for the WDM halos is clear and is out of the statistical dispersion of the \( c_{1/5} - M_v \) relation of the CDM halos. For completeness, the concentrations obtained when fitting the density profiles of the halos to a NFW profile \( c_{\text{NFW}} \) are also presented in the bottom panel of Figure 7 (only halos with \( D < 10\% \) were included). The four halos from the 60 h\(^{-1}\) Mpc CDM simulation, as well as a linear interpolation for the \( c_{\text{NFW}} - M_v \) relation obtained from the \( \Lambda \text{CDM} \) simulation presented in

Fig. 6.—Top: Density profiles of the four host halos from simulation \( \text{AWDM}_{30} \). The two lower profiles were shifted by \(-0.5 \) in the log in order to avoid overlapping. Dashed lines show the best NFW fit. For comparison, the profile of a CDM halo from Paper I is also plotted (open circles). The inner points and the normalization of the radius are as in Fig. 4.

Bottom: Same as the top panel, but for the two host halos from runs \( \text{AWDM}_{15} \) (crosses) and \( \text{AWDM}_{15} \) (squares), respectively. As one expects, the density profile of the halo from run \( \text{AWDM}_{15} \) is affected by the small size of the box (see § 3).

Avila-Reese et al. for halos whose density profiles are well described by the NFW shape, are also shown in this plot.

5. EFFECT OF THERMAL VELOCITY DISPERSION ON THE DENSITY PROFILE

There are two relevant conditions for the problem of halo formation, namely, the initial fluctuation density profile (which determines the kind of collapse the halo suffers) and the amplitude of the (tangential) velocity dispersion, \( v_{\text{rms}} \), of the collapsing particles. The CDM halos form hierarchically, and the \( v_{\text{rms}} \) is acquired dynamically since early epochs due to interactions of substructures with the global tidal field. In a WDM scenario, the first structures to collapse are those with masses close to \( M_f \); they assemble the mass \( M_f \) almost synchronously, followed by some mass accretion (quasi-monolithic collapse). It is possible that structures formed by fragmentation also suffer a quasi-monolithic collapse. Because the collapse of halos less massive than \( M_f \) is delayed and the amount of substructure is small (compared
to halos formed hierarchically), one expects that particles will acquire a lower tangential velocity dispersion than in the hierarchical case, although this is a question to be explored in the numerical simulations. Instead, WDM particles have an intrinsic (residual) thermal velocity dispersion.

An interesting question to explore is how the angular momentum of the particles—due to either a residual thermal momentum or to a $v_{\text{rms}}$, acquired dynamically—affects the inner structure of virializing systems. For halos formed hierarchically, the velocity dispersion is not relevant for the formation of a soft core (e.g., Avila-Reese et al. 1998; Huss, Jain, & Steinmetz 1999), but for halos formed monolithically it may be. In § 5.1 we present an heuristic (analytical) model of halo virialization, which allows us to understand the role that velocity dispersion plays in a monolithic collapse. We also find an expression to relate the soft core radius with $v_{\text{th}}$, using $N$-body simulations for hot monolithic collapse and virialization. In § 5.2 results from cosmological $N$-body simulations, including several values of $v_{\text{th}}$, are presented.

5.1. Thermal Monolithic Collapse and Soft Cores

Let us study the monolithic collapse of a sphere of mass $M$, radius $R_M$, and uniform density $\rho_M$ at maximum expansion. We can introduce some thermal energy assuming that all particles move on elliptical orbits with a given eccentricity $e$. The constancy of the density and $e$ with radius ensure that all particles have the same orbital period $\tau$ and that the pericenter-to-apocenter ratio is the same for all of them, respectively. Therefore, shell crossing is avoided. For purely radial motion ($e = 1$), the collapse reaches a singular point. For $e < 1$, at maximum concentration, i.e., when each particle is found at pericenter, the collapse leads to a uniform sphere with radius

$$r_c = R_M(1 - e)/(1 + e) \, ,$$

(5)

The existence of this maximum concentration state is at the base of the existence of a soft core in the virialized halo. Tangential velocity at apocenter is

$$v_{\text{th}}^2 = \frac{r^2 4\pi G \rho_M (1 - e)}{3} \, ,$$

(6)

while, at the maximum expansion, the total thermal energy $T$ and the potential energy $W$ are given by

$$T = \frac{8\pi^2 G \rho_M^2}{15} (1 - e) R_M^5 \, ,$$

(7)

$$W = \frac{16\pi^2 G}{15} \rho_M^2 R_M^5 \, ,$$

(8)

where the relationship between $e$, $T$, and $W$ is $2T/W = 1 - e$.

The simplest approximation to estimate the virialized density at a given radius (density profile) is to assume a time-average density at the same radius. This approach is based on the statistical hypothesis that the time-average density is representative of the virialized halo density. Using the conventional motion parametric equations,

$$r = \frac{R_M}{1 + e} (1 - e \cos \theta) \, ,$$

(9)

the time-average density at a radius $r$ between $r_c$ and $R_M$ is

$$\langle \rho(r) \rangle = \frac{\rho_M (1 + e)^3}{\pi} \int_0^\pi \frac{1}{J_n (1 - e \cos \theta)^2} d\theta \, ,$$

where $J_n = \int_0^\pi \cos^n \theta d\theta = \frac{\pi (1 - e^2)^{n/2}}{1 - e^2} \left[ \frac{\sqrt{1 - \frac{\eta}{e - \eta^2}}}{\sqrt{1 - \frac{\eta}{e + \eta^2}}} \right]$, (10)

while between 0 and $r_c$ (the soft core) the average density is

$$\rho_c = \rho_M \left( \frac{1 + e}{1 - e} \right)^{3/2} \, ,$$

(11)

where $\eta = 1 - (1 + e)r/r_0$ and $\cos \theta = \eta/e$. The constant density between 0 and $r_c$ is product of the particle angular momentum, which prevents the particles from reaching the center. The core mass is given by

$$M_c = M \left( \frac{1 - e}{1 + e} \right)^{3/2} \, .$$

(12)

Equations (5), (11), and (12) clearly show that the ellipticity of the orbits is at the basis of the soft core formation in a monolithic collapse.

From our analysis, we conclude that soft cores in virialized dark collisionless halos may result from only two simultaneous conditions: (1) an initial homogeneous density profile for the progenitor of the present structure (monolithic collapse), and (2) the presence of velocity dispersion with a tangential component. The angular momentum of kinetic motion avoids the migration of particles toward the center, limiting the central density.

In order to obtain a more quantitative relation between the core radius $r_c$ and the velocity dispersion $v_{\text{rms}}$, we resort to $N$-body simulations of monolithic spherical (top-hat) collapse with several values of $v_{\text{rms}}$ injected uniformly at the maximum of expansion. The public version of HYDRA, an adaptive P3M-SPH code (Couchman, Thomas, & Pearce 1995), was used. The simulations can be rescaled to any mass $M$ and radius at maximum expansion $R_M$ through the relations $M = m M_\odot$, $R_M = r R_M$, $t = (r^3/m)^{0.5} t$, and $v_{\text{rms}} = (m/r)^{0.5} v_{\text{rms}}$, where $m$ and $r$ are scaling parameters and the quantities with a hat are obtained in the simulation. We assume that $v_{\text{rms}}$ is related to a relict thermal velocity $v_{\text{th}}$, which decays adiabatically as $v_{\text{th}} \propto R^{-1}$ until the sphere attains its maximum expansion, $R = R_M$. We give $v_{\text{th}}$ when the fluctuation is still in its linear regime and calculate its value, $v_{\text{th},M}$, at the maximum expansion of the sphere of mass $M$. The latter is the stage from which we start the $N$-body simulation.

From a set of simulations with $M = 3.1 \times 10^{11} M_\odot$ (1.64 $\times$ 10$^4$ particles) and $R = 0.5$ Mpc, we have found how the core radius scales with the injected velocity dispersion. The softening radius is 500 pc. We define the core radius $r_c$ as the radius at which the central density decreased by a factor of 3. In Figure 8 we show the results from simulations of hot monolithic collapse (open triangles) and the results of the analytic model presented above (solid and dashed line).
The line is valid for high tangential velocity dispersion when shell-crossing effects are negligible. When $v_{\text{th}} \lesssim 6$ km s$^{-1}$, the linear relation $r_c \approx 3\text{(kpc)}v_{\text{th}}$ is a good approximation. Using the scaling relations mentioned above and the expression $R_M = \frac{\left(32GM/H_0^2\Omega_m 9\pi^2\right)^{1/3}}{(1 + z_M)}$, valid approximately for a top-hat sphere in a flat low-density universe, we obtain

$$r_c \approx \frac{2.5 h^{-1} \text{kpc} v_{\text{th},M}/\text{km s}^{-1}}{\sqrt{\Omega_m 0.3} (1 + z_M)^{3/2}}.$$  \hspace{1cm} (13)

The applicability of this formula is valid only for $v_{\text{th},M} \lesssim 2$ km s$^{-1}$ $\Omega_m 0.3(hM_10)^{1/3}(1 + z_M)^{1/2}$, where $M_{10}$ is the mass in units of $10^{10} M_\odot$. In the WDM case, $z_M$ refers to the redshift of maximum expansion of structures with masses close to $M_f$, and $v_{\text{th},M}$ refers to the relic thermal velocity dispersion these structures have at $z_M$. Note that the upper-limit velocity given above is much larger than the corresponding $v_{\text{th}}$. If the less massive halos, which form by fragmentation, also suffer a monolithic collapse at a redshift not much later than $z_M$ (see § 6.1), then their thermal core radii also can be roughly predicted with equation (13). Equation (13) is easy understood in light of the angular momentum analysis of Bode et al. (2001).

For a WDM model with $m_w = 0.6$ keV, then $M_f = 2.8 \times 10^{11} h^{-1} M_\odot$ and $v_{\text{th}} = 3.5$ km s$^{-1}$ at $z = 40$. According to the spherical top-hat model and using the respective WDM variance, the maximum expansion redshift for a $M_f = 2.7 \times 10^{11} h^{-1} M_\odot$ $2\sigma$ fluctuation is $z_M = 5.3$ (2.1 for 1 $\sigma$, for which we give values in parentheses below). At this redshift the thermal velocity of the top-hat sphere decreased adiabatically to $v_{\text{th},M} = 1$ km s$^{-1}$ (0.5 km s$^{-1}$). Then, according to equation (13), the core radius of halos with masses close to $M_f$ is $r_c \approx 150$ pc (215 pc). This radius should be approximately the same for smaller halos, unless these halos during the fragmentation process acquire large velocity dispersions. Therefore, the thermal soft cores in WDM models are only a very small fraction of the virial radius, at least for halos not much smaller than $M_f$.

5.2. Results from N-Body Simulations

Previous estimates showed that the warmon relic velocity dispersion $v_{\text{th}}$ is too small to influence the halo inner density profiles. Now we resort to cosmological simulations that include several values of $v_{\text{th}}$ in order to explore this question. For a warmon of $\sim 1$ keV, $v_{\text{th}}$ is small (at $z = 40$, $v_{\text{th}} = 2$ km s$^{-1}$, which is about 10 times smaller than the typical peculiar velocities at that epoch). A larger thermal velocity dispersion could be possible if warmons self-interact; in this case the same free-streaming scale $R_f$ can be attained with a smaller particle mass (Hogan 1999; Hannestad & Scherrer 2000) and, since $v_{\text{th}} \propto m_w^{-4/3}$, a smaller $m_w$ implies a larger $v_{\text{th}}$. According to Hannestad & Scherrer (2000), the mass of the warmons that produce a given free-streaming scale in the case of collisionless WDM could be 1.9 times smaller if warmons self-interact. Therefore, the thermal velocity dispersion could be up to 2.3 times larger in the latter case.

We have run two of the simulations presented in § 3 (runs $\Lambda$WDM and $\Lambda$WDM$_{50}$) introducing a thermal velocity component in the particles with several amplitudes. The thermal velocities were randomly oriented, and their magnitudes were drawn from a Fermi-Dirac phase-space distribution with a given rms velocity $v_{\text{th}}$. These velocities were added to the initial peculiar velocities computed using the Zeldovich approximation.

For our preferred model ($R_f = 0.2$ Mpc), we compare the $c_{1.5}$ concentrations obtained in runs $\Lambda$WDM ($v_{\text{th}} = 0$) and the same model but with $v_{\text{th}}$ twice as high as $v_{\text{th}}$, the warmon velocity corresponding to the given $R_f$ value ($\Lambda$WDM$_{2}$), echoing the paper by Hannestad & Scherrer cited above. In Figure 9 we plot $c_{1.5}$ versus $M_f$ for the only host halo and the more than a dozen guest halos obtained in both simulations at two different epochs, $z = 0$ and 1. There is not any obvious difference in the concentrations of halos from the
two runs. The density profiles of the host and guest halos with more than 1000 particles from run \( \Lambda WDM_{2} \) are shown in the bottom panel of Figure 4. As can be seen from Figures 4 and 9, the introduction of a relict velocity dispersion, even 2 times higher than \( v_{th} \), does not affect notably the structure of WDM halos, at least in the parts where we attain good resolution (down to \( \sim 0.04r_{c} \)).

In order to explore in more detail the effect of \( v_{th} \) on the inner halo structure, we have run model \( \Lambda WDM_{60} \) (its resolution is \( \sim 0.01r_{c} \)) with an ever increasing \( v_{th} \). In runs \( \Lambda WDM_{60}t2, \Lambda WDM_{60}t4, \) and \( \Lambda WDM_{60}t16, v_{th} \) was fixed to values 2, 4, and 16 times larger than the \( v_{th} \) corresponding to a warmon of mass 125 eV (\( R_{f} = 1.7 \) Mpc), respectively. For the latter case, the thermal velocities at \( z = 40 \) are \( \sim 7 \) times higher than the peculiar velocities. Figure 10 shows the density profiles for the four host halos (shifted vertically by \( -1 \) in the log) obtained in the series of runs \( \Lambda WDM_{60}, \Lambda WDM_{60}t2, \Lambda WDM_{60}t4, \) and \( \Lambda WDM_{60}t16. \) While for the simulations with \( v_{th} = 2 \) and 4 times \( v_{th} \), the inner density profiles still do not deviate significantly from the case with \( v_{th} = 0 \), in the simulation with \( v_{th} = 16v_{th} \), a soft core is already evident at radii smaller than \( \sim 0.03-0.04r_{c} \).

The redshifts at maximum expansion of the four halos from runs \( \Lambda WDM_{60}t2, \Lambda WDM_{60}t4, \) and \( \Lambda WDM_{60}t16 \) are roughly 1.7–1.3. Therefore, the core radii predicted with equation (13) for these halos are approximately 3.6–3.9, 7.2–7.8, and 28.6–31.4 \( h^{-1} \) kpc, respectively. Halos in the last two runs have roughly \( r_{c} = 10–15 \) and 31–33 \( h^{-1} \) kpc, respectively. One should take into account that in the cosmological simulations, the studied halos do not suffer a perfect monolithic collapse, and that some velocity dispersion can be acquired during the collapse of the pancakes and filaments.

In conclusion, the introduction of a warmon relict thermal velocity has no important effect on the density profiles and concentrations of WDM halos. This velocity would have to be much larger in order to produce noticeable soft cores, as equation (13) shows. We should note that simulations with and without a thermal velocity are not similar at all. The identity of the guest halos is not the same for all of them, and their spatial distribution is different in both simulations. However, in a statistical sense, neither the concentrations nor the satellite circular velocity function change.

6. DISCUSSION

6.1. The Formation of Low-Mass Halos in Simulations with a Damped Power Spectrum

When the power spectrum of fluctuations is damped above some wavenumber \( k_{f} \), “pancakes” of size \( \sim k_{f}^{-1} \) are the first structures to collapse, and smaller objects are expected to form later by fragmentation (e.g., Zeldovich 1970; Doroshkevich et al. 1980). The coupling between different modes in the nonlinear gravitational evolution drives a transfer of power, with power flowing from larger to smaller scales. This transfer is very efficient; numerical simulations previously confirmed such a behavior (e.g., Little, Weinberg, & Park 1991; Bagla & Padmanabhan 1997; White & Croft 2000).

Although our numerical experiments were not aimed at studying the structure formation process in a statistical sense (for this see Bode et al. 2001), we noticed that indeed the first structures to collapse are those with scales close to the filtering scale length \( k_{f} \). These structures form smooth, coherent filaments when they enter the nonlinear regime (Fig. 1; see also Melott & Shandarin 1990; Little et al. 1991, and references therein), instead of the chains of dense clumps seen in CDM simulations (Fig. 2).

In the left panel of Figure 3 one can appreciate the filamentary structure of the protohalo that is becoming nonlinear at \( z = 1 \). This structure collapses roughly at the same time (quasi-monolithic collapse), but obviously the initial conditions are not spherically symmetric. Nonetheless, matter flows toward the center of the filament, and some spherical symmetry is established there (see the virialized halo at \( z = 0 \)). Within the shrinking filament, substructure probably forms by fragmentation. Theoretical and numerical support for the idea that cosmological pancakes are unstable with respect to fragmentation and the formation of filaments can be found in Valinia et al. (1997) and more references therein.

Based on a visual inspection (see also Table 2 and discussion below), we can say that the collapse of the substructure within the filament is almost parallel to the collapse of the filament. As mentioned above, the power transfer from larger to smaller scales is very efficient. It also seems that the collapse epoch of the fragmented halos is independent of their masses. Nevertheless, all these halos collapse on average later than in a CDM simulation. This might explain
why the small WDM halos are less concentrated and why
their concentrations, although with a large scatter, do not
depend on mass (see Fig. 7). Because halos are less concen-
trated than in the CDM simulations, they are more easily
disrupted. As was shown in Paper I, the reduction of the
number of small halos due to this effect is comparable to the
effect that the power spectrum suppression has on this
number, and both work together to deliver a very small
number of satellites at \( z = 0 \).

Interestingly, our results suggest that on average and in a
first approximation, the density profiles of dark matter
halos are indeed universal, no matter how they form, either
by hierarchical clustering, by a monolithic collapse, or by
fragmentation (see also Moore et al. 1999b). The difference
seen in their concentrations can be explained just as an
effect of the formation epoch, which for small WDM halos is
delayed compared to the formation epoch of their CDM
counterparts. As we have shown in §5, one way to signifi-
cantly affect the inner density profile of halos in the case of a
monolithic collapse is by including a high (tangential) veloc-
ity dispersion. The thermal velocities of warmons of mass
\( \sim 1 \) keV are much smaller than this required velocity dis-

6.2. Viability of the WDM Scenario: Observational Tests

Because of the increasing evidence that the predictions of
the current standard ΛCDM model at small scales are in
conflict with observations, modifications to this scenario,
able to retain their successful predictions at large scales, have
recently been analyzed. Because the nature of dark matter
particles is still a mystery, it is tempting to exchange CDM
particles for WDM particles as the simplest modification to
the standard scenario. At least from the point of view of
particle physics, there is not an obvious preference for any
of these particles (e.g., Colombi, Dodelson, & Widrow
1996). But what are the advantages of the WDM scenario
with respect to the CDM one from the point of view of
structure formation? Here we present a list of what we
consider to be these advantages:

1. For a WDM model with \( R_f \gtrsim 0.1 \) Mpc \((m_w \lesssim 1 \) keV),
\( \Omega_m = 0.3 \), and \( \Omega_s = 0.7 \), the observed maximum circu-
lar velocity function of Milky Way and Andromeda satel-
lites is roughly reproduced (Paper I).

2. Although in the WDM scenario the halos with masses
close to or below \( M_f \) do not have a noticeable constant
density core, they are less concentrated and have a density
profile shallower in the center than their CDM counterparts
(§4 and 5). Recent observational studies have shown that
with the current data it is not possible to accurately con-
strain the halo inner density profiles of dwarf and LSB
galaxies; these profiles are probably not steeper than \( r^{-3} \),
and, if anything, the concentrations of these halos are lower
than those predicted in the CDM scenario (van den Bosch
et al. 2000; Swaters et al. 2000; van den Bosch & Swaters
2001). In Figure 11 we compare the \( c_{1/5} \) concentration of
guest WDM halos from runs AWDMt2 and AWDM with
the \( c_{1/5} \) concentration of dwarf and LSB galaxies, inferred
from observational data (see details in the figure caption).
Unfortunately, there is not an overlap between the theoreti-
cal and observational data; nonetheless, it can already be
appreciated that the guest WDM halos are in better agree-
ment with observations.

3. The formation of disks within WDM halos \((m_w \lesssim 1 \)
keV) in N-body + hydrodynamic simulations does not
seem to suffer from the disk angular momentum problem
(Sommer-Larsen & Dolgov 2001). This problem is also at
the basis of another difficulty reported by Steinmetz &
Navarro (1999): the predicted infrared Tully-Fisher (TF)
relation in their simulations is much brighter than the
observed one. The maximum circular velocity, \( V_{\text{max}} \), of
the system increases after disk formation. In the numerical
simulations of Steinmetz & Navarro, this increase is about a
factor of 2 larger than for models in which detailed angular
momentum conservation is assumed for the infalling gas
(e.g., Mo, Mao, & White 1998; Avila-Reese et al. 1998;
Avila-Reese & Firmani 2000; Firmani & Avila-Reese 2000).
A factor of 2 in the velocity translates into a factor of \( \sim 8 \) in
mass or luminosity, explaining why Steinmetz & Navarro
obtain a brighter TF relation. If the angular momentum
problem is alleviated as in the WDM scenario, then one
expects that the zero point of the TF relation predicted in
numerical simulations will be in agreement with observa-
tions (Sommer-Larsen & Dolgov 2001).

4. Several authors have shown that the TF relation of
normal disk galaxies in the infrared band is an imprint of
the mass-velocity \((M_f - V_{\text{max}})\) relation of CDM halos
(Firmani & Avila-Reese 2000, and references therein). For
masses larger than \( \sim 10 M_f \), there are not major differences
between the CDM and the WDM halos (Paper I; see also

| Epoch   | \( N_{A,100} \) | \( N_N \) | \( X_{\text{em}} \) (h^{-1} Mpc) | \( Y_{\text{em}} \) (h^{-1} Mpc) | \( Z_{\text{em}} \) (h^{-1} Mpc) | \( M_0 \) (h^{-1} M_{\odot}) |
|---------|--------------|---------|------------------|------------------|------------------|------------------|
| 1.000    | 1            | 13      | 7.193            | 5.236            | 1.406            | 1.4 \times 10^{12} |
| 0.879    | 1            | 11      | 7.191            | 5.287            | 1.424            | 1.3 \times 10^{12} |
| 0.753    | 1            | 17      | 7.175            | 5.349            | 1.443            | 1.1 \times 10^{12} |
| 0.630    | 2            | 24      | 7.153            | 5.453            | 1.452            | 6.6 \times 10^{11} |
| 0.504    | 4            | 29      | 7.139            | 5.705            | 1.516            | 4.9 \times 10^{11} |
| 0.354    | 5            | 26      | 7.155            | 5.740            | 1.528            | 2.9 \times 10^{11} |
| 0.205    | 3            | 15      | 0.379            | 5.837            | 1.515            | 1.8 \times 10^{11} |
| 0.152    | 2            | 6       | 7.117            | 6.623            | 1.055            | 1.1 \times 10^{11} |
| 0.115    | 2            | 2       | 7.135            | 6.148            | 1.406            | 5.2 \times 10^{10}  |
| 0.124    | 3            | 15      | 0.379            | 5.837            | 1.515            | 2.7 \times 10^{10}  |
that the satellite dwarf galaxies form later than the host large galaxies. The evolution of the substructure for our WDM simulation ΛWDM12 (R_f = 0.2 Mpc) is shown in Table 2. The scale factor normalized to 1 at present is given in the first column, while the number of halos with maximum circular velocity V_max greater than 100 km s^{-1} is presented in column (2). Columns (4), (5), and (6) give the coordinates of the center of mass of the system composed by these halos. The total number of halos with V_max > 15 km s^{-1} that contain more than 200 particles and the mass of the most massive halo are given in columns (3) and (7), respectively. We see that the first structure starts to assemble at z ~ 7; this seed has a mass of ~3 × 10^{10} h^{-1} M_⊙. Later, at z ~ 4, when the mass of the most massive halo has grown to M_f ~ 10^{11} h^{-1} M_⊙, the first substructures appear by fragmentation. Thus, the building blocks for large galactic and supragalactic structures are those with masses near to M_f; smaller structures (guest halos) form slightly later by fragmentation. We can thus say that the formation redshift of guest halos is approximately equal to or less than z_f(M_f). For the ΛWDM model with the power spectrum normalized to COBE used here and according to the spherical top-hat model, the typical formation redshift of ~10^{11} h^{-1} M_⊙ halos z_f(10^{11} h^{-1} M_⊙) ≈ 1.3 and 3.7 for 1 and 2 σ peaks, respectively. If galaxies form from high peaks, then the typical formation redshifts of galaxies of ~10^{11} h^{-1} M_⊙ will be larger than z = 2−3. For the ΛWDM models with R_f ≤ 0.2 Mpc, smaller galaxies (dwarfs) will form slightly later than these redshifts. Similar conclusions are obtained by Bode et al. (2001), who also remark that low-mass halos form solely within pancakes and filaments, and not in the voids, in contrast to the situation in the CDM scenario (see also Fig. 1). From the observational point of view, there are some pieces of evidence that dwarf galaxies formed later than bright galaxies. In the Local Group, all dwarf galaxies seem to have their oldest stellar population slightly younger than the oldest Milky Way halo population (Mateo 1998); for the Large Magellanic Cloud this age difference could be 2 Gyr, according to studies of the horizontal branch mor-

5. Unlike in the CDM scenario, for WDM one expects
phology of stellar clusters (Olszewski, Suntzeff, & Mateo 1996, and references therein), while for the Small Magellanic Cloud the difference should be even larger, since this galaxy seems to be younger than its neighborhood. On the other hand, comparisons of observed and modeled galaxy counts for $B$ dropout galaxies at $3.5 \leq z \leq 4.5$ suggest that some $L_*$ galaxies were already in place at $z \approx 4$, but dwarf galaxies may have formed later at $3 \leq z \leq 4$ (Metcalfe et al. 2001), in agreement with predictions of the WDM scenario.

6. The Ly$\alpha$ forest is a powerful probe of the linear power spectrum on galactic and subgalactic scales, on the understanding that it traces the underlying matter density. Since in a WDM scenario the small-scales modes are damped out, one might think that the observed Ly$\alpha$ forest should not be reproduced by this cosmology. However, because of the efficient power transfer from larger to smaller scales (see § 6.1), enough power on small scales is regenerated at $z \approx 3$–4. Narayanan et al. (2000) have shown that WDM models ($\Omega_m = 0.3$ and $\Omega_\Lambda = h = 0.7$) with $R_f \lesssim 0.155$ Mpc ($m_\text{w} \gtrsim 0.75$ keV) are able to reproduce the observed properties of the Ly$\alpha$ forest. This last point, more than an advantage of WDM with respect to CDM, is a test for the former scenario. The CDM scenario is also able to predict the properties of the Ly$\alpha$ forest.

As we have seen, the WDM scenario works better than the CDM one in several aspects. Nonetheless, both scenarios fail in predicting soft halo cores and the independence of the halo central density on its mass, as some inferences from observations seem to suggest (e.g., Firmani et al. 2000, 2001, and references therein). More observational data that explore in detail the inner structure of the halo component of galaxies are urgently needed; however, it is likely that the more decisive data will come from strong-lensing studies of clusters of galaxies (the largest virialized structures). The phase space density of dark matter halos derived from observations also offers an important test for the WDM scenario (Hogan & Dalcanton 2000; Sellwood 2000). If more detailed observational studies confirm the existence of soft halo cores with the scaling properties suggested in Firmani et al. (2000, 2001) and Sellwood (2000), the WDM scenario must be abandoned. Then alternatives such as the self-interacting CDM might become more appealing.

7. SUMMARY AND CONCLUSIONS

We have carried out high-resolution $N$-body cosmological simulations with the aim of studying the effect on the halo density profiles produced by (1) the damping of the power spectrum at small scales and (2) the introduction of a relict thermal velocity dispersion, as well as exploring the viability of the WDM scenario. We have also studied hot monolithic collapse by means of an analytical model and top-hat $N$-body simulations. Our main conclusions are:

1. For the halos studied here, with masses close to or up to $\sim 100$ times smaller than the filtering mass $M_f$, the density profiles are on average well described by the NFW shape; the density profiles were well resolved down to $\sim 0.01r_c$. The only differences between halos with masses below $M_f$ and their CDM counterparts is that the former have lower concentrations, and their innermost density profiles are not as steep as $r^{-1.5}$. Thus, although the cosmogony of the halos in both cases is different, the final virialized structures are not dramatically different.

2. The $c_{1/5}$ or $c_{NFW}$ concentrations of halos more massive than a few times the filtering mass $M_f$ are similar to those of their CDM counterparts. However, as the mass decreases, the concentrations on average remain almost constant (slightly decrease), while for CDM halos the concentration increase monotonically. The scatter of the concentration for a given mass is large in both cases. The difference in the concentrations of the small halos can be explained because the relatively late formation epoch of small halos in the simulations with damped power spectrum at small scales.

3. The relic thermal velocity dispersion of warmons, $v_\text{th}$, does not affect the density profiles of WDM halos, at least down to $0.01r_c$. For our simulations with $R_f = 0.2$ and 1.7 Mpc ($\Lambda$WDM$_{m2}$ and $\Lambda$WDM$_{60}$, respectively), we used a thermal velocity 2 times larger than $v_\text{th}$, and we did not find any significant difference in the halo concentrations and density profiles with respect to the case with $v_\text{th} = 0$. For the high-resolution run $\Lambda$WDM$_{60}$, we also experimented with $v_\text{th} = 4$ and 16 times $v_\text{th}$, finding resolved soft cores only for the last case (with radii at $\sim 0.03$–0.04 $r_c$).

4. The inner structure of dark matter halos formed through monolithic collapse can be affected only if the particles have a significant (tangential) velocity dispersion $v_\text{rms}$. The penetration degree toward the halo center is determined by the pericenter of the orbiting particle, or equivalently, the angular momentum forces the particles to avoid migration toward inner regions. From $N$-body simulations of the collapse of top-hat spheres with different amounts of $v_\text{th}$, we have found that the core radius scales linearly with $v_\text{th}$ up to velocities much larger than $v_\text{th}$. In order for WDM halos with masses close to $M_f$ to form noticeable cores, $v_\text{th}$ should be much larger than $v_\text{th}$.

We conclude that a flat $\Lambda$WDM model with $R_f \approx 0.16$ Mpc ($m_\text{w} \approx 750$ eV), $\Omega_m \approx 0.7$, and $h \approx 0.7$, has several advantages over its CDM model counterpart. The problem of excess of substructure is solved (Paper I); the halos of dwarf and LSB galaxies are less concentrated than in the CDM scenario, as observations suggest (Fig. 11); and the disk angular momentum problem is alleviated (Sommer-Larsen & Dolgov 2001). Furthermore, our preferred WDM model describes better than CDM the TF relation of dwarf galaxies (Fig. 12) and the formation epochs of these galaxies (§ 6), as well as their large-scale distribution (Bode et al. 2001). On the other hand, this model is not apparently in conflict with measurements of the power spectrum of Ly$\alpha$ forest (Narayanan et al. 2000) and with the reionization constraints (Bode et al. 2001). Nonetheless, the WDM model is ruled out if more observations confirm the existence of soft halo cores as well as the independence of their densities from mass; the crucial test is at cluster scales. No doubt, exciting questions remain to be answered in the near future.

We are grateful to A. Klypin and A. Kravtsov for kindly providing us a copy of the ART code in its version of multiple mass, and to H. Couchman for having made available his adaptive P3M-SPH code HYDRA. We also thank A.
Klypin for enlightening discussions. We thank the second referee for a thorough and accurate revision of the manuscript as well as for the suggestions which helped to improve the quality of this paper. The first referee is acknowledged for his (her) criticism that conducted us to carry out more simulations in order to study the influence of thermal velocity on halo density profiles. This work has received partial funding from CONACyT grant J33776-E to V. A. The work of E. D. was supported by Fondazione Cariplo, Italy. Our ART simulations were performed at the Dirección General de Servicios de Computo Académico, UNAM, using an Origin-2000 computer.

REFERENCES

Aguilar, L., & Merritt, D. 1990, ApJ, 354, 33
Avila-Reese, V., & Firmani, C. 2000, Rev. Mexicana Astron. Astrofis., 36, 23
Avila-Reese, V., Firmani, C., & Hernández, X. 1998, ApJ, 505, 37
Avila-Reese, V., Firmani, C., Klypin, A., & Kravtsov, A. V. 1999, MNRAS, 310, 527
Bagla, J. S., & Padmanabhan, T. 1997, MNRAS, 286, 1023
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Blumenthal, G. R., Pagels, H., & Primack, J. R. 1982, Nature, 299, 37
Bode, P., Ostriker, J. P., & Turok, N. 2001, ApJ, 556, 93
Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, ApJ, 539, 517
Burkert, A. 1995, ApJ, 447, L25
Cole, S., Lacey, C. G., Baugh, C. M., & Frenk, S. F. 2000, MNRAS, 319, 168
Colin, P., Avila-Reese, V., & Valenzuela, O. 2000, ApJ, 542, 622 (Paper I)
Colombi, S., Dodelson, S., & Widrow, L. M. 1996, ApJ, 458, 1
Couchman, H. M. P., Thomas, A., & Pearce, F. R. 1995, ApJ, 452, 797
de Blok, W. J. G., & McGaugh, S. S. 1997, MNRAS, 290, 533
de Blok, W. J. G., McGaugh, S. S., Bosma, A., & Rubin, V. C. 2001, ApJ, 552, L23
Doroshkevich, A. G., Kotok, E. V., Novikov, I. D., Polyudov, A. N., Shardar, S. F., & Sigov, Yu. S. 1980, MNRAS, 192, 321
Firmani, C., & Avila-Reese, V. 2000, MNRAS, 315, 457
Firmani, C., D’Onghia, E., Hrná, A. A., Kravtsov, A. V., & Valenzuela, O. 2000, ApJ, 542, 622 (Paper I)
Frenk, C. S., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Hannestad, S., & Scherrer, R. J. 2000, Phys. Rev. D, 62, 043522
Hogan, C. J. 1999, preprint (astro-ph/9912549)
Hogan, C. J., & Dalcanton, J. J. 2000, Phys. Rev. D, 62, 063511
Huss, A., Jain, B., & Steinmetz, M. 1999, ApJ, 517, 64
Kamionkowski, M., & Liddle, A. R. 1999, preprint (astro-ph/9911003)
Kaufmann, G., White, S. D. M., & Guideroni, B. 1993, MNRAS, 264, 201
Klypin, A. A., & Holtzman, J. 1997, preprint (astro-ph/9712217)
Klypin, A. A., Kravtsov, A. V., Bullock, J., & Primack, J. 2001, ApJ, in press (preprint astro-ph/0006343)
Klypin, A. A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJ, 522, 82
Kravtsov, A. V., Klypin, A. A., & Khokhlov, A. M. 1997, ApJS, 111, 73
Lin, W. B., Huang, D. H., Zhang, X., & Brandenberger, R. 2001, Phys. Rev. Lett., 86, 954
Little, B., Weinberg, D. H., & Park, Ch. 2000, Phys. Rev. D, 62, 063511
Huss, A., Jain, B., & Steinmetz, M. 1999, ApJ, 517, 64
Huss, A., Jain, B., & Steinmetz, M. 1999a, ApJ, 524, L19
Moore, B., Quinn, T., Governato, F., Lake, G., Stadel, J., & Tozzi, P. 1999b, ApJ, 524, L19
Moore, B., Quinn, T., Governato, F., Stadel, J., & Lake, G. 1999b, MNRAS, 310, 1147
Narayanan, V. K., Spergel, D. N., Davé, R., & Ma, Ch.-P. 2000, ApJ, 543, L103
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Olszewski, E. W., Suntzeff, N. B., & Mateo, M. 1996, ARA&A, 34, 511
Pierpaoli, E., Borgani, S., Masiero, A., & Yamaguchi, M. 1998, Phys. Rev. D, 57, 2089
Sellwood, J. A. 2000, ApJ, 540, L1
Shapiro, P. R., & Raga, A. C. 2000, in Rev. Mexicana Astron. Astrofis. Ser. de Conf. 9, Astrophysical Plasmas: Codes, Models, and Observations, eds. J. Arthur, N. Brickhouse, & J. Franco (Mexico, D.F.: UNAM), 292
Sommer-Larsen, J., & Dolgov, A. 2001, ApJ, 551, 608
Steinmetz, M., & Navarro, J. F. 1999, ApJ, 513, 555
Swaters, R. A., Madore, B. F., & Trewhella, M. 2000, ApJ, 531, L107
Tittley, E. R., & Couchman, H. M. P. 1999, preprint (astro-ph/9911365)
Tremaine, S., & Gunn, J. E. 1979, Phys. Rev. Lett., 42, 407
Valinia, A., Shapiro, P. R., Martel, H., & Vishniac, E. T. 1997, ApJ, 479, 46
van den Bosch, F., Robertson, B. E., Dalcanton, J. J., & de Blok, W. J. G. 2000, AJ, 119, 1579
van den Bosch, F. C., & Swaters, R. A. 2001, MNRAS, 325, 1017
White, M., & Croft, R. A. C. 2000, ApJ, 539, 497
Zeldovich, Y. B. 1970, A&A, 5, 84