The superconducting phase transitions for the asymmetrical FS superlattices with interelectronic interaction in ferromagnet layers

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Abstract. The superconducting and magnetic states coexistence in the FS superlattice, where F is ferromagnetic metal and S is superconductor, is investigated on base of microscopically derived boundary value problem for the Eilenberger function. The asymmetrical four-layered structure $FSF'S'$ is considered as elementary cell. The second order phase transitions are explored for case of ideal boundary transparencies and clean Cooper limit. Each layer is characterized by its own thickness and electronic structure. Materials are also differed by electron-electron interaction constants (note these constants for ferromagnets are nonzero!). It is shown, that 0- and $\pi$-phase superconducting states of clean thin superlattices FS are defined by value and sign of electronic correlations in all four layers of elementary cell. The competition between uniform BCS pairing and non-uniform Fulde-Ferrell-Larkin-Ovchinnikov pairing are also taken into account. We predict that the complex system under consideration may have up to 8 different states which are characterized by phase shifts between superconducting order parameters in $S(F)$ and $S'(F')$ and mutual orientation of magnetizations in the $F$ and $F'$ layers. The states with $\pi$-phase magnetism can fully explain surprising experimental behavior of short-range Gd/La superlattice, i.e the coincidence of superlattice critical temperature $T_c$ with $T_c(La) = 5$ K for different thickness of the Gd layers exceeding the La layer thickness.

1. Introduction
The ferromagnet metal/superconductor (FS) nanostructures represent artificial layered systems in which various not trivial phenomena may be observed (see reviews [1, 2, 3] and references therein). For example, the superconductivity of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) type [4, 5], triplet superconducting correlations, various types of dependencies of critical temperature from thickness of the ferromagnetic layers (monotonous decrease, output on plateau, oscillations, re-entrant superconductivity), $\pi$-phase superconductivity and $\pi$-phase magnetism. It is caused by a proximity effect due to electrical and quantum contact of ferromagnets and the superconductors.

In our recent works [6, 7] we have developed the proximity effect theory for the FS bilayers and symmetric $FSF'$ trilayers taking into account the electron-electron interaction in a ferromagnet (for the first time) and spatial variations of the Eilenberger function along FS border. The last one leads to appearance of two-dimensional FFLO states. Four various states which differ phases of superconducting $\Delta$ and magnetic $I$ order parameters in adjacent $F$ layers were found for the three-layered nanostructures $FSF'$ [7, 8]. For asymmetric trilayer $FSF'$ [8], where the thickness
of layer \( F' \) was used as the controlling parameter, the solitary re-entrant superconductivity was predicted.

As a rule a superconductivity occurs in the layered FS nanostructures at \( d_f \ll d_s \), where \( d_{f(s)} \) is the F(S) layer thickness. The typical case is the layered Gd/Nb and Fe/V structures [1, 2, 3]. The recently observed behavior of critical temperature in short period Gd/La superlattices [9, 10] also motivate our work. The critical temperature \( T_c \) measured for the short-period Gd/La superlattice with different ratio of Gd and La layers thicknesses \( (d_f \approx 2d_s \text{ [10]} \text{ and } d_f \approx 3d_s \text{ [9]} \) was 5 K under cooling in zero field; this value almost coincides with the \( T_c \) of bulk lanthanum. This can not be explained by existing one-dimensional theory of the proximity effect [1, 2].

We develop the proximity theory for the thin pure SF superlattice and we try to explain these facts. The unit cell of the asymmetric superlattice is four-layered structure SFS\'F' in which all four layers may have different thicknesses and other parameters.

2. Mathematical formalism

In vicinity of the second order phase transition point the critical temperature of the nonuniform superconductor may be found from the Gor'kov self-consistency equation [11] for an order parameter \( \Delta \)

\[
\Delta(\mathbf{r}) = \lambda(\mathbf{r}) \pi T \sum_{\alpha \neq \beta} \sum_{\omega} F_{\alpha\beta}(\mathbf{r}, \omega),
\]

(1)

where \( \lambda \) is the interelectronic interaction parameter, \( \omega = \pi T(2n+1) \) is the Matsubara frequency, the prime at the second sum means a cutoff at the Debye frequency \( \omega_D \), \( T \) is the temperature, and \( \hbar = k_B = \mu_B = 1 \) is taken hereinafter. The Gor'kov function \( F \) is defined [11]

\[
F_{\alpha\beta}(\mathbf{r}, \omega) = \frac{1}{\pi N(\mathbf{r})} \iint_{V} \left\langle G_{\alpha\alpha}(\mathbf{r}, \mathbf{r}', \omega) G_{\beta\beta}(\mathbf{r}, \mathbf{r}', -\omega) \right\rangle_{\text{imp}} \Delta(\mathbf{r}') d\mathbf{r}'.
\]

(2)

Here \( G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) \) is the Fourier transform of the Green function of normal metal, instead of frequency is substituted the Matsubara one, and averaging is conducted on all configurations of nonmagnetic impurities. The integral boundary-value problem for the Gor'kov function in the FS nanocontact may be obtained by the diagrams technics [1, 6, 11]

\[
F_{\alpha\beta}(\mathbf{r}, \omega) = \frac{1}{\pi N(\mathbf{r})} \iint_{V} K_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) \left[ \Delta(\mathbf{r}') + \frac{1}{2\tau(\mathbf{r}') F_{\alpha\beta}(\mathbf{r}', \omega)} \right] d\mathbf{r}'.
\]

(3)

Here \( K_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) = G_{\alpha\alpha}(\mathbf{r}, \mathbf{r}', \omega) G_{\beta\beta}(\mathbf{r}, \mathbf{r}', -\omega) \), integration is conducted on all contact volume, the scattering velocity on impurities is equal to zero \( (\tau^{-1} \equiv 0) \).

The Eilenberger function \( \Phi \) is connected with the Gor'kov function \( F \) by a relation [12]

\[
F(\mathbf{q}, z, \omega) = \frac{1}{4\pi} \iint \Phi(\mathbf{p}, \mathbf{q}, z, \omega) d^2\mathbf{p}
\]

(4)

Here \( \mathbf{p} \) is the unit vector moving at integration on a solid angle of the Fermi sphere. The Eilenberger function satisfies to the differential boundary-value problem [6, 7] including the equation of diffusion type

\[
\left[ 2\omega + i\Gamma - v_z(z)\frac{\partial^2}{\partial z^2} \right] \Phi(\mathbf{p}, \mathbf{q}, z, \omega) = 2\Delta(z)
\]

(5)

and boundary conditions at the sharp FS borders

\[
\xi_z(z) \frac{\partial \Phi(\mathbf{p}, \mathbf{q}, z, \omega)}{\partial z} \bigg|_{z=\pm0} = \frac{\sigma}{2(1-\sigma)} [\Phi(\mathbf{p}, \mathbf{q}, +0, \omega) - \Phi(\mathbf{p}, \mathbf{q}, -0, \omega)].
\]

(6)
Here σ is transparency of the boundary, v is Fermi velocity, the 2D wave vector q lies in the FS contact plane, the pair-breaking parameter Γ will be determined below, ξ is correlation length

$$\xi_z = \frac{v_z}{2\omega}; \ \omega = 2\omega + i(2I + qv_z).$$

(7)

3. Asymmetric SF superlattice: the basic assumptions and the boundary-value problem solution

Let’s consider the SF superlattice which consist of repeating four-layered block SFS′F′. In common asymmetrical case all layers of this elementary cell have own parameters: thickness dζ, Fermi velocities vζ, interelectronic interaction constants λζ, exchange fields Iζ, coherence lengths ξζ, etc (ζ = s, f, s′, f′ numbers layers in unit cell). To solve analytically the Eilenberger equations we follow the papers [6, 7] and make next assumptions for all metals and boundaries. First of them is ideal transparency (σ=1), the second is the Cooper limit (ξζ/dζ ≫ 1), at last there are no impurities (a clean limit). The novel assumption is the account of interelectronic interaction in the ferromagnet which in preceding theories [1, 2, 3] was supposed zero owing to a powerful exchange field. The constant λζ can be either negative and positive. The last case of concealed electron-electron interaction corresponds to the latent superconductivity suppressed in bulk ferromagnet by the strong exchange field.

The superlattice case differs from four-layered case, which will be considered in separate paper, by periodical translational conditions on the Eilenberger function (L is period of the FS superlattice)

$$\Phi(p, q, z, \omega) = \Phi(p, q, z + L, \omega); \ \ L = d_s + df + d_{s'} + d_{f'}.$$  

(8)

Boundary conditions into elementary cell SFS′F′ for the second FS′ boundary (let z = 0) are given by the equations

$$\Phi_s(p, q_s, +0, \omega) = \Phi_f(p, q_f, -0, \omega); \ \ ξ_{sz} \frac{\partial \Phi_s(p, q_s, z, \omega)}{\partial z} \bigg|_{z=+0} = ξ_{fz} \frac{\partial \Phi_f(p, q_f, z, \omega)}{\partial z} \bigg|_{z=-0}.$$  

(9)

Boundary conditions at third S′F boundary (z = d_s) look like

$$\Phi_{s'}(p, q_{s'}, d_{s'} - 0, \omega) = e^{iα_f} Φ_{f'}(p, q_{f'}, d_{s'} + 0, \omega);$$

$$ξ_{s'} \frac{\partial \Phi_{s'}(p, q_{s'}, z, \omega)}{\partial z} \bigg|_{z=d_{s'}-0} = e^{iα_f} ξ_{f'} \frac{\partial \Phi_{f'}(p, q_{f'}, z, \omega)}{\partial z} \bigg|_{z=d_{s'}+0}.$$  

(10)

Boundary conditions at the SF border (z = −df) looks like

$$\Phi_f(p, q_f, -df + 0, \omega) = e^{iα_s} \Phi_s(p, q_s, -df - 0, \omega);$$

$$ξ_f \frac{\partial \Phi_f(p, q_f, z, \omega)}{\partial z} \bigg|_{z=−df+0} = e^{iα_s} ξ_s \frac{\partial \Phi_s(p, q_s, z, \omega)}{\partial z} \bigg|_{z=−df−0}.$$  

(11)

The phase shifts αs and αf can be equal only 0 and π, and, correspondingly, they determine the cases with 0- and π-types of superconductivity [1, 2, 6]. Firstly π-types of superconductivity was proposed by Bulaevskii et al. [13], we following paper [6] introduce π-phase of superconductivity on the F layers, this case corresponds αf = π. The first three conditions (9)-(11) are equivalent to the analogous relations for asymmetrical four-layered system. The last boundary conditions connect the Eilenberger function and its derivative of fourth layer (F′) and the same values of first layer (S) owing to periodical translational conditions (8) and look as (here α3 = 0, π is the phase shift)

$$\Phi_s(p, q_s, -df - d_s + L + 0, \omega) = e^{iα_3} Φ_{f'}(p, q_{f'}, d_{s'} + d_{f'} - 0, \omega);$$

$$ξ_{sz} \frac{\partial \Phi_s(p, q_s, z, \omega)}{\partial z} \bigg|_{z=-df-d_s+L+0} = e^{iα_3} ξ_{f'} \frac{\partial \Phi_{f'}(p, q_{f'}, z, \omega)}{\partial z} \bigg|_{z=d_{s'}+d_{f'}-0}.$$  

(12)
For simplicity below we will use that the S and S' metals are the same superconductors as well as the F and F' metals are the same ferromagnets. As trial solutions of the Eilenberger equations we use the four common solutions centered at the middle of corresponding layer \( z_\zeta \)

\[
\Phi_\zeta = \frac{\Delta_{f0}}{\omega_f} + A_\zeta \cosh \left[ \frac{(z - z_\zeta) / \xi_{sz}}{\cosh (z / \xi_{sz})} \right] + C_\zeta \frac{\sinh \left[ (z - z_\zeta) / \xi_{sz} \right]}{\sinh \left( z / \xi_{sz} \right)},
\]

(13)

where \( \Delta_{f0} \) and \( \Delta_{f'} \) are initial virtual superconducting order parameters in metal F and F', correspondingly, if they were nonferromagnetic, i.e. \( I = I' = 0 \).

Owing to the collectivization of electron correlations and the paramagnetic effect of the exchange field, the mutual effect of F, S, F' and S' metals is essentially strong in the Cooper limit when their thicknesses are small, \( d_{f(s)} \ll \xi_{f(s)}, a_f \), where \( \xi_{f(s)} = \nu_{f(s)}/2\pi T \) is the coherence length and \( a_f = v_f/2I \) is the spin stiffness length. Substituting the trial Eilenberger functions in boundary conditions (9)-(12) and using periodical condition (8), we gain the nonuniform set of the linear equations on eight constants \( A_\zeta \) and \( C_\zeta \)

\[
\begin{align*}
\frac{\Delta_{s0}}{\omega_s} + A_s' - C_s' &= \frac{\Delta_{f0}}{\omega_f} + A_f + C_f, \\
-A_s \frac{d_s'}{2\xi_{s'z}} + C_s' \frac{2\xi_{s'z}}{d_s'} &= A_f \frac{d_f'}{2\xi_{f'z}} + C_f \frac{2\xi_{f'z}}{d_f'}, \\
\frac{\Delta_{s0}}{\omega_s} + A_s + C_s &= e^{i\alpha_f} \left( \frac{\Delta_{f0}}{\omega_f} + A_f - C_f \right), \\
A_s \frac{d_s'}{2\xi_{s'z}} + C_s' \frac{2\xi_{s'z}}{d_s'} &= e^{i\alpha_f} \left( -A_f \frac{d_f'}{2\xi_{f'z}} + C_f \frac{2\xi_{f'z}}{d_f'} \right), \\
\frac{\Delta_{f0}}{\omega_f} + A_f - C_f &= e^{i\alpha_s} \left( \frac{\Delta_{s0}}{\omega_s} + A_s + C_s \right), \\
-A_f \frac{d_f'}{2\xi_{f'z}} + C_f \frac{2\xi_{f'z}}{d_f'} &= e^{i\alpha_s} \left( A_s \frac{d_s}{2\xi_{sz}} + C_s \frac{2\xi_{sz}}{d_s} \right), \\
\frac{\Delta_{s0}}{\omega_s} + A_s - C_s &= e^{i\alpha_3} \left( \frac{\Delta_{f0}}{\omega_f} + A_f + C_f \right), \\
-A_s \frac{d_s}{2\xi_{sz}} + C_s \frac{2\xi_{sz}}{d_s} &= e^{i\alpha_3} \left( A_f \frac{d_f}{2\xi_{f'z}} + C_f \frac{2\xi_{f'z}}{d_f} \right).
\end{align*}
\]

(14)

Note that, in four-layered nanostructure case we had six unknown constants and six equations. After solving the set of Eqs.(14) we can get the Eilenberger functions. Due to given assumptions (Cooper limit, ideal transparencies, etc) and collectivization of electron correlations and exchange fields we obtain the same the Eilenberger function for all layers, but they do not similar for different set of phase shifts. In common case we have \( 2^3 = 8 \) different solution \( (\alpha_f, \alpha_s, \alpha_3) \). Owing to translation symmetry (see Eq. (8)) only four states of them are realized. The total phase shift on elementary cell \( (\alpha_f + \alpha_s + \alpha_3) \) should be equal to either zero or 2\( \pi \). Hence \( \alpha_3 \) can be called a compensation phase shift. So, we have only four different solution having physical meaning: \( (000) \), \( (0\pi\pi) \), \( (\pi0\pi) \), \( (\pi\pi0) \). Direct calculations confirm this consideration.

Taking into account the two possible combinations of the exchange field \( I \) in ferromagnets

\[
I' = \exp(i\chi)I,
\]

(15)

where \( \chi = 0(\pi) \) is the magnetic phase shift \([14] \) corresponding to (anti)parallel mutual orientation of magnetizations in adjacent F and F' layers, we obtain eight different states \([\alpha_f\alpha_s\alpha_3\chi] \).
The expression for the Eilenberger functions in the common case $[\alpha_f \alpha_s \alpha_3 \chi]$ follows

$$
\Phi_f = \Phi_{f'} = e^{i\alpha_f} \Phi'_f = e^{i\alpha_s} \Phi_s = \frac{2 (c_f + c_{f'} e^{i\alpha_f}) \Delta_{f0} + 2 (c_s e^{i\alpha_s} + c_{s'}) \Delta_{s0}}{2\omega + i\Gamma},
$$

where $\Gamma$ is pair-breaking factor and $c_\zeta$ are the relative weights of layers, i.e

$$
\Gamma = (c_s + c_{s'}) q_\alpha v_{s\perp} + (c_f + e^{i\chi} c_{f'}) (2I + q_f v_{f\perp}), \quad c_\zeta = \frac{d_\zeta / v_{\zeta x}}{\sum_{\eta} d_\eta / v_{\eta z}}.
$$

Note, we obtained surprising result that the states for asymmetric infinite superlattice $[\alpha_f \alpha_s \alpha_3 \chi]$ coincide completely with states for finite asymmetric four-layered structure FSSF'S' $\{\alpha_f \alpha_s \alpha_3 \chi\}$. Because of this, we will below use the truncated four-layered classification $\{\alpha_f \alpha_s \alpha_3 \chi\}$.

The superlattice critical temperature $T_c$ in the common case is determined from integral equation, because the Eilenberger functions depend on the polar angles in velocity space,

$$
\ln \frac{T_c}{T_{cs}} = \left\{ \lambda_s \int \frac{d\Omega}{4\pi} \left[ \lambda_s (c_{s'} + e^{i\alpha_s} c_s) + \lambda_f (c_f + e^{i\alpha_f} c_{f'}) \right] \right\}^{-1} * 
$$

$$
\times \left\{ \int \frac{d\Omega}{4\pi} \left[ \lambda_s (c_{s'} + e^{i\alpha_s} c_s - 1) + \lambda_f (c_f + e^{i\alpha_f} c_{f'}) \right] + \lambda_s \int \frac{d\Omega}{4\pi} \left[ \lambda_s (c_{s'} + e^{i\alpha_s} c_s) + \lambda_f (c_f + e^{i\alpha_f} c_{f'}) \right] \right\} \Re \left[ \psi \left( \frac{1}{2} \right) - \psi \left( 1 + \frac{\Gamma}{4\pi T_c} \right) \right],
$$

where $\psi(x)$ is the digamma function and $T_{cs}$ is the critical temperature of the isolated S layer.

The analysis of the obtained equations for the critical temperature is very complicated. So, we present the essentially simplified expression for the case when all metals have the same electronic structure (i.e. there is component equality of the Fermi velocity of metals), then $c_\zeta = d_\zeta / (d_f + d_{f'} + d_s + d_{s'})$ and

$$
\ln \frac{T_c}{T_{cs}} = \frac{\lambda_s (c_{s'} + e^{i\alpha_s} c_s - 1) + \lambda_f (c_f + e^{i\alpha_f} c_{f'})}{\lambda_s \lambda_s (c_{s'} + e^{i\alpha_s} c_s) + \lambda_f (c_f + e^{i\alpha_f} c_{f'})} + \psi \left( \frac{1}{2} \right) - \Re \int \frac{d\Omega}{4\pi} \psi \left( 1 + \frac{\Gamma}{4\pi T_{cs}} \right).
$$

Note, that in symmetrical case which was examined in experiments for the Gd/La superlattice [9, 10] we have $d_f = d_{f'} \Rightarrow c_f = c_{f'}$ and $d_s = d_{s'} \Rightarrow c_s = c_{s'}$ then expression (19) is further simplified. Especially it takes place for the $\pi$-magnetic states ($\chi = \pi$) when the exchange fields $I$ and $-I$ of the adjacent F layers completely compensate each other in $\Gamma$. Then, from the condition of the maximum of $T_c$, we obtain $q = \Gamma = 0$ and the critical temperatures $T_c$ is determined only by the first term in Eq.(19).

Observed unexpected facts for the zero field cooled samples of the symmetric Gd/La superlattice with antiparallel mutual magnetizations orientation [9, 10] (an existence of $T_c$ for $d_f > d_s$, an independence of $T_c$ in regard to the $d_f/d_s$ and extremely high value equal to critical temperature of bulk La) can be completely explained by the our formula for the case $\{00\pi\}$

$$
\ln t = - \frac{c_f (\lambda_s - \lambda_f)}{\lambda_s (c_s \lambda_s + c_f \lambda_f)}.
$$

So, we see that $T_c$ coincides with $T_{cs} = T_{La}$ if $\lambda_f = \lambda_s$. The value $\lambda_s = \lambda_{La} \simeq 0.28$ can be estimated from $T_{La} \simeq 5$ K using the standard BCS expression [7].

Corresponding phase diagrams are shown in Fig. 1. Here we use that 2D momenta of FFLO
pairs $q_f$ and $q_s$ are identical ($q_f = q_s = q$) [6]. The straight bold black horizontal line (e) is calculated from Eq. (20).

For asymmetric case ($d_{f'}$ is fixed) the samples with varied F layer thickness $d_f$ are investigated by Eq. (19). The phase diagram $T_c(d_f)$ looks like as separate peak with re-entrant superconductivity. Both peaks shown in the lower panel in Fig. 1 are characterized by the FFLO-BCS-FFLO competition for various $d_f/d_s$ values. The dotted lines (b,d) correspond to the FFLO states with pair wave vector $q \neq 0$ (see the upper panel for the corresponding $q(d_f)$ dependencies). The solid color curves (a,c) correspond to the BCS state with $q = 0$. We see that the FFLO state with pair amplitude oscillation in the FS interface plane is favorable on peak wings.

In conclusion, the proximity effect theory for clean asymmetric FS superlattice is build. The unconventional experimental $T_c$ behavior of the short-range Gd/La superlattice [9, 10] is explained and the value of interelectronic interaction for the Gd is estimated. The separate re-entrant superconductivity for asymmetric FS superlattice is predicted (the Gd/La superlattice is best candidate for experimental checking this effect).

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