An analysis of problem solving ability in linear equation systems with two variables

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Abstract. Problem-solving is one of the mathematical abilities that students must possess. This study aims to analyze and describe students’ problem-solving abilities in the linear equation systems with two variables as consideration for improving the quality of learning. This type of research is descriptive qualitative. The subjects of this study were 20 tenth grade students of SMA Negeri 1 Comal. They were taken by using purposive sampling method. This research variable is an indicator of problem-solving ability that consists of understanding the problem, identifying and planning problem-solving, solving problems according to plan, and re-examining the answers generated. The data were collected by using tests and interview methods. The result showed that 73.5% of the students were successful in understanding the problems, 59.5% of the students were successful in identifying and planning problem-solving, 38% of the students were successful in solving problems according to plan, and 26% of the students were successful in re-examining the answers generated. Most students were not able to model mathematical forms correctly and have difficulty in solving problems.

1. Introduction
Mathematics plays an important role in the field of education [1]. However, the fact that mathematics is considered as a difficult subject to understand causes mathematics achievement to be low [2]. Student involvement influences their willingness to participate actively in various mathematical tasks, enjoying and being aware of the correlation between mathematics and their own lives [3]. Mathematics is studied and developed to equip students with the ability to think logically, analytically, systematically, critically, and creatively [4]. Therefore, the importance of mathematics to be learned.

Mathematics is a subject that requires a long reasoning in solving problems [5]. One way to understand how mathematical content in learning is to frame it in narrative form [6]. Guiding in contextual learning directs students to be able to solve mathematical problems, thus it is expected that students’ mathematical abilities can increase with increasing students' problem solving abilities.

Problem solving is a process for overcoming difficulties encountered to achieve the expected goals [7]. Problem solving is the actions and / or methods used by students to understand and solve problem situations [8]. The aim of learning mathematics, especially problem solving is to build new mathematical knowledge and to be able to understand the problem solving process and be able to apply various strategies in solving problems [9]. The ability to solve problems students are trained through problem solving because by solving problems, students can solve every problem in their lives [10]. Therefore the problem-solving ability must be present in the process of learning mathematics. Four activities in problem solving are as follows; (1) understanding of the problem, (2) planning the problem solving, (3) carrying out the plan, and (4) checking the solution [11].

This study aims to analyze and describe students' problem solving abilities in a system of linear equations with two variables as considerations to improve the quality of learning. Analysis of problem solving abilities in this study is based on modified indicators of problem solving ability.
2. Method
This type of research is descriptive qualitative. The subjects of this study were 20 tenth grade students of SMA Negeri 1 Comal. They were taken by using purposive sampling method. This research focuses on analyzing students' problem solving abilities in solving two-variable linear equation system problems. Indicators of problem solving ability in this study are understanding the problem, identifying and planning problem solving, solving the problem according to plan, and re-examine all the steps that are done. Indicator descriptions are explained in the following table:

| No | Indicators                                      | Description                                                                                                                                 |
|----|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | Understand the problem                        | a. Students are able to write what is known from the problem<br>b. Students are able to write what is asked of the problem                   |
| 2  | Identify and plan problem solving             | a. Students are able to identify existing problems and are able to convert them into their mathematical models                              |
| 3  | Resolve the problem according to plan         | a. Students are able to process calculations that have been identified previously<br>b. Students are able to answer what is the question of the problem |
| 4  | Re-examine all steps undertaken               | a. Students are able to review the answers that have been found so that it can be known the truth of the answers that have been found |

Triangulation is used to confirm the validity of the data by comparing test results and interviews. The data analysis phase in this study included (1) providing a test of students' problem solving abilities regarding a two-variable linear equation system, (2) organizing and analyzing test results, (3) conducting interviews with several students of various errors in each indicator and (4) analyzing Interview result.

3. Results and Discussion
3.1. Results
Results of analysis of student test answers by using indicators of problem solving among 20 tenth grade students of SMA Negeri 1 Comal. The results showed that the percentage of students who mastered the indicators understood the problem by 73.5%, identified and planned solutions to the problem by 59.5%, solved the problem according to the plan by 38%, and re-checked all the steps taken by 26.5% . A summary of the results is shown in Table 2.

| Indicators                                      | Problem 1 | Problem 2 | Percentage |
|------------------------------------------------|------------|------------|------------|
| Understand the problem                        | 78         | 69         | 73.5       |
| Identify and plan problem solving             | 74         | 45         | 59.5       |
| Resolve the problem according to plan         | 50         | 27         | 38         |
| Re-examine all steps undertaken               | 34         | 18         | 26         |

Table 2 shows 2 problems to be analyzed based on indicators of problem solving ability. In Table 2, it can be seen that the percentage of students' success in understanding a problem is 73.5%, which means that most students are able to mention the problems that are known and asked of these problems. The percentage of students' success in identifying and planning problem solving is 59.5%, which means that most students are quite capable of modeling the mathematical form of a problem. The percentage of students' success in solving problems according to plan is 38%, which means that
most students are less able to operate the calculations so that questions from existing problems cannot be answered. The percentage of students' success in re-checking all the steps taken was 26.5% which means that most students were unable to prove the answers they had found or they were already sure of the answers they had found.

3.1.1. Analysis Problem Solving Ability of Problem 1

**Figure 1. Problem 1**

The first indicator in problem 1 in this study shows the percentage of students' success in answering correctly is 78% which means that most students are able to mention the problems that are known and asked of these problems. The following Figure 2 is the student test result to the first indicator.

**Figure 2. Test's result of S1 first indicator on problem 1**

S1 is one of the subjects who gets the lowest score on the first indicator of problem 1. Figure 2 shows that S1 does not write the statement that is known and asked correctly. Based on the results of interviews with S1, it was concluded that S1 did not write down exactly what was known and asked because S1 did not absorb information correctly based on question 1 and S1 considered trivial steps 1 and step 2.

The second indicator in problem 1 in this study shows the percentage of students' success in answering correctly is 74%, which means that most students are able to model the mathematical form of a problem. The following Figure 3 is an example of student work outcomes related to the second indicator.

**Figure 3. Test's result of S2 second indicator on problem 1**

Budi has a chicken and goat farm in his house. One day Budi got his father’s order to label the legs of his livestock. Budi got 100 labels from his father to mark all the legs of his farm animals. After all the feet were labeled on his feet, apparently there were still 28 of them. If we know the number of Budi's chickens there are 6 more than his goats. How many chickens and goats does Budi have?

Step 1: Write the known from the problem!
Label = 100
Remaining = 28
There are 6 chickens more than goats
Step 2. Write the asked about the problem!
Lots of chickens and goats
Step 3: Write a mathematical model based on the problem!
Example:
chicken = x
goat = y
mathematical model:
\[ 2x + 4y = 72 \quad \cdots (1) \]
\[ x + 6 = y \quad \cdots (2) \]
S2 is one of the subjects who gets the lowest score on the second indicator of problem 1. Figure 3 shows that S2 was not successful in modeling the mathematical form of problem 1. Based on the results of interviews with S2, it was concluded that S2 experienced a concept error in step 3 in modeling the shape mathematics of problem 1.

The third indicator in problem 1 in this study shows the percentage of students' success in answering correctly is 50%, which means some students are quite capable of operating calculations so that questions from existing problems cannot be answered. The following Figure 4 is the student test result to the third indicator.

S3 is one of the subjects who gets the lowest score on the third indicator of problem 1. Figure 4 shows that S3 was wrong in answering step 4 and step 5 because the model found by S3 in the previous step showed an incorrect answer. Based on the results of interviews with S3, it was concluded that S3 was unaware of the error in the previous step which resulted in mistaken work in step 4 and step 5.

The fourth indicator in problem 1 in this study shows the percentage of students' success in answering correctly is 34% which means that most students are unable to prove the answers they have found or they already feel sure of the answers they have found. The following Figure 5 is the student test result to the fourth indicator.

S4 is one of the subjects who gets the lowest score on the fourth indicator of problem 1. Figure 5 shows that S4 did not answer step 6. Based on the results of interviews with S4, it was concluded that S4 did not know how to check the answers that were found so S4 did not write the answer in step 6.

Step 4: Write the calculation process using the settlement method that you understand!
Substitution equation (2) to equation (1)
\[2x + 4(x + 6) = 72\]
\[2x + 4x + 24 = 72\]
\[6x = 96\]
\[x = 16\]
Substitution \(x = 16\) to equation (2)
\[16 + 6 = y\]
\[y = 22\]
Step 5: Write the answer to the problem!
So, \(x = 16\) dan \(y = 22\)

**Figure 4.** Test’s result of S3 third indicator on problem 1

Step 6: Write the results of the settlement that you get then substitute it into a predetermined equation!

**Figure 5.** Test’s result of S4 fourth indicator on problem 1
3.1.2. Analysis Problem Solving Ability of Problem 2

One day Irma was asked to help her mother to shop at the market in front of her house complex. The goods purchased by Irma are a quarter kilo of sugar and half a kilo of chicken eggs with a total price to be paid by Irma in the amount of Rp. 10,000.00. Two weeks later Irma was again asked to help her mother to shop at the same market as before for tomorrow's social gathering needs. The goods purchased by Irma were a little more than usual, namely two kilos of granulated sugar and four kilos of chicken eggs with a total price of Rp. 100,000.00. Can you determine the price of sugar and chicken eggs per kilogram that Irma bought?

**Figure 6. Problem 2**

The first indicator in problem 2 in this study shows the percentage of students' success in answering correctly is 69% which means that most students are able to mention the problems that are known and asked of these problems. The following Figure 7 is the student test result to the first indicator.

**Figure 7. Test’s result of S5 first indicator on problem 2**

S5 is one of the subjects who gets the lowest score on the first indicator of problem 2. Figure 7 shows that S1 does not write the statement that is known and asked correctly. Based on the results of interviews with S1, it was concluded that S1 did not write down exactly what was known and asked because S1 did not absorb information correctly based on questions 2 and S5 underestimated step 1 and step 2.

The second indicator in problem 2 in this study shows the percentage of students' success in answering correctly is 45%, which means that most students are unable to model the mathematical form of a problem. The following Figure 8 is the student test result to the second indicator.

**Figure 8. Test’s result of S6 second indicator on problem 2**
S6 is one of the subjects who gets the lowest score on the second indicator of problem 2. Figure 8 shows that S6 was not successful in modeling the mathematical form of problem 2. Based on the results of interviews with S6, it was concluded that S6 experienced a concept error in step 3 in modeling the shape the mathematics of problem 2. S6 is not successful in the concept of simplification in an equation so that the answers obtained are not correct.

The third indicator in problem 2 in this study shows the percentage of students' success in answering correctly is 27%, which means that most students are unable to operate the calculations so that questions from existing problems cannot be answered. The following Figure 9 is the student test result to the third indicator.

![Figure 9. Test's result of S7 third indicator on problem 2](image)

S7 is one of the subjects who gets the lowest score on the third indicator of problem 2. Figure 9 shows that S7 only writes a mathematical model and does not continue the calculation process in step 4, while step 5 S7 does not answer the problem. Based on the results of an interview with S7, it was concluded that S7 did not master the calculation of fractions and did not master the completion of the material being tested.

The fourth indicator on problem 2 in this study shows the percentage of students' success in answering correctly is 18% which means that most students are unable to prove the answers they have found or even they are not working on the problem on the fourth indicator. The following Figure 10 is the student test result to the fourth indicator.

![Figure 10. Test’s result of S8 fourth indicator on problem 2](image)

S8 is one of the subjects who gets the lowest score on the fourth indicator of problem 2. Figure 10 shows that S8 did not answer step 6. Based on the results of interviews with S8, it was concluded that S8 did not know how to check the answers that were found so S8 did not write the answer in step 6.

3.2. Discussion

Based on the description of some subjects taken based on the lowest score for each indicator of problem solving ability, there are several indicators of problem solving ability that are not yet fully mastered by students. Evidently there are several indicators of problem-solving ability in which the percentage of successes in answering correctly is still relatively low.

In the indicator of understanding the problem, S1 and S5 are able to process information from the existing problems appropriately, although not entirely optimal. Evidenced by the results of their work that does not mention what is known and asked of the problem correctly perfectly. In fact S5 only writes numbers of problems as a form of a linear equation.
In indicators identifying and planning problem solving, S2 and S6 are less able to model problems. They both are wrong in modeling the problem. The error occurred because they were confused in turning a problem into a mathematical form. Even S6 is wrong in modeling the problem by simplifying the equation directly so that the problem looks simpler which is actually a mistake.

On the indicator of solving the problem according to plan, S3 and S7 are not able to answer correctly, because the mathematical model that was done in the previous step was wrong first so it made the calculation operation step in the problem wrong. Even S7 does not operate mathematical models that are formed due to the inability to operate mathematical models that are formed.

On the indicator of reexamining all the steps carried out, S4 and S8 were unable to complete the commands given because they were not familiar with the step of checking the answers so they did not know how to do the steps. They both chose to leave the answer blank at this step due to their inability to complete the order.

Based on the results of the analysis of problem solving abilities for both problems, it was found that some students have been able to understand and identify existing problems. However, most students are still unable to solve and check answers to problems. Most students are not able to operate the calculations based on existing mathematical models and most students are not able to re-check the completion answers obtained because of students' understanding of the form of the order. One factor that affects is the way students think. The teaching patterns so far only provide examples and practice questions, but rarely in the form of story problems [12]. Therefore students have difficulty in forming mathematical models of problems that will be useful in operating calculations and the majority of students do not know and are not accustomed to what to do in the step of re-checking the answer to completion [13].

To overcome this problem, several attempts must be made by the teacher to improve students' problem solving abilities. Because developing gradually problem solving skills is the main goal of education in schools [14]. One effort that must be made is to link learning with contextual problems. This is in line with [15,16], learning strategies using contextual contexts are one of the learning strategies that can be used to improve students' understanding of concepts in the problem solving process. This is in line with [17,18], that contextual learning strategies provide a positive impact on students. Provision of learning using contextual contexts, will train students to process information needed to solve problems.

4. Conclusion
The results of the analysis explain the problem-solving ability of tenth grade students in SMA Negeri 1 Comal on the material system of two-variable linear equations shows that the percentage of students 'success in understanding the problem is 73.5%, the percentage of students' success in identifying and planning problem solving is 59.5%, the percentage of success students in solving problems according to plan is 38%, and the percentage of students' success in re-checking all the steps done is 26%.

The lack of student abilities in solving problems is students' lack of skill in processing information from problems, students' inability to change problems into mathematics, operating calculations of problems, and lack of student knowledge of proof of answers to the problems obtained. Therefore teachers should to provide learning in contextual contexts so that students are accustomed to gathering and processing information that is on the problem. And the teacher is should to give motivation to students on the importance of proving the truth of the results obtained before moving on to the next problem.

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