Generation of slow intense optical solitons in a resonance photonic crystal

Igor V. Mel’nikov¹, Anton Knigavko¹,², J. Stewart Aitchison³, and Clark A. Merchant³

¹ High Q Laboratories, Inc., 2 Gledhill Cres., Hamilton, Ontario L9C 6H4, Canada
² Department of Physics, Brock University, 500 Glenridge Ave., St. Catharines, Ontario L2S 3A1, Canada
³ Department of Electrical and Computer Engineering, University of Toronto, 10 King’s College Rd., Toronto, Ontario M5S 3G4, Canada

Received: date / Revised version: date

Abstract. We demonstrate interesting and previously unforeseen properties of a pair of gap solitons in a resonant photonic crystal which are predicted and explained in a physically transparent form using both analytical and numerical methods. The most important result is the fact that an oscillating gap soliton created by the presence of a localized population inversion inside the crystal can be manipulated by means of a proper choice of bit rate, phase and amplitude modulation. Developing this idea, we are able to obtain qualitatively different regimes of a resonant photonic crystal operation. In particular, a noteworthy observation is that both the delay time and amplitude difference must exceed a certain level to ensure effective control over the soliton dynamics.

PACS. 42.70.Qs Photonic bandgap materials – 42.65.Tg Optical solitons; nonlinear guided waves – 42.50.Md Optical transient phenomena – 03.65.Ge Solutions of wave equations: bound states

1 Introduction

There has been an extensive amount of research focused on the study on nonlinear periodic systems and in particular those described by the sine-Gordon equation. This is primarily due to a large and diverse range of physical phenomena which can be described by the soliton solutions of this equation. Resonance photonic crystals (RPhC) represent one such system where the sine-Gordon equation can be used to describe the propagation of light. Periodicity in photonics results in bandgaps, or regions in time or space, where the propagation of energy is forbidden [1,2,3]. The addition of nonlinearity to the system allows the possibility of locally detuning these band gaps with the result that energy can propagate in the form of a solitary wave, or gap soliton (GS) [4,5,6,7]. Such gap solitons have been experimentally observed in fiber Bragg gratings [8] and more recently in AlGaAs waveguides [9,10,11]. These experiments relied on the non-resonant, Kerr nonlinearity to locally detune the stop-bands associated with the periodic structure.

A recent resurgence of interest in photonics crystals with resonant nonlinearities is due to the advances in the manufacturing of periodic Bragg semiconductor nanostructures such as InGaAs/GaAs multiple quantum wells and rare-earth doped AlGaAs/GaAs structures [12,13,14], and understanding their linear and nonlinear optical properties [15,16,17]. This suggests that the building blocks of future all-optical processing systems, such as those based on the propagation of GSs might be experimentally feasible at moderate optical power levels of about 10 MW/cm². The inclusion of a gain or a local population inversion in the system opens up a range of additional, novel nonlinear effects [18].

Recently, we proposed an effective method for the control of speed of gap solitons based on the defect associated with a weak linear excitation, or incoherent pump inside an RPhC [19,20,21,22]. It turns out that a break in periodicity of the RPhC is not necessary for slowing a gap soliton down and its trapping because the localized field creates a potential that acts as a phase-sensitive trap. The GS interaction at such defect was first studied to determine the gain implication for slow light localization in the RPhC [19]. More recent work was focused on the effects of localized gain on the GS collision and comprehensive theory was elaborated [20,21]. Light trapping at the gain inside an RPhC with subsequent release due to the collision with another GS have also been predicted and corresponding memory device was also suggested [22]. In this paper, we look at the two separate effects associated with a localized population inversion inside RPhCs: higher-order soliton dynamics in the presence of such gain defects and generation of the gain defect itself. This paper is subdivided as follows. In Sect. ² the bi-directional Maxwell-Bloch model is formulated and analytical results are given for the exact
2 Analytical Formalism

In order to explore the basic features of interaction of resonant gap solitons in the presence of a localized gain or defect inside the photonic crystal we consider the simplest cases where the defect is created in the middle of the RPC using an incoherent pump with square hump-like spatial profile.

2.1 Coupled-Mode Equations

The 1D resonant photonic crystal is assumed to comprise of periodically positioned thin layers of two-level atoms. The electric field that appears in the Maxwell’s equations and couples the two levels is taken in the form of a superposition of the forward, \( E^+ \), and backward-propagating wave, \( E^- \), as follows:

\[
E(x, t) = \frac{1}{2} \left[ E^+(x, t) e^{i(kx - \omega t)} + E^-(x, t) e^{-i(kx + \omega t)} \right].
\]  

The direction of propagation is normal to the layers. The incident radiation has a wavelength \( \lambda \) which exactly satisfies the Bragg condition \( d = \lambda \), where \( d \) is the RPC period. We also assume that the field described by Eq. (1) is tuned into the exact resonance with atomic absorption of the two-level system (which can represent atoms, ions, or excitons). In this case the dynamics of a two-level system, can be described by the generalized Bloch equations as follows:

\[
P_i(x, t) = \mp \left[ \Omega^+(x, t) e^{i k_x x_i} + \Omega^-(x, t) e^{-i k_x x_i} \right],
\]

\[
n(x, t) = -\text{Re} \left\{ \bar{P}^* (x, t) \left[ \Omega^+(x, t) e^{i k_x x_i} 
\right.ight.
\]

\[
\left. + \Omega^-(x, t) e^{-i k_x x_i} \right] \right\},
\]

where \( \Omega^\pm = 2\tau_c \mu E^\pm/h \) with \( E^\pm \) from Eq. (1) and \( \tau_c = (8\pi c T_i / 3c \rho \lambda^2)^{1/2} \) is the cooperative time that is the mean photon lifetime in the structure, \( T_i \) is the excited state lifetime, \( c \) is the dielectric constant of the medium, \( \rho \) is the density of the resonant atoms, \( \mu \) is the transition matrix element, \( x_i \) is the position of the \( i \)-th resonant layer, \( c \) is the speed of light in vacuum, and the subscript \( t \) indicates the corresponding derivative. In turn, the resonant polarizations \( P \) defines the dynamics of the optical field \( \Omega^\pm \) inside the medium via the set of the two-mode equations:

\[
\Omega^\pm_t(x, t) \pm \Omega^\pm_x(x, t) = \sum_i \bar{P}^* (x, t) \delta(x - x_i),
\]

which after averaging over the structure period forms the closed set with Eq. (2) and Eq. (3). This is shown here as:

\[
\Omega^\pm_t + \Omega^\pm_x = P_t,
\]

\[
P_t = \mp \left[ \Omega^+(t) + \Omega^-(t) \right]
\]

\[
n_{t} = -\text{Re} \left\{ \bar{P}^* \left( \Omega^+ + \Omega^- \right) \right\},
\]

where the subscripts \( x \) and \( t \) stand for the corresponding partial derivative along the dimensionless time and propagation coordinate \( t = t/\tau_p \) and \( x = x/(c \tau_c) \). As input values, we use the following boundary and initial conditions:

\[
\Omega^+_t(x = 0, t) = \sum_{i=1}^2 \Omega_i \text{sech}(t - t_i)/\tau_p,
\]

\[
\Omega^- (x = L, t) = 0,
\]

\[
\Omega^+_x(x = 0, t) = 0,
\]

\[
P(x, t = 0) = 0,
\]

\[
n(x, t = 0) = \begin{cases} 1, & |x - L/2| \leq \ell/2; \\ -1, & |x - L/2| \geq \ell/2, \end{cases}
\]

where \( \tau_p \) is the normalized duration of the incident pulse. Eqs. (5)–(7) together with Eqs. (8)–(12) fully define the system under consideration.

2.2 Lagrangian and mechanical analogy

The invariable quantity of Eqs. (5)–(7) is accessible via a transformation to the Bloch angle \( \theta(x, t) \), the sum \( U = \Omega^+ + \Omega^- \) and the difference \( V = \Omega^+ - \Omega^- \), which leads to the following set of equations:

\[
U_t + V_x = -2 \sin \theta,
\]

\[
V_t + U_x = 0,
\]

\[
\theta_t = U.
\]

An invariable quantity \( \Gamma(x) \) associated with this set of equations can be obtained by substituting Eq. (15) into Eq. (14) and results in

\[
\Gamma(x) = V(x, t) + \theta_x(x, t).
\]

This conservation parameter is defined by the initial conditions and does not change as the pulse propagates through the RPC. However, it does describe the behavior of the GS in the sense that setting it equal to zero provides a GS whereas, a non-zero value of \( \Gamma(x) \) leads to a loss of stability of the GS and can result in trapping of the GS inside the RPC. It is readily seen that the substitution of the invariant function described by Eq. (16) into Eq. (13) transforms it into the perturbed Sine-Gordon equation for the Bloch angle \( \theta \) shown as:

\[
\theta_{xx} - \theta_{tt} = 2 \sin \theta + \Gamma_x.
\]

In order to elucidate the interaction between the exact GS and defect mode that is due to the localized gain, we apply the approach developed earlier in [23] where the coordinate of the GS center-of-mass \( (t) \) can be derived from a corresponding Lagrangian density:

\[
L = \frac{1}{2} \theta_t^2 - \frac{1}{2} (\theta_x - \Gamma)^2 - (1 - \cos \theta),
\]

with corresponding density of the Hamiltonian

\[
H = \frac{1}{2} \theta_t^2 + \frac{1}{2} \theta_x^2 - \Gamma \theta_x + \frac{1}{2} \Gamma^2 + (1 - \cos \theta),
\]
where the first four terms on the right-hand side define the energy density $(U^2 + V^2)/2$ of the forward and backward waves inside the RPhC.

Assume now that the solution of Eq. (17) can be approximated by that of the non-perturbed sine-Gordon equation:

$$\theta(x, t) = 4 \arctan \left( \exp \left[ \frac{-x + \zeta(t)}{\sqrt{1 - u^2(t)}} \right] \right), \quad (20)$$

where $\zeta(t)$ and $u(t)$ are the time-dependent coordinate of the GS center and velocity, correspondingly. Since the total energy of the localized solution is an invariable quantity, the energy of the interaction of the soliton of Eq. (20) with perturbation created by the localized gain, is defined by the overlap integral \cite{20,21,22,23}:

$$\Phi = \frac{1}{4} \int_{-\infty}^{+\infty} dx \Gamma_x \theta_x, \quad (21)$$

which in turn describes the GS propagation across the RPhC with a length of the gain inside as a classical motion of the particle with unit mass,

$$\zeta_{tt} = -\Phi \zeta. \quad (22)$$

Our initial conditions of the gain localized in the middle of the RPhC, namely:

$$\theta(x, t = 0) = \begin{cases} 0, & 0 \leq x < (L - \ell)/2, \\ \pi, & (L - \ell)/2 \leq x \leq (L + \ell)/2, \\ 0, & 0 \leq (L + \ell)/2 \leq x \leq L, \end{cases} \quad (23)$$

along with the absence of the propagating waves in the beginning, generates the following potential:

$$\Phi \sim -\text{sech} \left[ x - (L + \ell)/2 \right] + \text{sech} \left[ x - (L - \ell)/2 \right]. \quad (24)$$

The profile of this potential and corresponding phase trajectories are depicted in Fig. 1; it is readily seen that if the velocity of the particle (pulse) is considerably above the potential zero level, it experiences some minor variations while traversing the potential, or gain bump in our case. Lowering the velocity shifts the phase trajectory into vicinity of the saddle point (not shown in Fig. 1b) and the potential becomes repelling that implies a blockage of the GS free transport across the RPhC. Next, if the velocity is small enough, the particle experiences aperiodic oscillations inside the potential well on the left. That is, the slower GS is trapped by the gain and oscillates around the gain center, it is unsteady but stable. It is worth noticing that the energy gain of the GS that is due to the gain of Eq. (23) along with a modification of this gain by the GS can change the dynamics of the interaction to a very large extend.

### 3 Calculated Behavior and Discussion

The analysis of the previous Section is now used to explore the behavior of gap solitons in the presence of a
defect mode that is due to the total population inversion at the centre of the sample. The class of interactions we are concerned with here are those that produce a light stopping and release operation which can be exploited in order to achieve a memory cell with high-speed switching at a moderate consumption of the laser power.

We have previously shown that it is possible to trap a gap soliton on the potential of Fig. 1 [19,21]. Figure 2 shows a contour plot in the \((x,t)\)-plane, depicting the local inversion and the propagation and subsequent trapping of a \(2\pi\) pulse. We call the GS that is stopped and oscillating at such defect an information bit. It is important to note that only such stopped solitons can be used to switch subsequent pulses providing either true memory readout, higher-density storage, or limiting action so providing a secure readout. This trapping represents a basic optical memory whereby the GS is stored in the RPhC. However, in order to demonstrate its usefulness we need to investigate the read-out or release of information from such a memory device.

In Figs. 3a and 3b we show that a second \(2\pi\) gap soliton can be used to release the stored pulse and effectively read out the optical memory. The soliton readout happens owing to the collision of the out-of-phase pulse with the trapping area. Our simulations show that the soliton can be released by the readout pulse as long as the readout GS is timed to arrive when the trapped soliton is at its maximum deviation towards the front side of the defect. As the readout soliton arrives it interacts with the trapped pulse and is dragged into the defect. The trapped and readout solitons cross the center of the defect together and have sufficient energy to overcome the potential and are released. Naturally, the released pulse is a higher order breather (see Fig. 3c) and it propagates with a higher velocity through the resonant photonic crystal.
From this argument it is clear that if the readout GS is not synchronized with the trapped soliton then they can combine at the defect and can both be trapped. We illustrate this effect in Fig. 4, where the out-of-phase soliton used to trigger the read-out arrives while the trapped one is at its turning point on the opposite side of the defect. The resulting interaction causes the solitons to arrive at the center of the inversion region from opposite sides. The result is that the solitons combine to form a trapped mode at the inversion. This corresponds to a higher order trapped mode as can be seen from the higher frequency structure on the soliton envelope. In order to implement a practical optical memory it is essential that the solitons be timed correctly. This implies that the strength of the inversion in the photonic crystal should be such that the period of oscillation of the trapped mode is the same as the bit rate of the data stream. If this condition is not satisfied, the merging breather-like state turns out to be unstable, dissociates into a soliton trapped to the inversion and the read-out pulse rejected backward - a read-out failure. This observation (which is not shown in Figures) is extremely important from the information security standpoint. That is, since the oscillation period and amplitude determine the open time of the memory cell a pulse with the wrong phase cannot take away the information bit.

Since the process of the memory read-out is due to the energy imbalance between interacting solitons and the trapping defect, it is natural to assume that trapping and release of a gap soliton may also be controlled by means of its interaction with an in-phase, and more intense pulse. As a relevant example, Fig. 5 shows the results of numerical integration of the Maxwell-Bloch equations, Eqs. (5)–(7) for the case of interaction of two solitons with different input amplitudes. Above a certain threshold, the trailing GS with larger energy permits the escape of the GS from the trapping area (Fig. 5a). This is due to the repulsive action of the read-out pulse that is stronger than the trapping action provided by the defect. What might be interesting from a potential device feasibility standpoint is that the released soliton emerges the RPhC in the backward direction. Again, this happens only for a certain bit rate of the data stream. The readout failure is demonstrated in Fig. 5b and 5c. Here, the speed of the released light is so low that it practically creates another defect inside the crystal so rendering the original one available for the information storage.

The other important finding that follows from Fig. 5 is the existence of a stabilization of a higher-order solution to the perturbed sine-Gordon equation, Eq. (17), due to the presence of a local perturbation. This is in sharp contrast to the well-known fact of the breakup of the $2\pi n$-pulse (with $n$ being an integer) of the self-induced transparency into a train of isolated $2\pi$ pulses with different amplitudes, speeds, and durations [24].

We conclude this Section with indication of an alternative way of stopping light inside a RPhC. Contrary to the initial conditions of Eqs. (8)–(12) let us now assume that the gain (or total inversion of the atoms) is not squeezed in the middle of the sample but is distributed from the input facet at certain distance inside. Using initial conditions pertinent to spontaneous decay [25,26], also leads to interesting results. This is summarized in Fig. 6. The localized field oscillating around the physical boundary between the pumped and unpumped parts of the RPhC is clearly visible.

**4 Conclusions**

The results presented in this paper are based on studying gap solitons in resonance photonic crystals. However, the sine-Gordon equation can also be used to describe a broad and diverse range of physical systems; for example, the dissociation of the breather solution of the sine-Gordon equation that is caused by a perturbation of a long Josephson junction [27,28,29,30,31,32]. The problem of the stability of the perturbed sine-Gordon equation also results in the ratchet effect [33,34], the high-frequency field interaction with electron plasmas in a semiconductor superlattice [35], and the spatial localization in a nonlinear chain [36]. We would therefore expect that similar effects of energy stor-
In conclusion, we have demonstrated that the oscillating immobile gap solitons created by the presence of a localized region of the population inversion (or gain) inside a resonance photonic crystal can be manipulated by a proper choice of bit rate, phase and amplitude modulation. Developing this idea, we are able to obtain qualitatively different regimes of the RPhC operation. A noteworthy observation is that both the delay time and amplitude difference must exceed a certain level to ensure effective control over the entire soliton dynamics and its speed, in particular. A modification of the defect that accomplishes a soliton slowing-down and trapping can make the dynamics of soliton trains in the resonance photonic crystal with defects even more interesting. This comprises a subject of our future work.

5 Acknowledgements

We acknowledge helpful discussions with J. Band, S. Flach, W. Hoyer, M. Kira, G. Kurizki, T. Meier, T. Stroucken, and M. I. Tribelsky. This work was supported in parts by National Science and Engineering Research Council of Canada, Photonics Research Ontario, and Max-Planck-Gesellschaft.

References

1. V.P. Bykov, Sov. Phys. JETP 35, (1972) 269-273; V.P. Bykov, Sov. J. Quant. Electron. 4, (1975) 861-871.
2. E. Yablonovitch, Phys. Rev. Lett. 58, (1987) 2059-2062.
3. J. A. John, Phys. Rev. Lett. 58, (1987) 2486-2489.
4. W. Chen and D. L. Mills, Phys. Rev. Lett. 58, (1987) 160-163.
5. B. I. Mantsyzov, Phys. Rev. A 51, 4939-4943 (1995).
6. A. Kozhekin and G. Kurizki, Phys. Rev. Lett. 74, 5020-5023 (1995).
7. C. Conti, S. Trillo, and G. Assanto, Phys. Rev. Lett. 78, 2341-2344 (1997).
8. B. J. Eggelton, R. E. Slusher, C. M. de Sterke et al., Phys. Rev. Lett. 76, 1627-1630 (1996).
9. P. Millar, M. De La Rue, T. F. Krauss et al., Opt. Lett. 24, 685-687 (1999).
10. D. Mandelik, H. S. Eisenberg, Y. Silberberg et al., Phys. Rev. Lett. 90, 053902 (2003).
11. J.P. Prineas, C. Ell, E.S. Lee et al., Phys. Rev. B 61, 13863-13872 (2000).
12. J.P. Prineas, J.Y. Zhou, J. Kuhl et al., Appl. Phys. Lett. 81, 4332-4334 (2002).
13. O.B. Gusev, J.P. Prineas, E.K. Lindmark et al., J. Appl. Phys. 82, 1815-1823 (1997).
14. N. C. Nielsen, J. Kuhl, M. Schaarschmidt et al., Phys. Rev. B 70, 075306 (2004).
15. M. Schaarschmidt, J. Forstner, A. Knorr et al., Phys. Rev. B 70, (2004) 233302.
16. T. H. Z. Siederdissen, N. C. Nielsen, J. Kuhl et al., Opt. Lett. 30, 1384-1386 (2005).
17. M. Boroditsky, T. F. Krauss, R. Caccioli et al., Appl. Phys. Lett. 75, 1036-1038 (1999).
18. W. M. Mak, B. A. Malomed, and P. L. Chu, JOSA B 20, 725-735 (2003); Phys. Rev. E 67, 026608 (2003).
19. I. V. Mel'nikov, J. S. Aitchison, and B. I. Mantsyzov, Opt. Lett. 29, 289-291 (2004).
20. B. I. Mantsyzov, I. V. Mel'nikov, and J. S. Aitchison, Phys. Rev. E 69, 055602(R) (2004).
21. B. I. Mantsyzov, I. V. Mel'nikov, and J. S. Aitchison, IEEE J. Sel. Top. Quant. Electron. 10, 893-899 (2004).
22. I. V. Mel'nikov and J. S. Aitchison, Appl. Phys. Lett. 87, 201111 (2005).
23. B. I. Mantsyzov and R. A. Silnikov, JETP Lett. 74, 456-459 (2001); JOSA B 19, 2203-2207 (2002).
24. L. Allen and J. H. Eberly, Optical Resonance and Two-level Atoms (J. Wiley and Sons, New York, 1975) chapter 4, §4.
25. I. V. Mel'nikov, Phys. Rev. Lett. 77, 842-845 (1996).
26. I. V. Mel'nikov, J. S. Aitchison, and J. W. Haus, Optics Commun. 244, 279 (2005); IEEE J Select Top Quant Electron 12, 1135-1142 (2006).
27. J. P. Keener and D. W. McLaughlin, Phys. Rev. A 16, 777-790 (1977).
28. D. W. McLaughlin and A. C. Newell, Proc. R. Soc. Lond. A 361, 413-446 (1978).
29. D. W. McLaughlin and A. C. Scott, Phys. Rev. A 18, 1652-1680 (1978).
30. M. Inoue, J. Phys. Soc. Jpn. 47, 1723-1731 (1979).
31. V. I. Karpman, E. M. Maslov, and V. V. Solov'ev, ZhETP 84, 289 (1983) [in Russian].
32. M. V. Fistul, A. Wallraff, Y. Koval et al., Phys. Rev. Lett. 91, 257004 (2003).
33. M. O. Magnasco, Phys. Rev. Lett. 71, 1477 (1993).
34. M. Salerno and N. R. Quintero, Phys. Rev. E 65, 025602 (2002).
35. S. V. Kryuchkov and E. G. Fedorov, Opt. Spectr. 94, 225-229 (2003).
36. F. Geniet and J. Leon, Phys. Rev. Lett. 89, (2002) 134102.