Direction Dependence of the Deceleration Parameter

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Abstract

In this paper we study the possibly existing anisotropy in the accelerating expansion Universe by use of the full sample of Union2 data. Using the hemisphere comparison method to search for a preferred direction, we take the deceleration parameter \( q_0 \) as the diagnostic to quantify the anisotropy level in the \( \Lambda \)CDM model. We find that the maximum accelerating expansion direction is \((l, b) = (314^\circ_{+13^\circ}^{+20^\circ}, 28^\circ_{+33^\circ}^{+11^\circ})\), with the maximum anisotropy level of \( \Delta q_{0,\text{max}}/q_0 = 0.79_{+0.28}^{-0.28} \), and that the anisotropy is more prominent when only low redshift data \((z \leq 0.2)\) are used. We also discuss this issue in the CPL parameterized model, showing a similar result.

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1. INTRODUCTION

Since the discovery of cosmic acceleration in the late of last century [1], the type Ia supernovae (SNIa) have become an important tool in determining the cosmological parameters. Given certain cosmological models, one usually implements the joint analysis with SNIa in combination with other observations, such as large scale structure [2], the cosmic microwave background (CMB) radiation [3], and so on, to give constraints of cosmological parameters.

Two pillars of modern cosmology are general relativity and the cosmological principle. It is assumed that Einstein’s general relativity still holds on the cosmic scale, which means that the evolution of the universe is governed by general relativity. The cosmological principle [4] says that our universe is homogeneous and isotropic on the cosmic scale. Indeed, the assumption of homogeneity and isotropy is consistent with currently accurate data coming from the cosmic microwave background (CMB) radiation, especially from the Wilkinson Microwave Anisotropy Probe (WMAP) [5], the statistics of galaxies [6], and the halo power spectrum [7], etc. And current astronomical observations are in agreement with \( \Lambda \)CDM model [8].

However, the standard model is also challenged by some observations [9] (for more details see [10] and references therein). Thus it is necessary to revisit the assumption of the homogeneity and isotropy. As more and more supernovae data are released [11, 12], this study becomes possible.

From the theoretical point of view, the anisotropy may arise in some cosmological models. For example, a vector field may lead to anisotropy of the universe and gives rise to an anisotropic equation of state of dark energy [13]. Peculiar velocities are also associated with dipole-like anisotropies, triggered by the fact that they introduce a preferred spatial direction, and as a result of the drift motion, observers may find that the acceleration is maximized in one direction and minimized in the opposite [14]. By use of 288 SNIa [15], Davis et al. studied the effects of peculiar velocities on cosmological parameters, including our own peculiar motion, supernova’s motion and coherent bulk motion, and found that neglecting coherent velocities in the current sample would cause a systematic shift in the equation of state of dark energy, with deviation \( \Delta w = 0.02 \) [16]. In this paper, we focus on the issue related to the possible existence of anisotropy, and search for a preferred direction by using the SNIa Union2 (consisting of 557 SNIa) data [1]. Some previous works payed attention to this issue using the SNIa data and found no statistically significant evidence for anisotropies [17]. Using the Union2, some authors derived the angular covariance function of the standard candle magnitude fluctuations, searching for angular scales where the covariance function deviates from 0 in a statistically significant manner, and no such angular scale was found [18]. Yet on the other hand, using the SNIa data in the framework of an anisotropy Bianchi type I cosmological model and in the presence of a dark energy fluid with anisotropic equation of state, Ref. [19] found that a large level of anisotropy was allowed both in the geometry of the universe and in the equation of state of dark energy. Ref. [20] constructed a “residual” statistic, which is sensitive to systematic shifts in the SNIa brightness, and used this to search in different slices of redshift for a preferred direction on the sky, and found that at low redshift

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1 http://supernova.lbl.gov/Union/.
(z < 0.5) an isotropic model was barely consistent with the SNIa data at 2-3σ. In addition, Ref. [21] took use of the hemisphere comparison method to fit the ΛCDM model to the supernovae data on several pairs of opposite hemispheres, and a statistically significant preferred axis was found. More recently, Antoniou and Perivolaropoulos [22] have applied the hemisphere comparison method to the standard ΛCDM model and found that the hemisphere of maximum accelerating expansion is in the direction (l, b) = (309°, +23°, 18°, +10°) with Union2 data. This result is consistent with other observations, such as CMB dipole [23], CMB quadrupole [24], CMB octopole [25], large scale velocity flows [26] and large scale alignment in the QSO optical polarization data [27]. In these observations, all the preferred directions appear to be towards the North Galactic Hemisphere. They obtained the average direction of the preferred axes as (l, b) = (278° ± 26°, 45° ± 27°).

In this paper we study the dependence of this result on the dark energy model. We consider two models. One is the wCDM model; the other is a dynamical dark energy model with the CPL parametrization [28]. We take the present value of deceleration parameter q0 as the diagnostic to quantify the anisotropy level of two opposite hemispheres.

The paper is organized as follows. In the next section we give a general introduction to the hemisphere comparison method using q0 to quantify the anisotropy level and we apply it to wCDM model fitted by the Union2 dataset. In Sec. 3, we give the numerical results on the preferred directions from the SNIa data with different slices of redshift. We also give the result for the CPL model. Sec. 4 gives our conclusions.

2. HEMISPHERE COMPARISON METHOD USING THE UNION2 DATASET

Since its first release, the SNIa data have become an important tool for us to understand the evolution of the universe. And as time goes on, more accurately determined data will be released. For example, the future Joint Dark Energy Mission (JDEM) [2] is aimed to explore the properties of dark energy and to measure how cosmic expansion has changed over time. In this paper we take use of the Union2 dataset [11], which contains 557 type Ia SNIa data and uses SALT2 for SNIa light-curve fitting, covering the redshift range z = [0.015, 1.4] and including samples from other surveys, such as CfA3 [29], SDSS-II Supernova Search [30] and high-z Hubble Space Telescope.

We fit the SNIa data by minimizing the χ2 value of the distance modulus. The χ2sn for SNIa is obtained by comparing theoretical distance modulus μth(z) = 5 log10[dL(z)] + μ0, where μ0 = 42.384 − 5 log10 h is a nuisance parameter, with observed μob of supernovae:

$$\chi^2_{sn} = \sum_{i=1}^{557} \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}.$$  

For a flat FRW cosmological model, one has

$$d_L(z) = (1 + z) \int_0^z \frac{H_0}{H(z')} dz'.$$  (2.1)

In the case of wCDM model, we have

$$H^2(z) = H_0^2[\Omega_m0(1 + z)^3 + (1 - \Omega_m0)(1 + z)^{3+3w}] - 3w0z/(1 + z)],$$  (2.2)

and the present value of the deceleration parameter can be expressed as

$$q_0 = \frac{1}{2} + 3 \frac{w0}{2}(1 - \Omega_m0).$$  (2.3)

Since μ0 is a nuisance parameter, we can eliminate the effect of μ0 in the following way. We expand χ2sn with respect to μ0 [31]:

$$\chi^2_{sn} = A + 2B\mu_0 + C\mu_0^2,$$  (2.4)

where

$$A = \sum_i \frac{[\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)},$$

$$B = \sum_i \frac{\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)}{\sigma^2(z_i)},$$

$$C = \sum_i \frac{1}{\sigma^2(z_i)}.$$  

Eq. (2.4) has a minimum as

$$\tilde{\chi}^2_{sn} = \chi^2_{sn, min} = A - B^2/C,$$

which is independent of μ0. In fact, it is equivalent to performing a uniform marginalization over μ0, the difference between χ2sn and the marginalized χ2sn is just a constant [31]. We will adopt χ2sn as the goodness of fitting between theoretical model and SNIa data.

The directions to the SNIa we use here are given in Ref. [18], and are described in the equatorial coordinates (right ascension and declination). In order to use the hemisphere comparison method, we need to convert these coordinates to usual spherical coordinates (θ, φ), and finally to the galactic coordinates (l, b) [32].

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2. http://jdem.lbl.gov/.
We apply the hemisphere comparison method to find the possibly existing preferred axis. This method was first proposed in Ref. [21], and further developed in Ref. [22], while the difference between them is the choice of the direction which separates the data into two subsets. By rotating the poles at \( l \in [0^\circ, 180^\circ] \) and \( b \in [-90^\circ, 90^\circ] \) in every 1° step, the original method uses definite number of directions. With the developed method, one can have more random directions compared with the original one, by setting the repeating times (see below). Therefore we will take the developed method. The hemisphere comparison method involves the following steps.

1. Generate a random direction
   \[ \hat{n} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta) \] (2.7)
   where \( \phi \in [0, 2\pi] \) and \( \theta \in [0, 2\pi] \) are random numbers with uniform probability distribution. Note that the uniform probability distribution of these two variables \((\phi, \theta)\) will guarantee that every direction will be generated with the same probability, other priors on the distribution of \( \phi \) or \( \theta \) may lead to a priori preferred direction.

2. Split the dataset under consideration into two subsets according to the sign of the value \( \hat{n} \cdot \hat{n}_{\text{dat}} \), where \( \hat{n}_{\text{dat}} \) is the unit vector describing the direction of each SNIa in the dataset. Then we divide the data in two opposite hemispheres, denoted by up and down, respectively.

3. Find the best-fitting values of \((\Omega_{m0}, w)\) \((\Omega_{m0}, w_0)\) for CPL on each hemisphere. Naturally, one can take use of the deceleration parameter \( q_0 \) to quantify the anisotropy level through the normalized difference
   \[ \frac{\Delta q_0}{\bar{q}_0} = 2\left(q_{0,u} - q_{0,d}\right) / q_{0,u} + q_{0,d} \] (2.8)
   where the subscripts \( u \) and \( d \) denote the up and down hemispheres, respectively. Note that the deceleration parameter is a good diagnostic to quantify the anisotropy level. The hemisphere with larger \( q_0 \) is expanding slower than the opposite. Larger normalized difference in one direction means that the anisotropy level in this direction is more notable.

4. Repeat 400 times from step 1 to step 3, and find the maximum normalized difference defined by (2.8) for the Union2 data, thus one can obtain the corresponding direction of maximum anisotropy.

As illustrated in [22], in order to maximize the efficiency, the number of directions should be no less than the number of data points of SNIa on each hemisphere. The reason for this is that changing the direction of an axis, does not change the corresponding \( \Delta q_0 \) until a data point is crossed by the corresponding equator line. Such a crossing is expected to occur when the direction of an axis changes by the approximately mean angular separation between data points. Thus, using more axes than the number of data points in a hemisphere does not improve the accuracy of the determination of the maximum anisotropy direction. Given that the number of data points per hemisphere for the Union2 dataset is about 280, we have used 400 axes in our analysis, well above the value of 280.

3. RESULTS

Following the steps introduced in Sec. II, one can obtain the best-fitting values of \( q_0 \) in each direction, and then find the maximum anisotropy direction. But in order to obtain the 1σ errors of the maximal anisotropic direction and the anisotropy level, we need to get the errors of the parameters \((\Omega_{m0}, w)\) for \( wCDM \) and \((\Omega_{m0}, w_0)\) for CPL, which would propagate to \( q_0 \) and then to \( \Delta q_0 \), because \( q_0 \) is determined by \( w \) and \( \Omega_{m0} \) [see (2.3)] for \( wCDM \) model or \( w_0 \) and \( \Omega_{m0} \) [see (2.5)] for CPL model. The analysis here is performed by using the Monte Carlo Markov Chain in the multidimensional parameter space to derive 1σ errors on each hemispheres in the maximum anisotropic direction. Accordingly, the 1σ deviation from the maximum anisotropy level can be expressed as \( \Delta q_0 = \Delta q_{0,max} \pm \sigma_{\delta q} \), and correspondingly the direction axes with 1σ error can also be obtained.

Further we explore the possible redshift dependence of the anisotropy. We implement a redshift tomography of the data and take the same procedure as before for all the following redshift slices: 0-0.2, 0-0.4, 0-0.6, 0-0.8, 0-1.0. Our results for \( wCDM \) model are summarized in Table I. And we also show the result for the full sample of Union2 (0 < \( z \) < 1.4) in Figure 1.

| Redshift range | \( \Omega_{m0} \) [degree] | \( w \) [degree] | \( \Delta q_0 \) [degree] |
|----------------|----------------|----------------|---------------------|
| 0 - 0.2        | 332.53 ± 44  | -29.11 ± 42   | 5.109 ± 0.50        |
| 0 - 0.4        | 319.12 ± 40 | -28.24 ± 41   | 1.06 ± 0.47         |
| 0 - 0.6        | 300.53 ± 24 | -16.24 ± 24   | 0.94 ± 0.37         |
| 0 - 0.8        | 309.34 ± 21 | 24.34 ± 28    | 0.94 ± 0.29         |
| 0 - 1.0        | 311.20 ± 16 | 19.17 ± 20    | 0.94 ± 0.28         |
| 0 - 1.4        | 314.20 ± 11 | 28.33 ± 11   | 0.79 ± 0.28         |

Table 1: Directions of maximum anisotropy for several redshift ranges of the Union2 data fitting with the \( wCDM \) model. The last column corresponds to the 1σ errors propagated from \( \Omega_{m0} \) and \( w \), which are obtained by MCMC method.

The redshift tomography analysis here shows that the preferred axes are all located in a relatively small part of the North Galactic Hemisphere [around \( (l, b) = (314^\circ - 33^\circ, 28^\circ - 40^\circ) \)], which is consistent with the result in [22] at 1σ confidence level, thus indicates that under the assumption of \( wCDM \) model, the universe has a maximum acceleration direction. For different redshift slices, there are slight differences in the direction of preferred axes, and the difference between the two opposite hemispheres is extremely obvious for the low-redshift slice \( (z \leq 0.2) \).

Here we also give the best-fitting parameters of the \( wCDM \) model in Table I where the subscripts \( u \) and \( d \)
denote the up and down hemispheres. For the case with full SNIa data, see the last row in Table 2. Note that in the $\Lambda$CDM model, $\Omega_{m0} = 0.30$ and $\Omega_{d0} = 0.19$ [22].

| redshift range | $\Omega_{m0}$ | $\Omega_{d0}$ | $w_{m}$ | $w_{d}$ |
|----------------|--------------|--------------|---------|---------|
| 0 – 0.2        | 0.45         | 0.10         | −0.80   | −0.51   |
| 0 – 0.4        | 0.44         | 0.11         | −1.93   | −0.65   |
| 0 – 0.6        | 0.41         | 0.11         | −1.91   | −0.70   |
| 0 – 0.8        | 0.37         | 0.12         | −1.68   | −0.67   |
| 0 – 1.0        | 0.37         | 0.18         | −1.68   | −0.70   |
| 0 – 1.4        | 0.35         | 0.22         | −1.46   | −0.77   |

Table 2: Best-fitting parameters of the $w$CDM model for different redshift slices on the opposite hemispheres in the direction of the maximum anisotropy, where $u$ denotes the hemisphere corresponding to larger accelerations, while $d$ denotes the opposite hemisphere.

Note that $\Omega_{m0}$ is used as the diagnostic to the anisotropy level in [22]. Here we use $q_0$ as the diagnostic to the anisotropy level. We found here that $q_0$ is more sensitive to the errors of the data, since $q_0$ is relative to the second order derivative of the luminosity distance. And also because more parameters involved in the analysis will introduce more uncertainties when we try to determine the error of $q_0$, therefore the errors here are larger than those in [22]. For example, the dots colored according to the sign and magnitude of the anisotropy level will be more scattered on the unit sphere, as shown in Figure 1. But the meaning of using $q_0$ is obvious that the hemisphere with larger $q_0$ is expanding slower than the opposite, and that the direction with normalized difference $\Delta q_{q_0}$ means that the anisotropy level in this direction is more notable. What’s more, it is clear from Figure 1 that the sphere is divided into two distinct hemispheres, one with smaller accelerations and the other with larger accelerations.

With the same procedure we also study this issue for a dynamical dark energy model with the CPL parametrization. By use of all Union2 data points, we find that the direction of preferred axis is $(l,b) = (309^\circ \pm 30^\circ, 21^\circ \pm 25^\circ)$, and the corresponding maximum anisotropy is $\Delta q_{q_0,max} = 0.76 \pm 0.46$. The result shows that it is not much different from the case of the $w$CDM model. This means that the best-fitting values of the preferred direction is not very sensitive to the dark energy models.

### 4. CONCLUSIONS

From some astronomical observations and some theoretical models of the universe, there seemingly exists some evidence for a cosmological preferred axis [10]. If such a cosmological preferred axis indeed exists, one has to seriously consider an anisotropic cosmological model as a realistic model, instead of the FRW universe model.

In this paper we investigated the existence of anisotropy of the universe by employing the hemisphere comparison method and the Union2 SNIa dataset and found this preferred direction. We used the present value of the deceleration parameter $q_0$ to quantify the anisotropy level of the two hemispheres. For the $w$CDM model, the preferred direction is $(l,b) = (314^\circ \pm 13^\circ, 28^\circ \pm 33^\circ)$, and $\frac{\Delta q_0}{q_0} = 0.79 \pm 0.28$. While in the case of CPL model, the direction of preferred axis is $(l,b) = (309^\circ \pm 30^\circ, 21^\circ \pm 25^\circ)$, and correspondingly the maximum anisotropy level is $\frac{\Delta q_{0,max}}{q_{0,max}} = 0.76 \pm 0.46$. Comparing with the result given in [22], where the $\Lambda$CDM model is employed, our results are basically in agreement. This means that the best-fitting preferred direction is not much sensitive to the dark energy models.

Finally let us notice that it can be seen from Table 1 that the result is weakly dependent on redshift if the redshift tomography analysis is employed.

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