Analysis of student’s proof construction on exponents

F Nurfadilah¹, Hapizah¹*, and Scristia¹

¹Mathematics Education Universitas Sriwijaya, Palembang, South Sumatra, Indonesia

* Corresponding author’s email: hapizah@fkip.unsri.ac.id

Abstract. This research is a descriptive research that aims to describe the construction of student’s proof on exponents. Proof construction is compiling proof from a statement based on the definitions and rules used. The subject of research were students of class X IPA from SMAN 10 Palembang which consisted of 38 students. The learning process takes place according to the steps of direct learning. The technique of collecting data that used on this research was written test consisting of three problem descriptions. The collected data were analyzed by evaluating the proof of the student work consisting of three assessment categories as follows: Arguments are valid and are proof (K₁), general arguments are valid but not proof (K₂) and cannot strive to be a valid general argument that is an invalid or incomplete (K₃). The research result shows that the construction of student’s proof on exponents was still in the K₃ categories, with details as follows: Students categorized K₁ (Arguments are valid and are proof) are 38.6%, students categorized K₂ (general arguments are valid but not proof) are 17.5% and students categorized K₃ (cannot strive to be a valid general argument that is an invalid or incomplete) are 43.9%. Students who are categorized K₁ as a whole are able to perform constructive proof properly. Students categorized K₂ are still not able to perform constructive proof caused by mistakes in mathematical manipulation so that it shows the illogical thing. Students who are categorized K₃ are still not able to perform constructive proof caused by not being able to utilize the definitions and rules to be used in proving so that the proof done is invalid or unfinished.

1. Introduction
Exponents are mathematics learning materials that have an important role, because the exponent is one of the material that is tested on the national exam, exponents are also often reviewed in mathematics and other lessons In junior high School, it contains many ideas and abstract concepts that are organized systematically and the concept of exponent is widely used as a prerequisite material for other materials, such as logarithmic, root withdrawal, etc. And in daily life, the exponent is used in the calculation of biological growth and economic fields [1-3].

One of the essential abilities to master students on the material is the mathematical proving ability, this is because proof is an important component of the curriculum and mathematics learning where proof is one of five Mathematical skills that must be achieved in mathematics learning because the proof can know whether or not a mathematical statement so that the learning of mathematics will not be separated from the name of a proof [4-9].

One of the skills of mathematical proofs that the students must have is proof construction. Proof construction is very important for those who are in the field of mathematics, this is because construction is one of the eight roles of proof in mathematics, namely (1) verification, (2) Inventions, (3) systematization, (4) Explanation, (5) Communications, (6) Explorations, (7) Construction, and (8) Unification [10-12]. Proof construction is one of the ability to be considered in mathematics learning because little or many experiences that students have in proof construction in high school will have an
impact on proof construction ability when were in college [13]. According to the statement above, it can be concluded that constructive proof is important in mathematics learning.

Research related to proof construction of students has been done often in the field of mathematics study that is geometry at the student level as well as in the students [14-19]. In addition to the field of mathematics study that is geometry, there is also research on proof construction of students in the field of other mathematical studies namely trigonometry, set theory, number theory, and analysis. The field of mathematical studies is trigonometry in the research of Khoiriah [20]. The field of mathematics study is the theory of the association, especially in the research material found in the study of Nadlifah and Prabawanto [9]. The field of mathematics study is the number theory in the research of Nurrahmah [21]. The field of mathematics study is analysis, especially in real analysis material found in the research of Lestari [22] and Perbowo [23].

Based on the research results of Faruq [13] Dan Khoiriah [20], which provides advice to other researchers that should conduct advanced research on proof construction in the field of other mathematics studies in order to further develop constructive proof of Students, not only in the field of mathematical studies, namely geometry and trigonometry. Therefore, researchers want to do research on the material exponent, this is because the exponent is one of the lesson materials in one of the areas of mathematical studies, namely algebra [24], and also algebra which is one of the standard test contest PISA [25]. In addition, the need for construction of proof on exponent material where there are rules on exponent material that can be proven [2]. It can be concluded that the importance of proof of the material exponent. Further researchers want to do research on this material because there has not been any research into proof construction on the materials so that there is no data showing that the proof construction of students on exponents focused on the answers Students, where in evaluating students ' answers generally using proof evaluation. So, the purpose of this study was to determine the construction of students’s proof on exponents.

2. Method
The study was conducted for 3 times with 2 learning meetings by using direct learning and 1 time written test meeting. This type of research is a descriptive research which aims to describe the proof construction of student on the material exponent. The research was conducted in class X MIA 5 SMAN 10 Palembang consisting of 38 students in odd semester of school year 2019/2020.

The data collection techniques used tested. The test problem consists of 3 description-shaped questions aimed at knowing the proof construction of students on exponents by evaluating proof of the results of the student work consisting of three categories of assessments adopted from Stylianides [26]. As follows: Arguments are valid and are proof (K1), general arguments are valid but not proof (K2), cannot strive to be a valid general argument that is an invalid or incomplete (K3).

3. Result and Discussion
The implementation of the study conducted 2 times. The first learning was implemented with a learning indicator that explains the concepts of the exponent (a positive rounded exponent, a zero round exponent, and a negative rounded exponent) and classifies the rule exponent (number exponent rules). Second learning was implemented with a learning indicator that explains the concepts of the exponent (fractional exponent) and classifies the rules exponent (the rule of the real number generation with fractional exponents). The test to see proof construction of students on the exporting material held after 2 times of learning meeting. The tests given to students are 3 descriptions. The following will be presented by the student proof construction summary of exponents.

3.1 Summary Results of Constructive Proof of Students on Exponents.
Table 1 shows that the summary results of proof construction of students on the exporting material are analyzed by evaluating the proof of the outcome of the student's work consisting of three assessment categories described earlier.
Based on Table 1, obtained by a student with arguments are valid and are proof (K1), general arguments are valid but not proof (K2), cannot strive to be a valid general argument that is an invalid or incomplete (K3). Students who are categorized as having arguments are valid and are proof (K1), have essentially proved the statement correctly (applying the definitions and rules exponent). Students who are categorized have general arguments are valid but not proof (K2) have essentially proven statements correctly (applying definitions and rules exponents). Students who are categorized have cannot strive to be a valid general argument that is an invalid or incomplete (K3) have essentially proven statements correctly (applying definitions and rules exponents). However, from the student work there is a mathematical manipulation error so that the student's work shows the illogical thing. As for students who are categorized having cannot strive to be a valid general argument that is an invalid or incomplete (K3), it is generally not able to prove the statement correctly (there is still an error in applying the definitions and rules of exponent). The following will be proposed findings from students' work as follows:

### 3.2 Students Works

![Figure 1](image-url)

**Figure 1.** The results of ARF work on the first question.

Figure 1 shows that the work done by ARF in the first problem as a whole can already use definitions and rules in proving it, this can be seen from the students' answers from each step of the answer, which is as follows: In step 1, ARF can change \( \left( \frac{a^m}{a^n} \right) \left( \frac{a^p}{a^q} \right) \) becomes \( \left( \frac{1}{a^n} \right)^m \left( \frac{1}{a^q} \right)^p \), this shows that ARF has been able to use the definition of real number rank with the rank of fractions. Then in step 2, ARF writes the multiplication of \( a^\frac{1}{n} \) by \( m \) factors and the multiplication of \( a^\frac{1}{p} \) by \( p \) factor, this shows that ARF has been able to use the definition of a positive round rank. In step 3, ARF writes the multiplication \( a^\frac{1}{n} \) by \( m + p \) factors, this shows that ARF has been able to use the multiplication rules...
of numbers. In step 4, ARF writes \( \left( a^{\frac{1}{n}} \right)^{m+p} \), this shows that ARF has been able to use the definition of positive round power. In the last step (Step 5), ARF writes \( (a)^{m+p} \), this shows that ARF has been able to use the definition of real number rank with fraction rank. Because ARF has been able to use definitions and rules in proving the first problem, it can be concluded that ARF is capable of constructing evidence correctly. Thus the results of NA work on the first problem can be classified in the \( K_1 \) category (arguments are valid and are proof). If students can use definitions and rules in proving, the proofs that are made can be categorized as arguments are valid and are proof [26].

Figure 2 shows that the work done by NA on the first question that seems logical, but in step 2 there is a mathematical manipulation performed by NA indicating the illogical thing. Basically, in step 2 NA can use a positive round rank definition, but there are errors of mathematical manipulation in the repeated multiplication of \( a^{\frac{1}{n}} \). Initially NA was correct in doing repeated multiplication \( a^{\frac{1}{n}} \) which was done as much as \( m \) factor, but the next NA made a mistake. NA writes repeated multiplication \( a^{\frac{1}{n}} \) which is done as much as \( m + p \) factor. The correct answer is that the repeated multiplication of \( a^{\frac{1}{n}} \) is done as much as \( m \) factor. Therefore, it can be noted that NA is wrong in writing the rank or exponent, in this case the rank written NA is \( m + p \). However, the answer NA in the next step (step 3) is correct. By using multiplication rule number of the rank, NA writes repeated multiplication \( a^{\frac{1}{n}} \) done as much as \( m + p \) factor, this indicates that NA has been correct in writing rank or exponent, in this case the rank in question is \( m + p \). Therefore, the error NA in step 2 is an error in mathematical manipulation, in which case NA makes mistakes in writing the rank or exponent so that it indicates the illogical. The result of NA work on the first problem can be classified in category \( K_2 \) (general arguments are valid but not proof). If in the case of proof of a mathematical manipulation error then the evidentiary can be categorized general arguments are valid but not proof [26].
Figure 3. The results of MIS work on the first question.

Figure 3 shows that the work done by MIS on the first question is not yet able to use the rules in conducting proof, this can be seen from the MIS response in step 3 of the repeated multiplication of $a^n$ performed as much as $m \times p$ factor. Based on the answer, MIS encountered an error in doing repeated multiplication $a^n$. This indicates that MIS has not been able to use the ranking rule of numbers. In step 3, using the rule of multiplication number, the correct answer is repeated multiplication $a^n$ done as much as $m + p$ factor. Because MIS encountered an error in step 3, this caused the error in step 4 and step 5. In step 4, MIS has used a positive rounded rank definition, but since MIS has encountered an error in step 3, it is not yet able to use a multiplication rule number of the rank then the MIS response in step 4 encountered an error. In step 4 MIS writes $(a^n)^{m \times p}$. It should be the correct answer $(a^n)^{m + p}$. In step 5, MIS has used a real number generation definition with fractional rank, but just like the previous step because MIS encountered an error in step 3 then the MIS response in step 5 is also wrong. In step 5 MIS write $(a)^{n \times m \times p}$. The correct answer is $(a)^{n \times m + p}$. Because there is an error in step 3, 4 & 5 caused by not being able to use the rule of multiplication number of rank in step 3 then the result of MIS work on the first problem can be classified in category $K_3$ (cannot strive to be a valid general argument that is invalid or incomplete). If it is not yet able to use the rules in conducting substantiation the proof can be categorized as invalid or unfinished argument [26].

Figure 4. The results of IFC work on the second question.
Figure 4 shows that the work done by the IFC in the second question is not yet able to use the rules and definitions in conducting proof, this can be seen from the IFC's answer in step 3, indicating that IFC has not been able to use the rank of number of ranks. In addition, the proof done by the IFC is not yet completed, which should be resolved using a positive rounded rank definition. In step 3, on the part of the multiplications of IFC wrote a repeated multiplication of $a$ which was done as much as $m + p$ factor, in the denominator of IFC wrote a repeated multiplication of $b$ performed by as many as $n + p$ factors. Based on the answer, IFC encountered an error in performing repeated multiplication $a$ and $b$. This indicates that IFC has not been able to use the rank rule of number of ranks. Using the rank rule of the ranking number, the correct answer is as follows: On the part of the multiplications, should be the correct answer that is repeated multiplication $a$ done as much as $m \times p$ factor and on the denominator, should be the correct answer of repeated multiplication $b$ is done as much as $n \times p$ factor. In addition to making mistakes in step 3, the IFC has not yet finished in proving. Supposed to be after step 3, the proof can be solved using a positive rounded rank definition. By using a positive rounded rank definition can be obtained $\frac{a^{m+p}}{b^{n+p}}$. Thus, because IFC made an error in step 3 caused by not being able to use the rules using the rank rule of the rank number, in addition to that the IFC has not been able to complete the proof then the result of IFC work on the matter The second can be classified in category $K_3$ (cannot strive to be a valid general argument that is an invalid or incomplete). If it is not yet able to use the rules and definitions in conducting substantiation the proof can be categorized as invalid or unfinished argument [26].

Figure 4. The results of IFC work on the second question.

Figure 5 shows that the work done by SD in the third question as it is finished, but there is an incomplete proof of proof, in which the unproof is supposed to be obtained using Rules and definitions in conducting proof, this indicates that the proof of SD is not valid. Based on SD's answer in the first step, SD wrote $(a \times b) \left(\frac{1}{n}\right)^n$ indicating that SD can already use real number generation definitions with fractional rank. Furthermore, in step 2 SD writes repeated multiplication $(a \times b) \left(\frac{1}{n}\right)^n$ which is done as much as $M$ factor, it indicates that SD can already use positive round rank definition. In step 3, SD wrote $a^{\frac{m}{n}} \times b^{\frac{m}{n}}$, indicating that the proof performed by SD is complete. It should be before heading to the answer to step 3 or after the 2nd step answer, there is an answer to another proof step that is obtained by using rules and definitions. After the answer in step 2, the student should write a repeated multiplication of $a^{\frac{1}{n}} \times b^{\frac{1}{n}}$ done as much as $m$ factor, this answer is obtained using the rank rule of multiplication number. After that step, students should write $\left(\frac{a^{\frac{1}{n}} \times b^{\frac{1}{n}}}{m}\right)^m$ which is obtained using a positive rounded rank definition. After that step, using the real number generation rules with fractional rank can be obtained $a^{\frac{m}{n}} \times b^{\frac{m}{n}}$, such as students' answers in step 3. Thus, due to the proof that the SD done is incomplete so that indicates that the SD has not been able to use the rank rule of multiplication number and the definition of a positive round rank in conducting proof then the work of SD The third question can be classified in category $K_3$ (cannot strive to be a valid general argument that is an
invalid or incomplete). If it is not yet able to use the rules and definitions in conducting substantiation the proof can be categorized as invalid or unfinished argument [26].

4. Conclusion

Based on the results of proof evaluation on proof construction of students on exponents indicates that it is still in category K_3 (invalid or unfinished argument), with the following details: Students categorized K_1 (arguments are valid and are proof) are 38.6%, students categorized K_2 (general arguments are valid but not proof) are 17.5% and students categorized K_3 (cannot strive to be a valid general argument that is an invalid or incomplete) are 43.9%. Students who are categorized K_1 as a whole are able to perform constructive proof properly. Students categorized K_2 are still not able to perform constructive proof caused by mistakes in mathematical manipulation so that it shows the illogical thing. Students who are categorized K_3 are still not able to perform constructive proof caused by not being able to utilize the definitions and rules to be used in proving so that the proof done is invalid or unfinished.

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