Research on State Estimation of Hypersonic Glide Vehicle

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Abstract: In order to estimate the motion state of the hypersonic glide vehicle, a motion model based on the aerodynamic parameters of the prototype CAV-H and an observation model of GEO infrared early-warning satellite were established. The exponential weighted recursive least square method with forgetting factor was used to measure the early-warning satellite observation data with measuring error in angle to obtain a fitting parameter describing the polynomial of the target state. The method can focus on the local motion of the target, dilute the influence of the early observation data on the fitting parameters, reduce the estimation error caused by the target manoeuvring. The simulation result shows that the estimation of both position and velocity of the hypersonic glide vehicle is highly accurate.

1 Introduction
As a new concept weapon, Hypersonic Glide Vehicle (HGV) brings new challenges to existing air defense anti-missile systems with the character of long range, low ballistic, fast speed, strong mobility [1]. When studying its state estimation problem, Most scholars choose to use Interacting Multiple Model (IMM) filtering algorithm [2][3][4][5], but IMM needs parallel computing which is not conducive to rapid response in wartime. And the accuracy depends on the accuracy of the motion model. This paper describes the spatial motion of HGV with high order polynomials, fit discrete observations with errors from GEO infrared warning satellite with Exponential Weighted Recursive Least Squares (EW-RLS). Using interpolation to obtain the position and velocity of the observation time and its neighborhood, Provides foundation for rapid ballistic and tracking observations of HGV targets.

2 Simulation of typical trajectories
After HGV re-enters the atmosphere, it can use aerodynamic force to continuously transform kinetic energy and potential energy, Jump in vertical plane to get out of the warning system to track. Horizontal manoeuvres can also be used to circumvent the impact of defensive weapons [6]. In this section, a three-dimensional motion model of HGV reentry segment with control conditions is established based on the published aerodynamic parameters of the US Army’s CAV-H prototype. Typical reentry trajectories are simulated and analyzed, which will lay the foundation for subsequent research on reentry interception methods.

2.1 Reentry model of HGV
HGV has a flat lift body structure, using Bank-to-Turn (BTT) turning method [7]. If we ignore the effect of Earth’s rotation, think of the Earth as a uniform sphere, The reentry kinetic equation of HGV can be expressed as:

\[ m\ddot{r} = R + mg \] (1)
Where, \( m \) is quality, \( \mathbf{r} \) is position vector, \( \mathbf{R} \) is aerodynamic vector, \( \mathbf{g} \) is gravitational acceleration vector.

Projects items in the above equation into the semi-velocity coordinate system \([8]\), we have,

\[
\begin{align*}
\dot{v} &= \frac{\rho v^2 C_D S_r}{2 m} - g \sin \theta \\
\dot{\mathbf{r}} &= v \sin \theta \\
\dot{\mathbf{v}} &= \frac{\rho v^2 C_L S_r}{2 m v} \cos \gamma + \left( \frac{v}{r} \right) \cos \theta \\
\dot{\mathbf{v}}_\perp &= \frac{\rho v^2 C_L S_r}{2 m v \cos \theta} + \frac{v \tan \varphi \cos \theta \sin \sigma}{r} \\
\dot{\mathbf{v}}_\parallel &= \frac{\rho v^2 C_L S_r}{2 m v \cos \theta} + \frac{v \tan \varphi \cos \theta \sin \sigma}{r}
\end{align*}
\]

Where, \( v \) is velocity, \( \theta \) is ballistic inclination, \( \sigma \) is Ballistic declination, \( r \) is Geocentric distance, \( \lambda \) is longitude, \( \psi \) is latitude, \( \rho \) is atmospheric density, \( S_r \) is Reference pneumatic area, \( C_D \) is resistance coefficient, \( C_L \) is lift coefficient, \( \gamma \) is tilt angle.

According to the 1976 American Standard Atmosphere Model \( \text{USSA76} [9] \),

\[
\rho = \rho_0 \exp(-\beta h)
\]

Where, \( \rho_0 = 1.225 \text{ kg/m}^3 \), \( \beta = 1.389 \times 10^{-4} \).

CAV-H’s drag coefficient and lift coefficient can be expressed as a function of attack angle and speed. \([10]\):

\[
\begin{align*}
C_D &= 0.000724 \cdot \alpha^2 + 0.406 \exp(-0.00095 \cdot v) + 0.024 \\
C_L &= 0.0513 \cdot \alpha + 0.2945 \exp(-0.000302 \cdot v) - 0.2317
\end{align*}
\]

Where, \( \alpha \) is attack angle.

Simultaneous Formula (2) and Formula (4), Given initial position, initial speed, total mass, and control variables \( \alpha(t) \) and \( \gamma(t) \), the position (spherical coordinates) in the ECEF coordinate system and the speed (spherical coordinates) in the local coordinate system can be obtained.

Convert spherical coordinates to rectangular coordinates,

\[
\begin{align*}
\mathbf{r} &= \begin{bmatrix} r \cos \psi \cos \lambda \\ r \cos \psi \sin \lambda \\ r \sin \psi \end{bmatrix} \\
\mathbf{v}^d &= \begin{bmatrix} v \cos \theta \cos \sigma \\ v \cos \theta \sin \sigma \\ v \sin \theta \end{bmatrix}
\end{align*}
\]

Further, the speed in the ECEF coordinate system,

\[
\mathbf{v} = \Omega_d \cdot \mathbf{v}^d = \begin{bmatrix} -\sin \psi \cos \lambda & -\sin \psi \sin \lambda & \cos \psi \cos \lambda & \cos \psi \sin \lambda & \sin \psi \cos \lambda & \sin \psi \sin \lambda \\ \cos \psi \cos \lambda & \cos \psi \sin \lambda & \sin \psi \cos \lambda & \sin \psi \sin \lambda & \cos \psi \cos \lambda & \cos \psi \sin \lambda \\ -\sin \lambda & \cos \lambda & 0 & -\sin \lambda & \cos \lambda & 0 \end{bmatrix} \begin{bmatrix} v \cos \theta \cos \sigma \\ v \cos \theta \sin \sigma \\ v \sin \theta \end{bmatrix}
\]

2.2 Simulation Example

Guidance control and trajectory optimization of HGV reentry section are not the focus of this paper, we assume constant control conditions.

- Initial conditions: as shown in Table 1.

| parameter   | trajectory 1 | trajectory 2 | trajectory 3 |
|-------------|--------------|--------------|--------------|
| longitude   | 0°E          | 0°E          | 0°E          |
| latitude    | 0°N          | 0°N          | 0°N          |
| height      | 80km         | 80km         | 80km         |
Velocity & 6000m/s & 6000m/s & 5000m/s  
Inclination & -3° & -10° & -3°  
Declination & 30° & 45° & 60°  

- Control conditions: $\alpha(t) \equiv 8^\circ$, $\gamma(t) \equiv 0^\circ$.  
- Termination conditions: $v(t) = 1000m/s$.  
- Simulation results: as shown in Figure 1.

![Figure 1 Trajectory Simulation curve](image1)

We can see from Figure 1 that affected by aerodynamic drag, the speed of the HGV is declining overall. Especially when the HGV moves up to the position which is the minimum value of the ballistic altitude, the speed will be obviously reduced. In addition, the lift-drag ratio of HVG was maintained at more than 3 in the early stage of reentry, which represented good aerodynamic efficiency and was consistent with relevant documents. Due to the decrease of HGV's own speed in the later period, lift-drag ratio is getting lower.

Import ballistic data into STK software:

![Figure 2 Bullet point and Space trajectory](image2)

In Figure 2, the yellow trajectory represents the trajectory 1, the green trajectory represents the trajectory 2, and the red trajectory represents the trajectory 3.

3 Binary observation error

In the ballistic missile defense process, early warning missions and tracking missions are generally completed by space-based infrared early warning satellites and ground-based active phased array radars. Ground-based active phased array radar is relatively high in terms of positioning and tracking accuracy. However, a layer of plasma [11-12] that absorbs and scatters radar waves will appear on the surface when HGV performs hypersonic gliding in the near space. As a result, ground-based radars are difficult to
accurately locate HGV. On the other hand, when the ground-based radar detects a flying target of near space, the effective distance is only about 700km due to the curvature of the earth.

In response to this problem, many scholars have proposed the idea of using space-based early warning satellites to complete the tracking mission and studied Infrared detectability of HGV reentry. The conclusion seems to be more consistent, that is, The infrared characteristics of HGV reentry section are obvious and stable [13-14] and can be accurately recognized by space-based infrared early warning satellites [15-16]. References to U.S. Space Based Infra-Red System-high, (SBIRS-high), two GEO infrared warning satellite are used to observe HGV in this section.

3.1 Geometric positioning error

Figure 3 shows the definition of the line of sight angle in the satellite measurement coordinate system.

Observing azimuth $q_a$ is the angle between the projection of line-of-sight vector in $x, O, y, z$ plane and $x$, axis, $q_a \in [0^\circ, 360^\circ)$, when we see along $z$, axis, $q_a$ increases in clockwise. Observing pitch angle $q_b$ is the angle between line-of-sight vector and $x, O, y, z$ plane, $q_b \in [-90^\circ, 90^\circ]$, $q_b$ is negative when the line-of-sight vector is on the same side of the $x, O, y, z$ plane as $z$, axis.

![Figure 3 Double Star Positioning Principle](image)

The unit vector of the line of sight can be expressed in the ECEF coordinate system:

$$\mathbf{L} = \mathbf{L}' = \begin{bmatrix}
-\sin \lambda & 0 & -\cos \lambda \\
\cos \lambda & 0 & -\sin \lambda \\
0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
\cos q_b \cos q_a \\
\cos q_b \sin q_a \\
\sin q_b
\end{bmatrix}
$$

(7)

Suppose the two line-of-sight unit vectors are $\mathbf{L}_1'$ and $\mathbf{L}_2'$. In the absence of error in the measured value, the intersection of the two lines of sight is the target centroid $O_t$. However, there is measurement noise in actual observations, line-of-sights may not intersect in space, The midpoint $O_t'$ of the vertical line of the two line-of-sights can replace the target position approximately, as shown in Figure 3.

Geostationary satellites are stationary with respect to the Earth. Assume the location of two infrared early warning satellites in the ECEF coordinate system are $\mathbf{O}_{1t} = [x_1, y_1, z_1]^T$ and $\mathbf{O}_{2t} = [x_2, y_2, z_2]^T$, two line-of-sight vectors are $\mathbf{L}_1' = [a_1, b_1, c_1]^T$ and $\mathbf{L}_2' = [a_2, b_2, c_2]^T$, then the $O_t'$ point can be Calculated according to the following procedure [17]:

4
\[
\begin{bmatrix}
    a_1 & -a_2 & -b_1 & c_2 & c_1 \\
    b_1 & -b_2 & a_1 & -a_2 \\
    c_1 & -c_2 & a_1 & b_2 & -b_2
\end{bmatrix}
\begin{bmatrix}
    \lambda_1 \\
    \lambda_2 \\
    \lambda_3
\end{bmatrix}
= \begin{bmatrix}
    x_2 - x_1 \\
    y_2 - y_1 \\
    z_2 - z_1
\end{bmatrix}
\]
(8)

\[
\mathbf{r} = \begin{bmatrix}
    \frac{x_1 + a_1 \lambda_1 + x_2 + a_2 \lambda_2}{2} \\
    \frac{y_1 + b_1 \lambda_1 + y_2 + b_2 \lambda_2}{2} \\
    \frac{z_1 + c_1 \lambda_1 + z_2 + c_2 \lambda_2}{2}
\end{bmatrix}^T
\]
(9)

3.2 Simulation Example

- HGV trajectory: trajectory 1 in section 2.2
- Longitude of GEO satellite: $\lambda_{s1} = 10^\circ$ E, $\lambda_{s2} = 40^\circ$ E
- Early warning satellite detection frequency: 1Hz
- Early warning satellite measurement angle error: Gaussian distribution from a mean of 0° and a standard deviation of 0.001°

The simulation scenario set up in STK software is shown in Figure 4.

![Figure 4. 3D view of STK scene](image)

- GEO satellite measurement data

Using the ACESS Toolbox in STK Software to Generate the True Value of Observation Angle of Two Early Warning Satellites to HGV. Add Gaussian white noise with a mean of 0 and a standard deviation of 0.001° as the measured value, as shown in Figure 5:

![Figure 5 Satellite observation data](image)

- Geometric positioning error
By the formula (9), observation position $\bar{R}$ of HGV can be expressed in the ECEF coordinate system as:

$$\bar{R} = [f(1) \ f(2) \ f(3) \ \cdots \ f(n)]$$

By the formula (5), real position $R$ of HGV can be known, so the geometric positioning error can be expressed in the ECEF coordinate system as:

$$\bar{E} = \bar{R} - R = \begin{bmatrix} \bar{E}_x & \bar{E}_y & \bar{E}_z \end{bmatrix}^T$$

Geometric positioning error in the ECEF coordinate system is shown in Figure 6.

Figure 6. Geometric positioning error

4 State Estimation of HGV

Interpolation polynomial method is a common method for ballistic missile target state estimation. However, the reentry trajectory of HGV is highly nonlinear, if the conventional least squares method is used to process observation data, the resulting state estimation error will be very large. This paper use Exponential Weighted Recursive Least Square [18](EW-RLS) with forgetting factor to estimate the polynomial coefficient, EW-RLS is able to focus on the local fit of the target motion, inhibit the influence of HGV frequent maneuvers, obtain more accurate position and velocity estimates.

4.1 EW-RLS fitting method

Assuming the observation position with the error sequence

$$Z_s = \begin{bmatrix} z_s(1) & z_s(2) & \cdots & z_s(n) \end{bmatrix}^T$$

$w_s$ is a certain estimate for the vector to be sought

$$w_s = \begin{bmatrix} \tilde{a}_0 & \tilde{a}_1 & \tilde{a}_2 & \tilde{a}_3 & \tilde{a}_4 \end{bmatrix}^T$$

Residual square sum due to fitting can be expressed as

$$S_s = (Z_s - Uw_s)^T (Z_s - Uw_s)$$

Where,

$$U = \begin{bmatrix} 1 & (t_1 - t_0) & (t_1 - t_0)^2 & (t_1 - t_0)^3 & (t_1 - t_0)^4 \\ 1 & (t_2 - t_0) & (t_2 - t_0)^2 & (t_2 - t_0)^3 & (t_2 - t_0)^4 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & (t_n - t_0) & (t_n - t_0)^2 & (t_n - t_0)^3 & (t_n - t_0)^4 \end{bmatrix}$$

$$u(1)^T$$  
$$u(2)^T$$  
$$\cdots$$  
$$u(n)^T$$
According to Least Squares Theory, the $w_x$ that makes $S_x$ minimal is the Optimal estimate, therefore, $S_x$ is called cost function. Make improvements based on least squares, take the cost function as

$$S_x = (Z_x - Uw_x)^T Z_x - Uw_x$$

Where, $\xi(n,i)$ is weighting factor, meets $0 < \xi(n,i) < 1$. Weighting factors enable parameter estimates to “forget” old observations and focus on new observations, let:

$$\xi(n,i) = \theta^{n-i}, \quad i=1,2,...n$$

Where, $\theta$ is Forgetting factor, $\theta$ equals 1 in classical least squares method.

By the formula (16), optimal estimation based on new cost function $\tilde{S}_x$ is:

$$\hat{w}_x = (U^T BU)^{-1} U^T BZ_s$$

4.2 Simulation Example

Single estimate position of HGV can be expressed in the ECEF coordinate system as:

$$\hat{r}(t) = \begin{bmatrix} \hat{r}_x(t) \\ \hat{r}_y(t) \\ \hat{r}_z(t) \end{bmatrix}, \quad \hat{v}(t) = \begin{bmatrix} \hat{v}_x(t) \\ \hat{v}_y(t) \\ \hat{v}_z(t) \end{bmatrix}$$

Where, $\hat{r}_x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$; $\hat{r}_y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4$; $\hat{r}_z(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4$; $\hat{v}_x(t) = a_1 t + 2a_2 t^2 + 3a_3 t^3 + 4a_4 t^4$; $\hat{v}_y(t) = b_1 t + 2b_2 t^2 + 3b_3 t^3 + 4b_4 t^4$; $\hat{v}_z(t) = c_1 t + 2c_2 t^2 + 3c_3 t^3 + 4c_4 t^4$

Where, $a_i$, $b_i$ and $c_i$ are coefficient to be estimated.

Let:

$$w_x(0) = (U^T U)^{-1} U^T Z \quad p(0) = (\sum_{i=1}^{n} u(i) u^T(i))^{-1}$$

By the formula (23), estimate position of HGV and estimate velocity of HGV can be expressed in the ECEF coordinate system as:
\[
\hat{\mathbf{R}} = [\hat{\mathbf{r}}(1) \ \hat{\mathbf{r}}(2) \ \hat{\mathbf{r}}(3) \ \cdots \ \hat{\mathbf{r}}(n)] \quad \hat{\mathbf{V}} = [\hat{\mathbf{v}}(1) \ \hat{\mathbf{v}}(2) \ \hat{\mathbf{v}}(3) \ \cdots \ \hat{\mathbf{v}}(n)]
\]

(25)

By the formula (5) and (6), real position \( \mathbf{R} \) and real velocity \( \mathbf{V} \) of HGV can be known, so the estimated error of position and velocity can be expressed in the ECEF coordinate system as:

\[
\hat{\mathbf{E}}_r = \hat{\mathbf{R}}_r - \mathbf{R}_r = [\hat{\mathbf{E}}_{rx} \ \hat{\mathbf{E}}_{ry} \ \hat{\mathbf{E}}_{rz}]^T \quad \hat{\mathbf{E}}_v = \hat{\mathbf{V}}_r - \mathbf{V}_r = [\hat{\mathbf{E}}_{vx} \ \hat{\mathbf{E}}_{vy} \ \hat{\mathbf{E}}_{vz}]^T
\]

(26)

Estimated error of position and velocity in the ECEF coordinate system is shown in Figure 7 and Figure 8.

![Figure 7. Estimated error of position](image7)

![Figure 8. Estimated error of velocity](image8)

5 Analysis and discussion
Under the initial conditions and the control conditions, HGV has made several jumps in the near space with a range of up to 8,000 km, which is close to its published design specifications.

When the standard deviation of satellite measurement angle is 0.001°, the observation error of GEO infrared early-warning satellite for HGV reentry section is within 5km.

Whenever the HGV moves near to the position which is a minimum altitude or a maximum altitude, the absolute value of the estimated error in position and velocity will increase rapidly. This is due to the drastic changes in the stress conditions of the HGV. The smaller the forgetting factor, the smaller the
fluctuation of the absolute value of the error.

When the forgetting factor is taken as 0.9, the position estimation error for HGV is within 1km, and the speed estimation error is within 80m/s. When the forgetting factor is taken as 0.93, the position estimation error for HGV is within 400m, and the speed estimation error is 30m/s. Within, the accuracy is more ideal.

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