Elastoplastic analysis of normal stiffness of bore wall of the rock-socketed pile based on Hoek-Brown failure criterion and cavity expansion theory

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ABSTRACT

The research on the shaft resistance governing by the sliding and shearing mechanism between the pile-rock interfaces is always a hot issue in the research of rock-socketed pile. All existing theoretical methods assumed that the normal stiffness of the bore wall of rock around the pile is constant which based on the elastic solution of cylindrical cavity expansion in a finite medium during the increasing of the relative displacement of the pile-rock interface. However, rock is not an ideal elastic material. In order to illustrate this problem, this paper combined the cavity expansion theory and the original Hoek-Brown failure criterion conducted an elastoplastic analysis of rock-socketed piles during the relative displacement of the pile-rock interface increases and derived the stress field and displacement field in the elastic and plastic phase. And, based on a case validation illustrated the difference between the elastic and elastoplastic solution of the normal stress increment with different radial displacement of the pile-rock interface and stated that only using the elastic solution of normal stiffness will overestimate the increment of normal stress at the pile-rock interface, and then overestimate the shaft resistance which will cause the design unsafety.

Keywords: rock-socketed pile, Hoek-Brown failure criterion, cavity expansion theory, elastoplastic analysis

1 INTRODUCTION

Rock-socketed drilled piles are typically selected when the large loads of superstructures, such as long-span bridges, high-rise buildings, and tower structures, need to be transferred to competent bearing strata so as restrict deformations within the serviceability limits (Seidel and Collingwood, 2001, Sagong et al., 2007, Akgüner and Kirkit, 2012). There are significant advantages in the design of piles which carry their load by both shaft and base resistance. However, in practice, the mobilization of shaft and base resistances may not occur simultaneously (O’Neil and Reese, 1999), and shaft resistance generally dominates at service loads (Williams et al., 1980, Carter and Kulhawy, 1988) due to the shaft resistance is generally mobilized at significantly smaller displacement than base resistance and utilization of the base resistance requires cleanliness of the pile base. Therefore, the research on the shaft resistance which controlled by the sliding and shearing mechanism between the pile-rock interface is always a hot issue in the research of rock-socketed piles. Since 1970, many scholars have carried out extensive theoretical and shear tests research under constant normal stress (CNL) and constant normal stiffness (CNS) boundaries on the sliding and shearing mechanism between pile-rock interface, and proposed many pile-rock shear strength models, such as Johnston and Lams’s (1989) model, Seidel-Haberfield’s (2002) model, Serrano and Olalla’s (2004) method, Nonlinear triple curve method by Seol et al. (2008) and Dai et al.’s (2012) method. The outcome of above studies provides good perspectives for understanding the bearing mechanisms of shaft resistance and optimized design of rock-socketed piles; however, they also have some obvious limitations. With the relative displacement of the pile-rock interface increases, the normal stress of that will increase as the normal dilation, and this expansion will lead to the normal and tangential stress changing of the surrounding rock. All existing theoretical methods used the elastic solution of cylindrical cavity expansion in a finite medium to calculate the increment of the normal stress caused by the normal dilation, that is, the Lamé’s solution, and assumed that the normal stiffness of the bore wall of

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rock around the pile is constant during the increasing of the relative displacement of the pile-rock interface. However, rock is not an ideal elastic material. When the relative sliding and expansion between the asperities makes the normal stress at the pile-rock interface exceeds the critical pressure, a plastic zone will form around the cavity. The stiffness of the bore wall in the plastic stage will reduce compared to the elastic stage, and only using the elastic solution of normal stiffness will overestimate the shaft resistance.

In order to illustrate this problem, the present author based on the cavity expansion theory and the original Hoek-Brown failure criterion conducted an elastoplastic analysis of rock-socketed piles during radial expansion, and derived the stress field and displacement field in the elastic and plastic phase. Then, based on a case validation illustrates the difference between the elastic solution and the elastoplastic solution of the normal stress increment with different radial displacement of the pile-rock interface.

2 PROBLEM STATEMENTS

The problem of cylindrical cavity expansion at the axisymmetric pile-rock interface in infinite rock can be regarded as a plane strain problem (Yu, 2000). The geometry of the problem and the boundary conditions are depicted in Fig. 1. The initial far-field isotropic pressure is \( p_0 \), the normal pressure of the pile-rock interface is \( p \), and the initial socket radius is \( r_0 \). The normal pressure (cavity internal pressure) \( p \) at the pile-rock interface gradually increases from its initial value \( p_0 \), the radial displacement of the pile-rock interface presents by \( u \).

As \( p \) increases, the surrounding rock will initially behave in an elastic manner until the internal pressure \( p \) reaching a yield pressure \( p_y \), when the internal pressure \( p \) exceeds \( p_y \), a plastic zone with a plastic radius of \( r_p \) appears, the rock with the plastic radius of \( r_p \) is still in the elastic phase, and the radial and tangential stresses of the rock at the radius \( r \) are \( \sigma_r \) and \( \sigma_\theta \) respectively. Since many influential elastoplastic solutions in this field have been analysed by using the tensile stress as a positive, this paper also adopts that the tensile stress is positive. In the process of cylindrical cavity expansion, the order of stress is \( \sigma_\theta > \sigma_z > \sigma_r \).

The equation of equilibrium for the cavity problem is expressed in terms of radial and circumferential stresses as follows:

\[
\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0
\]

The radial and tangential strains for the cylindrical cavity are:

\[
\varepsilon_r = \frac{\partial u}{\partial r}
\]
\[
\varepsilon_\theta = \frac{u}{r}
\]

3 STRESS FIELD

3.1 Elastic Solution \((p < p_y)\)

For elastic materials, the stress-strain relations for plane strain cylindrical cavity expansion are:

\[
\begin{align*}
\sigma_r &= \frac{1 - \nu^2}{E} \left( \sigma_\theta - \frac{\nu}{1 - \nu} \sigma_r \right) \\
\sigma_\theta &= \frac{1 - \nu^2}{E} \left( \frac{\nu}{1 - \nu} \sigma_r + \sigma_\theta \right)
\end{align*}
\]

3.2 Elastoplastic solution \((p > p_y)\)

3.2.1 Stresses in outer elastic zone \( (r \geq r_p) \)

Similar to above, the stresses in the outer elastic region as following, in which, \( r_p \) is the radius of the elastic-plastic boundary.

\[
\begin{align*}
\sigma_r &= -p_0 - (p - p_0) \left( \frac{r_0}{r} \right)^2 \\
\sigma_\theta &= -p_0 + (p - p_0) \left( \frac{r_0}{r} \right)^2
\end{align*}
\]

Moreover, the radial and circumferential stresses at the elastic-plastic boundary \( (r = r_p) \) as follows.

\[
\begin{align*}
\sigma_r &= -p_y \\
\sigma_\theta &= p_y - 2p_0
\end{align*}
\]
rock, the original Hoek-Brown failure criterion (Hoek and Brown, 1980) as shown in Equation (9).

$$\sigma_i = \sigma_1 + \sqrt{m \sigma_1 \sigma_3} + s \sigma_1^2$$

in which, $\sigma_1$ is the major principal stress at failure, $\sigma_3$ is the minor principal stress, $\sigma_c$ is the uniaxial compressive strength of the intact rock, and $m$ and $s$ are Hoek–Brown constants that depend on the properties of rock. Equation (9) was based on the assumption that the compressive stress is positive, therefore, it is necessary to rewrite it in here according to the assumption of tensile stress positive. The expression and the envelope of the Hoek-Brown failure criterion are shown in Equation (10) and Fig. 2.

$$\sigma_i = \sigma_1 - \sqrt{s \sigma_1^2 - m \sigma_1 \sigma_3}$$

where, $A = \sqrt{s \sigma_1^2 + m \sigma_1 p}_p$, $B = (m \sigma_1^3) / 4$.

Equations (13) and (14) fully determine the stress fields in the plastic zone. Then, by combined Equations (7) and (13), the radius of the elastic-plastic boundary $r_p$ can be solved by using the continuity of the radial stress at the elastic-plastic boundary as Equation (15).

$$r_p = \frac{1}{2} \frac{m \sigma_1 p + s \sigma_1^2 - s \sigma_1^2 + m \sigma_1^2 M}{2 \sigma_1}$$

4 DISPLACEMENT FIELD

4.1 Elastic Solution ($p < p_y$)

Combined Equations (2), (3) and (4), it can easily determine the displacement field of elastic solution as Equation (16).

$$u = r^2 \frac{1 + \nu}{E} \left( p - p_0 \right) \left( \frac{r}{r_0} \right)^2 r$$

4.2 Elastoplastic solution ($p > p_y$)

4.2.1 Displacement in the outer elastic zone ($r_0 \leq r \leq \infty$)

When the internal pressure $p$ exceeds $p_y$, a plastic zone with a plastic radius of $r_p$ appears and the outer zone ($r > r_p$) remains elastic, combined Equations (2), (3) and (6) the displacement field of this elastic zone is expressed as Equation (17).

$$u = r^2 \frac{1 + \nu}{E} \left( p - p_0 \right) \left( \frac{r}{r_p} \right)^2 r$$

In particular, at the elastic-plastic interface the displacement can be calculated by Equation (18):

$$u \bigg|_{r=r_p} = \frac{1 + \nu}{E} (p - p_0) \frac{r_p}{r_p}$$

4.2.2 Displacement in the plastic zone ($r_0 \leq r \leq r_p$)

To determine the displacement field in the plastic zone, a plastic flow rule is needed. A non-associated flow rule with a constant dilatancy angle $\psi$ of the H–B material is adopted as Equation (19), the indices $e$ and $p$ represent the elastic and plastic parts of the total strain, respectively, in which, $\beta = (1 + \sin \psi) / (1 - \sin \psi)$.

$$\frac{\dot{\varepsilon}_e^p}{\dot{\varepsilon}_e^p} = \frac{\dot{\varepsilon}_e - \dot{\varepsilon}_p}{\dot{\varepsilon}_e - \dot{\varepsilon}_p} = \frac{1}{\beta}$$

Furthermore, the total strains are written as functions of elastic as follows:

$$\beta \dot{\varepsilon}_{te} = \beta \dot{\varepsilon}_{te}^e + \dot{\varepsilon}_{te}^p$$

Combined the above Equations (3) and (20), the following Equation (21) derived.
For small strain problems, the strains can be expressed in terms of inward radial displacement \( u \) as follows:

\[
e_r = \frac{du}{dr}, \quad e_\theta = \frac{u}{r} \tag{22}\]

Then, substituting Equation (22) into Equation (20) gives:

\[
\beta e_r + e_\theta = \beta \frac{du}{dr} + \frac{u}{r} = g(r) \tag{23}\]

Substituting Equation (21) into Equation (23) gives:

\[
g(r) = \frac{1}{E} \left[ \left( \beta - \frac{\beta v}{1-v} \right) \sigma_r + \left( 1 - \frac{\beta v}{1-v} \right) \sigma_\theta + \left( \frac{\beta v}{1-v} \right) p_0 \right] \tag{24}\]

Then, substituting Equations (13) and (14) (solution for radial and tangential stresses in the plastic zone) into Equation (24) gives:

\[
g(r) = D_1 + D_2 \ln \left( \frac{r}{r_0} \right) + D_3 \ln^2 \left( \frac{r}{r_0} \right) \tag{25}\]

where,

\[
D_1 = \frac{1 + \sqrt{\beta^2(1-2\nu)(p_0 - p) + A(1-\nu-v\beta)}}{E}, \quad D_2 = \frac{1 + \sqrt{\beta^2 (A + \nu A - 2B) - v(A + A\beta - 2B\beta)}}{E}, \quad D_3 = \frac{1 + \sqrt{\beta^2 (2\nu - 1)(1 + \beta)}}{E}.
\]

The general solution of the above first-order nonlinear differential Equation (23) can be written as:

\[
u = c_0 r^\frac{1}{\beta} + r^\frac{1}{\beta} c(r) \tag{26}\]

where, \( c(r) = \int \frac{1}{\beta} \left[ D_1 + D_2 \ln \left( \frac{r}{r_0} \right) + D_3 \ln^2 \left( \frac{r}{r_0} \right) \right] r^{\beta} \, dr \).

The solution of \( c(r) \) is shown in Equation (27).

Thus, the solution of displacement field as Equation (28), in which, \( c_0 \) is the integral constant.

Then, by combined the displacement at the elastic-plastic interface as Equation (18) and above Equation (28), the integral constant \( c_0 \) can be determined as following Equation (29), in which, \( m_1 = (D_1 - D_2 \ln r_0 + D_3 \ln^2 r_0), m_2 = (D_2 - 2D_3 \ln r_0). \)

By combining Equations (28) and (29), the displacement field in the plastic zone is fully defined.

The cavity wall displacement can then be obtained as a special case by setting \( r = r_0 \) in Equation (30), and the solution of that as following Equation (30).

### Table 1. Parameters for calculation

| case | \( r_0 \) (mm) | \( p_0 \) (kPa) | \( E \) (Gpa) | \( v \) | \( \sigma_c \) (Mpa) | \( s \) | \( m \) | \( \beta \) |
|------|----------------|----------------|-------------|-----|-------------------|-----|-----|-----|
| a    | 200            | 500            | 12.99       | 0.25| 30                | 2.05e-2 | 15.6 | 1.67 |
5 VALIDATIONS

In order to quantify the difference, the author has compiled a calculation program code by MATLAB to compare the results of solutions of elastic (Equations (4) and (16)) and elastoplastic (Equations (13), (14) and (28)) in calculating the normal stress increment, namely, $\Delta \sigma_{n}$, at the bore wall. A set of data of rock parameter reported by Hoek and Brown (1997) was adopted and listed in Table 1, and the results for different initial radius $r_0$ were all plotted in following Fig.3.

![Fig. 3. A comparison of elastic and elastoplastic solutions in calculating the increment of normal stress: (a) $r_0 = 200$ mm, (b) $r_0 = 400$ mm, (c) $r_0 = 600$ mm, (d) $r_0 = 800$ mm.](image)

Fig. 3 illustrates that for the cases of different socketed radius, the difference between the elastic solution and the elastoplastic solution increases with the radial displacement of the pile-rock interface. In here, assuming that the interface between the pile and rock is a regular triangular interface (the inclination angle of asperities is 21.5°) and the relative displacement between the pile and the rock is 10 mm. According to $u=\Delta r = |x\tan \theta|$, the radial displacement of the interface between the pile and rock is estimated to be about 3.93mm, for the cases above of different initial radius $r_0$ from 200 to 800 mm, the error between the elastic solution and the plastic solution is 94.8 MPa, 23.9 MPa, 7.6 MPa, 1.7 MPa, respectively. At the same time, it is also clear that as the ratio of the radial displacement of the pile-rock interface to the initial radius of socket increases, the error between the elastic and plastic calculation results gradually increases. Therefore, only using the elastic solution of normal stiffness into the shear model will overestimate the increment of normal stress at the pile-rock interface, and then overestimate the shaft resistance which will cause the design unsafety.

6 CONCLUSIONS

In this paper, the present author based on the cavity expansion theory and the original Hoek-Brown failure criterion conducted an elastoplastic analysis of rock-socketed piles during the relative displacement of the pile-rock interface increases and derived the stress field and displacement field in the elastic and plastic phase. Then, based on a case validation quantified the difference between the elastic solution and the elastoplastic solution of the normal stress increment with different radial displacement of the pile-rock interface and illustrated that...
only using the elastic solution of normal stiffness into the shear model will overestimate the increment of normal stress at the pile-rock interface, and then overestimate the shaft resistance which will cause the design unsafety.

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