Learning Heuristics for Template-based CEGIS of Loop Invariants with Reinforcement Learning

Minchao Wu\textsuperscript{1}, Takeshi Tsukada\textsuperscript{2}, Hiroshi Unno\textsuperscript{3,4}, Taro Sekiyama\textsuperscript{5} and Kohei Suenaga\textsuperscript{6}

\textsuperscript{1} Australian National University & Data61, CSIRO, Australia
Minchao.Wu@anu.edu.au
\textsuperscript{2} Chiba University, Japan tsukada@math.s.chiba-u.ac.jp
\textsuperscript{3} University of Tsukuba, Japan uhiro@cs.tsukuba.ac.jp
\textsuperscript{4} RIKEN AIP, Japan
\textsuperscript{5} National Institute of Informatics, Japan tsukiyama@acm.org
\textsuperscript{6} Kyoto University ksuenaga@fos.kuis.kyoto-u.ac.jp

Abstract. Loop-invariant synthesis is the basis of program verification. Due to the undecidability of the problem in general, a tool for invariant synthesis necessarily uses heuristics. Despite the common belief that the design of heuristics is vital for the performance of a synthesizer, heuristics are often engineered by their developers based on experience and intuition, sometimes in an ad-hoc manner. In this work, we propose an approach to systematically learning heuristics for template-based CounterExample-Guided Inductive Synthesis (CEGIS) with reinforcement learning. As a concrete example, we implement the approach on top of PCSat, which is an invariant synthesizer based on template-based CEGIS. Experiments show that PCSat guided by the heuristics learned by our framework not only outperforms existing state-of-the-art CEGIS-based solvers such as HoICE and the neural solver Code2Inv, but also has slight advantages over non-CEGIS-based solvers such as Eldarica and Spacer in linear Constrained Horn Clause (CHC) solving.

1 Introduction

Static formal verification is gaining more attention owing to the increasing impact of software malfunctions. For its application to real-world software, its performance is of paramount importance.

One of the major properties of interest is partial correctness: given a program $c$ and logical formulae $\varphi_{\text{pre}}$ and $\varphi_{\text{post}}$, deciding whether $\varphi_{\text{pre}}$ and $\varphi_{\text{post}}$ are the correct precondition and postcondition of $c$, respectively. A verification procedure needs to either prove or disprove that, if $c$ is executed from an initial state that satisfies $\varphi_{\text{pre}}$ and terminates, then the final state satisfies $\varphi_{\text{post}}$.

It is known that a key to solving a partial correctness problem is discovering an appropriate loop invariant, or simply an invariant \cite{33}. The importance of an invariant leads to considerable research interest in the methods for solving an invariant synthesis problem (ISP).
Since ISP is undecidable, a common way of solving the ISP is to heuristically search for an invariant. For example, CounterExample-Guided Inductive Synthesis (CEGIS) \cite{17,48,51} is a popular approach to the ISP; it repetitively guesses an invariant and proves or refutes the correctness of the guess by using an SMT solver such as Z3 \cite{36}. If a guess is refuted, then CEGIS makes another guess based on the counterexample.

A CEGIS procedure typically maintains a search space for candidate invariants from which a guess is made. For example, template-based CEGIS \cite{17,48,51} expresses the search space for candidate invariants using a template, which is a predicate that contains parameters. A guess is obtained by instantiating the parameters with concrete values so that the instantiated guess does not contradict the counterexamples obtained so far.

If all the candidates are refuted by the counterexamples, a CEGIS procedure heuristically expands the search space. For a template-based CEGIS procedure, if it turns out that any instantiation of the parameters contradicts the obtained counterexamples, the procedure heuristically updates the template to make it more expressive. Despite that the choice of heuristics for updating a template affects the performance of invariant synthesis, heuristics are often engineered by developers, sometimes in an ad-hoc manner — their design is usually based on the experience and intuition of experts, and is not systematically explored.

We propose a novel framework that learns heuristics for template-based CEGIS for ISP using reinforcement learning (RL). RL is a kind of machine learning techniques that aims at learning a policy of the behavior of an agent that maximizes the rewards it receives from an environment. RL has been successfully applied to the field of formal methods recently, including loop invariant synthesis \cite{45,46} and theorem proving \cite{25,54}.

To this end, we reformulate template-based CEGIS as an RL problem, in which an agent issues a sequence of actions following a policy. Each action tells the underlying synthesizer how to update the template of candidate invariants given the internal state of that synthesizer. The agent receives rewards depending on the performance of the synthesizer after executing the actions taken by the agent. RL algorithms are then implemented to learn a policy that maximizes the rewards. Such a policy is essentially a heuristic for expanding the search space that leads to a good performance of the synthesizer.

As a concrete example, we implement our framework on top of PCSat \cite{41,51}, which is an invariant synthesizer based on template-based CEGIS. We then conduct experiments using problems from standard benchmarks of invariant synthesis. The experiments show that PCSat guided by the learned heuristics not only outperforms state-of-the-art CEGIS-based solvers such as HoICE \cite{9,10} and the neural solver Code2Inv \cite{16}, but also has slight advantages over non-CEGIS-based solvers such as Eldarica \cite{20} and Spacer \cite{27} in linear CHC solving.

**Contribution.** The contribution of the present paper is summarized as follows.

- We reformulate the template-based CEGIS for ISP as an RL problem. Our reformulation models the behavior of an underlying synthesizer as a Markov
decision process, in which a state represents the internal state of the synthesizer and an action represents a command that expands the search space for the candidate invariants.

– We implement our framework on top of PCSat, which is an invariant synthesizer based on template-based CEGIS, and conduct experiments. We observe that PCSat using the learned heuristics outperforms existing CEGIS-based solvers. Experiments also show that PCSat using the learned heuristics even has slight advantages over non-CEGIS-based solvers in linear CHC solving.

Structure of the paper. The rest of this paper is structured as follows. Section 2 reviews the preliminaries; Section 3 presents the template-based CEGIS procedure in detail; Section 4 models template-based CEGIS as a Markov decision process; Section 5 describes the experimental results and ablation studies; Section 6 introduces related work; and Section 7 concludes.

2 Preliminaries

2.1 Notations

We write $X$ for a finite sequence $X_1, \ldots, X_n$. We use symbols $\varphi$ and $\psi$ for first-order formulae over integers. We often write $\varphi(x)$ to express that $\varphi$ may depend on the variables $x$. For this $\varphi$ and integers $n$, we write $\varphi(n)$ for the closed formula obtained by substituting $n$ for $x$ in $\varphi$.

Let $F(x)$ be a predicate variable. A constraint over $F$, denoted by symbols $\Phi(F(x))$ and $\Phi(F(x))$, is a first-order formula that contains $F$. For example, $\forall x. F(x) \Rightarrow F(x - 1)$ is a constraint over $F$; this constraint holds if we set $F$ to $x \leq 0$ whereas it does not hold if we set $F$ to $x \geq 0$. We write $\Phi(\varphi)$ for the formula that is obtained by substituting $\varphi$ for $F$ in $\Phi$. A ground constraint is a constraint that does not contain a term variable. For example, $F(0, 0) \land (F(1, 2) \Rightarrow F(2, 3)) \land \neg F(-1, 0)$ is a ground constraint.

We use symbol $\sigma$ for a mapping from variables to integers; this mapping represents an assignment of values to variables. We write $\sigma(x)$ for a value of $x$ in $\sigma$ and $\sigma(\varphi)$ for the formula obtained by replacing every variable in $\varphi$ with its values in $\sigma$. For example, if $\sigma := \{x \mapsto 0, y \mapsto 1\}$, then $\sigma(y) = 1$ and $\sigma(x < 5 \land y > 5) = (0 < 5 \land 1 > 5)$, which is false.

2.2 Invariant Synthesis as CHC solving

It is well known that an ISP is equivalent to solving a constraint $\Phi(F(x))$ over a predicate variable $F$, where $\Phi(F(x))$ is of the form

$$
\begin{align*}
(\forall x. \varphi_{\text{pre}}(x) & \Rightarrow F(x)) \\
\land (\forall x y. \varphi_{\text{trans}}(x, y) \land F(x) & \Rightarrow F(y)) \\
\land (\forall x. F(x) & \Rightarrow \varphi_{\text{post}}(x)).
\end{align*}
$$
For example, consider the program $c_1$:

\[
\text{while } y > 0 \text{ do } x \leftarrow x + 1; y \leftarrow y - 1 \text{ done}.
\]

with precondition $x = 0 \land y = z \land z \geq 0$ and postcondition $x = z$. Then, the CHC $\forall xyz.\Phi_1(F(x, y, z))$ that expresses $F(x, y, z)$ to be a loop invariant in the program $c_1$ can be defined using the following $\Phi_1(F(x, y, z))$

\[
\begin{align*}
x &= 0 \land y = z \land z \geq 0 & \implies & F(x, y, z) & \land \\
y > 0 \land F(x, y, z) & \implies & F(x + 1, y - 1, z) & \land \\
y \leq 0 \land F(x, y, z) & \implies & x = z.
\end{align*}
\]

The first conjunct ensures that $F(x, y, z)$ is implied by the initial condition $x = 0 \land y = z \land z \geq 0$; the second conjunct ensures that the predicate $F(x, y, z)$ is preserved by one iteration of the loop if the guard condition ($y > 0$) of the loop holds; and the third conjunct ensures that $F(x, y, z)$ implies the postcondition $x = z$ if the loop terminates. The loop invariant $x + y = z \land y \geq 0$ satisfies $\forall xyz.\Phi_1(F(x, y, z))$.

A constraint of this form is called a \textit{linear Constrained Horn Clause} (linear CHC) \cite{7}. A solution to the above constraint is a logical formula $\varphi_{sol}(x)$ such that $\Phi(\varphi_{sol})$ is true.

### 2.3 Reinforcement Learning

A Markov decision process, which models an environment in RL problems, is specified by the following elements:

- a set $\mathcal{S}$ of states,
- an initial state $s_0 \in \mathcal{S}$,
- a subset $\mathcal{R} \subseteq \mathbb{R}$ of reals, called rewards,
- a finite set $\mathcal{A}$ of actions, and
- a dynamics function $p : \mathcal{S} \times \mathcal{A} \times \mathcal{R} \times (\mathcal{S} \cup \{\ast\}) \rightarrow [0, 1]$.

We write $p(r, s'|s, a)$ for $p(s, a, r, s')$. The dynamics function specifies a probability distribution for each $(s, a) \in \mathcal{S} \times \mathcal{A}$, that means,

\[
\sum_{s' \in \mathcal{S} \cup \{\ast\}} \sum_{r \in \mathcal{R}} p(r, s'|s, a) = 1, \quad \text{for every } (s, a) \in \mathcal{S} \times \mathcal{A}.
\]

$\ast$ is a special symbol meaning termination; if the current state is $s$ and the action is $a$, then $\sum_{r \in \mathcal{R}} p(r, \ast|s, a)$ is the probability of termination at the next step.

\footnote{The invariant synthesis problem in this paper is thus defined in terms of three constrained Horn clauses for simplicity and $\Phi$ denotes the conjunction of the three. This does not lose generality as linear CHC with multiple predicate variables and clauses can always be converted to this form by replacing the predicate variables with a single predicate variable representing their direct sum and then merging the clauses.}
The behaviour of the agent is described by a policy, which is a function \( \pi : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1] \) such that \( \sum_{a \in \mathcal{A}} \pi(s, a) = 1 \) for every \( s \in \mathcal{S} \). If the current state is \( s \), the agent chooses \( a \) as the next action with probability \( \pi(s, a) \). We write \( \pi(a|s) \) for \( \pi(s, a) \).

A pair of a Markov decision process and a policy probabilistically generates a sequence

\[
  s_0, a_0, r_1, s_1, a_1, r_2, s_2, \ldots
\]

where \( a_i \) is a sample of \( \pi(-|s_i) \) and \( s_{i+1} \) is a sample drawn from the distribution specified by \( p(\cdot, \cdot|s_i, a_i) \). We assume that the interaction terminates at step \( n \), i.e. \( s_n = \star \). Then, the return is the sum of the rewards \( r_1 + r_2 + \cdots + r_n \) (or \( r_1 + \gamma r_2 + \cdots + \gamma^{n-1} r_n \) where \( \gamma \in [0, 1] \) is a discount factor).

The goal of RL is to find a policy that maximize the expected return. There are many learning algorithms that can achieve this. For example, our implementation for the experiments in Section 5 uses the first-visit on-policy Monte Carlo control 17 and the Advantage Actor-Critic 28.

3 Template-Based CEGIS for ISP

3.1 CounterExample Guided Inductive Synthesis (CEGIS) for ISP

CounterExample-Guided Inductive Synthesis (CEGIS) [48] solves a given CHC \( \Phi(F(x)) \) via the interaction between a synthesizer (S) and a validator (V). Its high-level workflow is described as follows.

- S maintains a ground constraint \( \mathcal{E}(F(x)) \); \( \mathcal{E}(F(x)) \) is called example instances. The constraint \( \mathcal{E}(F(x)) \) is a necessary condition for \( \Phi(F(x)) \), i.e., every solution of \( \Phi(F(x)) \) satisfies \( \mathcal{E}(F(x)) \) as well. The task of S is to synthesize a candidate solution \( \psi(x) \), which is a solution of \( \mathcal{E}(F(x)) \). Notice that S does not access the CHC \( \Phi(F(x)) \). S then sends \( \psi(x) \) to V.
- V checks whether the candidate solution \( \psi(x) \) sent from S is a real solution to \( \Phi(F(x)) \) by checking whether \( \varphi_{pre}(x) \land \neg \psi(x) \land \varphi_{trans}(x, y) \land \psi(x) \land \neg \psi(y) \) or \( \psi(x) \land \neg \varphi_{post}(x) \) is satisfiable. Most solvers implement this step by querying the satisfiability of the above formulas using an off-the-shelf solver such as Z3. If all of the formulas are unsatisfiable, then it follows that \( \Phi(\psi(x)) \) is true and thus \( \psi(x) \) is a real solution to the CHC \( \Phi(F(x)) \). Otherwise, one of the above formulas is satisfiable, e.g., \( \varphi_{trans}(c, d) \land \psi(c) \land \neg \psi(d) \) is true for some vectors \( c, d \) of constants. Then, V sends \( F(c) \Rightarrow F(d) \) to S as a new example instance to be satisfied. S updates \( \mathcal{E}(F(x)) \) to \( \mathcal{E}(F(x)) \land (F(c) \Rightarrow F(d)) \) and seeks a new candidate solution again.

3.2 Template-Based Approach to CEGIS

In each step of a CEGIS-based CHC solving, a synthesizer needs to find a candidate solution that satisfies all the constraints in the current example instance \( \mathcal{E}(F(x)) \). One of the strategies to implement the candidate-solution discovery is called a template-based approach [3, 17, 51].
A template-based synthesizer works as follows. It maintains an example instance $E(F(x))$ and a template $\psi(a, x)$, which is a predicate over $x$ with parameters $a$, and constructs a candidate solution by finding an appropriate assignment to $a$. An appropriate assignment to $a$ can be computed by using an SMT solver such as Z3: since $E(F(x))$ is a ground constraint, i.e., a formula with no quantifier nor variable, $C(a) := E(\psi(a, x))$ is a quantifier-free formula with free variables $a$ and an SMT solver gives a satisfying assignment $\sigma$ to $a$, provided that $C(a)$ is satisfiable. Then $\psi(\sigma(a), x)$ is a candidate solution. If $C(a)$ is not satisfiable, there is no candidate solution of the form $\psi(c, x)$, where $c$ is a vector of constants. Then, the synthesizer heuristically updates the template and uses it to discover a new candidate solution. The strategies for updating the templates is what we mean by heuristics below.

For Constraint $[\Box]$ a synthesizer would be able to discover the solution $x + y = z \land y \geq 0$ if it designates a template $a_1 x + b_1 y + c_1 z \geq 0 \land a_2 x + b_2 y + c_2 z \geq 0 \land a_3 x + b_3 y + c_3 z \geq 0$; the above solution is obtained once an SMT solver finds a solution $(a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3) = (1, 1, -1, -1, 1, 0, 0, 1)$.

Notice that a template $\psi(a, x)$ determines a set of candidate solutions, each member of which is an instantiation of $a$ in $\psi(a, x)$ to concrete values. Therefore, a template in the template-based approach determines the search space for a candidate solution of a given constraint. A template is said to be more expressive than another if the set of formulae that is obtained by instantiating the parameters in the latter is a subset of the former.

There is a trade-off between the expressiveness of a template and the efficiency of a synthesizer [57, 51]. If one uses an expressive template, there is more chance that there is a true solution that can be obtained by instantiating the template. However, the constraint $C(a)$ generated by using an expressive template tends to be complex, which incurs performance degradation of SMT solving and therefore a synthesizer. In deciding which template to be used, it is crucial to find a sweet spot that addresses this trade-off.

Algorithm $[\Box]$ is a typical template-based synthesizer for CHC solving. This procedure uses the following subprocedures:

- $SMT(C(a))$: Decides whether the predicate $C(a)$ is satisfiable or not by an SMT solver. If it is satisfiable, then it returns $Sat(\sigma)$ where $\sigma$ is a value assignment to $a$ in $C(a)$ such that $\sigma(C(a))$ is valid. If not, it returns $Unsat(I)$, where $I$ is a collection of various information on the behavior of the decision procedure (e.g., consumed time and memory). $I$ also includes explanations why $C(a)$ is unsatisfiable such as an unsat core, which is an unsatisfiable subconstraint of $C(a)$. We write $I.uc$ for the unsat core contained in $I$.
- $Validator(\varphi(x))$: Sends the candidate solution $\varphi(x)$ to the validator and let it decide whether it is a real solution. If $\varphi(x)$ is indeed a solution, then the validator returns $Valid$. Otherwise, the validator returns $Cex(E(F(x)))$, where $E(F(x))$ is the new example instance to be satisfied by solutions.
- $ChangeTempl(\psi(a, x), I)$: Heuristically updates the template $\psi(a, x)$ to a new one based on the information $I$ returned by the SMT solver.
Algorithm 1 Template-based synthesizer

1: $\psi(\mathbf{a}, \mathbf{x}) \leftarrow$ the initial template
2: $E(F(\mathbf{x})) \leftarrow \emptyset$
3: while Timeout is not reached do
4: $C(\mathbf{a}) \leftarrow E(\psi(\mathbf{a}, \mathbf{x}))$
5: $r_1 \leftarrow \text{SMT}(C(\mathbf{a}))$
6: if $r_1 = \text{Sat}(\sigma)$ then
7: Instantiate parameters
8: $c \leftarrow \sigma(\mathbf{a})$
9: Send the candidate solution to the validator
10: $r_2 \leftarrow \text{Validator}(\psi(c, \mathbf{x}))$
11: Check whether $\psi(c, \mathbf{x})$ is a real solution
12: if $r_2 = \text{Valid}$ then
13: Return $\psi(c, \mathbf{x})$ as a real solution
14: else if $r_2 = \text{Cex}(E'(F(\mathbf{x})))$ then
15: Cex is found; new example instance is received.
16: $E(F(\mathbf{x})) \leftarrow E(F(\mathbf{x})) \land E'(F(\mathbf{x}))$
17: end if
18: else if $r = \text{Unsat}(I)$ then
19: $\psi(\mathbf{a}, \mathbf{x}) \leftarrow \text{ChangeTempl}(\psi(\mathbf{a}, \mathbf{x}), I)$
20: end if
21: end while

3.3 Synthesizer Implemented in PCSat

PCSat [41, 51] is one of the tools for CHC solving based on template-based CEGIS. It uses a family of template $\psi_{N,P,Q}(\mathbf{a}, \mathbf{x})$, in which each template is determined by the values $P, Q$ and $\mathbf{N} = (N_1, \ldots, N_M)$ (of which the length is denoted by $M$). Assume that $\mathbf{x} := (x_1, \ldots, x_L)$. Then the parameter $\mathbf{a}$ used in these templates consists of the following parameters.

- **Coefficient parameters** $a_{ij}^{(k)}$ for each $1 \leq i \leq M, 1 \leq j \leq N_i$ and $1 \leq k \leq L$.
- **Constant parameters** $c_{ij}^{(k)}$ for each $1 \leq i \leq M$ and $1 \leq j \leq N_i$.

The template family $\psi_{N,P,Q}(\mathbf{a}, \mathbf{x})$ is defined as follows using these parameters:

$$\bigvee_{i=1}^{M} \bigwedge_{j=1}^{N_i} \sum_{k=1}^{L} a_{ij}^{(k)} x_k \geq c_{ij}^{(k)} \land \bigwedge_{i=1}^{M} \bigwedge_{j=1}^{N_i} \sum_{k=1}^{L} |a_{ij}^{(k)}| \leq P \land \bigwedge_{i=1}^{M} \bigwedge_{j=1}^{N_i} |c_{ij}^{(k)}| \leq Q.$$ 

We explain the above definition in the following.

- The subformula $\bigvee_{i=1}^{M} \bigwedge_{j=1}^{N_i} \sum_{k=1}^{L} a_{ij}^{(k)} x_k \geq c_{ij}^{(k)}$ is a boolean combination of linear inequalities $\sum_{k=1}^{L} a_{ij}^{(k)} x_k \geq c_{ij}^{(k)}$ expressed in a disjunctive normal form (DNF). The parameter $M$ (resp. $N_i$) is the number of disjuncts (resp. conjuncts in each disjunct) in this template; therefore, the larger they are, the more expressive the template is.
The subformula $\bigwedge_{i=1}^{M} \bigwedge_{j=1}^{N_i} \sum_{k=1}^{L} |a_k^{(ij)}| \leq P$ bounds the sum of the absolute values of the coefficients in each linear inequality (i.e., the $L^1$ norm of each $(a_k^{(ij)})_{1 \leq k \leq L}$). The bound $P$ is a natural number or $\infty$; if $P = \infty$, then the coefficients may be any number. The larger $P$ is, the more expressive the template is.

The subformula $\bigwedge_{i=1}^{M} \bigwedge_{j=1}^{N_i} |c^{(ij)}| \leq Q$ bounds the absolute value of the constant $c^{(ij)}$ in each linear inequality. The bound $Q$ is a natural number or $\infty$. The larger $Q$ is, the more expressive the template is.

The strategy of PCSat actually uses a subset of above-defined templates, namely the subset of templates of the form $([N] * M, P, Q)$ where $[N] * M$ means the list of length $M$ consisting only of $N$. In other words, $N_i = N$ holds for every template reachable by the hand-crafted strategy of PCSat.

As mentioned in Section 3.2, an update to a template happens when the constraint $C(a)$ on parameters $a$ is unsatisfiable. PCSat decides how to update a template using the unsat core of $C(a)$.

Concretely, the heuristic implemented by PCSat is as follows. $([N] * M, P, Q)$ are initialized to $(1, 1, 1, 0)$. When PCSat needs to update the current template, it increments $M$ or $N$ by 1 and it may increment $P$ and/or $Q$ depending on the unsat core. If $P$ occurs in the unsat core, then $P$ is incremented by 1. If $Q$ occurs in the unsat core and $Q < 3$, then $Q$ is incremented by 1; if $Q \geq 3$, then $Q$ is set to $\infty$.

4 Finding Heuristics with Reinforcement Learning

This section describes how we formulate the problem of learning heuristics as a reinforcement learning (RL) problem. In our setting, the environment is an implementation of the template-based CEGIS parameterized by heuristics. We learn a policy that represents a heuristic, which tells the environment the shape of the template that should be tried in order to guess the next candidate solution. The long-term goal is to find a solution of a given CHC, preferably in a short time.

We model the template-based CEGIS procedure as a Markov decision process (MDP) as follows, and implement RL algorithms to solve such an MDP.

States. A state of the MDP in our formalism is $([N, P, Q, f_1, f_2, z]$ where $([N, P, Q)$ are values that determine the current template, $f_1, f_2$ are boolean values that summarize the information of the unsat core for the current template, and $z$ is the number of candidate solutions that the learner found since the previous update of template. $f_1$ is true if the unsat core contains $P$, and $f_2$ is true if the unsat core contains $Q$. The information expressed by flags $f_1$ and $f_2$ is also used by heuristics engineered by the PCSat developers. The parameter $z$ is inspired by Code2Inv [49], which uses the number of example instances satisfied by the current state of the environment as part of a reward.
**Actions.** An action is a tuple \((n, p, q) \in \mathbb{N}^* \times (\mathbb{N} \cup \{\infty\}) \times (\mathbb{N} \cup \{\infty\})\), which updates the current template \((N, P, Q)\) to \((N + n, P + p, Q + q)\). Here \(N + n\) is the coordinate-wise sum; if the lengths of \(N\) and \(n\) are different, we append 0’s to the tail of the shorter one. For example, if \(N = (1, 1)\) and \(n = (0, 0, 1)\), then \(N + n = (1, 1, 1)\). Note that the length \(M\) of \(N\) can be increased by choosing sufficiently long \(n\).

**Rewards.** Since simply checking whether a solution is found or not may have the problem of sparse rewards [49, Section 17.4] that makes learning harder, we define the reward associated with each action to be \(-T\), where \(T\) is the time spent since the last invocation of the agent. The sum of the rewards is then naturally \(-T_{\text{total}}\), where \(-T_{\text{total}}\) is the total time spent for the run. Intuitively, the earlier a solution is found, the more reward is given. The smallest reward is given if the synthesizer eventually times out when guided by the agent.

## 5 Experiments

**Datasets and tasks.** We use the problems in the Inv-Track of the SyGuS-Comp [4] 2019 competition [9] as the data set of our experiments. A tool for this competition is supposed to return one of the three answers: \textsc{Sat}, indicating that the tool successfully synthesized an answer to the given problem; \textsc{Unsat}, indicating that there is no solution to the given problem; and \textsc{Timeout}, indicating that the tool fails to solve the given problem within a specified time limit. An answer of \textsc{Sat} must be accompanied by a witness, which is an invariant in the case of Inv-Track. The Inv-Track consists of 858 problems for evaluating the performance of invariant-synthesis tools. We randomly split the problems in the Inv-Track into training and test sets in an 80:20 ratio. The learning task is to train an agent that is capable of guiding PCSat to find a solution to each problem as soon as possible.

**Configurations.** We train the agent using two different reinforcement learning algorithms: the first-visit on-policy Monte Carlo (MC) control [47] and Advantage Actor-Critic (A2C) [28]. The former is a tabular method that relies on state-action value tables and the latter leverages deep learning. The state and action spaces are as described in Section 4.

For MC, in order to control the size of the state-action value table and make the learning tractable, we set up an upper bound for each parameter in the

---

8 The actions adopted in the present paper only increase the complexity of templates. Before settling on the current design, we tried actions that can reduce the complexity of templates and states that incorporate the timeout feedback from Z3 (as a hint for the complexity of the current template). We however omitted them in the present paper because early experiments showed that learning with Monte Carlo control became unstable and the performance decreased in such a setting.

9 The problems can be found at [https://github.com/SyGuS-Org/benchmarks](https://github.com/SyGuS-Org/benchmarks).
We train the agent with $\epsilon$-greedy action selection with $\epsilon = 0.05$ and evaluate it with the greedy policy. The discount factor is set to 1. The time limit for each problem during training is 120 seconds.

For A2C, given a trajectory $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_T, a_T, r_T)$, our objective is to maximize the following expected policy rewards with discount factor $\gamma = 0.99$

$$E_{\tau \sim \pi(\theta)} \left[ \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'} - V_\varphi(s_t) \right]$$

where $\theta$ is the parameters of the actor policy $\pi(\theta)$ and $\varphi$ is the parameters of the critic policy $V_\varphi$. Each policy is parameterized by a multi-layer perceptron with two hidden layers of dimensions 256 and 512. We use the loss

$$\mathcal{L}(\varphi) = \sum_{t} \left( \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'} - V_\varphi(s_t) \right)^2$$

to train the critic. Both policies are trained using RMSProp \[19\] with a learning rate of $5 \times 10^{-5}$. The time limit for each problem during training is 10 seconds.

For each experiment, we train the agent for 200 epochs on the training set. Among the learned policies, we chose the one that has the best performance on the training set and evaluate them on the test set. All the experiments are conducted on PCSat with SMT solver Z3 (version 4.8.9) on a 2.8GHz Intel(R) Xeon(R) CPU with 64 GB RAM and a Tesla V100 GPU with 16GM RAM.

5.1 Evaluation

In this section, we study the quality of the heuristics learned by our framework by comparing them with the heuristic engineered by human experts (i.e., the PCSat developers), existing non-learning-based solvers (CVC4 \[6, 7, 39\], LoopInvGen \[37\], HoICE \[9\], Eldarica \[20\], FreqHorn \[15\] and Spacer \[27\]) and the state-of-the-art neural solver Code2Inv \[45, 46\].

The action space here is restricted to $(n, p, q) \in \{0, 1\}^4 \times \{0, 1, \infty\} \times \{0, 1, \infty\}$. In particular, the number of conjuncts in templates is unbounded but the number of disjuncts is at most 4. The agent’s ability to observe a state is thus limited. For $N_i$, the agent does not distinguish 4 with greater values; for $P$ and $Q$ it does not distinguish between 5 with greater values. For example, the state $((2, 1, 4, 0), 7, 4)$ looks $((2, 1, \geq 4, 0), \geq 5, 4)$ to the agent, where $\geq 4$ and $\geq 5$ are symbols representing numbers greater than or equal to 4 and 5, respectively.

Technical terms of reinforcement learning that are not explained in this paper are used in this and the next paragraphs, in order to appropriately describe the experimental setting. See a textbook \[49\] for a reference.

We have also tried to use CLN2INV \[40\], which is a deep-learning-based invariant-synthesis tool, as a baseline. However, we excluded it from our baseline since the implementation that is made public is not fully automated. It uses hints that are given by human to solve certain problems.
If classified by the underlying approaches to ISP, LoopInvGen, HoICE, FreqHorn and Code2Inv are all CEGIS-based solvers. LoopInvGen is based on CEGIS with greedy set covering for synthesis. HoICE uses decision-tree based CEGIS while FreqHorn and Code2Inv use grammar-based CEGIS. Eldarica and Spacer are non-CEGIS-based solvers. Eldarica uses CounterExample-Guided Abstraction Refinement (CEGAR) with predicate abstraction, and Spacer is based on Property Directed Reachability (PDR).

LoopInvGen and CVC4 are respectively the winners of SyGuS-Comp 2018 and 2019. Eldarica and Spacer are participants of some of the linear CHC tracks and won the 1st and 2nd places.

Table 1. Performance of the learned heuristics and that of the baselines on the test set. PCSat/random refers to PCSat guided by a random policy, and PCSat/expert refers to PCSat using the heuristic engineered by its developers. PCSat/A2C and PCSat/MC refer to PCSat using the heuristics learned with the corresponding approaches, as described in Section 5. PCSat/A2C+PCSat/MC means that the two policies work jointly in parallel, and a problem is solved if one of them returns a solution. See footnote 14 for the solvers marked with *.

| Methods       | approach | sat | unsat | timeout | time(s) |
|---------------|----------|-----|-------|---------|---------|
| FreqHorn      | CEGIS    | 70  | 0     | 101     | 6863    |
| LoopInvGen*   | CEGIS    | 87  | 5     | 79      | 5086    |
| CVC4*         |          | 102 | 9     | 60      | 3873    |
| PCSat/random   | CEGIS    | 116 | 9     | 46      | 3383    |
| Eldarica      | CEGAR    | 122 | 9     | 40      | 3714    |
| PCSat/expert   | CEGIS    | 135 | 9     | 27      | 2130    |
| HoICE         |          | 141 | 8     | 22      | 1707    |
| PCSat/A2C     | CEGIS    | 145 | 9     | 17      | 1947    |
| PCSat/MC      | CEGIS    | 146 | 9     | 16      | 1550    |
| PCSat/A2C+PCSat/MC | CEGIS | 149 | 9     | 13      | 1460    |
| Spacer        | PDR      | 156 | 9     | 6       | 380     |

Comparison with non-learning-based solvers. Table 1 shows the number of problems in the test set solved by each method given a time limit of 60 seconds.

---

13 According to https://chc-comp.github.io/2021/presentation.pdf. Also note that our RL formulation itself is general so that it is also applicable to non-linear CHC, and so is PCSat. The present paper focuses on the effectiveness for linear CHC as a first step because it is a non-trivial class of practical importance.

14 Unfortunately, we were not able to reproduce the same level of performance as described in the competition report of LoopInvGen and CVC4, possibly due to the differences in versions and configurations used by the solvers. CVC4 (resp. LoopInvGen) solved 592 (resp. 512) instances out of 858 according to the competition.
It can be seen that the learning is effective — compared to PCSat guided by a random policy, the policy learned by MC solves significantly more problems (146 as opposed to 116). The learned heuristics are also better than the heuristic engineered by human experts. PCSat guided by MC solves 146, while it solves only 135 when using heuristic designed by its developers.

The difference in the number of solved problems between the two learned heuristics seems to be minimal. Nonetheless, we observed that PCSat/A2C solves three problems whose solutions were not found by PCSat/MC and PCSat/MC solved four problems not solved by PCSat/A2C. In fact, as illustrated by Figure 2, while the two policies are similar in terms of time spent on solving most of the problems, they start to disagree on some of the problems when a larger time limit is allowed, with one being able to solve them in a short period of time, and the other taking significantly longer. This suggests that the difference in learning algorithms may have indeed induced different heuristics, and using learned heuristics jointly may boost the performance of PCSat in practice.

PCSat guided by the learned heuristics outperforms most of the existing non-learning-based solvers given a time limit of 60 seconds, with the only exception of Spacer. The difference in performance between the solvers is reduced when a larger time limit is allowed. Figure 1 and Table 2 show the difference in performance when using a time limit of 600 seconds. It can also be seen that the learned heuristics achieved the best performance among all CEGIS-based solvers regardless of the time limit. Spacer is the only one that significantly outperforms PCSat with learned heuristics and every other solver. The performance of Spacer on the test set seems to be an outlier, for which we do not know the exact reason. Nonetheless, below we are forced to use a different test set in or-

---

**Table 2. Performance with a time limit of 600 seconds.**

| Methods           | approach | sat | unsat | timeout | time(s) |
|-------------------|----------|-----|-------|---------|---------|
| FreqHorn          | CEGIS    | 91  | 0     | 80      | 52933   |
| LoopInvGen*       | CEGIS    | 90  | 5     | 76      | 46445   |
| CVC4*             | -        | 106 | 9     | 56      | 34869   |
| PCSat/random       | CEGIS    | 121 | 9     | 41      | 25858   |
| PCSat/expert       | CEGIS    | 145 | 9     | 17      | 12537   |
| HoICE             | CEGIS    | 147 | 9     | 15      | 10253   |
| PCSat/MC          | CEGIS    | 150 | 9     | 12      | 8705    |
| Eldarica          | CEGAR    | 151 | 9     | 11      | 12866   |
| PCSat/A2C         | CEGIS    | 151 | 9     | 11      | 8319    |
| PCSat/A2C+PCSat/MC| CEGIS    | 154 | 9     | 8       | 6413    |
| Spacer            | PDR      | 156 | 9     | 6       | 3619    |
Consider to compare the performance of our framework with that of Code2Inv, and noticed that Spacer is not always the winner on every benchmark.

**Comparison with Code2Inv.** We now compare the learned heuristics with the neural solver Code2Inv. Ideally, we would like to evaluate Code2Inv on our test set. However, the tools accompanied by Code2Inv for generating the graph representations of given problems failed to correctly convert the problems in our benchmark into a form readable by Code2Inv. For fairness, we use all the common problems that occur in both our data set and the Code2Inv benchmark, in their original forms readable by each solver respectively. There are 81 such problems in total. Since our training set may contain problems from the Code2Inv benchmark, we re-train our agent on a training set containing 127 problems from the SyGuS-Comp 2018 competition, which does not overlap the set of 81 common problems.

Table 3 shows the performance of our agent and that of Code2Inv as an out-of-the-box solver given a time limit of 600 seconds. We also add the performance of Spacer as an indicator of the state-of-the-art performance on this set of problems. It can be seen that PCSat with learned heuristics not only outperforms Code2Inv by solving 50% more problems, but also slightly surpasses Spacer in terms of the number of solved problems. Figure 3a and Figure 3b illustrate the performance on individual problems. Noticeably, every problem solved by Code2Inv is solved by our agent in a shorter time. Our agent is also able to find four solutions that were not found by Spacer.
The limited performance of Code2Inv is possibly due to the need to learn every problem from scratch, because the shapes of the neural networks in Code2Inv’s algorithm depend on the graph representation of the target problem. Different problems may use different neural networks, which makes it difficult to apply the learned policy for one problem to another which has a completely different graph representation. In contrast, our approach does not have such a restriction—the learned policies are applicable to all the problems as long as the state and action spaces remain unchanged.

Table 3. Performance of PCSat guided by the learned heuristics and that of Code2Inv and Spacer, on the new test set.

| Methods       | approach | sat | unsat | timeout | time(s) |
|---------------|----------|-----|-------|---------|---------|
| Code2Inv      | CEGIS    | 45  | -     | 36      | 27289   |
| Spacer        | PDR      | 70  | 6     | 5       | 3009    |
| PCSat/A2C     | CEGIS    | 72  | 6     | 3       | 2373    |

5.2 Ablation: design of the state space

So far we have been using the state space as described in Section 4. In this section, we study how a different state space may affect the performance of the learned heuristics. In particular, we are interested in knowing whether embedding expert knowledge into the state space helps improve the performance. To see if the expert knowledge helps, we introduce a new design of state space as follows which reflects the human expert knowledge of invariant synthesis.
Recall that the strategy of PCSat only uses templates of the form \((N \times M, P, Q)\) where \([N] \times M\) is the list of length \(M\) consisting only of \(N\). An expert state is an abstraction obtained from the original state \((N \times M, P, Q, f_1, f_2, z)\). Concretely, an expert state is a tuple \((b_0, b_1, b_2, b_3, b_4, b_5, f_1, f_2, z')\) of the boolean values, each element of which summarizes a corresponding parameter of the original state as follows.

- \(b_0\) (resp. \(b_1\)) \(\iff M < N\) (resp. \(M = N\)).
- \(b_2\) (resp. \(b_3\)) \(\iff P = \infty \lor P \geq 2\) (resp. \(P = \infty \lor P \geq 5\)).
- \(b_4\) (resp. \(b_5\)) \(\iff Q = \infty \lor Q \geq 2\) (resp. \(Q = \infty \lor Q \geq 5\)).
- \(f_1, f_2\) are the same as in the original state.
- \(z' \iff z > 0\).

This design of the state space is inspired by the heuristic engineered by the developers of PCSat (i.e., the heuristic used in the baseline PCSat/expert in Section 5.1). For example, the boolean value \(b_1\) can be used to alternate incrementing \(M\) and \(N\). The baseline heuristic also uses the values of \(P\) and \(Q\) to increase \(P\) and \(Q\) gradually, and the developers of PCSat believe that 2 and 5 are good magic numbers that balance the expressiveness of the templates well. The developers also believe that whether \(z = 0\) or not should be the most significant information about \(z\).

The expert state space is much smaller than the original state space as it has eliminated “irrelevant” possible states as believed by human experts. The size of the expert state space is only 512. Such a state space is especially useful when training with tabular methods such as MC, as we do not need to worry about huge state-action value tables that are impossible to be fully explored. Given the expert state space, the corresponding expert action is then represented by
\((n, m, p, q) \in \{0, 1\} \times \{0, 1\} \times \{0, 1, \infty\} \times \{0, 1, \infty\}\), which updates \((\lceil N \rceil * M, P, Q)\) to \((\lceil N + n \rceil * (M + m), P + p, Q + p)\).

We train an agent using MC with the expert state space and expert action space, and compare its performance with that of the agents trained with the original state space defined in Section 4. We call states in the original state space raw states below. Table 4 shows the performance of the four agents. It can be seen that while MC using the expert state space learns well, the extra expert knowledge does not help solve more problems. Conversely, the slightly worse performance may be a result of the overly abstracted state space — an agent using this space may not be able to distinguish states that may have otherwise led to different updates of a template when using a richer representation of states such as the raw states.

On the other hand, Figure 4a shows that MC using expert states helped find solutions not found by MC using the raw states, and vice versa. Figure 4b and Figure 4c further shows that the performance of MC/raw working together with MC/expert is almost identical to that of it working together with A2C/raw, suggesting that the expert insight embedded in the expert states might be learnable by A2C using the raw states.

Table 4. Performance of the learned heuristics using the expert states, as indicated by MC/expert, and the learned heuristics using the raw states, as indicated by MC (resp. A2C)/raw.

| Methods       | sat | unsat | timeout | time(s) |
|---------------|-----|-------|---------|---------|
| random        | 121 | 9     | 41      | 25858   |
| MC/expert     | 148 | 9     | 14      | 9337    |
| MC/raw        | 150 | 9     | 12      | 8705    |
| A2C/raw       | 151 | 9     | 11      | 8318    |

6 Related Work

This paper focused on a template-based approach to CEGIS [3, 17, 51], while grammar-based synthesis [8, 12, 16, 26, 45, 46] (e.g., Code2Inv) and decision-tree learning [9, 14, 18, 29, 30, 55] (e.g., HoICE) are also popular approaches to guessing a candidate solution from gathered data in CEGIS. Although the template-based approach is advantageous in that it adaptively adjusts atomic formulae to be used in a candidate solution, it requires careful tuning of a heuristic for deciding the shape of a candidate solution. We have addressed this challenge by leveraging RL to learn an effective heuristic.

Si et al. [45, 46] proposed a framework called Code2Inv to learn loop invariants with deep reinforcement learning. Code2Inv uses graph neural networks to encode the information of a program and synthesizes loop invariants using
a syntax-directed encoder-decoder structure. In contrast to their approach that tries to synthesize an invariant directly from a program, we learn the heuristics that guide the search in existing tools for program verification. One restriction of Code2Inv is that the shape of the neural networks that parameterize its policy depend on the graph representation of the target problems. It is unclear how a policy learned for one problem can be easily applied to another that has a completely different graph representation. In contrast, our policy learned for heuristics is universal — it works with any problem readable by the base solver.

Code2Inv and our approach also differ in the evaluation of the obtained solver. Si et al. evaluated the performance of their solver in the numbers of queries to Z3 \[36\], instead of measuring the actual running time. Although the running

![Graphs showing comparison of time spent on each problem by different methods.](image)

**Fig. 4.** (a)(b) Comparison of the time spent on each problem by different methods. (c) Cumulative time of each method.
cost of Z3 is dominant in most of the program verification tasks, Code2Inv conducts online learning of a neural network, which incurs non-trivial additional cost to the performance. On the contrary, the performance of our solver is evaluated in the wall-clock time.

RL has also been applied to program synthesis [8, 12] and relational verification [11]. These works learn heuristics that select one inference rule or production rule from finitely many options, while we apply RL to aid prioritized search for an infinite set of candidate solutions. In other words, we learn heuristics to synthesize cut-formulas, which correspond to loop invariants in our problem setting. Approaches such as Concord-StandardPG [12] also need to retrain the policies for each problem just like Code2Inv [45, 46], while our approach does not have such limitation.

The benefits from learning heuristics for theorem proving has been demonstrated in various automated theorem proving techniques including CDCL for SAT [33, 43, 44], strategies for SMT [3], connection tableau [24], and incremental determinization for QBF [32, 43]. These previous studies applied machine learning to enhance the proof search. We cannot directly apply these theorem proving techniques to the ISP because we need to synthesize an appropriate predicate as a solution to the ISP, in addition to deciding the validity of a given formula.

Extensive research has been conducted on learning embedding of programs and logical formulae through neural networks such as LSTMs [21, 22], tree-based neural networks [13, 34, 35], graph neural networks [1, 31, 38, 52], and a path-based attention model [2]. The present work does not use learned embedding of invariants or programs. It is an interesting future direction to investigate whether using embedding serves for any further improvement.

7 Conclusion

We presented how to apply reinforcement learning to the task of learning effective heuristics for a template-based CEGIS procedure. To this end, we modeled the behavior of the procedure as an MDP and a heuristic as an agent that updates a template. We trained the agents using the first-visit on-policy Monte Carlo control (MC) and Advantage Actor-Critic (A2C); the learned heuristics are the best in its kind — they outperformed the heuristics engineered by human experts, and achieved the best performance among the CEGIS-based solvers. The learned heuristics have also demonstrated comparable (and sometimes superior) performance to that of the state-of-the-art non-CEGIS-based solvers, validating the effectiveness of our approach.

We have focused on programs that have only one loop, whose loop-invariant synthesis can be reduced to solving a linear CHC with a single predicate variable. Future work includes handling a broader class of constraints such as CHCs with multiple predicate variables for safety verification of a program with multiple loops and the class pfwCSP of constraints for relational [51] and branching-time temporal [50] verification.
Acknowledgments

This work was supported in part by JSPS KAKENHI Grant Numbers JP19K22842, JP19H04084, JP20H04162, and JP20H05703, JST CREST Grant Number JP-MJCR2012, Japan, and ERATO HASUO Metamathematics for Systems Design Project (No. JPMJER1603), JST.
Bibliography

[1] Allamanis, M., Brockschmidt, M., Khademi, M.: Learning to represent programs with graphs. In: ICLR ’18. OpenReview.net (2018)

[2] Alon, U., Zilberstein, M., Levy, O., Yahav, E.: Code2vec: Learning distributed representations of code. Proceedings of the ACM on Programming Languages 3(POPL) (Jan 2019)

[3] Alur, R., Bodik, R., Dallal, E., Fisman, D., Garg, P., Juniwal, G., Kress-Gazit, H., Madhusudan, P., Martin, M.M.K., Raghothaman, M., Saha, S., Seshia, S.A., Singh, R., Solar-Lezama, A., Torlak, E., Udupa, A.: Syntax-guided synthesis. In: Dependable Software Systems Engineering, pp. 1–25 (2015)

[4] Alur, R., Fisman, D., Padhi, S., Reynolds, A., Singh, R., Udupa, A.: SyGuS, https://sygus.org accessed on 15-Oct-2021

[5] Balunovic, M., Bielik, P., Vechev, M.T.: Learning to solve SMT formulas. In: NeurIPS ’18. pp. 10338–10349 (2018)

[6] Barbosa, H., Reynolds, A., Larraz, D., Tinelli, C.: Extending enumerative function synthesis via SMT-driven classification. In: FMCAD ’19. pp. 212–220 (Oct 2019)

[7] Barrett, C., Conway, C.L., Deters, M., Hadarean, L., Jovanović, D., King, T., Reynolds, A., Tinelli, C.: CVC4. In: CAV ’11. p. 171–177. Springer (2011)

[8] Bunel, R., Hausknecht, M., Devlin, J., Singh, R., Kohli, P.: Leveraging grammar and reinforcement learning for neural program synthesis. In: ICLR ’18 (2018), https://openreview.net/forum?id=H1Xw62kRZ

[9] Champion, A., Chiba, T., Kobayashi, N., Sato, R.: ICE-based refinement type discovery for higher-order functional programs. In: TACAS ’18. LNCS, vol. 10805, pp. 365–384. Springer (2018)

[10] Champion, A., Kobayashi, N., Sato, R.: HoIce: An ice-based non-linear horn clause solver. In: APLAS ’18. pp. 146–156. Springer (2018)

[11] Chen, J., Wei, J., Feng, Y., Bastani, O., Dillig, I.: Relational verification using reinforcement learning. Proceedings of the ACM on Programming Languages 3(OOPSLA) (2019)

[12] Chen, Y., Wang, C., Bastani, O., Dillig, I., Feng, Y.: Program synthesis using deduction-guided reinforcement learning. In: CAV ’20. pp. 587–610. Springer, Cham (2020)

[13] Evans, R., Saxton, D., Amos, D., Kohli, P., Grefenstette, E.: Can neural networks understand logical entailment? In: ICLR ’18. OpenReview.net (2018)

[14] Ezudheen, P., Neider, D., D’Souza, D., Garg, P., Madhusudan, P.: Horn-ICE learning for synthesizing invariants and contracts. Proceedings of the ACM on Programming Languages 2(OOPSLA), 131:1–131:25 (Oct 2018)

[15] Fedyukovich, G., Kaufman, S.J., Bodik, R.: Sampling invariants from frequency distributions. In: 2017 Formal Methods
Learning Heuristics for Template-based CEGIS with Reinforcement Learning

in Computer Aided Design (FMCAD). pp. 100–107 (2017). https://doi.org/10.23919/FMCAD.2017.8102247

[16] Fedyukovich, G., Prabhu, S., Madhukar, K., Gupta, A.: Solving constrained horn clauses using syntax and data. In: FMCAD ’18. pp. 1–9 (2018). https://doi.org/10.23919/FMCAD.2018.8603011

[17] Garg, P., Löding, C., Madhusudan, P., Neider, D.: ICE: A robust framework for learning invariants. In: CAV ’14. pp. 69–87. Springer (2014)

[18] Garg, P., Neider, D., Madhusudan, P., Roth, D.: Learning invariants using decision trees and implication counterexamples. In: POPL ’16. pp. 499–512. ACM (2016)

[19] Hinton, G., Srivatava, N., Swersky, K.: Lecture 6c: rmsprop: Divide the gradient by a running average of its recent magnitude (2012), lecture notes available from http://www.cs.toronto.edu/~hinton/coursera/lecture6/lec6.pdf

[20] Hojjat, H., Rümmer, P.: The Eldarica horn solver. In: FMCAD ’18. pp. 1–7. IEEE (2018)

[21] Irving, G., Szegedy, C., Alemi, A.A., Eén, N., Chollet, F., Urban, J.: DeepMath - deep sequence models for premise selection. In: NIPS ’16. pp. 2235–2243 (2016)

[22] Iyer, S., Konstas, I., Cheung, A., Zettlemoyer, L.: Summarizing source code using a neural attention model. In: ACL ’16. The Association for Computer Linguistics (2016)

[23] Kaliszyk, C., Chollet, F., Szegedy, C.: HolStep: A machine learning dataset for higher-order logic theorem proving. In: ICLR ’17. OpenReview.net (2017)

[24] Kaliszyk, C., Urban, J., Michalewski, H., Olsák, M.: Reinforcement learning of theorem proving. In: NeurIPS ’18. pp. 8836–8847 (2018)

[25] Kaliszyk, C., Urban, J., Michalewski, H., Olsák, M.: Reinforcement learning of theorem proving. In: Bengio, S., Wallach, H., Larochelle, H., Grauman, K., Cesa-Bianchi, N., Garnett, R. (eds.) Advances in Neural Information Processing Systems. vol. 31. Curran Associates, Inc. (2018). https://proceedings.neurips.cc/paper/2018/file/55acf8539596d25624059980986aa78-Paper.pdf

[26] Kalyan, A., Molita, A., Polozov, O., Batra, D., Jain, P., Gulwani, S.: Neural-guided deductive search for real-time program synthesis from examples. In: ICLR ’18 (2018), https://openreview.net/forum?id=rywDjg-RW

[27] Komuravelli, A., Gurfinkel, A., Chaki, S.: SMT-based model checking for recursive programs. In: CAV ’14. LNCS, vol. 8559, pp. 17–34. Springer (2014)

[28] Konda, V., Tsitsiklis, J.: Actor-critic algorithms. In: Solla, S., Leen, T., Müller, K. (eds.) Advances in Neural Information Processing Systems. vol. 12. MIT Press (2000)

[29] Krishna, S., Puhrsch, C., Wies, T.: Learning invariants using decision trees. CoRR abs/1501.04725 (2015)

[30] Kura, S., Unno, H., Hasuo, I.: Decision tree learning in CEGIS-based termination analysis. In: CAV ’21. pp. 75–98. Springer (2021)
[31] Kusumoto, M., Yahata, K., Sakai, M.: Automated theorem proving in intuitionistic propositional logic by deep reinforcement learning. CoRR abs/1811.00796 (2018)
[32] Lederman, G., Rabe, M., Seshia, S., Lee, E.A.: Learning heuristics for quantified boolean formulas through reinforcement learning. In: ICLR ’20 (2020)
[33] Liang, J.H., Oh, C., Mathew, M., Thomas, C., Li, C., Ganesh, V.: Machine learning-based restart policy for CDCL SAT solvers. In: SAT ’18. pp. 94–110. Springer (2018)
[34] Loos, S.M., Irving, G., Szegedy, C., Kaliszyk, C.: Deep network guided proof search. In: LPAR ’17. EPI-C Series in Computing, vol. 46, pp. 85–105. EasyChair (2017)
[35] Mou, L., Li, G., Zhang, L., Wang, T., Jin, Z.: Convolutional neural networks over tree structures for programming language processing. In: AAAI ’16. pp. 1287–1293. AAAI Press (2016)
[36] de Moura, L., Bjørner, N.: Z3: An efficient SMT solver. In: TACAS ’08. LNCS, vol. 4963, pp. 337–340. Springer (2008)
[37] Padhi, S., Millstein, T.D., Nori, A.V., Sharma, R.: Overfitting in synthesis: Theory and practice. In: CAV ’19. LNCS, vol. 11561, pp. 315–334. Springer (2019)
[38] Paliwal, A., Loos, S.M., Rabe, M.N., Bansal, K., Szegedy, C.: Graph representations for higher-order logic and theorem proving. In: AAAI ’20. pp. 2967–2974. AAAI Press (2020)
[39] Reynolds, A., Barbosa, H., Nötzli, A., Barrett, C., Tinelli, C.: cvc4sy: Smart and fast term enumeration for syntax-guided synthesis. In: CAV ’19. pp. 74–83. Springer, Cham (2019)
[40] Ryan, G., Wong, J., Yao, J., Gu, R., Jana, S.: CLN2INV: learning loop invariants with continuous logic networks. In: ICLR ’20. OpenReview.net (2020)
[41] Satake, Y., Unno, H., Yanagi, H.: Probabilistic inference for predicate constraint satisfaction. AAAI ’20 34(02), 1644–1651 (Apr 2020)
[42] Sekiyama, T., Suenaga, K.: Automated proof synthesis for the minimal propositional logic with deep neural networks. In: APLAS ’18. LNCS, vol. 11275, pp. 309–328. Springer (2018)
[43] Selsam, D., Björner, N.: Guiding high-performance SAT solvers with unsatcore predictions. In: SAT ’19. LNCS, vol. 11628, pp. 336–353. Springer (2019)
[44] Selsam, D., Lamm, M., Bünz, B., Liang, P., de Moura, L., Dill, D.L.: Learning a SAT solver from single-bit supervision. In: ICLR ’19. OpenReview.net (2019)
[45] Si, X., Dai, H., Raghothaman, M., Naik, M., Song, L.: Learning loop invariants for program verification. In: NeurIPS ’18. pp. 7762–7773. Curran Associates, Inc. (2018)
[46] Si, X., Naik, A., Dai, H., Naik, M., Song, L.: Code2Inv: A deep learning framework for program verification. In: CAV ’20. pp. 151–164. Springer (2020)
[47] Singh, S.P., Sutton, R.S.: Reinforcement learning with replacing eligibility traces. Mach. Learn. 22(1-3), 123–158 (1996)
[48] Solar-Lezama, A., Tancau, L., Bodik, R., Seshia, S., Saraswat, V.: Combinatorial sketching for finite programs. In: ASPLOS XII. pp. 404–415. ACM (2006)
[49] Sutton, R.S., Barto, A.G.: Reinforcement Learning: An Introduction. The MIT Press, second edn. (2018)
[50] Unno, H., Satake, Y., Terauchi, T., Koskinen, E.: Program verification via predicate constraint satisfaction modulo theories. CoRR abs/2007.03656 (2020), https://arxiv.org/abs/2007.03656
[51] Unno, H., Terauchi, T., Koskinen, E.: Constraint-based relational verification. In: CAV ’21. pp. 742–766. Springer (2021)
[52] Wang, M., Tang, Y., Wang, J., Deng, J.: Premise selection for theorem proving by deep graph embedding. In: NIPS ’17. pp. 2786–2796 (2017)
[53] Winskel, G.: The Formal Semantics of Programming Languages: An Introduction. MIT Press (1993)
[54] Wu, M., Norrish, M., Walder, C., Dezfooli, A.: Tacticzero: Learning to prove theorems from scratch with deep reinforcement learning. CoRR abs/2102.09756 (2021), https://arxiv.org/abs/2102.09756
[55] Zhu, H., Magill, S., Jagannathan, S.: A data-driven CHC solver. In: PLDI ’18. pp. 707–721. ACM (2018)