Black Holes and Cosmological Constant in Bosonic String Theory: Some Remarks

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ABSTRACT

In this work we attempt to give some suggestion about the black hole solutions coming from the bosonic string theory with cosmological constant in four dimensions.

In our study we are first interested in the effects on the metric of a tree level cosmological constant; then we make a perturbative calculation to be compared with the general relativistic results.
1. Introduction

There are essentially two questions we would like to find an answer to:

i) What about nontrivial dilaton physics in black hole solutions of the bosonic string theory with cosmological constant?

ii) What about the effects of a cosmological constant on the black hole temperature?

There are various possibilities to implement this attempt: the o-critical string theory gives $D=26$, so we can introduce it directly to the tree level; another possibility is to think to the first order in $0^{\text{string}}$.

Our most interesting results are:

1) Introducing a non-perturbative cosmological constant gives rise to a non-trivial dilaton: the solution $=\text{const}$ (the only one if we want black hole solutions) is not possible.

This means that if we are going to believe in the o-critical string physics we end in the Einstein framework the following results:

$$g_{00}=\text{const}$$

For completeness a parallel study with the general relativistic Schwarzschild-de Sitter solution is made.
2. Non-perturbative Analysis

The action we are interested in is the effective bosonic string action in 4D, with $\mathcal{P} = 0$ (the case with $\mathcal{P} = 0$ is already treated in Callan-Mayers-Perry $^1$ [CMP]):

$$S = \frac{1}{(16\,G)} \int Z \, d^4x \, \sqrt{g} \, e^2 \left( R + 4 (r)^2 \right) - 2 + \frac{k}{2} R_{abcd} R^{abcd};$$  \hspace{1cm} (2.1)

The last term in this action is the well known first perturbative correction coming from the string beta function equations.

The perturbative expansion parameter $k$ is related to $0_{\text{string}}$ by the relation

$$k = \frac{0_{\text{string}}}{2}$$  \hspace{1cm} (2.2)

Let us make a Weyl transformation so as to go into the so-called Einstein framework:

$$g \rightarrow e^2 \, g$$  \hspace{1cm} (2.3)

and let us assume first that it is not perturbative.

The transformed action to the zero order is

$$S_{E} = \frac{1}{(16\,G)} \int Z \, d^4x \, \sqrt{g} \, (R - 2 (r)^2 \left( 2 \, e^2 \right))$$  \hspace{1cm} (2.4)

and gives rise to the following equations:

$$R - 2r \, r \, g \, e^2 = 0$$  \hspace{1cm} (2.5)

$$r^2 \, e^2 = 0$$  \hspace{1cm} (2.6)

We are looking for spherically symmetric statical solutions, so we put
\[ ds^2 = e^2 \, dt^2 + e^2 \, dr^2 + r^2 \, d^2 \]

Substituting in (2.5) and in (2.6) and denoting with a prime the derivative with respect to \( r \), we end:

\[
0 \quad 0 \quad 0^2 + \frac{2}{r} \quad 0 \quad = e^2 \quad e^2 ; \\
0 \quad 0 \quad 0^2 + \frac{2}{r} \quad 0 \quad 2 \quad 0 \quad = e^2 \quad e^2 ; \\
r( \begin{array}{c} 0 \\ 0 \end{array} ) + e^2 = r^2 \quad e^2 \quad e^2 ; \\
0 + ( 0 \quad 0 + \frac{2}{r} ) \quad 0 \quad = e^2 \quad e^2 ;
\]  

(2.7)

Subtracting the first equation in (2.7) from the second one we have:

\[
( 0 + 0 ) \quad r^2 = 0; \\
\]  

(2.8)

From (2.8) one can realize that the choice

\[
= \quad , \quad 0^0 = 0^1 
\]  

(2.9)

is incompatible with (2.6):

\[
0 = 0 \quad ) \quad = 0 \quad \text{in} \quad (2.6) \\
\]  

(2.10)

So, taking a non-perturbative , we conclude that:

- a) the solution = const is not allowed.
b) \( g_{00} \neq g_{11} \). This non-triviality of the solutions is a cosmological constant effect: without a non-perturbative the only one black hole solution, as we shall show in the appendix, is a constant dilaton in a Schwarzschild metric. We are not able to find an analytical solution, so we cannot say anything about the effective existence of a black hole solution for a tree level cosmological constant; the relevant point is the obvious absence of matching with the General Relativity.

3. Perturbative Approach

Lacking any analytical black-hole solution for non-perturbative, let us put

\[
\begin{align*}
&k: \quad (3:1) \\
\end{align*}
\]

This corresponds to introduce the cosmological constant to the first order in the string Lagrangian.

To the tree level, as in CM P, the action is

\[
S_E = \frac{1}{(16 \pi G)} \int d^4 x \sqrt{-g} (R - 2 \nabla (r^2)) \\
\]

and the corresponding equations

\[
R - 2 \nabla \nabla r + r = 0 \quad (3:3) \\
\]

allow as the only spherically symmetric static solution under the reasonable boundary
condition

\[ (r)! 0 + \frac{A}{r} + O \left( \frac{1}{r^2} \right); \]  

(3.4)

the CMP solution:

Schwarzschild metric

\[ = 0 = \text{const} \]  

We can put without any loss of generality \( 0 = 0 \). The perturbative correction is

\[ S^{(1)} = \frac{1}{(16G)} \int d^4x \frac{\mathcal{P}}{g} \left( 2e^2 + \frac{1}{2} e^2 \left[ R_{abcd}R^{abcd} + \ldots \right] \right) \]  

(3.6)

The dots in (3.6) mean further terms involving higher derivative terms of the dilaton field; with a field redefinition of \( \tilde{r} \) to the first order in \( k \) it is possible to eliminate these terms; in the appendix we shall give further technical details on these stu.

In the sequel we will again indicate the redefined dilaton with \( \tilde{r} \).

The equations to the first order are

\[ R^{(1)} \quad g^{(0)} + [R] \frac{1}{4} g^{(0)} R^{(1)} = 0; \]

\[ r^{2(1)} \quad \frac{1}{4} R^{(1)} = 0 \]  

(3.7)

where

\[ [R] \quad R_{\text{abc}} R^{\text{abc}}; \]

\[ R^{(2)} \quad R_{\text{abcd}} R^{\text{abcd}}; \]

we put

\[ (r) = l(r) + k \ (r) \]
\[ (\tilde{r}) = l(\tilde{r}) + k \ (\tilde{r}) \]
\[ e^{21} = g(\tilde{r}) = 1 \quad \frac{2m}{r} \]

\[ (r) = 0 + k' (r) \quad 0 + k^{(1)} \]
substituting in (3.7) the equations become

\[
\frac{r}{r} \frac{2m}{r} + \frac{r}{r^2} \frac{m}{r^0} + \frac{r^2}{12m^2} = 0 \quad (3.8)
\]

and

\[
m^0 + (2r \cdot m)^0 + (2m \cdot r^0) + r^2 = 0
\]

(3m \cdot 2r)^0 + 3m + (r \cdot 2m)^0 + r^2 = 0

\[2 + (r \cdot 2m)^0 \quad (r \cdot 2m)^0 \quad r^2 = 0 \quad (3.9)\]

As physical boundary conditions we require the perturbations to be regular on the horizon \(r_H = 2m\); the cosmological term prevents us to require the asymptotic atness.

For the dilaton we end:

\[
\frac{\tau}{r} = \frac{\tau}{CM \cdot P} \left( r \right) + \frac{4}{3} \frac{m^2}{r} \log (r) + \frac{2}{3} \frac{m}{r} + \frac{1}{6} \frac{2r^2}{r^0} + \text{const};
\]

(3.10)

where

\[
\frac{\tau}{CM \cdot P} \left( r \right) \quad \frac{2m}{3r^3} \quad \frac{1}{2r^2} \quad \frac{1}{2mr};
\]

(3.11)

is the CM P solution.

The dilaton solution is regular on the horizon, but it is not asymptotically at.

The three equations for \((r); \quad (r)\) are functionally related by means of the Bianchi identity; subtracting the first equation in (3.9) from the second one it follows
\[(r - 2m)(0 + \frac{\partial}{\partial r}) = 0; \quad (3.12)\]

so

\[= \quad ; \quad (3.13)\]

and the third equation in (3.9) gives

\[(r) = -\frac{1}{6} (r^2 + 2m r + 4m^2); \quad (3.14)\]

The divergence of the perturbations for \(r > 1\) shows that exists a validity limit of the perturbative expansion.

Finally the metric components are

\[g_{00} = e^2 = g(r) \frac{1}{3} (r^2 + 2m r + 4m^2) + O(\varepsilon^2); \quad (3.15)\]

\[g_{11} = e^2 = \quad \frac{1}{g_{00}} = \frac{1}{g(r)} \frac{1}{3} (r^2 + 2m r + 4m^2) + O(\varepsilon^2); \]

The structure of the unperturbed space-time is consistently modified by the perturbative cosmological term; besides, if in the formula

\[g_{00} = g(r) (1 + 2k(r))\]

we can think to extend the validity of the results in the region where \(^1\)

\[j_0 > 1; \]

then for \(j_0 > 0\) there exists the possibility to have a second horizon: the new zeroes

\(^1\) Some reason to make this extension could come from the analogy with the Schwarzschild case, where a calculation perturbative in \(\varepsilon\) lets guess the existence of a cosmological horizon; see also sec. 5.
of $g_{00}$ given by

$$r^2 + 2mr + 4m^2 = 0 \quad (3.16)$$

which means

$$r_+ = m + \frac{r}{3} \frac{1}{1} m^2$$

$$r = m \frac{r}{3} \frac{1}{1} m^2 \quad (3.17)$$

The reality condition imposes

$$\frac{1}{m^2} \geq 0, \quad m \geq \frac{1}{p+} \quad (3.18)$$

with the inequality needed for to be $r_+ > r$. We actually have a new horizon only for $r_+ > 0$, that is

$$m \frac{p+}{3} \frac{1}{2} p+ \quad (3.19)$$

Making a series expansion of $r_+$ in for $! 0$

$$r_+ = m + \frac{p+}{3} \frac{p+}{2} m^2 + O ( ) \quad (3.20)$$

and taking small enough, it results $r_+ > r_H$, so one can identify $r_+$ as a cosmological horizon like the de Sitter horizon in the Schwarzschild-de Sitter solutions. Then, if (3.19) is satisfied, we nd, as in the Schwarzschild-de Sitter case, two horizons; it is reasonable to think that the absence of existence conditions for the black-hole horizon $r_H$ is a pure artifact of the perturbative approximation: in the Schwarzschild-de Sitter solution, as it is known, if $m \geq \frac{1}{3}$, there doesn't exist
any horizon and there is a naked singularity; something similar could happen also in our case.

The case where \( r < 0 \) is also interesting; we then have only the horizon \( r_H \); it is natural to make a parallel between our string solutions and the Schwarzschild-antide Sitter exact solutions of the General Relativity, in which the only horizon occurs for

\[
 r_{H}^{SAD} = \left( \frac{3m}{j^2} \right)^{\frac{1}{3}} \left( 1 + \frac{1}{9m^2j^2} \right)^{\frac{1}{3}} + \left( 1 + \frac{1}{9m^2j^2} \right)^{\frac{1}{3}} \tag{3.22}
\]

4. Semi-classical Aspects

The appearance of non-trivial perturbations affects the black-hole temperature by means of the cosmological contributions.

We remember that for \( r = 0 \) (cfr. CMP) in 4D there is no new contribution to the black-hole temperature beyond the Hawking one. There are two cases:

1)

\( r > 0 \)

with two subcases:

\[ \text{n} \] 

We stress anyhow that the appearance of a naked singularity, taking \( 3 \times 10^{52} \) meters \( ^2 \) (the experimental upper-bound for \( j^2 \), would occur for

\[
 m_{\text{critical}} = \frac{1}{3} \times 10^{22} \text{ solar masses} \tag{3.21}
\]

a value very near the universe mass; so from a physical point of view we could avoid to be worried about it.
a) \[ m < \frac{p_3}{2} \frac{1}{2} \] \hspace{1cm} (4.1)

b) \[ m > \frac{p_3}{2} \frac{1}{2} \] \hspace{1cm} (4.2)

In the case a), (4.1), the surface gravity \( K \) is to be calculated for the black-hole horizon and for the de Sitter horizon; we find respectively

\[ K_H = \frac{1}{4m} \frac{1}{3} \frac{1}{j} \frac{1}{4} m^2 j + O \left( \frac{2}{2} \right); \] \hspace{1cm} (4.3)

and

\[ K_C = \frac{p_3}{2} \frac{p_3}{2} \frac{1}{3} \frac{1}{j} \frac{1}{2} \frac{m}{j} + O \left( \frac{3}{2} \right); \] \hspace{1cm} (4.4)

then

\[ T_H = \frac{1}{8} \frac{1}{m} \frac{1}{3} \frac{1}{j} \frac{1}{4} m^2 j + O \left( \frac{2}{2} \right) \] \hspace{1cm} (4.5)

and

\[ T_C = \frac{p_3}{2} \frac{p_3}{2} \frac{1}{3} \frac{1}{j} \frac{1}{2} \frac{m}{j} + O \left( \frac{3}{2} \right); \] \hspace{1cm} (4.6)

We can compare these temperatures with those obtained from the Schwarzschild-de Sitter [SD] case in the limit \( \frac{1}{m} \rightarrow 0 \):

\[ T_H^{SD} = \frac{1}{8} \frac{1}{m} \frac{1}{3} \frac{1}{j} \frac{16}{3} m^2 j + O \left( \frac{2}{2} \right); \] \hspace{1cm} (4.7)

\[ T_C^{SD} = \frac{p_3}{2} \frac{p_3}{2} \frac{1}{3} \frac{1}{j} \frac{2}{p_3} \frac{m}{j} + O \left( \frac{3}{2} \right); \] \hspace{1cm} (4.8)

The contributions are very similar.
From a physical point of view, the temperature actually measured by one observer is given by a mixture of the black-hole and the cosmological radiations. In the case b), (4.2), there is only the black-hole horizon, with temperature given by (4.5).

In the Schwarzschild-de Sitter case, we know that both the temperatures decrease when increases, so they cannot diverge with as it seems by looking at our approximate formulas; anyway, the existence condition of the event horizons

\[ m < \frac{1}{3} \]

is such that

\[ \frac{16}{3} m^2 < 1 \]

\[ \frac{2}{3} p - \frac{2}{3} m < 1 \]  

(4.9)

so (4.7) and (4.8) cannot diverge with and they are actually decreasing functions of . We can see a similar mechanism provided by an horizon existence condition prevents that and diverge with also in our string framework in the case a).

In the case b), there is a divergence probably due to the perturbative approximation.

2)  

\[ T \]  

< 0

---

# 3 The cosmological temperature, that one observer should see coming from any direction in the universe, would have the following upper bound:

\[ T_C \frac{p - \frac{hc}{2}}{\sqrt{1 - \frac{2p}{3}}} \frac{k_B}{2} < 2.3 \times 10^{29} \text{ K} \]
It results:

$$K_H = \frac{1}{4m}(1 + 4m^2 j \frac{j}{j}) + O(2); \quad (4.10)$$

and

$$T_H = \frac{1}{8m}(1 + 4m^2 j \frac{j}{j}) + O(2); \quad (4.11)$$

whereas the Schwarzschild-anti de Sitter solution gives

$$T_{H}^{\text{SAD}} = \frac{1}{8m}(1 + \frac{16}{3}m^2 j \frac{j}{j}) + O(2) \quad (4.12)$$

The formulas (4.11) and (4.12) show a pathological behaviour for $m \rightarrow 1$: we cannot appeal to any principle to avoid the temperature divergence in the limit $m \rightarrow 1$.

We can say that this behaviour physically begins to occur for very large masses (order of the universe mass).

### 5. On General Relativity Analogies

We want to make the same perturbative approach in for the General Relativity although the exact solution are well known: we could learn something about our solutions.

Using exactly the same definitions as in section 3, we write the equations at the first order in $k$:

$$\frac{g^0}{2g} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \otimes \frac{g^0}{g} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$$\frac{g^0}{2g} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \otimes \frac{g^0}{g} \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} + \frac{r}{g} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} = 0 \quad (5.1)$$

$$r \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \frac{2}{g} \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix} \frac{r^2}{g} = 0$$
Subtracting the first equation in (5.1) from the second one we get

\[ \frac{2}{r} (0 + 0) = 0; \]  

(5.2)

so again

\[ = ; \]

and the third equation in (5.1) gives

\[ 2r g^0 + 2 r^2 = 0; \]  

(5.3)

which is the same equation we obtained in sec. 3: so we conclude that a perturbative approach to the Schwarzschild-de Sitter physics leads to the same results for the metric as from the (3.2) action.

Actually what is new in the string action in comparison with the General Relativity is the coupling with the dilaton field; it occurs in eq. 1) of sec. 2 by means of \( g^2 \) and \( e^2 \).

If we put

\[ = 0 + k' (r) \]  

(5.4)

(and this is consistent only for a perturbative ) then

\[ e^2 = ke^2 0 + O (k^2) = K + O (k^2); \]  

\[ g^2 = O (k^2); \]  

(5.5)

that is, apart from a renormalization of the coupling constant (but we can choose \( 0 = 0 \), as we did in sec. 3) the dilaton decouples from the equations for the metric perturbations to order \( k \).
So the only possibility to appreciate the presence of the dilaton field seems to be connected to the case of a non trivial to the tree level; this is possible for a non perturbative cosmological constant.

Nevertheless matching with the analogous general relativistic calculations is not yet automatic, because of the $R_{abcd}R^{abcd}$ stringy contribution; but what actually happens is that on-shell

$$ [R] = \frac{1}{4} g^{(0)} [R]^2 ;$$

so it disappears from the equations for and .

We can in this way understand the equivalence of the results obtained from the General Relativity and from the string action in the Einstein framework.

6. Quantum Aspects and Conclusions

Not being possible to quantize the four dimensional string action, taken in the Einstein framework, we can think to make a "dimensional reduction" of it: we choose a spherically symmetric ansatz for the 4D metric

$$ ds^2 = g_{ab}(\mathbf{x}^0;\mathbf{x}^1) dx^a dx^b + e^{2\phi(\mathbf{x}^0;\mathbf{x}^1)} d^2 a; b = 0; 1 $$

and

$$ = (\mathbf{x}^0;\mathbf{x}^1) $$

The zero order action in (3.2) becomes

$$ Z \left[ S_E \right] = \int d^2 x P e^{2\phi(\mathbf{x})} \left[ R + 2(\phi')^2 + 2e^{2\phi} - 2(\phi')^2 \right] $$

This reduced action is a variant of dilaton gravity minimally coupled with a scalar field which is the real 4D-dilaton field; there still exists a black-hole solution
of classical equations with cosmological perturbation which is equal to the four-dimensional one.

The idea is to quantize this action, following the common ansatz that the 4D angular degrees of freedom qualitatively don't modify the bidimensional picture; but unfortunately we cannot implement this program because with (6.3) we are not at the critical conformal point: the dilaton gravity action conformally invariant is the Callan, Giddings, Harvey and Strominger (CGHS)\(^5\) action:

\[
S_{\text{CGHS}} = \int d^2 x \sqrt{g} e^{2\phi} (\mathcal{R} + 4 (\phi')^2 + 4 \phi^2) \quad (6.4)
\]

The very crucial differences are sketched by the following rules

\[
\begin{align*}
\text{CGHS} & \quad ! \quad (6.3) \\
4 (\phi')^2 & \quad ! \quad 2 (\phi')^2 \text{ in (6.3)} \\
4 \phi^2 & \quad ! \quad 2e^{2\phi} \text{ in (6.3)}
\end{align*}
\]

Besides, we have another scalar making more intricate the study.

However, the major problem to be faced if one wants to carry on this program is the non Toda-like form of our action, and as a consequence one cannot use the standard conformal field theory methods employed in ref. 6 for the quantization of (6.3).

In this work we have taken into account the physical contributions of the cosmological constant\(^4\) in the framework of the bosonic string theory in four

\[\text{# 4 A f t e r t h i s w o r k w a s c o m p l e t e d, w e b e c a m e a w a r e o f R e f . 7, i n w h i c h an e x p l i c i t s o l u t i o n t o s t r i n g e q u a t i o n s o f m o t i o n w i t h a t r e e l e v e l c o s m o l o g i c a l c o n s t a n t , a l b e i t i n t h e e x t r e m e l y c h a r g e d c a s e , i s p r e s e n t e d a n d t h e p r o b l e m o f f i n d i n g a n e x a c t s o l u t i o n i n t h e u n c h a r g e d c a s e i s d i s c u s s e d.}\]
dimensions; particularly, we have looked for stationary spherically symmetric black hole solutions and we have carried out a parallel study with the General Relativity [GR] to conclude that:

i) a tree level gives no matching with [GR]

ii) a first order gives the same results as a first order in [GR].

A semiclassical treatment of the black hole evaporation in the limit \( \mu \to 0 \) has shown that there is a cosmological contribution to the black hole temperature and that in the case of \( \mu < 0 \) the temperature has a pathological behaviour for large masses.

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APPENDIX

I) Black hole solutions in dilaton-gravity theories

We show that the only static spherically symmetric solution of the equations (A); (B) of section 3 under suitable boundary conditions compatible with a black-hole structure of the space-time is \( r = \text{const} \); we want to stress that the demonstration is quite the same as for the scalar-tensor theories of gravity like Brans-Dicke theories. We have the equation

\[ r^2 = 0 \]

under the boundary condition

\[ (r) \equiv 0 + \frac{A}{r} + O \left( \frac{1}{r^2} \right) \]

Let us denote

\[ 0 \]

obviously

\[ r^2 = 0 \]

we have

\[ \frac{Z}{d^4x} \frac{\mathbf{g}}{g(r^2)} = 0 = \frac{Z}{d^4x} \frac{\mathbf{g}_{\alpha}(r^\alpha)}{g(r^2)} = \frac{Z}{d^4x} \frac{\mathbf{g}(r^2)}{g(r^2)} \]

\[ \mathbb{V} \]

\[ \mathbb{V} \]

\[ \mathbb{V} \]

\[ 18 \]
where stays for the hypersurfaces which bound the volume $V$ defined as follows:

$S = \text{spacelike hypersurface}$

$S^0 = \text{spacelike hypersurface translated along the timelike killing vector of the exterior geometry}$

$T = \text{timelike hypersurface at infinity}$

$H = \text{event horizon}$

The surface integral over $H$ is zero because $r_a$ has no components along the symmetry directions and the surface element on $H$ is entirely in a Killing direction; the surface integrals over $S$ and $S^0$ cancel each other because of the time symmetry; the surface integral over $T$ is zero too because of the boundary condition on $T$. So we have

$$
Z \frac{d^4 x}{g(r)} = 0
$$

Because the gradient of $Z$ can be only spacelike or zero the thesis follows.

II) On Weyl transformations

Under a conformal transformation

$$
g = e^{2\phi} g
$$

we have

$$
R_{abcd} = e^{2} \left( \mathcal{R}_{abcd} + \tilde{\gamma}_{abcd} \left( \mathcal{G} \right)^2 \right)
$$

where

$$
\tilde{\gamma}_{abcd} = g_{ad} g_{bc} + g_{ac} g_{bd}
$$

$$
\mathcal{G}_{abcd} = g_{ad} g_{bc} g_{ac} g_{bd}
$$

$$
X_{ab} = \tilde{\gamma}_a \tilde{\gamma}_b \left( \mathcal{G} \right)^2
$$
\begin{equation}
R_{abcd}R^{abcd} = e^{2} (R_{abcd}R^{abcd} + f (\phi))
\end{equation}

and \( f \) indicates a function of the dilaton derivatives; to eliminate this \( f \) we make the following transformation:

\[
! + k \ F ( )
\]

and choose \( F \) such that

\[
2 (\phi^2) + \frac{k}{2} e^2 f (\phi) + (\phi^2) + O (k^2)
\]
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