Self-interacting scalar dark matter with local $Z_3$ symmetry

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Abstract. We construct a self-interacting scalar dark matter (DM) model with local discrete $Z_3$ symmetry that stabilizes a weak scale scalar dark matter $X$. The model assumes a hidden sector with a local $U(1)_X$ dark gauge symmetry, which is broken spontaneously into $Z_3$ subgroup by nonzero VEV of dark Higgs field $\phi_X$ ($\langle \phi_X \rangle \neq 0$). Compared with global $Z_3$ DM models, the local $Z_3$ model has two new extra fields: a dark gauge field $Z'$ and a dark Higgs field $\phi$ (a remnant of the $U(1)_X$ breaking). After imposing various constraints including the upper bounds on the spin-independent direct detection cross section and thermal relic density, we find that the scalar DM with mass less than 125 GeV is allowed in the local $Z_3$ model, in contrary to the global $Z_3$ model. This is due to new channels in the DM pair annihilations open into $Z'$ and $\phi$ in the local $Z_3$ model. Most parts of the newly open DM mass region can be probed by XENON1T and other similar future experiments. Also if $\phi$ is light enough (a few MeV $\lesssim m_\phi \lesssim O(100)$ MeV), it can generate a right size of DM self-interaction and explain the astrophysical small scale structure anomalies. This would lead to exotic decays of Higgs boson into a pair of dark Higgs bosons, which could be tested at LHC and ILC.

Keywords: dark matter theory, dark matter experiments, dwarfs galaxies

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1 Introduction

Although Planck [1] has already given the dark matter (DM) relic density $\Omega h^2 = 0.1199 \pm 0.0027$ with a high precision, we still do not know particle physics nature of DM at all. So far all the compelling evidences for the existence of DM come from astrophysics and cosmology, due to its gravitational interaction. Still, many particle physics models for DMs have been proposed, and most of them have a stable collisionless cold DM (CCDM) candidate whose self-interaction can be ignored.

The collisionless cold DM has been very successful when explaining the large scale structure of our Universe. However, anomalies from the small scale astrophysical observations [2–4] indicate that DM may have strong interactions between themselves. Such self-interaction [5] would make DM have a flat core density profile rather than a cusp one predicted by CCDM. Recent simulations show that in order to flatten the cores of galaxies the cross section for DM scattering should be around $\sigma \sim M_X \times \text{barn GeV}^{-1}$ [6–8], which is in fact a huge cross section compared with typical weak-scale cross sections $\sigma \sim 10^{-12} \text{ barn or 1 pb}$. Some light particle mediator in the dark sector could be an origin of such strong self-interaction between DMs.

In this paper, we propose a scalar DM model with a local $Z_3$ symmetry. Unlike models based on global symmetries, local discrete symmetries can protect symmetry-breaking from quantum gravity effects and guarantee the longevity or absolute stability of DM particles. Also a light mediator can exist in the models with local symmetry, and generate the correct self-interaction for DM in explaining the anomalies mentioned in the previous paragraph.

The outline of this paper is as follows. In section II, we introduce the model with a local $Z_3$ symmetry, establish the convention for parameters and give the physical mass spectra. Then we discuss both theoretical and experimental constraints on the parameters.
in section III. Then in section IV, we discuss the relic density and DM direct searches, paying attentions to the semi-annihilation feature, and compare with the global $Z_3$ mode. In section V, we show that a light scalar mediator in our model can induce strong interaction for DM. Finally we summarize the results in section VI.

2 Local $Z_3$ model

Let us assume the dark sector has a local $\text{U}(1)_X$ gauge which is spontaneously broken into local $Z_3$ symmetry a la Krauss and Wilczek [9] (see ref. [10] for local $Z_N$ case). This can be achieved with two complex scalar fields

$$\phi_X \equiv (\phi_R + i\phi_I)/\sqrt{2}, \quad X \equiv (X_R + iX_I)/\sqrt{2}$$

in the dark sector with the $\text{U}(1)_X$ charges equal to 1 and $1/3$, respectively. Then one can write down renormalizable Lagrangian for the SM fields and the dark sector fields, $\tilde{X}_\mu, \phi_X$ and $X$:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} \tilde{X}_{\mu
u} \tilde{X}^{\mu\nu} - \frac{1}{2} \sin \epsilon \tilde{X}_{\mu
u} B^{\mu\nu} + D_\mu \phi_X^\dagger D^\mu \phi_X + D_\mu X^\dagger D^\mu X - V$$

$$V = -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 - \mu_3^2 \phi_X^\dagger \phi_X + \lambda_3 \left(\phi_X^\dagger \phi_X\right)^2 + \mu_X^2 X^\dagger X + \lambda_X \left(X^\dagger X\right)^2 + \lambda_{\phi H} \phi_X^\dagger \phi_H X^\dagger H + \lambda_{\phi X} \phi_X^\dagger X^\dagger H \left(\lambda_{\phi X} X^\dagger \phi_X + H.c.\right) \quad (2.1)$$

where the covariant derivative associated with the gauge field $X^\mu$ is defined as $D_\mu \equiv \partial_\mu - i\tilde{g}_X Q_X \tilde{X}_\mu$. The coupling $\lambda_3$ can be chosen as real and positive since it is always possible to redefine $X$ to absorb the phase.

We are interested in the phase with the following vacuum expectation values for the scalar fields in the model:

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_h \end{pmatrix}, \quad \langle \phi_X \rangle = \frac{v_\phi}{\sqrt{2}}, \quad \langle X \rangle = 0, \quad (2.2)$$

where only $H$ and $\phi_X$ have non-zero vacuum expectation values(vev). This vacuum will break electroweak symmetry into $\text{U}(1)_{\text{em}}$, and $\text{U}(1)_X$ symmetry into local $Z_3$, which stabilizes the scalar field $X$ and make it DM. The discrete gauge $Z_3$ symmetry stabilizes the scalar DM even if we consider higher dimensional nonrenormalizable operators which are invariant under $\text{U}(1)_X$. This is in sharp constrast with the global $Z_3$ model considered in ref. [11]. Also the particle contents in local and global $Z_3$ models are different so that the resulting DM phenomenology are distinctly different from each other.

Other vacuum configurations could exist, such as $\langle \phi_X \rangle \neq 0$ and $\langle X \rangle \neq 0$ which give rise to both broken $\text{U}(1)_X$ and $Z_3$ but also no dark matter candidate. The complete analysis of vacuum structure is beyond the scope of this work and we shall focus on the vacuum eq. (2.2) in this paper.

Expanding the scalar fields around eq. (2.2),

$$H \rightarrow \frac{v_h + h}{\sqrt{2}}, \quad \phi_X \rightarrow \frac{v_\phi + \phi}{\sqrt{2}}, \quad X \rightarrow \frac{x}{\sqrt{2}} e^{i \theta} \quad \text{or} \quad \frac{1}{\sqrt{2}} \left(X_R + iX_I\right), \quad (2.3)$$

– 2 –
the minimum conditions for the potential would give

\[
\left. \frac{\partial V}{\partial \phi} \right|_{x=0} = \phi \left( -\mu_\phi^2 + \lambda_\phi \phi^2 + \frac{1}{2} \lambda_{\phi H} h^2 \right) = 0, \tag{2.4} \\
\left. \frac{\partial V}{\partial h} \right|_{x=0} = h \left( -\mu_H^2 + \lambda_H h^2 + \frac{1}{2} \lambda_{\phi H} \phi^2 \right) = 0. \tag{2.5}
\]

Then one can solve them for the VEVs as follows:

\[
\langle H^2 \rangle = \frac{v_h^2}{2} \left( 2\lambda_H v_h^2 - \lambda_{\phi H} v_{\phi}^2 \right), \tag{2.6} \\
\langle \phi_X^2 \rangle = \frac{v_\phi^2}{2} \left( 2\lambda_{\phi X} v_\phi^2 - \lambda_H v_h^2 \right). \tag{2.7}
\]

The mass matrix for the two mixed scalars is

\[
M^2 = \begin{pmatrix}
2\lambda_H v_h^2 & \lambda_{\phi H} v_h v_\phi \\
\lambda_{\phi H} v_h v_\phi & 2\lambda_\phi v_\phi^2
\end{pmatrix}, \tag{2.8}
\]

in the \((h, \phi)\) basis. Diagonalizing the mass matrix gives the mass eigenstates \(H_1\) and \(H_2\)

\[
\begin{pmatrix}
H_1 \\
H_2
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
h \\
\phi
\end{pmatrix}. \tag{2.9}
\]

and the mixing angle

\[
\tan 2\alpha = \frac{2M_{12}^2}{M_{22}^2 - M_{11}^2} = \frac{\lambda_{\phi H} v_h v_\phi}{\lambda_\phi v_\phi^2 - \lambda_H v_h^2}, \text{ or } \sin 2\alpha = \frac{2\lambda_{\phi H} v_h v_\phi}{M_{H_2}^2 - M_{H_1}^2}. 
\]

Physical masses for \(H_1\) and \(H_2\) are

\[
M_{H_1,H_2}^2 = \lambda_H v_h^2 + \lambda_\phi v_\phi^2 \pm \sqrt{\left( \lambda_H v_h^2 - \lambda_\phi v_\phi^2 \right)^2 + \left( \lambda_{\phi H} v_h v_\phi \right)^2}. \tag{2.10}
\]

We shall identify \(H_1\) as the recent discovered Higgs boson with \(M_{H_1} \simeq 125\text{GeV}\) and treat \(M_{H_2}\) as a free parameter. \(H_2\) could be either heavier or lighter than \(H_1\). The mass for the scalar DM \(X\) is

\[
M_X^2 = \mu_X^2 + \lambda_{\phi X} v_\phi^2 + \lambda_{H X} v_h^2. \tag{2.11}
\]

After the EW and dark gauge symmetry breaking, the mass terms for gauge fields are derived from

\[
\frac{v_\phi^2}{2} g_X \tilde{\phi} \tilde{\phi} + \frac{v_h^2}{8} \left( g_1 \tilde{B}_\mu - g_2 \tilde{W}_3 \right)^2. \tag{2.11}
\]

We can redefine the abelian gauge fields

\[
\begin{pmatrix}
\tilde{B}_\mu \\
\tilde{X}_\mu
\end{pmatrix} = \begin{pmatrix}
1 - \tan \epsilon \\
0, 1/\cos \epsilon
\end{pmatrix}
\begin{pmatrix}
\tilde{B}_\mu \\
\tilde{X}_\mu
\end{pmatrix}, \quad \tilde{W}_\mu = \tilde{W}_\mu, \tag{2.12}
\]
in order to remove the kinetic mixing term between $\hat{B}_\mu$ and $\hat{X}_\mu$. We may also rescale the gauge coupling $\hat{g}_X = \hat{g}_X / \cos \epsilon$. Substituting with the hatted field gives the mass matrix for $\hat{B}, \hat{W}_3$ and $\hat{X}$, which we can diagonalize by rotating

$$
\begin{pmatrix}
\hat{B}_\mu \\
\hat{W}_3\mu \\
\hat{X}_\mu
\end{pmatrix} =
\begin{pmatrix}
c_{\hat{W}} & -s_{\hat{W}} c_{\xi} & s_{\hat{W}} s_{\xi} \\
s_{\hat{W}} & c_{\hat{W}} c_{\xi} & -c_{\hat{W}} s_{\xi} \\
0 & s_{\xi} & c_{\xi}
\end{pmatrix}
\begin{pmatrix}
A_{\mu} \\
Z_\mu \\
Z'_\mu
\end{pmatrix}.
\tag{2.13}
$$

Then the final mixing matrix for the starting fields in the lagrangian is

$$
\begin{pmatrix}
\hat{B}_\mu \\
\hat{W}_3\mu \\
\hat{X}_\mu
\end{pmatrix} =
\begin{pmatrix}
ts_\xi s_\epsilon + s_{\hat{W}} c_\xi & s_\hat{W} s_\xi - t_\epsilon c_\xi \\
t_\xi c_\epsilon & c_\hat{W} c_\epsilon & -c_\hat{W} s_\epsilon
\end{pmatrix}
\begin{pmatrix}
A_{\mu} \\
Z_\mu \\
Z'_\mu
\end{pmatrix}.
\tag{2.14}
$$

In eq. (2.13) and (2.14), we have defined the new parameters:

$$
c_{\hat{W}} = \cos \theta_{\hat{W}} = \frac{g_2}{\sqrt{g_2^2 + g_4^2}}, \quad \tan 2\xi = -\frac{m_Z^2 s_{\hat{W}}}{m_X^2 - m_Z^2 (c_\xi^2 - s_\xi^2 s_{\hat{W}})},
$$

$$
t_\epsilon \equiv \tan x, \quad c_\epsilon \equiv \cos x \quad \text{and} \quad s_\epsilon \equiv \sin x \quad \text{for} \quad x = \epsilon, \xi,
$$

$$
m_X^2 = \hat{g}_X^2 v_\phi^2, \quad m_Z^2 = \frac{1}{4} (g_2^2 + g_4^2) v_h^2.
\tag{2.15}
$$

From eq. (2.14) we can observe that the SM particles charged under SU(2)L and/or U(1)Y now also have interaction with $Z'_\mu$. And particles in the dark sector also have interaction with $Z_\mu$ due to the kinetic mixing between $\hat{B}_\mu$ and $\hat{X}_\mu$.

The physical masses for four vector bosons in our model are given by

$$
m_A^2 = 0,
$$

$$
m_W^2 = \frac{1}{4} g_4^2 v_h^2, \quad m_Z^2 = \frac{1}{4} (g_2^2 + g_4^2) v_h^2,
$$

$$
m_{Z'}^2 = \frac{m_X^2}{c_\xi^2 (1 + s_{\hat{W}} t_\epsilon c_\xi)}.
\tag{2.19}
$$

3 Constraints on $\lambda$’s and $\epsilon$

The dimensionless parameters $\lambda_i$’s can not be arbitrarily large in perturbative theory. Demanding $|\lambda_i| \lesssim 4\pi$ would be sufficient for our consideration. The scale where perturbativity breaks down can be determined by the renormalization group (RG) analysis using the RG equations summarized in appendix. Generally, $|\lambda_i| \lesssim 1$ at the TeV scale would give perturbativity up to $10^{15}$ GeV.

Besides the perturbativity, the potential should be bounded from below, which means at large field value the potential needs to be positive semidefinite, $V \geq 0$. In the limit of $\phi_i \rightarrow \infty$, we can neglect the quadratic terms and consider only the quartic part in the potential. Then in the case of $\lambda_3 = 0$ we have [12–14]

$$
\lambda_H \geq 0, \quad \lambda_\phi \geq 0, \quad \lambda_X \geq 0, \quad A_{\phi H} \equiv \lambda_{\phi H} + 2\sqrt{\lambda_\phi \lambda_H} \geq 0,
$$

$$
A_{\phi X} \equiv \lambda_{\phi X} + 2\sqrt{\lambda_\phi \lambda_X} \geq 0, \quad A_{H X} \equiv \lambda_{H X} + 2\sqrt{\lambda_H \lambda_X} \geq 0,
$$

$$
\sqrt{\lambda_H \lambda_\phi \lambda_X} + \lambda_{\phi H} \sqrt{\lambda_X} + \lambda_{\phi X} \sqrt{\lambda_H} + \lambda_{H X} \sqrt{\lambda_\phi} + \sqrt{A_{\phi H} A_{\phi X} A_{H X}} \geq 0.
\tag{3.3}
$$
For general $\lambda_3$, there is no transparent criteria for the positive semidefinite of $V_4$. However we could get useful necessary conditions by using the general positive criteria for quartic polynomial. For instance, consider the direction in the field space,

$$h = 0, \phi_X = y \times x, \ X = \frac{x}{\sqrt{2}},$$

and substitute in the quartic potential, we have

$$V_4 = \frac{1}{4} (\lambda_\phi y^4 + \lambda_\phi x y^2 + 2\lambda_3 y + \lambda_X) x^4.$$ 

Boundness from below gives the constraints on the coefficients for any non-negative $y$

$$\lambda_\phi y^4 + \lambda_\phi x y^2 + 2\lambda_3 y + \lambda_X \geq 0,$$

of which one sufficient condition is

$$0 < \beta \equiv \frac{\lambda_\phi x}{\sqrt{\lambda_\phi \lambda_X}} \leq 6 \ \&\& \ \gamma \equiv \frac{2\lambda_3}{(\lambda_\phi \lambda_X)^{\frac{3}{4}}} > -\frac{\beta^2 + 2}{2},$$

or $\beta > 6 \ \&\& \ \gamma > -2\sqrt{\beta - 2}.$

General conditions for positivity on a quartic polynomial of a single variable [15] is summarized in the appendix B and shall be imposed in all following investigations.

Similarly we can do the analysis in another directions. The direction $h = y \times x, \phi_X = 0, \ X = \frac{x}{\sqrt{2}}$ gives

$$\lambda_H y^4 + \lambda_{HX} y^2 + \lambda_X \geq 0,$$

leading constraints on $\lambda_H, \lambda_X$ and $\lambda_{HX}$ which are just those in eq. (3.1).

Constraints on the kinetic mixing parameter $\epsilon$ come from the muon ($g-2$), atomic parity violation, the $\rho$ parameter and electroweak precision tests (EWPTs) [16–19]. These could put an upper limit on $\epsilon$ as a function of $M_{Z'}$. Among these constraints, EWPTs provides the most stringent one:

$$\left( \frac{\tan \epsilon}{0.1} \right)^2 \left( \frac{250\text{GeV}}{M_{Z'}} \right)^2 \leq 0.1.$$ (3.4)

For $M_{Z'} \sim 250\text{GeV}$ we have $\epsilon \lesssim 0.03$. In the case of $\epsilon = 0$, there is no mixing between $Z$ and $Z'$, the whole connection between SM and dark sector comes from the scalar sector.

In the following numerical investigation, we have imposed all the relevant constraints discussed in this section.

4 Relic density and direct detection

4.1 semi-annihilation

The $X^3\phi_X$ term and the cubic term $X^3$ after U(1)$_X$ symmetry breaking lend the semi-annihilation channel possible and could have a significant effect in the freeze out of the DM [20–22]. We show the relevant Feynman diagrams in figure 1. In the presence of semi-annihilation the Boltzman equation that determines the number density $n_X$ is modified into [23]

$$\frac{dn_X}{dt} = -v\sigma^{X X' \rightarrow YY} (n_X^2 - n_{X \text{ eq}}^2) - \frac{1}{2} v\sigma^{X X' \rightarrow X Y} (n_X^2 - n_{X n_{X \text{ eq}}}^2) - 3Hn_X,$$ (4.1)
where $Y$ stands for any other particles and $v$ for the relative velocity. Due to the semi-annihilation, new contribution appears as the second term in the above equation. The numerical investigation is done with micrOMEGAs [23] and FeynRules [24]. We may define the fraction of the contribution from the semi-annihilation in terms of

$$r \equiv \frac{1}{2} \frac{v_\sigma XX \to X^* Y}{v_\sigma XX \to X^* XX + v_\sigma XX \to XX + 1/2 v_\sigma XX \to X^* Y}.$$

The full Feynman diagrams for semi-annihilation are presented in figure 1. Depending on the particles’ masses or couplings, only a fraction of these diagrams might be kinematically allowed or relevant. For example, only first four diagram are relevant for $\epsilon \simeq 0$, $\lambda \phi X \simeq 0$ and very heavy $Z'$. Then the cross section for $XX \to X^* H_i$ semi-annihilation process is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|p_f|}{|p_i|} |M|^2,$$

with $|p_f| = \frac{1}{2\sqrt{3}} \sqrt{s - (M_\chi + M_{H_i})^2} \left( s - (M_\chi - M_{H_i})^2 \right)$. For dark matter $p_i = M_\chi v_{vel}/2$ and $v_{vel}$ is the relative velocity between two annihilating particles. Matrix elements are given by

$$iM_d \propto -i3\sqrt{2}\lambda_3,$$

$$iM_{a+b+c} \propto -i3\sqrt{2}\lambda_3 v_\phi \left[ \frac{i}{s - M_\chi^2} + \frac{i}{t - M_\chi^2} + \frac{i}{u - M_\chi^2} \right] (-i\lambda_{HX} v_h),$$

respectively. If $\lambda_{HX} v_h v_\phi / M_\chi^2 \ll 1$ and $M_{H_i} < M_\chi$, then $M_d$ dominates and we have

$$\langle \sigma v \rangle_d = \frac{9\lambda_3^2 |p_f|}{16\pi M_\chi^2},$$

and $|p_f| \simeq \frac{3}{4} M_\chi$ for $M_\chi \gg M_{H_i}$. The relevant contribution $r$ from semi-annihilation is shown with different color in figure 2. It is evident that as $\lambda_{HX}$ gets smaller, $r$ becomes larger and the semi-annihilation becomes dominant. Meanwhile the cross section for $X$’s scattering off a nucleon gets smaller for direct searches. Some of these points may even not be probed by XENON1T [27].
Figure 2. Illustration of discrimination between global and local $Z_3$ symmetry. We have chosen $M_{H_3} = 20\text{GeV}$, $M_{2'} = 1\text{TeV}$, $\lambda_3 < 0.02$, $\epsilon \approx 0$ and $\lambda_{3H} \approx 0$ as an example. From up to down, three nearly straight lines mark the XENON100 [25], LUX [26] and expected XENON1T limits [27], respectively. Colors in the scattered triangles and circles indicate the relative contribution of semi-annihilation, $r$. The curved blue band, together with the circles, gives correct relic density of $X$ in the global $Z_3$ model. And the colored triangles appears only in the local $Z_3$ model. See text for detail.

4.2 Global $Z_3$ vs local $Z_3$

When the $U(1)_X$ breaking scale $v_\phi$ is much larger than the EW scale $v_h$ and the masses, $M_{2'}$ and $M_{H_3}$, are much heavier than those of other particles, we can get the low energy effective theory by integrating out the heavy degrees of freedom, $X'$ and $\phi$. The effective theory then describes the SM+$X$ with the residual global $Z_3$ symmetry. And in the effective potential the terms involving $X$ always appears as $X^\dagger X$, $X^3$ and $X^{13}$,

$$V_{\text{eff}} \simeq -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + \mu_X^2 X^\dagger X + \lambda_X \left(X^\dagger X\right)^2 + \lambda_{XH} X^\dagger X H^\dagger H + \mu_3 X^3$$

+ higher order terms $+ H.c$,  \hfill (4.2)

/
where $\mu_3 \equiv \frac{\lambda_3 v_\phi}{\sqrt{2}}$. In such a case, the effective theory can not tell whether the $Z_3$ symmetry is a global one or just residual of a gauge symmetry. In fact the renormalizable parts of $V_{\text{eft}}$ in eq. (4.2) is exactly the same as the scalar potential in global $Z_3$ model [11]. Therefore we can consider the renormalizable scalar DM model with global $Z_3$ symmetry as an effective theory of local $Z_3$ models in the limit $v_\phi \gg v_h$.

However there is an important difference in the higher dimensional operators even in this limit. Within the local $Z_3$ model, the discrete $Z_3$ gauge symmetry is respected by higher dimensionional operators, and the scalar DM $X$ shall be absolutely stable. This is not the case for global $Z_3$ model, since the higher dimensional operators due to quantum gravity could break global $Z_3$ symmetry, so that the DM stability is no longer guaranteed. For example one can consider

$$\frac{1}{\Lambda} X F_{\mu
u} F^{\mu
u},$$

which renders the scalar $X$ with EW scale mass decay immediately, and so the scalar $X$ cannot make a good DM candidate of the universe.

The difference between local and global $Z_3$ models become even more apparent and significant when $v_\phi \sim \text{TeV}$ or smaller. There is only one additional new particle $X$ in the global $Z_3$ model, while in the local $Z_3$ model there are two more particles, $Z'$ and $H_2$, compared with the global $Z_3$ model. The particle spectra are different, and the local $Z_3$ model enjoys much richer phenomenology. In figure 2 we show an example that could illustrate the differences between the global and local $Z_3$ models. For simplicity we use $M_{H_1} = 20\text{GeV}$, $M_{Z'} = 1\text{TeV}$, $\lambda_3 < 0.02$, $\epsilon \simeq 0$ and $\lambda_{\phi H} \simeq 0$. The curved blue band shows the parameter region in which only $XX^* \rightarrow \text{SM}+\text{SM}$ processes contribute to annihilation, namely, only $\lambda_{XH} X^\dagger X H^\dagger H$ in the potential is relevant and it also marks the upper bound for $\lambda_{XH}$ for giving the correct relic abundance of $X$ in both global and local $Z_3$ models. We can see that the low mass range $M_X < M_{H_1}$ is excluded by latest dark matter direct search limit from LUX [26], except the resonance region $M_X \simeq M_{H_1}/2$ which will be probed by XENON1T [27]. Colored circles, together with the very curved blue band, describe the parameter space for the global $Z_3$ model where $X^3$-term comes to play since semi-annihilation happens here only when $M_X > M_{H_1}$. However, unlike the global model, local $Z_3$ model allows ample parameter space in the low mass range, $M_X < M_{H_1}$, even if LUX limit is taken into account. This is shown as colored triangles in figure 2.

There could exist other differences between local and global $Z_3$ models. Depending on the exact value of $M_{Z'}$, $M_{H_2}$ and other physical parameters, the phenomena could be quite different. For instance, when $Z'$ or $H_2$ is light, $H_1$ can decay to them if $\epsilon \neq 0$ or $\lambda_{\phi H} \neq 0$ (see ref. [28] for extensive survey and ref. [29] for the comprehensive study of a singlet scalar $\phi$ mixing with the SM Higgs boson). Also, in local $Z_3$ model isospin-violating interaction between DM and nucleon can arise from $Z'$ exchange. On the other hand, only isospin-conserving couplings between DM and nucleon exist in global $Z_3$ model through the Higgs mediation, if we neglect small isospin violation from $m_u \neq m_d$. Therefore one can have two independent channels in the DM-nucleon scattering amplitude, which might be helpful to understand the recent data on direct detection of DM in the light WIMP region [30]. This is generic in models with local dark gauge symmetry which is spontaneously broken by dark Higgs field [31].

Finally, when $M_{Z'}$ or/and $M_{H_2}$ is about $O(\text{MeV})$, sizable DM self-interaction could be realized, which is motivated to solve the astrophysical small scale structure anomalies. We shall discuss this self-interacting DM scenario in section 5 in detail.
4.3 Comparison with the effective field theory (EFT) approach

In this subsection, we make a brief comparison of the renormalizable local $Z_3$ scalar DM model with the effective field theory (EFT) approach. Usual starting point for the EFT approach is to write down the operators for direct detections of DMs. For a complex scalar DM $X$ we are considering in this work, one can easily construct the following operators imposing $Z_3$ symmetry, to list only a few:

\begin{align}
U(1)_{\text{sym}} &: X^\dagger X H^\dagger H, ~ \frac{1}{\Lambda^2} \left(X^\dagger D_\mu X \right) \left(H^\dagger D^\mu H \right), ~ \frac{1}{\Lambda^2} \left(X^\dagger D_\mu X \right) \left(\bar{f} \gamma^\mu f \right), \text{ etc.} \quad (4.3) \\
Z_3 \text{ sym} &: \frac{1}{\Lambda} X^3 H^\dagger H, ~ \frac{1}{\Lambda^2} X^3 \bar{f} f, \text{ etc.} \quad (4.4) \\
&\quad \text{(or } \frac{1}{\Lambda^3} X^3 \bar{T}_L H f_R, \text{ if we imposed the full SM gauge symmetry)} \quad (4.5)
\end{align}

where $f$ is a SM fermion field and $\Lambda$ is a combination of new physics scale and couplings of the DM particle to new physics particle, and can differ from one operator to another. The usual story within the EFT is that the direct detection cross section due to the renormalizable operator $X^\dagger X H^\dagger H$ is strongly constrained so that the scalar DM can not be thermalized if it is light.

Note that within the EFT picture there is no room for $Z'$ or $H_2(\approx \phi)$ to enter and play important roles in direct and indirect detection or in the calculation of DM thermal relic density. This is because we do not know which fields are relevant (or dynamical) at the energy scale we are considering. Without constructing a full theory which is mathematically consistent and physically sensible, it would be difficult to guess which fields would be relevant beforehand within the EFT approach.

Also note that the usual complementarity does not work in this $Z_3$ models, since the EFT approach for direct detection based on eq. (4.3) does not capture the semi-annihilation channels for thermal relic density or indirect DM signatures described by eqs. (4.4) and (4.5), which is unique in the $Z_3$ models. This simple example shows that the DM EFT can be useful only if we know the detailed quantum numbers of DM particle, such as its spin and other (conserved) quantum numbers. Otherwise the complementarity does not work. Since we do not know anything about the DM quantum numbers as of now, the EFT approach and complementarity arguments should be taken with a great caution. Otherwise one would make erroneous conclusions.

More detailed discussions on the subtleties and limitations of EFT approach for DM physics will be discussed elsewhere.

5 Self-Interacting dark matter $X$

One more difference between local and global $Z_3$ models is that there can exist strong self-interaction between scalar DM $X$ in the local $Z_3$ model.\(^\dagger\) Traditional collisionless cold dark matter (CDM) can explain the large scale structure of the Universe. However, astrophysical anomalies in small scale structures motivate collisional CDM, which has self-interaction around $\sigma/M_X \sim 0.1 - 10 \text{ cm}^2/\text{g}$. This can be achieved in the local $Z_3$ model with $\mathcal{O}(\text{MeV}) H_2$ or $Z'$. A vector $Z'$ can mediate both attractive and repulsive forces, and has been considered in [33–41]. So here we shall only concentrate on the $\mathcal{O}(\text{MeV}) H_2$ case in which only attractive

\(^\dagger\)This feature is not unique to local $Z_3$ model, but could appear in many other DM models with dark gauge symmetries. Another example with local $Z_2$ symmetry will be presented elsewhere \[31\].
Figure 3. Scatter plots of various parameters that are consistent with relic density, LUX direct search bound and self-interaction \( \sigma_T/M_X \in [0.1,10] \) cm\(^2\)/g at Dwarf galaxies scale with \( v_{\text{rel}} \simeq 10 \) km/s, and \( \sigma_T/M_X \lesssim 0.5 \) cm\(^2\)/g at Milky Way and cluster scales with \( v_{\text{rel}} \simeq 220 \) km/s and \( v_{\text{rel}} \simeq 1000 \) km/s, respectively. We have used \( M_Z \simeq 200 \) GeV and \( \epsilon \ll 0.03 \) and scanned other parameters as illustration.

force is mediated for explanation of small scale structures. Other different phenomenologies of a light mediator can be found in [42–50].

Consider the \( XX^* \rightarrow XX^* \) elastic scattering process mediated by a t-channel scalar \( H_2 \), the differential cross section is

\[
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |p_f||p_i| |M|^2, \quad M \propto \frac{\lambda_{XX}^2 v_\phi^4}{(p_1 - p_3)^2 - M_{H_2}^2},
\]

\[
(p_1 - p_3)^2 = 2M_X^2 - 2(E_1E_3 - \vec{p}_1 \cdot \vec{p}_3) = -2|\vec{p}_1|^2(1 - \cos \theta).
\]

Since \( |p_f| = |p_i|, s \simeq 4M_X^2, E_1 = E_3 \) and \( |\vec{p}_1| = |\vec{p}_3| \) in the centre-of-mass system, then we have

\[
\sigma_{\text{SI}} = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{\lambda_{XX}^4 v_\phi^4}{64\pi M_X^2 M_{H_2}^4} \frac{1}{(4|\vec{p}_1|^2 + M_{H_2}^2)} \simeq \frac{\lambda_{XX}^4 v_\phi^4}{64\pi M_X^2 M_{H_2}^4}, \text{ for } |\vec{p}_1| \ll M_{H_2}. \quad (5.1)
\]
The more relevant quantity for quantifying the self-interaction of DMs is the momentum-transfer or transport cross section\(^2\)
\[
\sigma_T = \int d\Omega (1 - \cos \theta) \frac{d\sigma}{d\Omega},
\]
which regularizes the forward scattering\((\theta = 0)\) at which no momentum is transferred. In our case, we have for \(XX^* \rightarrow XX^*\) scattering
\[
\sigma_T = \frac{\lambda_{\phi X}^4 v^4_\phi}{32\pi M_{X}^4} \left( \frac{1}{4 |\vec{p}_1|^2} \right)^2 \left[ \ln (1 + R^2) - \frac{R^2}{1 + R^2} \right], \text{ where } R^2 = \frac{4 |\vec{p}_1|^2}{M_{H_2}^2}.
\]
(5.2)
This formula is consistent with \([33]\) where a vector mediator is considered. We may rewrite the above equation as
\[
\sigma_T = \frac{2\pi}{M_{H_2}^2} \beta^2 \left[ \ln (1 + R^2) - \frac{R^2}{1 + R^2} \right], \text{ where } \alpha_\phi \equiv \frac{\lambda_{\phi X}^2}{4\pi} \left( \frac{v_\phi}{2M_X} \right)^2 \text{ and } \beta \equiv \frac{2\alpha_\phi M_{H_2}}{M_X v_{rel}^2}.
\]
On the other hand, annihilation cross section for \(XX^* \rightarrow \phi \phi\) at the freezing out time is approximately
\[
\sigma_{\text{ann}} \simeq \frac{\lambda_{\phi X}^4 v^4_\phi}{64\pi M_{X}^2 M_{X}^2} \frac{3}{M_{H_2}^2},
\]
which is much suppressed by \(M_{H_2}/M_X\), compared with eq.s (5.1) and (5.2). Naive estimates suffice to show that if we have \(\sigma_{\text{ann}} \sim O(1)\) pb for \(M_X \sim O(1)\)GeV, then \(M_{H_2} \sim O(1) - O(100)\)MeV would give \(\sigma_{\text{SI}} \sim O(1)\) barn and \(\sigma_T/M_X \sim 1 \text{ cm}^2/\text{g}\), although more dedicated analysis would involve the velocity-averaged \(<\sigma_T>\) and non-perturbative effects when \(\alpha_\phi M_X > M_{H_2}\).

As an illustration, we show the scatter plots for \(M_{H_2}-M_X\). Since we focus on the light \(H_2\) here, we can fix \(M_{Z'} = 200\)GeV and impose the constrain from electroweak precision observable, \(\epsilon \ll 0.03\). Other parameters are scanned as indicated from the legend bar of individual plot.

\[
g_X \lesssim 1.2, \; \lambda_{\phi X} \lesssim 1, \; \lambda_{H X} \lesssim 0.1, \; \lambda_3 \lesssim 0.1, \; \text{and } \lambda_{\phi H} \simeq 0.
\]

Because of the velocity-dependent behavior of eq. (5.1) and (5.2), the transfer cross section over mass, \(\sigma_T/M_X\), can be around \([0.1, 10]\) \(\text{cm}^2/\text{g}\) at Dwarf scale with \(v_{rel} \simeq 10 \text{ km/s}\) while still satisfy the requirement \(\sigma_T/M_X \lesssim 0.5 \text{ cm}^2/\text{g}\) to be consistent with ellipticity constraints on Milky Way and cluster scales.

Before closing this section, we briefly discuss the CMB constraints which are quite strong. When \(\alpha_\phi M_X > M_\phi\), there would exist large non-perturbative effect in the low-velocity limit \((v \rightarrow 0)\) of DM particle, known as Sommerfeld enhancement, and there could be relevant astrophysical constraint from cosmic microwave background(CMB) for some parameter space we discussed above. Then \(XX^*\) annihilation is enhanced at CMB time and significant energy would be injected to photon-baryon bath, broadening the last scattering surface and leaving an imprint in CMB spectra \([51–58]\). Current data constrains the enhancement factor \(S \lesssim O(1000)\) for \(O(\text{TeV})\) DM with the exact value depending on the specific

\(^2\)If the scattering particles are identical, \(XX \rightarrow XX\) for instance, it may be more appropriate to use the \(\sigma_T = \int d\Omega (1 - \cos^2 \theta) \frac{d\sigma}{d\Omega}\) which regularizes both forward and backward scattering \([32]\) (see \([38]\) for similar discussion).
annihilation channel. As an illustration, taking parameters for large self-interactions for the DM’s such as
\[ M_X \simeq 1\text{TeV}, \; M_\phi \simeq 1\text{MeV}, \; \lambda_{\phi X} \simeq 0.1, \]
we find that the enhancement factor saturates at \( S \sim \mathcal{O}(50) \), which is well below the current limit, \( S \lesssim \mathcal{O}(1000) \). Therefore the discussions on self-interacting DM presented in this section are safe from the CMB constraints.

Finally let us add that this mechanism for enhancing the DM self-interactions could be realized not only by light scalar mediator \( \phi \) but also by a light vector mediator \( Z' \) between \( X \) and \( X^* \) (namely, between opposite dark charges). Thus this feature is not unique to local \( Z_3 \) models, and could be easily realized in other models too, such as local \( Z_2 \) models [31].

6 Summary

In this paper, we have proposed a self-interacting scalar DM model with a local dark \( Z_3 \) symmetry. Unlike global dark symmetries, local ones can guarantee that DM is absolutely stable even in the presence of higher dimensional nonrenormalizable operators due to the underlying local gauge symmetry. Then we discussed perturbativity constraints on the scalar potential and the experimental limit on the kinetic mixing. Compared with a global \( Z_3 \) model, our scenario has two new particles, \( Z' \) and \( H_2 \), and there are new channels in the DM pair annihilations for thermalizing DMs. Therefore much ampler parameter space is allowed including a light DM with \( M_X < 125 \text{GeV} \), most region of which can be probed with future DM direct searches. Also, motivated by the small scale astrophysical anomalies, we investigated the phenomenology of a MeV scalar \( H_2 \) in our model which has no counterpart in the minimal global \( Z_3 \) model. Thanks to the velocity dependence of DM self-interaction cross section, such a light \( H_2 \) can mediate strong interaction for DM scattering at Dwarf galaxy scale while satisfying Milky Way and cluster scale constraints. Similar arguments go for the light \( Z' \) as well. For such a light \( H_2 \) or \( Z' \), there could be exotic decays of the 126GeV Higgs boson, which could be studied in the upcoming LHC running and at future lepton colliders.

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A Useful Formulas

A.1 RGEs

For future reference, here we present the RGEs in the case of no kinetic mixing,
\[
\frac{d\lambda_H}{d\ln \mu} = \frac{1}{16\pi^2} \left[ 24\lambda_H^2 + \lambda_{3H}^2 + \lambda_{HXX}^2 - 6y_t^4 + 3 \left( 2g_1^2 + (g_1^2 + g_2^2)^2 \right) - \lambda_H \left( 9g_2^2 + 3g_1^2 - 12y_t^2 \right) \right],
\]
\[
\frac{d\lambda_\phi}{d\ln \mu} = \frac{1}{16\pi^2} \left[ 20\lambda_\phi^2 + 2\lambda_{3\phi}^2 + \lambda_{\phi XX}^2 + 6y_t^4 - 12\lambda_\phi g_2^2 \right],
\]
Now the question is shifted to the positivity of $\beta x$
With the replacement $x = u + vz$
coefficients. Positivity on any fixed interval $(u, v)$ with real coefficients, positive dλX
the positive reals through the transformation
mathematical details. For a general quartic polynomial
This section summarizes the positivity conditions for quartic polynomials, see ref. \[15\] for

\[
\frac{d\lambda_X}{d \ln \mu} = \frac{1}{16\pi^2} \left[ 20\lambda_X^2 + 2\lambda_{HX}^2 + \lambda_{\phi X}^2 + 9\lambda_\phi^2 + \frac{2}{27}g_X^4 - \frac{4}{3}\lambda_\phi g_X^2 \right],
\]
\[
\frac{d\lambda_{\phi H}}{d \ln \mu} = \frac{1}{16\pi^2} \left[ 4\lambda_{\phi H} (3\lambda_H + 2\lambda_\phi + \lambda_{\phi H}) + \lambda_{\phi X} \lambda_{H X} - \lambda_{\phi H} \left( \frac{9}{2}g_2^2 + \frac{3}{2}g_1^2 - 6g_1^2 + 6g_2^2 \right) \right],
\]
\[
\frac{d\lambda_{H X}}{d \ln \mu} = \frac{1}{16\pi^2} \left[ 4\lambda_{H X} (3\lambda_H + 2\lambda_\phi + \lambda_{H X}) + \lambda_{\phi H} \lambda_{\phi X} - \lambda_{H X} \left( \frac{9}{2}g_2^2 + \frac{3}{2}g_1^2 - 6g_1^2 + 2g_2^2 \right) \right],
\]
\[
\frac{d\lambda_{\phi X}}{d \ln \mu} = \frac{1}{16\pi^2} \left[ 2\lambda_{\phi X} (2\lambda_\phi + 2\lambda_\phi + \lambda_{\phi X}) + 2\lambda_{\phi H} \lambda_{H X} + 18\lambda_\phi^2 - \lambda_{H X} \left( 6g_2^2 + \frac{2}{3}g_3^2 \right) \right],
\]
\[
\frac{dg_X}{d \ln \mu} = \frac{1}{16\pi^2} \left( \frac{1}{3} + \frac{1}{27} \right) g_X^3,
\]
\[
\frac{d\lambda_3}{d \ln \mu} = \frac{1}{16\pi^2} \left[ \lambda_3 (2\lambda_X + \lambda_{\phi X}) \right].
\]

### A.2 Positive conditions for quartic polynomial

This section summarizes the positivity conditions for quartic polynomials, see ref. \[15\] for
mathematical details. For a general quartic polynomial

\[
f(z) = az^4 + bz^3 + cz^2 + dz + e,
\]

with real coefficients, positive $a$ and $e$, $f(z) \geq 0$ for $z > 0$ shall constrain the regions of
coefficients. Positivity on any fixed interval $(u, v)$ can be translated directly to positivity on
the positive reals through the transformation

\[
t = \frac{u + vz}{1 + z}.
\]

With the replacement $x^4 = \frac{a}{e}z^4$, the polynomial $f(z)/e$ then becomes $p(x) = x^4 + \alpha x^3 +
\beta x^2 + \gamma x + 1$, where we have defined

\[
\alpha = ba^{-\frac{3}{2}}e^{-\frac{1}{2}}, \beta = ca^{-\frac{1}{2}}e^{-\frac{1}{2}}, \gamma = da^{-\frac{1}{2}}e^{-\frac{3}{2}}.
\]

Now the question is shifted to the positivity of $p(x)$ for $x \geq 0$. Define

\[
\Delta = 4 \left[ \beta^2 - 3\alpha \gamma + 12 \right]^3 - \left[ 72\beta + 9\alpha \beta \gamma - 2\beta^3 - 27\alpha^2 - 27\gamma^2 \right]^2,
\]
\[
\Lambda_1 \equiv (\alpha - \gamma)^2 - 16(\alpha + \beta + \gamma + 2),
\]
\[
\Lambda_2 \equiv (\alpha - \gamma)^2 - \frac{4(\beta + 2)}{\sqrt{\beta - 2}} \left( \alpha + \gamma + 4\sqrt{\beta - 2} \right).
\]

Then $p(x) \geq 0$ for all $x \geq 0$ or $f(z) \geq 0$ for all $z > 0$ if and only if

\[
(1) \ \beta < -2 \text{ and } \Delta \leq 0 \text{ and } \alpha + \gamma > 0;
\]
\[
(2) \ -2 \leq \beta \leq 6 \text{ and } \begin{cases} \Delta \leq 0 \text{ and } \alpha + \gamma > 0, \\
\Delta \geq 0 \text{ and } \Lambda_1 \leq 0; \end{cases}
\]
\[
(3) \ 6 < \beta \text{ and } \begin{cases} \Delta \leq 0 \text{ and } \alpha + \gamma > 0, \\
\alpha > 0 \text{ and } \gamma > 0, \\
\Delta \geq 0 \text{ and } \Lambda_2 \leq 0. \end{cases}
\]
It is also useful to give the following sufficient conditions for positivity,

\[ \alpha > -\frac{\beta + 2}{2} \quad \text{and} \quad \gamma > -\frac{\beta + 2}{2} \quad \text{for} \quad \beta \leq 6, \quad (A.8) \]

\[ \alpha > -2\sqrt{\beta - 2} \quad \text{and} \quad \gamma > -2\sqrt{\beta - 2} \quad \text{for} \quad \beta > 6. \quad (A.9) \]

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