Improvement of the population competition model

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Abstract. Classic population competition model with fixed parameters can only be used under the condition of constant living environment, which is impossible in real life. So it has little practical significance in some cases. In this paper, the model will be improved so that it can be used in a changeable living environment. The objective is to establish the relationship between model parameters and independent variables of external conditions with multivariate regression. Growth curves of populations competing with each other are obtained by piecewise prediction. The data used in the regression are obtained by doing experiments. This paper will take wood rot fungus as an example to explain how to establish the relationship by doing experiments, and apply the new model and the classic one with artificial simulation data, so that two groups of predicted population competition curves are obtained. It has been found that the new model can more accurately describe population competition in the case of severe fluctuations in external environment, while the classic model has obvious errors. Finally, this paper briefly describes other advantages of the new model.

1. Introduction

Models of population growth exert great significance upon human development. Its important applications include population control, monitoring and allocation of social resources, improvement of ecological environment, conservation of species, development of aquaculture, etc.[1] For instance, the population competition model can be used for the location and layout of urban charging stations [2]. The study of the factors that affect the growth of biological quantity can not avoid the study of population competition. The classic population competition model is a interspecific competition model of two populations[3]. Later generations have made many improvements on it, for example, introducing differential equation representing factors of intraspecies competition [4], food amount [5], seasonal variation [6], etc. or adding exponential terms [7-8]. Population competition models are becoming more accurate. But it is noticed that existing models, including the classic model, are used under constant external conditions for their fixed parameters. But in real life, conditions such as temperature and humidity can not remain the same. So there is room for improvement in accuracy in this direction. Sometimes the effects of environmental change have to be considered, especially when time span is very large or environment changes dramatically. The purpose of this paper is to explore how to introduce independent variables of external environment into the population competition model. In order to make this article easier and clearer, the classic population competition model with simple structure is selected as research object. Emphasis will be placed on how to establish model parameter equations with external environment parameters as independent variables and its application effect, as well as on the comparison between the new model and the classic one.

According to citation [3], p222 to p224, the expression of the Logistic model is:
The $x(t)$ denotes the number of individuals of the population at the $t$ moment. The $x_m$ indicates environmental capacity. It represents the maximum number of individuals in a population that can be maintained in a given space without damage to environmental conditions. The $x_0$ is the number of individuals of the population at the beginning, namely, when $t$ is 0. The $r$ is the rate of natural increase. The assumption of this model is that the rate of natural increase of the population is a constant equal to the $r$ in the model when there is no environmental accommodation constraint, namely, when $x_m$ is $\infty$. A larger $r$ indicates a faster growth in the number of individuals. The Logistic model is a model that depicts the growth curve of a population only under the constraints of its environmental capacity. And the growth rate of the population is depicted by the following differential equation:

$$\frac{dx}{dt} = rx(1 - \frac{x}{x_m})$$

(2)

And its initial value $x(0)=x_0$. If a second population competes in the same environment, the growth rate of the first population satisfies the following equation:

$$\frac{dx_i}{dt} = r_i x_i (1 - \frac{x_i}{x_{i1m}} - s_{21} \frac{x_s}{x_{2m}})$$

(3)

This equation depicts the impact of environmental capacity and population competition on the growth rate of the first population. The $s_{ij}$ can be regarded as the coefficient of the inhibition of population No. $i$ on the growth of population No. $j$. And it can be intuitively understood that the consumption of food, which No.$j$ survives on, of per unit number of No.$i$’s individuals is $s_{ij}$ times the amount of that of No.$j$’s $[3]$. Similarly, the second population satisfies the equation (4). And the classic model consists of equation (3) and (4).

$$\frac{dx_2}{dt} = r_2 x_2 (1 - \frac{x_2}{x_{2m}} - s_{12} \frac{x_1}{x_{1m}})$$

(4)

2. Model improvement and application

2.1. Overall objectives and methodology

The assumption of the classic model is that external conditions are constant, which is obviously ideal and has little practical significance in some cases. So I wonder if it can be improved into a model that can be used under changeable external conditions. Parameters in the model need to be linked to independent variables in external conditions to describe the effects of changes in the external environment on biological growth and competition. The estimation of parameter values can be carried out by experiments, which creates conditions that are relatively more ideal and suitable for parameter regression. We will study this model with wood rot fungi.

2.2. Estimating parameter values through experiments

2.2.1. Design of experiments. The factors affecting the growth of wood rot fungi in the nature include temperature, humidity, PH value, organic matter content, etc. $[9]$ Therefore, there are many independent variables in external environment. Here we only choose temperature and relative humidity as influencing factors, and the other factors are regarded as the same and suitable. It is assumed that we have successfully separated two kinds of fungi and carried out our experiments.
The optimum temperature for most fungi is 25–30°C [10], and relative humidity 97–100%. The optimum relative humidity is different at different growth stages, some can grow at 60% relative humidity [11-12]. Accordingly, it is determined that the temperature of our study is 15, 20, 25, 30, 35 and 40°C, and the relative humidity 30, 45, 60, 70, 80, 90, 95 and 100%, which means that we get 40 combinations of temperature and relative humidity. Each combination is an experimental condition. Then we assume that the following experiments will be done: the two kinds of fungi are mixed and cultured under each experimental condition. The number of individuals of the two on the first day is suitable, and the number of individuals of each population is counted at the same time every day. The experiments last until the number of individuals of the two populations remains basically unchanged. From this we can know that a total of 40 groups of experiments will be done.

2.2.2. Acquisition of model parameter equations. Through the above experiments, the number of individuals per day for each type of fungus at a certain temperature and relative humidity will be obtained. So we can estimate the parameters in equation (3) and (4) by regression, and get the value of parameter $s_{12}$, $s_{21}$, $x_{im}$, $x_{2im}$, $r_1$ and $r_2$ under 40 experimental conditions that are relatively ideal. These 40 sets of data can be used to carry on binary regression to each parameter separately, then we can obtain parameter equations of each parameter with temperature and relative humidity as independent variables. Specific regression methods will not be introduced here. After obtaining the parameter equations, parameter values under other external conditions can be calculated.

2.3. Prediction of biological growth curves
The growth curve of a population is a graphical representation with time as x axis and the number of individuals as y axis. If the number of individuals of a given species at a certain moment is known, it can be used as an initial value $x_0$, and our new model can predict the value $x_i$, which is the number of individuals at the next step. Similarly, the $x_i$ value can be used as the initial value of the next prediction to predict $x_{i+1}$. The points can be connected by our new model with the help of MATLAB R2016a, and we can get the growth curve of the population. Furthermore, for the convenience of following description, we introduce the concept of “relative time”. It is assumed that the time interval between these moments is equal, and parameters during one interval is constant. One interval is called a unit of relative time. The length of the time interval can be changed as desired.

2.4. Model application with artificial data
2.4.1. Hypothesis of parameter equations. Citation [13] shows that: (1) The slow growing strains of fungi tend to be better to survive and grow in the presence of environmental changes with respect to moisture and temperature, while the faster growing strains tend to be less robust to the same changes. (2) Faster-growing fungal populations tend to be more competitive. Based on the above information, we can simulate the equations of each parameter. We can get : $r \propto s$. Generally, the smaller the $r$, the slower the $r$ changes with temperature and relative humidity. Then we created two kinds of fungi with the following characteristics (shown by Table 1 and Figure 1~4). The characteristics of these curves are assumed according to the characteristics of fungi in the information above.

| population name | $r_i$ | $x_{im}$ | $s_{ij}$ |
|-----------------|------|--------|--------|
| Fungus No.1     | -0.0005*(x-20)^2+0.05+0.000005*y^2-0.0021 | $r_1*50$ | $r_1*2$ |
| Fungus No.2     | -0.06*(x-25)^2+0.12+0.000015*y^2-0.06   | $r_2*100$ | $r_2*10$ |

*The $x$ is temperature (°C), and the $y$ is relative humidity (%).

*C The $x_{im}$ is the environmental capacity of Fungus No.i. In addition, we assume that if $x_{im}<0$, $x_{im}=0$.

*d The $s_{12}=r_1*2$ and the $s_{21}=r_2*10$. Additionally, we assume that if $s_{ij}<0$, $s_{ij}=0$. 
For convenience, we assume that the \( r_i \) is obtained by the superposition of the effects of temperature and relative humidity. The effect of temperature and relative humidity on \( r_i \) is shown in Figure 1–4. The \( r_i \) is the sum of the corresponding values in Figure 1 and 2. Also, other parameters take multiple of the \( r_i \).

![Figure 1. Hypothetical effects of temperature on \( r \).](image1)

![Figure 2. Hypothetical effects of relative humidity on \( r \).](image2)

![Figure 3. Hypothetical relationship between \( r \) and its independent variables of Fungus No.1.](image3)

![Figure 4. Hypothetical relationship between \( r \) and its independent variables of Fungus No.2.](image4)

2.4.2. Hypothetical temperature and relative humidity change in the environment. The temperature and relative humidity assumed vary with relative time as shown in Figure 5. As defined in section 2.3, relative time refers to a multiple of the duration of the time interval. Climate volatility has been assumed for the development of our case study. We assume that the two kinds of fungi compete in this environment, and then use the new model to predict the number of individuals in each population.

![Figure 5. Climate change curve assumed.](image5)

2.4.3. Predictions based on the classic model. Since the classic model parameters are fixed, the parameters will be estimated as follows: the mean value of temperature and relative humidity in Figure 5.
are calculated, and relevant expressions described in section 2.4.1. are substituted. After calculating the value of each parameter, we can get the curves in Figure 7 with the classic model.

Figure 6. Population growth curves obtained using the new model.

Figure 7. Population growth curves obtained using the classic model.

3. Results and analysis

3.1. Prediction results of growth curves

3.1.1. Information in Figure 6. The population of Fungus No.1 in Figure 6 decreases first and then rises. Combined with Figure 1, 2 and 5, it is found that Fungus No.1 begins to decline because of competition for Fungus No.2. However, at the relative time of 6, the competition pressure is reduced due to the rapid decline of Fungus No.2, and Fungus No.1 slightly increases. But then the number of Fungus No.1 declines because the climate is not suitable for its growth. Then the climate returns to its original state and Fungus No.1 rises again. For Fungus No.2, it grows rapidly at the beginning because of the suitable climate at first and the strong competitiveness of population No.2, but later, due to climate change, Fungus No.2 does not adapt to the environment and dies out. Population of Fungus No.2 becomes set to 0.

3.1.2. Information in Figure 7. In Figure 5, the average temperature over this period is 25.44°C and relative humidity 75.89%. According to the equations in section 2.4.1, the following values are obtained: $r_1=0.0789$, $r_2=0.1470$, $x_{1m}=0.7891$, $x_{2m}=2.9403$, $s_{21}=1.4702$ and $s_{12}=0.1578$, from which Figure 7 is obtained. These results show that Fungus No.2 is always competitive, since the number of individuals grows rapidly during the simulation. And population of Fungus No.1 is in decline.

3.2. Analysis

From the results we can clearly see that the curves in Figure 6 are much more complex than Figure 7. They can show the slight effect of small fluctuations in climatic conditions on the curves, thus making the description more accurate. And more importantly, it is more informative than using a fixed parameter model. When relative time is about 6, external climate fluctuations have made Fungus No.2 unable to adapt to the environment, so the population rapidly declines, and it is extinct when relative time reaches 11. Population of Fungus No.2 remains at 0 even when the climate returns to more friendly conditions to this species. However, the information shown in Figure 7 is biased. It shows that Fungus No.2 has been in existence and has been growing at a rapid rate over the period. Combined with Figure 1, 2 and 5, it is clear that this result is wrong. The growth at different times is certainly not exactly the same.

4. Conclusion and discussion

The structure of the new population competition model is as simple and clear as that of the classic one, which consists of equation (3) and (4). However, with parameter equations of the independent variables
of external environment, the prediction is more detailed and accurate, especially the influence of fluctuating external environment on the population competition. When external conditions fluctuate violently, the classic population competition model with fixed parameters can not accurately describe some information in the population competition. So the proposed model is more practical than the fixed parameter model when climate conditions are changing. In fact, we can increase the number of equations in the new model to predict competition of multiple populations. What needs to be done is to figure out the equation of each parameter by experiments. The number of external variables can be more than two, as long as experiments can be carried out. In addition, if the initial population value is known, as well as the values of external environmental variables, the independent variables of external conditions can be approximately regarded as the independent variables of the number of individuals of each population. In this way, a database of parameter equations may be established and then predict population competition by predicting weather. At the same time, knowing the initial value of population number and the trend of climate change, we can also directly predict population competition under different population combinations, even if the competition we want to study does not occur in the real world.

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