Abstract We study a solution to the Einstein field equations on an eight-dimensional pseudo-Riemannian manifold (a spacetime of four space dimensions and four time dimensions) that exhibits inflation of three of the four space dimensions and deflation of three of the four time dimensions. Hubble parameters ($H_4, H_8$) exist for each of the unscaled dimensions (time: $x^4$, space: $x^8$). The scale factor for the three ordinary space dimensions ($x^1, x^2, x^3$) is $a = a(x^4, x^8)$, and during inflation is functionally a product of exponentials $a = e^{H_4 x^4}/c e^{H_8 x^8}/c$ of each Hubble parameter times its respective unscaled coordinate ($c$ is the physical speed of gravitational waves in vacuum in the observed universe). We investigate the conjecture that every particle/field in our universe is carried along the $x^8$-axis in the same way that it is carried along the $x^4$-axis, that is, in a manner that is independent of its energy, and with no freedom to change either its rate of passage along the $x^8$-axis or its location within the $x^8$-dimension. The evolution of the particle/field through the unscaled dimensions $(x^4, x^8) \in \mathbb{R}^{1,1}$ in our universe is entrapped and entrained by geometry. We employ a line element of the form $ds^2 = -|g_{44}| (c_4 dx^4)^2 + g_{88} (dx^8)^2 + \cdots$ to define a physical time $t = x^0$ through the relation $|g_{tt}| (c dt)^2 = |g_{44}| (c_4 dx^4)^2 - g_{88} (dx^8)^2$. Here $c_4$ is a speed related parameter (in this paper $g_{tt} = g_{44} = -1$). This model may provide new insight into physical time.

Keywords cosmology with extra dimensions, inflation, deflation
Extra-Dimensional Bi-Inflation

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1 Introduction

In general, inflationary cosmology \cite{1,2,3} is a widely accepted and largely successful theory of the physics of the very early universe. Recent observational data from important experiments such as WMAP \cite{4} and the Sloan Digital Sky Survey \cite{5,6,7}, and others support \cite{8,9,10,11} the main predictions of inflation theory. Inflation employs a period of accelerated expansion at the beginning of the universe to solve the monopole, horizon and flatness problems. Here we discuss a new model of “inflation” that is based on the idea that our universe has as many time dimensions as space dimensions.

Let $X_{4,4}^8$ denote an eight-dimensional pseudo-Riemannian manifold that admits a spin structure, whose local tangent spaces are isomorphic to flat Minkowski spacetime $M_{4,4}^4$. $X_{4,4}^8$ is a spacetime of four space dimensions, with local co-moving coordinates $(x^1, x^2, x^3, x^8)$, and four time dimensions, with local co-moving coordinates $(x^4, x^5, x^6, x^7)$. All coordinates have dimensionality of length (time coordinates have been scaled with a speed parameter so that they have correct units). Let $g$ denote the pseudo-Riemannian metric tensor on $X_{4,4}^8$. The signature of the metric $g$ is assumed to be $(+ + + - - - - +)$. We assume that $g \leftrightarrow g_{\alpha\beta} = g_{\alpha\beta}(x^\nu)$ carries the Newton-Einstein gravitational degrees of freedom, and that the ordinary Einstein field equations (on $X_{4,4}^8$) $c^4 G_{\mu\nu} = 8 \pi G T_{\mu\nu}$ are satisfied. Here Greek indices run from 1 to 8; we are using the usual component notation in local charts. (Also, the covariant derivative with respect to the symmetric connection associated to the metric $g$ is denoted by a double-bar; $G$ denotes the Newtonian gravitational constant, and $c$ is the speed of gravitational waves in vacuum in the present universe.)

We assume that the $x^8$-dimension is not compact (which leads to ideas far outside the scope of this paper), and moreover assume that there is not an infinity of universes that is parameterized by $x^8$. Instead, as stated in

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the Abstract, we investigate the conjecture that particles and fields in our universe are carried along the $x^8$-axis in the same way that they are carried along the $x^4$-axis, that is, in a manner that is independent of their energy, and with no freedom to change either the rate of passage along the $x^8$-axis or the position $x^8$ on this axis. The evolution of every “entity” possessing an associated energy and momentum, through the unscaled dimensions $(x^4, x^8) \in \mathbb{R}^{1,1}$ in our universe, is assumed to be entrapped and entrained by geometry.

With this assumption, we may employ a line element of the form

$$ds^2 = -|g_{44}| (c_4 dx^4)^2 + g_{88} (dx^8)^2 + \cdots,$$

where $c_4$ is a positive non-dimensional speed-related parameter. If we write $x^4$ in terms of a temporal $t^4$ and a positive speed unit $c_4'$, $x^4 = c_4' t^4$. In this paper $g_{tt} = g_{44} = -1$ and $g_{88} = \xi^2$. This implies that

$$c_4^2 \left(\frac{dx^4}{dt}\right)^2 = c^2 + \xi^2 \left(\frac{dx^8}{dt}\right)^2$$

whenever $x^4 = x^4(t)$ and $x^8 = x^8(t)$ are defined. The parameterizations employed in this paper are

$$x^4 = \frac{c_4}{\xi} \sqrt{2} t \quad \text{and} \quad x^8 = \frac{c_4'}{\xi} t.$$  

We study a solution to the Einstein field equations on $\mathbb{R}^{4,4}$ that exhibits inflation/deflation: during “inflation”, the scale factor for the three space dimensions $(x^1, x^2, x^3)$ exponentially inflates, and the scale factor for the three time dimensions $(x^5, x^6, x^7)$ exponentially deflates; moreover, $x^4$ and $x^8$ do not scale. (both $x^4$ and $x^8$ are assumed to be non-compact). For brevity this phenomenon is simply called “inflation.”

2 Problem Formulation and Solution

A Mathematica file in which we perform most of the lengthy calculations for this paper, and graph some of the functions, is included in the archive for this paper [12]. Accordingly, many of the lengthy equations and intermediate results are omitted from this text.

The line element for inflation/deflation is assumed to be given by

$$\{ds\}^2 = \{(dx^4, x^8)\}^2 \left[(dx^4)^2 + (dx^5)^2 + (dx^6)^2\right] - (c_4 dx^4)^2 - c_8^2 \{(b(x^4, x^8))\}^2 \left[(dx^5)^2 + (dx^6)^2 + (dx^7)^2\right] + (\xi dx^8)^2.$$  

Here $\xi$ and $c_4$ are positive dimensionless parameters. We seek an inflationary solution to the field equations that are sourced by a constant vacuum energy.
density $\rho = \rho_0$, which we write as $\rho_0 = \frac{\kappa^2}{4 \pi}$, where $\kappa$ is a positive parameter with units of inverse time. We seek a solution of the form

$$a(x^4, x^8) = a_0 \, e^{\nu(x^4, x^8)}$$
$$b(x^4, x^8) = b_0 \, e^{\nu(x^4, x^8)},$$

Here, $c \, H_4$ and $c \, H_8$ are dimensionless positive parameters (recall that $c$ is the physical speed of gravitational waves in vacuum). The positive Hubble parameters $H_4$ and $H_8$ for this model are measures of the expansion rates of the universe, and defined according to (note that $\sqrt{3} c \frac{\kappa^2}{3} = \sqrt{\kappa^2 \rho_0}$)

$$H_4 = \frac{c}{a(x^4, x^8)} \left[ \frac{\partial}{\partial x^4} a(x^4, x^8) \right] = \sqrt{\frac{2}{3}} \, c \, c_4$$
$$H_8 = \frac{c}{a(x^4, x^8)} \left[ \frac{\partial}{\partial x^8} a(x^4, x^8) \right] = \sqrt{\frac{2}{3}} \, c \, c_8 \xi.$$

We assume that the stress-energy tensor $T^\mu_\nu$ is diagonal and given by

$$T^\mu_\nu = c^2 \, \text{diag}(p, p, -\rho, -\rho_2, -\rho_2, -\rho_2, p_2),$$

where $\varrho = \varrho(x^4, x^8)$ is the ordinary energy density/cosmological constant and the ordinary pressure is $p = p(x^4, x^8)$. For the case of inflation we set $\varrho = \rho_0 = \text{constant}$. The Equations of State (EOS) are assumed to be

$$p = w \varrho, \quad p_2 = \mathbb{W} \varrho \quad \text{and} \quad \rho_2 = \lambda \varrho.$$

Here, $w, \mathbb{W}, \lambda$ are parameters. We are employing the simplest not-unreasonable definitions of $T^\mu_\nu$ and the EOS; there is to date, of course, little empirical evidence for these definitions. The strongest support for them is the prediction to which they lead.

When the Einstein field equations are satisfied $T^\mu_\nu || \mu = 0$. We find that $T^\mu_\nu || \mu = 0$ if any of the four sets of relations is satisfied:

$$\mathbb{W} \in \mathbb{R}, \quad \mathbb{W} = \frac{c^2 (1 + w) \, H_4 \, H_6 \, (H_4 + H_6 + \mathbb{W})}{H_4 (H_4 + H_6 + \mathbb{W})} \quad \mathbb{W} = 0 \quad \frac{c^2 (1 + w) \, H_4 \, H_6 \, \mathbb{W}}{H_4 (H_4 + H_6 + \mathbb{W})} \quad \lambda = 0 \quad \frac{c^2 (1 + w) \, H_4 \, H_6 \, \mathbb{W}}{H_4 (H_4 + H_6 + \mathbb{W})}$$

Upon substituting, in turn, each of these relations into the Einstein field equations $c^4 \, G^\mu_\nu = 8 \, \pi \, G \, T^\mu_\nu$ and solving, we obtain a class of solutions that are parameterized by $c \, H_4 > 0$. We find that there is a common solution giving $c \, H_4$ as a function of $c \, H_8$ that is given by $c \, H_4 = 3 \, (c \, H_8) \pm \sqrt{1 + 8 \, (c \, H_8)^2}$. The particular values/ranges of $c \, H_8$ that satisfy the field equations, as well as the detailed relationships between the parameters $(w, \mathbb{W}, \lambda)$ depends on the case, of course. We remind ourselves that on $X_{4,4}$ that during “inflation”, the scale factor for the three space dimensions $(x^3, x^5, x^7)$ exponentially inflates, and the scale factor for the three time dimensions $(x^4, x^6, x^7)$ exponentially
deflates; moreover, $x^4$ and $x^8$ do not scale. Inflation of $a(x^4, x^8)$ and deflation of $b(x^4, x^8)$ is manifest for

$$c H_4 = \begin{cases} 
3 \left( c H_8 \right) + \sqrt{ -1 + 8 \left( c H_8 \right)^2 } & \text{for } \frac{1}{2\sqrt{2}} < c H_8 < \infty \\
3 \left( c H_8 \right) - \sqrt{ -1 + 8 \left( c H_8 \right)^2 } & \text{for } \frac{1}{2\sqrt{2}} < c H_8 < \frac{1}{\sqrt{2}} (1 + 2\sqrt{2}) \end{cases} \quad (9)
$$

For the first listed case of Eq. (8), assuming nonzero $w$, we find that

$$w = -1 + 4 \left( c H_8 \right)^2 \pm 2 \left( c H_8 \right) \sqrt{ -1 + 8 \left( c H_8 \right)^2 } \quad (10)$$

The Hubble parameters are independent of the value of $w$ and only depend on the value of $c H_8 > \frac{1}{2\sqrt{2}}$. One observes that the radicals $\pm \sqrt{ -1 + 8 \left( c H_8 \right)^2 }$ vanish when $\left( c H_8 \right) = \frac{1}{\sqrt{2}}$, which we temporarily assume, in order to briefly summarize the predictions of this case. This yields

$$c H_8 = \frac{1}{2\sqrt{2}}, \quad c H_4 = \frac{3}{2\sqrt{2}}, \quad w = -\frac{1}{2}, \quad \mathcal{W} = -\frac{7}{2}, \quad \lambda = \frac{5}{2} \quad (11)$$

gives, for this special case only,

$$\ln \frac{a(x^4, x^8)}{a_0} = \sqrt{\frac{2}{3} \kappa} \left( c H_4 c_4 x^4 + c_8 x^8 \right) = \frac{\kappa (3c_4 x^4 + \xi x^8)}{2\sqrt{3}c} = \frac{\left( 3\sqrt{2} + 1 \right)}{2\sqrt{3}} \kappa t$$

$$= 1.51342 \kappa t > 0$$

$$\ln \frac{b(x^4, x^8)}{b_0} = \sqrt{\frac{2}{3} \kappa} \left( c_8 x^4 - c_4 x^8 \right) = \frac{\kappa (c_4 x^4 - 3\xi x^8)}{2\sqrt{3}c} = -\frac{\left( 3 - \sqrt{2} \right)}{2\sqrt{3}} \kappa t$$

$$= -0.45777 \kappa t < 0. \quad (12)$$

For this case we find that the extra time dimensions become less observable through deflation.

Two other cases yield definite values for $c H_8$ as follows: For the first listed case of Eq. (8), but with $w = 0$,

$$c H_8 = \frac{1}{2} \sqrt{\frac{2}{\left( \sqrt{5} - 1 \right)}} \Rightarrow c H_4 = \frac{1}{2} \sqrt{\frac{2}{\left( 11 + 5\sqrt{5} \right)}}. \quad \text{We find that}$$

$$\ln \frac{a(x^4, x^8)}{a_0} = \sqrt{\frac{2}{3} \kappa} \left( c H_4 c_4 x^4 + c_8 x^8 \right) = \frac{\left( \sqrt{5} - 1 + \sqrt{22 + 10\sqrt{5}} \right)}{2\sqrt{3}} \kappa t$$

$$= 2.24363 \kappa t > 0$$

$$\ln \frac{b(x^4, x^8)}{b_0} = \sqrt{\frac{2}{3} \kappa} \left( c_8 x^4 - c_4 x^8 \right) = -\frac{1}{6} \left( \sqrt{33 + 15\sqrt{5}} - \sqrt{6 \left( \sqrt{5} - 1 \right)} \right) \kappa t$$

$$= -0.90566 \kappa t < 0. \quad (13)$$

For this case we again find that the extra time dimensions become less observable through deflation. For the last listed case of Eq. (8),

$$c H_8 = \frac{1}{2} \sqrt{\frac{2}{\left( 11 + 5\sqrt{5} \right)}} \Rightarrow c H_4 = \frac{1}{2} \sqrt{\frac{2}{\left( \sqrt{5} - 1 \right)}}. \quad \text{For this case we find that}$$

$$\ln \frac{a(x^4, x^8)}{a_0} = 1.81343 \kappa t \quad \text{and} \quad \ln \frac{b(x^4, x^8)}{b_0} = 1.60174 \kappa t \quad \text{so that the extra time dimensions inflate, and are observable.}$$
3 Conclusion

We have demonstrated an inflation model that depends exponentially on two expansion rates $H_4$ and $H_8$ (hence the descriptor "Bi-Inflation" in the title of this paper). We have discussed a solution to the Einstein field equations on an eight-dimensional pseudo-Riemannian manifold $\mathbb{X}_{4,4}$ (a spacetime of four space dimensions and four time dimensions) that exhibits inflation of three of the four space dimensions and deflation of three of the four time dimensions. Scaled Hubble parameters ($H_4, H_8$) have been defined for each of the unscaled dimensions ($x^4, x^8$). The scale factor $a = a(x^4, x^8)$ during inflation is a product of exponentials $a = e^{H_4 x^4} / c \; e^{H_8 x^8} / c$ of each Hubble parameter times its respective unscaled coordinate. Assuming that the $x^8$-dimension is not compact (which leads to ideas far outside the scope of this paper), and moreover assuming that there is not an infinity of universes that is parameterized by $x^8$, then this interpretation as a model of inflation of a homogeneous isotropic universe $\mathbb{X}_{4,4}$ makes most sense if we conjecture that every particle/field in our universe is carried along the $x^8$-axis in the same way that it is carried along the $x^4$-axis, that is, in a manner that is independent of its energy, and with no freedom to change either its rate of passage along the $x^8$-axis or its location within the $x^8$-dimension. The evolution of the particle/field through the unscaled dimensions $(x^4, x^8) \in \mathbb{R}^{1,1}$ in our universe is entrapped and entrained by geometry.

Assuming the validity of the conjecture, we may employ a line element of the form $ds^2 = -|g_{44}| \left(c_4 dx^4\right)^2 + g_{88} \left(dx^8\right)^2 + \cdots$, to define a physical time $t = x^0$ through the relation $|g_{tt}| \left(c dt\right)^2 = |g_{44}| \left(c_4 dx^4\right)^2 - g_{88} \left(dx^8\right)^2$. Here $c_4$ is a speed related parameter (in this paper $g_{tt} = g_{44} = -1$). This model imposes a structure on physical time.

Both the fourth space dimension, whose associated co-moving coordinate is $x^8$, as well as the extra time dimensions, evidently pose a challenge to observe, if they exist. Relative to the first three space dimensions, whose co-moving coordinates are $(x^1, x^2, x^3) \in \mathbb{R}^3$, the distance $\Delta X$ between two points $(x_0, x_0 + \Delta X)$ on the $x^8$-axis is expected to be exponentially smaller by about 60 $e$-folds than the distance between two points in $\mathbb{R}^3$ separated by the same coordinate difference $\Delta X$, but lying on, say, the $x^3$-axis; the distance between two points $(x_0, x_0 + \Delta X)$ lying on, say, the $x^7$-axis is even smaller smaller, since the extra time dimensions experience deflation. Moreover, the working hypothesis is that we have no ability to move at will along the $x^8$-axis, but are instead inextricably carried forward along this axis. Events in $\mathbb{X}_{4,4}$ are labeled by markers that include an $x^8$-coordinate, but it is not obvious how to measure the $x^8$-coordinate difference in these markers using only the incredibly weak gravitational interaction, which, to date, is the only interaction hypothesized to carry an $x^8$-dependence.

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URL: [http://www.xact.es/index.html](http://www.xact.es/index.html) xAct was employed to calculate the Einstein tensor for this paper.

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