The family of strange multiquarks related to the $D_s(2317)$ and $D_s(2457)$

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I study the $D_s(2317)$ and $D_s(2457)$ discovered at BABAR, CLEO and BELLE, and find that they belong to a class of strange $S = -1$ tetraquarks and pentaquarks, which is equivalent to the class of kaonic molecules bound by short range attraction. In this class of hadrons a kaon is strongly trapped by a s-wave meson or baryon. To describe this class of multiquarks the Resonating Group Method is applied to a standard quark model with chiral symmetry breaking, and the short range kaon-meson(baryon) interactions are extracted. A criterion is derived to classify the attractive channels. I conclude that the mesons $B^0(0^{++}), B^{1+},$ and the baryons $\Omega_{cc}, \Omega_{cb}, \Omega_{bb}$ clearly belong to the new hadronic class of the $D_s(2317)$ and $D_s(2457)$. The hadrons $f_0(980), \Lambda, \Sigma_c, \Sigma_b$ possibly belong to a related family.

I. INTRODUCTION

Recently new narrow scalar resonances $D_s(2317)$ and $D_s(2457)$ were discovered at BABAR [1], Cleo [2] and Belle [3]. They are not expected to be standard hadrons because their masses are close to hundred MeV smaller than the ones predicted in quark models for positive parity $D_s$ quark-antiquark mesons [4]. For instance the experimental candidate to the first quark-antiquark $D_s$ meson is the $D_s(2536)$ [6]. This is confirmed by lattice simulations [5] for quark-antiquark mesons. The existence of non-mesonic and non-hadronic multiquarks has been suggested form the onset of the quark model [5,8]. These positive parity $D_s$ resonances are interpreted by Beveren and Rupp [9] as poles in the S Matrix of the coupled channels of mesons and meson pairs. They were also predicted by Nowak, Rho and Zahed [10,11] as chiral partners of the well known negative parity pseudoscalar $D_s(1968)$ and $D_s(2112)$. They are as well interpreted as K-D molecules by Barnes, Close and Lipkin [12] similar to the Deusons anticipated by Tornqvist [13]. In the same way I find here that the new $D_s$ resonances can be understood as tetraquarks, or equivalently as s-wave $D-K$ molecules bound by the short range attraction.

The experimental discovery of these hadron also suggests the existence of a new class of multiquarks. In this paper I study the family of all possible narrow tetraquark and pentaquark resonances where the quark $s$, or the $S = -1$ Kaon play a crucial role. The strangeness $S = 1$ pentaquark $\Theta^+$ is more difficult to bind and was recently addressed with the same techniques of this paper in reference [14]. A chiral invariant framework [13,14,15] is used to compute microscopically, at the quark level, the masses of this new class of hadrons. The resulting mechanism which provides the binding of the $S = -1$ tetraquarks and pentaquarks is equivalent to the short range attraction of hadrons.

Here a standard Quark Model (QM) Hamiltonian is assumed,

$$H = \sum_i T_i + \sum_{i<j} V_{ij} + \sum_{ij} A_{ij} \quad (1)$$

where each quark or antiquark has a kinetic energy $T_i$ with a constituent quark mass, and the colour dependent two-body interaction $V_{ij}$ includes the standard QM confining and hyperfine terms,

$$V_{ij} = \frac{-3}{16} \bar{\lambda}_i \cdot \bar{\lambda}_j \left[ V_{\text{conf}}(r) + V_{\text{hyp}}(r)\vec{S}_i \cdot \vec{S}_j \right]. \quad (2)$$

Moreover the Hamiltonian includes a quark-antiquark annihilation term $A_{ij}$ which is the result of spontaneous chiral symmetry breaking.

This paper is organised in sections. In Section II the QM is reviewed, together with the Resonating Group Method (RGM) [19] which is adequate to study multiquark states, where several quarks overlap. The RGM, together with chiral symmetry, produces short range hadron-hadron potentials, which can be either repulsive (hard core repulsion) or attractive. In Section III a criterion is derived to discriminate which systems bind and which are unbound. This criterion is applied to find, among the s-wave hadrons, the candidates to trap a kaon. In Section IV the binding energy is computed for the selected positive parity mesons $f_0(980), D^0_s(0^{++}), D^+_s(1^{++}), B^+_s(0^{++}), B^+_s(1^{++})$ and negative parity baryons $\Sigma_c, \Xi_b, \Omega_{cc}, \Omega_{cb}, \Omega_{bb}$. Finally the results are presented and discussed in Section V.

II. STUDYING MULTIQURARKS WITH THE RGM

For the purpose of this paper the details of potential [11] are unimportant, only its matrix elements matter. The hadron spectrum constrains the hyperfine potential,

$$\langle V_{\text{hyp}} \rangle \simeq \frac{4}{3} (M_\Delta - M_N) \quad (3)$$

The quark-antiquark annihilation potential $A_{ij}$ is also constrained when the quark model produces spontaneous

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are determined, a very small mass. From the hadron spectrum and using $M$ where this result is correct for the annihilation of $N$

Recently it was shown [21] used the same RGM to show that, in non-resonance, preventing the collapse of nuclei. This

For instance this explains the [20] to show that in exotic hadron-hadron scattering, the

Moreover

the heavy quarkonium to the light pion mass). Moreover

FIG. 1: The Jacobi coordinates of the incoming four quark wave-function $\phi_A(\rho_A)\phi_B(\rho_B)\psi(\lambda_{A,B})$.

chiral symmetry breaking [12, 10]. The annihilation potential $A$ is present in the $\pi$ Salpeter equation,

$$
\begin{bmatrix}
2T + V & A \\
A & 2T + V
\end{bmatrix}
\begin{pmatrix}
\phi^+ \\
\phi^-
\end{pmatrix}
= M_{\pi}
\begin{pmatrix}
\phi^+ \\
-\phi^-
\end{pmatrix}
$$

where the $\pi$ is the only hadron with a large negative energy wave-function, $\phi^- \simeq \phi^+$. In eq. (4) the annihilation potential $A$ cancels most of the kinetic energy and confining potential $2T + V$. This is the reason why the pion has a very small mass. From the hadron spectrum and using eq. (1) the matrix elements of the annihilation potential are determined,

$$
\langle 2T + V \rangle_{S=0} \simeq \frac{2}{3}(2M_N - M_{\Delta})
$$

where this result is correct for the annihilation of $u$ or $d$ quarks. When a strange quark is present, the corresponding matrix element is smaller by a factor $\sigma$ which is a power of the constituent quark mass ratio $M_{u,d}/M_s$.

The QM of eq. (1) reproduces the meson and baryon spectrum with quark and antiquark bound-states (from the heavy quarkonium to the light pion mass). Moreover the RGM was first used in hadronic physics by Ribeiro [20] to show that in exotic hadron-hadron scattering, the quark-quark potential together with the Pauli repulsion of quarks produces a repulsive short range interaction. For instance this explains the $N - N$ hard core repulsion, preventing the collapse of nuclei. This $N - N$ hard core repulsion is supposed to also occur in several hadron-hadron interactions, and this explains why many multiquarks systems are not stable. However Deus and Ribeiro [21] used the same RGM to show that, in non-exotic channels, the quark-antiquark annihilation could produce a short core attraction. Recently it was shown that in the particular case of the low energy $\pi - \pi$ system in the chiral limit, the short range attraction and repulsion exactly cancel [17], resulting in an Adler Zero and the Weinberg theorem. Addressing a tetraquark system with the $\pi - \pi$ quantum numbers, it was shown that the QM also fully complies with the chiral symmetry, including the PCAC theorems [15]. Therefore the QM is adequate to address the new $D_s$ hadrons, which were predicted by Nowak, Rho and Zahed in an effective chiral model. In this paper the QM and the RGM are applied to $S = -1$ multiquarks.

The RGM [19] computes the effective multiquark energy using the matrix elements of the microscopic quark-quark interactions. Any multiquark state can be decomposed in combinations of simpler colour singlets, the baryons and mesons. This can be illustrated with the colour structure of a multiquark with two quark-antiquark pairs, $q_1, \bar{q}_2, q_3, \bar{q}_4$. Assuming that this tetraquark is a colour singlet, each quark-antiquark pair can either be a colour singlet or a colour octet. Nevertheless the octet-octet state can be described with an exchange operator and with the singlet-singlet state,

$$
\hat{S}_{1,2} \cdot \hat{S}_{3,4} = \frac{1}{2\sqrt{2}} \left(P_{13} - \frac{1}{3}\right)_{1,2} 1,3,4
$$

and there is only one anti-symmetrised state,

$$
(1 - P_{13})(1 - P_{24})_{1,2} 1,3,4 =
\frac{1}{\sqrt{2}}(1 - P_{13})(1 - P_{24})\hat{S}_{1,2} \cdot \hat{S}_{3,4}
$$

where $(1 - P_{13})(1 - P_{24})$ is the quark and antiquark anti-symmetrised. Therefore the multiquarks can be described with an anti-symmetrised basis of baryons and mesons. The multiquark may only be a bound state, or a narrow resonance, if these baryons and mesons are sufficiently attracted. Thus the problem of multiquark stability can be technically reduced to the problem of the binding of baryons or mesons.

The RGM produces both the energy of a multiquark state and the effective hadron-hadron interaction. The energy of the multiquark is computed with the matrix elements of the Hamiltonian [11]. The wave functions of quarks are arranged in anti-symmetrised overlaps of simple colour singlet hadrons. As an example, in Fig. 2 a tetraquark system is arranged in a pair of mesons $A$

FIG. 2: Examples of RGM overlaps are depicted, in (a) the norm overlap for the meson-baryon interaction, in (b) a kinetic overlap the meson-meson interaction, in (c) an interaction overlap the meson-meson interaction, in (d) the annihilation overlap for the meson-baryon interaction.
and $B$. Once the internal energies $E_A$ and $E_B$ of the two hadronic clusters are accounted,

$$\frac{\langle \phi_a \phi_b | H \sum_p (-1)^P | \phi_a \phi_b \rangle}{\langle \phi_a \phi_b | \sum_p (-1)^P | \phi_a \phi_b \rangle} = E_a + E_b + V_{ab}, \quad (8)$$

where $\sum_p (-1)^P$ is the anti-symmetrizer, the remaining energy of the meson-baryon or meson-meson system is computed with the overlap of the inter-cluster microscopic potentials,

$$V_{\text{bar}}_{\text{mes}} = \langle \phi_B \phi_A | -(V_{14} + V_{15} + 2V_{24} + 2V_{25})3P_{14} + 3A_{15}|\phi_A \phi_B \rangle$$

$$V_{\text{mes}}_{\text{mes}} = \langle \phi_B \phi_A | (1 + P_{AB})|-(V_{14} + V_{23} + V_{14} + V_{24}) \times P_{13} + A_{23} + A_{14}||\phi_A \phi_B \rangle$$

$$/\langle \phi_B \phi_A |(1 + P_{AB})(1 - P_{13})|\phi_A \phi_B \rangle, \quad (9)$$

where $P_{ij}$ stands for the exchange of particle $i$ with particle $j$, see Fig. 2.

III. A CRITERION FOR BINDING

In what concerns the annihilation potential, it only occurs in non-exotic channels. Then it is clear from eqs. 3 that the annihilation potential provides an attractive (negative) overlap. This confirms that the hard core can be attractive for non-exotic channels where annihilation occurs.

In what concerns the quark-quark(antiquark) potential, it may also contribute to exotic channels. Because the potential $V_{ij}$ is assumed to be proportional to the colour dependent $\vec{S}_i \cdot \vec{S}_j$, it is clear that it can only contribute together with an exchange interaction, which provides a color octet, see eq. 4. Moreover the spin independent part of the interaction vanishes. For instance in the meson-meson overlap of eq. 4, the overlap of $\vec{S}_i \cdot \vec{S}_j + \vec{S}_i \cdot \vec{S}_j$ essentially cancels with the overlap of $\vec{S}_i \cdot \vec{S}_j + \vec{S}_i \cdot \vec{S}_j$. The only potential which may contribute is the hyperfine potential, proportional to $\vec{S}_i \cdot \vec{S}_j \cdot \vec{S}_j$. In the present case where the kaon is a spin singlet, the minus phase from colour is inverted by a minus phase from spin. I find that the total colour and spin matrix element is a hyperfine splitting,

$$\langle P_{13}(V_{13} + V_{23} + V_{14} + V_{24}) \rangle = 4 \frac{2}{3}(M_\Delta - M_N). \quad (10)$$

The flavour trace is quite simple and the spatial integral provides a geometrical overlap. All the corresponding factors are positive, an therefore the quark-quark interaction results in a repulsive interaction.

These results are independent of the particular quark model that one chooses to consider, providing it is chiral invariant. Therefore two opposite classes of diagrams exist. The exchange diagrams produce a repulsive interaction, which turns out to be proportional to the hyperfine quark-quark(antiquark) interaction. The annihilation diagrams produce an attractive interaction, which turns out to be proportional to the spin-independent quark-quark(antiquark) interaction. I arrive at the attraction/repulsion criterion for the short range hadron-hadron interaction,

- whenever the two interacting hadrons have quarks (or antiquarks) with a common flavour, the repulsion is increased,
- when the two interacting hadrons have a quark and an antiquark with the same flavour, the attraction is enhanced.

This paper is dedicated to the class of resonances which can be understood as a S=-1 kaon $\bar{s}u$ or $sd$ strongly trapped by a s-wave hadron. The criterion shows that all hadrons with an antiquark $\bar{u}$ or $d$ or with a a quark $s$ allow the exchange overlap ($P_{13}$), and this would certainly contribute to repulsion. Although systems with both a short range repulsion and a short range attraction may still bind, I specialize here in systems where clearly there is no hard core repulsion. In what concerns attraction, a quark $u$ or $d$ is needed in the s-wave partner of the kaon, in order to produce annihilation. Therefore the isospin of the partner of the kaon needs to be close to the opposite isospin of the kaon. This excludes the mesons $\eta, \eta'$, $\omega, \phi$ ... Moreover I also specialise in systems which may achieve a remarkable stability, where the kaon partner is a hadronic resonance with a very narrow width. This not only essentially restricts the kaon partner to s-wave mesons and baryons, it also excludes the meson $\rho$ and the baryon $\Delta$ because they are very wide, due to the decay with a pion production. The pion is also excluded because it is too light to bind to the kaon, all that one may get is a very broad resonance, the kappa resonance, which has been recently confirmed by the scientific community. Therefore the hadrons which are best candidates to strongly bind the Kaons $s\ell$ are the s-wave hadrons with flavor $ls, l\bar{c}, l\bar{b}, ll, llc, llb, llc, l\bar{b}, llb$. This is expected to result in the $f_0(980)$, the $D_s(2320)$, the $D_s(2463)$, the $\Lambda(1405)$ and several other resonances.

IV. THE BINDING ENERGY

A convenient approximation for the meson and baryon s-wave wave-functions is the harmonic oscillator ground-state wave-function,

$$\phi^\alpha_{000}(p_\rho) = N_\alpha^{-1} \exp \left(-\frac{p_\rho^2}{2\alpha^2}\right), \quad N_\alpha = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{3/2}, \quad (11)$$

where, in the case of vanishing external momenta $p_A$ and $p_B$, the momentum integral in eq. 13 is simply $N_\alpha^{-2}$. The coordinates of the incoming $\phi_A \phi_B$ functions are illustrated in Fig. 4 while the coordinates of the outgoing
$g_0^2(E,1)$

**FIG. 3:** The matrix element of the Green function $g_0(E, \mu, \alpha)$ are plotted in dimensionless units of $\mu = \alpha = 1$. The general case scales as $g_0^2(E, \mu) = \frac{1}{2\pi}g_0^2(E, 1)$.

$\phi_1^3$ have the quark 1 and 3 exchanged. I summarise [17, 18, 22] the effective potentials computed for the different channels,

$$
v_{K-K} = (2 + \sigma)\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-D} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-D^*} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-B} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-B^*} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-N} = 3\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-\Sigma_c} = 7\frac{1}{36}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-\Sigma_b} = 7\frac{1}{36}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-\Xi_{cc}} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-\Xi_{cb}} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

$$
v_{K-\Xi_{bb}} = 2\frac{1}{6}\langle A\rangle N_{\alpha}^{-2},
$$

(12)

where the colour and spin factors contribute respectively with $1/3$ and $1/2$, $\langle A\rangle$ is of the order of 430 MeV and the geometrical factor is $N_{\alpha}^{-2}$. The remaining factor is the flavour factor. The parameter $\alpha$ is the only one that is model dependent, and it will be determined with a fit of experimental binding energies. The estimation of $\alpha$ is an important by-product of this method because the hadronic size can not be estimated directly by the hadronic charge radius which is masked by the vector meson dominance.

To study binding one has to proceed to the finite momentum case. Then the effective potentials in eq. (12) turn out to be multiplied by the gaussian separable factor, exp $\left[-\frac{p^2}{2\beta^2}\right]$ and $\exp\left[-\frac{p^2}{2\beta^2}\right]$. In the exotic channels with exchange diagrams only, this result can be proved [21], and moreover the new parameter $\beta = \alpha$. This occurs because the overlaps decrease when the relative momentum of the hadrons $A$ and $B$ increases. In the non-exotic channels with annihilation diagrams the present state of the art of the RGM does not allow a precise determination of the finite momentum overlap. Nevertheless, it is expected that eventually the overlap decreases due to the geometrical wave-function overlap in momentum space. Here the precise determination of $\beta$ does not affect the results. I order to reduce the number of parameters, and for simplicity, in this paper I assume that $\beta \approx \alpha$. For other approaches that also lead to a separable potential, see for instance Ref. [24]. This parametrisation of the Schrödinger equation in a separable potential,

$$
[(E - T_A - T_B)(1 + n)|\phi^\alpha > < \phi^\alpha|)
$$

$$
+ v|\phi^\alpha > < \phi^\alpha| |\psi_\lambda > = 0,
$$

(13)

enables the use of standard techniques [22] to exactly compute the scattering $T$ matrix,

$$
T = |\psi_\lambda > < \phi^\alpha| \frac{1}{1 - v^2/4\mu^2 + i\epsilon} |\phi^\alpha > .
$$

(14)

The binding occurs when the $T$ matrix has a pole for a negative relative energy,

$$
1 - \frac{v}{1 - n}g_0^2(E, \mu) = 0.
$$

(15)

In Fig. 3 the function $G_0$ is plotted. $G_0$ is real for a negative relative energy $E$ and is complex for positive $E$. The binding only occurs if,

$$
-4\mu v \geq \alpha^2.
$$

(16)

Using Fig. 3 it is possible to determine the parameter $\alpha$ which reproduces the experimentally measured binding energy of the $D_s(2320)$. With a binding energy of 46 MeV, the corresponding parameter $\alpha$ is 285 MeV. This corresponds to a radius of 0.7 Fm for the nucleon.

**V. RESULTS AND CONCLUSION**

I now compute the binding energies of the kaonic strongly bound molecules, or equivalently strange multiquarks. These multiquarks are divided into two different families which are respectively coupled and decoupled to pionic channels. The computations are straightforward and the results are displayed in Tables I and II.
A. A family of very narrow multiquark resonances

The $D_s^{(0+)}$ has a lower mass by a hundred or more MeV than the expected $1P_0$ excitation of the $D_s^{(0-)}$ (1968). In order to conserve the angular momentum and parity, it can only decay to a $D_s^{(1-)}$ (2112) with the creation of a pion, in a p-wave, with a very low energy which is suppressed by an Adler zero [17]. More importantly, this decay mode violates isospin conservation, and therefore it is quite suppressed. The same isola-
tion from other strong hadronic channels is common to other multiquarks, with a quark $q_s$ one or more heavy quarks and the isospin zero combination $uu + dd$. I find that the $D_s^{(0+)}$, $D_s^{(1+)}$, $B_s^{(0+)}$, $B_s^{(1+)}$ belong to the same class of tetraquark hadronic resonances. This class is equivalent to the picture of a kaon trapped by a s-wave meson with a short range attraction. In what concerns pentaquarks, where the heavy antiquark in the trapping meson is replaced by a pair of heavy quarks, the results of this paper predict that there is a similar binding with the quantum numbers of the $\Omega_{cc}$, $\Omega_{cb}$, and $\Omega_{bb}$. I conclude that the tetraquarks and pentaquarks $D_s^{(0+)}$, $D_s^{(1+)}$, $B_s^{(0+)}$, $B_s^{(1+)}$, $\Omega_{cc}$, $\Omega_{cb}$ and $\Omega_{bb}$ belong to the same family of very narrow, non-exotic multiquarks. The corresponding binding energies of the kaon-hadron system are shown in Table I.

B. A second family, coupled to pionic channels

A second family of tetraquarks and pentaquarks may exist, where the coupling to pionic channels does not violate the conservation of isospin. The only suppression that one may expect comes from the Adler Zero. The only tetraquark that we select in this class is the $I = 0$ $f_0$, which is coupled to the $\pi - \pi$ channel. A possible existing candidate is the $f_0(980)$, although it is not excluded that it is a simple $q\bar{q}$ meson [10]. In what concerns pentaquarks, when the baryon (that traps the kaon) has two or more light quarks, the corresponding pentaquark is again coupled to pionic channels. For instance the kaon can be trapped by a nucleon to produce a pentaquark $\Lambda$, and this is coupled to the s-wave $\Sigma - \pi$ channel, with the same isospin. A possible candidate to this state is the $A(1405)$. Similarly the kaon can be trapped by the $\Sigma$, or the $\Omega$ baryons to produce pentaquarks with the quantum numbers of negative parity $\Xi_{-}^+$, $\Xi_0^+$, $\Xi_{-}^0$, $\Xi_0^0$. These channels are respectively coupled to the $\pi - \Xi_3$, and $\pi - \Xi_3$ channels where the $\Xi_3$ and $\Xi_3$ have a positive parity. Nevertheless I compute the binding energies of this class of multiquarks, ignoring the effect of the coupled pionic channel. The results are displayed in Table II. I find that the binding energy may be excessively large in the $f_0$ and $\Lambda$ channel. This suggests that there is mixing between the two coupled multiquark states, and that this pushes the mass of the $f_0$ and $\Lambda$ up to the higher experimental results. For instance it been advocated by Oset and collaborators [28] that the $A(1405)$ results from the mixing of a singlet-singlet state with an octet-octet state.

C. Comparing multiquarks with molecules and with chiral excitations

The formalism used here is also convenient to address the question, are these new hadrons multiquarks, or molecules composed of standard hadrons, or chiral images of standard hadrons?

I find that, in a microscopic and chiral invariant calculation, the new hadrons appear as tetraquarks, or as pentaquarks. In a macroscopic hadron-hadron interaction perspective, the resulting mechanism which provides the binding is the short range strong attraction of hadrons. Technically is is not possible distinguish a microscopic multiquark from a macroscopic strongly bound s-wave molecule, because the hadrons totaly overlap. The purely molecular pattern only appears in different cases, when there is some repulsion which produces the clustering of quarks, and this may happen in the "pentaquark" family of exotics [14]. Nevertheless the molecular perspective is convenient to estimate the mass of the multiquark because the binding energies are not very large, and the

| channel | $\mu_{exp}$ | $v_{th}$ | $\alpha = \beta B_{th}$ | $B_{exp}$ |
|---------|-------------|-----------|---------------------------|-----------|
| $D_s(2317)$ | $K^- D^0 + K^0 D^+$ | 392 -143 | 285 | 46 | 46 |
| $D_s(2457)$ | $K^- D^0 + K^0 D^+$ | 398 -143 | 285 | 47 | 46 |
| $B_s = K^- D_s^0 + K^0 D_s^+$ | 453 -143 | 285 | 55 | - |
| $B_s = K^- D_s^0 + K^0 D_s^+$ | 454 -143 | 285 | 55 | - |
| $\Omega_{cc}^+$ = $K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 442 -143 | 285 | 53 | - |
| $\Omega_{cc}^0$ = $K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 466 -143 | 285 | 56 | - |
| $\Omega_{bb} = K^- \Xi_{bb}^0 + K^0 \Xi_{bb}^0$ | 475 -143 | 285 | 58 | - |

| channel | $\mu_{exp}$ | $v_{th}$ | $\alpha = \beta B_{th}$ | $B_{exp}$ |
|---------|-------------|-----------|---------------------------|-----------|
| $f_0(980) = K^- K^0 + K^0 K^-$ | 248 -143 | 285 | 65 | 12 | 10 |
| $A(1405) = K^- K^0 + K^0 K^-$ | 325 -143 | 285 | 88 | 30 | 4 |
| $\Xi_{-}^+ = K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 412 -167 | 285 | 68 | - |
| $\Xi_0^+ = 2K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 412 -167 | 285 | 68 | - |
| $\Xi_{-}^0 = K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 456 -167 | 285 | 75 | - |
| $\Xi_0^0 = 2K^- \Xi_{cc}^0 + K^0 \Xi_{cc}^0$ | 456 -167 | 285 | 75 | - |

TABLE I: This table summarises the parameters $\mu$, $v$, $\alpha$, $\beta$ and binding energies $B$ (in MeV) [10] for the channels closed to pion decay. The italic binding energy $B_{th}$ of the $D_s(1327)$ is fitted from experiment.
mass is essentially the sum of the kaon plus hadron mass, with a relatively small binding energy.

The chiral excitation perspective, present in the Chiral Soliton Model [10] or in the Chiral Lagrangian, may also be eventually equivalent to the multiquark, or to the two-hadron narrow bound molecule. Suppose that a flavour singlet quark-antiquark pair $u\bar{u} + d\bar{d}$ or $s\bar{s}$ is created in a given hadron $H$. When the resulting multiquark $H'$ remains bound, we have a state with an opposite parity to the original $H$. The reversed parity occurs due to the intrinsic parity of fermions and anti-fermions. Moreover, because s-waves are expected to have the lowest mass, this is equivalent to adding a pseudoscalar meson to the original $H$. The finite masses of the Kaon and of the pion prevent the existence of a large number of pseudoscalar mesons in the multiquark family of the $D_s(2317)$. In this family the mass shift is of the order of the kaon mass minus the binding energy minus the strange-light quark mass difference $M_K - B - (M_s - M_u)$, and this results in a mass shift close to 350 MeV. The strange-light quark mass difference of the kaon mass minus the binding energy minus the strange-light quark mass difference $M_K - B - (M_s - M_u)$, and this results in a mass shift close to 350 MeV. The strange-light quark mass difference

Moreover, because s-waves are expected to have the lowest mass, this is equivalent to adding a pseudoscalar meson to the original $H$. The presented results only depend on the hadronic size $\alpha$ and not on the details of the quark-quark interactions, because the assumptions are quite simple. I also assumed that the inverse radius parameters $\alpha$ and $\beta$ are identical for all channels, although small channel dependences are expected. Moreover I neglected the meson exchange interactions because they are expected to be smaller than the hard core interaction. In the same way I did not consider the s-channel coupling to a single meson or baryon. More importantly, in the second family of multiquarks, the coupling to pionic channels was neglected although this coupling conserves isospin. It would be particularly interesting to study these neglected effects in the second family studied in this paper, in order to check if they correct the computed binding which seems too strong. Because the research in this direction will be model dependent, this will be done elsewhere.

D. Outlook

The presented results only depend on the hadronic size $\alpha$ and not on the details of the quark-quark interactions, because the assumptions are quite simple. I also assumed that the inverse radius parameters $\alpha$ and $\beta$ are identical for all channels, although small channel dependences are expected. Moreover I neglected the meson exchange interactions because they are expected to be smaller than the hard core interaction. In the same way I did not consider the s-channel coupling to a single meson or baryon. More importantly, in the second family of multiquarks, the coupling to pionic channels was neglected although this coupling conserves isospin. It would be particularly interesting to study these neglected effects in the second family studied in this paper, in order to check if they correct the computed binding which seems too strong. Because the research in this direction will be model dependent, this will be done elsewhere.

The short range attraction studied in this paper is quite general and can also be applied to the different candidates to multiquarks which have been discovered quite recently [30, 31, 32, 33, 34, 35, 36]. This will also be applied elsewhere.

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