Approaches and collisions of asteroids with the Moon and planets

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Abstract. In this paper we discuss the characteristics of possible collisions of the asteroid Apophis with the Earth. We use the initial conditions published in mid-January 2021 (the nominal orbit NASA), i.e. new observations for Apophis. Estimates of the probability of collisions of dangerous asteroids with planets and the Moon were obtained using the Monte Carlo method.

1. Introduction
The objectives of this work are to find and study possible collisions and approaches of asteroids with the Earth, the Moon and other planets, as well as an estimate of the likelihood of these events. The solution of these problems is important for ensuring the Earth’s asteroid safety, which is one of the pressing problems today. A huge number of works are devoted to this topic, for example [1],[2],[3], [4], [5]. Lots of information about dangerous asteroids are listed on the regularly updated website cneos.jpl.nasa.gov/sentry/ (NASA website). Currently known about a thousand asteroids with a nonzero probability of collision with the Earth in the next hundred years. In the paper [6] a method for identifying the trajectories of collisions of asteroids with the Earth, developed at the Department of Celestial Mechanics of St. State University, examples of its application are given. Comparison of the results obtained with the data presented on the website NASA shows generally good agreement for the most of asteroids we have studied. We usually managed to find significantly more possible collisions with the Earth than previously known. At the same time, the developed method has a large computational complexity, and we were able to investigate in detail only a few dangerous asteroids. There were especially many results obtained by us from the famous asteroid Apophis. More recently, his new observations began. Below we present our first results for possible Apophis collisions using the software of its new initial data.

Recently, the Department of Celestial Mechanics of St.Petersburg State University has developed a new software package for predicting the movement of asteroids, finding possible approaches and collisions with planets, which has a high speed [7]. The performance is due to the fact that the same type of calculations performed, the possibility of parallelization, etc. are fully used. The article [7] provides an example of estimating the cumulative probability of collisions with the Moon and planets of the Solar System for two hundred dangerous asteroids (the size is more than 30 meters, the cumulative probability of collisions with the Earth from...
the NASA site is more than one hundred millionth) in the time interval up to 2132. In fact, the Monte Carlo method was used. $10^7$ trajectories are calculated for each asteroid. A comparison with NASA data for Earth impacts showed generally satisfactory agreement.

In this paper, we present and discuss some results obtained using the new Monte Carlo program. For a number of dangerous asteroids, cumulative estimates of the probabilities of close encounters at 100 and 10 radii of the corresponding planet were obtained.

The task of expedient change of the asteroid’s orbit, in particular, its withdrawal from collisions with the Earth, is important. The fundamental difficulty that arises in solving it is the huge amount of energy that must be applied to an asteroid several hundred meters in size. If you do not use the energy of an atomic (thermonuclear) explosion, in principle, you can use the effect of a gravitational maneuver, if there is a close approach to the Earth before the collision. In this case, small changes in the orbit before the approach increase by several orders of magnitude after the approach. The free and almost inexhaustible gravitational energy of the Solar system is used. Therefore, it is important to know whether there are previous close approaches on the trajectories of possible collisions, and how typical this phenomenon is.

2. Possible approaches and collisions with the Earth of the asteroid Apophis.

The Apophis asteroid is one of the most famous dangerous asteroids. The number of works devoted to him is immense, two important works are [3], [4]. Among other things, Apophis demonstrates a close connection between possible collisions and its previous approaches to the Earth. Discovered by American astronomers in the summer of 2004, in December it became the most dangerous with a probability of impact with the Earth in 2029 of about 3 percent and a size of about 400 meters. Radar observations in January 2005 allowed us to establish the impossibility of a collision and the presence of a close approach in 2029. Close proximity leads to a scattering of possible trajectories, including resonant returns and impacts with the Earth in the foreseeable future. The most dangerous was the possible impact in 2036. In 2006-2012, Apophis was almost inaccessible for observations. At this time, theoretical studies of possible dangerous trajectories of Apophis continued, especially those leading to a collision in 2036 and to resonant returns after 2036. It was opened many possible collisions after 2036. In 2012-13, observations of Apophis began, after which possible collisions in 2036, as well as all associated with resonant returns after 2036, became impossible. However, shortly before that, American researchers discovered near the nominal orbit of Apophis possible rendezvous with the Earth in 2051. It is curious that there can be no collision in 2051, but very close approaches are possible. These approaches and the corresponding resonant returns give a huge number of possible collisions of Apophis with the Earth today. We have found about a hundred of them in the current century. Why exactly the approaches in 2051 induce so many collisions is an intriguing mystery.

The methods we use to identify possible asteroid impacts and approaches to the Earth are described in our previous works, in particular [6], [8].

Thus, the currently known trajectories leading to Apophis’ collisions with the Earth in the current century have a close (38 thousand km) approach to it in 2029, as well as a close approach to the Earth in 2051. For the 13 most dangerous collisions, Table 6 shows the minimum geocentric distances in 2051 on the corresponding trajectories.

At the end of 2020, observations of the asteroid Apophis resumed. In mid-January 2021 its new nominal orbit and possible collisions with the Earth appeared on the NASA website. We also found new possible collisions using our v19 program, which is described in the article [6]. At the same time, we varied for each of the three coordinates and three speeds. Recall that earlier we usually varied when searching for collisions only by one of the coordinates. Here are the preliminary results. We still found more collisions than are listed on the NASA website. The main collisions with the largest hole sizes have stable characteristics and are usually ”visible”
when varying in any of the six variables. Tables 1-5 show the minimum geocentric distances and hole sizes found when varying for each of the 6 variables. For Apophis, the numerical characteristics of collisions are usually close to each other when varying over any variable. The main collisions found from the new initial data generally correspond to the main collisions found from the previous nominal value and their characteristics. In particular, the corresponding minimum geocentric distances in 2051 are well matched.

We also found ”new” possible collisions using the v19 program. At the same time, we varied by three coordinates and three speeds. Here are the preliminary results. We still found more collisions than are listed on the NASA website. The main collisions with the largest gap sizes have stable characteristics and are usually ”visible” when varying in any of the six variables. They generally correspond to the main collisions found at the previous nominal value and their characteristics. In particular, the corresponding minimum geocentric distances in 2051 are well matched.

| Table 1. The impact of Apophis with the Earth in 2055. |
| --- | --- | --- | |
| Direction | $r_{\text{min}}$ ($10^3$ km) | Hole size $da$ (m) |
| $dx/dt$ | 5.912 | 14.6 |
| $dy/dt$ | 5.912 | 13.8 |
| $dz/dt$ | 5.912 | 10.9 |
| $x$ | 5.913 | 11.9 |
| $y$ | 5.915 | 11.1 |
| $z$ | 5.901 | 15.6 |

| Table 2. The impact of Apophis with the Earth in 2056. |
| --- | --- | --- | |
| Direction | $r_{\text{min}}$ ($10^3$ km) | Hole size $da$ (m) |
| $dx/dt$ | 4.196 | 54.8 |
| $dy/dt$ | 4.197 | 69.2 |
| $dz/dt$ | 4.196 | 80.2 |
| $x$ | 4.197 | 59.6 |
| $y$ | 4.199 | 55.7 |
| $z$ | 4.186 | 72.9 |

Figure 1 [9] shows the relative positions of the holes leading to the approaches and collisions, and the corresponding minimum geocentric distances for the virtual Apophis. The positions are determined by the deviation of the large semi-axis from the nominal value at the initial moment. Apophis is called virtual, because the approaches and collisions are considered in a much wider area than the one in which the real Apophis can be located according to modern data. It is seen that the positions of the collisions and the positions of the convergence thickenings generally correspond to each other. You can see that the positions of the holes form a structure similar to a fractal. Obviously, this is due to resonant returns.
Table 3. The impact of Apophis with the Earth in 2060.

| Direction | $r_{\text{min}}$ ($10^3$ km) | Hole size $da$ (m) |
|-----------|-------------------------------|-------------------|
| $dx/dt$  | 4.396                         | 20.0              |
| $dy/dt$  | 4.396                         | 23.5              |
| $dz/dt$  | 4.396                         | 17.2              |
| $x$      | 4.397                         | 20.9              |
| $y$      | 4.399                         | 22.3              |
| $z$      | 4.385                         | 17.7              |

Table 4. The impact of Apophis with the Earth in 2068.

| Direction | $r_{\text{min}}$ ($10^3$ km) | Hole size $da$ (m) |
|-----------|-------------------------------|-------------------|
| $dx/dt$  | 0.140                         | 1821.             |
| $dy/dt$  | 0.139                         | 1805.             |
| $dz/dt$  | 0.139                         | 1148.             |
| $x$      | 0.138                         | 1792.             |
| $y$      | 0.134                         | 1664.             |
| $z$      | 0.142                         | 1867.             |

Table 5. The impact of Apophis with the Earth in 2076.

| Direction | $r_{\text{min}}$ ($10^3$ km) | Hole size $da$ (m) |
|-----------|-------------------------------|-------------------|
| $dx/dt$  | 0.705                         | 164.              |
| $dy/dt$  | 0.705                         | 167.              |
| $dz/dt$  | 0.706                         | 115.              |
| $x$      | 0.705                         | 150.              |
| $y$      | 0.704                         | 166.              |
| $z$      | 0.710                         | 155.              |

3. Approaches and collisions for some asteroids.

Studies of possible collisions and approaches were conducted at the Department of Celestial Mechanics of St. Petersburg State University for the asteroid 2008 EX5 ([9]). 59 possible collisions with the Earth in the current century were found. Figure 2 for asteroid 2008 EX5 is similar to Figure 1 for Apophis. It can still be seen that the positions of the collisions and the previous approaches correspond to each other. You can see that the positions of the slits form a structure similar to a fractal. Obviously, this is due to resonant returns.

For the asteroid 2008 EX5, three possible collisions with the Moon were obtained using a new high-speed program: 09.10.2050 with a probability of $0.31 \cdot 10^{-5}$, 09.10.2077 with a probability of $0.46 \cdot 10^{-5}$, 09.10.2093 with a probability of $0.14 \cdot 10^{-5}$. Earlier, 12 possible collisions of 2008 EX5 with the Moon [10] were found at the Department of Celestial Mechanics of St. Petersburg
Figure 1. Rendezvous and 553 collisions of the virtual asteroid Apophis with the Earth (hole positions and minimum geocentric distances)

Table 6. Minimum geocentric distances of Apophis (million km) in 2051 on the trajectories leading to the collision

| N | Year of impact | \(R_{\text{min}}(2051)\) (10^6 km) |
|---|----------------|-----------------------------------|
| 1 | 2076           | 1.23                              |
| 2 | 2068           | 0.763                             |
| 3 | 2077           | 0.745                             |
| 4 | 2091           | 0.219                             |
| 5 | 2078           | 0.209                             |
| 6 | 2065           | 0.198                             |
| 7 | 2066           | 0.183                             |
| 8 | 2074           | 0.134                             |
| 9 | 2055           | 0.0934                            |
| 10| 2060           | 0.129                             |
| 11| 2056           | 0.179                             |
| 12| 2064           | 1.77                              |
| 13| 2075           | 1.81                              |

State University using the old [6] method. Of these, the three main ones (i.e. with large hole sizes) coincide with the dates listed above. Large hole sizes correspond to high probabilities. The remaining 9 collisions have the dimensions of the holes on the 1-3 order of magnitude less.

For the asteroid 2003 UQ25, in addition to collisions with the Earth, a collision with Mars was obtained on 19.11.2112 with a probability of 1.0 \( \cdot \) 10^{-7}.

For the asteroid 2011 QF48, in addition to collisions with the Earth, a collision with Mars was obtained on 15.06.2117 with a probability of 1.0 \( \cdot \) 10^{-7}.

For the asteroid 2019 BE5, in addition to collisions with the Earth and the Moon, two collisions with Venus were obtained: 27.03.2047 with a probability of 1.0 \( \cdot \) 10^{-7}, 04.11.2071.
with a probability of $1.0 \cdot 10^{-7}$, and three collisions with Mercury: 20.07.2067 with probability $2.0 \cdot 10^{-7}$, 13.11.2078 with a probability of $1.0 \cdot 10^{-7}$, 31.12.2114 with a probability of $1.0 \cdot 10^{-7}$.

Tables 7-9 show the number of possible collisions and approaches to planets (at a distance of 10 and 100 planet radii) of three asteroids, out of $10^7$ virtual asteroids.

**Table 7.** Number of asteroid impacts and approaches 1998 DK36 (out of $10^7$)

|                |                |                |
|----------------|----------------|----------------|
| **Earth**      | **Earth 10R**  | **Earth 100R** |
| 4              | 2.1 $\cdot$ 10^4 | 1.7 $\cdot$ 10^5 |
| **Moon**       | **Moon 10R**   | **Moon 100R**  |
| 0              | 1.4 $\cdot$ 10^3 | 9.0 $\cdot$ 10^3 |
| **Venus**      | **Venus 10R**  | **Venus 100R** |
| 136            | 9.0 $\cdot$ 10^3 | 9.0 $\cdot$ 10^3 |
| **Mercury**    | **Mercury 10R**| **Mercury 100R**|
| 6              | 2.9 $\cdot$ 10^4 | 2.7 $\cdot$ 10^4 |

Table 10 shows the cumulative probability of impact with the Earth for ten of the most dangerous asteroids. The probabilities obtained by us are compared with those given on the NASA website.

Figure 3 shows the cumulative impact probabilities for 200 asteroids. The abscissa axis shows NASA’s estimates, and the ordinate axis shows our estimates.
Table 8. Number of asteroid impacts and approaches 1997 UA11 (out of $10^7$)

| Asteroid | $P \cdot 10^7$ |
|----------|----------------|
| Earth R  | 2              |
| Earth 10R| $4.3 \cdot 10^3$ |
| Earth 100R| $3.2 \cdot 10^5$ |
| Moon R   | 0              |
| Moon 10R | $8.0 \cdot 10^2$ |
| Moon 100R| $2.5 \cdot 10^3$ |
| Mars R   | 0              |
| Mars 10R | 0              |
| Mars 100R| 2              |
| Jupiter R| 12             |
| Jupiter 10R| 21             |
| Jupiter 100R| 35             |

Table 9. Number of asteroid impacts and approaches 2000 SG344 (out of $10^7$)

| Asteroid | $P \cdot 10^7$ |
|----------|----------------|
| Earth R  | $3.6 \cdot 10^4$ |
| Earth 10R| $2.0 \cdot 10^5$ |
| Earth 100R| $3.0 \cdot 10^6$ |
| Moon R   | 110            |
| Moon 10R | $1.7 \cdot 10^4$ |
| Moon 100R| $8.0 \cdot 10^5$ |

Table 10. Cumulative probabilities of impacts with the Earth

| Asteroid   | $P \cdot 10^7$ | $P \cdot 10^7$ (NASA) |
|------------|----------------|----------------------|
| 2000 SG344 | 36000          | 26000                |
| 2007 FT3   | 22             | 14                   |
| 1994 GK    | 660            | 690                  |
| 2019 ND7   | 46             | 32                   |
| 2008 UB7   | 570            | 350                  |
| 2000 SB45  | 1700           | 1600                 |
| 2012 QD8   | 79             | 65                   |
| 2007 DX40  | 690            | 620                  |
| 2008 EX5   | 500            | 480                  |
| 2005 ED244 | 5              | 26                   |

4. Conclusion
For many asteroids the collision trajectories contain the close approaches that precede them, so an economical escape from collisions is possible. Rendezvous also contributes to the early
refinement of the orbit. A high-speed program has been created that allows you to find possible collisions and estimate the probabilities of asteroids approaching and colliding with the Moon and planets using the Monte Carlo method.

The recent new observations of Apophis confirmed the existence of the main possible collisions with the Earth, previously known. The collision characteristics are stable with respect to the motion model and the initial data.

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References
[1] Shustov B M 2013 Solar System Research 47, 327-40.
[2] Bong Wie, Ben Zimmerman, Joshua Lyzhoft and George Vardaxis 2017 Astrodynamics 1, 3-21.
[3] Chesley S.R. 2006 Potential impact detection of near-Earth asteroids: the case of 99942 (2004 MN4) (Proc. 229th Symp. of the International Astronomical Union “Asteroids, Comets, Meteors”, Rio de Janeiro, Brazil, August 7–12, 2005) Cambridge, Cambridge Univ. Press, 215–28.
[4] Farnocchia D, Chesley S R, Chodas P W, Micheli M, Tholen D J, Milani A, Elliott G T and Bernardi F 2013 Yarkovsky-driven impact risk analysis for asteroid (99942) Apophis (Icarus, 224, N 1), 192–200.
[5] Aleksandrova A G, Galushina T Yu, Prishchepenko A B, Kholshevnikov K V and Chechetkin V M 2016 The preventive destruction of a hazardous asteroid (Astronomy Reports, 60, N 6), 613–21.
[6] Nikita Petrov, Leonid Sokolov, Elena Polyakhova, and Kristina Oschina. 2018 Predictions of asteroid hazard to the Earth for the 21st century (AIP Conference Proceedings 1959, 040012) doi:10.1063/1.5034615
[7] Balyaev I A 2020 Acceleration of Numerical Integration of the Equations of Motion of Asteroids (Solar System Research, 54, No. 6) 557-66.
[8] Sokolov L L, Bashakov A A and Pitjev N P 2008 Peculiarities of the Motion of Asteroid 99942 Apophis (Solar System Research, 42, No. 1), 18-27.
[9] Sokolov L, Kuteeva G, Petrov N and Eskin B 2019 Hazardous near-Earth asteroids approach (AIP Conference Proceedings 2171, 13019) https://doi.org/10.1063/1.5133286
[10] Sokolov L L, Balyaev I A, Kuteeva G A, Petrov N A and Eskin B B 2020 Possible Collisions and Approaches of Some Dangerous Asteroids with the Earth (Solar System Research, 54, No.6) 541-9.

Figure 3. Cumulative Earth impact probabilities for 200 asteroids (abscissa axis-NASA estimates, ordinate axis-our estimates)