Rotating mirror servo system control based on modified sliding mode-active disturbance rejection controller

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ABSTRACT
In order to improve the tracking accuracy and robustness of the rotating mirror servo system, a modified sliding mode-active disturbance rejection control (MSM-ADRC) strategy is proposed. Firstly, the structural and working principle of the rotating mirror servo system are analyzed, and its mathematical model is established to prepare for the design of the controller. Then, a MSM-ADRC is proposed to reduce the influence of unknown disturbance and improve the tracking accuracy. Among them, the modified sliding mode extended state observation (MSM-ESO) is designed by replacing the traditional nonlinear function with the designed optimal control function, which enhances the observation accuracy of the system state quantity and total disturbance. Meanwhile, an improved approach law is proposed, and an improved sliding mode nonlinear error feedback control law (MSM-NLSEF) is designed based on this approach law, which improves the convergence speed and accuracy of the control law. In addition, the stability of the designed MSM-ESO and MSM-NLSEF is proved. Finally, the proposed control method is validated by simulation and experimental comparison with other state-of-the-art controllers. Results reveal that the proposed control method has satisfying tracking performance and strong disturbance rejection ability.

1. Introduction
In the field of weapon shooting range testing and research, it is very important to obtain a clear flight image of the projectile when detecting various parameters of the projectile (see Li, 2016; Yu et al., 2020; Zhang et al., 2020). In order to capture the attitude of the high-speed moving projectile in real time, scholars have proposed a rotating mirror servo system that realized real-time tracking technology of bullets through the rotating mirror and camera. The key to this technique is that the rotation of the mirror has a high response speed and sufficient stability (see Hu et al., 2022; Lu & Zheng, 2020; Mashimo et al., 2020). However, the mirror of the reflector is directly fixed on the motor shaft of the DC motor, its position angle control is extremely susceptible to changes that in motor parameters and external interference. Therefore, when the traditional control algorithm cannot meet its requirements, the research of high performance control algorithm becomes particularly urgent (see Cao et al., 2021; Chen et al., 2019; Han et al., 2022; Ning et al., 2021; Shen & Zhang, 2019).

Active Disturbance Rejection Control (ADRC) is an improved nonlinear PID control technology proposed by Academician Han (see Han, 2009). It has the characteristics of strong anti-disturbance performance, good robustness and weak model dependence. These features, which are highly suitable for servo control systems have been widely used in various fields. Under rapidly changing disturbances, Lamraoui and Zhu (2019) proposed an improved ADRC for the path tracking of autonomous underwater vehicles, which caused by waves and ocean currents. Wang et al. (2021) proposed ADRC algorithm with sliding mode compensation in the outer loop of the lightweight flexible single-link manipulator. Under the control of this method, the loss of disturbance suppression ability and robustness are well resolved. For the quadrotor attitude system, Song et al. (2018) presented a fixed-time active disturbance rejection control approach by applying the feedback linearization technique. Long et al. (2017) applied ADRC to clinical analysis to track the human gait trajectory of the lower limb rehabilitation exoskeleton. Wei et al. (2021) applied ADRC to an
aerospace electromechanical servo control system to estimate and compensate for the total disturbance of the load and improve the stability of the system. However, only a few studies have applied ADRC to rotating mirror servo control at present. In a fast rotating mirror system, Dong et al. (2018) proposed an improved ADRC with ILC control to perform high-precision positioning for FSO communication driven by using a piezoelectric actuator. Deng et al. (2018) proposed a novel ADRC based on the adaptive control law to meet the high-precision requirements of the K mirror speed control system of the 2m telescope. This control strategy realizes the control performance with small overshoot and strong anti-interference ability. These research results show that ADRC has a certain effect on improving the anti-interference performance of the mirror servo system, but it also exposes the problems of its low tracking accuracy and insensitive response speed.

In addition, the traditional ADRC also has the problem of parameter tuning, which will directly affect the performance of the system. Many scholars have conducted research on this issue. At the earliest, Han (2009) gave an empirical formula for parameters based on the principle of ADRC’s separation. The controller that uses this parameter has a fast adjustment speed, but the adjustment range is too large, which affects the overall performance of the system. Later, on the basis of Academician Han, Gao (2003) simplified the traditional nonlinear ADRC into a linear form, which reducing the parameters that need to be tuned for the controller, and gave a tuning method of the ‘bandwidth method’ for the gain parameters of the extended state observer. The proposal of this method makes the research and application of linear active disturbance rejection control (LADRC) become the mainstream. However, LADRC still has shortcomings. Compared with traditional ADRC, its response speed, steady-state accuracy and anti-interference ability are slightly insufficient. Therefore, Liu et al. (2018) extended the ‘bandwidth method’ to the parameter setting of traditional ADRC. The idea is to obtain the parameters of linear ADRC according to the bandwidth method, estimate the parameters of non-linear ADRC in turn. Hu et al. (2021) proposed a firefly algorithm and introduced it into the ADRC algorithm to realize the self-setting of nonlinear feedback control law parameters. However, whether the parameters obtained by this estimation method are the most effective remains to be verified. In addition, some scholars simplified the parameter tuning by introducing sliding mode control into active disturbance rejection control (see Djeghali et al., 2021; Ji et al., 2020; Wang, Liu, et al., 2021). For example, Huang et al. (2015) used the idea of sliding mode variable structure to improve both the extended state observer and the non-linear error feedback control rate respectively, which greatly reduced the difficulty of parameter tuning. Zhang et al. (2019) introduced the SMC on the basis of LADRC and improved its non-linear feedback control rate in order to improve the response speed and robustness of the system. It can be seen from that, for the mirror servo system, the composite controller of sliding mode control and active disturbance rejection control has significant research value.

In order to solve the problems of a rotating mirror servo system, the design of its controller needs to meet the following conditions: (1) Strong anti-interference ability, (2) Fast transient response and high precision, and (3) A simple and tuning process. An MSM-ADRC strategy for the rotating mirror servo system is proposed in this paper. The contributions of this article are as follows: (1) An MSM-ESO is designed to improve the observation accuracy by using the optimal control function instead of the nonlinear function; (2) An MSM-NLSEF is designed to improve the convergence speed and accuracy of the system. The sliding mode control law based on the modified approaching law is introduced to nonlinearly combine the state errors; (3) Based on the MSM-ESO and MSM-NLSEF designed in this paper, an improved sliding mode active disturbance rejection is proposed; (4) Compared with the ADRC, the MSM-ADRC designed in this paper reduces the parameters that need to be tuned, and reduces the difficulty of parameter tuning. The experimental results show that the performance of rotating mirror servo system is well realized under the control of MSM-ADRC.

The rest organization of this paper are as follows. In the Section 2, the rotating mirror servo system is introduced, and the mathematical model of the rotating mirror servo system is constructed simultaneously. In the Section 3, the MSM-ADRC strategy is introduced, and the design analysis of MSM-ESO and MSM-NLSEF is carried out. In the Section 4, the theoretical simulation and the experimental results are introduced. Finally, in Section 5, the conclusions are presented.

2. Mathematical model of rotating mirror servo system

Rotating mirror servo system consists of a controller, a driver, a DC motor, a mirror and a high-speed camera. The high-speed camera is fixed, and the rotating part is the mirror which is directly fixed on the motor shaft of the DC motor. The output signal of the controller outputs the driving voltage through the driver, and the driving motor drives the mirror to rotate, so as to achieve the purpose of control. The driving method of the motor adopts H-type bipolar mode PWM driving. Its model can be simplified as: $G_d(s) = K_a$, $K_d$ takes as a constant 1. The working
The principle of the rotating mirror is shown in Figure 1 and the specific model framework of the system is shown in Figure 2, where $K_e$ is the back-electromotive force (EMF) coefficient; $\theta_0$ is the real-time input signal given by the host computer; $\theta$ is the output signal; and $\omega$ is the motor angular speed.

In the Figure 1, $l$ is the horizontal distance from the bullet to the rotating mirror; $\xi$ is the elevation angle of the high-speed camera; $\gamma$ is the rotation angle of the rotating mirror; $v_0$ is the speed of the simulated bullet.

In accordance with the working principle of a DC torque motor and Kirchhoff's law, a four-balance equation of a DC torque motor is presented as below.

\[
\begin{align*}
U_a &= I_a R_a + E + L_a \frac{dI_a}{dt} \\
T_e &= K_i I_a \\
J \frac{d\omega}{dt} &= T_e - T_c - B\omega \\
E &= K_e \omega
\end{align*}
\tag{1}
\]

where $U_a$ is the motor control voltage; $I_a$, $R_a$ and $L_a$ indicate motor armature current, resistance and inductance, respectively; $E$ is the back EMF of the motor armature; $T_e$ is the electromagnetic torque; $J$ is the moment of inertia; $K_i$ is the motor torque coefficient; $B$ is the coefficient of viscous friction; and $K_e$ is the back-EMF coefficient, $\omega$ is the motor angular speed.

Equation (1) is transformed into Laplace transform, and the frequency domain equations of a DC torque motor are obtained as follows:

\[
\begin{align*}
I_a(s) &= \frac{1}{L_a s + R_a} (U_a(s) - E(s)) \\
T_e(s) &= K_i I_a(s) \\
J \frac{d\omega(s)}{dt} + B\omega(s) &= T_e(s) - T_c(s) \\
E(s) &= K_e \omega(s)
\end{align*}
\tag{2}
\]

Ignore the influence of the friction torque during the operation of the rotating mirror, and $L_a$ is small and can be disregarded. Therefore, the dynamic mathematical model of the motor is given by:

\[
\frac{d^2 \theta}{dt^2} = -\frac{K_e K_a}{J R_a} \frac{d\theta}{dt} + \frac{K_t}{J R_a} U_a - \frac{1}{J} T_c
\tag{3}
\]

Let $x_1 = \theta$, $x_2 = \omega = \frac{dx_1}{dt}$, $b = \frac{K_e K_a}{J R_a}$ and $u = U_a$. In addition, taking into account the uncertainty of the system, take $b_0$ as the estimated value of $b$, and $b_1$ as the actual compensation value, then $b = b_0 + b_1$. In addition, $f_0 = -\frac{K_e K_a}{J R_a} \frac{dx_1}{dt} + b_1 u$ is the internal disturbance, $f_1 = -\frac{1}{J} T_c$ is the external disturbance, and $f = f_0 + f_1$ is the total disturbance. Therefore, equation (3) can be abbreviated as follows:

\[
\frac{d^2 \theta}{dt^2} = f + b_0 u
\tag{4}
\]

3. MSM-ADRC design

In this section, in order to enable the rotating mirror servo system to accurately capture the flight trajectory of the bullet, we propose a MSM-ADRC strategy. The structure of the MSM-ADRC mainly includes three parts: TD (Tracking Differentiator), MSM-ESO and MSM-NLSEF. This section mainly improves the performance of MSM-ESO and MSM-NLSEF. A block diagram of the MSM-ADRC strategy is shown in Figure 3.

### 3.1. Tracking differentiator design

In order to solve the contradiction between overshoot and fastness, a tracking differentiator is designed in ADRC. In addition, the selection range of the feedback gain of the error and feedback gain of its differential is amplified under the action of the tracking differentiator. A standard tracking differentiator in the traditional ADRC is used in this paper, and its algorithm is expressed as equation (5).

\[
\begin{align*}
fh &= \text{fhan}(v_1(k) - \theta_0(k), v_2(k), r_0, h_0) \\
v_1(k + 1) &= v_1(k) + hv_2(k) \\
v_2(k + 1) &= v_2(k) + fh
\end{align*}
\tag{5}
\]
the expression for the fastest synthesis function \( f_{han}(v_1, v_2, r_0, h_0) \) is expressed as formula (6):

\[
\begin{align*}
\delta &= r_0 h_0 \\
\delta_0 &= h_0 \delta \\
y &= v_1 + h_0 v_2 \\
\alpha_0 &= \sqrt{(\delta^2 + 8r_0|y|)} \\
\alpha &= \begin{cases} 
\frac{v_2 + (\alpha_0 - \delta)}{2} \text{sign}(y), & |y| > \delta_0 \\
\frac{v_2 + y}{\delta}, & |y| \leq \delta_0
\end{cases}
\end{align*}
\]

\[
\text{han} = \begin{cases} 
\frac{r_0 \text{sign}(\alpha)}{\alpha}, & |\alpha| > \delta \\
\frac{r_0}{\delta}, & |\alpha| \leq \delta
\end{cases}
\]

where \( \theta_0 \) is the input position angle signal, \( v_1 \) is the tracking signal of \( \theta_0 \), \( v_2 \) is the differential signal of \( v_1 \), \( h_0 \) is the filter factor, \( r_0 \) is the tracking speed factor, \( h \) is the integration step size.

### 3.2. MSM-ESO design

The ESO is the core component of the traditional ADRC, it is used to observe disturbances. The expressions are as follows (see Han, 2009):

\[
\begin{align*}
e_1 &= z_1 - \theta \\
z_1 &= z_2 - \beta_1 e_1 \\
z_2 &= z_3 - \beta_2 \text{fal}(e_1, \alpha_1, \delta) + b_0 u \\
z_3 &= -\beta_3 \text{fal}(e_1, \alpha_2, \delta)
\end{align*}
\]

SM-ESO is based on the traditional ESO, combined with sliding mode variable structure control, to optimize the design of (7). The MSM-ESO designed in this paper reconstructs the introduced sliding mode surface on the basis of it. At the same time, the sliding mode control based on the reaching law is used to design a new optimal control function to replace the traditional nonlinear function, and the further optimization design improves the observation performance of the state observer. The expressions are as follows:

\[
\begin{align*}
e_1 &= z_1 - \theta \\
z_1 &= z_2 \\
z_2 &= z_3 - \hat{\beta}_1 \phi(e) + b_0 u \\
z_3 &= -\hat{\beta}_2 \phi(e)
\end{align*}
\]

Here, \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) are the gains of MSM-ESO. The function \( \phi(e) \) is the optimal control function based on the sliding mode surface, where \( e = (e_1, e_2, e_3) \) and \( e_2 \) is the angular velocity tracking error, \( e_3 \) is the observation error of the total disturbance of the system.

In accordance with the MSM-ESO, we can obtain the position as well as the angle tracking error \( e_1 \) and the angular velocity tracking error \( e_2 \) of the rotating mirror. The error equation is given by:

\[
\begin{align*}
e_1 &= z_1 - \theta \\
e_2 &= z_2 - \hat{\theta} \\
e_3 &= z_3 - f
\end{align*}
\]

combining (4) and (8), the derivative of (9) can be obtained as follows:

\[
\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 - \hat{\beta}_1 \phi(e)
\end{align*}
\]

for the design of function \( \phi(e) \), the sliding mode surface function based on the system state observation is first
constructed as follow:

\[ s = c_1 e_1 + e_2 \]  
(11)

where \( c_1 \) is the system state observation sliding mode surface parameter, and \( c_1 > 0 \). Adjusting the size of \( c_1 \) can modify the speed at which the state approaches zero. When \( c_1 \) is larger, the adjustment speed is faster.

Combined with (10), the derivative of (11) is obtained as follow:

\[ \dot{s} = c_1 \dot{e}_1 + \dot{e}_2 = c_1 e_2 + (e_3 - \hat{\beta}_1 \phi(e)) \]  
(12)

select exponential approach law: \( \dot{s} = -p_1 \tanh(s) - q_1 s \). \( p_1, q_1 \) are the coefficients of the exponential approach term and \( p_1, q_1 > 0 \). Therefore, the sliding mode optimal control function \( \phi(e) \) based on the reaching law can be designed as:

\[ \phi(e) = \frac{1}{\beta} (c_1 e_2 + e_3 + p_1 \tanh(s) + q_1 s) \]  
(13)

In summary, the specific expression for MSM-ESO is as follows:

\[
\begin{align*}
\epsilon_1 &= v_1 - z_1 \\
\epsilon_2 &= v_2 - z_2 \\
u &= u_0 - \frac{z_3}{b_0} \\
u_0 &= \psi(e)
\end{align*}
\]  
(15)

where \( \epsilon_1 \) and \( \epsilon_2 \) are the error signal and error differential signal of the transition process, respectively, and \( u_0 \) is the control signal without disturbance compensation, \( u \) is the control signal after compensation and \( \frac{z_3}{b_0} \) is the real-time total disturbance compensation value of the position angle of the rotating mirror. Function \( \psi(e) \) is the sliding mode optimal control function based on the non-linear error feedback control law, where \( e = (\epsilon_1, \epsilon_2) \).

It can be seen from (4.3) that when MSM-ESO is stable, \( z_2 \) approaches \( x_2, z_3 \) approaches \( x_3 \), and \( \phi(e) \) approaches 0. Therefore, assume that \( \phi(e) = 0 \). At the same time, giving that \( v_2 = \dot{v}_1 \) and \( z_2 = \dot{z}_1 \), we can obtain the following equations:

\[
\begin{align*}
\dot{\epsilon}_1 &= \epsilon_2 \\
\dot{\epsilon}_2 &= v_2 - z_3 - b_0 u
\end{align*}
\]  
(16)

Similarly, in order to design the sliding mode optimal control function \( \psi(e) \) based on error feedback, the sliding mode surface function is constructed as follows:

\[ \dot{s} = c_2 \epsilon_1 + \epsilon_2 \]  
(17)

Meanwhile, the derivative function of equation (15) is as follows:

\[ \dot{s} = c_2 \dot{\epsilon}_1 + \dot{\epsilon}_2 = c_2 \epsilon_2 + \dot{v}_2 - b_0 \psi(e) \]  
(18)

where \( c_2 \) is the sliding mode surface parameter of the system error feedback, and \( c_2 > 0 \). This article proposes a modified approach rate and its expression is as follows:

\[
\dot{s} = \begin{cases} 
-k_1 \tanh(\hat{s}) - k_2 \hat{s} & |\hat{s}| > \delta \\
-k_3 \tan(\hat{s}) - k_4 \tanh(\hat{s}) |\hat{s}|^\alpha & |\hat{s}| \leq \delta 
\end{cases}
\]  
(19)

where \( k_1 \) and \( k_2 \) are the modified approaching law coefficients, and \( k_1, k_2 > 0 \). \( 0 < \delta < 1 \) and \( \alpha > 0 \).

Set \( f(\hat{s}) = \dot{\hat{s}}, f_1(\hat{s}) = -k_3 \tan(\hat{s}) - k_4 \tanh(\hat{s}) |\hat{s}|^\alpha, f_2(\hat{s}) = -k_1 \tan(\hat{s}) - k_2 \hat{s} \).

In order to satisfy the continuity and derivability of the modified approaching law, the following conditions should be met as follows:

\[
\lim_{\hat{s} \rightarrow -\delta^-} f_1(\hat{s}) = \lim_{\hat{s} \rightarrow +\delta^-} f_1(\hat{s}) = f(\delta) \\
\lim_{\hat{s} \rightarrow -\delta^-} f_2(\hat{s}) - f(\delta) = \lim_{\hat{s} \rightarrow +\delta^-} f_2(\hat{s}) - f(\delta)
\]  
(20)

for the continuity condition, the following conditions need to be satisfied.

\[ k_1 \tanh(\delta) + k_2 \delta = k_3 \tan(\delta) + k_4 \tanh(\delta) |\delta|^\alpha \]  
(21)

for the derivability condition, the following conditions need to be satisfied.

\[ \lim_{\hat{s} \rightarrow -\delta^-} \frac{f_2(\hat{s}) - f(\delta)}{\hat{s} - \delta} = k_1[1 - \tanh^2(\delta)] + k_2 \]  
(22)

\[ \lim_{\hat{s} \rightarrow +\delta^-} \frac{f_2(\hat{s}) - f(\delta)}{\hat{s} - \delta} = k_3 \sec^2(\delta) + k_4[1 - \tanh^2(\delta)] |\delta|^\alpha + \tanh(\delta) |\delta|^\alpha \]  
(23)
then we can get
\[
\begin{align*}
k_1[1 - \tanh^2(\delta)] + k_2 &= k_3\sec^2(\delta) \\
+ k_4[1 - \tanh^2(\delta)]|\delta|^\alpha \\
+ \tanh(\delta)\alpha|\delta|^\alpha-1
\end{align*}
\] (24)

Therefore, the following conditions exist as equation (25). Solutions have to be as equation (26).
\[
\begin{align*}
k_1 \tanh(\delta) + k_2 \delta &= k_3 \tan(\delta) + k_4 \tanh(\delta)|\delta|^\alpha \\
+ k_4[1 - \tanh^2(\delta)]|\delta|^\alpha + \tanh(\delta)\alpha|\delta|^\alpha-1
\end{align*}
\] (25)
\[
\begin{align*}
k_3 &= \frac{(k_1 \tanh(\delta) - k_1 - k_2) \cos^2(\delta) \tan(\delta) + k_2 \delta}{|\delta|^{\alpha-1} \tanh(\delta) (|\delta| - \alpha \cos^2(\delta) \tanh(\delta)} \\
+ \alpha|\delta|^{\alpha-1} \tanh(\delta) (k_1 \tanh(\delta) + k_2 \delta)\\n+ |\delta|^{\alpha-1} \tanh(\delta) (|\delta| - \alpha \cos^2(\delta) \tanh(\delta)} \\
+ |\delta|^{\alpha-1} \tanh(\delta) (|\delta| - \alpha \cos^2(\delta) \tanh(\delta)} \\
+ |\delta|^{\alpha-1} \tanh(\delta) (|\delta| - \alpha \cos^2(\delta) \tanh(\delta)}
\end{align*}
\] (26)

3.4. Stability analysis of MSM-ESO and MSM-NLSEF

3.4.1. Stability analysis of MSM-ESO

MSM-ESO stability analysis is the core problem of the MSM-ADRC controller. The following assumption is to study the convergence of the estimated error system.

Assumption 3.1: Any second-order perturbed system can be extended to the third-order linear system and \( \dot{x}_3 = f = w(t) \), where \( w(t) \) is bounded.

Remark 3.1: This assumption is reasonable in almost all physical and engineering systems.

Indeed, in actual application scenarios and control objects, the uncertainties and the disturbances cannot be infinite. So assuming \( w(t) \) is bounded is legitimate. Otherwise, to stabilize the system, the gains of the controller is required to be infinite, which is unrealistic (see Djehghali et al., 2021; Zhang et al., 2022).

Combined with Assumption 3.1, equation (4) can be rewritten as:
\[
\begin{align*}
x_1 &= \theta \\
x_1 &= x_2 \\
x_2 &= x_3 + b_0u \\
x_3 &= w_0
\end{align*}
\] (29)

Let the external interference of the system be 0, then \( w(t) = 0 \). Combining equation (14), the observation error equation of the system can be obtained as follows:
\[
\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -c_1 q_1 e_1 - (c_1 + q_1) e_2 - p_1 \tanh(s) \\
\dot{e}_3 &= -p_1 q_1 e_1 - \beta(c_1 + q_1) e_2 - e_3 - p_1 \tanh(s)
\end{align*}
\] (30)

The error formula can be transformed into the following form:
\[
\begin{align*}
\dot{E} &= AE + BT(s) \\
\dot{s} &= CTE
\end{align*}
\] (31)

where
\[
A = \begin{bmatrix}
0 & 1 & 0 \\
-c_1 q_1 & -(c_1 + q_1) & 0 \\
-\beta & 0 & -\beta
\end{bmatrix},
C = \begin{bmatrix}
c_1^T \\
1 \\
0
\end{bmatrix},
\text{and } T(s) = \tanh(s).
\]

Since matrix \( A \) is a Holwitz matrix, and \( 0 < sT(s) < +\infty \), the system (31) is the standard form of the direct Lurie control system. Therefore, the problem of MSM-ESO’s stability proof is transformed into a necessary and sufficient condition for finding the absolute stability of the system (31), that is, the Lurie problem.

Lemma 3.1: For system (31), if there is a real number \( \rho \) and it is satisfied with \( \text{Re}((1 + iw\rho)C^TA_{\text{lo}}^{-1}B) \leq 0 \), where \( A_{\text{lo}} = iwE - A \) and \( w \) are constants, and \( w \geq 0 \), then system (31) is absolutely stable.

Proof: The proof of Lemma 3.1 is available in (see Fernandes & Colón, 2021).

Lemma 3.2: Two of the many necessary conditions for absolute stability of the system (31) are as follows: (a) \( C^T B \leq 0 \); (b) If \( A \) is a Hurwitz matrix, then \( C^T A^{-1} B \geq 0 \).

Proof: The proof of Lemma 3.2 is available in (see Lan, 2008).
Theorem 3.1: If the matrix $A$ in system (31) can be reduced to the Jordan standard form, that is,

$$
A = \begin{bmatrix}
-\lambda_1 \\
-\lambda_2 \\
-\lambda_3
\end{bmatrix}
$$

where $\lambda_1 \neq \lambda_2 \neq \lambda_3$ and $\lambda_i > 0 (i = 1, 2, 3)$, then the necessary and sufficient conditions for the absolute stability of system (31) is $c_i b_i \leq 0 (i = 1, 2, 3)$.

Proof: According to the conditions in Lemma 3.1, we can get formula (32) as follows:

$$
C^T A_{iw}^{-1} B = (c_1 \ c_2 \ c_3) \begin{bmatrix}
\frac{1}{iw + \lambda_1} \\
\frac{1}{iw + \lambda_2} \\
\frac{1}{iw + \lambda_3}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{iw + \lambda_1} \\
\frac{1}{iw + \lambda_2} \\
\frac{1}{iw + \lambda_3}
\end{bmatrix}
$$

so we can get:

$$
\text{Re}((1 + iw \rho) C^T A_{iw}^{-1} B) = \frac{c_1 b_1 (\lambda_1 + \rho w^2)}{w^2 + \lambda_1^2} + \frac{c_2 b_2 (\lambda_2 + \rho w^2)}{w^2 + \lambda_2^2} + \frac{c_3 b_3 (\lambda_3 + \rho w^2)}{w^2 + \lambda_3^2}
$$

(33)

When $c_i b_i \leq 0 (i = 1, 2, 3)$, that is, $\rho \geq 0$ exists, so that $\text{Re}((1 + iw \rho) C^T A_{iw}^{-1} B) \leq 0$, which satisfies the condition of Lemma 3.1. Therefore, it can be obtained that $c_i b_i \leq 0 (i = 1, 2, 3)$ is a sufficient condition for the absolute stability of the system (31).

Then, from Lemma 3.2, when the system (31) is absolutely stable, the conditions can be obtained as follows:

$$
\begin{cases}
\quad c_1 b_1 + c_2 b_2 + c_3 b_3 < 0 \\
c_1 \lambda_1 b_1 + c_2 \lambda_2 b_2 + c_3 \lambda_3 b_3 \leq 0
\end{cases}
$$

(34)

where $\lambda_i > 0 (i = 1, 2, 3)$.

From equation (25), we know that $c_i b_i \leq 0 (i = 1, 2, 3)$. Therefore, when the matrix $A$ can be reduced to a standard equivalent, that is, $A = \begin{bmatrix}
-\lambda_1 \\
-\lambda_2 \\
-\lambda_3
\end{bmatrix}$, where $\lambda_1 \neq \lambda_2 \neq \lambda_3$ and $\lambda_i > 0 (i = 1, 2, 3)$, $c_i b_i \leq 0 (i = 1, 2, 3)$ is a necessary condition for the absolute stability of the system (31).

In the system (31), $c_i$ is the sliding mode surface parameter. Since the sliding mode surface parameter satisfies the Hurwitz condition, that is, $c_i \geq 0 (i = 1, 2, 3)$, $\beta, q_1$ and all numbers in the matrix $B$ are greater than 0, so $c_i b_i \leq 0 (i = 1, 2, 3)$. Therefore, it can be proved that the system (31) is absolutely stable, that is, MSM-ESO is absolutely stable.

3.4.2. Stability analysis of MSM-NLSEF

The stability of MSM-NLSEF is the basis for the realization of MSM-ADRC function. Its stability is analyzed as follows: First, the Lyapunov function is defined as:

$$
V = \frac{1}{2} \hat{s}^2
$$

(34)

Secondly, in accordance with Lyapunov’s stability theory, the stability of the MSM-NLSEF based on the approach law should fulfill the following conditions:

$$
\lim_{t \to 0} \tilde{V} = 0.
$$

Finally, comprehensive (19) and (34), $V$ can be obtained as follows:

$$
\dot{V} = \ddot{s} = \begin{cases}
-k_1 \dot{s} \tanh(\dot{s}) - k_2 \dot{s}^2 & |\dot{s}| > \delta \\
-k_1 \dot{s} \tanh(\dot{s}) - k_4 |\dot{s}|^a \tanh(\dot{s}) & |\dot{s}| \leq \delta
\end{cases}
$$

(35)

It can be seen from (19) that the coefficients $k_1$ and $k_2$ of the modified approaching law are both greater than 0. From equation (25), it can be known that $k_3$ and $k_4$ are also greater than 0. Therefore, $\ddot{s} < 0$ can be guaranteed. This means that the defined errors $\dot{e}_1$ and $\dot{e}_2$ reach the sliding surface $\dot{s} = 0$ within a finite time. In other words, under the proposed MSM-NLSEF control law, the tracking error of the system will gradually slide to the equilibrium point to achieve stability. This completes the proof.

4. Simulation and experimental results

4.1. Simulation results

To validate the performance of proposed MSM-ADRC controller, we carried out the simulation in Matlab/Simulink environment. At the same time, the control method in this article is compared with the traditional ADRC (see Han, 2009) and SM-ADRC (see Huang et al., 2015) to better verify the advantages of the control method (for convenience, the two control strategies are called ‘ADRC’ and ‘SM-ADRC’, respectively). At the same time, for better comparison and verification, the adopted parameters have been tuned many times, and the final parameter selection is shown in Table 1. The system parameters used in the simulation are as follows: $U_a =$
Table 1. Parameters of three control strategies.

|                    | ADRC  | SM-ADRC | MSM-ADRC |
|--------------------|-------|---------|----------|
| **TD**             |       |         |          |
| $h_0 = 0.015$      | $h_0 = 0.015$ | $h_0 = 0.015$ |
| $r = 2000$         | $r = 2000$      | $r = 2000$   |
| **ESO/SM-ESO/MSM-ESO** |     |         |          |
| $\beta_0 = 8000$  | $\delta_1 = 0.05$ | $\beta = 800$ |
| $\beta_0 = 1000$  | $k_1 = 1000$      | $c_1 = 100$  |
| $\beta_3 = 50$    | $c_1 = 8000$      | $c_2 = 800$  |
| $\delta = 0.01$   | $c_2 = 800$       |             |
| **NLSEF/SM-NLSEF/MSM-NLSEF** |     |         |          |
| $\beta_1 = 120$   | $\delta = 1$      | $c_2 = 200$  |
| $\beta_2 = 20$    | $k_1 = 500$       | $\alpha = 20$ |
| $\delta = 1$      | $c = 200$         |             |

12V, $L_2 = 0.0189H$, $R_o = 14.7\Omega$, $K_o = 0.119V/rpm$, $K_i = 1.136V/rpm$, $\omega_0 = 95rpm$ and $J = 0.000179kg \cdot m^2$.

In this section, in order to verify the optimized performance of MSM-ESO, MSM-NLSEF and the overall performance of MSM-ADRC, the simulation of this study mainly includes three parts:

(A) Performance verification of MSM-ESO. MSM-ESO is the core part of SM-ADRC, which is used to observe the total disturbance in the system. In order to analyze its observation performance, the results are compared with SM-ESO in (see Huang et al., 2015) by observing the state variables of the galvanometer system. For the extended state observer, it can be known: $z_1 \rightarrow \theta$, $z_2 \rightarrow \omega$, $z_3 \rightarrow f$. MSM-ESO combines sliding mode control on the basis of traditional ESO. Compared with the existing SM-ESO, the optimal control function based on sliding mode surface and sliding mode control law make the MSM-ESO have better observation effect. The simulation results are shown in Figure 4. It can be clearly seen from the figures that the observation accuracy of each state quantity of MSM-ESO is significantly better than that of SM-ESO. In Figure 4(a)(c)(e), the tracking accuracy of MSM-ESO for $\theta$, $\omega$, $f$ is higher than that of SM-ESO. In Figure 4(b, d, f), under the control of SM-ESO, the error of the observed values is significantly smaller than that of SM-ESO.

(B) Performance verification of MSM-NLSEF. In order to verify the performance of MSM-NLSEF, a simulation comparison with SM-NLSEF in (see Huang et al., 2015) was carried out, and the comparison results are shown in Figure 5. It can be seen from Figure 5(a) that the convergence speed and accuracy of the sliding mode surfaces of MSM-NLSEF are significantly better than SM-NLSEF. Figure 5(b) reflects the better convergence accuracy of MSM-NLSEF’s control law $u_0$. Figure 5(c) and (d) are the tracking effect diagrams of the output $v_1$ of the TD. It can be seen from the figures that the tracking error of MSM-NLSEF is smaller. Thus, the effect of the designed sliding mode nonlinear function is obvious.

(C) Performance verification of MSM-ADRC. The performance analysis of MSM-ADRC mainly includes: fast response performance, anti-interference performance, tracking accuracy effect and noise suppression performance. The position angle $\theta_0$ of the motor rotor in Figure 6 presents the positioning unit step input, and the sudden interference torque $T_c$ is given at 1s. Here, $\theta_0 = 1rad$ and $T_c = 0.1$. As shown by the response curve, the response time of the ADRC, SM-ADRC, and the MSM-ADRC control strategies are 0.2533 s, 0.3568 s and 0.1641 s, respectively. At the same time, the influence of disturbance on the MSM-ADRC control strategy is almost negligible. Both ADRC and SM-ADRC are more affected than the MSM-ADRC, and the recovery times are longer. The recovery times of the ADRC, SM-ADRC, and the MSM-ADRC control strategies are 0.6031 s, 0.6582 s, 0.0146 s, respectively. Therefore, combined with the optimized performance of MSM-ESO and MSM-NLSEF, the MSM-ADRC has faster response speed and better anti-disturbance performance than the ADRC and SM-ADRC control strategies. In order to better verify the control performance of the three control strategies, this paper also verifies the case that the input signals are sinusoidal signals and impulse signals, and its response curves and error response curves under disturbance are shown in Figures 7 and 8. The simulation results also show that under the condition of disturbance, the response curve of MSM-ADRC almost has no fluctuation, while SM-ADRC has slight fluctuation, and the response curve of ADRC has the largest fluctuation.

In order to better verify the tracking performance of MSM-ADRC, the sine signal and the pulse signal of the rotor position angle in Figure 9 is given to compare the tracking effects of the three control strategies. It can be seen from Figure 9 that under different input signals, the simulation results show that the tracking accuracy of MSM-ADRC is significantly higher than that of SM-ADRC and ADRC. Take the sine signal as an example, it can be seen from Figures 9 and 10 that the tracking accuracy of MSM-ADRC to the desired signal is related to the transition process of the TD arrangement. Without considering the impact of TD, it can be clearly seen from Figure 10 that the tracking accuracy of MSM-ADRC is also more accurate than traditional ADRC and SM-ADRC under different parameters. In order to consider the effect of overshoot on the mutation signal, the TD parameters used in this paper are $r = 2000$, $h_0 = 0.015$. 
Figure 4. Observation results of the MSM-ESO.

(a) Observed value of $\theta$
(b) Observation error of $\theta$
(c) Observed value of $\omega$
(d) Observation error of $\omega$
(e) Observed value of $f$
(f) Observation error of $f$

Figure 5. Convergence and tracking performance curve of MSM-NLSEF.

(a) Convergence of sliding surface $S$
(b) Convergence of control law $u_0$
(c) Tracking curve of TD output signal $v_i$
(d) Tracking error of $v_i$
Figure 6. Position angle tracking response curve of the step signal (left) and error of angle tracking response (right).

Figure 7. Position angle tracking response curve of the sine signal (left) and error of angle tracking response (right).

Figure 8. Position angle tracking response curve of the pulse signal (left) and error of angle tracking response (right).

Figure 9. Position angle tracking response curve of sine signal and pulse signal.
Figure 10. The position angle tracking response curve of the sinusoidal signal under different TD parameters.

Figure 11. Measurement noise response curve (left) and error response curve of noise (right).

Figure 11 compares the noise suppression capabilities of the three control strategies (ADRC, SM-ADRC and MSM-ADRC). As shown in the figures, the same white noise interference is applied to the three control strategies as the measurement noise signal. The response curve in the Figure 8 indicates that the noise suppression capability of the MSM-ADRC control strategy is significantly better than those of the two other control strategies.

In summary, the performance comparison of the three control strategies in various aspects can be obtained. Furthermore, the overall performance of MSM-ADRC is significantly better than those of the ADRC and SM-ADRC control strategies. The details are provided in Table 2.

4.2. Experimental results

The simulation results show the good performance of the controller. On this basis, we carried out experiments to verify the performance of the rotating mirror servo system, after which we built a simulation trajectory tracking experimental platform. Its working principle and hardware are shown in Figure 9, respectively.

| Table 2. Parameters of three control strategies. |
|-----------------------------------------------|
| Performance | Performance indicators | ADRC | SM-ADRC | MSM-ADRC |
| Quick response ability | Response time/s | 0.2533 | 0.3538 | 0.1641 |
| Ability to resist sudden interference | Overshoot/rad | 0.0218 | 0.0000 | 0.0000 |
| | Overshoot/rad | 0.6652 | 0.0818 | 0.0011 |
| Tracking performance | Recovery Time/s | 0.6031 | 0.6582 | 0.0146 |
| | Maximum tracking error/rad | 0.1104 | 0.0911 | 0.0909 |
| Noise suppression performance | Noise impact size | Smaller | Tiny | Almost no effect |

Figure 12. Schematic diagram of the simulated ballistic tracking experiment (left) and physical image of the simulated ballistic tracking experimental platform (right).
As shown in Figure 12, the platform mainly includes three parts: an analog launcher, an image processing module, and a rotating mirror servo system. The analog launcher is a self-designed adjustable-speed bullet launching device. Considering that the trajectory tracking experiment requires a PC for image processing and other influencing factors, we directly implement position angle tracking control on the rotating mirror system, given the initial velocity of the simulated bullet. Among them, the relevant parameters of the galvanometer system are as follows: \( \xi = 10, \gamma = 60, l = 1.73, v_0 = 8 \).

Figure 13 shows the corner position tracking curve. It can be seen from that the X coordinate in the figure is time (unit: ms), and the Y coordinate is CTS (encoder counting unit, 100 cts corresponds to a motor rotation angle of 1°). The rotating mirror angle corresponding to the exit of the analog launcher is set to 0°, and the rotation angle of the rotating mirror from the launch to the end of shooting is set to 60°.

As shown in Figure 13, we can clearly see that the tracking effect of MSM-ADRC is better than the other two control strategies. The specific values are shown in Table 3. In addition, according to the simulated ballistic tracking experimental platform, it can be known that the field of view reflected by the galvanometer is \( 20 \times 18 \text{ cm}^2 \), and the length of the simulated bullet is 7 cm. Furthermore, through calculation, if the bullet is to be completely within the field of view of the mirror, the position angle error cannot exceed 2.317°. Therefore, the experimental results indicate that the rotating mirror servo system based on the MSM-ADRC is better than the SM-ADRC and ADRC control strategies for real-time tracking of flying bullets.

### Table 3. Experimental results of the three control strategies.

| Controller | ADRC | SM-ADRC | MSM-ADRC |
|------------|------|---------|----------|
| Maximum error of position angle tracking/° | 2.89 | 1.73 | 0.62 |
| Response time of the system/ms | 59.82 | 28.67 | 18.71 |

### 5. Conclusion

In this paper, we proposed an MSM-ADRC strategy that combines sliding-mode control theory and ADRC to
improve the tracking accuracy and robustness of the rotating mirror servo system. Simultaneously, the optimization performance of MSM-ESO and MSM-NLSEF is verified respectively in this paper. In order to verify the overall performance of MSM-ADRC, we compared this with the SM-ADRC and the ADRC control strategies. The results of the simulations and experiments show that the rotating mirror servo system based on MSM-ADRC achieves better tracking performance and robustness. In addition, in simulated ballistic experiments, compared with ADRC and SM-ADRC, the MSM-ADRC has better tracking accuracy and can achieve the expected results. As an extension of this research project, our future work will adjust the parameters of the controller in real time to achieve work efficiency in a changeable and complex environment.

Disclosure statement

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