Quantization of superflow circulation and of magnetic flux are considered for systems, such as superfluid $^3$He-A and unconventional superconductors, having non scalar order parameters. The circulation is shown to be the anholonomy in the parallel transport of the order parameter. For multiply-connected samples free of distributed vorticity, circulation and flux are predicted to be quantized, but generically to nonintegral values that are tunably offset from integers. This amounts to a version of Aharonov-Bohm physics. Experimental settings for testing these issues are discussed.

PACS numbers: 03.65.Bz, 67.57.-z, 74.20.-z
Preprint Number: P-95-05-036-ii

Introduction: In superfluid $^4$He, as well as in conventional superconductors, for any configuration of the appropriate order parameter, and for any closed path through nonsingular regions of it, the superflow circulation $\kappa$ adopts one of only a discrete set of values. Quantum-mechanical in origin \cite{1,2}, this remarkable phenomenon is referred to as the quantization of circulation. In $^4$He, circulation occurs at integral multiples of $\hbar/M$, $M$ being the mass of the $^4$He atom. (In conventional superconductors, $M$ is the Cooper-pair mass). The magnetic flux enclosed in a ring of conventional superconductor is also quantized, in units of $\Phi_0 = 2.07 \times 10^{-15}$ We. In superfluid $^3$He, however, $\kappa$ is not necessarily quantized, owing to the anisotropy of certain of its superfluid phases \cite{3,4,5}.

For $^3$He-A, the standard view \cite{5} is that despite the absence of circulation quantization in the bulk, integral quantization, precisely as in $^4$He, occurs at surfaces, owing to the anchoring of the relative orbital angular momentum vector $\mathbf{l}$. In fact, we shall see that quantization at surfaces is by no means the rule, instead being the exception, with integral quantization being yet more exceptional. Indeed, the aim of this Letter is to demonstrate that, although quantization is not generic, there do exist experimentally achievable order parameter configurations for which circulation is quantized, but not to conventional (i.e., integral) values. Instead, what occurs is offset circulation quantization, i.e., integrally-spaced but nonintegral circulations: $\cdots, -1 + \omega, 0 + \omega, 1 + \omega, \cdots$, for all paths. The offset $\omega$ can be continuously tuned by varying the sample shape or certain external fields, both of which can determine the texture of the order parameter. Moreover, as we shall see, the Aharonov-Bohm-like physics \cite{6} responsible for offset circulation-quantization in $^3$He-A also leads to the possibility that a nonintegral number of magnetic flux quanta could be enclosed in a ring of unconventional superconductor. Furthermore, if the order parameter is in equilibrium then offset quantization has the striking corollary that arbitrary nonintegral equilibrium circulations can be obtained, in contrast with the standard possibilities: 0, 1/2, 1 (bulk); 0, 1/2, 1/3, 2, $\cdots$ (surface); see, e.g., \cite{3}.

We proceed as follows. First, we establish a general relationship between circulation, parallel transport and order parameter anholonomy. Next, we introduce Aharonov-Bohm-type concepts to this area of physics. Finally, we discuss experimental configurations for superfluid $^3$He-A, and propose tests involving the offset of magnetic-flux-quantization in unconventional superconductors and the phase-shift of Little-Parks oscillations

Geometric phase: The origin of the phenomena considered here is the geometric phase \cite{6,7}, i.e., the phase acquired by Cooper pairs as they propagate through the condensate that they self-consistently form, their relative (orbital or spin) angular momentum following the local orientation determined by the condensate. This geometric phase is an “anholonomy: the geometrical phenomenon in which nonintegrability causes variables to fail to return to their original values, when others, which drive them, are altered round a cycle” \cite{6}. Indeed, as we shall show, the circulation computed around any path in an anisotropic superfluid is a natural anholonomy in the parallel transport of the triads that characterize its order parameter.

Consider superfluid $^3$He-A, which is characterized by an order parameter matrix $d_{\mu j}$ of the form:

$$d_{\mu j} = \Delta_0 d_{\mu} (m_j + in_j) \exp i\chi.$$  (1)

Here, $\Delta_0$ is the (real) gap parameter for the condensate, $d_{\mu}$ is a (real) unit-vector associated with the spin angular momentum sector, $\{l, m, n\}$ is an orthonormal, right-handed, triad of (real) vectors associated with the relative orbital angular momentum sector, and $\chi$ is a (real) phase. This parametrization of $d_{\mu j}$ in terms of $\chi$ and $\{l, m, n\}$ is not unique: $d_{\mu j}$ is invariant under the simultaneous operations of rephasing $\chi$ (by $\delta \chi$) and
rotating \{m, n\} about \textbf{l} (by \(\delta \chi\)). Due to the broken relative gauge-orbital symmetry (for reviews, see Refs. [3,4,5,11]) the superfluidity is manifested not by a complex scalar, as in \(^4\)He, but instead by the complex vector \(t \equiv (m + in)/\sqrt{2}\).

This recognition, that the parametrization of \(d_{\mu j}\) in terms of \(\chi\) and \(\{l, m, n\}\) is not unique, is crucial [12]. At any point in (real) space one may choose, e.g., \(\chi = 0\), and thus select a particular \(\{l, m, n\}\). A choice for \(\chi\) and the pair \(\{m, n\}\) at some other point may, if one wishes, then be determined as follows: given a path between the two points, the orientation of \(\{m, n\}\) at the second point is obtained by the parallel transport (defined below) of the pair over the (curved) surface of orientations of \(\textbf{l}\) that are encountered along the real-space path. However, owing to the nonintegrability of this parallel-transport law, the orientation of \(\{m, n\}\) cannot be expressed as a single-valued function of \(\textbf{l}\) over the entire \(l\)-sphere. Thus, if the (real-space) path is closed then, although \(\textbf{l}\) returns to its original value, \(m\) and \(n\) and therefore \(\chi\) will, in general, not (i.e., there can be anholonomy).

The parallel transport of triads over the \(l\)-sphere is accomplished via the connection

\[
\text{Im} \ t^*_j \nabla_1 t_j = 0, \tag{2}
\]

the line integral of which turns out to be the solid angle swept out by \(\textbf{l}\) as the path is traversed [13], and gives the anholonomy mentioned above.

**Circulation and geometry:** Having discussed geometric issues [13] associated with the superfluid \(^3\)He order parameter matrix, we now turn to the relationship between these issues and the circulation \(\kappa\). The local superfluid velocity \(v^{(s)}\) [14] can be expressed in terms of the orbital triad \(\{l, m, n\}\) (or, equivalently, the complex vector \(t\)) via

\[
2Mv^{(s)}/h = m_j \nabla_r n_j + \nabla_r \chi = \text{Im} \ t^*_j \nabla_r t_j + \nabla_r \chi. \tag{3}
\]

That \(v^{(s)}\) can be interpreted as the superfluid velocity follows from its behavior under galilean transformations: if \(t \rightarrow t' = t \exp{(2iM \delta \cdot r/h)}\) then \(v^{(s)} \rightarrow v^{(s)} + \delta v\).

A local rephasing of \(\chi\) and rotating of \(\{m, n\}\) (such that \(d_{\mu j}\) is unchanged) amounts to a gauge transformation, and thus leaves \(v^{(s)}\) unchanged. The velocity \(v^{(s)}\) has the form of a gauge potential, like the London current in a superconductor [15]. In the present context, this gauge potential is referred to as the Berry connection. As \(\textbf{d}\) is a real vector in the \(A\)-phase of \(^3\)He, it produces no contribution to \(v^{(s)}\), even if \(\textbf{d}\) is inhomogeneous.

Now, \(\kappa\) is the line integral around a closed path \(C_r\) in real space: \(\kappa \equiv \oint_{C_r} \text{d} \cdot v^{(s)}\). By using Eq. (3) \(\kappa\) can be expressed as

\[
\kappa = (\hbar/2M) \oint_{C_r} \text{d} \cdot \{\text{Im} t^*_j \nabla_r t_j + \nabla_r \chi\} \tag{4a}
\]

\[
= (\hbar/2M) \oint_{C_r} \text{d} \cdot \{\text{Im} t^*_j \nabla_1 t_j + \nabla_1 \chi\}, \tag{4b}
\]

where \(C_r\) is the contour on the \(l\)-sphere traced out by the orbital vector \(l\) as the path \(C_r\) is circumnavigated. By adopting the parallel-transport connection, Eq. (3), we see that the penultimate term in Eq. (11) vanishes, so that \(\kappa\) is determined by the anholonomy \(\chi(C_r)\) in \(\chi\), which is necessary to compensate for the anholonomy in \(\{l, m, n\}\) induced by the parallel-transport connection:

\[
\kappa = (\hbar/2M) \chi(C_r). \tag{5}
\]

Hence, we see that the superfluid circulation is given by the anholonomy in the parallel transport of the order parameter, and that \(\kappa\) is the flux associated with the Berry connection. The computation of \(\chi(C_r)\) [10] gives

\[
\chi(C_r) = \Omega(C_r) + 2\pi p, \tag{6}
\]

where \(\Omega(C_r)\) is the solid angle subtended by \(C_r\) at the center of the \(l\)-sphere and \(p\) is an integer. Although \(\Omega(C_r)\) is defined only modulo \(4\pi\), the term \(2\pi p\) (including odd \(p\)) results in the standard circulation quantum.

For a nonsingular order parameter matrix in a simply-connected sample, Eq. (4) is of course the well-known Ho circulation theorem, obtained by Ho in a different manner [16]. The parallel-transport approach adopted here yields an extension to multiply-connected samples, first conjectured in Ref. [17]. The extra term, \(2\pi p\), allows for singularities, which render the sample multiply-connected. We emphasize the anholonomic essence of the circulation, even in simply-connected samples. We note that Eq. (4) holds whether or not the distributed vorticity \(\nabla \times v^{(s)}\) of the superflow, which is given by the Mermin-Ho [15] equation \(\nabla_r \times v^{(s)} = (\hbar/4M)\epsilon_{ijk} t_i (\nabla_r l_j \times \nabla_r l_k)\), is zero.

We now come to our primary observation. In a multiply-connected sample from which distributed vorticity is absent the circulation \(\kappa\) is quantized with an offset, in striking contrast with superfluid \(^4\)He. As we elucidate below, this amounts to Aharonov-Bohm physics stemming from geometry. The offset is determined by the solid angle \(\Omega(C_r)\) via Eqs. (3) and (4). Furthermore, the offset of \(\kappa\) results in truly stable (i.e., equilibrium) persistent flow. For a system with radius of order 1 mm the speed of this equilibrium flow will be of order \(3 \times 10^{-3} \text{mm/s}\). It is desirable to exclude distributed vorticity because whenever the area between nearby (real-space) paths is penetrated by distributed vorticity, a continuous variation of a path will lead to a continuous variation of \(\kappa\). (The presence of a small amount of distributed vorticity will smear the sharpness of the quantization, but not destroy it altogether.) By contrast, if the sample is simply-connected and there is distributed vorticity then \(\kappa\) is not quantized (i.e., a continuous variation in the path leads to a continuous variation in \(\kappa\)). However, contrary to the standard view, quantization is absent not only in the bulk but also at surfaces, owing to their geometry. For example, for surfaces with nonzero gaussian curvature (such as hyperboloids) there is no quantization. (If the sample is simply-connected and there is no distributed vorticity then \(\kappa\) is zero.)
What is the connection between Aharonov-Bohm physics and offset circulation-quantization? Due to the structure of the $^3$He-A order parameter, the distributed vorticity of the superflow velocity field $v^{(s)}$ is, in general, nonzero. Hence, in a generic sample a continuum of values of the circulation can be found. If the distributed vorticity vanishes and the sample is simply-connected then for all circulation-paths the circulation is zero. However, if the sample is multiply-connected then, even if the distributed vorticity vanishes, the circulation need not be zero. This is Aharonov-Bohm physics \[1\]. A close analogy with Aharonov-Bohm–type phenomena from mesoscopic physics is evident. Consider a thin-walled normal metal cylinder in a homogeneous axial magnetic field \[20\]. If the wall is sufficiently thin that a negligible amount of magnetic flux penetrates the metal itself then the conductance of the cylinder oscillates with a negligible amount of magnetic flux threading the interior of the cylinder because all topologically-equivalent Feynman (electron) paths enclose identical magnetic flux. If the wall is thick, so that a non-negligible amount of flux penetrates the metal, then the amplitude of the oscillations is diminished, as topologically-equivalent paths now enclose differing fluxes. In $^3$He-A, the superfluid velocity plays the role of the magnetic vector potential, and the distributed vorticity of the superflow plays the role of the magnetic field. A quantization offset, the analogue of a threading flux, requires a nontrivial $\Omega(C_j)$, which can be maintained, e.g., by a surface or an external field that induces an appropriate texture in $l$.

Illustrative settings: We now illustrate the general ideas presented above by discussing two novel and experimentally feasible settings in which offsets may be observed. We note that several authors have considered a variety of textures for $^3$He-A, with and without singularities; see, e.g., Refs. \[3-8\]. (Equilibrium implications of geometric phases in the mesoscopic context have been discussed in Ref. \[24\].) In the first setting, a sample of $^3$He-A is contained in the cavity between a pair of coaxial cylinders. A static, uniform magnetic field $B_m$ is applied to stabilize the $A_1$-phase (see \[3\]). In this phase, the spin-orbit interaction \[3\] tends to align (or anti-align) $\mathbf{f}$ and $\mathbf{l}$. If $\mathbf{l}$ is strongly anchored by the surface, as it would be in a sufficiently narrow cavity, then for $B_m = B_{m0}$ we have $\mathbf{f} = \mathbf{e}_\varphi \cos \beta - \mathbf{e}_z \sin \beta$, where $\tan \beta \equiv B_m/B_{so}$ and $B_{so} \approx 2.8$ mT is the magnitude of the effective magnetic field \[3\] due to the spin-orbit interaction \[27\]. The solid angle subtended by $\mathbf{f}$ is then given by $\Omega_f = 2\pi(1 - \sin \beta)$. This solid angle and, correspondingly, the persistent current, can be tuned by varying of $B_m$. By tilting $B_m$ away from the cylinder axis, $\Omega_f$ is reduced. In the present setting, the sense of the current is determined by the magnetic field, which explicitly breaks time-reversal symmetry. In the first setting, by contrast, time-reversal symmetry is spontaneously broken (i.e., $\mathbf{l}$ may point inward or outward), the sense of the current being determined by the exhibited choice. Spin textures may also be controlled via the interplay between $B_m$ and a second field (produced, e.g., by a axial current-carrying wire) that is inhomogeneous on the length-scale of the sample.

Flux-quantization: We now turn to implications of geometric phases in the context of flux-quantization in superconductors with nonscalar order parameters. In a

\[\Omega(C_j) = 2\pi,\] the axial component of net relative orbital angular momentum vanishes, and $\kappa = 0$. Now imagine varying $\alpha$ so that the cylinder becomes a cone, all the while maintaining cylindrical symmetry. Then, no axial torque acts during this variation, and the increase in the axial component of the net relative orbital angular momentum is compensated by the appearance of center-of-mass orbital angular momentum and, hence, circulation.

We remark that although the truncated cone and the cylinder are topologically equivalent they are, of course, geometrically inequivalent, this inequivalence being characterized by the vertical angle $\alpha$. We hope that the complication arising from the truncations of the cones will, at least in the case of long cones, not be too severe.

The second setting concerns a magnetic-field–induced spin-texture in the $A_1$ phase of superfluid $^3$He. As this phase is spin-polarized, so that only spin-projection +1 Cooper pairs form the condensate, mass supercurrents imply spin supercurrents. The parameter order matrix has the form $d_{ij} = \frac{1}{2} \Delta_{\parallel j} (d_{l} + im_{y}) (m_{y} + in_{y}) e^{i\chi}$. Here, $\Delta_{\parallel j}$ is the (real) gap parameter, \{$f,d,e$\} is an orthonormal triad of vectors associated with the spin angular momentum sector, and $\chi$ is a phase. In the $A_1$ phase, the orbital and spin sectors are identical in structure, and the superfluid velocity $v^l$ is given by

\[v^l = (h/2M) \{ \text{Im} \left( t^j_j \nabla_r t^j_l + t^j_l \nabla_r t^j_j + \nabla_r \chi \right) \}, \]

where $\tau \equiv (d + ie)/\sqrt{2}$. The result for $\kappa$ then reads:

\[\kappa = (h/2M) \{ \Omega_l(C_l) + \Omega_f(C_f) + 2\pi\tau \}. \]

Both $l$- and $f$-textures may lead to the offset of $\kappa$ and to a persistent equilibrium current. Consider a sample contained in the cavity between a pair of coaxial cylinders. A static, uniform magnetic field $B_m$ is applied to stabilize the $A_1$-phase (see \[3\]). In this phase, the spin-orbit interaction \[3\] tends to align (or anti-align) $\mathbf{f}$ and $\mathbf{l}$. If $\mathbf{l}$ is strongly anchored by the surface, as it would be in a sufficiently narrow cavity, then for $B_m = B_{m0}$ we have $\mathbf{f} = \mathbf{e}_\varphi \cos \beta - \mathbf{e}_z \sin \beta$, where $\tan \beta \equiv B_m/B_{so}$ and $B_{so} \approx 2.8$ mT is the magnitude of the effective magnetic field \[3\] due to the spin-orbit interaction \[27\]. The solid angle subtended by $\mathbf{f}$ is then given by $\Omega_f = 2\pi(1 - \sin \beta)$. This solid angle and, correspondingly, the persistent current, can be tuned by varying of $B_m$. By tilting $B_m$ away from the cylinder axis, $\Omega_f$ is reduced. In the present setting, the sense of the current is determined by the magnetic field, which explicitly breaks time-reversal symmetry. In the first setting, by contrast, time-reversal symmetry is spontaneously broken (i.e., $\mathbf{l}$ may point inward or outward), the sense of the current being determined by the exhibited choice. Spin textures may also be controlled via the interplay between $B_m$ and a second field (produced, e.g., by a axial current-carrying wire) that is inhomogeneous on the length-scale of the sample.

Flux-quantization: We now turn to implications of geometric phases in the context of flux-quantization in superconductors with nonscalar order parameters. In a
superconductor that exhibits a spin-triplet pairing state \[28\] and a corresponding order parameter matrix \(\Delta_{\mu j}\) the current is given by
\[
    j = \left(2e\hbar/M\right) \text{Im} \Delta^*_{\mu j} \left(-i\nabla + 2eA/h\right) \Delta_{\mu j},
\]
where \(e\) is the electronic charge. Following Ref. \[2\], we consider a ring of this superconductor, integrate over a path deep inside the ring (where the current is zero), and obtain the result that the enclosed magnetic flux is quantized with an offset. Similarly, we anticipate a phase-shift in Little-Parks oscillations \[6\]. Moreover, it would be interesting to test these ideas in the context of high-temperature superconductivity. Related effects may occur in the context of neutron star physics (see, e.g., Ref. \[7\], sec. 6.2.5).

**Conclusions:** We have considered implications of geometric phases in the context of nonscalar superfluidity and superconductivity. We have shown that the geometry of the order parameter can have fascinating quantum-mechanical ramifications, especially in the realm of circulation- and flux-quantization. In particular, we have shown that, in the absence of distributed vorticity, the possible values of the circulation are determined solely by the anholonomy in the parallel transport of the order-parameter triad along the circulation-path. We have also discussed certain experimental settings in which the effects proposed here might be observed. Thus, we have demonstrated that, even when rotational, superfluid in \(^3\)He-A is strikingly different from that in \(^3\)He and \(^4\)He. The present work may have implications for the experiments reported in Ref. \[23\].

It is a pleasure to thank I. Aleiner, J. Davis, D. Os-heroff, M. Zapotocky and, especially, A. J. Leggett and M. Stone for useful discussions. This work was supported by NSF via ECS91-08300 (YLG), DMR91-57018 and DMR94-24511 (PMG) and NSERC Canada (DL).

---

* E-mail address: lyanda@ceg.uiuc.edu† E-mail address: goldbart@uiuc.edu‡ E-mail address: dloss@sfu.ca

[1] F. London, Phys. Rev. 74, 562 (1948); R. Doll, M. Näbauer, Phys. Rev. Lett. 7, 51 (1961); B. Deaver, W. Fairbank, ibid., 7, 43 (1961).

[2] N. Byers, C. N. Yang, Phys. Rev. Lett. 7, 46 (1961).

[3] A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

[4] Helium Three, edited by W. P. Halperin, L. P. Pitaevskii (North-Holland, Amsterdam, 1990).

[5] D. Vollhardt, P. Wölfle, The Superfluid Phases of Helium Three (Taylor and Francis, London, 1990).

[6] Y. Aharonov, D. Bohm, Phys. Rev. 115, 485 (1959).

[7] For a review, see A. Shapere, F. Wilczek, Geometric Phases in Physics (World Scientific, Singapore, 1989).

[8] W. A. Little, R. D. Parks, Phys. Rev. Lett. 9, 9 (1962).

[9] M. V. Berry, Proc. R. Soc. Lon., Ser. A 392, 45 (1984).

[10] See M. V. Berry, in Ref. \[1\], p. 8 et seq.

[11] G. E. Volovik, Exotic Properties of Superfluid \(^3\)He (World Scientific, Singapore, 1992).

[12] One may say that at each point in the base space (the I-sphere) there is a fiber of orientations of the pair \([\mathbf{m}, \mathbf{n}]\) in the plane tangent to the point I. Together, the orientations and the I-sphere make a frame-bundle. See B. Simon, Phys. Rev. Lett. 51, 2167 (1983), reprinted in Ref. \[3\]; and, e.g., Ref. \[4\], Chap. 3.

[13] For other implications of geometric phases in superfluid \(^4\)He, see, e.g., W. E. Goff, F. Gaitan, M. Stone, Phys. Lett. A136, 433 (1989); A. Garg et al., Ann. Phys. (NY) 173, 149 (1987). The Aharonov-Casher effect is considered in A. Balatsky, B. L. Altshuler, Phys. Rev. Lett. 70, 1678 (1993).

[14] The superfluid current density in \(^3\)He-A contains two terms proportional to \(\nabla \times I\) as well as the term determined by the superfluid velocity \(v^{(s)}\). If \(\nabla \times I = 0\) then the superfluid current density is due solely to \(v^{(s)}\).

[15] See, e.g., J. R. Schrieffer, *Theory of Superconductivity* (Addison-Wesley, 1983), pp. 10-17.

[16] T.-L. Ho, Phys. Rev. B 18, 1144 (1978).

[17] H. E. Hall, J. R. Hook, *Prog. in Low Temp. Physics*, Vol. 9, D. Brewer (ed.) (North-Holland, 1986) p. 143.

[18] N. D. Mermin, T.-L. Ho, Phys. Rev. Lett. 36, 594 (1976).

[19] Superfluid quantized with an offset may be viewed as a macroscopic analog of the persistent current caused by magnetic flux in a mesoscopic metallic ring.

[20] B. L. Altshuler, A. G. Aronov, B. Z. Spivak, Zh. Eksp. Teor. Fiz. 33, 101 (1981) [JETP Lett. 33, 94 (1981)].

[21] V. Ambegaokar et al., Phys. Rev. A 9, 2676 (1974).

[22] P. Anderson, W. Brinkman, in *Helium Liquids*, J. Armitage, I. Farquhar (eds.), (Academic Press, London, 1974), p. 315; P. Anderson, G. Toulouse, Phys. Rev. Lett. 38, 508 (1977); A. Fetter, Phys. Rev. B 15, 1350 (1977); K. Maki, X. Zotos, Phys. Rev. B 31, 177 (1985); P. Muzikar, J. Low Temp. Phys. 46, 533 (1982).

[23] T. Arai, T. Soda, Prog. Theor. Phys. 69, 699 (1983).

[24] D. Loss et al., Phys. Rev. Lett. 65, 1655 (1990); D. Loss, P. M. Goldbart, Phys. Rev. B 45, 13544 (1992).

[25] For more general textures (e.g., for generalized cones), i.e., surfaces constructed by sweeping a straight generatrix fixed at one point around a closed path) the geometric phase viewpoint provides an efficient computational tool.

[26] Alternatively, there is a degenerate texture in which \(1\) points from the inner to the outer cone. For narrow cavities, \(I\) is perpendicular to the surfaces throughout the cavity. The favorability of similar textures between concentric cylinders has been considered in Ref. \[23\]. Excited states containing domain walls between textures with opposite \(I\) are expected to be rather costly in energy.

[27] Spin-orbit geometric phases in mesoscopic rings are discussed in A. Aronov, Y. Lyanda-Geller, Phys. Rev. Lett. 70, 343 (1993); Y. Lyanda-Geller, ibid., 71, 657 (1993).

[28] In the cubic heavy-fermion superconductors (such as \(U\)Be\(_{13}\)) an order parameter triad is symmetry-allowed. See, e.g., L. P. Gor’kov, Sov. Sci. Rev. A Phys. 9, 1-116 (1987), sec. 6.

[29] R. J. Zieve, Yu. Mukharsky, J. D. Close, J. C. Davis, R.
E. Packard, J. Low Temp. Phys. 89, 47 (1992); we thank
J. C. Davis for drawing our attention to this paper.

FIG. 1. a) Concentric truncated cones of vertical angle $\alpha$
with superfluid $^3$He-A in the cavity. The texture is given
by $\chi = 0$ and (for cylindrical polar coordinates $\{\rho, \varphi, z\}$)
$
\{l, m, n\} = \{-e_\rho \cos \alpha + e_z \sin \alpha, e_\rho \sin \alpha + e_z \cos \alpha, e_z\}.
$
The texture has $l$ pointing from outer to inner cone [24]. $C_r$ is a
path along which $\kappa$ is computed. b) $l$-sphere with tangent
plane containing $\{m, n\}$, and $C_l$ is the image of $C_r$. 
