Status and perspectives of sin 2\(\alpha\) measurements

H Sagawa

aKEK, Tsukuba

In the neutral \(B\) meson system, it is possible to measure the CKM angle \(\alpha\) using the decay mode \(b \to u\bar{d}d\) in the presence of pollution from gluonic \(b \to d\) penguin decays. Here the recent status of the measurements of CP-violating asymmetry parameters using time-dependent analyses in \(B^0 \to \pi^+\pi^-\) and \(B^0 \to \rho\pi\) decays and the perspectives of a sin 2\(\alpha\) measurement are presented.

1 Introduction

In 1973, Kobayashi and Maskawa (KM) proposed a model where CP violation is accommodated as an irreducible complex phase in the weak-interaction quark mixing matrix [1]. Recent measurements of the CP-violating asymmetry parameters \(\sin 2\beta = \sin 2\phi_1\) [2] by the Belle [3] and BaBar [4] Collaborations established CP violation in the neutral B meson system. Measurements of other CP-violating asymmetry parameters provide important tests of the KM model. Any mode with a contribution from \(b \to u\bar{d}d\) is a possible source of measurement of the Cabibbo-Kobayashi-Maskawa (CKM) angle \(\alpha\) (= \(\phi_2\)). Here the recent status of the measurements of CP-violating asymmetry parameters using time-dependent analyses in \(B^0 \to \pi^+\pi^-\) and \(B^0 \to \rho\pi\) decays [5] and the perspectives of a sin 2\(\alpha\) measurement are presented.

2 \(B^0 \to \pi^+\pi^-\) decays

The \(B^0 \to \pi^+\pi^-\) decay is one of the important modes for the measurement of sin 2\(\alpha\). The KM model predicts CP-violating asymmetries in the time-dependent rates for \(B^0\) and \(\bar{B}^0\) decays to a common CP eigenstate, \(f_{\text{CP}}\). In the decay chain \(\Upsilon(4S) \to B^0\bar{B}^0 \to f_{\text{CP}}f_{\text{tag}},\) in which one of the \(B\) mesons decays at time \(t_{\text{CP}}\) to \(f_{\text{CP}}\) and the other decays at time \(t_{\text{tag}}\) to a final state \(f_{\text{tag}}\) that distinguishes between \(B^0\) and \(\bar{B}^0\), the \(B^0 \to \pi^+\pi^-\) decay rate has a time-dependence given by

\[
\tau_{\pi\pi}^d(\Delta t) = \frac{e^{-\Delta m_d|\Delta t|}}{4\tau_{B^0}^2} \left[ 1 + q \cdot |S_{\pi\pi} \sin(\Delta m_d \Delta t) - C_{\pi\pi} \cos(\Delta m_d \Delta t)| \right],
\]

where \(\tau_{B^0}\) is the \(B^0\) lifetime, \(\Delta m_d\) is the mass difference between the two \(B^0\) mass eigenstates, \(\Delta t = t_{\text{CP}} - t_{\text{tag}},\) and the \(b\)-flavor charge \(q = +(-1)\) when tagging the \(B\) meson is a \(B^0(\bar{B}^0)\). The CP-violating asymmetry parameters \(S_{\pi\pi}\) and \(C_{\pi\pi}\) defined in Eq. (1) are expressed as

\[
S_{\pi\pi} = \frac{1 - |\lambda_{xx}|^2}{1 + |\lambda_{xx}|^2}, \quad C_{\pi\pi} = \frac{2 |\lambda_{xx}|}{1 + |\lambda_{xx}|^2},
\]

where \(\lambda_{xx}\) is a complex parameter that depends on both \(B^0-\bar{B}^0\) mixing and on the amplitudes for \(B^0\) and \(\bar{B}^0\) decay to \(\pi^+\pi^-\). A measurement of CP-violating asymmetries in the mode \(B^0 \to \pi^+\pi^-\) is sensitive to direct CP violation and the CKM angle \(\alpha\). If the decay proceeded only via a \(b \to u\) tree amplitude, \(S_{\pi\pi} = \sin 2\alpha\) and \(C_{\pi\pi} = 0\), or equivalently \(|\lambda_{xx}| = 1\). The situation is complicated by the possibility of significant contributions from gluonic \(b \to d\) penguin amplitudes that have a different weak phase and additional strong phases. In general, \(S_{\pi\pi}\) is given by \(\sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{\text{eff}}\). Here \(\alpha_{\text{eff}} - \alpha\) depends on the magnitudes and relative weak and strong phases of the tree and penguin amplitudes. As a result, \(S_{\pi\pi}\) may not be equal to sin 2\(\alpha\) and direct CP violation, \(C_{\pi\pi} \neq 0\), may occur.

Candidate \(B\) mesons are reconstructed kinematically using two variables, the energy difference \(\Delta E \equiv E_{\text{beam}}^{\text{cms}} - E_{\text{beam}}^{\text{iso}}\) and the beam-energy constrained mass \(M_{\text{BC}} \equiv \sqrt{(E_{\text{beam}}^{\text{cms}})^2 - (p_{\text{beam}}^{\text{iso}})^2}\) [6], where \(E_{\text{beam}}^{\text{cms}}\) is the cms beam energy, and \(E_{\text{beam}}^{\text{iso}}\) and \(p_{\text{beam}}^{\text{iso}}\) are the cms energy and momentum of the \(B\) candidate.

Charged tracks in \(B^0 \to h^+h^-\) candidates are identified as charged pions or kaons. Here \(h^+\) and \(h^-\) represent a pion or a K. The Belle Collaboration uses the likelihood ratio (KID) for a particle to be a \(K^\pm\) meson, which is based on the combined information from the Aerogel Cherenkov counter and CDC \(dE/dx\) measurement. Tracks are positively identified as pions with KID<0.4 for \(B^0 \to \pi^+\pi^-\) candidates. The BaBar Collaboration uses the Cherenkov angle measurement \(\theta_c\) from a detector of internally reflected Cherenkov light. The probability density function (PDF) from the difference between measured and expected values of \(\theta_c\) is used in the extended likelihood function for the fit to extract yields and CP parameters.

Background from the process \(e^+e^- \to q\bar{q}\) continuum (\(q = u, d, s, c\)) is suppressed by their event topology. The Belle Collaboration forms signal and background likelihood functions \(L_S\) and \(L_{BG}\) from a Fisher discriminant determined from six modified Fox-Wolfram moments [7] and the cms \(B\) flight direction with respect to the beam axis. The continuum background is reduced by impos-
ing requirements on the likelihood ratio \( LR = L_S/(L_S + L_{BG}) \) for different flavor-tagging dilution factor intervals. The BaBar Collaboration uses the angle \( \theta_R \) between the sphericity axis of the \( B \) candidate and the sphericity axis of the remaining particles in the cms frame, and cut on \( |\cos \theta_R| \). The shapes of Fisher discriminant \( F \) for signal and background events are included as PDFs in the maximum likelihood fit.

Leptons, kaons, and charged pions that are not associated with the reconstructed \( B \) candidate are used to identify the flavor of the accompanying \( B \) meson.

The vertex reconstruction algorithm is the same as that used for the \( \sin 2\beta \) ( \( \sin 2\theta_W \) ) analysis. The time difference \( \Delta t \) is obtained from the measured distance between the \( z \) positions along the beam direction of the \( B \) candidate and the sphericity axis of the \( B \) candidate are used to identify the charmless three-body \( B \) decay vertices and the boost factor \( \beta \gamma \) of the \( e^+e^- \) system.

Fig. 1 and Fig. 2 show distributions of \( \Delta E \) for events enhanced in signal \( \pi^+\pi^- \) and \( K^+\pi^- \) decays from the Belle Collaboration [10] and the BaBar Collaboration [11], respectively. The Belle and BaBar Collaborations obtained the following results using an unbinned maximum likelihood fit based on \( 85 \times 10^6 \) and \( 88 \times 10^6 B\bar{B} \) pairs, respectively:

\[
\begin{align*}
C_{\pi\pi} &= -0.77 \pm 0.27 \pm 0.08, \quad S_{\pi\pi} = -1.23 \pm 0.41^{+0.08}_{-0.07} \\
&\quad \text{(Belle)} \\
C_{\pi\pi} &= -0.30 \pm 0.25 \pm 0.04, \quad S_{\pi\pi} = -0.02 \pm 0.34 \pm 0.05 \\
&\quad \text{(BaBar)}
\end{align*}
\]

The first and the second errors correspond to statistical and systematic errors, respectively, unless otherwise stated. The average values of \( C_{\pi\pi} \) and \( S_{\pi\pi} \) are

\[
C_{\pi\pi} = -0.49 \pm 0.19, \quad S_{\pi\pi} = -0.47 \pm 0.26,
\]

and the difference of the results between the Belle and BaBar Collaborations is 2.2\( \sigma \) [12]. In Fig. 3 and Fig. 4 the \( \Delta t \) distributions for events enhanced in signal \( B^0 \to \pi^+\pi^- \) decays are shown for the Belle Collaboration [10] and the BaBar Collaborations [11], respectively.

Fig. 5 shows the two-dimensional confidence regions in the \( \mathcal{A}_{\pi\pi} = -C_{\pi\pi} \) vs. \( S_{\pi\pi} \) plane using the Feldman-Cousins frequentist approach [13]. In order to form confidence intervals, the \( \mathcal{A}_{\pi\pi} \) and \( S_{\pi\pi} \) distributions of the results of fits to MC pseudo-experiments for various input values of \( \mathcal{A}_{\pi\pi} \) and \( S_{\pi\pi} \) are used for the Belle result, and the obtained errors of \( \mathcal{A}_{\pi\pi} \) and \( S_{\pi\pi} \) are used for the BaBar result. The case that \( CP \) symmetry is conserved, \( \mathcal{A}_{\pi\pi} = S_{\pi\pi} = 0 \), is ruled out at the 99.93\% confidence level (C.L.), equivalent to 3.4\( \sigma \) significance for Gaussian errors from the Belle result. More data is necessary to clarify the difference between the Belle result and the BaBar result.
Figure 3. The raw, unweighted $\Delta t$ distributions for $B^0 \rightarrow \pi^+\pi^-$ candidates with $LR > 0.825$ in the signal region from the Belle Collaboration: (a) candidates with $q = +1$, i.e. the tag side is identified as $B^0$; (b) candidates with $q = -1$; (c) $B^0 \rightarrow \pi^+\pi^-$ yields after background subtraction; (d) the CP asymmetry for $B^0 \rightarrow \pi^+\pi^-$ after background subtraction. In Figs. (a) through (c), the solid curves show the results of the unbinned maximum likelihood fit to the $\Delta t$ distributions of the whole $B^0 \rightarrow \pi^+\pi^-$ candidates. In Fig. (d), the dashed (dotted) curve is the contribution from the cosine (sine) term.

The decay amplitudes for $B^0$ and $\bar{B}^0$ to $\pi^+\pi^-$ can be written by using the $c$-convention notation \cite{12}:

$$A(B^0 \rightarrow \pi^+\pi^-) = -(|T|e^{i\delta_T} + |P|e^{i\delta_P})$$

$$A(\bar{B}^0 \rightarrow \pi^+\pi^-) = -(|T|e^{i\delta_T} e^{-i\delta_P} + |P|e^{i\delta_P})$$

where $T$ and $P$ are the amplitudes for the tree and penguin graphs and $\delta_T$ and $\delta_P$ are their strong phases. The expressions for $S_{\pi\pi}$ and $A_{\pi\pi}$ are

$$S_{\pi\pi} = [\sin 2\phi_2 + 2|P/T| \sin(\phi_1 - \phi_2) \cos \delta - |P/T|^2 \sin 2\phi_1]/R_{\pi\pi}$$

$$A_{\pi\pi} = [-2|P/T| \sin(\phi_2 + \phi_1) \sin \delta]/R_{\pi\pi}$$

$$R_{\pi\pi} = 1 - 2|P/T| \cos \delta \cos(\phi_2 + \phi_1) + |P/T|^2$$

where the strong phase difference $\delta \equiv \delta_P - \delta_T$. When $A_{\pi\pi}$ is positive and $0^\circ < \phi_1 + \phi_2 < 180^\circ$, $\delta$ is negative. Fig. 6 shows the two-dimensional confidence regions in the $A_{\pi\pi}$ vs. $S_{\pi\pi}$ plane together with the pQCD prediction \cite{15} for various values of $\phi_2$ ($= \alpha$). Fig. 7 shows predictions for $C_{\pi\pi}$ ($= -A_{\pi\pi}$) and $S_{\pi\pi}$ for several analysis steps with experimental and theoretical constraints \cite{16}. In Fig. 8, the interpretation of the confidence regions of $A_{\pi\pi}$ vs. $S_{\pi\pi}$ is shown in terms of $\phi_2$ and $\delta$ for the Belle data \cite{16}. The range of $\phi_2$ that corresponds to the 95.5% C.L. region of $A_{\pi\pi}$ and $S_{\pi\pi}$ is $78^\circ \leq \phi_2 \leq 152^\circ$ for $\phi_1 = 23.5^\circ$ \cite{17} and $0.15 \leq |P/T| \leq 0.45$ \cite{18}. The result is in agreement with...
constraints on the unitarity triangle from other measurements [19]. Other interpretations for the current results can be found in ref. [16].

![Figure 5](image1.png)

**Figure 5.** Confidence regions for $A_{\pi\pi}$ and $S_{\pi\pi}$ from the Belle and BaBar results.

![Figure 6](image2.png)

**Figure 6.** Plot of $A_{\pi\pi}$ versus $S_{\pi\pi}$ for various values of $\phi_2$ with $\phi_1=24.0^\circ$, $0.18 < R_C < 0.30$, and $-41^\circ < \delta < -32^\circ$ in the pQCD method. Here $R_C = |P/T|$. Dark areas are allowed regions in the pQCD method for different $\phi_2$ values. The results of $A_{\pi\pi}$ and $S_{\pi\pi}$ from the Belle and BaBar Collaborations and the confidence regions from the Belle Collaboration are also shown.

![Figure 7](image3.png)

**Figure 7.** Predictions for $C_{ee} (=-A_{ee})$ and $S_{ee}$ for several analysis steps with experimental and theoretical constraints. The Belle and BaBar results are shown.

![Figure 8](image4.png)

**Figure 8.** The region for $\phi_2$ and $\delta$ which corresponds to the 68.3%, 95.5%, and 99.73% C.L. regions of $A_{\pi\pi}$ and $S_{\pi\pi}$ from the Belle result in Fig. 5. $\phi_1 = 23.5^\circ$ and $|P/T| = 0.45$. The horizontal dashed line corresponds to $\phi_2 = 180^\circ - \phi_1$. 
Using isospin relations [20], we can constrain the difference, $\theta$ between $\alpha_{eff}$ and $\alpha$. From the central values of the recent world average values of the branching ratios of $B^0 \rightarrow \pi^+ \pi^-$, $B^+ \rightarrow \pi^+ \pi^0$ and the 90% C.L. upper limit on the $B^0 \rightarrow \pi^0 \pi^0$ branching ratio [21] together with $C_{||}$, the upper limit on $\theta$ is 54°.

### 3 $B^0 \rightarrow \rho \pi \rightarrow \pi^+ \pi^- \pi^0$ decays

In principle, the CKM angle $\alpha$ can be measured in the presence of penguin contributions using a full Dalitz plot analysis of the final state. However, there are difficulties of combinatorics and lower efficiency in three-body topology with $\pi^0$ and large backgrounds from misreconstructed signal events and other decays. In order to extract $\alpha$ cleanly, data with large statistics are required.

Unlike $B^0 \rightarrow \pi^+ \pi^-$ decay, $B^0 \rightarrow \rho^+ \rho^-$ decay is not a $CP$ eigenstate, and four flavor-charge configurations ($B^0(\bar{B}^0) \rightarrow \rho^+ \rho^-$) must be considered. Following a quasi-two-body approach [22], the analysis is restricted to the two regions of the $\pi^0 \pi^0 h^0$ Dalitz plot ($h = \pi$ or $K$) that are dominated by $\rho^+ h^0$. The decay rate is given by

$$f_{\rho^+ h^0} (\Delta t) = (1 \pm A_{\rho h}^0) \frac{e^{-|\Delta t|/\tau_{\rho h}}}{4\tau_{\rho h}} \times [1 + q(S_{rho} + \Delta S_{rho}) \sin(\Delta m_d \Delta t) - q(C_{rho} - \Delta C_{rho}) \cos(\Delta m_d \Delta t)],$$

(5)

where $\Delta t = t_{\rho h} - t_{tag}$ as the time interval between the decay of $B_{\rho h}^0$ and that of the other $B^0$ meson in the event, $B_{\rho h}^0$ tag.

The time- and flavor-integrated charge asymmetries $A_{\rho h}^0$ and $A_{\rho h}^0$ measure direct $CP$ violation. For the $\rho \pi$ mode, the quantities $S_{\rho h}$ and $C_{\rho h}$ parameterize mixing-induced $CP$ violation related to the CKM angle $\alpha$, and flavor-dependent direct $CP$ violation, respectively. $\Delta C_{\rho h}$ describes the asymmetry between the rates $\Gamma(B^0 \rightarrow \rho^- \rho^+) + \Gamma(B^0 \rightarrow \rho^+ \rho^-)$ and $\Gamma(B^0 \rightarrow \rho^+ \rho^-) + \Gamma(B^{0*} \rightarrow \rho^- \rho^+)$). $\Delta S_{\rho h}$ is related to the strong phase difference between the amplitudes contributing to $B^0 \rightarrow \rho \pi$ decays. One finds the relations $S_{\rho h} = \Delta S_{\rho h} \sqrt{1 - (C_{\rho h} + \Delta C_{\rho h})^2} \sin(2\alpha_{eff} \pm \delta)$, where $2\alpha_{eff} = \arg(q/p) + \arg(A_{\rho h}^0)$, $\delta = \arg(A_{\rho h}^0)$, $q/p$ is the $B^0(\bar{B}^0)$ mixing phase, and $A_{\rho h}^0$ and $A_{\rho h}^0$ are the transition amplitudes of the processes $B^0(\bar{B}^0) \rightarrow \rho^+ \pi^- + B^{0*} \rightarrow \rho^- \pi^+$, respectively. The angles $\alpha_{eff}$ are equal to $\alpha$ if contributions from penguin amplitudes are absent. For the self-tagging $\rho$K mode, the values of the four time-dependent parameters are $C_{\rho K} = 0$, $C_{\rho K} = -1$, $S_{\rho K} = 0$, and $\Delta S_{\rho K} = 0$. The results on direct $CP$ violation can be expressed using the asymmetries

$$A_{+-} = \frac{N(B_{\rho h}^0 \rightarrow \rho^- \pi^+) - N(B_{\rho h}^0 \rightarrow \rho^+ \pi^-)}{N(B_{\rho h}^0 \rightarrow \rho^- \pi^+) + N(B_{\rho h}^0 \rightarrow \rho^+ \pi^-)} = \frac{A_{\rho h}^0 - C_{\rho h} - \Delta C_{\rho h}}{1 - \Delta C_{\rho h} - A_{\rho h}^0 - \Delta C_{\rho h}}$$

(6)

$$A_{+} = \frac{N(B_{\rho h}^0 \rightarrow \rho^- \pi^+) - N(B_{\rho h}^0 \rightarrow \rho^+ \pi^-)}{N(B_{\rho h}^0 \rightarrow \rho^- \pi^+) + N(B_{\rho h}^0 \rightarrow \rho^+ \pi^-)} = \frac{A_{\rho h}^0 + C_{\rho h} + \Delta C_{\rho h}}{1 + \Delta C_{\rho h} + A_{\rho h}^0 + \Delta C_{\rho h}}$$

(7)

With a data sample of 89 million $B\bar{B}$ pairs [23], the BaBar Collaboration found $428^{+34}_{-29}(stat) \rho \pi$ and $120^{+20}_{-20}(stat) \rho K$ events and the following measurements of the $CP$ violation parameters are obtained:

$$A_{\rho h}^0 = -0.18 \pm 0.08 \pm 0.03,$$

$$C_{\rho h} = +0.36 \pm 0.18 \pm 0.04,$$

$$S_{\rho h} = +0.19 \pm 0.24 \pm 0.03.$$

For the other parameters in the description of the $B^0(\bar{B}^0) \rightarrow \rho \pi$ decay-time dependence,

$$\Delta C_{\rho h} = +0.28 \pm 0.19 \pm 0.04,$$

$$\Delta S_{\rho h} = +0.15 \pm 0.25 \pm 0.03.$$
are found. For the asymmetries $A_+$ and $A_-$, which probe direct $CP$ violation,

$$A_+ = -0.62^{+0.24}_{-0.28} \pm 0.06, \quad A_- = -0.11^{+0.16}_{-0.17} \pm 0.04,$$

are measured. The raw time-dependent asymmetry in the tagging categories dominated by kaons and leptons is shown in Fig. 19.

## 4 Prospects

Table 1 shows the expected error on $CP$-violating parameters in $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ decays with accumulated luminosities of 140 fb$^{-1}$, 400 fb$^{-1}$, 3000 fb$^{-1}$ (3 ab$^{-1}$), and 30000 fb$^{-1}$ (30 ab$^{-1}$) in the future.

Fig. 10 shows the prospects of $\alpha_{\text{eff}} - \alpha$ for the ICHEP’02 central values of the $B \to \pi\pi$ branching ratios and the central values of $S_{\pi\pi}$ and $C_{\pi\pi}$ from the BaBar measurement at luminosities of the current and future B-factories (87 fb$^{-1}$, 500 fb$^{-1}$, 2 ab$^{-1}$, 10 ab$^{-1}$). The inner and outer borders can be obtained from the isospin analysis when $B^0 \to \pi^0\pi^0$ flavors are tagged and not tagged, respectively. Only a luminosity of around 10 ab$^{-1}$ allows to separate the solutions.

For $B \to \rho\pi$ decays, the isospin analysis is not feasible yet with the present statistics of the $B$ factories. In [24], the projections into the future full SU(2) analysis was demonstrated. If the branching fraction of $B^0 \to \rho^0\rho^0$ is below the experimental sensitivity, a strong constraint on $\alpha$ is expected above luminosity of around 2 ab$^{-1}$. In this workshop, theoretical problems such as form factors and $\sigma$ meson [25], and experimental problems for several sources of backgrounds were pointed out.

Detailed discussions can be found in [27] for $B^0 \to \pi^+\pi^-$ and in [24] for $B^0 \to \rho\pi$ at this workshop.

| parameters | 140 fb$^{-1}$ | 400 fb$^{-1}$ | 3 ab$^{-1}$ | 30 ab$^{-1}$ |
|------------|--------------|--------------|-------------|-------------|
| $\mathcal{R}_{\pi\pi}$ | 0.21 | 0.13 | 0.05 | 0.02 |
| $S_{\pi\pi}$ | 0.31 | 0.19 | 0.07 | 0.03 |
| $A_{CP}^{\pi\pi}$ | 0.07 | 0.04 | 0.02 | 0.008 |
| $C_{\rho\pi}$ | 0.14 | 0.09 | 0.03 | 0.013 |
| $S_{\rho\pi}$ | 0.18 | 0.11 | 0.04 | 0.014 |

Table 1. The errors of $CP$-violating parameters in $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ decays at 140 fb$^{-1}$, 400 fb$^{-1}$, 3 ab$^{-1}$, and 30 ab$^{-1}$, assuming that statistical and systematic errors are proportional to $1/\sqrt{L}$ and $1/\sqrt{2L}$, respectively. Here L is accumulated luminosity.

## 5 Summary

In summary, the Belle and BaBar Collaborations obtain the following measurements of the $CP$-violating asymmetry parameters in $B^0 \to \pi^+\pi^-$ decays:

$$\mathcal{R}_{\pi\pi} = +0.77 \pm 0.27 \pm 0.08, \quad S_{\pi\pi} = -1.23 \pm 0.41^{+0.08}_{-0.07}$$

(Belle),

$$\mathcal{R}_{\pi\pi} = +0.30 \pm 0.25 \pm 0.04, \quad S_{\pi\pi} = -0.02 \pm 0.34 \pm 0.05$$

(BaBar),

The following measurements of the $CP$-violating asymmetry parameters in $B^0 \to \rho\pi$ decays using a quasi two-body analysis are obtained by the BaBar Collaboration:

$$A_{CP}^{\pi\pi} = -0.18 \pm 0.08 \pm 0.03,$$

$$C_{\rho\pi} = +0.36 \pm 0.18 \pm 0.04, \quad \Delta C_{\rho\pi} = +0.28 \pm 0.19 \pm 0.04,$$

$$S_{\rho\pi} = +0.19 \pm 0.24 \pm 0.03, \quad \Delta S_{\rho\pi} = +0.15 \pm 0.25 \pm 0.03.$$  

For the asymmetries $A_+$ and $A_-$, which probe direct $CP$ violation,

$$A_+ = -0.62^{+0.24}_{-0.28} \pm 0.06, \quad A_- = -0.11^{+0.16}_{-0.17} \pm 0.04,$$

are obtained.

## References

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3. Belle Collaboration, K. Abe et al., Phys. Rev. D 66, 071102 (2002).
4. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 89, 218002 (2002).
5. Throughout this paper, the inclusion of the charge conjugate mode decay is implied unless otherwise stated.
6. \( C_{\text{re}} = -\mathcal{A}_{\text{re}}. \) The BaBar Collaboration uses \( C_{\text{re}} \) and the Belle Collaboration uses \( \mathcal{A}_{\text{re}}. \) Usually the partial-rate asymmetry or direct CP violation parameter is defined by \( |N(B \rightarrow f) - N(B \rightarrow f)\rangle|/|N(B \rightarrow f) + N(B \rightarrow f)\rangle|. \) Its sign is consistent with the sign of \( \mathcal{A}_{\text{re}}. \) Here \( B \) represents either a \( B^0 \) or \( B^* \) meson, \( f \) represents a flavor-specific final state, and \( \bar{B}^0 \) and \( f \) are their conjugates.
7. The Belle Collaboration uses the beam-energy constrained mass \( M_{\text{bc}}. \) The BaBar Collaboration uses the beam-energy substituted mass \( m_{\text{RS}} = \sqrt{(s/2 + p_B \cdot p_B)/E_s^2 - p_B^2}, \) where \( \sqrt{s} \) is the total cms energy, and the \( B \) momentum \( p_B \) and the four-momentum of the initial state \( (E_s, p_i) \) are defined in the laboratory frame.
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