QUASI-LOCAL EVOLUTION OF THE COSMIC GRAVITATIONAL CLUSTERING IN THE HALO MODEL

LONG-LONG FENG\(^1\,2\) AND LI-ZHI FANG\(^3\)

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ABSTRACT

We show that the nonlinear evolution of the cosmic gravitational clustering is approximately spatial local in the \(x\)-\(k\) (position-scale) phase space if the initial perturbations are Gaussian. That is, if viewing the mass field with modes in the phase space, the nonlinear evolution will cause strong coupling among modes with different scale \(k\), but at the same spatial area \(x\), while the modes at different area \(x\) remain uncorrelated or very weakly correlated. We first study the quasi-local clustering behavior with the halo model and demonstrate that the quasi-local evolution in the phase space is essentially due to the self-similar and hierarchical features of the cosmic gravitational clustering. The scaling of the mass density profile of halos ensures that the coupling between \((x - k)\) modes at different physical positions is substantially suppressed. Using high-resolution \(N\)-body simulation samples in the LCDM model, we justify the quasi locality with the correlation function between the discrete wavelet transform (DWT) variables of the cosmic mass field. Although the mass field underwent a highly nonlinear evolution and the DWT variables display significantly non-Gaussian features, there are almost no correlations among the DWT variables at different spatial positions. Possible applications of the quasi locality have been discussed.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The large-scale structure of the universe arose from initial fluctuations through the nonlinear evolution of gravitational instability. Gravitational interaction is of long range; therefore, the evolution of cosmic clustering is not localized in physical space. The typical processes of cosmic clustering, such as collapsing and falling into potential wells, the Fourier mode-mode coupling, and the merging of previrialized dark halos, are generally nonlocal. These processes lead to a correlation between the density perturbations at different positions, even if the perturbations at those positions initially are statistically uncorrelated. For instance, in the Zel’dovich approximation (Zel’dovich 1970), the density field \(\rho(x,t)\) at (Eulerian) comoving position \(x\) and time \(t\) is determined by the initial perturbation at (Lagrangian) comoving position, \(q\), plus a displacement \(S\):

\[
x(q,t) = q + S(q,t).
\]

The displacement \(S(q,t)\) represents the effect of density perturbations on the trajectories of self-gravitating particles. The intersection of particle trajectories leads to a correlation between mass fields at different spatial positions. Thus, the gravitational clustering is nonlocal even in the weakly nonlinear regime.

On the other hand, spatial locality has been employed in the Gaussianization technique for recovery of the primordial power spectrum (Narayanan & Weinberg 1998). Underlying this algorithm is the assumption that the relation between the evolved mass field and the initial density distribution is local, i.e., the high (low) initial density pixels will be mapped into high (low) density pixels of the evolved field (Narayanan & Weinberg 1998). Obviously, the localized mapping has difficulty reconciling the initially Gaussian field with the coherent nonlinear structures, such as halos with scaling behavior. It has been argued that the locality assumption may be a poor approximation to the actual dynamics because of the nonlocality of gravitational evolution (Monaco & Efstathiou 1999). Nevertheless, the localized mapping is found to work well for reconstructing the initial mass field and power spectrum from transmitted flux of the Ly\(\alpha\) absorption in QSO spectra (Croft et al. 1998). These results, together with the data of WMAP, have been used to determine the cosmological parameters (Spergel et al. 2003). However, the dynamical origin of the locality assumption remains a problem. It is still unclear under which condition the localized mapping is a good approximation.

This problem has been studied in the weakly nonlinear regime under the Zel’dovich approximation. The result showed that the cosmic gravitational clustering evolution is spatially quasi-localized in phase (\(x\)-\(k\)) space made by the discrete wavelet transform (DWT) decomposition (Pando, Feng, & Fang 2001). In this approach, each perturbation mode corresponds to a cell in the \(x\)-\(k\) space, \((x \leftrightarrow \Delta x, k \leftrightarrow \Delta k)\) with \(\Delta x\Delta k = 2\pi\), and the density perturbation of the mode \((x,k)\) is \(\delta(k,x)\). They demonstrated that, in the Zel’dovich approximation, if the initial density perturbations in each cell are statistically uncorrelated, i.e., \(\langle \delta_0(k_1,x_1)\delta_0(k_2,x_2) \rangle \propto \delta_{k_1,k_2}\delta_{x_1,x_2}\) where \(\delta_{k}\) denotes the Kronecker delta function, the evolved \(\delta(k_1,x_1)\) and \(\delta(k_2,x_2)\) will remain approximately spatially uncorrelated always, \(\langle \delta(k_1,x_1)\delta(k_2,x_2) \rangle \propto \delta_{x_1,x_2}\), which is just what we call spatial quasi locality in the dynamics of cosmic clustering. The spatial quasi locality implies a significant local mode-mode coupling (different scales \(k\) at the same position \(x\)) but very weak nonlocal coupling between the modes (different positions \(x\)). The nonlinear dynamical evolution is well developed along the direction of \(k\)-axis rather than \(x\)-axis in the phase space. This quasi locality has been justified in
weak nonlinear samples such as the transmitted flux of the QSO Lyα absorption spectrum (Pando et al. 2001). It places the dynamical base of recovering the initial power spectrum from the corresponding weakly evolved field via a localized mapping in phase space (Feng & Fang 2000).

This purpose of this paper is to extend the concept of quasi locality to the fully nonlinear regime. We try to show that the quasi locality of the cosmic clustering in phase space holds not only in the weak nonlinear regime, but also in nonlinear evolution. Since the nonlinear cosmic density field can be expressed by the semianalytical halo model (e.g., Cooray & Sheth 2002 and references therein), our primary interest is to study whether the quasi locality could be incorporated in the halo model. We first analytically derive the quasi locality from the halo model and then make a numerical test using high-resolution N-body simulation samples.

The outline of this paper is as follows. In § 2 we present the statistical criterion of the quasi-local evolution of a density field in the halo model and then make a numerical test using high-resolution N-body simulation samples. The conclusions and discussions will be given in § 5.

2. QUASI LOCALITY IN x-k SPACE

2.1. DWT Variables of the Mass Field

In physical space (x), the mode is the Dirac delta function δ0(x), and the cosmic mass density field variable is ρ(x), while in scale space (k), the mode is the Fourier bases e−ik·x, and the field variable is ̃ρ(k), which is the Fourier transform of ρ(x). In hybrid x-k phase space, one can use the complete and orthogonal bases of the discrete wavelet transform (DWT) as the mode function. The mass density field is then described by the DWT variables.

Without loss of generality, we introduce the DWT variables by considering a density field ρ(x) in a cubic box of 0 ≤ x ≤ L, i = 1, 2, 3 and volume V = L3. We first divide the box into cells with volume L3/2i+1×2i+1×2i+1, where j1,j2,j3 = 0, 1, . . . . For a given j ≡ (j1,j2,j3), there are 2j1+j2+j3 cells labeled by l ≡ (l1,l2,l3), and l = 0, 1, . . . , 2j−1. The cell (l1,l2,l3) occupies the spatial range [l1/2i, (l1+1)/2i] × [l2/2i, (l2+1)/2i] × [l3/2i, (l3+1)/2i], i.e., ΔxiΔk = 2π/(L/2i), i = 1, 2, 3. Accordingly, indices j and l denote, respectively, the scale and position of the cells. In each dimension, we have Δxi = L/2i and Δk = 2π/(L/2i), i.e., ΔxΔk = 2π, or the volume of all cells in the x-k space is (2π)3.

Each cell (j,l) supports two compact functions: the scaling function φj,l(x) and the wavelets ψj,l(x) (Daubechies 1992; Fang & Thews 1998; Fang & Feng 2000). Both φj,l(x) and ψj,l(x) are localized in cell (j,l). The scaling functions φj,l(x) are orthonormal with respect to index l as

\[ \int \phi_{j,l}(x)\phi_{j',l'}(x) \, dx = \delta_{j,j'}\delta_{l,l'}. \]  

The scaling function ψj,l(x) is a low-pass filter at cell (j,l). The scaling function coefficient (SFC) of the density field is defined by

\[ \epsilon_{j,l} = \int \rho(x)\phi_{j,l}(x) \, dx, \]  

which is proportional to the mean density in cell (j,l).

The wavelets ψj,l(x) are orthonormal with respect to both indices j and l,

\[ \int \psi_{j,l}(x)\psi_{j',l'}(x) \, dx = \delta_{j,j'}\delta_{l,l'}. \]  

The wavelets \( \{\psi_{j,l}(x)\} \) form a complete and orthogonal base (mode) in the phase space. Therefore, the density field can be described by the wavelet function coefficients (WFCs) defined as

\[ \tilde{\epsilon}_{j,l} = \int dx \, \rho(x)\psi_{j,l}(x). \]  

The WFCs \( \tilde{\epsilon}_{j,l} \) are the DWT variables of the density field. The DWT variable \( \tilde{\epsilon}_{j,l} \) is the fluctuation of the density field around \( \bar{\rho} \).

Accordingly, indices \( (j_{\ell}, l_{\ell}) \) are represented by orthogonality with respect to index \( l_{\ell} \)

\[ \int \phi_{j_{\ell},l_{\ell}}(x)\phi_{j_{\ell}',l_{\ell}'}(x) \, dx = \delta_{j,j'}\delta_{l,l'}. \]  

The wavelets \( \{\psi_{j_{\ell},l_{\ell}}(x)\} \) form a complete and orthogonal base (mode) in the phase space. Therefore, the density field can be described by the wavelet function coefficients (WFCs) defined as

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Because the set of wavelets is complete, \( \tilde{\epsilon}_{j_{\ell},l_{\ell}} \) give a complete description of the density field \( \rho(x) \); i.e., one can reconstruct \( \rho(x) \) or \( \delta(x) \) in terms of variables \( \tilde{\epsilon}_{j_{\ell},l_{\ell}} \) as

\[ \delta(x) = \frac{1}{\bar{\rho}} \sum_{j_{\ell},l_{\ell}} \tilde{\epsilon}_{j_{\ell},l_{\ell}}\psi_{j_{\ell},l_{\ell}}(x). \]  

2.2. Quasi Locality of Gaussian Fields

The initial density perturbation of the universe \( \delta(x,t_{i}) \) is believed to be a Gaussian random field with correlation matrix of the Fourier variables \( \delta(k,t_{i}) \),

\[ \langle \delta(k,t_{i})\delta(k',t_{i}) \rangle = P(k,t_{i})\delta^{k}_{k',k}, \]  

and all higher order cumulant moments of \( \delta(k,t_{i}) \) vanish. The function \( P(k,t_{i}) \) of equation (10) is the initial power spectrum. The Kronecker delta function \( \delta^{k}_{k',k} \), in equation (10) indicates that the initial perturbation for each mode \( k \) is independent or localized in k-space.
Generally, if the initial Fourier power spectrum, \( P(k, t) \), is colored, i.e., \( k \) dependent, the correlation matrix of variables other than the Fourier mode will no longer be diagonal. For instance, the correlation function in \( x \)-space will be \( \langle \delta(x, t) \delta(x', t) \rangle = \delta(x-x', t) \), which is the Fourier counterpart of \( P(k, t) \). However, the correlation matrix of the DWT variables of a Gaussian field is always diagonal or quasi-diagonal, regardless of whether the Fourier power spectrum \( P(k) \) is white or colored. That is, the correlation function of \( \tilde{e}_{ij} \) is localized with respect to \( (j, \ell) \)

\[
\langle \tilde{e}_{ij}(t_1) \tilde{e}_{ij'}(t_2) \rangle \simeq P_{ij}(t_1) \delta_{jj'} \delta_{\ell \ell'},
\]

(11)

where \( P_{ij} \) in equation (11) is the DWT power spectrum of the field.

The reason of the diagonality of equation (11) is as follows. First, the WFC \( \tilde{e}_{ij} \) is given by a linear superposition of the Fourier modes \( \hat{e}(k) \) in the waveband around \( k_i \approx 2\pi 2^l/L \) \((l = 1, 2, 3)\); \( \tilde{e}_{ij} \) with different \( j \) consist of the Fourier modes \( \delta(k) \) in different \( k \) bands. While for Gaussian fields, the Fourier modes in different wave bands are uncorrelated in general (eq. [10]), and therefore there might be no correlation between the DWT modes of \( j \) and \( j' \) if \( j \neq j' \). This yields the quasi locality of \( \tilde{e}_{ij} \). Second, the phases of the Fourier modes of a Gaussian field are independent and random. For a superposition of the random-phased Fourier modes \( \delta(k) \) in the band from \( k \) to \( k + \Delta k \), the spatial correlation length cannot be larger than that given by the uncertainty relation \( \Delta x \approx 2\pi/\Delta k \approx 2^{l/2}L \). Moreover, the nonzero regions of two DWT modes \( \psi_{ij} \) and \( \psi_{ij'} \) with \( l \neq l' \) have spatial distance \( \Delta x_i \approx 2^{l/2}L_1 - 2^{l'}/L_1 \). Consequently, all off-diagonal elements \( (l \neq l') \) or \( j \neq j' \) vanish or are much smaller than diagonal elements, i.e.,

\[
\left| \int \right| dx \int \frac{dx'}{x' x} \psi_{ij}(x-x', t) \psi_{ij'}(x') \right| < 1, \quad \text{if} \ l \neq l' \text{ or } j \neq j'.
\]

(12)

Thus, the correlation function of the DWT variables of a Gaussian field is rapidly decaying when \( |l-l'| \geq 1 \) and \( |j-j'| \geq 1 \). The diagonal correlation function described by equation (11) is a generic feature of a Gaussian field in the DWT representation.

It should be pointed out that the WFC correlation function equation (11) is different from the ordinary two-point correlation function \( \langle \delta(x) \delta(x') \rangle = \xi(x-x') \). The former is the correlation between two modes in \( x-k \) phase space, while the latter is for two modes in \( x \)-space. Explicitly, equation (11) describes the correlation of perturbation modes in the waveband \( k \rightarrow k + \Delta k \) between positions \( I \) and \( I' \), and so it is sensitive to the phases of modes. The ordinary two-point correlation is not sensitive to the phase of perturbations. In the DWT analysis, an analog of the ordinary two-point correlation function is defined by the convolution between SFCSs, i.e., \( \langle \tilde{e}_{ij}(t_1) \tilde{e}_{ij'}(t_2) \rangle \). Since the scaling function \( \psi_{ij} \) is a low-pass filter on scale \( j \) and position \( I \), the correlation function \( \langle \tilde{e}_{ij}(t_1) \tilde{e}_{ij'}(t_2) \rangle \) behaves in a similar way as \( \langle \delta_{k}(x) \delta_{k}(x') \rangle \), where \( \delta_{k}(x) \) is a filtered density field smoothed on the scale \( R \approx 2^{-l/2}L \). However, \( \psi_{ij} \) is a high-pass filter, the WFC covariance \( \langle \tilde{e}_{ij} \tilde{e}_{ij'} \rangle \) shows quite different statistical features from the SFC correlation, e.g., it is always quasi-diagonal or even fully diagonal for a Gaussian field. Generally, it has been shown for many analytically calculable random fields that the SFC correlation is significantly off-diagonal, while the WFC correlation is exactly diagonal (Greiner, Lip, & Carruthers 1995).

Moreover, within a given volume in the \( x-k \) space, such as \( V d^k \), the number of the DWT modes \( \{j, \ell\} \) is the same as that of the Fourier modes \( k \). Accordingly, \( P(k, t) \) can be expressed as a linear superposition of \( P_{ij}(t) \), and vice versa. Equivalently, the Fourier power spectrum can be replaced by the DWT power spectrum (Fang & Feng 2000).

2.3. Statistical Criteria of Quasi Locality

If the evolution of the cosmic mass field is localized, the evolved density field \( \delta(x) \) at a given spatial point is determined only by the initial density distribution \( \delta(x, t_0) \) at the same point. As emphasized in § 1, this locality is inconsistent with the nonlocal behavior of gravitational clustering. However, the evolution of the cosmic mass field can be quasi-localized in the sense that the correlation between the DWT variables of the evolved field is always spatial diagonal if the initial correlation function is diagonal, such as equation (11). For perturbation modes in a waveband \( k \rightarrow k + \Delta k \), the quasi-localized range is \( \Delta x \approx 2\pi/\Delta k \).

A quasi-localized evolution means that the autocorrelation function of the DWT variables is always diagonal, or quasi-diagonal \( \langle \tilde{e}_{ij} \tilde{e}_{ij'} \rangle \approx \langle \tilde{e}_{ij} \tilde{e}_{ij'} \rangle \) when \( I \neq I' \), if it is diagonal initially. Thus, one may place a statistical criterion for the quasi locality as

\[
\kappa_{jj'}(\Delta I) \ll 1, \quad \text{if} \ \Delta I \neq 0,
\]

(13)

where

\[
\kappa_{jj'}(\Delta I) = \frac{\langle \tilde{e}_{ij} \tilde{e}_{ij'} \rangle}{\langle \tilde{e}_{ij} \tilde{e}_{ij} \rangle^{1/2} \langle \tilde{e}_{ij'} \tilde{e}_{ij'} \rangle^{1/2}}.
\]

(14)

This is a normalized correlation function of the DWT modes, i.e., \( \kappa_{jj'}(\Delta I = 0) = 1 \).

The autocorrelation function of the DWT variables, \( \langle \tilde{e}_{ij} \tilde{e}_{ij'} \rangle \), measures the correlations between the perturbation modes on scales \( j = (j_1, j_2, j_3) \) and \( j' = (j'_1, j'_2, j'_3) \) at two cells with a vector distance given by \( \langle \ell_1 L/2^j - \ell_2 L/2^{j_2}, \ell_2 L/2^j - \ell_3 L/2^{j_3}, \ell_3 L/2^j - \ell_1 L/2^{j_1} \rangle \). In the case of \( \Delta I = 0 \), \( \kappa_{jj'}(0) \) gives the correlation between fluctuations on scale \( j \) and \( j' \) at the same physical area. Therefore, if the condition given by equation (13) holds for all redshifts, the dynamical evolution of the mass field is basically spatially localized in the DWT bases. Comparing the condition given by equation (13) with equation (12), we see that the cosmic field undergoing a local evolution is different from its Gaussian predecessor by the factor \( \delta_{k} \), but not \( \delta_{k} \). In other words, the evolution leads to the significant scale-scale coupling, rather than modes at different locations \( I \neq I' \).

One can also construct the criteria for the quasi locality using higher order correlations among the DWT variables \( \tilde{e}_{ij} \). For instance, a \( (p+q) \)-order statistical criterion is given by

\[
C^{pq}_{jj'}(\Delta I \neq 0) \approx 1,
\]

(15)

where

\[
C^{pq}_{jj'}(\Delta I) = \frac{\langle \tilde{e}_{ij}^p \tilde{e}_{ij'}^q \rangle}{\langle \tilde{e}_{ij} \rangle^p \langle \tilde{e}_{ij'} \rangle^q},
\]

(16)

where \( p \) and \( q \) can be any even number. Obviously, \( C_{jj'}^q \approx 1 \) for Gaussian fields. \( C_{jj'}^q(0) \neq 1 \) corresponds to a local...
scale-scale correlation, while $C^{pq}_{ij}(\Delta \xi \neq 0) \neq 1$ corresponds to a nonlocal scale-scale correlation. The quasi-local evolution of the cosmic mass field requires that the nonlocal scale-scale correlation is always small.

It should be pointed out that the statistical conditions given by equations (13)–(16) are not trivial because the DWT basis is not subjected to the central limit theorem. If a basis is subjected to the central limit theorem, the corresponding variables will be Gaussian even when the random field is highly non-Gaussian. In this case, equations (13) and (15) may be easily satisfied, but it does not imply that the evolution is localized or quasi-localized. Statistical measure subjected to the central limit theorem is unable to capture non-Gaussian features of the evolution.

3. QUASI-LOCAL EVOLUTION IN THE HALO MODEL

We show in this section that the statistical criteria of $\xi_j$ are fulfilled if the cosmic mass field can be described by the halo model.

3.1. The Halo Model

The cosmic clustering is self-similar and hierarchical, as the dynamical equations of collisionless particles (dark matter) do not have preferred scales, and admits a self-similar solution, and also the initial density perturbations are Gaussian and scale-free. The halo model further assumes that all mass in a fully developed cosmic mass field is bound in halos on various scales (Neyman & Scott 1952; Scherrer & Bertschinger 1991).

Thus, the cosmic mass field in the nonlinear regime is given by a superposition of the halos

$$\rho(x) = \sum_i \rho_i(x-x_i) = \sum_i m_i u(x-x_i, m_i)$$

where $\rho_i(x-x_i)$ is the density profile of a halo with mass $m_i$ at position $x_i$ and $u(x-x_i, m_i)$ is the density profile normalized by $\int dx \rho_i(x-x_i) = \int dx m_i u(x-x_i, m_i) = m_i$.

There are several different versions of the halo density profiles, such as $u(x, m_i) \propto 1/(r/r_i)^\alpha [1 + (r/r_i)]^\beta$ with $\alpha = 1$, $\beta = 2$ (Navarro, Frenk, & White 1996) and $u(x, m_i) \propto 1/(r/r_i)^\alpha [1 + (r/r_i)]^\beta$ with $\alpha = 3/2$, $\beta = 3/2$ (Moore et al. 1999). A common feature of the halo density profiles is self-similarity, which implies that the indices $\alpha$ and $\beta$ should be mass independent. The mass dependence is only given by $r_i$, which characterizes the size of the $m_i$ halo. The details of the profiles are indifferent for the problem we try to study below.

What is important for us is only that one can set a self-similar upper limit to the normalized halo density profile as

$$u(x, m_i) < C \left( \frac{r}{r_i} \right)^{-\gamma},$$

where $r = |x|$, $C$ is a constant, and the index $\gamma$ is mass independent. The $m_i$ dependence of $r_i$ is not stronger than a power law as $r_i \propto m_i^{1/2}$.

The halo model also assumes that the halo-halo correlation function on scales larger than the size of halos is given by the two-point correlation functions of the linear Gaussian field with a linear bias correction, or by the correlation functions of the quasi-linear field. Therefore, no assumption about higher order halo-halo correlations on large scales is needed.

In this model, the time dependence of the field is mainly given by the mass function of halos, $n(m, t)$, which is the number density of the halos with mass $m$ at time $t$. In the Press & Schechter (1974) formalism, the mass function is determined by the power spectrum of the initial Gaussian density perturbation. Moreover, the self-similarity of the halo density profiles ensures that equation (18) holds for all time. The cosmic evolution only leads to the parameters on the right-hand side of equation (18) being time dependent.

3.2. Quasi Locality of the DWT Correlation Function

With equation (17), the DWT variable of the cosmic mass field in the halo model is given by

$$\tilde{\xi}_{ij} = \int dx \rho(x) \psi_{ij}(x) = \sum_i m_i \int dx u(x-x_i, m) \psi_{ij}(x).$$

The autocorrelation function of the DWT variables is then

$$\langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle = \langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^h + \langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^{bh},$$

where the first and second terms on the right-hand side are usually called, respectively, one- and two-halo terms. They can be written in the explicit form

$$\langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^h = \int dx n(m) m^2 \int dx_1 u(x_1, m)$$

$$\int dx d' \psi_{ij}(x) u(x_1 + x - x', m) \psi_{ij}^{*}(x'),$$

$$\langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^{bh} = \int dm_1 n(m_1) \int dm_2 n(m_2) m_2 \int dx_1$$

$$\int dx_2 u(x_1, m_1) u(x_2, m_2) \int dx$$

$$\int dx' \psi_{ij}(x) \xi(x - x' + x_1 - x_2, m_1, m_2) \psi_{ij}^{*}(x'),$$

where $n(m) = (\sum_i \delta^D(m - m_i) \delta^D(x - x_i))$ is the number density of halos with mass $m$ and $\xi(x - x', m_1, m_2) = (\sum_i \delta^D(m_1 - m_i) \delta^D(x - x_i) \delta^D(m_2 - m_i) \delta^D(x' - x_i)) / n(m_1)n(m_2)$ is the halo-halo correlation function.

We show now that the DWT correlation function $\langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^h$ is quasi-diagonal, or fast decaying with respect to the spatial distance $l - l'$. First, consider the two-halo terms given by equation (22). According to the halo model, the two-point correlation function $\xi(x_1 - x_2, m_1, m_2)$ is determined by the linearly Gaussian density field. In fact, as has been discussed in $\xi_j$, the DWT correlation function of a Gaussian field is generally diagonal (eq. (12)), regardless of whether the Fourier power spectrum is white or colored. Therefore, the DWT correlation function $\langle \tilde{\xi}_i \tilde{\xi}_j^{*} \rangle^{bh}$ contributed from the two-halo term should be quasi-local.

The DWT integral in the two-halo term of equation (22) is not completely the same as equation (12), as the halo-halo correlation function in equation (22) is $\xi(x - x' + x_1 - x_2)$, while in equation (12) it is $\xi(x - x')$. For halos with size less than the scale $l$ considered, the factor $|x_1 - x_2|$ in correlation function $\xi^{bh}$ is smaller than the size of the DWT mode, $L/2$, and so the factor $x_1 - x_2$ can be ignored in comparison with the variable $x - x'$. Moreover, by definition of the halo model, the correlation function $\xi^{bh}$ does not include the contributions...
from halos with sizes larger than \(|x - x'|\). Thus, the cases of 
\(|x_1 - x_2| > |x - x'|\) can always be ignored. The two-halo term essentially follows equation (12) and is always approximately diagonal with respect to spatial indices \(I\) and \(I'\).

To analyze the one-halo term given by equation (21), we use the following theorem of wavelets (Tewfik & Kim 1992). Because the DWT basis is self-similar, for any one-dimensional power-law function \(f(x) \propto x^{-\gamma}\), we have

\[
\int dx \int dx' \psi_j(x)f(x-x')\psi_{j'}(x') < C'2^{-j}l - l'\left|^{-\gamma - 2M},
\]

(23)

where \(\psi_j(x)\) is a wavelet in one-dimensional space and \(C'\) is a constant. Therefore, while using wavelets with large enough \(M\) (eq. [5]), the integral given by equation (23) is quickly decaying with the spatial distance \(|l - l'|\). In other words, besides the two nearest positions of \(l - l' = \pm 1\), there is no correlation between modes at different positions \(l \neq l'\).

The three-dimensional integral \(I(l - l') = \int \int dx' \psi_{j_1}(x_1)\psi_{j_2}(x_2)\psi_{j_3}(x_3)\) in equation (21) has a similar structure as equation (23). Using the upper limit given by equation (18), we can expand the function \(\psi_{j_1}(x_1)\psi_{j_2}(x_2)\psi_{j_3}(x_3)\) in terms of \(|x_1|, |x_2|, |x_3|\). Thus, applying equation (23) term by term, we have the decaying behavior of \(I\) at least as fast as \(\sim 2^{-j_1-j_2-j_3}l - l'|^{-\gamma - 2M}\). Similarly, in the case of \(|x_1| > |x - x'|\), one can expand the function \(\psi_{j_1}(x_1)\psi_{j_2}(x_2)\psi_{j_3}(x_3)\) in terms of \(|x_1|, |x_2|, |x_3|\). Thus, applying equation (23) term by term, we have the decaying behavior of \(I\) at least as fast as \(\sim 2^{-j_1-j_2-j_3}l - l'|^{-\gamma - 2M}\). Accordingly, the one-halo term in the correlation between modes \(j, I\) and \(j', I'\) will generally decay as \(|l - l'|^{-2 \gamma - 2M}\) or \(|l - l'|^{-4M}\). The correlation \(\langle \xi_{j_1}^1 \xi_{j_2}^2 \xi_{j_3}^3 \rangle\) is then approximatively diagonal with respect to the spatial indices \(I\) and \(I'\). This result is largely valid as a result of \(r_i\) varying with \(m_i\) by a power law.

Proceeding in a similar way as above, we can also show the diagonality of the correlation function \(\langle \xi_{j_1}^1 \xi_{j_2}^2 \xi_{j_3}^3 \rangle\). In this case, instead of equation (23), we use the following theorem (Tewfik & Kim 1992):

\[
\int dx \int dx' \psi_{j_1}(x)f(x-x')\psi_{j'}(x') < C'2^{-j}l - \left(\frac{l'}{2^{j-j'}}\right)^{-\gamma - 2M}, \text{ if } j' > j.
\]

(24)

Consider that the cell \((j, I)\) has the same physical position as cell \((j', I')\) if \(l' = l2^{j-j'}\); the theorem given by equation (24) also yields that the one-halo term of the two modes \(j, I\) and \(j', I'\) will decline with the physical distance between the two modes as \(\sim (l - l'2^{-j-j'})^{-\gamma - 2M}\) or \(\sim (l - l'2^{-j-j'})^{-4M}\).

Based on above discussions, one may draw the conclusion that if the halo model is a good approximation to the cosmic density field \(\rho(x)\) in the nonlinear regime at all times, their correlation matrix in the DWT representation remains quasi-diagonal forever, and the evolution is quasi-local. This result is based on the self-similarity of the density profiles of halos and the weakly nonlinear correlation between halos. The self-similar scaling ensures that the non-local correlation among the DWT variables is uniformly suppressed, independent of the mass of halos. Mathematically, the correlations between the DWT modes of perturbations at different physical places are uniformly converging to zero with the increasing of \(M\) if the index \(\gamma\) is mass independent.

3.3. Quasi Locality of Higher Order Statistics

To show the quasi locality of a higher order statistical criterion, we use the hierarchical clustering or linked-pair relation (Peebles 1980), which is found to be consistent with the halo model. For the third-order correlations, the linked-pair relation is

\[
\langle \delta(x)\delta(x')\delta(x'') \rangle \simeq Q_3 \left[ \langle \delta(x)\delta(x') \rangle \langle \delta(x')\delta(x'') \rangle \right] + \text{ two terms with cyc. permutations},
\]

(25)

where the coefficient \(Q_3\) might be scale dependent. Subjecting equation (25) to a DWT by third-order basis \(\psi_{j_1}(x_1)\psi_{j_2}(x_2)\psi_{j_3}(x_3)\), we have

\[
\langle \xi_{j_1}^1 \xi_{j_2}^2 \xi_{j_3}^3 \rangle \simeq Q_3 \sum_{j_1,j_2,j_3} a_{j_1,j_2,j_3}^3 \left[ \langle \xi_{j_1}^1 \xi_{j_2}^2 \rangle \langle \xi_{j_2}^2 \xi_{j_3}^3 \rangle \right] + \text{ two terms with cyc. permutations},
\]

(26)

where \(a_{j_1,j_2,j_3}^3\) is given by the three-wavelet integral

\[
a_{j_1,j_2,j_3}^3 = \int \psi_{j_1}(x)\psi_{j_2}(x)\psi_{j_3}(x) dx.
\]

(27)

Since \(\psi_j(x)\) is localized in the cell \((j, I)\), \(a_3^3\) is significant only if the three cells \((j_1, I_1), (j_2, I_2), (j_3, I_3)\) coincide in the same physical area. Thus, by virtue of the locality of correlations \(\langle \xi_{j_1}^1 \xi_{j_2}^2 \rangle\) and \(\langle \xi_{j_2}^2 \xi_{j_3}^3 \rangle\) (§ 3.2), it is easy to see that \(\langle \xi_{j_1}^1 \xi_{j_2}^2 \xi_{j_3}^3 \rangle\) is small if the cells \((j_1, I_1), (j_2, I_2), (j_3, I_3)\) are disjoint in the physical space. Since \(Q_3\) does not depend on \(x\), the result of locality will remain valid when \(Q_3\) is scale dependent.

Obviously, the third-order result can be generalized to the nth-order DWT correlation function. The integral of \(n\) wavelets \(\psi_{j_1}(x), \psi_{j_2}(x), \ldots, \psi_{j_n}(x)\) is zero or very small; otherwise, the \(n\) cells \((j_1, I_1), (j_2, I_2), \ldots, (j_n, I_n)\) coincide in the same physical area. In addition, all cells on the right-hand side of the hierarchical clustering relation consist of a linked second-order DWT correlation function; the nth-order DWT correlation function \(\langle \xi_{j_1}^1 \xi_{j_2}^2 \ldots \xi_{j_n}^n \rangle\) should be localized. Thus, the criterion given by equation (15) and other higher order criteria will be satisfied in general.

In summary, if the cosmic density field is evolved self-similarly from an initially Gaussian field, the spatial quasi locality is true at all times, i.e., (1) the second-order and higher order correlation functions of the DWT variables \(\xi_{j_1}\) of the evolved field are quasi-diagonal with respect to the position index \(I\). For those types of fields, the possible non-Gaussian features with the DWT variables are mainly (2) the non-Gaussian one-point distribution of the DWT variables \(\xi_{j_1}\) and (3) local scale-scale correlation among the DWT variables. The above three points are the major theoretical results of this paper.

4. Testing with N-Body Simulation Samples

4.1. Samples

To demonstrate the quasi locality of the evolved cosmic density field, we use samples produced by Jing & Suto (2002). The samples are given by high-resolution N-body simulations, running with the vectorized-parallelized P^M code. The
cosmological model was taken to be the LCDM model specified by parameters \((\Omega_0, \Lambda, \sigma_8, \Gamma) = (0.3, 0.7, 0.9, 0.2)\). The primordial density fluctuation is assumed to obey the Gaussian statistics (this is important for us), and the power spectrum is of the Harrison-Zel’dovich type. The linear transfer function for the dark matter power spectrum is taken from Bardeen et al. (1986).

The simulation was performed in a periodic, cubical box of size 100 \(h^{-1}\) Mpc with 1200\(^3\) grid points for the particle-mesh (PM) force computation and 512\(^3\) particles. The short-range force is compensated for the PM force calculation at a separation less than \(\epsilon = 2.7H\), where \(H\) is the mesh cell size. The simulations are evolved by 1200 time steps from the initial redshift \(z_i = 72\). The force resolution is 20 \(h^{-1}\) kpc for the linear density softening form. It is noted that our statistic tests are performed on scales \(\geq 0.2 \, h^{-1}\) Mpc.

We have one realization. In the practical computations, we divide the 100 \(h^{-1}\) Mpc simulation box into eight subboxes each with size \(L = 50 \, h^{-1}\) Mpc. Accordingly, the ensemble average and 1 \(\sigma\) variance are obtained from those eight subboxes.

### 4.2. Two-Point Correlation Functions

Before showing the quasi locality, we first calculate the correlation function of the SFCs, i.e., \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\). As has been discussed in \S 2.2, the correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) is actually an analog to the ordinary two-point correlation function \(\xi(r)\), where \(r = x - x'\). Since \(\epsilon_{j_1}\) is a filtered density field smoothed by the scaling function on the scale \(j\), it is expected that the correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) will display similar features as \(\xi(r)\).

![SFC correlations](image)

**Fig. 1.—**SFC correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) vs. \(r\) for simulation data (hexagon) and one- \(j_1\) (square) and two-halo \(j_2\) (circle) terms. The scale \(j_1\) is taken to be \((7, 7, 7)\). The physical distance is given by \(r = |l - l'|/50/2 \, h^{-1}\) Mpc.

Figure 1 presents the \(r\) dependence of \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) at \(j = (7, 7, 7)\), corresponding to the smoothed field filtered on the linear scale 50/2\(^2\) Mpc. The spatial distance between the cells \(l\) and \(l'\) is \(r = |l - l'|/50/2 \, h^{-1}\) Mpc. In this calculation, we applied wavelet Daubechies 4 (Daubechies 1992). As expected, the \(r\) or \(|l - l'|\) dependence of the correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) shows the standard power law, \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle \propto r^{-\alpha}\) with the index \(\alpha \approx 2\). That is, this correlation function is not localized. Meanwhile in Figure 1, the contributions of the one- and two-halo terms are also plotted. The one-halo term is calculated from equation (21) with the NFW density profiles of halos, and the two-halo term is given by equation (22), in which the correlation function \(\xi(r)\) is given by the linear power spectrum. The nonlinear clustering is largely due to the one-halo term. Therefore, one can see that the nonlinear evolution of gravitational clustering causes correlations on scales of several million parsecs.

### 4.3. Justifying the Quasi Locality

We now study the quasi locality of the clustering with the correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\), which is used in the criterion \(\kappa_{j_1} \pmb{\langle} \Delta l \pmb{\rangle}\) (eq. [13]). First, we take the same parameter as Figure 1, \(j = j' = (7, 7, 7)\) and \(r = |l - l'|/50/2 \, h^{-1}\) Mpc, and also we used wavelet Daubechies 4 (Daubechies 1992), which has \(M = 2\). The result of DWT correlation function \(\langle \epsilon_{j_1} \epsilon_{j_2} \rangle\) is shown in Figure 2. The filled circle at \(r = 0\) in Figure 2 corresponds to \(\Delta l = 0\) or \(l = l'\), and other filled circles from left to right correspond, successively, to \(\Delta l = 1, 2, 3, \ldots\).
From Figure 2 we can see immediately that the shape of the $r$ or $|\Delta l|$ dependence of $\langle \tilde{c}_j \tilde{c}_{j'} \rangle$ is quite different from the "standard" power law. The correlation function $\langle \tilde{c}_j \tilde{c}_{j'} \rangle$ is nonzero mainly at point $r = 0$ or $|\Delta l| = 0$. At $|\Delta l| = 1$, the correlation function $\langle \tilde{c}_j \tilde{c}_{j, l \pm 1} \rangle$ drops to tiny values around $\sim 0$. For $|\Delta l| > 1$, the correlation function basically is zero. The correlation length in terms of the position index $l$ is approximately zero, namely, the covariance $\langle \tilde{c}_j \tilde{c}_{j', l} \rangle$ is diagonal. This is the spatial quasi locality. In Figure 2 we also plot the one- and two-halo terms, $\langle \tilde{c}_j \tilde{c}_{j', l} \rangle^{h}$ and $\langle \tilde{c}_j \tilde{c}_{j', l} \rangle^{hh}$. Although the one-halo term is dominated by massive halos, the covariance $\langle \tilde{c}_j \tilde{c}_{j', l} \rangle^{h}$ is also perfectly quasi-localized because of the self-similarity of density profiles of massive halos. The two-halo term is zero at $r = 0$ because, by definition, equation (22) does not contain the contribution of auto-correlations of halos.

Figure 3 presents $\kappa_{j,j'}(\Delta l)$ versus $r$, for $j = j' = (4, 4, 4)$, $(5, 5, 5)$, and $(6, 6, 6)$. The physical distance $r$ is the same as Figure 2, given by $r = |\Delta l| 50 / 2 = |l - l'| 50 / 2 \ h^{-1} \text{Mpc}$. The filled circle at $r = 0$ corresponds to $|\Delta l| = 0$, at which, by definition, we have the normalization $\kappa_{j,j}(0) = 1$. Other points from small to large values of $r$ correspond to $|\Delta l| = 1, 2, \ldots, 50$, successively. Clearly, all $\kappa_{j,j}(\Delta l)$ for $\Delta l > 0$ are less than $10^{-6}$, which is actually from the noises of the sample. The result implies that for all calculable points of $r \geq 0$, the correlation is negligible and satisfies the criterion given by equation (13).

From approximately zero, namely, the covariance distance is given by (4, 4, 4) ("standard" power law. The correlation function result implies that for all calculable points of The correlation length in terms of the position index two-halo term is zero at $r$ the self-similarity of density profiles of massive halos. The Figure 2, given by (5, 5, 5), and (6, 6, 6) (bottom). The physical distance is given by $r = |l - l'| 50 / 2 \ h^{-1} \text{Mpc}$. The physical distance is $r = |\Delta l| 50 / 2 \ h^{-1} \text{Mpc}$. Wavelets D4 ($M = 2$) and D6 ($M = 3$) are used.

We also calculated $\kappa_{j,j'}(\Delta l)$ for modes of $j = j' = (j_1, j_2, j_3)$ but $j_1 \neq j_2 \neq j_3$. Most of these cases show $\kappa_{j,j'}(\Delta l) \approx 0$ if $|\Delta l| > 0$. The only exception is for the cases of $j_1, j_2 < j_3$ and $|\Delta l| = |l_3 - l_3'| = 1$. As an example, Figure 4 presents $\kappa_{j,j}(\Delta l)$ for modes $j = (5, 5, 6)$ and $r = |l_1 - l_3| 50 / 2$.

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![Figure 3](image3.png)  
**Fig. 3.** Variable $\kappa_{j,j}$ vs. $r$ for the simulation data. The $j$-values are taken to be (4, 4, 4) (top), (5, 5, 5) (middle), and (6, 6, 6) (bottom). The physical distance is given by $r = |l - l'| 50 / 2 \ h^{-1} \text{Mpc}$.  

![Figure 4](image4.png)  
**Fig. 4.** Variable $\kappa_{j,j'}(\Delta l)$ vs. $r$ for the simulation data. The scales are $j = (5, 5, 6)$. The physical distance is $r = |\Delta l| 50 / 2 \ h^{-1} \text{Mpc}$. Wavelets D4 ($M = 2$) and D6 ($M = 3$) are used.

![Figure 5](image5.png)  
**Fig. 5.** Variable $\kappa_{j,j+1}(\Delta l)$ vs. $r$ for the simulation data. The scale indices are taken to be $j = (j_1, j_2, j_3) = (j + 1, j + 1, j + 1)$, and $j = 3, 4, 5, 6$, and $\Delta l = 0.1$. The physical distance is $r = |\Delta l| 50 / 2 \ h^{-1} \text{Mpc}$. 


It shows that \( h_{j,l} \sim C_1 \), \( j \neq 0 \). The nonzero value at \( |l_3 - l'_3| = 1 \) is about 20% of that at \( \Delta l = 0 \). This result is consistent with the theorem given by equation (23), which only requires the suppression for modes \( |l - l'| > 1 \), but may not work for \( |l - l'| = 1 \). It implies that the clustering may give rise to the correlations between nearest neighbor cells in phase space. However, the cell resolved by \( j_1, j_2 < j_3 \) is a rectangle in the physical space, and the shortest edge is given by \( j_3 \); the physical distance of \( |l_3 - l'_3| = 1 \) is still less than the whole size of the rectangle. Thus, it could be concluded that the covariance \( \langle \tilde{e}_{j,l} \tilde{e}_{j',l'} \rangle \) is always quasi-diagonal in the sense that all members with \( l \) and \( l' \) are almost zero if the distance \( r \) between \( l \) and \( l' \) is larger than the size of the cell \( (j_1, j_2, j_3) \) considered. In Figure 4 we show also a result calculated with wavelets Daubechies 6 (D6), for which \( M = 3 \). It yields about the same results as D4.

For the correlation between modes \( (j, l) \) and \( (j', l') \) with \( j \neq j' \), we use the criterion \( \kappa_{j,j'} \) of equation (14). In this case, the physical distance between two cells is \( r = |r| \) and \( r_3 = |l_3/2^j - l'_3/2^j| \). Figure 5 plots \( \kappa_{j,j'}(\Delta l) \) versus \( r \) for modes \( j = (j, j, j) \), \( j' = (j + 1, j + 1, j + 1) \), and \( j = 3, 4, 5, \) and 6. All the values of \( \kappa_{j,j'}(\Delta l) \) in Figure 5 are not larger than \( 10^{-5} \) and are much less than the diagonal terms \( \langle \tilde{e}_{j,l}^2 \rangle \) or \( \langle \tilde{e}_{j',l'}^2 \rangle \). We found that this result is generally true for all the cases of \( j \neq j' \). That is, the second-order correlation between two modes with different scales \( j \) and \( j' \) is always negligible, regardless of the indices \( l \) and \( l' \). In other words, the covariance of the WFC variables \( \langle \tilde{e}_{j,l} \tilde{e}_{j',l'} \rangle \) generally is quasi-diagonal.
As for the high-order statistics by the criteria of equations (15) and (16), we can cite some previous calculations of the nonlocal scale-scale correlation defined by

\[ C_{j,j}^{2,2}(\Delta l) = \frac{\langle \xi_{j}^{2} \tilde{\xi}_{j+\Delta l}^{2} \rangle}{\langle \xi_{j}^{2} \rangle \langle \tilde{\xi}_{j+\Delta l}^{2} \rangle}, \tag{28} \]

which is the criterion of equation (15) with \( p = q = 2 \). It has been shown that either for the APM bright galaxy catalog (Loveday et al. 1992) or mock samples of galaxy survey (Cole et al. 1998), the nonlocal scale-scale correlation always yields \( |C_{j,j}(\Delta l)| < 1 \) if \( \Delta l > 0 \) (Feng, Deng, & Fang 2000). Although this work was not for addressing the problem of the quasi locality, the result did support the quasi locality up to the fourth-order statistics.

4.4. Non-Gaussianity Revealed by DWT Variables

As discussed in \( \S \) 2.3, it is necessary to show that the random variables \( \tilde{\xi}_{j} \) are non-Gaussian for evolved fields. The possible non-Gaussian features with the DWT variables are (1) the non-Gaussian one-point distribution of \( \tilde{\xi}_{j} \) and (2) local scale-scale correlations (\( \S \) 3.3).

In Figure 6 we plot the one-point distribution on scales \( j = (j, j, j) \) and \( j = 5, 6, 7, \) and 8. It illustrates that the kurtosis of the one-point distribution is high. The probability distribution function (PDF) is approximately lognormal. The fourth-order local scale-scale correlation \( C_{j,j}^{2,2}(\Delta l) = 0 \) is plotted in Figure 7. It shows \( C_{j,j}^{2,2}(\Delta l = 0) \approx 1 \) on small scales \((j = 5, 6, \) and 7), while random data give \( C_{j,j}^{2,2}(\Delta l = 0) = 1 \) on all scales. The evolved field is highly non-Gaussian, although it is always quasi-localized.

5. DISCUSSIONS AND CONCLUSIONS

We showed that the cosmic clustering behavior is quasi-localized. If the field is viewed by the DWT modes in phase space, the nonlinear evolution will give rise to the coupling between modes on different scales but in the same physical area, and the coupling between modes at different positions is weak. The quasi-local evolution means that if the initial perturbations in a waveband \( k = \pm \Delta k/2 \) and at different space range \( \Delta x \) are uncorrelated, the evolved perturbations in this waveband at different space range \( \Delta x \) will also be uncorrelated, or very weakly correlated. In this sense, the nonlinear evolution has a memory of its initial spatial correlation in the phase space. This memory is essentially from the hierarchical and self-similar feature of the mass field evolution. The density profiles of massive halos obey the scaling law (eq. [18]); therefore, the contributions to the nonlocal correlation function from various halos are uniformly suppressed.

It was realized about 10 years ago that some random fields generated by a self-similar hierarchical process generally show locality of their autocorrelation function in the phase space, if the initial field is local, like a Gaussian field (Ramanathan & Zeitouni 1991; Tewfik & Kim 1992; Flandrin 1992). Later, this result was found to be correct for various models of structure formations via hierarchical cascade stochastic processes (Greiner et al. 1996; Greiner, Eggers, & Lipa 1998). These studies imply that the local evolution and initial perturbation memory seem to be generic of self-similar hierarchical fields, regardless of the details of the hierarchical process. It has been pointed out that models for realizing the self-similar hierarchical evolution of the cosmic mass field, such as the fractal hierarchy clustering model (Soneira & Peebles 1977), the block model (Cole & Kaiser 1988), and the merging cell (Rodrigues & Thomas 1996), have the same mathematical structures as hierarchical cascade stochastic models applied in other fields (Pando et al. 1998, 2001). Obviously, the local evolution can be straightforwardly obtained in those models.

The DWT analysis is effective in revealing the quasi locality in phase space. Such quasi locality is hardly described by the Fourier modes \( \delta(k) \), as the information of spatial positions is stored in the phases of all Fourier modes. Moreover, the Fourier amplitudes \( |\delta(k)| \) are subject to the central limit theorem and are insensitive to non-Gaussianity. The wavelet basis, however, is not subjected to the central limit theorem (Pando & Fang 1998), which enables us to measure all the quasi-local features with the statistics of \( \tilde{\xi}_{j} \).

The quasi-locality of the DWT correlation is essential for recovery of the primordial power spectrum using a localized mapping in phase space. Such mapping has been developed in recovering the initial Gaussian power spectrum from the evolved field in the quasi-linear regime (Feng & Fang 2000). By virtue of the quasi locality in fully developed fields, we would be able to generalize the method of localized mapping in phase space to the highly nonlinear regime.

The quasi-local evolution may also provide the dynamical base for the lognormal model (Bi 1993; Bi & Davidsen 1997; Jones 1999). The basic assumption of the lognormal model is that the nonlinear field can be approximately found from the corresponding linear Gaussian field by a local exponential mapping. The local mapping is supported by the quasi-local evolution. We see from Figure 6 that the PDF of the evolved field is about lognormal. Therefore, in the context of quasi-local evolution, a local (exponential) mapping from the linear Gaussian field to a lognormal field might be a reasonable sketch of the nonlinear evolution of the cosmic density field.

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