Propagating and evanescent waves in absorbing media

S. Anantha Ramakrishna  
School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom

A. D. Armour  
The Blackett Laboratory, Imperial College, London SW7 2BW, United Kingdom

We compare the behavior of propagating and evanescent light waves in absorbing media with that of electrons in the presence of inelastic scattering. The imaginary part of the dielectric constant results primarily in an exponential decay of a propagating wave, but a phase shift for an evanescent wave. We then describe how the scattering of quantum particles out of a particular coherent channel can be modeled by introducing an imaginary part to the potential in analogy with the optical case. The imaginary part of the potential causes additional scattering which can dominate and actually prevent absorption of the wave for large enough values of the imaginary part. We also discuss the problem of maximizing the absorption of a wave and point out that the existence of a bound state greatly aids absorption. We illustrate this point by considering the absorption of light at the surface of a metal.

I. INTRODUCTION

Analogies between the propagation of classical electromagnetic waves and the non-relativistic quantum mechanical motion of matter have been exploited for many years. The analogies stem from the fact that the wave equations for light and quantum particles have a very similar form. If the propagation of an electromagnetic wave preserves the polarization, then Maxwell’s equations in the steady state reduce to the Helmholtz equation for a scalar wave,

$$\nabla^2 \mathcal{E}(r) + (\omega/c)^2 n^2(r) \mathcal{E}(r) = 0,$$

where $\mathcal{E}$ is the complex wave amplitude and $n(r)$ is the refractive index of the medium. If we replace $\mathcal{E}$ by $\Psi$, the wavefunction for an electron, $k \equiv \sqrt{2mE/\hbar^2}$, and $n^2$ by $1 - V/E$, where $V$ is the potential and $E$ is the energy of the electron, we obtain the time independent Schrödinger equation for a scalar field, hence we see that the effects of the refractive index on electromagnetic waves and the effects of an electrostatic potential on electrons are similar.

Despite the close similarity between the equations describing the propagation of electromagnetic waves and electrons, there is by no means an exact mapping of properties between the two systems. A particularly tricky topic is the absorption of waves that takes place in the presence of dissipation, particularly for evanescent waves. Continuum electromagnetic theory adds an imaginary part to the dielectric permittivity or the magnetic permeability to model the microscopic processes of absorption. An imaginary potential is frequently adopted to model scattering of electrons into inelastic channels and is often referred to as an optical potential because of the analogy with electromagnetic waves. Such potentials are extensively used in nuclear physics to describe the scattering events in which particles are removed from the incident flux and to describe the formation of complex nuclei.

It is now possible to fabricate solid-state structures that are small enough for the transport of electrons to be largely coherent so that the wave nature of the electrons dominates. Such systems are known as mesoscopic and their current characteristics are modeled using a scattering formalism analogous to that used to calculate the transmission of light through one or more slabs of dielectric material. The propagation of electrons can be thought of as due to contributions from a number of different coherent channels that are essentially coherent wave modes. Complex potentials can be used in such systems to describe the removal of electrons from a coherent channel due to inelastic scattering processes and dephasing (scattering out of one wave mode into others and loss of coherence). However, although it is frequently used, this approach is over simplistic because it neglects the important fact that any potential (whether real or imaginary) also gives rise to additional scattering.

We will discuss some aspects of wave propagation in dissipative media that many students (and some teachers) do not appreciate after a first course on quantum mechanics. In Sec. II we begin by discussing the familiar cases of propagating and evanescent classical electromagnetic waves in an absorbing medium. In Sec. III, we analyze the analogous case for electrons: propagation in a region with an imaginary contribution to the potential. Such non-unitary Hamiltonians can formally cause the absorption of the particle and are frequently used to mimic absorption. We examine the consequences of an optical potential on both propagating and evanescent electron waves. Then in Sec. IV we use the example of scattering from a complex delta-function potential to explore the additional scattering that is encountered when using optical potentials in quantum mechanics. In Sec. V, we address the problem of how...
to construct a maximally absorbing material, a so-called perfect absorber. Intriguingly, a way of maximizing the absorption of light in non-magnetic materials can be derived by analogy with a mesoscopic system. Finally we draw conclusions in Sec. VI.

II. THE CASE OF LIGHT

It is possible to describe the propagation of light through an absorbing medium using a very simple prescription: just add an imaginary part to the refractive index of the medium, \( n = n_r + i n_i \), where \( n_i \), the imaginary part of the refractive index, is proportional to the degree of absorption in the medium. The ratio \( \kappa = n_i/n_r \) is often called the attenuation index. A complex refractive index implies that the wave vector of a wave in the medium will also be complex, \( \mathbf{k} \equiv 2\pi n/\lambda = 2\pi(n_r + i n_i)/\lambda \), and a propagating wave, \( e^{i k z} \) (in one dimension), decays in amplitude as it travels through the medium.

The imaginary part of \( n \) enters the continuum theory as an imaginary component of the dielectric constant. A material will typically be at least partly absorbing and so the dielectric constant will generally be complex, with the degree of absorption measured by the magnitude of its imaginary component (for non-magnetic materials). The refractive index is given by

\[
\begin{align}
n &= c\sqrt{\varepsilon} = c\sqrt{\mu(\varepsilon_r + i\varepsilon_i)} \\
&\approx c\sqrt{\mu\varepsilon_r}[1 + i\frac{\varepsilon_i}{2\varepsilon_r}] = n_r + in_i, 
\end{align}
\]

where \( n_i = \frac{c}{\sqrt{\mu}}\varepsilon_i \), and we have made the usual assumption that \( \varepsilon_i \ll \varepsilon_r \). The energy flow through the medium is determined by the magnitude of the time-averaged Poynting vector for the electromagnetic wave, \( \langle |S| \rangle_t \). For a dissipative medium,

\[
|\langle |S| \rangle_t| = \frac{c}{8\pi} |\mathbf{E} \times \mathbf{H}^*| = \frac{c}{8\pi\sqrt{\mu}} |\mathbf{E}_0|^2 e^{-4\pi n_i z/\lambda},
\]

where \( z \) is the distance propagated, \( |\mathbf{E}_0| \) is the amplitude of the electric field vector associated with the electromagnetic wave and \( \mathbf{H}^* \) is the complex conjugate of \( \mathbf{H} \), the magnetic field vector. The decay of the energy flow with distance corresponds to the energy dissipated into the medium (\( -dS/dz \)). The quantity \( \chi = 4\pi n_i/\lambda = 4\pi n_r \kappa/\lambda \) is called the absorption coefficient.

![FIG. 1: A schematic illustration of total internal reflection. A wave incident on a dielectric medium with a larger dielectric constant at an angle greater than the critical angle, \( \theta > \theta_c \), results in an exponentially decaying wave in medium 2. The plane of incidence is the \( x-z \) plane and the electric vector is taken to lie in the plane of polarization (p-polarization).](image)

Evanescent waves occur when the magnitude of the transverse incident wave vector, \( k_\parallel \), is larger than the wave vector in the medium, \( k_0 = 2\pi n/\lambda \). The physics of evanescent waves is readily understood by considering what happens
when light is shone on a non-absorbing medium at an angle greater than the angle for total internal reflection from a medium with lower refractive index. The geometry is illustrated in Fig. 1 where a wave with a wave vector \( k_{(1)}^x, \ 0, k_{(1)}^z \) in medium 1 is incident on medium 2. Maxwell’s equations require that \( (k_{(1)}^x)^2 + (k_{(1)}^z)^2 = \varepsilon_1 \mu (\omega/c)^2 \) in medium 1 and the continuity condition at the boundary requires \( k_{(2)}^x = k_{(1)}^x = k_x \). Therefore, if \( \varepsilon_2 < \varepsilon_1 \) and \( k_x \) is sufficiently large, \( k_{(2)}^z \) must become imaginary to satisfy the condition in medium 2, \( (k_{(2)}^x)^2 + (k_{(2)}^z)^2 = \varepsilon_2 \mu (\omega/c)^2 \). This condition is the origin of total internal reflection as there exists no propagating wave inside medium 2 for \( k_x > \sqrt{\varepsilon_2 \mu (\omega/c)^2} \).

The spatial dependence of the electromagnetic field in medium 2 takes the form

\[
e^{ik_{(2)}^z z} = e^{ik_{(2)}^x x - |k_{(2)}^x| z},
\]

and the amplitude of the wave decays exponentially with distance into the medium. However, in this case the decay of the wave amplitude does not imply the absorption of the wave energy because examination of the Poynting vector shows that no energy enters medium 2. Evanescent waves are the classic example of a wave in which exponential decay of the amplitude does not imply that dissipation is taking place.

The electromagnetic near-field of a radiation source and the electromagnetic fields inside a metal or plasma are also evanescent. The case of the field inside a metal or plasma is interesting because the dielectric constant is large, is almost entirely real, but with a negative sign. A negative dielectric constant makes it impossible to satisfy the dispersion relation, \( k_x^2 + k_y^2 + k_z^2 = \varepsilon \mu (\omega/c)^2 \), without making at least one component of the wave vector imaginary. Such a medium cannot support propagating waves at all and the wave amplitudes always decay exponentially with distance inside the medium. If the dielectric constant is a purely real and negative number, then the Poynting vector is identically zero, corresponding to a dissipationless metal. Purely evanescent waves do not transmit energy, and so cannot give rise to dissipation.

Things become a little more complicated if the medium supporting the evanescent wave has a dielectric constant which has not only a real part that is negative, but also a small imaginary part corresponding to absorption in the medium. In this case the magnetic field associated with the wave in the medium takes the form (assuming that the electric field of the incident radiation lies in the plane of incidence),

\[
H(r, t) = [0 \ 1 \ 0] \mathcal{H}_0 e^{ik_x x - |k_x^z| z} e^{i\omega t},
\]

where [0 1 0] denotes a unit vector in the y-direction. Because we are dealing with an evanescent mode with \( k_x^2 > (\omega/c)^2 \text{Re}[\varepsilon \mu] \), we have made the transformation \( k_z \rightarrow ik_z \) with real and imaginary parts \( k'_z \) and \( k''_z \), respectively. The electric field can be obtained from Maxwell’s equation, \( \mathbf{E} \times \mathbf{H} = -\varepsilon \varepsilon_0 \mathbf{E} \). Although the imaginary part of the dielectric constant does not affect the amplitude of the evanescent wave, it induces a phase shift as a function of the distance into the medium (that is, the \( z \)-direction). The Poynting vector is now finite and given by

\[
(S)_1 = [\varepsilon_r k_x, \ 0, \ v_x k'_z + \varepsilon_i k''_z] \frac{c^2|\mathcal{H}_0|^2}{8\pi \omega |\varepsilon|} e^{-2k_z z},
\]

where \( \varepsilon = \varepsilon_r + i\varepsilon_i \). Because \( k_z = \sqrt{k_x^2 + \varepsilon \varepsilon_0 (\omega/c)^2} \), we see that

\[
k''_z \approx -\frac{\mu \varepsilon_i (\omega/c)^2}{2\sqrt{k_x^2 + \varepsilon_r \mu (\omega/c)^2}},
\]

that is, \( k''_z \propto \varepsilon_i \). In other words, the Poynting vector is only finite if \( \varepsilon_i \neq 0 \). Hence the energy dissipation \( \sim -dS/dz \) arises, as before, from the imaginary part of the dielectric constant although the exponential decay of the Poynting vector is due to the real part.

The roles of the real and imaginary parts of the dielectric constant in affecting the amplitude of an evanescent wave are the opposite of those for a propagating wave. The decay in the amplitude of an evanescent wave is due to the real (and perhaps, negative) part of the dielectric constant. In contrast, the imaginary part of the dielectric constant, to first order in \( \varepsilon_i \), only causes a phase shift with respect to the distance into the medium. This phase shift can be measured experimentally under suitable conditions. The phase shift as a function of position is crucial: it means that the Poynting vector of the evanescent wave is finite and energy is absorbed by the medium.

III. THE CASE OF ELECTRONS

Matter waves, like electrons, are described by a wave function which, for non-relativistic energies, satisfies the unitary Schrödinger equation. The unitarity of Schrödinger’s equation implies that probability is preserved. This condition
reflects the fact that massive particles, in contrast to photons, cannot be physically absorbed. However, in mesoscopic systems there are many situations in which electrons in one coherent channel are scattered inelastically due to electron-electron and electron-phonon interactions into other channels. Thus when the transmission through the coherent channel is measured using an interferometer, it appears as if the electrons in the specified channel are absorbed. To describe this problem completely, a substantial generalization of the usual scattering formalism would be required to include all relevant interactions and processes. However, the problem can be simplified dramatically by modeling the apparent absorption of electrons using a phenomenological approach based on an imaginary potential.

Imaginary potentials are often introduced in the context of scattering theory and the optical theorem. The presence of an imaginary potential destroys the unitarity of Schrödinger’s equation. For a complex potential of the form $V(r) = V_r(r) - iV_i(r)$ (with $V_i(r)$ a positive definite function), the equation of motion for a wavepacket $Ψ(r, t)$ is

$$i\hbar \frac{∂Ψ(r, t)}{∂t} = \frac{ℏ^2}{2m} \nabla^2 Ψ(r, t) + [V_r(r) - iV_i(r)]Ψ(r, t). \quad (9)$$

The importance of the minus sign in the potential becomes apparent from the equation of motion for the probability density, $P(r, t) = Ψ^∗(r, t)Ψ(r, t)$,

$$\frac{∂P(r, t)}{∂t} + \nabla \cdot S(r, t) = -\frac{2V_i}{ℏ}P(r, t), \quad (10)$$

where $S$ is the particle flux. The imaginary potential clearly acts as a sink for particles.

The imaginary potential in the Schrödinger equation can be eliminated by writing the wavefunction as $Ψ(r, t) = \exp(-V_i t/ℏ)φ(r, t)$, where $φ(r, t)$ obeys the Schrödinger equation in the absence of the imaginary potential. Thus an imaginary potential in the Schrödinger equation is equivalent to a wavefunction that intrinsically decays in time. Thus it is not surprising that imaginary potentials have been widely used to construct phenomenological models for absorption in quantum mechanics.

If the imaginary potential is spatially uniform, then in the absence of any sources of particles, the decay of probability will be uniform in time. In this case it makes sense to examine the steady-state scattering solutions, that is, solutions of the form $Ψ(r, t) = ψ(r) \exp(-iEt)$, which obey (in one-dimension) the time-independent wave equation

$$-\frac{ℏ^2}{2m} \frac{d^2ψ(x)}{dx^2} + (V_r - iV_i)ψ(x) = Eψ(x), \quad (11)$$

where $E$ is the energy of the wavefunction. For constant potentials and $E > V_r$, $ψ(x)$, represents propagating waves of the form $ψ ∼ \exp(ikx) = \exp(ik′x - k''x)$, where

$$k = k′ + ik'' \simeq \sqrt{2m(E - V_r)/ℏ^2} \left[1 + i\frac{V_i}{2(E - V_r)}\right], \quad \text{(12)}$$

assuming $V_i ≪ E - V_r$. The amplitude of the wave decays with distance because of the imaginary component in the potential, in close analogy with the optical case where the imaginary part of the dielectric constant causes a wave to decay in amplitude.

In the classically forbidden region, where $E < V_r$, the wavefunction corresponds to tunneling rather than propagation, $ψ = ψ_0 \exp(-Kx) = ψ_0 \exp[-(K′ - iK'')x]$, where

$$K = K′ - iK'' = \sqrt{2m(V_r - E)/ℏ^2} \left[1 - i\frac{V_i}{2(V_r - E)}\right]. \quad \text{(13)}$$

This behavior is like the optical case of evanescent waves: the decay of the wavefunction with distance is controlled by the real part of the potential, while the imaginary part of the potential gives rise to just a change in the phase. However, the phase change is again important because it is responsible for absorption.

The absorption that arises from evanescent waves that undergo a position-dependent phase shift is nicely illustrated by considering the text-book example of tunneling through a rectangular barrier, shown schematically in Fig. 2. If the wave has energy $E > V_0$ so that it is propagating throughout, then the wavefunction inside the barrier region is given by $ψ = Ae^{ikx} + Be^{-ikx}$, where $k = \sqrt{2m(E - V_0)/ℏ^2}$ and the coefficients $A$ and $B$ are determined by the boundary conditions for the wavefunction. The particle flux takes the form

$$J = -\frac{ℏk}{2m} \left[ψ^∗ \frac{∂ψ}{∂x} - ψ \frac{∂ψ^∗}{∂x}\right] = \frac{ℏk}{m}(|A|^2 - |B|^2). \quad \text{(14)}$$
FIG. 2: Tunneling of a wave across a potential barrier. The dash-dotted lines inside the barrier schematically show the decaying and amplifying evanescent waves inside.

That is, for propagating waves the flux is given by the difference in amplitudes between the forward and backward travelling wave solutions.

For a wave of energy less than the barrier height, $0 < E < V_0$, the wavefunction inside the barrier has the form,

$$\psi(x) = A'e^{-Kx} + B'e^{Kx},$$

with $K = \sqrt{2m(V_0 - E)/\hbar^2}$, and consists of two parts that decay and amplify with $x$, respectively. In this case, the particle current is given by,

$$J = -i\hbar K m (A'^* B' - B'^* A').$$

Crucially, the particle current depends on the phase difference between $A'$ and $B'$, that is, between the decaying and growing solutions. For a barrier of finite width with $0 < E < V_0$, the coefficients $A'$ and $B'$ are complex and there is a finite particle current.

For an incident evanescent wave, the energy is less than zero so the coefficients $A'$ and $B'$ are real as long as $V_0$ is real. Hence there is no particle flux in the entire system — reaffirming that an evanescent wave does not give rise to a particle current. However, in the case of an evanescent wave in an absorbing medium with $\psi = \psi_0 \exp[-Kx] K$ given by Eq. (13), the particle current is

$$J = \frac{\hbar K''}{m} |\psi_0|^2 \exp(-2K'x).$$

Here the change of phase due to the imaginary part of the potential causes a finite amount of absorption, and there is a finite particle current corresponding to absorption in the medium.

IV. IMAGINARY POTENTIALS AND SCATTERING

As we have seen, an imaginary potential in Schrödinger’s equation can account phenomenologically for absorption in a flux of electrons. This approach has been applied to a number of problems in mesoscopics such as the effect of inelastic processes on tunneling resonances in one dimension. However, as Rubio and Kumar have pointed out, an imaginary potential causes scattering in addition to absorption and this means that it must be used with great care. The scattering due to an imaginary potential introduced to model inelastic processes is frequently neglected (see Refs. 7, 5, and 11), but such an approximation is hardly warranted because there is no guarantee that the scattering should be weak.

A localized imaginary potential causes scattering, just like a real potential, because of the inevitable mismatch in the overall potential where the imaginary potential begins and ends. We can see this effect more clearly by considering the example used in Ref. 2 of a complex scatterer located at the origin, $V(x) = (V_r - iV_i)\delta(x)$. We use a delta-function potential because of its simplicity, even though it doesn’t resemble the potential in any realistic mesoscopic system. However, the main features of this simple example carry over to much more realistic mesoscopic systems such as double barrier potentials.
FIG. 3: The absorption (A), reflection (|R|^2), and transmission (|T|^2) coefficients for a wave incident on a δ potential of strength \( V_r - iV_i \). The real part \( V_r = 1 \) and \( m/(\hbar^2 k) = 1 \) in the graph.

By solving the Schrödinger equation for a plane wave, \( \psi(x) \propto e^{ikx} \), incident from the left \( x = -\infty \), we can obtain the transmission (T) and reflection (R) coefficients associated with the potential. The absorption due to the potential, A = 1 - |T|^2 - |R|^2, is given by

\[
A = \frac{2mV_i/\hbar^2 k}{(1 + mV_r/\hbar^2 k)^2 + (mV_i/\hbar^2 k)^2}.
\]

Notice that the absorption goes to zero when \( V_i = 0 \). In contrast, \( R \neq 0 \) when \( V_r \) is zero, but \( V_i \) is finite. Indeed, the behavior of \( R, T, \) and \( A \) with increasing \( V_i \) is surprisingly complex, as shown in Fig. 3. Crucially, the absorption does not increase steadily with \( V_i \), but instead goes through a maximum and then decreases as \( V_i \) is increased further. Thus the idea that the magnitude of the imaginary potential is a simple measure of absorption is incorrect. These surprising properties are by no means peculiar to the δ-function potential, but apply quite generally to complex potentials. We should point out, however, that it is possible to engineer some complex (real + imaginary) spatially varying potentials that completely absorb a wave without reflection or transmission at selected wave numbers. Carefully engineered potentials not withstanding, the basic lesson remains: it is not possible to model inelastic processes with complete fidelity in mesoscopic systems by simply adding an imaginary part to the potential. Such an ansatz always gives rise to additional scattering, which is an artifact of the model rather than physically real.

For light, an imaginary component in the dielectric constant generally gives rise to scattering as well as absorption. However, unlike the case of electrons, the scattering is not an artifact of an oversimplified model of absorption, but is physically real. For electromagnetic systems we are interested in learning how we should tailor the properties of a material in order to observe certain types of behavior. In contrast, in the case of electrons we want to see how to tune the potential in order to faithfully model the physics of a particular system.

In electromagnetism there are exceptions to the link between absorption and scattering. In other words, we can conceive of a material in which there is absorption but no reflection. Consider, for example, the case of light passing from air to a medium with \( \varepsilon = \mu = 1 + i\Delta \) at normal incidence. The reflection amplitude at the interface between the media is \( R = (Z - 1)/(Z + 1) \) where \( Z = \sqrt{\mu/\varepsilon} \) is the impedance. Clearly there will be no reflection for \( \varepsilon = \mu \) as \( Z = 1 \). However, there will still be absorption because the refractive index, \( n = \sqrt{\varepsilon\mu} = 1 + i\Delta \), is complex. The same effect can be obtained by making the imaginary parts of both \( \varepsilon \) and \( \mu \) very large compared to the respective
real parts. This effect turns out to be the basis of the “stealth” materials: a material that absorbs electromagnetic radiation without reflection sends no signal back to a radar and so, in principle, is undetectable. The breaking of the link between absorption and reflection for electromagnetic waves is possible because there are more degrees of freedom to play with compared to the case of electrons. In the Schrödinger equation we are only able to make the potential complex, but in optical materials both the dielectric constant and the permeability constant can be made complex leading to a richer range of phenomena.

In passing, we note that optical media are amplifying when \( \varepsilon_i < 0 \). In contrast to electrons, coherent amplification of light is well-known and arises from stimulated emission — a characteristic unique to bosons. A mismatch in the imaginary part of the dielectric constant alone can cause resonant enhancement of the scattering coefficients when the scatterer is amplifying. For electrons, a positive imaginary component of the potential would formally correspond to the amplification of the particle flux. Indeed, if the potential is a purely imaginary \( \delta \)-function, \( V = -iV_i \delta(x) \), then the reflection and transmission coefficients diverge when \( V_i = -\hbar^2 k/m \), as is clear from Eqs. (13) and (14). However, there exists no analogous process for electrons that can give rise to coherent amplification of the particle flux.

V. PERFECT ABSORBERS

In practice, maximizing the absorption is surprisingly complicated: increasing the imaginary part of the potential (for electrons) or the imaginary part of the dielectric constant (for light) does not simply equate to increased absorption; rather the effect of absorption can be drowned out by scattering. In optics, the way to produce absorption without reflection is to exploit the magnetic properties of the medium. However, for electrons there is no easy way of eliminating reflection entirely, so the question becomes one of optimizing the amount of absorption relative to the amount of reflection.

The problem of maximizing the absorption for the canonical case of electrons incident on a (real) double barrier potential with a constant imaginary potential in the well between the barriers was considered in Ref. [7]. In this case electrons that have energies close to those of the levels inside the well undergo resonant transmission. In the absence of an imaginary potential the electrons spend a time \( \sim h/\Gamma_e \) in the well, where \( \Gamma_e \), the elastic resonance width, is determined by the size of the (real) potential barriers. The trick for maximizing absorption is to ensure that the electrons are trapped in the potential well for as long as possible, while also making \( V_i \) (which gives rise to an inelastic resonance width \( \Gamma_i \) \( \propto V_i \)) as large as possible so that the absorber can gradually “bleed” the probability. However, making the imaginary part of the potential too large leads to additional scattering which starts to destroy the resonance and thus reduce the time spent by the electron in the well. It turns out that the best compromise between dwell-time and absorption is achieved when the elastic and inelastic widths of the resonance are matched, \( \Gamma_e \approx \Gamma_i \).

For non-magnetic optical media, maximizing the amount of absorption is also non-trivial. One particularly interesting case is that of surface plasmon modes at an air-metal interface where the analogy between optical and electronic systems can be used to devise a way of maximizing the absorption of light by making use of a plasmon resonance. At a resonant frequency, \( \omega \), the real part of the metal’s dielectric constant, \( \varepsilon_m(\omega) = -\varepsilon_a \), where \( \varepsilon_a \) is the dielectric constant of air, and surface plasmon modes on the metal-air interface can be excited. These are purely longitudinal solutions of Maxwell’s equations and correspond to charge density oscillations on the metallic surface. A metal’s dielectric permittivity disperses with frequency and the resonance condition can be achieved at some other frequency by swapping air for another medium with a suitable dielectric constant. The surface plasmon modes can be thought of as bound states at the metal surface because their amplitude decays exponentially on both sides of the air-metal interface. However, for a dissipative metal the plasmons cannot exist as stationary states.

Under normal circumstances, the surface plasmon modes cannot be directly excited on a smooth surface by an incident propagating electromagnetic wave, because the wave vector of the surface plasmon is larger than the photon wave vector \( (|k_{sp}| > \omega/c) \). However, it is possible to excite the surface modes on a structured metal surface (grating) or a rough metal surface by a propagating wave. For a metal film on a dielectric medium, we can again excite surface plasmons on the air-metal surface by impinging light at certain angles from inside the dielectric medium. It is also possible to resonantly excite the surface plasmon modes by an incident evanescent wave arising, for example, from the near field modes of an object in air.

Let us consider the case of an evanescent mode incident on a metal surface from air, with dielectric constants \( \varepsilon_m = \varepsilon_m' + i\varepsilon_m'' \) and \( \varepsilon_a \), respectively. For \( p \)-polarized light (the electric field vector lies in the plane of incidence), the magnetic field on the air side of the interface is given by

\[
\mathcal{H} = [0 \ 1 \ 0] \mathcal{H}_0 e^{i(k_x x - \omega t)} \left[ e^{-k_{sp}^a z} + Re^{i(k_{sp}^a z)} \right].
\]
where,

\[
R = \frac{(k_z^{(a)}/\varepsilon_a - k_z^{(m)}/\varepsilon_m)}{(k_z^{(a)}/\varepsilon_a + k_z^{(m)}/\varepsilon_m)},
\]

(22)
is the Fresnel coefficient for reflection, which is obtained by matching the tangential components of the electric and magnetic fields at the interface. Similarly, the magnetic field on the metal side of the interface is given by,

\[
H = [0 1 0] H_0 e^{i(k_x x - \omega t)} \left[T e^{-k_z^{(m)}z}\right],
\]

(23)
where

\[
T = \frac{2k_z^{(m)} k_z^{(a)}/\varepsilon_m \varepsilon_a}{k_z^{(m)}/\varepsilon_m + k_z^{(a)}/\varepsilon_a},
\]

(24)
is the Fresnel coefficient for transmission, and \( k_z^2 - (k_z^{(m)})^2 = \varepsilon_m \left(\frac{c}{\omega}\right)^2 \).

If we calculate the Poynting vector on the metal side, we obtain an expression very similar to that in Eq. (7)

\[
⟨S⟩_t = \left[\varepsilon_m k_x, 0, \varepsilon_m' k_z'' + \varepsilon_m'' k_z'\right] \frac{c^2}{8\pi\varepsilon_m |\varepsilon_m|^2} |T|^2 |H_0|^2 e^{-2k_z'z},
\]

(25)
where \( k_z'' = \text{Im}[k_z^{(m)}] \) and \( k_z' = \text{Re}[k_z^{(m)}] \). Although the energy flow is predominantly along the surface for large \( k_x \), there is a component normal to the boundary that corresponds to the energy dissipated in the metal.

For large \( k_x \), \(|T|^2 \sim 1/(\varepsilon_m'')^2 \) at the resonance (when \( \varepsilon_m' = -1 \)). Under these conditions the absorption is maximized in the limit that the imaginary part of the dielectric constant goes to zero. Increasing the value of \( \varepsilon_m'' \) causes a decrease in absorption because the consequent mismatch in dielectric constants at the boundary causes reflection and prevents the wave from even entering the metal.

The conditions for maximizing absorption in this optical system turn out to be very similar to those for electrons traversing a double potential well. In either case absorption arises from a small imaginary part to the dielectric constant or potential in combination with a resonance due to the corresponding real part. In both cases, the resonance keeps the wave in the vicinity of the absorber long enough for it to suck energy (density) out of the wave.

VI. CONCLUSIONS

We have discussed aspects of light and electron wave motion in dissipative media. Specifically, we have explored the problems associated with propagating and evanescent light in absorbing media and by analogy the inelastic scattering of electrons in mesoscopic systems. Adding an imaginary part to the dielectric constant for light, or the potential for electrons, can be used to model absorption, but it also causes additional scattering. This scattering complicates the use of such phenomenological models to describe inelastic processes for electrons because the additional scattering is an unphysical artifact of the model. In fact, the spurious scattering due to the imaginary part can dominate and actually prevent absorption of the particle flux if it is large enough, which would clearly invalidate the model completely.

Finally, we discussed the problem of maximizing the absorption of a wave. The existence of a bound state for electrons greatly aids the absorption as it ensures that the electron remains in the region of absorption for a long time while a small amount of absorption slowly bleeds the wave without scattering it out of the bound state. In this case the analogy between electrons and light can be used to suggest a way of maximizing absorption of light in a non-magnetic medium by exploiting a resonant surface plasmon state.

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1 L. I. Schiff, *Quantum Mechanics* (McGraw Hill, Singapore, 1968).
Usually the absorption in a material varies with the frequency and is accompanied by dispersion in the real part of the material parameters as well. In fact, the real and imaginary parts of the material parameters are related by the well-known Kramers-Kronig relations. In this paper, however, we only consider a single well-chosen frequency/energy, which is a good approximation for the case when the frequency bandwidth is very small.

P. E. Hodgson, *The Optical Model of Elastic Scattering* (Oxford University Press, Oxford, 1963).

S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995).

A. D. Stone and P. A. Lee, “Effect of inelastic processes on resonant tunneling in one dimension,” Phys. Rev. Lett. **54**, 1196–1199 (1985).

P. E. Hodgson, *The Optical Model of Elastic Scattering* (Oxford University Press, Oxford, 1963).

S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995).

A. D. Stone and P. A. Lee, “Effect of inelastic processes on resonant tunneling in one dimension,” Phys. Rev. Lett. **54**, 1196–1199 (1985).

Y. Zohta and H. Ezawa, “Effect of inelastic scattering on resonant tunneling studied by the optical potential and path integrals,” J. Appl. Phys. **72**, 3584–3588 (1992).

A. Rubio and N. Kumar, “Reflection effect of a localized potential on non-resonant and resonant tunneling,” Phys. Rev. B **47**, 2420–2422 (1993).

M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, 1999).

For magnetic media, a similar contribution will arise from the magnetic polarizability.

S. Yang, J. H. Page, Z. Liu, M. L. Cowan, C. T. Chan, and P. Sheng, “Ultrasound tunneling through 3D phononic crystals,” Phys. Rev. Lett. **88**, 104301–104304 (2002).

M. Jonson and A. Grincwajg, “Effect of inelastic scattering on resonant and sequential tunneling in double barrier heterostructures,” Appl. Phys. Lett. **51**, 1729–1731 (1987).

A. M. Jayannavar, “Absorption-induced coherence in quantum systems,” Phys. Rev. B **49**, 14718–14721 (1994).

S. A. Ramakrishna, E. K. Das, G. V. Vijayagovindan, and N. Kumar, “Reflection of light from a random amplifying medium with disorder in the complex refractive index: Statistics of fluctuations,” Phys. Rev. B **62**, 256–261 (2000).

S. Brouard, D. Marcias, and J. G. Muga, “Perfect absorbers for stationary and wavepacket scattering,” J. Phys. A **27**, L439–L445 (1994).

H. Raether, *Surface Plasmons* (Springer, Berlin, 1988).

The dispersion of the surface plasmons on a smooth air-metal interface is given by $k_{sp} = \frac{\omega}{c} \left( \frac{\varepsilon_m \varepsilon_a}{\varepsilon_m + \varepsilon_a} \right)^{1/2}$. 

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