A Dynamic Model for Continuous Lowering Analysis of Deep-Sea Equipment, Based on the Lumped-Mass Method

Pan Gao 1,*, Keliang Yan 1, Mingchen Ni 2, Xuehua Fu 1 and Zhihui Liu 1

1 College of Ocean Science and Engineering, Shanghai Maritime University, Shanghai 201306, China; 201930410037@stu.shmtu.edu.cn (K.Y.); 201930410015@stu.shmtu.edu.cn (X.F.); 201830410003@stu.shmtu.edu.cn (Z.L.)
2 COTEC Inc., Beijing 102209, China; mcni@cotecinc.com
* Correspondence: pgao@shmtu.edu.cn; Tel.: +86-21-38284837

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Abstract: The installation of subsea equipment is a critical step in offshore oil and gas development. A dynamic model to evaluate the lowering process is proposed. The cable–payload system is discretized as a series of spring dampers with the lumped-mass method. For the first time, not only the lowering velocity but also the rope’s structural damping and the nonlinear loads, such as drag force and snap load, are considered. The lowering velocity of the cable is considered through a variable-domain technique. Snap loads are considered by setting the internal forces in the elements to be zero when the cable slacks. A series of simulations reveals that the lowering velocity has great effects on the dynamic force in the cable. However, the structural damping of the cable has little effect on the system response. The snap load may occur in the cable when subjected to rapid downward heave motion, and decreases with the lowered depth increasing. The cable stiffness affects the system’s resonance depth, but has little effect on the peak dynamic force. The present work should be a valuable reference for future subsea equipment installation analysis.

Keywords: subsea equipment installation; lumped-mass method; variable-domain method; lowering velocity; structural damping; cable stiffness; snap load

1. Introduction

The lifting installation method (LIM) is widely used to install subsea equipment in deep-sea oil and gas development [1,2]. Instead of the traditional installation with drilling strings, the LIM method installs subsea equipment through crane lifting and lowering (see Figure 1). The installation with drilling strings may have some limitations of equipment size by the moonpool dimension, and is usually expensive, due to the high day rate of drilling platforms. By overcoming these disadvantages, the LIM method is much more cost-effective and versatile [3,4]. Even so, it is still one of the more costly operations in offshore oilfield construction, mainly due to the high daily cost of dedicated vessels. To improve efficiency and reduce the cost, improvement of the methodology to give quicker and more accurate predictions of the system response and weather window is critical.

The first studies of subsea equipment lowering were mainly through models in the frequency domain, by linearizing the hydrodynamic force on both the cable and the equipment [5]. Although the frequency domain methods were cost-effective during the simulation time, the simplification of the problem is not practical. Later, the problem was solved in the time domain with single-degree-of-freedom (SDOF) models [6,7] and multi-degree-of-freedom (MDOF) models [8–10]. Richter et al. [11] proposed a MDOF model with the discrete lumped-mass method, incorporating structural rope damping and nonlinear effects,
such as snap loads and drag forces. These studies all analysed the problem with fixed rope length, which could save computational costs. Therefore, the fixed-length method is recommended in the widely-recognized recommended practice (DNV-RP-H103 [12]) for the modeling and analysis of offshore operations, and has been widely adopted for subsea structure lowering operations [13–16]. However, this method does not take into account the effect that the lowering velocity may have on the dynamics of the system.

In the present study, a numerical model to analyse the subsea equipment lowering is proposed based on the lumped-mass method, which is reported to be more effective and widely recognized [24]. In the model, the effects of the rope’s structural damping, lowering velocity, snap loads, and nonlinear drag forces are considered. By integrating all these factors into the model, it differs from previous models and shows some advantages. In addition, this model is solved with the variable-domain method. The present model is believed to provide a valuable tool for the subsea equipment installation analysis.

2. Mathematical Model and Solution

2.1. Mathematical Model

With the lifting installation method, the subsea equipment is usually lowered through a crane mounted on the dedicated vessel. The whole system is comprised of the vessel, the wire rope, and the equipment, forming a vessel–cable–payload system (see Figure 2a). Usually, the equipment’s mass...
is less than 1% of the vessel, so the vessel motion is considered to be independent from the wire rope and subsea equipment [12]. Generally, the cable–payload system, including the wire rope and equipment, is chosen as the research object, while the vessel motion is set as its boundary condition. Due to the relatively small equipment mass and the hydrodynamic damping on the cable-payload system in deep water, the coupling of the payload and the vessel in the horizontal direction is not likely to be significant. Previous work has confirmed that in the absence of time-varying currents or large horizontal excursions of the vessel, the ship and the equipment are only coupled vertically [9,25]. Therefore, only the vertical movement is considered in the present study.

As shown in Figure 2b, the cable–payload system is divided into N segments. In the discretized model, each segment is massless, as its mass is lumped equally to the two mass points at both ends. The system is comprised of a spring and a damper, taking into account the elastic and structural damping behavior of the wire rope. Each mass point is also externally connected to a damper modelling the hydrodynamic damping, e.g., the nonlinear drag force acting on the cable–payload system. The first mass point \( m_0 \) is located at the releasing point at the sea surface, e.g., the crane tip.

Based on the lumped-mass assumption, the mass of each point \( m_i \) is expressed as

\[
m_i = \begin{cases} 
\rho_l \Delta L & \text{for } i = 1, \ldots, N-1, \\
m_p + \frac{1}{2} \rho_l \Delta L & \text{for } i = N 
\end{cases},
\]

in which, \( \rho_l \) is the linear density of the wire rope, \( \Delta L \) is the segment length, and \( m_p \) is the payload.

The displacement and velocity at the crane tip (the \( m_0 \) mass point) are denoted as \( u_0 \) and \( \dot{u}_0 \), respectively, expressing the boundary conditions of the discrete system. In addition, the displacement and velocity of the \( i \)th mass point are denoted as \( u_i \) and \( \dot{u}_i \), respectively.

As shown in Figure 2c, the \( i \)th mass point bears internal forces from the springs and dampers connected at both sides, e.g., \( F_{k_i}^i \) and \( F_{c_i}^i \) from the upper side and \( F_{k_{i+1}}^{i+1} \) and \( F_{c_{i+1}}^{i+1} \) from the lower side. It also endures external forces due to the gravity, buoyancy, and hydrodynamic drag, e.g., the gravitational force \( F_{g_i}^i \), the buoyance force \( F_{b_i}^i \), and the drag force \( F_{d_i}^i \). The exception is that on the \( N \)th mass point, only the internal forces from the spring and damper connected at the upper side are considered.

**Figure 2.** Mechanical model of the subsea equipment installation. (a) Schematic diagram of the operation; (b) lumped-mass model; (c) force balance analysis.
For each segment, the spring and damper impose spring force and damping force on the mass points. The spring force in the \(i\)th spring is

\[
F_k^i = k_i(u_i - u_{i-1}),
\]

where \(k_i\), the axial stiffness of the \(i\)th segment, is defined as

\[
k_i = EA/\Delta L,
\]

in which \(E\) is the Young’s Modulus of the cable and \(A\) is the cross-section area of the cable.

The damping force \((F_c^i)\) in the \(i\)th segment is

\[
F_c^i = c_i(u_i - u_{i-1}),
\]

where \(c_i\), the damping coefficient of \(i\)th segment, is defined as

\[
c_i = 2\zeta \sqrt{k_i \rho_i \Delta L},
\]

in which, \(\zeta\) is damping ratio of the cable [26].

The external forces on the lumped-mass points are mainly the gravitational forces, the buoyancy forces, and the hydrodynamic forces. The temperature varies with water depth, resulting to the variation of physical and mechanical properties of the sea water and the cable–payload system, such as the density and viscosity. However, the variation of these properties is usually insignificant. For example, the sea water’s density variation from 0 to 30 °C is usually less than 1%, which is neglectable for a cable–payload system. Therefore, temperature variation of the seawater and its effects are neglected in the present study. A constant water density is assumed, and the gravitational \((F_g^i)\) and buoyancy \((F_b^i)\) forces on the \(i\)th mass point are expressed as

\[
F_g^i = \left\{ \begin{array}{ll}
\rho_i \Delta L g & \text{for } i = 1, \ldots, N - 1 \\
(m_p + \frac{1}{2} \rho_i \Delta L) g & \text{for } i = N
\end{array} \right.,
\]

\[
F_b^i = \left\{ \begin{array}{ll}
\rho_w \pi D_r \Delta L g & \text{for } i = 1, \ldots, N - 1 \\
\rho_w (V_p + \frac{1}{2} \pi D_r \Delta L) g & \text{for } i = N
\end{array} \right.,
\]

where \(\rho_w\) is the sea water density; \(g\) is the gravitational acceleration; \(D_r\) is the outer diameter of the rope; and \(V_p\) is the displaced water volume by the payload.

As the cross-sectional area of the cable is small, much smaller than the payload, the added mass of the cable is neglected in the model. The hydrodynamic drag force \((F_d^i)\) on each segment is also lumped, then, on each mass point:

\[
F_d^i = \left\{ \begin{array}{ll}
\frac{1}{16} C_{d,r} \rho_w \pi D_r \Delta L \left[ (\dot{u}_i + \dot{u}_{i+1}) \left| \dot{u}_i + \dot{u}_{i+1} \right| + (\dot{u}_{i+1} + \dot{u}_{i-1}) \left| \dot{u}_{i+1} + \dot{u}_{i-1} \right| \right] & \text{for } i = 1, \ldots, N - 1 \\
\frac{1}{16} C_{d,p} \rho_w \pi D_p \Delta L \left[ (\dot{u}_i + \dot{u}_{i+1}) \left| \dot{u}_i + \dot{u}_{i+1} \right| + (\dot{u}_{i+1} + \dot{u}_{i-1}) \left| \dot{u}_{i+1} + \dot{u}_{i-1} \right| + \frac{1}{2} C_{d,p} \rho_w A_p \dot{u}_i \dot{u}_i \right] & \text{for } i = N
\end{array} \right.,
\]

in which, \(C_{d,r}\) and \(C_{d,p}\) are the hydrodynamic drag coefficients of the cable and the payload, respectively, and \(A_p\) is the vertical projected area of the payload. The drag coefficient of the payload may vary with water depth. With a lack of drag coefficient data, it is assumed to be constant in the present study, in order to simplify the analysis. However, it should be noted that the present model is capable of considering varying drag coefficients without major modifications.

For the payload, there is also an inertia force acting on it, which is also known as the added mass effect. This force \((F_a^N)\) is given as

\[
F_a^N = -m_a \ddot{u}_N,
\]
in which, \( m_a \) is the added mass due to the inertia effect, and \( \ddot{u}_N \) is the acceleration of the payload. Applying Newton’s second law for each mass point, the equilibrium equations are given as

\[
m_i \ddot{u}_i = F^g_i - F^b_i - F^k_i - F^e_i + F^{i+1}_c - F^i_{d'}, \quad \text{for } i = 1, \ldots, N - 1, \tag{10}
\]

\[
(m_N + m_d) \ddot{u}_N = F^g_N - F^b_N - F^k_N - F^i_d, \quad \text{for } i = N \tag{11}
\]

To consider the lowering velocity, the boundary condition at the crane tip is defined as

\[
\begin{align*}
    u_0 &= v_l t + S(t), \quad (12) \\
    \dot{u}_0 &= v_l + \dot{S}(t), \quad (13)
\end{align*}
\]

in which, \( v_l \) is the lowering velocity, \( t \) is the time, and \( S(t) \) is the crane tip displacement. In the present study, the crane tip motion is not coupled with the cable–payload system to simplify the numerical model, and the neglect of coupling tends to give a conservative result. A sinusoidal movement is assumed for the crane tip, which is given as

\[
S(t) = H_s \sin(2\pi t/T_s), \tag{14}
\]

where \( H_s \) the crane tip motion amplitude, and \( T_s \) the period.

2.2. Numerical Solution

The discrete system is comprised of a series of elements (e.g., segments) and nodes (e.g., mass points). The above equations can be reformulated into a set of nonlinear ordinary differential equations. They can then be reformatted into the matrix form as Equation (15), which can be efficiently solved with the fourth-order Runge–Cutter algorithm implemented in MATLAB:

\[
[M][\ddot{u}] = [D][u] + [B_\nu][\dot{u}] - [B][\ddot{u}] \ast [\ddot{u}] + [f], \tag{15}
\]

in which, \([M]\) is the mass matrix; \([D]\) is the rigidity matrix; \([B_\nu]\) is the damping matrix; \([B]\) is the viscous matrix; \([f]\) is the load vector; \([u]\), \([\dot{u}]\), \([\ddot{u}]\) are the displacement, velocity, and acceleration vectors of the nodes, respectively; \(\ddot{u}\) is the vector of the average velocity of each element; and \(\ddot{u}\) is the vector of the absolute average velocity of each element. Detailed expressions of these matrices and vectors are listed in Appendix A.

Once the problem is numerically solved, the traction in the cable can be obtained. As half of the mass and the drag force of the first segment is lumped into \( m_0 \), the traction on the top of the cable \( (F_{top}) \) should be calculated by

\[
F_{top} = k_l (u_1 - u_0) + \frac{1}{2} \rho_l \Delta L + \frac{1}{16} C_{d,\nu} \rho w \pi D_r \Delta L (\ddot{u}_1 + \ddot{u}_0) |\ddot{u}_1 + \ddot{u}_0|, \tag{16}
\]

Similarly, the traction on the top of the payload should be calculated as

\[
F_{top} = k_l (u_N - u_{N-1}) - \frac{1}{2} \rho_l \Delta L - \frac{1}{16} C_{d,\nu} \rho w \pi D_r \Delta L (\ddot{u}_N + \ddot{u}_{N-1}) |\ddot{u}_N + \ddot{u}_{N-1}|, \tag{17}
\]

As the wire rope is released through the crane tip, the suspended cable length increases. Therefore, the cable discretization should be adjusted during the solution process. In the present study, the variable-domain method is adopted for the numerical solution. That is to say, the length of each element is increased as the cable length increases.

Each element has an initial length that is one \( N^{th} \) of the whole cable length at the beginning of the solution. Once the wire rope is released, the cable length increases, resulting in an increase of the
element length. At each increment of the solution, the element length should be adjusted according to the whole cable length. The solution technique involved is sketched in Figure 3, and the detailed solution process is as follows.

1. At \( t_0 \), the beginning of the solution, Equation (15) is solved with the fourth-order Runge–Cutter method to get the displacement and velocity of each mass point, e.g., \( u_i^{(0)} \) and \( \dot{u}_i^{(0)} \).
2. At \( t_0 + \Delta t \), the whole length of the cable is increased by \( v_i \Delta t \), and then the element length is increased by \( v_i \Delta t / N \). Equation (15) is updated according to Appendix A, and solved to get \( u_i^{(0+\Delta t)} \) and \( \dot{u}_i^{(0+\Delta t)} \). Then the traction in the cable can be calculated through Equations (16) and (17).
3. Step (2) is repeated until the dedicated lowering depth is achieved.

\[
F_{top} = k_i (u_N - u_{N-1}) - \frac{1}{2} \rho l \Delta L - \frac{1}{16} C_d r \rho w \pi D r \Delta L (u_i^{(1)} + u_i^{(0)}).
\]

\[
F_{bottom} = k_i (u_1 - u_0) + c_i (u_1 - u_0).\]

As the wire rope cannot be compressed, the snap load will be induced if the cable is slack. To calculate the possible snap load, the relative displacement between two connected nodes is carefully monitored during the solution. If the relative displacement between the two nodes of the \( i \)th element is negative, then the spring force \( F_{k,i} \) is set to zero.

2.3. Model Validation

To validate the proposed model, it is compared with the widely recognized commercial software Orcaflex. For the following comparison, as well as for later analysis, a typical subsea equipment installation configuration is used, of which the physical properties are listed in Table 1.

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**Figure 3.** Schematic diagram of the variant-domain method. (a) Initial configuration of the discrete system; (b) adjusted configuration of the discrete system with segment length increased.
Table 1. System parameters for the model validation and analysis.

| Parameter                  | Value         |
|----------------------------|---------------|
| **Cable**                  |               |
| Linear mass, \( \rho_l \) | 24.6 kg/m     |
| Cross-section area, \( A \) | 4098 mm²      |
| Axial stiffness product, \( EA \) | 315 MN |
| Equivalent diameter, \( Dr \) | 72.23 mm     |
| Drag coefficient, \( C_d, r \) | 0.7          |
| Damping ratio, \( \zeta \) | 0.1          |
| Mass, \( m_p \)           | 60 t         |
| Volume, \( V_p \)         | 7.63 m³      |
| Vertical projected Area, \( A_p \) | 56.95 m² |
| Added mass, \( m_a \)     | 300 t        |
| Drag coefficient, \( C_d, p \) | 7           |

In the validation case, the system’s response, considering a lowering velocity and a sinusoidal displacement imposed on the crane tip, is evaluated. The cable–payload system is discretized in 30 elements, and the hydrodynamic force on the payload is considered. The response obtained with the proposed model is compared with that from Orcaflex.

The imposed sinusoidal displacement oscillates, with a period \( (T_s) \) of 9 s and an amplitude \( (H_s) \) of 0.3 m. Figure 4a,b shows the equipment’s motion response at 1500 m and 3000 m water depth, respectively. Comparison of both the amplitude and the phase of payload’s response between the two models show that the present model agrees well with the widely recognized software Orcaflex. Figure 5 presents the traction force on the top of the cable, and on the payload when the equipment is lowered to 1500 m depth. It also shows good agreement between the two models. The traction in the cable oscillates as the cable–payload system is imposed with a sinusoidal excitation.

Figure 4. Comparison of the payload motion under the sinusoidal crane tip excitation \( (H_s = 0.3 \text{ m}, \ T_s = 9 \text{ s}) \) with a payout speed of 0.2 m/s at different water depths: (a) 1500 m depth, (b) 3000 m depth.
Figure 5. Comparison of the traction under the sinusoidal crane tip excitation \((H_k = 0.3 \text{ m}, T_r = 9 \text{ s})\) with a payout speed of 0.2 m/s at 1500 m depth: (a) at the top of the cable, (b) on the equipment.

Figure 6 represents the envelopes of the oscillating forces on the top of the cable and on the payload. As for the traction development in the cable, along with the payload lowering, the present model agrees well with the Orcaflex. It is found in Figure 6a that the traction on the top of the cable increases with the lowered depth. This is because the cable length increases when lowering the payload, increasing cable–payload system’s weight hung at the top of the cable. It can be seen in Figure 6b that the traction on the equipment significantly increases at around 1500 m depth. At this depth, the first-order natural frequency of the cable–payload system is about 0.12 Hz (period of 8.37 s), which is very close to the excitation frequency, so the resonance occurs for the system.

Figure 6. Traction envelopes under the sinusoidal crane tip excitation \((H_k = 0.3 \text{ m}, T_r = 9 \text{ s})\) with a payout speed of 0.2 m/s: (a) at the top of the cable, (b) on the equipment.

3. Results and Discussions

The proposed model features its capability of considering the lowering velocity, the structural damping, and the snap load. Effects of those parameters, as well as the cable stiffness, are evaluated in this section.

3.1. Lowering Velocity Effect

To analyze the lowering velocity effect, a series of simulations are conducted. In these simulations, the system is configured with the parameters listed in Table 1. The payload is lowered, with a payout speed ranging from 0.1–0.9 m/s, while the top of the cable is imposed with a sinusoidal excitation \((H_k = 0.3 \text{ m}, T_r = 9 \text{ s})\). Envelopes of the traction at the top of the cable, in the cases with payout speeds of 0.1, 0.3, and 0.7 m/s, are plotted in Figure 7. It is shown that the maximum top traction during the
lowering process decreases with the lowering velocity, and that the resonance depth changes with the lowering velocity.

![Figure 7](image-url)

**Figure 7.** Traction at the top of the cable in the lowering process, considering different lowering velocities (0.1, 0.3, or 0.7 m/s).

As pointed out by Tommasini et al. [17], the top traction consists of the static force and the dynamic force. The static force is considered to be the force acting on the cable when it is lowered, with the vessel remaining stationary. It is determined by the payout cable length and the lowering velocity. It also increases in a quadratic way with the lowering velocity. By subtracting the static force, the top traction is transformed into the dynamic force. The amplitude of the dynamic force is plotted against the lowered depth for different lowering velocities in Figure 8. It can be seen that the depth of the peak dynamic force decreases with the lowering velocity. However, the peak value tends to first decrease and then increase with the lowering velocity.

![Figure 8](image-url)

**Figure 8.** Influence of the lowering velocity on the dynamic force on the top of the cable.

3.2. **Snap Loads**

If slack occurs to the wire rope during the payload lowering, snap loads will be induced in the rope, bringing great threat to its structural integrity. In order to trigger slack conditions, a rapid downward displacement is imposed at the crane tip. The displacement is defined with a sigmoid function in Equation (18), making the crane tip move 1 m downwards in about 4 s. As mentioned previously, if slack occurs to a segment in the cable–payload system, the traction in the segment is set to zero. Once the segment is tensioned, the traction is restored.

\[
S(t) = \frac{1}{1 + \exp[-10 \times (t - 2)]},
\]  

(18)
To investigate the snap loads due to rapid movement of the crane tip, several cases with lowered depths of 100 m, 300 m, and 1500 m are simulated. In these cases, the payout speed is set as zero, and the sinusoidal vessel motion is neglected. The payload motion and the corresponding traction on the payload are plotted in Figure 9, in which all the cases are studied at the depth when the rapid movement starts. For the case with a 100 m lowered depth, the payload sinks independently from the crane tip when it moves rapidly. In this process, slack occurs to the rope, resulting in zero traction on the payload. After the crane tip motion stops, the payload keeps sinking, and then induces a large traction in the rope. Such large forces are very dangerous for the payload itself and the cable. It is also shown that the longer the suspended cable length is, the less slack is likely to occur, and the lower the snap load is.

![Figure 9](image.png)

**Figure 9.** System response under a rapid downwards crane tip motion for different lowered depths: (a) payload motion; (b) force on the payload.

### 3.3. Damping Effect

To study effects of the structural damping on the payload’s response to sinusoidal top excitation, five different cable damping ratios were considered, while the other parameters remain the same. As the decay test of wire rope is scarce, there is a lack of tested data on the damping ratio. In these cases, the damping ratio varies from 0.01 to 0.40, which is close to the tested value reported in [27]. In addition, the payload is lowered with a velocity of 0.2 m/s up to 3000 m depth, while the top of the cable is imposed with a sinusoidal excitation ($H_s = 0.3$ m, $T_s = 9$ s).

The envelope of the traction in the cable is plotted in Figure 10. It shows that the damping ratio of the wire rope has little effect on the traction on the top of the cable. This is mainly because the structural damping of the cable is relatively small compared to the hydrodynamic damping of payload. If the hydrodynamic force acting on the payload significantly decreases, the structural damping of the cable will have more significant effects on the system’s response.

![Figure 10](image.png)

**Figure 10.** Influence of the damping ratio on the top traction.
Unlike the forced vibration of the payload under a sinusoidal top excitation, the cable–payload system’s response to a rapid top motion is a kind of decaying vibration. The effects of both the hydrodynamic damping and the structural damping on the potential snap loads are studied. First, three cases with the payload’s drag coefficients of 0.7, 3.5, and 7.0 are compared. As shown in Figure 11, under the top excitation expressed in Equation (18), the payload’s drag coefficient has a significant effect on the traction in the cable.

![Figure 11. Influence of the payload’s drag coefficient on the snap load of the cable–payload system in 300 m depth, with a structural damping ratio of 0.1.](image)

Then several cases in which different structural damping ratios ranging from 0 to 0.8 are simulated. As shown in Figure 12a, for the cases with a payload drag coefficient of 0.7, the system’s response in the case of \( \zeta = 0.1 \) is very close to that of \( \zeta = 0 \). Also, if the structural damping ratio increases from 0 to 0.4, the maximum snap load decreases by about 2%. If the payload’s drag coefficient is larger—say, \( C_{dp} = 7.0 \)—the effect of structural damping is smaller, as shown in Figure 12b. Generally, effects of the structural damping on the system response is insignificant. However, consideration of the structural damping of the rope can give a more accurate prediction of the system response.

![Figure 12. Influence of the structural damping ratio on the snap load of the cable–payload system in 300 m depth, with different drag coefficients (\( C_{dp} \)) of the payload: (a) \( C_{dp} = 0.7 \); (b) \( C_{dp} = 7 \).](image)

3.4. Cable Stiffness Effect

In recent years, synthetic rope has been used to replace steel wire rope in some deep-sea and renewable energy applications [28]. Stiffness of these two kinds of ropes is quite different, which may have great influence on the system’s dynamic response. Three cases with different cable stiffnesses are simulated, with variable Young’s modulus and but other cross-section parameters constant.
The equipment is lowered with a velocity of 0.2 m/s to 3000 m water depth. The top of the cable is also imposed with a sinusoidal excitation ($H_0 = 0.3$ m, $T_s = 9$ s).

The motion of the equipment at a 1500 m depth is plotted in Figure 13a. For the cable with a stiffness of 315 MN, the equipment oscillates around its equilibrium position, which is the static elongation of the cable. As the cable stiffness decreases, the static elongation of the cable increases. For the case where $EA = 315$ MN, the depth zone around 1500 m is in the resonant zone of the cable–payload system under excitation, with a period of 9 s, but this is not true for the other two cases. Therefore, the payload’s vibration amplitude is the most significant. Correspondingly, the traction on the payload is the largest (see Figure 13b). It is notable that the cable stiffness also affects the phase of the payload’s response.

For cable–payload systems with different cable stiffnesses, the traction on the top of the cable is plotted in Figure 14. The resonance water depth increases with the cable stiffness. This is because that the natural frequency of the system is mainly determined by the ratio between the cable stiffness and the cable mass. The larger the cable stiffness is, the larger the resonance depth is. From Figure 14b, it can be seen that the peak of the dynamic force is not significantly affected by the cable stiffness.

![Figure 13](image1.png)

**Figure 13.** Comparison of the payload’s response with different cable stiffness coefficients for 1500 m depth: (a) motion of the payload, (b) traction on the payload.

![Figure 14](image2.png)

**Figure 14.** Influence of the cable stiffness on the traction on the top of the cable: (a) traction envelope, (b) dynamic force.

In general, analysis shows that the present model is capable of considering effects of the rope’s structural damping and lowering velocity, as well as nonlinear effects like drag forces and snap loads. In previous studies, none of the models has incorporated all these effects. The present model extends the application of lumped-mass method in continuous lowering analysis, and provides a practical tool for offshore installation design.
4. Conclusions

A dynamic model for continuous lowering analysis of deep-sea structures is proposed, based on the lumped-mass method. The structural damping effect is considered by incorporating the internal damping force in the governing equation. The solution with the variable domain method updates the element length at each time increment to consider the lowering velocity, without bringing any numerical instability. Generally, the proposed model can take into account all the effects of structural damping, lowering velocity, and snap load.

The dynamic force in the cable is significantly affected by the lowering velocity. However, the cable’s damping ratio has little effect on the system response, as the structural damping is usually much smaller than the hydrodynamic damping on the system. The snap load may be induced in the cable if the crane tip moves rapidly downwards. The probability of snap load decreases with the lowered depth. If this occurs, the snap load also decreases with the lowered depth. The cable stiffness has great effects on the system response, as it determines the system’s natural frequency. The resonance depth increases with the cable stiffness, but the peak dynamic force is not significantly affected by the cable stiffness.

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Appendix A

The problem of continuous lowering analysis is modelled with a set of differential equations in the matrix form. For Equation (15), the matrixes and vectors can be expressed with the parameters of the discrete system, as follows:

\[
[M] = \begin{bmatrix}
m_1 \\ \vdots \\ m_N
\end{bmatrix},
\]

\[
[D] = \frac{EA}{\Delta L} \begin{bmatrix}
1 & -2 & 1 \\ & \ddots & \ddots \\ 1 & -2 & 1 \\ 1 & -1
\end{bmatrix}_{N \times N+1},
\]

\[
[B_c] = 2\tau \sqrt{m'EA} \begin{bmatrix}
1 & -2 & 1 \\ & \ddots & \ddots \\ 1 & -2 & 1 \\ 1 & -1
\end{bmatrix}_{N \times N+1},
\]

\[
[B] = \frac{1}{16} \pi D\Delta L\rho_w c_{dr} \begin{bmatrix}
1 & 1 \\ & \ddots & \ddots \\ 1 & 1 \\ 1
\end{bmatrix}_{N \times N},
\]

\[
[u] = \begin{bmatrix}
u_0 \\ u_1 \\ \vdots \\ u_N
\end{bmatrix}^T,
\]

\[
[\dot{u}] = \begin{bmatrix}
\dot{u}_0 \\ \dot{u}_1 \\ \vdots \\ \dot{u}_N
\end{bmatrix}^T.
\]
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