Quantized current steps due to the a.c. coherent quantum phase-slip effect

The a.c. Josephson effect predicted in 1962 and observed experimentally in 1963 as quantized voltage steps (the Shapiro steps) from photon-assisted tunnelling of Cooper pairs is among the most fundamental phenomena of quantum mechanics and is vital for metrological quantum voltage standards. The physically dual effect, the a.c. coherent quantum phase slip (CQPS), photon-assisted tunnelling of magnetic fluxes through a superconducting nanowire, is envisaged to reveal itself as quantized current steps. The basic physical significance of the a.c. CQPS is also complemented by practical importance in future current standards, a missing element for closing the quantum metrology triangle. In 2012, the CQPS was demonstrated as superposition of magnetic flux quanta in superconducting nanowires. However, the direct flat current steps in superconductors, the only unavoidable basic effect of superconductivity to date, was unattainable due to lack of appropriate materials and challenges in circuit engineering. Here we report the direct observation of the dual Shapiro steps in a superconducting nanowire. The sharp steps are clear up to 26 GHz frequency with current values 8.3 nA and limited by the present set-up bandwidth. The current steps were theoretically predicted in small Josephson junctions 30 years ago. However, unavoidable broadening in Josephson junctions prevents their direct experimental observation. We solve this problem by placing a thin NbN nanowire in an inductive environment.

Quantum mechanical duality, a fundamental concept of physics, dictates that the phase and charge of a superconductor are quantum conjugate variables. Under appropriate conditions there is an exact duality between the dynamics of the two variables and their classical counterparts. In both cases, the nonlinear effects lead to the formation of steps on the current–voltage characteristics under microwave radiation. In conventional Josephson junctions, the well-known Shapiro steps appear at voltages and frequencies where the microwave frequency, , is an integer (Fig. 1a). The dual to this is the formation of current steps in coherent quantum phase slip (CQPS) junctions. The steps form at current values , where the charge quantum is the charge of a Cooper pair comprising two electrons of charge (Fig. 1b). To observe the Josephson effect in highly resistive superconductor–insulator–superconductor tunnel junctions, phase fluctuations should be suppressed, which can be done using a shunting capacitance, as shown in Fig. 1c (Supplementary Information). Similarly, for the observation of CQPS, a series inductance should be used. This results in suppression of charge fluctuations (Fig. 1d).

Theory predicts that the dual Shapiro steps (current steps) can be observed in JJs with resistance, , higher than the resistance quantum, , and Josephson energy, , close to the charging energy of the junction, . The Josephson energy, , is determined by the junction critical current , and the charging energy, , by its capacitance . Dual Shapiro steps have been experimentally observed in the differential resistance of JJs. However, the direct observation of the steps in the characteristics has proved difficult in such systems, as subsequent experiments have shown. This difficulty is probably due to hysteretic transport behaviour of junctions with and , and the relatively small value of the energy gap separating the lowest Bloch band from the excited states. The gap is roughly equal to the Josephson energy , where is the superconducting gap, facilitating Landau–Zener tunnelling to higher energy levels and promoting frequent switching between the superconducting and the resistive states. Such processes smear the dual Shapiro steps and significantly reduce the range of bias currents and frequencies at which they can be observed.

An alternative system in which dual Shapiro steps are predicted to occur is a superconducting nanowire, a tunnelling element for magnetic fluxes. In contrast to a JJ, the nanowire may have a high value of the re-trapping current, , below which the superconducting branch of the characteristics is stable. Additionally, the energy gap in the spectrum of an ideal nanowire is very high (approximately ). Moreover, CQPS in nanowires has already been used to demonstrate the superposition of flux states in different materials, and for the demonstration of the interference of CQPS tunnelling amplitudes (the Aharonov–Casher effect).
4.4 μH. The I30 μV (≪ V), where μV. (EY=\frac{\mu H}{\text{nm}}) is delivered via the two coupling capacitances. Most of the devices are measured with the four-probe technique. The leads with estimated sheet resistance of the NbN film with critical temperature of the circuit (Fig. 2a) is made of a 2.7 nm thick superconducting NbN nanowire. A capacitance nanowire. A capacitance c and a resistor R parallel to the Jj are replaced by an inductance L and an admittance Y in series to the CQPS junction.

Here we report the observation of distinct dual Shapiro steps in NbN nanowires under microwave drive. The entire superconducting part of the circuit (Fig. 2a) is made of a 2.7 nm thick superconducting NbN film with critical temperature Tc = 5.8 K grown by atomic layer deposition, with superconducting gap in the BCS limit Δ = 1 meV. The film, with estimated sheet resistance Rs ≈ 4 kΩ, is close to the superconductor–insulator transition point.

The CQPS junction is a constriction with geometrical width of approximately 20 nm and length of approximately 50 nm in a 100 nm wide NbN strip (Fig. 2a). Long, 100 nm wide NbN meanders either side of the constriction form a high kinetic inductance of total L′ ≈ 1.7 μH, with estimated sheet resistance Rs ≈ 10 kΩ, above which the nanowire switches to the normal state. The large excess current Iex = 100 nA suggests the absence of unwanted tunnel JJs inside the nanowire. Conversely, an expanded voltage axis reveals a current blockade at voltages |V| < Vb with Vb ≈ 2.3 μV (Fig. 2c).

Under microwave excitation, current steps develop and the I–V curve is drastically modified. Figure 3 shows steps in the measured I–V characteristic at frequencies of f1 = 14.924 GHz, f2 = 19.845 GHz and f3 = 25.963 GHz. The first two steps for each frequency appear at I = ± 1(2) × Qp, f = ± 2(4) × ef, in agreement with theory. The highest frequency at which the effect is still visible is 31 GHz (I = Qp, f = 10 kA) (Supplementary Information). The frequency is limited by the bandwidth of the transmission lines and can be further increased by optimizing the set-up. Out of a total of 32 measured samples, 9 exhibit dual Shapiro steps, 4 exhibit purely superconducting behaviour and 19 show blockade without signature of CQPS (no steps and most of them with Vb ≈ 50 μV). We have found that only samples with critical voltages in the range 0.2 μV < Vb < 30 μV (<Δ/e = 1 mV) exhibit the dual Shapiro steps.

In the CQPS-dominated regime, magnetic flux quanta tunnel across the nanowire with the net rate proportional to the bias voltage Vb/F0. This process is dual to the charge flow of supercurrent Jbc through an insulating barrier of a JJ with the critical current of JJs = 2πd/Φ0. Hereafter the superscript ‘d’ denotes the JJ case of Fig. 1a. In the CQPS regime, the critical current is replaced by the critical voltage Vb = 2πd/Qp, where Ep is the phase-slip tunnelling energy. The capacitance charge Q = CV is the applied d.c. voltage in the circuit of Fig. 1c, equivalent to the magnetic flux Φ = Lexc determined by the d.c. current in the circuit of Fig. 1d.

To explain the origin of this duality, we briefly summarize the theory of CQPS. It is convenient to characterize the system state by the total number, k, of the flux quanta Φ0 that have crossed the nanowire before a given time. Then, the adjacent states are coupled by the energy Ek. If a supercurrent of the form I(t) = Iexc, cos(ωt), where ω = 2nf, flows through the nanowire, the energy of the state |k) becomes Ek(t) = −kI(t)I0. Hence, the system is described by a simple Hamiltonian

\[ H = \sum_k E_k |k⟩⟨k| - \frac{E_0}{2} (|k⟩⟨k+1| + |k+1⟩⟨k|) \]

(1)

Fig. 1 | Principles of the microwave-induced transport in dual junctions. a. JJ transport. b, CQPS transport. In a and b the I–V characteristics without microwaves (blue curve) and with microwaves (red curve) are shown schematically. Insets show energy diagrams for the microwave-assisted transport between reservoirs separated by tunnel barriers (an insulator and a nanowire) biased by Qp, Vb. c, d, Effective electrical circuits for the transport measurements for Jj (c) and for CQPS (d). Tunneling of Cooper pairs in the JJ is replaced by tunneling of vortices through a CQPS nanowire. A capacitance and a resistor parallel to the Jj are replaced by an inductance L and an admittance Y in series to the CQPS junction.

Fig. 2 | Device and transport. a. The device layout. The superconducting 100 nm wide wire with a constriction of approximately 20 × 50 nm2 geometrical size (zoomed out) is embedded into the circuit with four compact series meandering inductances made of the NbN films with kinetic inductances \( L' = 1.7 \mu H, L'' = 0.5 \mu H \). Inductances are connected to series Pd resistances \( R' = 11.5 k\Omega \) and Pd contact pads. The circuit is connected to current, I/I', and voltage, \( V/V' \), leads. The microwave are delivered through an on-chip coplanar line, coupled to the circuit via capacitances Cc. An inset shows a CQPS junction: a small nanowire constriction. b. I–V characteristics in a wide voltage range demonstrate high re-trapping \( (I_{\text{exc}}) \) and excess \( (I_{\text{exc}}) \) currents. c. An I–V characteristic of the central part. A clear blockade is found with the re-trapping voltage \( V_b = 2.3 \mu V \).
which is widely used in condensed matter physics. For example, it accurately models Bloch oscillations in semiconducting superlattices\textsuperscript{24}. At the resonance condition, \(I_{dc} \Phi_0 = n \hbar \omega = n Q_f \Phi_0\), the flux tunnelling becomes synchronized with the microwave signal and a current step is formed. This is schematically depicted in the inset of Fig. 1b for \(n = 1\).

The eigenstates of the Hamiltonian at \(t \tau = 0\) form the Bloch band. The eigenstates of the Hamiltonian at \(t \tau = 0\) form the Bloch band. The eigenstates of the Hamiltonian at \(t \tau = 0\) form the Bloch band. (dots), together with the simulations shown by solid lines. These oscillations are primarily determined by the Bessel functions of equation (3) with an additional decay caused by temperature rise at increasing power. In Fig. 4d we plot the positions of the peaks in \(dV/dI\) versus frequency for several samples with different critical voltages. We fit these data with \(I_{dc} = Q_{c,f}\) and obtain \(Q_{c,f} = (3.20 \pm 0.01) \times 10^{-12}\), which agrees with the Cooper pair charge \(2e\). In the inset we plot the ratios \(I_{dc}/Q_{c,f}\) for all data points to reveal their scattering.

\[\tau \dot{\theta} + \theta + \omega \sin \theta = \omega_{dc} + \omega_{ac} \cos(\omega t),\]

where \(\theta = 2 \pi n Q_c, \tau = L/R, \omega_{dc} = 2\pi V_{ac}/RQ_c, \omega_{ac} = 2\pi V_{ac}/RQ_a\) and \(\omega = 2\pi V/Q_c\). According to this model, at \(V_{dc} = 0\) and at voltages below the critical voltage, \(|V_{dc}| < V_c\), the mean current \(\langle \dot{\phi} \rangle = 0\). This is indeed observed in the experiment (Fig. 2c). Outside the blockade region, equation (2) predicts a quick drop of the time-averaged voltage \(\langle V_s \sin \theta \rangle\) with increasing current\textsuperscript{1}, which is again consistent with our observations presented in Fig. 2c (see Supplementary Information for details).

Equation (2) also describes JJ dynamics of Fig. 1a,c in the so-called RCSJ model\textsuperscript{13} with the following substitutions \(\theta = \phi, \tau = CR, \omega = 2\pi n L/R\Phi_0, \omega_{dc} = 2\pi n L/R\Phi_0\) and \(\omega_{ac} = 2\pi n L/R\Phi_0\). Using this analogy, one can derive a universal expression for the voltage–current characteristic of a nanowire subject to microwave radiation,

\[V(t) = \sum_n f_n(I_{dc} - Q_{c,f}) V_0(I_{dc} - Q_{c,f}n).\]  

Here \(f_n(x)\) is the Bessel function and \(V_0(t)\) is the voltage–current dependence in the absence of radiation determined by the noise of the environment\textsuperscript{26}. Equation (3) is the dual version of the Tien–Gordon formula\textsuperscript{27}. The conventional Tien–Gordon formula accurately describes the \(I–V\) characteristics of JJs with small critical currents\textsuperscript{28,29}. Equation (3) is universal. It is not limited to the model of equation (2) and remains valid for any external impedance and noise provided \(V_c\) is sufficiently small.

We believe that the finite slope of the current plateaus in the experiment is mainly caused by thermal noise of the resistors, which are heated by the bias currents exerting the Joule power \(P_R = 10^{-8}\) W for the demonstrated steps. Assuming that electron–phonon coupling is the main cooling mechanism\textsuperscript{30}, we estimate the temperature of the resistors at the first plateaus caused by d.c. current in Fig. 3 as \(T = 0.2\) K (Supplementary Information). From the experiment we find the widths of the corresponding peaks in \(dV/dI\) at \(\Delta I \leq 2\) nA (Fig. 4a). The peak width is estimated to be \(\Delta I \approx \sqrt{(4kT/R)}/\sqrt{\Delta f_c} = \sqrt{\hbar Q_c/IT}\). In our LR circuit \(\Delta f_c = R/4L\), which gives \(\Delta f_c = 1\) nA. In the discussed sample, the experimentally observed peaks at non-zero bias are typically close but slightly wider than \(\Delta f_c\). The widths mainly depend on the a.c.-driving current amplitude.

Our estimates of the plateau slopes are valid when \(eV_c \lesssim k_BT\). However, the optimal regime for the steps is in the limit \(eV_c \approx k_BT\). In this case the current blockade at \(|V| < V_c\) becomes very strong, the zero bias resistance peak becomes extremely narrow, \(\Delta R \approx \exp(-2eV_c/k_BT)\), and the dual Shapiro steps become flat. Such a regime has been considered in the original paper on Bloch oscillations in a Josephson junction\textsuperscript{4}.

Simulations based on equation (3) are presented in Fig. 4b (see Supplementary Information for details). Figure 4c shows the oscillations of \(dV/dI\) versus \(I_{dc}\) at the current steps, that is, at \(I_{dc} = nQ_{c,f}\) (dots), together with the simulations shown by solid lines. These oscillations are primarily determined by the Bessel functions of equation (3) with an additional decay caused by temperature rise at increasing power. In Fig. 4d we plot the positions of the peaks in \(dV/dI\) versus frequency for several samples with different critical voltages. We fit these data with \(I_{dc} = Q_{c,f}\) and obtain \(Q_{c,f} = (3.20 \pm 0.01) \times 10^{-12}\), which agrees with the Cooper pair charge \(2e\). In the inset we plot the ratios \(I_{dc}/Q_{c,f}\) for all data points to reveal their scattering.
We demonstrate quantized current steps in photon-assisted tunnelling of magnetic vortices through a nanowire. The a.c. CQPS effect, physically dual to the a.c. Josephson effect, is achieved when the nanowire is in series with high but compact inductances. In addition to the fundamental nature of the phenomenon, the observation of the dual Shapiro steps will have an impact on metrology. Similarly to the ordinary Shapiro steps used for commercial voltage standards, the dual Shapiro steps can be utilized for quantum current standards. In 2018 the General Conference on Weights and Measures announced a redefinition of SI units, including the ampere. The electric current $I$ is required. Currently, there exist several devices exploiting transfer of individual electrons (not on coherent quantum phenomena) as a product of the electron charge by frequency.

So far, quantum standards exist for voltage $V$ and resistance $R$ and, to close the ‘metrology triangle’ (in one of its definitions) with three interrelated quantities $V–R–I$, the direct quantum current standard is required. Currently, there exist several devices exploiting transfer of individual electrons (not on coherent quantum phenomena) that realize such a primary standard\(^6\). The most advanced one is the single-electron pump, in which the electrons are transferred through a semiconducting quantum dot\(^7\). To date, a maximum current up to 1 nA has been reached\(^8\). Further increase of the current might be limited by non-adiabatic excitation of electrons to the higher levels in the dot, which also limits the accuracy of the device. We demonstrate a larger value of the current at the first plateau, approaching 10 nA, and it can be further increased. Importantly, there is also room for optimizing noise and filtering, which will result in flatter current plateaus. The drastic plateau flattening will take place, when the limit $eV_C \gg k_B T$ is achieved.

**Online content**

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24. Glück, M., Kolovsky, A. R. & Korsch, H. J. Wannier–Stark resonances in optical and semiconductor superlattices. Phys. Rep. **366**, 103–182 (2002).

25. Barone, A. & Paterno, G. Physics and Applications of the Josephson Effect Vol. 1 (John Wiley & Sons, 1982).

26. Averin, D. V., Nazarov, Y. V. & Odintsov, A. A. Incoherent tunneling of the Cooper pairs and magnetic flux quanta in ultrasmall Josephson junctions. *Physica B* **165**, 945–946 (1990).

27. Tien, P. K. & Gordon, J. P. Multiphoton process observed in the interaction of microwave fields with the tunneling between superconductor films. Phys. Rev. **129**, 647–651 (1963).

28. Roychowdhury, A., Dreyer, M., Anderson, J. R., Lobb, C. J. & Wellstood, F. C. Microwave photon-assisted incoherent Cooper-pair tunneling in a Josephson STM. Phys. Rev. Appl. **4**, 034011 (2015).

29. Kot, P. et al. Microwave-assisted tunneling and interference effects in superconducting junctions under fast driving signals. Phys. Rev. B **101**, 134507 (2020).

30. Giazotto, F., Heikkinen, T. T., Luukanen, A., Savin, A. M. & Pekola, J. P. Opportunities for mesoscopics in thermometry and refrigeration: physics and applications. Rev. Mod. Phys. **78**, 217 (2006).

31. Giblin, S. P. et al. Realisation of a quantum current standard at liquid helium temperature with sub-ppm reproducibility. Metrologia. **57**, 025013 (2020).

32. Yamahata, G., Giblin, S. P., Kataoka, M., Karasawa, T. & Fujiwara, A. High-accuracy current generation in the nanoampere regime from a silicon single-trap electron pump. Sci. Rep. **7**, 45137 (2017).

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Data availability
The datasets used to produce the plots are available in the Open Science Framework (OSF) repository, https://osf.io/4sn3w/.

Acknowledgements
This work was supported by European Union’s Horizon 2020 Research and Innovation Programme under grant agreement no. 862660/QUANTUM E-LEAPS and Engineering and Physical Sciences Research Council (EPSRC) grant no. EP/T004088/1.

Author contributions
O.V.A. proposed, simulated and planned the experiment and circuit design and analysed data. R.S.S. made the major contribution to the design and fabrication of various samples, planned the experiment and analysed data. R.S.S. made the experiments with an important contribution from K.H.K. J.W.D., I.V.A., V.N.A. and K.H.K. designed and fabricated samples, analysed data and prepared figures. S.L., M.Z. and E.V.I. developed technology and M.Z. and S.L. fabricated NbN films. D.S.G. provided theory and simulations of the experiment. O.V.A., E.V.I., D.S.G. and V.N.A. wrote the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41586-022-04947-z.

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Peer review information
Nature thanks Mikael Fogelstrom, Masaya Kataoka and the other, anonymous, reviewer(s) for their contribution to the peer review of this work. Peer reviewer reports are available.

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