Hybrid Influence Index Based on Endpoint Attribute Diversity in Link Prediction in Complex Networks

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ABSTRACT

Prediction of links in complex networks aims to study possibility of forming a link between unconnected nodes, with the information contained in network, mainly including nodes, links and topological structure. In recent years, it has caused amounts of attention in various fields and achieved great success, especially in the indices based on topological similarity. However, most topological similarity indices incline to predict links based on transmission paths rather than endpoints. In addition, they typically measure endpoint influence only by degree, ignoring the role of other attributes of endpoints. Therefore, considering the diversity between endpoint attributes, we propose a method named Hybrid Influence Index based on Endpoint Attribute Diversity (HED), to specifically locate the impact of endpoint attributes in the measurement of endpoint influence. Finally, experiments based on twelve datasets of real world account for the superiority of our model in accuracy.

KEYWORDS

Link Prediction, H-index, Node Degree, Hybrid Endpoint Influence, Diversity.

INTRODUCTION

Link prediction of complex networks investigates the possibility of forming a links between two nodes through information about known nodes, links and network structure [1]. In recent years, numerous researches have been done on the issue of complex networks, and formed a relatively complete theoretical system [2],[3], which provides a data tool for studying real network, such as recommendation system [4], industrial system [5], disease control [6], social networks [7], etc. And based on complex network theory, researchers have made great achievements in link prediction fields, such as discovering potential friends among strangers through existing friends in social networks [8], searching key information fast in collaborative filtering mechanism [9],[10], identifying potentially important connections in Internet topology network, air transport system, and power networks [11],[12], predicting theoretical threshold of epidemic outbreak in disease control [6],[13], and exploring the interactions between proteins and the relationships between food and metabolic tissue in bio-ecological networks [14].
Link prediction, with high practicability and research value, has aroused great interest of scholars from multiple research fields and achieved great achievements [15], especially the indices based on topological similarity [8],[15]. The methods calculate similarity of two unlinked nodes based on network topological structure, and predict links according to the similarity order. They believe that the possibility of forming a link between two nodes increases with the increase of node similarity.

The models have three castes for the division of length of transmission paths in the topological similarity based ones: local similarity based, global similarity based and quasi-local similarity based models. Indices based on local node similarity focus on the role of local transmission paths through neighbors connected to both endpoints, also known as common neighbors. For instance, Common Neighbors (CN) [16] models node similarity with the number of neighbors connected to both endpoints at the same time. Resource Allocation (RA) [17] and Adamic-Adar Index (AA) [18] take into account the heterogeneity of neighbors connected to both endpoints in transmission paths. What's more, Leicht-Holme-Newman Index (LHN) [19] and Sørensen Index [20] model node similarity with the suppression of role of endpoints possessing large degree. Indices based on global similarity consider all the information contained in the network. For example, Katz Index [21] takes emphasis on short path and computing all transmission paths between endpoints. However, local similarity indices possess low accuracy with only considering local paths, and global similarity methods possess high computational complexity with an overall consideration of network structure. Quasi-local similarity models balance accuracy and complexity, like Local Path Index (LP) [17], considering paths of two steps and three steps with two steps preferred, Local Random Walk Index (LRW) [22], considering special length of paths with degree as endpoint influence, while Superposed Random Walk Index (SRW) [22], on account of the idea of LRW, calculating the superposition of different lengths of step.

The topological similarity indices predict links based on network structure, which mainly includes nodes and links. However, most researchers pay more attention to transmission links in modeling, ignoring the role of endpoints. And for endpoint influence measurement, they usually neglect the impact of multiple attributes of endpoints, and measure it only by degree, like LHN [19], Sørensen Index [20], LRW and SRW [22]. Recently, a related study [23] indicates that node attribute, H-index possesses natural importance and advantage in measuring the role of endpoints. So, in order to specifically locate the impact of endpoint attributes in the measurement of endpoint influence, based on an excellent link prediction model Superposed Random Walk Index (SRW), which measures the effect of endpoints only by degree, we make a research and propose a model named as Hybrid Influence Index Based on Diversity of H-index and Degree (HED). Through experiments in twelve real networks, we find the model considering the diversity between H-index and degree of network datasets does improve accuracy of predicting links.

TOPOLOGICAL SIMILARITY BASED INDICES

Experimental Preparation of Link Prediction

As mentioned above, the complex networks can be described as sets of nodes and links, $G(V,E)$, among them, $V$ aggregates network nodes, and $E$ aggregates
network links. In order to measure the possibility of the existence of unconnected links, we derive score \( s_{xy} \), an index to measure similarity, of each pair of nodes, where \( x, y \in V \). Links between nodes with top \( L \) similarity scores exist with a more possibility. Moreover, we only make experiments in undirected and unweighted networks, and preprocess the datasets, deleting self-connected links and multiple connections between two nodes.

And more, we randomly select 90% links of each network as training set, named as \( E^T \), with the remaining 10% defined as testing set, which is named \( E^P \). To avoid contingency, we divide each dataset into 30 times and take the average result of the 30 divisions as the performance of indices.

**Classical Indices based on Topological Similarity**

In this paper, we list several classical similarity based methods, which are sown as follows, and compare our index with them:

1) Common Neighbors(CN) [16], the most basic similarity calculation index, equates node similarity with number of intermediate nodes which connect both endpoints,

\[
s_{xy}^{CN} = |\Gamma(x) \cap \Gamma(y)|,
\]

among them, nodes \( \in \Gamma(x) \) are intermediate ones connect endpoint \( x \) and nodes \( \in \Gamma(y) \) connect endpoint \( y \), and nodes \( \in \Gamma(x) \cap \Gamma(y) \) connect both \( x \) and \( y \).

2) Adamic-Adar Index(AA) [18], with the consideration of heterogeneity of common neighbors, computes similarity by

\[
s_{xy}^{AA} = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\log k_z},
\]

among them, \( z \) belongs to the connection of common neighbors of endpoint \( x \) and endpoint \( y \), and \( k_z \) refers to node degree of \( z \).

3) Resource Allocation(RA) [17], just like AA, also considers heterogeneity. The difference is that RA is heterogeneous in reciprocal of node degree, and AA is heterogeneous in logarithmic reciprocal of node degree, respectively. So, the similarity is

\[
s_{xy}^{RA} = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{k_z}.
\]

4) Local Path (LP) [17], mainly focusing on paths with length of 2 and 3 and defaulting 2 step paths priority over 3 step paths, models node similarity as

\[
S^{LP} = A^2 + \varepsilon A^3,
\]

where \( A \) is the adjacency matrix of the network and \( \varepsilon \) is uncertain parameter, generally much less than 1.
(5) Superposed Random Walk Index (SRW) [22] innovatively puts forward concept of endpoint influence and measures it by degree. On the basis of Markov chain, SRW models the similarity as

\[
s_{xy}^{SRW}(t) = \sum_{l=2}^{t} \frac{k_x}{2|E|} \pi_{xy}(l) + \frac{k_y}{2|E|} \pi_{yx}(l).
\]

where \(l\) is length of path changes from 2 to \(t\).

SIMILARITY INDEX BASED ON ENDPOINT ATTRIBUTE DIVERSITY

Superposed Random Walk Model: SRW

To be more logical, we first introduce the core idea of SRW [22], in which nodes transfer resource based on Markov chain. Assuming that each node in the network possesses a unit of resource and the resource is evenly allocated to the neighbors of the node, the resource transmission probability from \(x\) to \(y\) is \(p_{xy} = 1/k_x\), where \(k_x\) refers to degree of node \(x\). And \(p_{xy} = 0\) if there is no link between node \(x\) and \(y\).

According to the definition of Markov chain, the \(l\)-length transmission path between endpoints \(x\) and \(y\) can be described as a node sequence \(\{x = x_0 = y_1, x_1 = y_{l-1}, ..., x_{l-1} = y_{l-1}, x_l = y_0 = y\}\). So the resource transmission probability, \(\pi_{xy}(l)\), of \(l\) steps from node \(x\) to node \(y\) is \(\pi_{xy}(l) = \prod_{i=0}^{l-1} p_{x_i,x_{i+1}}\). Similarly, the probability of resource transfer of \(l\) steps from node \(y\) to node \(x\), \(\pi_{yx}(l)\), is \(\pi_{yx}(l) = \prod_{i=0}^{l-1} p_{y_i,y_{i+1}}\). In addition, SRW innovatively considers the role of endpoints on resource transmission, with degree as the metric to express endpoint influence. So, based on endpoint influence and Markov chain, the resource transfer probability between node \(x\) to \(y\) with a \(l\)-step path is

\[
s_{xy}(l) = \frac{k_x}{2|E|} \pi_{xy}(l) + \frac{k_y}{2|E|} \pi_{yx}(l),
\]

among them, \(\frac{k_x}{2|E|}\) and \(\frac{k_y}{2|E|}\) measure the normalized influence of endpoint \(x\) and \(y\), respectively.

Superposed Random Walk Index (SRW) calculates all paths within a specific length of step, and models the similarity as

\[
s_{xy}^{SRW}(t) = \sum_{l=2}^{t} s_{xy}(l).
\]
Hybrid Influence Index Based on Diversity of H-index and Degree: HED

Based on idea of endpoint influence and Markov chain, SRW achieves excellent performance in prediction accuracy. However, SRW measures endpoint influence only by degree, ignoring the role of multiple attributes on endpoint influence. Recently, researchers [23] have proved that the node attribute, H-index [24], performs well in measuring endpoint influence. So, we consider the diversity between endpoint H-index and endpoint degree, and model endpoint influence as \( k^{1-\alpha} h^\alpha \). Thus, we can know that node similarity of endpoint \( x \) and endpoint \( y \) through transfer \( l \)-step path based on Local Hybrid Influence Index Based on Diversity of H-index and Degree (LHED) is

\[
 s_{xy}^{LHED}(l) = \frac{k_x^{1-\alpha} h_x^\alpha}{2|E|}, \pi_{xy}(l) + \frac{k_y^{1-\alpha} h_y^\alpha}{2|E|}, \pi_{yx}(l)
\]  

(8)

where \( h_x \) denotes the H-index of node \( x \), \( \frac{k_x^{1-\alpha} h_x^\alpha}{2|E|} \) and \( \frac{k_y^{1-\alpha} h_y^\alpha}{2|E|} \) measure the normalized influence based on synthetical attribute diversity of endpoint \( x \) and \( y \), respectively, and \( \alpha \) is the weight factor, measuring the diversity of H-index and degree.

With the path length increasing from 2 to \( t \), node similarity modeled by Hybrid Influence Index Based on Diversity of H-index and Degree (HED) can be described as

\[
 s_{xy}^{HED}(t) = \sum_{l=2}^{t} s_{xy}^{LHED}(l).
\]  

(9)

Obviously, HED degenerates to SRW when \( \alpha = 0 \).

RESULTS AND DISCUSSIONS

Datasets

To verify that hybrid diversity of H-index and degree do work, we do research on twelve datasets which can described in the following:

(1) US Air97 (USAir), composed of American aerospace transport network, with nodes denoting airports and links referring to routes between airports; (2) Yeast PPI (Yeast), mapping interaction between yeast proteins with nodes referring to proteins and edges represent interactions; (3) Food Web (Food), carbon exchange network of cypress in South Florida Wetlands; (4) C.elegans (CE), describing neural network of the nematode worm C.elegans. Among them, nodes refer to neurons, links refer to the synapse or gap junction of neurons; (5) NetScience (NS), composed of scientists who have published papers on complex networks, with nodes denoting scientists, and links denoting cooperative relationships among scientists; (6) Jazz, communication among Jazz musicians; (7) Email network (Email), describing the communicative relations among students in University of Virginia (URV) in
Taragona via email; (8) Slavko, composed of communicative relations of Slavko Zitnik on Facebook; (9) UC (Uc-Social), illustrating the social relationship of students studied at the University of California, Irvine, based on their online communication; (10) Infectious (Infec), composed of Face-to-face communication among people attending the exhibition "Infectious Diseases: Far from" held at the Dublin Science Museum in 2009; (11) EuroSiS (ES), describing the connections between Science and sociologists from twelve European countries; (12) Power Grid (Power), western electric power network of the US, with nodes representing substations or converters and links representing high-voltage lines. And we illustrate some basic parameters of the networks in Table I.

TABLE I. SOME COMMON NETWORK TOPOLOGY PARAMETERS ARE SHOWN IN THIS TABLE, WHERE |V| DENOTES THE CALCULATION OF AMOUNTS OF NODES, <K> DENOTES AVERAGE DEGREE OF NODES IN THE NETWORK, H CALCULATES DEGREE HETEROGENEITY BY $H = \frac{(k^2)}{(\langle k \rangle)}$, |E| DENOTES THE CALCULATION OF AMOUNTS OF LINKS, <D> DENOTES THE AVERAGE DISTANCE, C INDICATES CLUSTERING COEFFICIENT, AND R IS ASSORTATIVITY COEFFICIENT.

| Nets  | |V| |<k>| |H| ||E| |<d>| |C| |r|
|-------|------|-----|----|-----|----|------|-----|----|-----|----|------|
| USAir | 332  | 12.81| 3.36| 2128| 2.74| 0.749| -0.208|
| Yeast | 2370 | 9.2  | 3.35| 10904| 5.16| 0.378| 0.469|
| Food  | 128  | 32.42| 1.24| 2075 | 1.78| 0.334| -0.112|
| CE    | 453  | 8.94 | 4.49| 2025 | 2.66| 0.655| -0.225|
| NS    | 1461 | 3.75 | 1.85| 2742 | 5.82| 0.878| 0.461|
| Jazz  | 198  | 27.7 | 1.4 | 2742 | 2.24| 0.633| 0.02 |
| Email | 1133 | 9.62 | 1.94| 5451 | 3.61| 0.254| 0.078|
| Slavko| 334  | 13.28| 1.62| 2218 | 3.05| 0.488| 0.247|
| UcSocial | 1893 | 14.62| 3.81| 13825 | 3.06| 0.138| -0.188|
| Infec | 410  | 13.49| 1.39| 2765 | 3.63| 0.467| 0.226|
| EuroSiS| 1272 | 10.15| 2.46| 6454 | 3.86| 0.382| -0.012|
| Power | 4941 | 2.669| 1.45| 6594 | 15.87| 0.107| 0.003|

Metrics

The performance of indices are measured by AUC in this paper. Each time, we randomly compare the scores of two links, one of which is testing link in $E'$ and the other is nonexistent link in non-exist set (links in $U \setminus E$, and $U$ refers to the collection of universal links). If the score of testing set is higher than that of nonexistent link, we mark this comparison as 1. And if the two scores are equal, we mark 0.5. To avoid accidental results and increase the reliability of measurements, we make $n$ independent repetition comparisons. So, with scores of testing links greater than and equal to the nonexistent links $n'$ and $n''$ times, respectively, the AUC is
Results and Analysis

At the end of the study, we illustrate the performance of HED measured by AUC on twelve datasets in accuracy, compared with several classical baselines. The results and discussions are shown below.

\[
AUC = \frac{n' + 0.5n''}{n}.
\]  

Figure 1. Dependence of AUC on the value of \( \alpha \). \( t = 6,10,15 \) are contrast lines, where data points of \( t = 6 \) represented by circles and connected by magenta dots, data points of \( t = 10 \) represented by triangles and connected by blue dash-dots, and data points of \( t = 15 \) represented by stars and connected by green dash-dot-dots, compared with the grids connected by red dashes, which denote the data points with the optimal step \( t \). And solid graphics are the optimal AUC at the given step.

Based on SRW, we propose a model named HED considering the diversity of endpoint attributes with weighting factor \( \alpha \), and illustrate the relationship between AUC and \( \alpha \) in Figure 1. In the figure, each date point is the average of the 30 divisions. And we choose step \( t = 6,10,15 \) as the contrast lines, where data points of \( t = 6 \) represented by circles and connected by magenta dots, data points of \( t = 10 \) represented by triangles and connected by blue dash-dots, and data points of \( t = 15 \) represented by stars and connected by green dash-dot-dots, compared with the grids.
connected by red dashes, which denote the data points with the optimal step $t$. What's more, solid graphics are the optimal AUC for the given step.

In Figure 1, we can find that AUC of our proposed index HED varies with the value of $\alpha$ and length of step. And the optimal AUC can be obtained at specific value of step and $\alpha$. Moreover, with the increase of length of step, the optimal $\alpha$ under a given step also presents a increasing trend in most datesets, especially in USAir, Jazz, Food, Email, EuroSiS, UcSocial, Slavko, CE, Infec and NS. That's attributable to the different connotations of H-index and degree. Node degree refers to the number of its neighbor nodes, and H-index of a node means that there are $h$ nodes in its $N$ neighbors possess at least $h$ neighbors, while the other $N-h$ nodes possess fewer than or equal to $h$ neighbors. So, it is obvious that degree plays a more important role in shorts paths, and H-index affects more in longer pathes.

What's more, we also list AUC of HED and the baselines CN, AA, RA, LP and SRW in Table II. The bold characters in the table refer to the optimal values of AUC of the dataset. The values in parentheses represent the optimal step $t$ of SRW, optimal step $t$ and value of $\alpha$ of HED, respectively. Obviously, HED shows excellent performance in prediction accuracy in most datasets. And most of the optimal $\alpha$ of each dataset is less than 0.5, more clearly confirming that degree affects more in endpoint influence measurement based on diversity than Figure 1.

### Table II. Performance of the Model in 12 Datasets Measured by AUC

| Dataset  | AUC    | CN   | AA   | RA   | LP   | SRW             | HED                        |
|----------|--------|------|------|------|------|-----------------|-----------------------------|
| USAir    | 0.977771 | 0.984248 | 0.986586 | 0.977503 | 0.989735(3) | **0.989947(3,0.3)** |
| Yeast    | 0.736946 | 0.737136 | 0.73706 | 0.742918 | 0.744316(3) | **0.744345(3,0.1)** |
| Food     | 0.616391 | 0.617368 | 0.619442 | 0.633358 | 0.770706(3) | **0.770706(3,0)** |
| Power    | 0.679613 | 0.679727 | 0.679622 | 0.763958 | 0.949572(14) | **0.949572(14,0)** |
| NS       | 0.990227 | 0.990342 | 0.990346 | 0.994158 | 0.995607(8) | **0.995608(8,0.1)** |
| Jazz     | 0.972242 | 0.976373 | 0.981301 | 0.970142 | 0.981323(2) | **0.982077(2,0.3)** |
| Email    | 0.881955 | 0.883291 | 0.882445 | 0.944844 | 0.956087(6) | **0.957493(12,0.7)** |
| Slavko   | 0.964026 | 0.965894 | 0.965701 | 0.968414 | 0.971663(4) | **0.971673(4,0.1)** |
| UcSocial | 0.813094 | 0.817374 | 0.817516 | 0.915291 | 0.950102(7) | **0.95212(11,0.6)** |
| Infec    | 0.962318 | 0.964185 | 0.964189 | 0.976616 | 0.980476(4) | **0.980693(4,0.3)** |
| EuroSiS  | 0.955269 | 0.956569 | 0.956009 | 0.980853 | 0.985383(5) | **0.98542(5,0.1)** |
| CE       | 0.951545 | 0.977094 | 0.979047 | 0.9578 | 0.985173(3) | **0.985411(4,0.5)** |
In general, considering the random walk based on Markov Chain and endpoint influence measurement based on node attributes diversity, HED reaches an excellent performance in prediction accuracy. CN, AA, RA and LP get low accuracy because they only consider local paths. And SRW ignores the diversity of node attributes in measuring endpoint influence, even if it considers long paths.

What's more, the time complexity can also reflect the performance of the indices. According to mathematical tools, it costs $O(N^3)$ time complexity to compute multiplication of two $N$-order matrices. Obviously, the time complexity of the matrix multiplication based indices, like CN, AA, RA, is $O(N^3)$. Furthermore, according to the definition of LP and SRW, time complexity of LP and SRW is $m \times O(N^3)$. Therefore, HED achieves better performance in accuracy with the same time complexity as SRW.

CONCLUSION

Link prediction models based on traditional similarity study the possibility of links between unconnected nodes according to network information. Generally speaking, traditional indices prefer link-based prediction, regardless of the role of endpoints. Recently, SRW achieves excellent performance in prediction, with innovatively measuring endpoint influence by degree. However, SRW still ignores the diversity of endpoint attributes in measuring endpoint influence. Fortunately, a related study by Lu et al. [23] shows that the H-index possesses natural importance and advantages in measuring the role of endpoints. So, based on SRW, we propose a model named Hybrid Endpoint Algorithm Based on Diversity of H-index and Degree(HED), considering diversity between endpoint attributes. And overall, HED reaches a better performance in accuracy, with the comparison of classical baselines, like SRW, CN, RA, AA and LP. In addition, it is believed that our research will provide some inspiration for future link prediction research.

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