Quasiparticle states for integer- and fractional-charged electron wave packets

X. K. Yue and Y. Yin

1Department of Physics, Sichuan University, Chengdu, Sichuan, 610065, China

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It is well-known that Lorentzian voltage pulses with integer quantum flux can inject integer-charged wave packets without electron-hole pairs. The wave packets are composed of soliton-like quasiparticles on top of the Fermi sea, which have been named as “levitons”. However, it is not clear what kind of charged quasiparticles can be injected by Lorentzian pulses with fractional quantum flux. To answer this question, we study the wave packets injected by a train of Lorentzian pulses with repetition period $T$. We introduce a set of single-body wave functions, within which the quantum state of the charged quasiparticles can be described for pulses with arbitrary quantum flux. We find that in general case the charged quasiparticles are injected with two different periods. For pulses with integer quantum flux, the two periods match each other. The charged quasiparticles are levitons in this case, which are injected with a single period $T$. For pulses with fractional quantum flux, the two periods mismatch. The charged quasiparticles can then be injected in a multi-periodic way. This makes the each quasiparticle can carry only a fractional electric charge into the quantum conductor within a single period $T$. These quasiparticles can have pronounced impact on the charge injection. In particular, they can lead to the cycle-missing event, in which the voltage pulse fails to inject an electron within a single period $T$. The cycle-missing event can be seen intuitively from the waiting time distribution between electrons above the Fermi sea, which exhibits a series of peaks at multiplies of the period $T$. By using the wave functions of the charged quasiparticles, we elucidate in detail how a leviton evolves as the flux of the pulse changes. In the meantime, we also clarify how additional $e\!h$ pairs can be excited.

I. INTRODUCTION

In the past decade, much effort has been devoted to the on-demand single-electron source, within which electron wave packets carrying single or few electric charges can be injected coherently into a quantum conductor [1–22]. In a simple way, such injection can be realized by applying a nanosecond pulse on the Ohmic contact of the conductor, as illustrated in Fig. 1. The injected charges $Q$ of the wave packet is decided by the flux $\varphi$ of the pulse, while the detailed quantum states of the wave packet can be controlled via fine-tuning the profile of the pulse. This offers a simple but feasible approach to archive the time-resolved quantum control of propagating electron wave packet in solid-state circuits [23–36].

![Fig. 1. (a) Schematic of the on-demand electron injection via the voltage pulse $V(t)$. By applying $V(t)$ on the contact of the quantum conductor, electron (hole) or $e\!h$ pairs from the reservoir (region I) can be injected into the quantum conductor (region III). The voltage drop is assumed to occur across a short interval at the interface (region II). (b) Schematic of the applied voltage pulse train. The pulse train is composed of identical Lorentzian pulses, which can be characterized by half width at half maximum $W$ and Faraday flux $\varphi$. These pulses are separated by a time interval $T$. Generally speaking, the wave packet is composed of charged quasiparticles in the Fermi sea ($|F\rangle$) of the conductor, which are usually accompanied by a neutral cloud of electron-hole ($e\!h$) pairs [37, 38]. Remarkably, it is](http://arxiv.org/pdf/2004.00743v2)

* Author to whom correspondence should be addressed; yin80@scu.edu.cn.
possible to inject a “clean” wave packet without eh pairs, which can be done by tuning the pulse to be a Lorentzian with integer quantum flux [3, 33]. In doing so, one obtains soliton-like quasiparticles propagating on top of the Fermi sea, which has been named as “levitons” [2, 8]. Each leviton carries an unit electric charge and has a well-defined wave function. A sequence of well-separated levitons can be injected by using a train of Lorentzian pulses, emerging as promising candidates for flying qubits in solid state circuits [40–44].

By using a Lorentzian pulse with fractional quantum flux, one can inject a wave packet carrying fractional charges, which has a quite different structure. On one hand, it contains a large amount of eh pairs, which is closely related to the dynamical orthogonality catastrophe [15, 16]. On the other hand, it can sustain charged quasiparticles carrying fractional charges. The structure of the wave packet has been demonstrated for the Lorentzian pulse with a half-quantum flux. In this case, the quantum state of the wave packet can be decomposed into two mixed states: one represents the neutral cloud of eh pairs, while the other one can be regarded as a zero-energy quasiparticle carrying an effective e/2 charge [47]. This makes the wave packet show distinctly different features from the wave packet built from levitons [29, 38, 39].

Intuitively, one expect that the fractional-charged quasiparticles can be injected in a similar way as levitons, providing an alternative approach to realize flying qubits. However, the nature of these quasiparticles has not been fully understood yet. In particular, it is not clear how a leviton can evolve into a fractional-charged quasiparticle as the flux of pulses changes. To answer this question, one needs to describe the quantum states for both integer- and fractional-charged wave packets in an unified manner, which has not been given yet.

In this paper, we attack this problem by examining the case when a Lorentzian pulse train with repetition period $T$ is applied on the Ohmic contact, as illustrated in Fig. 1(b). In this case, we show that the injected charges are carried by a train of of wave packets, whose quantum state can be given as

$$|\Psi_{\text{train}}\rangle = \prod_{l=0,\pm 1,\pm 2,...} |\Psi_l\rangle,$$

(1)

with $|\Psi_l\rangle$ representing the quantum state of the $l$-th wave packet. Each wave packet is composed of charged quasiparticles and neutral eh pairs, which can be described by a set of single-body wave functions $\psi^\alpha_{kl}(t)$, with $\alpha = c$ for the quasiparticles and $\alpha = e/h$ for the electron/hole component of the eh pairs. This allows one to introduce the corresponding creation operators as

$$C_{kl}^\dagger = \int_{-\infty}^{+\infty} dt \psi_{kl}^c(t) \hat{a}^\dagger(t),$$

$$(B_{kl}^c)^\dagger = \int_{-\infty}^{+\infty} dt \psi_{kl}^e(t) \hat{a}^\dagger(t),$$

$$(B_{kl}^h)^\dagger = \int_{-\infty}^{+\infty} dt \psi_{kl}^h(t) \hat{a}^\dagger(t),$$

(2)

with $\hat{a}(t) [\hat{a}^\dagger(t)]$ being the electron annihilation [creation] operator in the time domain. In doing so, the quantum state of the $l$-th wave packet can be described by the Slater determinant as

$$|\Psi_l\rangle = \prod_k C_{kl}^\dagger \prod_k \left[ \sqrt{1-p_k} + i \sqrt{p_k} (B_{kl}^c)^\dagger (B_{kl}^h)^\dagger \right] |F\rangle,$$

(3)

with $p_k$ representing the excitation probabilities of the eh pairs. Both the excitation probabilities $p_k$ and the single-body wave functions $\psi_{kl}^\alpha(t)$ can be extracted from the time-dependent scattering matrix, providing a general way to study the quantum state of both the integer- and fractional-charged wave packets.

As the charges $Q$ are injected with the period $T$, one may expect that the charged quasiparticles are also injected with the same period. Indeed, this picture holds when $Q/e$ takes integer values. This is illustrated in the inset of Fig. 2 corresponding to $Q/e = 1$. In this case, all the single-body wave functions of the charged quasiparticles exhibit the same profiles, which are separated from each other by the time interval $T$. They essentially correspond to a periodic train of levitons. The structure of the leviton train can be understood intuitively from the corresponding waiting time distribution $W(\tau)$ (WTD) [50], which is characterized by a strong peak around $\tau = T$ [see the green dashed curve in the main panel of Fig. 2]. This indicates that the voltage pulse tends to inject exactly one electron per period into the quantum conductor.

In contrast, the above picture is inapplicable when $Q/e$ takes fractional values. In this case, the charged quasiparticles are essentially injected with two different periods. Due to the mismatch between these two periods, the wave functions of the quasiparticles can exhibit different profiles, which are injected with an extended period longer than $T$. This is illustrated in the inset (b) of Fig. 2 corresponding to $Q/e = 2/3$. One can see that the wave functions can exhibit two types of profiles, which are plotted with thick and thin curves. They are separated from each other by the time interval $3T/2$. On average, each quasiparticle can carry only $2e/3$ charge into the quantum conductor within a single period $T$. This makes them behave effectively like quasiparticles carrying fractional charges. These quasiparticles can have pronounced impact on the charge injection. In particular, they lead to the cycle-missing event, in which the voltage pulse can fail to inject an electron within a single period $T$. Such event can be seen from the corresponding WTD $W(\tau)$.
which corresponds to a periodic train of Lorentzian pulses with width $W$ [see Fig. 1(b)]. The voltage drop $V(t)$ between the contact and the conductor is assumed to occur across a short interval, so that the corresponding dwell time $\tau_D$ satisfies: $k_B T_c < \hbar / T < \hbar / W < \hbar / \tau_D < E_F$, with $E_F$ representing the Fermi energy and $T_c$ representing the electron temperature. In this paper, we choose $E_F = 0$ and concentrate on the zero-temperature limit.

The scattering matrix of the system can be solely determined by the driving voltage $V(t)$ as

$$S(t, t') = \delta(t - t') \exp[-i \frac{e}{\hbar} \int_0^t dt' V(\tau)].$$

Given the scattering matrix, the electrons in the contact and the conductor can be related via the equation

$$\hat{b}(t) = \int dt' S(t, t') \hat{a}(t'),$$

where $\hat{a}(t)$ and $\hat{b}(t)$ represent the electron annihilation operators in the Ohmic contact and the quantum conductor, respectively.

In this setup, the injected current can be simply given as $I(t) = (e^2 / h) V(t)$. The charge $Q$ injected within a single period can be solely by the flux $\phi$ as

$$Q = \int_{-T/2}^{+T/2} dt I(t) = e \phi.$$

For simplicity, here we assume $Q/e > 0$ so that the wave packets carry negative charges.

The quantum state of the injected wave packets can be obtained from the Bloch-Messiah reduction, which extracts the many-body quantum state from the decomposition of the first-order correlation function $G(t, t')$ [31, 32]. In the zero-temperature limit, $G(t, t')$ can be given as

$$i G(t, t') = \langle F | \hat{b}^\dagger(t') \hat{b}(t) | F \rangle,$$

with $| F \rangle$ representing the Fermi sea. To find the many-body state corresponding to $G(t, t')$, the Bloch-Messiah reduction essentially seeks out the quantum state $| \Psi \rangle$, which satisfies

$$\langle \Psi | \hat{a}^\dagger(t') \hat{a}(t) | \Psi \rangle = \langle F | \hat{b}^\dagger(t') \hat{b}(t) | F \rangle.$$

This can be done by a proper decomposition of $G(t, t')$. Here we only present the outline and leave the details to Appendix A.

A. Decomposition in Floquet space

For the system under periodic driving, it is straightforward to perform such decomposition in Floquet space.
which can be generally written as

\[ iG(t, t') = \sum_k \int_0^\Omega \frac{d\omega}{\Omega} e^{-i\omega(t-t')} \psi_k^r(\omega, t) \psi_k^l(\omega, t')^* \]  

with asterisk denoting the complex conjugation. In the above expression, the quantity \( p_k(\omega) \) is real, which satisfies \( p_k(\omega) \in [0, 1] \). The functions \( \psi_k^r(\omega, t) \) and \( \psi_k^l(\omega, t) \) are complex, which are periodic in the time domain \( \psi_k^r(\omega, t) = \psi_k^r(\omega, t + T) \) with \( \alpha = c, e \) and \( h \). These functions can form orthonormal basis within a single period, i.e.,

\[ \int_{-T/2}^{T/2} dt [\psi_k^r(\omega, t) \psi_k^l(\omega, t)]^* = \delta_{\alpha,\alpha'} \delta_{k,k'}. \]

All these functions can be characterized by two indices \( \omega \) and \( k \). Here \( k \) is a discrete index, which can be described by (dimensionless) integer numbers. In contrast, the index \( \omega \) has the unit of frequency, which satisfies \( \omega \in [0, \Omega] \) with \( \Omega = 2\pi/T \) being the repetition rate of the pulses.

The function \( \psi_k^r(\omega, t) \) is closely related to the single-body wave function of the charged quasiparticle \( (\alpha = c) \) and the neutral ch pair \( (\alpha = e, h) \), while \( p_k(\omega) \) represents the excitation probability of the ch pair. Both \( \psi_k^r(\omega, t) \) and \( p_k(\omega) \) can be obtained from the polar decomposition of the scattering matrix. In general cases, they can exhibit a complicated dependence on \( \omega \). For the scattering matrix given in Eq. (5), we find that the \( \omega \)-dependence can be much simpler: First, the probabilities \( p_k(\omega) \) are independent on \( \omega \) and can hence be written as \( p_k \) for short. Second, \( \psi_k^r(\omega, t) \) can be written in the form of separation of variables as

\[ \psi_k^r(\omega, t) = U_k^r(t) F_k^Q(\omega), \]

where \( F_k^Q(\omega) \) is a real function defined in the region \( \omega \in [0, \Omega] \), while \( U_k^r(t) \) is a complex periodic function defined in the whole time domain \( t \in (-\infty, +\infty) \), which satisfies \( U_k^r(t) = U_k^r(t + T) \).

The function \( U_k^r(t) \) usually has to be obtained numerically, which is sensitive to the details of the scattering matrix. In contrast, the function \( F_k^Q(\omega) \) can be given analytically. To do this, it is convenient to describe the discrete index \( k \) by two non-negative integers \( n \) and \( m \) \([i.e., n, m = 0, 1, 2, \ldots]\). In doing so, we find that \( F_k^Q(\omega) \) can be written as

\[ F_k^Q(\omega) = \begin{cases} H[(Q/e - n + 1)\Omega - \omega], & \text{for } Q/e \in [n - 1, n], \\ H[\omega - (Q/e - n)\Omega], & \text{for } Q/e \in (n, n + 1], \\ 0, & \text{otherwise}. \end{cases} \]

with \( H(\omega) \) representing Heaviside step function [55]. Note that \( F_k^Q(\omega) \) is independent on the details of the scattering matrix and is solely decided by the charged \( Q \) of the wave packet.

| \( k \) | \( n \) | \( m \) |
|-----|-----|-----|
| 0   | 1   | 0   |
| 0   | 1   | 1   |
| 1   | 2   | 2   |
| 0   | 3   | 3   |

It is worth noting that the available parameter space of the index \( k = [n, m] \) is different for the charged quasiparticles \( (\alpha = c) \) and the ch pairs \( (\alpha = e, h) \): one has \( m < n \) for the charged quasiparticles, while \( m \geq n \) for the eh pairs. This can be demonstrated more intuitively in Table I.

### B. Decomposition in wave-packet representation

Given the decomposition of \( G(t, t') \) in Eq. (10), one can construct a set of single-body wave functions corresponding to the injected quasiparticles. The many-body state of the wave packets can then be described by using the Slater determinant built from them. However, one can construct different sets of single-body wave functions, which are related to each other via unitary transformations. Hence the detailed expression of the Slater determinant is not uniquely defined. As the driving voltage \( V(t) \) corresponds to a train of pulses [see Eq. (4)], it is favorable to express the single-body wave functions in a similar form. This can be done by defining a set of wave functions \( \psi_{kl}(t) \) from \( \psi_k^r(\omega, t) \) as

\[ \psi_{kl}(t) = \frac{1}{\sqrt{q_k}} \int_0^\Omega \frac{d\omega}{\Omega} e^{-i\omega(t-IT/q_k)} \psi_k^r(\omega, t), \]

with \( l = 0, \pm 1, \pm 2, \ldots \). Note that we have introduced a normalization factor \( q_k \) so that \( \psi_{kl}(t) \) can form an orthonormal basis set in the whole time domain \( t \in (-\infty, +\infty) \), which satisfies

\[ \int_{-\infty}^{+\infty} dt [\psi_{kl'}^r(t)]^* \psi_{kl}(t) = \delta_{\alpha,\alpha'} \delta_{k,k'} \delta_{l,l'}. \]

By substituting Eqs. (12), (13) and (14) into (15), it is straightforward to show that \( q_k \) can be given analytically as

\[ q_k = q_{[n,m]} = \begin{cases} Q/e - n + 1, & \text{for } Q/e \in [n - 1, n], \\ n + 1 - Q/e, & \text{for } Q/e \in (n, n + 1], \\ 0, & \text{otherwise}. \end{cases} \]
The wave functions $\psi_{kl}^c(t)$ can be regarded as Martin-Landauer-like wave packets [56], which offers an intuitive way to interpret the time-resolved behavior of the charged quasiparticles ($\alpha = c$) and eh pairs ($\alpha = e, h$). The decomposition of $G(t, t')$ can then be given as

$$iG(t, t') = \sum_{k,l} \psi_{kl}^c(t)[\psi_{kl}^c(t')]^* + \sum_{k,l} [\psi_{kl}^c(t), \psi_{kl}^c(t')] \times \left[ \frac{p_k}{i\sqrt{p_k[1-p_k]}} \frac{i\sqrt{p_k[1-p_k]}}{1-p_k} \right] [\psi_{kl}^c(t')]^*.$$  \hspace{1cm} (17)

For wave packets carrying integer and fractional charges, both the charged quasiparticles and eh pairs can show different natures, leading to wave functions with different features. To better demonstrate these differences, we shall first concentrate on two concrete examples: wave packets carrying an unit ($Q = e$) and two-thirds ($Q/e = 2/3$) electric charges.

III. WAVE PACKET WITH UNIT CHARGE

Let us start our discussion from the wave packet carrying an unit electric charge ($Q = e$). In this case, the decomposition of $G(t, t')$ takes a simple form:

$$iG(t, t') = \sum_l \psi_{[1,0],l}(t)[\psi_{[1,0],l}(t')]^*.$$  \hspace{1cm} (18)

This indicates that the each wave packet contain only one charged quasiparticle associated with the index $k = [1, 0]$. By introducing the creation operator

$$C_{[1,0]}^l = \int_{-\infty}^{+\infty} dt \psi_{[1,0],l}(t) \hat{a}^\dagger(t),$$  \hspace{1cm} (19)

the corresponding many-body state of the whole wave packet train can be expressed as

$$|\Psi_{\text{train}}\rangle = \prod_{l=0,1,2,\ldots} C_{[1,0],l}(F).$$  \hspace{1cm} (20)

Equation (20) essentially corresponds to a periodic train of levitons. Accordingly, the wave functions $\psi_{[1,0],l}(t)$ can be regarded as Martin-Landauer-like wave packets built from levitons. This can be seen more clearly by carrying out the integration in Eq. (14) [57]:

$$\psi_{[1,0],l}(t) = U_{[1,0],l}^c(t)e^{-i\Omega(t-lT)/2} \sinh[\frac{\Omega(t-lT)}{2\pi}],$$  \hspace{1cm} (21)

where the periodic function

$$U_{[1,0],l}^c(t) = \frac{\sqrt{\cosh(\pi W/T) \sinh(\pi W/T) / T}}{\sinh(\pi T/l - i\pi W/T)},$$  \hspace{1cm} (22)

represents the leviton train [58]. Each wave function $\psi_{[1,0],l}(t)$ exhibits a strong peak around $t = lT$, corresponding to a leviton injected in the $l$-th period. Wave functions with different $l$ can form a periodic sequence, providing an intuitive way to understand the structure of the wave packet train. This is illustrated in the inset of Fig. 3.

The wave functions $\psi_{[1,0],l}(t)$ can provide an orthonormal basis set in the time domain, within which various physical quantities can be expressed in a neat way. In particular, the current carried by the train of levitons can be written as [see Appendix C for details]

$$I(t) = \sum_{l=0,1,2,\ldots} e|\psi_{[1,0],l}(t)|^2.$$  \hspace{1cm} (23)

One notices that in Eq. (23), the current $I(t)$ is expressed as an incoherent summation of all the wave functions $\psi_{[1,0],l}(t)$, even if these functions can overlap with each other [see the inset of Fig. 3]. However, this does not mean that levitons contribute incoherently to the charge transport process. In fact, the overlap between the wave functions can enhance the fluctuations of the waiting time between successive electron injection. This effect can be seen more intuitively from the waiting time distribution (WTD) between electron above the Fermi sea [59-61].

The WTD can be calculated from the corresponding idle time probability $\Pi(t_s, t_e)$ [59]. It can be expressed as the determinant [see Appendix C for details]:

$$\Pi(t_s, t_e) = \det[\hat{1} - Q_{se}],$$  \hspace{1cm} (24)

where $\hat{1}$ denotes the unit operator and the operator $Q_{se}$ counts the number of electrons injected in the time interval $[t_s, t_e]$, whose energy is larger than the Fermi energy $E_F$. By introducing the Dirac notation $(t|1, 0; l) = \psi_{[1,0],l}(t)$, the matrix element of the operator $Q_{se}$ can be
given as

\[ \langle 1, 0; l | \hat{Q}_{sc} | 1, 0; l' \rangle = \int_{t_s}^{t_s} dt [\psi_{[1, 0]}^*(t)]^* \psi_{[1, 0]}(t). \quad (25) \]

For system under periodic driving, it is usually convenient to average the idle time probability over a single period:

\[ \Pi(\tau) = \int_{-T/2}^{T/2} dt_s \Pi(t_s, t_s + \tau). \quad (26) \]

In doing so, one obtains the time-averaged idle time probability \( \Pi(\tau) \), which only depends on the length of the time interval. The corresponding WTD can be given as

\[ W(\tau) = \langle \tau \rangle \partial_{\tau}^2 \Pi(\tau), \quad (27) \]

with \( \langle \tau \rangle \) being the mean waiting time.

The above equations offer a direct relation between the wave functions and WTD, where the overlap between levitons manifest itself as the off-diagonal elements in \( \Pi(l, s) \), and the waiting time.

A quite similar result has been obtained for the ideal single-electron source built from the mesoscopic capacitor [22]. The corresponding WTD \( W_c(\tau) \) calculated from \( \Pi_c(l, s) \) can exhibit a strong peak around the point \( \tau = T \) and drops rapidly to zero when \( \tau > 2T \), as illustrated by the black dashed curve in Fig. 3. This indicates that one injects exactly one electron per period, corresponding to the case of ideal single-electron injection. In realistic conditions, Eq. (28) can be regarded as a semi-classical approximation. The presence of the overlap between levitons can lead to a deviation between the exact WTD \( W(\tau) \) and the semi-classical approximation \( W_c(\tau) \). This can be seen by comparing the red solid curve \( [W(\tau)] \) to the black dashed one \([W_c(\tau)]\) in Fig. 3 which are calculated for \( W/T = 0.1 \). One can see that the peak in the WTD is slightly broadened due to the overlap, indicating an enhancement of the fluctuations of the waiting time.

In fact, the enhancement is not significant for \( W/T = 0.1 \). Moreover, it can be suppressed by decreasing \( W/T \). This is illustrated in Fig. 4 where we compare the WTDs for the width \( W/T = 0.05, 0.1 \) and 0.2. This indicates that the ideal single-electron injection can be approached in the limit \( W/T \rightarrow 0 \). Accordingly, the wave functions \( \psi_{[1, 0]}^*(t) \) are well-separated and can be treated as individual levitons in this limit.

The above results show that levitons can be well described by the single-body wave function \( \psi_{kl}^*(t) \). In the following section, we shall further demonstrate that the wave function \( \psi_{kl}^*(t) \) can also be used to describe the charged quasiparticles in the fractional-charged wave packet.

\[ \psi_{[1, 0]}^*(t) = U_{[1, 0]}^c(t) \times e^{-i q_{[1, 0]} \Omega(t - iT/q_{[1, 0]})/2} \frac{\sin[\xi_{[1, 0]}(t - iT/q_{[1, 0]})]}{2\pi}, \quad (29) \]

with the factor \( q_{[1, 0]} = 2/3 \). By comparing the wave function of levitons in Eq. (21), we show that there are two...
differences between the two cases: 1) The periodic function $U_{[-1,0]}^c(t)$ has to be obtained numerically in this case; 2) While the function $U_{[-1,0]}^c(t)$ has the period $T$, the sinc function in this case represents the wave packet localized around $t = l(3T/2)$. This indicates that the wave functions $\psi_{[l,0]}^c(t)$ correspond to the quasiparticles, which are injected with two different periods: $T$ and $3T/2$. It is the double periodicity, which makes the quasiparticles exhibit qualitatively different features from the ones of levitons.

The period $3T/2$ decides the charges carried by the quasiparticles. In fact, as the wave functions $\psi_{[l,0]}^c(t)$ with different $l$ are still orthogonal to each other [see Eq. (15)], one can still express the current as the incoherent summation of them, which has the same form as the one of levitons [see Eq. (23)]. However, as these wave functions are separated from each other by the time interval $3T/2$ [see the inset of Fig. 5], on average each quasiparticle can carry only $2e/3$ charge within a single period $T$, making them behave effectively like quasiparticles carrying fractional charges.

Note that in this case, the wave functions can exhibit two different profiles. For $l = -2, 0$ and 2 (red solid curves), the wave functions $\psi_{[l,0]}^c(t)$ can exhibit a strong peak, which is accompanied by two small shoulder peaks. In contrast, for $l = -1$ and 1 (green dashed curves), the wave functions $\psi_{[l,0]}^c(t)$ exhibit double peak structures. This is a direct consequence of the double periodicity of the wave functions. In fact, from Eq. (20), one can see that when the periods corresponding to $U_{[l,0]}^c(t)$ (with the period $T$) and the sinc function (with the period $T/q_{[l,0]}$) do not match, for $q_{[l,0]} = A/B$ (with $A$ and $B$ being coprime integers), the wave functions can exhibit $A$ different profiles, which are separated from each other with the extended period $BT/A$.

Due to the mismatch between the two periods, the wave functions are strongly overlapped with each other. This can be seen intuitively from the inset of Fig. 5. The overlap can induce a large fluctuation of the waiting time, which can be seen from the corresponding WTD [63]. This is illustrated by the red solid curves in the main panel of Fig. 5. One can see that the WTD exhibits a series of peaks at multiples of the repetition period $T$. This indicates the presence of the cycle-missing event, in which the voltage pulse fails to inject an electron within a single period $T$ [62,64].

As the overlap between the wave functions are rather large, the semi-classical approximation $W_c(\tau)$ of the WTD [Eq. (28)] is inapplicable. One can see that $W_c(\tau)$ largely overestimates the WTD around the point $\tau = 0$, which is illustrated by the black dashed curve in Fig. 5. In fact, $W_c(\tau)$ gives an unphysical value around this point: The WTD should be zero at $\tau = 0$ due to the Pauli principle. Unlike the case of levitons, the overlap between the wave functions cannot be eliminated by just decreasing the width $W/T$. As a consequence, the multiple-peak structure of the WTD preserves as $W/T$ decreases. This is illustrated in Fig. 6 corresponding to $W/T = 0.2, 0.1$ and 0.05.

The above results explains the nature of the fractional-charged quasiparticles: they are just quasiparticles injected with an extended period $T/q_k$, which is longer than the period $T$ of the driving pulses. The wave functions of these quasiparticles are always strongly overlapped with each other, manifesting themselves at multiple peaks in the corresponding WTD. The feature of these quasiparticles can be characterized by the factor $q_k$, indicating that each quasiparticle can carry $eq_k$ charges per period $T$, making them behave effectively as fractional-charge quasiparticles.
FIG. 7. (Color online) Wave functions of the charged quasiparticle \((k = [1, 0])\) and the eh pair \((k = [0, 0])\). The red curves represent the wave functions of the charged quasiparticle. The green and blue curves represent the wave functions for the electron and hole components of the eh pair, respectively. All the wave functions are calculated with \(W/T = 0.1\). The solid (dashed) curves from left to right correspond to \(l = -2, 0\) and \(2\) \((l = -1\) and \(1)\), respectively.

**B. Electron-hole pairs**

Now let us briefly discuss the eh pairs in the wave packet. For \(W/T = 0.1\), we find that each wave packet contains only one eh pair, which is associated with the index \(k = [0, 0]\). The corresponding excitation probability \(p_{[0,0]}\) is only 0.138. The other eh pairs are negligible due to their small excitation probabilities \(0.05\). The eh pairs can be described in a similar way as the charged quasiparticles. In fact, the wave functions of the electron and hole components can be expressed in a similar form as shown in Eq. (29):

\[
\psi_{[0,0]}^{e/h}(t) = U_{[0,0]}^{e/h}(t) \times e^{-i q_{[0,0]} \Omega(t - IT/q_{[0,0]})/2} \sqrt{q_{[0,0]}} \sin[(q_{[0,0]} \Omega(t - IT/q_{[0,0]}))/2] .
\]  

(30)

with the factor \(q_{[0,0]} = 1/3\). The corresponding wave functions \(\psi_{[0,0]}^{e/h}(t)\) are plotted by the green/blue curves in Fig. 7 where the wave function \(\psi_{[1,0]}^{e}(t)\) of the charged quasiparticles are also plotted by the red curves for comparison. One can see that in this case, the wave functions for the electron (hole) component exhibit only one type of profiles. They are separated from each other by the time interval \(3T\), making them behave as quasiparticles carrying \(e/3\) charges. Note that electron and hole components carry the same amount of charges but with opposite sign, which cannot contribute to the total charge \(Q\) of the wave packet.

By combining the information of both the charged quasiparticles and eh pairs, the quantum state of the whole train of wave packet can be written as

\[
\Psi_{\text{train}} = \prod_{l = 0, \pm 1, \pm 2, \ldots} C_{l}^{l} \left[ \sqrt{T - p_{[0,0]}} \right. \\
+ i \sqrt{p_{[0,0]}} \left( B_{f, [0,0]}^{e})^\dagger \right) \left( B_{f, [0,0]}^{h})^\dagger \right] \left| \Omega \right>,
\]

(31)

with

\[
C_{l}^{l} = \int_{-\infty}^{+\infty} dt \psi_{[0,0]}^{e}(t) a_{l}^{\dagger}(t), \\
(B_{f, [0,0]}^{e})^\dagger = \int_{-\infty}^{+\infty} dt \psi_{[0,0]}^{e}(t) a^{\dagger}(t), \\
(B_{f, [0,0]}^{h})^\dagger = \int_{-\infty}^{+\infty} dt \psi_{[0,0]}^{h}(t) a(t).
\]

(32)

This provides a full information of the injected electric wave packet. It allows us to elucidate how the quantum state of wave packets can evolve as the flux of the pulses changes. In the following section, we shall concentrate on the evolution of the charged quasiparticles. We shall show how levitons can emerge as the flux approaches an integer value.

**V. EVOLUTION OF CHARGED QUASIPARTICLE**

The evolution of the charged quasiparticles can be fully described by the single-body wave function \(\psi_{[0,0]}^{e}(t)\). This is illustrated in Fig. 8 corresponding to the index \(k = [1, 0]\). In the figure, we choose \(W/T = 0.1\) and \(Q/e \in (0.0, 2.0)\). Curves with different colors and line types correspond to wave functions \(\psi_{[1,0]}^{e}(t)\) with different \(l\). As the factor \(q_{[1,0]}\) can play an important role, we also show the corresponding \(q_{[1,0]}\) alongside the wave functions.

From the figure, one first notices that one has \(q_{[1,0]} = Q/e\) when \(Q/e \in (0.0, 1.0)\). For \(q_{[1,0]} = 1/4\), all the wave functions of the quasiparticles exhibit the same profile. These quasiparticles are injected with the extended period \(4T\), indicating that they can carry \(e/4\) charge within each period \(T\). As \(q_{[1,0]}\) increases from \(1/4\) to \(1/2\), the extended period is reduced to \(2T\), indicating that the quasiparticles evolve into the \(e/2\)-charged quasiparticles. As \(q_{[1,0]}\) further increases from \(1/2\) to \(3/4\), there can exist three types of quasiparticle, which are injected with the extended period \(3T\), leading to \(3e/4\) charges per period. As \(q_{[1,0]}\) reaches 1.0, all the quasiparticles can evolve into levitons, which are injected with the period \(T\).

For \(Q/e \in (1.0, 2.0)\), one has \(q_{[1,0]} = 2 - Q/e\). As \(Q/e\) increases in this region, \(q_{[1,0]}\) is dropping linearly to zero. Accordingly, the levitons can evolve back into fractional-charged quasiparticles, which are injected with the extended period \(T/q_{[1,0]}\). Note that one has \(T/q_{[1,0]} \rightarrow +\infty\) for \(q_{[1,0]} \rightarrow 0\). This implies that the corresponding quasiparticles cannot be injected in this limit, since the time
interval between successive quasiparticle injection tends to infinity. Generally speaking, quasiparticles associated with the index \( k = [n, m] \) can only be injected when \( Q/e \in [n - 1, n + 1] \), as shown in Eq. (16). This can be seen more clearly for Fig. 9.

The evolution of the quasiparticles can also be seen from the corresponding WTD, as illustrated in Fig. 10. One can see that for \( Q/e = 1/4 \), the waiting time has a rather wide distribution. This is because the corresponding wave functions of the quasiparticles are strongly overlapped, as shown in Fig. 8. As \( Q/e \) approaches 1.0, the WTD \( W(\tau) \) tends to exhibit a strong peak around \( \tau = T \), indicating the emergence of levitons. Hence the evolution of the wave functions for \( Q/e < 1.0 \) can also be tracked by using the corresponding WTD. As \( Q/e \) goes above 1.0, additional charged quasiparticles can be injected. From Fig. 9 one can see that two additional quasiparticles \( k = [2, 0] \) and \( [2, 1] \) can emerge. The evolution of these two quasiparticles are demonstrated in Fig. 11. By comparing to Fig. 8, one can see that they evolve in a similar way as the quasiparticle \( k = [1, 0] \). As these two quasiparticles can also contribute to the WTD, it is difficult to read the evolution of a single charged quasiparticles from the WTD when \( Q/e > 1.0 \).

FIG. 8. (Color online) Wave functions of the charged quasiparticle \( |\psi_{[1,0]}(t)|^2T \), corresponding to \( W/T = 0.1 \) and \( Q/e \in (0.0, 2.0) \). For each value of \( Q/e \), we plot the wave functions for \( l = -3 \sim 3 \). Curves with different colors and line types correspond to \( |\psi_{[1,0]}(t)|^2T \) with different \( l \).

FIG. 9. (Color online) Factor \( q_k \) as functions of \( Q/e \). The red solid curve represents \( q_{[1,0]} \). The green dashed curves represent \( q_{[2,0]} \) and \( q_{[2,1]} \). Note that one has \( q_{[2,0]} = q_{[2,1]} \), so the two curves are overlapped. Similarly, the blue dotted curves represent \( q_{[3,0]}, q_{[3,1]} \) and \( q_{[3,2]} \). The black solid curve represents the charge of the wave packet \( Q/e \), which satisfies \( Q = e \sum_k q_k \).

FIG. 10. (Color online) The WTDs between electrons above the Fermi sea, corresponding to \( Q/e \in (0.0, 2.0) \) and the width \( W/T = 0.1 \). Curves corresponding to different \( Q/e \) are shifted vertically for better visibility.
where

\[
S_c = S_0 \sum_k q_k, \\
S_{ex} = 2S_0 \sum_k q_k p_k.
\]

with \(S_0 = 2 \hbar^2 D(1 - D)h\Omega\) being the typical scale of the shot noise.

The first part corresponds to the contribution of the charged quasiparticles. It is solely decided by the charge \(Q\) of the wave packet, since one has \(\sum_k q_k = Q/e\) from Eq. (16). The second part is the excess shot noise, which has been used extensively to characterize the feature of eh pairs [2 67]. By using the information of the excitation probability \(p_k\) and the factor \(q_k\), one can decompose the excess shot noise \(S_{ex}\) into the contribution of individual eh pairs. This is illustrated in Fig. [12]. From the figure, one can identify the contribution of three eh pairs, corresponding to \(k = \{0, 0\}, k = \{1, 1\}\) and \(k = \{2, 2\}\). These eh pairs dominate the excess shot noise \(S_{ex}\) in different regions. Such decomposition makes it possible to extract the information of individual eh pairs from the excess shot noise. By combining the WTD with the shot noise, one can hence obtain the full information of the evolution of the quantum state of the wave packet.

**FIG. 11.** (Color online) Wave functions of the charged quasiparticle \(|\psi_{2,0}(t)|^2 T\) (a) and \(|\psi_{2,1}(t)|^2 T\) (b), corresponding to \(W/T = 0.1\) and \(Q/e \in \{1.0, 3.0\}\). For each value of \(Q/e\), we plot the wave functions for \(l = -3 \sim 3\). Curves with different colors and line types correspond to the wave functions with different \(l\).

**VI. EVOLUTION OF ELECTRON-HOLE PAIRS AND SHOT NOISE**

As levitons evolve into fractional-charged quasiparticle, additional eh pairs can be excited. Due to the small excitation probabilities, the eh pairs can have little contribution to the WTD between electrons above the Fermi sea [66]. In contrast, it can have pronounced impact on the shot noise, which has been extensively studied in previous works [2 34 35 40]. When the wave packet is partitioned at a localized scatter with transmission probability \(D\), both the charged quasiparticles and eh pairs can contribute to the shot noise \(S_N\). It can be decomposed into two part [see Appendix [5] for details]: \(S_N = S_c + S_{ex} \).
In summary, we have present a general approach to extract the quantum state of wave packets injected by Lorentzian pulse train with arbitrary flux. We show that the charged quasiparticles can be described by a set of single-body wave functions $\psi_{kl}(t)$. These wave functions can be regarded as Martin-Landauer-like wave packets, which offers an intuitive way to interpret their time-resolved behaviors. In integer-charged wave packets, the charged quasiparticles are levitons, which are injected with the same period as the pulse train. No $eh$ pairs can be injected in this case. In fractional-charged wave packets, the charged quasiparticles can be injected with two different periods. Due to the double periodicity, their wave functions can exhibit different profiles. They can form a periodic train, whose period is longer than the period of the pulse train. This makes them behave effectively as quasiparticles carrying fractional charges. We show that the evolution of the charged quasiparticles can be seen from the WTD between electrons above the Fermi sea. Our approach can also be used to describe the evolution of $eh$ pairs, which can be tracked by using the shot noise. Note that although our approach is demonstrated for the Lorentzian pulses, it is rather general and can be applied to pulses with arbitrary profiles. We expect our work will be helpful to explore the full potential of the voltage electron source.

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Appendix A: Bloch-Messiah reduction within the framework of scattering matrix theory

The basic idea is straightforward: In a non-interacting electron system, the many-body state can be expressed as a Slater determinant in the zero-temperature limit, which can be fully determined by the first-order correlation function. By calculating the correlation function based on the scattering matrix theory of quantum transport, one can reconstruct the Slater determinant from the scattering matrix.

FIG. 13. (a) A single-channel quantum conductor connected to two reservoirs A and B.
Such reconstruction can be demonstrated more clearly in a single-channel quantum conductor at zero temperature, as illustrated in Fig. 13. The incoming electrons are injected from the reservoir $A$ into the conductor, while the outgoing electrons from the conductor are fed into the reservoir $B$. Without interactions, the quantum transport of electrons in such system can be generally described by a single-body scattering matrix $S$. By introducing annihilation operators $\hat{a}(E)$ and $\hat{b}(E)$ for the incoming and outgoing electrons in the energy domain, one has

$$\hat{b}(E) = \sum_{E'} S(E, E') \hat{a}(E'), \quad (A1)$$

with $S(E, E')$ representing the matrix element of the scattering matrix $S$ in the energy domain.

It is convenient to introduce the polar decomposition of the scattering matrix, which has the form

$$S(E, E') = \sum_{j} \left[ \psi_{j}^{e}(E), \psi_{j}^{h}(E) \right] \times \left[ \sqrt{1 - p_j} - i \sqrt{p_j} \right] \left[ \phi_{j}^{e}(E'), \phi_{j}^{h}(E') \right]^{*}, \quad (A2)$$

where $\psi_{j}^{e}(E), \phi_{j}^{e}(E')$ are nonzero for $E > 0$, while $\psi_{j}^{h}(E), \phi_{j}^{h}(E')$ are nonzero for $E \leq 0$. These functions form orthonormal basis in the energy domain:

$$\int \frac{dE}{2\pi \hbar} <\alpha_{j'}|E> <E|\alpha_{j}> = \delta_{j, j'} \delta_{\alpha, \alpha'}, \quad (A3)$$

with $\delta_{j, j'}$ being the Kronecker delta. Note that we have introduce the Dirac notation $<E|\alpha_{j} > = \psi_{j}^{e}(E)$, with $\alpha = e, h$.

Now let us turn to discuss the many-body state of the electrons in such system. For the incoming electrons, the many-body state $|\Psi_{A}\rangle$ is just a Fermi sea $|\mathbf{F}\rangle$, whose Fermi level $E_F$ is decided by the reservoir $A$. Assuming $E_F = 0$, it can be expressed as

$$|\Psi_{A}\rangle = |\mathbf{F}\rangle = \prod_{\epsilon \leq 0} \hat{a}^{\dagger}(\epsilon)|\text{Vac}\rangle, \quad (A4)$$

with $|\text{Vac}\rangle$ being the vacuum state.

Alternatively, one can also describe the many-body state $|\Psi_{A}\rangle$ by using the corresponding first-order correlation function, which has the form in the energy domain

$$iG_{A}^{<}(E, E') = \langle \mathbf{F} | \hat{a}^{\dagger}(E') \hat{a}(E) | \mathbf{F} \rangle, \quad (A5)$$

or equivalently,

$$iG_{A}^{<}(E, E') = \langle \mathbf{F} | \hat{a}(E) \hat{a}^{\dagger}(E') | \mathbf{F} \rangle, \quad (A6)$$

which related to $G_{A}^{<}(E, E')$ as $G_{A}^{<}(E, E') + G_{A}^{>}(E, E') = i\delta(E - E')$.

By substituting Eq. (A4) into Eqs. (A5) and (A6), the correlation function $G_{A}^{<}$ can be expressed by using the single-body states $|\epsilon\rangle$ of electrons as

$$iG_{A}^{<}(E, E') = \sum_{\epsilon \leq 0} \langle \epsilon | \epsilon \rangle \langle \epsilon | E' \rangle, \quad (A7)$$

$$iG_{A}^{>}(E, E') = \sum_{\epsilon > 0} \langle \epsilon | \epsilon \rangle \langle \epsilon | E' \rangle, \quad (A7)$$

with $\langle \epsilon | \epsilon \rangle = \delta(E - \epsilon) \Delta E$. Here $\Delta E$ is the mesh size in the energy domain. The limit $\Delta E \to 0$ should be taken in the end of the calculation.

The many-body state $|\Psi_{B}\rangle$ of the outgoing electrons is usually not given explicitly in the scattering matrix theory. Instead, it is described by the first-order correlation function as:

$$G_{B}^{<}(E, E') = \langle \mathbf{F} | \hat{b}^{\dagger}(E') \hat{b}(E) | \mathbf{F} \rangle = \sum_{E_{1}, E_{1}'} \langle \mathbf{F} | S(E, E_{1}')^{*} S(E, E_{1}) \hat{a}^{\dagger}(E_{1}') \hat{a}(E_{1}) | \mathbf{F} \rangle. \quad (A8)$$

To find the explicit form of the many-body state $|\Psi_{B}\rangle$, we write $G_{B}^{<}(E, E')$ in a form analogous to Eqs. (A7). This can be done by using the polar decomposition of the scattering matrix given in Eq. (A2), which gives

$$G_{B}^{<}(E, E') = \sum_{j} \langle j | \gamma_{j}^{<} \rangle \langle \gamma_{j}^{<} | E' \rangle, \quad (A9)$$

with

$$\langle \gamma_{j}^{<} | = i \sqrt{p_{j}} | e_{j} \rangle + \sqrt{1 - p_{j}} | h_{j} \rangle, \quad (A10)$$

$$\langle \gamma_{j}^{>} | = i \sqrt{p_{j}} | h_{j} \rangle + \sqrt{1 - p_{j}} | e_{j} \rangle. \quad (A10)$$

This indicates that the many-body state $|\Psi_{B}\rangle$ can be expressed in a BCS-like form, corresponding to a neutral cloud of $e h$ pairs. The quantum state of the $e h$ pair can be described by the excitation probability $p_{j}$ and the single-body state $|\epsilon_{k}\rangle | h_{k}\rangle$ of the electron [hole] components. They can be obtained by solving the polar decomposition of the scattering matrix. In our previous works [51, 52], we have studied the quantum state of $e h$ pairs by using such decomposition.

It is possible for the polar decomposition to give solutions corresponding to either $|e_{j}\rangle = 0$ or $|h_{j}\rangle = 0$, when $p_{j} = 1.0$. This indicates that there also exist unpaired electrons or holes, which are just quasiparticles carrying negative or positive charges. Moreover, additional normalization factors can also emerge, representing the injection probability of the corresponding quasiparticles. This is the case we have encountered in this paper, when the corresponding first-order correlation function can be given as

$$G_{B}^{<}(E, E') = \sum_{j} q_{j} \langle j | \gamma_{j}^{<} \rangle \langle \gamma_{j}^{<} | E' \rangle. \quad (A11)$$

By taking these ingredients into consideration, one can express the many-body state as given in Eqs. (1), (2).
and \( [3] \). The correlation function in the time domain can be obtained from the transform
\[
G^\mathbb{R}(t, t') = \int \frac{dEdE'}{(2\pi\hbar)^2} e^{-iEt/h + iE't'/h} G^\mathbb{R}(E, E'). \tag{A12}
\]
Note that in the main text, we have expressed the first-order correlation function \( G^\mathbb{R}_B(t, t') = G(t, t') \) in the time domain [see Eq. [17]].

For the system we considered here, the scattering matrix has a simple structure in the time domain [see Eq. [5]]. By introducing the wave packet functions
\[
\psi_{kl}^{\epsilon/h}(t) = \int \frac{dE}{2\pi\hbar\sqrt{q_k}} e^{-iEt/h} \varphi_{kl}^{\epsilon/h}(E),
\]
\[
\varphi_{kl}^{\epsilon/h}(t) = \int \frac{dE}{2\pi\hbar\sqrt{q_k}} e^{-iEt/h} \varphi_{kl}^{e/h}(E), \tag{A13}
\]
with \( q_k \) being the normalization factor, the polar decomposition [Eq. (A2)] can be obtained by solving the equations
\[
e^{-i\phi(t)} \left[ \begin{array}{c} \varphi_{kl}^{\epsilon}(t) \\ \varphi_{kl}^{\epsilon}(t) \end{array} \right] = \sqrt{1 - p_k} \left[ \begin{array}{c} \psi_{kl}(t) \\ \psi_{kl}(t) \end{array} \right] + i\sqrt{p_k} \left[ \begin{array}{c} \psi_{kl}(t) \\ \psi_{kl}(t) \end{array} \right],
\]
where we have chosen the compound index \( j = [k, l] \). Here \( \phi(t) \) is the forward scattering phase, which can be written as \( \phi(t) = \frac{t}{\hbar} \int V(\tau) d\tau \).

Although \( V(t) \) is periodic, the forward scattering phase \( \phi(t) \) is non-periodic. In fact, it is possible to extract the periodic part from \( \phi(t) \) by introducing the relation
\[
\varphi = N_T + \frac{\omega_T}{\Omega}, \tag{A15}
\]
with \( N_T \) being integer and \( \omega_T \) being real number, which satisfies \( \omega_T \in [0, \Omega] \). By using \( N_T \) and \( \omega_T \), one can express \( \phi(t) \) as
\[
\phi(t) = \phi_n(t) + [\omega_T + (N_T - n)\Omega]t. \tag{A16}
\]
where \( \phi_n(t) \) represents the periodic part of the forward scattering phase, with \( n \) being integer.

Hence the integer \( n \) offers a natural index for the solutions. For a given \( n \), Eqs. (A13) and (A14) can be reduced to a singular value problem, whose solutions can be labelled by another integer \( m \). That is why we choose the index \( k = [n, m] \) in the main text. Note that for a given flux \( \varphi = Q/e \), we find that only the solutions related to \( n = N_T \) and \( n = N_T - 1 \) are relevant. The corresponding \( \psi_{kl}^{\epsilon/h}(t) \) can be expressed by the ansatz:
\[
\psi_{kl}^{\epsilon/h}(t) = U_{kl}^{\epsilon/h}(t) \int \frac{d\omega}{2\pi\sqrt{q_k}} F^Q_k(\omega) e^{-i\omega(t-T/q_k)}, \tag{A17}
\]
where \( F^Q_k(\omega) \) is the real function given in Eq. [13]. The function \( U_{kl}^{\epsilon/h}(t) \) is periodic in the time domain, which usually has to be obtained numerically from the singular value problem.

### Appendix B: Current and shot noise

Various observable quantities can be calculated directly from the expression of the many-body state \( |\Psi_B\rangle \) given in Appendix [A]. The current carried by the train of wave packets can be given as
\[
I(t) = e\langle \Psi_{\text{train}} | \hat{a}^\dagger(t) \hat{a}(t) | \Psi_{\text{train}} \rangle
\]
\[
= \sum_k \sum_{l=0, \pm 1, \pm 2, \ldots} I_k^{\epsilon}(t) + \sum_k \sum_{l=0, \pm 1, \pm 2, \ldots} I_k^{\phi}(t), \tag{B1}
\]
where \( I_k^{\epsilon}(t) \) represents the contribution from the charged quasiparticles, which has the form
\[
I_k^{\epsilon}(t) = e q_k |\psi_{kl}^{\epsilon}(t)|^2. \tag{B2}
\]
In contrast, \( I_k^{\phi}(t) \) represents the contribution from the eh pair, which can be written as
\[
I_k^{\phi}(t) = e q_k \frac{\pi}{\hbar} \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{\Omega}} \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t)^\dagger. \tag{B3}
\]
Note that for the electron source we considered here, one always has \( I(t) = (e^2/h)V(t) \).

When the train of wave packets is partitioned at a localized scatter with transmission probability \( D \), both the quasiparticles and eh pairs can contribute to the shot noise. The time-dependent shot noise can be expressed as
\[
S_{sn}(t, t') = e^2 D (1 - D) \frac{i}{2\pi} \frac{1}{t - t' + i\eta}
\]
\[
\times \sum_k q_k \sum_{l=0, \pm 1, \pm 2, \ldots} \left\{ |\psi_{kl}^{\epsilon}(t)|^2 |\psi_{kl}^{\epsilon}(t')|^2 + 2e q_k \frac{\pi}{\hbar} \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{\Omega}} \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t) \psi_{kl}^{\epsilon}(t)^\dagger \right\}. \tag{B4}
\]
The shot noise in the dc limit can be obtained as
\[
S_{sn}(t, t') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{sn}(t, t') dt dt' = \sum_{l=0, \pm 1, \pm 2, \ldots} [S_{c}(l) + S_{ex}(l)], \tag{B5}
\]
where \( S_c(l) \) and \( S_{ex}(l) \) representing the shot noise attributed to the charged quasiparticles and eh pairs in the \( l \)-th wave packet. We find that both of them are independent on the index \( l \), which can be written as
\[
S_c = S_0 \sum_k q_k, \tag{B6}
\]
\[
S_{ex} = 2S_0 \sum_k q_k p_k.
\]
with \( S_0 = 2\pi^2 D(1 - D)\hbar\Omega \) being the typical scale of the shot noise.
Appendix C: Waiting time distribution

To obtain the information of the many-body state in short time scales, we study the waiting time distribution between electrons above the Fermi sea. This can be obtained from the corresponding time-dependent full counting statistics (FCS) \[50\]. As far as the electrons above the Fermi sea are concerned, the characteristic function of the corresponding FCS in the time interval \([t_s, t_e]\) can be given as

\[
\chi(\lambda; t_s, t_e) = \langle \Psi_B | e^{i\lambda \hat{N}_{se}} | \Psi_B \rangle, \tag{C1}
\]

where the operator \(\hat{N}_{se}\) counts the number of electrons above the Fermi sea injected in the time interval \([t_s, t_e]\). It can be given as

\[
\hat{N}_{se} = \int_{t_s}^{t_e} dt \hat{a}_p^\dagger(t) \hat{a}_p(t), \tag{C2}
\]

with \(\hat{a}_p(t) = \int_{0}^{\infty} e^{-iEt/\hbar} \hat{a}(E) dE/(2\pi\hbar)\).

By using the expression of \(|\Psi_B\rangle\), the characteristic function can be expressed as

\[
\chi(\lambda; t_s, t_e) = \text{det}[\hat{1} + (e^{i\lambda} - 1)\hat{\Lambda}_{se} \hat{G}_B^\ominus]. \tag{C3}
\]

with \(\hat{G}_B^\ominus = \sum_j \langle \gamma_j^\ominus | \gamma_j^\ominus \rangle\) being the operator corresponding to the first-order correlation function. Here we have introduced a single-body operator \(\hat{\Lambda}_{se} = \int_{t_s}^{t_e} dt |t_p\rangle \langle t_p|\) corresponding to \(\hat{N}_{se}\), with \(|t_p\rangle = \int_{0}^{\infty} e^{iEt/\hbar} E dE/(2\pi\hbar)\). This allows us to calculate the FCS by using the information of the single-body states of the charged quasiparticles and eh pairs. By taking the limit \(\lambda \to i\xi\), the above equation can be reduced to the idle time probabilities of electrons above the Fermi sea [see also Eq. (24)]

\[
\Pi(t_s, t_e) = \text{det}[\hat{1} - \hat{Q}_{se}]. \tag{C4}
\]

with \(\hat{Q}_{se} = \hat{\Lambda}_{se} \hat{G}_B^\ominus\).

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