Selection and analysis of material models in copper jet penetration into water

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Abstract. A reliable choose of material models is quite important to correctly conduct the numerical simulation of copper jet into water. In the present paper, we compared three copper models, i.e., Steinberg Guinan, Piecewise JC and Zerilli Armstrong models and two water EOS of Mie-Grüneisen and Polynomial. With referencing the experimental results, the numerical simulations of copper jet into water were carried out and analysed by employing different material models in Euler algorithm of AUTODYN. It concluded that Zerilli Armstrong model was suitable for copper and Shock EOS was suitable for water.

1. Introduction

Experimental observation [1-5] is a very common way to study the motion of metal jet in water by highspeed photography, flash radiography or intermittent electric switch technology. With the technical limit of only a few photos obtained in each test, numerical simulations are usually conducted to get much more details of jet mechanism in the water. For examples, Shi et al [6] and Saroha et al [7] used Euler algorithm in AUTODYN to simulate, but there is a certain gap between their simulations and experimental results.

There are many constitutive models of copper used in existing literatures, e.g., Steinberg Guinan model is adopted by Shi et al [6] and Piecewise JC model is adopted by Saroha et al [7]. In general, Zerilli Armstrong model [8] is very commonly used for metals, which has excellent performance with low hardness such as copper and pure iron. Therefore, with referencing the experimental results, this paper aims to compare the simulations of the three models to determine the best one.

Shock and Polynomial equations are usually used as the equation of state for water in AUTODYN, especially they are commonly used in study of underwater explosion [9-11]. By the way, Shi et al [6] and Saroha et al [7] adopted Shock EOS.

The better models of copper and water are determined by the fitness of numerical simulation and experimental results in this paper, which will construct a research foundation for the compressibility of copper jet into water.
2. The Choice of Material Model
This section introduces equation of state and strength model of materials required for numerical simulation. Equation of state is necessary for all materials in numerical simulation. Strength model is one part of the constitutive model, and it is suitable for solid material. Basically, the constitutive model also includes failure criterion. Eulerian algorithm in AUTODYN only needs strength model, so failure criterion is no longer given.

2.1. Equation of state (EOS)
Mie-Grüneisen and Polynomial EOS of materials are employed in this paper. The former is used to describe solid and water, while the latter is only used to describe water.

2.1.1. Mie-Grüneisen EOS [9]. Mie-Grüneisen EOS is very common and most of solid and all fluid can be described by it in AUTODYN. Here the Mie-Grüneisen EOS is presented as

\[ E(P, v) = E_H + \frac{P - P_H}{\rho \Gamma} \]

where \( E, P, \rho, \) and \( v \equiv 1/\rho \) are specific internal energy, pressure, density and specific volume, respectively. \( \Gamma = v (\partial P/\partial E)_v \) is the Grüneisen parameter and it is assumed by the relation \( \rho \Gamma = \rho_0 \Gamma_0 \). \( E_H \) and \( P_H \) are specific internal energy and pressure under the Hugoniot shock state, respectively. In order to calculate \( P_H \), the quadratic Hugoniot relation between shock velocity \( U \) and particle velocity \( u_p \) is adopted as

\[ U = c_0 + s_1 u_p + \frac{s_2}{c_0} u_p^2 \]

where \( s_1 \) and \( s_2 \) are material constants and \( c_0 \) is initial bulk sound speed. Equation (1.2) and the conservation laws of the mass, momentum and energy at the shockwave are combined to get

\[ E_H = \frac{\xi P_H}{2 \rho_0} \]

\[ P_H = \begin{cases} \frac{p_0 c_0^2 \xi}{(1 - s_1 \xi)^2}, & s_2 = 0 \\ \frac{p_0 c_0^2}{4 \xi^3 s_2^2} \left[ 1 - s_1 \xi - \sqrt{(1 - s_1 \xi)^2 - 4 s_2 \xi^2} \right]^2, & s_2 \neq 0 \end{cases} \]

where \( \xi = (v_0 - v) / v_0 \) is the volumetric strain.

Actually, The Shock EOS in AUTODYN is a special case of Equation (1.2), i.e., \( s_2 = 0 \) and \( \Gamma \) is always zero. Therein the pressure of Shock EOS is only affected by the change of density, independent of internal energy, and further independent of the entropy increase of shock wave. Essentially, the Shock equation is still a state equation of "weak isentropic" hypothesis [11].

2.1.2. Polynomial EOS [13]. This is a special case of Mie-Grüneisen equation of state and it has different analytic forms for states of compression and tension. The equation is as follows:
where $\mu$ is a compression parameter, $\mu = \rho^l / \rho^0 - 1$ ($\rho$ and $\rho^0$ is material density at a certain time and initial density respectively). $\mu \geq 0$ means compression, $\mu < 0$ means tension and $\mu = 0$ means no stress respectively. $A_1$, $A_2$, $A_3$, $B_0$, $B_1$, $T_1$ and $T_2$ are material constants. Note that $A_1$ is often equivalent to the material bulk modulus, if $T_1$ is input as 0 it is reset to $T_1 = A_1$ in the solver, $E$ is internal energy per unit mass.

2.2. Material strength model

2.2.1. Steinberg Guinan model [12]. Steinberg et al proposed a constitutive equation for high strain rate ($> 10^4 \text{s}^{-1}$) in 1980. This model has a good description ability of shock wave loading and unloading wave profile. Shear modulus $G$ and yield strength $Y$ are taken as the functions of pressure $P$ and temperature $T$ in this model, and the specific expressions are as follows:

$$G = G_0 \left[ 1 + \left( \frac{G'}{G_0} \right) \frac{P}{\eta^{\frac{1}{\beta}}} + \left( \frac{G'}{G_0} \right) (T - 300) \right]$$

$$Y = Y_0 \left[ 1 + \beta (\varepsilon + \varepsilon_i) \right]^n \left[ 1 + \left( \frac{Y'}{Y_0} \right) \frac{P}{\eta^{\frac{1}{\beta}}} + \left( \frac{G'}{G_0} \right) (T - 300) \right]$$

At the same time, the restrictions are

$$Y_0 \left[ 1 + \beta (\varepsilon + \varepsilon_i) \right]^n \leq Y_{\text{max}}$$

$$G = 0, \quad Y = 0, \quad \text{when } T \geq T_m$$

where $\eta = \frac{V_s}{V}$ is the compression ratio of volume, $\beta$ and $n$ is the strain hardening parameter, $\varepsilon$ is the plastic strain, $\varepsilon_i$ is the initial plastic strain. $G_0$ is the shear modulus at normal temperature and pressure, and $Y_0$ is the initial yield strength of the highest strain rate, and $G^'$, $G^T$ and $Y^'$ are the first partial derivatives of $G$ and $Y$ to pressure $P$ or temperature $T$. $Y_{\text{max}}$ is the maximum value allowed for strain hardening, and $T_m$ is the melting temperature related to pressure.

2.2.2. Piecewise JC model. Johnson Cook model is a very classical constitutive model with a wide range of applications. It consists of two parts: the first part is stress-strain function, and the second part is fracture function. The fracture function is not given in this paper because Euler grid is used in this paper. The stress-strain function is as follows:

$$\sigma = \left( A + B \varepsilon^n \right) \left( 1 + C \ln \dot{\varepsilon}^* \right) \left( 1 - T^m \right)$$

where $\sigma$ is von-Mises flow stress, $A$ is initial yield stress at the reference strain rate and temperature, $B$ is strain hardening modulus, $n$ is hardening index, $C$ is strain rate hardening parameter, $m$ is thermal softening index of the material; $\varepsilon$ is equivalent plastic strain, $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$ is relative equivalent plastic
strain rate, taking \( \dot{\varepsilon}_0 = 1.0 \text{s}^{-1} \); \( T^* = (T - T_{R\text{oom}})/(T_{M\text{elt}} - T_{R\text{oom}}) \) is relative temperature, \( T_{R\text{oom}} \) and \( T_{M\text{elt}} \) is room temperature and melting point of material respectively.

Piecewise JC model is a modification of Johnson-Cook model, where the dependence on effective plastic strain represented by the term \( (A + B\varepsilon_p^n) \) is replaced by a piecewise linear function of yield stress \( Y \) versus effective plastic strain \( \varepsilon_p \) (as shown in figure 1.). The strain rate effect \((1 + C \ln \dot{\varepsilon}^*)\) and thermal softening effect \((1 - T^{*m})\) are the same as Johnson cook model.

![Flow Stress vs Effective Plastic Strain](image)

**Figure 1.** The Piecewise linear function about yield stress \( Y \) and effective plastic deformation \( \varepsilon_p \) in Piecewise JC Model [13].

2.2.3. Zerilli Armstrong model. It is widely used for Johnson Cook model because of simple and easily measuring its parameters. However, numerical simulation of copper and pure iron based on the JC model is quite different from experimental phenomenon especially in Taylor experiment. Zerilli et al [8] suggested a new model aimed at improving the simulation performance of copper and pure iron. This model considered dislocation dynamics, the effects of strain hardening, strain rate hardening, thermal softening (based on thermal activation analysis) and particle size. It included two types, FCC and BCC. As copper belongs to FCC metal and thus FCC model function is as follows:

\[
Y = Y_0 + C_2 \sqrt{\varepsilon} \exp\left[ -C_3 T + C_4 T \log \dot{\varepsilon} \right]
\]

where \( Y_0 \) is the initial yield stress, \( \varepsilon \) and \( \dot{\varepsilon} \) are consistent with Johnson-Cook model, \( T \) is absolute temperature, \( C_2, C_3 \) and \( C_4 \) are constants [8].

3. Comparison between numerical simulation and experiment

The experimental results of Shi et al [6] and numerical simulation are compared in order to determine the better EOS and strength model of materials. The copper adopts Piecewise JC model and Zerilli Armstrong model respectively, while water adopts Shock and Polynomial respectively. Other material models are same as selected by Shi et al [6], as shown in table 1. Ideal Gas EOS is used for air, JWL EOS is used for BR-HMX-8, Shock EOS and Johnson Cook model are used for 1006 steel. The better model of copper is determined first, and then the better EOS of water is selected.

| Material      | Equation of state | Strength          |
|---------------|-------------------|-------------------|
| Air           | Ideal Gas         | None              |
| Steel 1006    | Shock             | Johnson Cook      |
| BR-HMX-8      | JWL               | None              |

3.1. Selection of strength model for copper

First of all, we declare that water temporarily adopt Shock EOS in this section.
The shape of jet before entering water is directly related to cavity effect formed after entering water. At \( t = 23\mu s \) (at this time, jet has not entered the water), jet shape formed by three copper material models is shown in figure 3. It can be seen that jet head formed by Steinberg Guinan model (as shown in figure 3 (a)) is arrow cluster. However, the shape of jet head formed by Piecewise JC model is smoother than Steinberg Guinan model, but still sharper than Zerilli Armstrong model. Generally, the head of shaped jet is usually round and hemispherical before penetrating target in X-ray photos taken in research of shaped jet forming, so Zerilli Armstrong model is close to real jet.

![Figure 2](image)

**Figure 2.** Different jet forms of three models at \( t = 23\mu s \).

The diameter of cavity formed by jet of Steinberg Guinan model is too small compared with experimental pictures in figure 4. The reason is that the cross-sectional area of jet head formed by Steinberg Guinan model is too small, which is far from the real jet and causes cavity diameter to be small, so the simulation results are far from experimental photos. Therefore, it can be concluded that Steinberg Guinan model is not suitable for jet into water. The simulation results of two other models are similar to experimental photos in figure 4, but we use the same experimental zoom of jet in water [15] and continue to compare in figure 5 in order to find out the better model.
Figure 4. The comparison of experimental photos [6], results of Steinberg Guinan model, Piecewise JC model and Zerilli Armstrong model, respectively.
Figure 5. The comparison of zoom of jet in water [15], results of Piecewise JC model and Zerilli Armstrong model.

We can see more details by zooming in in figure 5. The picture shows that there is an obvious convergence in the wake of jet head (figure 5(a) red frame), and the front shape of jet head is relatively smooth (figure 5 (a) green frame). The reason why the wake converges is that jet enters water from air and causes the sudden change of medium. The sudden increase of jet head resistance will cause jet head speed to drop suddenly and the huge change of speed will inevitably cause the head wake to converge suddenly, finally it forms the situation in figure 5 (a). There is a necking in middle of the cavity (in the red frame), i.e., the inner wall of cavity in the green frame has an obvious crease and the whole inner wall is rough in figure 5 (b). The basic reason for these series of phenomena is that jet will break continuously in the process of penetration, resulting in the formation of a "gourd shaped" in cavity, which is similar to the segmented penetration of jet into metal target plate and high-pressure gas swell the cavity forming the image in figure 5 (b).

We can see that Piecewise JC model does not reflect these details, its jet head is relatively flat and its wake does not converge compared with two models. The main reason why this model cannot show figure 5 (a) is that the plastic performance of model is poor and it is difficult to show these details.

Moreover, the reflection of Piecewise JC model to jet cavity is not as good as Zerilli Armstrong model, especially the necking phenomenon in the middle of cavity in figure 5 (b). The diameter of cavity is obviously small and the inner wall of cavity is relatively smooth, which are not fitting with experimental pictures, but Zerilli Armstrong model can better reflect these details in the details in green frame of figure 5 (b). It still shows that poor plasticity of Piecewise JC model makes these details fail to perform, while Zerilli Armstrong model is better than Piecewise JC model because of its better plasticity. Generally, Piecewise JC model is not suitable, while jet shape of Zerilli Armstrong model is relatively close to experiment and the movement in water also has a high agreement with experiment.

3.2. Selection of EOS for water
The strength model of copper is determined in the previous section, two EOS of water are studied on this basis. The calculation model is still Shi et al. [6] model, which only changes EOS of water. The numerical simulation is still compared with pictures in literature, and the comparison is shown in figure 6.
It can be seen that both EOS can reflect the head convergence (as shown in red frame in figure 5 (a)) comparing the simulation results in figure 6 (a) and figure 5 (a). Jet head in water of Polynomial is too flat and it is quite different from green frame part of experimental picture in figure 5 (a). Shock EOS fits experimental picture well. The two EOS can well show the cavity shape at $t=39\mu s$ in figure 5 (b) and figure 6 (b). However, Polynomial has not been seriously true to the original with increase of simulation time at $t=54.1\mu s$ (figure 7). We believe that agreement of Polynomial and experimental photos in early stage of calculation is acceptable, but the agreement is getting worse with increase of calculation time, which is similar to situation of prediction distortion of medium and far field shock wave in underwater explosion research. Indeed, we can confirm that Polynomial is not suitable for situation of large deformation and long time. So Polynomial is also not suitable for the study on motion of jet in water for a long time. Generally speaking, Shock EOS is more suitable for simulation of jet into water.
4. Conclusions and shortcomings

The numerical simulation is carried out by AUTODYN and compared with experimental photos in this paper. It is determined that Zerilli Armstrong model is suitable for copper, and the same method is used to determine Shock EOS for water. The combination of two models makes numerical simulation fitting to experimental photos, especially gourd cavity. It establishes a foundation for future study of the influence of compressibility of water.

In addition, there are still some details cannot be displayed due to fuzzy experiment photos, so it is difficult to make a further comparison and judgment with numerical simulation. The reason is that water, air and explosive gas are transparent substances, which cannot be identified by X-ray.

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