Full $\mathcal{O}(\alpha)$ electroweak radiative corrections to $t\bar{t}\gamma$ and $e^-e^+\gamma$ productions at ILC with GRACE-Loop

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The full $\mathcal{O}(\alpha)$ electroweak radiative corrections to $t\bar{t}\gamma$ and $e^-e^+\gamma$ production at International Linear Collider (ILC) are presented in this paper. The computation is performed with the help of GRACE-Loop system. In the physical results, we discuss on the cross section, electroweak corrections, and the top quark forward-backward asymmetry ($A_{FB}$) which are the function of the center-of-mass energy.

1 Introduction

The main goals of future colliders, such as ILC, are not only to make precise measurements Higgs properties as well as top quarks, vector bosons interactions, but also to search for physics beyond the Standard Model (BSM). At ILC the measurements will be performed with high precision, with the statistical error below 0.1% typically. Therefore, when we talk about the processes at $e^-e^+$ collision the full one-loop electroweak radiative corrections should be important. In this paper, we present full $\mathcal{O}(\alpha)$ electroweak radiative corrections to the most important two reactions, $e^-e^+ \rightarrow t\bar{t}\gamma$ and $e^-e^+ \rightarrow e^-e^+\gamma$. In the numerical analysis, we examine the $A_{FB}$ of $t\bar{t}\gamma$ production as well as the corrections to the total cross section of the reactions.

The paper is organized as follow. In the section 2, we introduce the GRACE-Loop system and explain how to make the check of the calculations. Then the physical results of the calculations will be discussed in the section 3. The section 4 is devoted to the conclusion and future plan of the paper.

2 GRACE-Loop and test of the calculation

2.1 GRACE-Loop

GRACE-Loop is a generic program for the automatic calculation of scattering processes in High Energy Physics. In the system, the renormalization has been carried out with the on-shell renormalization conditions of the Kyoto scheme. The GRACE-Loop have also been implemented non-linear gauge fixing conditions. With the results being independent non-linear gauge parameters, the ultraviolet and the infrared finiteness, the system provides a power tool to check the calculation as discussed in detail in the next section.
Recently, we have implemented the axial gauge for external photon into the GRACE-Loop which brings two advantages. First it cures a problem with large numerical cancellation. This is very useful when we calculate the processes at small angle and energy cuts for the particles in the final state. Secondly it provides with a useful tool to check the consistency of the results which should be independent of the choice of the gauge.

2.2 Test of the calculation

In this subsection, we discuss on the test of the calculation in detail. We take the process $e^-e^+ \rightarrow t\bar{t}\gamma$ as an example. The corrected total cross section is composed of the tree and the full one-loop graphs together with the soft and hard bremsstrahlung contributions. In general the total cross section should be independent of ultraviolet cutoff parameter ($C_{UV}$), the fictitious photon mass ($\lambda$) as well as the minimum cut of hard photon energy ($k_c$) and five gauge parameters ($\tilde{\alpha}, \tilde{\beta}, \tilde{\kappa}, \tilde{\delta}, \tilde{\epsilon}$). The table (1) shows the test for $C_{UV}$, $\lambda$, and gauge parameters independence of the amplitude at one phase space point arbitrarily chosen. We find that the results are stable over 19 digits. In the table (2), the $k_c$-stability of the result is presented. This test is done at the cross section level, we find that the results are in agreement with an accuracy better than 0.1% when we vary $k_c$.

| $C_{UV} \lambda \{\tilde{\alpha}, \tilde{\beta}, \tilde{\kappa}, \tilde{\delta}, \tilde{\epsilon}\}$ | $2R(\mathcal{T}_{Tree}\mathcal{T}_{Loop})+soft$ contribution |
|---|---|
| $(0, 10^{-21}, \{0, 0, 0, 0, 0\})$ | $-2.5673606057317280692041118561248255 \cdot 10^{-3}$ |
| $(10, 10^{-22}, \{1, 2, 3, 4, 5\})$ | $-2.5673606057317280690196417213681083 \cdot 10^{-3}$ |

Table 2: Test of the $k_c$-stability of the result. We choose the photon mass to be $10^{-17}$ GeV and the center-of-mass energy is 1 TeV. The second column presents the hard photon cross-section and the third column presents the soft photon cross-section. The final column is the sum of both.

| $k_c$ [GeV] | $\sigma_H$ [pb] | $\sigma_S$ [pb] | $\sigma_{S+H}$ [pb] |
|---|---|---|---|
| $10^{-3}$ | $2.92668 \cdot 10^{-02}$ | $7.13173 \cdot 10^{-02}$ | $0.100584$ |
| $10^{-1}$ | $1.67899 \cdot 10^{-02}$ | $8.37731 \cdot 10^{-02}$ | $0.100563$ |

3 The physical results

In this section, we will present the numerical analysis of $t\bar{t}\gamma$ and $e^-e^-\gamma$ productions at ILC. Our input parameters for the calculation are as follows. The fine structure constant in the Thomson limit is $\alpha = 137.0359895$. The mass of the Z boson is $M_Z = 91.187$ GeV, the mass of W boson is $M_W = 80.3759$ GeV and $M_H = 120$ GeV. For the lepton masses we take $m_e = 0.51099891$ MeV, $m_\tau = 1776.82$ MeV and $m_\mu = 105.658367$ MeV. For the quark masses we take $m_u = 1.7$ MeV, $m_d = 4.1$ MeV, $m_c = 1.27$ GeV, $m_s = 101$ MeV, $m_t = 172.0$ GeV and $m_b = 4.19$ GeV.

3.1 The process $e^-e^+ \rightarrow t\bar{t}\gamma$

For this process, we impose an energy cut $E_{\gamma}^{cut} \geq 10$ GeV and an angle cut $10^\circ \leq \theta_{\gamma}^{cut} \leq 170^\circ$ on the photon. In Fig (1) the total cross-section, the full electroweak corrections as well as the genuine weak correction in the $\alpha$ scheme and $\mathcal{A}_{FB}$, which are shown as a function of the center-of-mass energy ($\sqrt{s}$), are presented. We vary $\sqrt{s}$ from 360 GeV to 1 TeV. We find that the cross-section is largest near the threshold, $\sqrt{s}$ around 550 GeV and it decreases when $\sqrt{s}$ increases. The Fig 1(b) shows clearly that the QED corrections is dominant in the low energy region. In the high energy region it is getting much smaller. In contrast to the QED corrections the weak corrections in the $\alpha-$scheme is less than 10% in low energy but reaches $-16\%$ at 1
TeV. In the Fig 1(c), we find that the top quark asymmetry in the full correction is smaller than that one at the tree level. Therefore, the electroweak corrections to this process plays important role for $A_{FB}$ measurement in the future collider. In the last Fig 1(d), we compare the $A_{FB}$ in $t\bar{t}\gamma$ to that one in $t\bar{t}$ $^4$. The former is greater than the latter case when $\sqrt{s} \geq 400\text{GeV}$. This must be clearly observed at ILC.

![Figure 1](image.png)

Figure 1: The total cross-section, the full electroweak corrections as well as the genuine weak correction in the $\alpha$ scheme and $A_{FB}$, which are shown as a function of the center-of-mass energy.

3.2 The process $e^-e^+ \rightarrow e^-e^+\gamma$

For the calculation of corrections for this process we impose two cuts on the final state particles: an energy cut $E_{cut}^\gamma \geq 10\text{ Gev}$ and an angle cut $10^\circ \leq \theta_{cut}^\gamma \leq 170^\circ$ off the beam line. Further in order to isolate the photon from the electron (and positron) we require the open angle cut between the photon and $e^-(e^+)$ to be $10^\circ$. Also the angle between the electron and the positron should be greater than $10^\circ$. In Fig (2) the cross section and the electroweak corrections as functions of $\sqrt{s}$ and also the invariant mass distributions of $e^-e^+$ at $\sqrt{s} = 250\text{ GeV}$ and $\sqrt{s} = 1\text{ TeV}$, are shown. The center-of-mass energy ranges from 250 GeV (the threshold of $M_Z + M_H$) to 1 TeV. The cross section decreases in the high energy region. The electroweak corrections are found to decrease from $-2\%$ to $-20\%$ when $\sqrt{s}$ varies from 250 GeV to 1 TeV. In Figs. 2(c) and 2(d) peaks appear near $M_Z$ and high mass side (it corresponds to radiative tail). From these distributions we see that the radiative corrections are visible and thus it should be important for luminosity monitor of ILC.

4 Conclusion

We have presented the full $\mathcal{O}(\alpha)$ electroweak radiative corrections to the process $e^+e^- \rightarrow t\bar{t}\gamma$ and $e^+e^- \rightarrow e^+e^-\gamma$ at ILC. The calculations were done with the help of the GRACE-Loop.
Concerning the top quark productions we found that the corrections give sizable contribution to $A_{FB}$ as well as the total cross section. Also the corrections to the reactions related to the Bhabha scattering are very important for high precision measurement and thus for luminosity monitoring at ILC.

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