Universal de Sitter solutions at tree-level

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Abstract

Type IIA string theory compactified on SU(3)-structure manifolds with orientifolds allows for classical de Sitter solutions in four dimensions. In this paper we investigate these solutions from a ten-dimensional point of view. In particular, we demonstrate that there exists an attractive class of de Sitter solutions, whose geometry, fluxes and source terms can be entirely written in terms of the universal forms that are defined on all SU(3)-structure manifolds. These are the forms $J$ and $\Omega$, defining the SU(3)-structure itself, and the torsion classes. The existence of such universal de Sitter solutions is governed by easy-to-verify conditions on the SU(3)-structure, rendering the problem of finding dS solutions purely geometrical. We point out that the known (unstable) solution coming from the compactification on SU(2) $\times$ SU(2) is of this kind.
1 Introduction

Flux compactifications of IIA string theory with O6-planes allow for supersymmetric (susy) anti-de Sitter (AdS) vacua with all moduli stabilised at tree-level in both the string coupling $g_S$ and $\alpha'$ [1]. A natural, cosmologically more relevant, extension of this would be to achieve moduli stabilisation at tree-level in a de Sitter (dS) background instead. We have to take care, though, to circumvent the no-go theorems\(^2\) that make the construction of a de Sitter solution a difficult task. Some of the ingredients required for circumventing de Sitter no-go theorems are already present in the setup of [1], such as non-zero Romans mass ($F_0$-flux) and O6-planes. A further necessary ingredient is negative curvature of the internal space [4, 5]. Negative curvature is not present in the original model of [1] since it assumed an internal Calabi–Yau space. Fortunately, the generalisation of the supersymmetric AdS solution of [1] turns out to be a compactification on an SU(3)-structure manifold [11–15], which is generically curved. As a consequence, de Sitter solutions in this generalised setup were found in three different papers [6–8]. We briefly discuss these three solutions as they play an important role in the forthcoming.

The solutions of [6] and [7] are very similar in nature. They are both established numerically by minimising the scalar potential in four dimensions. The internal space is a group manifold with orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ and four intersecting smeared O6-planes. The solution found in [6] uses the group manifold SU(2) $\times$ SU(2) while [7] contains two other examples, and the one that is most relevant to us is based on the solvable group $s1.2$. Although these examples are the first examples of classical 4d de Sitter solutions ever constructed, they are perturbatively unstable and therefore not useful as true (metastable) vacua.

Ref. [8] constructed a class of 4d de Sitter solutions, directly in 10 dimensions, with the same ansatz used for the susy AdS solution, albeit with different flux quanta, such that the solution breaks supersymmetry and has positive energy. The condition for having a solution to the ten-dimensional equations of motion translates in a simple condition on the torsion classes [8]. However [8] did not provide an explicit manifold with torsion classes that satisfies this condition in the case of a de Sitter-space. The aim of this paper is to amend this, which requires us to generalise the universal de Sitter solutions of [8]. This situation is similar to the general susy AdS solutions, where first the conditions were constructed in a general way [11], and later more geometries (next to the solutions of [16]), satisfying all the constraints on the torsion classes, where found [12,13].

This ten-dimensional approach based on universal forms has some advantages with respect to the four-dimensional approach. First, it is technically easier than minimising a multiple-variable function. Second, it is conceptually attractive since it describes a solution irrespective of the details of the geometry. It therefore describes a whole class of solutions\(^3\).

\(^2\)The first of these no-go theorems for de Sitter solutions was established in [2], requiring negative-tension sources, like O-planes, to circumvent it. It has been later extended in [3] by investigating the dependence of the 4d scalar potential on the dilaton and volume modulus. This has then been used to construct de Sitter no-go theorems tailored to various models [4–9]. See also [10] for a different approach.

\(^3\)Of course, issues like perturbative stability can only be addressed once a specific geometry is used.
Third, one is assured that the solution solves the ten-dimensional equations of motion and, on the same footing, issues like flux and charge quantisation require a lift to ten dimensions anyway.

The generalisation of the universal de Sitter solutions of [8] we describe in this paper, has a more general SU(3)-structure. The susy AdS solutions of [11–14] have vacuum expectation values for the geometric moduli for which only two torsion classes \((W_1, W_2)\) are non-zero. In this paper we generalise by allowing the third torsion class \((W_3)\) to be non-zero as well. The key result we have found is a geometry that explicitly satisfies the geometrical constraints for the universal de Sitter solutions, thereby proving that universal de Sitter solutions exist. Remarkably, we rediscover the known solution on the SU(2) \(\times\) SU(2) group manifold, thereby providing the lift of the solution to ten dimensions. But the main message of this paper is not about the details of that solution, rather, the proof of principle that the set of universal de Sitter solutions is non-zero. This probably indicates that there must be many more geometries that allow the universal de Sitter solutions, and we hope to report on a more thorough investigation of this soon [17].

However not all de Sitter solutions should be universal since, as we explain below, the solvmanifold solution [7] is not of this kind. Nonetheless, there is a simple argument why one expects the universal solutions to be more likely. The Ricci tensor of SU(3)-structure manifolds is entirely expressed in terms of the universal forms [18, 19]. From the Einstein equation this implies that the energy-momentum tensor should be written in terms of contractions of these forms. The most natural way to achieve this is to assume that all the fluxes and sources are given in terms of these forms. Indeed, not only the known susy AdS solutions are universal, recently also classes of non-susy AdS solutions were found using a universal ansatz [8, 15, 20–22]. In [22] it was even shown that sourceless solutions exist that can achieve moduli stabilisation.

This paper is organised as follows. In section 2 we establish our conventions for SU(3)-structure and the type IIA supergravity equations of motion. In section 3 we present the universal de Sitter solutions, whereas in section 4 we investigate the conditions for universal de Sitter solutions on group manifolds with SU(3)-structure and establish that the group SU(2) \(\times\) SU(2) fulfills all the constraints. The solution on SU(2) \(\times\) SU(2) is discussed in some detail in section 5. Finally in section 6 we end with a discussion.

2 Preliminaries

2.1 SU(3)-structure manifolds

A six-dimensional SU(3)-structure manifold can be characterised by a globally defined real two-form \(J\) and a complex decomposable three-form \(\Omega = \Omega_R + i\Omega_I\), satisfying a compatibility and a normalisation condition

\[\Omega \wedge J = 0, \quad \Omega \wedge \Omega^\ast = (4i/3) J \wedge J \wedge J = 8i \text{ vol}_6.\]  

(1)

This was furthermore used for constructing Lifshitz solutions in type II supergravity [23, 24]. In this case solutions were found in type IIA* supergravity and not in IIA supergravity [24].
From the real part of the three-form we can build an almost complex structure for which $J$ is of type $(1, 1)$ and $\Omega$ is of type $(3, 0)$. It is given by

$$I^l_k = c \varepsilon^{m_1m_2 \ldots m_5}(\Omega_R)_{km_1m_2}(\Omega_R)_{m_3m_4m_5},$$

(2)

where $\varepsilon$ is the Levi-Civita symbol, and the real scalar $c$ is such that $I$ is properly normalised: $I^2 = -1$. The metric then follows via

$$g_{mn} = -I^l_m J_{ln}.$$  

(3)

The torsion classes $W_1, \ldots, W_5$ correspond to the expansion of the exterior derivatives of $J$ and $\Omega$ in terms of SU(3)-representations [25]. In this paper we will put $W_4 = W_5 = 0$ and take $W_1, W_2$ real (in our conventions), leading to a so-called half-flat manifold. We will motivate this truncation in section 4.1. We find then

$$dJ = \frac{3}{2} W_1 \Omega_R + W_3, \quad \quad \text{(4a)}$$
$$d\Omega_R = 0, \quad \quad \quad \text{(4b)}$$
$$d\Omega_I = W_1 J \wedge J + W_2 \wedge J, \quad \quad \text{(4c)}$$

where $W_1$ is a real scalar, $W_2$ a real primitive $(1,1)$-form and $W_3$ a real primitive $(1,2) + (2,1)$-form. This means that

$$W_2 \wedge J \wedge J = 0, \quad W_3 \wedge J = 0, \quad \quad \text{(5a)}$$
$$W_2 \wedge \Omega = 0, \quad W_3 \wedge \Omega = 0. \quad \quad \text{(5b)}$$

Furthermore, we find that under the Hodge star, defined from the metric (3),

$$\star_6 \Omega = -i\Omega, \quad \quad \star_6 J = \frac{1}{2} J \wedge J, \quad \quad \star_6 W_2 = -J \wedge W_2. \quad \quad \text{(6)}$$

We will make use of the fact that the Ricci tensor can be expressed in terms of the torsion classes [18] (see also [19]). Let us first observe that any real symmetric two-tensor $T_{ij}$ splits as follows in representations of SU(3)

$$T_{ij} = \frac{s(T_{ij})}{6} g_{ij} + T^+_{ij} + T^-_{ij}.$$  

(7)

Here $s(T_{ij})$ is the trace, an SU(3)-invariant, and $T^+_{ij}$ and $T^-_{ij}$ transform as $8$ and $\bar{6} + \bar{6}$ respectively. The latter are traceless and have respectively index structure $(1,1)$ and $(2,0)+(0,2)$. The former are traceless and have respectively index structure $(1,1)$ and $(2,0)+(0,2)$. Then

$$T^+_{ij} g^{ij} = 0, \quad I^l_m T^+_{ij} P^l_j = T^+_{kl}, \quad \quad \text{(8a)}$$
$$T^-_{ij} g^{ij} = 0, \quad I^l_m T^-_{ij} P^l_j = -T^-_{kl}. \quad \quad \text{(8b)}$$

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Furthermore to $T^+_{ij}$ and $T^-_{ij}$ a primitive real (1,1)-form and a complex primitive (2,1)-form can be respectively associated

\begin{align}
P_2(T_{ij}) &= \frac{1}{2} J^k i T^+_{kj} \ dx^i \wedge dx^j, \\
P_3(T_{ij}) &= \frac{1}{2} T^-_{il} \Omega^l_{jk} \ dx^i \wedge dx^j \wedge dx^k. \tag{9b}
\end{align}

Using this decomposition it was shown in [18] that the Ricci tensor can be expressed as follows in terms of the torsion classes

\begin{align}
s(R_{ij}) &= \frac{15}{2} W_1^2 - \frac{1}{2} W_2^2 - \frac{1}{2} W_3^2, \tag{10a} \\
P_2(R_{ij}) &= -\frac{1}{4} \ast (W_2 \wedge W_2) - \frac{1}{2} \ast_6 d \ast_6 \left( W_3 - \frac{1}{2} W_1 \Omega_R \right), \tag{10b} \\
P_3(R_{ij}) &= 2 W_1 W_3 \Omega_{(2,1)} + 2 dW_2 \Omega_{(2,1)} - \frac{1}{4} Q_1(W_3, W_3), \tag{10c}
\end{align}

with

\begin{align}
Q_1(W_3, W_3) &= (\Omega^{ijk} W_3^i \wedge \Omega_{w} W_3^w)_{(2,1)}, \tag{11}
\end{align}

and where in the right-hand side of eqs. (10b)-(10c) the projection onto the primitive part is understood.

Finally it will be convenient to extract the norms of $W_2$ and $W_3$ as follows

\begin{align}
W_2 &= w_2 \hat{W}_2, \quad w_2 = \sqrt{(W_2)^2}, \tag{12a} \\
W_3 &= w_3 \hat{W}_3, \quad w_3 = \sqrt{(W_3)^2}, \tag{12b}
\end{align}

where we introduced $(W_2)^2 = \frac{1}{2} W_{2ij} W_2^{ij}$ and likewise $(W_3)^2 = \frac{1}{3!} W_{3ijk} W_3^{ijk}$.

### 2.2 IIA supergravity with calibrated sources

We have to solve the equations of motion of type IIA supergravity, for which we will use the string-frame equations listed in appendix B of [26], and put $\kappa^2_{10} = 1$. These equations contain source terms and are valid when the sources are calibrated (which must be the case if the sources are embedded supersymmetrically).

The dilaton $\Phi$ and the warp factor $A$ are taken to be constant (actually we take $A = 0$) and the fluxes $F_n$ ($n = 0, 2, 4, 6, 8, 10$ in the democratic formalism [27]) are decomposed as follows in an “electric” and a “magnetic” part

\begin{equation}
F = \hat{F} + \text{vol}_4 \wedge \hat{F}, \tag{13}
\end{equation}

where $\hat{F}$ and $\hat{F}$ have only internal indices. The self-duality constraint of the democratic formalism relates the electric and the magnetic flux so that it suffices to calculate the
magnetic part in the following. In the presence of calibrated D6/O6 sources $j$, we can write the non-trivial equations of motion as follows

$$d\hat{F}_2 + H\hat{F}_0 = -j,$$  \hspace{1cm} \text{(Bianchi }\hat{F}_2) \hspace{1cm} \text{(14a)}

$$d\ast_6\hat{F}_4 - H \wedge \ast_6\hat{F}_6 = 0,$$  \hspace{1cm} \text{(eom }\hat{F}_4) \hspace{1cm} \text{(14b)}

$$d(e^{-2\Phi}\ast_6H) - (\ast_6\hat{F}_2)\hat{F}_0 - (\ast_6\hat{F}_4) \wedge \hat{F}_2 - (\ast_6\hat{F}_6) \wedge \hat{F}_4 = 0,$$  \hspace{1cm} \text{(eom }H) \hspace{1cm} \text{(14c)}

$$2(R_4 + R_6) - H^2 - e^{\Phi}\ast_6(\Omega_I \wedge j) = 0,$$  \hspace{1cm} \text{(eom }\Phi) \hspace{1cm} \text{(14d)}

$$R_4 + e^{2\Phi}\sum_n(\hat{F}_2^n) + e^{\Phi}\ast_6(\Omega_I \wedge j) = 0,$$  \hspace{1cm} \text{(external Einstein) } \hspace{1cm} \text{(14e)}

$$-\frac{1}{2}H^2 + \frac{1}{4}e^{2\Phi}\sum_n(5-n)\hat{F}_2^n + \frac{3}{4}e^{\Phi}\ast_6(\Omega_I \wedge j) = 0,$$  \hspace{1cm} \text{(Tr Einstein/eom }\Phi) \hspace{1cm} \text{(14f)}

$$R_{ij} - \frac{1}{2}H_i \cdot H_j - \frac{1}{4}e^{2\Phi}\sum_n(\hat{F}_2^n) - \frac{3}{4}e^{\Phi}\ast_6(\Omega_I \wedge j) \{ -g_{ij}\ast_6(\Omega_I \wedge j) + 2\ast_6[(g_{k(\Omega_I \wedge j)}\Omega_I \wedge j)] \} = 0,$$  \hspace{1cm} \text{(Einstein/eom }\Phi) \hspace{1cm} \text{(14g)}

where we defined $\phi_i \cdot \phi_j = 4\phi_i \cdot \phi_j = 1$ for an $l$-form $\phi$. As compared to the equations of motions of [26], we found it convenient to take linear combinations of the Einstein equation, its trace and the dilaton equation of motion.

3 Universal de Sitter solutions in general

The basic idea of universal de Sitter solutions is to find solutions independent of the details of the compactification manifold. This can be done by writing down an ansatz in terms of form-fields that are defined on all SU(3)-structure manifolds. Then the equations of motion translate into simple conditions on these form-fields. Once these simple conditions are satisfied for a given manifold, one has an explicit solution in ten dimensions. In ref. [8] such an ansatz was presented\footnote{Compared to [8] we have absorbed some convenient dilaton factors in the definition of the flux parameters, ensuring that all dependence on the dilaton drops out of the equations of motion. This implies that the dilaton is a free parameter in the solutions we will present.}

$$e^{\Phi}\hat{F}_0 = a_1, \hspace{2cm} e^{\Phi}\hat{F}_2 = a_2J + a_3\hat{W}_2, \hspace{1cm} \text{(15a)}$$

$$e^{\Phi}\hat{F}_4 = a_4J \wedge J, \hspace{2cm} e^{\Phi}\hat{F}_6 = a_5\text{vol}_6, \hspace{1cm} \text{(15b)}$$

$$H = a_6\Omega_R, \hspace{2cm} e^{\Phi}j = j_1\Omega_R, \hspace{1cm} \text{(15c)}$$

which also covers the supersymmetric AdS solution\footnote{The susy AdS solution of [11] is given by $a_1 = e^{\Phi}m, a_2 = -W_1/4, a_3 = -w_2, a_4 = 3e^{\Phi}m/10, a_5 = 9W_1/4, a_6 = 2me^{\Phi}/5$ and $j_1 = -2e^{2\Phi}m^2/5 + 3(W_1^2 - 2w_2^2)/8$. The ansatz can be extended to also allow for a term proportional to $J \wedge \hat{W}_2$ in $\hat{F}_4$, which was used in [22] to construct non-susy AdS solutions.}

$$d\hat{W}_2 = c_1\Omega_R, \hspace{2cm} \hat{W}_2 \wedge \hat{W}_2 = c_2J \wedge J + d_2\hat{W}_2 \wedge J, \hspace{1cm} \text{(16)}$$
where \( c_1, c_2, d_2 \) are real proportionality coefficients. The coefficients \( c_1 \) and \( c_2 \) are fixed by internal consistency of the SU(3)-structure equations,

\[
c_1 = -\frac{w_2}{4}, \quad c_2 = -\frac{1}{6},
\]

while \( d_2 \) is specific to the geometry. The equations of motion then fix all geometrical quantities in terms of the flux parameters \( a_i \) and the charge \( j_1 \)

\[
W_1 = W_1(a_i, j_1), \quad w_2 = w_2(a_i, j_1), \quad d_2 = d_2(a_i, j_1).
\]

Furthermore, the parameters \( a_i \) and \( j_1 \) are related amongst themselves by the equations of motion, such that there is a slice in the space formed by the variables \( (a_i, j_1) \) that solves the equations of motion. We have explicitly demonstrated in [8] that there exist ranges of values for the parameters \( a_i \) and \( j_1 \) (on the parameter slice that solves the equations) such that the resulting cosmological constant is positive. However, up to now, we have not found a geometry that can reach these values for the geometrical quantities [18].

### 3.1 The ansatz

Because we have not yet found a suitable geometry that allows for the universal de Sitter solution (15) we propose a more general universal ansatz, which considers a half-flat manifold and includes \( W_3 \) as an expansion form. In a forthcoming paper [17] we will study this ansatz in general. It turns out, however, that one obtains a very interesting simplified case if one puts

\[
W_2 = 0,
\]

which we will consider in this paper. The ansatz for the fluxes is then

\[
e^\Phi \hat{F}_0 = f_1, \quad e^\Phi \hat{F}_2 = f_2 J, \quad e^\Phi \hat{F}_4 = f_3 J \wedge J, \quad e^\Phi \hat{F}_6 = f_4 \text{vol}_6, \quad H = f_5 \Omega_R + f_6 \hat{W}_3, \quad e^\Phi j = j_1 \Omega_R + j_2 \hat{W}_3.
\]

For dS-solutions the source \( j \) will correspond to O6-planes. When writing such an ansatz, one has to take into account the parity under the orientifold involutions. For O6-planes we have that the fluxes \( H, \hat{F}_2 \) and \( \hat{F}_6 \) are odd whereas \( \hat{F}_0 \) and \( \hat{F}_4 \) are even. Furthermore, if the O6-planes are supersymmetrically embedded and thus preserve the SU(3)-structure we have that \( J, \Omega_R \) and \( \hat{W}_3 \) are odd, therefore our ansatz is justified from that point of view.

Upon plugging the ansatz in the equations of motion one immediately finds certain constraints on the remaining torsion classes

\[
d \ast_{6} \hat{W}_3 = c_1 J \wedge J, \quad (\hat{W}_{3j} \cdot \hat{W}_{3j})^+ = 0,
\]

\[
(21a) \quad (21b)
\]
We will now study the solutions to the equations (22), in particular those with $c_3, j_1, j_2$. With the use of these constraints, the equations of motion (14) lead to a set of algebraic relations for the constants $f_i$ and $j_1, j_2$

\[
\begin{align*}
(\frac{2}{3} f_2 W_1 + f_1 f_5 + j_1) \Omega_R + (f_2 w_3 + f_1 f_6 + j_2) \tilde{W}_3 &= 0, \quad (22a) \\
(3 f_3 W_1 - f_4 f_5) \Omega_R + (2 f_3 w_3 - f_4 f_6) \tilde{W}_3 &= 0, \quad (22b) \\
(f_5 W_1 + \frac{1}{2} f_6 w_3 - \frac{1}{2} f_1 f_2 - 2 f_2 f_3 - f_3 f_4) J \wedge J &= 0 \quad (22c) \\
R_4 &= -\frac{15}{2} (W_1)^2 + \frac{1}{2} \left[ (w_3)^2 + (f_6)^2 \right] + 2 f_5^2 + 2 j_1, \quad (22d) \\
R_4 + (f_1)^2 + 3 f_2^2 + 12 (f_3)^2 + (f_4)^2 + 4 j_1 &= 0, \quad (22e) \\
-2 (f_5)^2 - \frac{1}{2} (f_6)^2 + \frac{1}{4} \left[ 5 (f_1)^2 + 9 (f_2)^2 + 12 (f_3)^2 - (f_4)^2 \right] + 3 j_1 &= 0, \quad (22f) \\
2 (W_1 w_3 - j_2 - 2 f_5 f_6) W_3 |_{(2,1)} - \frac{1}{4} \left[ w_3^2 Q_1 (\tilde{W}_3, \tilde{W}_3) + f_6^2 Q_2 (\tilde{W}_3, \tilde{W}_3) \right] &= 0, \quad (22g)
\end{align*}
\]

where

\[
Q_2 (\tilde{W}_3, \tilde{W}_3) = \left(\frac{1}{2} \tilde{W}_3 \eta_{imn} \tilde{W}_3 \eta^{mn} \Omega_{ijk} dx^i \wedge dx^j \wedge dx^k \right) |_{(2,1)}.
\]

We note that using the decomposition (17)-(19), eq. (14g) would normally have a two-form and a three-form part. The two-form part is satisfied using the constraint (21d), while the three-form part leads to (22g) above.

We see that the three-form piece of the internal Einstein equation leads to further constraints. We will put

\[
Q_1 (\tilde{W}_3, \tilde{W}_3) = c_2 Q_2 (\tilde{W}_3, \tilde{W}_3) = c_3 (W_3)_{2,1}.
\]

The equations (22) will then lead to relations between the coefficients $f_i, j_1, j_2$ and the parameters $w_3, c_2, c_3$ characterising the geometry.

### 3.2 The solutions

We will now study the solutions to the equations (22), in particular those with $R_4 > 0$, which are the dS solutions. We will demonstrate later on that with the O-plane configuration of section 4.1 one obtains

\[
Q_1 = Q_2 \Rightarrow c_2 = 1, \quad c_3 = \frac{8}{\sqrt{3}}.
\]

For these choices one can show that $f_3$ and $f_4$ have to be zero in order to find a de Sitter solution. Once $f_3 = f_4 = 0$ the general solution to the equations forms a two-dimensional set in the parameter space. The equations are however invariant under an overall rescaling

\[
f_1 \rightarrow \lambda f_1, \quad j_1 \rightarrow \lambda^2 j_1, \quad R_4 \rightarrow \lambda R_4, \quad W_1 \rightarrow \lambda W_1, \quad w_3 \rightarrow \lambda w_3.
\]

\[\text{De Sitter solutions to eqs. (22) also exist when } c_3 \neq \frac{8}{\sqrt{3}} \text{ and then one can take } f_3, f_4 \text{ non-zero. The geometry should then, however, fall outside the class studied in section 4 and we have not found such an example.}\]
Figure 1: $\Lambda/(f_1)^2$ as a function of $f_2/f_1$, for $c_3 = 8/\sqrt{3}$ and $f_3 = f_4 = 0$. The area close to the horizontal axis is shown in more detail such that the dS solutions (in green) are visible. AdS solutions are shown in red and Minkowski solutions in blue.

We will choose to factor out this overall scale by dividing each quantity by appropriate factors of $f_1$ (which is equivalent to considering solutions with $f_1 = 1$). We then end up with a one-dimensional set of solutions that include non-susy AdS solutions, non-susy Minkowski solutions and de Sitter solutions. This is made clear in figure 1, where we plot the value of the cosmological constant (vertical axes) against $f_2/f_1$ (horizontal axis). This plot contains all kinds of solutions, but to make the de Sitter solutions visible, in an inset we zoomed in on an area very close to the horizontal axis. Most essential in the figure is the interval with de Sitter solutions bounded from below by a Minkowski point at $f_2/f_1 = 0.965$ and this forms our key result.

For large values of $f_2/f_1$ the line of de Sitter solutions asymptotes to a line of Minkowski solutions, which are remarkably simple solutions taking the form:

$$f_5/f_1 = 1/4, \quad f_6/f_1 = \sqrt{3}/2, \quad W_1 = f_2/2, \quad w_3/W_1 = 3\sqrt{3}.$$

This phenomenon, where de Sitter solutions interpolate between Minkowski solutions in parameter space, was first observed in [28], which obtained similar results from a four-dimensional point of view. Here we gain the extra insight of a ten-dimensional interpretation. On one side the bounding four-dimensional Minkowski solutions are the remarkably simple solutions [27] with fluxes along the universal forms. On the other side we find a Minkowski solution where the curve of figure 1 crosses the x-axis (as displayed in the inset). For the de Sitter solutions itself the explicit forms of the parameters $f_i$ are not that insightful, so apart from the plots we do not present them explicitly.

Most abundant are the non-supersymmetric AdS branches in solution space. It seems a generic property of tree-level flux compactifications (allowing dS solutions) that the AdS solutions far outnumber the dS solutions.

Figure 2 displays the ratio $w_3/W_1$ on the vertical axis and $f_2/f_1$ on the horizontal axis. This plot reveals that the ratio $w_3/W_1$ is bounded from above for all solutions and that
the saturation value $w_3/W_1 = 3\sqrt{3}$ corresponds to the line of Minkowski solutions. Furthermore we read off from the plot that in order to obtain dS solutions one must find a geometry such that

$$4.553 < w_3/W_1 < 3\sqrt{3}.$$  

(28)

4 Universal de Sitter solutions on group manifolds

4.1 Group manifolds, orientifolds and SU(3)-structures

Group manifolds form an interesting class of candidates for the internal manifold. If the six-dimensional group manifold $G$ is not already compact we assume that there exists a discrete subgroup $\Gamma \subset G$, without fixed points, such that $G/\Gamma$ is a compact manifold. This $\Gamma$ does not always exist and we refer to [29] (and references therein) for a discussion on this. On $G$ we have six, globally-defined, left-invariant one-forms (a.k.a. Maurer–Cartan forms), $e^i$, with $i = 1, \ldots, 6$. These forms obey the Maurer–Cartan relations

$$de^i = \frac{1}{2} f^i_{jk} e^j \wedge e^k,$$

(29)

with $f$ the structure constants of the Lie algebra $\mathfrak{g}$ associated to $G$. We will restrict ourselves to the supergravity degrees of freedom that are expanded along these left-invariant forms, which are named the left-invariant degrees of freedom.

A compactification of a supergravity theory on such spaces leads to a lower-dimensional supergravity theory with the same amount of supersymmetries, if one restricts to the left-invariant degrees of freedom. Therefore, if we desire de Sitter solutions in an $N = 1$ supergravity theory we need at least three intersecting O6-planes (implying a fourth). Naively extending the Tseytlin rules for brane intersections in flat space to the case of a group manifold, leads to the following intersection of four O6-planes (we present only the internal directions)
where each entry denotes a left-invariant direction. This intersection is unique up to relabeling of the O6-planes and relabeling the Maurer–Cartan forms. From this we find that the smeared orientifold source is given by

$$j_6 = j_A e^{456} + j_B e^{236} + j_C e^{134} + j_D e^{125},$$

with the corresponding involutions

$$A : (e^4, e^5, e^6) \rightarrow -(e^4, e^5, e^6),$$
$$B : (e^2, e^3, e^6) \rightarrow -(e^2, e^3, e^6),$$
$$C : (e^1, e^3, e^4) \rightarrow -(e^1, e^3, e^4),$$
$$D : (e^1, e^2, e^5) \rightarrow -(e^1, e^2, e^5).$$

Note that $A.B.C = D$. (32)

This shows that three O6-involutions, in this setting, imply the fourth. Alternatively, one can look at this as one orientifold involution (say $A$) together with the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ generated by $AB$ and $BC$. Therefore our compactification space is

$$G = \frac{\Gamma \times \mathbb{Z}_2 \times \mathbb{Z}_2}{\Gamma \times \mathbb{Z}_2 \times \mathbb{Z}_2}.$$ (33)

When the orbifold singularities are blown up, we generate new moduli commonly denoted as the twisted sector. We do not discuss this any further, but a thorough analysis of moduli stabilization should also include these modes.

There are no one-forms that have the same parity under $A, B, C$ and $D$. Furthermore, the only two-forms with a fixed parity all have negative parity and are spanned by $\{e^{16}, e^{24}, e^{35}\}$. The odd three-forms are spanned by $\{e^{456}, e^{236}, e^{134}, e^{125}\}$. Since $J$ and $\Omega_R$ must be odd under orientifold involutions preserving the SU(3)-structure, we find that they must be of the form

$$J = ae^{16} + be^{24} + ce^{35},$$
$$\Omega_R = v_1 e^{456} + v_2 e^{236} + v_3 e^{134} + v_4 e^{125},$$ (34a) (34b)

with $a, b, c, v_1, \ldots, v_4$ real coefficients. With some abuse of language we name $a, b, c$ the Kähler moduli and $v_1, \ldots, v_4$ the complex structure moduli. Note that this implies the calibration conditions

$$j_6 \wedge \Omega_R = 0 = j_6 \wedge J.$$ (35)
The orientifold involutions also restrict the possible metric flux, which must be even, or equivalently the possible group manifolds. In particular, the Lie algebra should be of the following form

\[
\begin{align*}
de^1 &= f_{23}^1 e^{23} + f_{45}^1 e^{45}, \\
de^2 &= f_{13}^2 e^{13} + f_{56}^2 e^{56}, \\
de^3 &= f_{12}^3 e^{12} + f_{46}^3 e^{46}, \\
de^4 &= f_{36}^4 e^{36} + f_{15}^4 e^{15}, \\
de^5 &= f_{14}^5 e^{14} + f_{26}^5 e^{26}, \\
de^6 &= f_{34}^6 e^{34} + f_{25}^6 e^{25}.
\end{align*}
\]

(36)

As a consistency check one finds that \(dJ\) indeed gives only rise to odd three-forms and that \(d\Omega_R = 0\) automatically. Furthermore, the algebra is unipotent \((f_{ab} = 0)\) automatically. Unipotence is a necessary condition for having a compact group manifold (after the quotient by a discrete subgroup \(\Gamma\) if need be). The Jacobi identities, which are equivalent to the nilpotence \(d^2 e^i = 0\), impose further quadratic constraints on the \(f\)'s.

From

\[
J \wedge J \wedge J = -6abc e^{123456},
\]

(37)

we find that (for our choice of orientation) \(abc < 0\) rendering all or one of the coefficients \(a, b, c\) negative. In order to be able to properly normalise \(I^2 = -1\) with real \(c\) in (2) we need furthermore \(v_1 v_2 v_3 v_4 > 0\). From equation (3) we obtain the metric, which turns out to be diagonal, consistent with even parity under the orientifold involutions

\[
g = \frac{1}{\sqrt{v_1 v_2 v_3 v_4}} \left( av_3 v_4, -bv_2 v_4, cv_2 v_3, -bv_1 v_3, cv_1 v_3, av_1 v_2 \right). \tag{38}
\]

With the metric available we can compute \(\Omega_I = \ast \Omega_R\)

\[
\Omega_I = \sqrt{v_1 v_2 v_3 v_4} \left( v_1^{-1} e^{123} + v_2^{-1} e^{145} - v_3^{-1} e^{256} - v_4^{-1} e^{346} \right). \tag{39}
\]

The normalisation condition (1) leads to

\[
\sqrt{v_1 v_2 v_3 v_4} = -abc. \tag{40}
\]

The required parity under the orientifold involutions (31) will automatically imply that \(W_4 = W_5 = 0\) and \(W_1, W_2\) real so that we indeed obtain a half-flat \(SU(3)\)-structure as advertised. Furthermore, we can construct the remaining torsion classes from the identities

\[
\begin{align*}
W_1 &= -\frac{1}{6} \ast_6 (dJ \wedge \Omega_I), \\
W_2 &= - \ast d\Omega_I + 2W_1 J, \\
W_3 &= dJ - \frac{3}{2} W_1 \Omega_R.
\end{align*}
\]

(41)

The parity properties further imply the following relations

\[
\begin{align*}
dW_3 &= 0, & dW_2 \wedge W_3 &= 0, \\
W_2 \wedge W_3 &= 0, & W_2 \wedge \ast_6 W_3 &= 0.
\end{align*}
\]

(42)

\[\text{To understand how the torsion classes depend on all moduli is not too hard for these simple examples, but formulae for more general cases have been derived in [30].}\]
4.2 de Sitter no-go theorems

The above choice of orbifold/orientifold group is not unique and we chose it for simplicity. But, interestingly, all other choices of orbifold groups have shown not to admit any de Sitter solutions according to [7].

The explicit form of the de Sitter no-go theorem of [7], in our setup, implies that the matrix

\[
F = \begin{pmatrix}
    f_{45}^1 & f_{23}^1 & -f_{34}^6 & -f_{25}^6 \\
    f_{56}^2 & -f_{36}^4 & f_{13}^2 & f_{15}^4 \\
    -f_{46}^3 & f_{56}^5 & f_{14}^5 & f_{12}^3
\end{pmatrix}
\]

should have at least two columns non-zero and three rows non-zero.

It is not straightforward to classify all six-dimensional Lie-algebras that can be written as (36). But once an algebra is found it is easy to check the no-go theorem. We postpone a full classification of groups and cosets that violate the no-go theorem to the future. So far we have found four algebras which fulfill the conditions

\[
SO(4), \quad SO(3,1), \quad SO(2,2), \quad s1.2,
\]

where s1.2 is the solvable algebra that appears in e.g. the classification of [29]. Note that SO(4) = SU(2) × SU(2) and SO(2,2) = SU(1, 1) × SU(1, 1).

The first and the last correspond to the algebras used for the de Sitter solutions constructed in [6, 7]. The last three algebras correspond to non-compact groups and one has to demonstrate that they can be compactified. We do not discuss this here, but mention that this can depend on the explicit solution, since the symmetries of the metric depend on the expectation values for the metric scalars \(a, b, c, v_1, \ldots, v_4\).

The algebras are explicitly given by

\[
F_{SO(4)} = \frac{1}{2} \begin{pmatrix}
    +1 & +1 & +1 & -1 \\
    +1 & -1 & -1 & -1 \\
    +1 & -1 & +1 & +1
\end{pmatrix}, \quad F_{SO(2,2)} = \frac{1}{2} \begin{pmatrix}
    +1 & +1 & +1 & -1 \\
    -1 & +1 & +1 & +1 \\
    +1 & -1 & +1 & +1
\end{pmatrix},
\]

\[
F_{SO(3,1)} = \frac{1}{2} \begin{pmatrix}
    -1 & +1 & +1 & -1 \\
    -1 & -1 & -1 & -1 \\
    -1 & -1 & +1 & +1
\end{pmatrix}, \quad F_{s1.2} = \frac{1}{2} \begin{pmatrix}
    -1 & +1 & 0 & 0 \\
    -1 & -1 & 0 & 0 \\
    +1 & +1 & 0 & 0
\end{pmatrix},
\]

up to re-scalings, permutations, and other transformations in GL(6, \mathbb{R}) that preserve the form (36). We expect more algebras to exist that evade the no-go theorem of [7] and more details will be presented elsewhere [17].

4.3 Constraints on the torsion classes

With some explicit algebras at hand we can check the various conditions for allowing the universal de Sitter solution of the previous section. Let us recapitulate the conditions on
the torsion classes

\[ W_2 = 0, \]  

\[ d \ast_6 \hat{W}_3 = c_1 J \wedge J, \]  

\[ (\hat{W}_3^2)_{ij} = 0, \]  

\[ Q_1(\hat{W}_3, \hat{W}_3) = c_2 Q_2(\hat{W}_3, \hat{W}_3), \]  

\[ Q_1(\hat{W}_3, \hat{W}_3) = c_3 (\hat{W}_3)_{2,1}. \]

Taking into account the restricted set of odd three-forms under the orientifold involutions (see the discussion before (34)) we can verify that constraint (46c) is automatic as well as (46d) for \( c_2 = 1. \)

Likewise, constraint (46e) can be simplified using the O-plane ansatz. Let us write

\[ W_3 = \alpha_1 e^{456} + \alpha_2 e^{236} + \alpha_3 e^{134} + \alpha_4 e^{125}, \]  

where we note that the \( \alpha_i \) are not entirely arbitrary as \( W_3 \) needs to be a simple (2,1)+(1,2)-form. After some algebra one finds that (46e) leads to the following four simple equations

\[ 2 \left( \frac{3 \alpha_i^2}{v_i^2} - \sum_{j \neq i} \frac{\alpha_j^2}{v_j^2} \right) = c_3 w_3 \alpha_i/v_i. \]

One verifies that these equations are only consistent when

\[ c_3 = \pm \frac{8}{\sqrt{3}} \text{ or } c_3 = 0. \]

We do not consider the case \( c_3 = 0 \) as we have not found any interesting results with that value. Furthermore, we can always take the positive value for \( c_3 \) by allowing \( w_3 \) and \( \hat{W}_3 \) to flip sign (under which the physical \( W_3 \) does not change).

The remaining constraint equations need to be imposed and fix the geometric moduli \( a, b, c, v_i \). Furthermore we found from the analysis of section 3.2 that we need

\[ 4.553 < w_3/W_1 < 3\sqrt{3}. \]

We have investigated these constraints for the four algebras listed above and only for \( \text{SU}(2) \times \text{SU}(2) \) we found that the constraints lead to a solution. In all other cases one finds a complex solution or a solution with a metric that is not positive-definite anymore. For \( \text{SO}(3,1) \) we came close, but then (50) was violated.

This implies that the de Sitter solution of \( [7] \), which was found for the s1.2 algebra, is not captured with our ansatz. We have also checked that it can not be found from the more extended ansatz with \( W_2 \neq 0. \) It would be interesting to understand how many extra non-universal forms one needs to describe this solution from a ten-dimensional viewpoint.
Let us now demonstrate that on the group manifold SU(2) × SU(2) one can satisfy all the constraints (46a-46e, 49, 50) above and construct a dS solution. The geometric moduli values for which SU(2) × SU(2) obeys the constraints (46a-46e) are given by

\[ a = -b = c, \quad v_1 = v_2 = v_4, \quad v_3 = \frac{a^6}{(v_1)^3}. \]  

For these values we have explicitly

\[ g = \text{diag} \left( \frac{a^4}{(v_1)^2}, \frac{(v_1)^2}{a^2}, \frac{a^4}{(v_1)^2}, \frac{(v_1)^2}{a^2}, \frac{(v_1)^2}{a^2} \right), \]

\[ W_1 = \frac{a^6 + v_1^4}{4a^6v_1}, \quad W_2 = 0, \]

\[ W_3 = \frac{a^6 - 3(v_1)^4}{8a^5(v_1)^4} \left[ (v_1)^4(e^{456} + e^{236} + e^{125}) - 3a^6e^{134} \right], \quad w_3 = -\frac{\sqrt{3}(a^6 - 3(v_1)^4)}{4a^6v_1}, \]

where we have chosen the sign of \( w_3 \) such that \( c_3 = 8/\sqrt{3} \).

We find furthermore that

\[ \frac{w_3}{W_1} = \frac{\sqrt{3}(3(v_1)^4 - a^6)}{a^6 + (v_1)^4} \]

takes values in the interval \(-\sqrt{3} < \frac{w_3}{W_1} < 3\sqrt{3}\) so that we can satisfy (50) and find dS solutions (as well as a Minkowski solution at \( \frac{w_3}{W_1} = 4.553 \) and AdS solutions below that value). Note that since the endpoint \( \frac{w_3}{W_1} = 3\sqrt{3} \) would correspond to \( \frac{(v_1)^4}{a^6} \to \infty \) we cannot actually construct the Minkowski solution [27]. It is amusing to see the geometry of SU(2) × SU(2) seems to know about the type IIA supergravity equations of motion in that the upper bound for \( \frac{w_3}{W_1} \) on SU(2) × SU(2) corresponds to the upper bound for any type of solutions in figure 2.

Using \( \mathcal{N} = 1 \) supergravity techniques along the lines of [6] it is possible to calculate the scalar potential and obtain the eigenvalues of the (14 × 14) mass matrix for the left-invariant fluctuations around our dS solutions. We found that there are always unstable directions. In figure 3 we make a plot of the negative eigenvalues. In particular we have one tachyonic direction for \( f_2/f_1 < 2.16 \) and three tachyonic directions beyond this value, as shown in the figure 3. The two extra tachyons that appear beyond \( f_2/f_1 = 2.16 \) have the same negative (mass)² and since it is small compared to the other masses, we have zoomed in on it in the figure.

That this configuration satisfies the constraints (46a-46e) can be easily seen as follows. Consider the exchanges

\[ p_1 : e^1 \leftrightarrow e^4, e^6 \leftrightarrow e^2 \quad p_2 : e^1 \leftrightarrow e^3, e^6 \leftrightarrow e^5. \]  

One easily sees that the only even two-form under these interchanges is \( J \), while from the three-forms odd under the orientifold involutions, the ones also odd under this symmetry group are spanned by \( \Omega_R \) and \( W_3 \). Furthermore, the structure constants and thus the exterior derivative are also odd. The above constraints now follow from the transformation properties under these exchanges.
Figure 3: Negative eigenvalues of the mass matrix $M^2/(f_1)^2$ for our line of dS solutions.

6 Discussion

We have established the fact that the class of universal de Sitter solutions exists by at least providing one explicit example: an unstable de Sitter solution on an orientifold of $SU(2) \times SU(2)$. This solution turns out to be the ten-dimensional lift of the solution found in [4]. Furthermore, the other explicit solution, coming from (an orientifold) of the solvmanifold $s.2$ [7] is not universal, which demonstrates that the class of universal de Sitter solutions does not cover all of the classical de Sitter solutions. But, as we argued in the introduction, we believe that most natural solutions are of this form and we hope to report on more examples in a future work [17].

Obviously, the understanding of classical de Sitter solutions is very incomplete, and so far no conclusion can be drawn regarding the existence of phenomenologically viable solutions. The requirements for phenomenological viability are plenty, and some requirements can be dropped for other purposes. Especially if we want a simple de Sitter solution for the sake of understanding holography, or more general, quantum gravity in de Sitter space-time, we can drop the requirement for a small cosmological constant, a decoupling of KK modes, etc. The most important requirement is perturbative stability. The two examples coming from $SU(2) \times SU(2)$ and the solvmanifold $s.2$ are both perturbative unstable. This is not surprising since the lack of supersymmetry does not protect one from tachyons in the spectrum. Since most simple models have order 10 moduli, one has to be rather lucky that all of them have positive mass. In case one is allowed to think that, in the absence of susy, there is an equal chance for a field direction to be unstable or stable, then one is forced to conclude that stable solutions must exist if there are enough classical de Sitter solutions in the landscape. But, the existence of meta-stable non-susy AdS solutions seems to demonstrate that one should not apply a “50-50” reasoning for all field directions to be stable.

When it comes to stability with respect to the light degrees of freedom it is useful to consult the results that were obtained directly in four-dimensional supergravity theories, without the concern of a higher-dimensional origin. Investigations on this mainly focus on extended gauged supergravities [31–38], and some on $\mathcal{N} = 1$ supergravity [10, 28, 39, 40].
These results indicate that in theories with $\mathcal{N} > 2$ metastable solutions do not exist, while for theories with $\mathcal{N} \leq 2$ metastable solutions exist, but are not generic. For those metastable examples for which the ten-dimensional origin seems understood the solutions are non-geometric \cite{28}, which is problematic since the supergravity limit might be invalid. The known geometric examples \cite{6,7} are solutions of four-dimensional $\mathcal{N} = 1$ supergravity. Although the solutions are unstable, they do evade the known no-go theorems for stable de Sitter solutions in $\mathcal{N} = 1$ supergravity \cite{10,39}.

Finally a word on directions that deserve further investigation. Obviously, it would be interesting to find more explicit simple de Sitter solutions, in order to be able to understand how generic unstable solutions are and whether meta-stable solutions can be found at all. With the universal ansatz there is no reason to restrict to group and coset manifolds and we can also look at non-homogeneous manifolds. One could also be interested in simple de Sitter solutions in higher dimensions than four. In higher dimensions we expect less moduli and this simplifies the situation, and, for pure theoretical purposes de Sitter solutions in any dimension are of interest. Another way to obtain more classical de Sitter solutions would be by looking at IIB supergravity. Recently it has been shown in reference \cite{41} that moduli stabilisation at tree-level in IIB AdS vacua is possible as well, and more importantly, an unstable dS critical point of the effective potential was found. This opens up the interesting possibility of finding tree-level dS vacua in IIB.

Perhaps a more pressing problem than stability is the issue of the backreaction of the sources. Orientifolds are really localised objects and smearing them leads to lower-dimensional supergravity effective actions, but the consistency of smearing from a string theory point of view has not been shown \cite{42}. This issue arises for some of the AdS solutions as well, and it is important to sort it out.

Acknowledgements

We benefitted from useful discussions and ongoing collaboration with Shajid Haque, Gary Shiu and Timm Wrase. We also thank Timm Wrase for comments on the manuscript. T.V.R likes to thank Tine De Smedt for some help in editing the pictures.

U.D. is supported by the Swedish Research Council (VR) and the Göran Gustafsson Foundation. P.K. is a Postdoctoral Fellow of the FWO – Vlaanderen. The work of P.K. is further supported in part by the FWO – Vlaanderen project G.0235.05 and in part by the Federal Office for Scientific, Technical and Cultural Affairs through the ‘Interuniversity Attraction Poles Programme Belgian Science Policy’ P6/11-P. T.V.R. is supported by the Göran Gustafsson Foundation.

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