Restoration of superconductivity in high magnetic fields in UTe$_2$

Andrei G. Lebed

Department of Physics, University of Arizona,
1118 E. 4th Street Tucson, Arizona 85721, USA
lebed@physics.arizona.edu and
L.D. Landau Institute for Theoretical Physics,
RAS, 2 Kosygina Street, Moscow 117334, Russia

It was theoretically predicted more than 20 years ago [A.G. Lebed and K. Yamaji, Phys. Rev. Lett. 80, 2697 (1998)] that a triplet quasi-two-dimensional (Q2D) superconductor could restore its superconducting state in parallel magnetic fields, which are higher than its upper critical magnetic field, $H > H_{c2}(0)$. It is very likely that recently such phenomenon has been experimentally discovered in the Q2D superconductor UTe$_2$ by Nicholas Butch, Sheng Ran and their colleagues and has been confirmed by Japanese-French team. We review our previous theoretical results, using such a general method that it describes the reentrant superconductivity in the above mentioned compound as well as will hopefully describe the similar phenomena, which can be discovered in other Q2D superconductors.

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1. INTRODUCTION

The so-called reentrant superconductivity phenomenon, experimentally observed in quasi-two-dimensional (Q2D) organic superconductor $\lambda$-(BETS)$_2$FeCl$_4$ [1] as well as in ferromagnetic superconductors URhGe [2,3] and UCoGe [4], have been recently intensively studied both experimentally and theoretically. In the case of the above mentioned organic superconductor, the high-field superconducting phase has been prescribed to Jaccarino-Peter effect [5], whereas the physical origin of the reentrant phase in the ferromagnetic superconductors was prescribed to the existence of ferromagnetic fluctuations [6,7]. On the other hand, for layered Q1D [8-10] and for isotropic within the layers Q2D triplet superconductors [11], many years ago, there was suggested effect of reentrant superconductivity in a parallel magnetic field. It was later confirmed in Refs. [12-15]. Very recently, superconductivity and the reentrant superconductivity have been discovered [16-19] in the non-ferromagnetic Q2D [15,20] superconductor UTe$_2$. As was stressed in Ref. [15], the above mentioned reentrant superconductivity cannot be due to the ferromagnetic fluctuations and are likely due to the effect of two-dimensionalization of electron spectrum first theoretically predicted in Refs.[8,11].

2. GOAL

Our goal is to review Refs.[8,11,12], using the general method [12], that describes well the case of the Q2D superconductor UTe$_2$ [20]. We hope that it would describe also possible discoveries of the reentrant superconductivity, which may be done in the future in different Q2D and Q1D materials. In other words, we show that the reentrant superconductivity [8,11,12] appears in Q2D and layered Q1D superconductors due to two-dimensionalization of electron spectrum for arbitrary in-plane shapes of electron spectra and arbitrary triplet equal-spin in-plane superconducting electron interactions. It is important that our approach is qualitatively also applied to two-band superconductivity, which may exist in UTe$_2$ [20].

3. RESTORATION OF SUPERCONDUCTIVITY IN A GENERAL Q2D CASE

In this section, we consider a general Q2D case [12], where in-plane electron spectrum has an arbitrary shape and in-plane electron-electron interactions are of a general form and promote a triplet pairing, which is not sensitive to the Pauli spin-splitting effects against superconductivity.
3.1. Qualitative description of a general Q2D case

In this subsection, we suggest strong qualitative arguments why superconductivity restores in a Q2D triplet superconductor at very high magnetic fields, $H > H_{c2}$. We consider a layered superconductor with the following Q2D electron spectrum in a metallic phase:

$$\epsilon(p) = \epsilon_{\parallel}(p_x, p_y) + 2t_\perp \cos(p_z d), \quad t_\perp \ll \epsilon_F,$$

where arbitrary in-plane energy, $\epsilon_{\parallel}(p_x, p_y)$, corresponds to closed Fermi surface (FS), $t_\perp$ is the integral of overlapping of electron wave functions in a perpendicular to the conducting planes direction, $d$ is a distance between the conducting layers, and $\epsilon_F$ is the Fermi energy. For the further development, it is convenient to linearize Q2D electron spectrum (1) near the FS:

$$\epsilon^\pm(p) - \epsilon_F = \pm |v_x(p_y)| |p_x \mp |p_x(p_y)|| + 2t_\perp \cos(p_z d),$$

where $v_x(p_y)$ is $x$-component of the Fermi velocity on the FS, $p_x(p_y)$ satisfies the following condition:

$$\epsilon_{\parallel}[p_x(p_y), p_y] = \epsilon_F,$$

$\pm$ stands for $p_x(p_y) > 0$ [$p_x(p_y) < 0$].

In a magnetic field,$$
\mathbf{H} = (0, H, 0), \quad \mathbf{A} = (0, 0, -Hx),$$
electron quasiclassical motion on the FS occurs due to the following $z$-component of the Lorenz force:

$$\frac{dp_z}{dt} = \frac{e}{c} v_x(p_y)H.$$

Taking into account that

$$v_z(p_z) = \frac{2t_\perp \cos(p_z d)}{dp_z} = -2t_\perp d \sin(p_z d),$$

we find that the electron motion between the conducting planes is a trajectory oscillating in time,

$$z(t) = l_\perp(H, p_y) \cos[\omega_c(H, p_y)t].$$

with the frequency and amplitude being:

$$\omega_c(H, p_y) = \frac{e|v_x(p_y)|}{c} H d, \quad l_\perp(H, p_y) = \frac{d}{\omega_c(H, p_y)} 2t_\perp.$$ 

From Eqs.(7) and (8), it directly follows that the amplitudes of electron motion between the conducting planes in a magnetic field decrease with the increasing magnetic field (4). In very high magnetic fields, the electron amplitudes become less than the distance between the planes, $d$, for the majority of electrons. It happens when

$$H \geq H^* = \frac{2t_\perp c}{e v_F d},$$

where $v_F$ is a characteristic velocity of in-plane electron motion. In this case, the destructive Meissner currents perpendicular to the planes become small and superconducting state has to restore (see Fig.1). This is a qualitative explanation of the two-dimensionalization phenomena of electron spectrum which lead to the restoration of superconductivity in high magnetic fields [8-15]. We pay attention that the restoration of superconductivity has to occur for any in-plane anisotropic electron spectrum (1) and for any equal spin in-plane electron-electron interactions. Generalization of the two-dinsionalization phenomenon for two-band superconductivity is straightforward.
3.2. Quantitative description of a general Q2D case

Here, we use the Green’s functions method to quantitatively establish the two-dimensionalization phenomenon and the restoration of superconductivity in high magnetic fields in a triple Q2D superconductor with a general in-plane electron spectrum and with general in-plane electron-electron interactions[12,11]. In a magnetic field $\mathbf{H} \parallel \mathbf{y}$ in the gauge (4), we can use for the electron spectrum (2) the so-called Peierls substitution method,

$$
p_x \rightarrow -i \frac{d}{dx}, \quad p_z \rightarrow p_z - \frac{e}{c} A_z = p_z + \frac{e}{c} Hx.
$$

In this case, we obtain the following Schrödinger-like equation for non-interacting electron wave functions in the magnetic field (4):

$$
\left\{ \pm |v_x(p_y)| \left[ -i \frac{d}{dx} \mp |p_x(p_y)| \right] + 2t_\perp \cos \left( p_z d + \frac{eHdx}{c} \right) - 2\mu_B H \sigma \right\} \Psi(x, p_y, p_z; \sigma) = e\Psi(x, p_y, p_z; \sigma)
$$

The solutions of Eq.(11) for the electron wave functions are:

$$
\Psi_e(x, p_y, p_z; \sigma) = \frac{1}{\sqrt{|v_x(p_y)|}} \exp\left[ \pm \frac{ix}{|v_x(p_y)|} \right] \exp[\pm i |p_z(p_y)|x] \times \exp\left[ \pm \frac{2\mu_B H \sigma x}{|v_x(p_y)|} \right] \exp\left[ \pm \frac{\lambda(p_y)}{2} \sin \left( p_z d + \frac{eHdx}{c} \right) \right],
$$

where $\mu_B$ is the Bohr magneton, $\sigma = \pm \frac{1}{2}$ is $y$-component of the electron spin; $\lambda(p_y) = 4t_\perp c/|v_x(p_y)|Hd$. Let us define superconducting transition temperature in the magnetic field (4). To this end, it is convenient to introduce equation for temperature (Matsubara’s) Green’s function [21]. In accordance with Eq.(11) and Ref.[21], the Green’s functions, $G^{\pm}_{\omega_n}(x, x_1; p_y, p_z; \sigma)$, obey the following equation:

$$
\left\{ -i\omega_n \pm |v_x(p_y)| \left[ -i \frac{d}{dx} \mp |p_x(p_y)| \right] + 2t_\perp \cos \left( p_z d + \frac{eHdx}{c} \right) - 2\mu_B H \sigma \right\} \times G^{\pm}_{\omega_n}(x, x_1; p_y, p_z; \sigma) = \delta(x - x_1),
$$

where $\delta(x - x_1)$ is the Dirac delta-function. It is important that Eq.(13) can be analytically solved:

$$
G^{\pm}_{\omega_n}(x, x_1; p_y, p_z; \sigma) = -i \frac{sgn(\omega_n)}{|v_x(p_y)|} \exp\left[ \pm \frac{i\omega_n(x-x_1)}{|v_x(p_y)|} \right] \exp[\pm i |p_z(p_y)| |x - x_1|] \exp\left[ \pm \frac{2\mu_B H \sigma x}{|v_x(p_y)|} \right] \exp\left[ \pm \frac{\lambda(p_y)}{2} \sin \left( p_z d + \frac{eHdx}{c} \right) \right].
$$
\[ \Delta(p_x, p_y; x) = \int dp_y \int_{x-x_1>\mid v_x(p_y)\mid/\Omega}^{\infty} \frac{2\pi T dx_1}{v_x^2(p_y) \sinh \frac{2\pi T |x-x_1|}{|v_x(p_y)|}} \times J_0 \left( 2\lambda(p_y) \sin \frac{\epsilon H\delta(x-x_1)}{e} \sin \frac{\epsilon H\delta(x+x_1)}{e} \right) \times \{ U[p_x, p_y; |p_x^1(p_y)|, p_y^1|\Delta[|p_x^1(p_y)|, p_y^1; x_1]] + U[p_x, p_y; -|p_x^1(p_y)|, p_y^1|\Delta[-|p_x^1(p_y)|, p_y^1; x_1]} \right), \] 

where the order parameter, \( \Delta(p_x, p_y; x) \), depends on the position of a center of mass of the Cooper pair, \( x \), as well as on the position on the FS \( p_x \) and \( p_y \) satisfy the following condition: \( \epsilon(p_x, p_y) = \epsilon_F \); \( U[p_x, p_y; |p_x^1(p_y)|, p_y^1] \) is a matrix element of the electron-electron interactions; \( S = 0, 1 \) is the total spin of the Cooper pair; \( \Omega \) is a cutoff energy.[Note that, in Eq.(15), the Bessel function, \( J_0(\ldots) \), describes the orbital effects against superconductivity in a magnetic field, whereas \( \cos[\ldots] \) describes the destructive Pauli spin-splitting paramagnetic effects]. Below, we consider the case of triplet equal-spin pairing, therefore, \( S = 1 \) in Eq.(15) and, thus, the Pauli paramagnetic effects against superconductivity are absent.

Thus, we can rewrite Eq.(15) in the following form:

\[ \Delta(p_x, p_y; x) = \int dp_y \int_{x-x_1>\mid v_x(p_y)\mid/\Omega}^{\infty} \frac{2\pi T dx_1}{v_x^2(p_y) \sinh \frac{2\pi T |x-x_1|}{|v_x(p_y)|}} \times J_0 \left( 2\lambda(p_y) \sin \frac{\epsilon H\delta(x-x_1)}{e} \sin \frac{\epsilon H\delta(x+x_1)}{e} \right) \times \{ U[p_x, p_y; |p_x^1(p_y)|, p_y^1|\Delta[|p_x^1(p_y)|, p_y^1; x_1]] + U[p_x, p_y; -|p_x^1(p_y)|, p_y^1|\Delta[-|p_x^1(p_y)|, p_y^1; x_1]} \right). \] 

Then, by means of relationships,

\[ \frac{dp_y}{v_x(p_y)} = \frac{dp_l}{v_{l\perp}(p_l)}, \quad dp_l^2 = dp_x^2 + dp_y^2, \quad v_{l\perp}^2(p_l) = v_x^2(p_l) + v_y^2(p_l), \] 

we can express Eq.(16) as

\[ \Delta(p_l; x) = \int \frac{dp_l}{v_{l\perp}(p_l)} \int_{x-x_1>\mid v_x(p_l)\mid/\Omega}^{\infty} \frac{2\pi T dx_1}{|v_x(p_l)| \sinh \frac{2\pi T |x-x_1|}{|v_x(p_l)|}} \times J_0 \left( 2\lambda(p_l) \sin \frac{\epsilon H\delta(x-x_1)}{e} \sin \frac{\epsilon H\delta(x+x_1)}{e} \right) \times U[p_l; p_l^1|\Delta[p_l, p_l^1; x_1]], \] 

where integration in Eq.(18) is performed over the FS contour.

Let us introduce new variable \( z \),

\[ x_1 = x + z|v_x(p_l^1)|/v_F, \quad v_F = \mid v_x(p_l) \mid > p_l, \]
$\langle p \rangle_{\parallel}^i$ is an average value over the FS. In this new variable of integration the gap equation (18), can be rewritten in the following more convenient way:

$$\Delta(p; x) = \oint_{v_{\perp}(p)}^{p_{\parallel}} \frac{dp_{\parallel}^i}{2 \pi T d\Omega} \int_{|z| > v_F / \Omega} \frac{2 \pi T d\Omega}{v_F} \frac{2 \pi T d\Omega}{v_F}$$

$$\times J_0 \left\{ 2 \lambda(p_{\parallel}) \sin \left[ \frac{2 \pi T d\Omega}{v_F} \right] \sin \left[ \frac{2 \pi T d\Omega}{v_F} \right] \right\}$$

$$\times U[p_{\parallel}; p_{\parallel}] \Delta[p_{\parallel}, p_{\parallel}^i x + z | v_{\perp}(p_{\parallel}) / v_F],$$

(20)

The effect of the two-dimensionalization of the Q2D electron spectrum (1) and the restoration of superconductivity in a magnetic field are directly seen from Eq. (20), where

$$\lambda(p_{\parallel}) = \frac{2[l_{\perp}(p_{\parallel})]}{d}$$

(21)

is a dimensionless magnitude of electron trajectory in the perpendicular to the planes direction, expressed in terms of the inter-plane distance, $d$. If $H \geq H^*$, where the critical field $H^*$ is given by Eq.(9), then $|\lambda(p_{\parallel})| \leq 1$ for the significant part of electrons on the Q2D FS. In this case, the Bessel function $J_0(...) \approx 1$ in Eq. (20) and, therefore, Eq. (20) has the same solutions as without the magnetic field (4):

$$\Delta(p) = \oint_{v_{\perp}(p)}^{p_{\parallel}} \frac{dp_{\parallel}^i}{2 \pi T d\Omega} \int_{|z| > v_F / \Omega} \frac{2 \pi T d\Omega}{v_F} \frac{2 \pi T d\Omega}{v_F}$$

$$\times U[p_{\parallel}; p_{\parallel}] \Delta[p_{\parallel}, p_{\parallel}].$$

(22)

For this reason superconductivity restores in the triplet case at $H \geq H^*$ with the same transition temperature, as it has in zero magnetic field (see Fig.1):

$$T_c(H \gg H^*) \approx T_c(0).$$

(23)

As we mentioned in the previous subsection, the physical meaning of the restoration of superconductivity is that electrons are almost localized on the conducting planes and, therefore, the destructive Meissner currents are significantly suppressed at $|\lambda(p_{\parallel})| \leq 1$. In the case of s(d)-wave superconducting pairing [i.e., at $S = 0$ in the Eq.(15)], the above described phenomenon creates opportunity [8,11] for superconductivity to exist at $H > H_p(0)$ in the form of the so-called Larkin-Ovchinnikov-Fulde-Ferrel (LOFF) phase [25,26], where $H_p$ is the so-called paramagnetic limit [27,28].

4. RESTORATION OF SUPERCONDUCTIVITY IN AN IN-PLANE ISOTROPIC Q2D CASE

Let us consider an important limiting case with in-plane isotropic electron spectrum and the simplest in-plane triplet superconducting electron-electron interactions [11].

4.1. Qualitative description of an in-plane isotropic Q2D case

As usual, we start from qualitative description of the two-dimensionalization of in-plane isotropic Q2D electron spectrum and its consequence - the phenomenon of the restoration of superconductivity in high magnetic fields, $H > H_c2$. Instead of arbitrary electron spectrum, here we consider a layered superconductor with the following in-plane isotropic Q2D electron spectrum in a metallic phase:

$$\epsilon(p) = \frac{p_x^2 + p_y^2}{2m} + 2t_\perp \cos(p_z d), \quad t_\perp \ll \epsilon_F,$$

(24)

where isotropic in-plane energy corresponds to the in-plane closed FS, $t_\perp$ is the integral of overlapping of electron wave functions in a perpendicular to the conducting planes direction, $d$ is a distance between the conducting layers, and $\epsilon_F$ is the Fermi energy. For calculation of the quasi-classical electron trajectories, it is convenient, as usual, to linearize Q2D electron spectrum (24) near the FS:

$$\epsilon^\pm(p) - \epsilon_F = \pm v_F |\sin \phi| |p_x + p_F| |\sin \phi| + 2t_\perp \cos(p_z d),$$

(25)
where we count the polar angle \( \phi \) from y-axis. In Eq. (25), \( v_F \) is the Fermi velocity, \( p_F = m v_F \) is the Fermi momentum; \( v_F \sin \phi \) is \( x \)-component of the Fermi velocity, \( p_F \sin \phi \) is \( x \)-component of the Fermi momentum, which satisfies the following condition:

\[
\frac{p_F^2 \sin^2 \phi + p_F^2 \cos^2 \phi}{2m} = \frac{p_F^2}{2m} = \epsilon_F. \tag{26}
\]

In the external magnetic field (4), electron motion on the FS is due to the action of the following z-component of the Lorenz force:

\[
\frac{dp_z}{dt} = \frac{e}{c} v_F \sin \phi H. \tag{27}
\]

It is known that in the quasiclassical approximation the superconducting gap, \( \Delta(x, y) \), depends on the coordinate of a center of mass of the Cooper pair, \( x \), as well as on the position on the FS, where \( \phi \) is the polar angle between \( y \)-axes and two component vector \( \mathbf{p} = [p_x(p_y), p_y] \), where \( p_x^2 + p_y^2 = p_F^2 \). In this review, we consider the case, where electron-electron interactions depend only on in-plane momenta. In this section, in contrast to the previous one, we consider the following simplest case of triplet equal spin pairing:

\[
U(\phi_1, \phi_1) = g \ u(\phi) \ u(\phi_1). \tag{33}
\]
In this case, we can rewrite Eq.(33) in more simple form:

\[
\Delta(x) = \frac{g}{2} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{|x-x_1|>v_F} u^2(\phi_1) \frac{2\pi T dx_1}{v_F |\sin \phi_1| \sinh \frac{2\pi T |x-x_1|}{v_F |\sin \phi_1|}} \times J_0 \left\{ \frac{\lambda_1}{|\sin \phi_1|} \sin \left[ \frac{\phi_1 (x-x_1)}{2v_F} \right] \sin \left[ \frac{\phi_1 (x+x_1)}{2v_F} \right] \right\} \Delta(x_1),
\]

where the superconducting gaps in Eqs.(31) and (34) are

\[
\Delta(\phi, x) = u(\phi) \Delta(x),
\]

\( g \) is dimensionless constant of electron coupling. By introducing the more appropriate variable,

\[
x_1 - x = |z| \sin \phi_1.
\]

\[
\Delta(x) = \frac{g}{2} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{|z|>v_F / \Omega} \frac{2\pi T dz}{v_F |\sin \phi_1|} \sinh \frac{2\pi T |z|}{v_F |\sin \phi_1|} u^2(\phi_1)
\times J_0 \left\{ \frac{\lambda_1}{|\sin \phi_1|} \sin \left[ \frac{\phi_1 |z| \sin \phi_1}{2v_F} \right] \sin \left[ \frac{\phi_1 (2x+|z| \sin \phi_1)}{2v_F} \right] \right\} \Delta(x + z | \sin \phi_1|),
\]

Since \( \lambda \) is inversely proportional to a magnetic field [see Eq.(32)], it is clear that in high magnetic fields, \( H > H^* \) [see Eq.(9) and Fig.1], superconductivity has to restore with the zero-field transition temperature [see Eq.(23)].

5. RESTORATION OF SUPERCONDUCTIVITY IN A LAYERED Q1D CASE

In this section, we consider an important limiting case of layered Q1D superconductors with the simplest equal spin triplet superconducting electron-electron interactions [8].

5.1. Qualitative description of a layered Q1D case

As in the previous sections, we begin our consideration from the qualitative description of the phenomenon of the restoration of superconductivity in high magnetic fields, \( H > H_2 \). Instead of a Q2D electron spectrum, here we consider a layered superconductor with the following Q1D electron spectrum in a metallic state:

\[
\epsilon(p) = 2t_a \cos(p_a a/2) + 2t_b \cos(p_y b) + 2t_{1} \cos(p_z d), \quad t_{1} \ll t_b \ll t_a.
\]

This layered Q1D electron spectrum is realized in the so-called Bechgaard salts - compounds with chemical formular (TMTSF)$_2$X, where X=ClO$_4$, PF$_6$, AsF$_6$, etc. [We note that the compound (TMTSF)PF$_6$ is considered as a candidate for a triplet electron pairing.] The first term in Eq.(38) represents free-electron motion along the chains \((t_a \approx 2500K)\); whereas \( t_b \approx 250K \) and \( t_{1} \approx 3 - 5K \) are the overlapping integrals of electron wave functions in the perpendicular to the conducting chains directions. To calculate the quasi-classical electron trajectories in the magnetic field (4), as usual, it is convenient to linearize Q1D electron spectrum (38) near the Q1D FS’s:

\[
\epsilon^\pm(p) - \epsilon_F = \pm v_F [p_x \mp p_F] + 2t_b \cos(p_y b) + 2t_{1} \cos(p_z d),
\]

where \( v_F = t_a a \sin(p_F a/2) = t_a a / \sqrt{2} \) is the Fermi velocity, \( p_F = \pi / 2a \) is the Fermi momentum, \(+(-)\) stands for right(left) piece of the Q1D FS.

In the external magnetic field (4), electron motion on the right(left) piece of the Q1D FS (39) satisfies the conditions:

\[
\frac{dp_z}{dt} = \pm \frac{e}{c} v_F H.
\]

As usual, in the quasiclassical approximation

\[
v_z(p_z) = \frac{2t_{1} \cos(p_z d)}{dp_z} = -2t_{1} d \sin(p_z d)
\]

Instead of a Q2D electron spectrum, here we consider a layered superconductor with the following Q1D electron spectrum in a metallic state:
and we find that electron trajectory between the conducting planes is the following oscillating function in time,

$$z(t) = l_\perp(H) \cos[\omega_c(H)t],$$

(42)

with the frequency and amplitude being:

$$\omega_c(H) = \frac{ev_F H d}{c}, \quad l_\perp(H) = d \frac{2t_\perp}{\omega_c(H)}.$$

(43)

As directly follows from Eqs. (42) and (43) [compare to Eqs. (7) and (8)], the amplitude of electron motion between the conducting planes in the magnetic field (4) becomes less than the inter-plane distance for very high magnetic fields,

$$H \geq H^*,$$

(44)

where $H^*$ is given by Eq.(9). As usual, in this case, the destructive Meissner currents become small and superconductivity has to restore with $T_c(H \gg H^*) \approx T_c(0)$ (see Fig.1).

5.2. Quantitative description of a layered Q1D case

Below, we consider the simplest equal-spin triplet electron-electron pairing in a Q1D case, where the superconducting gap changes its sign on the different pieces of the FS. It corresponds to the following electron-electron interactions:

$$U(p_x, p_{x1}) = g \text{sign}(p_x) \text{sign}(p_{x1})$$

(45)

and to the following superconducting gap:

$$\Delta(p_x; x) = \text{sign}(p_x)\Delta(x).$$

(46)

In this case, Eq.(15) can be rewritten in the more simple way:

$$\Delta(x) = \frac{g}{2} \int_{|z| > v_F/\Omega} 2\pi T dz, \quad \omega_c(z) = \frac{2\pi T dz}{v_F \sinh \left[ 2\pi T |z| v_F \right]} \sin \left[ \frac{\omega_c(z)}{2v_F} \right] \Delta(x),$$

(47)

where the parameters $\lambda$ and $\omega_c$ are defined by Eq.(32). [Note that, in Eq.(47), the superconducting gap, $\Delta(x)$, depends only on the coordinate of a center of mass of the Cooper pair, $x$.] If we introduce, as usual, the more convenient variable,

$$x_1 - x = z,$$

(48)

then

$$\Delta(x) = \frac{g}{2} \int_{|z| > v_F/\Omega} 2\pi T dz, \quad \omega_c(z) = \frac{2\pi T dz}{v_F \sinh \left[ 2\pi T |z| v_F \right]} \sin \left[ \frac{\omega_c(z)}{2v_F} \right] \Delta(x + z),$$

(49)

Since $\lambda \sim \frac{1}{H}$, it is clear that, in high magnetic fields, $H \geq H^*$, where $H^*$ is given by Eq.(9), superconductivity is restored with the zero-field transition temperature, $T_c(H \gg H^*) \approx T_c(0)$ (see Fig.1).

6. CONCLUSION

In the review, we have discussed both qualitative and quantitative pictures of the two-dimensionalizations effect in layered Q2D and Q1D superconductors in a parallel magnetic field. We have concentrated our attention on an
important consequence of this effect - the restoration of triplet superconductivity phenomenon, first suggested by us in Q1D case in Ref.[8] and in Q2D case - in Ref.[11] (see Fig.1). Our qualitative description is very general one and is valid for arbitrary Q2D superconductors with arbitrary 2D electron-electron interactions, including two-band superconductors. Our quantitative calculations are done for a one-band arbitrary Q2D superconductor with arbitrary 2D electron-electron interactions. We hope that the suggested phenomenon describes not only the recently experimentally discovered reentrant superconductivity in the triplet superconductor UTe$_2$, but is useful for the future experiments.

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