Assessment of Characteristic Function Modulus of Vibroacoustic Signal Given a Limit State Parameter of Diagnosed Equipment

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Abstract. Improvement of distinguishing criteria, determining defects of machinery and mechanisms, by vibroacoustic signals is a recent problem for technical diagnostics. The work objective is assessment of instantaneous values by methods of statistical decision making theory and risk of regulatory values of characteristic function modulus. The modulus of the characteristic function is determined given a fixed parameter of the characteristic function. It is possible to determine the limits of the modulus, which correspond to different machine’s condition. The data of the modulus values are used as diagnostic features in the vibration diagnostics and monitoring systems. Using such static decision-making methods as: minimum number of wrong decisions, maximum likelihood, minimax, Neumann-Pearson characteristic function modulus limits are determined, separating conditions of a diagnosed object.

1. Introduction
The development of the system for defects characteristic criterion is based on a group of diagnostic signals features, which allow accurately determine machinery condition and its change cause [1, 11, 3] has been a recent problem for technical diagnostics. One of the essential parts of the diagnostics is distinguishing of possible technical conditions (diagnoses). In the particular case, one of the two diagnoses are to be distinguished (differential diagnosis or dichotomy), for instance: operating condition (ACCEPTABLE or ACTION REQUIRED conditions) and failure condition (UNACCEPTABLE condition) [4, 5, 6]. The solution of the problem is increasingly based on the static decision-making theory. The work objective is to assess regulatory values of characteristic function features, used as diagnostic features in systems for vibration diagnostics and monitoring, by methods of static decision-making and risk theory [7, 8, 9, 10, 11, 12, 19, 20, 21].

2. Statement of problem
One of the strategies for diagnostics is static decision-making method [4, 15, 16, 18]. Therewith, a decision procedure is selected on the basis of optimum conditions.

The objective is to select $x_0$ value of $x$ parameter, which is a failure diagnostic feature and distinguishes the characteristic function, particularly, it is the characteristic function modulus given the feature, so if $x < x_0$ it is required to stop the operation and if $x > x_0$ to continue. The features are distinguished on two classes: $D_1$ is operating condition, $D_2$ is failure condition. Then the decision procedure means the following:
Density of probability distribution of the x parameter for different objects condition is shown in figure 1.

The areas of operating ($D_1$) and failure ($D_2$) conditions are crossed, so the $x_0$ value cannot be determined, at which there would be no wrong decisions. The objective is to select the most suitable $x_0$, so the number of wrong decisions amounts to minimum.

![Figure 1](image)

**Figure 1.** Probability density distribution of diagnostic feature for operating $D_1$ and failure $D_2$ conditions; $x_{0_{\text{min}}}$, $x_{0_{\text{max}}}$ are the extremum points of average risk of wrong decisions; $P_{II}$ and $P_I$ are the probabilities of not detecting the defect and false alarm respectively.

### 3. Theory

There are the possible faults that could appear if decisions made: false alarm (error of first kind $P_I$): an operative object considered defected ($D_1$ considered as $D_2$); not detecting or missing of defect or failure (error of second kind $P_{II}$): defective object considered operative ($D_2$ considered $D_1$). Possible solutions according to the rule (hypothesis) are denoted as $H_{ij}$ ($ij = 1, 2$), where index $i$ corresponds to the accepted diagnosis; $j$ corresponds to the actual diagnosis.

Thereafter:
- $H_{21}$ is false alarm (a type I error or error of first kind);
- $H_{12}$ is missing (a type II error or error of second kind);
- $H_{11}$ is correct diagnosis (operative condition);
- $H_{22}$ is correct diagnosis (faulty condition).

The possibility of the false alarm equals to the possibility of two events: availability of operative condition and $x < x_0$ values for the operative condition is determined as follows:

$$P(H_{21}) = P(D_1) \cdot P(x < x_0 \mid D_1) = P_1 \cdot \int_{-\infty}^{x_0} f(x \mid D_1) \, dx = P_1 \cdot \left[1 - F(x_0 \mid D_1)\right]$$

(2)

The possibility of failure missing

$$P(H_{12}) = P(D_2) \cdot P(x > x_0 \mid D_2) = P_2 \cdot \int_{x_0}^{\infty} f(x \mid D_2) \, dx = P_2 \cdot F(x_0 \mid D_2).$$

(3)

where $f(x \mid D_1)$ is probability density of feature for operative condition; $f(x \mid D_2)$ is probability density of feature for faulty condition; $P_1 = P(D_1)$ и $P_2 = P(D_2)$ is prior probability of the $D_1$ (operative condition) and $D_2$ (faulty condition) relatively, which are considered to be known on the basis of preliminary static data: in this case, it is the condition probabilities given the x feature of the specified $x_0$ value [1]:

$F(x_0 \mid D_1) = \int_{-\infty}^{x_0} f(x \mid D_1) \, dx$ and $F(x_0 \mid D_2) = \int_{x_0}^{\infty} f(x \mid D_2) \, dx$ is possibilities of operative and faulty conditions relatively at the corresponding intervals from $-\infty$ to $x_0$ and from $x_0$ to $\infty$[4, 15, 16, 18].

The probability of making an erroneous decision is specified by the possibilities of false alarm and of not detecting the failure.
If failures are to be attributed to the costs, and the costs of the correct decisions are to be defined as $C_{11}$ and $C_{22}$, the formula for average risk (expected amount of loss) is deduced as follows [4, 15, 16, 18]:

$$R = C_{11}P_1 \int_{-\infty}^{x_0} f(x/D_1)dx + C_{22}P_2 \int_{-\infty}^{x_0} f(x/D_2)dx + C_{12}P_1 \int_{x_0}^{\infty} f(x/D_1)dx + C_{21}P_2 \int_{x_0}^{\infty} f(x/D_2)dx$$

(4)

where $C_{21}$ is cost of false alarm; $C_{12}$ is cost of not detecting the failure (the first index is accepted condition, the second index is current condition); the average formula is $C_{12} \gg C_{21}$.

Due to the costs of correct decisions $C_{11} = C_{22} = 0$, the formula for average risk is deduced as follows:

$$R = C_{12}P(H_{12}) + C_{21}P(H_{21}) = C_{12}P_2 \left[ F(x_0/D_2) \right] + C_{21}P_1 \left[ 1 - F(x_0/D_1) \right]$$

(5)

The $x$ value is a current (measured) value of a diagnostic feature, and is a random one, so the specified formulae amount to the average value (expectation value) of risk.

4. Outcome of experiment

In order to describe statistic prosperities of diagnostic features, it is required to obtain empiric and theoretical distribution functions and their possibilities for different conditions of diagnosed machinery [1, 11, 17]. The research is the first step to assess limits of diagnostic features, distinguishing machinery condition to the classes.

The Weibull-Gnedenko distribution is used for description of the statistic prosperities of diagnostic features. The distribution is quite cross-functional as if the features are varied it describes a broad range of probabilistic characteristic of different processes [1, 17].

Considering the fact, that some parameters of diagnostic features has Weibull-Gnedenko distribution rule [1], as the rule is transformed to different distribution rules given the different parameters, it is requires to aim at approximation of empiric distribution function (EDF) by two-parameter distribution of Weibull-Gnedenko distribution:

$$F(x) = \begin{cases} 0 & ; x \leq 0, \\ 1 - \exp \left\{ -\left(\frac{x}{c}\right)^b \right\} & ; x > 0 \end{cases}$$

(6)

In such case the probability density function is deduced as follows:

$$f(x) = \begin{cases} 0 & ; x \leq 0, \\ b \left(\frac{x}{c}\right)^{b-1} \exp \left\{ -\left(\frac{x}{c}\right)^b \right\} & ; x > 0 \end{cases}$$

(7)

Where $c$ is scale parameter, $b$ is shape parameter и $x$ is random variable.

Using the method [17] of function approximation distribution by experimental sampled values, the data was processed and theoretical distribution function (TDF) and probability density of characteristic functions parameters of instantaneous values of vibroacoustic signals was determined [22, 23, 24, 25], which was received from such assets of piston compressors as: intake and pressure valve, axial and radial direction of cylinder, slider-crank mechanism, base bearings – for such assets and details conditions (table 1) as UNACCEPTABLE (UAC), ACTIONS REQUIRED (ARQ) and ACCEPTABLE.

| TDF parameter | ACCEPTABLE | ACTIONS REQUIRED | UNACCEPTABLE |
|---------------|------------|------------------|--------------|
| $b$           | 5.050      | 5.99             | 5.04         |
| $c$           | 0.2849     | 0.1119           | 0.0415       |
Limit value of \( x_0 \) is determined by the formula:

\[
\frac{f(x_0 / D_1)}{f(x_0 / D_2)} = \frac{P_2 \cdot (C_{12} - C_{22})}{P_1 \cdot (C_{21} - C_{11})} = \lambda
\]  

(8)

where \( \lambda \) is likelihood ratio: decision threshold.

According to the formula (7) we get:

\[
f(x_0 / D_1) = \frac{b_1}{c_1} \left( \frac{x_0}{c_1} \right)^{b_1 - 1} \exp \left\{ - \left( \frac{x_0}{c_1} \right)^b \right\};
\]

\[
f(x_0 / D_2) = \frac{b_2}{c_2} \left( \frac{x_0}{c_2} \right)^{b_2 - 1} \exp \left\{ - \left( \frac{x_0}{c_2} \right)^b \right\}.
\]  

(9)

The \( x_0 \) is understood as a value of a diagnostic feature.

Substituting the values of probability density function in (7) and take up \( C_{11}=C_{22}=0 \) the following formula is deduced:

\[
\frac{b_1}{c_1} \left( \frac{x_0}{c_1} \right)^{b_1 - 1} \exp \left\{ - \left( \frac{x_0}{c_1} \right)^b \right\} = \frac{P_2 \cdot C_{12}}{P_1 \cdot C_{21}}.
\]  

(10)

There are the method factors for decision making when determining the threshold value of \( x_0 \):

– minimum risk method targeted on minimum achievement of average risk for likelihood ratio \( \lambda=[(C_{12}–C_{22})P_2]/[(C_{21}–C_{11})P_1] \);

– minimum number of wrong decision method targeted on the cost of false alarm and of not detecting the failure is equal when \( \lambda=[P_2]/[P_1] \);

– maximum likelihood method means the cost and possibility of not detecting the failure approximately equals to the cost and possibility of false alarm given \( \lambda=1 \);

– minimax method means the risk volume becomes minimum among the maximum values caused by “negative” \( P_i \) figure, in such case \( \lambda=[(C_{12}–C_{22})(1–P_i)]/[C_{21}–C_{11}]P_1] \);

– Neumann-Pearson method minimizes the possibility of not detecting the failure given the permissible \( A \) level of false alarm:

\[
P_2 \int_{x_0}^{\infty} f(x / D_i) dx \leq A
\]  

(11)

The calculation results of given the prior values \( P_1=0.97, P_2=0.03, C_{12}=1, C_{21}=0.01 \) for distinguishing the ACTIONS REQUIRED and UNACCEPTABLE conditions are denoted in table 2; the results for ACCEPTABLE and UNACCEPTABLE conditions are denoted in table 3.

**Table 2. Calculation results of feature threshold value (ACTIONS REQUIRED – ACCEPTABLE).**

| Methods                      | Probabilities and risks |
|------------------------------|------------------------|
|                              | \( x_0 \) | \( P(H_{21}) \) | \( P(H_{12}) \) | \( R \) |
| Minimum risk                 | 0.088   | 0.013   | 0.000682   | 0.000814 |
| Minimum number of wrong decisions | 0.077    | 0.000115 | 0.000126   | 0.000126 |
| Maximum likelihood           | 0.084   | 0.000409 | 0.000833   | 0.000874 |
| Minimax                      | 0.119   | 0.271   | 0.002259   | 0.004973 |
| Neumann-Pearson              | 0.093   | 0.049   | 0.000284   | 0.000773 |
Table 3. Calculation results of feature threshold value (ACTIONS REQUIRED – ACCEPTABLE).

| Methods                  | $x_0$   | $P(H_{21})$ | $P(H_{12})$ | $R$      |
|--------------------------|---------|-------------|-------------|----------|
| Minimum risk             | 0.216   | 0.017       | 0.000682    | 0.000748 |
| Minimum number of wrong decisions | 0.186   | 0.0001875   | 0.000126    | 0.000139 |
| Maximum likelihood       | 0.206   | 0.005637    | 0.000833    | 0.000823 |
| Minimax                  | 0.285   | 0.2         | 0.001686    | 0.003688 |
| Neumann-Pearson          | 0.227   | 0.043       | 0.000284    | 0.000636 |

5. Discussion

The calculation results (table 2, figure 2, 3) show that the minimum number of erroneous decision method provides the lowest values of decision-making risk $R$, the possibility of not detecting the failure $P(H_{12})$ and false alarm $P(H_{21})$ for characteristic function modulus $\Theta(0.8)$ of vibroacoustic signal, in case of selecting between UNACCEPTABLE and ACTIONS REQUIRED conditions. The second place takes the Neumann-Pearson method, which provides minimum values of decision-making risk $R$, the possibility of not detecting the failure $P(H_{12})$. The minimax method provides the highest values of decision-making risk $R$, the possibility of not detecting the failure $P(H_{12})$ and false alarm $P(H_{21})$. The third place takes the minimum risk method.

![Figure 2. Probability density function of characteristic function modulus value given parameter of characteristic function for different condition of diagnosed machine and modulus threshold values ($x_0$ from table 2 and 3), determined by different methods of decision-making.](image_url)

The minimum number of erroneous decision method provides the lowest values of the decision-making risk ($R$) and the possibility of not detecting the failure ($P(H_{12})$), when determining the threshold values for ACTIONS REQUIRED and ACCEPTABLE conditions (table 3, figure 2). The calculation by the Neumann-Pearson methods are reliable as well, and provide lower values of the decision-making risk and the possibility of not detecting the failure. The worst results are given by the minimax method.
6. Summary and conclusion

The results of the research show that in order to determine the limits of the characteristic function modulus of instantaneous value of vibroacoustic signals, corresponding to different conditions of diagnosed machinery; it is required to apply statistical methods for decision-making. That allows determining threshold values of diagnostic features on the basis of decision-making risk assessment. The suggested calculation method is targeted on the diagnostic features and based on the parameters of the characteristic function. Using the statistical method of decision-making the threshold values of the characteristic function modulus was obtained given the characteristic function parameter for distinguishing the ACCPERABLE and ACTION REQUIRED conditions, ACTIONS REQUIRED and UNACCEPTABLE conditions. The vibroacoustic signal and the characteristic function of its instantaneous value were used as the diagnostic signal.

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