SUM RULES AND POSITIVITY CONSTRAINTS ON NUCLEON SPIN STRUCTURE

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Abstract. The spin structure of nucleon at twist 2 and 3 levels is analyzed. The contribution of quark and gluon spins to nucleon spin are Lorentz invariant, while it is not sure for orbital angular momentum. The conserved fractional moments of transversity distribution are considered. The scenario of decoupled total angular momentum, determined predominantly by the unpolarized scattering, is discussed.

1 Introduction

The spin structure of nucleons is still one of the major puzzles of hadronic physics. Ten years after the first experimental data on polarized structure...
functions opened the spin crisis, much progress has been achieved in the QCD understanding of sum rules, $Q^2$ evolution of distribution functions and the orbital angular momentum contribution to the total spin. Moreover, transversity and off-forward distribution functions now appear as complementary sources of information. In this paper, we focus on spin sum rules and on positivity constraints which have been recently derived. We also address questions related to the contribution of orbital angular momentum to the total spin.

2 Spin sum rule

The fact that the total nucleon spin is just $1/2$ is usually expressed as a sum rule

$$J_q + J_G = \frac{1}{2} \sum_{q,\bar{q}} S_q + S_G + L_q + L_G = \frac{1}{2},$$

(1)

for the first moments $S$ of quark and gluon spin-dependent distributions.

$$S_q = \int_0^1 \Delta q(x) dx; \quad S_G = \int_0^1 \Delta G(x) dx.$$  

(2)

The orbital angular momentum $L$ is a necessary ingredient to make the sum rule (1) $Q^2$ independent. The different roles of $S$ and $L$ entering the sum rule (1) are manifested by the fact that the spin parts (3) are naturally measured in inclusive processes like deep inelastic scattering or high-$p_T$ direct photon production, while indirect access to $L$ requires exclusive reactions like deeply virtual Compton scattering, with essential experimental and theoretical difficulties accompanying its extraction from the data.

The possible resolution of this paradox might be the effective decoupling of the orbital and total angular momenta, resulting in the equal sharing of the momentum and total angular momentum of quarks and gluons. This would naturally explain the observed smallness of the isoscalar anomalous magnetic moment ($1.79 = \mu_p^A \sim -\mu_n^A = 1.91$), which is also manifested in the model calculations. The reason is that the anomalous magnetic moment related to the non-forward distribution $E$, leading the a different sharing of momentum and total angular momentum. The recently suggested
representation of the quark total angular momentum \[ J_q \],

\[ J_q = \frac{1}{2} \sum_{q,\bar{q}} \int_0^1 dx [xq(x) + E_q(x)], \] (3)

implying, by making use of the momentum and total angular momentum conservation, that

\[ J_G = \frac{1}{2} \int_0^1 dx [xG(x) - \sum_{q,\bar{q}} E_q(x)], \] (4)

with small \( E \) term, qualitatively supports this point of view.

3 Transverse spin sum rule

One may wonder, what is a counterpart of a spin sum rule for the transverse spin case, when twist 3 operators are involved. Projecting the Pauli-Lubanski vector on the transverse direction, the conservation of the total angular momentum leads to the sum rule

\[ \frac{1}{2} \sum_{q,\bar{q}} S_{Tq} + S_{TG} + L_{Tq} + L_{TG} = \frac{1}{2}, \] (5)

where transverse spin of quarks and gluons are given by the integrals:

\[ S_{Tq} = \int_0^1 g_{Tq}(x)dx; \quad S_{TG} = \int_0^1 \Delta G_T(x)dx. \] (6)

Here \( g_T = g_1 + g_2 \) is the natural measure of the quark contribution to transverse polarization, as given by the matrix element:

\[ g_T(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} < p, S | \bar{\psi}(0)\mathcal{S}_T\gamma_5\psi(\lambda n)|p, S >, \] (7)

while \( \Delta G_T \) is the similar quantity for gluons. Only operators containing two quark or gluon fields appear, while quark-gluon and three-gluon operators are related to them by the equations of motion. Here only chiral even quark operators contribute, which is the immediate result of the fact that the quark energy momentum tensor is chiral even, so that transversity does not enter this sum rule.
The important simplification of the transverse sum rule comes from the Burkhardt-Cottingham sum rule\[11\], which is valid for each quark flavour and gluons separately and states that the first moment of longitudinal and transverse distributions are equal,

\[
S_T^q = S_q; \quad S_T^G = S_G.
\] (8)

This just mean that the contributions of quarks and gluon spin to the proton spin behave like Lorentz invariant quantities, although generally speaking they are not \[12\]. Lorentz invariance comes from rotational invariance, guaranteed by the Burkhardt-Cottingham sum rule, and boost invariance.

As an important ingredient of the derivation of the Burkhardt-Cottingham sum rule is the locality of the operator, one may doubt the Lorentz invariance of the quark and gluon orbital momenta, although their sum is Lorentz invariant due to the total angular momentum conservation.

## 4 Transversity sum rule

As transversity distributions come from a chirally odd operator with its first moment (tensor charge) being subject to renormalization, one may look for a conserved quantity by considering fractional moments \[13\]:

\[
\int_0^1 x^\alpha h_1(x) dx = \text{const}
\] (9)

A direct calculation shows that at leading order \(\alpha = -0.345\), while at next-to-leading order \(\alpha = -0.49\) for \(Q^2 \sim 10 GeV^2\).

At the moment, it is difficult to find an interpretation of these numbers. They should be rather considered as phenomenological inputs, defining a conserved quantity. In particular, it is this quantity which seems to be the natural candidate for low energy calculations.

## 5 Positivity constraints

Important constraints for the nucleon spin structure may be derived from the positivity of the density matrix. Recall that non-diagonal elements of a density matrix are constrained by positivity as well as its diagonal elements.
This enables to derive inequalities originally proven at the level of the parton model [14], which were shown to be preserved by the QCD $Q^2$ evolution, up to next-to-leading order [13, 16]. They read

$$|h_1(x)| \leq q_+(x).$$  \hspace{1cm} (10)

where $q_+(x)$ is the quark distribution with helicity parallel to that of the nucleon.

For gluons, the similar bound [17] reads

$$|\Delta G_T(x)| \leq \sqrt{\frac{G(x)G_L(x)}{2}}$$  \hspace{1cm} (11)

where $G_L$ is the distribution of longitudinally polarized gluons in nucleon [18], for which (11) provides a lower bound.

Positivity leads also to constraints [19] for the off-forward gluon distribution $g(x_1, x_2)$:

$$x'g(x_1, x_2) \leq \sqrt{x_1x_2g(x_1)g(x_2)} \cdot \lambda[P(x_1), P(x_2)],$$  \hspace{1cm} (12)

with $x_1$, the light-cone fraction of the parton emitted by the proton target, $x'$, the fraction of the parton absorbed by the scattered proton (both fractions with respect to the initial proton momentum), and $x_2 = x'/(1 - x + x')$ the light-cone fraction of the absorbed parton with respect to its parent’s momentum, and

$$\lambda[P(x_1), P(x_2)] = \frac{\sqrt{(1 + P(x_1))(1 + P(x_2))} + \sqrt{(1 - P(x_1))(1 - P(x_2))}}{2}$$  \hspace{1cm} (13)

where one introduces the gluon polarization, defined as $P(x) = \Delta G(x)/G(x)$. This inequality offers, in principle, a possibility of extracting information on the gluon spin-dependent distribution $\Delta G$ from unpolarized diffractive processes.

To conclude, let us stress again that sum rules and positivity might be used in order to provide constraints on the spin structure of nucleon, including the most difficult transverse polarization case.
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