HARD-SOFT RENORMALIZATION OF THE
MASSLESS WESS-ZUMINO MODEL

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ABSTRACT

We show that in a Wilsonian renormalization scheme with zero-
momentum subtraction point the massless Wess-Zumino model
satisfies the non-renormalization theorem; the finite renormaliza-
tion of the superpotential appearing in the usual non-zero mo-
momentum subtraction schemes is thus avoided.

We give an exact expression of the beta and gamma functions
in terms of the Wilsonian effective action; we prove the expected
relation $\beta = 3g\gamma$.

We compute the beta function at the first two loops, finding agree-
ment with previous results.

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The perturbative renormalization of massless theories presents difficulties of a practical order, due to the presence of infrared singularities.

In theories with symmetries admitting an invariant regulator these difficulties can be avoided choosing an appropriate subtraction scheme, like the minimal subtraction scheme \([1]\).

In some theories, however, these procedures cannot be adopted and the Ward identities must be verified explicitly in presence of massless fields. This is for instance the case of chiral gauge theories.

A possible way of renormalizing these theories is to impose renormalization conditions at non-zero momentum subtraction points \([2]\); however this scheme is computationally awkward, since it is usually technically hard to satisfy the Ward identities at non-zero momenta, especially the Slavnov-Taylor identities.

In the hard-soft renormalization schemes, first introduced in \([3]\) with the purpose of studying in a simple way the renormalizability of massless theories with BPHZ, a splitting of the fields into hard and soft fields is made at a scale \(\Lambda_R\), in such a way that the renormalization conditions can be chosen at zero momentum.

A recent discussion of the hard-soft (HS) schemes in the Wilsonian approach \([4, 5]\) can be found in \([6, 7]\).

In massless QED the renormalization conditions can be chosen at zero momentum at a Wilsonian renormalization scale \(\Lambda_R\) \([6]\), satisfying effective gauge and axial Ward identities; the usual Ward identities follow automatically for any ultraviolet cut-off. The nice feature of this approach is that the effective Ward identities are easily satisfied, being the renormalization conditions imposed at zero momentum. Detailed one-loop computations are made in this scheme. In \([8]\) one-loop computations are made in a HS scheme in Yang-Mills, using dimensional regularization; in this way the gauge Ward identities are trivially satisfied.

The renormalization group equation can be easily obtained in the HS schemes, expressing the beta and gamma functions in terms of the Wilsonian effective action at \(\Lambda_R\); in \([7]\) this procedure has been applied to the case of massless \(g\phi^4\) and of massive \(g\phi^4\), renormalized in a mass-independent way.

In this letter we apply the HS renormalization scheme to the massless Wess-Zumino model \([9]\). In this case the supersymmetric Ward identities and the R-symmetry are trivially satisfied, choosing an ultraviolet momentum cut-off.

An interesting feature of the massive Wess-Zumino model is that, choosing zero-momentum renormalization conditions, the superpotential is not
renormalized \([11]\); as a consequence the simple relation \(\beta = 3g\gamma\Phi\) \([11]\) between the renormalization group functions holds. Using supergraph Feynman rules, these facts follow from the non-renormalization theorem \([12]\), stating that all 1PI graphs are of the form of a single integral in superspace; as a consequence all 1PI graphs contributing to the superpotential of the massive Wess-Zumino model vanish at zero momentum. In the massless case zero-momentum renormalization conditions cannot be chosen due to infrared singularities; using non-zero subtraction points on the two- and three-point Green functions one gets a finite renormalization of the superpotential \([13]\). In fact, starting from two loops, the three-point function is non-vanishing at generic momenta \([14, 15]\).

We show that in the HS scheme the non-renormalization property of the superpotential of the massless Wess-Zumino model is easily satisfied, and the relation \(\beta = 3g\gamma\Phi\) holds exactly. We compute the two-loop beta function in the HS scheme, finding agreement with \([16]\), where minimal subtraction was used.

Consider the massless \(g\Phi^3\) Wess-Zumino model \([9]\) in Euclidean four-dimensional space; the path-integral is

\[
Z_{0,\Lambda_0}[J, \bar{J}] = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} \exp \left\{ -S[\Phi, \bar{\Phi}] + \int d^6zdJ\Phi + \int d^6\bar{z}\bar{J}\bar{\Phi} \right\}
\]

with the bare action

\[
S[\Phi, \bar{\Phi}] = -\int d^8z\bar{\Phi}K_{\Lambda_0}^{-1}\Phi + S'[\Phi, \bar{\Phi}]
\]

\[
S'[\Phi, \bar{\Phi}] = \int d^8z c_1\Phi\bar{\Phi} + \int d^6z \frac{c_2}{3!}\Phi^3 + \int d^6\bar{z} \frac{c_2}{3!}\bar{\Phi}^3
\]

We use the superspace conventions of \([17]\). In the loop expansion, at tree level the bare coefficients are \(c_1^{(0)} = 0, c_2^{(0)} = g\). In momentum space the cut-off function \(K_\Lambda(p) = K(\frac{p^2}{\Lambda^2})\) satisfies \(K(0) = 1\) and goes to zero at least as fast as \(1/x\) for \(x \to \infty\). (In \([14, 15]\) \(K(x)\) was required to go to zero at least as fast as \(1/x^2\) for \(x \to \infty\); relying on the cancellation of quadratic divergences in supersymmetry, it is sufficient to impose the former weaker condition). \(\Lambda_0\) is the ultraviolet cut-off.

This cut-off is compatible with supersymmetry and the R-symmetry, so that the counterterms in \([3]\) are the only ones allowed by these symmetries.
Let us consider the path-integration on the hard modes

\[
Z_{\Lambda_0} [J, \bar{J}; \chi] = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} \exp \left\{ -S_{\Lambda_0} [\Phi, \bar{\Phi}; \chi] + \int d^6z J \Phi + \int d^6z \bar{J} \bar{\Phi} \right\} 
\]

with action

\[
S_{\Lambda_0} [\Phi, \bar{\Phi}; \chi] = -\int d^8z \bar{\Phi} K^{-1}\Lambda_0 \Phi + S^I [\Phi, \bar{\Phi}] - \chi \int d^8z \bar{\Phi} K\Lambda K^{-1}\Lambda_0 \Phi
\]

where the term with real parameter \(\chi\) has been introduced for later convenience; \(Z_{\Lambda_0} [J, \bar{J}] \equiv Z_{\Lambda_0} [J, \bar{J}; 0]\).

\(K_{\Lambda_0} \to (1 - K\Lambda)\) for \(\Lambda_0 \to \infty\). The cut-off functions \(K\Lambda\) and \(K_{\Lambda_0}\) are chosen to be analytic functions; an explicit representation for them will be given later.

The flow of the functional \(Z_{\Lambda_0}\) from \(\Lambda\) to zero can be represented as

\[
Z_{0\Lambda_0} [J, \bar{J}] = \exp \left\{ \int d^8z \frac{\delta}{\delta J} \left[ K^{-1}_\Lambda - K^{-1}_{\Lambda_0} - \chi K\Lambda K^{-1}_{\Lambda_0} \right] \frac{\delta}{\delta J} \right\} Z_{\Lambda_0} [J, \bar{J}; \chi]
\]

Observe that \(K_{\Lambda_0}(p)\) goes to zero as \(p^2/\Lambda^2\) for \(p^2/\Lambda^2 \to 0\), so that the Wilsonian Green functions generated by \(Z_{\Lambda_0} [J, \bar{J}; 0]\) are infrared finite for \(\Lambda > 0\) even at exceptional momenta.

The 1PI functional generator corresponding to \(Z_{\Lambda_0} = e^{W_{\Lambda_0}}\) is obtained by Legendre transformation

\[
\Gamma_{\Lambda_0} [\Phi, \bar{\Phi}; \chi] = -W_{\Lambda_0} [J, \bar{J}; \chi] + \int d^6z J \Phi + \int d^6z \bar{J} \bar{\Phi}
\]

with

\[
\Phi = \frac{\delta W_{\Lambda_0}}{\delta J}, \quad J = \frac{\delta \Gamma_{\Lambda_0}}{\delta \Phi}
\]

Let us introduce a renormalization scheme in which some renormalization scale \(\Lambda_R\) appears; according to general arguments the Gell-Mann and Low renormalization group equation on \(Z [J, \bar{J}] \equiv Z_{0\infty} [J, \bar{J}]\) holds

\[
\left\{ \Lambda_R \frac{\partial}{\partial \Lambda_R} + \beta \frac{\partial}{\partial g} + \gamma \left[ \int d^6z J \frac{\delta}{\delta J} + \int d^6z \bar{J} \frac{\delta}{\delta \bar{J}} \right] \right\} Z [J, \bar{J}] = 0
\]
where $\beta$ and $\gamma_\Phi$ are functions of $g$. From eq. (\ref{eq:1}) it follows that $Z_\Lambda [J, \bar{J}; \chi] \equiv Z_{\Lambda \infty} [J, \bar{J}; \chi] = e^{W_\Lambda [J, \bar{J}; \chi]}$ satisfies the effective renormalization group equation

$$\left\{ \Lambda_R \frac{\partial}{\partial \Lambda_R} + \beta \frac{\partial}{\partial g} + \gamma_\Phi \left[ \int d^6 z J \delta \frac{\delta}{\delta J} + \int d^6 z \bar{J} \delta \frac{\delta}{\delta \bar{J}} \right] \right\} Z_\Lambda [J, \bar{J}] = -2 \frac{\partial}{\partial \chi} Z_\Lambda [J, \bar{J}; \chi] \bigg|_{\chi=0} \quad (10)$$

$W_\Lambda [J, \bar{J}; \chi]$ satisfies the same equation (10) as $Z_\Lambda [J, \bar{J}; \chi]$. Making a Legendre transformation we get

$$\left\{ \Lambda_R \frac{\partial}{\partial \Lambda_R} + \beta \frac{\partial}{\partial g} - \gamma_\Phi \left[ \int d^6 z \Phi \delta \frac{\delta}{\delta \Phi} + \int d^6 z \bar{\Phi} \delta \frac{\delta}{\delta \bar{\Phi}} \right] \right\} \Gamma_\Lambda [\Phi, \bar{\Phi}] = 2 \gamma_\Phi \mathcal{T}_\Lambda [\Phi, \bar{\Phi}] \quad (11)$$

where

$$\mathcal{T}_\Lambda [\Phi, \bar{\Phi}] = -\frac{\partial}{\partial \chi} \Gamma_\Lambda [\Phi, \bar{\Phi}; \chi] \bigg|_{\chi=0} \quad (12)$$

Making a Volterra expansion $\Gamma_{\Lambda \Lambda_0} [\Phi, \bar{\Phi}] = \sum_{n, \bar{n} \geq 0} \Gamma_{n, \bar{n}}^{\Lambda \Lambda_0} [\Phi, \bar{\Phi}]$, where $\Gamma_{n, \bar{n}}^{\Lambda \Lambda_0}$ is the monomial of order $n$ ($\bar{n}$) in $\Phi$ ($\bar{\Phi}$) and making a similar expansion on $\mathcal{T}_\Lambda$ we get

$$\left\{ \Lambda_R \frac{\partial}{\partial \Lambda_R} + \beta \frac{\partial}{\partial g} - (n + \bar{n}) \gamma_\Phi \right\} \Gamma_{n, \bar{n}}^{\Lambda \Lambda_0} [\Phi, \bar{\Phi}] = 2 \gamma_\Phi \mathcal{T}_{n, \bar{n}}^{\Lambda} [\Phi, \bar{\Phi}] \quad (13)$$

where $\Gamma_\Lambda = \Gamma_{\Lambda \infty}$. The R-symmetry implies that $n - \bar{n} \equiv 0 (mod. 3)$. The only relevant vertices are $\Gamma_{1,1}, \Gamma_{3,0}, \Gamma_{0,3}$. In \cite{13} it was proven that all the 1PI graphs can be written as integrals over a single $\int d^4 \theta$; therefore

$$\Gamma_{1,1}^{\Lambda \Lambda_0} [\Phi, \bar{\Phi}] = \int d^4 \theta \int_p \Phi(\theta, -p) \Phi(\theta, p) F^{\Lambda \Lambda_0}(p^2) \quad (14)$$

where $\int_p \equiv \int \frac{d^4 p}{(2\pi)^4}$ and $F^{\Lambda \Lambda_0}(l)$ is regular in $p = 0$ for $\Lambda > 0$.

Using the same theorem one can arrive at

$$\Gamma_{3,0}^{\Lambda \Lambda_0} [\Phi] = \int d^2 \theta \int_{p_1, p_2} \Phi(\theta, p_1) \Phi(\theta, p_2) \Phi(\theta, -p_1 - p_2) G^{\Lambda \Lambda_0}(p_1, p_2) \quad (15)$$

where $G^{\Lambda \Lambda_0}(l)(p_1, p_2) - c_2^{(l)}$ goes to zero for $p_1, p_2 \to 0$ as a bilinear in $p_1$ and $p_2$, provided $\Lambda > 0$. 

\[4\]
This statement can be easily proven looking at each supergraph contributing to $\Gamma_{3,0}^{\Lambda_0(l)}$: apart from the $l$-loop counterterm $c_2^{(l)}$, which is trivially of the form of eq.(15), due to the R-symmetry there are exactly two $D$’s more than $\bar{D}$’s; using the manipulations on supergraphs explained in [12], these two $D$’s are pulled out of the supergraph and act on the external legs, where they combine with the two $\bar{D}$’s in $\int d^4\theta \simeq \int d^2\theta \bar{D}^2$ to give two external momenta; since the remaining bosonic Feynman integrals are clearly regular at zero momentum for $\Lambda > 0$, the assertion follows.

$T_{1,1}^\Lambda$ and $T_{3,0}^\Lambda$ have the same structure as $\Gamma_{1,1}^\Lambda$ and $\Gamma_{3,0}^\Lambda$ respectively:

$$ T_{1,1}^\Lambda [\Phi, \bar{\Phi}] = \int d^4\theta \int_p \bar{\Phi}(\theta, -p)\Phi(\theta, p)T^\Lambda(p^2) $$

(16)

and $T_{3,0}^\Lambda$ gives a vertex which is vanishing at zero momentum for $\Lambda > 0$.

Imposing Wilsonian renormalization conditions at $\Lambda = \Lambda_R > 0$ one can use the effective renormalization group equation (11) to give an expression for the beta and gamma functions in terms of the Wilsonian effective action.

Due to the infrared finiteness of the Wilsonian Green functions at a scale $\Lambda_R > 0$, one can choose the following standard set of Wilsonian renormalization conditions (HS scheme):

$$ F^{\Lambda_0 \Lambda}(0) = -1 \quad \quad \quad G^{\Lambda_0 \Lambda}(0,0) = \frac{g}{3!} $$

(17)

which imply the non-renormalization of the chiral superpotential: $c_2^{(l)} = G^{\Lambda_0 \Lambda}(0,0) = 0$ for all $l \geq 1$.

Using eqs.(13-17) we get

$$ \gamma_\Phi = \frac{1}{2[1 - T^\Lambda_R(0)]} \Lambda \frac{\partial}{\partial \Lambda} F^\Lambda(0) \bigg|_{\Lambda_R} $$

(18)

and

$$ \beta = 3g\gamma_\Phi $$

(19)

According to general arguments one expects that the beta function is scheme-independent at the first two loops. Using eq.(19) one can reduce the computation of beta to the computation of gamma. Let us compute $\gamma_\Phi$ at the first two loops. At one loop the only non-vanishing bare parameter is

$$ c_1^{(1)} = \frac{g^2}{2} \int_q D^2_{\Lambda_0}(q) $$

(20)
where \( D_{\Lambda R\Lambda_0}(q) \equiv K_{\Lambda R\Lambda_0}(q)/q^2 \) is the hard scalar propagator; this counterterm cancels the self-energy graph (see Fig.1a) at scale \( \Lambda_R \) and at zero momentum.

One has

\[
F^{\Lambda A_0(1)}(0) = -\frac{g^2}{2} \int q \left[D_{\Lambda A_0}^2 - D_{\Lambda R\Lambda_0}^2\right](q) \tag{21}
\]

At one loop not only \( c_2^{(1)} = G^{\Lambda R\Lambda_0(1)}(0,0) = 0 \), but also one has at arbitrary momentum \( G^{\Lambda R\Lambda_0(1)}(p_1, p_2) = 0 \).

Finally we get (with \( T^\Lambda \equiv T^{\Lambda \infty} \))

\[
T^{\Lambda R\Lambda_0(1)}(0) = -g^2 \int q K_{\Lambda R}(q) D_{\Lambda R\Lambda_0}^2(q) \tag{22}
\]

corresponding to the Wilsonian graph in Fig.1b, where the cross indicates the insertion of the \( \chi \)-term of eq.(5).

Let us consider the class of HS schemes characterized by a cut-off of the form

\[
K_{\Lambda A_0}(p) = p^2 \int_{\Lambda_0^2}^{\infty} d\alpha e^{-\alpha p^2} \rho(\alpha \Lambda^2) \tag{23}
\]

where the function \( \rho(x) \) satisfies \( \rho(0) = 1 \) and goes to zero fast enough for \( x \to \infty \). In the present case it is not necessary to add the condition \( \rho'(0) = 0 \) on the cut-off function (23), required in [6, 7], since in the Wess-Zumino model the quadratic divergences cancel.

From eq.(18,21) we get

\[
\gamma^{(1)}_{\Phi} = \frac{1}{2} \Lambda \frac{\partial}{\partial \Lambda} F^{\Lambda(1)}(0)_{|_{\Lambda_R}} = \lim_{\Lambda_0 \to \infty} -\frac{g^2}{16\pi^2} \int_{\Lambda_0^2}^{\infty} d\alpha_1 d\alpha_2 \frac{\alpha_1}{(\alpha_1 + \alpha_2)^2} \rho'(\alpha_1) \rho(\alpha_2) = \frac{1}{2} \frac{g^2}{16\pi^2} \tag{24}
\]
At two loops one has (see Fig.2)

\[ F^{\Lambda_0(2)}(0) = \frac{g^4}{2} \int_{pq} K_{\Lambda_0}(p) D_{\Lambda_0}^2(p) \left[ D_{\Lambda_0}(q) D_{\Lambda_0}(p + q) - D_{\Lambda_0}^2(q) \right] + c_{1}^{(2)} \]  

Observing that

\[ \Lambda \frac{\partial}{\partial \Lambda} \Big|_{\Lambda_R} \int_{pq} (1 - K_{\Lambda_0}(p)) D_{\Lambda_0}^2(p) \left[ D_{\Lambda_0}(q) D_{\Lambda_0}(p + q) - D_{\Lambda_0}^2(q) \right] \]  

vanishes in the limit \( \Lambda_0 \to \infty \) we find, using eqs.\([18,25]\),

\[ \gamma^{(2)}_{\Phi} = \frac{1}{2} \lim_{\Lambda_0 \to \infty} \Lambda \frac{\partial}{\partial \Lambda} \Big|_{\Lambda_R} \left[ F^{\Lambda_0(2)}(0) + T^{\Lambda_0(1)}(0) F^{\Lambda_0(1)}(0) \right] = \]  

\[ \frac{g^4}{4} \Lambda \frac{\partial}{\partial \Lambda} \Big|_{\Lambda_R} \int_{pq} D_{\Lambda_0}^2(p) \left[ D_{\Lambda_0}(q) D_{\Lambda_0}(p + q) - D_{\Lambda_0}^2(q) \right] = -\frac{1}{2} \left( \frac{g^2}{16\pi^2} \right)^2 \]

where in the last step the integral is the same as for computing \( \beta^{(2)} \) in \( g\phi^4 \) in the HS scheme \([7]\) so that we get the expression of the beta function at the first two loops

\[ \beta = 3g \left[ \frac{1}{2} \frac{g^2}{16\pi^2} - \frac{1}{2} \left( \frac{g^2}{16\pi^2} \right)^2 \right] \]  

in agreement with \([16]\).

Let us compare the HS renormalization scheme with a renormalization scheme at \( \Lambda = 0 \) with non-zero momentum subtraction points

\[ F^{\Lambda = 0\Lambda_0}(\mu^2) = -1 \quad G^{\Lambda = 0\Lambda_0}(p_1, p_2) = \frac{g}{3!} \]  

For generic non-zero momenta \( G^{\Lambda = 0\Lambda_0(l)}(p_1, p_2) \) is non-vanishing for \( l \geq 2 \); e.g. \( G^{\Lambda = 0\Lambda_0(l)}(0, p) \) has been evaluated in \([15]\), corresponding to the graph in
Fig. 3a; this chiral vertex gives also a non-vanishing contribution to the three-loop self-energy graph of Fig. 3b, which has been evaluated in [16, 18]. This shows that $G^{\Lambda=0\Lambda_0(\ell)}(s, p)$ is non-vanishing not only at $s = 0$, but also for small $s$. Observe furthermore that the finiteness of $G^{\Lambda=0\Lambda_0(\ell)}(0, p)$ for $p \to 0$ and $\ell = 2$ is accidental. In fact a simple renormalization group argument shows that at three loops it diverges as $\ln(p/\Lambda_R)$ for $p \to 0$, due to the diagram of Fig. 3a with a self-energy insertion.

Using the notation of [16], the bare coupling constant has the form

$$g_0 = \mu^\epsilon f(g)Z_\Phi^{-3/2}$$

(30)

where $\mu^\epsilon$ is the usual factor introduced in dimensional regularization and $f(g) = g + ...$ is an odd function of $g$, which is finite due to the non-renormalization theorem (in its ‘weak’ form assuring that the chiral supergraphs are superficially convergent) but not equal to $g$, due to the above-mentioned quantum corrections to the superpotential. One gets

$$\gamma_\Phi = \frac{1}{2} \mu \frac{\partial}{\partial \mu} \Big|_{g_0} \ln Z_\Phi$$

(31)

$$\beta = \mu \frac{\partial}{\partial \mu} \Big|_{g_0} g = 3\gamma_\Phi \left[ \frac{d}{dg} \ln f \right]^{-1}$$

(32)

The beta and gamma functions are still proportional, but their relation involves the function $[\ln f]'(g)$ which has to be computed order by order in perturbation theory in a generic momentum-subtraction scheme, so that in general $\beta^{(l)} \neq 3g^{(l)}\gamma^{(l)}$ for $l \geq 3$. In fact the same is true in the massive Wess-Zumino model [13]; only choosing renormalization conditions at zero momentum one gets the relation (19) in a natural way (i.e. without order-by-order fine tuning of the renormalization conditions).
In [16] a ‘momentum subtraction’ scheme is also considered, in which only the renormalization condition $F^{A=0}(\bar{p}^2) = -1$ is imposed, while the vertex is not renormalized, relying on a supersymmetric invariant regularization and imposing the condition $g_0 = \mu^{3/2}Z^{-3/2}$; then the relation (19) follows. This procedure is consistent due to the non-renormalization theorem, but it relies on the choice of an invariant regulator.

The advantage of a renormalization scheme in which all the relevant vertices are subjected to a renormalization condition is that the renormalized theory is completely defined, regardless of the choice of an invariant regulator. Using the HS scheme (with $\rho'(0) = 0$) with a generic (non supersymmetric invariant) ultraviolet cut-off, imposing that the renormalization conditions (17) are satisfied in the limit of infinite ultraviolet cut-off, one obtains the same renormalized Green functions as in the case previously studied, and hence the relation (19).

Using the Wilson-Polchinski [4, 5] flow equation technique for the Wilsonian effective functional, simple rigorous proofs of renormalizability and other important results in perturbation theory have been obtained [21]. It would be interesting to use this approach to study the massless Wess-Zumino model. A first step in this direction has been made in [22], where the flow equation has been written in the superfield formalism.

Let us make a comment on the question of the ‘holomorphic anomaly’. In this letter we chose the coupling constant $g$ to be real; taking it complex the conclusions are similar; choosing Wilsonian renormalization conditions at zero momentum the ‘holomorphic anomaly’ term $\int d^6 z g^3 \bar{g}^2 \Phi^3$ appearing at two loops, corresponding to the contribution of Fig. 3a [15] is avoided as long as the Wilsonian scale $\Lambda$ is different from zero. This is in agreement with [19], where it was observed that using a Wilsonian effective action these ‘anomalies’ are avoided, as first suggested in [20] in the context of supersymmetric gauge theories.

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