On supercorrelated systems and phase space entrainment

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December 2004 (revised)

Abstract

It is demonstrated that power-laws which are modified by logarithmic corrections arise in supercorrelated systems. Their characteristic feature is the energy attributed to a state (or value of a general cost function) which depends nonlinearly on the phase space distribution of the constituents. A onedimensional dissipative deterministic model is introduced which is attracted to a supercorrelated state (phase space entrainment). Extensions of this particular model may have applications in the study of transport and equilibration phenomena, particularly for supply and information networks, or for chemical and biological nonequilibrium systems, while the qualitative arguments presented here are believed to be of more general interest.

Keywords: power-law, correlation, nonequilibrium system, statistical mechanics

1 Introduction

Power-laws are omnipresent in natural or man-made systems [1, 2]. They arise in the areas of high-energy particle physics, condensed matter, complexity, sociology, and linguistics, to name a few. They are an important feature of Tsallis statistics [3, 4]. While numerous out-of-equilibrium statistical systems are known showing this behavior, only rarely the dynamics is understood that leads to it.

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In this letter we study systems that are correlation dominated, in a sense to be more accurately defined. We show that in this class of models the asymptotic spectra are essentially given by power-laws, however, modified by logarithmic corrections. They appear quite similar from a phenomenological point of view.

Recently, for example, most interesting transient dynamical behavior has been observed in relativistic heavy-ion collisions [5]. It is observed that apparently thermodynamics rules, with a local equation of state, in particular, and commencing at unexpectedly early times (of order \(0.3 - 0.9 \times 10^{-24}s\)) during the collision process. However, the experiments also indicate that typical one-particle observables are not correspondingly thermalized by that time. Theoretical investigations based on quantum field theoretical models support the view that such ‘prethermalization’ is indeed produced [6]. This is accompanied by spectra, which show considerable deviations from Fermi-Dirac, Bose-Einstein, or Boltzmann distributions, with uniform parameters, and which relax to the equilibrium form only on a much longer time scale.

However, a heuristic picture which allows us to understand essential features of the underlying processes generally has been lacking. Recently, we have shown that such behavior can be interpreted in terms of certain correlations generated dynamically and, in fact, power-laws can emerge as a consequence [7]. Presently, we start with a different class of models. In some limits analytical results can be obtained, the models can be easily generalized, and may be applied in quite different contexts. We demonstrate the emergence of log-modified power-laws as a robust feature in supercorrelated systems where correlations among the distributions of the constituents govern the statistical behavior.

Other recent works invariably invoke stochastic processes in the explanation of power-laws, e.g., in Refs. [7, 8, 9, 10, 11]. In distinction, we present a deterministic dissipative model where microscopic stochastic processes are subsumed in suitable (anti)damping terms of otherwise Hamiltonian equations of motion. We demonstrate that phase space entrainment leads to supercorrelation here and, thus, ultimately to the log-modified power-laws.

2 Supercorrelated systems

Specifically, we consider a two-dimensional ‘phase space’ spanned by a discrete ‘spatial’ coordinate \(x \in \{x_i, \ i = 1, \ldots, L\}\) (with periodic boundary condition) together with a discrete ‘energy’ coordinate \(E \in \{E_j, \ j = 1, \ldots, M\}\). We would like to understand the statistical behavior of a system consisting of \(L\) ‘particles’, which are distributed randomly over the energy \(E\), however, with exactly one particle per lattice site \(x\).

The interpretation assigned to the coordinates may vary according to circumstances. In particular, the ‘energy’ may generally represent any cost function.

Crucially, we assume that there is a characteristic energy \(E_j\) associated with the \(j\)-th state
of the system. Furthermore, it is the product of this energy times a correlation function $C_j$ which yields the relevant effective energy. Only the latter will determine the relative weight of terms in a sum over states. More precisely, the correlation function $C_j[p]$ is determined by a product of particle distributions $p$. The number of factors of $p$ involved will be called the correlation degree $c$. (Out of a sum of terms of different degree only the one with smallest $c$ matters asymptotically.)

Before we proceed, we define this correlation energy in various examples:

- If the correlation is between a particle at site $x$ and a second one a distance $\Delta x$ away, then the contribution to the total energy is:

$$\sum_j E_j C_j[p] \equiv \sum_{i,j} E_j p(x_i, E_j) p(x_i + \Delta x, E_j). \quad (1)$$

Note that all possible pairs ($c = 2$) are counted here exactly once, due to the periodic boundary condition.

- Another case of interest is that the correlation of the particles does not depend on the mutual distance, corresponding to the limit that the interaction length is larger than the system size, $\Delta x > x_L - x_1$:

$$\sum_j E_j C_j[p] \equiv \sum_{i_1 < i_2 < \ldots < i_c;j} E_j p(x_{i_1}, E_j) p(x_{i_2}, E_j) \ldots p(x_{i_c}, E_j), \quad (2)$$

with $c$ ordered factors of $p$, not restricted otherwise because of lattice periodicity.

- Finally, we may have an anticorrelation instead of the previous case:

$$\sum_j E_j C_j[p] \equiv \sum_{i_1;j} E_j p(x_{i_1}, E_j) \left(1 - \sum_{i_1 < i_2 < \ldots < i_c} p(x_{i_2}, E_j) \ldots p(x_{i_c}, E_j)\right). \quad (3)$$

Numerous further variants can be imagined.

It seems worth while to discuss various features here. First of all, it is important to realize that the energies $E_j$ need neither be related to single-particle nor to bound-state energy levels. The energy $E_j$, or a fraction thereof, is contributing here to the extent that the particle distribution maximizes the correlation function, or minimizes it, depending on the details of $C_j[p]$. Thus, we are studying the nonlinear problem of statistical distributions of particles (one per lattice site) over states $j$ when the effective energy depends itself on the distribution, $E_j \rightarrow E_j C_j[p]$; generally, it may as well depend on $j$ explicitly. Hence, we call such systems **supercorrelated**.

The relation between supercorrelated statistics and dissipative yet deterministic dynamics shall be discussed in the following section of this letter.
Furthermore, a little thought shows that the examples of Eqs. (1) and (2) will tend to enhance the probability of rare configurations of the system, relative to the case of Boltzmann-Gibbs equilibrium statistics. Similarly, the example (3) will enhance the frequent ones. These expectations will be confirmed by the following calculations.

We look for the least biased probability distribution \( p(x, E) \) describing an ensemble of such systems, subject to the additional constraints representing the average total energy \( \langle E \rangle \) and the fixed particle number \( L \). Optimizing the Shannon (information) entropy functional, we are lead to a variational determination of the probability distribution:

\[
\delta \left( \sum_{i,j} p(x_i, E_j) \ln[p(x_i, E_j)] + \beta \left[ \sum_j C_j[p] E_j - \langle E \rangle \right] + \sum_i (\lambda(x_i) - 1) \left( \sum_j p(x_i, E_j) - 1 \right) \right) = 0 .
\]

(4)

Here \( \beta \) and \( \lambda \) are Lagrange multipliers incorporating the average energy and fixed particle number constraints.

A remark is in order here. The mathematical foundations of statistical mechanics of super-correlated systems, as introduced in this work, certainly need further study. Presently, we do not presume a detailed relation to thermodynamics, even though some of the following results are suggestive in this direction (see also the discussion in Ref. [7]). However, the application of the variational principle for the Shannon entropy is justified, since the searched-for (normalized) distribution \( p(x, E) \) refers to a closed system which is thought to be realizable in a sufficiently large number of cases, thus defining the ensemble [12, 13]. What is nonstandard here, is the nonlinear (in \( p \)) constraint incorporating the correlation energy or cost function.\(^1\)

Since the present nonlinearities bear some resemblance to the ones of escort distributions, we anticipate that further developments will allow to relate the present approach to superstatistics [14], where such distributions can be successfully dealt with in a canonical framework [15].

We proceed to perform the variations and, thus, obtain the following set of equations:

\[
\sum_j p(x_i, E_j) = 1 ,
\]

(5)

\[
\sum_j C_j[p] E_j = \langle E \rangle ,
\]

(6)

\[
\ln[p(x_k, E_j)] + \frac{\beta E_j}{p(x_k, E_j)} \sum_{i_1 < \ldots < i_c} p(x_{i_1}, E_j) \ldots p(x_{i_c}, E_j) + \lambda(x_k) = 0 ,
\]

(7)

where we chose the example of Eq. (2) in the last equation; the prime on the sum indicates that the sum is only over those terms which contain the factor \( p(x_k, E_j) \).

\(^1\) Restriction to any particular form of the constraints is not required by the arguments leading to the variational principle for the Shannon entropy [12, 13].
Combining Eqs. (5) and (7), the Lagrange multiplier $\lambda$ can be eliminated to obtain implicitly the normalized distribution:

$$p(x_k, E_j) = Z^{-1}(x_k) \exp \left[ -\frac{\beta E_j}{p(x_k, E_j)} \sum_{i_1 < \ldots < i_c}^t p(x_{i_1}, E_j) \cdots p(x_{i_c}, E_j) \right],$$

(8)

with the partition function,

$$Z(x_k) \equiv \sum_j \exp \left[ -\frac{\beta E_j}{p(x_k, E_j)} \sum_{i_1 < \ldots < i_c}^t p(x_{i_1}, E_j) \cdots p(x_{i_c}, E_j) \right].$$

(9)

In principle, the Eq. (6) serves to fix $\beta$. However, instead, we may choose to work with the distribution as it is, considering $\beta$ as a macroscopic parameter characterizing the ensemble.  

In the absence of localized impurities or dissipative effects, the system cannot spontaneously break (lattice) translation invariance. With this simplification, the distribution has to be homogeneous in space,

$$p(x_k, E_j) = p_x(E_j),$$

(10)

i.e., the same at any chosen lattice site. This allows us to simplify Eq. (8):

$$p_x(E_j) = Z_x^{-1} \exp \left[ -\beta N_c E_j p_x^{-1}(E_j) \right],$$

(11)

where $Z_x$ is the correspondingly simplified partition function, cf. Eq. (9). The combinatorial factor is given by:

$$N_c \equiv c \left( \frac{L}{c} \right).$$

(12)

Note that for $c = 1$ we recover Boltzmann statistics, however, with a renormalized temperature, which is due to the presence of $L$ identical copies of the system, in this case.

The transcendental Eq. (11) is of the form $P = \exp(-\eta P)$, with $P \equiv (p_x Z_x)^{c-1}$ and $\eta \equiv (c - 1)\beta N_c E_j Z_x^{1-c}$. Its solution can be given in terms of the Lambert $W$-function, $P(\eta) = W(\eta)/\eta$. In the limit $\eta \ll 1$, one obtains the following approximation:

$$p_x(E_j) = Z_x^{-1} \frac{1}{\left( 1 + (c - 1)Z_x^{1-c}\beta N_c E_j \right)^{1/(c-1)}}, \quad \eta \ll 1 ,$$

(13)

which yields a power-law with exponent dictated by the correlation degree. This is a typical form of the Tsallis distribution [3, 4], or of Zipf’s law as generalized by Mandelbrot [1, 2]. In the opposite limit, a log-modified power-law results:

$$p_x(E_j) = Z_x^{-1} \left( \frac{F((c - 1)Z_x^{1-c}\beta N_c E_j)}{(c - 1)Z_x^{1-c}\beta N_c E_j} \right)^{1/(c-1)}, \quad \eta \gg 1 ,$$

(14)

This is analogous to working with a standard canonical ensemble, where $\beta$ would correspond to the inverse temperature, $\beta \equiv 1/T$, in units where $k_B = 1$.  

with the function $F$ defined by:

$$F(x) \equiv \ln(x) \left(1 + \ln(\ln(x))/\ln(x)\right)^{-1}.$$  \hspace{1cm} (15)

In the example of Eq. (11), we would find similar results in the homogeneous limit, differing only by the appropriate correlation degree and the combinatorial factor.

We remark that there is a considerable similarity between the Zipf-Mandelbrot law (13) and the distribution (14), when the former is also applied in the limit $\eta \gg 1$. In a log-log representation one could perhaps be mistaken for the other (for a limited range of data) by readjusting the parameters.

In any case, having obtained the probability distribution $p_x(E)$ for positive integer values of $c$, it is tempting to extend the definition of the correlation degree to all real positive numbers. In Eq. (12) this is implemented by employing the Gamma function instead of factorials, and it can obviously be done in Eq. (11), for $c > 1$; a comment concerning $c < 1$ will follow shortly. However, can we understand such analytic continuation also in terms of the picture of supercorrelated systems?

Physically, we expect that supercorrelations which are not fully developed somehow should correspond to noninteger correlation degree here. Indeed, let us focus on situations with $c$ sufficiently close to an integer. Here, an expansion provides a hint for the interpretation. For example, we may consider the phenomenologically interesting case $c = 1 + \gamma$, with $\gamma$ sufficiently small. We obtain:

$$p^{1+\gamma} = p \cdot \left(1 - \gamma q - \frac{1}{2!} \gamma (1 - \gamma) q^2 + O(q^3)\right).$$  \hspace{1cm} (16)

We recognize a particular form of anticorrelation. For example, this can correspond to a term $\propto p(x_i) [1 - \gamma q (x_i + \Delta x)]$, which presents a correlation with a ‘hole’ distribution, $q \equiv 1 - p$, modifying the leading Boltzmann term. This should be compared to the different correlation and anticorrelation defined in Eqs. (11) and (3), respectively.

To complement these remarks, we briefly recall the anticorrelation of Eq. (3), instead of Eq. (2). The result replacing Eq. (11) is simply:

$$p_x(E_j) = Z_x^{-1} \exp \left[-\beta E_j \left(1 - N_c p_{x}^{c-1}(E_j)\right)\right].$$  \hspace{1cm} (17)

Here, for $c > 1$, the second term in the exponent presents a small correction to the leading Boltzmann term, which vanishes exponentially for $\beta E_j \gg 1$.

An interesting situation arises for $c < 1$. – Both, the Eqs. (11) and (17) cease to have solutions for $p_x(E_j)$ for sufficiently large $\beta E_j$ in this case. This is reflected in the pole which arises in the power-law result of Eq. (13), when continued to $c < 1$. Similarly, for example, the power-law obtained here from Eq. (17),

$$p_x(E_j) = Z_x^{-1} \left(\frac{1 + (1 - c)Z_x^{1-c} \beta N_c E_j}{1 + (1 - c) \beta E_j}\right)^{1/(1-c)},$$  \hspace{1cm} (18)
breaks down, when unavoidably \( p_x(E_j) \rightarrow 1 \), for sufficiently large energy. Therefore, a cut-off on \( E_j \) is necessary here, as discussed in the applications of Tsallis statistics before [4].

3 Via phase space entrainment to supercorrelation

It is not the purpose of this section to develop the statistical mechanics of supercorrelated systems from the bottom up. Since the present approach is entirely new, our modest aim here is to show in a simple example, based on analyzing the asymptotic solutions of the equations of motion, how phase space entrainment occurs and leads to supercorrelations as defined in the previous section. – An extension of the simplest two-body system to a more realistic one-dimensional lattice model and its mean field analysis are contained in a first version of this letter [16].

Therefore, fundamental questions, such as related to the (non)thermalization of the Hamiltonian system presented in the following, to the H-theorem, or to the fluctuation-dissipation theorem, when the system is made dissipative in a particular way by incorporating phaenomenological (anti)damping terms in the equations of motion, must be deferred to future microscopic statistical mechanics studies of such systems.

Here, to begin with, correlation energies, such as defined in Eqs. (1)–(3), must reflect an underlying microscopic dynamics in a coarse-grained way. In order to further illustrate the notion of correlation energy, we consider here a two-body Hamiltonian, \( \mathcal{H}(\Pi_1, E_1; \Pi_2, E_2) \), describing the interaction of two ‘cells’ (called ‘particles’ before) localized at two given sites “1” and “2” of, for example, the previous onedimensional lattice. In distinction to ordinary Hamiltonian dynamics, we treat the local energies, \( E_{1,2} \), in place of a coordinate, together with the canonical momenta, \( \Pi_{1,2} \). Phase space consists of the 2x2-dimensional \((\Pi, E)\)-space \((E_{1,2} > 0)\).

The precise form of our Hamiltonian is solely dictated by mathematical convenience in modelling the following phaenomenological features:

- two subunits (cells, agents, etc.) of the system, i.e. sites “1” and “2”, exchange energy (adjust a potential difference across a biological membrane, trade a commodity, etc.) in an oscillatory way;

- the total amount of energy (base potential, commodity, etc.) distributed in the system is conserved;

- the ensuing Hamiltonian dynamics may be nonlinear;

- the exchange process is damped, while preserving the overall conservation of energy (etc.).
We will continue to use the term ‘energy’ henceforth. However, the versatility of such a model should be kept in mind. We believe that this type of systems is generic in that it produces some kind of supercorrelation.

Employing more convenient continuum notation, the average energy \( \langle \mathcal{E} \rangle \) corresponding to \( \mathcal{H} \) for a pair of cells is:

\[
\langle \mathcal{E} \rangle = (2\pi)^{-2} \int d\Pi_1 dE_1 d\Pi_2 dE_2 \mathcal{H}(\Pi_1, E_1; \Pi_2, E_2) p(\Pi_1, E_1; \Pi_2, E_2) , \tag{19}
\]

in terms of the phase space pair probability density \( p \), and where our model Hamiltonian is:

\[
\mathcal{H}(\Pi_1, E_1; \Pi_2, E_2) \equiv \frac{E_1 + E_2}{\xi \zeta} \exp[-\xi^{-2}(\Pi_1 - \Pi_2)^2 - \zeta^{-2}(E_1 - E_2)^2] = \mathcal{E} , \tag{20}
\]

with \( \xi, \zeta \) constant parameters.

Using variables \( \Pi_\pm \equiv \Pi_1 \pm \Pi_2 \) and \( E_\pm \equiv E_1 \pm E_2 \), and with the conserved total energy \( \mathcal{E} \) of Eq. (20), the equations of motion for this Hamiltonian can be combined to yield:

\[
\dot{E}_+ = 0 , \tag{21}
\]

\[
\dot{\Pi}_+ = -2\mathcal{E}E_+^{-1} , \tag{22}
\]

\[
\dot{E}_- = -4\mathcal{E}\zeta^{\frac{2}{2}}\Pi_- , \tag{23}
\]

\[
\dot{\Pi}_- = 4\mathcal{E}\zeta^{\frac{2}{2}}E_- , \tag{24}
\]

or, in particular, the second order equation:

\[
\ddot{E}_- + \omega^2 E_+ = 0 , \quad \omega^2 \equiv \left(\frac{4\mathcal{E}}{\xi \zeta}\right)^2 . \tag{25}
\]

Thus, we find a harmonic oscillator with a constant frequency. However, the frequency strongly depends on the initial conditions, being exponentially smaller when the two cells start out far away from each other in phase space (\( \Pi_- \) or \( E_- \) large), than when they are close. There is a rapid energy swapping in the latter case, while it is slow in the former.

We expect internal dissipative forces in coarse-grained descriptions of interacting microscopic systems. Presently, this dissipation is assumed to conserve the total energy \( \mathcal{E} \). Therefore, it has to change the energy sum \( E_+ \), at the expense of damping the swapping of energy between the cells, which is described by the oscillation of the energy difference \( E_- \) (and of \( \Pi_- \)). This can indeed be implemented by modifying only two of the Eqs. (21)–(24),

\[
\dot{E}_+ = -2\gamma (E_-/\zeta)^2 E_+ , \tag{26}
\]

\[
\dot{E}_- = -4\mathcal{E}\zeta^{\frac{2}{2}}\Pi_- - \gamma E_- , \tag{27}
\]
keeping the other two. This results in the damped oscillator equation: 
\[ \ddot{E} - \gamma \dot{E} + \omega^2 E = 0, \]
where \( \gamma \) is the damping constant, and with \( \omega^2 \) the same as before.

The following properties can be read off immediately from the equations of motion (22), (24), (26), and (27): (A) The amplitude of the oscillating energy difference \( E_\pm \) (and its derivative) is damped to zero \((\propto \exp[-\gamma t/2])\). – (B) The energy sum \( E_+ \) for the two cells decreases monotonically from its initial value and saturates exponentially (with rate \( \gamma \)) at its finite asymptotic value. – (C) The oscillating ‘momentum’ difference \( \Pi_\pm \) decreases exponentially to zero \((\propto \exp[-\gamma t/2])\). – (D) The corresponding sum \( \Pi_+ \) decreases monotonically \((\propto -t)\) for late times. Note that \( \Pi_+ \) does not enter the Hamiltonian.

From these observations we obtain the asymptotic time dependence of each cell’s ‘coordinate’ and ‘momentum’:

\[ E_{1,2}(t) = \frac{1}{2}(E_+ \pm E_-) \approx a_+(1 + ae^{-\gamma t}) \pm a_- e^{-\gamma t}, \quad (28) \]

\[ \Pi_{1,2}(t) = \frac{1}{2} (\Pi_+ \pm \Pi_-) \approx b_+ (b - t - b'e^{-\gamma t}) \pm b_- e^{-\gamma t}, \quad (29) \]

with constants depending on the initial conditions, besides \( \xi, \zeta \), and \( \gamma \) and \( \omega \) (“1” with upper signs, “2” with lower ones). We suppress oscillating factors here, thus emphasizing the presently important terms; using the full solutions, it can easily be verified that they do not affect the following argument.

The dynamics described by Eqs. (22), (24), (26), and (27) is dissipative. Obviously, there is a onedimensional attractor, see Eqs. (28)–(29). However, it is the pairing of variables which is most characteristic: the two cells considered are forced to follow similar trajectories in phase space, which converge asymptotically to one and the same (even with oscillation factors included). Simultaneously, the Hamiltonian is a constant of motion, \( \mathcal{H} = \mathcal{E} \), i.e., the overall energy (or other quantity represented by the cost function) is conserved.

This *phase space entrainment* is an essential ingredient for supercorrelated statistics in this illustrating example. Let us consider an ensemble of such pairs of cells with distributed initial conditions. Then, the Liouville equation, including the appropriate dissipative terms \((\propto \gamma)\), describes the evolution of the ensemble in phase space. *Any* solution of this equation for the probability density can be written in the form:

\[ p(\vec{\Pi}, \vec{E}; t) \equiv p \left( \vec{\Pi} - \vec{\Pi}(t), \vec{E} - \vec{E}(t) \right), \quad (30) \]

where the vectors collect the components “1,2”, and where \( \vec{\Pi}(t), \vec{E}(t) \) present a solution of the equations of motion, e.g., asymptotically as in Eqs. (28)–(29).

Considering initial distributions which are factorized, akin to Boltzmann’s ‘Stosszahlansatz’ (molecular chaos), \( p(\vec{\Pi}, \vec{E}; t) = p(\Pi_1 - \Pi_1(0), E_1 - E_1(0)) \cdot p(\Pi_2 - \Pi_2(0), E_2 - E_2(0)) \), one obtains correspondingly factorized solutions at all times.
Employing this fact, we further evaluate Eq. (19) together with (20). Since the Hamiltonian is a constant of motion, the average energy can be evaluated at any (late) time. Neglecting exponentially small corrections, see Eqs. (28)–(29), we obtain:

\[
\langle E \rangle = \int \frac{d\Delta\Pi d\Delta E}{2\pi} \frac{2}{\xi \zeta} \exp[-\xi^{-2} \Delta_{\Pi}^2 - \zeta^{-2} \Delta_{E}^2] \int \frac{d\Pi dE}{2\pi} \left( E + \Delta_{E}/2 \right) \\
\cdot p\left( \Pi - b_+(b - t), E - a_+ \right) \cdot p\left( \Pi - b_+(b - t) + \Delta_{\Pi}, E - a_+ + \Delta_{E} \right)
\]

\[
\approx \int \frac{d\Pi dE}{2\pi} E p(\Pi, E) p(\Pi, E),
\]

where we substituted \( \Pi_2 \equiv \Pi + \Delta_{\Pi} \) and \( E_2 \equiv E + \Delta_{E} \), and with \( \Pi, E \) instead of \( \Pi_1, E_1 \) earlier; the further approximation here consists in replacing the Gaussians by \( \delta \)-functions (for sufficiently small \( \xi, \zeta \)), where, more generally, gradient corrections come into play. The constant shift of the energy by \( -a_+ \) has been absorbed into a redefinition of the distribution \( p \), i.e., of the initial condition. Thus, we obtain a phase space generalization of the simplest case of supercorrelation (\( c = 2 \)) and corresponding correlation energy, cf. Eq. (1), in our two-cell example.

We remark that the intermediate result of Eq. (31) indicates a natural generalization of the correlation energies of Eqs. (1)–(3), namely allowing a certain spread of the energy when summing over states.

Having demonstrated the emergence of supercorrelations in an almost trivial example, which in turn form the starting point of the derivations of the previous section, we conclude here with two remarks concerning more realistic generalizations of this model.

A many-body system made up from subunits which interact pairwise, as investigated here, is bound to have rich transient dynamics, due to locally varying and time dependent effective oscillator frequencies [16]. In the two-cell example, the one frequency depends strongly on the initial condition. However, energy conservation prevents its variation. In general, local energy conservation will amount to only a global constraint and the frequencies vary locally, depending on the system variables. This may initially lead to the formation of ‘hot spots’, with rapid energy swapping between neighbouring cells, and relatively inactive regions elsewhere.

Furthermore, without adding further couplings to the basic Hamiltonian, the model cannot thermalize, i.e., sustain a certain amount of fluctuations. This can easily be changed in a future extension by modifying the description of damping of the relative motion between interacting cells and feedback of the dissipated energy into the system. For example, a damping term like for the van der Pol oscillator could be considered. This would raise the dimensionality of the underlying attractor and generate more interesting behavior, especially in a many-body system.
4 Conclusions

This letter has addressed the dynamical origin of power-laws. We have introduced the notion of supercorrelated systems where, as a result of interactions among its constituents, the energy or a more general cost function depend effectively on the distribution of the constituents.

This leads to a nonlinearity in the evaluation of sums over states, such as calculating the partition function, and in the determination of the “would-be-equilibrium” distributions. We showed that asymptotically the resulting distributions are (log-modified) power-laws, with parameters determined by the correlation degree characterising the correlation energy, see Eqs. (1)–(3), besides macroscopic parameters of the system.

These results follow from a simple statistical analysis and indicate that a wide range of models can be conceived which lead to such distributions. The crucial point consists in the identification of the relevant correlation energy, depending on the underlying dynamical model.

We presented a very simple deterministic dissipative model of interacting cells, where the interaction causes redistribution of the energy content among cells. While the overall energy of the system is conserved, the rate of change of gradients of the energy content of cells is damped by a dissipative force.

The model has interesting nonlinear features which deserve further study. Presently, we have considered only two interacting subunits of the system, while also a mean field approximation of a onedimensional lattice model has been studied with analogous results [16]. In its present form, the model cannot lead to thermodynamic equilibrium. However, an extension achieving this can easily be defined. For our purposes most important has been the fact that the dissipation leads to phase space entrainment, i.e. the oscillator degrees of freedom are forced to converge on a simple attractor in phase space.

While the effectively linear dynamics of the present model is rather trivial, it demonstrates how supercorrelations arise, which in turn generate (log-modified) power-laws.

It will be very interesting to learn about the time dependent behavior of the system when nonlinearities and further thermalizing couplings are introduced. Then, interesting questions may be raised concerning the (non)equilibrium character of the obtained distributions and the thermodynamic interpretation of the information entropy, on which our derivations are based. Thus, one may find a testing ground for ideas about the prethermalization mentioned in the Introduction. Our model presents a coarse-grained picture of microscopic dynamics, which may be difficult to access directly in complex systems. Thus, refined versions may have interesting applications in studies of transport and equilibration properties of condensed matter, polymer, or generally network structures.
Acknowledgements

We wish to thank C.E. Aguiar and T. Koide for discussions. One of us (HTE) wishes to thank P. Quarati and A. Lavagno for kind hospitality at Politecnico di Torino and for the interesting discussions, which were helpful in revising this paper. The anonymous referee is warmly thanked for his constructive criticism. – This work has been supported in part by CNPq, FINEP, FAPERJ, and CAPES/PROBRAL.

References

[1] G.K. Zipf, “Human Behavior and the Principle of Least Effort” (Addison-Wesley, Cambridge, MA 1949).

[2] B.B. Mandelbrot, “The Fractal Geometry of Nature” (Freeman, San Francisco 1983).

[3] C. Tsallis, J. Stat. Phys. 52, 479 (1988).

[4] “Nonextensive Statistical Mechanics and its Applications”, ed. by S. Abe and Y. Okamoto, Lecture Notes in Physics, Vol. 560 (Springer-Verlag, Berlin 2001); “Nonextensive Statistical Mechanics and Thermodynamics”, ed. by S.R.A. Salinas and C. Tsallis, Braz. J. Phys. 29, No. 1 (1999).

[5] R. Rapp, Theory Highlights of Quark Matter 2004, Theory Summary Talk at 17th Quark Matter Conference, Oakland (CA, USA), 11.-17.01.04, to be published in J. Phys. G

[6] J. Berges, S. Borsanyi, C. Wetterich, Prethermalization (2004), arXiv: hep-ph/0403234.

[7] T. Kodama, H.-T. Elze, C.E. Aguiar and T. Koide, Prethermalization and the Effects of Dynamical Correlations (2004), arXiv: cond-mat/0406732.

[8] S. Denisov, Phys. Lett. A235, 447 (1997).

[9] C. Beck, Phys. Rev. Lett. 87, 180601 (2001).

[10] T. Sherman and J. Rafelski, in: “Decoherence and Entropy in Complex Systems”, ed. by H.-T. Elze, Lecture Notes in Physics, Vol. 633 (Springer-Verlag, Berlin 2004), p. 377.

[11] T.S. Biro and A. Jakovac, Power-law tails from multiplicative noise (2004), arXiv: hep-ph/0405202.

[12] H. Haken, “Synergetics – An Introduction”, 3rd ed. (Springer-Verlag, Berlin 1983), Ch. 3.
[13] H.G. Schuster, “Deterministic Chaos – An Introduction”, 2nd ed. (VCH, Weinheim 1989), Appendix F.

[14] C. Beck and E.G.D. Cohen, Physica 322A, 267 (2003).

[15] C. Beck, Superstatistics, escort distributions, and applications (2003), arXiv: cond-mat/0312134.

[16] H.-T. Elze and T. Kodama, first version of this paper (2004), arXiv: cond-mat/0406736v1.