Time-Dependent Ginzburg–Landau Simulation of Critical Current Density Including z-axis Anisotropy

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Abstract. In this study, the three-dimensional time-dependent Ginzburg–Landau equations were numerically solved to visualize the motion of the flux lines in a superconductor under a transverse magnetic field. Pins were inserted into a superconducting rectangular parallelepiped, and the magnetic field dependence of the normalized critical current density $J_c$ was calculated. Anisotropy $\gamma_z$ of different magnitudes was introduced along the direction of the magnetic field (z-axis). Different pin shapes and orientations were also considered: columnar pins aligned parallel to the direction of either the magnetic field or the current flow, spherical pins, and a planar pin in the field–current plane. For the columnar pins aligned parallel to the field (along the flux lines), $J_c$ showed almost no dependence on $\gamma_z$. Additionally, a peak in the $J_c$-$B$ curve for this pin geometry was observed at normalized magnetic field, $B = 0.4$ for all considered $\gamma_z$. In contrast, $J_c$ was dependent on $\gamma_z$ for the columnar pins aligned parallel to the current flow (perpendicular to the flux lines) and the spherical pins. At low magnetic fields ($B = 0.1$), $J_c$ increased with increasing $\gamma_z$ in both these cases. In the case of the planar pin, $J_c$ showed no dependence on $\gamma_z$. In conclusion, when a pin was inserted parallel to the normalized magnetic field $B$, $J_c$ did not decrease even when the z-axis anisotropy $\gamma_z$ was large.

1. Introduction
The motion of flux lines has been calculated with the time-dependent Ginzburg–Landau (TDGL) equations [1–3]. In a previous study, we reported the angular dependence of the normalized critical current density $J_c$ in a superconductor with various pinning center shapes [1]. It was found that $J_c$ has almost no angular dependence in the case of star-shaped pins.

It is well known that the coherence length of the $ab$-plane is very different from the direction of the $c$-axis in high-temperature superconductors [4]. Anisotropy control is considered to be key to the development of new high-temperature superconducting devices [5]. However, there have been few theoretical calculations that incorporate anisotropy. In this study, anisotropy was introduced in superconductor simulations using models for the effective mass and the effective conductivity [6]. The magnitude of the z-axis anisotropy $\gamma_z$ was varied, and $J_c$ was computed with various pinning center shapes. The effect of $\gamma_z$ on $J_c$ in the case of different pin shapes is discussed.
2. Calculation methods

Figure 1 shows a superconducting rectangular parallelepiped with dimensions of $20\xi \times 10\xi \times 10\xi$, where $\xi$ is the coherence length. A magnetic field is applied in the direction of the z-axis, and a current with current density flows in the direction of the y-axis, as shown in Figure 1. Therefore, flux lines move in the direction of the x-axis.

Figure 2(a)–(d) shows schematics of the superconducting rectangular parallelepiped with spherical pins, planar pin in the yz-plane, columnar pins aligned parallel to the z-axis, and columnar pins aligned parallel to the y-axis, respectively. The diameters of the spherical pins in Figure 2(a) and the columnar pins in Figure 2(c) and (d) are all equal to $\xi$, and the thickness of the yz-planar pin in Figure 2(b) is $0.6\xi$. The distance $d$ between the spherical and columnar pins defined in Figure 2(a), (c), and (d) is $4\xi$.

In this study, the time-dependent Ginzburg–Landau (TDGL) equations with a reduced number of constants were used to solve the motion of the flux lines. The reduced TDGL equations are given as follows [7]:

$$\frac{\partial \Psi}{\partial t} + iV\Psi + (\mathbf{-i\nabla - A})^2\Psi - \Psi + |\Psi|^2\Psi = 0$$  \hspace{1cm} (1)

$$\sigma \nabla^2 \Psi = \frac{i}{2} (\Psi^*\nabla^2\Psi - \Psi\nabla^2\Psi^*) - \nabla \cdot (|\Psi|^2\mathbf{A}),$$  \hspace{1cm} (2)

where $\Psi$ is the order parameter, $V$ is the scalar potential, $\mathbf{A}$ is the vector potential, and $\sigma$ is the normal conductivity.

In addition, anisotropy was included in the computation by applying models for the effective mass and effective conductivity [6]. The anisotropy parameters along the x-, y-, and z-axes are defined as $\gamma_x$, $\gamma_y$, and $\gamma_z$, respectively. The effective mass $m^*$ and the effective conductivity $\sigma$ are given respectively by the following:

$$m^* \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma_z^2 \end{pmatrix} m^*$$  \hspace{1cm} (3)

$$\sigma \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma_z^2 \end{pmatrix} \sigma.$$  \hspace{1cm} (4)

Equation (3) or (4) is then introduced into the third term on the left-hand side of Equation (1) and included in the left- and right-hand sides of Equation (2) [6]. In the case represented by Equations (3) and (4), $\gamma_x$ and $\gamma_y$ are unity, and only the value of $\gamma_z$ is considered as a variable. In addition, the relationship between the anisotropy $\gamma_z$ and $I_c$ was investigated. Simulations were conducted with normalized external magnetic field ranging from $B = 0.10$ to $0.60$ in intervals of 0.10 and anisotropies of $\gamma_z = 1, 8, 64$, and 512. Here, the magnetic field is normalized by the upper critical field $B_{c2}$. The normalized current densities is in the range of $J = 0.01$ to $0.385$, with the upper limit here corresponding to the pair-breaking current density. The normalized critical current density $I_c$ is determined by using a criteria in $E$-$J$ curve.
3. Results and discussion

Figure 3 shows the magnetic field dependence of the critical current density for spherical pins with $\gamma_z = 1, 8, 64,$ and $512$. The results indicate that $J_c$ is weakly dependent on $\gamma_z$ at low magnetic fields. The value of $J_c$ at $\gamma_z = 512$ was larger than that at $\gamma_z = 1$ for $0 \leq B < 0.1$.

Figure 4 shows the magnetic field dependence of the critical current density in the case of the $yz$-planar pin with $\gamma_z = 1, 8, 64,$ and $512$. In this case, $J_c$ showed no dependence on $\gamma_z$.

Figure 5 shows the magnetic field dependence of the critical current density in the case of the $z$-axis columnar pins with $\gamma_z = 1, 8, 64,$ and $512$. Once again, $J_c$ showed no dependence on $\gamma_z$. However, a peak was observed at $B = 0.4$ for all $\gamma_z$. This is due to the peak effect [8], which occurs because the pin distance $d$ and the spacing $s$ of the flux lines are equal at $B = 0.4$.

Figure 6 shows the magnetic field dependence of the critical current density in the case of the $y$-axis columnar pins with $\gamma_z = 1, 8, 64,$ and $512$. In this case, $J_c$ showed weak dependence on $\gamma_z$ at low magnetic fields. The value of $J_c$ at $\gamma_z = 512$ was larger than that at $\gamma_z = 1$ for $0 \leq B < 0.1$. This is because the magnetic flux is connected in the $x$-direction at high $\gamma_z$, as shown in the inset of Figure 6. There was no significant difference among the $J_c$ values with different anisotropies $\gamma_z$ at high magnetic fields ($B \geq 0.4$).

The dependence of $J_c$ on $\gamma_z$ at a constant magnetic field was then compared for various pin shapes, as this comparison is not clearly observable from Figures 3–6. Figure 7 shows the critical current density $J_c$ plotted against the $z$-axis anisotropy $\gamma_z$ at $B = 0.1$ for the different pin shapes. In the case of the spherical pins, $J_c$ was highest at $\gamma_x = 64$ and $512$ and increased with increasing $\gamma_x$ for $\gamma_x < 64$. This is because the magnetic flux is connected in the $x$-direction at high $\gamma_x$. This is shown in the inset of Figure 3. It is considered that the pinning effect becomes large with connected magnetic flux lines. $J_c$ showed no $\gamma_x$ dependence in the case of the $z$-axis columnar pins or the $yz$-planar pin. In the case of $y$-axis columnar pins, $J_c$ increases as increasing $\gamma_x$. This is also explained by the connected magnetic flux along $x$-direction as shown in inset figure of Figure 6.

Figure 8 shows $J_c$ plotted against $\gamma_x$ at $B = 0.5$ for the different pin shapes. In the case of the spherical pins, $J_c$ decreased for $\gamma_x < 64$ and was constant for $\gamma_x \geq 64$. Once again, $J_c$ showed no $\gamma_x$ dependence in the case of the $z$-axis columnar pins or the $yz$-planar pin. In the case of the $y$-axis columnar pins, $J_c$ increases as increasing $\gamma_x$. This is also explained by the connected magnetic flux along $x$-direction as shown in inset figure of Figure 6.

Figure 1. Geometry of the superconducting rectangular parallelepiped.

Figure 2. Superconducting rectangular parallelepiped with pins of different shapes: (a) spherical pins, (b) $yz$-planar pin, (c) columnar pins parallel to the $z$-axis, (d) columnar pins parallel to the $y$-axis.
dependence in the case of the $z$-axis columnar pins or the $yz$-planar pin. This is because the effect of the anisotropy is lost if the pin is inserted parallel to the magnetic field. In the case of $y$-axis columnar pins, $J_c$ shows similar tendency of spherical pins, i.e., $J_c$ decreased as increasing $\gamma_z$.

In conclusion, it was confirmed that when a pin is inserted parallel to the magnetic field, $J_c$ does not decrease even under a large $z$-axis anisotropy $\gamma_z$.

![Figure 3](image1.png)  
**Figure 3.** Critical current density plotted against the magnetic field in the case of the spherical pins with $\gamma_z = 1, 8, 64, and 512$. The inset shows $\gamma_z = 512$ at $B = 0.1$.

![Figure 4](image2.png)  
**Figure 4.** Critical current density plotted against the magnetic field in the case of the $yz$-planar pin with $\gamma_z = 1, 8, 64, and 512$.

![Figure 5](image3.png)  
**Figure 5.** Critical current density plotted against the magnetic field in the case of the $z$-axis columnar pins with $\gamma_z = 1, 8, 64, and 512$.

![Figure 6](image4.png)  
**Figure 6.** Critical current density plotted against the magnetic field in the case of the $y$-axis columnar pins with $\gamma_z = 1, 8, 64, and 512$. The inset shows $\gamma_z = 512$ at $B = 0.1$. 

4. Conclusion

In this study, the z-axis anisotropy $\gamma_z$ dependence of the critical current density $J_c$ was numerically investigated using the three-dimensional TDGL equations.

In cases of the yz-planar pin and the z-axis columnar pins, $J_c$ showed no dependence on $\gamma_z$. However, a peak was observed at $B = 0.4$ for all $\gamma_z$ in the case of the z-axis columnar pins. In contrast, in the cases of the spherical and y-axis columnar pins, $J_c$ did show dependence on $\gamma_z$. At low magnetic fields ($B = 0.1$), $J_c$ increased with increasing $\gamma_z$ for both of these pin types. The results for the two types of pins are similar because of the overlapping volume between the flux lines and the pins along x-direction.

Therefore, it was confirmed that $J_c$ does not decrease even at high anisotropies if a pin is inserted parallel to the magnetic field. In conclusion, the magnetic field dependence of $J_c$ in the presence of anisotropy was clarified by numerical simulation using the TDGL equations.

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