Image Super Resolution Based on Gradient Constrained POCS Method

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Abstract A gradient constraint set based on edge detection is proposed to solve the problem of edge blurring in traditional projections onto convex sets (POCS) images reconstruction. Using the complete edge information of the original low-resolution (LR) image, the edge gradient constraint is applied to the reconstructed image, and the high-frequency information of the reconstructed image is retained to solve the problem of blurring the reconstructed image edge. And this paper uses the resolution test card to carry out simulation experiments, quantitative evaluation of the reconstructed image with respect to the LR images to improve the actual resolution of multiple. Simulation results show that the proposed method can reduce the edge blur of the reconstructed image, and the actual resolution is greatly improved compared with the traditional convex set projection method.

1. Introduction
In the process of images imaging, due to factors such as under-sampling, point diffusion of low-resolution (LR) sensors and optical blur, the quality and resolution of digital images obtained are not good. The super-resolution reconstruction technology can improve the image quality and resolution without changing the hardware conditions. Multi-frame super-resolution technology can be divided into two categories: frequency domain algorithm and space domain algorithm. The current super-resolution reconstruction algorithms mainly focus on the spatial domain to get high-resolution (HR) image. The commonly used algorithms include the maximum posterior probability estimation method (MAP)[1], the iterative back-projection method (IBP)[2], and the POCS method[3,4,5]. Among them, POCS is a widely used reconstruction algorithm based on set theory. Stark and Oskoui first applied this method to super-resolution image reconstruction, which has advantages such as good adaptability and strong prior information embedding ability.

2. Principles
2.1 Principle of the POCS
POCS method seeks feasible solutions that satisfy a series of prior information through iteration, including observation model, blackness model, noise statistical characteristics, and some characteristics of image itself, such as energy boundedness, smoothness and data reliability. Each constraint can be defined as a closed set of convex sets $C_m$, and the image solution exists in the intersection set of these convex sets, it is defined as:
The solution space of the super-resolution reconstructed image is formed by solving the intersection of these sets. Let $m$ be the number of convex set, and project the super-resolution estimated image $f_0$ to these solution spaces, and finally get an optimal solution. The corresponding projection calculations $P_1, P_2, ... P_n$ are defined by the convex set, and the process of image reconstruction is transformed into an iterative process. The initial value of the iterative selection is $f_0$, and the iterative process is as in Eq.(2) until the convergence is satisfied the optimal solution.

$$f_{n+1} = P_mP_{m-1}...P_2P_1f_0$$

Commonly used constraints are: energy finite constraints, regular constraints, amplitude constraints and similarity constraints, etc. The classic POCS method for refactoring according to these constraints is as follows:

1) Select one frame in the LR images as the reference frame,
2) Interpolating the reference frame to obtain an initial estimate $f_0$ of the HR image, and estimating a motion parameter between the LR images and the reference frame;
3) Projecting each LR image into the HR grid (ie, $f_0$) using the projection operator $P$, and correcting the estimated value of HR image;
4) Calculating a residual $\sigma$ of the reconstructed image according to the error constraint;
5) If the residual $\sigma$ is less than the set threshold condition, then it ends, otherwise it goes to step 3).

2.2 Principle of the proposed method

The traditional POCS method uses the bilinear interpolation method to construct the initial value $f_0$. The bilinear interpolation has the property of a low-pass filter, which will damage the high-frequency information and blur the edges of the image. Utilizing the integrity of the edge information of the original low-resolution image, the edge information gradient constraint is added in the reconstruction process, and the high-frequency information of the image texture and the edge is preserved, thereby reducing the edge blur of the reconstructed image.

Gradient constraints based on edge detection include three processes: edge detection, gradient calculation, and gradient error calculation.

Firstly, the edge detection of the LR images are obtained, and the edge position informations of the LR images are obtained. Then the edge gradient corresponding to the LR image is calculated to obtain the edge information gradient constraint set. Finally, the reconstructed image $f_i(i = 1, 2, ..., n)$ is constrained and corrected by the gradient constraint set. The algorithm flow for proposed method is:

1) Select one frame in the LR images as the reference frame,
2) Interpolating the reference frame to obtain an initial estimate $f_0$ of the HR image, and estimating a motion parameter between the LR images and the reference frame;
3) Projecting each LR image into the HR grid (ie, $f_0$) using the projection operator $P$, and correcting the estimated value of HR;
4) Gradient constraints based on edge detection;
5) Calculating the residual $\sigma$ of the reconstructed image according to the error constraint;
6) If the residual $\sigma$ is less than the set threshold condition, then it ends, otherwise it goes to step 3).

3. Test Results and Discussions

For the subjective and objective evaluation of the proposed reconstruction method, the LR images are reconstructed using bilinear interpolation, IBP, POCS and the proposed reconstruction method. The LR images are downsampled from the the 1951 USAF resolution test card. In the image downsampling process, the mean value blur with a kernel size of $4\times4$ is added, and the downsampling factor is 4, and the number of LR images is 16.
3.1 Subjective evaluation

Fig. 1 is the reconstruction images of Bilinear, IBP, POCS and the proposed reconstruction method. It can be seen that the reconstruction effect of Bilinear is not obvious. The details of IBP reconstructed images are prominent, and there is “ringing” in the image. POCS reconstructed image has better visual effects overall, but there is blurring phenomenon compared with the proposed method. Using our method exhibits comparatively higher quality i.e. the details in the image are outstanding, which explains that the proposed method reconstructed image has higher spatial resolution.

3.2 Objective analysis

In addition, for quantitative evaluation of the reconstructed images of different methods, the average gradient $V_G$, the information entropy $H$ and the image sharpness $S$ are employed, according to which, the higher the value of the index, the better the image quality. The assessment results are reported in Table 1 and the $V_G$, $H$ and $S$ are defined as

$$V_G = \frac{1}{(M-1)(N-1)} \sum_{i=2}^{M-1} \sum_{j=2}^{N-1} (\nabla f(i,j))^2 + (\nabla f(i,j))^2$$

$$H = -\sum_{i=0}^{255} p(i) \log_2 p(i)$$

$$S = \frac{1}{n} (\sum |f(i,j)-f(i,j-1)| + \sum |f(i,j)-f(i-1,j)|)$$

Table 1 exhibits that the proposed reconstruction method plays an important role in improving image quality of reconstruction procedure. Our method produces higher value compared with the method of Bilinear, IBP and POCS, which shows that the method in this paper not only improves the image clarity, but also enriches the image details and has a good visual.
Table 1 Objective parameters of reconstruction images

| Parameters   | VG   | H    | S    |
|--------------|------|------|------|
| Original LR  | 2.4677 | 6.1464 | 3.8424 |
| Bilinear     | 3.6258 | 6.1464 | 3.9367 |
| IBP          | 4.0890 | 6.1305 | 4.3348 |
| POCS         | 5.6641 | 6.2580 | 5.4912 |
| Proposed     | 6.2111 | 6.3648 | 5.7332 |

4. Conclusion
Aimed at the problem of image edge blurring caused by traditional POCS method reconstruction, the edge gradient constraint condition is added in the image reconstruction process to reduce the edge blur of reconstructed image. At the same time, the resolution test card image is used for experiments and reconstructing the images for supervisory and objective evaluation. The experimental results show that compared with other methods, the proposed method can obtain reconstructed images with better visual effects and the reconstructed images are more detailed and have better quality parameters.

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