Gravitational Waves as a New Probe of Bose-Einstein Condensate Dark Matter

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There exists a class of ultralight Dark Matter (DM) models which could form a Bose-Einstein condensate (BEC) in the early universe and behave as a single coherent wave instead of individual particles in galaxies. We show that a generic BEC DM halo intervening along the line of sight of a gravitational wave (GW) signal could induce an observable change in the speed of GW, with the effective refractive index depending only on the mass and self-interaction of the constituent DM particles and the GW frequency. Hence, we propose to use the deviation in the speed of GW as a new probe of the BEC DM parameter space. With a multi-messenger approach to GW astronomy and/or with extended sensitivity to lower GW frequencies, the entire BEC DM parameter space can be effectively probed by our new method in the near future.

INTRODUCTION

Although the existence of Dark Matter (DM) constituting about 27% of the energy budget of our Universe [1] is by now well established through various cosmological and astrophysical observations, very little is known about its particle nature and interactions. While the standard ΛCDM model with collisionless cold DM (CDM) successfully explains the large-scale structure formation by the hierarchical clustering of DM fluctuations [2, 3], there are some unresolved issues on galactic and sub-galactic scales, such as the core-cusp [4–7], missing satellite [8–11] and too big to fail [12–14] problems. All these small-scale structure anomalies can in principle be resolved if the DM is made up of ultralight bosons that form a Bose-Einstein condensate (BEC), i.e. a single coherent macroscopic wave function with long range correlation; for a review, see e.g., Ref. [15].

There are two classes of BEC DM, depending on whether DM self interactions are present or not. Without any self interactions, the quantum pressure of localized particles is sufficient to stabilize the DM halo against gravitational collapse only for a very light DM with mass \( m \sim 10^{-22} \) eV [16–20], whereas a small repulsive self-interaction can allow a much wider range of DM masses up to \( m \lesssim 1 \) eV [21–25].¹

Concrete particle physics examples for BEC DM are WISPs (Weakly Interacting Slim Particles) [31], which include the QCD axion or axion-like particles [32–39] and spin-1 hidden bosons ubiquitous in string theories [40–42], but our subsequent discussion will be generically applicable to any BEC DM with a repulsive self-interaction.²

The observational consequences on structure formation mentioned above cannot distinguish a BEC DM from an ordinary self-interacting DM [44]. Existing distinction methods include enhanced integrated Sachs-Wolfe effect [32], tidal torquing of galactic halos [39, 45, 46], and effects on cosmic microwave background matter power spectrum [47, 48]. In this paper, we propose a new method to probe the BEC DM parameter space using gravitational wave (GW) astronomy, inspired by the recent discovery of transient GW signal at LIGO [49, 50]. In particular, we show that if the GW passes through a BEC DM halo on their way to Earth, the small spacetime distortions associated with them could produce phononic excitations in the BEC medium which in turn induce a small but potentially observable change in the speed of GW, while the speed of light remains unchanged. This approach might be very effective if any of the future multi-messenger searches for gamma-ray, optical, X-ray, or neutrino counterparts to GW signal become successful. On the contrary, a lack of any observable deviation in the speed of GW will put stringent constraints on the BEC DM scenario. In fact, we find that even with the current LIGO sensitivity, it might be possible to completely rule out the BEC DM parameter space otherwise preferred by existing cosmological data.

SPEED OF GRAVITATIONAL WAVES INSIDE BEC

The cosmological dynamics of BEC DM can be described by a single quantum scalar field \( \phi \), with the effective Lagrangian

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \lambda \phi^4,
\]

(1)

analogous to the Ginzburg-Landau free energy density in a neutral superfluid. We have assumed that the DM particles do not have any interactions with the Standard Model (SM) particles, so a real scalar field will suffice for our discussion, but choosing it complex will not significantly change our results. In Eq. (1), we have considered a simple renormalizable scalar potential with only quadratic and quartic terms, the latter providing a repulsive quartic self-interaction for the DM, as required in addition to the quantum pressure of localized particles to stabilize the DM halo core against gravita-

¹ BEC configurations with heavier DM and/or an attractive self-interaction are usually unstable against gravity and are more likely to form local dense clumps such as Bose stars [26–30], unless the thermalization rate is faster than the Hubble rate to overcome the Jeans gravitational instability.

² Although the simplest models, where the scalar potential has an approximate symmetry to ensure the radiative stability of the ultralight scalar, usually give rise to an attractive self-interaction in the non-relativistic limit, it is possible to have realistic models with repulsive self-interaction [25, 43].
tional collapse. For $\lambda = 0$ (i.e. no self-interaction), the quantum pressure is sufficient only if $m \sim 10^{-22}$ eV, a scenario known as fuzzy DM [17]. In principle, we could also add a cubic term $-g\phi^3$ to Eq. (1); however, for the self-interaction to be repulsive in the non-relativistic limit, we must have $\lambda > 5g^2/2$ [25]. Similarly, we do not exclude any higher-dimensional operators, which are usually subdominant compared to the renormalizable ones considered here.

Using Eq. (1), we calculate the stress-energy tensor

$$T^{\mu\nu} = \frac{\partial L}{\partial (\partial_\mu \phi)} \partial_\nu \phi - g^{\mu\nu} L,$$  \hspace{1cm} (2)

where $g^{\mu\nu}$ is the spacetime metric. In presence of a GW, the linearized spacetime metric is usually written as $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$, where $\eta^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the flat Minkowski metric (in particle physics conventions) and $h^{\mu\nu}$ is a small perturbation. To leading order, the background mean field values of the energy density $p_0 \equiv T^{00}$ and pressure $p_0 \equiv T^{ii}$ of the BEC medium are related by the equation of state

$$p_0 = \frac{3}{2} \frac{\lambda}{m^4} \rho^2.$$  \hspace{1cm} (3)

However, the small spacetime distortions caused by the propagation of GW get amplified in the BEC medium. This is because almost all particles in the BEC system are condensed into the lowest energy available state with very long de Broglie wavelength, and gravity is a long-range force. Therefore, although gravitational interactions among particles are extremely weak, the massless phonon modes in the ground state of the scalar wave function can be excited by the GW [51], which then slowly propagate in the medium. The effective metric of the BEC phononic excitations on the flat spacetime metric can be written as [52, 53]

$$g_{\text{eff}}^{\mu\nu} = \frac{n^2_0}{c_s(p_0 + p_0)} \text{diag}(c_s^2, -1, -1, -1),$$  \hspace{1cm} (4)

where $n_0 \equiv \rho_0/m$ is the number density of the background mean field and

$$c_s = \left( \frac{\partial p_0}{\partial \rho_0} \right)^{1/2} \equiv \left( \frac{3\lambda \rho_0}{m^4} \right)^{1/2}$$  \hspace{1cm} (5)

is the speed of sound obtained from the background equation of state (3). Using the effective metric (4) in Eq. (2), we get the new equation of state

$$p = \frac{3n^2_0 \lambda \rho^2_0}{2c_s m^4[c_s n^2_0 + 2(p_0 + p_0)]} \equiv \frac{3}{2} \frac{\lambda}{m^4} \rho^2,$$  \hspace{1cm} (6)

where the effective change in the BEC energy density due to the phononic excitations is given by $\rho \equiv \kappa \rho_0$, with

$$\kappa = \left[ \frac{n_0}{c_s \left[ c_s n^2_0 + 2(p_0 + p_0) \right]} \right]^{1/2},$$  \hspace{1cm} (7)

by comparing the new equation of state (6) with the background equation of state (3).

Analogous to the case of light traveling through a dense medium, GW propagating in a medium of matter with density $\rho$ will experience an induced refractive index, arising from the coordinate-dependent gravitational potential corrections to the Newtonian metric [54], of the form

$$n_g = 1 + \frac{2\pi G\rho}{m^2 \omega^2} = 1 + \frac{\rho}{4M^2 \omega^2},$$  \hspace{1cm} (8)

where $G \equiv 1/8\pi M^2$ is Newton’s constant ($M^2 = 2.4 \times 10^{18}$ GeV being the reduced Planck mass), and $\omega = 2\pi f$ is the angular frequency ($f$ being the frequency) of the GW. Due to the Planck mass suppression in Eq. (8), the resulting effect on the propagation speed of GW, $c_g = 1/n_g$ (in units of $c = 1$), is unobservable [55] for a conventional DM halo with average density $\rho_{\text{halo}} \simeq 0.3$ GeV/cm$^3$. However, the key difference for a BEC medium is that the GW will experience an enhanced effective matter density $\rho \equiv \kappa \rho_0$ that amplifies the change in refractive index by a factor of $\kappa$ [cf. Eq. (7)], i.e.

$$\delta n_g \equiv n_g - 1 = \frac{\kappa \rho_0}{4M^2 \omega^2}.$$  \hspace{1cm} (9)

This can be understood qualitatively from energy conservation arguments, since the phononic excitation by the propagation of GW must have a back-reaction effect on the propagating gravitons, thus inducing an effective mass for the graviton and slowing them down by a factor $\delta n_g$. Therefore, a constraint on the speed of GW can be directly translated into a constraint on the $(m, \lambda)$ parameter space of BEC DM, as we show below.

**GW CONSTRAINTS ON BEC DM**

Let us assume that the GW produced at a distance $D$ encounters a spherical BEC DM halo of radius $R$ en route to Earth. Thus, the average distance they propagate through the DM halo with a reduced speed $c_g$ is given by $\langle D_{\text{halo}} \rangle = 4R/\pi$ and the corresponding average fraction of distance

$$x \equiv \frac{\langle D_{\text{halo}} \rangle}{D} = \frac{4R}{\pi D}.$$  \hspace{1cm} (10)

The effective speed of GW is then given by

$$c_{\text{eff}} \equiv \frac{D}{\Delta \tau} = \frac{c_g}{x + (1-x)c_g},$$  \hspace{1cm} (11)

where $\Delta \tau = xD/c_g + (1-x)D$ is the proper time elapsed between the emission and detection of the GW signal. So the change in the speed of GW from the speed of light in vacuum
due to its encounter with the BEC DM halo is given by
\[ \delta c_g \equiv 1 - c_{\text{eff}}^{\lambda} = \frac{x^2 n_g}{1 + x^2 n_g}. \]  
(12)

This is our key result that will be used to put new constraints on the BEC DM properties.

The radius of the gravitationally bound BEC with a repulsive self-interaction is independent of the central density and the mass of the system, and depends only on the physical characteristics of the particles in the condensate [56]:
\[ R = \left( \frac{\pi^2 a_s}{G m^3} \right)^{1/2}, \]  
(13)

where \( a_s \) is the s-wave scattering length which in the low-energy limit is defined by \( \lim_{k \to 0} \sigma(\phi_0 \to \phi_0) = 4 \pi a_s^2 \) (where \( k \) is the wavenumber). For the interaction Lagrangian given by Eq. (1), we find
\[ \sigma = \frac{9 \lambda^2}{\pi m^2}, \]  
(14)

and hence, from Eq. (13),
\[ R = 2 \pi \sqrt{3 \lambda} \frac{M_{\text{pl}}}{m^2}. \]  
(15)

When substituted in Eq. (10), this relates the fraction \( x \) to the microscopic properties of the BEC DM.

Similarly, the density distribution of a static, spherically-symmetric BEC DM halo can be obtained from the solution to the Lane-Emden equation in the weak field, Thomas-Fermi regime, given by the analytic form [56]
\[ \rho(r) = \rho_{\text{cr}} \frac{\sin kr}{kr}, \]  
(16)

where \( k = \sqrt{G m^3/a_s} = \pi/R \) and \( \rho_{\text{cr}} \) is the central density of the condensate. The average density of a BEC DM halo is thus given by
\[ \rho_0 \equiv \langle \rho \rangle = \frac{3 \rho_{\text{cr}} \pi^2}{2}, \]  
(17)

which will be used in our Eq. (9).

Using Eq. (12), we numerically calculate the maximum possible deviation in the speed of GW \( \delta c_g^{\text{max}} \) as a function of the microscopic BEC DM parameters \( m \) and \( \lambda \) for given values of the source distance \( D \) and the GW frequency \( f \). For illustration, we will fix the central core density at \( \rho_{\text{cr}} = 0.04 M_{\odot}/pc^3 \) (where \( M_{\odot} \) is the solar mass), which is within the range suggested by a recent N-body simulation of self-interacting DM [57]. We also take \( D = 400 \) Mpc and \( f = 35 \) Hz as representative values following the GW150914 event at LIGO [49]. The mass of the DM particle is varied in the range \( m \in \{10^{-3}, 1\} \) eV. The upper limit comes from the basic condition that the particle’s de Broglie wavelength, \( \lambda_{\text{DB}} = 2\pi/mv \) (where \( v \sim 10^{-3} \) is the virial velocity and we set \( h = 1 \)) should be larger than the inter-particle spacing, \( d = (m/\rho)^{1/3} \), such that the wave functions of the individual particles in the system overlap with each other to form a BEC. The de Broglie wavelength also sets a natural lower limit to the core size of equilibrium BEC-DM halos that can form; taking \( \lambda_{\text{DB}} \lesssim 1 \) kpc, the size of DM halo of a typical dwarf spheroidal (DShp) galaxy, we get the lower limit of \( m \gtrsim 5 \times 10^{-23} \) eV, which is saturated in the fuzzy DM scenario [17].

With this choice of parameters, we find the maximum \( \delta c_g \) that can be induced by BEC DM is at the level of \( 10^{-36} \). Thus we need the experimental sensitivity of \( \delta c_g^{\text{exp}} \) at this level or below to be able to put constraints on the BEC DM parameter space using our method. For comparison, the current best model-independent bound is \( \delta c_g \lesssim 2 \times 10^{-15} \) [58], deduced from the absence of gravitational Cherenkov radiation allowing for the unimpeded propagation of high-energy cosmic rays across our galaxy. Recently, assuming that the short gamma-ray burst above 50 keV detected by Fermi-GBM [59] just 0.4 seconds after the detection of GW150914 at LIGO [49] originated from the same location, more stringent limits on \( \delta c_g \) have been derived [60–63]. While a typical time-of-flight analysis [64] gives \( \delta c_g \lesssim 10^{-17} \) [60–62], using modified energy dispersion relations (typical of many quantum gravity models) with the quantum gravity scale \( E_G \gtrsim M_{\text{pl}} \) yields a much stronger limit of \( \delta c_g \lesssim 10^{-40} \) [65]. However, whether the Fermi-GBM event originates from the same astrophysical source responsible for GW150914 is a controversial issue [65–71] and according to a recent analysis [71], the GBM event is more likely a background fluctuation, which is consistent with the non-detection of similar gamma-ray events at SWIFT [72], INTEGRAL [73] and AGILE [74]. Nevertheless, after the detection of the second LIGO event GW151226 [50], the multi-messenger searches have become more intense and now include searches for gamma-ray [75–77], X-ray [76, 78], optical [79, 80] and neutrino [81–84] counterparts. With more GW events expected from LIGO in the near future, these multi-messenger searches are likely to detect events coming from the same source and improve the limits on \( \delta c_g \) significantly.

Since the change in refractive index in a BEC medium is inversely proportional to the square of the GW frequency, a future space-based GW interferometer, such as eLISA [85] with a lower operational frequency range of 0.1–100 mHz can further improve the sensitivity. For instance, for \( f = 1 \) mHz, \( D = 3 \) Gpc and \( \rho_{\text{cr}} \) same as above, the maximum \( \delta c_g \) that can be induced by BEC DM is at the level of \( 10^{-28} \). Pulsar timing arrays, such as the ones united under IPTA [86] and SKA [87], probe much lower frequencies around 1–10 nHz and are capable of bringing \( \delta c_g^{\text{max}} \) close to its current upper bound, irrespective of the success of multi-messenger searches.

In Figure 1, we translate the upper limit on \( \delta c_g \lesssim \delta c_g^{\text{exp}} \) to an exclusion region in the \((m, \lambda)\) plane using Eq. (12) for two values of \( \delta c_g^{\text{exp}} = 10^{-36} \) and \( 10^{-40} \). For comparison, we also show the region which gives \( \sigma/m = (0.01 − 1) \) cm²/g (blue shaded), as preferred by N-body simulations to explain
the small-scale structure anomalies, while being consistent with all observational constraints from colliding galaxy clusters [88–92] and halo shapes [57, 93–96]. Similarly, the viability of the BEC DM halo model (16) to fit the rotational curves of the most DM-dominated low surface brightness and DSph galaxies from different surveys implies $R \sim 0.5–10$ kpc [23, 56, 97–99], which can be translated to a preferred range of $m/\lambda^{1/4} \sim 4–18$ eV, as shown by the dark green shaded region in Figure 1. Most of this region is also consistent with the constraint on the total energy density of the relativistic species from Big Bang Nucleosynthesis (BBN) [99, 100]. Note that the region of intersection between the blue and green shaded areas gives the physically preferred value of $(m, \lambda) \simeq (10^{-4}$ eV, $10^{-19}$), as shown by the yellow point. For $\delta c_g \leq 10^{-36}$, this value is still allowed, whereas for $\delta c_g \leq 10^{-40}$, this is excluded. We find that for the LIGO frequency range $f = 10–350$ Hz, the physical region can be completely excluded for $\delta c_g \leq 10^{-36}–10^{-40}$, which should be soon achievable in the multi-messenger approach.

Apart from the observational constraints, one should also satisfy important theoretical constraints from the BEC formation requirements. The purple shaded region in Figure 1 is excluded as the relaxation time $t_{\text{relax}}$ in the virialized DM clumps due to the scattering process $\phi \phi \rightarrow \phi \phi$ exceeds the age of the Universe $t_{\text{universe}}$, which sets a lower limit on the self-interaction strength $\lambda \gtrsim 10^{-15} (m/\text{eV})^{7/2}$ for the BEC to form [28, 101, 102]. Similarly, the black shaded region in the top left part of the parameter space is disfavored, as the critical temperature $T_c = (24 m^2/\lambda)^{1/2}$ [103–105] below which a BEC can form, falls below the temperature of the universe at the source redshift of $z = 0.1$, which means the BEC DM halo could not have formed at the time the GW was emitted from the binary black hole merger event GW150914. If we assume the DM particles were in thermal equilibrium with the SM particles (through some additional interactions not shown in Eq. (1)), this constraint becomes more severe, as in this case, $T_c$ is required to be larger than the BBN temperature $T_{\text{BBN}} \sim 1$ MeV. Finally, an average halo size larger than about 1 Mpc seems unrealistic for self-interacting DM halos and disfavored by simulations [56, 94]; this excludes the cyan shaded region in Figure 1.

**GRAVITATIONAL LENSING**

In the multi-messenger approach, one way to confirm the existence of a BEC DM halo in the path of the GW is by studying the deflection of photons passing through the region where galactic rotation curves are flat. The deflection angle is given by the standard formula [106]

$$\delta \theta_{\text{def}} = \frac{4GM}{b},$$  \hspace{1cm} (18)

where $b$ is the impact parameter (i.e., distance of closest approach) for which we use the radius of the BEC DM halo from

![FIG. 1. Constraints of the quartic self-interaction as a function of the scalar mass for an intervening BEC DM halo along the line of sight of a GW signal, where the red and orange shaded regions can be excluded for an upper bound on $\delta c_g$ of $10^{-36}$ and $10^{-40}$ respectively. Here we have used $f = 35$ Hz for the GW frequency and $D = 400$ Mpc for the source distance as representative values from the event GW150914. The blue band corresponding to $\sigma/m = (0.01–1)$ cm$^2$/g is the preferred range for a self-interacting DM to solve the small-scale structure anomalies. The dark green band corresponds to a BEC DM halo with radius $R = 0.5–10$ kpc, as preferred by fits to DSph galaxy rotation curves. The overlap of the blue and green regions is physically favored, as shown by the yellow point which is completely within the exclusion region for $\delta c_g \leq 10^{-40}$. The purple shaded region in the bottom half is excluded as the relaxation time to form a BEC exceeds the age of the universe. The black shaded region above the cyan solid line, the average halo size $\langle D_{\text{halo}} \rangle$ exceeds 1 Mpc. The dark blue dashed line gives a light deflection angle of $\delta \theta_{\text{def}} = 10^{-7}$ due to gravitational lensing by the intervening BEC DM halo along the line of sight. See text for more details.](image-url)
messenger counterparts. In the geometrical optics approximation, treating the total gravitating mass as a point source, the time delay is the same for GW, photons and neutrinos, given by the general formula [108, 109]

$$\Delta t_{\text{Shapiro}} = (1 + \gamma)GM\ln \left( \frac{D}{b} \right),$$  \hspace{1cm} (21)

where $\gamma$ is a parametrized post-Newtonian parameter. However, this geometrical approximation breaks down for GW with wavelengths larger than the size of the lensing object, which corresponds to lens masses less than approximately $10^{-8}M_\odot (f/\text{Hz})^{-1}$, and it can induce an arrival time difference between the GW and photons/neutrinos of up to $0.1\text{sec} (f/\text{Hz})^{-1}$ [110]. For $R \sim 1$ kpc, we estimate the mass of BEC DM halo from Eq. (19) to be $M \sim 10^8 M_\odot$; so for the LIGO frequency range of 10-500 Hz, the geometrical optics approximation (21) remains valid and there is no relative Shapiro time delay to be considered in the multi-messenger analysis. However, for smaller frequencies, such as those relevant for eLISA, the additional time delay must be taken into account while deriving experimental bounds on $\delta c_g$.

**CONCLUSION**

We have proposed a new method to probe Bose-Einstein condensate Dark Matter using gravitational waves. We have shown that GWs passing through a BEC DM halo will get appreciably slowed down due to energy loss in phononic excitations. The effective refractive index depends only on the mass and quartic coupling of the DM particles, apart from the frequency of the propagating GW. Thus, an observable deviation $\delta c_g$ in the speed of GW can be used to put stringent constraints on the BEC DM parameter space, as demonstrated in Figure 1. The physically interesting region of BEC DM parameter space satisfying all existing constraints can be completely probed by this new method for $\delta c_g \leq 10^{-37} - 10^{-40}$ in the LIGO frequency range, which is soon achievable in a multi-messenger approach to GW astronomy.

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