A determination of gluon spin distribution from the deep inelastic scattering data

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Abstract
In order to determine polarized parton densities in nucleon we have made fits using all experimental data on spin asymmetries measured in the deep inelastic scattering on different nucleon targets. We have used in our analysis next to leading order QCD corrections in \( \overline{MS} \) renormalization scheme. The functional forms of polarized densities are based on the fit to unpolarized deep inelastic data made by the MRST group (MRST 99). We have concentrated on different models for gluon spin distributions. In the best fit we get for gluon polarization (at \( Q^2 = 1 \text{GeV}^2 \)) \( \Delta G = -1.0 \pm 0.6 \). The obtained result does not depend very much on various model assumptions.
The experimental data on deep inelastic scattering of leptons on nucleons enable us to determine parton densities in nucleons. For polarized particles such experiments were made in SLAC [1], CERN [2] and DESY [3]. The results were analyzed by many groups and several spin distributions and different gluon polarization were presented [4, 5, 6, 7]. In this paper we do not consider any new experimental data. All existing data on polarised deep inelastic scattering were already included in our latest fit [7] where we have determined polarized parton densities (performing next to leading order QCD analysis in $\overline{MS}$ renormalization scheme). However, since our method of determination of spin distributions depends very strongly on the parton distributions for unpolarized case we have performed a new analysis using unpolarized distributions obtained by Martin, Roberts, Stirling and Thorne (MRST) [8].

The unpolarized densities are the sums of $+\,$ and $-\,$ components (densities of quarks and gluons with helicity along or opposite to the helicity of the parent nucleon) whereas the polarized ones are given by the differences. The asymptotic behaviour of our polarized parton distributions is determined (up to the condition that the corresponding spin densities are integrable) by the fit to unpolarized data.

The determination of unpolarized parton densities performed by MRST [8] gives three solutions for gluon distribution. In this paper we want to follow the method used in our previous papers [3, 4] and to check how different functional form of gluon distributions influences our fits. We will use data for spin asymmetries at given $x$ and different $Q^2$. Altogether we will use in our analysis 431 experimental points. In our formulae for polarized densities we have included 12 parameters.

Experiments on unpolarized targets provide information on the spin averaged quark and gluon densities $q(x, Q^2)$ and $G(x, Q^2)$ inside the nucleon. In ref. [8] these distributions (at $Q^2 = 1$ GeV$^2$) are parametrized as follows:

$$q_i(x) = A_i x^{\lambda_i} (1 - x)^{\eta_i} (1 + \epsilon_i \sqrt{x} + \mu_i x),$$

(1)

where one assumes that sea antiquarks and quarks of the same flavour have identical distributions. The similar form one has for gluon density:

$$G(x) = B_G x^{\lambda_G} (1 - x)^{\eta_G} (1 + \epsilon_G \sqrt{x} + \mu_G x).$$

(2)

The values of constants $\lambda_i$ and $\eta_i$, used in eqs. (1) and (2), are given in Table 1 for all three solutions which are called lower (L), central (C) and higher
(H) gluon solution in ref.[8] (the differences are visible for big $x$ values). Our polarized densities for valence quarks (at $Q^2 = 1 \text{ GeV}^2$) are:

$$\Delta q_i(x) = x^{\lambda_i}(1 - x)^{\eta_i}(\alpha_i + \beta_i \sqrt{x} + \gamma_i x), \hspace{1cm} (3)$$

whereas for sea quarks and antiquarks one has:

$$\Delta q_j(x) = x^{\lambda_j + \frac{1}{2}}(1 - x)^{\eta_j}(\alpha_j + \beta_j \sqrt{x}) + c_i x^{\lambda_i}(1 - x)^{\eta_i + 2}\alpha_i(1 + \epsilon_i \sqrt{x} + \mu_i x), \hspace{1cm} (4)$$

where $c_i = +1(-1)$ for up (down) flavour and zero for strange quarks. With such parametrization of quark spin densities we get from the fit approximately $\alpha_\delta \approx 0$ and $\alpha_\tau \approx 2\alpha_\tau$, $\beta_\tau \approx \beta_\bar{\tau}$. Hence, our $a$ posteriori assumptions are:

$$\alpha_\delta = 0,$$

$$\alpha_\tau = \alpha_\bar{\tau} = 2\alpha_\tau,$$

$$\beta_\tau = \beta_\bar{\tau}. \hspace{1cm} (5)$$

Such choice is consistent with the SU(2) symmetry in the sea.

For polarized gluon distribution we assume:

$$\Delta G(x) = x^{\lambda_G}(1 - x)^{\eta_G}(\alpha_G + \beta_G \sqrt{x} + \gamma_G x). \hspace{1cm} (6)$$

The parameters $\alpha_i$, $\beta_i$ and $\gamma_i$ are determined from our fits and presented in Table 2.

**Table 1.** The values of constants which appear in eqs. (1-4,6) for three solutions of unpolarized gluon density. This figures determine the behaviour of distributions at $x = 0$ and $x = 1$.

| Constant | Lower gluon solution | Central gluon solution | Higher gluon solution |
|----------|----------------------|------------------------|-----------------------|
| $\lambda_{uv}$ | -0.527 | -0.583 | -0.560 |
| $\eta_{uv}$ | 3.44 | 3.43 | 3.45 |
| $\lambda_{dv}$ | -0.748 | -0.730 | -0.740 |
| $\eta_{dv}$ | 3.86 | 3.86 | 3.95 |
| $\lambda_s$ | -1.240 | -1.282 | -1.273 |
| $\eta_s$ | 7.63 | 7.65 | 8.30 |
| $\lambda_\delta$ | -0.356 | 0.183 | 0.157 |
| $\lambda_G$ | 0.131 | -0.031 | -0.005 |
| $\eta_G$ | 5.67 | 6.88 | 7.45 |
Table 2. The parameters of three fits calculated at $Q^2 = 1\text{ GeV}^2$.

| Parameter | Lower gluon solution | Central gluon solution | Higher gluon solution |
|-----------|----------------------|------------------------|-----------------------|
| $\alpha_{u,v}$ | 0.110 | 0.499 | 0.279 |
| $\beta_{u,v}$ | -3.50 | -4.71 | -4.22 |
| $\gamma_{u,v}$ | 13.3 | 14.0 | 13.9 |
| $\alpha_{d,v}$ | -0.055 | 0.028 | -0.059 |
| $\beta_{d,v}$ | -1.66 | -1.70 | -1.67 |
| $\gamma_{d,v}$ | 0.007 | -0.077 | -0.007 |
| $\alpha_{\pi}$ | -0.033 | -0.133 | -0.082 |
| $\beta_{\pi}$ | 1.54 | 1.37 | 1.58 |
| $\beta_{\pi}$ | -0.402 | -0.056 | -0.226 |
| $\alpha_{G}$ | -18.3 | -24.1 | -20.3 |
| $\beta_{G}$ | 52.5 | 117.2 | 86.7 |
| $\gamma_{G}$ | -48.8 | -173.5 | -127.9 |

The total quark distributions are given by:

$$
\begin{align*}
\Delta u &= \Delta_{u,v} + 2\Delta_{\pi}, \\
\Delta d &= \Delta_{d,v} + 2\Delta_{\bar{d}}, \\
\Delta s &= 2\Delta_{\pi},
\end{align*}
$$

(7)

whereas one has for axial charges:

$$
\begin{align*}
\Delta \Sigma &= \Delta u + \Delta d + \Delta s, \\
a_8 &= \Delta u + \Delta d - 2\Delta s, \\
a_3 &\equiv g_A = \Delta u - \Delta d.
\end{align*}
$$

(8)

In Table 3 one can find above quantities integrated over variable $x$ and calculated at $Q^2 = 1\text{ GeV}^2$. The axial charge $a_3$ is not fixed (usually one fixes it in an analysis) and comes out close to its experimental value. The quantity $a_8$ is in some sense fixed i.e., it is included in a fit as an additional experimental point. We put $a_8 = 0.58 \pm 0.1$, the value taken (with enhanced to $3\sigma$ error) from the data on hyperon $\beta$ decays.

The typical errors of our determination are: $\Delta G = -1.02 \pm 0.61$, $\Delta \Sigma = 0.04 \pm 0.18$, where we quote the figures for higher gluon solution. We use all
three MRST solutions keeping the quantity $a_8$ fixed. As it is seen from Table 3 the best $\chi^2$ we got for higher gluon solution.

Table 3. The integrated quantities from our three fits calculated at $Q^2 = 1$ GeV$^2$.

| Quantity | Lower gluon solution | Central gluon solution | Higher gluon solution |
|----------|----------------------|------------------------|-----------------------|
| $\Delta \Sigma$ | 0.19 | 0.02 | 0.04 |
| $a_8$ | 0.58 | 0.59 | 0.51 |
| $g_A$ | 1.25 | 1.29 | 1.31 |
| $\Delta u$ | 0.79 | 0.75 | 0.75 |
| $\Delta d$ | -0.46 | -0.54 | -0.55 |
| $\Delta s$ | -0.13 | -0.19 | -0.16 |
| $\Delta G$ | -0.77 | -1.15 | -1.02 |
| $\chi^2$ | 363.91 | 363.16 | 362.28 |

For higher gluon solution we have tried to make several fits, where we investigate other assumptions which slightly change gluon distribution. In one of the fits we put $\alpha_{GH} = 0$ at $Q^2 = 1$ GeV$^2$ (it means, that gluon density is less divergent at $x = 0$). In this case we get a fit which is of undistinguishable quality from the best fit (in both cases $\chi^2$ per degree of freedom is 0.86) and one gets $\Delta G = -0.63$. However, when we put $\alpha_{GH} = \beta_{GH} = \gamma_{GH} = 0$ (i.e. $\Delta G = 0$ at $Q^2 = 1$ GeV$^2$) we get $\chi^2 = 366.0$ (per degree of freedom we have 0.87) which gives not much worse fit. We have also performed other fits, one in which $a_8$ is treated as free parameter and in another we allow gluon distributions $G^+(x) = (G(x) + \Delta G(x))/2$ and $G^-(x) = (G(x) - \Delta G(x))/2$ not to be positively defined. In both cases we have $\Delta G \approx -1.0$ and $\chi^2$ is nearly as good as in the best fit. In all considered models we get negative polarization of gluons (the negative value of gluon polarization one can find also in [5]). The values of gluon polarization (from our best fit) at other $Q^2$ values are: $\Delta G(Q^2 = 5$ GeV$^2$) = $-2.0$ and $\Delta G(Q^2 = 100$ GeV$^2$) = $-3.9$. The variations of our model does not change significantly other quantities (see Table 3).

In figures 1 and 2 we present the shapes of gluon spin distribution obtained in our fits at $Q^2 = 1$ GeV$^2$. One can see that the curves does not differ very much.
Figure 1: The comparison of polarized gluon densities $x\Delta G(x)$ versus $x$ obtained from the fits corresponding to lower gluon (dotted line) central gluon (dashed line) and higher gluon (solid line) solutions from ref.[8].

Figure 2: The comparison of polarized gluon densities $x\Delta G(x)$ versus $x$ obtained from the fits corresponding to the higher gluon (solid line) and the same model with $\alpha_{G_H} = 0$ (dashed line).
We have made fits to precise data on spin asymmetries on proton, neutron and deuteron targets. Our model for polarized parton distributions is based on MRST 1999 fit [8] to unpolarized data. Gluon polarization comes out negative and relatively high. This conclusion does not depend very much on different assumptions (within our framework) about gluon spin distribution.

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