A Novel Viewpoint of Proton Decay

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Abstract

We update the standard model $d=6$ operators of Weinberg (1979) using a modified notation which accommodates the bilepton extension of the standard model. This may lead to an enhancement of the proton lifetime by orders of magnitude due to mixing first-family with third-family quarks. By contrast, $d=5$ operators which can provide Majorana neutrino masses retain the family structure of their counterparts in the standard model.
1 Introduction

Despite the fact that conservation of baryon number $B$ is not a local symmetry, no example of its violation has yet been established. The most readily observed example would be the decay of the proton which was predicted by the earliest grand unified theories (GUTs) invented in the 1970s. In 1974, minimal SU(5) \cite{1} predicted a partial lifetime for the expected dominant decay $p \to e^+\pi^0$ of $\tau_p \sim 10^{30.5}$ y. A decade later, in 1984, experiment refuted this prediction when the Irvine-Michigan-Brookhaven (IMB) collaboration \cite{2} established a lower bound $\tau_p > 10^{32.5}$ y. The present limit for this decay mode, provided by the Super-Kamiokande experiment \cite{3}, is $\tau_p > 10^{34.5}$ y. As an upper limit on $\tau_p$, it is reasonable to use the first estimate of the proton lifetime by Sakharov \cite{4} in 1967 based on gravitational interactions. It was $\tau_p \sim 10^{50}$ y, a lifetime beyond the reach of any terrestrial experiment using present technology.

Before introducing our novel viewpoint, we briefly review proton decay in the minimal SU(5) GUT. This is based on the standard model and the only matter present from the Fermi scale up to the GUT scale are the three quark-lepton families, treated sequentially throughout this large energy range. The renormalisation group equations (RGEs) suggested a value $M_{GUT} \sim 10^{15}$ GeV and a proton lifetime $\tau_p \sim 10^{30.5}$ y. The economy and simplicity of SU(5) was such that a majority of the particle theory community supported it for the decade 1974-84 until the IMB result, already mentioned, slapped it down. Our novel viewpoint will offer two related reasons for why SU(5) underestimated the proton lifetime by at least four orders of magnitude.

A useful analysis of $B$-violating higher-dimensional operators \cite{5} provides a more general approach not tied to SU(5) and we shall make use of it in the present article. SU(5) assumed matter was confined to the three-family standard model. Later we shall assume that the matter representations are enlarged at the TeV scale as in the bilepton model \cite{6} which offers a solution to the family puzzle and in which a $|Q| = |L| = 2$ gauge boson whose existence is subject to present LHC searches. We shall discuss how this could avoid the too-fast proton decay predicted by SU(5).

- The third family is no longer sequential, already at the TeV scale, with the first two, so families must be treated asymmetrically in a GUT.
• The d=6 operators suggest an enhancement factor $E > 10^4$ in proton lifetime because of quark mixing with the third family.

2 Modified Notation

We begin by reviewing the d=6 $|\Delta B| \neq 0$ operators in the standard model using Weinberg’s notation from [5]. For quarks we use

$$q_{iaL} \quad i = 1, 2 \quad SU(2)_L; \quad \alpha = 1, 2, 3 \quad (colour); a = 1, 2, 3 \quad (families). \quad (1)$$

and

$$u_{aaR}; \quad d_{aaR} \quad (2)$$

while, for leptons, we employ

$$l_{iaL} \quad i = 1, 2 \quad (SU(2)_L); \quad a = 1, 2, 3 \quad (families) \quad (3)$$

and

$$l_{aR}. \quad (4)$$

Then there are six d=6 B-violating operators, classified in [5] as follows

$$\mathcal{O}^{(1)}_{abcd} = (\bar{d}^c_{\alpha aR} u_{\beta bR}) (\bar{q}^c_{i\gamma cL} l_{j\delta L}) \epsilon_{\alpha \beta \gamma \epsilon_{ij}},$$

$$\mathcal{O}^{(2)}_{abcd} = (\bar{q}^c_{iaaL} q_{j\beta bL}) (\bar{u}^c_{\kappa cR} l_{\delta R}) \epsilon_{\alpha \beta \gamma \epsilon_{ij}},$$

$$\mathcal{O}^{(3)}_{abcd} = (\bar{q}^c_{iaaL} q_{j\beta bL}) (\bar{q}_{k\gamma cL} l_{\delta L}) \epsilon_{\alpha \beta \gamma \epsilon_{ij} \epsilon_{kl}},$$

$$\mathcal{O}^{(4)}_{abcd} = (\bar{q}^c_{iaaL} q_{j\beta bL}) (\bar{q}_{k\gamma cL} l_{\delta L}) \epsilon_{\alpha \beta \gamma} (\tau \epsilon)_{ij} (\tau \epsilon)_{kl},$$

$$\mathcal{O}^{(5)}_{abcd} = (\bar{d}^c_{\alpha aR} u_{\beta bR}) (\bar{u}^c_{\gamma cR} l_{\delta R}) \epsilon_{\alpha \beta \gamma},$$

$$\mathcal{O}^{(6)}_{abcd} = (\bar{u}^c_{\alpha aR} u_{\beta bR}) (\bar{d}^c_{\gamma cR} l_{\delta R}) \epsilon_{\alpha \beta \gamma}. \quad (5)$$

The well-known selection rule that $(B - L)$ must be conserved follows from Eq. (5), as do several other constraints.

To study the B-violating d=6 operators when the standard model is extended to the bilepton model [6], we introduce a modified notation as follows.

A=1,2 (1st 2 families); C=3 (3rd family); I=1,2,3 $SU(3)_L$

Superscript I denotes triplet and subscript I denotes antitriplet.
Modified notation for quarks

For the first two families we write

\[ q_{IaAL} \quad I = 1, 2, 3 \quad SU(3)_L; \quad \alpha = 1, 2, 3 \quad (\text{colour}); \quad A = 1, 2 \quad (\text{families}). \]  

while for the third family we use

\[ q^I_{\alpha 3L} \quad I = 1, 2, 3 \quad SU(3)_L; \quad \alpha = 1, 2, 3 \quad (\text{colour}) \]  

Finally we add the \( SU(3)_L \) singlets \( u_{aaR}; \quad d_{aaR}. \)

Modified notation for leptons

The three families are here treated sequentially

\[ L_{IaL} \quad I = 1, 2, 3 \quad SU(3)_L; \quad a = 1, 2, 3 \quad (\text{families}). \]  

3 d=6 Operators

Now we reconsider the six operators of Eq.(5). Rewriting the \( O^{(n)}_{abcd} \) in the new notation reveals that the available d=6 operators change significantly due to the new requirement that the operator be singlet under \( SU(3)_L. \)

For example, consider trying to make an operator \( O^{(1)}_{abcd} \) using only the first family. Such an operator \( O^{(1)}_{1111} \) does not exist because of the second parenthesis where the lepton field is in a \( \bar{3} \) of \( SU(3)_L \) as is the quark field. To create an overall \( SU(3)_L \) singlet is possible only if the quark field is in a \( 3 \) of \( SU(3)_L \) which requires it to be in the third family. Thus, \( O^{(1)}_{1131} \) exists, although \( O^{(1)}_{1111} \) does not.

A similar situation occurs, slightly differently, for \( O^{(2)}_{abcd} \). The lepton field is in a \( 3 \) of \( SU(2)_L \) and to make an overall \( SU(3)_L \) singlet the first two quark fields must be \( \bar{3} \times \bar{3} \) which requires the second quark field to be in the third family. Thus \( O^{(2)}_{1311} \) exists but \( O^{(2)}_{1111} \) does not.

The two operators \( O^{(3,4)}_{abcd} \) are similar to each other. The lepton field is in a \( 3 \) of \( SU(3)_L \) while if we stay in the first family the three quark fields are respectively in \( (3,3,3) \) of \( SU(3)_L \) and no singlet is possible. It becomes possible only when either the first or third quark belongs to the third family. Thus, \( O^{(3,4)}_{3111} \) and \( O^{(3,4)}_{1131} \) exist, but not \( O^{(3,4)}_{1111} \).
Finally, the operators $O_{abcd}^{(5,6)}$ do not exist as an $SU(3)_L$ singlet for any choice of families because the three quark fields are singlets and the lepton field is in a $\bar{3}$.

To summarise, in the bilepton model none of the six $O_{abcd}^{(n)}$ of Eq. (5) can form $SU(3)_L$ singlets when all four fermions are in the first or second family. With a quark field in the third family, $n = 1, 2, 3, 4$ do exist, while $n = 5, 6$ are excluded for any family content.

Weinberg also considered $d=5$ terms in the standard model:

$$f_{abmn}\bar{l}_aL_l\phi_k^{(m)}\phi_l^{(n)}\epsilon_{ik}\epsilon_{jl} + f'_{abmn}\bar{l}_aL_l\phi_k^{(m)}\phi_l^{(n)}\epsilon_{ij}\epsilon_{kl}$$

(9)

These terms violate lepton number and can contribute non-zero Majorana neutrino masses.

Unlike the $d=6$ operators discussed above, the transition from the standard model to the bilepton model does not make any essential difference in the family structure for the $d=5$ operators and Majorana neutrino masses can arise in the bilepton model just as discussed for the standard model in [5].

4 Proton decay

The minimal bilepton model [6] does not, to our knowledge, fit into a GUT with a simple gauge group\(^2\). Nevertheless, it is plausible to assume that there exist unknown additional states which permit unification as well as baryon-number-violating intermediaries.

In that case, proton decay will be generated by the $d=6$ operators discussed in the previous section similarly to how the operators in Eqs. (5) generate proton decay in SU(5). In the case of SU(5), as mentioned *ut supra*, the proton lifetime was predicted to be some four orders of magnitude shorter than the present experimental lower limit. Several theoretical attempts have been made to explain this discrepancy by some enhancement factor in the lifetime, although no convincing argument was previously found.

The bilepton model does provide a novel reason for a longer proton lifetime. The $d=6$ operators require the appearance of a third-family quark. To mix with the first family quarks in the proton this will lead, in the decay rate, to suppression by the square of the (13)-element of the CKM quark mixing

\(^2\)No such model is known to us. We cannot prove that such a GUT is impossible.
matrix. This element is of order $\sim \lambda^3$ where $\lambda$ is the sine of the Cabibbo angle $\lambda = \sin \Theta_C \simeq 0.22$. This provides a suppression of $\sim 10^{-2}$ in the matrix element, $\sim 10^{-4}$ in the decay rate and hence an enhancement $E \sim 10^4$ in the proton lifetime.

Since an explicit GUT is not at hand, further discussion necessarily involves an additional assumption. The proton lifetime will be proportional to $E/M_{GUT}^4$ so that, in principle, $E$ can be compensated by a change in $M_{GUT}$ but let us assume, for the moment, that this does not happen and that $M_{GUT}$ is close to its SU(5) value, $M_{GUT} \sim 10^{15}$ GeV. With such an assumption, the proton lifetime would be predicted to be approximately $\tau_p \sim 10^{34.5}$ Y which is at the lower limit provided by Super-Kamiokande and within reach of Hyper-Kamiokande. Proton decay would provide important information about far higher energies than available in colliders.

5 Discussion

The bilepton model [6] gives a special rôle to the third family and this could be reflected by the proton lifetime which is unexpectedly long from the viewpoint of minimal SU(5). At least the assumption of three sequential families $3(10 + \bar{5})$ between the Fermi and GUT scales is invalidated. In the above discussion, we saw a novel reason that may enhance the proton lifetime is that the $d=6$ operators require the costly mixing of first-family with third-family quarks.

Such a crucial rôle of families is reminiscent of the KM mechanism [7] for CP violation which is non-zero only if all the three families participate as shown by the Jarlskog determinant [8].

The appearance of families first seemed, when the muon was discovered in 1936, to provide a major theoretical challenge but now we begin to understand how families can play an elegant part in an emerging theory.
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