Analytical and Experimental Approaches for Controlling Unstable Oscillation by Changing Switching Timing in an Interrupted Electric Circuit

Shota Uchino and Hiroyuki Asahara

Faculty of Engineering, Okayama University of Science
1–1 Ridai-cho, Kita-ku, Okayama-shi, Okayama 700-0005, Japan
E-mail: shota@nonlineargroup.jp, asahara@ee.ous.ac.jp

Abstract

In this paper, we propose a method of controlling an unstable oscillation observed in an interrupted electric circuit with focus on the state-dependent switching action. We define the Poincaré map and calculate the differential forms by linearly differentiating the Poincaré map with respect to the initial values. The control gain is derived from the differential forms, and the validity of the proposed method is confirmed from mathematical and experimental viewpoints. The proposed method focuses on the switching timing and it advances or delays the switching action. Therefore, it makes it easy to realize a control algorithm in a microcomputer.

1. Introduction

Nonlinear phenomena can be observed in many power electric circuits with switching devices, because the switching action causes nonlinearity in the circuit dynamics. These switching circuits are categorized as interrupted electric circuits. In particular, it is well known that a feedback controller makes the circuit dynamics complex, and their rich nonlinear phenomena have been studied over the past few decades [1, 2].

The circuit behavior, i.e., inductor current or capacitor voltages, changes qualitatively with the circuit parameter, a phenomenon called bifurcation. Analyzing bifurcation is important for understanding the qualitative characteristic of a circuit in a wide parameter space. There are many studies that have analyzed bifurcation to develop circuit and bifurcation theories. On the other hand, controlling bifurcation is also important from an engineering viewpoint, because switching ripple often increases through bifurcation.

There are many papers that deal with control methods for reducing the switching ripple in an interrupted electric circuit. In particular, some methods focus on unstable oscillations, and a control method based on the Poincaré map has been proposed. Most of these methods required differential forms obtained by linearly differentiating the Poincaré map with respect to the initial and parameter values. For example, as indicated in Refs. [3, 4, 5] the DC power supply, the threshold in the controller, and the switching frequency were used as control parameters, respectively.

On the other hand, here we do not use the circuit parameters but focus on the switching timing. We reported the basic idea of this algorithm in Ref. [6]. However, the detailed discussion and experimental realization were insufficient. This paper rigorously describes the control method, which is confirmed experimentally. We advance or delay the switching timing to control unstable oscillations. This approach makes it easy to realize a control algorithm in a microcomputer.

2. System Description and Definition of the Control Gain

We consider a one-dimensional interrupted dynamical system and explain the method of controlling unstable oscillations. Assume that the system has two subsystems, which switch from one to the other depending on their state and periodic interval. The dynamic behavior of the system is described as

$$\frac{dx}{dt} = \begin{cases} f_1(x, \lambda_1) & \text{subsystem 1} \\ f_2(x, \lambda_2) & \text{subsystem 2} \end{cases}$$

where $\lambda_1$ and $\lambda_2$ are the parameters in subsystems 1 and 2, respectively. Assume that the clock pulse appears at every period of $T$, and $x_0$ is the initial value. Let the solution of Eq. (1) be

$$x(t) = \varphi(t, x, \lambda) = \begin{cases} \varphi_1(t, x_0, \lambda_1) & \text{subsystem 1} \\ \varphi_2(t, x_0, \lambda_2) & \text{subsystem 2} \end{cases}$$

Figure 1 shows the waveform behavior of the system. If the waveform $x(t)$ reaches the threshold $x_{\text{ref}}$, subsystem 1 switches to subsystem 2. Subsequently, the clock pulse is applied and subsystem 2 switches to subsystem 1. Note that the clock pulse is ignored if it appears while the system is in subsystem 1.

Let the waveform of period-1 be

$$\varphi(T, x_k, \lambda) - \varphi(0, x_k, \lambda) = 0$$

where $x_k$ denotes the initial value at $t = kT$.  

SELECTED PAPER AT NCSP’20
Next, we explain the stability of the period-1 waveform as an example. Figure 2 shows the period-1 and perturbed waveforms, where we use the symbol ∗ for the period-1 waveform. Let the perturbation of the waveform at $t = kT$ be $\Delta x_k$, where

$$\Delta x_k = x_k - x_k^*$$

(4)

Likewise, assume that the perturbation at $t = (k + 1)T$ is

$$\Delta x_{k+1} = x_{k+1} - x_{k+1}^*$$

(5)

Therefore, the waveform $x(t)$ is stable when $\Delta x_{k+1}/\Delta x_k < 1$. On the other hand, the waveform $x(t)$ is unstable when $\Delta x_{k+1}/\Delta x_k > 1$. It is our control target.

In the following, we discuss the control method. Here, we focus on the black waveform in Fig. 3, which is the perturbed waveform. We assume that the switching timing of the perturbed waveform is advanced or delayed by $\Delta t_1$, where

$$\Delta t_1 = g\Delta x_k$$

(6)

and $g$ is the control gain. For example, we assume that the switching timing of the perturbed waveform is delayed by $\Delta t_1$, and subsystem 1 switches to subsystem 2. In this case, the perturbed waveform is shown by the dotted gray waveform, where $x_{k+1}'$ is the solution at $t = (k + 1)T$. Moreover, let the perturbations of the waveform after applying the control method at $t = (k + 1)T$ be $\Delta x_{k+1}$ and $\Delta x_k$, as shown in Fig. 3. These perturbations are described as

$$\Delta x_{k+1} = x_{k+1} - x_{k+1}^*$$

(7)

and

$$\Delta x_k = x_k - x_k^*$$

(8)

Here, Eq. (8) can be redefined using $\Delta x_{k+1}$ and $\Delta x_k$ as

$$\Delta x_k = \Delta x_{k+1} + \Delta x_k$$

(9)

Eq. (9) is subsequently rewritten as

$$\Delta x_k = \Delta x_{k+1} + \Delta x_k$$

$$= \frac{\Delta x_{k+1}}{\Delta x_k} \Delta x_k + g\frac{\Delta x_{k+1}}{\Delta x_k} \Delta x_k$$

$$= \left(1 + g\frac{\Delta x_{k+1}}{\Delta x_k}\right) \Delta x_k$$

(10)

Let $\mu$ be the characteristic multiplier, which is described as

$$\frac{\Delta x_{k+1}}{\Delta x_k} = \mu$$

(11)

Substituting Eq. (11) into Eq. (10), we obtain

$$\frac{\Delta x_{k+1}}{\Delta x_k} + g\frac{\Delta x_{k+1}}{\Delta x_k} - \mu = 0$$

(12)

Because $x_{k+1}'$ overlaps with $x_{k+1}^*$ at $t = (k + 1)T$ when $\mu = 0$ is satisfied, the control gain $g$ can be given by

$$g = \frac{\frac{\Delta x_{k+1}}{\Delta x_k}}{\frac{\Delta x_{k+1}}{\Delta x_k} - \mu} \approx \frac{d x_{k+1}}{d t_1} \Delta x_k$$

(13)

Therefore, the control signal $\Delta t_1$ can be obtained from Eqs. (6) and (13).

3. Example of Application

We apply the control method to an interrupted electric circuit to confirm its validity. Figure 4 shows an interrupted
electric circuit. That simulates the switching action of a current-controlled DC-DC converter [7]. The position of the switch depends on the capacitor voltage and the appearance of the clock pulse. If the capacitor voltage \( v \) reaches the reference voltage \( v_{\text{ref}} \), the position of the switch changes from 1 to 2. Subsequently, the switch moves to position 2 when the clock pulse appears at every period of \( T \). The switching action is controlled by Arduino, and we realize the control method using Arduino.

The circuit parameters are fixed as:

\[
R = 1.0[\text{M} \Omega], \quad C = 1.0[\mu \text{F}], \quad E = 3.0[\text{V}], \quad T = 0.606[\text{s}]
\]

Moreover, the circuit equation is given by

\[
RC \frac{dv}{dt} = \begin{cases} -v + E & \text{switch 1} \\ -v & \text{switch 2} \end{cases}
\]

We use the dimensionless variables

\[
\tau = \frac{t}{RC}, \quad T' = \frac{T}{RC}, \quad x = v, \quad x_{\text{ref}} = v_{\text{ref}}
\]

Therefore, Eq. (15) is rewritten as

\[
\frac{dx}{d\tau} = \begin{cases} -x + E & \text{switch 1} \\ -x & \text{switch 2} \end{cases}
\]

In the following, we rewrite \( \tau \) and \( T' \) as \( t \) and \( T \), respectively, for simplicity. Therefore, the solution of Eq. (17) is derived as

\[
x(t) = \begin{cases} (x_0 - E)e^{-t} + E & \text{switch 1} \\ x_0e^{-t} & \text{switch 2} \end{cases}
\]

where \( x_0 \) is an initial value.

Assume the threshold that classifies the waveform behavior during the clock interval as

\[
D = (x_{\text{ref}} - E)e^{-T} + E
\]

The switch stays at position 1 if the initial \( x_k \) is smaller than the threshold \( D \). On the other hand, switching occurs if \( D < x_k \) is satisfied.

The Poincaré map for \( D < x_k \) is defined as

\[
x_{k+1} = x_k - E \frac{x_k - E}{x_{\text{ref}} - E} e^{-T}
\]

Therefore, the fixed point \( x_k^* \) can be described as

\[
x_k^* = \frac{x_{\text{ref}}Ee^{-T}}{x_{\text{ref}}(e^{-T} - 1) + E}
\]

The differential forms obtained by linearly differentiating the Poincaré map with respect to the initial and parameter values are given by

\[
\frac{dx_{k+1}}{dx_k} = \frac{x_{\text{ref}}}{x_{\text{ref}} - E} e^{-T}
\]

and

\[
\frac{dx_{k+1}}{dT} = x_{\text{ref}} e^{-(T - t)}
\]

The control gain is expressed as

\[
g = -\frac{1}{x_k - E} = \frac{1}{x_{\text{ref}} - E}
\]

Therefore, the perturbation of the switching timing is defined as

\[
\Delta t = -\frac{x_{\text{ref}}e^{-T} (x_k - E) + x_k (E - x_{\text{ref}})}{(x_k - E)(x_{\text{ref}} - E)(e^{-T} - 1) + E}
\]

Figures 5 and 6 respectively show the analytical and experimental results. We start to apply the control method at \( t = 4.85 \) in the analytical results. It is clear that the period-2, non-periodic, and period-3 oscillations are controlled to the unstable period-1 oscillation (see Fig. 5). The corresponding experimental results shown in Fig. 6 verify the analytical results.

The circuit model shown in Fig. 4 simulates the switching action of the current-mode controlled DC-DC converters. Therefore, we consider that the proposed method can be applied to this type of circuit under digital control.

4. Conclusion

We focused on the control of an unstable oscillation observed in an interrupted electric circuit both mathematically and experimentally. We focused on the state-dependent switching event and advanced or delayed the switching timing. This approach makes it easy to realize a control algorithm in a microcomputer. In the future, we will modify the method of controlling a high-dimensional interrupted dynamical system.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Numbers 18K18126 and 19H00772.
Figure 5: Analytical results

References

[1] S. Banerjee and G. C. Verghese: Nonlinear Phenomena in Power Electronics —Attractors, Bifurcations, Chaos, and Nonlinear Control, IEEE Press, 2001.

[2] C. K. Tse: Complex Behavior of Switching Power Converters, CRC Press, 2003.

[3] T. Kousaka, T. Ueta and H. Kawakami: Controlling chaos in a state-dependent nonlinear system, International Journal of Bifurcation and Chaos, Vol. 12, No. 5, pp. 1111-1119, 2002.

[4] D. Ito, T. Ueta, T. Kousaka, J. Imura and K. Aihara: Controlling chaos of hybrid systems by variable threshold values, International Journal of Bifurcation and Chaos, Vol. 24, No. 10, pp. 1450125-1-1450125-12, 2014.

[5] D. Ito, H. Asahara, T. Kousaka and T. Ueta: Clock pulse modulation for ripple reduction in buck-converter circuits, Chaos, Solitons and Fractals, Vol. 111, pp. 138-145, 2018.

[6] T. Sasada, D. Ito, T. Ueta, H. Ohtagaki, T. Kousaka and H. Asahara: Controlling unstable orbits via varying switching time in a simple hybrid dynamical systems, Proc. of 2015 International Symposium on Nonlinear Theory and Its Applications, pp. 475-478, 2015.

[7] T. Kousaka, T. Ueta, S. Tahara, H. Kawakami and M. Abe: Implementation and analysis of a simple circuit causing border-collision bifurcation, IEEJ Transactions on Electronics, Information and Systems, Vol. 122, No. 11, pp. 1908-1916, 2002.