X(5568) as a $su \bar{d}b$ tetraquark in a simple quark model

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Abstract

The $S$-wave eigenstates of tetraquarks of type $su \bar{d}b$ with $J^P = 0^+, 1^+$ and $2^+$ are studied within a simple quark model with chromomagnetic interaction and effective quark masses extracted from meson and baryon spectra. It is tempting to see if this spectrum can accommodate the new narrow structure X(5568), observed by the DØ Collaboration, but not confirmed by the LHCb Collaboration. If it exists, such a tetraquark is a system with four different flavors and its study can improve our understanding of multiquark systems. The presently calculated mass of X(5568) agrees quite well with the experimental value of the DØ Collaboration. Predictions are also made for the spectrum of the charmed partner $su \bar{d}c$. However we are aware of the difficulty of extracting effective quark masses, from mesons and baryons, to be used in multiquark systems.

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I. INTRODUCTION

The DØ Collaboration [1] has recently observed a narrow structure named \(X(5568)\) in the \(B_0^\pm\pi^\mp\) invariant mass spectrum with 5.1\(\sigma\) significance based on 10.4 fb\(^{-1}\) of \(p\bar{p}\) collisions data at \(\sqrt{s} = 1.96\) TeV. Its measured mass and width are \(M = 5567.8 \pm 2.9\) (stat)\(^{+0.9}_{-1.9}\) (syst) MeV and \(\Gamma = 21.9 \pm 6.4\) (stat)\(^{+5.0}_{-2.5}\) (syst) MeV, respectively. Its decay into the final state \(B_0^\pm\pi^\mp\) suggests that \(X(5568)\) could be a \(s\bar{u}\bar{d}\bar{b}\) (or \(s\bar{d}\bar{b}\bar{u}\)) tetraquark with four different flavors, among which one is heavy. The DØ Collaboration suggests that, with \(B_0^\pm\pi^\mp\) produced in an \(S\)-wave, the quantum numbers of \(X(5568)\) should be \(J^P = 0^+\) and that the resonance may be the heavy analogue of the isotriplet scalar \(a(980)\) with an \(s\) quark replaced by a \(b\) quark.

Shortly after the DØ Collaboration observation a search for the claimed \(X(5568)\) was performed by the LHCb Collaboration in \(pp\) collision data at \(\sqrt{s} = 7\) and 8 TeV, where no significant excess was found to confirm the existence of \(X(5568)\) [2]. More data are needed to cover a larger mass range and more decay channels in order to determine the properties of the \(s\bar{u}\bar{d}\bar{b}\) tetraquark, if it exists.

An advantage of studying a tetraquark with four different flavors is that there are no annihilation processes like in \(c\bar{c}q\bar{q}\) systems \((q = u,d,s)\). These are difficult to deal with theoretically.

The DØ Collaboration observation immediately stimulated theoretical interest. So far, approaches based on QCD sum rules [3–10] quark models [11–13] or rescattering effects [14] have been adopted. An SU(3) classification has also been made [15]. In Refs. [3–5] scalar tetraquarks were studied while Ref. [7] considered both scalar and axial tetraquark. Within quark models [11, 12] scalar, axial and tensor tetraquarks were analysed.

An incentive to studying tetraquarks can be found in a recent paper by Weinberg [16] based on large \(N_c\) QCD. It is quite natural to inquire about the existence of exotics at large \(N_c\) inasmuch as the \(1/N_c\) expansion method proposed by \(‘t\) Hooft [17] has been very successful for ordinary hadrons. According to Weinberg exotic mesons consisting of two quarks and two antiquarks are not ruled out in large \(N_c\) QCD. The real question is the decay rate of a tetraquark. The suggestion has been followed in subsequent papers as, for example, in Refs. [18–21].

For simplicity, in this work, we use the schematic model of Refs. [22, 24] based on the chromomagnetic interaction between quarks (antiquarks). In Ref. [22] it was shown that
X(3872) can be interpreted as a $c\bar{c}q\bar{q}$ tetraquark where the lowest $1^{++}$ state has a dominant color octet-octet component (0.9997) and a very small color singlet-singlet component (0.026) which may explain why this state decays with a very small width into $J/\psi + \rho$ or $J/\psi + \omega$, in agreement with the experimental value for the total width $\Gamma < 2.3$ MeV of X(3872) [25]. The analysis has been extended to $0^{++}$ and $2^{++}$ sectors and the problem of extraction the chromomagnetic strength from meson and baryons to be used in tetraquarks has been extensively discussed in Ref. [23]. The study of the entire spectrum of the $c\bar{s}s\bar{s}$ system in view of the interpretation of the Y(4140) resonance as a tetraquark has been later performed in Ref. [26] within the same model. The suggestion was that Y(4140) could be the strange partner of X(3872), because the corresponding wave function also has a very small color singlet-singlet component which may explain a narrow width in the $J/\psi + \phi$ channel. The model of Refs. [22, 23] has also been considered in Ref. [27].

The paper is organized as follows. In Sec. II we introduce the quark model used in this study. In Sec. III we recall the basis states in the diquark picture, the direct meson-meson channel useful for $B_0^s\pi^\pm$ decays and the exchange meson-meson channel useful for $B^+\pi^0$ decays.

In Sec. IV we shortly introduce the Hamiltonian matrix of the model, followed by the analytic forms we have obtained for the hyperfine interaction for $J^P = 0^+, 1^+$ and $2^+$ states in Secs. V, VI and VII respectively. In Sec. VIII we exhibit the spectrum and the structure of the lowest eigenstates of the tetraquark system $su\bar{d}\bar{b}$ resulting from the diagonalization of the matrices obtained for the chromomagnetic interaction. In Sec. IX we briefly present the spectrum of the $su\bar{d}\bar{c}$ tetraquark. The last section is devoted to conclusions.

II. THE MODEL

This study is based on the simple model of Refs. [22, 24] which can reveal the gross features of a tetraquark, in particular the structure of the wave functions.

In the next section we introduce the relevant basis states in the color-spin space, including both the color singlet-singlet and the octet-octet channels, the latter being called hidden color channels. There are no correlated quarks or diquarks, as in the diquark-antidiquark picture [28, 29].

According to Refs. [22, 24] the mass of a tetraquark is given by the expectation value of
the effective Hamiltonian

\[ H = \sum_i m_i + H_{\text{CM}}, \]  

(1)

where the chromomagnetic hyperfine interaction \( H_{\text{CM}} \) is described by

\[ H_{\text{CM}} = -\sum_{i,j} C_{ij} \lambda_i^c \cdot \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j. \]

(2)

The first term in Eq. (1) contains the effective quark masses \( m_i \) as parameters. The constants \( C_{ij} \) in (2) represent integrals in the orbital space of some unspecified radial forms of the chromomagnetic part of the one gluon-exchange interaction potential and of the wave functions.

A warning should be given to the way of determining the effective masses \( m_i \) to be used for multiquark systems. Besides the kinetic energy contribution, they incorporate the effect of the confinement, which is still an open problem [30]. Information from lattice calculations on tetraquarks (see i.e. [31, 32]) may lead to a better understanding of the effective masses to be used in simple models.

Presently, we use the compromise proposed in Refs. [22, 23] for \( m_q \) and \( m_s \), while for \( m_b \) we rely on a value compatible with heavy-light systems [24]. Therefore we have

\[ m_{u,d} = 320 \text{ MeV}, \quad m_s = 590 \text{ MeV}, \quad m_b = 4860 \text{ MeV}. \]

(3)

Due to the arbitrariness in the choice of effective masses of quarks, precise estimates of the absolute values of tetraquark masses is difficult to make. One can have an approximate idea about the range where the spectrum should be located. We stress that in this study we favor \( m_{u,d} = 320 \text{ MeV} \) instead of 450 MeV of Ref. [24] because this value brings the \( \rho \) and the \( \pi \) masses close to experiment (see below).

However, the relative distances between the eigenstates obtained from the chromomagnetic Hamiltonian (2) and the structure of its eigenstates do not depend on the effective masses, which is important for exploring the strong decay properties.

The parameters \( C_{ij} \) have been taken from Ref. [23], Table 1 for \( C_{q\bar{q}} \) and Table 2 for \( C_{qq} \) (\( q = u, d \)). The parameter \( C_{\bar{d}b} \), absent in Ref. [23], was taken identical to \( C_{db} \) of Ref. [24]. Therefore we have

\[ C_{ud} = 29.8 \text{ MeV}, \quad C_{sd} = 18.4 \text{ MeV}, \quad C_{us} = 13 \text{ MeV}, \]
\[ C_{\bar{u}b} = 2.1 \text{ MeV}, \quad C_{\bar{d}b} = 1.9 \text{ MeV}, \quad C_{sb} = 2.2 \text{ MeV}. \]

(4)
We should mention that the above parameters were extracted from a global fit to meson and baryon ground states. With these parameters the pion acquires a mass of 163 MeV and the $\rho$ meson 799 MeV, the experimental values being approximately 140 MeV and 770 MeV respectively.

As one can infer from these values, most of the hyperfine attraction will come from light $qq$ or $q\bar{q}$ pairs, as expected.

In Ref. [12] the same type of Hamiltonian has been used with hyperfine interaction parameters $C_{ij}$ obtained from a fit to light or heavy baryons and $B$ and $D$ mesons. The comparison of the values of the relevant common parameters of Ref. [12] with those of Ref. [23] shows that they are very close to each other, except for $C_{us}$ which is 8.8 MeV in Table VI Ref. [12] (for consistency with the present definition the values of Table VI Ref. [12] should be multiplied by a factor 3/32). In fact $C_{us} = 13$ MeV of Eq. (4) is the central value of this parameter varying between 12 MeV and 14 MeV in Ref. [23]. Taking $C_{us} = 8.8$ MeV (which fits well the $\Sigma^* - \Sigma$ splitting [12]), instead of 13 MeV, increases the mass of the lowest state by only 10 MeV.

However the most important parameter (see next sections), namely $C_{ud}$, is missing in Ref. [12]. The lack of $C_{ud}$ from Table VI Ref. [12] was justified by the fact that pseudoscalar mesons are influenced by chiral symmetry and its spontaneous breaking. By using a triquark-heavy antiquark basis states the masses of tetraquarks were calculated by avoiding this parameter, as any other $C_{ij}$ parameter of the chromomagnetic interaction between quarks and antiquarks, such contribution being claimed to vanish in the most significant case. In addition, the mixing of the four basis states constructed from the triquark-heavy antiquark basis was neglected which made an extra loss in the contribution of the hyperfine interaction. In our case $C_{ud}$ of Eq. (4) gives a $\rho - \pi$ splitting of 635.7 MeV as compared to the experimental value of 630 MeV.

We do not intend to make a fine tuning of the effective masses. We are mostly interested in the structure of the tetraquark wave functions which essentially depends on the hyperfine interaction as well as on the level sequence. We shall compare the calculated spectrum to the experimental thresholds.
FIG. 1: Three independent relative coordinate systems. Solid and open circles represent quarks and antiquarks respectively: (a) diquark-antidiquark channel, (b) direct meson-meson channel, (c) exchange meson-meson channel.

III. THE BASIS STATES

Here we use a basis vectors relevant for understanding the decay properties of tetraquarks. The total wave function of a tetraquark is a linear combination of these basis vectors. We suppose that particles 1 and 2 are quarks and particles 3 and 4 are antiquarks, see Fig. 1. Presently we take 1 = u, 2 = s, 3 = d and 4 = b.

In principle the basis vectors should contain the orbital, color, flavor and spin degrees of freedom such as to account for the Pauli principle. But, as we consider ℓ = 0 states the orbital part is symmetric and anyhow irrelevant for the effective Hamiltonian described in the previous section. Moreover, as the flavor operators do not explicitly appear in the Hamiltonian, the flavor part does not need to be specified. A detailed description of each of the three distinct bases corresponding to three choices of internal coordinates, shown in Fig. 1, was presented in Refs. [34, 35]. One can use any base, (a), (b) or (c).

In the color space the three distinct bases are: a) $|\mathbf{3}_{12}\mathbf{3}_{34}\rangle$, $|6_{12}\mathbf{6}_{34}\rangle$, b) $|1_{13}\mathbf{1}_{24}\rangle$, $|8_{13}\mathbf{8}_{24}\rangle$, and c) $|1_{14}\mathbf{1}_{23}\rangle$, $|8_{14}\mathbf{8}_{23}\rangle$, associated to the three distinct internal coordinate systems shown in Fig. 1. The 3 and $\mathbf{3}$ are antisymmetric and 6 and $\mathbf{6}$ are symmetric under interchange of quarks and antiquarks respectively. This basis is convenient for diquark-antidiquark models, where the color space is truncated to contain only $|\mathbf{3}_{12}\mathbf{3}_{34}\rangle$ states [28, 29]. This reduces each
\( J^{PC} \) spectrum to twice less states than allowed by the Pauli principle \([33]\) and influences the tetraquark properties. In the present context an example is Ref. \([11]\) where, although mixing across the basis states was allowed, only the color state \( |\bar{3}_{12}3_{34}\rangle \) was considered which may partly explain why the contribution of the hyperfine interaction is so small.

The sets \( b) \) and \( c) \) both contain a color singlet-singlet and a color octet-octet state. The amplitude of the latter vanishes asymptotically, when the mesons, into which a tetraquark decays, separate. These are called hidden color states by analogy to states which appear in the nucleon-nucleon problem, defined as a six-quark system \([36, 37]\). The contribution of hidden color states to the binding energy of light tetraquarks has been calculated explicitly in Ref. \([34]\). The coordinate sets \( b) \) and \( c) \) define the direct and the exchange meson-meson channels. The relation between the three different bases can be found in Ref. \([35]\).

As the quarks and antiquarks are spin 1/2 particles the total spin of a tetraquark can be \( S = 0 \), \( S = 1 \) or \( S = 2 \). In the following we shall use the notation \( P \) and \( V \) for pseudoscalar and vector subsystems respectively.

IV. MATRIX ELEMENTS

As already stressed, the Hamiltonian \([1]\) does not contain flavor operators so that the flavor part of the wave function does not need to be specified. Then the quantum numbers of the states can be defined in terms of the permutation properties of the spin and color parts of the basis vectors, as shown in Ref. \([38]\). The basis vectors can be written such as to have a good charge conjugation quantum number (see Appendix \( A \)). In particular, for two identical flavors, a ground state tetraquark can have \( J^{PC} = 0^{++}, 1^{++}, 1^{+-} \) and \( 2^{++} \). For four distinct flavors the situation is more complicated, as it will be seen below.

In the direct meson-meson channel, in each case a basis can be built with the quark-antiquark pairs \((1,3)\) and \((2,4)\) as subsystems, where each subsystem has a well defined color state, a singlet-singlet or an octet-octet. The other quark-antiquark pairs, \((1,4)\) and \((2,3)\) are needed to study the meson-meson exchange channels (see Fig. 1c). One can fix a basis in terms of the problem one looks at, but for convenience, in the calculations one can pass from one basis to another by an orthogonal transformation.
V. THE $J^P = 0^+$ STATES

To study the $J^P = 0^+$ spectrum we can use a basis constructed from products of color and spin states associated to Fig. 1b

$$\psi_{10^+}^1 = |1_{13}1_{24}P_{13}P_{24}\rangle, \psi_{0^+}^2 = |1_{13}1_{24}(V_{13}V_{24})_0\rangle,$$
$$\psi_{0^+}^3 = |8_{13}8_{24}P_{13}P_{24}\rangle, \psi_{0^+}^4 = |8_{13}8_{24}(V_{13}V_{24})_0\rangle. \tag{5}$$

The chromomagnetic interaction Hamiltonian with minus sign, $-H_{\text{CM}}$, acting on this basis leads to the following symmetric matrix

$$
\begin{pmatrix}
16(C_{13} + C_{24}) & 0 & 0 & -4\sqrt{2}\left(C_{12} + C_{23} + C_{14} + C_{34}\right) \\
-\frac{16}{3}(C_{13} + C_{24}) & -4\sqrt{3}\left(C_{12} + C_{23} + C_{14} + C_{34}\right) & \frac{8\sqrt{2}}{3}(C_{23} - C_{12} + C_{14} - C_{34}) & -2(C_{13} + C_{24}) \\
0 & \frac{8\sqrt{2}}{3}(C_{23} - C_{12} + C_{14} - C_{34}) & \frac{2}{\sqrt{3}}[2(C_{12} + C_{34}) - 7(C_{23} + C_{14})] & \frac{8}{3}(C_{12} + C_{34}) + \frac{28}{3}(C_{14} + C_{23}) + \frac{2}{3}(C_{13} + C_{24}) \\
0 & -2(C_{13} + C_{24}) & \frac{8}{3}(C_{12} + C_{34}) + \frac{28}{3}(C_{14} + C_{23}) + \frac{2}{3}(C_{13} + C_{24}) & \frac{8}{3}(C_{12} + C_{34}) + \frac{28}{3}(C_{14} + C_{23}) + \frac{2}{3}(C_{13} + C_{24})
\end{pmatrix} \tag{6}
$$

In the case of two distinct flavors, for example $s\bar{s}c\bar{c}$ systems, the following equalities hold

$$C_{14} = C_{23}, \quad C_{12} = C_{34}, \tag{7}$$
in which case the above matrix takes a simpler form, to be found in Ref. [26]. Note that the matrix element row 1 column 4 of $-H_{\text{CM}}$ has a correct - sign, a misprint has to be corrected in Ref. [26].

VI. THE $J^P = 1^+$ STATES

To study the $J^P = 1^+$ spectrum we find useful to start from a basis constructed from products of color and spin states associated to Fig. 1a (diquark basis). These are

$$\psi_{1+}^1 = |6_{12}\bar{6}_{34}(V_{12}V_{34})_{1}\rangle, \psi_{1+}^2 = |3_{12}3_{34}(P_{12}V_{34})_{1}\rangle,$$
$$\psi_{1+}^3 = |3_{12}3_{34}(P_{12}V_{34})_{1}\rangle, \psi_{0+}^4 = |6_{12}\bar{6}_{34}(V_{12}P_{34})_{1}\rangle,$$
$$\psi_{1+}^5 = |3_{12}3_{34}(V_{12}P_{34})_{1}\rangle, \psi_{0+}^6 = |6_{12}\bar{6}_{34}(P_{12}V_{34})_{1}\rangle. \tag{8}$$
Using this basis we now construct a new basis where every vector has a definite charge conjugation, following the observation made in the Appendix. They are defined as the linear combinations

\[ \psi_{1}^{1} = \psi_{1}^{1}, \quad \psi_{1}^{2} = \psi_{1}^{2}, \]
\[ \psi_{1}^{3} = \frac{1}{\sqrt{2}}(\psi_{3}^{1} - \psi_{5}^{1}), \quad \psi_{1}^{4} = \frac{1}{\sqrt{2}}(\psi_{4}^{1} - \psi_{6}^{1}), \]
\[ \psi_{1}^{5} = \frac{1}{\sqrt{2}}(\psi_{3}^{1} + \psi_{5}^{1}), \quad \psi_{1}^{6} = \frac{1}{\sqrt{2}}(\psi_{4}^{1} + \psi_{6}^{1}). \] (9)

In this basis the matrix of \(-H_{CM}\) for \(J^{+} = 1^{+}\) has the form

\[
\begin{pmatrix}
D_{11} & 2\sqrt{2}(C_{13} + C_{24} - C_{14} - C_{23}) & 4\sqrt{2}(C_{13} - C_{24}) & -20/3(C_{13} - C_{24}) & -4\sqrt{2}(C_{14} - C_{23}) & 20/3(C_{14} - C_{23}) \\
D_{22} & 8/3(C_{13} - C_{24}) & -4\sqrt{2}(C_{13} - C_{24}) & 8/3(C_{14} - C_{23}) & -4\sqrt{2}(C_{14} - C_{23}) & 16/3(C_{12} - C_{34}) & 0 \\
D_{33} & -2\sqrt{2}(C_{13} + C_{14} + C_{23} + C_{24}) & 16/3(C_{12} - C_{34}) & 0 & 8/3(C_{12} - C_{34}) & -2\sqrt{2}(C_{13} + C_{14} + C_{23} + C_{24}) \\
D_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{55} & -2\sqrt{2}(C_{13} + C_{14} + C_{23} + C_{24}) & 0 & 0 & 0 & 0 & 0 \\
D_{66} & -4\sqrt{2}(C_{13} + C_{24}) & -4\sqrt{2}(C_{13} - C_{24}) & -4\sqrt{2}(C_{14} - C_{23}) & -4\sqrt{2}(C_{14} - C_{23}) & -4\sqrt{2}(C_{14} - C_{23}) & -4\sqrt{2}(C_{14} - C_{23}) \\
\end{pmatrix}
\] (10)

with

\[
D_{11} = \frac{4}{3}(C_{12} + C_{34}) + \frac{10}{3}(C_{13} + C_{14} + C_{23} + C_{24}) \\
D_{22} = -\frac{8}{3}(C_{12} + C_{34}) + \frac{4}{3}(C_{13} + C_{14} + C_{23} + C_{24}) \\
D_{33} = \frac{8}{3}(C_{12} + C_{34}) + \frac{4}{3}(C_{13} - C_{14} - C_{23} + C_{24}) \\
D_{44} = -\frac{4}{3}(C_{12} + C_{34}) + \frac{10}{3}(C_{13} - C_{14} - C_{23} + C_{24}) \\
D_{55} = \frac{8}{3}(C_{12} + C_{34}) - \frac{4}{3}(C_{13} - C_{14} - C_{23} + C_{24}) \\
D_{66} = -\frac{4}{3}(C_{12} + C_{34}) - \frac{10}{3}(C_{13} - C_{14} - C_{23} + C_{24}) \] (11)

One can see again that for tetraquarks with two flavors, where the relations \([\text{7}]\) remain valid, the above matrix takes a quasidiagonal form, with one block \(4 \times 4\) for states with \(C = -1\) and a \(2 \times 2\) block for states with \(C = +1\) respectively. For \(s\bar{s}c\bar{c}\) systems, for example, their eigenvalues recover the \(1^{+-}\) and \(1^{++}\) spectra of Fig. 2 Ref. \([26]\), obtained here in another basis. The advantage of using the basis \([\text{9}]\) is that one can have a control
of the wave function components with a specific charge conjugation and this may be useful
in processes where the charge conjugation quantum number is conserved.

VII. THE J\(^P\) = 2\(^+\) STATES

For tensor tetraquark one can use a basis of color and spin states corresponding to Fig.
1b. This is

\[ \psi_{1++}^1 = |1_{13}1_{24}(V_{13}V_{24})_2\rangle, \quad \psi_{2++}^2 = |\bar{8}_{13}2_{24}(V_{13}V_{24})_2\rangle. \]  \(12\)

The corresponding \(-H_{CM}\) 2×2 matrix is

\[
\begin{pmatrix}
-\frac{16}{3} (C_{13} + C_{24}) & \frac{4\sqrt{2}}{3} (C_{12} + C_{34} - C_{14} - C_{23}) \\
-\frac{2}{3} (2C_{12} + 2C_{34} + 7C_{23} + 7C_{14} - C_{13} - C_{24}) & -\frac{16}{3} (C_{13} + C_{24})
\end{pmatrix}
\]  \(13\)

where one can use the relations (7) to recover the result for two flavors of Ref. [26].

We note that the above matrices can be used in any quark model containing a chromomagnetic interaction. In that case the parameters \(C_{ij}\) should be replaced by integrals containing the chosen form factor of the chromomagnetic interaction and the orbital wave functions of the model.

The matrices (6), (10) and (13) can be used to calculate the spectrum of either \(sud\bar{b}\) or its charm partner \(sud\bar{c}\) by implementing the corresponding parameters \(C_{ij}\). Below we shall show results for \(sud\bar{b}\) and in the next section we shall shortly describe the spectrum of \(sud\bar{c}\).

VIII. THE SPECTRUM OF \(sud\bar{b}\)

The calculated spectrum of the \(sud\bar{b}\) tetraquark is exhibited in Fig. 2. One can see that
the choice of the effective masses (3) and of the hyperfine interaction parameters (4) is quite
adequate, giving for the lowest state a mass of 5530 MeV, close to the observed value by
the DØ Collaboration [1], with a hyperfine contribution of - 560 MeV. Note that the state
\(\psi_{0+}^1\) alone gives - 512 MeV to this binding, as one can see from the first diagonal matrix.
element of Eq. (6), namely $16(C_{13} + C_{24})$, with the parameters of Eq. (4). This proves that the hyperfine contribution of the $u\bar{d}$ pair is dominant in the system which means that one cannot neglect it.

The hyperfine interaction is still attractive for $J^P = 1^+$ contributing with $-502$ MeV. Thus the lowest $J^P = 0^+$ and $J^P = 1^+$ are not degenerate, contrary to the diquark-antidiquark model [11].

However in the heavy quark limit, which can be simulated by taking $C_{i4} = 0$, which follows from $C_{i4} \propto 1/m_{\text{heavy}} \rightarrow 0$ when $m_{\text{heavy}} \rightarrow \infty$, one obtains degeneracy as follows.
The spectrum of $0^+$ coincides with the first, second, third and sixth states of $1^+$, with eigenvalues 5573 MeV, 5851 MeV, 6133 MeV and 6319 MeV respectively and the spectrum of $2^+$ coincides with the fourth and fifth states of $1^+$, having as eigenvalues 6172 MeV and 6250 MeV respectively.

The hyperfine interaction is repulsive for $J^P = 2^+$ states rising the pure mass term from 6090 MeV to 6183 MeV for the lowest state, as one can see from Fig. 2.

In the basis (5) the lowest $J^P = 0^+$ state has the amplitudes

$$(-0.9193, 0.0246, -0.0852, 0.3833)$$

The first number implies that this state can decay substantially into a PP channel, i.e. $B_s^0 + \pi^\pm$ (threshold 5506 MeV). The second number indicates a negligible coupling to the VV channel $B_s^{0*} + \rho^\pm$ which is consistent with the experiment. The third and fourth numbers show the hidden-color content of the lowest state. The latter numbers should decrease asymptotically when the two quark-antiquark pairs separate. As the PP channel component of the lowest eigenstate is quite large it must be another reason to produce a narrow width, contrary to the case of X(3872) where a tiny VV channel component was a reason to explain its narrow width into $J/\psi + \rho$ or $J/\psi + \omega$. The phase space could be an important factor.

Actually one can understand the reason why the two cases are so different. In the matrix produced by the chromomagnetic interaction for the $1^{++}$ case the key quantity was the off-diagonal matrix element $C_{23} - C_{12} \equiv C_{qe} - C_{qc} = 1.5$ MeV which is very small, so there is no much coupling between the only color singlet-singlet and the only hidden color states. Then the amplitude of this VV channel is negligible and the hidden color part, which does not decay, is dominant. Combined with the phase space of the decay of X(3872) into $J/\psi + \rho$ or $J/\psi + \omega$ the lowest $1^{++}$ acquires a very small width. An analogue situation occurs for Y(4140) where $C_{23} - C_{12} \equiv C_{sc} - C_{sc} = 1.7$ MeV is also very small.

In the present case there is no such tiny off-diagonal matrix element, which one can infer from the definitions of these elements and the values of $C_{ij}$ given in Eq.(4).

On the other hand, there is a large phenomenological difference between X(3872) and X(5568). While X(3872) is located only a few MeV below the $D\bar{D}$ threshold, the X(5568) resonance has a width about ten times larger and is quite far above the $B_s^0 + \pi^\pm$ threshold. This implies that X(5568) could be a more compact system, therefore a better candidate for
tetraquarks.

Note however that the $J^P = 2^+$ lowest state has more resemblance with the lowest $1^{++}$ state of $X(3872)$. Its amplitudes are

$$(-0.1343, 0.9909)$$

which shows that the hidden color component is dominant and its decay width can be diminished in this way.

One can also introduce the exchange channel. The corresponding amplitudes can in principle be obtained from the orthogonal transformation going from the direct meson-meson channel, Fig. 1b, to the exchange meson-meson channel, Fig. 1c.

Looking at Fig. 1c and recalling that we chose $1 = u$, $2 = s$, $3 = \bar{d}$ and $4 = \bar{b}$, for the exchange meson-meson channels we obtain

$$\psi_{0^+}^{1\text{ex}} = |1_{14}1_{23})P_{14}P_{23}⟩ = B^+K^0$$

$$\psi_{0^+}^{2\text{ex}} = |1_{14}1_{23})(V_{14}V_{23})_0⟩ = B^{++}K^{0*}$$

The orthogonal transformation between the direct and exchange channel bases can be found in Appendix A of Ref. [26]. But the tetraquark state $X(5568)$ can hardly decay into the exchange channel $B^+K^0$, the threshold being too high, at 5777 MeV.

Back to the lowest $1^+$ state, located at 5588 MeV, we can see that the channel $B_s^+ + \pi$ is kinematically allowed, being at 33 MeV above the threshold of 5555 MeV.

Finally, we note that the $su\bar{d}b$ system studied here forms together with $su\bar{u}b$ and $\frac{1}{\sqrt{2 + \alpha^2}}s(d\bar{d} - u\bar{u})\bar{b}$ an isospin triplet, all members being degenerate in the present approach. There is also an isosinglet partner $\frac{1}{\sqrt{2 + \alpha^2}}s(u\bar{u} + d\bar{d} + \alpha s\bar{s})\bar{b}$, with $\alpha$ for SU(3)-flavor breaking, which would have a slightly larger mass because the $s\bar{s}$ subsystem introduces a less attractive hyperfine contribution, the parameter $C_{s\bar{s}} = 8.6$ MeV [23] being smaller than $C_{q\bar{q}} = 29.8$ MeV ($q = u, d$) from Eq. (4). The isosinglet partner may decay into $B_s + \eta$ if the phase space allows. Useful considerations about the decay and observation of the isoscalar partner were made in Ref. [39].

**IX. THE SPECTRUM OF $su\bar{d}c$**

We have also calculated the spectrum of the $su\bar{d}c$ tetraquark using the matrices (6), (10) and (12). In addition to the parameters from row 1 Eq. (4), we need $C_{q\bar{c}}$ coefficients. They
FIG. 3: The spectrum of the $u\bar{d}\bar{c}$ tetraquark.

are

$$C_{uc} = 6.6 \text{ MeV}, \ C_{\bar{d}c} = 6.0 \text{ MeV}, \ C_{s\bar{e}} = 6.7 \text{ MeV}. \quad (18)$$

where the first and the third were found in Ref. [23] and the second was identified with $C_{\bar{d}c}$ of Ref. [24] where $\Sigma_c - \Lambda_c$ and $\Sigma_c^* - \Sigma_c$ splittings were fitted. We took $m_c = 1550 \text{ MeV}$ [22], like in our study on Y(4140) [26] and $m_{u,d}$ as in Eq. (3). This gives $\sum_i m_i = 2780 \text{ MeV}$. The lowest $0^+$ state has a mass of 2128 MeV from a hyperfine attraction of -652 MeV and the lowest $1^+$ state has a mass of 2278 MeV with a hyperfine attraction of -502 MeV. Thus there is a gap of 150 MeV between the lowest $1^+$ state and the lowest $0^+$. The resulting
masses for the lowest states are by about 200 MeV lower than in diquark-antidiquark studies of \textit{su}d\bar{c} \cite{11}. But of course, the choice of quark masses imposes some arbitrariness because they depend on the environment, in particular, on the confinement. As pointed out in Ref. \cite{23} the values of \(m_i\) extracted from baryons are usually higher than those from mesons.

The pattern of the whole spectrum of \textit{su}d\bar{c} and of \textit{su}d\bar{b} are very similar. The main difference is that the gap between the lowest \(1^+\) state and the lowest \(0^+\) state is about twice as large as that of \textit{su}d\bar{b} system. This gap will decrease in the heavy quark limit, making \(0^+\) and four of the \(1^+\) states degenerate. The \(2^+\) state raises from 2780 MeV to 2896 MeV due to a repulsive hyperfine contribution of 116 MeV and the highest \(1^+\) state is close to the highest \(2^+\) state.

In this description the lowest \(0^+\) state can decay into \(D_s + \pi\) the channel for which the threshold is at 2108 MeV and the lowest \(1^+\) state can decay into \(D_s^* + \pi\), the threshold being at 2252 MeV.

\textbf{X. CONCLUSIONS}

We have presented results in a simple tetraquark model to see whether or not this model is compatible with the observation of the X(5568) resonance announced by the DØ Collaboration. The parameters of the model were deduced from meson and baryon masses. The calculated mass of X(5568) so obtained is in agreement with the experimental mass range found by the DØ Collaboration. The large value of the chromomagnetic parameter \(C_{ud}\) and a complete color basis played key roles in obtaining a low mass for X(5568). The structure of the lowest \(0^+\) wave function indicates a large component in the \(B_s^0\pi^+\) channel. Besides the \(J^P = 0^+\) spectrum we also gave predictions for the \(J^P = 1^+\) and \(2^+\) sectors.

In the critical analysis of various possible interpretations of X(5568) (threshold, cusp, molecular, tetraquarks) performed in Ref. \cite{40} it has been argued that none of these interpretations seems a natural fit for X(5568). Although this resonance seems to be too light for a plausible tetraquark candidate, the authors of Ref. \cite{40} consider that the present approach seems most promising due to low quark masses and a large hyperfine contribution, taken fully into account in a complete color basis.

A more elaborate study of the \textit{su}d\bar{b} tetraquark system is worth by itself. The tetraquarks containing four different flavors may be more difficult to study than those with one light
and one heavy flavor. It may be a long way before understanding them, if confirmed. The presence of light quarks or antiquarks challenges our understanding of QCD [41].

Note added. The first version of this work was submitted to arXiv, prior to the LHCb announcement [2] described results only for the $J^P = 0^+$ states of the $su\bar{d}\bar{b}$ tetraquark. The present version extends the study of the spectrum to $J^P = 1^+$ and $2^+$ as well in order to clarify a few issues raised in the literature meanwhile, and includes predictions for the charmed partner.

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Appendix A: Charge conjugation

From Ref. [42] Ch. 10, one can see that the permutation (13)(24) leaves invariant the color basis vectors $|1_{13}1_{24}\rangle$ and $|8_{13}8_{24}\rangle$. Then, with the identification $1 = u$, $2 = s$, $3 = \bar{d}$ and $4 = \bar{b}$ the permutation (13)(24) is equivalent to the charge conjugation operator [38]. Thus all basis states introduced in this way have a definite charge conjugation, which is easy to identify.

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