Tully-Fisher relation, galactic rotation curves and dissipative mirror dark matter

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Abstract. If dark matter is dissipative then the distribution of dark matter within galactic halos can be governed by dissipation, heating and hydrostatic equilibrium. Previous work has shown that a specific model, in the framework of mirror dark matter, can explain several empirical galactic scaling relations. It is shown here that this dynamical halo model implies a quasi-isothermal dark matter density, $\rho(r) \simeq \rho_0 r_0^2/(r^2 + r_0^2)$, where the core radius, $r_0$, scales with disk scale length, $r_D$, via $r_0/kpc \approx 1.4 (r_D/kpc)$. Additionally, the product $\rho_0 r_0$ is roughly constant, i.e. independent of galaxy size (the constant is set by the parameters of the model). The derived dark matter density profile implies that the galactic rotation velocity satisfies the Tully-Fisher relation, $L_B \propto v_{\text{max}}^3$, where $v_{\text{max}}$ is the maximal rotational velocity. Examples of rotation curves resulting from this dynamics are given.

Keywords: dark matter theory, rotation curves of galaxies

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Small scale structure, that is, the structure of dark matter galaxy halos has been a long standing puzzle. If dark matter is dissipative then there is a possible solution. Dissipative dark matter leads to nontrivial dynamics: the halo of a spiral galaxy can then be governed by hydrostatic equilibrium, dissipation and heating [1]. Mirror dark matter (see [2–8] for reviews) offers a specific framework in which to study this possibility. Within the mirror dark matter context a previous study [9] has shown that this emerging picture of halo dynamics can potentially explain small scale structure as it leads to successful galactic scaling relations. The purpose of this article is to further explore this dynamics.

Mirror dark matter supposes the existence of a hidden sector exactly isomorphic to the standard model [10–12]. That is, the Lagrangian governing the fundamental properties of the elementary particles has the form:

\[
\mathcal{L} = \mathcal{L}_{SM}(e, \nu, u, d, \gamma, \ldots) + \mathcal{L}'_{SM}(e', \nu', u', d', \gamma', \ldots) + \mathcal{L}_{\text{mix}}.
\]

Such a theory features an exact $Z_2$ symmetry, which can be interpreted as the spacetime parity symmetry, which maps each ordinary particle onto a mirror partner, denoted with a prime ($\prime$). This $Z_2$ symmetry is assumed to be unbroken. It follows that the mirror particles each have the same mass and interactions among themselves as the particles in the ordinary sector. The mirror quarks form mirror baryons which together with mirror electrons constitutes the inferred dark matter in the Universe [2–8]. In addition to gravity, the mirror particles can interact with the ordinary particles via kinetic mixing interaction:

\[
\mathcal{L}_{\text{mix}} = \frac{\epsilon}{2} F_{\mu\nu} F'_{\mu\nu},
\]

where $F_{\mu\nu}$ ($F'_{\mu\nu}$) is the field strength tensor for the photon (mirror photon). Such kinetic mixing is gauge invariant and renormalizable and $\epsilon$ can be viewed as a fundamental parameter of the theory [13]. The physical effect of the kinetic mixing interaction is to induce a tiny ordinary electric charge ($\propto \epsilon$) for the hidden sector $U(1)'$ charged particles [14].

In this picture, galactic halos are composed, predominantly, of a plasma of such self interacting and dissipative dark matter [1, 9]. If this is indeed the case, then the halo could be modelled as a fluid, governed by Euler’s equations of fluid dynamics. At the current epoch, this fluid is presumed to have evolved to a hydrostatic equilibrium configuration where the energy being absorbed in each volume element is equal to the energy being radiated from the same volume element. Thus, in addition to the hydrostatic equilibrium condition we have the dynamical condition:

\[
\frac{d^2 E_{\text{in}}}{dt dV} = \frac{d^2 E_{\text{out}}}{dt dV}.
\]

In ref. [9] approximate formulas for the left and right-hand sides of the above equation have been derived, which we summarize below.

(a) dissipation. We assume that thermal bremsstrahlung is the main dissipative process. The rate at which bremsstrahlung energy is radiated per unit volume, per unit time at a particular point, $P$, in the halo is [15]:

\[
\frac{d^2 W}{dt dV} = \frac{16 \alpha^3}{3 m_e} \left( \frac{2 \pi T}{3 m_e} \right)^{1/2} \sum_j \left| Z_j^2 n_j n_e g_B \right|.
\]
where the index \( j \) runs over the mirror ions in the plasma (of charge \( Z_j \)) and \( \bar{g}_B \) is the frequency averaged of the velocity averaged Gaunt factor for free-free emission. We take \( \bar{g}_B = 1.2 \), which is known to be accurate to within about 20\% [15]. In our numerical work we approximate \( E_{\text{out}} = W \). See ref. [9] for further discussions.

**b) heating.** If kinetic mixing is nonzero (i.e. \( \epsilon \neq 0 \)) then light mirror particles, \( e^{'}, \bar{e}^{'} \), \( \gamma^{'} \) can be produced in a core-collapse supernova from processes such as \( e \bar{e} \rightarrow e^{'} \bar{e}^{'} \). In fact, mirror particle emission can be comparable to that of neutrinos for \( \epsilon \sim 10^{-9} \) [16–19]. The bulk of this energy is expected to be carried off by mirror photons, \( \gamma^{'} \), in the region around an ordinary supernova. The idea is that these mirror photons, with total energy up to around half the supernova core-collapse energy (\( \sim 10^{53} \text{ erg per supernova} \)) will replace the energy lost in the halo due to dissipation. This heating is achieved by interactions (photoionization) of \( \gamma^{'} \) with heavy mirror metal components, which is possible because these components retain their K-shell mirror electrons. As far as the heating of the halo is concerned, it might be sufficient (at least as a rough approximation) to include just the mirror iron (\( Fe^{'} \)) component provided that the proportion of the supernova \( \gamma^{'} \) energy contributed by \( \gamma^{'} \) with \( E_{\gamma^{'}} \) less than the \( Fe^{'} \) K-shell binding energy, \( I \simeq 9 \text{ keV} \), is small. [Although we consider a mirror metal component consisting of just \( Fe^{'} \), reasonable alternatives e.g. replacing \( Fe^{'} \) with mirror oxygen (\( O^{'} \)) do not significantly affect any of our results.]

The total photoelectric cross-section\(^{1} \) of \( Fe^{'} \) (\( Z = 26 \)) is given by (see e.g. [20]):

\[
\sigma_{PE}(E_{\gamma^{'}}) = \frac{g16\sqrt{2\pi}}{3m_e^6} \alpha^6 Z^5 \left( \frac{m_e}{E_{\gamma^{'}}} \right)^{7/2} \text{ for } E_{\gamma^{'}} \gg I .
\] (5)

Here \( g = 1 \) or 2 counts the number of K-shell mirror electrons present. The \( Fe^{'} \) number density can be parameterized in terms of the halo \( Fe^{'} \) mass fraction, \( \xi_{Fe^{'}} \):

\[
n_{Fe^{'}} = n_{He^{'}} \left( 1 + \frac{f}{4} \right) \left( \frac{m_{He}}{m_{Fe}} \right) \left( \frac{\xi_{Fe^{'}}}{1 - \xi_{Fe^{'}}} \right)
\] (6)

with \( f \equiv n_{He^{'}}/n_{He^{'}} = 0.4 \) suggested by early Universe cosmology [21].

The flux, \( F(r) \), of supernova \( \gamma^{'} \) at a particular point, \( P \), will deposit an energy per unit volume per unit time of:

\[
\frac{d^2E_{in}}{dt} = \int \frac{dF(r)}{dE_{\gamma^{'}}} n_{Fe^{'}}(r) \sigma_{PE} \ dE_{\gamma^{'}} .
\] (7)

If the (average) supernova flux traces the stellar mass density, \( \rho_D(r) \), assumed spherically symmetric for simplicity then:

\[
\frac{dF(r)}{dE_{\gamma^{'}}} = R_{SN} E_{\gamma^{'}} \frac{dN_{\gamma^{'}}}{dE_{\gamma^{'}}} \int_0^\infty \int_{-1}^1 \frac{\rho_D}{m_D} e^{-\tau r^2} 2d^2 \ d\cos \theta dr .
\] (8)

Here, \( R_{SN} \) is the frequency of type II supernova in the galaxy under consideration, \( d = \sqrt{r^2 + r'^2 - 2rr' \cos \theta} \) is the distance from the supernova to the point \( P \) and \( \tau \) is the optical depth along this path: \( \tau = \int_0^\infty n_{Fe^{'}(y)} \sigma_{PE} dy \). We define the spherically symmetric stellar mass density by requiring that the mass within a radius \( r \) is the same as that of the Freeman disk, with surface density \( \Sigma_s = \frac{m_D}{2\pi r_D^2} e^{-r/r_D} . \) This implies that

\[
\rho_D(r) = \frac{m_D}{4\pi r_D^2} e^{-r/r_D} .
\] (9)

Here \( m_D \) is the total stellar mass of the disk and \( r_D \) is the disk scale length.

\(^{1}\)Unless otherwise indicated, we use natural units with \( \hbar = c = 1 \).
An important ingredient is the frequency of supernova in a given galaxy, $R_{SN}$. In our previous study [9] we used the rough baryonic scaling relation \[ m_D \propto (L_B)^{1.3} \] and \[ R_{SN} \propto (L_B)^{0.73} \] from the supernova study [23]. These relations together imply \[ R_{SN} \propto (m_D)^{0.56} \]. Of course, the systematic uncertainty is significant, later we examine the effect of a variation of $\pm 0.15$ in the exponent of this relation.

To proceed, we can parameterize the mirror photon energy spectrum from an (average) single supernova via a power law, with a cut-off at \[ E_{\gamma'} = E_c^2 \]

\[ E_{\gamma'} \frac{dN_{\gamma'}}{dE_{\gamma'}} \equiv \kappa (E_{\gamma'})^{c_1} \quad (10) \]

With this parameterization, \[ R_{SN} \int_0^{E_c} \kappa (E_{\gamma'})^{c_1} dE_{\gamma'} \equiv L_{SN}' \] is the total $\gamma'$ energy produced (on average) by ordinary supernova per unit time in the galaxy under consideration. With the above definitions, and the assumed scaling, \[ R_{SN} \propto (m_D)^{0.5} \], we have:

\[ \kappa R_{SN} = \frac{1 + c_1}{(E_c^{1+c_1})} \left( \frac{m_D}{m_{MW}} \right)^{0.5} L_{SN}' \quad (11) \]

where \( L_{SN}' \) is the total $\gamma'$ energy produced (on average) by ordinary supernovae for a reference $\sim$ Milky Way sized spiral galaxy of stellar mass \( m_{MW} = 5 \times 10^{10} m_\odot \).

(c) hydrostatic equilibrium. Both the heating and cooling depend on the halo temperature, \( T(r) \). The halo temperature can be evaluated (for a given dark matter plasma density, \( \rho(r) \)) assuming hydrostatic equilibrium, where the force of gravity is balanced by the pressure gradient. That is,

\[ \frac{dP}{dr} = -\rho(g(r)) \quad (12) \]

Here, \( \rho(r) = \bar{m} n_T(r) \) where \( n_T(r) \) is the number density of the plasma mirror particle component\(^3\) with mean mass, \( \bar{m} \). For a fully ionized plasma, arguments from early Universe cosmology suggest that \( \bar{m} \simeq 1.1 \text{ GeV} \) [21]. The local acceleration due to gravity, \( g(r) \), is given in terms of Newton’s constant, \( G_N \):

\[ g(r) = \frac{v_{rot}^2}{r} = \frac{G_N}{r^2} \int_0^r [\rho(r) + \rho_{baryon}(r)] \, dV \quad (13) \]

Although in principle, the baryonic density contains both a stellar and gas component, in this work, we make the simplifying approximation of considering only the stellar mass contribution to \( \rho_{baryon} \). This can be justified as follows: the gas component of spirals is typically smaller than the stellar component and importantly its distribution is more (radially) extended.

\(^2\)Of course, the $\gamma'$ spectrum will not be a simple power law over all energies, however only the part of the spectrum with \( E_{\gamma'} \lesssim 30 \text{ keV} \) is important since galaxy halos are optically thin for $\gamma'$ with energies greater than around 30 keV [9].

\(^3\)As in our previous study [9], we neglect a possible small dark disk/compact object component made of old mirror stars, mirror white dwarfs etc, so that dark matter consists only of the plasma component.
Thus at any given radius, the gas contribution to $g(r)$ is generally much smaller than either the stellar contribution or the dark matter contribution.

To solve the hydrostatic equilibrium equation, we need to assume a boundary condition. We assume the boundary condition $dT/dr \to 0$ at large galactic radius, $R_{\text{gal}}$ (taken to be $50r_D$). Our numerical results are independent, to a very good approximation, of the particular value of $R_{\text{gal}}$ chosen so long as $R_{\text{gal}} \gg r_D$.

The conditions, eq. (3), (12), can be solved numerically. The strategy employed in our previous work [9] was to assume that the dark matter profile had the Burkert form: 

$$\rho(r) = \rho_0 r_0^3 \left( \frac{r^2}{r_0^2} + 1 \right)^{-\frac{3}{2}}.$$  

(14)

We then obtained $r_0, \rho_0$ by numerically minimizing the function:

$$\Delta (r_0, \rho_0) \equiv \frac{1}{10r_D} \int_{r_D}^{r_D} \left| 1 - \frac{d^2E_{\text{in}}}{dt} \frac{d^2E_{\text{out}}}{dt} \right| dr.$$  

(15)

It was found [9] that the Burkert profile provided a rough solution to the dynamical condition, eq. (3), with minimum value of $\Delta$ around 0.1 (roughly independently of $m_D$ in the range of interest: $10^9 m_\odot < m_D < 10^{12} m_\odot$). That is, the left and right-hand sides of eq. (3) agree to within about 10%.

The main reason the Burkert profile was adopted in [9] was to compare the derived values of $r_0, \rho_0$ with the corresponding values obtained by Salucci and others [26] which assumed that profile. In this work, we adopt a more general approach and assume that the dark matter distribution takes the form:

$$\rho(r) = \rho_0 \left( \frac{r_0^2}{r^2 + r_0^2} \right)^\beta.$$  

(16)

It turns out that this more general profile allows a near exact solution to the condition, eq. (3). The resulting $\Delta$ minimum obtained by varying $r_0, \rho_0$ and now $\beta$ is less than 0.01. That is, the left and right-hand sides of eq. (3) agree to better than 1%. We expect therefore that this profile, and the values of $r_0, \rho_0$ and $\beta$ derived by minimizing $\Delta$, should provide an accurate representation of the dark matter properties expected from this dynamics.

An empirical baryonic scaling relation is known which relates $m_D$ to $r_D$ for spiral galaxies [27, 28]:

$$\log \left( \frac{r_D}{\text{kpc}} \right) \approx 0.633 + 0.379 \log \left( \frac{m_D}{10^{11} m_\odot} \right) + 0.069 \left[ \log \left( \frac{m_D}{10^{11} m_\odot} \right) \right]^2.$$  

(17)

With this relation, the baryonic parameters of spirals are (roughly) specified by a single parameter which can be taken as either $m_D$ or $r_D$.

To continue, we must choose values of $L_{SN}^{MW}, c_1, \xi_F c'$ and then we can numerically solve the equations, minimizing $\Delta (\rho_0, r_0, \beta)$ to give values of $\rho_0, r_0$ and $\beta$ for a given spiral galaxy parameterized by $m_D, r_D$. We consider stellar disk masses in the range, $10^9 m_\odot \lesssim m_D < 10^{12} m_\odot$.

\footnote{We have also checked that $\Delta_{\text{min}} \approx 0.01$ holds true even when the range of integration of the integral in eq. (15) is extended from \{r_D \leq r \leq 11r_D\} to \{0.2r_D \leq r \leq 15r_D\}. Extending this integration range also has negligible effect on the derived values for $\beta, r_0$ and $\rho_0$.}
$m_D \lesssim 10^{12} m_\odot$, and parameterize $r_D$ via eq. (17). This mass range covers the typical $m_D$ values for spiral galaxies. The result of performing this numerical task is the following. We find that $\Delta_{\text{min}} \lesssim 0.01$ (independently of $m_D$) and

$$\beta \simeq 1.0$$

$$r_0 \simeq 1.4 \left( \frac{r_D}{\text{kpc}} \right) \text{kpc}$$

$$\rho_0 r_0 \simeq \left[ \frac{\xi_{Fe'}}{0.02} \right]^{0.8} \left[ \frac{L_{SN}^{\text{MW}}}{10^{45} \text{ erg/s}} \right]^{0.8} \left[ \frac{2}{c_1} \right] 50 \text{ m}_\odot/\text{pc}^2 .$$

(Eq. 18)

Evidently, this dissipative halo dynamics yields a quasi-isothermal dark matter density profile, a result that is not unexpected given earlier analytic work of refs. [1, 29]. In particular, it was shown in those references that the behaviour: $\rho(r) \propto 1/r^2$ leads to an approximate solution to the hydrostatic and energy balance equations for $r \gg r_D$ (where the supernova energy source can be modelled as a point source and the matter density is dominated by mirror dark matter). For $r \lesssim \text{few } r_D$, the distribution of the supernova energy source over a finite volume weakens the heating rate, and consequently, energy balance requires a cored halo [29].

We emphasize that the first two results in eq. (18) above, hold even when the parameters ($L_{SN}^{\text{MW}}$, $\xi_{Fe'}$, $c_1$) are varied. This is illustrated in figures 1, 2 and 3, where we give the results for $\beta$, $r_0$, $\rho_0 r_0$ obtained from minimizing $\Delta$ for fixed $c_1 = 2$, $\xi_{Fe'} = 0.02$, $E_c = 50 \text{ keV}$ but with an order-of-magnitude variation in $L_{SN}^{\text{MW}}$. [Alternatively fixing $L_{SN}^{\text{MW}}$ and varying $E_c$ is equivalent as both operations simply scale the supernova energy $\gamma'$ flux.]

Similar results hold when $\xi_{Fe'}$ is varied and are therefore not shown. Variation of $c_1$ also has little effect for the first two relations in eq. (18). Note that the derived scaling of the core radius with disk scale length is consistent with observations [30]. The third relation, the one for $\rho_0 r_0$, does depend on the parameters. However there is an approximate parameter degeneracy, since this quantity depends on only one combination of these parameters. Also, note that these parameters are likely to be approximately independent of galaxy size, so that a $\rho_0 r_0 \propto \text{constant}$ scaling is expected. Again such a scaling relation is consistent with observations [26].

The equations governing dissipation and heating depend on the ionization state of the halo. In particular the number of $Fe'$ K shell states occupied [the $g$ factor in eq. (5)] depends on the ionization state of $Fe'$ while the ion density in eq. (4) depends on the ionization state of $H'$, $He'$. The equations governing the ionization state have been given in [9] and depend only on the temperature of the mirror particle plasma. [These equations are, of course, included in our numerical work here.] Interestingly we find that the mean plasma temperature corresponding to the $m_D$ range $10^9 m_\odot \lesssim m_D \lesssim 10^{12} m_\odot$ is approximately $20 \text{ eV} \lesssim T \lesssim \text{keV}$. This is roughly the range for which (a) $H'$, $He'$ are typically fully ionized and (b) $Fe'$ has both K-shell states filled. This consistency of the ionization state of the halo, over the entire mass range of interest, $10^9 m_\odot \lesssim m_D \lesssim 10^{12} m_\odot$ is necessary to obtain the smooth behaviour of the derived relations, eq. (18). This would appear to be an important constraint if one were to think about replacing mirror dark matter with a more generic dissipative hidden sector model.

In figure 4 we plot the evaluated temperature versus radial distance for some examples. For each of these examples the equation governing hydrostatic equilibrium, eq. (12), is solved with $\beta$, $r_0$ and $\rho_0$ obtained by minimizing $\Delta$, eq. (15) [with reference parameters: $c_1 = 2$, $\xi_{Fe'} = 0.02$, $L_{SN}^{\text{MW}} = 10^{45} \text{ erg/s}$, $E_c = 50 \text{ keV}$ are assumed].
Figure 1. The dark matter density, slope, $\beta$ [eq. (16)], versus $m_D [m_\odot]$. The dashed-dotted, solid and dotted lines correspond to $L_{MW}^{SN} = 0.3 \times 10^{45}$ erg/s, $L_{MW}^{SN} = 1.0 \times 10^{45}$ erg/s and $L_{MW}^{SN} = 3.0 \times 10^{45}$ erg/s respectively.

Figure 2. Dark matter core radius, $r_0$, versus disk scale length, $r_D$. Parameters as per figure 1.
Figure 3. $\rho_0 r_0$ versus $m_D$. Parameters as per figure 1.

Figure 4. Halo mirror plasma temperature versus $r/r_D$ for the examples with (from bottom to top curves): $m_D = 10^9 \, m_\odot$, $m_D = 10^{10} \, m_\odot$, $m_D = 10^{11} \, m_\odot$, $m_D = 10^{12} \, m_\odot$. 
Figure 5. Rotation curve for NGC3198. The solid line is the ‘theoretical’ rotation curve derived from the assumed halo dynamics. Also shown (dashed curve) is the baryonic contribution. The data is obtained from [32].

Examples of the rotation curves predicted by this dynamics are given in figures 5 and 6. Consider first a specific example, for which we choose the galaxy NGC3198. This galaxy has stellar mass around $m_D = 3.0 \times 10^{10} m_\odot$ [31] and from eq. (17) we find $r_D = 2.8$ kpc. We use the measurement of the rotation curve from [32] which is consistent with other measurements such as the one in [31]. In figure 5 we give our result for the rotation curve, determining the dark matter parameters, $\beta, r_0, \rho_0$ by minimizing $\Delta$ inputting the above baryonic parameters for NGC3198. We find that the data could be roughly fit with the parameters taken to be: $c_1 = 2, \xi_{Fe} = 0.02, L_{SN}^{MW} = 2.2 \times 10^{45}$ erg/s and $E_c = 50$ keV. In figure 6 we show derived rotation curves obtained for representative examples with (from bottom to top curves) $m_D = 10^9 m_\odot, m_D = 10^{10} m_\odot, m_D = 3 \times 10^{10} m_\odot, m_D = 10^{11} m_\odot, m_D = 3 \times 10^{11} m_\odot$. The reference parameters taken are the same as per figure 4.

Tully and Fisher discovered some time ago that the luminosity of a spiral galaxy has a tight correlation with the maximum value of its rotational velocity [33]. Current estimates, e.g. [34–36], indicate that $L_B \propto v_{\text{max}}^{\alpha_1}$, with $\alpha_1 \approx 3.0 - 3.5$ ($L_B$ is the B-band luminosity). A baryonic Tully-Fisher relation is also known to approximately relate $m_D \propto v_{\text{max}}^{\alpha_2}$ with $\alpha_2 \approx 4.0 - 4.5$. Given that we can derive the dark matter profile via the assumed halo dynamics we can also work out the $L_B$ versus $v_{\text{max}}$ dependence. To do this, we obtain $L_B$ from the measured scaling, $R_{SN} \propto (L_B)^{0.73}$ [23]. As before we minimize the function $\Delta$ considering first the usual reference parameters: $L_{SN}^{MW} = 10^{45}$ erg/s, $c_1 = 2, \xi_{Fe} = 0.02, E_c = 50$ keV. The result of this numerical work is shown in figure 7 for $L_B$ versus $v_{\text{max}}$ and figure 8 for $m_D$ versus $v_{\text{max}}$.

Our numerical results can be excellently approximated by a power law: $L_B \propto v_{\text{max}}^{\alpha_1}$ and $m_D \propto v_{\text{max}}^{\alpha_2}$, with $\alpha_1 \approx 2.9 (\alpha_2 \approx 4.1)$. We have also investigated the dependence on the
slope parameter on variation of our parameters: $L^{MW}_{SN}$, $\xi_{Fe'}$, $c_1$. Considering $\alpha_1$ (similar results hold for $\alpha_2$) we find that an order-of-magnitude variation in $L^{MW}_{SN}$ or $\xi_{Fe'}$ around the reference value changes the slope $\alpha_1$ by $\pm 0.4$ (with larger values of $L^{MW}_{SN}$ or $\xi_{Fe'}$ steepening the slope). Variation of $c_1$ over the range: $1 \leq c_1 \leq 3$ modifies $\alpha_1$ by $\pm 0.2$. We also considered the effect of a possible variation of the exponent, $\delta$, in the relation $R_{SN} \propto (m_D)^{\delta}$, over the range $\delta = 0.50 \pm 0.15$. Variation of $\delta$ over this range affects the slope parameter $\alpha_1$ by $\pm 0.4$. Of course, the consistency of the derived slope parameter with the value $\alpha_1 \approx 3$ inferred from observations is not so surprising given that the galactic scaling relations, derived in part with the same types of data, were found to be satisfied with this halo dynamics in [9]. Nevertheless, working directly with relations such as those of Tully and Fisher might be useful to further scrutinize this dynamical halo model.

To summarize, galaxy structure has presented a fascinating puzzle for many years. Dissipative dark matter candidates such as mirror dark matter offer an interesting approach to a possible solution. In this picture the halo has nontrivial dynamics: the energy lost due to dissipation is replaced ultimately by heating from ordinary supernovae. Such a mechanism requires small kinetic mixing interaction, $\epsilon \sim 10^{-9}$, for which there is independent evidence from direct detection experiments [37, 38]. The heating from ordinary supernovae provides a direct coupling between the dark matter and ordinary matter in galaxies. In the framework of mirror dark matter such a dark matter picture is quite predictive: the dark matter distribution, $\rho(r)$, can be completely determined from the baryonic properties of galaxies. We found that $\rho(r) \approx \rho_0 r_0^2/(r^2 + r_0^2)$, where the core radius, $r_0$, scales with disk scale length, $r_D$, via $r_0/kpc \approx 1.4 (r_D/kpc)$. Additionally, the product $\rho_0 r_0$ is roughly constant, i.e. independent of galaxy size (the constant is set by the parameters of the model). We further show that such a dark matter distribution leads to rotation curves which satisfy the Tully-Fisher relation.
Figure 7. \( L_B \) versus \( v_{\text{max}} \), with halo profile derived from dissipative mirror dark matter.

Figure 8. Stellar disk mass \( m_D \) versus \( v_{\text{max}} \), with halo profile derived from dissipative mirror dark matter.
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