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To cite this article: Anita Bagora (Menaria) and Rakeshwar Purohit 2013 J. Phys.: Conf. Ser. 423 012022

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Conformally Flat Bulk Tilted Cosmological Model

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Abstract. Conformally flat plane symmetric bulk viscous fluid tilted cosmological model is investigated. We assume that $\zeta \Theta = K$ (constant), where $\zeta$ is the coefficient of bulk viscosity and $\Theta$ is the expansion in the model. It has been assumed that the supplementary condition between metric potential as $A = B^n$, where $n$ is constant. Special model is also investigated in the absence of bulk viscosity. The physical and geometrical aspects of the model with singularities in the presence and absence of bulk viscosity are also discussed.

1. Introduction

Cosmological models are studied as one of the most important applications of general relativity. Einstein [1918] in his basic researches based on the spherically symmetric, isotropic and homogeneous model of universe but without red shift [1] and hence non-expanding, whereas de-Sitter gave the expanding static model which was empty [1]. The non-static models were propounded by Friedmann [1924], Lemaître [1929], Robertson and Walker [1935] and started the process of searching for the various cosmological models which resembles the actual universe up to maximum extent [1]. Bondy and Gold [1948] initiated the steady state theory based upon the cosmological principle. At the present stage of evolution, the universe is spherically symmetric with homogeneous and isotropic matter distribution. But close to the big bang singularity, the assumptions of spherical symmetry or isotropy are not strictly valid and the researchers Randall [2], da Silva and Wang [3], Prakash[4], Pradhan and Pandey [5] etc. started considering plane symmetry. H. Takeno [6] defined the conformal curvature tensor $C_{ijkl}$ and the condition that a Riemannian space be conformally flat is given by vanishing of the conformal curvature tensor, i.e. $C_{ijkl} = 0$. A number of conformally flat, physically significant models are known like Schwarzschild interior solution. These were later on studied by Singh and Roy [7], Singh and Abdussattar [8], Roy and Bali [9], Pandey and Tiwari [10], Pradhan [11], Pradhan, Srivastava and Singh [12].

A considerable interest is being shown towards the tilted universes. A spatially homogeneous universe is said to be tilted if the fluid velocity vector is non-orthogonal to the group orbits. Ellis and Baldwin [13] have shown that the actual universe is likely to be tilted one. King and Ellis [14], Ellis and King [15], Collins and Ellis [16] studied the general dynamics of tilted models. Various aspects of tilted cosmological models have been studied by Dunn and Tupper [17], Matravers et al. [18], Hewitt et al. [19], Horwood et al. [20], Coley and Tupper [21], Beesham [22], Bali and Sharma [23].
In most treatments of cosmology, cosmic fluid is considered as perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe. Various authors [24–28] have shown that bulk viscosity leads to inflationary like solution and acts like a negative energy field in an expanding universe. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. A number of authors have discussed cosmological solutions with bulk viscosity in various context [29-33].

Tilted solutions containing perfect fluid have been obtained by Farnsworth [34] and Reboucas [35].Cosmological models with heat flow have been studied by various author viz. Novello and Reboucas [36], Ray [37], Reboucas and Lima [38], Roy and Banerjee [39-40],Singh et al. [41-42].have also investigated Bianchi type I homogeneous tilted cosmological models in different context. Bagora [43-44] obtained tilted Bianchi type I and III cosmological model for disordered radiation and stiff fluid distribution. Motivated by these studies, in this paper we propose to find conformally flat plane symmetric bulk viscous fluid tilted cosmological model. We assume that

\[ K = \zeta \theta (\text{constant}), \]

where \( \zeta \) is the coefficient of bulk viscosity and \( \theta \) is the expansion in the model. It has been assumed that the condition \( A = B^n \), between metric potentials. Special model is also investigated in the absence of bulk viscosity. The physical and geometrical aspects of the model in the presence and absence of bulk viscosity are also discussed.

2. The Metric and Field Equations

We consider plane symmetric metric in the form

\[ ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2, \quad (1) \]

where \( A \) and \( B \) are functions of \( t \) alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis [45] and for bulk viscosity given by Landau and Lifshitz [46] is given by

\[ T^i_j = (\epsilon + p)v_i v^j + pg_i^j + q_i v^j + v_i q^j - \zeta 0(g_i^j + v_i v^j), \quad (2) \]

together with

\[ g_{ij} v^i v^j = -1, \quad (3) \]

\[ q_i q^j > 0, \quad (4) \]

\[ q_i v^i = 0, \quad (5) \]

where \( p \) is the isotropic pressure, \( \epsilon \) the matter density and \( q_i \) the heat conduction vector orthogonal to \( v^i \). The fluid flow vector has the components \( \left(0, 0, -\frac{\sinh \lambda}{B}, \frac{\cosh \lambda}{B}\right) \) satisfying (3) and \( \lambda \) is the tilt angle.

The Einstein field equation

\[ \frac{1}{2} R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T^i_j, \quad (\text{units such that } c = G = 1) \]

For the line element (1) are

\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 C_4}{AB} = -8\pi (p - K), \quad (6) \]

\[ \frac{A_4^2}{A^2} + \frac{2A_4}{A} = -8\pi \left(\frac{(\epsilon + p) \sinh^2 \lambda + p + 2q_3}{B} \frac{\sinh \frac{\lambda}{B} - K \cosh^2 \lambda}{\sinh \lambda}ight), \quad (7) \]

\[ \frac{A_4^2}{A^2} + \frac{2A_4 B_4}{AB} = 8\pi \left(\frac{(\epsilon + p) \cosh^2 \lambda - p + 2q_3}{B} \frac{\sinh \frac{\lambda}{B} - K \sinh^2 \lambda}{\cosh \lambda}ight), \quad (8) \]

\[ -8\pi \left(\frac{(\epsilon + p)}{B} \frac{B \sinh \lambda \cosh \lambda + q_3 \cosh \lambda + q_2}{\cosh \lambda} \frac{\sinh^2 \lambda}{\cosh \lambda} - KB \sinh \lambda \cosh \lambda\right) = 0. \quad (9) \]
Here the suffix ‘4’ stands for ordinary differentiation with respect to cosmic time ‘t’ alone.

3. Solution of the Field Equations

Equations from (6)-(9) are four equations in six unknowns $A, B, \varepsilon, p, \lambda$ and $q_1$, therefore to determine the complete solution we require two more conditions.

Firstly, we assume that the space-time is conformally flat which leads to

\[
C_{1212} = \frac{1}{3} \left[ A^3 A_{44} - A^3 A_{4} - A^3 A_{4} + B_{4} \frac{B_{4}}{B} + A^4 \frac{B_{44}}{B} \right] = 0. \quad (10)
\]

Secondly, we consider relation between metric potential as

\[
A = B^n. \quad (11)
\]

Also, we assume that $\zeta\theta = K$. \quad (12)

The condition $\zeta\theta = K$ is due to the peculiar characteristic of the bulk viscosity. It acts like a negative energy field in an expanding universe (Johri and Sudharshan[47]) i.e $\zeta\theta = K$. According to that expansion is inversely proportional to bulk viscosity.

Equations (10) and (11) lead to

\[
B^{1} B_{44} + (n - 1)B^{3} B_{4} = 0. \quad (13)
\]

From equations (7), (8) and (11), we have

\[
n B_{44} + (n^2 - 2n) \frac{B_{4}}{B} = -4 \pi \left[ (\varepsilon + p) \cosh 2\lambda + 4q_3 \frac{\sinh \lambda}{B} - K \cosh 2\lambda \right]. \quad (14)
\]

and

\[
n B_{44} + 2n \left( \frac{B_{4}}{B} \right)^2 = 4\pi(\varepsilon - p + K). \quad (15)
\]

From equations (9) and (11), we have

\[
-16\pi q_3 \frac{\sinh \lambda}{B} = 4\pi(\varepsilon + p + K) \sinh 2\lambda \tan h 2\lambda. \quad (16)
\]

Again from equations (14) and (16), we have

\[
n B_{44} + (n^2 - 2n) \frac{B_{4}}{B} = -4\pi(\varepsilon + p + K) \frac{\cosh 2\lambda}{\cosh 2\lambda}. \quad (17)
\]

Equation (13) gives

\[
B = \sqrt{n(\ell + m)}, \quad (18)
\]

where '$\ell$' and 'm' are constants of integration.

Equation (18) leads to

\[
A^2 = n^2(\ell + m)^2. \quad (19)
\]

Hence the metric (1) reduces to

\[
\text{ds}^2 = -d\tau^2 + n^2(\ell + m)^2(dx^2 + dy^2) + n^2(\ell + m)^2 \text{d}z^2. \quad (20)
\]

By using the suitable transformations the metric (20) becomes

\[
\text{ds}^2 = \frac{d\tau^2}{\ell^2} + n^2 T^2(dX^2 + dY^2) + n^{\frac{2}{\ell}} T^2 \text{d}Z^2. \quad (21)
\]

4. Some Physical and Geometrical Features

The density and pressure for the model (21) are given by

\[
8\pi \varepsilon = \frac{8n^2\pi KT^2 - 1}{n^2 T^2}, \quad (22)
\]
The tilt angle $\lambda$ is given by
\[
\cosh\lambda = \sqrt{\frac{n^2 + 2n - 1 + 8\pi^2 KT^2}{2n}},
\]
(24)
\[
\sinh\lambda = \sqrt{\frac{n^2 - 1 + 8\pi^2 KT^2}{2n}}.
\]
(25)

The reality conditions
\[
\varepsilon + p > 0, \quad (ii) \varepsilon + 3p > 0,
\]
lead to
\[
4n^2\pi KT^2 - 1 + n^2 + n \geq 0.
\]
(26)

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by
\[
\theta = \frac{[8(1+3n)n^2\pi KT^2 + 4n^4 + 10n^3 - 2n]}{nT\sqrt{2n[n^2 + 2n - 1 + 8\pi^2 KT^2]}}.
\]
(27)

The components of fluid flow vector $v^i$ and heat conduction vector $q^i$, for the model (21) are given by
\[
v^3 = \frac{1}{(nT)^{\frac{1}{n}}} \sqrt{\frac{n^2 - 1 + 8\pi^2 KT^2}{2n}},
\]
(28)
\[
v^4 = \sqrt{\frac{n^2 + 2n - 1 + 8\pi^2 KT^2}{2n}},
\]
(29)
\[
q_3 = \frac{-n^{\frac{1}{2n}}}{8\pi^2 T^2} \left[\frac{n^2 + 2n - 1 + 8\pi^2 KT^2}{n^2 - 1 + 8\pi^2 KT^2}\right] \sqrt{\frac{n^2 - 1 + 8\pi^2 KT^2}{2n}},
\]
(30)
\[
q_4 = \frac{n^2 - 1 + 8\pi^2 KT^2}{8\pi^2 T^2} \left[\frac{n^2 + 2n - 1 + 8\pi^2 KT^2}{n^2 - 1 + 8\pi^2 KT^2}\right] \sqrt{\frac{n^2 - 1 + 8\pi^2 KT^2}{2n}}.
\]
(31)

The non-vanishing components of shear tensor ($\sigma_{ij}$) and rotation tensor ($\omega_{ij}$) are given by
\[
\sigma_{33} = \frac{n^{\frac{2}{n}} (1 - n)(n^2 + n - 1 + 8\pi^2 KT^2)}{3n^{\frac{2}{n}} T^{\frac{1}{1-n}}} \sqrt{\frac{n^2 + 2n - 1 + 8\pi^2 KT^2}{2n}},
\]
(32)
\[
\sigma_{34} = \frac{-((1 - n)(n^2 + n - 1 + 8\pi^2 KT^2)}{3n^{\frac{2}{n}} T^{\frac{1}{1-n}}} \sqrt{\frac{n^2 - 1 + 8\pi^2 KT^2}{2n}},
\]
(33)
\[
\omega_{34} = \frac{-n^2 - 1 + 8\pi^2 KT^2}{n^{\frac{2}{n}} T^{\frac{1}{1-n}}} \sqrt{\frac{2n(n^2 + 2n - 1 + 8\pi^2 KT^2)}{2n}}.
\]
(34)

Thus
\[
\sigma_{33} v^3 + \sigma_{34} v^4 = 0.
\]
(35)

Similarly $\omega_{33} v^3 + \omega_{34} v^4 = 0$.
(36)

In the absence of bulk viscosity, the above mentioned quantities lead to
\[
8\pi \varepsilon = -\frac{1}{n^2 T^2},
\]
(37)
\[
8\pi p = \frac{2n^2 + 2n - 1}{n^2 T^2},
\]
(38)
\[
\cosh\lambda = \sqrt{\frac{n^2 + 2n - 1}{2n}}.
\]
(39)
\[
\sinh \lambda = \sqrt{\frac{n^2 - 1}{2n}}, \quad (40)
\]
\[
\theta = \frac{2[2n^3 + 5n^2 - 1]}{T \sqrt{2n(n^2 + 2n - 1)}}, \quad (41)
\]
\[
\nu^3 = \frac{1}{(nT)^{\frac{1}{2n}}} \sqrt{\frac{n^2 - 1}{2n}}, \quad (42)
\]
\[
\nu^4 = \sqrt{\frac{n^2 + 2n - 1}{2n}}, \quad (43)
\]
\[
q_3 = -\frac{(n^2 + 2n - 1)}{8n n^{\frac{1}{2}}T^{\frac{1}{2}}} \sqrt{\frac{n^2 - 1}{2n}}, \quad (44)
\]
\[
q_4 = \frac{(n^2 - 1)}{8n n^{\frac{1}{2}}T^{\frac{1}{2}}} \sqrt{\frac{n^2 + 2n - 1}{2n}}, \quad (45)
\]
\[
\sigma_{33} = \frac{n^{\frac{1}{2}}(1-n)(n^2 + n - 1)}{3n^{\frac{3}{2}}T^{\frac{1}{2}-\frac{1}{2n}}} \sqrt{\frac{n^2 + 2n - 1}{2n}}, \quad (46)
\]
\[
\sigma_{44} = \frac{-(1-n)(n^2 + n - 1)}{3n^{\frac{3}{2}-\frac{1}{2}}T^{\frac{1}{2}-\frac{1}{2n}}} \sqrt{\frac{n^2 - 1}{2n}}, \quad (47)
\]
\[
\omega_{34} = \frac{-(n-1)}{n^{\frac{3}{2}}T^{\frac{1}{2}} \sqrt{2(n^2 - 1)}}. \quad (48)
\]

Thus
\[
\sigma_{33} \nu^3 + \sigma_{34} \nu^4 = 0. \quad (49)
\]

The physical significance of conditions (35), (36) and (49) are explained by Ellis[48]: The shear tensor \((\sigma_{ij})\) determines the distortion arising in the fluid flow, leaving the volume invariant. The direction of principal axis is unchanged by the distortion, but all other directions are changed. Thus we have \(\sigma_{ij} \nu^j = 0\),

which leads to
\[
\sigma_{33} \nu^3 + \sigma_{34} \nu^4 = 0 \quad (\because \nu_3 \neq 0, \nu_4 \neq 0)
\]

Shear \((\sigma)\) is given by
\[
\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}. \text{Thus } \sigma^2 \geq 0 \text{ and } \sigma = 0 \Leftrightarrow \sigma_{ij} = 0.
\]

The vorticity tensor \((\omega_{ij})\) determines a rigid rotation of cluster of galaxies with respect to a local inertial rest frame. Thus, we have
\[
\omega_{ij} = \eta_{ijkl} \omega^k \omega^l, \text{where } \eta_{ijkl} \text{ is pseudo tensor and } \omega^i = \frac{1}{2} \eta_{ikl} \nu^k \omega^l.
\]
Thus $\omega_{ij}v^i = 0$.

This leads to

$$\omega_{3i}v^3 + \omega_{4i}v^4 = 0 \quad (\because v_4 \neq 0, v_4 \neq 0)$$

The magnitude of $\omega_{ij}$ is $\omega$ and is defined as

$$\omega^2 = \frac{1}{2} \omega_{ij} \omega^{ij}.$$  

Also $\omega = 0 \Leftrightarrow \omega_{ij} = 0$.

The rates of expansion $H_i$ in the direction of x, y and z axes are given by

$$H_1 = \frac{1}{T}, \quad (50)$$

$$H_2 = \frac{1}{nT}, \quad (51)$$

$$H_3 = \frac{1}{nT}. \quad (52)$$

5. Conclusion

The model (21) represents a tilted model. The model starts with a big-bang at $T=0$ and the expansion in the model decreases as time increases. Also, $\sigma_{ij}v^i = 0$ and $\omega_{ij}v^i = 0$ are satisfied as $\sigma_{ij}v^i + \sigma_{4i}v^4 = 0$ and The model has point type singularity at $T=0$ (MacCallum [49]).

In the absence of bulk viscosity, $\epsilon \rightarrow \infty$ when $T \rightarrow 0$ and $\epsilon \rightarrow 0$ when $T \rightarrow \infty$ therefore $\epsilon$ is the decreasing function of time. The reality conditions $\epsilon + p > 0, \epsilon + 3p > 0$ given by Ellis [50] are satisfied when $n^2 > 1$. The model is an expanding universe in which the lines of flow of matter are geodesic, shearing and rotating. The rate of expansion is decreasing function of time and tilt angle is function of time. The metric potentials are constant at $t=1$ i.e. the space time reduces to flat space time. For particularly $n=1$ this model becomes non-shearing with no rotation. So that the universe is not tilted for $n>1$.

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