The rôle of star formation in the Tully-Fisher law

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ABSTRACT
We investigate the influence of the star formation rate on the Tully-Fisher relation. We find that a simple model which combines the empirically-determined star-formation rate with the expected properties of galaxy halos provides a remarkably good fit to the absolute magnitude-rotation speed correlation. We find that the power-law nature, its slope, normalisation and scatter, are all readily accounted for if the Universe has a low density parameter, with or without a cosmological constant and disks are assembled at \( z \sim 1 - 1.5 \). Moreover, this agreement is found simultaneously in 4 wavebands. An Einstein-de Sitter Universe produces disks which are too faint unless the disks are assembled at \( z \sim 0.5 \). The scatter in the relation is due to a combination of the expected range of spin parameters of the halos and the range of formation redshifts. The source of the scatter opens up possibilities of a better galaxy distance indicator, if spectroscopic observations of globular clusters can be used to determine the halo rotation.

The spectrophotometric evolution cannot be done accurately in an analytic way, so for this step we use a synthetic stellar population numerical code (Jimenez et al. 1998; Jimenez et al. 1999). Rather surprisingly, this simple model is able to account extremely well for the Tully-Fisher relation. We are able to reproduce the slope, normalisation and scatter of the relation, simultaneously in 4 wavebands, with very little freedom. The idea of the spin parameter influencing the galaxy properties is of course not new; it has been standard since Fall & Efstathiou (1980), and applications have been made for disk galaxies for example by Kashlinsky (1982), Dalcanton, Spergel & Summers (1997), Jimenez et al. (1997), Jimenez et al. (1998), as well as Mo, Mao & White (1998); van den Bosch (1998). The new element in the analysis here is the combination with the empirical dependence of star formation rate on surface density.

1 INTRODUCTION
The correlation between the luminosity and the rotation speed of spiral galaxies (\( L \propto v^\alpha \), with \( \alpha \sim 3 \)) has been used as a distance indicator for nearby galaxies (see e.g. Strauss & Willick 1995) since its discovery twenty years ago (Tully & Fisher 1977). In the very roughest terms, the correlation is explicable as a consequence of the virial theorem, but the fact that the slope of the relation is dependent on the waveband used suggests that the picture is at best slightly more complicated. There are clearly at least two potentially relevant ingredients in a model to explain the Tully-Fisher relation, these being the properties of the mass distribution (the halo) and the stellar properties of the baryonic disk material. Mo, Mao & White (1998) assume the disk mass-to-light ratio is constant, and therefore argue that it is the halo properties which control the Tully-Fisher relation. On the other hand Silk (1997) argues that self-regulated star formation is the controlling influence, and the Tully-Fisher relation has nothing to do with the halo. The picture may be complicated still further by effects such as energy feedback from stars, and fresh infall of gas over time, motivating an approach where all physical parameters; this is the main motivation for taking this analytic approach. It also appears to account for the relation extremely well. We model the halo with the theoretically known spin properties of collapsed objects, using the isothermal sphere for the density profile, partly for simplicity, and partly because the issue of the dark matter profile in collapsed haloes is not yet settled (Moore et al. 1998).

We model the star formation entirely empirically, using the Schmidt law relating star formation rate to disk surface density (Kennicutt 1998). With this simple model, we are able to obtain analytic expressions for the star formation rate as a function of time.

2 HALO AND STAR-FORMATION MODEL
We follow the notation and method of Mo, Mao & White (1998) in the modelling of the halo by an isothermal sphere, characterised by its mass \( M \), and circular velocity \( V_c \). These are related via the Hubble constant at the redshift \( z \) of formation \( H(z) \)

\[
M = \frac{V_c^2}{10GH(z)}
\]  

(1)

Note that \( V_c \) is not the actual circular velocity of the halo – it is the circular velocity required for centrifugal support in the potential of the halo. In terms of the present Hubble constant and the density parameter in non-relativistic matter and cosmological constant, \( H(z) = H_0 \left[ \Omega_M + (1 - \Omega_M - \Omega_\Lambda)(1 + z)^2 + \Omega_\Lambda(1 + z)^3 \right]^{1/2} \). The disk mass we assume to be \( M_d = m_d M \), where the fraction \( m_d = \Omega_b/\Omega_0 \) and the baryon density is set by nucleosynthesis.
In more astronomical units:

\[ R_d = \frac{\lambda V_c}{10\sqrt{2} H(z)} \]  

(2)

where we have assumed that the disk and halo have the same specific angular momentum (Mestel 1963), \( \lambda = |J/E|^{1/2} G^{-1} M^{-5/2} \)

and \( E \) is the total energy of the halo, and \( J \) its angular momentum. This sets the initial gas surface density, which we assume is accreted on a timescale which is short compared with a Hubble time. We also ignore any gas returned to the ISM by stars, or late infall of fresh gas.

We assume that the star formation rate is set by the empirical Schmidt law (Kennicutt 1998), dependent only on the local gas surface density \( \Sigma_g \), \n
\[ \Psi_{\text{SFR}} = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_g}{M_\odot \text{pc}^{-2}} \right)^{1.4 \pm 0.15} M_\odot \text{y}^{-1} \text{ kpc}^{-2}. \]  

(3)

The time-evolution of the gas surface density is therefore given in terms of the initial gas surface density \( \Sigma_{g0} \) by

\[ \Sigma_g(t) = \left( \frac{\Sigma_{g0}}{0.4Bt} \right)^{-2.5} \]  

(4)

where \( B = 9.5 \times 10^{-17} \) in SI units and \( t \) is measured from the assembly of the disk. From this and the star formation law, we can integrate over the disk to compute the total star formation rate:

\[ \dot{M}_* = \frac{2 \pi B R_d^2 \Sigma_{g0}}{(0.4)^2} \int_0^\infty dy y (\exp y + a)^{-3.5} \]  

(5)

(in kg s\(^{-1}\)), where \( a(t, \Sigma_{g0}) \equiv 0.4Bt \Sigma_{g0}^{0.4} \). This may be written in terms of a generalised hypergeometric function (e.g. Gradshteyn & Ryzhik 1980):

\[ M_*(t) = \frac{50 \pi B R_d^2 \Sigma_{g0}^{1.4}}{49} 3 F_2 (3.5, 3.5, 3.5; 4, 4.5; -a). \]  

(6)

A similar integral gives the remaining gas mass as a function of time as

\[ M_g(t) = 2 \pi R_d^2 \Sigma_{g0} 3 F_2 (2.5, 2.5, 2.5; 3.5, 3.5; -a). \]  

(7)

In more astronomical units:

\[ \dot{M}_* = 247h_z^{-0.6} \left( \frac{m_d}{0.05} \right)^{1.4} \left( \frac{V_c}{250 \text{ km s}^{-1}} \right)^{3.4} \left( \frac{\lambda}{0.05} \right)^{-0.8} \]  

\[ \times 3 F_2 (3.5, 3.5, 3.5; 4, 4.5; -a) M_\odot \text{y}^{-1} \]  

(8)

and

\[ a = 1.06h_z^{0.4} \left( \frac{t}{Gy} \right) \left( \frac{V_c}{250 \text{ km s}^{-1}} \right)^{0.4} \left( \frac{\lambda}{0.05} \right)^{-0.8} \left( \frac{m_d}{0.05} \right)^{0.4}. \]  

(9)

\( h_z \equiv H(z)/100 \text{ km s}^{-1} \text{ Mpc}^{-1}. \) We therefore have a complete description for the star formation rate as a function of time, for any cosmology and any halo parameters \( V_c, z, \lambda \). The relative star formation rate and remaining gas fraction are shown in Fig. 1. The gas fractions remaining depend sensitively on cosmology and circular velocity but are typically in the range 1 – 10% (cf. Roberts & Haynes (1994)). The star formation rate is then fed into a sophisticated spectrophotometric stellar evolution code, which has been described elsewhere (Jimenez et al. 1998; Jimenez et al. 1999).

### Figure 1

The star formation rate and remaining gas fraction, normalised to unity at the disk formation time, as a function of the parameter \( a \propto t \).

Broad-band magnitudes in \( B, R, I \) and \( K' \), for disk formation redshifts \( z=1,2,3 \) and spin parameters \( \lambda=0.025, 0.05, 0.1 \). These spin parameters represent the 10, 50 and 90 percentile points for a gaussian field for any reasonable power spectrum (e.g. Warren et al. 1992). Disks are subject to disruption by major merging events, and there are arguments that present-day disks may have been assembled at \( z \sim 1-2 \) (Weil, Eke & Efstathiou 1998), and there are arguments from disk sizes that argue for even lower assembly redshifts (Mo, Mao & White 1998). In Fig. 2, we show the results for a flat cosmological model with \( h = 0.65, \Omega_0 = 0.3, \Omega_{\Lambda 0} = 0.7 \). Superimposed is the data from the careful study of spiral galaxies in the loose clusters of Ursa Major and Pisces (Tully et al. 1998). The inclination-corrected FWHM of the lines has been converted there to \( W_d' \), which approximates to twice the circular velocity. From Figure 2 and also for the model of Figure 4, we find the following:

- The T-F relation is a good power-law, with about the right slope
- The T-F normalisation is reproduced for plausible formation redshifts
- The scatter is comparable to observation, if disk formation redshifts are not too widely distributed

Moreover, these successes apply simultaneously to all four colours (especially for \( R, I \), and \( K' \), which sample the old population). The blue data are a little more ragged; nevertheless our model provides a good fit leaving little room for any enhanced star formation due to late infall of gas. It is worth noting that the predicted level of star
formation agrees well with that found by Madau (1997); Steidel et al. (1998) since our model has typical initial SFR of 10-100 M\(_{$\odot$}\) y\(^{-1}\) with a highest possible value of 1000 M\(_{$\odot$}\) y\(^{-1}\) for the systems with the highest \(V_c\) and lowest \(\lambda\).

The magnitude is not necessarily a monotonic function of spin parameter, at fixed \(V_c\). Low-spin systems forming at given \(z\) will have high surface densities and high initial star formation rates. At first they will be brighter than the high-spin systems. However, the timescale over which they use up most of their gas is short, and the fading of stars over time may make them fainter than the high-spin systems at late times.

Fig. 3 shows results for an Einstein-de Sitter cosmology. This model forms disks with very high surface densities, which have high initial star formation rates, but which fade excessively unless their formation redshift is low (\(\sim 0.5\)). One could argue that we have a small amount of freedom in moving the curves left and right, since the observed relation uses the FWHM \(W\) of the lines, and we have assumed \(W = 2V_c\). This is probably most appropriate for the isothermal sphere model we assume, but one might allow some flexibility here. We have also investigated an open universe with zero cosmological constant; it fits almost as well as the \(\Lambda\) model.

In Fig. 4, we show the flat model, but we incorporate the peak height-spin parameter anticorrelation (approximately \(\lambda \propto \nu^{-1.4}\)) claimed from analytical studies by Heavens & Peacock (1988) (see also Steinmetz & Bartelmann 1994). There is some support for this from the numerical simulations of Ueda et al. (1994). Lemson & Kauffmann (1999) see no anticorrelation with density, but they present results only for density in a rather large sphere (10 h\(^{-1}\) Mpc). Since the density on galaxy scales (filter length \(\sim 0.5h^{-1}\) Mpc) is only weakly correlated with density on such large scales, it is hard to see how any small-scale correlation could be apparent in such a study. Assuming that the small-scale anticorrelation is present makes little difference to the Tully-Fisher relation, for a reasonable mean formation redshift, which we take to be 1.5 (Note that this refers to the mean collapse redshift of the haloes; the disks observable today may be younger and have a lower assembly redshift). Early-forming high peaks have systematically lower spin, which leads to higher surface density, brighter disks. The predicted scatter in the relation depends on the probability distribution of the disk assembly redshift. We can estimate a lower limit to the scatter by assuming a single formation redshift. For \(z = 1 - 1.5\), the scatter is approximately 0.3 magnitudes in \(K\) and 0.7 in \(B\); the observed values in Ursa Major and Pisces vary from 0.42 in \(R\), \(I\) and \(K\) to 0.55 in \(B\), although a sample of 12 clusters shows a smaller scatter of 0.35 in \(I\) (Tully 1998). Inspection of Fig. 4 shows that a range of formation redshifts can give good agreement with the observed scatter. Note that the uncertainty in the Kennicutt fit to the Schmidt law (equation 3) makes relatively little impact on the results; changing the normalisation of the star formation rate by 30 per cent either way alters the magnitudes by about 0.15. Further we note that the anticorrelation would ease the problem of disks being too small compared with observation (Mo, Mao & White 1998), since the later-forming haloes have typically larger spins than average. Late-forming haloes forming at redshift 1 (when the mean is 1.5) are 40% larger if the anticorrelation holds.

The thick lines in Fig. 4 show the theoretical limit for galaxies for forming stars according to the Toomre (1964) stability criterion (see also Kennicutt 1989). As discussed in Jimenez et al. (1997) halos with \(V_c \leq 40 \text{ km s}^{-1}\) fail to form stars for any value of \(\lambda\), while for halos with \(V_c \geq 120 \text{ km s}^{-1}\) stars are formed in the disk for any value of \(\lambda\) as shown in Fig. 4 of Jimenez et al. (1997). The absence of faint discs is also present in other samples (see e.g. Strauss & Willick 1995), although observational selection effects may be the explanation for current data (Tully, priv. comm.). To illustrate this we have also plotted the near-dark galaxies NGC2915 (diamond) and DDO154 (triangle) (see Meurer et al. 1996). These systems fall quite well in the scenario described in Jimenez et al. (1997) since only the central parts of these galaxies have formed stars, and have a dark HI disk. The reason why they fall below the theoretical TF law is due to the fact that most of its HI mass is not in stars (they have mass-to-light ratios of about 80), thus they are under-luminous.

Fig. 5 shows the predicted Tully-Fisher relation for disks observed at redshift 1. If the disk assembly is also at redshift 1, then the disks are extremely bright in all wavebands. For disk assembly at redshifts 1.5 and 2, the Tully-Fisher relation is generally a little brighter, the degree of brightening depending in detail on circular velocity, band and formation redshift. For massive systems forming at redshift 1.5, the systems are about 0.5 – 1.5 magnitude brighter in blue, somewhat larger than the observations of Vogt et al. 1997, although the samples are not large enough as yet for the situation.
to be clear (Rix et al. 1997, Hudson et al. 1998, Simard & Pritchet 1998).

The model predicts some dependence of the Tully-Fisher relation on the surface brightness of the galaxy, in the sense that higher surface brightness disks should have higher luminosities for given rotation speed. The effect is not large, as witnessed by the tightness of the predicted relations in Fig. 4, and there is also some overlap in the surface brightnesses at fixed $M$, $V_c$ arising from a spread in disk assembly redshifts. Thus the coincidence of TF relations of high and low surface brightness disks (Sprayberry et al. 1995; Zwaan et al. 1995) presents no great difficulty for the model. More serious is the bimodal surface brightness distribution of disks observed by Tully & Verheijen (1997) (but see Mihos, Spaans & McGaugh 1998 for an alternative view), which does not appear naturally in this model. The authors suggest that the high surface brightness disks may be self-gravitating, whereas the LSBs are dark-matter dominated. We cannot exclude such a possibility, as our model assumes a given halo potential. If the passage to self-gravitation means only more early star formation (from higher surface densities), then the results we present are virtually unchanged: at these wavebands, the distinction between a burst of star formation and the ongoing trickle which our model predicts is negligible for the high surface density disks. Observations at $U$ would be required to distinguish the histories.

Figure 3. As Fig. 2, but for an Einstein-de Sitter universe and formation redshifts of 0.5 (solid line), 1.0 (dotted line) and 1.5 (dashed line).

Figure 4. As Fig. 2, but including the halo peak height-spin parameter anticorrelation predicted by Heavens & Peacock (1988). The thick solid lines shows the expected theoretical threshold for halos that fail to form stars (Jimenez et al. 1997), for formation redshift 1.5. System below the line will fail to form any significant population of stars and thus will be ‘dark’. Comfortably enough, no galaxies are observed below the threshold in any 4 bands, except for system with stars confined to the central regions with few stars in the disk, such as NGC2915 (diamond) and DDO154 (triangle). Also plotted as open squares are the data for LSBs from Zwaan et al. (1995), this illustrates the point that LSB and HSB galaxies lie in the same TF law.

4 SUMMARY AND PROSPECTS FOR A BETTER DISTANCE INDICATOR

We have combined the theoretical properties of dark matter halos with the empirically-determined dependence of star formation rate on disk surface density to predict a correlation between luminosity and circular velocity which is compared with observation. Remarkably good agreement is found, in 4 wavebands, with the recent study of Tully et al. (1998), for a flat universe dominated by a cosmological constant if the disk assembly redshift is $z = 0.5$. This explanation of the Tully-Fisher relation falls between the halo-determined extreme of Mo, Mao & White (1998) and the halo-independent model of Silk (1997), however, it is only a partial explanation, as it relies on the empirical Schmidt law (see Silk 1997 for some relevant discussion). This study suggests that
there are two parameters which lead to scatter in the Tully-Fisher relation. These are the spin parameter of the halo, and the redshift at which the disk is accumulated. The age of the disc appears to be less important than the redshift, which controls the initial surface density of the disk and hence the star formation rate. The general effect of the formation redshift and the spin parameter is almost inevitable, and is likely to be more robust than the specific model calculations presented here. The relative influence of formation redshift and spin parameter is model-dependent. In particular, if, as has been claimed (Heavens & Peacock 1988; Ueda et al. 1994), there is an anticorrelation between peak overdensity and spin parameter, then the predicted Tully-Fisher relation is changed only slightly. The scatter might be narrowed by measurement of the age and spin parameter. The latter is a difficult quantity to measure, but could be attempted by looking at the systematic velocity of halo stars which are not formed dissipatively in the disk. Individual stars would be impossible to measure, especially with a bright disk present, but it is possible that for nearby galaxies the integrated light from globular clusters could be used.

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