Abstract

This research investigates how to determine whether two rankings can come from the same distribution. We evaluate three hybrid tests: Wilcoxon’s, Dietterich’s, and Alpaydin’s statistical tests combined with cross-validation, each operating with folds ranging from 5 to 10, thus altogether 18 variants. We have used the framework of a popular comparative statistical test, the Sum of Ranking Differences, but our results are representative of all ranking environments. To compare these methods, we have followed an innovative approach borrowed from Economics. We designed eight scenarios for testing type I and II errors. These represent typical situations (i.e., different data structures) that cross-validation (CV) tests face routinely. The optimal CV method depends on the preferences regarding the minimization of type I/II errors, size of the input, and expected patterns in the data. The Wilcoxon method with eight folds proved to be the best under all three investigated input sizes, although there were scenarios and decision aspects where other methods, namely Wilcoxon 10 and Alpaydin 10, performed better.

1 Introduction

Ranking objects is one of the most commonly applied computational tasks. Query results are ranked by search engines; universities are ranked by excellence; and sports teams are ranked according to various performance measures. Yet, there are still unresolved tasks regarding the statistics of rankings. One such task is to attribute uncertainty to ranks. This is an extremely difficult problem as the underlying numerical scale contains many variance elements (bias,
random errors, precision, accuracy, ruggedness, repeatability, reproducibility, etc.). It would be expedient to calculate the standard deviation of ranks, although it can be done in at least two ways: in terms of objectives/solutions and of periods/use cases. For instance, a university ranking is based on various objectives (number of published papers, student/staff ratio, etc.) and is different with regard to each objective. Furthermore, ranking in terms of a single objective is uncertain as it changes each year. In addition, aggregation itself is riddled with various paradoxes (Condorcet paradox, Arrow’s and Gibbard–Satterthwaite's impossibility theorem, discursive dilemma, etc.) (Moulin, 2016). Hence, it is no wonder that the scientists do not bother with the error of ranks but calculate them for the numerical scale. The uncertainty in a set of many alternative rankings, or in the raw data itself (from where rankings are generated) are moderately studied, see Section 2.

Another closely related problem is the comparison of rankings. How can we determine that two rankings are generated from the same distribution? One way to answer this question is by using cross-validation (CV) techniques: looking at the subsets of the positions of two rankings, measuring their similarity, and comparing the obtained values for different subsets.

A unique contribution of this paper is in comparing the efficacy of CV methods in a ranking environment. For the latter, we have used the framework of Sum of Ranking Differences (SRD) – a popular comparative statistical test. This is a convenient choice as SRD is a ranking-based test, that uses CV for grouping the compared methods. In this paper, we have aimed to kill two birds with one stone: provide a theoretical foundation for the validation element of SRD, with our results being applicable in a more general setting.

SRD, introduced in ref. (Héberger, 2010), is a relatively novel test that is especially suitable for comparing methods in multi-objective optimization environments. In recent years, there have been many published papers, with topics ranging from pharmacology (Vajna et al., 2012) and chromatography (Andrić et al., 2016), through multi-criteria decision-making (Gere et al., 2021) and machine learning (Moorthy et al., 2017), to political science (Sziklai and Héberger, 2020), and even sports (West, 2018), that apply or further extend the technique, showing its versatility.

SRD scores are computed by converting the input data into rankings, and then calculating the Manhattan distance of each method’s ranking and the reference ranking. The latter can be an external gold standard or an aggregate from the data (Héberger, 2010; Sziklai and Héberger, 2020). The SRD algorithm contains two validation steps:

- In the so-called permutation test, we compare the methods’ scores to the SRD scores of random rankings. A detailed overview of this method is available in ref. (Héberger and Kollár-Hunek, 2011).

- In the second step, we cross-validate the results by re-sampling the data. We then assign uncertainties to the SRD scores, which are deterministic by nature, using cross-validation (CV). It also allows us to group and compare the methods differently than the pure SRD scores.

SRD scores can be seen as a metric of the mean absolute error (on a rank scale) to the reference, which is the gold standard. It is easy to draw a parallel with machine learning where the CV of the chosen error metric is a standard technique. There are a lot of approaches in the literature about how to make conclusions from the many error scores calculated
on different subsamples of the data. Not all of these can be easily adapted to ranking frameworks.

Originally, by Kollár-Hunek and Héberger (2013), the Wilcoxon signed-rank test was proposed for CV purposes. Dietterich’s CV \( t \)-test (Dietterich, 1998) and Alpaydin’s CV \( F \)-test (Alpaydin, 1999) are popular in machine learning for CV. An original contribution of this paper is the adaptation of these tests to the ranking framework. In this paper, we have examined which CV technique performs better within a ranking framework. We have also proposed several scenarios for testing type I and II errors, see Section 4.1. Based on these scenarios, practitioners can decide which CV algorithm is more suitable for their use case. The scenario analysis reveals a mixed picture. Different CV methods prevail under different circumstances (type I/II errors, input sizes, data structure, etc.). The Wilcoxon method with eight folds seems to be the best compromise, but its 10-fold version as well as the Alpaydin test with 10 folds are also viable alternatives.

2 Literature Overview

Cross-validation Cross-validation is a general method of statistical analysis with various purposes, but mainly for model validation. It traditionally involves multiple rounds with three steps: the bi-partitioning of the data set, analysis of one partition, and validation/testing on the other one. Results from these rounds may be analyzed after some aggregation, or individually. There are multiple variants with tri-partitioning, to generate distinct validation and testing sets. For a general introduction to the topic, see the survey of Arlot and Celisse (2010). CV is most popular in predictive analytics to assess the accuracy of prediction on new (test) data, independent of the already-seen (training) data. The aggregation of the results yields a more accurate estimate of the model prediction error. It is also applied in model parameter selection, e.g., by Varma and Simon (2006), as an optimizer for classification parameters in SVM and shrunken centroid models. However, many statistical tools require modifications of the regular CV tools to adapt to different applications. For example, Szymańska et al. (2012) introduced a special CV scheme in partial least squares discriminant analysis (PLS-DA), and Bro et al. (2008) tested and reviewed the available CV techniques for PCA. Domains with dependent data sets also require a revision of the CV methods to address dependency issues. Hijmans (2012) addresses spatial sorting bias (a usual problem in species distribution models) using pairwise distance sampling. Roberts et al. (2017) examine various techniques in the case of dependent data sets and recommend that block CV should be used wherever dependence structures exist in a data set. Various problems arise during the parameter selection of the CV techniques themselves, e.g., Fushiki (2011) examines bias correction of the \( k \)-fold CV in the prediction setup for small \( k \)s with large samples. Bayesian modifications are also studied in CV, e.g., by Vehtari et al. (2017). Isaksson et al. (2008) state that CV and bootstrap are in fact unreliable in small samples, and promote Bayesian alternatives because the uncertainty introduced by the CV is too large. Different repetition techniques were also introduced on top of the CV methods to assess different problems, e.g., for parameter tuning in classification and regression problems by Krstajic et al. (2014), for optimizing the complexity of regression models for small data sets by Filzmoser et al. (2009), and for reducing the variability of the estimator by Kim
(2009). Domain-specific problems also require the study of the CV techniques, e.g., Wenger and Olden (2012) demonstrate the importance of considering model transferability in ecology based on different cross-validation approaches.

**Uncertainty of rankings** There are many approaches to measure the uncertainty among many rankings, but the most direct way to do so is to calculate the standard deviation of the rankings as if they were simply real vectors. For rankings $\pi_1, \ldots, \pi_m$ of $n$ elements, it is $SD = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (\pi_i - \bar{\pi})^2}$, where $\bar{\pi}$ is simply the mean of the sample (or objects) rankings as vectors. This $L_2$ approach has been applied in many papers, e.g. in refs. (Falivene et al., 2010; Palmer et al., 2009; Triantafilis et al., 2001; Farshadfar and Amiri, 2016). Aside from the above metric, Barlow and Ballin (1976) measure uncertainty with another metric taken from information theory. Rosander (1936) calculates the standard error of the mean ranking based on the rank-order correlation. In refs. (Lockwood et al., 2002; Zampetakis and Moustakis, 2010), the uncertainty of rankings is considered using Bayesian modeling of the raw data, from where the rankings originate. Zuk et al. (2007) tackle the uncertainty in a ranking by introducing noise to the raw data, and compare the original ranking to the noisy one with Top-K-List overlap and Kendall’s Tau measure.

Aggregating rankings to obtain an optimal one is a problem naturally arising in many fields of science, from multi-criteria decision-making and information fusion to social choice. In the previously cited ref. (Lockwood et al., 2002), the authors also dealt with the problem of finding an optimal ranking in their Bayesian framework. The approach of Pacheco and Krohling (2018) uses the mean and standard deviation of the raw data to determine a final ranking. Tavanaei et al. (2018) take into account the uncertainty of ranks by weights of a parameterized rank aggregation process. Klementiev et al. (2007) work with the rankings only, instead of using the properties of raw data, based on the principle of rewarding ranking agreements. Tehrani et al. (2012) use the Choquet integral as an underlying model for representing ranking before the aggregation. There are countless other techniques for ranking aggregation, with a lot of them coming from e.g. voting theory where preference and judgment aggregation are age-old problems (Endriss, 2018; Mongin, 2012).

The aforementioned references usually deal with the uncertainty of a set of many alternative rankings, or see the uncertainty in the raw data itself. A distinctive contribution of this paper is that we grasp the uncertainty of a single ranking by looking at its partial rankings coming from manyfold CV.

### 3 Methodology

SRD requires the input data to be arranged in a matrix form: rows $(1, \ldots, n)$ represent objects (statistical cases, compounds, features, etc., depending on the use case), whereas columns $(1, \ldots, m)$ represent the variables (solutions, models, methods, etc.) to be compared. There is one designated column containing a reference value for each object. These can be a previously established gold standard, an estimation, or even an aggregation from the input data (this last technique is also called data fusion). The input matrix is transformed into a ranking matrix by ranking the values in each column from the smallest to the largest.
element. The resulting \( n \times (m + 1) \) matrix contains \( m \) rankings \( \pi_1, \ldots, \pi_m \) associated to the variables and a reference ranking \( \pi_r \).

3.1 The Metric Utilized by SRD

The distance metric utilized in the SRD framework is simply the \( L_1 \) norm or city block (Manhattan) distance of the rankings; yet another name for this (specifically for rankings) is the Spearman’s footrule if no ties are present in the ranking. For rankings \( \pi_i, \pi_j \), we will denote it with \( d(\pi_i, \pi_j) := \sum_{k=1}^n |\pi_i(k) - \pi_j(k)| \). In particular, the distance of \( \pi_i \) from the reference will be denoted as \( \text{SRD}(\pi_i) := d(\pi_i, \pi_r) \).

Properties of the Spearman’s footrule have been intensively studied over the symmetric group \( S_n \), which contains the permutations over \( 1, \ldots, n \).

The maximal distance is easy to compute:

\[
M := \max_{i,j} d(\pi_i, \pi_j) = \begin{cases} 
\frac{n^2}{2} & \text{if } n \text{ is even} \\
\frac{n^2-1}{2} & \text{if } n \text{ is odd}
\end{cases}
\]

(1)

The distance is right-invariant to the composition operation, meaning

\[
d(\pi_i, \pi_j) = d(\pi_i \sigma, \pi_j \sigma) \quad \forall \sigma \in S_n.
\]

For convenience and interpretability, we usually normalize the distance by the maximal distance of Eq. 1:

\[
\text{SRD}(\pi) := \frac{\text{SRD}(\pi)}{M}.
\]

The normalized \text{SRD} values also make comparison possible if the numbers of objects are different.

Diaconis and Graham (1977) showed that the distance is asymptotically normal (as \( n \to \infty \)) if we choose two permutations uniformly from \( S_n \):

\[
d(\cdot, \cdot) \sim \mathcal{N} \left( \frac{1}{3} n^2 + O(n), \sqrt{\frac{2}{45}} n^3 + O(n^2) \right).
\]

Due to right-invariance, the distribution is the same if we fix one of the permutations, as it happens in the SRD framework where we have a fixed reference ranking. After normalization:

\[
\text{SRD}(\cdot) \sim \mathcal{N} \left( \frac{2}{3}, \sqrt{\frac{8}{45n}} \right).
\]

(2)

Based on the above properties, a hypothesis test can be created to answer the question: Is a specific model ranking \( (\pi_{\text{model}}) \) close enough to the reference ranking \( (\pi_r) \)? The null hypothesis \( H_0 \) is that the ranking is uniformly selected from \( S_n \). The test statistic is \( \text{SRD}(\pi_{\text{model}}) \), which is asymptotically normal for large \( n \) under \( H_0 \) by Eq. 2. If ties are present or \( n \) is small, the exact discrete distribution should be used (Héberger and Kollár-Hunek, 2011).
3.2 Uncertainty of SRD

As already mentioned, SRD values are deterministic by nature, but with CV, we can introduce uncertainty to them in order to compare two rankings with each other.

3.2.1 Wilcoxon Signed-Rank Test

The Wilcoxon signed-rank test was proposed for CV purposes in ref. (Kollár-Hunek and Héberger, 2013). Take random subsets \((A_1, \ldots, A_k)\) of size \(n - \lceil n/k \rceil\) from the \(n\) rows/objects. Let \(\text{SRD}_{j,i}\) denote the SRD (from the reference) on fold \(A_i\) for the ranking \(\pi_j\). This results in a paired sample of size \(k\) for the two models to compare:

\[
\{(\text{SRD}_{1,1}, \text{SRD}_{2,1}), (\text{SRD}_{1,2}, \text{SRD}_{2,2}), \ldots, (\text{SRD}_{1,k}, \text{SRD}_{2,k})\}.
\]

To this, we apply the signed-rank test. We take the sub-sample (of size \(k_r < k\)) containing the non-zero absolute differences \(|\text{SRD}_{1,i} - \text{SRD}_{2,i}|\), and then rank this subsample. The test-statistic \(W\) comes from

\[
W^+ = \sum_{i=1}^{k} \mathbb{I}(\text{SRD}_{1,i} > \text{SRD}_{2,i}) \cdot R_i, \quad \text{and} \quad W^- = \sum_{i=1}^{k} \mathbb{I}(\text{SRD}_{1,i} < \text{SRD}_{2,i}) \cdot R_i,
\]

where \(\mathbb{I}()\) is the indicator function, and \(R_i\) is the rank of the \(i\)th fold in the aforementioned subsample. \(W\) can be \(\min(W^+, W^-)\), \(W^+ - W^-\), or \(W^+\) itself. In each case, \(W\) has a specific distribution (depending on \(k_r\)) under the \(H_0\) that the difference between the pairs follows a symmetric distribution around zero.

3.2.2 Dietterich \(5 \times 2\) CV \(t\)-test

Dietterich (1998) proposed this test for determining whether there is a significant difference between the error rates of the two classifiers. Here, we show how to apply the test within the SRD framework for comparing the rankings of two models \((\pi_1, \pi_2)\).

We start by taking random subsets \((A_1, \ldots, A_k)\) and their complements \((A_1^c, \ldots, A_k^c)\) from the \(n\) rows (objects) of sizes \(\lceil n/2 \rceil\) and \(\lceil n/2 \rceil\). In the original paper and in most applications, \(k = 5\). Let \(\text{SRD}_{j,i}\) denote once again the SRD (from the reference) on fold \(A_i\) for the ranking \(\pi_j\), and let \(\text{SRD}_{j,i}^c\) denote the same on the complement \(A_i^c\).

First, look only at the fold \((A_i, A_i^c)\). Calculate the differences of the normalized SRDs in both subsets,

\[
\Delta_i := \text{SRD}_{1,i} - \text{SRD}_{2,i}, \quad \text{and} \quad \Delta_i^c := \text{SRD}_{1,i}^c - \text{SRD}_{2,i}^c,
\]

and then calculate their average and sample variance:

\[
\overline{\Delta}_i = \frac{1}{2} (\Delta_i + \Delta_i^c), \quad s_{\Delta,i}^2 = (\Delta_i - \overline{\Delta}_i)^2 + (\Delta_i^c - \overline{\Delta}_i)^2.
\]

For large enough \(n\), \(\Delta_i\) and \(\Delta_i^c\) are asymptotically normal under the \(H_0\) of uniformly selecting a ranking from \(S_n\), thus we can get an approximately \(t\)-distributed test statistic:

\[
\frac{\Delta_i}{\sqrt{\frac{1}{k} \sum_{i=1}^{k} s_{\Delta,i}^2}} \sim t_k.
\]
and the same applies to the complementing fold. This way, we have $2k$ different test statistics, of which we can choose any for the evaluation of the hypothesis test.

### 3.2.3 Alpaydin $5 \times 2$ CV $F$-test

The $5 \times 2$ CV $F$-test of Alpaydin (1999) was proposed as an improvement on the CV $t$-test of Dietterich (1998). Here, we show how to apply the test within the SRD framework for comparing the rankings of two models ($\pi_1, \pi_2$). Up until the calculation of the $\Delta_i, \Delta_{i\cdot}$ pairs and their sample variances for each fold, the setup is the same as in Section 3.2.2. However, the test statistic is approximately $F$-distributed:

\[
\frac{1}{2} \left( \frac{\Delta_i^2 + \Delta_{i\cdot}^2}{s^2_{\Delta,i}} \right) \sim F_{2,1}.
\]

Aggregating these for all folds results in:

\[
\frac{1}{2k} \sum_{i=1}^{k} \left( \frac{\Delta_i^2 + \Delta_{i\cdot}^2}{s^2_{\Delta,i}} \right) \sim F_{2k,k}.
\]

Note that this assumes the independence of the repeated measurements on the folds, and again, the independence of the numerator and denominator too. Instead of aggregating the statistics of the folds, one can aggregate the dependent $p$-values coming from the folds separately. The correct aggregation of $p$-values assumes the independence of only the numerator and denominator.

### 4 Evaluation

How can the best fitting option be selected in a complex decision situation where all the competing solutions seem fair in some way? A formal approach to this, commonly used in economics, is axiomatization. We characterize the methods by the properties they satisfy (Pareto optimality, symmetry, monotonicity, etc.). We clarify which properties are important for us and then choose the method that best suits our needs.

Statistical tests do not fit into this scheme readily. Their behavior is stochastic rather than binary, meaning none of them satisfy properties 100% of the time. Nevertheless, when choosing from tests, an axiomatic mindset can be useful; the idea is to come up with scenarios that the tests will likely face in practice, and then observe through a simulation which test is more apt in which scenario. This type of analysis is especially suitable for rankings, as differences between two rankings can be characterized fairly well.

We have evaluated three hybrid tests (Wilcoxon, Dietterich, and Alpaydin) under various parametrizations. The aim was to find the one that is the most efficacious in categorizing solutions. We have used the number of folds as parameters and varied it between 5 and 10.

#### 4.1 Scenarios

We analyzed eight scenarios under three assumptions on the size of the rankings ($n = 7$, $n = 13$, and $n = 32$). The size options aim to represent the typical data sizes. There is
Table 1: Ranking transformations used in the scenarios

| Identifier | Description |
|------------|-------------|
| x          | We apply $x$ number of random inversions (switching of neighboring elements) on the reference ranking. |
| xt         | We apply $x$ number of random inversions on the top $\lfloor n/2 \rfloor$ positions of the reference ranking. |
| xb         | We apply $x$ number of random inversions on the bottom $\lceil n/2 \rceil$ positions of the reference ranking. |
| 1u         | We select a random element (the underdog) from the bottom $\lceil n/2 \rceil$ positions of the reference ranking and switch it with the first element. |
| 4m         | We consecutively select four random positions between 1 and $n - \lceil n/4 \rceil$ from the reference ranking and switch the selected element at position $s$ with the element at position $s + \lceil n/4 \rceil$. |

no point in further increasing $n$ as anything that is true for $n = 32$ will probably stay true for larger row sizes. Also note that $n = 7$ is an odd number and almost too small for CV. However, since practitioners will not refrain from applying cross-validation for sub-optimal data sizes, we felt the need to test this case as well. For $n = 7$ and $k \geq 7$, the Wilcoxon reduces to leave-one-out CV. In particular, if the number of folds, $k$, exceeds seven, we are forced to use bootstrapping; some rows are left out more than once.

All rankings that come from the same distribution are alike, rankings that come from different distributions are different in their own way\(^{1}\)—hence we have looked at five scenarios for type II errors but only three for type I errors. These scenarios cover all typical situations, and other scenarios are unlikely to yield a new aspect as they would constitute a transition in between these ones.

In the following list, we have described the scenarios in detail. We started with a reference ranking, which is just the ordered list of numbers from 1 to $n$. Then, we composed two additional rankings, denoted by A and B, by making some transformation to the reference ranking. For checking type I error, we drew both rankings from the same distribution (Scenarios 1-3). In type II scenarios, the rankings were constructed in different ways (Scenarios 4-8). Table 1 displays the ranking transformations that were used to produce the rankings, and Figure 1 shows their average distance from the reference.

A scenario is defined as a pair $(a|b)$, where $a$ and $b$ refer to the transformations used to create rankings A and B respectively.

1. $(2n|2n)$: This scenario investigates what happens if both rankings are drawn from the same distribution and their distance to the reference ranking is relatively large.

2. $(n|n)$: Similar to the previous one but with fewer inversions, so the rankings are closer to the reference.

3. RT I.: We picked a transformation uniformly and randomly from the set $2n$, $n$, $(n/2)t$, $(n/2)b$, $1u$, $4m$. Both rankings A and B were drawn from the selected distribution. This scenario demonstrates what we can expect from a CV method regarding type I error when we do not have prior information on the rankings’ distribution.

\(^{1}\)We are slightly paraphrasing Lev Tolstoy’s Anna Karenina here.
4. \((2n | n)\): This scenario investigates what happens if the rankings are drawn from different distributions, that is, if their mean distance to the reference ranking is different, because the second ranking is a result of fewer inversions, so it is closer to the reference.

5. \(((n/2)\times t | (n/2)\times b)\): This scenario shows what happens when the data is structured. For instance, for data collected from two periods, the solutions perform differently on the first and second periods. Note that the expected distance from the reference is approximately the same for both A and B (cf. Figure 1).

6. \((2n | 1u)\): This scenario tests the presence of outliers. Rankings A and B are of the same distance from the reference. However, the former is constructed by applying many small inversions, while in ranking B, we only swap one pair of elements. To illustrate this scenario, let us borrow an example from sports. Ranking A shows how the actual result of a sporting event, e.g. the soccer World Cup, differs from the preliminary ranking. Ranking B is the same as the preliminary ranking except that Burkina Faso wins, relegating Brazil to the second half of the points table. The preferability of ranking A or B depends on the application, but a CV method should be able to distinguish between the two rankings.

7. \((x | 4m)\) Similar to the previous scenario, but having more, albeit less extreme, outliers. The number of inversions for ranking A, \(x\), is chosen in such way that the expected distance from the reference for both rankings is approximately the same.

8. RT II.: We picked a transformation uniformly and randomly from the set \(2n, n, (n/2)t,\)
(n/2)b, 1u, 4m. Ranking A was drawn from the selected distribution, and for B, we picked another transformation randomly. This scenario demonstrates what can we expect from a CV method regarding type II error when we do not have prior information on the rankings’ distribution.

4.2 Simulation Results

For each scenario and data size, we performed 10 simulation rounds, each comprising 100,000 runs. We randomly generated instances of rankings A and B, applied the CV technique, and noted whether it accepted or rejected the null hypothesis at 5% level of significance. Tables 2-4 show the average rate of rejections in the 10 rounds. In type I scenarios (1–3) the lower the rejection rate the better—the rankings come from the same distribution, so the CV method is expected to accept the null hypothesis. In type II scenarios (4–8), the opposite is true; the higher the rejection rate, the better. We also observed the standard error on the average rejection rate of the 10 rounds, which falls under 0.003 in all scenarios.

There are several conclusions that we can infer from Tables 2–4:

• There is no single best CV method. Depending on the kind of scenarios that are relevant for us, we may choose different techniques.

• The Dietterich test rarely rejects the null hypothesis. Thus, it excels in type I scenarios but performs poorly in type II scenarios.

• As expected, increasing the number of folds raises the rate of rejection for all methods. This results in a trade-off between the efficiency of methods in type I and type II scenarios.

As the tables show, different techniques excel in different scenarios. If practitioners have some preliminary knowledge about the distribution of the rankings that the investigated methods produce, they can choose the appropriate CV method. In many cases, however, these distributions are unknown. Therefore, to select the most suitable method for generic purposes, we need to dig deeper into the data.

4.3 Selection Criteria

Here, we have listed some aspects according to which the data can be assessed. We have ranked the methods according to each aspect, and then aggregated the rankings using the Borda count.

**Discriminative power (DISC):** We have taken the absolute difference between the rejection rate in the RT II and RT I scenarios; the underlying idea is that the number of folds

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2 Note that, if all the rigorous assumptions of the applied statistical tests held firm, these numbers would be around 5%. Both Dietterich (1998) and Alpaydin (1999) knew well that these assumptions are not satisfied in general but dismissed concerns based on the empirical success of the tests.
### Table 2: Null hypothesis testing for n = 32 – rejection rate in different scenarios

| CV method / number of folds | type I scenarios | RT I | type II scenarios | RT II |
|-----------------------------|------------------|------|-------------------|-------|
| Wilcoxon 5                  | 0.356569         | 0.303679 | 0.175987 | 0.741035 |
| Wilcoxon 6                  | 0.357112         | 0.303986 | 0.175987 | 0.741035 |
| Wilcoxon 7                  | 0.473411         | 0.429921 | 0.191380 | 0.831334 |
| Wilcoxon 8                  | 0.588461         | 0.533774 | 0.163902 | 0.825753 |
| Wilcoxon 9                  | 0.594213         | 0.534122 | 0.163902 | 0.825753 |
| Dietterich 5                | 0.178738         | 0.176794 | 0.163902 | 0.825753 |
| Dietterich 6                | 0.184617         | 0.179388 | 0.163902 | 0.825753 |
| Dietterich 7                | 0.201583         | 0.198745 | 0.163902 | 0.825753 |
| Dietterich 8                | 0.209073         | 0.207973 | 0.163902 | 0.825753 |
| Dietterich 9                | 0.210821         | 0.207912 | 0.163902 | 0.825753 |
| Dietterich 10               | 0.214646         | 0.211956 | 0.163902 | 0.825753 |
| Alpaydin 5                  | 0.205458         | 0.203899 | 0.163902 | 0.825753 |
| Alpaydin 6                  | 0.252986         | 0.244541 | 0.163902 | 0.825753 |
| Alpaydin 7                  | 0.308622         | 0.306652 | 0.163902 | 0.825753 |
| Alpaydin 8                  | 0.365651         | 0.355574 | 0.163902 | 0.825753 |
| Alpaydin 9                  | 0.420021         | 0.408763 | 0.163902 | 0.825753 |
| Alpaydin 10                 | 0.474681         | 0.462281 | 0.163902 | 0.825753 |

### Table 3: Null hypothesis testing for n = 13 – rejection rate in different scenarios

| CV method / number of folds | type I scenarios | RT I | type II scenarios | RT II |
|-----------------------------|------------------|------|-------------------|-------|
| Wilcoxon 5                  | 0.304751         | 0.252904 | 0.33819 | 0.656070 |
| Wilcoxon 6                  | 0.268702         | 0.217531 | 0.33819 | 0.656070 |
| Wilcoxon 7                  | 0.377361         | 0.315731 | 0.33819 | 0.656070 |
| Wilcoxon 8                  | 0.363692         | 0.302224 | 0.33819 | 0.656070 |
| Wilcoxon 9                  | 0.464543         | 0.406123 | 0.33819 | 0.656070 |
| Dietterich 5                | 0.179819         | 0.171503 | 0.14296 | 0.745924 |
| Dietterich 6                | 0.194771         | 0.184166 | 0.14296 | 0.745924 |
| Dietterich 7                | 0.20905          | 0.198805 | 0.14296 | 0.745924 |
| Dietterich 8                | 0.216319         | 0.205356 | 0.14296 | 0.745924 |
| Dietterich 9                | 0.221547         | 0.207893 | 0.14296 | 0.745924 |
| Dietterich 10               | 0.224842         | 0.211944 | 0.14296 | 0.745924 |
| Alpaydin 5                  | 0.177043         | 0.165078 | 0.14296 | 0.745924 |
| Alpaydin 6                  | 0.215996         | 0.202365 | 0.14296 | 0.745924 |
| Alpaydin 7                  | 0.260259         | 0.241922 | 0.14296 | 0.745924 |
| Alpaydin 8                  | 0.306552         | 0.284003 | 0.14296 | 0.745924 |
| Alpaydin 9                  | 0.351683         | 0.324057 | 0.14296 | 0.745924 |
| Alpaydin 10                 | 0.396722         | 0.366118 | 0.14296 | 0.745924 |

### Table 4: Null hypothesis testing for n = 7 – rejection rate in different scenarios

| CV method / number of folds | type I scenarios | RT I | type II scenarios | RT II |
|-----------------------------|------------------|------|-------------------|-------|
| Wilcoxon 5                  | 0.195301         | 0.153787 | 0.180724 | 0.537498 |
| Wilcoxon 6                  | 0.163101         | 0.123560 | 0.150414 | 0.321538 |
| Wilcoxon 7                  | 0.298591         | 0.241511 | 0.2430222 | 0.488997 |
| Wilcoxon 8                  | 0.303048         | 0.249167 | 0.24477 | 0.488997 |
| Wilcoxon 9                  | 0.428605         | 0.386654 | 0.451112 | 0.488997 |
| Dietterich 5                | 0.426488         | 0.378171 | 0.427973 | 0.736539 |
| Dietterich 6                | 0.185265         | 0.169846 | 0.13018 | 0.196852 |
| Dietterich 7                | 0.178824         | 0.161214 | 0.128726 | 0.196852 |
| Dietterich 8                | 0.221859         | 0.211266 | 0.166147 | 0.228643 |
| Dietterich 9                | 0.231075         | 0.221781 | 0.176007 | 0.238156 |
| Dietterich 10               | 0.237529         | 0.230241 | 0.182806 | 0.238156 |
| Alpaydin 5                  | 0.113434         | 0.094250 | 0.075262 | 0.127464 |
| Alpaydin 6                  | 0.124051         | 0.097556 | 0.083011 | 0.127464 |
| Alpaydin 7                  | 0.155288         | 0.132312 | 0.101960 | 0.189265 |
| Alpaydin 8                  | 0.185588         | 0.174387 | 0.120632 | 0.226551 |
| Alpaydin 9                  | 0.181246         | 0.176427 | 0.142352 | 0.263768 |
| Alpaydin 10                 | 0.241476         | 0.190664 | 0.154616 | 0.291209 |
largely explains the rejection rate of the methods. The amount a method rejects RT II instances more often than RT I instances, shows how well it can distinguish the two situations. We had no prior knowledge about the distribution of the rankings, hence we used random scenarios.

**Maximum distance from the best option (MAXDIFF):** In each scenario, we identified the best option, that is, the CV method with the best rejection rate (lowest for type I/highest for type II scenarios). Then, for each method, we measured the difference between that particular method’s performance and that of the best option. Finally, a method was evaluated by the maximum of the differences across all scenarios.

**Average distance from the best option (AVGDIFF):** Same as the previous, with the exception that we calculated the average difference (instead of the maximum) from the best option.

**Balancedness (BLNC):** We measured the difference of type I and type II errors using the RT I and RT II columns. The idea behind this is that we want to balance the errors – the smaller the difference, the better. Balancing the two types of error rates is a common method in biometrics, called crossover (or equal) error rate, see e.g. ref. (Conrad et al., 2017, Chapter 5).

**Sum of Ranking Differences (SRD):** We took the transpose of Tables 2–4, hence the CV methods correspond to the columns (solutions), and the scenarios to the rows (objects). Reference is the row minimum in type I scenarios (the lowest rejection rate) and the row maximum in type II scenarios (the highest rejection rate). SRD calculates how far each CV method falls from the reference.

**Pair-wise correlation methods (CEPWAVG/WTPWAVG):** Similarly to SRD, these methods work with the transposed data matrix (Rajkó and Héberger, 2001; Héberger and Rajkó, 2002). Two solutions (X1 and X2) were selected and checked to determine whether they were related to the reference (here the average), whether both of their difference was positive (A); one of them was positive and the other was negative (B), or *vice versa* (C). The frequencies were counted for all possible pairs of scenarios. Then, two statistical tests—the conditional exact Fisher’s test (CE), and Williams’ *t*-test (Wt)—decide whether the frequencies of events B and C are significantly different or not; i.e., one solution (say X1) is overriding X2, conversely X1 loses against X2, or no decision can be made (tie). After that, the solutions were compared pairwise with the reference, considering all possible combinations (W05, W06,..., and A10). The solutions were further ranked according to the number of wins minus the number of losses, but the ranking was adjusted in the present case: probability-weighted ranking (pW) was used, i.e. based on p(wins)-p(losses) scores.

There are other possible aspects in terms of which the CV methods can be compared. However, since the analysis already contains seven different decision criteria, a new aspect has a small chance to turn over the aggregated ranking. As for the aggregation method, there are certainly more sophisticated ways to evaluate the data. The advantage of the Borda
count is its conceptual simplicity. As Tables 5–7 show, the results are fortunately rather straightforward.

### 4.4 Comparing Selection Criteria

Before we proceed with discussing the results, it is worthwhile to look at how the chosen selection criteria relate to each other. To uncover this, we performed an SRD analysis using the data of all three input sizes. The average of the normalized performance values was taken as a reference. The idea is that using seven measures we can invoke the “wisdom of the crowd”: Not only the random errors but the systematic ones (biases) of different methods follow normal distribution. If the average is used as the reference, the errors cancel each other out, supported by the maximum likelihood principle and empirical evidence (Youden, 1997). Figure 2 displays the results.

![Comparison of ranks with random numbers](image)

**Figure 2:** Comparison of ranks with random numbers For sake of visualization, the colored bars’ height is equal to their standardized SRD values and they follow the same order as in the legend. The black curve is a continuous approximation of the cumulative distribution function of the random SRD values. All (standardized) SRD values fall outside the 5% threshold (XX1: 5% threshold, Med: Median, XX19: 95% threshold).

MAXDIFF turned out to be the closest to the average evaluation of the selection criteria, followed by SRD itself. In fact, all criteria fell outside the 5% threshold, that is, all of them are acceptable. CEPWAVG was the least similar to the average, meaning that it ranked the methods according to a different dimension than the other criteria. Another explanation for its relatively poor performance is because of the many ties it produces.

Notice that even the best criterion is far from the reference, indicating that there is a lot of room for improvement.
Table 5: Ranking of CV-methods in the $n = 32$ case

| Method  | DISCPOW | MAXDIFF | AVGDIFF | BLNC | SRD | CEPWAVG | WTPWAVG | Borda |
|---------|---------|---------|---------|------|-----|---------|---------|-------|
| Wilcoxon 5 | 4       | 7       | 8       | 3    | 3.5 | 5.5     | 3       | 92    |
| Wilcoxon 6 | 2       | 8       | 6       | 5    | 3.5 | 5.5     | 4       | 92    |
| Wilcoxon 7 | 1       | 4       | 4       | 7    | 3.5 | 2.5     | 12      | 92    |
| Wilcoxon 8 | 3       | 2       | 3       | 8    | 3.5 | 2.5     | 6       | 98    |
| Wilcoxon 9 | 5       | 3       | 2       | 10   | 3.5 | 2.5     | 15      | 85    |
| Wilcoxon 10 | 6      | 6       | 1       | 11   | 3.5 | 2.5     | 18      | 78    |
| Dietterich 5 | 18     | 18      | 18      | 18   | 14.5| 14.5    | 17      | 8     |
| Dietterich 6 | 17     | 17      | 17      | 17   | 8   | 8       | 5       | 37    |
| Dietterich 7 | 16     | 15      | 16      | 16   | 14.5| 14.5    | 13      | 21    |
| Dietterich 8 | 15     | 15      | 15      | 15   | 10  | 10      | 7       | 40    |
| Dietterich 9 | 14     | 14      | 14      | 14   | 8   | 8       | 2       | 53    |
| Dietterich 10 | 13    | 13      | 12      | 12   | 8   | 8       | 1       | 59    |
| Alpaydin 5 | 12      | 16      | 12      | 13   | 14.5| 14.5    | 16      | 28    |
| Alpaydin 6 | 11      | 11      | 11      | 9    | 14.5| 14.5    | 14      | 41    |
| Alpaydin 7 | 10      | 10      | 10      | 6    | 14.5| 14.5    | 9.5     | 51.5  |
| Alpaydin 8 | 9       | 9       | 9       | 4    | 14.5| 14.5    | 9.5     | 56.5  |
| Alpaydin 9 | 8       | 7       | 5       | 1    | 14.5| 14.5    | 9.5     | 66.5  |
| Alpaydin 10 | 7      | 5       | 2       | 2    | 14.5| 14.5    | 9.5     | 72.5  |

Table 6: Ranking of CV-methods in the $n = 13$ case

| Method  | DISCPOW | MAXDIFF | AVGDIFF | BLNC | SRD | CEPWAVG | WTPWAVG | Borda |
|---------|---------|---------|---------|------|-----|---------|---------|-------|
| Wilcoxon 5 | 4       | 8       | 9       | 2    | 3   | 9.5     | 3       | 87.5  |
| Wilcoxon 6 | 3       | 9       | 10      | 4    | 11  | 18      | 7       | 64    |
| Wilcoxon 7 | 2       | 3       | 6       | 3    | 3   | 9.5     | 5       | 94.5  |
| Wilcoxon 8 | 1       | 4       | 7       | 1    | 3   | 9.5     | 6       | 94.5  |
| Wilcoxon 9 | 6       | 5       | 4       | 5    | 3   | 9.5     | 17      | 76.5  |
| Wilcoxon 10 | 5      | 7       | 3       | 7    | 3   | 1       | 14      | 86    |
| Dietterich 5 | 18     | 18      | 18      | 18   | 11  | 9.5     | 18      | 15.5  |
| Dietterich 6 | 17     | 17      | 16      | 16   | 11  | 9.5     | 16      | 22.5  |
| Dietterich 7 | 16     | 16      | 15      | 15   | 11  | 9.5     | 12      | 31.5  |
| Dietterich 8 | 15     | 15      | 14      | 14   | 11  | 9.5     | 4       | 43.5  |
| Dietterich 9 | 14     | 14      | 13      | 13   | 11  | 9.5     | 2       | 49.5  |
| Dietterich 10 | 13    | 12      | 13      | 11   | 11  | 9.5     | 1       | 55.5  |
| Alpaydin 5 | 12      | 16      | 12      | 17   | 17.5| 9.5     | 15      | 27    |
| Alpaydin 6 | 11      | 11      | 12      | 12   | 17.5| 9.5     | 13      | 41    |
| Alpaydin 7 | 10      | 10      | 8       | 10   | 11  | 9.5     | 9.5     | 58    |
| Alpaydin 8 | 9       | 6       | 5       | 9    | 11  | 9.5     | 9.5     | 67    |
| Alpaydin 9 | 8       | 2       | 2       | 8    | 11  | 9.5     | 9.5     | 76    |
| Alpaydin 10 | 7      | 1       | 1       | 6    | 11  | 9.5     | 9.5     | 81    |

Table 7: Ranking of CV-methods in the $n = 7$ case

| Method  | DISCPOW | MAXDIFF | AVGDIFF | BLNC | SRD | CEPWAVG | WTPWAVG | Borda |
|---------|---------|---------|---------|------|-----|---------|---------|-------|
| Wilcoxon 5 | 3       | 3       | 10      | 5    | 7   | 17.5    | 5       | 75.5  |
| Wilcoxon 6 | 4       | 4       | 14      | 6    | 7   | 17.5    | 16      | 57.5  |
| Wilcoxon 7 | 2       | 2       | 5       | 4    | 2.5 | 7.5     | 10      | 93    |
| Wilcoxon 8 | 1       | 1       | 4       | 3    | 2.5 | 7.5     | 10      | 97    |
| Wilcoxon 9 | 6       | 11      | 1       | 2    | 14  | 16      | 10      | 66    |
| Wilcoxon 10 | 5      | 7       | 2       | 1    | 14  | 9       | 10      | 78    |
| Dietterich 5 | 17     | 15      | 15      | 14   | 14  | 12.5    | 15      | 23.5  |
| Dietterich 6 | 16     | 16      | 15      | 14   | 14  | 3       | 4       | 42    |
| Dietterich 7 | 15     | 12      | 12      | 12   | 14  | 12.5    | 14      | 34.5  |
| Dietterich 8 | 14     | 10      | 11      | 10   | 2.5 | 1.5     | 10      | 67    |
| Dietterich 9 | 12     | 9       | 9       | 9    | 2.5 | 1.5     | 10      | 73    |
| Dietterich 10 | 13    | 8       | 8       | 8    | 7   | 5       | 10      | 67    |
| Alpaydin 5 | 18      | 18      | 18      | 18   | 7   | 5       | 3       | 39    |
| Alpaydin 6 | 11      | 17      | 17      | 17   | 7   | 5       | 1       | 51    |
| Alpaydin 7 | 10      | 14      | 13      | 16   | 14  | 12.5    | 2       | 44.5  |
| Alpaydin 8 | 9       | 13      | 7       | 13   | 14  | 12.5    | 6       | 51.5  |
| Alpaydin 9 | 8       | 6       | 6       | 11   | 14  | 12.5    | 18      | 60.5  |
| Alpaydin 10 | 7      | 5       | 3       | 7    | 14  | 12.5    | 17      | 60.5  |
5 Discussion

We can evaluate our data on three levels and can select the best method based on the scenarios, the decision criteria, or the aggregated indicator.

If practitioners know which scenario they will face, they can choose the best CV method according to Tables 2–4. For instance, if they are working with a large data set, say \( n \geq 30 \), and are concerned that the result is driven by outliers, they can pick Alpaydin 10 or Wilcoxon 10, as these are the most successful methods for outlier detection (64\( |1u \) and 88\( |4m \) scenarios respectively). Alpaydin 10 is also the most efficient choice for structured data under all three input sizes. If practitioners would like to avoid type I errors, they can choose Wilcoxon 6 that has a fairly low rejection rate in type I situations while still maintaining the best discriminative power.

Although Dietterich is much better in type I situations than any other method, we cannot recommend it as almost all decision criteria ranks it as the worst method. The biggest problem is its low discriminative power. Its rejection rate is almost the same in type I and type II situations.

Unfortunately, we cannot establish a clear winner based on the scenarios alone; Alpaydin 10 outperforms Wilcoxon 10 in five out of eight scenarios. On the other hand, Wilcoxon 6 surpasses Alpaydin 10 in six scenarios, while Wilcoxon 10 in turn outperforms Wilcoxon 6 in five scenarios. Thus, if we rely on simple majority voting, we would end up with a cyclic preference (A10\( \succ W10 \succ W6 \succ A10 \)). Let us now turn toward more sophisticated approaches and evaluate the data according to the aforementioned aspects. What do the decision criteria recommend? Yet again, we see a mixed picture; different criteria prefers different solution.

According to MAXDIFF, recommended by the SRD analysis in Section 4.4, Alpaydin 10 is to be preferred for large \( n \) and Wilcoxon 8 for small input sizes. DISCPOW suggests Wilcoxon 6–8, and SRD is tied between Wilcoxon 7–8.

If practitioners have no preliminary knowledge on the data distribution and want to have a balance between type I and type II errors, then they may select Alpaydin 9, as it is the best method according to the BLNC criterion for large data sets. For smaller data sets, practitioners are better off with Wilcoxon 8 or 10.

If they have no preference over the decision criteria or wants to pick a method that is “satisfactory from every aspect”, they may choose Wilcoxon 8, which has the best aggregated score. In fact, Wilcoxon 8 performed the best under all three data size categories. Although that doesn’t mean that this method is a jack of all trades. For instance, in the 16t|16b scenario, Alpaydin 10 is much better, while maintaining a similar performance in type I scenarios as well.

Declaring a clear winner is difficult as different methods prevail in different scenarios. For some scenarios, even the best method is not that good. Perhaps a hybrid test or a method that uses a different distance metric could be a more efficacious validation tool.

Let us summarize our findings.

- Based on the aggregated indicator, Wilcoxon 8 performed the best, but Wilcoxon 10 and Alpaydin 10 prevailed in certain scenarios and under various decision criteria.

- Alpaydin 10 dominated other Alpaydin tests with lesser folds based on practically every
decision criteria. This is an interesting observation because Alpaydin is almost always used with five folds.

- We cannot recommend the Dietterich test due to its low discriminative power and otherwise poor performance under various aspects.

- None of the methods were particularly good. Therefore, future research is needed to discover a more efficacious combination of a statistical test and a suitable CV method.

6 Conclusion

We tested how successful the combinations of statistical tests and CV methods are in distinguishing rankings that come from various distributions. Despite the widespread usage of rankings, to our knowledge, this is the first paper that has tackled this problem. Our method of analysis is innovative, as we devised simulation scenarios to uncover the strengths and weaknesses of certain methods. This approach resembles axiomatic analysis, a common practice in theoretical economics, especially in social choice.

We investigated three tests, Wilcoxon’s, Dietterich’s, and Alpaydin’s, each combined with folds ranging from 5 to 10, making 18 variants altogether (Tables 2–4). The simulation data was ranked according to various decision criteria, and the rankings were then aggregated to choose a winner. Although the Wilcoxon method with eight folds proved to be the best under all investigated input sizes, there were scenarios and decision criteria where other methods, namely Wilcoxon 10, or preferably Alpaydin 10, performed better.

Another interesting observation is that Alpaydin 10 dominates the 5-fold Alpaydin test in every aspect (Tables 5–7), even though in practice, the latter is used almost always.

According to our analysis, there is a lot of room for improvement regarding to rejection rates. Even the best combination was not particularly good (Section 4.4)—there were scenarios where it performed poorly. Future research is needed to devise a more efficacious CV method.

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References

Alpaydin, E. (1999). Combined 5×2 cv F Test for Comparing Supervised Classification Learning Algorithms. Neural Computation 11, 1885–1892.
Andrić, F., S. Šegan, A. Dramićanin, H. Majstorović, and D. Milojković-Opsenica (2016). Linear modeling of the soil-water partition coefficient normalized to organic carbon content by reversed-phase thin-layer chromatography. *Journal of Chromatography A* 1458, 136–144.

Arlot, S. and A. Celisse (2010, January). A survey of cross-validation procedures for model selection. *Statistics Surveys* 4(none).

Barlow, G. W. and P. J. Ballin (1976, November). Predicting and assessing dominance from size and coloration in the polychromatic midas cichlid. *Animal Behaviour* 24(4), 793–813.

Bro, R., K. Kjeldahl, A. K. Smilde, and H. A. L. Kiers (2008, March). Cross-validation of component models: A critical look at current methods. *Analytical and Bioanalytical Chemistry* 390(5), 1241–1251.

Conrad, E., S. Misenar, and J. Feldman (2017, January). Chapter 5 - Domain 5: Identity and access management (controlling access and managing identity). In E. Conrad, S. Misenar, and J. Feldman (Eds.), *Eleventh Hour CISSP® (Third Edition)*, pp. 117–134. Syngress.

Diaconis, P. and R. L. Graham (1977, January). Spearman’s footrule as a measure of disarray. *Journal of the Royal Statistical Society: Series B (Methodological)* 39(2), 262–268.

Dietterich, T. G. (1998, October). Approximate Statistical Tests for Comparing Supervised Classification Learning Algorithms. *Neural Computation* 10(7), 1895–1923.

Endriss, U. (2018). Judgment aggregation with rationality and feasibility constraints. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems*, AAMAS ’18, pp. 946–954.

Falivene, O., L. Cabrera, R. Tolosana-Delgado, and A. Sáez (2010, April). Interpolation algorithm ranking using cross-validation and the role of smoothing effect. A coal zone example. *Computers & Geosciences* 36(4), 512–519.

Farshadfar, E. and R. Amiri (2016, October). In vitro application of integrated selection index for screening drought tolerant genotypes in common wheat. *Acta agriculturae Slovenica* 107(2), 335.

Filzmoser, P., B. Liebmann, and K. Varmuza (2009, April). Repeated double cross validation. *Journal of Chemometrics* 23(4), 160–171.

Fushiki, T. (2011, April). Estimation of prediction error by using K-fold cross-validation. *Statistics and Computing* 21(2), 137–146.

Gere, A., A. Rácz, D. Bajusz, and K. Héberger (2021). Multicriteria decision making for evergreen problems in food science by sum of ranking differences. *Food Chemistry* 344, 128617.

Héberger, K. (2010, January). Sum of ranking differences compares methods or models fairly. *TrAC Trends in Analytical Chemistry* 29(1), 101–109.
Héberger, K. and K. Kollár-Hunek (2011, April). Sum of ranking differences for method discrimination and its validation: Comparison of ranks with random numbers. *Journal of Chemometrics 25*(4), 151–158.

Héberger, K. and R. Rajkó (2002). Generalization of pair correlation method (PCM) for non-parametric variable selection. *Journal of Chemometrics 16*(8-10), 436–443.

Hijmans, R. J. (2012, March). Cross-validation of species distribution models: Removing spatial sorting bias and calibration with a null model. *Ecology 93*(3), 679–688.

Isaksson, A., M. Wallman, H. Göransson, and M. Gustafsson (2008, October). Cross-validation and bootstrapping are unreliable in small sample classification. *Pattern Recognition Letters 29*(14), 1960–1965.

Kim, J.-H. (2009, September). Estimating classification error rate: Repeated cross-validation, repeated hold-out and bootstrap. *Computational Statistics & Data Analysis 53*(11), 3735–3745.

Klementiev, A., D. Roth, and K. Small (2007). An Unsupervised Learning Algorithm for Rank Aggregation. In J. N. Kok, J. Koronacki, R. L. de Mantaras, S. Matwin, D. Mladenič, and A. Skowron (Eds.), *Machine Learning: ECML 2007*, Volume 4701, pp. 616–623. Berlin, Heidelberg: Springer Berlin Heidelberg.

Kollár-Hunek, K. and K. Héberger (2013). Method and model comparison by sum of ranking differences in cases of repeated observations (ties). *Chemometrics and Intelligent Laboratory Systems 127*, 139–146.

Krstajic, D., L. J. Buturovic, D. E. Leahy, and S. Thomas (2014, December). Cross-validation pitfalls when selecting and assessing regression and classification models. *Journal of Cheminformatics 6*(1), 10.

Lockwood, J., T. A. Louis, and D. F. McCaffrey (2002, September). Uncertainty in Rank Estimation: Implications for Value-Added Modeling Accountability Systems. *Journal of Educational and Behavioral Statistics 27*(3), 255–270.

Mongin, P. (2012). The doctrinal paradox, the discursive dilemma, and logical aggregation theory. *Theory and Decision 73*, 315–355.

Moorthy, N. H. N., S. Kumar, and V. Poongavanam (2017). Classification of carcinogenic and mutagenic properties using machine learning method. *Computational Toxicology 3*, 33–43.

Moulin, H. (2016). *Handbook of Computational Social Choice*. Cambridge University Press.

Pacheco, A. G. C. and R. A. Krohling (2018, March). Ranking of Classification Algorithms in Terms of Mean–Standard Deviation Using A-TOPSIS. *Annals of Data Science 5*(1), 93–110.
Palmer, D., B. Höck, M. Kimberley, M. Watt, D. Lowe, and T. Payn (2009, October). Comparison of spatial prediction techniques for developing Pinus radiata productivity surfaces across New Zealand. Forest Ecology and Management 258(9), 2046–2055.

Rajkó, R. and K. Héberger (2001). Conditional Fisher’s exact test as a selection criterion for pair-correlation method. Type I and Type II errors. Chemometrics and Intelligent Laboratory Systems 57(1), 1–14.

Roberts, D. R., V. Bahn, S. Ciuti, M. S. Boyce, J. Elith, G. Guillera-Arroyo, S. Hauenstein, J. J. Lahoz-Monfort, B. Schröder, W. Thuiller, D. I. Warton, B. A. Wintle, F. Hartig, and C. F. Dormann (2017, August). Cross-validation strategies for data with temporal, spatial, hierarchical, or phylogenetic structure. Ecography 40(8), 913–929.

Rosander, A. C. (1936, March). The standard error of a mean rank order. Journal of Educational Psychology 27(3), 193–196.

Sziklai, B. R. and K. Héberger (2020, March). Apportionment and districting by Sum of Ranking Differences. PLOS ONE 15(3), e0229209.

Szymańska, E., E. Saccenti, A. K. Smilde, and J. A. Westerhuis (2012, June). Double-check: Validation of diagnostic statistics for PLS-DA models in metabolomics studies. Metabolomics 8(S1), 3–16.

Tavanaei, A., R. Gottumukkalay, A. S. Maida, and V. V. Raghavan (2018, July). Unsupervised Learning to Rank Aggregation using Parameterized Function Optimization. In 2018 International Joint Conference on Neural Networks (IJCNN), Rio de Janeiro, pp. 1–8. IEEE.

Tehrani, A. F., Weiwei Cheng, and E. Hullermeier (2012, December). Preference Learning Using the Choquet Integral: The Case of Multipartite Ranking. IEEE Transactions on Fuzzy Systems 20(6), 1102–1113.

Triantafilis, J., I. Odeh, and A. McBratney (2001, May). Five Geostatistical Models to Predict Soil Salinity from Electromagnetic Induction Data Across Irrigated Cotton. Soil Science Society of America Journal 65(3), 869–878.

Vajna, B., A. Farkas, H. Pataki, Z. Zsigmond, T. Igricz, and G. Marosi (2012). Testing the performance of pure spectrum resolution from raman hyperspectral images of differently manufactured pharmaceutical tablets. Analytica Chimica Acta 712, 45–55.

Varma, S. and R. Simon (2006). Bias in error estimation when using cross-validation for model selection. BMC Bioinformatics 7(1), 91.

Vehtari, A., A. Gelman, and J. Gabry (2017, September). Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. Statistics and Computing 27(5), 1413–1432.
Wenger, S. J. and J. D. Olden (2012, April). Assessing transferability of ecological models: An underappreciated aspect of statistical validation: Model transferability. *Methods in Ecology and Evolution* 3(2), 260–267.

West, C. (2018). Statistics for analysts who hate statistics, Part VII: Sum of Ranking Differences (SRD)s. *LCGC North America* 36, 2–6.

Youden, W. J. (1997). *Statistical Manual of the Association of Official Analytical Chemists. Statistical techniques for collaborative test* (8th ed.). Gaithersburg, MD, USA: AOAC International.

Zampetakis, L. A. and V. S. Moustakis (2010, March). Quantifying uncertainty in ranking problems with composite indicators: A Bayesian approach. *Journal of Modelling in Management* 5(1), 63–80.

Zuk, O., L. Ein-Dor, and E. Domany (2007). Ranking under uncertainty. In R. Parr and L. C. van der Gaag (Eds.), *UAI 2007, Proceedings of the Twenty-Third Conference on Uncertainty in Artificial Intelligence, Vancouver, BC, Canada, July 19-22, 2007*, pp. 466–473. AUAI Press.