How to predict the orientation factor of non-rigid macro-synthetic fibre reinforced concrete

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Abstract. Polyolefin fibre reinforced concrete can meet the requirements set in the standards that enable to consider the residual strengths in structural design. Such residual load-bearing capacity of fibre reinforced concrete is assessed by flexural tensile tests in which the presence of fibres can bridge the crack formed and provide strengths that are directly related with the number of fibres and their positioning in the fracture surface. Therefore, the orientation and distribution of the fibres is decisive in the mechanical behaviour of fibre-reinforced concrete and this can be estimated by means of the orientation factor. Several classical models have been extensively used for the case of rigid steel fibres. The increasing interest in structural synthetic fibres that can bend demands new considerations in this matter. A probabilistic model that considers the previous research with stereographical assumptions has been performed allowing the use of fibres that can bend. This study has developed significant tools for design with the aim of predicting such number of fibres crossing a vertical surface using fibre reinforced concrete with steel and polyolefin fibres. The model has been verified with experimental data and represents with accuracy the existence of boundaries, the type of concrete and compaction methods used to cast the moulds.

1. Introduction and background

Given the increasing interest on the use of fibre reinforced concrete (FRC) and the advances on improved synthetic materials, macro polymer fibres that provide concrete with properties and behaviour analogous of those of steel fibre reinforced concrete (SFRC) have become an alternative to consider in structural design. Essentially, polyolefin-based macro-polymer fibres, can be used for structural design with lower dosages in terms of weight. Such polyolefin fibres (PF) are chemically stable and their promising properties have been the centre of significant amount of research in recent years [1, 2, 3, 4, 5]. Post-cracking behaviour of FRC has become the reference property in order to consider tensile strengths in structural design. Such behaviour is typically assessed by flexural tensile tests on notched specimens and the results have shown to be strongly influenced by the number and positioning of the fibres crossing the fracture surface [6, 7, 8]. In such a sense, with the aim of providing trustworthy tools for designers, it is necessary to find predictive models that may allow the prediction of the number and positioning of the fibres placed in the critical sections of a structural piece [9]. At the time of writing there was limited research about the orientation factor in PFRC elements. Some results [10] have shown that there are several differences that should be considered before assuming the validity of the models of SFRC. The most remarkable one is the effect of fibre-flexibility. Besides, macro-synthetic fibres are remarkably different in the physical properties with
lower density and flexibility when compared with steel fibres. Thus, models that deal with SFRC are questionable.

FRC in the modern concept entails by definition [11, 12] randomly added fibres to concrete during the mixing stage. Thereby, the position and orientation of each isolated fibre depends on the pouring process, formwork geometry and rheological properties of concrete. Some studies have assessed the problem by using stereological tools [13], statistical ones [14] or rheological measures [15]. Regarding the orientation of fibres, it should be noted that some models are based on the angles that fibres take in the concrete bulk material. These seem to be appropriate when using methods such as CT-scan [16], X-Ray [17], optical methods [18], methodologies which seek to imitate concrete by using a translucent fluid [19], and others that exploit the electrical properties of concrete in measuring real angles of the fibres [20]. However, most of the works performed for this purpose have measured the number of fibres placed in a sawn surface or in the resulting surfaces after fracture tests [21, 22]. In such cases, the orientation factor \( \theta \) proposed by Krenchel in 1975 [23] enables the use of a factor that offers a coupled value of the orientation and the distribution of the fibres. By means of \( \theta \), the models have even assessed the improvements in SFRC combined with self-compacting concrete (SCC) [24] and the effects of several types of vibration on conventional concrete (VCC) [25].

Fibres could be idealized as a number of fibres with the same cross section crossing the whole piece completing the final volume fraction \( V_f \) that was introduced in the mixer. Hence, the theoretical number of fibres crossing any section would be the one obtained by means of expression (1). Then, \( \theta \) is defined as the relation of the number of fibres counted \( n \) in a certain surface over its theoretical number. That is to say that it is possible to obtain the orientation factor once it has been counted by using the expression (2) with, \( A \) being the cross section of the sample, \( A_f \) the section of one fibre and \( n \) the number of fibres actually counted in situ.

\[
\begin{align*}
th &= \frac{A \cdot V_f}{A_f} \\
\theta &= \frac{n}{th} = n \frac{A_f}{V_f A}
\end{align*}
\]

If fibres were perfectly aligned, the number of fibres counted would match the theoretical value and, therefore, the ratio \( \theta \) would be the unity. Subsequently, the counting process supplies a value that deals with the orientation of the fibres and which is not a measure of fibre efficiency. The relation \( \theta \) takes values in general below one and the rotation angles of the fibres are strongly related with such reduction. That is to say, given certain distance from the gravity point of the fibre to a counting surface, such a fibre would cross the surface for limited rotations until reaching a critical angle. A summary of the values taken by previously mentioned models and the model presented in this study by Alberti, Enfedaque and Gálvez and with more detail in references [26, 27], presented in this paper, is summarized in Table 1. The orientation factor \( \theta \) is a common measure in research with values that ranging from 0.41 to 0.82 [28].

| Probabilistic boundaries | 2-D | 3-D |
|--------------------------|-----|-----|
| Buffon's needle problem  | \( 2/\pi = 0.637 \) | -   |
| Soroushian and Lee [28]  | 0.637 | 0.405 |
| Dupont and Vandewalle [29]| 0.600 | 0.500 |
| Alberti, Enfedaque and Gálvez [26, 27]| 0.637 | 0.500 |

With the aim of addressing flexible and lower density fibres in the existing models as well as making new considerations about the pouring and compaction methods given the fast development of SCC and PFRC, this study has developed new model built for the theoretical prediction of the positioning of fibres adapted for rigid and non-rigid fibres alike. The flexible nature of macro-synthetic fibres has been explicitly considered. The model is based on understandable geometrical...
concepts, solved by numerical integration, and not only fits the most accepted values of the orientation factor but also improves the possibilities of applying counting methods to assess the homogeneity distribution of fibres in concrete elements.

2. Proposed model in isotropic conditions

In isotropic conditions, the stereographical concept proposed in this study is to relate the sphere described by a single fibre that rotates around its gravity point and the intersection produced with a vertical plane. That plane would be the base of counting in a real specimen. The probability of a fibre intersecting a vertical plane would vary with the distance to the vertical plane as it is coupled with its rotation angle, as depicted in Figure 1. Thus, if the fibre is included in a cube cell of size \( l_f \), the intersection with the plane that moves along its horizontal axis becomes a circumference that reaches its maximum surface of probability in an equatorial circumference of diameter equal to \( l_f \). The complete mathematical apparatus can be seen in detail in reference [27]. As a matter of length, this study shows a brief overview of the main concepts and the most useful results obtained from the model.

An intermediate situation would describe a cone as having at its base the aforementioned circumference and a height of the distance between the plane and the fibre gravity point. With such an assumption, the probability of counting one fibre in a vertical plane could be given as a function of the two parameters: the fibre orientation angle (\( \theta \)) and distance of the projected fibre to the cracked or cut surface that would be represented by the plane. The extreme positions are an equatorial plane that contains fibres rotated 90º or at the cutting of a fibre in its extreme position by its last cross-section. The intermediate positions relate the probability of occurrence as the result of a fraction of the base of the cone of such a position and the equatorial one with the fibre length as its radius. Once the cone is defined, the graphical solution of the probability of occurrence of a single case would be the relation between the circumference described by the fibre in that case \( (A_2) \) and the base one of probability that would be the circumference \( (A_1) \) described by the fibre. These are described in Figure 2.

![Figure 1. Graphical description of the cones formed by the probability situations](image1)

![Figure 2. Surfaces of probability obtained as intersection of the vertical plane and the sphere](image2)

Therefore, the probability in a three-dimensional isotropic situation could involve the relation at a given distance of the surface drawn by the fibre that crosses the cut surface in all the angles of rotation and the total surface probability. If we consider the space probability as the average obtained in all the possible geometrical situations, expressions (3) would provide the solution. The area under the probability curve can be an average in the space probability in the value of 0.500. This value could be considered as the mean orientation factor found in the bulk area.

\[
P(\theta_{isotropic}) = \frac{\int_0^{\pi} \sin(\theta)^2 d\theta}{\pi} = 0.500
\]

3. Proposed model under real conditions: the presence of walls

Isotropic conditions would take place in the bulk zone of the specimen if the compaction is made with conventional times of vibration and no other of the previously summarized effects take place.
Concrete elements need to be poured in some mould or formwork. Therefore, the areas surrounding the walls should be considered separately due to the wall effect. The presence of a boundary condition impedes the fibre from taking certain positions, as can be noticed in Figure 3. If we consider the fibre distance to the wall from its gravity point, the symmetrical positions would not be reached. If the wall reaches the position of the centre of the sphere, in which the fibre would be located in the wall, given that the only possible rotations would be in the angle perpendicular to the paper the situation would be equal to a two-dimensional one. Therefore, the same conceptual basis were used in the presence of on wall but new geometrical problems appeared in order to compute expression (4).

If Figure 4 is analysed, two significant changes in the calculation emerge in order to compute expression (4). The first is that the surface of probability one \( (A_{1-1wall}) \) is smaller than in the isotropic case given that in the surface of the circumference two sectors have to be subtracted (one in the side of the wall and another in the symmetrical position). The second one is that there are two formulations for the drawing of the possible cut on the counting surface \( (A_{l-1wall}) \). In the positions of the cone not reaching a limit angle of rotation, the base of the cone remains as a circumference of similar dimensions. Once the distance allows rotation angles that would allow positions that are impeded by the existence of a boundary, the obstructed sector and its symmetrical position have to be subtracted.

\[
P(\theta_{wall}(h)) = \frac{A_{1-1wall}}{A_{1-1wall}}
\]

The numerical solution of this integral provides a value for each distance, varying from the case of a distance equal to half the fibre length to the one in which the fibre is in the wall plane. In such a sense, the probability distribution is unique for each distance. The area of probability when \( h = l_f / 2 \) produces a mean value of probability of \( 2 / \pi \). It is possible to build a profile of the orientation factor by giving values to \( h \) as a function of \( l_f \). In doing so, the maximum value at the edge of the wall is \( \theta_{wall} = 2 / \pi \), with the rest resulting in a continuously increasing curve from the value of the bulk up to what could be named as \( \theta_{wall} \). Table 2 offers the values of the orientation factor in several intermediate relative positions of the gravity point and the wall.
Regarding the presence of two walls, it would be merely an extension of the previous concept developed for one wall. Thus, analogous formulation would apply as can be seen in expression (5) although new geometrical problems must be solved. If the position of the first wall is fixed in a position at a certain distance \( h \) defined as a fraction of \( l_f/2 \), the position of the new wall needs to define a new parameter in order to integrate the possible positions. That is to say, the distance with the second wall, named \( u \) in the study, can be defined as a fraction of \( l_f/2 \) and varies \( u \) from 0.0 to 0.5. With this new parameter, it is possible to find the surfaces generated to compute the probability, since the position of the first wall remains fixed.

\[
P(\theta_{2\text{walls}}(h, u)) = \frac{A_{1-2\text{walls}}}{A_{1-2\text{walls}}}
\]  

(5)

| \( h(l_f) \) | \( \theta \) |
|------------|------------|
| 0.500      | 0.500      |
| 0.400      | 0.531      |
| 0.250      | 0.584      |
| 0.100      | 0.625      |
| 0.000      | 0.637      |

Table 2. Orientation factor with one wall

However, there is not only a new limit angle but also a key point for a general situation in which both planes intersect in a straight line idealized in a two-dimensional point. That point can be either outside or inside the circumference. Thus, the singular point \( (k) \) in which such a key point belongs to the circle, border of the probability one circumference, is the limit between the two situations. Therefore, the reference surface of probability one \( (A_{1-2\text{walls}}) \) changes in shape and it should be computed in parts. Then, probability is defined with analogous processes to the previous probabilities. With such assumptions, limits and solving of the numerical integration, the probability surfaces and the mean values that vary the distance to the second wall (and for each fixed value of the first) would show a three-dimensional orientation factor map. It is evident that for the lowest values of \( (u, h) \), the mean orientation factor has to tend to one and for values of \( (u, h) \) close to 0.500, the orientation factor has to be close to 0.500. All the mathematical elaboration of the model can be studied in more detail in references [26, 27].

4. Orientation profile from the model

As may be seen in previous explanations, the values that the orientation factor take for a decreasing distance to the wall are continuously increasing up to the values obtained in the edges. That is to say, the profile of orientation factor that considers the presence of only one wall varies from 0.500 in the bulk to 0.637. In the case of the values of two walls when the first wall is considered in the edge, it would vary from 0.637 to 1.000. It should be highlighted that the model presented in this paper can provide the full profile for each section by numerical integration which can be seen in Figure 5.
Research on this matter has provided a variety of functions representing these variations of the orientation factor as a function of the thickness of the piece (or the relation of fibre length with the thickness of the piece). Exponentially decreasing relations such as the presented in references [30] and [31] related the orientation factor with the specimen thickness \( (h) \). Nonetheless, Martinie [32] provided expressions and graphic descriptions that relate the orientation factor with the relation between fibre length and specimen thickness. Laranjeira [24] also proposed the use of a bilinear model to adjust the results of his theoretical model.

5. Non-rigid fibres: folding effect

The possible folding effects have been considered as a reduction of the sphere drawn by the fibre in the model. That is to say, the possible positions are the same but for a reduction of the projected fibres due to its own bending. This can be done by assuming a reduction of the diameter of the sphere considering the fibre length as the projected length when it is folded and introducing such value in the model. In such a way, the expression needed has a geometrical relation with the cord \( (l_{fr}) \) of the circumference drawn by the fibre in each plane. The expression (6) relates the fibre length with the bend radius, with the probability of the reduction of the fibre being obtained for a maximum bending radius limited with half the fibre length. Expression (7) provides the projected length of the fibre for each curvature angle \( (\varphi) \). The resulting reduction of probability is in the case of the average fibre having a length of 94.6% of the original length, as obtained by expression (8). These values were set as an initial hypothesis in order to propose the model as shown in Figure 6. The possible values would vary with the geometrical and material properties of each type of fibres and the procedure sought allows adjusting the model to any other type of non-rigid fibres. The procedure description for the adjustment to 60mm-long polyolefin fibres can be read in the verification of the model and in more detail in reference [27].

\[
R = \frac{l_f}{2\varphi} \quad (6)
\]

\[
l_{fr} = \frac{l_f}{\varphi} \sin \varphi \quad (7)
\]

\[
l_{fr} = \int_0^1 l_f \cdot \frac{1}{\varphi} \cdot \sin \varphi \cdot d\varphi = 0.946l_f \quad (8)
\]
6. Discussion and verification of the model

It should be highlighted that models proposed by Soroushian [28], Stroeven [13] or Dupont and Vandewalle [29] have been successfully used for the case of steel fibres and having suitable results, even though there were several computed values that were used adequately for a varied casuistry. In any event, there was a demand of an updated model which allowed considering the use of flexible fibres. In addition, some other facts such as the use of SCC, the relative positions of the walls, the sizes of the beams and notches or the treatment suggested for the topped surfaces could be reconsidered and revised.

Regarding the verification of the model, it is worth noting that the variety of solutions, models and procedures to assess the orientation factor, makes it difficult to use reported experimental data to validate theoretical models. Even though this reveals the need of a standard procedure that would boost the possibility of sharing data, the values provided by using the presented model with regard to steel fibres can be validated with the data published in references [29, 22, 21, 28]. In addition, for smaller sizes of 100x100 mm² square cross-section, and when using SCC with steel hooked-fibres 35 mm long, the experimental data in reference [26] provided an average orientation factor of 0.590. Computation of the orientation factor with the proposed model, with the assumption of the bulk zone, as having a boundary condition due to the flux (0.578 in the central sector) gives 0.593.

Regarding synthetic fibres, validation has been made in references [1, 2, 3, 26]. In such a sense, the factor that affects the fibre length obtained by means of expression (35) was initially set on 0.946. With such a value, the ratio computed over experimental number of fibres counted takes a value 1.009. Since the value of the projected chord required verification, it was possible to pitch the limits of the integration placed in expression (8). The limit for a maximum curvature radius that fits the model was 0.895 radians. Therefore, this change in expression (8) would be that presented in expression (9).

\[ l_{fr} = \int_{0}^{0.895} \frac{l_f \cdot \sin \varphi \cdot d\varphi}{\varphi} = 0.856l_f \quad (9) \]

One more validation may be of interest regarding synthetic fibres: the percentage of distribution in nine equal sectors counted. In order to provide practical tools, the two sizes of cross-sectional specimens placed on the test standards EN 14651 [33], RILEM TC 162-TDF [34] and ASTM C 1609 [35] have been taken as an example. The orientation factor in each of the nine equal sectors of a specimen with squared cross section and with a notched beam of sizes 100 mm² and 150 mm² and for fibres 35, 48, 60 and 70 mm long were published in reference [27]. With the values of orientation computed, it is possible to achieve a number of fibres to be counted in each of the sector by fixing the
fibre dosage and the cross-section surface of a single fibre. However, beyond such a simple and powerful possibility, a new concept appears that can be useful. Once the distribution of the orientation factor is not uniform, the number of fibres counted on similar surfaces would not be uniform either due to the orientation. In summary, a uniform distribution of the fibres across the cross-sectional surface and along the beam would lead to a non-proportional distribution of the fibres counted in equal squares, having an uneven percentage due to the distinct orientation of the sectors that are close to a wall or any other boundary condition. In addition to this, it is worth noting that the assumption of Laranjeira [24] establishes the value of the bulk in the case of SCC as being in the presence of one wall and thus idealizing the flux as taking place in a layer of fibre-length thickness. Figure 7 shows the results obtained in reference [26] for the sawn surface, which detailed counting results. The table shows the mean experimental distributions of fibres in 20 surfaces with a 150x150 mm² square cross-section produced with PFRC with 6 kg/m³ polyolefin fibres 60 mm long.

### Table

| PFRC with 6 kg/m³ of 60mm long polyolefin fibres | Mean distribution of fibres in VCC | Mean distribution of fibres in SCC | Theoretical distribution of fibres |
|-----------------------------------------------|----------------------------------|----------------------------------|----------------------------------|
|                                               | 11,20% 11,20% 9,50%             | 10,70% 10,20% 12,70%            | 10,80% 9,90% 10,80%            |
|                                               | 11,30% 10,00% 11,00%           | 10,20% 9,90% 10,20%            | 10,80% 9,90% 10,80%            |
|                                               | 13,70% 10,50% 11,60%          | 12,30% 11,10% 12,70%           | 13,00% 10,80% 13,00%           |

*Figure 7*. Experimental distributions of fibres in 150x150 mm² square cross-section produced with PFRC using 60 mm-long fibres.

### 7. Conclusions

The review of the previous models has allowed an initial validation of the model for rigid fibres. Moreover, the most accepted values for the orientation factor in distinct positions of an isolated fibre inside the concrete were fully complied with by the results obtained by means of a numerical integration. In comparison with other classical models that have been extensively used, the proposed model supplies the possibility of considering the bending of the non-rigid fibres. It allows the fitting of the model for any type of fibres that should not be considered as remaining straight inside the concrete piece. Although wider research on this matter would provide reliability, due to the typical scatter of fibre reinforced concrete, the proposed concept of reducing the overall fibre length by a bending factor would appear to be suitable.

Additionally, the model allowed the study to prepare curves with the profile of orientation when one and two walls affect the fibre positioning as a function of the relation of the fibre length and the distance to a wall.

The model presented in this study permits comparison of the number of fibres predicted to cross a vertical plane with the most common structural fibres and lengths. Beyond such a possibility, it allows this to be done for non-rigid fibres. Moreover, it is a powerful tool gathered with the developed distribution tables also presented to evaluate the behaviour of new types of fibres or shapes in research. This also applies to the assessment of the influence of the placing conditions, pouring methods and rheological properties of the concrete in the final positioning of the fibres. In summary, this achieves not only a more reliable use of the fibres but also provides a valid and sound measurement.

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