Bound States of the Heavy Flavor Vector Mesons and $Y(4008)$ and $Z_{1}^{+}(4050)$

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The $D^*\bar{D}^*$ and $B^*\bar{B}^*$ systems are studied dynamically in the one boson exchange model, where $\pi, \eta, \sigma, \rho$ and $\omega$ exchanges are taken into account. Ten allowed states with low spin parity are considered. We suggest that the $1^{--}, 2^{++}, 0^{++}$ and $0^{-+} B^*\bar{B}^*$ molecules should exist, and the $D^*\bar{D}^*$ bound states with the same quantum numbers very likely exist as well. However, the CP exotic $(1^{-+}, 2^{++}) B^*\bar{B}^*$ and $D^*\bar{D}^*$ states may not be bound by the one boson exchange potential.

We find that the $I = 0$ configuration is more deeply bound than the $I = 1$ configuration, hence $Z_{1}^{+}(4050)$ may not be a $D^*\bar{D}^*$ molecule. Although $Y(4008)$ is close to the $D^*\bar{D}^*$ threshold, the interpretation of $Y(4008)$ as a $D^*\bar{D}^*$ molecule is not favored by its huge width. $1^{--} D^*\bar{D}^*$ and $B^*\bar{B}^*$ states can be produced copiously in $e^+e^-$ annihilation, detailed scanning of the $e^+e^-$ annihilation data near the $D^*\bar{D}^*$ and $B^*\bar{B}^*$ threshold is an important check to our predictions.

PACS numbers: 12.39.Pn, 12.39.Jh, 12.40.Yx, 13.75.Lb

I. INTRODUCTION

In the past years, the observations of a number of charmonium-like "X, Y, Z" mesons at $B$ factories have stimulated the interest in the spectroscopy of the charmonium states again. There is growing evidence that at least some of these new states are non-conventional $c\bar{c}$ states, such as deuteron like hadronic molecules, tetraquark states or hybrid have been suggested [1]. Among these new mesons, some are very close to the threshold of two charmed mesons, such as $X(3872)$ [2] and $Z^+(4430)$ [3, 4]. This distinctive character inspires the molecular interpretation for these mesons. In particular, some new enhancements near the $D^*\bar{D}^*$ threshold have been observed recently.

The Belle collaboration reported a broad $\pi^+\pi^- J/\psi$ peak near 4008 MeV in addition to the well-known state $Y(4260)$ by studying the initial state radiation process $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^- J/\psi$ [5], its mass and width are fitted to be $M = (4008 \pm 40_{-28}^{+114})$ MeV and $\Gamma = (226 \pm 44 \pm 87)$ MeV respectively. We notice its width is huge. This peak was suggested to be related to the $D^*\bar{D}^*$ threshold and could be a $D^*\bar{D}^*$ molecule in Ref. [6]. The $\pi^+\pi^- J/\psi$ spectrum was studied further by the Babar collaboration. However, there was no evidence for this broad enhancement, and the an upper limit $\sigma(\pi^+\pi^- J/\psi)\Gamma_{e^+e^-} < 0.7eV$ at 90% C.L. was obtained [7]. $Y(4008)$ is far from being established so far, more experimental efforts are obviously needed.

Of special importance is the observation of the state carrying non-zero electric charge with hidden charm quarks. Since the observation of $Z^+(4430)$ [3, 4], the Belle collaboration reported two resonance-like structures $Z_{1}^{+}(4050)$ and $Z_{2}^{+}(4250)$ in the $\pi^+\chi_{c1}$ mass distribution in the exclusive process $B^0 \rightarrow K^-\pi^+\chi_{c1}$ [8]. Their masses and widths are determined to be $M_1 = (4051 \pm 14_{-41}^{+20})$ MeV, $\Gamma_1 = 82_{-17}^{+21} + 47$ MeV, $M_2 = (4248 \pm 44_{-29}^{+180})$ MeV and $\Gamma_2 = (177_{-39}^{+54} + 316)$ MeV respectively. Since $\pi^+$ is an isovector with negative G-parity, and $\chi_{c1}$ is an isospin singlet with positive G-parity, the quantum numbers of both $Z_{1}^{+}(4051)$ and $Z_{2}^{+}(4250)$ are $I^G = 1^-$. In Ref. [9], $Z_{1}^{+}(4050)$ was suggested to be possibly a $J^P = 0^{++} D^*\bar{D}^*$ molecule due to its closeness to the $D^*\bar{D}^*$ threshold. However, the QCD sum rule results indicated that the $D^*\bar{D}^*$ state is probably a virtual state, which is not related with the $Z_{1}^{+}(4050)$ resonance-like structure [10]. In addition, we demonstrated that $Z_{2}^{+}(4250)$ as a $D_1^0D^0$ or $D_0^0D^+D^0$ molecule is disfavored in Ref. [11].

Since the repulsive kinetic energy is greatly reduced by the heavy quark mass, the interaction between light quarks is strong enough so that the molecular states consisting of heavy flavor mesons very likely exist. In fact, the hadronic molecules consisting of two charm mesons were suggested long ago [12]. De Rujula, Georgi and Glashow proposed that the molecular states involving hidden $c\bar{c}$ pair do exist, and have a rich spectrum [13]. Possible new resonance near the $D^*\bar{D}^*$ threshold was suggested by Voloshin [14, 15]. Different from other possible exotic structures, there are uncontroversial evidences for hadronic molecule such as the deuteron, which is unambiguously a proton-neutron bound state. The deuteron has been studied in great details over the years [16, 17]. From these studies, we learn that the pion exchange determines most of the binding energy and the long range part of the deuteron wavefunction, and the S-D wave mixing effect plays a critical role in providing the binding. Guided by the binding of deuteron, Tornqvist performed a systematic study of possible deuteronlike two mesons bound states with long distance one pion exchange [18, 19]. At short distance, the interaction should be induced by the interactions among the quarks. However, a detailed and reliable modelling of the short range interaction is not a easy matter, and various phenomenological models have been proposed [20, 21], although one pion exchange is expected to be dominant for the hadronic molecule. Inspired

arXiv:0905.1188v2 [hep-ph] 12 Sep 2009
by the nucleon-nucleon interactions, we further extended the one pion exchange model to include the short distance contributions from the heavier bosons $\eta$, $\sigma$, $\rho$ and $\omega$ exchanges in Ref. [22]. We have also taken into account the contribution of the "$\delta$ function" term, which leads to a $\delta$ function term in the effective potential in configuration space when no regularization is used. This one boson exchange model gives a very good description of the weakly bound hadronic molecule. It has been successfully applied to exploring the possible heavy flavor pseudoscalar-vector molecular states [22], the $D^*_sD^*_s$ system and the molecular interpretation of $Y(4140)$ [23]. Motivated by the controversial states $Y(4008)$ and $Z^+_1(4050)$, we shall examine for which quantum numbers the boson exchange potential is attractive and strong enough so that bound states are expected, ten allowed $D^*_sD^*$ states with low spin parity are considered. Moreover, the $B^*\bar{B}^*$ system would be discussed as well.

The paper is organized as follows. In section II, the formalism of one boson exchange model is summarized. In section III, we apply the one boson exchange model to the $D^*_sD^*$ system, the quantum numbers of the $D^*_sD^*$ bound states which might exist, are suggested. The $B^*\bar{B}^*$ system is discussed along the same line in section IV. Finally we present our conclusions and some discussions in section V.

II. THE FORMALISM OF THE ONE BOSON EXCHANGE MODEL

In the one boson exchange model, the effective potential between two hadrons is obtained by summing the interactions between light quarks or antiquarks via one boson exchange. To leading order in the boson fields and their derivative, the effective interactions between the constituent quark and the exchanged boson are as follows [22, 24, 25, 26].

1. Pseudoscalar:

\[
\mathcal{L}_p = \frac{\mu_p^2}{4\pi} \left[ \frac{g_{pq}}{12m_q^2} \left( -H_1(\Lambda, m_p, \mu_p, r) \mathbf{\sigma} \cdot \mathbf{\sigma} + H_3(\Lambda, m_p, \mu_p, r) S_{ij}(\hat{r}) \right) \right]
\]

2. Scalar:

\[
\mathcal{L}_s = \frac{\mu_s^2}{4\pi} \left[ \frac{g_{pq}}{8m_q^2} H_1(\Lambda, m_s, \mu_s, r) + \frac{\mu_s^2}{2m_q^2} H_2(\Lambda, m_s, \mu_s, r) \mathbf{L} \cdot \mathbf{S}_{ij} \right]
\]

3. Vector:

\[
\mathcal{L}_v = \frac{\mu_v^2}{4\pi} \left\{ \left[ g_{pq} H_0(\Lambda, m_v, \mu_v, r) - (g_{pq} + 4g_{pq} f_{pq}) \frac{\mu_v^2}{8m_q^2} H_1(\Lambda, m_v, \mu_v, r) \right] \right. \\
\left. - (g_{pq} + f_{pq}) \frac{\mu_v^2}{12m_q^2} \left[ H_3(\Lambda, m_v, \mu_v, r) S_{ij}(\hat{r}) + 2H_1(\Lambda, m_v, \mu_v, r) (\mathbf{\sigma} \cdot \mathbf{\sigma}) \right] \right. \\
\left. - (3g_{pq} + 4g_{pq} f_{pq}) \frac{\mu_v^2}{2m_q^2} H_2(\Lambda, m_v, \mu_v, r) \mathbf{L} \cdot \mathbf{S}_{ij} \right\}
\]
where \( \mu^2 = m^2 - (m_{V1} - m_{V2})^2 \) approximately reflects the recoil effect with \( m \) being the exchanged vector meson mass. For \( I = 1 \) isovector boson exchange, the above three potentials in Eq. (2) - Eq. (4) should be multiplied by the operator \( \tau_i \cdot \tau_j \) in the isospin space.

The dimensionless functions \( H_0(\Lambda, m, \mu, r) \), \( H_1(\Lambda, m, \mu, r) \), \( H_2(\Lambda, m, \mu, r) \) and \( H_3(\Lambda, m, \mu, r) \) introduced in Eq. (2) - Eq. (4) are defined as follows

\[
\begin{align*}
H_0(\Lambda, m, \mu, r) &= \frac{1}{\mu r} (e^{-\mu r} - e^{-Xr}) - \frac{\Lambda^2 - m^2}{2\mu X} e^{-Xr} \\
H_1(\Lambda, m, \mu, r) &= -\frac{1}{\mu r} (e^{-\mu r} - e^{-Xr}) + \frac{X(\Lambda^2 - m^2)}{2\mu^2} e^{-Xr} \\
H_2(\Lambda, m, \mu, r) &= \left(1 + \frac{1}{\mu r}\right) \frac{1}{\mu^2 r^2} e^{-\mu r} - \frac{X}{\mu^2 r^2} e^{-Xr} - \frac{\Lambda^2 - m^2}{2\mu^2} e^{-Xr} \\
H_3(\Lambda, m, \mu, r) &= \left(1 + \frac{3}{\mu r}ight) \frac{1}{\mu^2 r^2} e^{-\mu r} - \frac{3}{X r^2} e^{-Xr} - \frac{\Lambda^2 - m^2}{2\mu^2} \left(1 + Xr\right) e^{-Xr}
\end{align*}
\]

with \( X^2 = \Lambda^2 + \mu^2 - m^2 \). In deriving the above effective potentials, we have introduced form factor at each interaction vertex to regularize the effective potential at short distance, and the form factor in momentum space is taken as

\[
F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}
\]

where \( \Lambda \) is the so-called regularization parameter, \( m \) and \( q \) are the mass and the four momentum of the exchanged boson respectively. This form factor suppresses the contribution of high momentum, i.e. small distance. The presence of such a form factor is dictated by the extended structure of the hadrons. The parameter \( \Lambda \), which governs the range of suppression, can be directly related to the hadron size which is approximately proportional to \( 1/\Lambda \). However, since the question of hadron size is still very much open, the value of \( \Lambda \) is poorly known phenomenologically, and it is dependent on the models and applications. In the nucleon-nucleon interactions, the \( \Lambda \) in the range of 0.8-1.5 GeV has been used to fit the data. For the present application to the heavy flavor vector mesons system, which have a smaller size than the nucleon, we would expect a larger regularization parameter \( \Lambda \). We have demonstrated that the binding energy and static properties of the deuteron are produced very well in the one boson exchange model, if \( \Lambda \) is chosen to be about 808 MeV [22]. In the case that all coupling constants except \( g_{NN} \) are reduced by half, \( \Lambda \) should be approximately 970 MeV. The extended structure of hadrons also has the following obvious consequence: because the mass of the exchanged meson determines the range of the corresponding contribution to the \( D^* D^* \) interactions, one should restrict oneself to meson exchange with the exchanged meson mass below a certain value, typically on the order of the regularization parameter \( \Lambda \). Since \( \pi, \eta, \sigma, \rho \) and \( \omega \) exchanges are considered in the present work, the value of \( \Lambda \) should be larger than the \( \omega \) meson mass.

In the present one boson exchange model, the input parameters include the masses of the exchanged bosons and heavy flavor vector mesons, and the effective coupling constants between the constituent quarks and the exchanged bosons. The meson masses are taken from the compilation of the Particle Data Group [28]: \( m_{\pi^\pm} = 139.57 \) MeV, \( m_{\pi^0} = 134.98 \) MeV, \( m_{\eta} = 547.85 \) MeV, \( m_{\sigma} = 600 \) MeV, \( m_{\rho} = 775.49 \) MeV, \( m_{\omega} = 782.65 \) MeV, \( m_{\eta'} = 5325.1 \) MeV. The constituent quark-meson coupling constants can be estimated from the phenomenologically known \( \pi NN, \eta NN, \sigma NN, \rho NN \) and \( \omega NN \) coupling constants via the well-known Goldberger-Treiman relation [22, 24].

\[
\begin{align*}
g_{\pi NN} &= \frac{3}{5} m_q \frac{m_N}{m_N} g_{\pi NN} \, \frac{m_q}{m_N} \, g_{NN} \\
g_{\rho NN} &= \frac{3}{5} m_q \, f_{\rho NN} \, g_{\rho NN} \, f_{\rho NN} \\
g_{\omega NN} &= \frac{1}{3} g_{\omega NN} \, g_{\omega NN} \, \frac{m_q}{m_N} \, f_{\omega NN} \, g_{\omega NN} \, f_{\omega NN} \\
g_{\sigma NN} &= \frac{1}{3} g_{\sigma NN} \, g_{\sigma NN} 
\end{align*}
\]

Among the above effective boson-nucleon coupling constants, only \( g_{NN} \) has been determined accurately from the pion-nucleon and nucleon-nucleon scatterings. As in our previous work [22, 23], the effective coupling constants are taken from the famous Bonn model [26]. The uncertainty of the coupling constants will be taken into account later, all the coupling constants except \( g_{NN} \) are reduced by half for demonstration, and the corresponding numerical results are presented as well.
For the system consisting of two vector mesons, the spatial parity is determined by $P = (-1)^L$ and the $C$ parity is $C = (-1)^{L+S}$, where $L$ is the relative angular momentum between two vector mesons, and $S$ is the total spin of the system. We cutoff the total angular momentum of the system to $J = 2$, the allowed states with low spin parity are listed in Table I. In the following, we shall study for which quantum numbers the one boson exchange potential is so attractive that bound states may exist.

$$V(r) = -V_\pi(r) + V_\eta(r) + V_\sigma(r) + V_\rho(r) - V_\omega(r)$$

$$= V_C(r) + V_S(r)(\sigma_i \cdot \sigma_j) + V_I(\mu_\rho, r)(\tau_i \cdot \tau_j) + V_F(r)(\hat{f}) + V_{SI}(\mu_\pi, \mu_\rho, r)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

$$+ V_{TI}(\mu_\pi, \mu_\rho, r)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) + V_{LS}(r)(L \cdot S_{ij}) + V_{LSI}(\mu_\rho, r)(L \cdot S_{ij})(\tau_i \cdot \tau_j)$$

where $V_M(r)$ ($M = \pi, \eta, \sigma, \rho$, and $\omega$) denotes the effective potential induced by the meson $M$ exchange between two quarks. The subscripts $i$ and $j$ are the indexes of light quark and antiquark. The spin operator(isospin operator) $\sigma_i$ or $\sigma_j$ ($\tau_i$ or $\tau_j$) only acts on the light quark and antiquark. The parameters $m_\pi, \mu_\pi$, and $\mu_\rho$ could take different sets of values due to the small mass splitting within the $D^*$ and $\pi$ isospin multiplets. For $D^{*0}D^{*0} \rightarrow D^{*0}D^{*0}$ and $D^{*+}D^{*-} \rightarrow D^{*+}D^{*-}$, we should choose $m_\pi = m_{\pi^0}$, $\mu_\pi = m_{\pi^0}$, and $\mu_\rho = m_\rho$. Whereas for the $D^{*0} \bar{D}^{*0} \rightarrow D^{*+}D^{*-}$

**III. THE POSSIBLE MOLECULAR STATES OF THE $D^* \bar{D}^*$ SYSTEM**

There is a sign difference $(-1)^G$ between the quark-quark interactions and quark-antiquark interactions, the magnitudes are the same, where $G$ is the $G$-parity of the exchanged meson. Consequently both $\pi$ and $\omega$ exchanges give opposite sign between quark-quark interactions and quark-antiquark interactions. In the present case, the effective potential is induced by one boson exchange between a pair of light quark and antiquark, and the diagram contributing to $D^* \bar{D}^*$ interactions is shown in Fig. 1. The effective potential is explicitly expressed as

| $J^P C$ | Channels |
|--------|----------|
| 0$^+$ | $^1S_0, ^3D_0$ |
| 1$^-$ | $^3S_1, ^3D_1$ |
| 0$^+$ | $^3P_0$ |
| 1$^+$ | $^5D_1$ |
| 1$^-$ | $^3P_2$ |
| 2$^-$ | $^3D_2$ |
| 1$^-$ | $^1P_1, ^5P_1, ^7P_1$ |
| 2$^+$ | $^1D_2, ^5S_2, ^7S_2, ^7G_2$ |
| 2$^-$ | $^3P_2, ^3F_2$ |
| 2$^-$ | $^5P_2, ^5F_2$ |

**TABLE I:** The allowed states of the system consisting of two vector mesons, where only states whose total angular momentum is smaller than 3 are listed.
and $D^{*+}D^{-} \rightarrow D^{*0}\bar{D}^{*0}$ processes, we should take $m_\pi = m_{\pi^+}$, $\mu_\pi = |m_{\pi^+} - (m_{D^{*+}} - m_{D^{*0}})|^{1/2} \equiv \mu_{\pi^+}$ and $\mu_\rho = |m_\rho - (m_{D^{*+}} - m_{D^{*0}})|^{1/2} \equiv \mu_{\rho}$. The eight potential functions $V_C(r)$, $V_S(r)$ etc are given by

$$V_C(r) = -\frac{g_{pq}^2 m_q}{4\pi} H_0(\Lambda, m_\sigma, m_\pi, r) + \frac{m_q^2}{8m_q^2} H_1(\Lambda, m_\sigma, m_\pi, r) - \frac{g_{pq}^2 m_\omega}{4\pi} H_0(\Lambda, m_\omega, m_\omega, r) + \frac{g_{pq}^2 + 4g_{pq}^2 f_{pq}}{4\pi} \frac{m_\pi^3}{8m_q^2} H_1(\Lambda, m_\omega, m_\omega, r)$$

$$V_S(r) = -\frac{g_{pq}^2 m_q^3}{4\pi} H_1(\Lambda, m_\sigma, m_\pi, r) + \frac{(g_{pq}^2 + f_{pq})^2 m_\pi^3}{6m_q^2} H_1(\Lambda, m_\omega, m_\omega, r)$$

$$V_T(\mu_\rho, r) = \frac{g_{pq}^2}{4\pi} \mu_\rho^2 H_0(\Lambda, m_\rho, m_\rho, r) - \frac{g_{pq}^2 + 4g_{pq}^2 f_{pq}}{4\pi} \frac{m_\rho^3}{8m_q^2} H_1(\Lambda, m_\rho, m_\rho, r)$$

$$V_T(\mu_\rho, r) = -\frac{g_{pq}^2 m_q^3}{4\pi} H_3(\Lambda, m_\sigma, m_\pi, r) + \frac{(g_{pq}^2 + f_{pq})^2 m_\pi^3}{12m_q^2} H_3(\Lambda, m_\omega, m_\omega, r)$$

$$V_{LS}(\mu_\rho, r) = -\frac{g_{pq}^2 m_q^3}{4\pi} H_2(\Lambda, m_\sigma, m_\pi, r) + \frac{3g_{pq}^2 + 4g_{pq}^2 f_{pq}}{4\pi} \frac{m_\pi^3}{2m_q^2} H_2(\Lambda, m_\omega, m_\omega, r)$$

$$V_{LS}(\mu_\rho, r) = -\frac{g_{pq}^2}{4\pi} \mu_\rho^3 H_2(\Lambda, m_\rho, m_\rho, r)$$

Since the threshold of $D^{*+}D^{-}$ is about 6.6 MeV higher than the $D^{*0}\bar{D}^{*0}$ threshold, the isospin symmetry is expected to be violated drastically for the $D^*D^*$ molecular states whose binding energy is of order a few MeV [22]. Isospin violation mainly comes from three aspects: the first is the different kinetic energies for $D^{*0}D^{*0}$ and $D^{*+}D^{-}$, the second is the different effective potentials from $\pi^0$ exchange and $\pi^\pm$ exchange, and the third is because of the different thresholds of $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{-}$. In the following, we will perform the same analysis as that for the deuteron and the possible heavy flavor molecules in Ref. [22, 23]. One can then determine which quantum numbers the one boson exchange potential is attractive and strong enough so that the $D^*D^*$ bound states are expected. Firstly we consider the $J^{PC} = 0^{++} D^* D^*$ states as an demonstration, the system can be in S wave or D wave similar to the deuteron. Taking into account the isospin violation effect, one has four coupled channels. For convenience, we choose the basis to be $| 1 \rangle \equiv |{^1S_0(D^{*0}\bar{D}^{*0})}\rangle$, $| 2 \rangle \equiv |{^3D_0(D^{*0}\bar{D}^{*0})}\rangle$, $| 3 \rangle \equiv |{^1S_0(D^{*+}D^{-})}\rangle$ and $| 4 \rangle \equiv |{^3D_0(D^{*+}D^{-})}\rangle$, then the wavefunction of the system is written as

$$| 0^{++}(D^*D^*) \rangle = \frac{u_1(r)}{\sqrt{r}} |{^1S_0(D^{*0}\bar{D}^{*0})}\rangle + \frac{u_2(r)}{\sqrt{r}} |{^3D_0(D^{*0}\bar{D}^{*0})}\rangle + \frac{u_3(r)}{\sqrt{r}} |{^1S_0(D^{*+}D^{-})}\rangle + \frac{u_4(r)}{\sqrt{r}} |{^3D_0(D^{*+}D^{-})}\rangle$$

where $u_1(r)$, $u_2(r)$, $u_3(r)$ and $u_4(r)$ are the spatial wavefunctions. The matrix elements of the light quark relevant operators $\sigma_i, \sigma_j, \bar{S}_{ij}(r)$ and $L \cdot S_{ij}$ etc in Eq. (5) can be calculated straightforwardly with the help of angular momentum algebra, and the results are given analytically in the Appendix of Ref. [22]. Consequently the one boson exchange potential for the $0^{++} D^* D^*$ state can be written in the matrix form as

$$V_{0^{++}}(r) = V_C(r) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + V_S(r) \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} + V_T(\mu_\rho, r) \begin{pmatrix} -1 & 0 & -2 \\ -2 & 0 & 1 \end{pmatrix}$$

$$+ V_T(\mu_\rho, r) \begin{pmatrix} 0 & -\sqrt{2} \\ -2 & 0 \\ 0 & 0 \\ 0 & -\sqrt{2} \end{pmatrix} + V_{LS}(m_\pi, \mu_\pi, \mu_\rho, r) \begin{pmatrix} 2 & 0 & 4 & 0 \\ 0 & -1 & 0 & -2 \\ 4 & 0 & 2 & 0 \\ 0 & -2 & 0 & -1 \end{pmatrix}$$

$$+ V_{LS}(m_\pi, \mu_\pi, \mu_\rho, r) \begin{pmatrix} 0 & \sqrt{2} \\ 2 & 0 \\ 0 & 2 \sqrt{2} \\ 2 \sqrt{2} \end{pmatrix} + V_{LS}(m_\pi, \mu_\pi, \mu_\rho, r) \begin{pmatrix} 0 & 0 \\ -3 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} + V_{LS}(m_\pi, \mu_\pi, \mu_\rho, r) \begin{pmatrix} 0 & 0 \\ 3 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} + V_{LS}(m_\pi, \mu_\pi, \mu_\rho, r) \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 6 \\ 0 \end{pmatrix}$$
For the up-left and down-right $2 \times 2$ matrix elements, we should choose $m_\pi = m_\omega$, $\mu_\pi = m_\rho$ and $\mu_\rho = m_\rho$. While for the off-diagonal $2 \times 2$ matrix elements, we should take $m_\pi = m_\omega$, $\mu_\pi = \mu_\omega$, and $\mu_\rho = \mu_\rho$. Taking into account the D wave centrifugal barrier and solving the coupled channel Schrödinger equation numerically, the numerical results are listed in the Table II. It is obvious that the binding energy and the static properties are rather sensitive to the regularization parameter $\Lambda$ and the effective coupling constants, this is common to the one boson exchange model [22, 23, 27]. We also find that the binding energy increases with $\Lambda$, this is because increasing $\Lambda$ increases the strength of the potential at short distance. For $\Lambda = 930$ MeV, a bound state with mass about 4013.80 MeV appears. We can see that the isospin symmetry is strongly broken especially for the states near the $D^*\bar{D}^*$ threshold. Fig. 2 displays the wavefunction of the bound state with mass 4004.40 MeV and $\Lambda = 950$ MeV. One notice that the $D^0\bar{D}^{*0}$ component dominates over the $D^*+\bar{D}^*$ component for both the S wave and D wave configurations, as could be expected. Because the wavefunctions $u_1(r)$ and $u_3(r)$ have the same sign, the same is true for $u_2(r)$ and $u_4(r)$. Therefore the $I = 0$ component is predominant for this state, it would be an isospin singlet in the isospin symmetry limit. The dominance of the $I = 0$ configuration is observed for all the states listed in Table II. From the numerical results, we can see that the D wave probability increases with the regularization parameter $\Lambda$, the importance of the tensor force is obvious. The uncertainties induced by the effective coupling constants are considered as well. All the coupling constants except $g_{\pi NN}$ are reduced by half, and the corresponding numerical results are presented in Table II. The same pattern of the static properties dependence on $\Lambda$ is found. Bound state solution appears if the regularization parameter $\Lambda$ is about 1100 MeV, the value of $\Lambda$ is still in the reasonable range. Since the molecular $D\bar{D}$ is strongly suppressed or even forbidden by the phase space. Consequently the $0^{++}$ $D^*\bar{D}^*$ molecule mainly decays into $D\bar{D}\gamma\gamma$, and $D\bar{D}\gamma\pi$, and the mode $D\bar{D}\pi\pi$ is strongly suppressed or even forbidden by the phase space.

![FIG. 2: The four components spatial wavefunctions of the 0++ D* D* state with $\Lambda = 940$ MeV.](image)

The one boson exchange potentials for the nine remaining states are listed in Appendix A. Following exactly the same method, the binding energy and the static properties can be predicted, and partial results are shown in Appendix B for illustration. The binding energy and the static properties are found to be rather sensitive the regularization parameter $\Lambda$. The spatial wavefunctions for the $D^0\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ components have the same sign, consequently the $I = 0$ component is dominant for all these states, and they are isospin singlets in the isospin symmetry limit. The same conclusion has been reached in the one pion exchange model [10], where the strength of the effective potential for the $I = 0$ state is about one third of that for $I = 1$. We shall discuss these states one by one in the following.

For the axial vector $1^{+-}$ state, there are four channels $^3S_1(D^{*0}\bar{D}^{*0})$, $^3D_1(D^{*0}\bar{D}^{*0})$, $^3S_1(D^{*+}\bar{D}^{*-})$ and $^3D_1(D^{*+}\bar{D}^{*-})$. The energy of the system is substantially lowered due to the S-D wave mixing effect. The coupling between the S wave and D wave has the same strength as the $0^{++}$ case, which is clearly seen from Eq. (11) and Eq. (14). It is obvious that the predictions for the binding energy and the static properties have similar pattern with the ones for the $0^{++}$ state, and the binding of the $1^{+-}$ state is less stronger than the $0^{++}$ one for the same $\Lambda$ value. For $\Lambda$ as large as 980 MeV, we can find a bound state with mass about 4012.43 MeV. If all coupling constants except $g_{\pi NN}$ are reduced by half, bound state begins to appear for $\Lambda \approx 1200$ MeV. We note that the unnatural spin parity forbids its decay into $D\bar{D}$, while the decay mode $D\bar{D}/D^*\bar{D}$ is allowed.

The pseudoscalar $0^{-+}$ $D^*\bar{D}^*$ state involve two channels $^3P_0(D^{*0}\bar{D}^{*0})$ and $^3P_0(D^{*+}\bar{D}^{*-})$. If the small isospin violation effect is neglected, the two coupled channel problem is reduced to a familiar one channel problem. Although
there is repulsive P wave centrifugal barrier, the one boson exchange potential is so strong that the P wave centrifugal barrier can be partly compensated, therefore bound state solutions can be found for reasonable values of $\Lambda$, as can be seen from Table III For $\Lambda = 950 - 1030$ MeV, we find that the binding energy with respect to the $D^{*0}\bar{D}^{*0}$ threshold is in the range of 2.6 to 137.2 MeV. The binding energy is more sensitive to $\Lambda$ than the $0^+$ and $1^+$ four coupled channels cases. The probabilities for the $^3P_0(D^{*0}\bar{D}^{*0})$ and $^3P_0(D^{*+}D^{*-})$ components are close to each other, as could be expected. The small difference is induced by the mass splitting within the $D^*$ and $\pi$ isospin multiplet.

The results for the $1^{++}$ state are similar to the $0^{+-}$ case. Because the D wave centrifugal barrier is higher than the P wave centrifugal barrier, the total effective potential for the $1^{++}$ state is less attractive than the $0^-$ state. If the coupling constants except $g_{\pi NN}$ are reduced by half, bound state solutions can be found only for $\Lambda$ larger than 1290 MeV. Since the numerical results indicate that larger value of $\Lambda$ is required to bind the $1^{++} D^*\bar{D}^*$ state, the $1^{++} D^*\bar{D}^*$ state is harder to be bound than the previous states considered.

For the CP exotic $1^{--}$ and $2^{+-}$ states, the system is in P wave and D wave respectively. Considering isospin violation effect, two channels are involved for both states. As has been shown in Eq. (3) and Eq. (4), the one boson exchange potentials for these two states are exactly the same, and they are less attractive than the potentials for the $0^{-+}$ and $1^{++}$ states. For the $1^{-+}$ state, bound state solution appears only for $\Lambda$ as large as 1300 MeV, and we can find $2^{+-}$ bound state only if the regularization parameter $\Lambda$ is larger than 1640 MeV. If we reduce all the coupling constants except $g_{\pi NN}$ by half, $\Lambda$ larger than 3010 MeV and 5110 MeV respectively for the $1^{-+}$ and $2^{+-}$ states is required to find bound state solutions. Because the value of $\Lambda$ is so large that it is far beyond the range of 0.8 to 1.5 GeV favored by the nucleon-nucleon interactions, we tend to conclude that the CP exotic $1^{--}$ and $2^{+-} D^*\bar{D}^*$ states can not be bound by the one boson exchange potential. This conclusion is consistent with the fact that no such CP exotic states have been observed so far.

We then come to the very interesting $1^{--} D^*\bar{D}^*$ state, there are six configurations $^1P_1(D^{*0}\bar{D}^{*0})$, $^5P_1(D^{*0}\bar{D}^{*0})$, $^5F_1(D^{*0}\bar{D}^{*0})$, $^1P_1(D^{*-D}^{*+})$, $^5P_1(D^{*-D}^{*+})$ and $^5F_1(D^{*-D}^{*+})$. Due to the substantially strong attraction of the effective potential, bound state solutions can be found for reasonable value of $\Lambda$ in spite of the P wave centrifugal barrier. The wavefunction of the bound state with mass about 4006.82 MeV and $\Lambda = 920$ MeV is displayed in Fig. 3. From the numerical results in Table IV, we see that the isospin symmetry is violated, especially for the states near the threshold. The $^5P_1(D^{*0}\bar{D}^{*0})$ and $^5P_1(D^{*-D}^{*+})$ are the dominant components, the $^5F_1$ components are strongly suppressed by the large P wave centrifugal barrier. For the same value of the regularization parameter $\Lambda$, we notice that the binding energy of $1^{--}$ state is the largest among the ten allowed $D^*\bar{D}^*$ states. Hence we suggest that the $1^{--} D^*\bar{D}^*$ molecular state should exist, this conclusion is consistent with the results obtained from the general quantum mechanical properties of unitarity and analyticity. The $1^{--} D^*\bar{D}^*$ state is remarkable, it can be directly produced via the $e^+e^-$ annihilation or with the help of the initial state radiation (ISR) technique at B factory. The existence of such a state can be either confirmed or rejected if more detailed $e^+e^-$ annihilation data near the $D^*\bar{D}^*$ threshold become available. We strongly urge the Babar and Belle collaboration to search for this state. The $1^{--} D^*\bar{D}^*$ molecule mainly decays into $D\bar{D}\gamma\gamma$ and $D\bar{D}\gamma\pi$ via the dissociation of $D^*$ and $\bar{D}$, the decays into $DD$ and $D\bar{D}^*/D^*\bar{D}$ are allowed as well. The width of the $1^{--} D^*\bar{D}^*$ molecule should be of the same order as the $D^*$ width. Therefore it would be a narrow state, and its width is expected to be of the order about 10 MeV. For the $1^{--}$ state $Y(4008)$ reported by the Belle collaboration, although it is close to the $D^*\bar{D}^*$ threshold, its width is huge, which is $\Gamma = (226 \pm 44 \pm 87)$ MeV. Consequently it seems unreasonable to interpret $Y(4008)$ as the $1^{--} D^*\bar{D}^*$ molecule.

![Graph](image-url)

**FIG. 3:** The spatial wavefunctions of the $1^{--} D^*\bar{D}^*$ state with $\Lambda = 920$ MeV.
For the $2^{++}$ state, eight channels $^1D_2(D^{*0}\bar{D}^{*0})$, $^5S_2(D^{*0}\bar{D}^{*0})$, $^5D_2(D^{*0}\bar{D}^{*0})$, $^5G_2(D^{*0}\bar{D}^{*0})$, $^1D_2(D^{*+}\bar{D}^{*-})$, $^5S_2(D^{*+}\bar{D}^{*-})$, $^5D_2(D^{*+}\bar{D}^{*-})$ and $^5G_2(D^{*+}\bar{D}^{*-})$ are involved. The effective potential in matrix form is given in Eq. (A7), which is more complex than the previous cases considered. It is obvious that both the tensor interaction and the spin-orbit interaction vanish in the $^5S_2$ configuration. However, bound state solution can be found for reasonable value of $\Lambda$ ($\Lambda$ should be larger than 860 MeV and 970 MeV respectively for the two sets of coupling constant values). The reason is that the mixing of $^5S_2$ with $^1D_2$, $^5D_2$ and $^5G_2$ under the tensor force increases the binding of the system considerably through higher order iterative processes. We show the wavefunction of the $2^{++}$ molecular state with mass 4010.82 MeV and $\Lambda = 860$ MeV in Fig. 4. Isospin violation is obvious, and the $^{1}S_{0}$ component is suppressed.

Finally two states with $J^{PC} = 2^{-+}$ and $2^{+-}$ respectively remain. For both states, one has four coupled channels, P wave and F wave configurations are involved. Because of the P wave and F wave centrifugal barrier, bound state begins to appear for $\Lambda$ as large as 1150 MeV and 1090 MeV respectively. If all the coupling constants except $g_{\pi NN}$ are reduced by half, the value of $\Lambda$ should be larger than 1630 MeV and 1480 MeV respectively in order to find bound state solutions. We tend to conclude that the one boson exchange potential may not support the $2^{+-}$ and $2^{+-}$ $D^{*}\bar{D}^{*}$ states.

In short summary, ten allowed $D^{*}\bar{D}^{*}$ states with low spin parity has been studied. We find that the isospin symmetry is violated, especially for the states near the $D^{*}\bar{D}^{*}$ threshold. The $I = 0$ configuration is dominant for all the lowest bound states, hence they would be isospin singlets in the isospin symmetry limit. Since $Z_{T}^{+}(4050)$ is an isospin vector, $Z_{S}^{+}(4050)$ as a $D^{*}\bar{D}^{*}$ molecule is not favored. This conclusion is consistent with the prediction from the QCD sum rule [10]. We suggest that the $1^{-+}$, $2^{++}$, $0^{++}$ and $0^{-+} D^{*}\bar{D}^{*}$ molecules should very likely exist, whereas the CP exotic $1^{--}$ and $2^{+}\bar{D}^{*}\bar{D}^{*}$ states, $2^{+-}$ and $2^{--} D^{*}\bar{D}^{*}$ states should not be bound by the one boson exchange potential. Although $Y(4008)$ is close to the $D^{*}\bar{D}^{*}$ threshold, its width is so large that it is not reasonable to identify $Y(4008)$ as a $D^{*}\bar{D}^{*}$ molecule. The $1^{--} D^{*}\bar{D}^{*}$ molecular state can be produced copiously in $e^{+}e^{-}$ annihilation or via the initial state radiation at $B$ factory, detailed scan of the $e^{+}e^{-}$ annihilation data near the $D^{*}\bar{D}^{*}$ threshold is crucial to confirming this prediction.

IV. THE MOLECULAR STATES OF THE $B^{*}\bar{B}^{*}$ SYSTEM

The mass difference between $B^{*+}$ and $B^{*0}$ is so small that it can be negligible [28]. If we take into account the mass splitting within the exchanged pion isospin multiplet, we should solve similar coupled channel problems as the $D^{*}\bar{D}^{*}$ case. The repulsive kinetic energy is greatly reduced due to the larger mass of $B^{*}$ meson, therefore the $B^{*}\bar{B}^{*}$ system should be more deeply bound than the $D^{*}\bar{D}^{*}$ system. Numerically solving the corresponding Schrödinger equation, we notice that the molecular states bound for reasonable $\Lambda$ value have definite isospin, and the $I = 0$ configuration is obviously much easier to be bound than the $I = 1$ configuration. We also find that the binding energy dependence...
on $\Lambda$ becomes less sensitive, if we reduce all the coupling constants except $g_{\pi NN}$ by half. In the case that the mass difference between $\pi^+$ and $\pi^0$ is neglected, the $B^*\bar{B}^*$ state is of definite isospin, and the dimensions of the coupled channel equations would be reduced by half. We have seriously calculated the binding energy and the static properties for both the $I = 0$ and $I = 1$ states. However, the numerical results are too lengthy to be listed in the manuscript. We find that introducing the pion mass splitting will modify the binding energy by at most 0.5 MeV.

For the $0^{-+}$ and $1^{+-}$ $B^*\bar{B}^*$ system, both the $I = 0$ and $I = 1$ bound states can be found for the same value of the regularization parameter $\Lambda$, whereas the two states behave in different way. The $I = 0$ state is generally more deeply bound than the $I = 1$ state. For the isospin singlet, the D wave components increase drastically with $\Lambda$. Whereas for the isospin vector, the S wave components are dominant, and they increase slightly with $\Lambda$. Because $B^*$ mainly decays into $B\gamma$ $[28]$, $BB\gamma\gamma$ is the leading decay mode of the $B^*\bar{B}^*$ molecule via the $B^*$ and $\bar{B}^*$ dissociation. For the eight remaining states, we also find that the $I = 0$ configuration is more tightly bound than the $I = 1$ configuration, hence it seems to be a universal result that the hadronic molecule prefers to a isospin singlet.

Since the mass splitting within the pion isospin multiplet only introduces minor modifications. The numerical results can be understood easily in the exact isospin symmetry limit. If the pion mass splitting is neglected, the attractive interaction in the intermediate range. Therefore $0^{-+}$ bound state can be found for reasonable values of $\Lambda$. For $\Lambda = 808 - 950$ MeV, we find the binding energy is in the range of 2.93 to 134.71 MeV. Because the D wave centrifugal barrier is partly compensated by the one boson exchange potential, and there remains a weak attractive interaction in the intermediate range. Therefore $0^{-+}$ bound state can be found for reasonable values of $\Lambda$. For $\Lambda = 900$ MeV, $1^{-+}$ $B^*\bar{B}^*$ bound state cannot be found, it is because the potential including the P wave centrifugal barrier is repulsive in this case. When $\Lambda$ is increased to about 1120 MeV, the total potential in Fig. 5b becomes attractive in the intermediate region, the $1^{-+}$ $B^*\bar{B}^*$ system is marginally bound. The one boson exchange potentials for the CP exotic $1^{-+}$ and $2^{-+}$ states are exactly the same. However, the $2^{-+}$ $B^*\bar{B}^*$ system is more loosely bound than the $1^{-+}$ system because of the D wave centrifugal barrier. Fig. 5d and Fig. 5e clearly show that the total potential of the $2^{-+}$ state is still repulsive for both $\Lambda = 900$ MeV and 1120 MeV. $2^{-+}$ $B^*\bar{B}^*$ bound state begins to appear for $\Lambda$ as large as 1260 MeV.

![FIG. 5: The potentials for the single channel $0^{-+}$, $1^{+-}$, $1^{-+}$ and $2^{+-}$ $B^*\bar{B}^*$ states with $I = 0$. (a) and (b) show the potentials for the $0^{-+}$ and $1^{+-}$ states with $\Lambda = 900$ MeV respectively. (c) and (d) are the potentials of the $1^{-+}$ states with $\Lambda = 900$ MeV and 1120 MeV respectively. (e) and (f) are the $2^{+-}$ states with $\Lambda = 900$ MeV and 1120 MeV respectively. The solid line represents the potential from one boson exchange, and the dashed line denotes the total potential including the centrifugal barrier.](image-url)
The predictions presented in the work could become more precise.

Detailed scanning of the independent part and $C_I$ multiplying the isospin relevant part, where the parameter $C_I$ is equal to -3 and 1 respectively for $I = 0$ and 1. Concretely for the $0^{-+}$ state, the isospin independent potential is $V_C(r) - V_S(r) - 2V_T(r) - V_{LS}(r)$, and the isospin relevant part is $V_I(\mu_+, r) - V_{SI}(\mu, \mu_+, \mu_+ r) - 2V_{TI}(\mu_+, \mu, \mu_+ r) - V_{LSI}(\mu, r)$. We plot both the isospin irrelevant and relevant potentials for these states in Fig. 6 it is obvious that the isospin irrelevant potential is usually attractive. For the $0^{-+}$ and $1^{++}$ states with $\Lambda = 900$ MeV, the isospin relevant potential is positive, hence it is easily understood why the $I = 0$ configuration is more attractive than $I = 1$. For the $1^{-+}$ state with $\Lambda = 900$ MeV, the isospin relevant part is negative, thus the potential for the $I = 1$ state is deeper than the one for $I = 0$. However, the corresponding potential still can not support a $1^{-+} B^*\bar{B}^*$ isovector state, such state can be bound only when $\Lambda$ is increased to about 1200 MeV. For $\Lambda$ as large as 1120 MeV, the isospin relevant potential becomes positive, accordingly the $I = 0$ configuration is more tightly bound than $I = 1$. In this case, a marginally bound $1^{-+} B^*\bar{B}^*$ isospin singlet appears with mass about 10635.39 MeV. If we reduce all the coupling constants except $g_{\pi NN}$ by half, the $1^{-+}$ and $2^{++}$ $B^*\bar{B}^*$ states can be bound only if $\Lambda$ is larger than 1700 MeV and 2790 MeV respectively. Therefore the one boson exchange potential may not support the CP exotic $1^{-+}$ and $2^{++}$ $B^*\bar{B}^*$ molecules.

![FIG. 6: The isospin relevant and irrelevant one boson exchange potentials for the $0^{-+}$, $1^{++}$ and $1^{-+} B^*\bar{B}^*$ states. (a) and (b) respectively show the potentials for the $0^{-+}$ and $1^{++}$ states with $\Lambda = 900$ MeV. (c) and (d) are the potentials for the $1^{-+}$ states with $\Lambda = 900$ MeV and 1120 MeV respectively. The one boson exchange potential for the $2^{++}$ state is exactly the same as the $1^{-+}$ case. The solid line represents the isospin irrelevant potential, and the dashed line denotes the isospin relevant part.](image)

Following the same method as the $D^*\bar{D}^*$ case and carefully examining the numerical results, we suggest that the $1^{-+}$, $2^{++}$, $0^{++}$ and $0^{-+}$ $B^*\bar{B}^*$ molecules should exist, and the $1^{++}$ $B^*\bar{B}^*$ bound state also very likely exists. Similar to the $1^{-+} D^*\bar{D}^*$ state, the $1^{-+} B^*\bar{B}^*$ molecule can be produced largely in $e^+e^-$ annihilation at Babar or Belle. Detailed scanning of the $e^+e^-$ annihilation data at the $B^*\bar{B}^*$ threshold is expected.

V. CONCLUSIONS AND DISCUSSIONS

Motivated by the charmonium-like state $Y(4008)$ and $Z_{1}^{0}(4050)$, the possible $D^*\bar{D}^*$ molecular states have been studied dynamically in the one boson exchange model, where $\pi, \eta, \sigma, \rho$ and $\omega$ exchanges are taken into account. Ten allowed states with low spin parity have been considered. We find that the binding energy and static properties are sensitive to the regularization parameter $\Lambda$ and the effective coupling constants. The binding energy increases with $\Lambda$, whereas the root of mean square radius decreases with $\Lambda$, this is because increasing $\Lambda$ increases the strength of the potential at short distance. Larger coupling constants are favorable to the formation of molecular states. If all the coupling constants except $g_{\pi NN}$ are reduced by half, larger value of $\Lambda$ is required to find bound state solutions. Isospin violation is expected, especially for the states close to the threshold, and the $I = 0$ component is dominant. The predominance of the $I = 0$ configuration over $I = 1$ can be clearly understood in the exact isospin symmetry limit, and the same conclusion is reached in the one pion exchange model. Hence the interpretation of $Z_{1}^{0}(4050)$ as a $D^*\bar{D}^*$ molecule is not favored.

Since the regularization parameter $\Lambda$ is poorly known so far, we are not able to precisely predict the binding energies for the possible molecular states bound by one boson exchange potential. Certainly, if the potential is strong enough one can be quite confident that such bound state must exist, but their exact binding energy always depends on the details of the regularization procedure. However, we can reliably predict which ones of the ten allowed states are much easier to be bound, and the prediction is rather stable even if the uncertainty of the coupling constants is considered, as is obvious from the numerical results in Appendix. Further research on X(3872) would put severe constraint on the parameters on the one boson exchange model, especially on the regularization parameter $\Lambda$, so that the predictions presented in the work could become more precise.

Our detailed numerical results indicate that the $1^{-+}$, $2^{++}$, $0^{++}$ and $0^{-+}$ $D^*\bar{D}^*$ bound states should very likely
exist, whereas the CP exotic $1^{-} +$ and $2^{++} D^*\bar{D}^*$ states, $2^{-} -$ and $2^{-} D^*\bar{D}^*$ states may not be bound by the one boson exchange potential. The $1^{-} -$ state can be directly produced in $e^+e^-$ annihilation, detailed $e^+e^-$ annihilation data near the $D^*\bar{D}^*$ threshold are important to confirm or refute the existence of such state. The $D^*\bar{D}^*$ molecule mainly decays into $D\bar{D}\gamma$ and $D\bar{D}\gamma\pi$ via the dissociation of $D^*$ and $\bar{D}^*$, the $DD\pi\pi$ mode is highly suppressed or forbidden by the phase space, and its width should be about tens of MeV. Although $Y(4008)$ is close to the $D^*\bar{D}^*$ threshold, its width is huge so that it is unreasonable to identify $Y(4008)$ as a $D^*\bar{D}^*$ molecule. The existence of $Y(4008)$ and $Z^+_1 (4050)$ has not been confirmed so far, more experimental efforts are urgently needed.

The possible $B^*\bar{B}^*$ molecular states have been discussed along the same line. The $B^*\bar{B}^*$ system is more deeply bound than the $D^*\bar{D}^*$ system. We find that the molecular states of the $B^*\bar{B}^*$ system have definite isospin, and the isospin singlet is much easier to be bound than the isospin vector for the states considered, the reason is analyzed in the isospin symmetry limit. If the small isospin mass splitting between the neutral and charged pion mesons is neglected, the dimension of the coupled channel problem is reduced by half, the situation is simplified greatly. It is observed that including the pion mass splitting modifies the binding energy by at most 0.5 MeV. We suggest that the $1^{-} -$, $2^{++}$, $0^{++}$ and $0^{--}$ $B^*\bar{B}^*$ molecules should exist, they would be narrow states, and they dominantly decay into $BB\gamma\gamma$.

Acknowledgments

We are grateful to Prof. Mu-Lin Yan and Dao-Neng Gao for stimulating discussions. This work is supported by the China Postdoctoral Science Foundation (20070420735), K.C. Wong Education Foundation, and KJCX2-YW-N29 of the Chinese Academy.

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APPENDIX A: THE ONE BOSON EXCHANGE POTENTIAL FOR THE $D^* \bar{D}^*(B^* \bar{B}^*)$ STATES INCLUDING THE ISOSPIN MASS SPLITTING

\[ V_{1+} = [V_C(r) - V_S(r)] \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + [-V_I(\mu, r) + V_{SI}(m, \mu, \mu, r)] \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix} \]

\[ + V_T(r) \begin{pmatrix} 0 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & \sqrt{2} & -1 \end{pmatrix} + V_{TI}(m, \mu, \mu, r) \begin{pmatrix} 0 & -\sqrt{2} & -\sqrt{2} & -2\sqrt{2} \\ -\sqrt{2} & 1 & -2\sqrt{2} & 2 \\ 0 & -2\sqrt{2} & 0 & -\sqrt{2} \\ -2\sqrt{2} & 2 & -\sqrt{2} & 1 \end{pmatrix} \]

\[ + V_{LS}(r) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 0 \end{pmatrix} + V_{LSI}(\mu, r) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} \end{pmatrix} \]

(A1)

\[ V_{0++} = [V_C(r) - V_S(r) - 2V_T(r) - V_{LS}(r)] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + [-V_I(\mu, r) + V_{SI}(m, \mu, \mu, r)] \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \]

(A2)

\[ V_{1++} = [V_C(r) + V_S(r) - V_T(r) - \frac{5}{2}V_{LS}(r)] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + [-V_I(\mu, r) - V_{SI}(m, \mu, \mu, r)] \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \]

(A3)

\[ V_{1--} = [V_C(r) - V_S(r) + V_T(r) - \frac{1}{2}V_{LS}(r)] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + [-V_I(\mu, r) + V_{SI}(m, \mu, \mu, r)] \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \]

(A4)

\[ V_{2+-} = [V_C(r) - V_S(r) + V_T(r) - \frac{1}{2}V_{LS}(r)] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + [-V_I(\mu, r) + V_{SI}(m, \mu, \mu, r)] \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \]

(A5)
\[ V_1(r) = V_C(r) + V_S(r) + V_I(\mu_\rho, r) + V_T(r) + V_I(m_\pi, \mu_\rho, r) + V_L(r) + V_{LS}(\mu_\rho, r) + V_{LSI}(\mu_\rho, r) \]
\[ V_{2++} = V_C(r) + V_S(r) \]

\[ + V_I(\mu_\rho, r) \]

\[ + V_{SI}(m_\pi, \mu_\pi, \mu_\rho, r) \]

\[ + V_{TT}(m_\pi, \mu_\pi, \mu_\rho, r) \]

\[ + V_{LS}(r) \]

\[ + V_{LSI}(\mu_\rho, r) \]

\[ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \]

\[ \begin{pmatrix} \sqrt{\frac{2}{5}} & 0 & -\sqrt{\frac{2}{5}} & \sqrt{\frac{3}{5}} & 0 & -\sqrt{\frac{2}{5}} & 0 & -\sqrt{\frac{2}{5}} \\ 0 & \sqrt{\frac{2}{5}} & 0 & \sqrt{\frac{3}{5}} & 0 & \sqrt{\frac{2}{5}} & 0 & \sqrt{\frac{2}{5}} \\ -\sqrt{\frac{2}{5}} & 2 \sqrt{\frac{2}{5}} & 0 & 2 \sqrt{\frac{2}{5}} & 0 & 2 \sqrt{\frac{2}{5}} & 0 & 2 \sqrt{\frac{2}{5}} \\ 2 \sqrt{\frac{2}{5}} & 2 \sqrt{\frac{2}{5}} & 0 & \sqrt{\frac{3}{5}} & 0 & \sqrt{\frac{3}{5}} & 0 & \sqrt{\frac{3}{5}} \end{pmatrix} \]

(A7)
In the above expressions, the parameters $m_\pi, \mu_\pi$ and $\mu_\rho$ could take two different sets of values. For $D^{*0} \bar{D}^{*0} \rightarrow D^{*0} \bar{D}^{*0}$ ($B^{*0} \bar{B}^{*0} \rightarrow B^{*0} \bar{B}^{*0}$) and $D^{*+} D^{-} \rightarrow D^{*+} D^{-}$ ($B^{*+} B^{-} \rightarrow B^{*+} B^{-}$), we should choose $m_\pi = m_{\pi^0}$, $\mu_\pi = m_{\pi^0}$ and $\mu_\rho = m_\rho$. Whereas for the processes $D^{*0} \bar{D}^{*0} \rightarrow D^{*+} D^{-}$ ($B^{*0} \bar{B}^{*0} \rightarrow B^{*+} B^{*-}$) and $D^{*+} D^{-} \rightarrow D^{*0} \bar{D}^{*0}$ ($B^{*+} B^{*-} \rightarrow B^{*0} \bar{B}^{*0}$), we should take $m_\pi = m_{\pi^*}$, $\mu_\pi = [m_{\pi^*}^2 - (m_{D^{*+}} - m_{D^{*0}})^2]^{1/2}$ ($\mu_\pi = m_{\pi^*}$) and $\mu_\rho = [m_\rho^2 - (m_{D^{*+}} - m_{D^{*0}})^2]^{1/2}$ ($\mu_\rho = m_\rho$) respectively.

**APPENDIX B: NUMERICAL RESULTS FOR THE $D^* D^*$ STATES**
| Λ(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{S}^{00}$ : $P_{D}^{00}$ : $P_{S}^{+}$ : $P_{D}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 960    | 3990.26| 1.03                 | 25.21:29.74:16.55:28.50                          |
| 970    | 3968.64| 0.86                 | 17.70:34.73:14.01:33.57                          |
| 980    | 3938.98| 0.76                 | 13.19:38.20:11.41:37.19                          |
| 990    | 3900.83| 0.68                 | 10.22:40.68:9.27:39.83                           |

All couplings are reduced by half except $g_{\pi NN}$

| A(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{S}^{00}$ : $P_{D}^{00}$ : $P_{S}^{+}$ : $P_{D}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 950    | 4011.77| 2.36                 | 64.96:12.17:10.77:12.10                          |
| 940    | 4011.72| 2.39                 | 70.69:11.01:7.43:10.88                          |
| 930    | 4004.40| 1.36                 | 39.99:22.32:16.38:21.31                          |
| 960    | 3990.26| 1.03                 | 25.21:29.74:16.55:28.50                          |
| 970    | 3968.64| 0.86                 | 17.70:34.73:14.01:33.57                          |
| 980    | 3938.98| 0.76                 | 13.19:38.20:11.41:37.19                          |
| 990    | 3900.83| 0.68                 | 10.22:40.68:9.27:39.83                           |

TABLE II: The predictions for the static properties of the $J^{P}=0^{++}$ $D^{*}\bar{D}^{*}$ hadronic molecule, where $M$ denotes the mass, $r_{\text{rms}}$ is the root of mean square radius, $P_{S}$ and $P_{D}$ represent the S state and D state probabilities respectively.

| A(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{S}^{00}$ : $P_{D}^{00}$ : $P_{S}^{+}$ : $P_{D}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 1110   | 4011.77| 2.36                 | 64.96:12.17:10.77:12.10                          |
| 1130   | 3998.69| 1.15                 | 30.86:26.00:18.08:25.06                          |
| 1160   | 3971.28| 0.85                 | 18.83:33.55:15.04:32.59                          |
| 1190   | 3927.61| 0.70                 | 13.03:38.09:11.59:37.29                          |
| 1220   | 3866.06| 0.61                 | 9.66:40.98:9.01:40.35                           |

All couplings except $g_{\pi NN}$ are reduced by half

| A(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{S}^{00}$ : $P_{D}^{00}$ : $P_{S}^{+}$ : $P_{D}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 1120   | 4012.74| 2.05                 | 60.78:39.22                                     |
| 1150   | 4004.34| 1.30                 | 54.77:45.23                                     |
| 1200   | 3980.14| 0.94                 | 52.20:47.80                                     |
| 1250   | 3940.92| 0.75                 | 51.28:48.72                                     |
| 1300   | 3883.95| 0.63                 | 50.83:49.17                                     |

TABLE III: The predictions about the mass, the root of mean square radius($r_{\text{rms}}$) and the probabilities of the different components for the $0^{-}+D^{*}\bar{D}^{*}$ molecule.

| A(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{1}^{00}$ : $P_{2}^{00}$ : $P_{1}^{+}$ : $P_{2}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 920    | 4006.82| 1.35                 | 4.68:50.36:0.42:3.59:40.55:0.40                  |
| 930    | 3997.17| 1.11                 | 3.79:49.19:0.47:3.21:42.89:0.45                  |
| 940    | 3984.57| 0.97                 | 3.21:48.71:0.51:2.86:44.19:0.50                  |
| 950    | 3968.85| 0.87                 | 2.80:48.47:0.57:2.57:45.04:0.55                  |
| 960    | 3949.86| 0.79                 | 2.49:48.33:0.62:2.33:45.61:0.61                  |
| 970    | 3927.45| 0.73                 | 2.25:48.21:0.70:2.13:46.02:0.69                  |
| 980    | 3901.44| 0.68                 | 2.06:48.11:0.79:1.97:46.29:0.78                  |

All couplings except $g_{\pi NN}$ are reduced by half

| A(MeV) | M(MeV) | $r_{\text{rms}}$(fm) | $P_{1}^{00}$ : $P_{2}^{00}$ : $P_{1}^{+}$ : $P_{2}^{+}$ (%) |
|--------|--------|----------------------|--------------------------------------------------|
| 1050   | 4007.15| 1.34                 | 5.36:49.41:0.49:4.16:46.10:0.48                  |
| 1080   | 3991.87| 1.01                 | 4.10:48.03:0.59:3.60:43.11:0.58                  |
| 1110   | 3969.82| 0.84                 | 3.38:47.66:0.67:3.12:44.51:0.66                  |
| 1140   | 3940.40| 0.73                 | 2.90:47.52:0.77:2.75:45.31:0.75                  |
| 1170   | 3903.00| 0.65                 | 2.56:47.43:0.88:2.46:45.79:0.87                  |

TABLE IV: The predictions about the mass, the root of mean square radius($r_{\text{rms}}$) and the probabilities of the different components for the $1^{-}+D^{*}\bar{D}^{*}$ molecule, where $P_{1}^{00}$ and $P_{2}^{00}$ denote the $1P_{1}$ and $5P_{1} D^{*0}\bar{D}^{*0}$ states probabilities respectively.
| $\Lambda$(MeV) | $M$(MeV) | $r_{\text{rms}}$(fm) | $P^{00}_{D0}$ : $P^{00}_{S}$ : $P^{00}_{D2}$ : $P^{0+}_{G}$ : $P^{±}_{-}$ : $P^{±}_{+}$ : $P^{±}_{G}$ (%) |
|-----------------|----------|---------------------|---------------------------------|
| 860             | 4010.82  | 1.96                | 0.39:67.73:1.95:0.01:0.38:27.59:1.96:0.01 |
| 890             | 4007.14  | 1.49                | 0.51:60.38:2.68:0.01:0.49:33.27:2.64:0.01 |
| 920             | 4001.88  | 1.26                | 0.64:55.35:3.60:0.02:0.61:36.24:3.51:0.02 |
| 950             | 3994.34  | 1.10                | 0.79:51.20:4.86:0.04:0.76:37.58:4.72:0.04 |
| 980             | 3983.27  | 0.98                | 0.98:47.19:6.63:0.09:0.94:37.63:6.43:0.09 |
| 1010            | 3966.58  | 0.88                | 1.21:42.85:9.11:0.22:1.17:36.37:8.85:0.22 |
| 1040            | 3940.41  | 0.79                | 1.51:37.53:12.49:0.69:1.47:33.44:12.18:0.68 |
| 1070            | 3894.04  | 0.69                | 2.01:29.16:16.55:3.55:1.96:27.02:16.22:3.53 |

**TABLE V:** The predictions about the mass, the root of mean square radius($r_{\text{rms}}$) and the probabilities of the different components for the $2^{++}$ $D^{*}\bar{D}^{*}$ molecule.

all couplings except $g_{\pi NN}$ are reduced by half

| $\Lambda$(MeV) | $M$(MeV) | $r_{\text{rms}}$(fm) | $P^{00}_{D0}$ : $P^{00}_{S}$ : $P^{00}_{D2}$ : $P^{0+}_{G}$ : $P^{±}_{-}$ : $P^{±}_{+}$ : $P^{±}_{G}$ (%) |
|-----------------|----------|---------------------|---------------------------------|
| 970             | 4009.21  | 1.66                | 0.54:61.73:2.69:0.01:0.53:31.79:2.69:0.01 |
| 1000            | 4006.34  | 1.41                | 0.63:57.50:3.18:0.02:0.60:34.90:3.15:0.02 |
| 1100            | 3991.66  | 1.01                | 0.89:49.31:5.04:0.05:0.87:38.86:4.94:0.05 |
| 1200            | 3965.34  | 0.81                | 1.21:43.80:7.63:0.14:1.18:38.43:7.48:0.14 |
| 1300            | 3918.43  | 0.68                | 1.59:38.25:11.19:0.47:1.56:35.48:11.01:0.46 |
| 1350            | 3882.03  | 0.62                | 1.82:34.98:13.32:0.95:1.79:33.04:13.15:0.95 |