Differences Between The $^3P_0$ and $C^3P_0$ model in the Charming Strange Sector

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Abstract.

The goal of this work is to establish a comparison between the very well studied $^3P_0$ model and a bound-state corrected version, the $C^3P_0$ model, obtained from applying the Fock-Tani transformation to the $^3P_0$ model, in the context of the charmed-strange meson sector ($D_{sJ}$ meson). In particular, we shall calculate the decay amplitudes and decay rates of the $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^0$ and $D_{s1}(2536)^+ \rightarrow D^*(2010)^+ K^0$, showing the differences between the two models.

1. Introduction

The Fock-Tani formalism is a field theoretic method appropriated for the simultaneous treatment of composite particles and their constituents. This technique was originally used in atomic physics [1] and later in hadron physics to describe hadron-hadron scattering interactions [2, 3, 4] and meson decay [5, 6].

The $^3P_0$ model is a typical decay model which considers only OZI-allowed decay processes. The model considers a quark-antiquark pair created with the vacuum quantum numbers which
interact with a meson in the initial state. It is described as the non-relativistic limit of a pair creation Hamiltonian [7].

The Fock-Tani transformation is applied to a \( \bar{q}q \) pair creation Hamiltonian, producing a characteristic expansion in powers of the wave function, where the \( 3^P_0 \) model is the lowest order term in the expansion. The corrected \( 3^P_0 \) model \( (C^3P_0) \) is obtained from higher orders terms in this expansion, where terms containing the bound state kernel appear [5].

Both the \( 3^P_0 \) model and \( C^3P_0 \) model have been widely used in the study of meson spectroscopy. Our motivation for this work is in a comparison of these models for mesons in the charmed-strange sector. In particular, we shall calculate the decay amplitudes and decay rates of \( D_s(2460)^+ \rightarrow D_s^{*+} \pi^0 \) and \( D_s(2536)^+ \rightarrow D_s^{*}(2010)^+ K^0 \).

2. Mesons in the Fock-Tani formalism: a brief outline

In the Fock-Tani formalism (FTf) we can write the meson creation operators in the following form:

\[
M^\dagger_\alpha y = \Phi^\mu_{\alpha y} q_\mu \bar{q}_\mu.
\]

A single particle state in second-quantization is

\[
|\alpha\rangle = M^\dagger_\alpha |0\rangle,
\]

where \( \Phi^\mu_{\alpha y} \) is the bound-state wave-function for two-quarks. The quark and antiquark operators obey the usual anticommutation relations. The composite meson operators satisfy non-canonical commutation relations

\[
[M_\alpha, M_\beta] = 0 \; ; \; [M_\alpha, M^\dagger_\beta] = \delta_{\alpha\beta} - \Delta_{\alpha\beta},
\]

where

\[
\Delta_{\alpha\beta} = \Phi^{*\mu\nu}\Phi_\beta^{\gamma\rho} q_\rho q_\mu + \Phi^{*\mu\nu}\Phi_\beta^{\gamma\rho} \bar{q}_\rho \bar{q}_\mu.
\]

The idea of the FTf is to make a representation change, where the composite particle operators are described by “ideal particle” operators that satisfy canonical commutation relations, i.e.,

\[
[m_\alpha, m_\beta] = 0 \; ; \; [m_\alpha, m^\dagger_\beta] = \delta_{\alpha\beta}.
\]

To implement this change of representation one can define a unitary transformation \( U \) that maps the composite state \( |\alpha\rangle \) into an ideal state \( |\alpha\rangle \). In the meson case, for example, we have

\[
U^{-1} M^\dagger_\alpha |0\rangle = m^\dagger_\alpha |0\rangle \equiv |\alpha\rangle,
\]

where \( U = \exp (tF) \) and \( F \) is the generator of the meson transformation given by

\[
F = m^\dagger_\alpha \tilde{M}_\alpha - \tilde{M}^\dagger_\alpha m_\alpha,
\]

with \( \tilde{M}_\alpha \) defined up to third order

\[
\tilde{M}_\alpha = M_\alpha + \frac{1}{2}\Delta_{\alpha\beta} M_\beta + \frac{1}{2} M^\dagger_\beta [\Delta_{\beta\gamma}, M_\alpha] M_\gamma.
\]

3. The Microscopic Model

The Hamiltonian used in this model is inspired in the \( 3^P_0 \) model, deduced in [7]:

\[
H_I = g \int d^3x \Psi^\dagger(\vec{x}) \gamma^0 \Psi(\vec{x})
\]
where $\Psi(x)$ is the Dirac quark field, one should note that the bilinear $\Psi^\dagger \gamma^0 \Psi$ leads to the decay $(q\bar{q})_A \rightarrow (q\bar{q})_B + (q\bar{q})_C$ through the $b^\dagger d^\dagger$ term. Introducing the following notation $b \rightarrow q; \ d \rightarrow q\ : \ \mu = (\vec{p}', s')$ e $\nu = (\vec{p}, s)$, after the expansion in the momentum representation, one obtains a compact notation for $H_I$:

$$H_I = V_{\mu\nu} \ q_\mu \ q_\nu$$

where the sum (integration) is applied over repeated indeces and

$$V_{\mu\nu} \equiv -\gamma \ \delta_{\alpha\beta} \ \delta_{\gamma\delta} \ \delta (\vec{p}_\mu + \vec{p}_\nu) \ \chi^\alpha_{\mu} \ \chi^\gamma_{\nu} \ \chi^\beta_{\nu} \ \chi^\delta_{\mu} . \ (2)$$

In Eq. (2) $\gamma$ is the free parameter pair production strength with $\gamma = g/2m_q$, where $m_q$ is the quark mass of the pair creation. Applying the Fock-Tani transformation to $H_I$ one obtains the effective Hamiltonian

$$H^{C_3 P_0}_T = U^{-1} H_I U = H_0 + \delta H_1$$

The decay amplitude $h_{fi}$ for $m_{\gamma} \rightarrow m_{\alpha} + m_{\beta}$, is given by

$$\langle f \ | \ H^{C_3 P_0}_T \ | \ i \rangle = \delta (P_{\gamma} - P_{\alpha} - P_{\beta}) \ h_{fi} \ (3)$$

where $|i\rangle = m_{\alpha} \ m_{\beta} \ |0\rangle$ and $|f\rangle = m_{\alpha} \ m_{\beta} \ |0\rangle$ and

$$h_{fi} = -V_{\mu\nu} \ \left\{ \Phi_{\beta}^{*\rho\mu} \Phi_{\alpha}^{*\mu\rho} + \Phi_{\beta}^{*\rho\mu} \Phi_{\alpha}^{*\mu\rho} \right\} \ \Phi_{\gamma}^{\rho\eta}$$

$$-\frac{1}{4} V_{\mu\nu} \ \left\{ \Phi_{\beta}^{*\rho\tau} \Phi_{\alpha}^{*\mu\eta} + \Phi_{\beta}^{*\rho\tau} \Phi_{\alpha}^{*\mu\eta} \right\} \ \Delta (\rho\eta; \ \lambda\nu) \ \Phi^{\lambda\tau}_{\gamma}$$

$$-\frac{1}{4} V_{\mu\nu} \ \left\{ \Phi_{\beta}^{*\rho\eta} \Phi_{\alpha}^{*\mu\tau} + \Phi_{\beta}^{*\rho\eta} \Phi_{\alpha}^{*\mu\tau} \right\} \ \Delta (\rho\mu; \ \lambda\xi) \ \Phi^{\lambda\xi}_{\gamma}$$

$$+\frac{1}{2} V_{\mu\nu} \ \left\{ \Phi_{\alpha}^{*\rho\sigma} \Phi_{\beta}^{*\sigma\tau} + \Phi_{\alpha}^{*\rho\sigma} \Phi_{\beta}^{*\sigma\tau} \right\} \ \Delta (\rho\eta; \ \mu\nu) \ \Phi^{\sigma\tau}_{\gamma} \ (4)$$

In the Eq. (4), the terms dependent on $\Delta$ are the bound state corrections, where the kernel represents an intermediate state of transition of the particle from the initial state to the final state. In this kernel a sum is performed over mesons with the quantum numbers of the final state. The meson wave function is defined as

$$\Phi_{\alpha}^{\mu\nu} = \chi_{S^2 \alpha} \ f_{f_1 f_2} \ C^{c_1 c_2} \ F_{n l} \ -\vec{p}_1 -\vec{p}_2 ,$$

where $\chi$ is spin; $f$ is flavor and $C$ are color coefficients. The spatial part is given by the SHO wave-functions [8].

4. Applications and Results

Now we shall consider some specific processes for a comparative study between the $3P_0$ and $C_3 P_0$ models. In particular, the decay processes studied are: $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^0$ and $D_{s1}(2536)^+ \rightarrow D^*(2010)^+ K^0$. The full expressions for the decay amplitudes $h_{fi}$ has the following form

$$h_{fi} = \left[ \frac{\gamma}{\pi \sqrt{4(\rho + 1)^2}} \right] \sum_{LS} C_{LS} Y_{LM}(\Omega) ,$$

where $\Psi(x)$ is the Dirac quark field, one should note that the bilinear $\Psi^\dagger \gamma^0 \Psi$ leads to the decay $(q\bar{q})_A \rightarrow (q\bar{q})_B + (q\bar{q})_C$ through the $b^\dagger d^\dagger$ term. Introducing the following notation $b \rightarrow q; \ d \rightarrow q\ : \ \mu = (\vec{p}', s')$ e $\nu = (\vec{p}, s)$, after the expansion in the momentum representation, one obtains a compact notation for $H_I$:

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$$\langle f \ | \ H^{C_3 P_0}_T \ | \ i \rangle = \delta (P_{\gamma} - P_{\alpha} - P_{\beta}) \ h_{fi} \ (3)$$

where $|i\rangle = m_{\alpha} \ m_{\beta} \ |0\rangle$ and $|f\rangle = m_{\alpha} \ m_{\beta} \ |0\rangle$ and

$$h_{fi} = -V_{\mu\nu} \ \left\{ \Phi_{\beta}^{*\rho\mu} \Phi_{\alpha}^{*\mu\rho} + \Phi_{\beta}^{*\rho\mu} \Phi_{\alpha}^{*\mu\rho} \right\} \ \Phi_{\gamma}^{\rho\eta}$$

$$-\frac{1}{4} V_{\mu\nu} \ \left\{ \Phi_{\beta}^{*\rho\tau} \Phi_{\alpha}^{*\mu\eta} + \Phi_{\beta}^{*\rho\tau} \Phi_{\alpha}^{*\mu\eta} \right\} \ \Delta (\rho\eta; \ \lambda\nu) \ \Phi^{\lambda\tau}_{\gamma}$$

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$$+\frac{1}{2} V_{\mu\nu} \ \left\{ \Phi_{\alpha}^{*\rho\sigma} \Phi_{\beta}^{*\sigma\tau} + \Phi_{\alpha}^{*\rho\sigma} \Phi_{\beta}^{*\sigma\tau} \right\} \ \Delta (\rho\eta; \ \mu\nu) \ \Phi^{\sigma\tau}_{\gamma} \ (4)$$

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$$h_{fi} = \left[ \frac{\gamma}{\pi \sqrt{4(\rho + 1)^2}} \right] \sum_{LS} C_{LS} Y_{LM}(\Omega) ,$$
where the coefficients $C_{LS}$ are polynomials which have a dependence on the momentum $P$ and in $\beta$ gaussian width of the mesons involved in the processes. The decay amplitude $h_{fi}$ can be combined with relativistic phase space to give the decay rate [5, 7]:

$$\Gamma_{A\rightarrow BC} = 2\pi P \frac{E_B E_C}{M_A} \left(\frac{\gamma}{\pi^{1/4}(\rho + 1)^2}\right)^2 \sum_{LS} (C_{LS})^2.$$ 

The experimental values are extracted from “Particle Data Group 2010” (PDG) [9] and the theoretical values obtained with $^3P_0$ and $C^3P_0$ model for these processes are shown in tables 1 and 2. In Tab. 1 we can see that the $^3P_0$ model is zero for the decay processes with final state $D_{SJ} \pi$. In Tab. 2 the two models obtain the equal results. For the theoretical results presented in tables the $\gamma$ and $\beta$ (in GeV) are $\gamma = 0.420$, $\beta_0 = 0.410$, $\beta_{K^0} = 0.399$, $\beta_{D^{*+}(2010)^+} = 0.280$, $\beta_{D_s^{*+}} = 0.200$ and for the intermediate state $\beta_6 = 0.100$ and $\beta_7 = 0.630$ is the state $1^1S_0$ and the intermediate state $\beta_8 = 0.300$ and $\beta_9 = 0.200$ is the state $1^3S_1$.

### Table 1. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2460)^+$.

| Process | Exp. (PDG) | $^3P_0$ | $C^3P_0$ |
|---------|-------------|----------|----------|
| $\Gamma_{D_s^{+} \rightarrow D^{+} \pi}$ | 0.48 ± 0.11 | 0 | 0.014 |

### Table 2. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2536)^+$.

| Process | Exp. (PDG) | $^3P_0$ | $C^3P_0$ |
|---------|-------------|----------|----------|
| $\Gamma_{D_s^{+}(2010)^{+} \rightarrow K^0}$ | 0.72 ± 0.05 ± 0.01 | 0.995 | 0.995 |

### 5. Conclusions

Briefly we presented a comparison of the $^3P_0$ model and the Corrected $^3P_0$ model applied for two meson decay processes of the charmed-strange sector. In this sector the decay processes are of two forms: $D_{SJ}^{\pm} \rightarrow D_{SJ} \pi$ and $D_{SJ}^{\pm} \rightarrow D^{*} K$. The first can not be obtained in the $^3P_0$ model, but the $C^3P_0$ model can be applied for both decay processes. The next step will be to consider the other $D$ decay channels in the Corrected $^3P_0$ model.

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