On hard exclusive reactions in the time-like region

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The proton form factor, two-photon annihilations into $p\bar{p}$ as well as exclusive charmonium decays are critically examined. It will be argued that the standard perturbative QCD analysis of these reactions fails, i.e. the need for additional contributions can convincingly be demonstrated. Possible dynamical mechanisms such as colour-octet admixtures to the charmonium states or diquarks inside baryons, will be discussed and compared to the data.

1. INTRODUCTION

At large Mandelstam $s$ and large momentum transfer the hard scattering approach (HSA) provides a scheme to calculate exclusive processes. Observables are described as convolutions of hadronic wave functions which embody soft non-perturbative physics, and hard scattering amplitudes $T_H$ to be calculated from perturbative QCD. In most cases only the contribution from the lowest-order pQCD approach in the collinear approximation using valence Fock states only (termed the standard HSA) has been worked out. Applications of the standard HSA to space-like exclusive reactions, as for instance the magnetic form factor of the nucleon, the pion form factor or Compton scattering off protons revealed that the results are only in fair agreement with experiment if strongly end-point region (where one of the quark momentum fractions, $x$, tends to zero) concentrated hadronic wave functions are used. As has been pointed out by several authors (e.g. $^{[4]}$), the results obtained from such wave functions are dominated by contributions from the end-point regions where perturbative QCD cannot readily be applied. Hence, despite the agreement with experiment, the predictions of the standard HSA are theoretically inconsistent for such wave functions. It should also be stressed that the large momentum transfer behaviour of the helicity-flip controlled Pauli form factor of the proton remains unexplained within the standard HSA.

Applications of the HSA to time-like exclusive processes fail in most cases (e.g. $G_M$, $F_\pi$, $\gamma\gamma \to p\bar{p}$). The predictions for the integrated $\gamma\gamma \to \pi\pi$ cross-section ($|\cos\theta| \leq 0.6$) are in fair agreement with the data whereas the predictions for the angular distribution fails. Exclusive charmonium decays constitute another class of time-like reactions. If the end-point concentrated wave functions are employed again, the standard HSA provides results in fair agreement with the data in many cases. Noteworthy are the failures for the decays of the $\eta_c$ and the $\chi_{c0}$ into $p\bar{p}$. The standard HSA predicts zero decay widths for these reactions while experimentally the decay widths and the branching ratios are of similar magnitude as those of the other charmonium decays into $p\bar{p}$. The reason for this failure is obvious: the perturbative mechanism produces only $p\bar{p}$ pairs with opposite helicities while the quantum numbers of the $\eta_c$ and the $\chi_{c0}$ require pairs with the same helicities. It should also be noted that in most calculations of exclusive charmonium decays $^{[3]}$ $\alpha_s$ values of the order of 0.2 - 0.3 are employed. Such values do not match with $\alpha_s$ evaluated at the charm quark mass, the characteristic scale for these decays ($\alpha_s(m_c = 1.5\text{ GeV}) = 0.37$ in one-loop approximation with $\Lambda_{QCD} = 200\text{ MeV}$). Since high powers of $\alpha_s$ are involved in charmonium decays a large factor of uncertainty is hidden in the predictions.

In this talk I am going to discuss higher Fock state corrections to the standard HSA. Constraining the pion wave function from the recent precise data on the $\pi\gamma$ transition form factor $^{[4]}$, one observes an order-of-magnitude discrepancy be-
between data and HSA predictions for charmonium decays into two pions. In contributions from the $c\bar{c}g$ Fock state are suggested as the solution of this puzzle. In order to cure the failure of the standard HSA for reactions involving protons a variant of the HSA has been proposed in which the proton is viewed as being composed of quarks and diquarks. The latter objects constitute a particle model for higher Fock state contributions.

2. THE $\pi\gamma$ FORM FACTOR

The apparent success of the end-point concentrated wave functions, in spite of the theoretical inconsistencies, prevented progress in understanding hard exclusive reactions for some time. Recently the situation has changed with the advent of the CLEO data on the $\pi\gamma$ transition form factor $F_{\pi\gamma}$. The leading twist result for that form factor, including $\alpha_s$-corrections, reads

$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{3}}{3} \langle x^{-1} \rangle \frac{f_{\pi}}{Q^2} \times \left[ 1 + \frac{\alpha_s(\mu_R)}{2\pi} K(Q^2, \mu_R) + \mathcal{O}(\alpha_s^2) \right].$$

(1)

The function $K$ has been calculated by Braaten in $f_{\pi}$ is the usual pion decay constant (130.7 MeV) and $\langle x^{-1} \rangle$ is the $1/x$ moment of the pion distribution amplitude (DA), $\phi$, which represents the light-cone wave function of the pion integrated over transverse quark momenta, $k_\perp$, up to a factorization scale, $\mu_F$, of order $Q$. The DA can be expanded upon Gegenbauer polynomials, $C_n^{3/2}$, the eigenfunctions of the evolution kernel

$$\phi(x, \mu_F) = \phi_{AS}(x) \left[ 1 + \sum_{n=2,4,\ldots} \frac{B_n(\mu_0)}{\alpha_s(\mu_0)} \frac{\gamma^n}{\alpha_s(\mu_0) C_n^{3/2}(2x-1)} \right]$$

(2)

where the asymptotic DA is $6x(1-x)$. The process-independent expansion coefficients $B_n$ embody the soft physics; they are not calculable at present. The $\gamma_n$ are the anomalous dimensions. $\mu_0$ is a typical hadronic scale, actually $\mu_0 = 0.5$ GeV. Any DA evolves into the asymptotic DA for $\ln Q^2 \to \infty$. Hence, the limiting behaviour of the transition form factor is

$$F_{\pi\gamma} \to \sqrt{2} f_{\pi} / Q^2$$

(3)

which is a parameter-free QCD prediction. As comparison with the CLEO data reveals, the limiting behaviour is approached from below. At 8 GeV$^2$ the data only deviate by about 10—15% from $F_{\pi\gamma}$. The leading twist result without (with) $\alpha_s$-corrections nicely fits the CLEO data for $B_{\pi}(\mu_0) = -0.39 \pm 0.05$ and $B_n = 0$ $(n \geq 4)$. I.e. the required DA is narrower than the asymptotic one in the momentum transfer region of a few GeV$^2$. The frequently used Chernyak-Zhitnitsky DA $[12]$, defined by $B_{\pi}(\mu_0) = 2/3$, $B_n = 0$ $(n \geq 4)$, is in clear conflict with the data and should, therefore, be discarded.

Recently a modified HSA has been proposed by Botts, Li and Sterman in which transverse degrees of freedom as well as Sudakov suppressions are taken into account. This approach has the advantage of strongly suppressed end-point regions. Hence, the perturbative contributions can be calculated self-consistently. Using a Gaussian for the $k_\perp$-dependence of the pion wave function

$$\Psi_{\pi}(x, k_\perp; \mu_F) = \frac{f_{\pi}}{2\sqrt{6}} \phi_{\pi}(x, \mu_F) \times N \exp \left( -a_2^2 \frac{k_\perp^2}{x(1-x)} \right)$$

(4)

where $N = 16\pi^2 a_2^2 / (x(1-x))$ and, for a DA with $B_n = 0$ for $n \geq 4$, $a_\pi = 1/3 f_{\pi} \sqrt{8(1 + B_2)}$ which automatically satisfies the $\pi^0 \to \gamma\gamma$ constraint, one finds perfect agreement with the CLEO data for $B_{\pi}(\mu_0) = -0.006 \pm 0.014$, i.e. the asymptotic wave function works very well if the modified HSA is used.

3. PION DECAYS OF CHARMONIUM

In view of the results for $F_{\pi\gamma}$ a fresh analysis of the decays $\chi_{cJ} \to \pi\pi$ is in order. Using the information on the $\pi$ wave function obtained from the analysis of $F_{\pi\gamma}$, one finds the following values for the partial widths

$$\Gamma(\chi_{c0(2)} \to \pi^+\pi^-) = 0.872 (0.011) \text{ keV}$$

(5)
For comparison the experimental values as quoted in \[1\] and reported in a recent paper of the BES collaboration \[13\] are

\[
\Gamma(\chi_{c0(2)} \to \pi^+\pi^-) = 8.22 \pm 0.41 \text{ keV}. \tag{6}
\]

One notes that both the theoretical results, \[6\] and \[8\], fail by at least an order of magnitude. To assess the uncertainties of the theoretical results one may vary the parameters, \(m_c\), \(B_2\) and \(\Lambda_{QCD}\). However, even if the parameters are pushed to their extreme values the predicted rates are well below data. Thus, one has to conclude that calculations based on the assumption that the \(\chi_{cJ}\) is a pure \(c\bar{c}\) state, are not sufficient to explain the observed rates. The necessary corrections would have to be larger than the leading terms. A new mechanism is therefore called for.

Recently, the importance of higher Fock states in understanding the production and the inclusive decays of charmonium has been pointed out \[13\]. It is therefore tempting to assume the inclusion of contributions from the \(|c\bar{c}s(3S_1)g\rangle\) Fock state to exclusive \(\chi_{cJ}\) decays as the solution to the failure of the HSA. The usual higher Fock state suppresion by powers of \(1/Q^2\) \[14\] where \(Q = m_c\) in the present case, does not appear as a simple dimensional argument reveals; both the contributions to the decay amplitude, the colour-singlet and the octet one, behave as \(1/m_c^3\). In \[6\] the colour-octet contributions to the exclusive \(\chi_{cJ}\) decays are estimated by calculating the hard scattering amplitude from the set of Feynman graphs shown in Fig. \[1\] and convoluting it with the asymptotic pion wave function. The colour-octet and singlet contributions are to be added coherently. The \(\chi_{cJ} \to \pi\pi\) decay widths are given in terms of a single non-perturbative parameter \(\kappa\) which approximately accounts for the soft physics in the colour-octet contribution. A fit of the data \[17,18\] yields \(\kappa = 0.16 \text{ GeV}^2\) and the widths

\[
\Gamma(\chi_{c0(2)} \to \pi^+\pi^-) = 49.85 \pm 3.54 \text{ keV}. \tag{8}
\]

Comparison with \[6\] reveals that the inclusion of the colour-octet mechanism brings predictions and data in generally good agreement. The value found for the parameter \(\kappa\) has a reasonable interpretation in terms of charmonium properties and the mean transverse momentum of the quarks inside the pions. Thus it seems that the colour-octet mechanism leads to a satisfactorily explanation of the decay rates of the \(\chi_{cJ}\) into two pions. Of course, that mechanism has to pass more tests in exclusive reactions before this issue can be considered as being settled.

Figure 1. Representatives of the various groups of colour-octet decay graphs.
4. REACTIONS INVOLVING PROTONS

The standard HSA runs into many difficulties with these exclusive reactions as mentioned in the introduction. In a series of papers [3–8] a variant of the HSA has been proposed in which baryons are assumed to be composed of quarks and diquarks. A diquark, being a cluster of two valence quarks and a certain amount of glue and sea quarks pairs, is regarded as a quasi-elementary constituent. In so far, a quark-diquark state represents an (unspecified) superposition of higher Fock states

\[ |B, \lambda > = \Psi_{qD}(x, k_\perp)|qD > \]

In the diquark model spin 0 (S) and spin 1 (V) colour-antitriplet diquarks are considered. Assuming zero relative orbital angular momentum between quark and diquark and taking advantage of the collinear approximation, the valence Fock state of a proton with helicity \( \lambda \) and momentum \( p \) can be written in a covariant fashion (omitting colour indices)

\[ |P; p, \lambda > = \int \Phi_S(x) B_S u(p, \lambda) \]

\[ + \frac{f_V}{\sqrt{3}} \Phi_V(x) B_V (\gamma^\alpha + p^\alpha / m) \gamma_5 u(p, \lambda) \]

where \( u \) is the proton’s spinor. The two terms in (10) represent configurations consisting of a quark and either a scalar or a vector diquark, respectively. The couplings of the diquarks with the quarks in a spin-isospin 1/2 baryon lead to the flavour functions

\[ B_S = u S_{[u,d]} \]

\[ B_V = [u V_{[u,d]} - \sqrt{2} d V_{[u,d]}] / \sqrt{3} \] (11)

In the diquark model the following DAs have been proven to work satisfactorily well in many applications [3–8]:

\[ \Phi_S(x) = N_S x(1 - x)^3 \exp \left[ -b_S^2 \frac{m_q^2}{x} + \frac{m_S^2}{1 - x} \right] \]

\[ \Phi_V(x) = N_V x(1 - x)^3 (1 + 5.8 x - 12.5 x^2) \times \exp \left[ -b_V^2 \left( \frac{m_q^2}{x} + \frac{m_u^2}{1 - x} \right) \right] \] (12)

These DAs are suitable adaptions of a meson DA obtained by transforming the harmonic oscillator wave function to the light cone. The constants \( N \) are fixed through the normalization convention \( (N_S = 25.97 \text{ and } N_V = 22.92) \). The exponentials in (12) guarantee a strong suppression of the end-point regions. The masses appearing in the exponentials are constituent masses since they enter through a rest frame wave function. For \( u \) and \( d \) quarks 350 MeV and for the diquarks 580 MeV are appropriate mass values. It is to be stressed that the quark and diquark masses only appear in the DAs [12]; in the hard scattering kinematics they are neglected. The transverse size parameter \( b \) is fixed from the assumption of a Gaussian transverse momentum dependence of the full wave function and the requirement of a value of 600 MeV for the mean transverse momentum (actually \( b = 0.498 \text{ GeV}^{-1} \)). The constituent masses and the transverse size parameter are not considered as free parameters since the final results only depend on them mildly.

Diquark-gluon and diquark-photon vertices appear in the Feynman graphs contributing to the hard scattering amplitude of a given process. Following standard prescriptions, these vertices are defined as

\[ \text{SgS} : \quad i g_s t^a (p_1 + p_2)_\mu \]

\[ \text{VgV} : \quad -i g_s t^a \left\{ g_{\alpha\beta}(p_1 + p_2)_\mu \right. \]

\[ \left. -g_{\beta\mu} [(1 + \kappa) p_2 - \kappa p_1], \right. \]

\[ \left. -g_{\mu\alpha} [(1 + \kappa) p_1 - \kappa p_2]_\beta \right\} \] (13)

where \( g_s = \sqrt{4\pi\alpha_s} \) is the QCD coupling constant. \( \kappa \) is the anomalous magnetic moment of the vector diquark and \( t^a = \lambda^a/2 \) the Gell-Mann colour matrix. For the coupling of photons to diquarks one has to replace \( g_s t^a \) by \( -\sqrt{4\pi\alpha} e_D \) where \( \alpha \) is the fine structure constant and \( e_D \) is the electrical charge of the diquark in units of the elementary charge. The couplings \( DgD \) are supplemented by appropriate contact terms required by gauge invariance.

The composite nature of the diquarks is taken into account by phenomenological vertex functions. Advice for the parameterization of the 3-point functions (diquark form factors) is obtained
from the requirement that asymptotically the diquark model evolves into the standard HSA. In so far the standard HSA and the diquark model do not oppose each other, they are not alternatives but rather complements. Interpolating smoothly between the required asymptotic behaviour and the conventional value of 1 at \( Q^2 = 0 \), the diquark form factors are actually parametrized as

\[
F_S^{(3)}(Q^2) = \frac{Q_S^2}{Q_S^2 + Q^2} \\
F_V^{(3)}(Q^2) = \left( \frac{Q_V^2}{Q_V^2 + Q^2} \right)^2
\]

in the space-like region. The asymptotic behaviour of the diquark form factors and the connection to the hard scattering model is discussed in more detail in [7,8]. In accordance with the required asymptotic behaviour the \( n \)-point functions for \( n \geq 4 \) are parametrized as

\[
F_S^{(n)}(Q^2) = a_S F_S^{(3)}(Q^2), \\
F_V^{(n)}(Q^2) = \left( a_V \frac{Q_V^2}{Q_V^2 + Q^2} \right)^{n-3} F_V^{(3)}(Q^2).
\]

The constants \( a_{S,V} \) are strength parameters. Indeed, since the diquarks in intermediate states are rather far off-shell one has to consider the possibility of diquark excitation and break-up. Both these possibilities would likely lead to inelastic reactions. Therefore, these possibilities are not considered in the diquark model explicitly but taken into account by the strength parameters. Since in most cases the contributions from the \( n \)-point functions for \( n \geq 4 \) only provide small corrections to the final results that recipe is sufficiently accurate.

The relations (14,15) represent effective parameterizations valid at large space-like \( Q^2 \). It is not possible to continue these parameterizations to the time-like region in a unique way since the exact dynamics of the diquark system is unknown. A suitable continuation to the time-like region is defined by the following prescription: \( Q^2 \) is replaced by \(-s\) in (14,15) which ensures the correct asymptotic behaviour and, in order to avoid the appearance of unphysical poles, the diquark form factors are kept constant once their absolute values have reached a certain value \((c_0 = 1.3)\).

\[\text{Figure 2. The magnetic form factor of the proton in the time-like and space-like (at } Q^2 = -s \text{) regions. The solid line represents the predictions of the diquark model 1. The time-like data (})\]

\[\text{are from 21,22, the space-like data (}}\]

\[\text{from 2.}\]

The analysis of electromagnetic nucleon form factors constitutes the simplest application of the diquark model and the most obvious place to fix the various parameters of the model. The Dirac and Pauli form factors (the necessary helicity flips are provided by the vector diquarks) of the nucleon are evaluated by convoluting the DAs (12) with the respective hard scattering amplitude (11). The parameters are determined from a best fit to the data in the space-like region. The following set of parameters

\[
f_S = 73.85 \text{ MeV}, \quad Q_S^2 = 3.22 \text{ GeV}^2, \quad a_S = 0.15, \\
f_V = 127.7 \text{ MeV}, \quad Q_V^2 = 1.50 \text{ GeV}^2, \quad a_V = 0.05, \\
\kappa = 1.39
\]

provides a good fit of the data (8). The parameters \( Q_S \) and \( Q_V \), controlling the size of the diquarks, are in agreement with the higher-twist effects observed in the structure functions of deep inelastic lepton-hadron scattering if these effects are modelled as lepton-diquark elastic scattering (21). The predictions for the magnetic form factor in both the space-like and the time-like regions, are compared to the data (21,22) in Fig. 2. Two-photon annihilation into \( pp \) pairs has also been investigated within the diquark model. The prediction for the integrated \( \gamma \gamma \rightarrow pp \) cross section is compared to the CLEO data (24) in Fig. 3. At large energies the agreement between predic-
Figure 3. The integrated $\gamma\gamma \rightarrow p\bar{p}$ cross section ($|\cos \theta| \leq 0.6$). The solid line represents the diquark model prediction [6]. Data are taken from CLEO [24].

The prediction for the angular distribution is in agreement with the CLEO data too.

The diquark model also allows to investigate the decay process $\eta_c \rightarrow p\bar{p}$. A calculation along the same lines as for the other two time-like processes, using the same DAs and the same set of parameters, leads to a decay width of 3.88 keV [6] which is in fair agreement with the data [17]. Note that in the pure quark HSA a zero width is obtained.

5. SUMMARY

The study of hard exclusive reactions is an interesting and challenging subject. The standard HSA, i.e. the valence Fock state contribution in collinear approximation to lowest order perturbative QCD, while asymptotically correct (at least for form factors), does not lead to a consistent description of the data. In many cases the predicted perturbative contribution to particular exclusive reactions are much smaller then the data. The observed spin effects do not find a comforting explanation. In some reactions agreement between prediction and experiment is found although at the expense of dominant contributions from the soft end-point regions rendering the perturbative analysis inconsistent.

In view of these observations it seems that higher Fock state contributions have to be included in the analysis. However, not much is known about them as yet. We are lacking systematic investigations of such contributions to exclusive reactions. A few examples of such contributions have been discussed in this talk, namely the colour octet model for exclusive charmonium decays and the diquark model. More work is needed.

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