CP Violation in Non-Leptonic $\Omega^-$ Decays

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Abstract

We estimate the size of the $CP$-violating rate asymmetry for the decay $\Omega^- \rightarrow \Xi \pi$. Within the standard model we find a value of $2 \times 10^{-5}$, and it could be as much as ten times larger if new physics is responsible for $CP$ violation. Even though our calculation suffers from the usual uncertainty in the estimate of hadronic matrix elements, we find a rate asymmetry that is significantly larger than the corresponding one for octet-hyperon decays.

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1 Introduction

In this paper we estimate the size of the $CP$-violating rate asymmetry for $\Omega^- \rightarrow \Xi\pi$ decays. As is well known, such rate asymmetries for octet-hyperon decays are small as a result of the product of three small factors: a ratio of $|\Delta I| = 3/2$ to $|\Delta I| = 1/2$ amplitudes; a small strong-rescattering phase; and a small $CP$-violating phase \cite{1}. We find that for the decay channel $\Omega^- \rightarrow \Xi\pi$ all of these factors are larger than their counterparts for octet-hyperon decays, and this results in a rate asymmetry that could be as large as $2 \times 10^{-5}$ within the minimal standard model. Physics beyond the standard model could enhance this rate asymmetry by a factor of up to ten. Our calculation suffers from typical hadronic uncertainties in the computation of matrix elements of four-quark operators and for this reason it should be regarded as an order-of-magnitude estimate.

2 $\Omega^- \rightarrow \Xi\pi$ decay

The measured decay distributions of these decays are consistent with the amplitudes being mostly P-wave \cite{2}. We parametrize the P-wave amplitude in the form

$$i\mathcal{M}_{\Omega^- \rightarrow \Xi\pi} = G_F m_\pi^2 \bar{u}_\Xi A_{\Omega^- \Xi\pi}(P) k_\mu u_\Omega \equiv G_F m_\pi^2 \frac{\alpha_{\Omega^- \Xi\pi}(P)}{\sqrt{2} f_\pi} \bar{u}_\Xi k_\mu u_\Omega,$$

where the $u$'s are baryon spinors, $k$ is the outgoing four-momentum of the pion, and $f_\pi$ is the pion-decay constant. The P-wave amplitude has both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ components which are, in general, complex. We write

$$\alpha_{\Omega^- \Xi^0}^{(P)} = \frac{1}{\sqrt{3}} \left( \sqrt{2} \alpha_{\Omega^- \Xi^0}^{(1)} e^{i\delta_1 + i\phi_1} - \alpha_{\Omega^- \Xi^0}^{(3)} e^{i\delta_3 + i\phi_3} \right),$$

$$\alpha_{\Omega^- \Xi^-}^{(P)} = \frac{1}{\sqrt{3}} \left( \alpha_{\Omega^- \Xi^-}^{(1)} e^{i\delta_1 + i\phi_1} + \sqrt{2} \alpha_{\Omega^- \Xi^-}^{(3)} e^{i\delta_3 + i\phi_3} \right),$$

where $\alpha_{\Omega^-}^{(1)}$ are real quantities, strong-rescattering phases of the $\Xi\pi$ system with $J = 3/2$, P-wave and $I = 1/2, 3/2$ quantum numbers are denoted by $\delta_1$, $\delta_3$, respectively, and $CP$-violating weak phases are labeled $\phi_1$, $\phi_3$. The corresponding expressions for the antiparticle decay $\Omega^+ \rightarrow \Xi\pi$ are obtained by changing the sign of the weak phases $\phi_1$, $\phi_3$ in Eq. (2).

Summing over the spin of the $\Xi$ and averaging over the spin of the $\Omega^-$, one derives from Eq. (1) the decay width

$$\Gamma(\Omega^- \rightarrow \Xi\pi) = \frac{|k|^3 m_\Xi}{6\pi m_\Omega} |A_{\Omega^- \Xi\pi}(P)|^2 G_F^2 m_\pi^4.$$

As was found in Ref. \cite{3}, using the measured decay rates \cite{4} and ignoring all the phases, we can extract the ratio $\alpha_{\Omega^-}^{(3)} / \alpha_{\Omega^-}^{(1)} = -0.07 \pm 0.01$. Final-state interactions enhance this value, but this
enhancement is not significant for the values of the scattering phases that we estimate in the following section. This ratio is higher than the corresponding ratios in other hyperon decays \[^4\], which range from 0.03 to 0.06 in magnitude, and provides an enhancement factor for the CP-violating rate asymmetry in this mode.

By comparing the hyperon and anti-hyperon decays, we can construct CP-odd observables. The one considered here is the rate asymmetry

$$\Delta(\Xi^0 \pi^-) \equiv \frac{\Gamma(\Omega^- \rightarrow \Xi^0 \pi^-) - \Gamma(\Omega^- \rightarrow \Xi^0 \pi^+)}{\Gamma(\Omega^- \rightarrow \Xi^0 \pi^-) + \Gamma(\Omega^- \rightarrow \Xi^0 \pi^+)}$$

\[ \approx \sqrt{2} \frac{\alpha_3^{(\Omega)}}{\alpha_1^{(\Omega)}} \sin(\delta_3 - \delta_1) \sin(\phi_3 - \phi_1), \tag{4} \]

where in the second line we have kept only the leading term in $\alpha_3^{(\Omega)}/\alpha_1^{(\Omega)}$. Similarly, $\Delta(\Xi^- \pi^0) = -2\Delta(\Xi^0 \pi^-)$. The current experimental results indicate that any D-waves are very small in these decays, and that the parameter $\alpha$ that describes P-wave–D-wave interference is consistent with zero: $\alpha(\Xi^0 \pi^-) = 0.09 \pm 0.14$ and $\alpha(\Xi^- \pi^0) = 0.05 \pm 0.21 \[^4\]$. For this reason we do not discuss the potential CP-odd asymmetry in this parameter.

### 3 $\Xi\pi$-scattering phases

There exists no experimental information on the $\Xi\pi$-scattering phases, and so we will estimate them at leading order in heavy-baryon chiral perturbation theory. The leading-order chiral Lagrangian for the strong interactions of the octet and decuplet baryons with the pseudoscalar octet-mesons is \[^5\]

$$\mathcal{L}^s = \frac{1}{4} f^2 \text{Tr} \left( \partial^\mu \Sigma \partial^\mu \Sigma^{\dagger} \right) + \text{Tr} \left( \vec{B}_v \cdot \mathcal{D} B_v \right) + 2D \text{Tr} \left( \vec{B}_v S^\mu_v \{ A_\mu, B_v \} \right) + 2F \text{Tr} \left( \vec{B}_v S^\mu_v \left[ A_\mu, B_v \right] \right) - 2 \bar{T}_v^\mu \cdot \mathcal{D} T_{\nu\mu} + \Delta m \bar{T}_v^\mu T_{\nu\mu} + \mathcal{C} \left( \bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T^\mu_v \right) + 2H \bar{T}_v^\mu S_v \cdot \mathcal{A} T_{\nu\mu}, \tag{5}$$

where we follow the notation of Ref. \[^4\].

The scattering amplitudes for $\Xi^0 \pi^- \rightarrow \Xi^0 \pi^-$ and $\Xi^- \pi^0 \rightarrow \Xi^- \pi^0$ are derived from the diagrams shown in Figure \[^5\]. Of these, the first two diagrams in Figure \[^5\](a) and the first one in Figure \[^5\](b) do not contribute to the $J = 3/2$ channel. From the rest of the diagrams, we can construct the amplitudes for the $I = 1/2$ and $I = 3/2$ channels,

$$\mathcal{M}_{I=1/2} = 2\mathcal{M}_{\Xi^0 \pi^- \rightarrow \Xi^0 \pi^-} - \mathcal{M}_{\Xi^- \pi^0 \rightarrow \Xi^- \pi^0}, \tag{6}$$

$$\mathcal{M}_{I=3/2} = -\mathcal{M}_{\Xi^0 \pi^- \rightarrow \Xi^0 \pi^-} + 2\mathcal{M}_{\Xi^- \pi^0 \rightarrow \Xi^- \pi^0}.$$
Figure 1: Diagrams for (a) $\Xi^0\pi^-\to\Xi^0\pi^-$ and (b) $\Xi^-\pi^0\to\Xi^-\pi^0$. The vertices are generated by $\mathcal{L}^s$ in Eq. (4). A dashed line denotes a pion field, and a single (double) solid line denotes a $\Xi$ ($\Xi^*$) field.

and project out the partial waves in the usual way. Calculating the $J = 3/2$ P-wave phases, and evaluating them at a center-of-mass energy equal to the $\Omega^-$ mass, yields

$$
\delta_1 \approx -\frac{|k|^3 m_\Xi}{24\pi f^2 m_\Omega} \left[ \frac{(D - F)^2}{m_\Omega - m_\Xi} + \frac{1}{7} C^2 \frac{1}{m_\Omega - m_\Xi^*} + \frac{1}{18} C^2 \frac{1}{m_\Omega - 2m_\Xi + m_\Xi^*} \right],
$$

$$
\delta_3 \approx -\frac{|k|^3 m_\Xi}{24\pi f^2 m_\Omega} \left[ -\frac{2(D - F)^2}{m_\Omega - m_\Xi} - \frac{1}{7} C^2 \frac{1}{m_\Omega - 2m_\Xi + m_\Xi^*} \right].
$$

The phases are dominated by the terms proportional to $C^2$ arising from the $\Xi^*\Xi\pi$ couplings. For this reason, we do not use the value $C \approx 1.5$ obtained from a fit to decuplet decays at tree-level \cite{5}, nor the value $C \approx 1.2$ obtained from a one-loop fit \cite{7}. Instead, we determine the value of $C$ from a tree-level fit to the width of the $\Xi^* \to \Xi\pi$ decay, which gives $C = 1.4 \pm 0.1$. Using $f = f_\pi \approx 92.4$ MeV, isospin-symmetric masses, and the values $D = 0.61$ and $F = 0.40$, we obtain

$$
\delta_1 = -12.8^\circ, \quad \delta_3 = 1.1^\circ.
$$

In Figure 2 we plot the scattering phases as a function of the pion momentum.

Our estimate indicates that the $J = 1/2$ P-wave phase for the $\Xi\pi$ scattering is larger than other baryon-pion scattering phases. Eq. (7) shows that this phase is dominated by the $s$-channel $\Xi^*$-exchange diagram. This is what one would expect from the fact that the $\Xi^*$ shares the quantum

\footnote{See, e.g., Ref. \cite{6}.}

\footnote{We have also computed the phases in chiral perturbation theory without treating the baryons as heavy, and found very similar results, $\delta_1 = -13.1^\circ$ and $\delta_3 = 1.4^\circ$.}
numbers of the channel. Notice, however, that the phase is not large due to the resonance because it is evaluated at a center-of-mass energy equal to the $\Omega^-$ mass, significantly above the $\Xi^*$ pole. The phase is relatively large because the pion momentum in $\Omega^-$ decays is large.$^3$

4 Estimate of the weak phases

Within the standard model the weak phases $\phi_1$ and $\phi_3$ arise from the $CP$-violating phase in the CKM matrix. The short-distance effective Hamiltonian describing the $|\Delta S| = 1$ weak interactions in the standard model can be written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_i C_i(\mu) Q_i(\mu) + \text{h.c.},$$

where the sum is over all the $Q_i(\mu)$ four-quark operators, and the $C_i(\mu) = z_i(\mu) + \tau y_i(\mu)$ are the Wilson coefficients, with $\tau = -V_{td}^* V_{ts} / V_{ud}^* V_{us}$. We use the same operator basis of Ref. $^9$ because our calculation will parallel that one, but we use the latest values for the Wilson coefficients from Ref. $^10$. To calculate the phases, we write

$$iM_{\Omega^- \rightarrow \Xi \pi} = -i\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_i C_i(\mu) \langle \Xi \pi | Q_i(\mu) | \Omega^- \rangle.$$  

$^3$In fact, this $\Xi \pi$-scattering phase is much larger than the corresponding P-wave $\Lambda \pi$-scattering phase $\delta_P \approx -1.7^\circ$ $^8$ because the pion momentum is much larger in the reaction $\Omega^- \rightarrow \Xi \pi$ than it is in the reaction $\Xi \rightarrow \Lambda \pi$. 

Figure 2: Scattering phases as a function of the center-of-mass momentum of the pion. The solid and dashed curves denote $\delta_1$ and $\delta_3$, respectively. The vertical dotted-line marks the momentum in the $\Omega^- \rightarrow \Xi \pi$ decay.
Unfortunately, we cannot compute the matrix elements of the four-quark operators in a reliable way. As a benchmark, we employ the vacuum-saturation method used in Ref. [9]. For $\Omega^- \to \Xi^0\pi^-$, we obtain

$$\mathcal{M}_{\Omega^- \to \Xi^0\pi^-} = -\frac{C_F}{\sqrt{2}} V_{ud}^* V_{us} \left( M_1^P + M_3^P \right) \langle \Xi^0 | \bar{u} \gamma^\mu \gamma_5 s | \Omega^- \rangle \langle \pi^- | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle ,$$

(11)

where we have used the notation

$$M_1^P = \frac{1}{3} (C_1 - 2C_2) - \frac{1}{2} C_7 + \xi \left[ \frac{1}{3} (-2C_1 + C_2) - C_3 - \frac{1}{2} C_8 \right] + \frac{2m_\pi^2}{(m_u + m_d)(m_u + m_s)} \left[ C_6 + \frac{1}{2} C_8 + \xi \left( C_5 + \frac{1}{2} C_7 \right) \right] ,$$

(12)

$$M_3^P = -\frac{1}{3} (1 + \xi)(C_1 + C_2) + \frac{1}{2} C_7 + \frac{1}{2} \xi C_8 + \frac{m_\pi^2}{(m_u + m_d)(m_u + m_s)} (\xi C_7 + C_8) .$$

(13)

The current matrix-elements that we need are found from the leading-order strong Lagrangian in Eq. (4) to be

$$\langle \Xi^0 | \bar{u} \gamma^\mu \gamma_5 s | \Omega^- \rangle = -C \bar{u}_\Xi u_\Omega^\mu , \quad \langle \pi^- | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle = i\sqrt{2} f_\pi k_\mu ,$$

(14)

and from these we obtain the matrix elements for pseudoscalar densities as

$$\langle \Xi^0 | \bar{u} \gamma_5 s | \Omega^- \rangle = \frac{C}{m_u + m_s} \bar{u}_\Xi k_\mu u_\Omega^\mu$$

(15)

$$\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle = i\sqrt{2} f_\pi \frac{m_\pi^2}{m_u + m_d} .$$

Numerically, we will employ $m_\pi^2 / [(m_u + m_d)(m_u + m_s)] \sim 10$, $\xi = 1/N_c = 1/3$, and the Wilson coefficients from Ref. [10] that correspond to the values $\mu = 1$ GeV, $\Lambda = 215$ MeV, and $m_t = 170$ GeV. Given the crudeness of the vacuum-insertion method, we use the leading-order Wilson coefficients. For the CKM angles, we use the Wolfenstein parameterization and the numbers $\lambda = 0.22$, $A = 0.82$, $\rho = 0.16$ and $\eta = 0.38$ [11]. Putting all this together, we find

$$\alpha_3^{(\Omega)} e^{i\phi_3} = -0.11 + 2.8 \times 10^{-6} i ,$$

(16)

$$\alpha_1^{(\Omega)} e^{i\phi_1} = 0.23 + 2.3 \times 10^{-4} i .$$

The $|\Delta I| = 3/2$ amplitude predicted in vacuum saturation is comparable to the one we extract from the data, $\alpha_3^{(\Omega)} = -0.07 \pm 0.01$. To estimate the weak phase, we can obtain the real part of the amplitude from experiment and the imaginary part of the amplitude from the vacuum-saturation estimate to get $\phi_3 \approx -4 \times 10^{-5}$. Unlike its $|\Delta I| = 3/2$ counterpart, the $|\Delta I| = 1/2$ amplitude is predicted to be about a factor of four below the fit [4]. Taking the same approach as that in estimating

\footnote{We note here that only the relative sign between $\alpha_1^{(\Omega)}$ and $\alpha_3^{(\Omega)}$ is determined, while the overall sign of either the predicted or experimental numbers is not.}
$\phi_3$ results in $\phi_1 \approx 3 \times 10^{-4}$. We can also take the phase directly from the vacuum-saturation estimate (assuming that both the real and imaginary parts of the amplitude are enhanced in the same way by the physics that is missing from this estimate) to find $\phi_1 = 0.001$.

For the decay of the $\Omega^-$, it is much more difficult to estimate the phases in quark models than it is for other hyperon decays. For instance, to calculate the phase of the $|\Delta I| = 1/2$ amplitude, we would need to calculate the matrix element $\langle \Xi^*^- | H_W | \Omega^- \rangle$, but this vanishes for the leading $|\Delta I| = 1/2$ operator because the quark-model wavefunctions of the $\Omega^-$ and the $\Xi^*$ do not contain $u$-quarks. Considering only valence quarks, these models would then predict that the phase is equal to the phase of the leading penguin operator or about $\phi_1 \sim 0.006$.

5 Results and Conclusion

Finally, we can collect all our results to estimate the $CP$-violating rate asymmetry $\Delta(\Xi^0 \pi^-)$. They are

$$\frac{\alpha_3^{(\Omega)}}{\alpha_1^{(\Omega)}} \approx -0.07,$$

$$|\sin(\delta_3 - \delta_1)| \approx 0.24,$$

$$|\sin(\phi_3 - \phi_1)| \approx 3 \times 10^{-4} \text{ or } 0.001,$$

where the first number for the weak phases corresponds to the conservative approach of taking only the imaginary part of the amplitudes from the vacuum-saturation estimate and the second number is the phase predicted by the model. The difference between the resulting numbers, $|\Delta(\Xi^0 \pi^-)| = 7 \times 10^{-6}$ or $2 \times 10^{-5}$, can be taken as a crude measure of the uncertainty in the evaluation of the weak phases. For comparison, estimates of rate asymmetries in the octet-hyperon decays result in values of less than $10^{-6}$.

A model-independent study of $CP$ violation beyond the standard model in hyperon decays was done in Ref. [13]. We can use those results to find that the $CP$-violating rate asymmetry in $\Omega^- \to \Xi^0 \pi^-$ could be ten times larger than our estimate above if new physics is responsible for $CP$ violation. The upper bound in this case arises from the constraint imposed on new physics by the value of $\epsilon$ because the P-waves involved are parity conserving.

In conclusion, we find that the $CP$-violating rate asymmetry in $\Omega^- \to \Xi^0 \pi^-$ is about $2 \times 10^{-5}$ within the standard model. Although there are significant uncertainties in our estimates, it is probably safe to say that the rate asymmetry in $\Omega^- \to \Xi \pi$ decays is significantly larger than the corresponding asymmetries in other hyperon decays.

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5 Early calculations obtain the amplitude as a sum of a bag model estimate of the penguin matrix element and factorization contributions [12].
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