Weak-scale Higgs inflation

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Abstract: The present measurement of the Standard Model (SM) parameters suggests that the Higgs effective potential has a maximum at the Higgs field value of approximately $10^{11}$ GeV, and the electroweak (EW) vacuum is not absolutely stable. To achieve absolute EW stability, a very large Higgs-Ricci scalar non-minimal coupling can be introduced. I study cosmic inflation driven by the Higgs field in this extension of the SM and refer to it as “weak-scale Higgs inflation” because the resulting inflationary Hubble parameter is around the weak scale. The Palatini formulation of gravity with a dimension 6 term is shown to drive successful inflation. I also argue for the UV (in)sensitivity of the predictions, and phenomenological implications. In the metric formulation case, the scenario for stable EW vacuum may be probed by measuring the Higgs coupling in future colliders.

Keywords: inflation, cosmology of theories beyond the SM

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1 Introduction

Cosmic inflation [1–13], which generates the primordial density perturbation, is strongly suggested from the recent cosmic-microwave background (CMB) data [14, 15]. From the field-theoretical point of view, there should be a scalar field, called inflaton, with a very flat potential to drive slow-roll inflation. The potential should not be completely flat for the inflation to end, after which the inflaton decays to reheat the Universe, leading to the big-bang cosmology. The particle origin of the inflaton is a leading mystery of cosmology.

The Higgs inflation [16, 17] (see also [18]), in which the Higgs boson is the inflaton, was considered as a minimal possibility for inflation. In the scenario, a non-minimal coupling between the Higgs-Ricci scalar, $\xi$, is introduced. When the Higgs field value is much larger than the Planck scale divided by $\sqrt{\xi}$, the Higgs field behaves as a dilaton, and the Higgs potential becomes flat due to the approximate scale invariance. It has also been pointed out that the Standard Model (SM) couplings may be different when the Higgs field is large due to UV-completion-dependent threshold effects [19, 20]. The model also violates the perturbative unitarity for the metric formulation of gravity, as the kinetic term of the Higgs boson is enhanced in the Einstein frame. Hilltop/inflection-point-type Higgs inflation, with certain smooth transitions of the SM couplings to arbitrary values at high energy scales, was considered in refs. [21–23]. In the Palatini formulation of gravity, in which the full affine connection and metric are independent geometrodynamics variables, the unitarity violation is not severe [19], but the spectral index is sensitive to higher dimensional terms [24]. The authors of previous studies have focused on relatively high inflation scales, and the Higgs inflation has only been found to be viable at relatively high inflation scales. This situation cannot be fully resolved by introducing higher dimensional operators [24–26].
The precision measurement of SM parameters, on the other hand, suggests that the Higgs quartic coupling, $\lambda$, turns negative at an instability scale of

$$\Lambda_I = 10^{9-12} \text{GeV}.$$  

(1.1)

This disfavors the minimal high-scale Higgs inflation without introducing UV-completion-dependent threshold effects on SM couplings. In addition, although the electroweak (EW) false vacuum has a lifetime above the age of the Universe within quantum field theory \cite{30-45}, various factors such as the inflationary fluctuation \cite{46}, the preheating after inflation \cite{28, 47, 48}, right-handed neutrinos \cite{45}, the present Universe’s small blackholes \cite{49, 50} and compact objects without a horizon \cite{51}, and even the pure gravitational effect \cite{52} may let it decay into the true vacuum within the age of the Universe. These considerations suggest that the EW vacuum may need to be a true vacuum, which implies the existence of some beyond SM physics. Some examples of such physics include \cite{53–56}.

Although it was not clearly pointed out in the literature, the simplest possibility to make the EW vacuum absolutely stable should be introducing an extremely large non-minimal coupling, $\xi$ (section 2), much larger than the conventional Higgs inflation.\footnote{This corresponds to $M_t = 172$ to 174 GeV, which is around the 2 $\sigma$ range of the direct measured $M_t^{\text{direct}} \simeq 172.76 \pm 0.3$ GeV averaged by particle data group, or within the 1 $\sigma$ range of the cross-section measurement of $M_t^{\text{crosssection}} \simeq 172.5 \pm 0.7$ GeV (see refs. \cite{27} and \cite{28}). It should be noted that there may be a possible uncertainty of 0.5 GeV in the direct measurement, as discussed in \cite{29}. The absolutely stable electroweak vacuum, with $M_t \lesssim 171$ GeV, is in 5-6 $\sigma$ (2-3 $\sigma$) tension with the direct measurement (cross-section measurement).} This makes the potential flat before the Higgs quartic coupling runs to a negative value. Thus, absolute stability can be obtained with negligible UV-completion-dependent threshold effects on the potential. In this paper, I will study the phenomenological implications of this scenario. An observation in the metric formulation is that we can have a change of the Higgs couplings below a few TeV scales, which could affect the EW phase transition and provide testability in colliders.

What I will mainly demonstrate is that the same setup can result in Higgs inflation with a $\Lambda_I$ scale potential and consistent CMB normalization if the non-minimal coupling is chosen appropriately so that the scale-invariant regime starts close to the would-be Higgs potential hilltop suggested by the SM (section 3.1). This occurs even with negligible UV-completion-dependent threshold effects on the SM parameters. The $\Lambda_I$ potential energy corresponds to a weak-scale Hubble parameter, and I therefore refer to it as weak-scale Higgs inflation.\footnote{See ref. \cite{57} for a study of the suppression of the vacuum decay rate through the use of a non-minimal coupling.} In the Palatini formulation of Higgs inflation, I will demonstrate that the spectral index can be further explained by introducing a dimension 6 term in the Jordan frame. The mass scale of the higher dimensional term is slightly larger than the usual Planck scale. If all the higher dimensional operators are represented by a single mass scale slightly larger than the Planck scale, the predictions from the viable parameter region are under control against higher order corrections. In this case, there is a significant running of the spectral index. This running can be probed in CMB-S4 \cite{59}, SPHEREx \cite{60}, and in combination with DESI \cite{61}, WFIRST \cite{62}, or SKA \cite{63} (section 3.2). The potential for successful inflation happens to lie very close to the critical parameter regime between eternal and non-eternal inflation, which will be discussed in section 4. The last section is devoted to conclusions and discussions.

\footnote{See ref. \cite{58} where the term “weak-scale inflation” is used in a similar sense.}
2 A simple possibility for absolute stability of EW vacuum

2.1 Mechanism for EW vacuum stability

Let me introduce the setup and point out the simple (perhaps simplest) way to make the EW vacuum absolutely stable. This is done by introducing a large non-minimal coupling, $\xi$. The non-minimal coupling in the Jordan frame is expressed as follows:

$$L \supset \frac{1}{2} M^2_{pl} \Omega^2 g^{\mu\nu} R_{\mu\nu},$$

(2.1)

where $R_{\mu\nu}$ is the Ricci curvature tensor, $\Omega \equiv \left( 1 + \frac{\xi}{M^2_{pl}} \right)^{1/2}$,

(2.2)

$h$ is the neutral component of the Higgs field, $M_{pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck scale, and $g_{\mu\nu}$ is the metric. The action is $S = \int d^4x \sqrt{|det g|} L$. A Weyl transformation can be performed to move to the Einstein frame. I will show that the Higgs 1PI potential, including the leading radiative corrections, has the following relationship (here I count the 1-loop contribution via the Higgs quartic coupling as higher order, as in [64], because it is small):

$$V_{EF}[h, \mu_H^2] = V_{JF} \left[ \frac{h}{\Omega}, \frac{\mu_H^2}{\Omega^2} \right],$$

(2.3)

where $\mu_H^2(>0)$ is the Higgs mass parameter, and $\Omega$ appears in the potential as if counting dimensions. Here, $V_{JF}[h, \mu_H^2]$ can be considered as an ordinary effective potential within the SM. It is important to note that $h$ is used as the Higgs field in the Einstein frame without normalizing the kinetic term, while $\phi$ will be used as the kinematically normalized Higgs field in that frame. At the tree-level with $V_{JF} = -\mu_H^2 h_{JF}^2 / 2 + \lambda h_{JF}^4 / 4$, this is well-understood because $V_{EF} = \Omega^{-4} V_{JF}[h, \mu_H^2] = -\frac{\mu_H^2}{\Omega^2} \left( \frac{h}{\Omega} \right)^2 / 2 + \lambda \left( \frac{h}{\Omega} \right)^4 / 4$. The feature is extended to the loop level of the 1PI effective potential without internal Higgs lines because then $\Omega$ essentially behaves as a dilaton that spontaneously breaks the scale invariance, as we will show soon. The contribution with internal Higgs lines will be discussed in the next subsection.

For a moment, let us consider a model with the Higgs field only coupled to a top quark pair, as described in [20]. Specifically, I will not take into account any Higgs components other than the radial mode, any other quarks besides the top, any leptons, or any gauge bosons. The discussion on higher-order corrections will be presented in the following subsection.

By integrating out the top quark and normalizing the kinetic term after switching to the Einstein frame, $\Omega$ appears in the quark Yukawa couplings as $\frac{y_t h}{\sqrt{2}} \bar{t}_L t_R \rightarrow \frac{y_t h}{\sqrt{2} \Omega} \bar{t}_L t_R$. By treating $h$ as the background field, we obtain the 1PI effective potential in the $\overline{\text{MS}}$ scheme

$$V_{EF} \approx -\frac{3 y_t^4 h^4}{64 \pi^2 \Omega^4} \left( \log \left[ \frac{y_t^2 h^2}{2 \Omega^2 \mu_{RG}} \right] - \frac{3}{2} + \frac{\lambda \mu_{RG}}{4 \Omega^4} h^4 - \frac{\mu_H^2 h^2}{2 \Omega^4} \right),$$

(2.4)

where $y_t[\mu_{RG} = 10^{9-12}$ GeV$] \sim 0.5$ is the top Yukawa coupling evaluated using the $\overline{\text{SMDR}}$ code [65], which includes partial contributions from 3, 4, and 5 loop effects. Here $\mu_{RG}$ is
the renormalization scale. This expression is essentially obtained from eq. (2.3) by using the ordinary 1PI potential in the Jordan frame. The Jordan frame potential is $V_{JF}[h_{JF}] \approx -\frac{3y_t^4 h_{JF}^4}{64\pi^2} \left( \log \left[ \frac{y_t^2 h_{JF}^2}{\mu_{RG}^2} \right] - \frac{3}{2} \right) + \frac{\lambda}{4} h_{JF}^4 - \frac{\mu_h^2 h_{JF}^2}{2}$. I emphasize, again, that the kinetic term of $h$ is not yet normalized at this point.

Let us fix the renormalization scale $\mu_{RG} = \Lambda_I$ which is defined by

$$\frac{1}{4} \lambda + \frac{9y_t^4}{128\pi^2} = 0,$$

which is consistent with my order counting: $\lambda$ is at the loop level. This is the renormalization scale at which the quartic coupling becomes zero at the precision of the tree-level. Then we get

$$V_{EF} \approx \frac{3y_t^4 h^4}{64\pi^2 \Omega^4} \log \left[ \frac{y_t^2 h^2}{2\Omega^2 \Lambda_I^2} \right] - \frac{\mu_h^2 h^2}{2\Omega^4}. \quad (2.6)$$

This is the potential that we will consider. We only take into account the leading logarithmic contribution. Thus, this potential is a good approximation around the field value $y_t h/\Omega \sim \Lambda_I$. For $y_t h/\Omega \ll \Lambda_I$ or $y_t h/\Omega \gg \Lambda_I$, we need to have the renormalization group (RG) improvement of the potential. At the 1-loop level, one can improve the potential by taking a field-dependent renormalization scale $\mu_{RG} = \gamma h/\Omega$. Here $\gamma$ is introduced so that the radiative correction is minimized in the field range of interest, for example, at the hilltop. By neglecting the radiative correction, we have the potential $V_{EF} \approx \frac{\lambda (\gamma h/\Omega)}{4} \left( \frac{h}{\Omega} \right)^4 - \frac{\mu_h^2 h^2}{2\Omega^4}$ as in [20, 23]. Within the validity of the approximations, the two descriptions should be equivalent because the potential, including the wave function renormalization of $h$, should not depend on the renormalization scale $\mu_{RG}$. Although eq. (2.6) (or the expansion eq. (3.3)) will be mostly used for presentation, we will also use the other description, similar to [20, 23], for confirming our conclusions. The potential has a special point corresponding to the hilltop in the Jordan frame

$$h/\Omega = h_{\text{hilltop,JF}} \equiv \sqrt{\frac{e^{-1/2} 2\Lambda_I^2}{y_t}} \quad (2.7)$$

at which $V_{JF}' \approx 0$, by neglecting the small $\mu_h^2$ term. This is also the hilltop in the Einstein frame due to the relationship given by eq. (2.3).

The 1PI potential, including logarithmic corrections, in the Einstein frame becomes independent of $h$ if $h \gtrsim \Lambda_{\text{flat}} \equiv \frac{M_{\text{pl}}}{\sqrt{\xi}}$. This is also valid at the full loop level if the UV model has (approximate) scale invariance at large $h$, which is often assumed in the context of Higgs inflation. Therefore, if

$$\Lambda_{\text{flat}} \lesssim \Lambda_I \Rightarrow \xi \gtrsim \frac{M_{\text{pl}}^2}{\Lambda_I^2}, \quad (2.8)$$

the EW vacuum in the Einstein frame is the true vacuum. As a result, simply taking $\xi$ to be extremely large solves the EW vacuum stability problem.

As we have emphasized several times, $h$ is not the canonically normalized field. The canonically normalized field, $\phi$, can be obtained by normalizing the wave function of $h$ in the Einstein frame,

$$\mathcal{L}_{\text{kin}} = -\frac{Z[h]}{2} \partial_{\mu} h \partial^{\mu} h, \quad (2.9)$$
Figure 1. The Higgs potential with non-minimal coupling is shown in the top solid black line for the Palatini formalism and in the bottom solid purple line for the metric formalism. The value of the non-minimal coupling constant $\xi$ is set to $\sqrt{\frac{y_t^2 M^2}{2\Lambda^2}}$, as discussed in the context of inflation. The red dashed line shows the SM Higgs potential without the non-minimal coupling, where the EW vacuum is not the absolute minimum. For this plot, we have fixed the values of $\Lambda_I$ to $10^{11}$ GeV, $y_t$ to 0.5, and $\mu^2_H$ to $(125 \text{ GeV})^2/2$.

via the relation $\frac{d\phi}{dh} = Z[h]^{1/2}$. Thus, $h$ in $V_{EF}$ should be understood as a function of $\phi$ determined by this relation. Our discussion so far applies to both the metric formulation

$$Z = \Omega^{-2} + \frac{3}{2} (M_{pl} d \log(\Omega^2)/dh)^2 \quad \text{(Metric)} \quad (2.10)$$

and the Palatini formulation

$$Z = \Omega^{-2} \quad \text{(Palatini)} \quad (2.11)$$

Examples of the Higgs potential satisfying the condition in eq. (2.8) in the Palatini formulation and the metric formulation are shown in figure 1.

2.2 Higgs potential, precisely

The higher-order corrections that have been omitted so far include gauge loop contributions and Higgs loop contributions. In particular, the latter results in non-trivial threshold corrections due to the non-renormalizable nature of the full Lagrangian. In this section, I will discuss the reasons why or when the corrections to the 1PI effective potential are in higher order than the terms in eq. (2.6) and how our simplification remains valid.
Gauge contributions. We may include contributions from other SM couplings in addition to the ones from the Higgs-top coupling. Slightly smaller contributions come from the gauge sector. At the leading logarithmic approximation, we obtain $\delta V_{EF} \simeq 3g_f^4 \hbar^4 \log \frac{g_f^2 h^2}{4\Omega^2 \mu_{\text{RG}}} + \frac{3(g_1^2 + g_2^2)^2 h^4}{16\pi^2} \log \frac{(g_1^2 + g_2^2) h^2}{4\Omega^2 \mu_{\text{RG}}} + \delta \lambda_0/4$, where $g_2$ and $g_Y$ are the gauge couplings of $\text{SU}(2)_L$ and $\text{U}(1)_Y$, respectively. $\delta \lambda_0/4$ is a correction term without a logarithmic factor. By including all the leading logarithmic contributions, we obtain

$$V_{EF} \simeq \frac{\hbar^4}{4\Omega^2} \left\{ (\lambda[\mu_{\text{RG}}] + \Delta \lambda) - \frac{3c^2}{64\pi^2} \log \frac{c h^2}{2\mu_{\text{RG}}^2} \right\} - \mu_H^2 \frac{h^2}{2\Omega^2} \quad (2.12)$$

with $c = \sqrt{y_f^2 - \frac{1}{16}(3g_2^2 + 2g_2^2g_Y^2 + g_Y^4)}$, $\Delta \lambda = -\frac{3c^4}{8\pi^2} \log \frac{g_Y^2}{16} + \frac{3g_2^4}{64\pi^2} \log \frac{g_2^2}{4\pi} + \frac{3g_2^4 + g_Y^4}{128\pi^2} \log \frac{g_2^2 + g_Y^2}{4\pi} + \delta \lambda_0$. This has the same form as eq. (2.4) by replacing $y_f^2$ with $c$ and by redefining $\mu_{\text{RG}} = \Lambda_I$, so that $\lambda[\mu_{\text{RG}}] = \Lambda_I + \Delta \lambda = 0$. This changes $\Lambda_I$ by $\mathcal{O}(1)$ which is much smaller than the uncertainty in eq. (1.1). $c$ can be different from $y_f^2$ by $\mathcal{O}(10\%)$. I will take into account the higher logarithmic corrections numerically to check that the simplified form of eq. (2.6) remains a good approximation for the conclusions of this paper.

Higgs contribution and matching of SM to chiral SM. The Higgs loop contribution is special in the presence of non-minimal coupling due to the non-renormalizable nature of the couplings for the kinematically normalized Higgs field, $\phi$. In this part, I mainly follow refs. [20, 23, 66] to discuss the matching and the threshold effects. When $h \gg M_{\text{pl}}/\sqrt{\xi}$, the model enters a chiral SM, in which the radial mode of the Higgs field decouples, and thus the model approaches to renormalizable one (other than the gravitational interaction). The parameters for the 1PI Higgs potential in the SM with $h \ll M_{\text{pl}}/\sqrt{\xi}$ should match those in the chiral SM by taking account of the threshold effects at $h \sim M_{\text{pl}}/\sqrt{\xi}$. To this end, we need to perform an order-by-order renormalization of the non-renormalizable model in the Einstein frame. At the vanishing $\mu_H^2$ limit, the Higgs field enters into the Einstein frame tree-level Lagrangian through positive powers of

$$F \equiv \frac{h}{\Omega} \left( \frac{h}{M_{\text{pl}}/\sqrt{\xi}} (M_{\text{pl}}^3/2\xi^{3/2} h^2) + \mathcal{O} \left( \frac{M_{\text{pl}}^5}{\xi^{5/2} h^4} \right) \right). \quad (2.13)$$

On the right-hand side, I have also shown the large $h$ expansion for later convenience. A $\phi$ internal line in the large $h$ regime contributes to the 1PI effective potential of the order

$$\left( \frac{1}{16\pi^2} \right)^2 \left( \partial_\phi \mathcal{O} \left( \frac{M_{\text{pl}}^3}{\xi^{3/2} h^2} \right) \right)^2 = \left( \frac{1}{16\pi^2} \right)^2 \mathcal{O} \left( \frac{M_{\text{pl}}^6}{h^{-4}} \right). \quad (2.14)$$

This is a 2-loop effect (notes again that $\lambda$ is 1-loop order, see eq. (2.5)). In the right-hand side, we omitted the dependence of $\xi$ because they are different in metric and Palatini formulations. The explicit forms for some loop contributions can be found in ref. [20]. They are confirmed to start from $\mathcal{O}(h^{-4})$ at large $h$. For instance, the finite part of the Higgs loop via self-interaction has the form

$$\delta V_{EF}[\phi] = \tilde{c} \frac{\lambda^2}{16\pi^2} F^4 \left( F_{\phi\phi}^2 + \frac{1}{3} F_{\phi\phi\phi} F \right), \quad (2.15)$$
where $\tilde{c}$ is a constant. In this paper, $X_Y$ represents the derivative of $X$ with respect to $Y$. It can be checked to be an $O(h^{-1})$ term.

Nevertheless, the renormalization procedure requires the inclusion of corresponding counterterms, as demonstrated in ref. [20]. This implies that the values of parameters, such as $\tilde{c}$, should be treated as free variables and interpreted as parameters of an effective theory. The values of these parameters may be fixed by considering a UV completion.

The threshold effects and the additional parameters contribute to the low-energy effective couplings of the SM, which should be matched to the chiral SM parameters. For example,

$$\lambda_{\text{SM}} \simeq \tilde{c} \frac{\lambda^2}{16\pi^2} + \lambda. \quad (2.16)$$

Since $\tilde{c}$ is influenced by the UV completion, a fine-tuning analysis may be conducted to motivate the range of this parameter. In the range defined by the inequalities

$$1 \lesssim |\tilde{c}| \lesssim \lambda \left[ M_{\text{pl}} / \sqrt{\xi} \right]^{-1} 16\pi^2 \quad (2.17)$$

the couplings, $\lambda_{\text{SM}}$ and $\tilde{c}$, are not obtained through fine-tuning. The upper bound ensures that $\lambda_{\text{SM}}$ is not obtained through the cancellation between the $\tilde{c}$ dependent term and $\lambda$, which is the quartic coupling in the chiral SM in eq. (2.16). The lower bound guarantees that radiative corrections to $\tilde{c}$ do not surpass $\tilde{c}$. In the range specified by equation (2.17), the threshold effect contributions to the effective potential at either $h \gg M_{\text{pl}} / \sqrt{\xi}$ or $h \ll M_{\text{pl}} / \sqrt{\xi}$ are subdominant. To simplify the analysis, I have neglected threshold effects as was done in early papers on Higgs inflation [16, 17]. I have instead used the matching $\lambda_{\text{chiral SM}}(\gamma F) \approx \lambda_{\text{SM}}(\gamma F)$ by setting $\mu_{\text{RG}} = \gamma F$, which is equivalent to using equation (2.6) at the leading logarithmic level. The same arguments can also be applied to other threshold corrections, which are all at the 2-loop level (see eq. (2.14)).

Note that if eq. (2.17) is satisfied, the inclusion of radiative corrections does not significantly alter the shape of the potential in figure 1, and the solution to the vacuum stability problem remains unchanged.

The threshold corrections may negatively impact the predictions for Higgs inflation in the metric formulation. This is because inflation occurs at $h \sim M_{\text{pl}} / \sqrt{\xi}$ as we will see. This does not mean that weak scale metric Higgs inflation is impossible but rather highlights the importance of a precise selection of the potential shape around $h \sim M_{\text{pl}} / \sqrt{\xi}$, based on a UV completion, for making predictions. On the other hand, in the Palatini formulation, the inflation will take place at $h \gg M_{\text{pl}} / \sqrt{\xi}$, where the contributions from the threshold effects can be neglected.

**Perturbative unitarity and low energy phenomena in metric formulation.** In the metric formalism, the kinetic term for $h \ll M_{\text{pl}} / \sqrt{\xi}$ includes a contribution of $Z(\partial h)^2 \supset 6\xi^2 \frac{\hbar^2}{M_{\text{pl}}^2} (\partial h)^2$, which leads to the violation of perturbative unitarity at the energy scale of $M_{\text{pl}} / \xi \lesssim 2 \text{TeV} \left( \frac{M_{\text{pl}}}{10^{13} \text{GeV}} \right)^2$. Therefore the regime, $M_{\text{pl}} / \xi \ll h \ll M_{\text{pl}} / \sqrt{\xi}$, may require a non-perturbative treatment, as discussed in refs. [20, 23]. Meanwhile, for $h \gg M_{\text{pl}} / \sqrt{\xi}$, the cut-off scale is field dependent and is given by $h \sqrt{\xi}$ in the Jordan frame, and $\sim M_{\text{pl}}$ in the Einstein frame, as described in [67]. Connecting the Higgs potential in these two regimes is crucial in
relating the SM and chiral SM. It has been proposed to achieve this connection by introducing a jump with a function of the form such as eq. (2.15), in the potential around \( h \sim M_{\text{pl}}/\xi \), where the theory becomes non-perturbative. According to \[20\], proper threshold effects with \(|\lambda_{\text{SM}} - \lambda_{\text{chiral SM}}| \gtrsim |\lambda_{\text{SM}}|\), can ensure EW vacuum stability if the would-be hilltop is at \( h \gg M_{\text{pl}}/\xi \). On the other hand, our mechanism in this paper for vacuum stability can apply when the jump of the parameters is not significant and even satisfies the condition (2.17).

Interestingly, future probes of this scenario may be possible in the metric formulation, due to the emergence of various higher-dimensional terms suppressed by the violation scale of perturbative Unitarity. The “new physics” scale \( M_{\text{pl}}/\xi \) is as small as TeVs. From the perturbative contribution, we can naively estimate the effect’s order of magnitude. From eq. (2.15), expanded at \( h \ll M_{\text{pl}}/\xi \), we derive a dimension 6 term:

\[
\delta V \supset -\frac{\xi^2}{\pi^2 M_{\text{pl}}^2} h^6. \tag{2.18}
\]

This term is proportional to \( \xi^2 \), and we neglect the subleading terms proportional to \( \xi \). This affects the Higgs self-interaction as a higher dimensional term of scale \( < (1-10) \text{ TeV} \), reflecting the scale violating the perturbative unitarity. The reason why I expect that there is no higher order coupling in \( \xi \) at a higher loop level is that in the original Lagrangian, we have only two kinds of parameters relevant to \( \xi \). One is \( 1/\Lambda_1^2 \equiv \xi/M_{\text{pl}}^2 \) which appears in \( \Omega \), and the other is \( 1/\Lambda_2^2 \equiv \xi^2/M_{\text{pl}}^2 \), which appears in the wave function \( Z \). Thus at any perturbative order of calculation of radiative corrections, we can only have the corrections relevant to \( \Lambda_1^{-2}, \Lambda_2^{-2} \), and from the dimensional argument, we get the term \( \propto \xi^2 \) as the highest order of \( \xi \) valid at any order of loops for this dimension 6 terms. More generically, the strongest higher dimensional terms are suppressed by the scale of \( \Lambda_2 \) in the low energy effective theory with small \( h \).

The higher dimensional Higgs self-coupling is rarely constrained in collider studies, both current and prospective. However, its interaction with the higher-dimensional contribution to the kinetic term may impact the phase transition of the EW symmetry, leading to gravitational wave production as a probe of the scenario.

Alternatively, we have the radiative corrections to a Yukawa coupling \( y \) \[20\]:

\[
\delta y \sim \frac{y^3}{16\pi^2}(F_{\phi}^2 - 1) \supset -3y^3 \frac{\xi^2 h^2}{8\pi^2 M_{\text{pl}}^2}. \tag{2.19}
\]

This yields dimension 6 terms relevant to the fermion mass. We again neglect terms with smaller power than \( \xi^2 \) and notice that this should be the largest contribution at any loop order. Given that this leads to at most a correction of \( O(0.1 - 1)\% \) to the Yukawa coupling, it remains consistent with recent precision measurement data and could be probed in future colliders. The corrections stemming from the Higgs self-energy would be challenging to affect the S,T, and U parameters, e.g., \[68\], and the gauge sector should not have a significant constraint from the measurement of the parameters. I also note that the discussion so far is based on the assumption that the (non-minimal) flavor violating parameters do not appear due to the non-perturbative effect from the Higgs-gravity sector, or, in other words, we assume the minimal-flavor/CP-violation.

\[4\] I express my gratitude to the referee for raising the question about the collider search.
Perturbative unitarity in Palatini formulation. In the Palatini formulation, there is no significant component proportional to $M_{\text{pl}}^2 (d \log \Omega^2 / dh)^2$ present in the wave function, $Z$. As a result, perturbative unitarity remains valid up to a scale of $\sim M_{\text{pl}} / \sqrt{\xi} \sim \Lambda_I$ for $h \ll M_{\text{pl}} / \sqrt{\xi}$. By having the kinematical normalization and including radiative corrections, the Higgs interactions are expected to have corrections with the factors of the form $(\xi h^2 / M_{\text{pl}}^2)^n \sim (h^2 / \Lambda_I^2)^n$ at the leading order of any loop level. Here $n$ is an integer. This is because, as before, in the original Lagrangian, we only have the parameter $\Lambda_I^{-2} \equiv \xi / M_{\text{pl}}^2$ (in contrast to the metric formulation, the term $\propto \xi^2 / M_{\text{pl}}^2$ does not exist in $Z$.) The form can be checked at the 1-loop level for the Higgs Coleman Weinberg potential and the Yukawa interaction. During inflation, the cutoff scale, which depends on $h$, is even higher, while the SM particle mass scales (the Hubble parameter) are (much) lower than the cutoff scale \cite{19}. Consequently, the model is always in a perturbative regime both during and after inflation. It is worth noting that the energy scale at which perturbative unitarity is violated is around $\sim \sqrt{16 \pi M_{\text{pl}} / \sqrt{\xi}}$, where $16 \pi$ arises from the partial wave decomposition. By contrast, when the Higgs field value is $h \sim M_{\text{pl}} / \sqrt{\xi}$, the relevant dynamical mass scale for discussing the Higgs potential corrections becomes $\sqrt{\lambda h / \Omega}$ or $y_t h / \Omega$, which falls in the order of $O(0.1) h$. This scale is marginally lower than the scale derived from perturbative unitarity considerations. As such, it appears reasonable to assume that a perturbative description remains valid for all values of the Higgs field, $h$.

3 Higgs inflation with extremely large $\xi$

The flat potential for absolute EW vacuum stability can drive cosmic inflation. In this section, we show that with specific $\xi$ values and higher dimensional terms, we can have successful inflation explaining the CMB data. Since we will work in the large $h$ expansion, we mainly consider Palatini as the formulation of gravity, which will have a well-controlled series expansion by $\Lambda_I / h$, as we will see.

3.1 Enhancement of curvature perturbation with SM hilltop

For slow-roll inflation to occur, we need both the slow-roll parameters $\varepsilon$ and $\eta$ to be less than 1, where $\varepsilon(\phi) \equiv \frac{M_{\text{pl}}^2}{2} (\frac{V_E F, z}{V_E F})^2 = \frac{M_{\text{pl}}^2}{2} (\frac{V_E F, h}{V_E F})^2 Z^{-1}$ and $\eta(\phi) \equiv \frac{M_{\text{pl}}^2}{2} \frac{V_E F, \phi \phi}{V_E F} = M_{\text{pl}}^2 Z^{-1/2} d(Z^{-1/2} V_E F, h) / dh$. It is well known that without a non-minimal coupling, the slow-roll condition cannot be satisfied due to the not-too-small SM couplings and the induced Coleman-Weinberg corrections. Thus, the focus of this paper will be on the parameter region surrounding the (would-be) hilltop at $h / \Omega \sim h_{\text{hilltop}, JF}$. Specifically, we will examine the regime in which

$$h \gg h_{\text{hilltop}, JF} = O(\Lambda_I),$$

where the r.h.s. is the hilltop value of $h$ in the absence of the non-minimal coupling. The inequality is due to the large value of $\Omega$. It is then convenient to use the parameterization,

$$\xi = \xi_0 + \delta \xi, \quad \xi_0 = \sqrt{e g_t^2} \frac{M_{\text{pl}}^2}{2 \Lambda_I^2}. \quad (3.2)$$

When $\delta \xi = 0$ and $\mu^2_H = 0$, we find \( \lim_{h \to \infty} \frac{h}{\Omega} \to h_{\text{hilltop}, JF} \).
By expanding the potential in terms of $\Lambda_I/h$, $\delta \xi/\xi_0$, and $\mu_H^2/\Lambda_I^2$, we obtain the expression,

$$V_{\text{EF}} \approx V_0 \left(1 + \kappa \Lambda_I^2 h^{-2} - \frac{8}{e y_0} \Lambda_I^2 h^{-4} + O(\Lambda_I^6 h^{-6})\right),$$

(3.3) where

$$V_0 \approx \frac{3 \Lambda_I^4}{32 e \pi^2}, \quad \kappa \approx -64 \pi^2 \left(\frac{\mu_H^2}{\Lambda_I^2} + 9 H_{\text{inf}}^2 \delta \xi/(4 e \pi^2)\right), \quad H_{\text{inf}}^2 \equiv \frac{V_0}{3 M_{\text{pl}}^2}. \tag{3.4}$$

Here, we present only the leading-order terms in $\delta \xi/\xi_0$ and $\mu_H^2/\Lambda_I^2$ in the coefficient for each order of $1/h$. It is important to note that $\mu_H^2$ is comparable to the inflationary Hubble parameter, $H_{\text{inf}}^2$, and thus, the Higgs mass term cannot be neglected in the inflationary dynamics.

The e-folding number can be calculated using the slow-roll equation, $3 H \dot{\phi} \approx -V_{\text{EF},\phi}$, as $N \approx \int_{\phi_{\text{end}}}^{\phi_{\text{end}}} \left(\frac{V_{\text{EF},\phi}}{3 H}\right)^{-1} d\phi \approx \int_{h_{\text{end}}}^{h_{\text{end}}} \left(\frac{V_{\text{EF}}}{V_{\text{EF},h} M_{\text{pl}}^2}\right) Zdh$, where $H \approx \sqrt{V_{\text{EF}}/(3 M_{\text{pl}}^2)}$ is the Hubble parameter and $\dot{X} \equiv dX/dt$ with $t$ being the cosmic time. The subscript $*$ denotes the quantity at the horizon exit. $\phi_{\text{end}}$ (or $h_{\text{end}}$) represents the field value at which one of the slow-roll conditions is violated, that is when $\max \{ |\xi(\phi_{\text{end}})|, |\eta(\phi_{\text{end}})| \} = 1$. For low-scale inflation, if it gives the correct amplitude of the power spectrum for the curvature perturbation, it is known that $|\xi|$ is much smaller than $|\eta|$. By assuming $\xi = \xi_0$ and $\kappa = 0$, we can estimate $\phi_{\text{end}}$ using the condition $|\eta(\phi_{\text{end}})| = 1$ for large enough $h_{\text{end}}$. As an order of magnitude estimate, from the equation $|\eta(\phi_{\text{end}})| \sim \frac{V_{\text{EF},\phi}}{3 H_{\text{inf}}} = 1$, we can obtain

$$h_{\text{end}} \sim h_s \sim \frac{\sqrt{M_{\text{pl}} \Lambda_I}}{\sqrt{y_t}} \quad \text{for the Palatini formulation} \tag{3.5}$$

(see figure 1 for the corresponding potential with a canonically normalized Higgs field, and figure 5 for the consistent numerical result). Here I used $h_s \sim h_{\text{end}}$ as $|\xi| \ll 1$ during low-scale inflation. In this case, the assumed large $h$ expansion is well-controlled since $h_{\text{end}}, h_s \gg \Lambda_I$ during inflation.

In the metric formulation of gravity, on the other hand, $h$ is given by $h_{\text{end}} h_s \sim \Lambda_I/y_t$. Since $\Lambda_I/h_{\text{end}}, h \sim O(1)$, the large $h$ expansion is not a good approach. In addition, precise knowledge of the SM potential and UV-completion-dependent threshold corrections [19, 20] is required (as discussed in section 2). Although certain potential shapes around $h \sim M_{\text{pl}}/\sqrt{\xi}$ can lead to weak-scale Higgs inflation explaining the CMB data, the prediction is very sensitive to the potential shape around $h \sim \Lambda_I/y_t$. This is the reason why we focus on Palatini Higgs inflation.

At the horizon exit, when the field value $\phi = \phi_s$, corresponding to the scale at which the power spectrum of the curvature perturbation, the spectral index, and its runnings are measured, the e-folding number is matched with the thermal history. Following inflation, reheating must occur quickly due to the fast Higgs interaction rate, leading to a radiation-dominated universe. This results in

$$N \approx 43 + \log \left[ \frac{\Lambda_I}{10^{11} \text{GeV}} \right], \tag{3.6}$$

which is lower than what is typically observed in Higgs inflation.

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5The QCD scale is around $\mu_H$ due to the presence of heavy quarks during inflation. The contributions can be neglected in comparison to the Higgs mass term because of the suppressed power of the Higgs field.
The power spectrum of the curvature perturbation, $\Delta_R^2(k)$, can be estimated by relating it to the inflaton potential shape, as given by the following equation:

$$\Delta_R^2(k) \simeq \left( \frac{H^2}{2\pi}\phi \right)^2 \simeq \frac{V_{EF}(\phi)^2}{12\pi^2 V'_{EF}(\phi)^2 M_{pl}^6}.$$  \hfill (3.7)

The CMB data can be used to match this power spectrum, and one such example is the measured CMB normalization [14, 15], given by:

$$\Delta_R^2,_{\text{CMB}}(k_*) \simeq 2.1 \times 10^{-9},$$  \hfill (3.8)

where the pivot scale is $k_* = 0.05 \text{Mpc}^{-1}$. The equation eq. (3.8) = eq. (3.7) provides a relationship between the potential height and slope at the horizon exit.

In the standard Higgs inflation scenario with a low-scale Hubble parameter, it is challenging to satisfy the CMB normalization because only the $O(h^{-2})$ term contributes to the slow roll around the horizon exit. The contribution to the power spectrum of the curvature perturbation, $\Delta_R^2$, for the Palatini formulation is given by

$$\Delta_R^2[\phi_*] \simeq \frac{\xi N^2 V_0}{3\pi^2 M_{pl}^4},$$  \hfill (3.9)

and is small since $V_0 \ll M_{pl}^4$. In our scenario, the value of $|\kappa|$ can be much smaller than the conventional one, leading to an important contribution from both the $h^{-2}$ and $h^{-4}$ terms to the classical motion of the inflaton field. This is illustrated in figure 2, where the power spectrum $\Delta_R^2$ is displayed for varying values of $\delta \xi$ in eq. (3.3). The plot was generated using values of $y_t = 0.66$, $\mu_H^2 = (125 \text{GeV})^2/2$, and $\Lambda_I = 10^{9,10,11,12} \text{GeV}$ from top to bottom. By taking $\delta \xi$ appropriately, we can enhance the contribution of $h^{-2}$ term. The running coupling $\lambda(\gamma h/\Omega)$ is also utilized to demonstrate similar behavior, as evidenced by the red data points shown in the figure. Further details on the analysis can be found in the appendix. The results indicate that the higher logarithmic contributions, higher loop contributions, and contributions from couplings other than top quarks are indeed subleading, as discussed in section 2. As a result, the enhancement of $\Delta_R^2$ to the required value is possible in the case of low-scale Higgs inflation.

### 3.2 Spectral indices and UV (in)sensitivity in weak-scale Palatini Higgs inflation

The predicted scalar spectral index $n_s$ in the previous weak-scale Palatini Higgs inflation, when $\Lambda_I = 10^{9-12} \text{GeV}$, tends to be lower than the measured value $n_{s,\text{CMB}} = 0.9665 \pm 0.0038$ [14, 15].\(^6\) For example, with $\Lambda_I = 10^{11} \text{GeV}$, the predicted value of $n_s$ is around 0.6, which is highly in tension.

To cure this problem, we will introduce higher dimensional terms in the Jordan frame. It has been shown that the prediction of $n_s$ in the Palatini formulation is sensitive to the addition of higher-dimensional terms suppressed by super-Planckian scale in the Jordan

\(^6\)For the Palatini (metric) formulation with $\Lambda_I \approx 10^{14} \text{GeV}$, (\(10^{16} \text{GeV}\)) we can have the spectral index to be consistent with the observation.
Figure 2. The CMB normalization is shown as a function of $\kappa$ in eq. (3.3) for Palatini Higgs inflation in purple lines. The results correspond to different values of $\Lambda_I$ with $\Lambda_I = 10^9, 10^{10}, 10^{11}, 10^{12} \text{GeV}$ from bottom to top and fixed values of $y_t = 0.66, \mu_0^2 = (125 \text{ GeV})^2 / 2$. We also show the corresponding result (red data points) by using the running coupling evaluated from $M_t = 172.16 \text{ GeV}$. It is fitted well with $\Lambda_I = 10^{10.97} \text{ GeV}$ with eq. (3.3) in a black dashed line. For comparison, the horizontal red dashed line is the measured value.

frame [24] (Other similar models to enhance $n_s$ include new inflation [69], multi-natural inflation [70–76], and very low scale inflation by an axion-like particle (ALP) [77–82]). Since we have infinity higher dimensional terms, we consider the following assumptions for the couplings: we use a single mass scale $M$ as the order parameter for representing the couplings of all higher dimensional corrections in the Jordan frame. This is similar to the usual effective quantum field theory approach, e.g., chiral perturbation theory, but is slightly unnatural since we will see that $M \gg M_{\text{pl}}$ gives the viable parameter region.\footnote{A similar unnatural assumption is usually adopted in large field inflation scenarios [83] by assuming the inflaton has an approximate shift symmetry. In our scenario, we assume $h$ has an approximate scale-invariance, whose explicit breaking is represented by the order parameter of $M$.} I call this assumption Jordan frame EFT assumption. By focusing on the field values within the range

$$ (h/\Omega \lesssim M_{\text{pl}}/\sqrt{\xi} \ll h \ll M_{\text{pl}}), $$

(3.10)

where inflation occurs, the effect of higher dimensional terms controlled by the scale $M \gtrsim M_{\text{pl}}$ have a well-controlled expansion.

Let me show that with the Jordan frame EFT assumption, the leading correction in the Einstein frame becomes

$$ \delta V_{\text{EF}} \sim \frac{V_0}{M^2} h^2 $$

(3.11)
One example of a model that leads to this form is the dimension six term, $3g^4_h^6/(128\pi^2M^2)$ in the Jordan frame Higgs potential [24]. Another example is obtained by correcting the bottom quark Yukawa coupling, where

$$\mathcal{L}_{JF} \supset -y_b \left(1 - \frac{y_b^2 h^2_{JF}}{4g^2_h M^2}\right) \frac{h_{JF}}{\sqrt{2}} b_L b_R$$  \hspace{1cm} (3.12)

leads to a logarithmic correction. The leading term in the expansion of this correction gives the desired form. We will see that a similar correction in the top Yukawa interaction is suppressed in the next paragraph.

There are also corrections that are irrelevant to the inflationary dynamics. One example of such a correction is the modification of the field $\Omega$. To see this, it is important to note that $\varepsilon$ is highly suppressed. Hence, $[\partial V_{EF}/\partial \Omega]$, which is equal to $h\Omega |V_{EF,h}| + O(\mu^2_H)$, is also suppressed due to the relationship between $h$ and $\varepsilon$ via eq. (2.3). I used $Z^{-1/2} = \Omega$ in the Palatini formalism. Therefore, a correction to $\Omega^2$ that is suppressed by the Planck scale is unlikely to affect the inflationary prediction significantly.\footnote{The correction to the smooth and monotonic wave function $Z$ is given by $d\log Z/d(\Omega^2) \times O(h^2_{\text{hilltop}, JF}/M^2_{pl})$, which is negligible in the calculation of $\mathcal{N}[\phi_\ast], \Delta^2_R[\phi_\ast]$, and $n_s[\phi_\ast]$.} The correction to the top Yukawa interaction is also suppressed. This is because by setting other couplings to zero and taking $\delta \xi \to 0$, it can be interpreted as a redefinition of $\Omega$. Additionally, the running effect of $\xi$ may induce a correction to $\Omega$, as shown in [66]. At large $h$, the correction takes the form $\delta(\Omega^2) \sim -\frac{3\sqrt{2}g^4_h}{16\pi^2\Lambda^2} h^2 - \frac{3\pi^2}{4\pi^2} + \frac{3\lambda^2}{4\pi^2} h^{-2} + O(h^{-4}, \delta\xi/\xi_0)$. The first two terms simply redefine the original $\Omega$, while the $O(h^{-2})$ term is highly suppressed and does not affect the shape of the potential around $h_\ast$.

Our analysis has shown that, at the leading order of various UV corrections, we can use the expression in eq. (3.11) with Jordan frame EFT assumption. This leads to a slight suppression of $|V_{EF,\phi\phi}|$ at the horizon exit, which also modifies the value of $\phi_\ast$. This enhancement of $n_s$ while keeping $\Delta^2_R[\phi_\ast] = \Delta^2_R\text{CMB}[k_\ast]$ intact is demonstrated. By utilizing the full potential of eq. (2.6), the third-order slow-roll expansions have been included in the estimation of $n_s$ and its running [84]. The predictions and parameter relationships are shown in figure 3 with $\Delta^2_R[\phi_\ast] = \Delta^2_R\text{CMB}[k_\ast]$ fixed. We see that $\delta \xi/\xi \sim O(\Lambda^0/M^2_{pl})$ can explain $n_s$. I checked that when $n_s$ is explained, there is an almost inflection point around the horizon exit, i.e., $V_{EF,h} \approx 0, V_{EF,hh} \approx 0$. This scenario, therefore, provides a consistent explanation for the CMB data. It has the prediction of a small tensor-to-scalar ratio,

$$r \approx 1.3 \times 10^{-27} \left(\frac{\Lambda_I}{10^{11}\text{GeV}}\right)^4$$  \hspace{1cm} (3.13)

which may not be accessible in the near future, but a sizable running of the spectral index,

$$\alpha_s = -(3-4) \times 10^{-3}$$  \hspace{1cm} (3.14)

with $\alpha_s \equiv \frac{dn_s}{d\log k_\ast}$ (see figure 4). This running can be probed by future CMB experiments such as CMB-S4 [59] and SPHEREx [60], which are expected to measure $\alpha_s$ with a precision of $10^{-3}$, along with other experiments such as DESI [61], WFIRST [62], or/and SKA [63]. The running of the running of the spectral index, $\beta_s = O(\eta\alpha_s)$, is expected to be more suppressed, with $|\beta_s| < O(10^{-4})$.\footnotemark
Figure 3. $n_s$ vs $\delta \xi / \xi_0$ [upper panel], and $M^{-2} M_{\text{pl}}^2$ vs $\delta \xi / \xi_0$ [lower panel]. We take $\Delta R^2_0[\phi_*] = \Delta R^2_{\text{CMB}}[k_*]$ by varying the one-dimensional combination of $\delta \xi$ and $M^{-2}$. Notice that the normalization of $\Lambda_I/M_{\text{pl}}$ are different for lines with different $\Lambda_I$. The dependency of $M^{-2} M_{\text{pl}}^2 / \Lambda_I$ on $\delta \xi / \xi_0 M_{\text{pl}} / \Lambda_I$ is almost irrelevant to $\Lambda_I$.

Although I used the full potential of eq. (2.6), I have verified that the higher-order terms in the expansions of eq. (3.3), namely, $\mathcal{O}(h^{-6})$ terms in the large $h$ expansion and $V_0 \mathcal{O}(h^4/M^4)$ terms in the small $h$ expansion, are insignificant in determining the relations between $\alpha_s$ and $n_s$. This is because inflation occurs in an intermediate range where $\Lambda_I \ll h \ll M_{\text{pl}}$. Numerical checks confirm that our results are not impacted by adding a term of $V_0 h^4/M^4$. This is because the contribution from this term is suppressed by $\Lambda_I^4(M_{\text{pl}}^2)^2$ compared to the $V_0 h^2/M^2$ term. This suppression is small enough that it does not affect the cancellation of terms necessary for a flat direction during slow-roll inflation thanks to Jordan frame EFT assumption.
\[ \Lambda_I = 10^{12}\text{GeV}, 10^{11}\text{GeV}, 10^{10}\text{GeV,} \]

\[ n_{\text{CMB}} = 10^9\text{GeV} \]

Figure 4. The prediction of the spectral index and its running with \( \Delta_2^2[\phi^*] = \Delta_2^2_{\text{CMB}}[k^*] \), by varying one-dimensional combination of \( \delta \xi, \delta M^{-2} \). \( \Lambda_I = 10^{12}, 10^{11}, 10^{10}, 10^9 \) GeV from top to bottom with fixed \( y_t = 0.5 \). The CMB measurement is also shown in the pink region.

I confirmed the prediction of the running of the spectral index by using the running coupling, \( V_{\text{EF}} = (\lambda[h/\Omega]h^4/4 - \mu^2 h^2/2) / \Omega^4 + (10^{12} \text{GeV})^4 \frac{h^2}{M^2} \) which goes beyond the leading logarithmic approximation. With \( M_t = 171.5 \text{GeV}, M^{-2} = 4 \times 10^{-8} M_{\text{pl}}^{-2} \), \( N \approx 46 \) and \( \xi \approx 2.63298590476443 \times 10^{11} \), I obtained the values \( \Delta_2^2 \approx 2.1 \times 10^{-9}, n_s \approx 0.965, \alpha_s \approx -0.00289, \beta_s \approx -0.0000471 \). I chose a slightly smaller value for \( M_t \) compared to the central value of \( M_t \) direct to reduce numerical cost (see the appendix for detailed analysis). The prediction of \( \alpha_s \) is in agreement with the leading logarithmic approximation that I used for plotting figure 4.

We may have threshold effects satisfying eq. (2.17), which is \( \delta V_{\text{EF}} = \frac{V_0}{(16\pi)^2} \mathcal{O}(\Lambda^4 h^{-4}) \) (as seen in eqs. (2.14) and (2.17)). This term is subdominant compared to the \( h^{-6} \) term in eq. (3.3) and can be accounted for by redefining \( \Lambda_I \) and \( y_t \) in the previous leading logarithmic estimations. The difference would only appear in \( \mathcal{O}(h^{-6}) \) terms. To assess the stability of the results against threshold effects, I have added an \( \mathcal{O}(h^{-6}) \) term to the previous potential: \( V_{\text{EF}} = (\lambda[h/\Omega]h^4/4 - \mu^2 h^2/2) / \Omega^4 + (10^{12} \text{GeV})^4 \frac{h^2}{M^2} + \frac{(10^{13} \text{GeV})^4}{(16\pi^2)^2} \left( 10^{13} \text{GeV} / h \right)^6 \), and performed the calculation with \( M^{-2} = 3.9 \times 10^{-8} M_{\text{pl}}^{-2} \) and \( \xi \approx 2.6329859465 \times 10^{11} \) using the same other inputs as before. The resulting values of \( n_s \approx 0.967, \alpha_s \approx -0.00282, \beta_s \approx -0.0000432 \) are consistent with the previous estimations without the \( h^{-6} \) term. I also confirmed that adding terms \( \delta V_{1F} = h_3^3 F / M^4 \) or/and \( \delta \Omega^2 = h_3^3 F / M_{\text{pl}}^4 \) do not affect the numerical results obtained using the running coupling.
As a result, given the Jordan frame EFT assumption, our analysis remains stable despite the presence of higher-order corrections, thanks to the range of fields $\Lambda_I \ll h \ll M$ indicated from eq. (3.5), which effectively reduces the impact of both the UV-completion-dependent threshold effects and higher dimensional terms. The required value of $M$ to affect $n_s$ is smaller than what was obtained in ref. [24]. This is because, in our setup, the inflationary Higgs field value is given by eq. (3.5), while the reference’s setup for ordinary Higgs inflation has $h \sim M_{pl}$.

I emphasize that our result relies on the Jordan frame EFT assumption. In general, this is not the only solution for explaining the CMB data. In the more generic theory that higher dimensional term couplings are not controlled by a single mass scale $M$, we may have different conclusions.\(^9\)

4 Discussions, future directions, and conclusions

Successful inflation is close to the criticality between eternal and non-eternal inflation. For successful inflation, careful adjustment of parameters is essential, just as it is in conventional low-scale inflation.\(^10\) In this discussion, I aim to demonstrate that when successful inflation is required, with the consistency of $\Delta \phi$ and $n_s$ at a scale of $\Lambda_I = 10^{-9}-12$ GeV, the system tends to be near the threshold between eternal and non-eternal inflation. As depicted in the top panel of figure 5, I have shown the behavior of the potential in the $h/M_{pl}$-$d \log V_{EF}/d \log h$ plane with $\delta \xi = \delta \xi_{\text{sample}} \equiv -3.888888889 \times 10^{-7} \xi_0$, $M^{-2} = 4.137259718 \times 10^{-6} M_{pl}^{-2}$ (See eq. (3.2) for the definition of $\xi_0$).\(^11\) The given parameter set yields $n_s \approx 0.967$, $\alpha_s \approx -0.0033$, $\beta_s \approx -0.000048$, and $r \approx 1.3 \times 10^{-27}$, which are in agreement with CMB observations and can be tested in the future through the measurement of $\alpha_s$. The end of inflation and the horizon exit are indicated by a blue circle and red star, respectively, and the internal dashed-purple line represents the slow-roll regime for the observable universe. $dV_{EF}/dh$ increases again for larger values of $h$. In this scenario, the classical motion of the canonical field, $|\Delta_{\text{classical}} \phi| \sim |dV_{EF}/d\phi| \times (3H_{\text{inf}}^2)^{-1} = |dV_{EF}/dh| Z^{-1/2} \times (3H_{\text{inf}}^2)^{-1}$, is much greater than the quantum diffusion, $\Delta_{\text{quantum}} \phi \sim H_{\text{inf}}/2\pi$, for any field value with $h < M$, as $|\Delta_{\text{quantum}} \phi| \sim 10$ GeV is much smaller than $|\Delta_{\text{classical}} \phi| \gtrsim 10^{-5}$--$10^{-6}$ GeV. The total number of e-folds in the range $h \lesssim M$, or equivalently $\phi \lesssim 10 M_{pl}/\sqrt{\xi}$, is at most $\sim 43$ for the observable universe. Therefore, eternal inflation does not occur for $h \lesssim M$.

\(^9\)In particular, one may consider the possibility that all the higher dimensional terms are controlled by the gravity-motivated coupling $M = M_{pl}$, while some terms are finely tuned to be small for inflation: $\delta V_{EF} = c_h h^3_{\phi}/M_{pl}^2 + c_h h^5_{\phi}/M_{pl}^6 + \cdots$. In this scenario, we only need to study the inflation dynamics with two parameters, $c_h$ and $\alpha_h$, which may be finely tuned to be much smaller than unity. The impact from the Planck suppressed terms with dimensions $\geq 10$ is negligible, if the inflationary field value does not change much from our previous analysis. Thus, a thorough study of the setup can be performed with the parameter scan, including these two couplings.

\(^10\)It is possible that this tuning can be explained through anthropic principle.

\(^11\)I would like to stress that the inputs required for successful inflation are highly sensitive to the SM parameters or approximations used for the Higgs potential. The values provided here are solely for the purpose of reproducing the results, in which case, it is necessary to use the same values for $y_\phi$, $M_{pl}$, etc., as I did. While a different choice of SM parameters may result in different values of $\xi$ and $M^{-2}$ for successful inflation, it does not alter our main conclusions or the proximity to criticality. I have confirmed that these results remain unchanged with various parameter sets.
In contrast, the bottom panel of figure 5 shows the results when $\delta \xi$ is decreased by $5 \times 10^{-6}\%$, which corresponds to an increase in $\xi$ by $2 \times 10^{-12}\%$, from the value in the top panel. As a result, $\delta \xi_R^2 \sim 10 \delta \xi_{\text{CMB}}^2$ at the horizon exit, making it inconsistent with the CMB data. This highlights the need for a precise tuning of parameters to achieve successful inflation, as seen in the top panel. However, this tiny change in the parameter value leads to a qualitative change in the potential shape, as indicated by the red curve. The potential is negative at two points where $V_{E_{\text{F,J}}h} = 0$: a hilltop and a false vacuum, from left to right. At both of these points, $|\Delta \phi_{\text{quantum}}| > |\Delta \phi_{\text{classical}}|$, allowing for eternal inflation to occur. As long as the explanation for the stability of the EW vacuum relies on an extremely large value of $\xi$, the false vacuum and hence the eternal inflation regime will still exist for $\delta \xi < (1 + \mathcal{O}(10^{-6})\%) \delta \xi_{\text{sample}}$ (where $\delta \xi_{\text{sample}} < 0$). This is because the positive $1/M^2$ term raises the potential at large $h$, while a sufficiently smaller $\xi$ than $\xi_0$ would result in a potential hilltop in the regime $h > M_{\text{pl}}/\sqrt{\xi_0}$. The near criticality behavior is also observed for the case of $\Lambda_I = 10^{9,10,12}$ GeV when $n_s = 0.96 - 0.97$. I conclude that successful inflation lies in a regime that is very close to the criticality between the Higgs potentials that lead to eternal and non-eternal inflation. Given that $\xi_0 = \mathcal{O}(10^{14})$ and the criticality of the potential is within a change of $\xi$ by $10^{-12}\%$, $\Delta \xi = \mathcal{O}(1)$ can reach the criticality.\footnote{From the degeneracy in $\kappa$ in eq. (3.4), this is equivalent to a change in $\mu_H^2$ while fixing $\xi$. Interestingly, we observe that the criticality can be reached by changing $\mu_H^2$ by $\Delta \mu_H^2 \sim (100 \text{ GeV})^2$ for $\Lambda_I \sim 10^{11}$ GeV. This change at the weak scale can result in the potential having a false vacuum. This is due to the intriguing coincidence between $N_I^2/M_{\text{pl}} \sim \mu_H$ as suggested by the SM parameter measurements. Furthermore, there is another coincidence for the QCD scale $\Lambda_{\text{QCD, JF}} \sim 10^{11}$ GeV [56]. This may point to new principles and mechanisms for the origin of these scales.} This may indicate a scenario for explaining the fine-tuned value of $\xi$, by introducing another light field. This scalar field slowly rolls and changes the relevant parameters during inflation. The change triggers the end of the “eternal inflation”, predicting the parameters around the criticality. Then most Universe after inflation and reheating would have the parameters around the criticality (cf. [85]).\footnote{This mechanism for alleviating the tuning may be generic in inflection-point inflation in which by explaining $n_s$ the inflation is non-eternal [80].}

Dark matter production from inflationary fluctuation. The inflation model is compatible with various dark matter scenarios that are produced through thermal processes after inflation, such as the WIMP scenario, without any modifications from the conventional ones.

There are also non-trivial connections between the inflation era and some dark matter scenarios. For example, it was proposed that if weak-scale inflation is triggered by the Higgs field excursion, the QCD axion with a decay constant near the Planck scale could be the dominant dark matter. This is due to the enhanced QCD scale of $\sim 100$ GeV caused by the SM quarks and the RG running at the scale of $\sim \Lambda_I$, as well as the stochastic behavior during eternal inflation [56]. This scenario is not possible in weak-scale (Palatini) Higgs inflation because the inflation is not long enough, and even if it were longer due to modifications, the QCD scale in the Einstein frame would be $\sim 0.1$ GeV $10^{13}$ due to the large $\Omega$ during inflation. The stochastic scenario for the QCD axion is also unlikely to work [86, 87]. The decay constant is likely to be around $10^{11}$ GeV to explain the axion dark matter abundance.
Slow-roll for observable Universe
Horizon exit
End of Inflation
\[ \delta \xi = \delta \xi_{\text{sample}} \equiv -3.888888889 \times 10^{-7} \xi_0 \]

Negative $V_{E/F,h}$
\[ \delta \xi = \delta \xi_{\text{sample}} (1 + 5 \times 10^{-8}) \]

Figure 5. $d \log V_{E/F} / d \log h$ vs $h/M_{\text{pl}}$. In both panel, $\Lambda_I = 10^{11} \text{GeV}$, $y_t = 0.5$, $\mu_H^2 = (125 \text{GeV})^2/2$, and $M^{-2} = 4.137259718 \times 10^{-6} M_{\text{pl}}^{-2}$. The difference is that, in the upper panel, I take $\xi = \delta \xi_{\text{sample}} \equiv -3.888888889 \times 10^{-7} \xi_0$, which explain the CMB data well, while, in the lower panel, $\xi = (1 + 5 \times 10^{-8}) \delta \xi_{\text{sample}}$, which cannot explain the CMB data.

Other light axionic or scalar dark matter can be produced through the inflationary fluctuation as well [88–90]. On the other hand, weak-scale inflation has been known to alleviate the cosmological moduli problem [58, 88]. It is important to consider the changes in scalar masses when estimating fluctuations in the inflationary era. On the contrary, this suppression of dark matter masses during inflation should alleviate the isocurvature problem for relatively heavy dark matter [91].

Another interesting possibility for light dark matter production is from the decay of the "inflaton", which has a smaller mass than the reheating temperature [92, 93]. In particular, the Higgs inflaton couples to all possible dark matter through a large non-minimal coupling, making the production of dark matter through this coupling an interesting topic for future research (cf. [28, 94]).

Neutrino mass and leptogenesis. Let us consider the seesaw mechanism to generate the neutrino mass [95–99] since the $h$ value during inflation can be around or beyond the seesaw scale. During inflation, the right-handed neutrino (RHN) masses, whose Majorana components are suppressed by $\Omega$, are dominantly induced by the Higgs-neutrino-RHN Yukawa interaction. By estimating the Coleman-Weinberg potential from the Higgs-neutrino-RHN system we can find that the contribution can be regarded as the redefinitions of $\kappa$ in eqs. (3.3) in the large $h$ limit while the $O(h^{-4})$ term rarely change as long as the RHN Yukawa coupling is much smaller than $y_t$. Our conclusions in the main part do not change.

This inflation scenario which has a reheating temperature, $T_R = 10^{8–11} \text{GeV}$, can have a successful leptogenesis [100–104] (see also refs. [105, 106]) when the RHNs are light enough after inflation.

On the other hand, after the inflation, the RHNs may get heavier than the reheating temperature due to the smaller $\Omega$, and thus the thermal or direct production from the
Higgs interaction to produce the RHN may be kinematically suppressed. In this case we can still have successful leptogenesis via the left-handed lepton oscillation [107] (See also refs. [108, 109]). Indeed there can be an enhancement in the scenario because soon after inflation, we should have an over-dense system during the first periods of reheating [109].

Conclusions. The recent precise measurements of the SM parameters and the renormalization group running have indicated the presence of a hilltop in the Higgs potential at an intermediate scale, causing the EW vacuum to metastable. This paper highlights the potential of a very large Higgs non-minimal coupling in stabilizing the EW vacuum and making it the true minimum. This same setup can also drive slow-roll inflation, which increases the density perturbation to the observed levels due to the suggested hilltop in the SM. We have conducted a study of this minimal inflation scenario, particularly in the Palatini formulation of gravity.

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A Estimation with running quartic coupling

We can go beyond the leading logarithmic approximation by taking the renormalization scale $\mu_{RG} = \gamma h/\Omega$, with $\gamma$ being a factor of order 1 to minimize the radiative correction at around the hilltop. We have

$$V_{EF} \simeq \lambda \left[ \frac{\gamma h}{\Omega} \right]^4 - \frac{\mu_{RG} h^2}{2\Omega^4}. \tag{A.1}$$

For simplicity, we take $\gamma = 1$, which is a good approximation at the 1-loop level. Since the threshold corrections are in the 2-loop order, we simply use $\lambda[\mu_{RG}]$ obtained in the SM (see the discussions in section 2.2).

For $M_t = 171.16$ GeV used in figure 2, the numerical result of $\lambda[\mu_{RG}]$ was obtained using the SMDR code [65], which partially includes 3, 4, and 5 loop effects. The result was fitted using the following analytical function:

$$\lambda[\mu_{RG}] = \frac{757t^6}{52642997920583} - \frac{4550t^5}{1447396579981} + \frac{74609t^4}{248272459617} - \frac{108464t^3}{6520077091} + \frac{247203t^2}{4192761488}$$

$$- \frac{57110021}{134444298t^2} - \frac{5068357t}{403070660} + \frac{48520615}{99963291t} + \frac{23970602}{241157271} \tag{A.2}$$

where $t = \log[\mu_{RG}/\text{GeV}]$. For clarity, rational numbers were used to eliminate ambiguity. To further confirm the discussions with a higher dimensional term in section 3.2, the data of $M_t = 171.5$ GeV around $\lambda = 0$ was fitted to obtain:

$$\lambda[\mu_{RG}] = \frac{39451546}{287737145} - \frac{23277978}{2194779941} t + \frac{6614676}{24070045651} t^2 - \frac{228011}{92262625053} t^3. \tag{A.3}$$
Figure 6. The black dots in the figure are obtained from the SMDR code in $\mu_{RG} - \lambda$ plane while the red-solid lines represent our fitted functions. In the top and bottom panels, the top quark mass is taken as 172.16 GeV and 171.5 GeV, respectively. The red dashed line represents the SM prediction without a non-minimal coupling, in which the EW vacuum is not the absolute minimum. The values for $\Lambda_I$, $y_t$, and $\mu^2_H$ are fixed at $10^{11}$ GeV, 0.5, and $(125 \text{ GeV})^2/2$, respectively.

The reason for choosing the data for $\lambda$ corresponding to the top quark mass slightly smaller than the 2σ range of $M_t^{\text{direct}}$ in the footnote 1 was due to the fact that simulating lower-scale inflation requires more precise estimations of the slow-roll dynamics, making it more difficult. For those reasons, the top masses used are slightly in tension with the $M_t^{\text{direct}}$. But the numerical results were intended to verify that the higher-order corrections are insignificant. In the running coupling estimations, I used the central values in PDG [27] for the other SM parameters.

References

[1] A.A. Starobinsky, *Spectrum of relict gravitational radiation and the early state of the universe*, JETP Lett. 30 (1979) 682 [inSPIRE].

[2] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, Phys. Lett. B 91 (1980) 99 [inSPIRE].
[3] A.H. Guth, *The inflationary universe: a possible solution to the horizon and flatness problems*, Phys. Rev. D 23 (1981) 347 [SPIRE].

[4] K. Sato, *First order phase transition of a vacuum and expansion of the universe*, Mon. Not. Roy. Astron. Soc. 195 (1981) 467 [SPIRE].

[5] D. Kazanas, *Dynamics of the universe and spontaneous symmetry breaking*, Astrophys. J. Lett. 241 (1980) L59 [SPIRE].

[6] A.D. Linde, *A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, Phys. Lett. B 108 (1982) 389 [SPIRE].

[7] V.F. Mukhanov and G.V. Chibisov, *Quantum fluctuations and a nonsingular universe*, JETP Lett. 33 (1981) 532 [SPIRE].

[8] S.W. Hawking and I.G. Moss, *Supercooled phase transitions in the very early universe*, Phys. Lett. B 110 (1982) 35 [SPIRE].

[9] G.V. Chibisov and V.F. Mukhanov, *Galaxy formation and phonons*, Mon. Not. Roy. Astron. Soc. 200 (1982) 535 [SPIRE].

[10] S.W. Hawking, *The development of irregularities in a single bubble inflationary universe*, Phys. Lett. B 115 (1982) 295 [SPIRE].

[11] A.H. Guth and S.Y. Pi, *Fluctuations in the new inflationary universe*, Phys. Rev. Lett. 49 (1982) 1110 [SPIRE].

[12] A. Albrecht and P.J. Steinhardt, *Cosmology for grand unified theories with radiatively induced symmetry breaking*, Phys. Rev. Lett. 48 (1982) 1220 [SPIRE].

[13] A.A. Starobinsky, *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*, Phys. Lett. B 117 (1982) 175 [SPIRE].

[14] Planck collaboration, *Planck 2018 results. X. Constraints on inflation*, Astron. Astrophys. 641 (2020) A10 [arXiv:1807.06211] [SPIRE].

[15] Planck collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641 (2020) A6 [Erratum ibid. 652 (2021) C4] [arXiv:1807.06209] [SPIRE].

[16] F.L. Bezrukov and M. Shaposhnikov, *The standard model Higgs boson as the inflaton*, Phys. Lett. B 659 (2008) 703 [arXiv:0710.3755] [SPIRE].

[17] F.L. Bezrukov, A. Magnin and M. Shaposhnikov, *Standard model Higgs boson mass from inflation*, Phys. Lett. B 675 (2009) 88 [arXiv:0812.4950] [SPIRE].

[18] J. Rubio, *Higgs inflation*, Front. Astron. Space Sci. 5 (2019) 50 [arXiv:1807.02376] [SPIRE].

[19] F. Bauer and D.A. Demir, *Higgs-Palatini inflation and unitarity*, Phys. Lett. B 698 (2011) 425 [arXiv:1012.2900] [SPIRE].

[20] F. Bezrukov, J. Rubio and M. Shaposhnikov, *Living beyond the edge: Higgs inflation and vacuum metastability*, Phys. Rev. D 92 (2015) 083512 [arXiv:1412.3811] [SPIRE].

[21] J. Fumagalli and M. Postma, *UV (in)sensitivity of Higgs inflation*, JHEP 05 (2016) 049 [arXiv:1602.07234] [SPIRE].

[22] S. Rasanen and P. Wahlman, *Higgs inflation with loop corrections in the Palatini formulation*, JCAP 11 (2017) 047 [arXiv:1709.07853] [SPIRE].

[23] V.-M. Enckell, K. Enqvist, S. Rasanen and E. Tomberg, *Higgs inflation at the hilltop*, JCAP 06 (2018) 005 [arXiv:1802.09299] [SPIRE].

[24] R. Jinno, M. Kubota, K.-Y. Oda and S.C. Park, *Higgs inflation in metric and Palatini formalisms: required suppression of higher dimensional operators*, JCAP 03 (2020) 063 [arXiv:1904.05699] [SPIRE].
[25] I.D. Gialamas and A.B. Lahanas, Reheating in $R^2$ Palatini inflationary models, *Phys. Rev. D* **101** (2020) 084007 [arXiv:1911.11513] [inSPIRE].

[26] I.D. Gialamas, A. Karam, A. Lykkas and T.D. Pappas, Palatini-Higgs inflation with nonminimal derivative coupling, *Phys. Rev. D* **102** (2020) 063522 [arXiv:2008.06371] [inSPIRE].

[27] Particle Data Group collaboration, Review of particle physics, *PTEP* **2022** (2022) 083C01 [inSPIRE].

[28] Q. Li, T. Moroi, K. Nakayama and W. Yin, Instability of the electroweak vacuum in Starobinsky inflation, *JHEP* **09** (2022) 102 [arXiv:2206.05926] [inSPIRE].

[29] A.H. Hoang, What is the top quark mass?, *Ann. Rev. Nucl. Part. Sci.* **70** (2020) 225 [arXiv:2004.12915] [inSPIRE].

[30] M. Sher, Electroweak Higgs potentials and vacuum stability, *Phys. Rept.* **179** (1989) 273 [inSPIRE].

[31] P.B. Arnold, Can the electroweak vacuum be unstable?, *Phys. Rev. D* **40** (1989) 613 [inSPIRE].

[32] G.W. Anderson, New cosmological constraints on the Higgs boson and top quark masses, *Phys. Lett. B* **243** (1990) 265 [inSPIRE].

[33] P.B. Arnold and S. Vokos, Instability of hot electroweak theory: bounds on $m_H$ and $M_t$, *Phys. Rev. D* **44** (1991) 3620 [inSPIRE].
[46] A. Kobakhidze and A. Spencer-Smith, *Electroweak vacuum (in)stability in an inflationary universe*, Phys. Lett. B 722 (2013) 130 [arXiv:1301.2846] [SPIRE].

[47] M. Herranen, T. Markkanen, S. Nurmi and A. Rajantie, *Spacetime curvature and Higgs stability after inflation*, Phys. Rev. Lett. 115 (2015) 241301 [arXiv:1506.04066] [SPIRE].

[48] Y. Ema, K. Mukaida and K. Nakayama, *Fate of electroweak vacuum during preheating*, JCAP 10 (2016) 043 [arXiv:1602.00483] [SPIRE].

[49] W.A. Hiscock, *Can black holes nucleate vacuum phase transitions?*, Phys. Rev. D 35 (1987) 1161 [SPIRE].

[50] V.A. Berezin, V.A. Kuzmin and I.I. Tkachev, *O(3) invariant tunneling in general relativity*, Phys. Lett. B 207 (1988) 397 [SPIRE].

[51] N. Oshita, M. Yamada and M. Yamaguchi, *Compact objects as the catalysts for vacuum decays*, Phys. Lett. B 791 (2019) 149 [arXiv:1808.01382] [SPIRE].

[52] N. Oshita, Y. Shoji and M. Yamaguchi, *Polychronic tunneling: new tunneling processes experiencing Euclidean and Lorentzian evolution simultaneously*, Phys. Rev. D 107 (2023) 045007 [arXiv:2112.10736] [SPIRE].

[53] O. Lebedev, *On stability of the electroweak vacuum and the Higgs portal*, Eur. Phys. J. C 72 (2012) 2058 [arXiv:1203.0156] [SPIRE].

[54] J. Elias-Miro et al., *Stabilization of the electroweak vacuum by a scalar threshold effect*, JHEP 06 (2012) 031 [arXiv:1203.0237] [SPIRE].

[55] K. Nakayama and W. Yin, *Hidden photon and axion dark matter from symmetry breaking*, JHEP 10 (2021) 026 [arXiv:2105.14549] [SPIRE].

[56] H. Matsui, F. Takahashi and W. Yin, *QCD axion window and false vacuum Higgs inflation*, JHEP 05 (2020) 154 [arXiv:2001.04464] [SPIRE].

[57] O. Czerwińska, Z. Lalak, M. Lewicki and P. Olszewski, *The impact of non-minimally coupled gravity on vacuum stability*, JHEP 10 (2016) 004 [arXiv:1606.07808] [SPIRE].

[58] L. Randall and S.D. Thomas, *Solving the cosmological moduli problem with weak scale inflation*, Nucl. Phys. B 449 (1995) 229 [hep-ph/9407248] [SPIRE].

[59] CMB-S4 collaboration, *CMB-S4 science book, first edition*, arXiv:1610.02743 [SPIRE].

[60] SPHEREx collaboration, *Cosmology with the SPHEREX all-sky spectral survey*, arXiv:1412.4872 [SPIRE].

[61] DESI collaboration, *The DESI experiment, a whitepaper for Snowmass 2013*, arXiv:1308.0847 [SPIRE].

[62] D. Spergel et al., *Wide-Field InfraRed Survey Telescope-Astrophysics Focused Telescope Assets WFIRST-AFTA 2015 report*, arXiv:1503.03757 [SPIRE].

[63] Cosmology SWG collaboration, *Cosmology from HI galaxy surveys with the SKA*, arXiv:1501.04035 [SPIRE].

[64] K. Endo and Y. Sumino, *A scale-invariant Higgs sector and structure of the vacuum*, JHEP 05 (2015) 030 [arXiv:1503.02819] [SPIRE].

[65] S.P. Martin and D.G. Robertson, *Standard model parameters in the tadpole-free pure MS scheme*, Phys. Rev. D 100 (2019) 073004 [arXiv:1907.02500] [SPIRE].

[66] F. Bezrukov and M. Shaposhnikov, *Standard model Higgs boson mass from inflation: two loop analysis*, JHEP 07 (2009) 089 [arXiv:0904.1537] [SPIRE].

[67] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, *Higgs inflation: consistency and generalisations*, JHEP 01 (2011) 016 [arXiv:1008.5157] [SPIRE].
[68] K. Sakurai, F. Takahashi and W. Yin, *Singlet extensions and W boson mass in light of the CDF II result*, *Phys. Lett. B* **853** (2022) 137324 [arXiv:2204.04770] [INSPIRE].

[69] F. Takahashi, *New inflation in supergravity after Planck and LHC*, *Phys. Lett. B* **727** (2013) 21 [arXiv:1308.4212] [INSPIRE].

[70] M. Czerny and F. Takahashi, *Multi-natural inflation*, *Phys. Lett. B* **733** (2014) 241 [arXiv:1401.5212] [INSPIRE].

[71] M. Czerny, T. Higaki and F. Takahashi, *Multi-natural inflation in supergravity*, *JHEP* **05** (2014) 144 [arXiv:1403.0410] [INSPIRE].

[72] M. Czerny, T. Higaki and F. Takahashi, *Multi-natural inflation and BICEP2*, *Phys. Lett. B* **734** (2014) 167 [arXiv:1403.0410] [INSPIRE].

[73] T. Higaki, T. Kobayashi, O. Seto and Y. Yamaguchi, *Axion monodromy inflation with multi-natural modulations*, *JCAP* **10** (2014) 025 [arXiv:1405.0775] [INSPIRE].

[74] D. Croon and V. Sanz, *Saving natural inflation*, *JCAP* **02** (2015) 008 [arXiv:1411.7809] [INSPIRE].

[75] T. Higaki and F. Takahashi, *Elliptic inflation: interpolating from natural inflation to R\(^2\)-inflation*, *JHEP* **03** (2015) 129 [arXiv:1501.02354] [INSPIRE].

[76] T. Higaki and Y. Tatsuta, *Inflation from periodic extra dimensions*, *JCAP* **07** (2017) 011 [arXiv:1611.00808] [INSPIRE].

[77] R. Daido, F. Takahashi and W. Yin, *The ALP miracle: unified inflaton and dark matter*, *JCAP* **05** (2017) 044 [arXiv:1702.03284] [INSPIRE].

[78] R. Daido, F. Takahashi and W. Yin, *The ALP miracle revisited*, *JHEP* **02** (2018) 104 [arXiv:1710.11107] [INSPIRE].

[79] IAXO collaboration, *Physics potential of the International Axion Observatory (IAXO)*, *JCAP* **06** (2019) 047 [arXiv:1904.09155] [INSPIRE].

[80] F. Takahashi and W. Yin, *ALP inflation and big bang on earth*, *JCAP* **07** (2019) 095 [arXiv:1903.00462] [INSPIRE].

[81] D.J.E. Marsh and W. Yin, *Opening the 1 Hz axion window*, *JHEP* **01** (2021) 169 [arXiv:1912.08188] [INSPIRE].

[82] F. Takahashi and W. Yin, *Challenges for heavy QCD axion inflation*, *JCAP* **10** (2021) 057 [arXiv:2105.10493] [INSPIRE].

[83] D. Baumann, *Cosmology*, Cambridge University Press, Cambridge, U.K. (2022) [DOI:10.1017/9781108937092] [INSPIRE].

[84] J.-O. Gong and E.D. Stewart, *The density perturbation power spectrum to second order corrections in the slow roll expansion*, *Phys. Lett. B* **510** (2001) 1 [astro-ph/0101225] [INSPIRE].

[85] W. Yin, *Small cosmological constant from a peculiar inflaton potential*, *Phys. Rev. D* **106** (2022) 055014 [arXiv:2108.04246] [INSPIRE].

[86] P.W. Graham and A. Scherlis, *Stochastic axion scenario*, *Phys. Rev. D* **98** (2018) 035017 [arXiv:1805.07362] [INSPIRE].

[87] F. Takahashi, W. Yin and A.H. Guth, *QCD axion window and low-scale inflation*, *Phys. Rev. D* **98** (2018) 015042 [arXiv:1805.08763] [INSPIRE].

[88] S.-Y. Ho, F. Takahashi and W. Yin, *Relaxing the cosmological moduli problem by low-scale inflation*, *JHEP* **04** (2019) 149 [arXiv:1901.01240] [INSPIRE].
[89] F. Takahashi and W. Yin, QCD axion on hilltop by a phase shift of $\pi$, JHEP 10 (2019) 120 [arXiv:1908.06071] [SPIRE].

[90] S. Nakagawa, F. Takahashi and W. Yin, Stochastic axion dark matter in axion landscape, JCAP 05 (2020) 004 [arXiv:2002.12195] [SPIRE].

[91] Y. Ema, K. Nakayama and Y. Tang, Production of purely gravitational dark matter, JHEP 09 (2018) 135 [arXiv:1804.07471] [SPIRE].

[92] T. Moroi and W. Yin, Light dark matter from inflaton decay, JHEP 03 (2021) 301 [arXiv:2011.09475] [SPIRE].

[93] T. Moroi and W. Yin, Particle production from oscillating scalar field and consistency of Boltzmann equation, JHEP 03 (2021) 296 [arXiv:2011.12285] [SPIRE].

[94] Q. Li, T. Moroi, K. Nakayama and W. Yin, Production of purely gravitational dark matter, JHEP 09 (2018) 135 [arXiv:1804.07471] [SPIRE].

[95] P. Minkowski, $\mu \rightarrow e\gamma$ at a rate of one out of $10^9$ muon decays?, Phys. Lett. B 67 (1977) 421 [SPIRE].

[96] T. Yanagida, Horizontal gauge symmetry and masses of neutrinos, Conf. Proc. C 7902131 (1979) 95 [SPIRE].

[97] S.L. Glashow, The future of elementary particle physics, NATO Sci. Ser. B 61 (1980) 687 [SPIRE].

[98] M. Gell-Mann, P. Ramond and R. Slansky, Complex spinors and unified theories, Conf. Proc. C 790927 (1979) 315 [arXiv:1306.4669] [SPIRE].

[99] R.N. Mohapatra and G. Senjanovic, Neutrino mass and spontaneous parity nonconservation, Phys. Rev. Lett. 44 (1980) 912 [SPIRE].

[100] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, Phys. Lett. B 174 (1986) 45 [SPIRE].

[101] A. Pilaftsis, Resonant CP violation induced by particle mixing in transition amplitudes, Nucl. Phys. B 504 (1997) 61 [hep-ph/9702393] [SPIRE].

[102] W. Buchmuller and M. Plumacher, CP asymmetry in Majorana neutrino decays, Phys. Lett. B 431 (1998) 354 [hep-ph/9710460] [SPIRE].

[103] E.K. Akhmedov, V.A. Rubakov and A.Y. Smirnov, Baryogenesis via neutrino oscillations, Phys. Rev. Lett. 81 (1998) 1359 [hep-ph/9803255] [SPIRE].

[104] T. Asaka and M. Shaposhnikov, The $\nu$MSM, dark matter and baryon asymmetry of the universe, Phys. Lett. B 620 (2005) 17 [hep-ph/0505013] [SPIRE].

[105] W. Buchmuller, R.D. Peccei and T. Yanagida, Leptogenesis as the origin of matter, Ann. Rev. Nucl. Part. Sci. 55 (2005) 311 [hep-ph/0502169] [SPIRE].

[106] S. Davidson, E. Nardi and Y. Nir, Leptogenesis, Phys. Rept. 466 (2008) 105 [arXiv:0802.2962] [SPIRE].

[107] Y. Hamada, R. Kitano and W. Yin, Leptogenesis via neutrino oscillation magic, JHEP 10 (2018) 178 [arXiv:1807.06582] [SPIRE].

[108] Y. Hamada and R. Kitano, Primordial lepton oscillations and baryogenesis, JHEP 11 (2016) 010 [arXiv:1609.05028] [SPIRE].

[109] S. Eijima, R. Kitano and W. Yin, Throwing away antimatter via neutrino oscillations during the reheating era, JCAP 03 (2020) 048 [arXiv:1908.11864] [SPIRE].