Toward a New Phenomenon: Super-Čerenkov Radiation

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In this letter a new coherent gamma emission mechanism, called Super-Čerenkov radiation, is introduced. The SCR is expected to take place when the charged particle is moving in a medium with a phase velocity $v_{xph}$ satisfying the super-coherent condition: $\cos \theta_{SC} = v_{xph}/v_{\gamma ph} \leq 1$. The results on an experimental test of SCR in RICH detector are presented.

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The electromagnetic Čerenkov radiation was first observed in the early 1900’s by the experiments developed by Marie and Pierre Curie when studying radioactivity emission. The first deliberate attempt to understand the phenomenon was made by Mallet [1] in 1926. He observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, not possessing the line or band structure characteristic of fluorescence. Only the exhaustive experimental work, carried out between years 1934-1937 by P. A. Čerenkov [2], characterized completely this kind of radiation. These experimental data preceded and are fully consistent with the classical electromagnetic theory developed by Frank and Tamm [3]. So, they showed that charges travelling faster than the speed of light in a substance with a frequency-independent refractive index $n_c(\omega_c)$ emit coherent radiation satisfying the Čerenkov coherence relation (we adopted the system of units $\hbar = c = 1$):

$$\cos \theta_C = v_{\gamma ph}/v_x \leq 1$$

(1)

where $\theta_C$ is the angle between the direction of motion and that of the electromagnetic wavefront, $v_x$ is the speed of the particle in medium. A quantum approach of the Čerenkov effect by Ginsburg [4] resulted in only minor modification to the classical theory. Now, the Čerenkov radiation (CR) is the subject of many studies related to extension to the nuclear media [4] as well as to other coherent particle emission via Čerenkov-like mechanisms [5-12]. The generalized Čerenkov-like effects based on four fundamental interactions has been investigated and classified recently in [10]. In particular, this classification includes the nuclear (mesonic, $\gamma$, weak boson)-Čerenkov-like radiations as well as the high energy component of the coherent particle emission via (baryonic, leptonic, fermionic) Čerenkov-like effects. Recent results on subluminal Čerenkov radiation [11] as well the result on anomalous Čerenkov rings [12] stimulated new theoretical investigations [13] leading us to the discovery that Čerenkov radiation is in fact only a component (low energy component) of a more general phenomenon called here Super-Čerenkov radiation (SCR) characterized by the Super-Čerenkov coherence condition

$$\cos \theta_{SC} = v_{xph} \cdot v_{\gamma ph} \leq 1$$

(2)

where $v_{xph}$ and $v_{\gamma ph}(\omega)$ are phase velocities of the charged particle and photon, respectively.

**Super-Čerenkov radiation.** While the complete mathematical theory (classical and quantum) can be constructed step by step as in the case of traditional Čerenkov radiation it is nevertheless appropriate at this beginning point to explain the basic principles of the Super-Čerenkov effect in qualitative manner. This should enable the reader to appreciate more fully the interpretation of earlier experiment as well as the synthesis which includes in a more general and exact form the recent results on the anomalous and subthreshold Čerenkov radiations. First, we consider that the propagation properties of particles in any medium (dielectric, nuclear, hadronic, etc.) are changed in agreement with their elastic scattering with the constituents of that medium. To be more precise, the phase velocity $v_{xph}(E_x)$ of any particle $x$ (with the total energy $E_x$ and rest mass $M_x$ in medium) is modified according to the relation (we underline again that we work in the units system $\hbar = c = 1$):

$$v_{xph}(E_x) = \frac{E_x}{\text{Re} n_x \sqrt{E_x^2 - M_x^2}}$$

(3)

while the refractive index $n_x(E_x)$ in a medium composed from the constituents “c” can be calculated in standard way by using the Foldy-Lax formula [14]

$$n_x^2(E_x) = 1 + \frac{4\pi \rho}{E_x^2 - M_x^2} \cdot C(E_x) \bar{f}_{xc \rightarrow xc}(E_x, 0^0)$$

(4)

where $\rho$ is the density of the constituents “c”, $C(E_x)$ is a coherence factor ($C(E_x) = 1$ when the medium constituents are randomly distributed), $\bar{f}_{xc \rightarrow xc}(E_x, 0^0)$ is the averaged forward xc-scattering amplitude. In order
to obtain a simple proof of the Super-Čerenkov condition we start with the kinematics of a general (in medium) decay $B_1 \rightarrow B_2 + \gamma$ where a photon $\gamma$ [with energy $\omega$, momentum $k = \omega \cdot v_1$ and refractive index $n_1$] is emitted in a medium by incident particle $B_1$ [with energy $E_1$, momentum $p_1 = \text{Re} n_1(E_1) \sqrt{E_1^2 - M^2}$, rest mass $M$, refractive index $n_1(E_1)$], that itself goes over into a final particle [with energy $E_2$, momentum $p_2 = \text{Re} n_2(E_2) \sqrt{E_2^2 - M^2}$, rest mass $M$, refractive index $n_2(E_2)$].

The super-Čerenkov relation (see Fig. 1a,b):

$$\cos \theta_{B_2 \gamma} = v_{B_2 \gamma} \cdot v_{\gamma ph} \leq 1$$

(5)
can be easy proved by using the energy-momentum conservation law for the “decay” $B_1 \rightarrow B_2 + \gamma$ to obtain

$$\cos \theta_{B_2 \gamma} = \frac{E_2}{p_2} \cdot \frac{\omega}{k} + \frac{D_1 - D_2 - D_3}{2p_2 k} \approx v_{B_2 \gamma} \cdot v_{\gamma ph}$$

(6)

Here $D_x, x \equiv B_1, B_2, \gamma$, are given by $D_x \equiv E_x^2 - p_x^2$ in medium. It is important to note that from the dual super-coherence conditions two important generalized Čerenkov-like limits follow: the photon Čerenkov limit: $v_{\gamma ph} \leq v_{B_1 \gamma}^{-1}$ (see Fig. 1a), and Coherent B-Čerenkov-like limit: $v_{B_2 \gamma} \leq v_{B_1 \gamma}^{-1}$ (see Fig. 1b).

Figure 1: Schematic description of Super-Čerenkov effect: (a) Čerenkov radiation sector: $\cos \theta_{SC} = v_{\gamma ph} \cdot v_{B_1 \gamma} \leq 1$; (b) Čerenkov-like bremsstrahlung radiation sector: $\cos \theta_{SC} = v_{B_2 \gamma} \cdot v_{B_2 \gamma} \leq 1$.

To be more specific let us consider a charged particle (e.g., $e^\pm$, $\mu^\pm$, $p$, etc.) moving in a (dielectric, nuclear or hadronic) medium and to explore the $\gamma$-coherent emission via Super-Čerenkov mechanism in that media. In the case of dielectric medium the Čerenkov radiation is a well established phenomenon widely used in physics and technology. Also, experimentally, the high energy $\gamma$-emission via coherent bremsstrahlung is well known as a channelling effect in many crystals.

So, an important problem now is if the high energy component of the Super-Čerenkov phenomenon can be identified with the coherent bremsstrahlung radiation. Hence, more experimental and theoretical investigations are needed since the usual Čerenkov radiation and Coherent bremsstrahlung radiations can be described in a unified way via the Super-Čerenkov as two-component generalized Čerenkov-like effects.

Now, the subthreshold rings observed experimentally can also be interpreted as Super-Čerenkov signatures since in the CR-sector $\theta_{SC} \equiv \theta_2 = \theta_1 \gamma$(Fig.1a) the number of photons emitted in the intervals $(x, x + dx)$, $(\omega, \omega + d\omega)$ in an nonabsorbent medium will be given by

$$\frac{d^2 N}{dx d\omega} = \alpha Z^2 B_1 \sin^2 \theta_{\gamma} = \alpha Z B_1 (1 - v_{\gamma ph}^2 v_{B_1 \gamma})$$

(7)

where $\alpha = 1/137$ is the fine structure constant and $Z_B$, the electric charge of the $B_1$ particle. Indeed, it is easy to see that the Super-Čerenkov coherence condition includes in a general and exact form the subthreshold Čerenkov-like radiation [11] since:

$$v_{\gamma ph}^2 v_{B_1 \gamma} \leq 1$$

(8) since both $\gamma$ and $P b$ after emission of high energy photon can produce secondary (anomalous) rings. The principal signatures of the Super-Čerenkov radiation for these two limiting sectors are given in Table 1.

Table 1: The SCR main predictions.

| Name | (SCR) Low \((\text{Fig.1a})\) | (SCR) High \((\text{Fig.1b})\) |
|------|-----------------|-----------------|
| 1 coherence relation | $v_{\gamma ph} \cdot v_{B_1 \gamma} \leq 1$ | $v_{\gamma ph} \cdot v_{B_1 \gamma} \leq 1$ |
| 2 coherence angle | $\cos \theta_{SC} \approx v_{\gamma ph} \cdot v_{B_1 \gamma}$ since $\theta_{SC} \equiv \theta_2 \approx \theta_{\gamma}$ | $\cos \theta_{SC} \approx v_{\gamma ph} \cdot v_{B_1 \gamma}$ since $\theta_{SC} \equiv \theta_2 \approx \theta_{\gamma}$ |
| 3 threshold velocity | $v_{x thr}(SC) = \frac{\omega_{\gamma ph}}{n_1 n_2}$ | $v_{x thr}(SC) = \frac{\omega_{\gamma ph}}{n_1 n_2}$ |
| 4 maximum emission angle | $\theta_{max}^{\gamma SC} = \arccos \frac{1}{n_1 n_2}$ | $\theta_{max}^{\gamma SC} = \arccos \frac{1}{n_1 n_2}$ |
| 5 spectrum | $N^{\gamma SC}_{\omega} = \alpha Z^2 L^2 \sin^2 \theta_{SC}$ | $N^{\gamma SC}_{\omega} = \alpha Z B_1 (1 - v_{\gamma ph}^2 v_{B_1 \gamma})$ |
| 6 polarization | $100\% (\vec{e} \cdot \vec{Q})$ | $100\% (\vec{e} \cdot \vec{Q})$ **

* L is the particle path length in the medium, ** Q is the SCR decay plane.

All these results can be proved in a quantum theory of the Super-Čerenkov effect. Here, we give only some final results. So, just as in the quantum theory of CR, the same interaction Hamiltonian $H_{int}$ with some modifications of the source fields in medium can also describe the coherent $\gamma$—emission in all sectors. Then, it is easy to see that intensity of the Super-Čerenkov radiation can
be given in the form
\[
\frac{d^2N}{dtd\omega} = \frac{\alpha Z^2}{v_1} \frac{1}{|n_{\beta_1}|^2 |n_{\beta_2}|^2 |n_{\eta}|^2} \frac{k}{dk} S \Theta (1 - \cos \theta_{SC})
\]
where the spin factor \( S \), for a two-body spin \((1/2^+ \rightarrow \gamma + 1/2^+)\) electromagnetic “decay” in medium, is given by
\[
S \equiv \frac{(E_1 + M)(E_2 + M)}{4E_1E_2} \left[ \frac{\vec{p}_1^2}{E_1 + M} + \frac{\vec{p}_2^2}{E_2 + M} + 2 \left( \frac{\vec{e}_k \cdot \vec{p}_1}{E_1 + M} - \frac{\vec{e}_k \cdot \vec{p}_2}{E_2 + M} \right) \right] \tag{8}
\]

Now, one can see that \( \Theta (1 - \cos \theta_{SC}) \) Heaviside function is 1 in two (or many) physical regions defined by the constraint: \( \cos \theta_{2\gamma} \approx v_{\gamma ph}(\omega)v_{2ph}(E_2) \leq 1 \). So, the low \( \gamma \)-energy sector is that where \( \theta_{SC} \approx \theta_{2\gamma} \approx \theta_{1\gamma} \), while the high \( \gamma \)-energy sector is where \( \theta_{SC} \approx \theta_{2\gamma} \approx \theta_{1\gamma} \). Hence, the low \( \gamma \)-energy sector can be identified as extended Čerenkov region \( |v_{\gamma ph}(\omega)| \) which includes the medium modifications on the propagation properties of charged particle, while high \( \gamma \)-energy sector can be identified as extended Čerenkov-like mechanism to the charged particle in the sense that the source spontaneously decays in a high-energy photon according to a Čerenkov-like relation: \( v_{\gamma ph}(E_1) \geq v_{2ph}(E_2) \). The spin factor \( S \) in Eqs. \((9-10)\) is defined just as in the usual quantum theory of CR with but the particle’s momenta \( P_i \), \( i = 1, 2 \) considered in medium. The vector \( \vec{e}_k \) is the photon polarization for a given photon momentum \( \vec{k} \). Now, if for a given \( \vec{k} \) one chooses two orthogonal photon spin polarization directions, corresponding to a polarization vector perpendicular and parallel to the plane given by \( \vec{p}_1 \) and \( \vec{k} \), respectively, the corresponding spin factors are given by
\[
S_\perp = \frac{(E_1 + M)(E_2 + M)}{4E_1E_2} \left( \frac{\vec{p}_1}{E_1 + M} - \frac{\vec{p}_2}{E_2 + M} \right)^2 \tag{10}
\]
and
\[
S_\parallel = v_1 \text{Re} n_1 v_2 \text{Re} n_2 \sin \theta_{1\gamma} \sin \theta_{2\gamma} \tag{11}
\]

Therefore, the relations \((8-11)\) includes in a general and unified way all the main predictions of the Super-Čerenkov radiation from which the results from Table 1 are obtained as two particular limiting cases. In the SC-Low energy sector we have 100% linear polarization \( \vec{e}_\parallel |Q \), while in the SC-High energy sector we get 100% linear polarization \( \vec{e}_\perp |Q \), where \( Q \) is the “SCR-decay” plane.

**Experimental tests of Super-Čerenkov effect.** Čerenkov radiation is extensively used in experiments for counting and identifying relativistic particles in the fields of elementary particles, nuclear physics and astrophysics. A spherical mirror focuses all photons emitted at Čerenkov angle along the particle trajectory at the same radius on the focal plane. Photon sensitive detectors placed at the focal plane detect the resulting ring images in a Ring Imaging Čerenkov (RICH) detector. So, RICH-counters are used for identifying and tracking charged particles. Čerenkov rings formed on a focal surface of the RICH provide information about the velocity and the direction of a charged particle passing the radiator. The particle’s velocity is related to the Čerenkov angle \( \theta_C \) (or to the Super-Čerenkov \( \theta_{SC} \) ) by the relation \((1) \) (or \((2) \), respectively). Hence, these angles are determined by measuring the radii of the rings detected with the RICH. In ref. \[1\] a \( C_4F_{10}Ar(75 : 25) \) filled RICH-counter read out was used for measurement of the Čerenkov ring radii. Fig. 2a shows the experimental values of the ring radii of electrons, muons, pions and kaons measured in the active area of this RICH-detector. The saturated light produced from electrons was a decisive fact to take an index of refraction \( n_\gamma = 1.00113 \) for the radiator material. The absolute values for excitation curves of electron, muon, pion and kaon, shown by dashed curves in Fig. 2a, was obtained by using this value of refractive index in formula: \( r_C(p) = (R/2) \tan \theta_C(p) \). The solid curves show the individual best fit of the experimental ring radii with eqn. \( r_{SC}(p) = (R/2) \tan \theta_{SC}(p) \) (see Table 1). For the particle refractive index we used the parametrization
\[
n_z^2(p) = 1 + a^2 / p^2, \quad v_\perp = p / \sqrt{p^2 + m^2} \tag{12}
\]

**Table 2:** The best fit parameters of experimental ring radii with the Super-Čerenkov prediction.

| Particle | Mass (MeV) | \( 10^3 \cdot a^2 \) (GeV/c) | \( \chi^2 / n_{\text{dof}} \) |
|----------|------------|---------------------------|-----------------|
| e        | 0.511      | -0.081 \pm 0.101         | 0.468           |
| µ        | 105.658    | 1.449 \pm 0.098          | 3.039           |
| π        | 139.570    | 2.593 \pm 0.167          | 0.234           |
| K        | 493.677    | 21.140 \pm 2.604         | \( < 10^{-14} \) |

We fitted all the 18 experimental data on the ring radii from ref. \[1\] with our Super-Čerenkov prediction formula
\[
r_{SC}(p/m) = \frac{R}{2} \tan \theta_{SC} = \frac{R}{2} \left[ n_\gamma^2 v_\perp^2 - 1 \right]^{1/2} = \frac{R}{2} \left[ n_\gamma^2 (p/m)^2 + (a/m)^2 \right]^{1/2} - 1 \tag{13}
\]
and we obtained the following consistent result (see Fig. 2b). The best fit parameters are as follow: \((a/m)^2 = 0.12109 \pm 0.00528\) and \( \chi^2 / n_{\text{dof}} = 1.47 \), where \( n_{\text{dof}} = 16 \) is the number of degree of freedom (dof). The \( r_{SC}(p/m) \) scaling function \((13)\) together with all experimental data on the ring radii of the electron, muon, pion and kaon, are plotted as a function of the scaling variable \((p/m)\) in Fig. 2b.

**Conclusions.** In this letter a description of a new dual coherent particle production mechanism, called Super-Čerenkov mechanism (SCR), is presented:
(i) The SČR-phenomenon, as generalized two-component Čerenkov-like effect, can be viewed as a continuous two body decays $B_1 \rightarrow \gamma + B_2$ in medium and is expected to take place when the phase velocities of the emitted photon $v_{\gamma ph}$ and that of particle source $v_{B_{1ph}}$ satisfy the dual super-coherence condition: $v_{\gamma ph} \cdot v_{B_{1ph}} \leq 1$. It is shown that the SCR includes in a general and exact form two coherent limiting phenomena: coherent Čerenkov emission (see Fig.1a) and a Čerenkov-like effect for the charged particles (see Fig. 1b, Table 1).

(ii) The results on experimental test of the super-coherence conditions are presented in Fig. 2a,b. These SČR-predictions are verified experimentally with high accuracy $\chi^2/n_{\text{dof}} = 1.47$ (see Fig.2b) by the data on the Čerenkov ring radii of electron, muon, pion and kaon, all measured with RICH detector.

(iii) We shown that the Super-Čerenkov phenomenon can explain not only subthreshold CR [11] but also the observed secondary rings (or anomalous Čerenkov radiation) at CERN SPS accelerator [12].

(iv) The influence of medium on the particle propagation properties is investigated and the refractive properties of electrons, muons, pions, in the radiator (iv) at CERN SPS accelerator [12].

The influence of medium on the particle propagation properties is investigated and the refractive properties of electrons, muons, pions, in the radiator $C_4F_{10}Ar$ are obtained. The refractive indices for this radiator are as follows: $n_\mu = 1.001449 \pm 0.000098$, $n_\pi = 1.0012593 \pm 0.000167$, $n_K = 1.0214 \pm 0.0026$, $n_p = 1.1066 \pm 0.046$. So, we proved that the refractive indices of the particles in medium are also very important for the RICH detectors, especially at low and intermediate energies.

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