A new mechanism of neutrino mass generation in the NMSSM with broken lepton number

Asmaa Abada $^{1,*}$, Gautam Bhattacharyya $^{2,†}$, Grégory Moreau $^{3,‡}$

$^1$Laboratoire de Physique Théorique, Université de Paris-sud XI
Bâtiment 210, 91405 Orsay Cedex, France
$^2$Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India
$^3$Centro de Física Teórica de Partículas, Departamento de Física
Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

Abstract

In the Minimal Supersymmetric Standard Model with bilinear $R$-parity violation, only one neutrino eigenstate acquires a mass at tree level, consequently experimental data on neutrinos cannot be accommodated at tree level. We show that in the Next-to-Minimal extension, where a gauge singlet superfield is added to primarily address the so-called $\mu$-problem, it is possible to generate two massive neutrino states at tree level. Hence, the global three-flavour neutrino data can be reproduced at tree level, without appealing to loop dynamics which is vulnerable to model-dependent uncertainties. We give analytical expressions for the neutrino mass eigenvalues and present examples of realistic parameter choices.

PACS Nos: 11.30.Fs, 12.60.Jv, 14.60.Pq, 14.80.Ly
Keywords: Neutrino mass, $R$-parity violation, NMSSM

I Introduction

In the Minimal Supersymmetric Standard Model (MSSM) with bilinear interactions in the superpotential explicitly violating the $R$-parity symmetry [1], a neutrino Majorana mass can be generated. Nevertheless, the rank 1 nature of the neutrino mass matrix suggests that only one eigenstate becomes massive at tree level, whereas the neutrino oscillation data require at least two non-zero mass eigenvalues [2]. The bilinear $R$-parity violating ($R_p$) soft masses induce one more non-zero eigenvalue, but only at one-loop order. What happens if one considers the next-to-minimal version of the MSSM, called ‘NMSSM’ [3], in the presence of bilinear $R_p$ terms in the superpotential? Here, the particle content is extended by one gauge singlet superfield. Our main result in this work is that two non-degenerate massive neutrino states now emerge at tree level. The upshot is that one can now reproduce the neutrino oscillation data with the superpotential parameters and gaugino masses just

*E-mail address: abada@th.u-psud.fr
†E-mail address: gb@theory.saha.ernet.in
‡E-mail address: greg@cftp.ist.utl.pt
from *tree level* physics. On the other hand, in the usual MSSM with bilinear $R_p$ terms, the generation of the second neutrino mass eigenvalue relies on the soft supersymmetry breaking scalar masses which in turn bring more uncertainties from the supersymmetry breaking mechanism; furthermore, uncertainties from loop dynamics creep in too.

An increasingly important virtue of the NMSSM [3] (see [4] for phenomenological studies) is that it ameliorates the ‘little hierarchy’ problem originating from the requirement of large soft supersymmetry breaking scalar masses compared to the electroweak scale (in order to sufficiently push the lightest Higgs mass beyond the LEP limit). The NMSSM also provides a solution to the so-called $\mu$-problem by arranging the vacuum expectation value (vev) of the gauge singlet scalar of the order of the supersymmetry breaking scale, so that the $\mu$ parameter turns out to be at the electroweak scale.

A recent paper [5], in the context of NMSSM with $R_p$ couplings, deals with the generation of neutrino masses where two eigenvalues arise at loop level. In another recent analysis [6], it has been shown that the NMSSM with bilinear $R_p$ terms offers a possible mechanism of neutrino mass suppression, thus significantly reducing the hierarchy between $\mu_i$ and $\mu$ (defined below). Besides, an alternative extension of the MSSM explicitly breaking the $R$-parity has been proposed in order to simultaneously address the $\mu$-problem and the issue of small neutrino masses [7].

In Section II, we present the superpotential of the model we consider. In Section III, we discuss the effective neutrino mass matrix. We present our numerical results in Section IV. Finally, we conclude in Section V.

### II Superpotential

The NMSSM superpotential contains two dimensionless couplings $\lambda$ and $\kappa$ in addition to the usual Yukawa couplings:

$$W_{\text{NMSSM}} = Y_{ij}^u Q_i H_u U^c_j + Y_{ij}^d Q_i H_d D^c_j + Y_{ij}^e L_i H_d E^c_j + \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

where $Y_{ij}^{u,d,e}$ are the Yukawa coupling constants ($i, j, k$ are family indexes), and $Q_i, L_i, U^c_i, D^c_i, E^c_i, H_u, H_d, S$ respectively are the superfields for the quark doublets, lepton doublets, up-type anti-quarks, down-type anti-quarks, anti-leptons, up Higgs, down Higgs, extra singlet under the standard model gauge group. An effective $\mu$ term, given by $\lambda(s) H_u H_d$, is generated via the vev of the scalar component $s$ of the singlet superfield $S$.

We now take note that in supersymmetric theories there is no deep underlying theoretical principle for the imposition of $R$-parity as a symmetry [8]. However, there exist strong constraints on the $R_p$ couplings coming from various phenomenological considerations [9] [10]. Limits on neutrino masses and mixings have also been translated into tight upper bounds for $R_p$ couplings [11].

In the present paper, we consider a generic NMSSM superpotential containing both the bilinear and trilinear $R_p$ terms:

$$W = W_{\text{NMSSM}} + \mu_i L_i H_u + \lambda_i S L_i H_u,$$

where $\mu_i$ ($\lambda_i$) are the dimension-one (dimensionless) $R_p$ parameters. Actually, the contribution of trilinear term $\lambda_i S L_i H_u$ was studied in Ref. [5]. Admittedly, the most generic NMSSM superpotential also contains the other renormalizable trilinear $R_p$ interactions, namely, $\lambda_{ijk} L_i L_j E^c_k$, $\lambda'_{ijk} L_i Q_j D^c_k$ and $\lambda''_{ijk} U^c_i D^c_j D^c_k$, which are not relevant so long as we stick to tree level neutrino mass matrix.
Normally, in the NMSSM, only trilinear couplings with dimensionless parameters (like $\lambda$ and $\kappa$) are kept in the superpotential, while dimensional parameters (like $\mu$) are generated from the vev $\langle s \rangle$. In this paper, the $R_p$ NMSSM superpotential (2) is assumed to arise in either one of the following three possible scenarios:

1. All possible renormalizable terms are included in the superpotential. Then both bilinear ($\mu_0 H_u H_d$, $\mu_i L_i H_u$) and trilinear ($\lambda S H_d H_u$, $\lambda_i S L_i H_u$) terms are admitted. However, even if one may start with a term $\mu_0 H_u H_d$, it can be rotated away by a redefinition of fields through a rotation on $L_\alpha = (H_d, L_\alpha) \ [\alpha = 0, \ldots, 3]$, since $H_d$ and $L_i$ have the same gauge quantum numbers. There is no reason why this redefinition would remove also the $\lambda_\alpha$ terms ($\lambda_\alpha = (\lambda, \lambda_i)$), since the corresponding $4 \times 4$ rotation matrix depends on the $\mu_\alpha$ parameters (the generic case is considered here, where $\mu_\alpha$ and $\lambda_\alpha$ are not proportional). The coefficient $\mu_s$ of the $S^2$ term is assumed to be zero which can be considered as a possible natural value for a superpotential parameter. It should be noted that in the standard NMSSM with conserved $R$-parity there is an accidental $Z_3$ discrete symmetry whose spontaneous breaking causes the domain wall problem. In our version of the NMSSM, the $R_p$ bilinear $L_i H_u$ term explicitly breaks that $Z_3$ symmetry. In this scenario, we simply assume the existence of the dimensionful $\mu_i$ terms, which as we will see later will be constrained from neutrino data. But we do not advance any theoretical reason as to why $\mu_i$ would be small.

2. Our second scenario is based on the ’t Hooft criteria of naturalness: the parameters $\mu_i$, $\mu_0$ and $\mu_s$ are naturally small if the symmetry of the theory increases as these parameters are set to zero. For instance, one can assume that somehow a weak breaking (compared to the electroweak scale $Q_{EW}$) of some symmetry (like e.g. a U(1) symmetry forbidding the bilinear terms) generates the bilinear terms in the superpotential associated to $\mu_i$, $\mu_0$, $\mu_s \ll Q_{EW}$. This small breaking would allow to address the $\mu$-problem. Indeed, the main contribution to the dimension-one coefficient of $H_u H_d$ here comes from $\lambda(s)$, as $\mu_0 \ll \mu = \lambda(s) \sim Q_{EW}$. The weak breaking of the symmetry is also responsible for the smallness of $R_p$ couplings and neutrino masses, since $\mu_i \ll \mu$. Thus, in such a scenario, the treatment of the $mu$-naturalness (à la NMSSM) and of the neutrino masses (à la $R_p$) are nicely connected via the weak breaking of a common symmetry. Admittedly, we do not provide any specific realisation of this weak breaking. We only hint at such a possibility that the bilinear $\mu_i, \mu_0, \mu_s$ couplings may arise from powers of some small spurion vev ($\ll Q_{EW}$).

3. Finally, we propose a scenario where the trilinear $\lambda_i$ terms in superpotential (2) are not present. This scenario relies on the $Z_3$ symmetry, where the chiral superfields transform by $\exp(i2\pi q/3)$, with the following charge assignments: $q = 0$ for $U^c$, $D^c$, $E^c$; $q = 1$ for $S$, $H_u$, $H_d$; and $q = 2$ for $Q$, $L$. Such a symmetry allows all couplings in Eqs. (11) and (2) except $SL_i H_u$. The other terms $S^2$ and $H_u H_d$ are also eliminated by this symmetry. A spontaneous breaking of this $Z_3$ symmetry admittedly creates the domain wall problem, as happens in the standard NMSSM with $R$-parity. In this scenario, the $\mu$ term is created from the vev of $S$, but the $\mu_i$ terms are present in the superpotential from the beginning only to be constrained by neutrino data.

### III Neutrino mass matrix

**Neutralino mass matrix:** Within our framework, the neutralino mass terms read as,

$$\mathcal{L}_{\chi^0}^{m} = -\frac{1}{2} \Psi^\dagger \mathcal{M}_{\chi^0} \Psi^0 + \text{h.c.}$$

(3)
in the basis $\Psi^{0T} = (\tilde{F}^0, \tilde{W}^0, \tilde{H}^0_u, \tilde{H}^0_d, \tilde{s}, \nu_i)^T$, where $\tilde{H}^0_{u,d}$ (\tilde{s}) are the fermionic components of the superfields $H^0_{u,d}$ ($S$) and $\nu_i$ [$i = 1, 2, 3$] denote the neutrinos. In Eq. (3), the neutralino mass matrix is given, in a generic basis (where $\langle \tilde{\nu}_i \rangle \equiv v_i \neq 0$, $\mu_i \neq 0$ and $\lambda_i \neq 0$), by

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} \mathcal{M}_{\text{NMSSM}} & \xi_{\bar{R}_p}^T \\ \xi_{\bar{R}_p} & 0_{3 \times 3} \end{pmatrix},$$

(4)

where $\mathcal{M}_{\text{NMSSM}}$ is the neutralino mass matrix corresponding to the NMSSM. While writing the latter mass matrix, we assume $v_i \ll v_{u,d}$ (the exact expression of $M_Z$ being given by $v^2 = v_u^2 + v_d^2 + \sum_{i=1}^3 v_i^2 = 2c_{\theta_\mu}^2 M_Z^2 / g'^2 \simeq (246 / \sqrt{2} \text{ GeV})^2$). Also, we use $s$ and $c$ to stand for sine and cosine, respectively.

$$\mathcal{M}_{\text{NMSSM}} = \begin{pmatrix} M_1 & 0 & -M_Z s_\theta \beta c_\beta & M_Z s_\theta \beta s_\beta & 0 \\ 0 & M_2 & M_Z c_\theta c_\beta & -M_Z c_\theta s_\beta & 0 \\ -M_Z s_\theta c_\beta & M_Z c_\theta c_\beta & 0 & -\mu & -\lambda v_u \\ M_Z c_\theta c_\beta & -M_Z c_\theta s_\beta & -\mu & 0 & -\lambda v_d + \sum_{i=1}^3 \lambda_i v_i \\ 0 & 0 & -\lambda v_u & -\lambda v_d + \sum_{i=1}^3 \lambda_i v_i & 2\kappa \langle s \rangle \end{pmatrix}.$$  \hspace{1cm} (5)

Above, $M_1$ ($M_2$) is the soft supersymmetry breaking mass of the bino (wino), $\tan \beta = v_u / v_d = \langle h^0_u \rangle / \langle h^0_d \rangle$, and $\mu = \lambda \langle s \rangle$. We assume for simplicity that $\lambda, \kappa$ and the soft supersymmetry breaking parameters are all real.

In Eq. (4), $\xi_{\bar{R}_p}$ is the $\bar{R}_p$ part of the matrix mixing neutrinos and neutralinos:

$$\xi_{\bar{R}_p} = \begin{pmatrix} -\frac{g_\mu}{\sqrt{2}} & \frac{v_\mu}{\sqrt{2}} & 0 & \mu_1 + \lambda_1 \langle s \rangle & \lambda_1 v_u \\ -\frac{g_\mu}{\sqrt{2}} & \frac{v_\mu}{\sqrt{2}} & 0 & \mu_2 + \lambda_2 \langle s \rangle & \lambda_2 v_u \\ -\frac{g_\mu}{\sqrt{2}} & \frac{v_\mu}{\sqrt{2}} & 0 & \mu_3 + \lambda_3 \langle s \rangle & \lambda_3 v_u \end{pmatrix}. \hspace{1cm} (6)$$

g and $g'$ being the SU(2) and U(1) gauge couplings.

**Effective neutrino mass matrix:** We restrict ourselves to the situation where $v_i / v_{u,d} \ll 1$ (as before), $|\mu_i / \mu| \ll 1$ and $|\lambda_i / \lambda| \ll 1$ so that (i) no considerable modifications of the NMSSM scalar potential are induced by the additional bilinear and trilinear term in superpotential (2), (ii) the neutrino-neutralino mixing is suppressed, leading to sufficiently small neutrino masses as shown later, and (iii) the effective neutrino mass matrix can be written to a good approximation by the following see-saw type structure,

$$m_\nu = -\xi_{\bar{R}_p} \mathcal{M}^{-1}_{\text{NMSSM}} \xi_{\bar{R}_p}^T.$$  \hspace{1cm} (7)

From Eqs. (5), (6) and (7), we deduce the analytical expression of the effective Majorana neutrino mass matrix:

$$m_{\nu_{ij}} = \frac{1}{|\mathcal{M}_{\text{NMSSM}}|} [\mu_i \mu_j \mathcal{F} + (\mu_i \Lambda_j + \mu_j \Lambda_i) \mathcal{G} + \Lambda_i \Lambda_j \mathcal{H}], \hspace{1cm} (8)$$

where $|\mathcal{M}_{\text{NMSSM}}|$ is the determinant of matrix (5), $\Lambda_i = \langle s \rangle (\lambda_i + \lambda_{\tilde{\nu}_d})$ and:

$$\mathcal{F} = \lambda^2 v^2 M_1 M_2 + \mathcal{X},$$

$$\mathcal{G} = \mathcal{X} + (\lambda v_d - \cos^2 \beta \sum_{i=1}^3 \lambda_i v_i) \mathcal{Y}, \hspace{1cm} (9)$$

$$\mathcal{H} = \mathcal{X} + 2 \cos^2 \beta (\lambda v_d - \sum_{i=1}^3 \lambda_i v_i) \mathcal{Y},$$

where $\mathcal{X} = M_1 M_2 + \mathcal{X},$
Neutrino mass eigenvalues at tree level: The eigenvalues of the effective neutrino mass matrix contributions are not so crucial for generating a viable neutrino mass spectrum. Still, they contribute in our scenario, but now that we have two tree-level masses, those loop-suppressed (for a series of analysis in a three-flavour framework, see [1,3]). The Grossman-Haber loops would grossman-Haber diagrams [12] which proceed through sleptons-Higgs and neutrino-neutralino mixings $\mu$ contributions originate at tree level, the $\mu_iB_j$ and $B_iB_j$ contributions arise at one-loop order through Grossman-Haber diagrams [12] which proceed through slepton-Higgs and neutrino-neutralino mixings (for a series of analysis in a three-flavour framework, see [13]). The Grossman-Haber loops would still contribute in our scenario, but now that we have two tree level masses, those loop-suppressed contributions are not so crucial for generating a viable neutrino mass spectrum.

Neutrino mass eigenvalues at tree level: The eigenvalues of the effective neutrino mass matrix (8) turn out to be $\{0, m_{\nu}^-, m_{\nu}^+\}$ with,

$$m_{\nu}^\pm = \frac{1}{2 |M_{\text{NMSSM}}|} \left[ \left( \sum_{i=1}^{3} \mu_i^2 \right) F + \left( \sum_{i=1}^{3} \Lambda_i \right) H + 2 \left( \sum_{i=1}^{3} \mu_i \Lambda_i \right) G \right]$$

$$\pm \left[ \left( \sum_{i=1}^{3} \mu_i^2 \right) \left( \sum_{i=1}^{3} \Lambda_i \right) - \left( \sum_{i=1}^{3} \mu_i \Lambda_i \right)^2 \right] I + \left[ \left( \sum_{i=1}^{3} \mu_i^2 \right) F + \left( \sum_{i=1}^{3} \Lambda_i \right) H + 2 \left( \sum_{i=1}^{3} \mu_i \Lambda_i \right) G \right]^2 \right]^{\frac{1}{2}},$$

(10)

where $F, G, H$ are defined in Eq. (9), and $I = 4(G^2 - FH)$.

Note that for either all $\mu_i = 0$ or all $\Lambda_i = 0$, the eigenvalue $m_{\nu}^-$ vanishes as expected since in this limit we recover the rank 1 form. An inspection of the form of Eq. (10) reveals that the coefficient of $I$ can be written as $\sum_{i \neq j} (\mu_i \Lambda_j - \mu_j \Lambda_i)^2$, which indicates that the misalignment between $\mu_i$ and $\Lambda_i$ is crucial in creating a non-vanishing $m_{\nu}^-$. Therefore, the condition for generating two non-vanishing and non-degenerate eigenvalues is to ensure $\mu_i \neq 0$ and $\Lambda_i \neq 0$ simultaneously. In other words, to achieve two non-zero eigenvalues, $\mu_i$ has to be non-zero always, but we can go to a basis of $L_\alpha$ fields where $v_i = 0$ (then generally $\lambda_i \neq 0$), or to the other extreme where $\lambda_i = 0$ (but $v_i \neq 0$), or any other basis in between basically maintaining $\Lambda_i \neq 0$.

On the contrary, if $\mu_i = 0$, only one neutrino eigenstate gets a mass different from zero, as was also found by the authors of Ref. [5]. But all the scenarios we considered in this paper, according to the above arguments, will yield two non-zero neutrino masses at tree level. We mention that within the first scenario, a rotation on the $L_\alpha$ fields has already been performed, whereas in the third one, no rotation is possible.

In Figures (11) and (2), we present the Feynman diagrams contributing to the Majorana neutrino mass...
Thus the present model predicts a hierarchical neutrino mass spectrum at tree level. This hierarchical pattern could be modified by the loop level contributions to neutrino masses. At least, the massless state acquires a mass once the loop contributions are considered.

It is not our aim to give detailed numerical fits in this short paper. We would just like to numerically demonstrate that our scenario can reproduce the neutrino data from the tree level neutrino mass matrix, with a choice of NMSSM parameters that corroborate the $\mu$-naturalness.

As an example, we take the following NMSSM parameters:

$$\lambda = 0.4, \quad \kappa = 0.3, \quad \mu = -200 \text{ GeV}, \quad \tan \beta = 30, \quad M_1 = 350 \text{ GeV}, \quad M_2 = 500 \text{ GeV}. \quad (11)$$

These parameters satisfy the general NMSSM constraints described below. These constraints are not expected to be significantly modified by the presence of $R_p$ interactions in superpotential \[2\] as we work under the assumption $|\mu_i/\mu| \ll 1$ and $|\lambda_i/\lambda| \ll 1$.

The $\mu$-naturalness forces one to restrict to $\langle s \rangle \lesssim 10 \text{ TeV}$, which translates into the condition $|\mu| [\text{GeV}] \times 10^{-4} \lesssim \lambda$. Furthermore, the absence of Landau singularities, for $\lambda, \kappa$, the top and bottom Yukawa coupling constants below the GUT energy scale, imposes [4] the typical bounds: $\lambda \lesssim 0.75, |\kappa| \lesssim 0.65$.
and $1.7 \lesssim \tan \beta \lesssim 54$. Finally, the LEP bound on the lightest chargino mass, namely $m_{\tilde{\chi}_1^+} > 103.5 \text{ GeV}$ [14], translates into $|\mu| \gtrsim 100 \text{ GeV}$.

Together with the values in Eq. (11), we take the following $R_p$ effective couplings,

$$
\mu_1 = 1 \cdot 10^{-4} \text{ GeV}, \quad \mu_2 = 2 \cdot 10^{-4} \text{ GeV}, \quad \mu_3 = 2 \cdot 10^{-4} \text{ GeV}
$$

$$
\Lambda_1/\langle s \rangle = 1.9 \cdot 10^{-5}, \quad \Lambda_2/\langle s \rangle = 1.4 \cdot 10^{-5}, \quad \Lambda_3/\langle s \rangle = 1.5 \cdot 10^{-5}.
$$

This set of parameters yield the following three neutrino mass eigenvalues at tree level:

$$
m_{\nu_1} = 0, \quad m_{\nu_2} = 0.0095 \text{ eV}, \quad m_{\nu_3} = 0.058 \text{ eV}.
$$

These values are in agreement with the three-flavour analyzes including results from solar, atmospheric, reactor and accelerator oscillation experiments which lead to (4$\sigma$ level): $6.8 \leq \Delta m^2_{21} \leq 9.3 \ [10^{-5} \text{eV}^2]$ and $1.1 \leq \Delta m^2_{31} \leq 3.7 \ [10^{-3} \text{eV}^2]$ [2]. Besides, the neutrino mass eigenvalues in (13) satisfy the bound extracted from WMAP and 2dFGRS galaxy survey (depending on cosmological priors):

$$\sum_{i=1}^{3} m_{\nu_i} \lesssim 0.7 \text{ eV}$$

Finally, these eigenvalues are perfectly compatible with the limits extracted from the tritium beta decay experiments (95% C.L.): $m_\beta \leq 2.2 \text{ eV}$ [Mainz] and $m_\beta \leq 2.5 \text{ eV}$ [Troitsk] [16], this effective mass being defined as $m^2_\beta = \sum_{i=1}^{3} |U_{ei}|^2 m^2_{\nu_i}$ where $U_{ei}$ is the leptonic mixing matrix.

Although we have chosen a particular set of input parameters for illustration, solutions exist over a wide range of parameter space. More realistic estimates can be obtained by switching on the soft scalar terms $B_i \tilde{\ell}_i h_u + \text{h.c.}$ plus the superpotential terms $\lambda_{ijk} L_i L_j E^c_k$ and $\lambda'_{ijk} L_i Q_j D^c_k$. All these terms contribute to the neutrino mass matrix at one-loop order. A combined fit of all these parameters is beyond the scope of this paper.
V Conclusion

In the NMSSM, which is a gauge singlet extension of the MSSM addressing the \(\mu\)-naturalness, two non-vanishing neutrino mass eigenvalues can arise at tree level when the lepton number violating bilinear terms \(\mu_i L_i H_u\) are present. One can then explain the neutrino oscillation data without essentially depending on the loop-generated masses which otherwise bring in more uncertainties from unknown dynamics. This result is in contrast with any other supersymmetric \(\mathcal{R}_p\) scenario, as those scenarios do not generate more than one massive neutrino eigenstate at tree level (except the scenario proposed in [7] where 3 right-handed neutrinos are added to the field content).

Acknowledgments: G. M. thanks U. Ellwanger for fruitful discussions. A. A. acknowledges the Agence Nationale Pour la Recherche (ANR), project NEUPAC. G. B. acknowledges hospitality at CFTP, Instituto Superior Técnico, Lisbon, when this work was initiated, and ICTP, Trieste, during the final phase of the work. G. M. acknowledges support from a Marie Curie Intra-European Fellowships (under contract MEIF-CT-2004-514138), within the 6th European Community Framework Program.

References

[1] G. Farrar and P. Fayet, Phys. Lett. B 76, 575 (1978); S. Weinberg, Phys. Rev. D 26, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. B 197, 533 (1982); C. Aulakh and R. Mohapatra, Phys. Lett. B 119, 136 (1982).

[2] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6, 122 (2004).

[3] H. P. Nilles et al., Phys. Lett. B 120, 346 (1983); J. M. Frère et al., Nucl. Phys. B 222, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. B 237, 307 (1984); J. R. Ellis et al., Phys. Rev. D 39, 844 (1989); M. Drees, Int. J. Mod. Phys. A 4, 3635 (1989); U. Ellwanger et al., Phys. Lett. B 315, 331 (1993); S. F. King and P. L. White, Phys. Rev. D 52, 4183 (1995); F. Franke and H. Fraas, Int. J. Mod. Phys. A 12, 479 (1997); M. Bastero-Gil et al., Phys. Lett. B 489, 359 (2000).

[4] P.N. Pandita, Phys. Lett. B 318, 338 (1993); P.N. Pandita, Z. Phys. C 59, 575 (1993); U. Ellwanger et al., Z. Phys. C 67, 665 (1995); U. Ellwanger et al., Nucl. Phys. B 492, 21 (1997); U. Ellwanger and C. Hugonie, Eur. Phys. J. C 13, 681 (2000); A. Dedes et al., Phys. Rev. D 63, 055009 (2001); U. Ellwanger et al., arXiv:hep-ph/0111179; D. G. Cerdeno et al., JHEP 0412, 048 (2004); U. Ellwanger et al., JHEP 0507, 041 (2005); U. Ellwanger and C. Hugonie, Phys. Lett. B 623, 93 (2005); G. Bélanger et al., JCAP 0509, 001 (2005); J. F. Gunion, D. Hooper and B. McElrath, Phys. Rev. D 73, 015011 (2006).

[5] M. Chemtob and P. N. Pandita, Phys. Rev. D 73, 055012 (2006).

[6] A. Abada and G. Moreau, JHEP 0608, 044 (2006).

[7] D. E. Lopez-Fogliani and C. Muñoz, Phys. Rev. Lett. 97, 041801 (2006).

[8] R. Barbier et al., Phys. Rep. 420, 1 (2005).
[9] G. Bhattacharyya, *Nucl. Phys. Proc. Suppl.* A 52, 83 (1997), arXiv:hep-ph/9608415; G. Bhattacharyya, Invited talk presented at ‘Beyond the Desert’, Castle Ringberg, Tegernsee, Germany, 8-14 June 1997, arXiv:hep-ph/9709395; H. K. Dreiner, “Perspectives on Supersymmetry”, Edited by G. L. Kane, World Scientific, arXiv:hep-ph/9707435; R. Barbier et al., arXiv:hep-ph/9810232; B. C. Allanach, A. Dedes and H. K. Dreiner, *Phys. Rev.* D 60, 075014 (1999); *Phys. Rev.* D 69, 115002 (2004); M. Chemtob, *Prog. Part. Nucl. Phys.* 54, 71 (2005).

[10] G. Bhattacharyya et al., *Mod. Phys. Lett.* A 10, 1583 (1995); G. Bhattacharyya and D. Choudhury, *Mod. Phys. Lett.* A 10, 1699 (1995); G. Bhattacharyya and A. Raychaudhuri, *Phys. Rev.* D 57, 3837 (1998); G. Bhattacharyya and A. Datta, *Phys. Rev. Lett.* 83, 2300 (1999); S. Raychaudhuri, arXiv:hep-ph/9905576.

[11] R. Hempfling, *Nucl. Phys.* B 478, 3 (1996); E. J. Chun and S. K. Kang, *Phys. Rev.* D 61, 075012 (2000); M. Hirsch et al., *Phys. Rev.* D 62, 113008 (2000); J. C. Romao, arXiv:hep-ph/0510411; S. Rakshit, G. Bhattacharyya and A. Raychaudhuri, *Phys. Rev.* D 59, 091701 (1999); G. Bhattacharyya, H. V. Klapdor-Kleingrothaus and H. Pas, *Phys. Lett.* B 463, 77 (1999); S. Rakshit, *Mod. Phys. Lett.* A 19, 2239 (2004); Y. Grossman and S. Rakshit, *Phys. Rev.* D 69, 093002 (2004).

[12] Y. Grossman and H. E. Haber, *Phys. Rev. Lett.* 78, 3438 (1997); *Phys. Rev.* D 59, 093008 (1999); *Phys. Rev.* D 63, 075011 (2001).

[13] S. Davidson and M. Losada, *JHEP* 0005, 021 (2000); A. Abada and M. Losada, *Nucl. Phys.* B 585, 45 (2000); A. Abada and M. Losada, *Phys. Lett.* B 492, 310 (2000); A. Abada, S. Davidson and M. Losada, *Phys. Rev.* D 65, 075010 (2002); S. Davidson and M. Losada, *Phys. Rev.* D 66, 075025 (2002); A. Abada, G. Bhattacharyya and M. Losada, *Phys. Rev.* D 66, 071701 (2002), see also arXiv:hep-ph/0305330.

[14] LEP SUSY Working Group, LEPSUSYWG Note/02-04.1.

[15] S. Hannestad, JCAP 0305, 004 (2003); K. Ichikawa, M. Fukugita and M. Kawasaki, *Phys. Rev.* D 71, 043001 (2005); J. Lesgourgues and S. Pastor, to be published in Phys. Rep., arXiv:astro-ph/0603494.

[16] Y. Farzan, O. L. G. Peres and A. Yu. Smirnov, *Nucl. Phys.* B 612, 59 (2001); J. Bonn et al., *Nucl. Phys. Proc. Suppl.* 91, 273 (2001); V. M. Lobashev et al., *Phys. Lett.* B 460, 227 (1999); *Nucl. Phys. Proc. Suppl.* 77, 327 (1999); 91, 280 (2001).