On the adequacy of the adaptive control model and the use of L-distribution of the rotary-conveyor milking process on "Karusel" type installations

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Abstract. The article reflects the degree of adequacy of the use of G-distribution and the constructed mathematical model of adaptive control, which allows increasing the productivity of conveyor-ring milking machines of the "Karusel" type. This paper discusses the results of research on the process of milking a herd of cows on a carousel-type milking machine. Based on the methods of variational statistics the influence of the parameters of the statistical distribution of the duration of milking performance of a milking machine is studied, a theoretical analysis of the timeline of its work is carried out. Models and algorithms for adaptive speed control of milking pipeline with the use of specialized software, including the system of analytical calculations Maple are developed. This software allows you to perform analytical transformations of large amounts of information and display the results of calculations in tabular graphical form and in animation mode.

1. Introduction

Scientist sand constructor shave been searching for theorganizingandtechnologicaldecisionsformilk- ingoperatorsformanyyears [1, 2].

In this case, you can study the information system-a herd of cows, using the process of mathematical modeling. This process is based on the following stages: full - scale experiment, calculation scheme of the milking process, mathematical model in the form of a system of equations (algebraic or differential), algorithm and software, identification of the structure and parameters of the model. The final stage is a computational experiment (analysis of the results of calculation on a computer). The methodology of mathematical modeling is widely recognized and covers many areas from the development of complex technical systems to the analysis of complex economic, biological, social and a number of other systems and processes. Mathematical modeling is developed in the bowels of the fundamental Sciences: mechanics, physics, biology, which are distinguished by the highest level of theoretical research. The main problem of the mathematical modeling process is the development of adequate mathematical models that correspond to the behavior of a real process or object. When servicing animals with different milking times, all companies offer almost identical ways to solve this problem. Analysis of the results of these studies has shown the need for extensive use of mathematical statistics and probability theory methods for making decisions about setting the optimal speed of the conveyor,
which guarantees high milking productivity on installations of the "Karusel" type. Some of the results of these studies in this direction are given in [3, 4].

2. Problem of the research

The key issue in developing of adaptive control model is determining the angular speed of platform rotation, which ensures an adequate milking cycle for each animal. One of the options to start creating an adaptive control algorithm is discussed below.

We get different options by assigning a platform turnover time equal to or greater than the mathematical expectation of milking time. In the latter case, it is necessary to stop the conveyor. In this case, the total downtime of the milking machine will be determined by the integral:

\[ T_{\Sigma_{idl}} = \int_{m}^{\max} f(x)dx, \]

or taking into account the characteristics of the distribution according to the logarithmic-normal law:

\[ f(x, \mu, \sigma, \xi) = \frac{1}{\sigma(x-\xi)^{1/2} \sqrt{2\pi}} \exp \left\{ -\frac{1}{2\sigma^2} \left[ \ln(x-\xi) - \mu \right]^2 \right\}, \]

with \( x \geq \xi \), \( -\infty < \xi < \infty \), \( \sigma > 0 \).

It can be written:

\[ T_{\Sigma_{idl}} = \int_{m}^{\max} \frac{1}{\sigma(x-\xi)^{1/2} \sqrt{2\pi}} \exp \left\{ -\frac{1}{2\sigma^2} \left[ \ln(x-\xi) - \mu \right]^2 \right\} dx. \]

Consistently solving the problem of reduction \( T_{\Sigma_{idl}} \rightarrow 0 \). It can be concluded that you should assign \( \tau_m \rightarrow t_{m,\max} \). In this case, to maintain a high rate of operation of the milking machine or a constant flow rate:

\[ r = \frac{\tau_m}{N_{st}}, \]

where \( r \) – flow rhythm, \( N_{st} \) – number of “Karusel” machines, we need to increase the number of machine-places \( N_{st} \).

In general, this relationship is easily determined:

\[ 2\pi R_{st} = N_{st} \cdot l_{st}, \]

where \( l_{st} \) – machine length.

In the case of a serial arrangement of platform – \( l_{st}^S \approx l_{c} \) – by the type «Tandem», in the case of oblique location by the type «Elotchka» – \( l_{st}^E \approx l_{c} \cdot \cos \alpha \), and in the case of radial location – it will be \( l_{st} = b \).

In the latter case, we have the maximum density of the location of animals and the rhythm of the flow. Given the above, it is obvious:

\[ r_{\min}, r_{\max} = \frac{\tau_m \cdot l_{st}[l_{c} \cdot \cos \alpha; b]}{2\pi R_{st}}. \]

Thus, a necessary condition for maintaining a constant rhythm \( r \) is the desire of the parameter \( N_{st} \rightarrow \max \).
Analysing an expression

\[ N_\mu = \frac{2\pi R_{\mu t}}{I_\mu} \]  

It is obvious that if it is needed to save the value \( R_{\mu t} \), we must decrease the value \( I_\mu \), then we’ll get the value \( N_\mu \) with the small value of platform radius and relative value \( r – \) flow rhythm.

From these three methods of animal distribution the biggest value of \( r – \) flow rhythm will be with radial animal distribution \( \mu_\mu = b \).

The main idea of this paper is to increase the productivity of the milking process (reducing the total time of milking \( F \)) of herd from \( N \) cows. In order to analyze in more detail the physiological condition of each cow from herd (taking into account the computer equipment of modern rotary conveyor milking system) it is necessary to take into account the characteristics of the milking process for the previous one or several milks.

3. Materials and methods

The main parameters of this research are: \( N \) – total cow number in a herd; \( t_{\mu i} \) – milking time of i-cow, s; \( t_{p,\mu} \) – time of preparing operations for machine milking, s; \( t_{f,\mu} \) – time of finishing operations for machine milking, s; \( n \) – number of machine places on milking platform, unit; \( n_\mu \) – the number of machine places that are not used in operation, intended for entering and exiting the cow from the platform, unit; \( N_{\mu,\mu} \) – number of slow-moving cows in a herd, heads; \( \alpha \) – coefficient that takes into account the excess of the cow’s milking time compared to the average value for the herd, \( \alpha \geq 2 \), b/r; \( t_{\mu,\mu} \) – time of platform stopping for entering and exiting the cow, s; \( t_{\mu,\mu,\mu} \) – maximum value of milking time of the cows from a herd, s; \( t_{\mu,\min} \) – minimum value of milking time of the cows from a herd, s; \( t_{\mu,\av} \) – average value of milking time of the cows from a herd, s; \( \omega_{\mu,\mu} \) – speed of platform moving, rad/s; \( \beta \) – coefficient characterizing change of angular speed, b/r.

The function of total milking time \( \Phi \) depends on the above-mentioned parameters: \( \Phi = \Phi(x_1, x_2, x_3, ..., x_{18}) \). This function should adequately reflect the process of milking a herd of cows for the type of milking machines under consideration and the process under study.

To get an adequate expression for a function \( \Phi (x_1, ..., x_{18}) \) let’s consider some statistical methods for processing sample data.

In the beginning, the stages of the algorithm for calculating the K. Pearson consent criterion for analyzing arrays of milking speed and time using statistical processing methods are described. The paper analyzes hypotheses about the distribution of these characteristics: L-distribution, normal distribution, and logarithmically normal distribution. From the physical formulation of the problem, it is natural to assume that the milking speed \( V \) (g / s) and the milking time \( T \) (s) are absolutely continuous non-negative random variables. In the future, the paper uses the main facts on the application of the criterion of consent of K. Pearson, given in [5, 6].

For calculations, we used the L-distribution of a random variable for which there is a distribution density (absolutely continuous). These dependencies are well known from the scientific and educational literature.

To test the hypothesis about the distribution of \( F(t) \) using the K. Pearson criterion, you can use the method of moments to estimate parameters from a sample of the function under consideration.

For each of the three hypotheses about the theoretical distribution of the considered random variables \( V \) and \( T \), this criterion is used to check their degree of agreement with the considered samples. Observing the value \( Xn \) times, we get a sample \( \bar{x}_1, ..., x_n \). For each of the hypotheses, an estimate of the
expectation and variance will be used. We use functions for this sample \( \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \) and 
\[
S^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 .
\]
For each hypothesis to be tested, you must create your own system of algebraic equations, for example, the method of moments – for the L-distribution:
\[
\begin{align*}
\lambda & = \bar{X} \\
\frac{a}{\lambda} & = S^2 .
\end{align*}
\]
(6)

From the given system, we find a solution for estimating the moments of each of the considered distributions and calculate the numerical values of The K. Pearson criterion. This information is compared with reference table values [7].

As a result, we get two functions from the selection \( \lambda \) and \( a \), which are estimates, respectively, for \( \lambda \) and \( a \). At the first stage, the hypothesis was tested \( H_0 \). This hypothesis is that the distribution function of a random variable in the observed process is some distribution function \( F(t) \) for any values \( x \).

To test this hypothesis, we apply the CarlPearson on consensus criterion. To implement the \( H_0 \) verification process, you must calculate this criterion:
\[
\chi^2 = \sum_{i=1}^{m} \left( r_i - n \cdot P_i \right)^2 \cdot n \cdot P_i,
\]
(7)

where \( n \) – number of all observations, \( P_i \) – possibility of that the value \( X \) gets into \( i \)-group interval, \( m \) – number of group interval of the selection, \( r_i \) – number of getting of the value \( X \) into \( i \)-group interval. It is known that the value \( \chi^2 \) comes close to the value \( \chi^2 \) \( (\chi^2 \) – distribution with \( \phi \) freedom degrees). By this case:
\[
\phi = m - 1 - l ,
\]
(8)

where \( l \) – number of assessed parameters.

For checking the hypothesis \( H_0 \), it is recommended to reject this hypothesis if the calculated value \( \chi^2 > C \), that is, the discrepancy between the values of the function is considered significant \( F(t) \) and empirical function of distribution. The value of \( C \) is found in the distribution tables \( \chi^2 \) \( c \) \( \phi \) the degrees of freedom given in [7]. In this case, the inequality must be met
\[
P\left( \chi^2 > C \mid H_0 \right) \leq \alpha ,
\]
(9)

where \( \alpha \) – the level of significance of the criterion, in other words, the probability of rejecting the correct hypothesis \( H_0 \). For this reason \( \alpha \) is chosen to be close to zero \( (\alpha \approx 0.05) \).

If the calculated values are \( \chi^2 \) K. Pearson’s criterion is greater than the reference value \( C_{cr} \), then the hypothesis being tested is rejected and, otherwise, accepted. Similar studies were conducted to test the normal distribution hypothesis. A random variable is described by a normal distribution if its density satisfies the equality:
\[
\varphi(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}} .
\]
(10)

It is known, mathematical expectation \( E(X) = a \), \( D(X) = \sigma^2 \). Assessments for \( a \) and \( \sigma^2 \) will be \( \bar{X} \) and \( C^2 \).
At the third stage, a statistical analysis was performed (similar to the previous one) for the lognormal (LG) distribution [3], the distribution density of which has the form:

\[
\int (x, \mu, \sigma, \xi) = \frac{1}{\sigma(x-\xi)^2\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}\left[\ln(x-\xi)-\mu\right]^2\right\},
\]

where \(x\) - observed value of a random variable \((x \geq \xi)\);
\(\mu\) - shift on the time axis \((\mu > \xi)\);
\(\sigma > 0\) - constant value that characterizes the value of fluctuations in sales on average;
\(\xi = 0\) - the beginning of the actual reference of the considered statistical sample.

The parameters of this distribution are calculated using integrals [7, 8]. Using the previous expressions, we create a system of equations of the moment method for finding numerical estimates of parameters \(\mu, \sigma\) in the following view:

\[
\begin{align*}
\bar{X} &= \exp\left(\frac{\sigma^2}{2} + \mu\right) \\
S^2 &= \left(e^{\sigma^2 - 1}\right)\cdot e^{\sigma^2 + 2\mu}
\end{align*}
\]

The assessment for \(\mu\) and \(\sigma\) in the view:

\[
\bar{\mu} = \ln\left(\frac{X^2}{\sqrt{S^2 + X^2}}\right), \quad \bar{\sigma} = \left(\ln\left(\frac{S^2 + X^2}{X^2}\right)\right)^{1/2}
\]

In the future, we calculate statistics \(\chi^2\) K. Pearson for the hypothesis of lognormal distribution, normal and L-distributions (table 1).

| Types of distributions | Normal | Gamma | Long-normal |
|------------------------|--------|-------|-------------|
| Milking time           | \(\chi^2\)| 6.59  | 3.18        | 58.64       |
| Milking speed          | \(\chi^2\)| 4.17  | 1.54        | 70.45       |

When testing hypotheses, the value \(\chi^2\) was compared with the critical values \(C_{\alpha}\) according to [3]. This value is found from the table \(\chi^2\)-distribution, which is subject to the number of degrees of freedom \(m\) (in our case \(m = 3\)) and at the significance level \(\alpha = 0.05\) of the K. Pearson criterion. Numerically, we obtained a better agreement of the L-distribution than the normal and lognormal distributions.

The graph of the considered distribution densities and histograms are shown in figure 1 (for milking speed) and figure 2 (for milking time). The obtained results of the computational experiment confirm that L-distribution is the most suitable method for statistical analysis of the milking process under consideration.
Different types of density of distribution

![Graph of distribution densities and a histogram of the milking time](image1)

**Figure 1.** Graphs of distribution densities and a histogram of the milking time

Different types of density of distribution

![Graph of distribution densities and a histogram of the milking time](image2)

**Figure 2.** Graphs of distribution densities and a histogram of the milking time

4. Discussion
It has been found that the employment of milking machines by animals and the corresponding productivity of the milking machine are directly affected by the total duration of milking cycle of each animal, and especially by the maximum duration of milking for the slowest cow, and others.

5. Results
The paper presents an analysis of the biotechnical system "man-machine-animal" using the theory of algorithms, factor analysis and methods of mathematical statistics, implemented based on high information technologies. This allowed us to develop a mathematical model of the service cycle for individual animals on a conveyor-ring milking machine. Prerequisites for the development of an adaptive control algorithm for the process under consideration are obtained. It was revealed that the majority of animals in the considered samples are subject to the L-distribution, which allows us distinguishing separate groups from the herd: slow-moving cows, cows with a short milking time, and others. This information will improve the adequacy of the algorithm for controlling the rotation of rotary conveyor milking machines of the "Karusel" type.
6. Conclusion

The mathematical models and the results of statistical research obtained in this work will form the basis for the design of new high-performance rotary conveyor milking systems in the future.

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