Abstract
Despite recent success in large language model (LLM) reasoning, LLMs still struggle with hierarchical multi-step reasoning like generating complex programs. In these cases, humans often start with a high-level algorithmic design and implement each part gradually. We introduce Parsel\(^2\), a framework enabling automatic implementation and validation of complex algorithms with code LLMs, based on hierarchical function descriptions in natural language. Parsel can be used across domains requiring hierarchical reasoning, e.g. code synthesis, theorem proving, and robotic planning. We demonstrate Parsel’s capabilities by using it to generate complex programs that cannot currently be automatically implemented from one description and backtranslating Python programs in the APPS dataset. Beyond modeling capabilities, Parsel allows problem-solving with high-level algorithmic designs, benefiting both students and professional programmers.

1. Introduction
It is widely believed by computer scientists that (informally) there are many problems for which it is easier to verify a solution than to write a solution (Fortnow, 2009). Certainly, for code large language models (LLMs) like Codex (Chen et al., 2021), generating solutions, rather than testing them, is often the bottleneck. As we try to generate longer programs, the chance of any sample yielding a complete, working solution dramatically decreases.

Consider a program with relatively independent parts, and suppose an LLM has a fixed chance of implementing each part correctly. As the number of parts grows, we need exponentially more samples for a chance of a working program: for 6 parts, with a 10% chance, we’d need a million samples for a 63% chance of a fully correct program. For Codex (Chen et al., 2021) at 1,000 tokens per solution, it would take 17.4 days to generate these samples via the API. Clearly, this approach – sampling and testing complete programs – cannot scale to arbitrarily complex programs.

On the other hand, human programmers have successful strategies for implementing complex programs. One fundamental strategy is to make a high-level algorithmic design, decomposing a program into abstract modular parts that can be further implemented independently, even by different people. In our example, it would take only 18 seconds to generate ten 200-token examples of each of the 6 components. Motivated by this, we study whether we can leverage LLMs with such algorithmic decompositions to compile complex programs specified in structured natural language. Further, we investigate the ability of LLMs to automatically generate and expand these designs.

We propose Parsel, a compiler framework to implement and extend a high-level algorithmic design for interacting with an external environment (e.g. a programming language interpreter, a set of parameterized robot actions, or a formal theorem-proving environment). We formalize the high-level algorithmic design as a hierarchical set of function descriptions and constraints on their implementations (e.g. unit tests) in natural language, translating to a fixed underlying grammar. Given such a design, a Parsel compiler considers a minimal combinatorial set of function implementations based on the function descriptions and identifies one implementation per function description such that all constraints are satisfied. In effect, Parsel aims to perform algorithmic problem-solving by mirroring a common pattern in human reasoning – making an abstract plan to the level that it can be solved automatically (Simon & Newell, 1971).

In short, we make the following contributions:
1. We propose Parsel, a framework for generating constraint-validated programs from hierarchical natural language function descriptions, as shown in Fig. 1.
2. We propose a compiler-inspired algorithm to efficiently identify programs that satisfy all constraints, by identifying the smallest parts that must be implemented together and implementing them from the bottom up.
3. We present some programs that can be implemented automatically by using Parsel together with Codex (Chen et al., 2021), which we could not implement from one description directly using Codex. We discuss why prior work may indicate that there are programs that require task decomposition to be soluble.
4. We demonstrate that it is possible to automatically backtranslate nontrivial Python programs, from the
code-focused APPS dataset (Hendrycks et al., 2021),
into Parsel. Parsel can then generate code satisfying the
original unit tests. For the sampled APPS programs
that we could backtranslate, the Parsel solution is 76-
90% shorter in terms of non-whitespace non-assert
lines of code, with an average reduction of 84%.

5. We highlight a variety of recent works (e.g. Huang
et al. (2022); Jiang et al. (2022); Zhou et al. (2022a))
which could be implemented and expressed in Parsel.

2. Specifying Programs in Parsel

To specify a high-level algorithmic design formally, we
develop a small language underlying Parsel. We design
it considering programmers, code LLMs, and students, as
discussed in Section 6 and inspired by many works, noted
in Section 5. In Parsel, each line contains a description, a
constraint, or a reference to a previous description. These obey
scoping rules and have some nuances per target language.

2.1. Descriptions

A function description is represented as a function name fol-
lowed by comma-separated input arguments in parentheses,
and optionally what the function returns, then a colon and
text describing the function to be implemented. For example

count_living_neighbors(grid, i, j): count the
number of living neighbors of the cell at
the index (i, j)

A function generated from a description can call either the
functions defined directly below the description in the func-
tion graph (indicated with indentation) or references directly
below the description\(^4\), both shown in Fig. iii.

2.2. Constraints

A constraint is represented as a function’s input values
comma-separated, optionally followed by an arrow and the
expected output of the function. Constraints are provided at
the same indentation level as the preceding description. For example, after the definition of count_living_neighbors,

\[ [(1, 0), [0, 1]], 0, 0 \rightarrow 1 \]
\[ [(1, 0), [0, 1]], 0, 1 \rightarrow 2 \]

This indicates that the function count_living_neighbors
should return 1 when called with the arguments \([(1, 0), [0, 1]], 0, 0 \) and 2 when called with \([(1, 0), [0, 1]], 0, 1 \).

Notably, to apply complex constraints on functions, one can
describe and constrain higher-order functions. For example,

\(^4\)A nuance here is the optional ability for undefined/out-of-
scope functions which are generated by the code language model
to also be implemented automatically.
one could write

def type_fn_output(fn, args):
    return the type of
    ↦ the output of a function called with args

def count_living_neighbors, (\([1, 0], [0, 1]\)], 0, 0)
    ↦ \(\rightarrow\) int

This indicates that the function count_living_neighbors
should return an integer when called with the input argu-
ments \([1, 0], [0, 1]\)], 0, 0.

Note that constraints can only be applied following descrip-
tions, and not to references.

What it means to satisfy constraints to validate a program
varies from language to language: in Python, one can check
that a program passes certain assert statements by evaluating
them; however, in a language like Lean, where the ability to
run a program without any sorry lines shows that a proof
holds, one would instead represent the formal proof state-
ment as the specified constraint (that is, that you are actually
proving what you set out to prove). For languages where
correctness can be checked without any unit tests, their
functions can be treated as also having implicit constraints.

2.3. References
A reference is simply the name of a function defined in the
current scope (see Subsection 2.4 for details) within the
function graph. A reference allows and encourages (via
prompt) the parent function to use the referenced function.
This allows for recursive function definitions and functions
called by multiple functions. Note a reference without a
description in scope is not valid.

For example, one can define an (overly verbose) version of
the Collatz conjecture as shown in Figure iii, where the final
line is a reference. We visualize the corresponding call graph
and its strongly connected components (SCC) in Figure 2.
In the Collatz functions, base_case is implemented first as
the collatz_recursion SCC depends on it.

2.4. Scoping
Scope in Parsel is defined by indentation. Specifically, the
scope \(S\) of a function \(f\) includes the set of functions that
can be used as a reference for a given function – that is,
all functions where the indentations between the current
function to the referenced function are strictly decreasing.

2.5. Variations Due to Target Language Requirements
For certain aspects, it is necessary to be mindful of nuances
of the language to which one is translating. As discussed
in Subsection 2.2, the meaning and representation of a con-
straint may vary by language. Moreover, every language
has a different evaluation function: executing Python is
different than compiling and running C++ code which is
different than checking a proof with Lean. In addition, ev-
ery language will likely require a different prompt for the
language model. Thus, we detail these particularities in
language-specific configuration files.

3. Implementing Programs in Parsel
There are several steps to compiling a Parsel program, which
we explicate in this section, and provide a high-level pseudo-
docode capturing the details in Figure 3.

3.1. Constrained Implementation
Every approach here relies on the same principle: with a
way of generating functions with a code LLM and testing
them, we can identify an implementation that satisfies each
of the constraints. To generate examples, we prompt a code
language model with a description and a function signature,
shown in Appendix B. In Python, we use this to mean pass-
ing assert statements. We can then treat the implementation
as “fixed” and leverage it in functions calling it. Note this
places a strong responsibility on the user – if constraints
are specified and Parsel finds an implementation passing the
constraints, it assumes the implementation is correct!

3.2. Leaves and Merges
How a function is implemented in Parsel depends on
whether it depends on any other functions. If a function
is a leaf node of its call graph, it is implemented by query-
ing the code language model using the description text as
a docstring and the description’s function name and argu-
ments for the signature, with prompts in Appendix B. If a
function is not a leaf node, it is implemented by refer-
encing the implementations of its direct children. In this
case, the descriptions and function signatures of the children
are aggregated (for Python, we generate a prompt as if the
child functions are imported and use their descriptions as
comments). We then query the language model similarly
to the leaf case, but with the function name and arguments
of the parent function. Crucially, this allows us to easily
interchange child implementations.
3.3. Sequential Case

Perhaps the most straightforward and simplest case of a Parsel program is one where all functions have constraints (e.g., unit tests), and no functions have recursive dependencies (e.g., Fig. 6). We start by considering this case. This defines a clear topological order of functions so they can be implemented sequentially. In this situation, Parsel implements functions with post-order traversal from the root, generating implementations and finding one passing the specified constraints for each function. In other words, without any cycles in the call graph, we can start by implementing the leaf functions first, then their parents, etc. until the program is implemented. However, in practice, many programs have more complex structures and constraints.

3.4. Strongly Connected Components of Functions

3.4.1. Recursion

One can imagine a strict version of Parsel where we require all functions to have accompanying constraints. However, even with this requirement, as long as recursion is possible, it is sometimes necessary to implement functions jointly — with cyclic dependencies, none can be tested alone. Further, there are many contexts in which one may only have access to constraints for some set of functions. In particular, to support automatic “elaboration” where a language model may automatically construct the Parsel subfunctions for a given function, it is necessary to allow for functions to be defined without constraints. This raises a key question: how?

One solution would be to consider all possible implementations of all functions defined in the Parsel program and then to iterate through combinations until a valid implementation is found. However, this is clearly intractable for large programs. Specifically, the number of possible implementations of a program with $k$ functions and $n$ implementations per function is $O(n^k)$, exponential in the number of functions.

Instead, we propose a more efficient solution, inspired by (Cocke & Kennedy, 1977): for cases of recursion, we reference the function call graph and identify all of its strongly connected components (e.g., Fig. 2). In other words, these are the sets of functions where their definitions depend on one another. For these cases, we apply the above-described technique of considering all possible sets of their implementations until one satisfies all of their constraints. Notably, we make the simplifying assumption that any statefulness across functions does not interfere with dependent asserts.

We evaluate each element of a shuffled list corresponding to the function implementation sets arising from the direct product of all the sets of function implementations in the strongly connected component. In other words, for possible implementations of functions $f, g, h$ which form a strongly connected component of a call graph, and $I(f)$ corresponds to the language-model-generated implementations of $f$, we consider uniformly random samples without replacement from $I(f) \times I(g) \times I(h)$. We use multiprocessing with a user-specified timeout and encourage the use of a sandboxed evaluation environment in order to allow for many fast solutions to be tested alongside slower solutions.

3.4.2. Functions with No Constraints

In the case of functions with no constraints, we can conveniently use the same approach as above by reformulating the call graph as a “test dependency graph.” That is, if a function has no constraints, it depends on all of its parents

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3With the caveat that we automatically remove exact duplicate implementations for a specific function. This could further be improved by identifying functionally-equivalent but not string-equivalent implementations, as in e-graphs (Ellis et al., 2021)

4As anticipating the number of steps that a solution will take universally is a version of the halting problem and thus intractable.
to enforce constraints on its implementations. This also allows us to automatically introduce new children via automatic decomposition, as it is nontrivial to also automatically introduce constraints for those children (see Subsection 3.8).

### 3.5. Caching

We cache responses from the language model with respect to the prompt and language model decoding parameters 1) to reduce the number of queries necessary and 2) to keep the programs generated mostly stable (i.e. a working function should continue working unless it or its children change). To this end, when the number of desired implementations increases for a pre-existing query with all other arguments fixed (temperature, number of decoding tokens, etc), we append the additional ones to those already generated.

### 3.6. Headers

We also support program headers, allowing global contexts, used when implementing all new functions within a program. This is indicated by a line containing an optional string of special characters (e.g. “###”) separating the body and the text and is passed as a prefix to all prompts.

### 3.7. Automatic Function Infilling

Sometimes, a function generated by a language model may call a function that is not yet implemented. In this case, we can (optionally) attempt to automatically generate and implement it based on its usage. The function is then incorporated into the call graph as a unit-test-less child of the function which calls it. To avoid infinite recursion and inefficient use of language model quota, we limit the number of times that this process can be applied to a function.

### 3.8. Automatic Decomposition

As indicated by a rapidly growing number of papers (Brohan et al., 2022; Huang et al., 2022), the task of decomposing a task into steps in natural language is one that language models are surprisingly capable of. Thus, we treat the ability to automatically decompose a Parsel function as a key feature of Parsel. This is an optional flag that prompts a language model to generate Parsel code corresponding to any additional subfunctions necessary when Parsel fails to implement a function. These proposed subfunctions are then added as child nodes to the decomposed function node. However, an additional consequence is that Parsel can thus be used to recursively decompose tasks into steps, by repeatedly identifying descriptions that cannot be directly implemented and attempting to decompose them.

### 4. Experiments

#### 4.1. APPS Dataset

##### 4.1.1. Backtranslation

We anticipate that there are many programs that LLMs can implement by first generating Parsel code. But, as Parsel is a new framework, while language models can sometimes generate Parsel programs with few-shot prompts, it is not a syntax they have previously encountered. Thus, we may want to use existing code in other languages to construct datasets of Parsel programs from other languages. This requires us to first extract the call graph from the code, generate descriptions for each of the functions, and then generate Parsel programs from the graph. This call graph representation is broadly convenient, so it is useful to have a bidirectional method to produce a graph from Parsel code and to produce Parsel code from the graph.

The APPS dataset is a collection of coding challenges (in long-form text) with solutions in Python (Hendrycks et al., 2021). We filter the dataset to problems with starter code (providing the name of the evaluated function) and unit tests (provided as input-output pairs). For those tasks, we select solutions that define and call at least three functions, with at least one over 4 lines long and none over 15 lines.

As a proof of concept, we show 10 Parsel solutions which we could automatically generate from the APPS solutions. We generated the descriptions by prompting Codex to explain each function and its inputs and outputs. From this, we use backtranslation to attempt to implement these solutions in Python. We then verify that they are correct by applying the original unit tests as constraints on the root function.

As mentioned in Section 1, the Parsel code is substantially shorter in terms of lines of code.

##### 4.1.2. Solutions

We also explored whether a code language model, given examples of Parsel code, a problem statement, and that problem’s asserts, could generate and implement new and complex solutions to APPS problems. We include one Parsel example in Figure 4 (with the resulting Python in the appendix), which returns the current leader from a chess board as either “Draw”, “White”, or “Black”.

#### 4.2. Case Studies

We include an example function we could not generate directly from the top-level description in Figure 5. The corresponding Python code (included in the appendix) is exactly 58 non-whitespace lines of code, including 17 lines of comments (3 corresponding to the descriptions), 2 asserts, and 39 lines implementing the three functions described as well as an automatically generated `get_number_of_active_cells_around_cell` func-
weight

weight_helper

weight_helper(board, row, col, weight):
  weight(board): weight takes a board and returns
  → the weight of the board.
  → weight_helper takes a board, a row, a
  column, and a weight and returns the
  → weight of the board.
  → weight_helper

Figure 4. A Parsel program generated by Codex, solving Problem 368 in the APPS test set. Note the references to weight and weight_helper at the bottom are redundant.

4.3. Theorem Proving in Lean

With the same framework, we can generate proofs in formal theorem-proving languages such as Lean, as in Figure ii. We include the translated version in the appendix. Note a nuance of Lean and theorem-proofing languages is that the ability to run Lean on proof with no errors/warnings indicates the proof is correct (but is not a guarantee that the proof statement matches our claim in language). Thus, each function in a Lean Parsel proof has an “implicit constraint.”

4.4. Robotic Planning in VirtualHome

We also perform a VirtualHome (Puig et al., 2018) case study to demonstrate that Parsel can also be used for complex robotic planning. VirtualHome is a simulation environment designed to allow complex interactions with the environment. It consists of households with various objects and agents to interact with.

To test the effectiveness of Parsel in this domain, we investigate whether a Parsel could generate programs to solve tasks in the VirtualHome environment, while using the environment to provide feedback on whether the plan is executable. Specifically, we use Parsel to generate a python program that can generate action plans in natural language similar to ones used in (Huang et al., 2022). In each specified constraint, the produced natural language action plan is translated to formal VirtualHome instructions with minimal regex matching and tested executability. If the instructions can be successfully executed, they are considered valid – however, one could describe object-relational constraints on the state of the world after instructions execution. We include an example of a Parsel program that successfully executed and decomposed a task in VirtualHome in Figure 1. Note we also used a header describing a valid action plan, shown in Figure 37.

In the future, incorporating asserts at the execution level (e.g., checking whether the agent is close to the microwave, as in Singh et al. (2022)), is a promising research direction.

5. Related Works

5.1. Step by Step Problem Solving with LMs

A large body of work shows that step-by-step reasoning benefits LLM performance (Rajani et al., 2019; Shwartz et al., 2020; Nye et al., 2021; Wei et al., 2022; Marasović et al., 2021; Lampinen et al., 2022) and correspondingly, that this performance can be improved with further guidance and tool use (Zhou et al., 2022a; Zelikman et al., 2022; Yao et al., 2022; Usato et al., 2022; Dua et al., 2022). One encouraging prior work is Acquaviva et al. (2022), showing that humans, when asked to explain how to solve problems in the Abstract Reasoning Corpus (Chollet, 2019), tended to provide step-by-step hierarchical descriptions with many verification steps. Moreover, Wies et al. (2022) presents a theoretical argument showing problems that can be learned efficiently when decomposed but require exponentially many examples w.r.t. length if not decomposed.

5.2. Program Synthesis

Program synthesis is the long-standing challenge of generating programs from high-level specifications (Gulwani et al., 2017), such as input-output examples (Bauer, 1979; Gulwani, 2016) and/or natural language descriptions (Raza et al., 2015; Yin & Neubig, 2017; Desai et al., 2016). Program synthesizers typically search the exponentially large space of programs. Consequently, synthesizing large, complex programs remains an open challenge. Recently, library learning has shown a way to make progress: even complex programs can be short in terms of the right high-level library. In turn, this library can be progressively induced from solutions to simpler synthesis problems. This idea is embodied in DreamCoder (Ellis et al., 2021; Bowers et al., 2022). Library learning requires a rich distribution of related tasks so that patterns emerge from solutions to simple problems. The idea is that patterns can be abstracted into useful library functions, enabling short solutions to more complex problems. Parsel similarly aims to synthesize complex programs by decomposing them into smaller functions. In Parsel, however, the user specifies the decomposition, so a family of related tasks is not required.

5.3. LMs for Formal Environment Multi-step Planning

Also encouragingly, several existing works can be expressed in Parsel. For example, Huang et al. (2022) and Brohan et al. (2022) showed that language models can be used to generate step-by-step algorithms for robotic agents in language automatically. In both cases, the generated language corresponds directly to pre-implemented low-level robotic
array_inversion(array) -> inverted_array: Invert a square array by flipping 0's and 1's

game_of_life_iteration(array_at_time_t) -> array_at_time_t_plus_1: Takes a board with active and inactive cells as a list of lists and returns the next iteration of the game of life, but with all values flipped

game_of_life_inversion_iteration(array_at_time_t): Takes a board and returns the next iteration of the game of life, but with all values flipped

array_inversion(array) -> inverted_array: Invert a square array by flipping 0’s and 1’s

Figure 5. An example Parsel program for Python that takes in a list of lists representing a state of Conway’s game of life (Games, 1970) and returns the next state, with all the values inverted.

abilities. This could be expressed by providing a description of the task and constraints that evaluate that the high-level task was completed successfully. In addition, Jiang et al. (2022) proposed a framework to generate formal proofs in formal theorem-proving languages from informal proofs by first generating an intermediate natural language proof sketch. This could be expressed in Parsel by generating each sketch step as a function and then using formal verification for each lemma as the Parsel validation step.

5.4. Programming Languages and Frameworks

Incorporating Language Models

Other works have explored programming languages that incorporate language models. For example, Cheng et al. (2022) explored the introduction of a language-model-based evaluation function, which would allow `f('North America?', 'U.S.')` to automatically return 'yes' by referencing the knowledge of the language model, and showed that they could also generate programs using this tool with a language model and Beurer-Kellner et al. (2022) explored a related SQL-style LM-querying language. In addition, Dohan et al. (2022) presents an inference-focused framework for language models more broadly for probabilistic graphical models composed of language model actions. Unlike LM Cascades (Dohan et al., 2022), we primarily focus on the constrained generation of programs, instead of leveraging language models as functions within a particular program.

5.5. Testing Code Language Model Outputs

Related works have explored the capacity of assert statements to constrain the generation space of LLMs for code on individual functions (Austin et al., 2021; Chen et al., 2021; Li et al., 2022). In particular, Merrill et al. (2021) proves essential constraints on what can be learned from assertions alone, and more crucially, what cannot.

6. Implications

Parsel is a natural language compiler framework that bridges the gap between natural language and programming language by allowing programmers to write high-level algorithmic designs in natural language and automatically compiling them into valid code. This has potential benefits for programmers, students, and code language models.

6.1. For Programmers

6.1.1. CURRENT LIMITATIONS

First, programming generation language models like Codex continue to be constrained primarily to individual functions, rarely exceeding a few dozen lines in practice (Chen et al., 2021; Tabachnyk & Nikolov, 2022). This is still a dramatic shift from foundational earlier works, which focused on the association between one line of natural language pseudocode with one line of code (Kulal et al., 2019) or a line of text to a StackOverflow snippet (Yin et al., 2018). Yet, these models perform worse the more unusual the desired functions are, and recent research suggests that people using these language models are more likely to introduce buggy code (Perry et al., 2022), although this is not yet conclusive (Sandoval et al., 2022).

6.1.2. POTENTIAL BENEFITS

On the other hand, results from Google and others indicate that professionals can write code more efficiently with large language models, and the benefits will likely only improve as they improve (Tabachnyk & Nikolov, 2022). Since Parsel requires constraints that ensure functions behave as expected, this should encourage bug-free programs and avoid the need for manually checking that specific underlying functions are correct. Furthermore, a function written in Parsel is likely to be more resilient to breaking changes in the target language, especially syntactic changes (e.g. Python2 to Python3). In addition, a natural extension would draw on work on automatic unit testing (Daka & Fraser, 2014) to suggest additional constraints where behavior is ambiguous between implementations of a function.

6.2. For Students

6.2.1. CURRENT LIMITATIONS

In addition, these language models pose serious challenges for programming pedagogy – existing introductory programming classes rely extensively on teaching syntax and how to implement algorithms over how to solve problems with them. Free language model-based tools like Copilot can essentially solve many of these introductory assignments directly, function by function. Those which cannot be solved currently will be increasingly solved (Denny et al., 2022).
6.2.2. Potential Benefits

Many students currently introduced to programming struggle with learning syntax and debugging unclear compiler or interpreter errors. However, abstracting away these details with a natural-language coding language will likely make learning to code more accessible to students who are just beginning to code. In addition, stepping away from implementation-focused assignments will allow a focus on higher-level problem-solving assignments earlier. These will allow for assignments that are more like those in mathematics. For example, for a problem like Figure 6, instead of choosing between requiring students to manually implement a problem-solving focused question like the top-level description of, or requiring teaching assistants to manually evaluate the reasoning for correctness, one could ask them to implement a solution in Parsel.

6.3. For Code Language Models

6.3.1. Current Limitations

Traditional programming languages result in some unique challenges for language models. For example, unlike natural languages, traditional programming languages are far less robust to slight variations in wording. In addition, traditional programming languages require many tokens for syntactic details and in some cases, may take many lines to express what can be expressed far more simply in language. For example, referring to a shortest-path algorithm or Conway’s game of life takes far fewer tokens than actually implementing them. However, even with fairly nonstandard problems, LLMs have shown remarkable algorithmic generalization ability (Liang et al., 2022; Xu et al., 2022; Anil et al.; Zhou et al., 2022b). One alternative that has been explored is conversational code generation (Nijkamp et al., 2022; Yin et al., 2022). However, these approaches have primarily focused on highly imperative programming structures. Moreover, they still require having the full program in context and do not clearly generalize to complex hierarchical programs with many functions.

6.3.2. Potential Benefits

Parsel allows code language models to stay closer to natural language when generating code, which corresponds more closely to their primary source of training data. Moreover, it allows complex but standard methods to be described concisely, requiring fewer tokens to generate. One exciting additional benefit is the potential to generate solutions recursively: if the Parsel compiler is unable to find a solution for a set of functions, it should be possible to prompt the model to define new helper functions. In fact, we find that often the model attempts to reference undefined auxiliary functions when defining complex functions (e.g. “count_living_neighbors(grid, i, j)” in Conway’s game of life), and as a result support an optional argument where the model can attempt to resolve NameErrors automatically by attempting to implement functions.

7. Limitations

There are several limitations to the current implementation of Parsel. First, Parsel relies on a code language model to generate implementations of individual functions, and the quality of these implementations can vary depending on the specific model used and the complexity of the function descriptions. In particular, Parsel may struggle to generate correct code for functions with complex behavior. However, this can be mitigated by decomposing the complex func-

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```python
1 select_airport_cities(city_road_cost, city_airport_cost): given a matrix representing the cost of
2 building a road between any two cities, and a list representing the cost of building an airport
3 in a city (where any two cities with airports are connected), return a list of the cities that
4 should have airports built in them to minimize the total cost of building roads and airports such
5 that all cities are connected. The list should be sorted in ascending order.

6    [[0, 3, 3], [3, 0, 3], [3, 3, 0]], [0, 0, 0] → [0, 1, 2]
7    [[0, 3, 3], [3, 0, 3], [3, 3, 0]], [10, 10, 10] → []
8    [[0, 10, 3], [10, 0, 11], [3, 11, 0]], [1, 4, 5] → [0, 1]

9 sky_city_cost(city_road_cost, city_airport_cost): given a list of lists representing the cost of
10 building a road between any two cities, and a list representing the cost of building an
11 airport in a city, return a new cost matrix with a new node corresponding to the sky.

12   [[1, 2, 3], [1, 2, 3], [1, 2, 3]], [4, 5, 6] → [[1, 2, 3, 4], [1, 2, 3, 5], [1, 2, 3, 6], [4, 5, 6, 0]]
13 minimum_spanning_tree(cost_matrix): given a list of lists representing the cost of each edge,
14 return an adjacency matrix corresponding to the minimum spanning tree.

15   [[0, 1, 3, 4], [1, 0, 2, 100], [3, 2, 0, 5], [4, 100, 5, 0]] → [[0, 1, 0, 1], [1, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0, 0]]
16 final_node_connectors(adjacency_matrix): given a list of lists representing an adjacency matrix,
17 return a list of the nodes connected to the final node. However, if only one node is connected
18 to the final node, return an empty list.

19   [[0, 1, 0, 1], [1, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0, 0]] → []
20   [[0, 1, 0, 1], [1, 0, 1, 0], [0, 1, 0, 1], [1, 0, 1, 0]] → [0, 2]

Figure 6. A potential programming assignment focused on problem-solving rather than implementation. The top-level function and asserts would be the assigned problem (which Codex does not seem to be able to solve directly (Chen et al., 2021)), while the other functions would be the student solution.
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We hope that Parsel provides a broadly useful framework for programming and reasoning. Additionally, the generated code may be less efficient or less readable than manually written code.

The current implementation of Parsel may struggle to generate correct code for functions with complex dependencies or without constraints. This is because the number of implementation combinations to consider grows exponentially with the size of the corresponding strongly connected components. As discussed throughout the paper, this can limit Parsel’s performance on some programs.

One limitation of code language models is that they may not perform well on languages that were underrepresented in their training data. This is because they have fewer examples to learn from and thus may struggle to generate correct code in these languages (Athiwaratkun et al., 2022). However, some large language models can adapt and learn new languages in context, allowing them to generate code in languages not in their training data (Athiwaratkun et al., 2022). These limitations can impact the quality and reliability of the code generated by Parsel.

In addition, the best open-source code language models like PolyCoder (Xu et al., 2022) currently available substantially underperform Codex while Codex is surprisingly competitive with other traditional language models on reasoning tasks (Liang et al., 2022). This allows the providers of closed language models to change behavior (e.g. rate limits or model implementations) without warning.

However, despite these limitations, the current Parsel implementation has shown promising results in generating correct code for a variety of functions and languages. Many limitations will likely be ameliorated as code LLMs improve.

8. Conclusion and Future Work

We hope that Parsel provides a broadly useful framework for several groups: for programmers, we hope Parsel provides a language for robust code generation without the need to evaluate the underlying code; for students, we hope Parsel allows the teaching of algorithmic reasoning with less emphasis on syntax and more emphasis on problem-solving, similarly to a mathematics curriculum; for language models that perform tasks requiring reasoning, we hope this allows for a general framework for hierarchical task decomposition.

In the future, we also hope to integrate automatic unit test generation (Daka & Fraser, 2014). One method would be to identify edge cases and check whether the set of functions that successfully solve all existing tests disagree on any new tests. This could permit automatic decomposition without exponential growth in implementation combinations.

In addition, we plan to incorporate ways of varying the “confidence threshold” of the language model. Ensuring that the descriptions are straightforward and unambiguous is important for more critical programs and parts of programs. In addition, when teaching students simpler concepts, requiring them to decompose the task further may be useful.

We would like to integrate value functions to allow decomposition to be done more methodically where no verification is possible. Specifically, automatically decomposing all functions that have not yet been implemented in an SCC is suboptimal and could be improved with a model of expected improvement due to expansion, as done for proof expansion in Polu & Sutskever (2020). In addition, when decomposing functions, we would like to permit the model to reference already-defined functions (rather than to just define new ones). We might even use the code language model to determine which function to next evaluate. Further, we aim to support more general reward functions for function implementations where multiple may be valid but we rank implementations based on a desired feature. These “soft” constraints may also allow new Parsel uses, e.g. planning stories in natural language (Ye et al., 2022).

Finally, we hope that it would be possible to use Parsel as a framework for bootstrapping increasingly complex program generation (e.g. Anthony et al. (2017); Zelikman et al. (2022); Odena et al. (2020)). That is, by 1) generating Parsel examples from a purely natural language specification and then reinforcing those which successfully compile, and 2) by reinforcing the model with each successfully compiled component, we would likely be able to iteratively improve performance with an arbitrarily large dataset of examples.

Another feature that would be valuable would be the ability to incorporate multiple base tools with different kinds of specialized models, inspired by Ibarz et al. (2022) and Dohan et al. (2022). That is, it would be valuable to allow a model to determine which target language to use, possibly combining them. For example, for large parts of the TensorFlow and PyTorch libraries, while their interfaces are written in Python, they depend heavily on large C++ codebases (Paszke et al., 2019; Abadi et al., 2015). Relatedly, Cobbe et al. (2021) showed that giving language models access to a calculator allowed them to solve more complex math word problems. This, combined with the observation that Parsel could also compile programs by generating language model prompts to be used as part of the program, may potentially allow the automatic generation of task-specific language model cascades (Dohan et al., 2022).

Another noteworthy addition would be the integration of Synchromesh (Yin et al., 2018), ensuring that each new word or token generated by the model is actually possible within the grammar of the given formal language.

Ultimately, we hope that this initial specification for Parsel is a jumping-off point for a new way of thinking about programming and reasoning.
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A. Parsel Pseudocode

We include a longer-form Parsel pseudocode in the style of Parsel. Note this pseudocode does not include backtranslation.
parsel(program, target_language, allow_autofill=False, allow_autodecomp=False): compile a program from a string specifying a Parsel program.

parse_program(program): parse the Parsel program string to a call graph

create_root_node(): create a root node as the current function node, without any constraints

parse_line(line, current_node, current_indent) -> function_graph: for each step up in indentation, set the current node to its parents, then, parse the definition, reference, or constraint.

parse_definition(line) -> name, args, rets, description: create a new function node, make it a child of the current node's parent, then assign it as current node. Description of the form "name(args) -> rets: description" if return variables are present else "name(args): description".

populate_fn_node(name, args, rets, description): populate the new node's name, arguments, description, and optionally a list of returned variables.

parse_reference(line): add reference as a child of current node if reference is an ancestor or a direct child of an ancestor

parse_constraint(line): add the constraint to the current node's constraints.

generate_dependency_graph(function_graph) -> dependency_graph: taking the function graph, create a copy where all nodes without constraints also depend on their parents unless the target language implicitly tests all functions.

identify_strongly_connected_components(dependency_graph): return SCCs of the dependency graph and the edges between the SCCs.

compile_scc(scc, scc_graph, allow_autofill, allow_autodecomp): compile any SCCs this SCC depends on and add them to the implementation string.

compile_children(scc, scc_graph, allow_autofill, allow_autodecomp): compile any SCCs this SCC depends on and add them to the implementation string.

direct_product_implementations(fn_implementations): return the direct product of the list of lists of fn_implementations

generate_implementations(scc, n, children_implementation_str): for each function in the SCC, generate n implementations of each function starting with the implementation string of the SCC's children.

fn_implementation(fn_node, n): prompt the language model to generate n implementations of a function

generate_prompt(fn_node): first prepend a string with all descriptions, names, arguments, and returns of fn_node's direct children, in a style idiomatic for the target language, then, add fn_node's description and function signature.

generate_constraints(fn_node): translate each of the constraints into an evaluation string idiomatic to the target language.

eval_str(scc, implementation_str, allow_autofill): evaluate an implementation including constraints by running it in a target-language executor. If allow autofill and the execution fails due to an undefined reference, attempt autofill

exec_implementation(implementation_str): run the implementation, including constraints/tests, in a target-language-specific executor, returning whether it was successful

attempt_autofill(scc, implementation_str, undefined_fn_use_example): create a new function node for the referenced function, then re-attempt to execute autofill

add_undefined_fn(scc, implementation_str, undefined_fn_caller, undefined_fn_use_example): create a new function node as a child of the function which calls it and add it to the scc and implementation string. Prompt the language model with the usage example as the description to generate a set of implementations.

eval_str: on fail(scc, scc_graph, allow_autofill, allow_autodecomp): if allowing autodecomposition, attempt to decompose. otherwise, raise an error highlighting the scc which could not be compiled

attempt_autodecomp(scc, scc_graph, allow_autofill, allow_autodecomp): prompt the language model to decompose each unimplemented function node.

prompt_model(scc, fn): prompt the language model, asking it to generate a "fn name(arg): desc" for each subfunction necessary to implement the function node. Add those functions to the scc, including a set of possible implementations for each.

fn_implementation: compile_scc
raise_error(scc): raise an error that Parsel could not compile the scc

Figure 7. Longer pseudocode of Parsel, including automatic infilling and automatic decomposition.
B. Parsel Prompts
def select_airport_cities(city_road_cost, city_airport_cost):

Figure 9. Codex Prompt for an example merge node

Figure 10. Prompt format to generate descriptions for backtranslation
C. APPS Backtranslation

We exclude the asserts in these examples for brevity - they correspond to those in the original dataset.
longest_palindrome(s): longest_palindrome takes a string s and returns the longest palindrome in s.

is_palindrome(s): is_palindrome returns True if the string s is the same forwards and backwards, and False otherwise.

check(li, ri, s): check takes a string s, a left index li, and a right index ri, and returns the longest palindrome that starts at or before li and ends at or after ri.

# longest_palindrome takes a string s and returns the longest palindrome in s.
def longest_palindrome(s):
    if len(s) <= 1:
        return s
    else:
        longest = s[0]
        for i in range(len(s)):
            for j in range(len(s)):
                if is_palindrome(check(i, j, s)) and len(check(i, j, s)) > len(longest):
                    longest = check(i, j, s)
        return longest

# is_palindrome returns True if the string s is the same forwards and backwards, and False otherwise.
def is_palindrome(s):
    if len(s) <= 1:
        return True
    else:
        return s[0] == s[-1] and is_palindrome(s[1:-1])

# check takes a string s, a left index li, and a right index ri, and returns the longest palindrome that starts at or before li and ends at or after ri.
def check(li, ri, s):
    while li >= 0 and ri < len(s) and s[li] == s[ri]:
        li -= 1
        ri += 1
    return s[li+1:ri]
def case_id(c_str):
    if is_snake(c_str) == True:
        return "snake"
    elif is_kebab(c_str) == True:
        return "kebab"
    elif is_camel(c_str) == True:
        return "camel"
    else:
        return "none"

def is_snake(s):
    if s[0].isalpha() and s[0].islower() and len(s) > 1:
        for char in s:
            if not (char.isalpha() or char == '_'):
                return False
        else:
            return True
    else:
        return False

def is_kebab(s):
    if s == '':
        return False
    if type(s) != str:
        return False
    if s != s.lower():
        return False
    for c in s:
        if c == '-' or s[0] == '-' or s[-1] == '-':
            return False
    for i in range(len(s)-1):
        if s[i] == '-' and s[i+1] == '-':
            return False
    return True

def is_camel(s):
    return s != s.lower() and s.find('_') == -1 and s.find('-') == -1

Figure 14. Train Problem 2892, Solution 7
find_2nd_largest(arr): find_2nd_largest takes a list of numbers and returns the second largest number in the list.

sec_big(a, b): sec_big takes two numbers and returns the smaller of the two.

sort(arr): sort takes an array of numbers and returns a sorted array of numbers.

is_diff(arr): is_diff takes an array of numbers and returns True if there are any two numbers in the array that are different, and False if all the numbers in the array are the same.

Happy numbers(n): happy_numbers takes a positive integer n and returns a list of all the happy numbers between 1 and n, inclusive.

_is_happy_number(number): _is_happy_number takes a positive integer and returns True if the number is a happy number, False otherwise.

_sum_squares(number): _sum_squares takes a non-negative integer and returns the sum of the squares of its digits.
# happy_numbers takes a positive integer n and returns a list of all the happy numbers between 1 and n, inclusive.

def happy_numbers(n):
    return [i for i in range(1, n + 1) if _is_happy_number(i)]

# _is_happy_number takes a positive integer and returns True if the number is a happy number, False otherwise.
def _is_happy_number(number):
    # We want to make sure that the number is positive
    if number < 0:
        return False
    # We want to make sure that the number is not 1
    if number == 1:
        return True
    # We want to keep track of the numbers we have used
    used_numbers = []
    # We want to loop through the number
    while number not in used_numbers:
        # We want to add the number to the list of used numbers
        used_numbers.append(number)
        # We want to find the sum of the squares of the digits of the number
        number = _sum_squares(number)
        # We want to check if the number is one
        if number == 1:
            # We want to return True
            return True
        # We want to return False
        return False

# _sum_squares takes a non-negative integer and returns the sum of the squares of its digits.
def _sum_squares(number):
    if number < 0:
        raise ValueError
    else:
        number = str(number)
        sum = 0
        for i in number:
            sum += int(i) ** 2
        return sum

Figure 18. Train Problem 3140, Solution 7

am_i_wilson(n): am_i_wilson(n) returns True if n is a prime number between 2 and 563, inclusive, and False otherwise.

is_prime(n): is_prime takes a positive integer n and returns True if n is prime and False otherwise.
factorial(n): factorial(n) returns the product of all integers from 1 to n, inclusive.

Figure 19. Train Problem 3229, Solution 26
# am_i_wilson(n) returns True if n is a prime number between 2 and 563, inclusive, and False otherwise.

def am_i_wilson(n):
    if is_prime(n) and 2 <= n and n <= 563:
        return (factorial(n-1) + 1) % (n**2) == 0
    else:
        return False

# is_prime takes a positive integer n and returns True if n is prime and False otherwise.

def is_prime(n):
    if n == 2:
        return True
    if n == 3:
        return True
    if n % 2 == 0:
        return False
    if n % 3 == 0:
        return False
    i = 5
    w = 2
    while i * i <= n:
        if n % i == 0:
            return False
        i += w
        w = 6 - w
    return True

# factorial(n) returns the product of all integers from 1 to n, inclusive.

def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

---

Figure 20. Train Problem 3229, Solution 26

Figure 21. Train Problem 3229, Solution 71
# am_i_wilson takes a positive integer \( n \) and returns True if \( n \) is prime and \((n-1)! + 1\) is divisible by \( n^2 \), and False otherwise.

```python
def am_i_wilson(n):
    return is_prime(n) and (fac(n-1) + 1) % n**2 == 0
```

# fac is a function that takes a positive integer \( n \) and returns the product of all integers from 1 to \( n \).

```python
def fac(n):
    if n == 0:
        return 1
    return n * fac(n-1)
```

# is_prime takes a positive integer \( n \) and returns True if \( n \) is prime and False otherwise.

```python
def is_prime(n):
    if n == 2:
        return True
    elif n < 2 or n % 2 == 0:
        return False
    for i in range(3, int(n**0.5)+1, 2):
        if n % i == 0:
            return False
    return True
```

---

**Figure 22.** Train Problem 3229, Solution 71

# evil(n) returns "It’s Evil!" if \( n \) is an evil number, otherwise it returns "It’s Odious!" The range of evil is the set of all integers, and the domain is the set of all strings.

```python
def evil(n):
    gen = evilometer(n)
    if sum(list(gen)) % 2 == 0:
        return "It’s Evil!"
    else:
        return "It’s Odious!"
```

# evilometer(n) is a generator that yields \( n \) times if \( n \) is even, and yields \( n//2 \) times if \( n \) is odd.

```python
def evilometer(n):
    while n:
        yield n
        if n % 2:
            n //= 2
        else:
            n -= 1
```

---

**Figure 23.** Train Problem 3321, Solution 33

# circular_prime(number) takes a number and returns True if it is a circular prime, and False otherwise.

# rotate(l, n) takes a list and an integer and returns a new list with the last \( n \) elements of the original list moved to the front.

# is_prime(n) returns True if \( n \) is a prime number, and False otherwise.

---

**Figure 24.** Train Problem 3321, Solution 33

**Figure 25.** Train Problem 3368, Solution 5
```python
# circular_prime takes a number and returns True if it is a circular prime, and False otherwise.
def circular_prime(number):
    if is_prime(number) == False:
        return False
    else:
        for i in range(len(str(number))-1):
            number = rotate(str(number), 1)
            if is_prime(int(number)) == False:
                return False
        return True

# rotate takes a list and an integer and returns a new list with the last n elements of the original list moved to the front.
def rotate(l, n):
    return l[-n:] + l[:-n]

# is_prime returns True if n is a prime number, and False otherwise.
def is_prime(n):
    if n < 2:
        return False
    for i in range(2, int(n ** 0.5) + 1):
        if n % i == 0:
            return False
    return True
```

Figure 26. Train Problem 3368, Solution 5

```python
# circular_prime(n): circular_prime(n) returns True if n is a circular prime, and False otherwise.
circular_prime(n)

# is_prime(n): is_prime takes a number n and returns True if n is prime and False if n is not prime.
is_prime(n)

# circul_num(n, l): circul_num takes an integer n and an integer l, and returns the integer that results from moving the first digit of n to the end of n, and then padding the result with zeros until it has length l.
circul_num(n, l)
```

Figure 27. Train Problem 3368, Solution 6
```python
# circular_prime(n) returns True if n is a circular prime, and False otherwise.
def circular_prime(n):
    # Check if n is a prime number.
    if not is_prime(n):
        return False
    else:
        # Get the length of n.
        length = len(str(n))
        # Get the first digit of n.
        first_digit = str(n)[0]
        # Get the number of rotations.
        rotations = length
        # Rotate n.
        while rotations > 0:
            # Get the new number.
            n = circul_num(n, length)
            # Check if n is a prime number.
            if not is_prime(n):
                return False
            # Decrement the number of rotations.
            rotations -= 1
        return True

# is_prime takes a number n and returns True if n is prime and False if n is not prime.
def is_prime(n):
    if n == 2 or n == 3:
        return True
    if n % 2 == 0 or n < 2:
        return False
    for i in range(3, int(n**0.5)+1, 2):
        if n % i == 0:
            return False
    return True

# circul_num takes an integer n and an integer l, and returns the integer that results from moving
# the first digit of n to the end of n, and then padding the result with zeros until it has length
# l.
def circul_num(n, l):
    n = str(n)
    digit = n[0]
    n = n[1:] + digit
    n = n.ljust(l, "0")
    return int(n)
```

---

```python
# list_squared(m, n): list_squared(m, n) returns a list of lists of integers, where each sublist
# contains an integer and its sum of squared divisors, and the integers are in the range [m, n] (inclusive).
def list_squared(m, n):
    # divisors_list(num): divisors_list takes a number and returns a list of all the divisors of that
    # number.
def divisors_list(num):
        # sum_squares(nums): sum_squares takes a list of numbers and returns the sum of the squares of those
        # numbers.
        # isSquarable(num): isSquarable takes a number and returns True if the number is a perfect square, and
        # False otherwise.
```

---

**Figure 28.** Train Problem 3368, Solution 6

**Figure 29.** Train Problem 3393, Solution 35
list_squared(m, n) returns a list of lists of integers, where each sublist contains an integer and 
its sum of squared divisors, and the integers are in the range [m, n] (inclusive).

def list_squared(m, n):
    result = []
    for num in range(m, n + 1):
        divisors = divisors_list(num)
        sum_of_squares = sum_squares(divisors)
        if isSquarable(sum_of_squares):
            result.append([num, sum_of_squares])
    return result

def divisors_list(num):
    divisors = []
    for i in range(1, num + 1):
        if num % i == 0:
            divisors.append(i)
    return divisors

def sum_squares(nums):
    sum = 0
    for i in range(len(nums)):
        sum += nums[i]**2
    return sum

def isSquarable(num):
    root = num ** (1/2)
    if root % 1 == 0:
        return True
    else:
        return False

Figure 30. Train Problem 3393, Solution 35
D. Compiled Examples
# Takes a board and returns the next iteration of the game of life, but with all values flipped

def game_of_life_inversion_iteration(array_at_time_t):
    # Your code here
    return game_of_life_iteration(invert_array(array_at_time_t))

# Takes a board and returns the next iteration of the game of life

def invert_array(array_at_time_t):
    return [list(map(lambda x: 1-x, row)) for row in array_at_time_t]

# Takes a board and returns the board with all values flipped

def game_of_life_iteration(array_at_time_t):
    # The array that will be returned
    array_at_time_t_plus_1 = []

    # Iterate through the rows of the array
    for i in range(0, len(array_at_time_t)):
        # The array that will contain the next row
        next_row = []

        # Iterate through the columns of the array
        for j in range(0, len(array_at_time_t[i])):
            # The number of neighbors
            num_neighbors = 0

            # Iterate through the neighbors of the cell
            for k in range(-1, 2):
                for l in range(-1, 2):
                    # Don’t count the cell itself
                    if k == 0 and l == 0:
                        continue

                    # Check if the neighbor is valid
                    if i + k >= 0 and i + k < len(array_at_time_t) and j + l >= 0 and j + l < len(array_at_time_t[i]):
                        # If the neighbor is alive, increment the number of neighbors
                        if array_at_time_t[i + k][j + l] == 1:
                            num_neighbors += 1

            # If the cell is alive, check if it should die
            if array_at_time_t[i][j] == 1:
                if num_neighbors < 2 or num_neighbors > 3:
                    next_row.append(0)
                else:
                    next_row.append(1)
            # If the cell is dead, check if it should become alive
            else:
                if num_neighbors == 3:
                    next_row.append(1)
                else:
                    next_row.append(0)

        # Add the next row to the array
        array_at_time_t_plus_1.append(next_row)

    # Return the next array
    return array_at_time_t_plus_1

assert game_of_life_inversion_iteration([[0, 0, 1], [1, 0, 0], [1, 0, 0]]) == [[1, 1, 1], [1, 0, 1], [1, 1, 1]]
assert game_of_life_inversion_iteration([[0, 1, 0, 0], [1, 0, 1, 0], [1, 0, 0, 1], [0, 1, 1, 0]]) ==

Figure 31. Compilation of the program taking Conway’s game of life (Games, 1970) and returning the next state, with the values inverted. In this case, we use automatic decomposition to generate the two subfunctions of game_of_life_iteration.
# Takes a board with active and inactive cells and returns the next iteration of the game of life, but with all the values flipped

def game_of_life_inversion_iteration(array_at_time_t):
    return array_inversion(game_of_life_iteration(array_at_time_t))

# Takes a board with active and inactive cells as a list of lists and returns the next iteration of the game of life

def game_of_life_iteration(array_at_time_t):
    array_at_time_t_plus_1 = []
    for row in range(len(array_at_time_t)):
        array_at_time_t_plus_1.append([])
    for column in range(len(array_at_time_t[row])):
        array_at_time_t_plus_1[row].append(0)
    for row in range(len(array_at_time_t)):
        for column in range(len(array_at_time_t[row])):
            if array_at_time_t[row][column] == 1:
                if count_neighbors(array_at_time_t, row, column) < 2:
                    array_at_time_t_plus_1[row][column] = 0
                elif count_neighbors(array_at_time_t, row, column) > 3:
                    array_at_time_t_plus_1[row][column] = 0
                else:
                    array_at_time_t_plus_1[row][column] = 1
            else:
                if count_neighbors(array_at_time_t, row, column) == 3:
                    array_at_time_t_plus_1[row][column] = 1
    return array_at_time_t_plus_1

# Invert a square array by replacing all 0's with 1's and vice versa

def array_inversion(array):
    inverted_array = []
    for i in range(len(array)):
        inverted_array.append([])
    for j in range(len(array[i])):
        inverted_array[i].append(1 - array[i][j])
    return inverted_array

# Count the number of neighbors

def count_neighbors(array_at_time_t, row, column):
    count = 0
    for i in range(row-1, row+2):
        for j in range(column-1, column+2):
            if i == row and j == column:
                continue
            if i < 0 or j < 0:
                continue
            if i >= len(array_at_time_t) or j >= len(array_at_time_t[0]):
                continue
            if array_at_time_t[i][j] == 1:
                count += 1
    return count

assert game_of_life_inversion_iteration([[0, 0, 1], [1, 0, 0], [1, 0, 0]]) == [[1, 1, 1], [1, 0, 1], [1, 1, 1]]
assert game_of_life_inversion_iteration([[0, 1, 0, 0], [1, 0, 1, 0], [1, 0, 0, 1], [0, 1, 1, 0]]) == [[0, 1, 0, 0], [1, 0, 1, 0], [1, 0, 0, 1], [0, 1, 1, 0]]
assert game_of_life_iteration([[0, 0, 1], [1, 0, 0], [1, 0, 0]]) == [[0, 0, 0], [0, 1, 0], [0, 0, 0]]
assert array_inversion([[1]]) == [[0]]
assert array_inversion([[0, 1], [1, 0]]) == [[1, 0], [0, 1]]

Figure 32. Compilation of the program taking Conway’s game of life (Games, 1970) and returning the next state, with the values inverted. In this case, we use automatic infilling to generate the count_neighbors function.
### Calls base_case if 1, otherwise recursion_rule

```python
def collatz_recursion(num, cur_list=list()):
    if num == 1:
        return base_case(num, cur_list)
    else:
        return recursion_rule(num, cur_list)
```

### Returns the list with the number appended to it

```python
def base_case(num, cur_list):
    cur_list.append(num)
    return cur_list
```

### Add num to list, collatz with 3n + 1 if odd or n / 2 if even

```python
def recursion_rule(num, cur_list):
    cur_list.append(num)
    if num % 2 == 0:
        return collatz_recursion(num / 2, cur_list)
    else:
        return collatz_recursion((3 * num) + 1, cur_list)
```

```python
assert collatz_recursion(19) == [19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1]
```

**Figure 33.** Compilation of the program generating a list corresponding to the Collatz conjecture.
1 -- if p ∧ q, then q ∧ p
2 lemma p_q_implies_q_p {p q: Prop}:
3  p ∧ q → q ∧ p :=
4 begin
5  intro h,
6  cases h with hp hq,
7  split,
8  exact hq,
9  exact hp,
10 end
11
12 -- Description: if p ∨ q, then q ∨ p
13 -- if q ∧ p, then p ∧ q
14 lemma q_p_implies_p_q {p q: Prop}:
15  (q ∧ p) → (p ∧ q) :=
16 begin
17  intro h,
18  split,
19  exact h.right,
20  exact h.left,
21 end
22
23 /-
24 Theorem:
25 -- if q ∧ p, then p ∧ q
26 /-
27 -- the and operator is commutative
28 lemma and_commute {p q: Prop}:
29  (p ∧ q → q ∧ p) ∧ (q ∧ p → p ∧ q) :=
30 begin
31  apply and.intro,
32  { apply p_q_implies_q_p },
33  { apply q_p_implies_p_q }
34 end
35
36 /-
37 -- Description: if p ∧ q, then p
38 -- Signature: p_and_q_implies_p {p q: Prop}
39
40 -- show (p ∧ q → q ∧ p) ∧ (q ∧ p → p ∧ q)

Figure 34. Generated proof of and_commute.
```python
# given a list of lists representing the cost of building a road between any two cities, and a list representing the cost of building
# an airport in a city, return a new cost matrix with a new node corresponding to the sky.
def sky_city_cost(city_road_cost, city_airport_cost):
    ***
    :param city_road_cost: list of lists representing cost of road between any two cities
    :param city_airport_cost: list representing cost of an airport in a city
    return new cost matrix with a new node corresponding to the sky
    ***
    # add new node for sky to cost matrix
um_cities = len(city_road_cost)
sky_city_cost = [[0 for _ in range(num_cities + 1)] for _ in range(num_cities + 1)]
for i in range(num_cities):
    for j in range(num_cities):
        sky_city_cost[i][j] = city_road_cost[i][j]
for i in range(num_cities):
    sky_city_cost[i][-1] = city_airport_cost[i]
sky_city_cost[-1][i] = city_airport_cost[i]
return sky_city_cost

def select_airport_cities(city_road_cost, city_airport_cost):
    # given a list of lists representing the cost of building a road between any two cities, and a list representing the cost of building
    # an airport in a city (where any two cities with airports are connected), return a list of the cities that should have airports built
    # in them to minimize the total cost of building roads and airports such that all cities are connected. The list should be sorted in
    # ascending order.
def select_airport_cities(city_road_cost, city_airport_cost):
    num_cities = len(city_road_cost)
sky_city_cost = [[0 for _ in range(num_cities + 1)] for _ in range(num_cities + 1)]
for i in range(num_cities):
    for j in range(num_cities):
        sky_city_cost[i][j] = city_road_cost[i][j]
for i in range(num_cities):
    sky_city_cost[i][-1] = city_airport_cost[i]
sky_city_cost[-1][i] = city_airport_cost[i]

# this is the adjacency matrix corresponding to the MST
adjacency_matrix = [[0 for _ in range(len(cost_matrix))] for _ in range(len(cost_matrix))]
while len(unvisited) > 0:
    # get the index of the minimum edge
    min_edge_index = -1
    min_edge_value = float('inf')
    for i in range(len(visited)):
        for j in range(len(unvisited)):
            if cost_matrix[visited[i]][unvisited[j]] < min_edge_value:
                min_edge_index = visited[i], unvisited[j]
                min_edge_value = cost_matrix[visited[i]][unvisited[j]]
    # add the minimum edge to our MST
    edges.append(min_edge_index)
    # add the unvisited vertex to the list of visited vertices
    visited.append(min_edge_index[1])
    # remove the unvisited vertex from the list of unvisited vertices
    unvisited.remove(min_edge_index[1])
    # add edges to the adjacency matrix
    for edge in edges:
        adjacency_matrix[edge[0]][edge[1]] = 1
        adjacency_matrix[edge[1]][edge[0]] = 1
    return adjacency_matrix

# given a list of lists representing an adjacency matrix, return a list of the nodes connected to the final node. However, if only one
# node is connected to the final node, return an empty list.
def final_node_connectors(adjacency_matrix):
    final_node = len(adjacency_matrix) - 1
    final_node_connectors = []
    for i in range(len(adjacency_matrix) - 1):
        if adjacency_matrix[i][final_node] == 1:
            final_node_connectors.append(i)
    if len(final_node_connectors) == 1:
        final_node_connectors.append(i)
    return final_node_connectors

# this is the adjacency matrix corresponding to the MST
adjacency_matrix = [[0 for _ in range(len(cost_matrix))] for _ in range(len(cost_matrix))]
while len(unvisited) > 0:
    # get the index of the minimum edge
    min_edge_index = -1
    min_edge_value = float('inf')
    for i in range(len(visited)):
        for j in range(len(unvisited)):
            if cost_matrix[visited[i]][unvisited[j]] < min_edge_value:
                min_edge_index = visited[i], unvisited[j]
                min_edge_value = cost_matrix[visited[i]][unvisited[j]]
    # add the minimum edge to our MST
    edges.append(min_edge_index)
    # add the unvisited vertex to the list of visited vertices
    visited.append(min_edge_index[1])
    # remove the unvisited vertex from the list of unvisited vertices
    unvisited.remove(min_edge_index[1])
    # add edges to the adjacency matrix
    for edge in edges:
        adjacency_matrix[edge[0]][edge[1]] = 1
        adjacency_matrix[edge[1]][edge[0]] = 1
    return adjacency_matrix

# given a list of lists representing the cost of each edge, return an adjacency matrix corresponding to the minimum spanning true.
def minimum_spanning_tree(cost_matrix):
    num_cities = len(cost_matrix)
sky_city_cost = [[0 for _ in range(num_cities + 1)] for _ in range(num_cities + 1)]
for i in range(num_cities):
    for j in range(num_cities):
        sky_city_cost[i][j] = cost_matrix[i][j]
for i in range(num_cities):
    sky_city_cost[i][-1] = city_airport_cost[i]
sky_city_cost[-1][i] = city_airport_cost[i]

# this is a list of the vertices that have been added to the MST
visited = [0]
# this is a list of the vertices that have not been added to the MST
unvisited = [i for i in range(1, len(cost_matrix))]
# this is a list of edges that are part of the MST
edges = []
# this is the adjacency matrix corresponding to the MST
adjacency_matrix = [[0 for _ in range(len(cost_matrix))] for _ in range(len(cost_matrix))]
while len(unvisited) > 0:
    # get the index of the minimum edge
    min_edge_index = -1
    min_edge_value = float('inf')
    for i in range(len(visited)):
        for j in range(len(unvisited)):
            if cost_matrix[visited[i]][unvisited[j]] < min_edge_value:
                min_edge_index = visited[i], unvisited[j]
                min_edge_value = cost_matrix[visited[i]][unvisited[j]]
    # add the minimum edge to our MST
    edges.append(min_edge_index)
    # add the unvisited vertex to the list of visited vertices
    visited.append(min_edge_index[1])
    # remove the unvisited vertex from the list of unvisited vertices
    unvisited.remove(min_edge_index[1])
    # add edges to the adjacency matrix
    for edge in edges:
        adjacency_matrix[edge[0]][edge[1]] = 1
        adjacency_matrix[edge[1]][edge[0]] = 1
    return adjacency_matrix
```

Figure 35. Generated solution for select_airport_cities.
# chess_game takes a board and returns the winner of the game.
def chess_game(board):
    if weight(board) == 0:
        return 'Draw'
    if weight(board) > 0:
        return 'White'
    if weight(board) < 0:
        return 'Black'

# weight takes a board and returns the weight of the board.
def weight(board):
    weight = 0
    for row in range(len(board)):
        for col in range(len(board[row])):
            if board[row][col] != '.':
                weight += weight_helper(board, row, col, 0)
    return weight

# weight_helper takes a board, a row, a column, and a weight and returns the weight of the board.
def weight_helper(board, row, col, weight):
    if board[row][col] == '.':
        return 0
    elif board[row][col] == 'Q':
        return weight + 9
    elif board[row][col] == 'R':
        return weight + 5
    elif board[row][col] == 'B':
        return weight + 3
    elif board[row][col] == 'N':
        return weight + 3
    elif board[row][col] == 'P':
        return weight + 1
    elif board[row][col] == 'q':
        return weight - 9
    elif board[row][col] == 'r':
        return weight - 5
    elif board[row][col] == 'b':
        return weight - 3
    elif board[row][col] == 'n':
        return weight - 3
    elif board[row][col] == 'p':
        return weight - 1
    else:
        return weight

assert repr(str(chess_game('...QK...
........
........
........
........
........
........
......'))) == repr('White')
assert repr(str(chess_game('rnbqkbnr
pppppppp
........
........
........
........
PPPPPPPP
........'))) == repr('Draw')
assert repr(str(chess_game('rppppppr
...k....
........
........
........
........
K...Q...
........'))) == repr('Black')
assert repr(str(chess_game('....bQ.K
.B......
.....P..
........
........
........
...N.P..
.....R..'))) == repr('White')

Figure 36. Generated solution for Problem 368 of the APPS test set, identifying the leader of a chess game from the board.
E. VirtualHome
An action plan is a list of strings that describes a sequence of steps to accomplish a task. To be correctly parsed, an action plan must be syntactically correct and contain only allowed actions and recognizable simple objects. Allowed actions: ‘close’ <arg1>, ‘cut’ <arg1>, ‘drink’ <arg1>, ‘drop’ <arg1>, ‘eat’ <arg1>, ‘find’ <arg1>, ‘grab’ <arg1>, ‘greet’ <arg1>, ‘lie on’ <arg1>, ‘look at’ <arg1>, ‘open’ <arg1>, ‘plug in’ <arg1>, ‘plug out’ <arg1>, ‘point at’ <arg1>, ‘pour’ <arg1> ‘into’ <arg2>, ‘pull’ <arg1>, ‘push’ <arg1>, ‘put’ <arg1> ‘on’ <arg2>, ‘put’ <arg1> ‘in’ <arg2>, ‘put back’ <arg1>, ‘take off’ <arg1>, ‘put on’ <arg1>, ‘read’ <arg1>, ‘release’, ‘rinse’ <arg1>, ‘run to’ <arg1>, ‘scrub’ <arg1>, ‘sit on’ <arg1>, ‘sleep’, ‘squeeze’ <arg1>, ‘stand up’, ‘switch off’ <arg1>, ‘switch on’ <arg1>, ‘touch’ <arg1>, ‘turn to’ <arg1>, ‘type on’ <arg1>, ‘wake up’, ‘walk to’ <arg1>, ‘wash’ <arg1>, ‘watch’ <arg1>, ‘wipe’ <arg1>. To satisfy the common-sense constraints, each action step in this action plan must not violate the set of its pre-conditions (e.g. the agent cannot grab milk from the fridge before opening it) and post-conditions (e.g. the state of the fridge changes from "closed" to "open" after the agent opens it)."

```python

#***# task_plan(): return a list of strings that represents an action plan to put a mug on the stall and bread on the desk.
> "executable"

put_object_on(object, place): return a list of strings that represents an action plan to put an object in a place.
> "mug", "stall" -> "executable"

Figure 37. Full Parsel program including header for Fig. 1 example, with the #***# as the header separator. Note that we essentially just took the executability definition in (Huang et al., 2022) and added the list of available actions.