$f(T)$ models with phantom divide line crossing

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Abstract

In this paper, we propose two new models in $f(T)$ gravity to realize the crossing of the phantom divide line for the effective equation of state, and we then study the observational constraints on the model parameters. The best fit results suggest that the observations favor a crossing of the phantom divide line.

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I. INTRODUCTION

Various observations \( [1-4] \) have confirmed the fact that our Universe is undergoing an accelerating expansion and it entered this accelerating phase only at the near past. The proposals that have been put forth to explain this observed phenomenon can basically be classified into two categories. One is to assume the existence of an exotic energy with negative pressure, named dark energy. The simplest candidate of dark energy is the cosmological constant with the equation of state \( w = -1 \) \( [5] \). It, however, suffers from two serious theoretical problems, i.e., the cosmological constant problem and the coincidence problem. Thus, some scalar field models, such as quintessence \( [6] \) and phantom \( [7] \), are proposed. For single scalar field models, it has been shown that the equation of state cannot cross the phantom divide line \( (w = -1) \). So, models with a combination of phantom and quintessence \( [8, 9] \), and scalar field models with scalar-dependent coupling in front of kinetic term \( [10] \) as well as fluid models \( [11] \) have also been constructed to realize the crossing of the phantom divide line, which still seems to be allowed by recent observations \( [12, 13] \).

Another alternative to account for the current accelerating cosmic expansion is to modify Einstein’s general relativity theory. One such modification is the \( f(R) \) theory \( [14] \) (see \( [15] \) for recent reviews), where the Ricci scalar \( R \) in the Einstein-Hilbert action is generalized to an arbitrary function \( f \) of \( R \). For this theory, it has been found that the effective equation of state can cross the phantom divide line from phantom phase to non-phantom one \( [16, 17] \).

Recently, a new modified gravity which can also explain the accelerating cosmic expansion \( [18] \), named \( f(T) \) theory, has spurred an increasing deal of interest. The \( f(T) \) theory is obtained by extending the action of teleparallel gravity \( [19] \) in analogy to the \( f(R) \) theory, where \( T \) is the torsion scalar. An important advantage of the \( f(T) \) theory is that its field equations are second order as opposed to the fourth order equations of \( f(R) \) gravity. More recently, Linder \( [20] \) proposed some concrete \( f(T) \) models (see also Ref. \( [21] \)). We placed observational constraints on the parameters of some of these models \( [20, 22, 23] \), in particular, and analyzed the dynamical properties of the \( f(T) \)
theory [24], in general. A reconstruction of the $f(T)$ theory from the background expansion history and the $f(T)$ theory driven by scalar fields were studied in [25], and the cosmological perturbations and growth factor of matter perturbations in the $f(T)$ theory were investigated in Refs. [26, 27]. In addition, the issue of local Lorentz invariance was examined in Refs. [28, 29]. It should be noted, however, that the analysis performed in Refs. [31, 32, 33, 34] indicate that models proposed so far in the $f(T)$ theory [20, 21] behave quintessence-like or phantom-like, and thus cannot realize the crossing of the phantom divide line for the effective equation state, although the observational data [12, 13] seems to indicate this crossing is still a possibility not ruled out. So, in this paper, we propose two new $f(T)$ models which can realize the crossing of $-1$ line, and then discuss the constraints on model parameters from recent observations. A remarkable feature of our models is that they realize the crossing of the phantom divide line from a non-phantom phase to a phantom phase in contrast to the viable $f(R)$ models where the phantom divide line is crossed the other way around [17]. It is interesting to note that a crossing of the phantom divide from the non-phantom phase to the phantom one is consistent with the recent cosmological observational data [12]. Finally, let us note that, recently, a new model with the crossing of phantom divide line is also proposed in [32].

II. THE $f(T)$ THEORY

The $f(T)$ theory is obtained by extending the action of teleparallel gravity to $T + f(T)$. The teleparallel theory of gravity is built on teleparallel geometry, which uses the Weitzenböck connection rather than the Levi-Civita connection. So, the spacetime has only torsion and is thus curvature-free.

Assuming that the universe is described by a flat homogeneous and isotropic Friedmann-Robertson-Walker metric

$$g_{\mu\nu} = \text{diag}(1, -a^2(t), -a^2(t), -a^2(t)) ,$$  

where $a$ is the scale factor, it has been found in Refs. [33, 34] that the torsion scalar in the teleparallel gravity can be expressed as

$$T = -6H^2 ,$$  

where
with \( H = \dot{a}a^{-1} \) being the Hubble parameter. In addition, the modified Friedman equations have the following form

\[
H^2 = \frac{8\pi G}{3} \rho - \frac{\dot{f}}{6} + \frac{T}{3} f_T,
\]

where a prime denotes a derivative with respect to \( \ln a \), the subscript \( T \) represents a derivative with respect to \( T \), \( \rho \) is the energy density and \( P \) is the pressure. Here we assume that there are both matter and radiation components in the Universe, thus

\[
\rho = \rho_m + \rho_r, \quad P = \frac{1}{3} \rho_r.
\]

If we rewrite the modified Friedmann equation (Eq. (3)) in the standard form as that in general relativity, we can define an effective dark energy, whose energy density can be expressed as,

\[
\rho_{\text{eff}} = \frac{1}{16\pi G} (-f + 2T f_T).
\]

Here \( \frac{2T f_T}{f} > 1 \) is required in order to have a positive value for \( \rho_{\text{eff}} \). This usually gives a constraint on physically meaningful models. Using energy conservation equation, \( \dot{\rho}_{\text{eff}} + 3H(1 + w_{\text{eff}})\rho_{\text{eff}} = 0 \), one can yield the effective equation of state \( w_{\text{eff}} \)

\[
w_{\text{eff}} = -\frac{f/T - f_T + 2T f_{TT} + \frac{1}{3} \frac{8\pi G \rho_r}{3H^2} (f_T + 2T f_{TT})}{(1 + f_T + 2T f_{TT})(f/T - 2f_T)}.
\]

The same expression can also be obtained using Eq. (4) to define an effective pressure \( p_{\text{eff}} \) and then deriving \( w_{\text{eff}} \).

### III. TWO NEW \( f(T) \) MODELS

In this section, we propose two new \( f(T) \) models, labeled as Model A and Model B, which can realize the crossing of the phantom divide line for the effective equation of state.
Model A

\[ f(T) = \alpha(-T)^n \tanh \frac{T_0}{T}, \quad (8) \]

where \( \alpha \) and \( n \) are two model parameters. The requirement of \( \frac{2T f'}{f} > 1 \), which ensures \( \rho_{eff} > 0 \), gives rise to \( 2n - 4x \cdot \text{csch}[2x] > 1 \) with \( x \equiv \frac{T_0}{T} \). Since \( 4x \cdot \text{csch}[2x] \leq 2 \), \( n \) must be greater than \( \frac{3}{2} \). Substituting Eq. (8) into the modified Friedmann equation, we have

\[ \alpha = -\frac{1 - \Omega_{m0} - \Omega_{r0}}{(6H_0^2)^{n-1}[2\text{sech}(1)^2 + (1 - 2n) \tanh(1)]}. \quad (9) \]

Here \( \Omega_{m0} \) and \( \Omega_{r0} \) are the present dimensionless density parameters of matter and radiation, respectively.

In Fig. (1), we show the evolutionary curves of the effective equation state with different values of \( n \) (right panel) and the cosmic evolution with \( n = 1.65 \) (left panel). From the right panel, one can see that the effective equation of state firstly crosses the phantom divide line from \( > -1 \) (non-phantom phase) to \( < -1 \) (phantom phase), and then evolves to \( > -1 \). So, it crosses the \(-1\) line twice. In order to illustrate why this phenomenon occurs, we plot a figure (Fig. 2) to show the regions \( w_{eff} < -1 \) in \( n - \Omega \) plan with \( \Omega_{m0} = 0.26 \), where \( \Omega = \frac{H}{H_0} \). From this figure, one can see that \( n \) must be smaller than a critical value, i.e. \( n < 1.686 \) when \( \Omega_{m0} = 0.26 \), to render \( w_{eff} \) cross \( -1 \), and, once \( w_{eff} \) cross the \(-1\) line, it must cross it twice. This makes the \( f(T) \) models distinct from the viable \( f(R) \) models where only a crossing from phantom phase to non-phantom one is allowed [17]. Finally, \( w_{eff} \) approaches to \(-1\), which means that the final state of our Universe is an exponential expansion phase. This result is consistent with what obtained in Ref. [24] where it has been found through the dynamical analysis that the Universe in the \( f(T) \) theory finally enters a de Sitter expansion phase. Furthermore, the right panel reveals that the Universe, in this model, has a long enough period of radiation domination to give the correct primordial nucleosynthesis and radiation-matter equality, and a matter dominated phase. In other words, the usual early universe behavior can be successfully obtained to agree with the primordial nucleosynthesis and the cosmic microwave background constraints.

Model B

\[ f(T) = \alpha(-T)^n(1 - e^{\beta T_0/T}) \]

(10)
FIG. 1: The evolutionary curves of the effective equation of state with different values of $n$ and $\Omega_{m0} = 0.26$ (left panel), and the cosmic evolution with $n = 1.65$, $\Omega_{m0} = 0.26$ and $\Omega_{r0} = 0.26/1200$ (right panel) for Model A. In the right panel, the dot-dashed, dashed, and solid lines represent the evolutionary curves of the dimensionless density parameters for the effective dark energy, radiation and matter, respectively.

FIG. 2: The regions of phantom and quintessence in $n - E$ plane with $\Omega_{m0} = 0.26$, where $E = H/H_0$. The red dashed line is the minimum value to which the universe can reach.

with three model parameters $\alpha$, $n$ and $p$. From $\frac{2Tf}{f} > 1$ given by the requirement of $\rho_{eff} > 0$, we obtain that $2n - \frac{2xe^x}{1+x^2} > 1$ with $x \equiv pT_0/T$. This leads to $n > 0.5$ since $-\frac{2xe^x}{1+x^2} \leq 0$. So, we now restrict our discussion to the case of $n > \frac{1}{2}$ for Model B. Using
the modified Friedmann equation, we have
\[
\alpha = \frac{(6H_0^2)^{1-n}(1 - \Omega_{m0} - \Omega_{r0})}{-1 + 2n + e^p(1 - 2n + 2p)}.
\] (11)

The exponential model given by Linder [20] is a special case, the \( n = 1 \) case to be exact, of the present model. When \( p = 0 \), our model reduces to the power low model \( f(T) \sim (-T)^n \), which has been studied in detail in Refs. [18, 20, 22–24]. Let us note that when \( n = 1 \) or \( p = 0 \), the crossing of phantom divide line is impossible as we have already pointed out [24]. This is also confirmed by the \( n = 1 \) case in Fig. (3) in the present paper. Fig. (3) shows the evolutionary curves of \( w_{eff} \) (left and middle panels) and the cosmic evolution (right panel) for model B. We find that, for the crossing of the \(-1\) line to occur, it is required that \( p \) and \( n - 1 \) should have the same sign. When \( p > 0 \) and \( n > 1 \), \( w_{eff} \) evolves from \( > -1 \) to \( < -1 \), while, when \( p < 0 \) and \( \frac{1}{2} < n < 1 \), the crossing direction is just the opposite. In addition, we also find that, the model behaves like quintessence when \( p > 0 \) and \( \frac{1}{2} < n < 1 \), and like phantom when \( p < 0 \) and \( n > 1 \). The right panel in Fig. (3) gives the cosmic evolution with \( n = 1.1 \) and \( p = 0.1 \), from which one can see that the usual early universe behavior can also be obtained just as Model A.

**FIG. 3**: The evolutionary curves of the effective equation of state with different values of \( p \) and \( n \) and \( \Omega_{m0} = 0.26 \) (left and middle panels), and the cosmic evolution with \( n = 1.1 \), \( p = 0.1 \) \( \Omega_{m0} = 0.26 \) and \( \Omega_{r0} = 0.26/1200 \) (right panel) for Model B. In the right panel, the dot-dashed, dashed, and solid lines represent the evolutionary curves of the dimensionless density parameters for the effective dark energy, radiation and matter, respectively.
IV. OBSERVATIONAL CONSTRAINTS

Now, we discuss the constraints on model parameters of Model A and Model B from recent observational data, including the Type Ia supernovae (Sne Ia), the baryonic acoustic oscillation (BAO) distance ratio and the cosmic microwave background (CMB) radiation. The Sne Ia data used in our analysis is the Union2 compilation released by the Supernova Cosmology Project collaboration recently [35], which consists of 557 data points and is the largest published sample today. Using the usual method, we constrain the theoretical model from the Sne Ia by minimizing the $\hat{\chi}^2$ value

$$\hat{\chi}^2_{\text{Sne}} = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_{u,i}^2},$$

where $\sigma_{\mu,i}^2$ are the errors due to the flux uncertainties, intrinsic dispersion of Sne Ia absolute magnitude and peculiar velocity dispersion. $\mu_{\text{obs}}$ is the observed distance moduli and $\mu_{\text{th}}$ is the corresponding theoretical one, which is defined as

$$\mu_{\text{th}} = 5 \log_{10} D_L - \mu_0.$$  

Here $\mu_0 = 5 \log_{10} (h + 42.38)$ with $h = H_0/100 \text{ km/s/Mpc}$, and $D_L$ is the luminosity distance,

$$D_L \equiv (1 + z) \int_0^z \frac{dz'}{E(z')} ,$$

with $E(z) \equiv H(z)/H_0$. In order to marginalize the nuisance parameter $\mu_0$ (or $h$), following the approach given in Ref. [36], we expand $\chi^2_{\text{Sne}}$ to $\chi^2_{\text{Sne}}(\mu_0) = A\mu_0^2 - 2B\mu_0 + C$ with $A = \sum 1/\sigma_{u,i}^2$, $B = \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L]/\sigma_{u,i}^2$, and $C = \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L]^2/\sigma_{u,i}^2$, and find that $\chi^2_{\text{Sne}}$ has a minimum value at $\mu_0 = B/A$, which is given by

$$\chi^2_{\text{Sne}} = C - \frac{B^2}{A}.$$  

Thus, we can minimize $\chi^2_{\text{Sne}}$ instead of $\hat{\chi}^2_{\text{Sne}}$ to obtain constraints from Sne Ia.

For the BAO data, the BAO distance ratio at $z = 0.20$ and $z = 0.35$ from the joint analysis of the 2dF Galaxy Redshift Survey and SDSS data [37] is used. This distance ratio

$$\frac{D_V(z = 0.35)}{D_V(z = 0.20)} = 1.736 \pm 0.065$$
is a relatively model independent quantity with $D_V(z)$ defined as

$$D_V(z_{BAO}) = \left[ \frac{z_{BAO}}{H(z_{BAO})} \left( \int_0^{z_{BAO}} \frac{dz}{H(z)} \right) \right]^{1/3}. \quad (17)$$

So, the constraint from BAO can be obtained by performing the following $\chi^2$ statistics

$$\chi^2_{BAO} = \frac{[D_V(z = 0.35)/D_V(z = 0.20) - 1.736]^2}{0.065^2} \quad (18)$$

Finally, we add the CMB data in our analysis. Since the CMB shift parameter $R$ contains the main information of the observations from the CMB, it is used to constrain the theoretical models by minimizing

$$\chi^2_{CMB} = \frac{[R - R_{obs}]^2}{\sigma_R^2}, \quad (19)$$

where $R_{obs} = 1.725 \pm 0.018$, which is given by the WMAP7 data, and its corresponding theoretical value is defined as

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_{CMB}} \frac{dz'}{E(z')}, \quad (20)$$

with $z_{CMB} = 1091.3$.

The constraints from a combination of Sne Ia, BAO and CMB can be obtained by calculating $\chi^2_{Sne} + \chi^2_{BAO} + \chi^2_{CMB}$. We find that, for Model A, the best fit values occur at $\Omega_{m0} = 0.282$ and $n = 1.65$ with $\chi^2_{Min} = 543.948$. The contour diagrams at the 68.3% and 95.4% confidence levels are given in Fig. (4). From this figure and Fig. (1), we conclude that the observation favors a crossing of phantom divide line.

For Model B, the best fit values of model parameters are $\Omega_{m0} = 0.267$, $p = 0.02$ and $n = 1.08$ with $\chi^2_{Min} = 544.213$. It is easy to see that the best fit value favors a crossing of the phantom divide line from $> -1$ (non-phantom phase) to $< -1$ (phantom phase). This is consistent with the recent observational data but is opposite to what was found in viable $f(R)$ models. Fig. (5) gives the constraints in the $n - p$ plane with $\Omega_{m0} = 0.267$ at the 68.3% and 95.4% confidence levels, and in this figure $n > \frac{1}{2}$ given by the requirement of $\rho_{eff} > 0$ has been taken into consideration. From Figs. (3, 5), one can see that all possible behaviors of $w_{eff}$ shown in the left and middle panels of Fig. (3) are allowed by observations.
FIG. 4: The constraint on $\Omega_m$ and $n$ at the 68.3% and 95.4% confidence levels for Model A from Sne Ia+BAO+CMB.

Now we consider the constraints on the $\Lambda$CDM model. The best fit result is $\Omega_m = 0.270$ with $\chi^2_{Min} = 544.403$. This $\chi^2_{Min}$ is slightly larger than that obtained in the above two $f(T)$ models. With the $\chi^2_{Min}/\text{dof}$ (dof: degree of freedom) criterion, the $\Lambda$CDM is slightly favored by observations.

FIG. 5: The constraint on $p$ and $n$ for Model B with $\Omega_m = 0.267$ at the 68.3% and 95.4% confidence levels from Sne Ia+BAO+CMB. $n > \frac{1}{2}$ given by the requirement of $\rho_{eff} > 0$ is considered.
V. CONCLUSION

The $f(T)$ theory is a new modified gravity, obtained by extending the teleparallel gravity, to account for the current accelerating cosmic expansion without the need of dark energy. In this paper, we have proposed two new $f(T)$ models in which the crossing of the phantom divide line is possible. A remarkable feature of the our models is that they realize the crossing of the phantom divide line from a non-phantom phase to a phantom phase in contrast to the viable $f(R)$ models where the phantom divide line is crossed the other way around \[17\]. It is interesting to note that a crossing of the phantom divide from the non-phantom phase to the phantom one is consistent with the recent cosmological observational data \[12\]. By studying the evolutionary curves of $w_{\text{eff}}$, we find that $w_{\text{eff}}$ can cross the $-1$ line in both models and it is crossed twice in Model A. Furthermore, we also find that both models can produce the usual early universe behaviors in the sense that they both allow a long enough period of radiation domination and a matter dominated phase to agree with the primordial nucleosynthesis and the cosmic microwave background constraints. We have also discussed the constraints on model parameters from recent observations including Sne Ia, BAO and CMB. Our results show that observations favor a crossing of the $-1$ line for Model A, whereas, for Model B, all possible evolutions for $w_{\text{eff}}$ given in Fig. \[3\] are allowed, although the best fit result favors the crossing. With the $\chi^2_{\text{Min}}/\text{dof}$ (dof: degree of freedom) criterion, we find that the $\Lambda$CDM is still favored slightly by observations.

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