Improving the accuracy of designing a delta robot for 3D printing

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Abstract. Currently, there is a need in the industry for design changes to existing installations, such as conveyor lines, various machine tools, 3D printers, and so on. Designing delta robots for 3D printers reveals the advantages of using delta robots as working parts of printers compared to traditional designs. In this article, the direct and inverse problems of the kinematics of the delta robot are solved in a geometric way. Also, dependencies for the search for angular velocities and accelerations of the input links were obtained, which allows in the future to design more accurate working bodies of 3D printers. The research was carried out through mathematical modeling.

1. Introduction

In connection with more and more new requests for increasing productivity, there is a need not only to modernize existing machines, but also to fundamentally change the mechanisms themselves performing the movement of objects. Quite often, industrial tasks involve mechanisms that represent open kinematic chains, that is, a number of consecutive links connected to each other in a movable way. Such mechanisms are simple, reliable and quite applicable for solving most applied problems. However, they have a number of disadvantages, which, first of all, include low rigidity and high inertia, which ultimately reduces the speed of movement of the executive link and the accuracy of its positioning [1].

One of the ways to eliminate these shortcomings is to use mechanisms of the parallel structure. They are closed kinematic chains and perceive the load as spatial trusses [2]. An example of such a mechanism is the delta robot invented by Raymond Clavel at the end of the last century. The main advantage of this mechanism is the high speed of manipulation of light objects, achieved due to the fact that heavy drives are located on a fixed base, and all moving links of the mechanism are made of light, often composite materials. Due to these characteristics, designing a 3D printer based on a delta robot reveals the advantages of operating the latter in the best possible way.
2. Main part

The typical design of the delta robot is shown in figure 1. The robot consists of a massive base 1, to which the drives 3 are attached, changing the positions of the levers 2, the planes of rotation of which are rotated 120° relative to each other. Each lever ends with two three-movable hinges. The mobile platform 4 also has two three-movable hinges on each of the delta robot's shoulders. The shoulder for this mechanism will be called the combination of a lever and two rods connected to it 3.

Moving the platform to a given point is carried out by turning the three levers at the desired angles. The motors that drive each of the levers are located on a fixed base, which makes the moving part lighter, that is, less inertial, and therefore faster. This is the reason for the main advantage of this design – the ability to develop high speeds and accelerations.

Determining the angles of rotation of the shoulders, at which the center of the mobile platform will have the specified coordinates, is called the inverse kinematic problem. The solution of this problem plays an important role, since in most cases the robot will be given a command to move its working body to a given point, and the control system will have to calculate the angles to which the shafts of each engine need to be rotated in order to execute this command.

There is also a direct kinematic problem, as a result of which, according to the known angles of rotation of the levers (they can be determined if rotation angle sensors are installed on the axes of rotation of the levers), it is possible to determine the position in which the working body is currently located. This may be necessary to determine the size and appearance of the workspace or adjust the position.

2.1. Inverse kinematic task

To solve the inverse kinematic task, we first determine which structural dimensions of the installation are known. The length of the side of the upper base $F$ (the triangle formed by the axes of rotation of the levers), the length of the lever $R_l$ (the distance from the axis of the lever to the common axis of the three-movable hinges), the length of the rod $R_r$ (the distance between the common axes for the three-movable hinges of the lever and the platform) and the length of the side of the platform $f$ (the triangle formed by the common axes of the hinges of the movable platform) will be considered known. In addition, the required coordinates of the platform center $V(x_V, y_V, z_V)$ are considered known. The angles of rotation of the levers $\theta_1$, $\theta_2$ and $\theta_3$, measured from the base plane, are subject to determination. These parameters are presented more clearly in figure 2.

First, it is necessary to introduce a simplification - replace two rods of each arm with one and assume that the planes of the base and platform will always be parallel, that is, in figure 2, segments $AD$ and $BC$ are replaced with $LM$.

A right-hand Cartesian coordinate system is introduced, the origin of which is located at the intersection of the heights of the base triangle $O$. The $Y$ axis is directed along one of these heights, the $X$ axis is located in the base plane, the $Z$ axis will then be perpendicular to the base plane. The choice of
the coordinate system, as well as all further considerations related to geometry, are illustrated in figure 3.

Figure 2. Robot design, 1-base; 2-lever; 3-drive; 4-platform.

Figure 3. Points of intersection of the trajectories of the ends of the lever and the rod.

A shoulder lying in the \(YOZ\) plane is considered. To determine the required angle \(\theta_1\), it is enough to know the coordinates of the points \(Q\) and \(L\), and it is possible to calculate the angle using the formula:

\[
\theta_1 = 180^\circ + \arctg \left( \frac{-z_L}{y_Q - y_L} \right)
\]  

(1)

where \(z_L\) – is the coordinate of the point \(L\) on the \(z\) axis; \(y_Q\) – is the coordinate of the point \(Q\) on the \(y\) axis; \(y_L\) – is the coordinate of the point \(L\) on the \(y\) axis.

The \(Y_Q\) coordinate of the point \(Q\) along the \(Y\) axis can be found using the formula known for an equilateral triangle to find the radius of an inscribed circle:

\[
Y_Q = -\frac{F\sqrt{3}}{6}
\]

(2)

where \(F\) – is the length of the side of the base triangle.

In order to find the coordinates of the point \(L\), it is necessary to analyze the trajectories of the movement of the ends of the lever and the rod. The point \(L\) belonging to the lever describes a circle in the \(YOZ\) plane with the center at the point \(Q\). The point \(M\) is a three-moving pair, so the point \(L\) belonging to the rod describes a sphere centered at the point \(M\). Therefore, to determine the coordinate of the point \(L\) of the entire mechanism, you need to find the intersection point of the sphere and the circle. There are two such intersection points – \(L\) and \(L_1\), you need to choose a point whose \(Y\)-axis coordinate is less. Accordingly, it is necessary to solve a system of two equations:

\[
\begin{cases}
(y_L - y_Q)^2 + (z_L)^2 = R_l^2 \\
x_M^2 + (y_L - y_M)^2 + (z_L - z_M)^2 = R_r^2
\end{cases}
\]

(3)

where \(x_k, y_k, z_k\) – are the coordinates of the points \(k\) along the corresponding axes; \(R_l\) – is the lever length, \(R_r\) – is the rod length.

The coordinate of the point \(M\) along the \(Y\) axis is calculated using the formula (4), similar to the formula (2):
$Y_M = -\frac{f\sqrt{3}}{6} + Y_V$  

(4)

where $f$ is the length of the side of the platform triangle. The remaining coordinates of point $M$ are equal to the corresponding coordinates of point $V$, which leads to the final system (5):

\[
\begin{align*}
(y_L - y_Q)^2 + (z_L)^2 &= R^2_l \\
x_L^2 + (y_L - y_V)^2 + (z_L - z_M)^2 &= R^2_l
\end{align*}
\]

(5)

By opening the brackets in the first and second equations and subtracting the second from the first equation, an equality is obtained from which $z_L$ can be expressed in a linear way by the formula:

$$z_L = \frac{2y_Ly_Q - 2y_Ly_M + \sigma}{2z_V}$$  

(6)

where $\sigma = R^2_l - R^2 + x_L^2 + y_L^2 - y_Q^2 + z_V^2$. Then, substituting this into the first equation of the system and reducing it to the form of a quadratic equation with respect to $y_L$, we can create a system (7) for calculating the coefficients of the quadratic equation:

\[
\begin{align*}
\begin{cases}
a = \frac{(y_M - y_Q)^2}{z_V^2} \\
b = -y_Q - \frac{(y_M - y_Q)\sigma}{z_V^2} \\
c = \frac{\sigma^2}{4z_V^2} + y_Q^2 - R^2_l
\end{cases}
\end{align*}
\]

(7)

Next, the smaller root of the quadratic equation is searched for by formula (8), and the resulting solution is substituted into formula (6), which leads to finding an unknown coordinate of the point $L$ along the $z$ axis:

$$y_L = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$  

(8)

However, there is another way to find the angle of rotation of the first shoulder. We can proceed to the plane problem, since it is known that the $YOZ$ plane intersects the spherical trajectory of the end of the rod along a circle with a cent at the point $N$, which is shown in figure 4.

Figure 4. Intersection of circles in the $YOZ$ plane.
The radius of this circle – $NL$ can be found from the right triangle $LMN$ by the Pythagorean theorem. Since the $NM$ side, in fact, is the coordinate of the point $V$ on the $X$ axis, and the $LM$ side is a rod of length $R$, we can get the formula:

$$NL = \sqrt{R^2 - x_V^2}$$

(9)

Moving to the $YOZ$ plane, we can consider the triangle $QLN$. The side of $NQ$ can be found by knowing the coordinates of the points $N$ and $Q$ by the formula:

$$NQ = \sqrt{(y_m - y_q)^2 + (-z_V)^2}$$

(10)

The lengths of the other sides are also known, therefore, according to the cosine theorem, we can find the angle $\alpha$:

$$\alpha = \arccos \left( \frac{R^2 + NQ^2 - NL^2}{2R \cdot NQ} \right)$$

(11)

The required angle $\beta$ is found as the arccosine by the formula:

$$\beta = \arccos \left( \frac{y_m - y_q}{NQ} \right)$$

(12)

Then the desired angle can be calculated as the difference between the total revolution and the found angles:

$$\theta_i = 360^\circ - \alpha - \beta$$

(13)

To find the two remaining angles of rotation of the levers, use the following technique. An auxiliary coordinate system is introduced, rotated by $120^\circ$ around the $Z$ axis relative to the main $X_{120}Y_{120}Z_{120}$ (figure 5):

![Figure 5. Rotation of the coordinate system.](image)

In the new coordinate system, the problem for determining the angle $\theta_2$ is reduced to the problem of determining the angle $\theta_1$ with the only difference that the coordinates of the point $V$ need to be translated from $XYZ$ to $X_{120}Y_{120}Z_{120}$ according to the known formulas:

$$x_{V_{120}} = x_V \cos 120^\circ - y_V \sin 120^\circ$$

$$y_{V_{120}} = x_V \sin 120^\circ + y_V \cos 120^\circ$$

(14)
where \( x_{V_{120}} \) и \( y_{V_{120}} \) - coordinates of the point \( V \) along the \( X \) and \( Y \) axes in the coordinate system \( X_{120}Y_{120}Z_{120} \). Similarly, it is possible to calculate the angle of rotation of the third arm by entering a coordinate system rotated \( 240^\circ \). These formulas are valid only for the case when we find the coordinates of a point in a system rotated clockwise relative to the main one.

2.2. Direct kinematic task

The initial data for solving the direct kinematic problem are the angles of rotation of the levers. Knowing these angles, you can find the coordinates of the point \( L_1 \) from the right-angled triangle \( Q_1W_1L_1 \), located in the \( YOZ \) plane (figure 6).

Using the previously found coordinate of the point \( Q \), the following formulas are obtained:

\[
y_{L_1} = \frac{-F\sqrt{3}}{6} - R_x \cos(\theta_1 - 180^\circ)
\]

\[
z_{L_1} = -R_y \sin(\theta_1 - 180^\circ)
\]

where \( y_k \) and \( z_k \) - coordinates of points \( k \) along the corresponding axes. To find the coordinates along the \( Z \) axis of the remaining points \( L_2 \) and \( L_3 \), similar formulas (16) are used.

\[
z_{L_2} = -R_y \cos(\theta_2 - 180^\circ)
\]

\[
z_{L_3} = -R_y \sin(\theta_3 - 180^\circ)
\]

(16)

On the other hand, to search for coordinates along the \( Y \) axis, it is correct to use the already known trick - to calculate the coordinates along this axis in coordinate systems rotated \( 120^\circ \) and \( 240^\circ \) relative to the main one, denoting them \( y'_{L_2} \) and \( y'_{L_3} \), and then recalculate them into the main coordinate system according to the formulas (17). In contrast to formulas (14), formulas are used for conversion into a system rotated counterclockwise relative to the main one.

\[
y'_{L_2} = \frac{-F\sqrt{3}}{6} - R_x \cos(\theta_2 - 180^\circ)
\]

\[
y'_{L_3} = \frac{-F\sqrt{3}}{6} - R_x \cos(\theta_3 - 180^\circ)
\]

\[
x_{L_2} = y'_{L_2} \sin 120^\circ
\]

\[
y_{L_2} = y'_{L_2} \cos 120^\circ
\]

\[
x_{L_3} = y'_{L_3} \sin 240^\circ
\]

\[
y_{L_3} = y'_{L_3} \cos 240^\circ
\]

(17)
It is known that points $L_1$, $L_2$, and $L_3$ are three-movable hinges, and if we take these points as centers of rotation, then their ends - points $M_1$, $M_2$, and $M_3$ describe 3 spheres. Next, each bar is shifted parallel to the $XOY$ plane so that points $M_1$, $M_2$, and $M_3$ are at points $V$, which is clearly shown in figure 7.

Now the spheres under consideration (already with centers at points $P_1$, $P_2$, and $P_3$) intersect just at point $V$. To find the coordinates of the new centers of the spheres, they move to the $XOY$ plane (figure 7). To find the coordinate of the point $P_1$, it is enough to shift the coordinate along the $Y$ axis of the point $L_1$ by $f\sqrt{3}/6$ in an upward direction.

![Figure 7. Intersection of spheres at point $V$.](image)

![Figure 8. Finding the coordinates of points $P_1$, $P_2$, and $P_3$.](image)

The coordinates of the remaining points are easily found by considering a right triangle $VM_3T$, the hypotenuse of which is also $f\sqrt{3}/6$. From it, the coordinates of points $P_2$ and $P_3$ are determined by the formulas:

\[
\begin{align*}
  x_{P_2} &= x_{L_1} + \frac{f\sqrt{3}}{6} \cos 30^\circ \\
  y_{P_2} &= y_{L_1} - \frac{f\sqrt{3}}{6} \sin 30^\circ \\
  x_{P_3} &= x_{L_1} - \frac{f\sqrt{3}}{6} \cos 30^\circ \\
  y_{P_3} &= y_{L_1} - \frac{f\sqrt{3}}{6} \sin 30^\circ
\end{align*}
\]

(18)

where $x_k$ and $z_k$ - coordinates of points $k$ along the corresponding axes.

After the above calculations, It is possible to find the necessary coordinates of the point $V$, as the solution to the system of equations (19) that is smaller along the $Z$ axis, which is the lower point of intersection of the three spheres.

\[
\begin{align*}
  (x_{P_1} - x_V)^2 + (y_{P_1} - y_V)^2 + (z_{P_1} - z_V)^2 &= R_1^2 \\
  (x_{P_2} - x_V)^2 + (y_{P_2} - y_V)^2 + (z_{P_2} - z_V)^2 &= R_2^2 \\
  (x_{P_3} - x_V)^2 + (y_{P_3} - y_V)^2 + (z_{P_3} - z_V)^2 &= R_3^2
\end{align*}
\]

(19)

To get a solution, the first step is to expand the brackets and move all known terms to the right. The right-hand sides of the equations in the system are immediately denoted by the constants $w_1$, $w_2$, and $w_3$. 


and then the second equation is subtracted from the first, and then the third from the first, thus it is possible to get the system:

\[
\begin{align*}
-2x_Px_V - 2y_Py_V - 2z_Pz_V + 2x_Px_V + 2y_Py_V + 2z_Pz_V &= w_1 - w_2 \\
-2x_Px_V - 2y_Py_V - 2z_Pz_V + 2x_Px_V + 2y_Py_V + 2z_Pz_V &= w_1 - w_3
\end{align*}
\]

(20)

In this system \(x_V, y_V, z_V\) are outside the brackets and the left and right parts are divided by 2:

\[
\begin{align*}
\left(x_P - x_V\right) + y_V\left(y_P - y_V\right) + z_V\left(z_P - z_V\right) &= \frac{w_1 - w_2}{2} \\
\left(x_P - x_V\right) + y_V\left(y_P - y_V\right) + z_V\left(z_P - z_V\right) &= \frac{w_1 - w_3}{2}
\end{align*}
\]

(21)

The designations \((x_{P2}-x_{P1}), (y_{P2}-y_{P1}), (z_{P2}-z_{P1})\) and \((w_1-w_2)/2\) for \(a_1, b_1, c_1\) and \(d_1\) respectively. In the second equation, similar designations are introduced, but with indices equal to 2. Next, we can express from the resulting system \(x_V\) and \(y_V\) in terms of \(z_V\), obtaining the equations:

\[
\begin{align*}
x_V &= \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} z_V - \frac{b_2d_2 - b_1d_1}{a_1b_2 - a_2b_1} \\
y_V &= \left(\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}\right) z_V + \frac{a_2d_2 - a_1d_1}{a_1b_2 - a_2b_1}
\end{align*}
\]

(22)

Then, substituting this into the first equation of system (19), we get a quadratic equation (23) with respect to the variable \(z_V\), calculating the smaller root of which is the required coordinate of the point \(V\) along the \(Z\) axis:

\[
a_qi z_V^2 + b_qi z_V + c_qi = 0
\]

(23)

where:

\[
\begin{align*}
a_{qi} &= e_1^2 + e_2^1 + 1 \\
b_{qi} &= 2e_1\left(f_1 - x_V\right) - 2z_P + 2e_2\left(f_2 - y_V\right) \\
c_{qi} &= z_P^2 + \left(f_1 - x_V\right)^2 + \left(f_2 - y_V\right)^2 - R_i^2 \\
e_1 &= \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}; f_1 = -\frac{b_2d_2 - b_1d_1}{a_1b_2 - a_2b_1} \\
e_2 &= -\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}; f_2 = \frac{a_2d_2 - a_1d_1}{a_1b_2 - a_2b_1}
\end{align*}
\]

(24)

Knowing the coordinate \(z_V\), the remaining coordinates of the point \(V\) are found according to the already derived formulas (24).

3. Conclusions

In the article, the direct and inverse problems of the kinematics of the delta robot are solved in a geometric way (for the direct kinematic problem, the angles of rotation of the shoulders are determined at which the center of the movable platform will have the given coordinates; for the inverse kinematic problem, the position in which the working body is located according to the known angles of rotation is determined). Dependencies for the search for angular velocities and accelerations of the input links are obtained.
The solution of these problems plays an important role, since most often the robot is given a command to move its working body to a given point, and the control system must calculate the angles by which the shafts of each motor must be turned in order to execute the command.

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