We discuss masses of stellar black holes found in binary systems and errors in their determination. The observed mass distribution has a broad shape within the range $4 - 16 M_\odot$ without visible concentration to some preferred value. On the other hand, the initial black hole mass function as inferred from observations of luminous X-ray sources in other galaxies shows a power-law form. We show that both dynamically obtained black hole mass function and derived from X-ray observations can be made consistent in the frame of the hypothesis of enhanced black hole evaporation on a RSII brane for reasonable values of the warp-factor of the Anti-de-Sitter bulk space.

1 Introduction

There is a growing number of dynamical mass determinations of invisible compact objects in binary systems that are believed to be black holes (BH) primarily for their large mass ($> 3 M_\odot$) and lack of evidence for having solid surface (see e.g. Orosz\(^1\), Cherepashchuk\(^2\) for recent reviews). The mass distribution of compact objects in binary systems shows that masses of neutron stars (NS) are concentrated within a narrow range $M_{NS} = (1 - 2) M_\odot$ while all stellar BH masses fall within a wide interval $\sim 4 - 16 M_\odot$ with a “mass gap” 2-4 $M_\odot$ (e.g. Bailyn et al.\(^3\),\(^4\) and references therein). Indirect information on BH masses in binary systems can also be obtained from examining properties of X-ray luminosity function of the most luminous (with X-ray luminosity $L_x > 10^{39}$ erg/s) X-ray point sources observed in external galaxies by X-ray satellites assuming these sources to contain stellar mass BH and their luminosities being at the Eddington level. While dynamical BH masses appear to have a nearly flat distribution within the 4-16 $M_\odot$ range $dN/dM \sim M^0$, the X-ray luminosity function evidences for a power-law BH mass distribution $dN/dM \sim M^{-1.9-2.2}$. Unless some selection effects shape the form of BH mass distribution in both cases, BH mass functions derived from dynamical and X-ray observations
are very different. Here we hypothesize that the above properties of the observed BH mass function can be explained assuming an initial power-law BH mass function \((dN/dM)_0 \sim M^{-\alpha}\) with \(\alpha \sim -2...-3\) and secular mass decrease of BH mass which is possible in the framework of some modern theories of multidimensional gravity.

2 Determination of black hole masses and their errors

The detailed analysis of systematic errors in determination of BH masses from observations which can affect the observed BH mass distribution is given elsewhere and we only briefly describe them here.

2.1 Dynamical BH mass determination in binary systems

The basic information on the mass of the invisible component in a binary system is obtained from the mass function of the optical companion

\[
f_v(m) = \frac{m_x^3 \sin^3 i}{(m_x + m_v)^2} = 1.038 \times 10^{-7} K_v^3 P(1 - e^2)^{3/2},
\]

where \(K_v\) is the semi-amplitude of the optical star radial velocity curve (in km/s), \(m_v\) and \(m_x\) are masses of the optical and invisible star (in solar units), respectively, \(P\) is the orbital period of the binary system (in days), \(e\) is the orbital eccentricity and \(i\) is the binary inclination angle with respect to the line of sight. The mass of the invisible companion as derived from the optical mass function is

\[
m_x = f_v(m) \left(1 + \frac{1}{q}\right)^2 \frac{1}{\sin^3 i}. \tag{2}
\]

where \(q = m_x/m_o\) is the mass ratio. The only value of \(f_v(m)\) immediately provides the lower limit \(m_x \geq f_v(m)\). More precise value of \(m_x\) requires the knowledge of \(q\) and \(\sin i\).

a) Optical star form effects.

If the mass ratio \(q < 1\) (high-mass X-ray binaries, HMXB, e.g. Cyg X-1, LMC X-1, SS 433)), the system’s barycenter lies within the optical star body and tidal distortions of the optical star form mostly influence the spectral line profiles which are used to determine the radial velocity curve. Another distortion of radial velocity curve and its semi-amplitude \(K_v\) in HMXB is due to variable selective absorption of light by strong stellar wind of the massive O-B optical star.

In low-mass X-ray binaries (LMXB) \(q > 1\) the system’s barycenter is outside the optical star so the effects of the star form can be neglected. Stellar wind effects from low-massive A-M dwarfs in these systems are also insignificant.

b) Stellar rotation effects.

The mass ratio \(q\) is usually derived from rotational broadening of absorption lines in the optical star spectrum. Indeed, assuming synchronous axial and orbital rotation, one can find

\[
v \sin i = 0.462 K_v q^{-1/3} \left(1 + \frac{1}{q}\right)^{2/3}. \tag{3}
\]

The determination of \(v \sin i\) from usual analysis of absorption line profiles can not be made better than to within the 10-20% errors due to X-ray heating of the optical star atmosphere. However, parameters \(q, i\) can be derived from orbital variability of the absorption line profiles using new method proposed by Antokhina and Cherepashchuk and Shahbaz. This allows the strong reducing of errors in the parameter \(q\) determination.

c) Binary inclination angle effects.

Uncertainties in the binary inclination angle \(i\) provide the largest error in the mass determination. The usual way of measuring \(i\) is from the ellipticity effect of the optical star. The
main uncertainty here comes from model-dependent contributions of other emitting structures (gaseous stream, accretion disk) into the total optical or infrared variability of the system and is especially important (> 50%) in LMXB (in which most BH candidates have been discovered). The new method of determination of parameters $q$, $i$ based on the analysis of the orbital variation of the absorption line profiles is independent of contribution from other gas structures in the binary system. However, it can be applied at a very high spectral resolution of 50000-100000 and can be realized only on very large telescopes.

So, with the 90% certainty we can state that masses of stellar BH measured dynamically in binary systems span a wide range 4-16 $M_\odot$ without visible concentration to some value, i.e. $dN/dM \sim M^\alpha$ with $\alpha \sim 0$.

2.2 X-ray luminosity function of ultra-luminous X-ray sources

Ultra-luminous X-ray sources (ULX) are point-like sources with persistent X-ray luminosity $L_x > 2 \times 10^{38}$ erg/s corresponding to a 1.4 $M_\odot$ neutron star. They have been discovered by several X-ray satellites (mostly by Chandra, see e.g. 12 and references therein) in other galaxies. Although some of them proved to be another galaxies seen through the observed galaxy disk (see 13 for a recent identification), they constitute the most luminous population of X-ray sources in galaxies and based on their X-ray spectral fitting can be identified with accreting stellar-mass BH in binary systems 14.

A general analysis of X-ray source populations in nearby galaxies was recently carried out by Grimm et al. 15. It revealed the universal power-law shape of the average X-ray luminosity function $dN/dL_x \sim L_x^{-1.6}$ in a wide range of X-ray luminosities $10^{34} - 10^{39}$ erg/s with a steeper decline $\propto L_x^{-2.2}$ for the most luminous sources $L_x > 2 \times 10^{38}$ erg/s. The universal shape of the X-ray luminosity function below $(2-4) \times 10^{38}$ erg/s can be explained on the very general grounds by accretion in binary systems 16. The cut-off in the luminosity function at $L_x > 10^{39}$ erg/s (i.e. for ultra-luminous X-ray sources) appears to be a general feature of X-ray source populations in galaxies (see e.g. a recent analysis of Chandra observations of NGC 5194 in 14) reflecting the presence of binaries with BH accreting at the Eddington limit. This allows us to assume that for BH in these systems $dN/dM = (dN/dL_x)(dL_x/dM) \sim L_x^{-1.9...-2.2} \sim M^{-1.9...-2.2}$. Uncertainties in this method of determining BH mass distribution are due to possible beaming of X-ray emission from binary systems (the value of the X-ray luminosity is usually obtained from the observed X-ray fluxes assuming spherical symmetry). At present, this uncertainty is hard to estimate, but similar shapes of X-ray luminosity functions of point-like sources in different galaxies and the very possibility of obtaining a universal power-law form for $dN/dL_x$ which is simply scaled by the star formation rate from galaxy to galaxy (as argued in 15) seem encouraging.

3 Black hole mass function and enhanced black hole evaporation

Clearly, the decreasing power-law shape of BH mass distribution is drastically different from the nearly flat form derived from dynamical BH mass determinations. First, we should note that if ULX are actually BH accreting at the Eddington limit, the mass accretion rates $\dot{M}$ in these binaries must be above $\sim 10^{-7} M_\odot$/year, i.e. such systems must be HMXB with ages $< 10^7$ years. In contrast, most BH with dynamically determined masses reside in transient LMXB (X-ray novae) which are old systems with ages $> 10^8$ years.

Let us consider the distribution function $f(M) \equiv dN/dM$ of a population of sources with changing mass. Let the initial mass distribution be $f_0(M)$ within the mass range $[M_{\text{min}}, M_{\text{max}}]$ and the law of the mass change be $M(t)$. In the stationary case the evolution of mass distribution
is described by the one-dimensional kinetic equation

$$\frac{\partial}{\partial M} \left[ f(M) \dot{M} \right] = f_0(M)$$

(4)

so the stationary distribution function for $\dot{M} > 0$ will be

$$f(M) = \frac{\int_{M_{\text{min}}}^{M} f_0(M') dM'}{M}, \quad M < M_{\text{max}}$$

(5)

while for $M \geq M_{\text{max}}$ the stationary distribution is independent of the initial mass function and is determined only by the mass change $\dot{M}$:

$$f(M) = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} f_0(M') dM'}{M} = \text{const}, \quad M \geq M_{\text{max}}$$

(6)

For example, assuming the initial power-law mass function $f_0(M) \propto M^{-\alpha_i}$ yields for $\alpha_i > 1$ $f(M) \sim (1 - (M/M_{\text{min}})^{-\alpha_i+1})/\dot{M} \sim 1/\dot{M}$ at $M > M_{\text{min}}$, and for $\alpha_i < 1$ $f_0(M) \sim [(M/M_{\text{min}})^{-\alpha_i+1} - 1]/\dot{M}$.

Since BH mass in accreting binaries can never increase faster than $\dot{M} \propto M$, the stationary distribution never (for any $\alpha_i$) decreases steeper than $1/\dot{M}$. So the observed steep BH mass cutoff (propto $M^{-1.9...-2.2}$ in ULX) apparently evidences that stationarity arguments for these systems are inapplicable due to short time of Eddington-limited accretion in these systems (typically $\sim 10^5$ years $< M/\dot{M}$) so that BH masses cannot change significantly. Then we stay with the possibility that the observed steep BH mass function reflects the initial BH mass distribution in these binaries.

In LMXB (in which most BH candidates are found) or for single BH (only two of them are suspected by gravitational microlensing experiments), the situation can be quite different. Small mean accretion rates $\dot{M} \sim 10^{-3} M_\odot / \text{yr}$ typical for these systems are insignificant for BH mass growth. Applying the same arguments as above, we could expect the initial BH mass function to have a nearly flat shape in this case. However, there is another possibility.

Let us consider the hypothesis of enhanced BH mass evaporation recently put forward by Tanaka and Emparan et al. These authors consider the RS II brane world model, in which our Universe is localized on a 4D-brane embedded in an Anti-de-Sitter bulk characterized by the warp factor (curvature length) $L$. Such models admit macroscopic values of $L$ up to 0.1 mm so as not to contradict the existing laboratory measurements. In this setup, the authors speculate that no stationary macroscopic BH can exist on the brane due to enhanced evaporation into (virtually unobservable) CFT-modes (KK-gravitons). This process has not yet been calculated properly so this hypothesis remains highly speculative (see e.g. Casadio et al. who obtained different results). Nevertheless, we try to see what happens should the macroscopic BH evaporation actually exist.

In this hypothesis, the evaporation time $\tau$ of a macroscopic BH localized on the brane (i.e. with the horizon size $> L$) is shorter than the classical Hawking evaporation time $\sim t_{\text{Pl}}(M/m_{\text{Pl}})^3$ by a huge factor $O(L/l_{\text{Pl}})^2$ (subscript $Pl$ stands for the Planck units)

$$\tau \simeq 10^2 [\text{yrs}] \left( \frac{M}{M_\odot} \right)^3 \left( \frac{1[\text{mm}]}{L} \right)^2$$

(7)

So the mass of an isolated BH decreases as $\dot{M} \sim M^{-2}$. It is easy to show that for $L \sim \propto r^{-3} - \propto r^{-\Delta}$ mm the BH evaporation rate exceeds the mean accretion rate in LMXB for $M \sim 10M_\odot$. So the present bounds $L < 10^5$ mm do not contradict this assumption.

Now for old BH in low-mass X-ray binaries we can find the stationary shape of BH mass distribution function. For evaporation $\dot{M} < 0$ so the integration of Eq. 4 yields

$$f(M) = \frac{\int_{M}^{M_{\text{max}}} f_0(M') dM'}{M}, \quad M > M_{\text{min}}$$

(8)

For example, assuming the initial power-law mass function $f_0(M) \propto M^{-\alpha_i}$ yields for $\alpha_i > 1$ $f(M) \sim (1 - (M/M_{\text{min}})^{-\alpha_i+1})/\dot{M} \sim 1/\dot{M}$ at $M > M_{\text{min}}$, and for $\alpha_i < 1$ $f_0(M) \sim [(M/M_{\text{min}})^{-\alpha_i+1} - 1]/\dot{M}$.
Figure 1: Schematic view of the expected stationary BH mass function (the solid curve) with the initial power-law form \( \frac{dN}{dM} \sim M^{-\alpha_i} \) (the dashed line) obtained in the enhanced BH evaporation model on the RSII-brane. The mass \( M_0 \) corresponds to a minimal BH mass that can evaporate in the Hubble time.

\[
\frac{\int_{M_{\text{min}}}^{M_{\text{max}}} f_0(M')dM'}{M} = \text{const} \frac{1}{M}, \quad M \leq M_{\text{min}}
\]

Assuming as above \( f_0(M) \sim M^{-\alpha_i} \) with \( \alpha_i < 1 \) we have \( f(M) \sim M^{-\alpha_i+3} \) for \( M > M_{\text{min}} \) and \( f(M) \sim M^2 \) for \( M \leq M_{\text{min}} \) (we always assume \( M \ll M_{\text{max}} \)). This is illustrated in Fig. 1. Clearly, if \( \alpha_i \sim 2 \), as can be inferred from ULX observations, the stationary BH mass distribution could be made much flatter by BH evaporation. In addition, the steep decrease of the stationary BH mass function \( f(M) \sim M^2 \) below \( M_{\text{min}} \) can be identified with apparent deficit of BH with masses below 4 \( M_{\odot} \).

4 Conclusions

The steadily growing number of mass determinations of stellar BH allows us to construct and study BH mass function. It has apparently broad and near flat shape within the interval 4–16 \( M_{\odot} \) without visible concentration at some value, which is opposite to what is observed for NS masses \( (M_{\text{NS}} = 1 - 2 M_{\odot}) \). Possible systematic errors in BH mass determination, which mostly rely on the radial velocity curve of the optical companion, can be reduced in future high-resolution spectroscopic observations.

BH mass function can also be inferred from observations of luminous X-ray binary systems (with \( L_x > 10^{39} \) erg/s) in other galaxies. The current Chandra observations suggest a steep decrease \( (dN/dL_x \sim L_x^{-2}) \) in the X-ray luminosity function of point-like objects in galaxies. If these sources are accreting BH at the Eddington limit, the initial BH mass function should have a similar slope \( f_0(M) \sim M^{-2} \), which disagrees with BH distribution found in galactic binary systems.

We show here that in the framework of the enhanced BH evaporation hypothesis which can in principle take place in some multidimensional gravity models (of RSII type), the initial steep power-law BH mass function can be made flatter to match the observed BH distribution in galactic binaries. This hypothesis also predicts low-mass BH (with \( M \) less than a few solar masses) should be very rare. Discovery of such low-mass BH would strongly limit this hypothesis.
Concluding, we note that it is unclear at present how close to reality such multidimensional gravity models are, so any test of their astrophysical predictions is very desirable. We hope that future precision measurements of BH masses (both single and in binaries) by traditional and new (e.g., gravitational waves from coalescing binaries with BH) astrophysical methods can be potentially very interesting in this respect.

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