Incoherent Eta Photoproduction from the Deuteron near Threshold

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Abstract

Very recent data for the reaction $\gamma d \rightarrow \eta np$, namely total cross sections, angular and momentum spectra, are analyzed within a model that includes contributions from the impulse approximation and next order corrections due to the $np$ and $\eta N$ interactions in the final state. Comparison between the calculations and the new data indicate sizable contributions from the $np$ and $\eta N$ final state interactions. Some systematic discrepancies between the calculations and the data are also found.
The reaction $\gamma d \rightarrow \eta np$ close to the meson production threshold offers an opportunity to investigate the final state interactions (FSI) between the outgoing particles, proton, neutron and $\eta$-meson. Provided the FSI between the nucleons is understood, the reaction allows to draw conclusions about the $\eta N$ interaction at low energies. In Refs. [1,2] we investigated incoherent $\eta$ photoproduction from the deuteron close to threshold taking into account that the reaction amplitude is given by the sum of the first order term, the impulse approximation (IA), and the terms of next higher order due to the final state interactions in the neutron-proton ($np$) the $\eta$-nucleon ($\eta N$) system.

When comparing our calculations to the data [3] available at that time we found that the few experimental points for the cross section of the reaction $\gamma d \rightarrow \eta np$ close to the reaction threshold require for an adequate description the additional contribution from the $\eta$-nucleon final state interaction.

In Fig.1 these old data points are indicated by open squares. It seemed clear that more precise measurements are necessary to further understand the interplay between the final state interactions of the two subsystems. Moreover, in [1] we concluded that due to the strong $np$ final state interaction the momentum spectra of the $\eta$-meson should be enhanced at high momenta if the reaction is considered close to threshold.

Very recently the TAPS Collaboration reported new data [4] for the incoherent photoproduction of $\eta$-mesons from the deuteron near threshold. These new data not only contain the total $\gamma d \rightarrow \eta np$ reaction cross section but also angular and momentum spectra of the $\eta$-meson. In this brief communication we study whether those new data [4] can shed some light on issues raised in our previous work [1,2].

For the sake of completeness we briefly summarize the main ingredients of our previous calculations [2] on incoherent photoproduction of $\eta$-mesons from the deuteron. Let us recall that for a given spin $S$ and isospin $T$ of the final nucleons the amplitude $\mathcal{M}_{IA}$ for the impulse approximation is written as

$$\mathcal{M}_{IA} = A^T(s_1)\phi(p_2) - (-1)^{S+T}A^T(s_2)\phi(p_1),$$

(1)

where $\phi(p_i)$ is the deuteron wave function [3], $p_i (i = 1, 2)$ is the momentum of the proton or neutron in the deuteron rest frame, and $A^T$ is the isoscalar or isovector $\eta$-meson photoproduction amplitude on a nucleon at the squared invariant collision energy $s_N$

$$s_N = s - m_N^2 - 2(E_\gamma + m_d)E_N + 2k_\gamma \cdot \vec{p}_i.$$  

(2)

The photon momentum is given by $k_\gamma$, and the invariant mass by $s=m_N^2+2m_dE_\gamma$. $E_N$ is the total nucleon energy and $m_N$ and $m_d$ are the nucleon and deuteron mass, respectively.

Now the amplitude $\mathcal{M}_{NN}$ for the $NN$ final state interaction is given by

$$\mathcal{M}_{NN} = m_N \int dk k^2 \frac{t_{NN}(q,k)A^T(s_N')\phi(p_N')}{q^2 - k^2 + i\epsilon},$$

(3)

where $q$ is the nucleon momentum in the final $np$ system and

$$\vec{p}_N = \vec{k} + \frac{\vec{k}_\gamma - \vec{p}_\eta}{2},$$

(4)
\[ p_\eta \text{ is the } \eta \text{-meson momentum and } p'_N = |\vec{p}'_N|. \] The half-shell \( np \) scattering matrix \( t_{NN}(q, k) \) in the \(^1S_0\) and \(^3S_1\) partial waves was obtained at corresponding off-shell momenta \( k \) from the CD-Bonn potential \([3]\). Finally, the amplitude \( \mathcal{M}_{\eta N} \) for the \( \eta N \) final state interaction is given as

\[
\mathcal{M}_{\eta N} = \frac{m_N m_\eta}{m_N + m_\eta} \int dk \frac{k^2}{q^2 - k^2 + i\epsilon} t_{\eta N}(q, k) A^T(s'_N) \phi(p'_N),
\]

where the \( \eta \)-meson momenta in the final and intermediate state of the \( \eta N \) system are indicated by \( q \) and \( k \), \( t_{\eta N}(q, k) \) is the half-shell \( \eta N \) scattering matrix in the \(^3S_1\) partial wave and

\[
\vec{p}'_N = \vec{k} + \frac{m_N (\vec{k}_N - \vec{p}_N)}{m_N + m_\eta}, \tag{6}
\]

where \( \vec{p}_N \) is the momentum of final proton or neutron in the deuteron rest frame and \( m_\eta \) is \( \eta \)-meson mass.

Furthermore, within the effective range approximation, the \( \eta N \) on-shell scattering matrix is related to the scattering length \( a_{\eta N} \) as

\[
\left[ iq - \frac{1}{a_{\eta N}} \right]^{-1} = \pi \frac{\sqrt{q^2 + m_N^2 \sqrt{q^2 + m_\eta^2}}}{\sqrt{q^2 + m_N^2} + \sqrt{q^2 + m_\eta^2}} t_{\eta N}(q, q). \tag{7}
\]

In our previous work \([1,2]\) we showed that within our approach the uncertainty of the calculations is dominated by the insufficient knowledge of the strength of the \( \eta N \) interaction at low energies, here represented by \( a_{\eta N} \). Moreover, possible effects due to higher order corrections from the multiple scattering expansion \([6,7]\) might be overshadowed by the sizable variation of \( a_{\eta N} \), which as a result of different model calculations or extractions can range from 0.25+i0.16 to 1.05+i0.27 fm. In our calculations we adopt \( t_{\eta N}(q, k) \) from the Jülich meson-baryon model \([8]\), which gives a scattering length \( a_{\eta N} = 0.42+i0.32 \) fm. When comparing our calculations \([1]\) with the old cross section data for the reaction \( \gamma d \to \eta np \) \([3]\), we concluded that the value of \( a_{\eta N} \) given by this model was consistent with the data.

A comparison between our full calculation, including NN and \( \eta N \) FSI, with the recent experimental information \([3]\) for the total cross section of the reaction \( \gamma d \to \eta np \) is shown in Fig. 1. Here the new data are indicated by filled circles. We can well describe the data close to the reaction threshold, while there is systematic underprediction of \( \approx 10\% \) of the experimental results between 660 and 680 MeV photon energy. We should not attribute this discrepancy to \( a_{\eta N} \), since we found in Ref. \([3]\) that the \( \eta N \) interaction acts predominantly very close to threshold. We also want to point out that our calculation matches up with the older data (open squares) at energies larger than 680 MeV.

The recent data of Ref. \([3]\) are more complete and contain not only total cross sections but also angular distributions of \( \eta \)-mesons in the photon-deuteron center-of-mass (c.m.) system at different photon energies. They are shown in Fig. 2 together with our calculations. The angular spectra at the lower energies, \( 630 \leq E_\gamma \leq 650 \) MeV, are quite sensitive to both final state interactions. Especially, the \( \eta N \) FSI is necessary to describe the data.
At photon energies $650 \leq E_\gamma \leq 689$ MeV our predictions show a stronger peaking at forward angles compared to the data, and a slight but systematic underestimation of the data at backward angles. The latter might be attributed to an additional contribution from rescattering mechanism with intermediate $\pi$-meson and $\pi N \rightarrow \eta N$ transition. However, we are aware that we can not make any final assessment about the discrepancies at the present stage.

The momentum spectra of the $\eta$-mesons in the $\gamma - d$ cm frame are shown in Fig. 3 and compared with our calculations. We would like to emphasize that the theoretical results displayed represent an average over a finite energy interval. This is done in order to make the predictions comparable to the experiment, where likewise an averaging over energy bins is made. Specifically for the momentum distribution of the $\eta$ meson this averaging has a significant influence on the result as shown in Fig. 4. The vertical arrows in Fig. 3 indicate the maximally allowed $\eta$ momentum, which is given by

$$p_\eta = \frac{\sqrt{s}}{2} \sqrt{\frac{\left(s-m^2_\eta\right)^2 - 4 \left(m_n + m_p\right)^2 m^2_\eta}{2 \sqrt{s}}},$$

and calculated for the maximal photon energy $E_\gamma$ indicated in the figure.

As Fig. 3 clearly indicates, for the lowest energy interval a substantial part of experimental points is located beyond the kinematical limit. This might stems from a larger experimental uncertainty in determining the $\eta$-meson momentum. These errors are not indicated in Fig. 3 by horizontal error bars. Unfortunately, due to this a clean comparison between our calculations and the data can not be made. As an aside, when shifting all data points by the same percentage inside the kinematically allowed region, the cross section points fall closer toward our calculation. However this can only serve as a guide to the eye and does not allow any further speculations.

In order to investigate the sensitivity of the $\eta$ momentum distribution to the $\eta N$ scattering length we calculate $\eta$-meson angular and momentum spectra at $E_\gamma = 630-640$ and 640-650 MeV with different values for $a_{\eta N}$ and compare our results with the data in Fig. 4. Here the solid lines show our calculations with $a_{\eta N} = 0.42+i0.32$ fm, the dashed lines the ones with $a_{\eta N} = 0.74+i0.27$ fm, and the dotted line the calculations with $a_{\eta N} = 0.25+i0.16$ fm. As it turns out, both observables are quite sensitive to the size of $a_{\eta N}$.

It can be instructive to consider different possibilities of analyzing the data to find a representation which may shed a different light on the reaction $\gamma d \rightarrow \eta np$. For this reason we consider the Dalitz plot representation, which is given as

$$\frac{d\sigma}{ds_{\eta p}ds_{np}} = \frac{|M_{IA} + M_{NN} + M_{\eta N}|^2}{256\pi^3 s (s-m^2_d)}. \quad (9)$$

Here $s_{\eta p}$ and $s_{np}$ denote the squared invariant mass of the $\eta p$ and $np$ subsystems.

This representation may be an additional tool to isolate the different FSI. In Fig. 5 we display the Dalitz plot projections calculated at photon energies $E_\gamma = 643$ and 681 MeV. For this calculation we employ the $\eta N$ FSI given by the Jülich model with $a_{\eta N} = 0.42+i0.32$ fm. The hatched areas indicate the contributions from the impulse approximation, the dotted lines stand for the the calculations with $np$ FSI alone, while the solid lines represent the full calculations. The difference between the impulse approximation and full calculation is not
only a result of the absolute sizes, but essentially the different shapes of the invariant mass spectra. At \( E_\gamma = 643 \text{ MeV} \) the low mass part of \( np \) spectrum is substantially enhanced by the \( np \) FSI. Thus the \( \eta p \) spectrum is shifted to higher masses. The difference between the calculations with \( \eta N \) and \( np \) FSI and that with \( np \) alone can be considered as an overall rescaling of the model results. This can be well understood through our findings in Ref. [2], namely that a quite weak \( \eta N \) interaction can manifest itself through the interference with the substantially stronger \( np \) FSI.

The results become more exciting at \( E_\gamma = 681 \text{ MeV} \). While the shape of the \( \eta p \) distribution is almost similar to that obtained at \( E_\gamma = 643 \text{ MeV} \), the \( np \) spectrum clearly shows a low mass structure due to the \( np \) and \( \eta N \) FSI, and the size of this enhancement is given by the coherent sum of the \( np \) and \( \eta N \) interactions. The production mechanism alone, or the contribution of the impulse approximation may well be isolated by imposing \( np \) invariant mass cuts.

We believe that an experimental observation of such double peaks structure might serve as direct evidence of FSI effects. Finally, we notice that the Dalitz plot analysis of the reaction \( pp \rightarrow pp\eta \) measured at COSY [10] indicates quite a similar structure in the \( pp \) invariant mass distribution. This finding may encourage further analysis of the new \( \gamma d \rightarrow \eta np \) data [4].

In conclusion, we presented a detailed comparison between our model for the reaction \( \gamma d \rightarrow \eta np \) developed in Refs. [1,2] and recently published experimental information [4] on total cross sections as well as angular and momentum \( \eta \)-meson spectra for this reaction. For our calculations we employ the \( \eta N \) FSI obtained from the Jülich meson-baryon model.

The comparison between the data [4] and our calculations shows reasonably agreement. The \( \gamma d \rightarrow \eta np \) data close to the reaction threshold require additional contributions from the \( \eta N \) FSI and are consistent with the size of \( a_{\eta N} = 0.42 + i 0.32 \text{ fm} \) given by the Jülich model.

The angular and momentum \( \eta \)-meson spectra at \( E_\gamma \leq 650 \text{ MeV} \) are very sensitive to the size of \( a_{\eta N} \).

However, we found some \( \simeq 10\% \) disagreement between our calculations and the new data [4] for the total cross section for the reaction \( \gamma d \rightarrow \eta np \) at photon energies \( 650 \leq E_\gamma \leq 689 \text{ MeV} \). Furthermore, at these energies the model predicts a stronger peaking of the angular distribution at forward angles and a slight but systematic underestimation of the data at backward angles. Further investigations are necessary in order to clarify whether this discrepancy stems from re-scattering mechanisms.

In addition, we found that the Dalitz plot analysis of the reaction \( \gamma d \rightarrow \eta np \) may serve as a very helpful tool for isolating FSI effects. The Dalitz plot projection on \( np \) invariant mass may show a clean double peak structure at \( E_\gamma = 681 \text{ MeV} \), while at low photon energy \( E_\gamma = 643 \text{ MeV} \) the \( np \) invariant mass spectrum is substantially enhanced at low masses.

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FIGURES

FIG. 1. The total cross section for inclusive photoproduction of \( \eta \) mesons off deuterium as function of the photon energy \( E_\gamma \). The open squares are old data [3], while the circles indicates new results [4]. The dotted line represents the IA calculation, while the dashed line is the result with the \( np \) final state interaction. The solid line shows the full calculation, including the \( \eta N \) final state interaction from the Jülich meson-baryon model [2].

FIG. 2. The angular spectra of \( \eta \)-mesons in the photon-deuteron c.m. system at different photon energies \( E_\gamma \). The data are from Ref. [4]. The dotted lines show IA calculations, while the dashed lines represent the results using only the \( np \) final state interaction. The solid lines show the full calculation, including the \( \eta N \) final state interaction from the Jülich meson-baryon model.

FIG. 3. The momentum spectra of \( \eta \)-mesons in the photon-deuteron c.m. system at different photon energies \( E_\gamma \). The data are from Ref. [4]. The lines show our calculations with notations similar to Fig.2. The arrows indicates the kinematical limit for \( \eta \)-meson momenta calculated by Eq.8.

FIG. 4. The angular (upper part) and momentum (lower part) spectra of \( \eta \)-mesons in the photon-deuteron c.m. system at photon energies \( E_\gamma =630-640 \) and \( 640-650 \) MeV. The data are from Ref. [4]. The lines show our calculations with different \( \eta N \) scattering lengths, namely \( a_{\eta N} =0.42+i0.32 \) (solid), \( 0.74+i0.27 \) (dashed) and \( 0.25+i0.16 \) fm (dotted). The arrows indicate the kinematical limit for \( \eta \)-meson momenta calculated by Eq.8.

FIG. 5. The invariant mass spectra of the \( \eta p \) (left) and \( np \) (right) subsystem produced in \( \gamma d \rightarrow \eta np \) reaction at photon energies \( E_\gamma =643 \) and \( 681 \) MeV. The hatched areas show calculations based on the IA, the dashed lines stands for the calculation including the \( np \) FSI only, while the solid lines represents the full calculations including both, \( np \) and \( \eta N \) FSI.
FIG. 3
FIG. 4
FIG. 5