Cosmologies with a time dependent vacuum

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Abstract. The idea that the cosmological term $\Lambda$ should be a time dependent quantity in cosmology is a most natural one. It is difficult to conceive an expanding universe with a strictly constant vacuum energy density, $\rho_\Lambda = \Lambda/(8\pi G)$, namely one that has remained immutable since the origin of time. A smoothly evolving vacuum energy density $\rho_\Lambda = \rho_\Lambda(\xi(t))$ that inherits its time-dependence from cosmological functions $\xi = \xi(t)$, such as the Hubble rate $H(t)$ or the scale factor $a(t)$, is not only a qualitatively more plausible and intuitive idea, but is also suggested by fundamental physics, in particular by quantum field theory (QFT) in curved space-time. To implement this notion, is not strictly necessary to resort to ad hoc scalar fields, as usually done in the literature (e.g. in quintessence formulations and the like). A “running” $\Lambda$ term can be expected on very similar grounds as one expects (and observes) the running of couplings and masses with a physical energy scale in QFT. Furthermore, the experimental evidence that the equation of state (EOS) of the dark energy (DE) could be evolving with time/redshift (including the possibility that it might currently behave phantom-like) suggests that a time-variable $\Lambda = \Lambda(t)$ term (possibly accompanied by a variable Newton’s gravitational coupling too, $G = G(t)$) could account in a natural way for all these features. Remarkably enough, a class of these models (the “new cosmon”) could even be the clue for solving the old cosmological constant problem, including the coincidence problem.

1. Introduction

The observed accelerated expansion of the universe [1, 2] is currently one of the central issues of observational and theoretical cosmology. It somehow came as a surprise to discover that the universe is not progressively slowing down its pace, but actually rocketing it up. The usual explanatory paradigm for this striking observation assumes the existence of the so-called dark energy (DE) – a mysterious cosmic component with negative pressure, which, in contrast to dark matter (DM), does not cluster (at least in a comparable way) and pervades all corners of the universe. An obvious candidate for the DE is a strictly constant cosmological term, $\Lambda$. Nevertheless, this option precludes the possibility of a time evolution of the DE (which might be appealing so as to face the so-called coincidence problem, namely the fact that the current matter density $\rho_M$ is so close to the DE density $\rho_\Lambda = \Lambda/(8\pi G)$, or, equivalently, the fact that the acceleration epoch started so recently). This drawback could be cured by admitting that the CC can be a dynamical quantity: $\Lambda = \Lambda(t)$, an option which is perfectly admissible by the Cosmological Principle.

Actually, the mere possibility of a variable CC approach was explored long ago on purely phenomenological grounds – cf. [3, 4] – although many of the models are little motivated. At
present, we know that the idea of a slowly running CC is in general compatible with the experimental data [5] and there are reasonable theoretical models grounded on fundamental physical principles that can support these ideas, meaning that there is no strict need to invoke ad hoc scalar fields to generate a time dependence for the DE – see [6] and references therein, see also [7, 8] for recent alternative approaches reinforcing this point of view. Still, the variable \( \Lambda = \Lambda(t) \) models can be dealt with in terms of a self-conserved DE fluid in the manner of a scalar field (as we cannot know a priori the very nature of the DE, and hence we may be naturally tempted to keep on using this simple scalar field approach). However, although this description in terms of scalar fields (which may be called the “DE picture” of the original \( \Lambda = \Lambda(t) \) model [9]) is perfectly possible, the corresponding effective equation of state (EOS) for the DE, \( \omega = p_{\text{DE}}/\rho_{\text{DE}} \), will be a non-trivial function of time or of the redshift, \( \omega = \omega(z) \), and in general it cannot be reduced to the standard forms proposed in the literature. In other words, the effect of having a variable cosmological term \( \Lambda = \Lambda(t) \) cannot be described by the simple parameterizations usually employed for \( \omega(z) \), characterized by two parameters (\( \omega_0, \omega_1 \)) that become constrained by the observations [10]. The effective EOS of a variable vacuum model is in general more complicated than that. This is shown in detail, with specific examples, in reference [9]. In particular, the vacuum models that we are going to consider cannot be described with these simple parameterizations. Therefore, the \( \Lambda = \Lambda(t) \) models must be studied on their own and constitute an independent class of DE models. It means that we should be more open-minded on the form that the effective EOS can adopt, and we should not limit ourselves to some admittedly popular (albeit perhaps too simple) forms utilized too often in the literature.

Remarkably, the variable \( \Lambda = \Lambda(t) \) approach can also be advantageously employed for solving (or highly palliating) the big or “old” CC problem [11], viz. the absolutely formidable task of trying to explain the value itself of the current vacuum energy density \( \rho_{\Lambda}^0 \sim 10^{-47} \text{ GeV}^4 \) on the face of its enormous input value \( \rho_{\Lambda}^0 \sim M_X^4 \) (with \( M_X \sim 10^{16} \text{ GeV} \)) left over in the very early times, presumably just after the inflation phase induced by a Grand Unified Theory (GUT). This option has been recently investigated in [12, 13], and we will also review it here. It is based on the idea of “relaxation” of the effective vacuum energy density \( \rho_{\Lambda}(t) = \Lambda(t)/(8\pi G) \). Relaxation offers indeed an entirely different perspective on the old CC problem; in particular, it provides the extremely appealing possibility to solve the CC problem dynamically, hence without fine tuning [13]. Moreover, the current value of the expansion rate \( H \) becomes determined by an approximate relation of the form \( H_0 \sim 1/(\rho_{\Lambda}^{1/n}) \). Interestingly enough, this relation tells us that \( H_0 \) is small just because the initial vacuum energy \( \rho_{\Lambda} \) was very large!

In the next sections, I summarize the general notion and properties of the models with time-dependent vacuum energy, and consider specific scenarios possessing very interesting features concerning the possible solution, or at least significant alleviation, of some of the most fundamental cosmological constant problems of modern times.

2. Two cosmological pictures: “CC picture” versus “DE picture”

Suppose that we are given a model where the vacuum energy density \( \rho_{\Lambda} = \rho_{\Lambda}(\xi(t)) \) is time evolving through its dependence on other cosmological function(s) of the cosmic time, e.g. \( \xi(t) = \rho_{\Lambda}(t), H(t), a(t), .... \) We take this starting “picture” as the original (physical) representation of the DE, and we call it “the CC picture”. In general, other fundamental “constants” can also be varying, in particular the gravitational coupling \( G \). Therefore, in the CC picture we will have in general two functions, at least, of the cosmic time or the cosmological redshift, namely

\[
\rho_{\Lambda}(z) = \rho_{\Lambda}(\rho_{\Lambda}(z), H(z), ...), \quad G(z) = G(\rho_{\Lambda}(z), H(z), ...).
\]
Of course other fundamental parameters could also be variable. For example, the fine structure constant has long been speculated as being potentially variable with the redshift. However, here we focus purely on the potential variability of the most genuine fundamental gravitational parameters of Einstein’s equations, to wit: $\Lambda$ and $G$.

For a cosmological model of the kind (1), the general Bianchi identity of the Einstein tensor leads to $\nabla^{\mu}[G(T_{\mu\nu} + g_{\mu\nu}\rho_{\Lambda})] = 0$. In the FLRW metric, and modeling the universe as a perfect fluid, it implies

$$\frac{d}{dt}[G(\rho_M + \rho_{\Lambda})] + 3G H_{\text{CC}}(\rho_M + p_M) = 0.$$  (2)

This “mixed” conservation law connects the variation of $\rho_{\Lambda}$ and $G$ with that of the matter density $\rho_M$. As a result, the matter fluid in the CC picture is generally non-conserved and its evolution may be non-canonical, as we will see later on in a specific example. The general expression for the Hubble function in the CC picture, assuming a spatially flat FLRW universe, is

$$H_{\text{CC}}^2(z) = \frac{8\pi G(z)}{3}(\rho_M(z) + \rho_{\Lambda}(z)) \equiv H_0^2 \left[ \Omega_M^0 f_M(z)(1 + z)^\alpha + \Omega_{\Lambda}^0 f_{\Lambda}(z) \right],$$  (3)

where $\alpha = 3(1 + \omega_m)$, with $\omega_m = 0, 1/3$ for the matter and radiation epochs respectively. Here $f_M$ and $f_{\Lambda}$ are certain functions of the redshift. They must satisfy $f_M(0) = 1$ and $f_{\Lambda}(0) = 1$, in accordance with the current cosmic sum rule $\Omega_M^0 + \Omega_{\Lambda}^0 = 1$ for a flat universe.

On the other hand, the above cosmological dynamics in the CC picture can also be described in terms of an alternative fluid with covariantly self-conserved densities ($\rho_{MS}, p_D$), therefore satisfying $\dot{\rho}_{MS} + \alpha H_{DE}(\rho_{MS}, p_D) = 0$ and $\dot{p}_D + 3H_{DE}(1 + \omega_{DE})p_D = 0$, where $\omega_{DE} = p_D/\rho_D$ will be, in general, a function of the redshift: $\omega_{DE} = \omega_{DE}(z)$. Of course $\rho_{MS} \neq \rho_M$ and $\rho_D \neq \rho_{\Lambda}$ in general.

The Hubble rate in the new picture takes on the form:

$$H_{DE}^2 = \frac{8\pi G_0}{3}(\rho_{MS}(z) + \rho_D(z)) = H_0^2 \left[ \Omega_M^0 (1 + z)^\alpha + \Omega_D^0 \right].$$  (4)

where

$$\Omega_D(z) = \Omega_D^0 \exp \left\{ 3 \int_0^z dz' \frac{1 + \omega_{DE}(z')}{1 + z'} \right\}.$$  (5)

We have defined $\Omega_M^0 = \rho_{MS}/\rho_c^0$, with $\rho_c^0 = 3H_0^2/(8\pi G_0)$ the current value of the critical density. Notice that the parameter $\Omega_M^0$ need not coincide with $\Omega_M^0$, as they are determined from two different parameterizations of the DE data. Similarly, we have defined $\Omega_D(z) = \rho_D(z)/\rho_c^0$ – and its current value $\Omega_D^0 = \Omega_D(0)$. Notice also that $G_0 = 1/M_P^2$ ($M_P$ being the Planck mass) is strictly constant, whereas $G$ in eq. (3) is in general a variable function of the redshift: $G = G(z)$. The cosmological picture characterized by the self-conserved pair of energy densities ($\rho_{MS}, \rho_D$) at fixed $G_0$ will be called the “DE picture” [9].

Since the two pictures are assumed to be equivalent descriptions of the same cosmological evolution, their matching requires that the expansion history of the universe is the same in both pictures, i.e. that their Hubble functions are numerically equal:

$$H_{DE}(z) = H_{CC}(z).$$  (6)

Recall that we take the original picture (1) as the physical one, whereas the DE picture plays the role of an effective description of the former in terms of the self-conserved DE component $\rho_D$ at fixed $G_0$. Therefore, the function $\omega_{DE}(z)$ defined above can be considered as the “effective EOS” (in the “DE picture”) of the original model (1). A straightforward calculation, using the matching condition (6) and the general Bianchi identity (2), allows to determine the effective
EOS function $\omega_{\text{eff}}(z)$ in terms of the fundamental parameters of the original theory (1), as follows [9]:

$$
\omega_{\text{eff}}(z) = -1 + \frac{1}{3} \frac{1 + z}{\rho_D} \frac{d\rho_D}{dz} = -1 + \frac{\alpha}{3} \left( 1 - \frac{\zeta(z)}{\rho_D(z)} \right),
$$

(7)

where we have defined $\zeta(z) \equiv (G(z)/G_0) \rho_\Lambda(z)$, and

$$
\rho_D(z) = (1 + z)^\alpha \left[ \rho_D(0) - \alpha \int_0^z \frac{dz' \zeta(z')}{(1 + z')(\alpha + 1)} \right].
$$

(8)

In particular, for a model of the kind (1) with fixed $G = G_0$ in the matter epoch ($\alpha = 3$), the effective EOS simply reads $\omega_{\text{eff}}(z) = -\rho_\Lambda(z)/\rho_D(z)$. The latter would shrink to just $\omega_{\text{eff}} = -1$ only in case of a truly cosmological constant $\rho_\Lambda = \text{const.}$ because then $\rho_D$ would coincide with it. With the help of the matching condition (6) of the two pictures, and the Bianchi identity, one can also show, with some effort, that

$$
\frac{d\rho_D(z)}{dz} = \alpha (1 + z)^{\alpha-1} \rho_c^0 \left( \Omega_M^0 f_M(z) - \tilde{\Omega}_M^0 \right).
$$

(9)

This is a remarkable relation, if we recall the conditions $f_i(0) = 1$ fulfilled by the functions $f_M$ and $f_\Lambda$ in eq. (3). Why is remarkable? As $\Omega_M^0$ and $\tilde{\Omega}_M^0$ must be very close, and $f_M(z)$ is presumably monotonous, eq. (9) shows that a value $z = z^*$ always exists for which the l.h.s. of eq. (9) vanishes and, therefore, the effective EOS $\omega_{\text{eff}}(z)$ crosses the phantom divide at that point: $\omega_{\text{eff}}(z^*) = -1$. Whether the crossing is in the recent past or in the immediate future, it will depend on the details of the original model (1). Notice, however, that if $\Omega_M^0 = \tilde{\Omega}_M^0$, then the crossing will be exactly at $z^* = 0$, i.e. now. This shows that a generic model (1) with time varying vacuum energy, when described in the DE picture (i.e. as if it were a cosmological self-conserved DE fluid) generally leads to a crossing of the phantom divide, thus providing a possible natural explanation for the observational data – which still admit a tilt in the phantom domain [2].

3. Time dependent vacuum in the running $\Lambda$CDM model

In contradistinction to purely phenomenological models of time dependent $\Lambda$ [3, 4], we adopt here the fundamental point of view that the $\Lambda$ term is a running parameter in QFT in curved space-time [14]. As suggested in [6], we should have a corresponding renormalization group (RG) equation of the general form

$$
(4\pi)^2 \frac{d\rho_\Lambda}{d\ln \mu} = \sum_{n=1}^{\infty} A_n \mu^{2n},
$$

(10)

in which $\mu$ is an arbitrary scale associated to the RG running. Of course the full effective action is perfectly scale independent (i.e. RG invariant)[6], and the running of the vacuum energy should ultimately reflect the dependence of the leading quantum effects with respect to some physical cosmological quantity $\xi = \xi(t)$ associated with $\mu$, hence $\rho_\Lambda = \rho_\Lambda(\xi)$. Despite it being a matter of debate, it has been argued by different methods [16, 17] that the physical scale $\xi$ pointed by the $\mu$-dependence is the Hubble rate $H$. Since $H = H(t)$ evolves with the cosmic time, the cosmological term inherits a time-dependence through its primary scale evolution with $H$. The coefficients $A_n$ in eq. (10) are obtained after summing over the loop contributions of fields of different masses $M_i$ and spins $\sigma_i$. The general behavior is $A_n \sim \sum M_i^{4-2n}$ [16, 17]. Therefore, for $\mu = H \ll M_i$, the series above is (for $n > 1$) an expansion involving powers of the small quantities $\mu = H$ and $H/M_i$. Given, however, that $A_n \sim \sum M_i^{2n}$, we see that the heaviest fields furnish the dominant contribution. This trait ("soft-decoupling") represents a generalization of the decoupling theorem in QFT. In addition, being $H_0 \sim 10^{-33} \, eV$, the condition $\mu \ll M_i$ is
amply met for all known particles, and the series on the r.h.s of eq. (10) converges extremely fast at present. Also important is the fact that only even powers of $\mu = H$ are consistent with general covariance, see [6]. Finally, the $n = 0$ contribution is absent because it corresponds to terms $\propto M_i^4$ that give an extremely fast evolution. Actually, from the RG point of view they are already excluded because, as noted above, $\mu \ll M_i$ for all known masses. In practice only the first term $n = 1$ is needed, with $M_i$ of the order of the highest mass available.

As the dominant masses $M_i$ will be of order of a GUT mass scale $M_X$ near the Planck mass $M_P$, it is convenient to introduce the ratio $\nu = \sigma/(12\pi) \left( M_X^2 / M_P^2 \right)$, in which $\sigma = \pm 1$ depending on whether bosons or fermions dominate in their loop contributions to (10). If the effective mass $M_X$ of the heavy particles is just $M_P$, the parameter $\nu$ takes the value $\sigma \nu_0$, with $\nu_0 \equiv 1/(12 \pi) \simeq 2.6 \times 10^{-2}$. In general we expect that $\nu$ will take values of this order or below.

Under the very good approximation $n = 1$, and with $\mu = H$, eq. (10) abridges to

$$\frac{d\rho_\Lambda}{d\ln H} = \frac{3\nu}{4\pi} M_P^2 H^2,$$

whose solution is

$$\rho_\Lambda = c_0 + c_1 H^2,$$  \hspace{1cm} (12)

with

$$c_0 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2 H_0^2, \quad c_1 = \frac{3\nu}{8\pi} M_P^2.$$  \hspace{1cm} (13)

Solving the model in the original CC picture [9] leads to a non-standard evolution law for the matter density as a function of the redshift: $\rho_M(z) = \rho_M^0 (1 + z)^{3(1 - \nu)}$, along with the following evolution of the vacuum energy density:

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu \rho_M^0}{1 - \nu} \left[ (1 + z)^{3(1 - \nu)} - 1 \right].$$  \hspace{1cm} (14)

For $\nu = 0$ we recover the $\Lambda$CDM results, of course. But for $\nu \neq 0$ we have a model in which the vacuum energy decays into matter (or vice versa) according to the generalized conservation law (2), with $G = \text{const}$ in this case. With this RG model of the cosmic evolution at hand, we may

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\(^1\) It is interesting to notice that this quadratic evolution law for the vacuum energy with the expansion rate has also been suggested recently by alternative QFT methods, see [15].
now use it to apply the procedure of section 2 for obtaining the effective properties of the model in the “DE picture” (in which matter and dark energy are separately conserved). Obviously, for this particular model, \( \zeta (z) = \rho_\Lambda (z) \). The corresponding effective EOS cannot be described with standard parameterizations [10]. Indeed, from equation (7) – using (8) and (14) – we find:

\[
\omega_{\text{eff}}(z)\big|_{\Delta \Omega \neq 0} = -1 + (1 - \nu) \frac{\Omega_M^0 (1 + z)^{3(1 - \nu)} - \tilde{\Omega}_M^0 (1 + z)^3}{\Omega_M^0 [(1 + z)^{3(1 - \nu)} - 1] - (1 - \nu) [\tilde{\Omega}_M^0 (1 + z)^3 - 1]},
\]

(15)

Here \( \Delta \Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0 \neq 0 \) is the difference of the cosmological mass parameters in the two pictures (corresponding to two different fits of the same data). For \(|\nu| \ll 1 \) we may expand the previous result to first order in \( \nu \). Assuming for simplicity that \( \Delta \Omega_M = 0 \), we find

\[
\omega_{\text{eff}}(z) \simeq -1 - 3 \nu \frac{\Omega_M^0}{\Omega_\Lambda^0} (1 + z)^3 \ln(1 + z).
\]

(16)

This result reflects the essential qualitative features of the analysis presented in the previous section, where in this case \( z^* = 0 \). Indeed, for \( \nu > 0 \) eq. (16) shows that we get an (effective) phantom-like behavior (\( \omega_{\text{eff}} \lesssim -1 \)), whereas for \( \nu < 0 \) we have an (effective) quintessence behavior (\( \omega_{\text{eff}} \gtrsim -1 \)). In other words, this variable CC model can give rise to two types of very different qualitative behaviors by changing the sign of a single parameter. In Fig. 1 we show also more general cases where \( z^* \gtrsim 0 \), corresponding to \( \Delta \Omega_M \neq 0 \). In contrast to the previous situation, here the variable CC model may exhibit phantom behavior for \( \nu < 0 \) (if \( \Delta \Omega_M < 0 \)), and manifests itself through the existence of a transition point \( z^* \) in our recent past – marked explicitly in the figure – as predicted by the general formalism of section 2.

Let us also examine another interesting situation. From eq. (2), we see that if \( G = G(t) \) is variable and we assume that matter is conserved (thus no transfer of energy with the time dependent vacuum \( \rho_\Lambda \)), we have \( d\zeta_\Lambda = -(\rho_M/G_0) dG = (\rho_M^0/G_0)(1 + z)^\alpha dG \). From eq. (8) it is then not difficult to show that

\[
\frac{d\rho_D}{dz} = \alpha (1 + z)^{-1} \frac{\rho_M^0}{G_0} [G(z) - G(z^*)].
\]

(17)

where \( z^* \) is the crossing point of the phantom divide, which satisfies \( \rho_D(z^*) = \zeta_\Lambda(z^*) \). It follows that if \( G \) is asymptotically free – that is to say, if \( G(z) \) decreases with redshift – we should observe quintessence behavior (\( d\rho_D/dz > 0 \)) for \( 0 < z < z^* \); whereas if \( G \) is “IR free” (i.e. it increases with \( z \), hence decreases with the expansion), we should then have phantom behavior (\( d\rho_D/dz < 0 \)) in that interval. An interesting example of model of this kind appears if we take once more the evolution law (12) for the vacuum energy, but assume now that matter is conserved [16]. The basic cosmological equations can be formulated in the following compact way:

\[
E^2(z) = g(z) [\Omega_M(z) + \Omega_\Lambda(z)] \quad \text{(Friedmann equation)}
\]

\[
(\Omega_M + \Omega_\Lambda)dg + g d\Omega_\Lambda = 0 \quad \text{(Bianchi identity)}
\]

\[
\Omega_\Lambda(z) = \Omega_\Lambda^0 + \nu [E^2(z) - 1] \quad \text{(running \( \rho_\Lambda \)),}
\]

(18)

2 One first shows that \( \rho_D \) in eq. (8) satisfies the differential equation \( d\rho_D(z)/dz = \alpha (\rho_D(z) - \zeta_\Lambda(z))/(1 + z) \). Next one notes that the previous equation is also solved by the alternative expression below:

\[
\rho_D(z) = \zeta_\Lambda(z) - (1 + z)^\alpha \int_z^\infty \frac{dz'}{(1 + z')^\alpha} \frac{d\zeta_\Lambda(z')}{dz'} \quad \Rightarrow \quad \frac{d\rho_D(z)}{dz} = -\alpha (1 + z)^{-\alpha} \int_z^\infty \frac{dz'}{(1 + z')^\alpha} \frac{d\zeta_\Lambda(z')}{dz'}.
\]
Figure 2. Features of the \(\Lambda XCDM\) model. (a) \(\Omega(z)\) for the total DE and its individual components, \(\Omega_X(z)\) and \(\Omega_\Lambda(z)\), for \(\Omega_0^M = 0.3\), \(\Omega_0^\Lambda = 0.75\), \(\omega_X = -1.85\), \(\nu = -\nu_0\). The stopping of the expansion at \(z = z_s\) is achieved here thanks to \(\Omega_X(z)\) becoming eventually negative; (b) Behavior of the effective EOS, eq.\((21)\), for the same \(\omega_X\) and \(\Omega_0^\Lambda\) and different values of \(\nu\).

where \(E(z) \equiv H(z)/H_0\) and \(g(z) \equiv G(z)/G_0\). It is now straightforward to solve them for the gravitational coupling, which becomes a logarithmically running quantity of the Hubble rate:

\[
G(H) = \frac{G_0}{1 + \nu \ln \left(\frac{H^2}{H_0^2}\right)}. \tag{19}
\]

We see that \(G\) is asymptotically free (resp. IR free) for \(\nu > 0\) (resp. \(\nu < 0\)). An alternative formulation of this running law for \(G\) and \(\Lambda\), motivated by the quantum physics of inflation, was presented in [17].

The previous considerations show that we have plenty of theoretical and phenomenological motivations to support the possibility of observing running cosmological parameters, specially through the study of time-evolving vacuum cosmologies. Ultimately this issue must of course be backed up by observations, and in this sense it is very important to try to find all possible phenomenological indications suggesting such a vacuum dynamical picture. It is also remarkable that one could even find signs of the running \(\Lambda\) in the astrophysical context; for example, through the study of the implications that a variable \(\Lambda\) distributed (homogeneously or inhomogeneously) in a large domain could have in the process of local structure formation, see [18].

Time-evolving vacuum models inspired by QFT principles could do even more: they could provide a possible solution to the cosmic coincidence problem, and ultimately to the “old cosmological constant problem”, namely the CC fine-tuning problem – the toughest cosmological conundrum of all times [11]. Possible avenues leading to a significant step in this direction are summarized in the next two sections.

4. The \(\Lambda XCDM\) model: a possible solution to the cosmic coincidence problem

The \(\Lambda XCDM\) model [19] makes a further step in our systematic approach to a cosmology with variable cosmological parameters. It is based on assuming that we have a composite DE energy density \(\rho_D = \rho_X + \rho_\Lambda\). In the canonical case where \(G = \text{const}\), it can be taken as covariantly conserved:

\[
\dot{\rho}_D + 3(1 + \omega_e) \rho_D H = \dot{\rho}_\Lambda + \dot{\rho}_X + 3(1 + \omega_X) \rho_X H = 0, \tag{20}
\]

and in these conditions matter is self-conserved too of course: \(\dot{\rho}_M + \alpha H \rho_M = 0\). Here \(\rho_\Lambda = \rho_\Lambda(t)\) is the energy density of the running \(\Lambda\). Moreover, \(\rho_X(t)\) is the energy density of a new dynamical
entity $X$ that we call the “cosmon”. Its dynamics is completely determined by the above local conservation law (20) once the evolution for $\rho_X(t)$ is given. As emphasized in [19], this implies that in general it will be an effective entity, not a fundamental one (i.e. $X$ is not supposed to be a scalar field $\phi$) because the dynamics imposed by (20) will in general make it incompatible with the scalar potential provided for $\phi$. The cosmon $X$, instead, embodies the behavior of terms in the effective action of QFT in curved space-time, and, far from being an elementary scalar field, it represents a convenient way to parametrize the effect of those terms in the cosmological evolution. The EOS of the cosmon is indicated by $\omega_X$ (assumed, for the moment, to be constant for simplicity). It should not be confused with the effective EOS of the total DE, which reads

$$\omega_e = \frac{p_D}{\rho_D} = -\rho_X + \omega_X \rho_X \rho_X = -1 + (1 + \omega_X) \frac{\rho_X}{\rho_D},$$

Next we will assume that $\rho_X = \rho_X(H)$ is the same function of $H$ as in the running $\Lambda$CDM model presented in section 3 – cf. eqs. (12) and (13). Notice that this does not mean that its redshift evolution is still given by eq. (14), as here the overall conservation law is no longer (2) but (20). After solving explicitly the details of this new cosmological model – which we refrain from reporting here [19] – the normalized energy densities $\Omega(z) = \rho(z)/\rho_c(\rho = \rho_M, \rho_X, \rho_D)$ with respect to the present critical density $\rho_c$ become determined as a function of the redshift (cf. Fig. 2a). We see that the cosmon density $\Omega_X$ can be negative, although this is not too surprising for an effective quantity that does not represent a fundamental field. On the other hand we observe that the total DE density $\Omega_D = \Omega_M + \Omega_X$ behaves properly, and its current value is the measured DE value, i.e. we have $\Omega_D^0 = 0.7$ with $\Omega_X^0 + \Omega_M^0 = 1$ (flat space). Furthermore, the EOS function $\omega_e = \omega_e(z)$ can display a rich variety of behaviors while being in agreement with the most recent data [2]. This is shown in Fig. 2b, where we see that, depending on the value of $\nu$, the EOS can be quintessence-like, or mimic (in some cases almost exactly) a pure CC term ($\omega_e \simeq -1$), and it can even exhibit a mild transition from the phantom to the quintessence regime. From Fig. 2b we recognize that this model is a spectacular example of cosmology, in which despite of the fact that the vacuum energy $\rho_X = \rho_X(H)$ is a running quantity, the effective EOS of the composite DE can approach the standard $\Lambda$CDM model in an arbitrary way, and may display only small “quintessence-like” or “phantom-like” departures from it, thus mimicking a scalar field even in situations where such field could not behave canonically ($\omega_e \lesssim -1$).

The truly nice feature of a model of this kind, however, is that it can cure or highly alleviate the “cosmic coincidence problem”. Recall that this problem is related to the behavior of the ratio $r(z) \equiv \rho_D(z)/\rho_M(z)$ between the dark energy density and the matter energy density. In the standard $\Lambda$CDM model, that ratio increases arbitrarily with the cosmic evolution because $\rho_D$ is just the strictly constant vacuum energy density $\rho_X^0$, whereas $\rho_M \to 0$ with the expansion, so there is no apparent reason why we should find ourselves precisely in an epoch where $r = \mathcal{O}(1)$. Or, to put in other terms: there is no obvious reason why the epoch where the universe started to accelerate is so recent. This is an unexplained coincidence puzzle in the standard model. Not so in the $\Lambda$XCDM model, in which, thanks to the presence of the cosmon $X$, the function $r = r(z)$ presents a maximum at the redshift [19]:

$$z_{\text{max}} = \frac{1}{\omega_X \left( \Omega_X^0 + \nu \Omega_M^0 - \epsilon (1 - \Omega_X^0) \right) + 1} - 1,$$

where we have defined $\epsilon \equiv \nu (1 + \omega_X)$. One can show that this point cannot be in our past [19], so it will generally be in the future (hence $-1 < z_{\text{max}} < 0$). Beyond that maximum, there is a turning point $z = z_r < z_{\text{max}}$ where the universe stops and reverses its evolution. At the stopping point, $\Omega_D(z_r) = -\Omega_M(z_r) < 0$. This is possible because we can have $\Omega_X < 0$, or $\Omega_X < 0$.

The following observation is now in order: notice, from eq. (20), that for $\nu = 0$ (which implies no running of $\rho_X$) the cosmon energy density is conserved; so, if in addition $\Omega_X^0 = 0$ (vanishing
current cosmon density), we must then have $\Omega_X(z) = 0$ at all times. Therefore, in the limit $\nu = \Omega_X^0 = 0$ we exactly recover the standard $\Lambda$CDM model. And it is precisely in this limit that the formula (22) for the redshift location of the maximum of the ratio $r(z)$ ceases to make sense, as it is evident by simple inspection of that formula. In contrast, when we make allowance for the parameter space of the $\Lambda X$CDM model, the maximum does exist and moreover the ratio $r(z)$ stays bounded – typically $r < \mathcal{O}(10)$ – for the entire history of the universe. This is clearly seen in Fig. 3, for various values of the parameters. So, in fact, the $\Lambda X$CDM model can provide a nice solution to the cosmic coincidence problem without necessarily departing from the $\Lambda$CDM model in a significant way. Furthermore, the detailed analysis of the cosmic perturbations in the $\Lambda X$CDM model indicate excellent compatibility with the present observational data on structure formation [20].

5. The “new cosmon” model: towards solving the “old CC problem”

The “Relaxed Universe” is a special realization of the $\Lambda X$CDM model which points to a more ambitious aim, to wit: solving the “old CC problem” [11], i.e. understanding how the Universe could start up its thermal history with a huge value of the initial vacuum energy $\rho_\Lambda^0 \sim M_X^4$ (e.g. triggered by a GUT with, say $M_X \sim 10^{16}$ GeV, presumably responsible for inflation) and then eventually end up with its current tiny value $\rho_\Lambda^0 < 10^{-47}$ GeV$^4 \sim 10^{-110} \rho_\Lambda^0$ ! The history of this profound theoretical conundrum, which keeps in check the fundamental physics principles since more than forty years ago, traces back to Zeldovich’s realization in the late sixties [21], that the vacuum fluctuations of the quantum fields should induce a very large value of the CC. Since then the problem stays with us, unbeaten and virtually invincible. The “traditional” fine-tuning solution [11] (i.e. the sheer adjustment by hand of the initial big CC value $\rho_\Lambda^0$ by a carefully chosen finite “counterterm”) is simply and fully inadmissible, especially at the quantum level – see [13] (particularly Appendix B of this reference) for a detailed account. A much more promising approach, instead, should be to find out a mechanism that neutralizes dynamically the presence of the initial vacuum energy, $\rho_\Lambda^0$, whatever it be its seed value in the early universe. We will call the cosmological model related to this relaxation mechanism, the “Relaxed Universe” [13].

For other recent alternative approaches aiming to palliate the CC problem, see e.g. [22].
As we would like that the expansion of the universe be responsible for the triggering of the relaxation dynamics, it is reasonable to assume that such mechanism should be closely related to the expansion itself, or more precisely, to the interaction governing the expansion, i.e. gravity. A note of caution, however: although we aim at a modified form of gravity, we cannot just content ourselves with a “late time effect” (as usually done in the literature on modified gravity [23, 24, 25]). We rather must have a persistent effect, fully active throughout the entire history of the universe. Indeed, in order to solve the old CC problem we need that some automatic mechanism sets to work at a time prior to the onset of the radiation epoch – at which time it must cancel most of the value of the original vacuum energy present in the early universe – and, most important, we also need that this cancelation is maintained for the rest of the universe lifetime, leaving a small value today and maybe also in the future.

It is not easy to devise such a mechanism, but some ideas on it are briefly explained here – see [12], and specially [13], for a detailed account; and [26, 27] for alternative forms. The effective action of the “Relaxed Universe” is proposed in the following way:

\[ S = \int d^4 x \sqrt{|g|} \left[ \frac{1}{16\pi G} R - \rho_\Lambda - F(R, G) + L_\nu (\text{matter fields}) \right], \]

(23)

in which we have an (arbitrarily large) initial vacuum energy \( \rho_\Lambda \), and we have included a functional (the “cosmon functional”) \( F = F(R, G) \) of the Ricci scalar \( R \) and the Gauss-Bonnet invariant \( G = R^2 - R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \). The structure of this functional is convenient in order to avoid the presence of ghost degrees of freedom and also to dodge the Ostrogradski instability, i.e. the appearance of vacuum states of negative energy [28]. The corresponding (generalized) Einstein equations read

\[ R_{\mu\nu} - \frac{1}{2} \delta_{\mu\nu} R = -8\pi G \left[ \rho_\Lambda \delta_{\mu\nu} + 2E^\mu_{\nu} + T^\mu_{\nu} \right], \]

(24)

where \( T^\mu_{\nu} \) is the ordinary energy-momentum tensor of the matter fields, and \( E^\mu_{\nu} \) is the new tensor in the field equations associated to the presence of the cosmon functional \( F \) in the action.

We assume that matter is covariantly conserved, and therefore \( E^\mu_{\nu} \) must also be conserved: \( \nabla^\mu E^\mu_{\nu} = 0 \). Clearly, our functional constitutes an extension of the Einstein-Hilbert’s action with CC, in which we have added the \( F(R, G) \) part. The next point is to make an appropriate choice for the functional \( F \) such that it can have a bearing on the cosmological constant problem.

We make the following Ansatz:

\[ F(R, G) = \beta F(R, G) = \frac{\beta}{B(R, G)}, \]

(25)

\[ B(R, G) = \frac{2}{3} R^2 + \frac{1}{2} G + (y R)^3, \]

(26)

where \( B(R, G) \) is a polynomial of the Ricci scalar and the Gauß-Bonnet invariant, and \((\beta, y)\) are dimensionful parameters to be determined later. This model is in fact of \( \Lambda XCDM \) type, in the sense that the cosmon \( X \) here is related to the energy density \( \rho_F = 2E^0_0 \) generated by \( F \) in the effective action, and so \( \rho_F \) plays the role of \( \rho_X \) in the \( \Lambda XCDM \) model. Explicitly, one finds

\[ \rho_F = 2 E^0_0 = \beta \left[ F(R, G) - 6(\dot{H} + H^2) F + 6H \dot{F} R - 24H^2 (\dot{H} + H^2) F^G + 24H^3 \dot{F}^G \right]. \]

(27)

In our case, \( F \) has the form \( F = 1/B \) as defined in (25), and \( F^Y \equiv \partial F/\partial Y \) are the partial derivatives of \( F = F(R, G) \) with respect to \( Y = R, G \). The parallelism with the \( \Lambda XCDM \) model becomes even more transparent if we notice that the mixed DE energy density defined in (20)
takes here the form $\rho_{\text{eff}}(t) = \rho_{\Lambda}^i + \rho_F(t)$, and is covariantly conserved. Indeed, since $\rho_{\Lambda}^i$ is constant, we have $\dot{\rho}_{\Lambda} + 3H(1 + \omega_F)\rho_F = 0$, where $\omega_F$ is the effective EOS of the cosmon, i.e. the analog of $\omega_X$ in section 4, except that here $\omega_F = \omega_F(t)$ is a non-trivial function of the cosmological evolution which is determined by the structure of the cosmon functional (25).

This is indeed a good example of the situation we have mentioned in section 4, in which we emphasized that the cosmon $X$ is not a fundamental field, but a non-trivial field-theoretical object generated from terms of the effective action of QFT in curved space-time. Evaluating it in the FLRW metric we obtain, for its denominator (26):

$$B = 24H^4(q - \frac{1}{2})(q - 2) + [6yH^2(1 - q)]^3,$$

(28)

where $q = -1 - \dot{H}/H^2$ is the deceleration parameter. The previous formula shows why the cosmon functional (25) can provide a dynamical mechanism for counterbalancing the value of $\rho_{\Lambda}^i$, whatever it be its size. The relaxation of the effective vacuum energy $\rho_{\text{eff}}(t) = \rho_{\Lambda}^i + \rho_F(t)$ in the radiation epoch originates from the $(1 - q)$ factor in the denominator (28) of the cosmon functional, as this term grows as $\sim H^6$ at large $H$ and dominates $B$ in the earlier epochs. The large $\rho_{\Lambda}^i$ left over in the immediate post-inflationary period drives the deceleration parameter $q$ to larger values until $q \rightarrow 1$, which corresponds to radiation-like expansion. In other words, the very existence of the radiation period is triggered automatically by the presence of this term, which can be thought of as a countermeasure launched by the universe against the presence of the large “residual” vacuum energy $\rho_{\Lambda}^i$ in the pre-radiation era. As a result, $\rho_F = 2E_0^0$ is driven dynamically to very large values, opposite in sign to $\rho_{\Lambda}^i$, and produce a small effective $\rho_{\text{eff}}$ in the radiation epoch, for an appropriate sign of the parameter $\beta$. Similarly, the $(q - 1/2)$ factor in the denominator (28) will be responsible for the relaxation in the matter era $(q \simeq 1/2)$, at lower values of $H$. This is possible because the term with the $(q - 1/2)$ factor is weighted by a power of $H$ smaller than the term carrying the $(1 - q)$ factor. The transition between the two regimes (i.e. from radiation to matter) is what fixes the parameter $y \sim H_{\text{eq}}^{-2/3}$, where $H_{\text{eq}} \sim 10^5 H_0$ is the Hubble rate just at the transition time between the two epochs.

A numerical example illustrating all these features can be appreciated in Fig. 4. The Relaxed Universe is seen to follow very closely the past history of the standard $\Lambda$CDM model (see especially the upper panel in that figure), and its effective EOS loiters around the $\Lambda$CDM value $\omega_{\text{eff}} \simeq -1$ near our time (see the second panel of that figure). In the far past, $\omega_{\text{eff}}$ turns out to track the values $\omega_{\text{eff}} \sim 0$ and $\omega_{\text{eff}} \sim 1/3$ in the matter and radiation epochs, respectively. Finally, when the universe leaves the matter epoch $q \simeq 1/2$, there is a kind of impasse, as there is no longer a special value for $q$ to pick dynamically in order to maintain the counterbalance of the large $\rho_{\Lambda}^i$ by the cosmon density $\rho_F$. What to do now? Under these circumstances, the last resort of the relaxation mechanism is to enforce (dynamically) a very low value of $H$. Since in the current universe we must have $|\rho_{\Lambda}^i + \rho_F(H)|/\rho_{\Lambda}^i \ll 1$, with $\rho_F \sim \beta H^4$, it follows that the value of $H$ that solves this equation is approximately given by

$$H_q \sim \left(\frac{\beta}{|\rho_{\Lambda}^i|}\right)^{1/4}.$$

(29)

For an appropriate choice of $\beta$, we may bring $H_q$ close enough to the current value $H_0$. As $\beta$ has dimension 8 of mass, we can rewrite it as $\beta \equiv M^8$. The relaxation condition requires $\rho_F \sim -\rho_{\Lambda}^i$, where $|\rho_{\Lambda}^i| \sim M^4_A$. Thus, $M^8 \sim |\rho_F|H_0^4 \sim M^4_A H_0^4$. Taking the standard GUT scale $M_X \sim 10^{16}$ GeV, we obtain $M$ in the ballpark of a light neutrino mass:

$$M \sim \sqrt{M_X H_0} \sim 10^{-4} \text{eV}.$$  

(30)
Figure 4. Deceleration parameter $q$, effective EOS $\omega_{\text{eff}}$, and relative energy densities for each energy component (normalized with respect to the corresponding critical density) $\Omega_n(z) = \rho_n(z)/\rho_c(z)$ of dark energy $\rho_{\Lambda \text{eff}}$ (orange thick curve), dark matter $\rho_M$ (black dashed-dotted) and radiation $\rho_r$ (red dashed) as functions of redshift $z$ in the relaxation model (25) with $y = 0.7 \times 10^{-3} H_0^{-2/3}$, $\rho^i_\Lambda = -10^{60}$ GeV$^4$, $\Omega^0_M = 0.27$, $\Omega^0_r = 10^{-4}$, $q_0 \approx -0.6$, $q_0 = -0.5 H_0$. In the $q$-plot, the thick orange curve corresponds to the Relaxed Universe, and the black dashed-dotted curve to $\Lambda$CDM.

Incidentally, this value for $M$ is the geometric mean of the two most extreme mass scales available in our universe below the Planck mass. It is rewarding to see that we do not obtain an extremely small mass scale of order $m_\phi \sim H_0 \sim 10^{-33}$ eV, as in the case of quintessence models.

From the above considerations, we see that the $F$-functional’s main mission is to dynamically counterbalance the big initial $\rho_\Lambda$ at all epochs, not just now; and in the last stage of this dynamical process the Hubble rate is driven to the very small value $H_0$ that we observe today. At the end of the day, what we get is not just a late time effect of the cosmological evolution (as in the traditional modified gravity models), but rather a truly perennial large scale influence on all periods of the cosmic history. One hope is that a relaxation mechanism of this sort should ultimately provide a dynamical solution to the tough fine tuning problem underlying the “old CC problem” [11]. For example, in the standard model of electroweak interactions such fine-tuning involves 55 decimal places, at least, and it must be retuned order by order in perturbation theory until reaching loop diagrams of order 20th! – see [13] for details. In the Relaxed Universe, in contrast, the tuning to all orders is automatic and performed dynamically by the universe’s expansion. This is why such field-theoretical object associated to the functional $F$ is called the “cosmon”, as its primary mission is to compensate for the huge initial CC throughout the entire cosmological evolution. Clearly, it has the same aim as the cosmon scalar field first introduced in [29], except that here the cosmon has nothing to do with scalar fields, and moreover is not subdued by Weiberg’s “no-go” theorem [11] – see [13] for a more detailed account.

While in the present version the cosmon is not a fundamental quintessence field [30], it can perfectly mimic quintessence-like behavior (cf. Fig. 4). The new cosmon, however, is a more
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complex object: it is a functional whose ultimate origin must be provided by a fundamental
theory (say string theory or any other theory aiming at a final explanation of quantum gravity).
In the meanwhile, what we have shown here is a practical working example, actually a class of
such functionals which are able to truly adjust the vacuum energy dynamically, whatever it be
its initial value, and without performing fine-tuning at any stage of the cosmological evolution.
As this was the main aim of the original cosmon model [29], it is naturally suggested that the
general class of these functionals is called the “new cosmon”. For more details, see [13].

Let us also mention that the Relaxed Universe based on the cosmon functional (25) and
extensions thereof (cf. Ref. [13]), provides also a possible natural solution to the cosmic
coincidence problem (mentioned in section 4). This additional bonus is not too surprising if
we recall that the Relaxed Universe is a particular implementation of the class of ΛXCDM
models of the cosmic evolution [19]. We have already mentioned that, in the Relaxed Universe,
there is a tracking behavior of matter/radiation and DE during the $q = 1$ (radiation) and $q = 1/2$
(matter epochs). However, when the matter epoch is left behind and the relaxation mechanism
is enforced (dynamically) to pick a very small value of $H$ (as we have seen above) in order to
insure the condition $|\rho_{\Lambda}^i + \rho_F(H)| \ll \rho_{\Lambda}^i$ to hold, this crucial event decoupled once and forever the
tracking behaviors of matter/radiation and DE, and since then the DE dominates the universe’s
expansion. Such breakdown of the tracking property at the end of the matter epoch, on the other
hand, pushed the subdominant DE density curve upwards until crossing the decaying DM one at some point after the matter epoch and hence near our time. This feature, which can
be very well appreciated in Fig. 4 (cf. third panel), encodes the very origin and the possible
exciting explanation (within the Relaxed Universe) of the “coincidence $\Omega_{\Lambda}^i \sim \Omega_M^0$” of the matter
and DE densities in our late time neighborhood. Not only so: we can also understand now the
smallness of both $\rho_{\Lambda}^0$ and $H_0$ in front of the huge initial vacuum energy density $\rho_{\Lambda}^i$. Indeed,
from (29) we predict $H_0 \sim M^2/M_X \sim 10^{-42}$ GeV, for $M$ not far from the $\sim$meV scale, which
is the mass scale itself of the current CC: $m_{\Lambda} \equiv (\rho_{\Lambda}^0)^{1/4} \sim 10^{-3}$ eV. We have thus arrived at
a most natural scenario (“The Relaxed Universe”) in which, at variance with the quintessence
formulation of the DE, there is no need to severely fine-tune any parameter of the model, and
moreover it is not required the existence of any extremely tiny mass scale.

6. Conclusions

We have shown that models with variable vacuum energy $\rho_{\Lambda}$ (and may also be with variable
$G$) generally lead to a non-trivial effective EOS when viewed from the point of view of a self-
conserved DE fluid at fixed $G$. As a result, these models can effectively appear as quintessence,
and even as phantom energy, without need of invoking the existence of fundamental quintessence
and phantom fields. Combining a time-evolving $\rho_{\Lambda}$ with a variable “cosmon” energy density $\rho_X$
(in general not representing any scalar field), we obtain the class of AXCDM models, which are
able to deal with the cosmic coincidence problem. Finally, one can provide a version of the
AXCDM model in which the new cosmon stands for a self-adjusting functional, namely one
which is able to counteract the effects of an arbitrarily large CC in the early universe, leaving a
small and completely innocuous net value of the CC at any stage of the cosmological evolution,
in particular the tiny value $\rho_{\Lambda}^0 \sim 10^{-47}$ GeV$^4$ that we observe at present. This particular cosmon
functional is thus able to reduce the value of the CC without fine-tuning, and may be eligible as
a prototype model (“The Relaxed Universe”) for eventually solving the old CC problem.

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