MOdified Newtonian Dynamics as an entropic force

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Under natural assumptions on the thermodynamical properties of space and using the holographic principle we reproduce a MOND-like behaviour of gravity on particular scales of mass and length, where Newtonian gravity requires modification or extension if no dark matter component is introduced in the description of gravitational phenomena. The result is directly obtained with the assumption that a fundamental constant of nature needs to be introduced into the problem. This calculation extends the one by Verlinde [24] in which Newtonian gravity is shown to be an emergent phenomenon and together with it, suggests that gravity at all scales is emergent.

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I. Introduction

The laws for black hole mechanics have suggested a remarkable similarity with the three laws of thermodynamics, in such a way that quantities associated to black holes properties have their corresponding thermodynamic equivalent interpretation [1–3]. In particular, the black hole area, which is determined by its horizon, is related to the associated black hole entropy, in the sense that it cannot decrease in time under any physical process on a closed system. The temperature of the black hole is given by the Hawking-Zeldovich temperature and is inversely proportional to the mass of the black hole [8, 23].

The well known interpretation of entropy as a quantity that offers a measure of non-available information or disorder in a system, has leaded directly to the idea that the increase in entropy, and therefore in area, is due to the lost of information when a particle crosses the horizon and it has no more causal relation with the rest of the universe [2].

All the above suggest the possibility for a deep relation between thermodynamics and gravity. This has been studied mainly in the relativistic regime under the concept of emergent gravity, considering thermodynamics as a more fundamental theory from which, general relativity can be derived [see e.g. 20 and references therein]. Using a metric treatment of the thermodynamic variables in a curved space-time, Jacobson [11] has been able to derive Einstein’s field equations. In the non-relativistic regime, Verlinde [24] used very simple assumptions about space, energy and information in order to show how a MOND-like gravity force can be obtained from thermodynamical arguments. In this work, we show how, using arguments about thermodynamics and information, it is possible to derive an equation for the gravitational force in a MOND-like modified gravity regime, which support the idea that gravity can be understood as an emergent force, i.e. a consequence of deeper fundamental principles. The article is organised as follows: in section II we use dimensional analysis arguments to find an expression for the number of bits contained inside a surface under the assumption that Milgrom’s acceleration constant $a_0$ is a fundamental constant of nature. In section III we perform a similar approach as the one made by Verlinde [24] in order to show how a MOND-like gravity force can be obtained from thermodynamical arguments. Finally, in section IV we discuss our main results.

II. Dimensional Analysis

One of the most important assumptions made by Verlinde [24], is that the information describing a physical system is stored on spatial surfaces, or screens, that are ruled by the holographic principle. Every surface behaves as a “stretched horizon” of a black hole and when a particle interacts with it, the entropy, and consequently, the amount of information gets affected. In principle we do not know the shape of the surfaces, so, for simplicity we can consider each screen as closed and spherical with radius $r$. Each surface contains $N$ bits of information. One can also think that on each fundamental Planck square area, the maximum information that can be stored is one
bit. This length is constructed with three fundamental constants of nature: (1) Newton’s constant of gravity $G$, (2) the velocity of light $c$ and (3) Planck’s constant $\hbar$. As shown by [4, 5, 10], the introduction of a new fundamental constant of nature $a_0 \approx 1.2 \times 10^{-10}$ m s$^{-2}$ allows for a general understanding of gravity on extended regimes where either Newtonian and/or relativistic standard gravity fails to explain different phenomena usually ascribed to the existence of an unknown dark matter component. For the same reason, when Milgrom’s acceleration constant $a_0$ is introduced in the description of gravitational phenomena, the precise way in which the information is stored on a particular screen must differ from Verlinde’s calculation in this extended regime of gravity.

Buckingham’s Π-theorem of dimensional analysis [21] gives a useful approach to the problem of finding the characteristic area in which the information is contained. With the introduction of a new fundamental quantity $a_0$, there are two considerations that become important: the first one is the choice of the independent variables, and the other one is the existence of a degree of freedom in the resultant system of equations. In order to begin visualising the dimensional problem, note that two characteristic lengths can be constructed:

$$l_p := \sqrt{\frac{G\hbar}{c^3}}, \quad \text{and} \quad \lambda := \frac{c^2}{a_0}$$

The characteristic length $l_p$ is Planck’s length and the length $\lambda$ is a characteristic length that appears in a natural way when the fundamental constant $a_0$ is introduced in the description of gravitational phenomena [16]. Since there are two natural lengths that appear in this extended description of gravity, it is not straightforward to know how to calculate the fundamental area storing one bit of information. To do so, note that the number of bits $N$ stored on a screen at a distance $r$ from the origin is given by a general unknown function $f$ as:

$$N = f(r, l_p, \lambda).$$

Assuming the validity of the holographic principle, the amount of information must be proportional to the area of the screen, i.e. to $r^2$. For the Newtonian case analysed by Verlinde [24], the amount of information $N \propto r^2/l_p^2$ which follows directly from Buckingham’s Π-Theorem since $a_0$ does not enter into the description of gravitational phenomena at the Newtonian level. With this, we can write equation (2) as:

$$N = \frac{r^2}{l_p^2} \left\{ \begin{array}{ll}
1, & \text{for Newtonian gravity.} \\
F(\lambda/l_p), & \text{for MOND-like gravity,}
\end{array} \right.$$ (3)

for a dimensionless unknown function $F(\lambda/l_p)$, which according to Buckingham’s Π-Theorem is given by $F \propto (\lambda/l_p)^b$. The precise value of the unknown exponent $b$ will be found by requiring a match with MOND’s force formula. In other words, in the extended regime of gravity we expect the number of bits of information to be:

$$N \propto \frac{r^2}{l_p^2} \left( \frac{\lambda}{l_p} \right)^b.$$ (4)

### III. Emergence of Modified Gravity

The main motivation by Verlinde [24] to think of gravity as a force related to entropy has its origin on the the restitutive force that acts on a polymer when it suffers a displacement $\Delta x$. This force tends to back the polymer to its original position since this configuration maximises the entropy. The link with gravitation consists on a similar idea for which there is an entropic force that emerges as a consequence of the system searching for a configuration of maximum entropy when a particle approaches to a particular screen. We assume that inside the screen, the dynamics allow us to define energy, and consequently, the associated mass $M$ and temperature $T$ are well defined quantities. With this, we can use the first law of thermodynamics to find the force $F$ associated to changes in the stored information, i.e. due to a change in entropy $\Delta S$ given by:

$$F\Delta x = T\Delta S,$$ (5)

for a constant volume. Let us now find the expression for the gravitational force by considering gravity as an entropic force. For this, we follow the approach by Verlinde [24] & Jacobson [11] analysing the behaviour of a mass $M$ particle near a black hole horizon. At a distance of one Compton length from the horizon, the particle can be considered to be part of the black hole and so its entropy is increased in the following way [24]:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x,$$ (6)

when a displacement $\Delta x$ occurs. In other words, a change in the particle position causes an increment on the system entropy, such that it tries to maximise it and as such, the horizon can be substituted by a screen. The other assumption to make is that the energy contained inside the surface satisfies the principle of equipartition, and that it can be transformed into a mass $M$. Thus, using these facts on equations (4), (5) and (6) it follows that:

$$F = \frac{4\pi a_0 \hbar^{1/2} \sqrt{G^3/2} m M}{r^2 c^{7/2}} \left( \frac{\lambda}{l_p} \right)^{-b}.$$ (7)
In this form, the last equation has a non-straightforward MOND-like representation and has a $1/r^2$ Newtonian-like behaviour. However, it can be written in a more convenient way using the fact that the total mass $M$ can be expressed in terms of Planck’s mass $M_P := (hc/G)^{1/2}$ as:

$$M = NM_P.$$  \hspace{1cm} (8)

Direct substitution of this last equation into relation 7 yields:

$$F = \frac{4\pi m \sqrt{a_0 GM}}{\zeta^{1/2}} \frac{\Lambda}{r} \left( \frac{\lambda}{l_P} \right)^b,$$  \hspace{1cm} (9)

in which the value $N$ has been substituted by the right-hand side of equation 4 and the constant $\zeta$ is the proportionality factor on that relation. The choice $b = 1$ and $\zeta = 16\pi^2$ gives the required absolute value of the force experienced by a point mass $m$ under the influence of the gravitational field produced by the point mass $M$ in the MONDian (or extended) regime of gravity:

$$F = m \frac{\sqrt{a_0 GM}}{r}.$$  \hspace{1cm} (10)

IV. DISCUSSION

As explained by Famaey and McGaugh 7, Mendoza 14 and in a more profound and empirical way by Mendoza and Olmo 17, if gravitational phenomena requires to be modified at a certain scales of mass and length, one needs to incorporate a new fundamental constant of nature relevant in all gravitational phenomena at those scales. As shown by Mendoza and Olmo 17 this gravitational constant is as important as Newton’s constant of gravity and can be mathematically manipulated as to have dimensions of acceleration which converge to Milgrom’s acceleration constant $a_0$. This is so since gravitational phenomena does not follow the standard Newtonian (or general relativistic) behaviour of gravity at scales which greatly differ from the ones in which precise gravitational experiments have been performed to test the validity of Newton’s law of gravitation (or Einstein’s general relativity cf. 25). The behaviour of gravity at those scales can be considered as independent of the behaviour of standard gravity and as such, a new fundamental constant of nature has to be introduced in the description of gravitational phenomena 21. In this article we have introduced this extra fundamental constant of nature $a_0$ in the description of gravity and used thermodynamic and information properties of space and time in order to show that a MONDian force law can be obtained by assuming the validity of the holographic principle.

A full non-relativistic theory of gravity can be constructed assuming a modification of inertia as described by Famaey and McGaugh 7, but as shown in this work the modification naturally appears in the force sector and not on the dynamical one. As such, the developments made by Mendoza et al. 16 in which the extensions of gravity are made in the force sector seem to be more appropriate. From basic fundamental principles and with no field equations in the description of gravitational phenomena, Mendoza and Olmo 17 have shown that such an extended theory of gravity can be relativistically constructed. An example of such a theory has been developed in the works of Bernal et al. 15, Mendoza 14 and Mendoza et al. 15. Furthermore, in recent years a growing number of independent observations have suggested that gravity requires modification 9, 10, 12, 13, 13, 22 and not the inclusion of unknown dark matter entities.

With a few natural assumptions about space and information, the main result of this article is to show that gravity can be considered an emergent phenomenon also in the MONDian regime. This suggests that the force of gravity on this extended regime is not a fundamental force of nature, but a consequence of the inherent properties of space and time. Since Verlinde 24 showed that Newtonian gravity emerges from the thermodynamical properties of space and time, this all suggests that gravitation is an emergent phenomenon at all scales of mass and length.

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