Actuator line simulations of a Joukowsky and Tjæreborg rotor using spectral element and finite volume methods

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Abstract. The wake structure behind a wind turbine, generated by the spectral element code Nek5000, is compared with that from the finite volume code EllipSys3D. The wind turbine blades are modeled using the actuator line method. We conduct the comparison on two different setups. One is based on an idealized rotor approximation with constant circulation imposed along the blades corresponding to Glauert’s optimal operating condition, and the other is the Tjæreborg wind turbine. The focus lies on analyzing the differences in the wake structures entailed by the different codes and corresponding setups. The comparisons show good agreement for the defining parameters of the wake such as the wake expansion, helix pitch and circulation of the helical vortices. Differences can be related to the lower numerical dissipation in Nek5000 and to the domain differences at the rotor center. At comparable resolution Nek5000 yields more accurate results. It is observed that in the spectral element method the helical vortices, both at the tip and root of the actuator lines, retain their initial swirl velocity distribution for a longer distance in the near wake. This results in a lower vortex core growth and larger maximum vorticity along the wake. Additionally, it is observed that the break down process of the spiral tip vortices is significantly different between the two methods, with vortex merging occurring immediately after the onset of instability in the finite volume code, while Nek5000 simulations exhibit a 2-3 radii period of vortex pairing before merging.

1. Introduction

Responding to the increased interest in wind turbines by the industry, the last decades have seen a rise in research activity focusing on a better understanding of the aerodynamics of horizontal axis wind turbines. To reduce installation and maintenance cost, wind turbines are often placed close to each other in wind farms. Here, turbines can experience an increased turbulence level due to wake effects which leads to a modified performance and life expectancy.

In our study we aim at a time-dependent representation of the tip and root vortices. Therefore we employ the actuator line (ACL) method, proposed by Sørenson and Shen [1], where the blades are substituted with discretized lines as sketched in figure 1(b). The body forces are then computed from the aerodynamic forces acting on the blades. Using a correlation between the local blade velocities and the forces at the actuator line, the blade forces are determined and included as extra terms in the Navier–Stokes (N–S) equations. The ACL technique together
with large eddy simulations has been extensively used to simulate wind turbine wakes. These simulations study the influences of the ACL model parameters in Mikkelsen [2], the influence of turbulence in Trolldborg [3] and the stability of the helical vortex system in Sarmast et al. [8].

To date, spectral element methods have rarely been employed to simulate flows around wind turbines. One study was performed by Peet et al. [5], which provides a brief qualitative validation of the helical tip vortex structure and a comparison of the mean velocity profile with the Tjæreborg turbine by Trolldborg [3]. Tamnøy et al. [6] further validated the performance parameters, power and thrust coefficient, with the work by Trolldborg [3], and Tamnøy et al. [7] introduced a neutral atmospheric boundary layer model in Nek5000. To the author’s knowledge, the current study provides for the first time a detailed comparison between a finite volume and a high-order spectral element code of a wind turbine wake simulation. Understanding the limitations of different codes is crucial for future simulations with multiple turbines.

2. Numerical methods and setup

Numerical simulations are performed for two setups, the constant circulation (CC) case and the Tjæreborg (TJ) wind turbine. For the CC setup the lift force $F_L$ at the actuator lines is derived from the Kutta–Joukowski theorem, where the lift force is directly related to the circulation $\Gamma$. In this configuration the drag force $F_D$ is assumed to be zero. This results in a lift force that is solely dependent on the velocity relative to the flow approaching the airfoil and the pre-defined circulation. The CC approximation holds for turbines operating at design conditions and corresponds to Glauert’s optimal working condition [10]. When modeling a real turbine the lift and drag forces are determined by the flow velocity and tabulated aerodynamic coefficients. Figure 1(c) visualizes a two-dimensional airfoil with the relevant parameters. These include the relative velocity of the approaching fluid $U_{rel}$, local angle of attack $\alpha$, twist $\Theta$, the normal and tangential velocities of the flow at the blade $u_n$ and $u_{\Theta}$ and the lift and drag forces $F_L$ and $F_D$.

The discrete two-dimensional ACL forces $\mathbf{F}_{2D} = (F_L, F_D)$ are distributed in a three-dimensional Gaussian manner across the grid points as shown in equations (1)–(2). Hereby, $d = |x_{ACL} - x_n|$ denotes the distance between the points on the actuator line and the grid.
Table 1. Definition of case parameters

|   | $\varepsilon$ | $Re_R$ | $\lambda$ | $\Gamma$ | $N$ |
|---|---|---|---|---|---|
| CC1 | $2.5\Delta r$ | 50000 | 7.07 | 0.133 | 9 |
| CC2 | $2.5\Delta r$ | 10000, 50000, 250000 | 7.07 | 0.133 | 7, 8, 9, 11 |
| CC3 | $1.5\Delta r$, $2.5\Delta r$, $3.5\Delta r$ | 50000 | 7.07 | 0.133 | 9 |
| TJ | $2.5\Delta r$ | 50000 | 7.07 | – | 9 |

points and $\varepsilon$ is a smearing parameter; $x, y, z$ denote the spatial coordinates and $t$ the time,

$$\mathbf{F}(x, y, z, t) = -\sum_{n=1}^{N} \mathbf{F}_{2D}(x_n, y_n, z_n, t) \eta_{\varepsilon}(|x_{ACL} - x_n|)$$  (1)

$$\eta_{\varepsilon}(d) = \frac{1}{\varepsilon^{3} \pi^{3/2}} \exp \left[ -\left( \frac{d}{\varepsilon} \right)^{2} \right].$$  (2)

The employed model uses uniform, non-turbulent inflow and neglects the effect of nacelle and tower on the flow, thus only representing the wind turbine using the actuator lines. We consider the rotating frame of reference with the blades located at a fixed position in the mesh.

The computational domain is non-dimensionalized with the turbine blade length $R^*$. It is cylindrical for Nek5000, with a dimensionless radius of $R_{rad} = 10$ to ensure very low blockage effects. The actuator lines are located at $R_{in} = 7$ downstream of the inlet, as shown in figure 1(a). The domain is extended in the streamwise direction to ensure $L_z \geq 40$. The computational domain of EllipSys3D differs in the hub area, where a hole of $R_h = 0.1$ exists (annular domain). It was established in EllipSys3D, that the hole does not impact the spiral tip vortex structure, which was the initial focus of this study. Therefore the chosen domains were considered adequate.

Nek5000 employs a stress-free boundary condition at the outlet with a sponge of length $5\Delta r$. It is 60% coarser resolution for Nek5000. EllipSys3D employs a Dirichlet boundary condition at the inlet and a slip boundary condition at the inner and outer annulus boundary. The actuator lines extend from $0.1 \leq r \leq 1$ and are discretized with a 91 point stencil. A comparison of a slice of the mesh perpendicular to the streamwise direction from $0.0 < r < 1.3$ revealed that Nek5000 uses approximately 73% of the resolution of EllipSys3D. Along the streamwise axis the Nek5000 resolution is 80% of EllipSys3D thus leading to a 60% coarser resolution for Nek5000.

The parameters for the constant circulation reference case are defined in table 1 and are adopted from Sarmast et al. [13]. Additionally, table 1 defines the parameter variations performed in section 3.3. Hereby, $Re_R = U_{\infty}^* R^* / \nu$ denotes the radius-based Reynolds number and $\lambda = \Omega R^* / U_{\infty}^*$ the tip speed ratio, with $U_{\infty}^*$ defined as the free stream velocity. These reference quantities are used to non-dimensionalize all results in the following. The smearing parameter $\varepsilon$ is scaled with the exact distance between the grid points along the actuator line $\Delta r$ in EllipSys3D. The Reynolds number used is much lower than that of real wind turbines, however it has been confirmed by Ivanell et al. [4] that the Reynolds number is expected to influence the wake structure only below a certain limiting value, above which the integral quantities, such as the circulation $\Gamma$ approach an asymptotic value. After the initial validation the influence of the smearing parameter and the Reynolds number on the flow were studied.

Large eddy simulations are carried out using EllipSys3D [11], a general three-dimensional flow solver using a subgrid-scale model developed by Ta Phuoc [15]. The code is based on...
a multiblock/cell-centered third order finite volume discretization of the incompressible N–S equations. The code is formulated in collocated primitive variables, i.e. in pressure and velocity variables. Rhie/Chow interpolation is used to avoid odd/even pressure decoupling.

The spectral element code Nek5000 for the incompressible three-dimensional N–S equations combines the geometric flexibility of a finite element solver with the exponential convergence of a global spectral code [9] and is highly scalable. In each spectral element the solution is expanded using Legendre polynomials on Gauss–Lobatto–Legendre quadrature points. The elements are loosely coupled with only $C_0$ continuity required. The non-linear terms are treated explicitly by third-order extrapolation (EXT3), while the viscous terms are discretized implicitly using a third-order backward differentiation scheme (BDF3). Filtering is applied to the highest modes to stabilize the numerical simulation.

3. Results
We conduct the validation by comparing the wake structure and blade loading produced by the different codes. Additionally, as validation against experimental data, the power coefficient is compared against data provided by experiments on the Tjæreborg wind turbine. Figure 2 shows the development of the wake behind the actuator line model in Nek5000. The influence of the actuator lines can be seen upstream of the rotor where they cause a deceleration of the flow. The wake is defined by a velocity deficit and initially stable and organized tip and root vortices, which after a certain length destabilize and break down.

In the following sections the wake expansion, vortex pitch (streamwise distance between two subsequent vortices), circulation and vortex core size are used to characterize the wake geometry and tip vortex structure. The vortex center is determined as the position of maximum vorticity magnitude. The swirl velocity $\omega_\theta$ at each vortex is extracted by subtracting the convection velocity in both streamwise and radial direction from the existing velocity field. The core size is defined as the position of maximum swirl velocity.

3.1. Constant circulation turbine
Figure 3 visualizes the placement of the helical tip and root vortices generated by the finite volume and spectral element code, respectively. Their structures agree closely with each other immediately downstream of the ACLs. We observe a destabilization of the tip vortices in both
codes, with the vortices computed by EllipSys3D destabilizing slightly further upstream at $z \approx 4.0$, where they immediately merge. In Nek5000 we observe vortex pairing at $z \approx 5.0$ and subsequently vortex merging $2 - 3$ radii further downstream. The helical vortex cores dissipate much quicker in EllipSys3D than for Nek5000.

The root vortices destabilize immediately downstream of the rotor in EllipSys3D and lose their strength quickly while in Nek5000 the vortices retain their initial strength and destabilize slightly upstream of the tip vortex instability. In Nek5000, the root vortices are stable until they undergo a similar pairing as in the tip vortices and then break down into small-scale turbulence. The discrepancy in the root vortex development is attributed to the domain differences in the nacelle area. The slip boundary condition at the inner ring of the finite volume setup forces the vorticity components in the streamwise and azimuthal direction to zero at the boundary, thus leading to the quick decrease in the vorticity magnitude in the streamwise direction for EllipSys3D. In Nek5000 the blade forces are not imposed below $r = 0.1$, resulting in a high speed jet at the rotor center (visible in figure 2).

While the wake expansion is almost identical as shown in figure 5(a), the increased numerical dissipation in EllipSys3D results in a larger vortex core growth, as shown in figures 4 and 5(b). The smaller core size in Nek5000 coincides with a slower decay of the swirl velocity due to the decrease in dissipation of the spectral element method, which also explains the difference in the vorticity magnitude development in figure 3. Figure 5(b) additionally compares the numerical results to the theoretical core growth of a Lamb–Oseen (L–O) vortex with a Gaussian velocity distribution. The L–O vortex represents the simplest analytical model of a two-dimensional vortex with a viscous core [14], with which it is possible to approximate the core growth. The helix pitch comparison (not shown) agrees closely between the two methods.

In figure 6 we analyze the azimuthally averaged streamwise and tangential velocities $u_z$ and $u_t$ along the wake at four different streamwise positions, beginning at the rotor plane ($z = 0$). The comparison of $u_z$ reveals excellent agreement in the near wake ($z < 4$). In the transition region the spectral element code obtains a different velocity gradient at the wake borders, due to the different wake break down mechanism. The tangential velocity agrees well.

To confirm that the differences in the wake behaviour are due to the different numerical methods, and not caused by the eddy viscosity introduced by the SGS-model employed in
Swirl velocity $u_\Phi$ of tip vortex at (a) position 1 and (b) position 2 in figure 3; EllipSys3D, EllipSys3D without SGS model, Nek5000, the arrow spans the core radius of position 2 for Nek5000; CC1.

Wake expansion $r$. (b) Core radius $r_{\text{core}}$; EllipSys3D, L-O vortex fitted to EllipSys3D; EllipSys3D without SGS, Nek5000 and L-O vortex fitted to Nek5000; CC1.

Azimuthally averaged streamwise velocity $u_z$ (left) and tangential velocity $u_t$ (right) at different positions in the wake. (a) $z = 0$, (b) $z = 2$, (c) $z = 4$ and (d) $z = 8$; Nek5000, EllipSys3D, EllipSys3D without SGS model; CC1.

EllipSys3D the SGS-model is disabled and the resulting flow is compared with the previous results. There is no significant difference in the wake behaviour (figures 4–6).

Finally we compare the wall-clock time per gridpoint for both methods to determine the speed of each code. For a comparable number of cores (about 2000) the reference case run with the finite volume code requires 78% of the time used for the spectral element method, thus confirming that the wall-clock time is of the same order in both setups. However, the spectral element simulation requires a time step which is about a factor 8 smaller than EllipSys3D. This is partly due to the non-equidistant spacing in the spectral element mesh, but also due to some small grid-spacing caused by fully meshing the cylindrical domain.

### 3.2. Tjæreborg turbine

To further validate the actuator line implementation in the spectral element code it is necessary to discuss the wake developing behind a real wind turbine, due to its use of tabulated airfoil data. Figures 7–8 show the development of the wake expansion, the core size, helix pitch and circulation in the wake. Overall, the comparison using the Tjæreborg turbine data gives a close agreement, the most prominent difference between the methods being the core size. The helix...
pitch is the same in both codes and the development of the circulation and wake expansion match well. Integrating the tangential force, the power coefficient $C_P$ is obtained. For tip speed ratio $\lambda = 7.07$ we compute power coefficients of $C_{P,Nek} = 0.50$ compared with the EllipSys3D $C_{P,ES3D} = 0.487$ and the experimentally determined value $C_{P,exp} = 0.49$ [16].

A comparison of the development of the vorticity magnitude in the streamwise direction in figure 9 shows larger similarity of the root vortices than for the constant circulation case. This is most probably related to the fact that the circulation distribution of the Tjæreborg wind turbine is not constant along the blade, instead decreasing towards the hub and tip, causing the root vortices to detach farther away from the rotor center. This decreases the influence of the slip boundary condition in EllipSys3D and enables the root vortices to develop analogous to the full domain used in Nek5000, with the root vortices destabilizing between $z \approx 4$ leading to root vortex merging. Additionally, it is observed that the tip vortices of the Tjæreborg wake simulated with the spectral element method still experience vortex pairing at approximately the same position as in the CC1 case, while for EllipSys3D the vortex pairing is delayed to $z = 6$. 

**Figure 7.** (a) Wake expansion $r$. (b) Core radius $r_{core}$; EllipSys3D; + Nek5000.

**Figure 8.** (a) Pitch $h$. (b) Circulation $\Gamma$; EllipSys3D; + Nek5000; Tjæreborg case.

**Figure 9.** Contours of vorticity magnitude. (a) Nek5000 and (b) EllipSys3D, the line at $r = 0.1$ denotes the inner domain boundary in EllipSys3D; Tjæreborg case.
3.3. Parametric studies of the constant circulation setup

One may hypothesize that in order to obtain better agreement between the codes, one should in fact compare a lower polynomial order in Nek5000 to compensate for the increased numerical dissipation in EllipSys3D. Figure 10 shows that the core size at polynomial order \( N = 8 \) increases while the maximum swirl velocity decreases thus approaching the results of EllipSys3D. Using polynomial order \( N = 8 \) we reduce the grid size to 42\% of the EllipSys3D grid in the near wake. The wake development is further visualized in figure 12. We observe a more similar behaviour as in EllipSys3D, with vortices merging after a very short pairing session and an increased smearing out of the tip vortices. However, the absolute strength of the vortices still decreases slower than in EllipSys3D. The lift and drag forces at the actuator lines are converged at polynomial order \( N = 8 \). Figures 10–11 additionally include the data for polynomial order \( N = 7 \).

To confirm resolution independency a study was conducted using the constant circulation setup. Four different mesh densities are compared. Figure 13 shows a comparison of the thrust and power coefficients for the mesh independency study. For polynomial order \( N = 9 \), the resolution used in subsequent comparisons with EllipSys3D, we receive converged performance coefficients with less than 2\% deviation from EllipSys3D. The remaining deviation is suspected to be caused by the differences in the chosen domain. The larger thrust coefficient in the EllipSys3D simulation explains the slightly larger wake expansion in the finite volume code.

To confirm the Reynolds number independency of the blade loading a variation of the Reynolds number between \( Re_R = 10000, 50000, 250000 \) is performed to investigate the resulting performance parameters. Figure 13 reveals that the performance parameters have also converged for the reference Reynolds number. An analysis of the circulation and the vortex core size for
We finally investigate the smearing parameter $\varepsilon$ in the range between $1.5\Delta r < \varepsilon < 3.5\Delta r$. It has been pointed out in Trolldborg [3] that to avoid numerical instabilities a choice of $\varepsilon > 2\Delta r$ is recommended. It can be observed that by using $\varepsilon = 1.5\Delta r$ some wiggles occur at the tip vortices as shown in figure 14, instead of obtaining the smooth vortices of figure 2. The blade forces are here being distributed over too few grid points. Distributing the forces over a larger number of grid points leads to an increase in the vortex core size, as shown in figure 15. For EllipSys3D the same trend is observed for $\varepsilon \leq 2.5\Delta r$. However, Nek5000 has higher velocity gradients and a larger maximum swirl velocity. For all $\varepsilon$ the core size is consistently larger in the finite volume simulations. With increasing $\varepsilon$ the power coefficient $C_P$ is seen to increase slightly (for $\varepsilon = 3.0$: $C_P = 57.7$; for $\varepsilon = 3.5$: $C_P = 58.1$) in the constant circulation setup.

4. Conclusions and Outlook

Based on the results presented in this paper, we conclude that the qualitative agreement between the two codes and implementations is good; quantitative differences can be traced back to the different amount of numerical dissipation and in some cases to the difference in the numerical domain, especially with regard to the root vortices in the constant circulation case. It is shown, that the agreement of the root vortex structure is improved, when using the Tjæreborg dataset, due to the more realistic circulation distribution along the blades which causes the root vortices to form further away from the rotor center and thus further away from the slip boundary in EllipSys3D. By removing the SGS model it is established, that the differences in the dissipation are due to the employed numerical methods. The parametric studies reveal convergence of the performance parameters and a close agreement with experimental data and the established values obtained by EllipSys3D. Finally, the effect of the smearing parameter was compared, resulting in the confirmation that the same qualitative behaviour is observed in both codes.

Using a lower polynomial order in Nek5000 a more similar wake behaviour to EllipSys3D can
be observed. This leads to the conclusion that the spectral element method is clearly adequate when going to more complex setups, e.g. multiple turbines and the subsequent wake interaction due to accurate representation of the solution. The spectral element method allows the use of fewer grid points to obtain an accurate solution. However, the non-aquidistant grid gives rise to lower possible time steps. The discrepancy in the required time step will be addressed by improving the grid and removing the time-step limiting grid points. Future studies will aim to further validate the setup against experimental data provided in Krogstad & Eriksen [12]. Additionally, the model will be extended to include a representation of the nacelle and the tower.

Computer time was provided by the Swedish National Infrastructure for Computing (SNIC).

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Figure 15. Swirl velocity for the $\varepsilon = 1.5\Delta r$, $\varepsilon = 2.5\Delta r$, $\varepsilon = 3.5\Delta r$, EllipSys3D: dashed lines, Nek5000: solid lines; CC3.

Figure 16. (a) Circulation $\Gamma$. (b) Core size $r_{\text{core}}$; $+ Re = 10000$ $+ Re = 50000$ and $+ Re = 250000$; Nek5000,CC2.