Mathematical Model of Rheological Behavior of Wood Plate Taking Into Account the Zone of Evaporation of Moisture

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Abstract. The subject of this paper is a mathematical model of heat-transfer in the process which takes place in capillary-porous materials during drying. The moving boundaries of phase transitions for media are also considering. The analytical-numerical method was developed for mathematical model. The difference schemes were constructed with consideration for the boundaries of the evaporation zone of heat exchange with the environment in the presence of a phase transition in the conditions. The numerical realization and software were developed for solving the Stefan problem. The fractal structures of the medium were taken into account for synthesization of mathematical models of viscoelastic deformation. Functional dependences of the temperature field and the motion of the phase transition boundary in media, depending on the value of fractional parameters, were received for modelling different technological processes of drying capillary-porous materials.

1. Introduction

The intensification of drying schedules for colloidal capillary-porous materials leads to further development of mathematical modeling of the processes of heat-and-mass transfer, phase transformation, and deformation, which would adequately describe the laws of moisture removal in the materials being dried. The presence of a moving boundary of phase transformations at the interface between phases with different thermophysical and mechanical characteristics considerably complicates mathematical models of the processes of deformation-relaxation and heat-and-mass exchange during the drying of capillary-porous materials. The simulation of heat-and-mass transfer with phase transitions in the drying process is reduced to solving Stefan's problems, which are the most complicated even for minor changes in the density of the material in the evaporation zone. Therefore, the mathematical modeling of such processes by generalizing the Stefan problem for media requires the development of new non-traditional methods and modeling tools.

The general approach to modeling the interconnected heat-and-mass transfer and the stress-strain state is based on the thermodynamics of irreversible processes. In research works [1,4] this approach is applied to the study of the drying process of capillary-porous materials. In the framework of the theory of small viscoelastic-plastic deformations, a one-dimensional problem of self-stresses in a thin free plate during symmetric drying on both sides was solved. In the work [8], the equations of heat-and-mass transfer in deformed elastic capillary-porous media were proposed, taking into account the dependence of surface properties on temperature. However, they do not describe the presence of irreversible deformations due to the shrinkage process.

In all these studies, the type of rheological behavior of the material is assumed to be independent of moisture. However, in many cases the rheological characteristics of the medium in a dry and a saturated state are significantly different. In this regard, it is possible to distinguish the series of researchers’ works [3].

The development of mathematical models of viscoelastic capillary-porous materials in the drying process is based on hereditary creep models [4]. The equation of the balance of energy of dissipation on the surface of the phase transition makes it possible to formulate the conditions for the deepening
zone of evaporation, taking into account the preservation of irreversible deformations during phase transitions. However, such models describe the rheological behavior of various media for a continuous history of deformation. If, in the process of deformation of capillary-porous media, phase transitions are implicit and the process of deformation at their boundary is characterized by jump discontinuities, it is necessary to take into account the influence of the prehistory of loading before the phase transition on the further development of the stress-strain state of the medium.

2. A mathematical model of the rheological behavior of a wood plate taking into account the moisture evaporation zone

The stress-strain state arising in the process of drying wood is the main factor in slowing down the intensification of the technological process. The complexity of calculating the components of stress and deformation is due, primarily, to the behavior of the material, the anisotropy of its structure, the capillary-porous structure of the medium. The process of drying wood, especially during a period of decreasing drying speed, is characterized by the presence of at least two zones of moisture evaporation with different technological characteristics.

The paper deals with the mathematical model of the rheological behavior of a wood plate in the process of convection drying, taking into account the moisture evaporation zone. In the moist area of the wood plate, the material is characterized by viscoelastic properties. To study the stress-strain state in the moist zone, the Kelvin-Voigt model is used. The dry zone is described by elastic behavior. One of the causes of stresses in the materials being dried is free drying conditioned by capillary processes of free (intercapillary) moisture removal and the occurrence of uneven drying over the thickness of the material.

The total deformation of wood under conditions of convection drying can be represented in the for

\[ \varepsilon = \varepsilon_f + \varepsilon_v + \varepsilon_w, \]

where \( \varepsilon_f \), \( \varepsilon_v \), \( \varepsilon_w \) are, respectively, elastic, visco-elastic, and shrinkage deformations.

To study elastic deformations, Hooke’s law is used, taking into account shrinkage deformations which are determined by a linear dependence on changes in moisture content. For the moist zone of the wood plate, the Kelvin-Voigt rheological model is described by the relation [5]

\[ GK \left( 1 + \tau_1 \frac{\partial}{\partial \tau} \right) \left( 1 + \tau_0 \frac{\partial}{\partial \tau} \right) \left( \varepsilon_{ij} - \frac{\varepsilon_w}{3} \delta_{ij} \right) = 3K \left( 1 + \tau_0 \frac{\partial}{\partial \tau} \right) \sigma_{ij} - \\
- \left( 3K \left( 1 + \tau_0 \frac{\partial}{\partial \tau} \right) \right) \frac{2}{3} G \left( 1 + \tau_1 \frac{\partial}{\partial \tau} \right) \sigma_{ii} \delta_{ij}, \tag{2} \]

where \( \varepsilon_{ij}, \sigma_{ij} \) are components of the strain and stress tensor; \( G, K \) are module of elasticity and volume expansion, \( \tau_0, \tau_1 \) are relaxation time, \( \delta_{ij} \) are Kronecker’s symbols.

In the case of a one-dimensional moisture field during the drying of a wood plate with a thickness of \( 2l \) taking into account the dependencies \( \sigma_i = 0, \sigma_\|=\sigma_i, \varepsilon_\|=\varepsilon_i = \varepsilon_i \) from (2) we get the mathematic model in the form:

\[ \tau_0 \tau_1 \frac{\partial^2 \varepsilon_w}{\partial \tau^2} (\tau) + (\tau_0 + \tau_1) \frac{\partial \varepsilon_w}{\partial \tau} (\tau) + \varepsilon_w (\tau) = \frac{4G \tau_1 + 3K \tau_0}{18GK} \frac{\partial \sigma_{\|}}{\partial \tau} + \frac{4G + 3K}{4G \tau_1 + 3K \tau_0} \sigma_{\|}. \tag{3} \]

The boundary conditions are characterized by the absence of stresses and deformations in the wood plate at the beginning of the drying process \( (\tau) = 0 \) and take the form \( \sigma = 0, \varepsilon = 0 \).

In (3) it was taken into account that in the moist zone, shrinkage of wood is insignificant, therefore \( \varepsilon_w = 0 \).
The stress-strain state of the wood plate in the dried zone, taking into account the shrinkage, according to the accepted assumptions, is described by law [5, 6]

\[
\sigma_{ij} = \frac{E}{1 + v} \left( \varepsilon_{ij} - \varepsilon_w \right) + \frac{v}{1 - 2v} \delta_{ij} \left( \varepsilon_{kk} - \varepsilon_w \right)
\]  

(4)

Since the one-dimensional problem of the drying process is considered, the compatibility equation [5] can be represented:

\[
\frac{d^2}{dt^2} \left( \sigma + \frac{E}{1 + v} \varepsilon_w \right) = 0; \quad \frac{\partial \varepsilon}{\partial z} = 0;
\]  

(5)

Taking into account (5) with (4), we obtain

\[
\sigma(t) = \frac{E}{1 - v} \left( \varepsilon(t) - \varepsilon_w \right).
\]  

(6)

According to [6, 7], the process of transition from the moist zone to the dried zone during the process of wood drying is characterized by almost constant values of shrinkage, then we assume that the value of shrinkage deformation \( \varepsilon_w \) is equal to the total deformation (1) at the time of transition \( * \), between elastic zones, that is \( \varepsilon_w = \varepsilon^0(\tau) = \varepsilon(\tau^*) \), \( \tau^* = \xi^*(x) \), where \( \xi(x) \) is the coordinate of the phase boundary surface which is parallel to the plane of the wood plate. Thus, the shrinkage deformation, in addition to the processes of deformation of the wood plate during the drying process, also takes into account the size of the phase transition \( \xi(x) \) depending on the technological modes of the drying process, in particular non-isothermal moisture transfer.

The boundary conditions on the surface of the wood plate, given that the moment of forces and total forces are equal to zero, take the form:

\[
\int_{x} \sigma(z) dz = 0, \quad \int_{x} z \sigma(z, \tau) dz = 0.
\]  

(7)

Thus, integrating equation (4) with zero boundary conditions \( (\tau = 0: \varepsilon = 0, \partial \varepsilon / \partial \tau = 0) \) for the moist zone and taking into account relations (6), (7) for the dried zone, we obtain a mathematical model for determining the stress-strain state of a wood plate during convection drying taking into account the moving zone of moisture evaporation in the form \( z \leq z^* \):

\[
\sigma(\tau) = A_o \varepsilon(\tau) + A_1 \frac{\partial \varepsilon(\tau)}{\partial \tau} + A_2 \int_0^{\tau} \varepsilon(\tau') \exp(-b_o(\tau - \tau')) d\tau',
\]  

(8)

where the following notation is introduced:

\[
A_o = \frac{E(\tau_o + \tau_1)b_1 - 3(1 - v)\tau \tau_0}{b_1^2}; \quad A_1 = \frac{E\tau_o \tau_1}{b_1}; \quad A_2 = \frac{E}{b_1^3} \left( b_1^2 - 3(1 - v)(\tau_i + \tau_0) b_1 + 9(1 - v)^2 \tau_o \tau_1 \right); \quad b_0 = \frac{3(1 - v)}{2(1 - 2v)\tau_i + (1 + v)\tau_0}; \quad b_1 = 2\tau_i(1 - 2v) + \tau_0(1 + v).
\]

For the dried zone \( z^* \leq z \leq l \) we can write down accordingly
\[ \sigma = \frac{E}{1 - \nu} \left( \varepsilon + \frac{\varepsilon_w}{3} \right) \]  \hspace{1cm} (9)

In (8), (9) by \( E \) and \( \nu \) denote the elastic module and Poisson's ratios for wood which in the general case are functions of moisture content and temperature, i.e. \( E = E(U, t) \), \( \nu = \nu(U, t) \).

The determination of these values characterizing the rheological behavior of wood is associated with the analysis of experimental data. For this purpose, we use the experimental dependences of the creep deformation curves for wood across the fibers for different temperatures and moisture content [6, 9]. Implementation features of complex structured composite materials effective mechanical characteristics finding subsystem that is based on microlevel cellular models usage are described in papers [11, 12].

To determine the deformations \( \varepsilon_w \) during the drying of the wood plate, which are included in (8), (9) taking account the moisture evaporation zone, we used the approach [7] and the results of temperature and moisture modeling from [13, 14]. In particular, for the case of one-dimensional mass-transfer fields, we have the following expressions for deformations in the moist and dried zones of the plate

\[ \varepsilon_w = \beta_1 \Delta U^{(1)} = \beta_1 \Delta (U_1(z, \tau) - U_0) \quad 0 < z \leq z^*; \]  \hspace{1cm} (10)

\[ \varepsilon_w = \beta_2 \Delta U^{(2)} = \beta_2 \Delta (U_2(z, \tau) - U_0), \quad z^* < z \leq l, \]

where \( \beta_1, U_i (i = 1, 2) \) are the corresponding coefficients of shrinkage and the corresponding values of moisture content, \( U_0 \) is moisture content at the beginning of the drying process. Moisture content \( U \) is defined as the ratio of the mass of moist wood to the mass of absolutely dry material.

The important point is to determine the shrinkage coefficients \( \beta_i \). For wood, the decisive coefficient is \( \beta_2 \) for the drying zone which is associated with the removal of moisture under the action of capillary forces. Therefore, the shrinkage coefficient is determined by the movement the boundary of the evaporation zone. Note that the geometric dimensions of the moisture evaporation zone are not taken into account. In view of this, the dependence [1] is accepted for quantitative description \( \varepsilon_w = \beta V_0(V + V_0) \), where \( \beta_0 \) is the shrinkage coefficient for low-intensive drying processes, as, for example, for a period of constant drying rate. It is assumed that [8] the speed of movement of the phase transition boundary is equal to \( V = -d z^*/d \tau = j/\rho \), where \( j \) is the flow of moisture that is inversely proportional to the size of the dried zone. It can be determined by the formula [4,8]: \( j = A/(1 - z^*/l) \), where \( A = l/2 \tau_1 \) is some constant which depends on the thermophysical characteristics of ambient humidity: \( A = [D]MP / RT l \). Such assumptions make it possible to make transition from time to coordinate of the evaporation boundary by the formula \( \xi_j = 1 - z^* / l = (2A \tau / l)^{1/2} \), since \( \xi_j(\tau) = \tau^{1/2} \), \( d \xi_j(\tau)/d \tau = (2 \xi_j)^{-1} \). In turn, this allows determining the deformations \( \varepsilon(\xi_j) \) at the phase transition boundary from the boundary conditions (7).

To determine the deformations, the rheological equation [10] is used in the form

\[ \partial^2 \varepsilon / \partial \tau^2 + \alpha_1 \partial \varepsilon / \partial \tau + \alpha_2 \varepsilon = \alpha_3 \]  \hspace{1cm} (11)

The parameter values in this equation take the form

\[ \alpha_1 = 2 \frac{\tau_1}{\tau_1 \tau_2} (1 - \xi_j/l_2) \left( \tau_1 + \tau_2 - (1 - \xi_j/l_2) \right) \left( 4 \tau_2 (1 - 2v) + 3 \tau_1 (1 + v) \right) / 2(1 - v), \]  \hspace{1cm} (12)
elastic behavior of the wood plate for various values of drying time. The change of stresses in the wood plate for various values of drying time, initial moisture content, and species of wood. For the numerical experiment, pine wood with a base density $ρ_0=450$ kg/m³ was selected for the study, using the mathematical models obtained above, the viscoelastic behavior of the wood plate during convection drying. The parameters of the drying agent were as follows: initial moisture content $u_0=0.4$ kg/kg; initial temperature $T_i=20$ °C; ambient temperature $t_a=70$ °C; relative humidity $φ_0=60\%$; the speed of movement of the drying agent $ν=2$ m/s.

Fig. 1 shows the graphical dependences of the stresses $σ$ on the thickness of the wood plate depending on the different values of the moisture evaporation boundary $ξ_e$ for different values of drying time. Fig. 2 characterizes the change in stresses during the drying of a wood plate at the evaporation boundary for various moisture content values. Fig. 3 show graphical dependences of the change of stresses in the wood plate for various values of drying time, initial moisture content, and different operating parameters of drying. Fig. 4 characterizes the change of relative deformations in the wood plate for various values of drying time (1-Fo=2,15, 2-Fo=1,15, 3-Fo=0,60, 4-Fo=0,45).

\[
\alpha_2 = \frac{\tau_2}{\tau_1} (1 - \frac{ξ}{l_2})^2 \cdot \left(1 + \frac{l_2}{ξ} \frac{10v + 14}{2(1 - 2v) + 6(1 + v)(1 - \frac{ξ}{l_2})} + \frac{4\tau_2(1 + v) + 3\tau_1(1 + v)}{2\tau_1(1 - 2v)(1 - \frac{ξ}{l_2})} \right).
\]

\[
\alpha_3 = \frac{4}{81} \frac{4\tau_2(1 - 2v) + 3\tau_1(1 + v)}{\tau_2(1 - 2v)} \cdot \frac{1}{\tau_1} \left(1 - \frac{ξ}{l_2}\right)^2 \cdot \frac{2(11 + 15v)(8\tau_1(1 - 2v) + 6\tau_1(1 + v))(1 - \frac{ξ}{l_2})(1 - \frac{ξ}{l_2}) + (\tau_2 / \tau_1)}{2(1 - \frac{ξ}{l_2})(1 - \frac{ξ}{l_2})^2} + \left(1 - \frac{ξ}{l_2} - \frac{l_2}{2\tau_1 \theta}\right) \ln \left(1 - \frac{ξ}{l_2} - \frac{l_2}{2\tau_1 \theta}\right) + \frac{\tau_2 / \tau_1}{2(1 - \frac{ξ}{l_2} + \frac{l_2}{2\tau_1 \theta})}.
\]

3. Simulation results

The dependencies for determining the shrinkage coefficients $β_i = β_2 = β$ take the form [8]. The value $β_i$ in the moist zone is constant and calculated according [8] for $U = 0.25 \div 0.3$ and $t = 20$ °C depending on the species of wood.

For the numerical experiment, pine wood with a base density $ρ_0=450$ kg/m³ was selected for the study, using the mathematical models obtained above, the viscoelastic behavior of the wood plate during convection drying. The parameters of the drying agent were as follows: initial moisture content $u_0=0.4$ kg/kg; initial temperature $T_i=20$ °C; ambient temperature $t_a=70$ °C; relative humidity $φ_0=60\%$; the speed of movement of the drying agent $ν=2$ m/s.

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![Figure 1](image1.png) Figure 1. Change in stresses $σ$ over the thickness of the wood plate for different values of $ξ_e$.

![Figure 2](image2.png) Figure 2. Change in stresses $σ$ in the wood plate for various values of drying time.
Conclusions

A new mathematical model of viscoelastic deformation of a capillary-porous plate under conditions of changing moisture transfer with taking into account the moisture evaporation zone is constructed. A study of stresses and strains in the moist and dried areas of the wood plate was conducted. It was shown that stresses and strains for various values of the evaporation zone are characterized by time non-monotony, a shift in the values of the maximum stresses appears in the moist zone with an increase in the drying time and a jump discontinuity at the phase boundary. The patterns of the development of stress components in wood during drying are determined depending on changes in temperature, moisture content, moisture evaporation zone and technological parameters of the drying agent. The presence of compressive stresses in the dried zone of the plate for the initial stages of the drying process was detected.

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