The business competitiveness of SMEs through mathematical modeling*

La competitividad empresarial de las PyMES a través del modelado matemático

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ABSTRACT

The objective of this research is to propose a functional application model for small and medium-sized enterprises (SMEs), with the purpose of determining the optimal production preferences of your specific production system. The problematic that originates this research is the methodology of projection of the costs that practice this type of companies in the market —which departs from a process of determination of the optimum batch of production—, which are based on models of mathematical determination. The applied methodology is of an analytical nature of a transversal type, the factorial study allows the application of descriptive statistics and the development of equations that determine variables of study of the ex ante model. The result of the research consists in the application to a scenario that represents the optimal level of transport costs of the SMEs products which interact in the market. It is concluded that mathematically it is possible use variables for determining the productivity and level of competitiveness.

Key Words: Small and medium enterprises, production, productive system.
RESUMEN

El objetivo de esta investigación es proponer un modelo funcional de aplicación para las pequeñas y medianas empresas (mipymes) con el propósito de determinar las preferencias de producción óptima de su sistema productivo en concreto. La problemática que origina esta investigación es la metodología de proyección de los costos que practican este tipo de empresas en el mercado —lo cual se aparta de un proceso de determinación del lote óptimo de producción—, que estén basados en modelos de determinación matemática. La metodología aplicada es de carácter analítico de tipo transversal, el estudio factorial permite la aplicación de estadística descriptiva y el desarrollo de ecuaciones que determinan variables de estudio del modelo ex ante. El resultado de la investigación consiste en la aplicación a un escenario que representa el nivel óptimo de costos de transporte de los productos de las mipymes que interactúan en el mercado. Se concluye que matemáticamente es posible usar variables para determinar la productividad y el nivel de competitividad.

Palabras clave: pequeñas y medianas empresas, producción, sistema productivo.

Introduction

Competition in the market has caused a constant price discrimination, which has made the market more competitive, in this sense companies around the world need alternatives that allow them to reduce their operating costs in order to remain competitive and profitable (Valencia, Lambán & Royo, 2014) at the same time, it is evident then that the optimization of resources is a key factor that generates profits and allows it to increase the corporate value of the organization.

According to Baykasoglu and Kapanoglu (2006) the productive performance of the production units depends on the management skills; in this sense, several investigations (Sanchez, Osorio & Baena, 2007; Vélez et al., 2008; Vera-Colina & Mora-Riapira, 2011) have determined several entropies that reduce the productivity levels of SMEs; which represents a constant problem for the countries of the region, in Latin America it is estimated that from the process of globalization of the economy, it has resulted in a loss of competitiveness in international markets, especially for those countries that face types of variable change in its economic system (Bada, Rivas & Littlewood, 2017).

In this same sense, the average performance standard of SMEs in Latin America is equivalent to less than 40 % of the productivity of the big company (Valencia, Lambán & Royo, 2014), which leaves behind the SMEs in front of several countries, such as those of the European Union and the United States, where the sector reaches an average of 60 % (Mora, Vera & Melgarejo, 2015).

Several researchers (Rubio & Aragón, 2006; De la Cruz Morales & Carrasco, 2006; Solleiro & Castañón, 2005; Quiroga, 2003) have developed new indicators that allow a more technical measurement of the competitiveness levels of SMEs; while other Martínez & Álvarez, 2006; Deniz, Livas & López, 2008; Santillán, 2010; Gómez, 2001; Membrillo, 2006 & Herrera, 2007). have made empirical applications to determine the competitiveness of this sector.
In this order of ideas, models of perfect competition can be cited in response to the processes in central assumptions, improving on average the productivity provided by the assumption of perfect information, namely, that all agents (companies, consumers) know how the prices set by all companies thus provide their participation in the production chain process and improve their activity and ability to compete.

Having made the previous observation, the assumption of atomicity and homogeneity of the production process, therefore, considers how the effective administration of business resources involves the way of managing both expenses and income in an appropriate manner, from which the effectiveness in business activity and its correct execution in the proper determination of the optimal production lot, improving productivity of SMEs.

However, in a perfectly competitive market, marginal revenues are equal to the price where; if the company sets a price above that of the other companies, then it sells nothing; if on the contrary the company sets a price below the other companies, then it receives all the market demand that; compared to its capacity, it is a large amount of production (Tarziján, 2006).

It means then that the assumption of equal access in the market that all companies have allow the access to all production technologies (Tarziján, 2006); finally, we conclude that the assumption of free entry namely allows any company to enter or exit the market as desired.

As a final summary, the work begins with the bibliographic and textual revision of perfect competition models that are related to the thesis of business competitiveness, continues with the delineation of the state on SMEs in the country, as well as the correspondence analysis between the performance of the commerce sector as an innovative aspect of proactive outputs to the problems that affect SMEs. The methodology part estimates the process of characterization of the relationship of variables that describe the levels of competitiveness of SMEs. Consecutively, in the final phase a discussion of the results is made and finalized with the conclusions.

**Bases for the determination of the mathematical model**

In this sense several models were reviewed, one of them is the perfect competition model; model that shows that competition is a good thing; specifically, because the balance under perfect competition is efficient. Made this consideration, the model is directed in two ways: first, where each company sets the level of efficient production, that is, the level of production in such a way that the price is equal to cost; a lower level of production would be less efficient, since it increases more than the cost; on the contrary, a higher level of production would also be inefficient. Second, the group of long-term active companies demonstrates a more efficient degree, due to their way of entering the market, where their price is equal to the minimum average cost; this gives a greater or lesser number of companies involved in a higher total cost for the same level of production.

In that same sense, a linear demand function can function as a useful approximation for local analysis, but not very likely to work in a global dynamic; except that a linear demand function results in negative values of all its variables; which almost always make no sense in the economy, since it contradicts basic economic theory, because it cannot be the solution to the maximization of any limited budgetary utility.
In this sense, we analyze the discriminatory monopoly markets where it is stated that q1 and q2 are amounts of prices determined by brand p1 and p2.

In the theory of market firms the quantities sold in the Cournot market are described as q1 and q2 which means that (Q1 + q1)(Q2 + q2) are the total amount of product sold by the two companies.

If we substitute in the budget constraint we have:

\[ p_1 q_1 + p_2 q_2 = y^* \]

Budget actions will not be added to a constant budget.

Suppose we assume:

\[ q_1 = A_1 - B_1 p_1 + C_1 p_2, \quad q_2 = A_2 - B_2 p_2 + C_2 p_1 \]

As suggested by replacing the budget constraint we will obtain:

\[ p_1 q_1 + p_1 q_1 = A_1 p_1 + A_2 p_2 - B_1 p_1^2 - B_2 p_2^2 + (C_1 + C_1) p_1 q_2 \]

To make sense of the model we match for all eligible p1 and p2 so that it does not produce a logical error in the model. In the Cournot market we only need the “inelastic” demand model as suggested by several authors (Puu and Norin, 2003). It is derived from a simple utility function Cobb-Douglas, therefore, using the suggested notation, it is expressed:

\[ U = Q_1 Q_2 \]

and maximizing under a usual budget constraint is represented:

\[ p_1 Q_1 + p_2 Q_2 = 1 \]

where the budget normalizes the unit.

The solutions to maximize the usefulness of Cobb-Douglas subject to linear budgetary restrictions such as the optimal budget quotas are constant, in this case symmetrical, half of each one, that is, it is up to consumers to decide which one can be chosen: the price or quantity.

The problem is quite logical, because a product cannot be homogeneous in the “Cournot” sense and not homogeneous in the “Bertrand” sense at the same time, as it is nothing that sellers can choose when devising their actions (Puu and Norin, 2003).

Then, suppose a slight modification of the utility function for Bertrand markets

\[ U = q_1 q_2 (q_1 + q_2) \]

It has a Cobb-Douglas type factor q1 and q2 although we multiply by a factor dependent on the sum (q1 + q2). There is a budget restriction again

\[ p_1 q_1 + p_2 q_2 = 1 \]
The economic purpose of this proposed model is to make a combination of Cournot and Bertrand, in this direction have been a bit confusing the ideas that duopolists could choose one or the other, quantity of offer or price, as an action parameter. However, it can only be consumers who decide whether they consider the product supplied to be homogeneous or not. What suppliers can do is just conclude and market the product to convince consumers that there are different brands with their defined advantages.

Several models have allowed the improvement of the relationship of intrinsic variables in the optimal batch process, among which we mention the Hall and Mendoza model that incorporate the cost of distribution in the calculation of optimal lots (Valencia, Lambán & Royo, 2014). Exploring in this direction the optimal production batch model is based on the “recognized EOQ model” (Economic Order Quantity) which optimizes the volume of the purchase or production lot by reducing the result of combining two important management costs on “Total Annual Cost of Inventory” that defines the sum of acquisition or purchase costs (D*C), order issuance costs (D/Q)*S and storage costs (Q/2)*H.

In the EOQ/EPQ model, it seeks to determine the optimum production lot (represented as Q*) from only the following data:

\[ H = \text{Annual unit cost of maintaining inventory.} \]

\[ D = \text{Demand.} \]

\[ S = \text{Fixed purchase or production cost.} \]

The EOQ Order Economic Size model formula represents the total cost of the inventory is as follows:

\[
CT = D \times C + \left(\frac{D}{Q}\right) \times S \left(\frac{Q}{2}\right) \times H \tag{1}
\]

Equation (1) allows to identify that the total of the sum of all costs is detailed by the absence of the logistic index, but, as the name implies, it should be considered only in the process of transport or storage of the merchandise.

The optimal lot is found by deriving and equalizing the management cost illustrated in the following equation to zero:

\[
Q = \sqrt{\frac{2DS}{H}} \tag{2}
\]

\[
\int_{Q} H = 2DS
\]

The logistic index was calculated with:

\[
I_{volume} = \left(\frac{\text{Reference volume}}{\mu \text{Volume of references}}\right)
\]
Several investigations identify that the economic order quantity can be established with the EOQ model, a model that establishes the optimal process for the quantity to be produced, or purchased; they affirm that in an organization it is known, representing a fixed quantity model which seeks to determine through quantitative equality the costs of ordering and maintenance costs at the lowest possible total cost (Hall, 1996; Jamal, Sarker and Mondal, 2004; Yuan et al., 2011).

As a result of this, different researchers have recently proposed modifications to the model to obtain optimal ones closer to the real ones; among the models developed in recent years are the Hall’s model (Hall, 1996) and Mendoza’s model (Mendoza and Ventura, 2008) who incorporate the cost of distribution in the calculation of optimal lots.

$\text{EOQ model} = \text{Variable cost of raw material} + \text{Fixed cost} + \text{Variable cost of production} + \text{Variable cost of rework} + \text{Variable cost of scrap disposal} + \text{Logistic index of transport} + \text{Variable cost of transportation to the customer} + \text{Variable cost of internal transport}$

+ Logistic index of storage (Variable cost of storage during production + Variable cost of storage during rework + Variable cost of storage during deliveries) + Variable cost of inspection after production + Variable cost of inspection after rework + Variable cost of maintenance by production + Variable cost of maintenance by rework.

Other outstanding works (Sarker and Khan, 1999; Jamal, Sarker and Mondal, 2004), include raw material costs, and identify the need to incorporate costs known as “Reprocessing”; defining the importance of these in the process of calculating the logistic index (Yuan et al., 2011).

$$C_q = rQ + Q + C \sum_{i=1}^{Q} t_i + C_R \sum_{j=1}^{Q} x_j = 1t_i + C_x \theta t_i Q + nK_i + I_{logA} [C_T H + C_{TT} H] +$$

$$l_{logA} [h \sum_{i=1}^{Q-1} = t_i + 1 + h_1 \sum_{i=1}^{Qx} = (t_i)(Qx - j) + hH_2 \sum_{i=1}^{Qx-1} = t_j + 1 + h(n - 1) |2nHT_3|] +$$

$$MQ + MQ_x + NQ + NxQ \ (3)$$

In equation (3) it is observed that not all costs are described so that they can influence the calculation of the volume of the logistic index, but, as the name implies, this should only be considered in processes such as transport or storage, which is represented below:
However, this model is still widely applied in several companies, several authors (Jaber, Bonney and Moualek, 2009) consider that the reduced calculation of the number of costs taken into account is too simplistic and therefore it makes the model inaccurate.

These contributions allow production times to be counted as non-constant, but to follow a normal distribution, as well as to integrate the logistic index, a cost inducer that allows logistic costs to be adjusted to a specific reference.

**Material and Methods**

The research determined as the subject of application and measurement instrument to the managers and owners of SMEs, the same that according to Ecuadorian legislation can be found observations on SMEs in articles 53 and 56 of the Organic Code of Production, Commerce and Investments, that speak of the definitions and the unique register of SMEs.

Next, the classification of SMEs in Ecuador is shown in the following table:
Table 1. Classification of SMEs in Ecuador

| Classification of the companies | Annual Sales Volumes     | Occupied staff |
|---------------------------------|--------------------------|----------------|
| Microenterprise                 | Less than or equal to 100,000 | 1 to 9         |
| Small Company                   | From 100,001 to 1'000,000  | 10 to 49       |
| Medium Company “A”              | From 1’000,001 to 2’000,000 | 50 to 99       |
| Medium Company “B”              | From 2’000,001 to 5’000,000 | 100 to 199     |
| Big Company                     | From 5’000,001 onwards     | 200 onwards     |

Source: Author’s elaboration.

Table 2. Number of companies by company size and national participation

| Company Size              | No. of Companies | % Total |
|---------------------------|------------------|---------|
| Total                     | 843,745          | 100,0 % |
| Micro Enterprise          | 763,636          | 90,5 %  |
| Small Company             | 63,400           | 7,5 %   |
| Medium Company “A”        | 7,703            | 0,9 %   |
| Medium Company “B”        | 5,143            | 0,6 %   |
| Big Company               | 3,863            | 0,5 %   |

Source: Author’s elaboration.

Of this number of companies, according to the National Institute of Statistics and Census, about 200 SMEs have obtained an ISO mark. Of these, about 100 are companies from Quito and 50 from Guayaquil; the rest comes from other cities in the country.

We proceed to establish the formula to estimate the sample size from companies with ISO certification and based on it we proceed to build the table where a sample is determined in relation to a population. The research elements to whom the measurement instrument was applied are the managers and owners of commercial SMEs.

The type of probabilistic sampling, based on the table by Krejcie and Morgan (Valencia, Lambán & Royo, 2014), where the population size and the number of errors determine the size of the randomly selected sample.

Formula:
A sample of 48 SMEs was determined. In this sense, a sample of 48 SMEs was obtained, placing the population in the table; a pilot test was applied to 20 SMEs.

The instrument that was designed for the measurement was based on the variables diagram of the expost model, which juxtaposes to the sagittal diagram of variables, where the methodological matrix of variables is defined in a conceptual, operational way, and by dimensions, which establish indicators, as well as the level of measurement.

\[
S = \frac{X^2 NP(1-P)}{d^2 (N-1)l + X^2 P(1-P)}
\]

Results
With respect to the determination of the level of reliability of the measuring instrument, the statistical test of the pilot test was performed where a Kaiser-Meyer-Olkin alpha = 0.874 was obtained, indicating a level of significant relationship of the variables of study.

Figure 2. Ex ante Model. Source: Author’s elaboration.
The measure of the Kaiser-Meyer-Olkin sample adequacy contrasts whether the partial correlations between the variables are small. Bartlett’s test of sphericity contrasts whether the correlation matrix is an identity matrix, which would indicate that the factorial model is inadequate.

R2 is the percentage of variation of the response variable that explains its relationship with one or more predictor variables.
Discussion

From the preliminary results it can be inferred that the logistics dimension has an obvious impact on the appreciation of the competitiveness of entrepreneurs, observing lower correlation coefficients in this second analysis; in addition to identifying that several of the relationships between the different variables lose significance statistics. This allows us to identify that associativity, in addition to contributing significantly to improve the competitiveness of SMEs, as it demonstrates that it contributes to the other dimensions established in the study harmonize and work according to the same objective.

Next, in Figure 3 the methodological congruence is observed, with the approach of the variables and the degree of relationship of each independent variable.

![Figure 3. Structural equation modeling. Source: Author’s elaboration.](image)

Figure 3 estimates the modeling of structural equations, where the dependent variable “Associativity” has a greater correlation in the development of the productive chain; it is explained by the Direct Actors, Logistics and Public Policies, which are in the parameter of 0.764, 0.7567 and 0.7608, correspondingly, as shown, through ovals of the variables of less significance that obtained a parameter of 0.30 and 0.040, the alphas of the variables do not contribute to the prediction of the independent variable, that is due to their low alpha and the size of the sample.

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3 The ovals are the latent variables or constructs, the arrows that join the ovals are beta coefficients and the small arrows that are next to the rectangles are estimation errors.
Conclusions

The purpose of the investigation determined that the economic aspects of the proposed models of Cournot and Bertrand, they apply duopolist ideas that could determine that the microentrepreneurs could choose one or the other, quantity of offer or price, as a parameter of action in the lower index impact for the logistic volume and optimal production lot.

However, it can only be consumers who decide whether they consider the product supplied to be homogeneous or not. What makes suppliers can do is just conclude and market the product to convince consumers that there are different brands with their defined advantages.

This is treated under the title that business competitiveness can apply mathematical procedures that allow measuring the risk of the application of price discrimination in standard microeconomics. And this is precisely what we propose to do, although we need three different markets; the common duopoly market (Cournot) and two discriminatory monopoly markets (Bertrand), one for each supplier.

However, there is a direct and significant relationship between the associativity and the dimensions of the competitiveness of SMEs in the commerce sector of the study area, so the logic can be rectified how competitors can influence the sizes of consumer groups to through several devices. Mathematically the model produces bifurcations this gives the possibility to study more variables much more intriguing before the exposed ones.

The Associativity variable of the productive chain is more accepted and explained by the SMEs of the sector than the other study variables established such as; direct actors, logistics and government policies.

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