Study of the dynamics and analysis of the effect of the position of the vibration motor to the oscillation of vibrating screen

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Abstract. A vibrating screen is a device for sorting materials, which is widely used in screening plants. The working ability of a vibrating screen relies on various parameters, such as dynamic parameters, technological parameters, and so forth. If it is necessary to find a reasonable set of parameters in order to enhance the machine’s performance, it is essential to set up the vibration equation of the vibrating screen. This is also a part of the research content that the article mentions. The next step is to establish the equation of motion of random point-P on the screen box, conduct the simulation, and discuss the effect of the position of the vibration motor on the oscillation of vibrating screens. The result reveals that the farther the point is from the focus of vibrating screen, the higher the effect of the vibration motor placement on the amplitude of vibration will be. The points are symmetrical with each other across the focus tend to create the rule of change is opposite. When the vibration motor is moving in the positive direction of the horizontal direction, the vertical oscillation amplitude of the points lying on the upper sieve will increase gradually, while the bottom points will decrease progressively. When the vibration motor is moving in the negative direction of the horizontal direction the change rule is opposite to the case of positive direction; if the vibration motor is 0.3 meters away from the focus of screen, the vertical vibration amplitude of the points on the sieve reaches the maximum value. The research results are of considerable significance to reasonably design parameters for vibrating screen working well.

1. Introduction
A vibrating screen is a sorting device which is used to separate bulk lump material into different sized particles by screen mesh [1]. Ther structure and working principle of vibrating screen as shown in figure 1, including vibration motor (2) with a screen mesh (3) placed on springs (4) and on a screen frame to install vibration motor (1). When the vibration motor (1) is operated, it will create a vibrating force that causes the screen frame to oscillate and implements the screening process [2].
Currently, vibrating screens are commonly used in a range of screening plants [3, 4, 5], and there have also been tons of studies on vibrating screen in the world [6-8]. However, how to improve the efficiency of the vibrating screen has been still a challenging question because the productivity, performance, and useful life of vibration screens depend on a lot of factors such as material properties (moisture content, mud content, particle size, density), dynamics of vibrating screen, technological parameters of the machine, etc. [9, 10, 11, 12]. In which the dynamic parameters in the working process plays a crucial role, that significantly affect the working ability of the device [13, 15-18]. Therefore, in this study, the author is going to dig deeper into the dynamic research of the vibrating screen and establish the oscillation equation of the vibrating screen, the equation of motion of random point on screen box, then applying Matlab software to build up the relationship between the influence of the position of vibration motor and the oscillation of the vibrating screen [14].

Materials and methods

**Figure 1.** The structure of a vibrating screen

**Figure 2.** Dynamic model of vibrating screen
2. Dynamics of the vibrating screen

2.1. Establishing the dynamic equation of vibrating screen

From the working principle and structure of the vibrating screen, as it is shown in figure 1, by dint of using some simplified rules like concentrated volume and concentrated hardness, the author built the dynamic model of vibrating screen as shown in figure 2.

Where,
- M - The total mass of screen refers to the focus C, kg;
- m - The mass of the eccentric plate, kg;
- r - The eccentric radius, mm;
- k_{x1}, k_{x2}, k_{y1}, k_{y2} - The spring hardness in the x and y directions, N/m;
- L_1, L_2 - The distance from the focus of screen to the springs in the x direction, mm;
- L_{ox}, L_{oy} - The distance from the focus of screen to the center of vibration motor, mm;
- $\alpha$ - Initial tilt angle of the screen, degree;
- $\varphi$ - Angle of rotation around the axis of eccentric plate, $\varphi = \omega t$ (rad);
- $\omega$ - Angular frequency of eccentric plate, rad/s;
- $\alpha_s$ - Position angle of the screen while oscillating, rad;

According to the moving theorem of an object in the plane, the general dynamic equation of the vibrating screen is expressed in formula 1 [11]

$$M \frac{d^2 r}{dt^2} = \sum F^{(e)}$$  \hspace{1cm} (1)

Based on the dynamic model of the vibrating screen (figure 2) and some transformations, we find the dynamic differential equation of vibrating screen as follows:

$$M \ddot{x} = A_o \cos \omega t - f_x \dot{x} - (k_{x1} + k_{x2})x$$
$$M \ddot{y} = A_o \sin \omega t - f_y \dot{y} - (k_{y1} + k_{y2})y - (k_{y1}L_1 - k_{y2}L_2)\alpha_s$$
$$J \ddot{\alpha}_s = A_o (L_{oy} \cos \omega t - L_{ox} \sin \omega t) - f_{ax} \dot{\alpha}_s - (k_{y1}L_1^2 + k_{y2}L_2^2)\alpha_s - (k_{y1}L_1^2 + k_{y2}L_2^2)\alpha_s \sin^2 \alpha - (k_{y1}L_1 - k_{y2}L_2)y$$  \hspace{1cm} (2)

Where $F^{(e)}$ is the main force caused by the eccentric plate.

In order for the oscillation of screen from the nonlinear to linear, we take $k_{y1}L_1 = k_{y2}L_2$ then formula (2) can be presented as follows:

$$(M + m)\ddot{x}_c + f_x \dot{x}_c + (k_{x1} + k_{x2})x_c = A_o \cos \omega t$$
$$(M + m)\ddot{y}_c + f_y \dot{y}_c + (k_{y1} + k_{y2})y_c = A_o \sin \omega t$$
$$J \ddot{\alpha}_c + f_{ax} \dot{\alpha}_c + (k_{y1}L_1^2 + k_{y2}L_2^2)\alpha_c = A_o (L_{oy} \cos \omega t - L_{ox} \sin \omega t)$$  \hspace{1cm} (3)

Let $\xi_1 = \frac{f_x}{2\omega_1(M + m)}$, $\xi_2 = \frac{f_y}{2\omega_2(M + m)}$, $\xi_3 = \frac{f_{ax}}{2\omega_3J}$

$$\omega_1 = \sqrt{\frac{k_{x1} + k_{x2}}{M + m}}$$,  \hspace{1cm} $$\omega_2 = \sqrt{\frac{k_{y1} + k_{y2}}{M + m}}$$,  \hspace{1cm} $$\omega_3 = \sqrt{\frac{k_{y1}L_1^2 + k_{y2}L_2^2}{J}}$$

Then formula (3) becomes:
\[ \ddot{x}_c + 2\xi_1 \omega_1 \dot{x}_c + \omega_1^2 x_c = \frac{A_o}{M + m} \cos \omega t \]
\[ \ddot{y}_c + 2\xi_2 \omega_2 \dot{y}_c + \omega_2^2 y_c = \frac{A_o}{M + m} \sin \omega t \]
\[ \ddot{\alpha}_s + 2\xi_3 \omega_3 \dot{\alpha}_s + \omega_3^2 \alpha_s = \frac{A_o}{J_c} (L_{\omega y} \cos \omega t - L_{\alpha \omega} \sin \omega t) \]

Where,
\( \omega_1, \omega_2 \) - the natural frequency of the screen in x and y directions;
\( \omega_3 \) - the natural shaking frequency in \( \alpha_s \) direction.

The oscillation of the screen is a function of the oscillator frequency, at steady state:
\[ x_c = X \cos(\omega t - \phi_1) \]
\[ y_c = Y \sin(\omega t - \phi_2) \]
\[ \alpha_s = \alpha_{01} \cos(\omega t - \phi_3) + \alpha_{02} \sin(\omega t - \phi_4) \]

Substituting Eq. (5) into (4), yielding:
\[ X = \frac{A_o \left[ (M + m)\omega_1^2 \right]}{\sqrt{\left[ 1 - \left( \frac{\omega}{\omega_1} \right)^2 \right]^2 + \left[ 2\xi_1 \frac{\omega}{\omega_1} \right]^2}} \]
\[ \phi_1 = \arctan \left( \frac{2\xi_1 \frac{\omega}{\omega_1}}{1 - \left( \frac{\omega}{\omega_1} \right)^2} \right) \]
\[ Y = \frac{A_o \left[ (M + m)\omega_2^2 \right]}{\sqrt{\left[ 1 - \left( \frac{\omega}{\omega_2} \right)^2 \right]^2 + \left[ 2\xi_2 \frac{\omega}{\omega_2} \right]^2}} \]
\[ \phi_2 = \arctan \left( \frac{2\xi_2 \frac{\omega}{\omega_2}}{1 - \left( \frac{\omega}{\omega_2} \right)^2} \right) \]
\[ \alpha_{01} = \frac{A_o L_{\omega y} / (J \omega_1^2)}{\sqrt{\left[ 1 - \left( \frac{\omega}{\omega_3} \right)^2 \right]^2 + \left[ 2\xi_3 \frac{\omega}{\omega_3} \right]^2}} \]
\[ \phi_3 = \arctan \left( \frac{2\xi_3 \frac{\omega}{\omega_3}}{1 - \left( \frac{\omega}{\omega_3} \right)^2} \right) \]
\[ \alpha_{02} = \frac{-A_o L_{\alpha \omega} / (J \omega_1^2)}{\sqrt{\left[ 1 - \left( \frac{\omega}{\omega_3} \right)^2 \right]^2 + \left[ 2\xi_3 \frac{\omega}{\omega_3} \right]^2}} \]
\[ \phi_4 = \phi_3 \]
2.2. Establishing equations of motion of a random point on the screen surface

Figure 3 shows that point C represents the initial position of the focus of screen. After a period of work the screen has a tendency to move constantly, then the focus C will have an approximate orbit of the circle centered at C with a radius as CP'. If we neglect the shake of the screen, the trajectory of point P will be an approximation of the circle centered at C' with a radius as C'P' (P is a random point on screen surface, P is about l focus away from C); If the sway factor of the screen is included, the trajectory of point P will be in an ellipse form. Hence, the position of point P can be expressed as formula (7).

\[
\begin{align*}
x_p &= x_C + l \alpha_s \sin \alpha \\
y_p &= y_C - l \alpha_s \cos \alpha
\end{align*}
\]  
(7)

Since point C' and C have similar motion trajectories, then:

\[
\begin{align*}
x_c &= X \cos(\omega t - \phi) \\
y_c &= Y \sin(\omega t - \phi) \\
\alpha_s &= \alpha_{01} \cos(\omega t - \phi_3) + \alpha_{02} \sin(\omega t - \phi_3)
\end{align*}
\]  
(8)

Substituting Eq. (8) into (7), yielding:

\[
\begin{align*}
x_p &= A_1 \cos(\omega t + \psi_1) \\
y_p &= A_2 \cos(\omega t + \psi_2)
\end{align*}
\]  
(9)

Where,

\[
\begin{align*}
A_1 &= \sqrt{a_1^2 + b_1^2} \\
A_2 &= \sqrt{a_2^2 + b_2^2}
\end{align*}
\]  
(10)

\[
\begin{align*}
a_1 &= X \cos \phi_1 + l \alpha_{01} \sin \alpha \cos \phi_3 - l \alpha_{02} \sin \alpha \sin \phi_4 \\
b_1 &= X \sin \phi_1 + l \alpha_{01} \sin \alpha \sin \phi_3 + l \alpha_{02} \cos \alpha \cos \phi_4 \\
a_2 &= -Y \sin \phi_2 - l \alpha_{01} \cos \alpha \cos \phi_3 + l \alpha_{02} \cos \alpha \sin \phi_4 \\
b_2 &= Y \cos \phi_2 - l \alpha_{01} \cos \alpha \sin \phi_3 - l \alpha_{02} \cos \alpha \cos \phi_4
\end{align*}
\]  
(11)

If \(a_1 \geq 0\) then \(\psi_1 = \arctan\left(-\frac{b_1}{a_1}\right)\); If \(a_1 < 0\) then \(\psi_1 = \arctan\left(-\frac{b_1}{a_1}\right) + \pi\)
If $a_2 \geq 0$ then $\psi_2 = \arctan\left(-\frac{b_2}{a_2}\right)$; If $a_2 < 0$ then $\psi_2 = \arctan\left(-\frac{b_2}{a_2}\right) + \pi$

3. Effect of the position of vibration motor on the oscillation of screen

In order to study the influence of the position of the vibration motor on the oscillation of screen, we would choose four points (S, R, Q, P) on the screen surface, which will be the research object as shown in figure 1.

Deriving from formula (9), (5) and (6) we consider $L_{ox}$ and $L_{oy}$ as variables and take the values of $L_{ox} = -0.5 \pm 0.5m$; $L_{oy} = -0.5 \pm 0.5m$. Other parameters have the following values: $l_{SC} = l_{SC} = 0.3m$; $l_{QC} = l_{RC} = 0.3m$; $L = 1.2m$; $B = 0.6m$; $M = 228.6kg$; $m = 30.4 kg$; $J = 17.8kg.m^2$; $f_x = 0.01$; $f_y = 0.005$; $f_\alpha = 0.02$; $\alpha = 25^o$; $A_0 = 2500N$; $\omega = 104.7$ rad/s; $L_1=L_2 = 0.5m$; $k_{x1} = k_{x2} = 100000N/m$; $k_{y1} = k_{y2} = 50000N/m$. Using Matlab to develop the correlation between the vibration motor position and the amplitude of vibration of points P, Q, R, S as shown in figure 4 and figure 5.

![Figure 4](image1.png)

(a) Correlation between $L_{ox}, L_{oy}$ and the amplitude of oscillation of point Q in the x direction

(b) Correlation between $L_{ox}, L_{oy}$ and the amplitude of oscillation of point P in the x direction

![Figure 4](image2.png)

(c) Correlation between $L_{ox}, L_{oy}$ and the amplitude of oscillation of point R in the x direction

(d) Correlation between $L_{ox}, L_{oy}$ and the amplitude of oscillation of point S in the x direction

Figure 4. Influence of the position change of excitation force the x-orientation amplitude of vibration
Figure 5. Influence of the position change on excitation force the y-orientation amplitude of vibration

From the results in figure 4, it is evident that:
- Figure 4a and figure 4b are graphs showing the characteristics of the two points Q and P above the screen surface above the centre of focus C. From the results we can see that the points on the screen surface in the x direction are farther away from the focus of screen, the effect of the amplitude on the position of vibration motor will be higher, and their rules of change are basically the same.
- Figure 4a and figure 4c are graphs showing the characteristics of the two points on the screen surface (Q, R) that are symmetrically identical through the focus of screen C. This indicates that the distribution of the amplitude of the two points is entirely opposite.
- Figure 4b and figure 4d are graphs showing the characteristics of the two points on the sieve (P, S) that are symmetric through the focus of screen C. The outcomes also reflect the same features as mentioned above (points Q and R). As a result, it can be affirmed that any two points are symmetrical through the focus; the rule of change follows the above rule.

From the results of figure 5, leading to:
- When the vibration motor is moving in the positive direction of the x-axis, the amplitude of the oscillation in the y-direction of the points on the screen surface above (Q, P) will increase gradually, while the points lying on the screen surface at the bottom (R, S) will decrease progressively. When L_{ox} approximately equals 0.3m, the amplitude of vibration in the y-direction of the points on screen surface approaches the maximum values (the point one on the above screen surface equals 0.0011m; the point on the bottom screen surface is 0.00088m) then go up and go down gradually;
- When the vibration motor is moving in the negative direction of the x-axis, the rule of change of the points on the screen is utterly opposite to the case where the vibration motor is moving in the positive direction of the x-axis;
- The farther the position of the points on the screen to the focus is, the greater the effect of changing the location of the vibration motor on the amplitude of vibration will be, (e.g., when L_{ox} > 0 then A_{Py} > A_{Qy}; A_{Py}, A_{Qy} is the oscillation amplitude of point P, Q according to the y-axis).
4. Conclusion
According to the working principle, the structure of the vibrating screens and some simplified rules, the author has built a dynamic model and vibration equation of vibrating screen, established the equation of motion of random point-P on screen box and conducted the simulation. Subsequently, the analysis demonstrates the effect of the position of the vibrating screen on the oscillation of vibrating screens. These results might help scientists in their design of vibrating screens.

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