Statistical analysis of sloshing-induced dissipative energy across a range of Froude numbers

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Abstract.

Fuel sloshing-induced damping is currently being studied extensively within the EU-funded SLOWD project as a means of passively reducing dynamic loads in aircraft wings. It is of interest to be able to determine which parameters have the greatest influence on the added damping from the sloshing motion. An uncertainty in the measured sloshing force has been observed when multiple consecutive and identical oscillation cycles are considered in sinusoidal excitation experiments, leading to variations in the measured energy dissipation. This current work considers liquid undergoing vertical sloshing motions for different fill level and excitation conditions (frequency and amplitude) leading to energy dissipation via several possible physical mechanisms. The sloshing dissipation is measured experimentally across a large number of excitation cycles and for each excitation amplitude, expressed in the form of Froude (Fr) numbers. Depending on the Fr number, distinct sloshing mechanisms dominate the dissipative effects and induce a particular variance across the identical cycles analysed. The sloshing-induced energy dissipation variation is quantified and correlated with different mechanisms depending on Fr number, helping to explain various non-stationary effects that are observed even in well-controlled experimental conditions. As well as improving the insights into the inherent dispersion nature of the studied phenomena, this research also establishes experimental characteristics suitable for future model validation and calibration.

1. Introduction

Airliner wings carry substantial amounts of fuel inside of their structure, however, the dynamic interaction between the two being relatively poorly understood and modelled. Historically, there have been studies involving fuel sloshing in wing-attached containers (see for example [1, 2, 3]), mainly focusing on the inertial effects added by the liquid mass. In this context, the violent vertical interaction between sloshing liquids and vibrating structures has recently attracted considerable attention. Most notably, current efforts to understand and model turbulent vertical liquid sloshing are carried out as part of the EU-funded Horizon 2020 project Sloshing Wing Dynamics (SLOWD), with the main aim of reducing unnecessary design conservatism by taking into account sloshing loads.

Previous experimental investigations most relevant to this current work have been conducted as part of the SLOWD project. Sloshing-induced damping was demonstrated in a continuous system by Titurus et al. [4] via experimental investigations in a beam with a tank attached to it. In order to gain a better understanding of the underlying dissipation-causing mechanisms,
as well as to avoid any nonlinearities that are inherent to mechanically complex systems, a series of single degree of freedom experiments have been designed and conducted. Constantin et al. [5] have presented a T-shape beam configuration with a tank attached to it, achieving lightly damped, high amplitude, vertical and linear transient motion of a liquid-filled tank. The sloshing-induced damping ratio was studied as a function of filling level and amplitude of excitation. Similar conclusions were reached independently in transient vertical experiments conducted by Martinez-Carrascal et al. [6]. In order to achieve a more controllable setup and to allow for detailed sloshing analysis inside individual tank cycles, harmonic-excitation experiments were conducted by Constantin et al. [7]. Energy dissipation was studied across a range of excitation amplitudes, with emphasis on detailed analysis of the sloshing force as a function of tank displacement and time, under sinusoidal vertical excitation. Based on the observed behaviour, a harmonic-ballistic model was proposed, which confirmed the fundamental kinematic reasoning behind the interaction between the sloshing liquid and the containing tank, at high amplitudes.

The current work builds upon the results presented in [5] and [7]. In [5], a series of transient experiments were conducted, at various filling levels and amplitudes of excitation. The experimental results are revisited here and variations in sloshing-induced damping for multiple repetitions are quantified. In [7], the authors presented an experimental setup consisting of a vertically-oriented electroactuator with a tank at its end. Under the hypothesis of steady-state behaviour of the sloshing liquid subject to sinusoidal excitation, the sloshing forces and sloshing-induced damping should have the same values in consecutive excitation cycles. However, deviations from the steady-state behaviour are experimentally observed mainly due to: (1) instabilities in the sloshing pattern at low amplitudes, (2) sources of nonlinearity at mid-range forcing amplitudes, as well as (3) chaotic behaviour of the liquid at large amplitudes when there is substantial interaction with the tank’s ceiling. Based on force and acceleration measurements across multiple consecutive excitation cycles, a statistical analysis is conducted on the sloshing-induced energy dissipation and sloshing force, in an attempt to quantify such deviations. This in turn leads to a better understanding of the expected sloshing-induced damping inside an average excitation cycle, as well as better calibration data for numerical models.

2. Experimental setups
2.1. Sinusoidal excitation setup
The experiment presented in [7] is designed to allow for the vertical sinusoidal excitation of a tank containing liquid, at large amplitudes and controlled frequencies. The setup consists of a linear electroactuator, at the end of which a tank filled with various amounts of deionised water is attached in line with a piezo-electric force sensor and accelerometer. Various filling levels were considered in this study: 30, 50 and 70% of the total tank volume (width $w = 30$ mm, height $h = 50$ mm and length $L = 100$ mm). A photograph of the experimental setup is shown in figure 1.

The vertically-oriented electroactuator is placed on a vibration-isolation table via a rigid frame. Both a force sensor and accelerometer are used in order to collect the force signal at the interface between the actuator and the tank, as well as the inertial component of the system. Sinusoidal excitation at a single frequency (8.3 Hz) was used throughout all of the test cases considered here and the forcing amplitude was varied. The frequency of excitation is approximately twice the first symmetric sloshing mode of the liquid at 50% filling level, chosen in order to parametrically excite only one sloshing mode at low amplitudes. Full details regarding this experimental setup are provided in [7].
Figure 1: Photograph of the forced vertical excitation experimental rig [7]

2.2. Step release setup
A T-shape beam configuration was presented in [5]. The system was loaded using a turnbuckle at various amplitudes in the Froude number range of 0.64 to 2.39 (amplitudes between 1 and 14 mm). Eleven different filling levels were considered, covering the full range from 0% (no liquid inside tank) to 100% (tank fully filled); tap water was used as the working liquid. The setup was shown to behave like a lightly damped single degree of freedom system, vibrating at approximately 10 Hz. Damping data was collected from the decay envelope of the acceleration of the vibrating tank. A photograph of the test setup is presented in figure 2.

3. Error quantification methodology
In sinusoidal excitation experiments, one of the most important quantities to be analysed is the hysteretic energy dissipation per tank cycle, since it encapsulates the damping-inducing action of the liquid. As presented in [7], the dissipative energy per cycle is calculated via the work done by the sloshing force (total force measured by force sensor minus total inertial force of the system) across one tank cycle. The sloshing force is numerically integrated with respect to tank displacement in order to obtain the sloshing-induced dissipation energy, and then normalized [7], giving

\[ E_c = \frac{E}{m_w \omega^2 A^2} \]  \hspace{1cm} (1)

where \( E \) is the dimensional dissipative energy, \( A \) is the displacement amplitude of the tank,
ω is the frequency of excitation and \( m_w \) is the water mass.

The Froude number can be used as a measure of normalized excitation acceleration amplitude, defined as \( Fr = \frac{\sqrt{A\omega^2}}{g} \), where \( g \) is the gravitational acceleration. Considering that only one frequency was studied here and that \( g \) is a constant, the Froude number in this case is a nondimensional measure of the excitation amplitude.

The energy data was presented in [7] in the form of mean, minimum and maximum values per cycle for each Froude number studied. However, considering the large number of available force-displacement cycles, much more information can be conveyed in alternative representations. One example is the violin plot [8], which uses a density kernel function on top of the common error bars showing the minimum/maximum values or standard deviation. Usually, the density kernel function is a smoothed histogram of the data, such that violin plots are capable of showing the structure and distribution of the data between the bounding values. A Matlab-based implementation of the violin plot was used in this research [9].

Considering that the sloshing-induced dissipation is calculated using the sloshing force, further insights into non-stationary effects can be gained by studying the statistical distribution of the force values across one cycle. To this end, the variation of the sloshing force was measured for each tank cycle. The curve representing the mean value of the sloshing force across all cycles can be used to obtain a representation of the average sloshing contribution for any position of the vibrating tank. The standard deviation of the force values, the minimum and maximum values at each tank position are calculated as well, offering a complete picture of the statistical variation of the sloshing force. These representations, when given as function of tank displacement, are directly connected to the calculation of the hysteresis dissipative energy.

Similar in-depth cycles analysis is not possible for the transient T-beam experiments, since force information was not collected at the tank - beam interface. However, variation in sloshing-induced damping can be quantified by running multiple transient experiments under the same conditions. Decaying acceleration timeseries and envelopes can then be compared in order to assess the expected variability in damping ratio caused by varying sloshing patterns. Data originating from the three 50% filling level experimental runs are shown in this work.
4. Results

For each of the three filling levels considered under sinusoidal excitation, the tank was excited for Fr number ranges between 0.46 and 1.86 at 8.3 Hz. Figure 3 shows the distributions of normalized energy dissipation per cycle $E_c$ (as defined in [7]) in the form of violin plots. Mean values of the energy are also shown with circle markers, as well as vertical bars showing the data range corresponding to $\pm 1$ standard deviation around the mean. A secondary axis is included on the right hand side corresponding to the square markers and dashed lines; it shows the values of energy standard deviation $\sigma(E_c)$ as the percentage of the corresponding mean values.

![Figure 3: Dissipation energy analysis. Left axis: violin plots of $E_c$ as a function of Froude number for the three filling levels; shown in figure: mean values, $\pm 1$ standard deviation (error bars) and distribution of energy values. Right axis: in dashed lines, values of $E_c$ standard deviation as percentage of mean value.](image)

The mean values in figure 3 correspond to the data that were shown in [7]. A first observation is that the energy distributions are not Gaussian; they can be skewed and multimodal (i.e. having more than one peak), depending on the interaction between different sloshing modes. Generally, it is observed that the greatest spread of the data is found between Froude numbers of approximately 1 and 1.5. Following the terminology used in [5] and [7], this range corresponds to a combination of R2 and R1 type of the liquid behaviours: there is a dominating sloshing mode, usually (2,0) for this frequency of excitation, combined with significant interaction between the liquid and the top of the tank (slamming). For these conditions, however, it was observed that the liquid did not impact the tank ceiling quite as consistently as it did at higher Fr numbers. This behaviour leads to a greater spread of the $E_c$ values in this range of Fr numbers. This fact is also seen in the energy standard deviation, which is generally higher than 10% of the mean value and can get as high as 50%.

The value of the energy standard deviation is not included in figure 3 for the Fr numbers lower than 0.8 because the mean $E_c$ is small, leading to $\sigma$ values that are very high when expressed as percentage points. Moreover, for these cases the signal-to-noise ratio in the sloshing force is also higher, since there is very little activity of the liquid. With these exceptions, it is observed that at lower Fr numbers (Fr < 1) the dissipative energy shows less variation. In this Fr range, the liquid does not impact the tank with the exception of the 30% fill cases, where an instability
triggers the out-of-plane sloshing mode (0,1) leading to substantial interaction with the tank ceiling.

The $E_c$ standard deviation is lowest at high test amplitudes ($Fr > 1.6$) for all fill levels, between 5% and 10% of the mean energy values. This region is characterised by strong vertical and turbulent interactions with the tank and this behaviour is found to be consistent across all cycles, leading to reduced spread of the data.

Figures 4-6 show examples of sloshing force – displacement cycles based on which the data shown in figure 3 was obtained. Hysteresis cycles are shown for three filling levels and two Froude numbers each. The amplitude of the tank is normalized to tank height ($h = 50$ mm). The mean force values are shown in black, averaged over multiple tank cycles. The standard deviation of the force is shown as blue curves and the light gray bars show the maximum and minimum values. A first observation is that higher frequency components are present in the force – displacement hysteresis cycles, stemming either from higher-frequency liquid sloshing components or from mechanical sources. The actuator is of a screw ball type, the contact between the internal
components representing a possible candidate for injecting higher frequency components into the recorded force responses. Second of all, the shapes of the cycles change with the filling level. This is indicative of differing modes of interaction between the liquid and the tank. For instance, at $Fr = 1.17$, the relative proximity between the liquid’s free surface and tank top at 70% filling level makes it easier for the liquid to vertically impact the tank at this lower $Fr$ number, changing thus the cycle shape.

Larger standard deviation values are seen for the 30% fill level at $Fr = 1.17$, correlating well with the distributions shown in figure 3. For higher Froude numbers ($Fr = 1.86$), the largest spread is seen around the points of liquid impact with the top and bottom of the tank, i.e. immediately after the tank’s minimum and maximum displacement. This happens due to the non-uniformity of the vertical interaction of the liquid with the tank walls. The mean value of the positive force peak is closer to the minimum $A/h$ for the higher filling levels, since the liquid has less of a distance to travel in order to meet the tank wall, the impact thus happens closer to the point where the tank changes the direction of motion.

The way the variation of the sloshing force inside different oscillation cycles translates into the transient responses is shown in figure 7. The mean decaying acceleration data in the transient experiments presented in [5] are shown in figure 7a, together with the confidence bounds (shown as the minimum and maximum acceleration value in each measured time instance). Damping ratio values were calculated for each of the experimental case. The insets show details from within various selected response regions. It is seen that, among the three repetitions considered, the repeatability is good. The initial amplitude is high enough such that the violent and turbulent vertical impacts between the liquid and the tank occur in the first part of the transient response. In a similar fashion to the variations of the sloshing force shown in figures 4 to 6, a lack of uniformity in the liquid-tank impacts leads to variations in the tank acceleration. This in turn leads to variations in the damping ratio, as shown in figure 7b, where the envelope of the acceleration signal is shown. The corresponding Froude numbers, shown for the different time instants as well, are also indicative of the transition points between different sloshing regimes (vertical dashed lines). Slightly higher variations of the damping ratio $\zeta$, expressed as percentage...
of the critical damping, are found in the second sloshing region. It is important to note that in transient experiments the cross-cycle dissipation variation is not as influential as in the forced excitation cases. The spread of the $\zeta$ values in the transient experiments is considerably less than the spread of the $E_c$ values in sinusoidal excitation experiments. The sloshing patterns appear to be more sensitive to small variations in the sloshing initial conditions than to the differences in the global characteristics representing the consecutive tank oscillation cycles. This correlates well with the sudden increase of the width of the red bands in figure 7b following the beginning of the second sloshing region.

5. Conclusions
Two different experiments were presented in this work involving both motion-controlled and free-vibration transient conditions. The sloshing force component was isolated and studied across multiple tank cycles, offering a statistical picture of the sloshing-induced damping. Depending on Froude number, sloshing patterns change and give rise to nonstationary phenomena. The highest variation in the dissipative energy was shown in the 1 to 1.5 Froude number range for the cases analysed here, where the standard deviation of the dissipative energy can get as high as 50% of its mean value. At large displacement amplitudes, variations in sloshing force are expected due to non-uniform interactions with the tank walls. The transient experiments show damping ratio variations across different tests. However, it is observed that the differences in the dissipative energy from one cycle to another do not strongly influence the uncertainty bounds of the identified values of the damping ratios. The piece-wise linear damping regions feature smoothed out representation of otherwise noticeable cross-cycle differences observed during the motion-controlled tests. From an induced damping point of view, the sloshing initial conditions following transition from R1 to R2 seem to be more relevant in coupled, transient cases.

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