Anomalous quartic $WW\gamma\gamma$ and $WWZ\gamma$ couplings through $W^+W^-Z$ production in $\gamma\gamma$ colliders

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Abstract

We find 95% confidence level limits on the anomalous coupling parameters $a_0$, $a_c$ and $a_n$ with an integrated luminosity of $500fb^{-1}$ and $\sqrt{s} = 0.5, 1$ and 1.5 TeV energies. We take into account incoming beam polarizations and also the final state polarizations of the gauge bosons in the cross-section calculations to improve the bounds. We show that polarization leads to a significant amount of improvement in the sensitivity limits.

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I. INTRODUCTION

Gauge boson self interactions are strictly constrained by the $SU_L(2) \times U_Y(1)$ gauge invariance. Therefore the direct study of gauge boson self interactions provide a crucial test of the gauge structure of the standard model (SM). Any deviation of the couplings from the expected values would indicate the existence of new physics beyond the SM. In the recent experiments at CERN $e^+e^-$ collider LEP and Fermilab Tevatron gauge boson self interactions have been studied experimentally through $e^+e^-, p\bar{p} \rightarrow W^+W^-, WZ, ZZ, Z\gamma, W^+W^-\gamma, Z\gamma\gamma$ gauge boson production processes [1, 2, 3, 4, 5]. It will be possible to produce a final state with three or more massive gauge bosons in the next generation of $e^+e^-$ colliders. After these future $e^+e^-$ colliders are constructed its operating modes of $e\gamma$ and $\gamma\gamma$ are expected to be designed [6].

Future $e^+e^-$ collider and its $e\gamma$ and $\gamma\gamma$ modes will have a great potential to probe anomalous quartic gauge boson vertices. We concentrate on genuine quartic gauge boson couplings which do not induce new trilinear vertices. Genuine quartic couplings are contact interactions, manifestations of the exchange of heavy particles. On the other hand non-genuine quartic gauge boson couplings emerge from an operator that induces both trilinear and quartic gauge boson couplings. Non-genuine couplings can be investigated much more efficiently through their trilinear counterpart. In this paper we assume that quartic couplings are modified by genuine anomalous interactions while the trilinear couplings are all given by their SM values.

In writing effective operators associated with genuinely quartic couplings we employ the formalism of [7]. Imposing custodial $SU(2)_W$ symmetry and local $U(1)_em$ symmetry and if we restrict ourselves to C and P conserving interactions, dimension 6 effective lagrangian for $W^+W^-\gamma\gamma$, $ZZ\gamma\gamma$ and $W^+W^-Z\gamma$ couplings are given by,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_c + \mathcal{L}_n$$

$$\mathcal{L}_0 = \frac{-\pi \alpha}{4\Lambda^2} a_0 F_{\mu\nu} F^{\mu\nu} W_{\alpha}^{(i)} W^{(i)\alpha}$$

$$\mathcal{L}_c = \frac{-\pi \alpha}{4\Lambda^2} a_c F_{\mu\alpha} F^{\mu\beta} W^{(i)\alpha} W^{(i)}\beta$$
\[ \mathcal{L}_n = \frac{i\pi\alpha}{4\Lambda^2} a_n \varepsilon_{ijk} W^{(i)} W^{(j)} W^{(k)\alpha} F^{\mu\nu} \]  

(4)

where \( W^{(i)} \) is the \( SU(2)_W \) triplet, and \( F_{\mu\nu} \) and \( W^{(i)}_{\mu\alpha} \) are the electromagnetic and \( SU(2)_W \) field strengths respectively. \( a_0, a_c \) and \( a_n \) are the dimensionless anomalous coupling constants. Effective lagrangians (2) and (3) give rise to anomalous \( W^+ W^- \gamma \gamma \) and also \( ZZ \gamma \gamma \) couplings. Effective lagrangian (4) give rise to \( W^+ W^- Z \gamma \) coupling. For sensitivity calculations to the anomalous couplings we set the new physics energy scale \( \Lambda \) to \( M_W \).

The vertex functions for \( W^+(k_1^\mu) W^-(k_2^\nu) \gamma (p_1^\alpha) \gamma (p_2^\beta) \) and \( W^+(p_3^\mu) W^- (p_3^\nu) \gamma (p_1^\alpha) \gamma (p_2^\beta) \) generated from the effective lagrangians (2), (3) and (4) are given respectively by

\[ i \frac{2\pi\alpha}{\Lambda^2} a_0 g_{\mu\nu} \left[ g_{\alpha\beta}(p_1 \cdot p_2) - p_2 \cdot p_1 \right] \]  

(5)

\[ i \frac{\pi\alpha}{2\Lambda^2} a_c \left[ (p_1 \cdot p_2) (g_{\mu\nu} g_{\alpha\beta} + g_{\mu\beta} g_{\alpha\nu}) + g_{\alpha\beta} (p_1 \cdot p_2 \cdot p_2 \cdot p_1) \right] 
- p_1 \cdot p_2 \left[ g_{\alpha\mu} p_2 \cdot p_1 + g_{\alpha\nu} p_2 \cdot p_1 \right] - 2p_2 \left[ g_{\beta\mu} p_1 \cdot p_2 + g_{\beta\nu} p_1 \cdot p_2 \right] \]  

(6)

\[ i \frac{\pi\alpha}{4 \cos \theta_W \Lambda^2} a_n \left[ g_{\mu\alpha} \left[ g_{\nu\beta} p_2 \cdot (p_1 - p_+) - p_2 \cdot (p_1 - p_+) \right] - g_{\nu\alpha} \left[ g_{\mu\beta} p_2 \cdot (p_1 - p-) - p_2 \cdot (p_1 - p-) \right] + g_{\mu\nu} \left[ g_{\alpha\beta} p_2 \cdot (p_+ - p-) - p_2 \cdot (p_+ - p-) \right] - p_1 \mu \left[ g_{\mu\beta} p_2 \cdot (p_1 - p_+) - p_1 \nu \left( g_{\mu\beta} p_2 \cdot (p_1 - p_+) \right) + p_1 \nu \left( g_{\mu\beta} p_2 \cdot (p_1 - p_+) \right) \right] - p_- \alpha \left( g_{\mu\beta} p_2 \cdot (p_1 - p_+) \right) + p_+ \alpha \left( g_{\mu\beta} p_2 \cdot (p_1 - p_+) \right) \right] 
- p_+ \nu \left( g_{\alpha\beta} p_2 \cdot (p_1 - p_+) \right) + p_- \nu \left( g_{\alpha\beta} p_2 \cdot (p_1 - p_+) \right) \]  

(7)

For a convention, we assume that all the momenta are incoming to the vertex. The anomalous \( ZZ \gamma \gamma \) couplings are obtained by multiplying (5), (6) by \( \frac{1}{\cos \theta_W} \) and making \( W \rightarrow Z \).

CERN \( e^+e^- \) collider LEP provide present collider limits on anomalous quartic \( W^+ W^- \gamma \gamma \) and \( W^+ W^- Z \gamma \) couplings. Recent results from OPAL collaboration for \( W^+ W^- \gamma \gamma \) couplings are given by \(-0.020 \text{ GeV}^{-2} < \frac{a_0}{\Lambda^2} < 0.020 \text{ GeV}^{-2} \), \(-0.052 \text{ GeV}^{-2} < \frac{a_c}{\Lambda^2} < 0.037 \text{ GeV}^{-2} \) at 95% C.L. assuming that the \( W^+ W^- \gamma \gamma \) couplings are independent of the \( ZZ \gamma \gamma \) couplings.
3. If it is assumed that $W^+W^-\gamma\gamma$ couplings are dependent on the $ZZ\gamma\gamma$ couplings then the 95% C.L. sensitivity limits are improved to $-0.002 \text{ GeV}^{-2} < \frac{a}{\Lambda^2} < 0.019 \text{ GeV}^{-2}$, $-0.022 \text{ GeV}^{-2} < \frac{a}{\Lambda^2} < 0.029 \text{ GeV}^{-2}$. Recent results from L3, OPAL and DELPHI collaborations for $W^+W^-Z\gamma$ coupling are given by $-0.14 \text{ GeV}^{-2} < \frac{a}{\Lambda^2} < 0.13 \text{ GeV}^{-2}$, $-0.16 \text{ GeV}^{-2} < \frac{a}{\Lambda^2} < 0.15 \text{ GeV}^{-2}$ and $-0.18 \text{ GeV}^{-2} < \frac{a}{\Lambda^2} < 0.14 \text{ GeV}^{-2}$ at 95% C.L. respectively [1, 2, 3].

In the literature there has been a great amount of work on anomalous quartic $W^+W^-\gamma\gamma$ and $W^+W^-Z\gamma$ couplings which focus on future linear $e^+e^-$ collider and its $e\gamma$ and $\gamma\gamma$ modes. Anomalous quartic $W^+W^-\gamma\gamma$ and $W^+W^-Z\gamma$ couplings have been studied through the reactions $e^+e^- \rightarrow VVV$ [8], $e^+e^- \rightarrow FFVV$ [9], $e\gamma \rightarrow VVF$ [10, 11], $\gamma\gamma \rightarrow VV$ [12], $\gamma\gamma \rightarrow VVV$ [13] and $\gamma\gamma \rightarrow VVVV$ [14] where $V = Z, W$ or $\gamma$ and $F = e$ or $\nu$. These couplings have also been studied at hadron colliders through the reactions $pp(\bar{p}) \rightarrow \gamma\gamma Z$, $\gamma\gamma W$ and $qq \rightarrow qq\gamma\gamma, qq\gamma\gamma Z$ [15].

In this work we have investigated anomalous quartic $W^+W^-\gamma\gamma$ and $W^+W^-Z\gamma$ couplings via the process $\gamma\gamma \rightarrow W^+W^-Z$. This process involve only interactions between the gauge bosons, making more evident any deviation from the SM predictions. It does not contains any tree-level Higgs contribution. Therefore it excludes all the uncertainties coming from the scalar sector, such as the Higgs boson mass. The same process was analyzed in [13] with unpolarized initial and final states. We take into account incoming beam polarizations and also the final state polarizations of the gauge bosons in the cross-section calculations to improve the bounds. We have showed that polarization leads to a significant amount of improvement in the sensitivity limits.

II. CROSS SECTIONS FOR POLARIZED BEAMS

The process $\gamma\gamma \rightarrow W^+W^-Z$ is described by twelve tree-level SM diagrams and two new diagrams that consist of an anomalous vertex $ZZ\gamma\gamma$ and a five-vertex $\gamma\gamma W^+W^-Z$ which is necessary to preserve the gauge invariance of the amplitude [13]. Feynman rules for this five-vertex is given in Ref. [13]. Since $W^+W^-\gamma\gamma$ and $W^+W^-Z\gamma$ couplings are non-zero in the SM, they contribute to the SM diagrams and modify SM amplitudes. The analytical expression for the cross section is quite lengthy and we have evaluated numerically. The phase space integrations have been performed by GRACE [16] which uses a Monte Carlo routine. As a check of our results we have confirmed the results of [13] for unpolarized
beams.

The most promising mechanism to generate energetic gamma beams in an $e^+e^-$ linear collider is Compton backscattering. The spectrum of Compton backscattered photons is given by \[17\]

\[
f_{\gamma/e}(y) = \frac{1}{g(\zeta)}[1 - y + \frac{1}{1 - y} - \frac{4y}{\zeta(1 - y)} + \frac{4y^2}{\zeta^2(1 - y)^2} + \lambda_0 \lambda_e r \zeta (1 - 2r)(2 - y)] \tag{8}
\]

where

\[
g(\zeta) = g_1(\zeta) + \lambda_0 \lambda_e g_2(\zeta)
\]

\[
g_1(\zeta) = (1 - \frac{4}{\zeta} - \frac{8}{\zeta^2}) \ln (\zeta + 1) + \frac{1}{2} + \frac{8}{\zeta} - \frac{1}{2(\zeta + 1)^2} \tag{9}
\]

\[
g_2(\zeta) = (1 + \frac{2}{\zeta}) \ln (\zeta + 1) - \frac{5}{2} + \frac{1}{\zeta + 1} - \frac{1}{2(\zeta + 1)^2} \tag{10}
\]

with \( r = y/[\zeta(1 - y)] \) and \( \zeta = 4E_e E_0/M_e^2 \). \( E_0 \) and \( \lambda_0 \) are the energy and helicity of initial laser photon and \( E_e \) and \( \lambda_e \) are the energy and the helicity of initial electron beam before Compton backscattering. \( y \) is the fraction which represents the ratio between the scattered photon and initial electron energy for the backscattered photons moving along the initial electron direction. Maximum value of \( y \) reaches 0.83 when \( \zeta = 4.8 \) in which the backscattered photon energy is maximized without spoiling the luminosity.

Backscattered photons are not in fixed helicity states their helicities are described by a distribution \[17\] :

\[
\xi(E_\gamma, \lambda_0) = \frac{\lambda_0 (1 - 2r)(1 - y + 1/(1 - y)) + \lambda_e r \zeta [1 + (1 - y)(1 - 2r)^2]}{1 - y + 1/(1 - y) - 4r(1 - r) - \lambda_e \lambda_0 r \zeta (2r - 1)(2 - y)} \tag{11}
\]

where \( E_\gamma \) is the energy of backscattered photons. The differential cross section for the subprocess is then written through the formula
\[ d\hat{\sigma}(\lambda_0^{(1)}, \lambda_0^{(2)}; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) = \frac{1}{4} (1 - \xi_1(E_\gamma^{(1)}, \lambda_0^{(1)}))(1 - \xi_2(E_\gamma^{(2)}, \lambda_0^{(2)}))d\hat{\sigma}(-, -; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) \]
\[ + \frac{1}{4} (1 - \xi_1(E_\gamma^{(1)}, \lambda_0^{(1)}))(1 + \xi_2(E_\gamma^{(2)}, \lambda_0^{(2)}))d\hat{\sigma}(-, +; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) \]
\[ + \frac{1}{4} (1 + \xi_1(E_\gamma^{(1)}, \lambda_0^{(1)}))(1 - \xi_2(E_\gamma^{(2)}, \lambda_0^{(2)}))d\hat{\sigma}(+,-; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) \]
\[ + \frac{1}{4} (1 + \xi_1(E_\gamma^{(1)}, \lambda_0^{(1)}))(1 + \xi_2(E_\gamma^{(2)}, \lambda_0^{(2)}))d\hat{\sigma}(+, +; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) \]  
(12)

Here \( d\hat{\sigma}(\lambda_0^{(1)}, \lambda_0^{(2)}; \lambda_{W^+}, \lambda_{W^-}, \lambda_Z) \) is the helicity dependent differential cross section in the helicity eigenstates; \( \lambda_0^{(i)} = +, - \) and \( \lambda_V = +, -, 0 \) \((V = W^+, W^- \text{ or } Z)\). Superscript (1) and (2) represent the incoming gamma beams and \( \xi_1(E_\gamma^{(1)}, \lambda_0^{(1)}) \) and \( \xi_2(E_\gamma^{(2)}, \lambda_0^{(2)}) \) represent the corresponding helicity distributions.

The integrated cross section for \( W^+W^-Z \) production via \( \gamma\gamma \) fusion can be obtained by the following integration:

\[ d\sigma(e^+e^- \rightarrow \gamma\gamma \rightarrow W^+W^-Z) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz \int_{y_{\text{min}}}^{y_{\text{max}}} dy f_{\gamma/e}(y) f_{\gamma/e}(z^2/y) d\hat{\sigma}(\gamma\gamma \rightarrow W^+W^-Z) \]  
(13)

where, \( d\hat{\sigma}(\gamma\gamma \rightarrow W^+W^-Z) \) is the cross section of the subprocess defined by (12) and center of mass energy of the \( e^+e^- \) system \( \sqrt{s} \), is related to the center of mass energy of the \( \gamma\gamma \) system \( \sqrt{\hat{s}} \) by \( \hat{s} = z^2 s \). We have calculated the cross sections imposing that the polar angles of the produced gauge bosons with the beam pipe are larger than 10°.

One can see from Fig.1-3 the influence of the final state polarizations on the deviations of the integrated total cross sections from their SM value for unpolarized initial beams. In these figures LO stands for ‘longitudinal’ and UNPOL stands for ‘unpolarized’. Transverse polarization configurations of the final bosons are almost insensitive to anomalous couplings. Therefore we omit them from the figures. It is clear from figures that longitudinally polarized cross sections are sensitive to the anomalous couplings. The most sensitive polarization configurations are \((\lambda_{W^+}, \lambda_{W^-}, \lambda_Z) = (\text{LO}, \text{LO}, \text{LO}) \) and \((\text{UNPOL, LO, LO})\). For instance in Fig.1 the cross sections at the polarization configurations \((\text{LO, LO, LO})\) and \((\text{UNPOL, LO, LO})\) increase by factors of 22 and 10 as \( a_0 \) increases from 0 to 0.02. But this increment is only a factor of 1.3 in the unpolarized case.
In Fig. 4-6 we see the influence of the initial state polarizations on the deviations of the integrated total cross sections from their SM value. We accept that initial electron beam polarizability is $|\lambda_e| = 0.9$. It can be shown from backscattered photon distribution (8) that photoproduction cross section for $\lambda_0\lambda_e > 0$ is very low at high energies [11]. Therefore we have only considered the case $\lambda_0\lambda_e < 0$ in the cross section calculations. Moreover interchanging backscattered photon polarizations ($\xi_1 \leftrightarrow \xi_2$) do not change the cross section due to the symmetry. During calculations we consider two different polarization combination; $(\lambda_0^{(1)}, \lambda_e^{(1)}, \lambda_0^{(2)}, \lambda_e^{(2)}) = (+1, -0.9, +1, -0.9)$ and $(+1, -0.9, -1, +0.9)$.

III. SENSITIVITY TO ANOMALOUS COUPLINGS

We estimate sensitivity of the $\gamma\gamma$ collider to anomalous couplings using simple one parameter $\chi^2$ criterion for the integrated luminosity of $500 fb^{-1}$ and $\sqrt{s} = 0.5, 1$ and $1.5$ TeV energies. In our calculations we ignore systematic errors and new physics energy scale $\Lambda$ is taken to be $M_W$.

We assume that W and Z polarizations can be measured. Indeed angular distributions of the W and Z decay products have clear correlations with the helicity states of them. For fixed W and Z helicities the polarized cross sections can be obtained from a fit to polar angle distributions of the W and Z decay products in the W and Z rest frames. Therefore we consider the case in which W and Z momenta are reconstructible. We restrict ourselves to leptonic decay channel of Z boson with a branching ratio $B(Z \to \ell\bar{\ell}) \approx 0.067$ ($\ell = e$ or $\mu$) and hadronic decay channel of W boson with a branching ratio $B(W \to q\bar{q}') \approx 0.676$. The number of events are given by $N = EB(Z \to \ell\bar{\ell})B^2(W \to q\bar{q}')L_{int}\sigma$ where $E$ is the efficiency and it is taken to be 0.85. Leptonic decay channel of W may not be appropriate since the momentum of the neutrino coming from the W decay can not be determined, and consequently, the W rest frame is unknown. For hadronic W decays, the quark charge is difficult to reconstruct experimentally and only the absolute value of the cosine of the decay angle can be measured. However, polar angle distribution can still be used to measure the transverse and longitudinal polarization states [18]. There have been several experimental studies in the literature for the measurement of W polarization. W boson polarization has been studied at CERN $e^+e^-$ collider LEP2 via the process $e^+e^- \rightarrow W^+W^- \rightarrow \ell vqq'$ [1, 18]. At Fermilab Tevatron, polarization of the W bosons produced in the top quark decay has
been measured by the CDF and D0 collaborations [19].

In Table I-III we show 95% C.L. sensitivity limits on the anomalous coupling parameters $a_0$, $a_c$ and $a_n$ for $\sqrt{s} = 0.5$, 1 and 1.5 TeV energies. In the tables, LO represents the longitudinal and UNPOL represents the unpolarized W and Z bosons. Transverse polarization states of the final gauge bosons are omitted since the limits for transverse polarization are weak. We see from these tables that limits on the anomalous couplings are usually very sensitive in $(\lambda_0^{(1)}, \lambda_c^{(1)}, \lambda_0^{(2)}, \lambda_c^{(2)}) = (+1, -0.9, +1, -0.9)$ initial and $(\lambda_W^+, \lambda_W^-, \lambda_Z) = (\text{LO, LO, LO, UNPOL, LO, LO})$ final polarization configurations. For instance, $(+1, -0.9, +1, -0.9)$ initial state polarization together with $(\text{LO, LO, LO})$ final state polarization (Table II) improve the lower bounds of the anomalous couplings $a_0$ and $a_c$ approximately a factor of 5 at $\sqrt{s} = 0.5$ TeV when compared with the unpolarized case (Table I). Same polarization configuration improve the upper bounds of $a_0$ and $a_c$ approximately factors of 4 and 3.3 at 1.5 TeV. Polarization improves also the sensitivity limits on $a_n$. $(+1, -0.9, +1, -0.9)$ initial state polarization together with $(\text{UNPOL, UNPOL, LO})$ final state polarization improves both the upper and lower bounds of $a_n$ approximately by a factor of 2 at 0.5 TeV. The most sensitive bounds on $a_n$ are obtained in $(\text{LO, LO, LO})$ at 1 and 1.5 TeV (Table II). This polarization configuration improves the limits on $a_n$ approximately by a factor of 3 at 1 TeV and 4 at 1.5 TeV.

IV. CONCLUSIONS

We have obtained a considerable improvement in the sensitivity bounds by taking into account incoming beam polarizations and also the final state polarizations of the gauge bosons. The subprocess $\gamma\gamma \rightarrow W^+W^-Z$ in the $\gamma\gamma$ mode of a linear collider with luminosity 500 $fb^{-1}$ probes the anomalous $WW\gamma\gamma$, $ZZ\gamma\gamma$ and $WWZ\gamma$ couplings with a far better sensitivity than the present collider LEP2 experiments. It improves the sensitivity limits by up to a factor of $10^5$ with respect to LEP2.

It is stated in Ref. [13] that bounds on the anomalous couplings $a_0$, $a_c$ and $a_n$ coming from $\gamma\gamma \rightarrow W^+W^-Z$ are better than the ones that can be obtained in an $e^+e^-$ collider. Our bounds on $a_0$ are a factor from 3.5 to 10 better than the bounds obtained in $e\gamma \rightarrow Z\gamma e$ and a factor from 1 to 5 better than the bounds obtained in $e\gamma \rightarrow W\gamma \nu$ depending on energy [10]. Our bounds on $a_c$ are approximately same with the bounds obtained in $e\gamma \rightarrow W\gamma \nu$
and little better than the bounds obtained in $e\gamma \to Z\gamma e$. On the other hand our bounds on $a_n$ are little worse than the ones coming from $e\gamma \to WZ\nu$ \[1\].

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FIG. 1: The integrated total cross section of $\gamma\gamma \rightarrow W^+W^-Z$ as a function of anomalous coupling $a_0$ for various final state polarizations stated on the figure. Initial beams are unpolarized and $\sqrt{s} = 1$ TeV.

FIG. 2: The same as Fig.1 but for anomalous coupling $a_c$. 
FIG. 3: The same as Fig. 2 but for anomalous coupling $a_n$.

FIG. 4: The integrated total cross section of $\gamma\gamma \rightarrow W^+W^-Z$ as a function of anomalous coupling $a_0$ for various initial state polarizations stated on the figure. Final state gauge bosons are unpolarized and $\sqrt{s} = 1$ TeV.
FIG. 5: The same as Fig. 4 but for anomalous coupling $a_c$.

FIG. 6: The same as Fig. 5 but for anomalous coupling $a_n$. 
TABLE I: Sensitivity of $\gamma\gamma \rightarrow W^+W^-Z$ to anomalous quartic $WW\gamma\gamma$ and $WWZ\gamma$ couplings at 95% C.L. for $\sqrt{s}=0.5, 1, 1.5$ TeV and $L_{int} = 500 fb^{-1}$. Initial beams are unpolarized. The effects of final state $W^+, W^-$ and $Z$ boson polarizations are shown in each row.

| $\sqrt{s}$ (TeV) | $\lambda_{W^+}$ | $\lambda_{W^-}$ | $\lambda_Z$ | $a_0$         | $a_c$         | $a_n$         |
|------------------|-----------------|-----------------|--------------|---------------|---------------|---------------|
| 0.5              | LO              | LO              | LO           | -0.130, 0.139 | -0.475, 0.488 | -1.261, 1.261 |
| 0.5              | UNPOL           | LO              | LO           | -0.113, 0.075 | -0.445, 0.250 | -1.089, 0.758 |
| 0.5              | LO              | LO              | UNPOL        | -0.210, 0.139 | -0.789, 0.459 | -1.441, 1.441 |
| 0.5              | UNPOL           | LO              | UNPOL        | -0.174, 0.089 | -0.713, 0.269 | -1.294, 0.859 |
| 0.5              | UNPOL           | UNPOL           | LO           | -0.153, 0.077 | -0.628, 0.237 | -0.878, 0.874 |
| 0.5              | UNPOL           | UNPOL           | UNPOL        | -0.291, 0.066 | -1.250, 0.181 | -1.076, 1.076 |
| 1                | LO              | LO              | LO           | -0.0044, 0.0043 | -0.0181, 0.0143 | -0.0626, 0.0626 |
| 1                | UNPOL           | LO              | LO           | -0.0045, 0.0034 | -0.0194, 0.0110 | -0.0675, 0.0616 |
| 1                | LO              | LO              | UNPOL        | -0.0059, 0.0041 | -0.0258, 0.0130 | -0.0766, 0.0766 |
| 1                | UNPOL           | LO              | UNPOL        | -0.0070, 0.0047 | -0.0303, 0.0148 | -0.0965, 0.0763 |
| 1                | UNPOL           | UNPOL           | LO           | -0.0071, 0.0049 | -0.0299, 0.0159 | -0.0857, 0.0860 |
| 1                | UNPOL           | UNPOL           | UNPOL        | -0.0118, 0.0060 | -0.0529, 0.0180 | -0.1127, 0.1127 |
| 1.5              | LO              | LO              | LO           | -0.0008, 0.0008 | -0.0035, 0.0027 | -0.0114, 0.0114 |
| 1.5              | UNPOL           | LO              | LO           | -0.0009, 0.0008 | -0.0040, 0.0024 | -0.0143, 0.0145 |
| 1.5              | LO              | LO              | UNPOL        | -0.0010, 0.0008 | -0.0046, 0.0026 | -0.0149, 0.0149 |
| 1.5              | UNPOL           | LO              | UNPOL        | -0.0014, 0.0011 | -0.0059, 0.0036 | -0.0216, 0.0199 |
| 1.5              | UNPOL           | UNPOL           | LO           | -0.0015, 0.0012 | -0.0061, 0.0041 | -0.0223, 0.0226 |
| 1.5              | UNPOL           | UNPOL           | UNPOL        | -0.0023, 0.0016 | -0.0096, 0.0053 | -0.0309, 0.0309 |
TABLE II: Sensitivity of $\gamma\gamma \rightarrow W^+W^-Z$ to anomalous quartic $WW\gamma\gamma$ and $WWZ\gamma$ couplings at 95% C.L. for $\sqrt{s}=0.5, 1, 1.5$ TeV and $L_{int} = 500 fb^{-1}$. Initial beam polarization is $(\lambda_0^{(1)}, \lambda_0^{(1)}, \lambda_0^{(2)}) = (+1, -0.9, +1, -0.9)$. The effects of final state $W^+, W^-$ and $Z$ boson polarizations are shown in each row.

| $\sqrt{s}$ (TeV) | $\lambda_{W^+}$ | $\lambda_{W^-}$ | $\lambda_Z$ | $a_0$     | $a_c$     | $a_n$     |
|------------------|-----------------|-----------------|------------|-----------|-----------|-----------|
| 0.5 LO           | LO              | LO              | -0.059, 0.064 | -0.234, 0.252 | -0.611, 0.611 |
| 0.5 UNPOL        | LO              | LO              | -0.068, 0.036 | -0.273, 0.140 | -0.666, 0.405 |
| 0.5 LO           | LO              | UNPOL           | -0.127, 0.066 | -0.506, 0.256 | -0.950, 0.950 |
| 0.5 UNPOL        | LO              | UNPOL           | -0.112, 0.044 | -0.459, 0.168 | -0.914, 0.506 |
| 0.5 UNPOL        | UNPOL           | LO              | -0.100, 0.039 | -0.408, 0.148 | -0.549, 0.546 |
| 0.5 UNPOL        | UNPOL           | UNPOL           | -0.217, 0.032 | -0.898, 0.119 | -0.737, 0.737 |
| 1 LO             | LO              | LO              | -0.0023, 0.0022 | -0.0095, 0.0082 | -0.0377, 0.0377 |
| 1 UNPOL          | LO              | LO              | -0.0026, 0.0018 | -0.0109, 0.0068 | -0.0416, 0.0364 |
| 1 LO             | LO              | UNPOL           | -0.0033, 0.0021 | -0.0139, 0.0076 | -0.0502, 0.0502 |
| 1 UNPOL          | LO              | UNPOL           | -0.0040, 0.0026 | -0.0170, 0.0097 | -0.0641, 0.0492 |
| 1 UNPOL          | UNPOL           | LO              | -0.0041, 0.0028 | -0.0174, 0.0104 | -0.0549, 0.0549 |
| 1 UNPOL          | UNPOL           | UNPOL           | -0.0069, 0.0035 | -0.0301, 0.0126 | -0.0765, 0.0765 |
| 1.5 LO           | LO              | LO              | -0.0005, 0.0004 | -0.0019, 0.0016 | -0.0073, 0.0073 |
| 1.5 UNPOL        | LO              | LO              | -0.0005, 0.0004 | -0.0022, 0.0016 | -0.0089, 0.0088 |
| 1.5 LO           | LO              | UNPOL           | -0.0006, 0.0004 | -0.0024, 0.0016 | -0.0099, 0.0099 |
| 1.5 UNPOL        | LO              | UNPOL           | -0.0008, 0.0006 | -0.0033, 0.0023 | -0.0141, 0.0128 |
| 1.5 UNPOL        | UNPOL           | LO              | -0.0009, 0.0007 | -0.0036, 0.0026 | -0.0140, 0.0143 |
| 1.5 UNPOL        | UNPOL           | UNPOL           | -0.0013, 0.0010 | -0.0055, 0.0035 | -0.0202, 0.0202 |
| $\sqrt{s}$ (TeV) | $\lambda_{W^+}$ | $\lambda_{W^-}$ | $\lambda_Z$ | $a_0$       | $a_c$       | $a_m$       |
|-----------------|-----------------|-----------------|-------------|-------------|-------------|-------------|
| 0.5             | LO              | LO              | LO          | -0.285, 0.302 | -0.597, 0.545 | -2.146, 2.146 |
| 0.5             | UNPOL           | LO              | LO          | -0.212, 0.162 | -0.571, 0.256 | -1.684, 1.173 |
| 0.5             | LO              | LO              | UNPOL       | -0.394, 0.308 | -0.897, 0.440 | -1.335, 1.335 |
| 0.5             | UNPOL           | LO              | UNPOL       | -0.307, 0.190 | -0.878, 0.232 | -1.303, 0.986 |
| 0.5             | UNPOL           | UNPOL           | LO          | -0.272, 0.166 | -0.782, 0.214 | -1.165, 1.160 |
| 0.5             | UNPOL           | UNPOL           | UNPOL       | -0.442, 0.146 | -1.462, 0.144 | -1.127, 1.127 |
| 1               | LO              | LO              | LO          | -0.0067, 0.0065 | -0.0239, 0.0135 | -0.0564, 0.0564 |
| 1               | UNPOL           | LO              | LO          | -0.0068, 0.0051 | -0.0255, 0.0101 | -0.0690, 0.0706 |
| 1               | LO              | LO              | UNPOL       | -0.0089, 0.0061 | -0.0346, 0.0113 | -0.0619, 0.0619 |
| 1               | UNPOL           | LO              | UNPOL       | -0.0105, 0.0069 | -0.0388, 0.0134 | -0.0885, 0.0755 |
| 1               | UNPOL           | UNPOL           | LO          | -0.0107, 0.0073 | -0.0377, 0.0150 | -0.0922, 0.0924 |
| 1               | UNPOL           | UNPOL           | UNPOL       | -0.0181, 0.0087 | -0.0671, 0.0161 | -0.1103, 0.1103 |
| 1.5             | LO              | LO              | LO          | -0.0012, 0.0011 | -0.0044, 0.0025 | -0.0092, 0.0092 |
| 1.5             | UNPOL           | LO              | LO          | -0.0013, 0.0011 | -0.0049, 0.0023 | -0.0134, 0.0146 |
| 1.5             | LO              | LO              | UNPOL       | -0.0015, 0.0011 | -0.0060, 0.0022 | -0.0115, 0.0115 |
| 1.5             | UNPOL           | LO              | UNPOL       | -0.0020, 0.0015 | -0.0072, 0.0033 | -0.0196, 0.0185 |
| 1.5             | UNPOL           | UNPOL           | LO          | -0.0021, 0.0017 | -0.0074, 0.0040 | -0.0227, 0.0231 |
| 1.5             | UNPOL           | UNPOL           | UNPOL       | -0.0033, 0.0023 | -0.0117, 0.0051 | -0.0297, 0.0297 |