An Improved Negatively Correlated Search Inspired by Particle Swarm Optimization

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Abstract. Negatively correlated search (NCS) is a latest intelligence algorithm which maintains multiple individual search processes in parallel and models the behavior of individual search processes using probability distributions. In this paper, an improved NCS inspired by particle swarm optimization (PSO) is proposed, namely PSO-NCS. This algorithm introduces the global best solution (gbest) and personal best solution (pbest) from PSO to calculate the Bhattacharyya Distance (BD) of individuals in the population, which greatly reduces the computational time. Experimental results on six 100 dimensional functions and 10 CEC2005 benchmark functions show that the proposed PSO-NCS has outstanding capability in solving the optimization problems and outperforms the conventional NCS and other methods on most of the test functions. The computational time has been greatly shortened, which saves about 65% in average compared to NCS.

1. Introduction
With the advent of the big data era, the computational models used in real world engineering fields are becoming more and more complicated. How to choose the optimal parameters to obtain higher accurate results in the field of complex system modeling can be treated as optimization problems which are often challenging due to such features as discontinuity, non-differentiability, multiple local minimums, and flat areas, etc. Conventional nonlinear optimization methods, such as the gradient descent method [1] and lagrangian relaxation [2] suffer from sever limitations in solving such challenging problems. For example, the conventional methods have difficulties in solving non-linear discontinuous objective functions or objective functions with no mathematical expressions. Further, the performance of conventional methods depend heavily on the selection of initial values. If the initial values are not well selected, they are easily trapped in the local extremums, and there is no generic solutions to guide the selection of initial values. In the last decades, a number of intelligent optimization algorithms such as the Genetic Algorithm (GA) [3], Particle Swarm Optimization (PSO) [4], the Differential Evolution (DE) [5] and Artificial Bee Colony algorithm (ABC) [6] have been proposed to solve such challenging optimization problems. These intelligent algorithms have the features of simple principle, strong robustness and no special requirements for the objective function and constraints, and they have become indispensable tools in solving the complex optimization problems.

The Negatively Correlated Search (NCS) [7] was a recent intelligent optimization tool that was originally proposed by Tang etc., and it has attracted a lot of attentions since 2016. The NCS
is characterized by its information sharing and cooperation scheme, which explicitly promotes negative correlation search behavior for more effective exploration in the search space. Empirical results shows that NCS has a stronger global searching capability and the higher robustness in comparison to other existing algorithms on complex benchmark problems. NCS has been applied in deep learning to decide the sizes of the deep belief network (DBN) and learning rates during the training processes, and it exhibits excellent performance [8].

Although NCS has a number of merits, it also has some shortcomings. NCS employs the Bhattacharyya distance (BD) to calculate the difference and the correlation, where each individual calculates the BD with all other individuals. Therefore, the algorithm spends a lot of time in calculation, and inefficiency therefore becomes a shortfall of the algorithm. In order to improve the efficiency of NCS, this paper proposes an improved NCS method based on the idea of PSO, namely PSO-NCS. Each individual does not need to calculate the Bhattacharyya distances with all other individuals and only needs to calculate the distance between the mean of global best solution (gbest) and personal best solution (pbest). For example, there are ten individuals in the population, the NCS needs to calculate the distance by 10*10 times, but the PSO-NCS only needs to calculate 10*1 times, which greatly reduces the computation time. The improved PSO-NCS not only significantly shortens the computation time, thus improving its efficiency, but also improves the performance. In order to verify the effectiveness, PSO-NCS is applied to test six 100 dimensional benchmark functions and 10 CEC2005 functions in comparison with other algorithms.

The reminder of this paper is organized as follows: In section 2, it introduces the conventional NCS and details the PSO-NCS algorithm. Section 3 presents the simulation results. Finally, Section 4 concludes the paper.

2. PSO-NCS

2.1. NCS (Negatively Correlated Search)

In 2016, Tang etc. proposed the NCS method motivated by the interpretation of human behavior of cooperation and avoiding duplication of same work. It adopts the Bhattacharyya Distance (BD) [9] to estimate the difference and the correlation between two continuous probability distributions. BD is defined in the Eq. (1).

\[ D_B(p_i, p_j) = -\ln\left(\sqrt{p_i(x)p_j(x)}dx\right) \] (1)

where \( p_i \) and \( p_j \) are the probability density functions of two distributions.

In addition, the distance between each individual and the associated group \( Corr(p_i) \) can be defined as (2):

\[ Corr(p_i) = \min_j \{D_B(p_i, p_j)|j \neq i\} \] (2)

A new solution \( x_i' \) is generated by Guassian mutation on a current solution as (3):

\[ x_{id}' = x_{id} + N(0, \sigma_i) \] (3)

where the \( N(0, \sigma_i) \) stands for a Guassian random variable of zero mean and standard deviation \( \sigma_i \) and \( x_{id}' \) is the dth element of \( x_i \).

The step size \( \sigma_i \) follows the 1/5 successful rule [10] for every iteration as (4):

\[ \sigma_i = \begin{cases} \sigma & \text{if } \frac{c}{\text{epoch}} > 0.2 \\ \sigma_i \ast r & \text{if } \frac{c}{\text{epoch}} < 0.2 \\ \sigma_i & \text{if } \frac{c}{\text{epoch}} = 0.2 \end{cases} \] (4)
where \( r \) is a parameter less than 1, and \( c \) is the times that \( x_i' \) is discarded in the past iterations. This indicates that the larger \( c \), the smaller search step-size, and the solution is closer to the global optimum. On the contrary, a large step size will more likely make the search being trapped in a local optimum.

In NCS, a smaller objective function value \( f(x_i') \) with a larger \( \text{Corr}(p_i') \) is preferred. To obtain a better population, strategy considering the \( f(x_i') \) and \( \text{Corr}(p_i') \) is used in NCS, which is defined as Eq. (5):

\[
x_{\text{new},i} = \begin{cases} x_i, & \text{if } f(x_i') \text{ Corr}(p_i') < \lambda \\ x_i', & \text{otherwise} \end{cases}
\]

where \( \lambda \) is a time-varying parameter which can influence the search process and the capability of NCS as Eq. (6):

\[
\lambda_{\text{iter}} = N(1, 0.1 - 0.1 \ast \frac{t}{T_{\text{max}}})
\]

where the \( T_{\text{max}} \) is the total number of iterations in executing the NCS, and \( t \) is the current iteration.

2.2. PSO-NCS

In the conventional NCS, each individual in the population needs to calculate its BD with all other individuals in the population, which is computationally intensive. In order to enhance the efficiency of NCS, this paper proposes an improved NCS, namely PSO-NCS. PSO is initialized with a group of random particles which are then updated using several operators to produce new generation. At every iteration, each particle is updated using two “best” values. The first one is the personal best solution (fitness) it has achieved so far. The fitness value is also stored. This value is called “pbest”. Another “best” value that is tracked by the particle swarm optimizer is the global best value, obtained so far by any particle in the population. This best value is a global best and called “gbest”. In PSO-NCS, each individual calculates the sum of distances between itself to the pbest and to the gbest. Therefore, the BD in PSO-NCS can be defined as (7) [11]:

\[
D_B(p_i, p_j) = \frac{1}{8}(x_i - x_q)^T(x_i - x_q) + \frac{1}{2} \ln \left( \frac{\det \sum}{\det \sum_i \sum_q} \right)
\]

where the \( x_q \) is the mean of gbest and pbest in the iteration, \( \sum = \sum_i + \sum_q \), \( \sum_i = \sigma_i^2 I \), and \( I \) is an n-dimensional identity matrix.

In Eq. (6), it can be found that for each individual, it calculates only two distances rather than all distances in NCS. The calculation time is therefore greatly reduced. Meanwhile, the introduction of the pbest and gbest can improve the solution quality.

3. Simulations And Results

In this section, two case studies are presented to validate the effectiveness and efficiency of PSO-NCS. In case 1, the proposed PSO-NCS is tested on six typical shifted benchmark functions with 100 dimensions which listed in Table 1 to verify its performance compared to conventional NCS and other well-known algorithms such as the PSO-CF [12], HS [13], and weight-PSO [14]. In case 2, the PSO-NCS is tested on the CEC2005 test functions in comparison to the conventional NCS. All the cases are run for 25 independent trials under the same number of function evaluations(FES) (case 1 is set to 1000000 and case 2 is set to 300000) for a fair comparison. The control parameters used in NCS follow the references in the literature while the parameters of the PSO-NCS are the same as the NCS. All the experiments are carried out using the MATLAB 2016a on a PC of 3.70GHz with 32GB RAM.
Table 1. Formulations of Benchmark functions

| Function               | Formulation                                                                 | Boundary    | $f_{\text{min}}$ |
|------------------------|-----------------------------------------------------------------------------|-------------|-----------------|
| Shifted Sphere         | $f(x) = \sum_{i=1}^{\text{Dim}} (x_i - o_i)^2$                             | [-100,100]  | 0               |
| Shifted Rosenbrock     | $f(x) = \sum_{i=1}^{\text{Dim}-1} (100((x_{i+1} - o_{i+1}) - x_i^2) + (x_{i+1} - o_{i+1} - 1)^2)$ | [-30,30]    | 0               |
| Shifted Rastrigin      | $f(x) = \sum_{i=1}^{\text{Dim}-1} ((x_i - o_i)^2 - 10 \cos(2\pi(x_i - o_i)) + 10)$ | [-5.12,5.12]| 0               |
| Shifted Griewank       | $f(x) = \sum_{i=1}^{\text{Dim}} (x_i - o_i)^2 - \prod_{i=1}^{\text{Dim}} \cos(\frac{x_i - o_i}{\sqrt{i}}) + 10$ | [-600,600]  | 0               |
| Shifted Axis           | $f(x) = \sum_{i=1}^{\text{Dim}} i(x_i - o_i)^2$                            | [-5.12,5.12]| 0               |
| Shifted De Jong        | $f(x) = \sum_{i=1}^{\text{Dim}} i(x_i - o_i)^4$                            | [-1.28,1.28]| 0               |

Table 2. Test results in case1

| Function               | PSO-CF   | W-PSO    | HS       | NCS      | PSO-NCS  |
|------------------------|----------|----------|----------|----------|----------|
| Shifted Sphere         | mean 7.42E02 | 9.08E01  | 4.87E03  | 4.76E03  | 3.11E-27 |
|                        | std 3.92E02 | 5.66E01  | 5.35E02  | 8.21E02  | 7.79E-28 |
| Shifted Rosenbrock     | mean 7.75E04 | 1.44E04  | 1.56E06  | 9.90E01  | 8.89E01  |
|                        | std 6.86E04 | 2.14E04  | 2.94E05  | 1.51E01  | 3.50E00  |
| Shifted Rastrigin      | mean 7.14E02 | 4.27E02  | 1.46E02  | 7.42E02  | 6.30E02  |
|                        | std 6.53E01 | 7.20E01  | 1.30E01  | 5.95E02  | 6.03E01  |
| Shifted Griewank       | mean 1.55E01 | 3.91E00  | 4.60E01  | 4.24E01  | 3.15E-03 |
|                        | std 5.78E00 | 1.87E00  | 5.78E00  | 7.51E00  | 5.53E-03 |
| Shifted Axis           | mean 4.42E00 | 9.38E-01 | 2.51E00  | 3.58E00  | 3.89E00  |
|                        | std 8.84E-01 | 2.79E-01 | 1.36E-01 | 4.75E-01 | 5.62E-01 |
| Shifted De Jong        | mean 9.60E00 | 2.47E00  | 4.47E00  | 1.10E00  | 5.63E-01 |
|                        | std 1.56E00 | 5.88E-01 | 4.22E-01 | 5.30E-01 | 3.78E-01 |

3.1. Case1: PSO-NCS in benchmark function test

Table 2 lists the means and the standard deviations of the 25 runs of the five different algorithms on the six benchmark functions with 100 dimensions. Two different types of benchmark functions (a) shifted sphere (unimodal) (b) shifted griewank (multimodal) are selected to demonstrate the convergence characteristics of five algorithms as illustrated in Figure. 1. It can be seen that the PSO-NCS is easier to jump out of local optimum and to avoid premature convergence than other algorithms regardless of the types of the benchmark functions.
3.2. Case 2: PSO-NCS in CEC2005 test

Multi-modal optimization problems are more attractive since they have motivated the invention of a number of population-based search methods. 10 multi-modal continuous functions proposed in the CEC2005 competition in [15] are used in the study. Table 3 shows that the means, the standard deviations and the executing time of the 25 runs of the PSO-NCS and the NCS in the Ref.[7] on the 10 CEC2005 functions with 30 dimensions. It can be found in the Table 3 that PSO-NCS outperforms the conventional NCS on most of the test functions. It should be pointed out that the computational time of PSO-NCS has been greatly decreased on the all test functions, achieving about 65% reduction in average.

| Function | NCS | Mean  | Std  | Time | PSO-NCS | Mean  | Std  | Time |
|----------|-----|-------|------|------|---------|-------|------|------|
| f6       | NCS | 2.08E01 | 3.61E00 | 1.35E00 | 1.98E01 | 2.23E00 | 2.05E00 | 28.68 |
|         | mean | 2.08E01 | 3.61E00 | 1.35E00 | 1.98E01 | 2.23E00 | 2.05E00 | 28.68 |
|         | std  | 1.35E00 | 2.05E00 | 3.61E00 | 1.98E01 | 2.23E00 | 2.05E00 | 28.68 |
|         | time | 28.68  | 10.32 | 10.6  | 11.68  | 130.52 | 130.52 | 130.52 |

Figure 1. Convergence curves for five algorithms.

Table 3. Test results in case 2

| Function | NCS | Mean  | Std  | Time | PSO-NCS | Mean  | Std  | Time |
|----------|-----|-------|------|------|---------|-------|------|------|
| f7       | NCS | 1.69E-02 | 1.38E00 | 1.22E-02 | 1.35E00 | 1.52E00 | 8.43E-01 | 11.68 |
|         | mean | 1.69E-02 | 1.38E00 | 1.22E-02 | 1.35E00 | 1.52E00 | 8.43E-01 | 11.68 |
|         | std  | 1.22E-02 | 1.38E00 | 1.69E-02 | 1.35E00 | 1.52E00 | 8.43E-01 | 11.68 |
|         | time | 11.68  | 10.32 | 10.6  | 11.68  | 130.52 | 130.52 | 130.52 |
4. Conclusion and future work
This paper proposes an improved NCS algorithm, namely PSO-NCS, which introduces the pbest and the gbest from the PSO to calculate the BD for each individuals in the population. The simulation results on six 100-dimensional benchmark functions and ten CEC2005 functions demonstrate that PSO-NCS has a stronger global search capability and a higher computational effectiveness than NCS.

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