Cosmological Constant and Fermi-Bose Degeneracy

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Abstract

We study some cases in $D = 4$ and in infinite-volume high dimensional theories when unbroken supersymmetry in the vacuum cannot guarantee Fermi-Bose degeneracy among excited states. In 4D we consider an example in which both supersymmetry and $R$ symmetry are unbroken in the vacuum, and the cosmological constant vanishes. However, theory admits solitons that do not allow existence of conserved supercharges. These objects are magnetically charged global monopoles which in some respect behave as point-like particles, but create a solid angle deficit which eliminates asymptotically covariantly constant spinors and lifts Fermi-Bose degeneracy of the spectrum. The idea is that in some ”dual” description monopoles with global topological number and gauge magnetic charge may be replaced by electrically charged particles with global Noether charges, e.g., such as baryon number. Alternatively theories with infinite volume extra dimensions may support unbroken bulk supersymmetry without Fermi-Bose degeneracy in the brane spectrum. We suggest a scenario in which brane is a source similar to a global monopole embedded in 3 infinite extra dimensions. Brane produces a deficit angle at infinity and may localize a meta-stable 4D graviton. Although bulk cosmological constant is zero, conserved supercharges can not be defined on such a background.

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A. Introduction.

Broken supersymmetry is hard to reconcile with vanishing of the cosmological constant. In the presence of gravity even unbroken supersymmetry alone does not suffice to guarantee the zero vacuum energy, but under reasonable assumptions, e.g. such as $R$ symmetry, this can be done. In this view perhaps the way for understanding the vanishing of the vacuum energy, is to look for the theories in which unbroken supersymmetry in vacuum cannot guarantee Fermi-Bose degeneracy among excited states. One may think of two possible strategies in this direction. One is to search for non-degenerate states directly in four-dimensional theories, generalizing earlier observation by Witten in $2 + 1$ dimensions [1].

Alternatively [2] [3], Fermi-Bose non-degeneracy may be compatible with unbroken bulk supersymmetry, if matter is localized on a brane embedded in infinite-volume extra dimensions. In this way cosmological constant may be controlled by bulk supersymmetry, while supersymmetry may be completely broken on the brane. Example of infinite-volume extra dimensions was recently suggested in [4]. In this framework four dimensional Newtonian gravity is reproduced by a meta-stable graviton localized on the brane [4,5,2].

In this letter we consider both possibilities. First we shall study some candidate solitonic states that make impossible the definition of unbroken supercharges in four-dimensions, although supersymmetry and $R$-symmetry are unbroken in vacuum. These objects are global monopoles that are known to create a solid angle deficit at infinity. Due to this solid angle deficit, the covariantly constant spinors do not exist on such a background and there are no conserved supercharges. Thus there is no reason for Fermi-Bose degeneracy in the monopole spectrum. In many respect the dynamics of the global monopoles is reminiscent of the one of the point-like particles with a finite mass localized in the tiny core. We show that this particles can carry gauge quantum numbers and in particular both Abelian or non-Abelian magnetic charges. In particular, with an appropriate choice of a “magnetic” gauge group $H_{\text{magnetic}}$, the magnetic charge assignment of the global monopoles can be made identical to the electric Yang-Mills charges of the standard model particles under $H_{\text{electric}} = SU(3) \otimes SU(2) \otimes U(1)$, much in the same way as for the gauge monopoles in $SU(5)$, as observed by Vachaspati [6]. However, we shall not restrict $H_{\text{magnetic}}$ by such a choice. In this way we end up with the objects that in certain sense are “dual” to ordinary particles. Instead of global Noether charge (baryon or lepton charge), they carry global topological charge (winding number), instead of electric charges (color and ordinary electric charge), they carry $H_{\text{magnetic}}$-magnetic charge, etc. An interesting fact about these states is that despite the unbroken supersymmetry in the vacuum, they are not Fermi-Bose degenerate. So the theory describes a toy Universe with zero cosmological term, but non-equal masses of fermions and bosons. The hope for more realistic model-building is that in some dual description solitons may be replaced by particles (topological charges with baryon or lepton numbers, etc..) which are not Fermi-Bose degenerated, but cosmological constant is still zero due to supersymmetry.

Secondly we consider infinite volume theories and suggest some new ways of quasi-localization of gravity without invoking negative tension branes. One possibility is that our brane originates from spontaneous breaking of global $O(3)$ symmetry in theories with three extra dimensions and has the structure of global monopole in transverse space.

Such an object produces a deficit angle at infinity and in a certain limit may support a
meta-stable graviton state. Since the brane appears as a solution of an underlying sigma model on a background with zero bulk cosmological constant, it may avoid problems with violation of the weak energy positivity conditions pointed in [3] (see also [2]).

B. Supersymmetry in the Global Monopole Background in Four Dimensions.

In the inspiring paper [1] Witten made an observation that unbroken supersymmetry (and thus zero cosmological constant) is not incompatible with Fermi Bose non-degeneracy in 2 + 1 dimensions.

The idea is roughly as follows. In supergravity theories the supercurrent in general is not conserved in the usual sense, but rather is covariantly conserved

$$D_\mu J^\mu = 0$$ (1)

However, in the presence of a covariantly constant spinor

$$D_\mu \epsilon = 0$$ (2)

the conserved current can be constructed

$$\bar{\epsilon} J^\mu$$ (3)

and thus one can define a globally conserved supercharge

$$Q = \int dx^3 \bar{\epsilon} J^0$$ (4)

Now in three dimensions any localized mass is known to produce a conical geometry at infinity [8]. Such a geometry makes in general impossible the existence of covariantly constant spinors and thus of unbroken supercharges [1, 1]. Thus although supersymmetry is unbroken in the vacuum and vacuum energy vanishes, there is no Fermi-Bose degeneracy among the excited states. Explicit examples along Witten’s idea were considered in some papers [7], [10]. Needless to say it would be very important to find some sort of generalization of this effect to four-dimensions.

In this letter we will consider some possible candidates that may generalize the three-dimensional behavior of point masses to four dimensions. We shall look for $N = 1 \ D = 4$ supergravity theories, in which both supersymmetry and $R$ symmetry are unbroken in the vacuum, but yet there is no Fermi-Bose degeneracy among certain excited solitonic states.

The model consists of four chiral superfields. Three of them, $\Phi_a$, $a = 1, 2, 3$ compose a triplet under an internal symmetry group $O(3)$, while the fourth superfield $X$ is a singlet. The superpotential is given by

$$W = X(\Phi_a^2 - \nu^2)$$ (5)

1In the presence of gauge fields, Killing spinors may still exist, since the deficit angle can be compensated by an Aharonov-Bohm phase. [2]
where \( v \) is a real mass parameter and for simplicity the coupling constant is set to one. This theory has a global \( O(3) \) invariance, plus an \( U(1) \) R-symmetry, under which \( X \) transforms in the same way as \( W \), whereas \( \Phi_a \)-s are invariant.

The vacuum of the theory is given by (below we shall denote chiral superfields and their scalar components by the same symbols)

\[
X = 0, \quad \Phi_a = v^2
\]

Thus \( O(3) \) is broken spontaneously to \( O(2) \), whereas both supersymmetry and \( R \) symmetry are unbroken. This ensures that vacuum energy is zero to all orders in perturbation theory.

Due to a nontrivial topological structure, however, this theory admits topological knots. These knots are global monopoles, which in spherical coordinates can be given by the solution

\[
X = 0, \quad \Phi_a = f(r) r_a
\]

where \( r_a \) are the components of the radius-vector \( r \) and \( f(r) \) is a smooth function such that

\[
f(0) = 0, \quad f(r)|_{r \to \infty} \to v
\]

As shown by Barriola and Vilenkin [11], with this ansatz, the metric takes the following form

\[
ds^2 = -a^2 dt^2 + b^2 dr^2 + r^2 d\Omega^2
\]

where asymptotically

\[
a^2 = b^{-2} = \left( 1 - \frac{v^2}{M_P^2} - \frac{2M_{\text{core}}}{M_P^2 r} \right)
\]

where \( M_P \) is the reduced Planck mass and \( M_{\text{core}} \sim v \) is an integration constant, which is a negative quantity [12]. Due to this fact the Newtonian potential of the monopole core is repulsive. This metric describes a space with a solid angle deficit \( 4\pi v^2/M_P^2 \). The \( \theta = \pi/2 \) surface is a cone with a deficit angle \( \delta = 2\pi v^2/M_P^2 \).

Of our interests are the supersymmetric transformations in the monopole background. Choosing the vierbein as

\[
e^\mu_a = \text{diag}(\frac{1}{a}, a, \frac{1}{r \sin \theta}), \quad a^2 = \left( 1 - \frac{v^2}{M_P^2} \right)
\]

(where \( \mu = t, r, \theta, \phi \) the only non-vanishing components of the spin connection are \( \Gamma_\theta = -\frac{a}{2} \gamma_1 \gamma_2 \) and \( \Gamma_\phi = -\frac{a}{2} \sin \theta \gamma_1 \gamma_3 - \frac{1}{2} \cos \theta \gamma_2 \gamma_3 \). On such a background, the conditions for Killing spinors

\[
\delta \psi_\theta = (\partial_\theta + \frac{a}{2} \gamma_1 \gamma_2) \epsilon = 0
\]

\[
\delta \psi_\phi = (\partial_\phi + \frac{a}{2} \sin \theta \gamma_1 \gamma_3 + \frac{1}{2} \cos \theta \gamma_2 \gamma_3) \epsilon = 0
\]

can not be satisfied and no conserved supercharges can be defined to control Fermi-Bose degeneracy in the monopole spectrum.
Alternatively, this can be seen in the language of zero modes. In the global supersymmetry limit the zero mode in the monopole background is

\[
\delta \psi_{X+} = \sqrt{2}(f^2(r) - v^2)\epsilon_+ \\
\delta \psi_{a+} = -i\sqrt{2}\partial_\mu \Phi^a \gamma^\mu \epsilon_-, \\
\]

where \(\epsilon_{\pm}\) is an eigenspinor of \(\gamma_5\). Note that \(\delta \psi_{X+}\) and \(\frac{r_a}{|r_a|} \delta \psi_{a+}\) are normalizable, whereas \(\delta \psi_{\theta}\) and \(\delta \psi_{\phi}\) are not. The reason behind this is the following. We can say that global supersymmetry is spontaneously broken in the monopole background. The order parameters for this breaking are the auxiliary (\(F\)) component of the chiral superfield \(X\)

\[
\langle F_X \rangle = (f(r)^2 - v^2) \\
\]

and also an auxiliary (\(D\)) component of a composite real superfield \(\Phi_a^* \Phi_a\)

\[
\langle (\Phi_a^* \Phi_a)_D \rangle = |\partial_j \Phi_a|^2 \]

Since both \(F_X\) as well as radial derivatives vanish away from the core at least as \(1/r^2\), the corresponding fermion variations \(\delta \psi_X\) and \(\frac{r_a}{|r_a|} \delta \psi_a\) are normalizable. Away from the core breaking is dominated by angular gradients, which only drop-off as inverse square of distance and goldstino is not normalizable. In fact this is also to be expected from the fact that monopole configuration spontaneously breaks both internal symmetry as well as coordinate rotations and leaves unbroken an \(O(3)\) symmetry of combined space and internal rotations, which leaves invariant a product \(r_a \Phi_a\). Non-normalizable fermionic zero modes are simply the supersymmetric partners of the Nambu-Goldstone fields corresponding to this breaking. Monopole configuration interpolates between the core where internal \(O(3)\) is restored, but coordinate symmetry and supersymmetry are broken maximally, to the outer region where the strength of supersymmetry and coordinate symmetry breakings weakens but internal symmetry is maximally broken. Since both coordinate and supersymmetries are broken by angular gradients which vanish away from the core, all energy densities must vanish as \(\sim 1/r^2\). Although locally energy density can be arbitrarily small, in the background of the global monopole the globally conserved supercharge linearly diverges together with the total energy of the configuration

\[
Q(\epsilon) \sim 4\pi \bar{\epsilon} \gamma_0 \epsilon v^2 R \]

where \(\epsilon\) is an arbitrary constant spinor and \(R\) is the distance from the core of the monopole. In the case of supergravity, the zero mode acquires an additional non-normalizable gravitino component \(\delta \psi_{\mu}\) which renders it unphysical.

C. Monopoles as Particles.

The global monopoles due to the divergent energy, are different from the ordinary localized sources. However, in many respect they can behave like objects with a finite mass localized in the core. We can treat the cores of the global monopoles as point-like objects of finite mass moving in the “clouds” of the Goldstone gradient energy produced by their
Goldstone field. In this way divergence in the total energy of the configuration is not important for the core dynamics, in a same way as the total infinite energy of the matter in the Universe is not important for the particle interactions at short distances.

Of course, the precise numbers do not work as well as in the case of the ordinary matter. For instance, roughly \( n \sim 10^{19} \) global monopoles, with the core mass equal to the proton mass, presented within the observable part of the Universe would be enough to produce an energy density equal to the critical \([\text{E}]\). However, we shall ignore this complications, since our goal is to understand whether in principle 4D theories with unbroken vacuum supersymmetry and Fermi-Bose non-degeneracy are possible. We see no reason to think that the “dual” Universe in which particles are described by monopoles must obey similar cosmological constraints.

To understand how far the analogy between particles and the global monopoles can be extended, we can estimate their interaction potential due to various forces.

We shall discuss gravitational effects first. The leading contribution comes from a tree-level one-graviton exchange

\[
\sim G \int d^4x \ d^4x' \ T_{\mu\nu}(x) \ G^{\mu\nu\alpha\beta}(x-x') \ T'_{\alpha\beta}(x'),
\]

where \( G \) is a Newtons constant and the graviton propagator is given by

\[
G^{\mu\nu\alpha\beta}(x-x') = \int \frac{dp^4}{(2\pi)^4} \frac{\frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) - \frac{1}{7}\eta^{\mu\nu}\eta^{\alpha\beta}}{p^2 - i\epsilon} e^{-ip(x-x')} \tag{20}
\]

For the rough estimate we shall ignore the effect of the curvature (which dies-off as \( \sim 1/r^2 \) away from the core) and use the flat space propagator. In the first approximation, we can assume that the structure of gravitating sources are not affected by simultaneous presence of two monopoles, and ignoring the effect of the core we can set:

\[
T^t_t = T^r_r = \frac{v^2}{|r|}, \quad T^t_t = T^r_r = \frac{v^2}{|r| - |R|},
\]

and all other components zero. Here \( R \) is a distance between the monopoles. In this case \( (19) \) is zero. So the gravitational interaction of monopoles will be governed by the Newtonian interactions between the two cores \([\text{E}]\)

\[
V(r) \sim G \frac{M_{\text{core}}^2}{r} \tag{22}
\]

This is sub-dominant with respect to the force mediated by the Goldstone field. For a monopole-anti-monopole pair the Goldstone-mediated interaction gives an attractive constant force

\[3\]This estimate assumes that all the monopoles within the observable part have the same charge (like baryons). If one assumes the equal number of monopoles and anti-monopoles within the Hubble horizon, the allowed number would be much higher.

\[3\]We shall ignore the tidal acceleration \( \sim 1/r^2 \), which may be very important for non-static sources moving with a nonzero impact parameter \([13]\).
\[ V(r) \sim v^2r \quad (23) \]

However, we are more interested in monopoles of the same charge, since a monopole-anti-monopole pair produces no deficit angle at infinity. Below we shall assume that vacuum topology is such that monopoles with the higher winding numbers are possible. In this case the long range interaction is repulsive (there still however can be an attractive short-range interaction between the cores). Due to this the \( n \)-monopole system is unstable. This fact can be understood qualitatively as follows. Consider \( n \) monopoles uniformly distributed inside the sphere of radius \( R \). For distances \( r \gg R \) the configuration of the Higgs field is similar to the one of the monopole with winding number \( n \), and gradient energy diverges as

\[ E_{\text{out}} \sim n^2 v^2 (r_{\text{max}} - R) \quad (24) \]

while inside the sphere it scales as

\[ E_{\text{in}} \sim n v^2 R \quad (25) \]

Therefore it is energetically favorable for \( R \) to grow.

However, the linear potential between monopoles can be modified in many cases. In particular, it is easy to imagine situation when this force vanishes because of the complex structure of the monopole. For instance, imagine that there are two broken global symmetries \( G \) and \( G' \) that produce monopoles. Let \( \Phi \) and \( \Phi' \) be the Higgs fields responsible for such breakings. We assume for simplicity that \( \Phi \) is trivial under \( G' \) and so is \( \Phi' \) under \( G \) respectively. Then the two fields produce independent monopoles which we shall refer to as \( \Phi \)-monopoles and \( \Phi' \)-monopoles respectively. If there is a \( G \otimes G' \)-invariant contact interaction in the potential

\[ -\Phi^* \Phi \Phi'^* \Phi' \quad (26) \]

the monopoles of the two sorts will tend to create the bound-states. Such interaction can be easily arranged by introducing the following couplings in the superpotential

\[ W = Y(h\Phi^2 - h'\Phi'^2) + ... \quad (27) \]

Where \( Y \) is a chiral superfield and \( h, h' \) are constants The equation of motion for \( Y \) then forces \( h\Phi^2 = h'\Phi'^2 \) and encourages \( \Phi \) and \( \Phi' \) to have coincident zeros. Thus \( \Phi \)-monopoles get bounded to \( \Phi' \)-monopoles. The strength of the binding force is set by the parameters \( h, h' \) and the magnitude of the two VEVs. Note that the binding interaction does not distinguish between monopoles or anti-monopoles in the opposite sectors. So formation of a monopole-monopole (\( MM \)) boundstate is as probable as the formation of a monopole-anti-monopole (\( M\bar{M} \)) one. Depending on the parameters of the theory, the Goldstone-mediated force between \( MM \) and \( M\bar{M} \) states may be either repulsive, attractive or zero. In the latter case the interaction will be dominated by a gravitational potential from the core.

In addition global monopoles can have long-range gauge interactions. In particular, as we shall show below, this long range interaction can be due to their magnetic charge.
D. Global Monopoles with Magnetic Charges.

Interestingly the global monopoles can carry magnetic charges under both Abelian or non-Abelian gauge groups. Let us consider first the Abelian case. For this, in addition to the spontaneously broken $O(3)$ global symmetry we introduce an unbroken $U(1)_m$ gauge symmetry in the theory. The question is whether a global monopole given by (7) can acquire a $U(1)_m$-magnetic charge. To answer this let us define an invariant two form

$$M_{\mu\nu} = \epsilon_{abc} \Phi^a \partial_\mu \Phi^b \partial_\nu \Phi^c$$

which determines a topological winding number

$$n = \frac{1}{4\pi} \int_{S^2} \frac{M_{\mu\nu}}{|\Phi|^3} dx^\mu dx^\nu$$

where integral is taken over a two-sphere surrounding the monopole. Let us now couple this two-form to a gauge-invariant $U(1)_m$-field strength

$$\frac{\lambda}{2} M_{\mu\nu} F^{\mu\nu}$$

We shall treat this term as the perturbation on the stable monopole background. The equation of motion then gives

$$\partial_\mu F^{\mu\nu} = \lambda \partial_\mu M^{\mu\nu}$$

This tells us that $U(1)_m$-magnetic field $(B)$ is radial, but does not fix its magnitude. The magnitude is determined by minimizing the $U(1)_m$-magnetic energy of the system

$$E_{\text{magnetic}} = -\lambda M B + \frac{B^2}{2}$$

where $M = \epsilon_{ijk} M_{jk}$. The energy is minimized by

$$B = \lambda v^3 \frac{r}{r^3}$$

In this way, the global monopole acquires $U(1)_m$ magnetic charge

$$Q_m = \frac{1}{4\pi} \int_{S^2} F_{\mu\nu} dx^\mu dx^\nu = N v^3 \lambda$$

proportional to the topological charge of its own. For this configuration not to be singular $U(1)_m$ must be embedded in some non-Abelian group. To do this, let us, instead of $U(1)_m$, introduce $SU(N)$ gauge symmetry, spontaneously broken to $SU(N - M) \otimes SU(M) \otimes U(1)$ by a Higgs field $\Sigma$ in the adjoint representation. We can define a gauge-invariant two-form

$$F_{\mu\nu} = \text{Tr} \Sigma F_{\mu\nu}$$

where $F_{\mu\nu}$ is the $SU(N)$ field-strength. Then the coupling of the form (35) will induce a magnetic charge of the global monopole. Effectively what happens is that $SU(N)$-gauge t’Hooft-Polyakov monopole gets trapped in the core of the global monopole.
Finally in case when gauge sector permits topologically stable monopoles (just like in above $SU(N)$ example), the global monopole can acquire magnetic charge by simply trapping the gauge monopoles inside its core due to a short-range Higgs interaction of the form similar to \[ W_{\text{cross}} = Y(h \Phi^a \Phi^a - h'Tr\Sigma^2) \] \hspace{1cm} (36)

This interaction tries to bring zeros of the two Higgs fields together and thus creates a bound-state of global and gauge monopoles. For instance, we can choose $SU(2)$ gauge symmetry broken by triplet $\Sigma^a$ and set its self-interaction term in the superpotential \[ W_\Sigma = Z(Tr\Sigma^2 - v_\Sigma^2) \] \hspace{1cm} (37)

where $Z$ is an additional superfield. The asymptotic form of the solutions for the “composite” monopole is

\[
Y = Z = 0, \quad \Phi^a = \pm \sqrt{h'/h} \Sigma^a r^a, \quad A^a_\mu = \epsilon_{\mu ab} r^b r^2
\] \hspace{1cm} (38)

and has the same magnetic charge as the elementary $SU(2)$ monopole.

E. Some speculations.

As we have seen, the global monopoles in $N = 1 \, D = 4$ supersymmetric theories exhibit some interesting properties. Due to the solid angle deficit at infinity they do not permit existence of conserved supercharges and, thus, there is no \textit{a priori} reason for a Fermi-Bose degeneracy in the spectrum, although supersymmetry is unbroken and cosmological constant vanishes. Despite the fact that the total energy of the configuration diverges, in some respect they behave like point-like particles of a finite mass localized at the core. These localized masses can carry gauge quantum numbers and, in particular, both Abelian as well as non-Abelian magnetic charges. If this toy picture can have any relation with the observed smallness of the cosmological term, we should expect that in some “dual” theory these monopoles are related to the ordinary particles carrying electric Yang-Mills charges.

It has been shown \cite{6} that the $SU(3) \otimes SU(2) \otimes U(1)$ magnetic charges of the gauge monopoles produced in the symmetry breaking \[ SU(5) \to SU(3) \otimes SU(2) \otimes U(1)/Z_6 \] \hspace{1cm} (39)

are in correspondence with the electric gauge charges of the standard model fermions. This interesting observation naturally leads to the idea of the dual standard model \cite{6}.

We have shown that global monopoles can carry any magnetic charge that can be carried by the local ones. Then the following toy scenario emerges. We take $N = 1$ supersymmetric theory with a symmetry group $G_g \otimes G_l$, where $G_g$ is global and $G_l$ is gauged. We assume that these symmetries are spontaneously broken to $H_g \otimes H_l$ by Higgs fields $\Phi$ and $\Sigma$ respectively. The breaking is such that $\pi_2(G_g/H_g) \neq 0$. This ensures the existence of stable global $\Phi$-monopoles. On the other hand $\pi_2(G_l/H_l)$ may or may not be empty, since as we have seen, the global monopoles can acquire magnetic charges even if the vacuum of the broken gauge
symmetry is topologically trivial. If however, also \( \pi_2(G_l/H_l) \neq 0 \), then there will be local monopoles formed by \( \Sigma \). If \( G_l \) is simply connected, then gauge monopoles are classified by non-contractable paths in \( H_l \) and carry different magnetic charges. There can be a finer classification \( [13] \) according to the representations of a dual magnetic group.

Now the global monopoles can acquire magnetic charges under \( H_l \) via one of the above discussed mechanisms, either by “kinetic” mixing \( [13] \) or by forming the boundstate due to the contact Higgs interaction \( [27] \). In the latter case for each stable gauge monopole there will be a composite stable global monopole with identical gauge charge. These objects carry a global topological charge (“dual baryon number”) and a local magnetic charge (“dual electric charge”). They move in a cloud of Goldstone field which creates a solid angle deficit at infinity. No conserved supercharges can be defined on such a background. On the other hand supersymmetry and \( R \) symmetry are unbroken in the vacuum and cosmological constant vanishes. Then one may expect that in the dual picture this monopoles are replaced by electrically charged particles (“baryons” and “leptons”) of some more conventional theory in which fermions and bosons will not be degenerate but vacuum energy will be zero because of (hidden) unbroken supersymmetry. Such a duality probably has to exchange the global topological winding number with the global Noether charge (baryon or lepton numbers).

**F. Infinite-Volume Extra Dimensions.**

An alternative way of controlling the cosmological constant by supersymmetry may be by going to a “brane world” scenario. In the standard brane world picture the ordinary matter is localized on a brane embedded in \( N \) large extra dimensions. These can be as large as millimeter with the fundamental Planck mass \( (M_{Pl}) \) around TeV \( [14] \). The volume of extra dimensions is finite, either because these dimensions are compact or because the warp factor decays exponentially fast away from the brane \( [16] \). The compactification combined with non-trivial warp factors in co-dimension two spaces were considered \( [17] \). Compactification may take place due to singularity at finite distance from the brane as, for instance, in \( [18] \).

In either case there is an upper bound around \( \sim \) mm on the volume of extra dimensions from gravitational measurements. We shall define this volume as

\[
V_{extra} = \int d^{N}_{extra} \sqrt{-g_{extra}g^{00}(x_{extra})} \tag{40}
\]

where the integration is taken over an \( x_{extra} \)-dependent part of the metric. This volume factor sets the normalization of the 4D zero mode graviton and thus the relation between fundamental and observable Planck scales

\[
M^2_P = M^{2+N}_{Pl}V_{extra} \tag{41}
\]

\[^{4}\text{If } H_l = SU(3) \otimes SU(2) \otimes U(1)/\mathbb{Z}_6 \text{ then according to } [10] \text{ the magnetic charges of these monopoles will match the } SU(3) \otimes SU(2) \otimes U(1)-\text{electric charges of the standard model fermions. This choice however is not necessary since the dual “electric” group } \tilde{H}_l \text{ need not be isomorphic to } H_l.\]
In this set-up, it is very difficult to control the value of the cosmological constant by bulk supersymmetry. Indeed, since the extra volume is finite, the extra coordinates can be integrated over and at large distances, the effective four-dimensional description should be valid. Then by four-dimensional general covariance the brane and bulk states should be gravitationally connected through the zero mode graviton. Now, supersymmetry must be broken among the brane states at least at scale $\text{TeV}$. This breaking then will universally get transmitted to all bulk modes by gravity. This transmission is at best volume suppressed, so that by dimensional argument the mass splitting among the bulk states is at least $\Delta m^2 \sim (\text{TeV})^4/M^2_p$ and the resulting cosmological constant is at least $\sim \Delta m^2 \Lambda^2_{\text{cut-off}}$.

The crucial point is that in theories with infinite $V_{\text{extra}}$ there is no a priori reason for the above inconsistency [2,3]: since the volume is infinite, the theory is never four-dimensional and effective 4D description is never valid. In other words, supersymmetry broken on the brane may not get transmitted in the bulk. So the high-dimensional theory can be supersymmetric even though brane observers will not see any Fermi-Bose degeneracy.

An interesting model of infinite-volume extra dimensions was invented in [4] as modification of RS scenario [16]. (Modification of gravity at large distances was suggested earlier in [20].) In their case the warp factor in five dimensional metric

$$ds^2 = A(y)ds_4^2 - dy^2$$  \hspace{1cm} (42)

instead of vanishing as $y \to \pm \infty$, was asymptotically approaching a tiny constant value. As a result the space is infinite and asymptotically flat in $y$ direction. This structure was achieved by expense of introducing a combination of positive and negative tension branes. Despite this fact a correct four-dimensional Newtons law is reproduced at intermediate scales on the brane. As was shown in [4,5,6], this can be explained by the existence of meta-stable resonant graviton localized on the brane. There are two potential problems with this scenario. One, to be put aside in the present paper, is the fact that 4D gravity on the brane is mimicked by massive spin-2 fields, which can not reproduce the predictions of Einsteins theory [2].

Second is the violation of the weak energy condition, which has to do with the existence of AdS portion bounded by negative tension branes embedded in Minkowski space [3]. In this respect it is very important to obtain a source that quasi-localizes gravity as a solution of the underlying theory. We want to note that the branes which appear from spontaneous

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5This is true whenever dominant contribution to 4D gravity comes from a massive spin-2 state(s). Some way out of this problem in the context of [20] was suggested in [21].

6In [2,19] it was noted that the unwanted contributions to one-graviton exchanges may be canceled by unconventional states like ghost (we understand that this was essentially confirmed by ref [22,23]), but then it is hard to make sense of the theory, even if this states only dominate exchanges at finite distances. This goes in contrast to the conclusions of [24] and [25] in which it was argued that such states can solve the problem (conclusion about their long-distance behavior is opposite in these two references). We think that existence of the physical ghost is a problem at any distances. For instance, even if canceled in all one-particle exchanges, it is not clear what can prevent their appearance in the final state.

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breaking of non-Abelian global symmetry may have this property. We consider co-dimension 3 case. In this case the brane is a global monopole embedded in three extra dimensions. Consider a theory in 7 dimensions with a spontaneously broken global $O(3)$ symmetry, by the VEV of a triplet scalar field $\Phi_a$. The potential is
\[
V = (\Phi_a \Phi_a - v^2)^2
\] (43)

Due to topological arguments this theory has co-dimension 3 objects, independent of four space-time coordinates, described by eq(7), where $r$ has to be understood as the radial coordinate perpendicular to the brane. The general spherically symmetric ansatz for the metric can be written
\[
ds^2 = a^2(r)\eta_{\mu\nu}dx^\mu dx^\nu - b^2(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)
\] (44)

Much in the same way as for the global monopole, away from the core we can set $f(r) = v^2$ and the only contribution to the source will come from the angular derivatives. As a result
\[
T^\mu_\mu = T^r_r = \frac{v^2}{r^2} + ...
\] (45)

and all other components zero. The ellipses stand for the sub-leading correction. With this source, there is a straightforward generalization of the Barriola-Vilenkin solution, which (in $M_{fp}$ units) reads:
\[
a^2 = b^{-2} = 1 - v^2 = \alpha
\] (46)

As in the four-dimensional case, there is a deficit angle $4\pi v^2$. In principle, by tuning the parameter $v$, one can make angular deficit arbitrarily close to $4\pi$. Now let us study the issue of graviton localization.

First let us ignore the subleading corrections in (45). That means assume that $a$ goes to it asymptotic constant value fast enough. Then the volume of the transverse space
\[
V_{extra} = 4\pi \int dr a^2 br^2
\] (47)

is infinite unless $a$ vanishes asymptotically. This volume has two contributions
\[
V_{extra} = V_{core} + 4\pi \int_{r_{core}}^{\infty} ar^2 dr
\] (48)

The second term diverges, unless we tune $\alpha = 0$ in which case the volume is finite and is given by $V_{core}$. There is no reason to expect in $\alpha \to 0$ limit any pathological behavior in the core. Assuming this the theory must have a finite volume and must support a localized zero mode graviton in the core of the monopole. The following fluctuation about the background metric

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7 From a different perspective the higher co-dimension global defects were studied by Vilenkin and Olasagasti [26] and by Cho. I am grateful to these authors for correspondence and discussions.
\[ ds^2 = a^2(r)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - b^2(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \] (49)

is a zero mode. This mode is normalizable if \( V_{\text{extra}} \) is finite. Now, we can make \( \alpha \) to be nonzero, but arbitrarily small. Now the volume becomes infinite and the zero mode becomes non-normalizable. However, if \( \alpha \) is small enough, the system by continuity should still allow a localized meta-stable mode, with an arbitrarily long life-time.

A problem with this argument is that it relies on an oversimplified anzats for the monopole core. For a realistic monopole, there are sub-leading corrections \( \sim \frac{1}{r^2} \) to eq(45) (recall that \( (f^2 - v^2) \) dies away as \( \frac{1}{r^2} \)). So the volume for \( r \to \infty \) is divergent, which makes existence of quasi-localized mode unclear. More precise conclusion depends on the model of monopole core which will not be discussed here.

Summarizing, in order to avoid the violation of the weak energy condition one may try to quasi-localize gravity on a brane which is a solution of an underlying sigma model. Under certain assumption regarding the core structure, this brane may supports a meta-stable graviton because of the deficit angle at infinity. The next step would be to supersymmetrize the initial theory in such a way that supersymmetry gets restored in the bulk. However, this is not so easy, since due to the same deficit angle we will not be able to define globally conserved supercharges in such a background. However, this is not necessarily a problem, since as in four-dimensions, the bulk vacuum energy density is controlled by the order parameters that break supersymmetry. These are angular gradient densities that die away from the core as \( \frac{1}{r^2} \). Thus all the energy densities in the bulk must be controlled by the distance from the brane and get zero at infinity. Although the total transverse energy is linearly divergent, this is not of any problem, since the four-dimensional metric on the brane is flat.

An alternative approach one can try is to reintroduce Killing spinors, by adding gauge fields and canceling the deficit angle by Aharonov-Bohm-type phases, just as in the three dimensional case of ref [7]. In this way some of the supercharges may be unbroken, but act trivially on the brane.

G. conclusions.

It is not clear how the 4D scenario discussed in this paper can be related to an observable Universe. Yet it has some ingredients for this connection and is an example of 4D supersymmetric theory with vanishing cosmological constant, in the background that makes impossible definition of unbroken supercharges. Probably the way this framework may be related to more conventional ones is through some sort of a duality, which relates solitons with particles, magnetic charges with electric charges and in addition global topological charges (winding number) to conserved global Noether charges (such as baryon or lepton number).

Among many open questions, not addressed in this letter, there are cosmological issues. For instance with a rough estimate the number of the global monopoles that would contribute \( \rho_{\text{baryon}} \) energy density within the observable part of the Universe is much less than the number of baryons. So one may ask how this fast fits in the above picture.

We do not see any reason why two theories related through duality should have similar cosmological history. We expect that duality relates spectra and quantum numbers, but
not the actual number of occupied states, which is determined by the cosmological initial conditions.

Alternatively infinite volume extra dimensional theories may provide example of unbroken supersymmetry compatible with four-dimensional Fermi-Bose non-degeneracy. We have suggested that gravitating sources similar to global topological defect, producing angular deficit in the extra space, may under certain conditions localize $4D$ meta-stable graviton.

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