Criteria of Strength and Plasticity of Asphalt Concrete with the Account of Effect of Accumulation of the Damage while Influence of Re-Load

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Abstract. The paper presents a method for modifying the strength criteria and plasticity conditions, based on the fundamental properties of the algebra of matrices. The essence of the method is that applying the basic property of the algebra of matrices, the stress tensor in the solid body is multiplied by the damage function. As a result of this mathematical operation, a stress tensor is obtained in the damaged body. Then, from the component of stress tensor of the damaged body, its invariant characteristics are determined and formed, which are substituted into the known criteria of strength and plasticity. Thus, the new criteria contain damage. The modification of both simple two or three invariant criteria and modern multi-surface criteria capable of changing the shape of the limit surface depending on the value of one of the material parameters is considered.

1. Introduction

The layers of road clothes of monolithic materials are traditionally calculated to the criterion of the resistance to tension from bending, at the heart of which lies the first theory of strength. Due to the fact that this criterion does not take into account the influence of other stress tensor components, the strength of the material is evaluated incorrectly. Specialists of the road industry have done a lot of work in the field of research the impact of various factors on the strength of the asphalt concrete during bending. In the last regulatory document, the fatigue coefficient is determined by an empirical formula describing the results of tests performed with asphalt on different bitumen. In accordance with this dependence, asphalt concrete withstand as many loads as you want. Therefore, the work aimed at the study of fatigue processes and accumulation of damage in the asphalt concrete is relevant. In addition, there is no doubt the relevance of the work aimed at developing the strength criterion of the asphalt concrete, taking into account the ability to accumulate damage. Earlier regulatory documents regulated calculation of asphalt concrete coverings on resistance to shift, but only in places of stops and parking of vehicles. Currently, on the road pavement is detected by the track, which is often accompa-
nied by side bulging, which clearly shows the shift of asphaltic concrete pavement. As a result, the criterion for calculating the coatings to shear resistance must be revived, but it must be based on the modern condition of plasticity.

In connection with the analysis the tasks of the publications is to improve the criterion of strength and plasticity conditions, which consists in taking into account the dynamics of accumulation of damage in the structure of the material.

2. Methods

In mechanics are used, two fundamental concepts of the damage theory and its measures [1]. Both areas have become widespread in the evaluation of stress-strain state (SSS) of materials. According to the first direction, the measure of the damage theory is the ratio of the number of loads applied to the material, to its durability, which is the number of the same loads that the material must withstand to failure at a given value of the cyclic stress [1]. The measure of damage is determined by the formula:

\[ D = \frac{n}{N} \]

where \( n \) and \( N(\sigma) \) – the applied number of loads and its limit value that the material can withstand at a given cyclic stress \( \sigma \).

The damage accumulated by the material from the impact of repeated loads is determined by summation principle, and its methods can be divided into three groups:

1 – linear summation or the Palmgren-Miner Rule;
2 – bilinear summation or the principle of S.S. Manson [2];
3 – nonlinear, accumulation of damage, or a hypothesis of Richart-Newmark [3].

At figure 1 presents a graphical interpretation of these models, the accumulation of damage.

![Figure 1. Principles of damage summation in the calculation of damage: a - linear summation; b - bilinear summation; c – nonlinear summation](image)

The hypothesis of fatigue failure, which is the basis of methods in this direction, was formulated by Palmgren and gave her a simple mathematical equation

\[ \sum_{i=1}^{n} D_i = \sum_{i=1}^{n} \frac{n_i}{N(\sigma)} = 1, \]
where \( i \) and \( n \) – number and total number of values (amplitudes) of stresses.

This direction of the damage theory was demanded by specialists of road branch \([4, 5]\). E.V. Uglova applied the principle of linear summation of damages to account for the kinetics of their accumulation in the operation of asphalt concrete pavement \([4]\)

\[
D = \frac{\sum n_i \text{calc}}{N_i \text{calc}}, \quad (3)
\]

In experiments on the resistance of materials to deformation under the influence of cyclic loads and the accumulation of damage came to the conclusion that the Palmgren – Miner formula needs modification. Therefore, in 1966 S. S. Manson \([2]\) proposed a bilinear model of damage accumulation. This model uses a set of two straight lines intersecting at one common point at figure 1, \( b \).

When applying the bilinear model, the greatest difficulties are caused by the determination of the fracture point, of two straight segments. To solve this problem, you can use the results of the study of Manson S.S., Halford G.R. \([6]\), in which a series of samples have been tested. The results of these tests are approximated by formulas describing the location of the fracture point depending on the ratio of the limit number of loads for I and II phase \( N_1/N_2 \). In general, Manson – Halford formulas can be written as follows

\[
\frac{n_1}{N_1} = a \cdot \left( \frac{N_1}{N_2} \right)^b; \quad \frac{n_2}{N_2} = c \cdot \left( \frac{N_1}{N_2} \right)^b, \quad (4)
\]

where \( a, b \) and \( c \) – parameters of material. For steel the coefficient \( a=0.35 \), and the parameter \( b=0.35 \). For asphalt concrete, these parameters must be determined experimentally.

One of the first attempts to predict fatigue damage using nonlinear models, the accumulation of damage, were made in 1948 by F. E. Richart Jr., and N. M. Newmark \([3]\). In this work, it is accepted that the continuous curve will more accurately reflect the impact of the load on the damage of material. For a long time the hypothesis of Richart-Newmark remained an assumption that does not have a mathematical description. This disadvantage persisted until the appearance of the work of Manson S.S., Halford G.R. \([6]\), in which, along with the model \((6)\) was proposed a non-linear formula that describes the accumulation of damage when exposed to repeated loads. This model has the form \([6, 7]\)

\[
D = \left( \frac{n}{N_f} \right)^{\left( \frac{N_f}{N_{\text{ref}}} \right)^b}, \quad (5)
\]

where \( N_f \) – the model parameter according to the number of loads; \( b \) – parameter of material; \( N_{\text{ref}} \) – limit number of loads, withstanding material, or life cycle.

In work \([8]\) Manson S.S., Halford G.R. improved the nonlinear model by introducing a linear term in it in order to be able to shift the curve relative to the x-axis.

\[
D = \frac{n}{N} \left\{ q_1^\gamma + 1 - q_1^\gamma \left[ \frac{n}{N} \right]^{\gamma \cdot q_2^{-1}} \right\}^{\frac{1}{\gamma}}, \quad (6)
\]

where \( n \) – number of cycles applied at a given load level; \( N \) – the number of load cycles of the same level, which must be applied to destroy the material; \( \gamma \) – a constant representing two intersecting straight lines that can be replaced by a single curve; \( q_1 \) and \( q_2 \) – the model parameters which are determined using the formulas by work \([8]\).
Calculation of parameter $q_1$ and $q_2$ made according to the formulas.

In addition to the models we have considered, a sufficiently large number of empirical and analytical dependencies are known for calculating damage $D$. An overview of these models starts from work S. M. Marco, W. L. Starkey [9], in which the principle of linear summation described by the dependency (2) replaced by the nonlinear summation. As a nonlinear model used the degree function [9, 10], which we will write in the form

$$D = \sum r_i = \sum_{i=1}^{m} \left( \frac{n_i}{N_{ref}} \right)^{x_i},$$

(7)

where $x_i$ – this is a variable value, associated with $i$ load, able to account to its nominal value.

Another option for improving the principle of summation of damage is the model of Corten-Dolon [11], incorporating stress dependence, i.e., a function of stresses and the interaction effects. In this model, it is assumed that the number of damage cores, which are understood to be components, is caused by the occurrence of higher stresses. These stresses have an impact on the increase of damage. Model Corten-Dolon has the form [11]

$$D = \frac{n_1}{N} + \frac{n_2}{N} \left( \frac{\sigma_{a2}}{\sigma_{a1}} \right)^d + \frac{n_3}{N} \left( \frac{\sigma_{a3}}{\sigma_{a1}} \right)^d + \ldots + \frac{n_m}{N} \left( \frac{\sigma_{am}}{\sigma_{a1}} \right)^d,$$

(8)

Where $n_1, n_2, n_3, \ldots, n_m$ – number of cycles, applied loads with appropriate amplitudes $\sigma_{a1}, \sigma_{a2}, \sigma_{a3}, \ldots, \sigma_{am}$, moreover $\sigma_{a1} > \sigma_{a2} > \sigma_{a3} > \ldots > \sigma_{am}$; $d$ – material parameter.

It should be noted that the determination of the limit number of loads sustained by the asphalt concrete before its destruction is devoted to a sufficiently large number of works. In these works empirical formulas linking the life cycle of asphalt concrete with different parameters are obtained. Such empirical formulas are given in table 1.

Another direction of accounting for the decrease in the continuity of the material is the application of measures by L.M. Kachanov and Y.N. Rabotnov. In accordance with L. M. Kachanov proposal, the state of the material is characterized by a parameter called continuity [22] and determined by the ratio stresses in the continuous $\sigma$ and damaged material $\sigma_D$.

$$\psi = \sigma / \sigma_D .$$

(9)

For an undamaged environment $\psi=1$, and for the material with defects of structure the parameter $\psi<1$ and decreases as damage accumulates [22]. Damage kinetics by L. M. Kachanov and stress growth is described by the degree equation, includes two more parameters in addition to continuity. Y. N. Rabotnov proposed as a measure a parameter called damage $\omega$ [23]. Under the damage $\omega$ it is necessary to understand the total area of defects, expressed as a fraction of the geometric area of the whole nondestructive section. Y. N. Rabotnov writes [23]: «If the geometric cross-sectional area is $F$, then the effective area receiving the load is $F(1-\omega)$». As a result, the true stress is determined by the ratio of the load $N$ perceived by the cross section to the effective area $F(1-\omega)$. Load ratio $N$ to the geometric area of the section $F$ determines value the voltage $\sigma$ in solid cross-section. Therefore, the dependence of the true stresses arising in the damaged section $\sigma_D$, at the Y. N. Rabotnov is determined by the equation $\sigma_D = \sigma / (1-\omega)$ [23]. If this dependence is to substitute in the equation L. M. Kachanov, defining the continuity $\psi$, we get
Table 1. Formulas for determining the maximum number of loads for asphalt concrete

| Specialists who used the formula | Formula |
|----------------------------------|---------|
| P.S. Pell [12]                  | \[ N_p = K_1 \left( \frac{1}{\varepsilon_r} \right)^{K_2}, \] |
|                                 | where \( \varepsilon_r \) – tensile deformation; \( K_1 \) and \( K_2 \) – coefficients of the model |
| F.N. Fin [13, 14]               | \[ N_p = K_1 \left( \frac{1}{\varepsilon_r} \right)^{K_2} \cdot E^{K_3}, \] |
|                                 | where \( E \) – asphalt stiffness (modulus of elasticity); \( K_1 \), \( K_2 \) and \( K_3 \) – model parameters |
| F. Zhou et al. [15]             | \[ N_p = a \cdot 10^{b \left( \frac{V_b}{V_b + V_a - c} \right)} \cdot \varepsilon_r \cdot d \cdot E \cdot f, \] |
|                                 | where \( V_a \) – air porosity, \%; \( a, b, c, d \) and \( f \) – model parameter |
| F. Bonnaure et al. [16]         | \[ N_p = A_f \cdot a \cdot PI + b \cdot PI \cdot V_b + c \cdot V_b - d^5 \] |
|                                 | where \( A_f \) – correction factor, which takes into account the difference between operating conditions and laboratory conditions; \( PI \) – penetration index; \( V_b \) – bitumen content by volume, \%; \( a, b, c \) and \( d \) – model parameter |
| S.H. Carpenter [17, 18]         | \[ N_p = a \cdot \exp \left( b \cdot VBF \cdot \varepsilon_r^c \cdot S^d \right), \] |
|                                 | where \( VBF \) – the percentage of voids filled with bitumen; \( S \) – initial loss stiffness of the mixture, measured at bending |
| H. Di Benedetto [19], C. De La Roch [20] | \[ N_p = 10^6 \cdot \left( \frac{\varepsilon_{cal}}{a \cdot \varepsilon_6} \right)^b, \] |
|                                 | where \( \varepsilon_{cal} \) – strain calculated for a single load application; \( \varepsilon_6 \) – deformation, resulting in the destruction of a sample with temperature 0 °C, after application 10^6 loads |
| G.W. Maupin [21]                | \[ N_p = 10^{a + b \cdot R_p} \cdot \left( \frac{1}{\varepsilon_r} \right)^{c + d \cdot R_p}, \] |

\[
\psi = \sigma \cdot \frac{1 - \omega}{\sigma} = 1 - \omega. \tag{10}
\]

From (10) follows:

\[
\omega = 1 - \psi. \tag{11}
\]

Equations (10) and (11) Express a relationship between the two parameters of the theory of damage: continuity \( \psi \) by L. M. Kachanov and damage \( \omega \) by Y. N. Rabotnov. As a consequence from (9) or
we can get:

\[ \omega + \psi = 1. \quad (12) \]

From (12) it follows the difference between the continuity and damage parameters from each other, consisting in the fact that each of them characterizes the total relative area occupied by the defects (damage \( \omega \)), and without defects (continuity \( \psi \)). The sum of these parameters (15) equal to one, that is, the sum of these measures is nothing more than the geometric area of the section, expressed as one.

Damage detection is performed by the ratio of different physical and mechanical characteristics of the material in the damaged and intact specimen [24, 25]. Such characteristics include the size of defects, Young modulus and elasticity modulus of the material, porosity, density, etc. [24, 25].

3. Results and Discussion

When developing the criteria of strength and plasticity conditions of the damaged body, use L. M. Kachanov hypothesis (9), but written in accordance with the property of matrix algebra in the form of the ratio of the stress tensor in the continuous medium to the measures of the damage theory. Then the components of the tensor of the damaged body \( \sigma_{ik} \) can be determined through the components of the stress tensor of the solid body \( \sigma_{ij} \):

\[
\sigma_{ik} = \frac{\sigma_{ij}}{1-\omega} = \frac{\sigma_{ij}}{\psi}. \quad (13)
\]

Using the formula (13), any characteristic of the stress tensor can be determined. According to equation (13), the principal stresses in the damaged body are determined by formulas:

\[
\sigma_1 = \frac{\sigma_1}{1-\omega} = \frac{\sigma_1}{\psi}; \quad \sigma_2 = \frac{\sigma_2}{1-\omega} = \frac{\sigma_2}{\psi}; \quad \sigma_3 = \frac{\sigma_3}{1-\omega} = \frac{\sigma_3}{\psi}. \quad (14)
\]

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) – the principal stresses in the intact body

Using formulas (14) we can calculate the maximum tangent stress in the damaged environment \( \tau_{\text{max}} \), octahedral normal and tangent stress \( \sigma_{\text{oct}} \) and \( \tau_{\text{oct}} \); and intensity of R. Mises. These characteristics are calculated according to the formulas:

\[
\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2(1-\omega)} = \frac{\sigma_1 - \sigma_3}{2\psi}. \quad (15)
\]

\[
\sigma_{\text{oct}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_{\text{oct}}}{\psi}; \quad \tau_{\text{oct}} = \frac{\tau_1 + \tau_2 + \tau_3}{3} = \frac{\tau_{\text{oct}}}{\psi}. \quad (16)
\]

\[
\sigma_{ik} = \frac{\sigma_i}{1-\omega} = \frac{\sigma_i}{\psi}; \quad \tau_{ik} = \frac{\tau_i}{1-\omega} = \frac{\tau_i}{\psi}. \quad (17)
\]

Invariants of the stress tensor (first \( I_{10} \), second \( I_{20} \) and third \( I_{30} \)) of damaged bodies are defined by formulas:

\[
I_{10} = \frac{I_1}{1-\omega} = \frac{I_1}{\psi}; \quad I_{20} = \frac{I_2}{1-\omega} = \frac{I_2}{\psi}; \quad I_{30} = \frac{I_3}{1-\omega} = \frac{I_3}{\psi}. \quad (18)
\]

The second and third invariants of the deviator of stresses are determined by formulas:
Substituting formulas (14) – (20) in the criteria of strength and plasticity conditions of solid bodies can be performed their modification, so that it will be possible to take into account the accumulation of damage in the bodies. Substituting equation (14) in the criterion, we obtain the formula:

\[
J_{20} = \frac{1}{1-\omega} \left( I_2 - \frac{1}{3} I_1 \right) = \frac{1}{\psi^2} \left( I_2 - \frac{1}{3} I_1 \right).
\]  

(19)

\[
J_{30} = \frac{1}{1-\omega} \left( I_3 - \frac{1}{3} I_1 I_2 + \frac{2}{27} I_1^3 \right) = \frac{1}{\psi^3} \left( I_3 - \frac{1}{3} I_1 I_2 + \frac{2}{27} I_1^3 \right).
\]  

(20)

The strength criterion (21) can be used to calculate the asphalt concrete layers on tension in bending.

When calculating the shear resistance, it is possible to use the modified plasticity condition obtained in the works [26, 27]. If in this criterion is introduce dependencies to calculate the principal stresses of damaged body, expressed through damage or continuity, we obtain

\[
\frac{1}{2} \left( \sigma_1 - \frac{R_s}{R_c} \cdot \sigma_3 \right) \cdot \frac{1}{1-\omega} = R_s
\]

or

\[
\frac{1}{\psi} \left( \sigma_1 - \frac{R_s}{R_c} \cdot \sigma_3 \right) = R_s.
\]  

(21)

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\[
\frac{1}{2} \cdot \frac{1}{1-\omega_N} \left[ \sigma_1 \left( \frac{1-\sin \phi}{1+\sin \phi} \right) - \left( \frac{1+\sin \phi}{1-\sin \phi} \right) \cdot \sigma_3 \right] = c.
\]  

(22)

\[
\frac{1}{2 \cdot \psi_N} \left[ \sigma_1 \left( \frac{1-\sin \phi}{1+\sin \phi} \right) - \left( \frac{1+\sin \phi}{1-\sin \phi} \right) \cdot \sigma_3 \right] = c.
\]  

(23)

where \( \omega_N \) and \( \psi_N \) – damage and continuity after exposure to N-th number of design loads, percentage of units; \( \sigma_1 \) and \( \sigma_3 \) – maximum and minimum principal stresses arising in the asphalt concrete at high summer temperatures, causing its three-axis compression over the entire thickness of the layer, MPa; \( \phi \) and \( c \) – angle of internal friction and adhesion of asphalt concrete, which are the parameters of the limiting straight Coulomb-Mohr, degree (radian) and MPa; \( d \) – parameter introduced into one of the forms of recording Coulomb – Mohr conditions [41], taking into account the value of deformation of the sample, taken as a limit for three-axis tests of the material.

To calculate the damage theory measures, let us assume that in the conditions of repeated load influence this process is continuous and has hereditary character. This nature is manifested in the fact that the magnitude of the modulus of elasticity observed under the influence of the n-th load is due to the application of both this and all previous loads. Therefore, to determine the function of changing the modulus of elasticity, you can use the integral equations of fatigue theory. A subintegral function defining the reduction of the modulus of elasticity from the application of the load with number \( n \) is given in the form of a degree equation:

\[
\Delta E_D = a \cdot n^b,
\]  

(24)

where \( a \) and \( b \) – parameters of the material.

Write the integral equation in the form of

\[
E_{DN} = E \cdot \left( 1 - a \cdot \int_1^N n^b \, dn \right).
\]  

(25)

Taking the integral (25), get
where $N$ – the total number of applied calculation loads.

To calculate the damage measures, we accept the deformation equivalence hypothesis, according to which the damage and continuity are calculated by formulas [28–30]:

$$E_D = E \cdot (1 - \omega) ; \quad \omega = 1 - \frac{E_D}{E} ; \quad \psi = \frac{E_D}{E}.$$  \hspace{1cm} (27)

Thus, the damage and continuity are determined through the ratio of the modulus of elasticity of the damaged and continuous medium.

Place (26) in formulas (27), we get

$$\omega = a \cdot \frac{N^{b+1} - 1}{b+1} ; \quad \psi = 1 - a \cdot \frac{N^{b+1} - 1}{b+1}.$$  \hspace{1cm} (28)

4. Conclusions

In conclusion, it is advisable to give recommendations on the application of the article and further ways of developing the study.

1. The paper proposed a method of modifying strength criteria and conditions of plasticity, which consists in the substitution in the equations of limit state the characteristics of stress state of a damaged body instead of their analogue in the continuum. In these criteria, the physical meaning of the theory of Kachanov – Rabotnov remains, according to which the increase in the damage of the material or the decrease in its continuity leads to the growth of the stress tensor components, and, consequently, all other characteristics of the stress state. The proposed criteria and conditions can be applied both for calculating the tensile bending and shear resistance.

2. Known solutions of continuum mechanics allow to calculate the magnitude of the main stresses at different stress states, which allows to quickly put into effect the proposed criteria of strength and plasticity conditions of damaged bodies.

3. Modern laboratory equipment allows to determine the kinetics of accumulation of damage by the material in a complex stress state. As a result there is the possibility of developing mathematical models describing the dependence of damage $\omega$ and continuity $\psi$ body from the magnitude of the principal stresses and the number of loads. The authors consider the formulation of such experiments and the selection of empirical formulas describing their results to be their further research.

References

[1] Sosnovskij L and Shherbakov S 2011 Material Damage Concepts Visnik TNTU Special Issue vol 1 pp 14-23 (In Ukrainian)

[2] Manson S S, Freche J C and Ensign C R 1967 Application of a double linear damage rule to cumulative fatigue ASTM STP 415 pp 384-412

[3] Richart Jr F E and Newmark N M 1948 An hypothesis for determination of cumulative damage in fatigue Proc. ASTM. vol 48 pp 767-800

[4] Uglova E V 2009 Theoretical and methodological framework for the assessment of residual fatigue life of asphalt concrete pavement of roads PhD thesis (Volgograd) p 38 (In Russian).

[5] Aleksandrova N P and Chysow V V 2016 The usage of integral equations hereditary theories for calculating changes of measures of the theory of damage when exposed to repeated loads Magazine of Civil Engineering № 2(62) pp 69-82
[6] Manson S S and Halford G R 1981 Practical implementation of the double linear damage rule and damage curve approach for treating cumulative fatigue damage Int. J. Fatigue vol 17 pp 169-192
[7] Goodin E, Kallmeyer A and Kurath P 2004 Evaluation of nonlinear cumulative damage models for assessing HCF/LCF interactions in multiaxial loadings Proceedings of the 9th national turbine engine high cycle fatigue (HCF) conference
[8] Halford G R and Manson S S 1985 Reexamination of cumulative fatigue damage laws Structure Integrity and Durability of Reusable Space Propulsion Systems, NASA CP-2381 pp 139-145
[9] Marco S M and Starkey W L 1954 A concept of fatigue damage Transactions of the ASME vol 76 pp 627-632
[10] Banjara N K and Sasmal S 2012 Evaluation of Fatigue Remaining Life of Typical steel plate girder Bridge under Railway Loading SL vol 8 № 1 pp 37-52
[11] Corten H T and Dolon T J 1956 Cumulative fatigue damage In Proceedings of the International Conference on Fatigue of Metals, Institution of Mechanical Engineering and American Society of Mechanical Engineers pp 235-246
[12] Pell P S 1987 Pavement Materials Proc., Sixth International Conference on The Structural Design of Asphalt Pavements. Vol. 2 pp 36-70
[13] Fin F N 1973 Relation between cracking and performance. // Special Rep. No. 140, Highway Research Board
[14] Fin F, Saraf C, Kulkarni R, Nair K, Smith W and Abdullah A 1977 The uses of distress prediction subsystems for the design of pavement structures Proc., 4th Int. Conf. on Structural Design of Asphalt Pavements, International Society for Asphalt Pavements (ISAP) pp 3-38
[15] Zhou F, Fernando E and Scullion T 2008 A review of performance models and test procedures with recommendations for use in the texas ME design program Report 0-5798-1. Texas transportation institute
[16] Bonnaure F, Gravois A and Udron J 1980 A New Method of Predicting the Fatigue Life of Bituminous Mixes Journal of the Association of Asphalt Paving Technologists vol 49 pp 499-529
[17] Carpenter S H and Freeman T J 1986 Characterizing Premature Deformation in Asphalt Concrete Placed over Portland Cement Concrete Pavements TRB, Transportation Research Record 1070
[18] Carpenter S H 2006 Fatigue performance of IDOT mixtures Research Report FHWA-ICT-07-007 Illinois Center for Transportation
[19] Di Benedetto H, Soltani A A and Chaverot P 1996 Fatigue Damage for Bituminous Mixtures: A Pertinent Approach Proceedings of Association of Asphalt Paving Technologists vol 65 pp 142-158
[20] De La Roche C, Odeon H, Simoncelli J-P and Spernol A 1994 Study of the Fatigue of Asphalt Mixes Using the Circular Test Track of the Laboratoire Central des Ponts et Chausses in Nantes, France Transportation research record vol 1436 pp 17-27
[21] Maupin G W 1977 Test for Predicting Fatigue Life of Bituminous Concrete Transportation Research Record № 659 pp 32-37
[22] Kachanov L M 1986 Introduction to continuum damage mechanics (Dordrech: Martinus Nijhoff Publishers) 136 p
[23] Rabotnov Y N 1979 Mechanics of deformable solids (Moscow: Science) 744 p (In Russian)
[24] Lemaitre J A 2005 Course on Damage Mechanics (Verlag Berlin Heidelberg: Springer) p 380
[25] Aleksandrova N P, Aleksandrov A S and Chusov V V 2017 Application of Principles of Theory of Damage Accumulation to Calculation of Asphalt-Concrete Coatings IOP Conf. Ser.: Mater. Sci. Eng. Vol 262
[26] Aleksandrov A S and Kalinin A L 2015 Improvement of shear strength design of a road structure Part 1 Deformations in the Mohr – Coulomb plasticity condition Inzhenerno-stroitel’nyi zhurnal № 7(59) pp 4-17 (In Russian)
[27] Aleksandrov A S, Dolgih G V and Kalinin A L 2016 Improvement of shear strength design of a road structure Part 2 Modified models to calculate the principal and shear stresses Inzhenerno-stroitel’nyj zhurnal № 2(62) pp 51-68 (In Russian)

[28] Ambroziak A and Klosowski P 2006 Survey of modern trends in analysis of continuum damage mechanics Task Quarterly № 4 pp 437-454

[29] Granda Marroquin L E et al. 2008 Cumulative Damage Evaluation under Fatigue Loading Applied Mechanics and Materials vols 13-14 pp 141 – 150

[30] Tsiloufas S P and Plaut R L 2012 Ductile Fracture Characterization for Medium Carbon Steel Using Continuum Damage Mechanics Materials Sciences and Applications № 3 pp 745-755