A semi-empirical approximation of static hysteresis for high flux densities in highly grain-oriented silicon iron

C Carrander, G Engdahl
Department of Electromagnetic Engineering, Royal Institute of Technology, Drottning Kristinas väg 6, 100 44 Stockholm, Sweden
claesca@kth.se

Abstract. In calculations and simulations regarding magnetic materials, it is important to have a highly accurate model of the hysteresis loop. The major loop, in particular, is used in many simulations. However, it is generally not possible to measure the true major loop, and it must therefore be approximated using a minor loop. There are several methods available for approximating magnetization curves, but they are primarily designed for paramagnetic materials, and are poorly suited to the highly grain-oriented steels used in modern transformers. Therefore, we propose two expressions for approximating the magnetization curves of grain-oriented silicon-iron steels. Both methods give close agreement with measurements and can be extrapolated to in order to describe the major loop.

1. Introduction
When modelling the magnetization of electromagnetic equipment such as transformers, it is important to have a hysteresis model that is accurate over the whole magnetization range. Over this range, the relative permeability varies from its nominal value, which in transformer steels is in the tens of thousands, to unity at saturation; a variation by more than four orders of magnitude.

There are several existing models for static magnetization, and many of the most common models [1] are based on the Brillouin function [2], with angular moments $m=-J,-J+1,\ldots,0,\ldots,J-1,J$:

$$B_f(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{1}{2J} x\right).$$

For large $J$ (i.e. for a large sample with a large number of possible states), the ratio of the magnetization to the saturation magnetization $M_S$ is the Langevin function with fitting parameter $\alpha$:

$$\frac{M}{M_S} = \mathcal{L}\left(\frac{B}{a}\right) = \coth\left(\frac{B}{a}\right) - \frac{a}{B}.$$  

The Brillouin function only holds for paramagnets, and does not account for hysteresis. This can partly be taken care of by replacing $B$ in eq. (2) with an effective field $B_e = \mu_0(H + fM)$, where $f$ is a constant and $\mu_0$ is the permeability of free space. Still, the underlying physics hold only when the exchange energy is negligible compared to the Zeeman energy. As a result, using the Langevin function for ferromagnets tends not to give very good curve fits, as will be shown further on in this paper. In power transformers, it is often desirable to calculate the losses to within a few percent accuracy or better, and the Langevin approximation cannot deliver this level of accuracy. Additionally, it is impractical to have an expression where the magnetization appears in both the left and right hand terms.
Finally, in engineering applications, it is often more useful to express $B(H)$ than $M(B)$. This can be done without much loss of generality since the derivation in [2] is based on minimization of energy. This minimization can be done, provided that the sample is large, using the per-volume energy $BBH$ instead of the per-electron magnetic energy $μB B$, where $μB$ is the Bohr magneton. In the rest of this paper, $B(H)$ will be given. With this convention, the magnetization curve is made hysteretic by introducing the coercive force $H_c$, and $H − H_c$ thus fulfills the role of the effective field. In transformer steels, the two branches of the major loop do not have the same shape (see Figure 1) and must be expressed separately. Thus, by introducing fitting parameters $k_1, k_2, a_1, and a_2$, Eq. (2) becomes:

$$\frac{B}{B_{sat}} - μ_0 H = \begin{cases} k_1 \left( \coth \frac{H - H_c}{a_1} - \frac{a_1}{H - H_c} \right), & \text{for } \frac{dB}{dH} < 0 \\ k_2 \left( \coth \frac{H - H_c}{a_2} - \frac{a_2}{H - H_c} \right), & \text{for } \frac{dB}{dH} > 0 \end{cases}$$

(3)

where $B_{sat}$ is the saturation flux density. It should be noted here that although $k_2$ should be equal to one, this does not, in our experience, give the best fit, and both $k_1$ and $k_2$ have therefore been left as variables. This modified Langevin function will be used as the benchmark for the other two magnetization expressions described in this paper.

2. Methods

The modified Langevin function in eq. (3) does not have the fat-tailed behavior of measured magnetization curves. This could be corrected by adding a constant slope, but doing so gives the wrong asymptotic behavior, as the slope of the $B-H$ curve should tend toward $μ_0$ at saturation.

Instead, we propose two alternative methods. Both use $B_1(x)$, the Brillouin function with $J=1$. The choice of $J=1$ is arbitrary, but is not very significant. The first method applies the correct tail behavior directly in the Brillouin function and uses an exponential function to get the correct low-flux behavior:

$$\frac{B}{B_{sat}} - μ_0 H = \begin{cases} - \left( 1 - \exp \left( - \frac{H - H_c}{c_1} \right) \right) A_1 B_1 \left( d_1 (-H + H_c)^{b_1} \right), & \text{for } \frac{dB}{dH} < 0 \\ - \left( 1 - \exp \left( - \frac{H - H_c}{c_2} \right) \right) A_2 B_1 \left( d_2 (H - H_c)^{b_2} \right), & \text{for } \frac{dB}{dH} > 0 \end{cases}$$

(4)

Here, $A_1, A_2, b_1, b_2, c_1, c_2, d_1, and d_2$ are fitting parameters.

The second model uses a probability distribution function to get the correct tail behavior:

$$\frac{B}{B_{sat}} - μ_0 H = A_1 S_{CDF}(-H; \alpha, 1, \gamma_1, -H_c; 1) B_1 \left( q_1 (H - H_c) \right) + A_2 S_{CDF}(H; \alpha, 1, \gamma_2, H_c; 1) B_1 \left( q_2 (H - H_c) \right),$$

(5)

where $A_1$ and $A_2$ are amplitude factors, and $S_{CDF}(H; \alpha, \beta, \gamma, \delta; p)$ is a stable cumulative distribution function [3] with stability parameter $\alpha$, skewness parameter $\beta$, shape parameter $\gamma$, location parameter $\delta$, and parametrization $p$. In both methods, since the function must approach $B_{sat}$ when $H$ goes to infinity, $A_2$ must be equal to one, and $A_1$ can be deduced since the two branches must be equal at the reversal point. This gives a total of seven and six unknown fitting parameters, respectively.

3. Results and discussion

The static hysteresis curve of a highly grain-oriented electrical steel (27ZDKH) was measured at peak flux densities 1.7 T, 1.8 T, and 1.9 T.

The modified Langevin expression and the two alternative methods in Eqs. (4) and (5) were fitted to the measured curves. The best fit parameters are shown in Table 1. The table also shows the RMS deviation from the measured values. As can be seen in the table, the parameters vary only slightly depending on the peak flux density. Low variation means that the values are easy to extrapolate to higher peak flux densities. The large difference between the two branches of the hysteresis curve (i.e. between subscripts 1 and 2) also shows the need for the modified Langevin function as compared to
the unmodified. The fat-tailed behavior of the magnetization curve is particularly marked close to the “knee” of the curve, as can be seen in Figure 1.

**Table 1:** Best fit parameters for the static hysteresis, using the two proposed models and the modified Langevin expression.

|               | Model 1  | Model 2  | Langevin |
|---------------|----------|----------|----------|
|               | 1.9 T    | 1.8 T    | 1.7 T    | 1.9 T    | 1.8 T    | 1.7 T    | 1.9 T    | 1.8 T    | 1.7 T    |
| $H_c$         | 6.4      | 6.2      | 5.5      | 6.4      | 6.2      | 5.5      | 6.4      | 6.2      | 5.5      |
| $b_1$         | 0.25     | 0.35     | 0.4      | $y_1$    | 0.35     | 0.35     | 0.5      | $k_3$    | 0.92     | 0.89     | 0.86     |
| $b_2$         | 0.2      | 0.2      | 0.2      | $y_2$    | 2        | 2        | 2        | $k_2$    | 0.93     | 0.9      | 0.87     |
| $c_1$         | 0.9      | 1.7      | 1.9      | $q_1$    | 0.86     | 0.45     | 0.45     | $a_1$    | 0.98     | 1.76     | 1.83     |
| $c_2$         | 3.5      | 3.5      | 3.5      | $q_2$    | 0.45     | 0.45     | 0.45     | $a_2$    | 2.99     | 2.63     | 2.69     |
| $d_1$         | 1.09     | 0.86     | 0.8      | $\alpha$ | 0.45     | 0.45     | 0.45     |          |          |          |
| $d_2$         | 0.82     | 0.82     | 0.82     |          |          |          |          |          |          |          |

$\Delta B_{rms}$ [mT] 30 50 33 23 31 21 44 44 28

**Figure 1:** Measured static hysteresis at 1.9 T peak flux density and approximated curves using the modified Langevin expression and using model 2. Note that the two branches do not have the same shape.

**Figure 2:** Deviation from measured static hysteresis at 1.9 T peak flux density for approximated curves using the modified Langevin expression and the proposed models.

The deviation of the fits from the measured values are shown in Figure 2. Here, it can be seen that the Langevin function gives an incorrect slope at high fields, while Model 2 agrees with the measurement to within 0.02 T for most values of $H$. Model 1 deviates more from the measured than Model 2 does, and the slope is not correct at high fields, as evidenced by the non-constant difference. All models give large errors close to the zero-crossing. This is because the slope is very large, and a small error in the model or in the measurement gives a large deviation.

It should be emphasized that both these methods, while based on valid theoretical models for paramagnets, are not intended to explain the true magnetization behavior of the steel, but merely to provide a highly accurate fit to measured values. In this sense, Model 2 is the most successful.

**References**

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