Abstract

We study the semileptonic decays $B_c \rightarrow B_n \ell^+ \nu_\ell$ where $B_{c(n)}$ is the anti-triplet-charmed (octet) baryon with the $SU(3)_f$ flavor symmetry and helicity formalism. In particular, we present the decay branching ratios of $B_c \rightarrow B_n \ell^+ \nu_\ell$ in three scenarios: (a) an exact $SU(3)_f$ symmetry with equal masses for the anti-triplet-charmed (octet) baryon states of $B_c$ ($B_n$), (b) $SU(3)_f$ parameters without the baryonic momentum-transfer dependence, and (c) $SU(3)_f$ with baryonic transition form factors in the heavy quark limit. We show that our results are all consistent with the existing data. Explicitly, we predict that $B(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e) = (11.9 \pm 1.3, 9.8 \pm 1.1, 10.7 \pm 0.9) \times 10^{-2}$ and $B(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.0 \pm 0.3, 2.4 \pm 0.3, 2.7 \pm 0.2) \times 10^{-2}$ in the scenarios (a), (b) and (c) agree with the data of $(14.0^{+8.3}_{-8.5}) \times 10^{-2}$ and $(5.6 \pm 2.6) \times 10^{-2}$ from the CLEO Collaboration, respectively. In addition, we obtain that $B(\Lambda_c^+ \rightarrow ne^+ \nu_e) = (2.8 \pm 0.4, 4.9 \pm 0.4, 5.1 \pm 0.4) \times 10^{-3}$ in (a), (b) and (c).

We also examine the longitudinal asymmetry parameters of $\alpha(B_c \rightarrow B_n \ell^+ \nu_\ell)$, which are sensitive to the different scenarios with $SU(3)_f$. Some of the decay branching ratios and asymmetries can be observed by the ongoing experiments at BESIII and LHCb as well as the future searches by BELLEII.
I. INTRODUCTION

Very recently, the absolute branching ratio of \( B(\Xi_c^0 \to \Xi^- \pi^+) = (1.8 \pm 0.5) \times 10^{-2} \) has been measured for the first time by the Belle collaboration [1], which is the golden mode in \( \Xi_c^0 \) decays. In fact, there have been significant experimental progresses in observing weak decays of charmed baryons [2]. It is no doubt that we are witnessing a new era of charm physics. On the other hand, theoretical studies of charmed baryon decays have faced many difficulties. For instance, the complicated structures of these baryons with large non-perturbative effects of the quantum chromodynamic (QCD) make us impossible to reliably calculate their decays amplitudes from first principles. Fortunately, there is a very powerful tool to explore the charmed baryon decays based on the flavor symmetry of \( SU(3)_f \) in the quark model, which is a model independent way to connect various decay channels. Recently, several theoretical analyses of two and three-body non-leptonic processes of charmed baryons have been performed in the literature [3–10] based on the newly measured decay branching fractions. In particular, the prediction of \( B(\Xi_c^0 \to \Xi^- \pi^+) = (1.6 \pm 0.1) \times 10^{-2} \) from the \( SU(3)_f \) approach [6] is consistent with the measurement by Belle [1]. As a result, we are confidence that the use of \( SU(3)_f \) is a good method to examine the weak decays of charmed baryons.

It is known that the semileptonic decays of charmed baryons are the cleanest processes as they can be calculated through the QCD factorization approach. Hence, these decays are good platforms to test \( SU(3)_f \) and identify the corresponding breaking effects. Besides the total branching ratios of these semileptonic decays, the angular distribution asymmetries, which contain the information of the underlined dynamics of the decays, can also constrain the theoretical QCD models. In Refs. [11–14], the helicity formalism has been used to analyze the angular properties of both parent and daughter baryons in the baryonic decays. However, theoretical considerations for the asymmetrical parameters in the semileptonic decays of charmed baryons with the \( SU(3)_f \) symmetry have not been systematically examined yet even though they could be well measured experimentally. Currently, there are only three experimental data for the semileptonic decays of \( \Lambda_c \), given by [2]

\[
B(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.6 \pm 0.4) \times 10^{-2}, \\
B(\Lambda_c^+ \to \Lambda \mu^+ \nu_\mu) = (3.5 \pm 0.5) \times 10^{-2}, \\
\langle \alpha \rangle(\Lambda_c^+ \to \Lambda e^+ \nu_e) = -0.86 \pm 0.04 ,
\]

(1)
where $\alpha$ is the longitudinal asymmetrical parameter. The decay of $\Lambda_c^+ \to \Lambda e^+ \nu_e$ has been extensively studied in the literature. In particular, its decay branching ratio has been found to be $(1.42, 1.63, 2.78, 2.96, 3.6, 3.8) \times 10^{-2}$ by the heavy quark effective theory (HQET) along with the non-relativistic quark model (NRQM) [15], QCD light front (LF) approach [17], covariant quark model (CQM) [18, 19], MIT bag model (MBM) [20], NRQM [20] and lattice QCD (LQCD) [22, 23]. Clearly, the predicted values in the models except NRQM and LQCD are inconsistent with the experimental data in Eq. (1). The $SU(3)_f$ structures for the semileptonic decay amplitudes of charmed baryons are quiet simple. In fact, all decay branching ratios and asymmetries are related by one $SU(3)_f$ parameter, which can be determined by the experimental data in Eq. (1) [3, 4]. In this study, besides imposing $SU(3)_f$ in the decay amplitudes [3, 10], we also consider $SU(3)_f$ to be held in each baryonic transition form factor, which describes the non-perturbative QCD effect in the decay processes to include the mass effect in phase space integration. We will systematically examine all semileptonic decays of the anti-triplet charmed baryons.

This paper is organized as follows. In Sec. II, we write down decay widths and asymmetrical parameters in terms of the helicity formalism. In Sec. III, after including the mass corrections in the phase space integrations and imposing the spin symmetry in the heavy quark limit to reduce the $SU(3)_f$ parameters, we show our numerical results. We present the conclusions in Sec. IV.

II. DECAY BRANCHING RATIOS AND ASYMMETRIES

We concentrate on the semileptonic decays of $B_c \to B_n \ell^+ \nu_\ell$, where $B_c$ and $B_n$ are anti-triplet charmed and octet baryon states under the $SU(3)_f$ flavor symmetry, defined by

$$B_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c),$$

$$B_n = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda \end{pmatrix},$$

while $\ell = e, \mu$ and $\nu_\ell$ are the charged and neutral leptons, respectively. The decay transition amplitudes are written as

$$\mathcal{A}(B_c \to B_n \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq} \langle B_n | J_{\mu}^{(V-A)} | B_c \rangle \bar{u}_\nu \gamma^\mu (1 - \gamma_5) \nu_\ell,$$
where \( J^{(V-A)}_\mu = \bar{q} \gamma_\mu (1 - \gamma_5)/2c, q = d, s, \bar{u}_\nu \) and \( \nu_\ell \) are Dirac bispinors, \( G_F \) is Fermi constant, and \( V_{cq} \) are the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements.

Since the lepton part can be traced back to the off-shell W-boson (\( W_{os} \)), we can introduce a set of helicity vectors \( \epsilon^\mu(\lambda_W) \) in the \( B_c \) rest frame, given by

\[
\epsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}} (0, \pm 1, -i, 0) \\
\epsilon^\mu(0) = \frac{1}{\sqrt{q^2}} (p, 0, 0, -q_0)
\]

where \( \lambda_W = (\pm 1, 0) \) and \( \lambda_W = t \), denoting \( J = 1 \) and 0 parts of \( W_{os} \), respectively, and \( q^\mu = (q_0, 0, 0, p) \) is the four momentum of \( W_{os} \) with \( m_\ell^2 < q^2 < (M_{B_c} - M_{B_n})^2 \) and

\[
p = \frac{1}{2M_{B_c}} \sqrt{Q_+ Q_-}, \\
Q_\pm = (M_{B_c} \pm M_{B_n})^2 - q^2,
\]

with \( M_{B_c(n)} \) being the mass of \( B_c(n) \). Hence, we can decompose the total transition amplitudes into helicity ones \[14\], given by

\[
A(B_c \rightarrow B_n \ell^+ \nu_\ell) = \sum_{\lambda_W, \lambda_W'=\pm 1, 0, t} \frac{G_F}{\sqrt{2}} V_{cq} H_{\lambda_2 \lambda_W} \bar{u}_\nu \gamma_\beta \frac{(1 - \gamma_5)}{2} \nu_\ell \epsilon^{*\beta}(\lambda_W) g_{\lambda_W \lambda_W'},
\]

\[
H_{\lambda_2 \lambda_W} = H_{\lambda_2 \lambda_W}^V - H_{\lambda_2 \lambda_W}^A, \quad H_{\lambda_2 \lambda_W}^{V(A)} = \langle B_n | J^{(V(A))}_\mu | B_c \rangle \epsilon^\mu(\lambda_W),
\]

where \( \lambda_{2(W)} \) corresponds to the helicity of the daughter baryon (\( W_{os} \)). Note that the helicity of the parent baryon is fixed by \( \lambda_1 = \lambda_2 - \lambda_W \), while \( H_{\lambda_2 \lambda_W}^{V(A)} = (-)H_{-\lambda_2 - \lambda_W}^{V(A)} \) under the parity transformation. We can also write the helicity amplitudes in terms of the invariant baryonic transition form factors \[14\], given by

\[
H_{\frac{1}{2}1}^V = \sqrt{2Q_-} \left( -F_1^V - \frac{M_{B_c} + M_{B_n}}{M_{B_c}} F_2^V \right), \\
H_{\frac{1}{2}0}^V = \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left( (M_{B_c} + M_{B_n}) F_1^V + \frac{q^2}{M_{B_c}} F_2^V \right), \\
H_{\frac{1}{2}1}^A = \sqrt{2Q_+} \left( F_1^A - \frac{M_{B_c} - M_{B_n}}{M_{B_c}} F_2^A \right), \\
H_{\frac{1}{2}0}^A = \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left( -(M_{B_c} - M_{B_n}) F_1^A + \frac{q^2}{M_{B_c}} F_2^A \right), \\
H_{\frac{1}{2}1}^A = \sqrt{\frac{Q_-}{q^2}} \left( -(M_{B_c} + M_{B_n}) F_1^A + \frac{q^2}{M_{B_c}} F_2^A \right) (7)
\]
where $F_{1,2,3}^{V,A}$ are defined by

$$
\langle B_{n}|J_{\mu}^{V}|B_{c}\rangle = \bar{u}_{B_{c}}(p_{B_{c}}) \left[ F_{1}^{V}(q^{2})\gamma_{\mu} - \frac{F_{2}^{V}(q^{2})}{M_{B_{c}}} i\sigma_{\mu\nu}q^{\nu} + \frac{F_{3}^{V}(q^{2})}{M_{B_{c}}} q_{\mu} \right] u_{B_{c}}(p_{B_{c}}),
$$

$$
\langle B_{n}|J_{\mu}^{A}|B_{c}\rangle = \bar{u}_{B_{c}}(p_{B_{c}}) \left[ F_{1}^{A}(q^{2})\gamma_{\mu} - \frac{F_{2}^{A}(q^{2})}{M_{B_{c}}} i\sigma_{\mu\nu}q^{\nu} + \frac{F_{3}^{A}(q^{2})}{M_{B_{c}}} q_{\mu} \right] \gamma_{5} u_{B_{c}}(p_{B_{c}}).
$$

where the $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_{\mu}, \gamma_{\nu}]$. We now parameterize baryon states and quark operators into SU(3) tensor forms, while the polarization vectors $\epsilon_{\mu}$ are invariant under SU(3) transformation. As a result, the transition operators of $(\bar{q}c)^{(V)}_{\lambda_{W}}$ are transformed as an anti-triplet (3), denoted as $T(3) = (0, 1, 1)$, under SU(3). Consequently, the helicity amplitudes can be rewritten as

$$
H_{\lambda_{2}\lambda_{W}}^{V(A)} = a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2}) \langle B_{n}\rangle_{j}^{i} T^{j}(3)_{i}.
$$

With the SU(3) symmetry, the connections between the helicity amplitudes of different channels are presented in Table I. Since the baryon transition matrix of $\langle B_{n}|(J_{\mu}^{V-A})|B_{c}\rangle$ is invariant under the CP transformation, all SU(3) parameters $a_{\lambda_{W}}^{V(A)}(q^{2})$ with the same helicity quantum number are real up to an overall phase.

**TABLE I.** Helicity amplitudes of $B_{c} \rightarrow B_{n}\ell^{+}\nu_{\ell}$ with SU(3).

| channel | $H_{\lambda_{2}\lambda_{W}}^{V(A)}$ |
|----------|----------------------------------|
| $\Lambda_{c}^{+} \rightarrow \Lambda\ell^{+}\nu_{\ell}$ | $-\sqrt{\frac{3}{2}} a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Xi_{c}^{+} \rightarrow \Xi^{0}\ell^{+}\nu_{\ell}$ | $-a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Xi_{c}^{0} \rightarrow \Xi^{-}\ell^{+}\nu_{\ell}$ | $a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Lambda_{c}^{+} \rightarrow n\ell^{+}\nu_{\ell}$ | $a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Xi_{c}^{+} \rightarrow \Sigma^{0}\ell^{+}\nu_{\ell}$ | $\sqrt{\frac{3}{2}} a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Xi_{c}^{+} \rightarrow \Lambda\ell^{+}\nu_{\ell}$ | $-\sqrt{\frac{1}{3}} a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |
| $\Xi_{c}^{0} \rightarrow \Sigma^{-}\ell^{+}\nu_{\ell}$ | $a_{\lambda_{2}\lambda_{W}}^{V(A)}(q^{2})$ |

The differential decay widths of the semileptonic processes can be written as analytic forms, given by

$$
\frac{d\Gamma}{dq^{2}} = \frac{1}{3} \frac{G_{F}^{2}}{(2\pi)^{3}} |V_{qe}|^{2} \frac{(q^{2} - m_{q}^{2})^{2}p}{8M_{B_{c}}^{2} q^{2}} \left( 1 + \frac{m_{q}^{2}}{2q^{2}} \right)
$$

$$
\left( |H_{\frac{1}{2}_{1}}^{2}|^{2} + |H_{\frac{1}{2}_{-1}}^{2}|^{2} + |H_{\frac{1}{2}_{0}}^{2}|^{2} + |H_{\frac{5}{2}_{0}}^{2}|^{2} + |H_{\frac{3}{2}_{\frac{1}{2}}}^{2}|^{2} \right) + \frac{3m_{q}^{2}}{2q^{2}} \left( |H_{\frac{5}{2}_{0}}^{2}|^{2} + |H_{\frac{3}{2}_{-\frac{1}{2}}}^{2}|^{2} \right).
$$

1 The formula of invariant form factors used by CLEO can be found in Ref. 11, which have opposite signs in front of $F_{2}^{V,A}$ compared with Eq. (8), so that there is a sign difference between our result and that by CLEO for $r = F_{2}/F_{1}$. 

5
In the limit of \( m_\ell = 0 \), \( H_{\lambda_2\lambda_W}^{V(A)} \) has no contribution to the branching ratios due to the helicity suppression by the factor of \( m_\ell^2/M_{B_c}^2 \) as given by Refs. [13, 14]. Since all \( H_{\lambda_2\lambda_W}^{V(A)} \) for the decays correspond to the same \( SU(3)_f \) parameters of \( a_{\lambda_2\lambda_W}^{V(A)}(q^2) \), there are several direct relations for the differential decay widths before integrating over the \( q^2 \) dependences, given by

\[
\begin{align*}
d\Gamma(\Lambda_c^+ \to \Lambda\ell^+\nu_\ell) & = \frac{2p_{\Lambda}}{3p_{\Xi^+}} d\Gamma(\Xi_c^+ \to \Xi^0\ell^+\nu_\ell) = \frac{2p_{\Lambda}}{3p_{\Xi^+}} d\Gamma(\Xi_c^0 \to \Xi^-\ell^+\nu_\ell), \\
d\Gamma(\Lambda_c^+ \to n^0\ell^+\nu_\ell) & = \frac{2p_n}{p_{\Sigma^0}} d\Gamma(\Xi_c^+ \to \Sigma^0\ell^+\nu_\ell) = \frac{6p_n}{p_{\Lambda}} d\Gamma(\Xi_c^+ \to \Lambda\ell^+\nu_\ell) \equiv \frac{p_n}{p_{\Xi^-}} d\Gamma(\Xi_c^0 \to \Sigma^-\ell^+\nu_\ell),
\end{align*}
\]

(11)

for the \( c \to (s, d) \) transitions, respectively, where the masses of the charmed baryons are set to be equal, \( p_{B_n} \) is defined in Eq. [5] with different masses for the octet baryon. Under the exact \( SU(3)_f \) limit, \( M_{\Lambda_c} = M_{\Xi_c^+} = M_{\Xi^0} \) and \( M_\Lambda = M_n = M_\Sigma = M_{\Sigma^0} = M_{\Xi^+} = M_{\Xi^-} = M_{\Xi^0} \) so that \( p_{B_n}/p_{B_\Lambda} = 1 \) along with the same phase space volume. Hence, without knowing the \( q^2 \) dependences of \( a_{\lambda_2\lambda_W}^{V(A)}(q^2) \), we can use one experimental data to derive all other branching ratios.

In the semileptonic decay of \( B_c \to B_n\ell^+\nu_\ell \), one can write the asymmetrical parameter \( \alpha(B_c \to B_n\ell^+\nu_\ell) \) [11, 13], which is also known as the longitudinal polarization of the daughter baryon, defined by

\[
\alpha(q^2) = \frac{(1 + \frac{m_\ell^2}{2q^2}) \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 - |H_{\frac{1}{2}\frac{1}{2}}|^2 + |H_{\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2 \right) + \frac{3m_\ell^2}{2q^2} \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 - |H_{\frac{1}{2}0}|^2 \right)}{(1 + \frac{m_\ell^2}{2q^2}) \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 - |H_{\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2 \right) + \frac{3m_\ell^2}{2q^2} \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 + |H_{\frac{1}{2}0}|^2 \right)} \cdot (13)
\]

Consequently, from Eq. (10) one can also define the integrated (averaged) asymmetry by [19, 21]

\[
\langle \alpha \rangle = \frac{\int dq^2 \left( \frac{q^2 - m_\ell^2}{8m_{B_c}^2 q^2} \right) \left( (1 + \frac{m_\ell^2}{2q^2}) \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 - |H_{\frac{1}{2}\frac{1}{2}}|^2 + |H_{\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2 \right) + \frac{3m_\ell^2}{2q^2} \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 - |H_{\frac{1}{2}0}|^2 \right) \right)}{\int dq^2 \left( \frac{q^2 - m_\ell^2}{8m_{B_c}^2 q^2} \right) \left( (1 + \frac{m_\ell^2}{2q^2}) \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 + |H_{\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2 \right) + \frac{3m_\ell^2}{2q^2} \left( |H_{\frac{1}{2}\frac{1}{2}}|^2 + |H_{\frac{1}{2}0}|^2 \right) \right)}.
\]

If \( SU(3)_f \) is an exact symmetry, the parameter of \( \alpha \) is equal for all decay modes. Clearly, it is a good observable to test the \( SU(3)_f \) flavor symmetry.

III. NUMERICAL RESULTS

We first show our numerical results based on the exact \( SU(3)_f \) flavor symmetry with the same mass for all anti-triplet charmed (octet) baryon states in the second column of
TABLE II. Decay branching ratios of $B_c \rightarrow B_n \ell^+ \nu_\ell$ with $SU(3)_f$ for (a) equal masses of $B_c$ ($B_n$), (b) $a^{V(A)}_{\lambda W}(q^2) = \text{constant}$ and (c) the baryonic transition form factors in the heavy quark limit.

| Branching ratio | $SU(3)_f$ | $SU(3)_f$ | $SU(3)_f$ | HQET | LF | MBM(NRQM) | LQCD | Data |
|-----------------|-----------|-----------|-----------|------|----|-----------|------|------|
| $10^2 B(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e)$ | $3.6 \pm 0.4$ | $3.6 \pm 0.4$ | $3.2 \pm 0.3$ | 1.42 | 1.63 | 2.96(3.60) | 3.80(3.60) | 3.6 ± 0.4 |
| $10^2 B(\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu)$ | $3.5 \pm 0.5$ | $3.6 \pm 0.4$ | $3.2 \pm 0.3$ | - | - | - | 3.69 ± 0.22 | 3.5 ± 0.5 |
| $10^2 B(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e)$ | $11.9 \pm 1.3$ | $9.8 \pm 1.1$ | $10.7 \pm 0.9$ | - | 5.39 | 1.33(1.01) | - | 14.0^{+1.8}_{-1.7} |
| $10^2 B(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu)$ | $11.6 \pm 1.7$ | $9.8 \pm 1.1$ | $10.8 \pm 0.9$ | - | - | - | - |
| $10^2 B(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)$ | $3.0 \pm 0.3$ | $2.4 \pm 0.3$ | $2.7 \pm 0.2$ | 0.86 | 1.35 | 0.40(0.30) | - | 5.6 ± 2.6 |
| $10^2 B(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu)$ | $2.9 \pm 0.4$ | $2.4 \pm 0.3$ | $2.7 \pm 0.2$ | - | - | - | - |
| $10^3 B(\Lambda_c^+ \rightarrow ne^+ \nu_e)$ | $2.8 \pm 0.4$ | $4.9 \pm 0.5$ | $5.1 \pm 0.4$ | - | 2.01 | 2.20(3.40) | 4.10(3.09) | - |
| $10^3 B(\Xi_c^+ \rightarrow \Sigma^0 e^+ \nu_e)$ | $3.1 \pm 0.4$ | $3.8 \pm 0.4$ | $4.6 \pm 0.4$ | - | 1.87 | 4.42(4.42) | - | - |
| $10^4 B(\Xi_c^+ \rightarrow \Lambda e^+ \nu_e)$ | $10.3 \pm 1.5$ | $16.6 \pm 1.8$ | $21.8 \pm 1.8$ | - | 8.22 | 8.84(8.84) | - | - |
| $10^4 B(\Xi_c^0 \rightarrow \Sigma^- e^+ \nu_e)$ | $15.7 \pm 2.2$ | $19.2 \pm 2.1$ | $23.2 \pm 1.9$ | - | 9.47 | 2.24(1.12) | - | - |

Table III. We then consider the mass effects from the $q^2$ integrations, in which the helicity amplitudes still preserve $SU(3)_f$. We treat the $SU(3)_f$ parameters as constants without the $q^2$ dependences, i.e. $a^{V(A)}_{\lambda W}(q^2) = \text{constant}$, and present the decay branching ratios in the third column of Table III.

In order to get more precise numerical values, we use the relations between the helicity amplitudes and invariant form factors in Eq. (17). Since these form factors have the same relations as those in the exact $SU(3)_f$ limit, they also preserve the $SU(3)_f$ symmetry. When we treat the charm quark to be much heavier than other quarks in $B_c$, we can apply the spin symmetry in this heavy quark limit (HQL) to derive $F_i^V(q^2) = F_i^A(q^2) = F_i(q^2)$ in the $B_c \rightarrow B_n$ transitions [11, 24]. In the following discussions, we will take the HQL in our numerical calculations. To illustrate our results, we choose the dipole behavior for $F_i(q^2)$ as used by CLED in Ref. [16], given by

$$F_i(q^2) = \frac{F_i}{(1 - q^2/M_V^2)^2} ,$$

where $F_i \equiv F_i(q^2 = 0)$ and $M_V = 2.061$ GeV, which is the average mass of the lowest excited $D$ and $D_s$ mesons with quantum numbers of $J^P = 1^-$. By using the experimental
data of $\langle \alpha \rangle (\Lambda_c^+ \to \Lambda e^+\nu_e) = -0.86 \pm 0.04$ in Eq. (1), we can fix the ratio of $r = F_2/F_1$, which is plotted in Fig. 1 with two possible solutions. Since the absolute value of $r$ is expected to be smaller than 1, we select the solution $r = 0.40^{+0.12}_{-0.11}$. We also perform the minimum $\chi^2$ method to fit $F_1$ and $F_2$ with two measured branching ratios of $\Lambda_c \to \Lambda \ell^+\nu_\ell$ ($\ell = e, \mu$) and one average asymmetry parameter in Eq. (1) along with $B(\Xi^0 \to \Xi^- e^+\nu_e) = (2.4 \pm 0.3) \times 10^{-2}$ predicted by $SU(3)_f$ given by the first and second columns in Table II. We obtain that $(F_1, F_2) = (0.62 \pm 0.03, 0.25 \pm 0.08)$ and $\chi^2/d.o.f = 1.2$ with $d.o.f = 2$. It is interesting to see that the ratio of $r = 0.40 \pm 0.11$ from the $\chi^2$ fitting is consistent with the value from the direct calculation. Our results for the decay branching ratios with $SU(3)_f$ and the baryonic transition form factors in the HQL are shown in the 4th column of Table II.

In Table II the data of $B(\Xi^0 \to \Xi^- e^+\nu_e) = (5.6 \pm 2.6) \times 10^{-2}$ is derived from the ratio of $B(\Xi^0 \to \Xi^- e^+\nu_e)/B(\Xi^0 \to \Xi^- \pi^+) = 3.1 \pm 1.1$ given by the CLEO Collaboration.\footnote{We note that as $F_1$ and $F_2$ are correlated, the correlation coefficient is found to be 0.622, which will be used to evaluate the errors of our results in the fit.}
TABLE III. Averaged decay asymmetries of $\mathbf{B}_c \to \mathbf{B}_n \ell^+ \nu_\ell$ with $SU(3)_f$ and the baryonic transition form factors in the heavy quark limit.

| Channel       | Asymmetry $\langle \alpha \rangle$ |
|---------------|-------------------------------------|
| $\Lambda_c^+ \to \Lambda \ell^+ \nu_\ell$ | $-0.86 \pm 0.04$ |
| $\Xi_c^+ \to \Xi^0 \ell^+ \nu_\ell$     | $-0.83 \pm 0.04$ |
| $\Xi^0_c \to \Xi^- \ell^+ \nu_\ell$     | $-0.83 \pm 0.04$ |
| $\Lambda_c^+ \to n \ell^+ \nu_\ell$     | $-0.89 \pm 0.04$ |
| $\Xi_c^+ \to \Sigma^0 \ell^+ \nu_\ell$  | $-0.85 \pm 0.04$ |
| $\Xi_c^+ \to \Lambda \ell^+ \nu_\ell$   | $-0.86 \pm 0.04$ |
| $\Xi^0_c \to \Sigma^- \ell^+ \nu_\ell$  | $-0.85 \pm 0.04$ |

and the recent measurement of $\mathcal{B}(\Xi^0_c \to \Xi^- \pi^+) = 1.8 \pm 0.5$ by Belle [1], while that of $\mathcal{B}(\Xi^+_c \to \Xi^0 e^+ \nu_e) = (14.0^{+8.3}_{-8.0}) \times 10^{-2}$ is resulted from the data of $\mathcal{B}(\Xi^+_c \to \Xi^0 e^+ \nu_e)/\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = 2.3^{+0.7}_{-0.8}$ [2, 25], and the extracted value of $\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (6.2 \pm 3.1) \times 10^{-2}$ [8] from the data of $\mathcal{B}(\Xi^+_c \to \Sigma^- \pi^+ \pi^+)/\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = 0.18 \pm 0.09$ [2].

As shown in Table III all of results with $SU(3)_f$ agree well with the existing data. We note that the predicted value of $\mathcal{B}(\Lambda_c^+ \to ne^+ \nu_e) = (2.93 \pm 0.34) \times 10^{-3}$ with $SU(3)_f$ in Ref. [4] corresponds to our result of $(2.8 \pm 0.4) \times 10^{-3}$ in Table II(a) with the exact $SU(3)_f$ symmetry, while that of $4.1 \times 10^{-3}$ by the LQCD is consistent with our results in Table II(b) and (c). In addition, it is interesting to see that, after multiplying a factor of 2 the LF results in Ref. [17], which apparently do not agree with the experimental data, almost match up our predicted values in Table II. Moreover, we remark that our value for the inclusive decay branching ratio of $\mathcal{B}(\Lambda_c^+ \to Xe^+ \nu_e) \sim 3.9 \times 10^{-2}$ is also consistent with the recent data of $\mathcal{B}(\Lambda_c^+ \to Xe^+ \nu_e) = (3.95 \pm 0.35) \times 10^{-2}$ measured by BESIII [26].

Our results for $\langle \alpha \rangle(\mathbf{B}_c \to \mathbf{B}_n \ell^+ \nu_\ell)$ based on the $SU(3)_f$ symmetry and baryonic form factors with the HQL are listed in Table III. From Table III it is interesting to see that the asymmetry parameters in the muon modes are quite different from the corresponding electron ones. We note that in the scenarios of (a) and (b) with $SU(3)_f$, all values of $\langle \alpha \rangle(\mathbf{B}_c \to \mathbf{B}_n \ell^+ \nu_\ell)$ are found to be $-0.86 \pm 0.04$ fitted by the data of $\langle \alpha \rangle(\Lambda_c^+ \to \Lambda e^+ \nu_e)$ in Eq. (1) because of the cancellation between numerator and denominator integration values.
IV. CONCLUSIONS

We have studied the semileptonic decays $B_c \to B_n \ell^+ \nu_\ell$ with the $SU(3)_f$ flavor symmetry and helicity formalism. We have considered the baryonic transition form factors to include the mass effects in the $q^2$ integrations. In particular, we have concentrated on three scenarios:

(a) an exact $SU(3)_f$ symmetry, in which the masses of the anti-triplet-charmed (octet) baryon states of $B_c$ ($B_n$) are equal, (b) $SU(3)_f$ parameters without the baryonic momentum-transfer dependence, i.e., $a^{V(A)}_A(\mu^2) = \text{constant}$, and (c) $SU(3)_f$ with baryonic transition form factors in the HQL, so that $F_i^V(q^2) = F_i^A(q^2) = F_i(q^2)$. We have demonstrated that our results are all consistent with the current data. Explicitly, we have found that $B(\Xi^+_c \to \Xi^0 \ell^+ \nu_\ell) = (11.9 \pm 1.3, 9.8 \pm 1.1, 10.7 \pm 0.9) \times 10^{-2}$ and $B(\Xi^+_c \to \Xi^- e^+ \nu_e) = (3.0 \pm 0.3, 2.4 \pm 0.3, 2.7 \pm 0.2) \times 10^{-2}$ in the scenarios (a), (b) and (c), which are lower than but consistent with the data of $(14.0^{+8.3}_{-8.7}) \times 10^{-2}$ and $(5.6 \pm 2.6) \times 10^{-2}$ from the CLEO Collaboration, respectively. We have predicted that $B(\Lambda^+_c \to n e^+ \nu_e) = (2.8 \pm 0.4, 4.9 \pm 0.4, 5.2 \pm 0.4) \times 10^{-3}$ in (a), (b) and (c), which agree with the previous analysis with $SU(3)_f$ in Ref. [4] as well as that the LQCD [22, 23]. This mode can be observed at the experiments by BELLE and BESIII.

We have also explored the longitudinal polarization asymmetries in $\langle \alpha \rangle (B_c \to B_n \ell^+ \nu_\ell)$. These asymmetries are good observables to test $SU(3)_f$ as they are sensitive to the different scenarios. We have given that $\langle \alpha \rangle (B_c \to B_n \ell^+ \nu_\ell)$ are around $-0.83 \sim -0.89$ for both $\ell = (e, \mu)$ [19, 21].

Finally, we remark that the semileptonic decays of $\Xi^{+0}_c$ are accessible to not only the current experimental charmed facilities, but the future ones, such as BELLEII and upgraded BESIII.

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