Natural CP Violation Criteria for the Minimal Supersymmetric Standard Model

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Abstract

CP violating effects in the Minimal Supersymmetric Standard Model (MSSM) can lead to excessively large contributions to the neutron EDM ($d_n$). We write criteria which ensure that the low energy supergravity (SUGRA) parametrization of the MSSM does not require fine-tunings or large mass scales to evade the constraint from $d_n$, and consider the implications on SUGRA theories. In particular, we show that in the Polonyi model, two of the mass scales are in general complex, meaning that model does not naturally avoid a large $d_n$ as is sometimes claimed.

1 Introduction

Supersymmetric (SUSY) theories are perhaps the most widely considered extensions to the Standard Model (SM) [1]. This is in large measure due to the fact that they are the only known perturbative solutions to the naturalness problem [3]. Since this means SUSY theories are attractive partially because they remove fine-tunings present in the SM (in quadratically divergent radiative corrections to the Higgs mass), we believe that new fine-tunings should
not be introduced to satisfy other phenomenological constraints. We also would like to be able to break $SU_2 \times U_1$ radiatively. Both of these considerations lead one to conclude that large superpartner masses are disfavored.

It is well known [5]-[16] that for moderate mass scales, SUSY predicts a neutron electric dipole moment ($d_n$) of order $10^{-22-23}\tilde{\varphi}$ e cm, where $\tilde{\varphi}$ is some combination of SUSY phases. Since the experimental upper bound on $d_n$ is extremely small, now below $10^{-25}$e cm [17], this constitutes a fine-tuning problem. One would like a set of criteria which ensures that a given model avoids this ‘SUSY $d_n$ problem’ naturally. Our approach is to write these criteria in a way which is useful for supergravity (SUGRA) model building. Using our criteria, it will be easy to show that the Polonyi model of SUGRA does not naturally avoid a large $d_n$ as is sometimes claimed.

We write the superpotential of the Minimal Supersymmetric Standard Model (MSSM) as

$$W = W_Y + \mu H_u \times H_d,$$

(1)

where $H_u$ and $H_d$ are Higgs doublet superfields, and $W_Y$ contains the Yukawa sector of the theory. The Higgs mixing coefficient $\mu$ can be complex. There are also complex phases in the soft breaking potential ($\mathcal{L}_{soft}$). We will use the low energy supergravity parametrization of $\mathcal{L}_{soft}$ [18]:

$$-\mathcal{L}_{soft} = |m_i|^2|\varphi_i|^2 + \left(\frac{1}{2} \sum_\lambda \tilde{m}_\lambda \lambda \lambda + A m_0^*[W_Y]_\varphi + B m_0^* [\mu H_u \times H_d]_\varphi + h.c.\right).$$

(2)

where $\varphi_i$ are the scalar superpartners, $\lambda$ are the gauginos, and $[\ ]_\varphi$ means take the scalar part. The parameters $A$ and $B$, their associated mass scale
$m_0$, and the common gaugino mass $\tilde{m}_\lambda$, are in general complex, and thus contribute to $CP$ violating effects. The phase of $m_0$ is often overlooked. We could lump $m_0$ into the definitions of $A$ and $B$, by defining $\tilde{A} \equiv Am_0^*$ and $\tilde{B} \equiv Bm_0^*$, but that obscures the fact that $A$ and $m_0$ come from different places in the SUGRA theory, while $A$ and $B$ are simply related by $B = A - 1$ [13]. As a dramatic illustration of this point, we will show in Section 4 that the Polonyi model of SUGRA gives $A$ and $B$ real, but $m_0$ is complex, so that $\tilde{A}$ and $\tilde{B}$ are both complex and thus contribute to $d_n$.

The largest SUSY contributions to $d_n$ tend to come from squark mixing in gluino-squark loops, as in Figure 1. The LR mixing pieces of the down squark mass matrix can be written as $M_{LR}^2 = (A^*m_0 - \mu v_u/v_d) \tilde{M}_D$, where $v_{u,d}$ are the Higgs vacuum expectation values (VEVs), and $\tilde{M}_D$ is the diagonal down quark mass matrix. Note that several key references omit the $\mu$ term [10, 14, 19, 20, 21]. Figure 1 gives [14]

$$d_n(\tilde{g}) = \left(\frac{8 e\alpha_s}{27\pi}\right) I\left(\frac{\tilde{m}_g^2}{\tilde{m}_d^2}\right) \frac{\text{Im}[(A^*m_0 - \mu v_u/v_d) \tilde{m}_g]}{\tilde{m}_d^4} m_d,$$

where $\tilde{m}_g$ is the gluino mass, $m_d$ is the current down quark mass, and $\tilde{m}_d$ is the average down squark mass. The integral $I(X)$ [22],

$$I(X) = \frac{1}{(1 - X)^2} \left[\frac{1}{2} (1 + X) + \frac{X}{1 - X} \ln X\right],$$

appears incorrectly in [14] but is corrected in [20]. Note that some papers which cite [14] quote the incorrect integral [10, 11]. Also note that [10, 14] write $d_n$ in a way which gives the appearance that $d_n$ is proportional to the mass squared difference between the squarks. Their expressions are correct,
but misleading, because the mixing angle between the squarks goes as the inverse of that mass difference, leaving $d_n$ of the form given above.

The two phases which come into (3) are $\varphi_{A\lambda} \equiv \text{Arg}(A^* m_0 \tilde{m}_\lambda)$, and $\varphi_{\mu\lambda} \equiv \text{Arg}(\mu v_u/v_d \tilde{m}_\lambda)$. We have used the fact that the phase of the gluino mass is that of the common gaugino mass $\tilde{m}_\lambda$. We define a new SUSY mass scale, $\tilde{M}^2 \equiv \tilde{m}_d^4/|m_0|\tilde{m}_g$, so as to group all of the SUSY mass scale behavior in one place. We expect this $\tilde{M}$ to be of order the weak scale, because larger values can cause fine-tuning problems. For example, Ross and Roberts find they run into fine-tuning problems if the SUSY mass scale is greater than about $3M_Z$. Finally, we take $\alpha_s = 0.12, m_d = 10$ MeV, define $\tan \beta \equiv |v_u/v_d|$, and normalize $\tilde{M}$ to 100 GeV to obtain

$$d_n \simeq 5 \times 10^{-23} \text{e cm} \left(\frac{100\text{GeV}}{\tilde{M}}\right)^2 \left[|A| \sin \varphi_{A\lambda} - \frac{|\mu|}{|m_0|} \tan \beta \sin \varphi_{\mu\lambda}\right]. \quad (5)$$

There are other contributions to $d_n$ from up squark, chargino and neutralino mixing, which have different combinations of these phases (e.g. the up squark contribution goes as $\cot \beta$ instead of $\tan \beta$), so that a cancellation between the separate pieces would be a fine-tuning.

It is clear there are three ways we can make (5) satisfy the experimental bound on $d_n$, which is now below $10^{-25}$ e cm:

\footnote{There will also be strong CPV effects, but these occur in the SM as well and presumably a solution to the strong $CP$ problem will not change our conclusions about weak CPV.}
(i) fine-tune the phases (individually or in combination) to order $10^{-2} - 10^{-3}$.

(ii) require a large scale $\tilde{M}$, of order a few TeV.

(iii) restrict ourselves to models in which $\sin \phi_{A\lambda}$ and $\sin \phi_{\mu\lambda}$ are naturally zero.

As we said, (i) is simply unacceptable. Several authors use (ii) [7, 8, 23]. In addition to the fine-tuning problems mentioned above, such models may also cause cosmological problems: if all the sfermion masses exceed about 400 GeV, the lightest supersymmetric partner (LSP) annihilation cross section will be too small, leaving an LSP relic density with $\Omega > 1$ [24]. We have also examined the data from models of Kane, Kolda, Roszkowski and Wells [25] which satisfy all other important phenomenological constraints. We find that fine-tunings between $10^{-2}$ to $10^{-3}$ are required in almost all of these models. So we choose to explore option (iii), and develop criteria which ensure that $\sin \phi_{A\lambda}$ and $\sin \phi_{\mu\lambda}$ are naturally zero.

2 Phases Criteria

Let us henceforth assume that we are working in a MSSM with a scale $\tilde{M}$ of order the weak scale, and that we do not accept fine-tunings as a solution to the SUSY $d_n$ problem. These assumptions lead us to conclude that our model must satisfy the criteria in (iii) of the last section, i.e. we need the two physical phases to be zero, i.e. $\sin \phi_{A\lambda}$ and $\sin \phi_{\mu\lambda} = 0$. Setting these phases to zero has been discussed in several references; the purpose of this section is to rewrite these criteria in a way most useful to model builders. We will then be able to explore the implications of these criteria on SUGRA models.
slightly different approach was taken by Kurimoto [26], which is equivalent to the first part of our discussion.

The first thing we can do is rotate away the phase of $\mu$. The $\mu$ dependence comes from two sources: the soft breaking term in (2), which is responsible for the $\mu_{12}^2$ Higgs mixing term in the scalar potential [27], and the $F$ term, which contributes to everything else. We can rotate the relative phase of the Higgs superfields $H_u$ and $H_d$ so that the $\mu_{12}^2$ term is real. The Higgs potential will be real, so that $v_u/v_d$ will be real, the phase of $\mu$ in all the $F$ terms will be changed to that of $B^*m_0$, and the Yukawa couplings will absorb the phase after a quark field rotation. Thus there is no remaining trace of the original phase of $\mu$.

Crucial to this procedure is the assumption that the $\mu_{12}^2$ term comes from the superpotential. If the soft breaking scalar potential Higgs mixing term were put in by hand, then $\mu_{12}^2$ would be unrelated to $\mu$, and the phase of $\mu$ would contribute to $CP$ violation. We discuss this below.

At this point, we have established that one can write the squark mixing contribution to $d_n$ in terms of $\text{Arg}(A^*m_0\tilde{m}_\lambda)$ and $\text{Arg}(B^*m_0\tilde{m}_\lambda)$. In SUGRA theories, one often obtains $B = A - 1$. If $B = A - 1$ holds\footnote{If this relation does not hold (as with an effective $B$ described below), one simply adds the condition that $B$ be real to (6) while (7) is unchanged.} then one can write our criteria as:

\begin{equation}
A \& (m_0\tilde{m}_\lambda) \text{ must be real.} \tag{6}
\end{equation}

These criteria should be satisfied at the high energy scale. The parameters will remain real as they evolve down to the weak scale.
There is one more degree of freedom we can rotate: the phase of the Grassmann variable $\theta$. This freedom allows us to rotate away the phase of $m_0^* \tilde{m}_\lambda$, which means that if $m_0 \tilde{m}_\lambda$ is real, we can always arrange to have $m_0$ and $\tilde{m}_\lambda$ separately real. In addition, we know that the phase of $\mu$ can be rotated away. Thus if our criteria of (6) are satisfied, we can always write the theory such that

$$A, B, m_0, \tilde{m}_\lambda, \& \mu \text{ are real.}$$

It is clear that our criteria in (6) are sufficient because (7) says that all SUSY parameters are real—there is no new SUSY contribution to $d_n$.

Important special cases are those in which one or more of the parameters are zero. Then one must worry about other phenomenological constraints—for example if $\mu$, $B$ or $m_0$ were zero, then $\mu_{12}^2$ would be zero and the theory would develop an unacceptable massless axion [28]. To avoid this, one could relax the assumption that the bilinear soft breaking term comes from the superpotential, and put $\mu_{12}^2$ in by hand, though this may not fit into a SUGRA derived theory. Thus there are two cases to consider:

**Case I:** $\mu_{12}^2 = B m_0^* \mu$, i.e. the soft breaking Higgs mixing term comes from the Higgs mixing term in the superpotential.

**Case II:** $\mu_{12}^2$ is not related to $\mu$, e.g. the soft breaking terms are put in by hand.

In case II, the phase of $\mu$ cannot be rotated away, but we can lump its phase into an effective parameter $\tilde{B}_{\text{eff}} \equiv \mu_{12}^2/\mu$ (which one identifies with $\tilde{B} \equiv B m_0^*$)

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3This is equivalent to an R-rotation with all superfields having R-character zero. It does not assume anything about the R-invariance of the Lagrangian.
of case I), and proceed as if we were in case I, keeping $\bar{A} \equiv A m_0^*$ and $B_{\text{eff}}$ as separate parameters. The criteria in case II are best written:

$$\bar{A}, \ B_{\text{eff}}, \ & \tilde{m}_\lambda \text{ must be real.} \quad (8)$$

Of course the effective $B$ prescription fails if $\mu = 0$, but that case yields a massless higgsino [4] and is thus ruled out. The criteria in (8) also imply that we can write the theory such that (7) holds (with $B$ replaced by $B_{\text{eff}} \equiv B_{\text{eff}}/m_0$), i.e. there is no new SUSY CP violation.

The only remaining possibilities (in either case I or case II) are for $\bar{A}$ and/or $\tilde{m}_\lambda$ to be zero. Having $\tilde{m}_\lambda = 0$ allows one to rotate away one phase, leaving one physical SUSY phase. For example, if $B = A - 1$, the only requirement to avoid a large $d_n$ would be that $A$ is real, which is satisfied in some SUGRA models. The case $\tilde{m}_\lambda = 0, \bar{A} = 0$ has no physical SUSY phases (we can rotate away the phase of $\bar{B}_{\text{eff}}$), and thus would also solve the SUSY $d_n$ problem. Unfortunately for both of these solutions, $\tilde{m}_\lambda = 0$ means that the gluino is massless at tree level, leaving the gluino with a loop generated mass which is far too small.

The remaining possibility, $\bar{A} = 0$ (but $\tilde{m}_\lambda \neq 0$), offers no improvement over the criteria in (8), for one still has to make $\bar{B}_{\text{eff}}$ and $\tilde{m}_\lambda$ real.

In summary, there are no phenomenologically viable solutions to the

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4We note in passing that the sign of $\mu$ before rotation is not physical. After rotation of the Higgs superfields, the sign of the Higgs mixing $F$-terms is that of $\bar{B}_{\text{eff}}$ from before the rotation. For example, a SUGRA model (in case I) which gives a $B > 0 \ (B < 0)$, will after rotation give a positive (negative) coefficient $\mu$ (in the basis of positive $\mu_{12}^2$), regardless of the original sign of $\mu$.

5Recently there has been a revival of the concept of a ‘light gluino window’ [29], but this possibility is almost certainly ruled out experimentally [30].

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8
SUSY $d_n$ problem obtained from setting any of the parameters zero which are not included explicitly in (6) or (8), and each of these imply that there is no new SUSY $CP$ violation in the MSSM. Of course model builders may find it easier to make a parameter zero rather than simply real, but one must be careful that no other phenomenological constraints are violated.

3 Radiative Effects

We must be sure that radiative effects do not change our conclusions. If the parameters are initially real, they will remain so when evolved to low energy [31, 32, 33]. There are CKM dependent terms induced into the squark mass matrix, but these were found to give $d_n$ of order $10^{-31}$ e cm [13].

However, this does not include possible finite effects to the squark LR mixing of the form

$$\delta M^2_{LR} = k\bar{A}V^\dagger_L XV_L \hat{M}_D,$$

where $X$ can be off-diagonal. The contribution to (9) from Figure 2 gives

$$X = \sum_{n=1}^{2} \Omega^\dagger_{Ln} I(n) \Omega_{Ln},$$

where $I(n)$ is a diagonal $2N_F \times 2N_F$ matrix of integrals, and

$$U^\dagger_{II} \equiv \begin{pmatrix} \Omega_{L1} & \Omega_{R1} \\ \Omega_{L2} & \Omega_{R2} \end{pmatrix}$$

diagonalizes the up squark mass matrix. The $3 \times 3$ matrix $\Omega_{L1}$ has large off-diagonal parts, but the off-diagonal parts of $X$ turn out to be proportional
to squark mass splittings, and the contribution to $d_n$ from these finite pieces is negligible, of order $10^{-37}$ e cm. Finite CKM corrections to the gluino mass were addressed in [15] and also found to be small.

We note that one could have spontaneous $CP$ violation induced through loop effects to the Higgs potential in the MSSM [34], though this possibility has been ruled out because it gives a $CP$ odd Higgs scalar which is too light [35]. So we conclude that radiative effects do not affect our criteria.

4 Supergravity Criteria

We would like to see how SUGRA theories (for a review see for example [18, 19, 21, 36]) fare with respect to the SUSY $d_n$ problem. We consider models of the superhiggs effect, where the VEVs of a hidden sector break supersymmetry, and the soft SUSY breaking parameters are determined by inputs to the underlying SUGRA theory. Using our criteria from Section 2, we derive criteria for prospective SUGRA models which would naturally solve the SUSY $d_n$ problem. We then examine the argument that a particular SUGRA model, the Polonyi model [37], provides such a solution. While it turns out that the soft trilinear coefficient $A$ is real [10, 11, 12, 13, 21, 22, 38, 39], the argument that this is a solution [10, 23] to the SUSY $d_n$ problem is incorrect, since the Polonyi model cannot in general require $m_0$ or $\tilde{m}_\lambda$ to be real.
4.1 The Superhiggs Effect

We begin with the SUGRA scalar potential \[ V = e^{K/M^2} \left[ (D_i w) (D_j w^*) g^{ij} - 3 \frac{|w|^2}{M^2} \right] + \frac{1}{2} f_{\alpha \beta} D^{\alpha} D^{\beta}, \quad (12) \]

where \( M \) is \((8\pi)^{-1/2}\) times the Planck mass, \( w \) is the superpotential, \( D^\alpha \) are the auxiliary fields, \( K \) is the Kähler potential, and \( g^{ij} = \partial^2 K / \partial \phi_i \partial \phi^*_j \). We define the Kähler derivative, \( D_i \), as

\[ D_i w = \frac{\partial w}{\partial \phi_i} + \frac{1}{M^2} \frac{\partial K}{\partial \phi_i} w. \quad (13) \]

SUSY is broken by \( \langle D_i w \rangle \), giving a common gaugino mass \[ \tilde{m}_\lambda = e^{<K>/2M^2} \left\langle g^{ij} \right\rangle \left\langle f_{ij} \right\rangle \langle D_i w^* \rangle, \quad (14) \]

where \( f_{ij} = \partial f(\varphi^i) / \partial \varphi^j \) and we have assumed that the gauge kinetic metric \( f_{\alpha \beta} \) is diagonal in its gauge indices, i.e., \( f_{\alpha \beta}(\varphi_i) = f(\varphi_i) \delta_{\alpha \beta} \). The gauginos are massless at tree level unless \( f \) is a non-trivial function of \( \varphi_i \).

To analyze the \( CP \) violating effects of SUGRA models, we must find the low energy scalar potential. We make the usual assumption that \( w \) can be divided into a sum of a "hidden sector" and a "visible sector", \[ w(z_i, y_a) = h(z_i) + g(y_a), \quad (15) \]

where \( y_a \) are visible scalar fields, which interact with Standard Model particles, and \( z_i \) are hidden scalar fields, which interact with Standard Model particles only through gravity. The VEVs of the \( z_i \) break SUSY and give a
mass to the gravitino and to the scalars in $V_{soft}$. Plugging (15) into (12) and using the definition (13), we find

$$V = e^{K/M^2} \left[ \left( h, i + \frac{h + g}{M^2} K, i \right) \left( h^*, j + \frac{h^* + g^*}{M^2} K^*, j \right) g^{ij*} 
+ \left| g, a + \frac{h + g}{M^2} y_a \right|^2 - \frac{3|h + g|^2}{M^2} \right] + \frac{1}{2} f_{a\beta}^{-1} D^a D^\beta. \quad (16)$$

Now we want to break SUSY by letting $z_i$ have a VEV of order $M$ \[19\],

$$\langle z_i \rangle = b_i^0 M, \quad \langle K \rangle = \bar{b}^2 M^2, \quad \langle K, i \rangle = b_i^* M, \quad (17)$$

where $b_i^0$ is some complex constant of order one. We have defined $\bar{b}^2$ and $b_i$ such that in the flat case (where $K, i = z_i^*$), $\bar{b}^2 \rightarrow |b_i^0|^2$ and $b_i \rightarrow b_i^0$. We need a hidden sector potential, $h(z_i)$, which contains a small scale $m$:

$$\langle h \rangle = m M^2, \quad \langle h, i \rangle = a_i^* m M, \quad (18)$$

where $a_i$ is some other complex constant of order one. Using (17) and (18) in (16), the condition for no Planck scale cosmological constant is

$$\langle a_i^* + b_i^* \rangle \langle a_j + b_j \rangle \langle g^{ij*} \rangle = 3, \quad (19)$$

which for the flat case just gives the usual \[19\] expression $|a_i + b_i|^2 = 3$.

Let us define the low energy superpotential $[W]_\varphi \equiv e^{\frac{1}{2} \bar{b}^2} g$, and mass

$$m_0 \equiv e^{\frac{1}{2} \bar{b}^2} \langle h \rangle = e^{\frac{1}{2} \bar{b}^2} \frac{\langle h \rangle}{M^2}, \quad (20)$$

which in the flat case has a magnitude of the gravitino mass, $m_{3/2}$. We further define the parameter
\[ A \equiv b_i^* (a_{j^*} + b_{j^*}) \left\langle g^{ij} \right\rangle, \]  
(21)

which is a generalized definition of the usual flat Kähler metric definition \( A \to b_i^*(a_i + b_i) \). Putting these definitions into (16), we obtain the low energy supergravity scalar potential:

\[
V(\varphi_i) = |F_i|^2 + \frac{1}{2} D_a D^a + V_{soft},
\]

\[
V_{soft} = |m_0|^2 |\varphi_i|^2 + \left( A m_0^* \left[ W^{(3)} \right] + B m_0^* \left[ W^{(2)} \right] + h.c. \right), \tag{22}
\]

where the parameter \( B = A - 1 \). Here \( W^{(2)} \) and \( W^{(3)} \) are the quadratic and cubic terms in the superpotential.

Now we can write the criteria from Section 2 in terms of SUGRA parameters. Since \( \mu^2_{12} = B m_0^* \mu \) (Case I), and \( B = A - 1 \), we can use the criteria in (6). The first criterion, that \( A \) must be real, is satisfied by (21) if \( a_i \) and \( b_i \) are relatively real, and if \( \left\langle g^{ij} \right\rangle \) is real. The latter is true in the flat case. We will see that the former is true at least in the Polonyi model.

The other criterion of (6) is that \( (m_0 \tilde{m}_\lambda) \) must be real. It turns out that \( \tilde{m}_\lambda \) in (14) can be written proportional to \( m_0^* \), so that

\[
m_0 \tilde{m}_\lambda = |m_0|^2 \left\langle M f, i \right\rangle (a_{j^*} + b_{j^*}) \left\langle g^{ij} \right\rangle, \tag{23}
\]

which is real if the coefficients \( a_i \) and \( b_i \) are real (not just relatively real), and if \( \left\langle g^{ij^*} \right\rangle \) and \( \left\langle f, i \right\rangle \) are real. These contain the criteria which make \( A \) real. Thus our criteria (6) can be written:
where \( a_i \) and \( b_i \) are defined by (18) and (17) respectively. If, for example, \( f \) can be written in the simple form \( f(z_i) = c_n z_i^n / M^n \), then the only phases in \( \langle f, i \rangle \) will be those of \( c_n \) and \( b_i^0 \), so that a sufficient condition for solving the SUSY \( d_n \) problem in the flat case would be:

\[
\langle f, i \rangle = 0, \quad \langle g^{ij} \rangle = 0
\]

Finally we note that if \( \langle f, i \rangle = 0 \), the criteria in the flat case would simplify to requiring \( a_i \) and \( b_i \) relatively real, but this possibility is excluded because it gives \( \tilde{m}_\lambda = 0 \), and thus massless gauginos at tree level.

4.2 The Polonyi Model

Let us examine the implications of our criteria on a specific model. The Polonyi model \[37\] is a simple SUGRA model, with a flat Kähler potential, and only one hidden field \( z \) whose VEV breaks supersymmetry. Using this information, we write the quantities defined in Section 4.1 as:

\[
\langle z \rangle = b M, \quad \langle h \rangle = m M^2, \quad \langle h' \rangle = a^* m M, \\
A = b^*(a + b), \quad m_0 = e^{1/2|b|^2} m.
\]

In the Polonyi model, the hidden potential \( h(z) \) has the specific form:

\[
h(z) = m' M (z + \beta M),
\]
where \( \beta \) and \( m' \) are in general complex. The parameters \( m \) and \( m' \) are universally defined as one \( \equiv m \). On the surface this seems silly, because it is trivial to see that \( m = m'(b + \beta) \), and \( b \) and \( \beta \) are both arbitrary complex numbers! However if one minimizes \( V \) and uses the \( \Lambda = 0 \) condition \( |a + b|^2 = 3 \), one finds that \( a, b \) and \( \beta \) are necessarily relatively real, and \( |b + \beta| = 1 \), so that \( |m| = |m'| \). But this does not mean that \( m = m' \), because they still differ by the phase of \( \beta \). Notice that since \( a \) and \( b \) are relatively real, \( \beta \) is manifestly real. This is the basis of the claim that the Polonyi model solves the SUSY \( d_n \) problem. Unfortunately this satisfies only the first criterion in (3). The problem is that \( m_0 \) is not in general real,

\[
\text{Arg} m_0 = \text{Arg} m = \text{Arg} \beta m',
\]

and neither is the product \( m_0 \bar{m}_\lambda \), whose phase we can find using (23):

\[
\text{Arg} \left[ m_0 \bar{m}_\lambda \right] = \text{Arg} \left[ \beta \left( \frac{\partial f}{\partial z} \right) \right].
\]

Both masses are invariant under a redefinition of \( z \), so the phase in (29) cannot be rotated away. If we can write \( f(z) = c_n z^n / M^n \), then one needs to have \( \beta \) and the \( c_n \) real. We know of no mechanism to achieve this naturally. As we said above, \( \bar{m}_\lambda = 0 \) \( \langle \partial f / \partial z \rangle = 0 \) solves the CPV problem, but gives an unacceptable mass spectrum.

We note that the Polonyi model has another naturalness problem coming from the parameter \( \beta \). When we used the condition \( |a + b|^2 = 3 \) to make the cosmological constant \( \Lambda \) vanish, we had no mechanism to enforce it. We had
to arbitrarily choose $|\beta|$ to be exactly $2 - \sqrt{3}$ \cite{21}. In fairness, the cosmological constant is a problem in all theories, and at least SUGRA models allow for $\Lambda = 0$, whereas global SUSY models do not \cite{12}.

5 Concluding Remarks

The SUSY contribution to the electric dipole moment of the neutron ($d_n$) is quite large unless one allows fine-tunings, or one has a large SUSY mass scale, or one naturally sets the phases of the relevant parameters to zero. Our criteria for the soft breaking parameters employ the latter method. We showed that these criteria can be written so that the MSSM gives no new contribution to $d_n$, we investigated the cases where the parameters were zero, and we derived forms for the criteria in a large class of SUGRA models. This allowed us to show that the Polonyi model does not naturally solve the SUSY $d_n$ problem. We also showed that CKM induced finite loop effects in the squark mass matrices give non-zero but negligible contributions to $d_n$, and thus do not affect our conclusions.

We believe that $CP$ violating observables such as $d_n$ should be viewed as important phenomenological constraints, and that therefore any serious model of supersymmetry should be consistent with the limit on $d_n$, and any model of SUSY breaking should provide a solution to this ‘SUSY $d_n$ problem’. For minimal supersymmetric models which give small to moderate superpartner masses (of order the weak scale), our phases criteria should be looked upon as tools for model builders for solving this SUSY $d_n$ problem. Non-minimal models will in general have more phases, so in most cases our
criteria will be a subset of the criteria in MSSM extensions.

As we said, there is no new CPV in a MSSM which satisfies our criteria. Is this an acceptable situation? It would mean that $d_n$ and $d_e$ would be unobservably small. This in and of itself is not unacceptable—it would place SUSY in the same position as the SM—though that may be disheartening for some experimentalists. However, if a non-zero electric dipole moment were observed in the near future, a MSSM satisfying the criteria would be unable to account for it. A complete understanding of the strong $CP$ problem would be needed before conclusions could be drawn about supersymmetry, but it would be useful to know if a supersymmetric model satisfying our criteria could explain an observable $d_n$ without the need to appeal to a small amount of strong CPV. A non-zero $d_e$ would demand such a mechanism. There is also the possibility of generating the baryon asymmetry of the universe at the electroweak scale [43], which requires a source of $CP$ violation [44]. Some recent models of baryogenesis [45] make use of a moderate amount of CPV in the Higgs sector of a two doublet model, which would not be present in a MSSM satisfying our criteria.

One might thus consider ways of generating moderate amounts of CPV in models which have real tree level MSSM parameters. We explored such a mechanism, which uses simple extensions to the MSSM to generate $CP$ violating contributions to the Higgs potential that are naturally suppressed by the size of loop effects [46]. This mechanism could generate $d_n$ and $d_e$ near their experimental bounds without resorting to fine-tunings or large mass scales, and may be able to provide sufficient CPV for baryogenesis [46]. It may also be possible to induce moderate amounts of CPV in SUGRA mod-
els through the spontaneous breaking of horizontal symmetries \cite{47}. Also, an interesting model of baryogenesis was recently proposed \cite{48} using $CP$ violation at finite temperature that might work with a MSSM satisfying our criteria, though it is unclear that even the small explicit phases they need could be supplied by radiative corrections involving the CKM phase. They might need a separate mechanism for this small amount of CPV, such as that of \cite{46}. Of course the baryon asymmetry could be generated at the GUT scale, and may have nothing to do with weak scale $CP$ violation.

Moderate mass scales come out of most reasonable SUSY models. To avoid the SUSY $d_n$ fine-tuning problem, such models should satisfy our criteria. They would then tend to give a negligible $d_n$ and $d_e$. However, such models would not be immediately ruled out by the observation of a non-zero electric dipole moment, because there may be ways of naturally reintroducing a moderate amount of $CP$ violation into the theory.

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Figure Captions

Figure 1: Gluino mediated contribution to the electric dipole moment of the neutron. The ‘X’ indicates LR mixing of the down squarks is needed.

Figure 2: Diagram which can give a finite contribution to $\delta M^2_{LR}$ in (3). Here the ‘X’ indicates a mass insertion. Note that the $H^+ \tilde{U}_L \tilde{D}_L$ vertex is unsuppressed \[27\].