12th International Conference on Vibration Problems, ICOVP 2015

Effect of gravity and magnetism on surface wave propagation in heterogeneous earth crust

Nirmala Kumaria*, Amares Chattopadhyayb

a Ph.D.student, Department of Applied Mathematics, Indian School of Mines, Dhanbad, 826004, Jharkhand, India
b Professor, Department of Applied Mathematics, Indian School of Mines, Dhanbad, 826004, Jharkhand, India

Abstract

This paper aims to study the propagation of surface wave in two initially stressed heterogeneous magnetoelastic transversely isotropic media lying over a transversely isotropic half-space under the action of gravity. Heterogeneities of both the layers are caused due to exponential variation in elastic parameters. Dispersion relation is obtained in closed form by using Whittaker’s asymptotic expansion. Magnetoelastic coupling parameters, heterogeneity, horizontal compressive initial stress and gravity parameters have remarkable effect on the phase velocity of surface wave. The obtained dispersion relation is found to be in well agreement with the classical Love-wave equation. Comparative study and graphical illustration has been made to exhibit the outcomes.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Keywords: Heterogeneity; magnetoelastic; transversely isotropic; surface wave; Dispersion equation; initial stress.

1. Introduction

The crust is heterogeneous, which makes the study of wave propagation much practical considering the heterogeneous layers. There are different types of vertical heterogeneity persist in crustal layers in the form of

* Corresponding author. Tel.: +91 8271979433.
E-mail address: nirmala.ism@gmail.com
exponential function, linear function etc. The study of wave propagation in layered elastic media with different boundaries helps to understand and predict the seismic behaviour at the different margins of earth, which makes it applicable in the field of geophysics, civil, mechanical, and other engineering branches. Many researchers had widely studied the theory of Love wave/shear wave propagation in a medium where the velocity, rigidity and density are functions of depth. Kar [1] worked on the propagation of Love-type waves in a non-homogeneous internal stratum of finite thickness lying between two semi-infinite isotropic media. Love waves in different heterogeneous layered media were studied by Gogna [2]. Chattopadhyay [3] discussed the propagation of SH-waves in a sandwiched heterogeneous layer lying between two semi-infinite homogeneous elastic media where the heterogeneity in the sandwiched layer was taken in the form of linearly varying function of depth in the rigidity and density was kept constant. Due to the presence of many physical factors, a large amount of initial stress evolves in a medium which have a pronounced influence on the propagation of waves as shown by Biot [4]. Chattopadhyay and Singh [5] have discussed the propagation of a crack due to magnetoelastic shear waves in a self-reinforced medium. In this paper we have discussed the propagation of shear waves in two initially stressed non-homogeneous magnetoelastic transversely isotropic media. The both media are lying over a transversely isotropic half-space which is under gravity. The dispersion equation is obtained in closed form.

2. Formulation and solution of the problem

We consider two heterogeneous magnetoelastic transversely isotropic media (M1 and M2) of thickness \(H_1\) and \(H_2\). The layers are under horizontal initial stresses \(P_1\) and \(P_2\) overlying a semi-infinite transversely isotropic medium under Gravity. The \(x\)-axis is taken in the direction of wave propagation, \(z\)-axis as vertically downwards and origin has taken at the interface of layers and half-space. The rigidity and density of both the layers are taken in the form of exponential variation with heterogeneities parameter \(a\) and \(b\) in transversely isotropic media.

The rigidity and density for transversely isotropic half-space under gravity are considered as \(\{L_1, N_1\}, \rho_1\) and \(P_3 = -\rho_3 g z\) (Fig.1). If \(u_j, v_j\) and \(w_j\) are the displacement components in \(x, y\) and \(z\) directions respectively, then for shear wave propagating along the \(x\)- direction and causing displacement in the \(y\)- direction only, we consider the displacement components as \(u_j = 0, w_j = 0, v_j = v_j(x, z, t)\) and \(\partial / \partial y \equiv 0\), where \(j = 1, 2, 3\).

The only non-vanishing equation of motion for the propagation of shear wave in initially stressed heterogeneous magnetoelastic transversely isotropic medium is

\[
\frac{\partial}{\partial x} \left[ N_j \frac{\partial v_j}{\partial x} \right] + \frac{\partial}{\partial z} \left[ L_j \frac{\partial v_j}{\partial z} \right] + \left( \frac{P_j}{2} + \mu_j^{(i)} \rho_j \phi \cos^2 \phi \right) \frac{\partial^2 v_j}{\partial x^2} + \mu_j^{(i)} \rho_j \sin^2 \phi \frac{\partial^2 v_j}{\partial z^2} + \mu_j^{(i)} \rho_j \phi \sin^2 \phi \frac{\partial^2 v_j}{\partial x \partial z} + \mu_j^{(i)} \rho_j \phi \sin^2 \phi \frac{\partial^2 v_j}{\partial t^2}.
\]

On simplifying the above Eq. (1), reduces to

\[
\left[ \frac{N}{L_i} + \frac{P_i}{2L_i} + t_H^{(i)} \cos^2 \phi \right] \frac{\partial^2 v_j}{\partial x^2} + \left[ 1 + t_H^{(i)} \sin^2 \phi \right] \frac{\partial^2 v_j}{\partial z^2} + \left[ t_H^{(i)} \sin 2\phi \right] \frac{\partial^2 v_j}{\partial x \partial z} = 0.
\]

Where \(t_H^{(i)} = \frac{\mu_i^{(i)} H_0^2}{L_i}, \ \beta_i = \sqrt{L_i / \rho_i}\).
\( \rho_i \) are the densities of the media, \( N_i \) and \( L_i \) can be identified as the shear elastic modulus in the transverse and longitudinal direction of the transversely isotropic media (for all \( i = 1, 2, 3 \)). \( v_j \) is the displacement components along \( y \) directions for all \( j = 1, 2, 3 \). \( \mu_e^{(i)} \) \{for all \( i = 1, 2, 3 \)\} are the magnetic permeability for media (\( M_1, M_2 \) and \( M_3 \)) respectively. \( H_0 \) be the induced magnetic (primary) field.

\[
\begin{align*}
  z &= -H \quad \text{Initial Stressed Heterogeneous Magnetoelastic Transversely Isotropic Medium (M1)} \\
  z &= -H_1 \quad \text{Initial Stressed Heterogeneous Magnetoelastic Transversely Isotropic Medium (M2)} \\
  z &= 0 \quad \text{Transversely Isotropic gravitational half-space (M3)}
\end{align*}
\]

Fig. 1 Geometry of the problem

2.1. The equation of motion for medium (\( M_1 \))

Putting \( i = j = 1 \) in eq. (1) the equation of motion for the medium (\( M_1 \)) can be written as

\[
\frac{\partial}{\partial x} \left[ N_1 \frac{\partial v_1}{\partial x} \right] + \frac{\partial}{\partial z} \left[ L_1 \frac{\partial v_1}{\partial z} \right] + \left( \frac{P_i}{2} + \mu_e^{(1)} H_0^2 \cos^2 \varphi \right) \frac{\partial^2 v_1}{\partial x^2} + \mu_e^{(1)} H_0^2 \sin^2 \varphi \frac{\partial^2 v_1}{\partial z^2} + \mu_e^{(1)} H_0^2 \sin 2\varphi \frac{\partial^2 v_1}{\partial x \partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}.
\]

(3)

The heterogeneity parameters for this medium (\( M_1 \)) are considered as

\[
\begin{align*}
  N_i &= N_i^{(0)} e^{az}, \quad L_i = L_i^{(0)} e^{az}, \quad \rho_i = \rho_i^{(0)} e^{az}, \\
  \mu_e^{(i)} &= \mu_e^{(0)} e^{az} \quad \text{and} \quad P_i = P_i^{(0)} e^{az}
\end{align*}
\]

(4)

where \( a \) is the heterogeneity parameter having dimension inverse of length and \( \left\{ N_i^{(0)}, L_i^{(0)}, \rho_i^{(0)}, P_i^{(0)}, \mu_{e_i}^{(0)} \right\} \) are constants of medium M1. With the help of Eq. (4), the Eq. (3) becomes
\[ Q_1 \frac{\partial^2 v_1}{\partial x^2} + R_i \frac{\partial^2 v_1}{\partial z^2} + S_i \frac{\partial^2 v_1}{\partial x \partial z} + \frac{\partial v_1}{\partial z} = \frac{1}{\beta_1^2} \frac{\partial^2 v_1}{\partial t^2}, \]  

(5)

where

\[ Q_i = \frac{N_i^{(0)}}{L_i^{(0)}} + \frac{P_i^{(0)}}{2L_i^{(0)}} + i^{(1)}_\mu \cos^2 \phi, \quad R_i = 1 + i^{(1)}_\mu \sin^2 \phi, \quad S_i = i^{(1)}_\mu \sin 2\phi, \quad \beta_1^{(1)} = \frac{\mu_0^{(0)} H_0^2}{L_i^{(0)}} \]

and \( \beta_1 \) is the shear wave velocity of medium \( M_1 \).

Assuming

\[ v_1(x, z, t) = V_1(z) e^{ik(x-ct)} \]  

(6)

With the help of eqns. (5) and (6), we get the displacement components of medium \( M_1 \) as

\[ v_1(x, z, t) = e^{-(\alpha_i/2)z} \left[ A_1 \cos(T_i z) + B_1 \sin(T_i z) \right] e^{ik(x-ct)} \]  

(7)

where \( \alpha_i = ikS_i + a \) and \( T_i = k \sqrt{\frac{S_i^2}{4R_i^2} + \left( \frac{c^2}{\beta_1^2} - Q_i \right)} \). \( A_1 \) and \( B_1 \) are arbitrary constants.

2.2. The equation of motion for medium \( M_2 \)

Putting \( i = j = 2 \) in Eq. (1), the equation of motion for medium \( M_2 \) becomes

\[ \frac{\partial}{\partial x} \left[ N_2 \frac{\partial v_2}{\partial x} \right] + \frac{\partial}{\partial z} \left[ L_2 \frac{\partial v_2}{\partial z} \right] + \left( \frac{P_2}{2} + \mu_2^{(2)} H_0^2 \cos^2 \phi \right) \frac{\partial^2 v_2}{\partial x^2} + \mu_2^{(2)} H_0^2 \sin^2 \phi \frac{\partial^2 v_2}{\partial z^2} + \mu_2^{(2)} H_0^2 \sin 2\phi \frac{\partial^2 v_2}{\partial x \partial z} = \rho_2 \frac{\partial^2 v_2}{\partial t^2}. \]  

(8)

The heterogeneity parameters in exponential variation for the intermediate layer \( M_2 \) are considered as

\[ \begin{align*}  
N_2 &= N_2^{(0)} e^{bz}, & L_2 &= L_2^{(0)} e^{bz}, \\
\rho_2 &= \rho_2^{(0)} e^{bz}, & P_2 &= P_2^{(0)} e^{bz}, & \mu_2 &= \mu_2^{(0)} e^{bz} \end{align*} \]

(9)

where \( b \) is the heterogeneity parameter of medium \( M_2 \) and \( \left\{ N_2^{(0)}, L_2^{(0)}, \rho_2^{(0)}, P_2^{(0)}, \mu_2^{(0)} \right\} \) are constants.

In view of Eq. (9), Eq. (8) can be written as
\[
Q_2 \frac{\partial^2 v_2}{\partial x^2} + R_2 \frac{\partial^2 v_2}{\partial z^2} + S_2 \frac{\partial^2 v_2}{\partial x \partial z} + b \frac{\partial v_2}{\partial z} = \frac{1}{\beta_1^2} \frac{1}{\partial^2 v_2}{\partial t^2},
\]  

(10)

where

\[
Q_2 = \frac{N_2^{(0)}}{L_2^{(0)}} + \frac{P_2^{(0)}}{2L_2^{(0)}} + t_h^{(2)} \cos^2 \varphi, \quad R_2 = 1 + t_h^{(2)} \sin^2 \varphi, \quad S_2 = t_h^{(2)} \sin 2 \varphi, \quad t_h^{(2)} = \frac{\mu_2^{(0)} H_0}{L_2^{(0)}}
\]

and \( \beta_2 = \sqrt{\frac{L_2^{(0)}}{\rho_2^{(0)}}} \) is the shear wave velocity of intermediate layer M2.

Using \( v_2(x,z,t) = V_2(z) e^{i(k-x-c)t} \) in Eq. (10) we get the displacement components of medium M2 as

\[
v_2(x,z,t) = e^{-i(\alpha_2/2)z} \left[ A_2 \cos(T_2 z) + B_2 \sin(T_2 z) \right] e^{i(k-x-c)t}
\]

(11)

where \( \alpha_2 = ikS_2 + b \) and \( T_2 = k \sqrt{\frac{S_2^2}{4R_2^2} + \frac{\left( c^2 / \beta_2^2 - Q_2 \right)}{R_2}} \), \( A_2 \) and \( B_2 \) are arbitrary constants.

2.3. The equation of motion for half-space (M3)

The equation of motion for the transversely isotropic half space (M3) can be written as

\[
\left[ N_3 - \frac{1}{2} \rho_3 g z \right] \frac{\partial^2 v_3}{\partial x^2} + \left[ L_3 - \frac{1}{2} \rho_3 g \frac{A_3}{c^2} \right] \frac{\partial^2 v_3}{\partial z^2} - \frac{1}{2} \rho_3 g \frac{\partial v_3}{\partial z} = \rho_3 \frac{\partial^2 v_3}{\partial t^2}
\]

(12)

where \( N_3, L_3, \rho_3, v_3, g \) and \( c \) are the shear elastic modulus in transverse and longitudinal directions, the density of the medium, displacement component, acceleration due to gravity and common wave velocity of the half-space (M3).

Using \( v_3(x,z,t) = V_3(z) e^{i(k-x-c)t} \) in the above eq. (12), we have

\[
\frac{d^2 V_3(z)}{dz^2} + \frac{b_1}{(a_1 + b_1 z)} \frac{dV_3(z)}{dz} + k^2 \left[ \frac{c_1^2}{(a_1 + b_1 z)^2} - 1 \right] V_3(z) = 0
\]

(13)

Considering \( \sigma_1(z) = -2k/b_1(a_1 + b_1 z), s = -c_1^2 k/2b_1 \) and using the transformation \( V_3(z) = \psi(z)/\sqrt{a_1 + b_1 z} \) the above Eq. (13) takes the form

\[
\frac{d^2 \psi}{d \sigma_1^2} + \left[ \frac{1}{4} + \frac{s}{\sigma_1} + \frac{1}{4 \sigma_1^2} \right] \psi(\sigma_1) = 0.
\]

(14)

Eq. (14) is the form of Whittaker’s equation and its solution can be written as
\[ \psi(\sigma_1) = A_1 W_{s,0}(\sigma_1) + B_1 W_{s,0}(-\sigma_1), \]  

(15)

where \( W_{s,0}(\sigma_1) \) and \( W_{s,0}(-\sigma_1) \) are Whittaker’s functions of first and second kind of order \( s \) and \( 0 \).

In view of condition \( \psi(\sigma_1) \rightarrow 0 \) as \( z \rightarrow \infty \) the appropriate solution of eq. (15) becomes

\[ \psi(\sigma_1) = A_1 W_{s,0}(\sigma_1). \]

(16)

Hence the solution for the displacement component of the half space reduces to

\[ v_3(x,z,t) = A_1 \left( \frac{2a_1 - gz}{2} \right)^{1/2} W_{s,0}\left[-\left(\frac{4}{G} - 2kz\right)\right]e^{ik(x-cz)}, \]

(17)

where \( a_1 = L_3/\rho_3 \), \( b_1 = (-g/2) \), \( c_1^2 = c^2 + a_1(1 - N_3/L_3) \) and \( G = g/ka_1 \) is the Universal gravitational constant (Biot’s gravity parameter). The asymptotic expansion (Whittaker and Watson [6]) of Whittaker’s function for large argument and retaining up to the second term \( W_{s,0}\left[-\left(\frac{4}{G} - 2kz\right)\right] \), may be approximated as

\[ W_{s,0}\left[-\left(\frac{4}{G} - 2kz\right)\right] \approx e^{-\left(\frac{4z-2G}{G}\right)} \left(\frac{2kz - 4/G}{s + 0.5}\right)^s \left[1 - \frac{(s + 0.5)^2}{(2kz - 4/G)}\right]. \]

(18)

3. Boundary conditions

For the shear wave propagation, the following boundary conditions must be satisfied:

\[
\begin{align*}
\text{i) } & \tau_{yz}^{(1)} = 0 \text{ at } z = -H_2, \\
\text{ii) } & v_1 = v_2 \text{ at } z = -H_1, \\
\text{iii) } & \tau_{yz}^{(1)} = \tau_{yz}^{(2)} \text{ at } z = -H_1, \\
\text{iv) } & v_2 = v_3 \text{ at } z = 0, \\
\text{v) } & \tau_{yz}^{(2)} = \tau_{yz}^{(3)} \text{ at } z = 0.
\end{align*}
\]

(19)

Using the displacement components \( v_1(x,z,t) \), \( v_2(x,z,t) \) and \( v_3(x,z,t) \) from eqns. (7), (11) and (17) in the above five boundary conditions of eq. (19), we get

\[ \left\{ \begin{array}{l}
- \frac{\alpha}{2} \cos(T_1H_2) + T_1 \sin(T_1H_2) \\
\end{array} \right\} A_1 + \left\{ \begin{array}{l}
\frac{\alpha}{2} \sin(T_1H_2) + T_1 \cos(T_1H_2) \\
\end{array} \right\} B_1 = 0, \]

(20)

\[ e^{\frac{\alpha_1 - \alpha}{2}H_1} \left\{ \begin{array}{l}
A_1 \cos(T_1H_1) - B_1 \sin(T_1H_1) \\
\end{array} \right\} - \left\{ \begin{array}{l}
A_2 \cos(T_2H_1) - B_2 \sin(T_2H_1) \\
\end{array} \right\} = 0, \]

(21)
\[
\left( \frac{L_1^{(0)}}{L_2^{(0)}} \right) e^{-(b-a) H_i} \left\{ \left( -\frac{\alpha_1}{2} \cos(T_1 H_i) + T_1 \sin(T_1 H_i) \right) A_1 + \left( \frac{\alpha_1}{2} \sin(T_1 H_i) + T_1 \cos(T_1 H_i) \right) B_1 \right\} \\
= \left\{ \left( -\frac{\alpha_2}{2} \cos(T_2 H_i) + T_2 \sin(T_2 H_i) \right) A_2 + \left( \frac{\alpha_2}{2} \sin(T_2 H_i) + T_2 \cos(T_2 H_i) \right) B_2 \right\}
\]

(22)

\[
A_2 = \frac{A_3}{\sqrt{a_1}} \left\{ W_{-s,0} \left[ -\left( \frac{4}{G} - 2kz \right) \right] \right\} \quad \text{at } z=0
\]

(23)

and

\[
\left( \frac{L_2^{(0)}}{L_1^{(0)}} \right) \left( -\frac{\alpha_1}{2} A_2 + T_2 B_2 \right) = A_3 \left[ \frac{d}{dz} \left( \frac{2a_i - gz}{2} \right)^{1/2} W_{-s,0} \left[ -\left( \frac{4}{G} - 2kz \right) \right] \right] \quad \text{at } z=0
\]

(24)

where \(A_1, B_1, A_2, B_2\) and \(A_3\) are arbitrary constants. Eliminating arbitrary constants \(A_1, B_1, A_2, B_2\) and \(A_3\) from Eq. (20) to (24), we find the dispersion equation as

\[
\frac{L_1^{(0)} e^{(b-a) H_i}}{L_2^{(0)}} = \left[ T_1 \frac{\alpha_2}{2} + \sqrt{a_1 L_0} \right] \left[ \frac{d}{dz} \left( \frac{2a_i - gz}{2} \right)^{1/2} W_{-s,0} \left[ -\left( \frac{4}{G} - 2kz \right) \right] \right] \quad \text{at } z=0
\]

\[
= 1 - \frac{\alpha_i/2 - T_1 \tan(T_1 H_2)}{\alpha_i/2 \tan(T_1 H_2) + T_1} \tan(T_1 H_1)
\]

(25)

With the help of Whittaker’s asymptotic expansion (18), Eq. (27) reduces to
\[
\frac{L_1^{(0)} e^{(b-o)H_i}}{L_2^{(0)}} \left[ \frac{1 - \left( \frac{\alpha_2}{2} + Q \right) \tan \left( T_2 H_i \right)/T_2}{1 - \left( \frac{\alpha_i}{2} - T_i \tan \left( T_i H_i \right) \right) \tan \left( T_i H_i \right) + T_i \tan \left( T_i H_i \right) + T_i} \right]
\]

\[
= \frac{\left( T_2 \tan \left( T_2 H_i \right) + \left( \frac{\alpha_2}{2} \right)^2 \tan \left( T_2 H_i \right)/T_2 \right) + Q \left( 1 + \left( \frac{\alpha_2}{2} \right) \tan \left( T_2 H_i \right)/T_2 \right)}{T_1 \tan \left( T_i H_i \right) - \tan \left( T_i H_i \right)}
\]

(26)

where \( Q = \left( L_1/L_2^{(0)} \right) \sqrt{a_i} \left[ \frac{g}{4a_i} + k \left( \frac{Gs}{2} - 1 + \frac{G^2(s + 0.5)^2}{2 \left( 4 + G(s + 0.5) \right)^2} \right) \right] \).

Equation (26) is the dispersion equation for shear wave in the heterogeneous magnetoelastic transversely isotropic media under initial stresses lying over a transversely isotropic half-space under gravity.

4. Numerical results and discussions

We consider the following data of elastic coefficients of equivalent transversely isotropic models of the Upper Mantle (olivine model and petrofabric model) from Anderson [7]:

Fig. 2 Variation in dimensionless phase velocity \((c/β)\) against dimensionless wave number \((kH_i)\) for different values of transversely isotropic magnetoelastic coupling parameter \((t_m^{(0)})\) when both heterogeneous layers (M1 and M2) are under initial stress and half space (M3) is under the action of gravity.
Fig. 3 Variation in dimensionless phase velocity \( \frac{c}{\beta} \) against dimensionless wave number \( \frac{kH}{c} \) for different values of transversely isotropic magnetoelastic coupling parameter \( \gamma^2 \) when both heterogeneous layers (M₁ and M₂) are under initial stress and half space (M₃) is under the action of gravity.

Fig. 4 Variation in dimensionless phase velocity \( \frac{c}{\beta} \) against dimensionless wave number \( \frac{kH}{c} \) for different values of heterogeneity parameter \( aH \) when both magnetoelastic transversely isotropic layers (M₁ and M₂) are under initial stress and half space (M₃) is under the action of gravity.

Fig. 5 Variation in dimensionless phase velocity \( \frac{c}{\beta} \) against dimensionless wave number \( \frac{kH}{c} \) for different values of heterogeneity parameter \( bH \) when both magnetoelastic transversely isotropic layers (M₁ and M₂) are under initial stress and half space (M₃) is under the action of gravity.
5. Conclusions

The present paper deals with the propagation of shear waves in two initially stressed heterogeneous magnetoelastic transversely isotropic medium overlying a transversely isotropic half-space under the action of gravity has been studied. Closed form expression for dispersion equation in terms of various affecting parameters viz. heterogeneity parameter, transversely isotropic magnetoelastic coupling parameter, horizontal compressive initial stress of the layers and universal gravitational constant of the half-space has been established. The magnetoelastic coupling parameters and heterogeneity parameters of the layers (M₁ and M₂) give the significant effect on the phase velocity of shear waves. The following outcomes can be accomplished through this study:

- The wave number affects phase velocity substantially. More precisely, the phase velocity of dispersion curves decreases with the increase of wave number.
- As heterogeneity grows in the uppermost layer it increases the phase velocity of shear wave. Moreover the heterogeneity grows in the sandwich layer give the adverse effect on the phase velocity of shear wave.

Acknowledgement

The authors convey their sincere thanks to Indian School of Mines, Dhanbad for providing JRF to Ms. Nirmala Kumari and also facilitating us with its best facility.

References

[1] B.K. Kar, On the propagation of Love-type waves in a non-homogeneous internal stratum of finite thickness lying between two semi-infinite isotropic media, Gerlands Beitr Geophys. 86(5) (1977) 407-412.
[2] M.L. Gogna, Love waves in heterogeneous layered media. Geophys J R Astr. Soc. (1976) 357-370.
[3] A. Chattopadhyay, On the propagation of Love type waves in an intermediate non- homogeneous layers lying between two semi-infinite homogeneous elastic media, Gerlands Beitr Geophysik, 84(3/4) (1975) 327-334.
[4] M.A. Biot, Mechanics of Incremental Deformations, John Wiley and Sons Inc., New York, 1965.
[5] A. Chattopadhyay, A.K Singh, Propagation of a Crack due to Magnetoelastic shear waves in a self-reinforced medium, J. Vib. Cont. 20(3) (2014)406-420.
[6] E.T. Whittaker, G.N. Watson, A Course of Modern Analysis, India Universal Book Stall, New Delhi, 1991.
[7] Don. L. Anderson, Theory of the Earth, Boston: Blackwell Scientific Publications, 1989.