Research Article

Engineering Classification of Jointed Rock Mass Based on Connectional Expectation: A Case Study for Songta Dam Site, China

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Engineering classification of complex jointed rock mass is influenced and controlled by many factors with random, nonlinear, and unascertained characteristics, which is an extremely complicated problem. This paper introduces a comprehensive method to classify the rock mass with complex joints. Firstly, evaluation indexes are described by the interval number theory. Secondly, the weight values of the evaluation indexes are determined by the analytic hierarchy process (AHP). Thirdly, the connectional expectation between interval numbers is analyzed and the classification grade of jointed rock mass quality is identified by the set pair analysis theory. The new method can not only describe the dynamic evolution trend of various influencing factors, but also simplify the analysis process of the relationship between interval numbers. The Songta dam abutment rock mass is selected as a study case to verify the rationality of the new method. The classification results of rock mass quality obtained by the new method are in accordance with the actual situation and are consistent with the results provided by the RMR classification.

1. Introduction

Jointed rock mass quality is mainly affected by its own structural characteristics (integrity, rock strength, weathering degree, etc.) and its surrounding environment (in situ stress and groundwater), which can generally reflect the engineering geological properties of rock mass within a certain spatial extent [1–3]. Therefore, the classification of rock mass quality should be based on the above influencing factors, and specific indexes should be used to evaluate rock mass properties by qualitative or quantitative methods. In large-scale water conservancy and hydropower projects, reasonable and accurate determination of rock mass classification is not only conducive to the correct selection of mechanical parameters of various rock masses, but also benefit to the optimization of the engineering design and the determination of a reasonable foundation surface [4, 5].

The classification of rock mass quality started from underground engineering at first and then gradually expanded to dam foundation engineering. Early methods mostly focused on qualitative or quantitative evaluation of a single index, such as [6] former Soviet Union Ф.М. Серженски classification (1937), Н. Н.Мисю classification (1941), Terzaghi classification [7], the dam foundation rock mass classification proposed by the former Soviet Union concrete gravity dam design specification, and the RQD classification introduced by Deere in the United States [8].

In the 1970s, the engineering classification of rock mass gradually developed from the single index to multiindex, qualitative to quantitative evaluation methods. The typical rock mass classification methods for underground engineering are as follows: RSR classification proposed by Wickham et al. [9], the American scholars. RMR
classification proposed by Bieniawski [10, 11], a Dutch scholar from South Africa, and Q system classification proposed by Barton et al. [12], the Norwegian scholars. These methods are relatively perfect and widely applied to practical engineering.

Compared with underground engineering, rock mass quality classification for dam foundation engineering is not perfect and is still in exploration. The representative rock mass quality classification methods for dam foundation engineering abroad as follows [6]: R. P. Miller’s classification scheme, the classification scheme for inhomogeneous rock mass proposed by the Spanish scholars Kikuchi et al. [13], and the dam foundation rock mass classification system introduced by a Japanese scholar Kikuhiro.

In China, the research on rock mass classification of dam foundation engineering started relatively late. The representative classification methods are as follows: rock mass quality coefficient $Z$ classification proposed by Gu and Huang [14], rock mass quality index $M$ classification proposed by Yang [15], the YZP classification of the Three Gorges Project proposed by the Yangtze River Commission [16], standard for engineering classification of rock masses (GB50218-94) [17], and code for water resources and hydropower engineering geological investigation (GB50287-99) [18] are proposed, respectively.

Generally speaking, the rock mass classification is gradually developing from qualitative to quantitative on the basis of engineering practice. GB50287-99 is successfully applied to the Laxiwa Hydropower Station, and the dam foundation rock mass is reasonably divided into 5 major grades and 7 subgrades [19, 20]. Due to the particularity of columnar jointed basalt developed in the Baihetan Hydropower Station, RMR, Q, GB50218-94, and GB50287-99 are synthetically used for rock mass quality classification. However, the results obtained by the above four methods are not exactly consistent. Finally, the quality grade of the rock mass is determined through comprehensive analysis and comparison [21, 22]. With the increase for the scale of engineering rock mass and the complexity of its occurrence environment, the practicability of the traditional rock mass classification method is limited.

Since the 1990s, researchers at home and abroad have gradually realized that the influencing factors of rock mass quality have the characteristics of fuzziness. Habibagahi and Katebi adopt the fuzzy set theory to classify rock mass quality [23]. Yuan et al. proposed a multiindex rock mass quality evaluation method based on extension theory [24]. Cao and Zhang introduced the idea of variable weight processing into rock mass quality evaluation and established a fuzzy evaluation method of rock mass quality based on variable weight [25]. In the engineering classification of jointed rock mass, the aforementioned method fully considers the fuzziness of influencing factors. However, rock mass quality evaluation is to evaluate the engineering geological properties of rock mass in a certain space, which contains a large number of stochastic joints. The geometric, mechanical, and hydraulic properties of these joints often change in a certain range. That is to say, the factors affecting rock mass quality classification have the characteristics of interval numbers. Therefore, this paper attempts to use interval numbers to express the influencing factors of rock mass quality and then applies set pair analysis theory to analyze the connectional expectation between interval numbers and determine the quality grade of rock mass.

2. Theory for the Connectional Expectation

2.1. Interval Number Theory. Objectively speaking, the development of the fractures in complex rock mass is due to anisotropy and randomness. In addition, limited field outcrop and incomplete survey information will lead to the lack of data. Hence, the original data indicating the engineering geological properties of complex fractured rock mass are often not a certain number but some interval numbers. Subjectively speaking, engineers have a deeper understanding for fractured rock mass and will not stay at a single point. Therefore, engineers need to use interval numbers to quantify the engineering geological properties of the fractured rock mass.

Usually, an interval number can be used to represent a certain attribute of the research object, which is defined as follows [26–28].

Assume that any $x^−$ and $x^+$ belong to the set $R$ of real numbers, and $x^− ≤ x^+$. Then, a standard interval number can be expressed as $[X] = [x^−, x^+]$, where $x^−$ is the minimum value of the interval number and $x^+$ is the maximum value of the interval number. If $x^+ > 0$, then $[X]$ is called a positive interval number. If $x^+ < 0$, then $[X]$ is called a negative interval number. If $x^- < 0$ and $x^+ > 0$, then $[X]$ is called a difference interval number. If $x^- = x^+$, then the interval number $[X]$ degenerates to an ordinary real number $X$.

The expectation of interval numbers can be expressed as:

$$E([X]) = x_1p_1 + x_2p_2 + \cdots + x_tp_t,$$

where $x_1, x_2, \ldots, x_t$ are the measured values describing a certain attribute of the research object, $x_1, x_2, \ldots, x_t \in [X]$, and $p_1, p_2, \ldots, p_t$ are the probability values of the measured value $x_1, x_2, \ldots, x_t$.

The study of interval number theory is not perfect, and its application is not very mature. Especially the problem of comparison and connection between two or more interval numbers is very difficult to solve. This study will try to use set pair analysis theory to analyze the connection problem between interval numbers.

2.2. Set Pair Analysis Theory. In nature, the same thing has both certainty and uncertainty. From a philosophical point of view, they are a pair of contradictions, both opposition and same. That is to say, certainty and uncertainty of things can be transformed into each other under certain conditions. Set pair analysis theory is precisely the mathematical theory used to deal with the interaction between certainty and uncertainty. This theory was proposed by Zhao, a mathematician in China. Set pair and connection degree are the main concepts of the set pair analysis theory. If the known sets $X$ and $Y$ have a certain connection, then the two sets can be integrated into a pair, which is expressed as a set pair
\( \tilde{H} = (X, Y) \) [29]. The same, difference, and opposition between the two sets can be expressed by the connection degree \( \mu_{(X,Y)} \). The specific formula is as follows [30]:

\[
\mu_{(X,Y)} = a + bi + cj, \tag{2}
\]

where \( i \) is the coefficient of difference, \(-1 \leq i \leq 1\); \( j \) is the coefficient of opposition, generally \( j = -1 \); \( a, b, \) and \( c \) separate represent the same degree, difference degree, and opposition degree between the evaluation index \( X_p \) of sample \( X_p \) and rock mass quality grade \( k \), and \( a + b + c = 1 \) (Figure 1).

In the uncertain evaluation of things, the interval number form of the \( n \)th index value of the evaluation sample \( m \) is \([X_{mn}] = [x_{mn}, x_{mn}^+]\) and the corresponding expectation can be calculated according to formula (1). Similarly, the grading standard of evaluation index can also be expressed by the interval number. The interval number form of the \( k \)th grade of the \( n \)th index is \([Y_{nk}] = [y_{nk}, y_{nk}^+]\), and the corresponding expectation is \( E[Y_{nk}] \). The measured data and grade interval of the evaluation index can form a set pair. The evaluation criteria for the same, difference, and opposition of the set pair is as follows: if the measured data \([X_{mn}]\) are completely within the grade interval \([Y_{nk}]\), i.e., \( x_{mn} < y_{nk}^+ \) and \( x_{mn}^+ < y_{nk} \), then the sample \([X_{mn}]\) and the grade \([Y_{nk}]\) are in the same relationship. If the measured data \([X_{mn}]\) are completely outside the grade interval \([Y_{nk}]\), i.e., \( x_{mn}^+ < y_{nk}^+ \) or \( x_{mn} < y_{nk} \), then the sample \([X_{mn}]\) and the grade \([Y_{nk}]\) are in opposition relationship (Figure 2). In addition to the above two relationships, the sample \([X_{mn}]\) and the grade \([Y_{nk}]\) are in difference relationship [31].

Usually, the evaluation indexes of the sample are divided into two types, namely, cost-type and benefit-type. The benefit-type index refers to the index whose value increases in the same direction as the grade increases. However, the cost-type index is just the opposite. Regarding the connectional expectation, the cost-type index can be calculated according to formula (3) and the benefit-type index can be calculated according to formula (4) [32]:

\[
\mu([X_{mn}], [Y_{nk}]) = \begin{cases} 
E([X_{mn}]) - y_{nk}^- & \text{if } y_{nk}^- \leq E([X_{mn}]) < y_{nk}^+ \\
\frac{E([X_{mn}]) - y_{nk}^-}{y_{nk}^- - y_{nk}^{+1}} & \text{if } y_{nk}^- \leq E([X_{mn}]) < E([Y_{nk}]) \\
E([Y_{nk}]) - y_{nk}^- & \text{if } y_{nk}^- \leq E([X_{mn}]) < y_{nk}^+ \\
\frac{E([X_{mn}]) - y_{nk}^-}{y_{nk}^- - y_{nk}^{+1}} & \text{if } y_{nk}^- \leq E([X_{mn}]) < E([Y_{nk}]) \\
-1 & \text{other},
\end{cases} \tag{3}
\]

\[
\mu([X_{mn}], [Y_{nk}]) = \begin{cases} 
E([X_{mn}]) - y_{nk}^- & \text{if } y_{nk}^- \leq E([X_{mn}]) < y_{nk}^+ \\
\frac{E([X_{mn}]) - y_{nk}^-}{y_{nk}^- - y_{nk}^{+1}} & \text{if } y_{nk}^- \leq E([X_{mn}]) < E([Y_{nk}]) \\
E([Y_{nk}]) - y_{nk}^- & \text{if } y_{nk}^- \leq E([X_{mn}]) < y_{nk}^+ \\
\frac{E([X_{mn}]) - y_{nk}^-}{y_{nk}^- - y_{nk}^{+1}} & \text{if } y_{nk}^- \leq E([X_{mn}]) < E([Y_{nk}]) \\
-1 & \text{other}. \tag{4}
\end{cases}
\]
where \( \mu (\{X_{mn}\}, \{Y_{n,k}\}) \) is the connectional expectation of the \( n \)th index of the \( m \)th sample with respect to the evaluation grade \( k \). When the weight value of the \( n \)th index is \( W_{n} \), then the integrated connectional expectation of the \( m \)th sample with respect to the evaluation grade \( k \) is:

\[
\mu_{m,k} = \sum_{n=1}^{N} W_{n} \cdot \mu (\{X_{mn}\}, \{Y_{n,k}\}).
\]  

(5)

If

\[
\mu_{m,k_{0}} = \max \{ \mu_{m,k} \mid k = 1, 2, \ldots, k \}.
\]  

(6)

Then, the evaluation grade of this sample is \( k_{0} \).

3. The Case Study

3.1. Study Area. The Songta Hydropower Station will be built on the main stream of the Nu River, which is the first cascade hydropower station in the hydropower development scheme for the Nu River. The dam site is located in Songta Village, Chayu County, Tibet in China, about 7 km distance from the boundary between Yunnan and Tibet along the main stream of the Nu River. The flow direction of the Nu River at the dam site is 188° SW (Figure 3).

The upstream basin of the dam site is vast and the water flow is large. The area of the basin reaches 1035,000 km², and the annual runoff reaches 39.1 billion m³. The concrete double-curved arch dam with a design height of 318 m is planned to be built. The total storage capacity is 4.547 billion m³, and the installed capacity is 3600 MW.

The dam site is a typical mountain-canyon geomorphology, and the valley exhibits an asymmetric “V” shape. The overall slope of the river bank is approximately 50° (Figure 4(a)). The river bank at the dam site is composed by two types of lithology: biotite monzonitic granite and plagioclase amphibolite from the Yanshanian (Cretaceous) period. The biotite monzonitic granite is the predominant lithology, which is primarily comprised of quartz, plagioclase, potassium feldspar, and biotite. The plagioclase amphibolite is intruded as dykes with a width 0.05–5 m into the biotite monzonitic granite. Under the action of extrusion, some stochastic joints are formed within the above rock masses (Figure 4(b)).

3.2. Data Acquisition and Analysis. In order to ascertain the engineering geological condition of the rock mass at the dam site, some adits are excavated at different elevations of the dam abutment. The strike of these adits is basically perpendicular to the flow direction of the Nu River. The commonly used window sampling method is adopted to investigate the joint information outcropped within the adit. Joint information collected includes orientation, rock strength, spacing, RQD, roughness, aperture, weathering, and groundwater.

In the study, the adits PDS1, PD222, PD224, and PD226 located at the right dam abutment are chosen as the study
case for the rock mass classification. The adits PDS1, PD222, PD224, and PD226 are located at elevations of 1716.7 m, 1765.9 m, 1814.9 m, and 1863.9 m, respectively. The lengths of adits PDS1, PD222, PD224, and PD226 are 200 m, 200 m, 150 m, and 150 m, respectively. The distribution map of the above adits is shown in Figure 5(a).

The adit outcrops display that the properties of joints developed within the rock mass varied with the horizontal distance from the valley slope. According to the engineering geological condition of the outcrop surface within adits and the joint formation mechanism of the unloading zone in the high slope of the river valley [33], the rock mass around the adits is divided into three sections from the slope surface to the slope interior (Figure 5(b)):

(i) Section 1: this area is located on the surface section of the valley slope. In the section, in situ stress is significantly reduced and the rock mass is highly weathered. Joints are abundantly developed within the rock mass. Joints inclined to the slope surface at gentle dip angles have a dominant advantage, and these joints are filled with clay mud.
Section 2: *in situ* stress increases gradually, and rock mass is moderately and slightly weathered. Joints develop randomly within the rock mass, and joint density decreases gradually. Only a small amount of joints with gentle dip angles are filled with clay.

Section 3: *in situ* stress is basically stable, and rock mass is fresh. Joints seldom develop within the rock mass.

To ascertain the dominant orientation of the joints within each section of each adit, the joint sets within each section are identified according to an improved FCM method proposed by Song et al. [34]. The dominant joint sets of each section in each adit are shown in Figure 6. The figure exhibits that the joints in each section are divided into three groups. Set 1 is the joint dipping towards the slope surface at a gentle dip angle. Set 2 and Set 3 are joints with a steep dip angle. In section 1 of each adit, the number of joints in Set 1 is much larger than that in Set 2 and Set 3. In section 3 of each adit, the number of joints in each group is basically equal.

3.3. Rock Mass Quality Evaluation Model Based on Connectional Expectation. In this paper, engineering classification of jointed rock mass based on connectional expectation is the comprehensive method. The new method combines three mathematical methods to solve complex decision-making problems affected by various uncertainties. Firstly, interval number theory is adopted to represent the evaluation indexes of rock mass quality. Secondly, AHP is utilized to determine the weight values of evaluation indexes. Thirdly, set pair analysis theory is used to analyze the connectional expectation between interval numbers and determine the quality grade of rock mass. The flowchart of the new method is shown in Figure 7.

3.4. Selection of Evaluation Index. In order to obtain more reasonable and accurate results for rock mass quality evaluation, it should be as comprehensive and scientific as possible when selecting the evaluation index. Table 1 lists the main indexes considered in the 7 typical methods for rock mass quality evaluation at home and abroad. It can be seen from the table that the indexes considered by the 7 typical methods are not identical. This indirectly indicates that the above indexes are the main indexes affecting the engineering geological characteristics of jointed rock mass.

Referring to the main influencing factors of rock mass quality evaluation commonly used by experts at home and abroad and combining with the rock mass structure characteristics of the Songta dam site, this paper synthetically selected eight indexes to classify the quality of dam abutment rock mass. These eight indexes include rock strength, joint spacing, RQD, roughness, aperture, weathering, groundwater, and dip difference. The dip difference refers to the difference between the dominant dip angle of the joint set dipping towards the slope surface and the dip angle of the slope surface.

When collecting joint information, roughness, aperture, weathering, and groundwater are qualitatively described. These qualitative indexes are inconvenient to participate in the calculation of rock mass classification grade. Hence, roughness, aperture, weathering, and groundwater are quantified according to qualitative description. The quantitative values of the above indexes are shown in Table 2. On this basis, evaluation indexes of rock mass in adits PDS1, PD222, PD224, and PD226 are expressed by the interval number and are shown in Table 3.

3.4.1. Determination of Evaluation Grade Criteria. Before determining the quality grade of rock mass, it is necessary to establish a single index evaluation system for influencing factors of rock mass quality. This paper mainly refers to the classification criteria commonly used at home and abroad,
such as RQD classification, RMR classification, China’s national code for water resources, and hydropower engineering geological investigation (GB50287-99) [8, 10, 18], and comprehensively considers the development characteristics of joints in the study area. In addition, combining the quantitative values of each index in Table 2, a single

Figure 6: The dominant joint sets of each section in each adit. (a) PDS1. (b) PD222. (c) PD224. (d) PD226.
3.4.2. Calculation for Weight of Evaluation Index. Through the analysis of eight evaluation indexes, it can be seen that each evaluation index has a very important impact on rock mass quality, but the impact degree of each evaluation index is different. That is to say, the weight value of each evaluation index is different. Only when the impact degree (weight value) of the index is taken into account in the evaluation of rock mass quality, can a reasonable evaluation result be obtained. In this section, analytic hierarchy process (AHP) will be used to consider the impact of each evaluation index on rock mass quality and to calculate its weight value.

AHP is one of the most commonly used methods of weight assignment. It is a multiobjective decision analysis method that combines qualitative and quantitative research as proposed by an American scholar Saaty [35]. The basic idea of this method is to hierarchize and quantify complex decision problems according to human thinking process and then make multicriteria decision-making on this basis. Its characteristic is that less quantitative information is used to mathematize the decision-making process under the premise of fully excavating the essence of complex decision problems. The major steps of the AHP method are as follows [36, 37]:

1. According to the factors involved in a complex decision problem and its membership relations, the decision problem is divided into the component factors and a hierarchical structural model is established. The hierarchical structural model of rock mass classification established in this paper is shown in Figure 8.
2. Assigning numerical values to each factor based on the subjective judgment for the relative importance of each factor, a pairwise comparison matrix of decision factors is constructed. The construction standard of the comparison matrix is based on the 1–9 scale method, which is shown in Table 5. When the factor on the vertical axis is more important than the factor on the horizontal axis, the value varies between 1 and 9. Conversely, the value varies between the reciprocals 1/2 and 1/9. The pairwise comparison matrix for evaluation indexes of rock mass quality is constructed, as shown in Table 6.
3. The maximum eigenvalue of the comparison matrix and its corresponding eigenvector are calculated, and the eigenvector is normalized to be the weight vector. After calculation, the maximum eigenvalue of the comparison matrix consisting of the evaluation indexes is 8.125, and the corresponding eigenvectors are [0.205, 0.161, 0.164, 0.070, 0.069, 0.073, 0.088, and 0.179]. Thus, the corresponding weight values for the evaluation indexes of rock mass quality are shown in Table 7.
4. Determining whether the comparison matrix satisfies the consistence test. If it does not, go back and redo the pairwise comparison matrix. Usually, the consistency ratio CR is used to measure the quality of
Table 1: A list of influencing factors considered in typical rock mass classification methods.

| Method                        | RSR         | RMR         | Q   | Z   | YZP | ET   | China national standard | Number of factors |
|-------------------------------|-------------|-------------|-----|-----|-----|------|------------------------|-------------------|
| Proposed age                  | 1972        | 1973        | 1974| 1979| 1985| 1985 | 1994                   | 1                 |
| Number of joint sets          | √           |             |     |     |     |      |                         |                   |
| Joint spacing                 | √           | √           |     |     |     |      |                         | 6                 |
| Joint state                   | √           | √           |     |     |     |      |                         | 6                 |
| RQD                           | √           |             |     |     |     |      |                         | 5                 |
| Rock mass structure           |             |             |     |     |     |      |                         | 1                 |
| Integrity                     | √           |             |     |     |     |      |                         | 5                 |
| Weathering                    |             |             |     |     |     |      |                         | 3                 |
| In situ stress                |             |             |     |     |     |      |                         | 3                 |
| Groundwater                   | √           | √           |     |     |     |      |                         | 6                 |
| Geological structure          |             |             |     |     |     |      |                         | 1                 |
| Rock strength                 | √           | √           | √   |     |     |      |                         | 7                 |
| Joint shear strength          |             |             |     |     |     |      |                         |                   |
| Rock mass deformation modulus |             |             |     |     |     |      |                         |                   |
| Rock deformation model        |             |             |     |     |     |      |                         |                   |
| Rock mass elastic wave velocity|             |             |     |     |     |      |                         |                   |
| Rock wave velocity            |             |             |     |     |     |      |                         |                   |
| Joint orientation             | √           | √           | √   |     |     |      |                         | 5                 |
| Classification method         |             |             |     |     |     |      |                         |                   |
| Classification grade          |             |             |     |     |     |      |                         |                   |
| Engineering application       |             |             |     |     |     |      |                         |                   |
| Tunnel support                | 5 grades    | 5 grades    | 9 grades | 5 grades | 5 grades | 5 grades | 5 grades | Underground and ground slope |
| Tunnel mining                 |             |             |     |     |     |      |                         |                   |
| Tunnel cavern                 |             |             |     |     |     |      |                         |                   |
| Dam foundation of underground engineering |             |             |     |     |     |      |                         |                   |
| Rock mass of dam foundation   |             |             |     |     |     |      |                         |                   |
| Rock mass of dam foundation   |             |             |     |     |     |      |                         |                   |
| Underground and ground slope  |             |             |     |     |     |      |                         |                   |

Table 2: Quantitative table for the evaluation index of rock mass quality.

| Roughness               | Quantitative value | Aperture          | Quantitative value | Weathering                   | Quantitative value | Groundwater         | Quantitative value |
|-------------------------|--------------------|-------------------|--------------------|------------------------------|--------------------|---------------------|--------------------|
| Toothed rough           | 1                  | Tightly close     | 1                  | Fresh                        | 1                  | Dry                 | 1                  |
| Toothed slightly rough  | 2                  | Close             | 2                  | Slightly weathered           | 2                  | Moist               | 2                  |
| Toothed smooth           | 3                  | Microopen         | 3                  | Moderately weathered         | 3                  | Wet                 | 3                  |
| Wavy rough               | 4                  | Open              | 4                  | Highly weathered             | 4                  | Soaking water       | 4                  |
| Wavy slightly rough      | 5                  | Medium open       | 5                  | Completely weathered         | 5                  | Dripping            | 5                  |
| Wavy smooth              | 6                  | Wide open         | 6                  |                              |                    | Linear drip         | 6                  |
| Flat rough               | 7                  |                   |                    |                              |                    | Flowing water       | 7                  |
| Flat slightly rough      | 8                  |                   |                    |                              |                    |                     |                    |
| Flat smooth              | 9                  |                   |                    |                              |                    |                     |                    |
Table 3: Evaluation indexes of rock mass in each adit are expressed by the interval number.

| Evaluating index | Interval number | PDS1 | PDS2 | PDS3 | PD221 | PD222 | PD223 | PD224 | PD225 | PD226 |
|------------------|----------------|------|------|------|-------|-------|-------|-------|-------|-------|
| Rock strength (MPa) | Minimum       | 47.4 | 25.7 | 73.9 | 34.8  | 67.3  | 51.3  | 62.6  | 60.2  | 87.8  | 49.3  |
|                   | Maximum       | 163.6| 241.5| 138.9| 83.0  | 131.6 | 99.4  | 145.3 | 189.1 | 192.0 | 130.1 |
|                   | Expectation   | 102.2| 96.8 | 106.0| 56.9  | 116.1 | 94.0  | 114.4 | 111.9 | 148.1 | 85.0  |
| Joint spacing (m)  | Minimum       | 0.007| 0.006| 1.311| 0.003 | 0.112 | 0.020 | 0.033 | 0.277 | 0.044 | 0.032 |
|                   | Maximum       | 4.573| 7.349| 7.311| 8.016 | 5.233 | 18.781| 11.155| 7.524 | 33.844| 4.699 |
|                   | Expectation   | 0.802| 1.164| 4.046| 0.791 | 0.999 | 4.294 | 0.981 | 0.875 | 7.097 | 0.846 |
| RQD (%)           | Minimum       | 54   | 35   | 94   | 14    | 78    | 22    | 54    | 45    | 92    | 34.3  |
|                   | Maximum       | 85   | 100  | 100  | 98    | 98    | 100   | 98    | 100   | 100   | 100   |
|                   | Expectation   | 74   | 84   | 98   | 68    | 91    | 96    | 74    | 92    | 98    | 66    |
| Roughness         | Minimum       | 1    | 1    | 4    | 4     | 4     | 4     | 4     | 4     | 4     | 4     |
|                   | Maximum       | 9    | 7    | 7    | 8     | 8     | 9     | 8     | 9     | 9     | 7     |
|                   | Expectation   | 5.70 | 4.89 | 4.67 | 5.49  | 5.18  | 5.25  | 6.32  | 6.78  | 4.54  | 4.57  |
| Aperture          | Minimum       | 1    | 1    | 1    | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
|                   | Maximum       | 6    | 6    | 3    | 6     | 4     | 3     | 6     | 5     | 5     | 6     |
|                   | Expectation   | 2.43 | 1.79 | 1.33 | 1.83  | 1.42  | 1.15  | 2.00  | 2.01  | 1.73  | 2.07  |
| Weathering        | Minimum       | 2    | 2    | 1    | 2     | 2     | 1     | 1     | 1     | 1     | 1     |
|                   | Maximum       | 4    | 4    | 4    | 3     | 2     | 4     | 3     | 4     | 3     | 4     |
|                   | Expectation   | 2.46 | 2.67 | 1.94 | 2.51  | 2.18  | 1.97  | 2.96  | 1.72  | 1.77  | 2.85  |
| Groundwater       | Minimum       | 1    | 1    | 1    | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
|                   | Maximum       | 4    | 4    | 4    | 3     | 2     | 4     | 5     | 5     | 5     | 4     |
|                   | Expectation   | 1.58 | 1.92 | 1.72 | 1.49  | 1.25  | 1.13  | 1.92  | 2.07  | 1.50  | 1.28  |
| Dip difference (°)| Minimum       | −44  | −45  | −23  | −44   | −27   | −39   | −32   | −42   | −34   | −36   |
|                   | Maximum       | 6    | 17   | 10   | 19    | 15    | 6     | 11    | 33    | −5    | 14    |
|                   | Expectation   | −33.31| −19.94| −2.50| −11.70| −6.90 | −12.47| −16.16| −9.36 | −24.62| −16.10| −5.88 |

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When CR < 0.1, the logical consistency of judgment thinking is very good and the weight value determined by its comparison matrix is reasonable. When CR > 0.1, there are contradictions in logical thinking, and it is necessary to readjust the quantitative value of the

Table 4: Classification standard for the evaluation index of rock mass quality [8, 10, 18].

| Evaluating index          | Classification grade of rock mass |
|---------------------------|-----------------------------------|
|                           | 1  | 2  | 3  | 4  | 5  |
| Rock strength (MPa)       | 250~1000 | 100~250 | 50~100 | 25~50 | 0~25 |
| Joint spacing (m)         | 3~100 | 1~3  | 0.3~1  | 0.05~0.3 | 0~0.05 |
| RQD (%)                   | 90~100 | 75~90 | 50~75  | 25~50 | 0~25 |
| Roughness                 | 1~3  | 3~5  | 5~7   | 7~8   | 8~9 |
| Aperture                  | 1~2  | 2~3  | 3~4   | 4~5   | 5~6 |
| Weathering                | 1~1.5 | 1.5~2.0 | 2.0~2.5 | 2.5~3.0 | 3.0~4.0 |
| Groundwater               | 1~2  | 2~3  | 3~4   | 4~5   | 5~7 |
| Dip difference (°)        | 20~35 | 5~20  | ~5~5  | ~20~5  | ~35~20 |

Figure 8: The hierarchical structural model of rock mass classification.

Table 5: The fundamental scale of AHP (Saaty [36]).

| Intensity of importance | Definition                     | Explanation                                                                                       |
|-------------------------|--------------------------------|--------------------------------------------------------------------------------------------------|
| 1                       | Equal importance               | Two activities contribute equally to the objective                                                |
| 2                       | Weak                           | —                                                                                                 |
| 3                       | Moderate prevalence of one over another | Experience and judgment slightly favor one activity over another |
| 4                       | Moderate plus                  | Experience and judgment strongly favor one activity over another                                 |
| 5                       | Strong or essential prevalence | —                                                                                                 |
| 6                       | Strong plus                    | An activity is favored very strongly over another; its dominance demonstrated in practice        |
| 7                       | Very strong or demonstrated prevalence | The evidence favoring one activity over another is of the highest possible order of affirmation |
| 8                       | Very very strong               | A reasonable assumption                                                                          |
| 9                       | Extremely high prevalence      | If consistency were to be forced by obtaining n numerical values to span the matrix               |

Reciprocals of above

Rational

If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.
relative importance of each factor in the comparison matrix. The consistency ratio CR can be calculated according to the following formula:

$$CR = \frac{CI}{RI},$$

(7)

where RI is a random consistency index, which was given by Saaty, as shown in Table 8, and CI is a consistency index and can be calculated according to the following formula:

$$CI = \frac{\lambda_{\text{max}} - N}{N - 1},$$

(8)

where $\lambda_{\text{max}}$ is the maximum eigenvalue of the comparison matrix and $N$ is the order of the comparison matrix.

After calculation, the consistency ratio $CR$ of the comparison matrix is 0.0127, which is obviously less than 0.1. It shows that the logical consistency of the comparison matrix is good, and the weight value calculated by the comparison matrix is reasonable.

3.5. Determination of Rock Mass Quality Grade. Firstly, the interval number expectation of each index in the sample to be evaluated and the evaluation criteria are calculated according to formula (1). Secondly, each index of the sample to be evaluated and the evaluation classification criteria are integrated into a set pair, and the connectional expectation for each index is calculated. Since rock strength, joint spacing, RQD, and dip difference belong to cost-type indexes, the corresponding connectional expectation is calculated according to formula (3). The other indexes are benefit-type indexes, and the connectional expectation can be calculated according to formula (4). Then, combined with the weight value of each index, the integrated connectional expectation of each sample to be evaluated with respect to

**Table 6: The pairwise comparison matrix for the evaluation index of rock mass quality.**

| Evaluation index | Rock strength | Joint spacing | RQD | Roughness | Aperture | Weathering | Groundwater | Dip difference |
|------------------|---------------|---------------|-----|-----------|----------|------------|-------------|----------------|
| Rock strength    | 1             | 1             | 2   | 3         | 3        | 3          | 2           | 1              |
| Joint spacing    | 1             | 1             | 1   | 2         | 2        | 2          | 2           | 1              |
| RQD              | 1/2           | 1             | 1   | 2         | 3        | 3          | 2           | 1              |
| Roughness        | 1/3           | 1/2           | 1/2 | 1         | 1        | 1          | 1/2         | 1/2            |
| Aperture         | 1/3           | 1/2           | 1/3 | 1         | 1        | 1          | 1/3         | 1/2            |
| Weathering       | 1/3           | 1/2           | 1/3 | 1         | 1        | 1          | 1           | 1/2            |
| Groundwater      | 1/2           | 1/2           | 1/2 | 2         | 1        | 1          | 1           | 1/2            |
| Dip difference   | 1             | 1             | 1   | 2         | 3        | 2          | 2           | 1              |

**Table 7: Weight values for the evaluation index of rock mass quality.**

| Evaluation index | Rock strength | Joint spacing | RQD | Roughness | Aperture | Weathering | Groundwater | Dip difference |
|------------------|---------------|---------------|-----|-----------|----------|------------|-------------|----------------|
| Weight value     | 0.205         | 0.161         | 0.164| 0.070     | 0.069    | 0.073      | 0.088       | 0.179          |

**Table 8: The random consistency index (Saaty [36]).**

| N    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| RI   | 0 | 0 | 0.58| 0.90| 1.12| 1.24| 1.32| 1.41| 1.45| 1.49 |

**Table 9: The results of rock mass classification.**

| Adit     | $\mu_{m,1}$ | $\mu_{m,2}$ | $\mu_{m,3}$ | $\mu_{m,4}$ | $\mu_{m,5}$ | The new method | RMR |
|----------|--------------|--------------|--------------|--------------|--------------|----------------|-----|
| PDS1     | -0.681       | -0.259       | -0.130       | -0.726       | -0.534       | 3              | 3   |
| Section 1| -0.602       | -0.309       | -0.320       | -0.896       | -0.459       | 3              | 3   |
| Section 2| -0.309       | -0.245       | -0.360       | -0.754       | -0.284       | 3              | 3   |
| Section 3| -0.369       | -0.340       | -0.425       | -0.754       | -0.623       | 3              | 3   |
| PD222    | -0.733       | -0.401       | -0.579       | -0.915       | -0.735       | 2              | 2   |
| Section 1| -0.733       | -0.401       | -0.579       | -0.915       | -0.735       | 2              | 2   |
| Section 2| -0.373       | -0.280       | -0.287       | -0.754       | -0.975       | 2              | 2   |
| Section 3| -0.436       | -0.240       | -0.320       | -0.639       | -0.950       | 2              | 2   |
| PD224    | -0.654       | -0.262       | -0.250       | -0.601       | -0.806       | 3              | 3   |
| Section 1| -0.654       | -0.262       | -0.250       | -0.601       | -0.806       | 3              | 3   |
| Section 2| -0.410       | -0.161       | -0.320       | -0.639       | -0.950       | 2              | 2   |
| Section 3| -0.083       | -0.137       | -0.767       | -0.856       | -0.725       | 1              | 1   |
| PD226    | -0.703       | -0.327       | 0.017        | -0.469       | -0.823       | 3              | 3   |
| Section 1| -0.703       | -0.327       | 0.017        | -0.469       | -0.823       | 3              | 3   |
| Section 2| -0.115       | 0.043        | -0.522       | -0.801       | -0.990       | 2              | 2   |
| Section 3| -0.064       | 0.166        | -0.679       | -1.000       | -1.000       | 2              | 2   |
Finally, the evaluation grade of each sample to be evaluated is determined according to formula (6); that is, the grade corresponding to the maximum value of integrated connectional expectation is the classification grade of rock mass.

The integrated connectional expectations of the section 1 in adit PDS1 with respect to the evaluation grades 1, 2, 3, 4, and 5 are −0.6274, −0.2493, −0.1340, −0.7609, and −0.7917, respectively. It is shown that the integrated connectional expectation of the section 1 in adit PDS1 with respect to the grade 3 of rock mass quality is the largest. Therefore, the rock mass quality of the section 1 in adit PDS1 is classified as grade 3. Similarly, the classification results of the new method in other homogeneous regions are shown in Table 9.

To verify the reliability of the new method, the RMR method is selected as the benchmark. The classification results of rock mass quality obtained by the new method are in accordance with the actual situation and are consistent with the results provided by the RMR classification. The comparison results show that the new method is reliable and can easily deal with the evaluation indexes of rock mass classification expressed by the interval number. In order to more intuitively display the classification results of the Songta dam abutment rock mass, a spatial division map of rock mass quality is drawn as shown in Figure 9.

4. Conclusions

The stochastic joints developed within the rock mass are the key factors affecting rock mass quality. Besides, the geometric, mechanical, and hydraulic properties of stochastic joints usually change in a certain range. Hence, using interval numbers to represent the influencing factors of rock mass quality can not only reduce uncertainty, but also make the evaluation results more reasonable.

This paper introduces a comprehensive method to classify the rock mass with complex joints. Firstly, interval number theory is adopted to represent the evaluation indexes of rock mass quality. Secondly, analytic hierarchy process is utilized to determine the weight values of evaluation indexes. Thirdly, set pair analysis theory is used to analyze the connectional expectation between interval numbers and determine the quality grade of rock mass. The new method combines three mathematical methods to solve complex decision-making problems affected by various uncertainties, which can not only describe the dynamic

![Figure 9: Spatial division of rock mass quality in the Songta dam site.](image-url)
evolution trend of various influencing factors, but also simplify the analysis process of the relationship between interval numbers.

The new method is applied to evaluate the quality grade of Songta dam abutment rock mass. The classification results of rock mass quality obtained by the new method are in accordance with the actual situation and are consistent with the results provided by the RMR classification. The quality grade of the section 1 in adits PDS1, PD222, PD224, and PD226 is grade 3. The quality grade of the section 2 in adits PDS1, PD222, PD224, and PD226 and the section 3 in adit PD226 is grade 2. The quality grade of the section 3 in adits PDS1, PD222, and PD224 is grade 1.

Data Availability

The data used to support this research article are available from the first author on request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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