Upper Limit on the molecular resonance strengths in the $^{12}\text{C}+^{12}\text{C}$ fusion reaction

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Abstract. Molecular resonances complicate the prediction of the carbon fusion rate at the energies of astrophysical relevance. By comparing the cross sections of three carbon isotope fusion reactions, $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$, an empirical relationship among the three systems is found. With this relationship and the Equivalent Square Well (ESW) model, for the first time, the resonant cross sections in the $^{12}\text{C}+^{12}\text{C}$ fusion reactions are quantitatively described. Consequently, an upper limit on the molecular resonance strengths in $^{12}\text{C}+^{12}\text{C}$ fusion reactions is established. With this limit, we claim that the strong resonances at 1.5 MeV and 2.14 MeV are not realistic. This upper limit posts an additional constraint for the superburst models.

1. Introduction
In 1960 Almqvist, Kuehner and Bromely discovered several resonances in collisions between $^{12}\text{C}$ nuclei. For at least three energies, $E_{\text{c.m.}}$=5.68, 6.00 and 6.32 MeV, they observed resonances in the yield of collisional byproducts: $p$, $\alpha$, $n$ and $\gamma$. These resonances have characteristic widths of about 100 keV. The resonances were identified as signatures of the formation of nuclear molecules$^{[1, 2]}$. In the following years, as measurements were pushed down towards lower energies, the discoveries of such resonances continued as far as the lowest measured energies. For instance, the most recent published measurement of the $^{12}\text{C}+^{12}\text{C}$ fusion reported a strong resonance at $E_{\text{c.m.}}=2.14$ MeV$^{[3]}$.

Apart from the interesting molecular resonances, the $^{12}\text{C}+^{12}\text{C}$ fusion reaction plays a crucial role in a number of important astrophysical scenarios$^{[4]}$. The important energy range spans from 1 MeV to 3 MeV in the center of mass frame, which is only partially covered by the measurements of the $^{12}\text{C}+^{12}\text{C}$ fusion reaction. An extrapolation is the only resource available to obtain the fusion reaction rate for the astrophysical applications. The currently adopted reaction rate is established based on the modified S factor $S^*(E)$$^{[5]}$, which is defined as,

$$S^*(E) = \sigma(E)E^{87.21/\sqrt{E}+0.46E}.$$  \hspace{1cm} (1)

An averaged $S^*$ factor of $3 \times 10^{16}$ MeV$^*\text{b}$ was obtained by fitting the data measured by Patterson$^{[5]}$, Spinka$^{[6]}$ and Becker$^{[7]}$. This averaged value was extrapolated down to lower energies with the simple assumption that the averaged modified S factor is constant at sub-barrier energies$^{[5, 8]}$. At present, there is nothing known about the exact energies and strengths
of these resonances in the energy region of astrophysical relevance. Therefore, the current carbon fusion rate based on the averaged $S^*$ factor is highly uncertain [9].

This uncertainty leaves an ambiguous space in astrophysical models and prevents us from a precise understanding of the underlying physics. For example, superbursts are long, energetic, and rare thermonuclear flashes on accreting neutron stars in low-mass X-ray binaries[10]. These bursts are considered to be triggered by the unstable $^{12}$C burning in the ash left over from the rp-process on the surface of a neutron star. The ash is heated up with the heat generated in the crust of a neutron star and ignites the carbon in the ash. However, with the currently adopted carbon fusion rate, superburst modes fail to explain the ignition of carbon at the column depth inferred from observations [11]. Another difficulty is that the amount of $^{12}$C in the ash does not seem to be sufficient to trigger the superburst. Inspired by the strong resonance observed at $E_{c.m.}=2.14$ MeV, a hypothetical resonance at $E_{c.m.}=1.5$ MeV with a similar strength was proposed to enhance the fusion reaction rate at $T=0.5\times10^9$ K by a factor of over two orders of magnitude and thereby alleviate the discrepancy between superburst models and observations[10].

2. An Empirical relationship among the carbon isotope fusion reactions

In contrast to the striking resonances in the $^{12}$C+$^{12}$C fusion reaction, other carbon isotope fusion reactions, such as $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C, behave more regularly. Only minor resonance features have been observed in these two systems [12, 13]. The shape of the fusion cross sections at sub-barrier energies is primarily dominated by the Coulomb barrier penetration effect. To remove this effect and reveal more details of nuclear interaction, the cross sections of all three carbon fusion systems are converted into the $S^*$ factors using eq.1. The advantage of using this conversion over the traditional astrophysical S factor is that the cross section ratios among the three systems are preserved.

The experimental $S^*$ factor for all three carbon isotope fusion systems are shown in Fig.1. There are several important features in these carbon isotope fusion systems:

(i) The $^{12}$C+$^{12}$C cross sections are bound from above by the cross sections of the other two carbon isotope fusion systems.

(ii) The major resonant cross sections of $^{12}$C+$^{12}$C ($E_r=3.08, 4.28, 4.92, 5.67, 5.96$ and $6.26$ MeV) match remarkably well with the other two carbon isotope fusion cross sections, especially $^{13}$C+$^{13}$C.

(iii) Overall, the $^{12}$C+$^{13}$C cross sections are the highest among the three carbon isotope fusion systems. The deviation between $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C occurs in the energy of 3.5 to 5 MeV (excluding the 4.25 MeV resonance in $^{12}$C+$^{13}$C) and is less than 30%.

(iv) The 4.25 MeV resonance in $^{12}$C+$^{12}$C is also present in $^{12}$C+$^{13}$C.

In the $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C fusion reactions, the valence neutron of $^{13}$C introduces an additional degree of freedom for excitation, besides the $^{12}$C core excitation. Despite this complication, the maximum deviation among the $^{12}$C+$^{12}$C peak cross sections and the $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C fusion cross sections is less than 30%. Based on this strong correlation, we claim:

(i) In the $^{12}$C+$^{13}$C fusion reaction, all the major resonances with maxima matching the $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C cross sections should have the same reaction mechanism;

(ii) This reaction mechanism should also be present in the fusion of $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C;

(iii) The effect arising from the valence neutron in $^{13}$C is not strong compared to this reaction mechanism but it is strong enough to make the resonances disappear in the fusion of $^{12}$C+$^{13}$C and $^{13}$C+$^{13}$C.

One possible mechanism is the coupling effect between the incident elastic channel and the excited channels proposed by Imanishi and others[15, 2]. In the Nogami-Imanishi model, the
The experimental S* factors of three carbon isotope fusion reactions, $^{12}\text{C}+^{12}\text{C}$[3, 5, 7, 14], $^{12}\text{C}+^{13}\text{C}$ from this work (green points) and Ref.[12] and $^{13}\text{C}+^{13}\text{C}$[13]. The averaged S* factor of $^{12}\text{C}+^{12}\text{C}$(S*=3×$10^{16}$ MeVb), labeled as CF88(the red line), is recommended based on the data of Becker, Patterson and Spinka (not shown here)[8]. There are some data with large error bars in Spillane’s data. In this paper, only those with relative errors less than 70% are shown. The hypothetical resonance proposed by Copper et al. is also shown. The explanations for CRC-AW1,2 and ESW1,2 are available in the section 3.

Effect of the coupling to the $^{12}\text{C}(2^+, 4.44$ MeV) state is crucial for the formation of the $^{12}\text{C}+^{12}\text{C}$ molecular resonances. The coupling effect leading to the resonances only takes place when the energy matches the condition of the molecular states[15]. In the other two carbon fusion systems, such a coupling effect should also exist. The presence of valence neutron significantly increases the level density of compound nuclei[16]. As a result, the matching condition becomes valid at any energies and the coupling effect can take place at any energy. This may be the reason for the observed correlations among these three carbon fusion systems.

3. Upper limit on the resonant strengths

The correlations among the three carbon isotope systems provide a possibility to establish an upper limit for the important $^{12}\text{C}+^{12}\text{C}$ reaction. If all the major resonances in $^{12}\text{C}+^{12}\text{C}$, including those to be discovered at lower energies, are driven by the same reaction mechanism, one can expect to establish an extrapolation based on the resonant cross sections observed at higher energies. To test the predictive power of extrapolating models and achieve a more reliable extrapolation within the energy range of 1 to 3 MeV, it is necessary to push the measurements of the carbon isotope fusion reactions towards lower energies.

The recent study of $^{12}\text{C}+^{13}\text{C}$ at Notre Dame provides a great opportunity to test the predictive power of extrapolating models. In this experiment, the lowest measured cross section has been pushed down from 1 µb [12] to 20 nb [17]. The dominant component in the error bars is the systematic uncertainty of the theoretical branching ratio which is estimated as 20% [12]. Two different models have been applied to test their predictive power at low energies. The first one is the standard coupled-Reaction-Channels calculation[19] using the the Akyüz-Winther potential (CRC-AW). In the calculation, the $^{13}\text{C}(3/2^−, 3.684$ MeV) state is included. The other is the Equivalent Square Well (ESW) potential, which has been chosen traditionally to model the sub-barrier fusion cross sections and provide extrapolation within the energy ranges of astrophysical interest[9]. Using the new $^{12}\text{C}+^{13}\text{C}$ data at lower energies, it is demonstrated that CRC-AW...
calculation (labeled as CRC–AW2 in Fig. 1) over-predicts the fusion cross sections by a factor of 3.3 at $E_{c.m.}=2.640$ MeV. This observation agrees with the hindrance study[18]. The ESW extrapolation taken from Ref.[12] (labeled as ESW2 in Fig. 1) proves its superior prediction power by matching the new data at the energies below 3.1 MeV within their quoted error bars.

After the validation, these two models are used in $^{12}\text{C}+^{12}\text{C}$ to establish the upper limit for the resonant cross sections. The ESW model extrapolation, labeled as ESW1 in Fig. 1, is obtained by fitting those major peak cross sections ( $E_r=3.08, 4.28, 4.92, 5.67, 5.96$ and $6.26$ MeV) which match the $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$ fusion cross sections. Even though the fit is based on the peak cross sections above 3 MeV, the extrapolation to lower energies provide an upper bound on most of Spillane’s data as well, except the strong resonance observed at 2.14 MeV which is about a factor of 40 higher than the ESW extrapolation. The preliminary result from the $^{12}\text{C}+^{12}\text{C}$ experiment at Naples[20] found that $S^*=(1.4\pm3.3)\times10^{16}$ MeV b at 2.15 MeV, which disagrees with the strong resonance found in Ref.[3] but agrees well with our upper limit. Another safer upper limit can be estimated using CRC-AW which has been proven to over-predict the fusion cross sections at extreme sub-barrier energies. In this calculation, the coupling to the $^{12}\text{C}(2^+,4.44\text{ MeV})$ state is included. The result is shown as CRC–AW1 in Fig. 1. The two resonances at 1.5 MeV and 2.14 MeV are about a factor of 10 higher than the CRC-AW calculation.

Even though we do not know the exact location of the potential resonance in the unexplored energy range, with the established upper limit we can claim the real $^{12}\text{C}+^{12}\text{C}$ fusion rate is much less than Cooper et al. proposed. Taking the over-predicted $S^*$ factors calculated with CRC-AW, the real rate should not be more than a factor of 6 stronger than the currently adopted fusion rate based on the $S^*$ factor in the energy range of 1 to 3 MeV. The upper limit established with the ESW fit, the best fit we have achieved, is only 20% higher than the currently adopted rate in the range of 1 to 2.5 MeV. As shown in Fig. 1, the averaged $S^*$ factor of Spillane’s data tends to suggest a smaller average than the currently adopted value. Therefore, the factor of more than 40 enhancement proposed in Ref.[10] is not realistic. The upper limit establish in this work posts an additional constraint on the current superburst models. Since the carbon burning rate cannot be as high as Cooper et al. proposed, it is likely that some unknown physics process is not included in the current superburst models.

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