Determination of the $D^0 \rightarrow K^-\pi^+\pi^0$ and $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ Coherence Factors and Average Strong-Phase Differences Using Quantum-Correlated Measurements

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Abstract

The first measurements of the coherence factors ($R_{K\pi\pi^0}$ and $R_{K\pi^3}$) and the average strong-phase differences ($\delta_{DK\pi^0}$ and $\delta_{DK3\pi}$) for $D^0 \rightarrow K^-\pi^+\pi^0$ and $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ are presented. These parameters can be used to improve the determination of the unitarity triangle angle $\gamma$ in $B^- \rightarrow DK^-$ decays, where $D$ is a $D^0$ or $\bar{D}^0$ meson decaying to the same final state. The measurements are made using quantum-correlated, fully-reconstructed $D_0\bar{D}_0$ pairs produced in $e^+e^-$ collisions at the $\psi(3770)$ resonance. The measured values are: $R_{K\pi\pi^0} = 0.84 \pm 0.07$, $\delta_{DK\pi^0} = (227^{+14}_{-17})^\circ$, $R_{K\pi^3} = 0.33^{+0.20}_{-0.23}$, and $\delta_{DK3\pi} = (114^{+26}_{-23})^\circ$. These results indicate significant coherence in the decay $D^0 \rightarrow K^-\pi^+\pi^0$, whereas lower coherence is observed in the decay $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$. The analysis also results in a small improvement in the knowledge of other $D$-meson parameters, in particular the strong-phase difference for $D^0 \rightarrow K^-\pi^+$, $\delta_{DK^\pi}$, and the mixing parameter, $y$.

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This paper presents the first determination of the coherence factors and the average strength-phase differences for $D^0 \to K^-\pi^+\pi^0$ and $D^0 \to K^-\pi^+\pi^-\pi^-$ made using quantum-correlated, fully-reconstructed (double-tagged) $D^0\bar{D}^0$ pairs produced in $e^+e^-$ collisions at the $\psi(3770)$ resonance. Knowledge of these parameters improves the sensitivity of measurements of the unitarity triangle angle $\gamma$ using $B$-meson decays to these $D$-meson final states. Although $CP$-violation involving $B$-mesons has been clearly established experimentally [1], and existing results are in good agreement with Standard Model predictions, additional and improved measurements are required to overconstrain the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [2] and probe for the effects of non-Standard Model physics. An important ingredient in this program will be a precise determination of the angle $\gamma$.

Several methods to determine $\gamma$ using $B^- \to DK^- [3]$ decays have been proposed [4,5,6]. Here, $D$ refers to either a $D^0$ or $\bar{D}^0$ meson. All these methods exploit the fact that a $B^-$ can decay into $D^0K^-$ and $\bar{D}^0K^-$ final states via $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions, respectively. The weak phase between these two transitions is equal to $-\gamma$. Therefore, the amplitudes are related by: $A(B^- \to D^0K^-)/A(B^- \to D^0K^-) = r_B e^{i(\delta_B - \gamma)}$, where $r_B \sim 0.1$ is the absolute amplitude ratio and $\delta_B$ is the strong-phase difference. The two amplitudes interfere with one another if the $D^0$ and $\bar{D}^0$ decay to the same final state, which can lead to direct $CP$-violation between the $B^-$ and $B^+$ decay rates if $\gamma$ is non-zero.

The Atwood-Dunietz-Soni (ADS) method [3] uses common flavor-specific final states such as $D \to K^-\pi^+$ to determine $\gamma$. The rates are given by:

$$\Gamma(B^+ \to D(K^+\pi^\pm)K^+) \propto 1 + (r_B r_{D}^{K \pi^})^2 + 2r_B r_D^{K \pi^} \cos(\delta_B - \delta_D^{K \pi^} \mp \gamma) \quad (1)$$

and

$$\Gamma(B^+ \to D(K^\pm\pi^0)K^+) \propto (r_B)^2 + (r_D^{K \pi^0})^2 + 2r_B r_D^{K \pi^0} \cos(\delta_B + \delta_D^{K \pi^0} \mp \gamma) \quad (2)$$

where $r_D^{K \pi}$ is the absolute amplitude ratio of the doubly-Cabibbo-suppressed (DCS) decay $D^0 \to K^+\pi^-$ to the Cabibbo-favored (CF) decay $D^0 \to K^-\pi^+$, and $\delta_D^{K \pi}$ is the strong-phase difference between these two amplitudes, which is defined as: $A(D^0 \to K^+\pi^-)/A(D^0 \to K^-\pi^+) = r_D^{K \pi} e^{-i\delta_D^{K \pi}}$. Present measurements give $r_D^{K \pi} = 0.0579 \pm 0.0007$ [7], therefore, the terms on the righthand side of Eq. (2) are all of the same order, which allows significant changes to $\Gamma(B^+ \to D(K^\pm\pi^0)K^+)$ depending on the values of $\gamma$ and the strong phases. The suppressed decays $B^+ \to D(K^\pm\pi^+)K^+$ have not yet been observed [8,9]. The measurement of $\delta_D^{K \pi}$ has been made in quantum-correlated $D^0\bar{D}^0$ decays [10] in a similar manner to the analysis reported in this paper.

The flavor-specific final states $D \to K^-\pi^+\pi^-\pi^-(D \to K^-3\pi)$ and $D \to K^-\pi^+\pi^0$ have significantly larger branching fractions than $D \to K^-\pi^+\pi^0$ [11]. However, for three- or four-body $D$ decay the amplitude ratio and strong-phase difference vary over phase space. For such $D$ decays, for example $D \to K^-\pi^+\pi^0\pi^0$, Eq. (2) is modified as follows [12]:

$$\Gamma(B^+ \to D(K^\pm\pi^0\pi^0)K^+) \propto (r_B)^2 + (r_D^{K \pi^0\pi^0})^2 + 2r_B r_D^{K \pi^0\pi^0} \cos(\delta_B + \delta_D^{K \pi^0\pi^0} \mp \gamma) \quad (3)$$

where $R_D^{K \pi^0\pi^0}$, $\delta_D^{K \pi^0\pi^0}$ and $r_D^{K \pi^0\pi^0}$ are defined as:

$$R_D^{K \pi^0\pi^0} e^{-i\delta_D^{K \pi^0\pi^0}} = \frac{\int A_{K^-\pi^+\pi^0}(x)A_{K^+\pi^-\pi^0}(x)dx}{A_{K^-\pi^+\pi^0}A_{K^+\pi^-\pi^0}} \quad \text{and}$$

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Here $A_{K^+\pi^+\pi^0}(\mathbf{x})$ is the amplitude for $D^0 \to K^+\pi^+\pi^0$ at a point in multi-body phase space described by parameters $\mathbf{x}$, and $A_{K^+\pi^+\pi^0}^2 = \int |A_{K^+\pi^+\pi^0}(\mathbf{x})|^2 d\mathbf{x}$. (The expressions for $D \to K^-\pi^+\pi^+\pi^-$ take the same form and involve the parameters $r_D^{K3\pi}$, $R_{K3\pi}$ and $\delta_D^{K3\pi}$.)

The parameter $R_{K^+\pi^0}$ is known as the coherence factor and can take any value from zero to one. A small value of $R_{K^+\pi^0}$ indicates a lack of coherence between the intermediate states involved in the decay, a situation expected when there are many resonances contributing; a value close to one occurs when the resonances are largely in phase, or one state dominates. Decays to two-body final states, such as $D^0 \to K^-\pi^+$, and to $CP$ eigenstates have a coherence factor equal to one. Even if the coherence is small the rate described by Eq. (3) is still useful, because it possesses high sensitivity to the parameter $r_B$.

The coherence factors $R_F$ and average strong-phase difference, $\delta_D$, where $F = K^-\pi^+\pi^0$ or $K^-\pi^+\pi^+\pi^-$, can be determined using double-tagged $D^0\bar{D}^0$ pairs produced in $e^+e^-$ collisions at the $\psi(3770)$ resonance. The two mesons are produced in a $C$-odd eigenstate and their decays are quantum-correlated. The rate for the two $D$ mesons to decay to states $F$ and $G$ is given by (12):

$$\Gamma(F|G) = \Gamma_0 \int \int |A_F(\mathbf{x})A_G(\mathbf{y}) - A_F(\mathbf{x})A_G(\mathbf{y})|^2 d\mathbf{x} d\mathbf{y}$$

$$= \Gamma_0 [A_F^2A_G^2 + A_F^2A_G^2 - 2R_FR_GB_FA_FG\cos(\delta_D^G - \delta_D^F)],$$

(4)

where $A_F(\mathbf{x})$ ($A_G(\mathbf{y})$) and $A_F(\mathbf{x})$ ($A_G(\mathbf{y})$) are the amplitudes of $D^0 \to F$ ($D^0 \to G$) and $D^0 \to F$ ($D^0 \to G$) at points $\mathbf{x}$ ($\mathbf{y}$) in phase space, respectively, and $\Gamma_0 = \Gamma(\psi(3770) \to D^0\bar{D}^0)$. From Eq. (4) the following double-tagged rates arise:

$$\Gamma(F|CP) = \Gamma_0 A_F^2A_{CP}^2[1 + (r_D^F)^2] = \Gamma_0 [A_F^2A_{CP}^2 + A_F^2A_{CP}^2 - 2R_FR_GB_FA_FG\cos(\delta_D^G - \delta_D^F)],$$

(5)

$$\Gamma(F|F) = \Gamma_0 A_F^2A_F^2[1 - R_F^2],$$

(6)

$$\Gamma(F|K^-\pi) = \Gamma_0 A_F^2A_{K^-\pi}^2[(r_K^{K\pi})^2 + (r_D^F)^2 - 2r_K^{K\pi}r_D^F\cos(\delta_D^F - \delta_D^F)],$$

(7)

and

$$\Gamma(K^+\pi^0|K^+\pi^0) = \Gamma_0 A_{K^+\pi^0}^2A_{K^+\pi^0}^2[(r_D^{K3\pi})^2 + (r_D^{K\pi^0})^2 - 2r_D^{K3\pi}r_D^{K\pi^0}R_{K3\pi}R_{K\pi^0}\cos(\delta_D^{K3\pi} - \delta_D^{K\pi^0})].$$

(8)

Here $CP$ denotes a $CP$ eigenstate with eigenvalue $\lambda_{CP} = \pm 1$. The final states described by Eqs. (5), (7), and (8) are referred to as ‘like-sign’ (LS) on account of the charges of the two kaons involved. Furthermore, the following relations are noted: $\Gamma(F|CP) = \Gamma(F|CP)$, $\Gamma(F|F) = \Gamma(F|F)$ and $\Gamma(F|K^-\pi) = \Gamma(F|K^-\pi)$; these expressions ignore $CP$-violation in $D$ decay, which is well motivated theoretically and by current experimental limits (13).

To relate the amplitudes in Eqs. (5) to (8) to branching fractions the effects of charm mixing must be included. Charm mixing is commonly characterised by the parameters $x = (M_+ - M_-)/\Gamma$ and $y = (\Gamma_+ - \Gamma_-)/2\Gamma$, where $M_\pm$ and $\Gamma_\pm$ are the masses and widths of the $\lambda_{CP} = \pm 1$ neutral $D$ meson mass eigenstates, respectively, and $\Gamma = (\Gamma_+ + \Gamma_-)/2$. 

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The relations between amplitudes and branching fractions, following Ref. [14], are given in Table I.

The best constraints on $x$, $y$ and $\delta_{D}^{K\pi}$ come from the combination of several measurements [13]. These constraints [7] are included in the analysis reported here to improve the determination of $R_{F}$ and $\delta_{D}^{K\pi}$. However, the analysis is also sensitive to these parameters so results are presented without the external constraints as well.

**TABLE I: Relations between branching fractions, $B$, and amplitudes including the effects of charm mixing.** The DCS and $CP$ expressions are quoted to $O((x/r_{D})^2; (y/r_{D})^2)$ and $O(y)$, respectively. The corrections due to mixing in the CF amplitude are negligible (< 1%).

| Mode | $B$ |
|------|-----|
| $D^{0} \rightarrow CP$ | $A_{CP}^{2}(1 - \lambda_{y})$ |
| $D^{0} \rightarrow F$ | $A_{F}^{2}$ |
| $D^{0} \rightarrow F$ | $A_{F}^{2}[1 - (y/r_{D}^{F})] R_{F} \cos \delta_{D}^{F}$ $(x/r_{D}^{F}) R_{F} \sin \delta_{D}^{F} + (y^{2} + x^{2})/2(r_{D}^{F})^{2}$ |
| $D^{0} \rightarrow K^{-}\pi^{+}$ | $A_{K^{-}\pi^{+}}^{2}$ |
| $D^{0} \rightarrow K^{+}\pi^{-}$ | $A_{K^{+}\pi^{-}}^{2}[1 - (y/r_{D}^{K^{+}\pi^{-}})] \cos \delta_{D}^{K^{+}\pi^{-}}$ $(x/r_{D}^{K^{+}\pi^{-}}) \sin \delta_{D}^{K^{+}\pi^{-}} + (y^{2} + x^{2})/2(r_{D}^{K^{+}\pi^{-}})^{2}$ |

An 818 pb$^{-1}$ data set of $e^{+}e^{-}$ collisions produced by the Cornell Electron Storage Ring (CESR) at $E_{cm} = 3.77$ GeV and collected with the CLEO-c detector is analysed. The CLEO-c detector is described in detail elsewhere [13]. Table I lists the reconstructed $D^{0}$ and $D^{0}$ final states, with $\pi^{0} \rightarrow \gamma\gamma$, $K_{S}^{0} \rightarrow \pi^{+}\pi^{-}$, $\omega \rightarrow \pi^{+}\pi^{-}\pi^{0}$, $\phi \rightarrow K^{+}K^{-}$, $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^{+}\pi^{-}\pi^{0}$ and $\eta' \rightarrow \eta(\gamma\gamma)\pi^{+}\pi^{-}$. When required in the analysis, reconstruction efficiencies are calculated from simulated samples of signal $D$ decays. Backgrounds from other $DD$ decays are estimated from a simulated sample of generic $D\bar{D}$ decays.

**TABLE II: $D$ final states reconstructed in this analysis.**

| Type               | Final states                                      |
|--------------------|--------------------------------------------------|
| Flavored           | $K^{+}\pi^{\pm}$, $K^{+}\pi^{\pm}\pi^{\pm}$, $K^{+}\pi^{\pm}\pi^{0}$, $K_{S}^{0}\pi^{0}$, $K_{S}^{0}\pi^{0}$, $K_{S}^{0}\omega$ |
| $CP$-even          | $K_{S}^{0}\pi^{0}$, $K_{S}^{0}\omega$, $K_{S}^{0}\phi$, $K_{S}^{0}\eta$, $K_{S}^{0}\eta'$ |
| $CP$-odd           |                                                  |

The $\pi_{0}$, $K_{S}^{0}$, $\omega$ and $\eta \rightarrow \gamma\gamma$ reconstruction is identical to that used in Ref. [10]. Candidates for $\eta \rightarrow \pi^{+}\pi^{-}\pi^{0}$, $\eta'$, and $\phi$ mesons are considered if their masses are within the intervals [506, 590] MeV/$c^{2}$, [950, 964] MeV/$c^{2}$, and [1009, 1033] MeV/$c^{2}$, respectively. Final states that do not contain a $K_{L}^{0}$ meson are fully reconstructed via two kinematic variables: the beam-constrained candidate mass, $M_{bc} \equiv \sqrt{E_{cm}^{2}/(4c^{4})} - \mathbf{p}_{D}^{2}/c^{2}$, where $\mathbf{p}_{D}$ is the $D$ candidate momentum, and $\Delta E \equiv E_{D} - E_{cm}/2$, where $E_{D}$ is the sum of the $D$ daughter candidate energies. The double-tagged yield is determined from counting events in signal and sideband regions of $M_{bc}$. Fig. I(a) shows the distribution of $M_{bc}$ for $D \rightarrow K^{-}\pi^{+}\pi^{0}$ candidates tagged by $D \rightarrow K_{S}^{0}\pi^{0}$ decays for data and simulated background events. The selection and yield determination procedures are similar to those presented in Ref. [10]. For modes that were not considered in Ref. [10] the values of the $\Delta E$ criteria are identical to those used in Ref. [10]. In
addition, to suppress background from $D^0 \to K^0 L\pi^0 \pi^0$ misidentified as $D^0 \to K^0 \pi^+ \pi^- \pi^0 \pi^0$ for the like-sign $K^3\pi$ or $K^-\pi^+ \pi^- \pi^0 \pi^0$ tagged by $K^\pm \pi^\mp$, which is also estimated from simulated sample. However, this contamination is treated as a separate source of systematic uncertainty because it is the dominant source for some measurements. The measured event yields after background subtraction are given in Table III.

The results of the analysis are presented in terms of the observables $\rho_{CP \pm}$, $\rho_{LS}$, $\rho_K$, $\rho_{K^\pm \pi^\mp \pi^0}$, and $\rho_{K^\pm \pi^\mp \pi^0 L\pi^0}$, which are the ratios of the measured values of $\Gamma(F|CP)$, $\Gamma(F|F)$, $\Gamma(F|\pi^\mp \pi^\pm)$ and $\Gamma(K^\mp \pi^\pm |K^\mp \pi^\pm \pi^\pm \pi^\mp)$ to the expected rates, on the assumption that the two $D$ mesons
TABLE III: Measured double-tagged signal yields.

| Mode          | \(K^+\pi^+\pi^+\pi^+\) | \(K^+\pi^+\pi^0\) | \(K^+\pi^0\) | \(K^+\pi^\pm\) |
|---------------|--------------------------|---------------------|---------------|---------------|
| \(K^+\pi^+\pi^+\pi^+\) | 4,044 ± 64               | –                   | –             | –             |
| \(K^+\pi^+\pi^+\pi^\pm\) | 29.1 ± 5.9               | –                   | –             | –             |
| \(K^+\pi^+\pi^0\) | 9,594 ± 99               | 7,342 ± 87          | –             | –             |
| \(K^+\pi^+\pi^0\) | 63.6 ± 8.8               | 12.5 ± 4.1          | –             | –             |
| \(K^+\pi^0\) | 5,206 ± 72               | 7,155 ± 85          | –             | –             |
| \(K^+\pi^\pm\) | 35.6 ± 6.2               | 7.3 ± 3.3           | –             | –             |
| \(K^+\pi^-\) | 536 ± 23                 | 764 ± 28            | –             | –             |
| \(K^0\pi^0\) | 283 ± 18                 | 406 ± 21            | 221 ± 15      | –             |
| \(K^0\pi^0\) | 827 ± 30                 | 1,236 ± 38          | 689 ± 28      | –             |
| \(K^0\phi\) | 296 ± 18                 | 449 ± 22            | 251 ± 17      | –             |
| \(K^0\pi^0\) | 705 ± 27                 | 891 ± 30            | 473 ± 22      | –             |
| \(K^0\phi\) | 319 ± 19                 | 389 ± 21            | 183 ± 14      | –             |
| \(K^0\phi\) | 53.0 ± 7.5               | 90.9 ± 9.9          | 42.8 ± 6.9    | –             |
| \(K^0\eta(\gamma\gamma)\) | 128 ± 12                | 116 ± 11            | 65.5 ± 8.3    | –             |
| \(K^0\eta(\pi^+\pi^-\pi^0)\) | 35.9 ± 6.5              | 36.3 ± 7.2          | 27.2 ± 5.4    | –             |
| \(K^0\eta'\) | 35.7 ± 6.0               | 60.6 ± 7.8          | 30.0 ± 5.5    | –             |

decay in an uncorrelated fashion or have zero coherence. Therefore significant deviation of any of the \(\rho\) parameters from a value of one can only come about through the quantum-correlated nature of \(D \bar{D}\) production at the \(\psi(3770)\) and a non-zero coherence in the \(D\) decay. The \(\rho\) observables are related to the background and efficiency corrected signal yields, \(S\), as follows:

\[
\rho_{LS}^F = \frac{S(F|F) + S(\bar{F}|\bar{F})}{2N_{D^0\bar{D}^0}\mathcal{B}(D^0 \to F)\mathcal{B}(D^0 \to \bar{F})}, \tag{9}
\]

\[
\rho_{K\pi,LS}^F = \frac{S(F|K^-\pi^+) + S(\bar{F}|K^+\pi^-)}{2N_{D^0\bar{D}^0}[\mathcal{B}(D^0 \to F)\mathcal{B}(D^0 \to K^+\pi^-) + \mathcal{B}(D^0 \to \bar{F})\mathcal{B}(D^0 \to K^-\pi^+)]}, \tag{10}
\]

\[
\rho_{CP,\pm}^F = \frac{[S(F|CP) + S(\bar{F}|\bar{CP})]}{2N_{D^0\bar{D}^0}\mathcal{B}(D^0 \to CP)}
\]

\[
\left[\mathcal{B}(D^0 \to F) + \mathcal{B}(D^0 \to \bar{F})\right], \tag{11}
\]

and

\[
\rho_{K\pi^0,LS}^{K\pi^0} = \frac{[S(K^-\pi^0|K^-3\pi) + S(K^+\pi^-\pi^0|K^+3\pi)]}{2N_{D^0\bar{D}^0}[\mathcal{B}(D^0 \to K^-\pi^+\pi^0)\mathcal{B}(D^0 \to K^+3\pi) + \mathcal{B}(D^0 \to K^+\pi^-\pi^0)\mathcal{B}(D^0 \to K^-3\pi)]}, \tag{12}
\]

where \(N_{D^0\bar{D}^0}\) is the total number of \(\psi(3770) \to D^0\bar{D}^0\) events.

In the extraction of each like-sign observable, the product of \(N_{D^0\bar{D}^0}\) and the reconstruction efficiency is determined from the background-subtracted yield in the corresponding opposite-sign samples, taking the values of the branching fractions reported in Ref. [11]. For example,
in the case of $\rho_{LS}^{F}$, the observable is given by

$$\rho_{LS}^{F} = \frac{N(F|F) + N(F|\bar{F}) B(D^0 \rightarrow F)}{2N(F|F) B(D^0 \rightarrow F)} ,$$

(13)

where $N$ are the background-subtracted yields without any efficiency corrections applied. For the majority of the $CP$ double-tags an alternative normalization procedure is exploited, whereby knowledge of $N_{D0J0}$, the reconstruction efficiency and the branching ratio of the $CP$ mode, which in many cases is poorly known, is accommodated by a comparison with double-tag events involving the $CP$-tag against $D \rightarrow K^-\pi^+$ decays. The good knowledge of $\delta_{D}^{K\pi}$ [7, 10, 18] allows the contribution from quantum-correlations in these normalization events to be accounted for. Small corrections are applied related to the differing environment arising from the finite size of the assigned systematic uncertainties. The most important systematic uncertainties are those directly from Ref. [11], together with measurements of the reconstruction efficiency and $S(F|K^+\pi^-)$.

Table [IV] shows the measured value of each observable. In the case of $\rho_{CP}^{F,\pm}$ the results from the individual $CP$-tags are found to be consistent and are therefore combined into mean values for $CP$-even and $CP$-odd, taking full account of the correlations among the assigned systematic uncertainties. The most important systematic uncertainties are those arising from the finite size of the $K^-\pi^+$ vs. $CP$ double-tag samples (0.018), residual corrections associated with this normalization procedure (0.008), and knowledge of the CF $D^0 \rightarrow K^-\pi^-\pi^-\pi^+$ and $D^0 \rightarrow K^-\pi^0\pi^0$ branching ratios (0.010). For the $\rho_{LS}^{K3\pi}$ ($\rho_{LS}^{K\pi0}$) observable the dominant uncertainty of 0.082 (0.021) comes from the knowledge of the $CP$ branching ratio; this is also a significant component for $\rho_{K,\pi,LS}^{K3\pi}$ ($\rho_{K,\pi,LS}^{K\pi0}$), where the uncertainty is 0.034 (0.003), with further important contributions arising from the knowledge of the $D^0 \rightarrow K^+\pi^-$ branching ratio of 0.024 (0.005) and the rate of misidentification of $D^0 \rightarrow K^-\pi^+$ as $D^0 \rightarrow K^+\pi^-$ of 0.016 (0.026). For $\rho_{K,\pi,LS}^{K\pi0}$ the largest uncertainty is 0.065 from the DCS branching fractions. For all observables uncertainties are also assigned to account for non-uniform acceptance across phase-space; this uncertainty is only found to be significant for $\rho_{LS}^{K3\pi}$, $\rho_{K,\pi,LS}^{K3\pi}$ and $\rho_{K,\pi,LS}^{K\pi0}$ where it is 0.051, 0.040 and 0.037, respectively. The results in Table [IV] suggest significant coherence in the $D \rightarrow K^-\pi^+\pi^0$ decay, but much less so in the case of $D \rightarrow K^-\pi^-\pi^-\pi^+$.

The relationships between the like-sign kaon observables and the physics parameters are given by:

$$\rho_{LS}^{F} = \frac{1 - R_F^2}{1 + \frac{x^2+y^2}{2(r_D)^2} - \frac{R_F}{r_D} (y \cos \delta_D - x \sin \delta_D)} ,$$

(14)

$$\rho_{K,\pi,LS}^{F} = \frac{[1 + \frac{(x^2+y^2)}{2(r_D)^2} - \frac{2x}{r_D} R_F \cos (\delta_{K\pi} - \delta_D)] B_{K,\pi,LS}^{F}}{[1 + \frac{(x^2+y^2)}{2(r_D)^2} - \frac{R_F}{r_D} (y \cos \delta_{K\pi} - x \sin \delta_D)} ,$$

(15)

and

$$\rho_{K,\pi,LS}^{K\pi0} = \frac{[1 + \frac{(x^2+y^2)}{2(r_D)^2} - \frac{2x}{r_D} R_{K,\pi0} R_{K,3\pi} \cos (\delta_{K\pi0} - \delta_{K3\pi})] B_{K,\pi,LS}^{K\pi0}}{[1 + \frac{(x^2+y^2)}{2(r_D)^2} - \frac{R_{K,3\pi}}{r_D} (y \cos \delta_{K3\pi} - x \sin \delta_{K3\pi})} ,$$

(16)
TABLE IV: Measured $\rho$ observables, where the first uncertainty is statistical and the second systematic.

| Observable | Measured Value |
|------------|----------------|
| $\rho_{CP+}^{K\pi}$ | 1.077 ± 0.024 ± 0.029 |
| $\rho_{CP-}^{K\pi}$ | 0.933 ± 0.027 ± 0.046 |
| $\rho_{LS}^{K\pi}$ | 1.112 ± 0.226 ± 0.102 |
| $\rho_{K\pi,LS}^{K\pi}$ | 0.971 ± 0.169 ± 0.062 |
| $\rho_{CP+}^{K\pi}$ | 1.073 ± 0.020 ± 0.035 |
| $\rho_{CP}^{K\pi}$ | 0.868 ± 0.023 ± 0.049 |
| $\rho_{LS}^{K\pi}$ | 0.388 ± 0.127 ± 0.026 |
| $\rho_{K\pi,LS}^{K\pi}$ | 0.170 ± 0.072 ± 0.027 |
| $\rho_{K\pi}^{3\pi}$ | 1.221 ± 0.169 ± 0.080 |

where $B_{K\pi,LS}^F = B(D^0 \rightarrow F)B(D^0 \rightarrow K^+\pi^-)/(B(D^0 \rightarrow F)B(D^0 \rightarrow K^+\pi^-) + B(D^0 \rightarrow F)B(D^0 \rightarrow K^-\pi^+))$ and $B_{K\pi,LS}^{3\pi} = B(D^0 \rightarrow K^-\pi^+\pi^0)B(D^0 \rightarrow K^+\pi^0\pi^0)/(B(D^0 \rightarrow K^-\pi^+\pi^0)B(D^0 \rightarrow K^+\pi^0\pi^0)B(D^0 \rightarrow K^-3\pi)$. In making use of the $\rho_{CP\pm}$ observables it is convenient to define the $CP$-invariant observable, $\Delta_{CP}^F$:

$$\Delta_{CP}^F \equiv \lambda_\pm (\rho_{CP\pm}^F - 1) = y - 2\tau_D R_F \cos \delta_F^r. \quad (17)$$

Some anticorrelated systematic uncertainties on $\rho_{CP+}^F$ and $\rho_{CP-}^F$ cancel when computing $\Delta_{CP}^F$. It is found that $\Delta_{CP}^{K\pi} = 0.077 \pm 0.018 \pm 0.022$ and $\Delta_{CP}^{K\pi} = 0.097 \pm 0.015 \pm 0.023$, with $\chi^2/d.o.f$ values of 7.3/10 and 5.7/10, respectively.

The values of $R_{K\pi\pi}$, $R_{K3\pi}$, $\delta_{D}^{K\pi\pi,0}$ and $\delta_{D}^{K3\pi}$ are obtained by a $\chi^2$ fit to $\rho_{LS}^F$, $\rho_{K\pi,LS}^F$, $\rho_{K\pi,LS}^{K\pi}$ and $\Delta_{CP}^F$. In addition, the fit has $x$, $y$, $\delta_D$ and the CF and DCS branching fractions for $D \rightarrow K^-\pi^+$, $K^-\pi^+\pi^0$ and $K^-3\pi$ as free parameters. The values of the $D$-mixing parameters and branching fractions are constrained to those reported in Refs. [7] and [11], respectively; this procedure is referred to as the mixing-constrained fit. The values of the constraints are given in Table IV. Correlations amongst all free parameters are accounted for. The results of the mixing-constrained fit are given in Tab. IV. The best fit values of the coherence factors and average strong-phase differences are: $R_{K\pi\pi} = 0.84 \pm 0.07, \delta_{D}^{K\pi\pi} = (227^{+10}_{-17})^\circ, R_{K3\pi} = 0.33^{+0.20}_{-0.23}$, and $\delta_{D}^{K3\pi} = (114^{+26}_{-23})^\circ$. There are also small improvements in the precision of $\delta_{D}^{K\pi}$, $y$ and the DCS and CF $D \rightarrow K^-3\pi$ branching fractions compared to the external constraints. The uncertainties are those arising from the statistical and systematic uncertainties on the observables. The $\chi^2/d.o.f$ for the mixing-constrained fit is 7.3/3. The correlations amongst the parameters are presented in Ref. [13].

Figures 2(a) and 2(b) show the 1σ, 2σ and 3σ allowed regions of $(R_{K\pi\pi}, \delta_{D}^{K\pi\pi})$ and $(R_{K3\pi}, \delta_{D}^{K3\pi})$ parameter space from the mixing-constrained fit, respectively. The likelihood is computed as $L = e^{-(\chi^2-\chi^2_{min})/2}$ at a point in parameter space; the fit is repeated at each point with the values of $R_F$ and $\delta_{D}^F$ fixed. The 95% confidence level (CL) intervals for the parameters are found by integrating one-dimensional likelihood scans within the physically allowed region. The following 95% CL intervals are found: $0.70 < R_{K\pi\pi} < 0.95$, $167^\circ < \delta_{D}^{K\pi\pi} < 249^\circ$ and $R_{K3\pi} < 0.62$. There is no bound on $\delta_{D}^{K3\pi}$ at the 95% CL.

The fit is repeated with the constraints on $x$, $y$ and $\delta_{K\pi}$ removed to estimate these pa-
TABLE V: Results of the mixing-constrained and unconstrained fits to the observables. Values of external constraints are listed. The uncertainties are those arising from the statistical and systematic uncertainties on the observables.

| Parameter                  | Mixing constrained | Mixing unconstrained | External input |
|----------------------------|--------------------|----------------------|----------------|
| $R_{K^{\pi\pi^0}}$        | 0.84 ± 0.07        | 0.78$^{+0.11}_{-0.25}$ | --             |
| $\delta_D^{K^{\pi\pi^0}}$ (°) | 227$^{+14}_{-17}$  | 239$^{+32}_{-30}$    | --             |
| $R_{K3\pi}$               | 0.33$^{+0.26}_{-0.23}$ | 0.36$^{+0.24}_{-0.30}$ | --             |
| $\delta_D^{K3\pi}$       | 114$^{+26}_{-23}$  | 118$^{+62}_{-53}$    | --             |
| $x$ (%)                   | 0.96 ± 0.25        | -0.8$^{+2.9}_{-2.7}$  | 1.00 ± 0.25    |
| $y$ (%)                   | 0.81 ± 0.16        | 0.7$^{+2.4}_{-0.7}$   | 0.76 ± 0.18    |
| $\delta_D^{K\pi}$        | -151.5$^{+9.6}_{-9.5}$ | -130$^{+38}_{-28}$  | -157.5$^{+10.4}_{-11.0}$ |
| $B(D^0 \to K^-\pi^+)$ (%) | 3.89 ± 0.05        | 3.89 ± 0.05          | 3.89 ± 0.05    |
| $B(D^0 \to K^+\pi^-)$ (10$^{-4}$) | 1.47 ± 0.07      | 1.47 ± 0.07          | 1.47 ± 0.07    |
| $B(D^0 \to K^-\pi^-\pi^0)$ (%) | 13.8 ± 0.5        | 13.8 ± 0.5           | 13.9 ± 0.5     |
| $B(D^0 \to K^+\pi^-\pi^0)$ (10$^{-4}$) | 3.05 ± 0.17       | 3.05 ± 0.17          | 3.05 ± 0.17    |
| $B(D^0 \to K^-\pi^+\pi^-\pi^-)$ (%) | 7.96 ± 0.19       | 8.03 ± 0.19          | 8.10 ± 0.20    |
| $B(D^0 \to K^+\pi^-\pi^-\pi^-)$ (10$^{-4}$) | 2.65 ± 0.19       | 2.63 ± 0.19          | 2.62 ± 0.20    |

FIG. 2: The 1σ, 2σ and 3σ allowed regions of (a) ($R_{K^{\pi\pi^0}}, \delta_D^{K^{\pi\pi^0}}$) and (b) ($R_{K3\pi}, \delta_D^{K3\pi}$) parameter space.

Parameters from the data; this procedure is referred to as the mixing-unconstrained fit. The $\Delta^F_{CP}$ observables are dependent on the value of $\delta_D^{K\pi}$ and its uncertainty from the normalisation method that used the measured values of $S(CP|K^-\pi^+)$. Therefore, initially the value and uncertainties of $\Delta^F_{CP}$ are recalculated assuming $\cos\delta_D^{K\pi} = 0 \pm 1$ and the mixing-unconstrained fit is performed. The resulting value of $\delta_D^{K\pi}$ is used to recalculate $\Delta^F_{CP}$ and the mixing-unconstrained fit is repeated. This procedure is iterated until the parameter values returned by the fit no longer changed within the quoted precision. The results of the final iteration are shown in Tab. V. The best-fit values of x, y, and $\delta_D^{K\pi}$ are: $x = (-0.8^{+2.9}_{-2.5})\%$, $y = (0.7^{+2.4}_{-2.7})\%$, and $\delta_D^{K\pi} = (-157.5^{+10.4}_{-11.0})\%$, respectively.
\[ y = (-0.7^{+2.4}_{-2.7})\%, \text{ and } \delta_{D_\pi} = (-130^{+38}_{-28})\% \text{.} \] There is an ambiguity in the solution of the unconstrained-fit if the signs of \( \delta_{D_\pi}, \delta_{D_3\pi}, \delta_{D_\pi^0} \) and \( x \) are all reversed. The correlations amongst the fit parameters maybe found in Ref. [19].

In summary, the first determination of the coherence factors and average strong-phase differences for \( D^0 \to K^-\pi^+\pi^0 \) and \( D^0 \to K^-3\pi \) has been presented. The results show significant coherence for \( D^0 \to K^-\pi^+\pi^0 \), but no significant coherence for \( D^0 \to K^-3\pi \). These results will improve the measurement of the unitarity triangle angle \( \gamma \) and the amplitude ratio \( r_B \) in \( B^- \to DK^- \) decays, where the \( D \) decays to \( K^-\pi^+\pi^0 \) and \( K^-3\pi \). The preliminary result for \( R_{K_3\pi} \) and \( \delta_{D_3\pi} \) [20] combined with CLEO-c’s measurement of \( \delta_{D_\pi} \) [10] was shown to improve the expected sensitivity to \( \gamma \) at LHCb in a combined ADS analysis of \( K\pi \) and \( K3\pi \) final states by up to 40\% [21]. The sensitivity of these data to \( y \) and \( \delta_{D_\pi} \) is also presented.

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TABLE VI: Correlation matrix for the mixing-constrained fit. Only elements above the diagonal are shown.

|         | $\delta^{K_3\pi}_{D}$ | $R_{K_{3\pi}}$ | $\delta^{K_{\pi\pi}}_{D}$ | $\delta^{K_{\pi\pi}}_{D}$ | $x$ | $y$ | $\delta^{K_{\pi}}_{D}$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|---------|------------------------|----------------|---------------------------|---------------------------|-----|-----|------------------------|-------|-------|-------|-------|-------|-------|
| $R_{K_{3\pi}}$ | -0.067 | 0.078 | 0.045 | -0.082 | -0.020 | -0.014 | 0.002 | 0.008 | 0.071 | 0.325 | -0.134 | 0.051 |
| $\delta^{K_3\pi}_{D}$ | -0.127 | 0.256 | -0.008 | 0.140 | 0.188 | -0.023 | 0.096 | 0.244 | -0.031 | -0.126 | -0.032 |
| $R_{K_{\pi\pi}}$ | 0.455 | 0.080 | -0.059 | -0.046 | -0.014 | 0.060 | 0.018 | 0.098 | -0.138 | 0.150 |
| $\delta^{K_{\pi\pi}}_{D}$ | -0.033 | 0.377 | 0.467 | 0.004 | -0.027 | 0.142 | 0.131 | -0.295 | 0.114 |
| $x$ | -0.189 | -0.188 | -0.001 | 0.005 | -0.037 | 0.001 | 0.047 | -0.006 |
| $y$ | 0.945 | 0.004 | -0.015 | 0.107 | -0.014 | -0.146 | 0.012 |
| $\delta^{K_{\pi}}_{D}$ | 0.005 | -0.004 | 0.121 | -0.002 | -0.071 | 0.008 |
| $B_1$ | 0.006 | -0.005 | 0.008 | 0.001 | -0.002 |
| $B_2$ | 0.005 | -0.028 | -0.024 | 0.008 |
| $B_3$ | 0.104 | 0.047 | -0.001 |
| $B_4$ | -0.054 | -0.006 |
| $B_5$ | 0.028 |

Key of branching fractions ($B$):
- $B_1 \equiv B(D^0 \to K^-\pi^+)$
- $B_2 \equiv B(D^0 \to K^+\pi^-)$
- $B_3 \equiv B(D^0 \to K^-\pi^+\pi^-)$
- $B_4 \equiv B(D^0 \to K^+\pi^-\pi^+)$
- $B_5 \equiv B(D^0 \to K^-\pi^+\pi^0)$
- $B_6 \equiv B(D^0 \to K^+\pi^-\pi^0)$
TABLE VII: Correlation matrix for the mixing-unconstrained fit. Only elements above the diagonal are shown.

|        | $\delta_{D}^{K_3\pi}$ | $R_{K_3\pi\rho}$ | $\delta_{D}^{K_\pi\pi}$ | $x$ | $y$ | $\delta_{D}^{K\pi}$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|--------|------------------------|-------------------|--------------------------|-----|-----|----------------------|-------|-------|-------|-------|-------|-------|
| $R_{K_3\pi}$ | -0.093                | -0.293            | -0.546                   | -0.558 | -0.175 | -0.505               | 0.002 | 0.009 | 0.038 | 0.245 | -0.079 | 0.069 |
| $\delta_{D}^{K_3\pi}$ | —                     | 0.763             | -0.179                   | 0.577 | -0.819 | 0.012               | -0.002 | 0.014 | 0.231 | -0.166 | -0.121 | -0.045 |
| $R_{K_3\pi\rho}$ | —                     | —                 | -0.049                   | 0.802 | -0.596 | -0.108               | -0.005 | 0.028 | 0.058 | -0.124 | -0.006 | 0.029 |
| $\delta_{D}^{K_\pi\pi}$ | —                     | —                 | —                        | —     | 0.175 | 0.504               | 0.692 | 0.002 | -0.026 | 0.194 | -0.003 | -0.308 | 0.034 |
| $x$ | —                     | —                 | —                        | —     | —     | —                   | -0.232 | 0.092 | 0.003 | -0.003 | 0.035 | -0.132 | 0.072 | -0.042 |
| $y$ | —                     | —                 | —                        | —     | —     | —                   | 0.255 | 0.004 | 0.015 | -0.081 | 0.144 | 0.029 | 0.008 |
| $\delta_{D}^{K\pi}$ | —                     | —                 | —                        | —     | —     | —                   | 0.006 | -0.034 | 0.211 | -0.157 | -0.223 | -0.038 |
| $B_1$ | —                     | —                 | —                        | —     | —     | —                   | —     | —     | —     | —     | —     | —     | —     | 0.005 | -0.003 | 0.006 | -0.000 | -0.002 |
| $B_2$ | —                     | —                 | —                        | —     | —     | —                   | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | 0.007 | -0.018 | -0.012 | 0.008 |
| $B_3$ | —                     | —                 | —                        | —     | —     | —                   | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | 0.075 | -0.013 | 0.000 |
| $B_4$ | —                     | —                 | —                        | —     | —     | —                   | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | -0.017 | -0.005 |
| $B_5$ | —                     | —                 | —                        | —     | —     | —                   | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | 0.017 |