Relativistic Continuum Quasiparticle Random Phase Approximation in Spherical Nuclei

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We have calculated the strength distributions of the dipole response in spherical nuclei, ranging all over the periodic table. The calculations were performed within two microscopic models: the discretized quasiparticle random phase approximation (QRPA) and the quasiparticle continuum RPA, which takes into account the coupling of the single-particle continuum in an exact way. Pairing correlations are treated with the BCS model. In the calculations, two density functionals were used, namely the functional PC-F1 and the functional DD-PC1. Both are based on relativistic point coupling Lagrangians. It is explicitly shown that this model is capable of reproducing the giant as well as the pygmy dipole resonance for open-shell nuclei in a high level of quantitative agreement with the available experimental observations.

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I. INTRODUCTION

The investigation of the isovector giant dipole resonance (IVGDR) is one of the fundamental problems in nuclear physics and astrophysics. These collective resonances can be studied experimentally by photon scattering (γ,γ′) or photodissociation (γ,n) processes, as well as by means of nuclear resonance fluorescence using linearly polarized and unpolarized bremsstrahlung [1]. Theoretical investigations are based mainly on self-consistent microscopic approaches, such as the Random Phase Approximation (RPA) or the Quasiparticle RPA (QRPA), which is a straightforward generalization of the RPA including pairing correlations.

In solving the RPA equations one should take into account all the particle-hole pairs that contribute to the excitation. The most common but computationally expensive way of doing this is by discretizing the basis and introducing a truncation (cut-off) parameter for the otherwise infinite configuration space. However, despite the fact that these models, in principle, enable one to reproduce experimental mean energies and total strengths of the giant resonances, they fail to describe their finite structure. One of the reasons is that RPA or QRPA do not provide a mechanism that produces the escape width Γ\(^\uparrow\), which gives a considerable contribution to the total width of the Giant Multipole Resonances. In addition, the space truncation which is required by the numerical calculations leads to model deficiencies, such as deviations of the resonance energy or mixing of spurious states coming from symmetry breaking with the physical states.

There is the alternative method of continuum RPA \(^2\) which avoids a basis truncation by treating the single-particle continuum explicitly. As a consequence, the entire configuration space is included effectively, without the need of energy cut-offs. Despite the fact that continuum RPA (CRPA) is proven to be a more complete and numerically faster approach than any other RPA method available, it has been applied in the past mostly in the non-relativistic framework \(^2\)–\(^18\). Applications of relativistic continuum RPA have been restricted to cases without pairing correlations, i.e. to doubly magic nuclei \(^19\)–\(^22\). Recently a CRPA approach based on relativistic point coupling models has been developed to study giant multipole resonances for spherical double magic nuclei in the entire periodic table \(^22\).

Apart from the well studied GDR, the pygmy resonance (IVPR) which appears as a soft low-lying mode in the dipole spectrum of neutron rich nuclei has still a lot to reveal \(^24\). A concept that attracts most of the attention is the picture of an oscillation of the excess neutrons against the isospin saturated proton-neutron core. Furthermore, low-lying E1 strength in unstable nuclei is currently also discussed in the context of the astrophysical r-process nucleosynthesis \(^23\), since they can affect the neutron capture rates, which are important quantities in the determination of the r-process path. The main difficulty in the study of this mode is due to the fact that its excitation energy is very close and sometimes even below the particle emission threshold, hindering in that way its experimental identification, specially when a photodissociation (γ,n) process is used.

In the present paper we investigate theoretically the dipole strength distribution for several even-even nuclei all over the nuclear chart. The calculations are performed within the framework of continuum QRPA (CQRPA) as well as of conventional QRPA, where the continuum is discretized. Pairing correlations in open-shell nuclei are treated within the BCS model. Introducing finite occupation probabilities for the single-particle spectrum of relativistic mean field (RMF) theory is a very successful scheme for treating pp-correlations in these nuclei. In addition dynamical pairing is applied on the RPA level.

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with an additional $pp$-term in the effective interaction. In this way, the approach is fully self-consistent and spurious states due to restoration of symmetries, such as translation and particle number invariance are properly separated from the rest of the spectrum.

It has to be emphasized, however, that the BCS model faces serious problems in nuclei with large neutron excess in the neighborhood of the drip line where the Fermi level is close to the continuum. In these cases, one has to use full Hartree Fock Bogoliubov or Relativistic Hartree Bogoliubov (RHB) theory which treats pairing correlations in a more consistent way. But, since we concentrate here on stable nuclei far from the drip lines, BCS is a suitable model.

The paper is organized as follows. In section II we briefly describe the Continuum Quasiparticle RPA based on relativistic point coupling Lagrangians. In section III we perform calculations on isovector dipole and pygmy resonances for several Sn isotopes and compare the results using the various successful parameter sets. Conclusions are drawn in Sec. IV.

### II. THE POINT COUPLING MODEL

As in all relativistic models, the nucleons are described as point like Dirac particles. In contrast to the Walecka model, however, where these particles interact by the exchange of effective mesons with finite mass, point coupling models neglect mesonic degrees of freedom and consider only interactions with zero range. In principle, these models are similar to the Nambu Jona-Lasinio model used extensively in hadron physics. There is, however, an important difference: in order to obtain a satisfactory description of the nuclear surface properties one also needs gradient terms in the Lagrangian simulating a finite range of the interaction. In this work we use two different point coupling Lagrangians: the set PC-F1 introduced by Buervenich et al. in Ref. [36] and the more recent set DD-PC1 by Nikšić et al. [37]. In both cases, the Lagrangian is represented in terms of the nucleon scalar ($S$), vector ($V$) and isovector-vector ($TV$) fields:

$$
\mathcal{L} = \bar{\psi}(i\gamma \cdot \partial - m)\psi - \sum_i \frac{1}{2} \alpha_i(\bar{\psi}\Gamma_i\psi)(\bar{\psi}\Gamma_i\psi) - \frac{1}{2} \delta_i(\partial_\nu \bar{\psi}\Gamma_i\psi)(\partial^\nu \bar{\psi}\Gamma_i\psi) - e^2(\bar{\psi}\Gamma_i C\psi)(\bar{\psi}\Gamma_i C\psi),
$$

(1)

where the Dirac vertices $\Gamma_i$ ($i = S, V, TV$) and the electromagnetic vertex $\Gamma_C$ have the explicit form:

$$
\Gamma_S = 1, \quad \Gamma_V = \gamma_\mu, \quad \Gamma_{TV} = \gamma_\mu \bar{\tau}, \quad \Gamma_C = \gamma_\mu \frac{(1 - \tau_3)}{2}.
$$

(2)

The coupling constants $\alpha_i$ depend on the density and this density dependence is different for the two forces. While, for PC-F1 each $\alpha_i(\rho)$ in the various spin-isospin channels depend on the corresponding local densities $\rho$, for DD-PC1 all the couplings $\alpha_i(\rho)$ depend on the baryon density $\rho = \rho_V$ alone. In particular, one has:

$$
\alpha_i(\rho) = \begin{cases} 
  a_i + b_i \rho + c_i \rho^2 & \text{for PC-F1}, \\
  a_i + (b_i + c_i) x e^{-d_i x} & \text{for DD-PC1},
\end{cases}
$$

(3)

where $x = \rho/\rho_{sat}$ denotes the nucleon density in units of the saturation density in symmetric nuclear matter.

In addition, DD-PC1 unlike its predecessors has not been adjusted to spherical nuclei but to ab-initio calculations together with nuclear matter data and to a large set of axially deformed nuclei. The two sets are listed in Table I and they have been tested in the calculation of many ground-state properties of spherical and deformed nuclei all over the periodic table. The results are very well comparable with reasonable effective meson-exchange interactions.

However, the success of a particular interaction relies on the ability to reproduce, apart from the static properties of a broad range of nuclei, also their dynamical properties, as for instance the properties of collective multipole resonances via the microscopic RPA approaches, which we will discuss in the next section.

| Parameter | PC-F1       | DD-PC1     |
|-----------|-------------|------------|
| $a_S$     | -14.935894  | -10.0462   |
| $b_S$     | 22.994736   | -9.1504    |
| $c_S$     | -66.769116  | -6.4273    |
| $d_S$     | 1.3724      |            |
| $\delta_S$| -0.634576   | -0.8149    |
| $a_V$     | 10.098025   | 5.9195     |
| $b_V$     | 0.0         | 8.8637     |
| $c_V$     | -9.817323   | 0.0        |
| $d_V$     | 0.6584      |            |
| $\delta_V$| -0.180746   | 0.0        |
| $a_{TV}$  | 1.350268    | 0.0        |
| $b_{TV}$  | 0.0         | 1.8360     |
| $c_{TV}$  | 0.0         | 0.0        |
| $d_{TV}$  | 0.6403      |            |
| $\delta_{TV}$ | -0.063680 | 0.0        |

TABLE I: The coupling constants for the density functionals PC-F1 and DD-PC1 resulting from the fitting procedure in Ref. [36] and Ref. [37] respectively. The parameters $d_i$ of DD-PC1 are dimensionless.
Linear response theory

If a nucleus is exposed to an external field, such as in the photoabsorption process, the strength function

\[ S(\omega) = -\frac{1}{\pi} Im \sum_{\alpha \beta} F_{\alpha}^* R_{\alpha\beta}(\omega) F_{\beta} \]  \hspace{1cm} (4)

measures the change on the nuclear density due to the influence of this field. The Greek indices \( \alpha, \beta \) indicate the various degrees of freedom of a nucleus \((r,L,S,T)\) while \( F \) is the operator of the external field.

If we consider only small amplitude variations of the density taking into account only one particle-one hole \((1\phi h)\) excitations, the response function \( R(\omega) \) can be deduced from the linearized Bethe-Salpeter equation:

\[ R_{\alpha\beta}(\omega) = R_{\alpha\beta}^0(\omega) + \sum_{\gamma\delta} R_{\alpha\gamma}^0(\omega) V_{\gamma\delta}^{ph} R_{\delta\beta}(\omega). \]  \hspace{1cm} (5)

This method is usually referred to as a response function formalism of the Random Phase Approximation, to distinguish from the more frequently used configuration space formalism \([42]\). The reason we chose this formalism is because it is essential for an exact treatment of the continuum coupling.

The residual interaction

As explained in textbooks \([3, 4]\), the residual interaction \( V_{\alpha\beta}^{ph} \) of Eq. (1) is connected to the static problem via the second derivative of the energy functional

\[ V_{\alpha\beta}^{ph} = \frac{\delta^2 E[\rho]}{\delta \rho_\alpha \delta \rho_\beta} \]  \hspace{1cm} (6)

and hence, it depends on the same coupling constants \( \alpha_i[\rho] \) and their derivatives with respect to the densities. In this way, one ensures a fully self-consistent treatment of the dynamical problem. The point coupling scheme allows one to write \( V_{\alpha\beta}^{ph} \) as a sum of separable terms, simplifying considerably the numerical solution of the Bethe-Salpeter equation. In particular, one can write:

\[ V_{\alpha\beta}^{ph}(1, 2) = \sum_c \int_0^\infty dr Q_c^{(1)}(r) v_{c,c'}(r) Q_c^{(2)}(r), \]  \hspace{1cm} (7)

where the upper indices \((1)\) and \((2)\) indicate that these operators act on the ”coordinates” \(1 = (r_1 \Omega_1 s_1 t_1)\) and \(2 = (r_2 \Omega_2 s_2 t_2)\).

Each separable term is characterized by \((c, r)\), the channel index \(c\) given by the discrete numbers \(\{D, S, L, J, T\}\) and the radial mesh point \(r\). In point coupling models under investigation, there are overall seven channels; one scalar \(S\), three isoscalar \(V\) and three isovector \(TV\) vectors. Assuming a coordinate mesh of 50 points, the final size of the interaction matrix is not larger than \(350 \times 350\). The corresponding channel vertices \(Q_c^{(1)}(r)\) are local single-particle operators

\[ \hat{Q}_c^{(1)}(r) = (-)^{s_c} \frac{\delta (r - r_1)}{rr_1} \hat{c}_c^\dagger Y_L(\Omega_1). \]  \hspace{1cm} (8)

Therefore, the Bethe-Salpeter equation (5) becomes:

\[ R_{cc'}(\omega) = R_{cc'}^0(\omega) + \sum_{c_1c_2} R_{cc_1}^0(\omega) v_{c_1,c_2} R_{c_2c'}(\omega), \]  \hspace{1cm} (9)

which has the same formal solution as in Eq. (4). For the continuous variables of the channel index \(c\), such as the radial coordinate \(r\), Eq. (9) is an integral equation.

The interaction \( v_{c,c'} \) can be expressed as a matrix in the spin-isospin space, indicated by the index \(c\). Apart from the direct terms \( \alpha_i[\rho] \) of the Lagrangian, \( v_{c,c'} \) also consists of the so called rearrangement terms, as a result of the double variation of Eq. (10). Furthermore, the gradient term is described by the operator \([23]\):

\[ \Delta = r^2 \nabla_r \frac{1}{r^2} \nabla_r + \frac{L(L+1)-2}{r^2}. \]  \hspace{1cm} (10)

In Table II and III, the exact form of the interaction matrices is displayed for the two parameter sets PC-F1 and DD-PC1. Further details can be found also in Refs. [23, 45].

Apart from the Coulomb force

\[ v_C(r, r') = \frac{4\pi e^2}{2L+1} \cdot \frac{r_L^L}{r_L^L + 1} \]  \hspace{1cm} (11)

all other interaction terms are diagonal in \(r\). Here, \(r_<\) and \(r_>\) are the smaller and the greater of \(r\) and \(r'\). This force breaks isospin symmetry. Hence, one has to expand
\(v_C\) in its spin-isospin components:

\[
V_C(1, 2) = \left( \frac{1}{2} + \gamma_3 \right) \left( \frac{e^2}{|r_1 - r_2|} \right) \frac{1}{2} (1 + \gamma_3) (2) \tag{12}
\]

\[
= \frac{1}{4} \frac{e^2}{|r_1 - r_2|} \times \left( (1)^2 (1) - \gamma_3 (1) \gamma_3 (2) - (1) \gamma_3 (1) (2) + (1) \gamma_3 (1) (2) \right).
\]

This means that in spin-isospin space, the Coulomb interaction can be split into four parts, but thankfully not into four additional channels. In particular, the two diagonal terms are added to the already existing isoscalar and isovector vector channels, while the other two correspond to off-diagonal terms. This leads to a Coulomb matrix \(v_C(r, r')\) as shown in Table IV.

The free response function

The free response function \(R_0^{cc'}(\omega)\) is the key point for a successful description of the RPA problem, since it includes all the microscopic properties of the nucleus under investigation. In the discrete quasiparticle space, i.e. in the spectral representation, it can be written as:

\[
R_0^{cc'} = \sum_{\alpha \leq \beta} \frac{1}{1 + \delta_{\alpha \beta}} \eta^{S}_{\alpha \beta} (\alpha || Q^\dagger \alpha || |c\rangle r, \eta^{S}_{\alpha \beta} (\alpha || Q_{c'} || |\beta\rangle r, \times \left( \frac{1}{\omega - E_{\alpha \beta} \pm i\eta} - \frac{1}{\omega + E_{\alpha \beta} \pm i\eta} \right), \tag{13}
\]

where the indices \(\alpha\) and \(\beta\) refer to the two-quasiparticle states in the excitation, while \(|\alpha\rangle_r = |\alpha(r)\rangle\) and \(|\beta\rangle\) are the corresponding radial single-particle wave functions. The factors \(\eta^{S}_{\alpha \beta} = u_{\alpha} v_{\beta} + (-)^{\delta_{\alpha \beta}} u_{\alpha} v_{\beta}\) include the BCS occupations \(u_{\alpha}\) and \(v_{\alpha}\). The energy \(E_{\alpha \beta} = E_{\alpha} + E_{\beta}\) is the sum of two quasiparticle energies. In addition, the smearing parameter \(\eta\) is used in Eq. (13) in order to avoid a divergence of the free response function and its value is often adjusted to the observed width of the giant resonances. It has to be emphasized that according to the no-sea approximation in relativistic RPA the sum in Eq. (13) includes also transitions to negative energy solutions \(\eta^{S}_{\alpha \beta} = u_{\alpha} v_{\beta}\) since the vacuum polarization is neglected.

One of the important questions which arises by using the spectral representation of Eq. (13) is the question of completeness of the basis. Numerical limitations require a basis truncation, which can play a role on the final solution. In practical calculations, an energy cutoff \(E_{\text{cut}} = 500\) MeV for the particles and a corresponding \(E_{\text{cut}} = -2000\) MeV for the antiparticles is required to achieve a converged solution. This fact, combined with the additional excitations to the anti-particles states leads to a very large configuration space \(\Sigma\).

In continuum QRPA however, the situation is very different. Here, one makes use of the non-spectral representation of the free response function, which is given by:

\[
R_0^{cc'}(\omega) = \sum_{\alpha} u_{\alpha}^{2} \langle \alpha(r) | Q^\dagger \alpha || c \rangle (\omega - \epsilon_{\alpha}) Q_{c'} |\alpha(r')\rangle + \langle \alpha(r) | Q^\dagger \alpha || c \rangle (\omega - \epsilon_{\alpha}) Q_{c'} |\alpha(r')\rangle,
\tag{14}
\]

where the relativistic Green’s function is:

\[
g_{\kappa}(E) = \begin{cases} |w_{\kappa}(r)||u_{\kappa}^{*}(r')\rangle/W & \text{for } r > r' \\ |u_{\kappa}(r)||w_{\kappa}(r')\rangle/W & \text{for } r < r' \end{cases}
\tag{15}
\]

The two-dimensional spinors \(|u_{\kappa}(r)\rangle = (f_{\kappa}(r)_{g_{\kappa}(r)})\) and \(|w_{\kappa}(r)\rangle = (f_{\kappa}(r)_{g_{\kappa}(r)})\) are the regular and the irregular wave functions of a continuum state with the quantum number \(\kappa = (ij)\) and the energy \(E\). Further details are given in Ref. [25]. Since \(|w_{\kappa}(r)\rangle\) behaves as an outgoing wave at \(r \to \infty\), it is a complex quantity. In this way, the strength function in Eq. (14) has a non-vanishing imaginary part without the need of any smearing parameter, as in Eq. (13). The denominator \(W\) describes the Wronskian of the system:

\[
W = \langle w_{\kappa}(r)||u_{\kappa}^{*}(r)\rangle = f_{\kappa}(r)g_{\kappa}(r) - g_{\kappa}(r)f_{\kappa}(r)
\tag{16}
\]

and is independent of \(r\). In the particular case where the energy of the continuum state meets the value of an eigenstate \(E = \omega - \epsilon_{\alpha} = \epsilon_{\beta}\), the two wavefunctions \(|u_{\kappa}\rangle\) and \(|w_{\kappa}\rangle\) become identical and the Green’s function diverges, similar to the case in Eq. (13). So, the two expressions of the free response functions in Eqs. (13) and (14) are in principle identical, but in practice and due to the space truncation of the spectral representation, differences are expected.

Consequently, one of the big advantages of the non-spectral or continuum representation Eq. (14) is the inclusion of the entire configuration space, without the need of any truncation technique. This is achieved by summing over blocks of well defined quantum numbers \(\kappa\). The wavefunctions are exact and obey the proper properties and better agreement with the experimental data [23].

Despite its simplicity, a problem arises when pairing correlations are included. From Eq. (14), we see that all levels lying in the continuum have \(u_{\beta} = 1\), i.e. they are

|   | S | V | TV |
|---|---|---|----|
| S | 0 | 0 | 0 |
| V | 0 | \(\frac{1}{2}v_C\) | \(-\frac{1}{2}v_C\) |
| TV | \(-\frac{1}{2}v_C\) | \(\frac{1}{2}v_C\) |

TABLE IV: The structure of the channel matrix \(v_C^{(c,c')}\) for the Coulomb interaction.
all considered as pure particles. But this is not the case in the QRPA model, where levels close to the Fermi surface (some of them above the single-particle continuum limit) must be treated as quasiparticles. That implies that in order to be consistent, we need to modify the continuum representation to properly account for the states within the space, where pairing is active.

In Fig. 1, we schematically illustrate our technique, which has been used in the past to define the continuum QRPA in non-relativistic models [9, 10]. In this figure, the allowed transitions are displayed as a sum of three different terms. At first, the continuum term \( R^0_{\text{cont}} \) of Eq. (14) includes the transitions to the entire single-particle space.

The two-quasiparticle excitations within the pairing active space are treated by the two-quasiparticle response function Eq. (13) up to the pairing cut-off, which does not exceed the \( E_p = +20 \text{ MeV} \).

However, in order to avoid double counting of the excitations in the active pairing space, one needs to subtract a correction term, which describes transitions from a quasiparticle to a pure particle using the expression:

\[
R^0_{\text{corr}} = \sum_{\alpha \leq \beta} \frac{1}{1 + \delta_{\alpha \beta}} \left( \langle \alpha || Q^\dagger || \beta \rangle_r \langle \alpha || Q || \beta \rangle_{r'} \right) \\
\times \left\{ \frac{v^2_\alpha}{\omega - \Omega_{\alpha, \beta} + i\eta} - \frac{v^2_\alpha}{\omega + \Omega_{\alpha, \beta} + i\eta} \right\} \left(17\right)
\]

where \( \Omega_{\alpha, \beta} = E_\beta + \varepsilon_\alpha - \lambda \). The indices \( \alpha \) and \( \beta \) run only over the partially occupied states below the continuum limit (\( \varepsilon_\alpha < 0 \)). Consequently, a proper treatment of the quasiparticle CRPA requires that the response function

\[
R^0(r, r' \omega) = R^0_{\text{cont}}(r, r' \omega) + R^0_{\text{2qp}}(r, r' \omega) - R^0_{\text{corr}}(r, r' \omega) \quad \left(18\right)
\]

is used in the Eq. (5).

It has to be emphasized that all the particle states above the pairing window as well as the states in the Dirac sea do not participate in the above calculation, since they are completely taken into account in the continuum part \( R^0_{\text{cont}}(r, r' \omega) \) in an exact way.

### III. RESULTS AND DISCUSSIONS

In the following we show the IVGDR results for various spherical open-shell nuclei, using both the continuum and the discrete QRPA models.

In a first step, the ground state of the nucleus is determined by solving the self-consistent RMF equations for the parameter sets PC-F1 and DD-PC1 given in Table I. Pairing correlations are treated within the BCS model with constant gap. For the pairing gap, either the empirical expression \( \Delta_{\text{np}} = 12.0/\sqrt{A} \) or the gaps deduced from nuclear mass tables [1] can be used, with the difference to be insignificant to the final outcome.

Using the single-particle wave functions and the corresponding quasiparticle energies of this static solution, we determine the free response \( R^0 \) of Eq. (18). Finally, we solve the Bethe-Salpeter equation (9) to get the strength distribution \( S(\omega) \).

At the same time, we perform similar calculations using the conventional RPA approach, where the continuum is fully discretized. For those calculations, we have used an energy cut-off \( |\epsilon_p - \epsilon_h| < E_{cut}^{ph} = 300 \text{ MeV} \) for the configurations with particles above the Fermi sea and \( |\epsilon_a - \epsilon_h| < E_{cut}^{ph} = 2000 \text{ MeV} \) for configurations with anti-particles in the Dirac sea.

In the left panel of Fig. 2, we show the cross section for the IVGDR in the open shell nucleus \(^{124}\text{Sn}\) for the two parameter sets PC-F1 (red solid) and DD-PC1 (blue dashed), as well as the experimental values from Ref. [17]. The cross section is linear to the E1 strength of Eq. (4).
FIG. 2: (Color online) Panel (a): The IVGDR strength distributions for $^{124}$Sn using the forces PC-F1 (red solid line) and DD-PC1 (blue dashed line). The experimental results are taken from Ref. [47]. Panel (b): Neutron and proton transition densities for the GDR at $E = 14.40$ MeV.

According to:

$$\sigma(E) = \frac{16\pi^2 e^2}{9\hbar c} E S(E) \left[ \text{fm}^2 \right] = 4.022 E S(E) \left[ \text{mb} \right]. \quad (19)$$

The two parameterizations PC-F1 (red solid) and DD-PC1 (blue dashed) perform very well in reproducing the Giant Dipole resonance, as compared to experimental data [47]. The energy-weighted sum rule for the two forces are found at $m_1 = 2043.47$ and $m_1 = 2151.85$ [mb · MeV] respectively. These results are in agreement with the discrete QRPA and, as usual, somewhat (13.7% and 19.7%) larger than the classical Thomas-Reiche-Kuhn (TRK) sum rule:

$$m_{\text{TRK}} = 60.0 \frac{N Z}{A} = 1782.32 \left[ \text{mb MeV} \right]. \quad (20)$$

Due to the exact coupling to the continuum, the escape width $\Gamma^\uparrow$ of the IVGDR is automatically taken into account. However, in heavy nuclei, $\Gamma^\uparrow$ is very small, due to the high Coulomb and centrifugal barriers, which prevent the excited nucleon from escaping. The rest of the total width comes mainly from the coupling to more complex configurations (2ph, 3ph, etc.). Here, this part is treated approximately by an additional smearing parameter, which can be energy or temperature dependent [49], according to the expression:

$$\Gamma(E) = \Gamma_0 \frac{E^2 + 4\pi^2 T^2}{E_{GDR}^2}. \quad (21)$$

In our study, we have calculated the width by using $\Gamma_0 = 1.4$ MeV and $T = 0$. On the right panel of Fig. 2 we give the proton and neutron transition densities $\delta\rho(r)$ associated with the GDR peak at $E = 14.40$ MeV. We see that the excitation at this energy has a pure isovector character, since the neutrons are coherently oscillating against the protons.

In Fig. 3 the centroid energies of the IVGDR are revealed for several Sn isotopes, which have been well identified experimentally [1, 47, 50]. The centroid energy $\bar{E} = m_1/m_0$ is calculated in the same energy window as the one used in the experimental analysis, i.e. between 13-18 MeV. It can be clearly seen that both discrete and continuum QRPA approaches predict a similar decrease of the centroid energies, with respect to the mass number of the isotope, but the agreement with the experimental data is better reproduced with the continuum approach.

FIG. 3: (Color online) Centroid energies for several Sn isotopes using the DRPA (blue boxes) and the CRPA (red squares) approaches. The experimental results are deduced from Refs. [1, 47, 50].
the neutron emission threshold at exactly the same isotope, as observed in both the continuum and the discrete approach. In the right panel of Fig. 6, the proton transition densities at the peak energy is used to show the pygmy character of this mode, i.e. the oscillation of a relatively pure neutron skin against the isospin-saturated proton-neutron core.

This can be clearly seen in Fig. 5, where we compare the E1 strength of $^{124}$Sn derived from the discrete RPA approach with various energy cut-offs with results from continuum RPA. For the sake of the present discussion, a very small smearing parameter $\Gamma$ has been used. It is very interesting to see how the position of the pygmy mode moves to lower energies, as we increase the configuration space and also that it approaches the position of continuum RPA, in which the entire configuration space is included.

Therefore, it becomes evident that a proper treatment of the continuum seems to be very important in the calculation of the low-lying collective phenomena. It has to be emphasized that till now in all the previous relativistic investigations this has not been possible. In Fig. 6 we go further and show the details of the PDR in the isotope $^{132}$Sn, which has only recently been identified experimentally [52].

The dashed line correspond to the discrete RPA, while the solid one shows the continuum RPA calculations. Although the experimental uncertainties are still large in this neutron rich nucleus, we find a nice agreement with the Relativistic RPA model, in particular in the continuum approach. In the right panel of Fig. 6 the proton and neutron transition densities at the peak energy is used to show the pygmy character of this mode, i.e. the oscillation of a relatively pure neutron skin against the isospin-saturated proton-neutron core.

In Table V results for the two density functionals PC-F1 and DD-PC1 are compared using the continuum approach. We have studied a series of stable spherical nuclei, which have experimentally well defined collective properties. The centroid energies have been calculated in the area between 11 – 18 MeV.

### Table V: Excitation Energy of the isovector dipole resonance for several nuclei, using both DD-PC1 and PC-F1 self-consistent interactions. The centroid energies have been calculated in the area between 11 – 18 MeV.

|核 | $E_0$ | $\bar{E}$ | Exp. [MeV] |
|---|---|---|---|
|$^{70}$Zn | 17.50 | 16.70 | 17.25 ± 0.08 [51] |
| | | | |
|$^{94}$Zr | 16.60 | 15.60 | 16.67 ± 0.07 [52] |
| | | | |
|$^{124}$Sn | 15.40 | 14.40 | 14.67 ± 0.08 [53] |
| | | | |
|$^{130}$Te | 15.30 | 14.60 | 14.53 ± 0.13 [53] |
| | | | |
|$^{138}$Ba | 15.20 | 14.40 | 15.29 ± 0.15 [54] |
| | | | |
|$^{144}$Sm | 15.10 | 14.50 | 15.37 ± 0.13 [55] |
| | | | |
|$^{208}$Pb | 13.60 | 12.80 | 13.50 ± 0.19 [56] |

FIG. 4: (Color online) Panel (a): The ratio of the PDR strength to the TRK sum rule for various Sn isotopes. The blue circles indicate discrete RPA results, while the red triangles are Continuum RPA results. The calculated PDR is integrated up to 11 MeV. In both cases, the force PC-F1 has been used. Panel (b): The neutron emission threshold for the same range of Sn isotopes. Here the force PC-F1 is used.
IV. CONCLUSIONS

Starting from a point coupling Lagrangian, we have used the relativistic continuum QRPA approach to examine the E1 collective excitation spectra of spherical open-shell nuclei. This non-spectral method has several advantages. The use of exact scattering wavefunctions with proper boundary conditions, instead of expansions in a harmonic oscillator basis in the continuum allows for the simultaneous inclusion of the entire continuum and the contributions of the Dirac sea. Furthermore, large sums over unbound states are avoided, which is extremely important in relativistic model, since the unbound states in the Dirac sea are the root for computationally very expensive calculations.

In these investigations the ground state properties are calculated using a relativistic point coupling Lagrangian with the parameter sets PC-F1 and DD-PC1. The RMF equations are solved in $r$-space self-consistently. For open-shell nuclei, the BCS model is applied to treat the pairing correlations properly.

The residual particle-hole interaction used in the QRPA calculations is derived from the same parameter sets as the second derivative of the energy functional with respect to the density. In this way no additional parameter is required for the study of the dynamical problem. One has current conservation and an exact separation of the spurious modes and one is able to reproduce the collective properties, as for instance the multipole giant resonances, in a fully self-consistent way.

The calculations are carried out by using two different point-coupling forces, namely the PC-F1 and DD-PC1. The interaction then includes the basic zero range terms, rearrangement terms due to density dependance, the derivative terms which simulate the finite range of the nuclear interaction, the various current-current terms and finally the Coulomb interaction between protons.

We have used the continuum QRPA approach to study collective properties, which are initiated by photoabsorption processes. We have shown that the model performs well in describing both the giant dipole and the soft pygmy resonance. In particular, although there are not large differences in the details between the continuum and the discrete RPA calculations as far as the GDR is concerned, there is still some systematic difference close to the neutron separation threshold of stable nuclei and an even larger one for the extreme neutron-rich nuclei.

New ($\gamma,\gamma'$) experiments on this low-lying area \cite{58} are expected to be of paramount importance to the understanding of the real contribution of the exact coupling to the continuum.

Finally, this approach accounts for nuclei far from the drip lines where no level in the continuum is occupied and thus the BCS models can be safely applied. For nuclei up to the drip line, the relativistic Hartree-Bogoliubov approximation would be required. Investigations in this direction are in progress.

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