Influence of Varying Temperature and Concentration on (MHD) Peristaltic Transport for Jeffrey Fluid through an Inclined Porous Channel

Dheia Gaze Salih Al–Khafajy
Department of Mathematics, College of Science, University of Al-Qadisiyah, Diwaneyah, Iraq.
E-mail: dr.dheia.g.salih@gmail.com, dheia.salih@qu.edu.iq

Abstract: This paper aims to create a mathematical model that studies the effects of the peristaltic transfer of the Jeffrey fluid with a magnetic field, a change in temperature, and concentration through a slanted porous channel. Assuming that the viscosity of the fluid is variable by the effect of temperature, we found a solution for the momentum equation by using perturbation series method and assuming the long-wavelength which leads to small Reynolds number, and then we studied the effect of effective variables on the movement of the fluid.

Keywords: Jeffrey fluid; Variable viscosity; Peristaltic flow; MHD, Inclined porous channel.

1- INTRODUCTION

The presence of the magnetic field generated from an electric current is one of the important topics in fluid kinematics through a channel, which has invited many researchers to write about their uses in multiple sciences, including natural, health, and others. Many researchers have studied the peristaltic flow of a fluid through a channel under the influence of a magnetic field, Reddy in (1), found that the fluid velocity increases with increasing permeability parameter, while the fluid velocity decreases with
increasing magnetic parameter. Hayat et al. in (2), found that when the magnetic parameter increased the fluid velocity increased near the flow channel wall while the fluid velocity decreased in the center of the channel. In (3), Imran et al. found that the fluid velocity decreases with magnetic parameter. Ahmed and Dheia in (4), found that when the magnetic parameter is increased the fluid velocity increases when the half of the flow channel is less than 0.2 while the fluid velocity decreases when half of the flow channel is greater than 0.2. Riaz et al. in (5), found that the fluid velocity decreases with magnetic parameter.

Peristalsis movement is important in the flow of fluids in the human body, especially the movement of blood in the arteries and veins (blood circulation) in addition to the movement of water and food waste in the intestine (food cycle). There are other uses of peristaltic motions through oil or water pumping pipelines and many other uses. There are multiple studies in the scientific literature on this flow, some of which are in (1, 2, 4, 5, 6, 7).

Recently, interest began to study the effect of temperature and focus on fluid movement across the channel, as most researchers agree that increasing the temperature increases the velocity of the fluid while the fluid's velocity changes in an unclear manner with the difference in concentration and according to the location of the fluid in the channel, see (8-11) for more details.

Most of the flow channels, especially in the human body, are oblique channels, and for this, there was great interest in the recent period of fluid flow through oblique channels as many researchers presented mathematical models in the flow of different fluids in the oblique channels. Numerically analyzed the simulation of buoyancy peristaltic flow of Johnson-Segalman nanofluid in an inclined channel has been investigated by Hayat et al. (11). Mohammed and Ahmed (12) studied the effect of the heat and mass transfer on inclined MHD peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined asymmetric channel. Al-Khafajy (13) described the radiation and mass transfer effects on MHD oscillatory flow for Carreau fluid through an inclined porous channel.

This study aimed to analyze the mathematical model of the effect of peristalsis flow from the MHD of a Jeffrey fluid through a cylindrical polar coordinates system for the slanted porous channel at different temperatures and concentrations with variable viscosity for the fluid. This article consists of five sections, and this is the first section "Introduction". The second section includes the formulation of governing equations with the fluid type used and the boundary conditions of the problem, as well as introducing dimensional transformations to facilitate governing equations by assuming a very small Reynolds's number or a very large wavelength to solve the problem. In the third section, the dimensionless momentum equations are analytically solved by the perturbation technique. The third section includes the effects of various emerging parameters that are discussed through graphs in detail. The fourth section discusses the trapping phenomenon and the parameters that affect the increase and decrease, appear or disappear of the trapping bolus. The last section briefly reviews the most important
parameters (Schmidt number, Grashof number, Prandtl number, Darcy number, magnetic parameter) that affect the movement of the fluid.

2- MATHEMATICAL FORMULATION

Consider the peristaltic flow of a non-Newtonian incompressible Jeffrey fluid through an inclined co-circular tube. We use a cylindrical polar coordinate system \((R, Z)\), where \(R\) represents the length of the tube radius and \(Z\) is the tube axes as shown in Figure 1. A uniform magnetic field \(B_0\) is imposed and acting along axis, with a difference in concentration and temperature on both sides of the flow channel.

\[ H(\tilde{Z}, \lambda) = a + b \sin\left(\frac{2\pi}{\lambda}(\tilde{Z} - c\tilde{t})\right) \]  

(1)

The geometry of wall:

\[ \eta(\tilde{Z}, \lambda) = \eta + \eta_0 \sin(\lambda \tilde{Z}) \]

(2)

The constitute equation for Jeffrey fluid with variable viscosity is [4]:

\[ \dot{\gamma} = -\dot{P} + \dot{S}, \]

(2)

\[ \dot{S} = \frac{\mu(\gamma)}{1+\lambda_1} (\gamma + \lambda_2 \dot{\gamma}), \]

(3)

Let \( \dot{V} = (u, v, w) \) be the velocity vector in the cylindrical coordinates \((r, \theta, z)\). The relevant equations for present flow are:

\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \]

(4)

\[ \rho \left( \frac{\partial u}{\partial r} + \tilde{w} \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r\eta S_{rr}) + \frac{\partial}{\partial z} (S_{zz}) + \frac{\partial \tilde{S}_{\theta r}}{\partial r}, \]

(5)

Fig.1 Physical model

The geometry of wall: \( H(\tilde{Z}, \lambda) = a + b \sin\left[\frac{2\pi}{\lambda}(\tilde{Z} - c\tilde{t})\right] \)
\[ \rho \left( \frac{\partial w}{\partial t} + \vec{u} \cdot \nabla w \right) = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \tilde{S}_{zz}}{\partial r} \right) + \frac{\partial}{\partial z} \left( \tilde{S}_{zz} \right) + \rho \beta_T (T - T_r) \sin(\sigma) + \rho g \beta_C (C - C_r) \sin(\sigma) - \sigma B_0^2 \sin^2(\sigma) \vec{w} - \frac{u(T)}{k} \vec{w} + \rho g \sin(\beta), \]

(6)

\[ \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \rho c_p \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} T + \frac{\partial^2 T}{\partial z^2} \right) - \frac{16 \alpha T}{3 \kappa c_p \rho r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \frac{Q}{\rho c_p}, \]

(7)

\[ \frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = \frac{D_m}{\alpha_2} \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2} \right) + \frac{\partial m_k}{\partial t} \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2} \right), \]

(8)

with boundary conditions:

\[ \vec{w} = -1, \quad \vec{u} = 0, \quad T = T_{r1}, \quad C = C_r \quad \text{at} \quad r = r_1 = \epsilon \]

\[ \vec{w} = -1, \quad \vec{u} = 0, \quad T = T_{r2}, \quad C = C_r \quad \text{at} \quad r = r_2 = 1 + \phi \sin(2\pi z), \]

(9)

To solve the equation system (4-8) need the dimensionless transformations as follows:

\[ \vartheta = \frac{T - T_{r1}}{C_r - C_{r1}}, \quad \varphi = \frac{r - r_1}{C_r - C_{r1}}, \quad \varrho = \frac{a_2}{2}, \quad \mu = \frac{c_p}{\rho c_p}, \quad \phi = \frac{a_2}{a_2}, \]

(10)

Non-dimensional equations are:

\[ \left( \frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial u}{\partial z} \right) = 0, \]

\[ (u \frac{\partial \vartheta}{\partial r} + (w + 1) \frac{\partial \vartheta}{\partial z}) = -\frac{\partial \varrho}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) + \frac{\partial^2}{\partial z^2} \left( S_{zz} \right) - \frac{\delta \partial^2 \vartheta}{r} - \frac{\partial^2 \vartheta}{r^2} u - \delta \frac{\partial \vartheta}{\partial z}, \]

(11)

\[ R_e \delta \left( u \frac{\partial \vartheta}{\partial r} + (w + 1) \frac{\partial \vartheta}{\partial z} \right) = -\frac{\partial \varrho}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) + \frac{\partial^2}{\partial z^2} \left( S_{zz} \right) - \left( \frac{M_e^2 + \mu(\vartheta)}{\mu_0^2} \right) \vartheta + \frac{\partial \vartheta}{\partial z}, \]

(12)

\[ \delta \left( u \frac{\partial \vartheta}{\partial r} + w \frac{\partial \vartheta}{\partial z} \right) = \frac{1}{r} \frac{\partial^2 \vartheta}{\partial z^2} + \frac{1}{r^2} \frac{\partial^2 \vartheta}{\partial z^2} + \frac{1}{r} \frac{\partial^2 \vartheta}{\partial z^2} + \frac{1}{r} \frac{\partial^2 \vartheta}{\partial z^2}, \]

(13)

Where

\[ S_{rr} = \frac{2 \mu(\vartheta)}{1 + \lambda_1} \left[ 1 + \frac{c \lambda_2}{a_2} \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \right] \left( \frac{\partial u}{\partial r} \right), \]

\[ S_{rz} = \frac{\mu}{1 + \lambda_1} \left[ 1 + \frac{c \lambda_2}{a_2} \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \right] \left( \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right), \]

\[ S_{\vartheta r} = \frac{2 \mu(\vartheta)}{1 + \lambda_1} \left[ \frac{u}{r} + \frac{c \lambda_2}{a_2} \left( \frac{u}{\partial r} - \frac{u^2}{r^2} + \frac{w}{\partial z} \right) \right], \]

\[ S_{\vartheta z} = \frac{2 \mu(\vartheta)}{1 + \lambda_1} \left[ 1 + \frac{c \lambda_2}{a_2} \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \right] \left( \frac{\partial w}{\partial z} \right), \]

(16)

(17)

(18)

(19)

The assumption of long-wavelength is often achieved in physical situations such as blood flow in the narrow arteries and the transport of food through the small intestine, where half the width of the intestine is small compared to the wavelength of peristaltic wavelength. Thus, after using the long-wavelength (\( \delta \ll 1 \)) and small Reynolds number (\( R_e \rightarrow 0 \)), one gets:
\[
\frac{\partial \rho}{\partial t} = 0,
\]
\[
\frac{\partial \rho}{\partial x} = \frac{1}{r} S_{rx} + \frac{\partial}{\partial r} \left( S_{r \theta} \right) - \left( M_{\phi}^2 + \frac{\mu(\theta)}{\lambda_1} \right) w + Gr \theta \sin(\sigma) + Gc \phi \sin(\sigma) + \frac{R_{e}}{r} \sin(\beta)
\]
\[- \left( M_{\phi}^2 + \frac{\mu(\theta)}{\lambda_1} \right),
\]
\[
\left( \frac{1}{R_{e} \phi} + \frac{4}{3R_n} \right) \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} - \Omega \phi = 0,
\]
\[
\frac{1}{S_c} \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} = -S_r \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right).
\]

where \( S_{rr} = S_{r \theta} = S_{xx} = 0 \) and \( S_{rz} = \frac{\mu(\theta)}{1+\lambda_1} \left( \frac{\partial w}{\partial r} \right). \) By replacing \( S_{rz} \) in equation (21), we have:
\[
\frac{\partial \rho}{\partial x} = \frac{\mu(\theta)}{1+\lambda_1} \left( \frac{\partial w}{\partial r} \right) + \frac{\mu(\theta)}{1+\lambda_1} \left( \frac{\partial w}{\partial r} \right) - \left( M_{\phi}^2 + \frac{\mu(\theta)}{\lambda_1} \right) w + Gr \theta \sin(\sigma) + Gc \phi \sin(\sigma)
\]
\[+ \frac{R_{e}}{r} \sin(\beta) - \left( M_{\phi}^2 + \frac{\mu(\theta)}{\lambda_1} \right).
\]

With the following dimensionless boundary conditions:
\[w = -1, u = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad r = r_1 = \varepsilon \]
\[w = -1, u = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{at} \quad r = r_2 = 1 + \phi \sin(2 \pi z).
\]

And the stream function \( \psi \) which shows the phenomenon of the trapping bolus is the following two equations \( u = -\frac{1}{r} \left( \frac{\partial \psi}{\partial y} \right) \) and \( w = \frac{1}{r} \left( \frac{\partial \psi}{\partial r} \right).

### 3. SOLUTION PROCEDURE

This section is divided into three parts:

#### 3.1 Solution of the heat equation and concentration equation

The solutions of heat equation (22) and concentration equation (23), respectively, are (see 4 for details):
\[
\theta = c_1 J_0 \left[ \sqrt{\alpha} r \right] + c_2 Y_0 \left[ \sqrt{\alpha} r \right],
\]
where \( \alpha = -\frac{\alpha}{R_{e} \pi} \), \( c_1 = \frac{Y_0[\sqrt{\alpha}]}{J_0[\sqrt{\alpha}]} \), and
\[
c_2 = \frac{J_0[\sqrt{\alpha}]}{J_0[\sqrt{\alpha}]} - \frac{J_0[\sqrt{\alpha}]}{J_0[\sqrt{\alpha}]} - \frac{J_0[\sqrt{\alpha}]}{J_0[\sqrt{\alpha}]}
\]
\[
\phi = \frac{S_0}{S_0} \frac{r^2}{2} \frac{\partial \phi}{\partial r} + c_3 \ln(r) + c_4
\]
where \( c_3 = \frac{S_0}{S_0} \frac{r^2}{4 \ln(r_2/r_1)} \) and \( c_4 = c_3 \ln(r_2). \)

#### 3.2 Solution of the momentum equation

To solve the momentum equation (24), using the Reynolds model for variation viscosity with temperature (see 8). Let:
\[
\mu(\theta) = e^{-\alpha \theta} = 1 - \alpha \theta, \alpha << 1
\]
Equation (20) yield \( p(r, z) = p(z) \). Substituting equation (29) into equation (24), obtain:

\[
\begin{align*}
\frac{dp}{dz} = & \left( 1 - \frac{1}{\lambda_1} \right) \frac{1}{r} \left( \frac{\partial w}{\partial r} \right) + \left( 1 - \frac{1}{\lambda_1} \right) \frac{\partial}{\partial r} \left( \frac{\partial w}{\partial r} \right) - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) w + Gr \partial \sin(\sigma) + Gc \partial \sin(\sigma) \\
& - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) \frac{R_e}{Fr} \sin(\beta)
\end{align*}
\]

(30)

It is difficult to find the solution of the above equation, therefore using the perturbation technique method to solve the equation assuming

\[
\begin{align*}
w = & w_0 + \alpha w_1 + o(\alpha^2) \\
p = & p_0 + \alpha p_1 + o(\alpha^2)
\end{align*}
\]

(31)

substituting equations (31) into equations (30), and equalize the powers of \( \alpha \).

\[
\begin{align*}
\frac{1}{1 + \lambda_1} \frac{\partial w_0}{\partial r} + & \frac{1}{1 + \lambda_1} \frac{\partial^2 w_0}{\partial r^2} - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) w_0 + Gr \partial \sin(\sigma) + Gc \partial \sin(\sigma) - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) \frac{R_e}{Fr} \sin(\beta) \\
& - \frac{d p_0}{dz} + \alpha \left[ \frac{1}{1 + \lambda_1} \frac{\partial w_1}{\partial r} + \frac{\partial}{\partial a} \right] \frac{\partial^2 w_1}{\partial r^2} - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) w_1 + \frac{\partial}{\partial a} \frac{d p_1}{dz} \right] = 0
\end{align*}
\]

(32)

the following set of equations are obtained.

(A) **Zeros-Order System** \((\alpha^0)\)

\[
\begin{align*}
& \frac{\partial^2 w_0}{\partial r^2} + \frac{1}{r} \frac{\partial w_0}{\partial r} - \left( 1 + \lambda_1 \right) \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) w_0 = (1 + \lambda_1) \left( \frac{d p_0}{dz} + \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) \frac{R_e}{Fr} \sin(\beta) \right) \\
& Gc \partial \sin(\sigma) - \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) \frac{R_e}{Fr} \sin(\beta)
\end{align*}
\]

(33)

The boundary conditions are: \( w_0(r_1) = w_0(r_2) = -1 \).

(B) **First-Order System** \((\alpha^1)\)

\[
\begin{align*}
& \frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r} \frac{\partial w_1}{\partial r} - \left( 1 + \lambda_1 \right) \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) w_1 = (1 + \lambda_1) \left[ \frac{\partial}{\partial a} \frac{\partial^2 w_0}{\partial r^2} + \left( M_{\sigma}^2 + \frac{1}{\partial_a} \right) \frac{R_e}{Fr} \sin(\beta) \right] + \frac{d p_1}{dz}
\end{align*}
\]

(34)

The boundary conditions are: \( w_1(r_1) = w_1(r_2) = 0 \).

Discussed the trapping phenomena after solving the equation \( \psi = r \int w(z, r) dz \). The mean volume flow over a period is obtained as \( q_1 = q + \frac{1}{2} \left( 1 - e^2 + \frac{\alpha^3}{2} \right) \), where \( q = 2 \int_{r_1}^{r_2} r w(z, r) dr \) is the instantaneous volume flow rate, see 4 for details.

4. **NUMERICAL RESULTS AND DISCUSSION**
This section is divided into two parts:

4.1 The velocity and the pressure gradient

This section includes discussing the behavior of the fluid movement with the pressure gradient generated by this movement, and we have not discussed the behavior of solving the heat and concentration equations that were previously discussed in 4. Analytical solutions are acquired for motion equation with the perturbation technique. Discussed graphically the solution obtained under variations of different parameters relevant in this section. Figs 2-10, illustrate the effect of the parameters \( D_a, M, G_r, \beta, G_c, \sigma, \alpha, \lambda_1, \Omega, R_n, R_e, F_r, P_r, q_1, S_r, S_c, \epsilon \) and \( \emptyset \) on the velocity of the fluid and pressure gradient \( (dp/dz) \) generated by this velocity, respectively.

Fig 2 illustrates the influence of the parameters \( D_a \) and \( M \) on the velocity distribution in part (a) and pressure gradient \( (dp/dz) \) in part (b). It is found that the velocity profile and pressure gradient rising with the increasing \( M \), while the velocity and pressure decreases with increasing \( D_a \). Fig 3 shows the velocity profile and pressure gradient behavior under the variation of \( G_r \) and \( \beta \). It is mentioned that the velocity and pressure go down with the increasing effects of both the parameters, respectively. Fig 4 contains the velocity profile and pressure gradient behavior under the difference of \( G_c \) and \( \sigma \), it is aforesaid that the velocity and pressure decreases with the increasing effects of both the parameters, respectively. Fig 5 illustrates the influence of the parameters \( \alpha \) and \( \lambda_1 \) on the velocity distribution in (a) and pressure gradient in (b). It is found that the velocity profile goes down with the increasing effects of both parameters while the pressure gradient rising with the increasing effects of both parameters. The effect of the two parameters \( \Omega \) and \( R_n \) is similar to the effect of the parameters \( \alpha \) and \( \lambda_1 \) on low velocity and pressure rise, this is observed in Fig 6. Fig 7 illustrates the influence of the parameters \( R_e \) and \( F_r \) on the velocity distribution in and pressure gradient. It is found that the velocity profile and pressure gradient rising with the increasing \( F_r \), while the velocity and pressure decreases with increasing \( R_e \). Figs 8 and 9 shows the velocity profile and pressure gradient behavior under the variation of the parameters \( q_1, P_r, S_c \) and \( S_r \). It is mentioned that the velocity and pressure rise with the increasing effects of these parameters, respectively. Finally, Fig 10 shows the behavior of the velocity distribution and pressure gradient under the variation of \( \epsilon \) and \( \emptyset \), one can depict here that the velocity distribution increases with the increasing of \( \emptyset \) and decreases with the increasing of \( \epsilon \). The pressure gradient increases with the increasing \( \epsilon \), while the pressure gradient increases with the increasing \( \emptyset \) when \( 0 < z < 0.5 \), and decreases otherwise.
Fig 2: (a) velocity distribution, (b) pressure gradient distribution; for various values of $D_a$ and $M$ with $\alpha = 0.05$, $D = 0.5$, $\epsilon = 0.25$, $\varnothing = 0.2$, $\lambda_3 = 0.2$, $R_e = 3$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $G_r = 2$, $G_c = 2$, $F_r = 0.8$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 3: (a) velocity distribution, (b) pressure gradient distribution; for various values of $\beta$ and $G_r$ with $\alpha = 0.05$, $D = 0.5$, $\epsilon = 0.25$, $\varnothing = 0.2$, $\lambda_3 = 0.2$, $R_e = 3$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $D_a = 0.8$, $M = 1.1$, $G_c = 2$, $F_r = 0.8$, $\sigma = \pi/4$.

Fig 4: (a) velocity distribution, (b) pressure gradient distribution; for various values of $\sigma$ and $G_c$ with $\alpha = 0.05$, $D = 0.5$, $\epsilon = 0.25$, $\varnothing = 0.2$, $\lambda_3 = 0.2$, $R_e = 3$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $G_r = 2$, $D_a = 0.8$, $M = 1.1$, $F_r = 0.8$, $\beta = \pi/4$. 
Fig 5: (a) velocity distribution, (b) pressure gradient distribution; for various values of $\lambda_1$ and $\alpha$ with $\beta = 0.5$, $\epsilon = 0.25$, $\emptyset = 0.2$, $R_e = 3$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 2$, $F_r = 0.8$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 6: (a) velocity distribution, (b) pressure gradient distribution; for various values of $R_n$ and $\Omega$ with $\alpha = 0.05$, $\epsilon = 0.25$, $\emptyset = 0.2$, $R_e = 3$, $\lambda_1 = 0.2$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 2$, $F_r = 0.8$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 7: (a) velocity distribution, (b) pressure gradient distribution; for various values of $F_r$ and $R_e$ with $\alpha = 0.05$, $\Omega = 0.5$, $\epsilon = 0.25$, $\emptyset = 0.2$, $\lambda_2 = 0.2$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.5$, $S_c = 0.3$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 2$, $\sigma = \pi/4$, $\beta = \pi/4$. 
4.2 Trapping phenomena

The effects of the parameters $\Phi$, $\epsilon$, $\Omega$, $\alpha$, $R_\text{e}$, $F_r$, $\lambda_1$, $R_\text{n}$, $M$, $D_\text{h}$, $G_r$, $G_c$, $S_r$, $S_c$, $q_1$, $\beta$ and $\sigma$ on trapping bolus can be seen through Figs 11–27. Fig 11 shows that the flow wave turns into a trapped bolus and presses from both ends along the channel to turn into an oval and then fade away with the increasing of $\Phi$. Effect of the parameter $\epsilon$ opposes the effect of the parameter $\Phi$ on trapped bolus in Fig 12 where we notice the increase in the size of the trapped bolus and expand to turns into a flow wave along the flow channel.
with the increasing of $\epsilon$. In Fig 13 notice that the trapped bolus generated by the fluid movement in the flow channel decreases with the increase of the parameter $\Omega$ and turns from a homogeneous oval shape into a deformed shape compressed from both ends of the channel wall, then the trapped bolus opens from the top to connect to the wave near the upper wall of the channel and the bolus disappears turning into a wave. Fig 14 shows that the flow wave turns into a trapped bolus and decreases in size until it fades and disappears with an increase $\alpha$. Effect of the parameter $R_e$ similar to the effect of the parameter $\alpha$ on trapped bolus in Fig 15. Fig 16 illustrates the influence of the parameter $F_r$ on the trapped bolus where we observe the creation of trapped bolus and expands to be converted to a flow wave along the flow channel with the increase $F_r$. Effect of the parameter $\lambda_1$ on trapping bolus is similar to the effect of the parameters $R_e$ and $\alpha$ on trapping bolus which can be seen in Fig 17. Effect of the parameter $R_m$ on trapping bolus is similar to the effect of the parameters $\Omega$ on trapping bolus which can be seen in Fig 18. Fig 19 shows the effect of the parameter $M$ on the size of the trapped bolus, where we notice an increase in the size of the trapped bolus and its transformation from an oval shape to a shape close to circular along the flow channel with an increase $M$. Fig 20 illustrates the effect of the parameter $D_a$ on the size of the trapped bolus, where we notice an increase in the size of the trapped bolus and its transformation from a shape close to circular to an oval shape along the flow channel with an increase $D_a$. The effect of the parameters $G_r$ and $G_c$ on the trapped bolus created during fluid flow through the channel is illustrated in Figs 21 and 22, respectively, where we notice their similar effects on the trapped bolus, but the effect of $G_r$ is greater than the effect of $G_c$ on the bolus. The trapped bolus growth and expands which is opening from four sides and turning it into a wave with an increase one of them $G_r$ or $G_c$. The effect of the parameters $S_r$ and $S_c$ on the trapped bolus is illustrated in Figs 23 and 24, respectively, where we notice their effects on the trapped bolus, but the effect of $S_c$ is greater than the effect of $S_r$ on the bolus. The bolus trapped in the middle of the flow channel expands and grows and is pointed from its end near the upper wall, where it merges with the wave near the upper wall of the channel with an increase of one $S_r$ or $S_c$. Fig 25 illustrates the influence of the parameter $q_1$ on the trapped bolus where we observe the creation of trapped bolus and expands in the flow channel with the increase $q_1$. The effect of inclination for the gravitational angle $\beta$ on the trapped bolus is shown in Fig 26 where we notice the bolus shrinkage and its approach to the lower channel wall where it disappears. The effect of deviation angle $\sigma$ for the temperature and concentration on the trapped bolus is shown in Fig 27 were noticed the effects $\sigma$ similar to the influence of the parameter $G_c$ on the bolus.

Fig 11: Streamlines for three various values of $\phi$: (a) $\phi = 0.1$, (b) $\phi = 0.125$ and (c) $\phi = 0.15$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\Omega = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_m = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $D_a = ...
\[ 0.8, \, M = 1.1, \, G_r = 2, \, G_c = 1, \, F_r = 0.2, \, \sigma = \pi/4, \, \beta = \pi/4. \]

**Fig 12:** Streamlines for three various values of \( \epsilon \); (a) \( \epsilon = 0.2 \), (b) \( \epsilon = 0.225 \) and (c) \( \epsilon = 0.25 \), at \( \alpha = 0.05, \, \phi = 0.15, \, \Omega = 0.5, \, R_e = 2, \, \lambda_1 = 0.15, \, R_n = 1, \, P_r = 2, \, q_1 = 0.5, \, S_r = 0.3, \, S_c = 0.1, \, D_n = 0.8, \, M = 1.1, \, G_r = 2, \, G_c = 1, \, F_r = 0.2, \, \sigma = \pi/4, \, \beta = \pi/4. \]

**Fig 13:** Streamlines for three various values of \( \Omega \); (a) \( \Omega = 0.5 \), (b) \( \Omega = 1.5 \) and (c) \( \Omega = 2.5 \), at \( \alpha = 0.05, \, \phi = 0.15, \, R_e = 2, \, \lambda_1 = 0.15, \, R_n = 1, \, P_r = 2, \, q_1 = 0.5, \, S_r = 0.3, \, S_c = 0.1, \, D_n = 0.8, \, M = 1.1, \, G_r = 2, \, G_c = 1, \, F_r = 0.2, \, \sigma = \pi/4, \, \beta = \pi/4. \]

**Fig 14:** Streamlines for three various values of \( \alpha \); (a) \( \alpha = 0.025 \), (b) \( \alpha = 0.005 \) and (c) \( \alpha = 0.1 \), at \( \epsilon = 0.2, \, \phi = 0.15, \, \Omega = 0.5, \, R_e = 2, \, \lambda_1 = 0.15, \, R_n = 1, \, P_r = 2, \, q_1 = 0.5, \, S_r = 0.3, \, S_c = 0.1, \, D_n = 0.8, \, M = 1.1, \, G_r = 2, \, G_c = 1, \, F_r = 0.2, \, \sigma = \pi/4, \, \beta = \pi/4. \]
Fig 15: Streamlines for three various values of $R_c$: (a) $R_c = 1.75$, (b) $R_c = 2$ and (c) $R_c = 2.25$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\varnothing = 0.15$, $\Omega = 0.5$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 16: Streamlines for three various values of $F_r$: (a) $F_r = 0.175$, (b) $F_r = 0.2$ and (c) $F_r = 0.225$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\varnothing = 0.15$, $\Omega = 0.5$, $R_c = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 17: Streamlines for three various values of $\lambda_1$: (a) $\lambda_1 = 0.1$, (b) $\lambda_1 = 0.15$ and (c) $\lambda_1 = 0.2$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\varnothing = 0.15$, $\Omega = 0.5$, $R_c = 2$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$. 
Fig 18: Streamlines for three various values of \( R_n \): (a) \( R_n = 1 \), (b) \( R_n = 2 \) and (c) \( R_n = 3 \), at \( \alpha = 0.05 \), \( \epsilon = 0.2 \), \( \theta = 0.15 \), \( \Omega = 0.5 \), \( R_c = 2 \), \( \lambda = 0.15 \), \( P_r = 2 \), \( q_1 = 0.5 \), \( S_r = 0.3 \), \( S_c = 0.1 \), \( D_a = 0.8 \), \( M = 1.1 \), \( G_r = 2 \), \( G_c = 1 \), \( F_r = 0.2 \), \( \sigma = \pi/4 \), \( \beta = \pi/4 \).

Fig 19: Streamlines for three various values of \( M \): (a) \( M = 0.9 \), (b) \( M = 1 \) and (c) \( M = 1.1 \), at \( \alpha = 0.05 \), \( \epsilon = 0.2 \), \( \theta = 0.15 \), \( \Omega = 0.5 \), \( R_c = 2 \), \( \lambda = 0.15 \), \( R_n = 1 \), \( P_r = 2 \), \( q_1 = 0.5 \), \( S_r = 0.3 \), \( S_c = 0.1 \), \( D_a = 0.8 \), \( G_r = 2 \), \( G_c = 1 \), \( F_r = 0.2 \), \( \sigma = \pi/4 \), \( \beta = \pi/4 \).

Fig 20: Streamlines for three various values of \( D_a \): (a) \( D_a = 0.7 \), (b) \( D_a = 0.8 \) and (c) \( D_a = 0.9 \), at \( \alpha = 0.05 \), \( \epsilon = 0.2 \), \( \theta = 0.15 \), \( \Omega = 0.5 \), \( R_c = 2 \), \( \lambda = 0.15 \), \( R_n = 1 \), \( P_r = 2 \), \( q_1 = 0.5 \), \( S_r = 0.3 \), \( S_c = 0.1 \), \( M = 1.1 \), \( G_r = 2 \), \( G_c = 1 \), \( F_r = 0.2 \), \( \sigma = \pi/4 \), \( \beta = \pi/4 \).
Fig 21: Streamlines for three various values of $G_r$: (a) $G_r = 1$, (b) $G_r = 2$ and (c) $G_r = 3$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\emptyset = 0.15$, $\Omega = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $M = 1.1$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 22: Streamlines for three various values of $G_c$: (a) $G_c = 1$, (b) $G_c = 2$ and (c) $G_c = 3$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\emptyset = 0.15$, $\Omega = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $M = 1.1$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 23: Streamlines for three various values of $S_r$: (a) $S_r = 0.1$, (b) $S_r = 0.3$ and (c) $S_r = 0.5$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\emptyset = 0.15$, $\Omega = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$. 
Fig 24: Streamlines for three various values of $S_c$: (a) $S_c = 0.1$, (b) $S_c = 0.2$ and (c) $S_c = 0.3$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\phi = 0.15$, $D = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 25: Streamlines for three various values of $q_1$: (a) $q_1 = 0.3$, (b) $q_1 = 0.5$ and (c) $q_1 = 0.7$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\phi = 0.15$, $D = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $S_r = 0.3$, $S_c = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$, $\beta = \pi/4$.

Fig 26: Streamlines for three various values of $\beta$: (a) $\beta = \pi/6$, (b) $\beta = \pi/4$ and (c) $\beta = \pi/2$, at $\alpha = 0.05$, $\epsilon = 0.2$, $\phi = 0.15$, $D = 0.5$, $R_e = 2$, $\lambda_1 = 0.15$, $R_n = 1$, $P_r = 2$, $q_1 = 0.5$, $S_r = 0.3$, $S_c = 0.1$, $D_a = 0.8$, $M = 1.1$, $G_r = 2$, $G_c = 1$, $F_r = 0.2$, $\sigma = \pi/4$. 

5. Conclusions

We have study the influence of MHD peristaltic flow of Jeffrey fluid with varying temperature and concentration through a porous medium an inclined channel. Summary from the discussions above the following results:

1- Increase of the parameters $M$, $F_r$, $Q_1$, $P_r$, $S_c$, $S_r$ and $\Phi$ leads to velocity fluid rise.

2- Increase of the parameters $D_a$, $G_r$, $\beta$, $G_c$, $\sigma$, $\lambda_4$, $\Omega$, $R_n$, $R_e$ and $\epsilon$ leads to velocity fluid decreased.

3- Increase of the parameters $M$, $\alpha$, $\lambda_4$, $F_r$, $Q_1$, $P_r$, $S_c$, $S_r$ and $\epsilon$ leads to pressure gradient rise.

4- Increase of the parameters $D_a$, $G_r$, $\beta$, $G_c$, $\sigma$, $\lambda_4$, $\Omega$, $R_n$, $R_e$ and $\epsilon$ leads to pressure gradient decreased.

5- Increase of the parameters $M$, $D_a$ and $Q_1$ leads to trapped bolus growth and expands.

6- Increase of the parameters $\epsilon$, $F_r$, $G_r$, $G_c$, $S_r$ and $S_c$ leads to trapped bolus expand to turns into a flow wave.

7- Increase of the parameters $\Omega$, $\lambda_4$, $R_n$ and $\sigma$ leads to trapped bolus shrinkage.

8- Increase of the parameters $\Phi$, $\alpha$, $R_e$ and $\beta$ leads the flow wave turns into a trapped bolus to turn into an oval and then fade away.

References

1. M.G. Reddy, Heat and mass transfer on magnetohydrodynamic peristaltic flow in a porous medium with partial slip, Alexandria Eng. J. (2016). http://dx.doi.org/10.1016/j.aej.2016.04.009.

2. T. Hayat, Sabia Asghar, Anum Tanveer, Ahmed Alsaeedi; Chemical reaction in peristaltic motion of MHD couple stress fluid in channel with Soret and Dufour effects, Results in Physics 10 (2018) 69–80. https://doi.org/10.1016/j.rinp.2018.04.040.

3. M.A. Imran, Fizza Miraj, I. Khan, I. Tili; MHD fractional Jeffrey’s fluid flow in the presence of thermo diffusion, thermal radiation effects with first order chemical reaction and uniform heat flux, Results in Physics 10 (2018) 10–17. https://doi.org/10.1016/j.rinp.2018.04.008.

4. Ahmed A. H. Al-Aridhee, Dheia G. S. Al-Khafaji. Influence of MHD Peristaltic Transport for Jeffrey Fluid with Varying Temperature and Concentration through Porous Medium, IOP Conf. Series: Journal of Physics: Conf. Series 1294 (2019)
5. Riaz, A. Zeeshan, S. Ahmad, A. Razaq, M. Zubair; Effects of External Magnetic Field on non-Newtonian Two Phase Fluid in an Annulus with Peristaltic Pumping, Journal of Magnetics 24(1), 1-8 (2019). https://doi.org/10.4283/JMAG.2019.24.1.XXX.

6. Dheia G. S. Al-Khafajy, Abdulhadi AM. Effects of wall properties and heat transfer on the peristaltic transport of a Jeffrey fluid through porous medium channel. Math. Theory and Modeling, 2014;4(9):86–99.

7. Ellahi R., Hussain F., Ishtiaq F., Hussain F. Peristaltic transport of Jeffrey fluid in a rectangular duct through a porous medium under the effect of partial slip: An application to upgrade industrial sieves/filters. Pramana - J Phys 93, 34 (2019). https://doi.org/10.1007/s12043-019-1781-8.

8. N. Alvi, T. Latif, Q. Hussain, S. Asghar. Peristalsis of nonconstant viscosity Jeffrey fluid with nanoparticles, Results in Physics 6 (2016) 1109–1125. http://dx.doi.org/10.1016/j.rinp.2016.11.045.

9. M.A. Imran, Fizza Miraj, I. Khan, I. Tlili; MHD fractional Jeffrey’s fluid flow in the presence of thermo diffusion, thermal radiation effects with first order chemical reaction and uniform heat flux, Results in Physics 10 (2018) 10–17. https://doi.org/10.1016/j.rinp.2018.04.008.

10. M. Ahmad, M. A. Imran, Maryam Aleem, I. Khan; A comparative study and analysis of natural convection flow of MHD non-Newtonian fluid in the presence of heat source and first-order chemical reaction, Journal of Thermal Analysis and Calorimetry, Springer 2019. https://doi.org/10.1007/s10973-019-08065-3.

11. T. Hayat, Sadia Ayub, Ahmed Alsaedi, Bashir Ahmad, Numerical simulation of buoyancy peristaltic flow of Johnson-Segalman nanofluid in an inclined channel, Results in Physics 9 (2018) 906–915. https://doi.org/10.1016/j.rinp.2018.03.037.

12. Mohammed R. Salman and Ahmed M. Abdulhadi, 2018, Influence of heat and mass transfer on inclined (MHD) peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined asymmetric channel, IOP Conf. Series, J. Phys. 1032 012043.

13. Dheia G. Salih Al-Khafajy; Radiation and Mass Transfer Effects on MHD Oscillatory Flow for Carreau Fluid through an Inclined Porous Channel, Iraqi Journal of Science, 61(6), 1426-1432., 2020. https://doi.org/10.24996/ijs.2020.61.6.21.
