Exploiting modal interaction during run-up of a magnetically supported Jeffcott rotor

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Abstract. Introducing a parametric anti-resonance in a vibrating system couples two of the many vibration modes and enables an energy exchange between those two. This feature is employed during the run-up of a \textsc{Jeffcott} rotor supported by two active magnetic bearings. The vibration performance at bearing stiffness modulation is compared to the well-known performance at nominal bearing characteristics for a simple run-up at a constant acceleration. It is shown that by introducing a specific periodic change of the bearing stiffness coefficients, a mode coupling between two selected modes is activated. This coupling impacts the maximum amplitude developed during passage through resonance. At each critical speed transient vibrations of the corresponding mode are excited. Due to the modal coupling, if one mode is excited at a critical speed then energy is exchanged with the coupled mode. On one hand, the maximum amplitude at the first critical speed is decreased by modulation since some vibration energy is transferred to the highly damped second mode where it is partly dissipated. On the other hand, the maximum amplitude at the second critical speed is increased by modulation since some vibration energy is transferred to the lightly damped first mode. The concept is outlined in detail based on numerical studies.

1. Introduction

The beneficial effect of a parametric anti-resonance on self-excited vibration was discovered by Tondl in his pioneering work \cite{1}. This concept was then transferred to general dynamic systems \cite{2}, was interpreted physically as an energy transfer between the vibration modes of the original system and was validated experimentally for simple systems including a flexible rotor \cite{3}. A recent summary on this topic can be found in \cite{4}. A parametric anti-resonance is a specific parametric combination resonance which does not lead to a parametric instability but enables an increased dissipation of vibration energy.

Active magnetic bearings as discussed in this paper offer the possibility to apply a desired time-periodic variation of the bearing characteristics with an accuracy that enables a parametric anti-resonance phenomenon. In rotating machines fluid film bearings are commonly used which inspired investigations on applying a variation of the fluid film bearing characteristics via a bearing shell of variable geometry \cite{5, 6, 7}. Journal bearings of variable geometry aim to control the fluid film characteristic passively or actively. The recent works on the passive adjustment of the bearing geometry and thus of fluid film characteristics confirmed a displacement reduction during passage through resonance. The promising concept of a variable geometry bearing and its physical realisation via a moving bearing shell is followed in very recent studies \cite{8, 9} and...
combined with the concept of parametric anti-resonance. Adjusting bearing properties in a time-periodic manner has several benefits, like increased stability limits that allow a larger operation range and decreased vibration amplitudes when passing through critical speeds.

In the present investigation, a flexible rotor supported by bearings is investigated whose dynamic properties are controlled semi-actively. Such a concept was proposed for active magnetic bearings in [10, 11] and validated theoretically and experimentally. It allowed the steady-state operation of active bearings beyond the stability limit of the implemented PID controller. The rotor configuration in [11, 12] is summarised here. The basic effect that a parametric excitation reduces the response amplitude during run-up was already proven in [12]. The present work analyses the dependency of the run-up characteristic at parametric anti-resonance for different run-up accelerations.

2. System modelling

Preliminary theoretical investigations [13, 14] showed that introducing a periodic change in the bearing stiffness is capable of increasing the rotor speed limit of a simple Jeffcott rotor under the influence of a destabilising self-excitation. This approach is followed to enhance the effective damping of an already stable flexible rotor shaft under the action of unbalance and electromagnetic forces [10]. The experimental realisation is shown in Fig. 1: A slender, flexible rotor shaft supported by two active magnetic bearings (AMBs). The shaft is torsionally rigid and isotropic. A rigid disc (D) is attached to the shaft center and two bearing studs (AMB1, AMB2). The discs are unbalanced. The total length of the shaft is 680 mm. The main system parameters are listed in Table 1.

![Figure 1. Jeffcott rotor supported by two magnetic bearings [11]: (left) experimental test rig, (right) corresponding dynamic model (taken from [11]).](image)

| Table 1. Properties of the Jeffcott rotor. |
|------------------------------------------|
| bending stiffness of rotor shaft          | 41.4 Nm  |
| total rotor length and diameter          | 680 mm, 8 mm  |
| mass and axial moment of inertia of disc| 1.20 kg, 1.40 · 10^-3 kg m^2  |
| mass of studs in AMB1 and AMB2           | 0.88 kg  |
| radial bearing clearance                 | 0.8 mm  |
| specific load capacity                   | 13 N/cm^2 |

The electromagnetic forces generated in the AMBs depend on the rotor deflection and the magnetic field. The magnetic field can be changed in a wide range by the current (bias and control current) provided to the electromagnets. The actual position of the rotor shaft is measured by inductive sensors. These signals are processed by the real-time controller hardware dSPACE which implements decentralised PID controllers to regulate the currents provided by
power amplifiers to each of the electromagnets and to levitate the rotor, see Fig. 2 for more details.

The electromagnetic force generated by the AMB depends on its geometry parameters (cross-section of the pole shoes, size of the air gap) and its electromagnetic properties (number of turns, permeability) and is a strongly nonlinear function of these parameters. In practice, however, the resulting force can be linearised close to a certain operation point. With the rotor displacement from the centre position \( r \) and the initial gap width \( s_0 \), the linearised force-current-displacement relationship of the magnetic bearing becomes

\[
F_{m}^{\text{lin}} = 4 k_m \left( \frac{i_0}{s_0^2} r - \frac{i_0^2}{s_0^3} r^2 \right) = k_i i_c - k_s r.
\]  

Herein, \( k_i \) is the current-force constant and \( -k_s \) the negative bearing stiffness,

\[
k_i = 4 k_m \frac{i_0}{s_0^2} \quad \text{and} \quad k_s = 4 k_m \frac{i_0^2}{s_0^3},
\]

see [15] for more details. The magnetic bearing constant \( k_m \) depends on geometry parameters (cross-section of the pole shoes, size of the air gap) and electromagnetic properties (number of turns, permeability). The nonlinearities of the magnetic force are generally reduced by adding a high bias current \( i_0 \) to the control current \( i_c \). It is common to set the bias current to half of the maximally available current (saturation or limitations of the power amplifier) in order to exploit the full operation range.

The most widely used control concept for an AMB is a PID controller. The proportional \((k_P)\) and the derivative actions \((k_D)\) constitute the stiffness and damping characteristics of the bearing while the integral action \((k_I)\) assures that the radial rotor deflection \( r \) keeps track with a predefined setpoint,

\[
i_c = k_P r + k_D \dot{r} + k_I \int r \, dt.
\]  

Inserting into eq. (1) leads to

\[
F_{m}^{\text{lin}} = c_m r + d_m \dot{r} + k_i k_I \int r \, dt, \quad c_m = k_i k_P - k_s, \quad d_m = k_i k_D,
\]

with the active stiffness and active damping coefficients \( c_m \) and \( d_m \). Adjusting the control parameters \( k_P \) and \( k_D \) determines the dynamic properties of the AMB. Both AMBs are isotropic.

The parametric anti-resonance is implemented in parallel to this PID control by a periodic open-loop control of the proportional action \( k_P(t) \),

\[
k_{P}^{\text{pe}}(t) = k_P (1 + \varepsilon \sin \nu t) \quad \rightarrow \quad k_m^{\text{pe}}(t) = k_i k_{P}^{\text{pe}}(t) - k_s.
\]
This realises a periodic change in the active bearing stiffness which is implemented in both AMBs in Fig. 1 at a certain variation amplitude \( \varepsilon \) and excitation frequency \( \nu \).

2.1. Equations of motion of a continuous shaft with discrete discs

The flexible, continuous shaft is discretised using finite beam elements. For a finite beam element of length \( l_i \), bending stiffness \( EI \), cross-section \( A \) and density \( \rho \), the inertia and stiffness matrix become [16]

\[
M_{bi}^{z/y} = \frac{\rho A l_i}{420} \begin{bmatrix}
156 & \mp 22l_i & 54 & \pm 13l_i \\
\mp 22l_i & 4l_i^2 & \mp 13l_i & -3l_i^2 \\
54 & \mp 13l_i & 156 & \pm 22l_i \\
\pm 13l_i & -3l_i^2 & \pm 22l_i & 4l_i^2
\end{bmatrix},
\]

(6)

and

\[
C_{bi}^{z/y} = \frac{2EI}{l_i^3} \begin{bmatrix}
6 & \mp 3l_i & -6 & \mp 3l_i \\
\mp 3l_i & 2l_i^2 & \pm 3l_i & l_i^2 \\
-6 & \pm 3l_i & 6 & \pm 3l_i \\
\pm 3l_i & l_i^2 & \pm 3l_i & 2l_i^2
\end{bmatrix},
\]

(7)

with respect to the element coordinate vector in \( z \)-direction and \( y \)-direction, respectively,

\[
q_i^z = \begin{bmatrix} z_i & \varphi_{yi} & z_{i+1} & \varphi_{yi+1} \end{bmatrix}^T, \quad q_i^y = \begin{bmatrix} y_i & \varphi_{zi} & y_{i+1} & \varphi_{zi+1} \end{bmatrix}^T.
\]

(8)

Rigid discs of mass \( m_r \) and moment of inertia \( \Theta_r \) are attached at discrete positions along the shaft. Their symmetry axis is aligned with the central rotary axis. For rigid discs that are attached to the end points of a finite beam element, the corresponding mass and stiffness matrices have diagonal form and read

\[
M_r = \begin{bmatrix} m_{ri} & \Theta_{ri} \\
\Theta_{ri} & m_{ri+1}
\end{bmatrix},
\]

(9)

The element matrices in eqs. (6), (7) and (9) are assembled to the global system matrices \( M_b^{z/y}, C_b^{z/y} \) and \( M_r \) with respect to the global coordinate vectors

\[
q^z = \begin{bmatrix} z_1 & \varphi_{y1} & \cdots & z_n & \varphi_{yn} \end{bmatrix}^T, \quad q^y = \begin{bmatrix} y_1 & \varphi_{z1} & \cdots & y_n & \varphi_{zn} \end{bmatrix}^T.
\]

(10)

Adjusting the control parameters \( k_P \) and \( k_D \) determines the dynamic properties of an AMB. With the mechanical properties in eq. (4), the stiffness and damping matrices with respect to the global coordinate vector have diagonal form with entries at the location of the AMBs,

\[
C_m = \begin{bmatrix} 0 & c_{m1} & 0 & \cdots & 0 \\
0 & 0 & c_{m2} & \cdots & 0 \\
\end{bmatrix}, \quad D_m = \begin{bmatrix} 0 & d_{m1} & 0 & \cdots & 0 \\
0 & 0 & \cdots & d_{m2} & 0 \\
\end{bmatrix}
\]

(11)

The rotor system is excited by unbalance forces originating from eccentricities \( \varepsilon_i \) of the five rigid discs of mass \( m_i \) (including the bearing studs). The unbalance force vectors can be written as

\[
f_z = - (\cos \varphi(t))^{\nu} \begin{bmatrix} m_1 \varepsilon_1 & 0 & m_2 \varepsilon_2 & 0 & \cdots & m_5 \varepsilon_5 & 0 \end{bmatrix}^T,
\]

\[
f_y = - (\sin \varphi(t))^{\nu} \begin{bmatrix} m_1 \varepsilon_1 & 0 & m_2 \varepsilon_2 & 0 & \cdots & m_5 \varepsilon_5 & 0 \end{bmatrix}^T,
\]

(12)
where $\varphi(t)$ is the rotary angle of the shaft counted positively in negative $x$-axis according to the definitions in [17].

The element matrices from eqs. (6), (7) and (9) are assembled to global system matrices with respect to the global coordinate vectors in eq. (10). Together with the global system matrices in eq. (11) describing the electromagnetic actions and the unbalance forces in eq. (12), the equations of motion of the rotor system at constant speed $\Omega$ with respect to the global coordinate vector $q = [q_x^r, q_y^r]^T$ become

$$M \ddot{q} + D \dot{q} + C q = f$$

with the global force vector $f = [f_x^r, f_y^r]$ and the assembled coefficient matrices

$$M = \begin{bmatrix} M^b_x + M_r & 0 \\ 0 & M^b_y + M_r \end{bmatrix}, 
D = \begin{bmatrix} D_m & -\Omega G_r \\ \Omega G_r & D_m \end{bmatrix}, 
C = \begin{bmatrix} C^b_x + C_m & 0 \\ 0 & C^b_y + C_m \end{bmatrix}.$$  \hspace{1cm} (14)

The only source of damping is the control strategy in the AMBs. The lateral vibrations in $y$- and $z$-directions are coupled by gyroscopic effects of the rigid discs, however, the influence of gyroscopic effects is negligible for the considered speed range. For safety reasons, retainer bearings acting at discrete positions along the rotary shaft are applied but these are not in the focus of the present study. The operational deflection of the present rotor is assumed to be sufficiently small such that rotor-stator contacts are excluded and the linearisation of the electromagnetic force in eq. (1) remains valid.

A time-periodic stiffness variation in the rotor system is realised in the AMBs by introducing a time-dependent proportional action $k_P(t)$ in the PID controllers [10]. This control parameter is changed periodically for both AMBs simultaneously according to eq. (5) resulting in the global time-periodic stiffness matrix

$$C^{pe}(t) = C + \varepsilon C_l \sin \nu t,$$ \hspace{1cm} (15)

herein $C$ and $C_l$ are constant coefficient matrices.

### 3. Parametric anti-resonance during run-up

The run-up characteristics of the Jeffcott rotor supported by two active magnetic bearings is discussed in detail in the following. The first three natural frequencies obtained from an eigenvalue analysis of the undamped system in eq. (13) at rest are listed in Table 2 together with the corresponding parametric anti-resonance and resonance frequencies. Evaluating the analytical predictions in [14] reveals that for the present system, parametric anti-resonance can be only achieved at the difference type $\nu \approx |\omega_1 - \omega_2|/n$. Numerical calculations performed in [4, 11, 12] revealed that a strong parametric anti-resonance exists at $\nu \approx 170$ rad/s which corresponds to the parametric anti-resonance frequency $|\omega_1 - \omega_3|$.

| natural frequencies          | main parametric resonance frequencies | main parametric anti-resonance frequencies |
|------------------------------|---------------------------------------|--------------------------------------------|
| $\omega_1 = 164$ rad/s      | $2\omega_1 = 328$ rad/s               | $|\omega_1 - \omega_2| = 108$ rad/s        |
| $\omega_2 = 272$ rad/s      | $2\omega_2 = 544$ rad/s               | $|\omega_1 - \omega_3| = 174$ rad/s        |
| $\omega_3 = 339$ rad/s      | $\omega_1 + \omega_2 = 436$ rad/s    | $|\omega_2 - \omega_3| = 67$ rad/s         |

**Table 2.** First natural and parametric frequencies of the Jeffcott rotor at rest, $\Omega = 0$. 

The discretised continuous shaft is excited by unbalance forces originating from the rigid disc. The equations of motion in eq. (13) are solved by direct numerical integration including the PID controllers in the AMBs with a constant or with periodic proportional control action according to eq. (5). The resulting response deflections are evaluated as radial deflections \( r = \sqrt{y^2 + z^2} \) of the disc D and the bearings.

The passage through resonance at constant accelerations \( \alpha \) are shown in Figs. 3 and 4 for a light and intermediate bearing damping. At constant, nominal AMB characteristics (\( \varepsilon = 0 \) in eq. (5)), the maximum deflections are reached during passage through resonance. This maximum is a function of the run-up acceleration \( \alpha \) and the modal damping of the corresponding mode. The maximum displacement can be reduced in two independent ways (see [17]):

- A larger derivative action \( k_D \) (higher modal damping). This is confirmed by comparing the peak displacements of the black time histories in the left and the right plot in Fig. 3 or Fig. 4.
- A higher run-up acceleration \( \alpha \) (faster passage through resonance). This is confirmed by comparing the black time histories in Fig. 3 with the corresponding time histories in Fig. 4.

Note that the radial deflections are described in a coordinate system that is fixed to the disc. Consequently, the frequency components observed are modulated by the rotor speed \( \Omega \).

![Figure 3](image1.png)

**Figure 3.** Passage through resonance at constant acceleration \( \alpha = 5 \text{rad/s}^2 \): (top) speed characteristic, (bottom) time histories of radial deflections at nominal bearing characteristics (black lines) and for induced parametric anti-resonance at 170 rad/s and \( \varepsilon = 10\% \) (green: disc, blue/red: bearing studs) for small and medium bearing damping.

![Figure 4](image2.png)

**Figure 4.** Passage through resonance speeds at constant acceleration \( \alpha = 40 \text{rad/s}^2 \): (top) speed characteristic, (bottom) time histories of radial deflections at nominal bearing characteristics (black lines) and for induced parametric anti-resonance at 170 rad/s and \( \varepsilon = 10\% \) (green: disc, blue/red: bearing studs) for small and medium bearing damping.
At constant run-up acceleration $\alpha$, the first critical speed is passed showing a major peak displacement and the second critical speed a small peak displacement. Activating the optimum parametric anti-resonance $\omega_3 - \omega_1$ with a strength of $\varepsilon = 10\%$, modal coupling is introduced which activates an energy transfer between the corresponding modes 1 and 3. This coupling leads to an equal distribution of the maximum amplitudes at both critical speeds during run-up. At each critical speed transient vibrations of the corresponding mode are excited. Due to modal coupling, the vibration energy of the initially excited mode is transferred to the coupled mode and back to the initial mode. Part of the vibration energy is dissipated during each modal energy transfer. This mechanism ensures that the maximum amplitude at the first critical speed is decreased due the highly damped higher mode. However, the maximum amplitude at the second critical speed is increased due to the lightly damped first mode. Note that the decrease of maximum amplitude at the first critical speed is not only achieved at the disc position but also at the stud positions. The vibration energy is truly dissipated and not simply shifted from one rotor location to the other.

![Graph](image)

**Figure 5.** Dependency of the maximum displacements of $r_D$ on the run-up acceleration and the damping at nominal AMB characteristics (without PE) and periodic proportional action (with PE): peak displacement at first critical speed (left) and second critical speed (right).

The dependency of the maximum displacements $r_D$ on the run-up acceleration and the damping is summarised in Fig. 5. The maximum displacement at nominal bearing characteristic ($\varepsilon = 0$) shows a large deflection at first critical speed and a small one at the second critical speed. For the AMBs with periodic proportional action ($\varepsilon \neq 0$), the maximum displacements at the critical speeds are brought to a comparable level which result from the induced modal interaction between the corresponding mode shapes. It has to be highlighted that the parametric anti-resonance is most effective at low system damping. There is only little benefit to introduce a periodic proportional action $k_P$ for intermediate damping values $k_D$ above 1.5. At low damping, introducing a parametric anti-resonance with an excitation amplitude of 10% is more efficient than increasing the run-up acceleration 2.5 to 80 rad/s$^2$. This example highlights the benefit of a parametric anti-resonance in rotordynamic systems with low damping.

4. Conclusions
During run-up of a rotor, transient vibrations are introduced when passing through critical speeds which excite the corresponding vibration mode. A mode interaction is artificially achieved by employing a parametric anti-resonance via the bearing controller. Due to this mode coupling, if one mode is excited at a critical speed then energy is transferred to the other mode, too. On one hand, the maximum amplitude at the first critical speed is decreased by modulation since
some vibration energy is transferred to the highly damped second mode where it is partly dissipated. On the other hand, the maximum amplitude at the second critical speed is increased by modulation since some vibration energy is transferred to the lightly damped first mode. It has to be highlighted that the decrease of maximum amplitude is not only achieved at the disc position but also at the journal positions.

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