GLUEBALLS AND HYBRID MESONS

Chris Michael

Theoretical Physics, Dept. of Mathematical Sciences, University of Liverpool,
Liverpool L69 3BX, UK

Abstract

The status of non-perturbative QCD calculations for mesons with gluonic excitation is presented. Lattice results for the glueball spectrum are reviewed. For hybrid mesons, the heavy quark results are summarised and new results are presented for light quarks. Preliminary results for the spectrum of light-quark hybrid mesons indicate substantial mixing with quark model states for non-exotic $J^{PC}$. For the exotic $J^{PC}$ hybrid mesons, the $J^{PC} = 1^{-+}, 0^{+-}$ and $2^{+-}$ states are explored.

\footnote{presented at the Rencontre de Physique ‘Results and Perspectives in Particle Physics’, La Thuile, March 4 1996}
1 Introduction

Since the advent of QCD as a theory of hadronic interactions, there have been experimental searches for unambiguous evidence of gluonic excitations in mesons. These searches need to be guided by theoretical input. The theoretical exploration involves non-perturbative methods and lattice QCD has become the most reliable tool. Here we review the status of glueball mass determinations from the lattice. The main aspect of topicality comes from a widely publicised claim [1] that the lattice work uniquely targets a particular experimental candidate. We discuss this claim and put it in context.

Another area which is promising for a study of gluonic excitations is that of hybrid mesons. These have a gluonic field in a non-trivial representation so that it is truly excited. We review lattice results for this spectrum for the case of heavy quarks. New results for light quarks are presented. These preliminary results give strong evidence for the splitting among the many possible hybrid meson states. The states with exotic quantum numbers (ie not allowed by the naive quark model: $J^{PC} = 1^{-+}, 0^{+-}$ and $2^{+-}$) are studied and their spectrum is estimated.

2 Glueball Masses

The difficulty in isolating glueball candidates experimentally comes from the indirect methods that have to be used to deduce if a given resonance is composed primarily of gluons or of quarks. Lattice QCD allows the quark masses to be varied at will. In the simplest case, the quenched approximation, the dynamical quark mass is taken as large so that no quark loops are present in the vacuum. In this approximation, glueballs are stable and do not mix with quark-antiquark mesons. This approximation is very easy to implement in lattice studies: the full gluonic action is used but no quark terms are included. This corresponds to a full non-perturbative treatment of the gluonic degrees of freedom in the vacuum. A systematic lattice study of the neglected quark loop effects can be made in principle - though no comprehensive treatment has yet been made.

The glueball mass can be measured on a lattice through evaluating the correlation $C(t)$ of two closed colour loops (called Wilson loops) at separation $t$ lattice spacings. This correlation has contributions from all glueballs of the given symmetry, with the ground state contribution dominating at large $t$. In practice, sophisticated methods are used to choose loops such that the correlation $C(t)$ is dominated by the ground state glueball. By using several different loops, a variational method can be used to achieve
this effectively. Even so, it is worth keeping in mind that upper limits on the ground state mass are obtained in principle.

The method also needs to be tuned to take account of the many glueballs: with different $J^{PC}$ and different momenta. On the lattice the Lorentz symmetry is reduced to that of a hypercube. Non-zero momentum states can be created (momentum is discrete in units of $2\pi/L$ where $L$ is the lattice spatial size). The usual relationship between energy and momentum is found for sufficiently small lattice spacing. Here we shall concentrate on the simplest case of zero momentum (obtained by summing the correlations over the whole spatial volume).

For a state at rest, the rotational symmetry becomes a cubic symmetry. The lattice states will transform under irreducible representations of this cubic symmetry group (called $O_h$). These irreducible representations can be linked to the representations of the full rotation group SU(2). Thus, for example, the five spin components of a $J^{PC} = 2^{++}$ state should be appear as the two-dimensional $E^{++}$ and the three-dimensional $T^{++}_2$ representations on the lattice, with degenerate masses. This degeneracy requirement then provides a test for the restoration of rotational invariance - which is expected to occur at sufficiently small lattice spacing.

The results of lattice measurements [2, 3, 4, 5] of the $0^{++}$ and $2^{++}$ states are shown in fig 1. Since the lattice observables, such as the glueball mass $\hat{m}$, are not in physical units, it is necessary to form dimensionless ratios of lattice observables to compare with experiment. Fig 1 shows the dimensionless combination of the lattice glueball mass $\hat{m}_0$ with a lattice quantity $\hat{r}_0$, which is a well measured quantity (given by $\hat{r}^2d\hat{V}/d\hat{r} = 1.65$ at $\hat{r} = \hat{r}_0$ where $\hat{V}(\hat{r})$ is the lattice interquark potential at separation $\hat{r}$) on the lattice that can be used to calibrate the lattice spacing and so explore the continuum limit. The quantity plotted, $\hat{m}_0\hat{r}_0$, is expected to be equal to the product of continuum quantities $m_0r_0$ up to corrections of order $a^2$. This behaviour near the continuum limit is indeed found as shown by the linear dependence of fig 1. The extrapolation to the continuum limit ($a \to 0$) can now be made with confidence. Note that older lattice data were only available at larger values of $a^2$ which explains why a smaller $0^{++}$ glueball mass was favoured at that time.

The lattice results in fig 1 from the UKQCD and GF11 groups have signals which are of comparable statistical significance and which are consistent with each other. However, their published values [4, 5] of the $0^{++}$ glueball mass are different (1550 versus 1740 MeV). The GF11 group chose to extrapolate $\hat{m}_0/\Lambda a$ to the continuum limit. This ratio has the disadvantage that there can be corrections both of order $a$ and of order $(\ln a)^n$ while they assume in their extrapolation that only order $a^2$ effects are significant. They determine
Figure 1: The value of mass of the $J^{PC} = 0^{++}$ and $2^{++}$ glueball states from refs[2, 3, 4, 5] in units of $r_0$. The restoration of rotational invariance is shown by the degeneracy of the $T_2$ and $E$ representations that make up the $2^{++}$ state: shown by octagons and diamonds respectively. The straight lines show fits describing the approach to the continuum limit as $a \to 0$. 
Λa from their own results for \( \hat{m}_\rho \) which yields a 0\(^{++} \) mass of 1740 ± 71 MeV leading them to claim \([5, 1]\) that the \( f_J(1710) \) meson is a preferred glueball candidate. Their error estimate on the glueball mass does not take into account fully the systematic errors in the extraction of the continuum limit or those due to quenching.

Using instead the best determined \textit{continuum} quantity from the lattice results, we need to determine a physical value for \( r_0 \). From the interquark potential as determined in spectroscopy, the value of \( r_0 \) in physical units is about 0.5 fm and we will adopt a scale equivalent to \( r_0^{-1} = 0.372 \) GeV. This information yields lattice predictions for the glueball masses based on all lattice data of around 1.6 GeV and 2.2 GeV for the 0\(^{++} \) and 2\(^{++} \) glueballs respectively. Setting the scale in a quenched lattice calculation is inherently imprecise because ratios of lattice observables are found to disagree with experimental values (unquenched) by different amounts for different ratios. Thus no common scale determination is possible for the quenched lattice. It is prudent to assign a systematic error of at least 10\% to the scale. Since this dominates the statistical error, the conservative conclusion is a 0\(^{++} \) glueball mass of 1600 ± 160 MeV. This is an energy range consistent with promising experimental 0\(^{++} \) glueball candidates such as \( f_0(1500) \) - for a review see ref\([7]\). A candidate for a 2\(^{++} \) glueball at 2230 MeV has also been reported recently \([7]\).

The predictions for the other \( J^{PC} \) states are that they lie higher in mass and the present state of knowledge is summarised in fig 2. Note that the lattice gives a clear indication that no light pseudoscalar glueball should exist. Remember that the lattice results are strictly upper limits. For the \( J^{PC} \) values not shown, these upper limits are too weak to be of use.

Since quark - antiquark mesons can only have certain \( J^{PC} \) values, it is of special interest to look for glueballs with \( J^{PC} \) values not allowed for such mesons: 0\(^{--} \), 0\(^{+-} \), 1\(^{-+} \), 2\(^{--} \), etc. Such spin-exotic states, often called “oddballs”, would not mix directly with quark - antiquark mesons. This would make them a very clear experimental signal of the underlying glue dynamics. Various glueball models (bag models, flux tube models, QCD sum-rule inspired models,...) gave different predictions for the presence of such oddballs (eg. 1\(^{-+} \)) at relatively low masses. The lattice mass spectra clarify these uncertainties but, unfortunately for experimentalists, do not indicate any low-lying oddball candidates. The lightest candidate is from the \( T_2^{++} \) spin combination. Such a state could correspond to an 2\(^{++} \) oddball. Another interpretation is also possible, however, namely that a non-exotic 3\(^{+-} \) state is responsible (this choice of interpretation can be resolved in principle by finding the degenerate 5 or 7 states of a \( J = 2 \) or 3 meson). The overall conclusion at present is that there is no evidence for any oddballs of mass less than 3 GeV.
Figure 2: The mass of the glueball states with quantum numbers $J^{PC}$ from ref[4]. The scale is set by $\sqrt{\sigma} \approx 0.44$ GeV which yields the right hand scale in GeV. The solid points represent mass determinations whereas the open points are upper limits.
Glueballs are defined in the quenched approximation and, hence, they do not decay into mesons since that would require quark-antiquark creation. It is, nevertheless, still possible to estimate the strength of the matrix element between a glueball and a pair of mesons within the quenched approximation. For the glueball to be a relatively narrow state, this matrix element must be small. A very preliminary attempt [3, 4] has been made to estimate the size of the coupling of the 0^{++} glueball to two pseudoscalar mesons. A relatively small value is found. Furthermore, they see indications for a dependence on the pseudoscalar mass of the reduced decay matrix element. These conclusions imply that the quenched glueball mass determination was of relevance to the experimental situation since the mixing with other mesons would be small. Further work needs to be done to investigate this in more detail, in particular to study the mixing between the glueball and 0^{++} mesons since this mixing may be an important factor in the decay process.

In principle, it is possible to study on a lattice the glueball spectrum in full QCD vacua with sea quarks of mass $m_D$. For large $m_D$, the result is just the quenched result described above. For $m_D$ equal to the experimental light quark masses, the results should just reproduce the experimental meson spectrum - with the resultant uncertainty between glueball interpretations and other interpretations. The lattice enables these uncertainties to be resolved in principle: one obtains the spectrum for a range of values of $m_D$ between these limiting cases, so mapping glueball states at large $m_D$ to the experimental spectrum at light $m_D$.

3 Hybrid Mesons

In order to set the scene for the study of mesons with gluonic excitations, it is worthwhile to summarise briefly the simple constituent quark model. This model of massive quark and antiquark bound by a potential is only justified theoretically for $b\bar{b}$ and to a lesser extent $c\bar{c}$, but it is still useful guide for light quark states. The mesonic states that can be made from $q\bar{q}$ with spatial wavefunction with orbital angular momentum $L$ and total spin $S^{PC} = 0^{-+}$ or $1^{--}$ have $J^{PC}$ values of

$$L^{PC} = 0^{++}, 1^{--}, 2^{++} \quad J^{PC} = 0^{--}, 1^{++}, 2^{++}, 3^{--}. $$

Since the gluon can introduce no flavour quantum numbers, the $J^{PC}$ assignments will be of importance. Of special interest in the following will be the absence of certain $J^{PC}$ values in the above list. These states are known as spin-exotic and include $0^{--}, 0^{+-}, 1^{++}, 2^{+-}$. 
Figure 3: The lattice static quark potential for the ground state and first excited state from ref [9] with the scale given by a lattice spacing corresponding to $a_{6.0} = 0.50$ GeV$^{-1}$. The energy difference between the excited potential and the ground state is seen to be well approximated [9] by a string model expression ($\approx \pi/R$ as shown by the continuous line). Also shown are some of the lower lying $b\bar{b}$ states in these potentials obtained from the Schrödinger equation in the adiabatic approximation. The lattice potentials share a common self-energy so that the energy difference between the lowest hybrid level and the $\Upsilon$ meson is determined directly (1.36 GeV). The dotted curve shows the modification to the quenched lattice ground-state potential needed to give the experimental spectrum. This gives an estimate of the systematic error from quenching.
We define a hybrid meson as a $q\bar{q}$ system with additional gluonic excitation. The definition of a hybrid meson is less clear than for a glueball since even the basic quark model mesons have a gluonic component which is responsible for the binding force. So we must establish that the gluonic component is excited before labelling a state as a hybrid meson. This is straightforward for the case of static quarks at separation $R$. The ground state potential will then have cylindrical symmetry about the interquark axis while less symmetric configurations correspond to various excitations of the gluonic flux joining the sources. A pioneering study using lattice techniques [8] found that the first excited gluonic state arises from transverse gluonic flux excitations of the form $\sqcap \sqcup$. Such spatial excitations of the distribution of the colour flux from quark to antiquark correspond to gluonic fields with $J_z \geq 1$ about the interquark axis and so are clearly hybrid states.

In molecular physics, it is common to assume that the electronic degrees of freedom adjust themselves with a much shorter timescale than that of the rotation of the molecule as a whole - this is the adiabatic approximation. For hybrid mesons, this will be valid if the gluonic degrees of freedom have a much shorter time-scale than those associated with the quarks. This will be a plausible approach since we find gluonic excitations with energies exceeding 1 GeV while quark model excitations (orbital and radial) have smaller energies (of a few hundred MeV). Then the allowed $J^{PC}$ values of hybrid mesons bound in this excited gluonic potential can be easily determined within this adiabatic approximation using the Schrödinger equation. The lowest lying hybrid states are found [8] to have $J^{PC}$ values arising from two spatial symmetries

\[
L^{PC} = 1^{+-}, J^{PC} = 1^{--}, 0^{-+}, 1^{-+}, 2^{--} \\
L^{PC} = 1^{--}, J^{PC} = 1^{++}, 0^{+-}, 1^{+-}, 2^{-+}
\]

The first group corresponds to the states accessible from a “magnetic gluon” excitation with spatial symmetry $L^{PC} = 1^{+-}$ while the second group are from an “electric gluon” with $L^{PC} = 1^{--}$. The lattice determination [3] of the ground state and excited potentials is illustrated in fig 3 for the quenched case. At large interquark separation $R$, a hadronic string picture is expected to be a reasonable model and we see that the string model excitation energy ($\pi/R$ in the simplest version - but see ref [4]) gives a good description. Also shown is the spectrum of mesons in these potentials obtained in the adiabatic approximation for $b$ quarks. Although the absolute value of the bound state energy is not accessible because of lattice self-energy effects, the difference between energies of bound states in the ground and excited potential is completely predicted. The hybrid level shown is the lowest such level and has the above eight degenerate $J^{PC}$ values. Thus there will be mesons with exotic quantum numbers at this energy which provide a prediction that
can be checked by experiment.

The ground state potential in the quenched approximation does not correctly reproduce the experimental $\Upsilon$ spectrum. The simplest explanation is that in full QCD the short distance (Coulombic) component of the potential would be enhanced (by $33/(33 - 2N_f)$ at leading order in perturbation theory) and such an enhancement is illustrated in fig 3 by the dotted ground state potential that does indeed reproduce the experimental spectrum. Using this prescription, but taking into account uncertainties from different approaches to modifying the quenched approximation, the lattice prediction [9] is for the lightest hybrid meson excitation to be at $4.19(15)$ GeV for $c\bar{c}$ and $10.81(25)$ GeV for $b\bar{b}$. These energy values lie above the open $D\bar{D}$ and $B\bar{B}$ thresholds. An alternative procedure, within the quenched approximation, is to focus [10] on the energy difference between the hybrid meson and the $B\bar{B}$ threshold. This suggests that the lowest hybrid level may lie below the threshold.

These lattice predictions do not take account of splitting of the degeneracy of the hybrid levels due to spin-spin and spin-orbit effects. Indeed, going beyond the adiabatic approximation, the non-exotic hybrid mesons can mix with states in the usual $L$-excited quark model and may thus be modified substantially. Evidence also exists from lattice studies [11] at small separation $R$ that the $L^{PC} = 1^{+-}$ excitation is a few hundred MeV lighter than the $L^{PC} = 1^{++}$ excitation. This would imply that the degenerate hybrid levels would be split with the lightest exotic state having $J^{PC} = 1^{+-}$. The experimental detection of such states depends crucially on whether they lie above or below the open quark threshold. The quenched lattice estimate [9] shows that, for both $b$ and $c$ quarks, the lightest hybrid levels lie above threshold. The uncertainties due to the level splitting effects described above, combined with the uncertainty in interpretation of the quenched spectrum, both point to the possibility that a narrow spin exotic hybrid meson could exist close to the open quark threshold. It is important to search carefully for such states.

In principle the lattice approach allows a study of hybrid mesons formed from light ($u$, $d$ and $s$) quarks. Preliminary results have recently been obtained [12] by the UKQCD collaboration. The method builds on the experience with static quarks and uses operators to create hybrid mesons in which the quark and antiquark are joined by colour flux which is excited in the transverse plane as $\square - \square$. Excitations of this kind are clearly non-trivial gluonic contributions and the mesonic states in such excited potentials include exotic $J^{PC}$ values. The lattice analysis is a fully relativistic analysis of propagating mesons. The approximations used are that of the quenched approximation for the vacuum and, in this preliminary study, we used a light quark mass corresponding to the strange quark
Figure 4: The lattice effective mass for the $J^{PC} = 1^{-+}$ hybrid meson versus time separation $t$. The source used was a U-shaped path of size $6 \times 6$ while the sinks were combinations of U-shaped paths of size $6 \times 6$ (diamonds), $3 \times 3$ (crosses) and $1 \times 1$ (squares). The fit shown has a ground state mass of 0.98(26) in lattice units at $\beta = 6.0$ with tadpole-improved clover action for hopping parameter $\tilde{K} = 0.137$. This corresponds to mesons made of strange quarks so $m(1^{-+})/m(\phi) = 1.8(5)$. 
Figure 5: Preliminary results for the ordering of the hybrid meson levels for strange quarks [12]. The states with burst symbols are $J^{PC}$ exotic. The dashed lines represent $L$-excited quark model states as determined on the lattice. The strong mixing of the states created by our hybrid operators with these is apparent for the pseudoscalar and vector meson cases.
(since lighter quarks are computationally more demanding). From the results we will be able to explore the splitting among hybrid meson levels as well as the mixing between non-exotic hybrid mesons and \( q\bar{q} \) mesons.

Preliminary results \cite{12} come from 70 lattices of size \( 16^3 \times 48 \) at \( \beta = 6.0 \) which is only a small fraction of the eventual statistical sample. We used a SW-clover fermionic action with clover coefficient \( c = 1.4785 \) and hopping parameter \( K = 0.1370 \). To establish our methods, we have studied the \( L \)-excited quark model mesons for S, P and D waves and successfully determined their energy levels. The signal for the hybrid mesons is weaker and our present data sample does not give precise estimates of the hybrid meson masses. For example, our results for the \( J^{PC} = 1^{-+} \) meson are shown by the fit in fig 4 and are consistent with a mass ratio \( m/m_\phi = 1.8(5) \) which shows the large errors remaining. What is somewhat better determined, however, are the splitting effects. We see in fig 5 significant mixing of non-exotic hybrids created using our hybrid operators with \( q\bar{q} \) mesons of the same quantum numbers. We intend to explore this more fully in future. We find that the exotic hybrids are all at comparable masses with the state with spatial excitation \( L^{PC} = 1^{-+} \) slightly lower in mass so that the lightest exotic hybrid meson would have \( J^{PC} = 1^{-+} \).

The preliminary study presented here was conducted using \( s \)-quarks. Quenched lattice studies of the QCD spectrum suggest that quark mass effects in the meson mass (or mass squared) are well described by a term linear in the quark mass. Experimental meson masses are consistent with the ansatz that \( M_{s\bar{s}}^2 - M_{q\bar{q}}^2 \approx 0.5 \) GeV\(^2 \) where \( q \) means \( u \) or \( d \). This suggests that the \( u, d \) mesons will be around 160 MeV lighter than the \( s\bar{s} \) mesons for masses around 1.5 GeV. Note that these lattice studies are for mesons made from two quarks of equal mass which are thus eigenstates of \( C \). For unequal masses (eg. strange mesons) the lack of \( C \) invariance masks the identification of spin exotic states.

There have been several experimental claims for hybrid mesons - for reviews see refs\cite{7, 13}. Our results suggest that non spin-exotic candidates may need re-appraisal since big mixing effects are possible. For the exotic mesons, the favoured candidate to lie lowest will have \( J^{PC} = 1^{-+} \) and several experimental hints of such states have been reported.

4 Conclusions

We have summarised quenched lattice predictions for glueballs and hybrid mesons. Recent developments include an estimate of glueball decay widths and a first study of light quark hybrid mesons. The study of the light quark hybrid mesons with considerable greater
statistics is currently under way. The qualitative features from such predictions are an essential guide to the experimental exploration of such mesons. Lattice studies with dynamical quarks will enable better control of the systematic error from quenching - this is also in progress.

5 Acknowledgements

I acknowledge the contributions made by my colleagues in the UKQCD collaboration, especially Pierre Lacock, to the study of hybrid mesons with light quarks.

References

[1] J. Sexton, A. Vaccarino and D. Weingarten, Phys.Rev.Lett.75 (1995) 4563.
[2] P. De Forcrand, et al., Phys. Lett. 152B (1985) 107
[3] C. Michael and M. Teper, Nucl. Phys. B314 (1989) 347
[4] UKQCD collaboration, G. Bali, K. Schilling, A. Hulsebos, A. C. Irving, C. Michael and P. Stephenson, Phys. Lett. B309 (1993) 378-84.
[5] H. Chen, J. Sexton, A. Vaccarino and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 357.
[6] J. Sexton, A. Vaccarino and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 27
[7] F. E. Close, Proc. Hadron Conference, Manchester 1995; [hep-ph/9509245].
[8] L. A. Griffiths, P. E. L. Rakow and C. Michael, Phys. Lett. B 129 (1983) 351.
[9] S. Perantonis and C. Michael, Nucl. Phys. B347 (1990) 854.
[10] R. Sommer, heplat/9401037 (to be published in Phys. Rep.).
[11] I. H. Jorysz and C. Michael, Nucl. Phys. B 302 (1987) 448.
[12] P. Lacock, C. Michael et al., UKQCD Collaboration (in preparation).
[13] S. U. Chung, These proceedings.