Implementing Agent-Based Systems via Computability Logic CL2

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Computability logic (CoL) is a powerful computational model. In this paper, we show that CoL naturally supports multi-agent programming models where resources (coffee for example) are involved. To be specific, we discuss an implementation of the Starbucks based on CoL (CL2 to be exact).

Keywords: Computability logic, multi-agent Programming, Distributed Artificial Intelligence.

1 Introduction

The design and implementation of multi-agent systems is recognized as a key component of general AI. Yet it remains the case that existing approaches – classical logic, π-calculus, linear logic, etc – are too simplistic to encode real-world multi-agent systems. Implementing the Starbucks in AI is such an example.

Computability logic (CoL) [2]-[6], is an elegant theory of (multi-)agent computability. In CoL, computational problems are seen as games between a machine and its environment and logical operators stand for operations on games. It understands interaction among agents in its most general — game-based — sense. There are many fragments of CoL. To represent resources such as coffee, we choose CL2 – a basic fragment of CoL – as our target language. CL2 is obtained by adding to CL1 a second kind of atoms called general atoms. A general atom models an arbitrary interactive computing problem such as a coffee machine.

In this paper, we discuss a web-based implementation of multi-agent programming based on CL2 [4]. We assume the following in our model:

- Each agent corresponds to a web site with a URL. An agent’s resource-base(RB) is described in its homepage.

- There are three kinds of agents: God, resource providers/consumers and regular agents. A resource provider for a resource $R$, written as $*R$, is an agent who is given a resource manual by God. It can thus
produce as many copies of resource R using the manual. A resource consumer is an agent who gives the resource to God.

- God is both the ultimate provider for every resource and its ultimate consumer.

- Our goal here is to program every agent including resource providers/consumers. For this, we assume that a resource provider has a machine’s manual/heuristic h for creating a resource R. Similarly, the counterstrategy of a resource consumer - the consumer’s script – is preprogrammed in the environment’s strategy ¯s. Note that, unlike the machine’s strategy, a consumer’s counterstrategy varies from a resource customer to a resource customer. To represent these manuals/scripts, we extend general atom P to P{h} for resource providers, general atom P to P{¯s} for resource consumers.

In this paper, we present CL2ψ which is a web-based implementation of CL2. This implementation is rather simple and straightforward. What is interesting is that CL2ψ is an appealing multi-agent programming model where resources are involved.

2 Preliminary

We review the basic relevant concepts of CoL, and some basic notational conventions. A reader may want to consult [3] for further details.

CoL understands the interactive computational problems as games between two players: machine and environment. The symbolic names for these two players are ⊤ and ⊥, respectively.

A move means a finite string over the keyboard alphabet. A labmove is a move prefixed with ⊤ or ⊥. A run is a (finite or infinite) sequence of labmoves, and a position is a finite run. Runs will be delimited by “(" and ")”. ⟨⟩ denotes the empty run.

The following is a brief definition of the concept of a constant game.

Definition 2.1 A constant game is a pair A = (LrA, WnA), where:

1. LrA is a set of runs satisfying the condition that a finite or infinite run is in LrA iff all of its nonempty finite — not necessarily proper — initial segments are in LrA (notice that this implies ⟨⟩ ∈ LrA). The elements of LrA are said to be legal runs of A.
2. \( W^n_A \) is a function that sends every run \( \Gamma \) to one of the players \( \top \) or \( \bot \).

Unfortunately, the above definition is not sufficient to represent games equipped with some kind of heuristics. AlphaGo is such an example. For this reason, we introduce a new game which we call a constant game with heuristics, denoted by \( A\{h\} \) where \( h \) is a heuristic function. For example, AlphaGo can be represented by \( Go\{h\} \) where \( h \) represents the powerful heuristics of the AlphaGo.

**Definition 2.2** A constant game with heuristics is a pair \( A\{h\} = (L^r_A, W^n_A, h_A) \), where:

\( h_A \) is a heuristic function for the machine to follow, i.e., the machine’s strategy for the game \( A \). \( h_A \) typically depends on the run of the game.

Often we need to preprogram the environment’s strategy as well. For this reason, we introduce a new game which we call a constant game with environment’s strategy, denoted by \( A\{\bar{s}\} \) where \( \bar{s} \) describes the environment’s strategy for the game.

### 3 CL2\( ψ \)

We review the propositional computability logic called CL2 [3].

As always, there are infinitely many elementary atoms in the language, for which we will be using the letters \( p, q, r, \ldots \) as metavariables. There are also infinitely many general atoms in the language, for which we will be using the letters \( P, Q, R, \ldots \). We introduce general atoms with machine’s strategy/heuristics, denoted by \( P\{h\}, \ldots \) and general atoms with environment’s strategy, denoted by \( P\{\bar{s}\}, \ldots \).

The two atoms: \( \top \) and \( \bot \) have a special status in that their interpretation is fixed. Formulas of this language, referred to as CL2-formulas, are built from atoms in the standard way:

**Definition 3.1** The class of \( CL2 \)-formulas is defined as the smallest set of expressions such that all atoms are in it and, if \( F \) and \( G \) are in it, then so are \( \neg F, F \land G, F \lor G, F \rightarrow G, F \sqcap G, F \sqcup G \).

Now we define \( CL2\( ψ \), a slight extension to CL2 with environment parameters. Let \( F \) be a CL2-formula. We introduce a new env-annotated formula \( F^\omega \) which reads as ‘play \( F \) against an agent \( \omega \).’ For an \( \sqcap \)-occurrence
O (or an occurrence of a general atom) in $F^\omega$, we say $\omega$ is the matching environment of $O$. For example, $(p \cap (q \cap r))^w$ is an agent-annotated formula and $w$ is the matching environment of both occurrences of $\cap$. We extend this definition to subformulas and formulas. For a subformula $F'$ of the above $F^\omega$, we say that $\omega$ is the matching environment of both $F'$ and $F$.

In introducing environments to a formula $F$, one issue is whether we allow ‘env-switching’ formulas of the form $(F[R]^u)^w$. Here $F[R]$ represents a formula with some occurrence of a subformula $R$. That is, the machine initially plays $F$ against agent $w$ and then switches to play against another agent $u$ in the course of playing $F$. For technical reasons, we focus on non-‘env-switching’ formulas. This leads to the following definition where $h$ is a heuristic function:

**Definition 3.2** The class of $\mathbf{CL2}^\Psi$-formulas is defined as the smallest set of expressions such that (a) For any $\mathbf{CL2}$-formula $F$ and any agent $\omega$, $F^\omega$ are in it and, (b) if $H$ and $J$ are in it, then so are $\neg H$, $H \land J$, $H \lor J$, $H \rightarrow J$.

**Definition 3.3** Given a $\mathbf{CL2}^\Psi$-formula $J$, the skeleton of $J$ – denoted by $\text{skeleton}(J)$ – is obtained by replacing every occurrence $F^\omega$ by $F$.

For example, $\text{skeleton}((p \cap (q \cap r))^w) = p \cap (q \cap r)$.

We often use $F$ instead of $F^\omega$ when it is irrelevant.

The following definitions come from [4]. They apply both to $\mathbf{CL2}$, and $\mathbf{CL2}^\Psi$.

Understanding $E \rightarrow F$ as an abbreviation of $\neg E \lor F$, a positive occurrence of a subformula is one that is in the scope of an even number of $\neg$’s. Otherwise, the occurrence is negative.

A surface occurrence of a subformula means an occurrence that is not in the scope of a choice ($\sqcup$ or $\sqcap$) operator.

A formula is elementary iff it does not contain the choice operators and general atoms.

The elementarization of a formula is the result of replacing, in it, every surface occurrence of the form $F_1 \sqcup \ldots \sqcup F_n$ by $\bot$, every surface occurrence of the form $F_1 \sqcap \ldots \sqcap F_n$ by $\top$, every positive surface occurrence of each general atom by $\bot$, and every negative surface occurrence of the form each general atom by $\top$.

A formula is stable iff its elementarization is valid in classical logic, otherwise it is unstable.

$F$-specification of $O$, where $F$ is a formula and $O$ is a surface occurrence in $F$, is a string $\alpha$ which can be defined by:
• $F$-specification of the occurrence in itself is the empty string.

• If $F = \neg G$, then $F$-specification of an occurrence that happens to be in $G$ is the same as the $G$-specification of that occurrence.

• If $F$ is $G_1 \land \ldots \land G_n$, $G_1 \lor \ldots \lor G_n$, or $G_1 \rightarrow G_2$, then $F$-specification of an occurrence that happens to be in $G_i$ is the string $i.\alpha$, where $\alpha$ is the $G_i$-specification of that occurrence.

The proof system of $\text{CL}^2_\Psi$ is identical to that $\text{CL}^2$ and has the following three rules, with $H$, $F$ standing for $\text{CL}^2_\Psi$-formulas and $\vec{H}$ for a set of $\text{CL}^2_\Psi$-formulas:

Rule (A): $\vec{H} \vdash F$, where $F$ is stable and, whenever $F$ has a positive (resp. negative) surface occurrence of $G_1 \cap \ldots \cap G_n$ (resp. $G_1 \cup \ldots \cup G_n$) whose matching environment is $\omega$, for each $i \in \{1, \ldots, n\}$, $\vec{H}$ contains the result of replacing in $F$ that occurrence by $G_i^\omega$.

Rule (B): $H \vdash F$, where $H$ is the result of replacing in $F$ a negative (resp. positive) surface occurrence of $G_1 \cap \ldots \cap G_n$ (resp. $G_1 \cup \ldots \cup G_n$) whose matching environment is $\omega$ by $G_i^\omega$ for some $i \in \{1, \ldots, n\}$.

Rule (C): $F' \vdash F$, where $F'$ is the result of replacing in $F$ one negative surface occurrence of some general atom $P$ and one positive surface occurrence of some general atom $P$ by a nonlogical elementary atom that does not occur in $F$.

Example 3.4 $\text{CL}^2_\Psi \vdash (C \land C) \rightarrow (C \lor C)^\omega$

where $\omega$ is an agent. Note that $\omega$ play no roles in the proof procedure. Similarly, the machine’s manual and the environment’s script play no role in the proof procedure.

1. $(p \land q) \rightarrow (p \lor q)^\omega$, rule A, 0
2. $(p \land C) \rightarrow (p \lor C)^\omega$, rule C, 1
3. $(C \land C) \rightarrow (C \lor C)^\omega$, rule C, 2

4 Hyperformulas

To facilitate the execution procedure, we modify $\text{CL}^2_\Psi$ to obtain $\text{CL}^{\omega}_\Psi$. Unlike $\text{CL}^2_\Psi$, this new language allows any hyperformulas. Its rules are Rules (a) and (b) of $\text{CL}^2_\Psi$ plus the following Rule $(c^\omega)$ instead of the old
Rule (c):

Rule (C'): $F' \vdash F$, where $F'$ is the result of replacing in $F$ one negative surface occurrence of some general atom $P$ and one positive surface occurrence of some general atom $P$ by a hybrid atom $P_q$.

In the above, we introduced hybrid atoms. Each hybrid atom is a pair consisting of a general atom $P$, called its general component, and a non-logical elementary atom $q$, called its elementary component. Hybrid atoms were introduced in [4] to distinguish elementary atoms introduced in Rule (c) from all other elementary atoms.

Now atoms can be of one of the three (elementary, general or hybrid) sorts. All the terminologies and definitions of the previous section extends well to hyperformulas. One exception is that in the elementarization of a hyperformula, every surface occurrence of each hybrid atom must also be replaced by the elementary component of that atom.

We can easily convert $\text{CL2}^\omega$ proof to a modified one: if $q$ is obtained from $P$ by Rule (c), replace all occurrences of $q$ by $P_q$. Apply this procedure to all of its descendants in the proof tree as well.

**Example 4.1**  $\text{CL2}^\omega, \Psi \vdash (C \land C) \rightarrow (C \lor C)^\omega$

where $\omega$ is an agent. Note that $\omega$ play no roles in the proof procedure.

1. $(C_p \land C_q) \rightarrow (C_p \lor C_q)^\omega$, rule A, 0
2. $(C_p \land C) \rightarrow (C_p \lor C)^\omega$, rule C, 1
3. $(C \land C) \rightarrow (C \lor C)^\omega$, rule C, 2

5 Execution Phase

The machine model of $\text{CL2}$ is designed to process only one query/formula at one time. In distributed systems such as $\text{CL2}^\Psi$, however, it is natural for an agent to receive/process multiple queries. For this reason, our machine processes multiple formulas one by one.

Multiple queries cause some complications, as the RB of the machine evolves to RB' in the course of solving a query. In such a case, subsequent queries must be solved with respect to RB'. To be specific, it maintains a queue $Q = \langle Q_1, \ldots, Q_n \rangle$ for storing multiple incoming queries. We assume that the machine processes $Q_1, \ldots, Q_n$ by executing the following $n$ procedures sequentially:
Here $RB_1$ is the original RB associated with the machine. We assume here that, for $1 \leq i \leq n$, $RB_i$ evolves to $RB_{i+1}$ after solving $Q_i$.

It leads to the following definition:

procedure EXEC(K, Q): $K$ is RB of the agent and $Q$ is a queue of incoming queries.

- If $K = \varnothing$ and $Q = (Q_1, \ldots, Q_n)$ then we do the following:
  
  In this case, the machine tries to solve the first query by invoking $\text{Exec}(\varnothing \rightarrow Q_1)$ and then EXEC(\( \varnothing', (Q_2, \ldots, Q_n) \)).

- Else ($Q$ is empty): wait for new incoming service calls.

Below we will introduce an algorithm that executes a formula $J$. The algorithm is a minor variant of the one in [4] and contains two stages:

Algorithm Exec(J): $J$ is a $\text{CL2}^\Psi$-formula

1. First stage is to initialize a temporary variable $E$ to $J$, a position variable $\Omega$ to an empty position $\langle \rangle$. Activate all the agents specified in $J$.

2. The second stage is to play $J$ according to the following mainloop procedure (which is from [4]):

procedure mainloop(Tree): $Tree$ is a proof tree of $J$

Case $E$ is derived by Rule (B):

Let $H$ be the premise of $E$ in the proof. $H$ is the result of substituting, in $E$, a certain negative (resp. positive) surface occurrence of a subformula $G_1 \cap \ldots \cap G_n$ (resp. $G_1 \sqcup \ldots \sqcup G_n$) by $G_i^\omega$ for some $i \in \{1, \ldots, n\}$. Here, we assume that $\omega$ is the matching environment of that occurrence. Let $\gamma$ be the $E$-specification of that occurrence. Then make the move $\gamma_i$, update $E$ to $H$. Then inform $\omega$ of the move $\gamma_i$. repeat mainloop
Case $E$ is derived by Rule (C$^0$):

Let $H$ be the premise of $E$ in the proof. $H$ is the result of replacing in $E$ some positive surface occurrence $\pi$ and some negative surface occurrence $\nu$ of a general atom $P$ by a hybrid atom $P_q$. Let $\langle \bot \pi_1, \ldots, \bot \pi_n \rangle$ and $\langle \bot \nu_1, \ldots, \bot \nu_m \rangle$ be $\Omega^\pi$ and $\Omega^\nu$, respectively. Here $\Omega^\pi$ is the subrun of the occurrence $\pi$ and $\Omega^\nu$ is the subrun of the occurrence $\nu$ of the hybrid atom introduced. Then: make the $m + n$ moves $\pi\nu_1, \ldots, \pi\nu_m, \nu\pi_1, \ldots, \nu\pi_n$ (in this order); update $\Omega$ to $\langle \Omega, \top \pi\nu_1, \ldots, \top \pi\nu_m, \top \nu\pi_1, \ldots, \top \nu\pi_n \rangle$. Update $E$ to $H$; repeat mainloop.

Case $E$ is derived by Rule (a):

Follow the procedure innerloop described below. Below, “the environment makes a move” means that either the environment makes a move or $\top$ makes a move for the environment using a given heuristic function.

innerloop: Keep granting permission until the environment makes a move $\alpha$.

Subcase (i): $\alpha = \gamma\beta$, where $\gamma$ $E$-specifies a surface occurrence of a general atom. Then update $\Omega$ to $\langle \Omega, \bot \gamma\beta \rangle$ and repeat innerloop.

Subcase (ii): $\alpha = \gamma\beta$, where $\gamma$ $E$-specifies a surface occurrence of a hybrid atom. Let $\sigma$ be the $E$- specification of the other occurrence of the same hybrid atom. Then make the move $\sigma\beta$, update $\Omega$ to $\langle \Omega, \bot \gamma\beta, \top \sigma\beta \rangle$ and repeat innerloop.

Subcase (iii): $\alpha = \gamma i$, where $\gamma$ $E$-specifies a positive (negative) surface occurrence of a subformula $G_1 \sqcap \ldots \sqcap G_n$ ($G_1 \sqcup \ldots \sqcup G_n$) and $i \in \{1, \ldots, n\}$. Let $H$ be the result of substituting, in $E$, a certain negative (resp. positive) surface occurrence of a subformula $G_1 \sqcap \ldots \sqcap G_n$ (resp. $G_1 \sqcup \ldots \sqcup G_n$) by $G_i^\omega$ for some $i \in \{1, \ldots, n\}$. Here $\omega$ is the matching environment of that occurrence. Then update $E$ to $H$, and repeat mainloop.

If $\alpha$ does not satisfy the conditions of any of the above Subcases (i),(ii),(iii), ignore it.

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$^1\Omega^\pi$ and $\Omega^\nu$ may be programmed in $h$ in $P\{h\}$. 

8
6 Examples

As an example of multi-agent system, we will look at the Starbucks. This example introduces several interesting concepts such as how service flows among agents. It is formulated with the God, the Folger coffee maker (coffee provider), the Starbucks owner, a user and the bank (dollar provider). We assume the following:

- In our example, God provides the coffee-making manual to the Folger and collects $10. It also provides the dollar-making manual to the bank and collects ten coffees.
- God is not actually implemented. Instead, the Folger and the bank play the role of God whenever necessary.
- The store owner plays the roles of barista and cashier.
- The owner tries to borrow $8 from the bank and pay 8 coffees to it. He also tries to pay $10 and gets ten coffees from the Folger.
- Each coffee costs a dollar.
- The user tries to get two coffees by paying two dollars to the owner. He also tries to get two dollars by paying two coffees to the bank.
- The user is active from the beginning.

Now we want to implement the above. The first task is to determine the representation of a coffee. A coffee is represented by a (imaginary or real, depending on your need) coffee machine. We assume that the owner has a coffee manual/heuristic which provides the ‘rules of thumb’ to make a good coffee.

A coffee machine – similar to an ATM machine – can be seen as a game between the owner with a manual and its customer with a sequence of interactions. Assume we have a particular coffee machine with LCD monitor where

1. The user of the machine selects the $x (= 1, 2, \ldots)$ grams of sugar, and then the $y (= 1, 2, \ldots)$*10cc of milk
2. The owner selects the $z (= 1, 2, \ldots, 10)$ spoons of coffee.

For simplicity, we assume that the owner uses the following heuristic evaluating function

$$h(x, y, z) = |z - xy - 1|.$$
In other words, if it selects \( z \) such that \( z = xy + 1 \), then it knows that he/she makes good coffees.

One simplest way of representing this device is to represent it as a general atom \( C\{h\} \) with the above heuristic \( h^2 \).

Similarly, the consumer’s preference in coffee can be programmed in the user’s scripts. Below illustrates some user’s scripts \( \bar{c}_0, \bar{c}_1 \) used in the example below in coffee making.

\[
\bar{c}_0 = \{(\bot, 3), (\bot, 1)\}. \text{ } \% \text{ 3 grams of sugar, 10cc of milk}
\]

\[
\bar{c}_1 = \{(\bot, 4), (\bot, 2)\}. \text{ } \% \text{ 4 grams of sugar, 20cc of milk}
\]

As in the case of coffee, the same approach can be employed to represent a dollar, i.e., as a credit-card paying machine or a POS machine. A credit-card paying machine can be seen as an interactive constant game. To make things simple, we assume the bank is a provider for one dollar and \( r \) is a manual for making a dollar.

An example is provided by the following \(*C, o, u, *1\) agents. In \( 1\{\bar{d}_0\} \) of the \(*C\) agent, \( \bar{d} \) describes a preprogrammed God’s requirements in making the first dollar. Similarly, in \( C\{\bar{c}_0\} \) of the bank agent, \( \bar{c}_0 \) describes a preprogrammed requests in making the first coffee.

Now consider \( C\{h\} \) in \(*C\). Here \( h \) is a heuristic function for making a coffee. That is, \( h \) is a coffee-making manual.

\( \text{agent } *C. \text{ Folger coffee provider} \)
\( \bar{d}_0 = \ldots \text{ God’s requirements in the first dollar} \)
\( \vdots \)
\( \bar{d}_9 = \ldots \text{ God’s requirements in the tenth dollar} \)
\( h(x, y, z) = \ldots \text{ coffee-making manual} \)
\( ((1\{\bar{d}_0\} \land \ldots \land 1\{\bar{d}_9\}) \rightarrow C\{h\})^{God}. \text{ the coffee manual costs ten (customized) dollars.} \)

\( \text{agent } o. \text{ Starbucks owner} \)
\( ((C \land \ldots \land C) \rightarrow (1 \land \ldots \land 1)^k). \text{ pay 8 coffees and get } $8 \text{ from bank.} \)
\( ((1 \land \ldots \land 1) \rightarrow (C \land \ldots \land C))^p. \text{ pay } $10 \text{ and get 10 coffees from Folger.} \)

\( \text{agent } u. \text{ the client} \)

\footnote{Coffee machine can be represented without using general atoms but it is cumbersome.}
$$((C \land C) \rightarrow (1 \land 1))^k \text{. } \% \text{ pay 2 coffees and get }$2 from bank.
$$((1 \land 1) \rightarrow (C \land C))^\omega \text{. } \% \text{ pay two dollars and get two coffees from owner.}

agent^* 1. \% \text{ the bank}
\bar{c}_0 = \ldots \% \text{ God’s requirements in the first coffee}
\vdots
\bar{c}_9 = \ldots \% \text{ God’s requirements in the tenth coffee}
h(x, y, z) = \ldots \% \text{ coffee-making manual}
r(\ldots) = \ldots \% \text{ dollar-making manual}
$$((C\{\bar{c}_0\} \land \ldots \land C\{\bar{c}_9\}) \rightarrow 1\{r\})^{God} \text{. } \% \text{ dollar-making manual costs 10 (customized) coffees.}

Now consider the user agent \(u\). The user is active from the beginning and tries to do the following: (1) obtain two coffees from the owner and pass it along to the bank, and by (2) obtaining two dollars and then passing them along to the owner. The task (2) easily succeeds, as \(u\) makes two dollars by copying the moves of the bank (The bank makes moves according to the recipe \(r\)). From this, the agent \(u\) successfully pays the owner \(o\) two dollars. The owner \(o\) pays $10 to \(\ast C\) ($2 from \(u\), $8 from \(\ast 1\)) all using the copy-cat method. Upon request, \(\ast C\) makes ten (real or imaginary) coffees using the “coffee manual” \(h\). \(o\) makes ten coffees by copying \(\ast C\). Note that the user can make two coffees and \(\ast 1\) can make 8 coffees both by copying \(o\).

7 Conclusion

In this paper, we proposed a multi-agent programming model based on CL2\(^\Psi\). Unlike other formalisms such as LogicWeb\([8]\) and distributed logic programming\([1]\), this model does not require any centralized control. Our next goal is to replace CL2\(^\Psi\) with much more expressive CL12\([5]\).

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