Response of a reinforced concrete cantilever beam subject to impulse impact loads

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Abstract. To study the response of a RC cantilever beam subjected to impulse excitations is based computational model. Static and dynamic analysis of the computational model was performed using a software product based on the Finite Element Method. Two numerical models have been builded for static analysis. In the first model, the concrete modelled with a linear - elastic material model and the reinforcement as deformable bars. In the second model, the concrete modelled using a nonlinear model of concrete - the material model of Willam and Warnke, and the reinforcement as deformable bars. For dynamic analysis, concrete modelled as a linearly elastic material and reinforcement as deformed bars. Under impulse loading, the applied force acts for a very short but limited time interval. This article presents five cases of impulse duration. The dynamic time-history analysis used to determine the dynamic response of the RC cantilever beam under the action of time variables. This type of analysis used to determine time-varying displacements, stresses, and internal forces caused by a combination of static and dynamic (pulse and harmonic) loads. A dynamic time-history analysis used to study the dynamic response to time-varying loads. The time-determined displacements, deformations, stresses and internal forces give the behaviour and reaction of the structure at any time in the studied interval.

1. Introduction

Transient or time-history dynamic analysis is a technique used to determine the dynamic response of a structure under the action of time-dependent excitations. Can use this type of analysis to determine the time-varying displacements, strains, stresses, and forces in a structure as it responds to any combination of static, transient, impulse and harmonic loads [1].

The basic equation in Finite Element Method of motion solved by a transient dynamic analysis is (1):

\[
[M]{\ddot{u}} + [C]{\dot{u}} + [K]{u} = {F(t)}
\]

(1)

where:

[M] – Mass matrix;
[C] – Damping matrix;
[K] – Stiffness matrix;
\{\ddot{u}\} – Nodal acceleration vector;
\{\dot{u}\} – Nodal velocity vector;
\{u\} – Nodal displacement vector;
\{F(t)\} – Load vector.
2. Response to impulse excitation – unit impulse
Where very large force acts for a very short time but with a time integral that is finite is called impulsive force. As shown in Figure 1 – force $F(t) = \frac{1}{\Delta t}$, with time duration $\Delta t$ starting at time $t_1$. Where value of $\Delta t$ approaches to zero, force becomes infinite, however, the magnitude of impulse, defined by the time integral of $F(t)$ remains equal to unity. Such a force in the limit case $\Delta t \rightarrow 0$ is unit impulse [3].

![Figure 1. Impulse excitation – unit impulse](image)

According to Newton’s Second law of motion, if a force $F$ acts on a body with mass $m$, the rate of change of momentum of the body is equal to applied force (2):

$$\frac{d}{dt}(mu) = F$$

(2)

If mass is constant: $F = m \ddot{u}$.
Integrating both sides with respect to $t$ and gives (3):

$$\int_{t_1}^{t_2} pdt = m(\dot{u}_2 - \dot{u}_1) = m\Delta \dot{u}$$

(3)

The integral at the left side is magnitude of impulse, and product of mass and velocity is momentum [3].

3. Numerical models
The concrete and reinforcement material properties are according to Eurocode 2. [4]

![Figure 2. a) Scheme and load area b) Position of reinforced bars](image)

The concrete properties are: C20/25, $f_{cm} = 28$ MPa, $E_{cm} = 30$ GPa, Poisson’s ratio is 0.2.
Reinforcement diameters and position is shown on Figure 2b).
Cross section of the cantilever beam – $b=0.25$ m, $h=0.40$ m, $L=1.00$ m
3.1. Static analysis. Linear elastic concrete model.

First numerical model (M1) is modeling by ANSYS (Figure 4). The load stamp size is 0.25/0.25 m. The static load of 370 kN/m$^2$ is applied on the load stamp. The cantilever beam is modelled by SOLID 65 (CONCRET 65) finite elements as linear elastic concrete material model – Table 1. Reinforcement bars is modelled with BEAM188 finite elements – Table 2.

### Table 1. Concrete properties

| Concrete material model for C20/25 |  
|-----------------------------------|---
| Linear isotropic                 |  
| Young’s modulus $E_c=3\times10^7$ kN/m$^2$ |  
| Poisson’s ratio $\nu=0.2$        |  

### Table 2. Reinforcement properties

| Reinforcement steel B500 model |  
|--------------------------------|---
| Linear isotropic              |  
| Young’s modulus $E_s=2\times10^8$ kN/m$^2$ |  
| Poisson’s ratio $\nu=0.3$     |  

3.2. Static analysis. Nonlinear inelastic concrete model.

Second model (M2) is modelling with material properties according to Table 3, are presented. The boundary conditions adopted are restraints applied on the left side of the cantilever beam - displacements and rotation by axes X, Y and Z. The load stamp size are 0.25/0.25 m. The applied force start at 160 kN/m$^2$ by step of 160 kN/m$^2$. The beam is modelled by SOLID 65 (CONCRET 65) finite elements nonlinear plasticity material model. Different constants for application of the Willam and Warnke’s material model [5] in ANSYS are require (Table 3). Reinforcement bars is modeled with BEAM188 finite elements – Table 2. In Figure 4 is shown the distributtion of the normal stresses in the support zone and displacements in the free end of the cantilever beam. In Figure 5 is shown formed cracks at model M2.

### Table 3. Concrete material nonlinear inelastic model in tension zone

| 1. Shear transfer coefficient for an open crack ($\beta_k$) | 0.2 |
| 2. Shear transfer coefficient for an close crack ($\beta_k$) | 0.9 |
| 3. Uniaxial cracking stress ($f_r$) | 2800 kN/m$^2$ |
| 4. Uniaxial crushing stress ($f_c$) | -1 |

Tensile strength of bending concrete - the average tensile strength in bending of reinforced concrete elements depends on the average tensile strength and the height of the cross section. [8]

3.3. Dynamic analysis. Impulse load.

The most important point in the study of building structures of dynamic impacts is the choice of a suitable dynamic model. [6]

The numerical model of cantilever beam is modeling by ANSYS. Applied load at the end of the beam on the load stamp (Figure 2a). The load stamp size is 0.25/0.25 m. The stamp load case is presented. The impulse force with magnitude of 370 kN/m$^2$ is applied. The concrete is modelled by SOLID 65 (CONCRET 65) finite element as linear elastic concrete material model – Table 1. Reinforcement bars is modelled with BEAM188 finite elements – Table 2. Five load cases are presented – Table 5.
Figure 3. Impulse excitation, applied on the load stamp

Table 4. Impulse load cases

| №  | Load magnitude, kN/m² | Duration Δt, s | Duration t, s |
|----|----------------------|----------------|---------------|
| 1  | q=370                | 0,005s         | 2,00 s        |
| 2  | q=370                | 0,025s         | 2,00 s        |
| 3  | q=370                | 0,05s          | 2,00 s        |
| 4  | q=370                | 0,10s          | 2,00 s        |
| 5  | q=370                | 0,20s          | t=2,00 s      |

4. Numerical results and discussion

4.1. Static analysis
In Figure 4b) displacement $u_y$ is presented and in Figure 4a) presented distribution of $\sigma_x$ at load of 370 kN/m². The results from static analysis at node 596 are presented in Table 5.

4.1.1. Linear elastic material models

Figure 4. a) Displacement $u_y$, b) Distribution of normal stress $\sigma_x$ at the cantilever beam, q=370 kN/m²
4.1.2. Nonlinear material model

Figure 5. Nonlinear concrete material model. Crack initiation at applied force $q=370 \text{ kN/m}^2$

Table 5. Results from static analysis at node 596 and node 227, $q=370 \text{ kN/m}^2$.

| Parameter | Linear elastic material model (M1) | Plasticity material model (M2) |
|-----------|----------------------------------|-------------------------------|
| $|u_y| - \text{node 596}$ | 0.000161757 m | 0.00102674 m |
| $\sigma_x - \text{node 227}$ | 2911.42 kN/m$^2$ | 2517.98 kN/m$^2$ |

Figure 6. Displacement - Load relation in node 596 by nonlinear static analysis.

Figure 7. Normal stress $\sigma_x$ - Load relation in node 227 by nonlinear static analysis.
4.2. Modal analysis
In the Table 6 natural frequencies from modal analysis are presented.

| Mode | Frequency, s\(^{-1}\) |
|------|---------------------|
| 1    | 42,994              |
| 2    | 65,236              |
| 3    | 148,393             |

4.3. Dynamic analysis
In the Figure 8 is shown variations of displacement \(u_y\) in the node 596 at time duration of impulse load: \(\Delta t=0,005\) s, \(\Delta t=0,025\) s, \(\Delta t=0,05\) s, \(\Delta t=0,10\) s and \(\Delta t=0,20\) s for first numerical model with material properties from Table 1 and Table 2 (M3).

**Figure 8.** Displacement \(u_y\) - time \(t\) relation at node 596, \(q=370\) kN/m\(^2\); a) \(\Delta t=0,005\) s, \(t=2,00\) s; b) \(\Delta t=0,025\) s, \(t=2,00\) s; c) \(\Delta t=0,05\) s, \(t=2,00\) s; d) \(\Delta t=0,10\) s, \(t=2,00\) s; d) \(\Delta t=0,20\) s, \(t=2,00\) s

**Figure 9.** Response of the impulse load excitations. Relation \(u_y - \Delta t\) in the node 596.
Table 7. Results for displacements at node 596 and normal stress at node 227: static vs. dynamic analysis

| Parameter | Static analysis | Dynamic analysis |
|-----------|----------------|------------------|
|           | Linear elastic |                  |
|           | material model |                  |
| max. \(|u_z| \times 10^4, m| node 596 | 1.61757 | 2.17199 | 2.61146 | 2.70347 | 2.93618 | 2.50516 |
| Difference from | - | 34.27% | 61.44% | 67.13% | 81.52% | 54.87% |
| max \(\sigma_y\), kN/m² | node 227 | 2911.42 | 3931.40 | 4894.65 | 5076.84 | 5426.91 | 4616.71 |
| Difference from | - | 35.03% | 68.11% | 74.38% | 86.40% | 58.57% |

5. Conclusions

Normal stresses at magnitude load considerably exceed those applied static load the same size – from 68 to 87% - Table 7. The stress state near the load stamp characterized by stresses increasing.

With the applied impulse action, the displacements in a node from the free end of the cantilever beam increase smoothly, as their maximum value is reached at a duration of action of the impulse force of \(\Delta t = 0.1\) s (Figure 9). Then, with a decrease in the time interval of action of the impulse force, a gradual decrease in displacements is observed. The maximum values of displacements increase by about 82% compared to the value of displacements from static load. It follows that multiplying the values of displacements / respectively stresses and strains / coefficient equal to 1.9 get the values of displacements from the impulse action.

The FEM also provides opportunities for the study of stress and strain state of the constructive members subjected to local concentrated static load and dynamic excitations, taking into account the material and geometric nonlinearity. The results obtained by these calculation models are much closer to the actual behavior of the studied systems and constructive members. The conclusion is that by multiplying the values of displacements coefficient, we will obtain the values of displacements from impulse effects.

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