Deterministic Identification for Molecular Communications Over the Poisson Channel

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Abstract—Various applications of molecular communications (MC) are event-triggered, and, as a consequence, the prevalent Shannon capacity may not be the right measure for performance assessment. Thus, in this paper, we motivate and establish the identification capacity as an alternative metric. In particular, we study deterministic identification (DI) for the discrete-time Poisson channel (DTPC), subject to an average and a peak molecule release rate constraint, which serves as a model for MC systems employing molecule counting receivers. It is established that the number of different messages that can be reliably identified for this channel scales as $2^{(n \log n)R}$, where $n$ and $R$ are the codeword length and coding rate, respectively. Lower and upper bounds on the DI capacity of the DTPC are developed. The obtained large capacity of the DI channel sheds light on the performance of natural DI systems such as natural olfaction, which are known for their extremely large chemical discriminatory power in biology. Furthermore, numerical results for the empirical miss-identification and false identification error rates are provided for finite length codes. This allows us to characterize the behaviour of the error rate for increasing codeword lengths, which complements our theoretically-derived scale for asymptotically large codeword lengths.

Index Terms—Channel capacity, deterministic identification, molecular communication, Poisson channel.

I. INTRODUCTION

MOLECULAR communication (MC) is a new paradigm in communication engineering where information is transmitted via signaling molecules [2], [3]. In particular, information can be embedded into the type [4], concentration [5], release time [6], and spatial release pattern [7] of signaling molecules. Over the past decade, synthetic MC has been extensively studied in the literature from different perspectives including channel modeling [8], modulation and detection design [9], biological building blocks for transceiver design [10], and information-theoretical performance characterization [11], [12]. Moreover, several proof-of-concept implementations of synthetic MC systems have been reported in the literature, see, e.g., [13], [14], [15]. Furthermore, the ongoing progress in synthetic biology [10], [16] is expected to enable sophisticated MC systems in the future, capable of performing complex sensing, computation, and communication tasks needed for realizing the Internet of Bio-nano Things [17].

A. Capacity Characterization of MC Systems

Information-theoretical analysis of MC systems is useful not only for the characterization of their performance limits, but also for guiding MC system design and assessing the efficiency of practical designs against these performance limits. In this context, a mathematical foundation for information-theoretical analysis of diffusion-based MC is established in [18] where a channel coding theorem is proved. The information rate capacity of diffusion-based MC is studied in [19] where both channel memory and molecular noise are taken into account. For diffusion-based MC, the capacity limits of molecular timing channels are investigated in [20] and lower and upper bounds on the corresponding capacity are reported. In [21], a new characterization of capacity limits and capacity achieving distributions for the particle-intensity channel are studied. Capacity bounds for point-to-point communication are

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the presence of extremely large numbers of different detect-olfactory systems of animals have the capability of MC systems. It is widely accepted in the literature that the example of the identification problem appears in olfactory problems can also be found in various natural MC systems. An exceeds a critical threshold. Moreover, identification prob-lem can also be found in various natural MC systems. An may be interested in whether or not the pH value of the blood not a specific cancer biomarker is present around the target e.g., a nano-device’s objective may be to identify whether or or not synchronization (required to avoid interference with the spatial and temporal channel uses. For the latter, different channel uses for each codeword are realized by releasing the the same type of molecules in different time instances, whereas, for the former, a different type of signaling molecule is used for each channel use. The spatial channel use can be employed to model molecule-mixture communications in mammalian and insect olfactory systems, where a given mixture of different types of molecules represents a codeword [42], [47].

Identification in MC Systems

Various applications of MC within the framework of sixth generation wireless networks (6G) and beyond [30], [31] are associated with event-triggered systems, where Shannon’s message transmission capacity, as considered in [11], [12], [18], [19], [20], [21], [24], [25], [26], [27], [28], [29], may not be the appropriate performance metric. In particular, in event-detection scenarios, where the receiver wishes to decide about the occurrence of a specific event in terms of a reliable Yes/No answer, the so-called identification capacity is the relevant performance measure [32].

Specific examples of the identification problem in the con- text of MC can be found in certifying synchronization between nano-devices, where, e.g., a nano-transmitter seeks to verify whether or not synchronization (required to avoid interference in time-slotted communication settings) with respect to a target nano-receiver is established [33]; targeted drug delivery [33], [34] and cancer treatment [35], [36], [37], where, e.g., a nano-device’s objective may be to identify whether or not a specific cancer biomarker is present around the target tissue; in health monitoring [38], [39], where, e.g., one may be interested in whether or not the pH value of the blood exceeds a critical threshold. Moreover, identification problems can also be found in various natural MC systems. An example of the identification problem appears in olfactory MC systems. It is widely accepted in the literature that the olfactory systems of animals have the capability of detecting the presence of extremely large numbers of different molecule mixtures (e.g., pheromones, odors, etc) [40], [41], which has motivated researchers to use them as inspiration for the design of synthetic MC systems [42]. The communication goal may involve the recognition of a specific type of secreted odor/pheromone which corresponds to an odor/pheromone identification task. Examples include the determination of whether or not a certain nutrient is present or the identification of whether or not an individual of the opposite gender is present in the surrounding territory for the exchange of genetic material. Considering the above discussion, in this paper, we investigate the fundamental performance limits of identification in MC systems, which can be modelled by the DTPC.

C. Contributions

In this work, we consider MC systems employing molecule counting receivers, where the received signal has been shown to follow the Poisson distribution when the number of released molecules is large, see [8, Sec. IV], [25], [46] for details. Our main objective is to study the fundamental performance limits of deterministic identification (DI) problem over the DTPC. Specifically, this paper makes the following contributions:

- Identification Model: We formulate the problem of DI over the DTPC under average and peak molecule release rate constraints to account for the limited molecule production/release rates of the transmitter. Moreover, to model the diverse applications of coded identification in the context of MC, we introduce two major approaches to realize the channel uses within each codeword, namely spatial and temporal channel uses. For the latter, different channel uses for each codeword are realized by releasing the same type of molecules in different time instances, whereas, for the former, a different type of signaling molecule is used for each channel use. The spatial channel use can be employed to model molecule-mixture communications in mammalian and insect olfactory systems, where a given mixture of different types of molecules represents a codeword [42], [47].

- Codebook Scale: We derive lower and upper bounds on the DI capacity of the DTPC, which are the main results of this paper. In particular, as a key finding, we establish that the codebook size for the DTPC with deterministic encoding scales as $2^{(n \log n) R}$, where $n$ and $R$ are the codeword length and coding rate, respectively. Although the general mathematical approach used in this paper to derive the bounds on the capacity is similar to those for Gaussian channels in [48], the explicit mathematical techniques used in the achievability (sphere packing, decoder, error analysis, etc.) and converse analysis are different. Our result for the DTPC is in contrast to the scaling of the codebook size for conventional transmission (i.e., $2^{nR}$ [49]) and randomized-encoder identification (RI) (i.e., $2^{nR}$ [32]). The enlarged codebook size of the identification problem compared to the transmission problem may have interesting implications for MC system design. For instance, it may help explain the extremely large identification capability of natural olfactory systems and guide the design of olfactory-inspired synthetic MC systems [42] by, e.g., determining the maximum number of identifiable molecule mixtures.

- Technical Novelty: To obtain the proposed lower bound, we exploit the existence of an appropriate sphere packing within a hyper cube where the distance between the centers of the spheres does not fall below a certain value. Unlike for the Gaussian channels [48], here the radius of the packed spheres diverges to infinity as the codeword length increases. For the proposed upper bound, we assert

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1 Besides MCs, the DTPC has also been used to model other types of communication such as optical communication systems with direct-detection receivers [43], [44], [45].
a stricter criterion for the codebook compared to that for the Gaussian channel [48], namely, we impose a symbol-wise distance for every pair of codewords. Consequently, this condition leads to more involved analysis and gives a larger upper bound.

D. Organization

The remainder of this paper is structured as follows. Section II provides background information and reviews previous results on the identification problem. In Section III, scenarios for application of DI in the context of MC are discussed and the required preliminaries regarding DI codes are established. Section IV provides the main contributions and results on the message identification capacity of the DTPC. Section V presents simulation results for the empirical type I and type II error rates. Finally, Section VI of the paper concludes with a summary and directions for future research. We use the notations provided in Appendix A.

II. BACKGROUND ON IDENTIFICATION PROBLEM

In this section, we establish the background for our work and introduce the identification problem. Furthermore, we review the previous results on the DI capacity for different channels.

A. Identification Problem

In Shannon’s communication paradigm [49], a sender, Alice, encodes her message in a manner that will allow the receiver, Bob, to reliably recover the message. In other words, the receiver’s task is to determine which message was sent. In contrast, in the identification setting, the coding scheme is designed to accomplish a different objective [32]. The decoder’s main task is to determine whether a particular message was sent or not, while the transmitter does not know which message the decoder is interested in. Ahlswede and Dueck [32] introduced an RI scheme, in which the codewords are tailored according to their corresponding random source (distributions). It is well-known that such distributions do not increase the transmission capacity for Shannon’s message transmission task [50]. On the other hand, Ahlswede and Dueck [32] established that given local randomness at the encoder, reliable identification is accomplished with a codebook size that is double-exponential in the codeword length, i.e., \( \sim 2^{2nR} \) [32], where \( R \) is the coding rate. This behavior differs radically from the conventional message transmission setting, where the codebook size grows only exponentially, with the codeword length, i.e., \( \sim 2^{nR} \). Therefore, RI yields an exponential gain in the codebook size compared to the transmission problem. The construction of RI codes has been considered in previous works [51], [52]. For example, in [51], a binary code is constructed based on a three-layer concatenated constant-weight code. Nevertheless, realizing RI codes can be challenging in practice since they require the implementation of a random mapping function. Therefore, from a practical point of view, it is of interest to consider the case where the codewords are not selected based on a distribution but rather by means of a deterministic mapping\(^2\) from the message set to the channel input space.

B. Related Work on Deterministic Identification Capacity

DI may be preferred over RI in complexity-constrained applications of MC systems where the generation of random codewords in a controlled manner according to a specific distribution is challenging.\(^3\) In the deterministic coding setup for identification, for discrete memoryless channels (DMCs), the codebook size grows only exponentially in the codeword length, similar to the conventional transmission problem [32], [53], [55], [56], [61], [62]. However, the achievable identification rates are significantly higher compared to the transmission rates [55], [57]. Deterministic codes often have the advantage of simpler implementation and simulation [63], [64] and explicit construction [65]. In our recent works [48], [55], [57], we have considered DI for channels with an average power constraint, including DMCs and Gaussian channels with fast and slow fading, respectively. In the Gaussian case, we have shown that the codebook size scales as \( 2^{(n \log n)R} \) by deriving bounds on the DI capacity. DI for Gaussian channels is also studied in [66], [67]. Furthermore, DI for typical MC channel models, such as the DTPC with inter-symbol interference (ISI) and the Binomial channel is studied in [59], [68], [69], where the correct scale of the size of the codebook is proved to be \( 2^{(n \log n)R} \). Deterministic K-identification\(^4\) (DKI) for slow fading channels subject to an average power constraint and a codebook size of super-exponential scale, i.e., \( \sim 2^{(n \log n)R} \), is studied in [71], [72], where capacity bounds are derived. Also, a full characterization of the DKI capacity for the binary symmetric channel subject to a Hamming weight constraint is established in [73]. To the best of the authors’ knowledge, the DI capacity of the DTPC with emphasis on MC aspects has not been studied in the literature.\(^5\) Yet, Therefore, our main objective in this paper is to investigate the fundamental performance limits of DI over the DTPC.

III. SYSTEM MODEL, MC SCENARIOS FOR DI, AND PRELIMINARIES

In this section, we present the adopted system model, introduce MC scenarios for DI, and establish some preliminaries regarding DI coding.

\(^2\)In the literature, this approach is also referred to as identification without randomization [53] or deterministic identification (DI) [1], [48], [54], [55], [56], [57], [58], [59].

\(^3\)On the other hand, we note that the biological hardware of MC systems (e.g., reaction networks) features an inherent stochastic nature [60] which can potentially be exploited for realizing RI.

\(^4\)In the case of standard DI or RI problems [32], [57], the receiver is interested in identifying a single message. However, for the K-identification problem [70], the receiver is provided with an arbitrary set of K messages and it aims to identify whether or not the sent message belongs to such a set. The K-identification scenario may be interpreted as a generalization of the standard DI or RI problems in the sense that the target message at the receiver is substituted with a set of K messages where \( K \geq 1 \). That is, the K-identification problem with \( K = 1 \) corresponds to the standard DI problem considered in this paper.

\(^5\)The DI problem over the DTPC is so far only studied in [1], [74], where MC aspects were not investigated in detail.
A. System Model

We focus on an identification setup, where the decoder wishes to reliably determine whether or not a particular message was sent by the transmitter, while the transmitter does not know which message the decoder is interested in, see Figure 1. To achieve this objective, we establish coded communication between the transmitter and the receiver over $n$ channel uses of an MC channel by modulating the molecule concentration. In particular, we consider a stochastic release model, where for the $t$-th channel use, the transmitter releases molecules with rate $x_t$ (molecules/second) over a time interval of $T_{rls}$ seconds into the channel [11]. These molecules propagate through the channel via diffusion and/or advection, and may even be degraded in the channel via enzymatic reactions [8]. We assume a counting-type receiver which is able to count the number of received molecules. Examples include the transparent (perfect monitoring or passive) receiver, which counts the molecules at a given time within its sensing volume [15], the fully absorbing (perfect sink) receiver, which absorbs and counts the molecules hitting its surface within a given time interval [75], and the reactive (ligand-based) receiver which counts the number of molecules bound to the ligand proteins on its sensing surface at a given time [76].

Assuming that the release, propagation, and reception of individual molecules are statistically similar but independent of each other, the received signal follows Poisson statistics when the number of released molecules is large, i.e., $x_t T_{rls} \gg 1$ [8, Sec. IV]. Let $X \in \mathbb{R}_{\geq 0}$ and $Y \in \mathbb{N}_0$ denote random variables (RVs) modeling the rate of molecule release by the transmitter and the number of molecules observed at the receiver, respectively. For the DTPC, the channel output $Y$ is related to the channel input $X$ according to

$$
Y = \text{Pois}(\rho X + \lambda),
$$

where $\rho X$ is the mean number of observed molecules due to their release by the transmitter, $\rho = p_{ch} T_{rls}$, and $p_{ch} \in (0, 1]$ denotes the probability that a given molecule released by the transmitter is observed at the receiver. The value of $p_{ch}$ depends on the propagation environment (e.g., diffusion, advection, and reaction processes) and the reception mechanism (e.g., transparent, absorbing, or reactive receiver) as well as the distance between transmitter and receiver, see [8, Sec. III] for the characterization of $p_{ch}$ for various setups. Moreover, $\lambda \in \mathbb{R}_{>0}$ is the mean number of observed interfering molecules originating from external noise sources which employ the same type of molecule as the considered MC system.

The letter-wise conditional distribution of the DTPC output is given by

$$
W(y|x) = \frac{e^{-(\rho x + \lambda)}(\rho x + \lambda)^y}{y!},
$$

Standard transmission schemes employ strings of letters (symbols) of length $n$, referred to as codewords, that is, the encoding schemes use the channel in $n$ consecutive times to transmit one message. As a consequence, the receiver observes a string of length $n$, referred to as output vector (received signal). We assume that different channel uses are orthogonal. This assumption is justified for different MC scenarios in Section III-B. Therefore, for $n$ channel uses, the transition probability law reads

$$
W^n(y|x) = \prod_{t=1}^{n} W(y_t|x_t) = \prod_{t=1}^{n} \frac{e^{-(\rho x_t + \lambda)}(\rho x_t + \lambda)^{y_t}}{y_t!},
$$

where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ denote the transmit codeword and the received signal, respectively. The codewords are subject to peak and average molecule release rate constraints as follows $0 \leq x_t \leq P_{\text{max}}$ and $n^{-1} \sum_{t=1}^{n} x_t \leq P_{\text{avg}}$, respectively. In general, $P_{\text{max}} > 0$ and $P_{\text{avg}} > 0$ constrain the rate of molecule release per channel use and over the entire $n$ channel uses in each codeword, respectively. We note that while the average power constraint for the Gaussian channel is a non-linear (square) function of the symbols (signifying the signal energy), here for the DTPC, it is a linear function (signifying the number of released molecules) [11].

B. Spatial vs. Temporal Channel Uses

The coded communication considered in this paper requires $n$ independent uses of the MC channel; however, how the MC channel is accessed for each channel use may depend on the application of interest. In the following, we introduce two application scenarios, which employ spatial and temporal channel uses, respectively.

Spatial Channel Use: In olfactory-based communications, signalling chemicals are, e.g., odorants and pheromones. Odorants form a large class of chemical substances with a high diversity of chemical features that share a certain degree of volatility and both polar and nonpolar properties. They can be as diverse as being esters, alcohols, thiols, terpenoids, and aromatic substances to name a few [41]. Pheromones are chemical...
An immediate memory and proper measures have to be taken to ensure the dispersive nature of the diffusive MC channel, the channel has to be equipped with memory for generation and processing of 

Temporal Channel Use: For spatial channel uses, the complexity of transmitter and receiver may be high as they have to be able to generate and detect different types of molecules, respectively. To avoid this complexity, one may employ only one type of molecule but of a mixture of various molecule types [47]. Therefore, olfactory-based communication using odorants and pheromones is also referred to as “molecule mixture” communication [42]. The Ahlswede-Dueck identification problem applies to natural olfaction since each molecular mixture conveys a particular message that the receiver may be interested in. Thereby, the molecule mixtures may be interpreted as codewords since the structural composition of these mixtures enables their reliable identification by the olfactory systems even at very low concentrations [41]. Motivated by this, we consider a communication scenario, where the transmitter releases a mixture of different types of molecules to convey a message to the receiver, see Figure 2. The receiver is equipped with a dedicated type of receptor for each type of molecule, which ensures the orthogonality of the channel uses. The receiver’s task is to determine whether or not a desired message (molecular mixture) has been sent by the transmitter.

7In order to have identical channel statistics across different spatial channel uses, the release, propagation, and reception mechanisms for different types of molecules should be identical. For example, the different adopted types of molecules should have the same diffusion coefficients. The study of the identification problem for an MC system which employs different types of molecules that have non-identical diffusion coefficients (i.e., different channel uses follow different channel statistics) constitutes an interesting topic for future research.

C. DI Coding for the DTPC

The definition of a DI code for the DTPC is given below.

Definition 1 (Poisson DI Code): An \((L(n, R), n, \lambda_1, \lambda_2)\) DI code for a DTPC \(W\) under average and peak molecule release rate constraints of \(P_{\text{ave}}\) and \(P_{\text{max}}\), respectively, and for integer \(L(n, R)\), where \(n\) and \(R\) are the codeword length and coding rate, respectively, 8 is defined as a system \((U, D)\) which consists of a codebook \(U = \{u_i\}_{i \in [L]} \subset \mathbb{R}^n\), such that

\[
0 \leq u_{i,t} \leq P_{\text{max}} \quad \text{and} \quad \frac{1}{n} \sum_{i=1}^{n} u_{i,t} \leq P_{\text{ave}},
\]

\(\forall i \in [L], \forall t \in [n]\), and a collection of decoding regions \(D = \{D_i\}_{i \in [L]}\) with \(\bigcup_{i=1}^{L(n,R)} D_i \subset \mathbb{N}^n_0\). Given a message \(i \in [L]\), the encoder transmits \(u_i\), see Figure 1, the decoder’s aim is to answer the following question: Was a desired message \(j\) sent or not? There are two types of errors that may occur, which are

\[
\text{Type 1 Error:} \quad P_{\text{type\_1}} = \mathbb{P}(i \neq j|U = i) = \mathbb{P}(\text{Wrongly identified message})\]

\[
\text{Type 2 Error:} \quad P_{\text{type\_2}} = \mathbb{P}(i = j|U \neq i) = \mathbb{P}(\text{False positive})\]

The eligible (non-trivial) pair of average and peak constraints fulfill \(0 < P_{\text{ave}} \leq P_{\text{max}}\): See Section IV-B for further details.
occur: Rejection of the true message or acceptance of a false message. These errors are referred to as type I and type II errors, respectively.

The corresponding error probabilities of the identification code \((U, \mathcal{D})\) are given by

\[
P_{e,1}(i) = 1 - \sum_{y \in \mathcal{D}_i} W^n(y | u_i) \text{ (Miss - DI)},
\]

and satisfy the following bounds

\[
P_{e,1}(i) \leq \lambda_1 \quad \text{and} \quad P_{e,2}(i, j) \leq \lambda_2, \quad \forall i, j \in \mathcal{I} \setminus \{L\} \quad \text{and every} \quad \lambda_1, \lambda_2 > 0.
\]

A rate \(R > 0\) is called achievable if for every \(\lambda_1, \lambda_2 > 0\) and sufficiently large \(n\), there exists an \((L(n, R), n, \lambda_1, \lambda_2)\) DI code. The operational DI capacity of the DTPC is defined as the supremum of all achievable rates, and is denoted by \(C_{DI}(W, L)\).

IV. DI CAPACITY OF THE DTPC

In this section, we first present our main results, i.e., lower and upper bounds on the achievable identification rates for the DTPC. Subsequently, we provide the detailed proofs of these bounds.

A. Main Results

The DI capacity theorem for the DTPC is stated below.

**Theorem 1:** The DI capacity of the DTPC \(W\) subject to average and peak molecule release rate constraints of the form \(n^{-1}\sum_{t=1}^{n} u_{i,t} \leq P_{ave}\) and \(0 \leq u_{i,t} \leq P_{max}\), respectively, in the super-exponential scale, i.e., \(L(n, R) = 2^{(n \log n)R}\), is bounded by

\[
\frac{1}{4} \leq C_{DI}(W, L) \leq \frac{3}{2}.
\]

**Proof:** The proof of Theorem 1 consists of two parts, namely the achievability and the converse proofs, which are provided in Sections IV-B and IV-C, respectively.

In the following, we highlight some insights obtained from Theorem 1 and its proof.

**Scale:** Theorem 1 shows a different behavior compared to the traditional scaling of the codebook size with respect to codeword length \(n\). The bounds given in Theorem 1 are valid in the super-exponential scale of \(L = 2^{(n \log n)R}\) which is in between the conventional exponential and double exponential codebook sizes (see Figure 3). In other words, Theorem 1 reveals that for the capacity to assume informative non-zero finite values, the coding rate should be defined in the following scale as a function of codebook size and codeword length:

\[
R = \frac{\log L}{n \log n}.
\]

The capacity values in the standard codebook sizes, i.e., exponential and double exponential, are infinite (\(\lim_{n \to \infty} \frac{\log L}{n} = \infty\)) and zero (\(\lim_{n \to \infty} \frac{\log L}{n} = 0\)), respectively [55, see Rem. 1].

**Budget for Molecule Release:** The proposed capacity bounds in the super-exponential scale are independent of the values of \(P_{ave}\) and \(P_{max}\) as long as a the codeword length \(n\) grows sufficiently large, i.e., \(n \to \infty\). However, for finite \(n\), the codebook size is indeed a function of \(P_{ave}\). This can be readily seen from the achievability proof, where the codebook size in its raw form (see (20)) before division by the dominant term reads

\[
L(n, R) = 2^{(n \log n)R + n(\log \frac{P_{ave}}{\sqrt{n}}) + o(n)},
\]

where \(P_{ave} > 0\) and \(a > 0\) is a parameter of the codebook construction, cf. (12). In other words, the codebook size increases as \(P_{ave}\) increases; however, since \(P_{ave}\) appears in a term that is exponential in \(n\), i.e., \(\sim 2^{n(\log \frac{P_{ave}}{\sqrt{n}})}\), the influence of \(P_{ave}\) becomes negligible compared to the dominant super-exponential term, i.e., \(2^{(n \log n)R}\) as \(n \to \infty\). While the proof of Theorem 1 mainly concerns the asymptotic regime of \(n \to \infty\), we are still able to get some insight for finite \(n\), too. For instance, the error constraints in (7) can be met by the proposed achievable scheme even for finite \(n\) if \(P_{ave}\) is sufficiently large and \(n = \Omega(A^4)\), cf. (28), (37), and (38). A comprehensive study of the achievable DI rates for finite \(n\) constitutes an interesting research topic for future work, but is beyond the scope of this paper.

**Adopted Decoder:** For the achievability proof, we adopt a decoder that upon observing an output sequence \(y\), declares that the message \(j\) was sent if the following condition is met

\[
\left\| y - \mathbb{E}(y | u_j) \right\|^2 - \left\| y \right\|_1 \leq n \delta_n,
\]

where \(u_j = [u_{j,1}, \ldots, u_{j,n}]\) is the codeword associated with message \(j\) and \(\delta_n\) is a decoding threshold. In contrast to

\[9\]
the popular distance decoder used for the Gaussian channels [48] that includes only the distance term \( \| y - \mathbb{E}(Y|u_j) \| \), the proposed decoder in (11) comprises the additional correction term \( \| y \|_1 \). This choice stems from the fact that the noise in the DTPC is signal dependent [8]. Therefore, the variance of \( \| y - \mathbb{E}(Y|u_j) \| \) depend on the adopted codeword \( u_j \) which implies that unlike the Gaussian channel, here the radius of the decoding region is not constant for all the codewords. To account for this fact, we include a correction term of \( \| y \|_1 \).

**B. Achievability**

Consider DTPC \( \mathcal{W} \). We show achievability of (8) using a packing of hyper spheres and a distance decoder. We pack hyper spheres with radius \( r \sim n^{-\frac{1}{3}} \) inside a larger hyper cube. While the radius of the spheres in a similar proof for Gaussian channels vanishes, as \( n \) increases [48], the radius here diverges to infinity. Yet, we can obtain a positive rate while packing a super-exponential number of spheres satisfying the input and error constraints in (4)-(7). A DI code for the DTPC \( \mathcal{W} \) is constructed as follows.

**Codebook Construction:** In the following, we restrict ourselves to codewords that meet the condition \( 0 \leq u_{i,t} \leq P_{\text{ave}} \), \( \forall i \in \{[L]\}, \forall t \in \{[n]\} \), which ensures that both constraints in (4) are met for \( P_{\text{ave}} > P_{\text{max}} \) and \( P_{\text{ave}} \leq P_{\text{max}} \):

1. \( P_{\text{ave}} > P_{\text{max}} \): Then, the condition \( 0 \leq u_{i,t} \leq P_{\text{max}}, \forall i \in \{[L]\}, \forall t \in \{[n]\} \), gives \( n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{ave}} \). Thus, the average constraint trivially holds and we exclude this scenario from the analysis.

2. \( P_{\text{ave}} \leq P_{\text{max}} \): Then, the condition \( 0 \leq u_{i,t} \leq P_{\text{ave}}, \forall i \in \{[L]\}, \forall t \in \{[n]\} \), gives both \( 0 \leq u_{i,t} \leq P_{\text{max}} \) and \( n^{-1} \sum_{t=1}^{n} u_{i,t} \leq P_{\text{ave}} \).

Hence, in the following, we restrict our considerations to a hyper cube with edge length \( P_{\text{ave}} \). We use a packing arrangement of non-overlapping hyper spheres of radius \( r_{0} = \sqrt{n} \epsilon_{n} \) in a hyper cube with edge length \( P_{\text{ave}} \), where

\[
\epsilon_{n} = \frac{a}{n^{\frac{1}{2}}(1-b)},
\]

and \( a > 0 \) is a non-vanishing fixed constant and \( 0 < b < 1 \) is an arbitrarily small constant.\(^{10}\)

Let \( \mathcal{S} \) denote a sphere packing, i.e., an arrangement of \( L \) non-overlapping spheres \( S_{u_i}(n, r_{0}) \), \( i \in \{[L]\} \), that are packed inside the larger cube \( Q_{0}(n, P_{\text{ave}}) \) with an edge length \( P_{\text{ave}} \), see Figure 4. As opposed to standard sphere packing coding techniques [86], the spheres are not necessarily entirely contained within the cube. That is, we only require that the centers of the spheres are inside \( Q_{0}(n, P_{\text{ave}}) \) and are disjoint from each other and have a non-empty intersection with \( Q_{0}(n, P_{\text{ave}}) \). The packing density \( \Delta_{n}(\mathcal{S}) \) is defined as the ratio of the saturated packing volume to the cube volume, i.e.,

\[
\Delta_{n}(\mathcal{S}) \triangleq \frac{\text{Vol}
(\bigcup_{i=1}^{L} S_{u_i}(n, r_{0}))}{\text{Vol}(Q_{0}(n, P_{\text{ave}}))}.
\]

Sphere packing \( \mathcal{S} \) is called saturated if no spheres can be added to the arrangement without overlap. Specially, we use

\[\text{Vol}(S_{u_i}(n, r_{0})) = \frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2} + 1\right)} \cdot r_{0}^{n}.\]

Hence, if the radius of the small spheres is doubled, the volume of \( \bigcup_{i=1}^{L} S_{u_i}(n, \sqrt{n} \epsilon_{n}) \) is increased by \( 2^{n} \). Since the spheres with radius \( 2r_{0} \) cover \( Q_{0}(n, P_{\text{ave}}) \), it follows that the original \( r_{0}-\text{radius packing} \) has a density of at least \( 2^{-n} \). We assign a codeword to the center \( u_i \) of each small sphere. The codewords satisfy the input constraint as \( 0 \leq u_{i,t} \leq P_{\text{ave}}, \forall i \in \{[n]\}, \forall t \in \{[L]\} \), which is equivalent to \( \| u_i \|_{\infty} \leq P_{\text{ave}} \).

\[\text{Since the volume of each sphere is equal to Vol}(S_{u_i}(n, r_{0}))\]

\[\text{We note that the proposed proof of the lower bound in (15) is non-constructive in the sense that, while the existence of the respective saturated packing is proved, no systematic construction method is provided.}\]
and spheres’ centers lie inside the cube, the total number of spheres is bounded from below by

\[
L = \frac{\text{Vol} \left( \bigcup_{i=1}^{L} S_{u_i} (n, r_0) \right)}{\text{Vol} \left( S_{u_i} (n, r_0) \right)} \geq \Delta_n \left( \frac{L}{P_0} \right) \cdot \text{Vol} \left( S_{u_i} (n, P_{\text{ave}}) \right) \geq 2^{-n} \cdot \frac{P_{\text{ave}}}{\text{Vol} \left( S_{u_i} (n, r_0) \right)},
\]

where the first inequality holds by (13) and the second inequality holds by (15). The above bound can be further simplified as follows

\[
\log L \geq \log \left( \frac{P_{\text{ave}}}{\text{Vol} \left( S_{u_i} (n, r_0) \right)} \right) - n \geq n \log \left( \frac{P_{\text{ave}}}{\sqrt{\pi} r_0} \right)
+ \log \left( \Gamma \left( \frac{n}{2} + 1 \right) \right) - n \geq n \log P_{\text{ave}} - n \log r_0
+ \log \left( \Gamma \left( \frac{n}{2} \right) \right) - n = - \frac{n}{2} \log e + o \left( \frac{n}{2} \right) - n,
\]

where (a) exploits (16) and (b) follows by exploiting Stirling’s approximation, i.e., \( \log n! = n \log n - n \log e + o(n) \) [88, p. 52] with substitution of \( n \left\lfloor \frac{n}{2} \right\rfloor \in \mathbb{Z} \), and since

\[
\Gamma \left( \frac{n}{2} + 1 \right) \geq \frac{n}{2} \Gamma \left( \frac{n}{2} \right) \geq \left\lceil \frac{n}{2} \right\rceil \Gamma \left( \frac{n}{2} \right) \triangleq \left\lceil \frac{n}{2} \right\rceil !,
\]

where (a) holds by the recurrence of the Gamma function [89] for real \( \frac{n}{2} \) and (b) follows from \( \left\lceil \frac{n}{2} \right\rceil \leq \frac{n}{2} \leq \left\lfloor \frac{n}{2} \right\rfloor \) and the monotonicity of the Gamma function [89] for \( \left\lceil \frac{n}{2} \right\rceil \geq 1.46 \) which is implied for \( n \geq 4 \). Now, for \( r_0 = \sqrt{\frac{\log e}{n}} = \sqrt{\frac{\log n}{n}} \), we obtain

\[
\log P_{\text{ave}} \geq 2^{-n} \log e + n \log \left( \Gamma \left( \frac{n}{2} \right) \right) - n = - \frac{n}{2} \log e + o \left( \frac{n}{2} \right) - n
+ \log \left( \Gamma \left( \frac{n}{2} \right) \right) - n = - \frac{n}{2} \log e + o \left( \frac{n}{2} \right) - n
\]

where (a) follows by \( \left\lceil \frac{n}{2} \right\rceil > \frac{n}{2} - 1 \) and (b) holds since \( \log \left( \frac{n}{2} \right) \geq \log \left( \frac{n}{2} - 1 \right) - 1 \) for \( \frac{n}{2} \geq 2 \) and \( \left\lceil \frac{n}{2} \right\rceil \leq \frac{n}{2} \) for integer \( n \). Observe that the dominant term in (20) is of order \( n \log n \). Hence, for obtaining a finite value (see Appendix B for further details) for the lower bound of the rate, \( R \), (20) induces the scaling law of \( L \) to be \( 2^{(n \log n) R} \). Therefore, we obtain

\[
R \geq \frac{1}{n \log n} \left( \frac{n}{2} - 1 \right) \log n + n \log n \log \left( \frac{P_{\text{ave}}}{\sqrt{\pi} r_0} \right) + O(n),
\]

which tends to \( \frac{1}{4} \) when \( n \to \infty \) and \( b \to 0 \).

Encoding: Given message \( i \in [L] \), transmit \( x = u_i \).

Decoding: Let \( \delta_n = c^2 r^2 e \frac{2}{\log (b^{1/2} - 1)} \), where \( 0 < b < 1 \) is an arbitrarily small constant and \( 0 < c < 2 \) is a constant. To identify whether message \( j \in \mathcal{M} \) was sent, the decoder checks whether the channel output \( y \) belongs to the following decoding set:

\[
D_j = \{ y \in \mathbb{Y}^n : D (y; u_j) \leq \delta_n \},
\]

where \( D (y; u_j) = n^{-1} \sum_{t=1}^n (y_t - \left( \rho u_{j,t} + \lambda \right))^2 - y_t \), is referred to as the decoding metric evaluated for observation vector \( y \) and codeword \( u_j \).

Error Analysis: Consider the type I errors, i.e., the transmitter sends \( u_i \), yet \( Y \notin D_i \). For every \( i \in [L] \), the type I error probability is bounded by

\[
P_{e,1} (i) = \Pr \left( | \{ Y| u_i \} > \delta_n \right) \leq \frac{1}{\delta_n^2},
\]

where the condition means that \( x = u_i \) was sent. In order to bound \( P_{e,1} (i) \), we apply Chebychev’s inequality, namely

\[
\Pr \left( | \{ Y| u_i \} - E [ D (Y; u_i) | u_i ] \right) > \delta_n u_i \right) \leq \frac{\text{var} \{ D (Y; u_i) | u_i \}}{\delta_n^2}.
\]

First, we derive the expectation of the decoding metric as follows

\[
E \{ D (Y; u_i) | u_i \} = \frac{1}{n} \sum_{t=1}^n \left[ \text{var} \{ Y_t | u_i \} - E [ Y_t | u_i ] \right] = 0.
\]

Now, since the channel is memoryless, we can compute the variance as follows

\[
\text{var} \{ D (Y; u_i) | u_i \}
= \frac{1}{n} \sum_{t=1}^n \left[ \text{var} \{ Y_t - (\rho u_{i,t} + \lambda) \} | u_i \right] - E [ \text{var} \{ Y_t | u_i \} ]
\]

where (a) holds since \( \text{var} \{ Y_t | u_i,t \} = \rho u_{i,t} + \lambda \) and (b) follows since \( u_{i,t} \geq 0 \), \( \forall t \in [n] \), \( \forall i \in [L] \). Next, to establish an upper bound on the first sum in (26), we present a useful lemma.

Lemma 1: Let \( Z \sim \text{Pois} (\lambda Z) \) be a Poisson RV with mean \( \lambda Z \). The following inequality holds

\[
E \left( Z - \lambda Z \right)^4 \leq 7 (\lambda^2 + \lambda^3) Z^2 + \lambda Z.
\]

Proof: The proof is provided in Appendix C.

Using the above lemma, we bound the variance of the decoding metric as follows:

\[
\text{var} \{ D (Y; u_i) | u_i \}
\leq \text{var} \{ D (Y; u_i) | u_i \}
\leq \frac{1}{n} \left[ \text{var} \{ Y_t - (\rho u_{i,t} + \lambda) \} | u_i \right] - \frac{\text{var} \{ Y_t | u_i \} }{\delta_n^2}
\]

where (a) follows since \( E \{ D (Y; u_i) | u_i \} = 0 \), (b) follows since \( \text{var} \{ Z \} \leq E \{ Z^2 \} \), and (c) holds by letting \( Z = (Y_t - (\rho u_{i,t} + \lambda))^2 \) and exploiting an upper bound on the fourth non-central
moment of a Poisson random variable (see Appendix C). Therefore, exploiting (24), (25) and (27), we can bound the type I error probability in (23) as follows

\[
P_{e,1}(i) = \Pr(D(Y; u_i)) > \delta_n | u_i) \leq \frac{7((\rho \lambda)^3 + (\rho \lambda + \lambda)^2 + \rho \lambda^2)}{\sqrt{\lambda}} \leq \lambda_1,
\]

(28)

for sufficiently large \( n \) and arbitrarily small \( \lambda_1 > 0 \).

Next, we address type II errors, i.e., when the transmitter sent \( u_i \). Then, for every \( i, j \in [L] \), where \( i \neq j \), the type II error probability is given by

\[
P_{e,2}(i, j) = \Pr(|D(Y; u_j)| \leq \delta_n | u_i),
\]

(29)

where \( D(Y; u_j) = \beta - \alpha, \) with

\[
\beta \triangleq \frac{1}{n} \sum_{t=1}^{n} (Y_t - (\rho u_{i,t} + \lambda) + \rho(u_{i,t} - u_{j,t}))^2
\]

(30)

and \( \alpha \triangleq \frac{1}{n} \sum_{t=1}^{n} Y_t \). Observe that term \( \beta \) can be expressed as \( \beta = \beta_1 + \beta_2 \), where

\[
\beta_1 \triangleq \frac{1}{n} \left[ \|Y - (\rho u_i + \lambda_0)\|_2^2 + \|\rho(u_i - u_j)\|_2^2 \right]
\]

\[
\beta_2 \triangleq \frac{2\rho}{n} \sum_{t=1}^{n} (u_{i,t} - u_{j,t})(Y_{i,t} - (\rho u_{i,t} + \lambda))
\]

(31)

Then, define the following events

\[
\mathcal{H}_i = \{ |\beta - \alpha| \leq \delta_n | u_i \}, \text{ and } \mathcal{E}_i = \{|\beta_2| > \delta_n | u_i \}
\]

(32)

Exploiting the reverse triangle inequality, i.e., \(|\beta - \alpha| \leq |\beta - \alpha|\), we obtain the following upper bound on the type II error probability

\[
P_{e,2}(i, j) = \Pr(\mathcal{H}_i) = \Pr(|\beta - \alpha| \leq \delta_n | u_i)
\]

(33)

where \( a \) follows since \( \alpha \geq 0 \) and \( \beta \geq 0 \). Now, applying the law of total probability to event \( \mathcal{B} = (\beta - \alpha \leq \delta_n | u_i) \) over \( \mathcal{E}_0 \) and its complement \( \mathcal{E}_0^c \), we obtain

\[
P_{e,2}(i, j) \leq \Pr(\mathcal{B} \cap \mathcal{E}_0) + \Pr(\mathcal{B} \cap \mathcal{E}_0^c)
\]

(34)

where inequality \( a \) follows from \( \mathcal{B} \cap \mathcal{E}_0 \subset \mathcal{E}_0 \) and inequality \( b \) follows from \( \Pr(\mathcal{B} \cap \mathcal{E}_0^c) \leq \Pr(\mathcal{E}_1) \), which is proved in the following. Observe,

\[
\Pr(\mathcal{B} \cap \mathcal{E}_0) = \Pr\left\{ |\beta_1 - \alpha| \leq \delta_n \right\} \cap \{|\beta_2| \leq \delta_n \}
\]

(35)

where inequality \( a \) holds since \( \delta_n - \beta_2 \leq 2\delta_n \) conditioned on \( |\beta_2| \leq \delta_n \). We now proceed with bounding \( \Pr(\mathcal{E}_0) \). By Chebyshev’s inequality, the probability of this event can be bounded as follows

\[
\Pr(\mathcal{E}_0) \leq \frac{\text{var} \sum_{t=1}^{n} (u_{i,t} - u_{j,t})(Y_{i,t} - (\rho u_{i,t} + \lambda)) | u_i}{\sqrt{n} \delta_n^2 / (4\lambda)}
\]

\[
= \frac{4\lambda^2 \sum_{t=1}^{n} (u_{i,t} - u_{j,t})^2 \cdot \text{var} \{Y_t | u_i\}}{\sqrt{n} \delta_n^2 / (4\lambda)}
\]

\[
= \frac{4\lambda^2 \sum_{t=1}^{n} (u_{i,t} - u_{j,t})^2 \cdot (\rho u_{i,t} + \lambda)}{\sqrt{n} \delta_n^2 / (4\lambda)}
\]

\[
\leq \frac{4\lambda^2 (\rho \lambda + \lambda) \|u_i - u_j\|^2}{\sqrt{n} \delta_n^2 / (4\lambda)}.
\]

(36)

Observe that

\[
\|u_i - u_j\|^2 \leq \left( \|u_i\| + \|u_j\| \right)^2 \leq \left( \sqrt{n} \right) |u_i| \leq \left( \sqrt{n} \right) |u_j| \leq \left( \sqrt{n} \right) \lambda
\]

(37)

for sufficiently large \( n \), where \( \lambda_0 > 0 \) is an arbitrarily small constant.

We now proceed with bounding \( \Pr(\mathcal{E}_1) \) as follows. Based on the codebook construction, each codeword is surrounded by a sphere of radius \( \sqrt{n} \), that is \( \|u_i - u_j\|^2 \geq 4n \varepsilon_n \). Thus, we can establish the following upper bound for event \( \mathcal{E}_1 \):

\[
\Pr(\mathcal{E}_1) = \Pr \left\{ \frac{1}{\sqrt{n}} \left[ \|Y - (\rho u_i + \lambda_0)\|_2^2 + \|\rho(u_i - u_j)\|_2^2 \right] - \sum_{t=1}^{n} Y_t \leq 2\delta_n | u_i \right\}
\]

(38)

where \( a \) follows since \( \alpha \geq 0 \) and \( \beta \geq 0 \). Now, applying the law of total probability to event \( \mathcal{B} = (\beta - \alpha \leq \delta_n | u_i) \) over \( \mathcal{E}_0 \) and its complement \( \mathcal{E}_0^c \), we obtain

\[
\Pr(\mathcal{B} \cap \mathcal{E}_0) \leq \Pr(\mathcal{E}_0) + \Pr(\mathcal{B} \cap \mathcal{E}_0^c)
\]

(39)

where inequality \( a \) follows from \( \mathcal{B} \cap \mathcal{E}_0 \subset \mathcal{E}_0 \) and inequality \( b \) follows from \( \Pr(\mathcal{B} \cap \mathcal{E}_0^c) \leq \Pr(\mathcal{E}_1) \), which is proved in the following. Observe,

\[
\Pr(\mathcal{B} \cap \mathcal{E}_0) \leq \Pr\left\{ |\beta_1 - \alpha| \leq \delta_n \right\} \cap \{|\beta_2| \leq \delta_n \}
\]

(40)

where inequality \( a \) holds since \( \delta_n - \beta_2 \leq 2\delta_n \) conditioned on \( |\beta_2| \leq \delta_n \).
C. Converse Proof

We show that the capacity is bounded by $C_{DI}(W, L) \leq \frac{3}{2}$. The derivation of this upper bound for the achievable rate of the DTFC is more involved than that of the Gaussian case [48]. In our previous work on Gaussian channels with fading [48], the converse proof was based on establishing a minimum distance between each pair of codewords. Here, we use the stronger requirement that the ratio of the letters for every pair of codewords is different from 1 for at least one index. We begin with the following lemma on the ratio of the letters of pair of codewords.

Lemma 2: Suppose that $R$ is an achievable rate for the DTFC. Consider a sequence of $(L(n, R), n, \lambda(n), \lambda'(n))$ codes $(U(n), D(n))$ such that $\lambda(n)$ and $\lambda'(n)$ tend to zero as $n \to \infty$. Then, given a sufficiently large $n$, the codebook $U(n)$ satisfies the following property. For every pair of codewords, $u_{i_1}$ and $u_{i_2}$, there exists at least one letter $t \in [n]$ such that

$$1 - \frac{\rho u_{i_2,t} + \lambda}{\rho u_{i_1,t} + \lambda} > \epsilon'_t \tag{39}$$

for all $i_1, i_2 \in [L]$, with $i_1 \neq i_2$ and $\epsilon'_t = P_{ave}/n^{1+b}$, where $b > 0$ is an arbitrarily small constant.

Proof: The proof is given in Appendix D.

Next, we use Lemma 2 to prove the upper bound on the DI capacity. Observe that since $v_{i,t} = \rho u_{i,t} + \lambda > \lambda$, Lemma 2 implies $\rho |v_{i_1,t} - v_{i_2,t}| = |v_{i_1,t} - v_{i_2,t}| \geq \epsilon'_t v_{i_1,t} > \lambda e'_t$, where (a) follows by (39) and (b) holds since $v_{i,t} = \rho u_{i,t} + \lambda > \lambda$. Now, since $\|u_{i_1} - u_{i_2}\| \geq |u_{i_1,t} - u_{i_2,t}|$, we deduce that the distance between every pair of codewords satisfies $\|u_{i_1} - u_{i_2}\| \geq \lambda e'_t/\rho$. Thus, we can define an arrangement of non-overlapping spheres $S(n, \lambda e'_t/2\rho)$, i.e., spheres of radius $\lambda e'_t/2\rho$ that are centered at the codewords $u_i$. Since all codewords belong to a hyper cube $Q_0(n, P_{ave})$ with edge length $P_{ave}$, it follows that the number of packed small spheres, i.e., the number of codewords $L$, is bounded by

$$L \leq \frac{\text{Vol}(U_L)}{\text{Vol}(S_u(n, n))} \geq \Delta_n(\mathcal{V}) \cdot \text{Vol}(Q(n, P_{ave})) \leq 2^{-0.599n} \left( \frac{P_{ave}}{\text{Vol}(S_u(n, n))} \right), \tag{40}$$

where the last inequality follows from inequality (15).

Therefore,

$$\log L \leq \log \left( \frac{P_{ave}}{\text{Vol}(S_u(n, n))} \right) - 0.599n = n \log P_{ave} - n \log r_0 - n \log \sqrt{\pi} + \frac{1}{2} n \log n + \frac{n}{2} - n \log e + o(n) - 0.599n,$$

where the dominant term is again of order $n \log n$. Hence, for obtaining a finite value for the upper bound of the rate, $R$, (41) induces the scaling law of $L$ to be $2^{(n \log n)/R}$. Hence, by setting $L(n, R) = 2^{(n \log n)/R}$ and $r_0 = \lambda e'_t/2\rho = \lambda P_{ave}/2\rho n^{1+b}$, we obtain

$$R \leq \frac{1}{n \log n} \left[ \log P_{ave} - n \log r_0 - n \log \sqrt{\pi} + \frac{1}{2} n \log \frac{n}{2} - \frac{n}{2} \log e + o(n) - 0.599n \right] = \frac{1}{n \log n} \left( \frac{1}{2} + (1 + b) \right) \times n \log n - n \left( \log \frac{\lambda e'_t}{2\rho} + 1.0599 \right) + o(n), \tag{42}$$

which tends to $\frac{3}{2}$ as $n \to \infty$ and $b \to 0$. This completes the proof of Theorem 1.

V. SIMULATION RESULTS

Before reporting on our numerical simulations, we first emphasize that the main result of this paper is the characterization of fundamental performance bounds in terms of the DI capacity for the DTFC (cf. Theorem 1), which by definition holds for asymptotically large codewords, i.e., as $n \to \infty$. Explict code construction for the DTFC is not the main focus of this paper. Hence, the purpose of this section is not the evaluation/verification of our achievability proof in Section III.[12] Nonetheless, we are interested in studying whether our key finding, i.e., the possibility of reliable identification for super-exponentially large codebook sizes, also holds for a heuristically-designed (structure-less) finite-length code.

A. Heuristic Codebook Construction

For the simulation results reported in this section, we adopt a heuristic codebook construction, which is briefly sketched in the following. At first, codewords are generated uniformly, that is, the value of each symbol is chosen uniformly distributed between 0 and $P_{ave}$, Next, in order to realize the minimum distance property of the codebook, once a codeword is created, before adding it to the codebook, it is verified whether it has at least a minimum Euclidean distance of $2\sqrt{\pi e}$ from all previously generated codewords or not. In the course of codeword generation, if a codeword violates the minimum distance property, it is discarded and a new codeword is generated and the procedure is repeated until the desired codebook size is obtained. To simulate the receiver’s task, the distance decoder in (11) is implemented and the empirical type I and type II error rates for finite codeword lengths are obtained via Monte Carlo simulation. We focus on a range of small codeword lengths, i.e., $19 \leq n \leq 28$, since the above simple look-up table code construction and full search decoding are not scalable for large $n$. Moreover, since rates $R \geq \frac{1}{4}$ are achievable by the proposed scheme only as $n \to \infty$, we choose a smaller rate, i.e., $R=0.1$, for codebook generation for finite $n$. However, we study a codebook with super-exponential size in $n$, i.e., $L = 2^{(n \log n)/R}$, which is the key element of Theorem 1. Without loss of generality, we assume that the transmitter sends message $i = 1$ and denote the empirical type I and type II error rates (average and maximum) by $\bar{e}_1(i)$ and $\bar{e}_2(i)$, respectively. The values of the parameters used in

\[12\] In fact, our achievability proof in Section IV-B shows only the existence of codes and does not provide any explicit construction of the codebook.
TABLE I
PARAMETERS OF THE SIMULATIONS

| Description                                      | Notation | Value                          |
|--------------------------------------------------|----------|--------------------------------|
| Peak and Average Molecule Release Rate Constraint| $P_{\text{max}}, P_{\text{ave}}$ | 1200,1000 molecules/s          |
| Prob. Molecules Reaching the Receiver, Release time| $p_{\text{ch}}, T_{\text{ch}}$ | 0.01, 1 s                      |
| # of Iterations, Expected Number of Interfering Molecules | $n_{\lambda}$ | $7 \times 10^2, 0.2$ |
| Codeword Length, Coding Rate                     | $n, R$   | [19 - 28], 0.1                 |
| Codebook Size, Codebook Parameters               | $L = 2^n (\log n) R, a, b, c$ | [268 - 11273], $10^6$, 0.99, $\frac{1}{3}$ |

Fig. 5. Impact of codeword length on the empirical type I and type II error rates. Larger lengths decrease the empirical rates.

the proposed simulation setup and codebook construction are summarized in Table I.

B. Results and Discussions

Figures 5(a) and 5(b) show respectively the empirical type I and type II error rates versus the codeword length. The results in Figures 5(a) and 5(b) show that fast-decaying error rates are attainable for the considered codebook which has a super-exponentially large size in codeword length $n$ even though the code construction is sub-optimal and $n$ is finite. This is an interesting observation given the fact that our theoretical results only prove that asymptotically as $n \rightarrow \infty$, reliable identification with super-exponentially large codebook size in $n$ is achievable. Furthermore, the simulation results in Figures 5(a) and 5(b) show the general trend of the empirical error rates as functions of the codeword length is well captured by the analytical upper bounds. However, the achieved error rates for the constructed code with $R = 0.1, 0.2$ decay faster than the theoretical upper bounds provided in (28), (37), and (38) evaluated for $b = 0.99$. We note that since our theoretical bounds are derived for $n \rightarrow \infty$, it is not contradictory if the slopes of the empirical error rates are slightly higher for the simulated curves at finite $n$.

VI. SUMMARY AND FUTURE DIRECTIONS

In this paper, we studied the identification problem for MC channels, which is relevant for applications involving event-triggered tasks, e.g., synchronization, olfactory communication, targeted drug delivery, and health condition monitoring. In particular, we considered MC systems with molecule counting receivers, modeled by the DTPC, and focused on deterministic encoders, i.e., DI. For this setting, we derived lower and upper bounds on the DI capacity of the DTPC subject to average and peak molecule release rate constraints for a codebook size of $L(n, R) = 2^n (\log n) R = n^R$. Our results revealed that the super-exponential scale of $n^R$ is the appropriate scale for the DI capacity of the DTPC, which was proved by finding a suitable sphere packing arrangement embedded in a hyper cube. We emphasize that this scale is sharply different from the ordinary scales in transmission and RI settings, where the codebook size grows exponentially and double exponentially, respectively.

The results presented in this paper can be extended in several directions, some of which are listed in the following as potential topics for future research:

- In our system model, we assume that for spatial channel uses, the diffusion coefficients of all signalling particles are identical. However, in practice, different types of molecules may have different diffusion coefficients which result in different arrival times at the receiver. Therefore, a possible topic for future research is to study decoders that account for the propagation delay for molecules with different speeds of propagation.
- Latency is an important metric for an identification setting and can originate from a number of different sources including the encoding, molecule release, molecule propagation, transmission, and decoding. One source of latency is the codeword length, which is relevant for temporal channel uses. Furthermore, there is a trade-off between latency and the type I and type II error probabilities. Therefore, a potential topic for future research is to study decoders that account for the propagation delay for molecules with different speeds of propagation.
Another interesting research topic is to investigate the behavior of the DI capacity in the sense of Fekete’s Lemma [90], that is, to verify whether the pessimistic (\( C = \lim_{n \to \infty} \frac{\log L(n,R)}{n \log n} \)) and optimistic (\( \overline{C} = \limsup_{n \to \infty} \frac{\log L(n,R)}{n \log n} \)) capacities [91] are equal or not.

We note that to fully characterize the asymptotic behavior of the decoding errors as a function of the codeword length for every value of the rate \( 0 < R < C \), knowledge of the corresponding channel reliability function (CRF) is required [92]. To the best of the authors’ knowledge, the CRF for DI has not been studied in the literature so far, neither for the Gaussian channel [57] nor the Poisson channel [11, 58, 74]. We note that even for the conventional message transmission problem, the characterization of the CRF is difficult, as the corresponding channel reliability function is not Turing computable [92].

Our main focus in this paper was the establishment of fundamental performance limits of the DI problem for the DTPC, where an explicit code construction was not considered. Hence, interesting directions for future research include the explicit construction of identification codes and the development of low-complexity encoding and decoding schemes for practical applications. The efficiency of these designs can be evaluated against the performance bounds derived in this paper.

**Appendix A**

**Notations**

We use the following notations throughout this paper: Calligraphic letters \( \mathcal{X}, \mathcal{Y}, \mathcal{Z}, \ldots \) are used for finite sets. Lower case letters \( x, y, z, \ldots \) stand for constants and values of random variables, and upper case letters \( X, Y, Z, \ldots \) stand for random variables. Lower case bold symbol \( x \) indicates a row vector of size \( n \), that is, \( x = (x_1, \ldots, x_n) \). Bold symbol \( 1_n \) indicates the all-one row vector of size \( n \). The distribution of a random variable \( X \) is specified by a probability mass function (PMF) \( p_X(x) \) over a finite set \( \mathcal{X} \). All logarithms and information quantities are for base 2. The set of consecutive natural numbers from 1 through \( n \) is denoted by \([1,n] \). The set of whole numbers is denoted by \( \mathbb{N} \). The gamma function for non-positive integer \( n \) is denoted by \( \Gamma(n) \) and is defined as \( \Gamma(n) = (x-1)! \), where \( (x-1)! \triangleq (x-1) \times (x-2) \times \cdots \times 1 \). We use the small O notation, \( f(n) = O(g(n)) \), to indicate that \( f(n) \) is dominated by \( g(n) \) asymptotically, that is, \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \). The big O notation, \( f(n) = \Theta(g(n)) \), is used to indicate that \( f(n) \) is bounded above by \( g(n) \) (up to constant factor) asymptotically, that is, \( \lim \sup_{n \to \infty} \frac{f(n)}{g(n)} < \infty \). We use the big Omega notation, \( f(n) = \Omega(g(n)) \), to indicate that \( f(n) \) is bounded below by \( g(n) \) asymptotically, that is, \( g(n) = O(f(n)) \). The \( \ell_2 \)-norm and \( \ell_\infty \)-norm are denoted by \( \|x\|_2 \) and \( \|x\|_\infty \), respectively. Furthermore, we denote the \( n \)-dimensional hyper sphere of radius \( r \) centered at \( x_0 \) with respect to the \( \ell_2 \)-norm by \( S_{x_0}(n,r) = \{ x \in \mathbb{R}^n : \|x - x_0\|_2 \leq r \} \). An \( n \)-dimensional hyper cube with center \((A/2, \ldots, A/2)\) and a corner at the origin, i.e., \( 0 = (0, \ldots, 0) \), whose edges have length \( A \) is denoted by \( Q_0(n,A) = \{ x \in \mathbb{R}^n : 0 \leq x_t \leq A, \forall t \in [\lceil n \rceil] \} \).

**Appendix B**

**Volume of a Hyper Sphere With Growing Radius**

To solidify the idea of packing spheres within a hyper cube, we reveal and explain a counter-intuitive phenomenon regarding the packing of hyper spheres with growing radius in the codeword length inside a hyper cube. We observe that despite the fact that the hyper sphere’s radius tends to infinity as the codeword length goes to infinity \( \sim n^{\frac{1}{2}} \) its volume tends to zero super-exponentially inverse, i.e., \( \sim n^{-\frac{2}{3}} \). This allows us to accommodate super-exponential number of such hyper spheres inside the hyper cube. The ratio of the spheres in our construction grows with \( n \), as \( \sim n^{\frac{1}{2}} \). It is well-known that the volume of an \( n \)-dimensional unit sphere (radius \( r_0 = 1 \)) tends to zero, as \( n \to \infty \) [86, Ch. 1, eq. (18)].

Nonetheless, we prove that the volume of a small hyper sphere still tends to zero for a radius of \( r_0 = n^c \), where \( 0 < c < \frac{1}{2} \). More precisely, observe that in order to derive the number of small spheres that can be packed inside the hyper cube, we calculate the log-ratio of the volumes as follows

\[
\log\left(\frac{\text{Vol}(Q_0(n,P_{ave}))}{\text{Vol}(S_{11}(n,r_0))}\right) = n \log P_{ave} - n \log r_0 - n \log \sqrt{n} + \frac{1}{2} n \log n - O(n) \\
\quad + n \log \left(\frac{P_{ave}}{\sqrt{n} \epsilon}\right) + O(n),
\]

(43)

where (a) follows by (16), and (b) holds by setting \( r_0 = n^c \) and the same argument as provided for the derivations given in (18) and (20). Now, since the dominant term in (43) involves \( n \log n \), we deduce that codebook size should be \( L(n,R) = 2^{(n \log n)R} \) thereby by (17) we obtain

\[
R \geq \frac{1}{n \log n} \cdot \log\left(\frac{\text{Vol}(Q_0(n,P_{ave}))}{\text{Vol}(S_{11}(n,r_0))}\right) \\
= \frac{1}{n \log n} \left(\left(\frac{1}{2} - c\right) n \log n + n \log P_{ave} + O(n)\right),
\]

which tends to \( \frac{1}{2} - c \) when \( n \to \infty \).

**Remark 2:** Observe that the behavior of the volume of a hyper cube \( Q_0(n,P_{ave}) \) with edge length \( P_{ave} \) in the asymptotic, i.e., for \( n \to \infty \) is given by

\[
\lim_{n \to \infty} \text{Vol}(Q_0(n,P_{ave})) = \lim_{n \to \infty} P_{ave}^n = \begin{cases} 0 & P_{ave} < 1, \\ 1 & P_{ave} = 1, \\ \infty & P_{ave} > 1. \end{cases}
\]

(44)
Therefore, even for the case that $P_{\text{ave}} < 1$, as (43) implies, the term containing $P_{\text{ave}}$ is negligible, that is, the volume $P_{\text{ave}}^n$ still is capable of incorporating super exponentially many small hyper spheres as long as the exponent of their radius remains strictly less than $1/2$.

APPENDIX C
MOMENT GENERATING FUNCTION
OF POISSON RANDOM VARIABLE

The moment-generating function (MGF) of a Poisson variable $Z \sim \text{Pois} (\lambda Z)$ is $G_Z (\alpha) = e^{\lambda Z (e^\alpha - 1)}$. Hence, for $X = Z - \lambda Z$, the MGF is given by $G_X (\alpha) = e^{\alpha Z (e^\alpha - 1)}$. Since the fourth non-central moment equals the fourth order derivative of the MGF at $\alpha = 0$, we have

$$\mathbb{E} \left[ X^4 \right] = \frac{d^4}{d\alpha^4} G_X (\alpha) \bigg|_{\alpha = 0}$$

$$= \lambda Z \left( \lambda^2 e^{-3\alpha} + 6\lambda^2 Z e^{-2\alpha} + 7\lambda^2 Z e^{-\alpha} + 1 \right) e^{\alpha Z (e^\alpha - 1)} \bigg|_{\alpha = 0}$$

$$= \lambda^2 Z + 6\lambda^2 Z^2 + 7\lambda^2 Z^3 + (\lambda^2 Z + \lambda^2 Z^2 + \lambda Z).$$

APPENDIX D
PROOF OF LEMMA 2

In the following, we provide the proof of Lemma 2. The method of proof is by contradiction, namely, we assume that the condition given in (39) is violated and then we show that this leads to a contradiction (sum of the type I and type II error probabilities converge to one).

Fix $\lambda_1, \lambda_2 > 0$. Let $\kappa, \delta > 0$ be arbitrarily small constants. Assume to the contrary that there exist two different messages $i_1$ and $i_2$, meeting the error constraints in (7), such that for all $t \in [n]$:

$$\left| 1 - \frac{v_{i_2,t}}{v_{i_1,t}} \right| \leq \epsilon_n,$$  

where $v_{i,k,t} = \rho u_{i,t} + \lambda$, $k = 1, 2$. In order to show contradiction, we will bound the sum of the two error probabilities, $P_{e,1}(i_1) + P_{e,2}(i_2, i_1)$, from below. To this end, define

$$B_{i_1} = \left\{ y \in D_{i_1} : n^{-1} \sum_{t=1}^{n} y_t \leq \rho P_{\text{ave}} + \lambda + \delta \right\}. $$  

Then, observe that

$$P_{e,1}(i_1) + P_{e,2}(i_2, i_1)$$

$$= 1 - \sum_{y \in D_{i_1}} W^n (y | u_{i_1}) + \sum_{y \in D_{i_2}} W^n (y | u_{i_2})$$

$$\geq 1 - \sum_{y \in D_{i_1}} W^n (y | u_{i_1}) + \sum_{y \in D_{i_1} \cap B_{i_1}} W^n (y | u_{i_2}).$$

Now, consider the sum over $D_{i_1}$ in (47),

$$\sum_{y \in D_{i_1}} W^n (y | u_{i_1}) = \sum_{y \in D_{i_1} \cap B_{i_1}} W^n (y | u_{i_1})$$

$$+ \sum_{y \in D_{i_1} \cap B_{i_1}} W^n (y | u_{i_1}) \leq \sum_{y \in D_{i_1} \cap B_{i_1}} W^n (y | u_{i_2})$$

$$+ \Pr \left( n^{-1} \sum_{t=1}^{n} Y_t > \rho P_{\text{ave}} + \lambda + \delta | u_{i_1} \right).$$

Next, we bound the probability on the right hand side of (48) as follows

$$\Pr \left( n^{-1} \sum_{t=1}^{n} Y_t - n^{-1} \sum_{t=1}^{n} \mathbb{E} \{ Y_t \} > \rho P_{\text{ave}} + \delta \right)$$

$$= \frac{\var{ n^{-1} \sum_{t=1}^{n} Y_t | u_{i_1} } }{\left( \rho P_{\text{ave}} + \delta - \frac{1}{n} \sum_{t=1}^{n} \mathbb{E} \{ Y_t \} \right)^2}$$

$$\leq \left( \rho P_{\text{ave}} + \lambda / n \delta^2 \leq \kappa, \right.$$  

for sufficiently large $n$, where inequality (a) follows from Chebyshev’s inequality, for equality (b), we exploited $\var{ Y_t | u_{i_1} } = E \{ Y_t | u_{i_1} \} = \rho u_{i_1,t} + \lambda$, and for inequality (c), we used the fact that $u_{i_1,t} \leq P_{\text{ave}}, \forall t \in [n]]$.

Returning to the sum of error probabilities in (47), exploiting the bound (49) leads to

$$P_{e,1}(i_1) + P_{e,2}(i_2, i_1)$$

$$\geq 1 - \sum_{y \in D_{i_1} \cap B_{i_1}} \left[ W^n (y | u_{i_1}) - W^n (y | u_{i_2}) \right] - \kappa. $$  

Now, let us focus on the summand in the square brackets in (50). By (3), we have

$$W^n (y | u_{i_1}) - W^n (y | u_{i_2})$$

$$= W^n (y | u_{i_1}) \cdot \left[ 1 - \frac{W^n (y | u_{i_2})}{W^n (y | u_{i_1})} \right]$$

$$= W^n (y | u_{i_1}) \cdot \left[ 1 - \prod_{t=1}^{n} e^{-\left( \frac{v_{i_2,t} - v_{i_1,t}}{v_{i_1,t}} \right)^{\rho \lambda^2 \delta}} \right]$$

$$= W^n (y | u_{i_1}) \cdot \left[ 1 - \prod_{t=1}^{n} e^{-\left( \frac{v_{i_2,t} - v_{i_1,t}}{v_{i_1,t}} \right)^{\rho \lambda^2 \delta}} \right], $$

where for the last inequality, we employed $v_{i_2,t} - v_{i_1,t} \leq \epsilon_n v_{i_1,t}$ and $1 - v_{i_2,t}/v_{i_1,t} \leq \left| 1 - v_{i_2,t}/v_{i_1,t} \right| \leq \epsilon_n$, which follow from (45). Now, we bound the product term in (51) as follows:

$$\sum_{t=1}^{n} e^{-\epsilon_n v_{i_1,t} \left( 1 - \epsilon_n^\delta \right)} = e^{-\epsilon_n \sum_{t=1}^{n} v_{i_1,t} \cdot \left( 1 - \epsilon_n^\delta \right) \sum_{t=1}^{n} y_t}$$

$$\geq e^{-n \epsilon_n^\delta} \cdot e^{-n \epsilon_n^\delta} \cdot \left( 1 - \epsilon_n^\delta \right) \sum_{t=1}^{n} P_{\text{ave}} + \lambda + \delta$$

$$\geq e^{-n \epsilon_n^\delta} \cdot e^{-n \epsilon_n^\delta} \cdot \left( 1 - \epsilon_n^\delta \right) \sum_{t=1}^{n} P_{\text{ave}} + \lambda + \delta$$

$$\geq 1 - 3 \rho P_{\text{ave}} + \lambda + \delta \text{ by } n \geq \kappa - 1,$$  

for sufficiently large $n$. For inequality (a), we used $v_{i_1,t} \leq \rho P_{\text{ave}} + \lambda$, $\forall t \in [n]]$, and $\sum_{t=1}^{n} y_t \leq n (\rho P_{\text{ave}} + \lambda + \delta)$, where the latter inequality follows from $y \in B_{i_1}$, cf. (46). For (b), we used Bernoulli’s inequality $(1 - x)^r \geq 1 - rx$ for all $x > -1$ and $r > 0$ [93, see Ch. 3. For (c), we exploited $e^{-x^2} > 1$ and the following definition:

$$f(x) = e^{-ex} (1 - x)^c,$$
\begin{align*}
\text{Combining, (50), (51), and (54) yields}
\begin{align*}
P_{\text{e,1}}(i_1) + P_{\text{e,2}}(i_2, i_1) &= \left(\frac{1}{\kappa}\right) \\
&\geq 1 - \sum_{y \in \mathcal{B}_i} \left[ W^n(y | u_{1,i}) - W^n(y | u_{2,i}) - \kappa \right] \\
&= 1 - \sum_{y \in \mathcal{B}_i} \left[ \kappa \cdot W^n(y | u_{1,i}) - \kappa \geq 1 - 2\kappa, \right]
\end{align*}
\end{align*}
\right.}
\right.}

where for (a), we replaced \( y \in \mathcal{B}_i \cap \mathcal{D}_i \) by \( y \in \mathcal{B}_i \) to enlarge the domain and for (b), we used \( \sum_{y \in \mathcal{B}_i} W^n(y | u_{1,i}) \leq 1 \). Clearly, this is a contradiction since the error probabilities tend to zero as \( n \to \infty \). Thus, the assumption in (45) is false. This completes the proof of Lemma 2.
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