Application of firefly algorithm intelligent optimization particle filter in dynamic harmonic detection of power system

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Abstract. The harmonic control is one of the important ways to improve the power quality, and the fast and accurate detection of the amplitude and phase of the harmonic in the power system is the prerequisite for the realization of the harmonic control of the power system. In this paper, based on the operation mechanism of particle filter (PF), the optimization method of the firefly algorithm (FA) is modified and the intelligent optimization particle filter of the firefly algorithm (FAPF) is proposed, which is used to detect the amplitude and phase of the dynamic harmonic in the power system. The method improves the overall quality of particle swarm and preserves its unique advantages in dealing with nonlinear, non-Gauss system of parameter estimation and state filtering. The experimental results show that, under the nonlinear and non-Gauss noise environment, the FAPF algorithm has the advantages of high accuracy and good real-time performance when estimating the dynamic harmonic amplitude and phase of the power system.

1. Introduction

Due to the widespread application of nonlinear loads in power systems, the phenomenon of harmonic pollution in power systems is becoming more and more serious. Timely and accurate grasp of the actual situation of harmonic in power system is conducive to assessment of power quality and effective measures are taken to control it. As the starting point of harmonic pollution control, harmonic detection has become an important research object of scholars at home and abroad. It has become an important research focus on how to detect the harmonic content in the power system with high precision and real-time.

Calculating the amplitude and phase of harmonics in the range of effective accuracy is the main task of harmonic analysis. The main methods of harmonic analysis include fast Fourier transform and its improved algorithm¹², particle swarm optimization based detection method³⁴, d-q coordinate transformation detection method⁵ and improved phase difference correction method based on Hanning window⁶. These methods have a good effect on harmonic detection in the usual sense, but the application of these methods will be limited to the detection of more complex harmonics such as dynamic harmonics and inter-harmonics.

2. Harmonic analysis model

The $N$ subharmonic signal with noise and attenuation can be expressed as:
\[ y(k) = \sum_{n=1}^{N} A_n \sin(\omega_n k T_s + \phi_n) + A_d \exp(-\alpha_d k T_s) + \mu_k \]  \hspace{1cm} (1)

Among them, \(N\) is the frequency of harmonics; \(A_n\) is the amplitude; \(\omega_n = n2\pi f_0\), \(f_0\) is the frequency of the fundamental harmonic; \(\phi_n\) is the phase of the harmonic; \(T_s\) is the sampling period; \(k = t/T_s\); \(A_d \exp(-\alpha_d k T_s)\) is the attenuation part, \(A_d\) and \(\alpha_d\) is constant; \(\mu_k\) is the random noise added. The \(A_d \exp(-\alpha_d k T_s)\), expanded by Taylor's formula as \(\exp(\alpha) = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!}\). Then,

\[ y(k) = \sum_{n=1}^{N} A_n \sin(\omega_n k T_s + \phi_n) + A_d \alpha_d k T_s + \mu_k \]  \hspace{1cm} (2)

Formula (2) can be expressed as \(y(k) = H(k)X(k) + \mu(k)\)  \hspace{1cm} (3)

The \(H(k)\) is measurement of transfer matrix; The state quantity can be expressed as

\[ X(k) = [X_1(k) \ X_2(k) \ \cdots \ X_{2N-1}(k)]^T \]  \hspace{1cm} (4)

Here \(X_{2N-1}(k) = A_N \cos \phi_N\), \(X_N(k) = A_N \sin \phi_N\). According to formula (4), the equation of state of harmonic signals can be expressed as \(X(k+1) = F(k)X(k)\)

From the state quantity, the amplitude and phase of all the harmonics can be obtained

\[ A_n = \sqrt{X_{2n}^2 + X_{2n-1}^2} \]  \hspace{1cm} (5)

\[ \phi_n = \arctan \left( \frac{X_{2n}}{X_{2n-1}} \right) \]  \hspace{1cm} (6)

3. Particle filter

It is assumed that the nonlinear dynamic process of particle filter is expressed as \[^6\]

\[ x(k) = f(x_{k-1}, v_{k-1}) \]  \hspace{1cm} (7)

\[ y(k) = f(x_k, w_k) \]  \hspace{1cm} (8)

Among them, \(x_k\) is the state value, \(f(\cdot)\) is the state function, \(v_{k-1}\) is the system noise, \(y_k\) is the observation value, \(h(\cdot)\) is the observation function, \(\alpha_k\) is the measurement noise.

Hypothesis state initial probability density function is \(p(x_0 | y_0) = p(x_0)\), State prediction equation is

\[ p(x_k | y_{k-1}) = \int p(x_k | x_{k-1})p(x_{k-1} | y_{k-1})dx_{k-1} \]  \hspace{1cm} (9)

The renewal equation of the state is

\[ p(y_k | y_{k-1}) = \int p(y_k | x_k)p(x_k | y_{k-1})dx_k \]  \hspace{1cm} (10)

Hypothesis the known and easily sampled importance function as \(q(x_0 | y_{k-1})\) to rewrite it as

\[ q(x_{0:k} | y_{k-1}) = q(x_0) \prod_{j=1}^{k} q(x_j | x_{0:j-1}, y_{1:j}) \]  \hspace{1cm} (12)

Weight formula is \(\omega_k = \frac{p(y_k | x_k)p(x_0 | y_{k-1})}{q(x_k | x_{0:k-1}, y_{k-1})q(x_{0:k-1}, y_{k-1})} = \omega_{k-1} \frac{p(y_k | x_k)p(x_k | x_{k-1})}{q(x_k | x_{0:k-1}, y_{k-1})} \)  \hspace{1cm} (13)

Approximate calculation of the number of effective particles by resampling of formula is

\[ N_{eff} = \frac{1}{\sum_{i=1}^{N} (\omega_k)^2} \]  \hspace{1cm} (14)

According \(p(x_{k-1} | y_{1:k-1})\) to sample \(N\) points \(\{x_{k-1}^i\}_{i=1}^{N}\), probability density is
\[
p(x_{k-1} | y_{1:k-1}) = \sum_{i=1}^{N} \alpha_k^i \delta(x_{k-1} - x^i_{k-1})
\]

Probability density renewal formula is
\[
\alpha_k = \alpha_{k-1} \frac{p(y_k | x^i_k) p(x^i_k | x^i_{k-1})}{q(x^i_k | x^i_{k-1}, y_k)}
\]

The estimated of state is
\[
x_k = \sum_{i=1}^{N} \alpha_k^i x^i_k
\]

4. Firefly algorithm
The firefly algorithm contains two elements, namely brightness and attraction. The luminance reflects the position of the firefly and determines its direction of movement. The attraction determines the distance of the firefly movement. The target optimization is achieved through the continuous updating of the brightness and attraction. The main parameters of the firefly algorithm are described as follows from the mathematical angle.

The relative luminance of fireflies is
\[
I = I_0 e^{-\gamma r_{ij}}
\]

Among them, \(I_0\), the maximum luminosity of the firefly is related to the target function value, the better objective function value, the brighter its own brightness. Absorption coefficient of light intensity is \(\gamma\), fluorescence will gradually weaken with the increase of distance and the absorption of the media; \(r_{ij}\) is the space distance between firefly \(i\) and \(j\).

The attractiveness of the firefly is
\[
\beta = \beta_0 e^{-\gamma r_{ij}^2}
\]

Maximum attraction is \(\beta_0\); \(\gamma\) is absorption coefficient of light intensity. The firefly \(i\) is attracted to the location of the firefly \(j\) mobile update formula is
\[
x_j = x_j + \beta (x_j - x_i) + \alpha (rand - \frac{1}{2})
\]

5. FA-PF
The traditional particle filter resampling method can avoid the phenomenon of particle shortage by removing the small weight particle set, but after many iterations, it will bring the problem of particle dilution. In this study, the idea of optimizing the particle filter using the firefly algorithm is proposed \([9]\).

The implementation and steps of the algorithm:
Step1: At the initial time, Sampling \(N\) particles \(\{x_0^i, i = 1, \cdots, N\}\) as the algorithm’s initial particle, the importance density function representation is
\[
x_k^i \sim q(x_k^i | x_{k-1}^i, z_k) = p(x_k^i | x_{k-1}^i)
\]

Step2: Attracting behavior and mobile behavior of simulated firefly optimization.
1) The attraction between the calculated particle \(i\) and the global optimal value.
\[
\beta = \beta_0 e^{-\gamma r_{ik}^2}
\]

Among them, maximum attraction is \(\beta_0\); \(\gamma\) is absorption coefficient of light intensity; \(r_{ik}\) is the spatial position between the particle \(i\) and the global optimum value \(g_{best_k}\).

2) Update the position of the particle according to the degree of attraction
\[
x_k^i = x_k^i + \beta (g_{best_k} - x_k^i) + \alpha (rand - \frac{1}{2})
\]
Step 3: Calculate and contrast the brightness value and update the global optimum.

\[
g_{best_k} \in \left\{ x_1^k, x_2^k, x_3^k, \ldots, x_N^k \mid I(x) \right\} = \max \left\{ I(x_1^k), I(x_2^k), I(x_3^k), \ldots, I(x_N^k) \right\}
\]  

(24)

Step 4: From the calculation formula of the fluorescence degree, it can be seen that the value of the fluorescence degree changes in the opposite direction with the predicted value and the real observation value, and the iterative termination threshold is 0.01. When the value of the fluorescence function is greater than 0.01, the algorithm stops iterating, otherwise the algorithm continues to iterate to the maximum number of iterations. When the algorithm meets the set threshold \( \varepsilon \), the particle has been explained. When the distribution is near the real value or the maximum number of iterations is reached, the optimization is stopped at this time. Otherwise, step 2.

Step 5: Weight compensation and update. The core idea of the combination of the firefly algorithm and the PF is to perform the iterative optimization of each particle in the particle filter, making the particle moving in the direction of higher posterior density the target state, and improving the accuracy of the particle estimation for the state. However, the firefly algorithm changes the position of each particle in the state space, and the particle set is expressed at this time.

\[
\omega_k = \frac{p(z_k = s_k^i \mid x_k = s_k^i)}{q(s_k^i)} \rho(z_k = s_k^i)
\]

(25)

Step 6: Weight normalization is

\[
\omega_k' = \frac{\omega_k}{\sum_{i=1}^{N} \omega_k^i}
\]

(26)

Step 7: State estimation is

\[
\hat{x} = \sum_{i=1}^{N} \omega_k^i x_k^i
\]

(27)

6. Firefly intelligent optimization particle filter for dynamic harmonic detection in electric power system

Step 1. According to the above harmonic model, the initial value of the state quantity in the formula (3) is set and the \( M \) particles are sampled from it, and the state noise covariance \( Q \), the measurement noise covariance \( R \) and the state error covariance \( P \).

Step 2. For each particle, first calculate the weight of each particle according to formula (7) to formula (13), and then calculate the optimal estimate \( \hat{x} \) of state quantity according to formula (21) to (27).

Step 3. The components of \( \hat{x} \) in the optimal estimation of state quantities are calculated in terms of formula (5) and formula (6) respectively for the harmonic amplitude and phase of \( k \) moment. Order \( k + 1 \) to estimate the parameters at the next moment.

7. Experimental analysis

In order to verify the accuracy and real-time performance of FAPF for dynamic harmonic detection of power system, MATLAB software is used to analyze dynamic harmonic detection, and the sampling frequency is 2.5KHz.

The mean square error (MSE) is used to measure the amplitude and phase of harmonic. The mean square error of \( i \) moment is defined as

\[
MSE = \frac{1}{N} \sum_{i=1}^{N} \left( y_i' - y_i \right)^2
\]

(28)

Among them, \( N \) is the dimension of the state quantity; \( y_i' \) and \( y_i \) are the \( i \) components of the predicted value and the theoretical value, the smaller the mean square error, the better the result of the detection.
7.1 Detection of dynamic harmonic amplitude in power system based on FAPF algorithm

It is assumed that the voltage signal of the dynamic harmonic of the power system (Amplitude changes with time) is described as

\[
y(t) = [1.2 + a_1(t)]\sin(\omega t + 75^\circ) + [0.7 + a_2(t)]\sin(3\omega t + 55^\circ) + [0.3 + a_3(t)]\sin(5\omega t + 45^\circ) + \mu(t)
\]

Among them \(\omega = 2\pi f; f = 50\text{ Hz}\); \(\mu(t)\) is random Gauss noise, mean value is 0, variance is 1.

\[
a_1(t) = 0.12\sin(2\pi f_1 t) + 0.07\sin(2\pi f_2 t)
\]

\[
a_2(t) = 0.07\sin(2\pi f_3 t) + 0.03\sin(2\pi f_4 t)
\]

\[
a_3(t) = 0.03\sin(2\pi f_5 t) + 0.05\sin(2\pi f_6 t)
\]

Among them, \(f_1 = 2.0\text{ Hz}; f_3 = 5.0\text{ Hz}; f_5 = 7.0\text{ Hz}\)

As shown form Figure 3 to figure 5, although only the amplitude estimation of the 3rd and 5th dynamic harmonics in the power system is given, the method is also applicable when it is extended to any higher harmonics.

![Figure1. Estimation of fundamental harmonic amplitude.](image1)

![Figure2. 3rd Harmonic amplitude estimation.](image2)

![Figure3. 5th Harmonic amplitude estimation.](image3)

| Harmonic       | Actual Value | FAPF | PF  | KF |
|----------------|--------------|------|-----|----|
|                | Amplitude    | MSE  | Amplitude | MSE | Amplitude | MSE |
| Fundamental Harmonic | 1.2480       | 3.8970e-3 | 1.2490 | 3.9490e-3 | 1.2570 | 9.8730e-3 |

Table1. The results contrast (t = 0.1s).
It can be seen from the simulation diagram of fundamental harmonic, 3rd harmonic and 5th harmonic amplitude that the prediction of real values by FAPF algorithm is higher than PF and KF. The prediction accuracy more well, the real-time performance is better, and the MSE value is smaller and more stable.

The table 1 adds non-Gauss noise $u(t) = \Gamma(0.3, 0.2)$ into the power system at 0.1s moment, the results of dynamic amplitude estimation of FAPF, PF and KF are compared. From table 1, it is known that the amplitude of dynamic fundamental harmonic, 3rd harmonic and 5th harmonic of power system based on FAPF method is more approximate to real value, and its MSE value is also less than the other two methods.

### 7.2 Detection of dynamic harmonic phase in power system based on FAPF algorithm

It is assumed that the voltage signal of the dynamic harmonic of the power system (Phase change with time) is described as [7]:

$$y(t) = 1.2 \sin(\omega t + 75^\circ) + 0.5 \sin(3\omega t + 55^\circ) + 0.3 \sin(5\omega t + 45^\circ) + \omega(t)$$

where $\omega = 2\pi f$; $f = 50\text{Hz}$; $\mu(t)$ is random Gauss noise, mean value is 0, variance is 1.

$$a_1(t) = 0.12 \sin(2\pi f_1 t) + 0.07 \sin(2\pi f_2 t)$$  
$$a_2(t) = 0.07 \sin(2\pi f_1 t) + 0.03 \sin(2\pi f_2 t)$$  
$$a_3(t) = 0.03 \sin(2\pi f_1 t) + 0.05 \sin(2\pi f_2 t)$$

Among them, $f_1 = 2.0\text{Hz}$; $f_2 = 5.0\text{Hz}$; $f_5 = 7.0\text{Hz}$.

FAPF, PF and KF are used to estimate the phase of dynamic fundamental, 3rd harmonic and 5th harmonic of noisy power system and calculate their corresponding MSE values. As shown from Figure 4 to Figure 6, although only the estimation of the phase of the power system with 3rd and 5th dynamic harmonics is given, but the method is also applicable when it is extended to any high order harmonics.

![Figure 4. Fundamental wave phase estimation.](image1)

![Figure 5. 3rd harmonic phase estimation.](image2)
Figure 6. 5th harmonic phase estimation.

Table 2. The results contrast (t = 0.1s).

| Harmonic      | Actual Value | FAPF Phase(°), MSE | PF Phase(°), MSE | KF Phase(°), MSE |
|---------------|--------------|--------------------|-----------------|-----------------|
| Fundamental Harmonic | 75.0500      | 75.0532 4.6960e-3 | 75.0613 6.9480e-3 | 75.0749 13.6800e-3 |
| 3rd harmonic  | 55.0400      | 55.0431 4.4470e-3 | 55.0567 10.0900e-3 | 55.0632 13.3300e-3 |
| 5th harmonic  | 44.9820      | 44.9822 0.3985e-3 | 44.9913 8.9680e-3 | 45.0000 13.7300e-3 |

From the above chart, we can see that the FAPF algorithm is superior to PF and KF in the dynamic harmonic phase estimation of power system. After adding the non-Gauss noise \( u(t) = \Gamma(0.3, 0.2) \), for the estimation of the phase of the dynamic harmonic signal, the FAPF algorithm has higher accuracy and better curve fitting than PF and KF for the estimation of the phase of the dynamic harmonic signal. From the above simulation chart, we can see that the FAPF algorithm has higher estimation accuracy for dynamic harmonic phase of power system, smaller mean square error and better tracking prediction effect.

8. Conclusion

In this paper, a method of detecting dynamic harmonic in power system by intelligent optimization particle filter is proposed in this paper. The method has better convergence and higher detection precision than PF and KF. The theoretical analysis and simulation show that, under the non-Gauss noise, the method has high accuracy in estimating the amplitude and phase of the dynamic harmonic in the power system, with good real-time, small mean square error and the estimation precision of the method can be further improved with the increasing of the number of sample particles.

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