Euclidean Supergravity and Multi-Centered Solutions

W. A. Sabra

Centre for Advanced Mathematical Sciences and Physics Department
American University of Beirut
Lebanon

Abstract

In ungauged supergravity theories, the no-force condition for BPS states implies the existence of stable static multi-centered solutions. The first solutions to Einstein-Maxwell theory with a positive cosmological constant describing an arbitrary number of charged black holes were found by Kastor and Traschen. Generalisations to five and higher dimensional theories were obtained by London. Multi-centered solutions in gauged supergravity, even with time-dependence allowed, have yet to be constructed. In this letter we construct supersymmetry-preserving multi-centered solutions for the case of $D = 5$, $N = 2$ Euclidean gauged supergravity coupled to an arbitrary number of vector multiplets. Higher dimensional Einstein-Maxwell multi-centered solutions are also presented.
1 Introduction

In Newtonian gravity, one can obtain a system of point particles, each having a charge equal to its mass, in static equilibrium by balancing the mutual attractive gravitational and repulsive electrostatic forces. In Einstein theory of general relativity and quite surprisingly the analogue situation first appeared in the early work of Weyl, Majumdar and Papapetrou [1]. The Majumdar-Papapetrou (MP) solutions were found to describe a system of multi-centered extremal Reissner-Nordström black holes in thermal and mechanical equilibrium [2]. The MP metrics are the static limits of the Israel-Wilson-Perjés (IWP) solutions [3]. If one considers Einstein-Maxwell theory as the bosonic sector of the theory of $N = 2$, $D = 4$ supergravity, then the MP metrics turn out to be solutions admitting half of the supersymmetry [4]. A systematic classification by Tod [5] also demonstrated that the IWP metrics are the unique solutions with time-like Killing vector admitting supercovariantly constant spinors. Analogues of the MP solutions were also found for black holes with a dilaton field in [6]. In [7] general half-supersymmetric solutions, which can be considered as generalisation of the MP and IWP metrics, to the theories $N = 2$, $D = 4$ supergravity with vector multiplets were found. BPS solutions in five-dimensional Einstein-Maxwell theory were considered in [8]. The metric in this case is of the Tanghehini form [9]. Moreover, electric and magnetic BPS solutions breaking half of the supersymmetry in ungauged five-dimensional supergravity coupled to vector multiplets were constructed in [10].

The first multi-centered solutions asymptotic to de Sitter space were obtained in four dimensions in [11]. These are non-static solutions to the Einstein-Maxwell equations in the presence of a positive cosmological constant. They describe an arbitrary number of charged black holes in motion due to the positive cosmological constant. As should be expected, these solutions reduce to MP solutions in the limit of vanishing cosmological constant. Multi-centered solutions to $d$-dimensional Einstein-Maxwell theory with a positive cosmological constant were given in [12]. In the five-dimensional case and with an imaginary coupling $g$ (with the cosmological constant being proportional to $-g^2$), this theory may be viewed as the bosonic sector of pure de Sitter $D = 5$, $N = 2$ supergravity. Within this fake supergravity framework, it was shown in [12] that the multi-centered solutions preserve...
some supersymmetry through the explicit construction of the corresponding Killing spinors. Multi-centered solutions to $D = 5, N = 2$ gauged supergravity coupled to an arbitrary number of vector multiplets were considered in [13][14]. In the fake de Sitter supergravity case, one obtains rotating multi-centered solutions. However, in the standard Anti-de Sitter cases, the multi-centered solutions have a complex space-time metric.

In this letter, we will be mainly concerned with the gauged version of the five-dimensional Euclidean supergravity theory which was recently constructed in [15]. It will be demonstrated through the analysis of the Killing spinor equations that this theory admits multi-centered solutions with real space-time metric and real fields unlike in the Lorentzian theory where the solution is complex. The results obtained may be useful as many investigations of the AdS/CFT conjecture [16] have in fact been performed in Euclidean space.

We organise our work as follows. In section two we briefly present the Euclidean five-dimensional theory and its special geometry structure relevant for our subsequent analysis. Section three contains the analysis and construction of the multi-centered solution for these theories. In the last section we present multi-centered solutions to $d$-dimensional Einstein-Maxwell theories and end with a summary.

2 Euclidean $D = 5, N = 2$ Gauged Supergravity

The theory of $D = 5, N = 2$ Euclidean supergravity coupled to vector multiplets was recently constructed in [15]. The new Euclidean theory has the same bosonic fields content of the Lorentzian theory with scalar fields parametrizing a projective special real target manifold [17]. The Lagrangian of the Euclidean theory differs from the Lorentzian one in that the terms of the Euclidean gauge fields appear with the opposite sign. Upon dimensional reduction on a circle, the Euclidean $D = 5, N = 2$ bosonic Lagrangian produces the bosonic Lagrangian of $D = 4, N = 2$ Euclidean supergravity with the ‘wrong’ sign in front of the gauge terms. However, in four dimensions with Euclidean signature, theories with different signs in front of their gauge kinetic terms can be mapped to one another by a duality transformation [15][18].

In the five-dimensional Euclidean supergravity cases, unlike in four dimensions, only one sign is allowed in front of the gauge field terms in the Lagrangian.

As the scalar fields structure in the Euclidean theory is unaltered, a scalar potential $V$
can be added to the theory resulting in the gauged Euclidean $N = 2$, $D = 5$ supergravity coupled to an arbitrary number $n$ of abelian vector supermultiplets. The action is given by

$$S = \frac{1}{16} \int (R - 2g^2V) \ast 1 - G_{IJ} \left( dX^I \wedge \ast dX^J - F^I \wedge \ast F^J \right)$$

$$- \frac{C_{IJK}}{6} F^I \wedge F^J \wedge A^K,$$

where $I, J$ take values $1, \ldots, n$, $R$ is the scalar curvature, $F^I = dA^I$ denote the abelian field-strengths two-forms. The constants $C_{IJK}$ are symmetric in all indices and the coupling matrix $G_{IJ}$ is invertible and remains the same as in the Lorentzian case. The fields $X^I$ are functions of $(n-1)$ unconstrained scalars $\phi^i$.

Some useful relations which from very special geometry which will be used in our analysis are

$$G_{IJ} = \frac{1}{2} \left( 9X^I X_J - C_{IJK} X^K \right),$$

$$\frac{1}{6} C_{IJK} X^I X^J X^K = X^I X_I = 1,$$

$$dX_I = - \frac{2}{3} G_{IJ} dX^J, \quad X_I = \frac{2}{3} G_{IJ} X^J,$$

$$X_I dX^I = X^I dX_I = 0.$$  (2.2)

The scalar potential of the theory is given by

$$V = 9 V_I V_J \left( X^I X^J - \frac{1}{2} G^{IJ} \right),$$

where the $V_I$ are constants. The Killing spinor equations are given by

$$\left[ \nabla_\mu + \frac{3}{2} g V_I A^I_\mu - \frac{1}{8} X_I (\gamma_\mu^{\nu\rho} - 4 \delta_\mu^{\nu} \gamma_\rho) F^I_{\nu\rho} + \frac{1}{2} g X^I V_I \gamma_\mu \right] \epsilon = 0,$$

$$\left( 3 \partial_\mu X_I \gamma^\mu - G_{IJ} F^J_{\mu\nu} \gamma^{\mu\nu} + 6 g V_I \right) \partial_\nu X^I \epsilon = 0,$$  (2.5)

where $\partial_\nu$ denotes differentiation with respect to the scalars $\phi^i$.

3 Supersymmetric Multi-centered Solutions

In this section we construct multi-centered solutions to Euclidean theory described by (2.1). Before proceeding to construct the solutions of the gauged theory, we start with the analysis

1Our conventions are as follows: We use the metric $\eta^{ab} = (+, +, +, +, +)$ and Clifford algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$. The covariant derivative on spinors is $\nabla_\mu = \partial_\mu + \frac{i}{2} \omega_{\mu ab} \gamma^a \gamma^b$ where $\omega_{\mu ab}$ is the spin connection. Finally, antisymmetrization is with weight one, so $\gamma^{a_1 a_2 \ldots a_n} = \frac{1}{n!} \gamma^{[a_1 a_2 \ldots a_n]}$.  

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of the solutions of the ungauged theory, i.e., for the cases when $g = 0$. As in [10], we start
with the following metric ansatz

$$ds^2 = e^{-4U}(d\tau + w)^2 + e^{2U}ds_4^2,$$

(3.1)

with $U$ and $w = w_m dx^m$ independent of the $\tau$ coordinate. The four-dimensional base space
described by $ds_4^2$ is flat Euclidean with coordinates $x^m$. The spin-connections components
can be extracted from the vanishing of torsion conditions

$$de^0 + \omega^{0a} \wedge e^a = 0, \quad de^a + \omega^{ab} \wedge e^b - \omega^{0a} \wedge e^0 = 0,$$

(3.2)

where

$$e^0 = e^{-2U}(d\tau + w), \quad e^a = e^U \delta_m^a dx^m,$$

(3.3)

and are given by

$$\omega^0_\tau = -2e^{-3U} \delta^{am} \partial_m U,$$
$$\omega^0_n = -e^{-3U} \delta^{am} \left(\frac{1}{2}w_{nm} + 2w_n \partial_m U\right),$$
$$\omega^{ab}_\tau = \frac{1}{2} e^{-6U} \delta^{nb} \delta^{ma} w_{nm},$$
$$\omega^{ab}_n = (\delta^{mb} \delta^a_n - \delta^{ma} \delta^b_n) \partial_m U + \frac{1}{2} e^{-6U} w_n \delta^{pb} \delta^{ma} w_{pm},$$

(3.4)

where $w_{nm} = (\partial_n w_m - \partial_m w_n)$. Plugging into the Killing spinor equation (2.4) for $g = 0$,
requiring that the Killing spinor satisfies the projection condition $\gamma^0 \epsilon = \epsilon$, and making use
of the identity $\gamma_{abc} = -\epsilon_{abcd} \gamma^{d} \gamma_0$ and special geometry relations we obtain the conditions

$$\partial_\tau \epsilon = 0,$$
$$F^I_{\tau m} = \partial_m (X^I e^{-2U}),$$
$$F^I_{mn} = \partial_n (e^{-2U} X^I w_m) - \partial_m (e^{-2U} X^I w_n),$$

(3.5)

and

$$(\partial_n + \partial_m U) \epsilon = 0,$$
$$\gamma^b \left(w_{ab} + \frac{1}{2} \epsilon_{abcd} w^{cd}\right) \epsilon = 0.$$

(3.6)

(3.7)
Therefore we get

\[ dw = - * dw, \]  

(3.8)

thus implying that the two-form \( \phi = dw \) satisfies

\[ d\phi = d * \phi = 0, \]  

(3.9)

and therefore is a harmonic two-form. Equation (3.6) implies that the Killing spinor is given by

\[ \epsilon = e^{-U} \epsilon_0, \quad \gamma_0 \epsilon_0 = \epsilon_0, \]  

(3.10)

where \( \epsilon_0 \) is a constant spinor. It can then be shown that the equation (2.5) for \( g = 0 \) is satisfied for scalars independent of \( \tau \). The Bianchi identities for our solution are identically satisfied. Using (3.5) and (3.8) we find that the Maxwell equations

\[ d \left( G_{IJ} * F^J \right) = \frac{1}{4} C_{IJK} F^J \wedge F^K \]  

(3.11)

are satisfied provided

\[ X_I = \frac{1}{3} e^{-2U} H_I, \]  

(3.12)

where \( H_I \) are a set of harmonic functions,

\[ H_I = h_I + \sum_{j=1}^N \frac{q_{IJ}}{|x^I - x^J|^2}. \]  

(3.13)

Here \( h_I \) are related to scalar values at infinity and \( q_{IJ} \) are electric charges. As in [10], we define the rescaled coordinates

\[ Y_I = e^{2U} X_I, \quad Y^I = e^U X^I, \]  

(3.14)

then the solution for \( U \) is given by

\[ e^{3U} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K, \quad \frac{1}{2} C_{IJK} Y^J Y^K = H_I. \]  

(3.15)

To get explicit solutions for a given model one needs to solve for the equations (3.15) which depend on the intersection numbers \( C_{IJK} \). However one can get a closed general solution
when the scalar fields take values in a symmetric space where one have the useful condition

\[ C_{IJK}C_{J'LM}C_{PQK'}\delta^{I'I'}\delta^{KK'} = \frac{4}{3}\delta_{I(L}C_{M PQ)}, \quad \text{(3.16)} \]

In this case we have the identity

\[ X^I = \frac{9}{2}C_{IJK}X_JX_K, \quad \text{(3.17)} \]

where \( C_{IJK} = \delta^{II'}\delta^{JJ'}\delta^{KK'}C_{I'J'K'} \), then the solution (3.15) implies

\[ e^{6U} = \frac{1}{6}C_{IJK}H_IH_JH_K. \quad \text{(3.18)} \]

We now move on to construct multi-centered solutions for Euclidean five-dimensional supergravity theories with non-trivial gauge and scalar fields. It will be shown that in the Euclidean case multi-centred solutions with real space-time metric do exist. Motivated by the results of [13], we take as an ansatz for our solution the metric

\[ ds^2 = e^{-4U}(d\tau + e^{2g\tau}w)^2 + e^{-2g\tau}e^{2U}ds^2_4, \quad \text{(3.19)} \]

where \( U = U(x, \tau), \, w = w_m(x)dx^m \) depends on the base coordinates only. The spin-connections components can be extracted from the vanishing of torsion conditions and are given by

\[
\begin{align*}
\omega^{0a}_\tau &= e^{g\tau-U}\delta^{am}\left(\partial_m\left(e^{-2U}\right) - \partial_\tau Q_m\right), \\
\omega^{0a}_n &= -e^{3U-g\tau}(U-g)\delta^a_n - \frac{1}{2}e^{3(\tau-g\tau)}\delta^{am}w_{nm} \\
&\quad + e^{g\tau+U}\delta^{am}Q_n\left(\partial_m\left(e^{-2U}\right) - \partial_\tau Q_m\right), \\
\omega^{ab}_\tau &= \frac{1}{2}e^{4g\tau-6U}\delta^{mb}\delta^{ma}w_{nm}, \\
\omega^{ab}_n &= (\delta^{mb}\delta^a_n - \delta^{ma}\delta^b_n)\left(\partial_mU + \frac{1}{2}e^{2U}\partial_\tau Q_m\right) + \frac{1}{2}e^{4(g\tau-U)}Q_n\delta^{mb}\delta^{ma}w_{pm}, \quad \text{(3.20)}
\end{align*}
\]

where \( Q_m = e^{2g\tau-U}w_m \). Plugging this into the Killing spinor equation (2.4) and as in the ungauged case we require that the Killing spinor satisfies \( \gamma^0\epsilon = \epsilon \), we then get from the \( \tau \)-component the conditions

\[ (\partial_\tau - gV_Ie^{-2U}X^I)\epsilon = 0, \quad \text{(3.21)} \]
and

\[ X_I F^I_{\tau m} = \partial_m \left( e^{-2U} \right) - \partial_{\tau} Q_m, \]
\[ X_I F^I_{mn} = \partial_n Q_m - \partial_m Q_n. \]  

(3.22)

Using the special geometry relations \( X_I dX^I = 0 \) and \( X^I X_I = 1 \), we can write

\[ F^I_{\tau m} = \partial_m \left( X^I e^{-2U} \right) - \partial_{\tau} \left( X^I Q_m \right), \]
\[ F^I_{mn} = \partial_n \left( X^I Q_m \right) - \partial_m \left( X^I Q_n \right), \]  

(3.23)

and therefore the gauge field can be given by

\[ A^I_m = -X^I Q_m, \quad A^I_{\tau} = -e^{-2U} X^I. \]  

(3.24)

The rest of the components of (2.4) give, in addition to the conditions obtained in the ungauged case (3.6) and (3.8), the following condition

\[ gX^I V_I + e^{2U} (-g + \dot{U}) = 0. \]  

(3.25)

The equations (3.21) and (3.25) then imply

\[ \left( \partial_{\tau} - g + \dot{U} \right) \epsilon = 0, \]  

(3.26)

which together with (3.6) imply that the Killing spinor equations are solved by

\[ \epsilon = e^{g_{\tau} - U} \epsilon_0, \quad \gamma_0 \epsilon_0 = \epsilon_0. \]  

(3.27)

Turning to the second equation (2.5) and substituting the equations obtained so far, we obtain the condition

\[ (3e^{2U} \partial_{\tau} X_I + 6 g V_I) \partial_i X^I = 0. \]  

(3.28)

Using special geometry relations, the \( \gamma^a \) and the \( \gamma^{ab} \) terms vanish identically. Note that \( F^I = dA^I \), where the gauge fields one-forms are given by

\[ A^I = -X^J e^0. \]  

(3.29)
then the Bianchi identities hold automatically.

After some analysis it can be shown that the condition (3.28) and the Maxwell’s equations are satisfied for our solution provided that the scalars satisfy

\[ e^{2U} X_I = \frac{1}{3} H_I, \]

\[ H_I(t, \vec{x}) = 3V_I + e^{2\tau} \sum_{j=1}^{N} \frac{q_{IJ}}{|\vec{x} - \vec{x_j}|^2} \]

(3.30)

together with the condition

\[ d *_4 w = 0. \]

(3.31)

This completes the construction of the multi-centered solutions to \( D = 5, N = 2 \) gauged Euclidean supergravity theories.

4 \( d \)-dimensional Solutions

In this section we present general multi-centered solutions to \( d \)-dimensional Einstein-Maxwell theory with a cosmological constant. We start first by writing the \( d \)-dimensional de Sitter space-time metric in terms of the so-called cosmological coordinates, this is given by

\[ ds^2 = -d\tau^2 + e^{-2\tau} ds^2_{(d-1)}, \]

(4.1)

where \( ds^2_{(d-1)} \) is the metric of \( (d-1) \)-dimensional flat Euclidean space. The metric (4.1) is a solution of \( d \)-dimensional Einstein gravity with a positive cosmological constant with

\[ R_{\mu\nu} = (d-1)t^2 g_{\mu\nu}. \]

(4.2)

Moreover, the metric

\[ ds^2 = d\tau^2 + e^{-2\tau} ds^2_{(d-1)}, \]

(4.3)

is a solution of \( d \)-dimensional Euclidean Einstein gravity with a negative cosmological constant with

\[ R_{\mu\nu} = -(d-1)t^2 g_{\mu\nu}. \]

(4.4)

We now consider a general \( d \)-dimensional Einstein-Maxwell with a cosmological constant, with Lagrangian density
\[ e^{-1} \mathcal{L}_d = R + \eta F_{\mu\nu} F^{\mu\nu} - \Lambda. \]  

(4.5)

Here \( \eta \) can be either +1 or -1. The Einstein gravitational equations of motion are given by

\[ R_{\mu\nu} = -2\eta \left( F_{\mu\lambda} F^{\nu}_{\lambda} - \frac{1}{2(d-2)} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) + \frac{g_{\mu\nu}}{d-2} \Lambda. \]

(4.6)

If one considers the solution

\[ ds^2 = \frac{\eta}{H^2} d\tau^2 + H^{2/(d-3)} e^{-2\tau} ds_{d-1}^2 \]  

(4.7)

then it can be shown that this is a solution of (4.6) provided that

\[ \Lambda = -\eta (d-1)(d-2)^2, \]

\[ F_{\tau_i} = \sqrt{\frac{(d-2)}{2(d-3)}} \partial_i H, \]

\[ H = 1 + \sum_{j=1}^{d-1} \frac{q_j}{|\vec{x} - \vec{x}_j|^{d-3}} e^{(d-3)\tau}. \]  

(4.8)

Here \( \partial_i \) represents differentiation with respect to the coordinates of the base space described by \( ds_{d-1}^2 \). For \( \eta = -1 \), we reproduce the solutions of [11, 12], i.e., multi-centered solutions with a positive cosmological constant. For \( \eta = 1 \), we obtain new multi-centered solutions for the Euclidean theory with a negative cosmological constant at the expense of introducing a Lagrangian with the opposite sign of the gauge terms. The systematic analysis of the Euclidean four-dimensional cases was treated in [18].

To summarise, we have constructed multi-centered solutions for the gauged Euclidean supergravity with vector multiplets as well as for \( d \)-dimensional Einstein-Maxwell theories. It must be noted that the cosmological Kastor-Traschen solution was obtained as a class of pseudo-supersymmetric solutions in the systematic analysis of minimal \( N = 2, D = 4 \) supergravity coupled to vector multiplets were considered in [19].

These solutions deserve further analysis.
de Sitter supergravity [20]. In five dimensions, the systematic analysis of [21] revealed that solutions admitting Killing spinors in five-dimensional ungauged and de Sitter supergravity have respectively a hyper-Kähler and hyper-Kähler torsion (HKT) manifold as a four dimensional space. The multi-centered solutions of [12–14] are special cases. A systematic classification for the solutions of the gauged Euclidean theories should be carried out and we hope to report on this in the future. Finally, it remains an open question to construct true multi-centered solutions of the standard Lorentzian gauged Anti-de Sitter supergravity theories.

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