The research status and development trend of stochastic resonance

Lei Xu *, Yueping Peng, Man Liu
Graduate Management Unit, Engineering University of PAP, China

*Corresponding author e-mail: 734052869@qq.com

Abstract. The synergistic reaction under specific conditions of the nonlinear system, weak driving signal and moderate noise can make noise to be advantageous in a certain extent, so as to achieve the purpose of signal enhancement, this seemingly anomalous phenomenon is defined as stochastic resonance. In this paper, the weak signal detection under strong noise background is the main line. The principle of white noise to counteract external noise is expounded, and the present research situation and development trend of stochastic resonance are reviewed in that paper, it also pointed out the direction of further research of stochastic resonance technology.

1. Introduction
The vast majority of weak signal detection used today is how to suppress noise as the solution. This makes the system restrain the noise while the signal to be measured has been suppressed or destroyed to some extent. In signal processing, noise removal may weaken the useful signals. The universality and application limitation of detection technique based on the weak features encourage people to explore and research of new theory and method of weak signal detection ceaselessly, so as to realize the more accurate and effective detection of useful signals from strong background noise. The study found the increase of noise can increase the local signal-to-noise ratio of the output greatly and enhances signal output instead of reducing output effect of signal in certain nonlinear systems.

Figure 1. The relation schema between output performance and input noise intensity of nonlinear system
So this paper devoted in the research of weak signal detection based on stochastic resonance, it also summarized and analyzed research status and development trend of stochastic resonance.

2. Research status of stochastic resonance

The concept of stochastic resonance was first proposed by Benzi and his team members in 1981 to study the periodic occurrence of paleo meteorological glacier problems [1] [2]. Then they built up a stochastic resonance model system using nonlinear conditions of the earth and random forces acting on the earth in order to explain cyclical phenomenon. His model system is shown in Fig 2, with a double well potential function represented the earth's climate to make one of the minimum value to indicate the temperature at which the earth is covered by large areas of ice. It uses another minimum to represent the temperature when the earth is in hot period and uses a small periodic driving force to indicate a slight change in orbital eccentricity of the earth. In that model, Gauss noise is used to simulate the short-term frequent climate fluctuations, such as fluctuations of annual solar radiation. According to the theory of Benzi, when the noise intensity is adjusted to satisfy a certain condition, the synchronization between cold and warm situations with its synchronous climate will significantly enhance the earth's climate response for minor stimulation caused by adjusting the orbital eccentricity. In the bistable circuit experiment with Schmidt trigger Fauve et al found the stochastic resonance phenomenon for the first time in 1983[3]. The bistable circuit with Schmidt trigger has two steady output and the system’s steady state depends on the input signal and initial conditions. They observed the peak curve and with the increasing of system input noise intensity, the output SNR of the system has a tendency to increase first and then decrease [4]. Then the relevant scholars began to recognize and study on the positive effects of noise.

![Figure 2: The nonlinear bistable system model](image)

In practical work, the collected noise signals will inevitably have a certain amount of noise which are mostly white background or colored noise in signal acquisition, and a small part is the internal noise within the system. The traditional detection theory about weak signal mainly consists of the time domain average method, correlation detection technology and filtering technology. But when the noise intensity is larger than the excitation signal intensity, the excitation signal will be completely or heavily submerged in background noise to not be identified. When the time-frequency characteristics are similar, the ability to detect weak signals will decline more obvious. Stochastic resonance theory can effectively compensate for the shortcomings of traditional methods. Through continuous learning and understanding of stochastic resonance, we can get a more efficient method of weak signal detection. With decades of development, the scope of the study of stochastic resonance theory has involved Mechanical and electrical equipment fault detection, circuit fault detection[5], pictorial information
enhancing signal detection in the background of ocean noise, biomedicine and other engineering field, covering Physics, Chemistry, Electronics, and other aspects.

\[
\begin{align*}
\text{Signal} & \rightarrow \text{Nonlinear system (Signal processing unit )} \rightarrow \text{Export} \\
\text{Noise} & \rightarrow
\end{align*}
\]

**Figure 3.** The general structural block diagram of stochastic resonance

The foundations of stochastic resonance theory are derived from stochastic resonance approximation theory (or linear response theory), but the fundamental point of adiabatic approximation theory of stochastic resonance (or linear response theory) is that the research object must be small signal parameters, that is to say, the amplitude, frequency and noise intensity of input signal of the stochastic resonance system are small parameters.

\[\omega << \tau_k^{-1}, A << 1, D << 1\]

(1)

\(\omega\) is the signal input frequency, \(A\) is the input signal amplitude, \(D\) is the input noise intensity, \(\tau_k\) is the system characteristic time.

The simplest stochastic resonance model can be expressed by the Langevin equation:

\[x(t) = \alpha x(t) - bx^3(t) + A \sin(\Omega t + \varphi) + n(t)\]

(2)

With the use of stochastic resonance of adiabatic approximation method, the signal-to-noise ratio can be simplified:

\[\text{SNR} = \frac{\sqrt{2\mu^2} A^2 e^{-\mu^2/4D}}{4D^2}\]

(3)

So when the system input parameter is not a small parameter, such as the signal with large frequencies, the stochastic resonance theory of small parameter will fail and cannot be used. In recent years, the processing algorithm for the input signal with high frequency are: normalized scaling transformation, stochastic resonance with variable metric, Adaptive stochastic resonance and etc [7]. The normalized model of Stochastic resonance is equivalent to the measured signal and noise multiplied the scale factor at the same time, the specific operation process of variable scale algorithm is: determine a frequency compression scale ratio \(R\) firstly, then define a compressed sampling frequency according to the \(R\), and solve the response output of bistable system. Since the emergence of adaptive stochastic resonance, people continue to study the parameter optimization methods of stochastic resonance, which mostly take the signal-noise ratio and other indicators as evaluation index, but the parameter selection problem is difficult to solve without the help of prior knowledge [7].

3. **The principle of white noise to counteract external noise**

That belongs to the areas of physics and signal processing and it is called stochastic resonance. In a nutshell, when the original signal intensity is very weak, especially under the level of detection signal, we can achieved the purpose of enhancing the intensity of the signal to be measured by adding a certain
amount of white noise to the signal. But the ideal white noise has the same intensity in the whole frequency domain, which is not exist actually.

After the signal with equal strength or close to the strength in the whole frequency domain are added to the nonlinear system, the signal band which is compared with the original signal will resonate with original signal to increase the intensity of original signal and when it rise to the level can be detected, the signal intensity of white noise has remained unchanged at the same time. For the signal processing mechanism of brain and signal processing equipment, white noise can be easily filtered out through the band-pass filter method, this shows that the increasing of input signal’s intensity will not affect the characteristics of the signal, which means fidelity. This phenomenon perhaps contrary to people’s intuition, but it is not a shelter or offset by other signals. If a weak signal cannot reach the detection threshold, we usually add a DC component that equal to the overall elevation to achieve the detection threshold because white noise is similar with the DC power in frequency domain. The experiment has shown that adding white noise in the original signal is equivalent to raising the signal-noise ratio by about ten dB, which is a very big progress.

4. Application and development trend of stochastic resonance in other fields
At present, stochastic resonance has been widely used in biology and medicine and it can be used to explain how organisms using the noise to enhance the signal transmission in the nervous system. For the human nervous system and biological system, the single neuron is often taken as an ideal system for demonstrating stochastic resonance, because it can be considered as a signal detector that is excited when the input signal exceeds a certain threshold. As early as 1991, Bulsara has proposed a single neuron model containing stochastic resonance and it can prove that the stochastic resonance can make weak signal-noise ratio increase up to fifteen dB through the simulation experiments [8].

The scientists conducted the experiment by using the sciatic nerve of toad and stimulating neurons with exogenous signals [9]. What is needed to explain is that the sciatric nerve is a kind of system which structure and electronic features are similar to those of human auditory nerve. The experimental results showed that when a certain amount of noise is added to the input weak signal for the simulation of electronic excitation, the signal-noise ratio of the input signal will reach the maximum. So people guess, for deaf patients, a weak signal that originally failed to excite neurons may cause the stochastic resonance to improve the signal-noise ratio through the input noise, it will be identified by neurons and cause the neurons to be excited [10]. Thus stochastic resonance technology will become a practical tool for the treatment of hearing injury.

5. Conclusion
This paper takes the weak signal detection under strong noise background as the main line, it expounds the principle of white noise canceling external noise in stochastic resonance and summarizes the principle of weak signal detection relating to the stochastic resonance and its application in other fields briefly. It mainly introduces the aspect that noise benefits mankind, looking forward to attracting more researchers’ attention to stochastic resonance technology. Due to stochastic resonance is still in the initial stage in signal processing field, many aspects of research are not involved in this paper. For example, study on the effect of stochastic resonance under different types of noise, whether other types of noise can counteract the external noise, whether stochastic resonance still exists in linear systems. There are still a series of problems need to be studied and proved to give full play to the unique advantages of stochastic resonance technology.

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