Two Nonhomogeneous Boundary Value Problems for a Rectangle: Exact Solutions

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Abstract. In the paper, for the first time we give exact solutions to two nonhomogeneous boundary value problems of the theory of elasticity for a rectangle with free long sides. Inside the rectangle there are applied two equal concentrated forces directed oppositely along the horizontal axis (even-symmetric deformation). The method of solution is based on the use of the solution to the biharmonic problem for a smooth semi-strip and the method of the integral Fourier transform. In the first problem, the short sides of the rectangle are free; in the second, they are rigidly clamped. The solutions to both problems are constructed on the superposition principle in the form of the sum of integrals and series in trigonometric functions and Papkovich–Fadle eigenfunctions. The coefficients of these expansions are determined by simple formulas as the Fourier integrals of given boundary functions.

1. Introduction
Much fewer publications are devoted to nonhomogeneous boundary value problems of the theory of elasticity than to homogeneous ones. Typically, they are more complicated and laborious than homogeneous ones. In textbooks on the theory of elasticity, as a rule, only a solution to a nonhomogeneous boundary value problem for an unbounded plane is given (we are talking about two-dimensional boundary value problems). Solutions for finite domains, for example, for a rectangle, obtained on the basis of approximate analytical and numerical methods, are also extremely small. Meanwhile, these problems are very important in engineering.

In the paper [1], a closed form analytical solution to a nonhomogeneous boundary value problem for a semi-strip clamped at the end was constructed. In this paper, we consider a nonhomogeneous boundary value problem of the theory of elasticity for a rectangle. The long sides of the rectangle are free; inside the rectangle there are applied two equal concentrated forces directed oppositely along the horizontal axis. We will consider only even-symmetric deformation with respect to the central axes. The method of solution is based on the solution to the biharmonic problem for a smooth semi-strip [2–5] and the method of the integral Fourier transform [6]. Two cases of boundary conditions at the short sides of the rectangle are considered. In the first case, the short sides of the rectangle are free, while, in the second, these sides are rigidly clamped. The solutions to both problems are constructed on the superposition principle in the form of the sum of integrals and series in trigonometric functions and Papkovich–Fadle eigenfunctions. The coefficients of these series are determined exactly by means of functions that are biorthogonal to the Papkovich–Fadle eigenfunctions. These biorthogonal functions are found with the help of the Borel transform in the class of quasi-entire functions of exponential type [7].
2. Formulation of boundary value problems

Let us consider the rectangle \( \Pi: |y| \leq 1, |x| \leq d \). We will assume that the longitudinal sides \( y = \pm 1 \) are free, i.e.
\[
\sigma_y(x, \pm 1) = \tau_{xy}(x, \pm 1) = 0.
\]

Suppose that, inside the rectangle at the points \( x = a \) and \( x = -a \), two equal concentrated forces \( P \) directed oppositely along the \( x \)-axis are applied, i.e.
\[
\sigma_y(\pm a, 0) = P\delta(x \mp a), \quad \tau_{xy}(\pm a, 0) = 0,
\]
where \( \delta \) is the delta function.

Let us consider two cases of boundary conditions at the short edges of the rectangle:

1. The short edges of the rectangle are free (figure 1), i.e.
\[
\sigma_y(\pm d, y) = \tau_{xy}(\pm d, y) = 0;
\]
2. The short edges of the rectangle are rigidly clamped (figure 2), i.e.
\[
u(\pm d, y) = 0.
\]

3. Solution to the first boundary value problem

The solution of the problem is constructed on the superposition principle and consists of the following stages.

I. First, we consider a nonhomogeneous periodic problem for a plane in which two equal forces \( P \) directed oppositely along the \( x \)-axis are applied at the points \( x = a \) and \( x = -a \). Take for definiteness \( P = 1 \). Then, the solution to the problem has the following form [1]:
\[
U^i(x, y) = \frac{1}{8} \sum_{k=1}^{\infty} \left[ (1 + \nu) \left| x - a \right| + \frac{3 - \nu}{k\pi} \right] e^{-1/4\pi^2 k\pi} \cos k\pi y - \frac{1}{8} \sum_{k=1}^{\infty} \left[ (1 + \nu) \left| x + a \right| + \frac{3 - \nu}{4k\pi} \right] e^{-1/4\pi^2 k\pi} \cos k\pi y,
\]
\[
V^i(x, y) = \frac{1}{8} \sum_{k=1}^{\infty} \left[ (1 + \nu) (x - a) e^{-1/4\pi^2 k\pi} \sin k\pi y - \frac{1}{8} \sum_{k=1}^{\infty} \left[ (1 + \nu) (x + a) e^{-1/4\pi^2 k\pi} \sin k\pi y,
\right.
\]
\[
\sigma_x^i(x, y) = -\frac{1}{4} \left[ \text{sgn}(x - a) - \text{sgn}(x + a) \right] - \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x - a \right| + \text{sgn}(x - a) \right] e^{-1/4\pi^2 k\pi} \cos k\pi y
\]
\[
+ \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x + a \right| + \text{sgn}(x + a) \right] e^{-1/4\pi^2 k\pi} \cos k\pi y,
\]
\[
\sigma_y^i(x, y) = -\frac{1}{4} \left[ \text{sgn}(x - a) - \text{sgn}(x + a) \right] - \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x - a \right| + \nu \text{sgn}(x - a) \right] e^{-1/4\pi^2 k\pi} \cos k\pi y
\]
\[
+ \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x + a \right| + \nu \text{sgn}(x + a) \right] e^{-1/4\pi^2 k\pi} \cos k\pi y,
\]
\[
\tau_{xy}^i(x, y) = -\frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x - a \right| + \frac{1 - \nu}{2} \right] e^{-1/4\pi^2 k\pi} \sin k\pi y + \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{k\pi}{1 + \nu} \left| x + a \right| + \frac{1 - \nu}{2} \right] e^{-1/4\pi^2 k\pi} \sin k\pi y.
\]
II. Let us cut the strip $\{\Omega: |x|<\infty, |y|\leq 1\}$ out of the plane and replace the action of the discarded part with the normal load (the tangential stress here is equal to zero) defined by formula (5) for $y = \pm 1$, i.e. $Y_0(x) = -\sigma_1'(x, \pm 1)$. For that, we add to solution I the solution for the strip $\Omega$ in which the normal stress $Y_0(x)$ is given on the longitudinal sides. Thus, we will obtain zero values of normal and tangential stresses on the longitudinal sides of the strip.

We find the Fourier transform of the function $Y_0(x)$:

$$
\tilde{Y}_0(\lambda) = -\frac{1}{2} \sum_{k=1}^{\infty} \left[ (-1)^k \left( -\frac{1 + \nu}{4} (k\pi)^2 - \frac{16\lambda \sin \lambda a}{(\lambda^2 + (k\pi)^2)^2} + \nu\frac{4\lambda \sin \lambda a}{\lambda^2 + (k\pi)^2}\right) + \nu\frac{\sin \lambda a}{\lambda} \right].
$$

The solution for the strip $\Omega$ in which the normal load is equal to $Y_0(x)$ has the form [1]:

$$
U^2(x, y) = \frac{1}{\pi} \int_0^\infty \left[ \frac{1 - \nu}{2} \sinh \lambda - \frac{1 + \nu}{2} \cosh \lambda \right] \cosh \lambda y + \frac{1 + \nu}{2} \lambda y \sinh \lambda y \frac{\tilde{Y}_0(\lambda) \sin \lambda x d\lambda}{(1 + \nu)\lambda \sinh \lambda \cosh \lambda},
$$

$$
V^2(x, y) = \frac{1}{\pi} \int_0^\infty \left[ \frac{1 + \nu}{2} \cosh \lambda + \sinh \lambda \right] \sinh \lambda y - \frac{1 + \nu}{2} \lambda y \cosh \lambda y \frac{\tilde{Y}_0(\lambda) \cos \lambda x d\lambda}{(1 + \nu)\lambda \sinh \lambda \cosh \lambda},
$$

$$
\sigma^x_0(x, y) = \frac{1}{\pi} \int_0^\infty (1 + \nu) \frac{\lambda}{(1 + \nu)\lambda \cosh \lambda} \left[ \left( \sinh \lambda - \lambda \cosh \lambda \right) \cosh \lambda y + \lambda y \sinh \lambda y \right] \frac{\tilde{Y}_0(\lambda) \cos \lambda x d\lambda}{\lambda \sinh \lambda \cosh \lambda},
$$

$$
\sigma^y_0(x, y) = \frac{1}{\pi} \int_0^\infty (1 + \nu) \frac{\lambda}{(1 + \nu)\lambda \cosh \lambda} \left[ \left( \cosh \lambda + \lambda \sinh \lambda \right) \cosh \lambda y - \lambda y \sinh \lambda y \right] \frac{\tilde{Y}_0(\lambda) \cos \lambda x d\lambda}{\lambda \cosh \lambda \sinh \lambda},
$$

$$
\tau^x_0(x, y) = -\frac{1}{\pi} \int_0^\infty \left( \frac{1 + \nu}{2} \lambda^2 \left( \cosh \lambda \sinh \lambda y - y \sinh \lambda \cosh \lambda y \right) \frac{\tilde{Y}_0(\lambda) \sin \lambda x d\lambda}{(1 + \nu)\lambda \cosh \lambda \sinh \lambda},
$$

III. Let us relieve the stresses $\sigma^x(\pm d, y) = \sigma^x(y)$ and $\tau^x(\pm d, y) = \tau^x(y)$ defined by formula (5) for $x = \pm d$. To perform this, we need to use the solution to a boundary value problem for the free rectangle $\Pi$ in which the normal stress $-\sigma^x(y)$ and tangential stress $-\tau^x(y)$ are specified at the short edges. This solution is written in the form of expansions in Papkovich–Fadle eigenfunctions [8]:

$$
U^3(x, y) = \sum_{k=1}^{\infty} \text{Re} \left[ \frac{\xi(\lambda_k y)}{\lambda_k M_k} \left[ \sigma^x_k \text{Im}(\bar{\lambda}_k \sinh \lambda_k d \sinh \lambda_k x) + \tau^x_k \text{Im}(\bar{\lambda}_k \cosh \lambda_k d \sinh \lambda_k x) \right] \right],
$$

$$
V^3(x, y) = \sum_{k=1}^{\infty} \text{Re} \left[ \frac{\chi(\lambda_k y)}{M_k} \left[ -\sigma^x_k \text{Im}(\bar{\lambda}_k \sinh \lambda_k d \cosh \lambda_k x) - \tau^x_k \text{Im}(\bar{\lambda}_k \cosh \lambda_k d \cosh \lambda_k x) \right] \right],
$$

$$
\sigma^y_0(x, y) = \sum_{k=1}^{\infty} \text{Re} \left[ \frac{s^y_k(\lambda_k y)}{\lambda_k M_k} \left[ \sigma^y_k \text{Im}(\bar{\lambda}_k \sinh \lambda_k d \cosh \lambda_k x) - \tau^y_k \text{Im}(\bar{\lambda}_k \cosh \lambda_k d \cosh \lambda_k x) \right] \right],
$$

$$
\sigma^y_0(x, y) = \sum_{k=1}^{\infty} \text{Re} \left[ \frac{s^y_k(\lambda_k y)}{\lambda_k M_k} \left[ \sigma^y_k \text{Im}(\bar{\lambda}_k \sinh \lambda_k d \cosh \lambda_k x) - \tau^y_k \text{Im}(\bar{\lambda}_k \cosh \lambda_k d \cosh \lambda_k x) \right] \right],
$$

$$
\tau^y_0(x, y) = \sum_{k=1}^{\infty} \text{Re} \left[ \frac{t^y_k(\lambda_k y)}{\lambda_k M_k} \left[ \sigma^y_k \text{Im}(\bar{\lambda}_k \sinh \lambda_k d \sinh \lambda_k x) - \tau^y_k \text{Im}(\bar{\lambda}_k \cosh \lambda_k d \sinh \lambda_k x) \right] \right],
$$

where $M_k = \cos^2 \lambda_k$, the numbers $\lambda_k, \bar{\lambda}_k$ (Re $\lambda_k < 0$) are the complex zeros of the entire function $L(\lambda) = \lambda(\lambda + \sin \lambda \cos \lambda)$.
Here the functions $\xi(\lambda, y)$, $\chi(\lambda, y)$, $s_\nu(\lambda, y)$, $s_\nu(\lambda, y)$, $t_\nu(\lambda, y)$ are Papkovich–Fadle eigenfunctions and have the following form:

$$\xi(\lambda, y) = \frac{1 - \nu}{2} \sin \lambda_k + \frac{1 + \nu}{2} \lambda_k \cos \lambda_k \cos \lambda_k y - \frac{1 + \nu}{2} \lambda_k \sin \lambda_k \sin \lambda_k y,$$

$$\chi(\lambda, y) = \frac{1 + \nu}{2} \lambda_k \sin \lambda_k \sin \lambda_k y + \sin \lambda_k \cos \lambda_k \cos \lambda_k y,$$

$$s_\nu(\lambda, y) = (1 + \nu) \lambda_k \{\sin \lambda_k - \lambda_k \cos \lambda_k \cos \lambda_k y - \lambda_k \sin \lambda_k \sin \lambda_k y\},$$

$$s_\nu(\lambda, y) = (1 + \nu) \lambda_k \{\sin \lambda_k + \lambda_k \cos \lambda_k \cos \lambda_k y + \lambda_k \sin \lambda_k \sin \lambda_k y\},$$

$$t_\nu(\lambda, y) = (1 + \nu) \lambda_k^2 \{\cos \lambda_k \sin \lambda_k y - \sin \lambda_k \cos \lambda_k y\}.$$

The equations to determine the biorthogonal systems of functions $\{U_k(y)\}_{k=1}^\infty$, $\{V_k(y)\}_{k=1}^\infty$, $\{X_k(y)\}_{k=1}^\infty$, $\{Y_k(y)\}_{k=1}^\infty$ and $\{T_k(y)\}_{k=1}^\infty$ corresponding to the Papkovich–Fadle eigenfunctions (9) appear as follows [2–4]:

$$\int_{-\infty}^{\infty} \xi(\lambda, y)U_k(y)dy = \frac{\lambda L(\lambda)}{\lambda_k^2 - \lambda_k^2}, \int_{-\infty}^{\infty} \chi(\lambda, y)V_k(y)dy = \frac{L(\lambda)}{\lambda_k^2 - \lambda_k^2}, \int_{-\infty}^{\infty} s_\nu(\lambda, y)X_k(y)dy = \frac{\lambda L(\lambda)}{\lambda_k^2 - \lambda_k^2}, \int_{-\infty}^{\infty} t_\nu(\lambda, y)Y_k(y)dy = \frac{\lambda L(\lambda)}{\lambda_k^2 - \lambda_k^2} (k \geq 1).$$

The numbers $\sigma_k$ and $\tau_k$ are determined from equations (10) as $\lambda \to 0$:

$$\sigma_k = \frac{1}{2} \sum_{m=1}^{\infty} \frac{m\pi(1 + \nu)(d - a)}{2} + \operatorname{sgn}(d - a) \int_{-\infty}^{\infty} e^{m\pi\nu \lambda_k} \frac{L(m\pi)}{[m\pi^2 - \lambda_k^2][(1 + \nu)(m\pi)^2 \cos m\pi]} \frac{\lambda L(m\pi)}{[m\pi^2 - \lambda_k^2][(1 + \nu)(m\pi)^2 \cos m\pi]}$$

$$\tau_k = \frac{1}{2} \sum_{m=1}^{\infty} \frac{m\pi(1 + \nu)(d + a)}{2} + \frac{1 - \nu}{2} \int_{-\infty}^{\infty} e^{m\pi\nu \lambda_k} \frac{m\pi L(m\pi)}{[m\pi^2 - \lambda_k^2][(1 + \nu)(m\pi)^2 \cos m\pi]}$$

$$+ \frac{1}{2} \sum_{m=1}^{\infty} \frac{m\pi(1 + \nu)(d + a)}{2} + \frac{1 - \nu}{2} \int_{-\infty}^{\infty} e^{m\pi\nu \lambda_k} \frac{m\pi L(m\pi)}{[m\pi^2 - \lambda_k^2][(1 + \nu)(m\pi)^2 \cos m\pi]} (k \geq 1).$$

IV. Let us relieve the stresses $\sigma_\nu(\pm d, y) = \sigma_\nu(y)$ and $\tau_\nu(\pm d, y) = \tau_\nu(y)$ defined by formula (7) for $x = \pm d$. In order to do this, we should use the solution to a boundary value problem for the free rectangle $\Pi$ when the normal stress $-\sigma_\nu(y)$ and tangential stress $-\tau_\nu(y)$ are specified at the short edges. This solution is described by the same formulas (8), but with different coefficients:

$$\sigma_k = \frac{1}{(1 + \nu)\pi} \int_{\lambda_k}^{\infty} \frac{\lambda Y(\lambda)}{\lambda_k^2 + \lambda_k^2} \cos(\lambda d)d\lambda, \tau_k = \frac{1}{(1 + \nu)\pi} \int_{\lambda_k}^{\infty} \frac{\lambda Y(\lambda)}{\lambda_k^2 + \lambda_k^2} \sin(\lambda d)d\lambda.$$

Thus, the complete solution to the problem is the sum of solutions I–IV. Figures 3 and 4 show the graphs illustrating the solution. It is assumed that $d = 2$, $a = 0.6$, $\nu = 1/3$. 

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4. Solution to the second boundary value problem

The algorithm for solving the problem is as follows.

I–II. The first two stages are the same as in the first problem.

III. Next, we need to relieve the displacements $U^i(\pm d, y) = u_i(y)$ and $V^i(\pm d, y) = v_i(y)$ defined by formula (5) for $x = \pm d$. For this, we have to use the solution to a boundary value problem for the free rectangle $\Pi$ in which the longitudinal displacement $-u_i(y)$ and transverse displacement $-v_i(y)$ are given at the short edges. This solution is written in the form of expansions in Papkovich–Fadle eigenfunctions [8]:

$$U^i(x, y) = \sum_{k=1}^{\infty} 2\text{Re} \left\{ \frac{\xi(\lambda_k, y)}{\lambda_k M_k} \left[ -\lambda_k v_k \frac{\text{Im}(\lambda_k \cos \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \cos \lambda_k d \cos \lambda_k d} + u_k \frac{\text{Im}(\lambda_k \sin \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \sin \lambda_k d \sin \lambda_k d} \right] \right\},$$

$$V^i(x, y) = \sum_{k=1}^{\infty} 2\text{Re} \left\{ \frac{\chi(\lambda_k, y)}{M_k} \left[ v_k \frac{\text{Im}(\lambda_k \cos \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \cos \lambda_k d \cos \lambda_k d} + u_k \frac{\text{Im}(\lambda_k \sin \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \sin \lambda_k d \sin \lambda_k d} \right] \right\},$$

$$\sigma^i_x(x, y) = \sum_{k=1}^{\infty} 2\text{Re} \left\{ \frac{s(\lambda_k, y)}{M_k} \left[ \lambda_k v_k \frac{\text{Im}(\lambda_k \cos \lambda_k d \cos \lambda_k x)}{\text{Im}(\lambda_k) \cos \lambda_k d \cos \lambda_k d} + u_k \frac{\text{Im}(\lambda_k \sin \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \sin \lambda_k d \sin \lambda_k d} \right] \right\},$$

$$\sigma^i_y(x, y) = \sum_{k=1}^{\infty} 2\text{Re} \left\{ \frac{t(\lambda_k, y)}{M_k \lambda_k} \left[ \lambda_k v_k \frac{\text{Im}(\lambda_k \cos \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \cos \lambda_k d \cos \lambda_k d} + u_k \frac{\text{Im}(\lambda_k \sin \lambda_k d \sin \lambda_k x)}{\text{Im}(\lambda_k) \sin \lambda_k d \sin \lambda_k d} \right] \right\}. $$

Here the numbers $\lambda_k$, $M_k$ and Papkovich–Fadle eigenfunctions are the same as in the first problem, while the numbers $u_k$ and $v_k$ are determined from equations (10) as $\lambda \to 0$:

$$v_k = -\frac{1}{4} \sum_{m=1}^{\infty} (d-a) e^{-\pi \nu |d-a|} \frac{L(m\pi)}{m\pi \cos m\pi (m\pi)^2 - \lambda_k^2} + \frac{1}{4} \sum_{m=1}^{\infty} (d+a) e^{-\pi \nu |d+a|} \frac{L(m\pi)}{m\pi \cos m\pi (m\pi)^2 - \lambda_k^2},$$

$$u_k = \frac{1}{4} \sum_{m=1}^{\infty} \left[ (1+\nu)|d-a| + \frac{3-\nu}{m\pi} \right] e^{-\pi \nu |d-a|} \frac{m\pi L(m\pi)}{(m\pi)^2 - \lambda_k^2} \left[ -(1+\nu)m\pi \cos m\pi \right]$$

$$-\frac{1}{4} \sum_{m=1}^{\infty} \left[ (1+\nu)|d+a| + \frac{3-\nu}{m\pi} \right] e^{-\pi \nu |d+a|} \frac{m\pi L(m\pi)}{(m\pi)^2 - \lambda_k^2} \left[ -(1+\nu)m\pi \cos m\pi \right],$$

IV. Next, we need to relieve the displacements $U^i(\pm d, y) = u_i(y)$ and $V^i(\pm d, y) = v_i(y)$ defined by formula (7) for $x = \pm d$. In order to do this, we ought to use the solution to a boundary value
problem for the free rectangle Π when the longitudinal displacement \( -u_1(y) \) and transverse displacement \( -v_2(y) \) are given at the short edges. This solution is described by the same formulas (13), but with different coefficients:

\[
v_i = \frac{1}{(1 + \nu)\pi} \int_0^\infty \frac{\tilde{Y}_i(\lambda)}{\lambda^2 + \lambda_i^2} \cos(\lambda d) d\lambda, \quad u_i = \frac{1}{(1 + \nu)\pi} \int_0^\infty \frac{\lambda \tilde{Y}_i(\lambda)}{\lambda^2 + \lambda_i^2} \sin(\lambda d) d\lambda.
\]

The complete solution to the problem is also the sum of the four solutions.

5. Conclusion

In this paper, we have considered examples of solving two nonhomogeneous boundary value problems for a rectangle with free horizontal sides, inside which there are applied two concentrated forces that are equal in magnitude and directed oppositely along the horizontal axis (even-symmetric deformation relative to the central axes). In the first problem, the short edges of the rectangle are free, and in the other, they are rigidly clamped. The solutions to the problems are exact because they are not reduced to infinite systems of algebraic equations, but are represented in the form of Fourier integrals and series whose coefficients are found by closed formulas. The solution for a rectangle clamped at the short edges does not have, at the angular points, the singularity that appears in the solution to the corresponding problem for an infinite wedge. This is due to the fact that the type of boundary conditions does not change along one coordinate line (as in the wedge problem), but along two mutually perpendicular coordinate directions [9]. The obtained solutions can be generalized to a rectangle of arbitrary width and to any load applied inside the rectangle.

Acknowledgments

This work was supported by the Russian Science Foundation, Grant No. 19-71-00094.

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