The $\bar{p}p \rightarrow \phi\phi$ reaction in an effective Lagrangian approach

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We investigate the $\bar{p}p \rightarrow \phi\phi$ reaction within an effective Lagrangian approach. We show that the inclusion of either a scalar meson $f_0$ or a tensor meson $f_2$ in the $s$-channel can lead to a fairly good description of the bump structure of the total cross section around the invariant $\bar{p}p$ mass $W \simeq 2.2$ GeV, which cannot be reproduced with only the “background” contributions from $t$- and $u$-channel $N^*(1535)$ resonance as studied in a previous work. From the fits, we infer the properties of the involved scalar or tensor resonances.

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I. INTRODUCTION

According to the naive constituent quark model, the $\phi$ meson is believed to be an almost pure $\bar{s}s$ state, \footnote{In the quark model, the physical isoscalars $\phi$ and $\omega$ are mixtures of the SU(3) wave function $\psi_s$ and $\psi_1$: \[ \phi = \psi_s\cos\theta - \psi_1\sin\theta, \] \[ \omega = \psi_s\sin\theta + \psi_1\cos\theta, \] where $\theta$ is the nonet mixing angle and: \[ \psi_s = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \] \[ \psi_1 = \frac{1}{\sqrt{3}}(u\bar{d} + d\bar{u} + s\bar{s}). \] For ideal mixing, $\tan\theta = 1/\sqrt{2}$ (or $\theta = 35.3^0$), the $\phi$ meson becomes pure $\bar{s}s$ state.} while there are only up and down quarks (antiquarks) in the nucleon (antineutron). Thus the $\bar{p}p \rightarrow \phi\phi$ reaction, with its disconnected quark lines, should be suppressed according to the Okubo-Zee-Iizuka (OZI) rule \footnote{Experimentally, the mixing angle $\theta$ is 36.4$^0$.}. However, even the OZI rule is strictly enforced by nature, the $\bar{p}p$ reaction can still proceed through the non-strange quark component of the $\phi$ meson, because of the slight discrepancy from the ideal mixing of the vector meson singlet and octet \footnote{Electronic address: xiejunjun@impcas.ac.cn}. \footnote{This is the reason why the $\bar{p}p \rightarrow \phi\phi$ reaction could yield valuable information on the strangeness of the neutron and nucleon resonances.} With this small discrepancy, one can determine an upper limit for the total cross section of $\bar{p}p \rightarrow \phi\phi$ reaction by comparison to the total cross section of the related $\bar{p}p \rightarrow \omega\omega$ reaction. This yields a cross section for $\bar{p}p \rightarrow \phi\phi$ at the order of 10 nb \footnote{We propose to introduce $s$-channel contributions via extrinsic interaction between quarks \cite{12}. In Refs. \cite{3,6,13}, considerable admixture of $\bar{ss}$ components in the nucleon was proposed to explain the large OZI violation in $\bar{p}p$ annihilation. As a result, it is often advocated that study of the $\bar{p}p \rightarrow \phi\phi$ reaction could be explained by considering the two-step hadronic loops in which each individual transition is OZI-allowed \cite{14,13}. Based on this, the role played by two-meson ($KK$) and antihyperon-hyperon ($\Lambda\Lambda$) intermediate states in the $\bar{p}p \rightarrow \phi\phi$ reaction have been studied by Lu et al. \cite{16} and Mull et al. \cite{17}, respectively. All the aforementioned models are able to predict the order of magnitude of the cross section, but not the detailed shape of the observed spectrum \cite{2}, where there is a bump around the invariant $\bar{p}p$ mass $W \simeq 2.2$ GeV, which might hint at a sizable contribution from a scalar or tensor meson in the $s$-channel.}.

Recently, Shi et al. \cite{18} extended the work of Ref. \cite{19} to study the $\bar{p}p \rightarrow \phi\phi$ reaction by including the contributions from the $N^*(1535)$ resonance in the $t$- and $u$-channel. They showed that this new mechanism may give significant contributions to the $\bar{p}p \rightarrow \phi\phi$ reaction, especially for the invariant $\bar{p}p$ mass $W$ above 2.3 GeV. However, the bump structure below 2.3 GeV could not be reproduced, hinting at the necessity of including contributions from the $s$-channel.

In the present work, we reanalyze the $\bar{p}p \rightarrow \phi\phi$ reaction within an effective Lagrangian approach and the isobar model. In addition to the “background” contributions from the $N^*(1535)$ resonance studied in Ref. \cite{18}, we propose to introduce $s$-channel contributions via either a scalar meson $f_0$ or a tensor meson $f_2$. Given the fact that the information about the $f_0$ and $f_2$ meson with mass around 2.2 GeV is scarce \cite{2}, we take the masses, the
total decay width, and the coupling constants, \( g_{f_0\bar{p}p}g_{f_0\phi\phi} \)
and \( g_{f_2\bar{p}p}g_{f_2\phi\phi} \) as free parameters, which will be fitted to the experimental data on the \( \bar{p}p \to \phi \phi \) reaction \[5\]. In this respect, we show in this work how the experimental study on the \( \bar{p}p \to \phi \phi \) reaction may lead to the discovery of a strangeness scalar or tensor resonance, or both around 2.2 GeV.

This paper is organized as follows. In Section II, we present the formalism and ingredients of our calculation. Numerical results and discussions are given in Section III, followed by a short summary in Section IV.

II. FORMALISM AND INGREDIENTS

The effective Lagrangian method is an important theoretical tool in describing the various processes around the resonance region. In this section, we introduce the theoretical formalism and ingredients to study the \( \bar{p}p \to \phi \phi \) reaction by using the effective Lagrangian method.

The basic tree level Feynman diagrams for the \( \bar{p}p \to \phi \phi \) reaction are depicted in Fig. 1. In addition to the “background” diagrams, such as the t-channel [Fig. 1 (b)] and u-channel [Fig. 1 (c)] \( N^*(1535) \) resonance exchange which have been considered in the previous calculation \[18\], we include the s-channel diagram [Fig. 1 (a)] through either a scalar meson \( f_0 \) or a tensor meson \( f_2 \) in our present calculation.

![Feynman diagrams for \( \bar{p}p \to \phi \phi \) reaction](image)

**FIG. 1:** Feynman diagrams for \( \bar{p}p \to \phi \phi \) reaction. The contributions from t- and u-channel \( N^*(1535) \) resonance exchange, and s-channel \( f_0 \) or \( f_2 \) resonance are considered.

The invariant scattering amplitudes that enter our model for the calculation of the total and differential cross sections for the reaction

\[
\bar{p}(p_1, s_1) p(p_2, s_2) \to \phi(p_3, \lambda_1) \phi(p_4, \lambda_2)
\]

are defined as

\[
- iT_i = \bar{\nu}(p_1, s_1) A_i^{\mu\nu} u(p_2, s_2) \epsilon_\nu^\ast(p_3, \lambda_1) \epsilon_\mu^\ast(p_4, \lambda_2),
\]

where \( \nu(p_1, s_1) \) and \( u(p_2, s_2) \) are Dirac spinors for anti-proton and proton, respectively, while \( \epsilon_\mu(p_3, \lambda_1) \) and \( \epsilon_\nu(p_4, \lambda_2) \) are polarization vectors for the \( \phi \) mesons. The subscript \( i \) stands for the s-channel \( f_0 \) or \( f_2 \) process, the t- and u-channel \( N^*(1535) \) resonance exchange.

The explicit expressions for the reduced \( A_i^{\mu\nu} \) amplitudes can be found in Ref. \[18\]. Here, we only give details about the s-channel \( f_0 \) and \( f_2 \) amplitudes, \( A_i^{\mu\nu} \), associated to the diagram of Fig. 1 (a). They are obtained from the following effective interaction Lagrangian \[20\]–\[22\]:

\[
L_{f_0\bar{p}p} = g_{f_0\bar{p}p} \bar{\psi}_\bar{p} f_0 \psi_p + h.c.,
L_{f_0\phi} = g_{f_0\phi\phi} m_\phi \phi_\mu f_0^\mu,
L_{f_2\bar{p}p} = g_{f_2\bar{p}p} \bar{\psi}_\bar{p} (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi_p f_2^{\mu\nu} + h.c.,
L_{f_2\phi} = g_{f_2\phi\phi} m_\phi \phi_\mu f_2^{\mu\nu}.
\]

With the above Lagrangians, the reduced \( A_i^{\mu\nu} \) amplitudes in Eq. (6) can be easily obtained,

\[
A_i^{\mu\nu} = -g_{f_0\bar{p}p} g_{f_0\phi\phi} m_\phi G_{f_0}(q_s) g^{\mu\nu} f_s,
A_i^{\mu\nu} = i g_{f_2\bar{p}p} g_{f_2\phi\phi} m_\phi [\gamma_\rho(p_1 - p_2)_\sigma + \gamma_\sigma(p_1 - p_2)_\rho] \times G_{f_2}^{\rho\sigma}(q_s) f_s,
\]

where the propagators for the scalar meson \( f_0 \) and the tensor meson \( f_2 \) are, respectively,

\[
G_{f_0}(q_s) = \frac{i}{s - M_{f_0}^2 + i M_{f_0} \Gamma_{f_0}},
G_{f_2}^{\rho\sigma}(q_s) = \frac{i}{s - M_{f_2}^2 + i M_{f_2} \Gamma_{f_2}} \rho^{\rho\sigma}(q_s),
\]

and

\[
P^{\rho\sigma}(q_s) = \frac{1}{2}(\bar{g}^{\rho\sigma} g^{-\rho\sigma} + \bar{g}^{\rho\sigma} g^{-\rho\sigma}) - \frac{1}{3} \bar{g}^{\rho\sigma} g^{\rho\sigma},
\bar{g}^{\rho\sigma} = -g^{\rho\sigma} + \frac{q^\rho q^\sigma}{s},
\]

with \( q_s = p_1 + p_2 \) the momentum of \( f_0 \) or \( f_2 \) and \( s = q_s^2 \) the invariant mass square of the \( \bar{p}p \) system.

As can be seen from Eqs. (11) and (12), in the tree-level approximation, only the products, \( g_{f_0\bar{p}p} g_{f_0\phi\phi} \) and \( g_{f_2\bar{p}p} g_{f_2\phi\phi} \) enter the invariant amplitudes, \( M_{f_0} \) (\( M_{f_2} \)) and \( \Gamma_{f_0} \) (\( \Gamma_{f_2} \)) are the mass and the total decay width of the \( f_0 \) (\( f_2 \)) meson. We take them as free parameters and determine them by fitting to the total cross section of the \( \bar{p}p \to \phi \phi \) reaction \[3\] using MINUIT.

In Eqs. (11) and (12), we have also included the relevant off shell form factors \[3\] for \( f_0 \) and \( f_2 \) mesons. We

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3 We take the following form factor for t- and u-channel \( N^*(1535) \) (\( \equiv N^* \)) resonance exchange as in Ref. \[18\]:

\[
F_{N^*(1535)} = \frac{\Lambda_{N^*}^2 - M_{N^*}^2}{\Lambda_{N^*}^2 - q_{N^*}^2},
\]

with \( q_{N^*}^2 \) the 4-momentum of the exchanged \( N^*(1535) \) resonance. In general, the cutoff parameter \( \Lambda_{N^*} \) for \( N^*(1535) \) resonance should be at least a few hundred MeV larger than the \( N^*(1535) \) mass, and thus in the range of 2 to 4 GeV.
adopt here the common scheme used in many previous works,

\[ f_s = \frac{\Lambda_i^4}{\Lambda_i^4 + (s - M_i^2)^2}, \quad i = f_0, f_2. \]

(17)

The cutoff parameters, \( \Lambda_{f_0} \) and \( \Lambda_{f_2} \), are constrained between 0.6 and 1.2 GeV. This way, we can reduce the number of free parameters.

III. NUMERICAL RESULTS AND DISCUSSION

The differential cross section for \( \bar{p}p \rightarrow \phi \phi \) reaction at the center of mass (c.m.) frame can be expressed as

\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi s} \left| \vec{p}_{f}^{c.m.} \right| \left( \frac{1}{4} \sum_{s_1 t_2 s_2 t_2} \left| T \right|^2 \right),
\]

where \( \theta \) denotes the angle of the outgoing \( \phi \) meson relative to the beam direction in the c.m. frame, while \( \vec{p}_{f}^{c.m.} \) and \( \vec{p}_{i}^{c.m.} \) are the 3-momentum of the initial \( \bar{p} \) and final \( \phi \) meson.

First, by including the contributions from the s-channel scalar meson \( f_0 \) \(^4\) and t- and u-channel \( N^*(1535) \) resonance (corresponding to \( T = T_{f_0} + T_{N^*(1535)} \)), with fixed cutoff parameters \( \Lambda_{f_0} \) and \( \Lambda_{N^*(1535)} \), we perform a \( \chi^2 \) fit (Fit I) to the total cross section data for \( \bar{p}p \rightarrow \phi \phi \) \([5]\). There are a total of 20 data points.

By constraining the value of the cutoff parameter \( \Lambda_{f_0} \) between 0.6 and 1.2 GeV and \( \Lambda_{N^*(1535)} \) around 3.0 GeV based on the results of Ref. \([18]\), we obtain a minimal \( \chi^2 / \text{d.o.f.} = 2.1 \) with \( \Lambda_{f_0} = 0.6 \) GeV and \( \Lambda_{N^*(1535)} = 3.05 \) GeV. The fitted parameters are: \( g_{f_0\bar{p}p}g_{f_0\phi\phi} = 0.45 \pm 0.08, \)

\( M_{f_0} = 2174 \pm 3 \) MeV, and \( \Gamma_{f_0} = 167 \pm 27 \) MeV.

Second, instead of a scalar meson, we study the case of a tensor meson \( f_2 \) in the s-channel and t- and u-channel \( N^*(1535) \) resonance (corresponding to \( T = T_{f_2} + T_{N^*(1535)} \)), and we perform a second \( \chi^2 \) fit (Fit II). In this case, we get a minimal \( \chi^2 / \text{d.o.f.} = 1.4 \) with \( \Lambda_{f_2} = 0.65 \) GeV and \( \Lambda_{N^*(1535)} = 3.05 \) GeV. The fitted parameters are: \( g_{f_2\bar{p}p}g_{f_2\phi\phi} = -0.12 \pm 0.02, \)

\( M_{f_2} = 2192 \pm 4 \) MeV, and \( \Gamma_{f_2} = 177 \pm 30 \) MeV.

Based on the value of the \( \chi^2 / \text{d.o.f.}, \) Fit II is preferred to Fit I. It seems to indicate that the \( \bar{p}p \rightarrow \phi \phi \) reaction is dominated by the exchange of a strange tensor meson with quantum number \( J^{PC} = 2^{++} \) in the s-channel, in agreement with the study of Ref. \([5]\). In addition, a partial-wave analysis of the \( \pi^+ p \rightarrow \phi \phi n \) reaction shows that the \( \phi \phi \) system is dominant by two \( J^{PC} = 2^{++} \)

\[ \text{states} \]

\[ \text{one an S wave and the other a D wave. The mass of the S wave state is } M = 2160 \pm 50 \text{ MeV, with a decay width } \Gamma = 310 \pm 70 \text{ MeV. The mass is in agreement with our fitted result for the tensor meson.} \]

Next, we show the corresponding fitted results for the total cross sections in Fig. 2 in comparison with the experimental data from Ref. \([3]\). In Fig. 2 the dashed curve stands for the contributions from the t- and u-channel \( N^*(1535) \) resonance, and the dash-dotted and dotted lines stand for the contributions from the s-channel scalar meson \( f_0 \) and tensor meson \( f_2 \), respectively, while the total results of Fit I and Fit II are shown by dash-dot-dotted and solid curves. From Fig. 2 one can see that the experimental total cross section can be described fairly well by including the contributions from both the \( N^*(1535) \) resonance and the scalar meson \( f_0 \) or tensor meson \( f_2 \). The contributions from \( N^*(1535) \) resonance dominates above \( W = 2.25 \) GeV, while the bump structure around \( W = 2.2 \) GeV can be well reproduced by considering the contributions from the strange mesons \( f_0 \) and \( f_2 \).

With the above fitted parameters, the corresponding differential cross sections for \( \bar{p}p \rightarrow \phi \phi \) reaction at the energy around the fitted masses of \( f_0 \) and \( f_2 \), \( W = 2.15 \) GeV, \( W = 2.20 \) GeV, and \( W = 2.25 \) GeV, are shown in Fig. (a), Fig. (b), and Fig. (c), respectively. From Fig. (a) we see that the shapes of the angular distributions are similar, mainly because both the scalar meson and the tensor meson decay to \( \phi \phi \) in the s-wave. But, there are still a little bit difference in the two cases, especially for the energies of \( W = 2.20 \) GeV and \( W = 2.25 \) GeV, because the production of a scalar meson \( f_0 \) from \( pp \) is in s-wave, while the \( pp \) to the tensor meson \( f_2 \) is in the D-wave. These predictions can be checked by future experimental measurements.
IV. SUMMARY

In this paper, we have phenomenologically reanalyzed the $\bar{p}p \rightarrow \phi\phi$ reaction within an effective Lagrangian approach and the isobar model. In addition to the “background” contributions from $t$- and $u$-channel $N^*(1535)$ resonance, we studied the role of scalar meson ($f_0$) and tensor meson ($f_2$) in the $s$-channel. Unfortunately, the information about the $f_0$ and $f_2$ meson with mass around 2.2 GeV is scarce [2]. Thus, in the present work, we have taken the masses, the total decay widths, and the coupling constants, $g_{f_0\bar{p}p\phi}$ and $g_{f_2\bar{p}p\phi}$ as free parameters, and we fitted them to the experimental data on the $\bar{p}p \rightarrow \phi\phi$ reaction in Ref. [5]. The fitted results are: $M_{f_0} = 2174 \pm 3$ MeV, $\Gamma_{f_0} = 167 \pm 27$ MeV, $M_{f_2} = 2192 \pm 4$ MeV, and $\Gamma_{f_2} = 177 \pm 30$ MeV. The fitted results are shown that the $\bar{p}p \rightarrow \phi\phi$ reaction is dominated by the exchange of a strange tensor meson with quantum number $J^{PC} = 2^{++}$ in the $s$-channel, which is in agreement with the previous analysis [3]. In this respect, we have shown how the experimental measurements for the $\bar{p}p \rightarrow \phi\phi$ reaction could lead to valuable information on scalar and tensor mesons with masses around 2.2 GeV.

Finally, we would like to stress that due to the important role played by the resonant contribution in the $\bar{p}p \rightarrow \phi\phi$ reaction, the bump structure around $W = 2.2$ GeV in the total cross section can be well reproduced, and more accurate data on this reaction can be used to improve our knowledge on the strange mesons $f_0$ and $f_2$, which is at present poorly known. This work constitutes a first step in this direction.

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