The Chaplygin dark star

O. Bertolami, J. Páramos
Instituto Superior Técnico, Departamento de Física,
Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal and
E-mail addresses: orfeu@cosmos.ist.utl.pt; zjorge@fisica.ist.utl.pt

(Dated: July 23, 2018)

The Chaplygin dark star

We study the general properties of a spherically symmetric body described through the generalized Chaplygin equation of state. We conclude that such object, dubbed generalized Chaplygin dark star, should exist within the context of the generalized Chaplygin gas model of unification of dark energy and dark matter, and derive expressions for its size and expansion velocity. A criteria for the survival of the perturbations in the GCG background that give origin to the dark star are developed, and its main features are analyzed.

PACS numbers: 98.80.-k, 97.10.-q Preprint DF/IST-6.2005

I. INTRODUCTION

The generalized Chaplygin gas (GCG) model has lately drawn some attention, mainly because it allows for a unified description of dark matter and dark energy, the first dominating at early times and gradually transferring energy to the dark energy component [1, 2]. The GCG model is consistent with various classes of cosmological tests, such as the Cosmic Microwave Background Radiation [3], supernovae [4], gravitational lensing [5] and gamma-ray bursts [6]. As with other competing candidates to explain the overwhelming energy density of the present Universe, GCG is naturally constrained through cosmological observables.

It is quite interesting that the GCG equation of state is that of a polytropic gas [9], although one with a negative polytropic index. This hints that one could look for astrophysical implications of the model, and hence hope for yet another approach to the problem of constraining the allowed space for its parameters (see, e.g. Ref. [10]). In this work we argue that a GCG dark star may arise from a density fluctuation in the cosmological background. In what follows we shall characterize these objects, look at their evolution and account for their initial probability of appearance within the GCG background.

II. THE GENERALIZED CHAPLYGIN GAS

The GCG model is based on the equation of state

$$P_{ch} = \frac{A}{\rho_{ch}^\alpha},$$

where $A$ and $\alpha$ are positive constants and $0 \leq \alpha \leq 1$ (see however Ref. [11] for reasons to consider $\alpha > 1$); the negative pressure hints that the GCG is related to a cosmological constant. The case $\alpha = 1$ corresponds to the Chaplygin gas [12]. In a Friedmann-Robertson-Walker cosmology, the relativistic energy conservation yields

$$\rho_{ch} = A + \frac{B}{a^{3(1+\alpha)}}$$

where $a$ is the scale factor of the Universe and $B$ a positive integration constant. This result shows a striking property of the GCG, namely that at early times it behaves as non-relativistic dark matter ($\rho_{ch} \propto a^{-3}$), while at late times it acts as a cosmological constant ($\rho_{ch} \simeq \text{const.}$). One can algebraically decompose the GCG into a dark matter and a dark energy component, which evolve such that the transferring of energy occurs from the former to latter [7]. This can be used to show that, while the dark energy component is spatially homogeneous, the dark matter component allows for structure formation at early times, when it dominates [2, 7, 8].

For convenience, one defines the parameter $A_s \equiv A/\rho^{1+\alpha}_{ch0}$, where $\Omega_{de0}$ is the dark energy density, $\rho_{cr0}$ the critical density and $\rho_{ch0}$ the GCG energy density, all at the present. Assuming, as observations suggest, the condition $\Omega_{dm0} + \Omega_{de0} = 1$, where $\Omega_{dm0}$ is the dark matter density (dropping the small baryon contribution), and taking $a_0 = 1$, yields the constraint $B = \Omega_{dm0}\rho_{cr0}\rho_{ch0}$.

III. POLYTROPIC STARS

In order to deal with stellar structure in general relativity, one considers that the spherical body behaves as a perfect fluid, characterized by the energy-momentum tensor

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + Pg^{\mu\nu},$$

where $g^{\mu\nu}$ is a spherically symmetric Birkhoff metric. The Bianchi identity implies that $\nabla_\mu T^{\mu\nu} = 0$, from which follows the relativistic Tolman-Oppenheimer-Volkov equation [9],

$$\frac{dP}{dr} = -\frac{G(P + \rho)}{r^2} [m + 4\pi r^3 P] \left(1 - \frac{2Gm}{r}\right)^{-1},$$

where $m = 4\pi a^3 \rho/3$ and $a$ is the object radius.
where \( m(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \). This equation collapses to the classical Newtonian hydrostatic equilibrium equation

\[
\frac{dP}{dr} = -\frac{4Gm(r)\rho}{r^2}, \tag{5}
\]

if the following conditions are satisfied:

\[
Gm(r)/r \ll 1, \quad 4\pi r^3 P(r) \ll m(r), \quad P(r) \ll \rho(r). \tag{6, 7, 8}
\]

The polytropic gas model for stellar like structure assumes an equation of state of the form \( P = K\rho^{n+1}/n \), where \( n \) is the polytropic index, which defines intermediate cases between isothermic and adiabatic thermodynamical processes, and \( K \) is the polytropic constant, defined as

\[
K = N_n GM^{(n-1)/n} R^{(3-n)/n} \tag{9}
\]

with

\[
N_n = \left[ \frac{n+1}{(4\pi)^{1/n}} \xi^{(3-n)/n} \left( \frac{n}{\xi^2} \frac{d\theta}{d\xi} \right)^{(n-1)/n} \right]^{-1}, \tag{10}
\]

where \( R \) is the star’s radius, \( M \) its mass and \( \xi_1 \), defined by \( \theta(\xi_1) \equiv 0 \), corresponds to the surface of the star (cf. below). Actually, this definition states that all quantities tend to zero as one approaches the surface.

This assumption leads to several scaling laws for the relevant thermodynamical quantities,

\[
\rho = \rho_c \theta^n(\xi), \tag{11}
\]

\[
T = T_c \theta(\xi), \tag{12}
\]

\[
P = P_c \theta(\xi)^{n+1}, \tag{13}
\]

where \( \rho_c, T_c \) and \( P_c \) are the density, temperature and pressure at the center of the star [9]. Notice that the scaling law for temperature requires the assumption that the gas behaves as an ideal one. This is not the case of the GCG.

The function \( \theta \) is a dimensionless function of the dimensionless variable \( \xi_1 \), related to the physical distance to the star’s center by \( r = \beta \xi \), where

\[
\beta = \left[ \frac{(n+1)K}{4\pi G \rho_c^{(1-n)/n}} \right]^{1/2}. \tag{14}
\]

The function \( \theta(\xi) \) obeys a differential equation arising from the equilibrium condition of Eq.(5), the Lane-Emden equation:

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n. \tag{15}
\]

Notice that the physical radius and mass of the spherical body appear only in the polytropic constant, and hence the behavior of the scaling function \( \theta(\xi) \) is unaffected by these. Therefore, the stability of a star is independent of its size or mass, and different types of stars correspond to different polytropic indices \( n \). This scale-independence manifests in the symmetry of the Lane-Emden equation (15) under homology transformations. The first solar model ever considered, developed by Eddington in 1926, was that of an \( n = 3 \) polytropic star. Although somewhat incomplete, this simplified model gives rise to relevant constraints on the physical quantities.

In what follows, we shall use the Lane-Emden equation to derive the properties of a generalized Chaplygin dark star, given that conditions (6)-(8) are shown to be fulfilled. This enables the use of the Newtonian approximation (5), which asides its simplicity allows for a prompt interpretation of the GCG as a polytropic gas subject to the Lane-Emden equation of motion. The generality of this procedure can be used in various cases of physical interest, as for instance, when studying the effect of scalar fields on the stellar structure [10].

IV. THE GENERALIZED CHAPLYGIN DARK STAR

As already discussed, the GCG model is cosmological in nature, and most bounds on the parameters \( \alpha \) and \( A_s \) are derived from cosmological tests. However, a quick look at the GCG equation of state (1) indicates that it corresponds to a polytrope with a negative polytropic constant and a negative pressure. At first glance, this seems to indicate that no valid analysis of a GCG at an astrophysical context can proceed, since a spherical body constituted by such exotic gas would experience an outward pressure that would prevent it from being stable. However, the following argument shows that such an objection is circumvented by the presence of the cosmological GCG background.

The first logical step for the construction of a symmetric body with the GCG equation of state should be, as for all polytropes, the solution of the related Lane-Emden equation. Firstly one notes that this stems from the hydrostatic equation (5). As already seen, this is directly derived from the general relativity equations, assuming the energy-momentum tensor of a perfect fluid: no assumption whatsoever is made concerning the pressure or density, nor the equation of state relating these quantities. Hence, one can use this equation and, through the usual derivation, the related Lane-Emden equation; as stated before the only concern is if one can neglect the higher-order relativistic terms present in Eq.(4), thus
working in the Newtonian limit. This will be explicitly shown in the case under investigation.

The polytropic equation of state \( P = \rho^{n+1/n} \) shows that the GCG can be assigned a negative polytropic index \( n = -1/(1 + \alpha) \). Next, a comparison with Eq. (1) yields \( K = -A \), since the pressure is negative. This requires some caution: indeed, the direct application of the coordinate transformation between the physical radial coordinate \( r \) and the coordinate \( \xi \) given by by \( r = \beta \xi \), with \( \beta \) defined in Eq. (14) yields

\[
\beta \equiv D \left[ (1 + n)K \right]^{1/2} = D \left( \frac{\alpha}{1 + \alpha} \right)^{1/2},
\]

where \( D = \left[ \frac{\rho_c^{(1-n)/n}}{4\pi G} \right]^{1/2} \); since \( \alpha > 0 \) and \( K < 0 \), the above quantity is imaginary. To avoid this, we define the coordinate \( \xi \) through the same equation \( r = \beta \xi \), but with \( K \) replaced by \( |K| = A > 0 \) in Eq. (14), obtaining

\[
\beta = \left[ \frac{A}{4\pi G \left( 1 + \alpha \right)} \right]^{1/2} \rho_c^{-1+(1/2 - \alpha/2)}. \tag{17}
\]

The negative sign of \( K \) will, of course, manifest itself in the terms of the Lane-Emden equation; explicitly, one gets

\[
\frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial \theta}{\partial \xi} \right) = \theta^n. \tag{18}
\]

As in the usual Lane-Emden equation, one has as boundary conditions \( \theta(0) = 1 \) and \( \theta'(0) = 0 \). This gives rise to a positive derivative for \( \xi > 0 \), indicating that \( \theta \) is a smoothly increasing function. This is related not to the negative polytropic index, but to the negative pressure of the GCG; as a consequence, Eqs. (11) and (13) indicate that the pressure inside the dark star increases (in absolute value), while the density decreases. This is key to our study, since it shows that a GCG spherical body accretes, as expected for a star.

In the usual Lane-Emden equation, the criteria concerning the size of a star is given by \( \theta(\xi_1) \equiv 0 \), corresponding to a surface of zero density, pressure and temperature. In a GCG dark star, the question is more convoluted: a vanishing density yields infinite pressure (and conversely), which are rather unphysical choices for the boundary of any astrophysical object. Furthermore, since the function \( \theta \) is increasing, the density \( \rho \propto \theta^n \) vanishes at an infinite distance, while the pressure \( P \propto -\theta^{1+n} \) does not vanish at all.

As a solution for this issue, one recalls that the GCG object is embedded in a cosmological background. Hence, a GCG dark star should not be taken as an isolated body, but rather as a spike on the overall cosmological background of density \( \rho_{ch} \) and negative pressure \( P_{ch} \). Therefore, its boundary should be signalled by the matching of the inner and outer pressures and densities, as indicated in Fig. 1. Both conditions are, of course, equivalent, given the GCG equation of state (1). From Eqs. (11) and (13), this equates to

\[
\theta(\xi_1) \equiv \delta^{-1/n} = \delta^{1+\alpha}, \tag{19}
\]

where one defines the ratio between central and the background density, \( \delta \equiv \rho_c/\rho_{ch} \). Hence, one gets a correspondence between the central density of the dark star, that is, the height of the energy density fluctuation, and its radius. This argument shows that the Chaplygin dark star cannot be taken merely as an isolated body having a common equation of state with the GCG, but must be viewed instead as a perturbation to the flat GCG background. This is advantageous, since one can use the constraints available on the GCG model to ascertain its properties. The only constraint affecting this quantity is \( \delta \gg 1 \), since we assume that the density perturbation must be large enough so we can view it as a physical object, not merely a small fluctuation on the GCG background.

Notice that, since the scaling function \( \theta(\xi) \) is completely specified by the two boundary conditions \( \theta(0) = 1 \) and \( \theta'(0) = 0 \), neither \( \rho_c \) nor \( \rho_{ch} \) affect each other: \( \rho_c \) is “scaled out” of the problem through the definition (11) and (13) and \( \rho_{ch} \) merely sets the criteria for the surface of the star, through Eq. (19). Hence, although \( \rho_{ch} \) varies with time, there is no contradiction in assuming a constant central density \( \rho_c \); this simplifies our study, since Eq. (17) then yields a constant coordinate scaling coefficient \( \beta \). Furthermore, given that the cosmological background density \( \rho_{ch} \) is decreasing, a constant central density \( \rho_c \) indicates that the density ratio \( \delta \) increases and, therefore, \( \xi_1 \) expands towards a final radius \( r_\infty = \beta \xi_\infty \), acquiring mass in the process. One can argue that, due to energy conservation, the background density should
decrease in order to compensate this, but this is a minor effect that can be neglected. Also, since the Chaplygin dark star is not an isolated object, it is reasonable to assume that neither its mass nor radius should be held constant, but instead must vary as it dilutes itself on the overall cosmological background; instead, the central density \( \rho_c \) arises as the natural candidate for distinguishing between these objects.

The absolute magnitude of the perturbation results from the dynamics ruling the generation of a perturbation, via a probability law arising from the fundamental physics underneath the GCG model; this, of course, should naturally disfavor very large perturbations. Furthermore, since any relative perturbation to the homogeneous energy density profile is local, its occurrence should not depend on cosmological quantities and the probability distribution should depend only on the relative perturbation \( \delta \), and not explicitly on the scale factor \( a \). A putative candidate could be a normal probability distribution given by

\[
f(\delta) = f_0 \exp\left[ -g\delta^2 \right],
\]

(20)

where \( f_0 \) is a normalization factor, \( g \) a parameter dependent on \( A_s \) and \( \alpha \). Of course, more complicated expressions for \( f(\delta) \) are possible, depending on the inner workings of the fundamental physics behind the GCG model.

The expansion velocity of a dark star can be shown to be given by

\[
v_e \equiv \dot{r}_1 = \frac{3(1 + \alpha)\beta B}{\theta'(\xi_1)} \left( \frac{\rho_c}{\rho_{ch}} \right)^{1+\alpha} \frac{H}{\alpha a^{3(1+\alpha)}},
\]

(21)

where the prime denotes differentiation with respect to \( \xi \) and \( H = \dot{a}/a \) is the rate of expansion of the cosmological background. The Friedmann equation allows one to write the latter as

\[
H^2 = H_0^2 \left[ \Omega_{de0} + \Omega_{dm0} a^{-3(1+\alpha)} \right] \frac{1}{a^{3(1+\alpha)}}.
\]

(22)

One can see that, as \( \rho_{ch} \) approaches a constant value at late times, the expansion tends to zero. Also, one can derive the dependence of \( v_e \) on \( \delta \) by noticing that the scaling coefficient \( \beta \) runs with \( \delta^{-(1+\alpha)/2} \) and the term in brackets in Eq. (21) scales with \( \delta \), while \( \theta'(\xi_1) \) is found numerically to always be of order unity. Hence, one concludes that the expansion velocity depends weakly on \( \delta \), \( v_e \propto \delta^{-\alpha/2} \sim \delta^{-0.1} \), given the chosen value of \( \alpha = 0.2 \) [7, 13]. By the same token, since numerically one finds that \( \xi_1 \propto \delta^{-\alpha/2} \sim \delta^{-0.1} \).

Given that the GCG tends to a smooth distribution over space, most density perturbations tend to be flattened within a timescale related to their initial size and the characteristic speed of sound \( v_s = (\partial P/\partial \rho)^{1/2} \). Inside the dark star, the equation of state (1) and the definitions (11) and (13) yield

\[
v_{s, in} = \sqrt{\frac{\alpha A_0 \theta(\xi)}{\rho_{ch}^\alpha}} = \sqrt{\alpha A_s \frac{\theta'(\xi_1)}{\rho_{ch}}} \left( \frac{\rho_{ch0}}{\rho_{ch}} \right)^{(1+\alpha)/2}.
\]

(23)

which at the surface amounts to

\[
v_{s, ch} = \sqrt{\frac{\alpha A_s}{\rho_{ch}^{1+\alpha}}} = \sqrt{\alpha A_s} \left( \frac{\rho_{ch0}}{\rho_{ch}} \right)^{(1+\alpha)/2}.
\]

(24)

One sees that the maximum sound velocity occurs at the surface of the star; since \( \alpha \lesssim 0.6 \) and \( 0.6 \leq A_s \leq 0.8 \) (see first references in [3]), this should be smaller than the present value of \( v_{s, max} = 0.693 \) (in units of \( c \)). A plausible criteria for the survival of an initial perturbation is given by \( v_{s, ch} < v_{e,0} \), where \( v_{e,0} \) is the initial expansion velocity. Equating Eqs. (21) and (24) yields, after a little algebra

\[
\theta'(\xi_1) \delta^{-\alpha/2} < \frac{3}{2} \frac{\alpha}{\pi G \rho_{ch}^2} \frac{H_0}{H} \frac{\Omega_{dm0} H_0}{a^{3(1+\alpha)}}.
\]

(25)

Numerically, one finds that the left-hand side of Eq. (25) is approximately constant. At early times, the GCG behaves as cold dark matter, with \( \rho_{ch} \propto a^{-3} \) and \( H \propto a^{-3/2} \), and the right hand side of condition (25) is constant. At late times, when the GCG acts as a cosmological constant, both \( \rho_{ch} \) and the expansion rate \( H \) are constant, and the rhs then scales as \( a^{3(1+\alpha)} \), and thus decreases with cosmic time. Thus, most dark stars are created at early times; this is consistent with the usual interpretation of the GCG as a model where dark matter dominates at early times, allowing for structure formation, while at late times the dark energy component takes over.

Eq.(24) and the GCG equation of state (1) allows one to rewrite condition (8) as

\[
\left| \frac{P(r)}{\rho(r)} \right| = \frac{A}{\rho(r)^{1+\alpha}} = A_s \left( \frac{\rho_{ch0}}{\rho_{ch}} \right)^{1+\alpha} < 1,
\]

(26)

where we have used \( \rho(r) > \rho_{ch} > \rho_{ch,\infty} \) for any redshift \( z \), with \( \rho_{ch,\infty} = A_1^{1+\alpha} \) the limit for the background cosmological density when \( a \rightarrow \infty \). Relativistic corrections could be important if the pressure and density are of the same order of magnitude. However, one can use the bound

\[
\left| \frac{P(r)}{\rho(r)} \right| < A_s \left( \frac{\rho_{ch0}}{\rho_{ch}} \right)^{1+\alpha},
\]

(27)
to ascertain that, for redshifts typical for structure formation, $z = z_c = 15$ and a set of GCG parameters $\alpha = 0.2$, $A_s = 0.7$, one gets $|P(r)/\rho(r)| < 10^{-4}$; higher redshifts provide an even lower upper bound. For a much recent $z = 1$, the same set of parameters yields $|P(r)/\rho(r)| < 0.188$, which still validates the Newtonian approximation, although to a lesser extent. This is not troublesome, since most dark stars are assumed to nucleate at an early age. Also, since the above condition only provides an upper limit for $P(r)/\rho(r)$, a more complete calculation can still validate the Newtonian approximation, depending on the value of $\delta$.

Assuming that all dark stars have expanded up to their final size (which follows from the stabilization of the GCG as a dark energy), one can write the mass contribution of those created when the Universe had a size $a(t)$:

$$\frac{M(a)}{4\pi} = \int_0^{r(a)} \int_0^{r_1(\rho_c)} \rho(r, \rho_c) f(\rho_c) r^2 dr d\rho_c \tag{28}$$

$$= \beta^3 \int_0^{r(a)} \int_0^{\xi_1(\rho_c)} \rho_c \theta^n(\xi, \rho_c) f(\rho_c) \xi^2 d\xi d\rho_c \ ,$$

where $r_1 = \beta \xi_1$ and the dependence on the integration variables is made explicit. Integrating over time one gets the mass contribution of all generations of dark stars, $M_{DS} = \int_0^{a_0} M'(a) da$, where $M'(a) \equiv dM/da$.

A comparison between known observational bounds and numerical integration of the above results can then be used to constraint the GCG parameters $A_s$ and $\alpha$, namely through supernovae data, gravitational lensing results and other dark matter searches. This will be considered elsewhere.

V. NUMERICAL RESULTS

In order to substantiate our arguments, in this section we present some numerical examples; we shall study the proposed scenario for the “typical” values of the GCG model; one takes $\alpha = 0.2$ and $A_s = 0.7$. A future study based on this results could embrace a wider range of parameters and probe the creation of dark stars at early stages of the Universe, providing further refinement to the already known bounds (see, e.g. Ref. [13] for a summary of the existing constraints).

A numerical integration of the modified Lane-Emden equation (18) produces the results plotted in Fig. 1. In Table I we draw different scenarios, in order to ascertain the dimensions of dark stars nucleated at different ages of the Universe. One can see that at a redshift of $z = z_c = 15$, presumably typical for structure formation, even a small perturbation $\delta = 5$ produces an overwhelmingly large object, with about 3000 times the mass and 20 times the diameter of the Milky Way. Since these dimensions scale with $\beta$, which decreases with $\rho_c$, one probes higher redshifts in order to obtain smaller dark stars. Therefore, at $z = 50$ and $\delta = 10$, one obtains an object with approximately the size and double the mass of our galaxy. A larger perturbation $\delta = 100$ yields approximately the same size, but a ten-fold increase in mass.

Going further back in time, a darkstar born at $z = 100$ with $\delta = 10$ (100) has about one-hundredth (one-tenth) the mass of the Milky Way and one-tenth its diameter. Finally, $\delta = 100$ and an extremely high redshift of $z = 500$, deep within the so-called “dark ages”, yield a dark star with $1.6 \times 10^8$ solar masses and a radius of 7.8 pc, dimensions similar to those ascribed to supermassive black holes in active galactic nuclei.

The above discussion is by no means definitive, and only serves to illustrate the concept developed in this study. Nevertheless, one might be surprised by the unphysically large size of a dark star hypothetically nucleated at the redshift typical for structure formation, $z_c = 15$. However, notice that this describes the condensation of baryonic matter interacting gravitationally with dark matter. The dark star scenario poses quite a different mechanism, where the GCG (in an era where its dark matter component dominates) is the sole constituent of the spherical body. Hence, it is reasonable to assume that bodies of astrophysical dimensions can arise much earlier in the history of the Universe. A precise description would of course imply the nucleation probability distribution $f(\delta)$, since this is the fundamental quantity ruling the onset of perturbations nucleation on the GCG background.

To ascertain the validity of the Newtonian limit from which the Lane-Emden equation is derived, conditions (6) and (7) can now be checked with a simple calculation. An inspection of the Table I shows that $\xi_1$ is of the order of $\delta$. Also, Fig. 1 shows that while $\theta'(\xi_1)$ slowly increases and is of order unity, the scaling function $\theta(\xi)$ grows regularly; hence, one can use the power-law approximation $\theta(\xi) \approx a \xi^b$, where $a$ and $b$ are coefficients of order unity, since it does not introduce large deviations from a full numerical calculation. Using Eq. (11), this yields

$$m(r) \sim \frac{4\pi}{3 + nb} \rho(r) r^3 \ , \tag{29}$$

and condition (6) becomes

$$\frac{4\pi}{3 + nb} G \rho(r) r^2 \ll 1 \ . \tag{30}$$

Since $\rho(r) \propto r^{-nb}$, the left-hand side of Eq. (30) scales with $r^{2+nb}$ and, since $|nb| \sim 1$, is an increasing function. Therefore, it is majorised by its value at $r = r_1$, amounting to $\sim 4\pi G \rho_{b} r_1^2$. Using Table I one finds that it attains a maximum value of $5.12 \times 10^{-5}$ for $\delta = 5$, $z = z_c = 15$, thus concluding that condition (6) is verified. Notice that, for a fixed redshift, this ratio is approximately independent of $\delta$. This is due to the very weak
sculpting of the physical radius with the relative perturbation, $r_1 \propto \delta^{-\alpha/2} \sim \delta^{-0.1}$, for the chosen value $\alpha = 0.2$.

In a similar fashion, the power-law approximation allows one to write condition (7) as

$$P(r) \ll \rho(r) \frac{1}{3 + nb} ,$$

which, since $|nb| \sim 1$, is equivalent to condition (8) and hence also satisfied at early ages. Thus, the Newtonian approximation implicit in the Lane-Emden equation is valid for the cases studied and, given the smallness of the values encountered in its evaluation, it is also applicable in a broader range of the nucleation redshift $z$ and relative densities $\delta$.

Given the indicated values for the initial expansion velocity $v_{e0}$ and the surface sound velocities $v_s$ (which depends only on the redshift) the inequality (25) is valid for the chosen values, and thus the corresponding dark stars do not collapse at birth.

### VI. CONCLUSIONS

In this study we have analyzed the properties of spherical bodies with a polytropic equation of state of negative index, in the context of the GCG dark energy/dark matter unification model. We have considered the associated Lane-Emden equation and looked at the qualitative behavior of its solution; amongst the results we find the conditions for fluctuations to be attenuated or to develop as dark stars. Our criteria is based on the condition that the sound velocity does not exceed the expansion velocity of the dark star when it nucleates $v_{s,ch} < v_{e,0}$. This enables the computation of the mass contribution of the dark stars at present times, which can then be used to constraint the GCG parameters $A_p$ and $\alpha$, providing another testing ground for this fascinating model.

**Note added:** While finalizing this work we became aware of the study of stable dark energy objects [14] and halos of k-essence [15]. Even though the motivation of both works are somewhat similar, our approaches are quite different.

**Acknowledgments**

JP is sponsored by the Fundação para a Ciência e Tecnologia under the grant BD 6207/2001.

---

[1] A. Kamenshchik, U. Moschella and V. Pasquier, *Phys. Lett. B* **511** (2001) 265.
[2] M.C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev. D* **66** (2002) 043507.
[3] M.C. Bento, O. Bertolami and A. A. Sen, *Phys. Lett. B575* (2003) 172; *Phys. Rev. D* **67** (2003) 063003; *Gen. Relativity and Gravitation* **35** (2003) 2063; D. Carturan and F. Finelli, *Phys. Rev. D* **68** (2003) 103501; L. Amendola, F. Finelli, C. Burigana and D. Carturan, *JCAP 0307* (2003) 005.
[4] See M.C. Bento, O. Bertolami, A. A. Sen and N. Santos, *Phys. Rev. D* **71** (2005) 063501 and references therein.
[5] P.T. Silva and O. Bertolami, *Ap. J.* **599** (2003) 829; A. Dev, D. Jain and J. S. Alcaniz, *Astron. and Astrophys. 417* (2004) 847.
[6] O. Bertolami and P. T. Silva, astro-ph/0507192; to appear in *Mon. Not. R. Ast. Soc.*.
[7] M. C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev. D* **70** (2004) 083519.
[8] N. Bilić, G. B. Tupper and R. D. Viollier, *Phys. Lett. B*
[9] “Textbook of Astronomy and Astrophysics with Elements of Cosmology”, V.B. Bhatia (Narosa Publishing House, Delhi 2001).
[10] O. Bertolami and J. Páramos, Phys. Rev. D 71 (2005) 023521.
[11] O. Bertolami, A. A. Sen, S. Sen and P. T. Silva, Mon. Not. R. Ast. Soc. 353 (2004) 329.
[12] S. Chaplygin, Sci. Men. Moscow Univ. Math. 21 (1904) 1.
[13] O. Bertolami, astro-ph/0403310.
[14] F. Lobo, gr-qc/0508115.
[15] C. Armendariz-Picon and E. A. Lim, JCAP 0508 (2005) 007.