HM-EH-RT: hybrid multimodal energy harvesting from rotational and translational motions

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This paper presents a novel hybrid multimodal energy harvesting device consisting of an unbalanced rotary disk that supports two transduction methods, piezoelectric and electromagnetic. The device generates electrical energy from oscillatory motion either orthogonal or parallel to the rotary axis to power electronic devices. Analytical models for the electromagnetic and piezoelectric systems were developed to describe the mechanical and electrical behavior of the device. From these models, numerical simulations were performed to predict power generation capabilities. The device was fabricated, and several components were optimized experimentally. The energy harvester was then experimentally characterized using a modal shaker in several different orientations. The device generates a maximum RMS power output of 120 mW from the electromagnetic system at 5 Hz and 0.8 g, and 4.23 mW from the piezoelectric system at 20.2 Hz and 0.4 g excitation acceleration. The device is 180 mm in diameter and 45 mm thick including the rotor height. Further size optimization will produce an energy harvester capable of being used as a wearable device to power mobile electronics for multiple applications.

Keywords: energy harvesting; piezoelectric; electromagnetic; wearable device; multimodal

1. Introduction and background

Recent advancements in technology have resulted in the miniaturization of electronic devices allowing for the development of smart phones, tablets, laptops, and other personal electronics. This has also introduced the possibility of wireless sensor networks (WSN) for constant monitoring in bridges and tunnels, medical diagnostics, animal tracking, smart buildings, and other industrial applications. In many of these applications, the recharging or replacing of batteries would be inconvenient. Furthermore, batteries dominate a large portion of space and cost in current mobile electronics, as well as add inconvenient maintenance requirements. To overcome these issues, an increasing amount of research has been dedicated to develop small-scale devices that generate electrical power from energy sources such as light, heat, motion, fluid flow, and vibration.

Two of the most popular methods for powering personal electronics and WSN in industrial applications are piezoelectric and electromagnetic (PEM) energy harvesters. A piezoelectric material generates a voltage when a stress is applied on the material, whereas...
EM generation occurs when a permanent magnet is passed over or through a coil. Since both energy harvesting techniques are powered by mechanical motion, theoretically, they are ideal for use as wearable devices to power personal electronics. However, because many piezoelectric energy harvesters particularly at microelectromechanical systems (MEMS) scale have resonance frequencies above 200 Hz [1], they are unable to operate in the frequency range of human motion, typically 1–10 Hz [2]. The high resonance frequencies are due to the practical geometrical values of the harvesters, which made the stiffness higher. Most piezoelectric harvesters also tend to have a very narrow bandwidth, meaning they can only generate power at certain frequency values. The problem that many EM energy harvesters encounter is their size. As a magnet is scaled down, the electromagnetic effect on a coil is greatly reduced. The performance of an energy harvester is best measured by power density, which is defined as the power generated divided by the volume the generator. Thus, there is a crucial balance between added volume and increased power output when developing improved energy harvesters. If two or more energy harvesting (EH) techniques are combined, the volume increase by the additional system or technique must produce enough additional power to result in an increase in power density. In addition, the increase in power density must justify the added cost of the device. One of the most effective ways to enhance the power density of EH devices is to use multiple transduction methods on one device, called a hybrid energy harvester, allowing them to harvest two or more energy sources at the same time. This paper proposes a novel design that incorporates PEM generation in a way that has not been done before. The device was designed with a dominant operational range of 1–5 Hz, the frequency of human motion. An extended range of up to 30 Hz was given to the device to fit into the frequency range a person experiences while riding in a car [2]. The initial design of the device was presented in our previous work [3]. In this paper, an improved design, analytical model, and simulations are presented along with the results from experimental optimization and characterization.

This paper is organized into various sections. The first section will be a brief review of current piezoelectric, electromagnetic, and hybrid energy harvesters. Then, we will describe the design, structural analysis, and analytical models of the proposed energy harvester, including the working principles of its electrical and mechanical systems. The experimental setup, simulation results, and experimental results for the optimization and characterization of the generator are then presented, followed by the discussion and conclusions.

1.1. Electromagnetic energy harvesters

A comprehensive book on energy harvesting technologies was presented by Priya and Inman [4] covering various aspects. Several review articles on varieties of domains in micro- and meso-scale energy harvesting systems were explored by researchers and can be found in the literature [5–7]. One of the most powerful and widely used energy harvesting techniques is electromagnetic (EM) energy harvesters. Many electromagnetic energy harvesters have been developed in recent years for powering electronic devices by using ambient vibrations or inertial forces.

Vibration-driven EM energy harvesters consisting of unique architectures were presented in the literature. Lee et al. [8] developed a low-frequency vibrational electromagnetic energy harvester consisting of a cylindrical NdFeB permanent magnet, suspended by two FR4 springs, that oscillates within a three EM coils wound around a hollow cylinder. The size of the device was 5 cm long with a diameter of 3 cm, and the resonance
frequency was 16 Hz, which is within the bandwidth of human motion. The maximum power obtained was 1.52 mW at an applied acceleration of 0.2 g, giving the device a power density of 43 µW cm$^{-3}$. Liu et al. [9] proposed a 36 mm$^3$ MEMS electromagnetic energy harvester chip driven by 3D vibrations. To find the power generated, the device was tested at its first three vibration modes, which were found to be 1285 Hz out-of-plane, 1470 Hz in-plane at 60° to horizontal, and 1550 Hz in-plane at 150° to horizontal. The maximum power densities at these three modes were 0.44, 0.24, and 0.13 µW cm$^{-3}$, respectively. In another study, an EM energy harvester driven by the linear motion of a foot during walking was proposed [10]. The device consisted of two parallel rectangular stators with parallel slots that have coils wound between them in a wave pattern. The stators were separated by a translator made of several permanent magnets with opposing magnetic polarizations. The device generated 674 mW at 1.75 Hz with amplitude of 40 cm, acceleration of 0.49 g, and maximum velocity 4.5 m s$^{-1}$. The power density was 3.4 mW cm$^{-3}$. Morais et al. [11] also explored energy from walking using a vibrational EM generator. Their design consists of a long tube with two coils wound around the outside. Driven by external inertial forces, a large cylindrical permanent magnet connected to a spring oscillates within the tube, passing through the coils. Placed inside a hip replacement, the completed generator occupied a total volume of 3.76 cm$^3$. For an average walking frequency of 1.3 Hz, the device was able to generate 109 µW giving a power density of 29 µW cm$^{-3}$.

Several EM energy harvesters recently developed convert linear motion into rotational motion by using an unbalanced rotor. Romero et al. [12] proposed such a device for powering biomedical devices. The 2 cm$^3$ generator consisted of a stator of stacked microfabricated radial planar coils, a circular rotor disk made of multiple permanent magnets, and an eccentric half-circle mass. The eccentric mass causes the rotor to rotate when a linear acceleration is applied perpendicular to the axis of rotation. The device was tested at six body locations, and it was found that the ankle had the highest power generation, producing 472 µW at a speed of 4 mph. It was predicted that when placed at the ankle the device would have a maximum power density of 236 µW cm$^{-3}$. In Shields’ thesis work [13], a micro-electromechanical rotary EM generator consisting of an unbalanced rotor made of magnetized nickel rotating over evaporated gold micro coils was presented. They found that the use of electroplated nickel as a magnetic rotor is not a feasible solution. However, the coil and rotor designs proposed meet the EM orientation requirements to induce current flow for a generator.

### 1.2. Piezoelectric energy harvesters

As discussed previously, when a stress is applied to a piezoelectric material, a charge is created. For energy harvesting applications, they are most often used as a cantilever beam or bonded to another cantilever beam to harness vibrations. Often, a proof mass is added to the end of the beam to decrease the natural frequency and exert a bending stress. There has been much attention given to reducing the natural frequency of a piezoelectric cantilever beam by using appropriate geometry and tip mass. One alternative is to convert low frequency to high frequency using a separate low-resonance frequency cantilever to drive the piezoelectric cantilever. Minami and Nakamachi [14] developed such a device. The device consisted of a piezoelectric (PVDF) thin film laminated on a thin metal cantilever beam with a permanent magnet tip mass and another moveable magnet attached to a vibrating base. The power generated was found to be 1.2 µW when an oscillatory motion was applied at 2 Hz.
A wearable energy harvester based on a thin film piezoelectric strip was proposed by Yang and Yun [15]. The generator was made of polyvinylidene fluoride (PVDF) film attached to a 2 × 0.2 cm² piece of curved polyester film and was designed to be stitched into fabric. The polyester film had previously been heat treated to give it a shell structure. Power is generated during fast state transition of the shell structure. In the study, a 4.2 cm² elbow sleeve consisting of ten strip generators was designed, fabricated, and tested. The single strip design generated 0.87 mW, power density of 2.18 mW cm⁻², when folded to an angle of 80° at a frequency of 3.3 Hz, 9.3 rad s⁻¹, while the elbow sleeve generated 0.21 mW, power density of 0.05 mW cm⁻², for a bending velocity of 5 rad s⁻¹. One off-the-shelf PZT energy harvester is MicroGen Systems’ BOLT R0100 Micro-Power Generator [16]. It is simply a packaged, wide, thin-film PZT cantilever. It harvests ambient vibrations from industrial sources such as buildings, bridges, or machinery. Its resonant frequency is 100 Hz with a maximum power generation of 58 µW at less than 0.1 g acceleration. With a volume of 1.2 cm³, the device’s power density is 41.7 µW cm⁻³.

1.3. Hybrid energy harvesters

One of the most effective ways to enhance the power density of energy harvesting devices is to use multiple transduction methods on one device. Such devices are called hybrid or multimodal energy harvesters. Depending on the transducers used, the device will either have multiple generators driven by the same energy source or will harvest from more than one energy source at the same time. However, when designing a hybrid energy harvester, there is a crucial balance between added volume and increased power output. The volume increase from the added components must produce enough additional power to result in an increase in power density. In addition, the increase in power density must justify the added cost of the device. For these reasons, the most effective hybrid generators are often those that combine transduction methods that harvest energy from the same source, such as hybrid PEM generators, allowing them to be combined into one device. The most common and simple PEM energy harvester consists of a piezoelectric cantilever beam with a magnet proof mass and electromagnetic coils that are placed near the magnet. As the beam vibrates due to linear oscillations, stresses are induced in the piezoelectric beam, producing electricity, and the magnet moves away from and toward the coil, inducing a current through the wire. Several variations of this device have been published. One such device was developed by Sang et al. [17] in 2012. They developed a vibration-driven hybrid energy harvester for wireless sensor systems. Four designs were presented, each consisting of a cantilever beam with PZT plates bonded to either side of the beam, an NdFeB permanent magnet, and electromagnetic coils. The fourth design was chosen for its convenience to implement and the high power to volume ratio. It consisted of the block magnet mounted to the tip of the cantilever and two coils mounted to the base on either side of the magnet. A maximum power of 10.7 mW at 50 Hz and 0.4 g was achieved. With an approximate volume of 35 cm³, the power density of the device is then ~0.3 mW cm⁻³.

Another vibration-driven PEM was proposed by Khaligh et al. [18] in 2008. Two permanent magnets are mounted to the top of a movable mass on either side of a center hole inside of which is an EM coil that is fixed to the base. The movable mass is suspended by four serpentine-piezoelectric springs and inertial forces applied to the device act on the mass causing it to oscillate, inducing stress in the piezoelectric springs and moving the magnets in relation to the coil. The S-shape of the piezoelectric springs results
in several bending moments and points of maximum stress when a force is applied as opposed to a beam that will only have one. This greatly magnifies the power that can be generated by one piezoelectric in a given space. Output power for the electromagnetic and piezoelectric parts of the device was found to be 37 mW and 6 mW, respectively, when an oscillating linear motion of frequency 2 Hz and amplitude 3 cm was applied to the structure. The volume occupied by the device is 19.6 cm$^3$, which gives a total power density of 2.2 mW cm$^{-3}$.

Chen and Hu [19] developed a PEM vibrational hybrid energy harvester in 2011. The generator consisted of a U-shaped aluminum beam with four NdFeB magnets mounted to the ends of the beam. Two stainless steel S-beams were bonded to the top and bottom of the U shaped beam, with the length of all four beams parallel. Both beams were fixed with one PZT patch at the tip for harvesting impact forces and one PZT patch slightly lower for harvesting vibrational forces. The final assembly was an impact body and a coil mounted to the tip of a cantilever beam. External forces or vibrations caused the cantilever beam to vibrate. The impact body/coil would oscillate past the permanent magnets mounted to the U-shaped beam and then strike the top or bottom impact PZT patches bonded to the S-beam. The resulting vibrations in the S-beam would induce a stress in the vibrational PZT patches. The fabricated hybrid generator prototype had a volume of 68 cm$^3$. The device was driven by an oscillatory motion with a frequency of 13 Hz and a maximum acceleration of 3 g. The impact piezos generated 1.9 µW, the vibration piezos generated 14.3 µW, and the electromagnetic component generated 218 µW of energy, resulting in a combined power density of 3.8 µW cm$^{-3}$.

2. Design and analysis

2.1. Design

The design of the proposed hybrid multimodal energy harvester in this paper is shown in Figure 1. Three piezoelectric cantilever beams having permanent magnets at the ends are connected to a base through a slip ring. Stationary electromagnetic coils are installed in the base and connected in series. The beams are oriented over every other coil and act as an unbalanced rotor in order to convert linear motion into rotational motion. The beams

![Figure 1](attachment:image.png)

Figure 1. (a) Isometric, (b) top, and (c) side views of the proposed energy harvester with linear motion, $X(t)$, and vertical vibrations, $Y(t)$, shown. (d) Connection diagram used for characterizing the power generation ($V_{pi}/R_{pi} = \text{load voltage/resistor from } i\text{th piezo}, V_{Tem}/R_L = \text{EM load voltage/resistor, } R_{Tint} = \text{internal resistance of all the coils}$).
were originally oriented over three consecutive coils, but the magnetic repulsive forces between the magnets caused the beams to deflect up or down drastically; therefore, the rotor was redesigned with an increased gap between the beams. Each beam has two NdFeB permanent magnet attached to the free end acting as a proof mass on the end of a cantilever. The cantilever beams rotate over ten electromagnetic coils that are oriented directly under the magnets. The operating mechanism of the harvester is described by Figure 1. Whenever the device is driven by a linear oscillation, \( X(t) \), parallel to the base, with amplitude \( A_x \) and frequency \( \omega_x \), the unbalanced rotor will rotate generating energy across the stationary coils in the base. In an opposite situation, if the device is driven by a linear oscillation, \( Y(t) \), perpendicular to the base, with amplitude \( A_y \) and frequency \( \omega_y \), a stress will be induced within the cantilever beams generating energy across the piezoelectric materials. The generator will be used as a wearable device for soldiers in the field or for hikers, campers, and adventurers. It will recharge batteries for GPS, mobile phones, and other personal devices. This is a novel design that incorporates PEM generation in a way that has not been done before. While there have been similar rotary electromagnetic energy harvesters [13], no one has used piezoelectric beams as the rotor. This design allows for easy scaling of the system as well as the ability to tune the operating frequency of the beams to operate well during any human activity by increasing or decreasing the size and weight of the magnets. The device will also be able to generate energy irrespective of the direction of the motion in relation to the generator. This is because accelerations applied parallel to the base will actuate the rotor, whereas accelerations applied perpendicular to the base will actuate the piezo beams.

The initial prototype was designed for ease of manufacturing, assembly, and implementation. Thus, the design requirements for each component were as follows: (1) all components off-the-shelf or 3D printed with ABS plastic, (2) piezoelectric beams pre-wired with mounting holes, and (3) coils of magnet wire wound in house using a custom fabrication process. The piezo beams also needed to be of reasonable geometry and size for resonance frequency tuning and ease of implementation. The Volture V21B Piezoelectric Vibration Energy Harvester and the Piezo Systems D220-A4-203YB Double Quick-Mount Bending Generator were selected and compared using SolidWorks Simulation failure and frequency analysis. Both are bimorph piezoelectric beams made of PZT5A piezoceramic (PZT) material and are pre-wired for parallel operation. The D220-A4-203YB has a mounting plate on either end of the piezoelectric element, and the overall size is 63.5 × 6.3 × 0.5 mm³. The V21B is a packaged PZT beam that uses FR4 epoxy for increased robustness and flexibility and is 61 × 14 × 0.87 mm³. The PZTs (V21B and D220) are very close in length so the dimensions of several components were chosen. First, a magnet width/length of 25.4 mm (1") was chosen because of size and availability. Considering the length of the beam to the end of the magnet (~85 mm), the base was designed with a diameter of 180 mm and ten coils that have an outer diameter of 30.5 mm and a thickness of ~2.5 mm.

### 2.2. Stress analysis

SolidWorks Simulation was used to perform static and frequency analysis on the Piezosystems D220-A4-203YB and Volture V21B piezoelectric vibration generators. From these results, the two PZT beams were compared with respect to strength, resonance frequency, and beam deflection. Figure 2(a)–2(d) shows the results of the static analysis of the beams with two 1" × 1" × 1/8" (25.4 × 25.4 × 3.2 mm³) thick magnets. Figure 2(a) and 2(b); shows the displacement plot, and Figure 2(c) and 2(d); shows the stress plot for
V21B and D220, respectively. The bending strength of the D220 PZT beam is 50 MPa, but the exact bending strength of V21B is unknown. However, the V21B is a composite beam of PZT and FR4 epoxy, which have bending strengths of ~50 MPa (assuming similar to D220), and 345 MPa, respectively. Thus, the V21B will easily be able to support anything below 50 MPa. As can be seen in Figure 2, the simulation was performed with the clamped area of each beam fixed and an acceleration of 0.4 g, in addition to the gravitational force, applied to the face of the beams. The purpose of the additional acceleration was to simulate roughly the acceleration amplitude of the excitation oscillations. The actual stress in the beams when driven at resonant frequency would be considerably higher. Thus, the piezo beam had to have a bending strength several times greater than the maximum stress found in this analysis. Figure 2(b) and 2(d) shows the stress plot for the V21B and D220 beams. Simulation results showed that the maximum stress in the D220 would be 50 MPa, while the maximum stress in the V21B would only be 10.9 MPa. With both having (approximate) bending strengths of 50 MPa, the D220 is too narrow, while the wider and more robust V21B is more than strong enough. Thus, the V21B piezoelectric vibration energy harvester was chosen as the piezoelectric beams for the initial prototype. The displacement at the end of the magnet was then found to be 0.94 mm for the V21B beam as shown in Figure 2(a). The actual displacement, or amplitude, of the magnet will be much greater when the beam is driven at its resonant frequency. Thus, the rotor mount (Figure 1a) was designed with a spacer so that the end of the magnet was 7 mm from the base with no beam deflection.

Frequency analysis was then done to find the natural frequency of the V21B with the magnet tip mass from the static analysis. Figure 3 shows the results of the frequency analysis done on beam V21B. It was found that for the geometry and material properties the first three natural frequencies are 25 Hz, 135 Hz, and 298 Hz. The first natural

![Figure 2](image2.png)

**Figure 2.** Static analysis of beams V21B and D220 respectively: (a, b) maximum displacement at end of magnet, and (c, d) maximum stress at bending point.

![Figure 3](image3.png)

**Figure 3.** Frequency analysis of V21B beam with two 1/8" thick magnet: (a) mode shape 1: 25 Hz; (b) mode shape 2: 135 Hz; (c) mode shape 3: 298 Hz.
frequency was within the design requirement (<30 Hz), while the second and third modes were out of the range of human motion. Thus, the piezoelectric beams were modeled discretely for the first resonance frequency.

3. Analytical model

The proposed device is a hybrid PEM energy harvester. The power generation capabilities of the device’s two transduction methods must be optimized before the device can be evaluated for energy harvesting from human motion. To optimize the energy harvester effectively, the mechanical and electrical behaviors of the device had to be characterized analytically and experimentally. Thus, an analytical model of the device was developed to better understand its mechanical and electrical working principles, as well as to aide in the optimization and characterization of the device. Figure 1 shows the design and working principles of the proposed device where piezoelectric generation is driven by linear oscillations $Y(t)$ perpendicular to the base, while EM generation is driven by linear oscillations $X(t)$ parallel to the base. These can be defined as their principal modes of operation, which are orthogonal to each other. However, there are some coupling dynamics between the two transduction methods, even when the device is actuated strictly in the $X$ or $Y$ direction. When the piezoelectric beams vibrate due to $Y(t)$, the permanent magnets oscillate perpendicular to the base. If the magnets are positioned directly over the coils, a current will be induced through the system. The coupled transduction methods will increase the total damping in the system, decreasing the power generated by either transducer individually with respect to their non-coupled power. However, if the magnets are positioned exactly midway between two coils, the current generated by either coil will be in opposite directions, resulting in no power generation. When the rotor spins due to $X(t)$, the centripetal forces will also induce stress within the piezoelectric beams, generating a voltage. It is clear that the piezos will not increase the mechanical damping of the spinning rotor. In addition, the power generated during spinning mode by the piezos is miniscule in comparison to the EM power and several orders of magnitude lower than the power generated by the piezos in vibrating mode. These results will be shown in Section 6.6 for experimental combined generation. Thus, to optimize the energy harvester, the electromagnetic and the piezoelectric transducers were treated separately for their respective principal mode of operation with coupling effects. The increase in mechanical damping was assumed to be negligible.

The device is to be used as a wearable device and will be subjected to forces in any direction. The main objective of modeling the system was not to accurately predict its power generation from human motion but rather to assist in optimizing the device and then characterize its optimal power generation capabilities. Thus, for simplicity the following assumptions were made: (1) the increase in total damping from coupling is negligible, and (2) if each transducer is optimized for its principal mode of operation, then it will be optimized for any direction. Thus, the dynamics and power generated by the hybrid energy harvester were analyzed using two decoupled models. An extensive experimental study on the hybrid energy harvester when used as a wearable device is currently being conducted and will be presented in the future.

3.1. Electromagnetic generation

The equations of motion of the electromagnetic generator when the generator is horizontally and vertically oriented can be derived by considering an unbalanced rotary disk as
shown in Figure 4. An oscillating linear motion, $X(t)$, is applied to the base of the generator perpendicular to the axis of rotation. Let $F_x$ be the resultant force acting on the center of mass of the rotor, $m$ be the mass of the rotor, $a$ be the distance from the axis of rotation to the center of mass of the rotor, $l_r$ be the radius of the rotor, $A_x$ be the amplitude of the applied motion, $J$ be the moment of inertia about the center of gravity, $\theta$ be the rotation angle of the rotor, $\omega$ be the angular frequency of the applied oscillatory motion, and $C_e$ be electromechanical damping caused by the current through the coils. From Figure 4(a), the equation of motion of the rotor when the generator is horizontal can be derived as

$$\left(ma^2 + J\right)\ddot{\theta} + C_e\dot{\theta} = maA_x\omega^2 \sin \theta \sin \omega t$$

(1)

From Figure 4(b), the equation of motion of the rotor when the generator is vertical can be derived. Let $g$ be the gravitational acceleration and the equation of motion will be

$$\left(ma^2 + J\right)\ddot{\theta} + C_e\dot{\theta} = maA_x\omega^2 \sin \theta \sin \omega t - mga \cos \theta$$

(2)

According to Sasaki et al. [20], the initial conditions for self-excited rotation are

$$|\dot{\theta}|_{t=0} > \omega$$

(3)
Equation (3) implies that in order to have continuous self-excited rotation, the rotor needs to have an initial angular velocity greater than the applied oscillatory angular frequency. Also, inspection of Equation (4) indicates that the electromechanical damping must be small enough to allow self-excited rotation. From [21], the open loop induced EMF and effective length can be determined by

\[
V_{Tem} = 3Bl_{eff}l_\theta \dot{\theta}
\]

(5)

\[
l_{eff} = \frac{l_w}{\pi}
\]

(6)

where \( B \) is the residual magnetic field, \( l_w \) is the length of the coil wire, and \( l_{eff} \) is the effective length. Total length of the spiral is estimated by

\[
l_w = \pi N_T \frac{D_o + D_i}{2}
\]

(7)

where \( N_T \) is the total number of turns, defined as the number of turns per layer multiplied by the number of layers, \( D_o \) is the outer diameter of the coil, and \( D_i \) is the inner diameter of the coil. However, the residual magnetic field \( B \) must be recalculated for the distance away from the coil as shown in Figure 4(d). From [22], the magnetic field a distance \( z \) from the magnet can be determined by

\[
B(z) = \frac{B}{2} \left( \frac{t_m + z}{\sqrt{r_1^2 + (t_m + z)^2}} - \frac{z}{\sqrt{r_1^2 + z^2}} \right)
\]

(8)

where \( t_m \) is the thickness of the magnet and \( r_1 \) is the radius of the magnet. Figure 4(c) shows the circuit diagram for electromagnetic generation. The coils are connected together in series and the internal resistance will therefore act as 10 individual resistors in series. However, since there are only three magnets, the voltage is treated as three voltage sources in series. Therefore, power delivered to the load resistance is defined as

\[
P_{Tem} = \frac{\left( \frac{R_{L}}{R_L + R_{Tint}} \right) V_{Tem}^2}{R_L}
\]

(9)

where \( R_L \) is the external load resistance, and \( R_{Tint} \) is the internal resistance of the coils. Internal resistance of the coil is calculated from the equation for resistance of a wire. Let \( \rho \) be the resistivity of the wire, \( l_w \) be the coil wire length, \( N_c \) be the number of coils, and \( A \) be the cross-sectional area of the wire:

\[
R_{Tint} = N_c \frac{\rho l_w}{A}
\]

(10)
It can be shown that the maximum power output is generated at an optimal load resistance. It can also be shown that the electromagnetic damping is dependent on the load resistance. Thus, the power output can be defined in terms of electromagnetic damping. In order to find the damping, we will first find the torque caused by electromagnetic forces between the coils and magnets. Figure 4(e) shows the force diagram for the magnet moving over a coil that is connected to load resistance. From Equation (1), electromagnetic torque is defined as

$$\tau_{em} = C_e \dot{\theta}$$  \hspace{1cm} (11)

The torque can also be defined as power over an angular frequency. Thus, the torque caused by electromagnetic damping can be defined as

$$\tau_{em} = \frac{V_{Tem}^2}{(R_{Tint} + R_L)\dot{\theta}} = \frac{(3Bl_{eff}l)^2}{R_{Tint} + R_L} \dot{\theta}$$  \hspace{1cm} (12)

Setting Equations (11) and (12) equal to each other, the electromagnetic damping is found to be

$$C_e = \frac{(3Bl_{eff}l)^2}{R_L + R_{Tint}}$$  \hspace{1cm} (13)

This shows that the electromagnetic damping is dependent on, and inversely proportional to, the load resistance of the energy harvesting circuit. The angular velocity of the rotor can then be found by inserting Equation (13) back into the equations of motion, Equation (1) or (2). The power generated by the system can then be found by plugging the calculated angular velocity into Equation (9). Thus, the optimum electromagnetic power generation of the system at a certain applied oscillatory motion can be found by varying load resistance.

### 3.2. Piezoelectric generation

The analytical model for piezoelectric power generation described in this section was adapted from Roundy and Wright [23]. The piezoelectric generation was modeled only for the vertical orientation of the device since the maximum power generation is expected at this orientation. Figure 5(a) shows the clamped piezoelectric beam modeled as a pin-pin mounted beam with a linear oscillation, $Y(t)$, applied to the base perpendicular to the beam. While the PZTs are expected to generate some power when the base is parallel to the oscillations, it is miniscule as compared to the electromagnetic component. For predicting and optimizing the piezoelectric generation of the system, the PZT beams were modeled individually as spring mass damper systems coupled with a piezoelectric structure, as in Figure 5(b). The piezoelectric is connected to a load resistor in parallel for energy harvesting. The oscillatory motion, $Y(t)$, applied to the base acts on effective mass $m$. In the Roundy and Wright model, the mechanical structure elements are coupled with the electrical system as shown in Figure 5(c). From this, the system equations were developed and are defined in Equations (14) and (15):
\[ \sigma_{in} = L_m \dot{S} + R_b \dot{S} + \frac{S}{C_k} + n V_{pi} \]  \hspace{1cm} (14)

\[ i_{pi} = C_b \dot{V}_{pi} + \frac{V_{pi}}{R_{pi}} \]  \hspace{1cm} (15)

where \( L_m \) is the equivalent inductor representing the mass or inertia of the generator, \( R_b \) is the equivalent resistance for the mechanical damping, \( \sigma_{in} \) is an equivalent stress generator for the stress caused by the input vibrations, \( C_k \) is the equivalent capacitor for the mechanical stiffness, and \( \dot{S} \) is the strain rate. The piezoelectric coupling is represented by a transformer with an equivalent turns ratio \( n \). The electrical elements of the piezoelectric beam are the voltage \( V_{pi} \) across the piezo device and the capacitance of the piezoelectric bender \( C_b \). It can be shown that from Equations (14) and (15), the voltage and power produced by the piezo device is defined as [23]

\[ V_{pi} = \frac{j \omega \frac{2 \zeta d_{31} t_p A_y}{k_c} a_y}{\sqrt{\frac{\omega_n^2}{\omega R_{pi} C_b} - \omega^2 \left( \frac{1}{\omega R_{pi} C_b} + 2 \zeta \omega_n \right) + j \omega \left( \frac{\omega_n^2}{2} \left( 1 + k_{31}^2 \right) + \frac{2 \zeta \omega_n}{R_{pi} C_b} - \omega^2 \right)}} \]  \hspace{1cm} \hspace{1cm} (16)

\[ P_{pi} = \frac{\left( 1 \right)^2}{\left( \frac{\omega_n^2}{\omega R_{pi} C_b} - \omega^2 \left( \frac{1}{\omega R_{pi} C_b} + 2 \zeta \omega_n \right) \right)^2} + \left( \frac{\omega_n^2}{2} \left( 1 + k_{31}^2 \right) + \frac{2 \zeta \omega_n}{R_{pi} C_b} - \omega^2 \right)^2 \]  \hspace{1cm} \hspace{1cm} (17)

\[ \omega_n = \sqrt{\frac{k}{m}} \quad k = \frac{c_p}{k_1 k_2} \quad k_1 = \frac{b(4 l_b + 3 l_m)}{4 l} \quad k_2 = \frac{l_b (l_c + l_b)}{3 b} \]  \hspace{1cm} \hspace{1cm} (18)

where \( \omega \) is the frequency of the driving linear oscillations, \( \omega_n \) is the resonant frequency of the piezo device, \( a_y \) is the RMS amplitude of the input acceleration, \( w \) is the width of the beam, \( l_b \) is the length of the beam, \( l_m \) is the length of the magnet, \( l_c \) is the length of the clamped area of the beam, \( t_p \) is the thickness of one PZT plate, \( t_{FR4} \) is the thickness of the
FR4, \( m \) is the mass of the tip magnets, \( \zeta \) is the damping ratio, \( j \) is the imaginary number, \( k_1 \) is the expression relating average stress to input force, and \( k_2 \) is the geometric constant relating strain to deflection. In addition, \( C_b \) is the capacitance of the PZT \((C_b = \alpha^2 w l c / 2 t c)\), and \( R_{pi} \) is the load resistance of the \( i \)th beam, \( a = 2 \) for PZT layers in parallel connection \( d_{31} \) and \( k_{31} \) are piezoelectric coefficients \((k_{31} = d_{31}^2 c_p / \varepsilon)\), \( c_p \) is the elastic constant of the PZT, \( c_f \) is the elastic constant of FR4, and \( \varepsilon \) is the dielectric constant of the PZT. If driving frequency is equal to resonant frequency \((\omega = \omega_n)\), then Equation (17) can be reduced to

\[
P_{pi} = \frac{1}{2 \omega_n^2} \times \frac{R_{pi} C_b^2 \left( \frac{2 \omega_n d_{31} l_c}{k_{31} a c} \right)^2 \alpha_y^2}{\left( 4 \zeta^2 + k_{31}^4 \right) \left( R_{pi} C_b \omega_n \right)^2 + 4 \zeta k_{31}^2 (R_{pi} C_b \omega_n) + 4 \zeta^2}
\]

By differentiating Equation (19) with respect to \( R_{pi} \) and setting the result to zero, the optimal load resistance can be defined as

\[
R_{opt} = \frac{1}{\omega_n C_b} \sqrt{\frac{2 \zeta}{\sqrt{4 \zeta^2 + k_{31}^4}}}
\]

### 4. Numerical results

Using the characteristic equations from Section 3.1, computational models and simulations of the system were developed in Matlab/Simulink. The simulations predicted the electrical and mechanical behavior of the energy harvesting system to reasonable accuracy. The simulations were used for proof of concept, in support of experimental optimization, and predicted power generation capabilities.

#### 4.1. Electromagnetic simulation

In order to characterize and optimize the electromagnetic generation of the system before fabrication and testing of the device, the EM power generation must be predicted. Inspection of Equation (9) shows that the only unknown is the angular velocity of the rotor. Thus, using Equations (1) and (2) a Simulink model of the spinning rotor was developed for horizontal and vertical orientation of the system with respect to gravity. It is to be noted that for both of these orientations the direction of motion is always parallel to the base. In order to find the optimal load resistance and power generation, a Matlab program was written that would find and plot the power generation for a range of load resistance values. Table 1 lists the parameters used for the EM simulations. The program first finds electromagnetic damping using Equation (13) for the first load resistance value. The Simulink simulation is then run using this electromagnetic damping value, and a plot of the rotational velocity of the rotor is extracted back into Matlab. A mean rotational velocity is found, and the power generated is then calculated from Equation (9). The program then repeats itself for the remaining resistance values.

The modal shaker that is used for experimental characterization has a maximum amplitude of \(~10\) mm. Equation (4) indicates that if the amplitude is kept constant, there is a minimum driving frequency under which the rotor will only swing. For an amplitude of 10 mm, the device cannot maintain spinning under 3 Hz. Thus, all the experiments and simulations for EM characterization were conducted at frequencies 3 Hz.
4 Hz, and 5 Hz with amplitude ~7–10 mm. The initial angular velocity of the rotor was set to 20 rad s\(^{-1}\) (3.1 Hz), 26 rad s\(^{-1}\) (4.2 Hz), and 32 rad s\(^{-1}\) (5.1 Hz), as described by Equation (3) for the three frequencies, respectively. Figure 6(a) shows the RMS electromagnetic power generated with load resistance for actuation frequencies 3 Hz, 4 Hz, and 5 Hz in horizontal orientation. The simulations were done for coils with 28 AWG wire and for beams with proof mass 1″ × 1″ × 1/8″ (25.4 × 25.4 × 3.2 mm\(^3\)) bottom magnet and 1″ × 1″ × 3/16″ (25.4 × 25.4 × 4.8 mm\(^3\)) top magnet. These were chosen based on experimental optimization results presented later in the experimental results. The calculated power generated during vertical orientation is not shown because the results were nearly identical to results for horizontal orientation when spinning. This was expected since as long as the rotor is spinning the angular velocity will be equal to the driving frequency. As expected, the maximum power for all three frequencies occurs approximately when the load resistance is equal to the internal resistance of the coils at 80 Ω.

From Equation (13), the load resistance is inversely proportional to the electromagnetic damping. In addition, Equation (4) indicates that the EM damping must be small enough to allow the rotor to spin. Combining these two relations, there is a load resistance below which the rotor will not be able to overcome damping and will not spin. The results of the simulations show this behavior very clearly.

From Figure 6(a), the power generated at low load resistance values is approximately zero. This is because the electromagnetic damping torque acting on the rotor is too high for it to overcome causing the rotor to stop spinning. This behavior is described in Figure 6(b), which shows the time domain of the angular position and velocity of the rotor from the Simulink simulation when the load resistance was 30 Ω. As can be seen, the angular position increases for a short time and then levels out, while the angular velocity jumps around before leveling out at zero. This indicates that the rotor begins spinning from the initial angular velocity but cannot overcome the damping forces, causing the rotor to stop spinning. This behavior was confirmed experimentally and can be seen in the supplemental movies.

Another behavior that was discovered is the rotor swinging that occurs when there is no initial angular velocity applied to the rotor. This is described in Figure 6(c), which shows the time domain for the angular position and velocity of the rotor from the Simulink simulation. The simulation was for vertical orientation with an initial velocity and position of zero, rotor perpendicular to gravity, and a load resistance of 80 Ω. Looking back to Figure 6(a), as the resistance increases there is a point for each frequency where the power jumps up considerably. This happens when the rotor is able to overcome the electromagnetic damping and continually spins. This behavior is described in Figure 6(d), which

| Term       | Definition                                      | Value     |
|------------|-------------------------------------------------|-----------|
| M          | Mass of rotor                                   | 0.13 kg   |
| A          | Rotor CoM offset from axis of rotation          | 33 mm     |
| l_r        | Radius of rotor                                 | 68 mm     |
| J          | Moment of inertia                               | 5.6 kg cm\(^2\) |
| A_x        | Amplitude of oscillations                       | 7 mm      |
| l_w        | Length of coil                                  | 16 m      |
| R_{int}    | Coil internal resistance                        | 80 Ω      |
| R_L        | Range of load resistance                        | 30–175 Ω  |
| \(\omega_d\)| Driving frequency                               | 3–5 Hz    |
shows the time domain of the angular position and velocity of the rotor from the Simulink simulation when the load resistance is $80 \, \Omega$. As can be seen, the cumulative angular position continually increases over the duration of the simulation while the angular velocity stays constant, indicating that the rotor is continually spinning. This behavior was confirmed experimentally and is shown in the supplemental movies.

### 4.2. Piezoelectric simulation

In this section, the results of the piezoelectric simulations are presented. The simulations were developed in Matlab using Equations (16) and (17), for piezoelectric voltage and power generation, respectively, to calculate the theoretical capabilities and characteristics of the piezoelectric generation of the system, as well as some optimized values. Table 2 lists the parameters used for the piezoelectric simulations. The natural frequency of the

| Term   | Definition               | Value    |
|--------|--------------------------|----------|
| $W$    | Width of beam            | 14 mm    |
| $T$    | Thickness of beam        | 0.73 mm  |
| $c_p$  | Young’s modulus          | 30.5 GPa |
| $k_{31}$ | Coupling factor         | 0.36     |
| $\varepsilon_r$ | Relative permittivity | 1700     |
| $g_{31}$ | Voltage constant       | $-11.3 \text{ V mm N}^{-1}$ |
| $t_p$  | Thickness of piezo       | 0.25 mm  |
| $b$    | Length of beam           | 33 mm    |
| $l_m$  | Length of magnet         | 19 mm    |
| $A_y$  | Magnitude of acceleration| 0.37 g   |
| $Z$    | Damping coefficient      | 0.07     |

Figure 6. Results of EM simulation: (a) RMS power from excitation of 3 Hz, 4 Hz, and 5 Hz with load resistance. Angular velocity and cumulative angular position of the rotor with (b) low load resistance (~30 $\Omega$), (c) swinging in vertical orientation: rotor starting from rest at horizontal, and (d) spinning with load resistance of 80 $\Omega$. 

Table 2. Piezoelectric simulation parameters.
piezo beam with a 1” × 1” × 1/8” (25.4 × 25.4 × 3.2 mm³) and a 1” × 1” × 3/16” (25.4 × 25.4 × 4.6 mm³) permanent magnet as the proof mass was calculated, using Equation (18), and found to be 20.2 Hz. By definition, the maximum power generated by the piezoelectric generator occurs when the driving frequency of the oscillations is equal to the resonant frequency. For this condition, the optimal load resistance resulting in maximum power, or resonant optimal load resistance, was found to be 120 kΩ using Equation (20). The first simulation predicted the power generated from transversal oscillations of frequency 18–24 Hz and acceleration 0.6 g, shown in Figure 7(a). As can be seen, the optimal load resistance varies greatly with driving frequency. In addition, the power generated increases while optimal load resistance decreases when the driving frequency is in the vicinity of the resonance frequency, as expected. From Figure 7(a), the piezoelectric beams were predicted to generate a maximum of 1.76 mW from a driving frequency of 20.2 Hz at the optimal load resistance (120 kΩ).

While the optimal load resistance varies with changing excitation frequency, the load resistance does not affect the resonant frequency. This can be seen in Figure 7(b), which shows the power generated with respect to the driving frequency load resistances of 10–130 kΩ. As expected, the maximum power generated at each load resistance occurs at the resonant frequency, 20.2 Hz, and increases as the load resistance approaches the optimal load resistance.

5. Experimental setup

A low force vibration system from Spectral Dynamics Inc. was used to test and characterize the electromagnetic and piezoelectric power generation capabilities. The system consisted of modal shaker (model SD-P11) powered by the PA-900 Power Amplifier, a four-channel Bobcat Analyzer (model 2308-9700), Bobcat analysis software, and a mini PC from SP Peripheral. The experimental setup is shown in Figure 8. Figure 8(a) shows the PC, Bobcat Analyzer, and amplifier. Figure 8(b) shows the experimental setup for piezoelectric testing (transversal), where the generator is excited with an oscillating motion perpendicular to the base and parallel to gravity. The setups for electromagnetic testing for the generator in vertical and horizontal orientation are shown in Figure 8(c) and 8(d), respectively. As can be seen, the shaker excites the generator with the base perpendicular to gravity and parallel to the oscillating motion in Figure 8(d) and with the base parallel to both gravity and the oscillating motion in Figure 8(c). The base accelerations were measured using a PCB Piezoelectronics Shear Accelerometer (model...
with sensitivity 101.4 mV/g. The acceleration signal from bobcat was filtered out using Butterworth low-pass filter with cutoff frequency 50 Hz.

In order to mount the generator to the actuating rod of the shaker, two custom brackets had to be made. The bracket for electromagnetic testing, shown in Figure 9(a), very simply screws onto the threaded actuating rod and then bolts onto the side of the base. The bracket for piezoelectric testing is more complex, as can be seen in Figure 9(b). The bracket is a cone shape with a large disk on the top that is bolted onto the underside of the base bottom with the screws that attach the slip ring to the base. The smaller disk on the bottom then is screwed onto the threaded actuating rod of the shaker. Three arms extruded radially outward from the bottom disk are bolted to the outer side walls for added stability.

6. Experimental results

6.1. Coil optimization

The initial prototype design had several variables that could be optimized. The first was the EM coil wire diameter. Wire diameters of 24 AWG (American Wire Gauge), 26 AWG, 30 AWG, and 32 AWG were tested against each other to find the wire diameter that works best with the coil winding process used, resulting in a high packing factor, and that generates the most power. Figure 10(a) shows the experimental setup of the coil wire diameter optimization. Two coils of each diameter were made and placed across from each other on the base in order to let the two outer beams pass over both the coils at the same time.
time. Each coil pair is connected in series and then connected to a load resistance equal to the internal resistance of the coil pair. The generator is actuated as described in Section 4 for electromagnetic generation, with the applied oscillation parallel to the base. Figure 10 (b) shows the RMS power and voltage with wire diameter. The 30 AWG generated the most power and was the easiest to work with. Thus, 30 AWG wire was chosen for the electromagnetic coils of the generator.

6.2. Magnet optimization
The second component to be optimized was the permanent magnets mounted on the ends of the piezoelectric beams. The effects of magnet size on the EM generation were characterized experimentally and numerically. All magnets tested had a length/diameter of 1". The magnets were optimized in terms of thickness and shape for both PEM generation. Five different magnets were chosen for testing: 1/8" and 3/16" block magnets and 1/8", 3/16", and 2/10" disk magnets. Only the thickness of the top magnet is varied since the generator was initially designed with only enough clearance for a 1/8" magnet on bottom. For the numerical simulation, Equation (8) was used to determine the residual magnetic field at the corresponding distance from the coil. SolidWorks was used to find the mass, center of gravity, and moment of inertia of the rotor for each magnet combination. The electromagnetic generation was then characterized for each top magnet with input oscillations of 5 Hz and 7 mm. Since the block magnets have more surface area than a disk magnet of the same dimensions, it was expected that the block magnets would generate more power. The results of this simulation and experiment can be as seen in Figure 11(a) and 11(b), respectively. While the simulation predicted that the 2/10" thick disk magnet would generate the most power, the experimental results showed that the 3/16" thick block magnet generated the highest power by approximately 5 mW. Thus, the 1" x 1" x 3/16" magnet was chosen for optimized EM generation.

Once the top magnet for optimized EM generation was selected, the effect of changing the proof mass size on the piezoelectric system was investigated. In order to characterize the effect of proof mass size on the natural frequency of the piezo beams, the power output of the piezos with three different top magnets, 1/8" block, 3/16" block, and 2/10" disk, was found experimentally over a wide range of excitation frequencies. The generator was excited using the experimental setup for piezoelectric testing, as shown in Figure 8 (b). When the 2/10" magnet was used, the deflection of the piezoelectric beams was too
large and the piezos, or rather the magnet proof mass, would strike the base of the generator violently. Thus, the power generated by the piezos with respect to frequency was only found for the smaller two magnets, the results of which are shown in Figure 12. As can be seen, the increase in mass results in a shift in the power curve to the left on the frequency axis, or a decrease in the resonant frequency. Figure 12 also indicates that the power generated with the larger mass is less than the power generated by the smaller mass for this experiment. Even though the change in power is only 0.1 mW, which is almost a 10% decrease, this result is opposite of what we would expect. One reason is that the optimal resistance value of the piezo beam changed slightly with the increase in proof mass and since the load resistance was kept constant, the power output with the new mass is less than the maximum power output.

The second reason is that the data points were taken over a wide range at increments of 2 Hz with only a few points taken near resonance. The experiment was conducted to show that, and find out by how much, the resonant frequency of the beam shifts with an increase in magnet mass, not to find the maximum power output from each magnet. To show that the piezoelectric power generation will increase with the larger top magnet, two additional experiments were conducted to find the maximum power generated by the device with the 1/8” top magnet and with the 3/16” top magnet. The piezoelectric simulation calculated the resonant frequency of the beams with a 1/8” top magnet and

![Figure 11. (a) Simulation and (b) experimental results of EM generation driven at 5 Hz and 7 mm with magnet tip masses: case 1: two 1/8” disk magnets. Case 2: two square 1/8”. Case 3: one 1/8” disk and one 3/16” disk. Case 4: one square 1/8” and one square 3/16”. Case 5: one 1/8” disk and one 2/10” disk.](image)

Figure 11. (a) Simulation and (b) experimental results of EM generation driven at 5 Hz and 7 mm with magnet tip masses: case 1: two 1/8” disk magnets. Case 2: two square 1/8”. Case 3: one 1/8” disk and one 3/16” disk. Case 4: one square 1/8” and one square 3/16”. Case 5: one 1/8” disk and one 2/10” disk.

![Figure 12. Experimental PZT power vs. frequency for 3/16” and 1/8” thick top magnets.](image)

Figure 12. Experimental PZT power vs. frequency for 3/16” and 1/8” thick top magnets.
with a 3/16" top magnet to be 22.5 Hz and 20.2 Hz with optimal load resistance 107 kΩ and 120 kΩ, respectively. Thus, using the experimental setup shown in Figure 8(b) and the driving frequencies and load resistances calculated by the piezoelectric simulation, the maximum combined piezoelectric power generated by the device was found to be 4.2 mW for the 3/16" top magnet and 2.1 mW for the 1/8" top magnet from a 0.4 g transversal excitation. Therefore, the 1" × 1" × 3/16" magnet was chosen as the top magnet for the device as it will provide higher power at a lower resonant frequency as compared to the 1" × 1" × 1/8" magnet.

6.3. Piezoelectric damping

For the output of the piezoelectric beams to be accurately calculated, the total damping coefficient of the piezoelectric system had to be found experimentally. The most direct way to determine the damping present in a vibrating system is to measure the rate of decay of the oscillations, which can be approximated using logarithmic decrement [24]. Furthermore, it can be shown from Equation (16) that the voltage generated in a piezoelectric beam is directly proportional and dependent on the vertical displacement of the beam. Thus, the damping coefficients were determined experimentally by exciting the device for piezoelectric generation (Figure 8b), with a step input and recording the voltage generated by each piezoelectric across a load resistance of 100 kΩ. The step response of the voltage for Beam 3 is shown in Figure 13(a). By assuming that the rate of decay is linear, the damping can be determined using logarithmic decrement, defined by

\[
\delta = \frac{1}{n} \ln \frac{A_0}{A_n}
\]

and

\[
\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}}
\]

where \(A_0\) is the first amplitude of the step response, \(A_n\) is the amplitude \(n\) peaks away, and \(\zeta\) is the damping coefficient. Since the decay between each peak is different, the approximation can be made more accurate by finding the average damping coefficient over a

![Figure 13](image-url) (a) Step response of the voltage generated by Beam 3 with load resistance 100 kΩ. (b) Ratio of decaying amplitude for Beam 3.
range of amplitudes. From Equation (21), the log ratios and corresponding curve fit of the first seven amplitudes are shown in Figure 13(b). The damping coefficient was found to be $\zeta = 0.07$ from Equation (22) using the slope of the curve fit as $\delta$.

6.4. Electromagnetic power generation

The maximum optimized power generated by the EM component of the device was obtained experimentally using the setup shown in Figure 8(d). As stated previously, at its maximum excitation amplitude of $\sim 10$ mm the modal shaker is unable to excite the rotor into continuous spinning below 2 Hz. Thus, the EM generation was characterized experimentally for input oscillations of 3 Hz, 4 Hz, and 5 Hz at a driving acceleration of 0.8 g for each. The resistance sweep was started at 60 $\Omega$, and the amplitude of the oscillations was increased until the rotor would continuously spin. Any resistances below 60 $\Omega$, for all three frequencies, and the rotor would not be able to overcome the electromagnetic damping and would stop spinning, as predicted by the analytical model.

As can be seen in Figure 14(a), the peak EM power generated was 75 mW at 3 Hz, 155 mW at 4 Hz, and 210 mW at 5 Hz with each occurring at an optimal resistance of 80 $\Omega$. The analytical model for EM generation, developed in Section 3.1, was then verified by comparing the experimental results with the results of the simulations performed in Section 3.2. Figure 14(b) compares the RMS power obtained from

![Figure 14](image-url)

Figure 14. Experimental EM power vs. resistance at 3 Hz, 4 Hz, and 5 Hz, with a driving acceleration of 0.8 g: (a) peak experimental power and (b) experimental and theoretical RMS power.
experimentation with the RMS power predicted by the numerical simulations. From the figure, the simulated power agrees well with the measured power, with both showing the optimal load resistance to be 80 Ω. The accuracy of the EM model will be discussed further in Section 7.

6.5. Piezoelectric power generation

The power generation characteristics of the piezoelectric component was optimized experimentally by first finding the natural frequency of the beams, followed by finding the optimal load resistance for each beam at this frequency, resulting in the maximum piezoelectric power generated. Using experimental setup for piezoelectric generation, shown in Figure 8(b), a load resistor is applied across each PZT and the voltage across each load resistor is measured. Figure 15(a) shows the RMS power generated by each beam with over a range of excitation frequencies applied to the generator. As can be seen, a large peak occurs first at 15 Hz but only for Beam 2, the middle beam. This was attributed to the generator base experiencing an eccentric rocking motion around this frequency. The rocking motion is most likely caused by a combination of a non-transversal resonant frequency, geometric conditions such as clearance between components, or related variables occurring at 15 Hz made worse by the unbalanced nature of the

Figure 15. Experimental piezoelectric RMS power at 0.6 g input acceleration: (a) frequency response of each beam with 37 kΩ load resistance, and (b) experimental and simulation power vs. load resistance at 20.6 Hz.
generator. Further detailed investigations are required to determine the actual cause. As can be seen, the transversal natural frequency of the beams is found to be 20.6 Hz. This corresponds well with the calculated value using Equation (22), which was found to be 20.2 Hz.

An experiment was then conducted to find the optimal load resistance and maximum power generated by each beam when excited at the experimental resonance frequency. Figure 15(b) shows the power generated from an excitation frequency of 20.6 Hz at 0.6 g. The RMS power generated by each PZT beam at this frequency was found to be 1.13 mW at 156 kΩ for Beam 1, 1.25 mW at 100 kΩ for Beam 2, and 1.25 mW at 168 kΩ for Beam 3. An additional experiment was conducted to find the maximum power generated by each beam when excited at the numerical resonance frequency and with optimal load resistance, 100 kΩ. When excited at 20.2 Hz and 0.4 g, the maximum RMS power was found to be 1.90 mW for Beam 1, 0.53 mW for Beam 2, and 1.79 mW for Beam 3. From these two experiments, the resonance frequency of the beams varies slightly due to placement. The resonance frequency of the middle beam, Beam 2, appears to be 20.6 Hz, while the resonance frequency of the outer two beams appears to be 20.2 Hz. Further characterization at the numerical resonance frequency is needed to make a definite conclusion.

6.6. Combined generation

For simplicity in developing the analytical model, the power generated by one of the transduction methods was assumed to be negligible. However, in the practical case the vibration source could act in any direction or could have a narrow range of frequency. Therefore, the combined harvesting capabilities of both transduction systems were characterized when actuated for either principal mode of operation. Figure 16(a) and 16(b) shows the power generated by all three beams from the rotor spinning at 4 Hz, and 5 Hz driven by horizontal oscillations of 0.8 g, respectively. The experimental setup can be seen in Figure 8(d). The combined maximum power from all three beams was 332 µW at 4 Hz and 245 µW at 5 Hz. Figure 17 shows the time domain plots for the voltage generated by each beam and the electromagnetic transducer at 5 Hz. From the PZT voltages, the beams experience undesirable harmonic distortions, which can be attributed to the low excitation frequency (off resonant conditions). The electromagnetic voltage completes five cycles in 0.2 s (5 Hz), corresponding to the magnets moving over ten coils each cycle.
A second experiment using the setup in Figure 8(b), consisting of two separate tests, was conducted measuring both transduction systems for a transversal excitation at 20.2 Hz. In both tests, the piezoelectric beams and EM generator had a load resistance of 100 kΩ, each, and 80 Ω, respectively. The first test was performed with the rotor oriented with the magnets directly over coils, allowing for EM generation. The time domain plot for the excitation accelerations, EM voltage, and the voltages generated by each beam is shown in Figure 18(a), with the same time domain plot for only the acceleration and EM voltage shown in Figure 18(b). As can be seen in Figure 18(a), Beams 1 and 3 were driven at their natural frequency and resulting in almost identical voltage signals with peak voltages 20.7 V and 19.9 V and RMS voltages of 13.6 V and 13.3 V, respectively. However, Beam 2 only generated a peak voltage of 10.4 V and RMS voltage 7.3 V. It is clear from the results that the magnetic interactions and dynamics of the device cause the middle beam to have a different resonant frequency from the outer two beams. This was verified by reordering the placement of the beams and running the same experiments, which gave identical results regardless of order. From Figure 18(b), the driving acceleration had amplitude ~0.4 g, and the EM voltage reached amplitude of 0.75 V with RMS voltage 0.47 V.

![Figure 17](image1.png)

**Figure 17.** Time domain plot of the voltage generated by each beam and the electromagnetic transduction when rotor is spinning at 5 Hz with driving oscillation acceleration of 0.8 g.

![Figure 18](image2.png)

**Figure 18.** Experimental results from transversal excitation at 20.2 Hz with the magnets directly over coils. Load resistance of 80 Ω across the EM coils and 100 kΩ across each beam. Time domain plot: (a) excitation acceleration, each PZT voltage, and EM voltage, and (b) only excitation acceleration and EM voltage.
The second test was performed with the same conditions as the previous test, but with the rotor oriented so that each magnet was exactly halfway between two coils. The resulting time domain plots are shown in Figure 19(a) and 19(b). As can be seen, the driving acceleration was again ~0.4 g, and the piezoelectric voltages are all nearly the same as the previous test: Beam 1, Beam 2, and Beam 3 had peaks 19.2 V, 10.8 V, and 19.9 V and RMS voltages 13.6 V, 7.3 V, and 13.2 V. From Figure 19(b), the EM voltage reached a peak of only 74 mV with an RMS voltage of 18 mV. The RMS power generated in both tests by EM generation, each piezoelectric beam, and the combined piezoelectric power was calculated and is presented in Figure 20. As can be seen, the EM power generated was substantial when the magnets are directly over the coils was 2.77 mW, almost 1 mW more than the power from Beam 1 or Beam 3. However, when the magnets were in-between the coils, the EM power generated was 4.4 µW. The piezoelectric power generated in this test was three orders of magnitude higher, making the EM power generated negligible. These results will be discussed further in the following section.

Figure 19. Experimental results from transversal excitation at 20.2 Hz with the magnets in-between coils. Load resistances of 80 Ω across the EM coils and 100 kΩ across each beam. Time domain plot: (a) excitation acceleration, each PZT voltage, and EM voltage, and (b) only excitation acceleration and EM voltage.

Figure 20. Power generated from transversal excitations at 20.2 Hz with the magnets directly over the coils and in-between coils.
Experimental optimization of the coil wire diameter, magnet shape, and size was performed. Based on the experimental results, the optimized magnet proof mass chosen consisted of a 1" × 1" × 1/8" (25.4 × 25.4 × 3.2 mm³) magnet on bottom and a 1" × 1" × 3/16" (25.4 × 25.4 × 4.8 mm³) magnet on top. The electromagnetic power generation capabilities of the optimized device were then characterized experimentally. The results showed the maximum RMS power generated by electromagnetic transduction with driving oscillations of 5 Hz, 4 Hz, and 3 Hz at 0.8 g to be 120 mW, 80 mW, and 42 mW, respectively, with a load resistance of 80 Ω. These results matched well with the simulation results with a percent error of 9.3%, 7.7%, and 8.9% for 3 Hz, 4 Hz, and 5 Hz, respectively. As discussed previously, the main purpose of the analytical models developed for the generator was not to precisely model the entire coupled dynamics of the device, but rather to assist in optimizing the device and then characterize its optimal power generation capabilities. Thus, the model provided insightful information and was reasonable for the proposed device.

One design consideration for future versions of the device is the quality of the slip ring connector that was used to connect the piezos with the energy harvesting circuit. The slip ring used in this device was fairly cheap, and as the slip ring was used more, the piece that the rotor was connected tends to wobble, making it very difficult to get consistent results from test to test. This showed how dependent the power generated is on the distance between the magnet and coil. While these results are not shown in this paper, if the distance between was increased/decreased by even as much as a millimeter, the power generated would change as much as 10 mW. Another design consideration is the consistency of the coil dimensions. Closer inspection of the electromagnetic voltage time plot in Figure 17 shows considerable variations from peak to peak. This is caused by the variations in the diameter, width, and packing factor from coil to coil. While the fabrication method used for winding the coils is effective, it is not very accurate or consistent. However, for the purposes of proof of concept and laboratory prototype, the variations in amplitude are acceptable.

The piezoelectric power generation capabilities were also characterized experimentally for the optimized device under various scenarios and parameters. The power generated by each piezoelectric beam was measured individually so that the behavior of each device could be characterized accurately. The experimental setup for driving piezoelectric generation is shown in Figure 8(b), where an excitation oscillation is applied perpendicular to the base. The resonance frequency of the beams was found to be 20.2 Hz from numerical analysis, Equation (18). To verify this, the voltage generated by each beam was measured while the device was actuated over a range of driving frequencies (10–30 Hz). The frequency response of each beam can be seen in Figure 15(a). There are a couple of differences between the beams, the first being the voltage spike of Beam 2 at 15 Hz. This spike is attributed to an eccentric rocking motion that the generator experiences when excited at 15 Hz. Since the location of the center of mass of the rotor is in the very middle of Beam 2, as can be seen in Figure 4(a) and 4(b), it experiences much greater amplitudes than the outer beams from the rocking motion. The reason for the rocking motion is not completely understood; however, the unbalanced nature of the device certainly amplifies the initial cause of the behavior. As can be seen in Figure 15(a), Beam 2 also generated the most power at 20.6 Hz as compared to Beams 1 and 3. This was initially thought to be caused solely from the increased forces felt by Beam 2, as the center of mass of the rotor coincides with the middle of Beam 2. However, it was found that while 20.6 Hz is the resonant frequency of the
center beam, this is slightly off for the outer two beams. Further experiments were conducted to show that slight variations in resonance frequency affect the performance of the PZT beams significantly. The first step was to find the optimal load resistance where the maximum power occurs. In order to find the optimal load resistance, the power generation capabilities of the piezoelectric system were characterized for the experimental resonant frequency. The power generated by each piezoelectric was recorded when the device was excited at 20.6 Hz with acceleration amplitude 0.6 g over a range of load resistances. Figure 15(b) shows the results of the experiment along with the simulation results for a driving frequency of 20.6 Hz. The maximum experimental power was 1.25 mW and was generated by Beams 2 and 3 with a load resistance of 100 kΩ and 168 kΩ, respectively. As can be seen in Figure 15(b), the simulation corresponds very well with this result, with a maximum predicted power of 1.34 mW occurring at 134 kΩ. From these points and only taking power into account, an error of 6.7% was calculated for the piezoelectric power generation model. Thus, the model is more than accurate for our purposes. The beams were then characterized for the numerical resonant frequency, 20.2 Hz. The three beams generated a combined maximum power of 4.23 mW at a frequency of 20.2 Hz and 0.4 g acceleration amplitude. Beams 1 and 3 generated 1.7 mW and 1.9 mW, respectively, and Beam 2 only generated 0.5 mW. Thus, Beams 1 and 3 have a resonance frequency closer to 20.2 Hz, while Beam 2 is closer to 20.6 Hz. The load resistance of each beam in this experiment was 100 kΩ.

The electromagnetic subsystem was characterized under horizontal excitation using a modal shaker. As in Figure 8(d), the shaker drives the harvester with a linear oscillation of 5 Hz. When an initial velocity was given to the rotor larger than the 5 Hz, as described by Equation (3), the rotor would start to spin. The rotor would continuously spin as long as the EM load resistance was big enough to satisfy Equations (4) and (13), >60 Ω. If the load resistance was too small, <60 Ω, the rotor would stop spinning once the circuit was closed, as can be seen in the supplemental movies. This behavior was described in the theoretical and experimental results, as can be seen in Figure 6(b), associated with the energy generation capabilities.

8. Future work and applications

There are several specific applications for the proposed electromagnetic and piezoelectric energy harvesting device. As a commercial product or military product, the device could be used for the high adventuring type, such as hikers, soldiers, mountain climbers, cross-country skiers, marathon runners, etc. The device would be worn on the ankle or wrist, charging a battery that would be used to power their electronics in case of an emergency. One promising application is for bicyclists and mountain bikers. With the generator worn on the ankle, the rotational motion from pedaling would excite the unbalanced rotor enabling the electromagnetic power generation, while the relatively high frequencies and impact forces encountered when mountain biking would power the piezoelectric beams. The third possible application would be for cell phones and other personal electronics. However, the device would need to be scaled down to a much smaller size, introducing new set of limitations. As the generator is scaled down, the EM generation decreases rapidly while the piezoelectric resonance frequency increase greatly. The geometry and tip mass of the cantilever beam can be modified for decreased resonance frequency. However, there is a limit to how much the width of the beam can be decreased or the mass of the magnet can be increased before the beam will break.
Future work is to be done on the hybrid energy harvesting device presented here. Currently, a scaled-down prototype is being built that will undergo extensive experimental tests, focusing on practical applications. Several tests with the current prototype for harvesting from walking have been conducted. We will also study other methods to investigate the electromagnetic modeling by considering the radial magnetic flux as described by Sodano et al. [25,26] and other approaches. All these studies will be presented in a future work.

9. Conclusions

In this paper, we briefly discussed the state-of-the-art hybrid multimodal energy harvesting device and systems for use in powering electronic device, and presented a novel energy harvesting system that converts multiple excitation sources. The device uses translational and transversal motions or oscillations and slight self-excitation to continuously spin and generate energy simultaneously from electromagnetic and piezoelectric system in one device. Several experimental results were presented, which were confirmed with theoretical modeling and simulations. Further optimization will lead to immense use of the device. We hope that the device will get applications in various systems to power consumer electronics in the health care, military, hobbyist, heath monitoring of structures, biomedical device, and many more.

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