A novel black-hole mass scaling relation based on coronal gas, and its dependence with the accretion disc

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ABSTRACT
Using bona-fide black hole (BH) mass estimates from reverberation mapping and the line ratio [Si vi] 1.963µm/Brγ broad as tracer of the AGN ionising continuum, a novel BH-mass scaling relation of the form log(MBH) = (6.40 ± 0.17) − (1.99 ± 0.37)× log ([Si vi]/[Brγ broad]), dispersion 0.47 dex, over the BH mass interval, 10^6 − 10^8 M⊙ is found. Following on the geometrically thin accretion disc approximation and after surveying a basic parameter space for coronal lines production, we believe one of main drivers of the relation is the effective temperature of the disc, which is effectively sampled by the [Si vi] 1.963µm coronal line for the range of BH masses considered. By means of CLOUDY photoionisation models, the observed anti-correlation appears to be formally in line with the thin disc prediction Tdisc ∝ MBH⁻¹/₄.

Key words: accretion, accretion discs – radiation mechanisms: thermal – galaxies: active – techniques: spectroscopic – quasars: supermassive black holes – quasars: emission lines

1 INTRODUCTION
The determination of black hole (BH) masses is a major focus in the community. Most accurate determinations rely on dynamical analysis, for galaxies including our Milky-Way. That approach, though, gets restricted to relative nearby objects, where high angular observations resolving the BH radius of influence or extensive variability monitoring programs of the broad line region in active galactic nuclei (AGN) are possible (e.g. Peterson 1993; Genzel et al. 2010; Kormendy & Ho 2013; Bentz & Katz 2015). Most BH mass estimates are based however on powerful correlations between the BH mass and the bulge stellar velocity dispersion - the M-σ relation (e.g. Ferrarese & Merritt 2000; Gültekin et al. 2009), or the AGN continuum luminosity - so called mass-luminosity relation by which the optical, UV or X-ray luminosity are found to correlate with the Broad-Line-Region (BLR) size (e.g. Koratkar & Gaskell 1991; Kaspi et al. 2000, 2005; Landt et al. 2013, and references therein).

While the use of the M-σ relation requires the measurement of σ, it is not always easy to determine, particularly in AGN. In these objects, the strong continuum from the nuclear region dilutes the stellar absorption lines. In order to overcome this difficulty, a number of alternative scaling relations using emission lines such as [O iii] λ5007 to measure the mass of the bulge (e.g. Nelson & Whittle 1996), [O i] λ3727 (e.g. Salviander et al. 2006), Hβ or Hα (e.g. Kaspi et al. 2005; Greene & Ho 2005) to infer on the BLR size, or [Fe ii] in the near-infrared (e.g. Riffel et al. 2013) to infer on the stellar σ, have been proposed.

Overall, the scatter of the scaling relations in the literature is 40% or larger. A fraction of the scatter should be intrinsic, inherent to the nature of the parameters used in the relations and their variations between objects. The Hubble type and chiefly, the presence of bulges or pseudo bulges affect the M-σ relation ( Gültekin et al. 2009). An intrinsic scatter of up 40% in the continuum luminosity - BLR size scaling relation is inferred by Kaspi et al. (2005), mostly introduced by changes in the optical - UV continuum shape with increasing AGN luminosity. Other factors such as intrinsic differences in the BLR density and the ionisation parameter are also expected to play a role (Collin-Souffrin et al. 1988; Marziani et al. 2019; Panda 2020).

With the start of operations of facilities optimised for the near - mid Infrared (IR) region, the James-Webb Space Telescope (e.g. Gardner et al. 2006) and the Vera Rubin Observatory’s LSST (e.g. Ivezić et al. 2019), the use of scaling-relations focused on spectral features centred in that interval will be an asset. This paper presents a novel approach to estimate BH masses as a function of high ionization IR emission lines, namely the coronal lines, after normalisation to HI broad line emission, so far for Type I AGN only.

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The motivation behind relies on the high ionisation potential, IP, of coronal lines, above 50 eV up to few hundreds eV which makes them excellent tracers of the ionising continuum. Hence, their ability to sample the peak - or highest temperature, of the ionising continuum whether this is dominated by a thermal component, an accretion disc or black body modified spectrum.

Coronal lines (CL) spread over the X-rays, optical and IR spectrum. Although often fainter than the classical medium-ionisation lines used for photoionisation diagnosis, high angular resolution in nearby AGN has shown that CL particularly in the near-IR are among the most conspicuous features (e.g. Marconi et al. 1994; Reunanen et al. 2003; Prieto et al. 2005; Rodríguez-Ardila et al. 2006; Müller-Sánchez et al. 2011; Rodríguez-Ardila et al. 2017; Gravity Collaboration et al. 2020).

In this work, we explore possible dependencies of the BH mass with optical and infrared coronal emission of different IP after normalising of the coronal emission to the nearest H$eta$ broad line emission. The CLs employed are the most common and brightest ones in AGN (Reunanen et al. 2003; Rodríguez-Ardila et al. 2011; Lamperti et al. 2017). A tight correlation between BH mass and the CL ratio [Si vi] 1.96 μm/Bry is observed. Possible explanations for the observed correlation (and the absence of it for some other CLs) are examined in the context of accretion theory and photoionisation model predictions.

The paper is organised as follows. Sect. 2 describes the observations and data reduction employed in this work; Sect. 3 deals with the diagnostics diagrams developed by us to weigh the BH mass using CL; Sect. 4 examines the scaling relationship in the framework of accretion theory and photoionisation predictions. The implications and limitations of the results are presented in Sect. 5. Throughout this paper, the following cosmology is adopted: $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$; $\Omega_M = 0.30$; $\Omega_{\Lambda} = 0.70$.

2 DATA SELECTION

Objects in this work are selected by having BH masses determined by reverberation mapping and single epoch optical and/or near-IR spectra with accurate CL measurements. The first criterion restricts the sample to Type I sources only. The second avoids variability issues. Although we give preference to sources with both optical and near-IR spectra available, this final criterion could not always be fulfilled.

The CL used are [Fe vi] 6087 Å in the optical and [S vi] 1.091 μm, [Si x] 1.432 μm and [Si vi] 1.964 μm in the near-IR. They are among the strongest CL in AGN (Reunanen et al. 2003; Rodríguez-Ardila et al. 2011; Lamperti et al. 2017) and span a wide IP range. 100 - 350 eV. In addition, H u lines of Hγ, Paβ and Bry are employed. The whole sample sets the ionising continuum over the 13.6 - 351 eV range. Near-IR CL were preferred because of their reduced extinction. Optical CL [Fe vi, Si vi] was also selected because of its strength, still moderate extinction, and IP close to that of [Si vi].

The final working sample of objects has 31 AGN (Table 1). It includes BH masses, most from Bentz & Katz (2015) compilation, coronal line ratios measured from spectra in this work or from spectra already described in other publications, and data sources. For a sub-sample, optical and near-IR data are presented in this work for the first time. Table 2 lists these later sources with details of the observations, including telescope/instrument employed, date of observation, airmass and exposure time.

2.1 Optical Spectroscopy

Optical spectra were taken from a variety of sources, as indicated in the last column of Table 1. In more than half of the sample, spectra from the Sloan Digital Sky Survey (SDSS) data release 7 (Abazajian et al. 2009) were employed. SDSS delivers fully wavelength and flux calibrated spectra. Therefore, data reduction for these objects will not be discussed here. Similarly, archival flux-calibrated spectra for Ark 564 taken by the Faint Object Spectrograph (FOS) on-board the Hubble Space Telescope (HST) were employed. Details of observations and reduction of this target will not be addressed here. NGC 4051 employs archival spectroscopy available from the NASA’s Extragalactic Database (NED). Details of that observation can be found in Moustakas & Kennicutt (2006). Mrk 335 was observed using the 2.15 m telescope at the Complejo Astronómico El Leoncito (CASLEO). Details of the observations and data reduction are in Rodríguez-Ardila et al. (2002). Fully reduced spectra for Fairall 9, NGC 4151, and Mrk 509 were extracted from the AGN Watch Project¹.

The second major source of optical data is the 4.1 m Southern Observatory for Astrophysical Research (SOAR) Telescope at Cerro Pachon, Chile. The observations were carried out using the Goodman Spectrograph (Clemens et al. 2004), equipped with a 600 l/mm grating and a 0.8 arcsec slit width, giving a resolution R ~ 1500. In addition to the science frames, standard stars (Baldwin & Stone 1984) were observed for flux calibration. The average seeing of the observations was ~ 1 arcsec. None of the nights were photometric, implying that the flux calibration is relative. Accordingly, no attempt was made to put the optical and NIR data on the same flux scale. Hg-Ar arc lamps were taken after the science frames for wavelength calibration. Daytime calibrations include bias and flat field images.

The data were reduced using standard IRAF tasks. It includes subtraction of the bias level and division of the science and flux standard star frames by a normalised master flat-field image. Thereafter, the spectra were wavelength calibrated by applying the dispersion solution obtained from the arc lamp frames. Finally, the spectra of standard stars were extracted and combined to derived the sensitivity function, later applied to the 1D science spectra. The final products are wavelength and flux calibrated optical spectra.

In all cases above, the final spectra were corrected for Galactic extinction using the extinction maps of Schlafly & Finkbeiner (2011) and the Cardelli et al. (1989) extinction law. Figures A1 to A4 show the optical spectra employed in this work.

2.2 NIR Spectroscopy

Most of the NIR emission line flux ratios employed in this work were extracted from Riffel et al. (2006). For targets not reported in that publication, observations were obtained using either the Gemini Near-Infrared Spectrograph (GNIRS) attached to the Gemini North Telescope or the ARCOIRIS spectrograph, mounted on either Blanco or SOAR Telescopes. Note that ARCOIRIS was installed on Blanco since 2017 and up to 2019, when it was then moved to SOAR with no modifications regarding their setup. Below we describe the observations and data reduction procedures, noting that no distinction between Blanco and SOAR is made. Both data collection and treatment is made employing the same observing strategy and reduction pipelines.

¹ http://www.astronomy.ohio-state.edu/agnwatch/
Figures A5 to A7 show the final reduced NIR spectra for those galaxies not reported in Riffel et al. (2006).

2.2.1 ARCoIRIS Blanco/SOAR data

NIR spectra of Fairall 9, 3C 120, Mrk 707, NGC 3783, Mrk 1310, Mrk 841 and NGC 6814 were obtained using the ARCoIRIS spectrograph attached to either the 4 m Blanco Telescope or the 4.1 m SOAR telescope. The science detector employed is a 2048 × 2048 Hawaii-2RG Hg–Cd–Te array with a sampling of 0.41 arcsec/pixel. The slit assembly is 1.1 arcsec wide and 28 arcsec long. The delivered spectral resolution R is ~3500 across the different dispersion orders. Observations were done nodding in two positions along the slit. Right before or after the science target, a telluric star, close in airmass to the former, was observed to remove telluric features and to perform the flux calibration. Cu–Hg–Ar frames were also observed at the same position as the galaxies for wavelength calibration.

The spectral reduction, extraction and wavelength calibration procedures were performed using spec2D (v4.1), an IDL-based software developed and provided by the SpeX team (Cushing et al. 2004) with some modifications specifically designed for the data format and characteristics of ARCoIRIS, written by Dr. Katelyn Allers (private communication). Telluric features removal and flux calibration were done using xTellCor (Vacca et al. 2003). The different orders were merged into one single 1D spectrum from 1 to 2.4 μm using the xMergedOrders routine. We then corrected these data for Galactic extinction using the Cardelli et al. (1989) law and the extinction maps of Schlafly & Finkbeiner (2011).

2.2.2 GNIRS/Gemini spectroscopy

Near-infrared spectra of NGC 4395 and Ark 564 were collected using GNIRS (Elias et al. 2006) in the cross-dispersed mode. It allows simultaneous z+H, H and K band observations, covering the spectral range 0.8 – 2.5 μm in a single exposure. GNIRS science detector consists of an ALADDIN 1k × 1k In-Sb detector. The instrument setup includes a 32 l/mm grating and a 0.8x7 arcsec slit, giving a spectral resolution of R ~1300 (or 320 km s⁻¹ FWHM). Individual exposures were taken, nodding the source in a ABBA pattern along the slit. Right after the observation of the science frames, an A0V star was observed at a similar airmass, with the purpose of flux calibration and telluric correction.

The NIR data were reduced using the XDGNIRS pipeline (v2.0)², which delivers a fully reduced, wavelength and flux calibrated, 1D spectrum with all orders combined (Mason et al. 2015). Briefly, the pipeline cleans the 2D images from radiative events and prepares a master flat constructed from quartz IR lamps to remove pixel to pixel variation. Thereafter, the s-distortion solution is obtained from daytime pinholes flats and applied to the science and telluric images to rectify them. Argon lamp images are then used to find the wavelength dispersion solution, followed by the extraction of 1D spectra from the combined individual exposures. The telluric features from the science spectrum are removed using the spectrum of a A0V star. Finally, the flux calibration is achieved assuming a black body shape for the standard star (Pecaut & Mamajek 2013) scaled to its K-band magnitude (Skrutskie et al. 2006). The different orders are combined in to a single 1D spectrum and corrected for Galactic extinction using the Cardelli et al. (1989) law and the extinction maps of Schlafly & Finkbeiner (2011). The spectra of these two sources are not presented here as they are shown in Mason et al. (2015).

In order to measure the flux of the lines for the sub-sample of objects described above, we modelled the observed profiles with a suitable function that best represents them and then integrated the flux under that function. To this purpose we employ the LINER routine (Pogge & Owen 1993). This software performs a least-square fit of a model line profile (Gaussian, Lorentzian, or Voigt functions) to a given line or set of blended lines to determine the flux, peak position and FWHM of the individual components. Typically, one or two Gaussian components were necessary to represent the coronal lines. For the permitted lines of Hλ a broad component associated to the BLR was employed. In this process, the underlying continuum emission was approximated by a linear fit.

For the optical part, the measurement of the Hβ flux was preceded by the removal of the underlying power-law continuum and the pseudo-continuum produced by the Fe lines that contaminates Hβ. This was done following the prescription of Boroson & Green (1992).

Table 1 shows the measured optical emission line flux ratio between [Fe v]1.6087 and the broad component of Hβ (column 3) and the NIR ratios for [Si v] 1.964 μm/Bry (column 4), [Si x] 1.431 μm/Paβ (column 5), and [S vi] 0.9914 μm/Paβ (column 6). For the later three ratios the flux associated to the broad component of the Bracket or Paschen lines was employed. Table B1 in the Appendix lists the individual fluxes of all lines employed in this work. Note that because the optical and NIR spectra were taken on different dates and in most cases, different telescopes, the intrinsic line ratios Hβ/Paβ and Hβ/Bry may depart from their theoretical value. This, however, does not affect our results as we do not use line ratios that combine both spectral regions. However, for consistency, the reported fluxes for the broad component of Hβ, Paβ and Bry are compared to those reported by Landt et al. (2008) for the objects in common. We found the values agree within a factor of 2, with most cases the difference being not larger than 30%. Moreover, our measured fluxes for NGC 4151 and NGC 5548 are in excellent agreement to those reported by Landt et al. (2015a) and Landt et al. (2015b) for these two objects, respectively.

3 CORONAL LINE DIAGNOSTIC DIAGRAMS

The use of coronal lines as a proxy of black hole measurements was theoretically explored by Cann et al. (2018) via photoionisation simulations. They show that for intermediate to low BH mass sources (10² – 10⁵ M⊙), CL of very high IP are favoured with respect to those of lower IP. Their approach, though, could not be assessed because of the lack of suitable data for AGNs with BH masses in that regime. Here, we expand the photoionisation modelling to the high BH mass range 10⁶ – 10⁹ M⊙, and confront the predictions with the data gathered in Table 1.

Fig. 1 presents new diagnostic diagrams in which the BH mass for the objects in the sample is plotted against a given CL flux normalised to the closest in wavelength H1 broad emission. The first plot – upper left panel, involving [Si v] 1.9641 μm/Bry broad-IP [Si v] = 167 eV, shows a clear trend with M BH over three orders of magnitude in BH mass. A linear regression yields:

\[
\log M_{BH} = (6.40 \pm 0.17) - (1.99 \pm 0.37) \times \log \left( \frac{[\text{Si} \ \text{VI}]}{\text{Bry broad}} \right),
\]  

and a 1σ dispersion of 0.47 dex in BH mass. The regression analysis

² Based on the Gemini IRAF packages
Table 1. Black hole mass and Coronal Line ratios for the galaxy sample.

| Galaxy     | log $M_{BH}$ | [Fe v] / Hβ | [Si v] / Brγ | [Si x] / Paβ | [S v] / Paβ |
|------------|-------------|-------------|-------------|-------------|-------------|
| Mrk 335    | 7.23±0.04   | 0.077±0.009 | 0.40±0.09^e| 0.04±0.01   | 0.02±0.006  |
| Fairall 9  | 8.29±0.09   | 0.038±0.003 | 0.11±0.02^f| 0.08±0.01   | 0.04±0.01   |
| NGC 683    | 7.57±0.06   | ...         | 0.24±0.07^e| ...         | ...         |
| 3C 120     | 7.74±0.04   | ...         | 0.33±0.06^e| 0.08±0.02   | 0.03±0.01   |
| Mrk 707    | 6.50±0.10^a| 0.025±0.002 | 0.38±0.04^f| ...         | ...         |
| Mrk 110    | 7.29±0.10   | 0.05±0.002  | ...         | ...         | ...         |
| NGC 3227   | 6.78±0.10   | ...         | 0.75±0.20  | 0.012±0.004 | ...         |
| Mrk 142    | 6.29±0.10   | 0.02±0.004  | ...         | ...         | ...         |
| SBS 1116+583A | 6.56±0.09 | 0.01±0.002  | ...         | ...         | ...         |
| PG 1126-041| 8.08±0.03^b| ...         | 0.28±0.02  | 0.04±0.01   | 0.025±0.002 |
| NGC 3783   | 7.37±0.08   | 0.052±0.002 | 0.42±0.09^f| 0.05±0.01   | 0.023±0.003 |
| Mrk 1310   | 6.21±0.08   | 0.032±0.002 | 0.57±0.17^f| 0.06±0.01   | 0.06±0.01   |
| NGC 4051   | 6.13±0.12   | 0.123±0.012 | 0.96±0.11  | 0.33±0.02   | 0.205±0.030 |
| NGC 4151   | 7.55±0.05   | 0.02±0.001  | 0.51±0.05  | 0.05±0.01   | 0.057±0.003 |
| Mrk 202    | 6.13±0.17   | 0.033±0.004 | ...         | ...         | ...         |
| Mrk 766    | 6.82±0.05   | 0.03±0.000^d| 0.78±0.10  | 0.05±0.01   | 0.045±0.002 |
| Mrk 50     | 7.42±0.06   | 0.005±0.001 | ...         | ...         | ...         |
| NGC 4395   | 5.45±0.13   | 0.092±0.005 | 1.18±0.10^f| 0.02±0.01   | 0.053±0.006 |
| Mrk 771    | 7.76±0.20   | 0.03±0.002  | ...         | ...         | ...         |
| NGC 4748   | 6.41±0.11   | ...         | 0.93±0.06  | 0.06±0.02   | 0.137±0.042 |
| PG 1307+085| 8.54±0.13   | 0.01±0.002  | ...         | ...         | ...         |
| MGC-6-30-15| 6.60±0.12   | 0.017±0.002 | ...         | ...         | ...         |
| NGC 5548   | 7.72±0.02   | 0.044±0.003 | 0.61±0.09  | 0.11±0.01   | 0.122±0.011 |
| PG1448+273 | 6.97±0.08   | ...         | 0.57±0.11  | ...         | ...         |
| Mrk 200    | 7.28±0.02   | 0.023±0.002 | ...         | ...         | ...         |
| Mrk 841    | 8.10±0.02^c| 0.008±0.002 | 0.20±0.06^f| 0.02±0.01   | 0.024±0.002 |
| 3C 390.3   | 8.64±0.04   | 0.018±0.001 | ...         | ...         | ...         |
| NGC 6814   | 7.04±0.06   | ...         | 0.13±0.02^f| ...         | ...         |
| Mrk 509    | 8.05±0.04   | ...         | 0.17±0.02  | ...         | ...         |
| Ark 564    | 6.59±0.17   | 0.056±0.007 | 1.08±0.10  | 0.30±0.01   | 0.101±0.006 |
| NGC 7469   | 6.96±0.05   | 0.02±0.001  | 0.60±0.05  | 0.07±0.01   | 0.037±0.006 |

All the line ratios are normalised to the broad component of Hβ. Individual line fluxes are listed in Table B1.1. Masses are from Benz & Katz (2015) unless stated otherwise: (a) – Park et al. (2017); (b) – Daszyra et al. (2007); (c) – Woo & Urry (2002). 2. [Fe v] / Hβ ratio is determined in this work unless stated otherwise: (d) – Rodríguez-Ardila et al. (2005). 3. The NIR emission line flux ratios are from Riffel et al. (2006) except when indicated: (e) – Rodríguez-Ardila et al. (2002); (f) – This work.

Table 2. Sub-sample of galaxies with observations first presented in this work

| Galaxy     | RA hh:mm:ss | DEC deg:mm:ss | Redshift (z) | Telescope/ Instrument | Date of Observation | A_V mag | Airmass | Nexp x Texp (sec) |
|------------|-------------|---------------|--------------|-----------------------|---------------------|---------|---------|-------------------|
| Fairall 9  | 01:23:45.8  | -58:48:21    | 0.04614      | SOAR/TSpec4           | 2019 Aug 08         | 0.071   | 1.22    | 20x180            |
| 3C 120     | 04:33:11.1  | 05:21:16     | 0.03301      | SOAR/TSpec4           | 2020 Feb 10         | 0.816   | 1.28    | 12x180            |
| Mrk 707    | 09:37:01.03 | 01:05:43.48  | 0.05025      | Blanco/ARCOIRIS       | 2017 Apr 09         | 0.189   | 1.21    | 28x180            |
| NGC 3783   | 11:39:01.7  | -37:44:19    | 0.00973      | Blanco/ARCOIRIS       | 2017 Mar 11         | 0.332   | 1.05    | 16x180            |
| Mrk 1310   | 12:01:14.3  | -03:40:41    | 0.01956      | Blanco/ARCOIRIS       | 2017 Apr 08         | 0.083   | 1.18    | 20x180            |
| MGC-6-30-15| 13:35:53.7  | -34:17:44    | 0.00749      | SOAR/Goodman           | 2017 Mar 21         | 0.165   | 1.05    | 3x1200            |
| Mrk 841    | 15:04:01.2  | 10:26:16.15  | 0.03642      | Blanco/ARCOIRIS       | 2017 Apr 08         | 0.082   | 1.42    | 12x180            |
| NGC 6814   | 19:42:40.6  | -10:19:25    | 0.00521      | Blanco/ARCOIRIS       | 2017 Apr 08         | 0.509   | 1.38    | 20x180            |
| NGC 7469   | 23:03:15.6  | +08:52:26    | 0.01632      | SOAR/Goodman           | 2010 Nov 11         | 0.188   | 1.34    | 3x900             |
follows the LtsFrr package (Cappellari et al. 2013), which accounts for the errors in all variables. The Pearson correlation coefficient is $r = -0.76$, with a null probability of $P_r = 3.8 \times 10^{-5}$.

A weak trend if any ($r = -0.5$) when involving $[\text{Fe vii}], \text{IP} (100 \text{ eV})$, and no trend for the higher IP ($> 260 \text{ eV}$) CL $[\text{Si x}]$ and $[\text{S vii}]$ are found. The correlation index for $[\text{S vii}]/\text{Pa } \beta_{\text{broad}}$ is $r = -0.44$, that for $[\text{Si x}]/\text{Pa } \beta_{\text{broad}}$, is $r = -0.3$.

To construct these diagrams, several considerations were made. First, line ratios are chosen close in wavelength to minimise reddening. Second, the normalisation of the CL emission is to the broad $\text{H } \alpha$ line, as only in this case the correlation $(\text{Si vi}/\text{Br } \gamma_{\text{broad}})$ with BH mass was found. Normalisation to the $\text{H } \alpha$ narrow line emission was equally evaluated, yet not clear trend with BH mass could be recovered, the dispersion being too large. We believe that reasons for that are the much larger narrow-$\text{H } \alpha$ emitting volume as compared with that of the coronal region and the fact that $\text{H}$ is subjected to additional ionisation sources beside the AGN, e.g. star formation, whereas coronal gas is an unambiguous AGN tracer. The coronal gas is found in the inner parsecs of the central engine, at the boundary of the BLR clouds, as recently shown in spatially resolved observations of the broad and coronal clouds by GRAVITY Collaboration et al. (2021). The CL high critical density, $n_e > 10^8 \text{ cm}^{-3}$, warrants its survival at the inner regions, possibly the nearest gas to the BLR - not at the BLR where densities are at least one order of magnitude higher. Hence, by normalising to broad $\text{H } \alpha$, the proposed CL ratios in this work become the closest possible tracer, perhaps a genuine one, of the conditions at the inner parsecs next to the accretion disc.

The derived correlation when involving the $[\text{Si vi}]$ line over al-

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**Figure 1.** Observed CL emission normalised to the broad component of $\text{H } \alpha$ versus black hole mass for the objects in sample. The black line is the best linear fit to the data and the red-dashed and -dotted lines show the 1$\sigma$ and 2$\sigma$ deviation.
most three order of magnitude in BH mass, the absence of an equivalent
dependence when involving the higher IP lines, prompted us to examine
the impact of an accretion-disc continuum in the coronal
gas production and, in turn, its dependence with disc temperature
and BH mass.

The CL in Fig. 1 are sensitive to different energy ranges of
the ionising continuum. This is illustrated in Fig 2, which shows a
parameterisation of an AGN ionising continuum as a combination
of a standard Shakura-Sunyaev (SS) accretion disc (Shakura &
Sunyaev 1973) which accounts for the rising of the spectrum at UV-
soft X-rays, and a power-law with a low and high energy cut-off to
account for the rising of the continuum at high energies. It follows
the equation (CLOUDY C13.1 formalism is used - Ferland et al.
2013):

$$ F_{ν} = ν^{α_{UV}} \exp \left( \frac{-hν}{kT_{SS}} \right) \exp \left( \frac{-kT_{IR}}{hν} \right) + aν^{α_{S}} $$ (2)

The first term is the parameterisation of the SS disc, repre-
sented by an exponential function with a cutoff at the disc effective
temperature, $T_{SS}$, and a power law with $α_{UV} = 0.33$ accounting for
the low energy tail of the disc. The low energy limit of the disc is
set by the IR-exponential with cutoff at 0.01 Ryd. The high energy
range is represented by a broken power law with spectral index $α_{S} =
-1$, and a cutoff at 100 keV. The scaling of the SS disc relative to the
high energy power law is controlled by the parameter "a" in eq. 2,
which refers to the ratio of the luminosities at 2 keV and at 2500 A
given by a power law with spectral index, so called $α_{SS}$, set here to
-1.4. With the exception of $α_{UV} = 0.33$, all other parameters quoted
above follow the generic AGN continuum used in CLOUDY C13.1-
sect. 6.2.

The IP of the CL and of H\,\text{n} are marked on the ionising contin-
num, Fig. 2, to illustrate their both location and energy coverage in a
typical AGN ionising spectrum. They sample the bulk of the ioniz-
ing spectrum over the 13.6 - 351 eV energy range. The figure shows
different SS accretion disc continuum for different BH masses and
spin (see Sec. 4).

4 CORONAL EMISSION AS PROXY OF THE BLACK HOLE MASS

The peak emission in the ionising continuum in Fig. 2 effectively
samples $T_{disc}$, with the disc getting hotter as BH mass decreases
as predicted by standard accretion disc theory. The IPs of the CL
fall within the range of peak temperatures, closer or slightly further
from the peak depending on BH mass. If the accretion disc is the
main source of photons to ionise the coronal gas, a trend between
CL strength and $T_{disc}$ may be expected. Because of the dependence
of $T_{disc}$ with BH mass, a correlation between coronal line strength
and BH mass may be ensued. This possibility is investigated below.

Following on the thin accretion disc approximation, for a Kerr
BH, the disc $T_{disc}$ can be approximated as (Frank et al. (2002)
formalism is used):

$$ T_{disc} = 3.4 \times 10^{5} K \left( \frac{M_{BH}}{10^{6} M_{\odot}} \right)^{1/4} \times \left( \frac{\dot{M}}{0.1} \right)^{1/4} \times \left( \frac{\eta}{0.26} \right)^{-1/4} \times \left( \frac{R_{in}}{1.4} \right)^{-3/4} $$ (3)

where, $M_{BH}$ is the BH mass, $(\dot{M}/dt)$ is the accretion rate in
Eddington units, $\eta$ is the BH accretion efficiency, $R_{in}$ is the
inner-most stable circular orbit in terms of the gravitational radius
$R_{G} = G M_{BH}/c^{2}$ ($G$ is the Gravitational constant, $c$ is the velocity
of light). The equation is normalised to $M_{BH} = 10^{6} M_{\odot}$, accretion
rate in Eddington units, $\dot{M}/dt \sim 0.1 (\dot{M}/dt)_{\text{Edd}}$, and a radiation
efficiency, $\eta = 0.26$ corresponding to a BH spin of 99% , i.e. $a =
0.99 G M_{BH}/c^{2}$ (hereafter a = 0.99), and co-rotation is adopted.
Current estimates of BH spins from different methods point to values
close to 1 (see Reynolds 2019, for a compilation). With increasing
spin, the inner-most stable orbit becomes smaller and $T_{disc}$ increases
accordingly. In the case of co-rotation, the disc temperature reaches
the highest values.

The ionising continua in Fig. 2 follow the $T_{disc}$ - BH mass
approximation described in Equation 3. Curves are presented for
three different BH mass and two spin, 0.8 and 0.99. Spins below
0.8 produce a marginal difference in the coronal gas production and, in turn, its dependence with disc temperature and BH mass.

The CL in Fig. 1 are sensitive to different energy ranges of
the ionising continuum. This is illustrated in Fig 2, which shows a
parameterisation of an AGN ionising continuum as a combination
of a standard Shakura-Sunyaev (SS) accretion disc (Shakura &
Sunyaev 1973) which accounts for the rising of the spectrum at UV-
soft X-rays, and a power-law with a low and high energy cut-off to
account for the rising of the continuum at high energies. It follows
the equation (CLOUDY C13.1 formalism is used - Ferland et al.
2013):

$$ F_{ν} = ν^{α_{UV}} \exp \left( \frac{-hν}{kT_{SS}} \right) \exp \left( \frac{-kT_{IR}}{hν} \right) + aν^{α_{S}} $$ (2)

The first term is the parameterisation of the SS disc, repre-
sented by an exponential function with a cutoff at the disc effective
temperature, $T_{disc}$, and a power law with $α_{UV} = 0.33$ accounting for
the low energy tail of the disc. The low energy limit of the disc is
set by the IR-exponential with cutoff at 0.01 Ryd. The high energy
range is represented by a broken power law with spectral index $α_{S} =
-1$, and a cutoff at 100 keV. The scaling of the SS disc relative to the
high energy power law is controlled by the parameter "a" in eq. 2,
which refers to the ratio of the luminosities at 2 keV and at 2500 A
given by a power law with spectral index, so called $α_{SS}$, set here to
-1.4. With the exception of $α_{UV} = 0.33$, all other parameters quoted
above follow the generic AGN continuum used in CLOUDY C13.1-
sect. 6.2.

The IP of the CL and of H\,\text{n} are marked on the ionising contin-
num, Fig. 2, to illustrate their both location and energy coverage in a
typical AGN ionising spectrum. They sample the bulk of the ioniz-
ing spectrum over the 13.6 - 351 eV energy range. The figure shows
different SS accretion disc continuum for different BH masses and
spin (see Sec. 4).

Figure 2. Generic AGN ionising continuum used in this work as per Eq. 2
for the BH mass range in the AGN sample. Each curve corresponds to a BH
mass with an associated Tdisc following the SS disc approximation in Eq.
3. Two spin values, $a = 0.8$ (dashed curves) and $a = 0.99$ (solid curves), and
three BH masses, i.e. $M_{BH}$ (in units of $M_{\odot}$) = $10^{6}$ (purple), $10^{7}$ (green) and
$10^{8}$ (red) are shown. Vertical dashed lines mark the IPs of the lines used in
the analysis.
issues that may affect the estimate of this CL ratio: 1) the high variability of Hβ as compared with the much more stable near-IR Paschen and Brackett lines (Landt et al. 2011); 2) the continuum underlying broad Hβ, affected by a strong Fe ii pseudo-continuum, makes difficult the estimate of broad Hβ; 3) differential reddening between Hβ and [Fe ii]. Nonetheless, this CL ratio is included in the photoionisation modelling below.

4.1 Testing Coronal Line diagnosis diagrams with photoionisation models

To probe the CL emission as a proxy of the disc temperature, we make use of the photoionisation code CLOUDY (v17.02 Ferland et al. 2017). The goal is to test whether the CL line ratios used in Fig. 1 show a dependence with $T_{\text{disc}}$, this being an input to CLOUDY via the ionising continua shown in Fig. 2. A range of electron densities, $n_e$, and cloud distances to the centre, $r$, in line with CL observations are input to CLOUDY. These parameters are selected as follows.

Coronal gas extends at most up to a few tens of parsec (Prieto et al. 2005). The bulk of the emission is mostly nuclear (Müller-Sánchez et al. 2011), spreading over sub-parsec scales (e.g. Gravity Collaboration et al. 2020; GRAVITY Collaboration et al. 2021) and can be explained by photoionisation (e.g. Ferguson et al. 1997; Contini & Viegas 2001; Rodríguez-Ardila et al. 2006). For the present test, the bulk of CL in the sample is assumed to be nuclear and powered by photoionisation from the accretion disc. Spatially resolved nuclear CL by GRAVITY interferometry reveals the coronal region extending up to 0.3 pc from the centre. Thus, a range of $r$ between 0.3 and 30 pc are tested.

Gas densities, $n_e$, in the $10^4 \leq n_e \leq 10^5 \text{ cm}^{-3}$ range are probed, the upper limit set by the critical density of the CL probed - $n_e > 10^8 \text{ cm}^{-3}$, the lower limit is set by the average densities inferred from the mid-IR [Ne v] lines, $\text{IP}=97 \text{ eV}$, $10^3 - 10^4 \text{ cm}^{-3}$ (e.g. Moorwood et al. 1996; Fernández-Ontiveros et al. 2016).

The input ionising continuum is set by Eq. 2 (Fig. 2), normalised to an adopted value of $L_{\text{bol}}/L_{\text{edd}}=0.1$. The present sample covers a range of Eddington ratio in the range $0.01 \leq L_{\text{bol}}/L_{\text{edd}} \leq 0.2$ as reported in the literature. These values are nonetheless subject to uncertainties up to an order of magnitude due to the evaluation methods and assumptions made to estimate $L_{\text{bol}}$ (see e.g. Richards et al. 2006). For a few sources, reliable $L_{\text{bol}}$ are available from integrating the parsecs-scale SED (e.g. Prieto et al. 2010). This is the case for NGC 7469 and NGC 3783, for which $L_{\text{bol}}/L_{\text{edd}}$ are 0.25 and 0.06, respectively. A few other sources have $L_{\text{bol}}$ estimated from hard X-ray data (Winter et al. 2012), re-normalising the Eddington ratios inferred by these authors to the BH masses in Table 1, yields $L_{\text{bol}}/L_{\text{edd}}$ in the 0.01 - 0.1 range. For the present test, a nominal $L_{\text{bol}}/L_{\text{edd}}=0.1$ is adopted, in line with the median found in larger quasar distributions, such as the SDSS DR7 quasar catalogue (Shen et al. 2011; Pandya et al. 2018). The impact of changing $L_{\text{bol}}/L_{\text{edd}}$ in the model results is discussed in next section.

4.2 Predictions vs observations

Figs. 3 to 6 show CLOUDY predictions for the CL ratios in this work as a function of $T_{\text{disc}}$ (in the right axis), and of BH mass (on the left axis, following the transformation in Eq. 3 for $L_{\text{bol}}/L_{\text{edd}} = 0.1$). Models are shown for a range of densities, $10^4 \leq n_e \leq 10^5$, clouds distance to the centre, 0.3 pc $\leq r \leq 30$ pc, and two spins, 0.8 and 0.99 - following discussion in Sect. 4.1. Given the BH spin, the two left free parameters in Eq. 3, accretion efficiency ($\eta$) and the location of the innermost stable circular orbit $R_{\text{in,}\beta}$, are set uniquely.

As discussed, the CL data is normalised to H i broad emission. Yet, CLOUDY predictions are derived for densities at least two orders of magnitude below that of the broad line region to cope with the lower CL critical densities. Thus, in comparing both a caveat is introduced, which mainly relates to the different volume emissivity of H i in the broad and in the coronal line region. To account for the difference a correction factor was required to shift CLOUDY line ratio predictions on top the plotted ratios. We find a factor of 15 as best compromise to get the predicted [Si v]/Brγ, for the whole range of $n_e$, $r$ and spin considered, on top of the plotted [Si v]/Brγ data. Having fixed the scale factor for this ratio, the scaling for the other CL ratios were derived by imposing theoretical H i recombination ratios. Accordingly, a factor of 25 is applied to CLOUDY’s [Fe v]/Hβ, and 90 for both [S vi]/Paα and [Si x]/Paβ. CLOUDY models in Figs. 3, 4, 5 and 6 are all shifted in the X-axis by these factors.

Focusing on the results for [Si v]/Brγ broad. Fig. 3, the models that best account for the whole range of observed ratios and their trend with BH mass are for cloud distances, $r$, of 3 pc and densities in the range $10^5 \leq 10^7 \text{ cm}^{-3}$ (middle panels). The case of spin 0.8 gives the best match, the spin 0.99 case provides an envelope of the scatter in the data. Models with cloud distances higher or lower than $r=3$ pc are more restrictive in covering the whole range of CL ratios, or BH mass. Models with distances as large as $r=30$ pc provide a fair account of the trend for the larger BH masses provided the densities are in the lower range as expected for clouds at those large distances. This model nonetheless is presented as a limit case as in the present context, the CL to BL emission comparison may not be applicable. Models for $r=0.3$ pc provide moderate account for the lowest mass range. Overall, no single model reproduces the whole range of observed CL ratios, but changes in cloud density appear to reproduce the general [Si v]/Brγ broad vs BH mass trend particularly for $r=3$ pc. The observed correlation shows a scatter of 0.47 dex (Fig. 3, Sect. 3), and part of it should be intrinsic, reflecting the different physical properties of the objects in the sample in terms of spin, and accretion rate - evaluated below - and also the CL region both density and distance to the centre.

Focusing on [Fe v]/Hβ broad, the observations show a rather scatter dependence with BH mass (Fig. 4). It is noteworthy though that the models that best account for the data spread are those that best account for the [Si vi] ratio trend, the $r=3$ pc models. All other models provide a poor representation of the loci of the data. The fact that a favoured model, $r=3$ pc, is hinted for both [Fe v] and [Si v] line ratios is somewhat expected given that the IP of both lines sample properly $T_{\text{disc}}$ for the range of BH mass considered (Fig. 2). It is also interesting that for the [Fe v] case, the models show an also erratic dependence with BH mass as do the data, that contrasts with the better behaved trend shown for the [Si vi] line case. This result and particularly the observational one need to be further explored.

Regarding the higher IP CL ratios [Si x]/Paα broad and [S vi]/Paα broad, as discussed in sect. 4, none of the two show a dependence with BH mass. None of the tested models, included the favoured one $r=3$ pc even account for the loci of the data, with the exception of the $r=0.3$ pc case (left panels in both Figs. 5 and 6).

In the context of the present analysis, this result is what is expected for two reasons: 1), the lack of a dependence of BH mass with $T_{\text{disc}}$ for these CLs is attributed to their IP falling at the high energy end of the disc spectrum (Fig. 2), hence $T_{\text{disc}}$ is not as well sampled as...
Figure 3. CLOUDY predictions for $[\text{Si} \, \text{VI}]/\text{Br}$ vs BH mass using as ionising continuum Eq. 2. Models are run for densities $n_e = 10^3 \, \text{cm}^{-3}$ in red, $10^5 \, \text{cm}^{-3}$ (green) and $10^6 \, \text{cm}^{-3}$ (blue). Each subplot shows the model results for distance $r$ to the ionising source, $r=0.3, 3$ and $30$ pc, depicted from left to right. Two spins, $a = 0.8$ (upper panel) and $a = 0.99$ (lower panel) are considered. The corresponding $T_{\text{disc}}$ per each BH mass - after eq. 3 - are depicted on the y-axis, right-side. Data points as in Fig. 1 are black squares.

for the case of lower IP CL for the range of BH mass considered. The lack of dependence with $T_{\text{disc}}$ is also indicated by the CLOUDY predictions, particularly for the $[\text{Si} \, \text{VI}]/\text{Br}$ case which shows an almost straight line along the temperature axis, as do the data, whereas a more erratic behaviour is predicted for the $[\text{Si} \, \text{x}]$ case, as also seen the data; 2) the higher IP of $[\text{S} \, \text{vii}]/[\text{S} \, \text{x}]$ naturally leads to the formation of these ions at the innermost edge of the CL region, hence, models with $r$ closer to the central engine should be favoured, as it appears to be the case. It may also be considered to decrease the density to foster the production of these ions at larger $r$, yet, this severely penalises the line emissivity, proportional to $n_e^2$, as it can be inferred from the results (Figs. 5 and 6).

We finally test the impact of an accretion rate different from the standard 10% Eddington used in the models above. An evaluation of the $L_{\text{bol}}/L_{\text{Edd}}$ for some of the objects in the sample points to a range between 1% and a few 10% (sect 4.1). Narrow Line Seyferts type-I are also predicted to have high Eddington ratios (e.g. Kuraszkiewicz et al. 2000; Panda et al. 2019). Decreasing $L_{\text{bol}}/L_{\text{Edd}}$ to 1% lowers $T_{\text{disc}}$ normalisation in Eq. 3 by factor $\sim 1.8$, which is about the same factor reduction introduced by change of spin from 0.99 to 0.8. The net effect in the models would be equivalent to that produced by the change of spin shown in all the figures, Figs. 3 to 6. Effectively, the decrease in $T_{\text{disc}}$ implies CL ratios progressively smaller, but the trend of the models keeps similar particularly for cloud distances $r \geq 3$ pc. Conversely, increasing $L_{\text{bol}}/L_{\text{Edd}}$ would shift the models in opposite direction towards higher $T_{\text{disc}}$. An increase in Eddington ratio by factor 3 yields an increase in $T_{\text{disc}}$ by 1.3. Focusing on the $[\text{Si} \, \text{VI}]/\text{Br}$ vs BH mass correlation, lowering (increasing) the accretion rate or spin just provide the envelope to enclose the scatter of the data. The dispersion in the observed correlation could largely be attributed to the range of spin and Eddington accretion rate in the sample.

4.3 Testing the relative-contribution / shape of the soft X-rays component on coronal emission

4.3.1 Modifying $\alpha_{\text{ox}}$

The IP of the CLs in this work extends over the soft X-rays, spreading over the 100 – 350 eV range. The generic ionising continuum used up to now (eq. 2, Fig.2), is sampled with a SS disc joined with a power law at the high energies whose relative contribution is fixed with the $\alpha_{\text{ox}}$ parameter set to -1.4. Obviously the contribution of the hard energy spectrum, which is modulated by this parameter, affects in different degrees the CL production, particularly for the highest IP lines. This effect is evaluated bellow.

$\alpha_{\text{ox}}$ in AGN falls in the -1.0 to -2.0 range (Avni & Tananbaum 1986; Wilkes et al. 1994; Lusso & Risaliti 2017), with a typical value about -1.4 (Zamorani et al. 1981; Ferland et al. 2013). A new
Figure 4. As in Fig. 3 but for $[\text{Fe VII}]/\text{H} \beta$.

4.3.2 Adding a warm Comptonisation component

A soft X-rays excess below ~1 keV on top of the nominal high energy power-law spectrum is an often feature in AGN spectra, with NLS1 being the most clear representatives (c.f. Boller et al. 1996). Its origin has been interpreted as reprocessed emission from the hot disc corona or as an additional warm Comptonising component dominating the soft X-rays (e.g. Fabian et al. 2013; Kubota & Done 2018, and references therein).

For the purpose of this work, to assess the effect of an additional soft X-ray excess component in the generic AGN ionizing continuum in this work - Eq. 2, the warm corona component as described in Kubota & Done (2018) is taken as a reference and incorporated as additional component to the generic continuum. The warm corona models are extracted from AGNSED model (Kubota & Done 2018) using xspec (Arnaud 1996). The new ionizing continuum is shown in Fig. 8 along with the generic one, Eq. 2, used in this work. The new model is calculated for Eddington accretion rate of 10%, as the generic case, but only the spectrum for spin = 0.8 is shown. In comparing both spectrum, it can be seen in the figure that with the addition of the warm component the peak of the ionizing continuum moves toward cooler temperatures. And because of the new shape of the spectrum, the IP of the CLs, particularly of the reference ones, $[\text{Si vi}]$ and $[\text{Fe vii}]$, fall in a region of the spectrum rather flat with little change with BH mass.

Figure 9 shows the new model results for the four CL ratio vs BH mass relations tested in this work. As in sect. 4.3.1, models are run for a restricted parameter range, i.e. $n = 10^4 \text{ cm}^{-3}$, $r=3$ pc, and spin $a=0.8$, with all other parameters as in the generic case (sect. 4.1), included $\alpha_{\text{ox}}$ set to -1.4.

The new models provide rather similar behaviour in CL ratio vs BH mass as those produced with the change of $\alpha_{\text{ox}}$. The inclusion of the warm soft component predicts lower values for the $[\text{Si vi}]$- and $[\text{Fe vii}]$- ratios, as compared with the effect of changing $\alpha_{\text{ox}}$, but otherwise the generic ionising continuum still provides the best envelope of the observed $[\text{Si vi}]$ ratio vs BH mass relation.

Not major impact is found for the $[\text{Fe vii}]$ vs BH mass diagram,
if any the BH mass range is slightly better covered by the generic model. Results are very similar among all models regarding the high IP CL ratios vs BH mass diagrams. Either the range of CL ratios or of BH masses are poorly covered in all cases. There are two aspects to emphasise here. 1) as discussed in sect. 4.2 the high IP CL ratios show no dependance with BH mass, nor any of the generic models hinted for a dependance either. It was argued as possible cause the high IP of the lines which are barely sampled by the SS disc for the range of BH mass considered. It is not surprising finding similar result with the new soft X-ray enhanced ionising continuum because of the flattening of spectrum at the relevant energies region for all the BH masses. 2) the new models are run for CL cloud distances \( r = 3 \) pc whereas smaller \( r \) cover best the bulk of the data as found with the generic ionizing continuum (best coverage is found for \( r = 0.3 \) pc, sect. 4.2). Yet, because of 1) models for \( r = 0.3 \) pc will not provide a distinct result from that obtained with the generic case.

### 5 OVERALL VIEW: A CORONAL GAS - BH MASS CALIBRATION

Using bona-fide BH mass estimate from reverberation mapping and the line ratio [Si vi] 1.963\( \mu m/Bry_{\text{broad}} \) as a genuine tracer of the AGN ionising continuum, a BH-mass scaling relation over almost three orders of magnitude in BH mass, \( 10^6 - 10^8 \) \( M_\odot \), is found (Fig. 1). The dependence follows a linear regression in log scale \( M_{BH} \propto ([\text{Si vi]}/Br_{\text{broad}}]^{1.99\pm0.37} \), with a dispersion in BH mass of 0.47 dex (Sec. 3). Following on the thin accretion disc approximation and after surveying a basic parameter space for coronal gas production, we believe one of the key parameters driving this correlation is the effective temperature of the accretion disc, the correlation being formally in line with the thin disc prediction \( T_{\text{disc}} \propto M_{BH}^{-1/4} \).

On these bases, on the assumption of a thin disc as the dominant component of the ionizing continuum (Fig. 2), and a suitable range of densities \( n_e \sim 10^4–10^6 \) cm\(^{-3} \) and cloud distances, \( 0.3 < r < 30 \) pc, for CL survival, photoionisation models provide a fair representation of the \( M_{BH} \) and [Si vi]/Bry\(_{\text{broad}} \) correlation.

No correlation is recovered when normalising the CL emission to narrow H\(_\text{i} \) gas. This may be due to two facts: 1) the much larger emission region covered by H\(_\text{i} \) as compared with that from CL gas; 2) H\(_\text{i} \) may be subjected to additional ionisation sources other that the AGN which is not the case for CL gas. However, the normalisation to broad H\(_\text{i} \) introduces a complication when comparing with photoionisation models, these been evaluated for narrow H\(_\text{i} \) gas. To account for this difference, a scaling factor between broad-measured and narrow-predicted H\(_\text{i} \) gas is introduced. This scaling factor should be understood as a way to account for the different volume emissivity and density that characterise the Broad- and the Coronal- line region. This scaling factor was found in a interactive form, by shifting the models predictions for [Si vi]/Bry on top of the observed ratios normalised to \( Br_{\gamma_{\text{broad}}} \). A factor around 15, unique for all models and objects in the sample, is found as the best compromise for this CL ratio. All other CL - H\(_\text{i}\)-broad ratios
Figure 6. Similar to Fig. 3 but for \([\text{Si} \times]/\text{Pa}\beta\) -BH mass scaling.

used in this work were consequently scaled down from that factor following recombination values.

No correlation involving the higher IP lines, \([\text{Si} \times]\) and \([\text{S} \text{vii}]\), IP > 250 eV is seen (Fig. 1). We believe this a natural consequence of the \(T_{\text{disc}} - \text{BH mass}\) dependence. Higher IP lines sample hotter discs, as compared with the \([\text{Si} \text{vi}]\) IP, and in turn smaller BH masses, not covered by this sample, hence the absence of a positive trend for the range of masses in this work. It follows that a lack of dependence with \(T_{\text{disc}}\) should then be expected. This is consistent with the photoionisation predictions as inferred from the same set of models used for the \([\text{Si} \text{vi}]\) case (Figs. 5 and 6). The high IP CL are however expected to show a dependence with BH masses below \(10^6 \text{M}_\odot\) (Fig. 2, Eq. 3), making them suitable BH mass scaling indicators for intermediate BH masses, as suggested by (Cann et al. 2018). Testing this low mass end is currently limited due to the unavailability of accurate BH masses and suitable CL data.

Thus, the BH mass - CL dependence appears sensitive to the ionisation potential of the CL employed. The use of \([\text{Si} \text{vi}]\) 1.963 \(\mu\)m, IP = 167 eV, restricts the dependence to BH masses in the range \(10^6 - 10^8 \text{M}_\odot\), presumably because of the disc temperature, for these masses, favours the production of \(S_{\text{[Si]}}^{7+}\) best (Fig. 2). In the same line of reasoning, an equivalent dependence involving other CL with IP close to the disc peak emission for these range of BH mass should be expected. Such is the case of e.g. \([\text{Fe} \text{vii}]\) 16087Å, IP =100 eV, but a scatter relation is found instead. Photoionisation predictions do not hint for a dependence with Temperature either (Fig. 4). Thus no clear conclusion for the potential of this line as gas temperature can be assessed.

Other suitable CL are those of \(\text{Ne}^{4+}\) IP = 97 eV. Yet, the UV lines are much subjected to reddening, those in the mid-IR are optimal but available for few sources with accurate BH mass determination. The analysis of these few sources shows indeed a trend with BH mass but the statistic is insufficient to establish a correlation, the analysis is in progress.

Above \(10^8 \text{M}_\odot\) BH mass, disc temperatures are foreseen in the \(10^5 \text{K}\) regime in the disc approximation. Lower ionisation lines would then be more favoured than the higher ones. The \([\text{Si} \text{vi}]/\text{Br}\beta_{\text{broad}}\) -BH mass correlation points in that direction, with \([\text{Si} \text{vi}]/\text{Br}\beta_{\text{broad}}\) decreasing with increasing BH mass (Fig. 1). The high BH mass tail in the local universe is the realm of elliptical and bulge dominated objects often associated with LINER (Low Ionisation Nuclear Emitting Regions) activity. The parsec-scale spectral energy distribution of some of these sources strictly limit the temperature of the disc to \(< 10^5 \text{K}\) (Fernandez-Ontiveros et al. submitted). Parsec-scale near-IR observations have so far proven the elusiveness of CL emission in a few of these cases (Müller-Sánchez et al. 2013; Mazzalay et al. 2014), in line with that prediction. These sources would appear as upper limits at the high mass range in Fig. 1.

Because of the different physical properties of the objects in the sample, some of the scatter in the proposed BH mass scaling relation should be intrinsic. In the proposed scenario, prime properties that should differ between objects are the accretion rate and spin. They affect the observed CL ratios dependence with
Figure 7. CLOUDY predictions for the four CL ratio vs BH mass diagrams in this work. The ionising continuum is that of eq. 2 but for different values of $\alpha_{\text{ox}}$: red refers to $\alpha_{\text{ox}} = -1.4$ - default value used in this work, sect. 4.2 - green for $\alpha_{\text{ox}} = -1$, blue for $\alpha_{\text{ox}} = -2$. Models are run for a suit of parameters that best account for the [Si vi] - BH mass correlation (Fig. 3): density $n_e = 10^3 \text{ cm}^{-3}$, distance $r = 3 \text{ pc}$, spin $a = 0.8$. All other parameters are as in the default case. Data points as in Fig. 1 are black squares.

BH mass in different degrees. Best agreement with the observed [Si vi] $1.963 \mu\text{m}/\text{Br}_\gamma$ - BH mass trend is achieved for spins above 80% and Eddington accretion rate of 10% (Fig. 3). A increase in spin, or decrease in accretion rate, translates into a similar effect, namely a progressive shift to higher, or lower, $T_{\text{disc}}$ for same BH mass, by factor of about 1.8 from spin = 0.8 to 0.99 or Eddington accretion rate from 10% to 1% (Fig. 3). Higher Eddington rates up to 30%, in line with some objects in the sample, would lead to a slight increase in $T_{\text{disc}}$ by 1.3. In practise, the decrease in $T_{\text{disc}}$ makes [Si vi]/Br$\gamma$ progressively less sensitive to lower BH mass as the peak temperature of the disc moves away from the optimal energy for producing $S_{\text{Si}^5}$ whereas the increase in $T_{\text{disc}}$ by similar factor would favour the diagnosis for lower BH masses.

Finally, changes on the relative contribution of a soft X-ray excess component on the CL ratio - BH mass dependences was evaluated. This was done by either changing the $\alpha_{\text{ox}}$ parameter or adding a warm X-ray corona component in the standard ionising continuum used in this work. None of these variants provide a significative account of the BH mass - [Si VI] ratio correlation (Fig. 7, 9). Nor, a significant result was found for the higher IP CLs, this even though the modification of the ionising spectrum at the relevant energies to produce these lines. An important result from the photoionisation modelling is that to produce the range of CL ratios observed at these high IPs, clouds should be less than few tenth of parsec from the centre (Figs. 5, 6). Still, to verify the potential of these high IP lines as BH mass scale tracers, the lower BH mass range has to be tested. The detection of coronal [Fe X], IP = 240 eV, in a large sample of dwarf galaxies by Molina et al. (2021) opens an exciting avenue.

With a final compendium of 21 objects, the dispersion in BH mass in the proposed calibration is 0.47 dex ($1\sigma$). In comparison, a dispersion of 0.44 dex is inferred from the $M - \sigma$ relation in 49 galactic bulges with direct dynamical BH mass estimate (Gültekin
et al. 2009). The intrinsic scatter in the mass - luminosity relations is in the 40% range (Kaspi et al. 2005), mostly driven by differences in optical - UV continuum shape. In the present [Si VI] - BH mass correlation, variations in the ionising continuum shape e.g. the soft X-ray excess slope and contribution, should contribute to the scatter but we consider that a minor effect as the production of the CL gas is mostly driven by the peak energy in the UV.

The present BH mass scaling relation is restricted to Type 1 AGN including narrow line Type I. The limitation is driven by the imposition of including bona-fide BH masses only, and the need to normalise to broad H i gas. We are nonetheless examining possibilities to extend it to Type 2. Source variability or changes in instrumental setup are not an issue. We found that most of the scatter is chiefly driven by differences in BH spin and accretion rate. The new scaling offers an economic, and physically motivated alternative for BH estimate using single epoch spectra, avoiding large telescope time (reverberation mapping) or absolute flux calibration (the continuum luminosity method). With James Webb Space Telescope and big surveys in the IR region, large samples of AGNs could be weighted using this approach.

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DATA AVAILABILITY

Data products will be shared on reasonable request to the corresponding author.

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Figure A1. Optical Spectra of the AGN sample in rest wavelength. For each galaxy, the left panel shows the full spectrum. The two smaller panels to the left shows [Fe VII] $\lambda$6087 and the H$\beta$ line.
Figure A2. Cont. Fig. A1.
Figure A3. Cont. Fig. A1.
Figure A4. Cont. Fig. A1.
Figure A5. NIR spectra of Fairall 9 (top panels), 3C 120 (middle panels) and Mrk 707 (bottom panels) in rest wavelength. For each AGN, the larger panel displays the observed spectrum in the 9000–23000 Å range. The smaller panels in the following row show a zoom around the most relevant lines to this work. The shaded areas mark regions of bad atmospheric transmission.
Figure A6. Same as Figure A5 for Mrk 1310 (top panels), NGC 4395 (middle panels) and Mrk 841 (bottom panels).
Figure A7. Same as Figure A5 for NGC 6814.
| Galaxy       | Hβ       | Paβ      | Brγ      | [Fe v1] | [S v1] | [Si v] |
|--------------|----------|----------|----------|---------|--------|--------|
| Mrk 335     | 712.6±44.13 | 170.5±2.13 | 26.7±3.1 | 55.1±5.5 | 3.76±1.00 | 7.45±2.00 |
| Fairall 9    | 490.2±30.12 | 107.7±4.80 | 43.1±4.7 | 18.4±1.33 | 4.23±0.52 | 8.16±0.67 |
| NGC 863      | 49.3±2.94   | 10.4±2.94 | ...      | ...      | ...     | 2.54±0.14 |
| 3C 120       | 222.9±5.94  | 46.3±6.55 | ...      | 2.97±0.17 | 9.96±2.47 | 15.48±1.96 |
| Mrk 707      | 107.6±3.31  | 4.76±0.39  | 2.60±0.23 | 1.4±0.27 | 3.18±0.31 | 1.82±0.09 |
| Mrk 110      | 52.2±1.43   | 2.43±0.08  | ...      | ...      | ...     | ...    |
| NGC 3227     | 168.7±10.23 | 20.0±3.92   | ...      | 1.97±0.6 | ...     | 14.9±2.6 |
| Mrk 142      | 8.65±0.70   | ...        | 1.67±0.35 | ...      | ...     | ...    |
| SBS 1116+583A| 47.2±1.93   | ...        | 0.46±0.08 | ...      | ...     | ...    |
| PG 1126-041  | 101.8±3.7   | 13.3±0.73  | ...      | 2.58±0.21 | 4.32±0.45 | 3.71±0.17 |
| NGC 3783     | 469.3±157.0 | 34.7±9.06  | 60.4±9.82 | 242±6.48 | 8.12±0.86 | 17.51±2.67 |
| Mrk 1310     | 34.9±0.72   | 11.4±1.1  | 1.4±0.4  | 1.10±0.05 | 0.79±0.10 | 0.83±0.19 |
| NGC 4051     | 48.2±2.44   | 66.6±1.7  | 13.1±0.8 | 5.95±0.51 | 13.7±2.0 | 22.2±1.1 |
| NGC 4151     | 718.5±79   | 712.4±8.77 | 125.0±10.8 | 151.0±3.47 | 40.5±2.1 | 37.7±1.6 |
| Mrk 202      | 41.9±2.14 | ...        | ...      | 1.41±0.15 | ...     | ...    |
| Mrk 766      | 829±30     | 117.8±1.8  | 20.0±2.26 | 24.0±1.20 | 5.3±0.2 | 6.3±0.4 |
| Mrk 50       | 160±5.0    | ...        | ...      | 0.77±0.15 | ...     | ...    |
| NGC 4395     | 11.0±0.2   | 30.5±1.1  | 1.9±0.2  | 1.03±0.06 | 1.6±0.2 | 0.57±0.09 |
| Mrk 771      | 125.2±2.44  | ...        | ...      | 3.88±0.23 | ...     | ...    |
| NGC 4748     | 60.1±2.2   | 9.7±0.5   | ...      | 8.2±2.5 | 3.4±1.1 | 9.0±0.3 |
| PG 1307+085  | 172.0±2.4  | ...        | ...      | 1.65±0.28 | ...     | ...    |
| MGC-6-30-15  | 1100±4.0  | ...        | ...      | 18.8±2.15 | ...     | ...    |
| NGC 5548     | 314.0±9.77 | 49.3±2.9  | 16.3±2.0 | 13.9±0.65 | 6.0±0.4 | 5.6±0.4 |
| PG1448+273  | 239.0±5.25 | ...        | ...      | 5.44±0.36 | ...     | ...    |
| Mrk 290      | 435.5±12.0 | 146.7±10.4 | 26.1±7.5 | 3.42±0.94 | 3.5±0.2 | 2.2±0.5 |
| 3C 390.3     | 433.1±2.7  | ...        | ...      | 7.82±0.39 | ...     | ...    |
| NGC 6814     | 81.0±13.3 | 11.9±3.5  | ...      | 0.55±0.23 | ...     | 2.00±0.29 |
| Mrk 509      | 1824.7±77.9 | 349.0±21.8 | ...      | ...      | ...     | 58.8±5.1 |
| Arp 564      | 160±2.2    | 59.0±1.5  | 5.7±0.4  | 9.0±1.14 | 5.9±0.4 | 17.9±0.3 |
| NGC 7469     | 854.5±12.0 | 153.1±7.3 | 20.7±1.4 | 17.9±0.96 | 5.6±0.8 | 11.4±1.3 |

Table B1. Measured broad line Hβ and coronal line fluxes, in units of 10^{-15} erg cm^{-2} s^{-1}, for the galaxy sample.