Reconciling BICEP2 and Planck results with right-handed Dirac neutrinos in the fundamental representation of grand unified $E_6$

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Abstract. The tensor-to-scalar ratio ($r = 0.20^{+0.07}_{-0.05}$) inferred from the excess B-mode power observed by the Background Imaging of Cosmic Extragalactic Polarization (BICEP2) experiment is almost twice as large as the 95% CL upper limits derived from temperature measurements of the WMAP ($r < 0.13$) and Planck ($r < 0.11$) space missions. Very recently, it was suggested that additional relativistic degrees of freedom beyond the three active neutrinos and photons can help to relieve this tension: the data favor an effective number of light neutrino species $N_{\text{eff}} = 3.86 \pm 0.25$. Since the BICEP2 ratio implies the energy scale of inflation ($V_4^{1/4} \sim 2 \times 10^{16}$ GeV) is comparable to the grand unification scale, in this paper we investigate whether we can accommodate the required $N_{\text{eff}}$ with three right-handed (partners of the left-handed standard model) neutrinos living in the fundamental representation of a grand unified exceptional $E_6$ group. We show that the superweak interactions of these Dirac states (through their coupling to a TeV-scale $Z'$ gauge boson) lead to decoupling of right-handed neutrino just above the QCD cross over transition: $175 \text{ MeV} \lesssim T_{\text{dec}}^{\nu_R} \lesssim 250 \text{ MeV}$. For decoupling in this transition region, the contribution of the three right-handed neutrinos to $N_{\text{eff}}$ is suppressed by heating of the left-handed neutrinos (and photons). Consistency (within $1\sigma$) with the favored $N_{\text{eff}}$ is achieved for $4.5 \text{ TeV} < M_{Z'} < 7.5 \text{ TeV}$. The model is fully predictive and can be confronted with future data from LHC14.

Keywords: cosmological neutrinos, particle physics - cosmology connection

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1 Introduction

The concordance model of cosmology, with dark energy (Λ), cold dark matter (CDM), baryons, and three flavors of left-handed (i.e. one helicity state \( \nu^L \)) neutrinos (along with their right-handed antineutrinos \( \bar{\nu}^R \)), provides a consistent description of the late early universe: big-bang nucleosynthesis (BBN), at \( \sim 20 \) minutes, the cosmic microwave background (CMB), at \( \sim 380 \) Kyr, and the galaxy formation epoch, at \( \gtrsim 1 \) Gyr [1]. Inflationary cosmology extends the ΛCDM model by postulating an early period where the scale factor of the universe expands exponentially: \( a \propto e^{Ht} \), where \( H = \dot{a}/a \) is the Hubble parameter [2]. If the interval of exponential expansion satisfies \( \Delta t \gtrsim 60/H \), a small casually connected region can grow sufficiently to accommodate the observed homogeneity and isotropy, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hyper-surfaces (i.e., \( \Omega = 8 \pi \rho / M_{Pl}^2 H^2 \rightarrow 1 \), where \( M_{Pl} = G^{-1/2} \) is the Planck mass and \( \rho \) the energy density; throughout we use natural units, \( c = \hbar = 1 \)).

The simplest inflationary models adopt Einstein gravity sourced by a scalar field \( \phi \) and a potential \( V(\phi) \) [3–6]. In co-moving coordinates an homogeneous scalar field with minimal coupling to gravity has the equation of motion

\[
\ddot{\phi} + 3H \dot{\phi} + V' = 0,
\]

where \( V' = dV/d\phi \). The phase of quasi-de Sitter expansion \( (H \approx \text{const.}) \), when the scalar field rolls slowly down the potential, can only be sustained for a sufficient long period of time if

\[
\frac{1}{2} \dot{\phi}^2 \ll |V| \quad \text{and} \quad \left| \frac{\ddot{\phi}}{3H \dot{\phi}} \right| \ll 1.
\]

These conditions imply

\[
\epsilon \equiv \frac{M_{Pl}^2}{16\pi} \left( \frac{V'}{V} \right)^2 \ll 1 \quad \text{and} \quad \eta \equiv \frac{M_{Pl}^2}{8\pi} \left| \frac{V''}{V} - \frac{1}{2} \left( \frac{V'}{V} \right)^2 \right| \ll 1,
\]

respectively.

Quantum fluctuations in de Sitter space causally generate large-scale density fluctuations, which are necessary for the formation of galaxies and large-scale structure. As a bonus, small perturbations \( (h_{ij}, \text{with } h_i^j = \partial^j h_{ij} = 0) \) in the metric of space-time,

\[
d s^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a (\delta_{ij} + h_{ij}) dx^i dx^j,
\]
become redshifted out to the horizon \cite{7–9}. The gravity-wave fluctuations are nearly frozen on super-Hubble scales and their B-mode power spectrum,

\[ P_h = A_t \left( \frac{k}{k_*} \right)^{n_t+\frac{1}{3} \alpha_t \ln \left( \frac{k}{k_*} \right) + \cdots} \] (1.5)

\[ \approx \frac{128 V}{3 M_{Pl}^4} \left[ 1 - \left( 2C + \frac{5}{3} \right) \epsilon \right] \left( \frac{k}{k_*} \right)^{n_t+\frac{1}{3} \alpha_t \ln \left( \frac{k}{k_*} \right) + \cdots}, \] (1.6)

can be imprinted in the CMB temperature and polarization. Here, the pivot \( k_* = aH \) typifies scales probed by the CMB, \( C = \gamma_E + \ln 2 - 2 \approx -0.7296 \). To second order in \( \epsilon \) the spectral index and its running are given by

\[ n_t \simeq -2 \epsilon + \frac{8}{3} + 4C \epsilon \eta - \frac{2}{3} (7 + 6C) \epsilon^2 \quad \text{and} \quad \alpha_t \equiv \frac{dn_t}{d \ln k} \simeq -4 \epsilon (\epsilon - \eta), \] (1.7)

respectively \cite{10}. On the other hand, the power spectrum of curvature perturbations is given by

\[ P_\chi = A_s \left( \frac{k}{k_*} \right)^{n_s-1+\frac{1}{3} \alpha_s \ln \left( \frac{k}{k_*} \right) + \cdots} \]

\[ \simeq \frac{8V}{3 M_{Pl}^4} \left[ 1 - (4C + 1) \epsilon + \left( 2C - \frac{2}{3} \right) \eta \right] \left( \frac{k}{k_*} \right)^{n_s-1+\frac{1}{3} \alpha_s \ln \left( \frac{k}{k_*} \right) + \cdots}, \] (1.8)

where

\[ n_s \simeq 1 - 4 \epsilon + 2 \eta + \left( \frac{10}{3} + 4C \right) \epsilon \eta - (6 + 4C) \epsilon^2 + \frac{2}{3} \eta^2 - \frac{2}{3} (3C - 1) \left( 2 \epsilon^2 - 6 \epsilon \eta + \xi^2 \right), \] (1.9)

\[ \alpha_s \equiv \frac{dn_s}{d \ln k} \simeq -8 \epsilon^2 + 16 \epsilon \eta - 2 \xi^2, \quad \text{and} \quad \xi^2 \equiv \frac{M_{Pl}^4 V'' V'''}{64 \pi^2 V^2}. \] (1.10)

For single field inflation with canonical kinetic term, the tensor spectrum shape is not independent from the other parameters. Slow-roll expansion implies a tensor-to-scalar ratio at the pivot scale of

\[ r \equiv \frac{A_t}{A_s} \simeq 16 \epsilon + 32 \left( C - \frac{1}{3} \right) \epsilon (\epsilon - \eta). \] (1.11)

Very recently, the BICEP2 collaboration reported the measurement of low-multipole B-mode polarization \cite{11}. The observed B-mode power spectrum is well-fit by a ΛCDM + \( r \) model, with \( r = 0.20^{+0.07}_{-0.05} \), and is inconsistent with the null hypothesis, \( r = 0 \), at a significance of 7σ. Such unexpectedly large value of \( r \) corresponds to a Hubble rate, \( H \simeq 1.1 \times 10^{14} \text{GeV} \), constraining the energy scale of inflation: \( V_*^{1/4} \sim 2 \times 10^{16} \text{GeV} \). The BICEP2 dataset then provides the first experimental evidence for the existence of a new physics scale in between the electroweak and Planck scales, which is astonishingly closed to the grand unification scale (determined by extrapolation of the running coupling constants \( \alpha_{\text{QCD}}, \alpha_{\text{QED}}, \) and \( \alpha_{\text{weak}} \) to a common, “unified” value).

BICEP2 data, however, is in significant tension with Planck’s 95% CL upper limit, \( r < 0.11 \), from the temperature anisotropy spectrum in the simplest inflationary ΛCDM + \( r \)
model [12]. The conflict is a result of the fact that the large angle temperature excess foreshadowed by the gravitational waves is not observed. This apparent mismatch cannot be resolved by varying parameters in this very restrictive, seven parameter model: \{\Omega_{\text{CDM}}h^2, \Omega_b h^2, \tau, \Theta_s, A_s, n_s, r\}, where \Omega_{\text{CDM}}h^2 is the CDM energy density, \Omega_b h^2 is the baryon density, \Theta_s is the ratio between the sound horizon and the angular diameter distance at decoupling, and \tau is the Thomson scattering optical depth of reionized intergalactic medium.

Several explanations have been put forward to help reconcile Planck and BICEP2 measurements (see e.g. [14–17]) . Of particular interest here, it was pointed out that the tension can be relaxed if extra light species (e.g. massive sterile neutrinos) contribute to the effective number of relativistic degrees of freedom (r.d.o.f.) [18–20]. In this work we take a somewhat related approach to investigate the possibility of relaxing the tension by considering extra massless neutrino species. Specifically, we associate the extra r.d.o.f. with the right-handed partners of three Dirac neutrinos, which interact with all fermions through the exchange of a new heavy vector meson \(Z'\).

2 Constraints on cosmological parameters from CMB data

As the BICEP2 collaboration carefully emphasized [11], the measurement of \(r = 0.2^{+0.07}_{-0.05}\) (or \(r = 0.16^{+0.06}_{-0.05}\) after foreground subtraction, with \(r = 0\) disfavored at 5.9σ) from the B-mode polarization appears to be in tension with the 95% CL upper limits reported by the WMAP \((r < 0.13)\) and Planck \((r < 0.11)\) collaborations from the large-scale CMB temperature power spectrum. These upper limits, which favor inflationary models with concave \((V'' < 0)\) plateau-like inflaton potentials, were derived on the basis of \(\alpha_s = 0\).

The discrepancy can be formulated in terms of the tilt of \(r\)

\[
\mathcal{T}_r = \frac{d \ln \mathcal{P}_h}{d \ln k} - \frac{d \ln \mathcal{P}_\chi}{d \ln k} = n_t - (n_s - 1).
\]

Planck data favor \(n_s = 0.9603 \pm 0.0073\) [12], whereas slow-roll inflationary models yield the “consistency relation,” \(n_t = -r/8\) [21], which is a red-tilt for gravity waves. Consequently, for such inflationary models, \(n_t\) is negative and of order \(\mathcal{O} \left(10^{-2}\right)\). However, BICEP2 and Planck reconciliation requires \(\mathcal{T}_r \geq 0.16\), which clearly shows the tension between standard slow-roll models with Planck+BICEP2 data [14].

As shown in figure 1, extension of the 7-parameter model to include non-zero running of the spectral index ameliorates the tension. However, the combination of Planck and BICEP2 data favors \(\alpha_s < 0\) at almost the 3σ level, with best fit value around \(\alpha_s = -0.028\pm0.009\) (68% CL) [11]. This is about 100 times larger than single-field inflation would predict [22]. Such a particular running can be accommodated, however, if \(V'''/V\) is roughly 100 times larger than the natural expectation from the size of \(V'/V \sim (10M_{\text{Pl}})^{-1}\) and \(V''/V \sim (10M_{\text{Pl}})^{-2}\) [17].

We previously noted in the Introduction that a higher effective number of relativistic species can relieve the tension. To accommodate new physics in the form of extra r.d.o.f., it is convenient to account for the extra contribution to the standard model (SM) energy density, by normalizing it to that of an “equivalent” neutrino species. The number of “equivalent” light neutrino species,

\[
N_{\text{eff}} = \frac{\rho_R - \rho_\gamma}{\rho_{\nu_L}},
\]

\(^1\)BICEP2 data is also in tension with the 95% CL upper limit, \(r < 0.13\), reported by the WMAP collaboration [13].
quantifies the total “dark” relativistic energy density (including the three left-handed SM neutrinos) in units of the density of a single Weyl neutrino:

\[
\rho_{\nu_L} = \frac{7\pi^2}{120} \left( \frac{4}{\Pi} \right)^{4/3} T_\gamma^4,
\]

where \(\rho_\gamma\) is the energy density of photons (with temperature \(T_\gamma\)) and \(\rho_R\) is the total energy density in relativistic particles [23]. Any relativistic degree of freedom originating from physics beyond SM is included in \(N_{\text{eff}}\). Allowing for the fact that the SM neutrinos do not decouple instantaneously, which enables them to share some of the energy released by \(e^+e^-\) annihilations, leads a small increase of the contribution from the SM neutrino flavors to the number of “equivalent” light neutrino species, \(N_{\text{eff}} = 3.046\) [24].

Cosmological observations are sensitive to the total neutrino mass, that is to the sum of the three active neutrino masses

\[
\Omega_{\nu} h^2 94.1 \text{ eV} = (3.046/3)^{3/4} \sum m_\nu.
\]

Laboratory neutrino oscillation experiments indicate that at least two species must be massive, and the sum of all three species must be \(\sum m_\nu > 0.055\text{ eV}\) [1]. Measurements from the Planck temperature power spectrum in combination with low-\(\ell\) polarization measurements from WMAP 9-year data [25] (WMAP + Planck) yield a 95% CL upper limit on the sum of the three active neutrino masses of \(\sum m_\nu < 0.933\text{ eV}\) [26]. When data from the Atacama Cosmology Telescope (ACT) [27–29] and the South Pole Telescope (SPT) [30–32] are incorporated into the analysis the 95% CL neutrino mass upper limit is considerably improved, \(\sum m_\nu < 0.663\) eV [26]. Finally, with the addition of Baryon Acoustic Oscillation (BAO) measurements from the Sloan Digital Sky Survey (SDSS)-II Data Release 7 [33, 34], from the WiggleZ survey [35], from the Baryon Acoustic Spectroscopic Survey (BOSS) [36] (one of the four surveys of SDSS-III [37] Data Release 9 [38]), and from 6dF Galaxy Survey [39] the constraint on the neutrino mass is strongly tightened: \(\sum m_\nu < 0.230\) eV at 95% CL [26].

Before proceeding we note that the Planck CMB temperature anisotropy spectrum is also in conflict with measurements of the local Universe. Unexpectedly, the best multi-parameter fit of Planck data yields a Hubble constant \(h = 0.674 \pm 0.012\) [26], a result which
deviates by more than $2\sigma$ from the value obtained with the Hubble Space Telescope (HST), $h = 0.738 \pm 0.024$ [40]. (Herein we adopt the usual convention of writing the Hubble constant at the present day as $H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$.) The impact of the Planck $h$ estimate is particularly important in the determination of $N_{\text{eff}}$. Combining observations of the CMB with data from BAO, the Planck collaboration reported $N_{\text{eff}} = 3.30 \pm 0.27$ [26]. However, if the value of $h$ is not allowed to float in the fit, but instead is frozen to the value determined from the maser-cephheid-supernovae distance ladder, $h = 0.738 \pm 0.024$, the Planck CMB data then gives $N_{\text{eff}} = 3.62 \pm 0.25$, which suggests new neutrino-like physics (at around the $2.5\sigma$ level) [26].

In figures 1 and 2 we compare aftermath of the multiparameter fit of ref. [20], generalized here for extra massless r.d.o.f. (i.e., $\{\Omega_{\text{CDM}} h^2, \Omega_b h^2, \tau, \Theta_s, A_s, n_s, r, N_{\text{eff}}, \sum m_\nu\}$), with the results reported by the Planck [12, 26] and BICEP2 [11] collaborations. The data sets consider in the 9-parameter fit include various CMB observations: temperature power spectrum from Planck [26], WMAP 9-year polarization [25], ACT/SPT high multipole power spectra [29–31], and the BICEP2 BB and EE polarization band powers [11]. In addition a collection of local data sets has been included: the $H_0$ inference from the maser-cepheid-supernovae distance ladder and BAO measurements [34, 35, 38]. From figure 1 we can see that for the 9-parameter fit, the 68% CL values of the parameter which controls the amplitude of the gravitation wave signal are roughly equal to those reported by the BICEP2 collaboration [11]. However, the allowed region of the $r-n_s$ plane is shifted to larger values of the scalar spectral index, and it is consistent with exact scale invariance ($n_s = 1$) [41–43]. In contrast to the results from Planck in the absence of running ($\alpha_s = 0$) the outcome of the multiparameter
fit favors inflationary models with $V'' > 0$. The allowed region of the $h - r$ parameter space is in agreement with the value of $h$ measured by both Planck and HST [26]. In the upper left panel of figure 2 we show the allowed 68% and 95% confidence regions for $(h - N_{\text{eff}})$. Unlike the previous results reported by the Planck collaboration on the basis of CMB data alone (which are consistent with $N_{\text{eff}} = 3.046$) the multiparameter fit favors extra r.d.o.f. The 68% CL contour yield $N_{\text{eff}} = 3.86 \pm 0.25$. The associated 95% CL region for the sum of the three active neutrino masses saturates current limits (see upper right panel of figure 2.) Finally, the higher effective number of r.d.o.f. is in agreement with the scalar spectral index (figure 2 lower left) and the tensor-to-scalar ratio (figure 2 lower right).

In closing, we note that the constraint on $N_{\text{eff}}$ derived above is in excellent agreement with the value inferred from BBN observations: $N_{\text{eff}} = 3.71^{+0.47}_{-0.45}$ [44]. We take this agreement as evidence favoring extra r.d.o.f. both during BBN and CMB epochs. In the next section we interpret the observed excess, $\Delta N = 0.81 \pm 0.25$, in the context of the $E_6$ grand unified model, which is largely well motivated by the measured scale of inflation.

3 Right-handed neutrinos with milli-weak interactions

In addition to the $2.984 \pm 0.009 \nu_L$ species measured from the width for invisible decays of the $Z$ boson [45] there could also exist $\nu_R$ states that are sterile, i.e. singlets of the SM gauge group and therefore insensitive to weak interactions. Such sterile states are predicted in models involving additional TeV-scale $Z'$ gauge bosons, which allow for milli-weak interactions of the $\nu_R$. If the $\nu_R$ carry a non-zero U(1) charge, then the U(1) symmetry forbids them from obtaining a Majorana mass much larger than the U(1)-breaking scale. Therefore, in most of these models there are no Majorana mass terms and the $\nu_R$ states, which are almost massless, become the Dirac partners of the SM $\nu_L$ species.

In this section we pursue a study to correlate the expected increase in the universe expansion rate due to the presence of such light Dirac neutrinos with ongoing searches of $Z'$ gauge bosons at the CERN’s LHC. A critical input for such an analysis is the relation between the relativistic degrees of r.d.o.f. and the temperature of the primordial plasma. This relation is complicated because the temperature which is of interest for right-handed neutrino decoupling from the heat bath may lay in the vicinity of the quark-hadron cross-over transition. To connect the temperature to an effective number of r.d.o.f. we make use of some high statistics lattice simulations of a QCD plasma in the hot phase, especially the behavior of the entropy during the changeover [46].

3.1 Theoretical considerations

To develop our program in the simplest way, we follow [47–52] and make use of extra U(1) symmetries embedded in a grand unified exceptional $E_6$ group, with breaking pattern

$$E_6 \rightarrow \text{SO}(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\psi \times U(1)_\chi.$$  \hspace{1cm} (3.1)$$

In $E_6$, each family of left-handed fermions is promoted to a fundamental 27-plet, which decomposes under $E_6 \rightarrow \text{SO}(10) \rightarrow SU(5)$ as

$$27 \rightarrow 16 + 10 + 1 \rightarrow (10 + 5^* + 1) + (5 + 5^*) + 1,$$  \hspace{1cm} (3.2)$$

as described in table 1 [58, 59]. In addition to the SM fermions, each 27-plet contains two SM singlets, $\nu^c$ and $S$, which may be charged under the extra U(1) symmetries. The $\nu^c$
In line with our stated plan, we now use the favored value of $N_{\text{eff}}$ to calculate the range of decoupling temperature. We begin by first establishing the contribution of right-handed neutrinos to $N_{\text{eff}}$, that is $\Delta N_\nu$ as a function of the $\nu_R$ decoupling temperature. Taking into account the isentropic heating of the rest of the plasma between $T^\text{dec}_{\nu_R}$ and $T^\text{dec}_{\nu_L}$ decoupling temperatures we obtain [65]

$$
\Delta N_\nu = 3 \left( \frac{g_s (T^\text{dec}_{\nu_R})}{g_s (T^\text{dec}_{\nu_L})} \right)^{4/3},
$$

where $g_s(T)$ is the effective number of interacting (thermally coupled) r.d.o.f. at temperature $T$; e.g., $g_s (T^\text{dec}_{\nu_R}) = 43/4$. At energies above the deconfinement transition towards the quark gluon plasma, quarks and gluons are the relevant fields for the QCD sector, such that the total number of SM r.d.o.f. is $g_s = 61.75$. As the universe cools down, the SM plasma transitions to a regime where mesons and baryons are the pertinent degrees of freedom. Precisely, the relevant hadrons present in this energy regime are pions and charged kaons, such that $g_s = 19.25$ [66]. This significant reduction in the degrees of freedom results from the rapid annihilation or decay of any more massive hadrons which may have formed during the transition. The quark-hadron crossover transition therefore corresponds to a large redistribution of entropy into the remaining degrees of freedom. Concretely, the effective number of

| $\text{SO(10)}$ | $\text{SU(5)}$ | $2\sqrt{10}Q_\chi$ | $2\sqrt{6}Q_\psi$ | $2\sqrt{15}Q_\eta$ | $2Q_l$ | $2\sqrt{10}Q_N$ | $2\sqrt{15}Q_S$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|----------|----------------|----------------|
| 16 \[ (u, d, w^c, e^+) \] | 10 \[ (u, d, w^c, e^+) \] | $-1$ | $1$ | $-2$ | $0$ | $1$ | $-1/2$ |
| \[ $5^* (d^c, \nu, e^-)$ \] | $\nu^c$ | $3$ | $1$ | $1$ | $-2$ | $2$ | $4$ |
| \[ $15^* \text{(v)} \] | $5^* (D^c, H^c)$ | $-2$ | $-2$ | $1$ | $1$ | $-3$ | $-7/2$ |
| $1$ | $1$ \[ $S$ \] | $0$ | $4$ | $-5$ | $-1$ | $5$ | $5/2$ |

**Table 1.** Decomposition of the $E_6$ fundamental representation of left-handed fermions 27 under SO(10) and SU(5), and the U(1) charges $Q_i$ for particular choices of $\theta_{E_6}$; U(1)$_\chi$ with $\theta_{E_6} = 0$, U(1)$_\psi$ with $\theta_{E_6} = \pi/2$, U(1)$_{\eta}$ with $\theta_{E_6} = \pi - \arctan \sqrt{5/3} \approx 0.71 \pi$, inert U(1)$_I$ with $\theta_{E_6} = \arctan \sqrt{3/5} \approx 0.21 \pi$, neutral-N U(1)$_N$ with $\theta_{E_6} = \arctan \sqrt{15} \approx 0.42 \pi$, and secluded sector U(1)$_S$ with $\theta_{E_6} = \arctan \sqrt{15/9} \approx 0.13 \pi$ [53–57].

 can be interpreted as the conjugate of the right-handed neutrino. There is also an exotic color-triplet quark $D$ and its conjugate $D^c$, both of which are SU(2) singlets, and a pair of color-singlet SU(2)-doublet exotics, $H_u = \left( \begin{array}{c} H^+_u \\ H^0_u \end{array} \right)$ and $H_d = \left( \begin{array}{c} H^0_d \\ H^-_d \end{array} \right)$. All the exotic fields are singlets or non-chiral under the SM, but may be chiral under the extra U(1) symmetries.

It is usually assumed that the gauge sector contains only one U(1) symmetry at low energies. Thus, there is a continuum of possible models where the new gauge boson couple to a linear combination of $Q_\chi$ and $Q_\psi$ parametrized by a mixing angle $\theta_{E_6}$. The resultant U(1)$_I'$ charge is then

$$
Q_i = Q_\chi \cos \theta_{E_6} + Q_\psi \sin \theta_{E_6}.
$$

In this work we focus on the particular case $\theta_{E_6} = 0$, in which $S$ does not couple to the $Z'$. This model provides a test basis for $Z'$ searches at the ATLAS [60, 61] and CMS [62–64] experiments.

### 3.2 Confronting neutrino cosmology with LHC data

In line with our stated plan, we now use the favored value of $N_{\text{eff}}$ to calculate the range of decoupling temperature. We begin by first establishing the contribution of right-handed neutrinos to $N_{\text{eff}}$, that is $\Delta N_\nu$ as a function of the $\nu_R$ decoupling temperature. Taking into account the isentropic heating of the rest of the plasma between $T^\text{dec}_{\nu_R}$ and $T^\text{dec}_{\nu_L}$ decoupling temperatures we obtain [65]

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Figure 3. Left. The parametrization of the entropy density given in eq. (3.6) (dashed line) superposed on the result from high statistics lattice simulations [46] (solid line). Right. Comparison of $g_s(T)$ obtained using eq. (3.7) (dashed line) and the phenomenological estimate of [67, 68] (solid line).

interacting r.d.o.f. in the plasma at temperature $T$ is given by

$$g_s(T) \simeq r(T) \left( N_B + \frac{7}{8} N_F \right),$$

(3.5)

with $N_B = 2$ for each real vector field and $N_F = 2$ for each spin-$\frac{1}{2}$ Weyl field. The coefficient $r(T)$ is unity for leptons, two for photon contributions, and is the ratio $s(T)/s_{SB}$ for the quark-gluon plasma. Here, $s(T)$ ($s_{SB}$) is the actual (ideal Stefan-Boltzmann) entropy shown in figure 3. For $150 \text{ MeV} < T < 500 \text{ MeV}$, we parametrize the entropy rise during the confinement-deconfinement changeover by

$$\frac{s}{T^3} \simeq \frac{42.82}{\sqrt{392\pi}} e^{-\frac{(T_{\text{MeV}}-151)^2}{392}} + \left( \frac{195.1}{T_{\text{MeV}}-134} \right)^2 18.62 \left[ e^{195.1/(T_{\text{MeV}}-134)} - 1 \right]^2.$$  

(3.6)

For the same energy range, we obtain

$$g_s(T) \simeq 47.5 \ r(T) + 19.25.$$  

(3.7)

In figure 3 we show $g_s(T)$ as given by (3.7). Our parametrization is in very good agreement with the phenomenological estimate of [67, 68].

If relativistic particles are present that have decoupled from the photons, it is necessary to distinguish between two kinds of r.d.o.f.: those associated with the total energy density $g_\rho$, and those associated with the total entropy density $g_s$. Since the quark-gluon energy density in the plasma has a similar $T$ dependence to that of the entropy (see figure 7 in [46]), we take $g_\rho(T) \simeq r(T) \left( N_B + \frac{7}{8} N_F \right)$.

The right-handed neutrino decouples from the plasma when its mean free path becomes greater than the Hubble radius at that time

$$\Gamma \left( T_{\text{dec}}^{\nu_R} \right) = H \left( T_{\text{dec}}^{\nu_R} \right),$$

(3.8)

where

$$\Gamma \left( T_{\text{dec}}^{\nu_R} \right) = \mathcal{K} \frac{1}{8} \left( \frac{7}{M_{Z'}} \right)^4 \left( T_{\text{dec}}^{\nu_R} \right)^5 \sum_{i=1}^{6} \mathcal{N}_i ,$$

(3.9)
is the $\nu_R$ interaction rate,

$$H(T^{\text{dec}}_{\nu_R}) = 1.66\sqrt{g_\rho} \left( \frac{T^{\text{dec}}_{\nu_R}}{M_{\text{Pl}}} \right)^2 ,$$

is the Hubble expansion rate at the $\nu_R$ decoupling temperature

$$\mathcal{g} = \left( \sum_{i=1}^{6} N_i g_i^2 \frac{g_0^2}{g_2} \right)^{1/4} ,$$

$N_i$ is the number of chiral states, $g_i = g_0 Q_i$ are the chiral couplings of the $Z'$ for the 6 relevant species (see below), and the constant $\mathcal{K} = 0.5$ (2.5) for annihilation (annihilation + scattering) [69]. In the second line of (3.10) we set $g_s \simeq g_\rho$. In conformity with grand unification we follow [50] and choose

$$g_0 = \sqrt{\frac{5}{3}} g_2 \tan \theta_W \sim 0.46 ,$$

with $g_2$ the SU(2)$_L$ coupling. (Note that for the N model, the effective coupling $\mathcal{g}$ has a similar strength.)

The physics of interest takes place in the quark gluon plasma itself so that we will restrict ourselves to the following fermionic fields in the visible sector, $[3u_R] + [3d_R] + [3s_R] + [3\nu_L + e_L + \mu_L] + [e_R + \mu_R] + [3u_L + 3d_L + 3s_L] + [3\nu_R]$, and their contribution to $g_\rho$. This amounts to 28 Weyl fields, translating to 56 fermionic r.d.o.f.: $\sum_{i=1}^{6} N_i = 28$.

Substituting (3.9) and (3.10) into (3.8) we obtain

$$\frac{\mathcal{g}}{M_{Z'}} = \left( \frac{3}{\Delta N_{\nu}} \right)^{3/2} \left( \frac{13.28}{g_\rho T^{\text{dec}}_{\nu_R}} \right)^{1/4} \left( \frac{g_s}{M_{\text{Pl}}} \right) \mathcal{K} \right( \sum_{i=1}^{6} N_i \right)^{3/4}$$

and

$$\Delta N_{\nu} = \left[ \frac{5.39 \times 10^{-6}}{\mathcal{K} \sum_{i=1}^{6} N_i} \left( \frac{M_{Z'}}{\text{TeV}} \right)^4 \left( \frac{\text{GeV}}{T_{\text{dec}}^{\nu_R}} \right)^3 \right]^{8/3} .$$

In figure 4 we show the region of the parameter space allowed from decoupling requirements to accommodate contributions of $\Delta N_{\nu} = 0.81 \pm 0.25$. This region is in agreement with LHC experimental limits on $M_{Z'}$ for null signals for enhancements in dilepton or dijet searches [60–64]. The model also provides a testable prediction, $\mathcal{g} \simeq 0.46$ and $4.5 \text{ TeV} \lesssim M_{Z'} \lesssim 7.5 \text{ TeV}$, within the LHC14 discovery reach.

4 Conclusions

Aside from exhibiting temperature fluctuations of one part in $10^5$, the CMB is partially polarized. The parity-odd polarization, or B-mode, arises from primordial tensor fluctuations, which manifest as gravitational waves. The temperature map provided by the Planck mission constrains the value of tensor-to-scalar perturbations, $r < 0.11$ at 95 CL. However, recent BICEP2 B-mode polarization data is in tension with this constrain, as imply a value $r = 0.20^{+0.07}_{-0.05}$. Very recently, it was suggested that additional relativistic degrees of freedom beyond the three active neutrinos and photons can help to relieve this tension: the data favor an effective number of light neutrino species $N_{\text{eff}} = 3.86 \pm 0.25$. 
Figure 4. The (yellow) shaded area shows the 68% confidence region allowed from decoupling requirements to accommodate $\Delta N = 0.81 \pm 0.25$. The horizontal line dictates the effective coupling $\bar{g}$ for $Z'_\chi$; the cross-hatched part of the line reflects the LHC experimental limits on the mass of the gauge boson. We have taken $K = 0.5$ (left) and $K = 2.5$ (right).

We have shown that we can accommodate the required $N_{\text{eff}}$ with the contribution from the right-handed partners of the three, left-handed, SM neutrinos (living in the fundamental representation of $E_6$). The six additional fermionic r.d.o.f. can be suppressed to levels in compliance with the favored $N_{\text{eff}}$, because the milli-weak interactions of these Dirac states (through their coupling to a TeV-scale $Z'$ gauge boson) may allow the $\nu_R$’s to decouple much earlier, at a higher temperature, than their left-handed counterparts. If the $\nu_R$’s decouple during the quark-hadron crossover transition, they are considerably cooler than the $\nu_L$’s and contribute less than 3 extra “equivalent neutrinos” to the early Universe energy density. For decoupling in this transition region, the 3 $\nu_R$ generate $\Delta N_{\nu} = 3 \left( \frac{T_{\nu_R}^{\text{dec}}}{T_{\nu_L}^{\text{dec}}} \right)^4 < 3$, extra relativistic degrees of freedom. These requirements strongly constrain the mass of the heavy vector field. Consistency (within 1σ) with $N_{\text{eff}}$ is achieved for an effective coupling $\bar{g} = 0.46$ and $Z'$ mass in the range $4.5 \text{ TeV} < M_{Z'} < 7.5 \text{ TeV}$. The model is fully predictive and can be confronted with future data from LHC14.

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