Librational solution for dust particles in mean motion resonances under the action of stellar radiation

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ABSTRACT

This paper presents a librational solution for evolutions of parameters averaged over a synodic period in mean motion resonances in planar circular restricted three-body problem (PCR3BP) with non-gravitational effects taken into account. The librational solution is derived from a linearization of modified Lagrange’s planetary equations. The presented derivation respects properties of orbital evolutions in the mean motion resonances within the framework of the PCR3BP. All orbital evolutions in the PCR3BP with the non-gravitational effects can be described by four varying parameters. We used the semimajor axis, eccentricity, longitude of pericenter and resonant angular variable. The evolutions are found for all four parameters. The solution can be applied also in the case without the non-gravitational effects. We compared numerically and analytically obtained evolutions in the case when the non-gravitational effects are the Poynting-Robertson effect and the radial stellar wind. The librational solution is good approximation when the libration amplitude of the resonant angular variable is small.

Subject headings: Interplanetary dust – Mean motion resonances – Celestial mechanics – Poynting-Robertson effect, Stellar wind

1. Introduction

Oscillations are often present when a physical system is near a stable state. The gravity of a star and a planet (or a planet and a satellite) that move according to a solution of the two body problem can perturb the motion of a body with negligible mass (restricted three-body problem). The dynamic of the body with negligible mass includes in this case also the so called mean motion resonances. In a mean motion resonance a ratio of orbital periods of the two minor bodies oscillates near a ratio of two natural numbers.

The perturbed motion of the body captured in the mean motion resonance can be studied using a disturbing function. The disturbing function is often expanded using Fourier series of the Laplacian type (e.g., Murray & Dermott 1999). Usability of this access is limited by two factors. First, a finite order of the expansion makes it practically unusable for large eccentricities. The
second limit is the approximation of the averaged disturbing function by a single periodic resonant term. Murray & Dermott (1999) substituted the truncated averaged Fourier series in the time derivatives of orbital elements given by Lagrange's planetary equations. By integrating of these equations individually they obtained dependencies of orbital elements on time. To derive the time dependencies they assumed that the only time-varying quantities in the equations for the time derivatives of orbital elements are in the trigonometric arguments of the single periodic resonant term and that the longitude of pericenter increases linearly with time at a constant rate determined by secular theory. The obtained results were applied for an asteroid in the 1/2 interior resonance with Jupiter (Murray & Dermott 1999, p. 259). Another example of the time dependence obtained using the Fourier series expansion of the disturbing function can be found in Greenberg (1973). Lagrange’s planetary equations for the planar circular restricted three-body problem (PCR3BP) including tidal dissipation were solved simultaneously using several crude approximations. The solution of model was applied to the 4/3 exterior resonance between Saturn’s satellites Titan and Hyperion. It has not been possible to match exactly the behavior of Titan-Hyperion resonance using this model because of the approximations used (Greenberg 1973). Even so, the model provided insight into the underlying mechanisms.

For the mean motion resonances in the PCR3BP canonical equations can be written using Hamiltonian formalism and averaged ever a synodic period. This access gives a system with one degree of freedom that is integrable. However, the obtained system is difficult to solve and analytical solution in general case was not found. The dynamics of dust particles captured in the mean motion resonances and simultaneously affected by non-gravitational effects was frequently investigated during the last three decades. Influence of electromagnetic radiation of a star on the dynamics of dust particles can be described by the Poynting–Roberson (PR) effect (Poynting 1904; Robertson 1937; Klačka 2004; Klačka et al. 2014). In Beaugé & Ferraz-Mello (1994) the equations of motion of a dust particle captured in a mean motion resonance in the PCR3BP with the PR effect were written in a near canonical form. Beaugé & Ferraz-Mello (1994) transformed the near canonical equations to a system of equations suitable for searching of librational points. In order to determine the stability of a librational point they mentioned a linearization performed around the libration point and a solution of obtained characteristic equation of the system. The stability of the libration points was determined with a similar procedure also in Šidlichovský & Nesvorný (1994). In both previously mentioned papers specific procedure used for solving of the linearized system cannot be found. Even more, the linearized system in Beaugé & Ferraz-Mello (1994) and Šidlichovský & Nesvorný (1994) is different. In Beaugé & Ferraz-Mello (1994) the system has four equations and in Šidlichovský & Nesvorný (1994) has only three equations.

In this paper we derive the solution of the linearized system of equations describing evolutions
of dust particles captured in mean motion resonances in the PCR3BP with non-gravitational effect. We apply the derived solution in the case when the non-gravitational effects are the electromagnetic radiation of a star and its radial stellar wind.

2. Averaged resonant equations

A secular evolution of a dust particle in an orbit around the Sun is significantly influenced also by non-gravitational effects. In order to describe the secular evolution in the most of practical cases time derivatives of orbital elements averaged over some time interval can be used. The averaged time derivatives of orbital elements of the dust particle’s orbit caused by the non-gravitational effects can be calculated using the Gaussian perturbation equations of celestial mechanics [Danby 1988 Murray & Dermott 1999]. The secular time derivatives determine the secular evolution in the case without the gravitational influence of a planet. In order to obtain a system of equations describing the secular evolution in a mean motion resonance with a planet we use Lagrange’s planetary equations (Brouwer & Clemence 1961 Danby 1988). The non-gravitational effects for which is possible to calculate the secular time derivatives of orbital elements can be included in Lagrange’s planetary equations. In a planar case (when the dust particle’s orbit lies in the planet orbital plane) modified Lagrange’s planetary equations which include the non-gravitational effects are

\[
\begin{align*}
\frac{da}{dt} &= \frac{2a}{L} \frac{\partial R_G}{\partial \sigma_b} + \left\langle \frac{da}{dt} \right\rangle_{EF}, \\
\frac{de}{dt} &= \frac{\alpha^2}{Le} \frac{\partial R_G}{\partial \sigma_b} - \frac{\alpha}{Le} \frac{\partial R_G}{\partial \omega} + \frac{\left\langle \frac{de}{dt} \right\rangle}{EF}, \\
\frac{d\omega}{dt} &= \frac{\alpha}{Le} \frac{\partial R_G}{\partial e} + \left\langle \frac{d\omega}{dt} \right\rangle_{EF}, \\
\frac{d\sigma_b}{dt} + t \frac{dn}{dt} &= -\frac{2a}{L} \frac{\partial R_G}{\partial a} - \frac{\alpha^2}{Le} \frac{\partial R_G}{\partial e} + \left\langle \frac{d\sigma_b}{dt} + t \frac{dn}{dt} \right\rangle_{EF}.
\end{align*}
\]

(1)

Here, \(a\) is the semimajor axis of the particle orbit, \(e\) is the eccentricity, \(\omega\) is the longitude of pericenter, and the angle \(\sigma_b\) is defined so that the mean anomaly can be computed from \(M = nt + \sigma_b\) (Bate, Mueller & White 1971). Where \(n = \sqrt{\mu(1 - \beta)/a^3}\) is the mean motion of the particle, \(\mu = G_0M_*\), \(G_0\) is the gravitational constant, and \(M_*\) is the mass of the star. The parameter \(\beta\) is defined as the ratio between the electromagnetic radiation pressure force and the gravitational force between the star and the particle at rest with respect to the star. \(L = \sqrt{\mu(1 - \beta)a}\). \(R_G\) is the disturbing function

\[
R_G = G_0 M_P \left( \frac{1}{|\vec{r} - \vec{r}_P|} - \frac{\vec{r} \cdot \vec{r}_P}{r_P^3} \right).
\]

(2)

In the disturbing function: \(M_P\) is the mass of the planet, \(\vec{r}_P\) is the position vector of the planet with respect to the star, \(r_P = |\vec{r}_P|\), and \(\vec{r}\) is the position vector of the dust particle with respect to the star. The partial derivative of the disturbing function with respect to the semimajor axis in the
last equation in Eqs. (1) is calculated with an assumption that the mean motion of the particle \( n \) is not a function of the semimajor axis. All terms in Eqs. (1) represent quantities averaged over a synodic period. \( \langle da/dt \rangle_{EF}, \langle de/dt \rangle_{EF}, \langle d\tilde{\omega}/dt \rangle_{EF}, \text{ and } \langle d\sigma_v/dt + t \, dn/dt \rangle_{EF} \) are caused by the non-gravitational effects only. Eqs. (1) already describe the secular evolution of the dust particle which is captured in the mean motion resonance and simultaneously affected by the non-gravitational effects. For the study of a specific mean motion resonance, it is convenient to define a resonant angular variable \( \sigma = \frac{p + q}{q} \lambda_P - s\lambda - \tilde{\omega} \),

where \( p \) and \( q \) are two integers (resonant numbers), \( \lambda_P \) is the mean longitude of the planet, \( \lambda = M + \tilde{\omega} = nt + \sigma_b + \tilde{\omega} \) is the mean longitude of the dust particle, and \( s = p/q \). In the mean motion resonance \( \sigma \) is librating rather than circulating. The resonant angular variable is not yet averaged. For the time derivative of \( \sigma \) we obtain from its definition

\[
\frac{d\sigma}{dt} = \frac{p + q}{q} n_P - sn - s \left( \frac{d\sigma_v}{dt} + t \frac{dn}{dt} + \frac{d\tilde{\omega}}{dt} \right) - \frac{d\tilde{\omega}}{dt} .
\]

The above equation can be averaged over the synodic period. If we use in the averaged result the last two averaged equations in Eqs. (1), then we get

\[
\frac{d\sigma}{dt} = -\frac{\alpha}{L} \left[ 1 + s \left( 1 - \alpha \right) \right] \frac{\partial R_G}{\partial e} + \frac{2sL}{\mu(1-\beta)} \frac{\partial R_G}{\partial a} + n_P \frac{p + q}{q} - ns \\
- \frac{p + q}{q} \langle \frac{d\tilde{\omega}}{dt} \rangle_{EF} - s \left( \frac{d\sigma_v}{dt} + t \frac{dn}{dt} \right)_{EF} .
\]

The averaged disturbing function \( R_G \) can be rewritten as a function of the variables \( a, e, \tilde{\omega}, \) and \( \sigma \). In this case the following relations hold

\[
\frac{\partial R_G}{\partial \sigma_v} = -s \frac{\partial R_G}{\partial \sigma} ,
\]

\[
\frac{\partial R_G}{\partial \tilde{\omega}} = -\frac{p + q}{q} \frac{\partial R_G}{\partial \sigma} .
\]

We can use in the first two equations in the system of equations given by Eqs. (1) relations Eqs. (6). The last equation in Eqs. (1) can be replaced with equivalent Eq. (5). By this we obtain a system that enables to study the secular orbital evolution of the dust particle captured in the specific mean motion resonance given by the resonant numbers \( p \) and \( q \) under the action of the non-gravitational effects,

\[
\frac{da}{dt} = -\frac{2sa}{L} \frac{\partial R_G}{\partial \sigma} + \langle \frac{da}{dt} \rangle_{EF} ,
\]

\[
\frac{de}{dt} = \alpha \frac{1 + s \left( 1 - \alpha \right)}{Le} \frac{\partial R_G}{\partial e} + \langle \frac{de}{dt} \rangle_{EF} ,
\]

\[
\frac{d\tilde{\omega}}{dt} = \alpha \frac{\partial R_G}{\partial e} + \langle \frac{d\tilde{\omega}}{dt} \rangle_{EF} ,
\]
\[
\frac{d\sigma}{dt} = -\frac{\alpha}{Le} [1 + s (1 - \alpha)] \frac{\partial R_G}{\partial e} + \frac{2sa}{L} \frac{\partial R_G}{\partial a} + np \frac{p + q}{q} - ns \\
- \frac{p + q}{q} \left\langle \frac{d\omega}{dt} \right\rangle_{EF} - s \left\langle \frac{d\sigma_b}{dt} + t \frac{dn}{dt} \right\rangle_{EF}.
\] (7)

The last term in the angle brackets, despite of its complicated meaning, can be straightforwardly obtained using (Bate et al. 1971)

\[
\left\langle \frac{d\sigma_b}{dt} + t \frac{dn}{dt} \right\rangle_{EF} = \left\langle 1 - e^2 \left[ a_R \left( \frac{\cos f}{e} - \frac{2}{1 + e \cos f} \right) - a_T \frac{\sin f}{e} \frac{2 + e \cos f}{1 + e \cos f} \right] \right\rangle_{EF},
\] (8)

where \(f\) is the true anomaly, \(a_R\) and \(a_T\) are the radial and transversal components of the acceleration caused by the non-gravitational effects.

The system of equations given by Eqs. (7) is different from systems considered in Beaugé & Ferraz-Mello (1994) and Šidlichovský & Nesvorný (1994). The system in Beaugé & Ferraz-Mello (1994) does not take into account the non-gravitational effects for which the secular variations of the particle orbit depend on the orientation of the orbit in space. The system in Šidlichovský & Nesvorný (1994) contains only three equations.

3. Linearization of averaged resonant equations

No general method exists for solving nonlinear differential equations in the system Eqs. (7). In practice, the best that can be accomplished is to study a linearization based upon initial conditions for the function and its derivatives. In the vicinity of an initial point \(a_0, e_0, \tilde{\omega}_0, \) and \(\sigma_0\) we use notation

\[
\delta_a = a - a_0, \\
\delta_e = e - e_0, \\
\delta_{\tilde{\omega}} = \tilde{\omega} - \tilde{\omega}_0, \\
\delta_{\sigma} = \sigma - \sigma_0.
\] (9)

The time will be measured from an initial time \(t_0\). Hence

\[
\delta_t = t.
\] (10)

The linearization of averaged resonant equations (Eqs. 7) finally gives

\[
\frac{d\delta_a}{dt} = \left( -\frac{s}{L_0} \frac{\partial R_G}{\partial \sigma} - \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial a \partial \sigma} + \frac{\partial}{\partial a} \left\langle \frac{da}{dt} \right\rangle_{EF} \right) \delta_a + \left( -\frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial e \partial \sigma} + \frac{\partial}{\partial e} \left\langle \frac{da}{dt} \right\rangle_{EF} \right) \delta_e \\
+ \left( \frac{p + q}{q} \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial \sigma^2} + \frac{\partial}{\partial \tilde{\omega}} \left\langle \frac{da}{dt} \right\rangle_{EF} \right) \delta_{\tilde{\omega}} + \left( -\frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial \sigma^2} + \frac{\partial}{\partial \sigma} \left\langle \frac{da}{dt} \right\rangle_{EF} \right) \delta_{\sigma} \\
+ \frac{\partial}{\partial t} \left( \frac{da}{dt} \right) t + \left( \frac{da}{dt} \right)_0,
\]
\[
\frac{d\delta_e}{dt} = \left\{ \frac{-\alpha_0}{2a_0 L_0 e_0^2} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial e} + \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial a \partial e} + \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \right\} \delta_a \\
+ \left\{ \frac{1}{L_0 e_0^2} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial e} + \frac{s}{L_0} \frac{\partial R_G}{\partial e} + \frac{\alpha_0}{L_0 e_0^2} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial e^2} \right\} \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \\
+ \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \right\} \delta_e + \left\{ \frac{p + q}{q} \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial e^2} \right\} \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) + \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \right\} \delta_\omega \\
\frac{d\delta_\omega}{dt} = \left\{ \frac{\alpha_0}{2a_0 L_0 e_0^2} \frac{\partial R_G}{\partial e} - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial e^2} \right\} \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \\
+ \frac{3s\alpha_0}{2a_0} - \frac{p + q}{q} \frac{\partial}{\partial e} \left( \frac{d\omega}{dt} \right) \right\} \delta_a \\
- \frac{1}{L_0 e_0^2} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial e} - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial e^2} - \frac{s}{L_0} \frac{\partial R_G}{\partial e} + \frac{2s\alpha_0}{L_0} \frac{\partial^2 R_G}{\partial e^2} \\
- \frac{p + q}{q} \frac{\partial}{\partial e} \left( \frac{d\omega}{dt} \right) \right\} \delta_e + \left\{ \frac{p + q}{q} \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial e^2} \right\} \frac{\partial}{\partial e} \left( \frac{de}{dt} \right) \\
- \frac{p + q}{q} \frac{2s\alpha_0}{L_0} \frac{\partial^2 R_G}{\partial e^2} - \frac{p + q}{q} \frac{\partial}{\partial e} \left( \frac{d\omega}{dt} \right) \right\} \delta_\omega
\]

The terms in parenthesis before the deltas on the right-hand sides in Eqs. (11) are constant therefore we can simply write

\[
\dot{\delta}_a = A\delta_a + B\delta_e + C\delta_\omega + D\delta_\sigma + Et + F \\
\dot{\delta}_e = G\delta_a + H\delta_e + I\delta_\omega + J\delta_\sigma + Kt + L \\
\dot{\delta}_\omega = M\delta_a + N\delta_e + O\delta_\omega + P\delta_\sigma + Qt + R \\
\dot{\delta}_\sigma = S\delta_a + T\delta_e + U\delta_\omega + V\delta_\sigma + Wt + X
\]

This system describes solution of system Eqs. (7) during a short time interval after the initial time. It is possible to obtain an equation for one chosen delta by an elimination of the remaining deltas.
using all equations in the system. The obtained equations for the separated deltas are

\[
\begin{align*}
\dddot{\delta} a &+ \Lambda a_3 \dot{\delta} a + \Lambda a_2 \ddot{\delta} a + \Lambda a_1 \dot{\delta} a + \Lambda a_0 \delta a + \Lambda a t t + \Lambda a = 0, \\
\dddot{\delta} e &+ \Lambda e_3 \dot{\delta} e + \Lambda e_2 \ddot{\delta} e + \Lambda e_1 \dot{\delta} e + \Lambda e_0 \delta e + \Lambda e t t + \Lambda e = 0, \\
\dddot{\delta} \varphi &+ \Lambda \varphi_3 \dot{\delta} \varphi + \Lambda \varphi_2 \ddot{\delta} \varphi + \Lambda \varphi_1 \dot{\delta} \varphi + \Lambda \varphi_0 \delta \varphi + \Lambda \varphi t t + \Lambda \varphi = 0, \\
\dddot{\delta} \sigma &+ \Lambda \sigma_3 \dot{\delta} \sigma + \Lambda \sigma_2 \ddot{\delta} \sigma + \Lambda \sigma_1 \dot{\delta} \sigma + \Lambda \sigma_0 \delta \sigma + \Lambda \sigma t t + \Lambda \sigma = 0.
\end{align*}
\]

(13)

One possible way how we can obtain the separated equation for \(\delta a\) in Eqs. (13) is to calculate the following time derivatives of the first equation in Eqs. (12).

\[
\begin{align*}
\dot{\delta} a &= A \delta a + B \delta e + C \delta \varphi + D \delta \sigma + E t + F , \\
\ddot{\delta} a &= \alpha_2 \delta a + \beta_2 \delta e + \gamma_2 \delta \varphi + \delta_2 \delta \sigma + \epsilon_2 t + \zeta_2 , \\
\dddot{\delta} a &= \alpha_3 \delta a + \beta_3 \delta e + \gamma_3 \delta \varphi + \delta_3 \delta \sigma + \epsilon_3 t + \zeta_3 , \\
\dddot{\delta} a &= \alpha_4 \delta a + \beta_4 \delta e + \gamma_4 \delta \varphi + \delta_4 \delta \sigma + \epsilon_4 t + \zeta_4 .
\end{align*}
\]

(14)

here \(\alpha_1, \beta_1, \gamma_1, \delta_1, \epsilon_1, \) and \(\zeta_1\) for \(l = 2, 3, 4\) are determined by the constants in Eqs. (12). We can substitute Eqs. (14) in the first equation in Eqs. (13). Now, when we realise that the first equation in Eqs. (13) should be valid for arbitrary deltas, we obtain the following system of equations

\[
\begin{align*}
\alpha_4 + \Lambda a_3 \alpha_3 + \Lambda a_2 \alpha_2 + \Lambda a_1 A + \Lambda a_0 &= 0, \\
\beta_4 + \Lambda a_3 \beta_3 + \Lambda a_2 \beta_2 + \Lambda a_1 B &= 0, \\
\gamma_4 + \Lambda a_3 \gamma_3 + \Lambda a_2 \gamma_2 + \Lambda a_1 C &= 0, \\
\delta_4 + \Lambda a_3 \delta_3 + \Lambda a_2 \delta_2 + \Lambda a_1 D &= 0, \\
\epsilon_4 + \Lambda a_3 \epsilon_3 + \Lambda a_2 \epsilon_2 + \Lambda a_1 E + \Lambda a t &= 0, \\
\zeta_4 + \Lambda a_3 \zeta_3 + \Lambda a_2 \zeta_2 + \Lambda a_1 F + \Lambda a &= 0.
\end{align*}
\]

(15)

The solution of Eqs. (15) gives unknown Deltas in the separated equation for \(\delta a\). In the used notation we obtain for all separated equations in Eqs. (13)

\[
\Lambda_3 = \Lambda a_3 = \Lambda e_3 = \Lambda \varphi_3 = \Lambda \sigma_3 = - A - H - O - V ,
\]

(16)

\[
\begin{align*}
\Lambda_2 &= \Lambda a_2 = \Lambda e_2 = \Lambda \varphi_2 = \Lambda \sigma_2 = AH + AO + AV + HO \\
&\quad + HV + OV - BG - CM \\
&\quad - DS - IN - JT - PU ,
\end{align*}
\]

(17)

\[
\begin{align*}
\Lambda_1 = \Lambda a_1 = \Lambda e_1 = \Lambda \varphi_1 = \Lambda \sigma_1 = - A B C &\quad - A B D \\
G H I &\quad - G H J \\
M N O &\quad - S T V.
\end{align*}
\]
We can simplify the equations above using properties of the mean motion resonances in the PCR3BP. Each of the equations in Eqs. (7) can be written as a sum of two terms

\[
\frac{d\Psi}{dt} = F_1(R_G) + F_2(\tilde{\omega}) ,
\]

(22)
where $\Psi$ denotes one of the variables ($a$, $e$, $\tilde{\omega}$, $\sigma$), $F_1$ is produced by the gravitational forces in the considered system and $F_2$ is caused by the non-gravitational effects. For the non-gravitational effects with a rotational symmetry around the star $F_2(\tilde{\omega})$ is a constant. For partial derivatives of Eq. (22) with respect to $\tilde{\omega}$ and $\sigma$ we obtain

$$
\frac{\partial}{\partial \tilde{\omega}} \frac{d\Psi}{dt} = \frac{\partial F_1}{\partial \sigma} \frac{\partial}{\partial \sigma} \frac{\partial}{\partial \tilde{\omega}} + \frac{\partial F_2}{\partial \tilde{\omega}} \frac{\partial}{\partial \tilde{\omega}} \frac{\partial}{\partial \sigma},
$$

(23)

From Eq. (3) we have

$$
\frac{\partial \sigma}{\partial \tilde{\omega}} = \frac{-p + q}{q}, \quad \frac{\partial \tilde{\omega}}{\partial \sigma} = -\frac{q}{p + q}.
$$

(24)

Substitution of Eqs. (24) in Eqs. (23) leads to the relation

$$
\frac{\partial}{\partial \tilde{\omega}} \frac{d\Psi}{dt} = \frac{p + q}{q} \frac{\partial}{\partial \sigma} \frac{d\Psi}{dt}.
$$

(25)

This relation implies

$$
C = -\frac{p + q}{q} D,
$$

$$
I = -\frac{p + q}{q} J,
$$

$$
O = -\frac{p + q}{q} P,
$$

$$
U = -\frac{p + q}{q} V.
$$

(26)

Therefore, determinants for $\Lambda_0$, $\Lambda_{at}$ and $\Lambda_{et}$ have linear proportionality between columns. In this case properties of determinants reduce corresponding equations in Eqs. (19)-(20) to

$$
\Lambda_0 = \Lambda_{at} = \Lambda_{et} = 0.
$$

(27)

General solution of Eqs. (13) with substituted $\Lambda_0 = 0$ has form

$$
\delta_\psi = \frac{A_{\psi_1}}{\lambda_1} e^{\lambda_1 t} + \frac{A_{\psi_2}}{\lambda_2} e^{\lambda_2 t} + \frac{A_{\psi_3}}{\lambda_3} e^{\lambda_3 t} - \frac{\Lambda_{\psi t}}{2\Lambda_1} t^2 + \frac{\Lambda_2 \Lambda_{\psi t} - \Lambda_1 \Lambda_{\psi}}{\Lambda_1^2} t + B_\psi.
$$

(28)

Here, $A_{\psi_i}$ are complex constants, $B_\psi$ are real constant numbers (as we will see later), and $\lambda_i$, $i = 1, 2, 3$ are all roots of a cubic equation with real coefficients

$$
\lambda^3 + \Lambda_3 \lambda^2 + \Lambda_2 \lambda + \Lambda_1 = 0.
$$

(29)

The roots of any cubic equation with real coefficients are always three real numbers or one real number and two complex numbers which are complex conjugate to each other. The roots of Eq. (29) can be written in the following form

$$
\lambda = \sqrt[3]{u} + \sqrt[3]{v} - \frac{\Lambda_2}{3},
$$

(30)
where $u$ and $v$ are the roots of quadratic equation
\[
y^2 + \left( \frac{2\Lambda_3^3}{27} - \frac{\Lambda_3\Lambda_2}{3} + \Lambda_1 \right) y - \frac{1}{27} \left( \Lambda_2 - \frac{\Lambda_3^2}{3} \right)^3 = 0 .
\] (31)

Eqs. (27) and (28) imply that the semimajor axis ($\delta_a$) and the eccentricity ($\delta_e$) cannot have terms varying quadratically with the time for this linearized system. The condition that the particle is in a mean motion resonance implies that the semimajor axis cannot have also a term linearly proportional to the time. Hence

\[
\Lambda_a = - \begin{vmatrix} B & C & E & \xi_1 & B & D & E \\ H & I & K & \xi_2 & H & J & K \\ N & O & Q & \xi_3 & T & V & W \end{vmatrix} = 0 ,
\] (32)

where also Eqs. (26) were used. Any dependence between rows of the determinants in Eq. (32) is physically worthless. The dependence between columns in the determinants suggests that Eq. (32) is satisfied with identities

\[
E = \xi_1 B + \xi_2 C + \xi_3 D ,
K = \xi_1 H + \xi_2 I + \xi_3 J ,
Q = \xi_1 N + \xi_2 O + \xi_3 P ,
W = \xi_1 T + \xi_2 U + \xi_3 V ,
\] (33)

where $\xi_i$ are real numbers. Adding of the column proportional to the partial derivatives with respect to semimajor axis ($A, G, M, S$) on the right-hand side of Eqs. (33) would give $\Lambda_a \neq 0$ in Eq. (32). In this case the semimajor axis would have the term varying linearly with the time. Hence, Eqs. (33) without these terms are consistent with the resonant behavior of the evolution of the semimajor axis. Eqs (26) and (33) cause that also the longitude of pericenter and the resonant angular variable cannot have terms varying quadratically with time (Eqs. (20)). We come to conclusion that for the sought for solution of the linearized system must hold

\[
\Lambda_{at} = \Lambda_{et} = \Lambda_{\tilde{\omega}t} = \Lambda_{\sigma t} = 0 .
\] (34)

Remaining equations in Eqs. (21) are simplified with substitutions of Eqs. (26) and (33) into the following form

\[
\Lambda_e = \xi_1 \begin{vmatrix} A & C & B & \xi_1 & A & D & B \\ G & I & H & & G & J & H \\ M & O & N & & S & V & T \end{vmatrix} ,
\] (35)

\[
\Lambda_{\tilde{\omega}} = \begin{vmatrix} A & B & F & D & A & B & C & A & B & D \\ G & H & L & J & G & H & I & G & H & J \\ M & N & R & P & M & N & O & M & N & P \\ S & T & X & V & S & T & X & V & S & T & X \end{vmatrix} - \xi_2 \begin{vmatrix} A & B & C & \xi_2 & A & B & D & A & B & D \\ G & H & I & & G & H & I & G & H & I \\ M & N & O & & M & N & O & M & N & O \\ S & T & X & V & S & T & X & V & S & T & X \end{vmatrix} + \xi_1 \begin{vmatrix} A & D & B & A & D & B \end{vmatrix} ,
\] (36)
Our solution of the linearized system allows the eccentricity, the longitude of pericenter and the resonant angular variable have the term varying linearly with time. However, after substitution of the solution (Eq. 28) in Eqs. (12) all terms varying linearly with time on the right-hand sides must cancel each other otherwise we would obtain on the left-hand sides of Eqs. (12) terms varying linearly with time. Thus, we can write

\[
\Lambda_\sigma = \begin{vmatrix} A & B & C & F \\ G & H & I & L \\ M & N & O & R \\ S & T & U & X \end{vmatrix} - \xi_2 \begin{vmatrix} A & B & C \\ G & H & I \\ M & N & O \\ S & T & U \end{vmatrix} - \xi_3 \begin{vmatrix} A & B & D \\ G & H & J \\ M & O & N \\ S & T & V \end{vmatrix} - \xi_1 \begin{vmatrix} A & C & B \\ G & H & I \\ M & O & N \\ S & U & T \end{vmatrix}. \tag{37}
\]

Eqs. (33) and Eqs. (38) will be satisfied when

\[
\xi_1 = \frac{\Lambda_e}{\Lambda_1}, \quad \xi_2 = \frac{\Lambda_\omega}{\Lambda_1}, \quad \xi_3 = \frac{\Lambda_\sigma}{\Lambda_1}. \tag{39}
\]

If we use Eqs. (26) in Eq. (18), then we obtain for \( \Lambda_1 \)

\[
\Lambda_1 = -\begin{vmatrix} A & B & C \\ G & H & I \\ M & N & O \\ S & T & U \end{vmatrix} - \begin{vmatrix} A & B & D \\ G & H & J \\ M & O & N \\ S & T & V \end{vmatrix}. \tag{40}
\]

A comparison of Eq. (35) with Eq. (40) yields that the first equation in Eqs. (39) is trivially satisfied for all \( \xi_1 \). The second equation in Eqs. (39) gives

\[
\xi_2 = \frac{1}{\Lambda_1} \left( \frac{q}{p + q} \begin{vmatrix} A & B & C & F \\ G & H & I & L \\ M & N & O & R \\ S & T & U & X \end{vmatrix} - \xi_2 \begin{vmatrix} A & B & C \\ G & H & I \\ M & O & N \\ S & U & T \end{vmatrix} - \xi_3 \begin{vmatrix} A & B & D \\ G & H & J \\ M & O & N \\ S & T & V \end{vmatrix} + \xi_1 \begin{vmatrix} A & D & B \\ M & P & N \\ S & V & T \end{vmatrix} \right), \tag{41}
\]

where also Eqs. (26) were used. The last equation is equivalent with

\[
\begin{vmatrix} A & B & C \\ G & H & I \\ M & N & O \\ S & T & U \end{vmatrix} - \xi_1 \begin{vmatrix} A & C & B \\ M & O & N \\ S & U & T \end{vmatrix} - \xi_2 \begin{vmatrix} A & B & C \\ G & H & I \\ M & O & N \\ S & T & U \end{vmatrix} + \xi_3 \begin{vmatrix} A & B & C \\ G & H & I \\ M & O & N \end{vmatrix} = 0. \tag{42}
\]
If we use Eq. (37) in Eq. (42), then we obtain

\[
\Lambda_\sigma = -\xi_3 \begin{vmatrix} A & B & C \\ G & H & I \\ M & N & O \end{vmatrix} - \xi_3 \begin{vmatrix} A & B & D \\ G & H & J \\ S & T & V \end{vmatrix} = \xi_3 \Lambda_1 .
\] (43)

In other words, if \( \xi_2 = \Lambda_\varpi_1 / \Lambda_1 \), then \( \xi_3 = \Lambda_\sigma / \Lambda_1 \). Hence, the obtained solution of the linearized system is self consistent (see Eqs. (39)). We have obtained three free parameters \( \xi_1, \xi_2 \) and \( \xi_2 \). These parameters determine unknown partial derivatives with respect to time in Eqs. (11). Substituting Eqs. (38) in Eqs. (12) one obtains

\[
\frac{da}{dt} = A a + B \left( e + \frac{\Lambda_\nu}{\Lambda_1} t \right) + C \left( \dot{\varpi} + \frac{\Lambda_\varpi}{\Lambda_1} t \right) + D \left( \sigma + \frac{\Lambda_\sigma}{\Lambda_1} t \right) + F^* ,
\]

\[
\frac{de}{dt} = G a + H \left( e + \frac{\Lambda_\nu}{\Lambda_1} t \right) + I \left( \dot{\varpi} + \frac{\Lambda_\varpi}{\Lambda_1} t \right) + J \left( \sigma + \frac{\Lambda_\sigma}{\Lambda_1} t \right) + L^* ,
\]

\[
\frac{d\varpi}{dt} = M a + N \left( e + \frac{\Lambda_\nu}{\Lambda_1} t \right) + O \left( \dot{\varpi} + \frac{\Lambda_\varpi}{\Lambda_1} t \right) + P \left( \sigma + \frac{\Lambda_\sigma}{\Lambda_1} t \right) + R^* ,
\]

\[
\frac{d\sigma}{dt} = S a + T \left( e + \frac{\Lambda_\nu}{\Lambda_1} t \right) + U \left( \dot{\varpi} + \frac{\Lambda_\varpi}{\Lambda_1} t \right) + V \left( \sigma + \frac{\Lambda_\sigma}{\Lambda_1} t \right) + X^* .
\] (44)

This suggests validity of the following equations

\[
\frac{\partial}{\partial t} \left( \frac{d\Psi}{dt} \right) = \frac{\partial}{\partial e} \left( \frac{d\Psi}{dt} \right) \frac{\partial}{\partial t} \left( \frac{\Lambda_\nu}{\Lambda_1} t \right) + \frac{\partial}{\partial \varpi} \left( \frac{d\Psi}{dt} \right) \frac{\partial}{\partial t} \left( \frac{\Lambda_\varpi}{\Lambda_1} t \right) + \frac{\partial}{\partial \sigma} \left( \frac{d\Psi}{dt} \right) \frac{\partial}{\partial t} \left( \frac{\Lambda_\sigma}{\Lambda_1} t \right) .
\] (45)

Said another way, the direct time dependence always appears at some of the variables \( e, \varpi \) and \( \sigma \). We can speculate if this property belongs only to our linearized system or it is a heritage of full solution of the system given by Eqs. (7). For completeness, Eqs. (45) are equivalent with Eqs. (38). The partial derivatives with respect to time in Eqs. (11) are unknown and \( \xi_i \) cannot be determined (see also Eqs. (21)). We can guess \( \xi_i \) values to be negatively taken, over a libration period averaged, slope of the evolution of the eccentricity, the longitude of pericenter and the resonant angular variable. If this assumption is usable, we obtain oscillations of a variable modulated on a line that has the slope equal to the time derivative of the variable averaged over the libration period. If there is rapid advance in the longitude of pericenter during a capture of the dust particle in a mean motion resonance in the PCR3BP with the non-gravitational effects, then the most natural choice seems

\[
\frac{\Lambda_\nu}{\Lambda_1} = 0 , \quad \frac{\Lambda_\varpi}{\Lambda_1} = -\int_0^{T_L} \frac{d\varpi}{dt} \, dt , \quad \frac{\Lambda_\sigma}{\Lambda_1} = 0 .
\] (46)

We have adjusted the properties of the sought for solution according to the properties of the evolutions of dust particles in the mean motion resonances with the influence from the non-gravitational effects taken into account. After omitting the zero terms remaining terms in the solved system are (compare to Eq. (13))

\[
\ddot{\delta}_a + \Lambda_3 \dot{\delta}_a + \Lambda_2 \delta_a + \Lambda_1 \delta_a = 0 ,
\]
The sought for solution of the system given by Eqs. (47) has the form
\[ \ddot{\delta}_e + A_3 \dot{\delta}_e + A_2 \ddot{\delta}_e + \Lambda_1 \dot{\delta}_e + \Lambda_e = 0 , \]
\[ \ddot{\delta}_\omega + A_3 \dot{\delta}_\omega + A_2 \ddot{\delta}_\omega + \Lambda_1 \dot{\delta}_\omega + \Lambda_\omega = 0 , \]
\[ \ddot{\delta}_\sigma + A_3 \dot{\delta}_\sigma + A_2 \ddot{\delta}_\sigma + \Lambda_1 \dot{\delta}_\sigma + \Lambda_\sigma = 0 . \] (47)

The sought solution of the system given by Eqs. (47) has the form
\[
\begin{align*}
\delta_a &= \frac{A_{a1}}{\lambda_1} e^{\lambda_1 t} + \frac{A_{a2}}{\lambda_2} e^{\lambda_2 t} + \frac{A_{a3}}{\lambda_3} e^{\lambda_3 t} + B_a , \\
\delta_e &= \frac{A_{e1}}{\lambda_1} e^{\lambda_1 t} + \frac{A_{e2}}{\lambda_2} e^{\lambda_2 t} + \frac{A_{e3}}{\lambda_3} e^{\lambda_3 t} - \frac{\Lambda_e}{\Lambda_1} t + B_e , \\
\delta_\omega &= \frac{A_{\omega1}}{\lambda_1} e^{\lambda_1 t} + \frac{A_{\omega2}}{\lambda_2} e^{\lambda_2 t} + \frac{A_{\omega3}}{\lambda_3} e^{\lambda_3 t} - \frac{\Lambda_\omega}{\Lambda_1} t + B_\omega , \\
\delta_\sigma &= \frac{A_{\sigma1}}{\lambda_1} e^{\lambda_1 t} + \frac{A_{\sigma2}}{\lambda_2} e^{\lambda_2 t} + \frac{A_{\sigma3}}{\lambda_3} e^{\lambda_3 t} - \frac{\Lambda_\sigma}{\Lambda_1} t + B_\sigma .
\end{align*}
\] (48)

In the next step we determine the complex constants \( A_{ai} , A_{ei} , A_{\omega i} \) and \( A_{\sigma i} \) as well as \( B_a , B_e , B_\omega \) and \( B_\sigma \) from initial conditions. We have assumed that in time zero \( \delta_a(0) = \delta_e(0) = \delta_\omega(0) = \delta_\sigma(0) = 0 . \) Therefore
\[
\begin{align*}
\delta_a(0) &= \frac{A_{a1}}{\lambda_1} + \frac{A_{a2}}{\lambda_2} + \frac{A_{a3}}{\lambda_3} + B_a = 0 , \\
\delta_e(0) &= \frac{A_{e1}}{\lambda_1} + \frac{A_{e2}}{\lambda_2} + \frac{A_{e3}}{\lambda_3} + B_e = 0 , \\
\delta_\omega(0) &= \frac{A_{\omega1}}{\lambda_1} + \frac{A_{\omega2}}{\lambda_2} + \frac{A_{\omega3}}{\lambda_3} + B_\omega = 0 , \\
\delta_\sigma(0) &= \frac{A_{\sigma1}}{\lambda_1} + \frac{A_{\sigma2}}{\lambda_2} + \frac{A_{\sigma3}}{\lambda_3} + B_\sigma = 0 .
\end{align*}
\] (49)

Equalling of Eqs. (12) and the differentiation of Eqs. (48) with respect to time in the time zero gives
\[
\begin{align*}
\dot{\delta}_a(0) &= A_{a1} + A_{a2} + A_{a3} = F , \\
\dot{\delta}_e(0) &= A_{e1} + A_{e2} + A_{e3} - \frac{\Lambda_e}{\Lambda_1} = L , \\
\dot{\delta}_\omega(0) &= A_{\omega1} + A_{\omega2} + A_{\omega3} - \frac{\Lambda_\omega}{\Lambda_1} = R , \\
\dot{\delta}_\sigma(0) &= A_{\sigma1} + A_{\sigma2} + A_{\sigma3} - \frac{\Lambda_\sigma}{\Lambda_1} = X .
\end{align*}
\] (50)

Equalling of the differentiation of Eqs. (12) with respect to time and the second differentiation of Eqs. (48) with respect to time in the time zero gives
\[
\begin{align*}
\ddot{\delta}_a(0) &= A_{a1}\lambda_1 + A_{a2}\lambda_2 + A_{a3}\lambda_3 = AF + BL + CR + DX + E , \\
\ddot{\delta}_e(0) &= A_{e1}\lambda_1 + A_{e2}\lambda_2 + A_{e3}\lambda_3 = GF + HL + IR + JX + K , \\
\ddot{\delta}_\omega(0) &= A_{\omega1}\lambda_1 + A_{\omega2}\lambda_2 + A_{\omega3}\lambda_3 = MF + NL + OR + PX + Q .
\end{align*}
\]
\[ \ddot{\delta}_\sigma(0) = A_{\sigma_1}\lambda_1 + A_{\sigma_2}\lambda_2 + A_{\sigma_3}\lambda_3 = SF + TL + UR + VX + W. \]  

Finally, equalling of the second differentiation of Eqs. (12) with respect to time and the third differentiation of Eqs. (48) with respect to time in the time zero yields

\[ \ddot{\delta}_i(0) = A_{ai}\lambda_i^2 + A_{ai}\lambda_i^2 + A_{ai}\lambda_i^2 = A (AF + BL + CR + DX + E) \]

\[ + B (GF + HL + IR + JX + K) \]

\[ + C (MF + NL + OR + PX + Q) \]

\[ + D (SF + TL + UR + VX + W), \]

\[ \ddot{\delta}_e(0) = A_{ei}\lambda_e^2 + A_{ei}\lambda_e^2 + A_{ei}\lambda_e^2 = G (AF + BL + CR + DX + E) \]

\[ + H (GF + HL + IR + JX + K) \]

\[ + I (MF + NL + OR + PX + Q) \]

\[ + J (SF + TL + UR + VX + W), \]

\[ \ddot{\delta}_i(0) = A_{\dot{\gamma}}\lambda_{\dot{\gamma}}^2 + A_{\dot{\gamma}}\lambda_{\dot{\gamma}}^2 + A_{\dot{\gamma}}\lambda_{\dot{\gamma}}^2 = M (AF + BL + CR + DX + E) \]

\[ + N (GF + HL + IR + JX + K) \]

\[ + O (MF + NL + OR + PX + Q) \]

\[ + P (SF + TL + UR + VX + W), \]

\[ \ddot{\sigma}_i(0) = A_{\sigma i}\lambda_{\sigma i}^2 + A_{\sigma i}\lambda_{\sigma i}^2 + A_{\sigma i}\lambda_{\sigma i}^2 = S (AF + BL + CR + DX + E) \]

\[ + T (GF + HL + IR + JX + K) \]

\[ + U (MF + NL + OR + PX + Q) \]

\[ + V (SF + TL + UR + VX + W). \]  

From equation Eqs. (50)-(52) \( A_{ai}, A_{ei}, A_{\dot{\gamma}i} \) and \( A_{\sigma i} \) can be obtained. The obtained equations are

\[ A_{\psi 1} = \frac{\ddot{\psi}(0) - \ddot{\psi}(0) (\lambda_2 + \lambda_3) + \left( \ddot{\psi}(0) + \frac{\dot{\psi}}{\lambda_1} \right) \lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}, \]

\[ A_{\psi 2} = \frac{\ddot{\psi}(0) - \ddot{\psi}(0) (\lambda_1 + \lambda_3) + \left( \ddot{\psi}(0) + \frac{\dot{\psi}}{\lambda_2} \right) \lambda_1 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)}, \]

\[ A_{\psi 3} = \frac{\ddot{\psi}(0) - \ddot{\psi}(0) (\lambda_1 + \lambda_2) + \left( \ddot{\psi}(0) + \frac{\dot{\psi}}{\lambda_3} \right) \lambda_1 \lambda_2}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}. \]  

\( A_{\psi i} \) in Eqs. (53) are related to \( \lambda_i \) in such a way that if \( \lambda_1 \) and \( \lambda_2 \) are complex conjugate to each other and \( \lambda_3 \) is a real number, then also \( A_{\psi 1} \) and \( A_{\psi 2} \) are complex conjugate to each other and \( A_{\psi 3} \) is a real number. This property holds for any permutation of not equal indexes \( i \). From Eqs. (49) we can obtain \( B_{\alpha}, B_{\dot{\gamma}}, B_{\dot{\gamma}} \) and \( B_{\sigma} \). Eqs. (49) gives

\[ B_{\psi} = \frac{\ddot{\psi}(0) - \ddot{\psi}(0) (\lambda_1 + \lambda_2 + \lambda_3) + \left( \ddot{\psi}(0) + \frac{\dot{\psi}}{\lambda_1} \right) (\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3)}{\lambda_1 \lambda_2 \lambda_3}. \]

\( B_{\psi} \) are always real numbers.
4. Stellar radiation and its influence on orbital evolution of cosmic dust

In astrophysical modelling of the orbital evolution of dust grains it is often assumed that a spherical particle can be used as an approximation to real, the arbitrarily shaped, particle. Influence of electromagnetic radiation on the motion of a homogeneous spherical dust particle can be described using the Poynting-Robertson (PR) effect ([Poynting]1904 | [Robertson]1937 | [Klačka]2004 | [Klačka et al.]2014). The acceleration of the dust particle caused by the PR effect in a reference frame associated with the source of radiation (star) is

\[
\frac{d\vec{v}}{dt} = \beta \frac{\mu}{r^2} \left[ \left( 1 - \frac{\vec{v} \cdot \vec{e}_R}{c} \right) \vec{e}_R - \frac{\vec{v}}{c} \right],
\]

where \(r\) is the radial distance between the star and the dust particle, \(\vec{e}_R\) is the unit vector directed from the star to the particle, \(\vec{v}\) the velocity of the particle with respect to the star and \(c\) is the speed of light in vacuum. For the parameter \(\beta\) we have from its definition after Eqs. (1)

\[
\beta = \frac{3L_* Q'_{pr}}{16 \pi c \mu R_d \varrho},
\]

where \(L_*\) is the stellar luminosity, \(Q'_{pr}\) is the dimensionless efficiency factor for the radiation pressure averaged over the stellar spectrum and calculated for the radial direction (\(Q'_{pr} = 1\) for a perfectly absorbing sphere), and \(R_d\) is the radius of the dust particle with mass density \(\varrho\). Expanding solar corona is continuous flux of solar wind inside the heliosphere. Stellar winds were also observed in the vicinity of other solar-like stars ([Wood et al.]2002 | 2005). The motion of the dust particles orbiting such a star can be affected by the stellar wind. It is possible to derive the acceleration caused by wind corpuscles impinging on the dust particle using the relativity theory ([Klačka & Saniga]1993 | [Klačka et al.]2012). A radial stellar wind produces the following acceleration of the dust particle in accuracy to first order in \(v/c\) (\(v\) is the speed of the dust particle with respect to the star), first order in \(u/c\) (\(u\) is the speed of the stellar wind with respect to the star) and first order in \(v/u\)

\[
\frac{d\vec{v}}{dt} = \frac{\eta}{Q'_{pr}} \frac{u}{c} \frac{\mu}{r^2} \left[ \left( 1 - \frac{\vec{v} \cdot \vec{e}_R}{u} \right) \vec{e}_R - \frac{\vec{v}}{u} \right].
\]

\(\eta\) is to the given accuracy the ratio of the stellar wind energy to the stellar electromagnetic radiation energy, both radiated per unit time

\[
\eta = \frac{4 \pi R^2 \mu}{L_*} \sum_{j=1}^{N} n_{swj} m_{swj} \varrho c^2,
\]

where \(m_{swj}\) and \(n_{swj}\), \(j = 1 \to N\), are the masses and concentrations of the stellar wind particles at a distance \(r\) from the star (\(u = 450\) km/s and \(\eta = 0.38\) for the Sun, [Klačka et al.]2012). We are interested in the motion of a dust particle in the frame of reference associated with the star. When we add the gravitational accelerations from the star and the planet, we obtain the final equation of
motion of the dust grain in the PCR3BP with the electromagnetic radiation and the stellar wind in the reference frame associated with the star

\[
\frac{d\vec{v}}{dt} = -\frac{\mu}{r^2} (1 - \beta) \vec{e}_R - \frac{G_0 M_P}{|\vec{r} - \vec{r}_P|^3} (\vec{r} - \vec{r}_P) - \frac{G_0 M_P}{r_P^3} \vec{r}_P - \beta \frac{\mu}{r^2} \left(1 + \frac{\eta}{Q_{pr}}\right) \left(\vec{v} \cdot \vec{e}_R \vec{e}_R + \frac{\vec{v}}{c}\right). \quad (59)
\]

At summation of Eq. (55) and Eq. (57) in Eq. (59) is assumed that \((\eta/\bar{Q}_{pr}) (u/c) \ll 1\).

Using the last term in Eq. (59) as a perturbation of the orbital motion in a Keplerian potential \(-\mu (1 - \beta)/r\), we obtain from the Gaussian perturbation equations of celestial mechanics (e.g. Danby 1988) the following averaged values in Eqs. (7)

\[
\langle \frac{da}{dt}\rangle_{EF} = -\frac{\beta \mu}{ca^3} \left(1 + \frac{\eta}{Q_{pr}}\right) (2 + 3e^2),
\]

\[
\langle \frac{de}{dt}\rangle_{EF} = -\frac{\beta \mu}{2ca^2} \left(1 + \frac{\eta}{Q_{pr}}\right) 5e,
\]

\[
\langle \frac{d\tilde{\omega}}{dt}\rangle_{EF} = 0,
\]

\[
\langle \frac{d\sigma}{dt} + t \frac{dn}{dt}\rangle_{EF} = 0. \quad (60)
\]

The substitution of the partial derivatives of Eqs. (60) with respect to \(a, e, \tilde{\omega}\) and \(\sigma\) in Eqs. (11) determines corresponding coefficients in Eqs. (12). The coefficients are shown in Appendix A.

5. Numerical results

In this section we compare analytical and numerical results. Real resonant libration for a spherical dust particle can be obtained from numerical solution of Eq. (59) in any specific mean motion resonance. Equations in the system of equations given by Eqs. (7) are averaged over the synodic period. All parameters substituted in Eq. (A1) should be averaged over the synodic period. Therefore, we averaged all parameters obtained from the numerical solution of equation of motion (Eq. 59) over the synodic period. The averaged partial derivatives of the disturbing function \(R_G\) in Eq. (A1) can be calculated numerically. If we know the semimajor axis, eccentricity, longitude of pericenter and the resonant angular variable, then we can substitute these values in Eqs. (A1) in order to determine the coefficients in Eqs. (16)-(19). The calculation of \(\Lambda_3, \Lambda_2\) and \(\Lambda_1\) using Eqs. (16)-(18) gives \(\lambda_i\) as roots of the cubic equation Eq. (29). If all roots are real, then the linearization approximation does not hold well. In the mean motion resonance should be always the oscillations of the semimajor axis which maintain the ratio of periods near the ratio of two small natural numbers. Therefore, the only usable roots for the librational solution are one real number and two complex numbers which are complex conjugate to each other. The imaginary part of the two complex \(\lambda_i\) determines an angular frequency of the libration.
Fig. 1.— The resonant angular variables obtained from the resonant condition $da/dt = 0$ for nine exact exterior mean motion resonances in the PCR3BP with the PR effect and the radial stellar wind. Dust particles in the mean motion resonances with the Earth in a circular orbit around the Sun are considered. The resonant angular variables are calculated for the eccentricity of particle orbit equal to the universal eccentricity $e_{lim}$. The minimal sizes of the dust particles are determined by existences of valid solutions of the linearized system of equation given by Eqs. (12). The star in the third plot denotes the position of the evolution depicted in Figure 3.

It is possible to define an exact mean motion resonance. In the exact mean motion resonance averaged mean motion of the particle and mean motion of the planet have a ratio that is exactly equal to the ratio of two natural numbers. For the exact resonance in the PCR3BP with radiation is the semimajor axis equal to $a = a_p \left(1 - \beta\right)^{1/3} \left[M_\star/(M_\star + M_P)\right]^{1/3} \left[p/(p + q)\right]^{2/3}$. Periodicity and stability of evolutions in the exact resonances in the PCR3BP with radiation is discussed in Pastor.
An evolution of eccentricity of the particle’s orbit during a capture in an exterior resonance approaches the universal eccentricity in the PCR3BP with radiation (Beaugé & Ferraz-Mello 1994; Liou & Zook 1997). The universal eccentricities are given by

\[
1 - \frac{3e_{\text{lim}}^2 + 2}{2(1 - e_{\text{lim}}^2)^{3/2}} \frac{p + q}{p} = 0 .
\] 

\(\frac{2\pi}{T_f} \left[ \text{year}^{-1} \right] \)
Fig. 3.— A comparison of the results obtained from the numerical solution of the equation of motion (solid black curve) with predictions from the analytical theory (dashed red curve) for the evolutions of semimajor axis $a$, eccentricity $e$, longitude of pericenter $\tilde{\omega}$, resonant angular variable $\sigma$ and $kh$ point for a dust particle with $\beta = 0.025$ and $Q_{pr}' = 1$ captured in an exterior mean motion orbital 4/3 resonance with the Earth under the action of the PR effect and the radial solar wind. The initial eccentricity of the particle orbit is equal to the universal eccentricity $e_{\lim}$ (Eq. 61) and as a consequence the $kh$ point librates around an libration center.

Astrophysical problems with a rotational symmetry around the star lead to the secular time derivatives of orbital elements caused by the considered non-gravitational effects that do not depend on the longitude of pericenter. The stellar radiation with radial stellar wind also has the rotational
Fig. 4.— The same parameters in the same PCR3BP with radiation as in Figure 3 are compared. However, in this case the dust particle is captured in an exterior mean motion orbital 3/2 resonance. The black solid curve corresponds to the evolution obtained from the numerical solution of equation of motion. The dashed red curve corresponds to the librational solution given by Eqs. (48). The collisions in the \( kh \) plane occur on the dashed curves. The initial eccentricity of the particle orbit is smaller than the universal eccentricity and the eccentricity asymptotically increases to the universal eccentricity. In this case there is not libration of the \( kh \) point around an libration center.

Symmetry and \( \langle da/dt \rangle_{EF} \), in the first equation in Eqs. (7), does not depend on the longitude of pericenter \( \bar{\omega} \). \( \bar{\omega} \) has single occurrence in the averaged disturbing function \( R_G \) inside the resonant angular variable \( \sigma \). Therefore, from a resonant condition \( da/dt = 0 \) (substituted in the first equation
in Eqs. 7, we obtain for the stellar radiation a resonant $\sigma$ which is equal for all values of $\tilde{\omega}$.

Knowing of the resonant $\sigma$ is necessary if we want to determine the angular frequency of libration. We calculated the resonant $\sigma$ for nine exact exterior mean motion resonances at the universal eccentricities in the PCR3BP with radiation which comprises the Earth and the Sun as the two major bodies. The librational solution with complex $\lambda_\ast$ does not exist for all found $\sigma$. The maximal $\beta$ (smallest size of the dust particle) on the red curves in Figure 1 corresponds to the maximal $\beta$ for which the librational solution exists on these curves. This rule determines the top boundary for $\beta$ in Figure 1. The angular frequency of libration for the problem solved in Figure 1 are shown in Figure 2. For $\beta = 0$ (the PCR3BP) the 2/1, 3/1 and 4/1 exterior mean motion resonances in Figure 1 have the asymmetric libration centers (see Beaugé 1994; Beaugé & Ferraz-Mello 1994). The asymmetric libration centers are not located at $\sigma = 0$ or in the middle of the allowed interval for $\sigma$. In Figure 1, we can see how the red curves rapidly go to zero at the maximal $\beta$ after initial increasing at smaller values of $\beta$. The angular frequency for the resonant angular variable depicted with the green curves in Figure 1 always increases with increasing $\beta$. The angular frequency for the blue curves decreases with increasing $\beta$.

In Figure 3, are compared evolutions of four variables describing the secular orbital evolution of the dust particle in the mean motion resonance and evolutions of a $kh$ point for circular Sun-Earth-dust system with the solar electromagnetic radiation and the radial solar wind taken into account. Coordinates of the $kh$ point are the non-canonical variables $k = e \cos \sigma$ and $h = e \sin \sigma$ (Beaugé & Ferraz-Mello 1993; Sidlichovský & Nesvorný 1994; Pástor 2014). Collisions of the particle with the planet occur on dashed curves in the $kh$ plane. The dashed curves cannot be crossed by the $kh$ point during the evolution in a mean motion resonance. Properties of the used dust particle are $\beta = 0.025$ and $\bar{Q}'_{pr} = 1$. The particle is placed in an exact exterior mean motion orbital 4/3 resonance, initially. The initial eccentricity equal to the universal eccentricity $e_{\text{lim}}$ is used. The initial resonant variable is determined from the resonant condition $da/dt = 0$ (substituted in the first equation in Eqs. 7) for the exact resonance with the universal eccentricity. For the numerical solution of the equation of motion (Eq. 59), the initial positions of the planet and the particle are necessary. The planet was initially located on the positive $x$-axis and the dust particle was initially located in the pericentre. The initial positions of the planet and the particle together with the initial resonant angular variable determine the initial longitude of pericenter, according to Eq. (3), as $\tilde{\omega}_{\text{in}} = -q/(p+q)\sigma_{\text{in}}$. The black solid curve corresponds to the evolution obtained from the numerical solution of the equation of motion. The dashed red curve corresponds to the librational solution given by Eqs. (48). Since the longitude of pericentre is advancing we used the term linearly proportional to the time (Eqs. 48) only in the longitude of pericentre (Eqs. 46). As can be seen in Figure 3, the librational solution describes the evolution of all parameters sufficiently in this case.

Figure 4 shows a comparison of the numerical solution of the equation of motion with the analytical librational solution for the same dust particle under the action of the same forces as in Figure 3. However, the particle is captured in this case in an exterior mean motion orbital 3/2 resonance. The initial semimajor axis corresponds to the exact resonance. The initial eccentricity
of the particle orbit is 0.2. The initial resonant angular variable is 293.8°. The planet was initially located on the positive x-axis and the dust particle was initially located in the pericentre. Similarly to Figure 3 for the initial longitude of pericentre holds \( \tilde{\omega} = -\frac{q}{p + q} \sigma \). The initial eccentricity 0.2 for the motion solved in Figure 4 is not equal to the universal eccentricity and the evolution of eccentricity monotonically approaches the universal eccentricity, as can be seen in Figure 4. We have used the terms varying linearly with time for the eccentricity and the longitude of pericenter in this case. Again the librational solution well describes the evolutions of all parameters in Figure 4. The position of the \( kh \) point decreases more quickly in the analytical solution mainly due to differences in the evolutions of the eccentricity between the numerical and the analytical solution.

6. Discussion

Figures 3 and 4 depict cases when the linearization approximation holds well. Such cases are characterized with small variations in all evolving parameters \( a, e, \tilde{\omega} \) and \( \sigma \) averaged over the synodic period. There exist cases when the linearization approximation is not usable. The orbital evolutions in mean motion resonances in the PCR3BP with radiation have relatively easy characteristic properties. The semimajor axis always oscillates. The eccentricity has oscillations with small amplitude that are modulated on the asymptotic approach of the eccentricity to the universal eccentricity for the exterior resonances and for the interior resonances the oscillations are modulated on a monotonic decrease of the eccentricity to zero (Pástor et al. 2009; Pástor 2013). The longitude of pericenter usually varies monotonically (but not always, see Pástor, Kláčka & Kómar 2009). The resonant angular variable has oscillations with an amplitude smaller than \( \pi \) modulated on slower variations. We found that the amplitude of oscillations in the evolution of the resonant angular variable has largest influence on the applicability of the librational solution. The librational solution is not usable for large amplitude oscillations in the resonant angular variable. We have obtained the analytical solution only using an assumption for values of \( \xi \) that are unknown. If the partial derivatives with respect to time in Eqs. (11) could be exactly determined, then the accordance between the analytical and the numerical results should be better. The solution of the linearized system of equations (Eqs. 11) can be easily applied for the mean motion resonances without influence of the non-gravitational effects (i.e. in the PCR3BP) by setting corresponding partial derivatives to zero.

7. Conclusion

We have obtained a librational solution for a body with negligible mass captured in a mean motion resonance in the PCR3BP with non-gravitational effects included. The solution describes the evolutions of over a synodic period averaged semimajor axis, eccentricity, longitude of pericentre and resonant angular variable. According to averaged Lagrange’s planetary equations with the non-gravitational effects included the evolutions of these four parameters determine the orbital
evolution in the mean motion resonance in full. In order to obtain the solution we performed
a linearization of the averaged equations of motion around an initial point. In the linearization
partial derivatives of the averaged equations of motion with respect to time are unknown. They can
be considered analytically as known during the analytical calculation of the librational solution.
However, for a comparison with real results their values must be found. The calculated analytical
solution implicates that a linear proportionality to the time can be present in the evolutions of
the eccentricity, longitude of perihelion and resonant angular variable. A coefficient at the time
depends on the unknown partial derivatives with respect to time. We assumed that the value of
the coefficient can be approximated by over a libration period averaged slope of the eccentricity,
longitude of pericenter and resonant angular variable. The libration period can be determined
without knowing of the partial derivatives with respect to time. We compared the analytical and the
numerical results for the PR effect and the radial stellar wind used as the non-gravitational effects.
Agreement between the compared results was good. The librational solution can be used in the
cases when the librational amplitude is small i.e. in the cases when the linearization approximation
can be used. The comparison of analytical and numerical results for very small libration amplitudes
is difficult because the small libration can be lost in the numerical averaging process.

A. Constant coefficients

This appendix presents coefficients for linearized system of equations (Eqs. 12) describing the
orbital evolutions of the dust particles captured in the mean motion resonances in the PCR3BP
with radiation.

\[
A = -s \frac{\partial R_G}{\partial \sigma} - 2s \frac{a_0 \partial^2 R_G}{\partial a \partial \sigma} + \frac{\beta \mu}{ca_0^2 \alpha_0} \left( 1 + \frac{\eta}{Q'_p} \right) \left( 2 + 3e_0^2 \right),
\]

\[
B = -2s \frac{a_0 \partial^2 R_G}{\partial e \partial \sigma} - 3\frac{\beta \mu e_0}{ca_0^3 \alpha_0} \left( 1 + \frac{\eta}{Q'_p} \right) \left( 4 + e_0^2 \right),
\]

\[
C = \frac{p + q}{q} \frac{2sa_0 \partial^2 R_G}{\partial \sigma^2},
\]

\[
D = -2s \frac{a_0 \partial^2 R_G}{\partial \sigma^2},
\]

\[
E = \frac{\Lambda_e}{\Lambda_1} B + \frac{\Lambda_o}{\Lambda_1} C + \frac{\Lambda_s}{\Lambda_1} D,
\]

\[
F = -2s \frac{a_0 \partial R_G}{\partial \sigma} \left( 1 + \frac{\eta}{Q'_p} \right) \left( 2 + 3e_0^2 \right),
\]

\[
G = -\frac{\alpha_0}{2a_0 L_0 e_0} \left[ 1 + s \left( 1 - \alpha_0 \right) \right] \frac{\partial R_G}{\partial \sigma} + \frac{\alpha_0}{L_0 e_0} \left[ 1 + s \left( 1 - \alpha_0 \right) \right] \frac{\partial^2 R_G}{\partial a \partial \sigma} + \frac{\beta \mu}{ca_0^3 \alpha_0} \left( 1 + \frac{\eta}{Q'_p} \right) 5e_0,
\]

\[
H = -\frac{1}{L_0 e_0^5 \alpha_0} \left[ 1 + s \left( 1 - \alpha_0 \right) \right] \frac{\partial R_G}{\partial \sigma} + \frac{s}{L_0} \frac{\partial R_G}{\partial \sigma} + \frac{\alpha_0}{L_0 e_0} \left[ 1 + s \left( 1 - \alpha_0 \right) \right] \frac{\partial^2 R_G}{\partial e \partial \sigma} - \frac{5\beta \mu}{2ca_0^2 \alpha_0} \left( 1 + \frac{\eta}{Q'_p} \right),
\]
\[
I = - \frac{p + q}{q} \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial \sigma^2},
\]
\[
J = \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial \sigma^2},
\]
\[
K = \frac{\Lambda_\omega}{\Lambda_1} H + \frac{\Lambda_\omega}{\Lambda_1} I + \frac{\Lambda_\sigma}{\Lambda_1} J,
\]
\[
L = \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial \sigma} - \frac{5\beta \mu}{2c_0^2 \alpha_0} \left(1 + \frac{\eta}{Q_{pr}}\right) e_0,
\]
\[
M = - \frac{\alpha_0}{2a_0 L_1 e_0} \frac{\partial R_G}{\partial e} + \frac{\alpha_0}{L_0 e_0} \frac{\partial^2 R_G}{\partial a \partial e},
\]
\[
N = - \frac{1}{L_0 e_0} \frac{\partial R_G}{\partial e} + \frac{\alpha_0}{L_0 e_0} \frac{\partial^2 R_G}{\partial e^2},
\]
\[
O = - \frac{p + q}{q} \frac{\alpha_0}{L_0 e_0} \frac{\partial^2 R_G}{\partial \sigma \partial e},
\]
\[
P = \frac{\alpha_0}{L_0 e_0} \frac{\partial^2 R_G}{\partial \sigma \partial e},
\]
\[
Q = \frac{\Lambda_\omega}{\Lambda_1} N + \frac{\Lambda_\omega}{\Lambda_1} O + \frac{\Lambda_\sigma}{\Lambda_1} P,
\]
\[
R = \frac{\alpha_0}{L_0 e_0} \frac{\partial R_G}{\partial e},
\]
\[
S = \frac{\alpha_0}{2a_0 L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial e} - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial a \partial e} + \frac{s}{L_0} \frac{\partial R_G}{\partial a} + \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial a^2} + \frac{3sn_0}{2a_0},
\]
\[
T = \frac{1}{L_0 e_0} \frac{\partial R_G}{\partial e} - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial a \partial e} - \frac{s}{L_0} \frac{\partial R_G}{\partial a} + \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial a^2},
\]
\[
U = \frac{p + q}{q} \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial \sigma \partial e} - \frac{p + q}{q} \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial \sigma \partial a},
\]
\[
V = - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial^2 R_G}{\partial \sigma \partial e} + \frac{2sa_0}{L_0} \frac{\partial^2 R_G}{\partial \sigma \partial a},
\]
\[
W = \frac{\Lambda_\omega}{\Lambda_1} T + \frac{\Lambda_\omega}{\Lambda_1} U + \frac{\Lambda_\sigma}{\Lambda_1} V,
\]
\[
X = - \frac{\alpha_0}{L_0 e_0} [1 + s (1 - \alpha_0)] \frac{\partial R_G}{\partial e} + \frac{2sa_0}{L_0} \frac{\partial R_G}{\partial a} + \frac{p + q}{q} n_{pr} - sn_0. \tag{A1}
\]

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