Singleton physics*

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Abstract

We review the developments in the past twenty years (which are based on our deformation philosophy of physical theories) dealing with elementary particles composed of singletons in anti De Sitter space-time. The study starts with the kinematical aspects (especially for massless particles) and extends to the beginning of a field theory of composite elementary particles and its relations with conformal field theory (including very recent developments).

1 Introduction: Deformations and Singleton Physics

Physical theories have their domain of applicability mainly depending on the velocities and distances concerned. But the passage from one domain (of velocities and distances) to another one does not appear in an uncontrolled way. Rather, a new fundamental constant enters the modified formalism and the attached structures (symmetries, observables, states, etc.) deform\cite{Fl82, Fl98} the initial structure; namely, we have a new structure which in the limit when the new parameter goes to zero coincides with the old formalism. In other words, to detect new formalisms we have to study deformations of the algebraic structures attached to a given formalism.

The only question is in which category we perform this search for deformations. Usually physics is rather conservative and if we start e.g. with the category of associative or Lie algebras, we tend to deform in this category. This is the case of traditional quantization\cite{BFFLS, St98} (deforming classical mechanics to quantum mechanics by introducing a new parameter \(\hbar\), keeping the same algebra of observables but deforming their composition law).

*This review is dedicated to our friend of 32.5 years, Ludwig Faddeev, upon the occasion of his 65\textsuperscript{th} birthday. It was initiated by the first author; the frame and a first draft were discussed between all three authors and the final formulation was completed by the latter two after the sudden and untimely death of Moshé Flato. To be published in a special issue of Proceedings of Steklov Mathematical Institute.
The same is true of the passage from Galilean physics to special relativity (new parameter $c^{-1}$, where $c$ is the speed of light) and thence to physics in De Sitter space-time (the new parameter being the curvature). It is this last aspect which we shall present here.

In this paper we touch recent developments in field theories based on supergravity, conformal field theories, compactification of higher dimensional field theories, string theory, M-theory, p-branes, etc. for which people rediscovered the efficiency and advantages of anti De Sitter theories (which are stable deformations of Poincaré field theories in the category of Lie groups; see however [FHT] for quantum groups, at roots of unity in that case). There are many reasons for the advantages of anti De Sitter (often abbreviated as AdS) theories among which we can mention that AdS field theory admits an invariant natural infrared regularization of the theories in question and that the kinematical spectra (angular momentum and energy) are naturally discrete. But in addition AdS theories have a great bonus: the existence of singleton representations discovered by Dirac [Di63] for SO(2, 3), corresponding to a “square root” of AdS massless representations. We discovered that fact around 20 years ago [FF78, FF80] and developed rather extensively its physical consequences in the following years [AF78, AFFS, BFFI, BFFS, BFI, FaF, FF81, FF84, FF86, FF86f, FF87, FF88, FF89, FF91, FF98, FF86, FF86f, FF87, FF88, FF89, FF91, FF98, FF86, FF86f, FF87, FF88, FF89, FF91, FF98, FFG86, FFS88, FHT, Fr79, Fr82, Fr88, FH87, IFF].

Singleton theories are topological in the sense that the corresponding singleton field theories live naturally on the boundary at infinity of the De Sitter bulk (boundary which has one dimension less than the bulk). They are new types of gauge theories which in addition permit to consider massless particles, e.g. the photon, as dynamically composite AdS particles [FF88, FF98, FFS88]. Some of the beautiful properties of singleton theories can be extended to higher dimensions, and this is the main point of the recent huge interest in these AdS theories, which touched a large variety of aspects of AdS physics. More explicitly, in several of the recent articles among which we can mention [Ma97, Wi98, FeFr, FFZ, FeZ, FKPZ, FMMR, SS98], the new picture permits to study duality between CFT on the boundary at infinity and the corresponding AdS theory in the bulk. That duality, which has also interesting dynamical aspects in it, utilizes among other things the great notational simplifications permitted by singleton physics.

2 Kinematics: one massless particle = two Dirac singletons

In order to give a flavor of the basic features of the theory of singletons in the (2+3) anti-De Sitter space-time $\text{AdS}_4$ and their relation to massless particles, we shall in this section and in the following indicate some of these features, referring to the quoted literature for a more detailed presentation. The theory can be extended to other dimensions (higher or lower). In $\text{AdS}_3$ one gets essentially the same features [IFF]: the main difference being (as is well known) that the (2+2) De Sitter group SO(2, 2) is not the full (infinite-dimensional) conformal group of $1+1$ space-time (one has then to study Witt and Virasoro algebras; cf. e.g. [BH86, Mi99, It98]). In space-times of dimension $\geq 5$, some care is needed [AL98, La98] as to the definition of masslessness and of singletons (the very nice properties of dimension 4
are not all preserved, a fact sometimes overlooked in the recent literature); we shall not enter here into this discussion.

The maximal compact subalgebra of \( \mathfrak{so}(2,3) \) is \( \mathfrak{so}(2) \oplus \mathfrak{so}(3) \). We then have minimal weight (positive energy, which is one of the reasons for choosing AdS) unitary irreducible representations (UIRs) of a corresponding Lie group. In the following we consider mainly the twofold covering \( SO(2,3) \) of the connected component of the identity of \( SO(2,3) \), and denote by \( D(E_0, s) \) these minimal weight representations. Here \( E_0 \) is the minimal \( SO(2) \) eigenvalue and the half-integer \( s \) is the spin. These irreducible representations are unitary (belonging to the discrete series above the limit of unitarity) provided \( E_0 \geq s + 1 \) for \( s \geq 1 \) and \( E_0 \geq s + \frac{1}{2} \) for \( s = 0 \) and \( s = \frac{1}{2} \).

At the limit of unitarity (i.e. when \( 2E_0 \), which is an integer for \( SO(2,3) \) but can take any value for the universal covering, tends to the limit from above), the Harish Chandra module \( D(E_0, s) \) becomes indecomposable and the physical UIR appears as a quotient, a hallmark of gauge theories. For instance, for \( s \geq 1 \), we get in the limit an indecomposable representation denoted here by \( ID(s) \) or more explicitly by \( D(s + 1, s) \to D(s + 2, s - 1) \), a shorthand notation \([FSSS]\) for what mathematicians would write as a short exact sequence of modules \( 0 \to D(s + 1, s) \to ID(s) \to D(s + 2, s - 1) \to 0 \).

Now in gauge theories one needs extensions involving more than two UIRs. A typical situation is the case of flat space electromagnetism where one has the classical Gupta-Bleuler triplet which, in our shorthand notations, can be written \( Sc \to Ph \to Ga \). Here \( Sc \) (scalar modes) and \( Ga \) (gauge modes) are massless zero-helicity UIRs \( h(0,0) \) of the Poincaré (inhomogeneous Lorentz) group \( \mathcal{P}_{1+3} = SO(1,3) \cdot \mathbb{R}^4 \) while \( Ph \) is the module of physical modes, transforming under \( h(0,1) \oplus h(0,-1) \), where \( h(0,s) \) is the UIR of \( \mathcal{P}_{1+3} \) with mass 0 and helicity \( s \in \mathbb{Z} \). The scalar modes can be suppressed by a gauge fixing condition (e.g. the Lorentz condition) but then one is left with a nontrivial extension \( Ph \to Ga \) on the vector space \( Ph \oplus Ga \) which has no invariant nondegenerate metric and cannot be quantized covariantly. However the above Gupta-Bleuler triplet, a nontrivial successive extension \( Sc \to (Ph \to Ga) \), is an indecomposable representation on a space which admits an invariant nondegenerate (but indefinite) Hermitian form and it must be used in order to obtain a covariant quantization of this gauge theory. We shall meet here a similar situation, which in fact cannot be avoided. Indeed a general result \([ArS\S]\) says in particular that if an extension \( U^1 \to U^2 \), with \( U^2 \) a UIR, has a nondegenerate Hermitian form, then \( U^1 \) is equivalent to an extension \( U^3 \to U^2 \) for some representation \( U^3 \) and the original extension is in fact a triplet \( U^2 \to U^3 \to U^2 \).

The **massless representations** of \( SO(2,3) \) are defined (for \( s \geq \frac{1}{2} \)) as \( D(s + 1, s) \) and (for helicity zero) \( D(1,0) \oplus D(2,0) \). There are many justifications to this definition, among which we can mention \([AFFS]\):

a) The representations \( D(s + 1, s) \) contract smoothly, in a precise mathematical sense, to either one of the two massless representations \( h(0, \pm s) \) of \( \mathcal{P}_{1+3} \). Each of the latter has an operationally unique extension to a UIR of \( SO(2,4) \) (a 4-fold covering of the conformal group), the restriction of which to the \( SO(2,3) \) subgroup is exactly the representation we started with. Moreover each \( D(s + 1, s) \) can be extended (also...
The representations $D(E, j)$ can be realized as field theories on $\text{AdS}_4$ but, at the limit of unitarity $D(s + 1, s)$ with $s \geq 1$, they are accompanied by extensions. As a consequence we get a gauge theory, quantizable only by use of an indefinite metric and a Gupta-Bleuler triplet. For $D(s + 1, s)$ with $s \geq 0$, the physical signals propagate on the $\text{AdS}_4$ light cone $\text{FFFG86}$.

For $s = 0$ and $s = \frac{1}{2}$, the above mentioned gauge theory appears not at the level of the massless representations $D(1, 0) \oplus D(2, 0)$ and $D\left(\frac{3}{2}, \frac{1}{2}\right)$ but at the limit of unitarity, the singletons $\text{Rac} = D\left(\frac{1}{2}, 0\right)$ and $\text{Di} = D\left(1, \frac{1}{2}\right)$. These UIRs remain irreducible on the Lorentz subgroup $\text{SO}(1, 3)$ and on the $(1+2)$ dimensional Poincaré group $\mathcal{P}_{1+2}$, of which $\text{SO}(2, 3)$ is the conformal group. On $\mathcal{P}_{1+2}$ they give $\text{Bi82}$ the only massless (discrete helicity) representations. The singleton representations have a fundamental property:

$$
(\text{Di} \oplus \text{Rac}) \otimes (\text{Di} \oplus \text{Rac}) = (D(1, 0) \oplus D(2, 0)) \oplus 2 \bigoplus_{s=\frac{1}{2}}^{\infty} D(s + 1, s).
$$

Note that all the representations that appear in the decomposition are massless representations. Thus, in contradistinction with flat space, in $\text{AdS}_4$, massless states are “composed” of two singletons. The flat space limit of a singleton is a vacuum (a representation of $\mathcal{P}_{1+3}$ which is trivial on the translations) and, even in $\text{AdS}_4$, the singletons are very poor in states: their $(E, j)$ diagram has only a single trajectory (hence their name). The $(E, j)$ spectra of the massless and singleton representations is:

- $D(s + 1, s)$, $s > 0$: $E - j = 1, 2, \ldots; j - s = 0, 1, \ldots$
- $D(1, 0)$: $E - j = 1, 3, \ldots; j = 0, 1, \ldots$
- $D(2, 0)$: $E - j = 2, 4, \ldots; j = 0, 1, \ldots$
- $\text{Rac} = D\left(\frac{1}{2}, 0\right)$: $E - j = \frac{1}{2}$; $j = 0, 1, 2, \ldots$
- $\text{Di} = D\left(1, \frac{1}{2}\right)$: $E - j = \frac{1}{2}$; $j = \frac{1}{2}, \frac{3}{2}, \ldots$

In $\text{AdS}_3$, where $\mathfrak{so}(2, 3)$ is the conformal Lie algebra and the anti-De Sitter Lie algebra is $\mathfrak{so}(2, 2) \equiv \mathfrak{so}(1, 2) \oplus \mathfrak{so}(1, 2)$, the “physical” massless representations are $\text{Di}$ and $\text{Rac}$ and the analogue of singletons are the metaplectic representations $D\left(\frac{1}{2}\right)$ and $D\left(\frac{3}{2}\right)$ of $\mathfrak{so}(1, 2)$. The sum of the two latter is the harmonic oscillator representation and its tensor square is $\text{Di} \otimes \text{Rac}$, so we have an exact analogue of $\text{FFFG86}$. There is however a potentially important difference $\text{FF80}$: the $\text{AdS}_3$ algebra $\mathfrak{so}(2, 2)$ is no more the whole conformal algebra of the $1+1$ space time, since that algebra is well-known to be infinite dimensional. We shall not elaborate on this point here.]

In normal units a singleton with angular momentum $j$ has energy $E = (j + \frac{1}{2})\rho$, where $\rho$ is the curvature of the $\text{AdS}_4$ universe. This means that only a laboratory of cosmic dimensions can detect a $j$ large enough for $E$ to be measurable since the cosmological constant (of the order of $\rho$) is very small. At the flat space limit, the singletons become vacua (representations of $\mathcal{P}_{1+3}$ with vanishing energy and momentum) so that they carry no energy at all. Furthermore local observation of a free singleton field is prevented by
gauge invariance (we shall come back briefly to this point below). We thus have what can be called "kinematical confinement" of singletons \[FF80\], which suggests that they can be a viable alternative for quarks as fundamental constituents of matter. Elementary particles would then be composed of two, three or more singletons and/or anti singletons (the latter being associated with the contragredient representations). As with quarks, several (three so far) flavors of singletons (and anti singletons) should eventually be introduced to account for all elementary particles. In order to pursue this point further we need to develop a field theory of singletons and of particles composed of singletons.

3 Field Theory

In this section we shall give a very short overview of the many developments already achieved with singleton field theory and interactions of singletons. A first attempt to quantize the singleton field was based on the De Sitter covariant Klein-Gordon equation \((\Box - \frac{5}{4}\rho)\phi = 0\) where \(\rho = 3\Lambda\), \(\Lambda\) the cosmological constant. An appropriate choice of boundary conditions, \(\lim_{r \to \infty} r^\frac{5}{2}\phi \leq \infty\), leads to a space of solutions that carries the singleton representation \(D(\frac{1}{2},0)\) but not as an invariant subspace. Instead, \(D(\frac{1}{2},0)\) is induced on a quotient space of solutions, where the ignorable invariant subspace consists of the solutions that satisfy \(\lim_{r \to \infty} r^\frac{5}{2}\phi \to 0\). This is a difficulty even in the context of classical field theory, for it means that there is no invariant propagator that includes the contributions from the singleton modes. An invariant propagator does exist, but an examination of its asymptotic properties reveals that all its Fourier modes fall off as \(1/r^\frac{5}{2}\) at infinity; these modes constitute an invariant subspace on which the space time symmetry group acts by \(D(\frac{5}{2},0)\).

It is very significant that this representation on the ignorable "gauge" modes is of the ordinary massive type, while the singleton representation is highly degenerate. The energy levels of the former are degenerate and the spectrum of angular momentum is limited from above by the energy (in units of \(\rho\)). The energy levels of \(D(\frac{1}{2},0)\) are much more degenerate: \(l = E - \frac{1}{2}\). This suggests that the physical, singleton modes are swamped by the gauge modes and that any interaction designed to detect singletons will fail to be gauge invariant and hence non-unitary.

In the idiom of representation theory, the space of solutions of the equation \((\Box - \frac{5}{4}\rho)\phi = 0\) satisfying the boundary condition \(r^\frac{5}{2}\phi \leq \infty\) as \(r \to \infty\), carries the non-decomposable representation \(D(\frac{1}{2},0) \to D(\frac{5}{2},0)\). Quantization needs a non-degenerate, invariant symplectic structure. This requires the introduction of additional modes, canonically conjugate to the gauge modes (compare the situation in electrodynamics where Maxwell theory has no momentum conjugate to gauge modes), to give to the total space the symmetric form

\[
D(\frac{5}{2},0) \to D(\frac{1}{2},0) \to D(\frac{5}{2},0)
\]

(2) or " scalar \to transverse \to gauge". Initially, this was done by admitting logarithmic solutions of the Klein-Gordon equation above. Afterwards it was discovered that the dipole
equation \((\Box - \frac{5}{4} \phi) \, \phi^2 = 0\) with the same boundary conditions, provides a much more interesting solution to the problem.

It is remarkable that this particular instance of the dipole equation, in marked contrast with what is the case in flat space, and also in anti De Sitter space with any other value of the mass parameter, actually contains physical propagating modes. (In all the other cases the representation takes the form “scalar→gauge”, with no physical section in between.) What is even more remarkable is that this theory is a topological field theory; that is, the physical solutions manifest themselves only by their boundary values at \(r \to \infty\): \(\lim_{r \to \infty} \phi \) defines a field on the 3-dimensional boundary at infinity. There, on the boundary, gauge invariant interactions are possible and make a 3-dimensional conformal field theory. This is a 4-dimensional analogue of the 5-dimensional anti DeSitter/4-dimensional conformal field theory duality discovered recently by Maldacena [Ma97].

However, if massless fields (in 4 dimensions) are singleton composites, then singleton must come to life as four dimensional objects, and this requires the introduction of unconventional statistics. The requirement that the bilinears have the properties of ordinary (massless) bosons also tells us that the statistics of singletons must be of another sort.

The basic idea is [FF88, FFS88] that we can decompose the singleton field operator as \(\phi(x) = \sum_{-\infty}^{\infty} \phi^{j}(x) a_{j}\) in terms of positive energy creation operators \(a^{*j} = a_{-j}\) and annihilation operators \(a_{j}\) (with \(j > 0\)) without so far making any assumptions about their commutation relations. The choice of commutation relations comes later, when requiring that photons, considered as two-Rac fields (using the full tensor product of the two singleton triplets) be Bose-Einstein quanta. The singletons are then subject to unconventional statistics [FF88] (which is perfectly admissible since they are naturally confined) and an appropriate Fock space can be constructed. Based on these principles, a (conformally covariant) composite QED theory was constructed [FF88], with all the good features of the usual theory. In addition one can show [FF87] that the BRST structure of singleton gauge theory induces the BRST structure of the electromagnetic potential. Conformal covariance is based [BFH] on the indecomposable \(\mathfrak{so}(2,4)\) Gupta-Bleuler triplet \(D(1, \frac{1}{2}, \frac{1}{2}) \to [D(2,1,0) \oplus D(2,0,1) \oplus \text{Id}] \to D(1, \frac{1}{2}, \frac{1}{2})\) which gives by restriction two inequivalent De Sitter triplets \(D(3,0) \to D(2,1) \to D(3,0)\) and \(D(1,1) \to [D(2,1) \oplus \text{Id}] \to D(1,1)\), both of which appear in the direct product of \(D(\frac{1}{2}, \frac{1}{2}) \to D(\frac{1}{2}, 0) \to D(\frac{1}{2}, 0)\) by itself.

This procedure can be (and has in great part been) extended to the spinor singleton (the Di) and both Di and Rac can be combined to give a superfield formulation covariant under the superalgebra \(\mathfrak{osp}(4|1)\) [Fr88, FF98]. This will permit to include Yang-Mills fields, quantum gravity, supergravity and models of QCD, all based on singletons as fundamental constituents.

The latest contribution [FF98] to this interpretation of massless fields as singleton composites deals with gravitons, giving an explicit expression for the weak gravitational potential in terms of singleton bilinears. If this idea is introduced in the context of bulk/boundary duality, then it is natural to relate massless fields on the bulk to conserved currents on the boundary. But we are interested in the composite nature of massless fields on space time (the bulk), and a direct current-field identity is then inappropriate, since currents are con-
served by virtue of the field equations while massless fields are divergenceless only on the physical subspace defined by gauge fixing. In the paper [FF98] it was shown that the dipole formulation provides a natural construction of all massless fields in terms of bilinears that are conserved only by virtue of the gauge fixing condition on constituent singleton fields.

Now remember that the “massless” De Sitter representation $D({\frac{3}{2}, \frac{1}{2}})$ (in contrast with other $D(s+1, s)$ representations), has spin above the limit of unitarity, a fact that singles it out among massless AdS$_4$ particles. It can be obtained as one of the two, $\gamma_5$-related, irreducible representations that constitute the space of solutions of the corresponding Dirac equation. By developing a field theory of composite neutrinos along the lines explained above (neutrinos composed of singleton pairs, with three flavors of singletons) it might be possible to correlate the recently observed oscillations between the two or three kinds of neutrinos (that suggests they should have a mass and gives some estimates of it) with the AdS$_4$ description of these “massless” particles. To avoid misunderstandings we want to stress that we are fully aware of the fact that any reasonable estimate of the value of the cosmological constant rules out a direct connection to the value of experimental parameters like PC violation coupling constants or neutrino masses. (PC violation is a feature of composite QED, though no estimate of its strength has been made.) What we are saying is that the structure of Anti De Sitter field theory, and more especially the structure of singleton field theory, may provide a natural framework for a description of neutrino oscillations.

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