Beam Tracking Using Least Mean Squares Algorithm for single-cell Massive MIMO Communication System

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Abstract. Millimetres wave (mm-wave) is an attractive option for high data rate applications in the 5G wireless communication systems that require proper beamforming and channel tracking. In this paper, we study, analyse and compare the performance of two closely related stochastic gradient descent-based approaches, namely the least mean square (LMS) algorithm, and the normalized least mean square (NLMS) algorithm, for tracking the transmit array beam in addition to the channel status. These adaptive filters usually result in a trade-off between convergence and accuracy. We found that the quality of the tracking results, measured in mean squared error (MSE) sense, are heavily dependent on the present step-size of the gradient descent.

1. Introduction

Recently, Massive multiple-inputs multiple-outputs (MIMO) frameworks are used in the fifth generation of mobile communication systems (5G) to enhance system efficiency and to reduce interference from multiple users as a large number of antennas are provided to the base station [1]. With the increasing demand for high spectral efficiency and high bandwidth, millimetre-wave (mm Wave) technology is adopted with massive MIMO technology in 5G. Millimetre wave's short-wavelength helps shorten the spacing of antennas. Beamforming is a major enabler of mm Wave communication [2][3] The beams must be synchronized between the base station (BS) and the associate user equipment (UEs) due to the high directivity when using beam modulation [4]. To get hold advantage of beamforming technology, in (mm- wave) range, accurate angle tracking is required. As the wireless user is moving in different directions, it is important to monitor the channel gain and the beam angle simultaneously. As the angle of the beam follows a non-linear function, non-linear tracking systems need to be used [5].

One way to track the beam is via the Ray tracing system. However, these methods involve a high computational complexity. Another way to track the beam is through some filters. These filters can be divided into two types: adaptive filter and fixed filter. The static filter is helpful when the signal and channel parameters are known. On the other hand, adaptive filters are useful when the dynamics of a signal or channel are unpredictable and change over time. An adaptive filter is a filter that works under the control of some algorithms such as the least mean square (LMS), and the recursive least square (RLS) [6]. Researchers Md. Faisal Rahman and A.H.M. Asadul Huq developed an adaptive noise canceling device by applying LMS and NLMS algorithms. Because LMS has a fixed step size, it is not suitable...
for working in an unstable environment. But for NLMS, it is suitable for working in an unstable environment as well as a static environment [7], Dhaval N. Patel, and B.J. Makwana used algorithms least mean square (LMS), Sample matrix inversion (SMI), Recursive least square (RLS). It was observed that with an increase in the number of elements, an improvement in beamwidth and reduced interference between users could be obtained [8]. Researchers Vutha Va and Haris Vikalo proposed the Kalman beam tracking algorithm as a solution to reduce beam training overhead. An interesting effect was the size of the matrix. At the same SNR, the array must be chosen to be large enough for optimal performance [9]. Researchers Saurabh R. Prasad1 and Bhalchandra B. Godbole2 studied the effect of step size and the number of iterations on filter performance such as squared error, estimation accuracy, etc. The decrease in step size reduces steady-state error but at the same time increases convergence time.[6], in the mm Wave mobile communication case, Yavuz Yapici and Ismail Guvence derived the LMS algorithm and the Bidirectional least mean square BiILMS extension algorithm is derived from the steeper main algorithm. The numerical results show that LMS and BiLMS rapidly converge with the best performance for increased SNR. On the other hand, with an increasing SNR, the RLS algorithm exhibits a relatively slow convergence [10].

In this paper, the algorithms of (LMS, NLMS) to track the channel in communications (mm- wave) were studied and analysed. The results show that the algorithm of (LMS) is less complex compared to the algorithm of (NLMS) it reduces the difference between the desired signal and the actual signal of an unknown system where the MSE standard is used, and the time of implementation of the algorithm. The remainder of this paper is organized as follows in Section II, the system model is presented with beam forming, in Section III, the algorithm (LMS, NLMS) is derived, in Section IV, we produced the numerical result, finally, some conclusions are present in Section V.

2. System models

In this section, we present the channel and the beamforming model that we used in this work.

2.1. Time-Varying (mm-wave) Channel Model

Here, the geometrical time-varying channel model can be used to model the mm Wave channel due to its high directional nature. In a given coherence time, represented by time index k, it can be represented as follows [10]:

\[
H_k = \sum_{l=1}^{L} a_{k,l} a_r(\theta_{k,l}) a_T(\phi_{k,l})^T,
\]

where L is the number of paths received by the antenna array at a time k, \(a_{k,l}\) is the complex path gain, and here we assume that it follows a Gaussian distribution with zero mean and covariance \(R_R\), \(\theta_{k,l}\) and \(\phi_{k,l}\) respectively, the angle of departure (AOD) and the angle of arrival (AOA) of the \(l^{th}\) path at time index \(k\).

Assuming a uniform linear array (ULA) along the Z-axis, the response vector of the receive arrays \(a_r(.)\) and transmit array \(a_T(.)\) can be expressed as follows [9]:

\[
a_r(\theta) = \frac{1}{\sqrt{M}} [e^{-j2\pi d \cos(\theta)} \ldots e^{-j2\pi d (M-1) \cos(\theta)}]^T,
\]

\[
a_T(\phi) = \frac{1}{\sqrt{N}} [e^{-j2\pi d \cos(\phi)} \ldots e^{-j2\pi d (N-1) \cos(\phi)}]^T,
\]

Where \([.]^T\) is the complex conjugate of \([.].\) \(\lambda\) is the signal wavelength, d is the inter-element spacing, N is the number of transmit antennas and M is the number of receive antennas.
Using the channel model in (1), there are four-channel parameters that are possibly changing over time and can be grouped into a state vector, \( x = [R(\alpha), J(\alpha), \theta, \phi]^T \), where \( R(\cdot) \) and \( J(\cdot) \) are the real and imaginary operators. Assuming that \( x \) is a realization of a random-walk process, the dynamics model for tracking \( x \) can be stated as follows:

\[
x_{k,1,l} = yx_{k,l} + u_k
\]  

Where \( y \) is a constant diagonal matrix, and given as:

\[
y = \text{diag}(\rho \rho 1 1)
\]  

and \( \rho \) is constant that represents the amount of correlation between the path gains at two consecutive times [3][5]. \( u_k \) is the amount of additive noise at time index \( k \), and is assumed to follow a zero-mean Gaussian distribution with covariance.

\[
R_{uk} = \text{diag} \{ \sigma^2_{\alpha k} \sigma^2_{\phi k} \sigma^2_{\theta k} \sigma^2_{\phi k} \}.
\]

2.2. Formatting Beamforming Model

In massive MIMO communication systems, transmit and receive combining are normally used to steer the beam into a known user location, in addition to making the equivalent channel scalar. A beamforming vector \( f \), and the combiner \( w \), which are steering vectors at pre-defined pointing angles, are used for this purpose. Mathematically, for a ULA, these vectors are defined as follows:

\[
f = \frac{1}{\sqrt{M}} [1 e^{-2j\pi\frac{d}{\lambda} \cos \bar{\theta}} \ldots e^{-2j\pi\frac{d}{\lambda}(M-1) \cos \bar{\theta}}]^T,
\]

\[
w = \frac{1}{\sqrt{N}} [1 e^{-2j\pi\frac{d}{\lambda} \cos \bar{\phi}} \ldots e^{-2j\pi\frac{d}{\lambda}(N-1) \cos \bar{\phi}}]^T.
\]
Where $\tilde{\theta}$ and $\Phi$ are the pointing angles of the receive and transmit pointing beams, respectively.

3. Tracking of The Adaptive Beam and Channel
In this section, the algorithms (LMS) and (NLMS) will be presented to estimate unknown mm wave channel parameters AOA, AOD, and path gain.

3.1. Least Mean Square Algorithm (LMS)
In signal processing applications, the least mean square (LMS) adaptive algorithm commonly utilizes the gradient vector estimation from the steepest descent. To determine the optimal weight vector, an iterative process is required to update the weight vector in direction of the negative gradient vector, the mean square error value is minimized by this algorithm, it is relatively easy and does not require any matrix inversion calculation or correlation function calculation [11].

Instead of estimating the weight, we will use the (LMS) algorithm to adaptively track the states $x_k$, which represents the measured states of the beam directions $\theta$ and $\Phi$ the components of the complex channel gain: the real $\alpha_R$ and the imaginary $\alpha_I$ as

$$x_k = [\alpha_R^T \alpha_I^T \theta_k \phi_k^T]$$

In order to track the channel and adaptive beam for the scenario of the communication, we will first consider the (LMS) algorithm, in the state vector $x_k$ since the measurement is not linear, we, therefore, want to derive (LMS) using the original algorithm of the steepest descent algorithm given as [10].

$$\hat{x}_{k+1} = \hat{x}_k - \mu \nabla_{\hat{x}_k} J_k$$

(9)

where $\hat{x}_k$ is the estimation of $x_k$. We reflect the algorithm’s adaptation step-size in equation (9) by the diagonal matrix $\mu = [\mu_a 1_{2L} \mu_\theta 1_L \mu_\phi 1_L]$, where $\mu_a$, $\mu_\theta$ and $\mu_\phi$ represent the particular channel path gain, AOD, and AOA respectively, the gradient operator is $\nabla$, the mean square error at time k is $J_k$.

The estimation error is $e(k) = y(k) - h(\hat{x}_k)$, has a real part $e_{R,k}$ and an imaginary part $e_{I,k}$, the MSE gave as $J_k = E(\|e_k\|^2)$ with $e_k = [e_{R,k} e_{I,k}]^T$ in our paper we will define the cost function, and we can express as:

$$\text{cost function} = \arg \min (J_k)$$

The gradient of the MSE is given as [10]:

$$\nabla_{\hat{x}_k} J_k = -2e_k^T \frac{\partial h(\hat{x}_k)}{\partial \hat{x}_k}$$

(10)

$$Z_k = \frac{\partial h(\hat{x}_k)}{\partial \hat{x}_k}$$

(11)

$$\hat{x}_{k+1} = \hat{x}_k + 2\mu e_k^T Z_k$$

(12)

3.2. Normalizing Normalized Least Mean Square Algorithm (NLMS)
The normalize least mean square (NLMS) algorithm can be considered to be a special implementation of the (LMS) algorithm that takes account of the change in signal level at the filter input, the relationship is the same for adaptively changing states in the (LMS) algorithm, in the least mean square it’s difficult to select a learning rate that ensures algorithm stability. The only adjustment in (NLMS) is to normalize the input in order to solve the stated problem [12].

Normalizing the input vector provides the (NLMS) the ability to solve the problem by letting the step size adapt with the status of the vector:

$$\text{cost function} = \arg \min (J_k)$$
\[ \hat{x}_{(k+1)} = \hat{x}_k + \frac{\mu}{\|x_k\|^2} e^T Z e_k \]

Where \( \mu \) is the step size of the adaptive filter, \( \|x_k\|^2 \) is the normalization norm vector of \( x_k \).

4. Numerical results and discussion

Depending on the program (MATLAB) to simulate the performance of the (LMS) and (NLMS) algorithms in the (mm-wave) beam tracking in this section. we suppose the multi-antenna system with \( N=16, M=16, \text{ and SNR}=30\, \text{dB}, \) we assume multipath \( L=1 \) for a narrow physical beam this is very likely, the time evolution of the channel underlying mm wave its regulated by \( \rho = 0.995 \) and \( \sigma^2 = (0.1)^2 \), and the vector of the precoder and combiner the arbitrary direction of \( 45^\circ \) is pointed out \( (\hat{\theta} = \hat{\phi} = 45^\circ) \). For (LMS, NLMS, RLS) for the adaptive \( \mu_\alpha = 0.1 \) and \( \mu_\beta = \mu_\gamma = 0.0001 \) we use the better step-size value \( \mu \), and the wavelength \( \lambda = \frac{c}{f} = c = 3 \times 10^3 \text{ meters/sec} = 0.9 \text{ cm}. \)

5. Results and discussion

![Figure 2. Comparison of the cost function of LMS algorithm for different values of \( \mu \).](image)

Figure 2 shows the cost function for the algorithm (LMS) for different values of the step size \( \mu \), where \( \mu \) is the step size parameter, and it controls the convergence characteristics of the LMS algorithm, and it is necessary to use an appropriate value for the performance of the LMS algorithm. from the figure 2, we can notice that the algorithm (LMS) at the lower value of \( \mu \), the algorithm converges slowly, then the cost function become high and gives high error rate, but for a large value of \( \mu \) the algorithm converges faster and gives a lower cost function with a lower error rate.
During complete execution, one of the main disadvantages of the (LMS) algorithm is having a fixed step size parameter. These calls for an understanding of the input signal statistics before the adaptive filtering process starts.

An extension of the LMS algorithm that bypasses this problem by choosing a different step size value \( \mu \) for each iteration of the algorithm is the normalized least mean square (NLMS). The increase in step size leads to faster convergence, from the figure it can be seen that the algorithm (NLMS) converges faster than the algorithm (LMS) at the same step size parameter and the value of the MSE after converges will be low and higher stability can be obtained.

Figure 3. Compare performance between (LMS and NLMS) algorithm.

To tracking the best beam, we can use the algorithm that has the least error, In figure 4, we can note that the tracking algorithm LMS have a cost function at SNR=30 less than the cost function at

Figure 4. compares between cost function of the LMS algorithm at different values for SNR.
SNR=20, this means that the LMS algorithm at high SNR has a better performance with lower error and gives higher stability.

6. Conclusions
In this paper, channel tracking and adaptive beam formation were studied using adaptive algorithms (LMS, NLMS) algorithms (LMS, NLMS), the algorithms were derived under a non-linear control model. The results showed that the algorithm (LMS) is one of the easiest and simplest types of algorithms and needs large step size values to converge faster and thus obtain a stable system. The algorithm (NLMS) is considered a special implementation of the algorithm (LMS) when choosing a normal step size leads to a more stable and convergent adaptive algorithm.

Acknowledgments
Authors wishing to acknowledge the University of Mosul.

7. References
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