Prediction of flow curves of very thin brass sheets incorporating size effect in hardening model

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Abstract. Due to the size effect, the plastic deformation behavior of very thin sheets is dependent on the ratio of specimen thickness to grain size. The general hardening models used for bulk materials and thick sheets are not applicable for representing the plastic flow behavior of very thin sheets accurately. In the present work, the grain size effect (ratio of sheet thickness to grain size \(t/d\)) on flow behavior in uni-axial tension has been studied. Very thin sheet specimens of CuZn36 brass of thickness 50\(\mu\)m, 100\(\mu\)m, 150\(\mu\)m and 200\(\mu\)m have been used in this study. The brass sheets were annealed with different combinations of annealing temperature and time to achieve a large range of \(t/d\) ratio (1 to 17). A new hardening model has been developed incorporating the grain size effect on flow stress by determination of strength coefficient as a function of \(t/d\). The predicted curves using the developed hardening model agreed well with the experimental results.

1. Introduction

In the recent years, there has been a growing demand for micro technical products in various fields of science and engineering such as micro electro-mechanical systems, telecommunication, medicine, bio-technology, automotive industry etc. [1]. Because of the growing demand for micro and macro metallic parts from very thin sheets, study of forming behavior of very thin sheet metals and the variables (mainly the size effects) which affect their forming characteristics has become very important. When specimen size is scaled down from conventional scale to the sub millimeter range, some aspects of the work piece remain unchanged, such as the microstructure and the surface topology. This causes the ratio between the dimensions of the part and parameters of the microstructure or surface to change, and is commonly known as the “size effect” [2]. Because of the size effects, the material behavior changes significantly with miniaturization.

The influence of miniaturization on the flow stress has been investigated by geometrically similar tensile tests, upsetting tests, air bending experiments and punching experiments [3, 4]. The experiments have shown that the flow stress decreased with the increasing miniaturization. The decrease of flow stress can be explained by the ‘Surface Layer Model’ [5]. The surface layer model is based on the fact that on micro scales, the ratio of blank / billet dimensions to grain size (ratio of sheet thickness ‘t’ to grain size ‘d’ in case of thin sheets) is a decisive factor. The share of grains representing the surface layer becomes higher compared to grains entirely surrounded by other grains for micro-parts (i.e. increase in the ratio of surface to volume grains) as shown schematically in Figure 1. During plastic
deformation process, the grains located at the specimen surface and the grains located within the specimen volume are expected to behave differently because of the lower forces of constraint in the surface area of the specimen than in the interior. With increase in ratio of surface to volume grains, the overall resistance to deformation decreases and as a result, the flow stress decreases with increasing miniaturization [6-8]. However, when the thickness to grain size ratio (t/d) decreased to very low values (in the range of 1-5), it was observed that flow stress increased with decreasing t/d ratio as the sheet which contains very few grains in thickness direction cannot be considered as homogeneous continuum any more. A summary of the effect of the ratio of sheet thickness to grain size (t/d) on the flow stress reported in the literature is presented in Figure 2[9].

The general hardening models applicable in the case of deformation at macro-scales cannot be applied to very thin sheets. A few researchers have made attempts to develop new hardening models to describe the material flow behavior of very thin sheets during plastic deformation incorporating the effect of sheet thickness and/or grain size. Michel and Picart [10] proposed a model for flow stress of very thin sheets based on the regression analysis of the experimental data observed from tensile tests on brass sheets (CuZn36) with various thicknesses. By introducing a scaling function, \( F(\lambda, \varepsilon) \), Swift model was modified. Peng et al. [11] developed a double linear elastic-plastic hardening model for CuZn36 sheets which also considered the thickness of sheets. However, the effect of grain size or ratio of thickness to grain size has not been incorporated in both these models. Yeh et.al. [12] generated a new constitutive equation for thin sheets of any thickness and grain size in micro-forming. Wang et al. [13] investigated the influence of thickness/average grain size on yield stress and tangent modulus in micro tensile tests of CuZn36 sheets. A double linear elastic–plastic constitutive equation was developed. Another hardening model considering the size effect (t/d) has been developed for thin sheets by Wang Yun et al. [14]. This model combines the Hollomon equation (power law of strain hardening) and Hall–Petch relation.

The present work is aimed at an investigation of size effects on tensile properties and flow stress of very thin brass sheets and improved prediction of flow curves incorporating the size effect in the hardening model of the material.

![Figure 1. Surface layer model [5]](image)

![Figure 2. Effect of t/d ratio on flow stress [9]](image)

### 2. Material characterization

Thin sheet materials of brass (CuZn36) of thickness 50μm, 100μm, 150μm and 200μm have been used for investigation of size effect. Brass has excellent mechanical and electrical properties which are essential for electronic devices. The chemical composition of materials (by weight %) used in the present study has been analyzed by spectroscopy as per ASTM- E-1507. The brass sheets were received in 1/4 hard temper. A large range of grain size (d) is required to study the effect of thickness to grain size ratio on size effect. In view of this, the samples of brass sheets were annealed with different combinations of annealing temperature and time in a furnace in normal atmosphere. Three different combinations of annealing parameters i.e. 400° C, 3 h; 500° C, 3 h and 600° C, 2 h were chosen for each thickness (total 12 cases) to study the effect of t/d on flow stress. Microstructures of all annealed sheets have been analyzed which showed α solid solution with clear grain boundaries in most cases. In specimens annealed at 600°C for 2h, the microstructures show fully recrystallized grains of uniform size with
considerable grain growth (grain size 40-50μm) where as in specimens annealed at 400° C and 500° C for 3h, the microstructure consists of a mixture of fine and medium size grains and it could be due to incomplete recrystallization and absence of grain growth due to lower annealing temperature (Figure 3). Using these microstructures, the average grain size was found by linear intercept method. The average grain size varied in the range of 9µm-50µm for sheets of thickness 50µm to 200µm. This resulted in specimens with a large range of thickness to grain size ratio (t/d), approximately 1 to 17, which was required to study the grain size effect.

![Figure 3. Microstructure of brass sheet of thickness 150µm annealed at (a) 400° C, 3h (b) 500° C, 3h and (c) 600° C, 2h](image)

3. Tensile tests
   Specimens of annealed brass sheets of all thicknesses were prepared as per ASTM standard E345–93 (ASTM Standard, 2008) for uni-axial tensile testing. From the tensile tests, load-elongation data was acquired and it was converted into engineering stress-engineering strain curves. The standard tensile properties such as yield stress (YS), ultimate tensile stress (UTS) and total elongation were determined from the engineering stress-engineering strain data. Typical engineering stress-engineering strain curves of brass specimens and True stress-true strain curves obtained from engineering stress-engineering strain curves are shown in Figure 4. Among the sheet samples of four thicknesses used in the present work, both strength and ductility have decreased with thickness though 150μm thick sheet has higher strength than 200μm thick sheet for annealing temperatures of 400° C and 500° C. Similarly, 100μm thickness sheet showed higher ductility than 150μm thick sheet for annealing temperatures of 400° C and 500° C. These differences are possibly due to the relative thickness to grain size ratio and the different behavior of very thin sheets cannot be attributed to the effect of thickness alone. The decreasing flow stress with the increasing miniaturization is related to t/d which is an important parameter for flow behavior in the case of very thin sheets. Therefore, to analyze the plastic deformation behavior of very thin sheets (i.e. variation of flow stress as a function of strain), it is important to consider the effect of t/d ratio on flow stress.

4. Effect of thickness to grain size ratio on flow stress
   The grain size effect on flow stress of thin annealed brass sheets has been studied using the specimens with 12 different t/d values obtained after annealing. The variation of 0.2% offset yield stress (YS) with t/d ratio is shown in Figure 5. When the t/d ratio values are large (10-18), the YS increased with increase in t/d ratio. This is in accordance with Hall -Petch relation [15] for a polycrystalline material. As the grain size decreases, the YS increases due to higher strengthening from larger grain boundary area network. However, here it is assumed that there are a large number of grains in the thickness direction, which is generally true for bulk materials and thick sheets. The effect of t/d on YS in range of 5-10 (specimen size effect) with decreasing specimen size for the same microstructure (size invariant), the share of the surface grains increases. The surface grains deform easier than the grains inside leading to lower resistance to deformation and less hardening and hence flow stress decreases. During plastic deformation process, the grains located at the specimen surface and the grains located within the specimen volume are expected to behave differently because of the lower forces of constraint in the
surface area of the specimen than in the interior. With increase in ratio of surface to volume grains, the overall resistance to deformation decreases and as a result, the flow stress decreases with increasing miniaturization. But, in Figure 5, it can be seen that for smaller values of t/d (< 5), YS decreases with increasing t/d ratio. This effect can be explained by the presence of very few grains in thickness direction and the material cannot be treated as a continuum any more. In this case, the yield strength depends on the orientation and strength of individual grains. The trend agrees well observations in the literature that the flow stress decreases with decreasing t/d value down to about 2-4, after which the flow stress starts to increasing as t/d decreases further [6,10,16,17]. Therefore, use of general hardening models for very thin sheets to predict flow stress without considering the size effect (effect of grain size relative to thickness) could lead to significant error in the flow stress values. The result clearly demonstrates the need for modification in the existing hardening models when they are applied to very thin sheets.

![Graph showing engineering stress-strain curves and true stress-true strain curves of brass sheets](image1)

**Figure 4.** (a) Engineering stress-engineering strain and (b) true stress-true strain curves of brass sheets

![Graph showing variation of yield stress with t/d ratio](image2)

**Figure 5.** Variation of yield stress of brass sheets with t/d ratio

5. Development of new hardening model

The results presented in the earlier section clearly showed dependence of flow stress on t/d ratio of very thin sheets. In view of this, a hardening model is proposed in this work to represent the plastic deformation behavior of very thin brass sheets considering the grain size effect.

The flow stress can be expressed by general Hollomon equation:

\[ \sigma = K \varepsilon^n \]  

(1)

where, \( \sigma \) is true stress, \( \varepsilon \) is true strain, \( K \) is strength coefficient and \( n \) is strain-hardening coefficient. \( K \) is a material constant in the conventional power law of strain hardening in which flow stress is a function of strain only. However, since in the case of very thin sheets, the flow stress also depends on
the grain size effect (t/d), the above equation can be rewritten incorporating the grain size effect on flow stress as given below:

\[ \sigma = f(\varepsilon, \phi) = K' \varepsilon^n = F(\phi)\varepsilon^n \]  

(2)

In the above equation, \( K' \) is a function describing the size effect on flow stress and \( \phi \) is t/d. From the flow curves obtained experimentally for the brass sheets, \( K' \) has been determined for 7 specimens with different t/d ratios (from log \( \sigma \) - log \( \varepsilon \) plots) and it is plotted as a function of t/d as shown in Figure 6. The correlation between \( K' \) and \( \phi \) has been obtained by curve fitting as follows:

\[ K' = F(\phi) = (-0.1847 \phi^3 + 5.0132 \phi^2 - 21.664 \phi + 751.2) \]  

(3)

It can be seen that the variation of the strain-hardening coefficient ‘n’ in different tests is very small. Therefore, the average value of the n values obtained from the tensile tests of the specimens with 7 t/d ratios has been taken as the modified value of ‘n’. By combining equations (2) and (3), the new hardening model for very thin brass sheets considering the size effects for thin sheets forming has been obtained as follows:

\[ \sigma = (-0.1847 \phi^3 + 5.0132 \phi^2 - 21.664 \phi + 751.2) \varepsilon^{0.367} \]  

(4)

The above equation has been validated with the experimental flow curves of another four specimens of different t/d ratios as shown in Figure 7. The predicted curves agreed well with the experimental results. The predicted curves have also been compared with flow curves predicted using the hardening model developed by Yun et al. [14] for very thin sheets of copper and it considers the effect of both specimen thickness and grain size.

The Hall–Petch equation and the internal grain boundary model and the surface layer model were combined to determine the strength coefficient in the Hollomon equation. Hall-Petch equation is given as:

\[ \sigma = \sigma_0 + k \sqrt{d} = \sigma_0 + k/\sqrt{t} \]

\[ \sigma \propto \sqrt{\phi} \]  

(5)

where, \( \sigma_0 \) and k are constants at a specific strain and d is average grain size, t is thickness and \( \phi = t/d \). The internal grain boundary model and surface layer model are expressed as:

\[
\left\{
\begin{array}{l}
\alpha \approx \frac{1}{2} \frac{1}{\phi} \\
\beta \approx 1 - \frac{2}{(2 \phi + 1)}
\end{array}
\right.
\implies \sigma \propto \phi^{-1} \]  

(6)

where, \( \alpha \) is share of surface grain and \( \beta \) is the ratio of the internal grain boundary length to the total length of grain boundary.

![Figure 6. Variation of \( K' \) with t/d](image)
Figure 7. Comparison of predicted flow curves using the developed hardening model with experimental curves for brass specimens

Incorporating the effect of $\varphi$, the Hollomon equation for true stress-true strain relationship is rewritten as:

$$\sigma = f(\epsilon, \varphi) = F(\varphi) \epsilon^n$$  \hspace{1cm} (7)

$F(\varphi)$ is a function describing the size effect on flow stress. By combining equations (5) and (6), $F(\varphi)$ is written as:

$$F(\varphi) = a + b\sqrt{\varphi} + c\varphi^{-1}$$  \hspace{1cm} (8)

where, $\varphi = t/d$ and $a$, $b$ and $c$ are material coefficients.

The material coefficients are obtained by uniaxial tensile tests on samples of different $t/d$ ratio to determine the flow curves.

In the present work, the above model has been used to predict the flow curves by determining the material coefficients ($a$, $b$, and $c$) from the tensile test data of 3 brass sheet specimens with different $t/d$ ratios. Since the variation of the strain-hardening coefficient $n$ in different tests is very small, the average value of the $n$ obtained in the three tests with respective $t/d$ ratios has been taken as the modified value of $n$. Using the determined material coefficients $a$, $b$, $c$ and $n$ in equation (7), the following equation has been obtained.

$$\sigma = (5378 + (-1106)\sqrt{\varphi} - 10626 \varphi^{-1})\epsilon^{0.336}$$  \hspace{1cm} (9)

where, $\sigma$ is flow stress, $\varphi$ is $t/d$ and $\epsilon$ is true strain.

The flow curves of specimens with 4 other $t/d$ values have been used to validate the equation (9). Flow curves of these specimens predicted by using the hardening model developed in this work (equation (4)) and the hardening model developed by Wang Yun et.al (2010) (equation (9)) are compared with experimental flow curves in Figure 8 for 4 different $t/d$ ratios. It can be observed from the figures that the flow curves predicted using the equation (4) developed in the present work agreed more closely with the experimental curves than the flow curves predicted using equation (9).

The main reason for this discrepancy could be the assumed theoretical relation between strength coefficient ($F(\varphi)$) with $t/d$ in equation (8) where as in the present work, correlation between the strength coefficient ($K'$) and $\varphi$ has been obtained from the actual variation of experimental values of $K'$ with $t/d$ ratio (Figure 6).
Figure 8. Comparison of experimental curves with developed curves predicted by equation (4) and equation (10) for different t/d ratios (a) 17.1, (b) 16.1, (c) 1.9 and (d) 2.5

6. Conclusions
Since the flow stress of very thin sheets depends on the grain size effect (t/d ratio), a new hardening model has been developed incorporating the grain size effect on flow stress of very thin brass sheets. Flow stress increases with increasing t/d ratio for larger values of t/d (5-18) and it decreases with t/d when the values of t/d ratio are less than 5. In the range 5-10, the effect can be explained by the surface layer model. It has been found the strength coefficient in the constitutive equation is a function of t/d ratio. Combining this with Hollomon equation, the flow curves have been predicted which agreed closely with the experimental curves reducing the error in prediction of flow stress for very thin brass sheets.

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