Lazy Pointer Analysis

Uday P. Khedker\textsuperscript{1}, Alan Mycroft\textsuperscript{2}, and Prashant Singh Rawat\textsuperscript{1}\textsuperscript{*}

\textsuperscript{1} Indian Institute of Technology Bombay
\{uday,prashantr\}@cse.iitb.ac.in
\textsuperscript{2} University of Cambridge
Alan.Mycroft@cl.cam.ac.uk

Abstract. Flow- and context-sensitive pointer analysis is generally considered too expensive for large programs; most tools relax one or both of the requirements for scalability. We formulate a flow- and context-sensitive points-to analysis that is lazy in the following sense: points-to information is computed only for live pointers and its propagation is sparse (restricted to live ranges of respective pointers). Our analysis also: (i) uses strong liveness, effectively including dead code elimination; (ii) afterwards calculates must-points-to information from may-points-to information instead of using a mutual fixed-point; (iii) uses value-based termination of call strings during interprocedural analysis (which reduces the number of call strings significantly).

A naive implementation of our analysis within GCC-4.6.0 gave analysis time and size of points-to measurements for SPEC2006. Using liveness reduced the amount of points-to information by an order of magnitude with no loss of precision. For all programs under 30kLoC we found that the results were much more precise than gcc’s analysis. What comes as a pleasant surprise however, is the fact that below this cross-over point, our naive linked-list implementation is faster than a flow- and context-insensitive analysis which is primarily used for efficiency. We speculate that lazy flow- and context-sensitive analyses may be not only more precise, but also more efficient, than current approaches.

1 Introduction

Interprocedural data flow analysis extends the scope of analysis across procedure boundaries to incorporate the effect of callers on callees and vice-versa. The efficiency and scalability of such an analysis is a major concern. The precision of such an analysis requires flow-sensitivity (associating different information with distinct control flow points) and context-sensitivity (computing information depending upon the calling context). Sacrificing precision for scalability is a common trend in interprocedural data flow analysis. This is more prominent in pointer analysis in which the size of information could be large. Flow- and context-sensitive pointer analysis is considered prohibitively expensive and most methods relax one or both of the requirements for scalability.

We formulate a flow- and context-sensitive points-to analysis that is lazy: points-to information is computed only for the pointers that are live and the propagation of points-to information is sparse in that it is restricted to live ranges of respective pointers. We

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main()
{  
x = &y;
  
w = &x;
  
p();
  
  print z;
}

p()
{
  if (...)
  {
    z = w;
    p();
  
    z = *z;
  }
}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{A motivating example for lazy points-to analysis, its supergraph representation and some observations. The solid edges in the supergraph represent intraprocedural control flow while dashed edges represent interprocedural control flow.}
\end{figure}

use strong liveness which identifies the pointers that are directly used or are used in defining pointers that are strongly live. Thus strong liveness incorporates the effect of dead code elimination on liveness and is more precise than simple liveness.

Fig. 1 provides a motivating example for lazy pointer analysis. By printing \( z \), the main procedure makes \( z \) live at node 12 after the call to procedure \( p \). This makes \( z \) in node 12 live. This in turn makes \( w \) live in node 9 and then in 3 resulting in the points-to pair \( (w, x) \). This pair is propagated to 12 giving the pair \( (z, x) \). When this information becomes available in 12, \( x \) becomes live. This liveness is propagated to 2 giving the pair \( (x, y) \). Eventually we get the pair \( (z, y) \) in 12. Figures 8 and 9 give fuller detail after formulating lazy pointer analysis interprocedurally. Here we observe the following:

- **Lazy computation.** Points-to pairs are computed when the pointers become live.
- **Sparse propagation.** Pairs \( (x, y) \) and \( (w, x) \) are not propagated beyond the call to \( p \) in the main procedure in spite of the fact that \( x \) or \( w \) are not modified in \( p \).
- **Flow sensitivity.** Points-to-information is different for different control flow points.
- **Context sensitivity.** \( (z, x) \) holds only for the inner call to \( p \) made from within \( p \) but not for the outer call to \( p \) made from within the main procedure. Thus in spite of \( z \) being live in 6, \( (z, x) \) is not propagated to 6 but \( (z, y) \) is.

We propose a novel data flow framework that employs an interdependent formulation for discovering strong liveness and points-to-information for pointer variables. This framework computes must-points-to-information from may-points-to-information without requiring an additional fixed-point computation. At the interprocedural level, flow- and context-sensitivity is ensured by using value-based termination of call strings.

Our findings conclusively demonstrate that instead of achieving scalability by compromising on precision, it is far better to contain the explosion of information by clearly distinguishing between the information that is relevant from the information that is not
relevant. Since pointer information is required to uncover the data items that are accessed indirectly or functions that are invoked indirectly, it is relevant only when there is some use of a pointer. We show that this change in perspective provides significant benefits in terms of time and space requirements of pointer analysis.

The rest of the paper is organised as follows: Section 2 reviews the background. Section 3 formulates the mutually dependent liveness and points-to analysis at the intra-procedural level. Section 4 formalises and proves some important properties of our analysis. It is lifted to interprocedural level in Section 5. Section 6 discusses the related work while Section 7 presents the empirical data. Section 8 concludes the paper.

2 Background

This section reviews intra- and interprocedural data flow analysis and pointer analysis.

Overview of Data Flow Analysis. Data flow analysis is formulated in terms of data flow equations that describe how the required data flow information can be computed for a statement. The set of data flow values, the functions to compute them and the operation to merge them are described by a data flow framework.

Unlike the classical view that treats a data flow framework and its instance as distinct[12345], we view a data flow framework parameterised by a program because the data flow values and the functions that manipulate them depend on the program being analysed. Formally, a data flow framework is a tuple \( \langle L_G, \sqcap_G, F_G \rangle \) where \( G \) is an unspecified graph representing a program, \( L_G \) is a meet semilattice representing the data flow values relevant to the analysis, and \( F_G \) is a set of admissible flow functions from \( L_G \) to \( L_G \). We require all strictly descending chains in \( L_G \) to be finite. \( \sqcap_G \) is the meet operator of \( L_G \). We require the flow functions in \( F_G \) to be monotonic.

At the intraprocedural level, a procedure is represented by a control flow graph (CFG) whose nodes represent program statements and edges represent control transfers. A CFG for procedure \( p \) must satisfy the following requirements: there must be a unique entry node \( \text{start}_p \) with no predecessor and a unique exit node \( \text{end}_p \) with no successor, each node \( n \) must be reachable from \( \text{start}_p \), and \( \text{end}_p \) should be reachable from each node. At the interprocedural level, a program is represented by a supergraph which connects the CFGs by interprocedural edges. A call to procedure \( p \) at call site \( i \) is split into a call node \( c_i \) and a return node \( r_i \) with a call edge \( c_i \rightarrow \text{start}_p \) and a return edge \( \text{end}_p \rightarrow r_i \). In examples we number nodes in the CFG in reverse post-order and assign contiguous numbers across procedures. Fig. 1 provides an example of a supergraph.

| Forward Analysis (\( \text{In}_n \) influences \( \text{Out}_n \)) | Backward Analysis (\( \text{Out}_n \) influences \( \text{In}_n \)) |
|----------------------------------------------------------|----------------------------------------------------------|
| \( \text{In}_n = \begin{cases} 
\text{BL} & n = \text{start}_p \\
\bigcap_{p \in \text{pred}(n)} \text{Out}_p & \text{otherwise} 
\end{cases} \) | \( \text{In}_n = f_\pi(\text{Out}_n) \) |
| \( \text{Out}_n = f_n(\text{In}_n) \) | \( \text{Out}_n = \begin{cases} 
\text{BL} & n = \text{end}_p \\
\bigcap_{s \in \text{succ}(n)} \text{In}_s & \text{otherwise} 
\end{cases} \) |

Fig. 2. Typical data flow equations for some procedure \( p \).
Data flow equations (Fig. 2) define data flow variables $l_n$ and $o_n$, which represent the data flow information associated with the entry and exit points of node $n$. $l_n, o_n \in L_G$ and $f_n \in F_G$. The boundary information $Bl$ represents the data flow information at the procedure entry for forward analysis and procedure exit for backward analysis. Its value is governed by the semantics of the information being discovered. Interprocedural analysis eliminates the need for a fixed $Bl$ (except for arguments to main) and computes it from the calling contexts during the analysis.

Iterative methods solve the data flow equations by refining the values starting from a conservative initialisation of $\top$. Round robin methods traverse the CFG in a fixed order; work list methods maintain a list of the nodes whose values are to be recomputed.

Interprocedural Data Flow Analysis. A supergraph contains control flow paths which violate nestings of matching call return pairs (e.g. 1-2-3-4-8-13-11 for the supergraph in Fig. 1). Such paths correspond to infeasible contexts. An interprocedurally valid path is a feasible execution path containing a legal sequence of call and return edges.

A context-sensitive analysis retains sufficient information about calling contexts to distinguish the data flow information reaching a procedure along different call chains. This restricts the analysis to interprocedurally valid paths and ensures propagation of information from a callee to appropriate call sites. A context-insensitive analysis does not distinguish between valid and invalid paths effectively merging data flow information across calling contexts. Although the resulting information is provably safe, it is often imprecise. Recursive procedures have potentially infinite contexts, yet context-sensitive analysis is decidable for data flow frameworks with finite lattices and it is sufficient to maintain a finite number of contexts for such frameworks. However, this number is combinatorially large even for non-recursive programs. Flow-insensitive approaches disregard intraprocedural control flow for efficiency. Instead of information being associated with each program point, a single summary is computed. Although the summary information is provably safe, it is imprecise. A flow-sensitive analysis honours the control flow and computes data flow information separately for each program point.

We use a flow- and context-sensitive approach called the call-strings method [7,6,8]. It embeds context information in the data flow information and ensures the validity of interprocedural paths by maintaining a history of calls in terms of call strings. A call string at node $n$ is a sequence $c_1c_2\ldots c_k$ of call sites corresponding to unfinished calls at $n$ and can be viewed as a snapshot of the call stack. $\lambda$ denotes an empty call string. Some call strings for our supergraph in Fig. 1 are: $\lambda, c_1, c_1c_2, c_1c_2c_2$ etc.

Call string construction is governed by interprocedural edges. Let $\sigma$ be a call string reaching procedure $p$. For an intraprocedural edge $m \rightarrow n$ in $p$, $\sigma$ reaches $n$ unmodified. For a call edge $c_i \rightarrow \text{start}_p$ where $c_i$ belongs to $p$, call string $\sigma c_i$ reaches $\text{start}_p$. For a return edge $\text{end}_p \rightarrow r_j$ where $r_j$ belongs to a caller of $p$, if the last call site in $\sigma$ is $c_j$ then the longest prefix that excludes $c_j$ reaches the call site corresponding to $r_j$. If the last call site in $\sigma$ is not $c_j$, the call string and its associated data flow value is not propagated to the call site corresponding to $r_j$. This ensures that the data flow information is only propagated to appropriate call sites. In a backward analysis, the call string grows on traversing a return edge and shrinks on traversing a call edge.
Fig. 3. An example of flow-sensitive intraprocedural points-to analysis.

The augmented data flow information is a pair \( (\sigma, d) \) where \( d \) is the data flow value propagated along call string \( \sigma \) and is modified by an intraprocedural edge only. A worklist-based iterative algorithm is used to perform the data flow analysis. The process terminates when no new pair \( (\sigma, d) \) is computed; merging the data flow values associated with all call strings reaching node \( n \) gives the final data flow value at \( n \). This method computes a safe and precise solution because it matches call and return nodes in a path thereby excluding interprocedurally invalid paths and traversing valid paths only.

In non-recursive programs, since the call strings are acyclic (no call site occurs multiple times), their number is finite and all of them are generated during analysis. However, in recursive programs, new call strings are generated with every visit to a call node involved in recursion. In such cases, the number of call strings considered must be bounded using explicit criteria. For computing a safe and precise solution, the full call-strings method [7] requires construction of all call strings of length up to \( K \times (|L| + 1)^2 \) where \( K \) is the maximum number of distinct call sites in any call chain and \( L \) is the lattice of data flow values. For bit-vector frameworks, we need to consider only those call strings in which a call site appears at most thrice [9]. Since these numbers are very large for practical programs and we use a recent variant in which the termination of call-string construction is based on the equivalence of data flow values instead of precomputed length bounds [8][9]. This allows us to discard call strings where they are redundant, and regenerate them when required. For cyclic call strings representing paths in recursion, regeneration facilitates computation of data flow values without explicitly constructing most of the call strings. This reduces the space and time requirements of the analysis dramatically without compromising on safety or precision.

**Pointer Analysis** Two forms of pointer analysis are extant: alias analysis identifies pairs of address expressions that both hold the address of a given location. Points-to analysis identifies locations whose addresses are held by pointers. May- and must- variants of both exist. This paper restricts itself to points-to analysis.
Points-to relations are computed by identifying locations corresponding to the left- and right-hand sides of a pointer assignment and taking their cartesian product \([10,11]\). The points-to pairs of locations that are modified are removed. May-points-to information at \(n\) contains the points-to pairs that hold along some path reaching \(n\) whereas must-points-to information contains the pairs that hold along every path reaching \(n\) (hence a pointer can have at most one pointee) \([11]\). Fig. 3 provides an example of flow-sensitive points-to analysis. For this example, an inclusion-based flow-insensitive analysis \([12]\) concludes that \((p, r), (p, s), (q, r), (r, s), (s, r)\) hold at all program points. An equality-based flow-insensitive analysis \([13]\) additionally computes \((q, s)\).

3 Lazy Pointer Analysis

We consider the four basic pointer assignment statements: \(x = \& y, \quad x = y, \quad x = *y, \quad *x = y\) using which other pointer assignments can be rewritten. We also assume a use \(x\) statement to model other uses of pointers (such as in conditions).

3.1 Notation and Basic Definitions

Let \(V\) denote the set of variables (i.e. “named locations”). Some of these variables (those in \(P \subset V\)) can hold pointers to members of \(V\). Other members of \(V\) hold non-pointer values. These include variables of non-pointer type such as \(\text{int}\). NULL is similarly best regarded as a member of \(V - P\); finally a special value '?' in \(V - P\) denotes an undefined location. This represents the value of an uninitialised pointer declaration, e.g. \(\text{int } *x\); At the moment it is simplest to think of '?' as being NULL as in Java rather than C, so that indirecting on it terminates execution (Section 3.4 explains this).

Points-to information is a set of pairs \((x, y)\) where \(x \in P\) is the pointer of the pair and \(y \in V\) is a pointee of \(x\) and is also referred to as the pointee of the pair. The pair \((x, \?)\) being associated with program point \(n\) indicates that \(x\) may not contain a valid address along some potential execution path from \(\text{start}_p\) to \(n\).

The liveness information for statement \(n\) is denoted by the data flow variables \(L_{in, n}\) and \(L_{out, n}\), the may-points-to information is denoted by \(A_{in, n}\) and \(A_{out, n}\), and the must-points-to information is denoted by \(U_{in, n}, U_{out, n}\). Instead of being calculated as a mutual fixed point with \(A_{in, n}, U_{out, n}\), in our framework \(U_{in, n}, U_{out, n}\) are computed afterwards from \(A_{in, n}, A_{out, n}\). Note that liveness propagates backwards (transfer functions map \textit{out} to \textit{in}) while points-to propagates forwards.

Let \(\mathcal{P}(S)\) denote the powerset of \(S\). Then \(\mathcal{L} = \langle \mathcal{P}(P), \supseteq \rangle\) is the lattice of liveness information. Note that this means that we do not track the liveness of non-pointer variables because their liveness is not relevant to points-to analysis. The lattice of may-points-to information is \(\mathcal{A} = \langle \mathcal{P}(P \times V), \supseteq \rangle\). The overall lattice of our data flow values is the product \(\mathcal{L} \times \mathcal{A}\) having partial order \(^3\)

\[\forall \langle l_1, a_1 \rangle, \langle l_2, a_2 \rangle \in \mathcal{L} \times \mathcal{A}, \langle l_1, a_1 \rangle \subseteq \langle l_2, a_2 \rangle \Leftrightarrow (l_1 \subseteq l_2) \land (a_1 \subseteq a_2) \Leftrightarrow (l_1 \supseteq l_2) \land (a_1 \supseteq a_2)\]  

\(^3\) We use the original data flow greatest fixpoint formulation where \(\top\) constitutes the initial value rather than the abstract-interpretation-style least fixpoint formulation which iterates from \(\bot\).
The ⊤ element of the lattice $L \times A$ is $\langle \emptyset, \emptyset \rangle$ and the ⊥ element is $\langle P, P \times V \rangle$.

We use standard algebraic operations on points-to relations:

- For a given relation $R \subseteq P \times V$ and some set $X$, relation application ($R \times X$) is defined as $R \times X = \{v \mid u \in X \land (u, v) \in R\}$ and relation restriction ($R|_X$) is defined as $R|_X = \{(u, v) \in R \mid u \in X\}$.
- Given relations $S \subseteq A \times B$ and $T \subseteq B \times C$, relation composition $T \circ S \subseteq A \times C$ is defined as $T \circ S = \{(u, w) \mid (u, v) \in S \land (v, w) \in T\}$.

However, since $R \subseteq P \times V$, we need to take a little more care formalising $R \circ R$ because of the mismatch between the sets. We adopt the conventional approach of using the inclusion map: since $P \subseteq V$, by inclusion of relations we regard the leftmost $R$ as being a subset of $V \times V$ (effectively coercing $P \times V$ into $V \times V$). To distinguish it from the usual composition, we denote it as $R \cdot R$. Note that the result is a subset of $P \times V$.

Consider $V = \{a, b, c, d, e, f, g, ?\}$ and $P = \{a, b, c, d, e\}$. Let relation $R \subseteq P \times V$ be $\{(a, b), (a, c), (b, d), (c, e), (c, g), (d, a), (d, g), (e, ?)\}$. Consider $Z = \{a, c\}$. Then:

$$R \cdot Z = \{b, c, e, g\}$$
$$R|_Z = \{(a, b), (a, c), (c, e), (c, g)\}$$
$$R^2 = R \cdot R = \{(a, d), (a, c), (a, g), (b, a), (b, g), (c, ?), (d, b), (d, c)\}$$

### 3.2 What is Lazy Pointer Analysis?

We formulate liveness and points-to analysis so that points-to information is computed relative to liveness. In particular a points-to pair for a pointer is generated only if the pointer is live. Hence $\{x=\&y; \text{ return 3;}\}$ and $\{x=\&z; \text{ return 3;}\}$ calculate the same (empty) points-to information as $x$ is dead after the assignment. Further, liveness information is similarly computed relative to points-to information, using strong liveness [5] instead of the more common simple liveness. In strong liveness, the variables read in an assignment statement are considered live only if any of the variables defined by the statement are known to be live. In simple liveness all variables that are read are considered live regardless of the liveness of the variables that are defined. This formulation directly allows a joint liveness-and-points-to analysis.

- The propagation of points-to information is sparse in the CFG; a points-to pair $(x, y)$ is propagated only along those live ranges of $x$ that include the statement in which this pair is generated. In contrast, the propagation of liveness information is dense because it is propagated to all possible program points.
- Must points-to information is computed incrementally from the may points-to information without requiring an interdependent fixed point computation. This is quite unlike [10][11].

We use the example of Fig. 3 as a motivating example for our intraprocedural formulation and make the following observations:

- $p$ is live at the exit of 2 because of its use in 3 and 4. $q$ becomes live because it is used in defining $p$ in 2 and $p$ is live at the exit of 2. Hence $(q, r)$ should be generated in 1 and should be propagated everywhere except in 7 where $q$ is not live.
– Since \((q,r)\) holds in 2, \((p,r)\) should be generated and should be propagated to only 3 and 4 because \(p\) is not live anywhere else.

– \(r\) becomes live in 3 because \((p,r)\) holds in 3. Its liveness is propagated to 1, 2, and 6. It is not live in 4, 5, and 7. \((r,s)\) should be generated in 6 and should be propagated to 2 and 3 but not beyond because \(r\) is not live beyond 2 and 3.

– \(s\) is not live anywhere hence \((s,r)\) should be not generated.

### 3.3 Defining Lazy Pointer Analysis

Fig. 4 provides the data flow equations for lazy pointer analysis. They resemble the standard data flow equations of liveness analysis and pointer analyses [6]. However, there are three major differences:

– liveness and may-points-to analyses depend on each other (bi-directional),

– lazy computation and sparse propagation are directly captured in the equations, and

– must-points-to information is computed from may-points-to information (i.e. fixed-point computation is not performed for must-points-to analysis).

The initial value (\(\top\) of the corresponding lattices) used for computing the fixed point is \(\emptyset\) for both liveness and may-points-to analyses. For liveness \(Bl\) is \(\emptyset\) and defines \(Lout\); whereas points-to analysis, \(Bl\) is \(Lin \times \{?\}\) and defines \(Ain\). This reflects that no pointer is live on exit or holds a valid address on entry to a procedure.

**Extractor Functions.** The flow functions occurring in Equations (4) and (6) use extractor functions \(Def_n\), \(Kill_n\), \(Ref_n\), and \(Pointee_n\) which extract the relevant pointer variables for statement \(n\) from the incoming pointer information \(Ain_n\). These extractor functions are inspired by similar functions in [10,11].

\(Def_n\) examines the left hand side an assignment statement to find the pointer variables which may be defined by the statement to hold new addresses. \(Pointee_n\) computes potential pointees by examining the right hand side of \(n\). Thus the new points-to pairs generated for statement \(n\) are \(Def_n \times Pointee_n\) (Equation 6). Sparse propagation of points-to pairs is ensured by restricting the collected points-to pairs to live pointers. \(Ref_n\) computes the variables that become live in statement \(n\). Condition \(Def_n \cap Lout_n\) ensures that \(Ref_n\) computes strong liveness rather than simple liveness. As an exception to the general rule, \(x\) is considered live in statement \(*x = y\) regardless of whether the pointees of \(x\) are live otherwise, the pointees of \(x\) would not be discovered. For example, given \{\(x=&a; y=3; *x=y; return;\}\}, \((x,a)\) cannot be discovered unless \(x\) is marked live. Hence liveness of \(x\) cannot depend on whether the pointees of \(x\) are live. By contrast, statement \(y = *x\) uses the liveness of \(y\) to determine the liveness of \(x\).

\(Kill_n\) identifies pointer variables that are definitely modified by the execution of statement \(n\). This information is used for killing liveness as well as points-to information. For statement \(*x = y\), \(Kill_n\) depends on \(Ain_n\) which is filtered using the function \(Must\). The filtering criteria ensures that when no points-to information for \(x\) is available, we conservatively assume that all pointers are modified by statement \(*x = y\). This is consistent with the initial values of may-points-to information and liveness both of which are \(\emptyset\). Given some points-to information for \(x\), \(Must\) uses the number of pointees
Given relation $R \subseteq P \times V$ (either $A_{in}^n$ or $A_{out}^n$) we first define an auxiliary extractor function

$$
\text{Must}(R) = \bigcup_{x \in P} \{x\} \times \begin{cases}
V & (R_x = \emptyset) \lor (R_x = \{(x,?)\}) \\
\emptyset & \text{otherwise}
\end{cases}
$$

(2)

**Extractor functions for statement $n$**

Notation: we assume that $x, y \in P$ and $a \in V$. $A$ abbreviates $A_{in}^n$.  

| Stmt. | $\text{Def}_n$ | $\text{Kill}_n$ | $\text{Ref}_n$ | Pointee, $\text{Pointee}_n$ |
|-------|---------------|----------------|----------------|---------------------|
| $\text{use } x$ | $\emptyset$ | $\emptyset$ | $\{x\}$ | $\emptyset$ |
| $x = \& a$ | $\{x\}$ | $\{x\}$ | $\emptyset$ | $\{a\}$ |
| $x \equiv y$ | $\{x\}$ | $\{x\}$ | $\{y\}$ | $\emptyset$ $A\{y\}$ |
| $x \equiv *y$ | $\{x\}$ | $\{x\}$ | $\{y\} \cup (A\{y\} \cap P)$ | $\emptyset$ $A^2\{y\}$ |
| $*x = y \ (A\{x\} \cap P \ (\text{Must}(A\{x\}) \cap P)$ | $\{x, y\}$ | $\{x\}$ | $A\{y\}$ |
| other | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

Domains:

| Extractor Functions | Data Flow Values |
|---------------------|-----------------|
| $\text{Pointee}_n \subseteq V$ | $\text{Lin}_n, \text{Lout}_n \subseteq P$ |
| $\text{Kill}_n, \text{Def}_n, \text{Ref}_n \subseteq P$ | $\text{Ain}_n, \text{Aout}_n \subseteq P \times V$ |

$\text{Lout}_n = \begin{cases}
\emptyset & n \text{ is end}_p \\
\bigcup_{s \in \text{succ}(n)} \text{Lin}_s & \text{otherwise}
\end{cases}$

(3)

$\text{Lin}_n = (\text{Lout}_n - \text{Kill}_n) \cup \text{Ref}_n$

(4)

$\text{Ain}_n = \begin{cases}
\text{Lin}_n \times \{?\} & n \text{ is start}_p \\
\bigcup_{p \in \text{pred}(n)} \text{Aout}_p \bigg| \text{Lin}_n & \text{otherwise}
\end{cases}$

(5)

$\text{Aout}_n = ((\text{Ain}_n - (\text{Kill}_n \times V) \cup (\text{Def}_n \times \text{Pointee}_n)) \ | \text{Lout}_n)$

(6)

Fig. 4. Intraprocedural formulation of lazy pointer analysis.

To determine whether to perform a weak update or a strong update: when $x$ has multiple pointees we cannot be certain which one will be modified because $x$ points to different locations along different execution paths reaching $n$. In this case we employ weak update which does not allow any data flow information to be killed. By contrast, when $x$ has a single pointee other than ‘?’, it indicates that $x$ points to the same location along all execution paths reaching $n$ and a strong update can be performed.

\[\textbf{Note:} \text{this conclusion is only possible because } BI = Lin_n \times \{?\}. \text{ This value of } BI \text{ ensures that if there is a definition-free path from the start}_p \text{ statement to statement } n, \text{ we get } (x, ?).\]
Second round of liveness and points-to
provision of highlights why must-points-to analysis need not be performed explicitly. With the Must pointer that generally in node 6. Hence must-points-to information can be extracted from may-points-to information by points-to analysis, we discover pair our motivating example of Fig. 3. After the first round of liveness analysis followed towards the end. A comparison with the result of the default may-point-to analysis in node 6. Note that the result is consistent with the observation made in Section 3.2 second round of liveness analysis. This then enables discovering the points-to pair. For such nodes that are not reached by the analysis because no pointer has been found to be live.

Motivating Example Revisited. Fig. 5 gives the result of lazy pointer analysis for our motivating example of Fig. 3. After the first round of liveness analysis followed by points-to analysis, we discover pair (p, r) in Ain. Thus r becomes live requiring a second round of liveness analysis. This then enables discovering the points-to pair (r, s) in node 6. Note that the result is consistent with the observation made in Section 3.2 towards the end. A comparison with the result of the default may-point-to analysis (Fig. 3) shows that our analysis eliminates many redundant points-to pairs.

Independent Must-points-to Analysis is Redundant. The explanation of Kill and Must highlights why must-points-to analysis need not be performed explicitly. With the provision of Lin × {?} as Bl, if we have a single points-to pair (x, y) with y ≠ ? for pointer x in Ain or Out, it is guaranteed that x must point to y. Conversely multiple points-to pairs associated with a given variable means that the must-points-to information for this variable is empty. Hence must-points-to information can be extracted from may-points-to information by Un = Must(Ain) and Out = Must(Aout). Note that generally Un ⊆ Ain and Out ⊆ Aout; the only exception would be for nodes that are not reached by the analysis because no pointer has been found to be live. For such nodes, Out, Out are P × V whereas Aout, Aout are ∅; this matches

at n and so a solitary pair (x, z) also reaching n is not incorrectly treated as a must-points-to pair.
int f(int a, int b)
{
    int *x, *y = &a;  // x = '?'
    int c = a * b;
    p: *x = 5;        // Illegal write
    q: *y = 6;        // Should kill avail(a * b)
    return c + (a * b);
}

– Def_{p} = {∅}.
– Liveness of y is killed.
– Points-to information of y is empty
  (monotonicity of Must requires this).
– Statement q cannot kill availability of a * b.
– The return value will be mis-optimised into
c + c.

Fig. 6. Motivating the need for a sanity check for programs with undefined behaviour.

3.4 Design Choices in Formulating Lazy Pointer Analysis

We have chosen not to compute liveness of non-pointer variables (keeping them in $V - P$) primarily for simplicity and efficiency of implementation. While we could in principle regard these as members of $P$ which may be live but can never point to anything, this does not help discovering points-to information.

In our formulation, data flow value '?' plays an ambiguous role: it represents an uninitialised pointer value, but it is left unclear whether this means “points to some variable in $V$” or “may be a wild pointer as in C”. We have formulated the analysis as if for Java; an assignment $*x=y$ can be assumed only to write to a non-'?’ member of the points-to set of $x$—writes to '?' raise an exception. In C however, writes via uninitialised pointers can write to any location in memory, including all user variables. There are two ways to address this. Firstly, we can treat '?' as not standing for a single pointer value, but instead being a set of locations including $V$. While formally most correct, this requires modification of data flow equations in Fig. 4, for example to Must.

Alternatively, and this is the course we have followed (because it gives more optimisation opportunities), we can optimise a C-like program as if no dereferences or assignments via invalid pointer may occur at run-time, but add a “sanity check” to stop invalid optimisations when an illegal pointer assignment necessarily happens. This is possible because the semantics of both C and Java is that code after an assignment via an illegal pointer is effectively unreachable. In Java an exception is raised so the following code is not reached. In C the behaviour is “undefined” so the code can do what it wants, which includes “being mis-optimised” as a special case. However, the possibility of mis-optimisations arising out of a wild write can be detected. We observe that such a situation cannot arise in programs in which every pointer is defined along some path before being dereferenced—it is just the guaranteed dereference of '?' which causes the problem. Corollary [1] in Section 4 asserts this formally. Hence as the final step in our analysis we perform a sanity check: there must be no statement $*x = y$ for which $Def_{n}$ is $\emptyset$—otherwise optimisation is disabled. Fig. 6 provides an example that motivates this sanity check as a form of data-flow-anomaly warning (for indirect assignments via a variable with no valid pointees).
Since $a$ is live at the exit of 2 and 3, pairs $(a, b)$ and $(a, c)$ are generated which causes $b$ and $c$ to be marked live in 5. Hence $L_{out}^2 = L_{out}^4 = \{a, b, c\}$.

However, $b$ is not live along path 1-3-4-5 because $(a, b) \notin A_{out}^4$. Similarly, $c$ is not live along path 1-2-5 because $(a, c) \notin A_{out}^2$.

Due to this imprecision in liveness, we generate the pair $(b, e)$ in 3 which is then propagated to 5. This is spurious because there is no use of $b$ anywhere along this path.

![Fig. 7. Imprecision in lazy pointer analysis due to indirect liveness.](image)

In our formulation, liveness is generated from points-to information as indicated by the presence of $Ain_n\{y\}$ in $Ref_n$ for the statement $x = *y$ in Fig. 4. This leads to imprecision in liveness information which in turn leads to imprecision in points-to information as illustrated in Fig. 7. This imprecision can be avoided by exploiting the mutual dependence of liveness and points-to information: i.e. propagation of indirect liveness should also be restricted to appropriate points-to propagation paths—in the same way that propagation of points-to pairs is restricted to liveness paths. We omit this formulation (which is not used in the implementation) for space reasons.

### 4 Properties of Lazy Pointer Analysis

In this section we show that lazy pointer analysis is monotonic, sparse, and discovers all pointees of a pointer variable where it is used. Proofs are provided in Appendix A.

**Monotonicity of Lazy Pointer Analysis** The extractor functions $Def_n$, $Pointee_n$, $Kill_n$ (and $Must$) use points-to information. Besides, $Ref_n$ uses liveness information also. In order to argue about monotonicity, we parameterise the extractor functions with the required information and drop the subscript $n$.

Recall that the lattice of points-to information is $A = \langle \mathcal{P}(\mathcal{P} \times V), \supseteq \rangle$. The following results hold $\forall v \in V$ and $\forall a_1, a_2 \in A$ such that $a_1 \subseteq a_2$ (i.e. $a_1 \supseteq a_2$):

\[
\begin{align*}
    a_1 \{v\} & \supseteq a_2 \{v\} \quad \text{(follows from the definition)} \tag{7} \\
    (a_1 \circ a_1) \{v\} & \supseteq (a_2 \circ a_2) \{v\} \quad \text{(follows from the definition)} \tag{8} \\
    Def(a_1) & \supseteq Def(a_2) \quad \text{(follows from (7))} \tag{9} \\
    Pointee(a_1) & \supseteq Pointee(a_2) \quad \text{(follows from (8))} \tag{10} \\
    (Must(a_1)) \{v\} & \subseteq (Must(a_2)) \{v\} \quad \text{(follows from (2) and (7))} \tag{11}
\end{align*}
\]

**Theorem 1.** Define the liveness flow function $f_L : \mathcal{L} \times A \rightarrow \mathcal{L}$ (Equation 4) and the may-points-to flow function $f_A : \mathcal{L} \times A \rightarrow A$ (Equation 6) as:

\[
\begin{align*}
    f_L(l, a) & = (l - Kill(a)) \cup Ref(l, a) \\
    f_A(l, a) & = (a - (Kill(a) \times V)) \cup (Def(a) \times Pointee(a))|_{l}
\end{align*}
\]

Then, $f_L$ and $f_A$ are monotonic.
Sparseness of Lazy Pointer Analysis Lazy pointer analysis is a form of sparse data flow analysis: the analysis is only done on live ranges rather than everywhere (contrast the previous use of the term to mean “along def-use chains”).

Equations (3) and (4) identify liveness paths for variables representing control flow paths in the program along which variables are live. For a given variable $x$, a liveness path is defined as a maximal sequence of statements $s_1, s_2, \cdots, s_k$ satisfying the following conditions:

- $x \in \text{Ref}_{s_k}$. (L1)
- $s_{i+1} \in \text{succ}(s_i), 1 \leq i < k$. (L2)
- $x \notin \text{Kill}_{s_i}, x \in \text{Lin}_{s_i}, 1 < i \leq k$. (L3)
- $x \in \text{Lout}_{s_i}, 1 \leq i < k$. (L4)

(L1) represents generation of liveness of $x$, (L2) insists on the sequence being a control flow path, while (L3) and (L4) ensure that the sequence is a modification free path.

Equations (5) and (6) identify propagation paths for points-to pairs representing control flow paths in the program along which points-to pairs are propagated. For a given points-to pair $(x, y)$, a propagation path is defined as a maximal sequence of statements $s_1, s_2, \cdots, s_k$ satisfying the following conditions:

- $(x, y) \in (\text{Def}_{s_1} \times \text{Pointee}_{s_1})$. (A1)
- $s_{i+1} \in \text{succ}(s_i), 1 \leq i < k$. (A2)
- $(x, y) \notin \text{Kill}_{s_i}, x \notin \text{Lin}_{s_i}, 1 < i \leq k$. (A3)
- $(x, y) \in \text{Aout}_{s_i}, x \in \text{Lout}_{s_i}, 1 \leq i < k$. (A4)

(A1) represents generation of the pair $(x, y)$, (A2) ensures that the sequence is a control flow path, while (A3) and (A4) ensure propagation along a modification free path.

Theorem 2. Every propagation path for a points-to pair $(x, y)$ is a suffix of some liveness path for $x$.

Sufficiency of Lazy Pointer Analysis At the program point of every use of a pointer variable, lazy pointer analysis discovers all pointees of the pointer variable. We first observe a useful relationship between $\text{Kill}_n$ and $\text{Def}_n$.

Lemma 1. $((\text{Kill}_n \neq \emptyset) \land (\text{Kill}_n \neq P)) \Rightarrow (\text{Kill}_n = \text{Def}_n)$.

Theorem 3. If $x \in P$ holds the address of $z \in (V - \{\text{?}\})$ along some execution path reaching node $n$, then $x \in \text{Ref}_n \Rightarrow (x, z) \in \text{Ain}_n$.

Corollary 1. If all pointer variables are initialised with values of proper types before they are used, then for every indirect assignment $*x = y$, $\text{Def}_n \neq \emptyset$.

5 Interprocedural Lazy Pointer Analysis

We use the call-strings method (Section 2) to ensure flow- and context-sensitivity. Since it is a generic method orthogonal to any particular analysis, lifting an intraprocedural formulation of an analysis to interprocedural level is straightforward. In our case, $\text{Lin}_n, \text{Lout}_n$ and $\text{Ain}_n, \text{Aout}_n$ become sets of pairs $(\sigma, a), a \in \mathcal{A}$ and $(\sigma, a), l \in \mathcal{L}$ at the interprocedural level where $\sigma$ is a call string reaching node $n$. The final values of $\text{Ain}_n, \text{Aout}_n$ are computed by merging the values along all call strings.
Terminating Call String Construction. We use data flow values [8] to terminate call-string construction instead of using a precomputed length as proposed originally [7]. This approach discards redundant call strings at \( \text{start}_p \) and regenerates them at \( \text{end}_p \) for forward flows as follows (and the other way round for backward flows):

- **Representation.** If two call strings \( \sigma \) and \( \sigma' \) have identical data flow values at \( \text{start}_p \), both need not be propagated within the body of \( p \) because the data flow values of both \( \sigma \) and \( \sigma' \) will undergo the same change and will remain identical at \( \text{end}_p \). More formally, \( \text{Out}_{\text{start}_p} \) is now computed as follows:

\[
\text{Out}_{\text{start}_p} = \{ \text{Rep}(\langle \sigma, x \rangle, p) \mid \langle \sigma, x \rangle \in \text{In}_{\text{start}_p} \}
\]

where,

\[
\text{Rep}(\langle \sigma, x \rangle, p) = \begin{cases} 
\langle \sigma', x \rangle & \langle \sigma', x \rangle \in \text{In}_{\text{start}_p}, |\sigma'| \leq |\sigma| \\
\langle \sigma, x \rangle & \text{otherwise}
\end{cases}
\]

- **Regeneration.** At \( \text{end}_p \), we examine the representation performed at \( \text{start}_p \). If \( \sigma \) represents \( \sigma' \), the data flow value associated with \( \sigma \) is copied to \( \sigma' \). Thus, \( \text{Out}_{\text{end}_p} \) is computed as follows:

\[
\text{Out}_{\text{end}_p} = \{ \langle \sigma', y \rangle \mid \langle \sigma, y \rangle \in \text{In}_{\text{end}_p}, \langle \sigma', y \rangle \in \text{Reg}(\langle \sigma, y \rangle, p) \}
\]

\[
\text{Reg}(\langle \sigma, y \rangle, p) = \{ \langle \sigma', y \rangle \mid \text{Rep}(\langle \sigma', x \rangle, p) = \langle \sigma, x \rangle \}
\]

Representation partitions call strings into equivalence classes based on the data flow values associated with them. Regeneration recreates the represented call string and recovers their values based on the partitions they belong to.

Matching Contexts for Liveness and Points-to Analysis. Since points-to information should be restricted to live ranges, it is propagated along the call strings constructed during liveness analysis. However, in the presence of recursion, we may need additional call strings for which liveness information may not be available. We explain below how this is handled.

Let \( \sigma_a \) denote an acyclic call string (i.e. a call string for an interprocedural control flow path with no unfinished recursive calls). Let \( \sigma_c \alpha^i \) denote a cyclic call string which corresponds to an interprocedural control flow path with unfinished recursive calls; \( \alpha \) denotes an acyclic sequence of call sites corresponding to unfinished recursive calls and \( i \) denotes the depth of recursion in the path. Then:

- The partitioning information for every \( \sigma_a \) is available because either \( \langle \sigma_a, x \rangle \) has reached node \( n \) in procedure \( p \) or \( \sigma_a \) has been represented by some other call string.
- Assume that the data flow values of \( \sigma_c \alpha^i \) are different for \( i \leq k \) for some \( k \geq 0 \) and the data flow values of \( \sigma_c \alpha^k \) and \( \sigma_c \alpha^{k+j}, j \geq 1 \) are identical. Then the partitioning information is available for only \( \sigma_c \alpha^k \) and \( \sigma_c \alpha^{k+1} \) because the call strings \( \sigma_c \alpha^{k+j}, j \geq 1 \) are not constructed.

Consider a call string \( \sigma' \) reaching node \( n \) during points-to analysis. From the above observations about partitioning it is clear that, if \( \sigma' \) is an acyclic call string then its partitioning information and hence its liveness information is available. If \( \sigma' \) is a cyclic call string, its value may not be available if it happens to be \( \sigma_c \alpha^{k+j}, j > 1 \). However, it is sufficient to locate the longest prefix of \( \sigma_c \alpha^{k+j} \) and use its liveness information. This is illustrated below in our motivating example.
Motivating Example Revisited. For brevity, let $I_n$ and $O_n$ denote the entry and exit of node $n$. In the first round of liveness (Fig. 8), $z$ becomes live at $I_6$ as $(\lambda, z)_L$, reaches 13, 12, and 11 as $(c_1, z)_L$, becomes $(c_1c_2, z)_L$ at $I_{11}$, reaches $O_{13}$ and gets represented by $(c_1, z)_L$. Hence $(c_1c_2, z)_L$ is not propagated within the body of $p$. $(c_1c_2, z)_L$ is regenerated at $I_8$, becomes $(c_1, z)_L$ at $I_{10}$, becomes $(c_1, w)_L$ at $I_9$. At $O_8$, it combines with $(c_1, z)_L$ propagated from $I_{13}$ and becomes $(c_1, w, z)_L$. Thus $c_1c_2$ is regenerated as $(c_1c_2, w, z)_L$ at $I_8$. $(c_1, w, z)_L$ reaches 4 and becomes $(\lambda, w, z)_L$.

In the first round of points-to analysis (Fig. 8), since $z$ is live at $I_1$, $Bl = (\lambda, (z, ?))_A$. $(\lambda, (w, x))_A$ is generated at $O_3$. Thus $(c_1, (w, x), (z, ?))_A$ reaches $I_8$. This becomes $(c_1, (w, x), (z, x))_A$ at $O_9$ and reaches as $(c_1c_2, (w, x), (z, x))_A$ at $I_8$. Since $z$ is not live at $I_9$, $(c_1c_2, (w, x))_A$ is propagated to $I_9$ which causes $(c_1c_2, (w, x), (z, x))_A$ to be generated at $O_10$ which reaches $I_9$ and is represented by $(c_1c_2, (w, x), (z, x))_A$. This is then regenerated as $(c_1c_2, (z, x))_A$ at $O_{11}$ because only $z$ is live at $O_{13}$. Note that we do not have the liveness information along $c_1c_2c_2$ but we know that it must be the same as the liveness information along $c_1c_2$. We get $(c_1c_2, (z, x))_A$ and $(c_1, (z, x))_A$ at $O_{11}$. Since we have no points-to information for $x$, we get $(c_1c_2, \emptyset)_A$ and $(c_1, \emptyset)_A$ at $O_{12}$.

The second round of liveness and points-to analysis is presented in Fig. 8. We leave it for the reader to verify that $x$ becomes live due to $z = \mathit{sz}$ in 12, reaches 2 and causes $(\lambda, x)_A$ to be generated. As a consequence, we get $(z, y)$ in 12.
This result corresponds to the observations in Section 1. Note that \((z, x)\) cannot reach 6 along any interprocedurally valid path. However, the method of [10] which is considered most precise flow- and context-sensitive method, computes \((z, x)\) at 6.

6 Related Work

The benefits of flow- and context-sensitivity have been found to vary from marginal to large in the literature [14, 15, 16, 17]. It has also been observed that an increase in precision could increase efficiency. However, studies have been inconclusive by and large and a large number of investigations relax flow- or context-sensitivity (or both) in their pursuit of efficiency in pointer analysis. Our premise is that the use of liveness enhances the effectiveness of flow- and context-sensitivity significantly. A flow-insensitive approach cannot benefit from liveness. The use of liveness in context-insensitive approaches has not been investigated.

We focus on approaches that are both flow- and context-sensitive. A memoisation-based functional approach observes that the number of possible pointer patterns that reach a procedure are small and hence it is beneficial to use partial transfer functions [18] instead of the usual full transfer functions. An alternative functional approach creates full transfer functions but contains the complexity of computing transfer functions by making them sensitive to the “level” of a pointer (i.e. the possible depth of its indirection) [19]. Transfer functions for a given level are defined in terms of lower-level transfer functions. The invocation-graph-based approach unfolds a call graph in terms of call chains [10]. Our work is inspired by this approach but we have incorporated
strong liveness and manage contexts very differently. Finally, a radically different approach proceeds in the opposite direction and begins with flow- and context-insensitive information which is refined systematically in cascaded steps to restrict it to flow- and context-sensitive information [20].

The above approaches summarise points-to information in recursive contexts using fixed-point iteration. This merges the information across different levels of nesting and all recursive calls receive the same summarised information. The call-strings approach maintains distinct data-flow values for each nesting depth of recursion. The partial-transfer-function-based approach [18] is slightly more precise than the invocation-graph-based approach [10] because it distinguishes the outer call to a recursive procedure from the calls inside the recursion. For example, in our motivating example, \((z, x)\) holds only in the recursive calls of \(p\). When recursion unwinds fully, \(z\) does not point to \(x\). Our approach discovers this correctly but [10] cannot do so. Fig. 9.6 (page 305) in [6] contains an example for which the methods in [18,10] compute imprecise results.

GCC uses a context-insensitive analysis which acquires limited flow sensitivity due to the effect of SSA representation—a half-way house. However, SSA form does not apply to pointers directly and interleaved SSA construction and pointer analysis are required [21] which is not done in GCC. Appendix [8] shows by example that the points-to information in GCC is effectively flow-insensitive.

7 Implementation and Empirical Measurements

We have implemented interprocedural lazy points-to analysis in GCC 4.6.0. It requires the command line switches \(-flto -flto-partition=none -flipta\) to invoke GCC’s Link Time Optimisation (LTO), pass on the control flow and call graphs, and finally perform lazy points-to analysis on the constructed supergraph. This implementation is available for download [3].

We have executed our implementation on SPEC CPU2006 Integer benchmarks as well as some programs from SPEC2000 benchmarks on a machine with 16 GB RAM running 8 processors (64-bit intel i7-960 CPU at 3.20GHz). The results of measurements are presented in Fig. 10. We compare three implementations: lazy points-to analysis (lpta), simple points-to analysis (spta) and GCC’s points-to analysis (gpta). The only difference between lpta and spta is that lpta uses liveness whereas spta does not—both are flow- and context-sensitive and use call strings with value-based termination. gpta is flow- and context-insensitive (see Section [3] for more details about GCC’s points-to analysis). All three methods use the same approach of handling arrays, heap locations, pointer arithmetic, function pointers, and field sensitivity.

Both lpta and spta are naive implementations that use linked lists and linear searches within them. The main goal of these implementations was to find out whether liveness increases the precision of points-to information. Our measurements confirm this hypothesis beyond doubt. Surprisingly, the time measurements exceeded our expectations because we had not designed these implementation for time/space efficiency or scalability. We were able to run our implementations on programs of around 30kLoC but not on the larger programs. It is evident from the measurements that:

5 http://www.cse.iitb.ac.in/grc/index.php?page=lipta
| Program     | lLoC  | Call Sites | Time in milliseconds | Points-to pairs | | Max#cs |
|-------------|-------|------------|---------------------|-----------------|--------|
|             |       |            |                     | lpta           | spta   | gpta   | lpta | spta | gpta |        |
| lbm         | 0.9   | 33         | 0.55                | 0.52            | 1.9    | 5.2    | 12   | 307  | 1911 | 4      |
| mcf         | 1.6   | 29         | 1.04                | 0.62            | 9.5    | 3.4    | 41   | 367  | 2159 | 4      |
| libquantum  | 2.6   | 258        | 2.0                 | 1.8             | 5.6    | 4.8    | 49   | 119  | 2701 | 55     |
| bzip2       | 3.7   | 233        | 4.5                 | 4.8             | 28.1   | 30.2   | 60   | 210  | 8.8×10⁴ | 70   |
| parser      | 7.7   | 1123       | 1.2×10⁴             | 145.6           | 4.3×10⁷ | 422.12 | 531  | 4196 | 1.9×10⁴ | 4619 |
| sjeng       | 10.5  | 678        | 858.2               | 99.0            | 3.2×10⁷ | 38.1   | 267  | 818  | 1.1×10⁴ | 4649 |
| hmmer       | 20.6  | 1292       | 90.0                | 62.9            | 2.9×10⁵ | 246.3  | 232  | 5805 | 1.9×10⁶ | 554   |
| gap         | 35.6  | 5312       | 4.6×10⁴             | 1.3×10⁵         | 1.0×10⁷ | 1.7×10⁴ | 421  | 1271 | 2.5×10⁷ | 1203 |
| h264ref     | 36.0  | 1992       | 2.2×10⁴             | 2.0×10⁴         | 7.4×10³ | 1685   | 274  | 1.6×10⁸ | 46600 |

Fig. 10. Empirical measurements. A “?” indicates that the analysis ran out of memory. Max#cs denotes maximum number of call strings at any program point for lpta.

- Lazy computation of points-to pairs reduces the number of points-to pairs dramatically. Although we could observe this for programs of approximately 30kLoC, we have no reason to believe that the situation would be different for larger programs.
- Lazy computation and sparse propagation of points-to pairs reduces execution time too and lpta out-performs gpta for most programs smaller than 30kLoC. That a flow- and context-sensitive analysis could be faster than flow- and context-insensitive analysis comes as a surprise to us. lpta shows that the actual data that we can gainfully use is much smaller than what is generally thought to be.
- A reduction in the number of data flow values enhances the effectiveness of value based termination of call strings and in most cases the number of contexts required for precise analysis is not exponentially large. Further, the maximum length of any call string never exceeded two digits.

The hypothesis that our implementation suffers because of linear search in linked lists was confirmed by an accidental discovery: in order to eliminate duplicate pairs in gpta, we used our data structure and function from lpta that adds points-to pairs in a linked list and maintains a unique entry for each pair in the list. With this addition, gpta executed for well over an hour on the hmmer program whereas originally gpta needed 246.3 milliseconds only! Since lpta uses linked lists to represent sets, it has to maintain uniqueness at each stage and this seems to be the primary reason why we could not execute it on the larger programs: gobmk, perlbench, and gcc.

Eager liveness computation to reduce points-to analysis work could also be a source of inefficiency: a new round of liveness is invoked when a new points-to pair for y is discovered for x = *y putting on hold the points-to analysis. This explains the unusually large time spent in liveness analysis compared to points-to analysis for programs parser and sjeng. The number of rounds of analysis required for these programs was much higher than in other programs of comparable size.

Our implementation can be improved many ways.
We can use efficient data structures (vectors or hash tables) supported by GCC. Alternatively, we can use BDDs to efficiently maintain sets of data flow values.

The LTO framework could be modified to load CFGs on demand. Currently, LTO gives one large program with all CFGs or just a call graph without CFGs. This results in a very large supergraph in memory—affecting locality (cache misses) partly explaining the 30kLoC threshold.

Our implementation performs full computations of liveness and points-to analysis. Revisiting a statement typically causes only a small additional amount of information to be generated. We posit significant savings by exploiting this third dimension of laziness: compute information incrementally on revisits.

Apart from improving the implementation, another route to scalability lies in the observation that 30kLoC seems to be a cross-over point: If we can preprocess programs to identify chunks of around 30kLoC which are very loosely coupled as far as pointer usage is concerned, we can expect this method to scale to much larger programs.

### 8 Conclusions and Future Work

We have described a data-flow analysis which jointly calculates points-to and liveness information. It does this in a flow- and context-sensitive way, using recent developments of the “call strings” approach. One novel aspect to our approach is that it is effectively bi-directional (such analysis seem relatively rarely exploited).

Initial results from our naive prototype implementation were impressive: unsurprisingly our analysis produced much more precise results, but by an order of magnitude (in terms of the size of the calculated points-to information). The reduction of this size allowed our naive implementation also to run faster than GCC’s points-to analysis at least for programs up to 30kLoC. This is significant because GCC’s analysis compromises both on flow and context sensitivity. This confirms our belief that separating relevant information from irrelevant information can have significant benefits and is a promising direction for further investigations.

We would like to take our work further by exploring the following:

- Improving our implementation: e.g. using efficient data structures such as vectors or hash tables, or perhaps BDDs. Improving the interface to GCC’s LTO framework by allowing the call graph to be loaded as a single unit, but then loading individual CFGs on demand so as not to keep the whole-program supergraph in memory at one time.
- Exploring the reasons for the 30kLoC speed threshold; while interprocedural analyses are very likely to be super-linear in terms of the number of procedures, perhaps there are ways in practice to partition most bigger programs (around loosely-coupled boundaries) without significant loss of precision.
- Currently our use of incremental computation is solely to avoid computing useless and imprecise data-flow information. However, we note that data-flow information often only slightly changes when revisiting a node compared to the information produced by the first iteration. We plan to explore incremental formulations of our lazy points-to analysis.
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20
A Proofs of Lemmas and Theorems

Theorem 1. Define the liveness flow function \( f_L : \mathcal{L} \times \mathcal{A} \rightarrow \mathcal{L} \) (Equation 4) and the may-points-to flow function \( f_A : \mathcal{L} \times \mathcal{A} \rightarrow \mathcal{A} \) (Equation 6) as:

\[
\begin{align*}
    f_L(l, a) &= (l - \text{Kill}(a)) \cup \text{Ref}(l, a) \\
    f_A(l, a) &= (a - (\text{Kill}(a) \times V)) \cup (\text{Def}(a) \times \text{Pointee}(a))
\end{align*}
\]

Then, \( f_L \) and \( f_A \) are monotonic.

Proof. Monotonicity of \( f_L \) can be proved by showing that \( \forall (l_1, a_1), (l_2, a_2) \in \mathcal{L} \times \mathcal{A}, \)

\[
(l_1, a_1) \subseteq (l_2, a_2) \Rightarrow (l_1 - \text{Kill}(a_1)) \supseteq (l_2 - \text{Kill}(a_2)) \quad (12)
\]

\[
(l_1, a_1) \subseteq (l_2, a_2) \Rightarrow \text{Ref}(l_1, a_1) \supseteq \text{Ref}(l_2, a_2) \quad (13)
\]

(12) follows from (11) while (13) follows from (7). Monotonicity of \( f_A \) can be proved by showing that \( \forall (l_1, a_1), (l_2, a_2) \in \mathcal{L} \times \mathcal{A}, \)

\[
(l_1, a_1) \subseteq (l_2, a_2) \Rightarrow (a_2 - (\text{Kill}(a_1) \times V)) \supseteq (a_1 - (\text{Kill}(a_2) \times V)) \quad (14)
\]

\[
(l_1, a_1) \subseteq (l_2, a_2) \Rightarrow (\text{Def}(a_1) \times \text{Pointee}(a_1)) \supseteq (\text{Def}(a_2) \times \text{Pointee}(a_2)) \quad (15)
\]

(14) follows from (11) while (15) follows from (9) and (10). □

Theorem 2. Every propagation path for a points-to pair \((x, y)\) is a suffix of some liveness path for \(x\).

Proof. Consider an arbitrary propagation path \(\rho_a\) for \((x, y)\). Since \(L2 \Leftrightarrow A2, A3 \Rightarrow L3,\) and \(A4 \Rightarrow L4\), it is easy to see that every statement \(n\) along \(\rho_a\) must also be part of a liveness path for \(x\). Let this liveness path be \(\rho_l\). Then the proof obligation reduces to showing that the last statement of \(\rho_a\) must also be the last statement of \(\rho_l\). In other words, we need to show that

C1. \(\rho_a\) does not end somewhere in the middle of \(\rho_l\), and
C2. \(\rho_a\) does not extend beyond \(\rho_l\).

We prove these by contradiction. For case (C1), assume that the last statement of \(\rho_a\) appears somewhere in the middle of \(\rho_l\) on position \(j\). Consider statements \(s_j\) and \(s_{j+1}\) in \(\rho_l\) such that \(s_j\) also appears in \(\rho_a\). From (L3) and (L4), in \(x \in \text{Lin}_{s_j}, x \in \text{Lout}_{s_j},\) and \(x \in \text{Lin}_{s_{j+1}}\). Also, \(x\) is neither in \(\text{Kill}_{s_j}\) nor \(\text{Kill}_{s_{j+1}}\) from (L3). Further \((x, y) \in \text{Ain}_{s_j}\) from (A2).

\[
(x, y) \in \text{Ain}_{s_j} \land x \notin \text{Kill}_{s_j} \land x \in \text{Lout}_{s_j} \Rightarrow (x, y) \in \text{Aout}_{s_j}
\]

\[
(x, y) \in \text{Aout}_{s_j} \land x \in \text{Lin}_{s_{j+1}} \Rightarrow (x, y) \in \text{Ain}_{s_{j+1}}
\]

Thus (A3) is satisfied for \(s_{j+1}\) also. Hence \(\rho_a\) is not maximal and can be extended to include \(s_{j+1}\). This leads to contradiction.

For case (C2) assume that the last statement of \(\rho_l\) appears somewhere in the middle of \(\rho_a\) on position \(j\). Consider statements \(s_j\) and \(s_{j+1}\) in \(\rho_a\) such that \(s_j\) also appears in \(\rho_l\). Then by conditions (A3) and (A4), \(x \notin \text{Kill}_{s_j}, x \in \text{Lout}_{s_j},\) and \(x \in \text{Lin}_{s_{j+1}}\). Thus \(\rho_l\) is not maximal and can be extended to include \(s_{j+1}\). This leads to contradiction. □
Lemma 1. \(((\text{Kill}_n \neq \emptyset) \land (\text{Kill}_n \neq \text{P})) \Rightarrow (\text{Kill}_n = \text{Def}_n)\).

Proof. The lemma trivially holds for all statements other than indirect assignment \(\ast x = y\). For the latter,

\[
((\text{Kill}_n \neq \emptyset) \land (\text{Kill}_n \neq \text{P})) \Rightarrow ((\text{Ain}_n|_x \neq \emptyset) \land ((\text{Ain}_n|_x) \neq \{(x, ?)\}) \Rightarrow (\text{Ain}_n = \{(x, z)\} \land (z \in \text{P})
\]

Hence \(\text{Kill}_n = \text{Def}_n = \{z\}\). \(\square\)

Theorem 3. If \(x \in \text{P}\) holds the address of \(z \in (\text{V} \setminus \{?\})\) along some execution path reaching node \(n\), then \(x \in \text{Ref}_n \Rightarrow (x, z) \in \text{Ain}_n\).

Proof. Let the execution path reaching node \(n\) be denoted by \(\rho \equiv s_0, s_1, \ldots, s_k\) where \(s_0 = \text{start}_p\) and \(s_k = n\). We prove the theorem by induction on path length \(k\). The basis is \(k = 2\) where \(s_1\) assigns the address of \(z\) to \(x\) and \(s_2\) uses it. Since \(x \in \text{Lin}_{s_2}\), the sequence \(s_1, s_2\) is trivially both a liveness path as well as points-to propagation path. Thus, \(x \in \text{Ref}_n \Rightarrow (x, z) \in \text{Ain}_n\).

Assume that the inductive hypothesis holds for \(k = i\). Consider the case when \(k = i + 1\). Note that \(x \in \text{Lout}_{s_i}\). Statement \(s_i\) could influence \(x\) in the following ways:

- \(x \notin \text{Kill}_{s_i}\). Assume that the last node in path \(\rho\) in which \(x\) is assigned a value is \(s_m\), \(m < i\). Statement \(s_m\) either directly assigns \&\(z\) to \(x\), or does so through some variables in \(\text{Ref}_{s_m}\). By inductive hypothesis, the pointees of every variable in \(\text{Ref}_{s_m}\) have been discovered in \(\text{Ain}_{s_m}\). Thus points-to analysis would discover that \((x, z) \in \text{Aout}_{s_m}\). The suffix of \(\rho\) from \(s_m\) to \(s_{i+1}\) is both a liveness path for \(x\) and points-to propagation path for \((x, z)\). Hence \((x, z) \in \text{Ain}_n\).

- \(x \in \text{Kill}_{s_i}\). In this case, \(\text{Kill}_{s_i}\) could be \(\text{P}\) if statement \(s_i\) is an indirect assignment \(\ast w = y\). Since \(w \in \text{Lin}_{s_i}\), by inductive hypothesis \((w, u) \in \text{Ain}_{s_j}\) such that \(u \neq \?\). Hence the first condition of \(\square\) cannot be satisfied. Thus this case is ruled out and \(\text{Kill}_{s_i} \neq \text{P}\). However since \(\text{Kill}_{s_i} \neq \emptyset\), \(\text{Kill}_{s_i} = \text{Def}_{s_i} = \{x\}\) from Lemma \(\square\) By a reasoning similar to that of node \(s_m\) in the previous case, \((x, z) \in \text{Aout}_{s_i}\). Since \(x \in \text{Lin}_{s_{i+1}}\), the sequence \(s_i, s_{i+1}\) is trivially both a liveness path as well as points-to propagation path. Thus, \((x, z) \in \text{Ain}_n\).

Thus the theorem holds because the inductive hypothesis holds for \(k = i + 1\). \(\square\)

Corollary 1. If all pointer variables are initialised with values of proper types before they are used, then for every indirect assignment \(\ast x = y\), \(\text{Def}_n \neq \emptyset\).

Proof. Since \(x \in \text{Ref}_n\), \(\exists (x, z) \in \text{Ain}_n\) such that \(z \neq \?\) from Theorem \(\square\) Thus \(\text{Def}_n\) cannot be \(\emptyset\). \(\square\)

\(\text{When statement } n \text{ uses } \ast x, \text{ the minimum length should be } k = 3 \text{ so that the pointee of pointee of } x \text{ is also defined but this is not relevant at the moment.}\)
B Flow Insensitivity in GCC’s Points-to analysis

Consider the following program:

```c
#include <stdio.h>
int a, b, c, *e;
int main()
{
    if (a == b)
        e = &c; /* statement n1 */
    else
        e = &b; /* statement n2 */
    e = &a; /* statement n3 */
p();
}
p()
{
    printf("%d", e);
}
```

In a flow sensitive analysis the points-to set of `e` will not contain `a, b, c` at the same time. There should be four different points-to sets associated with `e`: After `n1` and `n2`, it should be `{c}` and `{b}` respectively whereas it should be `{b, c}` before `n3` and `{a}` after it. However, GCC computes a single points-to set for `e` that contains all three of them. The relevant fragment from GCC’s dump is as follows:

Points-to sets

```plaintext
NULL = { }
ANYTHING = { ANYTHING }
READONLY = { READONLY }
ESCAPED = { READONLY ESCAPED NONLOCAL a b c }
NONLOCAL = { ESCAPED NONLOCAL }
CALLUSED = { }
STOREDANYTHING = { }
INTEGER = { ANYTHING }
e.0_1 = same as e
e = { ESCAPED NONLOCAL a b c }
a.1_1 = { ESCAPED NONLOCAL }
a = same as a.1_1
b.2_2 = { ESCAPED NONLOCAL }
b = same as b.2_2
c = { ESCAPED NONLOCAL }
```