Clothoid curve optimization for parallel parking path planning and tracking control

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Abstract. In order to solve the problem of abrupt curvature change at the connection between arcs and straight lines in circle-line-circle (C-L-C) combined parallel parking paths, curvature optimization was carried out by using a cycloid curve, and the trajectory curvature and heading Angle were made to meet the pose requirements of parallel parking. By establishing the kinematics model state equation and taking the optimized C-L-C trajectory as the reference trajectory, the error model was obtained. The model predictive controller based on the error model was designed, and the effectiveness of the path planning and model predictive controller was verified on Carsim and Matlab/Simulink co-simulation platform.

1. Introduction
In recent years, with the rapid growth of car ownership, the narrow parking space poses challenges to many drivers. Therefore, drivers are in urgent need of automatic parking system to liberate them from parking. At present, automatic parking technology is mainly divided into three aspects: environment perception, path planning and path tracking[1]. Sungwoo etc[2], zhang, etc[3] and the Su Bo[4], in order to adjust the vehicle posture at the end of parking, tangent arc is used for planning. This method has high computational efficiency and simple planning method, but it has the problem of curvature mutation at the tangent point. Li hong etc[5], by establishing the collision and steering wheel constraint functions in the parking process, b-spline curve is solved to obtain the parking path with continuous curvature. However, the b-spline solution requires a large amount of calculation and the selection of constraint function is complicated. Huang Jiang etc[6], used quintic polynomial to optimize C-L-C parking path. In order to ensure that the curvature of the whole path does not exceed the maximum curvature, the quintic multiple optimization method needs to adjust the initial parking position and increase the parking space to achieve. The curvature of any point on the line changes with the length of the line. This paper uses this characteristic to optimize the path curvature at the curvature mutation of the arc and the line in the arc-linear-arc combination parallel parking trajectory, and uses model predictive control to track and plan the path in track tracking. Finally, simulation experiments are carried out in Carsim and Matlab/Simulink co-simulation platform.

2. Vehicle kinematics model
Because the kinematic characteristic of the vehicle is greater than the dynamic characteristic in the parking process, lateral slip of the wheel is not needed to be considered. The vehicle is regarded as a rectangular rigid body, and the maximum external dimension of the vehicle is the shape parameter of the rectangle. Based on the Ackerman steering principle, the vehicle kinematics model is established in
the geodetic coordinate system, as shown in Fig 1.

Establish vehicle kinematics model:

\[
\begin{align*}
\dot{x}_r &= V_r \cos \varphi \\
\dot{y}_r &= V_r \sin \varphi \\
\dot{\varphi} &= \left( V_r \tan \delta \right) / L
\end{align*}
\]

(1)

\(x_r\) and \(y_r\) respectively represent the horizontal and vertical coordinates of the rear axle center point of the vehicle; \(V_r\) is the velocity of the center point of the rear axle of the vehicle; \(\varphi\) is yaw angle; \(\delta\) is equivalent vehicle front wheel angle; \(L\) is for wheelbase of vehicle.

Fig 1. Vehicle kinematics model

3. Parallel parking path planning and optimization

In this paper, the C-L-C combination method is adopted to design the parallel parking path, which is composed of two arcs with the same radius and a straight line segment tangent to the arc, as shown in Fig 2. As \(P_1\) the starting point, the vehicle heading angle is zero, the vehicle is put into reverse gear, \(\varphi_1\) the steering center \(R_{\text{min}}\) is the radius, and \(\varphi_1\) the driving angle reaches \(\varphi_1\); Arrive drive \(P_2\) along a straight line arrive \(P_3\), then consider \(\varphi_1\) the center of the circle \(R_{\text{min}}\) as the radius of \(\varphi_1\) the driving Angle arrive \(P_4\) complete parking. In this paper, the planning method of literature [6] is adopted to carry out preliminary planning for C-L-C parking paths and determine the range of parking starting points.

Fig 2. Parallel parking process
The parallel parking path designed based on C-L-C combination method has curvature mutation at the point of arc and straight line connection, which will produce in-situ steering, resulting in non-uniform tire wear, and increase the probability of tire explosion accident at high speed. Because the ground friction of in-situ steering tire is large, the working load of steering power motor is increased. Therefore, in this paper, based on the smoothing principle of cycloid curve as shown in Figure 3, the connection between the arc and the straight segment of parallel parking path is optimized to realize the transition of curvature smoothing at the connection between the arc and the straight segment.

As shown in Fig 3, the curvature of the arc $\overline{EFFE}$' is a constant value $\rho = 1/R_{\text{arc}}$, $\angle EOE' = \varphi_1$, $\angle EOF = \angle EOF' = \varphi_2$. The curve is adopted to design the transition path NF where the curvature increases linearly from zero to a fixed value $\rho$, and the Angle between the tangent line of $\overline{EFFE}'$ at point F and the straight line $\overline{NO}$ is $\varphi_1$. Symmetrically, the cycloid curve is used to design the transition path $\overline{FN}$ where the curvature decreases linearly from a fixed value $\rho$ to zero, and the Angle between the tangent line of the arc $\overline{EFFE}'$ at the point $F'$ and the line $\overline{NO}'$ is $\varphi_2$. After smoothing the two ends of $\overline{EFFE}'$ with the clothoid curve, the endpoints $E$ are modified as $N$, and the endpoints $E'$ are modified as $N'$. Since the movement of $\Delta NO, N'$ along the direction $\overline{OJ}$ can be overlapped with $\Delta J0, J'$, the end point N is further modified as point J after the curvature optimization of $\overline{EFFE}'$ by using the clothoid curve, and the end point $J'$ is further modified as $J''$. It can be seen from the analysis that the curvature optimization can be realized at the cost of increasing parking space by using the curve to design the transition path.

By calculating the length of line segment $\overline{JE}$ and $\overline{EJ}'$, the parking space can be increased quantitatively. As shown in Fig 3, coordinate system XNY is established. Since the curvature of any point on the rondo line changes linearly with the curve length, the curve length from point N to point F of the rondo curve is:

$$s = 1/c$$

Where, $c$ is the rate of change of curvature.

According to the calculation formula of coordinates and tangent Angle of any point on the rondo line, coordinate $(x_r, y_r)$ of point F and Angle $\varphi$ of tangent line and line $\overline{NO}$ of arc $\overline{EFFE}$ point F can be obtained as:
\[ x_y = \sum_{i=0}^{\infty} (-1)^i \frac{\phi^{i+1}}{(i+1)(2i+1)!} \]
\[ y_y = \sum_{i=0}^{\infty} (-1)^i \frac{\phi^{i+1}}{(i+1)(2i+1)!} \]
\[ \phi = \frac{c}{2} \frac{s_y}{s_x} \]

Where, \( Z \) is the approximate order of the clothoid curve.

According to Equation (3) and (5), the lateral offset of point N and point E is:
\[ l_1 = x_y - R_x \sin \phi_2 \]  

According to Equations (4) and (5), the longitudinal offset of point N and point E is:
\[ l_2 = y_y - R_y \sin (1 - \cos \phi_2) \]

The offset line segment JE and the length of the line segment \(JE'\) can be obtained from (6) and (7):
\[ l_4 = |JE| = |JE'| = l_1 + l_2 \tan \left( \frac{\phi_2}{2} \right) \]

The optimized parking path is shown in the dotted line segment in Fig 4. The parallel parking path curve \(PPPP_2'P_1'\) composed of arc \(PP_2'\), line \(P_2P_3\) and arc \(PP_1'\) is modified, and point \(P_1\) is modified to \(P_1'\), point \(P_2\) to \(P_2'\), point \(P_3\) to \(P_3'\), and point \(P_4\) to \(P_4'\). The coordinates of each point after modification are:
\[ P_1' = (x_{P_1} + l_1 \cos \phi_1, y_{P_1}) \]
\[ P_2' = (x_{P_2} + l_2 \cos \phi_2, y_{P_2}) \]
\[ P_3' = (x_{P_3} + l_3 \cos \phi_3, y_{P_3}) \]
\[ P_4' = (x_{P_4} + l_4 \cos \phi_4, y_{P_4}) \]

End point \(P_4\) is the limit position from the left edge of the parking space cd. In order to facilitate calculation, all the increased parking space is accumulated to the right, so curve \(PP_2'P_1'P_4'\) is shifted \(l_0\) length to the right, so that the parking end point \(P_4'\) and \(P_4\) coincide. According to the calculation, the distance of line \(PP_3'\) is greater than \(2l_0\), and the starting area of parking after translation shifts \(2l_0\) length to the right.

**Fig 4. Planning path transformation**

4. Simulation analysis

4.1. Path Planning Simulation

According to the collision and motion constraints of vehicles in literature [6] and the curvature
optimization algorithm mentioned above, a parallel parking simulation model is established in MATLAB in combination with the given size parameters of a certain vehicle. According to the urban road engineering design code\cite{7} the dimensions of parallel parking Spaces are set as 7.2m×2m, road width $w = 3.5m$, and safe distance $s = 0.2m$ between vehicles and parking Spaces. The starting upper limit $(9.49, 1.48)$ and lower limit $(8.71, 1.15)$ coordinates of parking were selected as the starting points of parking simulation, and the two tracks were marked as track 1 and track 2.

As shown in Fig 5 and 6, the solved two tracks enable vehicles to complete parallel parking without collision. As shown in Fig 7 and 8, the curvature and heading Angle change continuously, and the curvature is between -0.243 and 0.243, ensuring that the vehicle does not turn in place during parking. The heading Angle of the initial position and the end position of the trajectory are both 0, and the curvature of the initial point is 0, which meets the requirements of parking pose.

![Fig 5. track 1](image1)

![Fig 6. track 2](image2)

![Fig 7. Yaw angle of planning path](image3)

![Fig 8. Curvature of the planned path](image4)

4.2. Path tracking simulation

In order to verify whether the optimized path of the cyclotron curve meets the requirements of the controller, this paper builds a model predictive control algorithm (MPC) in Simulink, the vehicle speed is set to -1m/s, and Carsim builds a joint simulation platform. The vehicle parameter $L = 2.77$ (m), $L_r = 0.76$ (m), $\delta_{mc} = 34^\circ$ and $L_s = 1.147$ (m), $\delta_s = 1.78$ (m) the parking trajectory is trajectory 2, and the simulation result is shown in Fig 9.
As shown in Fig 9, using MPC for parallel parking control, the vehicle can enter the parking space smoothly without colliding with the edge of the parking space, meeting the parking safety requirements; as shown in Fig 10, when the controller is used to control parking, the actual trajectory of the center point of the rear axle of the vehicle basically coincides with the planned trajectory, and it can be seen from Fig 11 that the tracking error in the Y-axis direction remains within 0.012m~0.003m.

5. Conclusion
This paper to solve the combination of arc-line-parallel parking path curvature mutations in circular and linear problem, using clothoid curvature mutation based on curvature optimization, implementation circular and linear joint curvature smooth transition, in turn, makes planning path curvature, working condition of spin around to effectively avoid the parking process. In the aspect of trajectory tracking, the model predictive controller is designed based on the error model. Experimental results show that the optimized parking path meets the parking requirements.

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