**RESEARCH ARTICLE**

Predicting the radial heat transfer in the wellbore of cryogenic nitrogen fracturing: Insights into stimulating underground reservoir

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**Funding information**
Natural Science Foundation of China, Grant/Award Number: 51974173; Natural Science Foundation of Shandong Province, Grant/Award Number: ZR2018BEE002; Opening Fund of Key Laboratory of Unconventional Oil & Gas Development (China University of Petroleum (East China)), Ministry of Education; Fundamental Research Funds for the Central Universities; Opening Fund of Key Laboratory of Mining Disaster Prevention and Control, Grant/Award Number: MDPC201908

**Abstract**
Cryogenic nitrogen fracturing is an attractive method for stimulating underground reservoir, since it could favorably induce complex fracture due to the huge temperature difference with lower injection pressure and with the replacement of current water-based fracturing fluid. However, the concern about whether cryogenic nitrogen would be overheated remains unrevealed in the engineering environment with large wellbore length. In addition, reservoir stimulation results are also related with the pressure state at bottom hole. Therefore, in this study, a mathematical model was proposed to predict the radial heat transfer and its influence on vertical pressure transmission in the wellbore with cryogenic nitrogen as fracturing fluid. The model fully couples the heat transfer, hydraulics, and the compressibility of nitrogen, and then, the calculation results were presented and analyzed through a case study. According to the results, the temperature of nitrogen increases too fast under conventional engineering conditions, and it changes into gaseous state at the depth lower than 100 m. Finally, the temperature difference between nitrogen and formation rock becomes too minimal to induce thermal stress at bottom hole. Due to the fast temperature increase, the density of nitrogen decreases too much, and the vertical pressure increasing rate by liquid nitrogen (1.66 MPa/km) is merely 18.2% that in carbon dioxide fracturing (9.13 MPa/km). The results indicate that utilization of special casing with much larger thermal resistance is an indispensable approach to realize the feasibility and advantages of cryogenic nitrogen fracturing.

**KEYWORDS**
cryogenic nitrogen, fracturing, heat transfer efficiency, hydraulics calculation, temperature profile
1 | INTRODUCTION

Unconventional gas resources, with huge reserve as shale gas, coalbed methane, and tight sandstone gas, have been regarded as the most promising alternative options for conventional fossil fuels,\(^1\)\(^2\) which can also significantly reduce environmental pollution if they are more efficiently exploited at a commercial scale.\(^3\) However, unconventional gas resources are generally reserved in unfavorable reservoirs with low permeability,\(^4\)\(^5\) and stimulations are always necessary to achieve economical production and recovery.\(^6\)\(^\text{--}8\) In recent years, the technical development in multistage hydraulic fracture in horizontal well has made it accessible to exploit shale gas commercially, which also facilitates to mitigate the energy crisis and cut its cost.\(^9\) On the other side, water-based fracturing fluid has meet many limitations due to the huge consume of water and potential damages to the environment and underground water. In addition, the aqueous phase could induce swelling of clay minerals to block the gas flow channels.\(^10\)\(^11\) Some European countries have even forbidden the application of hydraulic fracturing. Therefore, the introduction of nonaqueous fracturing fluid becomes more urgent for petroleum engineering.

Both scholars and engineers have investigated the applicability of utilizing cryogenic nitrogen,\(^11\)\(^\text{--}13\) supercritical carbon dioxide,\(^14\)\(^\text{--}16\) or liquefied petroleum gas\(^17\)\(^\text{--}19\) as fracturing fluid. In 1990s, cryogenic nitrogen was firstly utilized as fracturing fluid to stimulate unconventional reservoirs.\(^11\)\(^12\) Theoretically, the fluid's extremely cold temperature will induce thermal tensile stresses to cause the fracture face to fragment,\(^20\)\(^21\) therefore, self-propping fractures could be created in the warm formation by the thermal shock and enhanced gas production rate was also successfully achieved in the mentioned field application. Since carbon-steel alloys used for surface-iron wellhead configuration and wellbore tubulars cannot withstand exposure to cryogenic temperature, all the pressing equipment and pipelines are made of stainless steel to protect them from thermal shock damage.\(^12\) Although cryogenic nitrogen fracturing had shown significant potential for stimulating unconventional reservoirs, its application is still limited at a certain small scale. The main concern lies in that whether nitrogen could maintain in ultra-cold state at bottom hole, since it would inevitably absorb thermal from formation rock due to the huge temperature difference. In addition, it is always necessary to predict or control the pressure state at bottom hole, since it affects the fracture initiation and propagation significantly. Recently, the rapid increase in energy price has drawn back the interest to develop this technology.

The compressibility of nitrogen is coupled with heat transfer and pressure transmission during it flows along the wellbore, which increases the difficulty to predict the wellbore flow field when utilizing nitrogen as fracturing fluid. The density, viscosity, and heat capacity would all change much with varying temperature and pressure,\(^22\)\(^23\) and then, they would further influence the heat transfer and pressure transmission. Due to the difficulties, there has been no reported research revealing this meaningful topic until now.

This paper firstly calculates the varying properties of nitrogen with changing temperature and pressure and then initiates to propose a mathematical model to calculate the temperature and pressure distribution along the wellbore according to the actual fracturing engineering conditions. The closed model fully couples the wellbore heat transfer, hydraulics, flow friction, and varying properties of nitrogen. Finally, the flow field in the wellbore is presented and analyzed aiming to lay theoretical foundation for better application of the actual technology.

2 | THE VARYING FLUID PROPERTIES OF NITROGEN

As depicted earlier, the wellbore heat transfer process would be directly impacted by the varying heat capacity of nitrogen, and the pressure transmission along the wellbore is affected by the varying density and viscosity; therefore, it is necessary to select accurate equations of state to quantitatively calculate the varying properties of nitrogen before modeling the wellbore field.

2.1 | The density of nitrogen

Based on high accuracy data from single- and dual-sinker apparatuses which improve the accuracy of the representation of the \(P\rho T\) surface of gaseous, liquid, and supercritical nitrogen, including the saturation states, Span\(^24\) has proposed a reference equation of state for the thermodynamic properties of nitrogen, with the uncertainty in density no larger than 0.02% at pressures less than 30 MPa. Sophisticated procedures for the optimization of the mathematical structure of equations of state and special functional forms for an improved representation of data in the critical region were also proposed. This equation of state has been cited by the national institute of standards and technology of American (NIST) and also used in this study to calculate the density at varying temperature and pressure.

The general functional form of the new equation for nitrogen, which is formulated in terms of the dimensionless Helmholtz energy \(\alpha\) with the reduced density and inverse reduced temperature as independent variables, reads

\[
P = \rho RT \left[ 1 + \delta \left( \frac{\partial \alpha'}{\partial \alpha} \right) \right] \tag{1}
\]
where $\rho$ represents the density of nitrogen, kg/m$^3$; $T$ is the temperature, K; $P$ is pressure, Pa; $R$ stands for the specific gas constant, $R = 0.2968$ kJ/(kg·K); $\delta = \rho / \rho_c$ is the reduced density; and $\tau = T_c / T$ is the inverse reduced temperature. The critical temperature ($T_c$) for nitrogen is 126.192 K, and the critical density ($\rho_c$) is 313.300 kg/m$^3$. The detailed solution process can be found in the mentioned reference; based on Span model, we calculated and provide the varying density of nitrogen in Figure 1.

As depicted in Figure 1, the density of nitrogen generally decreases with increasing temperature, and the decreasing rate reaches maximum as nitrogen changes from liquid state into vapor state at 3 MPa. When the pressure is larger than critical value (3.40 MPa), state change from liquid into supercritical would no longer induce abrupt decrease in density, and temperature where the decreasing rate reaches maximum increases with increasing pressure. The density of nitrogen is always in positive correlation with pressure although its state changes.

### 2.2 The heat capacity of nitrogen

The specific heat capacity is a physical quantity that measures the ability of a substance to store heat as its temperature changes, and it is a dominating factor to calculate the temperature profile in wellbore under cryogenic nitrogen fracturing. The isobaric heat capacity ($c_p$) of nitrogen can be calculated by:

$$c_p = -\tau^2 \left[ \left( \frac{\partial^2 u}{\partial \tau^2} \right)_0 - \left( \frac{\partial^2 u}{\partial \tau^2} \right)_\delta \right] + \frac{\delta + \left( \frac{\partial u}{\partial \delta} \right)_\tau - \delta \tau \left( \frac{\partial u}{\partial \tau} \right)_\tau}{1 + 2 \delta \left( \frac{\partial u}{\partial \delta} \right)_\tau + \delta^2 \left( \frac{\partial^2 u}{\partial \delta^2} \right)_\tau}$$

(2)

Figure 2 presents the isobaric heat capacity of nitrogen with varying temperature. For a certain pressure, there is a specific temperature where the heat capacity reaches maximum, and the specific temperature increases with the increasing pressure. As the pressure gets closer to the critical value (3.40 MPa), the maximum heat capacity increases significantly. While the pressure is much larger than 3.40 MPa, the change in isobaric heat capacity gets smaller.

### 2.3 The viscosity of nitrogen

The viscosity of nitrogen mainly affects the pressure transmission in the wellbore. Stephan and Krauss have presented an explicit model to calculate the viscosity and thermal conductivity of nitrogen in a temperature range from 70 to 1100 K and pressures up to 70 MPa. The model asserts that the viscosity $\eta(\rho, T)$ at a given density and temperature can be split into a term $\eta_0(T)$ and a residual part $\Delta \eta_R(\rho)$. Hence,

$$\eta(\rho, T) = \eta_0(T) + \Delta \eta_R(\rho)$$

(3)

where $\eta_0(T)$ depends on temperature only and is equal to that at the zero-density limit and $\Delta \eta_R(\rho)$ is only density dependent, up to a certain density limit. The calculation process has been presented in detail in the mentioned reference.

As depicted in Figure 3, when the pressure is smaller than the critical value (3.40 MPa), the viscosity of nitrogen decreases as the temperature increases in liquid state, and then, it would increase with the increasing temperature in vapor state, which means that the viscosity reaches minimum at the temperature where state change occurs. After the pressure exceeds 3.40 MPa, the minimum viscosity would no longer appear at...
the temperature where state change occurs, and actually, the critical temperature corresponding to minimum viscosity would increase with increasing pressure. Overall, the maximum changing rate in viscosity decreases with increasing pressure.

### 3 | MATHEMATICAL MODELS FOR HEAT TRANSFER IN PRODUCTION WELL

Under actual engineering conditions, cryogenic nitrogen should be pressurized first and then injected into the cased wellbore. Due to the huge temperature difference between cryogenic nitrogen and formation rock, convective heat transfer is inevitable near the casing surface as cryogenic nitrogen flows downward. Accompanied with changes in well depth, both the temperature and pressure would change, which will result in changes in properties of compressible nitrogen consequently. The changes in nitrogen properties will further affect the heat transfer and pressure transmission in the production well, and in this way, the heat transfer and hydraulics are finally coupled. Although the boundary conditions might vary in different wells, the typical geometry model can be represented by Figure 4.

#### 3.1 | Governing equations

Eulerian method, as a finite volume method, is suitable for modeling the compressible flow with heat transfer.\footnote{26 In this model, the temperature and pressure is assumed to be constant in every flow field element, and it is mainly composed of continuity equation, momentum equation, and energy equation.}

The simplified continuity equation for compressible flow can be presented as

$$\text{div}(\rho \vec{v}) = 0 \quad (4)$$

where $\vec{v}$ is the flow velocity vector, m/s.

The modified momentum equation is given by

$$\text{div}(\rho \vec{v} \vec{v}) = -\text{div}(P) + \text{div}(\vec{\tau}) + \rho \vec{g} \quad (5)$$

where $v_i$ represents the component of $\vec{v}$ on $i$ axis, m/s.

The energy equation for steady flow can be expressed as

$$\text{div} \left( \rho \vec{v} h \right) - \text{div} \left[ \lambda \text{grad} (T) \right] = 0 \quad (6)$$

where $h$ is the specific enthalpy, J/kg; and $\lambda$ stands for thermal conductivity, W/(m·K).

The equations of state are obviously involved in the governing equations, and they have been presented in Section...
3. The turbulence equations are introduced to close the governing equations. The standard \(k-\varepsilon\) model \(^{26}\) is suitable for compressible flow and is given by

\[
\begin{align*}
\frac{\partial}{\partial x_j} \left( \rho \frac{\partial k}{\partial x_j} \right) - (\mu + \mu_t) \frac{\partial k}{\partial x_j} & = \tau_{ij} S_{ij} - \rho \varepsilon + Q_k \\
\frac{\partial}{\partial x_j} \left( \rho \mu_t \varepsilon \right) - (\mu + \mu_t) \frac{\partial \varepsilon}{\partial x_j} & = \frac{1}{2} \left( \tau_{ij} S_{ij} - 1.92f_2 \mu_t \varepsilon^{3/2} \right) + \frac{\rho}{k} Q_{\varepsilon}
\end{align*}
\] (7)

where \(\tau_{ij} = 2\mu_t (S_{ij} - \rho \delta_{ij}/3) - 2\rho k \delta_{ij}/3 \) and \(\mu_t\) stands for eddy viscosity and is expressed as \(\mu_t = 0.09f_r \rho_k^{2/3}/\varepsilon\). The near-wall attenuation functions are calculated by \(f_r = e^{(-3.2/5(1+0.02Re)^2)}\) and \(f_2 = 1 - 0.3 e^{(-Re_\varepsilon)}\), where \(Re_\varepsilon = \frac{\rho \varepsilon}{\mu}\). The wall terms are given as \(Q_k = 2\mu (\frac{\partial \sqrt{k}}{\partial y})^2\) and \(Q_{\varepsilon} = 2\mu \varepsilon (\frac{\partial \mu_t}{\partial y})^2\). \(S_{ij}\) is the mean-velocity strain-rate tensor, and \(\delta_{ij}\) represents the Kronecker delta.

### 3.2 Solution procedure

In actual field application, the fluid temperature of nitrogen at inlet, the mass flow rate (related with the pump rate), and outlet pressure (related with fracture pressure of formation rock) are given as boundary conditions in the solution procedure. The temperature of nitrogen in every flow field element is initialized as the same with that at inlet, and then, the iteration begins with assuming the pressure at inlet a value. In the Eulerian model, the temperature and pressure are regarded as constant in every flow element; thus, the properties of nitrogen at inlet can be obtained based on Equations (1)-(3), and then, the heat transfer and pressure transmission are calculated according to Equations (4)-(7). The iteration carries on from inlet to outlet along the well, and by modifying inlet pressure based on the difference between assigned outlet pressure and calculated outlet pressure, the iteration would reach convergence and the flow field in the tubing will finally be obtained.

The radial heat transfer between formation rock and flowing nitrogen in the casing can be calculated by Equation (8), and it involves thermal conductivity in the body of steel casing and convection between flowing fluid and casing wall.

\[
Q_{cr} = \frac{T - T_r}{\frac{1}{2\pi \hat{h}r} + \frac{1}{2\pi \lambda} \ln \frac{r_o}{r_i}}
\] (8)

where \(T_r\) represents the temperature of surrounding rock, K; \(\hat{h}\) is the convective heat transfer coefficient between the casing wall and flowing nitrogen in the casing, W/(m²·K); \(\lambda\) is the thermal conductivity coefficient of casing body, W/(m·K); and \(r_i\) and \(r_o\) stand for the inner and outer diameter of casing respectively, m.

Therefore, the total heat transfer coefficient \(H\) can be calculated by

\[
H = \frac{1}{2\pi \hat{h}r} + \frac{1}{2\pi \lambda} \ln \frac{r_o}{r_i}
\] (9)

The convective heat transfer coefficient can be calculated by

\[
Nu = \frac{\hat{h}l}{\lambda} = \frac{x/8 \times (Re - 1000) \times Pr}{12.7 \times \sqrt{x/8 \times (Pr^{2/3} - 1) + 1}} [1 + (d/l)^{2/3}]
\] (10)

where \(Nu\) stands for the Nusselt number, dimensionless; and \(\lambda\) represents the thermal conductivity of nitrogen, W/(m·K), and it can also be calculated based on reference 25. Dimensionless \(x\) is given by

\[
x = \frac{1}{(1.82 \ln Re - 1.64)^2}
\] (11)

The dimensionless Prandtl number is calculated by

\[
Pr = \frac{\eta C_p}{\lambda}
\] (12)

And the dimensionless \(Re\) is calculated by

\[
Re = \frac{\rho \nu d}{\eta}
\] (13)

since \(Re\) is related with the density and viscosity of nitrogen, which reflects the coupling correlation between nitrogen properties and pressure transmission.

After the heat transfer is calculated, the temperature change could be obtained by

\[
\Delta T = \frac{Q}{c_p m_u}
\] (14)

where \(m_u\) stands for the mass in an infinitesimal element, kg.

In the calculation of hydraulics, the pressure loss of carbon dioxide flow along the well is obtained based on Darcy-Weisbach formula,

\[
h_f = \frac{\lambda}{d} \frac{v^2}{2g}
\] (15)

where \(d\) is the equivalent diameter, m; \(g\) represents the gravity and it equals 9.81, m/s²; \(\lambda\) stands for the flow
friction coefficient of nitrogen in cylinder, dimensionless; and $\lambda$ can be calculated based on the variation in Reynolds number $Re$

$$\frac{1}{\sqrt{\lambda}} = 1.8 \log \left[ \frac{6.9}{Re} + \left( \frac{\Delta}{3.7d} \right)^{1.11} \right]$$ (16)

where $\Delta$ represents the absolute roughness of pipe, and it is assumed as 0.2 mm in this paper for steel pipe.

The geometric model was divided into 471,993 tetrahedral cells based on rectangular coordinate system, where the meshes nearby wellbore were refined. The pressure-velocity coupling calculation was conducted with SIMPLE method, and the spatial discretization of pressure was conducted with standard method; other spatial discretization was based on second-order upwind method to get enhanced accuracy.

4 | RESULTS AND DISCUSSIONS

As an example of reservoir stimulation with cryogenic nitrogen as fracturing fluid, the well depth is set as 1500 m, and the geothermal gradient is set as constant at 0.028 K/m. The inner and outer radii of the casing are set as 0.0543 m and 0.0635 m, respectively. The mass flow rate is given as
10 kg/s for the compressible fluid flow, and the temperature of cryogenic nitrogen at inlet is set as 173.15 K. It is reported that the utilization of cryogenic nitrogen as working fluid would significantly decrease the fracture pressure of formation rock. In this view, the outlet pressure is set as 20 MPa, which means that fracture would appear at 20 MPa as channel to allow working fluid to flow out of the wellbore. The thermal conductivity of steel casing is set as 45 W/(m·K).

4.1 Analysis of the pressure transmission

As the pressure state at bottom hole is highly related to the initiation and propagation of fracture, knowledge about how nitrogen transports pressure energy from pump at surface to bottom hole is of essential importance for manual control of reservoir stimulation. Figure 5 presents the calculation results of pressure profile in the wellbore. After the cryogenic nitrogen is injected into the wellbore, the pressure increases fast at first, and soon, the increasing rate of pressure profile decreases to a linear trend. As a contrast, the increasing gradient of pressure profile in cryogenic nitrogen (1.66 MPa/km) is merely 18.2% that in carbon dioxide fracturing (9.13 MPa/km) overall. In addition, the pressure transfer efficiency of carbon dioxide is close to that of pure water. It can be concluded that the pressure transfer efficiency of nitrogen is quite poor. The pressure transmission in the vertical wellbore is positively dominated by the integral of density along depth and also negatively affected by the viscosity profile.

Figures 6 and 7 are the density profile and viscosity profile of nitrogen in the wellbore, respectively. After injected into the wellbore, both the density and viscosity of nitrogen decrease abruptly. Since larger density would facilitate faster increase, the increasing gradient of pressure profile near the inlet is larger than that in deeper well section. While larger viscosity would prohibit faster increase in pressure profile, the increasing gradient of pressure profile near the inlet is merely insignificant larger (Figure 5). After the well depth increases to about 100 m, the pressure profile would then decrease in an insignificant and linear trend. The results indicate that the changing trend of density profile is always dominated by the increasing temperature. The density of nitrogen is smaller than 200 kg/m³ after the depth reaches 100 m or larger, while the density of supercritical CO₂ could be larger than 900 kg/m³. The smaller density of nitrogen would facilitate to induce faster temperature increase of nitrogen than carbon dioxide along the wellbore. As for the viscosity profile, it begins to increase at about 50 m, and then, the increasing rate decreases to a linear method. It can be concluded that the changing viscosity profile is firstly dominated by the increasing temperature when the well depth is no larger than 50 m, and then, the increasing pressure becomes (Figure 5) the dominating factor.

Since the pressure transfer capability of nitrogen is inefficient, the feasibility of cryogenic nitrogen fracturing mainly lies in the fact that thermal stress, induced by the huge difference between nitrogen andformation rock, would facilitate the fracture to initiate and propagate significantly. Therefore, the reservoir stimulation results also depend highly on the radial heat transfer.

Figure 8 presents the temperature profile. The results show that the temperature of nitrogen increases significantly after injected into the wellbore. While the depth varies from 0 to 100 m, huge temperature change might induce threat to the strength of casing. After the well depth reaches 100 m approximately, the temperature difference between nitrogen and formation rock becomes negligible and is not large enough to induce thermal stress. The results indicate that the utilization of special casing with much larger thermal resistance is the only approach to realize the feasibility and advantages of cryogenic nitrogen fracturing.

The changing trend of temperature profile is high related with the total heat transfer coefficient profile (Figure 9) and heat...
As is shown in Figure 9, larger heat transfer coefficient at shallow well section would benefit to result in larger temperature change. After the well depth reaches 100 m, the heat transfer coefficient begins to decreases lightly and linearly. According to Equation (9), the total heat transfer coefficient is in positive correlation with the convective heat transfer coefficient. The total heat transfer coefficient also considers the influence of thermal resistance of casing, and they are in negative correlation. Based on Equation(9), the thermal resistance of casing $R_c$ can be presented as

$$R_c = \frac{1}{2\pi \lambda_i l} \ln \frac{r_o}{r_i}$$

Both Equations (9) and (17) indicate that larger thermal resistance or smaller thermal conductivity of casing would facilitate to increase radial heat transfer, resulting in faster temperature increase of nitrogen along the wellbore. The equations also indicate that utilization of special casing with larger thermal resistance would benefit to achieve necessary temperature difference at bottom hole.

As depicted in Figure 10, the changing trend of heat capacity is quite similar to that of heat transfer coefficient, and it firstly decreases fast and then begins to decrease in a lighter and linear trend after the well depth reaches 10 m. As a contrast, the changing heat capacity of liquid nitrogen (1241-2010 J/kg·K) is merely 29.5%-47.7% that of water (4210 J/kg·K).
kg·K). Smaller heat capacity is beneficial for faster temperature change (Figure 8) and smaller temperature difference between working fluid and formation rock at deep well section compared to that of supercritical carbon dioxide fracturing.28

The feasibility of cryogenic nitrogen fracturing depends highly on the temperature difference between nitrogen and formation rock at target zone. Since the radial heat transfer efficiency is unfavorably too high to maintain enough temperature difference, the calculation results indicate that the feasibility of cryogenic nitrogen fracturing depends on the block of radial heat transfer; therefore, the utilization of special casing with larger thermal resistance is necessary.

5 | CONCLUSIONS

A coupled mathematical model is presented to investigate the vertical pressure transmission and radial heat transfer in the wellbore of cryogenic nitrogen fracturing. The results present and analyze the pressure profile, temperature profile, and physical properties profiles in the wellbore. According to the results and discussions, the following conclusions can be obtained:

1. The vertical pressure transmission efficiency by liquid nitrogen is unfavorably poor, that is 1.66 MPa/km or merely 18.2% that in carbon dioxide fracturing (9.13 MPa/km).
2. The temperature of nitrogen increases significantly after injected into the wellbore. After the well depth reaches 100 m or larger, the temperature difference between nitrogen and formation rock becomes negligible and is not large enough to induce thermal stress. Utilization of special casing with much larger thermal resistance is the only approach to realize the feasibility and advantages of cryogenic nitrogen fracturing
3. The vertical pressure transmission and radial heat transfer are highly coupled with changing properties of liquid nitrogen.

ACKNOWLEDGEMENTS

The financial supports from National Natural Science Foundation of China (51974173), Natural Science Foundation of Shandong Province (ZR2018BEE002), the Opening Fund of Key Laboratory of Mining Disaster Prevention and Control (MDPC201908) and the Opening Fund of Key Laboratory of Unconventional Oil & Gas Development (China University of Petroleum (East China)), Ministry of Education are highly appreciated.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| $P$ | Pressure, Pa |
| $\rho$ | Density, kg/m$^3$ |
| $R$ | Specific gas constant, $R = 0.2968$ kJ/(kg·K) |
| $T$ | Temperature, K |
| $\delta$ | Reduced density, dimensionless |
| $\tau$ | Inverse reduced temperature, dimensionless |
| $\Phi(\delta, \tau)$ | Helmholtz energy, dimensionless |
| $\Phi'_\delta$ | Partial derivative of the Helmholtz energy, dimensionless |
| $c_p$ | Isobaric heat capacity, $J/(kg·K)$ |
| $\eta$ | Viscosity, $\mu Pa·s$ |
| $\eta_0$ | Viscosity in the zero-density limit, $\mu Pa·s$ |
| $\Delta\eta_R$ | Excess viscosity, $\mu Pa·s$ |
| $v$ | Flow velocity vector, m/s |
| $v_i$ | The component of $v$ on $i$ axis, m/s |
| $h$ | Specific enthalpy, J/kg |
| $\lambda$ | Thermal conductivity, W/(m·K) |
| $\delta_{ij}$ | The symmetric part of local speed gradient tensor, dimensionless |
| $F_i$ | Mass force component on $i$ axis, m/s$^2$ |
| $\rho$ | Stress, Pa |
| $\delta_{ij}$ | Kronecker delta |
| $Q_{cr}$ | Heat transferred from surrounding rock to nitrogen, J |
| $\lambda_c$ | Thermal conductivity of casing body, W/(m·K) |
| $\lambda_f$ | Thermal conductivity of nitrogen, W/(m·K) |
| $l$ | Length of finite unit, m |
| $T_r$ | Temperature of formation rock, K |
| $\hat{h}$ | Convective heat transfer coefficient between casing wall and flowing working fluid in wellbore, W/(m$^2$·K) |
| $r_i$ | Inner diameter of the casing, m |
| $r_o$ | Outer diameter of the casing, m |
| $m_u$ | The mass in an infinitesimal unit, kg |
| $m$ | Mass flow rate, kg/s |
| $\lambda$ | Flow friction coefficient of carbon dioxide in the production well, dimensionless |
| $d$ | Equivalent diameter, m |
| $g$ | Gravity, m/s$^2$ |
| $Re$ | Reynolds number, dimensionless |
| $Pr$ | Prandtl number, dimensionless |

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How to cite this article: Song W, Shi X, Wang C, Xu J, Chen S, Chen Z. Predicting the radial heat transfer in the wellbore of cryogenic nitrogen fracturing: Insights into stimulating underground reservoir. *Energy Sci Eng*. 2020;8:582–591. [https://doi.org/10.1002/ese3.479]