Flavor changing top decays to charm and Higgs with $\tau\tau$ at the LHC

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Abstract

We investigate the prospects of discovering the top quark decay into a charm quark and a Higgs boson ($t \rightarrow ch^0$) in top quark pair production at the CERN Large Hadron Collider (LHC). A general two Higgs doublet model is adopted to study flavor changing neutral Higgs (FCNH) interactions. We perform a parton level analysis as well as Monte Carlo simulations using PYTHIA 8 and DELPHES to study the flavor changing top quark decay $t \rightarrow ch^0$, followed by the Higgs decaying into $\tau^+\tau^-$, with the other top quark decaying to a bottom quark ($b$) and two light jets ($t \rightarrow bW \rightarrow bjj$). To reduce the physics background to the Higgs signal, only the leptonic decays of tau leptons are considered, $\tau^+\tau^- \rightarrow e^\pm\mu^\mp + \slashed{E}_T$, where $\slashed{E}_T$ represents the missing transverse energy from the neutrinos. In order to reconstruct the Higgs boson and top quark masses as well as to reduce the physics background, the collinear approximation for the highly boosted tau decays is employed. Furthermore, the energy distribution of the charm quark helps set the acceptance criteria used to reduce the background and improve the statistical significance of the signal. We study the discovery potential for the FCNH top decay at the LHC with collider energy $\sqrt{s} = 13$ and 14 TeV as well as a future hadron collider with $\sqrt{s} = 27$ TeV. Our analysis suggests that a high energy LHC at $\sqrt{s} = 27$ TeV will be able to discover this FCNH signal with an integrated luminosity $\mathcal{L} = 3$ ab$^{-1}$ for a branching fraction $\mathcal{B}(t \rightarrow ch^0) \gtrsim 1.4 \times 10^{-4}$, which corresponds to a FCNH coupling $|\lambda_{tch}| \gtrsim 0.023$. This FCNH coupling is significantly below the current ATLAS combined upper limit of $|\lambda_{tch}| = 0.064$.

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I. INTRODUCTION

The discovery of the Higgs boson in 2012 [1, 2] completes the experimental observation of the particle spectrum predicted by the Standard Model (SM). A primary goal of the high luminosity and higher energy Large Hadron Collider (LHC) is the precision testing of the SM and the search for physics beyond the Standard Model (BSM), especially the interactions of the Higgs boson, the top quark, and sources of CP violation. Several experimental searches [3–6] are being performed to improve the understanding of Higgs boson interactions with SM particles and to search for possible extensions of the Higgs sector.

There are some deviations from the SM such as the presence of baryon asymmetry in the Universe [7] requiring CP violation beyond that predicted by the SM. The muon anomalous magnetic moment measurements at BNL [8] and Fermilab [9] show approximately 4.2σ deviation from the SM [10–12]. In addition, there might be possible flavor anomalies among the quarks and leptons [13–17]. The Standard Model with one Higgs doublet cannot explain these anomalies [18] thus requiring BSM physics. A general two Higgs doublet model (2HDM) provides a simple extension to the SM. It consists of two scalar SU(2) doublets, which after electroweak symmetry breaking (EWSB) leads to five physical Higgs bosons: two CP-even scalars \( H^0 \) (heavier) and \( h^0 \) (lighter), one CP-odd pseudoscalar \( A^0 \) and a pair of charged Higgs boson \( H^\pm \). General 2HDMs can provide additional sources of CP violation [19, 20], and generate tree-level flavor changing neutral Higgs (FCNH) interactions that can enhance the branching fractions of flavor changing neutral currents, especially \( t \to c\phi^0 \) [21], \( \phi^0 \to t\bar{t} + \bar{t}c \) [22, 23], and \( \phi^0 \to \tau^\pm \mu^\mp \) [24–27], where \( \phi^0 = H^0, h^0 \) and \( A^0 \). The SM expectation is \( \mathcal{B}(t \to ch^0) \approx 10^{-14} \) [28–30], which is significantly less than current and near term experiments can observe. If this FCNH signal \( t \to ch^0 \) is observed at the LHC or HL-LHC, it would imply BSM physics [31–44].

We adopt the Yukawa Lagrangian in a general two Higgs doublet model [45, 46] as

\[
\mathcal{L}_Y = \frac{-1}{\sqrt{2}} \sum_{F=U,D,L} \bar{F} \left\{ \begin{array}{c}
\kappa_F c_{\beta - \alpha}^F + \rho_F c_{\beta - \alpha}^F \\
\kappa_F c_{\beta + \alpha}^F - \rho_F c_{\beta + \alpha}^F
\end{array} \right\} h^0 + \left[ \kappa_F c_{\beta - \alpha}^F - \rho_F c_{\beta + \alpha}^F \right] H^0
+ \text{H.c.}
\]

where \( P_{L,R} \equiv (1 \mp \gamma_5)/2 \), \( c_{\beta - \alpha} \equiv \cos(\beta - \alpha), s_{\beta - \alpha} \equiv \sin(\beta - \alpha), \alpha \) is the mixing angle between neutral Higgs scalars, \( \tan \beta \equiv v_2/v_1 \) [47] is the ratio of the vacuum expectation values of the two Higgs doublets, \( Q_F \) is the fermion charge, and the \( \kappa \) matrices are diagonal and fixed by fermion masses to \( \kappa_F = \sqrt{2}m_F/v \) with \( v \approx 246 \text{ GeV} \), while the matrices \( \rho \) contain both diagonal and off-diagonal elements with free parameters. In addition, \( F,U,D,L \) represent elementary fermions, up-type quarks, down-type quarks, and charged leptons, respectively. The matrix elements \( \rho \) are the FCNH couplings to the fermions. Almost all experimental data are consistent with the Standard Model [48, 49], which implies all two Higgs doublet models must be in the decoupling [50] or the alignment limit [51, 52] with one SM-like light scalar \( (h^0) \) that has a mass of 125 GeV.

Recently the ATLAS Collaboration [40] combined several channels to search for \( t \to ch^0 \) with \( h^0 \to bb, h^0 \to \tau\tau \) with at least one hadronic tau decay, \( h^0 \to WW^*, ZZ^*, \tau^+\tau^- \) (same sign \( 2\ell, 3\ell \)), and \( h^0 \to \gamma\gamma \), and put a strong constraint on the branching fraction \( \mathcal{B}(t \to ch^0) \leq 1.1 \times 10^{-3} \). This leads to an upper limit on the FCNH Yukawa coupling \( |\lambda_{tch}| \)

\[
\lambda_{tch} \leq 0.064 ,
\]

(2)
for the effective Lagrangian,
\[ \mathcal{L} = -\frac{\lambda_{tch}}{\sqrt{2}} c\theta^0 + H.c. \]  
with the relation between \( \lambda_{tch} \) and the \( t \rightarrow c\theta^0 \) branching fraction [53] being
\[ \lambda_{tch} \approx 1.92 \times \sqrt{B(t \rightarrow c\theta^0)}. \]  

In this article, we investigate the discovery potential of the top quark decay into a charm quark and a Higgs boson \((t \rightarrow c\theta^0)\) followed by the Higgs boson decaying into \( \tau^+\tau^- \) in top quark pair production at the CERN Large Hadron Collider (LHC). To investigate the discovery potential of a flavor changing neutral Higgs boson signal with low physics background, we consider only the leptonic decays of the tau leptons, \( \tau^+\tau^- \rightarrow e^\pm\mu^\pm + E_T \), where \( E_T \) is the missing transverse energy in the event from the neutrinos. This is complementary to the ATLAS searches for same charge dileptons.

We perform a parton level analysis as well as a Monte Carlo simulation using Pythia 8 [54] and Delphes [55] to study the FCNH decay of one top quark while the other top quark decays hadronically to a bottom quark \((b)\) and two light jets: \( pp \rightarrow t\bar{t} \rightarrow bW^\pm c\theta^0 \rightarrowbjjc\tau^+\tau^- + X \). We have calculated the production rates using the full tree level matrix elements including the Breit-Wigner resonance for both signal and background process. In addition, we optimize our acceptance using a standard selection based technique, as well as using a boosted decision tree to improve the signal to background ratio and statistical significance.

Since we did not apply charm tagging, our analysis is suitable for a general search for \( t \rightarrow q\theta^0, q = u,c \). Many previous studies have adopted the Cheng-Sher Ansatz [56] as the benchmark Yukawa coupling
\[ \lambda_{tq\theta} = \frac{\sqrt{2m_t m_q}}{v} \]  
where \( q = u,c \) and \( v \approx 246 \text{ GeV} \) is the Higgs vacuum expectation value. The FCNH couplings as the geometric mean for top and charm quarks is
\[ \lambda_{tch}(CS) = \frac{\sqrt{2m_t m_c}}{v} \approx 0.0895, \]
which has been excluded by recent ATLAS experiment [40]. For simplicity, we assume \( \lambda_{tch} \gg \lambda_{tuh} \) and focus on the search for \( t \rightarrow c\theta^0 \). To verify that the associated quark is a charm, we will need to apply charm tagging.

There are several aspects to note in this analysis. To reconstruct the Higgs boson and the top quark, the collinear approximation of tau decays [57] is used. The collinear approximation for tau decays with physical momentum fractions \( x_i (0 < x_i < 1) \), where \( x_i = p(\ell_i)/p(\tau), i = 1,2 \), more effectively reduced the physics background than the centrality requirement suggested in Refs. [37, 40]. Furthermore, the energy of the charm quark in the top quark rest frame provides good acceptance for the FCNH top signal while rejecting background [32, 58]. Promising results are presented for the LHC with \( \sqrt{s} = 14 \text{ TeV} \) and \( 27 \text{ TeV} \).

II. HIGGS SIGNAL AND EVENT SELECTIONS

This section presents the cross section for the FCNH signal \( t \rightarrow c\theta^0 \) from top quark pair production and outlines our search strategy for this signal at the LHC. We focus on
the discovery channel with one top quark decaying hadronically \((t \to bjj)\), while the other top quark decays into a charm quark and a Higgs boson \((h^0)\) followed by \(h^0 \to \tau^+\tau^- \to e^\pm \mu^\mp + \not{E}_T\). Unless explicitly specified, \(q\) generally denotes a quark \((q)\) or an anti-quark \((\bar{q})\) and \(\ell^\pm\) will represent an \(e^\pm\) or \(\mu^\pm\). This means our FCNH signal has the final state of \(pp \to t\bar{t} \to bjjej\bar{e}^\pm \mu^\mp + \not{E}_T + X\), where \(X\) represents all other particles produced in \(pp\) collisions. Since the mass of the Higgs boson is much greater than the tau lepton’s mass \((M_h \gg m_\tau)\), the tau leptons are highly boosted. Therefore, the collinear approximation of the tau decay \([57]\) is employed to reconstruct the Higgs boson mass and the top quark mass.

At parton level, our analysis employs MadGraph5-aMC-NLO \([59]\) to calculate tree-level cross sections for the full process \(pp \to t\bar{t} \to bjjejh^0 \to bjjej\tau^+\tau^- + X\) along with the collinear approximation of tau decays \([57]\). The parton level cross section is evaluated using the CT14LO parton distribution functions (PDFs) \([60]\). For simplicity, the factorization scale \(\mu_F\) and the renormalization scale \(\mu_R\) are chosen to be the invariant mass of the top quark pair \((M_{t\bar{t}})\). With the above scale choices and PDFs, our current estimates suggest a \(K\)-Factor of \(\approx 1.8\), and is approximately the same for all three energies \((\sqrt{s} = 13, 14,\) and \(27\) TeV), investigated for top quark pair production at the LHC. The \(K\)-factors are calculated using TOP++ \([61]\).

This analysis employs the full tree-level matrix elements to evaluate the cross section for the FCNH signal and physics background. In addition, a consistency check for the tree-level signal cross section has been performed in the narrow width approximation by calculating the cross section \(\sigma(pp \to t\bar{t} \to tch^0 \to jjc\ell^\pm_1 \ell^\mp_2 \not{E}_T + X)\) as the product of cross section times branching fractions:

\[
\sigma(pp \to t\bar{t} \to bjjej + X) \times \mathcal{B}(t \to ch^0) \times \mathcal{B}(h^0 \to \tau^+\tau^-) \times \mathcal{B}(\tau^+ \to \ell^+_1 \nu_{\ell_1} \nu_\tau) \times \mathcal{B}(\tau^- \to \ell^-_2 \nu_{\ell_2} \nu_\tau). \tag{7}
\]

To evaluate the branching fraction of \(t \to ch^0\), the effective Lagrangian in Eq. 3 is employed. The resulting decay width is then obtained as

\[
\Gamma(t \to ch^0) = \frac{\left|\lambda_{tch}\right|^2}{32\pi} \times (m_t) \times [(1 + r_c)^2 - r_t^2] \times \sqrt{1 - (r_h + r_c)^2} \sqrt{1 - (r_h - r_c)^2}, \tag{8}
\]

with \(r_h = M_h/m_t\) and \(r_c = m_c/m_t\). Assuming that the total decay width of the top quark is

\[
\Gamma_t = \Gamma(t \to bW) + \Gamma(t \to ch^0), \tag{9}
\]

the branching fraction of \(t \to ch^0\) is

\[
\mathcal{B}(t \to ch^0) = \frac{\Gamma(t \to ch^0)}{\Gamma_t}. \tag{10}
\]

Comparing this with the Yukawa Lagrangian in Eq. 1, we can express

\[
\lambda_{tch} = \tilde{\rho}_{tc} \cos(\beta - \alpha) \tag{11}
\]

with

\[
\tilde{\rho}_{tc} = \sqrt{\frac{\left|\rho_{tc}\right|^2 + \left|\rho_{ct}\right|^2}{2}}. \tag{12}
\]

To present the results, the more convenient free parameters \(\tilde{\rho}_{tc}\) and \(\cos(\beta - \alpha)\) are chosen for the FCNH Yukawa couplings.
In the event level analysis, parton level samples are generated from MadGraph using TauDecay-UFO \cite{62} to model $\tau$ decays, and then the sample is processed with PYTHIA 8 \cite{54} and DELPHES \cite{55} to generate events with hadronization, showering, and detector effects. In addition, the MLM-matching/merging \cite{63} algorithm is applied to match the additional hadronized jets in each event with partons to avoid double counting jets that are generated by parton showering from final state radiation for all background processes.

To provide a realistic estimate for production rates at the LHC, we evaluate the cross section for the FCNH signal and physics background in $pp$ collisions with the proper tagging and mistagging efficiencies. The ATLAS tagging efficiencies \cite{64} are adopted to evaluate the cross section for the FCNH signal and physics background. The $b$ tagging efficiency is 0.7, the probability that a $c$-jet is mistagged as a $b$-jet ($\epsilon_c$) is approximately 0.14, while the probability that a light jet ($u, d, s, g$) is mistagged as a $b$-jet ($\epsilon_j$) is 0.01.

### A. Event Selections

Our FCNH signal comes from top quark pair production with one top quark decaying into a charm quark and a Higgs boson while the other top quark decays to an all hadronic final state. Every event is required to contain at least four jets, including exactly one that is identified as a $b$ jet. In addition, there are two opposite charge leptons of different flavor ($e^\pm \mu^\mp$) with missing transverse energy from neutrinos.

We adopt the following basic requirements, which are similar to the ATLAS and CMS $h^0 \rightarrow \tau^+ \tau^-$ studies \cite{69}.

1. Four jets including one $b$-jet with $P_T(b,j) \geq 20$ GeV,
2. $|\eta(b,j)| \leq 2.5$,
3. Two opposite charge leptons with $P_T(\ell) \geq 10$ GeV, and $|\eta(\ell)| \leq 2.5$,
4. $P_T$(leading $\ell) \geq 20$ GeV,
5. $\Delta R(\ell\ell, jj, bj, lj, lb) \geq 0.4$,
6. $E_T \geq 25$ GeV,
7. All events containing more than one tagged $b$-jet with $P_T \geq 20$ GeV and $|\eta| < 2.5$ are rejected.

Since the $b$-quark jet is selected through tagging, this leaves three jets to be identified as two light-flavor jets and a $c$-quark jet. The two light-flavor jets, $j_1, j_2$, are selected by minimizing $|M_{jj} - m_W|$ and $|M_{bjj} - m_t|$. The remaining jet is labeled as the $c$-quark jet. For the event to be correctly reconstructed, $j_1$ and $j_2$ must result from the decay of a $W$ boson, therefore their invariant mass distribution $M_{j_1j_2}$ peaks at $M_W \approx 80.4$ GeV and $M_{bj_1j_2}$ has a peak at $m_t \approx 173.2$ GeV. Using the ATLAS mass resolution \cite{65}, the reconstructed $W$ and top quark masses are required to lie in the mass windows $\Delta M_{j_1j_2} = 0.20 M_W$ and $\Delta M_{bj_1j_2} = 0.25 m_t$. 
B. Higgs Mass Reconstruction

For the FCNH signal, \( t \to c\tau^+\tau^- \to c\ell^+\mu^- + E_T \), the reconstruction is performed two ways: (a) using the invariant \( \tau^+\tau^- \) mass from the Higgs decay and the invariant mass of \( c\tau^+\tau^- \) from the top quark decay, which have sharp peaks near \( M_H \) and \( m_t \), and (b) as in Ref. [41], using the cluster transverse masses of \( \ell^+\ell^- \) and \( c\ell^+\ell^- \), which have broad peaks near \( M_H \) and \( m_t \).

The Higgs boson mass can be reconstructed by applying the collinear approximation [66–68] on the \( \tau \) decay products

\[
p_{\ell_i} = x_ip_{\tau_i}, \quad i = 1, 2 \quad \text{with} \quad p_T(\ell_1) > p_T(\ell_2),
\]

and the missing transverse momentum 2-vector, \( \not{p}_T \). Taking \( x_i \) to be the momentum fractions carried away from the decay by \( \ell_i, i = 1, 2 \), we have:

\[
\left( \frac{1}{x_1} - 1 \right) p_T(\ell_1) + \left( \frac{1}{x_2} - 1 \right) p_T(\ell_2) = \not{p}_T.
\]

This yields two equations for \( x_1 \) and \( x_2 \) that can be solved to reconstruct the two original \( \tau \) 4-momenta \( p_{\mu_{\tau_i}} = p_{\mu_{\tau_1}}/x_1, p_{\mu_{\tau_2}}/x_2 \). Thus \( M_H^2 = (p_1/x_1 + p_2/x_2)^2 \) where \( p_1 = p(\ell_1) \) and \( p_2 = p(\ell_2) \). Physically \( x_i \) is constrained to \( 0 < x_i < 1, i = 1, 2 \), which reduces the background. Figure 1 presents the invariant mass distributions \( M_{\text{col}}(\tau\tau) \) and \( M_{\text{col}}(c,\tau\tau) \), which have sharp peaks near the Higgs boson and top quark masses, respectively. We require the reconstructed Higgs boson mass and top quark mass to lie in the mass windows \( \Delta M_{\tau\tau} = 0.20M_H \) and \( \Delta M_{c\tau\tau} = 0.25m_t \) using the ATLAS mass resolution [69]. We note that improvements in the discovery potential are possible by reducing the \( \tau \) pair mass resolution \( \Delta M_{\tau\tau} \).

![FIG. 1: Invariant mass distributions](image)

FIG. 1: Invariant mass distributions \( d\sigma/dM_{\tau\tau} \) (green dotted dashed) and \( d\sigma/dM_{c\tau\tau} \) (blue solid) for the FCNH signal \( (t \to ch^0) \) at (a) parton level, and (b) event level with detector simulation in \( pp \) collisions. Also shown are the invariant mass distributions \( d\sigma/dM_{\tau\tau} \) (magenta dotted) and \( d\sigma/dM_{c\tau\tau} \) (red dashed) for the dominant background from \( ttjj \).

Furthermore, we employ the cluster transverse mass distributions for \( e^+\mu^- \) and \( ce^\pm\mu^\mp \) with missing transverse energy \( (E_T) \) from the neutrinos to confirm the Higgs boson mass
and top quark mass reconstruction. These distributions have broad peaks near $M_h$ and $m_t$, respectively, as the kinematic characteristic of $t \rightarrow ch^0 \rightarrow c e^\pm \mu^\mp + \not{E}_T$. The cluster transverse mass [70] is defined as

$$M_T^2(C, \not{E}_T) = \left( \sqrt{p_T^2(C) + M_C^2 + \not{E}_T} - (\vec{p}_T(C) + \not{E}_T)^2 \right)^2,$$

where $C = \ell^+ \ell^-$ or $c \ell^+ \ell^-$, $p_T(\ell \ell)$ or $p_T(c \ell \ell)$ is the total transverse momentum of all the visible particles, while $M_{\ell \ell}$ and $M_{c\ell \ell}$ are the invariant cluster masses.

In most cases, the physics background can be reduced and the statistical significance for the Higgs boson signal enhanced if we apply a suitable requirement on the cluster transverse mass distributions [41] $M_T(\ell \ell, \not{E}_T)$ and $M_T(c\ell \ell, \not{E}_T)$. We have found that acceptance requirement on $M_{\tau \tau}$ and $M_{c\tau\tau}$ is much more effective than mass requirement on the cluster transverse masses. After the mass selection on the collinear invariant mass, the effects of additional requirements on the cluster transverse mass are negligible.

C. Centrality of Missing Transverse Energy

To further suppress the physics background, the authors of Refs. [37, 40] suggest the use of the centrality of the missing transverse energy ($C_{\text{MET}}$)

$$C_{\text{MET}} = (x + y)/\sqrt{x^2 + y^2},$$

with

$$x = \frac{\sin(\phi_{\text{MET}} - \phi_1)}{\sin(\phi_2 - \phi_1)}, y = \frac{\sin(\phi_2 - \phi_{\text{MET}})}{\sin(\phi_2 - \phi_1)},$$

where $\phi_{1,2}$ are the azimuthal angles of the two leptons ($e$ or $\mu$) in the transverse plane, and $\phi_{\text{MET}}$ is the azimuthal angle of the transverse missing energy. Figure 2 shows the centrality $C_{\text{MET}}$ for the FCNH signal from $t \rightarrow ch^0$ and the dominant background $t\bar{t}jj$. This is found to be less stringent than the requirement on the physical momentum fractions $0 < x_i < 1, i = 1, 2$, which leads to $C_{\text{MET}} > 1$.

III. THE PHYSICS BACKGROUND

The dominant background to the signal is from $t\bar{t}jj$, $j = q$ or $g$. Here both top quarks decay leptonically ($t \rightarrow b\ell\nu$) to the desired final state combination of leptons. This comprises more than 95% of the total background. The other dominant contribution is from $pp \rightarrow b\bar{b}jj\tau\tau \rightarrow b\bar{b}jj e^+ \mu^- + \not{E}_T + X$ and $pp \rightarrow b\bar{b}jj W^+ W^- \rightarrow b\bar{b}jj e^+ \mu^- + \not{E}_T + X$ (without a $t\bar{t}$ contribution) as well as $t\bar{t}W^\pm$ and $t\bar{t}Z$. For all of the backgrounds, one $b$ jet is selected while the other $b$ jet is mis-identified as a light jet. Events with two $b$-jets having $p_T(b) > 20$ GeV and $|\eta(b)| < 2.5$ are vetoed [71, 72]. We calculate the cross section for each of the backgrounds separately using MadGraph and apply the same event selection procedure as for the signal. The irreducible background from $pp \rightarrow t\bar{t}Z + X$ and $pp \rightarrow t\bar{t}h^0 + X$ with the subsequent decay of $Z \rightarrow \tau^+ \tau^-$ and $h^0 \rightarrow \tau^+ \tau^-$ are negligible after all acceptance requirements and the two $b$ veto are imposed.
FIG. 2: Distribution for the centrality of missing transverse energy ($d\sigma/dC_{\text{MET}}$) for the FCNH signal from $t \rightarrow c h_0 \rightarrow \tau \tau \rightarrow e^\pm \mu^\mp + E_T$ (blue solid) at the LHC with $\sqrt{s} = 13$ TeV. Requiring the momentum fraction to be physical $0 \leq x_i \leq 1$, we effectively select $C_{\text{MET}} > 1$ that is the region on the right hand slide of the vertical dash line. Also shown is the centrality distribution of the dominant physics background from $ttjj$ (red dotted dashed).

![Graph showing distribution](image)

TABLE I: K-Factors at NLO for $t\bar{t}W$ and $t\bar{t}Z$ produced at the LHC.

| Process \ $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV |
|-----------------------|--------|--------|--------|
| $t\bar{t}W$           | 1.64   | 1.66   | 1.70   |
| $t\bar{t}Z$           | 1.46   | 1.49   | 1.50   |

We scale our backgrounds to NLO using K-factors of 1.8 for $t\bar{t} + 2j$, $b\bar{b}jj\tau\tau$, and $bbjjW^+W^-$ for all energies i.e. $\sqrt{s} = 13$, 14, and 27 TeV. For $t\bar{t}W$ and $t\bar{t}Z$ we use the following K-factors calculated with MadGraph5-aMC-NLO.

After applying the event acceptance criteria, we reconstruct the invariant mass variables $M_{j_1j_2}$, $M_{b_1j_2}$, $M_{\tau\tau}$, and $M_{c\tau\tau}$, as discussed in the previous section. In addition, the energy of the charm quark ($E_c$) in the rest frame of the top quark is reconstructed to discriminate the $t \rightarrow ch_0$ signal from background [32, 58]. For the flavor changing top decay of $t \rightarrow ch_0$, the $E_c$ distribution exhibits a peak at the following value,

$$E_c^* = \frac{m_t}{2} \left[ 1 + \frac{m_c^2}{m_t^2} - \frac{m_h^2}{m_t^2} \right] \approx 41.43 \text{ GeV}. \quad (18)$$

Requiring $29 \text{ GeV} < E_c < 54 \text{ GeV}$, the background is significantly reduced while most of the signal is maintained. Figure 3 presents the energy distributions of the charm quark in the top quark rest frame.

From the invariant mass and the charm quark energy distributions at the parton and event levels, the following mass requirements are deduced

(i) $|M(j_1, j_2) - m_W| \leq 0.20 \times m_W$ and $|M(b, j_1, j_2) - m_t| \leq 0.25 \times m_t$,

(ii) $|M_{\text{col}}(\tau, \tau) - m_h| \leq 0.20 \times m_h$ and $|M_{\text{col}}(c, \tau, \tau) - m_t| \leq 0.25 \times m_t$,
FIG. 3: Energy distribution for the charm quark \( \frac{d\sigma}{dE_c} \) in the top rest frame for the Higgs signal in \( pp \) collisions with \( \sqrt{s} = 13 \) TeV, from \( t \to ch^0 \) (blue solid) at (a) parton level, and (b) event level with detector simulations. Also shown is the charm quark energy distribution for the dominant background \( ttjj \) (red dashed).

(iii) \( 40 \) GeV \( \leq M_T(\ell, \ell, E_T) \leq 140 \) GeV and \( 80 \) GeV \( \leq M_T(c, \ell, \ell, E_T) \leq 180 \) GeV, and

(iv) \( 29 \) GeV \( \leq E_c \leq 54 \) GeV.

These requirements are chosen to remove the physics background in a manner that maximizes the statistical significance of the FCNH signal.

IV. DISCOVERY POTENTIAL AT THE PARTON LEVEL

Applying all the selection criteria at parton level for \( \sqrt{s} = 13, 14 \) and 27 TeV, our signal cross sections for \( \lambda_{tch} = 0.064 \) are shown in Table II. The cross sections with \( \lambda_{tch} = 0.01 \) are also presented for a simple estimate to find the cross sections for other values of this FCNH Yukawa coupling. The cross sections for the backgrounds after applying the selection requirements are presented in Table III.

| \( \sqrt{s} \) (TeV) | \( \lambda_{tch} = 0.01 \) | \( \lambda_{tch} = 0.064 \) |
|-----------------------|-----------------|-----------------|
| 13                    | 0.0096          | 0.39            |
| 14                    | 0.012           | 0.46            |
| 27                    | 0.043           | 1.72            |

TABLE II: Signal cross section in fb after all cuts, scaled with b-tagging = 0.7.

Figure 4 presents the estimated statistical significance \( N_{SS} \) as a function of \( \lambda_{tch}/\sqrt{2} \) for the parton level analysis, where \( N_{SS} \) is calculated using [75],

\[
N_{SS} = \sqrt{2 \times (N_S + N_B) \ln(1 + N_S/N_B) - 2 \times N_S}. \tag{19}
\]

Here \( N_S \) and \( N_B \) are number of signal and background events, respectively.
TABLE III: Background cross sections in fb after applying the mass selection at the parton level.

| √s (TeV) | ttjj | ttjj(+) | bbjjττ | bbjjWW | ttjj(+)ττ | ttV | Total |
|----------|------|---------|--------|--------|----------|-----|-------|
| 13       | 0.45 | 0.21    | 0.021  | 3.2×10^{-4} | 0.012    | 3.5×10^{-3} | 0.68 |
| 14       | 0.52 | 0.25    | 0.025  | 3.8×10^{-4} | 0.014    | 3.8×10^{-4} | 0.8  |
| 27       | 1.96 | 0.9     | 0.074  | 1.3×10^{-3} | 0.05     | 9.8×10^{-3}  | 2.99 |

FIG. 4: Cross section of \( pp \rightarrow tch^0 \rightarrow tcττ \rightarrow bj jce^{±}μ^{±} + E_T + X \) (blue solid) in fb as a function of \( \lambda_{tch} \) for \( √s = (a) 14 \text{ TeV} \) and (b) 27 TeV. Also shown are the cross section required for 3\( σ \) (cyan dotted dotted dashed) and 5\( σ \) (green dotted dashed) as well as the cross section of physics background (red dashed).

Table IV presents a comparison between this study and our previous study for \( t \rightarrow ch^0 \rightarrow cWW^∗ \rightarrow ce^{±}μ^{±} + E_T \) [41]. This analysis suggests that \( h^0 \rightarrow ττ \) is much cleaner, because the Higgs boson mass is fully reconstructed and the energy of the charm quark in the top quark rest frame improves the statistical significance using the optimized requirements.

| √s (TeV) | \( h^0 \rightarrow WW^∗ \) | \( h^0 \rightarrow τ^+τ^- \) |
|----------|----------------------------|------------------|
| 13       | 0.060                      | 0.033            |
| 14       | 0.057                      | 0.031            |
| 27       | 0.041                      | 0.023            |

TABLE IV: Minimum \( \lambda_{tch} \) at \( L = 3000 \text{ fb}^{-1} \) for 5 \( σ \).

Figure 5 presents the 5\( σ \) discovery reach at the LHC for (a) \( √s = 14 \text{ TeV} \) and (b) \( √s = 27 \text{ TeV} \) at the parton level in the [cos(\( β − α \)), \( \tilde{ρ}_c \)] plane. We have chosen \( L = 300 \) and 3000 fb\(^{-1} \). It is clear that the high energy LHC at \( √s = 27 \text{ TeV} \) with a high luminosity \( L = 3000 \text{ fb}^{-1} \) significantly improves the discovery potential of \( t \rightarrow ch^0 \) for \( \lambda_{tch} \geq 0.038 \).
beyond the current ATLAS limit [40] $\lambda_{tch} = 0.064$.

FIG. 5: The parton level $5\sigma$ discovery contours at the LHC in the $[\cos(\beta-\alpha), \tilde{\rho}_{tch}]$ plane for (a) $\sqrt{s} = 14$ TeV and (b) $\sqrt{s} = 27$ TeV with $L = 30 fb^{-1}$ (dark green dotted dashed), $300 fb^{-1}$ (medium green solid) and $L = 3000 fb^{-1}$ (light green dashed) Also shown is the current limit on $\lambda_{tch} = \tilde{\rho}_{tch} \cos(\beta - \alpha)$ (red dotted dashed) set by ATLAS [40].

V. EVENT LEVEL ANALYSIS WITH BOOSTED DECISION TREES

In this section, we present the event level analysis using the event generator PYTHIA 8 [54] and the detector simulation program DELPHES [55]. From this analysis, the cross sections for the FCNH signal and the backgrounds are shown in Table V after applying the selection requirements.

| Process  | Cross-section |
|----------|---------------|
| $t\bar{t}jj$ | 1.30          |
| $b\bar{b}jj\tau\tau$ | 0.07          |
| $t\bar{t}W$ | 0.008         |
| $t\bar{t}Z$ | 0.001         |
| $t\bar{t}h^0$ | 0.0002       |
| $b\bar{b}jjWW$ | 0.001       |
| Total Background | $\approx 1.4$ |
| Signal($\lambda_{tch} = 0.01$) | 0.00098 |
| Signal($\lambda_{tch} = 0.064$) | 0.040       |

TABLE V: Event level cross section of signal and backgrounds in fb at the LHC with $\sqrt{s} = 13$ TeV and all selection requirements.

For the event level analysis, the mass resolutions are worse than at the parton level. Therefore, the mass selection window is relaxed and the selected events are used to train
and test the boosted decision trees (BDT) classifier to increase the background rejection relative to signal acceptance. The Root [73] TMVA [74] package is used to perform the signal and background classification. We apply the following requirements on the sample

(i) $65 \text{ GeV} \leq M(j_1, j_2) \leq 100 \text{ GeV}$,

(ii) $40 \text{ GeV} \leq M_T(\ell, \ell) \leq 300 \text{ GeV}$,

(iii) $M_{\text{col}}(\tau, \tau) \leq 200 \text{ GeV}$ and $M_{\text{col}}(c, \tau, \tau) \leq 300 \text{ GeV}$, and

(iv) $20 \text{ GeV} \leq E_c \leq 70 \text{ GeV}$,

and then process it through the BDT, which contains 1000 trees at a depth of 5. The BDT response is shown in Fig 6. The BDT is employed to optimize the selection requirements and improve the statistical significance.

Event selection using the BDT classifier improves the statistical significance of the analysis relative to using an event based selection on kinematic and acceptance variables only. Table VI shows that the BDT analysis improves the statistical significance by more than a factor of two.

| $\sqrt{s}$ (TeV) | Cut-Based | BDT |
|------------------|-----------|-----|
| 13               | 1.2       | 2.7 |
| 14               | 1.3       | 3.2 |
| 27               | 2.2       | 5.5 |

TABLE VI: A comparison of the statistical significance at $\lambda_{\text{tch}} \approx 0.064$ and $\mathcal{L} = 3000 fb^{-1}$ between a kinematic variable selection analysis (Cut-Based) and BDT analysis.

Table VII presents the 95% confidence level limits on $\lambda_{\text{tch}}$ at $\sqrt{s} = 13, 14$ and 27 TeV using an integrated $\mathcal{L} = 300$ and 3000 fb$^{-1}$. In addition, the minimum $\lambda_{\text{tch}}$ for 5σ discovery at the LHC is presented in Table VIII. We conclude that it will be difficult to discover this channel at 13 and 14 TeV colliders in this channel, but a 27 TeV high energy collider holds promise for this signature.

Figure 7 presents the 5σ discovery reach at the LHC for (a) $\sqrt{s} = 14$ TeV and (b) $\sqrt{s} = 27$ TeV at the event level in the $[\cos(\beta - \alpha), \hat{p}_t]$ plane. We have chosen $\mathcal{L} = 300$ and
\[ \sqrt{s} \text{ (TeV)} \quad \mathcal{L} = 300 \text{fb}^{-1} \quad \mathcal{L} = 3000 \text{fb}^{-1} \]

| \sqrt{s} (TeV) | \mathcal{L} = 300 \text{fb}^{-1} | \mathcal{L} = 3000 \text{fb}^{-1} |
|----------------|-------------------------------|-----------------------------|
| 13             | 0.099                         | 0.055                       |
| 14             | 0.092                         | 0.051                       |
| 27             | 0.068                         | 0.038                       |

TABLE VII: 95 % C.L Limits on \( \lambda_{tch} \) at different collider energies and integrated luminosities.

\[ \sqrt{s} \text{ (TeV)} \quad \mathcal{L} = 300 \text{fb}^{-1} \quad \mathcal{L} = 3000 \text{fb}^{-1} \]

| \sqrt{s} (TeV) | \mathcal{L} = 300 \text{fb}^{-1} | \mathcal{L} = 3000 \text{fb}^{-1} |
|----------------|-------------------------------|-----------------------------|
| 13             | 0.21                          | 0.088                       |
| 14             | 0.16                          | 0.082                       |
| 27             | 0.11                          | 0.061                       |

TABLE VIII: Minimal \( \lambda_{tch} \) for \( 5\sigma \) discovery at different collider energies and integrated luminosities.

3000 fb\(^{-1}\). It is clear that the high energy LHC at \( \sqrt{s} = 27 \text{ TeV} \) with a high luminosity \( \mathcal{L} = 3000 \text{ fb}^{-1} \) significantly improves the discovery potential of \( t \rightarrow ch^0 \) beyond the current ATLAS limit [40] \( \lambda_{tch} = 0.064 \).

We have illustrated the improvement in statistical significance achieved by using a boosted decision trees classifier relative to a cut based analysis. To avoid overtraining the BDT due to low statistics in the event level analysis, the more restrictive parton level invariant mass requirements are relaxed. We then rely on the BDT to optimize the selections on the kinematic variables. Our goal is to improve the significance by using the BDT to set the requirements on the invariant masses and the charm-quark energy, which is a strong signal to background discriminant. We encourage our experimental colleagues to include the charm-quark energy as an effective discriminant to further improve the potential of detecting this FCNH signature at the LHC.

FIG. 7: The event level \( 5\sigma \) discovery contours at the LHC in the \( [\cos(\beta - \alpha), \tilde{\rho}_{tc}] \) plane for (a) \( \sqrt{s} = 14 \text{ TeV} \) and (b) \( \sqrt{s} = 27 \text{ TeV} \) with 300 fb\(^{-1}\) (medium green solid) and \( \mathcal{L} = 3000 \text{ fb}^{-1} \) (light green dashed), as well as the event level discovery contours and \( 3\sigma \) contour (yellow dashed) Also shown is the current limit on \( \lambda_{tch} = \tilde{\rho}_{tc} \cos(\beta - \alpha) \) (red dotted dashed) set by ATLAS [40].

We have illustrated the improvement in statistical significance achieved by using a boosted decision trees classifier relative to a cut based analysis. To avoid overtraining the BDT due to low statistics in the event level analysis, the more restrictive parton level invariant mass requirements are relaxed. We then rely on the BDT to optimize the selections on the kinematic variables. Our goal is to improve the significance by using the BDT to set the requirements on the invariant masses and the charm-quark energy, which is a strong signal to background discriminant. We encourage our experimental colleagues to include the charm-quark energy as an effective discriminant to further improve the potential of detecting this FCNH signature at the LHC.
VI. CONCLUSIONS

Many beyond the Standard Model theories contain tree-level contributions to FCNH interactions, especially for the third generation fermions. These contributions arise naturally in models with additional Higgs doublets, such as the special two Higgs doublet model for the top quark (T2HDM) [77], or a general 2HDM [45, 46]. In the decoupling [50] or the alignment limits [51, 52], the light Higgs boson ($h^0$) resembles the standard model Higgs boson with a mass less than the top quark. This could engender the rare decay $t \rightarrow ch^0$.

We investigated the prospects for such a discovery at the LHC, focusing on the $t\bar{t}$ production channel and their subsequent decay, where one decays hadronically and the other through the FCNH mode. The primary background for this signal is $t\bar{t}jj$ with both top quarks decaying leptonically. This background contains one $b$ jet mis-identified as a $c$ jet, and two additional light jets, along with two leptons and missing transverse energy. Taking advantage of the available kinematic information, the $h^0$ and top quark masses in the signal can be reconstructed and much of the background rejected.

Based on our parton level analysis, we find that the LHC at $\sqrt{s} = 14$ TeV, with $\mathcal{L} = 3000$ fb$^{-1}$, can probe to as low as $B(t \rightarrow ch^0) \approx 2.5 \times 10^{-4}$ with $\lambda_{tch} \approx 0.033$. At $\sqrt{s} = 27$ TeV, the reach is $B(t \rightarrow ch^0) \approx 1.4 \times 10^{-4}$ with $\lambda_{tch} \approx 0.023$. The event level analysis implies that there are technical challenges to reach the discovery potential of the parton level analysis, especially, improving efficiencies and mass reconstruction with high precision for final states with missing transverse energy from neutrinos.

In summary, we have made several significant contributions to search for charming top decays with an associated Higgs boson:

(i) The $t \rightarrow ch^0 \rightarrow \tau^+\tau^- \rightarrow ce^\pm\mu^\mp$ has not been previously investigated as a dedicated discovery channel.

(ii) We demonstrate the effectiveness of reconstructing the Higgs boson and the top quark masses by applying the collinear approximation to the tau decays.

(iii) We show that the requirement on the momentum fractions $0 \leq x_i \leq 1, i = 1, 2$ is more effective at removing background and improving the significance than the requirement on centrality ($C_{MET} > 0$).

(iv) Our requirement on the energy of the charm quark ($E_c$) in the top quark rest frame significantly reduces the background and improves the significance.

(v) We have performed the first investigation of the discovery potential of $t \rightarrow ch^0 \rightarrow \tau^+\tau^- \rightarrow ce^\pm\mu^\mp$ for a high energy $pp$ collider at $\sqrt{s} = 27$ TeV.

There are two useful features in the $\tau^+\tau^-$ channel: (a) the reconstruction of $M_h$ and $m_t$ invariant masses applying the collinear approximation, and (b) the selection requirement on the charm quark energy in the top quark rest frame for reducing the physics background. This leads to the $\tau^+\tau^-$ discovery channel having a better reach in $\lambda_{tch}$ by a factor of approximately two over the $W^+W^-$ channel.

We look forward to being guided by new experimental results as we explore the interesting physics of EWSB and FCNH. While the properties of the Higgs boson undergo scrutiny as data is accumulated, dedicated FCNH searches for $t \rightarrow ch^0$ and $\phi^0 \rightarrow t\bar{c} + \bar{t}c, \phi^0 = H^0, A^0$ should be undertaken for the upcoming high luminosity LHC and future high energy $pp$ colliders.
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[1] G. Aad et al. [ATLAS], Phys. Lett. B 716, 1-29 (2012) doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].
[2] S. Chatrchyan et al. [CMS], Phys. Lett. B 716, 30-61 (2012) doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].
[3] A. M. Sirunyan et al. [CMS], Phys. Rev. Lett. 125, no.6, 061801 (2020) doi:10.1103/PhysRevLett.125.061801 [arXiv:2003.10866 [hep-ex]].
[4] G. Aad et al. [ATLAS], Phys. Rev. Lett. 125, no.5, 051801 (2020) doi:10.1103/PhysRevLett.125.051801 [arXiv:2002.12223 [hep-ex]].
[5] A. M. Sirunyan et al. [CMS], Phys. Lett. B 805, 135425 (2020) doi:10.1016/j.physletb.2020.135425 [arXiv:2002.06398 [hep-ex]].
[6] G. Aad et al. [ATLAS], JHEP 07, 108 (2020) doi:10.1007/JHEP07(2020)108 [arXiv:2001.05178 [hep-ex]].
[7] A. D. Sakharov, Sov. Phys. Usp. 34, no.5, 392-393 (1991) doi:10.1070/PU1991v034n05ABEH002497
[8] G. W. Bennett et al. [Muon g-2], Phys. Rev. D 73, 072003 (2006) doi:10.1103/PhysRevD.73.072003 [arXiv:hep-ex/0602035 [hep-ex]].
[9] B. Abi et al. [Muon g-2], Phys. Rev. Lett. 126, no.14, 141801 (2021) doi:10.1103/PhysRevLett.126.141801 [arXiv:2104.03281 [hep-ex]].
[10] S. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato and K. K. Szabo, et al. Nature (2021) doi:10.1038/s41586-021-03418-1 [arXiv:2002.12347 [hep-lat]].
[11] T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè and G. Colangelo, et al. Phys. Rept. 887, 1-166 (2020) doi:10.1016/j.physrep.2020.07.006 [arXiv:2006.04822 [hep-ph]].
[12] P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104
[13] J. P. Lees et al. [BaBar], Phys. Rev. D 88, no.7, 072012 (2013) doi:10.1103/PhysRevD.88.072012 [arXiv:1303.0571 [hep-ex]].
[14] M. Huschle et al. [Belle], Phys. Rev. D 92, no.7, 072014 (2015) doi:10.1103/PhysRevD.92.072014 [arXiv:1507.03233 [hep-ex]].
[15] R. Aaij et al. [LHCb], Phys. Rev. Lett. 115, no.11, 111803 (2015) [erratum: Phys. Rev. Lett. 115, no.15, 159901 (2015)] doi:10.1103/PhysRevLett.115.111803 [arXiv:1506.08614 [hep-ex]].
[16] R. Aaij et al. [LHCb], JHEP 08, 055 (2017) doi:10.1007/JHEP08(2017)055 [arXiv:1705.05802 [hep-ex]].
[17] A. M. Sirunyan et al. [CMS], Phys. Lett. B 781, 517-541 (2018) doi:10.1016/j.physletb.2018.04.030 [arXiv:1710.02846 [hep-ex]].
[18] A. Crivellin, C. Greub and A. Kokulu, Phys. Rev. D 86, 054014 (2012) doi:10.1103/PhysRevD.86.054014 [arXiv:1206.2634 [hep-ph]].
[hep-ph]].
[43] N. Castro, M. Chala, A. Peixoto and M. Ramos, JHEP 10, 038 (2020)
doi:10.1007/JHEP10(2020)038 [arXiv:2005.09594 [hep-ph]].
[44] Y. J. Zhang and J. F. Shen, Eur. Phys. J. C 80, no.9, 811 (2020)
doi:10.1140/epjc/s10052-020-8374-z
[45] S. Davidson and H. E. Haber, Phys. Rev. D 72, 035004 (2005) [erratum: Phys. Rev. D 72, 099902 (2005)]
doi:10.1103/PhysRevD.72.099902 [arXiv:hep-ph/0504050 [hep-ph]].
[46] F. Mahmoudi and O. Stal, Phys. Rev. D 81, 035016 (2010)
doi:10.1103/PhysRevD.81.035016 [arXiv:0907.1791 [hep-ph]].
[47] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, The Higgs Hunter's Guide, (Addison-Wesley, Redwood City, 1990) ISBN 978-0201509359.
[48] A. M. Sirunyan et al. [CMS], Eur. Phys. J. C 79, no.5, 421 (2019)
doi:10.1140/epjc/s10052-019-6909-y [arXiv:1809.10733 [hep-ex]].
[49] G. Aad et al. [ATLAS], Phys. Rev. D 101, no.1, 012002 (2020)
doi:10.1103/PhysRevD.101.012002 [arXiv:1909.02845 [hep-ex]].
[50] J. F. Gunion and H. E. Haber, Phys. Rev. D 67, 075019 (2003)
doi:10.1103/PhysRevD.67.075019 [arXiv:hep-ph/0207010 [hep-ph]].
[51] N. Craig, J. Galloway and S. Thomas, [arXiv:1305.2424 [hep-ph]].
[52] M. Carena, I. Low, N. R. Shah and C. E. M. Wagner, JHEP 04, 015 (2014)
doi:10.1007/JHEP04(2014)015 [arXiv:1310.2248 [hep-ph]].
[53] ATLAS Collaboration, ATL-PHYS-PUB-2013-012.
[54] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen and P. Z. Skands, Comput. Phys. Commun. 191, 159-177 (2015)
doi:10.1016/j.cpc.2015.01.024 [arXiv:1410.3012 [hep-ph]].
[55] J. de Favereau et al. [DELPHES 3], JHEP 02, 057 (2014)
doi:10.1007/JHEP02(2014)057 [arXiv:1307.6346 [hep-ex]].
[56] T. P. Cheng and M. Sher, Phys. Rev. D 35, 3484 (1987)
doi:10.1103/PhysRevD.35.3484
[57] K. Hagiwara, A. D. Martin and D. Zeppenfeld, Phys. Lett. B 235, 198-202 (1990)
doi:10.1016/0370-2693(90)90120-U
[58] T. Han, J. Jiang and M. Sher, Phys. Lett. B 516, 337-344 (2001)
doi:10.1016/S0370-2693(01)00949-2 [arXiv:hep-ph/0106277 [hep-ph]].
[59] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, JHEP 06, 128 (2011)
doi:10.1007/JHEP06(2011)128 [arXiv:1106.0522 [hep-ph]].
[60] S. Dulat, T. J. Hou, J. Gao, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin, C. Schmidt, D. Stump and C. P. Yuan, Phys. Rev. D 93, no.3, 033006 (2016)
doi:10.1103/PhysRevD.93.033006 [arXiv:1506.07443 [hep-ph]].
[61] M. Czakon and A. Mitov, Comput. Phys. Commun. 185, 2930 (2014)
doi:10.1016/j.cpc.2014.06.021 [arXiv:1112.5675 [hep-ph]].
[62] K. Hagiwara, T. Li, K. Mawatari and J. Nakamura, Eur. Phys. J. C 73, 2489 (2013)
doi:10.1140/epjc/s10052-013-2489-4 [arXiv:1212.6247 [hep-ph]].
[63] S. Hoeche, F. Krauss, N. Lavesson, L. Lonnblad, M. Mangano, A. Schallocke and S. Schumann, doi:10.5170/CERN-2005-014.288 [arXiv:hep-ph/0602031 [hep-ph]].
[64] G. Aad et al. [ATLAS], Eur. Phys. J. C 79, no.11, 970 (2019)
doi:10.1140/epjc/s10052-019-7450-8 [arXiv:1907.05120 [hep-ex]].
[65] G. Aad et al. [ATLAS], JHEP 11, 150 (2019)
doi:10.1007/JHEP11(2019)150 [arXiv:1905.02302 [hep-ex]].
[66] R. K. Ellis, I. Hinchliffe, M. Soldate and J. J. van der Bij, Nucl. Phys. B 297, 221-243 (1988) doi:10.1016/0550-3213(88)90019-3

[67] D. L. Rainwater, D. Zeppenfeld and K. Hagiwara, Phys. Rev. D 59, 014037 (1998) doi:10.1103/PhysRevD.59.014037 [arXiv:hep-ph/9808468 [hep-ph]].

[68] T. Plehn, D. L. Rainwater and D. Zeppenfeld, Phys. Rev. D 61, 093005 (2000) doi:10.1103/PhysRevD.61.093005 [arXiv:hep-ph/9911385 [hep-ph]].

[69] M. Aaboud et al. [ATLAS], Phys. Rev. D 99, 072001 (2019) doi:10.1103/PhysRevD.99.072001 [arXiv:1811.08856 [hep-ex]].

[70] V. D. Barger and R. J. N. Phillips, COLLIDER PHYSICS, (Addison-Wesley, Redwood City, 1987) ISBN 978-0201058765.

[71] M. Aaboud et al. [ATLAS], Eur. Phys. J. C 78, no.3, 186 (2018) doi:10.1140/epjc/s10052-018-5649-8 [arXiv:1712.01602 [hep-ex]].

[72] A. M. Sirunyan et al. [CMS], JHEP 10, 117 (2018) doi:10.1007/JHEP10(2018)117 [arXiv:1805.07399 [hep-ex]].

[73] R. Brun and F. Rademakers, Nucl. Instrum. Meth. A 389, 81-86 (1997) doi:10.1016/S0168-9002(97)00048-X

[74] A. Hocker, P. Speckmayer, J. Stelzer, J. Therhaag, E. von Toerne, H. Voss, M. Backes, T. Carli, O. Cohen and A. Christov, et al. [arXiv:physics/0703039 [physics.data-an]].

[75] N. Kumar and S. P. Martin, Phys. Rev. D 92, no.11, 115018 (2015) doi:10.1103/PhysRevD.92.115018 [arXiv:1510.03456 [hep-ph]].

[76] M. Aaboud et al. [ATLAS], Phys. Rev. D 98, no.3, 032002 (2018) doi:10.1103/PhysRevD.98.032002 [arXiv:1805.03483 [hep-ex]].

[77] A. K. Das and C. Kao, Phys. Lett. B 372, 106-112 (1996) doi:10.1016/0370-2693(96)00031-7 [arXiv:hep-ph/9511329 [hep-ph]].