Analysis and Applications of Bonferroni Mean Operators and TOPSIS Method in Complete Cubic Intuitionistic Complex Fuzzy Information Systems

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Abstract: This article manages vagueness, asymmetric data, and risk demonstrated in awkward information. The ambiguity is handled with the help of possibility and strategic decision-making theory. A MADM (multi-attribute decision-making) tool, the sub-part of the strategic decision theory, plays an important role in the circumstances of fuzzy data. The major influence of this analysis is to initiate the mathematical ideology of cubic intuitionistic complex fuzzy (CICF) information and its well-known properties such as algebraic laws, score values, and accuracy values. It is also to determine various inequalities for finding the relation between any two CICF numbers (CICFNs).

Further, we know that the Bonferroni mean (BM) operator is more generalized than the simple averaging/geometric aggregation operators due to parameters involved in the mathematical form of BM operators. Keeping the supremacy and consistency of BM operators, the idea of CICF Bonferroni mean (CICFBM) and CICF weighted BM (CICFWBM) operators are diagnosed. We try to describe their well-known results and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases. Further, we investigate three different sorts of decision-making procedures such as MADM tool, TOPSIS (Technique for order of preference by similarity to ideal solution) method using similarity measures, and TOPSIS method using aggregation operators to enhance the quality of the decision-making process. This analysis expressed how to make decisions when there is asymmetric data about companies. Finally, we compute the comparative analysis of the diagnostic approaches with various existing theories to demonstrate the feasibility and flexibility of the exposed work to try to illustrate with the help of geometrical expressions.

Keywords: cubic intuitionistic complex fuzzy sets; Bonferroni mean operators; TOPSIS methods; decision-making techniques

1. Introduction

MADM analysis is a tool for making options when there are massive awkward dilemmas involving various criteria. It is also used when there are many experts or parties who may be impacted by the outcomes of the options. They are the subparts of the decision-making strategy of operation research that deeply evaluates various conflicting criteria in decision-making. A certain decision-making procedure has been unsuccessful using crisp sets due to their structure because they have only two possibilities such as “0” or “1.” All scholars have tried to find the beneficial optimal procedure from the family of alternatives for crisp sets, which have lost a lot of information during decision-making procedures. Hence, achieving a beneficial decision is an enormously problematic task for individuals. Hence, the mathematical and theoretical concept of fuzzy set [1] was offered to compute MADM regards in complexity and ambiguity, employing the truth grade (TG).
to demonstrate the level of a term involving an FS. MADM tools, a procedure for deciding between the considered alternatives that are provided by many attributes, are important for enhancing decision-making in a large space from proficient to organizational to political. This instruction manual recommends social scholars an encapsulated view of the MADM technique, its advantages, efficiencies, and the technique for evaluating MADM dilemmas. To enhance accuracy and effectiveness in the data shown of demonstration criteria and feasibility of strategic decision-making results, scholars have diagnosed techniques using distinct extensions of FSs for different application frameworks, including the application of soft sets [2], N-soft sets [3], fuzzy soft sets [4], fuzzy N-soft sets [5], bipolar soft sets [6], and the decision-making process [7,8].

The above-cited theory has received much attention from different scholars, but there are various dilemmas in FS for managing awkward information. As such, the dominant theory of intuitionistic FS (IFS) that began by Atanassov [9] included two parts of functions, called TG and falsity grade (FG). In IFS, the value of the sum of the duplet classes lies between unit intervals. To enhance accuracy and effectiveness in the data shown of demonstration criteria and feasibility of strategic decision-making results, scholars have diagnosed techniques using distinct extensions of IFSs for different application frameworks. This includes the concept of t-norm and t-conorm by Xu and Yager [10] who diagnosed the theory of geometric operators using IFSs. Further, Zhao et al. [11] enhanced the aggregation operator to diagnose the generalized aggregation operators using IFS and described their application, where the simple aggregation operator for IFS was exposed by Xu [12]. To demonstrate the closeness among any number of attributes is very challenging for decision-makers. To resolve these issues, certain scholars have diagnosed different sorts of measures based on IFS, proposed by Dengfeng and Chuntian [13], Hung and Yang [14], and Liang and Shi [15]. The hybrid operators also play an important role in the environment of fuzzy set theory. To this, we also revised various prevailing theories such as Bonferroni mean operators for IFSs [16], geometric Heronian mean operators for IFSs [17], and simple Heronian mean operators for IFSs [18]. Further, the fundamental theory of interval-valued IFS was diagnosed by Atanassov [19]. It contains simple two grades in the shape of intervals whose upper parts of the duplet should lie in the unit interval. Additionally, Kaur and Garg [20] combined the theory of IFS and interval-valued IFS to propose the novel concept of cubic IFS. Moreover, aggregation operators using cubic IFS were diagnosed by Kaur and Garg [21].

From the above-cited theory, it is noted that the strategic MADM tool, pattern recognition, clustering algorithm, and computer networking have been diagnosed for FS, IFS, CIFS, and cubic IFS. These theories are incompetent to manage time-periodic dilemmas and two-dimension data simultaneously in a single set. Therefore, Ramot et al. [22] diagnosed complex FS (CFS) theory, addressed by TG, whose real and unreal parts are constructed in the shape of intervals. Further, many discussions were diagnosed based on CFS in the environment of aggregation operators [23], using similarity measures [24], methods [25], and decision-making scenarios [26]. Further, in various dilemmas, the theory of CFS has been neglected for managing awkward information such as in the shape of yes or no. As such the dominant theory of complex IFS (CIFS) was begun by Alkouri and Salleh [27]. The obtained value from the characteristic, the sum of the real part (also for the imaginary part) of the duplet is less than or equal to 1, of CIFS should lie between unit intervals. To enhance accuracy and effectiveness in the data shown of demonstration criteria and feasibility of strategic decision-making results, scholars have diagnosed techniques using distinct extensions of CIFSs for different application frameworks, including the novel concept of the complex intuitionistic fuzzy soft set that was diagnosed by Ali et al. [28]. Further, the fundamental theory of complex intuitionistic fuzzy relation was diagnosed by Jan et al. [29]. Yet many complications are working in the availability of the above-cited work. For this, the novel theory of complex interval-valued IFS (CIVIFS) was diagnosed by Garg and Rani [30].
The notion of CICF settings and BM operators are massively closely linked to the idea of symmetry. Under the symmetry, we can talk about the mixture of both theories. The TOPSIS tool is also very valuable for computing the beneficial option using various similarity measures or aggregation operators. Various diagnoses are described, for instance, the TOPSIS technique based on F5 was diagnosed by Chen [31]. The Analytic Hierarchy Process (AHP) and TOPSIS technique for F5 were also investigated by Junior et al. [32]. Further, Shen et al. [33] utilized the novel concept of the TOPSIS tool in the environment of IF settings and described its application in the region of MADM techniques. Li [34] improved the quality of the TOPSIS tool and utilized it in the region of interval-valued IFSs. Finally, Garg and Kaur [35] diagnosed the extended form of the TOPSIS tool using cubic IFSs and described their application in the environment of decision-making strategy. Yet, no one combined the theory of CIFS, and complex interval-valued IFS was to propose the theory of CICF setting and their important laws. The major contribution of this analysis is to analyze the novel theory of CICF setting and their laws to improve the quality of the research work. The proposed work in this analysis is described here.

1. To initiate the mathematical ideology of CICF information and their well-known properties such as algebraic laws, score values, and accuracy values and to determine various inequalities for finding the order between any two CICFNs.
2. To diagnose the idea of CICFBM, CICFWBM operators are diagnosed and trying to describe their well-known results and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases.
3. To explore the TOPSIS tool, which is also very valuable for computing the beneficial option using various similarity measures or aggregation operators.
4. To investigate three different sorts of decision-making procedures such as MADM tool, TOPSIS method using similarity measures, and TOPSIS method using aggregation operators to enhance the quality of the decision-making process.
5. To compute the comparative analysis of the diagnostic approaches with various existing theories to demonstrate the feasibility and flexibility of the exposed work and try to illustrate this with the help of geometrical expressions.

The proposed analysis is organized in the shape: In Section 2, we recall the mathematical ideology of CIFS, CIVIFS, and their well-known properties such as algebraic laws, score values, accuracy values. We determined various inequalities for finding the order between any two CIFS and CIVIFSs. Further, we recall the mathematical ideology of the BM operator. In Section 3, we initiate the mathematical ideology of CICF information and their well-known properties such as algebraic laws, score values, and accuracy values. We determined various inequalities for finding the order between any two CICFNs. In Section 4, we know that the BM operator is more generalized than the simple averaging/geometric aggregation operators, due to the parameters involved in the mathematical form of BM operators. Keeping the supremacy and consistency of BM operators, the idea of CICFBM and CICFWBM operators are diagnosed. We try to describe their well-known results and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases. In Section 5, we investigate three different sorts of decision-making procedures such as the MADM tool, TOPSIS method using similarity measures, and TOPSIS method using aggregation operators to enhance the quality of the decision-making process. In Section 6, we compute the comparative analysis of the diagnostic approaches with various existing theories to demonstrate the feasibility and flexibility of the exposed work. We try to illustrate with the help of geometrical expressions. Section 7 describes the concluding remarks.

2. Preliminaries

In this analysis, we recall the mathematical ideology of CIFS, CIVIFS, and their well-known properties such as algebraic laws, score values, and accuracy values. We determined various inequalities for finding the order between any two CIFS and CIVIFSs. Further, we
recall the mathematical ideology of the BM operator. Moving forward, the term \( \mathcal{X}_M \), is used for universal sets.

**Definition 1 ([27]).** A CIFS \( \overline{C}_r \) is diagnosed by:

\[
\overline{C}_r = \left\{ \left( \overline{M}_{\overline{C}_r}(x_E), \overline{N}_{\overline{C}_r}(x_E) \right) : x_E \in \mathcal{X}_M \right\}
\]  

(1)

where \( \overline{M}_{\overline{C}_r}(x_E) = \overline{M}_{\overline{R}} e^{2\pi \left( \overline{M}_{\overline{R}} \right)} \) and \( \overline{N}_{\overline{C}_r}(x_E) = \overline{N}_{\overline{R}} e^{2\pi \left( \overline{N}_{\overline{R}} \right)} \), stated the TD and FD with \( 0 \leq \overline{M}_{\overline{R}}(x_E) + \overline{N}_{\overline{R}}(x_E) \leq 1 \) and \( 0 \leq \overline{M}_{\overline{R}}(x_E) + \overline{N}_{\overline{R}}(x_E) \leq 1 \). The expression \( \overline{N}_{\overline{C}_r}(x_E) = \overline{N}_{\overline{R}}(x_E) e^{2\pi \left( \overline{N}_{\overline{R}} \right)} = \left( 1 - \left( \overline{M}_{\overline{R}}(x_E) + \overline{N}_{\overline{R}}(x_E) \right) \right) e^{2\pi \left( 1 - \left( \overline{M}_{\overline{R}}(x_E) + \overline{N}_{\overline{R}}(x_E) \right) \right)} \), called neutral grade. The complex intuitionistic fuzzy number (CIFN) is diagnosed by:

\[
\overline{C}_{r_1} = \left( \overline{M}_{\overline{R}_1} e^{2\pi \left( \overline{M}_{\overline{R}_1} \right)}, \overline{N}_{\overline{R}_1} e^{2\pi \left( \overline{N}_{\overline{R}_1} \right)} \right), \tau = 1, 2, \ldots, \Omega.
\]

**Definition 2 [27].** By considering any two CIFNs \( \overline{C}_{r_1} = \left( \overline{M}_{\overline{R}_1} e^{2\pi \left( \overline{M}_{\overline{R}_1} \right)}, \overline{N}_{\overline{R}_1} e^{2\pi \left( \overline{N}_{\overline{R}_1} \right)} \right), \tau = 1, 2, then

\[
\overline{C}_{r_1} \oplus \overline{C}_{r_2} = \left( \overline{M}_{\overline{R}_1} + \overline{M}_{\overline{R}_2}, \overline{M}_{\overline{R}_1} - \overline{M}_{\overline{R}_2}, \overline{N}_{\overline{R}_1} + \overline{N}_{\overline{R}_2}, \overline{N}_{\overline{R}_1} - \overline{N}_{\overline{R}_2}, \right)
\]

(2)

\[
\overline{C}_{r_1} \odot \overline{C}_{r_2} = \left( \left( \overline{M}_{\overline{R}_1} \overline{M}_{\overline{R}_2} \right) e^{2\pi \left( \overline{M}_{\overline{R}_1} \overline{M}_{\overline{R}_2} \right)}, \left( \overline{N}_{\overline{R}_1} + \overline{N}_{\overline{R}_2} - \overline{N}_{\overline{R}_1} \overline{N}_{\overline{R}_2} \right) e^{2\pi \left( \overline{N}_{\overline{R}_1} \overline{N}_{\overline{R}_2} \right)} \right)
\]

(3)

\[
\overline{C}_{r_1} = \left( 1 - \left( \overline{M}_{\overline{R}_1} \right) \right) e^{2\pi \left( 1 - \left( \overline{M}_{\overline{R}_1} \right) \right)}
\]

(4)

**Definition 3 ([27]).** By considering any CIFNs \( \overline{C}_{r_1} = \left( \overline{M}_{\overline{R}_1} e^{2\pi \left( \overline{M}_{\overline{R}_1} \right)}, \overline{N}_{\overline{R}_1} e^{2\pi \left( \overline{N}_{\overline{R}_1} \right)} \right), \tau = 1, \) then the SV and AV are diagnosed by:

\[
\overline{S}_{SV}(\overline{C}_{r_1}) = \frac{1}{2} \left( \overline{M}_{\overline{R}_1} + \overline{M}_{\overline{R}_1} - \overline{N}_{\overline{R}_1} - \overline{N}_{\overline{R}_1} \right), \overline{S}_{SV}(\overline{C}_{r_1}) \in [-1, 1]
\]

(6)

\[
\overline{H}_{AV}(\overline{C}_{r_1}) = \frac{1}{2} \left( \overline{M}_{\overline{R}_1} + \overline{M}_{\overline{R}_1} + \overline{N}_{\overline{R}_1} + \overline{N}_{\overline{R}_1} \right), \overline{H}_{AV}(\overline{C}_{r_1}) \in [0, 1]
\]

(7)

**Definition 4 ([27]).** By considering any CIFNs \( \overline{C}_{r_1} = \left( \overline{M}_{\overline{R}_1} e^{2\pi \left( \overline{M}_{\overline{R}_1} \right)}, \overline{N}_{\overline{R}_1} e^{2\pi \left( \overline{N}_{\overline{R}_1} \right)} \right), \tau = 1, 2, then

1. When \( \overline{S}_{SV}(\overline{C}_{r_1}) > \overline{S}_{SV}(\overline{C}_{r_2}) \) \( \Rightarrow \) \( \overline{C}_{r_1} > \overline{C}_{r_2} \);
2. When \( \overline{S}_{SV}(\overline{C}_{r_1}) < \overline{S}_{SV}(\overline{C}_{r_2}) \) \( \Rightarrow \) \( \overline{C}_{r_1} < \overline{C}_{r_2} \);
3. When \( \overline{S}_{SV}(\overline{C}_{r_1}) = \overline{S}_{SV}(\overline{C}_{r_2}) \) \( \Rightarrow \);
   (i) When \( \overline{H}_{AV}(\overline{C}_{r_1}) > \overline{H}_{AV}(\overline{C}_{r_2}) \) \( \Rightarrow \) \( \overline{C}_{r_1} > \overline{C}_{r_2} \);
   (ii) When \( \overline{H}_{AV}(\overline{C}_{r_1}) < \overline{H}_{AV}(\overline{C}_{r_2}) \) \( \Rightarrow \) \( \overline{C}_{r_1} < \overline{C}_{r_2} \);
(iii) When $\overline{H_{AV}}(\overline{C}_{e_1}) = \overline{H_{AV}}(\overline{C}_{e_2}) \Rightarrow \overline{C}_{e_1} = \overline{C}_{e_2}$.

Definition 5 ([30]). A CIVIFS $\overline{C}_e$ is diagnosed by:

$$\overline{C}_e = \left\{ \left( \overline{M}_{e}^{\omega}(\overline{x}_E), \overline{M}_{e}^{\Omega}(\overline{x}_E) \right) : \overline{x}_E \in \mathcal{X}_U \right\}$$

(8)

where $\overline{M}_{e}^{\omega}(\overline{x}_E) = \left[ \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E), \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right] e^{2\pi i \left( \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right)}$ and

$\overline{M}_{e}^{\Omega}(\overline{x}_E) = \left[ \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E), \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right] e^{2\pi i \left( \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right)}$, stated the IVTD, IVFD with $0 \leq \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \leq 1$ and $0 \leq \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \leq 1$. The expression

$\overline{M}_{e}^{\omega}(\overline{x}_E) e^{2\pi i \left( \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right)} = \left( 1 - \left( \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right) \right) e^{2\pi i \left( 1 - \left( \overline{M}_{\overline{x}}^{\omega}(\overline{x}_E) + \overline{M}_{\overline{x}}^{\Omega}(\overline{x}_E) \right) \right)}$, called neutral grade. The complex interval-valued intuitionistic fuzzy number (CIVFN) is diagnosed by:

$$\overline{C}_e = \left( \overline{M}_{e}^{\omega}, \overline{M}_{e}^{\Omega} \right) e^{2\pi i \left( \overline{M}_{\overline{x}}^{\omega} + \overline{M}_{\overline{x}}^{\Omega} \right)}$$

(9)

Definition 6 [30]. By considering any two CIVFNs $\overline{C}_{e_1}, \overline{C}_{e_2} = \left( \overline{M}_{e_1}^{\omega}, \overline{M}_{e_1}^{\Omega} \right) e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_1}}^{\Omega} \right)}$, $\tau = 1, 2, \ldots, \Omega$.

$$\overline{C}_{e_1} \oplus \overline{C}_{e_2} = \left( \begin{array}{c}
\left[ \overline{M}_{e_1}^{\omega} + \overline{M}_{e_2}^{\omega} - \overline{M}_{e_1}^{\omega} \overline{M}_{e_2}^{\Omega} \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_2}}^{\Omega} - \overline{M}_{\overline{x_1}}^{\omega} \overline{M}_{\overline{x_2}}^{\Omega} \right)} \\
\left[ \overline{M}_{\overline{x_1}}^{\omega} - \overline{M}_{\overline{x_2}}^{\Omega} \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_2}}^{\Omega} \right)}
\end{array} \right)$$

(10)

$$\overline{C}_{e_1} \odot \overline{C}_{e_2} = \left( \begin{array}{c}
\left[ \overline{M}_{e_1}^{\omega} \overline{M}_{e_2}^{\omega} - \overline{M}_{e_1}^{\omega} \overline{M}_{e_2}^{\Omega} \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_2}}^{\Omega} - \overline{M}_{\overline{x_1}}^{\omega} \overline{M}_{\overline{x_2}}^{\Omega} \right)} \\
\left[ \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_2}}^{\Omega} \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{M}_{\overline{x_2}}^{\Omega} \right)}
\end{array} \right)$$

(11)

$$\overline{\psi}_S \overline{C}_{e_1} = \left( \begin{array}{c}
\left[ \overline{M}_{e_1}^{\omega} \overline{\psi}_S - \overline{M}_{e_1}^{\omega} \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{\psi}_S \right)} \\
\left[ \overline{M}_{\overline{x_1}}^{\omega} \overline{\psi}_S \right] e^{2\pi i \left( \overline{M}_{\overline{x_1}}^{\omega} + \overline{\psi}_S \right)}
\end{array} \right)$$

(12)
Definition 7 ([30]). By considering any CIVIFNs
\[ \overline{\mathbb{C}_i} = \left( \begin{array}{c} \mathcal{M}_{R^i}^c, \mathcal{M}_{R^i}^d \\ \mathcal{N}_{R^i}^c, \mathcal{N}_{R^i}^d \end{array} \right) e^{2\pi i (\mathcal{M}_{R^i}^c \cdot \mathcal{N}_{R^i}^d)}, \] \( \tau = 1, \) then the SV and AV are diagnosed by:
\[ \overline{S_{SV}}(\overline{\mathbb{C}_1}) = \frac{1}{4} \left( \mathcal{M}_{R_1}^{\tau c} + \mathcal{M}_{R_1}^{\tau d} + \mathcal{M}_{R_1}^{\tau c} - \mathcal{N}_{R_1}^{\tau c} - \mathcal{N}_{R_1}^{\tau d} - \mathcal{M}_{R_1}^{\tau c} - \mathcal{N}_{R_1}^{\tau d} \right), \overline{S_{SV}}(\overline{\mathbb{C}_1}) \in [-1, 1] \] (13)
\[ \overline{H_{AV}}(\overline{\mathbb{C}_1}) = \frac{1}{4} \left( \mathcal{M}_{R_1}^{\tau c} + \mathcal{M}_{R_1}^{\tau d} + \mathcal{N}_{R_1}^{\tau c} + \mathcal{N}_{R_1}^{\tau d} \right), \overline{H_{AV}}(\overline{\mathbb{C}_1}) \in [0, 1] \] (14)

Definition 8 ([30]). By considering any CIVIFNs
\[ \overline{\mathbb{C}_i} = \left( \begin{array}{c} \mathcal{M}_{R^i}^c, \mathcal{M}_{R^i}^d \\ \mathcal{N}_{R^i}^c, \mathcal{N}_{R^i}^d \end{array} \right) e^{2\pi i (\mathcal{M}_{R^i}^c \cdot \mathcal{N}_{R^i}^d)}, \] \( \tau = 1, 2, \) then
\begin{enumerate}
  \item When \( \overline{S_{SV}}(\overline{\mathbb{C}_1}) > \overline{S_{SV}}(\overline{\mathbb{C}_2}) \Rightarrow \overline{\mathbb{C}_1} > \overline{\mathbb{C}_2}; \)
  \item When \( \overline{S_{SV}}(\overline{\mathbb{C}_1}) < \overline{S_{SV}}(\overline{\mathbb{C}_2}) \Rightarrow \overline{\mathbb{C}_1} < \overline{\mathbb{C}_2}; \)
  \item When \( \overline{S_{SV}}(\overline{\mathbb{C}_1}) = \overline{S_{SV}}(\overline{\mathbb{C}_2}) \Rightarrow \overline{\mathbb{C}_1} = \overline{\mathbb{C}_2}; \)
\end{enumerate}

Definition 9 ([36]). By considering any positive term \( \overline{\mathbb{C}_{i1}}, \overline{\mathbb{C}_{i2}}, \ldots, \overline{\mathbb{C}_{i\Omega}} \) with \( \mu, \psi \geq 0 \) then the BM operator is diagnosed by:
\[ BM^{\mu, \psi} (\overline{\mathbb{C}_{i1}}, \overline{\mathbb{C}_{i2}}, \ldots, \overline{\mathbb{C}_{i\Omega}}) = \left( \begin{array}{c} 1 \\ \Omega(\Omega - 1) \sum_{\zeta, \tau = 1}^{\Omega} \overline{\mathbb{C}_{i\zeta}} e^{2\pi i \mathcal{M}_{R^\zeta} \cdot \mathcal{M}_{R^\tau}} \end{array} \right)^{\frac{1}{\Omega - 1}} \] (15)

3. Cubic Intuitionistic Complex Fuzzy Sets

In this analysis, we initiate the mathematical ideology of CICF information and their well-known properties such as algebraic laws, score values, and accuracy values. We determined various inequalities for finding the order between any two CICFNs.

Definition 10. A CICFS \( \overline{\mathbb{C}} \) is diagnosed by:
\[ \overline{\mathbb{C}} = \left\{ \left( \mathcal{M}_{R^\zeta}^c (\overline{\mathbb{C}}), \mathcal{N}_{R^\zeta}^c (\overline{\mathbb{C}}) \right), \left( \mathcal{M}_{R^\zeta}^d (\overline{\mathbb{C}}), \mathcal{N}_{R^\zeta}^d (\overline{\mathbb{C}}) \right) : \overline{\mathbb{C}} \in \mathcal{R}^{\zeta} \right\} \] (16)
where
\[ \mathcal{M}_{R^\zeta}^c (\overline{\mathbb{C}}) = \left( \mathcal{M}_{R^\zeta}^c (\overline{\mathbb{C}}), \mathcal{M}_{R^\zeta}^d (\overline{\mathbb{C}}) \right) e^{2\pi i (\mathcal{M}_{R^\zeta}^c \cdot \mathcal{M}_{R^\tau}^d (\overline{\mathbb{C}}))}, \]
\[ \mathcal{N}_{R^\zeta}^c (\overline{\mathbb{C}}) = \left( \mathcal{N}_{R^\zeta}^c (\overline{\mathbb{C}}), \mathcal{N}_{R^\zeta}^d (\overline{\mathbb{C}}) \right) e^{2\pi i (\mathcal{N}_{R^\zeta}^c \cdot \mathcal{N}_{R^\tau}^d (\overline{\mathbb{C}}))}, \]
\[ \mathcal{M}_{R^\zeta}^d (\overline{\mathbb{C}}) = \mathcal{M}_{R^\zeta}^c (\overline{\mathbb{C}}), \]
\[ \mathcal{N}_{R^\zeta}^d (\overline{\mathbb{C}}) = \mathcal{N}_{R^\zeta}^c (\overline{\mathbb{C}}), \]
\[ 0 \leq \mathcal{M}_{R^\zeta} (\overline{\mathbb{C}}) + \mathcal{N}_{R^\zeta} (\overline{\mathbb{C}}) \leq 1, 0 \leq \mathcal{M}_{R^\zeta} (\overline{\mathbb{C}}) + \mathcal{N}_{R^\zeta} (\overline{\mathbb{C}}) \leq 1 \] and
0 \leq \overline{\mathcal{M}}_{\Lambda}^{\text{fl}}(\vec{x}_E) + \overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E) \leq 1, \quad 0 \leq \underline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E) \leq 1. \quad \text{The expression}

\overline{\mathcal{M}}_{\Lambda}^{\text{fl}}(\vec{x}_E)e^{2\pi i(\overline{\mathcal{M}}_{\Lambda}^{\text{fl}}(\vec{x}_E))} = \left(1 - \frac{\overline{\mathcal{M}}_{\Lambda}^{\text{fl}}(\vec{x}_E) + \overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E)}{1 - (\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E))}\right) e^{2\pi i(1-(\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E)))},

\overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E)e^{2\pi i(\overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E))} = \left(1 - \frac{\overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E) + \overline{\mathcal{M}}_{\Lambda}^{\text{fl}}(\vec{x}_E)}{1 - (\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E))}\right) e^{2\pi i(1-(\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E)))},

\overline{\mathcal{N}}_{\Lambda}^{\text{fl}}(\vec{x}_E) = \underline{\mathcal{M}}_{\Lambda}(\vec{x}_E)e^{2\pi i(\underline{\mathcal{N}}_{\Lambda}(\vec{x}_E))} = \left(1 - \frac{\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E)}{1 - (\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E))}\right) e^{2\pi i(1-(\overline{\mathcal{M}}_{\Lambda}(\vec{x}_E) + \underline{\mathcal{N}}_{\Lambda}(\vec{x}_E)))},

\text{called neutral grade. The cubic intuitionistic complex fuzzy number (CICFN) is diagnosed by:}

\overline{\mathcal{C}}_{\mathcal{C}_\tau} = \left(\begin{array}{c}
\overline{\mathcal{M}}_{\Lambda}^{\Omega}, \overline{\mathcal{M}}_{\Lambda}^{\text{fl}} \\
\overline{\mathcal{N}}_{\Lambda}^{\Omega}, \overline{\mathcal{N}}_{\Lambda}^{\text{fl}} \\
\overline{\mathcal{R}}_{\Lambda}^{\Omega}, \overline{\mathcal{R}}_{\Lambda}^{\text{fl}}
\end{array}\right), \quad \tau = 1, 2, \ldots, \Omega.

\text{Definition 11. By considering any two CICFNs}

\overline{\mathcal{C}}_{\mathcal{C}_\tau} = \left(\begin{array}{c}
\overline{\mathcal{M}}_{\Lambda}^{\Omega}, \overline{\mathcal{M}}_{\Lambda}^{\text{fl}} \\
\overline{\mathcal{N}}_{\Lambda}^{\Omega}, \overline{\mathcal{N}}_{\Lambda}^{\text{fl}} \\
\overline{\mathcal{R}}_{\Lambda}^{\Omega}, \overline{\mathcal{R}}_{\Lambda}^{\text{fl}}
\end{array}\right), \quad \tau = 1, 2, \text{then}

\overline{\mathcal{C}}_{\mathcal{C}_1} \oplus \overline{\mathcal{C}}_{\mathcal{C}_2} = \left(\begin{array}{c}
\overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{M}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{M}}_{\Lambda}^{\text{fl}}, \overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{N}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{N}}_{\Lambda}^{\text{fl}}, \overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{R}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{R}}_{\Lambda}^{\text{fl}}
\end{array}\right),

\overline{\mathcal{C}}_{\mathcal{C}_1} \otimes \overline{\mathcal{C}}_{\mathcal{C}_2} = \left(\begin{array}{c}
\overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{M}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{M}}_{\Lambda}^{\text{fl}}, \overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{N}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{N}}_{\Lambda}^{\text{fl}}, \overline{\mathcal{M}}_{\Lambda}^{\Omega} + \overline{\mathcal{R}}_{\Lambda}^{\text{fl}} - \overline{\mathcal{R}}_{\Lambda}^{\text{fl}}
\end{array}\right),

\overline{\psi S} \overline{\mathcal{C}}_{\mathcal{C}_1} = \left(\begin{array}{c}
1 - \left(1 - \overline{\mathcal{M}}_{\Lambda}^{\Omega}\right) \overline{\mathcal{S}}_{\Lambda}^{\Omega}, 1 - \left(1 - \overline{\mathcal{M}}_{\Lambda}^{\Omega}\right) \overline{\mathcal{S}}_{\Lambda}^{\Omega}, 1 - \left(1 - \overline{\mathcal{M}}_{\Lambda}^{\Omega}\right) \overline{\mathcal{S}}_{\Lambda}^{\Omega}
\end{array}\right).
\[
\overline{c}_{e_1} = \left( \begin{array}{c}
1 - \left( 1 - \frac{\overline{C}_{R_1}}{R_1} \right)^{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} e^{2\pi i \left( \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} \right)}
\end{array} \right)
\right), \tau = 1,
\]

\textbf{Definition 12.} By considering any CICF\(Ns\)

\[
\overline{c}_{e_1} = \left( \begin{array}{c}
1 - \left( 1 - \frac{\overline{C}_{R_1}}{R_1} \right)^{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} e^{2\pi i \left( \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} \right)}
\end{array} \right)
\right), \tau = 1,
\]

\text{agnoised by:}

\[
\overline{S}_{SV} (\overline{c}_{e_1}) = \frac{1}{2} \left[ \frac{1}{4} \left( \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}}}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} \right)
\right.
\frac{1}{2} \left( \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}}}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} + \frac{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} - \overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} \right)
\right)
\end{array} \right)
\]

\textbf{Definition 13.} By considering any CICF\(Ns\)

\[
\overline{c}_{e_1} = \left( \begin{array}{c}
1 - \left( 1 - \frac{\overline{C}_{R_1}}{R_1} \right)^{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} e^{2\pi i \left( \frac{\overline{M}_{R_1}^\frac{e}{\overline{M}_{R_1}^\frac{M}{\overline{M}_{R_1}^\frac{M}{L}}} \right)}
\end{array} \right)
\right), \tau = 1, 2,
\]

1. When \(\overline{S}_{SV} (\overline{c}_{e_1}) > \overline{S}_{SV} (\overline{c}_{e_2})\) \(\Rightarrow\) \(\overline{c}_{e_1} > \overline{c}_{e_2}\);
2. When \(\overline{S}_{SV} (\overline{c}_{e_1}) < \overline{S}_{SV} (\overline{c}_{e_2})\) \(\Rightarrow\) \(\overline{c}_{e_1} < \overline{c}_{e_2}\);
3. When \(\overline{S}_{SV} (\overline{c}_{e_1}) = \overline{S}_{SV} (\overline{c}_{e_2})\) \(\Rightarrow\);
   (i) When \(\overline{A}_{AV} (\overline{c}_{e_1}) > \overline{A}_{AV} (\overline{c}_{e_2})\) \(\Rightarrow\) \(\overline{c}_{e_1} > \overline{c}_{e_2}\);
   (ii) When \(\overline{A}_{AV} (\overline{c}_{e_1}) < \overline{A}_{AV} (\overline{c}_{e_2})\) \(\Rightarrow\) \(\overline{c}_{e_1} < \overline{c}_{e_2}\);
   (iii) When \(\overline{A}_{AV} (\overline{c}_{e_1}) = \overline{A}_{AV} (\overline{c}_{e_2})\) \(\Rightarrow\) \(\overline{c}_{e_1} = \overline{c}_{e_2}\);

4. BM Operators for Cubic Intuitionistic Complex Fuzzy Sets

We know that the BM operator is more generalized than the simple averaging/geomeric aggregation operators due to parameters involved in the mathematical form of BM operators. Keeping the supremacy and consistency of BM operators, the idea of CICFB\(M\) and CICFW\(B\)M operators are diagnosed. We try to describe their well-known results.
and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases.

**Definition 14.** By considering any collection of CICFNs

\[ \mathbf{c}_{\mathbf{e}, \tau} = \left( \begin{array}{c} \mathbf{N}_{\mathbf{e}}^\mu, \mathbf{N}_{\mathbf{e}}^\Omega \\ \mathbf{e}^{2\pi \mathbf{m}((\mathbf{N}_{\mathbf{e}}^\mu, \mathbf{N}_{\mathbf{e}}^\Omega))} \end{array} \right), \quad \tau = 1, 2, \ldots, \Omega, \]  

then the mapping \( \mathbf{eI}_{\mathbf{FBM}}^{\mu, \psi} : \mathbb{N}^\Omega \to \mathbb{N} \) is defined by:

\[ \mathbf{eI}_{\mathbf{FBM}}^{\mu, \psi} \left( \mathbf{c}_{\mathbf{e}_1}, \mathbf{c}_{\mathbf{e}_2}, \ldots, \mathbf{c}_{\mathbf{e}_\Omega} \right) = \frac{1}{\Omega(\Omega - 1)} \sum_{\zeta \neq \tau} 1 \left( \mathbf{c}_{\mathbf{e}_\zeta} \otimes \mathbf{c}_{\mathbf{e}_\tau} \right)^{\frac{1}{\mu + \psi}} \]  

(23)

Represented the CICFBM operator for \( \mu, \psi \geq 0 \).

**Theorem 1.** Evaluating the Equation (23) based on Definition 7, we interrogate

\[ \mathbf{eI}_{\mathbf{FBM}}^{\mu, \psi} \left( \mathbf{c}_{\mathbf{e}_1}, \mathbf{c}_{\mathbf{e}_2}, \ldots, \mathbf{c}_{\mathbf{e}_\Omega} \right) = \left( \begin{array}{c} \prod_{\zeta \neq \tau} 1 \left( 1 - \mathbf{N}_{\mathbf{e}_\zeta}^\mu \mathbf{N}_{\mathbf{e}_\tau}^\psi \right)^{\frac{1}{\mu + \psi}} \right) \right) ^{\frac{1}{\mu + \psi}} \]

(24)

Proof of Theorem 1 is given in Appendix A.

Further, we described their well-known results and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases.
Property 1. If \( \overline{\mathcal{E}_\tau} = \overline{\mathcal{E}} = \left( \left[ \mathcal{M}^\xi_R, \mathcal{M}^{\xi\eta}_R \right]^2, e^{2\pi i (\mathcal{M}^\xi_R \cdot \mathcal{M}^{\xi\eta}_R)} \right) \), then

\[
\mathcal{E}I\mathcal{F}BM^{\mu,\varphi}\left( \overline{\mathcal{E}_{\tau_1}}, \overline{\mathcal{E}_{\tau_2}}, \ldots, \overline{\mathcal{E}_{\tau_{\Omega}}} \right) = \overline{\mathcal{E}}
\]

(25)

Proof of Property 1 is given in Appendix B.

Property 2. If \( \overline{\mathcal{E}_\tau} \leq \overline{\mathcal{E}_\tau} \), that is \( \mathcal{M}^\xi_R \leq \mathcal{M}^\xi_R \), \( \mathcal{M}^{\xi\eta}_R \leq \mathcal{M}^{\eta\xi}_R \), \( \mathcal{M}^\xi_R \leq \mathcal{M}^{\xi\eta}_R \), \( \mathcal{M}^{\xi\eta}_R \leq \mathcal{M}^{\eta\xi}_R \), \( \mathcal{M}^{\xi\eta}_R \geq \mathcal{M}^{\eta\xi}_R \), \( \mathcal{M}^{\xi\eta}_R \geq \mathcal{M}^{\eta\xi}_R \), then \( \mathcal{E}I\mathcal{F}BM^{\mu,\varphi}\left( \overline{\mathcal{E}_{\tau_1}}, \overline{\mathcal{E}_{\tau_2}}, \ldots, \overline{\mathcal{E}_{\tau_{\Omega}}} \right) \leq \mathcal{E}I\mathcal{F}BM^{\mu,\varphi}\left( \overline{\mathcal{E}_{\tau_1}}, \overline{\mathcal{E}_{\tau_2}}, \ldots, \overline{\mathcal{E}_{\tau_{\Omega}}} \right) \)

(26)

Proof of Property 2 is given in Appendix C.

Property 3. If \( \overline{\mathcal{E}_\tau} = \left( \left[ \min_t \mathcal{M}^\xi_R, \min_t \mathcal{M}^{\xi\eta}_R \right]^2, e^{2\pi i (\min_t \mathcal{M}^\xi_R \cdot \min_t \mathcal{M}^{\xi\eta}_R)} \right) \) and \( \overline{\mathcal{E}_\tau} = \left( \left[ \max_t \mathcal{M}^\xi_R, \max_t \mathcal{M}^{\xi\eta}_R \right]^2, e^{2\pi i (\max_t \mathcal{M}^\xi_R \cdot \max_t \mathcal{M}^{\xi\eta}_R)} \right) \), then

\[
\overline{\mathcal{E}_\tau} = \mathcal{E}I\mathcal{F}BM^{\mu,\varphi}\left( \overline{\mathcal{E}_{\tau_1}}, \overline{\mathcal{E}_{\tau_2}}, \ldots, \overline{\mathcal{E}_{\tau_{\Omega}}} \right) \leq \overline{\mathcal{E}_\tau}
\]

(27)

Proof of Property 3 is given in Appendix D.

Definition 15. By considering any collection of CICFNs \( \mathcal{E}I\mathcal{F}BM^{\mu,\varphi} : \mathcal{H}^\Omega \rightarrow \mathcal{H} \) is defined by:

\[
\mathcal{E}I\mathcal{F}BM^{\mu,\varphi}\left( \overline{\mathcal{E}_{\tau_1}}, \overline{\mathcal{E}_{\tau_2}}, \ldots, \overline{\mathcal{E}_{\tau_{\Omega}}} \right) = \frac{1}{\Omega(\Omega - 1)} \bigoplus_{\zeta, \tau = 1}^{\Omega} \left( \left( \mathcal{E}_\zeta \overline{\mathcal{E}_{\tau}} \right)^\mu \otimes \mathcal{E}_\tau \mathcal{E}_{\tau'} \right)^\varphi
\]

(28)
Represented the CICFWBM operator for \( \mu, \psi \geq 0 \), where \( \Xi_1, \Xi_2, \ldots, \Xi_\zeta \) expressed the weight vector with \( \sum_{\zeta=1}^{\Omega} \Xi_\zeta = 1 \).

**Theorem 2.** Evaluating the Equation (28) based on Definition 7, we interrogate

\[
\mathcal{CICFWBM^{\mu,\psi}}(\bar{\xi}, \bar{\xi}_2, \ldots, \bar{\xi}_{(1)}) = \left( \left( \begin{array}{c} \mathcal{M}_R^e, \mathcal{M}_R^H \\ e^{2\pi i (\mathcal{M}_R^e, \mathcal{M}_R^H)} \\ \mathcal{M}_R^{e^{2\pi i (\mathcal{M}_R^e, \mathcal{M}_R^H)}} \end{array} \right) \right)_{(1)}^{(1)}
\]

\[
\mathcal{M}_R^e = 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \mathcal{M}_R^e \right)^{\Xi_\zeta} \right)^{\mu} \left( 1 - \left( 1 - \mathcal{M}_R^e \right)^{\Xi_\tau} \right)^{\psi} \left( \frac{1}{\Xi_\zeta - \Xi_\tau} \right)
\]

\[
\mathcal{M}_R^H = 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \mathcal{M}_R^H \right)^{\Xi_\zeta} \right)^{\mu} \left( 1 - \left( 1 - \mathcal{M}_R^H \right)^{\Xi_\tau} \right)^{\psi} \left( \frac{1}{\Xi_\zeta - \Xi_\tau} \right)
\]

\[
\mathcal{M}_R^e = 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \mathcal{M}_R^e \right)^{\Xi_\zeta} \right)^{\mu} \left( 1 - \left( 1 - \mathcal{M}_R^e \right)^{\Xi_\tau} \right)^{\psi} \left( \frac{1}{\Xi_\zeta - \Xi_\tau} \right)
\]

\[
\mathcal{M}_R^H = 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \mathcal{M}_R^H \right)^{\Xi_\zeta} \right)^{\mu} \left( 1 - \left( 1 - \mathcal{M}_R^H \right)^{\Xi_\tau} \right)^{\psi} \left( \frac{1}{\Xi_\zeta - \Xi_\tau} \right)
\]
\[
\overline{N}^l = 1 - \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \overline{N}^l R^\zeta \right)^\mu \right) \left( 1 - \overline{N}^l T^\tau \right)^\psi \right) \frac{1}{\rho^{1-\tau}}
\]

\[
\overline{M}^l = 1 - \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \overline{M}^l R^\zeta \right)^\mu \right) \left( 1 - \overline{M}^l T^\tau \right)^\psi \right) \frac{1}{\rho^{1-\tau}}
\]

\[
\overline{M}^R = 1 - \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \overline{M}^R R^\zeta \right)^\mu \right) \left( 1 - \overline{M}^R T^\tau \right)^\psi \right) \frac{1}{\rho^{1-\tau}}
\]

\[
\overline{N}^R = \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \overline{N}^R R^\zeta \right)^\mu \right) \left( 1 - \overline{N}^R T^\tau \right)^\psi \right) \frac{1}{\rho^{1-\tau}}
\]

\[
\overline{N}^T = \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \left( 1 - \overline{N}^T R^\zeta \right)^\mu \right) \left( 1 - \overline{N}^T T^\tau \right)^\psi \right) \frac{1}{\rho^{1-\tau}}
\]

**Proof.** Trivial. □

5. Applications

We know that the procedure of the MADM technique has several implementations in the field of fuzzy sets theory. However, one of the main and major troubles in the MADM technique is demonstrating evaluations that individuals will use but that does not submerge the essentialness of difficult compromise questions. There are massive various techniques aimed at solving the MADM tool. The major theme of this analysis is to develop a new tool, called the TOPSIS technique instead of the MADM technique. The TOPSIS tool is also very valuable for computing the beneficial option using various similarity measures or aggregation operators. Then, we investigate three different decision-making procedures such as MADM (multi-attribute decision-making) tool, TOPSIS method using similarity measures, and TOPSIS method using aggregation operators to enhance the quality of the decision-making process. Finally, we compute the comparative analysis of the diagnostic approaches with various existing theories to demonstrate the feasibility and flexibility of the exposed work. We try to illustrate this with the help of geometrical expressions.

5.1. TOPSIS Method-1

The major theme of this method is to diagnose new techniques for finding the best optimal option from the group of options. The main stages of the TOPSIS technique are diagnosed below.
Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives \( \overline{c}_{e_1}, \overline{c}_{e_2}, \ldots, \overline{c}_{e_m} \) and their attributes \( G_1, G_2, \ldots, G_\Omega \) with weight vector \( \Xi_1, \Xi_2, \ldots, \Xi_\zeta \) expressed the weight vector with \( \sum_{\zeta=1}^{\Omega} \Xi_\zeta = 1 \).

Stage 2: Using the available information, we compute the positive and native ideal solutions based on Equations (30) and (31), such that

\[
\overline{c}_e^+ = \left( \begin{array}{c}
\max_{R_t} \left( \min_{e_i} \hat{M}_{R_t}^t e, \max_{e_i} \hat{M}_{R_t}^{ul} t \right) e^{2 \pi i \left( \left[ \min_{R_t} \hat{N}_{R_t}^t e, \max_{R_t} \hat{N}_{R_t}^{ul} t \right] \right)} \\
\min_{R_t} \left( \max_{e_i} \hat{M}_{R_t}^t e, \min_{e_i} \hat{M}_{R_t}^{ul} t \right) e^{2 \pi i \left( \left[ \max_{R_t} \hat{N}_{R_t}^t e, \min_{R_t} \hat{N}_{R_t}^{ul} t \right] \right)}
\end{array} \right),
\]

(30)

And

\[
\overline{c}_e^- = \left( \begin{array}{c}
\min_{R_t} \left( \max_{e_i} \hat{M}_{R_t}^t e, \min_{e_i} \hat{M}_{R_t}^{ul} t \right) e^{2 \pi i \left( \left[ \max_{R_t} \hat{N}_{R_t}^t e, \min_{R_t} \hat{N}_{R_t}^{ul} t \right] \right)} \\
\max_{R_t} \left( \min_{e_i} \hat{M}_{R_t}^t e, \max_{e_i} \hat{M}_{R_t}^{ul} t \right) e^{2 \pi i \left( \left[ \min_{R_t} \hat{N}_{R_t}^t e, \max_{R_t} \hat{N}_{R_t}^{ul} t \right] \right)}
\end{array} \right),
\]

(31)

Stage 3: Further, using the positive and negative ideal solution, we determine the discrimination degree using Equations (32) and (33), such that

\[
D\left( \overline{c}_e^+, \overline{c}_e^- \right) = \frac{1}{6} \left| Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \right|
\]

(32)

\[
Z_1 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
Z_2 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
Z_3 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
Z_4 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
Z_5 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
Z_6 = \sum_{\tau=1}^{\Omega} \Xi_\tau \left( \max_{R_t} \hat{M}_{R_t}^t e \ln \left( \frac{2 \hat{M}_{R_t}^t e}{\hat{M}_{R_t}^t e + \hat{M}_{R_t}^{ul} t} \right) + \min_{R_t} \hat{N}_{R_t}^t e \ln \left( \frac{2 \hat{N}_{R_t}^t e}{\hat{N}_{R_t}^t e + \hat{N}_{R_t}^{ul} t} \right) \right),
\]

\[
D\left( \overline{c}_e^+, \overline{c}_e^- \right) = \frac{1}{6} \left| y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \right|
\]

(33)
\[ y_1 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{c}}{R_{t}} \ln \left( \frac{2N_{t}^{c}}{M_{t}^{c} + N_{t}^{c}} \right) + \frac{N_{t}^{c}}{R_{t}} \ln \left( \frac{2N_{t}^{c}}{M_{t}^{c} + N_{t}^{c}} \right) \right) \]

\[ y_2 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{d}}{R_{t}} \ln \left( \frac{2N_{t}^{d}}{M_{t}^{d} + N_{t}^{d}} \right) + \frac{N_{t}^{d}}{R_{t}} \ln \left( \frac{2N_{t}^{d}}{M_{t}^{d} + N_{t}^{d}} \right) \right) \]

\[ y_3 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{e}}{R_{t}} \ln \left( \frac{2N_{t}^{e}}{M_{t}^{e} + N_{t}^{e}} \right) + \frac{N_{t}^{e}}{R_{t}} \ln \left( \frac{2N_{t}^{e}}{M_{t}^{e} + N_{t}^{e}} \right) \right) \]

\[ y_4 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{f}}{R_{t}} \ln \left( \frac{2N_{t}^{f}}{M_{t}^{f} + N_{t}^{f}} \right) + \frac{N_{t}^{f}}{R_{t}} \ln \left( \frac{2N_{t}^{f}}{M_{t}^{f} + N_{t}^{f}} \right) \right) \]

\[ y_5 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{g}}{R_{t}} \ln \left( \frac{2N_{t}^{g}}{M_{t}^{g} + N_{t}^{g}} \right) + \frac{N_{t}^{g}}{R_{t}} \ln \left( \frac{2N_{t}^{g}}{M_{t}^{g} + N_{t}^{g}} \right) \right) \]

\[ y_6 = \sum_{t=1}^{\Omega} \Xi_t \left( \frac{M_{t}^{h}}{R_{t}} \ln \left( \frac{2N_{t}^{h}}{M_{t}^{h} + N_{t}^{h}} \right) + \frac{N_{t}^{h}}{R_{t}} \ln \left( \frac{2N_{t}^{h}}{M_{t}^{h} + N_{t}^{h}} \right) \right) \]

Stage 4: Finally, we determine the closeness coefficient of each alternative, using the values of discrimination, mentioned in Equations (32) and (33) with help of Equation (34), such that

\[ \epsilon_{e} = \frac{D(\overline{\epsilon}_{e}, \overline{\epsilon}_{e}^{+})}{D(\overline{\epsilon}_{e}, \overline{\epsilon}_{e}^{+}) + D(\overline{\epsilon}_{e}, \overline{\epsilon}_{e}^{-})} \]  

(34)

Stage 5: Using the values of closeness coefficient, we determine the final ranking values to explore the beneficial option from the family of alternatives.

5.2. TOPSIS Method-2

The major theme of this method is to diagnose new techniques for finding the best optimal option from the group of options. The main stages of the TOPSIS technique based on the operator are discussed below.

Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives \( \overline{\epsilon}_{e1}, \overline{\epsilon}_{e2}, \ldots, \overline{\epsilon}_{em} \) and their attributes \( G_1, G_2, \ldots, G_{\Omega} \) with weight vector \( \Xi_1, \Xi_2, \ldots, \Xi_{\Omega} \) expressed the weight vector with \( \sum_{\xi=1}^{\Omega} \Xi_{\xi} = 1 \).

Stage 2: Using the available information, we compute the positive and native ideal solutions based on Equations (35) and (36), such that

\[ \overline{\epsilon}_{e}^{+} = \left( \begin{array}{c} [1, 1]e^{2\pi i[1, 1]}, [0, 0]e^{2\pi i[0, 0]} \\ 1e^{2\pi i(1)}, 0e^{2\pi i(0)} \end{array} \right), \left( \begin{array}{c} [1, 1]e^{2\pi i[1, 1]}, [0, 0]e^{2\pi i[0, 0]} \\ 1e^{2\pi i(0)}, 0e^{2\pi i(0)} \end{array} \right), \ldots, \right) \]

\[ \left( \begin{array}{c} [1, 1]e^{2\pi i[1, 1]}, [0, 0]e^{2\pi i[0, 0]} \\ 1e^{2\pi i(1)}, 0e^{2\pi i(0)} \end{array} \right) \]

(35)

And
\[
\overline{\varepsilon_{c\ell}} = \left( \left( \begin{array}{c} [0,0]e^{2\pi(0,0)}, [1,1]e^{2\pi(1,1)} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \end{array} \right), \left( \begin{array}{c} [0,0]e^{2\pi(0,0)}, [1,1]e^{2\pi(1,1)} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \end{array} \right), \ldots, \right)
\]

(36)

Stage 3: Further, using the positive and negative ideal solution, we determine the discrimination degree using the Equations (37) and (38), such that

\[
D\left(\overline{\varepsilon_{c\ell}}, \overline{\varepsilon_{c\ell}}^+\right) = \frac{1}{6} |Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6|
\]

(37)

\[
Z_1 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
Z_2 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right),
\]

\[
Z_3 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
Z_4 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right),
\]

\[
Z_5 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
Z_6 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right).
\]

(38)

\[
y_1 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
y_2 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right),
\]

\[
y_3 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
y_4 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right),
\]

\[
y_5 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^c} \ln \left( \frac{2\overline{M_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{M_{c\ell}^c} + \overline{N_{c\ell}^c}} \right) \right) + \overline{N_{c\ell}^c} \ln \left( \frac{2\overline{N_{c\ell}^c}}{\overline{M_{c\ell}^c} + \overline{N_{c\ell}^c} + \overline{N_{c\ell}^c}} \right),
\]

\[
y_6 = \sum_{r=1}^{\Omega} \Xi_r \left( \overline{M_{c\ell}^{ul}} \ln \left( \frac{2\overline{M_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right) \right) + \overline{N_{c\ell}^{ul}} \ln \left( \frac{2\overline{N_{c\ell}^{ul}}}{\overline{M_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}} + \overline{N_{c\ell}^{ul}}} \right).
\]
Stage 4: Finally, we determine the closeness coefficient of each alternative, using the values of discrimination, mentioned in Equations (37) and (38) with help of Equation (39), such that

$$C_{C_\zeta} = \frac{D\left(\overline{C_{C_\zeta}}, \overline{C}\right)}{D\left(\overline{C_{C_\zeta}}, \overline{C}^+\right) + D\left(\overline{C_{C_\zeta}}, \overline{C}^-\right)}$$ \hspace{1cm} (39)

Stage 5: Using the values of closeness coefficient, we determine the final ranking values to explore the beneficial option from the family of alternatives.

5.3. MADM Techniques

MADM technique plays a very useful role in the circumstances of FS theory. The major theme of this method is to diagnose a new technique for finding the best optimal option from the group of options. The main stages of the MADM technique based on the operator are diagnosed below.

Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives $C_{C_1}, C_{C_2}, \ldots, C_{C_m}$ and their attributes $G_1, G_2, \ldots, G_\Omega$ with weight vector $\Xi_1, \Xi_2, \ldots, \Xi_\zeta$ expressed the weight vector with $\Omega \sum_{\zeta=1}^\zeta \Xi_\zeta = 1$.

Stage 2: Using Equations (24) and (29), we aggregate the suggested information.

Stage 3: Using Equation (21), we find the SV of the accumulated values.

Stage 4: Using the demonstrated SVs, we described the ranking results to find the best option.

5.4. Illustrated Example

To describe the above-cited work, we demonstrate it with the below practical example: Suppose a tycoon agrees to buy a novel strategy for his enterprise from an expert maker $X$. The expert maker gives data on five models of strategy $C_{C_1}, C_{C_2}, C_{C_3}, C_{C_4}$ and $C_{C_5}$ with distinct production dates for each model. The tycoon agrees to suggest four criteria namely $G_1$ Consistency, $G_2$ Comforts, $G_3$ Productivity of strategy, and $G_4$ Feasibility. Conferring to the variations on the fabrication date for a similar type of strategy, these changes also affect the model. The major analysis of this application is to find the best mechanism strategy in the availability of the different criteria. For this, experts give their opinion in the form of weight vector: 0.3, 0.3, 0.3, and 0.1 for four different criteria. Then, we compute the best option using the diagnostic approaches to demonstrate the feasibility and flexibility of the exposed work and to try to illustrate it with the help of geometrical expressions.

5.4.1. TOPSIS Method-1

The major theme of this method is to diagnose new techniques for finding the best optimal option from the group of options. The main stages of the TOPSIS technique are diagnosed below.

Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives $\overline{C_{C_1}}, \overline{C_{C_2}}, \ldots, \overline{C_{C_m}}$ and their attributes $G_1, G_2, \ldots, G_\Omega$ with weight vector $\Xi_1, \Xi_2, \ldots, \Xi_\zeta$ expressed the weight vector with $\Omega \sum_{\zeta=1}^\zeta \Xi_\zeta = 1$, described in Table 1.
Table 1. The decision matrix includes the CICF information.

|       | $G_1$                  | $G_2$                  |
|-------|------------------------|------------------------|
| $\xi_{C_1}$ | \[0.2, 0.3\] & \[0.2, 0.3\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.41, 0.51\] & \[0.41, 0.51\] & \[0.41, 0.51\] & \[0.41, 0.51\] |
|       | \[0.4, 0.5\] & \[0.4, 0.5\] & \[0.41, 0.51\] & \[0.41, 0.51\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] |
|       | \[0.4\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.3)}\] & \[0.4\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.3)}\] & \[0.4\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.3)}\] & \[0.4\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.3)}\] |

| $\xi_{C_2}$ | \[0.4, 0.5\] & \[0.4, 0.5\] & \[0.41, 0.51\] & \[0.41, 0.51\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] |
|       | \[0.2, 0.3\] & \[0.2, 0.3\] & \[0.41, 0.51\] & \[0.41, 0.51\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] |
|       | \[0.5\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.4)}\] & \[0.5\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.4)}\] & \[0.5\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.4)}\] & \[0.5\times e^{-2\pi(0.3)}, 0.2\times e^{2\pi(0.4)}\] |

| $\xi_{C_3}$ | \[0.5, 0.7\] & \[0.5, 0.7\] & \[0.51, 0.71\] & \[0.51, 0.71\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] |
|       | \[0.1, 0.2\] & \[0.1, 0.2\] & \[0.51, 0.71\] & \[0.51, 0.71\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] |
|       | \[0.7\times e^{-2\pi(0.1)}, 0.1\times e^{2\pi(0.4)}\] & \[0.7\times e^{-2\pi(0.1)}, 0.1\times e^{2\pi(0.4)}\] & \[0.7\times e^{-2\pi(0.1)}, 0.1\times e^{2\pi(0.4)}\] & \[0.7\times e^{-2\pi(0.1)}, 0.1\times e^{2\pi(0.4)}\] |

| $\xi_{C_4}$ | \[0.1, 0.3\] & \[0.1, 0.3\] & \[0.11, 0.31\] & \[0.11, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.21, 0.31\] |
|       | \[0.2, 0.3\] & \[0.2, 0.3\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.41, 0.61\] & \[0.41, 0.61\] & \[0.41, 0.61\] & \[0.41, 0.61\] |
|       | \[0.6\times e^{-2\pi(0.2)}, 0.2\times e^{2\pi(0.5)}\] & \[0.6\times e^{-2\pi(0.2)}, 0.2\times e^{2\pi(0.5)}\] & \[0.6\times e^{-2\pi(0.2)}, 0.2\times e^{2\pi(0.5)}\] & \[0.6\times e^{-2\pi(0.2)}, 0.2\times e^{2\pi(0.5)}\] |

| $\xi_{C_5}$ | \[0.1, 0.3\] & \[0.1, 0.3\] & \[0.21, 0.31\] & \[0.21, 0.31\] & \[0.41, 0.61\] & \[0.41, 0.61\] & \[0.41, 0.61\] & \[0.41, 0.61\] |
|       | \[0.2, 0.3\] & \[0.2, 0.3\] & \[0.41, 0.61\] & \[0.41, 0.61\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] & \[0.11, 0.21\] |
|       | \[0.1, 0.2\] & \[0.1, 0.2\] & \[0.51, 0.71\] & \[0.51, 0.71\] & \[0.31, 0.21\] & \[0.31, 0.21\] & \[0.31, 0.21\] & \[0.31, 0.21\] |

Stage 2: Using the available information in Table 1, we compute the positive and native ideal solutions based on Equations (30) and (31), described in Table 2.
Table 2. Includes positive and negative ideal solutions.

|                | Positive Ideal Solution | Negative Ideal Solution |
|----------------|-------------------------|-------------------------|
| $G_1$          | $\begin{pmatrix} 0.5, 0.71 \end{pmatrix}^{2\pi([0.4,0.5]), 0.12} \begin{pmatrix} 0.02939 \\ 0.01289 \end{pmatrix}$ | $\begin{pmatrix} 0.1, 0.31 \end{pmatrix}^{2\pi([0.1,0.2]), 0.12} \begin{pmatrix} 0.04086 \\ 0.4 \end{pmatrix}$ |
| $G_2$          | $\begin{pmatrix} 0.51, 0.71 \end{pmatrix}^{2\pi([0.4,0.5]), 0.12} \begin{pmatrix} 0.02939 \\ 0.01289 \end{pmatrix}$ | $\begin{pmatrix} 0.11, 0.31 \end{pmatrix}^{2\pi([0.1,0.2]), 0.12} \begin{pmatrix} 0.04086 \\ 0.4 \end{pmatrix}$ |
| $G_3$          | $\begin{pmatrix} 0.52, 0.72 \end{pmatrix}^{2\pi([0.4,0.5]), 0.12} \begin{pmatrix} 0.02939 \\ 0.01289 \end{pmatrix}$ | $\begin{pmatrix} 0.12, 0.32 \end{pmatrix}^{2\pi([0.1,0.2]), 0.12} \begin{pmatrix} 0.04086 \\ 0.4 \end{pmatrix}$ |
| $G_4$          | $\begin{pmatrix} 0.53, 0.73 \end{pmatrix}^{2\pi([0.4,0.5]), 0.12} \begin{pmatrix} 0.02939 \\ 0.01289 \end{pmatrix}$ | $\begin{pmatrix} 0.13, 0.33 \end{pmatrix}^{2\pi([0.1,0.2]), 0.12} \begin{pmatrix} 0.04086 \\ 0.4 \end{pmatrix}$ |

Stage 3: Further, using the positive and negative ideal solutions, we determine the discrimination degree using Equations (32) and (33), such that

$$D(\xi_{e_1}, \xi_{e^+}) = 0.04575, D(\xi_{e_2}, \xi_{e^+}) = 0.01289, D(\xi_{e_3}, \xi_{e^+}) = 0.01391, D(\xi_{e_4}, \xi_{e^+}) = 0.03158, D(\xi_{e_5}, \xi_{e^+}) = 0.01331$$

$$D(\xi_{e_2}, \xi_{e^-}) = 0.0006, D(\xi_{e_3}, \xi_{e^-}) = 0.04086, D(\xi_{e_4}, \xi_{e^-}) = 0.07144, D(\xi_{e_5}, \xi_{e^-}) = 0.00335, D(\xi_{e_1}, \xi_{e^-}) = 0.02939$$

Stage 4: Finally, we determine the closeness coefficient of each alternative, using the values of discrimination, mentioned in Equations (32) and (33) with help of Equation (34), such that

$$\xi_{e_1} = 0.98705, \xi_{e_2} = 0.23979, \xi_{e_3} = 0.163, \xi_{e_4} = 0.90399, \xi_{e_5} = 0.31173$$

Stage 5: Using the values of closeness coefficient, we determine the final ranking values to explore the beneficial option from the family of alternatives, stated here,

$$\xi_{e_1} \geq \xi_{e_2} \geq \xi_{e_3} \geq \xi_{e_4} \geq \xi_{e_5}$$

Using the TOPSIS-Method-1, we get the best optimal option as a $\xi_{e_1}$. Further, we describe the TOPSIS-Method-2, which is explained below.

5.4.2. TOPSIS Method-2

The major theme of this method is to diagnose new techniques for finding the best optimal option from the group of options. The main stages of the TOPSIS technique based on the operator are diagnosed below.

Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives $\Xi_{e_1}, \Xi_{e_2}, \ldots, \Xi_{e_m}$ and their attributes $G_1, G_2, \ldots, G_N$ with weight vector $\Xi_1, \Xi_2, \ldots, \Xi_\zeta$ expressed the weight vector with $\sum_{\zeta=1}^{N} \Xi_\zeta = 1$, described in Table 1.
Stage 2: Using the available information, we compute the positive and negative ideal solutions based on Equations (35) and (36), such that

\[
\overrightarrow{\mathbf{e}}^+ = \left( \begin{array}{c} [1, 1]e^{2\pi[(1,1)]}, [0, 0]e^{2\pi[(0,0)]} \\ 1e^{2\pi(1)}, 0e^{2\pi(0)} \\ [1, 1]e^{2\pi[(1,1)]}, [0, 0]e^{2\pi[(0,0)]} \\ 1e^{2\pi(1)}, 0e^{2\pi(0)} \end{array} \right), \quad \left( \begin{array}{c} [1, 1]e^{2\pi[(1,1)]}, [0, 0]e^{2\pi[(0,0)]} \\ 1e^{2\pi(1)}, 0e^{2\pi(0)} \\ [1, 1]e^{2\pi[(1,1)]}, [0, 0]e^{2\pi[(0,0)]} \\ 1e^{2\pi(1)}, 0e^{2\pi(0)} \end{array} \right)
\]

And

\[
\overrightarrow{\mathbf{e}}^- = \left( \begin{array}{c} [0, 0]e^{2\pi[(0,0)]}, [1, 1]e^{2\pi[(1,1)]} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \\ [0, 0]e^{2\pi[(0,0)]}, [1, 1]e^{2\pi[(1,1)]} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \end{array} \right), \quad \left( \begin{array}{c} [0, 0]e^{2\pi[(0,0)]}, [1, 1]e^{2\pi[(1,1)]} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \\ [0, 0]e^{2\pi[(0,0)]}, [1, 1]e^{2\pi[(1,1)]} \\ 0e^{2\pi(0)}, 1e^{2\pi(1)} \end{array} \right)
\]

Stage 3: Further, using the positive and negative ideal solutions, we determine the discrimination degree using the Equations (37) and (38), such that

\[
D\left(\overrightarrow{\mathbf{e}}_1, \overrightarrow{\mathbf{e}}^+\right) = 0.03665, D\left(\overrightarrow{\mathbf{e}}_2, \overrightarrow{\mathbf{e}}^-\right) = 0.04466, D\left(\overrightarrow{\mathbf{e}}_3, \overrightarrow{\mathbf{e}}^+\right) = 0.16631, D\left(\overrightarrow{\mathbf{e}}_4, \overrightarrow{\mathbf{e}}^+\right) = 0.02383, D\left(\overrightarrow{\mathbf{e}}_5, \overrightarrow{\mathbf{e}}^+\right) = 0.08986
\]

\[
D\left(\overrightarrow{\mathbf{e}}_1, \overrightarrow{\mathbf{e}}^-\right) = 0.00127, D\left(\overrightarrow{\mathbf{e}}_2, \overrightarrow{\mathbf{e}}^-\right) = 0.01871, D\left(\overrightarrow{\mathbf{e}}_3, \overrightarrow{\mathbf{e}}^-\right) = 0.25431, D\left(\overrightarrow{\mathbf{e}}_4, \overrightarrow{\mathbf{e}}^-\right) = 0.05181, D\left(\overrightarrow{\mathbf{e}}_5, \overrightarrow{\mathbf{e}}^-\right) = 0.12749
\]

Stage 4: Finally, we determine the closeness coefficient of each alternative, using the values of discrimination, mentioned in Equations (37) and (38) with help of Equation (39), such that

\[
\mathbf{c}_{\mathbf{e}_1} = 0.96644, \mathbf{c}_{\mathbf{e}_2} = 0.70471, \mathbf{c}_{\mathbf{e}_3} = 0.39539, \mathbf{c}_{\mathbf{e}_4} = 0.31509, \mathbf{c}_{\mathbf{e}_5} = 0.41344
\]

Stage 5: Using the values of closeness coefficient, we determine the final ranking values to explore the beneficial option from the family of alternatives, stated here,

\[
\mathbf{c}_{\mathbf{e}_1} \geq \mathbf{c}_{\mathbf{e}_2} \geq \mathbf{c}_{\mathbf{e}_3} \geq \mathbf{c}_{\mathbf{e}_4}
\]

Using the TOPSIS-Method-2, we get the optimal option as \(\mathbf{e}_{\mathbf{e}_1}\). Further, we describe the procedure of the MADM tool, which is explained below.

5.4.3. MADM Techniques

MADM technique plays a very useful role in the circumstances of FS theory. The major theme of this method is to diagnose a new technique for finding the optimal option from the group of options. The main stages of the MADM technique based on the operator are diagnosed below.

Stage 1: In the consideration of CICF information, we develop a matrix that includes alternatives \(\overrightarrow{\mathbf{e}}_{\mathbf{e}_1}, \overrightarrow{\mathbf{e}}_{\mathbf{e}_2}, \ldots, \overrightarrow{\mathbf{e}}_{\mathbf{e}_m}\), and their attributes \(G_1, G_2, \ldots, G_\Omega\) with weight vector \(\Xi_1, \Xi_2, \ldots, \Xi_\xi\) expressed the weight vector with \(\sum_{\xi=1}^{\Omega} \Xi_\xi = 1\), described in Table 1.

Stage 2: Using Equations (24) and (29), we aggregate the suggested information, described in Table 3 for \(\mu = \psi = 1\).
Table 3. Aggregated values are expressed in this matrix.

| SVs          | CICFBM Operator | CICFWBM Operator |
|--------------|-----------------|------------------|
| $\bar{c}_C^1$ | $\begin{pmatrix} 0.1528, 0.2255 \epsilon_2 \pi \left(0.0813, 0.1528\right) \\ 0.5653, 0.6457 \epsilon_2 \pi \left(0.4789, 0.5653\right) \\ 0.5653 \epsilon_2 \pi \left(0.4789\right), 0.1528 \epsilon_2 \pi \left(0.2255\right) \end{pmatrix}$ | $\begin{pmatrix} 0.04009, 0.06179 \epsilon_2 \pi \left(0.0203, 0.04009\right) \\ 0.8629, 0.8938 \epsilon_2 \pi \left(0.8256, 0.8629\right) \\ 0.8629 \epsilon_2 \pi \left(0.8256\right), 0.0409 \epsilon_2 \pi \left(0.06179\right) \end{pmatrix}$ |
| $\bar{c}_C^2$ | $\begin{pmatrix} 0.30028, 0.3778 \epsilon_2 \pi \left(0.1528, 0.2255\right) \\ 0.3831, 0.4789 \epsilon_2 \pi \left(0.4789, 0.5653\right) \\ 0.6457 \epsilon_2 \pi \left(0.4789\right), 0.06179 \epsilon_2 \pi \left(0.08601\right) \end{pmatrix}$ | $\begin{pmatrix} 0.08601, 0.11358 \epsilon_2 \pi \left(0.0409, 0.06179\right) \\ 0.7775, 0.8256 \epsilon_2 \pi \left(0.8256, 0.8629\right) \\ 0.8938 \epsilon_2 \pi \left(0.8256\right), 0.06179 \epsilon_2 \pi \left(0.08601\right) \end{pmatrix}$ |
| $\bar{c}_C^3$ | $\begin{pmatrix} 0.3778, 0.5485 \epsilon_2 \pi \left(0.30028, 0.3778\right) \\ 0.2687, 0.3830 \epsilon_2 \pi \left(0.3830, 0.4789\right) \\ 0.7964 \epsilon_2 \pi \left(0.2687\right), 0.0813 \epsilon_2 \pi \left(0.30028\right) \end{pmatrix}$ | $\begin{pmatrix} 0.1135, 0.1854 \epsilon_2 \pi \left(0.0860, 0.1135\right) \\ 0.7061, 0.7775 \epsilon_2 \pi \left(0.7775, 0.9256\right) \\ 0.9439 \epsilon_2 \pi \left(0.7061\right), 0.0203 \epsilon_2 \pi \left(0.08601\right) \end{pmatrix}$ |
| $\bar{c}_C^4$ | $\begin{pmatrix} 0.0813, 0.2255 \epsilon_2 \pi \left(0.1528, 0.3778\right) \\ 0.3830, 0.4789 \epsilon_2 \pi \left(0.3830, 0.4789\right) \\ 0.7232 \epsilon_2 \pi \left(0.3830\right), 0.1528 \epsilon_2 \pi \left(0.3778\right) \end{pmatrix}$ | $\begin{pmatrix} 0.0203, 0.06179 \epsilon_2 \pi \left(0.0409, 0.06179\right) \\ 0.7775, 0.8256 \epsilon_2 \pi \left(0.8256, 0.8629\right) \\ 0.9204 \epsilon_2 \pi \left(0.7775\right), 0.0409 \epsilon_2 \pi \left(0.1135\right) \end{pmatrix}$ |
| $\bar{c}_C^5$ | $\begin{pmatrix} 0.30028, 0.4598 \epsilon_2 \pi \left(0.1528, 0.2255\right) \\ 0.2687, 0.3830 \epsilon_2 \pi \left(0.4789, 0.5653\right) \\ 0.6457 \epsilon_2 \pi \left(0.6457\right), 0.2255 \epsilon_2 \pi \left(0.1528\right) \end{pmatrix}$ | $\begin{pmatrix} 0.0860, 0.1458 \epsilon_2 \pi \left(0.0400, 0.06179\right) \\ 0.7061, 0.7775 \epsilon_2 \pi \left(0.8256, 0.8629\right) \\ 0.8938 \epsilon_2 \pi \left(0.8938\right), 0.06179 \epsilon_2 \pi \left(0.0409\right) \end{pmatrix}$ |

Stage 3: Using Equation (21), we find the SV of the accumulated values, stated in Table 4.

Table 4. SVs included in this matrix.

| Score Values | CICFBM Operator | CICFWBM Operator |
|--------------|-----------------|------------------|
| $\bar{c}_C^1$ | 0.03888         | 0.01371          |
| $\bar{c}_C^2$ | 0.04351         | 0.01913          |
| $\bar{c}_C^3$ | 0.18223         | 0.06241          |
| $\bar{c}_C^4$ | 0.03288         | 0.01475          |
| $\bar{c}_C^5$ | 0.15858         | 0.06664          |

Stage 4: Using the demonstrated SVs, we described the ranking results to find the best option, described in Table 5.

Table 5. Ranking values are shown in this matrix.

| Methods                  | Ranking Values |
|--------------------------|----------------|
| CICFBM operator          | $\bar{c}_e^1 \geq \bar{c}_e^3 \geq \bar{c}_e^2 \geq \bar{c}_e^4 \geq \bar{c}_e^5$ |
| CICFWBM operator         | $\bar{c}_e^1 \geq \bar{c}_e^3 \geq \bar{c}_e^2 \geq \bar{c}_e^4 \geq \bar{c}_e^5$ |

It is clear from Table 5, we obtain the same ranking results under the consideration of two different concepts, the beneficial optimal option is $\bar{c}_e^5$.

5.5. Influence of Parameters

The variation of the complete score values, with the help of parameters $\mu$ and $\psi$, is analyzed so that the diagnosed operators are massively powerful. Despite this, we will try to find the dominancy of the diagnosed operators. Using the information in Table 1, the influence of parameters $\mu$ and $\psi$, are illustrated in Table 6. We fixed the value of
one parameter $\mu = 1$, and tried to change the value of $\psi$ to check the variations of the parameters, described in Table 6.

**Table 6.** Includes influence of parameter for $\mu=1$.

| Methods | Score Values | Ranking Values |
|---------|--------------|----------------|
| $\psi=1$ | $0.03888, 0.04351, 0.18223, 0.03288, 0.15858$ | $c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\psi=3$ | $0.04295, 0.04463, 0.2022, 0.03336, 0.16646$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\psi=5$ | $0.01691, 0.02316, 0.0775, 0.0179, 0.07069$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\psi=7$ | $0.01963, 0.0268, 0.09019, 0.02073, 0.09328$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\psi=10$ | $0.01864, 0.02546, 0.08559, 0.01969, 0.08863$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |

Using the information in Table 1, the influence of parameters $\mu$ and $\psi$, is illustrated in Table 7. We fixed the value of one parameter $\psi = 1$, and tried to change the value of $\mu$ to check the variations of the parameters, described in Table 7.

**Table 7.** Includes the influence of parameter for $\psi=1$.

| Methods | Score Values | Ranking Values |
|---------|--------------|----------------|
| $\mu=1$ | $0.03888, 0.04351, 0.18223, 0.03288, 0.15858$ | $c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\mu=3$ | $0.04294, 0.0447, 0.20209, 0.03345, 0.16665$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\mu=5$ | $0.01731, 0.02382, 0.07913, 0.01849, 0.08292$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\mu=7$ | $0.01899, 0.02612, 0.08689, 0.02032, 0.09082$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |
| $\mu=10$ | $0.04661, 0.04711, 0.2214, 0.0354, 0.1765$ | $c_{e_3} \geq c_{e_5} \geq c_{e_2} \geq c_{e_1} \geq c_{e_4}$ |

Tables 6 and 7 contain almost the same results because both operators provided different ranking results. The beneficial optimal option is $c_{e_5}$, and for some values, we obtain the value of diagnosed operators in the shape: $c_{e_5}$. After checking the influence of parameters, we enhance the quality of the diagnosed operators with the help of comparative analysis, described below.

6. Comparative Analysis

We know that without comparing diagnosed work there is no worth in the environment of qualitative research work. The major theme of this analysis is to demonstrate the key factor of the proposed approaches with the help of comparative analysis. For this, we suggest some prevailing theories whose information is described here: aggregation operators for cubic IFS, diagnosed by Kaur and Garg [20]; generalized aggregation operators for cubic IFS, also diagnosed by Kaur and Garg [21]; and aggregation operators for complex
interval-valued IFS, diagnosed by Garg and Rani [30] with proposed techniques. Using data in Table 1, the comparative analysis is described below.

1. We suggest that the aggregation operators based on cubic IFS, diagnosed by Kaur and Garg [20], have a lot of deficiencies due to its structure. This is because the mathematical form of cubic IFS manages only one-dimensional information which affected the result and was a major reason for losing information during the decision-making process. For this, we diagnosed the novel theory of CICF settings and their BM operators, which was more powerful and massively dominant as compared to the work of Kaur and Garg [20].

2. We suggest that the generalized aggregation operators based on cubic IFS, diagnosed by Kaur and Garg [21], have a lot of deficiencies due to its structure. This is because the mathematical form of cubic IFS manages only one-dimensional information which affected the result and was a major reason for losing information during the decision-making process. For this, we diagnosed the novel theory of CICF settings and their BM operators, which is more powerful and massively dominant as compared to the work of Kaur and Garg [21].

3. We suggest that the aggregation operators based on complex interval-valued IFS, diagnosed by Garg and Rani [30], have a lot of deficiencies due to its structure. This is because the mathematical form of complex interval-valued IFS has managed two-dimensional information, but the rule of complex interval-valued IFS is very weak. Similarly, the sum of the real part (imaginary part) of the duplet is limited to the unit interval, which has affected the result and was a major reason for losing information during the decision-making process. For this, we diagnosed the novel theory of CICF settings and their BM operators, which is more powerful and massively dominant as compared to the work of Garg and Rani [30].

Therefore, our diagnosed work has massive application in the region of computer science, engineering science, networking systems, and pattern recognition. In the future, we will employ it in the environment of the above-cited work.

7. Conclusions

In this analysis, we initiate the mathematical ideology of CICF information and their well-known properties like algebraic laws, score values, and accuracy values. We determined various inequalities for finding the order between any two CICFNs. Further, we know that the BM operator is more generalized than the simple averaging/geometric aggregation operators due to the parameters involved in the mathematical form of BM operators. Keeping the supremacy and consistency of BM operators, the idea of CICFBM and CICFWBM operators are diagnosed. We try to describe their well-known results and properties such as idempotency, monotonicity, commutativity, and boundedness with various specific cases. Similarly, the TOPSIS tool is also very valuable for computing the beneficial option using various similarity measures or aggregation operators. Then, we investigate three different sorts of decision-making procedures such as the MADM tool, the TOPSIS method using similarity measures, and the TOPSIS method using aggregation operators to enhance the quality of the decision-making process. Finally, we compute the comparative analysis of the diagnostic approaches with various existing theories to demonstrate the feasibility and flexibility of the exposed work. We try to illustrate this with the help of geometrical expressions.

In the future, we will revise various old theories and try to diagnose new theories such as cubic q-rung orthopair complex fuzzy sets and cubic T-spherical complex fuzzy sets, using the existing theories such as complex q-rung orthopair fuzzy sets [37], complex spherical fuzzy sets [38], T-spherical fuzzy sets [39], linear Diophantine fuzzy sets [40,41], and decision-making [42,43].
Author Contributions: Conceptualization, Z.A. and T.M.; methodology, Z.A.; software, Z.A., T.M. and S.B.; validation, T.M., S.B. and R.C.; formal analysis, S.B. and R.C.; investigation, Z.A., T.M., S.B. and R.C.; resources, S.B. and R.C.; data curation, T.M., S.B. and R.C.; writing—original draft preparation, Z.A.; writing—review and editing, T.M., S.B. and R.C.; visualization, T.M. and R.C.; supervision, T.M.; project administration, S.B. and R.C.; funding acquisition, S.B. and R.C. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Proof of Theorem 1. Suggested Definition 7, we have

\[\overline{\xi}^\mu = \left( \begin{array}{c}
\left[ \frac{\mathcal{M}^{\mu \nu}}{R_x}, \frac{\mathcal{M}^{\mu \mu}}{R_x} \right] e^{i2\pi \left( \frac{\psi}{\epsilon} - \frac{\mu}{\epsilon} \right)} \\
1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\mu, 1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\mu e^{i2\pi \left( 1 - \frac{1}{\epsilon} - \frac{\mu}{\epsilon} \right)}
\end{array} \right),
\]

\[\overline{\psi} = \left( \begin{array}{c}
\left[ \frac{\mathcal{M}^{\psi \psi}}{R_x}, \frac{\mathcal{M}^{\psi \mu}}{R_x} \right] e^{i2\pi \left( \frac{\psi}{\epsilon} - \frac{\mu}{\epsilon} \right)} \\
1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\psi, 1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\psi e^{i2\pi \left( 1 - \frac{1}{\epsilon} - \frac{\mu}{\epsilon} \right)}
\end{array} \right),
\]

Then,

\[\overline{\xi}^\mu \otimes \overline{\psi} = \left( \begin{array}{c}
\left[ \frac{\mathcal{M}^{\mu \nu}}{R_x}, \frac{\mathcal{M}^{\mu \mu}}{R_x}, \frac{\mathcal{M}^{\psi \psi}}{R_x}, \frac{\mathcal{M}^{\psi \mu}}{R_x} \right] e^{i2\pi \left( \frac{\psi}{\epsilon} - \frac{\mu}{\epsilon} \right)} \\
1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\mu, 1 - \left( 1 - \frac{\mathcal{M}}{R_y} \right)^\mu e^{i2\pi \left( 1 - \frac{1}{\epsilon} - \frac{\mu}{\epsilon} \right)}
\end{array} \right),
\]

Further,
\[ \oplus_{\zeta, \tau = 1}^{\Omega} \left( \frac{\zeta''}{\zeta'} \otimes \frac{\zeta''}{\zeta'} \right) = \left( \begin{array}{c} 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \frac{\zeta''}{\zeta'} \right)^{\phi} \end{array} \right) \left( \begin{array}{c} 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \frac{\zeta''}{\zeta'} \right)^{\mu} \end{array} \right) \right)^{\Omega} e^{i2\pi \left( (1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \frac{\zeta''}{\zeta'} \right)^{\phi}) \right.}^{(1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \frac{\zeta''}{\zeta'} \right)^{\mu})} \left. \right] \]
\[
\frac{1}{\Omega(\Omega - 1)} \prod_{\zeta \neq \tau}^{\Omega} (e^{\zeta} \otimes e^{\tau}) = \left( \begin{array}{c}
\prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - \frac{\Omega-1}{\Omega} \right) ^{\Omega} \\
\prod_{\zeta \neq \tau}^{\Omega} \left( 1 - \frac{\Omega-1}{\Omega} \right) ^{\Omega} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)}
\end{array} \right) \\
\left( 1 - \prod_{\zeta \neq \tau}^{\Omega} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)} \right) \left( 1 - \prod_{\zeta, \tau = 1}^{\Omega} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)} \right) \left( 1 - \prod_{\zeta \neq \tau}^{\Omega} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)} \right)
\right) \left( 1 - \prod_{\zeta \neq \tau}^{\Omega} \left( 1 - M_{\zeta}^{\mu} M_{\tau}^{\mu} \right) ^{\Omega(\Omega-1)} \right)
\]
\[
\epsilon^{\text{elemFM}}(\overline{e_1, e_2, \ldots, e_n}) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( \frac{1}{1 - \Omega \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right) \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]

\[
\epsilon^{\text{elemFM}}(\phi, \mathcal{N}, \zeta) = \left( \prod_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \left( 1 - \frac{\sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}}}{\Omega} \sum_{\mathcal{C}_\mathcal{T} = 1}^{\mathcal{T} \neq \mathcal{T}} \right)^{\phi} \right)^{\frac{1}{\mathcal{T}_{\mathcal{T}}}},
\]
Appendix B

Proof of Property 1. We suggest that $\overline{e_c_1} = \overline{e_c}$, then using Equation (24), we interrogate

$$e_{\mathcal{FB}}\mu,\psi\left(\overline{e_c_1}, \overline{e_c_2}, \ldots, \overline{e_{c_\Omega}}\right) = \left(\frac{1}{\Omega(\Omega-1)} \oplus \Omega\right)_{\zeta, \tau = 1 \atop \zeta \neq \tau} 1 \left( \overline{e_c^{\mu}} \otimes \overline{e_c^{\psi}} \right)_{\frac{1}{\Omega}}^{1-1}$$

$$= \left(\frac{1}{\Omega(\Omega-1)} \oplus \Omega\right)_{\zeta, \tau = 1 \atop \zeta \neq \tau} 1 \left( \overline{e_c^{\mu}} \otimes \overline{e_c^{\psi}} \right)_{\frac{1}{\Omega}}^{1-1} = \left( \overline{e_c^{\mu+\psi}} \right)_{\frac{1}{\Omega}}^{1-1} = \overline{e_c}.$$

$\square$

Appendix C

Proof of Property 2. By suggested information, if $\overline{e_c_1} \leq \overline{e_c_1}$, that is $\overline{\mathcal{M}_{\mathcal{R}_1}} \leq \overline{\mathcal{M}_{\mathcal{R}_1}}$, $\overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \leq \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1}}$, $\overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \leq \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1}}$, $\overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \geq \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}}$, and

$$\overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \geq \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}}$$

then

$$\overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \leq \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \leq 1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \leq 1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}}$$

$$\Leftrightarrow \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau} \left(1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \right)_{\frac{1}{\Omega(\Omega-1)}} \leq \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau} \left(1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \right)_{\frac{1}{\Omega(\Omega-1)}}$$

Similarly, we get

$$\prod_{\zeta, \tau = 1 \atop \zeta \neq \tau} \left(1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \right)_{\frac{1}{\Omega(\Omega-1)}} \leq \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau} \left(1 - \overline{\mathcal{M}_{\mathcal{R}_1}^{\Omega_1} \mathcal{M}_{\mathcal{R}_1}^{\Omega_1}} \right)_{\frac{1}{\Omega(\Omega-1)}}$$
\[
\left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}} \leq \left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}}
\]

And also, for simply FG, we have

\[
\left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}} \leq \left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}}
\]

And

\[
\left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}} \leq \left(1 - \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\frac{1}{\text{dim}(\tau)}} \right)^{\frac{1}{\text{dim}(\tau)}}
\]

Further,

\[
\left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi} \geq \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi}
\]

\[
\Leftrightarrow \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi} \leq \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi}
\]

\[
\Leftrightarrow \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi} \leq \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi}
\]

\[
\Leftrightarrow \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi} \leq \prod_{\zeta, \tau = 1 \atop \zeta \neq \tau}^{1} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\mu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\mu} \left(1 - \frac{\Omega}{\Omega_{R_{\tau}}} \frac{\Lambda^\nu}{\Lambda_{R_{\tau}}} \psi_{R_{\tau}}^{\Omega} \right)^{\psi}
\]
\[\begin{align*}
&1 - \left( \prod_{\zeta \neq \tau} \frac{1}{\left(1 - \frac{1}{\zeta^2} \right)^\mu \left(1 - \frac{1}{\tau^2} \right)^\nu} \right) ^{\frac{1}{\mu + \nu}} \\
&\geq 1 - \left( \prod_{\zeta \neq \tau} \frac{1}{\left(1 - \frac{1}{\zeta^2} \right)^\mu \left(1 - \frac{1}{\tau^2} \right)^\nu} \right) ^{\frac{1}{\mu + \nu}} \\
&\text{Similarly, we get}
\end{align*}\]

Then, by using the information given in Equations (21) and (22), we get

\[\epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1}, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}) \leq \epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1^+, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}}).\]

\[\square\]

Appendix D

Proof of Property 3. Using Property 1 and Property 2, we know that

\[\epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1}, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}) \geq \epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1^-}, \bar{\mathcal{C}}_{e_2^-}, \ldots, \bar{\mathcal{C}}_{e_\Omega}) = \bar{\mathcal{C}}_{e_r^-}\]

\[\epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1}, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}) \leq \epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1^+, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}}) = \bar{\mathcal{C}}_{e_r^+}\]

Then, we get

\[\bar{\mathcal{C}}_{e_r^-} \leq \epsilon_{IEFBM}^{\mu, \nu}(\bar{\mathcal{C}}_{e_1}, \bar{\mathcal{C}}_{e_2}, \ldots, \bar{\mathcal{C}}_{e_\Omega}) \leq \bar{\mathcal{C}}_{e_r^+}.\]

\[\square\]
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