Three-body molecules $\bar{D}D^*\Sigma_c$: understanding the nature of $T_{cc}$, $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$

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The nature of the three pentaquark states, $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$, discovered by the LHCb Collaboration in 2019, is still under debate, although the $D^{(*)}\Sigma_c$ molecular interpretation seems to be the most popular. In this work, by adding a $D$ meson into the $D^*\Sigma_c$ pair, we investigate the mass and decay width of the three-body molecules $\bar{D}D^*\Sigma_c$ and explore the correlation between the existence of the $\bar{D}D^*\Sigma_c$ molecules with the existence of $D^{(*)}\Sigma_c$ and $D^*\bar{D}$ two-body molecules. The latter can be identified with the doubly charmed tetraquark state $T_{cc}$ recently discovered by the LHCb Collaboration. Based on the molecular nature of $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, and $T_{cc}$, our results indicate that there exist two three-body bound states of $\bar{D}D^*\Sigma_c$ with $I(J^{P}) = 1(1/2^+)$ and $I(J^{P}) = 1(3/2^+)$, and binding energies $37.24$ MeV and $29.63$ MeV below the $D^*\Sigma_c$ mass threshold. In addition, we find that the mass splitting of these two three-body molecules are correlated to the mass splitting of $P_c(4440)$ and $P_c(4457)$, which offers a non-trivial way to reveal the molecular nature of these states. The partial widths of two $\bar{D}D^*\Sigma_c$ molecules decaying into $J/\psi p\bar{D}$ and $J/\psi p D^*$ are found to be several MeV. We recommend the experimental searches for the $\bar{D}D^*\Sigma_c$ molecules in the $J/\psi p\bar{D}$ and $J/\psi p D^*$ invariant mass distributions.

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I. INTRODUCTION

In terms of the constituent quark model proposed by Gell-Mann [1] and Zweig [2,3], hadrons can be classified either as mesons made of a pair of quark and anti-quark or baryons made of three quarks, the property of which can be well described in the conventional quark model [4, 5]. However, more and more the so-called exotic states beyond the traditional quark model have been discovered experimentally, starting from $X(3872)$ in 2003 [6]. To clarify the nature of these exotic states, a lot of theoretical interpretations were proposed, such as hadronic molecules, compact multiplet quark states, kinetic effects, and so on (for recent reviews, see Refs. [7, 16]). Among them, the hadronic molecular picture is rather popular because many (if not all) of these states are located near the mass threshold of a pair of conventional hadrons. Nevertheless, how to confirm the molecular nature of these exotic states remains a big challenge for both experiments and theory.

To confirm an exotic state as a hadronic molecule, one needs to be able to describe its production rate, decay width, mass, spin-parity, and other relevant properties consistently in the molecular picture. However, most approaches can only describe part of the relevant properties. This motivates us to find alternative methods to help achieve this goal. In nuclear physics, the existence of light nuclei, such as triton or $^3H$ serves as a non-trivial check on the two-body bound-state nature of the deuteron. Along this line, assuming $D_{s0}^*(2317)$ as a $DK$ bound state, we have studied the few-body systems of $D^0 K$ and $DDDK$, the existence of which indeed support the molecular nature of $D_{s0}^*(2317)$ [17]. In this work, assuming $T_{cc}$ and the three pentaquark states [P$_c$(4312), P$_c$(4440) and P$_c$(4457)] as $DD^*$ and $D^*\Sigma_c$ bound states, respectively, we investigate the related three-body system $\bar{D}D^*\Sigma_c$ and explore the correlation of the three-body bound states $\bar{D}D^*\Sigma_c$ with the related two-body bound states.

The hidden charm pentaquark states, $P_c(4380)$ and $P_c(4450)$, were firstly discovered by the LHCb Collaboration in 2015 [18]. With a statistics 10 times larger, the $P_c(4450)$ state splits into $P_c(4440)$ and $P_c(4457)$, in addition a new state $P_c(4312)$ appears, all of which lie close to the mass thresholds of $D^{(*)}\Sigma_c$ [19]. In our previous work [20], we have employed a contact-range effective field theory (EFT) to assign $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ as $D^{(*)}\Sigma_c$ hadronic molecules dictated by heavy quark spin symmetry (HQSS), which was confirmed by many other groups [21-32]. Even so, there exist other explanations, such as, hadro-charmonium [33], compact pentaquark states [34, 35], virtual states [36, 42] and double triangle singularities [43]. The existence of three-body bound states $D\bar{D}^*\Sigma_c$ will further verify the molecular nature of the pentaquark states, where $D\bar{D}^*\Sigma_c$ system can be viewed as a cluster of a $D$ meson and the $\bar{D}\Sigma_c$ pair or a $D^*$ meson and the $\Sigma_c$ pair.

In addition, the $\bar{D}D^*\Sigma_c$ system can be regarded as a cluster of a $\Sigma_c$ baryon and the $D^*\bar{D}$ pair, which is related to the doubly charmed tetraquark state $T_{cc}^+$ discovered by the LHCb Collaboration [44]. The mass of $T_{cc}^+$ is below the mass threshold of $D^0 D^{**}$ by only several hundred keV, and its decay width from the unitary analysis is rather small, only a few tens of...
where $T_{cc}$ state is a $D\bar{D}^*$ bound state, its mass and decay width can be described in the hadronic molecular model [46–55]. Although the molecular interpretation seems to be the most popular, the interpretation of a compact tetraquark state cannot be ruled out. The study of the three-body system $\bar{D}D^+\Sigma_c$ could also be helpful to verify its molecular nature.

The three-body system $\bar{D}D^+\Sigma_c$ is particularly interesting for a number of reasons. First of all, there is no annihilation of a pair of light quark and its anti-quark. This indicates that such a state, if exists, has a minimum quark content $\bar{c}cqqq$, which is explicitly exotic. Second, the interactions of the subsystems $D\Sigma_c$, $D^+\Sigma_c$, and $D^*\bar{D}$ can be precisely determined by reproducing the masses of their corresponding molecular candidates using the one-boson-exchange (OBE) potential, which largely reduces the uncertainty of the so-obtained binding energy of the $\bar{D}D^+\Sigma_c$ state. Such exotic states, if discovered by experiments, will help verify the molecular nature.

This paper is organized as follows. In Sec. II we briefly explain how to solve the three-body Schrödinger equation by GEM. Next in Sec. III we present the binding energies of the three-body bound states $\bar{D}D^+\Sigma_c$ and calculate the partial widths of the $\bar{D}D^+\Sigma_c$ molecules decaying into $J/\psi p\bar{D}^*$ and $T_{cc}\Lambda_\pi$. Finally, this paper is ended with a short summary in Sec. IV.

II. FORMALISM

To obtain the binding energy of the $\bar{D}D^+\Sigma_c$ system, we need to solve the three-body Schrödinger equation by GEM, which has been widely applied to investigate few-body systems in nuclear physics [56] and hadron physics [57, 58]. The three-body Schrödinger equation reads

$$[T + V^1(r_1) + V^2(r_2) + V^3(r_3) - E]\Psi_{J_M}^{total} = 0,$$

where $T$ is the kinetic-energy operator, $V^i(r_i)$ is the potential between the $j_{th}$ and $k_{th}$ particle pair ($i, j, k = 1 - 3$), and the $1_{st}$, $2_{nd}$, and $3_{rd}$ particle refer to the $D$ meson, $D^*$ meson, and $\Sigma_c$ baryon, respectively. The total wave function $\Psi_{J_M}^{total}$ is expressed as a sum of three component functions:

$$\Psi_{J_M}^{total} = \sum_{i=1}^{3} C_{i, \alpha} \Psi_{J_M, \alpha}^{c_{i}(r_1, r_2)},$$

where $C_{i, \alpha}$ are the expansion coefficients of relevant basis, $i = 1, 2, 3$ denotes the three channels of Fig. 1 and $\alpha = \{nl, N, L, \lambda, \Sigma, s, T, t\}$. Here $l$ and $L$ are the orbital angular momentum of the coordinates $r$ and $R$, and $s$ and $\lambda$ are the isospin and spin of the two-body subsystem in each channel, and $\lambda$, $\Sigma$ and $T$ are the total orbital angular momentum, spin and isospin, respectively. The wave function of each channel is

![FIG. 1: Three Jacobi coordinates of the $\bar{D}D^+\Sigma_c$ system](image)

expressed as

$$\Phi_{J_M, \alpha}^{c_{i}(r_1, r_2)} = [\Phi_{L, \lambda}^{c_{i}\Sigma}(r_1, r_2)]_{JM} H_{TT},$$

where $\Phi_{L, \lambda}^{c_{i}}$ is the spatial wave function, and $\Omega_{LM}^{c_{i}}$ is the spin wave function. The total isospin wave function $H_{TT}$ in each channel are written as

$$H_{TT}^{c_{i}=1} = [\eta_1(\bar{D}^*)\eta_1(\Sigma_c)],$$

$$H_{TT}^{c_{i}=2} = [\eta_2(\bar{D}^*)\eta_2(\Sigma_c)],$$

$$H_{TT}^{c_{i}=3} = [\eta_3(\bar{D}^*)\eta_3(\Sigma_c)],$$

where $\eta$ is the isospin wave function of each particle. The spatial wave function $\Phi_{L, \lambda}^{c_{i}}$ can be expanded as

$$\Phi_{L, \lambda}^{c_{i}} = [\phi_{nl,L_{\eta_1}(r_1)}^{\lambda_1} \psi_{N,L_{\eta_2}(R)}^{\lambda_2}(r_2)],$$

$$\phi_{nl,L_{\eta_1}(r_1)}^{\lambda_1} = N_{nl} R_{l} e^{-\nu_\eta r_1^2} Y_{lm}(r_1),$$

$$\psi_{N,L_{\eta_2}(R)}^{\lambda_2}(R) = N_{nL} R_{l} e^{-\lambda_N R^2} Y_{LM}(R),$$

where $N_{nl}$ (or $N_{N,L}$) is the normalization constant, and the relevant parameters $\nu_\eta$ and $\lambda_N$ are given by

$$\nu_\eta = \frac{1}{r_n^2}, \quad r_n = r_{2n}^{n-1}, \quad (n = 1 - n_{max}),$$

$$\lambda_N = \frac{1}{R_N^2}, \quad R_N = R_1 A^{n-1}, \quad (N = 1 - N_{max}),$$

where $\{n_{max}, r_{min}, a \text{ or } r_{max}\}$ and $\{N_{max}, R_{min}, A \text{ or } R_{max}\}$ are Gaussian basis parameters given in Table I.

| $I(J^P)$ | $c$ | $l$ | $\lambda$ | $s$ | $\Sigma$ | $t$ | $n_{max}$ | $r_{min}$ | $r_{max}$ |
|----------|-----|-----|------------|-----|---------|-----|---------|---------|---------|
| 1(2^+)  | 1   | 0   | 0          | 0   | 1/2     | 1/2 | 2/3     | 10      | 0.1     | 20.0    |
| 1(2^+)  | 2   | 0   | 0          | 0   | 1/2     | 1/2 | 2/3     | 10      | 0.1     | 20.0    |
| 3       | 0   | 0   | 1          | 0   | 0       | 0   | 1/2     | 10      | 0.1     | 20.0    |
TABLE II: Binding energies (in units of MeV), expectation values of the Hamiltonian (potential and kinetic energies) (in units of MeV) and root-mean-square radii (in units of fm) of the three-body system $\bar{D}\bar{D}^*\Sigma_c$, obtained in the three cases detailed in the main text.

| Case | $\Lambda_T$ | $\Lambda_P$ | $I(J^P)$ | $B$ | $T$ | $V_{D^*\Sigma_c}$ | $V_{D\Sigma_c}$ | $V_{D\bar{D}}$ | $\bar{r}_{D^*\Sigma_c}$ | $\bar{r}_{D\Sigma_c}$ | $\bar{r}_{D\bar{D}}$ |
|------|-------------|-------------|----------|-----|----|-----------------|----------------|----------------|----------------|----------------|----------------|
| Case I | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 0.998$ GeV | $1(\frac{1}{2}^+)$ | 10.86 | 65.41 | -19.64 | -21.69 | -34.94 | 1.42 | 1.41 | 1.36 |
| Case I | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 0.998$ GeV | $1(\frac{3}{2}^+)$ | 7.06 | 52.18 | -19.66 | -10.46 | -29.12 | 1.62 | 1.81 | 1.64 |
| Case II | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 1.16$ GeV | $1(\frac{1}{2}^+)$ | 37.24 | 116.16 | -41.53 | -72.44 | -39.43 | 1.00 | 0.88 | 1.03 |
| Case II | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 1.16$ GeV | $1(\frac{3}{2}^+)$ | 29.63 | 92.50 | -81.32 | -21.67 | -19.15 | 0.91 | 1.36 | 1.40 |
| Case III | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 1.16$ GeV | $1(\frac{1}{2}^+)$ | 63.07 | 169.01 | -52.14 | -66.03 | -113.91 | 0.83 | 0.82 | 0.75 |
| Case III | $\Lambda_T = 0.998$ GeV | $\Lambda_P = 1.16$ GeV | $1(\frac{3}{2}^+)$ | 46.94 | 141.01 | -61.84 | -25.27 | -100.84 | 0.91 | 1.02 | 0.86 |

III. RESULTS AND DISCUSSIONS

First we discuss the quantum numbers of the $\bar{D}\bar{D}^*\Sigma_c$ system. Considering only $S$-wave interactions, the total angular momentum of the $\bar{D}\bar{D}^*\Sigma_c$ system is either $J = 1/2$ or $J = 3/2$. The isospin of $\bar{D}\bar{D}^*$ is either 0 or 1. In the OBE model, the interaction in isospin 0 is much stronger than that in isospin 1, to such an extent that $T_{cc}$ can be understood as an isospin 0 $\bar{D}\bar{D}^*$ bound state. As a result, the total isospin of the $\bar{D}\bar{D}^*\Sigma_c$ system is taken to be 1. Therefore, in this work we investigate the two $\bar{D}\bar{D}^*\Sigma_c$ configurations with $I(J^P) = 1(\frac{1}{2}^+) + I(J^P) = 1(\frac{3}{2}^+)$. The relevant quantum numbers of the $\bar{D}\bar{D}^*\Sigma_c$ system are given Table II.

In this work, we employ the OBE model to construct the potentials of $\bar{D}\Sigma_c$, $D^*\Sigma_c$, and $D^*\bar{D}$ through the effective Lagrangians describing the interactions between charmed hadrons and light mesons $\pi$, $\rho$, $\sigma$ and $\omega$. For details, we refer to Refs. [25, 59]. Since the $S$-wave interaction plays a dominant role in forming hadronic molecules, we only consider the $S$-wave interaction in this work. To estimate the impact of the finite size of hadrons on the OBE potentials, we adopt a monopole form factor $\frac{\Lambda^2-m^2}{\Lambda^2-q^2}$ for the relevant meson-baryon vertices, which introduces an unknown parameter $\Lambda$. To decrease the uncertainty of the OBE potential induced by the cutoff, we determine it by reproducing the masses of some well known molecular candidates. Assuming $P_{c}(4312)$, $P_{c}(4440)$, and $P_{c}(4457)$ as $D(\star)\Sigma_c$ bound states, the corresponding cutoff (denoted by $\Lambda_P$) is fixed to be 1.16 GeV, while the cutoff of the $\bar{D}\bar{D}^*$ system (denoted by $\Lambda_T$) is fixed to be 0.998 GeV if $T_{cc}$ is regarded as a $\bar{D}\bar{D}^*$ bound state. Therefore, we take three sets of cutoff values to search for three-body bound states in the $\bar{D}\bar{D}^*\Sigma_c$ system: Case I: $\Lambda_T = \Lambda_P = 0.998$ GeV; Case II: $\Lambda_T = 0.998$ GeV, $\Lambda_P = 1.16$ GeV; and Case III: $\Lambda_T = \Lambda_P = 1.16$ GeV. As the OBE interaction increases with the cutoff, we anticipate that Case III will yield the largest binding energies while Case I the smallest ones.

In case I, the cutoff of the $D(\star)\Sigma_c$ potential is taken the same as that of the $\bar{D}\bar{D}^*$ potential. For such potentials, there exist two three-body bound states $\bar{D}\bar{D}^*\Sigma_c$ with $I(J^P) = 1(\frac{1}{2}^+) + I(J^P) = 1(\frac{3}{2}^+)$. One can see that the three-body system does not bind as long as $T_{cc}$ is a $\bar{D}\bar{D}^*$ bound state. As a result, case I indicates that there exist two three-body $\bar{D}\bar{D}^*\Sigma_c$ bound states even the $D(\star)\Sigma_c$ system does not bind as long as $T_{cc}$ is a $\bar{D}\bar{D}^*$ bound state.

In case II, we change the cutoff of the $D(\star)\Sigma_c$ potential from 0.998 GeV to 1.16 GeV, while keep the cutoff of the $\bar{D}\bar{D}^*$ potential unchanged. In this case, the strength of the $D(\star)\Sigma_c$ potential becomes stronger, resulting in two three-body states with larger binding energies 37.2 MeV and 29.6 MeV. One can see that assuming $T_{cc}$ as a $\bar{D}\bar{D}^*$ bound state and $P_{c}(4312)$, $P_{c}(4440)$ and $P_{c}(4457)$ as $D(\star)\Sigma_c$ bound states, we obtain two three-body bound states below the $\bar{D}\bar{D}^*\Sigma_c$ mass threshold.

In case III, we change the cutoff of the $\bar{D}\bar{D}^*$ potential from 0.998 GeV to 1.16 GeV, which naturally results in two bound states with even larger binding energies as shown in Table II. In Fig. 2 we present the binding energies of the $\bar{D}\bar{D}^*\Sigma_c$ system as a function of $\Lambda$. One can see that the three-body system $\bar{D}\bar{D}^*\Sigma_c$ remains bound even when both $\bar{D}\bar{D}^*$ and $D(\star)\Sigma_c$ are unbound, with binding energies of the order of several MeV. If the $\bar{D}\bar{D}^*\Sigma_c$ bound states are observed experimentally in the future, it will help verify the molecular nature of $P_{c}(4312)$, $P_{c}(4440)$, $P_{c}(4457)$ and $T_{cc}$ in terms of the results shown in Fig. 2.

It is interesting to note that the mass splitting of the three-body $\bar{D}\bar{D}^*\Sigma_c$ doublet in case III is larger than that of case II, which implies that the strength of the $\bar{D}\bar{D}^*$ potential affects the splitting. In Fig. 3 we present the mass splitting as a function of the cutoff of the $\bar{D}\bar{D}^*$ potential. It is obvious that the mass splitting increases with the strength of the $\bar{D}\bar{D}^*$ potential. Interestingly, the mass splitting is positive, which means that the mass of the spin $\frac{3}{2}$ $\bar{D}\bar{D}^*\Sigma_c$ bound state is
larger than that of its spin 1/2 counterpart. We note that in this case the \( J^P = \frac{3}{2}^- \) \( \bar{D}^*\Sigma_c \) system is more bound than the \( J^P = \frac{3}{2}^- \) \( \bar{D}^*\Sigma_c \) system. It indicates that the mass splitting of the three-body \( \bar{D}\bar{D}^*\Sigma_c \) doublet is oppositely correlated to the mass splitting of the two-body \( \bar{D}^*\Sigma_c \) bound states, which offers a non-trivial way to check the molecular nature of the involved states. We note in passing that in Ref. [60], we found that the mass splitting of \( P_c(4440) \) and \( P_c(4457) \) is correlated to the mass splitting of the \( \Xi^{(*)}\Sigma_c^{(*)} \) doublet via heavy anti-quark diquark symmetry.

From our above study we conclude that there exist two three-body molecules \( \bar{D}\bar{D}^*\Sigma_c \) with \( I(J^P) = 1(\frac{1}{2}^+) \) and \( I(J^P) = 1(\frac{3}{2}^+) \). In the following, we denote \( P_c(4312) \), \( P_c(4440) \), and \( P_c(4457) \) as \( P_{c1} \), \( P_{c2} \), and \( P_{c3} \), and the \( I(J^P) = 1(\frac{1}{2}^+) \) and \( I(J^P) = 1(\frac{3}{2}^+) \) \( \bar{D}\bar{D}^*\Sigma_c \) bound states as \( Hq_1 \) and \( Hq_2 \), respectively. We will discuss their possible decay modes and calculate the decay widths via the effective Lagrangian approach. Such \( \bar{D}\bar{D}^*\Sigma_c \) bound states can be regarded as three kinds of quasi two-body bound states, \( P_{c2}(P_{c3})\bar{D}, P_{c1}\bar{D} \), and \( T_{cc} \). One should note that the particles, \( P_{c2/c3} \), \( P_{c1} \), and \( \Sigma_c \), should be viewed as unstable particles in contrast to the particles, \( \bar{D} \), \( \bar{D}^* \) and \( T_{cc} \), as shown in Fig. 4. Since the minimum number of valence quarks of the \( \bar{D}\bar{D}^*\Sigma_c \) states is 7, it will not couple to a pair of traditional hadrons, which indicates that they can only decay into at least three traditional hadrons. In other words, the decay mechanism shown in Fig. 4 should be the dominant ones.

In the following, we will show how to calculate the partial decay widths of the \( \bar{D}\bar{D}^*\Sigma_c \) bound states in the effective Lagrangian approach. It should be noted that in the following study, we focus on Case II, among the three cases studied, because it can reproduce the pentaquark states and \( T_{cc} \). The effective Lagrangians describing the interactions between the three-body bound states and their constituents have the following form

\[
\mathcal{L}_{Hq_1\bar{P}_{c3}\bar{D}} = g_{Hq_1\bar{P}_{c3}\bar{D}} Hq_1(x) \int dy \bar{D}(x + \omega_{P_{c3}} y) P_{c3}(x + \omega_{\bar{D}} y) \Phi(y^2),
\]

\[
\mathcal{L}_{Hq_2\bar{P}_{c2}\bar{D}} = g_{Hq_2\bar{P}_{c2}\bar{D}} Hq_2(x) \int dy \bar{D}(x + \omega_{P_{c2}} y) P_{c2}(x + \omega_{\bar{D}} y) \Phi(y^2),
\]

\[
\mathcal{L}_{Hq_1\bar{P}_{c1}\bar{D}^*} = g_{Hq_1\bar{P}_{c1}\bar{D}^*} Hq_1(x) \int dy \bar{D}^{\ast\mu}(x + \omega_{P_{c1}} y) \gamma_\mu \gamma_5 P_{c1}(x + \omega_{\bar{D}^*} y) \Phi(y^2),
\]

\[
\mathcal{L}_{Hq_2\bar{P}_{c1}\bar{D}^*} = g_{Hq_2\bar{P}_{c1}\bar{D}^*} Hq_2(x) \int dy \bar{D}^{\ast\mu}(x + \omega_{P_{c1}} y) P_{c1}(x + \omega_{\bar{D}^*} y) \Phi(y^2),
\]

\[
\mathcal{L}_{Hq_1T_{cc}\Sigma_c} = g_{Hq_1T_{cc}\Sigma_c} Hq_1(x) \int dy T_{cc}^{\mu}(x + \omega_{\Sigma_c} y) \gamma_\mu \gamma_5 \Sigma_c(x + \omega_{T_{cc}} y) \Phi(y^2),
\]

\[
\mathcal{L}_{Hq_2T_{cc}\Sigma_c} = g_{Hq_2T_{cc}\Sigma_c} Hq_2(x) \int dy T_{cc}^{\mu}(x + \omega_{\Sigma_c} y) \Sigma_c(x + \omega_{T_{cc}} y) \Phi(y^2),
\]

where \( \Phi(y^2) \) denotes the Gaussian form factor, \( g \) with different subscripts represent the relevant coupling constants, and \( \omega_i = \frac{m_i}{m_i + m_j} \) is the kinematical parameter with \( m_i \) and \( m_j \) being the masses of the involved hadrons. We rely on the compositeness condition to estimate the above couplings, which is an effective approach to estimate the couplings between bound states and their constituents [62]. The condition implies that the coupling constants can be determined from the fact that the renormalization constant of the wave function of a composite particle should be zero. Following our previous works [49] [63], with the cutoff \( \Lambda = 1 \) GeV and the masses of the three-body bound states \( \bar{D}\bar{D}^*\Sigma_c \) the couplings are determined as shown in Table III. The details about deriving the couplings can be found in Ref. [63].

The Lagrangian describing the secondary decay process...
with the so-obtained amplitudes, one can easily calculate the partial decay width

\[ d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{2J+1} \frac{|M|^2}{32m_{H_{1(2)}}^2} dm_{12}dm_{23}. \]  

In Table IV we present the partial decay widths of the two three-body bound states, \( H_{1(2)} \) and \( H_{2(3)} \). We find that the bound state with \( J = 1/2 \) dominantly decays into \( J/\psi p D \),

\[
\begin{align*}
\mathcal{L}_{\pi\Lambda_c\Sigma_c} &= \frac{g_{\pi\Lambda_c\Sigma_c}}{f_\pi} \bar{\Lambda}_C \gamma^\mu \gamma^5 \partial_\mu \Phi_\pi \cdot \bar{\Sigma}_C,
\end{align*}
\]

\[
\begin{align*}
\mathcal{L}_{P_1 J/\psi p} &= g_{P_1 J/\psi p} \bar{P}_1 \gamma_\mu \gamma^5 J/\psi p, \\
\mathcal{L}_{P_2 J/\psi p} &= g_{P_2 J/\psi p} \bar{P}_2 \gamma_\mu \gamma^5 J/\psi p, \\
\mathcal{L}_{P_3 J/\psi p} &= g_{P_3 J/\psi p} \bar{P}_3 \gamma_\mu \gamma^5 J/\psi p.
\end{align*}
\]

are expressed as

\[ \mathcal{L}_{P_1 J/\psi p} = g_{P_1 J/\psi p} \bar{P}_1 \gamma_\mu \gamma^5 J/\psi p, \]

\[ \mathcal{L}_{P_2 J/\psi p} = g_{P_2 J/\psi p} \bar{P}_2 \gamma_\mu \gamma^5 J/\psi p, \]

\[ \mathcal{L}_{P_3 J/\psi p} = g_{P_3 J/\psi p} \bar{P}_3 \gamma_\mu \gamma^5 J/\psi p, \]

\[ \mathcal{L}_{\pi\Lambda_c\Sigma_c} = \frac{g_{\pi\Lambda_c\Sigma_c}}{f_\pi} \bar{\Lambda}_C \gamma^\mu \gamma^5 \partial_\mu \Phi_\pi \cdot \bar{\Sigma}_C, \]

where \( u \) and \( \bar{u} \) represent the corresponding spinor function denoted by the subscripts, \( \bar{\varepsilon} \) is the polarization vector, and

\[ S_{\mu\nu} = g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{\gamma^\mu p^\nu - \gamma^\nu p^\mu}{2m^2} - \frac{3p^\mu p^\nu}{2m^2} \]

with \( m \) being the corresponding mass of an spin-3/2 particle.

With the above relevant Lagrangians the corresponding amplitudes of the strong decays of Fig. 4 are

\[
\begin{align*}
\mathcal{M}_{a,(J=1/2)} &= ig_{H_{1(2)} D_{P_3}} g_{P_3 J/\psi p} \bar{u} H_{1(2)} \frac{1}{k - m_{P_3}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_2) u_p, \\
\mathcal{M}_{a,(J=3/2)} &= ig_{H_{1(2)} D_{P_2}} g_{P_2 J/\psi p} \bar{u} H_{1(2)} \frac{1}{k - m_{P_2}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_2) u_p, \\
\mathcal{M}_{b,(J=1/2)} &= ig_{H_{1(2)} D_{P_3}} g_{P_3 J/\psi p} \bar{u} H_{1(2)} \frac{1}{k - m_{P_1}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_2) u_p, \\
\mathcal{M}_{b,(J=3/2)} &= ig_{H_{1(2)} D_{P_2}} g_{P_2 J/\psi p} \bar{u} H_{1(2)} \frac{1}{k - m_{P_3}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_2) u_p, \\
\mathcal{M}_{c,(J=1/2)} &= ig_{H_{1(2)} T_{\Sigma_c}} \frac{g_{\pi\Lambda_c\Sigma_c}}{f_\pi} \bar{u} H_{1(2)} \frac{1}{k - m_{\Sigma_c}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_3) u_{\Lambda_c}, \\
\mathcal{M}_{c,(J=3/2)} &= ig_{H_{1(2)} T_{\Sigma_c}} \frac{g_{\pi\Lambda_c\Sigma_c}}{f_\pi} \bar{u} H_{1(2)} \frac{1}{k - m_{\Sigma_c}} \gamma_\mu \gamma^5 \gamma^\mu \gamma^5 (p_3) u_{\Lambda_c},
\end{align*}
\]
much larger than that of the $J = 3/2$ state. Therefore, the $J/\psi p \bar{D}$ mode is a golden channel to discriminate the spin of $Hq_1$ and $Hq_2$ molecules. $Hq_1$ and $Hq_2$ decay almost equally into $J/\psi p \bar{D}^*$, while the decay into $T_{cc} \Lambda_c \pi$ is rather small in contrast to the other two decay modes. One should note that the partial decay widths of the three pentaquark states decaying into $J/\psi p$ are not very precisely known [35], leading to some uncertainties about the partial decay widths given in Table [IV]. Nonetheless, we suggest to search for them in the $J/\psi p \bar{D}$ or $J/\psi p \bar{D}$ mass distributions.

Table IV: Partial decay widths of the $\bar{D} \bar{D}^* \Sigma_c$ molecules.

| Modes of Fig. | Hq₁ → J/ψpD | Hq₂ → J/ψpD |
|---------------|--------------|--------------|
| Value (MeV)   | 12.3         | 0.9          |

| Modes of Fig. | Hq₁ → J/ψpD⁺ | Hq₂ → J/ψpD⁺ |
|---------------|--------------|--------------|
| Value (MeV)   | 3.7          | 3.9          |

| Modes of Fig. | TₐccΛcπ → J/ψpD⁺ |
|---------------|-------------------|
| Value (keV)   | 0.2               |

IV. SUMMARY AND CONCLUSION

The exotic states, $T_{cc}$, $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, discovered by the LHCb Collaboration recently, have been suggested to be $D\bar{D}^*$ and $D^*(\Sigma_c)$ hadronic molecules. However, their molecular nature is difficult to be confirmed either experimentally or theoretically. In this work, we have investigated these exotic states in the three-body $\bar{D}D^*\Sigma_c$ system, which is equivalent to adding a $\bar{D}$ meson into the $D^*\Sigma_c$ system. The OBE interactions of the sub-systems, $\bar{D}^*(\Sigma_c)$ and $\bar{D}D^*$, are determined by reproducing the masses of the molecular candidates, i.e., the three pentaquark states $[P_c(4312), P_c(4440) \text{ and } P_c(4457)]$. After solving the three-body schrödinger equation, we obtained two three-body bound states, $I(J^P) = 1(\frac{1}{2}^+)$ $D\bar{D}^*\Sigma_c$ and $I(J^P) = 1(\frac{3}{2}^+)$ $D\bar{D}^*\Sigma_c$, with binding energies 37.2 MeV and 29.6 MeV, respectively. In particular, we explored the correlation between the existence of $D\bar{D}^*\Sigma_c$ molecules with the existence of $D^*(\Sigma_c)$ and $D^*\bar{D}$ molecules. If the $D\bar{D}^*\Sigma_c$ bound states can be observed experimentally in the future, the correlation can help to test the molecular nature of $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and $T_{cc}$. The mass splitting of the three-body doublet is found to be correlated to that of the $D^*(\Sigma_c)$ doublet. Assuming $P_c(4457)$ and $P_c(4440)$ as $J = 1/2$ and $J = 3/2 \bar{D}^*\Sigma_c$ bound states, respectively, we find that the mass splitting between $I(J^P) = 1(\frac{1}{2}^+)$ and $I(J^P) = 1(\frac{3}{2}^+)$ $D\bar{D}^*\Sigma_c$ bound states is positive.

At last, we employed the effective Lagrangian approach to calculate the partial decay widths of $D\bar{D}^*\Sigma_c$ bound states. We find that $J = 1/2$ and $J = 3/2 \bar{D}D^*\Sigma_c$ bound states mainly decay into $J/\psi p\bar{D}$ and $J/\psi p\bar{D}^*$, respectively, while the decay into $T_{cc} \Lambda_c \pi$ is small. We strongly recommend experimental searches for such three-body bound states in the $J/\psi p\bar{D}$ and $J/\psi p\bar{D}^*$ mass distributions, which can help verify the molecular nature of $T_{cc}$, $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$.

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