Realizing Fractional-Quantum-Hall Gravitons on Quantum Computers

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Introduction. While a universal fault-tolerant quantum computer with thousands of qubits remains elusive, noisy intermediate-scale quantum (NISQ) devices with a few qubits are already operational [1–3], albeit with limitations due to a lack of reliable error-correction [4]. This progress has stirred a flurry of research activity to identify problems that can take advantage of this recently developed quantum technology [5]. Utilizing NISQ systems as digitized synthetic platforms to study physics phenomena challenging to investigate otherwise has emerged as a critical frontier [6]. Realization of the fractional quantum Hall (FQH) “graviton” [7, 8] – a high-energy excitation arising in strongly-correlated topological quantum fluids, reminiscent of its quantum-gravity counterpart (albeit with important differences such as being massive and nonrelativistic [9, 10]) – is a major open problem in condensed matter physics, which combines the challenges associated with strong correlations, topological order, and non-equilibrium dynamics.

FQH states are strongly correlated phases of electrons in two dimensions which exhibit unique topological excitations with fractional charge [11, 12] and fractional statistics [13, 14]. Recently, FQH states have come into focus due to their universal geometric features, which include the so-called Hall viscosity [15–17] and the collective excitation known as the Girvin-MacDonald-Platzman magnetoroton mode [18, 19]. In the long-wavelength limit, the magnetoroton forms a spin-2 degree of freedom representing a quantum metric, ˜g, whose fluctuations can be exposed by breaking rotational symmetry [20]. Intuitively, ˜g can be understood as the shape of magnetic flux attachment in a FQH fluid [20], see Fig. 1(a). Formally, ˜g is a symmetric unimodular matrix, akin to the fluctuating space-time metric in a theory of quantum gravity, thus it is often referred to as FQH graviton [7, 8].

Intermediate-scale quantum technologies provide unprecedented opportunities for scientific discoveries while posing the challenge of identifying important problems that can take advantage of them through algorithmic innovations. A major open problem in quantum many-body physics is the table-top generation and detection of emergent excitations analogous to gravitons – the elusive mediators of gravitational force in a quantum theory of gravity. In solid state materials, fractional quantum Hall phases are one of the leading platforms for realizing graviton-like excitations. However, their direct observation remains an experimental challenge. Here, we generate these excitations on the IBM quantum processor. We first identify an effective one-dimensional model that captures the geometric properties and graviton dynamics of fractional quantum Hall states. We then develop an efficient, optimal-control-based variational quantum algorithm to simulate geometric quench and the subsequent graviton dynamics, which we successfully implement on the IBM quantum computer. Our results open a new avenue for studying the emergence of gravitons in a new class of tractable models that lend themselves to direct implementations on the existing quantum hardware.

Direct experimental evidence of FQH gravitons comes from the studies of the magnetoroton at large momenta, k∼ℓ−1, via inelastic light scattering [21–24]. Recent theoretical works [25, 26] have proposed that the long-wavelength properties of the graviton can be probed by quenching the metric of “space”, i.e., by suddenly making the FQH state anisotropic (see also alternative proposals [27–29]). Surprisingly, it was found that such geometric quenches induce coherent dynamics of the FQH graviton [25], even though the graviton mode resides at finite energy densities above the FQH ground state. In contrast, near the FQH liquid-nematic phase transition [30, 31], the graviton is expected to emerge as a gapless excitation [32–34].

In this paper, we generate the FQH graviton in a synthetic NISQ system – the IBM open-access digitized quantum processor – and observe its out-of-equilibrium dynamics. To achieve this, we have made two theoretical advances. (i) We map the problem onto a one-dimensional quantum spin chain in the limit FQH state is stretched into a thin cylinder [38–41]. While topological properties of FQH states have previously been studied in this limit [42–44], we show that this limit also captures the geometric properties of FQH systems, in particular their quench dynamics. (ii) We demonstrate that the obtained spin model can be efficiently implemented on NISQ devices.

In particular, we developed an efficient optimal-control-based [45–47] variational quantum algorithm [48–
FIG. 1. (a) Geometric quench probes the fluctuations of the quantum metric $\tilde{g}$ [20]. Here we focus on the $\nu=1/3$ Laughlin state, where $\tilde{g}$ describes the shape of the droplet corresponding to three magnetic orbitals surrounding each electron. This droplet can be efficiently represented by a qubit in a one-dimensional model in Eq. (5). (b) The post-quench dynamics can be accurately approximated by a two-parameter variational ansatz $|\psi_{\text{var}}(\alpha, \beta)\rangle$, which spans a small region of the Hilbert space. For every time $t$, there are optimal variational parameters $\alpha^*$ and $\beta^*$ for which the nonequilibrium wave function has a high overlap with the variational ansatz. The optimal parameters have a smooth dependence on time and weak dependence on system size, providing excellent extrapolation to the thermodynamic limit [35–37]. (c) The variational ansatz for the optimal parameters can be implemented in an efficient algorithm of linear circuit depth.

51] analogous to the Quantum Approximate Optimization Algorithm (QAOA) [52–57], which utilizes a two-parameter ansatz to accurately approximate the post-quench dynamics of the graviton, see Fig. 1(b)-(c). Our ansatz provides a quantum algorithm with a circuit depth that does not grow with the evolution time, thus circumventing the problems associated with the naive simulation of the dynamics based on Trotter decomposition, which leads to a prohibitively large number of entangling gates, even with drastic approximations due to a finite number of steps. Such hybrid classical-quantum approaches are the method of choice for quantum simulations on NISQ devices [58, 59]. They have the advantage of using hybrid classical-quantum software to physically create a quantum wave function in a digitized general-purpose quantum device without the need to develop single-purpose emulsion hardware.

Anisotropic Laughlin state near the thin-cylinder limit. We focus on the $\nu=1/3$ Laughlin FQH state [11] whose Hamiltonian near the thin-cylinder (TC) limit is given by [42]

$$\hat{H} = \sum_j V_{1,0} \hat{n}_j \hat{n}_{j+1} + V_{2,0} \hat{n}_j \hat{n}_{j+2} + V_{3,0} \hat{n}_j \hat{n}_{j+3} + V_{2,1} c_{j+1}^\dagger c_{j+2} c_{j+3} c_j + \text{h.c.}$$  \hspace{1cm} (1)

Here the operators $c_j, c_j^\dagger (\hat{n}_j \equiv c_j^\dagger c_j)$ destroy or create an electron in a Landau level (LL) orbital localized around $2\pi n^2/L_2^2$. We assume the system is defined on a cylinder of size $L_1 \times L_2$ containing $N$ electrons, such that the filling factor $\nu=N/N_B=1/3$, with magnetic flux $N_B=(L_1 L_2)/(2\pi L_B^2)$. The near-TC limit corresponds to $L_1 \gg L_2$ with the area ($N_B$) fixed, which allows us to neglect longer-range interaction terms beyond those in Eq. (1). Importantly, the Hamiltonian above describes a 2D system with strong spatial anisotropy, as opposed to a strictly 1D limit $L_2=0$, thus allowing the emergence of the graviton mode. The interaction matrix elements are given by

$$V_{k,m} = (k^2 - m^2) e^{-2\pi^2 (k^2 + m^2 - 2 k m g_{12})/L_2^2 g_{11}} \approx 0. \hspace{1cm} (2)$$

which we have generalized to the case of an arbitrary electron mass tensor $g_{ab}$, $a, b = 1, 2$. The mass tensor must be symmetric and unimodular (det$g=1$) [20], hence we can generally write it as $g = \exp(Q)$ where $Q = Q(2\hat{a}_i \hat{b}_j - \delta_{a,b})$ is a Landau-de Gennes order parameter and $\mathbf{d} = (\cos(\phi/2), \sin(\phi/2))$ is a unit vector [60]. Parameters $Q$ and $\phi$ intuitively represent the stretch and rotation of the metric, respectively.

Since the Hamiltonian in Eq. (1) is positive semidefinite, it has a unique (unnormalized) ground state with zero energy [42]

$$|\psi_0\rangle = \prod_j \left(1 - \sqrt{\frac{V_{3,0}}{V_{1,0}}} \right)^{e^{x+2 i k_B g_{12} 2 g_{11}}} \hat{S}_j |\ldots100100\ldots\rangle \hspace{1cm} (3)$$

where $\hat{S}_j=e_{j+1}^\dagger e_j^\dagger e_{j+2} e_{j+3} c_j$ is an operator that “squeezes” two neighbouring electrons while preserving their center-of-mass position [61]. The product state $|\ldots100100\ldots\rangle$ is the ground state in the limit $L_2 \rightarrow 0$. In previous works [42–44], the ground state of the model in Eq. (1) and its neutral excitations were studied on isotropic cylinders, $g_{11}=g_{22}=1, g_{12}=0$. In particular, it was found that the state in Eq. (3) has $\approx 98\%$ overlap with the ground state of the full Hamiltonian in the range of circumboranges $5\ell_B \lesssim L_2 \lesssim 7\ell_B$, justifying the use of the truncated model Eq. (1) in this regime [42]. We have confirmed that the same conclusions continue to hold in the presence of mass anisotropy [62].

Geometric quench. We now show that, in addition to the ground state, the effective model in Eq. (1) captures the high-energy excitations that govern the graviton dynamics in the FQH phase. We consider the FQH system initially prepared in the ground state $|\psi_0\rangle$ in Eq. (3) with isotropic metric ($g_{11}=g_{22}=1, g_{12}=0$). At time $t=0$, we instantaneously introduce diagonal anisotropy, $g_1^a=1/g_{22}^a>1$, and let the system evolve under unitary dynamics generated by the post-quench anisotropic Hamiltonian. As the system is taken out of its equilibrium state, we are interested in the dynamical fluctuations of its quantum metric $\tilde{g}$ in Fig. 1(a).

Note that even though $g$ and $\tilde{g}$ are related to one another, $\tilde{g}$ is an emergent property of a many-body state
and not necessarily equal to $g$. Nevertheless, we can parameterize $\bar{g}$ in the same manner using the parameters $\tilde{Q}$ and $\tilde{\phi}$, representing the stretch and rotation of the emergent metric. In order to determine the equations of motion for $\tilde{Q}(t)$ and $\tilde{\phi}(t)$, we maximize the overlap between $|\psi(t)\rangle$ and the variational family of states in Eq. (3) [63]. When this overlap is close to unity, we can be confident that we found the optimal metric parameters $\tilde{Q}$ and $\tilde{\phi}$ describing the state $|\psi(t)\rangle$.

In Fig. 2, we present the graviton dynamics in the model in Eq. (1) when anisotropy is suddenly changed from $Q=0$ to $Q=0.18$, keeping $\phi=0$. Fig. 2(a), presents the resulting dynamics of $\tilde{Q}$ and $\tilde{\phi}$ for different system sizes $N$. The dynamics is in excellent agreement with predictions of the bimetric theory in the linear regime [64],

$$\tilde{Q}(t) = \pm 2A \sin \frac{E_g t}{2}, \quad \tilde{\phi}(t) = \pm \frac{\pi}{2} - \frac{E_g t}{2},$$

where $E_g$ is the energy of the graviton mode in units of $V_{1,0}$. As can be seen in Fig. 2(a), the numerical data can be accurately fit using Eqs. (4). The fit yields the oscillation frequency $E_g=1.29$. Note that this energy is much higher than the first excited energy of the quench Hamiltonian. We identify this energy with the graviton state as evidenced by the sharp peak in the spin-2 spectral function $I_{1,2}(\omega)$ [65]. The later spectral function is designed to detect the characteristic $d$-wave symmetry of the graviton. Analogous to an oscillating space-time metric induced by a gravitational wave, $I_{1,2}(\omega)$ is the associated transition rate due to the dynamics of the oscillating mass-tensor [65]. Thus, the model in Eq. (1) reproduces the graviton oscillation as described by the bimetric theory.

**Mapping to a quantum spin chain.** We use the reduced registers scheme introduced in Ref. [41] to map the model (1) to a spin chain (see also [66]). The reduced register is a block of three consecutive orbitals that encodes whether or not the block is “squeezed” with respect to the root state $|100,100,\ldots\rangle$. For each block of three sites, the state of the reduced register is $\pm 1$ if it is squeezed (i.e., 011) or 0 if not (i.e., either 000 or 100). In the root state, none of the blocks are squeezed and it maps to $|0,0,0,\ldots\rangle$. If we apply the squeezing operator to one block of the root state, we obtain, e.g., $|100,011,000,\ldots\rangle \rightarrow |0,1,0,\ldots\rangle$. In terms of reduced registers, squeezing acts as flip of 0 to 1, so it can be viewed as the Pauli $X$ matrix. However, there is an important difference in that the Hilbert space is not a tensor product of reduced registers, since the squeezing can never generate two neighboring $\ldots 11 \ldots$ configurations of the reduced registers [67]. This type of constrained Hilbert space arises, e.g., in the Fibonacci anyon chain [68]. The inverse mapping is constructed as follows: for any 1 we make a 011 block. A 0 that follows a 1 (0) gives a 000 (100) block. With this mapping of states, we can show that the Hamiltonian (1) maps to a local spin-chain Hamiltonian

$$\hat{H} = \sum_\ell \{(V_{1,0} - 3V_{3,0})N_\ell + V_{3,0}N_\ell N_{\ell+2} + (1 - N_\ell-1)|\text{Re}(V_{2,1})X_\ell - \text{Im}(V_{2,1})Y_\ell|(1 - N_{\ell+1})\},$$

where we omitted the boundary terms for simplicity, and we introduced the occupation number $N_\ell=|\langle\ell|1\rangle|^2$, Pauli $X_\ell=|\langle\ell|1\rangle|^2 + |\langle\ell|0\rangle|^2$, and Pauli $Y_\ell=-i|\langle\ell|1\rangle|^2 + i|\langle\ell|0\rangle|^2$ operators [62].

**Variational quantum algorithm.** In the geometric quench, the initial state is the ground state $|\psi_0\rangle$ in Eq. (3) with the identity metric $(g_{1,2}=0)$. Ref. [41] found an efficient algorithm to generate $|\psi_0\rangle$ with a linear-depth circuit. Working in the reduced space, we can use only stage 1 of the circuit in Ref. 41 identifying qubits 1+3n with the reduced registers. The state after the quench is generated by the unitary evolution operator $U(t)=\exp(-itH')$, where $H'$ is the anisotropic post-quench Hamiltonian. Thus, combining an algorithm that generates $U(t)$ with the the ground-state preparation algorithm yields the post-quench state $|\psi(t)\rangle=U(t)|\psi_0\rangle$.

The standard approach for generating an evolution operator $U(t)$ is through the Trotter decomposition. However, a large number of Trotterization steps is needed.
to obtain accurate results [62]. Furthermore, terms like 
\((1-\mathcal{N}_{\ell-1})X_{\ell}(1-\mathcal{N}_{\ell+1})\), which appear in Eq. (5), require
\(R_c\) gates controlled by two other qubits, in turn requiring
six CNOT gates for each control gate. In order to overcome
these difficulties, we leverage the idea of QAOA [52] to approximate \(\psi(t)\) using the ansatz:

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_{\ell} e^{-i\alpha_{\ell}e^{i\beta(1-\mathcal{N}_{\ell-1})X_{\ell}}}|00\ldots\rangle,
\]

where on each reduced register \(\ell\), we apply alternating
\(\mathcal{N}_{\ell}\) and \((1-\mathcal{N}_{\ell-1})X_{\ell}\) (on the very first site, due to open boundary condition, we use \(X_{1}\) instead of \((1-\mathcal{N}_{\ell-1})X_{\ell}\) for \(\ell=1\)). The optimal parameters \(\alpha^*, \beta^* \in [0, 2\pi]\) are determined at each time step \(t\) using
the dual annealing algorithm to maximize the overlap,
\(\langle \psi_0|U^\dagger(t)|\psi_{\text{var}}(\alpha, \beta)\rangle\), with the exact state.

\[\begin{array}{c}
\includegraphics[width=1\textwidth]{fig3.png}
\end{array}\]

FIG. 3. (a)-(b): Optimal variational parameters \(\alpha^*\) and \(\beta^*\)
for \(N = 7-13\) particles and their extrapolation to \(N \to \infty\)
(solid black line). Optimal parameters vary smoothly with
time and exhibit weak finite-size effects. (c) Comparison of
the variational ansatz against TEBD simulation for \(N=60\)
for the overlap of the time evolved state with the root state.
The parameters of the variational ansatz are extrapolated to the
same system size, \(N=60\). The ansatz with extrapolated pa-
terms exhibits excellent agreement with the TEBD results.
TEBD simulations were performed using a bond dimension 20
with a time step \(\Delta t=0.01\), resulting in truncation error \(10^{-5}\).

The ansatz in Eq. (6) has several key advantages that
reduce its implementation cost on NISQ hardware [62],
while at the same time it captures any local observable
expectation values in the bulk of the system, in particu-
lar it faithfully reproduces the quantum metric dynamics,
\(Q(t)\) and \(\phi(t)\). Importantly, the optimal parameters
\(\alpha^*, \beta^*\) are found to exhibit a simple oscillatory behavior
as a function of time, as well as weak dependence on the
system size as shown in Fig. 3(a)-(b). Note that these fig-
ures show data for system sizes \(N = 7, \ldots, 13\) that almost
collapse on top of each other, allowing a smooth extrapo-
lation to the thermodynamic limit \((N \to \infty)\), shown as the
solid black line. Finally, in Fig. 3(c), we have checked using
time-evolved block decimation (TEBD) [69] that the
extrapolated parameters obtained from classical simulations
for up to \(N=13\) produce excellent agreement with
direct TEBD calculation of \(\langle \psi(t)\rangle\) for larger systems.
Although we utilize classical optimization for finding the
parameters of our quantum circuit, we find that the
weak system-size dependence of the variational parameters
eliminates the need to directly perform the classical
optimization for the actual size of the system, providing
access to system sizes for which the classical optimization
is infeasible.

**Results on the IBM Quantum Processor.** The
preparation of variational ansatz states in quantum cir-
cuits involves rotation gates such as \(R_x\), \(R_z\) and con-
trolled \(R_y\) – see Fig. 1 (c) for the variational ansatz
with translation symmetry for \(N=5\). We implemented
this ansatz on IBM’s ibmq_santiago processor [70].

\[\begin{array}{c}
\includegraphics[width=0.5\textwidth]{fig4.png}
\end{array}\]

FIG. 4. The comparison of \(\langle n_0(t)\rangle\), \(C_{0,2}(t)\), and overlap with
the root state \(|\ldots10100\ldots\rangle\) for \(N=5\) computed by different
schemes. The green curves show the exact diagonalization
results. The blue dots are results from classical simulation of
quantum circuits. The blue curves are computed from opti-
imized variational ansatz. The orange dots are results from
quantum circuits executed in IBM-Santiago processor.
ibmq_santiago consists of a linear array of five qubits with averaged readout error, CNOT noise and T2 dephasing time of roughly 1.5%, 0.6% and 120 \mu s, respectively [70]. The noise level varies throughout the day and depends on calibration rounds. We use qiskit, an open source library that provides basic gate implementations, classical simulation of quantum circuits, and execution of quantum circuits in IBM quantum processors. By comparing overlaps of each state in the Fock basis, the integrity of the quantum circuit can be established by reaching a perfect agreement between the classical simulations of quantum circuits and variational ansatz computations.

The out-of-equilibrium states prepared with the variational ansatz in a quantum computer are further post-processed to discard non-physical states. Specifically, we discount states having two consecutive 1 sites in the reduced register representation, because the structure of quantum circuits shown in Fig. 1(c) can never produce such a state. We then convert quantum states in reduced-register basis back to fermionic basis following the rules given above Eq. (5). The properties of the variational ansatz states prepared with IBM-Santiago processor are shown in Fig. 4. For each state, 40960 shots are executed to have sufficient statistics. We characterize the dynamics by measuring local densities, \langle n_i \rangle, equal-time density-density correlation functions in the fermionic basis, \text{C}_{i,j}(t) = \langle n_i(t)n_j(t) \rangle - \langle n_i(t) \rangle \langle n_j(t) \rangle, and the overlap with the root state, \vert \langle \psi_{\text{var}}(\alpha(t), \beta(t)) | 100100 \ldots \rangle \rangle^2. By comparing state characteristics from classical simulation and from quantum computer execution of quantum circuits shown in Fig. 4, we conclude that the variational ansatz allows us to prepare the out-of-equilibrium states with good fidelity in a noisy qubit device. In contrast, the Trotterized quantum circuits from the unitary evolution with Hamiltonian (5) fails to produce representative quench dynamics with the same device [62].

Conclusions. We showed that quantum-geometrical features of FQH states can be realized in an effective 1D model that has an efficient quantum-circuit representation. Our 1D model makes efficient use of resources, as each qubit corresponds to three Landau orbitals, reminiscent of holographic quantum simulation [71]. As a proof of principle, utilizing the quantum-circuit mapping, we developed an efficient quantum algorithm that allowed us to simulate graviton dynamics on IBM quantum processors. In contrast to traditional algorithms based on the Trotter decomposition, our variational algorithm has a linear circuit depth independent of the evolution time. We expect these results will motivate further analytical investigations into tractable models of graviton dynamics in condensed matter systems, as well as their realizations on NISQ devices.

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Supplemental Material for “Realizing Fractional-Quantum-Hall Gravitons on Quantum Computers”

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In this Supplementary Material, we give a detailed derivation of the FQH Hamiltonian on a thin cylinder and in the presence of mass anisotropy. We briefly review the bimetric theory of FQH states and present additional evidence that the model studied in the main text captures the graviton dynamics. Finally, we discuss the Trotterization approach to simulating quantum dynamics in this model, and demonstrate its inadequacy in the currently available NISQ devices.

**FQH HAMILTONIAN AND THE THIN-CYLINDER LIMIT**

In this Section, we derive the parent Hamiltonian for the Laughlin \(\nu=1/3\) state on a thin cylinder with general mass metric. We pick the Landau gauge, in which the single-electron orbitals are given by

\[
\phi_j^c(r) = \frac{1}{\sqrt{\pi \ell_B^2}} e^{i \phi q r} e^{-\frac{i (r - x_j \ell_B)^2}{2 \ell_B^2}}, \quad \text{(S1)}
\]

where \(j\) denotes the orbital index within the lowest Landau level (LLL) and \(L_2\) is the circumference of the cylinder, with \(\kappa = 2\pi / L_2\) (we work in units \(\ell_B=1\)). The number of available single-electron orbitals is given by the magnetic flux as \(N_\phi = (L_1 L_2) / (2\pi)\).

The field operator creating an electron at position \(r\) is given by

\[
\Psi^\dagger(r) = \sum_j \phi_j^c(r) c_j^\dagger, \quad \text{(S2)}
\]

where \(c_j^\dagger\) is the creation operator which creates an electron at orbital \(j\) within LLL (and similarly for the annihilation operator). From the field operator, the density operator in momentum space is given by

\[
\hat{\rho}(q) = \int d^2r \, \Psi^\dagger(r) \Psi(r) e^{iq \cdot r} = e^{-\sum_j q_j^2 / 2} \sum_j e^{i q \cdot \kappa_j} c_j^\dagger c_j = F(q) \hat{\rho}(q), \quad \text{(S3)}
\]

where, we introduced the LLL form factor \(F(q) = e^{-\sum_j q_j^2 / 2}\) and \(\hat{\rho}(q)\) is the projected density operator.

Finally, the FQH Hamiltonian in momentum space takes the form

\[
\hat{H} = \frac{1}{N_\phi} \sum_q \hat{\rho}(-q) \hat{V}(q) \hat{\rho}(q), \quad \hat{V}(q) = [F(q)]^2 v(q), \quad \text{(S4)}
\]

where \(v(q)\) is the interaction potential. For the \(\nu = 1/3\) Laughlin state, the potential is given by \(V_1\) Haldane pseudopotential and it takes the form \(1 - |q|^2\) in momentum space, which translates into the Trugman-Kivelson real space potential \(\nabla^2 \delta(r)\).

In the previous derivations, we have implicitly assumed that the electron band mass tensor,

\[
g = g_m = \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}, \quad \text{(S5)}
\]

is isotropic, i.e., \(g_{11} = g_{22} = 1\), \(g_{12} = g_{21} = 0\). In a more general case, mass anisotropy modifies the single-electron wave functions in Eq. (S1), which in turn modifies the effective interaction matrix elements. In momentum space, the effect of band mass metric is given by:

\[
\hat{V}(g_m, q) = (1 - g_{11} q_x^2 - g_{22} q_y^2 - 2 g_{12} q_x q_y) \cdot \exp\left(\frac{-g_{11} q_x^2 + g_{22} q_y^2 + 2 g_{12} q_x q_y}{2}\right). \quad \text{(S6)}
\]

To obtain the cylinder Hamiltonian for a general metric, we integrate out \(q_x\) in Eq. (S4) and use the above expression for \(\hat{V}(g_m, q)\). Dropping the overall multiplicative constant and making use of translation invariance, we obtain

\[
\hat{H} = \sum_{l,m,j} V_{l,m} c^\dagger_j c_j + m + c^\dagger_{j+1} c_j, \quad \text{(S7)}
\]

\[
V_{l,m} = \kappa^2 \frac{m^2 (g_{11} q_{22} - g_{12} q_{12})^2}{g_{11}^2} \cdot e^{-\kappa^2 (m^2 + l^2 - 2 g_{12} l m) / g_{11}}, \quad \text{(S8)}
\]

which can be simplified due to the unimodular property of the mass tensor, \(g_{11} q_{22} - g_{12} q_{12} = 1\), due to the magnetic flux through the system being fixed (det \(g = 1\)). We will use the following convenient reparametrization of \(g\)

\[
g = \begin{pmatrix} \cosh Q + \cos \phi \sinh Q & \sin \phi \sinh Q \\ \sin \phi \sinh Q & \cosh Q - \cos \phi \sinh Q \end{pmatrix}. \quad \text{(S9)}
\]
in terms of real numbers $Q$ and $\phi$, which represent the stretch and rotation in the plane. With these simplifications, the final Hamiltonian can be written as

$$
\hat{H} = \sum_{j=0}^{N_0-1} \sum_{k>|m|} V_{k,m} c_{j+m}^\dagger c_j + V_{k,m} \exp \left( -\kappa^2 \frac{(k^2 + m^2 - 2ikmg_{12})}{2g_{11}} \right).
$$

(S10)

In the thin torus limit, we can truncate the Hamiltonian in Eq. (S10) by dropping long-range scattering processes, which are exponentially suppressed in the small parameter $\exp(-\kappa^2/2)$, arriving at the Hamiltonian studied in the main text,

$$
\hat{H} = \sum_j V_{1,0} \hat{n}_{j} \hat{n}_{j+1} + V_{2,0} \hat{n}_{j} \hat{n}_{j+2} + V_{3,0} \hat{n}_{j} \hat{n}_{j+3} + V_{2,1} c_{j+1}^\dagger c_j + \text{h.c.},
$$

(S11)

where $\hat{n}_j \equiv c_j^\dagger c_j$ and $V_{2,1} = 3e^{-\kappa^2\frac{5+4i\lambda_{12}}{2g_{11}}}$. Comparison of the dynamics generated by the Hamiltonian in Eq. (S10) and the truncated Hamiltonian of Eq. (S11) are given in Fig S1. We find that both models give approximately the same dynamics near the thin-cylinder limit, where we expect the truncation to be justified.

The Hamiltonian in Eq. (S11) is positive semi-definite for general $g_{12}$, so we can write it in the form with

$$
\hat{H} = \sum_j (Q_j^\dagger Q_j + P_j^\dagger P_j),
$$

(S12)

$$
Q_j = \sqrt{V_{1,0}^2 c_j c_{j+1}} + \sqrt{V_{3,0}^2 e^{i\frac{s+2i\lambda_{12}}{2} g_{12}} \sqrt{g_{11}} c_j c_{j+3}},
$$

(S13)

$$
P_j = \sqrt{V_{2,0}^2 \sqrt{g_{11}}} c_j c_{j+2},
$$

(S14)

where $V_{1,0}$, $V_{2,0}$ and $V_{3,0}$ are given in Eq. (S10) and $e^{i\frac{s+2i\lambda_{12}}{2} g_{12}}$ is the complex phase due to anisotropy parameterized by the previously mentioned tensor $g$. The ground state of Hamiltonian of Eq. (S12) is given by the following expression given in the main text:

$$
|\psi_0\rangle = \mathcal{N} \prod_j \left( 1 - \sqrt{\frac{V_{1,0}}{V_{3,0}}} e^{i\frac{s+2i\lambda_{12}}{2} g_{12}} \hat{S}_j \right) |100100\ldots\rangle,
$$

(S15)

where $\hat{S}_j = c_{j+1}^\dagger c_j + 2 c_j c_{j+3}$ is the ‘squeezing’ operator mentioned in the main text, and $\mathcal{N}$ is a normalization constant. It is evident that $Q_j|\psi_0\rangle = P_j|\psi_0\rangle = 0$ for all sites $j$, and $|\psi_0\rangle$ is the ground state with zero energy. The geometric dependence enters our wave function in Eq. (S15) through the complex phase.

**GRAVITON EXCITATION NEAR THE THIN-CYLINDER LIMIT**

The Hamiltonian in Eq. (S10) has several symmetries, most importantly due to the specific form of $V_{k,m}$, it conserves the center-of-mass position of the electrons, $K = \sum_j j \hat{n}_j$. This is because each $V_{k,m}$ term destroys two particles, initially separated by $|k-m|$ orbitals, and creates two particles at a distance $k+m$ orbitals (and vice versa). This means that, in the Landau gauge, the total momentum along the circumference is conserved. Thus, we can simultaneously label our energy states as momentum eigenstates denoted by quantum number $K$ (in the units of $2\pi/L_2$). It is convenient to label the orbitals $j = 0, \pm 1, \ldots, \pm \frac{N_0-1}{2}$ such that $\sum j \hat{n}_j |\ldots100100\ldots\rangle = 0$, i.e. the root state lies in the zero momentum sector.

Before presenting the energy spectra of the models in Eqs. (S10)-(S11), we explain the construction of their bases in Fock space. In the case of the model in Eq. (S10), we work in the full Fock basis corresponding to $N$ spinless fermions residing in $N_0 - 2$ orbitals (note that the last two are always unoccupied sites). However, when considering the truncated Hamiltonian in Eq. (S11), the effective Hilbert space is much smaller: it consists of all configurations obtained by applying all possible squeezes to the root state. For example, the squeezed basis for $N=4$ electrons is $|100100100100\rangle$, $|011000100100\rangle$, $|100011000100\rangle$, $|100100011000\rangle$ and $|011000011000\rangle$. In general, the number of states in the squeezed basis is given by the Fibonacci number. Hence, the Hilbert space

![FIG. S1. Comparison of the geometric quench dynamics between the full Hamiltonian, Eq. (S10) and the truncated Hamiltonian, (S11), for cylinder circumference $L_2=6.245$ and $N=6$ electrons. The quench is driven by changing $Q=0 \to Q=0.26$. Fit is against the bimetric theory prediction in Eq. (S19) below.](image-url)
dimension still grows exponentially with system size, but it is much smaller than the full Fock basis. This is due to the special structure in the Hamiltonian (S11) which causes it to fracture into many dynamically-disconnected sectors [1].

The energy spectrum $E_K$ of the quench Hamiltonian is plotted in Fig. S2. We notice that the dominant state ($E_K \approx 1.29$) contributing to the dynamics is much higher than the first excited state in $K=0$ momentum sector, for both the squeezed basis and the full Fock basis. Note that the excited states with momenta $K \neq 0$ do not contribute to the dynamics since our quench preserves translation symmetry (hence, $K$ is conserved). In the main text, we identified $E_g \approx 1.29$ as the frequency of the emergent graviton mode which lies in the zero momentum sector.

**BREAKDOWN OF THE BIMETRIC THEORY IN THE STRICT 1D LIMIT**

For the observation of graviton oscillations in the main text, it was crucial that the FQH system is not in the strict 1D limit, $L_2 \to 0$. We require a sufficiently large $L_2$, typically $L_2 \gtrsim 5\ell_B$, such that the system can accommodate the essential correlations in neutral excitations underpinning the graviton mode. Taking $L_2 \to 0$ destroys these correlations, the graviton oscillations disappear and the bimetric theory description breaks down, as we illustrate in Fig. S3.

In Fig. S3 we contrast the dynamics of $\tilde{Q}(t)$ and $\phi(t)$ for two values of the cylinder circumference, $L_2 = 3.15\ell_B$ (a) and $L_2 = 2.75\ell_B$ (b). While in the first case the dynamics still largely follows the bimetric theory prediction (albeit with visible deviations), in the second case the dynamics no longer conforms to the bimetric theory. In the second case, we are in the regime of the 1D Tao-Thouless limit, where the dynamics is trivial due to the initial state being close a product state and an eigenstate of the post-quench Hamiltonian. Thus, we conclude that finite $L_2$ is necessary to observe the graviton oscillation.

We can independently estimate the minimal value of the cylinder circumference, necessary to observe the graviton oscillations, by studying the entanglement entropy scaling of the 1/3 Laughlin state. Fig. S4 shows the entanglement entropy of the 1/3 Laughlin state obtained using the matrix product state method of Ref. 2. The cylinder is infinite in one direction and has a circumference $L_2$ (in units of $\ell_B$).

In a 2D FQH system, the entropy is expected to obey
FIG. S4. Entanglement entropy of the 1/3 Laughlin state on an infinite cylinder [obtained using the matrix product state method of Ref. [2] as a function of the cylinder circumference $L_2$ (in units of $\ell_B$). $P_{\text{max}}$ is the parameter which controls the bond dimension of the matrix product state.

The area law, $S_{\text{ent}} = cL_2 - \gamma + O(e^{-L_2})$. The subleading term $\gamma$ is the topological entanglement entropy, for which linear extrapolation gives $-0.547$ – very close to the theoretical value, $-\frac{1}{2}\ln 3$. We see that the entropy area law scaling is obeyed only for circumferences $L_2$ greater than about $5\ell_B$. This value coincides with the smallest $L_2$ for which we observe the graviton oscillations following the bimetric theory dependence. Thus, our near thin-cylinder limit, $L_2 \approx 5.5 - 7\ell_B$ captures the essential information about the physics of the 2D FQH system.

GEOMETRIC QUENCH IN THE BIMETRIC THEORY

In FQH systems, “geometry” appears in three distinct guises. First, the form factor $F_m(q) = \exp(-g^2m q_a q_b \ell_B^2/4)$ is a function of the band mass tensor $g_m$, where we use the Einstein summation convention. As discussed in Eq. (S9), the symmetric, unimodular tensor $g_m$ can be conveniently parametrized by $Q$ and $\phi$. Second, the interaction potential $V_{\alpha}$ in general depends on another rank-2 tensor $g_i$, characterizing the underlying solid state material ($g_i$ originates from the dielectric tensor of the material for the Coulomb interaction). Both $g_m$ and $g_i$ are set by extrinsic experimental conditions. For simplicity, in this paper we study the model Hamiltonian for the Laughlin state, where we take $g_m = g_i$.

In the presence of extrinsic tensors $g_m$ and $g_i$, a FQH state develops its own, intrinsic geometric degree of freedom. This intrinsic degree of freedom defines the shape of particle-flux composite droplets in the FQH state and can be thought of as a metric $\tilde{g}$, which is similarly parametrized by Eq. (S9). Intrinsic metric $\tilde{g}$ is a “com-promise” between $g_m$ and $g_i$ [3], because $g_m$ and $g_i$ are physically independent and in general different from each other. The dynamics of $\tilde{g}$ can be induced by tilting the magnetic field, which can be exactly modelled by $2 \times 2$ anisotropic mass tensor for parabolic confining potential [3–5]. This forms the key component of the geometric quench protocol [6].

Bimetric theory [7] describes gapped dynamics of a single spin-2 degree of freedom which is present in the collective excitation spectrum of all gapped FQH states. In the long-wavelength limit, this degree of freedom has a variational description in terms of the Girvin-MacDonald-Platzman (GMP) neutral excitation [8]. The bimetric theory assumes the spin-2 excitation mode couples to an external electro-magnetic field and ambient geometry. Thus, the dynamical degree of freedom is the vielbein $\hat{e}_i^a$ [9] that “squares” to give the intrinsic metric introduced above, $g_{ij} = \hat{e}_i^a \hat{e}_j^b \delta_{ab}$. The inverse metric is given by $\tilde{G}^{ij} = \tilde{E}_a^i \tilde{E}_b^j \delta^{\alpha\beta}$, where $\tilde{E}_a^i$ is the inverse vielbein, satisfying $\tilde{E}_a^i \delta_i^j = \delta_i^j$. The unimodular condition on $\tilde{g}_{ij}$ reads $\sqrt{\tilde{g}} = \sqrt{\tilde{g}}$, where $\sqrt{\tilde{g}}$ is the determinant of the ambient metric $g_{ij}$. Quadrupolar anisotropy is introduced by modifying the ambient metric, $g_{ij}^m = m_{AB} e_i^A e_j^B$, where $e_i^A$ are the vielbeins that describe the ambient geometry (we assume ambient space is flat, i.e., $e_i^A = \delta_i^A$ and $m_{AB}$ is a unimodular matrix, assumed to be of the form diag $[e_i^A, -e_i^A]$, where $A$ is the effective anisotropy.

In a homogeneous magnetic field, the bimetric Lagrangian is [7]

$$\mathcal{L} = \frac{\nu\zeta}{2\pi\ell_B^2} \dot{\omega}_0 - \frac{m}{2} \left( \frac{1}{2} g^{ij} g_{ij}^m - \gamma \right)^2,$$

where $\dot{\omega}_0 = \frac{i}{2} \tilde{E}_a^i \tilde{E}_b^j \partial_\alpha \tilde{e}_a^\alpha$ is the temporal component of the (dynamic) Levi-Civita spin connection. The phenomenological coefficient $m$ sets the energy scale which determines the gap of the spin-2 mode, $E_2 = 2\Omega(1 - \gamma)$, where $\Omega = (m/\zeta)/(2\pi\ell_B^2/\nu)$, and the quantized coefficient $\zeta$ is determined by the “shift” [10]. Parameter $\gamma$ is used to tune the theory close to the nematic phase transition in the gapped phase $\gamma < 1$, where the GMP description is exact.

In order to compute the dynamics of $\tilde{g}$, we parametrize it in terms of $\bar{Q}$ and $\bar{\phi}$ as in Eq. (S9). Both $\bar{Q}$ and $\bar{\phi}$ are functions of time but not space, since we consider global (homogeneous) quench. The equations for $\bar{\phi}(t)$ and $\bar{Q}(t)$ have been shown to take the form [6]

$$\ddot{\bar{\phi}} \sinh \bar{Q} = -2\Omega \left( \sinh A \cosh \bar{Q} \cos \bar{\phi} - \cosh A \sinh \bar{Q} \right) \times \left( \gamma + \sinh A \sinh \bar{Q} \cos \bar{\phi} - \cosh A \cosh \bar{Q} \right),$$

$$\dot{\bar{Q}} \sinh \bar{Q} = -2\Omega \sin \bar{\phi} \sinh \bar{Q} \sinh A \left( \gamma + \sinh A \sinh \bar{Q} \cos \bar{\phi} - \cosh A \cosh \bar{Q} \right).$$

This non-linear classical system governs the universal
dynamics of FQH states following a geometric quench. When anisotropy is weak, we can assume both $A$ and $Q$ are close to 0. By Taylor expanding Eqs. (S17) and (S18) in $A$ and $Q$, it can be shown that for a particular initial condition $\tilde{Q}(0) = 0$, the analytical solution is given

$$\tilde{Q}(t) = \pm 2A \sin \left( \frac{E_{g}t}{2} \right), \quad \tilde{\phi}(t) = \pi \mp \frac{E_{g}t}{2},$$

(S19)

which was used for fitting the dynamics data in the main text. Note that there is only one linearly independent solution because the system is invariant under $Q \rightarrow -Q$ and $\phi \rightarrow \phi + \pi$. Thus, we can focus on the $Q \geq 0$ part and consider $\phi$ mod $2\pi$. By inspection of $\tilde{Q}(t)$, we see the solution alternates between two branches, which doubles the frequency from $E_g/2$ to $E_g$. The overall prefactor (written as 2A) is expected to be proportional to the anisotropy of the post-quench Hamiltonian.

Ref. [6] established a good quantitative agreement between the numerically exact quench dynamics and the above prediction of the bimetric theory. The agreement is observed in the regime where the ground state before and after the quench remains in the Laughlin phase. Intuitively, the agreement is due to the fact that the quench involves an exponentially vanishing fraction of eigenstates that contribute to the dynamics, which a posteriori justifies the fundamental assumption of the bimetric theory that assumes a single spin-2 degree of freedom.

**DERIVATION OF THE SPIN-CHAIN HAMILTONIAN**

Here we demonstrate that the Hamiltonian in Eq. (1) in the main text can be mapped onto the spin model in Eq. (5) in terms of the reduced registers. As discussed in the main text, each reduced register corresponds to a block of three consecutive sites, where the reduced register $\ell$ contains three fermionic sites $j = 3\ell - 3, 3\ell - 2, 3\ell - 1$ as shown below:

![Diagram](Image)

To derive the mapping, note that it is sufficient to restrict to the subspace of states obtained by repeated action of the Hamiltonian on the root state, i.e., $H|\ldots 100100 \ldots \rangle, H^2|\ldots 100100 \ldots \rangle$, etc., or, equivalently, states obtained by repeated action of the squeezing operators $\hat{S}_j = c_{j+1}^\dagger c_{j+2}^\dagger e_{j+3}^\dagger e_j$ on the root state. We refer to this subspace as the squeezing subspace. Within this subspace, the terms $\hat{n}_j \hat{n}_{j+2}$ can be dropped as none of the states generated by the application of $\hat{S}_j$ on $|\ldots 100100 \ldots \rangle$ have second neighbor 1s, hence $\hat{n}_j \hat{n}_{j+2}$ trivially vanishes in this subspace.

Next, we consider the $\hat{n}_j \hat{n}_{j+1}$ terms. We focus on open boundary conditions, where the last block is never squeezed, i.e., $\mathcal{N}_N = 0$. Therefore $\ell = N - 1$ is effectively the last register of the spin chain and register $\ell = N$ can be treated a ghost site. It is easy to observe that each squeezed block creates a pair of occupied nearest neighbors. For example, the state $|101000 \rangle \sim |011, 001, 011, 000, 100 \rangle$ has exactly two 1 registers and two nearest neighbor occupied pairs. Therefore,

$$\sum_{j=0}^{3N-2} \hat{n}_j \hat{n}_{j+1} \rightarrow \sum_{\ell=1}^{N-1} \mathcal{N}_\ell.$$

We next consider the $\hat{n}_j \hat{n}_{j+3}$ terms. The root state $|00000 \rangle = |100, 100, 100, 100, 100 \rangle$, shown for $N=5$, has $\sum_j \hat{n}_j \hat{n}_{j+3} = N - 1$. Now let us consider adding a squeezed block $\ell$, somewhere in the middle of the chain, surrounded by unsqueezed blocks:

$$|00000 \rangle = |100, 100, 100, 100, 100 \rangle$$

$$|00100 \rangle = |100, 011, 011, 011, 011 \rangle$$

(S20)

This operation changes blocks $\ell$ and $\ell + 1$, reducing $\hat{n}_j \hat{n}_{j+3}$ by 3 as the bonds between blocks ($\ell - 1, \ell$), ($\ell, \ell + 1$), and ($\ell + 1, \ell + 2$) no longer contribute to $\sum_j \hat{n}_j \hat{n}_{j+3}$. Therefore, we must subtract $3 \sum_\ell \mathcal{N}_\ell$ from the sum. However, if we squeeze two second-neighbor blocks $\ell$ and $\ell + 1$ as

$$|00100 \rangle = |100, 011, 011, 011, 011 \rangle$$

instead of reducing $\hat{n}_j \hat{n}_{j+3}$ by $2 \times 3 = 6$, we reduce it by 5 as the ($\ell + 1, \ell + 2$) bond is double counted. We thus need to add a term $\sum_\ell \mathcal{N}_\ell \mathcal{N}_{\ell+2}$. The only other correction involves $\ell=1$ and $\ell=N-1$, which respectively do not have the ($\ell - 1, \ell$), and ($\ell + 1, \ell + 2$) blocks and therefore only reduce the sum by 2 instead of 3. We then obtain

$$\sum_{j=0}^{3N-4} \hat{n}_j \hat{n}_{j+3} \rightarrow N - 1 - 3 \sum_{\ell=1}^{N-1} \mathcal{N}_\ell + \sum_{\ell=1}^{N-3} \mathcal{N}_\ell \mathcal{N}_{\ell+2} + \mathcal{N}_1 + \mathcal{N}_{N-1}.$$

Finally, we map the squeezing (off-diagonal) term. Unless $j$ is the first site of a reduced block, the operator $V_{2,1} c_j^\dagger c_{j+1}^\dagger c_{j+2} c_{j+3}^\dagger c_{j+4}^\dagger + V_{2,1}^* c_j^\dagger c_{j+3}^\dagger c_{j+2} c_{j+1}^\dagger$ annihilates the state. Thus we focus on an individual term corresponding to block $\ell$

$$W_\ell = V_{2,1} c_{j+1}^\dagger c_{j+2}^\dagger c_{j+3} c_{j+4}^\dagger + V_{2,1}^* c_j^\dagger c_{j+3}^\dagger c_{j+2} c_{j+1}^\dagger,$$

where $j = 3\ell - 3$ is the first site of the block $\ell$. We notice that if either of the neighboring blocks of an unsqueezed block $\ell$ are squeezed, $W_\ell$ annihilates a state:

$$W_\ell|\ldots 10100 \ldots \rangle = W_\ell|\ldots 011, 001, 011 \ldots \rangle = 0,$$

$$W_\ell|\ldots 0011 \ldots \rangle = W_\ell|\ldots 100, 100, 011 \ldots \rangle = 0,$$

$$W_\ell|\ldots 1001 \ldots \rangle = W_\ell|\ldots 011, 001, 011 \ldots \rangle = 0.$$  

(S21)

We can implement this annihilation by including a factor $(1 - \mathcal{N}_\ell)(1 - \mathcal{N}_{\ell+1})$. Blocks $\ell = 1$ and $\ell = N - 1$ are
special because they have only one neighboring block. If block \( \ell \) is already squeezed it cannot have a squeezed neighbor, in which case

\[
W_\ell |00\ldots0\rangle = V_{2,1} |\ldots100,011,000\ldots\rangle = V_{2,1}^* |\ldots0\underline{1}0\ldots\rangle,
\]

\[
W_\ell |0\underline{1}\ldots0\rangle = V_{*2,1}^* |\ldots100,100,100\ldots\rangle = V_{2,1}^* |\ldots00\underline{1}0\ldots\rangle.
\]

Therefore if the neighbors are 0 the action of \( W_\ell \) in the \(|0\rangle, |1\rangle\) basis is given by the matrix

\[
W_\ell = \begin{pmatrix} 0 & V_{2,1}^* \\ V_{2,1} & 0 \end{pmatrix} = \text{Re}(V_{2,1})X - \text{Im}(V_{2,1})Y.
\]

We then represent the full squeezing term for open boundary condition as

\[
\sum_{j=0}^{3N-4} V_{2,1} c_{j+1}^\dagger c_{j+2} c_{j+3} c_j + V_{2,1}^* c_j^\dagger c_{j+2} c_{j+3} c_{j+1}
\]

\[
\to \left[ \text{Re}(V_{2,1}) X_1 - \text{Im}(V_{2,1}) Y_1 \right]{(1 - N_\ell)}
\]

\[
+ (1 - N_{N-2}) \text{Re}(V_{2,1}) X_{N-1} - \text{Im}(V_{2,1}) Y_{N-1}
\]

\[
+ \sum_{\ell=2}^{N-2} (1 - N_{\ell-1}) \text{Re}(V_{2,1}) X_\ell - \text{Im}(V_{2,1}) Y_\ell \left(1 - N_{\ell+1}\right).
\]

**EXTRAPOLATION OF VARIATIONAL PARAMETERS**

In the main text we introduced the following variational ansatz to accurately approximate the time-evolved state:

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_\ell e^{-i\alpha N_\ell} e^{-i\beta(1-N_{\ell-1}) X_\ell} |000\ldots\rangle.
\]

The ansatz consists of applying alternating gates \( N_\ell \) and \( (1 - N_{\ell-1}) X_\ell \) on each reduced register \( \ell \) (on the very first site, due to open boundary condition, we use \( X_1 \) instead of \( (1 - N_{N-1}) X_\ell \) for \( \ell = 1 \)). The parameters \( \alpha \) and \( \beta \), which are limited to windows of length \( 2\pi \) due to the integer eigenvalues of operators \( N_\ell \) and \( (1 - N_{\ell-1}) X_\ell \), are variationally determined to maximize the overlap, \( \langle |\psi_0| U^\dagger(t) |\psi_{\text{var}}(\alpha, \beta)\rangle \), with the exact state at each time step \( t \).

One advantage of the ansatz in Eq. (S22) is that it relies only on two-qubit control gates, while ensuring no forbidden states (with neighboring \( \underline{11} \) registers) are generated. Another advantage is that we can obtain the final state by acting on the trivial root state \(|000\ldots\rangle\) rather than the ground state \( |\psi_0\rangle \). Since all registers are zero in the root state, if we start from the left end of the chain and apply controlled rotations, the qubit to the right is always zero before the application of the unitary operator, allowing us to use two-qubit control gates instead of three-qubit ones, with each register controlled only by its left neighbor. Finally, note that in general the parameters \( \alpha, \beta \) are expected to be site-dependent; however, we have numerically found that imposing translation invariance on the variational parameters produces high overlaps (~0.99) with the exact state and is sufficient for capturing any local observable expectation values in the bulk of the system, in particular it faithfully reproduces the quantum metric parameters \( \tilde{Q} \) and \( \phi \), as well as the full wave function obtained from exact diagonalization (ED) for \( \mathcal{N}=9 \) electrons.

For large \( \mathcal{N} \), the higher-order terms in the expansion are unimportant. As seen in Fig. S6, there is good agreement between the cubic and quadratic fits, with the largest difference on the order 1% for \( \beta^* \). Furthermore, the cubic fits are in excellent agreement with TEBD results for large systems. Therefore, the parameters \( p_0^* (t) \) obtained from the cubic fit provide a smooth, accurate extrapolation of the optimal variational parameters to the thermodynamic limit, \( N \to \infty \).

**SIMULATING QUANTUM DYNAMICS VIA TROTTERIZATION**

In the main text, we used a variational quantum algorithm of linear depth to simulate the geometric quench. The circuit approximated the action of the evolution operator on the pre-quench ground state as \( e^{-iH\tau} |\psi_0\rangle \). In principle, one could implement the evolution operator

![Image](https://via.placeholder.com/150)

FIG. S5. Variational ansatz in Eq. (S22) successfully captures the time-dependent quantum metric parameters, \( \tilde{Q} \) and \( \phi \), as well as the full wave function obtained from exact diagonalization (ED) for \( \mathcal{N}=9 \) electrons.
\( e^{-iH't} \) using Trotterization instead. However, such an approach is not fruitful on current quantum hardware, as the circuit depth necessary to simulate long times quickly exceeds the capabilities of current physical qubits, resulting in pure noise. This issue is exacerbated by the fact that, in contrast to the variational circuit, which acted on the root state \( |000\ldots\rangle \), the circuit which implements a Trotterized form of the propagator must be preceded by a circuit which prepares the pre-quench ground state, increasing the total circuit depth and negatively affecting the fidelity of measurements on the hardware. Here, we derive a circuit which implements a Trotterized time evolution of our Hamiltonian and demonstrate such a scheme is not feasible on current NISQ hardware.

The spin-chain Hamiltonian (neglecting constant terms that give a global phase) is given by

\[
\hat{H} = \sum_{\ell=1}^{N-2} H_\ell + \frac{(V_{1,0} - 2V_{3,0})N_0 + V_{2,1}X_0(1 - N_1)}{H_0} + \frac{(V_{1,0} - 2V_{3,0})N_{N-1} + V_{2,1}(1 - N_{N-2})X_{N-1}}{H_{N-1}},
\]

\[(S25)\]

where

\[
H_\ell \equiv (V_{1,0} - 3V_{3,0})\ell + V_{3,0}\ell N_{\ell-1}\ell + V_{2,1}(1 - N_{\ell-1})X_\ell(1 - N_{\ell+1}).
\]

\[(S26)\]

We decompose the evolution operator as follows:

\[
e^{-iH't} \approx \left( \prod_l e^{-iH_l t/k} \right)^k = \left( \prod_l U_l(t/k) \right)^k. \tag{S28}\]

The approximation improves in the limit of small \( \delta t = t/k \). However, we will see that only \( k=1 \) can be run without significant noise on IBM’s quantum hardware, and thus we can only hope to accurately simulate very short times; late times will be dominated by Trotter error. Note that \( V_{2,1} \) is real for the Trotterization circuit, as we assumed \( g_{12} = 0 \) in the quench Hamiltonian \( H' \).

We now explicitly write the \( U_l \) operators. Ignoring the commutations of the terms in the exponent in the limit of small \( \delta t \), we can write

\[
U_0(\delta t) \approx e^{-i\frac{V_{1,0} - 2V_{3,0}}{2}(I + Z_0)\delta t} e^{-iV_{2,1}X_0(1 - N_1)\delta t},
\]

where \( I \) is the identity. Ignoring a global phase, the first terms is a \( z \) rotation and the second term is a controlled \( x \) rotation, resulting in the circuit element of Fig. S7. Similarly the unitary at the right boundary can be written as

\[
U_{N-1}(\delta t) \approx e^{-i\frac{V_{1,0} - 2V_{3,0}}{2}(I + Z_{N-1})\delta t} e^{-iV_{2,1}(1 - N_{N-2})X_{N-1}\delta t},
\]

giving the circuit element shown in Fig. S8.

In the bulk of the spin chain, the unitary operator up a global phase has the following structure

\[
U_l(\delta t) \approx e^{-\frac{V_{1,0}}{4}Z_{l-1}\delta t} e^{-\frac{V_{1,0} - 3V_{3,0}}{2}Z_l\delta t} e^{-\frac{V_{1,0}}{4}Z_{l+1}\delta t} \times e^{-\frac{V_{1,0}}{4}Z_{l-1}Z_{l+1}\delta t} e^{-iV_{2,1}(1 - N_{l-1})X_l(1 - N_{l+1})\delta t}.
\]
Noting that the operator \( e^{-iV_{3,0}Z_{\ell-1}Z_{\ell+1}\delta t} \) can be implemented by a \( z \) rotation acting on qubit \( \ell + 1 \) sandwiched between two CNOT gates with qubit \( \ell - 1 \) as the control bit, we find the circuit element of Fig. S9 for implementing \( U_\ell(\delta t) \). The unitary constructed above should then act on the prequench ground state. We use the circuit of Ref. 11 to create the Laughlin \( \nu = 1/3 \) fractional quantum Hall state as the initial state.

We simulate the result of the Trotterization scheme we have described above classically, in order to determine the circuit depth \( k \) necessary to accurately capture the dynamics at late times. We find \( k = O(10) \) provides an accurate simulation up to \( t \approx 3 \), whereas \( k = O(100) \) is necessary to accurately simulate up to \( t \approx 10 \). Note that when implementing this circuit on real hardware, it is only multiple-qubit entangling gates which are affected by the fidelity of the qubits, as single-qubit gates do not greatly contribute to noise. The only multiple-qubit entangling gates our Trotterization circuit contains are CNOTs, thus the number of CNOTs in the circuit can be understood as a measure of how well we expect our circuit to perform — in general, more CNOTs means more noise. For example, from practical experience with the IBM platform, we find circuits start to suffer from noise when they contain \( \propto 25 \) CNOTs. Fig. S10 shows the maximum simulation time for which we can obtain accurate results as a function of the number of CNOTs with Trotterization, \( \tau_{acc} \). Fig. S10 also shows simulated dynamics for various \( k \). We find already at \( k=2 \) we exceed 25 CNOTs, yielding \( \tau_{acc} \approx 1 \). Therefore we expect noisy results on ibmq already at \( k>2 \), thus the Trotterized approach is limited to only accurately simulating times \( O(1) \).

Fig S11 displays results obtained from ibmq Santiago quantum computer, implementing Trotterized time evolution from the pre-quench ground state. We observe that while the hardware result for \( k=1 \) agrees with the simulation, such a shallow circuit is dominated by trotter error very quickly, and only accurately captures very short times. Already at \( k=2 \) we observe the hardware result does not agree with the simulated result. Thus, we conclude Trotterization is not a viable approach to simulating geometric quenches on current hardware.

\[ \text{FIG. S9. Circuit implementation of the trotterized unitary } U_j \text{ in the bulk.} \]

\[ \text{FIG. S10. Expected performance of Trotterization with increasing circuit depth. Left: In order to accurately reproduce exact dynamics, we require a very deep circuit (} k = O(100) \text{), which is not feasible on current quantum hardware due to qubit noise. Right: Maximum time } \tau_{acc} \text{ reliably simulated as a function of the number of CNOTs. Vertical line marks } 25 \text{ CNOTs, the approximate current limit of IBM hardware.} \]

\[ \text{FIG. S11. Hardware performance of two shallow-depth Trotterizations with } k=1 \text{ and } k=2. \]

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