Next-to-leading Corrections
at Small $x$ from Quark Evolution

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Abstract

Deep inelastic processes at small $x$ are discussed in the framework of perturbative QCD at high energy. New results are presented on the quark anomalous dimensions beyond the leading logarithmic approximation, and their relevance to the structure functions being measured at HERA is pointed out.
1. Introduction

Quantitative tests of QCD and searches for new physics at present and future hadron colliders are carried out at an increasingly large centre-of-mass energy $\sqrt{S}$. In this regime a new kinematic region opens up, characterized by small values of the ratio $x = Q^2/S$ between the typical momentum $\sqrt{Q^2}$ transferred in the process and the total energy $\sqrt{S}$.

The most striking phenomenological feature at low $x$ is the rise of the deep inelastic cross sections. This behaviour is qualitatively expected in QCD, and has been observed by the HERA collaborations \[1\], who have measured the structure function $F_2$ down to $x$-values as low as $10^{-3} \div 10^{-4}$. The observed rise is indeed steeper than any rise one finds in total cross sections for soft hadronic processes. Therefore it can only be accounted for by a hard QCD component, i.e., QCD interactions occurring at distance scales much smaller than $\Lambda_{QCD}^{-1}$. Such scales are controlled by perturbation theory. Let us then briefly recall the main features of QCD perturbation theory in the region of small $x$.

Perturbative QCD predictions rely upon the factorization theorem of collinear mass singularities \[2\]. The use of this theorem is normally based on two approximations. First, one picks out the leading-twist contribution and expresses the dimensionless cross section $F(x, Q^2) \sim Q^2 \sigma(x, Q^2)$ for a hard hadronic process as a convolution of process-dependent coefficient functions $C_a$ and universal parton densities $f_a$ (Fig. 1), as follows

$$F(x, Q^2) = C_a(x, \alpha_S(\mu^2), Q^2/\mu^2) \otimes f_a(x, \mu^2) + \mathcal{O}(\Lambda_{QCD}^2/Q^2) \ .$$

(1)

Here higher twists, i.e. contributions from multi-parton initial states (Fig. 1b), are neglected, since they are suppressed by inverse powers of the hard scale $Q^2$, and the leading-twist parton densities $f_a(x, \mu^2)$ fulfil the renormalization group evolution equations

$$\frac{d f_a(x, \mu^2)}{d \ln \mu^2} = P_{ab}(\alpha_S(\mu^2), x) \otimes f_b(x, \mu^2) \ ,$$

(2)

where $P_{ab}(\alpha_S, x)$ are the generalized splitting functions, whose Mellin transforms define the anomalous dimensions $\gamma_{ab,N}$

$$\gamma_{ab,N}(\alpha_S) \equiv \int_0^1 dx \ x^N \ P_{ab}(\alpha_S, x) = P_{ab,N+1}(\alpha_S) \ ,$$

(3)

$N$ being the moment conjugate to $x$.

Second, one considers the perturbative expansion for the kernels $P_{ab}$ of the $\mu^2$-evolution

$$P_{ab}(\alpha_S, x) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{2\pi} \right)^n P_{ab}^{(n-1)}(x) \ ,$$

(4)

as well as the analogous expansion for the coefficients $C_a$, and evaluates them to fixed order in $\alpha_S$. Typically, QCD perturbation theory is at present under control up to two loops, that is, the terms $P^{(0)}$ \[3\] and $P^{(1)}$ \[4,5\] in the splitting functions \[4\] are fully known, and the coefficient functions for most of the relevant processes at colliders are also known to the corresponding accuracy \[4\].

In the regime of low $x$, however, both the leading-twist approximation and the fixed-order truncation of evolution kernels and coefficient functions become critical. On the one hand, the leading-twist treatment violates the unitarity bound on the deep inelastic
cross section at asymptotic energies. This is related to the fact that, as the cross section increases, higher-twist contributions from parton recombination and rescattering \cite{7,8} become corrections of relative order $O(1)$ in the formula (1). Therefore they cannot be neglected, and in fact they are essential for asymptotic unitarity to be restored. On the other hand, large high-energy contributions of the type $\alpha_S^n \ln^m 1/x$ affect the perturbative expansions (1) (as well as the analogous expansions for the hard coefficients) to all orders in $\alpha_S$. They may spoil the convergence of the perturbative series at small $x$, as higher powers of $\alpha_S$ associated with multiple hard-jet emission may be compensated by large enhancing factors in $\ln(1/x)$. These corrections have to be identified and summed to all perturbative orders.

Throughout this paper we will not discuss the issue of unitarization, and stick to the leading-twist framework. Phenomenological support for this attitude is provided by the absence of any signal of unitarity corrections in HERA data, as well as by estimates \cite{7,9} showing that the onset of such corrections is expected to be well beyond the range of present colliders. We will instead focus on the high-energy logarithmic effects in perturbation theory. Such effects arise from multiple radiation of space-like gluons in the $t$-channel of the process. Each two-gluon intermediate state (such as that in Fig. 2) does indeed contribute a logarithmic factor $\ln x$ for $x \to 0$, or equivalently, in the moment space of Eq. (3), a pole $1/N$ for $N \to 0$. The resulting higher-order structure of the splitting functions $P_{ab}$ is

$$
P_{ab}^{(n-1)}(x) \sim \frac{1}{x} \left[ \ln^{n-1} x + O(\ln^{n-2} x) \right], \quad x \to 0.
$$

(5)

Analogous terms show up in the coefficient functions $C_a$, but here we will concentrate on the case of the splitting functions. The key approach to dealing with these potentially large corrections is based on perturbative resummation. One can consider improved perturbative expansions, which systematically sum classes of leading, next-to-leading, etc., small-$x$ logarithms (or $N$-poles) to all orders in $\alpha_S$. For instance, in the moment space one has the following expansion for the anomalous dimensions

$$
\gamma_{ab,N}(\alpha_S) = \sum_{k=1}^{\infty} \left[ \left( \frac{\alpha_S}{N} \right)^k A_{ab}^{(k)} + \alpha_S \left( \frac{\alpha_S}{N} \right)^k B_{ab}^{(k)} + \ldots \right].
$$

(6)

The coefficients $A^{(k)}$ (leading), $B^{(k)}$ (next-to-leading), etc., define the logarithmic hierarchy at high energy (i.e., small $x$). Once these coefficients are known, they can be combined with expansions of the type (3) (after subtracting the resummed logarithmic terms in order to avoid double counting), to obtain a prediction throughout the region of $x$ where $\alpha_S \ln(1/x) \lesssim 1$ (or $\alpha_S/N \lesssim 1$), which is much larger than the domain $\alpha_S \ln(1/x) \ll 1$ where the $\alpha_S$-expansions (1) are applicable.

The outline of this paper is as follows. In Sect. 2 we review the present leading-order knowledge of QCD at small $x$, and discuss the role of sub-leading effects. In Sect. 3 we present the results of a study of next-to-leading corrections \cite{10,11}. Sect. 4 contains the summary and some prospects.

2. Power counting at small $x$

On the basis of the definition (3), let us briefly go through a simple power counting at high energy. It is worth recalling first that no super-leading terms $\alpha_S^n/N^k$, $2n \geq k > n$, are present in the anomalous dimensions. This effect is associated with the coherence of gluon
radiation in space-like processes at small $x$ \cite{12-14}. Super-leading terms do contribute to exclusive final-state distributions, but they cancel out in inclusive quantities. The issue of the exclusive structure at small $x$ has been studied in Ref.\cite{13} (some phenomenological investigations have been carried out in Ref.\cite{15}), and will not be touched upon here. We will then be concerned with single-logarithmic corrections at small $x$.

Note next that flavour non-singlet observables do not couple to pure-gluon intermediate states. Therefore they are always less singular than $1/x$ at small $x$, i.e., in the moment space, they are regular for $N \to 0$ order-by-order in $\alpha_S$. All the high-energy contributions $\alpha_S^n/N^k$ ($n \geq k \geq 1$) are thus associated with the flavour singlet sector.

The singlet evolution equations are

$$\frac{d}{d \ln \mu^2} \left( \frac{\tilde{f}_S}{\tilde{f}_g} \right) = \left( \begin{array}{cc} 2N_f \gamma_{qq}^S & 2N_f \gamma_{qg} \\ \gamma_{qq} & \gamma_{gg} \end{array} \right) \left( \begin{array}{c} \tilde{f}_S \\ \tilde{f}_g \end{array} \right),$$

where we have used standard definitions for parton densities and anomalous dimensions, and neglected regular terms at $N \to 0$ in the quark-quark entry of the matrix.

It is a fundamental result which can be traced back to the work of Lipatov and collaborators \cite{12} that only the gluon channel (lower entries to the anomalous dimension matrix in Eq. (7)) contributes to the leading order ($A^{(k)}$-type coefficients in Eq. (6)), and that the leading-order resummation can indeed be carried through by means of an integral equation for the off-shell gluon Green function (BFKL equation). The resulting anomalous dimensions read as follows ($\bar{\alpha}_S \equiv C_A \alpha_S/\pi$)

$$\gamma_{gg,N}(\alpha_S) = \gamma(\bar{\alpha}_S/N) + \mathcal{O}(\alpha_S(\alpha_S/N)^k), \quad \gamma_{qg,N}(\alpha_S) = \frac{C_F}{C_A} \gamma_{gg,N}(\alpha_S) + \mathcal{O}(\alpha_S(\alpha_S/N)^k),$$

where the BFKL anomalous dimension $\gamma(\bar{\alpha}_S/N)$ is determined by the implicit equation

$$1 = \bar{\alpha}_S \chi(\gamma(\bar{\alpha}_S/N)), \quad \chi(\gamma) \equiv 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),$$

$\psi$ being the Euler $\psi$-function. By solving Eq. (9) as a power series in the coupling constant one finds the leading $(\alpha_S/N)^k$ contributions to the gluon anomalous dimension to all orders in $\alpha_S$. The first perturbative terms are

$$\gamma(\bar{\alpha}_S/N) = \frac{\alpha_S}{N} + 2\zeta(3) \left( \frac{\alpha_S}{N} \right)^4 + 2\zeta(5) \left( \frac{\alpha_S}{N} \right)^6 + \mathcal{O} \left( \left( \frac{\alpha_S}{N} \right)^7 \right),$$

$\zeta$ being the Riemann $\zeta$-function ($\zeta(3) \simeq 1.202$, $\zeta(5) \simeq 1.037$). Note that due to strong cancellations between real emission diagrams and virtual corrections the coefficients of $\alpha_S^2$, $\alpha_S^3$ and $\alpha_S^5$ in Eq. (10) vanish, and therefore the BFKL anomalous dimension departs from its one-loop approximation rather slowly. It is also worth recalling \cite{16} that by virtue of the collinear regularity of the BFKL integral kernel the leading-order anomalous dimensions \cite{9}-\cite{11} are factorization scheme invariant, i.e., they do not depend on the explicit procedure to regularize and factorize collinear mass singularities. The physical features of the BFKL resummation may be schematically described considering the asymptotic behaviour of the gluon density $f_g$. In the fixed-coupling limit we find for $x \to 0$

$$f_g(x, Q^2/Q_0^2) \sim \exp \left( 2\sqrt{\alpha_S} \ln 1/x \ln Q^2/Q_0^2 \right) \quad \text{(one-loop)}$$

(11)
\[ f_g(x, Q^2/Q_0^2) \sim x^{-\tilde{N}} \left( Q^2/Q_0^2 \right)^{1/2}, \quad \tilde{N} \sim 4 \ln 2 \bar{\alpha}_S \quad \text{(BFKL-resummed)} \] (12)

The all-order resummation of the perturbative $N$-poles builds up a branch point singularity at a value $\tilde{N}$ of the moment proportional to $\alpha_S$. As a result, the gluon density increases at small $x$ like a power rather than like $\exp(\sqrt{\ln(1/x)})$. Also, the associated scaling violations are stronger in the resummed case ($\left( Q^2/Q_0^2 \right)^{1/2}$ vs. $\exp[\sqrt{\ln(Q^2/Q_0^2)}]$).

The BFKL analysis predicts a growth of the anomalous dimension with respect to the fixed-order case, and eventually its saturation at the asymptotic value $1/2$. The evaluation of small-$x$ contributions to the anomalous dimension matrix beyond the leading logarithmic approximation has been the object of many efforts over the past fifteen years. As in any perturbative expansion, one needs to know sub-leading corrections in order to be able to assess the stability of the expansion and set its limits of validity. In particular, one may wonder whether the asymptotic singularity may change beyond the leading order. Moreover, since running coupling effects mix with sub-leading corrections, it is essential to know the latter in order to be able to carry out the renormalization group analysis consistently at low $x$.

It is worth noting that beyond the leading order quarks start to contribute on the same footing as gluons: corrections $O(\alpha_S(\alpha_S/N)^k)$ ($B^{(k)}$-type coefficients in Eq. (6)) affect the lower as well as the upper entries to the anomalous dimension matrix in Eq. (7). This remark is relatively trivial, but it has been long overlooked in the literature, and most studies have focused on the pure-gauge sector. Let us give a perturbative example at fixed order. Consider the off-shell quark Green function (Fig. 3), and its $\alpha_S$-expansion. In the lowest non-trivial order $O(\alpha_S^2)$, the remark above simply amounts to the well-known fact that the diagram in Fig. (3b) gives rise to a leading-order contribution ($\alpha_S/N$) to the gluon anomalous dimension (times a term $O(\alpha_S)$ in the coefficient function) when evaluated in the region of ordered momenta $q^2 \gg k^2 \gg p^2$, but it contributes a next-to-leading-order correction $\alpha_S^2/N$ to the quark anomalous dimension from the “disordered” configuration $q^2 \sim k^2 \gg p^2$. A mechanism of this kind holds in higher-order diagrams as well.

Actually, from a phenomenological viewpoint, one may argue that the knowledge of the next-to-leading quark anomalous dimensions is even more relevant than that of the corresponding corrections to the gluon anomalous dimensions. The reason for this is that the most accurate information on small-$x$ QCD is coming from HERA data on deep inelastic structure functions, which couple to quarks directly, and to gluons via a $O(\alpha_S)$-suppressed coefficient function.

The next-to-leading contributions $O(\alpha_S(\alpha_S/N)^k)$ to the quark anomalous dimensions (upper entries to the matrix in Eq. (6)) have recently been computed to all orders in $\alpha_S$ [10,11]. In the next section we present the results of this calculation. The next-to-leading corrections to the gluon anomalous dimensions are in contrast still unknown at present. A calculational program is however being pursued by Fadin and Lipatov [17].

3. Next-to-leading resummed results

Unlike the leading-order analysis, the resummation of next-to-leading logarithms at small $x$ is sensitive to the specific procedure one uses to regularize and factorize the collinear mass singularities arising from the low-momenta region. One then needs to develop a formalism in which high-energy resummation is consistently matched with the all-order
collinear analysis. This can be accomplished [1] by virtue of a property of factorization at high energy, which holds for off-shell Green functions and supplements the one due to the renormalization group.

We have performed the resummation of the next-to-leading corrections to the quark anomalous dimensions in two of the most commonly used factorization schemes, the DIS- and the \( \overline{\text{MS}} \)-scheme. The DIS-scheme is defined by setting the coefficient functions for the structure function \( F_2 \) equal to unity in the quark channel and zero in the gluon channel, modulo an additional condition which is needed to uniquely fix the gluon density to all loops [9,11]. In this scheme the resummed result for the next-to-leading quark anomalous dimensions is parametrized as follows

\[
\gamma_{qg,N}(\alpha_S) = \gamma_{qg,N}^{(\text{DIS})}(\alpha_S) + \mathcal{O}(\alpha_S^2), \quad \gamma_{qg,N}^{(\text{DIS})}(\alpha_S) = \frac{C_F}{C_A} \left[ \gamma_{qg,N}(\alpha_S) - \frac{\alpha_S}{2\pi} T_R \frac{2}{3} \right] + \mathcal{O}(\alpha_S^2),
\]

where the anomalous dimension \( \gamma_{qg,N}^{(\text{DIS})} \) can be expressed in a particularly compact form:

\[
\gamma_{qg,N}^{(\text{DIS})}(\alpha_S) = \frac{\alpha_S}{2\pi} T_R \frac{2 + 3\gamma_N - 3\gamma_N^2}{3 - 2\gamma_N} \frac{\Gamma^3(1 - \gamma_N) \Gamma^3(1 + \gamma_N)}{\Gamma(2 + 2\gamma_N) \Gamma(2 - 2\gamma_N)} R(\gamma_N). \tag{14}
\]

Here \( \gamma_N \) denotes the BFKL anomalous dimension [3], and the normalization factor \( R(\gamma_N) \) is given by

\[
R(\gamma_N) = \left\{ \frac{\Gamma(1 - \gamma_N) \chi(\gamma_N)}{\Gamma(1 + \gamma_N) [-\gamma_N \chi'(\gamma_N)]} \right\}^{1/2} \cdot \exp \left\{ \gamma_N \psi(1) + \int_0^{\gamma_N} d\gamma \frac{\psi'(1) - \psi'(1 - \gamma)}{\chi(\gamma)} \right\}, \tag{15}
\]

\( \chi, \chi' \) being the characteristic function in Eq. (9) and its first derivative, respectively. The resummation of the contributions \( \alpha_S(\alpha_S/N)^k \) to all orders in \( \alpha_S \) is incorporated in Eq. (14) through the explicit \( \gamma_N \)-dependence and the \( \alpha_S/N \)-dependence of \( \gamma_N \) (known from the BFKL equation (9)). For instance, using the expansion (10), one can compute the first perturbative terms of the anomalous dimension (14):

\[
\gamma_{qg,N}^{(\text{DIS})} = \frac{\alpha_S}{2\pi} T_R \frac{2}{3} \left\{ 1 + \frac{13}{6} \bar{\alpha}_S - \left( \frac{71}{18} - \zeta(2) \right) \left( \bar{\alpha}_S/N \right)^2 + \left[ \frac{233}{27} - \frac{13}{6} \zeta(2) + \frac{8}{3} \zeta(3) \right] \left( \bar{\alpha}_S/N \right)^3 \right\} \nonumber \\
+ \left[ \frac{1276}{81} - \frac{71}{18} \zeta(2) + \frac{91}{9} \zeta(3) - 6\zeta(4) \right] \left( \bar{\alpha}_S/N \right)^4 + \ldots \tag{16}
\]

\[
\approx \frac{\alpha_S}{2\pi} T_R \frac{2}{3} \left\{ 1 + 2.17 \frac{\bar{\alpha}_S}{N} + 2.30 \left( \bar{\alpha}_S/N \right)^2 + 8.27 \left( \bar{\alpha}_S/N \right)^3 + 14.92 \left( \bar{\alpha}_S/N \right)^4 + \ldots \right\}.
\]

Here the coefficients of the first two terms in the curly bracket agree with the known one- and two-loop anomalous dimensions in the DIS scheme [3,20], whereas the higher-order terms represent new sub-leading information at small \( x \).

Observe that Eq. (14) does not introduce any singularity in the \( N \)-moment space above the leading one (see Eq. (12)). This means that the position of the leading singularity is not changed by next-to-leading corrections in the quark sector. However, the approach to it is made faster, as a consequence of the positive sign of the corrections, and the resulting scaling violations are stronger.
It is worth noting that, as long as the gluon anomalous dimensions are unknown in next-to-leading order, the DIS-scheme may be convenient for phenomenological investigations of the structure function $F_2$ at small $x$, because by definition it decouples $F_2$ from gluons. The knowledge of the quark anomalous dimensions in this scheme (Eqs. (13)-(14)) may therefore be particularly useful.

As for the $\overline{\text{MS}}$-scheme, the resummed result is expressed by relations of the type (13) in terms of the analogous anomalous dimension $\gamma_{qg,N}^{(\overline{\text{MS}})}(\alpha_s)$. This is determined implicitly as a function of $\alpha_s$ and $N$ by an algebraic equation (see Ref. [11] for full details), and its perturbative expansion reads

$$\gamma_{qg,N}^{(\overline{\text{MS}})}(\alpha_s) = \frac{\alpha_s}{2\pi} T_R \frac{2}{3} \left\{ 1 + \frac{5}{3} \frac{\alpha_s}{N} + \frac{14}{9} \left( \frac{\alpha_s}{N} \right)^2 + \left[ \frac{82}{81} + 2 \zeta(3) \right] \left( \frac{\alpha_s}{N} \right)^3 + \frac{122}{243} \right. $$

$$+ \left. \frac{25}{6} \zeta(3) \left( \frac{\alpha_s}{N} \right)^4 + \ldots \right\} \right. $$

$$ \simeq \frac{\alpha_s}{2\pi} T_R \frac{2}{3} \left\{ 1 + 1.67 \frac{\alpha_s}{N} + 1.56 \left( \frac{\alpha_s}{N} \right)^2 + 3.42 \left( \frac{\alpha_s}{N} \right)^3 + 5.51 \left( \frac{\alpha_s}{N} \right)^4 + \ldots \right\} .$$

Note that the coefficients in Eq. (16) are systematically larger than those in Eq. (17). This is related to the fact that the difference between the two anomalous dimensions is proportional to the $\overline{\text{MS}}$-coefficients of the structure function $F_2$, which are quite sizeable at small $x$ [11].

One may ask what the consequences of resummation are in terms of the stability of the logarithmic expansion at low $x$. One must first notice that the numerical implementation of QCD evolution based on resummed anomalous dimensions is not available yet, and more work is needed. Moreover, a fully consistent analysis to next-to-leading logarithmic order obviously requires also the computation of the gluon anomalous dimensions, which are still unknown to this accuracy. However, one can already make some remarks based on fixed-order features. First, the cancellations in orders $O(\alpha_s^2)$, $O(\alpha_s^3)$, $O(\alpha_s^5)$ that we observe in the leading gluon anomalous dimensions (10) do no longer occur in the next-to-leading quark anomalous dimensions (16) (or (17)). Thus at large but finite energies (such as those at HERA) we may expect the latter to be of comparable importance to the former from the numerical point of view, in spite of their being formally sub-leading [11]. Second, a detailed analysis [21] of the third-loop term of the anomalous dimension (12) has recently been performed, which confirms this expectation. The authors of Ref. [21] have considered the full two-loop evolution of parton densities (as in standard sets of distributions, like the MRS’ [22]), and the evolution including the $O(\alpha_s^3)$ term in eq. (17) and the $O(\alpha_s^4)$ term in eq. (10). They have compared the predictions for $F_2$ in the two cases. They have done this both in the case of a flat input gluon distribution at low momenta (as in the set MRSD0) and a steep one (as in the set MRSD). The outcome is that the impact of the higher-order corrections heavily depends on the choice of the input. If the input gluon distribution is flat, the effect may be sizeable already in the range of $x$-values accessible at HERA. This suggests a possible instability of the perturbative series, and calls for a careful evaluation of sub-leading contributions to all orders. On the other hand, a very small effect is observed in the case of a steep input gluon: this simply means that if the input is more singular than any rise that may ever be generated in QCD, then the input dominates the evolution, and the output at large momentum scales essentially reproduces what has been assumed as an input at low scales.
4. Conclusions

QCD anomalous dimensions at small $x$ are resummed to leading logarithmic accuracy by the BFKL equation. Sub-leading corrections to this approximation are needed for both theoretical and phenomenological reasons. In this paper we have reported on a study aimed at investigating such corrections.

The analysis of the small-$x$ region beyond the leading order brings in the novel feature of the interplay between high-energy logarithms and collinear mass singularities. To deal with this, a method has been set up in which the small-$x$ resummation is carried out in a manner which can unambiguously be matched with the leading-twist collinear factorization to all orders.

Sub-leading corrections to the BFKL analysis involve the quark and the pure-gauge sector on an equal footing. The main achievement of the work presented here is the resummation of the next-to-leading corrections to quark anomalous dimensions. Explicit results have been obtained in the ${\overline{\text{MS}}}$ and DIS factorization schemes. On the other hand, the corresponding contributions in the gluon sector are still unknown.

Preliminary numerical studies indicate that the size of the next-to-leading effects discussed in this paper may be large enough to affect the deep inelastic measurements at HERA. This suggests that further and more detailed numerical investigations should be pursued (including the full implementation of resummed results), as well as further theoretical analyses on sub-leading contributions (in this respect, the calculation of the next-to-leading corrections to gluon anomalous dimensions represents a major challenge). Also, the procedure of matching mentioned above between the high-energy formalism and the collinear factorization should be used systematically to combine small-$x$ resummed results with finite-$x$ non-logarithmic contributions computed in fixed-order perturbation theory.

Acknowledgments. The work presented in this paper has been carried out in collaboration with S. Catani. Many useful discussions with M. Ciafaloni, G. Marchesini and B. Webber are gratefully acknowledged.

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