Merger rate distribution of primordial black hole binaries with electric charges

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\textbf{Abstract.} We consider gravitational radiation and electromagnetic radiation from point mass binary with electric charges in a Keplerian orbit, and calculate the merger rate distribution of primordial black hole binaries with charges and a general mass function by taking into account gravitational torque and electromagnetic torque by the nearest primordial black hole. We apply the formalism to the extremal charged case and find that $\alpha = -(m_i + m_j)^2 \partial^2 \ln R(m_i, m_j) / \partial m_i \partial m_j = 12/11$, which is independent of the mass function.
1 Introduction

Primordial black holes (PBHs) [1–3] are those black holes which are formed in the very early Universe. There are some mechanisms which have been proposed to produce PBHs, such as large curvature perturbations generated during inflation [4–8], domain walls [9, 10], bubble collisions [11–13], preheating instability [14], sound speed resonance [15] and parametric amplification of curvature perturbations [16]. Since LIGO detected black hole binary mergers, PBHs, as a promising candidate for dark matter (DM), have recently attracted much attention [17–33]. It is believed that the gravitational wave (GW) events observed by the LIGO detectors [34] could be explained by the coalescence of PBH binaries.\(^1\) By calculating the late-time merger rate of PBHs which formed binaries in the late Universe, Refs [37, 38] claim that the PBH merger rate could match the merger rate detected by LIGO if PBHs could account for all of the DM. In fact, there are two kinds of mechanisms proposed for PBH binary formation. One is that PBH binary formed in the late Universe [37–39] while the other is that PBH binary formed in the early Universe [40–53], that is expected to make the dominant contribution to the PBH merger rate today.

The merge rate of PBH binaries with monochromatic mass function is estimated through the three-body interaction [40–42]. Later, the merger rate of PBH binaries is improved in [45] by taking into account the torques exerted by all PBHs, but it is also assumed that all PBHs have the same mass. The mechanism has recently been developed for a general mass function by taking into account the torques from the all PBHs [46, 48, 49]. A formalism to estimate the effect of merger history of PBHs on merger rate distribution has been developed in [50]. Those works consider the merger rate distribution of PBHs binaries by assuming that PBHs are Schwarzschild black holes. However, in general case, PBHs have spin and charges. In this paper, we analyze the merger rate distribution of PBH binaries with charges and a general mass function by taking into account gravitational torque and electromagnetic torque by the nearest PBH. We find that \(\alpha = - (m_i + m_j)^2 \partial^2 \ln \mathcal{R}(m_i, m_j)/\partial m_i \partial m_j = 12/11\) for the extremal charged black hole binary with a general mass function.

Charged black holes have attracted much attention not only in theoretical study of Hawking radiation and Schwinger effect but also in recent observations of GWs. A non-extremal charged black hole emits all species of particles, neutral or charged, according to the Bose-Einstein or Fermi-Dirac distribution with the Hawking temperature ([54] for a review). The Hawking temperature vanishes for extremal charged black holes, which may

\(^1\)LIGO black holes also can be explained by stellar-origin black holes [35, 36].
literally cease the evaporation. The Schwinger mechanism, however, triggers pair creation of charged particles for extremal black holes [55]. The leading Boltzmann factor is given by the effective temperature for accelerating charges in the electric field on the horizon [56], whose near-horizon geometry has a factor of AdS$_2$ space.

When the horizon size of a PBH is smaller than the Compton wavelength or classical radius of a charged particle, the PBH cannot emit that particle and may be a candidate for dark matter [57]. For (near-)extremal charged black holes, this is equivalent to the Breitenlohner-Friedmann (BF) bound since the AdS$_2$ geometry near the horizon gives the bound $|R_{AdS}|/2 \geq (qE_H/m)^2$ against the Schwinger mechanism [56, 58], which in turn gives the BH size $2|q| \geq R_H$ for the charge $q$ [59]. These extremal PBHs have small masses and may also be a candidate for dark matter. On the other hand, in the early universe and beyond the standard model, a dark quantum electrodynamics with heavy dark electrons and massless dark photons, which couple to electrons and photons of the standard model at renormalization level, suppresses the Schwinger effect and allows the extremal PBHs whose life time is longer than the age of the universe [60]. In this paper we assume such scenarios for extremal PBHs.

The paper is organized as follows. In the next section, we calculate gravitational radiation and electromagnetic radiation from point masses with charges in a Keplerian orbit. In Sec. 3, we derive the merger rate distribution of PBH binaries with charges and a general mass function by taking into account gravitational torque and electromagnetic torque by the nearest PBH. In Sec. 4, we consider a specific cases of extremal charged PBH binaries, we find that $\alpha = -(m_i + m_j)^2 \partial^2 \ln R(m_i, m_j)/\partial m_i \partial m_j = 12/11$, which is independent of the mass function. The last section is devoted to conclusions and discussions.

In this paper, we choose units of $c = \varepsilon_0 = \mu_0 = 1$. Whenever relevant, we adopt the values of cosmological parameters from the Planck 2018 results [61] and the scale factor $s(t)$ is normalized to be unity at the matter-radiation equality.

## 2 Electromagnetic radiation and gravitational radiation

The point masses $m_1$ with charge $Q_1$ and $m_2$ with charge $Q_2$ have coordinates $(d_1 \cos \psi, d_1 \sin \psi)$ and $(-d_2 \cos \psi, -d_2 \sin \psi)$ in the $x$-$y$ plane, as shown in Fig. 1. Choosing the origin to be the center of mass, we have

$$d_1 = \left(\frac{m_2}{m_1 + m_2}\right) d, \quad d_2 = \left(\frac{m_1}{m_1 + m_2}\right) d,$$

(2.1)

where $d = d_1 + d_2$ is the distance between the two point masses. The total energy is given by

$$E = -\frac{Gm_1 m_2}{2a} + \frac{1}{4\pi} \frac{Q_1 Q_2}{2a} = -\frac{Gm_1 m_2}{2a} (1 - \lambda),$$

(2.2)

where $a$ is the semi-major axis and

$$\lambda = \frac{1}{4\pi} \frac{Q_1 Q_2}{Gm_1 m_2}.$$

(2.3)

Because the point masses make up a bound system, we have $\lambda < 1$. For the Kepler motion, the orbit equation, angular velocity and angular momentum are given by

$$d = \frac{a (1 - e^2)}{1 + e \cos \psi},$$

(2.4)
\[ \psi = \left[ \frac{G(m_1 + m_2) a (1 - e^2) (1 - \lambda)}{d^2} \right]^{1/2}, \]  
(2.5)

\[ L = \frac{\sqrt{a\sqrt{1 - e^2}} \sqrt{G\sqrt{1 - \lambda m_1 m_2}}}{\sqrt{m_1 + m_2}}, \]  
(2.6)

where \( e \) is the eccentricity. Firstly, we compute the total power radiated in electromagnetic waves. In our reference frame where the orbit is in the \( x-y \) plane, the electric dipole is given by

\[ p \equiv Q_1 x_1 + Q_2 x_2 = \frac{m_2 Q_1 - m_1 Q_2}{m_1 + m_2} d \cos \psi \hat{x} + \frac{m_2 Q_1 - m_1 Q_2}{m_1 + m_2} d \sin \psi \hat{y}, \]  
(2.7)

where \( \hat{x} \) is the unit vector along \( x \) and \( \hat{y} \) is the unit vector along \( y \). The Lagrangian density of the electromagnetic field is

\[ L_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (E^2 - B^2). \]  
(2.8)

The electric field \( E \), magnetic field \( B \) and vector potential \( A \) at \( r \) \((r \gg d)\) are

\[ E(r, t) \approx \frac{1}{4\pi r} [(\hat{r} \cdot \hat{p}(t'))\hat{r} - \hat{p}(t')] = \frac{1}{4\pi r} [\hat{r} \times (\hat{r} \times \hat{p})], \]  
(2.9)

\[ B(r, t) \approx \frac{1}{4\pi r} [\hat{r} \times \hat{p}], \]  
(2.10)

\[ A(r, t) \approx \frac{1}{4\pi} \frac{\hat{p}(t')}{r}, \]  
(2.11)

where \( \hat{r} \) is the unit vector along \( r \) and \( t' = t - r \). Because of emitting electromagnetic radiation, the system loses energy and angular momentum. From

\[ \partial_{\mu} T_{\mu\nu}^{EM} = 0, \quad E_{EM} = \int_V dx^3 T_{00}^{EM}, \]  
(2.12)

the rate of energy emission due to electromagnetic radiation is

\[ \frac{dE_{EM}}{dt} = -\int_V dx^3 \partial_{t} T_{00}^{EM} = -\int_S dA \cdot \vec{n} T_{00}^{EM} = -\frac{e^2}{6\pi}, \]  
(2.13)

where \( T_{\mu\nu}^{EM} \) is the energy-momentum tensor of the electromagnetic field. By using Eq. (2.7), we have the average energy loss over an orbital period \( T \) due to electromagnetic radiation

\[ \left\langle \frac{dE_{EM}}{dt} \right\rangle \equiv \frac{1}{T} \int_0^T dt \frac{dE_{EM}}{dt} = \frac{(e^2 + 2) G^2 (1 - \lambda)^2 (m_2 Q_1 - m_1 Q_2)^2}{12\pi a^4 (1 - e^2)^{5/2}}, \]  
(2.14)

where

\[ T = \int_0^{2\pi} d\psi \psi^{-1} = \frac{2\pi a^2}{\sqrt{aG(1 - \lambda)(m_1 + m_2)}}. \]  
(2.15)
Figure 1. A schematic picture of point masses with charges in a Keplerian orbit.

The angular momentum of the electromagnetic field along the i axis is then given by
\[ J_i^{EM} = (1/2) \epsilon^{ijk} J_{EM}^{jk}. \]
From the Noethers theorem, we have
\[ J_{EM}^{jk} = \int d^3x \left[ \frac{\partial L_{EM}}{\partial (\partial_0 A_i)} \right] \left( a^{\nu(jk)} \partial_\nu A_i - F_i^{(jk)} \right) - a^{0(jk)} L_{EM}, \quad (2.16) \]
where
\[ a^{\mu}_{(\rho\sigma)} = \delta^\mu_\rho x_\sigma - \delta^\mu_\sigma x_\rho, \quad F_i^{(jk)} = \delta^{ij} A^k - \delta^{ik} A^j. \quad (2.17) \]
After a straightforward computation, we obtain
\[ J_i^{EM} = \int d^3x \left[ -\epsilon^{ikl} (\partial_0 A_j) x^k \partial_l A_j + \epsilon^{ikl} A_k \partial_0 A_l \right], \quad (2.18) \]
where the first term is the orbital angular momentum and the second term is the spin part.

The density of the angular momentum of the electromagnetic field is given by
\[ j_i^{EM} = -\epsilon^{ikl} (\partial_0 A_j) x^k \partial_l A_j + \epsilon^{ikl} A_k \partial_0 A_l. \quad (2.19) \]
Let us consider electromagnetic waves propagating outward from the two point masses. At time t we consider a portion of the wave front covering a solid angle \( d\Omega \) at radial distance r from our source, and then at time \( t + dt \), this portion of the wave front has swept the volume \( d^3x = r^2 drd\Omega = r^2(dt)d\Omega \). Since the angular momentum of electromagnetic waves per unit volume is \( J_i^{EM} \), the angular momentum carried away by electromagnetic waves is given by
\[ dJ_i^{EM} = r^2 dt d\Omega j_i^{EM}. \quad (2.20) \]
Therefore the rate of angular momentum emission due to electromagnetic waves is obtained by
\[
\frac{dJ_{EM}^i}{dt} = - \int r^2 d\Omega(-\epsilon^{ikl}(\partial_k A_j)x^k \partial^j A_j + \epsilon^{ikl}A_k \partial_0 A_l). \tag{2.21}
\]

Using Eqs. (2.9), (2.10) and (2.11), we obtain
\[
\frac{dJ_{EM}^i}{dt} = - \frac{\epsilon^{ikl}}{6\pi} \dot{p}_k \dot{p}_i. \tag{2.22}
\]

For the orbit in the $x$-$y$ plane, we have $L_z = L$, $L_x = L_y = 0$. Using Eqs. (2.7) and (2.5), one has
\[
\dot{p}_1 = \frac{\sqrt{G} \sqrt{1 - \lambda} \sin(\psi)(m_2 Q_1 - m_1 Q_2)}{\sqrt{a} \sqrt{1 - e^2} \sqrt{m_1 + m_2}}, \tag{2.23}
\]
\[
\dot{p}_2 = \frac{\sqrt{G} \sqrt{1 - \lambda}(e + \cos(\psi))(m_1 Q_2 - m_2 Q_1)}{\sqrt{a} \sqrt{1 - e^2} \sqrt{m_1 + m_2}}. \tag{2.25}
\]
\[
\dot{p}_2 = \frac{G(1 - \lambda)\sin(\psi)(e \cos(\psi) + 1)^2(m_2 Q_1 - m_1 Q_2)}{a^2(1 - e^2)^2}. \tag{2.26}
\]

The rate of angular momentum emission due to electromagnetic radiation is given by
\[
\frac{dJ_{EM}}{dt} = - \frac{1}{6\pi} (\dot{p}_2 \dot{p}_1 - \dot{p}_1 \dot{p}_2) = - \frac{G^3/2(1 - \lambda)^3/2(e \cos(\psi) + 1)^3(m_2 Q_1 - m_1 Q_2)^2}{6\pi a^{5/2} (1 - e^2)^{5/2} \sqrt{m_1 + m_2}}. \tag{2.27}
\]

For the angular momentum loss due to electromagnetic radiation averaged one orbital period $T$, we have
\[
\left\langle \frac{dJ_{EM}}{dt} \right\rangle \equiv \frac{1}{T} \int_0^T dt \frac{dJ_{EM}}{dt} = - \frac{G^3/2(1 - \lambda)^3/2(m_2 Q_1 - m_1 Q_2)^2}{6\pi a^{5/2} (1 - e^2)^{5/2} \sqrt{m_1 + m_2}}. \tag{2.28}
\]

The electromagnetic field or gravitational field carries away a total angular momentum $J$, which is made of a spin contribution and of an orbital angular momentum contribution. This total angular momentum is drained from the total angular momentum of the source, which, for our binary system or any macroscopic source, is a purely orbital angular momentum. So, the loss rate of the angular momentum in the system due to electromagnetic radiation is given by
\[
\left\langle \frac{dL_{EM}}{dt} \right\rangle = \left\langle \frac{dJ_{EM}}{dt} \right\rangle = - \frac{G^3/2(1 - \lambda)^3/2(m_2 Q_1 - m_1 Q_2)^2}{6\pi a^{5/2} (1 - e^2)^{5/2} \sqrt{m_1 + m_2}}. \tag{2.29}
\]

Now, we begin to compute the total radiated power in GWs. In our reference frame where the orbit is in the $x$-$y$ plane, the second mass moment is given by a $2 \times 2$ matrix
\[
M_{ab} = \mu a^2 \begin{pmatrix}
\cos^2 \psi & \sin \psi \cos \psi \\
\sin \psi \cos \psi & \sin^2 \psi
\end{pmatrix}_{ab}, \tag{2.30}
\]
where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass and subscripts $(a, b = 1, 2)$ are indices in the $x$-$y$ plane. Following [62], the radiated power of GWs can be expressed in a rotation invariant form

$$P(\psi) = \frac{2G}{15} \left[ (\ddot{M}_{11} + \ddot{M}_{22})^2 - 3(\ddot{M}_{11} \ddot{M}_{22} - \ddot{M}_{12}^2) \right],$$

(2.31)

where the first term is the square of the trace and the second term is the determinant of the matrix of $\dot{M}_{ij}$. Using Eqs. (2.5) and (2.30), one has the components of the matrix

$$\ddot{M}_{11} = \frac{G^3/2(1 - \lambda)^{3/2} m_1 m_2 \sqrt{m_1 + m_2} \sin(2\psi)(e \cos(\psi) + 1)^2(3e \cos(\psi) + 4)}{a^{5/2} (1 - e^2)^{5/2}},$$

$$\ddot{M}_{12} = -\frac{G^3/2(1 - \lambda)^{3/2} m_1 m_2 \sqrt{m_1 + m_2} (e \cos(\psi) + 1)^2(5e \cos(\psi) + 3e \cos(3\psi) + 8 \cos(2\psi))}{2a^{5/2} (1 - e^2)^{5/2}},$$

$$\ddot{M}_{22} = -\frac{G^3/2(1 - \lambda)^{3/2} m_1 m_2 \sqrt{m_1 + m_2} \sin(\psi)(e \cos(\psi) + 1)^2(e(3 \cos(2\psi) + 5) + 8 \cos(\psi))}{a^{5/2} (1 - e^2)^{5/2}},$$

(2.32)

and the trace square and the determinant

$$\left( \ddot{M}_{11} + \ddot{M}_{22} \right)^2 = \frac{G^3(1 - \lambda)^{3/2} m_1^2 m_2^2 (m_1 + m_2) (e \cos(\psi) + 1)^4}{a^5 (1 - e^2)^5} \times (2e \sin(\psi))^2,$$

$$\ddot{M}_{11} \ddot{M}_{22} - \ddot{M}_{12}^2 = \frac{G^3(1 - \lambda)^{3/2} m_1^2 m_2^2 (m_1 + m_2) (e \cos(\psi) + 1)^4}{a^5 (1 - e^2)^5} \times (-16(e \cos(\psi) + 1)^2).$$

(2.33)

So, we get

$$P(\psi) = \frac{4G^4(1 - \lambda)^3 m_1^2 m_2^2(m_1 + m_2) \left( 11 e^2 \cos(2\psi) + 13 e^2 + 48 e \cos(\psi) + 24 \right) (e \cos(\psi) + 1)^4}{15a^5 (1 - e^2)^{7/2}}.$$

(2.34)

The energy of GWs is only well-defined by taking an average over several periods. In our case, a well-defined quantity is the average of $P(\psi)$ over one period $T$. So we can perform this time average to get the total radiated power

$$\bar{P} \equiv \frac{1}{T} \int_0^T dt P(\psi) = \frac{(37e^4 + 292e^2 + 96) \ G^4(1 - \lambda)^3 m_1^2 m_2^2(m_1 + m_2)}{15a^5 (1 - e^2)^{7/2}}.$$  

(2.35)

The average energy loss over an orbital period $T$ is given by

$$\left\langle \frac{dE_{GW}}{dt} \right\rangle = -\bar{P} = -\frac{(37e^4 + 292e^2 + 96) \ G^4(1 - \lambda)^3 m_1^2 m_2^2(m_1 + m_2)}{15a^5 (1 - e^2)^{7/2}}. 

(2.36)

Following [63], the rate of angular momentum emission due to GW is given by

$$\frac{dL_{GW}^i}{dt} = -\frac{2G}{5} e^{ikl} \left\langle \dot{M}_{ka} \dot{M}_{la} \right\rangle. $$

(2.37)
Similarly, if electromagnetic radiation is dominated, the coalescence time and gravitational radiation are given by

\[
\frac{dL_{GW}}{dt} = \frac{4G}{5} \langle \dot{M}_{12} (\dot{M}_{11} - \dot{M}_{22}) \rangle = -\frac{8G^7/2(\lambda - 1)^{5/2}m_1^2m_2^2\sqrt{m_1 + m_2}\sin^2(\psi)(e\cos(\psi) + 1)^2}{5a^{7/2}c^2(1 - e^2)^{7/2}} \times (e(3\cos(2\psi) + 4) + 8\cos(\psi))(e(\cos(2\psi) + 3) + 4\cos(\psi)),
\]

where

\[
\dot{M}_{12} = -\frac{G(1 - \lambda)m_1m_2\sin(\psi)(e(\cos(2\psi) + 3) + 4\cos(\psi))}{a(1 - e^2)}.
\]

For the angular momentum loss averaged one orbital period \(T\), we have

\[
\langle \frac{dL_{GW}}{dt} \rangle = \frac{1}{T} \int_0^T \frac{dL_{GW}}{dt} \, dt = -\frac{4(7e^2 + 8) G^7/2(1 - \lambda)^{5/2}m_1^2m_2^2\sqrt{m_1 + m_2}}{5a^{7/2}(1 - e^2)^2}.
\]

The total rate of energy and angular momentum emission due to electromagnetic radiation and gravitational radiation are given by

\[
\langle \frac{dE}{dt} \rangle = \langle \frac{dE_{EM}}{dt} \rangle + \langle \frac{dE_{GW}}{dt} \rangle,
\]

\[
\langle \frac{dL}{dt} \rangle = \langle \frac{dL_{EM}}{dt} \rangle + \langle \frac{dL_{GW}}{dt} \rangle.
\]

Whatever \(e \approx 0\) or \(e \approx 1\), we have

\[
\frac{\langle dE_{GW} \rangle}{\langle dE_{EM} \rangle} \approx \frac{\langle dL_{GW} \rangle}{\langle dL_{EM} \rangle} \sim \frac{24\pi(7e^2 + 8) G^2(1 - \lambda)m_1^2m_2^2(m_1 + m_2)}{5a(1 - e^2)(m_2Q_1 - m_1Q_2)^2}.
\]

The system spends most of the decay time in a state for which \(a \approx a_0\). For a given \(a_0\) and \(e_0\), the total rate of energy and angular momentum emission is dominated by gravitational radiation or electromagnetic radiation which depends on \(m_1, m_2, Q_1\) and \(Q_2\). If gravitational radiation is dominated, the coalescence time

\[
\tau_{GW} \approx \begin{cases} 
\frac{5a_0^4}{256c^4(1-\lambda)^2m_1m_2(m_1+m_2)}, & \text{for } e_0 \approx 0, \\
\frac{3a_0^4(1-e_0^2)^{7/2}}{85c^2(1-\lambda)^2m_1m_2(m_1+m_2)}, & \text{for } e_0 \approx 1.
\end{cases}
\]

Similarly, if electromagnetic radiation is dominated, the coalescence time

\[
\tau_{EM} \approx \begin{cases} 
\frac{\pi a_0^4m_1m_2}{C(1-\lambda)(m_2Q_1 - m_1Q_2)^2}, & \text{for } e_0 \approx 0, \\
\frac{4\pi a_0^4(1-e_0^2)^{5/2}m_1m_2}{C(1-\lambda)(m_2Q_1 - m_1Q_2)^2}, & \text{for } e_0 \approx 1.
\end{cases}
\]

The coalescence time for two point masses with charges can be approximated as

\[
\tau \equiv \min(\tau_{GW}, \tau_{EM}).
\]
3 Merger rate distribution of primordial black hole binaries with charges

Let us consider the condition two nearest PBHs with masses $m_i$, $m_j$ and charges $Q_i$, $Q_j$ decouple from the expanding Universe, assuming negligible initial peculiar velocities in what follows. The total energy and angular momentum of the bound system are

$$E = -\frac{G m_i m_j}{2a} + \frac{1}{4\pi} \frac{Q_i Q_j}{2a} = -\frac{G m_i m_j}{2a} (1 - \lambda), \quad (3.1)$$

$$L = \frac{\sqrt{a} \sqrt{1 - \epsilon^2} \sqrt{G} \sqrt{1 - \lambda m_i m_j}}{\sqrt{m_i + m_j}}. \quad (3.2)$$

Considering the gravitational force, electromagnetic force and the expansion of the Universe, the equation of motion for their proper distance $r$ in Newtonian approximation is given by

$$\ddot{r} - \left( \dot{H} + H^2 \right) r + \frac{m_b}{r^2 |r|} \left( 1 - \lambda \right) = 0, \quad (3.3)$$

where $m_b = m_i + m_j$ is total mass of the PBH binary and the dot denotes the differentiation with respect to the proper time. By defining $\chi \equiv r/x$, we can rewrite Eq. (3.3) as

$$\chi'' + \frac{sh'}{s^2 h} \left( s \chi' - \chi \right) + \frac{1}{\lambda} \frac{1}{(sh)^2} \frac{1}{\chi^2} \frac{\chi}{|\chi|} = 0, \quad (3.4)$$

where primes denote differentiation with respect to the scale factor $s$, $h(s) \equiv H(s)/\left( \frac{8\pi \rho_{eq}}{3} \right)^{1/2} = \sqrt{s^{-3} + s^{-4}}$, $\rho_{eq}$ is the energy density of the Universe at the matter-radiation equality, and $x$ is the comoving separation between these two nearest PBHs. Here, the dimensionless parameter $\tilde{\lambda}$ is given by

$$\tilde{\lambda} = \frac{8\pi \rho_{eq} x^3}{3 m_b (1 - \lambda)}. \quad (3.5)$$

The solution of Eq. (3.4) derived in [45] implies the semi-major axis $a$ of the formed binary is given by

$$a \approx 0.1 \tilde{\lambda} x. \quad (3.6)$$

When the two PBHs which could form a bound system come closer and closer, the surrounding PBHs, especially the nearest PBH, will exert torques on the PBH binary. The tidal force from other PBHs will provide an angular momentum to prevent this system from direct coalescence and form a highly eccentric binary. The angular momentum $L$ of the binary is estimated by multiplying the exerted total torques $\ell$ from the nearest PBH with mass $m_l$ and charge $Q_l$ by the free-fall time

$$L \approx |t_{ff} \times \ell|, \quad (3.7)$$

where the free-fall time is given by

$$t_{ff} \approx \frac{\pi}{2} \frac{x^2}{\sqrt{2Gm_b(1 - \lambda)}}. \quad (3.8)$$
The exerted total torque $\ell$ is made of the torque from the gravitational force $\ell_{GW}$ and the torque from electromagnetic force $\ell_{EM}$. As illustrated in Fig. 2, $y \gg x$ is the comoving distance from the third PBH to the PBH binary and $\theta$ is the angle between $x$ and $y$. Here, we introduce a dimensionless charge $k$

$$k = \frac{Q}{\sqrt{4\pi Gm}}.$$ (3.9)

so, $k_i$ represents $\frac{Q_i}{\sqrt{4\pi Gm_i}}$. The torque the from gravitational force $\ell_{GW}$ is given by

$$\ell_{GW} = \ell_{GW}^i + \ell_{GW}^j = -\frac{Gm_im_jxy\sin\theta}{R_i^3m_b} + \frac{Gm_im_jxy\sin\theta}{R_j^3m_b}.$$ (3.10)

Using

$$R_i = (y^2 \sin^2 \theta + (y \cos \theta + \frac{m_j}{m_b}x)^2)\frac{1}{2} \approx y(1 + \frac{m_jx \cos \theta}{m_by}),$$ (3.11)

$$R_j = (y^2 \sin^2 \theta + (y \cos \theta - \frac{m_i}{m_b}x))\frac{1}{2} \approx y(1 - \frac{m_ix \cos \theta}{m_by}),$$ (3.12)

we can rewrite Eq. (3.10) as

$$\ell_{GW} \approx \frac{3Gm_im_jx^2 \sin \theta \cos \theta}{m_by^3}.$$ (3.13)

Similarly, the torque from the electromagnetic force $\ell_{EM}$ is given by

$$\ell_{EM} \approx -\frac{Gm_im_jx \sin \theta}{m_by^2}(k_jk_l - k_lk_j) + \frac{3x \cos \theta}{m_by} (m_i k_jk_l + m_jk_i k_l).$$ (3.14)
The total torque $\ell$ is given by
\[
\ell = -\frac{Gm_im_jx\sin \theta}{m_0y^2}F,
\] (3.15)
where
\[
F = k_jk_l - k_ik_l + \frac{3x\cos \theta}{m_0y}(m_ik_jk_l + m_jk_ik_l - m_0).
\] (3.16)
Now, we introduce a dimensionless angular momentum
\[
j \equiv \sqrt{1 - e^2}.
\] (3.17)
By solving
\[
L \approx |t_{ff} \times \ell|,
\] (3.18)
\[
L = \frac{\sqrt{x}j\sqrt{G\sqrt{1 - \lambda m_im_j}}}{\sqrt{m_i + m_j}},
\] (3.19)
we can get
\[
j \approx \frac{x^2m_l\sin \theta}{y^2m_0(1 - \lambda)} |F|.
\] (3.20)
For Schwarzschild black holes where $k_i = k_j = k_l = 0$, we have
\[
j = 3\frac{m_lx^3\sin \theta \cos \theta}{m_0y^3},
\] (3.21)
which is consistent with the result in [46]. The coalescence time of PBH binaries derived in Sec. 2 can be estimated as
\[
\tau = \text{Min}(\frac{3a^4j^7}{85G^3(1 - \lambda)^2m_im_j(m_i + m_j)}; \frac{a^3j^5}{G^2(1 - \lambda)m_im_j(k_i - k_j)^2}).
\] (3.22)
The probability distribution function of PBH masses and charges $P(m, k)$ is normalized to be
\[
\int_{-1}^{+1} \int_0^{\infty} dm dk P(m, k) = 1.
\] (3.23)
The abundance of PBHs with charges in the mass interval $(m, m + dm)$ is
\[
fp(m)dm,
\] (3.24)
where
\[
P(m) \equiv \int_{-1}^{+1} dk P(m, k).
\] (3.25)
The fraction of PBHs in DM, \( f_{\text{pbh}} \), is related to the total abundance of PBHs in non-relativistic matter \( f \) by \( f_{\text{pbh}} \equiv \Omega_{\text{pbh}}/\Omega_{\text{dm}} \approx f/0.85 \). The average number density of PBHs in mass interval \((m, m + dm)\) at the matter-radiation equality is given by

\[
 n(m) dm = \frac{f P(m) dm \rho_{\text{eq}}}{m},
\]

while the comoving total average number density of PBHs, \( n_T \), is defined by

\[
 n_T \equiv f \rho_{\text{eq}} \int_0^\infty dm \frac{P(m)}{m}.
\]

For simplicity, we could define \( m_{\text{pbh}} \) as

\[
 \frac{1}{m_{\text{pbh}}} \equiv \int_0^\infty dm \frac{P(m)}{m}.
\]

So, \( n(m)/n_T = P(m) m_{\text{pbh}}/m \) is the fraction of the average number density of PBHs with mass \( m \) in the total average number density of PBHs.

To calculate the merger rate of PBH binaries, we have to know the spatial distribution of PBHs. Assuming that the spatial distribution of PBHs is random one, for the comoving distances, \( x \) and \( y \), in the intervals \((x, x + dx)\) and \((y, y + dy)\), PBH masses, \( m_i \), \( m_j \) and \( m_l \), in the intervals \((m_i, m_i + dm_i)\), \((m_j, m_j + dm_j)\) and \((m_l, m_l + dm_l)\), PBH charges, \( k_i \), \( k_j \) and \( k_l \), in the intervals \((k_i, k_i + dk_i)\), \((k_j, k_j + dk_j)\) and \((k_l, k_l + dk_l)\), and the angle \( \theta \), in the intervals \((\theta, \theta + d\theta)\), the probability is given by

\[
 dP = \frac{m_{\text{pbh}}^3}{m_i m_j m_l} P(m_i, k_i) dm_i dk_i P(m_j, k_j) dm_j dk_j P(m_l, k_l) dm_l dk_l \times 4\pi x^2 n_T dx 2\pi y^2 \sin(\theta) n_T dy d\theta e^{-\frac{4\pi y^3 n_T}{3}} \Theta(y - x),
\]

The fraction of PBHs that have merged before the time \( t \) is given by

\[
 G(t, m_i, m_j, m_l, k_i, k_j, k_l) = \int dxdy d\theta \frac{dP}{dxdydm_i dm_j dm_l dk_i dk_j dk_l d\theta} \Theta(t - \tau).
\]

The merger rate density \( \mathcal{R}(t, m_i, m_j, m_l, k_i, k_j, k_l) \) is given by

\[
 \mathcal{R}(t, m_i, m_j, m_l, k_i, k_j, k_l) = \frac{1}{2} \frac{n_T}{(1 + z_{\text{eq}})^3} \times \lim_{dt \to 0} \frac{G(t + dt, m_i, m_j, m_l, k_i, k_j, k_l) - G(t, m_i, m_j, m_l, k_i, k_j, k_l)}{dt},
\]

where the factor 1/2 accounts for that each merger event involves two PBHs. The merger rate distribution of PBH binaries with charges are given by

\[
 \mathcal{R}(t, m_i, m_j) = \int dm_l dk_l dk_j d\theta \mathcal{R}(t, m_i, m_j, m_l, k_i, k_j, k_l).
\]
4 Extremal charged PBH binaries

Ref [64] showed that a black hole evaporate or radiate as an ideal thermal blackbody and the temperature of black hole is only related to its surface gravity $\kappa$ via $T = \kappa/(2\pi)$, while $\kappa$ only depends on three parameters: mass $M$, electric charge $Q$, and angular momentum $L$. A Schwarzschild black hole ($Q = L = 0$) with mass $M < M_\ast \sim 5 \times 10^{14} \text{g}$ has temperature $T = 1/(8\pi M)$ and a lifetime less than the age of the Universe. For a rotating black hole, the angular momentum is emitted much faster than energy, so a rapidly rotating black hole will quickly become a nearly non-rotating state before most of its mass has been given up [65]. In this section, we focus on PBHs with masses $M$ much smaller than $M_\ast$. Because of Hawking radiation, those PBHs will quickly become extremal charged black holes. Their mass function could be arbitrary and only depend on their initial charge distribution. Thus we choose the probability distribution function of PBH masses and charges $P(m,k)$ as

$$P(m,k) = \delta(k - 1) + \delta(k + 1) 2P(m).$$ (4.1)

Only two PBHs with opposite charge ($\lambda = -1$) could form a bound system. The semi-major axis $a$, the dimensionless angular momentum $j$ and coalescence time $\tau$ of the formed binary can be approximated as

$$a \approx 0.1 \frac{4\pi \rho_c x^4}{3m_b},$$ (4.2)

$$j \approx \frac{x^2 m_i \sin \theta}{y^2 m_b},$$ (4.3)

$$\tau \approx \frac{a^3 j_5}{8G^2 m_i m_j}.$$ (4.4)

Applying the formalism in Sec. 3, the merger rate distribution of PBH binaries with charges are given by

$$\mathcal{R}(t, m_i, m_j) = \int dm_l \mathcal{R}(t, m_i, m_j, m_l).$$ (4.5)

where

$$\mathcal{R}(t, m_i, m_j, m_l) \approx P(m_i) P(m_j) P(m_l) \left( \frac{t}{t_0} \right)^{-\frac{10}{11}} \times 4.69 \times 10^6 (M_\odot)^{\frac{10}{11}} (m_i m_j)^{-\frac{19}{11}} (m_l)^{-\frac{17}{11}} (m_{\text{pbh}})^{\frac{16}{11}} \left( m_i + m_j \right)^{\frac{12}{11}} f_{\text{pbh}}.$$ (4.6)

which can be interpreted as the merger rate density in unit of $\text{Gpc}^{-3} \text{yr}^{-1} M_\odot^{-2}$. So, $\alpha = -(m_i + m_j)^2 \partial^2 \ln \mathcal{R}(t, m_i, m_j)/\partial m_i \partial m_j = 12/11$, which is independent of the PBH mass function. By contrast, for uncharged PBH binaries, $\alpha = 36/37$ derived in [48, 49].
5 Conclusions and discussions

We have calculated gravitational radiation and electromagnetic radiation from point masses with charges in a Keplerian orbit and applied the result to work out the merger rate distribution of PBH binaries with charges and a general mass function by taking into account gravitational torque and electromagnetic torque by the nearest PBH. For the extremal charged case, we find that
\[ \alpha = -\left( m_i + m_j \right)^2 \partial^2 \ln R(m_i, m_j) / \partial m_i \partial m_j = 12/11, \]
which is independent of the mass function. PBHs are a natural DM candidate without requiring physics beyond the standard model. A wide mass range of extremal charged black hole from the Planck mass scale to about $10^{12}$ g is still allowed by the experiment constraints. Such extremal charged black hole could be tested as DM.

In our calculation, we assumed that the spatial distribution of PBHs is random one. An additional consideration in calculating the merger rate is the cluster of PBHs which could considerably change the merger rate [38, 66–68]. This is an interesting topic, but it is believed that, for Gaussian initial conditions, the spatial distribution of PBHs is Poisson distributed with no additional clustering.

Finally, we discussed physical properties of charged black holes and their formation scenarios. In an asymptotically flat spacetime the Einstein-Maxwell theory has the Kerr-Newman black hole with an angular momentum $L = jM$ and an electric charge $Q$ and/or magnetic charge $P$ as well as a mass $M$. The extremal condition of $M^2 = j^2 + (Q^2 + P^2)$ is achieved for a charged non-rotating black hole when $M = Q$. In the present universe with a small Hubble constant, the Schwinger mechanism from astrophysical charged black holes has been proposed for gamma-ray bursts [69–72] and an argument was raised against the mechanism because of $q \gg m$ for emitted pairs in the standard model [73].

In the early universe, there may be possibly some statistical fluctuations that segregate the high density plasma into opposite charges against the electric repulsion, which collapse to form pairs of Kerr-Newman black holes with opposite charges. Then, a Kerr-Newman black hole with mass $M$ less than $M_*$ can quickly lose its angular momentum, become a Reissner-Nordström black hole, and finally become an extremal charged black hole through Hawking radiation [65]. A black holes with electric and magnetic charges through pure Hawking radiation ends up as an extremal one [74] if the Schwinger effect is not considered from the (near-)extremal black hole, whose BF bound allows an extremal black hole stable against both the Schwinger effect and Hawking radiation [55]. Such an extremal charged black hole is stable and may be a candidate of DM. The merger event of two light extremal charged black holes with opposite charges can generate a light and non-extremal charged black hole whose fast Hawking radiation can be detected by various experiments. The charged black hole has lots of implications to astrophysics and cosmology [75].

In the early universe with a large Hubble constant, non-rotating charged black holes have both the event horizon and the cosmological horizon, which depend on the de Sitter radius, and the effect of the de Sitter space on Hawking radiation cannot be neglected [76, 77]. The Schwinger mechanism from charged black holes in de Sitter space also differs from that of charged black holes in the asymptotically flat spacetime in that the Hubble radius affects the effective temperature for Schwinger mechanism and the emission of charges from the cosmological horizon affects that from the event horizon itself [78]. The detailed description requires a quantitative study.

Another scenario beyond the standard model for extremal PBHs is the dark quantum electrodynamics with dark electrons and photons, whose suppressed Schwinger effect gives
the life time of PBHs longer than the age of the universe [60]. Still another scenario is the formation of black holes from gauge fields during the inflation [79]. We leave all these topics for future works.

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