Prediction of Flutter Boundary Using Flutter Margin for The Discrete-Time System

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Abstract. Flutter testing in a wind tunnel is generally conducted at subcritical speeds to avoid damages. Hence, the flutter speed has to be predicted from the behavior some of its stability criteria estimated against the dynamic pressure or flight speed. Therefore, it is quite important for a reliable flutter prediction method to estimates flutter boundary. This paper summarizes the flutter testing of a wing cantilever model in a wind tunnel. The model has two degree of freedom; they are bending and torsion modes. The flutter test was conducted in a subsonic wind tunnel. The dynamic data responses was measured by two accelerometers that were mounted on leading edge and center of wing tip. The measurement was repeated while the wind speed increased. The dynamic responses were used to determine the parameter flutter margin for the discrete-time system. The flutter boundary of the model was estimated using extrapolation of the parameter flutter margin against the dynamic pressure. The parameter flutter margin for the discrete-time system has a better performance for flutter prediction than the modal parameters. A model with two degree freedom and experiencing classical flutter, the parameter flutter margin for the discrete-time system gives a satisfying result in prediction of flutter boundary on subsonic wind tunnel test.

1. Introduction

The most important aeroelastic phenomenon is flutter. Flutter is self-excited oscillation of the elastic structure under the action of the aerodynamic loads. Flutter instabilities often exhibit an explosive behavior that cause a sudden change in stability despite a small change in flight condition. Further, the aeroelastic vibrations that occur at all flight regimes have a strong impact on the fatigue life of the structure. Since flutter is the self excited vibration which causes a fatal damage to a wing, engineers have to pay attention to the occurrence of flutter in the airplane design and development. At the final stage of development, therefore, they have to conduct the flutter tests to check that flutter does not occur in the flight envelope, and to evaluate the flutter boundary. Since the flutter tests are carried out only in the safety range far below the critical point, the flutter boundary is predicted from the behavior of some stability measure plotted against the flight speed or the dynamic pressure [1].

Flutter boundary prediction during wind tunnel and flight tests is one of the most important, but very difficult tasks imposed in the process of development of a new airplane [2]. One difficult aspect of flutter testing is that a flutter mode can suddenly become unstable with a small increase in dynamic pressure. Furthermore, there may be little or no indication of the approach of the instability in a plot of modal damping against dynamic pressure. Although such a sudden onset of instability is not always the case, it occurs in practice frequently enough that flutter prediction based on the projection of modal damping...
may not be adequate [3]. As an alternative approach, several researchers have attempted to propose more suitable parameters for the flutter prediction than the modal damping. Among them, the flutter margin introduced by Zimmermann and Weissenburger [4] is the most famous and successful one.

The flutter margin decreases almost monotonically toward zero as the dynamic pressure increases, which is a very suitable property for the flutter prediction, though it is applicable only to binary-flutter [5]. Currently, measurement and analysis of test data are generally performed on a digital computer, where sampled data are processed to define the characteristics of the systems. System identification gives powerful and useful procedures to construct a linear discrete-time model, known as an Auto Regressive Moving Average (ARMA) model. The ARMA model consists of autoregressive (AR) terms of measurement, that is linear combination of present and the finite number of past data, and the moving average (MA) of white noise. The AR part correspond to the characteristic polynomial of the system and gives information on stability [6]. In the discrete-time domain, Matsuzaki and Ando have proposed to use Jury’s stability parameter as the indicator of stability margin, which is the stability criterion for the discrete-time system and are calculated from the Auto Regressive Moving Average (ARMA) model identified. Then Jury’s parameters and introduced the new parameter called the Flutter Margin for the Discrete-time System (FMDS), which is approximately equivalent to the flutter margin introduced. Though the FMDS has superior properties as the flutter prediction parameter, it is applicable to the data which include only two coupling modes [1]. They have shown that the stability parameter is a more effective index for the flutter prediction than modal damping through numerical and experimental studies [7].

Generally, flutter testing on a wind tunnel carried out at subcritical speeds to avoid facilities damage. Hence, the flutter speed has to be predicted from the behavior some of its stability criterions estimated against the dynamic pressure or flight speed. Therefore, it is quite important for a reliable flutter prediction method to estimates flutter boundary speed. A flutter prediction method has to estimates the flutter boundary speed as accurate as possible. This paper summarize flutter testing of a wing cantilever model in a wind tunnel. The model has two degree of freedom, are bending and torsion modes. The flutter test was conducted in the subsonic wind tunnel. The measurement was repeated by increasing the wind speed stepwise. The dynamic responses was measured by two accelerometers that were mounted on leading edge and center of wing tip. Dynamic responses were used to determine parameter stability FMDS. Flutter boundary of the model were estimated using extrapolation of FMDS against dynamic pressure. The predicted flutter speed using FMDS were compared with actual flutter speed to evaluate the accuracy of FMDS method. The goal of this study is to determine alternative method that is accurate and effective for predicting flutter boundary in a wind tunnel.

2. Method

2.1. Flutter Margin for Discrete-Time System (FMDS)

FMDS defined based on the time-series model, hence that it is suitable for the digital data processing, and it doesn’t need the modal damping and frequency. After preprocessed through the band-pass filter so as to include only the coupling mode, sampled data is identified by the ARMA model

\[
y_t + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \alpha_4 y_{t-4} = e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_{t-m} e_{t-m}
\]

(1)

where \(y_t\) is the data observed at time \(t\), and \(e_t\) a white noise. The order, \(m\), in the right hand side of Eq.(1) is determined by the Akaike Information Criteria (AIC). The left hand side corresponds to the characteristic equation of the system:

\[
G(z) = z^4 + \alpha_1 z^3 + \alpha_2 z^2 + \alpha_3 z + \alpha_4
\]
Where $z_i^*$ is complex characteristic roots. It is obvious that $i$ represents the number of the modes consisting of the aeroelastic system and $T$ is sampling rate of measurement. The modal frequencies, $\omega_i$, and the damping ratio, $\xi_i$, of the $i$-th mode are respectively given as [8]:

$$\xi_i = -(\ln|z_i|/\ln|z|)$$  \hspace{1cm} (2)

$$\omega_i = \frac{1}{T}\ln|z_i|$$  \hspace{1cm} (3)

The stability margin of the system is measured by [1]

$$F_z = \frac{\det(X_3 - Y_3)}{(1 - \alpha_4)^2}$$  \hspace{1cm} (4)

Where

$$X_3 = \begin{bmatrix} 1 & \alpha_1 & \alpha_2 \\ 0 & 1 & \alpha_1 \\ 0 & 0 & 1 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_3 & \alpha_4 & 0 \\ \alpha_4 & 0 & 0 \end{bmatrix}$$

The value of $F_z$ decreases monotonically and almost linearly toward zero in the subcritical range with the increase of the dynamic pressure [1].

2.2. Wind-Tunnel Flutter Testing

The Flutter test was conducted at Indonesia Low Speed Wind Tunnel (ILST). ILST is a close circuit wind tunnel. This tunnel has a close rectangular test section with size 4 x 3 m. Installation the model in the wind tunnel is depicted in Figure 1. The wind speeds around model were measured by a pitot tube that was installed close to the model.

![Pitot tube](image)

**Figure 1.** The Flutter Model in wind tunnel

Flutter model is depicted in Figure 2. The model is cantilever wing with two degree of freedom, are bending and torsion mode. Data responses were measured by two accelerometers that placed on leading edge and center of wing tip. Both are inside the wing. So that they don’t disturb the flow over model
surface. During each measurement, the wind speed was kept constant and the wing was excited by impulsive disturbance at the same time.

Figure 2. Flutter Model

Figure 3 describes the flow chart of flutter test. The flutter test conducted in the subsonic wind tunnel. The measurement was repeated by increasing the wind speed. The model excited by an impulsive force. The purpose of excitation are to get sufficient response to distinguish modes from noise. The data response was measured by accelerometers and then was digitized in a data acquisition. The digital data are then processed to eliminate low-frequency noise and higher modes using digital filter. Hence, the filtering data that consists only the lowest two modes. Then, filtering data identified with ARMA model, to estimate ARMA coefficients. Flutter margin parameter $F_z$ were determined using ARMA coefficients, and then the flutter boundary speed was predicted by linear fitting of flutter margin $F_z$ [8].

Figure 3. Flow Chart of Flutter Test

3. Results and Discussions

3.1. Modal Testing

The Modal testing was conducted to determine natural frequency of the flutter modes. Bending and torsion modes are the flutter modes. Attention must be paid to those frequencies in the wind tunnel test. The Modal testing was carried out using single input multiple outputs (SIMO) method. Excitation or inputs was provided by an impact hammer that is equipped by a force sensor on the tip. The dynamic response were measured by eight accelerometers. Those accelerometers were installed on the model surface. Installation of sensor is depicted in Figure 4.
The result of modal testing is presented in Table 1. Each vibrational mode shape corresponds to a single frequency. The first vertical bending and torsion mode are the important modes contributed to the occurrence of flutter.

**Table 1. Natural Frequencies**

| No. | Mode shape               | Frequency (Hz) |
|-----|--------------------------|----------------|
| 1   | First Vertical Bending   | 3.2            |
| 2   | First Inplane Bending    | 5.03           |
| 3   | First Torsion            | 8.6            |

### 3.2. Wind Tunnel Test

In the wind tunnel test, impulsive disturbance was applied to the model. The model was impacted by a hammer to create impulsive force and model oscillated freely toward a stable condition afterward. Transient responses of the model is depicted in Figure 5, the disturbance effect to the model was damped out and it returned to the original stable condition.

![Figure 4. Modal Testing of Model](image)

![Figure 5. Transient Response](image)

Modal parameters were extracted at each wind speed. The modal dampings that were extracted at each test point are given in Figure 6(a). The damping data indicates that that one mode is becoming less stable, whereas the other mode is more stable. The critical speed for flutter is happened if the total damping of the model is going to zero [9]. As shown in Figure 6(a), the modal damping is not possible to predict flutter boundary speed. The modal frequencies that were extracted at each test point are given in Figure 6(b). These data shows the indication that the model is experiencing a classical bending-
torsion flutter. The natural frequencies appear to be coalescing, as expected for a classical flutter until a possible coalescence at the airspeed very close to the onset of flutter [10].

![Figure 6(a). Measured Modal Damping](image1)

![Figure 6(b). Measured Modal Frequency](image2)

The flutter boundary is predicted by parameter flutter margin discrete systems ($F_z$). Parameters $F_z$ was determined using Eq.(4). As shown in Figure 7, the curve $F_z$ against dynamic pressure ($q$) is almost linear. Therefore, a linear fit is convenient to predict flutter boundary for this case. The parameters $F_z$ decrease monotonically toward zero with increasing dynamic pressure.

![Figure 7. The parameters $F_z$ against Dynamic Pressures ($q$)](image3)

The parameter $F_z$ has significant linear correlation against dynamic pressure ($q$) that is shown by value correlation coefficient ($R^2$) close to one. The critical speed or dynamic pressure occurs when the value of $F_z$ is zero or the point of fitted line crossing the horizontal coordinate. The Applying extrapolation the fitted line gives the critical pressure dynamic of $q_f = 972.5$ Pa or the wind speed of $v_f = 41.7$ m/s. Based on the experiment, the actual flutter boundary speed is $v_{f\text{act}} = 40.5$m/s, hence the error predicted flutter is $\text{error}=3\%$.

4. Conclusions
The flutter test of a half wing model at Indonesia Low Speed Wind Tunnel (ILST) had been carried out. This experiment has proved that the modal damping and frequency cannot predict flutter boundary.
accurately. The model was experiencing a classical bending-torsion flutter. It seems that the natural frequencies appear to be coalescing, as expected for a classical flutter until a possible coalescence at the airspeed very close to the onset of flutter. The parameter flutter margin for discrete-time systems (FMDS) has a significant correlation against dynamic pressure. Therefore, the parameter FMDS has the better performance for flutter prediction than the modal parameters. The application to a model with two degree freedom that experiencing a classical flutter, the FMDS gives a satisfying result in prediction of flutter boundary on subsonic wind tunnel test.

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