Global Dynamic Surface Control for a Class of Nonlinear System

Zhou Yang*, Dong Wenhan, Liu Zongcheng, Chen Yong, Zhang Wenqian, Feng Haoming, Sun Caihao

Aeronautics Engineering College, Air Force Engineering University, Xi’an 710038, China

*Corresponding author, email: 312201309012@kgd.mtn

Abstract. The traditional dynamic surface control (DSC) method can only guarantee the final boundedness of semi-global consistency of the system. Furthermore, the selection of filter time constant has significant influence on the DSC control performance. This paper presents a global dynamic surface control (GDSC) method, which constructs an automatically-updated filter time constant by using a first-order sliding mode differential estimator. Based on the GDSC method, a tracking controller is proposed for a class of nonlinear systems, and it is proved by the Lyapunov theorem that the controller can guarantee the global uniformly ultimately boundedness (GUUB) of the system.

1. Introduction

In recent years, the backstepping control method proposed by Saberi et al [1] has made great breakthroughs in the field of nonlinear system control. Nevertheless, there are some limitations in the backstepping control method: 1) the control law is highly complex and its complexity increases explosively with the increase of system order or relative order, which is called “explosion of terms”; 2) the tracked signal is generally required to be n-order differentiable, which is too strict in practical application.

In order to overcome the disadvantages of backstepping control method, the DSC method has been proposed by Gerdes [2]. To avoid repeated differentiations of some nonlinear functions in the recursive control design process, a low-pass-filter was used in the backstepping method designing [3,4]. Although the DSC design provides an effective process for the control of nonlinear systems, the existing work has the following limitations: 1) the selection of filter time constant has significant influence on the control performance, which may make the system go unstable if the constant is not choose appropriately; 2) the DSC method can only guarantee the final boundedness of semi-global consistency of the system, which is a trade-off in performance between ease in control law design and implementation, and global stability[5,6].

Based on the above discussion, this article first proposes a GDSC method for a class of nonlinear systems. By introducing a first-order sliding mode differential estimator, an automatically-updated filter time constant is obtained. The GDSC method eliminates restrictions on the initial conditions of system variables, so that the GUUB can be guaranteed.

2. Problem formulation

Consider a class of nonlinear strict-feedback systems given by
\[
\begin{align*}
\dot{x}_i &= x_{i+1} + f_i(x_i) + \Delta f_i(x_i), \\
\dot{x}_n &= g u, y = x_i,
\end{align*}
\]  
(1)

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) denotes the states of the system, \( f_i(x_i) \) is a known continuous function and \( \Delta f_i(x_i) \) is an unknown continuous function \( i = 1, 2, \ldots, n \), and \( g \) is a known positive constant.

The same assumption as [9] is considered as follows:

**Assumption 1** [2]: 
\[
|\Delta f_i(x_i)| \leq \rho_i(x_i)
\]
where \( \rho_i \) is a continuous differentiable function in its arguments.

**Assumption 2**: 
\[
\alpha_i(x_i) \text{ is a smooth function, and } \alpha_i(0, \ldots, 0) = 0.
\]

**Assumption 3**: The desired trajectory \( y_d \) is sufficiently smooth function of \( t \), and \( y_d, \dot{y}_d \) and \( \ddot{y}_d \) are bounded, that is, there exists a positive constant \( B_0 \) such that \( (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \leq B_0 \).

**Lemma 1**[7]: 
\[
|\text{tanh}(x)| \leq 1/2 \text{sign}(x) - \left( x \text{ tanh}\left( \frac{x}{\mu} \right) \right)^{1/2} \text{ tanh}\left( \frac{x}{\mu} \right) \leq \gamma
\]
where \( \mu \) and \( \gamma \) are unknown positive constants.

### 3. Global DSC design

Here, a class of global DSC approach is proposed, and based on this method, an novel adaptive control of the system (1) is proposed.

To start, consider the following change of coordinates:

\[
\begin{align*}
e_i &= x_i - y_d \\
e_j &= x_j - \alpha_{ij}
\end{align*}
\]

where

\[
\tau_{i+1} \dot{\alpha}_{i+1} + \alpha_{i+1} = \alpha_i, \quad \alpha_{i+1}(0) = \alpha_i(0)
\]

By defining the output error of this filter as \( y_{i+1} = \alpha_{i+1} - \dot{\alpha}_i \), it yields \( \dot{\alpha}_{i+1} = -y_{i+1}/\tau_{i+1} \).

**Lemma 3**: Let \( \frac{1}{\tau_{i+1}} \left| \dot{y}_{i+1} \right|^{1/2} + \epsilon_i \), where \( \epsilon_i \) and \( \kappa \) are positive design constants and \( \hat{\alpha}_i \) is the estimate of \( \alpha_i \) and we will define \( \hat{\alpha}_i \) later. Then,

\[
\left| y_{i+1} \right| \leq y_{i+1}^* \leq \gamma
\]

where \( y_{i+1}^* \) is some positive constant.

**Proof**: Consider the candidate:

\[
V_{y_{i+1}} = \frac{1}{2} y_{i+1}^2
\]

The derivative of \( V_{y_{i+1}} \) is

\[
\dot{V}_{y_{i+1}} = y_{i+1} \dot{y}_{i+1} = y_{i+1} (\dot{\alpha}_{i+1} - \dot{\alpha}_i) = -\frac{y_{i+1}^2}{\tau_{i+1}} - y_{i+1} \dot{\alpha}_i
\]

In order to avoid complicated calculation, the following first-order sliding mode differential estimator by using Lemma 1 and [9] is cited to approximate the term \( \dot{\alpha}_{i+1} \):

\[
\begin{align*}
\dot{\rho}_0 &= \xi_{i+1} - \dot{\delta}_0 |\rho_0 - \alpha_i|^{1/2} \text{ sign}(\rho_0 - \alpha_i) + \rho_1 \\
\dot{\rho}_1 &= \dot{\delta}_0 \text{ sign}(\rho_1 - \xi_{i+1}) \quad \text{ where } \dot{\delta}_0, \dot{\delta}_1 \text{ are positive.}
\end{align*}
\]

With the aid of the approximation characteristics of the first-order sliding mode differentiator, we can get
\[ |\zeta_{i0} - \hat{\alpha}_i| \leq \epsilon_{i0} \]  
(9)

where \( \epsilon_{i0} \) is positive.

Define
\[ \hat{\alpha}_i = -\hat{\phi}_0((\rho_{i0} - \alpha_{i0}) \tanh(\frac{\rho_{i0} - \alpha_{i0}}{\mu_i}))^\frac{1}{2} \tanh(\frac{\rho_{i0} - \alpha_{i0}}{\mu_i}) + \rho_i \]  
(10)

where \( \hat{\alpha}_i \) is the approximation of \( \zeta_{i0} \).

We know from Lemma 2 that
\[ |\zeta_{i0} - \hat{\alpha}_i| \leq \epsilon_{i0} \]  
(11)

where \( \epsilon_{i0} \) is any positive constant.

Then we can further have
\[ |\hat{\alpha}_i - \hat{\alpha}_i| \leq |\hat{\alpha}_i - \zeta_{i0} + \zeta_{i0} - \hat{\alpha}_i| \leq \epsilon_{i0} + \epsilon_{i0} = \epsilon_{i0} \]  
(12)

Substituting \( y_{i+1} = \alpha_{i+1} - \alpha_i \) and \( 1/\tau_{i+1} = 2|\hat{\alpha}_i|/\left(|y_{i+1}| + \kappa\right) + \epsilon_i \) into (7), we can have
\[ \dot{y}_{i+1} = -y_{i+1}^2 \left|\frac{2|\hat{\alpha}_i|}{|y_{i+1}| + \kappa} + \epsilon_i\right| - y_{i+1}\hat{\alpha}_i \]  
(13)

Two cases are discussed as follows.

Case 1 \( |y_{i+1}| \leq \kappa \). This directly means the boundedness of \( |y_{i+1}| \) and thus Lemma 3 holds.

Case 2 \( |y_{i+1}| > \kappa \). Then, we have
\[ 1 \leq \frac{2|y_{i+1}|}{|y_{i+1}| + \kappa} \leq 2 \]  
(14)

and thus
\[ -2|\hat{\alpha}_i| \frac{y_{i+1}^2}{|y_{i+1}| + \kappa} - y_{i+1}\hat{\alpha}_i \leq -2|\hat{\alpha}_i| \frac{|y_{i+1}| - y_{i+1}\hat{\alpha}_i}{|y_{i+1}| + \kappa} \leq \epsilon_{i0} |y_{i+1}| \]  
(15)

Substituting (15) into (13), we can further have
\[ \dot{y}_{i+1} = -y_{i+1}^2 \left|\frac{2|\hat{\alpha}_i|}{|y_{i+1}| + \kappa} + \epsilon_i\right| - y_{i+1}\hat{\alpha}_i \leq \epsilon_{i0} |y_{i+1}| \leq -(\epsilon_{i0} - \frac{1}{2}) y_{i+1}^2 + \epsilon_{i0}^2 \leq -C_1 |V_{i+1}| + C_2 \]  
(16)

where \( C_1 = 2\epsilon_{i0} - 1, C_2 = \frac{\epsilon_{i0}^2}{2} \). This means \( |y_{i+1}| \) is bounded. Thus Lemma 3 holds.

**Step** \( i (1 \leq i \leq n-1) \): Denote \( \alpha_{i+1} = y_d \). We have
\[ \hat{\alpha}_i = \dot{x}_i - \dot{\alpha}_d = x_{i+1} + f_i(\bar{x}) + \Delta f_i(\bar{x}) - \dot{\alpha}_d \]  
(17)

Consider the following candidate:
\[ V_i = \frac{1}{2} e_i^2 \]  
(18)

The derivative of \( V_i \) is
\[ \dot{V}_i = e_i \dot{e}_i = e_i (x_{i+1} + f_i(\bar{x}) + \Delta f_i(\bar{x}) - \dot{\alpha}_d) \]  
(19)

From Young’s inequality and Assumption 1
\[ e_i |\Delta f_i(\bar{x})| \leq \frac{e_i^2 \rho_i^2}{2a} + \frac{a}{2} \]  
(20)

Then, we design the virtual control law \( \alpha_i \) as follows:
\[ \alpha_i = -k e_i - f_i(\bar{x}) - \frac{e_i \rho_i^2}{2a} + \dot{\alpha}_d \]  
(21)

Noting \( x_{i+1} = \alpha_{i+1} + \alpha_{i+1} \) and \( y_{i+1} = \alpha_{i+1} - \alpha_i \), we have
Substituting (20), (21) and (22) into (19), we can have

\[ \dot{V} \leq \frac{e_i(x_{i+1} + f_i(x))}{a} \leq -\frac{\dot{\alpha}}{2} - e_i + e_i(x_{i+1} + y_{i+1}) + \frac{a}{2} \]

(23)

**Step n:** Noting that \( e_i = x_i - \alpha_i \) and \( \dot{x}_i = Gu \), we have

\[ \dot{e}_i = \dot{x}_i - \dot{\alpha}_i = gu - \dot{\alpha}_i \]

(24)

Consider the following quadratic Lyapunov function candidate:

\[ V_n = \frac{1}{2} e_n^2 \]

(25)

The time derivative of \( V_n \) along (24) is

\[ \dot{V}_n = e_i \dot{e}_n = e_i(gu - \dot{\alpha}_i) \]

(26)

Design \( u \) as

\[ u = g^{-1}(-k_i e_i + \alpha_i) \]

(27)

Substituting (27) into (26), we get

\[ \dot{V}_n = e_i \dot{e}_n = -k_i e_i^2 \]

(28)

**4. Stability analysis**

Notice that the global boundedness of \( y_{i,1} \) has been proved in Lemma 3, therefore we consider the Lyapunov function as follows

\[ V = \sum_{i=1}^{n} V_i \]

(29)

We have the following theorem for system (1) with the GDSC method.

**Theorem 1:** Consider the nonlinear system (1) under Assumptions 1-3. The first order sliding mode differentiator is designed as (8), (10). The virtual control laws are determined as (21), and the actual adaptive controller is constructed by (27). There exist \( k_i, a_i, \dot{0}_i, \dot{1}_i, \) and \( e_{i,1} \) which can make that:

1) all the signals in the system are GUUB;
2) \( e_i \) can converge within an arbitrarily small range

**Proof:**

1) Take the derivative of \( V \) with respect to time:

\[ \dot{V} = \sum_{i=1}^{n} \dot{V}_i \leq \sum_{i=1}^{n} [-k_i e_i^2 + e_i(e_{i,1} + y_{i,1}) + \frac{a}{2}] - k_i e_i^2 \]

(30)

By using Young’s inequality, we have

\[ V \leq -\sum_{i=1}^{n} k_i e_i^2 + \sum_{i=1}^{n} e_i^2 + \frac{\sum_{i=1}^{n} e_{i,1}^2}{2} + \frac{a}{2} \leq \sum_{i=1}^{n} (k_i - \frac{3}{2}) e_i^2 + \sum_{i=1}^{n} \frac{e_{i,1}^2}{2} + \frac{a}{2} \]

(31)

According to Lemma 3, (31) can be further written as

\[ V \leq \sum_{i=1}^{n} (k_i - \frac{3}{2}) e_i^2 + \sum_{i=1}^{n} \frac{e_{i,1}^2}{2} + \frac{a}{2} \leq -c_1 V + c_2 \]

(32)

which implies that

\[ V(t) \leq (V(0) - c_1) e^{-c_1 t} + c_2 \leq V(0) + c_3 \]

(33)

where \( c_1 = \min_{i=1,2,\ldots,n} \{k_i - \frac{3}{2} \} \), \( c_2 = \sum_{i=1}^{n} \frac{e_{i,1}^2}{2} + \frac{a}{2} \) and \( c_3 = c_2 / c_1 \) are positive constant.

It is noticeable that \( c_1 \) can be made arbitrarily small by reducing \( a \) and meanwhile increasing \( k_i \).

For \( e_i = x_i - y_d \) and \( y_d \) being bounded, \( x_i \) is bounded. Since \( \alpha_i \) is a function of bounded signals \( e_i, y_d, \) and \( y_{i,1}, \alpha_i \) is also bounded.

2) Since \( V_i = e_i^2 / 2 \), by using the first inequality in (33), we have
\[
\lim_{t \to \infty} |e_i| \leq \lim_{t \to \infty} \sqrt{2F(t)} \leq \sqrt{2\epsilon_i}
\]  

(34)

Note that \(\epsilon_i\) related to \(k_i, a, \delta_0, \delta_1\) and \(\epsilon_i\). Therefore, by appropriately adjusting the design parameters, the tracking error can be adjusted to a smaller range.

Proof completed.

5. Simulation results

Consider the following nonlinear system:

\[
\begin{align*}
\dot{x}_1 &= 5x_1 + x_2 + x_1 \cos(x_1) \\
\dot{x}_2 &= 5t_{\text{ref}} \quad y = x_1
\end{align*}
\]  

(35)

with \(\delta_0 = 100, \delta_1 = 1, \epsilon_{11} = 1\), and the virtual and actual controllers are designed as (21), (27) with \(k_1 = 5\), \(k_2 = 1\) and \(a = 1\).

For comparing with traditional DSC method, we have designed the DSC controller according to [2] with the same parameters \(k_1 = 5, k_2 = 1,\) and \(a = 1\). The filter time constant is chosen as \(\tau = 0.17\). Let \(y_d = \sin(t), y_r(0) = 0\).

In order to demonstrate the advantages of our method, we set two cases to compare the GDSC with DSC method:

Case 1: \([x_1(0) \ x_2(0)]^T = [0 \ 0]^T\); Case 2: \([x_1(0) \ x_2(0)]^T = [20 \ 0]^T\).

The simulation results are shown in Figs. 1-3 in Case 1 and Figs. 4-6 in Case 2. It can be obviously observed from Figs. 1-3 that the GDSC method can not only guarantee the global uniformly ultimately boundedness (GUUB) of all the signals in system, but also obtain the better tracking performance than DSC method. It can be seen from Figs. 4-6 that, even if the initial states of the system is very large, the GDSC method can still make the system stable in a short time and approximate the desired trajectory with a small error, while the DSC method is difficult to make the system stable in a short time.

6. Conclusions

This paper proposes a GDSC method based on the idea of DSC method. With the help of the first-order sliding mode differential estimator, an automatically-updated filter time constant is obtained, which make the filter error render to arbitrary small globally. Then a novel controller is proposed for a class of nonlinear systems. Finally, from the results, it is ensured that the signal in the closed-loop system is...
GUUB, and it is proved that the system output converges to a smaller neighborhood of the desired trajectory. The results prove the feasibility and effectiveness of our method.

Acknowledgement
This work was supported by Natural Science Basic Research Program of Shaanxi (Program No. 2019JQ-711).

References
[1] Saberi A, Kokotovic P V, Sussnam H J. Global stabilization of partially linear composite systems. SIAM Journal on Control and Optimization, 1990, 128(6): 1491–1503.
[2] Swaroop D, Hedrick J K, Yip P P. Dynamic surface control for a class of nonlinear systems. IEEE Transactions on Automatic Control, 2000, 45(10): 1893-1890
[3] Liu Z C, Dong X M, Xue J P, Li H B, Chen Y. Adaptive neural control for a class of pure-feedback nonlinear systems via dynamic surface technique. IEEE Trans. Neural Netw. Learning Syst. 2016; 27(9): 1969-1975.
[4] Liu Z C, Dong X M, Xie W J, Xue J P, Li H B. Adaptive fuzzy control for pure-feedback nonlinear systems with non-affine functions being semi-bounded and in-differentiable. IEEE Trans. Fuzzy Syst. 2018; 26(2): 395-408.
[5] Wang D, Huang J. Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form. IEEE Transactions on Neural Networks, 2005, 16(1): 195-202.
[6] Yoo S J, Jin B P. Neural-network-based decentralized adaptive control for a class of large-scale nonlinear systems with unknown time-varying delays. IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics, 2009, 39(5): 1316-1323.
[7] Zhou, Y; Dong, W; Dong, S; Chen, Y; Zuo, R; Liu, Z. Robust Adaptive Control of MIMO Pure-Feedback Nonlinear Systems via Improved Dynamic Surface Control Technique. IEEE Access. 2019; 7: 96672–96685.
[8] Chen M, Ge S S, How B V. Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities. IEEE Trans. Neural Netw. 2010; 21(5): 796-812.
[9] Leventa A. Robust Exact Differentiation via Sliding Mode Technique. Automatica 1998; 34(3): 379-384.