Simple mechanism that breaks Hall effect linearity at low temperatures

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Hall resistance $R_{xy}$ is commonly suggested to be linear-in-magnetic-field $B$, provided the field is small. We argue here that at low temperatures this linearity is broken due to weak localization/antilocalization phenomena in inhomogeneous systems, while in a uniform medium these effects do not affect the linear field dependence of $R_{xy}(B)$. We calculate the Hall resistance for different two-component media using a mean-field approach and show that this non-linearity is experimentally observable.

Hall resistance $R_{xy}$ is broadly used to calculate the charge carriers density $n$ since commonly accepted that $R_{xy} \propto 1/n$. The latter relation assumes that $R_{xy}$ depends linearly on the value of the transverse magnetic field $B$ and the carrier density $n$ is usually estimated at some fixed small magnetic field. In other words, one assumes that the Hall coefficient $R_{xy}/B$ is independent of $B$ in the low magnetic field range. However, there are a number of observations that the Hall coefficient at low temperatures and at low magnetic fields depends on $B$ even in the systems without magnetic impurities [1–6]. This means, in particular, that the value of the charge carriers density determined using Hall resistance may be misleading. Several microscopic models were suggested to explain the dependence $R_{xy}(B)$ in the low magnetic field range. In Ref. [4], this was attributed to the higher-order corrections to the Drude conductivity in $(k_F l)^{-1}$ (here $k_F$ is the Fermi momentum and $l$ is a mean free path). The memory effect in the electron scattering could also be a reason for the dependence of $R_{xy}$ on $B$ in low field, as it has been shown in Ref. [7]. A non-linearity in $R_{xy}(B)$ due to superconducting fluctuations was suggested in Ref. [8].

In this paper we suggest a different mechanism of $R_{xy}(B)$ non-linearity for two-dimensional (2D) systems, which also can be valid for 3D systems close to the metal-insulator transition. This mechanism is simple and rather general. We argue that due to tensor nature of the magnetoresistivity, the observed non-linearity directly follows from the weak localization/antilocalization if the system is inhomogeneous.

Weak localization and antilocalization phenomena, that is, quantum interference effects, lead to a steep low-field magnetoresistance. Let us consider a 2D isotropic system in the limit of low temperatures and low transverse magnetic field $\mu B \ll 1$, where $\mu$ is the mobility measured in the inverse Tesla. According to Hikami-Larkin-Nagaoka formula [9], magnetoresistivity due the weak localization [that is, $\Delta \sigma(B) = \sigma(B) - \sigma(0)$] for a uniform system can be expressed as

$$\Delta \sigma(B) = \alpha \frac{e^2}{2\pi^2 h} \left[ \psi \left( \frac{1}{2} \right) + \frac{h}{4eB_l \phi} \right] - \ln \left( \frac{h}{4eB_l \phi} \right)$$

(1)

Here $\psi$ is the digamma function, $e$ denotes elementary charge, $\alpha$ is some constant (typically from $-1$ to $1$), and $l_\phi$ is the phase breaking length, which typically is proportional to $\sqrt{k_F l}/T$, see Ref. [10].

In Refs. [11] and [12] it was shown that the quantum correction Eq. (1) does not contribute to the Hall effect since off-diagonal terms of the resistivity tensor are independent of $\Delta \sigma(B)$. In particular, in the 2D case corresponding resistivity tensor has a form

$$\hat{\rho} = \begin{pmatrix} 1/[(\sigma(0) + \Delta \sigma)] & -B/ne \sigma(0) \Delta \sigma/(\sigma(0) + \Delta \sigma) \\ -B/ne \sigma(0) \Delta \sigma/(\sigma(0) + \Delta \sigma) & 1/[(\sigma(0) + \Delta \sigma)] \end{pmatrix},$$

(2)

where $\sigma(0) = ne \mu$. We may introduce a modified field-dependent mobility

$$\tilde{\mu}(B) = \mu + \frac{\Delta \sigma(B)}{ne},$$

(3)

and write down the conductivity tensor (inverted resistivity) in a standard Drude form

$$\sigma = \frac{ne \tilde{\mu}^2}{1 + \tilde{\mu}^2} \left( \begin{array}{cc} 1 & \tilde{\mu}B \\ -\tilde{\mu}B & 1 \end{array} \right).$$

(4)

We consider here a 2D inhomogeneous medium. For simplicity, we assume that it consists of two species (or phases) of a 2D electron gas with different densities and mobilities, $(n_1, \mu_1)$ and $(n_2, \mu_2)$. In general, the mobilities $\mu_1$ and $\mu_2$ varies differently with the magnetic field. Therefore, the transport current redistributes between these species when the applied magnetic field changes. As a result, the Hall coefficient becomes field-dependent. In general, a solution of the problem of a current flow redistribution in a two-component media is rather complicated. However, in some special cases an exact or approximate analytical result can be obtained.

The simplest case of the inhomogeneous system is an array of strips and the current flow parallel to them, as shown in Fig. 1. The electric field along the current flow is the same in both components of the system (dark and light gray strips). Summing up Hall voltages for all strips and dividing the result by the total current, we get

$$R_{xy} = \frac{B}{e} \frac{p \mu_1 + (1-p) \mu_2}{pm_1 \mu_1 + (1-p)n_2 \mu_2},$$

(5)
Here $p$ denotes the fraction of the dark gray component ($n_1, \mu_1$). According to this formula, the Hall coefficient $R_{H1}(B)$ is not a constant in the low magnetic fields if $\mu_1$ and $\mu_2$ are field-dependent.

Another model system that we consider here is a regular array of circular inclusions in the conductive matrix, as shown in Fig. 1b. The carrier density in the inclusion is $n_1$ and mobility is $\mu_1$, while in the matrix these values are $n_2$ and $\mu_2$. To obtain an approximate solution to this model, we apply a self-consistent mean-field theory [13, 14]. In this approach we approximate each element of the array as a circular unit consisted of the inclusion in the center covered by the matrix shell. We place this unit in the media with an effective conductivity tensor, solve corresponding electromagnetic problem, average the calculated electric field self-consistently, and obtain mean-field equations for the effective conductivity. Details of such calculations are described in details in Appendix A. The obtained result can be presented in the form

$$R_{xy} = \frac{B}{n_2 e} \frac{S^2 + 2p[2n_1 n_2 \mu_1 (\mu_1 - \mu_2) - D^2] + p^2 D^2}{(S - pD)^2}, \quad (6)$$

where $D = n_2 \mu_2 - n_1 \mu_1$, and $S = n_1 \mu_1 + n_2 \mu_2$. The mobilities in the circular islands, $\mu_1$, and in the remaining 2D gas, $\mu_2$, are calculated using Eq. (5).

We apply the same mean-field approach to calculate the effective conductivity tensor in the case of random mixture of two 2D electron phases. We approximate the regions with different conductivities by a circular inclusions with different radii. The analytical solution to the problem is tremendous and we present it only in Appendix B with the details of calculations. Note, that the mean-field theory usually gives a good result, when the fraction content $p$ is not close to percolation threshold $p_c$ [13, 14]. For isotropic 2D systems $p_c = 0.5$, while for isotropic 3D structures $p_c \approx 0.15$.

Now let us analyze a possible value of non-linearity of the Hall resistance due to system inhomogeneity. This non-linearity arises due to the difference in the carriers’ mobility in different parts of the sample. The larger is this difference, the greater would be the non-linearity. Naively, according to Eqs. (1) and (6) there is no limitation on relative mobility variations with the magnetic field. Indeed, the effect would be high, if the system contains low-$n$ regions, since correction to the mobility is inversely proportional to $n$, see Eq. (6). However, in the low-$n$ regions the conductivity itself is low, then, the higher order corrections in $(ne\mu)^{-1}$ come into play, and suppress the magnetoresistance [15]. As a result, a realistic estimate of the relative variation of $\mu$ due to weak localization could be maximum 50% or so for 2D systems and for 3D systems, as well [3, 16].

Note here, that Eq. (1) describes magnetic-field induced dephasing in the diffusive limit of long interference loops. This formula is valid only in the low magnetic field limit $B \ll \hbar/(e l^2) \ll 1/\mu$. In higher magnetic fields, $B \sim \hbar/(e l^2)$, the logarithmic asymptotics of Eq. (1) should be replaced by $\propto 1/\sqrt{B}$ [17, 18]. In this regime, called ballistic, there is no a simple analytical expression for the magnetoconductivity. Below, we use Eqs. (1) and Eq. (6) for the qualitative analysis of the non-linear Hall effect.
The magnetic field dependence of the mobility and the Hall coefficient for the outlined above three inhomogeneous systems (parallel layers, ordered and disordered circular inclusions) are illustrated in Fig. 2. We assume that at zero magnetic field the mobilities in both fractions are equal, μ₁ = μ₂, while the carrier density are different and n₁ = n₂/5. This difference in the charge carrier densities gives rise to difference in the phase breaking length \( l_\phi \) by a factor \( \sqrt{5} \). We also put \( p = 0.3 \) for all calculated curves in Fig. 2. The effect of the weak localization on the charge mobility in two different fractions of the considered systems is illustrated in Fig. 3. The values \( \mu_1 \) and \( \mu_2 \) are calculated using Eqs. (1) and (3). The variation of the mobility with the magnetic field in the low-\( n \) phase is appreciably stronger than in the high-\( n \) one. Correspondingly, the current density increases in the low-\( n \) phase with an increase of \( B \). The current flow pattern varies giving rise to the dependence of the Hall coefficient on the magnetic field, as illustrated in Fig 2.

As it is seen from Fig. 2, the Hall coefficient increases with the growth of \( B \). This increase is higher for the systems with the circular inclusions than for the parallel strips. This behavior has a simple physical explanation. In the system with parallel strips the transport current mainly passes through the highly-conductive strips since they occupy 70% of the sample and their conductivity is five times higher. As a result, the increase of the conductivity of low-conductive phase with the growth of \( B \) results in a slight redistribution of transport flow. In the system with circular inclusions, the transport current flows mainly around the low-\( n \) inclusions. This way is long, and as the conductivity of the low-\( n \) phase regions drops with magnetic field, the path though them becomes more preferable, the fraction of transport current in the low-\( n \) islands increases giving rise to a more pronounced change in the Hall coefficient. The effect is the strongest in the structure with ordered circular inclusions.

A similarity of the obtained results for three different model systems implies that the particular type of inhomogeneity is not crucial for a qualitative picture of the effect.

In the special case \( p = 0.5 \), the Hall resistance in Eq. (5) is similar to that for a two-liquid model. Within this model, the existence of two different types of charge carriers with different conductivities \( \sigma^{(1)} \) and \( \sigma^{(2)} \) is postulated and the conductivity tensor of the system is the sum \( \sigma = \sigma^{(1)} + \sigma^{(2)} \). The two-band model is used to describe multiband systems, e.g., doped topological insulators [19], semimetals [20, 21], etc. A similar model can be also applicable for a bilayer shown schematically in Fig. 4. A question is, why the non-linearity of the Hall resistance is not observed in all inhomogeneous and multicomponent electron systems? We believe that a crucial factor is a scattering between different types of charge carriers. Indeed, as it was shown for multiband compounds [22], multivalley systems [23, 24], and topological insulators [25], if we take into account the scattering between different types of the quasi-particles, a multicomponent system effectively reduces to a single-component one from the viewpoint of the weak localization/antilocalization.

The macroscopic spatial separation of different phases guarantees that the weak localization occurs in them independently justifying our approach. If a typical scale of the inhomogeneity is comparable or smaller than the electron wave function phase breaking length \( l_\phi \), the weak localization in different parts of the inhomogeneous system can not be considered as independent, since the charge carrier passes through the regions with different mobilities and electron densities during dephasing time. This effect results in the diminishing of the corrections to the Hall coefficient due to the system inhomogeneity. This effect is especially important when \( T \to 0 \). However, further microscopic study of the weak localization is necessary since dephasing itself depends on the system inhomogeneity [26].

Another reason why there are no many reported manifestations of the magnetic field dependence of the Hall coefficient is a so-called “a textbook paradigm”, which unequivocally affirms that the low-field Hall voltage is linear in \( B \). Common methods of the Hall effect study in low magnetic field include: measurements in a fixed field \( \pm B \) with subsequent anti-symmetrization of the results; a sample rotation in a constant magnetic field [28]; low-amplitude AC-technique with subsequent averaging the signal over a small-field range [29]; a simple linear extrapolation of \( R_{xy}(B) \) dependence. It is worth to mention, that the magnetic field dependence of \( R_{xy}(B) \) is visually indistinguishable from the straight line even if \( B \) changes by several tens %. As a result, the low-field Hall non-linearity was reported in a few experiments when this effect was looked for intentionally [11, 30]. One of the goals of this paper is to motivate experimentalists to look for the low magnetic field non-linearity in the Hall effect.

The above considerations are not applicable to the Nernst effect - a “thermoelectric brother” of the Hall effect - since the correction due to weak localization to the Nernst coefficient is significant even in the homogeneous system [30].

The discussed above mechanism of the Hall effect non-linearity is simple and robust against the system specific details. We believe that it should be widely observed. The main idea is equally applicable to weak localization and antilocalization in 2D and 3D systems. In general, the carrier density fluctuations exist in any system. In order to observe Hall resistance non-linearity due to intrinsic disorder, two conditions should be fulfilled: (i) the spatial scale of the inhomogeneity should be larger than the phase breaking length, (ii) phase breaking length should be larger than electron mean free path for the weak localization to emerge.

The discussed effects could be also observed in the systems with a tendency to formation of spatially inhomogeneous state [31, 33], or structurally non-uniform systems, e.g., like mixture of single-layer and bilayer graphene, which naturally obtained in the chemical vapour deposition growth process [34]. A promising idea is to prepare a tunable 2D inhomogeneous system using independent gate electrodes controlling different parts of the 2D electron gas [35, 36]. Tunability of the system components may strongly enhance the effects under discussion. In particular, we can prepare the components with the carriers having different sign of the charge (electron-hole mixture). In so doing, according Eqs. (4) and (6), we can turn to zero the denominators in Eqs. (4) and (6). This regime is similar to the case of compensated semimetal, in which a
global current redistribution occurs [37].

We believe that the proposed mechanism might be relevant to explain some observations of the low-field Hall effect non-linearity. Indeed, in Ref. [2], this non-linearity was observed in disordered amorphous films of indium oxide, where domains with different properties may form. Refs. [18] are devoted to diluted semiconductors close to metal-insulator transition, where manifestation of the weak localization is especially strong and fluctuations of the dopant concentrations are possible. The sign of the effect in Refs. [1, 3] agrees with what we expect from Fig. 2. In LaAlO$_3$/SrTiO$_3$ interface (see Fig. 2b of Ref. [5]) low-field Hall non-linearity is also observed. The sign of this non-linearity is opposite to that plotted in Fig. 2. However, our explanation may remain reasonable since in this material a weak anti-localization was observed instead of weak localization (see Fig. 2c of Ref. [5]).

Conclusions. In this paper we argue that in inhomogeneous systems the weak localization or antilocalization lead to low-field non-linear magnetic field correction to Hall resistance. This effect arises due to transport current redistribution between components of the inhomogeneous system with the change of the magnetic field. The effect is measurable and can be observed in two- and three-dimensional systems.

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Appendix A: Mean field calculations for a regular two-component media

An approximate analytical expression for the conductivity of the regular array of equivalent circular islands embedded into a conductive matrix can be obtained analytically by means of effective media approach [14]. We consider infinite 2D array of conductive circular islands with radius $R$ and period $d$, see Fig. 5. The input parameters are magnetic field directed perpendicular to the plane of the sample and the conductivity tensors of the islands $\hat{\sigma}_1$ and the residual 2D electron gas with $\hat{\sigma}_2$. The conductivity tensors have a structure given by Eq. 4. We set a boundary condition that DC transport current with given average density $j_0$ flows through the sample. Our goal is to calculate an effective conductivity tensor of the inhomogeneous system $\hat{\sigma}^e$. From the symmetry consideration this tensor must have the following form

$$\hat{\sigma}^e = \begin{pmatrix} \sigma^e_{xx} & \sigma^e_{xy} \\ -\sigma^e_{yx} & \sigma^e_{yy} \end{pmatrix}$$

We have two independent quantities $\sigma^e_{xx}$ and $\sigma^e_{xy}$. These values have to be expressed through $\sigma^{(n)}_{xx}$, $\sigma^{(n)}_{xy}$ of islands (n=1) and the remaining 2D gas (n=2), and and geometrical factor $p = \pi R^2/d^2$ that denotes the fraction of the system that occupied by the islands.

We treat the periodical system within Wigner-Szetz approach, that is, we replace the square unit cell by a circular one, see left panel in Fig 3. This circular unit cell consists of the island with radius $R$ and conductivity $\hat{\sigma}_1$ in the center and the 2D electron gas shell with radius $R_1 = R/\sqrt{2}$ and conductivity $\hat{\sigma}_2$. Following the mean-field approach [13, 14], we put the system unit cell to the effective media with conductivity tensor $\hat{\sigma}^e$. Then, we solve corresponding electromagnetic problem setting the condition that the current density $j$ far from the center of the circular cell is equal to $j_0$. Finally, we use the self-consistency condition that the average transport current density in the unit cell is equal to $j_0$.

In each part of our inhomogeneous system the current conservation condition $\text{div } j = 0$ and Ohm’s law $j = \hat{\sigma}E$ are fulfilled. We introduce the electrical potential $\phi$, where $E = -\nabla \phi$. The potential $\phi$ evidently obeys the Laplace’s equation

$$\Delta \phi = 0,$$  \hspace{1cm} (A2)

where operator $\Delta$ is taken in 2D with the coordinates $x$ and $y$. The solution $\phi$ in different media are matched on the borders using the conditions of continuity of the electrical potential and radial component of the current $j_r$. The solutions must satisfy the boundary conditions $j_r = j_0$ and $j_r = 0$ at $x, y \rightarrow -\infty$ or in terms of the electrical potential

$$\sigma^e_{xx} \frac{\partial \phi}{\partial x} + \sigma^e_{xy} \frac{\partial \phi}{\partial y} = -j_0, \quad \sigma^e_{xy} \frac{\partial \phi}{\partial y} - \sigma^e_{yx} \frac{\partial \phi}{\partial x} = 0, \quad x, y \rightarrow -\infty.$$  \hspace{1cm} (A3)

In the polar coordinates $(r, \theta)$ the solution to Eqs. (A2) and (A3) reads

$$\phi = r(a \cos \theta + b \sin \theta), \quad 0 < r < R,$$  \hspace{1cm} (A4)

$$\phi = \left( d_1 r + \frac{d_2}{r} \right) \cos \theta + \left( c_1 r + \frac{c_2}{r} \right) \sin \theta, \quad R < r < R_1,$$  \hspace{1cm} (A5)

$$\phi = \frac{f \cos \theta + g \sin \theta}{r} - j r \left( \frac{\sigma^e_{xx} \cos \theta + \sigma^e_{xy} \sin \theta}{\sigma^e_{xx} \sigma^e_{yy} - \sigma^e_{xy} \sigma^e_{yx}} \right), \quad r > R_1,$$

where eight constants $a, b, d_1, c_1, f, g$ are determined from eight continuity conditions of $\phi$ and $j_r = j_0 \cos \theta + j_0 \sin \theta$ at the boundaries $r = R$ and $r = R_1$. To express $\hat{\sigma}^e$ through $\hat{\sigma}^{1,2}$ and $p$, we should add self-consistency conditions

$$\hat{\sigma}^e \cdot \mathbf{E} = \begin{pmatrix} j_0 \\ 0 \end{pmatrix}.$$  \hspace{1cm} (A5)
After rather cumbersome but straightforward algebra we derive

\[
\frac{\sigma_{xx}^{(2)}}{\sigma_{xx}^{(1)}} = \left[ \frac{(1-p)(1+\alpha^2) + \beta p(1-\alpha \gamma)}{(1-p)^2(1+\alpha^2) + \beta p[2(1-p) + \beta p]} + \right.
\]

\[
\frac{\gamma \sigma_{xx}^{(2)} - 1}{2} \left[ \frac{2}{1+\gamma^2} \right]^{-1}
\]

\[
\sigma_{xy}^e = -\gamma \sigma_{xx}^e,
\]

\[
\gamma = -\left\{ \frac{\sigma_{xx}^{(2)}}{2\sigma_{xx}^{(1)} + \beta p(1-p)(1+\alpha^2) + \beta p[2(1-p) + \beta p]} \right\}
\]

where

\[
\sigma_{(i)}^{(i)} = \frac{n_i \mu_i e}{1+(\mu_i B)^2}, \quad \sigma_{xy}^{(i)} = \frac{n_i \mu_i^2 Be}{1+(\mu_i B)^2},
\]

\[
\alpha = \frac{\sigma_{(i)}^{(i)} - \sigma_{xy}^{(i)}}{\sigma_{xx}^{(i)} + \sigma_{xx}^{(i)}}, \quad \beta = \frac{2\sigma_{xx}^{(2)}}{\sigma_{xx}^{(1)} + \sigma_{xx}^{(2)}}.
\]

In the low-field limit, \( \mu_1 B, \mu_2 B \ll 1 \), we can neglect quadratic in \( B \) terms, and obtain the Hall resistance in the low temperature and low field limit \( R_{xy} = -\gamma / \sigma_{xx}^e \), where \( R_{xy} \) obeys Eq. (A6).

**Appendix B: Mean field calculations for a random two-component media**

We consider an isotropic random two-component mixture of phases with different conductivity tensors \( \sigma_{(i)}^{(i)} \) and \( \sigma_{(i)}^{(i)} \). We approximate the inclusions of different phases by rings with different radii \( R \). Following the mean-field approach \([13, 14]\), we consider the inclusion with a radius \( R \) with \( \hat{\sigma}_i^{(i)} \) (where \( i = 1, 2 \)) placed in the matrix with the effective conductivity \( \hat{\sigma}_e \). We solve corresponding electric problem and, then, we average the electric field over the sample volume and obtain the self-consistency conditions, similar to that performed in Appendix [A]. Note, the present calculations can be easily generalized on the case of several components with different conductivities.

The solution for the electric potential \( \phi \) for each phase is obtained similar to Appendix [A]. This solution corresponds to a simple case \( R = R_1 \). In so doing, we get

\[
\phi = r(a_i \cos \theta + b_i \sin \theta), \quad r < R,
\]

\[
\phi = \frac{f_i \cos \theta + g_i \sin \theta}{r} - \frac{\mu r [\sigma_{xx}^e \cos \theta + \sigma_{xy}^e \sin \theta]}{\sigma_{xx}^{e2} + \sigma_{xy}^{e2}}, \quad r > R.
\]

The constants \( a_i, b_i, f_i, \) and \( g_i \) are obtained from the matching \( \phi \) and \( j_r \) at \( r = R \). The self-consistency condition remains the same, Eq. (A5), while for the average electric field now we have

\[
\pi R^2 E = p \int_0^R \int_0^{2\pi} d\theta E_1(r) + (1-p) \int_0^R \int_0^{2\pi} d\theta E_2(r), \quad (B2)
\]

where \( E_i \) is the electric field in the phase with conductivity tensor \( \hat{\sigma}_i \).

After rather cumbersome but straightforward algebra, we derive equation system for the components of the effective conductivity

\[
\frac{p(\sigma_{xx}^{(1)} + \sigma_{xx}^e)}{\sigma_{xx}^{(1)} + \sigma_{xx}^e + (\sigma_{xx}^{(1)} - \sigma_{xx}^e)^2} + \frac{(1-p)(\sigma_{xx}^{(2)} + \sigma_{xx}^e)}{(\sigma_{xx}^{(2)} + \sigma_{xx}^e + (\sigma_{xx}^{(2)} - \sigma_{xx}^e)^2)^2} = \frac{1}{2\sigma_{xx}^e}, \quad (B3)
\]

\[
\frac{p(\sigma_{xy}^{(1)} - \sigma_{xy}^e)}{(\sigma_{xx}^{(1)} + \sigma_{xx}^e + (\sigma_{xx}^{(1)} - \sigma_{xx}^e)^2)^2} + \frac{(1-p)(\sigma_{xy}^{(2)} - \sigma_{xy}^e)}{(\sigma_{xx}^{(2)} + \sigma_{xx}^e + (\sigma_{xx}^{(2)} - \sigma_{xx}^e)^2)^2} = 0. \quad (B4)
\]

In the limit of low magnetic field, we can neglect the terms with \( \sigma_{xy}^{(1)} \), which are quadratic in \( B \). In this case we have

\[
R_{xy} = \sigma_{xy}^e / \sigma_{xx}^e. \quad \text{The first of Eqs. (B3) reduces to quadratic one. We solve it and obtain}
\]

\[
R_{xy} = \frac{\sigma_{xy}^e}{\sigma_{xx}^e}. \quad \text{The first of Eqs. (B3) reduces to quadratic one. We solve it and obtain}
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\[
R_{xy} = \frac{\sigma_{xy}^e}{\sigma_{xx}^e}. \quad \text{The first of Eqs. (B3) reduces to quadratic one. We solve it and obtain}
\]
\[\sigma_{xx}^e = (0.5 - p)(\sigma_{xx}^{(2)} - \sigma_{xx}^{(1)}) + \sqrt{(0.5 - p)^2(\sigma_{xx}^{(2)} - \sigma_{xx}^{(1)})^2 + \sigma_{xx}^{(1)} \sigma_{xx}^{(2)}},\]
\[\sigma_{xy}^e = \frac{p\sigma_{xy}^{(1)}(\sigma_{xx}^{(2)} + \sigma_{xx}^{(1)})^2 + (1 - p)\sigma_{xy}^{(2)}(\sigma_{xx}^{(1)} + \sigma_{xy}^{(1)})^2}{p(\sigma_{xx}^{(2)} + \sigma_{xx}^{(1)})^2 + (1 - p)(\sigma_{xx}^{(1)} + \sigma_{xy}^{(1)})^2}.\]

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