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Partial orders on conjugacy classes in the Weyl group and on unipotent conjugacy classes.
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Summary: Let $G$ be a reductive group over an algebraically closed field and let $W$ be its Weyl group. In a series of papers, Lusztig introduced a map from the set $[W]$ of conjugacy classes of $W$ to the set $[G_u]$ of unipotent classes of $G$. This map, when restricted to the set of elliptic conjugacy classes $[W_e]$ of $W$, is injective. In this paper, we show that Lusztig’s map $[W_e] \to [G_u]$ is order-reversing, with respect to the natural partial order on $[W_e]$ arising from combinatorics and the natural partial order on $[G_u]$ arising from geometry.

MSC:
20G07 Structure theory for linear algebraic groups
06A07 Combinatorics of partially ordered sets
20F55 Reflection and Coxeter groups (group-theoretic aspects)
20E45 Conjugacy classes for groups

Keywords:
reductive groups; Weyl groups; conjugacy classes; partial orders

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