The spectrum of light scattered from an extended atomic wave packet is calculated. For a wave packet consisting of two spatially separated peaks moving on parallel trajectories, the spectrum contains Ramsey-like fringes that are sensitive to the phase difference between the two components of the wave packet. Using this technique, one can establish the mutual coherence of the two components of the wave packet without recombining them.

I. INTRODUCTION

The typical operation of a matter-wave interferometer [1,2] involves a beam splitter that separates an incoming atomic beam into a set of states having different center-of-mass momenta, and mirrors which recombine the beams. The atom density of the recombined beams exhibits interference fringes resulting from different phases acquired during free motion along the different arms of the interferometer. In this paper, we address a question of fundamental importance, Is it possible to establish the spatial coherence of the wave packet without recombining the beams?

To accomplish this goal, we propose a method involving Rayleigh scattering. It has already been shown [3] that the scattered signal, integrated over frequency, is not sensitive to the spatial coherence of atom wave packets. A similar conclusion was reached for the correlation properties of the field emitted in spontaneous emission [4]. On the other hand, it was shown that the spectrum of spontaneous emission was sensitive to the spatial form of the wave packet [5]. In effect, if one measures a frequency integrated spectrum, information on the momentum distribution of the atom wave packet is lost. The scattering cross section is then a sum over contributions from each position in the wave packet, with no interference. On the other hand, a measure of the scattered spectrum is equivalent to a measure of the momentum of the atom and all information on the position is lost. In this way contributions from different spatial positions can interfere. Similar conclusions can be reached on the modification of fringe contrast in an atom interferometer resulting from light scattering, but the localization in that case is on the order of a wavelength [3,6,7].

It is shown below that the spectrum of radiation scattered from the atom wave packets in two arms of an interferometer allows one to probe the spatial coherence between the beams. In other words, it is not necessary to recombine the beams to observe the interference between the beams. For a two-peaked initial wave packet, the scattered signal, as a function of the frequency difference between incident and scattered fields, exhibits a type of Ramsey fringe structure.

II. SPECTRUM

We consider the scattering of classical radiation by an atom having center–of-mass wave function \( \psi(r) \) and internal state wave function \( \psi_g \). Radiation from a mode \( \{k_i, \omega_i\} \) of the incident field is scattered into a mode \( \{k_f, \omega_f\} \) of the vacuum field. Scattering occurs via an off-resonant intermediate internal state \( n \) of the atom. The atom remains in its initial internal state following the scattering, but the momentum of the atom changes from \( p \) to \( (p - \hbar q) \), where \( q = k_f - k_i \). If the momentum state wave function is denoted by \( \Phi(p) \), then the scattering cross section is given by the Kramers-Heisenberg expression

\[
\frac{d\sigma}{dn_f} = P(\Delta) d\omega_f dn_f,
\]  
(2.1)

where

\[
P(\Delta) = \int dp |\Phi(p)|^2 \delta(\Delta + \omega_q - q \cdot p/m),
\]  
(2.2)

and
In these expressions, $\Delta = \omega_f - \omega_i$ is the frequency detuning between scattered and incident field modes, $\mathbf{n}_f = \mathbf{k}_f/k_f$, $\mathbf{d}_{gn}$ is a dipole moment matrix element, $\mathbf{e}_f$ and $\mathbf{e}_i$ are polarization vectors, and $\omega_q = \hbar \mathbf{q}^2/(2m)$ is a recoil frequency associated with the change in atomic momentum $\mathbf{q}$. The center-of-mass energy has been neglected in the denominators in Eq. (2.4). The function $P(\Delta)$ determines the scattering spectrum. Since $\int P(\Delta) d\Delta = 1$, the integrated spectrum does not depend on the form of the center-of-mass wave function, in agreement with Ref. [3].

For a one-dimensional wave packet consisting of a superposition of two identical packets $\psi_r(x)$ having extent $a$, phase difference $\phi$, distanced from one another by a large distance $L \gg a$, and having relative momentum $p_0$, i.e. $\psi_r(x) = 2^{-1/2} [\psi_r(x) + e^{-i\phi}e^{ip_0x/\hbar}\psi_r(x-L)]$, the wave function in momentum space is given by

$$\Phi(p) = 2^{-1/2} \{\Phi_a(p) + \Phi_a(p - p_0) \exp [-i(\phi + (p - p_0) L/\hbar)]\},$$

where $\Phi_a(p)$ is the Fourier transform of $\psi_r(x)$ and it has been assumed that $\psi_r(x)$ is real and an even function of $x$. The spectrum is given by

$$P(\Delta) = \frac{m}{2q_x} \{\Phi_a [(\Delta + \omega_q) m/q_x] + |\Phi_a [(\Delta + \omega_q) m/q_x - p_0]|^2$$
$$+ 2\Phi_a [(\Delta + \omega_q) m/q_x] \Phi_a [(\Delta + \omega_q) m/q_x - p_0] \cos [\phi + (\Delta + \omega_q) Lm/\hbar q_x - p_0 L/\hbar]\}$$

In the three-dimensional case, one can choose a double-peaked, Gaussian packet

$$\psi(r) = 2^{-1/2} (2/\pi a^2)^{3/4} A \left\{e^{-r^2/a^2} + e^{ip_0 \cdot r/\hbar - i\phi} e^{-(r-L)^2/a^2}\right\},$$

where

$$A = \left[1 + \cos \left(\phi - \frac{p_0 \cdot L}{2\hbar}\right) \exp \left(-\frac{L^2}{2a^2} - \frac{p_0^2 a^2}{2\hbar^2}\right)\right]^{-1/2}$$

is a normalization factor [Eqs. (2.6) and (2.7) are valid for arbitrary ratios of $L/a$]. For this packet one finds

$$P(\Delta) = \frac{am A^2}{2\sqrt{2\pi} \hbar q} \left\{\exp \left[-\left(\frac{m (\Delta + \omega_q)}{q}\right)^2 \frac{a^2}{2\hbar^2}\right] + \exp \left[-\left(\frac{m (\Delta + \omega_q)}{q} - \frac{p_0 \cdot q}{q}\right)^2 \frac{a^2}{2\hbar^2}\right]\right\}$$
$$+ 2 \exp \left[-L^2 \sin^2(\theta)/2a^2 - \left(\frac{m (\Delta + \omega_q)}{q}\right)^2 \frac{a^2}{2\hbar^2} - \left(\frac{p_0 \cdot q}{q}\right)^2 \frac{a^2}{8\hbar^2} - \frac{m (\Delta + \omega_q)}{q} (p_0 \cdot \mathbf{q})^2 \frac{a^2}{2\hbar^2}\right]$$
$$\times \cos \left[\phi - \frac{p_0 \cdot L+(p_0 \cdot \mathbf{q}) \mathbf{L} \cdot \mathbf{q}}{2\hbar} + \frac{(\Delta + \omega_q) m L \cos(\theta)}{\hbar q}\right]\}$$

where $\mathbf{q} = q/q$ is a unit vector along $\mathbf{q}$ and $\theta$ is the angle between $\mathbf{q}$ and $\mathbf{L}$.

III. DISCUSSION

It is clear from Eq. (2.2) that the spectrum is simply the momentum distribution of the entire wave packet, evaluated at momenta determined by the resonance condition

$$\mathbf{q} \cdot \mathbf{p}/m = \Delta + \omega_q.$$  

(3.1)

For a double peaked wave function, the momentum distribution oscillates as a function of $\mathbf{p}$, and this oscillation can be mapped into the spectrum of the scattered radiation. For the interference term to contribute, it is necessary that $m a^2 / \hbar^2 \lesssim 1$. Let us see how this condition applies in an atom interferometer.

A well-collimated atomic beam is incident on a beam splitter that splits the beam into two momentum components. We can imagine that a momentum difference in the $x$ direction, $p_0 = 2\hbar k$, is produced via frequency controlled Bragg scattering from two counterpropagating fields [3] or some equivalent process. The quantity $k = 2\pi/\lambda$ is the field propagation constant. For the Bragg field to split the beam into two distinct packets it is necessary that

$$\mathbf{q} \cdot \mathbf{p}/m = \Delta + \omega_q.$$  

(3.1)
Two, separated beams, the scattering techniques described above cannot be used to reveal the coherence of the wave packet since the interference term vanishes! A way around this is to apply a second Bragg pulse after the beams are separated. By a proper choice of the Bragg pulse it is possible to return the relative velocity of the split beams to a value \( p_0 = 0 \). The beams will still be spatially separated, but moving on parallel trajectories. As such, one can analyze scattering from a two-peaked packet when the relative momentum \( p_0 = 0 \).

With \( p_0 = 0 \), Eq. (2.8) reduces to

\[
P(\Delta) = \frac{a m A^2}{\sqrt{2\pi} h q} \exp \left[ -\left( \frac{m (\Delta + \omega_q)}{q} \right) \right] \left\{ 1 + \exp \left[ -L^2 \sin^2 (\theta)/2a^2 \right] \cos \left[ \frac{\Delta + \omega_q}{h q} m L \cos (\theta) \right] \right\} \tag{3.2}
\]

Owing to recoil effect \( \Delta \) the spectrum as a whole is shifted from \( \Delta = 0 \) by the recoil frequency \(-\omega_q\). The spectral width \( \gamma \) of the envelope of the signal is of order

\[
\gamma \sim \omega_q/q a
\]

The interference of the two momentum state wave packets represented in Eq. (3.2) translates in frequency space into oscillations having period

\[
\gamma_R = 4\pi \omega_q/q L \cos (\theta). \tag{3.4}
\]

To observe these oscillations, one must have \( \gamma_R < \gamma \) or, equivalently, \( L \cos (\theta) > a \). The oscillations in frequency space have the same structure encountered in Ramsey fringes. The central fringe occurs for \( \Delta = -\omega_q - hq/Lm \cos (\theta) \Delta \phi \). If, instead of a coherent superposition of two spatially separated wave packet components, one had chosen an incoherent sum of two separated wave packets, the interference term would be absent (corresponding to an average over \( \phi \)). Thus, in principle, one can establish the mutual spatial coherence of the spatially separated wave packet components without recombin- ing them.

A similar effect has been predicted previously in the spectrum of spontaneous emission from an extended wave packet. However, in order to resolve the effects related to the size of the wave packet in that case, it is necessary that the width of the atomic wave function in momentum space \( \delta \rho \sim h/a \) be larger than \( m\Gamma \lambda \), where \( \Gamma \) is the upper state decay rate and \( \lambda \) is the wavelength of the transition. This requirement restricts the wave packet size to \( a \lesssim (\omega_k/\Gamma) \lambda \), which is typically much smaller (10\(^{-2}\) to 10\(^{-3}\)) than an optical wavelength.

In our case, the size of the wave packet is limited only by the requirement that the wave packet be coherent over a distance \( a \). The spectral resolution needed to observe the coherence effects represented in Eq. (3.2) is of order \( \omega_q/q a \sim (\lambda/4\pi a) \omega_{2k} \) for backward scattering when \( q = 2k \). Using a well-collimated atomic beam or a released Bose condensate, one finds that a resolution of order \( 2\pi \times 1.0 \) kHz is needed. The experimental challenge is great to say the least. The scattered signal can be detected using heterodyne techniques, but the signal strength is small, the collection angle is small, and long integration times can be anticipated. The signal to noise can be improved if, instead of measuring the scattered spectrum, one adds a probe field and monitors the probe field absorption or index change as a function of probe-pump field detuning.

It may also be possible to reduce the resolution requirements by considering scattering from a multicomponent wave packet rather than a two-peaked wave packet. For example, if one scatters an atomic beam from a resonant standing wave field (resonant Kapitza-Dirac effect) or from a microfabricated structure having grating period \( d \), the momentum space density, \( |\Phi(p)|^2 \), following the interaction consists of a set of narrow peaks centered at \( p = n h q_r \), where \( q_r = 2\pi d / m \) and \( n \) is an integer whose maximum value is determined by the strength of the interaction. For this momentum distribution, one finds from Eq. (2.2) that the spectrum consists of a set of peaks (recoil components) centered at

\[
\Delta = -\omega_q + n q \cdot q_r/m. \tag{3.5}
\]

If atoms are scattered by a standing wave and \( q = q_r \), the recoil components are distanced from one another by \( 2\omega_q \), which for the \( D_2 \) line in Na, is equal to \( 2\pi 208 \) KHz. For \( n > 1 \) the spectral width is correspondingly larger. Note that it is still necessary to have a spectral width of order \( \omega_q/q a \sim (\lambda/4\pi a) \omega_{2k} \) to resolve the interference pattern of Eq. (3.3); the larger resolution quoted above refers to the spectral width of the entire scattered signal.
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