Quantum Groups, Strings and HTSC materials II

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Abstract

Previously we have indicated the relationship between quantum groups [Phys. Lett A272, (2000)] and strings via WZWN models. In this note we discuss this relationship further and point out its possible applications to cuprates and related materials. The connection between quantum groups and strings is one way of seeing the validity of our previous conjecture [i.e. that a theory for cuprates may be constructed on the basis of quantum groups]. The cuprates seem to exhibit statistics, dimensionality and phase transitions in novel ways. The nature of excitations [i.e. quasiparticle or collective] must be understood. The Hubbard model captures some of the behaviour of the phase transitions in these materials. On the other hand the phases such as stripes in these materials bear relationship to quantum group or string-like solutions. One thus expects that the relevant solutions of Hubbard model may thus be written in terms of stringy solutions. In short this approach may lead to the non-perturbative formulation of Hubbard and other condensed matter Hamiltonians. The question arises that how a 1-d based symmetry such as quantum groups can be relevant in describing a 3-d [spatial dimensions] system such as cuprates. The answer lies in the key observation that strings which are 1-d objects can be used to describe physics in \(d\) dimensions. For example gravity [which is a 3-d [spatial] plus time] phenomenon can be understood in terms of 1-d strings. Thus we expect that 1-d quantum group object induces physics in 2-d and 3-d which may be relevant to the cuprates. We present support for our contention using [numerical] variational Monte-Carlo [MC] applied to 2d d-p model. We also briefly discuss others ways to formulate a string picture for cuprates, namely by exploiting connection between gauge theories and strings and t’Hooft picture of quark confinement.
One of the central points of our contention is:

- Relevant charge degrees of freedom are 1-d.
- The spin degrees of freedom also exhibit a 1-d behavior.

Moreover the phonon-electron coupling can be treated in terms of a 1-d model, such Hubbard Holstein. The 1-d behavior of magnetic fluctuation was predicted by our theory before the experiment! The fluctuations associated with charge were regarded as 1-d, whereas magnetic fluctuations were regarded as 2-d. However it was predicted and experimentally shown that magnetic fluctuations are also 1-d. Thus in our scenario all relevant degrees of freedom are 1-d instead of quasiparticle in the sense of Landau. Superconductivity arises as a dressed stripe phase. We think it may be possible to write down transformations equivalent [such as one writes in BCS, i.e. Bogoliubov] which can map the 1-d states onto the 3-d superconducting phase. Let us now discuss some of the rationale behind our thinking.

In a previous work one of us has advanced the conjecture that one should attempt to model the phenomena of antiferromagnetism and superconductivity by using quantum symmetry group. Following this conjecture to model the phenomena of antiferromagnetism and superconductivity by quantum symmetry groups, three toy models were proposed, namely, one based on SO_q(3) the other two constructed with the SO_q(4) and SO_q(5) quantum groups. Possible motivations and rationale for these choices were outlined. In a model to describe quantum liquids in transition from 1d to 2d dimensional crossover using quantum groups was outlined. In the classical group SO(7) was proposed as a toy model to understand the connections between the competing phases and the phenomenon of pseudogap in High Temperature Superconducting Materials [HTSC]. Then we proposed in an idea to construct a theory based on patching critical points so as to simulate the behavior of systems such as cuprates. To illustrate our idea we considered an example discussed by Frahm et al., . The model deals with antiferromagnetic spin-1 chain doped with spin-1/2 carriers. In the connection between Quantum Groups and 1-dimensional [1-d] structures such as stripes was outlined. The main point of is to emphasize that 1-d structures play an important role in determining the physical behaviour [such as the phases and types of phases these materials are capable of exhibiting] of cuprates and related materials.

The question arises that how a 1-d based symmetry such as quantum groups can be relevant in describing a 3-d [spatial dimensions] system such as cuprates. The answer lies in the key observation that strings which are 1-d objects can be used to describe physics in d dimensions. For example gravity [which is a 3-d [spatial] plus time] phenomenon can be understood in terms of 1-d strings. Thus we expect that 1-d quantum group object induces physics in 2-d and 3-d which may be relevant to the cuprates. However we like to point out that the notion of string is more general and does not have to be restricted to quantum groups. The reason for our choice are several. For one quantum groups are intimately connected with WZWN models and to strings. In turn WZWN based models are relevant to disordered systems. The exact solution of Hubbard model in 1-d is governed by quantum
group symmetry. In turn the Hubbard model captures much of the physics of cuprates. We note that the t-J model and its various generalizations are also advocated for the study of cuprates. However the t-J model is but a limit of Hubbard model \([i.e. \ U/t \rightarrow \infty]\).

Quantum critical points may be naturally understood in terms of quantum groups.

Let us briefly comment on three different general aspects of the cuprates and related materials:

- **Statistics and fractionalization:**
  The cuprates seem to exhibit exotic statistics and charge fractionalization. Indications for electron fractionalization from Angle-Resolved Photoemission Spectroscopy \([ARPES]\) have been reported. We have suggested to measure fractionalization by using SET based experiments \([9]\).

  We note that the Fermi liquid is characterized by sharp fermionic quasiparticle excitations and has a discontinuity in the electron momentum distribution function. In contrast the Luttinger liquid is characterized by charge \(e\) spin 0 bosons and spin 1/2 charge 0 and the fermion is a composite of these [i.e. fractionalization]. It is well-known that transport properties are defined via correlation functions. The correlation functions of a Luttinger liquid have a power law decays with exponents that depend on the interaction parameters. Consequently the transport properties of a Luttinger liquid are very different from that of a Fermi liquid. Photoemission experiments on Mott insulating oxides seems to indicate the spinon and holon excitations of a charge Luttinger liquid. However the experimental signatures of Luttinger liquid are not totally convincing. To this end we propose SET based experiments to determine the Luttinger liquid behaviour of the cuprates.

- **Mixed dimensionality:**
  The cuprates and related materials seem to exhibit mixed dimensionality. This can be seen in many materials. We give an example of \(Ca_{2+x}\ Y_{2-x}\ Cu_5\ O_{10}\ [11]\, a\ material\ which\ simple\ [i.e. simplicity\ in\ structural\ aspect\ and\ controlled\ hole\ doping]\ and\ exhibits\ mixed\ dimensionality\ and\ can\ provide\ insight\ into\ the\ spin\ and\ charge\ dynamics\ of\ more\ complicated\ HTSC\ material.\ As\ a\ consequence\ of\ mixed\ dimensionality\ this\ material\ exhibits\ ferromagnetism\ and\ antiferromagnetism,\ introduction\ of\ holes\ leads\ to\ novel\ spin-charge\ dynamics\ in\ this\ magnetically\ frustrated\ system.

- **Phases:**
  A variety of phases have been reported in cuprates, for example, antiferromagnetic insulator, superconducting, metallic, strange metal, spin-glass, and insulating.

*After about a week or so later of our submission a paper by Sentil and Fisher “Detecting fractions of electrons in the high-\(T_c\) cuprates”\[cond-mat/0011345\] appeared without reference to our work. Some mathematical details were worked out but it used or was in line with basically our idea.
It has been known that the symmetry of Hubbard model is $SU(2) \times SU(2)/Z_2 \equiv SO(4)$, this can be taken as a motivation to consider a gauge model or NLSM for Hubbard based on $SU(2)$, $SU(2) \times SU(2)$, $SO(4)$ or even $Z_2$ or many combinations and generalizations thereof depending on one’s point of view. We take the following point of view. It is known that quantum groups in the context of usual HH are tied to 1-d. This is used as an argument against the application of quantum group to higher dimensions. We take the opposite attitude. Using the string notion instead of going from 1-d, to 2-d via the usual route, we assume that fundamental entities are 1-d objects [subject to quantum group symmetry] i.e. strings, for example we introduce the notion of Hubbard string. And attach the known symmetry of Hubbard Hamiltonian i.e. $SU(2) \times SU(2)/Z_2 \equiv SO(4)$ for example as a Chan-Paton factor. We conjecture that the solution of 2-d and 3-d Hubbard Hamiltonian can be written in terms of the 1-d Hubbard string solutions. This conjecture can be tested by looking at specific examples. We thus have provided a general framework for strongly correlated electron systems, such as Quantum Hall systems and HTSC materials and related materials. The recent data supports our earlier conjecture of important role of 1-d systems tied to quantum groups.

P.W. Anderson and others have tried to prove spin charge separation in 2-d. In our framework this is not necessary and follows naturally as a consequence of the Hubbard string. It is clear that spin charge separation takes place on the 1-d Hubbard string, the dynamics of the string leads to spin charge separation in 2-d and restricted confinement as appears in the form of stripes observed in experiment. The ARPES data shows a significant Fermi surface with hotspots, this cannot be easily explained in terms of Fermi Liquid or arranged non Fermi Liquid behaviour. The collective dynamics of 1-d strings can provide a suitable explanation. For example one can interpret that the holes Fermi surface are the regions where the collective or luttinger liquid-like behaviour dominates. Let us try to see this point intuitively, string theory has several conservation laws, which follows due to various symmetries. Now some of these symmetries are broken in the target space, for example in context of gravity conformal invariance is broken in the target space by gravity and preserved on the 2-d surface generated by string and yet it is the condition of conformal invariance on string generated surface that gives Einstein’s equation coupled to a scalar field in the target space. Thus in an analogous manner one may have obtain a situation where Luttinger liquid symmetry is preserved in some regions of the Fermi surface and badly broken in others. As the Hubbard strings fluctuate they generate a Fermi surface where Luttinger correlations are preserved in only some regions. The area or volume where non-Fermi liquid persists is

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† Chan-Paton factor:- An open string has boundaries, i.e. endpoints, in quantum with distinguished endpoints it is natural to associate degrees of freedom with these in addition to fields propagating in bulk. Moreover it is natural and necessary to have simply boundary conditions. For example in string theory of strong interactions, one introduces SU(3) flavor quantum numbers, the endpoints have quark-antiquark attached connected by ‘color-electric flux tube
expected to be proportional to the number of strings, which is turn is determined by physical parameters such as doping etc. A challenging problem is to define precisely a measure for the “correlations” in the system.

In short we expect that by formulating the underling theory in terms of collective excitations such as strings [which are indicative of Luttinger liquids] we can represent a general theory for strongly correlated systems in 2-d and 3-d. Yet another support for our point of view comes from data of Uemura, where he finds 2-d bose like behaviour, and these fit reasonably with Berezinskii-Kosterlitz-Thouless [BKT] type of phase transitions. Within our framework this is natural since conformal invariance is intimately tied with strings. It would be interesting to look into the details of the relationship between Uemura’s data and the BKT-type phase transitions.

We now comment briefly on the choice of Hamiltonian and naively on one kind of duality in this context. As already mentioned and as is well-known the t-J model is a special limit of Hubbard model. Keeping this point in mind we want to seek a non-trivial unification of Hubbard Hamiltonian and the t-J model using ‘duality’ of couplings in a new way. To achieve this unification we start with a small step. Let us explain. To this end let us first recall the usual argument based on perturbation theory which allows us to see the t-J model as the special limit of the Hubbard Hamiltonian [HH]. The HH reads

\[ H = -t \sum_{<ij>\sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}, \]  

(1)

where \( <ij> \) means sum over nearest neighbours, i.e. hybridization between neighbouring atoms. \( c_{i\sigma}^\dagger \) (\( c_{i\sigma} \)) are the creation (annihilation) operators of electrons at site \( i \) of spin \( \sigma \) (\( \sigma \equiv \uparrow, \downarrow \)). An important limit of HH is half-filled and strong coupling , i.e. \( U \gg t \).

The intermediate state has one doubly occupied atom and the effective interaction [second-order] is simply

\[ H = t \sum_{<ij>\sigma'} c_{i\sigma'}^\dagger c_{j\sigma'} - \frac{1}{U} \sum_{\sigma} c_{i\sigma}^\dagger c_{j\sigma}, \]  

(2)

which is the Hamiltonian of antiferromagnetically coupled spin-\( \frac{1}{2} \) Heisenberg model with coupling \( J = 2t^2/U \) per bond. Naively one can regard in some sense the Hamiltonian in Eq. 2 as a ‘dual’ with respect to the coupling \( U \) of the Hamiltonian in Eq. 1 This argument is made in the context of degenerate perturbation theory and the assumptions stated above. We now abandon any assumption of perturbation theory and simply assume to start with a Hamiltonian which is the sum of the above two Hamiltonians, i.e.

\[ H = -t \sum_{<ij>\sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\[ + J \sum_{<ij>\sigma'} c_{i\sigma'}^\dagger c_{j\sigma'} - \sum_{\sigma} c_{i\sigma}^\dagger c_{j\sigma}, \]

\[ J = -g^2/U, \]  

(3)
where $g$ is some suitable coupling.

In our scenario of HTSC theory we consider superconductivity arising from the dressing of 1-d stripe or string phase. The stripe arises due to a line of quantum critical points. This is supported by Monte-Carlo calculations [among other reasons] of 2D d-p Hubbard model where stripes and superconductivity exists, see below. The correlation length in HTSC material is short compared to their conventional cousins, since here superconductivity arises due to 1-d stripes.

In order to prove our conjecture we must consider realistic Hamiltonians which exhibit stringy or stripe solutions even in the superconducting state. One such Hamiltonian is the 2d d-p model \[^{12}\],

$$
H = \varepsilon_d \sum_{i, \sigma} d_i^\dagger d_i + U_d \sum_i d_i^\dagger d_i^\dagger d_i^\dagger d_i + \varepsilon_p \sum_{i, \sigma} p_i^\dagger p_i p_i^\dagger p_i + p_i^\dagger \hat{y}/2 p_i \hat{y}/2 + p_i^\dagger \hat{x}/2 p_i \hat{x}/2 + h.c. \\
+ t_{dp} \sum_{i, \sigma} \{ d_i^\dagger (p_i^\dagger \hat{z}/2 + p_i + \hat{y}/2, \sigma - p_i - \hat{z}/2, \sigma - p_i + \hat{y}/2, \sigma) \} + h.c. \\
+ t_{pp} \sum_{i, \sigma} \{ -p_i^\dagger \hat{y}/2 p_i + \hat{z}/2, \sigma + p_i^\dagger \hat{y}/2 p_i - \hat{z}/2, \sigma + p_i^\dagger \hat{y}/2 p_i - \hat{z}/2, \sigma - p_i^\dagger \hat{y}/2 p_i + \hat{z}/2, \sigma + h.c. \} \tag{4}
$$

For definitions of various terms see \[^{12}\]. It is not easy to consider and clarify the ground state because of the strong correlations between the d and p electrons. The Variational Monte Carlo is used \[^{12}\] to examine the overall structure of the phase diagram from weakly to strongly correlated regions. Some of the results of the MC calculations are summarized in Fig. 1-3. Fig. 1 shows spin density and hole density. This is calculated the for 16x16 d-p model. The doping ratio is 1/8, $U_d=8$ and the level difference between d and p orbitals is 2. In Fig. 2 the spin distribution in the ground state is given when the stripe is stable. The lengths of arrows are proportional to the magnitudes of spins. Finally in Fig. 3 the energy of stripe and commensurate SDW states is given. The calculations are carried out on 16x4 lattice at $U_d=8$ and the doping ratio=1/8. Squares denote the energy for commensurate SDW state, and circles and triangles indicate them for 4-lattice and 8-lattice stripes, respectively. Here for example, 4-lattice stripe means that there is one stripe per 4 ladders. It is found in \[^{12}\] that a picture of hole doping case which emerges from the MC evaluations is that the stripe state is stable at low doping and changes into the d-wave superconductivity. Thus we can see from MC the underlying connection between stripe [stringy] and superconductivity, which is a numerical support of our conjecture, namely that superconductivity is related to stripes [i.e. 1-d structures] and/or can be understood in terms of such 1-d systems. It would be interesting and useful to consider the electron-phonon interaction in the 2D d-p model, and see if the evaluations remain consistent with the current results.

Yet another support for the validity of our conjecture can be found in t’Hooft’s idea of phase transition in context of quark confinement \[^{13}\]. It was shown by this author that in quantized gauge theories one can introduce sets of operators that modify the gauge-topological structure of the fields but whose physical effect is in essence local. In 2+1 dimensions it was shown for non-abelian gauge theories that these operators form scalar
fields, and when local gauge symmetry is not broken spontaneously then these topological fields develop a vacuum expectation value and their mutual symmetry breaks spontaneously. Given a gauge group SU(N) we have a non-trivial center Z(N) of this group. Then one is lead to the concept that topologically defined operators which create or destroy topological quantum numbers can be thought of disorder parameters. Here we again have a dual relation between order and disorder. In most simple interpretation a superinsulator is the dual of superconductor, where the former is characterized by disorder.

In conclusion we have indicated new ways of looking at the physics of HTSC and related materials and in general strongly correlated systems. Although in its current form quantum groups are restricted to 1-d, we don’t see this as a problem but a solution. This 1-d restriction led us to think that we can solve a 2-d or 3-d problem by using string theory as is done in case of gravity. Gravity is a 4-d phenomenon, but can be understood elegantly in terms of string theory [1-d objects]. This approach is fundamentally different from the one where one tries to go from 1-d to higher dimensions directly. Recent developments in string and topological field theories can further help us to understand the physics of strongly correlated systems. In the context of strongly correlated systems such as HTSC it is the necessity to replace the Landau quasiparticle by something else which can lead to a better formulation and understanding of these sytems that has led us naturally to consider quantum groups, strings and topology. We have also presented numerical support for our conjecture from variational MC evaluations.

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REFERENCES

[1] Sher Alam, *Quantum Group based Modelling for the description of high temperature superconductivity and antiferromagnetism*, Phys. Lett. A272, (2000), 107-112.

[2] H.A.Mook et al., Nature 404 (2000), 730, cond-mat/0004362.

[3] Sher Alam, *A Conjecture for possible theory for the description of high temperature superconductivity and antiferromagnetism*. Proceedings of Quantum Phenomena in Advanced Materials at High Magnetic Fields, 4th International Symposium on Advanced Physical Fields [APF-4]. KEK-TH-607, KEK Preprint 98-xxx, cond-mat/9812060. KEK-TH-613, KEK Preprint 98-xxx, cond-mat/9903038.

[4] Sher Alam et al., *Theoretical modeling for quantum liquids from 1d to 2d dimensional crossover using quantum groups*. KEK-TH-619. KEK Preprint 99-xxx, cond-mat/990345.

[5] Holger Frahm et al., Phys. Rev. Lett. 81, (1998), 2116.

[6] Sher Alam et al., *The Choice of symmetry group for cuprates*, cond-mat/0004269.

[7] Sher Alam et al., *The patching of critical points using quantum groups*, cond-mat/0004350.

[8] Sher Alam, *Quantum group conjecture and Stripes* cond-mat/0005168.

[9] Sher Alam et al., *SET based experiments for HTSC materials* cond-mat/0011037.

[10] M. Kaku, *Strings, Conformal Fields, and Topology: An Introduction*, Springer-Verlag, 1991.

[11] H. Yamaguchi et al., Physica. C320, (1999), 167-172.

[12] T. Yanagisawa et al. and K. Yamaji et al., please see the the two articles by these authors in: “Physics in Local Lattice Distortions”, Eds. :Hiroyuki Oyanagi and Antonio Bianconi, AIP Conf. Proc. No. 554, 2000.

[13] G. t’Hooft, Nucl. Phys. B138, (1978), 1-25.
FIGURES

FIG. 1. The spin density and hole density. This is calculated for 16x16 d-p model. The doping ratio is 1/8, $U_d=8$ and the level difference between d and p orbitals is 2.

FIG. 2. The spin distribution in the ground state when the stripe is stable. The lengths of arrows are proportional to the magnitudes of spins.

FIG. 3. The energy of stripe and commensurate SDW states. The calculations are carried out on 16x4 lattice at $U_d=8$ and the doping ratio=1/8. Squares denote the energy for commensurate SDW state, and circles and triangles indicate them for 4-lattice and 8-lattice stripes, respectively. Here for example, 4-lattice stripe means that there is one stripe per 4 ladders.
$\left(-1\right)^{L-1} S_z (L)$

Hole density
$E - E_{\text{normal}} = 2.4$