Multicriterion Control Charts for Electrostatic Separation Processes

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Abstract. The variability of the output is often deplored by the users of electrostatic separators. The goal of this paper is to point out the effectiveness of multicriterion control charts for monitoring the output variables of electrostatic separation processes. The experiments were carried out on samples of chopped electric cable wastes, similar to those currently treated by the recycling industry. The two output variables considered in the study were the masses of product recovered in the middling and conductive compartments of the collector. When the separation process was under control, the two variables were correlated, and a $\chi^2$-type control chart could be established. A simulated out-of-control situation was detected by the multicriterion control chart, though each output variable taken independently remained within the control limits.

1. Introduction
The electrostatic separation of the granular mixtures is a mature technology with various applications: waste treatment, mineral beneficiation, food processing [1-4]. For each such application, the quality of the products is a crucial issue, and the statistic control techniques [5] proved to be an effective tool for ensuring stable process performances [6, 7].

Classic univariate statistic process control schemes have the advantage of simplicity, but are not very accurate when the output variables are correlated, as the case is with the electrostatic separation [8, 9]. The example in Figure 1 clearly points out the limits of independent monitoring of correlated variables: an “out-of-control” situation may not be detected by the two control charts.

Multivariate statistic process control (SPC), which was first proposed by Hotelling, overcome this limitation, but involves extensive use of matrix calculus [10, 11]. However, due to the large availability of SPC software, this does no longer represent a major obstacle against wide industry application of multicriterion control charts. The aim of this paper is to demonstrate the advantage of using one type of such control charts for monitoring an electrostatic separation process.

2. Experimental set-up and material
A laboratory roll-type electrostatic separator was used for the experimental study (Figure 2). In order to simulate an industrial electrostatic separation process, the high-voltage $U = 32$ kV, the roll speed $n = 75$ rpm; the angular ($\phi_1 = 30^\circ$) and radial ($d_1 = 40$ mm) positions of the corona electrode, the angular ($\phi_2 = 70^\circ$) and radial ($d_2 = 70$ mm) positions of the static electrode, as well as the angular positions $\gamma_1 = 30^\circ$ and $\gamma_2 = -6^\circ$ of the collector splitters were maintained fixed for all the experiments [7, 9].
The tests are carried out on granular samples, resulting from the chopped electric cable wastes provided by RIPS-ALCATEL, France. The mass of each sample was 100 g (5 g of copper and 95 g of PVC), with the average granular size ranging between 1 and 2 mm. The products are collected in three different compartments: conductor, non-conductor and middling.

Two output variables were monitored in this study: mass of middling product \((M_m)\) and mass of conducting product \((M_c)\). These products were weighed with an electronic balance (precision: 0.01 g).

Every 30 min, a set of three 100 g samples were subjected to the separation process, then analyzed. Twelve such sets of experiments were performed with all the input variables under control and at constant ambient conditions: 20.2 – 22.1°C; RH = 27.5 – 29.7%. The mean value \(\bar{x}_j\), the variance \(S_{jj}\), and the covariance \(S_{jh}\) of each output variable \(j\) were calculated for each set of experiments [10, 11]:

\[
\bar{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n}; \quad S_{jj} = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ih} - \bar{x}_h)}{n-1}; \quad X_j = [\bar{x}_j], \quad j = 1, \ldots, k
\]

where \(n\) is the sample size, and \(k\) is the number of variables (in the present study: \(n = 3\) and \(k = 2\)).

The results computed for the \(m = 12\) sets of experiments were then employed for setting the average vector \(\overline{X}\) and the covariance matrix \(\Sigma\):

\[
\overline{x}_j = \frac{\sum_{i=1}^{m} x_{ij}}{m} \Rightarrow \overline{X} = [\overline{x}_j], \quad j = 1, \ldots, k
\]

\[
S_{jh} = \frac{\sum_{i=1}^{m} S_{ijh}}{m} \Rightarrow \Sigma = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}
\]

Thus, it was possible to compute the scalar \(\chi^2_j\) for each of the \(m = 12\) sets of experiments:

\[
\chi^2_j = n(\overline{X} - \overline{X})^T \Sigma^{-1}(\overline{X} - \overline{X})
\]

and establish a \(\chi^2\)-type control chart with the upper limit given by the \(\chi^2\)-law table for a given risk \(\alpha\) and \(v = n-1\) degrees of freedom [10, 11]: \(UCL = \chi^2_{a,v}\).
In order to simulate an “out-of-control” situation [12], an experiment was performed at a reduced applied high-voltage \( U = 29 \text{ kV} \), instead of \( U = 32 \text{ kV} \), with three 100 g samples of the same granular material and under similar ambient conditions.

3. Results and discussion

3.1. Average vector and covariance matrix. The results of the 3x12 electrostatic separation tests are given in Table 1. The representation of the average masses of the middling and of the conducting product obtained in the 12 sets of experiments indicate that a control ellipse similar to that in Figure 1 can be drawn for this process, which means that a correlation exists between the two output variables. The use of a multicriterion control chart is thus fully justified.

| Experiment | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|------------|----|----|----|----|----|----|----|----|----|----|----|----|
| M\(_m\)    | 0.72 | 0.71 | 0.73 | 0.69 | 0.7 | 0.74 | 0.68 | 0.72 | 0.68 | 0.72 | 0.7 | 0.73 |
| M\(_c\)    | 0.71 | 0.73 | 0.71 | 0.73 | 0.73 | 0.72 | 0.71 | 0.68 | 0.73 | 0.69 | 0.68 | 0.69 |
| \(\overline{X}_m\) | 0.713 | 0.720 | 0.717 | 0.720 | 0.707 | 0.717 | 0.700 | 0.703 | 0.700 | 0.707 | 0.700 | 0.710 |
| M\(_c\)    | 3.15 | 3.21 | 3.18 | 3.2 | 3.32 | 3.4 | 3.32 | 3.33 | 3.43 | 3.36 | 3.33 | 3.1 |
| M\(_c\)    | 3.34 | 3.1 | 3.22 | 3.16 | 3.21 | 3.1 | 3.14 | 3.28 | 3.36 | 3.43 | 3.39 | 3.35 |
| \(\overline{X}_c\) | 3.167 | 3.147 | 3.150 | 3.197 | 3.250 | 3.250 | 3.247 | 3.297 | 3.343 | 3.347 | 3.300 | 3.223 |
| \(\chi^2\) | 1.819 | 2.093 | 2.415 | 2.962 | 1.154 | 1.797 | 2.135 | 0.935 | 2.262 | 2.695 | 1.707 | 0.029 |

Table 1: Results of the 12 sets of electrostatic separation experiments.

The average vector and the covariance matrix calculated with (2) and (3) are:

\[
\overline{X} = \begin{bmatrix} 0.7094 \\ 3.2631 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 0.0001 & -0.0006 \\ -0.0006 & 0.0064 \end{bmatrix}
\]  

(5)

The standard deviations of the two output variables being:

\[
\sigma_m = 0.007 \quad \text{and} \quad \sigma_c = 0.07
\]  

(6)

The respective upper and lows control limits are [10, 11]:

\[
UCL_m = 0.733, \quad LCL_m = 0.686, \quad UCL_c = 3.473 \quad \text{and} \quad LCL_c = 3.053
\]  

(7)

The values of \(\chi^2\) for the 12 sets of experiments are given in the last line of Table 1.

3.2. Multicriterion control chart. The previously computed values of \(\chi^2\) are represented on the control chart (Figure 3) with the upper control limit \(UCL = \chi^2_{\alpha, \nu} = 5.99\) given in [10] for \(\alpha = 0.05\) and \(\nu = 2\).

3.3. Simulation of an “out-of-control” situation. The results of the “out-of-control” experiment can be summarized as follows: \(M_{m1} = 0.74 \text{ g}; \ M_{c2} = 0.72 \text{ g}; \ M_{c3} = 0.72 \text{ g}; \overline{X}_m = 0.727 \text{ g}; \ M_{c1} = 2.86 \text{ g}; \ M_{c2} = 3.15 \text{ g}; \ M_{c3} = 3.22 \text{ g}; \overline{X}_c = 3.077 \text{ g}\). The respective value of the \(\chi^2\) is 6.538 is beyond the \(UCL\), as shown in Figure 4, though both \(\overline{X}_m\) and \(\overline{X}_c\) are within the control limits (7). The multivariate control chart was able to detect an “out-of-control” situation (the voltage decreased to \(U = 29 \text{ kV}\)) that would have passed unnoticed on univariate control charts.
4. Conclusion
In the case of the complex processes such as the electrostatic separation of mixed granular solids, it is necessary to use control charts that reflect simultaneously two or several output variables. Such control charts are able to take into account the correlations that exist between these variables, using them as sources of information about the global performances of the process.

The present study pointed out the importance of multicriterion control charts for the efficient monitoring of a specific electrostatic separation processes. The approach presented above can and should be adopted for the statistic process control of other electrostatic processes involving several correlated output variables.

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