A Novel Approach of Interface Stiffness Identification Based on Wave Propagation Method

Bo Yuan¹, Xiaokai Mu¹*, Yunlong Wang¹, Chao Zhang¹, Qingchao Sun** and Wei Sun¹

¹ School of Mechanical Engineering, Dalian University of Technology, Dalian, Liaoning Province, 116023, China
*Corresponding author’s e-mail: muxiaokai@dlut.edu.cn
**Corresponding author’s e-mail: qingchao@dlut.edu.cn

Abstract. The contact joint in mechanical system will bring uncertainty, which leads to the uncertainty of dynamic response. Identification of interface stiffness is particularly important in the assembly process, such as bolt connection and rivet connection. In this paper, the contact joint can be regarded as a kind of structural discontinuity, and the contact joint in two beams is modeling with shear stiffness and bending stiffness. The wave theory is integrated into the Euler-Bernoulli beam theory to establish the relationship between the power reflection coefficient and the interface stiffness. The proposed method is verified by establishing a finite element model. The results show that the interface stiffness is accurately identified. In addition, the influence of interface stiffness on power reflection coefficient is discussed. This method has better local characteristics and avoids the disadvantages of identification by natural frequency method.

1. Introduction
Mechanical joints are the important forms of equipment structure, which are widely used in aviation, aerospace, machine tool and other major equipment. The parts are combined by mechanical joint fasteners such as bolts, solid rivets and blind rivets [1]. Mechanical joints are regarded as discontinuous structure. Compared with the whole structure, the structure discontinuities show the weakening of the mechanical characteristics [2]. There is stiffness loss in the discontinuous structure, so it is necessary to design the connection parameters of the discontinuous structure reasonably to ensure the stability of the whole system. It has been shown that the parameters of discontinuous structure have a great influence on the overall performance of the equipment. Therefore, in the process of structural analysis, it is not appropriate to simply equivalent it to rigid connection, however, it is necessary to identify the parameters of the connection structure effectively.

It is well known that the state of the contact joint in the structure usually changes the dynamic response of the structure. Therefore, the frequency response function (FRF) based methods have been widely studied in decades [3]. In most of the FRF based joint identification methods, the basic strategy is to use FRFs of individual substructures without joints and those of assembly system to obtain information about joint performance [4]. Shi et al. [5] proposed a contact parameter identification method based on contact resonance. However, the method is based on a single degree of freedom dynamic system, which may lead to theoretical errors in identification results. Mehrpouya et al. [6] and Serife et al. [7] all take the cantilever beam with single bolt as the research object, and obtain the FRFs
of substructure and the FRF of the whole structure respectively. However, this method can only identify the parameters of a single joint, which is very difficult for multiple joints.

In addition to the FRF method, many researchers also consider using modal parameters to identify the interface stiffness. Ahmadian et al. [8] measured the free-free condition of the specimen, and updated the stiffness parameters of the contact joint to match the theoretical value with the test value. Considering the practicability of measurement, Iranzad et al. [9] and Zhao et al. [10] used a cantilever beam with one end fixed by bolts for modal analysis. Virtual material elements and elastic thin-layer elements are used to model the contact joint, and the natural frequencies of the model and ones of the test are minimized by using the optimization algorithm to obtain the interface stiffness. However, the modal information of the end will be incorporated into the identification results, thus affecting the correctness of the results.

Contact interface can be regarded as a kind of the structural discontinuity, which breaks the continuity of internal force [11]. From the perspective of waveguide structure and multi-mode propagation, many researchers consider the wave propagation in the beam model [15]. In a rectangular beam with attached blocking masses, Muggleton [16] proposed to estimate the stiffness, the mass and the second moment of inertia of the discontinuity by using the measured power reflection coefficients. Fan [17] uses high-order parameters to model the discontinuity, and these parameters are identified by using reflection coefficients at the discontinuity. However, the discontinuity stiffness identified by these studies is based on the ground and is not suitable for the identification of interface stiffness. Furthermore, these studies assume that the displacement is continuous, which does not conform to the characteristics of contact joint. The problem of interface stiffness identification is far from being solved. Generally, the state of the contact joint in the structure will in some way change the motion of the wave. Therefore, the change of the wave will in turn influence the reflection characteristics at the contact joint. Based on above analysis, the research on the propagation characteristics of the contact joint will be of great value for detection of interface stiffness.

The aim of this paper is to investigate the identification of interface stiffness by using the power reflection coefficient of beams with contact joint. Considering the characteristics of beam structure propagation, this paper extends the interface stiffness model from the normal interface stiffness of one-dimensional wave equation to the interface bending stiffness and interface shear stiffness of beam model. Beam model extends from mass discontinuity to contact joint discontinuity.

2. Formulation

2.1. Euler-Bernoulli beam theory based on wave propagation method

We consider a Euler-Bernoulli equation for a uniform elastic beam for a 1D system. The free vibration equation of the beam can be written as

\[ EI \frac{d^4v(x,t)}{dx^4} + \rho A \frac{d^2v(x,t)}{dt^2} = 0 \]  

(1)

where \( E \) is Young’s modulus, \( I \) is the second moment of section, \( \rho \) is the density of beam and \( A \) is cross-sectional area.

The general solution form of beam theory can be obtained by Fourier transform of time variable and Fourier transform of space variable for the control equation.

\[ \hat{v}(x,w) = Ae^{-ikx} + Be^{-kx} + Ce^{ikx} + De^{kx} \]  

(2)

where the first term is the wave solution in +X direction, the second term is the attenuation solution in +X direction, the third term is the wave solution in -X direction, and the fourth term is the attenuation solution in -X direction.

The boundary conditions are expressed by the derivatives of different orders of \( \hat{v}(x, w) \), and their specific forms are as follows

\[ W_j = \begin{bmatrix} \hat{v}_j(x, w) \\ \hat{\phi}_j(x, w) \end{bmatrix} = \begin{bmatrix} \hat{v}_j(x, w) \\ \frac{\partial \hat{v}_j(x, w)}{\partial x} \end{bmatrix} \]  

(3)
\[ Q_j = \begin{bmatrix} \hat{F}_j(x, w) \\ \hat{M}_j(x, w) \end{bmatrix} = \begin{bmatrix} -EI \frac{d^3 \hat{\phi}_j(x, w)}{dx^3} \\ EI \frac{d^2 \hat{\psi}_j(x, w)}{dx^2} \end{bmatrix} \] (4)

where \( \hat{\phi}_j(x, w) \) is the transverse displacement, \( \hat{\psi}_j(x, w) \) is the rotation angle, \( \hat{F}_j(x, w) \) is the shear force and \( \hat{M}_j(x, w) \) is the bending moment. For beams of rectangular section, \( l = bh^3/12 \), where \( b \) is the width and \( h \) is the height.

According to Eqs. (2), (3) and (4), the displacement vector \( \mathbf{W}_j \) and force vector \( \mathbf{Q}_j \) on the beam can be expressed by the wave amplitudes as

\[ \begin{bmatrix} \mathbf{W}_j \\ \mathbf{Q}_j \end{bmatrix} = \begin{bmatrix} \mathbf{A}^+ \\ \mathbf{A}^- \end{bmatrix} \begin{bmatrix} \mathbf{a}^+_j \\ \mathbf{a}^-_j \end{bmatrix} \] (5)

where the displacement transfer matrices \( \mathbf{A}^+_j, \mathbf{A}^-_j \), the force transfer matrices \( \mathbf{\Psi}^+_j, \mathbf{\Psi}^-_j \) and the vector of wave amplitudes \( \mathbf{a}^+_j, \mathbf{a}^-_j \) can be written as

\[ \mathbf{A}^+ = \begin{bmatrix} 1 & 1 \\ -ik & -k \end{bmatrix}, \quad \mathbf{A}^- = \begin{bmatrix} 1 & 1 \\ ik & k \end{bmatrix} \] (6)

\[ \mathbf{\Psi}^+ = \begin{bmatrix} -iElk^3 & Elk^3 \\ -Elk^2 & Elk^2 \end{bmatrix}, \quad \mathbf{\Psi}^- = \begin{bmatrix} iElk^3 & -Elk^3 \\ -Elk^2 & Elk^2 \end{bmatrix} \] (7)

\[ \mathbf{a}^+_j = \begin{bmatrix} a^+_{pj} \\ a^+_{ej} \end{bmatrix}, \quad \mathbf{a}^-_j = \begin{bmatrix} a^-_{pj} \\ a^-_{ej} \end{bmatrix} \] (8)

2.2. Reflection coefficients of contact joint based on wave propagation method

We consider applying the above theory to the contact joint between two beams. In this case, the contact joint is equivalent to a set of springs. As shown in Figure 1, beams \( a \) and \( b \) are connected by the contact joint. The boundary condition is characterized by displacement discontinuity and force continuity. To simplify the problem, the mass effect of joint interface can be ignored. The boundary conditions of the problem are defined

\[ F_a = F_b, \quad M_a = M_b \] (9)

In addition, compatibility requirements are defined by considering the contact joint. In this paper, the related theory and analysis are based on the small excitation amplitude, so the nonlinear problems can be ignored. Therefore, the shear stiffness of the contact joint is expressed using a linear translational spring \( K_v \), and the bending stiffness can be modelled by considering a linear torsional spring \( K_\theta \) for the contact joint.

\[ F_a = -K_v(v_a - v_b), \quad M_a = K_\theta(\psi_a - \psi_b) \] (10)

Figure 1. The joint structure of beam including displacements and forces
Then the relationship between displacement vectors $\mathbf{W}_a, \mathbf{W}_b$ and the force vectors $\mathbf{Q}_a, \mathbf{Q}_b$ can be obtained by using Eqs. (6) and (7). By applying these the relationship, the displacement and force on both sides of the contact joint can be related by the transfer matrix $\Omega$ as

$$
\begin{bmatrix}
\mathbf{W}_a \\
\mathbf{Q}_a
\end{bmatrix} =
\begin{bmatrix}
\Omega_{11} & \Omega_{12} \\
\Omega_{21} & \Omega_{22}
\end{bmatrix}
\begin{bmatrix}
\mathbf{W}_b \\
\mathbf{Q}_b
\end{bmatrix}
$$

(11)

where the elements of the transfer matrix $\Omega_{ij}$ (i, j = 1, 2) can be written as

$$
\Omega_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Omega_{12} = \begin{bmatrix} -\frac{1}{K_v} & 0 \\ 0 & -\frac{1}{K_g} \end{bmatrix}, \quad \Omega_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Omega_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

(12)

Now we consider two beams $a$ and $b$ which are connected. The expression is obtained by substituting Eq. (5) into Eq. (11).

$$
\begin{bmatrix}
\mathbf{a}_a \\
\mathbf{b}_a
\end{bmatrix} =
\begin{bmatrix}
\Lambda_a^- & \Lambda_b^- \\
\psi_a^- & \psi_b^-
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}_a \\
\mathbf{b}_a
\end{bmatrix}
$$

(13)

The relationship between the wave amplitudes on both sides of the contact joint can be determined by the wave-amplitude transfer matrix as

$$
\begin{bmatrix}
\mathbf{a}_a^- \\
\mathbf{b}_a^-
\end{bmatrix} =
\begin{bmatrix}
\Lambda_a^- & \Lambda_b^- \\
\psi_a^- & \psi_b^-
\end{bmatrix}
\begin{bmatrix}
\Lambda_a^+ & \Lambda_b^+ \\
\psi_a^+ & \psi_b^+
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}_a^+ \\
\mathbf{b}_a^+
\end{bmatrix}
$$

(14)

Then, the incoming waves ($\mathbf{a}_a^+, \mathbf{b}_a^+$) and the outgoing waves ($\mathbf{a}_a^-, \mathbf{b}_a^-$) at contact joint can be related by the reflection matrices ($R_{aa}, R_{bb}$) and transmission matrices ($T_{ab}, T_{ba}$) as

$$
\begin{bmatrix}
\mathbf{a}_a^- \\
\mathbf{b}_a^-
\end{bmatrix} =
\begin{bmatrix}
R_{aa} & T_{ba} \\
T_{ab} & R_{bb}
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}_a^+ \\
\mathbf{b}_a^+
\end{bmatrix}
$$

(15)

The reflection and transmission coefficient matrix can be calculated by combining Eqs. (14) and (15) as

$$
\begin{bmatrix}
R_{aa} & T_{ba} \\
T_{ab} & R_{bb}
\end{bmatrix} =
\begin{bmatrix}
\Lambda_a^- & \Lambda_b^- \\
\psi_a^- & \psi_b^-
\end{bmatrix}
\begin{bmatrix}
\Lambda_a^+ & \Lambda_b^+ \\
\psi_a^+ & \psi_b^+
\end{bmatrix}
$$

(16)

3. Results and discussion

3.1. Numerical example

In order to verify the method, the simulation is performed for two Euler-Bernoulli beams to which the translational spring and torsional spring are attached. The translational spring represents shear stiffness, and the torsional spring represents bending stiffness. The FEM model in Figure 2 will be employed here to simulate the contact joint. The distance of excitation force is $L_x = 700\text{mm}$. The response measurement distances are set to be $L_a = L_b = 300\text{mm}$, and the transducer spacing is $\Delta = 50\text{mm}$. Then, Table 1 lists the properties of the beams and the interface stiffness. The setting of interface stiffness parameters is referred to [7].
Considering that the Euler-Bernoulli beam theory is used for analysis, the power reflection coefficients in the frequency domain below 100kHz can be used for parameter identification of contact joint. Hence, the frequency range used is 0kHz to 100kHz.

### Table 1. Properties of the beams and the interface stiffness

| Properties of the beams | Interface stiffness parameters |
|-------------------------|-------------------------------|
| Length | Density | Young’s modulus | Width × thickness | Shear stiffness | Bending stiffness |
| 1000 mm | 7850 kg/m³ | 200 GPa | 60mm × 5mm | $1 \times 10^5$ N/m | $1 \times 10^5$ N·m/rad |

#### 3.2. Identification of interface stiffness parameters

The structure is excited with sweep signal with amplitude of 1N. The power reflection coefficients and the power transmission coefficients are shown in Figure 3. It can be found that the power reflection coefficient and power transmission coefficient change with frequency. In addition, the sum of power coefficients $|r_p|^2$ and $|t_p|^2$ is mostly close to 1, which is further ensure the correctness of the results.

The power reflection coefficients calculated by using the simulated data and the ones calculated by using the identified interface stiffness parameters are shown in Figure 4. With the increase of frequency, the power reflection coefficient increases. The good agreement between the results of the identified model and the simulated model is achieved as shown in Figure 4. The results indicate that the identification effect of the proposed method is almost the same as that of the reference[7].
Figure 4. Power reflection coefficient for simulation model and identified model

The parameters of interface stiffness, i.e. the shear stiffness $K_v$ and the bending stiffness $K_\theta$, are identified using power reflection coefficients. The parameters are achieved by minimizing the differences between the simulated power reflection coefficients and the modeled ones. As shown in Table 2, the identification errors of the shear stiffness and the bending stiffness are 12.51% and 8.74%, respectively.

| Interface stiffness parameters | Nominal value | Identification result | Relative error |
|--------------------------------|---------------|-----------------------|---------------|
| Shear stiffness $K_v$ (N/m)    | $1\times10^5$ | $8.729\times10^4$     | 12.71%        |
| Bending stiffness $K_\theta$ (N·m/rad) | $1\times10^5$ | $9.187\times10^4$     | 8.13%         |

3.3. Relationship between interface stiffness and power reflection coefficients

On the basis of this method, we can further analyze the effect of bending stiffness and shear stiffness on the power reflection coefficient. By controlling one of them not to change, we can analyze their respective influence on wave propagation.

As shown in Figure 5(a), the shear stiffness is maintained and the bending stiffness is increased. It can be seen that the main influence frequency range of bending stiffness is high frequency, while low frequency range has little influence. The larger the bending stiffness is, the smaller the adjacent difference of power reflection coefficient is. So, there is a saturation phenomenon. We can find that the power reflection coefficient is close to a value at higher frequency when the bending stiffness increases to a certain value.
As shown in Figure 5(b), the shear stiffness increases while the bending stiffness remains unchanged. With the increase of shear stiffness, the curve of power reflection coefficients tends to move to the right. No matter how the shear stiffness changes, the power reflection coefficient tends to be the same finally. In addition, the shear stiffness has a great influence in the low frequency range, but has little influence in the high frequency range.

4. Conclusions
This paper has presented a strategy for identifying interface stiffness in which the model is updated to match observed wave behavior of reflection. A model for Euler–Bernoulli beam with contact joint is developed. Considering the continuous displacement and discontinuous force of contact joint, the modeled power reflection coefficient is presented by using wave propagation method. Estimates for the effective shear stiffness and bending stiffness are in reasonably close agreement with directly simulated values. Identification for the interface stiffness are plausible. The effects of bending stiffness and shear stiffness on power reflection coefficients are discussed.

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