Resolved virtual photons in the small $x$ domain

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Abstract

It has been found that recent results on forward jet production from deep inelastic scattering can neither be reproduced by models which are based on leading order $\alpha_s$ QCD matrix elements and parton showers nor by next-to-leading order calculations. The measurement of forward jet cross sections has been suggested as a promising probe of new small $x$ parton dynamics and the question is whether these data provide an indication of this. The same question arises for other experimental data in deep inelastic scattering at small $x$ which can not be described by conventional models for deep inelastic scattering. In this paper the influence of resolved photon processes has been investigated and it has been studied to what extent the inclusion of such processes in addition to normal deep inelastic scattering leads to agreement with data. It is shown that two DGLAP evolution chains from the hard scattering process towards the proton and the photon, respectively, are sufficient to describe effects, observed in the HERA data, which have been attributed to BFKL dynamics.

1 Introduction

Experimental data from deep inelastic scattering (DIS) in a kinematic region where new parton dynamics is expected to become noticeable i.e. at small values of the scaled proton momentum, $x$, are not described by models based on interactions with pointlike photons. In a previous paper [1] we have demonstrated that the addition of interactions through resolved photons offers a possible explanation to the observed discrepancies and leads to good agreement with all available data.

This paper is devoted to a more detailed discussion of the resolved photon concept and comparisons with data on forward jet production in DIS since the forward jet cross section has been advocated as a particularly sensitive measure of small $x$ parton dynamics [2, 3]. Analytic calculations based on the BFKL equation in the leading logarithmic approximation (LLA) are in fair agreement with data. However, recent calculations of the BFKL kernel in the next-to-leading logarithmic approximation (NLLA) [4] have given surprisingly large corrections, and it remains to be shown whether the data can still be reasonably described.

Monte Carlo generators based on direct, point-like photon interactions (DIR model), calculated from leading order (order $\alpha_s$) QCD matrix elements, and leading log parton
showers based on the DGLAP evolution do not take any new parton dynamics in the small $x$ region into account and are therefore not expected to fit the experimental data. Recent results from the H1 \cite{5} and ZEUS \cite{6} experiments on forward jet production exhibit significant deviations from the predictions of such models. Also next-to-leading order calculations (NLO i.e. order $\alpha_s^2$) predict too small a cross section compared to data.

The study of forward jet production with contributions from direct as well as resolved photon processes has been performed using the RAPGAP 2.06 \cite{7-8} Monte Carlo event generator.

# 2 Resolved Photons in DIS

In electron-proton scattering the internal structure of the proton as well as of the exchanged photon can be resolved provided the scale of the hard subprocess is larger than the inverse radius of the proton, $1/R_p^2 \sim \Lambda_{QCD}^2$, and the photon, $1/R_\gamma^2 \sim Q^2$, respectively. Resolved photon processes play an important role in photo-production of high $p_T$ jets, where $Q^2 \approx 0$, but they can also give considerable contributions to DIS processes \cite{9-10} if the scale $\mu^2$ of the hard subprocess is larger than $Q^2$, the inverse size of the photon.

In the following we give a brief description of the model for resolved virtual photons used in the Monte Carlo generator RAPGAP. Given the fractional momentum transfer of the incoming electron to the exchanged photon, the Equivalent Photon Approximation provides the flux of virtual transversely polarized photons \cite{8, 7, and references therein}. The contribution from longitudinally polarized photons has been neglected. The partonic structure of the virtual photon is defined by parameterizations of the parton densities, $x_\gamma f_\gamma(x_\gamma, \mu^2, Q^2)$, which depend on the two scales $\mu^2$ and $Q^2$\cite{11-12, 13}. The following hard subprocesses are considered (RES model): $gg \to q\bar{q}$, $gg \to gg$, $qg \to qg$, $q\bar{q} \to gg$, $q\bar{q} \to q\bar{q}$, $qq \to qq$. Parton showers on both the proton and the photon side are included. The generic diagram for the process $q_\gamma g_p \to qg$ including parton showers is shown in Fig.\ref{fig:1}.

Since the photon structure function depends on the scale, $\mu^2$, of the hard scattering process, the cross section of resolved photon processes will consequently also depend on the choice of this scale. It has to be carefully considered in which range of $\mu^2/Q_0^2$ the photon-parton cross section can be factorised into a parton-parton cross section convoluted with the parton density of the photon. The parton density of the photon is evolved from a starting scale $Q_0^2$ to the scale $\mu^2$, the virtuality at the hard subprocess, giving a resummation to all orders.

## 2.1 Parton Distribution Functions

Due to factorization of the cross sections the parton densities of both the virtual photon and the proton enter into the calculations. The proton structure function, $F_2$, has been measured to high accuracy and therefore the various parameterizations only give marginal differences in the measurable kinematic region. Two parameterizations of the parton distribution in the proton, GRV 94 HO (DIS) and CTEQ4D have been considered, which both give good agreement with the proton structure function data \cite{14-15}. It was found
that the produced results were identical within the percent level, when keeping $\Lambda_{QCD}$ fixed. In the following we use only GRV HO.

The photon can interact via its partons either in a bound vector meson state or as decoupled partons if the $p_T$ of the partons is high enough. The splitting $\gamma \rightarrow q\bar{q}$ is called the anomalous component of the photon. The structure function of virtual photons has only recently been measured but by far not to the same precision as the proton structure function. However, it turns out that data are in good agreement with the parameterization of Schuler and Sjöstrand (SaS) [12]. The SaS parameterization offers a choice of $Q^2_0$ values at which the anomalous part becomes effective. We have studied these choices resulting in different magnitudes of the parton densities, and consequently of the cross sections. For the SaS parameterization we have used $Q^2_0$ as given by eq.(12) of ref. [12] ($IP2 = 2$). This choice is also suitable for a description of other hadronic final state properties (not considered in this paper), like energy flow, forward particle spectra and jet cross sections.

In a previous paper [1] we have shown that the ansatz of Drees and Godbole [13] gave very similar results to those obtained from the SaS parameterization why we restrict ourselves to the SaS parton distributions in this study.

The hadronic contribution to the virtual photon structure function decreases rapidly with increasing $Q^2$, which means that the main contribution at large $Q^2$ comes from the anomalous piece in the photon splitting. This is completely calculable in pQCD and leads to an expected agreement between the parameterization of Glück - Reya - Stratman [11] and that of Schuler - Sjöstrand [12] but also with the simple ansatz of Drees - Godbole [13]. However differences exist in the way the hadronic part of the structure function is
matched to the pointlike part, which just reflects the theoretical uncertainty.

2.2 Choice of Scale

In leading order \( \alpha_s \) processes the renormalization scale \( \mu_R \) and factorization scale \( \mu_F \) are not well defined which allows a number of reasonable choices. There are essentially two competing effects: a large scale suppresses \( \alpha_s(\mu^2) \) but gives, on the other hand, an increased parton density, \( x f(x, \mu^2) \), for a fixed small \( x \) value. The net effect depends on the details of the interaction and on the parton density parameterization.

In previous papers \([16, 1]\) we have tried different scales like \( \mu^2 = 4 \cdot p_T^2 \) and \( \mu^2 = Q^2 + p_T^2 \), and found that these choices gave similar results. However, in resolved virtual photon processes the choice of the scale \( \mu^2 \), at which the photon is probed, is severely restricted \([17]\). In a partonic process \( a + b \rightarrow c + d \), where \( a, b, c, d \) denote four-vectors and where parton \( a \) has the virtuality \( Q^2 \), the transverse momentum \( p_T^2 \) of parton \( c \) is given in the small angle limit \( -\hat{t} \ll \hat{s} \) by: \( p_T^2 = \hat{s}(-\hat{t})/(\hat{s} + Q^2) \), with \( \hat{s} \) and \( \hat{t} \) being the usual Mandelstam variables. In a \( t \) channel process the virtuality is given by \( \mu^2 = -\hat{t} \). Thus we have:

\[
\mu^2 = -\hat{t} = p_T^2 + Q^2 \cdot \frac{p_T^2}{\hat{s}} < Q^2 + p_T^2 \tag{1}
\]

From eq.(1) we see, that the scale \( \mu^2 \) is always larger than the transverse momentum squared of the hard partons and less than \( Q^2 + p_T^2 \). In the following we will use \( \mu^2 = Q^2 + p_T^2 \) as scale for both resolved virtual photon processes and for direct photon processes. This choice of scale provides a smooth transition from the kinematic region of normal DIS into the range where resolved photons start contributing and to the photo-production region. The same scale has also been used in NLO calculations including resolved photons in deep inelastic scattering \([18]\).

A basic test that the scale is reasonable is that the parton shower evolution scheme should not be able to produce partons with transverse momenta larger than those produced by the matrix element for the hard scattering process.

3 Forward Jets

HERA has extended the available \( x \) region down to values below \( 10^{-4} \) where new parton dynamics might show up. Based on calculations in the LLA of the BFKL kernel, the cross section for DIS events at low \( x \) and large \( Q^2 \) with a high \( p_T^2 \) jet in the proton direction (a forward jet) \([2, 3]\) is expected to rise more rapidly with decreasing \( x \) than expected from DGLAP based calculations. New results from the H1 \([5]\) and ZEUS \([6]\) experiments have recently been presented. The data can be described neither by conventional DIR Monte Carlo models nor by a NLO calculation, while comparisons to analytic calculations of the LLA BFKL mechanism has proven reasonable agreement.

It should be kept in mind that both the NLO calculations and the BFKL based calculations are performed on the parton level whereas the data are on the level of hadrons.

\(^1\)We are grateful to T. Sjöstrand for pointing out this simple explanation
Figure 2: The forward jet cross section as a function of $x$ for $p_{T\,\text{jet}} > 3.5$ GeV (a.) and $p_{T\,\text{jet}} > 5$ GeV (b.). Also shown are the RAPGAP predictions for the sum of direct and resolved processes (solid line) as well as the resolved photon contribution alone (dashed line).

In Fig. 2 the forward jet cross section as measured by the H1 collaboration [5] is compared to the prediction of the RAPGAP Monte Carlo generator for both, resolved photon process alone (labeled RES) and for the sum of direct and resolved processes (labeled DIR+RES). The calculation is performed with a scale $\mu^2 = Q^2 + p_{T\,\text{jet}}^2$ used in the parton densities and $\alpha_s$ for both direct and resolved photon processes. The measurement is well described by the Monte Carlo, including direct and resolved photon processes. The forward jet cross section as measured by the ZEUS experiment [6] can be equally well described by this Monte Carlo program using the same structure functions and parameter...
setting.

Figure 3: The ratio $R = q_T^2/p_T^2$ of the transverse momenta $q_T^2$ of partons from the initial state cascade to the transverse momentum $p_T^2$ of the partons from the hard scattering process. The solid line corresponds to the scale $\mu^2 = Q^2 + p_T^2$, the dotted line to $\mu^2 = 4 \cdot p_T^2$ and the dashed line to $\mu^2 = p_T^2$. The distribution is normalized to the total number of entries. Please note the logarithmic scale on the $y$-axis.

For the forward jet analysis it is important to check that the reconstructed jets really stem from the hard scattering and not from the initial state parton cascade. In Fig. 3 the ratio of the transverse momentum of any initial state parton $q_T^2$ to the transverse momentum $p_T^2$ of the hard scattering process is shown in the $\gamma^* p$ CMS for events which satisfy the forward jet analysis criteria. The solid line corresponds to $\mu^2 = Q^2 + p_T^2$, the dotted line to $\mu^2 = 4 \cdot p_T^2$ and the dashed line to $\mu^2 = p_T^2$. One can see, that essentially all partons coming from the initial state cascade have transverse momenta smaller than the partons of the hard scattering $p_T$, which is expected in a DGLAP type evolution, where the transverse momenta are ordered in $q_T$ towards the hard scattering process. Thus we conclude that the scale $\mu^2 = Q^2 + p_T^2$ which has been used for this study fullfils the
requirements of a DGLAP type initial state cascade.

Figure 4: Different contributions to the total cross section of resolved virtual photons within the cuts of the forward jet analysis [5]. In a. is shown the ratio $R = \frac{\sigma_i}{\sigma_{\text{res. tot}}}$, i.e. the pointlike part (solid line) and the hadronic part (dashed line), respectively, of the resolved virtual photon cross section divided by the the total resolved photon cross section as a function of $x$. In b. is shown the ratio $R = \frac{\sigma_i}{\sigma_{\text{res. tot}}}$ as a function of $x$ for different subprocesses $i$: $qg \rightarrow qg$ (dotted line), $qq \rightarrow qq$ and $qq \rightarrow q\bar{q}q$ (solid line), $gg \rightarrow gg$ (dashed line), $q\bar{q} \rightarrow gg$ (dashed-dotted line) and $gg \rightarrow q\bar{q}$ (solid line).

In Fig. 4 the different contributions of the total resolved photon cross section are shown separately within the cuts of the forward jet analysis. From Fig. 4a it is observed that the hadronic part of the virtual photon structure function as expected gives a negligible
contribution to the measured cross section, since it dies off rapidly with increasing $Q^2$. Fig. 4 shows that the subprocess $q\gamma g \rightarrow qg$ contributes the most to the resolved photon cross section in the forward jet region ($\sim 60\%$) and that the subprocesses $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ and $gg \rightarrow gg$ each give a contribution of the order of $10\%$.

A small fraction of the DIS events, fulfilling the selection criteria for forward jets, actually contains two identified jets. Analytic calculations (in LLA) [19] have been performed in the same kinematic region and with the same jet selection as defined for the one-jet sample. The predicted ratio varies from $3\%$ to $6\%$ as $x$ increases from $0.5 \cdot 10^{-3}$ to $x = 3 \cdot 10^{-3}$. Our previously reported prediction from the RAPGAP generator [1] including both direct and resolved photon processes was that about $1\%$ of the total forward jet sample contains two forward jets. This is about a factor of 3 lower than the prediction from the BFKL calculations but a large part of this discrepancy could be due to hadronization effects which would reduce the prediction of the parton level BFKL calculation.

Recently this ratio has been measured by the H1 experiment [5] to be $1\%$, in excellent agreement with the prediction of RAPGAP. This gives further confidence in the basic concept of resolved photons even at large $Q^2$.

The ZEUS collaboration has presented a measurement of the forward jet cross section as a function of $p_T^2/Q^2$ [20] in the kinematic region $Q^2 > 10 \text{ GeV}^2$, $y > 0.1$, $E_e > 10 \text{ GeV}$, $\eta_{\text{jet}} < 2.6$, $x_{\text{jet}} > 0.036$, $E_{T,\text{jet}} > 5 \text{ GeV}$, $p_{z,\text{breit}} > 0 \text{ GeV}$, $2.5 \cdot 10^{-4} < x < 8 \cdot 10^{-2}$ but without implementing the DGLAP suppression cut $0.5 < E_T^2/Q^2 < 2$. The results are compared to the predictions from different Monte Carlo programs, and the conclusion is that only the RAPGAP DIS generator including interactions through resolved virtual photons can describe the data over the full range in $E_T^2/Q^2$. In Fig. 5a the ZEUS results are shown together with the prediction of RAPGAP and in Fig. 5b the contributions coming from the hadronic and the pointlike part of the virtual photon structure function are presented separately.

4 Summary and Discussion

Recent experimental data on forward jet production show deviations from traditional LO Monte Carlo models assuming directly interacting point-like photons. It is tempting to assume that the observed effects could be explained by BFKL dynamics.

In the present study we have shown that the addition of resolved photon processes to the direct interactions in DIS leads to good agreement with the data. This agreement does not depend on any specific choice of scale or tuning of any other parameters in the RAPGAP generator. The best evidence of the universality of this approach is that, with the same parameter setting, it is possible to describe a wide range of other data like the transverse energy flow [21], transverse momentum spectra of single particles [22], the $(2+1)$ jet rate [23] and single inclusive jet cross sections [3], as we have shown in [1].

We have observed that the dominant contributions to the resolved photon processes come from order $\alpha_s^3$ diagrams with the hard subprocess $q\gamma g \rightarrow qg$ (see Fig. 1). Since the partons which form the photon remnant per definition have smaller $p_T$ than the partons involved in the hard scattering, a situation with non $q_T$ ordering is created.
Figure 5: The cross section of forward jets as a function of $E_T^2/Q^2$. In a. is shown the measurement of ZEUS [20] and the prediction of RAPGAP. The solid line shows the sum of direct and resolved virtual photon contributions, whereas the dashed line shows the resolved photon contribution alone. In b. is shown the part of the cross section coming from the anomalous component (solid line) and the one coming from the hadronic component of the virtual photon separately.

In the LO DIR model the ladder of gluon emissions is governed by DGLAP dynamics giving a strong ordering of $q_t$ for emissions between the photon and the proton vertex. The models describing resolved photon processes and BFKL dynamics are similar in the sense that both lead to a breaking of this ordering in $q_t$. The BFKL picture, however, allows for complete dis-ordering in $q_t$, while in the resolved photon case the DGLAP ladder is split into two shorter ladders, one from the hard subsystem to the proton vertex, and one
to the photon vertex, each of them ordered in $q_t$ (see Fig. 1). Only if the ladders are long enough to produce additional hard radiation it might be possible to separate resolved photon processes from processes governed by BFKL dynamics. Thus the resolved photon approach may be a “sufficiently good” approximation to an exact BFKL calculation and the two approaches may prove indistinguishable within the range of $x$ accessible at HERA.

It should be emphasized again that a NLO calculation assuming point-like virtual photons contains a significant part of what is attributed to the resolved structure of the virtual photon in the RES model [18].

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