Modelling of stress fields during LFEM DC casting of aluminium billets by a meshless method

B Mavrič and B Šarler

1 Institute of Metals and Technology, Lepi pot 11, SI-1000 Ljubljana, Slovenia
2 University of Nova Gorica, Vipavska 13, SI-5000 Nova Gorica, Slovenia

Abstract. Direct Chill (DC) casting of aluminium alloys is a widely established technology for efficient production of aluminium billets and slabs. The procedure is being further improved by the application of Low Frequency Electromagnetic Field (LFEM) in the area of the mold. Novel LFEM DC processing technique affects many different phenomena which occur during solidification, one of them being the stresses and deformations present in the billet. These quantities can have a significant effect on the quality of the cast piece, since they impact porosity, hot-tearing and cold cracking. In this contribution a novel local radial basis function collocation method (LRBFCM) is successfully applied to the problem of stress field calculation during the stationary state of DC casting of aluminium alloys. The formulation of the method is presented in detail, followed by the presentation of the tackled physical problem. The model describes the deformations of linearly elastic, inhomogeneous isotropic solid with a given temperature field. The temperature profile is calculated using the in-house developed heat and mass transfer model. The effects of low frequency EM casting process parameters on the vertical, circumferential and radial stress and on the deformation of billet surface are presented. The application of the LFEM appears to decrease the amplitudes of the tensile stress occurring in the billet.

1. Introduction

The stress fields during the DC casting process have important influence on the quality of the obtained billet. They can cause large deformations which influence the heat transfer and can lead to residual stress. Unwanted hot tearing and cracking [1] can occur if certain conditions are met [2], causing whole billet to be scrapped. Numerical models of the casting process, which allow us to develop better understanding of the stress fields during DC casting, are therefore of great importance for producing high quality aluminium billets.

Mathematical modelling of casting processes has been of great interest for many years and is reasonably well established [2,3]. Its developments mainly rely on the ever more sophisticated physical modelling, fast development of computer hardware and development of efficient numerical methods, especially the finite element method (FEM). The use of this method can be traced from the first modelling attempts [4,5] up to the sophisticated contemporary achievements [1,6]. In parallel, many new numerical methods have been developed, which aim to address various deficiencies of FEM. One class of the promising new methods are the meshless methods, where the space discretisation (meshing) is much simplified by removing the need for time consuming polygonization of the computational domain and circumventing the mesh distortion problems. The absence of the...
need for polygonization means that the nodes can be removed or added as necessary to achieve the highest possible computational efficiency at demanded precision and also that the complex geometries can be described with ease.

Local Radial Basis Function Collocation Method (LRBFCM) [7] is applied, as an alternative to FEM, to the problem of stress formation during Low-Frequency Electromagnetic Direct Chill (LFEM DC) casting in this paper. The considered alternative method interpolates the unknown solution over the localized subsets of nodes by using radial basis functions (RBF). The RBF interpolant is then used to estimate the differential operators present in the strong form of the elastostatic governing equation. This method has been already successfully applied to diffusion [7], solidification [8] and fluid flow problems, both with and without the presence of external electromagnetic fields [9]. Previous applications of the proposed method to thermoelasticity are rare and mainly focused on simple academic solid mechanics problems [10,11]. Present paper deals with application of the method in modelling of the industrial LFEM DC casting of cylindrical and axisymmetric aluminium billets and complements our previous fluid mechanics developments [12–14].

2. Physical model

Although in literature many examples of sophisticated thermomechanical physical models of DC casting can be found [4,15,16], the model used in this article is a simple one, allowing us to focus on the methodology. A linear thermoelastic model, incorporating the stress equilibrium, Hooke’s law for inhomogeneous isotropic body and linear isotropic thermal expansion is considered. Since the DC casting is assumed to be a stationary process, boundary value problem is considered. The stationary governing equation, stated in terms of the deformation field $\mathbf{u}$, is given as

$$G \nabla^2 \mathbf{u} + (G + \lambda) \nabla \cdot \mathbf{u} + \nabla \alpha \nabla T + \nabla G (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) = \nabla \left( \beta (T - T_{\text{ref}}) \right) - \mathbf{f}. \quad (1)$$

In the governing equation $G$ stands for shear modulus, $\lambda$ for Lamé parameter and $\mathbf{f}$ for the body force. The coupling with the temperature field is described by the coefficient $\beta$ defined as $\beta = (3\lambda + 2G)\alpha$, where $\alpha$ is the coefficient of linear thermal expansion. $T_{\text{ref}}$ stands for reference temperature at which the thermal expansion is considered to be zero. All the material properties are allowed to vary over the computational domain in continuous manner.

For the description of the problem setting, the displacement, symmetry and traction boundary conditions are needed. The deformation of the top part of the billet is restricted by the mold. The boundary conditions in this area are nonlinear and are therefore satisfied by iteration.

In the computational domain, the material undergoes solidification and therefore significantly changes the elastic properties. The temperature dependence of alloy properties can be obtained from JMatPro database for each alloy considered [17].

3. Method formulation

3.1. Domain decomposition

In the present work, only regular geometries are considered, which allows for a regular node arrangement on the computational domain and its boundary. For each interior node $l$, a domain of influence is determined by a predefined constant number $N$ of nearby nodes. In this work nearest neighbor search is used. By the choice of the domain of influence, a mapping $s(i) : i \rightarrow l$ from subdomain node index $i$, to the global index $l$ is defined.

3.2. Construction of the interpolant

Over each domain of influence defined in the previous step, the RBF interpolant is constructed. We are using multiquadric RBFs, defined as
Here $\epsilon$ is the shape parameter and $r_j$ is the position of the node on which the basis function is centred.

The scale of the basis function is given by the subdomain size $h_i$, defined as

$$h_i = \sqrt{\frac{\sum_{l=1}^{N} |r_{m(l)} - r_i|}{jN - 1}}.$$

The interpolation basis is expanded by constant and linear augmentation monomials. The interpolant is formally given as

$$u_x(r) = \sum_{i=1}^{N} \alpha_i \Phi_{r_i}(r) + \sum_{i=N+1}^{3N+2} j \alpha_i \psi_i(r) = \sum_{i=1}^{3N+2} j \alpha_i \psi_i(r),$$

(2)

where $a_{N}$ is the number of monomials used and has for two-dimensional problems value 3. The expansion coefficients $\alpha_i$, $\psi_i$ are determined by collocation.

In this step also the boundary conditions are taken into account. The boundary conditions are assumed to be linear and given by $B_{\gamma_i} u_x(r_j) = b_{\gamma_i}^j$. In case an interior point has a member of influence domain that lies on the boundary, the interpolation equation for this node is replaced by the appropriate boundary condition. The system of equations obtained in this manner can be compactly written in matrix form as

$$\sum_{i=1}^{jN} jA_{\gamma_i \Delta\xi} \alpha_i = j\gamma_{\Delta\xi},$$

with the right-hand side vector $j\gamma$ and the collocation matrix $jA$, given by

$$jA_{\gamma_i \Delta\xi} = \begin{cases} 
\Psi_{\gamma_i}(r_{i\gamma(j)}) \delta_{\gamma i} & \text{if } r_{i\gamma(j)} \in \Omega \\
B_{\gamma_i}(r_{i\gamma(j)}) \Psi_{\gamma_i}(r_{i\gamma(j)}) & \text{if } r_{i\gamma(j)} \in \Gamma,
\end{cases} \quad \gamma_{j\Delta\xi} = \begin{cases} 
u_x(r_{i\gamma(j)}) & \text{if } r_{i\gamma(j)} \in \Omega \\
b_{\gamma_i}(r_{i\gamma(j)}) & \text{if } r_{i\gamma(j)} \in \Gamma,
\end{cases} \quad 0 \quad \text{if } j > jN$$

3.3. Discretization of the governing equation

The interpolation of the field given in node points can be used to estimate differential operators. Since the expansion coefficients are assumed to be constant, the differential operators act only on the basis functions. This fact can be used to discretize partial differential equations. Let us assume a governing equation is given by $\nabla^2 u_x = g_x$. By replacing the unknown solution $u$ with the interpolation given by equation (2), the governing equation at every interior node can be stated in terms of the interpolation coefficients. The interpolation coefficients are further replaced by the components of vector $j\gamma$, thus expressing the governing equation at every interior node by the unknown solution values and given boundary conditions for nodes that belong to the influence domain, centred on the considered node. The resulting governing equation is for each interior node $l$ stated as

$$\sum_{k_{l\xi}} \gamma^l_{kl} \mathbf{u}_{l_{k\xi} l_{k\xi}} \sum_{i_{l\xi}} jA^{-1}_{\Delta\xi} D_{\gamma i} \psi_i(r_l) = g_{l\xi} - \sum_{k_{l\xi}} j\gamma_{k\xi} b_{k\xi} \sum_{i_{l\xi}} jA^{-1}_{\Delta\xi} D_{\gamma i} \psi_i(r_l)$$

(3)

In this expression the boundary and the domain indicators $\gamma^l_{k\xi}$ and $\gamma^l_{j\xi}$ are used to achieve efficient notation. The indicators evaluate to one, if the corresponding point $j$ belongs to the set under consideration and to zero otherwise.
The set of linear equations for the unknown solution values stated in (3) is sparse and can be solved efficiently by specialized solvers. The study of numerical performance and detailed statement of the method can be found in our recent publications [18,19].

4. Application to DC casting
The briefly described thermomechanics solver forms a part of the multiphysics and multiscale DC modelling framework, that is being developed in our group [13]. All the models are formulated by meshless methods. Currently, the modelling capabilities include heat and fluid flow, electromagnetic field and species transport. A special Point Automata (PA) meshless technique has been developed for simulation of the microstructure evolution [20]. The LFEM DC casting arrangement for 140 mm billet diameter is modelled from the top of the mould to 80 cm below the mould.

The computational domain is determined from the results of the heat and mass transport model. The computational domain and boundary conditions are given in Figure 1. Only the area below the highest point reached by the liquidus isotherm is considered. On the top surface, the metalostatic pressure is applied. The top part of the domain is constrained by the mould, while the rest of the outer surface is assumed to be free. In the centre, the symmetry boundary conditions are applied, and on the bottom the vertical deformation is set to zero, while zero traction is assumed in radial direction.

The computational domain consists of $10^5$ discretization points arranged in nodes of a rectangular grid. The shape parameter was set to $2 \cdot 10^{-2}$ with the domains of influence consisting of ten nodes. The temperature fields, which form the driving term of the governing equation (1) are obtained by the heat and mass transfer model and are shown in Figure 2 for different casting speeds.

Results obtained for different casting speeds in the absence of the electromagnetic field are shown in Figures 5, 6, and 7. In general features, the structure of the stress fields remains the same. Compressive state forms in the centre of the billet and tensile state occurs in the circumferential direction near the surface. The increase in casting speed only stretches those features along the casting direction due to the change of the sump depth.

More illustrative is the comparison of the surface deformations given in Figure 3. Since the transient zone in which the temperature drops from liquidus temperature to the ambient temperature is the thinnest in the case of the smaller casting speed, also the surface shrinks the fastest. The
differences in surface deformation between the two faster speeds are small; however the rate of shrinkage does not monotonously decrease with increasing speed, since the two curves cross each other. The images illustrating the influence of the casting speed on the radial stress, illustrate large stresses that are present in the contact between the solidifying shell and the mould surface, implying the need for lubrication of the area if high quality surface is required.

The effect of electromagnetic stirring on the stress state is shown in Figures 9 and 8. The small current amplitudes, displayed in this work do not have significant impact on the stress state in the billet. The changes are small but their effect remains the same if the current amplitude and frequency are varied. The application of the electromagnetic fields results in the increase of all three stress components in the centre of the billet. This increase reduces the magnitude of radial and vertical stress components and increases the value of the circumferential stress in the billet centre. Near the mould contact, the differences of all stress components are negative. This reduces the magnitudes of all three stress components, thus reducing the possibility of cracking or other stress-related defects.

**Figure 2** The temperature field at different casting speeds. From left to right: 90, 80 and 70 mm/min
Figure 3 The deformation of the surface of the billet at different casting speeds.

Figure 4 The dependence of the material properties on the temperature for the alloy AA6082 as given by the JMatPro database.

Figure 5 Radial stress at different casting speeds.

Figure 6 Vertical stress at different casting speeds.
5. Conclusions

A thermoelastic model of LFEMC DC casting of aluminium cylindrical billets, employing the meshless LRBFCM solution procedure has been developed. This method has been most probably for the first time applied to an industry related engineering problem.

The model calculates the stress field for a given thermal profile, which is calculated by a heat and mass transfer model that uses the equivalent meshless numerical method. The resulting stress fields are presented in this paper, together with the demonstration of the effect of the driving current parameters on the stress state. The application of the electromagnetic stirring to the casting process reduces the amplitude of the circumferential and radial stress, thus possibly reducing the cracking probability. The increase in the casting speed stretches the stress field along the casting direction.

The elaborated numerical method is straightforward to implement, can be easily extended to three spatial dimensions, and allows for simple applications to the cases when the material properties continuously vary. Because of the meshless nature of the LRBFCM, many potential improvements can be devised. The method can easily incorporate node refinement and can be used to solve problems defined on more complicated domains.

We next plan to extend the model with a more sophisticated material law, which will incorporate viscoplastic deformation of the mushy zone and plastic deformation of the solidified billet in the future.
6. Acknowledgements
This paper is supported by the Slovenian Funding Agency through grants L2-6775, P2-0379 and Young Researcher Programme.

References
[1] Lalpoor M, Eskin D G, Ruvalcaba D, Fjær H G, Ten Cate A, Ontijt N and Katgerman L 2011 Cold cracking in DC-cast high strength aluminum alloy ingots: An intrinsic problem intensified by casting process parameters Mater. Sci. Eng. A 528 2831–42
[2] Eskin D G 2008 Physical Metallurgy of Direct Chill Casting of Aluminum Alloys (Taylor & Francis)
[3] Grandfield J 2013 Direct-chill casting of light alloys: science and technology (Hoboken, New Jersey: TMS-Wiley, a John Wiley & Sons, Inc. Publication)
[4] Fjaer H G and Mo A 1990 ALSPEN-A mathematical model for thermal stresses in direct chill casting of aluminum billets Metall. Trans. B 21 1049–61
[5] Drezet J M, Rappaz M and Krähenbühl Y 1995 Thermomechanical effects during direct chill and electromagnetic casting of aluminum alloys. Part II: numerical simulation (TMS Publ.)
[6] Kumar P P, Nallathambi A K, Specht E and Bertram A 2011 Mechanical Behavior of Mushy Zone in DC casting using a Viscoplastic Material Model Tech. Mech. 32 342–57
[7] Šarler B and Vertnik R 2006 Meshfree explicit local radial basis function collocation method for diffusion problems Comput. Math. Appl. 51 1269–82
[8] Kosec G and Šarler B 2014 Simulation of macrosegregation with mesosegregates in binary metallic casts by a meshless method Eng. Anal. Bound. Elem. 45 36–44
[9] Mramor K, Vertnik R and Šarler B 2013 Simulation of natural convection influenced by magnetic field with explicit local radial basis function collocation method CMES Comput. Model. Eng. Sci. 92 327–52
[10] Simonenko S, Bayona V and Kindelan M 2014 Optimal shape parameter for the solution of elastostatic problems with the RBF method J. Eng. Math. 85 115–29
[11] Lee C K, Liu X and Fan S C 2003 Local multiquadric approximation for solving boundary value problems Comput. Mech. 30 396–409
[12] Vertnik R and Šarler B 2009 Solution of incompressible turbulent flow by a mesh-free method Comput. Model. Eng. Sci. CMES 44 65
[13] Košnik N, Vertnik R and Šarler B 2014 Simulation of low frequency electromagnetic DC casting Mater. Sci. Forum 790-791 390–5
[14] Mramor K, Vertnik R and Šarler B 2013 Low and intermediate Re solution of lid driven cavity problem by local radial basis function collocation method CMES Comput. Model. Eng. Sci. 92 327–52
[15] M’Hamdi M, Mo A and Fjaer H G 2006 TearSim: A two-phase model addressing hot tearing formation during aluminum direct chill casting Metall. Mater. Trans. A 37 3069–83
[16] Nallathambi A K, Bertram, Albrecht, Specht, Eckehard, Tyagi M, Specht E and Bertram A 2011 Thermomechanical simulation of direct chill casting Trans. Indian Inst. Met. 64 13–9
[17] Saunders N, Guo U K Z, Li X, Miodownik A P and Schillé J-P 2003 Using JMatPro to model materials properties and behavior JOM 55 60–5
[18] Mavrič B and Šarler B 2014 A collocation meshless method for linear thermoelasticity in 2D ThermaComp20014 (Bled) pp 279–82
[19] Mavrič B and Šarler B Local radial basis function collocation method for linear thermoelasticity in two dimensions Int. J. Numer. Methods Heat Fluid Flow (In review)
[20] Lorbiecka A Z and Šarler B 2010 Simulation of dendritic growth with different orientation by using the point automata method Comput. Mater. Contin. CMC 18 69