A comparison between the $P_c$ and $P_{cs}$ systems

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We construct the effective potentials of the $P_c$ and $P_{cs}$ states based on the SU(3)$_f$ symmetry and heavy quark symmetry. Then we perform the coupled-channel analysis of the lowest isospin $P_c$ and $P_{cs}$ systems. The coupled-channel effects play different roles in the $P_c$ and $P_{cs}$ systems. In the $P_c$ systems, this effect gives minor corrections to the masses of the $P_c$ states. In the $P_{cs}$ system, the $\Lambda_c D_s - \Xi_c \bar{D}$ coupling will shift the mass of the $P_{cs}(4338)$ close to the $\Xi_c \bar{D}$ threshold. The $\Lambda_c \bar{D}^{(*)} - \Xi_c \bar{D}^{(*)}$ coupling will also produce extra $P_{cs}$ states. We discuss the correspondence between the $P_c$ and $P_{cs}$ states. Our results prefer that the SU(3) partners of the observed $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ in the $P_{cs}$ system have not been found yet.

I. INTRODUCTION

Very recently, the LHCb Collaboration announced the observation of a $P_{cs}(4338)$ signal from the $J/\Psi \Lambda$ mass spectrum in the $B^- \rightarrow J/\Psi \Lambda \bar{p}$ process [1]. The mass and width of this new pentaquark candidate were measured to be

$$M_{P_{cs}} = 4338.2 \pm 0.7 \pm 0.4 \text{ MeV},$$

$$\Gamma_{P_{cs}} = 7.0 \pm 1.2 \pm 1.3 \text{ MeV}. \quad (1)$$

Meanwhile, the amplitude analysis prefers the $\frac{1}{2}^-$ spin-parity quantum numbers. The central value of the mass of $P_{cs}(4338)$ is above the $\Xi_c \bar{D}$ threshold. Thus, this state can not be directly assigned as the $\Xi_c \bar{D}$ molecular state. However, the authors of Ref. [2] pointed out that the lineshape of this resonance could be distorted from the conventional Breit-Wigner distribution if it lies very close to and strongly couples to the threshold.

Besides the newly observed $P_{cs}(4338)$, the $P_{cs}(4459)$ was observed at LHCb [3] as a candidate of a $\Xi_c \bar{D}^*$ molecular state, which agrees well with the prediction from the chiral effective field theory in Ref. [4]. The strange hidden-charm states were also discussed in Refs. [5–13] and reviewed extensively in Refs. [14–19].

The mass of the $P_{cs}(4459)$ is about 19 MeV below the $\Xi_c \bar{D}$ threshold. In Ref. [20], the author argued that from heavy quark symmetry, the $[\Xi_c \bar{D}]^{1/2}$, $[\Xi_c \bar{D}^*]^{1/2}$, and $[\Xi_c \bar{D}^{(*)}]^{3/2}$ channels should share identical potentials and have comparable binding energies. However, the heavy quark symmetry is drastically violated in the charm system due to the rather small charm quark mass. With the assignment of the $P_{cs}(4338)$ and $P_{cs}(4459)$ as the $[\Xi_c \bar{D}]^{1/2}$ and $[\Xi_c \bar{D}^*]^{1/2}$ ($[\Xi_c \bar{D}^{(*)}]^{3/2}$) molecular states, the degeneracy of the $[\Xi_c \bar{D}]^{1/2}$ and $[\Xi_c \bar{D}^{(*)}]^{3/2}$ channels is removed by the coupled-channel effects and recoil corrections.

Another novel phenomenon from the $M_{J/\Psi \Lambda}$ invariant spectrum [1] is that there seems to be a structure around $M = 4254$ MeV. To understand this signal, the LHCb checked the $m(J/\Psi \Lambda)$ distribution close to the $\Lambda_c^+ D_s^-$ threshold and found that this signal is not statistically significant. Nevertheless, the authors in Ref. [21] investigated the $P_{cs}(4338)$ and $P_{cs}(4255)$ pole positions from a unitary $\Xi_c \bar{D} - \Lambda_c D_s$ coupled-channel scattering amplitude. Besides, the $P_{cs}(4255)$ pole was also found in a model with the coupling between the meson-baryon molecule and the compact five-quark state [22]. The $P_{cs}(4255)$ state was also suggested in an effective field theory framework [23].

The analogy between the observed ($P_c(4312)$, $P_c(4440)$, $P_c(4457)$) [24, 25] and ($P_{cs}(4338)$, $P_{cs}(4459)$) states are discussed in Ref. [26–28]. However, since the $\Sigma_c$ and $\Xi_c$ belong to different SU(3)$_f$ multiplets, the relations between the discussed $P_c$ and $P_{cs}$ states are not clear. Besides, the $P_{N_{cs}}$ pentaquark states as the partners of the $P_c$ and $P_{cs}$ states are investigated in Ref. [29].

If the $P_{cs}$ states and $P_c$ states can be related via SU(3)$_f$ symmetry, it is important to investigate the similarities and differences between these two sets of molecular candidates. In Ref. [30, 31], we discussed the symmetry properties of different heavy flavor molecular systems via a quark level Lagrangian. We proposed that the interactions of different heavy flavor molecules can be related via a generalized flavor-spin symmetry [30]. This framework provides a suitable tool to discuss the similarities between the $P_c$ and $P_{cs}$ states.

We also notice an important difference between the $P_c$ and $P_{cs}$ states. The minimal quark components of the $P_c$ and $P_{cs}$ states are $c\bar{c}u$ and $c\bar{c}d$ ($n = u$, $d$), respectively. For the charmed/charmed-strange mesons and baryons, the SU(3)$_f$ symmetry breaking effects are reflected on their physical masses, and we need to distinguish the $s$ quark from $u$, $d$ quarks when we study the $P_{cs}$ systems. Unlike the $P_c$ pentaquarks, the $P_{cs}$ states can couple to two sets of channels, i.e., the cns-cn type and cns-cs type channels. In Table I, we list the possible open-charm channels and their thresholds for the $P_c$ and $P_{cs}$ systems.

In this work, we will take the $P_{cs}(4338)$ as a molecular candidate and discuss the following three issues:

1. Can we understand the minor binding energy of the $P_{cs}(4338)$ (close to the $\Xi_c \bar{D}$ threshold) through a $\Xi_c \bar{D} - \Lambda_c D_s$ coupled-channel effect?

2. Can we produce a $P_{cs}(4254)$ bound state by including the $\Xi_c \bar{D} - \Lambda_c D_s$ coupled-channel effect with the potential constrained from SU(3)$_f$ symmetry?

3. What is the correspondence between the $P_c$ and $P_{cs}$ states if the interactions of the $P_c$ and $P_{cs}$ states obey a generalized flavor-spin symmetry?
TABLE I. The thresholds of the meson-baryon channels associated with the $P_c$ and $P_{cs}$ systems, we adopt the isospin averaged masses for the ground charmed mesons and baryons [32]. All values are in units of MeV.

|       | $P_c$       | $P_{cs}$     |
|-------|-------------|--------------|
| $\Lambda_c D$ | 4153.7     | $\Lambda_c D_s$ | 4255.5 ξ $\bar{c} D$ | 4336.7 |
| $\Lambda_c D^*$ | 4295.0     | $\Lambda_c D_s^*$ | 4398.7 ζ $\bar{c} D^*$ | 4478.0 |
| $\Sigma_c D$ | 4320.8     | $\Sigma_c D_s$ | 4422.5 ξ $\bar{c} D$ | 4466.0 |
| $\Sigma_c D^*$ | 4385.4     | $\Sigma_c D_s^*$ | 4487.1 ζ $\bar{c} D^*$ | 4513.2 |
| $\Sigma_s D$ | 4461.2     | $\Sigma_s D_s$ | 4565.7 η $\bar{c} D^*$ | 4587.4 |
| $\Sigma_s D^*$ | 4526.7     | $\Sigma_s D_s^*$ | 4630.3 ζ $\bar{c} D^*$ | 4654.5 |

This paper is organized as follows. We present our theoretical framework in Sec. II and the corresponding numerical results and discussions in Sec. III. Sec. IV is the summary.

II. FRAMEWORK

In Ref. [31], we proposed an isospin criterion and pointed out that the $P_c$ and $P_{cs}$ states with the lowest isospin numbers are more likely to form bound states. Based on the same Lagrangian, we only focus on the $P_c$ and $P_{cs}$ states with isospin numbers $I = 1/2$ and 0, respectively. Thus, we will not include the $\Sigma_c(1^+) D_s(1^-)$ channels listed in Table I for the $P_{cs}$ system.

For the $I = 1/2 P_c$ states, we consider the following channels for the $J = 1/2$ and 3/2 states

\[ J = \frac{1}{2} : \Lambda_c D, \Lambda_c D^*, \Sigma_c D, \Sigma_c D^*, \Sigma_s D, \Sigma_s D^*, \]

\[ J = \frac{3}{2} : \Lambda_c D^*, \Sigma_c D^*, \Sigma_s D^*, \Sigma_s D^*, \]

Similarly, for the $I = 0 P_{cs}$ states, we include the following channels for the $J = 1/2$ and 3/2 states

\[ J = \frac{1}{2} : \Lambda_c D_s, \Lambda_c D_s^*, \Xi_c D, \Xi_c D^*, \Xi_s D, \Xi_s D^*, \eta_s D^*, \]

\[ J = \frac{3}{2} : \Lambda_c D_s^*, \Xi_c D^*, \Xi_s D^*, \eta_s D^*, \]

The result of the $P_c$ ($P_{cs}$) state with $J = 5/2$ can be obtained from a single-channel calculation and was predicted in Ref. [31] in the same framework. Thus, we will not discuss them further in this work.

A. Lagrangians for the baryon-meson systems

To describe the $S$-wave interactions between the ground charmed/charmed-strange baryons and mesons, we introduce the following quark-level Lagrangian [4, 31, 33, 34]

\[ \mathcal{L} = g_s \bar{q} S q + g_a \bar{q} \gamma_\mu \gamma_5 A^\mu q. \]

Here, $q = (u, d, s)$, $g_s$ and $g_a$ are two independent coupling constants that describe the interactions from the exchanges of the scalar and axial-vector meson currents. They encode the nonperturbative low energy dynamics of the considered heavy flavor meson-baryon systems.

From this Lagrangian, the effective potential of the light quark-quark interactions reads

\[ \mathcal{V} = \bar{g}_s \lambda_1 \cdot \lambda_2 + \bar{g}_a \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2. \]

Here,

\[ \lambda_1 \cdot \lambda_2 = \lambda_1^i \lambda_2^i + \lambda_1^i \lambda_2^i + \lambda_1^i \lambda_2^i, \]

where $i$ and $j$ sum from 1 to 3 and 4 to 7, respectively. The operators $\lambda_1^i \lambda_2^i (\lambda_1^i \lambda_2^i, \sigma_1 \cdot \sigma_2, \lambda_1^i \lambda_2^i (\sigma_1 \cdot \sigma_2))$ arise from the exchanges of the isospin singlet, triplet, and two doublets light scalar (axial-vector) meson currents, respectively. The redefined coupling constants are $\bar{g}_s \equiv g_s^2/m_s^2$ and $\bar{g}_a \equiv g_a^2/m_a^2$.

The Lagrangian in Eq. (7) allows the exchanges of two types of scalar and axial-vector mesons that have quantum numbers $I(J^P) = (0^+, 1^0)$, $1/2(0^+)$ and $I(J^P) = (1^+, 1^+), 1/2(1^+)$, respectively. At present, we can not specifically pin down the coupling parameter of each exchanged meson in the above six meson currents. Alternatively, since the mesons in each meson current have identical interacting Lorentz structure, we use the coupling constant $\bar{g}_s (\bar{g}_a)$ to collectively absorb the total dynamical effects from the exchange of each scalar (axial-vector) meson current. In addition, the couplings $\bar{g}_s (\bar{g}_a)$ for the scalar (axial-vector) meson currents with different isospin numbers are the same in the SU(3) limit.

The effective potential between the $i$-th baryon-meson channel $B_i M_i$ and the $j$-th baryon-meson-channel $B_j M_j$ with total isospin $I$ and total angular momentum $J$ can be calculated as

\[ v_{ij} = \langle [B_i M_i]_j [V [B_j M_j]_j] \rangle. \]

Here, the $\langle [B_i M_i]_j \rangle$ is the quark-level flavor-spin wave function of the considered $i$-th channel baryon-meson system

\[ \langle [B_i M_i]_j \rangle = \sum_{m_{1_i}, m_{1_j}} C_{1_i m_{1_i} 1_j m_{1_j}} B_i^{1_j} M_i^{1_j} \phi_{1_i, m_{1_i}} \phi_{1_j, m_{1_j}} \]

\[ \otimes \sum_{m_{2_i}, m_{2_j}} C_{S_{1_i} m_{2_i} S_{2_i} m_{2_j}} \phi_{S_{1_i}, m_{2_i}} \phi_{S_{2_i}, m_{2_j}}. \]

In Eq. (11), the $\phi_{S_{1_i}, m_{2_i}}$ and $\phi_{S_{2_i}, m_{2_j}}$ are the spin wave functions of the baryon and meson, respectively. The total spin wave function can be obtained with the help of SU(2) CG coefficient $C_{S_{1_i} m_{2_i} S_{2_i} m_{2_j}}$. For the flavor wave functions of the considered baryons ($\phi_{B_i} B_i$) and mesons ($\phi_{M_i} M_i$), their explicit forms have been given in Ref. [30]. When constructing the total flavor wave functions of the considered baryon-meson systems, we use the SU(2) CG coefficient and take the $s$ quark as a flavor singlet.
The coupled-channel Lippmann-Schwinger equation (LSE) reads

$$T(E) = \mathbb{V} + \mathbb{V}G(E)T(E),$$

(12)

with

$$\mathbb{V} = \begin{pmatrix}
v_{11} & \cdots & v_{1i} & \cdots & v_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
v_{i1} & \cdots & v_{ji} & \cdots & v_{jn} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
v_{n1} & \cdots & v_{ni} & \cdots & v_{nn}
\end{pmatrix},$$

(13)

$$T(E) = \begin{pmatrix}
t_{11}(E) & \cdots & t_{1i}(E) & \cdots & t_{1n}(E) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
t_{i1}(E) & \cdots & t_{ji}(E) & \cdots & t_{jn}(E) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
t_{n1}(E) & \cdots & t_{ni}(E) & \cdots & t_{nn}(E)
\end{pmatrix},$$

(14)

and

$$\mathbb{G}(E) = \text{diag}\{G_1(E), \ldots, G_i(E), \ldots, G_n(E)\}.$$ 

(15)

Here,

$$G_i = \frac{1}{2\pi^2} \int dq \frac{q^2}{E - \sqrt{m_i^2 + q^2} - \sqrt{m_j^2 + q^2}}.$$ 

(16)

The $m_{i1}$ and $m_{i2}$ are the masses of the baryon and meson in the $i$-th channel, respectively. In our previous work [30, 31], we use a step function to exclude the contributions from higher momenta to perform the single-channel calculation. In the coupled-channel case, we need to further suppress the contributions from the channels that are far away from the thresholds of the considered channels. Thus, we introduce a dipole form factor $u(\Lambda) = (1 + q^2/\Lambda^2)^{-2}$ with regular parameter $\Lambda = 1.0$ GeV [21, 35, 36].

The pole position of Eq. (12) satisfies $||1 - \mathbb{V}\mathbb{G}|| = 0$. For the bound state below the lowest channel, we search the bound state solution in the first Riemann sheet of the lowest channel. For the quasi-bound state between the thresholds of the $i$-th and $j$-th channels, we adopt the complex scaling method and replace the integration variable $q$ by $g \rightarrow g \times \exp(-i\theta)$ and maintain $0 < \theta < \pi/2$ to find the quasi-bound state solution in the first Riemann sheet of the higher $j$-th channel and the second Riemann sheet of the lower $i$-th channel. [37].

### III. NUMERICAL RESULTS

#### A. Determination of $\tilde{g}_s$ and $\tilde{g}_a$

We first determine the parameters $\tilde{g}_s$ and $\tilde{g}_a$ in our model. We collect the matrix elements of $\langle \lambda_1 \cdot \lambda_2 \rangle$, $\langle \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 \rangle$ for the $P_c$ and $P_{cs}$ states in Tables II and III, respectively.

We can directly obtain the effective potentials associated with the $P_c$ and $P_{cs}$ states from Tables II and III, respectively.

For example, the explicit form of the effective potential matrix for the $J = 3/2$ $P_c$ states is

$$\mathbb{V}_{3/2}^{P_c} = \begin{pmatrix}
\frac{2}{3}g_s & -2\sqrt{3}g_a & 2g_a & 2\sqrt{5}g_a \\
-2\sqrt{3}g_a & -\frac{10}{3}g_s & -\frac{18}{5}g_a & -\frac{10}{3}g_a \\
2g_a & -\frac{10}{3}g_s & -\frac{10}{5}g_a & -\frac{20}{9}g_a \\
2\sqrt{5}g_a & -\frac{10}{5}g_s & \frac{10}{5}g_a & -\frac{10}{9}g_a + \frac{20}{9}g_a
\end{pmatrix}. $$

(17)

Similarly, the effective potential matrixes $\mathbb{V}_{1/2}^{P_c}$, $\mathbb{V}_{3/2}^{P_c}$, and $\mathbb{V}_{3/2}^{P_{cs}}$ can also be obtained directly from Table II and III.

We use the masses of the observed $P_c$ states as input to determine the coupling constants $\tilde{g}_s$ and $\tilde{g}_a$. In our previous work, we find that the Lagrangian in Eq. (7) can give a satisfactory description of the observed $T_{cc}$ [38, 39], $P_c$, and $P_{cs}$ states if we assign the $P_c(4440)$ and $P_c(4457)$ as the $I(J^P) = 1/2(1/2^-)$ and $1/2(3/2^-)$ states. For consistency, we still adopt this set of assignments and use the masses of the $P_c(4440)$ and $P_c(4457)$ as inputs. In the coupled-channel formalism, the bound/quasi-bound states in the $J^P = 1/2^-$ and $3/2^-$ $P_c$ systems satisfy the following equations

$$\text{Re} \left| 1 - \mathbb{V}_{1/2}^{P_c} G_{1/2}^{P_c} \right| = 0,$$

(18)

$$\text{Im} \left| 1 - \mathbb{V}_{1/2}^{P_c} G_{1/2}^{P_c} \right| = 0,$$

(19)

$$\text{Re} \left| 1 - \mathbb{V}_{3/2}^{P_c} G_{3/2}^{P_c} \right| = 0,$$

(20)

$$\text{Im} \left| 1 - \mathbb{V}_{3/2}^{P_c} G_{3/2}^{P_c} \right| = 0.$$ 

(21)

These four equations can be solved numerically and we get 

$$\tilde{g}_s = 8.28 \text{ GeV}^{-2}, \quad \tilde{g}_a = -1.46 \text{ GeV}^{-2}. $$

(22)

The imaginary part of the pole positions of the $P_c(4440)$ and $P_c(4457)$ can also be obtained from Eqs. (18-21).

#### B. Flavor-spin symmetry of the $P_c$ and $P_{cs}$ systems in the single-channel formalism

With the determined parameters $\tilde{g}_s$ and $\tilde{g}_a$, we first present our single-channel results for the considered $P_c$ and $P_{cs}$ systems and demonstrate that we can relate the $P_c$ and $P_{cs}$ systems from their interactions constrained by the SU(3) and heavy quark symmetries.

Although the $\Xi, D$ and $\Sigma, \bar{D}$ belong to different multiplets, in Ref. [30] we proposed that there exists a generalized flavor-spin symmetry between two-body heavy-flavor systems. For two different heavy-flavor meson-baryon systems, if they both possess the same flavor ($\langle H^+_1 H^+_2 | \lambda_1 \cdot \lambda_2 | H^-_1 H^-_2 \rangle$) and spin ($\langle H^+_1 H^+_2 | \sigma_1 \cdot \sigma_2 | H^-_1 H^-_2 \rangle$) matrix elements, they will still have identical effective potentials in the SU(3) and heavy quark limits.
In the single-channel formalism, we present the masses and binding energies of the $P_c$ and $P_{cs}$ states in Table IV. The theoretical uncertainties are introduced by considering the experimental errors of the masses of the $P_c(4440)$ and $P_c(4457)$. We collect the $P_c$ and $P_{cs}$ states that share identical effective potentials in the same row. As listed in Table IV, the $P_c$ and $P_{cs}$ states have similar binding energies in the same row and can be related via a flavor-spin symmetry.

### C. The masses of $P_c$ states in the multi-channel formalism

Then we explore how the coupled-channel effect influences the masses of the $P_c$ states. As can be seen from Eq. (8), the effective potential consists of two parts, i.e., the central term ($g_s \lambda_1 \cdot \bar{\lambda}_2$) and the spin-spin interaction ($g_s \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2$) term. Since the determined $g_s$ is much larger than $g_0$, the central term dominates the total effective potential, and therefore determines whether the considered system can form a bound state.

As given in Table II, for the matrix elements in the $P_c$ system, all the diagonal matrix elements have central terms, and some of them have corrections from the spin-spin interaction terms. The off-diagonal terms only consist of the spin-spin interaction terms. Thus, before we perform a practical multi-channel calculation of the $P_c$ system, we may anticipate that the coupled-channel effect would have small corrections to the masses of the $P_c$ states.

As discussed in Sec. II, we include five and four channels to study the $J = 1/2$ and $J = 3/2$ $P_c$ states, respectively. For the $J = 1/2$ channels, according to their thresholds, we consider five energy regions

\[ E \leq m_{\Lambda_c D^*} \]

\[ m_{\Lambda_c D^*} < E \leq m_{\Sigma_c D^*} \]

\[ m_{\Sigma_c D^*} < E \leq m_{\Sigma_c \bar{D}}^* \]

\[ m_{\Sigma_c \bar{D}}^* < E \leq m_{\Sigma_c \bar{D}^*} \]

We search the bound (quasi-bound) state solutions below the higher threshold in each energy region on the first Riemann sheet. The bound (quasi-bound) state solutions of the $J = 3/2$ $P_c$, $J = 1/2$ and $3/2$ $P_{cs}$ states can be found by repeating the same procedure. We present the obtained $P_c$ states in Table V. We do not find any bound states below the $\Lambda_c \bar{D}$ threshold. Thus, all the obtained resonances ($E_{R}$) listed in Table V should refer to quasi-bound states and have imaginary parts ($\Im(E_{R})$). Since we only include the two body open-charm decay channels, the estimated widths ($\Gamma$) in Table V are smaller than experimental widths. By comparing the masses of $P_c$ states in Table IV and V, we find that the coupled-channel effect indeed have small influences to the masses of the $P_c$ states.
TABLE IV. In the single-channel formalism, the binding energies of the $P_c$ and $P_{cs}$ states that share the same effective potentials in the SU(3) and heavy quark limits. All results are in units of MeV.

| $P_\xi$ | Mass | BE  | $P_{cs}$ | Mass | BE  | $\text{V}$ |
|---------|------|-----|---------|------|-----|----------|
| $[\Xi_0 \bar{D}]_{\frac{1}{2}}$ | $^{+4.1}_{-2.6}$ | $-8.1^{+4.1}_{-2.6}$ | $[\Xi_0 \bar{D}]_{\frac{3}{2}}$ | $^{+4.1}_{-2.7}$ | $-8.2^{+4.1}_{-2.7}$ |
| $[\Sigma_0 \bar{D}]_{\frac{1}{2}}$ | $^{+4.2}_{-2.7}$ | $-8.5^{+4.2}_{-2.7}$ | $[\Xi'_0 \bar{D}]_{\frac{5}{2}}$ | $^{+4.5}_{-2.8}$ | $-8.8^{+4.3}_{-2.8}$ |
| $[\Sigma_0 \bar{D}]_{\frac{3}{2}}$ | $^{+4.2}_{-2.7}$ | $-8.5^{+4.2}_{-2.7}$ |

TABLE V. The results of $P_c$ states obtained in the coupled-channel formalism. Here, $\Gamma = -2\text{Im}(E_R)$, all the results are in units of MeV.

| $\text{State}$ | $J^P$ | Mass | $\Gamma$ | Mass | Width |
|--------------|-------|------|---------|------|-------|
| $P_c(4312)$ | $\frac{1}{2}^-$ | 4308.21$^{+2.6}_{-4.5}$ | 2.6$^{+2.4}_{-1.7}$ | 4311.9$^{+7.0}_{-10.9}$ | 10 $\pm$ 5 |
| $P_c(4440)$ | $\frac{1}{2}^-$ | 4440.3$^{+4.0}_{-5.0}$ (input) | 9.8$^{+4.6}_{-5.8}$ | 4440.3$^{+4.0}_{-5.0}$ | 21$^{+10}_{-11}$ |
| $P_c(4547)$ | $\frac{1}{2}^-$ | 4457.7$^{+4.0}_{-5.0}$ (input) | 2.0$^{+1.4}_{-0.8}$ | 4457.3$^{+4.0}_{-5.0}$ | 6.4$^{+5.4}_{-2.2}$ |
| $P_c(4380)$ | $\frac{1}{2}^-$ | 4373.3$^{+3.4}_{-6.8}$ | 5.2$^{+2.0}_{-2.5}$ | – | – |
| $P_c(4500)$ | $\frac{1}{2}^-$ | 4501.4$^{+5.0}_{-6.2}$ | 8.8$^{+17.2}_{-5.9}$ | – | – |
| $P_c(4510)$ | $\frac{1}{2}^-$ | 4513.4$^{+5.8}_{-3.4}$ | 7.6$^{+9.4}_{-0.0}$ | – | – |

D. A numerical experiment on the $(\Lambda_c \bar{D}^{(*)}, \Xi_c \bar{D}^{(*)})$ coupled-channel systems

There exists an important difference between the effective potential matrices in the $P_c$ and $P_{cs}$ systems. As presented in Tables II and III, the diagonal matrix elements in the $P_{cs}$ system are very similar to those of the $P_c$ system. But for the off-diagonal matrix elements, the effective potentials of the $\Lambda_c \bar{D}_s - \Xi_c \bar{D}$ and $\Lambda_c \bar{D}^*_s - \Xi_c \bar{D}^*$ channels in the $P_{cs}$ system with $J = 1/2$ or $3/2$ consist of central terms. These terms may give considerable corrections to the spectrum of the $P_{cs}$ states.

For the $J = 1/2$ and $J = 3/2$ $P_{cs}$ systems, as given in Eq. (5-6) we need to perform seven and five coupled-channel calculations. Before we perform such complete calculations, we first perform a detailed discussion on the $(\Lambda_c \bar{D}_s, \Xi_c \bar{D})$ and $(\Lambda_c \bar{D}^*_s, \Xi_c \bar{D}^*)$ coupled-channel systems.

The effective potential matrices of the $J = 1/2$ ($\Lambda_c \bar{D}_s, \Xi_c \bar{D}$), ($\Lambda_c \bar{D}^*_s, \Xi_c \bar{D}^*$) systems, and the $J = 3/2$ ($\Lambda_c \bar{D}^*_s, \Xi_c \bar{D}^*$) system share the same expressions in the heavy quark limit. From Table III, we obtain the corresponding effective potential matrix

$$
\Psi = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}
$$

with

$$
v_{11} = -\frac{4}{3} \tilde{g}_s, \quad v_{22} = -\frac{10}{3} \tilde{g}_s
$$

$$
v_{12} = v_{21} = 2\sqrt{2} \tilde{g}_s \tilde{g}_x.
$$

Here, for the diagonal matrix elements listed in Table II and III, their dominant components are from the exchange of the non-strange light scalar meson currents. Since the interactions of the off-diagonal channel $\Lambda_c \bar{D}^{(*)} - \Xi_c \bar{D}^{(*)}$ are introduced via the exchange of the strange scalar meson currents, we further introduce a factor $g_x$ to estimate the SU(3) breaking effects. Compared with the exchange of the non-strange light scalar meson currents, the off-diagonal matrix elements should be suppressed by the mass of strange mesons. Thus, we assume $0 \leq g_x \leq 1$. This factor also reflects the coupling strength of the $\Lambda_c \bar{D}_s - \Xi_c \bar{D}$ channel. With $g_x = 0$, the $\Lambda_c \bar{D}^{(*)}$ does not couple to the $\Xi_c \bar{D}^{(*)}$ channel. With $g_x = 1$, the $\Lambda_c \bar{D}^{(*)}$ couples to the $\Xi_c \bar{D}^{(*)}$ channel and its coupling strength is set to be the value in the SU(3) limit.

In Fig. 1 (b), we present the variation of the masses for the bound states $P_{cs}(4338)$ and $P_{cs}(4255)$ as the parameter $g_x$ increases. The masses of the $P_{cs}(4338)$ and $P_{cs}(4255)$ are denoted with black lines. At $g_x = 0$, the $\Lambda_c \bar{D}_s$ channel itself has a weak attractive force $v_{11} = -4/3 \tilde{g}_s$, and this force is too weak to form a $\Lambda_c \bar{D}_s$ bound state. On the contrary, the $\Xi_c \bar{D}$ channel can form a bound state and its mass is about $M = 4329$ MeV, slightly smaller than the experimental value. As the $g_x$ increases, the attractive force of the $P_{cs}(4338)$ decreases and its mass moves closer to the $\Xi_c \bar{D}$ threshold. In a very narrow region $0.62 \leq g_x \leq 0.64$, the attractive force is just enough to form a $P_{cs}(4338)$ bound state at the $\Xi_c \bar{D}$ threshold and the weak attractive channel $\Lambda_c \bar{D}_s$ starts to form a bound state due to the $\Xi_c \bar{D} - \Lambda_c \bar{D}_s$ coupling. Only in this very narrow region, the $P_{cs}(4338)$ and $P_{cs}(4255)$ can coexist as quasi-bound states. At $g_x > 0.64$, the $\Xi_c \bar{D} - \Lambda_c \bar{D}_s$ coupling further weakens the attractive force of the $\Xi_c \bar{D}$ channel and the $P_{cs}(4338)$ no longer exists as a quasi-bound state, while the attractive force of the $\Lambda_c \bar{D}_s$ channel becomes stronger and its mass will decrease. The observation of the $P_{cs}(4338)$ by LHCb seems to exclude the parameter region $0.64 < g_x < 1.0$. 


we find the role of the predicted $P_c (4477)$ and $P_c (4398)$ with $J^{P} = 1/2^-$ are very similar to those of the $P_c (4338)$ and $P_c (4255)$ with $J^{P} = 1/2^-$, respectively.

E. The results of $P_c$ system in the coupled-channel formalism

![Diagram showing variations of masses for the possible bound states in the $(\Lambda_c\bar{D}_s(\pi), \Xi_c\bar{D}_s(\pi))$ two-channel system as the parameter $g_x$ increases.](image)

Here, we also check the pole position of the $P_c (4338)$ at $g_x > 0.64$ in the energy region slightly above the $\Xi_c \bar{D}$ threshold. We find that the pole of the $P_c (4338)$ still exists in the first Riemann sheet. This is mainly due to the fact that the $\Lambda_c \bar{D}_s = \Xi_c \bar{D}$ coupling leads the $P_c (4338)$ to be a state that has a considerable width, thus the central value of the $P_c (4338)$ mass may cross the $\Xi_c \bar{D}$ threshold. In this case, the $P_c (4338)$ should be interpreted as a quasi-bound state above the $\Xi_c \bar{D}$ threshold. Nevertheless, in this work, we restrict our scope to the case that the masses of the bound/quasi-bound states are below their corresponding thresholds.

To understand why the $g_x$ region that allows the $P_c (4338)$ and $P_c (4255)$ states to coexist is so narrow, we further check the role of the $\Lambda_c \bar{D}_s$ channel in our two-channel model. We allow the effective potential of the $\Lambda_c \bar{D}_s$ channel to have a 20% shift, i.e.,

$$v_1' = v_{11} + v_{11}', \quad v_{11}' = 0, \quad \frac{1}{5} v_{11},$$

and further check how the masses of the $P_c (4338)$ and $P_c (4255)$ change as we increase the $g_x$.

The channel $\Lambda_c \bar{D}_s$ itself has a weak attractive force, as presented in Fig. 1 (a), at $g_x = 0$. After we increase this force by 20%, this single-channel still can not form a bound state. But the $g_x$ region that allows these two $P_c$ states to coexist becomes broader. On the contrary, as illustrated in Fig. 1 (c), if we decrease the attractive force of the $\Lambda_c \bar{D}_s$ channel by 20%, the $P_c (4338)$ and $P_c (4255)$ can not coexist no matter how we adjust the off-diagonal $\Lambda_c \bar{D}_s = \Xi_c \bar{D}$ coupling. Thus, the narrow $g_x$ region that the $P_c (4338)$ and $P_c (4255)$ can co-exist is due to the fact that the $\Lambda_c \bar{D}_s$ channel has a small but non-negligible attractive force.

The results for the $J = 1/2$ and $3/2$ $(\Lambda_c \bar{D}_s^\pi, \Xi_c \bar{D}^\pi)$ coupled-channels are presented in Fig. 1 (d-f). We find that the roles of the predicted $P_c (4477)$ and $P_c (4398)$ with $J^{P} = 1/2^-$ or $3/2^-$ are very similar to those of the $P_c (4338)$ and $P_c (4255)$ with $J^{P} = 1/2^-$, respectively.

![Diagram showing variations of masses for the possible bound states in the $(\Lambda_c\bar{D}_s(\pi), \Xi_c\bar{D}_s(\pi))$ two-channel system as the parameter $g_x$ increases.](image)

We present our complete multi-channel calculations on the $J = 1/2$ and $J = 3/2$ $P_c$ systems in Fig. 2. We find that only the bound states close to the $\Lambda_c \bar{D}_s^{\pi}$ and $\Xi_c \bar{D}^{\pi}$ channels have significant dependence on the $g_x$, since these bound states can couple to the $\Lambda_c \bar{D}_s^{\pi}$ and $\Xi_c \bar{D}^{\pi}$ channels through non-negligible central terms, while the bound states that can only couple to the $\Lambda_c \bar{D}_s^{\pi}$ and $\Xi_c \bar{D}^{\pi}$ channels via the spin-spin interaction terms have very tiny dependence on the parameter $g_x$.

To further present our numerical results, we fix the parameter $g_x$ at 0.5 and 0.62. We denote these two cases as case 1 and case 2. The case 1 and case 2 correspond to the results that the
possible $P_{cs}(4255)$ signal does not/does exist. The results of these two cases are listed in Table VI.

Comparing the masses of the $P_{cs}$ states calculated in the single-channel formalism (Table IV) with the results obtained in the coupled-channel formalism (Table VI), we infer that the off-diagonal channels that only consist of the spin-spin interaction terms have small influence on the masses of the $P_{cs}$ states, which is very similar to the $P_c$ system. From Table VI, we find that there exist three extra $P_{cs}$ states below the $\Lambda_c D_{s}^{(*)}$ thresholds in the case 2.

For the $P_{cs}(4338)$ state, due to its strong coupling to the $\Lambda_c D_{s}$ channel, the width of this state is broader than the result given by LHCb. Note that in our calculation, we only include the open-charm two body meson-baryon channels. Thus, the width predicted by our model should be regarded as the lower limit of the experimental width. Since the $P_{cs}(4338)$ is reported in the $B \rightarrow J/\Psi \Lambda \bar{b}$ channel, the narrow width of the $P_{cs}(4338)$ found by the LHCb may be due to the small phase space of this $B$ meson decay process. Thus, confirming the $P_{cs}(4338)$ in other decay processes is important to pin down its resonance parameters.

Besides, we also find that the $P_{cs}$ states that are close to the $\Xi_{c} \bar{D}^{(*)}$ states are broader than the other $P_{cs}$ states due to its strong coupling to the $\Lambda_c \bar{D}_{s}^{(*)}$ channel. Thus, our results suggest that there exist two $J^{P}=1/2^{-}$ and $J^{P}=3/2^{-}$ quasi-bound states near the $\Xi_c D^*$ region. This region is close to the reported $P_{cs}(4459)$, and the two-peak structure in this region has been discussed in many literatures [4, 8, 13, 40, 41]. The results from our model provide a new possibility, i.e., the two $P_{cs}$ structures in this region may have a significant overlap in the $J/\Psi \Lambda$ invariant spectrum due to their considerable widths. The decay behaviors of the $P_{cs}(4459)$ have been discussed in Refs. [11, 29, 42-44]. The decay widths and decay patterns are valuable in identifying the structure of the $P_{cs}(4459)$. Further investigations on the total and partial decay widths will be crucial to accomplish a thorough understanding on the $P_c$ and $P_{cs}$ states.

F. The correspondence between the $P_c$ and $P_{cs}$ systems

Finally, we compare the masses of the $P_c$ and $P_{cs}$ states obtained from our multi-channel model. Since the mass of constituent $s$ quark is heavier than that of the $u$, $d$ quarks by about 100 MeV. Thus, we shift the mass plot of the $P_{cs}$ system by 100 MeV to check the similarities between the $P_c$ and $P_{cs}$ states. We present the multi-channel results for the $P_{cs}$ system in Fig. 3 (a), and the multi-channel results for the $P_{cs}$ system calculated at $g_x = 0.5$ and $g_x = 0.62$ are given in Fig. 3 (b). As can be seen from Fig. 3 (a) and (b), the meson-baryon thresholds in the $P_c$ and $P_{cs}$ systems have the following analogies

$$m_{\Lambda_c D^{(*)}} \leftrightarrow m_{\Lambda_c \bar{D}_{s}^{(*)}},$$

$$m_{\Xi_{c} \bar{D}^{(*)}} \leftrightarrow m_{\Xi_{c} \bar{D}_{s}^{(*)}}.$$  

We denote these thresholds with the blue-dotted lines in Fig. 3. Besides, there exist two extra meson-baryon thresholds $\Xi_{c} \bar{D}$ and $\Xi_{c} \bar{D}^*$ in the $P_{cs}$ system. These two channels can not directly correspond to the meson-baryon channels in the $P_c$ system. We denote these two thresholds with the green-dotted lines.

As can be seen from Fig. 3, there exist six $P_c$ states with $J^{P}=1/2^{-}$ or $3/2^{-}$. These six states can correspond to the six states in the $P_{cs}$ system. We denote the masses of the central values of these 12 states with black lines and their uncertainties are denoted with red rectangles. According to Fig. 3, the experimentally observed $P_c$ states and the predicted $P_{cs}$ states should have the following analogies

$$P_c(4312) \leftrightarrow P_{cs}(4434),$$

$$P_c(4440) \leftrightarrow P_{cs}(4564),$$

$$P_c(4457) \leftrightarrow P_{cs}(4582).$$

As indicated in Fig. 3, if we replace the $\Lambda_c \bar{D}_{s}^{(*)}$ and $\Xi_{c} \bar{D}$ channels with the $\Xi_{c} \bar{D}$ and $\Xi_{c} \bar{D}^{*}$ channels, respectively, we can reluctantly obtain the following analogies

$$P_c(4312) \leftrightarrow P_{cs}(4472),$$

$$P_c(4440) \leftrightarrow P_{cs}(4564),$$

$$P_c(4457) \leftrightarrow P_{cs}(4582).$$

The predicted $P_{cs}(4472)$ may correspond to the reported $P_{cs}(4459)$. However, such an analogy indicates a considerable SU(3) breaking effect. In both sets of analogies, the $P_{cs}(4338)$ can not directly correspond to the lowest $P_c(4312)$ state.

There exist three and six extra $P_{cs}$ states that can not correspond to the states in the $P_c$ system at $g_x = 0.50$ and $g_x = 0.62$, respectively. We denote the masses of the central values of these states with the black lines and their uncertainties are denoted with red rectangles. Further experimental explorations on the $P_{cs}$ system may help us to distinguish which case should be preferred.

IV. SUMMARY

Motivated by the recently discovered $P_{cs}(4338)$ from the LHCb Collaboration, we have performed a multi-channel calculation of the $I = 1/2$ $P_c$ and $I = 0$ $P_{cs}$ systems and presented a comparison between the interactions of the $P_c$ and $P_{cs}$ states in the SU(3)$_f$ limit and heavy quark limit.

Unlike the $\bar{c}u - \bar{c}n$ ($u = u, d$) type meson-baryon channels in the $P_c$ system, we need to consider two types of channels when we study the $P_{cs}$ system, i.e., the $\bar{c}u - \bar{c}n$ and $\bar{c}s - \bar{c}n$ meson-baryon channels. This difference will lead to extra states in the $P_{cs}$ systems.

The effective potentials of the $P_c$ and $P_{cs}$ states are collectively obtained via a quark-level Lagrangian, which allows us to construct the correspondence between the $P_c$ and $P_{cs}$ systems.

We use the masses of the $P_c(4440)$ and $P_c(4457)$ as input to determine the coupling parameters $\tilde{g}_s$ and $\tilde{g}_a$ in our model. We first study the masses of the $P_c$ states in the single-channel and coupled-channel formalisms. Since all the off-diagonal
TABLE VI. The results of the $P_{cs}$ states calculated at $g_x = 0.50$ and $g_x = 0.62$ in the coupled-channel formalism. All results are in units of MeV.

| States  | $g_x = 0.50$ | $g_x = 0.62$ |
|---------|--------------|--------------|
|         | Mass (MeV)   | $\Gamma$    | BE (MeV) | Mass (MeV)   | $\Gamma$    | BE (MeV) |
| $[\Lambda, D_1]^{\pm}$ | $4255.5^{+0.0}_{-0.7}$ | 0.0 | $-0.9^{+0.0}_{-0.7}$ |
| $[\Lambda_c, D_1^+]^2$ | $4398.1^{+0.2}_{-1.5}$ | 0.0 | $-0.6^{+0.2}_{-1.5}$ |
| $[\Lambda_c, D_1^-]^2$ | $4398.3^{+0.4}_{-1.3}$ | 0.0 | $-0.4^{+0.4}_{-1.3}$ |
| $[\Xi_c, D_1^-]^2$ | $4335.9^{+0.7}_{-1.5}$ | 0.0 | $-0.7^{+0.7}_{-1.5}$ |
| $[\Xi_c, D_1^+]^2$ | $4477.1^{+0.5}_{-0.8}$ | 33.0^{+6.0}_{-8.0} | $-0.9^{+0.5}_{-0.8}$ |

FIG. 3. The mass spectra of the $P_c$ and $P_{cs}$ states in the multi-channel formalism, we present the results of $P_{cs}$ system at $g_x = 0.50$ and $g_x = 0.62$. The meson-baryon thresholds $m_{\Lambda_c, D_1^+(s)} - m_{\Lambda_c, D^+_1(s)}$, $m_{\Lambda_c, D^-_1(s)} - m_{\Lambda_c, D^0(s)}$, $m_{\Xi_c, D_1^0(s)} - m_{\Xi_c, D^-_1(s)}$, $m_{\Xi_c, D_1^+(s)} - m_{\Xi_c, D^+_1(s)}$ are illustrated with the blue-dotted lines. The two extra thresholds $\Xi_c, D$ and $\Xi_c, D^*$ in the $P_{cs}$ system are denoted with the green-dotted lines. We use the black lines to denote the central values of the obtained $P_c$ and $P_{cs}$ states, their uncertainties are illustrated with the green and red rectangles. The six green states in the $P_c$ system can directly correspond to the six green states in the $P_{cs}$ system.

terms in the effective potential matrices consist of the spin-spin interaction terms, the coupled-channel effect provides very small corrections to the masses of the $P_c$ states.

There exists an important difference between the $P_c$ system and $P_{cs}$ system. In the $P_{cs}$ system, the off-diagonal terms $\Lambda_c, D_{1^+(s)} - \Xi_c, D^0(s)$ in the effective potential matrices consist of the central terms and will have considerable corrections to the mass spectrum of the $P_{cs}$ states. To clarify the role of the $\Lambda_c, D_{1^+(s)} - \Xi_c, D^0(s)$ coupling, we have performed a numerical
experiment on the \((\Lambda_c, \bar{D}^s), \Xi_c, \bar{D}^\ast\) coupled-channel system. Our results suggest that the mass of the \(P_{cs}(4338)\) may shift very close to the \(\Xi_c, \bar{D}^\ast\) threshold by adjusting the coupling between the \(\Xi_c, \bar{D}^\ast\) and \(\Lambda_c, \bar{D}_c\) channels. This coupling may also lead to a \(P_{cs}(4255)\) state in a reasonable \(g_s\) region.

Then we present our complete multi-channel calculations of the \(P_{cs}\) systems. Since the \(P_{cs}(4255)\) is not confirmed by experiment, we present our numerical results with \(g_s = 0.50/0.62\), corresponding to the case that the \(P_{cs}(4255)\) does not/do exist, respectively. Due to the strong \(\Lambda_c, \bar{D}_c - \Xi_c, \bar{D}_c\) couplings, our predicted width of \(P_{cs}(4338)\) is broader than the experimental value. The reported narrower width may be due to the small phase space of the \(B\) meson decay process. Confirming the \(P_{cs}(4338)\) state in other processes will be helpful to pin down its resonance parameters. There exist two \(P_{cs}\) states with \(J^P = 1/2^+\) and \(J^P = 3/2^-\) below the \(\Xi_c, \bar{D}_c^\ast\) threshold. The masses of these two states are close to the mass of the reported \(P_{cs}(4459)\). Due to the \(\Lambda_c, \bar{D}_c^\ast - \Xi_c, \bar{D}_c^\ast\) coupling, these two states should have considerable widths and may have significant overlap in the \(J/\Psi\Lambda\) invariant spectrum. Further experimental exploration would be important to test our predictions.

Finally, we present a complete correspondence between the \(P_c\) and \(P_{cs}\) states. The observed \(P_c(4312), P_c(4440),\) and \(P_c(4457)\) do not directly correspond to the observed \(P_{cs}(4338)\) and \(P_{cs}(4459)\). It is particularly interesting to find the SU(3) \(P_c\) states that may correspond to the observed \(P_c\) states, and to investigate if such a correspondence does exist. Further experimental researches on these topics will be helpful to fulfill a complete picture on the spectra of the \(P_c\) and \(P_{cs}\) systems.

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