Research Article
Linear Prediction Using Refined Autocorrelation Function

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This paper proposes a new technique for improving the performance of linear prediction analysis by utilizing a refined version of the autocorrelation function. Problems in analyzing voiced speech using linear prediction occur often due to the harmonic structure of the excitation source, which causes the autocorrelation function to be an aliased version of that of the vocal tract impulse response. To estimate the vocal tract characteristics accurately, however, the effect of aliasing must be eliminated. In this paper, we employ homomorphic deconvolution technique in the autocorrelation domain to eliminate the aliasing effect occurred due to periodicity. The resulted autocorrelation function of the vocal tract impulse response is found to produce significant improvement in estimating formant frequencies. The accuracy of formant estimation is verified on synthetic vowels for a wide range of pitch frequencies typical for male and female speakers. The validity of the proposed method is also illustrated by inspecting the spectral envelopes of natural speech spoken by high-pitched female speaker. The synthesis filter obtained by the current method is guaranteed to be stable, which makes the method superior to many of its alternatives.

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1. INTRODUCTION

Linear predictive autoregressive (AR) modeling [1, 2] has been extensively used in various applications of speech processing. The conventional linear prediction methods, however, have been known to possess various sources of limitations [2–4]. These limitations are mostly observed during voiced segments of speech. Linear prediction method seeks to find an optimal fit to the log-envelop of the speech spectrum in least squares sense. Since the source of voiced speech is of a quasiperiodic nature, the peaks of linear prediction spectral estimation are highly influenced by the frequency of pitch harmonics (i.e., fundamental frequency, \(F_0\)). In high-pitched speaking, such estimation is very difficult due to the wide spacing of harmonics. Unfortunately, in order to study the acoustic characteristics of either the vocal tract or the vocal fold, the resonance frequencies of the vocal tract must be estimated accurately. Consequently, researchers long have attempted numerous modifications to the basic formulation of linear prediction analysis. While a significant number of techniques for improved AR modeling have been proposed based on the covariance method, improvements on the autocorrelation method are rather few.

Proposals based on the covariance method include analyzing only the interval(s) included within a duration of glottal closure with zero (or nearly zero) excitations [5–7]. However, it is very difficult to find such an interval of appropriate length on natural speech especially on speech uttered by females or children. Even if such an interval is found, the duration of the interval may be very short. The closed-phase method has been shown to give smooth formants contours in cases where the glottal close phase is about 3 milliseconds in duration [6]. If the covariances are computed from an extremely short interval, they could be in error, and the resulting spectrum might not accurately reflect the vocal tract characteristics [8]. In [9], Lee considered the source characteristics in the estimation process of AR coefficients by weighting the prediction residuals, where more weight is given to the bulk of smaller residuals while down-weighting the small portion of large residuals. A more general method, of course, was proposed earlier by Yanagida and Kakusho [10] where the weight is a continuous function of the residual. System identification principle [11–14] has also been exploited using least square method where an estimate of input is obtained in the first pass which is then used in the second-pass together with the speech waveform as output. Thus the estimated spectrum is assumed to be free from the influence of \(F_0\). Obtaining a good estimate of the input from natural speech is, however, a very complicated process and so is the formant estimation process. Instead of using existing
assumptions about glottal waves, Deng et al. [15] estimated
glottal waves containing detail information over closed glot-
tal phases that yield unbiased estimates of vocal tract filter
coefficients. Results presented on sustained vowels are quite
interesting.

In an autocorrelation based approach, Hermansky et al. [16] attempt to generate more frequency samples of the
original harmonic peaks and then fit an all-pole model to the new
sets of frequency points. Motivated by knowledge of the
auditory system, Hermansky [17] proposed another spectral
modification approach that accounted for loudness percep-
tion. Vahro and Alku proposed another variation of linear
harmonic peaks and then fit an all-pole model to the new
sets of frequency points. Motivated by knowledge of the au-
dition between two adjacent samples was the motivation of this
approach. The higher formants were shown to be estimated
more precisely by the new technique. However, the lower for-
mants are well known to be mostly affected by the pitch
harmonics.

In this paper, we consider the effect of periodicity of
excitation from a signal processing viewpoint. For the lin-
ear prediction with autocorrelation (LPA) method, when a
segment is extracted over multiple pitch periods, the ob-
tained autocorrelation function is actually an aliased version
of that of the vocal tract impulse response [3]. This is be-
cause copy of the autocorrelation of vocal tract impulse re-
sponse is repeated periodically with the periodicity equiva-
lent to pitch period, which overlaps and alters the underly-
ing autocorrelation function. However, the true solutions of
the AR coefficients can be obtained only if the autocorrela-
tion sequence equals that of the vocal tract impulse response.
This true solutions can be achieved approximately at a large
value of pitch period. As the pitch period of high-pitched
speech is very short, the increased overlapping causes the
low-order autocorrelation coefficients considerably different
from those of vocal tract impulse response. This leads to the
fact that the accuracy of LPA decreases as \( F_0 \) increases. To
alize the true solutions thus the aliasing must be removed.
The problem is greatly solved by the discrete-all-pole (DAP)
model in [3], where the aliasing is minimized in an iterative
way. But it sometimes suffers from spurious peaks between
the pitch harmonics. An improvement over DAP has been
proposed in [19] where a choice needs to be made depend-
ing on whether the signal is periodic, aperiodic, or a mixture
of both. This choice and the iterative computing are the dis-
advantages of the DAP methods.

As we will see in Section 2, the autocorrelation function of
the speech waveform gets aliased due to a convolution op-
eration of the autocorrelation function of vocal tract im-
pulse response with that of the excitation pulses. The prin-
cipal problem then is to eliminate the excitation contribu-
tion from the aliased version of autocorrelation function of the
speech waveform. Homomorphic deconvolution techne-
ique [20] has long history of successful applications in separat-
ing the periodic component from a nonlinearly combined sig-
nal. In this paper, we employ homomorphic deconvolution
method in the autocorrelation domain [21] to separate the
contribution of periodicity and thus obtain an estimate of the
autocorrelation of vocal tract impulse response which is
(nearly) free from aliasing. Unlike DAP methods, the pro-
posed solution is noniterative in nature and more straight-
forward. Experimental results obtained from both synthetic
and natural speech show that the proposed method can pro-
vide enhanced AR modeling especially for the high-pitched
speech where LPA provides only an approximation.

We organize the paper as follows. We define the problem
in Section 2 and we propose our method in Section 3. Sec-
tions 4 and 5 describe the results obtained using synthetic
and natural speeches, respectively. Finally, Section 6 is on the
concluding remarks.

2. PROBLEMS OF LPA

Though LPA is known to lead an efficient and stable solution
of the AR coefficients, this method inherits a different source
of limitation. For an AR filter with impulse response:

\[
h(n) = \sum_{k=1}^{p} a_k h(n-k) + \delta(n),
\]

where \( \delta(n) \) is an impulse and \( p \) is the order of the filter, the
normal equations can be shown as (see [22])

\[
\sum_{k=1}^{p} a_k r_n(i-k) = r_n(i), \quad 1 \leq i \leq p,
\]

where \( r_n(i) \) is the autocorrelation function of \( h(n) \). For a pe-
riodic waveform \( s(n) \), (2) can be expressed as

\[
\sum_{k=1}^{p} a_k r_n(i-k) = r_n(i), \quad 1 \leq i \leq p,
\]

where \( r_n(i) \) is the autocorrelation function of the windowed
\( s(n) \) \((s(n) \) is constructed to simulate voiced speech by con-
volving a periodic impulse train with \( h(n) \)).

For such periodic signal, El-Jaroudi and Makhoul [3] have
shown that \( r_n(i) \) equals the recurring replicas of \( r_n(i) \) as
given by

\[
r(i) = \sum_{l=\infty}^{\infty} r_n(i-IT), \quad \forall l,
\]

where \( T \) is the period of excitation and \( r_n(i) \) can be con-
sidered as an equivalent of \( r(i) \) for a finite-length speech
segment. The effect of \( T \) on \( r_n(i) \) is shown in Figure 1. When
the value of \( T \) is large, the overlapping is insignificant; ident-
cal values of \( r_n(i) \) (Figure 1(a)) and \( r_n(i) \) (Figure 1(b) at
\( T = 12.5 \) milliseconds) at the lower lags result in almost ident-
cal solutions when put in (2) and (3). However, as the pitch
period \( T \) decreases, \( r_n(i) \) (Figure 1(c) at \( T = 4 \) milliseconds)
suffers from increasing overlapping. For female speakers with
higher pitch, this effect leads to severe aliasing in the autocor-
relation function causing the low-order coefficients to dif-
siderably from those in \( r_n(i) \). The solutions of (3) are
then only the approximations of those of (2).
The autoregression of the vocal tract impulse response, $r_h(i)$; (b) autocorrelation of a periodic waveform at $T = 12.5$ milliseconds (at $F_0 = 80$ Hz); (c) autocorrelation of a periodic waveform at $T = 4$ milliseconds (at $F_0 = 250$ Hz).

3. HOMOMORPHIC DECONVOLUTION IN THE AUTOCORRELATION DOMAIN

From Section 2, it is now obvious that true solutions can be obtained only if the autocorrelation function in the normal equations equals $r_h(i)$. In this section, we propose a straightforward way to derive an estimate of $r_h(i)$ from its aliased counterpart $r_n(i)$.

We can write (4) as

$$r(i) = r_h(i) * r_p(i),$$

where $*$ stands for convolution and $r_p(i)$ is the autocorrelation function of the impulse train, which is also periodic with period $T$. Thus, $r(i)$ is a speech-like sequence and homomorphic deconvolution technique can separate the component $r_h(i)$ from the periodic component $r_p(i)$. This requires transforming a sequence to its cepstrum. The (real) cepstrum is defined by the inverse discrete Fourier transform (DFT) of the logarithm of the magnitude of the DFT of the input sequence. The resulting equation for the cepstrum of the autocorrelation function $r_n(i)$ corresponding to a windowed speech segment is given as

$$c_{rn}(i) = \frac{1}{N} \sum_{k=0}^{N-1} \log \left| R_n(k) e^{i(2\pi/N)ki} \right|, \quad 0 \leq i \leq N - 1,$$

where $R_n(k)$ is the DFT of $r_n(i)$ and $N$ is the DFT size. A 1024-point DFT is used for the simulations in this paper. It is noted that the term $R_n(1 : N/2) = R_n(N - 1 : N/2 + 1)$.

The term $\log \left| R_n(k) \right|$ in (6) can be expressed using (5) as

$$\log \left| R_n(k) \right| = \log \left| R_h(k) R_p(k) \right| = \log \left| R_h(k) \right| + \log \left| R_p(k) \right|$$

Thus an inverse DFT operation on $\log \left| R_n(k) \right|$ separates the contribution of the autocorrelation function of the vocal tract and source in the cepstrum domain. The contribution of $r_h(i)$ on the cepstrum $c_{rh}(i)$ can now be obtained by multiplying the real cepstrum by a symmetric window $w(i)$:

$$c_{rh}(i) = w(i) c_{rn}(i).$$

Application of an inverse cepstrum operation to $c_{rh}(i)$ converts it back to the original autocorrelation domain. The resulting equation for the inverse cepstrum is given as

$$\hat{r}_h(i) = \frac{1}{N} \sum_{k=0}^{N-1} \exp \left( C_{rh}(k) e^{i(2\pi/N)ki} \right), \quad 0 \leq i \leq N - 1,$$

where $C_{rh}(k)$ is the DFT of $c_{rh}(i)$. Clearly, the estimate $\hat{r}_h(i)$ is a refined version of $r_n(i)$, which results in accurate spectral estimation.
As an example, the deconvolution of the autocorrelation sequence in Figure 1(c) is shown in Figure 2. It is seen that the refined version of the autocorrelation function \( \hat{r}_h(i) \) (thin solid line) obtained through deconvolution of \( r_n(i) \) is indeed a good approximation of the autocorrelation function of the true impulse response \( r_h(i) \) (thick solid line).

The overall method of improved linear prediction using refined autocorrelation (LPRA) function is outlined in the block diagram of Figure 3. Real cepstrum is computed from the autocorrelation function \( r_n(i) \) of the windowed speech waveform. The low-time gating (i.e., truncation of the cepstral coefficients residing in an interval less than a pitch period) of the cepstrum followed by an inverse cepstral transformation produces the refined autocorrelation function \( \hat{r}_h(i) \), which closely approximates the true autocorrelation coefficients especially in lower lags that are the most important for formant analysis with linear prediction.

The LPA and LPRA spectral envelopes obtained using the autocorrelation sequence in Figures 1(b) and 1(c) (at \( F_0 = 80 \) and 250 Hz) are plotted in Figures 4(a) and 4(b), respectively, together with the true spectrum. The frequencies/bandwidths of the three formants in the “true” spectrum are (400/80, 1800/140, 2900/240) Hz. Both the LPA and LPRA methods produce perfect spectra at \( F_0 = 80 \) Hz (as overlapped with the “true” spectrum in Figure 4(a)). At \( F_0 = 250 \) Hz, however, the LPA spectrum, especially the first formant frequency and bandwidth, is considerably deviated from the “true” spectrum, where the spectrum estimated using the refined version of the autocorrelation function at \( F_0 = 250 \) Hz closely approximates the “true” spectrum (in Figure 4(b)). The formant frequencies/bandwidths estimated using LPA and LPRA spectra at \( F_0 = 250 \) Hz are (431/170, 1773/123, 2907/304) and (399/94, 1811/142, 2894/256) Hz, respectively.

Though impulse train used in the above demonstration does not exactly represent the glottal volume velocity, the example is a good representative to show the goodness of the method. In Section 4, we present the results in more detail taking the glottal and lip radiation effects into account.

### 3.1. Cepstral window selection

The standard cepstral technique [20] is employed here as the deconvolution method because of its straightforwardness in implementation over the others (e.g., [23–25]). Fixed length cepstral window independent of the pitch period of the underlying speech signal is the simplest form of cepstral truncation used in homomorphic deconvolution. Unfortunately, it may not be possible to define such an unique window which is equally suitable for both the male and female speeches. Fixed length cepstral window reported in literature is presented commonly for analyzing the typical male speech signals. Oppenheim and Schafer [20], for example, used the first 36 cepstral coefficients (i.e., 3.6 milliseconds in length) for spectrum estimation. This window, however, suits male speech better than (upper-range) female speech. Again, a shorter cepstral window is more proper for female speech and causes the spectral envelope of male speech smoother.
which may widen the formant peaks. If the application of interest is known a priori (or based on a logic derived from estimated $F_0$), using two different cepstral windows, one for analyzing the male speech and the other for the female speech, is more rational. In that case, 3.6 milliseconds and 2.4 milliseconds (36 and 24 cepstral coefficients in case of 10 kHz sampling rate) cepstral windows are good approximations for male (supposing $F_0 \leq 200$ Hz) and female speeches (supposing $F_0 > 200$ Hz), respectively.

Detail results on synthetic speech using two fixed-length cepstral windows (according to the $F_0$ value of the underlying signal) are presented in Section 4.

3.2. Stability of the AR filter

The standard autocorrelation function $r_n(i)$ is well known to produce stable AR filter [26, 27]. Thus, if the refined version of autocorrelation sequence $\hat{r}_n(i)$ can be shown to retain the property of $r_n(i)$, it can be said that the AR filter resulted by the LPRA method is stable. Since $r_n(i)$ is real, log magnitude of its Fourier transform, $\log |R_n(k)|$ at the right-hand side of (6), is also real and even. Thus, the DFT operation following $\log |R_n(k)|$ is essentially a cosine transformation. Then, the symmetric cepstral window (for low-time gating) followed by a DFT operation retains the nonnegative property of $\log |R_n(k)|$ in $C_{rh}(k)$ of (9). An estimate of the refined autocorrelation sequence $\hat{r}_n(i)$ derived from the positive spectrum $C_{rh}(k)$ therefore produces a positive semidefinite matrix like $r_n(i)$ [26], which guarantees the stability of the resulting AR filter.

4. RESULTS ON SYNTHETIC SPEECH

The proposed LPRA method is applied for estimating the formant frequencies of five synthetic Japanese vowels with varying $F_0$ values. The Liljencrants–Fant glottal model [28] is used to simulate the source which excites five formant resonators [29] placed in series. The filter $(1 - z^{-1})$ is operated on the output of the synthesizer to simulate the radiation characteristics from lip. The synthesized speech is sampled at 10 kHz. To study the variations of formant estimation against varying $F_0$, all the other parameters of the glottal model (open phase, close phase, and slope ratio) are kept constant. The formant frequencies used for synthesizing the vowels are shown in Table 1. Bandwidths of the five formants of all the five vowels are set fixed to 60, 100, 120, 175, and 281 Hz, respectively. The analysis order is set to 12. A Hamming window of length 20 milliseconds is used. The speech is preemphasized by a filter $(1 - z^{-1})$ before analysis. A 1024-point DFT is used for cepstral analysis.

4.1. Accuracy in formant frequency estimation

Formant values are obtained from the AR coefficients by using the root-solving method. In order to obtain a well-averaged estimation of the formants, analysis is conducted on twenty different window positions. The arithmetic mean of all the results is taken as a formant value.

The relative estimation error (REE), $E_{F_i}$, of the $i$th formant is calculated by averaging the individual $F_i$ errors of all the five vowels. Thus we can express $E_{F_i}$ as:

$$E_{F_i} = \frac{1}{5} \sum_{j=1}^{5} |\hat{F}_{ij} - F_{ij}| / F_{ij},$$

where $F_{ij}$ denotes the $i$th formant frequency of the $j$th vowel and $\hat{F}_{ij}$ is the corresponding estimated value.

Finally, the REE of the first three formants of all the five vowels are summarized as follows:

$$E = \frac{1}{15} \sum_{j=1}^{5} \sum_{i=1}^{3} |\hat{F}_{ij} - F_{ij}| / F_{ij}.$$

As mentioned earlier in Section 3.1, two fixed length cepstral windows of length 3.6 milliseconds and 2.4 milliseconds are used to estimate formant frequencies for $F_0 \leq 200$ Hz and $F_0 > 200$ Hz, respectively. The REEs of the first, second, and first three formants estimated using LPA, DAP, and LPRA methods are shown in Figure 5. The code for DAP has been obtained from an open source MATLAB library for signal processing: http://www.sourceforge.net/projects/matsig. The code has been verified to work correctly.

The first and second formants are mostly affected by $F_0$ variations at higher $F_0$s (because of increased aliasing in the autocorrelation function). It is seen that REE of $F_1$ estimated using LPA can exceed 15% depending on $F_0$s. Since LPRA reduces aliasing in the autocorrelation function occurred due to the periodicity of voiced speech, this method results in very smaller REE and affected slightly by the $F_0$ variations. The DAP modeling results in much accurate estimation of second and third formants, but accuracy of first formant estimation suffers from large errors. The normalized formant frequency error averaged over all the pitch frequencies for each vowel separately is shown in Table 2.

From Table 2, it is obvious that the LPRA technique proposed in this paper can be useful in reducing aliasing effects occurred due to the excitation in the autocorrelation function.

4.2. Dependency on the length of analysis window

The proposed algorithm has been observed to perform better at relatively smaller size of analysis window. The effect of a longer window (40 milliseconds) is shown in Figure 6, where REE of the first formant frequency (estimated similarly as in Figure 5(a)) is plotted. It is seen that the accuracy

| Table 1: Formant frequencies used to synthesize vowels. |
|---|---|---|---|---|---|
| vowel | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ Hz |
| /a/ | 813 | 1313 | 2688 | 3438 | 4438 |
| /i/ | 375 | 2188 | 2958 | 3438 | 4438 |
| /u/ | 375 | 1063 | 2188 | 3438 | 4438 |
| /e/ | 438 | 1813 | 2688 | 3438 | 4438 |
| /o/ | 438 | 1063 | 2688 | 3438 | 4438 |
Figure 5: Relative estimation error (REE) of formant frequencies: (a) REE of $F_1$; (b) REE of $F_2$; (c) REE of $F_1$, $F_2$, and $F_3$ together.

Figure 6: REE of first formant frequency when frame size is 40 milliseconds.

Figure 7: Bandwidth error of first three formants.

of LPRA has changed significantly (with respect to the results obtained using 20-milliseconds frame in Figure 5(a)) as compared with that of LPA method. For longer analysis window, the increase in the correlation coefficients at the pitch-multiples result in larger cepstral coefficients around the pitch lags. Thus the convolution effect gets stronger for longer window. The dependency of cepstral deconvolution on window length has been discussed in [25] where it is shown that better deconvolution takes place when the frame length is about three pitch periods. A 40-milliseconds long frame extracted from 250-Hz pitch speech signal contains ten pitch periods of signal which is much longer than the expected length.

4.3. Accuracy in formant bandwidth estimation

The absolute difference between the actual and estimated bandwidths averaged over the first three formant bandwidths is shown in Figure 7. Bandwidths are estimated in a similar way as formant frequencies. Though the improvement in estimating formant bandwidths is not as significant as that achieved in formant frequencies, it still shows nice improvements for high-pitched speakers as compared to other methods.

5. RESULTS ON REAL SPEECH

Performance of the proposed method on natural speech is demonstrated in Figures 8 and 9, where we show the spectral envelopes obtained from several voiced segments. The speech materials used in Figures 8(a), 8(b), and 8(c) are extracted from vowel sound /a/ at $F_0 = 300$ Hz, from /o/ in CV sound /bo/ at $F_0 = 250$ Hz, and from /ea/ in /bead/ at $F_0 = 256$ Hz, respectively. The LPRA spectra shown in Figure 8 are obtained using a cepstral window of length 2.4 milliseconds. In
Table 2: Normalized formant error (in %) for each vowel.

| Vowel | Method | LPRA | DAP | LPA |
|-------|--------|------|-----|-----|
|       |        | $F_1$ error | $F_2$ error | $F_3$ error | $F_1$ error | $F_2$ error | $F_3$ error | $F_1$ error | $F_2$ error | $F_3$ error |
| /a/    |        | 2.13   | 1.30 | 0.42 | 1.05   | 1.47   | 0.59   | 3.24   | 1.99   | 0.73  |
| /i/    |        | 3.08   | 0.51 | 0.45 | 8.22   | 0.67   | 0.36   | 7.68   | 1.15   | 0.82  |
| /u/    |        | 2.81   | 1.33 | 0.60 | 8.05   | 1.32   | 0.76   | 8.68   | 2.49   | 1.04  |
| /e/    |        | 2.86   | 0.48 | 0.46 | 6.19   | 0.69   | 0.35   | 8.61   | 0.95   | 0.67  |
| /o/    |        | 2.94   | 1.38 | 0.42 | 2.04   | 0.96   | 0.37   | 8.97   | 2.77   | 0.63  |

Figure 8: Analysis of natural voiced segments (a) from /a/ at $F_0 = 300$ Hz; (b) from /o/ in /bo/ at $F_0 = 250$ Hz; (c) from /ea/ in /bead/ at $F_0 = 256$ Hz.

Figure 9: Analysis of natural vowel /o/ at $F_0 = 352$ Hz (a) using LPA method; (b) using DAP method; (c) using LPRA method.

the LPA spectra, especially the lower, formants are not resolved with accurate bandwidths. The second formant bandwidth in Figure 8(a) is widened, while it is constricted in Figure 8(b). The second and third formants in LPA spectrum of Figure 8(c) remain unresolved. The LPA spectral estimation is affected due to the inclusion of pitch information with vocal tract filter coefficients. The LPRA spectra, on the other hand, exhibit accurate formant peaks in all the cases where the influence due to the pitch harmonics is not significant. The DAP spectrum in Figure 8(a) is estimated well, but the spectra in Figures 8(b) and 8(c) are more or less identical with the LPA spectra. Running spectra estimated from a prolonged vowel sound /o/ at very high pitch ($F_0 = 352$ Hz) using the LPA, DAP, and LPRA methods are shown in Figures 9(a), 9(b), and 9(c), respectively. The
improvement obtained by the current method is obvious in Figure 9, where the closely located lower formants (first and second) are perfectly estimated in the LPRA spectra. These examples indicate the reduction of aliasing in the autocorrelation function achieved through the deconvolution measure.

6. CONCLUSION

In this paper, we proposed an improvement to the linear prediction with autocorrelation method for spectral estimation. The autocorrelation function of voiced speech is distorted by the periodicity in a convolutive manner which can greatly be removed using the homomorphic filtering approach. The method works noniteratively and is suitable for analyzing high-pitched speech. The standard cepstral analysis [20] employed here, of course, introduces some distortion due to windowing and cepstral truncation. Use of an improved deconvolution method that takes the windowing effects into account (e.g., [25]) can compensate the problem. Furthermore, the straightforward deconvolution method does not account for the time-varying glottal effects. Thus, the performance of the LPRA method can be improved by eliminating the effects due to glottal variations [15].

One of the greatest concerns for speech synthesis is the stability of the linear prediction synthesis filter. Unfortunately, most of the well-known methods [6, 7, 9–11, 14] emerged so far for analyzing high-pitched speech are based on covariance method which cannot guarantee the stability of the resulted AR filter. The proposed method, on the other hand, is guaranteed to produce a stable synthesis filter.

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REFERENCES

[1] B. S. Atal and S. L. Hanauer, “Speech analysis and synthesis by linear prediction of the speech wave,” The Journal of the Acoustical Society of America, vol. 50, no. 2B, pp. 637–655, 1971.
[2] J. Makhoul, “Linear prediction: a tutorial review,” Proceedings of the IEEE, vol. 63, no. 4, pp. 561–580, 1975.
[3] A. El-Jaroudi and J. Makhoul, “Discrete all-pole modeling,” IEEE Transactions on Signal Processing, vol. 39, no. 2, pp. 411–423, 1991.
[4] G. K. Vallabha and B. Tuller, “Systematic errors in the formant analysis of steady-state vowels,” Speech Communication, vol. 38, no. 1-2, pp. 141–160, 2002.
[5] D. Y. Wong, J. D. Markel, and A. H. Gray Jr., “Least squares glottal inverse filtering from the acoustic speech waveform,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 27, no. 4, pp. 350–355, 1979.
[6] A. Krishnamurthy and D. G. Childers, “Two-channel speech analysis,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 34, no. 4, pp. 730–743, 1986.
[7] Y. Miyoshi, K. Yamato, R. Mizoguchi, M. Yanagida, and O. Kakusho, “Analysis of speech signals of short pitch period by a sample-selective linear prediction,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 35, no. 9, pp. 1233–1240, 1987.
[8] N. B. Pinto, D. G. Childers, and A. L. Lalwani, “Formant speech synthesis: improving production quality,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37, no. 12, pp. 1870–1887, 1989.
[9] C.-H. Lee, “On robust linear prediction of speech,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 36, no. 5, pp. 642–650, 1988.
[10] M. Yanagida and O. Kakusho, “A weighted linear prediction analysis of speech signals by using the given’s reduction,” in Proceedings of the IASTED International Symposium on Applied Signal Processing and Digital Filtering, pp. 129–132, Paris, France, June 1985.
[11] Y. Miyanaga, N. Miki, N. Nagai, and K. Hatori, “A speech analysis algorithm which eliminates the influence of pitch using the model reference adaptive system,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 30, no. 1, pp. 88–96, 1982.
[12] H. Fujisaki and M. Jungqvist, “Estimation of voice source and vocal tract parameters based on ARMA analysis and a model for the glottal source waveform,” in Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP ’87), pp. 637–640, Dallas, Texas, USA, April 1987.
[13] W. Ding and H. Kasuya, “A novel approach to the estimation of voice source and vocal tract parameters from speech signals,” in Proceedings of the 4th International Conference on Spoken Language Processing (ICSLP ’96), vol. 2, pp. 1257–1260, Philadelphia, Pa, USA, October 1996.
[14] M. S. Rahman and T. Shimamura, “Speech analysis based on modeling the effective voice source,” IEICE Transactions on Information and Systems, vol. E89-D, no. 3, pp. 1107–1115, 2006.
[15] H. Deng, R. K. Ward, M. P. Beddoes, and M. Hodgson, “A new method for obtaining accurate estimates of vocal-tract filters and glottal waves from vowel sounds,” IEEE Transactions on Audio, Speech, and Language Processing, vol. 14, no. 2, pp. 445–455, 2006.
[16] H. Hermansky, H. Fujisaki, and Y. Sato, “Spectral envelope sampling and interpolation in linear predictive analysis of speech,” in Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP ’84), vol. 9, pp. 53–56, San Diego, Calif, USA, 1984.
[17] H. Hermansky, “Perceptual linear predictive (PLP) analysis of speech,” Journal of the Acoustical Society of America, vol. 87, no. 4, pp. 1738–1752, 1990.
[18] S. Varho and P. Alku, “Separated linear prediction—a new all-pole modelline technique for speech analysis,” Speech Communication, vol. 24, no. 2, pp. 111–121, 1998.
[19] P. Kabal and B. Kleijn, “All-pole modelling of mixed excitation signals,” in Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP ’91), vol. 1, pp. 97–100, Salt Lake City, Utah, USA, May 2001.
[20] A. Oppenheim and R. Schafer, “Homomorphic analysis of speech,” IEEE Transactions on Audio and Electroacoustics, vol. 16, no. 2, pp. 221–226, 1968.
[21] M. S. Rahman and T. Shimamura, “Linear prediction using homomorphic deconvolution in the autocorrelation domain,” in Proceedings of IEEE International Symposium on Circuits and
[22] T. F. Quatieri, *Discrete-Time Speech Signal Processing: Principles and Practice*, Prentice-Hall, Upper Saddle River, NJ, USA, 2002.

[23] J. S. Lim, “Spectral root homomorphic deconvolution system,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 27, no. 3, pp. 223–233, 1979.

[24] T. Kobayashi and S. Imai, “Spectral analysis using generalised cepstrum,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 32, no. 6, pp. 1235–1238, 1984.

[25] W. Verhelst and O. Steenhaut, “A new model for the short-time complex cepstrum of voiced speech,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 34, no. 1, pp. 43–51, 1986.

[26] S. M. Kay, *Modern Spectral Estimation: Theory and Application*, Prentice-Hall, Upper Saddle River, NJ, USA, 1988.

[27] P. Stoica and R. L. Moses, *Introduction to Spectral Analysis*, Prentice-Hall, Upper Saddle River, NJ, USA, 1997.

[28] G. Fant, J. Liljencrants, and Q. G. Lin, “A four parameter model of glottal flow,” Quarterly Progress and Status, pp. 1–13, Speech Transmission Laboratory, Royal Institute of Technology, Stockholm, Sweden, October-December 1985.

[29] D. H. Klatt, “Software for a cascade/parallel formant synthesizer,” *Journal of the Acoustical Society of America*, vol. 67, no. 3, pp. 971–995, 1980.