The Search For Primordial Tensor Modes

George Efstathiou and Sirichai Chongchitnan

Institute of Astronomy, Madingley Road, Cambridge, CB3 OHA. England.

We review the prospects for detecting tensor modes generated during inflation by CMB polarization experiments and by searching for a stochastic gravitational wave background with laser interferometers in space. We tackle the following two questions: (i) what does inflation predict for the tensor fluctuations? (ii) is it really worth building experiments that can cover only a small range of tensor amplitudes?

§1. Introduction

Inflation is an extremely attractive idea that has gained widespread support. However, even a cursory glance at the literature will reveal a plethora of inflationary models. Inflation theory is mired in phenomenological model building (often involving ‘unnatural’ fine-tunings) rather than emerging in a compelling way from fundamental physics.

From the observational point of view, inflation has received strong support from observations of the cosmic microwave background (CMB) anisotropies, and from studies of the large-scale distribution of galaxies. In particular, the beautiful CMB results from WMAP\textsuperscript{1} are consistent with primordial adiabatic fluctuations with a nearly scale invariant spectrum, as expected in the simplest inflationary models. A key prediction of inflationary models is the existence of a stochastic background of gravitational waves. Such a background has not yet been observed, but its detection would provide incontrovertible evidence that inflation actually occurred and would set strong constraints on the dynamics of inflation. It is therefore no surprise that a vigorous effort is underway to detect tensor modes from inflation.

In this article, we will first review what can be learned about inflationary models from the detection of tensor modes. We will then discuss the prospects for detecting tensor modes from observations of the polarization of the CMB and by direct detection using interferometers in space. We will then tackle the following thorny questions:

(i) What does inflation predict for the amplitude of the tensor fluctuations?

(ii) If theory does not constrain the amplitude of the tensor mode to within many orders of magnitude, is it really worth the effort to build experiments that can only cover a small range?

Our perspective on point (i) is very different to that presented in a recent paper\textsuperscript{2}. Point (ii) is clearly important in assessing the case for a post-Planck satellite dedicated to CMB polarization measurements\textsuperscript{3}. Unless otherwise stated, we will assume the ‘concordance’ $\Lambda$-dominated cold dark matter cosmology, with cosmic
densities as given in\(^1\).

\section{Gravitational Waves from Inflation}

For the most part, we will consider only the simplest single field inflationary models, characterised by a potential \(V(\phi)\) and Hubble constant \(H(\phi)\), where \(\phi\) is the inflaton field. Following the normalizations of\(^4\)\(^5\), the amplitudes \(A(k)\) of scalar and tensor power spectra generated during inflation are given, to lowest order, by

\begin{align}
A_S(k) &\approx \frac{4}{5} \frac{H^2}{m_{\text{Pl}}^2 |H'|} \bigg|_{k=aH}, \\
A_T(k) &\approx \frac{2}{5 \sqrt{\pi}} \frac{H}{m_{\text{Pl}}} \bigg|_{k=aH},
\end{align}

where \(S\) and \(T\) denote scalar and tensor components respectively, \(m_{\text{Pl}}\) is the Planck mass, and primes denote derivatives with respect to \(\phi\). The amplitudes in (2.2) are evaluated when each mode \(k\) is equal in scale to the Hubble radius, \(i.e.\) \(k = aH\). The spectral indices \(n_S\) and \(n_T\) are defined by

\begin{align}
n_S - 1 &= \frac{d \ln A_S^2(k)}{d \ln k}, \\
n_T &= \frac{d \ln A_T^2(k)}{d \ln k},
\end{align}

evaluated at a ‘pivot’ scale \(k_0\) (which we will take to be \(k_0 = 0.002\ \text{Mpc}^{-1}\), following\(^6\)). To first order in the ‘slow roll’ parameters,

\begin{align}
\epsilon &= \frac{m_{\text{Pl}}^2}{4\pi} \left(\frac{H'}{H}\right)^2, \\
\eta &= \frac{m_{\text{Pl}}^2}{4\pi} \left(\frac{H''}{H}\right),
\end{align}

one finds that the spectral indices are given by

\begin{align}
n_S - 1 &\approx 2\eta - 4\epsilon, \\
n_T &\approx -2\epsilon.
\end{align}

Following the convention of\(^6\), we define the tensor-scalar ratio \(r\), as

\begin{align}
r &= 16 \frac{A_T^2}{A_S^2} \approx 16\epsilon,
\end{align}

where the last relation applies to first order in ‘slow-roll’ parameters. Note that the definition of a ‘tensor-scalar’ ratio varies widely in the literature. For instance, it is often defined\(^7\) as the ratio of the tensor and scalar CMB quadrupoles \(r_2 = C_T^2/C_S^2\). Such a definition is dependent on cosmology, especially on the dark energy density \(\Omega_A\). (See\(^8\) for a relation between \(r_2\) and \(r_\).)

As is well-known, the Thomson scattering of an anisotropic photon distribution leads to a small net linear polarization of the CMB anisotropies. (See\(^9\) for an introductory review and references to earlier work.) This polarization signal can be decomposed into scalar \(E\)-modes and pseudo-scalar \(B\)-modes. The separation
of a polarization pattern into $E$ and $B$ modes is of particular interest since scalar primordial perturbations generate only $E$ modes while tensor perturbations generate $E$ and $B$ modes of roughly comparable amplitudes\(^{10,11}\).

The $T, TE, E$, CMB power spectra for the concordance $\Lambda$CDM cosmology are shown in Figure 1. An $E$-mode polarization signal was first discovered by DASI\(^{12,13}\). Exquisite measurements of the temperature-$E$-mode cross power spectrum have been reported by the WMAP team\(^{14}\). Measurements of the $E$-mode power spectrum have been reported by the CBI experiment\(^{15}\) and by the 2003 flight of Boomerang\(^{16}\). Primordial $B$-mode anisotropies have not yet been detected in the CMB. The best current upper limits come from model fitting to CMB data and observations of the matter power spectrum at low redshift\(^{17}\), leading to the 95% upper limit of

$$r \lesssim 0.36.$$  \hspace{1cm} (2.8)

A primordial $B$-mode of this amplitude would produce an \textit{rms} anisotropy signal of only $\sim 0.35 \mu K$, \textit{i.e.} about a factor of 20 times smaller than the \textit{rms} anisotropy in $E$ modes (Figure 1). Evidently, the detection of primordial tensor modes presents a formidable experimental challenge.

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**Fig. 1.** Temperature and polarization power spectra for the concordance $\Lambda$CDM model. The current (indirect) upper limit of\(^{17}\) leads to an upper limit on \textit{rms} polarization anisotropy of $\lesssim 0.35 \mu K$. For comparison, the \textit{rms} signals for the $T$ and $E$ anisotropies are given. Lensing of $E$ modes by intervening matter leads to small scale $B$ modes\(^{18}\) as shown. The dashed line the white-noise level for an experiment with a sensitivity to $B$-modes of $r \sim 10^{-2}$.\n
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Primordial Tensor Modes
In the next Section, we will review briefly the prospects for experimentally detecting primordial tensor modes. We will then turn to theoretical implications and address the two questions posed in the Introduction.

§3. Prospects for Detecting Tensor Modes

3.1. WMAP and Planck

As mentioned above, the first year of data from WMAP have been used to measure the $T E$ power spectrum\textsuperscript{14). Direct detection of electric polarization, via its power spectrum $C_E^\ell$, is more challenging than statistical detection using the cross-correlation with the temperature anisotropies, since the expected $E$-polarization signal is much weaker than the correlated part of the temperature (Figure 1). At the time of writing, results for $C_E^\ell$ from the first three years of data from WMAP are eagerly awaited.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The left hand panel shows forecasts for the $\pm 1\sigma$ errors on the electric polarization power spectrum $C_E^\ell$ from WMAP after 4 years of observation. (Forecasts for the Boomerang (2003) experiment, labelled B2K, are also plotted). The right hand panel shows forecasts for Planck. For WMAP and B2K, flat band powers are estimated with $\Delta \ell = 150$ (with finer resolution on large scales for WMAP in the inset). For Planck, flat band powers are estimated with $\Delta \ell = 20$ in the main plot and with $\Delta \ell = 2$ in the inset. (Figures computed by A, Challinor, reproduced from\textsuperscript{19}).}
\end{figure}

Figure 2 shows forecasts\textsuperscript{19}, using the WMAP instrument sensitivities and beam widths, for what we might expect in an idealised case in which polarized emission from the Galaxy can be neglected over 65% of the sky. The concordance $\Lambda$CDM model has been assumed with a high optical depth for secondary reionization of $\tau = 0.17$. After four years, WMAP may make a detection in a few broad bands around $l = 400$, and on the largest scales where reionization dominates. Evidently, WMAP barely has the sensitivity to measure the $E$-polarization signal let alone the much smaller signal expected for primordial tensor modes.

The Planck satellite\textsuperscript{19}, scheduled for launch in August 2007, has higher sensitiv-
ity to polarization as shown by the right hand panel. Planck should be able to map out $C^E_l$ on all scales up to and beyond the global maximum at $l \sim 1000$. However, even at the sensitivities expected for Planck, the detection of $B$-modes will pose a formidable challenge.

![Planck forecast](image)

**Fig. 3.** Forecasts for the $\pm 1\sigma$ errors on the magnetic polarization power spectrum $C^B_l$ from Planck. Above $l \sim 150$ the primary spectrum is swamped by weak gravitational lensing of the $E$-modes\(^{18}\) (Figure computed by A. Challinor, reproduced from\(^{19}\)).

This is illustrated by Figure 3 which shows the errors on, $C^B_l$, expected from Planck for a model with $r$ arbitrarily set to 0.1. Note that with such early reionization almost 50 per cent of the total power in primordial $B$-polarization is generated at reionization, so confirmation of the high optical depth suggested by WMAP\(^{14}\) will have important implications for Planck. The figure suggests that for $r = 0.1$ Planck can characterise the primordial power spectrum in around four bands. The $B$-polarization signal generated by weak gravitational lensing\(^{18}\) (which is, of course, independent of $r$) dominates the primary signal above $l \sim 150$. At the very least, the weak lensing signal should be detectable by Planck. However, even if systematic errors and polarised Galactic emission can be kept under control, Planck will at best only be able to detect tensor modes from inflation if the tensor-scalar ratio is greater than a few percent.

### 3.2. More Sensitivity?

In case the reader finds the discussion above a little depressing, it is important to stress that constraining $r$ to within a few percent via CMB polarization would be a considerable achievement. Nevertheless, one can ask whether it is possible to do better. The key to achieving even higher sensitivities than Planck is to build experiments with large arrays of polarization sensitive detectors. Two such experiments, both ground-based, are known to the authors. Clover\(^{20}\) (the ‘Cl-Observer’) which...
will use large bolometer arrays and QUIET∗ which will use large arrays of coherent detectors.

Fig. 4. The expected errors from Clover on the $B$-mode power spectrum. The upper panel has tensor-scalar ratio $r = 0.36$ (cf. equation 2.8), the middle panel is for $r = 0.15$ and the lower panel is for $r = 0.011$. The smaller (magenta) error boxes are the contribution from instrument noise and the larger (blue) boxes also include sample variance, include the contribution from weak lensing. (Figure computed by A. Challinor).

Both of these experiments are still under development, and so there are uncertainties in forecasting what they might see. At present, it is envisaged that Clover will operate at three frequencies, 97, 150 and 230 GHz, with a resolution of 10 arcminutes. Each focal plane will be populated by a hexagonal array of corrugated single-mode feed horns, 160 at 97 GHz, 260 at each of 150 GHz and 230 GHz, whose outputs will be detected using novel arrays of voltage-biased Transition Edge Sensors. Figure 4 shows forecasts computed by Anthony Challinor, for various values of the gravitational wave amplitude, assuming one-year of integration with the full instrument and including the effects of foreground subtraction. This shows that Clover should have sufficient thermal sensitivity that its measurement of the $B$-mode power spectrum should be limited by lensing variance up to $l = 200$, covering the range where gravity waves contribute. These calculations suggest that a tensor-scalar ratio of $\sim 0.01$ should be detectable with this instrument at about the $3\sigma$ level. There are, therefore, good prospects for achieving high precision limits on primordial $B$-modes from future ground-based experiments.

∗) QU Imaging Experiment, http://quiet.uchicago.edu
3.3. Direct Detection with Interferometers in Space

Another way of detecting tensor modes generated during inflation is to search for a stochastic gravitational-wave background using laser interferometers (see, for example, the review by Cooray\cite{21} and references therein; for recent discussions see\cite{22,23,24}). The expected gravitational wave spectrum for wavenumbers \( k \gg k_{\text{equ}} \),

\[
\Omega_{gw}(k) \approx \frac{375}{4H_0^2k_{\text{equ}}^2\tau_0^4} \langle |A_T(k)|^2 \rangle,
\]

(3.1)

where \( \tau_0 \) is the conformal time at the present day and \( k_{\text{equ}} = \tau_{\text{equ}}^{-1} \) is the wavenumber that equals the Hubble radius at the time that matter and radiation have equal densities.

\[\text{Fig. 5. Plots of gravitational wave spectrum } \omega_{gw} \text{ against tensor-scalar ratio } r \text{ for a large number of models evolved with the inflationary flow equations. Square (red) points indicate models satisfying the observational constraints on } n_s \text{ and } dn_s/d\ln k \text{ given by}\ (3.3). \text{ The solid line shows the bound given by Equation}\ (3.4).\]

Evaluating equation (3.1) at wavenumbers characteristic of space-based gravitational wave interferometers \( (k \sim 6 \times 10^{13} \text{ Mpc}^{-1}, \text{ corresponding to frequencies } f \sim 0.1 \text{ Hz}) \) requires a large extrapolation of the tensor power spectrum of about 16 orders of magnitude in scale from CMB scales \( (k_0 \sim 0.002 \text{ Mpc}^{-1}) \). Linking predictions for direct gravitational wave detection to CMB polarization constraints is therefore model dependent. Figure 5 shows results from\cite{25} based on the ‘inflationary flow’ approach\cite{26,27} in which the Hubble constant is parameterised as a polynomial,

\[
H(\phi) = H_0 [1 + a_1 \phi + a_2 \phi^2 + \ldots + a_{n+1} \phi^{n+1}],
\]

(3.2)
truncated at finite $n$ (in our case, $n = 10$) with the coefficients $a_1, \ldots, a_{n+1}$, drawn at random from some assumed distribution (see Eq. (25) for details). The figure shows $\omega_{gw} \equiv \Omega_{gw} h^2$ (where the Hubble constant, $H_0 = 100 h$ km s$^{-1}$Mpc$^{-1}$) evaluated at $N = 20$ e-folds from the end of inflation, corresponding to the time when perturbations with scales relevant to direct detection experiments were equal to the Hubble radius. The tensor-scalar ratio $r$ plotted in Figure 5 is equation (2.7) evaluated $N = 60$ e-folds from the end of inflation, corresponding to CMB scales $\sim k_0$.

The (red) square points in Figure 5 show the subset of models that satisfy the $2 \sigma$ observational constraints\textsuperscript{17,28) on $n_s$ and $dn_s/d \ln k$,

$$0.92 \lesssim n_s \lesssim 1.06, \quad -1.04 \lesssim dn_s/d \ln k \lesssim 0.03,$$

and the (green) line shows the bound derived by assuming that the Hubble parameter $H(\phi)$ remains constant between $N = 60$ and $N = 20$:

$$\omega_{gw} \approx 4.36 \times 10^{-15} r.$$

Also shown is the rough sensitivity range\textsuperscript{29,30) for the proposed ‘post-LISA’ space-based interferometers BBO and DECIGO\textsuperscript{*). The sensivities of these proposed missions are highly uncertain and depend on the precise experimental configuration and useable bandwidth (for example, unresolved white-dwarf binaries could dominate the signal at frequencies below $\sim 0.2$ Hz significantly reducing the sensivities). The sensitivity range shown in Figure 5 is meant to be indicative only.

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\textsuperscript{*) Laser Interferometer Space Antenna (LISA); Big-Bang Observer (BBO); Deci-Hertz Interferometer Gravitational-wave Observer (DECIGO).
Evidently, equation (3.4) provides a strict upper bound to the gravitational wave spectrum independent of the shape of the inflationary potential. However, the majority of models satisfying the observational constraints (3.3) give $\omega_{\text{gw}} \lesssim 3 \times 10^{-16}$. It is difficult to exceed this value, even if the tensor-scalar ratio is high at CMB scales, unless the shape of the potential is adjusted to produce a sharp decline in $H(\phi)$ within the last 20 e-folds of inflation. For $r \ll 1$ most models lie very close to the limit (3.4) and even more fine-tuning of the inflaton potential shape is required to generate models that lie well below this bound (see Figure 6). Notice that the models shown in Figure 6b have a sharp downturn in $H(\phi)$ at the very end of inflation, it can be argued that all models with $r \ll 1$ require fine-tuning. We will return to this point in the next Section (which contains some cautionary remarks on the concept of fine-tuning applied to inflationary models).

Adopting an optimistic sensitivity for a BBO/DECIGO mission of $\omega_{\text{gw}} \sim 10^{-17}$, Figure 5 shows that a detection of gravitational waves from inflation is not expected unless $r \gtrsim 0.002$ on CMB scales. This is not too different from the sensitivity that seems feasible from the ground based CMB experiments (Section 3.2), on a much shorter timescale and at a tiny fraction of the cost. There has been some discussion in the literature of an ‘ultimate’ DECIGO interferometer, with a sensitivity of $\omega_{\text{gw}} \sim 10^{-20}$ limited by quantum noise for 100 kg test masses. Such a sensitivity, if it could be achieved, would probe models with $r \gtrsim 10^{-6}$, far below the levels likely to be reached with the CMB. Is it worth investing huge resources to reach this type of limit? This is discussed in the next Section.

§4. Theoretical Implications

In this Section, we will address the two questions raised in the Introduction.

4.1. What does inflation predict for the amplitude of the tensor fluctuations?

It is well known that the amplitude of the tensor mode CMB anisotropy fixes the energy scale of inflation \(^{31}\)

$$V^{1/4} \approx 3.3 \times 10^{16} r^{1/4} \text{GeV}. \quad (4.1)$$

The current upper limit of $r \lesssim 0.36$ gives the constraint $V^{1/4} \lesssim 2.6 \times 10^{16} \text{GeV}$, or equivalently $V \lesssim 2.2 \times 10^{-11} m_{\text{Pl}}^4$. At present, there are no compelling theoretical arguments to favour any particular energy scale. Since this energy scale depends only on the quarter power of $r$, our experimental colleagues have to work extremely hard to tighten the bounds:

| Experiment       | tensor-scalar limit | $V^{1/4}$ (GeV) |
|------------------|---------------------|-----------------|
| Planck           | $r \sim 0.1$        | $1.8 \times 10^{16}$ |
| Clover/QUIET     | $r \sim 0.01$       | $1.0 \times 10^{16}$ |
| BBO/DECIGO       | $r \sim 10^{-3}$    | $5.9 \times 10^{15}$ |
| ultimate DECIGO  | $r \sim 10^{-6}$    | $1.0 \times 10^{15}$ |

\(^{31}\) For a summary of other physical mechanisms that could give rise to a cosmological background of gravitational waves, see \(^{25}\).
Yet, given what we know about fundamental physics, the energy scale of inflation could easily be $\sim 10^{14}$ GeV or less, giving $r < \sim 10^{-10}$ which is well below the limit of any conceivable experiment.

What then, should we make of the wide spectrum of opinion amongst theorists, some of whom\textsuperscript{32,33} argue that the tensor-scalar ratio should be measurably high and others\textsuperscript{7} who argue that it should be immeasurably small? We will review some of the arguments:

(i) Initial Conditions: Let us imagine that inflation begins in a patch of about the Planck size, at Planck energies, and with an entropy (in Planck units) $S \sim 1$. We know empirically (equation 4.1) that the classical fluctuations that we see in the Universe today were frozen at much lower energy. So let us ‘connect’ the Planck scale to this lower energy scale by assuming simple power-law potential:

$$V(\phi) = \lambda m_{\text{Pl}}^4 \left( \frac{\phi}{m_{\text{Pl}}} \right)^\alpha.$$ \hfill (4.2)

Inflation occurs for field values, $\alpha/(4\pi)^{1/2} \lesssim \phi/m_{\text{Pl}} \lesssim \lambda^{-1/\alpha}$, and the amplitude of the scalar fluctuations in our Universe can be explained for suitably small values of the parameter $\lambda$. (For example, for the quartic potential, we require $\lambda \sim 4 \times 10^{-14}$). Notice that the field values vastly exceed the Planck scale initially (though the energy density is, by construction, always less than the Planck scale). In fact, the field value at the time that the fluctuations on CMB scales were equal to the Hubble radius is $\sim (N\alpha/4\pi)^{1/2}m_{\text{Pl}}$ (at $N$ e-folds before the end of inflation). The tensor-scalar ratio and scalar spectral index in such models are,

$$r \approx \frac{4\alpha}{N} \approx \frac{\alpha}{15}, \quad n_s \approx 1 - \frac{2 + \alpha}{2N} \approx 1 - \frac{2 + \alpha}{120},$$ \hfill (4.3)

where we have assumed $N \approx 60$. It is worth noting that there is already observational pressure on this type of model. The quartic potential, $\alpha \approx 4$, is marginally excluded by observations\textsuperscript{17} (mainly because of the large tilt in $n_s$) but a quadratic potential provides an acceptable fit to the data.

The initial conditions provide the main motivation for this class of models, since the Planck scale is linked to the much lower energy scale at which the fluctuations on CMB scales were frozen. For potentials with $\alpha$ of order unity, the tensor amplitude must necessarily be high. There is, therefore, a very good prospect of excluding this class of model with the next generation of CMB polarization experiments.

(ii) Fine-Tuning: Another set of arguments that have been used to favour inflationary models with a high tensor amplitude is based on fine-tuning\textsuperscript{33,2}. In a simple single field inflationary model, inflation ends when the ‘slow-roll’ parameter $\epsilon = 1$, yet the fluctuations on CMB scales were frozen $N \sim 60$ e-folds from the end of inflation when the field was rolling slowly. This introduces ‘natural’ values for the gradients of the inflaton potential. Steinhardt\textsuperscript{33} argues that the most natural values for the gradients are

$$\frac{V'}{V} \sim \frac{V''}{V'} \sim \frac{1}{N},$$ \hfill (4.4)
leading to the expectations,

\[ r \sim \frac{14}{N}, \quad n_s \sim 1 - \frac{3}{N}, \quad (4.5) \]

similar to the chaotic inflation values of equation (4.3). An attempt to quantify the degree of fine-tuning required to violate (4.5) involves counting the zeros that \( \epsilon \) or \( \eta \) undergo during the last 60 e-folds of inflation\(^{23}\). An impression of the fine-tuning involved is given by panel (b) of Figure 6. All of the models shown here, with a tensor-scalar ratio \( r \sim 10^{-3} \), show a sharp downturn in \( H(\phi) \) within the last few e-foldings of inflation.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Contours of the tensor-scalar ratio \( r \) for inflation with the potential (4.6) (starting from \( \phi \approx 0 \)) as a function of the parameters \( p \) and \( \phi_e \). Models with parameters in the shaded (blue) region have an unacceptably red scalar spectral index \( n_s < 0.90 \), or do not inflate by 60 e-folds.}
\end{figure}

How impressed should we be with this type of fine-tuning argument? Consider the potential\(^{34)}\)

\[ V(\phi) = V_0 \left( 1 - \left( \frac{\phi}{\phi_e} \right)^p \right). \quad (4.6) \]

We have solved the Hamilton-Jacobi equation,

\[ \left( H'(\phi) \right)^2 - \frac{12\pi}{m_{Pl}^2} H^2(\phi) = -\frac{32\pi^2}{m_{Pl}^4} V'(\phi) \quad (4.7) \]

\(^{*) This potential violates condition (4) of Boyle and collaborators\(^{2}\), in that it does not evolve smoothly to an analytic minimum with \( V \approx 0 \). This condition seems overly restrictive to us since complex physics is inevitable as \( V'(\phi) \) plummets from the energy scale of inflation to the present vacuum energy scale of a few milli-eV.\]
starting from $\phi \approx 0$, evaluating 'observables' ($n_s, r$ etc.) 60 e-folds from the end of inflation (when the field $\phi$ is always in the slow-roll regime). Figure 7 shows contours of the tensor-scalar ratio as a function of the parameters $p$ and $\phi_e$. Models in the shaded region either do not inflate for $N = 60$ e-folds, or produce a scalar spectral index with an unacceptably red tilt of $n_s < 0.9$. Evidently, the tensor-scalar ratio is largely controlled by the parameter $\phi_e$, which can be adjusted to produce tensor-scalar ratios that are unobservably small. It might be argued that for $p > 2$ the absence of a $\phi^2$ mass term at small field values requires some form of fine-tuning or special symmetry, but is the shape of the potential unreasonably contrived? We let the reader judge.

Another fine-tuning argument goes as follows\textsuperscript{33}: The amplitude of the scalar fluctuation spectrum requires an energy scale of inflation of

$$V^{1/4} \sim 10^{-5/2} (1 + w)^{1/4} m_{\text{Pl}}, \quad (4.8)$$

where $w$ is the equation of state parameter $w = p/\rho$. Now $1 + w$ must necessarily be much smaller than unity during the inflationary phase, but special fine-tuning is required to give $V^{1/4} \ll m_{\text{Pl}}$. For example, $V^{1/4} \sim 10^{-6} m_{\text{Pl}}$ requires $(1 + w) \sim 10^{-14}$. But in slow-roll inflation, the equation of state parameter is related to the gradient of the potential:

$$(1 + w) \approx \frac{2}{3} \epsilon_V, \quad \epsilon_V \equiv \frac{m_{\text{Pl}}^2}{16\pi} \left(\frac{V'}{V}\right)^2, \quad (4.9)$$

and the number of e-folds of inflation is given by,

$$N(\phi) \approx \frac{8\pi}{m_{\text{Pl}}^2} \int \frac{V}{V'} \, d\phi = \frac{2\sqrt{\pi}}{m_{\text{Pl}}} \int d\phi \sqrt{\epsilon_V(\phi)}. \quad (4.10)$$

Models with very small $\epsilon_V$ will therefore inflate by many e-folds, and so weighting by volume will strongly favour models with very flat potentials. But should we weight by volume? This is an example of the measure problem that plagues cosmology (see, for example, the papers by Tegmark\textsuperscript{35} and Vilenkin\textsuperscript{36}, and references therein). Until we have a more complete understanding of the measure problem, it seems dangerous to place much weight on fine-tuning arguments such as those discussed here.

(iii) Agnosticism: Chaotic inflation is an example of what are sometimes called 'high-field' models of inflation\textsuperscript{37}, since field values must necessarily exceed the Planck scale. It has been argued\textsuperscript{38} that this is an unattractive feature, since quantum gravity corrections would render an effective field theory out of control at $\phi \gtrsim m_{\text{Pl}}$. However, as Linde\textsuperscript{39,40} has emphasised, quantum gravity corrections to $V(\phi)$ should become large only for $V(\phi) \gtrsim m_{\text{Pl}}^4$. (Linde also discusses some phenomenological models with high field values in the context of supergravity, in which the potential displays a shift-symmetry $\phi \to \phi + \text{constant}$). Although the fundamental physics behind high-field inflationary models is poorly understood, it is premature to exclude them at this stage.

Inflationary models with small field values $\phi \ll m_{\text{Pl}}$ always produce a negligible tensor amplitude\textsuperscript{38,41}. Thus, some authors who approach inflationary model
building from a more ‘traditional’ particle-physics perspective\textsuperscript{7,42}) argue that the tensor-scalar ratio should be negligibly small. But this type of argument is unpersuasive because, as we have stressed above, insisting on a ‘controllable’ effective field-theory may just reflect our lack of knowledge of fundamental physics.

In our view, the prudent position at this stage is agnosticism. We simply do not know whether inflation is high-field, or low-field, high-energy or low-energy. For example, in the influential ‘string-inspired’ brane-inflation construction of Kachru and collaborators,\textsuperscript{43} (the KLM\textsuperscript{MT} scenario) the specific example given has parameters:

\[
\begin{align*}
V^{1/4} &\sim 10^{14} \text{ GeV}, \\
r &\sim 10^{-10}, \\
n_s &\sim 0.97, \\
\end{align*}
\]

and so produces a negligible tensor amplitude. Of course, the KLM\textsuperscript{MT} model is speculative, but it illustrates that there is no guarantee that the tensor modes will lie within a range accessible to experiment.

4.2. \textit{Is it really worth building experiments that can only cover a small range of tensor amplitudes?}

In the previous Sections we have argued that it is feasible to design experiments (at relatively low cost) to probe a tensor-scalar ratio of \(r \sim 10^{-2}\). A failure to detect tensor modes from inflation at this level would rule out the chaotic inflationary models described above and other examples of ‘high-field’ inflation. This is a well motivated and achievable goal.

But if we fail to detect tensor modes at this level, what then? Do we continue the search with a next-generation ‘CMBpol’ satellite designed to reach\textsuperscript{1}) \(r \sim 10^{-4}\)? The range of energy scales probed by such an experiment is so miniscule that the case would seem weak unless there are strong theoretical reasons to favour this narrow range. There are no such reasons at present.

However, a failure to detect tensor modes at a level of \(\sim 10^{-2}\) surely points to flat potentials and to an abrupt end to inflation. It therefore seems more sensible to design experiments to test for signatures associated with this abrupt end, rather than to focus single-mindedly on a search for inflationary tensor modes that will, in all likelihood, prove fruitless.

Producing an abrupt end to inflation is one of the main motivations for hybrid inflationary models\textsuperscript{7}). The archetypal hybrid inflation model\textsuperscript{44}) uses two coupled scalar fields with a potential,

\[
V = V_0 + \frac{1}{2}m_\phi^2 \phi^2 - \frac{1}{2}m_\psi^2 \psi^2 + \frac{1}{4}\lambda \psi^4 + \frac{1}{2}\lambda' \psi^2 \phi^2.
\]

In this model, most of the energy density during inflation is supplied by the field \(\psi\). Once \(\phi\) rolls down to a critical value \(\phi_c = m_\psi/\sqrt{\lambda'}\), the \(\psi\) field is destabilised and rolls down to its true vacuum, ending inflation. In some realisations of hybrid inflation heavy cosmic strings may be formed during a phase transition at the end

\textsuperscript{1}) Assuming that the Galactic polarization and lensing signals can be subtracted accurately at this level.
of inflation that could produce a CMB anisotropy of comparable amplitude to the scalar fluctuations\textsuperscript{45,46}. Cosmic strings could, in principle, provide an observable signature of the end of inflation even if the tensor-scalar ratio, $r$, is unobservably small.

More recently, the realisation that our Universe might be confined to a brane has stimulated a lot of research on whether interactions between branes can produce inflation. (See Quevedo\textsuperscript{47}, for a review). These models (of which the KLM\textsuperscript{T} scenario is an example) are speculative at present, but they share some features that may be generic. Firstly, in these models the inflaton field is identified with the separation between brane, or brane-anti-brane, pairs within a 5-D ‘bulk’. Thus inflation acquires a geometrical interpretation. Secondly, open strings must end on brane pairs. Below some critical separation, the mass of an open string mode becomes negative (tachyonic) at which point inflation ends abruptly. This mechanism for producing a sharp end to inflation is an attractive feature of brane inflation. Thirdly, cosmic superstrings (or ordinary gauge strings) can form at the end of inflation. The expected tensions of cosmic superstrings are model dependent, but may plausibly lie in the range\textsuperscript{48,49,50}

$$10^{-11} \lesssim G\mu \lesssim 10^{-6}. \quad (4.13)$$

If the string tension lies towards the upper end of this range\textsuperscript{*}), then strings should be easily detectable in the CMB, both as non-Gaussian signatures in the temperature maps and through their distinctive $B$-mode polarization power spectrum (which is very different\textsuperscript{48}) from the $B$-mode spectrum expected from tensor modes, peaking at $\ell \sim 1000$).

Recently Damour and Vilenkin\textsuperscript{53,54} have calculated the gravitational wave spectrum from bursts associated with cusps and kinks in loops of cosmic superstrings as a function of the theoretically uncertain intercommutation probability\textsuperscript{**}). They conclude the gravitational wave bursts from cosmic superstrings with tensions as low as $G\mu \sim 10^{-14}$ should be detectable by LISA and may even be observable by ground-based detectors such as LIGO if $G\mu \gtrsim 10^{-10}$ and the reconnection probability is small. As Polchinski\textsuperscript{49} has emphasised, cosmic superstrings could be the brightest objects visible in gravitational wave astronomy.

In summary, CMB experiments designed to probe tensor-scalar ratios as low as $r \sim 10^{-2}$ are feasible and well motivated. They could rule out chaotic inflation and many other versions of ‘high-field’ inflation. If tensor modes are not detected at this limit, then this suggests flat inflationary potentials and an abrupt end to inflation. Designing an experiment to probe as low as $r \sim 10^{-4}$ is a formidable daunting prospect, and yet this would improve the limit on the energy scale of inflation by only a factor of $\sim 3$. This seems poorly motivated, because there is no particular reason to expect the energy scale of inflation to lie in the narrow range $3.3 \times 10^{15}$ GeV $\lesssim V^{1/4} \lesssim 1 \times 10^{16}$ GeV. The energy scale of inflation could easily be

\textsuperscript{*} Note that constraints from pulsar timing may already overlap with this upper bound\textsuperscript{51}).

\textsuperscript{**} Fundamental strings differ from ordinary gauge strings in their reconnection properties, since they can miss each other in higher dimensions. The reconnection probability may therefore be very much less than unity for fundamental strings\textsuperscript{52}), whereas it is very close to unity for gauge strings.
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$\sim 10^{14}$ GeV or less. It may be more profitable to search for signatures associated with an abrupt end to inflation, such as non-Gaussianity in the CMB from multi-field inflation, cosmic strings formed at the end of inflation, B-mode polarization in the CMB associated with cosmic strings, and gravitational wave bursts from cosmic string cusps.

Acknowledgements

This work is supported by the UK Particle Physics and Astronomy Research Council. SC acknowledges the award of a Dorothy Hodgkin studentship. We thank Anthony Challinor for contributing Figures 1-4. We thank Anthony Challinor and Antony Lewis for comments on the manuscript.

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