Numerical Analysis of Distributed Acoustic Sensing Data Full-waveform Inversion

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Abstract. Distributed Acoustic Sensing (DAS) is a new type of seismic technique developed rapidly in recent years. Compared with conventional geophones, DAS receives strain rate signals with a continuous density survey. It has the advantages of low cost, high resolution, and wide application range. This abstract uses the elastic wave finite-difference method to simulate the DAS signal response and analyze surface wave dispersion characteristics. Then, we carry out 1-D dispersion curve inversion and revised 2-D DAS full-waveform inversion (FWI) tests on the DAS data under different gauge length conditions. The results show that our proposed DAS 1D dispersion inversion and 2D FWI method can provide a reliable velocity inversion result similar to conventional geophone data.

1. Introduction
As the urban population continues to rise, urban construction is also facing unprecedented challenges. The development and utilization of underground space have become an imperative development trend. The shallow surface velocity structure is of great significance in underground space utilization and cavity identification. Carrying out high-density observations in larger areas faces a series of practical difficulties, including instrument deployment, instrument safety, and observation costs. In recent years, a new seismic observation system (Distributed Acoustic Sensing, DAS) utilizes a large number of unused optical cables in cities obtaining seismic signals, realizing high-density dynamic monitoring at a low cost, has broad application prospects in urban underground space development and monitoring [1].

In this abstract, we try to use the elastic-wave finite difference method to simulate the DAS signal in the seismic survey. We compare DAS signals' response characteristics and dispersion characteristics with conventional geophone signals and developed the DAS full-waveform inversion (FWI) method. The effectiveness of the approach is verified by model testing, and it provides a good foundation for the next step of research on observed data.

2. Theory

2.1. Theory of DAS
DAS is based on the Rayleigh scattering in the optical fiber to obtain the seismic wavefield signal around the optical fiber. When the seismic signal is transmitted to a particular optical fiber position, it will cause local strain in the optical fiber, which will cause the phase change of the back Rayleigh scattered light in the optical fiber. The ground instrument receives the Rayleigh scattered light and
finally obtains the seismic wavefield information around the optical fiber through demodulation [2]. The DAS response is the average strain rate $\sigma_{xx,g}$ within the gauge length $g$:

$$\sigma_{xx,g} = \frac{1}{g} \int_{-g/2}^{g/2} \sigma_{xx} \, dx$$  \hspace{1cm} (1)

### 2.2. Conversion of DAS data to particle velocity
The relation between strain rate and velocity:

$$\sigma_{xx} = e \varepsilon_{xx}, \quad \varepsilon_{xx} = \frac{g}{\Delta t} \frac{\partial v}{\partial x} \frac{\partial u}{\partial x}$$  \hspace{1cm} (2)

Where $\sigma_{xx}$ is the strain rate, $\varepsilon$ is the strain, $u$ is the displacement, and $v$ is the velocity. According to the relationship between velocity and strain:

$$v_x = \frac{\partial u_x}{\partial x} - \frac{\partial u_x}{\partial x} = \pm \frac{\partial u_x}{\partial x} = \pm \frac{\omega}{k} \frac{\partial u_x}{\partial x} = \pm \frac{\omega}{k} \varepsilon_{xx}$$  \hspace{1cm} (3)

The strain rate can be integrated into the time domain into strain, take multi-track records and use two-dimensional Fourier transform to $f - k$ domain, multiply by the coefficient $(\omega + \sigma)/(k + \sigma)$, where $\sigma$ is the threshold to avoid instabilities in the division by zero frequency and wavenumber values [3], and finally use the two-dimensional inverse Fourier transform to the space-time domain to get the velocity $v$:

$$V_{(\omega,k)} = \pm \frac{(\omega + \sigma)E(\omega,k)}{(k + \sigma)}$$  \hspace{1cm} (4)

### 2.3. DAS data full-waveform inversion
According to the first-order velocity-stress equation:

$$\begin{align*}
\rho \frac{\partial v_x}{\partial t} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \\
\rho \frac{\partial v_z}{\partial t} &= \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \\
\frac{\partial \sigma_{xx}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} + s_{xx} \\
\frac{\partial \sigma_{zz}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} + s_{zz} \\
\frac{\partial \sigma_{xz}}{\partial t} &= \mu \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right)
\end{align*}$$  \hspace{1cm} (5)

This equation can be written in matrix form as

$$\begin{bmatrix}
\rho \frac{\partial}{\partial t} \\
0 \\
-\pi \frac{\partial}{\partial x} \\
-\lambda \frac{\partial}{\partial x} \\
-\mu \frac{\partial}{\partial z}
\end{bmatrix} \begin{bmatrix}
0 & -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{\partial z} \\
\rho & 0 & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial z} \\
0 & \rho & 0 & -\frac{\partial}{\partial x} \\
-\pi & -\lambda & 0 & \frac{\partial}{\partial t} \\
-\lambda & -\pi & 0 & \frac{\partial}{\partial t}
\end{bmatrix} \begin{bmatrix}
v_x \\
v_z \\
\sigma_{xx} \\
\sigma_{zz} \\
\sigma_{xz}
\end{bmatrix} = f$$  \hspace{1cm} (6)

The objective function is

$$\varepsilon(m) = \frac{1}{2} (f \cdot w(m) - d, f \cdot w(m) - d)$$  \hspace{1cm} (7)

Where $m$ is the model parameter, $d$ is the DAS observation data, and $f$ is the reciprocal spatial matrix.
The gradient of the objective function:

\[
J = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & 0 \\
0 & \frac{\partial}{\partial x} & 0 & 0 \\
0 & 0 & \frac{\partial}{\partial x} & 0 \\
0 & 0 & 0 & \frac{\partial}{\partial x}
\end{bmatrix}
\]  

(8)

The gradient of the objective function:

\[
\frac{\partial e(m)}{\partial m_i} = -\begin{pmatrix}
\text{exploding reflector} & \text{backpropagated residual} \\
\frac{\partial A(m)}{\partial m_i} w(m) & \{A(m)^{-1}\}^T J \Delta d(m)
\end{pmatrix}
\]  

(9)

3. Analysis of DAS signal response characteristics

In this section, we build a shear-wave velocity model to test the DAS signal response characteristic. The actual velocity model is shown in Figure 1. The longitudinal wave velocity is given by a fixed Poisson's ratio (\(V_p=1.732V_s\)). Figures 2a and 2b are the conventional geophones and DAS signal single-shot simulation data, respectively. We can see that the DAS recording signal is similar to the conventional geophone in amplitude and phase. The dispersion curves obtained by the Radon transform (RT) shown in 2c and 2d also have good similarities, indicating that the method described can perform the conversion between DAS signals and conventional geophone data. The 1-D dispersion inversion is shown in Figure 2e, which obtain stable S-velocity inversion results.

We also build the complex velocity model shown in Figure 3a to compare the DAS and geophone inversion results. The FWI inversion results after converting the DAS data to conventional geophone data shown in Figure 3c. There is some error in the velocity tomogram result. It demonstrates that the converted geophone data FWI did not match the fundamental elastic wave FWI theory. We present the DAS FWI theory to carry out the DAS data FWI test. Figure 3d is the DAS FWI result, which is in good agreement with the full waveform inversion result of the conventional geophone result.

![Figure 1. An actual velocity model](image-url)
Figure 2. (a) Conventional geophone shot gather; (b) DAS shot gather; (c) Dispersion curve of geophone data; (d) Dispersion curve of DAS data; (e) Results of dispersion curve inversion
4. Conclusions
This abstract presents the DAS signal analysis and velocity inversion based on the elastic wave finite-difference method. The results show that the overall trend of the DAS signal's dispersion curve and the geophone signal are the same, and only low frequencies are slightly different. The 1D dispersion curve inversion of the DAS data is consistent with the geophone data. The conventional converted DAS signal has some error in the FWI tomogram result. The revised DAS FWI method can invert the velocity model from the DAS data. The model test demonstrates that the modified DAS FWI method could provide a reliable inversion strategy for DAS data inversion.

Acknowledgments
This work is supported by the Natural Science Foundation of China (41874134), the Natural Science Foundation of Jilin Province (20200201216JC), and the Jilin Excellent Youth Fund (20190103142JH).

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