On charm scalar resonances

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A brief overview of charm scalar resonances is given. It is demonstrated that experimental data on the $D_s^+ \gamma$ and $D_s^+ \pi^0$ decays of $D_{s0}^+(2317)$ favor its assignment to the $I_3 = 0$ component of the iso-triplet $[c\bar{n}]$ mesons. The observed broad bump just below a large peak arising from the well-known tensor $D_2$ meson in a $D\pi$ channel is also studied.

I. INTRODUCTION

A charm-strange scalar meson $D_{s0}^+(2317)$ was discovered by the BABAR collaboration [1], and its existence has been confirmed by the BELLE [2], CLEO [3] and FOCUS [4] collaborations. Its mass and width are now compiled as

$$m_{D_{s0}} = 2317.4 \pm 0.9 \text{ MeV}, \quad \Gamma_{D_{s0}} < 4.7 \text{ MeV} \quad (1)$$

by the particle data group 2004 (PDG04) [5]. It has been observed in the $D_s^+ \pi^0$ channel but no signal in the radiative $D_s^+ \gamma$ has been detected at the BABAR and at the CLEO, so that the CLEO collaboration [3] has given a severe constraint,

$$\frac{\Gamma(D_{s0}^+(2317) \to D_s^+ \gamma)}{\Gamma(D_{s0}^+(2317) \to D_s^+ \pi^0)}_{\text{CLEO}} < 0.059, \quad (2)$$

For non-strange charm scalar mesons $D_0$, two independent observations of a broad bump just below a large peak arising from the well-known tensor $D_2$ in each of $(D\pi)^{0+}$ mass distributions have been reported [6,7], and it has been interpreted as the conventional scalar $D_0^0\{c\bar{n}\}$, $(n = u, d)$ meson. Their measured masses and widths are

$$m_{D_0^+} = 2308 \pm 60 \text{ MeV}, \quad \Gamma_{D_0^+} = 276 \pm 99 \text{ MeV} \quad (3)$$

by the BELLE collaboration and

$$m_{D_0^+} = 2407 \pm 56 \text{ MeV}, \quad \Gamma_{D_0^+} = 240 \pm 114 \text{ MeV},$$

$$m_{D_0^0} = 2403 \pm 49 \text{ MeV}, \quad \Gamma_{D_0^0} = 283 \pm 58 \text{ MeV}$$

by the FOCUS. In spite of the large difference between the central values of $m_{D_0}$ by the BELLE and by the FOCUS, we consider that they are of the same origin because they are consistent with each other within their large errors. We will discuss these broad bumps later.

The mass of $D_{s0}^+(2317)$ was considerably lower than theoretical predictions of mass of the conventional scalar $\{c\bar{s}\}$ by the potential model [8, 9], the lattice QCD [10, 11, 12], etc. Therefore, many authors have tried to assign it to various hadron states as listed in Table 1, where (a) the conventional scalar $\{c\bar{s}\}$ which is (b) the chiral partner of $D_s^+$ [14, 15] in the heavy charm quark picture, prior to the observation of $D_{s0}^+(2317)$, and (c) a mixed state of the scalar $\{c\bar{s}\}$ and a DK molecule [16], (d) a mixed state of the scalar $\{c\bar{s}\}$ and an iso-singlet four-quark meson [17], (e) an iso-singlet four-quark state [18], (f) an iso-triplet four-quark meson [19], and (g) a dynamically generated resonance [20, 21, 22], etc., after the observation. The ratios, $R(D_{s0}^+)$ and $R(D_s^{*+})$, in the table are given by

$$R(D_{s0}^+) = \frac{\Gamma(D_{s0}^+ \to D_s^+ \gamma)}{\Gamma(D_{s0}^+ \to D_s^+ \pi^0)} \quad \text{and} \quad R(D_s^{*+}) = \frac{\Gamma(D_s^{*+} \to D_s^+ \gamma)}{\Gamma(D_s^{*+} \to D_s^+ \pi^0)}. \quad (4)$$

For their experimental data, we take

$$R(D_{s0}^+)_{\text{exp}} < 0.059 \quad \text{and} \quad R(D_s^{*+})_{\text{exp}}^{-1} = 0.062 \pm 0.006 \pm 0.005 \quad (5)$$

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The former has been provided by the CLEO collaboration as mentioned before and the latter has been improved recently by the BABAR. Models which cannot satisfy both of these data should be excluded.

The ratios $R(D^{+}_{s0})$ for $D^{+}_{s}$ as the chiral partner of $D^{+}_{s}$ and as a DK molecule have been calculated in Ref. as listed in Table 1. The assignment to the DK molecule may be excluded because the result of $R(D^{+}_{s0}) \sim 3$ which is far beyond the experimental upper bound in Eq. (2). In this type of models, one would have an additional broad resonance (above the DK threshold) whose dominant component is the scalar $\{cs\}$. As seen in Table 1, the assignment to the conventional scalar or the chiral partner of $D^{+}_{s}$ seems to be difficult to satisfy $R(D^{+}_{s0})_{\text{exp}}$ and $R(D^{+}_{s})$ simultaneously. When $D^{+}_{s0}(2317)$ is assigned to the iso-singlet $D_{0s} \sim \{cnn\}_{I=0}$ as in the case (e), it should be observed as a narrow peak on a broad ($\gtrsim 100$ MeV) bump arising from $D^{+}_{s0}$ in the $D^{+}_{s0}\pi^0$ mass distribution if $D^{+}_{s0}$ is sufficiently produced. Besides, the conventional scalar $\{cs\}$ might be observed as an additional (broad) resonance in the DK channel, if its mass is higher than the DK threshold. In the case (g), $D^{+}_{s0}(2317)$ has been considered as one of dynamically generated resonances. However, in this type of theories, their radiative decays have not yet been investigated.

Table 1. Various assignments of $D^{+}_{s0}(2317)$ and tentative comments on them. $R(D^{+}_{s0})$ and $R(D^{+}_{s})$ are given in the text.

| Assignments | Comments |
|-------------|----------|
| $\{cs\}$ Scalar | $R(D^{+}_{s0}) \sim 0.13$ (§) $R(D^{+}_{s}) \sim 0.08$ (†), $R(D^{+}_{s})^{-1} \sim 0.018$ (†) |
| $\chi$-partner of $D^{+}_{s}$ | |
| Mixed state of $\{cs\}$ and DK molecule, or $\{cq\bar{q}s\}$ | $R(D^{+}_{s0}) \sim 3$ (§) and Broad $D^{+}_{s0}$ |
| Narrow $\bar{D}^{+}_{0s}$, broad $\bar{D}^{+}_{0s}$ and $D^{+}_{s0}$ | |
| $\hat{F}^{+}_{I} \sim [cn][\bar{s}\bar{n}]_{I=1}$ | Narrow $\hat{F}^{+}_{I}$, $D$, $D^{*}$; $F^{+}_{0}$, $E^{0}$ and Broad $D^{+}_{s0}$ |
| Dynamically generated | $R(D^{+}_{s0}) = ?$, $R(D^{+}_{s})^{-1} = ?$ |

In this article, we study charged scalar four-quark mesons and their two-body decays, and then the decays of charm-strange scalar mesons into $D^{+}\pi^0$ and $D^{+}\gamma$ final states in consistency with the $D^{+}_{s0} \rightarrow D^{+}\pi^0$ and $D^{+}_{s}\gamma$ decays. To this, we assign the observed scalar nonet, $\sigma(600)$, $f_{0}(980)$, $a_{0}(980)$, $\kappa(800)$, to the scalar four-quark $[qq][\bar{q}\bar{q}]$ mesons as suggested long time ago and as supported by many analyses, for example, in Refs. and adopt the observed rates $\Gamma(\sigma(980) \rightarrow \eta\pi)^{\text{exp}}$ and $\Gamma(\phi \rightarrow a_{0}(980)\gamma)^{\text{exp}}$ in Ref. as the input data. In this way, we see later that the experimental data on the $D^{+}_{s}\pi^0$ and $D^{+}\gamma$ decays favor the assignment of $D^{+}_{s0}(2317)$ to the $I_{3} = 0$ component of iso-triplet scalar four-quark mesons with charm and strangeness. It may be considered as an evidence for existence of scalar four-quark mesons with charm.

In the next section, we will give a very brief review on four-quark mesons with light flavors and extend them to the ones with charm. Then, we will study why $D^{+}_{s0}(2317)$ is so narrow. It is predicted that four-quark $[cq][\bar{q}\bar{q}]$ mesons are narrow or rather stable in III. In IV and V, radiative decays and isospin non-conserving decays of $D^{+}_{s0}(2317)$ will be studied, respectively. The results will be compared with the experimental constraint, Eq. (2). The broad bump in the $D\pi$ mass distribution observed by the BELLE and by the FOCUS will be discussed in VI. A brief summary will be given in the final section.

II. FOUR-QUARK MESONS

Before studying four-quark mesons, we review very briefly potentials,

\[ V_{qq}(r) = \sum \Lambda_{i}A_{i}v(r) \quad \text{and} \quad V_{\bar{q}q}(r) = -\sum \Lambda_{i}A_{i}v(r), \]

between two quarks and between a quark and an anti-quark, respectively, mediated by a vector meson with an extra SU(3) degree of freedom corresponding to the "color". The results are summarized in Table 2, because they are still instructive although they have been studied much earlier than the discovery of the color.
As seen in Table 2, the force between two quarks (or between two anti-quarks) is attractive when they are of \( \bar{3}_c \) (or \( 3_c \)) but repulsive when they are of \( 6_c \) (or \( \bar{6}_c \)) while the force between a quark and an anti-quark is attractive and much stronger when they are of color singlet.

Table 2. Potentials mediated by a vector meson with SU(3) "color".

| SU(3) | \( \bar{3}_c \) | \( 6_c \) | \( 8_c \) | \( 1_c \) |
|-------|---------------|---------------|---------------|---------------|
| Potential | \(-\frac{8}{3}(v)\) | \(-\frac{4}{3}(v)\) | \(\frac{2}{3}(v)\) | \(-\frac{16}{3}(v)\) |

Now, four-quark meson states can be classified into the following four groups \(^{27}\),

\[
\{qq\bar{q}q\} = \{qq|\bar{q}q\} \oplus (qq)(\bar{q}q) \oplus \{|qq|q\bar{q}\} \pm (qq)|q\bar{q}\}
\]

where ( ) and [ ] denote symmetry and anti-symmetry, respectively, under the exchange of flavors between them. The first two on the right-hand-side (r.h.s.) of Eq. 4 can have \( J^{P(C)} = 0^+ (+) \) while the last two have \( J^P = 1^+ \). We are now interested only in scalar mesons, so that we consider the first two. Each of them is again classified into two classes since there are two different ways to produce color singlet \( \bar{3}_c \times \bar{3}_c \) and \( 6_c \times \bar{6}_c \) representations to produce totally colorless states. Because of the property of the forces between two quarks (or anti-quarks) discussed before, it is expected that the scalar \( |qq|\bar{q}q \) mesons of \( \bar{3}_c \times \bar{3}_c \) of SU(3) can be the lowest lying four-quark mesons. However, these two states of \( \bar{3}_c \times \bar{3}_c \) and \( 6_c \times \bar{6}_c \), in general, can mix with each other. Nevertheless, we neglect hereafter the small mixing of \( 6_c \times \bar{6}_c \) in the lighter class of \( |qq|\bar{q}q \) for simplicity. The MIT bag model with the bag potential and a spin-spin force arising from one gluon exchange shows that the \( |qq|\bar{q}q \) mesons which are dominantly of \( \bar{3}_c \times \bar{3}_c \) and make a \( 9_F \)-plet of the flavor SU(3) are the lowest lying states. \(^{27}\) The wave functions of the \( |qq|\bar{q}q \) mesons with respect to flavors and their mass values which have been calculated in Ref. \(^{27}\) are listed in Table 3. As seen in the table, the mass relations among the \( |qq|\bar{q}q \) nonet members seem to be well realized by the observed candidates, although the calculated mass values are systematically a little larger than the observed ones. In this way, we can easily understand the approximate degeneracy between \( f_0(980) \) and \( a_0(980) \), and the mass hierarchy in the nonet.

Table 3. Ideally mixed scalar \( |qq|\bar{q}q \) mesons (with \( q = u, d, s \)) and their candidates, where \( S \) and \( I \) denote strangeness and isospin quantum numbers. The calculated mass values have been taken from Ref. \(^{27}\).

| \( S \) | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) | Mass(GeV) | Candidate |
|------|-----------|-----------|-----------|---------|-----------|
| 1    | \( \bar{c} \sim [ud][\bar{n}s] \) | \( \bar{\sigma}^* \sim [ns][\bar{n}s]_{I=0} \) | 0.90     | \( \kappa(800) \) |
| 0    | \( [ns][\bar{n}s]_{I=1} \) \( \sim \delta^* \) | \( \bar{\sigma} \sim [ud][\bar{u}d] \) | 1.10     | \( a_0(980), f_0(980) \) |

The \( (qq)(\bar{q}q) \) mesons also can have \( J^{P(C)} = 0^+ (+) \). However, their masses which have been estimated in the same way as the above are much higher than those of the \( |qq|\bar{q}q \) mesons \(^{27}\), so that we do not consider the \( (qq)(\bar{q}q) \) any more in this article.

Table 4. Ideally mixed scalar \( |cq|\bar{q}q \) mesons (with \( q = u, d, s \)), where \( C, S \) and \( I \) denote charm, strangeness and isospin quantum numbers. Their mass values are estimated in the text.

| \( C \) | \( S \) | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) | Mass(GeV) |
|------|------|-----------|-----------|-----------|---------|
| 1    | 0    | \( F^{d,+,+}_{1} \sim [cn][\bar{s}n]_{I=1} \) | \( F^d_{0} \sim [cn][\bar{s}n]_{I=0} \) | 2.32(†)  |
| -1   | 0    | \( D^{s,+,0} \sim [cs][\bar{s}n] \) | \( \bar{D}^{s,+,0} \sim [cn][\bar{u}d] \) | 2.42     |
|      |      | \( E^{0} \sim [cs][\bar{u}d] \) |                                      | 2.22     |

(†) Input data

Extension to charm scalar four-quark mesons is straightforward \(^{19}\), i.e., to replace a light quark \( q \) in the \( |qq|\bar{q}q \) by a charm quark \( c \). Open charm scalar four-quark \( |cq|\bar{q}q \) mesons are listed in Table 4. However, their mass values are not yet definite, although there have been various efforts \(^{30} \) \( ^{31} \) \( ^{32} \) \( ^{33} \) \( ^{34} \) to estimate their masses. Therefore, we
list tentative results from a simple quark counting with \(\Delta_s = m_s - m_n \approx m_{D_s} - m_D \approx 0.10\) GeV around 2 GeV scale, where we have assigned \(D_{s0}^+(2317)\) to the iso-triplet \(\tilde{F}_1^+\) meson and have taken \(m_{D_{s0}} = 2.32\) GeV as the input data.

Next, we study isospin conserving decays of scalar \([cq][\bar{q}\bar{q}]\) mesons, assigning \(D_{s0}^+(2317)\) to the iso-triplet scalar \(\tilde{F}_1^+ \sim [cn][\bar{s}\bar{n}]\) meson and comparing them with the observed decay \(a_0(980) \rightarrow \eta\pi\), where the observed \(\sigma(600), a_0(980)\) and \(f_0(980)\) are assigned to the scalar \([cq][\bar{q}\bar{q}]\) mesons, \(\sigma, \, \kappa, \, \delta_s^*\) and \(\delta_s^*\), respectively, as discussed before. In this way, we see, below, why \(\tilde{F}_1^+\) is so narrow, and the other decays will be studied in the later sections.

To this, we write the rate for the \(A(p) \rightarrow B(p')\pi(q)\) decay as

\[
\Gamma(A \rightarrow B\pi) = \left( \frac{1}{2J_A + 1} \right) \frac{q_e}{\delta \pi m_A} \sum_{\text{spins}} |M(A \rightarrow B\pi)|^2 ,
\]

where \(J_A, q_e\) and \(M(A \rightarrow B\pi)\) are the spin of the parent particle \(A\), the momentum of the final particles in the rest frame of the parent \(A\), and the amplitude for the decay, respectively. To calculate the amplitude, we use the PCAC (partially conserved axial vector current) hypothesis and a hard pion approximation in the infinite momentum frame (IMF), i.e., \(|p| \rightarrow \infty\). It is an innovation of the old current algebra. In this approximation, the amplitude is evaluated at a little unphysical point, i.e., \(m_{\pi}^2 \rightarrow 0\),

\[
M(A \rightarrow B\pi) \approx \left( \frac{m_{A}^2 - m_{B}^2}{f_\pi} \right) \langle B|A_\pi|A \rangle ,
\]

where \(A_\pi\) is the axial counterpart of the isospin. The asymptotic matrix element of \(A_\pi\) (matrix element of \(A_\pi\)) taken between single hadron states with infinite momentum, \(\langle B|A_\pi|A \rangle\), gives the dimensionless coupling strength.

The \(\tilde{F}_1^+ \rightarrow D_{s0}^+\pi^0\) decay is only one decay of \(\tilde{F}_1^+\) which conserves isospin, so that the narrow width of \(\tilde{F}_1^+\) means that the rate for the decay is small, i.e., overlap of wavefunctions between the final \(D_{s0}^+\pi^0\) and the initial \(\tilde{F}_1^+\) is small. In the present case, it corresponds to a small value of \(\langle D_{s0}^+|A_\pi|\tilde{F}_1^+ \rangle\). Such a small overlap of wavefunctions is possible in decays of heavy (charm) four-quark mesons into two pseudoscalar mesons \([42\, 43\, 44]\) as seen below. The scalar four-quark \([qq][\bar{q}\bar{q}]\) meson state can be decomposed into a sum of products of two \(|q\bar{q}\rangle\) states with various color and spin:

\[
||qq\rangle_{3c}^1[\bar{q}\bar{q}\rangle_{5c}^1]_1^s = -\frac{\sqrt{1}}{4}\sqrt{\frac{1}{3}}\times\sqrt{\frac{1}{3}}(qq\rangle_1^s\bar{q}\bar{q}\rangle_1^s)_{1c}^1 + \frac{\sqrt{3}}{4}\times\sqrt{\frac{1}{3}}(qq\rangle_1^s\bar{q}\bar{q}\rangle_3^s)_{1c}^1 - \frac{\sqrt{1}}{4}\sqrt{\frac{1}{3}}\times\sqrt{\frac{1}{3}}(qq\rangle_1^s\bar{q}\bar{q}\rangle_3^s)_{1c}^1 + \frac{\sqrt{3}}{4}\times\sqrt{\frac{1}{3}}(qq\rangle_3^s\bar{q}\bar{q}\rangle_3^s)_{1c}^1 ,
\]

where the first and second coefficients of each term on the r.h.s. of the above equation are given by the crossing matrices for spin and color \([27]\), respectively. The superscripts \(1_s\) and \(3_s\) denote the spin singlet and triplet, and the subscripts \(1_c\) and \(5_s\) the color singlet and octet. Since the \(|q\bar{q}\rangle\) pairs in the second line of the r.h.s. are of \(5_s\), they are unable to go into a final state of two physical mesons, unless a gluon is exchanged between them. When a gluon is exchanged, however, the states in the second line can supply the states with the configuration of color and spin in the first line, i.e., the configuration of color and spin given in Eq. \([10]\) will be reshuffled by a gluon exchange, and then the states in the second line can go to two pseudoscalar and two vector meson states.

Such a gluon exchange will be more effective at lower energy scale than at higher energy scale. Since it is known that the s-quark is considerably "slim" at the 2 GeV scale \([12]\), gluon couplings around the scale of charm meson mass will be rather perturbative and therefore it is expected that gluon exchanges between the two \(|q\bar{q}\rangle_8\)'s on the r.h.s. will be less effective in the charmed four-quark mesons. In contrast, in the case of light four-quark mesons, they will be non-perturbative and the gluon exchange between the two \(|q\bar{q}\rangle_8\)'s will be highly effective, so that the configurations of color and spin can be very easily reshuffled.

Now we compare the \(\tilde{F}_1^+ \rightarrow D_{s0}^+\pi^0\) with the experimentally known \(\delta_s^+ \rightarrow \eta\pi^+\). The \(\eta\) can be decomposed into a sum of \(n\bar{n}\) and \(s\bar{s}\) components (\(\eta^n\) and \(\eta^s\), respectively) as

\[
\eta = \cos \Theta \cdot \eta^n - \sin \Theta \cdot \eta^s
\]

where \(\Theta = \chi + \theta_P\) with \(\cos \chi = \sqrt{1/3}\) and the usual \(\eta^n\) mixing angle \(\theta_P \approx -20^\circ\). To see difference of wavefunction overlap between the above two decays, we introduce a parameter given by a ratio of the asymptotic matrix elements,

\[
\beta \equiv \frac{\sqrt{2}\langle D_{s0}^+|A_\pi|\tilde{F}_1^+ \rangle}{\langle \eta^s|A_\pi|\delta_s^+ \rangle}.
\]
A naive asymptotic $SU_f(4)$ symmetry \[46\] in which different configurations of color and spin are not cared and overlap of spatial wavefunctions is assumed to be in the symmetry limit implies that $\beta = 1$. However, the fact that a four-quark $[qq][q\bar{q}]$ state can be decomposed into a sum of products of two $[qq]$ pairs with various configurations of color and spin is very important in decays of four-quark mesons as seen above. For simplicity, we here consider the limiting case that no reshuffling in the $\hat{F}_I^+ \rightarrow D_s^+\pi^0$ (at higher energy scale) but the full reshuffling in the $\hat{\delta}^{s+} \rightarrow \eta\pi^+$ decay (at lower energy scale). In this limit, the overlapping factor $\beta$ with respect to the color and spin wavefunctions could be given by

$$|\beta|^2_{SU_f(4)} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{12},$$

(13)

when the overlap of the spatial wavefunctions is in the $SU_f(4)$ symmetry limit.

Taking tentatively $\Gamma(\hat{\delta}^{s+} \rightarrow \eta\pi^+) \sim 70$ MeV from the experimental data \[5\] that $\Gamma_{a_0(980)} = 50 - 100$ MeV and $a_0(980) \rightarrow \eta\pi$ dominates the decays of $a_0(980)$ and using Eq. \[8\] with Eq. \[9\], we estimate

$$|\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle| \sim 0.80$$

(14)

The above value of $|\langle\eta^s|A_{\pi^-}|\hat{\delta}^{s+}\rangle|$ and Eq. \[13\] lead us to

$$\Gamma(\hat{F}_I^+ \rightarrow D_s^+\pi^0)_{SU_f(4)} \sim 9 \text{ MeV},$$

(15)

which dominates the width of $\hat{F}_I^+$. The above value of the rate is a little larger than the experimental upper bound of width of $D_{s0}^+(2317)$. However, it is not serious because deviation from the $SU_f(4)$ symmetry limit of spatial wavefunction overlap is not considered at this stage. $SU_f(4)$ symmetry of spatial wavefunction overlap overestimates asymptotic matrix elements of charges, usually by about $20 - 30\%$, as seen below.

A measure of the (asymptotic) flavor symmetry breaking (in overlap of spatial wavefunctions) will be seen in deviations from unity of values of form factor, $f_+(0)$, of related vector current at zero momentum transfer squared. The estimated values of $f_+(0)$ are

$$f_+^{(\pi K)}(0) = 0.961 \pm 0.008,$$

(16)

$$f_+^{(RD)}(0) = 0.74 \pm 0.03,$$

(17)

$$\left[ f_+^{(\pi D)}(0) \right] / \left[ f_+^{(K D)}(0) \right] = 1.00 \pm 0.11 \pm 0.02,$$

(18)

$$= 0.99 \pm 0.08,$$

(19)

where the values in Eqs. \[16\] - \[19\] have been taken from Refs. \[41\] - \[50\], respectively. They imply that the asymptotic flavor $SU_f(3)$ symmetry works well while the $SU_f(4)$ overestimates by about $20 - 30\%$. This statement is confirmed by the observation that the asymptotic symmetry has predicted the rates \[11\], \[51\]. $\Gamma(D^{*+} \rightarrow D^0\pi^+)_{SU_f(4)} \simeq 96$ keV and $\Gamma(D^{*+} \rightarrow D^{0}\pi^+)^{\text{exp}} = 65 \pm 18$ keV, which are larger by about $50\%$ than the observed values, $\Gamma(D^{*+} \rightarrow D^{0}\pi^+)^{\text{exp}} = 30 \pm 8$ keV, obtained from the measured decay width \[52\], $\Gamma_{D^{*+}} = 96 \pm 4 \pm 22$ keV, and the branching fractions compiled in Ref. \[5\]. The above suggests that asymptotic $SU_f(4)$ symmetry overestimates the rate for the $D^{*} \rightarrow D\pi$ decays, by about $50\%$, as expected from the above values of the form factor $f_+(0)$. However, for simplicity, we will use asymptotic $SU_f(4)$ symmetry relations among asymptotic matrix elements of $A_\pi$ and $A_K$ in our estimates of decay rates. When we take account for the symmetry breaking, we will note it.

The above asymptotic $SU_f(4)$ symmetry breaking reduces the result in Eq. \[15\] by about $40 - 60\%$ because the amplitude is proportional to the asymptotic matrix element of axial charge, i.e.,

$$\Gamma(\hat{F}_I^+ \rightarrow D_s^+\pi^0) \sim 4 - 5 \text{ MeV}.$$

(20)

It is consistent with the experimental data on the width of $D_{s0}^+(2317)$ in Eq. \[14\]. It seems to imply that the above limiting situation providing $|\beta|^2_{SU_f(4)} = 1/12$ is approximately realized and that the asymptotic $SU_f(4)$ symmetry breaking arising from the overlap of spatial wavefunctions including a four-quark state is not very far from the one between the conventional meson states.

### III. ISOSPIN CONSERVING DECAYS

In this section, we study isospin conserving two-body decays of the scalar $[cq][q\bar{q}]$ mesons. $SU_f(3)$ relations of asymptotic matrix elements of $A_\pi$ are given by

$$\langle D_s^+|A_\pi^-|\hat{F}_I^{++}\rangle = \sqrt{2}\langle D_s^+|A_\pi^0|\hat{F}_I^+\rangle = \langle D_s^+|A_\pi^+|\hat{F}_I^0\rangle$$
Inserting Eq. (3) with Eq. (21) into Eq. (8) and using the width, \( \Gamma(\hat{F}_I^+ \to D_s^+\pi^0) \approx 4.5 \text{ MeV} \), from Eq. (20) as the input data, we obtain the results in Table 5. As seen in the table, the iso-triplet \( \hat{F}_I \) mesons have the same width as the input. The iso-doublet \( \hat{D} \) mesons are broader by about 50 % than the input data because they have two possible decay modes through isospin conserving strong interactions. Another iso-doublet \( \hat{D}^\ast \) mesons will be just on the threshold of \( D\eta \) decay, if the true mass of \( D^\ast \) is close to the estimated one. It is only one strong decay which is kinematically allowed. Since its rate is sensitive to the mass of \( D^\ast \) whose precise value has not been known, however, we here do not calculate it. Nevertheless, it is expected that its width is much narrower than the input.

The iso-singlet \( \hat{F}_0^+ \) meson will decay dominantly into \( D_s^+\gamma \) as will be studied later, if its mass is really lower than the \( D\bar{K} \) threshold. The exotic \( \hat{E}_0^0 \) would decay through weak interactions unless its mass is higher than the \( D\bar{K} \) threshold. Even if it is more massive than \( m_D + m_K \), it would be as narrow as the other members of the scalar \([cq][\bar{q}q]\) mesons.

Table 5. Dominant decays of scalar \([cq][\bar{q}q]\) mesons. The input data, \( \Gamma(\hat{F}_I^+ \to D_s^+\pi^0) \approx 4.5 \text{ MeV} \), is taken from the estimated width, Eq. (20), in the text.

| Parent (Mass in GeV) | Final State | Decay Rate (in MeV) |
|---------------------|-------------|---------------------|
| \( \hat{F}_I^+ \) (2.32) | \( D_s^+\pi^0 \) | \( \sim 4.5 \) |
| \( \hat{F}_I^+ \) (2.32) | \( D_s^+\pi^0 \) | \( \sim 4.5 \) |
| \( \hat{F}_I^0 \) (2.32) | \( D_s^+\pi^- \) | \( \sim 4.5 \) |
| \( \hat{D}^+ \) (2.22) | \( D^0\pi^+ \) | \( \sim 4.5 \) |
| \( \hat{D}^+ \) (2.22) | \( D^+\pi^0 \) | \( \sim 2.3 \) |
| \( \hat{D}^+ \) (2.22) | \( D^+\pi^- \) | \( \sim 4.5 \) |
| \( \hat{D}^+ \) (2.22) | \( D^0\pi^0 \) | \( \sim 2.3 \) |
| \( \hat{D}^0 \) (2.42) | \( D\eta \) (or \( D^{++}\gamma \)) | \( \ll 4.5 \) |
| \( \hat{F}_0^+ \) (2.32) | \( D_s^{++}\gamma \) | \( \sim 0.005(\ast) \) |
| \( \hat{E}_0^0 \) (2.32) | \( \langle DK \rangle \) | (weak int.) |

\( \ast \) Discussed later.

IV. RADIATIVE DECAYS

Now we study the radiative decays of \( D_s^{++} \) and \( D_s^{0} \) under the vector meson dominance (VMD). The results will be compared with \( D_s^{++}\pi^0 \) decays later. Amplitudes for the radiative decays of vector and scalar mesons are written as

\[
M(V \to P\gamma) = \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu}(\gamma) G_{\alpha\beta}(V) A(V \to P\gamma)
\]  
(22)

and

\[
M(S \to V\gamma) = F_{\mu\nu}(\gamma) G^{\mu\nu}(V) A(S \to V\gamma),
\]
(23)

respectively. The amplitudes \( A(V \to P\gamma) \) and \( A(S \to V\gamma) \) are given by

\[
A(V \to P\gamma) = \sum_{V' = \rho, \omega, \phi, \psi} \frac{X_{V'}(0)}{m_{V'}^2} A(V \to PV')
\]
(24)

and

\[
A(S \to V\gamma) = \sum_{V' = \rho, \omega, \phi, \psi} \frac{X_{V'}(0)}{m_{V'}^2} A(S \toVV')
\]
(25)

under the VMD, where \( X_V(0) \) is the \( \gamma V \) coupling strength on the photon-mass-shell. The OZI rule selects possible vector mesons which can work as a pole in the radiative decays. The photon-momentum-square dependence of
$X_V$ has been studied in Ref. [56]. The values of $X_V(0)$ have been measured by using photoproductions of vector mesons on various nuclei [57]. For the $\psi$ photoproduction, both the measured differential cross section at $t = 0$, $d\sigma(\gamma N \to \psi N)/dt|_{t=0} \approx 20 \text{ nb/GeV}^2$ around $\sim 20 \text{ GeV}$, and the $\psi N$ total cross section, $\sigma_T(\psi N) = 3.5 \pm 0.8 \text{ nb}$, measured in $A$-dependence of photoproduction cross sections [58] still have large uncertainties, where $t$, $N$, and $A$ are a momentum transfer squared, a nucleon, and a mass number of nucleus. $\psi$ denotes the usual $J/\psi$. The values of $X_V(0)$ and $X_V(m^2_V)$ on the vector meson mass-shell are listed in Table 6, where $X_V(m^2_V)$ has been estimated from rates for leptonic decays of vector mesons. Their sign is determined by using the quark model. It should be noted that the above $|X_\phi(0)|$ and $|X_\psi(0)|$ are considerably smaller than $|X_\phi(m^2_V)|$ and $|X_\psi(m^2_V)|$, respectively.

Table 6. Photon-vector meson coupling strengths in GeV$^2$. The sign of $X_V$ is determined by using the quark model.

| $V$ | $X_V(k^2 = 0)$ in GeV$^2$ | $X_V(k^2 = m^2_V)$ in GeV$^2$ |
|-----|-----------------|-----------------|
| $\rho^0$ | 0.033 $\pm$ 0.003 | 0.0357 $\pm$ 0.0008 |
| $\omega$ | 0.011 $\pm$ 0.001 | 0.0109 $\pm$ 0.0002 |
| $\phi$ | -0.018 $\pm$ 0.004 | -0.0238 $\pm$ 0.0003 |
| $\psi$ | $\sim$ 0.054 | 0.380 $\pm$ 0.013 |

Before going to the radiative decays of scalar mesons, we study radiative decays of vector mesons. We first estimate $A(\omega \to \pi^0 \rho^0)$ as one of $A(V \to PV')$ coupling strengths from the measured rate for the $\omega \to \pi^0 \gamma$ [3],

$$\Gamma(\omega \to \pi^0 \gamma)_{\text{exp}} = 0.734 \pm 0.035 \text{ MeV}. \tag{26}$$

In this decay, the OZI-allowed vector meson pole is given by the $\rho^0$. Putting $V' = \rho^0$ in the amplitude, Eq. (24), inserting it to Eq. (22) and using Eq. (3), we obtain

$$|A(\omega \to \pi^0 \rho^0)| \approx 18 \text{ (GeV)}^{-1} \tag{27}$$

which will be used below as an input data.

Now we study the $D_s^{\ast+} \to D_s^+ \gamma$ decay comparing with the $\omega \to \pi^0 \gamma$. The OZI-allowed poles are given by $\phi$ and $\psi$ mesons. Use of the $SU_f(4)$ relation of the $VPV'$ coupling strengths,

$$2A(D^{*0} \to D^0 \rho^0) = 2A(D^{*0} \to D^0 \omega) = \sqrt{2}A(D^{*0} \to D^0 \psi)$$

$$= -2A(D^{*+} \to D^+ \rho^0) = 2A(D^{*+} \to D^+ \omega) = \sqrt{2}A(D^{*+} \to D^+ \psi)$$

$$= \sqrt{2}A(D_s^{\ast+} \to D_s^+ \phi) = \sqrt{2}A(D_s^{\ast+} \to D_s^+ \psi) = A(\omega \to \pi^0 \rho^0), \tag{28}$$

leads to

$$\Gamma(D_s^{\ast+} \to D_s^+ \gamma)_{SU_f(4)} \approx 0.8 \text{ keV}, \tag{29}$$

which is consistent with the experimental constraint [3], $\Gamma(D_s^{\ast+} \to D_s^+ \gamma) < 1.8 \text{ MeV}$.

We here estimate uncertainties arising from the $SU_f(4)$ relation for the $VPV'$ coupling strengths. In the same way as the above, we can estimate rates for the $D^* \to D \gamma$ decays as

$$\Gamma(D^{++} \to D^+ \gamma)_{SU_f(4)} \approx 2.4 \text{ keV} \quad \text{and} \quad \Gamma(D^{*0} \to D^0 \gamma)_{SU_f(4)} \approx 19 \text{ keV}, \tag{30}$$

which can be compared with the measured ones [3].

$$\Gamma(D^{+} \to D^+ \gamma)_{\text{exp}} = 1.5 \pm 0.5 \text{ keV} \quad \text{and} \quad \Gamma(D^{*0} \to D^0 \gamma)_{\text{exp}} < 800 \text{ keV}. \tag{31}$$

In particular, the ratio

$$\frac{\Gamma(D^{*+} \to D^+ \gamma)_{SU_f(4)}}{\Gamma(D^{++} \to D^+ \gamma)_{\text{exp}}} \approx 1.6 \tag{32}$$

implies that the $SU_f(4)$ symmetry again overestimates the rate for radiative decays by $\approx 60\%$ with large errors.

The radiative $D_{s0}^+(2317) \to D_{s+}^+ \gamma$ decay is now in order. Possible three cases, i.e., (i) $D_{s0}^+(2317)$ is assumed to be the iso-singlet four-quark meson $F_{0+}^{*}$, (ii) $D_{s0}^+(2317)$ is assumed to be the conventional scalar meson $D_{s0}^+$, and (iii)
$D_s^0(2317)$ is assumed to be the iso-triplet four-quark meson $\hat{F}^+_I$, are studied. In the case (iii), the result will be compared with the rate, $\Gamma(\hat{F}^+_I \to D_s^+ \pi^0)_{SU_f(4)} \sim 9$ MeV in Eq. \(16\) or $\Gamma(\hat{F}^+_I \to D_s^+ \pi^0) \sim 4 - 5$ MeV, in Eq. \(20\) and it will be seen that the experiments favor the assignment of $D_s^0(2317)$ to $\hat{F}^+_I$. In the cases (i) and (ii), the results will be compared with the isospin non-conserving decays in the next section. We, first, consider the case (iii) $D_s^0(2317) = \hat{F}^+_I \sim [cn][\bar{s}n]_{I=1}$. In the $\hat{F}^+_I \to D_s^+ \gamma$, the OZI-allowed pole is given by $\rho^0$. The $SU_f(4)$ relation for the $SVV'$ couplings is

$$2A(\hat{F}^+_I \to D_s^+ \rho^0) = 2A(\hat{F}^+_0 \to D_s^+ \omega) = A(\phi \to \delta^s \rho^0)(\beta')_{SU_f(4)},$$

(33)

where the overlapping factor is now $\beta'$ (in place of $\beta$) because the mesons which couple to the $[cq][\bar{q}q]$ meson are now two vector mesons. Use of the measured rate \(3\),

$$\Gamma(\phi \to a_0(980)\gamma)_{\text{exp}} = 0.32 \pm 0.03 \text{ keV},$$

(34)
as the input data and the $SU_f(4)$ relation, Eq. \(33\), with $|\beta'|_{SU_f(4)}^2 \sim 1/4$ provides

$$\Gamma(\hat{F}^+_I \to D_s^+ \gamma)_{SU_f(4)} \simeq 45 \text{ keV}.$$  

(35)

Then its ratio to the rate $\Gamma(\hat{F}^+_I \to D_s^+ \pi^0)_{SU_f(4)} \sim 9$ MeV in Eq. \(16\) is $R(\hat{F}^+_I)_{SU_f(4)} \sim 0.005$. It will be left intact even if the $SU_f(4)$ symmetry breaking arising from overlap of spatial wavefunctions which has been discussed to be approximately the same in both cases is taken into account, i.e.,

$$R(\hat{F}^+_I) = \frac{\Gamma(\hat{F}^+_I \to D_s^+ \gamma)}{\Gamma(\hat{F}^+_I \to D_s^+ \pi^0)} \sim 0.005,$$

(36)

which satisfies well the experimental constraint, Eq. \(2\). Therefore the experiment favors the assignment of the new resonance $D_s^0(2317)$ to the $I_3 = 0$ component $\hat{F}^+_I$ of the iso-triplet four-quark mesons.

Next, we study radiative decays of iso-singlet scalar mesons in the cases (i) and (ii). In the case (i) $D_s^0(2317) = \hat{F}^+_0 \sim [cn][\bar{s}n]_{I=0}$, the OZI-allowed pole is given by $\omega$. When the $SU_f(4)$ relation, Eq. \(38\), with $|\beta'|_{SU_f(4)}^2 \sim 1/4$ and Eq. \(21\) as the input data are taken, the resulting rate for the radiative decay of $\hat{F}^+_I$ is

$$\Gamma(\hat{F}^+_I \to D_s^+ \gamma)_{SU_f(4)} \simeq 4.7 \text{ keV},$$

(37)

which is smaller by about an order than the above $\Gamma(\hat{F}^+_I \to D_s^+ \gamma)_{SU_f(4)} \simeq 45 \text{ keV}$ because of $X_{\rho0}(0) \simeq 3X_{\omega}(0)$. In the case (ii) $D_s^0(2317) = D_s^+ \sim \{c\bar{s}\}$, the OZI-allowed poles are given by $\phi$ and $\psi$ mesons. The $SU_f(4)$ relation of the $SVV'$ couplings is now

$$2A(D_s^+ \to D_s^+ \phi) = 2A(D_s^+ \to D_s^+ \psi) = A(\chi_{c0} \to \psi\psi),$$

(38)

where $\chi_{c0}$ denotes the $^3P_0 \{c\bar{s}\}$. The $SU_f(4)$ relation, Eq. \(38\), reads

$$\Gamma(D_s^+ \to D_s^+ \gamma)_{SU_f(4)} \simeq 35 \text{ keV}.$$  

(39)

Here $\Gamma(\chi_{c0} \to \psi\psi)_{\text{exp}} = 119 \pm 15 \text{ keV}$ \(2\) has been taken as the input data. The above result is much larger than the one of the constituent quark models \(25, 57\), in which the rate for the radiative decay has been strongly suppressed by taking a large strange $(s)$ quark mass, $m_s \simeq (3 - 4)m_s$, far from the heavy charm quark picture $(m_c \gg m_s)$. Such a "fat" $s$-quark reduces drastically the dipole moment of the $\{c\bar{s}\}$ system. However, it might be unnatural since a more slim $s$-quark, $m_s \simeq 0.9 \text{ GeV}$, seems to be favored by a semi-relativistic quark model \(60\) in the $\{c\bar{s}\}$ meson spectroscopy and by the recent result (at $\sim 2 \text{ GeV}$ scale) from the lattice QCD \(15\) as mentioned before. If $m_c \gg m_s$, the rate for the radiative decay will be much larger than the ones in Refs. \(25\) and \(57\), although, in this case, the $s$-quark might not behave non-relativistically. We will compare the results in Eqs. \(37\) and \(39\) with rates for the isospin non-conserving decays in the next section.

V. ISOSPIN NON-CONSERVING DECAYS

When $D_s^0(2317)$ is assigned to an iso-singlet state, the $D_s^0(2317) \to D_s^+ \pi^0$ decay does not conserve isospin. It is assumed to proceed through a tiny $\pi^0\eta$ mixing as usual \(61, 62\), where the physical $(\pi^0)_{\text{phys}}$ is an admixture of the pure iso-triplet $\pi^0$ and the iso-singlet $\eta$, i.e.,

$$\pi^0_{\text{phys}} \simeq \pi^0 + c\eta, \quad (|c| \ll 1).$$

(40)
To use the hard pion technique in the IMF, we replace the above mixing by

\[(A_{\pi^0})_{\text{phys}} \simeq A_{\pi^0} + \epsilon A_\eta, \quad (|\epsilon| \ll 1).\]  

(41)

The isospin non-conserving parameter \(\epsilon\) is estimated below. The non-diagonal element of \(\pi^0\)-\(\eta\) mass matrix was given by

\[\langle \pi^0 | \delta m^2 | \eta \rangle = \sqrt{\frac{1}{3}} \left\{ (m_{\pi^0}^2 - m_{\pi^+}^2) - (m_{K\eta}^2 - m_{K^+}^2) \right\},\]  

(42)

long time ago by assuming that the mixing is caused by the electromagnetic interactions. Then the \(\pi^0\)-\(\eta\) mixing parameter was estimated as \(61\)

\[\epsilon = -\frac{\langle \pi^0 | \delta m^2 | \eta \rangle}{m_{\pi^0}^2 - m_{\pi^+}^2} = 0.0105 \pm 0.0013,\]  

(43)

by inserting the well-known mass values of the related mesons. The estimated value of \(\epsilon\) is of the order of the fine structure constant \(\alpha\), i.e., \(\epsilon \sim O(\alpha)\) as expected. It implies that the isospin non-conserving interactions are much weaker than the electromagnetic ones.

With the above value of \(\epsilon\), we estimate the rate for the isospin non-conserving \(D_s^{++} \rightarrow D_s^{+} \pi^0\) decay. The \(\pi^0\)-\(\eta\) mixing, the OZI rule and the asymptotic \(SU_f(4)\) symmetry lead to

\[\langle D_s^{++} | (A_{\pi^0})_{\text{phys}} | D_s^{++} \rangle \simeq -\frac{1}{2} \epsilon \sin \Theta \langle \pi^+ | A_{\pi^+} | \rho^0 \rangle,\]  

(44)

where \(\eta = \cos \Theta \eta^* - \sin \Theta \eta^*\) because of the \(\eta\)-\(\eta^*\) mixing. Using \(|\langle \pi^+ | A_{\pi^+} | \rho^0 \rangle| \simeq 1.0\) from \(\Gamma(\rho \rightarrow \pi \pi)_{\exp} \simeq 150\) MeV \(5\), \(\Theta \simeq 35^\circ\) as before, and \(\epsilon \simeq 0.0105\) estimated above, we obtain

\[\Gamma(D_s^{++} \rightarrow D_s^+ \pi^0)_{SU_f(4)} \simeq 50\) eV \(45\)

which is much smaller than the \(\Gamma(D_s^{++} \rightarrow D_s^+ \gamma)_{SU_f(4)} \simeq 0.8\) keV estimated in Eq. \(46\) before. The ratio of these two rates is given by

\[R(D_s^{++})^{-1} \simeq 0.06\]  

(46)

which satisfy well the measured ratio in Eq. \(45\).

Next, we consider isospin non-conserving decays of scalar mesons. We study possible two cases, (i) \(D_{s0}^+(2317)\) as the iso-singlet four-quark meson \(F_0^0\), and (ii) \(D_{s0}^0(2317)\) as the conventional \(D_{s0}^+\) meson. In the case (i), the \(\pi^0\)-\(\eta\) mixing and the OZI rule lead to

\[\langle D_s^+ | (A_{\pi^0})_{\text{phys}} | F_0^+ \rangle \simeq \epsilon \cos \Theta \langle D_s^+ | A_{\eta^*} | F_0^+ \rangle,\]  

(47)

where \(\Theta \simeq 35^\circ\) and \(\epsilon \simeq 0.0105\) as before. The asymptotic \(SU_f(4)\) relates \(\langle D_s^+ | A_{\eta^*} | F_0^+ \rangle\) to \(\langle \eta^* | A_{s^-} | \delta^{s+} \rangle\) whose size has been estimated in Eq. \(13\), i.e.,

\[2\langle D_s^+ | A_{\eta^*} | F_0^+ \rangle = \sqrt{\frac{1}{2}} \langle \eta^* | A_{s^-} | \delta^{s+} \rangle \beta.\]  

(48)

Using Eq. \(15\) with Eq. \(14\) and \(|\beta|^2_{SU_f(4)} \simeq 1/12\), we get

\[\Gamma(F_0^+ \rightarrow D_s^+ \pi^0)_{SU_f(4)} \simeq 0.7\) keV, \(49\)

which is much smaller than the rate for the radiative decay in Eq. \(37\) as expected. In the case (ii), where \(D_{s0}^+(2317)\) is assigned to the conventional scalar meson \(D_{s0}^+\), the \(\pi^0\)-\(\eta\) mixing and the OZI rule lead to

\[\langle D_s^+ | (A_{\pi^0})_{\text{phys}} | D_{s0}^+ \rangle \simeq -\epsilon \sin \Theta \langle D_s^+ | A_{\eta^*} | D_{s0}^+ \rangle,\]  

(50)

where \(\Theta \simeq 35^\circ\) and \(\epsilon \simeq 0.0105\) again. The asymptotic \(SU_f(4)\) symmetry relates the asymptotic matrix element \(\langle D_s^+ | A_{\eta^*} | D_{s0}^+ \rangle\) to the experimentally known \(\langle K^+ | A_{s^+} | K_0^{*0}(1430) \rangle\),

\[\langle D_s^+ | A_{\eta^*} | D_{s0}^+ \rangle = \langle K^+ | A_{s^+} | K_0^{*0}(1430) \rangle,\]  

(51)
where $K_{s}^{*+}(1430)$ has been assigned to the conventional scalar \{$ns$\} meson as usual \cite{31}. The size of the matrix element in the r.h.s. of the above equation is estimated as

$$|\langle K^+ |A_{\pi^+} |K_0^{*0}(1430) \rangle| \simeq 0.29$$  \hspace{1cm} (52)

by using the mass $m_{K^*} = 1412 \pm 6$ MeV and the measured rate $\Gamma(K_0^{*0}(1430) \to K^+\pi^-)_{\text{exp}} = 182 \pm 24$ MeV from $\Gamma_{K_0^{*}} = 294 \pm 23$ MeV and $Br(K_0^{*0} \to K^+\pi^-) = 93 \pm 10$ % compiled by the PDG04 \cite{54}. With help of Eqs. (51) and (52), the rate for the $D_{s0}^{*+} \to D_{s}^{+}\pi^{0}$ is estimated as

$$\Gamma(D_{s0}^{*+} \to D_{s}^{+}\pi^{0})_{SU_f(4)} \simeq 0.6 \text{ keV},$$  \hspace{1cm} (53)

which is again much smaller than the rate for the radiative decay of $D_{s0}^{*+}$, Eq. (30), as expected.

Here we summarize the ratios of rates for the radiative $D_{s0}^{*+}\gamma$ decay to the $D_{s}^{+}\pi^{0}$ in Table 7. The experimental data to be compared is $R(D_{s0}^{*+})_{\text{exp}} < 0.059$ in Eq. (2). As seen in Table 7, $R(\tilde{F}_0^{+})$ and $R(D_{s0}^{*+})$ are much larger than the experimental upper bound. However, it is quite natural as discussed before. Therefore, the assignment of the iso-singlet state ($\tilde{F}_0^{+}$ or $D_{s0}^{*+}$) will be excluded, although there have been many efforts \cite{24,25,64,65} to reconcile the assignments with the experimental constraint, Eq. (2). In fact, it is very strange that the ratio is predicted to be much smaller than unity when $D_{s0}^{*+}(2317)$ is assigned to the conventional $D_{s0}^{*+}$, i.e., it means that a higher order term overcomes a lower order term in perturbation theory, since the rate for the isospin non-conserving decay is proportional to $|\epsilon|^2 \sim O(\alpha^2)$ while the one for the radiative decay is of the order of $\alpha$. When $D_{s0}^{*+}(2317)$ is assigned to the iso-triplet $\tilde{F}_0^{+} \sim [cn][\bar{s}\bar{n}]_{I=1}$, however, the ratio $R(\tilde{F}_0^{+})$ satisfies well the experimental constraint. It is again quite natural since the electromagnetic interactions are much weaker than the isospin conserving strong interactions. Therefore, the assignment of $D_{s0}^{*+}(2317)$ to $\tilde{F}_0^{+}$ is favored by the experiment, while the assignments to the iso-singlet states are inconsistent with it.

Table 7. Ratio of the rates for the radiative $D_{s0}^{*+}\gamma$ decay to the $D_{s}^{+}\pi^{0}$ of the scalar ($S = \tilde{F}_0^{+}$, $\tilde{F}_0^{+}$ and $D_{s0}^{*+}$) mesons. The experimental data is given in Ref. \cite{32},

\[
\begin{array}{c|c|c|c|c}
\text{Scalar meson (S)} & \tilde{F}_0^+ & \tilde{F}_0^+ & D_{s0}^{*+} \\
R(S) & \sim 0.005 & \sim 7 & \sim 50 \\
R(D_{s0}^{*+})_{\text{exp}} & < 0.059 & & \\
\end{array}
\]

VI. CONVENTIONAL SCALAR MESONS WITH CHARM

We have studied scalar four-quark $[cq][\bar{s}\bar{q}]$ mesons with charm in the previous sections. In this section we study the conventional charm scalar mesons, $D_{0}^{*} \sim \{c\bar{c}\}$ and $D_{s}^{*+} \sim \{c\bar{s}\}$, comparing with the light strange scalar meson $K_{0}^{*}$ which is usually assigned to the scalar \{n\bar{s}\} meson \cite{31}.

Since the mass of $D_{0}^{*}$ has not definitely been known, we take tentatively $m_{D_{0}^{*}} \simeq 2.35$ GeV which is close to the average of the central values of the broad bumps in the $D_{0}$ mass distributions and is consistent with predictions from various approaches, for example, potential models \cite{8,3}, lattice QCD sum rules \cite{10,11,12}, calculations based on QCD sum rule \cite{60}, etc.

The asymptotic $SU_f(4)$ symmetry relates asymptotic matrix elements of axial charges, $A_{\pi}$ and $A_{K}$, taken between a pseudoscalar meson state and a conventional scalar state to each other, for example, $A_{\pi}$, with the isospin $SU_f(2)$ symmetry and Eq. (8) with Eq. (9) leads to the following width of $D_{0}^{*}$ which is dominated by the $D_{0}^{*} \to D_{\pi}$ decays, $(\Gamma_{D_{0}^{*}})_{SU_f(4)} \simeq (\Gamma(D_{0}^{*} \to D_{\pi})_{SU_f(4)}) \simeq 90$ MeV. When the $SU_f(4)$ symmetry breaking is taken into account, the results will be reduced by about 50 %, i.e.,

$$\Gamma_{D_{0}^{*}} \sim 50 \text{ MeV}.$$  \hspace{1cm} (55)

The above width $\Gamma_{D_{0}^{*}}$ is still rather broad but much narrower than the widths of the bumps in the $D_{0}$ mass distributions observed by the BELLE and by the FOCUS collaboration. Therefore, we propose \cite{44,53,58} that they should be re-interpreted as four-quark mesons and conventional mesons. We do so because of the evidence that the charm-strange scalar meson $D_{s0}^{*}(2317)$ should be regarded as a four-quark state (as studied previous sections) and because there
would be no room of charm non-strange scalar four-quark mesons if each bump were saturated by the conventional $D^*_0$. In addition, we expect $m_{D^*_0}$ to be in this mass region below 2460 MeV in consistency with other approaches as discussed before. Therefore, we consider that the bump has a structure comprising a broader ($\Gamma_{D^*_0} \sim 50 - 90$ MeV) conventional $D^*_0$ in the region of its upper half and a narrower ($\Gamma_D \sim 5 - 10$ MeV) four-quark $\bar{D}$ in the region of its lower tail.

For charm-strange scalar mesons, we have discussed $D^+_{s0}(2317)$ as a possible evidence for existence of charmed four-quark mesons in the previous sections. Its width is very narrow; the measured one has been compiled as $\Gamma_{D^+_{s0}(2317)} < 4.7$ MeV and our estimate has been $\Gamma_{\bar{D}^0} \sim 4 - 5$ MeV. When $m_{D^+_{s0}} \sim 2.35$ GeV is taken, the mass of strange counterpart, $D^0_{s0}$ of the conventional $D^*_0$ is crudely estimated as $m_{D^*_0} \sim 2.45$ GeV by using a simple quark counting. Here we take the above value of $D^0_{s0}$, although there have been efforts to reconcile its value with the measured $m_{D^0_{s0}} \sim 2317$ MeV. This is because assigning $D^+_{s0}(2317)$ to the scalar $\{c\bar{s}\}$ meson is inconsistent with the experimental constraints, Eq. (5), as seen in the previous section. Our $m_{D^*_{s0}} \sim 2.45$ GeV is beyond the threshold of the $DK$ decays. Using a hard kaon approximation ($m_K^2 \rightarrow 0$) in place of the hard pion ($m_\pi^2 \rightarrow 0$), we can estimate the width of $D^0_{s0}$ which is dominated by the $\bar{D}^0 \rightarrow (DK)^0$ decays, i.e., $\Gamma(D^0_{s0} \rightarrow (DK)^0)_{SU_f(4)} \sim 70$ MeV, where the asymptotic $SU_f(4)$ relation, Eq. (24), has been used. When we take account for the $SU_f(4)$ symmetry breaking as before, we obtain

$$\Gamma_{D^0_{s0}} \sim 40 \text{ MeV} \quad (56)$$

which is again rather broad in contrast with the four-quark $\bar{F}^+_I$ meson. Of course, the contribution of possible isospin non-conserving $D^0_{s0} \rightarrow D^+_s \pi^0$ decays will be negligibly small, i.e., $\Gamma(D^0_{s0} \rightarrow D^+_s \pi^0)_{SU_f(4)} \sim 0.6$ keV estimated in Eq. (63). This is consistent with the fact that no scalar resonance has been observed in the region above the $D^0_{s0}(2317)$ resonance up to $\sim 2.7$ GeV in the $D^+_s \pi^0$ mass distribution [1]. It should be noted that the CLEO collaboration [7] have observed a peak around 2.39 GeV in the $DK$ mass distribution but it has been taken away as a false peak arising from the decay, $D_{s1}(2536) \rightarrow DK \rightarrow D[\pi^0]K$, where the $\pi^0$ has been missed. However, we hope that it might involve true signals of a resonance corresponding to $D^0_{s0}$ or that the resonance could be observed in the $DK$ channel in the region of $2.4 - 2.5$ GeV by experiments with higher statistics and resolution.

VII. SUMMARY

We have studied classification of charm scalar mesons and their decays into two pseudoscalar mesons and into the $D^*_0\gamma$. The two body decays has been calculated by using a hard pion (or kaon) technique in the infinite momentum frame which is an innovation of the old current algebra. The radiative decays have been studied under the vector meson dominance hypothesis. When the new resonance $D^+_0(2317)$ is assigned to the $I = 3$ component $\bar{F}^+_I$ of the iso-triplet four-quark mesons, the $\bar{F}^+_I \rightarrow D^+_s \pi^0$ decay can proceed through isospin conserving strong interactions. Therefore, the radiative $\bar{F}^+_I \rightarrow D^*_0\gamma$ decay is much weaker than the $\bar{F}^+_I \rightarrow D^+_s \pi^0$. On the other hand, when it is assigned to an iso-singlet state (the four-quark $\bar{F}^+_I \sim \{cn\}/\{\bar{c}\bar{n}\}$ or the conventional scalar $D^+_s \sim \{c\bar{s}\}$), its decay into the $D^+_s \pi^0$ final state does not conserve isospin. Such an isospin non-conservation has been assumed to proceed through the $\pi^0-\eta$ mixing as usual. We have used the value of the isospin violating parameter $\epsilon \sim 0.0105$ which was estimated long time ago and is of the order of the fine structure constant $\alpha$, i.e., $\epsilon \sim O(\alpha)$. Therefore the isospin non-conserving decays should be more strongly suppressed than the radiative decays. In this way, we have seen that the experimental constraint on the ratio of the decay rates for the $D^+_{s0}(2317) \rightarrow D^*_0\gamma$ to the $D^+_{s0}(2317) \rightarrow D^+_s \pi^0$ favors the assignment of $D^+_{s0}(2317)$ to the iso-triplet four-quark meson $\bar{F}^+_I$. However, it is difficult to reconcile its assignment to an iso-singlet state with the constraint, even if the calculated mass values of the iso-singlet states ($D^+_s$ and $F^+_I$) could reproduce well the observed one, $m_{D^0_{s0}} = 2317.4 \pm 0.9$ MeV. Therefore, $D^+_{s0}(2317)$ can be considered as an evidence for existence of four-quark mesons with charm.

Conventional charm scalar mesons also have been studied in relation to the broad enhancements in the $D_f$ mass distributions which have been observed by the BELLE and by the FOCUS collaboration, independently. We have pointed out that each bump is unlikely to be saturated by a single scalar $D^*_0 \sim \{c\bar{n}\}$ state, and we expect that each enhancement has a structure including at least two peaks, one arising from the four-quark $\bar{D} \sim \{cn/\bar{u}\}$ and the other from the conventional $D^* \sim \{c\bar{n}\}$, although the experiments have claimed that each bump is saturated by a conventional scalar meson. By comparing the decays of $D^*_0$ with the well-known $K^*_0(1430) \rightarrow K\pi$, the widths of $D^*_0$ have been predicted to be rather broad, $\Gamma_{D^*_0} \sim 90$ MeV (or $\sim 50$ MeV when the asymptotic $SU_f(4)$ symmetry breaking has been taken into account), but it is not enough to saturate the whole bump. In comparison, the four-quark $\bar{D}$ mesons have been expected to have a width of at most $\sim 5 - 10$ MeV. Therefore we have proposed that each of
the observed enhancements will consist of a broader $\hat{D}_0^*$ in the region of its upper half and a narrower $\hat{D}$ in the region of its lower tail.

The strange counterpart $\hat{D}_{s0}^* \sim \{c\bar{s}\}$ of the conventional scalar $\hat{D}_0^*$ is expected to have a mass around $m_{\hat{D}_{s0}^*} \sim 2.45$ GeV from many different approaches. Its width is expected to be approximately saturated by the $\hat{D}_{s0}^{*+} \rightarrow (DK)^+$ decays and is predicted to be $\sim 70$ MeV (or $\sim 40$ MeV when the asymptotic $SU_f(4)$ symmetry breaking has been taken into account). Therefore, we hope that experiments will observe a rather broad peak around $\sim 2.4 - 2.5$ GeV in the $DK$ mass distribution.

One of remaining problems is that the neutral and doubly charged partners $\hat{F}_0^I$ and $\hat{F}_{++}^I$ of $\hat{F}_I^+$ have not been observed. To solve this problem, we need to know production mechanism of $\hat{F}_I^{0,+,+++}$ mesons. It will be one of our future projects.

We emphasize that the values of the masses and widths of the scalar resonances which have been estimated in this article should not be taken too strictly since the width of $\hat{D}_{s0}^*(2317)$ which has been used as the input data is narrow but its absolute value is still uncertain, and since possible mixing between $\hat{D}_0^*$ and $\hat{D}$ through their common decay channels which might have considerable effects on the masses and widths of the mixed states has been neglected. Such a mixing will depend on the details of hadron dynamics including four-quark mesons, and it will be a subject for future studies.

Finally, we point out that it is desirable that the measured broad bumps in the $D\pi$ mass distributions be (re)analyzed by using an amplitude including at least two scalar resonances. It is also expected that the charm-strange scalar, $\hat{D}_{s0}^{*+} \sim \mathbf{3}_{P0}\{c\bar{s}\}$, will be observed in the $DK$ channels by experiments with high statistics and resolution.

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