Low-speed control of PMSM based on ADRC + FOPID

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\textbf{ABSTRACT}

In this study, a new low-speed control strategy based on active disturbance rejection control (ADRC) + fractional-order proportion integration differentiation (FOPID) is proposed to realize fast and accurate control of permanent magnet synchronous motor (PMSM) at low speed. First, a new nonlinear function with enhanced smoothness and continuity around the origin is designed. On the basis of the above nonlinear function, an improved extended state observer (ESO) is developed. Then, the parameters of ESO are adjusted online by the designed fuzzy rules. Considering the improved dynamic performance of the FOPID controller, the nonlinear state error feedback control law module of ADRC is replaced by FOPID to calculate the proportion, differential, and integral of the error signal. Finally, the simulation analysis of the new low-speed control system based on ADRC + FOPID is carried out. The results show that the proposed algorithm has faster response speed and better anti-interference ability in the low-speed control of PMSM than the traditional ADRC.

\textbf{ARTICLE HISTORY}

Received 7 July 2020
Accepted 9 December 2020

\textbf{KEYWORDS}

Active disturbance rejection control; low-speed; nonlinear function; permanent magnet synchronous motor; fractional order proportion integration differentiation; parameter tuning

\section{1. Introduction}

Permanent magnet synchronous motor (PMSM) is a complex object with multiple variables, strong coupling, nonlinearity, and variable parameters. It is widely used on various occasions owing to its small size, high efficiency, good servo performance, and large electromagnetic torque. The servo performance of PMSMs directly affects the development of many fields. The low speed or even ultra-low speed index is a major index of the servo system. However, the motor speed is easily affected by uncertain factors, such as motor parameters and load disturbance. Therefore, the use of an advanced control strategy guarantees the stable operation of the motor.

With the development of control theory, local and international researchers conducted numerous studies on the low-speed control method of PMSM. For example, Tian et al. (2016), Chen et al. (2017), Scicluna et al. (2020), and Antonello et al. (2017) proposed a sensorless control. However, the above control method requires the system parameters to be known accurately. Guo et al. (2017), Deng et al. (2016), Ma (2016), and Xia et al. (2020) used a sliding film control method. Despite its robustness, solving the jitter problem using the above method is difficult. Gao et al. (2018) proposed a large-capacity low-speed LSPMSM rotor structure fed by the stator three-phase winding AC voltage and the compensation winding DC, and a simulation verified its feasibility. Yu et al. (2016) proposed a nonlinear backstepping control method on the basis of the combination of recursive least square and differential evolution. The above method has good control performance and can stabilize a rotor that works at low reference speed to ensure the tracking reference signal of the stator current. Saleh and Rubaai (2018) proposed a direct torque control method with a stable, fast, dynamic, and accurate performance. The above method also exhibits less sensitivity to changes in terms of load torque and driving speed. Active disturbance rejection controller (ADRC) is increasingly and widely used in controlling PMSM systems because of its strong robustness, anti-interference, and low model dependence.

ADRC technology is a nonlinear control method proposed by Han Jingqing of the Chinese Academy of Sciences in the 1980s. This control method applies many nonlinear functions that can solve the control problems that most other controllers cannot solve efficiently, rapidly, and without overshoot. It also does not need to deal with the nonlinear functions of the system itself on a large scale (see Gao, 2013). ADRC consists of three parts, namely, the tracking differentiator (TD), extended state observer (ESO), and nonlinear state error feedback (NLSEF). TD is used to arrange the transition process and provide high-quality input, ESO is used to observe the internal and expanded states to improve system controllability, and NLSEF is used to provide efficient...
control quantities. ADRC has received extensive attention and research from local and international scholars, who attained several results. Rong and Hang (2016), Wang et al. (2019), Hou et al. (2019), Zhao and Tang (2019), and Li et al. (2019) proposed a method that combined auto disturbance rejection control and sliding mode control. The above method can suppress the internal and external disturbances and has strong robustness to the changes in parameters. Zhou et al. (2018), Jin et al. (2018), Liang et al. (2015), and He et al. (2019) improved the parameter tuning of ADRC, which enriched the theory and method of its parameter tuning. Kuang et al. (2019) proposed a torque feedforward compensation method on the basis of ADRC, which has good speed response under the condition of uncertain modelling parameters and load disturbances.

In recent years, fractional-order proportion integration differentiation (FOPID) control has been widely concerned by the academic community. By introducing two additional parameters of FO differential and fractional integral, the traditional integer-order PID control is extended to the FO domain, and the dynamic regulation range of the system is expanded. Therefore, it has better control performance than the traditional PID control (see Yang et al., 2018). Liu et al. (2019) proposed a trajectory tracking control method for robot arm on the basis of FO fuzzy ADRC to realize real-time optimization of FOPID parameters and improve the overall control performance of the system. Zhang et al. (2019) proposed an HPSO-FOPID algorithm to solve the problem of seam tracking performance in the robot welding process and achieved the expected control purpose. Sun et al. (2017) designed a fuzzy FOPID controller. Fuzzy rules are used to adjust the parameters of FOPID to improve the response speed and robustness of the FOPID controller.

In this study, a new nonlinear function with improved smoothness near the origin is used to replace the traditional nonlinear function to solve the high-frequency chattering problem of the traditional ADRC near the origin. A fuzzy controller is used to perform the real-time online self-tuning of the three parameters to facilitate the parameter setting of the expanded state observer. Moreover, ADRC’s strong anti-internal and external disturbances are combined with the excellent dynamic characteristics of the FOPID controller to design an improved ADRC and facilitate its use and improve the dynamic and static control performance of PMSMs. The main work of this study is presented as follows: First, a new type of nonlinear function that has better derivability, continuity, and smoothness than the traditional nonlinear function near the origin is designed. Second, an improved ADRC is developed on the basis of a new ESO and FOPID controller. Third, a fuzzy controller is used to tune the parameters of ESO. Finally, system simulation research is conducted.

2. PMSM speed loop disturbance analysis

The motion equation is expressed as

$$T_e - T_L = \frac{J}{p_n} \cdot \frac{d\omega_r}{dt} + \frac{B}{p_n} \omega_r + g(t),$$

(1)

where $T_L$ is the load torque, $\omega_r$ is the electrical angular velocity, $J$ is the rotational inertia, $B$ is the viscous friction coefficient, and $g(t)$ is the other unknown interference. The dynamic equation of PMSM velocity obtained by Formula (2) is

$$\dot{\omega}_r = \frac{1.5p_n^2 \psi_r}{J} l_q - \frac{B}{J} \omega_r - \frac{p_n}{J}(T_L + g(t))$$

$$= b i_q^* + b(i_q - i_q) - \frac{B}{J} \omega_r - \frac{p_n}{J}(T_L + g(t))$$

$$= b i_q^* + d(t).$$

(2)

Among them,

$$b = \frac{1.5p_n^2 \psi_r}{J}$$

$$d(t) = b(i_q - i_q^*) - \frac{B \omega_r}{J} - \frac{p_n(T_L + g(t))}{J},$$

(3)

where $d(t)$ is the total disturbance that includes $q$-axis current loop tracking error, friction, external load, and other unknown disturbances. For the above first-order system with $i_q^*$ as input and electrical angular velocity $\omega_r$ as output, the change in parameters, such as permanent magnet flux linkage and rotational inertia, can be regarded as the internal disturbance of the system. By contrast, the load torque can be regarded as the external disturbance of the system.

3. Design of the proposed ADRC + FOPID

The structure of the PMSM speed servo system is shown in Figure 1. The transition process is arranged by the TD, and the differential signal of this process is given, thereby enabling the system to respond quickly without overshoot. The function of the improved ESO cannot only obtain each of the observed values of the state variables but also the observations of system disturbances, such as rotation inertia and stator resistance, disturbances caused by inductance, load disturbances, and other unknown disturbances, by using the FOPID controller to control the speed error dynamically.

3.1. TD design

The TD achieves a smooth approximation of the generalized derivative of the input signal through a nonlinear
function in ADRC. The effect is to arrange the transition process following the reference input and the control object to obtain a smooth input signal. TD’s algorithm is expressed as follows:

\[
\begin{align*}
    v_1(t + h) &= v_1(t) + hv_2(t) \\
    v_2(t + h) &= v_2(t) + h\text{fhan}(v_2(t) - v(t), v_2(t), r, h)
\end{align*}
\]

where \( v(t) \), \( h \), \( r \), and \( \text{fhan} \) are the input signal, integration step, factor of the tracking rate, and the nonlinear function, respectively.

The expression is shown as follows:

\[
\text{fhan} = \begin{cases} 
    -r\frac{a}{d} & |a| \leq d \\
    -r\text{sgn}(a) |a| & |a| > d
\end{cases}
\]

\[
a = \begin{cases} 
    v_2 + \frac{y}{h_0} & |y| \leq d_0 \\
    v_2 + \frac{\text{sgn}(y)(a_0 - d)}{2} & |y| > d_0
\end{cases}
\]

\[
a_0 = \sqrt{d^2 + 8r|y|},
\]

\[
d = rh_0,
\]

\[
d_0 = h_0d,
\]

\[
y = v_1 - v + hv_2.
\]

### 3.2. Improved ESO design

#### 3.2.1. New nonlinear function design

ESO is the core part of ADRC. Borrowing the idea of the state observer, the disturbance that affects the output of the controlled object is expanded into a new state variable, and a special feedback mechanism is used to establish ESO. ESO can observe the real-time effects of unknown external disturbances and system models and use the feedback method to compensate. It does not depend on the generated disturbance model, nor does it require direct measurement to observe disturbances and obtain estimated values. The realization of the specific functions of the ESO requires the use of nonlinear functions. Thus, nonlinear functions are the core content of the ESO design. At present, the nonlinear function in the traditional ADRC is the \( \text{fal}(e, \alpha, d) \) function, and the expression of the \( \text{fal}(e, \alpha, d) \) function is expressed as follows:

\[
\text{fal}(e, \alpha, d) = \begin{cases} 
    |e|^\alpha \text{sgn}(e), |e| > d \\
    e/\delta^{1-\alpha}, |e| \leq d
\end{cases}, \quad d > 0.
\]

The \( \text{fal}(e, \alpha, d) \) function was simulated in two cases to verify its performance. First, the value of \( d \) remained as \( d = 0.02 \). The value of \( \alpha \) changed to \( \alpha = 0; 0.3; 0.6; 1.2 \), and the effect of the change in the value of \( \alpha \) on the function performance is evident (Figure 2[a]). Second, the value of \( \alpha \) remained as \( \alpha = 0.3 \). The value of \( d \) changed to \( d = 0.02; 0.06; 0.12; 0.18 \), and the effect of the change in the value of \( d \) on the performance of the function is evident (Figure 2[b]). The value of \( \alpha \) affects the degree of nonlinearity of the \( \text{fal}(e, \alpha, d) \) function, as shown in Figure 2(a). The smaller the value of \( \alpha \), the stronger the nonlinearity of the \( \text{fal}(e, \alpha, d) \) function. By contrast, the larger the value of \( \alpha \), the stronger the linearity of the \( \text{fal}(e, \alpha, d) \) function. \( \alpha \) generally takes a value between 0 and 1. The value of \( d \) corresponds to the width of the linear interval of the function, which is related to the error range, as shown in Figure 2(b). If the value of \( d \) is too small, then the high-frequency tremor phenomenon of the \( \text{fal}(e, \alpha, d) \) function becomes serious.

Two segment points are found in the \( \text{fal}(e, \alpha, d) \) function, namely, \( d \) and \( -d \). Derivate the function at the segmentation point \( d \). This process verifies its derivability at the segmentation point as follows:

\[
\text{fal}'(e, \alpha, d) = \begin{cases} 
    \alpha e^{d-1}, & e > d \\
    1/d^{1-\alpha}, & 0 < e \leq d
\end{cases}, \quad d > 0
\]

where \( \alpha e^{d-1} \neq 1/d^{1-\alpha}, e = d \).

The left and right derivative values vary at the segmentation point, and the same is true at the other segmentation points. Therefore, the original function is not derivable at the segmentation point. Although the
The interpolation method of combining polynomial and trigonometric functions is selected because $d$ in the $\text{newfal}(e, \alpha, d)$ function is generally less than 1. In this interval, $\sin e$ shows better stability than $e$, and $\arcsin e$ shows better convergence than $e^3$. The above interpolation method ensures that the function is continuous and derivable at the origin.

The fitting process needs to satisfy the condition of derivative continuity.

\begin{align*}
\text{newfal}(e, \alpha, d) &= d^{\alpha}, e = d \\
\text{newfal}(e, \alpha, d) &= -d^{\alpha}, e = -d \\
\text{newfal}'(e, \alpha, d) &= \alpha d^{\alpha-1}, e = d \\
\text{newfal}'(e, \alpha, d) &= \alpha d^{\alpha-1}, e = -d
\end{align*}

Then, the expression of the $\text{newfal}(e, \alpha, d)$ function can be obtained as

\begin{align*}
\text{newfal} = \begin{cases}
|e|^\alpha \text{sign}(e), & |e| > d; \\
\left[\frac{\alpha \cdot (\alpha-1) + d^2}{\alpha \cdot (\alpha-1) + d^2 + d^2} \cdot \arcsin d + d \cdot \frac{\alpha \cdot (\alpha-1) + d^2}{\alpha \cdot (\alpha-1) + d^2 + d^2} \cdot \arcsin d + d^2}\right]^{\frac{1}{2}} \cdot \sin e + \frac{\alpha \cdot (\alpha-1) + d^2}{\alpha \cdot (\alpha-1) + d^2 + d^2} \cdot \arcsin e, & |e| \leq d
\end{cases}
\end{align*}
The *newfal*(e, α, d) function was simulated in two cases to verify its performance. First, the value of *d* remained as *d* = 0.02. The value of *α* changed to *α* = 0; 0.3; 0.6; 1.2, and the effect of the change in the value of *α* on the function performance is evident (Figure 3[a]). Second, the value of *α* remained as *α* = 0.3. The value of *d* changed to *d* = 0.02; 0.06; 0.12; 0.18, and the effect of the change in the value of *d* on the performance of the function is

**Figure 3.** Response curve of the *newfal*(e, α, d) function.
The simulation of $\text{newfal}(e, \alpha, d)$ was compared with that of $\text{fal}(e, \alpha, d)$ to verify the excellent performance of the $\text{newfal}(e, \alpha, d)$ function (Figure 3[c]). The $\text{newfal}(e, \alpha, d)$ functions around the origin show better smoothness than the $\text{fal}(e, \alpha, d)$ functions.

### 3.2.2. ESO design

The analysis of the algebraic expression of the $\text{newfal}(e, \alpha, d)$ function shows that the coefficient of $e^2$ term set during interpolation fitting is 0, and only two terms, namely, $p \sin e$ and $r \arcsin e$, exist in the equation. The new function has better convergence than the expected $\text{Sex}$ after interpolation fitting. The improved ESO is expressed as follows:

\[
\begin{align*}
\varepsilon_1 &= Z_{21} - y \\
Z_{21} &= Z_{22} - \beta_1 \text{newfal}(\varepsilon_1, \alpha_1, d) \\
Z_{22} &= Z_{23} - \beta_2 \text{newfal}(\varepsilon_1, \alpha_2, d) + b_0 u \\
Z_{23} &= -\beta_3 \text{newfal}(\varepsilon_1, \alpha_3, d)
\end{align*}
\]  

where $Z_{21}$ is the tracking signal of the input signal $y$, $\varepsilon_1$ is the observation error, $Z_{22}$ is the differential signal of the input signal $y$, and $Z_{23}$ is the observation signal of the total disturbance from ADRC. $u$ is the control output. $\beta_1$, $\beta_2$, and $\beta_3$ are the gains of ESO. $\alpha_1$, $\alpha_2$, and $\alpha_3$ are the nonlinear factors. $d$ is the linear interval width of the $\text{newfal}(e, \alpha, d)$ function, and $d = 0.02$ and $b_0$ are the compensation coefficients. Parameters $\beta_1 = 30$, $\beta_2 = 300$, $\beta_3 = 1000$ and $\alpha_1 = 0.5$, $\alpha_2 = 0.25$, $\alpha_3 = 0.125$, and $b_0 = 2.485$ are taken for simulation to verify the performance of the improved ESO (Figure 4).

![Figure 4: Response curve of the improved ESO.](image-url)
The improved ESO can observe the internal and external disturbances of the system model and compensate well, as shown in Figure 4.

3.2.3. Parameter tuning of ESO

The parameter setting of ESO is similar to that of PID. In the face of different internal and external states, which are not conducive to the adjustment of temporary parameters, the size of each parameter needs to be adjusted manually. In this study, a fuzzy controller is introduced. On the basis of the error $\varepsilon_1$ and its derivative as the input of the fuzzy system, the fuzzy rules are used. This process is conducted to adjust the ESO parameters online to approximate the optimal $\beta_1, \beta_2,$ and $\beta_3$ automatically to meet the requirements of the controller parameters at different times (see Huang et al., 2013).

The selected fuzzy variables are $\varepsilon_1$, $\dot{\varepsilon}_1$, and $\Delta \beta_i$ ($i = 1, 2, 3$). Moreover, five language subsets are defined in their domain as ‘negative big’ (NB), ‘negative small’ (NS), ‘zero’ (ZO), ‘positive small’ (PS), and ‘positive large’ (PB). The membership function of input $\varepsilon_1$ and $\dot{\varepsilon}_1$ is selected as gaussmf, and the membership function of output $\Delta \beta_i$ ($i = 1, 2, 3$) is trimf. The basic domains of $\varepsilon_1$ and $\dot{\varepsilon}_1$ in this study are $[-3, 3]$ and $[-30, 30]$, respectively, and the basic domains of $\Delta \beta_i$ ($i = 1, 2, 3$) are $[-0.3, 0.3], [-0.1, 0.1],$ and $[-0.01, 0.01], 1$. The fuzzy reasoning uses Mamdani.

Following the membership table, after fuzzy reasoning and defuzzification, the coefficient of $\Delta \beta_i$ ($i = 1, 2, 3$) can be modified as

$$
\begin{align*}
\beta_1 &= \beta'_1 + \Delta \beta_1, \\
\beta_2 &= \beta'_2 + \Delta \beta_2, \\
\beta_3 &= \beta'_3 + \Delta \beta_3
\end{align*}
$$

where $\beta'_1, \beta'_2,$ and $\beta'_3$ are the initial values of the nonlinear state feedback.

3.3. FOPID design

Fractional order control theory is complementary to integer-order control methods. FOPID extends the traditional integer-order PID, and its expression is presented as follows:

$$
G_C(s) = K_p + \frac{K_i}{s^\mu} + K_ds^\nu, (\lambda, \mu > 0)
$$

where $K_p$ is a proportional constant, $K_i$ is an integral constant, and $K_d$ is a differential constant. Moreover, integral order $\lambda$ and differential order $\mu$ enable FOPID to have the ability to control the object more than the traditional PID. Considering that FOPID is difficult to apply in actual control situations, the Oustaloup filter is usually used. In the limited frequency domain segment $(\omega_b, \omega_h)$, the calculus operator is approximated to the form of an integer-order transfer function by frequency domain fitting.

$$
{s^\alpha} \approx K \prod_{k=-N}^{N} \frac{s + \omega_k}{s + \omega_k'},
$$

where $N$ is the filter order, $\omega_k'$ is the zero point, $\omega_k$ is the pole, $K$ is the gain, and $0 < \alpha < 1$ is the fractional differential order.

$$
\omega_k' = \omega_b \left( \frac{\omega_b}{\omega_b} \right)^{\frac{k+N+1/2-\alpha/2}{2N+1}},
$$

$$
\omega_k = \omega_b \left( \frac{\omega_b}{\omega_b} \right)^{\frac{k+N+1/2+\alpha/2}{2N+1}},
$$

$$
K = \omega_b^n.
$$

In this study, FOPID is used to replace the NLSEF control law.

3.3.1. Particle swarm optimization of FOPID parameters

The FOPID parameters $K_p, K_i, K_d, \lambda$, and $\mu$ are taken as the five components of a particle. Then, the parameters are optimized in the 5D space.

Parameter tuning is essentially a parameter optimization method based on the given objective function. The time integral performance index of absolute error value is added to the objective function to make the controlled object obtain satisfactory dynamic characteristics. The square term of controller input is added to the fitness function to prevent excessive control.

The penalty function is applied to restrain overshoot. When an overshoot occurs in the control system, it is added to the objective function as follows:

$$
J = \int_0^{\text{max}} (w_1|e(t)| + w_2u^2(t))dt + w_3\sigma,
$$

The fitness function is set to $\text{fitness} = J$.

$e(t)$ is the sampling deviation value, $\sigma$ is the overshoot of the control system, $u(t)$ is the input of the controller, and $w_1, w_2$, and $w_3$ is the set weight coefficients. The integration of deviation and the balance between controller input and overshoot are adjusted following the corresponding coefficients. In this study, $w_1 = 0.999, w_2 = 0.001$, and $w_3 = 100$ are considered.

(1) The steps of particle swarm optimization for FOPID parameters are indicated as follows:

(1) Initialization of particle swarm optimization, including the size of particle swarm, the range of search range, inertia weight, acceleration coefficient, the maximum number of iterations, and the speed and position of particle swarm optimization.
(2) Using the designed fitness function, the fitness function value of each particle is calculated.

(3) For each particle in the particle swarm, its current fitness value is compared with the individual most fitness value. If it is better, then its current maximum speed and individual most fitness value will be updated. Subsequently, the fitness value is compared with the global optimal fitness value. If it is better, then the global optimal fitness value will be updated.

(4) Following their fitness, the particles of the particle swarm are sorted. Then, some particles are eliminated and replaced with newly generated particles. For adaptive adjustment, the inertia weight of the particle swarm is also changed in accordance with the fitness value of the particle and the following formula: \( w(i) = 0.4 + 0.5 \times \frac{\text{wmax}(i)}{\text{Num}} \), where \( \text{wmax}(i) \) is the sequence number of the particles in the particle swarm in accordance with the fitness value, and \( \text{Num} \) is the size of the particle swarm.

(5) Using

\[
V_{in}(k + 1) = w(k + 1) V_{in}(k) + c_1 r_1 (P_{in}(k) - X_{in}(k)) + c_2 r_2 (P_{gn}(k) - X_{in}(k)) \quad \text{and} \quad X_{in}(k + 1) = X_{in}(k) + V_{in}(k + 1),
\]

the current speed and position of the particle swarm were updated. \( V_{in} \) and \( X_{in} \) are the velocity and position of the \( i \) particle. Among them, \( 1 \leq n \leq N \) and \( N \) are the dimensions of the search range, \( w \) are inertia weights, \( c_1 \) and \( c_2 \) are positive acceleration coefficients, \( r_1 \) and \( r_2 \) are two independent random numbers between \([0,1]\), and \( c_1 r_1 \) and \( c_2 r_2 \) randomly control the particle swarm motion speed. \( P_{in} \) is the current optimal position of the \( i \) particle, and \( P_{gn} \) is the current optimal position of the particle swarm.

(6) If the end condition is met, then exit; otherwise, return to Step (2) to continue the iteration.

### 3.4. System stability analysis

The above motor model is written as follows:

\[
s = m(t) + U_1. \tag{18}
\]

The second order ESO is designed as follows:

\[
\begin{aligned}
\dot{e} &= z_1 - s \\
z_1 &= z_2 - b_1 e + U_1 \\
\dot{z}_2 &= -b_2 \text{newfal}(\varepsilon, \alpha, d)
\end{aligned} \tag{19}
\]

Let \( s_1 = s, s_2 = m(t) \), and \( w(t) = -\dot{m}(t) \), of which 4 is bounded. Equation (18) is expressed as follows:

\[
\begin{aligned}
\dot{s}_1 &= s_2 + U \\
\dot{s}_2 &= -w(t)
\end{aligned} \tag{20}
\]

Let \( e_1 = z_1 - s_1, e_2 = z_2 - s_2 \), and \( e = e_1 \). The state error equation is

\[
\begin{aligned}
\dot{e}_1 &= e_2 - b_1 e_1 \\
\dot{e}_2 &= w(t) - b_2 \text{newfal}(e_1, \alpha, d_1)
\end{aligned} \tag{21}
\]

The following dynamic feedback compensation is applied to the system:

\[
U_1 = U_{10} - z_2. \tag{22}
\]

The controller selection of FOPID is

\[
\begin{aligned}
e_0 &= s_1^* - z_1 \\
U_{10} &= b_0 \text{newfal}(e_0, \alpha_0, d_0)
\end{aligned} \tag{23}
\]

The improved ESO adopts a nonlinear structure, which is difficult to analyze using traditional observer design theory. Therefore, the stability and estimation error of ADRC are analyzed by combining the self-stability theory introduced (see Huang, 2000) and the constructed \( \text{newfal}(\varepsilon, \alpha, d) \) function.

The following functions are defined to analyze the stability of ADRC+FOPID:

\[
V_1 = \frac{1}{2} \sum_{i=1}^{n} p_2^2(e_{1i}, e_{2i}), \tag{24}
\]

\[
p_{2i} = (e_{1i}, e_{2i}) = \begin{cases} |q_{2i}(e_{1i}, e_{2i})| & \text{if } |q_{2i}(e_{1i}, e_{2i})| > p_{1i}(e_{1i}) \\ p_{1i}(e_{1i}) & \text{if } |q_{2i}(e_{1i}, e_{2i})| \leq p_{1i}(e_{1i}) \end{cases}, \tag{25}
\]

\[
q_{2i}(e_{1i}, e_{2i}) = e_{2i} - b_1 e_{1i} + k p_{1i}(e_{1i}) \text{sign}(e_{1i}), \tag{26}
\]

\[
p_{1i}(e_{1i}) = \frac{b_2}{kb_{1}} |\text{newfal}(e_{1i}, \alpha, d_1)|, k > 1,
\]

\[
i = 1, 2, 3 \ldots n, \tag{27}
\]

**Lemma 1:** Region \( P_2 = \{(e_{1i}, e_{2i}) : |q_{2i}(e_{1i}, e_{2i})| \leq p_{1i}(e_{1i})\} \) is the self-stable region of the system (see Huang, 2000).

**Theorem 3.1:** For the system represented by Equation (18), the extended state quantity \( m(t) \) is assumed to be bounded to the time variation \(-m(t) = [-m_1(t) \ldots -m_n(t)]^T\), i.e., \( |\dot{m}_i(t)| < w, \quad i = 1, 2 \ldots n, w > 0 \), if the system satisfies the following conditions:

\[
b_1^2 > (1 + k) c_2 b_2 c_1^{m-1}, \tag{28}
\]

\[
b_1 p_{2i} > \frac{c_2}{c_2 - 1} w (c_2 > 1, i = 1, \ldots, n). \tag{29}
\]

Then, \( \dot{V} < 0 \), and the system is stable.

**Proof:** Let

\[
\dot{V}_1 = \sum_{i=1}^{n} \left[ p_{2i}(e_{1i}, e_{2i}) \dot{p}_{2i}(e_{1i}, e_{2i}) \right],
\]

when

\[
\dot{V}_1 = \sum_{i=1}^{n} \left[ p_{2i}(e_{1i}, e_{2i}) \dot{p}_{2i}(e_{1i}, e_{2i}) \right],
\]

\[
\dot{V}_{1i} = p_{2i}(e_{1i}, e_{2i})
\]
Equations (28) and (29) are substituted into

\[
p_{2i}(e_{1i}, e_{2i}) = q_{2i}q_{2i} = q_{2i}\left[\frac{\partial q_{2i}}{\partial \epsilon_{2i}} \cdot \epsilon_{2i} + \frac{\partial q_{2i}}{\partial \epsilon_{1i}} \cdot \epsilon_{1i}\right]
\]

\[
< q_{2i}\left[-b_{2}newfal(e_{1i}, \alpha_{1}, d_{1}) + \frac{\partial q_{2i}}{\partial \epsilon_{1i}}(q_{2i} - kp_{1i}sign(e_{1i}))\right]
\]

\[
= q_{2i}w + \frac{\partial q_{2i}}{\partial \epsilon_{1i}}q_{2i}^{2}
\]

\[
- q_{2i}\left(b_{2}\frac{newfal(\epsilon_{1i})}{p_{1i}} + \frac{\partial q_{2i}}{\partial \epsilon_{1i}}p_{1i}sign(e_{1i})\right)
\]

\[
= q_{2i}w + \frac{\partial q_{2i}}{\partial \epsilon_{1i}}q_{2i}^{2} + kp_{2i}^{2}p_{1i}\frac{dp_{1i}}{de_{1i}}
\]

\[
\leq q_{2i}w - b_{1}q_{2i}^{2} + kp_{2i}^{2}p_{1i}\frac{dp_{1i}}{de_{1i}} + k^{2}q_{2i}p_{1i}\frac{dp_{1i}}{de_{1i}} .
\]

Equations (28) and (29) are substituted into

\[
\dot{V}_{1i} \leq \frac{(C_{2} - 1)}{C_{2}} b_{1}p_{2i}q_{2i} - b_{1}q_{2i}^{2} + \frac{(1 + k)q_{2i}^{2}b_{2}}{b_{1}} \left|newfal(\epsilon, \alpha, d)_{1i}\right|\]

\[
= \frac{1}{c_{2}} b_{1}q_{2i}^{2} + \frac{(1 + k)q_{2i}^{2}b_{2}}{b_{1}} \left|newfal(\epsilon, \alpha, d)_{1i}\right| .
\]

(31)

Thus, Theorem 4.1 is proved.

4. Simulation and result analysis

The modified ADRC and the traditional ADRC were simulated separately under the same input conditions to verify the performance of the proposed ADRC + FOPID. Following the tuning principle of ADRC, the parameter gain of NLSEF is taken as 5, 25, and 75. Then, the filter factor is taken as 1. The FOPID value of the proposed ADRC + FOPID is shown in Table 2. The same input value of the two controllers is presented in Table 3. The MATLAB simulation model of the PMSM speed vector control system is shown in Figure 5.

4.1. Low-speed control simulation of the motor under no-load conditions

Under the condition that the motor is not loaded, the running curves of the proposed ADRC + FOPID and the traditional ADRC when the motor speed of 100r/min, 200

| $\dot{e}_{1i}$ | NB | NS | ZO | PS | PB |
|-------------|----|----|----|----|----|
| $\dot{e}_{1i}$ | NB/PB/PS | NS/P5/NS | NS/P5/NS | NS/P5/NS | NS/P5/NS |
| $K_{b}$ | 0.04 | 0.11 | 0.5 | $\lambda$ | $\mu$ |
| $K_{d}$ | 1 | | | | |
### Table 3. ADRC parameters.

| Component of ADRC | Parameter name | Symbol | Numerical value |
|-------------------|----------------|--------|-----------------|
| TD                | Speed factor   | $r_0$  | 10              |
|                   | Filter factor  | $h_0$  | 0.01            |
| ESO               | Gain           | $\beta'_1$ | 30          |
|                   |                | $\beta'_2$ | 300          |
|                   |                | $\beta'_3$ | 1000         |
|                   | Nonlinear factors | $\alpha_0$ | 0.5         |
|                   |                | $\alpha_1$  | 0.25           |
|                   |                | $\alpha_2$  | 0.125          |

$r/\text{min}, 300r/\text{min},$ and $400 \, r/\text{min}$ are shown in Figure 6. The running speed of the motor at a given speed of $400 \, r/\text{min}$ is shown in Figure 6(a). The proposed ADRC + FOPID reached the given speed at 0.0458 s and ran steadily without overshoot. The traditional ADRC reached the given speed at 0.0489 s and had a stable operation, and the overshoot was 0.5%. The running speed of the motor at a given speed of $300 \, r/\text{min}$ is shown in Figure 6(b). The proposed ADRC + FOPID reached the given speed at 0.0365 s and ran steadily without overshoot. The traditional ADRC reached the given speed for

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**Figure 5.** MATLAB model of the PMSM vector control system.

**Figure 6.** Running response curve of motors with different speeds under no-load conditions.
Table 4. Running status of two controllers at different speeds.

| Speed (r/min) | Controller  | Response time (s) | Overshoot | Running condition |
|--------------|-------------|------------------|-----------|------------------|
| 400          | Traditional ADRC | 0.0489           | 0.5%      | Steadily         |
|              | ADRC + FOPID  | 0.0458           | without   | Steadily         |
|              |              |                  |           | Slightly fluctuation |
| 300          | Traditional ADRC | 0.0431           | /         | Steadily         |
|              | ADRC + FOPID  | 0.0365           | without   | Violent fluctuation |
| 200          | Traditional ADRC | /                | /         | Steadily         |
|              | ADRC + FOPID  | 0.0159           | without   | Violent fluctuation |
| 100          | Traditional ADRC | /                | /         | Slight fluctuation |
|              | ADRC + FOPID  | 0.0078           | without   | Steadily         |

The first time at 0.0431 s. When the given speed was 300 r/min, the traditional ADRC had been fluctuating. The operating condition of the motor at a given speed of 200 r/min is shown in Figure 6(c). The proposed ADRC + FOPID reached the given value at 0.0159 s and maintained a stable operating state without overshoot. The traditional ADRC jittered violently at 200 r/min and could not run stably. The operating condition of the motor at a given speed of 100 r/min is shown in Figure 6(d). The proposed ADRC + FOPID reached the given value at 0.0078 s and maintained a stable operating state without overshoot. The traditional ADRC shook violently at 100 r/min and could not run stably.

The data in Table 4 can be obtained from Figure 3, and the analysis results show that the proposed ADRC + FOPID has a faster response speed and better running stability at low speed than the traditional ADRC.

4.2. Variable speed control simulation of the motor at low speed under no-load conditions

Figures 7(a) and 7(c) are the response curves of motor speed change and the response curve of adding white noise during speed change, respectively. The response speed of the proposed ADRC + FOPID is significantly faster than that of traditional ADRC, as shown in Figure 7(a) and 7(c). The anti-interference ability of the ADRC + FOPID has a faster response speed and better running stability at low speed than the traditional ADRC.

Figure 7. Speed change response curve of the motor under no-load conditions.

Table 5. Performance table of the control system when λ takes different values.

| λ Value | Rise time (s) | Overshoot | Stabilization time (s) | Steady-state error |
|---------|---------------|-----------|------------------------|-------------------|
| λ = 0.7 | 0.0856        | without   | 0.0856                 | 0.025             |
| λ = 1   | 0.0346        | without   | 0.0346                 | 0                 |
| λ = 1.3 | 0.0086        | 0.25      | 0.0287                 | 0.035             |
| λ = 1.7 | 0.0088        | 0.19      | 0.0241                 | 0.015             |

Table 6. Performance table of the control system when μ takes different values.

| μ Value | Rise time (s) | Overshoot | Stabilization time (s) | Steady-state error |
|---------|---------------|-----------|------------------------|-------------------|
| μ = 0.7 | 0.0746        | without   | unstable               | unstable          |
| μ = 1   | 0.0346        | without   | 0.0346                 | 0                 |
| μ = 1.3 | 0.0091        | 0.19      | 0.0286                 | 0.035             |
| μ = 1.7 | 0.0088        | 0.20      | 0.0364                 | 0.015             |
proposed ADRC + FOPID is better than that of traditional ADRC, as shown in Figures 7(b) and 7(d).

### 4.3. Analysis of the influence of controller parameters

When \( \lambda \leq 1 \), the larger the \( \lambda \), the shorter the adjustment time of the system, as shown in Figure 8. When \( \lambda > 1 \), \( \lambda \) is larger, the overshoot of the system is larger, and the adjustment time is longer.

Increasing the value of \( \mu \) can improve the dynamic quality of the system and shorten the adjustment time, as shown in Figure 9. However, excessively increasing its value will cause instability in the system. If the value of \( \mu \) is excessive or inadequate, the system will oscillate.

The change of the tracking error of the system when the speed of the system is 300 \( r/\text{min} \), which only changes \( \lambda \) or \( \mu \), is shown in Figure 10. At \( \mu, \lambda \leq 1 \), the system error decreases as the value of \( \mu, \lambda \) increases, as shown in Figure 10(a) and 10(b). When \( \mu, \lambda > 1 \), the error becomes larger as the value of \( \mu, \lambda \) increases.

### 4.4. Low-speed control simulation of the motor under load

The given motor speed is 600 \( r/\text{min} \), and the motor is loaded with 1 \( N \cdot m \). The torque load is at 0.2 s after a no-load start. The operating status of the motor is shown in Figure 11. When the torque load is added at 0.2 s, the control performance of the proposed ADRC + FOPID is better and more robust than that of the traditional ADRC.
Figure 10. Error curve of $\lambda$ and $\mu$ value change system.

Figure 11. $1N \cdot m$ load output curve.
5. Conclusion

An ADRC for PMSM low-speed control system was designed in this study. The proposed ADRC was composed of three parts, namely, TD, an improved ESO, and FOPID. The improved ESO adopted a new nonlinear function with improved origin smoothness. It also fully estimated the system state and ‘total disturbance.’ TD arranged the transition process of the system. FOPID calculated the proportion, differentiation, and integral of the error signal and obtained good control efficiency and effect. The ADRC control strategy was compared with the traditional ADRC control strategy to verify the overall performance of the PMSM low-speed control based on the proposed ADRC + FOPID. The results showed that the PMSM low-speed control system based on the proposed ADRC + FOPID had a faster response speed, better noise immunity, and better dynamic characteristics than the PMSM low-speed control system based on the traditional ADRC.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research is supported by Anhui Natural Science Foundation (no. 1808085MF182), Anhui Provincial Key Research and Development Plan Project-Special Scientific and Technological Cooperation with Foreign Countries (no. 1804b06020368), National Natural Science Foundation of Anhui Polytechnic University (no. Xjky02201906), Graduate Innovation Project of Anhui Polytechnic University (no. 2019YQ06), and Anhui Provincial Natural Science Foundation (no. 1808085QET69).

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