Features of spinning gravity in particle physics: 
supersymmetric core of the Kerr-Newman electron

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Abstract. The model of electron based on regularized Kerr-Newman solution shows that 
spinning gravity is consistently united with particle physics. Extremely high spin of the 
electron creates topological deformation of space on the Compton scale, and we show that 
supersymmetric Higgs model regularizes this space, forming a nonperturbative solution as 
the bag-like core of the regularized Kerr-Newman electron model. Conflict between gravity 
and quantum interior of the bag is eliminated by the domain wall boundary of the bag 
which interpolates between the external exact Kerr-Newman solution and the free from 
gravity superconducting interior of the bag, forming vacuum state necessary for action of 
the perturbative quantum theory. Similar to typical bag models, the Kerr-Newman bag is 
deformable and creates the controlled by gravity stringy structure.

We show that contrary to the widely-discussed confrontation, spinning gravity cooperates 
with quantum theory, playing fundamental role in the structure of the dressed electron.

1. Introduction

Unification of Gravity with Quantum theory represents principal problem of the modern 
theoretical physics. The both theories are confirmed experimentally with great precision. 
Nevertheless, they contradict and cannot be combined in a unified theory.

In this paper we show how this problem is solved for the Kerr-Newman (KN) solution, which 
is the known solution for a charged and rotating black hole. Fifty years ago, in 1968, Carter 
noticed that KN solution has gyromagnetic ratio $g = 2$ just the same as that of the Dirac electron 
[1], which gave rise to study of the electron model based on the KN solution [1, 2, 3, 4, 5, 6] and 
so on.

The KN model of electron is consistent with gravity by nature, and there appears the question, 
how the known insuperable contradictions between gravity and quantum physics is solved in this 
model.

In this paper, and in the series of previous works [7, 8], we show that the reason of this 
discrepancy lies in the delusion about weakness of gravity, and in underestimation of the role of 
spin in gravitational interactions.

Treatment of the electron model becomes principal for two reasons i) the electron is the most 
studied of the particles from theoretical and experimental point of view, ii) the KN electron 
model displays natural consistency with gravity, shedding the light on the features of spinning 
gravity and on the related problems of interpretation of the quantum theory.
1.1. Compton scale of the spinning Kerr gravity

Usually, it is recognized that the cause of the contradiction lies in gravity, which is very weak, but not subordinated to quantum theory and does not allow to be quantized.

However, the classical gravity, which has its own high self-esteem as the space-time arena for action of quantum theory, does not accept the quantum conception of the particles, in which the electron is considered as pointlike and even structureless. Gravity requires an extended description of the particle in configuration space in terms of the real fields, in particular, as a nonperturbative solution of the field equations. It needs a stress-energy tensor for the right hand side of the Einstein equations, \( G_{\mu\nu} = 8\pi T_{\mu\nu} \).

The modern theories of electron do not satisfy this requirement. In the Dirac theory, QED and in the Standard model the electron model is pointlike. Although, the point-like electron is confirmed experimentally at the high energy scattering, it does not exclude the existence of some thin structure (similar to strings) which cannot be seen at the high energy scattering, and we show further that this is indeed the case, and the KN model is extended as a "dressed" electron on the Compton region in the form of a circular string model. The development of the KN electron model along the line of publications \[1, 2, 3, 4, 5, 6\] leads us to a Supersymmetric Bag model \[9, 10\] which contains the important elements of the stringy structure.

The transfer from pointlike particles to strings was revolutionary step in the problem of unification, and so far as the string theory raised a claim on unification with gravity, this step was correlated with weakness of gravity. However, this stringy structure and the related extra dimensions are "invisible" experimentally, and following to the old argumentations of the five-dimensional Kaluza-Klein theory (which is predecessor of the string theory), the extra dimensions are "invisible" because they are extremely small, and we read in \[11\] "...The Planck scale is the natural first guess for a rough estimate of the fundamental string length scale as well as the characteristic size of compact extra dimensions. Experiments at energies far below the Planck energy cannot resolve distances as short as the Planck length. Thus, at such energies, strings can be accurately approximated by point particles." It means that particles are not point-like indeed, but their extended structure is "invisible" above the Planck scale \( \approx 10^{-33} \text{cm} \).

However, as it was mentioned by John Schwarz, \[12\], "...Since 1974 superstring theory stopped to be considered as particle physics... " and "... a realistic model of elementary particles still appears to be a distant dream ...

The extended particles, representing localized concentration of energy are described as solitons. These are just what is necessary for association with gravitation. However, gravity is ignored usually in the solitonic models, as it is conventionally assumed as very weak and not essential on the scale of electroweak interactions.

Indeed, mass of the elementary particles is very small concerning the scale of masses in cosmology. However almost all of the elementary particles have spin, which distort space as spinning Kerr’s gravity. Spin has only discrete values of order \( \hbar \), and typically, its value with respect to masses of elementary particles is extremely high, about \( 10^{20} - 10^{22} \) (in the dimensionless plank’s units \( G = \hbar = c = 1 \)).

Another very plausible arguments about weakness of gravity were given John C Baez, who is based on the Schwarzschild solution and doesn’t take into account Kerr’s distortion of the space caused by spin of elementary particles \[13\]: "... general relativity says that associated to any mass \( m \) there is a length called the Schwarzschild radius, \( l_s \), such that compressing an object of mass \( m \) to a size smaller than this results in the formation of a black hole. The Schwarzschild radius is roughly the distance scale at which general relativity becomes crucial for understanding the behavior of an object of a given mass. Now, ignoring some numerical factors, we have

\[
l_c = \frac{\hbar}{mc}
\]
and

\[ l_s = \frac{Gm}{c^2}. \]  

(2)

These two lengths become equal when \( m \) is the Planck mass. And when this happens, they both equal the Planck length...”

In this sentence \( l_c \) is the Compton length of the dressed quantum particle, and comparison of the lengths \( l_c \) and \( l_s \) is indeed comparison of the influence of quantum theory and Schwarzschild gravity. Author shows that these lengths become equal on the Planck scale. This comparison is well known and very popular, but similar comparison based on the Kerr solution alters this conclusion resolutely.

Gravitational field of the Kerr solution becomes strong at the Kerr singular ring, which has the radius \( a = \frac{Jm}{nc} \), where \( J \) is angular momentum of the Kerr solution. For parameters of the spinning particles one has to use \( J \approx \bar{h} \), which gives for characteristic length of the Kerr solution

\[ l_{Kerr} = a \approx \frac{\bar{h}}{mc} = l_c, \]  

(3)

which shows that \( l_{Kerr} \) and \( l_c \) are approximately equivalent for any values of the mass \( m \), and i) the given by John C Baez arguments on the exclusive role of the Planck mass and the Planck length do not work for the spinning Kerr gravity, 2) equality (3) shows explicitly that \( l_{Kerr} \) corresponds to the Compton length scale, which for electron is about \( 10^{-11} \text{cm} \), what is much more than the Planck length \( l_p \approx 10^{-33} \text{cm} \).

1.2. Two-sheeted Kerr space as Einstein-Rosen bridge

The Kerr singular ring is the branch line of the Kerr space into two sheets, and the Compton scale is the characteristic length at which Kerr’s gravity creates two-sheeted topology.

Note that the Kerr solution with parameters of elementary particles is not black hole because in the dimensionless units we have \( a \gg m \), which is condition for disappearance of the horizons. The Kerr singular ring and topologically two-sheeted space are naked – not covered by horizon, and there appeared the attempts to consider the two-sheeted Kerr solution as a "wormhole" or the famous Einstein-Rosen bridge [14, 15] – the old proposals on the related with gravity models of elementary particles.

By the condition (3) the Kerr gravity and Quantum theory are at the same scale and the conflict related with their incompatibility must be increased.

For electron, the two-sheeted Kerr’s background forms in space a hole of the Compton radius \( a = \frac{\bar{h}}{2m} \). Size of this hole corresponds to the region of the quantum virtual processes – the Compton zone of dressed electron. However, this space is singular and topologically different from the required Minkowski space, so that neither the Dirac theory nor perturbative QED can be applied, and to resolve this conflict, some special mechanism of regularization is necessary. Over the last fifty years many works were devoted to the problem of source of Kerr solution, and we note here only several of them, which are relevant to our direction of the research [3, 4, 5, 6, 14, 18, 19, 20, 21, 22]. Moving along this line, we arrive at the bag model of the KN source [9, 16, 17], which unifies the bubble and solitonic models of the KN source with string-like models, and also allows us to resolve the conflict between gravity and quantum theory without modification of the Einstein-Maxwell equations, by using a supersymmetric scheme of phase transition which is equivalent to supersymmetric Higgs model of symmetry breaking. Note, that supersymmetry becomes really necessary here in the form of the supersymmetric Landau-Ginzburg or Wess-Zumino field model (the concept of superparticles is not used here). In this model singular region of KN solution is replaced by the flat internal space of the Compton size. This region is free from gravity and forms interior of the bag, such so the external exact Kerr-Newman gravitational solution leaves untouched.
In our previous treatments [9, 16, 17] we found the corresponding non-perturbative BPS-saturated solution in frame of the generalized Landau-Ginzburg (LG) field, which is very close to true supersymmetric LG model, but nevertheless has some features (we use here term "generalized Landau-Ginzburg theory" for the case when the superpotential is not holomorphic but depends also on the complex conjugate anti-chiral fields). Boundary of the bag is formed by the domain wall (DW) interpolating between the external KN gravity and superconducting vacuum state inside the bag. Similar to the known MIT and SLAC bag models [23, 24], the KN bag becomes deformable, displaying consistency with external gravitational and electromagnetic (EM) KN field. In fact, the corresponding solutions of the Einstein-Maxwell fields equations determine the size, shape and dynamics of the bag. As a result, the mass, spin and charge of the electron fixe boundary of the bag as a highly oblate ellipsoidal surface. Bag forms a superconducting disk of the Compton radius, and the KN vector potential forms a closed Wilson loop, inducing the closed string of superficial currents placed at the sharp border of the oblate superconducting disk.

In this work we generalize the previous treatments by using the true supersymmetric Landau-Ginzburg model, which is equivalent to supersymmetric Higgs model and specifies the associated with bag stringy structure. This supersymmetric field model becomes equivalent to the Wess-Zumino SuperQED model [25], revealing the bridge between the non-perturbative bag-like solutions and the conventional perturbative technics used in QED.

Contrarily to general point of view that gravitation conflicts with quantum theory, we obtain that Kerr’s spinning gravity actively interacts with quantum theory, playing a fundamental role in the structure of elementary particles.

2. Basic features of the Kerr-Newman solution
In the Kerr-Schild coordinates, metric of the KN solutions is [2]

\[ g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \]

where \( \eta_{\mu\nu} \) is metric of an auxiliary Minkowski space \( M^4 \), (signature \((-+++)\)), and \( H \) is the scalar function which for the KN solution takes the form

\[ H_{KN} = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \]

and the KN vector potential is

\[ A_\mu = \frac{-er}{(r^2 + a^2\cos^2\theta)}k_\mu. \]

The non-linear part of the KN metric along with vector potential are collinear with null vector field \( k_\mu \), \( (k_\mu k^\mu = 0) \), forming Principal Null Congruence of the algebraically special solutions [2].

The used in [2] oblate spheroidal coordinates \( r, \theta \) and \( \phi_K \) are related to Cartesian coordinates as follows

\[ x + iy = (r + ia)\exp\{i\phi_K\}\sin\theta, \]
\[ z = r\cos\theta, \quad \rho = r - t, \]

where \( \rho = r - t \) is the light cone coordinate adapted to null direction

\[ k_\mu dx^\mu = dr - dt - a\sin^2\theta d\phi_K, \]

Kerr’s congruence \( k^\mu \) forms a vortex of the null field which propagates analytically from negative sheet of Kerr metric, \( r < 0 \), to positive one, \( r > 0 \), where the ingoing field \( k^-_\mu \) turns
The metrics on the ingoing and outgoing \((\pm)\)-sheets become different \(g_{\mu\nu}^{\pm} = \eta_{\mu\nu} + 2Hk_{\mu}^{\pm}k_{\nu}^{\pm}\), and there is a freedom in choice of the \(k_{\mu}^{+}\) or \(k_{\mu}^{-}\) for our "basic" space [26].

We take as the "basic" the outgoing field \(k_{\mu}^{+}\), that corresponds to the classical physical picture with the retarded electromagnetic field, and we chose as positive rotation the anticlockwise direction of the Kerr congruence near the Kerr singular ring.

Since electron and other spinning particles have the great spin/mass ratio \(J/m = \hbar/2m\) (about \(10^{20} - 10^{22}\) in the dimensionless units), the black hole horizons of the Kerr solution disappear, and the Kerr singular ring becomes naked.

Acting as a branch line of the Kerr space, the Kerr singular ring spoils space topologically, creating in space-time hole of the Compton radius \(a = \hbar/J/m\) (for seldom exclusion we use in this paper the dimensionless units \(G = c = \hbar = 1\)). This hole opens the pass to another sheet of the Kerr space, which was interpreted recently as a model of the worm-hole or the Einstein-Rosen bridge [14, 15].

It seems that emergence of this hole in the background intensifies the conflict between gravity and quantum theory and makes their coexistence absolutely incompatible. However, as it was shown in [9, 16] this confrontation can be removed by supersymmetry, more precisely, by the nonperturbative bag model based on the supersymmetric scheme of phase transition. Still more, this confrontation turns into a cooperation, and this gravitational hole emerges as an important part of the KN electron model.

### 3. Bag-like source of Kerr-Newman solution as a semi-classical electron model

**Supersymmetry** resolves conflict between the broken by Kerr’s gravity background and quantum theory without modifications of the Einstein-Maxwell equations. In the supersymmetric bag model, singular region of KN solution is covered by flat interior of the bag, which is formed by a supersymmetric vacuum state of the energy

\[
< 0 | \mathcal{H} | 0 > = 0.
\]
The corresponding nonperturbative bag-like solution is built as a domain wall phase transition interpolating between the supersymmetric vacuum state inside the bag and the external gravitational KN solution.

3.1. Emergence of the Compton wave length $\lambda_c$, classical radius $r_e$ and fine structure constant $\alpha$

Typical features of the bag models are the softness and deformability of the bags under influence of the field distributions, and in particular, under rotation. The KN super-bag is also soft and elastic, displaying a compliance with the external gravitational and electromagnetic KN field.

The Compton wave length appears as radius of the KN spinning particle automatically from basic relation of the Kerr geometry [26]

$$J = ma. \quad (10)$$

Setting for an electron $J = \hbar/2$, we obtain that radius of the Kerr singular ring corresponds to the reduced Compton wave length

$$a = \hbar/2mc \quad (11)$$

which could be wrapped twice around the Kerr singular ring.

The shape and dynamics of the bag are fully defined by matching of the bag boundary with a special surface (López, 1986 [4]), defined by condition

$$H_{KN}(r) = 0. \quad (12)$$

Near this surface metric becomes flat, $g_{\mu\nu} = \eta_{\mu\nu}$, and external gravitational field can be matched continuously with flat space inside the bag.

![Figure 3. Ellipsoidal shape of the Kerr-Newman bag defined by the boundary surface $r = r_e = e^2/2m$.](image1)

![Figure 4. Kerr’s coordinate $\phi_K = const.$ Singular ring drags the KN vector potential, forming a closed Wilson loop along edge border of the bag.](image2)

It gives the additional parameter – classical electron radius. According (5), it defines position of the bag border at

$$r = r_e = e^2/2m, \quad (13)$$

and the relations (7) show that this bag takes the form of a thin ellipsoidal disk of the thickness $2r_e = e^2/m$, and radius

$$r_c = \sqrt{r_e^2 + a^2}, \quad (14)$$
what is slightly more than the reduced Compton wave length \( a = \frac{\hbar}{2mc} \). So that the ratio

\[
\frac{r_e}{a} = \alpha \approx 1/137, \tag{15}
\]

becomes the fine structure constant, and the ratio \( r_e/r_c = \frac{r_e}{a}(1 + \frac{r_e^2}{a^2})^{-1/2} = \alpha(1 + \alpha^2)^{-1/2} \), which characterizes the degree of oblateness of the bag, is very close to \( \alpha \approx 1/137 \). Bag covers singular ring, and its flat interior regularizes KN space, where the Cartesian distance \( \delta \) works as cutoff for the KN electromagnetic and gravitational field, see Fig.6.

3.2. Quantization of the angular momentum

Vector-potential of the external KN solution (6)

\[
A_\mu dx^\mu = -\frac{cr}{r^2 + a^2 \cos^2 \theta}(dr - dt - a \sin^2 \theta d\phi) \tag{16}
\]

grows near the core and takes maximal value at the Kerr radial distance \( r = r_e = e^2/2m, \cos \theta = 0 \),

\[
A_{\mu}^{\text{max}} dx^\mu = -\frac{2m}{e}(dr - dt - ad\phi). \tag{17}
\]

Note, that the component \( A_r \) is a perfect differential (see for example [2]) and can be ignored. At the boundary \( r = r_e \), the dragged by the Kerr congruence component \( A_{\phi}^{\text{max}} \) is

\[
A_{\phi}^{\text{max}} d\phi = \frac{2m}{e}a d\phi. \tag{18}
\]

Taking the Wilson loop integral along the bag border for the period \( \phi \in [0, 2\pi] \), we obtain

\[
e \oint A_{\phi}^{\text{max}} d\phi = 4\pi ma = 4\pi J, \tag{19}
\]

where the last equality corresponds to the basic relation \( J = ma \) (10). Therefore, for the solutions with integer or half-integer angular momentum, \( J = \frac{1}{2}, 1, \frac{3}{2}, \ldots \), vector potential (17) forms closed Wilson loop along border of the bag.

3.3. Stringy structure created by spinning bag model

It was noticed long ago [5, 6], that the Kerr singular ring can be considered as a string, which is similar to string-like filament appearing in superconductivity [27] and in dual string models [28]. Kerr’s string forms a waveguide for traveling electromagnetic and gravitational waves, and it was shown in [29], that the structure of the electromagnetic and gravitational fields near the Kerr singular ring is close to structure of the Sen heterotic string solutions to low energy string theory [30].

Bag models are relatives of the string models. Bags are soft and flexible, and emergence of the string-like structures in the bag models is usually related with their deformations under influence of strong rotation [31, 32]. Typically, the rotating bag takes the form of a flux-tube with lines of the electric force stretched between the charges (quarks) placed at the ends. In toroidal models of the bags [33], the flux-tubes form closed loops of the electric lines. These strings have much in common with the known Nambu-Goto string model, and also with solitonic string models [34] and pp-wave solutions [35]. These strings differ from the famous higher-dimensional critical strings of the superstring theory because they are four-dimensional and also because their longitudinal excitations play important role. The string of the KN bag
model is formed by the Higgs mechanism of symmetry breaking. It is similar to the Nielsen-Olesen string model, formed as a vortex line in superconducting media [27], and to the Vilenkin-Witten superconducting cosmic string model [36], which was developed by Morris in [37] to a supersymmetric string model.

Similar to other bag models, the bag-like source of the KN solution is also deformed under influence of rotation and takes the shape of a very thin disk of the ellipsoidal form. Circular string emerges in equatorial plane ($\cos \theta = 0$) on the edge of the superconducting disk, instead of the Kerr singular ring and very close to its former position.

As a consequence of the strong concentration of the electromagnetic field, the superficial circular current appears at the border of the superconducting bag.

The Kerr string is regular and is build of the longitudinal mode of the vector potential, which forms the closed Wilson loop wrapped around the border. Structure of the Kerr-Newman electromagnetic field shows that there is also the transverse electromagnetic field in the form of traveling waves.

According to Carter [1], the original KN solution has magnetic momentum $\mu = \mu_{Dir} = ea$, which is in full correspondence with the Dirac electron model. The Kerr-Newman solution has singular ring, and therefore, it can be considered as a non-regularized electron model. So far as $\mu = \mu_{Dir}$, the anomalous magnetic momentum in the exact KN solution is zero. So far as the regularized bag model of the KN solution deforms initial model, and we can expect the emergence of the anomalous magnetic moment, which true value is $\alpha/2\pi$. We show that the effect of order $\alpha$ emerges in the considered regular bag model.

The related with regularization string is created very close to the lightlike Kerr singular ring, which grads the KN vector potential in the lightlike circular direction. In our previous works [29, 22, 9, 17] we considered approximate model, in which this string was perfectly lightlike and described in the auxiliary Minkowski background as the surface $x^\mu(\phi, t)$ parametrized by the cylindric coordinates $\phi, t$.

The Cartesian distance $\delta$ between border of the bag and singular ring is very small with respect to radius of the ring $a$, and this string can be considered as tangential to the Kerr singular ring with the lightlike direction

$$k^+_{\mu} dx^\mu = (dt - ad\phi), \tag{20}$$

In this case $\delta$ works as a cut-off parameter. However, there is one more essential effect – the tangent to string direction $d\phi$ deviates from the lightlike direction of the Kerr congruence $k_{\mu}^+$, and the real velocity of the bag border becomes smaller than the speed of the light.

![Figure 5. Decomposition $A_{max}^{\text{max}}$ in the left and right modes $A^+$ and $A^-$. Contribution of the small lightlike right mode $A^-$ reduces speed and increases length of the relativistic string.](image-url)
This deviation can be described as a small lightlike component $A^-_\mu$ in direction $k^-_\mu dx^\mu = (dt + ad\phi)$, which is opposite (antipodal) to $k^+_\mu$.

When the initiate basic potential

$$A^{max}_\mu dx^\mu = A^-_\mu |_{re} dx^\mu = -\frac{2m}{e} (dt - ad\phi),$$

(21)
is completed by antipodal components $A^-$, the resulting potential

$$A^{max}_\mu dx^\mu = A^+_\mu dx^\mu + A^-_\mu dx^\mu$$

(22)
is composed proportionally to contributions of the amplitudes $A^+$ and $A^-$, where according Fig.5

$$A^+_\mu dx^\mu = -\frac{2m}{e} \cos \theta (dt + ad\phi), \quad A^-_\mu dx^\mu = -\frac{2m}{e} \sin \theta (dt - ad\phi).$$

(23)

On the other hand, kinematic relations presented in Fig.6 show that the angle is

$$\theta_r = \arctan \frac{r_e}{a} = \arctan \alpha,$$

(24)

which is approximately equal to the fine structure constant $\alpha$.

We see that influence of the oppositely directed mode $A^-_\mu$ is proportional to

$$\tan \theta_r = r_e/a = \alpha.$$  

(25)

Physically, it leads to deceleration and to the corresponding lengthening of the string and expansion of the relevant current loop. The fields $A^+_\mu (t + \sigma)$ and $A^-_\mu (t - \sigma)$ can be considered now as the left and right modes which complete the full stringy system.

Deviation from the basic lightlike direction $k^+_\mu$, caused by additional contribution from the direction $k^-_\mu$ is small, because this mode is directed against the principal null congruence of
the KN solution, which is prohibited by the KN gravitational field. It can impact only in the zone near the bag border where the regularized space is flat. \( g_{\mu\nu} \approx \eta_{\mu\nu} \). Velocity of the originally lightlike string is reduced, and its length \( l = 2\pi a \) increases proportionally to \( \theta_r = \arctan \frac{r_e}{a} \approx \alpha \).

Identification of the angles \( \theta \) and \( \theta_r \) could give solution of the old problem – origin of the anomalous magnetic moment of the electron [38]. In the report to 12th Solvay Congress Feynman noted [39]: "We have no physical picture by which we can easily see that the correction is roughly \( \alpha/2\pi \), in fact, we do not even know why the sign is positive (other than by computing it). " Unfortunately, we have not found so far solution of this problem. We see definitely that \( \theta \) is proportional to \( \theta_r \), but we didn’t explained so far the emergence of the factor \( \frac{1}{2\pi} \).

We see that modes \( A_\mu^+(t + \sigma) \) and \( A_\mu^-(t - \sigma) \) have opposite direction of rotation, which corresponds to the opposite sign of \( a \) and is required by supersymmetry (see later (38)). Geometric origin of this difference is nontrivial.

We note, that although the supersymmetric phase transition regularized the Kerr-Newman space-time background, and the negative sheet of the Kerr metric is removed, the electromagnetic field remained two-valued, as far as it is described by the two-valued Kerr coordinate system (7). One can expect that emergence of the the factor \( \frac{1}{2\pi} \) can be related with the pole in the potential, which must be bypassed in the process of integration over the domain wall layers \( r \in [-r_e, +r_e] \).

From (6) and (8) we see that potential has the form

\[
A_\mu dx^\mu = -e \frac{er}{r^2 + a^2 \cos^2 \theta} (dr - a \sin^2 \theta d\phi_K),
\]

and the \((t, \phi)\) components are extended smoothly to negative values of \( r \), where the potential changes the sign and direction. However, at the "negative" border of the bag, \( r = -r_e \), potential forms oppositely oriented loop. Orientation of the string is retained when this transformation is completed by the replacement \( e \rightarrow -e \). The resulting string retains orientation, but changes direction of the rotation \( a \). The potential is continued analytically to negative \( r \), where the boundary of the bag-like source is completed by an anti-domain wall boundary. Therefore, the string mode formed by the potential \( A^- \) should be placed on the "negative" border of the anti-domain wall boundary. Such two-valued structure of the Kerr source, forming the "Domain Wall – Anti-Domain Wall" combination similar to the known kink-antikink solutions, sometimes was called as "breather". In the source of the KN solution it was at first noticed in [17].

4. Supersymmetric phase transition

It was noticed in several works that source of the KN solution must be superconducting [4, 19, 21], similar to many soliton models, the famous Nielsen-Olesen dual string model [27] and the bag models [23, 24]. Superconductivity is close related with Higgs mechanism of symmetry breaking [27] and in the simplest case it is described by the "minimal" Landau-Ginzburg model

\[
\mathcal{L}_{NO} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (D_\mu \Phi)(D^\mu \Phi)^* - V(|\Phi|),
\]

where \( D_\mu = \nabla_\mu + ie A_\mu \), and \( F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} \).

The typical form of the scalar potential \( V = \lambda (\Phi^4 - \eta^2)^2 \), cannot be matched with gravity, because it breaks gauge symmetry in external space. In particular, in the most of superconducting string models and in the bag models, strings are created as a vortex in superconductor [27], as well as the bags appear as "cavities in superconductor" [23, 24]. The consistency with gravity requires opposite construction – the bag must be superconducting itself, leaving unbroken the external gravitational and electromagnetic fields. As it was first noticed by Witten [36], this cannot be realized with one Higgs field, and a special form of the potential is
required which has a few Higgs-like fields. Morris [37] suggested to use "pseudo-supersymmetric" model where potential $V$ is formed through superpotential

$$ W(\Phi_i) = Z(\Sigma^+ \Sigma^- - \eta^2) + (Z + \mu) H \bar{H}, \quad (28) $$

depending on several chiral fields $\Phi_i$, ($\mu$ and $\eta$ are real constants). The notations $(H, Z, \Sigma) \equiv (\Phi_1, \Phi_2, \Phi_3)$ identify the complex Higgs field $H = |H|e^{i\chi}$, as the first field $\Phi_1$.

The scalar potential is formed as follows [25] (we use the sign "star" for complex conjugation)

$$ V(r) = \sum_i F_i F_i^* \quad (29) $$

where

$$ F_i = \partial W/\partial \Phi_i \equiv \partial_i W. \quad (30) $$

$D_{i\mu} = \nabla_{1\mu} + ieA_{1\mu}$, where $A_{1\mu}$ is the external KN vector field. The fields $\Phi_2$ and $\Phi_3$ are assumed uncharged, and $D_{i\mu} = \nabla_{i\mu}$ for $i = 2, 3$. The corresponding Lagrangian differs from (27) only by summation over the fields $\Phi_i$. This model was used in our previous works [9, 16, 17]. It is called as "pseudo-supersymmetric", because the true superpotential must indeed be holomorphic function of the all superfields [25, 37],

$$ W(\Phi_i) = Z(\Sigma^+ \Sigma^- - \eta^2) + (Z + \mu) H^+ H^-, \quad (31) $$

and therefore, the number of independent chiral fields $\Phi_i$ must be increased to five:

$$ (H^+, H^-, Z, \Sigma^+ , \Sigma^-) \equiv (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5). \quad (32) $$

The auxiliary fields $F_i^* = \partial W/\partial \Phi_i = (F_+^*, F_-^*, F_+^* - F_-^*, F^*_Z)$ take the form

$$ F_+^* = (Z + \mu) H^+, \quad (33) $$
$$ F_-^* = Z \Sigma^+, \quad (34) $$
$$ F_Z^* = \Sigma^+ \Sigma^- + H^+ H^- - \eta^2, \quad (35) $$

Vacuum expectation values of fields $\Phi_i$ for which $F_i^* = 0$ give minima of the potential $V = 0$ corresponding to supersymmetric vacuum states. Just as in case (28), we obtain Domain Wall with two isolated vacua:

Internal: $H^- H^+ = \eta^2$, $Z = -\mu$, $\Sigma^+ = \Sigma^- = 0$, and

External: $H^- H^+ = 0$, $Z = 0$, $\Sigma^+ \Sigma^- = \eta^2$, which are separated by Domain Wall – zone (R), formed by of the positive scalar potential

$$ V = |\Sigma^+ \Sigma^- + H^+ H^- - \eta^2|^2 + |(Z + \mu) H^+|^2 + |(Z + \mu) H^-|^2 + |Z|^2(|\Sigma^+|^2 + |\Sigma^-|^2). \quad (36) $$

As in the considered previously cases [9, 16, 17] the domain wall forms ellipsoidal boundary of the bag. The both Higgs fields $H^-$ and $H^+$ are concentrated inside the bag.

The most important difference between the supersymmetric model and the considered earlier pseudo-supersymmetric Landau-Ginzburg model is the relation

$$ H^- H^+ = \eta^2, \quad (37) $$

which shows that the chiral Higgs fields $H^- = |H^-|e^{i\chi^-}$ and $H^+ = |H^+|e^{i\chi^+}$ can have different amplitudes $|H^-| \neq |H^+|$, but their phases must be correlated

$$ \chi^+ = -\chi^- = \chi, \quad (38) $$

which is necessary to form Domain Wall corresponding to profile of the curved bag boundary $r = r_e$ (see Fig.4.) and provide BPS stability, similar to results in [9, 16, 17].
5. Superficial currents

According supersymmetric Landau-Ginzburg model, inside the bag the chiral fields $\Sigma^\pm$ vanish, (4), and potential on the border of the bag (17) interacts only with the Higgs fields $H^+$ and $H^-$, through covariant derivatives $D^+_\mu = \nabla_\mu + ieA^+_\mu$ and $D^-_\mu = \nabla_\mu - ieA^-_\mu$, which appropriate to opposite $U(1)$ charges and correspond to the Wess-Zumino field model of supersymmetric QED [25]. One can expect that the summarized potential (22), $A_{\mu}^{max} dx^\mu = A_{\mu}^+ dx^\mu + A_{\mu}^- dx^\mu$ creates on the border of the bag two surface currents

$$\Box A_{\mu}^{max} = J_\mu^+ + J_\mu^-,$$

where

$$J_\mu^\pm = e^\pm |H^\pm|^2 (\chi_{\pm,\mu}^\pm + e^\pm A^\pm_\mu).$$

(40)

The right mode $A^-_\mu$ is placed at the "negative" border $r = -r_e$, where is to be used opposite sign of the charge $e^- = -e^+ = -e$, and sign of the phase $\chi^- = -\chi^+$ of the Higgs fields $H^\pm$, according to (38).

In the core of superconducting disk we have $J_\mu^+ = 0$, and the equation $\chi^+_{\mu} - eA^+_{\mu} = 0$ gives

$$\chi^+_{,t} - eA^+_t = 0, \quad \chi^+_{,\phi} - eA^+_\phi = 0.$$  

(41)

From (23) we obtain $\chi^+_{,t} = eA^+_t = 2m \cos \theta$, $\chi^+_{,\phi} = eA^+_{\phi} = 2ma \cos \theta$, and integration gives

$$\chi^+ = 2m \cos \theta (t - t_0 + a \phi),$$

(42)

where $t_0$ is the constant of integration which is omitted later.

Similar calculations for $\chi^-_{\mu} + eA^-_{\mu} = 0$ (with change $e \to -e$ and (23)) give in components

$$\chi^-_{,t} = -eA^-_t = -2m \sin \theta, \quad \chi^-_{,\phi} = -eA^-_{\phi} = -2ma \sin \theta,$$

(43)

and integration gives

$$\chi^- = -2m \sin \theta (t - a \phi).$$

(44)

Since $\chi^+ = -\chi^-$, we have to set for the left and right modes equivalent frequencies and opposite directions of rotation, which demands to use the different parameters $a = a^+ = -a^-$, and the different masses $m^+ = m / \cos \theta$, $m^- = m / \sin \theta$. Using (42) and (44), we obtain

$$\chi^+ = 2m^+ \cos \theta (t + a^+ \phi) = -\chi^- = 2m^- \sin \theta (t - a^- \phi),$$

(45)

6. SuperBag as nonperturbative solution of the SuperQED model

Two oppositely charged Higgs fields $H^+$ and $H^-$ give rise to kinetic part of the Wess-Zumino SuperQED model [25],

$$-\frac{1}{4} W^a W_a + \Phi_+^e e^V \Phi_+ |_{\bar{\theta} \bar{\theta} \bar{\theta} \bar{\theta}} + \Phi_-^e e^{-eV} \Phi_- |_{\bar{\theta} \bar{\theta} \bar{\theta} \bar{\theta}},$$

(46)

where $V$ is vector superfield, and $W^a = -\frac{i}{2} \bar{D} \bar{D}_a V$. In the same time, the potential part (29), (30) and (31) corresponds to the most general renormalizable supersymmetric Lagrangian and gives rise to nonperturbative generalization of the SuperQED model.

The chiral superfields $\Phi_\pm$, can be expressed in the component form

$$\Phi_\pm(y) = H^\pm (y^\mu) + \sqrt{2} \theta \psi_\pm (y^\mu) + \theta \theta F_\pm (y^\mu),$$

(47)

as functions of the chiral coordinates $y^\mu = x^\mu + i\theta \sigma^\mu \bar{\theta}$ and $\theta$. 

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Interaction of the oppositely charged Higgs fields \( H^\pm \) with vector potential in zone (I) is defined by (39), (40) and yields

\[
H^\pm = |H^\pm|e^{\pm i\chi}, \quad \bar{H}^\pm = |H^\pm|e^{\mp i\chi}, \quad \chi = 2mt + 2am\phi,
\]

(48)

where the fields \( \bar{H}^\pm \) are scalar components of the antichiral fields

\[
\Phi^\pm_\pm(y^\mu) = \bar{H}^\pm(y^{+\mu}) + \sqrt{2}\bar{\theta}\bar{\psi}^\pm(y^{+\mu}) + \bar{\theta}\bar{\theta}\bar{F}^\pm(y^{+\mu}),
\]

(49)

as functions of the antichiral coordinates \( y^{+\mu} = x^\mu - i\theta\sigma^\mu\bar{\theta} \) and \( \bar{\theta} \). The corresponding nonperturbative solution with doubled Higgs fields (48) can be obtained similar to [16].

In the Wess-Zumino SuperQED model, the two Weyl spinors \( \psi^\pm \) in (47) combine into one massive Dirac spinor of the electron – superpartner of the Higgs doublet \( H^\pm \), [25].

Super-bag solution generates in the core of spinning particle the free from gravity Compton zone, which is flat and supersymmetric, representing necessary conditions for work of the perturbative SuperQED model, while the "miraculous cancellations" of the component supergraphs [25] forms the link to perturbative QED.

7. Conclusion

We have considered principal features of the spinning Kerr-Schild geometry which specify the bag model as a consistent with gravity way to particle physics and electroweak sector of the Standard Model. Two of these features are principally new, relative to the widespread belief:

– the spinning KN gravity is not weak, and becomes very strong at the Compton scale of the particle physics,

– compatibility between Quantum and Gravity can be achieved by means of supersymmetric generalization of the matter sector, without modification of the Einstein-Maxwell equations.

Due to extreme high spin/mass ratio, impact of the gravitational KN field on the structure of space-time becomes very strong, and the necessary for regularization nonperturbative bag-like solution becomes very sensible to the external Einstein-Maxwell field. As a result, the Kerr gravitational field determines many important parameters of the electron, such as the Compton wave length, the classical radius of the electron and the fine structure constant, creating the bag as a thin superconducting disk of the Compton radius with the thickness equal to classical radius of the electron. Simultaneously, the consistent with gravity bag forms the necessary stringy structure together with the quantum Wilson loop, which defines quantization of the angular momentum.

All that, along with the wonderful consistency of the Kerr geometry with solutions of the Dirac equation in the Weyl form [16, 40] and the known Carter result [1] that the exact Kerr-Newman solution has the gyromagnetic ratio of the Dirac theory, indicate that Kerr’s gravity participates actively in formation of the electron structure, contrary to wide-spread opinion that it is only a weak field theory which for some reason resists to be quantized. The spinning Kerr gravity deforms background topologically, as it is necessary to create quantum particle.

In conclusion we must answer the typical question: Why this bag-like and string structures are not seen experimentally, and electron looks like a point? Answer follows from the features of the Lorentz contraction for the relativistically rotating systems [41]. *The large left mode of the relativistic Kerr string shrinks to point by Lorentz contraction* [42].

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