Depolarizing power and polarization entropy of light scattering media: experiment and theory

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Abstract

We experimentally investigate the depolarizing power and the polarization entropy of a broad class of scattering optical media. By means of polarization tomography, these quantities are derived from an effective Mueller matrix, which is introduced through a formal description of the multi-mode detection scheme we use, as recently proposed by Aiello and Woerdman (arXiv:quant-ph/0407234). This proposal emphasized an intriguing universality in the polarization aspects of classical as well as quantum light scattering; in this contribution we demonstrate experimentally that this universality is obeyed by a surprisingly wide class of depolarizing media. This, in turn, provides the experimentalist with a useful characterization of the polarization properties of any scattering media, as well as a universal criterion for the validity of the measured data.

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I. INTRODUCTION

Characterization of optically transparent media with polarized-light scattering methods, is important for communication technology, industrial and medical applications. When polarized light is incident on an optically random medium it suffers multiple scattering and, as a result, it may emerge partly or completely depolarized. The amount of depolarization can be quantified by calculating either the entropy \( E_F \) or the degree of polarization \( P_F \) of the scattered field. It is simple to show that the field quantities \( E_F \) and \( P_F \) are related by a single-valued function: \( E_F(P_F) \). For example polarized light \( (P_F = 1) \) has \( E_F = 0 \) while partially polarized light \( (0 \leq P_F < 1) \) has \( 1 \geq E_F > 0 \). When the incident beam is polarized and the output beam is partially polarized, the medium is said to be depolarizing. An average measure of the depolarizing power of the medium is given by the so called index of depolarization \( D_M \). Non-depolarizing media are characterized by \( D_M = 1 \), while depolarizing media have \( 0 \leq D_M < 1 \). A depolarizing scattering process is always accompanied by an increase of the entropy of the light, the increase being due to the interaction of the field with the medium. An average measure of the entropy that a given random medium can add to the entropy of the incident light beam, is given by the polarization entropy \( E_M \). Non-depolarizing media are characterized by \( E_M = 0 \), while for depolarizing media \( 0 < E_M \leq 1 \). As the field quantities \( E_F \) and \( P_F \) are related to each other, so are the medium quantities \( E_M \) and \( D_M \). In a previous paper we showed the existence of a universal relation \( E_M(D_M) \) between the polarization entropy \( E_M \) and the index of depolarization \( D_M \) valid for any scattering medium. More specifically, \( E_M \) is related to \( D_M \) by a multi-valued function, which covers the complete regime from zero to total depolarization. This universal relation provides a simple characterization of the polarization properties of any medium, as well as a consistency check for the experimentally measured Mueller matrices. We emphasize that the results found in apply both to classical and quantum scattering processes, and might therefore become relevant for quantum communication optical applications, where depolarization is due to the loss of quantum coherence.

In this contribution, we present an experimental study of the depolarizing properties of a large set of scattering media, ranging from milk to multi-mode optical fibers. The results confirm the theoretical predictions for the bounds of the multi-valued function \( E_M(D_M) \).

The manuscript is divided as follows: in Section II we review the Mueller-Stokes formalism...
and show the differences between deterministic (non-depolarizing) and non-deterministic (depolarizing) scattering media. We also discuss the statistical nature of a depolarizing process resulting from the average (either spatial or temporal) performed in a multi-mode detection scheme. Furthermore, in order to describe the transverse spatial average present in our multi-mode detection set-up, we formally introduce the concept of an effective Mueller matrix ($M_{\text{eff}}$). In Section III we describe the experimental scheme for polarization tomography that was used to characterize the different scattering samples. These can be divided into two categories: (a) non-stationary (samples which fluctuate during the measurement time) and (b) stationary (samples which do not fluctuate). We then show the experimental results obtained for these samples followed by a brief discussion of the interesting structures in the ($E_M, D_M$) plane, that were revealed by the experiments. Finally, in Section IV we draw our conclusions.

II. DEPOLARIZING AND NON-DEPOLARIZING MEDIA

In the Introduction we stressed the fact that passive optical systems may be grouped in two broad classes: depolarizing and non-depolarizing systems. To the first class belong all media which decrease the degree of polarization $P_F$ of the impinging light, while to the second one belong all media which do not decrease $P_F$. In this Section we want to make the discussion more quantitative by using the Mueller-Stokes formalism which is widely used for the description of the polarization state of light beams.

A. Mueller-Stokes formalism

Consider a quasi-monochromatic beam of light of mean angular frequency $\omega$. Let us denote with $x, y, z$ the axes of a Cartesian coordinate system, with the $z$-axis along the direction of propagation of the beam whose angular spread around $z$ is assumed to be small enough to satisfy the paraxial approximation. Let

$$E_x(x, y, z_0, t_0) \equiv E_0 e^{-i\omega t_0}, \quad E_y(x, y, z_0, t_0) \equiv E_1 e^{-i\omega t_0},$$

be the component of the complex paraxial electric field vector in the $x$- and $y$-direction respectively, at the point $(x, y)$ located in the transverse plane $z = z_0$ at time $t_0$. If the field
is uniform on the transverse plane, then $E_x$ and $E_y$ will be, in fact, independent of $x$ and $y$ and a complete description of the field can be achieved in terms of a doublets $\mathbf{E}$ of complex variables (with possibly stochastic fluctuations):

$$\mathbf{E} = \begin{pmatrix} E_0 \\ E_1 \end{pmatrix},$$  \hspace{1cm} (2)

where $E_0$ and $E_1$ are now complex-valued functions of $z_0$ and $t_0$ only. A complete study of the propagation of $\mathbf{E}$ along $z$ can be found, e.g., in [8], however, for our purposes the main result we need is that propagation through non-depolarizing media can be described by a deterministic Mueller (or Mueller-Jones) matrix $M^J$, while to describe the propagation of a light beam through a depolarizing medium it is necessary to use a non-deterministic Mueller matrix $M$.

1. Deterministic Mueller matrix $M^J$

In a wide-sense, a deterministic linear scatterer as, e.g., a quarter-wave plate, a rotator or a polarizer, is an optical system which can be described by a $2 \times 2$ complex Jones matrix

$$J = \begin{pmatrix} J_{00} & J_{01} \\ J_{10} & J_{11} \end{pmatrix}. $$  \hspace{1cm} (3)

With this we mean that if $\mathbf{E}$ and $\mathbf{E}'$ describe the polarization state of the field immediately before and immediately after the scatterer respectively, then they are linearly related by the matrix $J$:

$$\mathbf{E}' = J\mathbf{E}. $$  \hspace{1cm} (4)

An alternative description can be given in terms of the Stokes parameters of the beam. To this end let $C$ be the covariance matrix of the field defined as

$$C_{ij} = \langle E_i E_j^* \rangle, \hspace{1cm} (i, j = 0, 1), $$  \hspace{1cm} (5)

where the brackets denote the statistical average over different realizations of the random fluctuations of the field. Then the four Stokes parameters $S_\mu$ ($\mu = 0, \ldots, 3$) of the beam are defined as

$$S_\mu = \text{Tr}\{C_\sigma \mu\}, \hspace{1cm} (\mu = 0, \ldots, 3), $$  \hspace{1cm} (6)
were the symbol \(\text{Tr}\{\cdot\}\) denote the trace operation and the \(\{\sigma_\mu\}\) are the normalized Pauli matrices:

\[
\sigma_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\]

\[
\sigma_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\] (7)

Now, if with \(S_\mu\) and \(S'_\mu\) we denotes the Stokes parameters of the beam before and after the scatterer respectively, it is easy to show that that they are linearly related by the real-valued \(4 \times 4\) Mueller-Jones matrix \(M^J\) as

\[
S'_\mu = M^J_{\mu\nu} S_\nu,
\]

where summation on repeated indices is understood and

\[
M^J = \Lambda^\dagger (J \otimes J^*) \Lambda,
\] (9)

where the symbol “\(\otimes\)” denotes the outer matrix product and the unitary matrix \(\Lambda\) is defined as

\[
\Lambda = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -i & 0 \\ 0 & 1 & i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}.
\] (10)

From the structure of \(M^J\) follows that a deterministic medium does not depolarize, that is \(P_F(S) = P_F(S')\) where the degree of polarization \(P_F\) of the field is defined as

\[
P_F(S) = \sqrt{S_1^2 + S_2^2 + S_3^2}.
\] (11)

Let us conclude by noticing that for deterministic media the two descriptions in terms of \(J\) or \(M^J\) are completely equivalent in the sense that the 16 real elements of \(M^J\) do not contain more information than the 4 complex elements of \(J\).

2. Non-deterministic Mueller matrix \(M\)

A non-deterministic scatterer is, in a wide-sense, an optical systems which cannot be described by a Mueller-Jones matrix. In this class fall all the depolarizing optical system as,
e.g., multi-mode optical fibers, particles suspensions, etc.. It has been shown \cite{10, 11} that it is possible to describe a non-deterministic optical system as an ensemble of deterministic systems, in such a way that each realization $\mathcal{E}$ in the ensemble is characterized by a well-defined Jones matrix $J(\mathcal{E})$ occurring with probability $p_{\mathcal{E}} \geq 0$. Then, the Mueller matrix $M$ of the system can be written as

$$M = \Lambda^\dagger (J \otimes J^\ast) \Lambda$$  \hspace{1cm} (12)$$

where the $\overline{\quad}$ symbol denotes the average with respect to the ensemble representing the medium:

$$\overline{J \otimes J^\ast} = \sum_{\mathcal{E}} p_{\mathcal{E}} J(\mathcal{E}) \otimes J^\ast(\mathcal{E}).$$  \hspace{1cm} (13)$$

At this point it is useful to introduce the auxiliary $4 \times 4$ Hermitian matrix $H$ defined as

$$H_{\mu\nu} = J_{ij} J_{kl}^\ast, \quad (\mu = 2i + j, \nu = 2k + l),$$  \hspace{1cm} (14)$$

which is, by definition, positive semidefinite, that is all its eigenvalues $\{\lambda_0, \lambda_1, \lambda_2, \lambda_3\}$ are non-negative. Then, it is possible to show that the depolarization index $D_M$ and the polarization entropy $E_M$ can be written as

$$D_M = \left[ \frac{1}{3} \left( 4 \sum_{\nu=0}^3 \lambda_\nu^2 - 1 \right) \right]^{1/2},$$  \hspace{1cm} (15)$$

$$E_M = - \sum_{\nu=0}^3 \lambda_\nu \log_4(\lambda_\nu).$$  \hspace{1cm} (16)$$

From ref. \cite{5} we know that $E_M$ is a multi-valued function of $D_M$ and that this dependence determines some physical bounds to any polarization scattering process. The function $E_M(D_M)$ shows thus character of universality. In the next Section we shall confirm this theoretical prediction with experimental results.

\section*{B. Unpolarized light and depolarizing media}

In classical optics, a light beam appears to be depolarized when its polarization direction varies rapidly with respect to other degrees of freedom that are not measured during the experiment (e.g. wavelength, time or position of the beam) \cite{12}. Moreover, depolarization occurs also when a single-mode polarization input beam is coupled with a multi-mode (either
spectral or spatial) system as, e.g., an optical fiber. In fact it is possible to identify two basic depolarizing processes, (a) one intrinsic to the medium, and (b) one due to the measurement scheme. In the first case (a) non-stationary temporal fluctuations of the optical properties of the medium, for instance due to the Brownian motion of suspended particles in a liquid, cause depolarization even when a single-mode detection scheme is employed (the time average performed during the measurement is responsible for the depolarization). On the other hand, type (b) stationary depolarizers (i.e. glass fibers) do not fluctuate in time and produce light depolarization only in the presence of a multi-mode detection scheme. In this case it is simple to build explicitly the ensemble of Mueller-Jones matrices representative of the medium, that we introduced in the previous subsection.

To this end, let us consider the case of a scattering process in which a coupling between polarization and spatial modes of the field occurs and a multi-mode detection scheme is employed. This is, in fact, the case occurring for the optical fibers we used. The Mueller-Stokes formalism, is suitable for a single-mode description of the field; however, it is possible to extend this formalism to the case in which $N_{in}$ spatial modes of the field impinge on the scatterer, $N_{out}$ leave from it and $D$ modes are eventually detected. We make the assumption that different spatial modes of the field are uncorrelated, that is we do not consider interference phenomena which are ruled out by the required linearity with respect to the intensities of the field. Moreover, without loss of generality, we assume $N_{in} = N_{out} = N$.

Let $S(j) \equiv \{S_0(j), S_1(j), S_2(j), S_3(j)\}$ be a generic 4-D Stokes vector defined with respect to the mode $j$, where $j \in \{1, \ldots, N\}$. For a $N$-mode field we have a collection of $N$ of these 4-D Stokes vectors that we can arrange in a single $4N$-D “super” vector $\mathbf{S} = \{\mathbf{S}(1), \ldots, \mathbf{S}(N)\}$. When the $N$-mode light beam undergoes a polarization-sensitive scattering, then, in general, the Stokes vectors $\{\mathbf{S}_{in}(j)\}$ of the input beam are related to the set of vectors $\{\mathbf{S}_{out}(j)\}$ of the output beam by:

$$\mathbf{S}_{out}(j) = \sum_{j_0=1}^{N} M^{j}(j, j_0)\mathbf{S}_{in}(j_0), \quad (j = 1, \ldots, N),$$

where $M^{j}(j, j_0)$ is the $4 \times 4$ Mueller-Jones matrix that describes the scattering from the input mode $j_0$ to the output mode $j$. If we introduce a “super” $4N \times 4N$ Mueller matrix
defined as
\[
\mathbf{M} \equiv \begin{pmatrix}
M^J(1,1) & \cdots & M^J(1,N) \\
\vdots & \ddots & \vdots \\
M^J(N,1) & \cdots & M^J(N,N)
\end{pmatrix},
\]
where each block \(M^J(j_1,j_2)\) is a 4 \(\times\) 4 Mueller-Jones matrix, then we can rewrite Eq. (17) in a compact form as
\[
\mathbf{S}_{\text{out}} = \mathbf{M} \cdot \mathbf{S}_{\text{in}},
\]
After the scattering process took place, we have to detect its products. We recently showed [14] that when \(D < N\) modes of the field are detected, a mode-insensitive polarization analyzer, put in front of a bucket-detector, can be described by a 4\(N\) \(\times\) 4\(N\) block-diagonal matrix \(\mathbf{A}\):
\[
\mathbf{A} \equiv \begin{pmatrix}
A(1) \\
\vdots \\
A(D) \\
0
\end{pmatrix},
\]
where \(A(j)\), \((j = 1, \ldots, D)\) are 4 \(\times\) 4 real-valued positive semi-definite matrices (in fact, projectors), and 0 is a null \((N - D) \times (N - D)\) matrix. In the paraxial limit \((D << N)\) each \(A(j)\) reduces to the 4 \(\times\) 4 identity. Then, the polarization state of the scattered beam after the analyzer, is described by the super Stokes vector \(\mathbf{S}_D\) given by
\[
\mathbf{S}_D = \mathbf{A} \cdot \mathbf{M} \cdot \mathbf{S}_{\text{in}},
\]
Finally, because of the mode-insensitive detection, the sum over all the detected modes reduces the number of degrees of freedom of the field from 4\(N\) to 4, producing the detected 4-D Stokes vector \(\mathbf{S}_D\)
\[
\mathbf{S}_D = \sum_{j=1}^{N} \mathbf{S}_{D}(j) = M_{\text{eff}} \mathbf{S}_{\text{in}}(1),
\]
where we have assumed that the input light beam is prepared in the single mode \(j_0 = 1\), so that \(\mathbf{S}_{\text{in}}(j_0) = \delta_{j_01} \mathbf{S}_{\text{in}}(1)\) and with \(M_{\text{eff}}\) we have denoted an effective 4 \(\times\) 4 Mueller matrix defined as
\[
M_{\text{eff}} = \sum_{j=1}^{D} A(j)M^J(j,1),
\]
which is written as a sum of \(D\) Mueller-Jones matrices. It is important to notice that while the product of Mueller-Jones matrices is still a Mueller-Jones matrix (in physical terms:
a cascade of non-depolarizing optical elements is still a non-depolarizing optical system), a sum, in general, is not. This causes depolarization. Moreover, since the “matrix coefficients” $A(j)$ are non-negative, the matrix $M_{\text{eff}}$ in Eq. (23) is an explicit version of the Mueller matrix written in Eq. (12). Then we have shown, by an explicit derivation, how to build the statistical ensemble representing the depolarizing medium, for this particular case.

III. DEPOLARIZATION EXPERIMENTS

A. Experimental scheme for polarization tomography

In order to measure the effective Mueller matrix $M_{\text{eff}}$ and thus the index of depolarization $D_M$ and the entropy $E_M$ of a scattering medium, it is straightforward to follow a tomography procedure: The light to be scattered by the sample is successively prepared in the four polarization basis states of linear ($V, H, +45^\circ$) and circular ($RHC$) polarization, which are represented by four independent input Stokes vectors $S_{\text{in}}$. For each of these input fields the corresponding Stokes vector $S_{\text{out}}$, that represents the output field, is obtained by measuring the intensities of the scattered light in the same four polarization basis states. This procedure provides the $4 \times 4$ independent parameters required to determine the 16 elements $M_{\mu\nu}$ of the Mueller matrix from Eq. (8). Note that we actually employ two additional polarization basis states ($-45^\circ$, $LHC$) in our experiments and perform $6 \times 6$ measurement, which allows us to reduce experimental errors by averaging within the over-complete data set.

The experimental scheme is illustrated in Fig. 1. The light source is a power-stabilized
He-Ne laser at 633 nm wavelength. The input field is prepared by the polarizer unit (PU), consisting of a fixed polarizer (P1), a half-wave plate (H1), and a quarter-wave plate (Q1). A microscope objective (MO, ×50/0.55) couples the light into the sample. The scattered light is collimated by a standard photographic objective (PO, 50 mm/1.9), followed by an adjustable pinhole (PH) that defines the amount of transverse spatial average to be performed in the light detection. The analyzer unit (AU) consists of a quarter-wave plate (Q2) and a polarizer (P2). Together with a focusing lens (L) and a photodiode (PD), it probes the polarization state of the scattered output field. As an estimation of the systematic error of the set-up, mainly due to imperfections of the used retarders, we measured the Mueller matrix of air (i.e. the identity matrix) and of well-known deterministic optical elements such as wave-plates. In all these cases, we found the deviations from the theoretically predicted matrix elements limited to $|\Delta M_{\mu\nu}| \leq \pm 0.04$.

B. Collection of scattering media

The various scattering media we investigated can be divided into (a) non-stationary samples where, e.g., Brownian motion induces temporal fluctuations within the detection integration time, and (b) stationary samples without such fluctuations, most notably multi-mode polymer and glass optical fibers. More specifically, we chose our scatterers from:

(a) Non-stationary media:

- polystyrene microspheres (2 µm dia., suspended in water, Duke Scientific Co., USA).
- diluted milk;

(b) Stationary media:

- Zenith™ polymer sheet diffusers (100 µm thick, SphereOptics Hoffman GmbH, Germany);
- holographic light shaping diffusers (0.5°, 1°, 5°, and 10° scattering angle, Physical Optics Co., USA);
- quartz/silica wedge depolarizers and quartz Lyot depolarizers, Halbo Optics, UK);
- step-index polymer optical fiber (NA=0.55, core dia. 250 µm, 500 µm, 750 µm ESKA CK type, Mitsubishi Rayon, Japan);
- step-index glass optical fiber (NA=0.48, core dia. 200 µm, 400 µm, 600 µm FT-x-URT type, distributed by Thorlabs, Inc., USA);
- step-index glass optical fiber (NA=0.22, core dia. 50 µm, ASF50 type, distributed by Thorlabs, Inc., USA);
- graded-index glass optical fiber (NA=0.27, core dia. 62.5 µm, GIF625 type, distributed by Thorlabs, Inc., USA).

C. Experimental results

For a large collection of different samples, Fig. 2 shows the measured polarization entropy $E_M$ vs. the corresponding index of depolarization $D_M$. The black lines represent the calculated analytical boundaries in the $(E_M, D_M)$ plane, whose functional dependence $E_M(D_M)$ was derived in Ref. [5]. These boundaries provide universal constraints to the possible values $(E_M, D_M)$ for any physical scattering system, that is, the range of admissible values is restricted to the rather limited grey-shaded area within the boundaries. As it is apparent from the experimental data, our choice of samples allowed us to widely fill in the range of values $(E_M, D_M)$, in good agreement with the prediction from Ref. [5]. For rather different scattering media, we observed similar values of the pairs $(E_M, D_M)$, which display the universality in this quantitative description of the depolarizing properties. We found samples throughout the full range of values in entropy and depolarizing power, $0 \leq E_M, D_M \leq 1$. However, note that the region below the curve connecting the points A and C in the $(E_M, D_M)$ plane is not covered by any data so far. Work is in progress to investigate this peculiarity.

Several scatterer-specific tuning parameters allowed us to realize this wide range of depolarizing systems and to reveal details of the depolarizing properties for the various media, as will be discussed in the following subsection.

The most versatile scatterers used to acquire data in the $(E_M, D_M)$ plane, were the multi-mode optical fibers. For them, the depolarization is caused by multiple reflections within the cylindrical light-guide together with mode mixing. We selected fibers of various lengths that displayed the full range between non-depolarizing and completely depolarizing prop-
FIG. 2: Measured entropy ($E_M$) vs. index of depolarization ($D_M$) for (a) non-stationary and (b) stationary scattering media. The maximal possible parameter range in the ($D_M, E_M$) plane is indicated by the grey-shaded area. Lines correspond to analytical boundaries predicted by theory. Cuspidal points are given by $A = (0, 1)$, $B = (1/3, \log_4 3)$, $C = (1/\sqrt{3}, 1/2)$, and $D = (1, 0)$ [5].

Fibers shorter than about 2 cm showed negligible depolarization ($D_M \approx 1$). In the case of the glass fibers complete depolarization ($D_M \approx 0$) was observed for lengths of $\approx 5$ m, whereas in the case of the polymer fibers this was achieved already for lengths of only $\approx 50$ cm. The reason is, presumably, the significant Rayleigh scattering at density fluctuations in the polymer material [15].

In our experimental scheme, the aperture of the pinhole (PH) defines the region of spatial of average in the scattered light detection (see Fig. [1]). By choosing the pinhole diameter between 2 mm and 13 mm, we realized scattering systems which are described by different
FIG. 3: Measured entropy ($E_M$) vs. index of depolarization ($D_M$) for (a) step-index polymer optical fibers ($♦$); (b) step-index glass optical fibers ($●$), graded-index glass optical fibers ($○$); (c) microspheres ($▲$), milk ($▼$), polymer diffuser ($□$), holographic diffusers with wave plate ($■$), wedge depolarizers ($△$), and Lyot depolarizers ($▽$). The analytical boundaries are indicated by the continuous lines.

effective Mueller matrices $M_{\text{eff}}$. In fact, a small pinhole, corresponding to an average over a small set of modes $j$ in Eq. (23), leads to a large index of depolarization. However, due to the huge optical mode volume in our fibers, the pinhole adjustment allowed only for small
changes, within $|\Delta D_M| \leq \pm 0.05$. With a lower limit of 2 mm in the pinhole diameter, special care was taken to select a sufficiently large number of speckles in the scattered output field. (The step-index glass fiber with 50 $\mu$m core showed the largest speckles of about 500 $\mu$m FWHM.) This is necessary in order to average out interference effects, generated by the coherent source (He-Ne laser), so that the assumption of uncorrelated modes in the derivation of $M_{\text{eff}}$ holds [16].

D. Discussion

In Fig. 3, we separately show the results for (a) step-index polymer fibers, (b) step-index and graded-index glass fibers, and (c) other scattering media. For the polymer fibers it is apparent that most of the data fall on the upper curve connecting the points A and D. This curve corresponds to Mueller matrices that have an associated operator $H$ with its four eigenvalues of the degenerate form $\{\lambda, \mu, \mu, \mu\}$. This degeneracy can be associated with isotropic depolarizers, which is obviously a good description for the polymer fibers. Contrarily, in Fig. 3(b) (glass fibers), we fill in also the allowed $(E_M, D_M)$ domain below the isotropy curve. These domains correspond to anisotropic media, the anisotropy being supposedly due to stress-induced birefringence in the glass fibers. It was actually this birefringence which we used as an additional tuning parameter accounting for changes of a few percent in the index of depolarization. The data obtained for long fiber samples, both in polymer and glass, are close to the cuspidal point A ($\lambda = \mu = 1/4$), which corresponds to total depolarizers. Contrarily, the data for very short fiber samples are close to the cuspidal point D ($\lambda = 1, \mu = 0$), which corresponds to a deterministic (i.e. non-depolarizing) system. In case of watery suspensions such as milk and microspheres, we observed purely isotropic depolarization, i.e. all data are found on the isotropic curve, similar to the polymer optical fibers (see Fig. 3(c)). We adjusted the depolarizing power by varying the concentration of the scatterers. The 100 $\mu$m thick Zenith™ polymer diffuser sheet was found to be almost completely depolarizing, whereas the holographic diffusers, when used alone, did not cause any significant depolarization. The latter effect is due to the absence of multiple scattering in the transmission of light through these surface-optical elements. In combination with a subsequent wave-plate we could, however, couple the scattered light angular spectrum to the polarization degrees of freedom, and thus achieve depolarization.
Finally, the data for standard wedge and Lyot depolarizers are also shown in Fig. 3(c). Wedge depolarizers are designed to completely depolarize a well defined linear input polarization, whereas our tomographic measurement procedure represents an average over all independent input polarizations. This results in a non-zero index of depolarization. The Lyot depolarizer is, within the experimental error, non-depolarizing since it is designed to depolarize a broadband light source while we operated with a monochromatic laser source.

IV. CONCLUSIONS

By means of polarization tomography we have experimentally characterized the depolarizing properties for a large set of scattering optical media. We describe these media with both the index of depolarization $D_M$ and the polarization entropy $E_M$ that is added by the medium to the scattered field. These quantities are derived from a measured effective Mueller matrix, which we formally introduce in the description of scattering systems subject to a multi-mode detection, as is the case of our experimental configuration. The set of studied media ranges from non-stationary scatterers such as milk and polystyrene microspheres to stationary scatterers such as multi-mode optical fibers, diffusers, and standard wedge depolarizers.

In Ref. [5] a universality was predicted for the possible values of $E_M$ and $D_M$, these values being restricted to a limited domain described by a set of analytical boundaries. The collected experimental data for our scatterers fill in this domain almost completely and give evidence of the predicted depolarization universality in light scattering. A certain range of the predicted $(E_M, D_M)$ values is, however, not covered by the scatterers we investigated so far. Work on this issue is currently under progress.

Furthermore, the quantities $D_M$ and $E_M$ provide insights into the particular depolarization mechanisms of the various media, as well as a consistency check for the measured data (see [16]), and may provide a useful classification of optical scatterers for quantum applications, where depolarization stands for decoherence [7]. In this spirit, an extension to twin photon quantum scattering experiments is relatively straight-forward. Work along this line is also under progress in our group.

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