Abstract

The gauge dependence problem of alternative flow equation for the functional renormalization group is studied. It is shown that the effective two-particle irreducible effective action depends on gauges at any value of IR parameter $k$. The situation with gauge dependence is similar to the standard formulation based on the effective one-particle irreducible effective action.

Keywords: Gauge dependence, functional renormalization group, effective two-particle irreducible action

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1 Introduction

Recently [1] it has been proved that in the case of gauge theories the effective average action of the functional renormalization group (FRG) [2, 3] found as a solution to the flow equation depends on gauges at any value of IR cutoff $k$ that makes impossible a physical interpretation of results obtained. The main reason of this situation is related to the fact that standard formulation of the FRG violates the gauge invariance of an initial classical action and the BRST symmetry [4, 5] of quantum action. In turn within standard perturbation approach to Quantum Field Theory it leads to the gauge dependence of effective average action even on-shell in contrast with corresponding property of effective action constructed by the Faddeev-Popov rules [6]. In this connection it is necessary to mention a redefinition of standard FRG approach in the form respecting the BRST symmetry [7, 8, 9]. It was proposed to arrive at this nice feature in contrast to the standard FRG approach with the help of regularization of an initial classical action which remains gauge invariant. Realization of this program in practice meets with serious difficulties [10]. Strongly speaking up to now a consistent and constructive procedure to realize explicitly the program does not exist because in [7] it was claimed a possibility to construct a regularized gauge invariant classical action perturbatively and in [8] it was just assumed existence of regularized interaction action.

Some years ago it was proposed [11] to realize main ideas of the FRG approach using concept of effective action with composite operators or, in another words, effective two-particle irreducible effective action introduced in paper [12] and then studied for gauge theories from point of view the gauge dependence in [13]. Quite recently the alternative flow equation using slightly different way in comparison with [11] has been introduced [14]. In standard perturbation approach to quantum theory of gauge fields the effective actions with composite operators used in [11, 14] are gauge independent on-shell. But by itself this property does not guarantee the gauge independence of the effective action with composite operators found as non-perturbative solution to the flow equation as it was shown in the case of standard FRG approach [1]. In the present paper we are going to analyze the derivation and the gauge dependence of alternative flow equation for the FRG proposed in [14].

The paper is organized as follows. In Sec. 2 basic properties of effective action with composite operators [14] in the framework of standard perturbation approach to gauge theories are considered. In Sec. 3 the derivation of alternative flow equation for the effective action with composite operators is given. In Sec. 4 the gauge dependence of the alternative flow equation and the effective action with composite operators is investigated. Finally, in Sec. 5 the results obtained in the paper are discussed.

We use the DeWitt’s condensed notations [15]. We employ the notation $\varepsilon(A)$ for the Grassmann parity of any quantity $A$. The functional derivatives with respect to fields and sources are considered as right and left correspondingly. The left functional derivatives with respect to fields are marked by special symbol ” $\rightarrow$ ”. Arguments of any functional are enclosed in
square brackets $[]$, and arguments of any function are enclosed in parentheses, $(\ )$. The symbol $F_{\alpha}[\phi,\ldots]$ means the right derivative of $F[\phi,\ldots]$ with respect to field $\phi^A$.

2 Ward identity and gauge dependence of 2PI effective action

Let us start from some initial classical action $S_0[A]$ of the fields $A^i$, with Grassmann parities $\varepsilon(A^i) \equiv \varepsilon_i$, being invariant under the gauge transformations $\delta X = \varepsilon_i \delta A^i$,

$$\delta A^i = R^i_\alpha(A)\xi^\alpha, \quad S_0[A]R^i_\alpha(A) = 0, \quad (2.1)$$

where $\xi^\alpha$ are arbitrary functions with Grassmann parities $\varepsilon(\xi^\alpha) \equiv \varepsilon_\alpha$, $\alpha = 1, 2, \ldots, m,$ and $R^i_\alpha(A)$, $\varepsilon(R^i_\alpha(A)) = \varepsilon_i + \varepsilon_\alpha$ are generators of gauge transformations. We restrict yourself by the case of Yang-Mills type of gauge theories when the algebra of generators $R^i_\alpha(A)$ has the form:

$$R^i_\alpha(A)R^j_\beta(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta} R^j_\alpha(A)R^i_\beta(A) = -R^i_\gamma(A)F^\gamma_{\alpha\beta}, \quad (2.2)$$

where $F^\gamma_{\alpha\beta} = (-1)^{\varepsilon_\alpha\varepsilon_\beta} F^\gamma_{\beta\alpha}$ are structure functions not depending on the fields $A^i$ and the generators $R^i_\alpha(A)$ form a set of linear independent operators with respect to the index $\alpha$. Standard quantization of the theory under consideration in the Faddeev-Popov method \[6\] operates with the action

$$S_{FP}[\phi] = S_0[A] + \Psi[A][\phi]R^A(\phi), \quad (2.3)$$

where $\Psi[\phi]$ is a gauge fixing functional and $R^A(\phi)$, $\varepsilon(R^A(\phi)) = \varepsilon_A + 1$ are the generators of BRST transformations of fields $\phi^A$ [1, 5],

$$R^A(\phi) = (R^i_\alpha(A)C^\alpha, 0, -\frac{1}{2}(-1)^{\varepsilon_\beta} F^\gamma_{\beta\gamma} C^\gamma C^\beta, B^\alpha(-1)^{\varepsilon_\alpha}). \quad (2.4)$$

In (2.3) $\phi = \{\phi^A\}$, $\phi^A = (A^i, B^\alpha, C^\alpha, \bar{C}^\alpha)$, $\varepsilon(\phi^A) = \varepsilon_A$ is full set of fields in the Faddeev-Popov quantization with the Faddeev-Popov ghost and anti-ghost fields $C^\alpha, \bar{C}^\alpha (\varepsilon(C^\alpha) = \varepsilon(\bar{C}^\alpha) = 1, \ gh(C^\alpha) = -gh(\bar{C}^\alpha) = 1)$, respectively, and the Nakanishi-Lautrup auxiliary fields $B^\alpha (\varepsilon(B^\alpha) = 0, \ gh(B^\alpha) = 0)$. The action (2.3) is invariant under global supersymmetry (BRST symmetry) [4, 5]

$$\delta_B\phi^A = R^A(\phi)\mu, \quad S_{FP}[\phi][A]R^A(\phi) = 0, \quad (2.5)$$

where $\mu$ is a constant anti-commuting parameter.

From technical point of view the standard FRG approach involves instead of $S_{FP}[\phi]$ the action

$$S_{Wk}[\phi] = S_{FP}[\phi] + S_k[\phi], \quad (2.6)$$

where $S_k[\phi]$ is so-called regulator action being quadratic in fields $\phi$,

$$S_k[\phi] = \int dx \frac{1}{2} R_k[AB](x,y)\phi^B(y)\phi^A(x) = \frac{1}{2} R_k[AB]\phi^B\phi^A. \quad (2.7)$$
The regulators $R_{k|AB}$ depend on IR parameter $k$ and obey the properties
\[
\lim_{k \to 0} R_{k|AB} = 0, \quad R_{k|AB} = R_{k|BA}(-1)\epsilon^A\epsilon^B, \quad \epsilon(R_{k|AB}) = \epsilon_A + \epsilon_B.
\] (2.8)

Standard choice of $R_{k|AB}(x, y)$ in the FRG reads
\[
R_{k|AB}(x, y) = z_{AB} \Box \exp\left\{-\frac{\Box}{k^2}\right\} \delta(x - y), \quad \Box = \partial_\mu \partial^\mu, \quad z_{AB} = \text{const},
\] (2.9)

and modifies behavior of all propagators in IR region making finite all Feynman diagrams. Nevertheless the standard FRG cannot be considered as a qualitative quantization method for gauge theories because at any value of the IR parameter $k$ the effective average action depend on gauges[1]. This conclusion is valid within the perturbation theory as well as on the level of solutions to the flow equation. Improvement of the FRG in the form of BRST exact renormalization group [7] remains a big question due to the lack of an explicit procedure for constructing a regularized gauge-invariant classical action [10].

Alternative approach to the FRG proposed in [11] is based on idea to consider Lagrangian density $\mathcal{L}_k(\phi)(x, y)$ of action (2.7),
\[
\mathcal{L}_k(\phi)(x, y) = \frac{1}{2}R_{k|AB}(x, y)\phi^B(y)\phi^A(x),
\] (2.10)
as composite operator in formalism [12] with introduction of additional external source $\Sigma(x, y)$ so that the generating functional of Green functions, $Z_k[J, \Sigma]$, is defined with the help of action $S_{FP}[\phi] + J_A\phi^A + \Sigma\mathcal{L}_k$. As a profit of this reformulation the effective action with composite operators does not depend on gauges on-shell in the perturbation theory. Recently [14], slightly different approach to formulate the FRG with the help of 2PI effective action has been proposed. Main difference with [11] is that the regulators $R_{k|AB}$ are considered as sources to composite fields $\phi^A, \phi^B$. We are going to study the gauge dependence problem in the proposed approach [14] because this very important issue did not touch in original paper at all.

In what follows specific forms of the initial classical action $S_0[A]$ and the gauge fixing functional $\Psi[\phi]$ are not essential and we consider them in general setting. Starting point of paper [14] is definition of generating functional of Green functions with composite operators $Z_k = Z_k[J, R_k]$, and generating functional of connected Green function with composite operators, $W_k = W_k[J, R_k]$, in the form
\[
Z_k[J, R_k] = \int D\phi \exp\left\{\frac{i}{\hbar}\left[S_{FP}[\phi] + J\phi + \frac{1}{2}R_k\phi\phi\right]\right\} = \exp\left\{\frac{i}{\hbar}W_k[J, R_k]\right\}.
\] (2.11)
Making use the change of integration variables in the form of BRST transformations (2.5), taking into account the invariance of $S_{FP}$ under these transformations (2.5) and triviality of corresponding Jacobian, we derive the Ward identity
\[
(J_A - i\hbar R_{k|AB}\partial_{\beta})R^A(-i\hbar\partial_{\beta})Z_k = 0.
\] (2.12)
In terms of functional $W_k$ the identity (2.12) rewrites as

$$(J_A + R_k|_{AB}(\partial_{j_B} W_k - i h \partial_{j_B}) ) R^A(\partial_j W_k - i h \partial_j) \cdot 1 = 0. \tag{2.13}$$

The 2PI effective action, $\Gamma_k[\Phi, \Delta - k]$, is introduced through the Legendre transformation of $W_k$,

$$\Gamma_k[\Phi, \Delta] = W_k[J, R_k] - J_A \Phi^A - R_k|_{AB} \left( \frac{1}{2} \Phi^B \Phi^A + h \Delta^B_k \right), \tag{2.14}$$

where

$$\frac{\delta W_k}{\delta J_A} = \Phi^A, \quad \frac{\delta W_k}{\delta R_k|_{AB}} = \frac{1}{2} \Phi^B \Phi^A + h \Delta^B_k. \tag{2.15}$$

From (2.14) and (2.15) it follows

$$\frac{\delta \Gamma_k}{\delta \Phi^A} = - J_A - R_k|_{AB} \Phi^B, \quad \frac{\delta \Gamma_k}{\delta \Delta^B_k} = - h R_k|_{BA}. \tag{2.16}$$

Then the Ward identity in terms of $\Gamma_k$ reads

$$\left( \frac{\delta \Gamma_k}{\delta \Phi^A} + i h \frac{\delta \Gamma_k}{\delta \Delta^B_k} (G_k^{-1})^{B|A} \partial_{F,A} \right) R^A(\hat{\Phi}) \cdot 1 = 0, \tag{2.17}$$

where the following notations

$$\hat{\Phi}^A = \Phi^A + i h (G_k^{-1})^{A|B} \partial_{F,B}, \tag{2.18}$$

$$J_A = (J_A, h R_k|_{AB}), \quad F^A = (\Phi^A, h \Delta^A_k), \tag{2.19}$$

$$(G_k^{-1})_{A|B} = - \frac{\delta}{\delta J_A} \mathcal{F}(\mathcal{J}), \quad (G_k^{-1})^{A|B} = - \frac{\delta \mathcal{F}(\mathcal{J})}{\delta J_A} \tag{2.20}$$

$$\mathcal{D}^{A|C}_k (G_k^{-1})^{C|B} = \delta^A_B, \tag{2.21}$$

$$\mathcal{J}_A(\mathcal{F}) = \left( - \frac{\delta \Gamma_k}{\delta \Phi^A} + \frac{1}{h} \frac{\delta \Gamma_k}{\delta \Delta^B_k} \Phi^B - \frac{\delta \Gamma_k}{\delta \Delta^A_k} \right), \tag{2.22}$$

$$\mathcal{F}^A(\mathcal{J}) = \left( \frac{\delta W_k}{\delta J_A} \frac{\delta W_k}{\delta R_k|_{AB}} - \frac{1}{2} \frac{\delta W_k}{\delta J_A} \frac{\delta W_k}{\delta J_B} \right), \tag{2.23}$$

are used.

Now we want to study the gauge dependence of functionals introduced in (2.11) and (2.14). To do this we consider infinitesimal variation of gauge fixing functional, $\Psi[\phi] \rightarrow \Psi[\phi] + \delta \Psi[\phi]$, and corresponding generating functional of Green functions using a temporary designation for the functional (2.11), $Z_k[J, R_k] = Z_{\Psi/k}[J, R_k]$, 

$$Z_{\Psi + \delta \Psi/k}[J, R_k] = \int D\phi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}[\phi] + \delta \Psi_{,A}[\phi] R^A(\phi) + J \phi + \frac{1}{2} R_k \phi \phi \right] \right\}. \tag{2.24}$$

From (2.24) it follows the variation of $Z_{\Psi/k}[J, R_k]$,

$$\delta Z_{\Psi/k} = \frac{i}{\hbar} \frac{\delta Z_{\Psi/k}}{\delta J_A} R^A(-i h \partial_j) Z_{\Psi/k}, \quad Z_{\Psi/k} = Z_{\Psi/k}[J, R_k]. \tag{2.25}$$
There exists an equivalent representation of gauge dependence of $Z_{\Psi|k}$ following from the BRST symmetry of functional $S_{FP}[\phi]$ (for details see [1]),

$$
\delta Z_{\Psi|k} = \left(\frac{i}{\hbar}\right)^2 (J_A - i\hbar R_k|_{AB} \partial_j) R^B(-i\hbar \partial_j) \delta \Psi[-i\hbar \partial_j] Z_{\Psi|k},
$$

(2.26)

From the equation (2.26) it follows that the vacuum functional $Z_{\Psi|k}[0] = Z_{\Psi}[J = 0, R_k = 0]$ does not depend on gauges

$$
\delta Z_{\Psi|k}[0] = 0.
$$

(2.27)

Due to this fact we will omit the subscript $\Psi$ in the functionals $Z_{\Psi|k} = Z_k, W_{\Psi|k} = W_k$.

The gauge dependence of functional $W_k$ is described by the equation

$$
\delta W_k = \frac{i}{\hbar} (J_A + R_k|_{AB}(\partial_j W_k - i\hbar \partial_j)) R^B(\partial_j W_k - i\hbar \partial_j) \delta \Psi[\partial_j W_k - i\hbar \partial_j] \cdot 1,
$$

(2.28)

or, equivalently,

$$
\delta W_k = \delta \Psi_{,A}[\partial_j W_k - i\hbar \partial_j] R^A(\partial_j W_k - i\hbar \partial_j) \cdot 1.
$$

(2.29)

Taking into account that due to the properties of Legendre transform one has

$$
\delta \Gamma_k = \delta W_k,
$$

(2.30)

the gauge dependence of the 2PI effective action is described by the following equation

$$
\delta \Gamma_k = -\frac{i}{\hbar} \left( \frac{\delta \Gamma_k}{\delta \Phi^A} + \frac{\delta \Gamma_k}{\delta \Delta_{AB}^k} (G_k^{\mathcal{F}^{-1}})^{B}[A] \partial_{\mathcal{F}^A} \right) R^A(\hat{\Phi}) \delta \Psi[\hat{\Phi}] \cdot 1.
$$

(2.31)

Therefore the 2PI effective action (2.14) repeats the property of the 2PI effective action introduced in [11] namely it does not depend on gauges when calculating with use of the equations of motion,

$$
\delta \Gamma_k \bigg|_{\partial_{\mathcal{F}} \Gamma_k = 0} = 0.
$$

(2.32)

In turn it allows to state that the 2PI effective action (2.14) defined as a solution to the following functional integro-differential equation,

$$
\exp \left\{ \frac{i}{\hbar} \left( \Gamma_k[\Phi, \Delta_k] - \frac{\delta \Gamma_k[\Phi, \Delta_k]}{\delta \Delta_{AB}^k} \Delta_{AB}^k \right) \right\} = \int D\phi \ exp \left\{ \frac{i}{\hbar} \left( S_{FP}[\Phi + \phi] - \frac{\delta \Gamma_k[\Phi, \Delta_k]}{\delta \Phi^A} \phi^A - \frac{1}{2} \frac{\delta \Gamma_k[\Phi, \Delta_k]}{\delta (\Delta_{AB}^k)} \phi^A \phi^B \right) \right\},
$$

(2.33)

leads to the gauge independent S-matrix due to the equivalence theorem [16]. Expanding the action $S_{FP}[\Phi + \phi]$ in Taylor series with respect to $\phi^A$ one can find the action $\Gamma_k = \Gamma_k[\Phi, \Delta_k]$ perturbatively,

$$
\Gamma_k = \Gamma_k^{(0)} + \hbar \Gamma_k^{(1)} + O(\hbar^2).
$$

(2.34)
For the zero-loop approximation, $\Gamma^{(0)}_k$, from (2.33) it follows
\[ \Gamma^{(0)}_k = S_{FP}[\Phi]. \] (2.35)

The one-loop approximation, $\Gamma^{(1)}_k$, satisfies the functional Clairaut-type equation
\[ \Gamma^{(1)}_k - \frac{\delta \Gamma^{(1)}_k}{\delta \Delta^B_k} \Delta^A_k = \frac{i}{2} \text{Str} \ln \left( S''_{FP}[\Phi] - \frac{\delta \Gamma^{(1)}_k}{\delta \Delta_k} \right), \] (2.36)
\[ S''_{FP}[\Phi] = \{ S''_{FP}[\Phi] \}, \quad S''_{FP}[\Phi] = \frac{\delta^2 S_{FP}[\Phi]}{\delta \Phi^B \delta \Phi^A}. \] (2.37)

Singular solutions to the ordinary and functional Clairaut-type equations have been studied in papers [17, 18]. For the type of equation (2.36) it was found that the solution can be presented in the form
\[ \Gamma^{(1)}_k[\Phi, \Delta_k] = S''_{FP}[\Phi] \Delta^{AB}_k - \frac{i}{2} \text{Str} \ln \Delta_k, \] (2.38)
up to some constant quantity.

As a general conclusion, in the perturbation theory the approach to quantum theory of gauge fields based on concept of the 2PI effective action is consistent method for describing physical results. In its turn, from the beginning the FRG approach is considered as non-perturbative method to describe quantum properties of gauge fields in terms of the effective average action found as solutions to the flow equation. As it was mentioned above the effective average action considering within perturbation theory is ill-defined because it is gauge dependent functional even on-shell [11]. But for the FRG it is the more critical fact that the effective average action found as a solution to the flow equation remains ill-defined due to the gauge dependence at any value of IR parameter $k$ (see Ref. [1]) that makes impossible of physical interpretations of obtained results within this method. In this connection the study of gauge dependence of the 2PI effective action for the FRG looks like as very important and actual task.

3 An alternative flow equation
The flow equation in the FRG is the basic relation describing the dependence of the effective average action on the IR parameter $k$. Let us derive an alternative flow equation for the 2PI effective action. Differentiating the functional $Z_k = Z_k[J, R_k]$ (2.11) with respect to $k$ and taking into account that only quantities $R_k|_{AB}$ depend on $k$, we obtain
\[ \partial_k Z_k = \frac{\hbar}{2i} \partial_k R_k|_{AB} \frac{\delta^2 Z_k}{\delta J_B \delta J_A}, \] (3.1)
or, equivalently,
\[ \partial_k Z_k = \partial_k R_k|_{AB} \frac{\delta Z_k}{\delta R_k|_{AB}}. \] (3.2)
In terms of the functional $W_k = W_k[J, R_k]$ the equations (3.1) and (3.2) rewrite in the form

\[
\partial_k W_k = \frac{1}{2} \partial_k R_k|_{AB} \left( \frac{\hbar}{i} \delta^2 W_k + \frac{\delta W_k}{\delta J_B} \frac{\delta W_k}{\delta J_A} \right), \tag{3.3}
\]

\[
\partial_k W_k = \partial_k R_k|_{AB} \frac{\delta W_k}{\delta R_k|_{AB}}. \tag{3.4}
\]

Due to the Legendre transform (2.15) variables $\Phi^A$ are functions of $J_A$ and $R_k|_{AB}$ and therefore they depend on IR parameter $k$ through $R_k|_{AB}$. Then we have

\[
\partial_k \Phi^A = \partial_k \frac{\delta W_k}{\delta J_A} = \frac{\delta \partial_k W_k}{\delta J_A} = \partial_k R_k|_{BC} \frac{\delta^2 W_k}{\delta R_k|_{BC} \delta J_A}. \tag{3.5}
\]

In deriving Eq. (3.5) we took into account that the partial derivative $\partial_k$ commutes with functional derivatives $\delta / \delta J_A$ because variables $J_A$ do not depend on $k$. Due to properties of the Legendre transform one has

\[
\partial_k \Gamma_k = \partial_k W_k, \quad \Gamma_k = \Gamma_k[\Phi, \Delta_k]. \tag{3.6}
\]

Then from Eqs. (3.4), (3.6), (2.15), (2.16) we derive the alternative flow equation for the 2PI effective action in the FRG,

\[
\partial_k \Gamma_k = -\left( \partial_k \frac{\delta \Gamma_k}{\delta (\hbar \Delta_k|_{AB})} \right) \left( \frac{1}{2} \Phi^B \Phi^A + \hbar \Delta_k|_{AB} \right). \tag{3.7}
\]

The equation (3.7) differs from the alternative flow equation proposed in [14]. The reason is the additional assumption $\partial_k \Phi^A = 0$ used. From (3.5) we see that this relation can be arrived at the requirement $\partial_k R_k|_{AB} = 0$ which contradicts with (2.9).

4 Gauge dependence of alternative flow equations

Now we are going to investigate the gauge dependence of the flow equation (3.7). The variation of gauge fixing functional, $\delta \Psi[\phi]$, does not touch upon a $k$-dependence of the functional $Z_k = Z_k[J, R_k]$. It allows us to derive the equation describing the gauge dependence of the flow equation for functional $Z_k$ as

\[
\delta \partial_k Z_k = \partial_k R_k|_{AB} \frac{\delta (\delta Z_k)}{\delta R_k|_{AB}}. \tag{4.1}
\]

Using the equation (2.25) the equation (4.1) can be presented in the form

\[
\delta \partial_k Z_k = \frac{i}{\hbar} \partial_k R_k|_{AB} \delta \Psi,_{C}[\partial_j \Psi, C, \partial_j 2W_k + \partial_j W_k] R^C \left( -i \hbar \partial_j \right) \frac{\delta Z_k}{\delta R_k|_{AB}}. \tag{4.2}
\]

Variation of the flow equation for functional $W_k = W_k[J, R_k]$ reads

\[
\delta \partial_k W_k = \frac{i}{\hbar} \left( \partial_k R_k|_{AB} \delta \Psi,_{C}[\partial_j W_k - i \hbar \partial_j] R^C \left( \partial_j W_k - i \hbar \partial_j \right) \frac{\delta W_k}{\delta R_k|_{AB}} - \partial_k W_k \delta W_k \right). \tag{4.3}
\]
Taking into account Eq. (2.29) we obtain the presentation of (4.3)
\[
\delta \partial_k W_k = \frac{i}{\hbar} \partial_k R_{k|AB} \left[ \delta \Psi_C \left[ \partial_j W_k - i\hbar \partial_j \right] R^C \left( \partial_j W_k - i\hbar \partial_j \right), \frac{\delta W_k}{\delta R_{k|AB}} \right] \cdot 1
\] (4.4)
containing the commutator of \( \delta \Psi_C \left[ \partial_j W_k - i\hbar \partial_j \right] R^C \left( \partial_j W_k - i\hbar \partial_j \right) \) and \( \delta W_k/\delta R_{k|AB} \). According to properties of the Legendre transform one has
\[
\delta \partial_k \Gamma_k = \delta \partial_k W_k.
\] (4.5)
Therefore the gauge dependence of alternative flow equation for the 2PI effective action can be described by the following equation
\[
\delta \partial_k \Gamma_k = -\frac{i}{\hbar} \left( \partial_k \frac{\delta \Gamma_k}{\delta (\hbar \Delta^A_{AB})} \right) \left[ \delta \Psi_C \left[ \hat{\Phi} \right] R^C \left( \hat{\Phi} \right), \frac{1}{2} \Phi^B \Phi^A + \hbar \Delta_k^{BA} \right] \cdot 1,
\] (4.6)
where the operators \( \hat{\Phi}^A \) are defined in (2.18)-(2.23). We conclude that flow equations for functionals \( Z_k, W_k, \Gamma_k \) depend on gages at any finite value of IR parameter \( k \). The same statement is valid for solutions to these equations as well. As to the case when \( k \to 0 \) the arguments given in [1] allow us to confirm the gauge dependence of the 2PI effective action at the fixed point too.

5 Summary

In the paper we have analyzed a reformulation of the FRG approach based on using instead of the effective average action (the 1PI effective action) [2, 3] the 2PI effective action introduced in [14]. It was shown that in contrast with the effective average action the 2PI effective action within the perturbation theory obeys the important property of gauge independence when it is calculated with using the equations of motion. In turn, thanks to the equivalence theorem [16] it leads to gauge independence of S-matrix. The same statement is valid for slightly different reformulation of the FRG approach proposed early in [11].

But the FRG approach by itself has been introduced as a non-perturbative method to study quantum properties of field theories. The effective average action should be found as non-perturbative solution to the flow equation which controls the dependence of effective average action on the IR parameter \( k \). Quite recently it has been proved the gauge dependence of the effective average action at any scale of \( k \) [1] when the FRG approach is applied to gauge theories. It means that in the case of gauge theories the standard FRG approach has no physical meaning because all obtained results within this method depend on gauges. For the first sight it seems that the reformulations of the FRG with the help of the 2PI effective action given in [11] [13] is more suitable due to good properties of these approaches in the perturbation theory. Unfortunately, this expectation does not come true.

We have derived the alternative flow equation in general setting without using an additional restriction on the \( k \)-dependence of fields \( \Phi^A, \partial_k \Phi^A = 0 \), appearing in the Legendre transform
because it contradicts to properties of regulators $R_{k|AB}$. The equation describing the gauge dependence of the alternative flow equation has been found. Analysis of this equation leads to conclusion that the 2PI effective action is gauge dependent in any scale of the IR parameter $k$. Therefore the FRG approach based on the effective average action $[2,3]$ or on the 2PI effective actions $[11,14]$ cannot be considered as quantization scheme of gauge fields having physical meaning.

At the moment the last hope to have consistent non-perturbative quantization procedure of gauge fields is connected with the BRST exact renormalization group $[7]$ where only the absence of an explicit procedure for constructing a regularized gauge invariant initial action prevents us from talking about the completeness of this method $[10]$.

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