Silicon nitride Mach-Zehnder interferometer for on-chip quantum random number generation

A Prokhodtsov1,2, V Kovalyuk1,3, P An2,3, A Golikov2,4, R Shakhovoy5,6, V Sharoglazova5,6, A Udaltsov5,6, Y Kurochkin5,6,7, G Goltsman1,2,3

1National Research University Higher School of Economics, Moscow 101000, Russia
2Department of Physics, Moscow State Pedagogical University, 119992, Russia
3Zavoisky Physical-Technical Institute of the Russian Academy of Sciences, 420029, Russia
4Moscow Institute of Physics and Technology (State University), 141700, Russia
5Russian Quantum Center, 45 Skolkovskoye shosse, Moscow, Russia
6QRate, 100 Novaya str., Skolkovo, Russia
7NTI Center for Quantum Communications, National University of Science and Technology MISiS, 4 Leninsky prospekt, Moscow, Russia

Abstract. In this work, we experimentally studied silicon nitride Mach-Zehnder interferometer (MZI) with two directional couplers and 400 ps optical delay line for telecom wavelength 1550 nm. We achieved the extinction ratio in a range of 0.76-13.86 dB and system coupling losses of 28-44 dB, depending on the parameters of directional couplers. The developed interferometer is promising for the use in a compact random number generator for the needs of a fully integrated quantum cryptography system, where compact design, as well as high generation speed, are needed.

1. Introduction
Quantum random number generator (QRNG) is an essential ingredient of quantum key distribution (QKD) systems. QKD is a secure communication method, one of the important functions of which is a secure transfer of a secret key created with a QRNG from Alice to Bob through a quantum communication channel. There have been demonstrated a number of optical QRNGs based on various quantum effects, including phase fluctuation in a laser diode [1,2], photon-pair interference [3], spontaneous Raman scattering [4], vacuum fluctuation [5]. Among all of these schemes, the QRNGs based on laser phase fluctuations could provide the highest generation rates, up to 68 Gbps. To date, most of the QRNGs were implemented in free space or using optical fiber and suffer from limitations due to their size and stability, which strongly limits their practical use. To reduce physical dimensions, some QRNGs have been demonstrated on various substrates like silicon-on-insulator (SOI) [6], indium phosphide (InP) [7], lithium niobate (LN) [8]. Different platforms have pros and cons, for example, LN provides increased generation speed but the relatively large waveguide of LN can limit the dimensions of the on-chip integration, SOI suffers from the two-photon absorption, while InP suffers from large optical losses. Here, we used the silicon nitride (Si3N4) platform, which combines low optical absorption in the infrared (IR) region and good mechanical properties [9]. The main element of the QRNG is the Mach-Zehnder interferometer (MZI) with directional couplers and a delay line in one of the arms. At the initial stage of creating an MZI for an on-chip QRNG, we examined in detail the splitting coefficient of a directional coupler, which is necessary for increasing the visibility of interference at the output of the device. At the final stage, based
on the obtained data, we fabricated and characterized the Mach-Zehnder interferometer with directional couplers and more than 5 cm delay line in one of the arms.

2. Device design and fabrication
One of the main elements of MZI is an on-chip beam splitter with a changeable splitting ratio based on the directional coupler that provides propagation light into different channels between nearby waveguides due to evanescent coupling within the framework of the coupled-mode theory (CMT) [10]. By adjusting the coupling rate between the neighboring waveguides, the splitting ratio can be tuned while maintaining high splitting uniformity.

Figure 1 (a-e). (a, b) Simulated $E_z$-profiles of the symmetric (even) and antisymmetric (odd) TE-like supermodes, respectively; (c) Calculated effective refractive index of the even (blue) mode and the odd (red) mode versus the gap between the two waveguides; (d) Calculated coupling length $L_c$ versus the gap between the two waveguides; (e) Top view FEM simulation of normalized electric field inside a directional coupler.

2.1. Directional coupler simulation
The cross-section of a directional coupler, based on rib silicon nitride waveguides is shown in Figure 1 (a). Directional coupler consists of two rib waveguides located close to each other, which leads to the overlap of the modes and, consequently, to a power mode coupling. The disappearing connection between the waveguides causes a periodic exchange of energy, characterized by the coupling length $L_c$, which is defined as the distance necessary for the complete transfer of energy. Therefore, the light associated with the lower input waveguide will be completely transferred to the upper waveguide after $L_c$, which will result in zero output optical power in the lower output waveguide. Different splitting ratios can be obtained by choosing a coupling length shorter than $L_c$ [11].

According to CMT, $L_c$ depends on the coupling coefficient $k$ but can also be expressed in terms of the effective mode indices by equation 1:

$$L_c = \frac{\pi}{2|k|} = \frac{\lambda}{2|n_{eff,odd} - n_{eff,even}|},$$

where $\lambda$ is the central wavelength, $n_{eff,odd}$ and $n_{eff,even}$ is the antisymmetric (odd) and symmetric (even)
TE-like supermodes refractive indexes.

Equation 1 more convenient than the evaluation of overlap integrals used in CMT, because the effective mode indices can be quickly extracted with a variety of numerical methods [11]. We simulated the coupling lengths for directional couplers by calculating the different mode indices with the Finite element method (FEM), realized in COMSOL Multiphysics.

Simulated $\varepsilon_{\text{E}}$-profiles for the symmetric (even) and antisymmetric (odd) TE-like supermodes presented in Figure 1 (a, b). We gradually varied the gap between the waveguides and obtained the values of the effective indices for the even and odd modes, which are presented in Figure 1 (c). After substituting the values in equation 1, the coupling length was extracted. As predicted by CMT, an increase in a gap decrease the modes overlap of two waveguides and leads the need to increase an interaction length. The calculated dependence of the coupling length on the gap for semi-etched silicon nitride waveguides of 1 $\mu$m wide is shown in Figure 1 (d).

![Figure 2 (a-c). (a) Optical micrograph a part of 2D array of the MZI with different parameters of directional couplers; (b) SEM image of the directional coupler.](image)

For the design of the MZI interferometer with a 400 ps delay line in one of the arms, the delay lines were studied previously [12]. For a half etched rib waveguide cross-section of 0.45×1 $\mu$m, we founded the group index of $n_g = 2.037 @ \lambda = 1.55 \mu$m, which corresponds to 5.8635 cm long delay line [13].

2.2. Fabrication of nanophotonic devices
Commercially available wafers on a silicon (Si) substrate with a thickness of 450 $\mu$m had 2.6 $\mu$m silicon buried oxide (SiO$_2$) and 450 nm silicon nitride (Si$_3$N$_4$) layer atop were used as a platform for nanophotonic device fabrication. We used one step of e-beam lithography with a high-contrast positive resist ZEP 520A and dry reactive-ion-etching (RIE) in CHF$_3$-Ar mixture to finalized both types of MZIs with and without delay lines.

To test directional coupler simulation, we fabricated a 2D array of the MZIs without delay line (Figure 2 (a)), with various parameters of directional couplers: interaction lengths in a range of 10 to 20 $\mu$m and gaps from 0.6 to 1.25 $\mu$m. We used a difference in the arm lengths of the interferometer of $\Delta L = 400 \mu$m to achieve a free spectral range of about $\approx 3$ nm. SEM image of one of the fabricated directional coupler shown in Figure 2 (b).

Figure 4 (a) shows an optical micro-photo one of the fabricated MZI with a long spiral delay line and short arm, two focusing grating couplers for input/output light, two directional couplers with equal parameters of interaction length and gap.

3. Experimental setup and results
To characterize MZI optical transmission we used an experimental setup, which includes tunable laser source (1510-1620 nm), motorized $x$, $y$, $z$ - rotation stage, fiber array with separated by 250 $\mu$m SMF-28 optical fibers, as well as an optical microscope for preliminary alignment [12]. The transmitted power
was measured by a low-noise photodetector and recorded by Ni-DAQ system.

3.1. Study of directional coupler splitting coefficient

The measured transmission spectra of several devices with a fixed interaction length of 10 μm, but differ in gaps (0.6, 1, 1.25 μm) are presented in Figure 3 (a). The envelope of the spectrum corresponds to focusing grating couplers optical transmission used to input the output of the telecom light, while resonances with FSR of ≈ 3 nm are associated with light interference and directly depends on the splitting coefficient of the directional coupler. Under the condition of small losses in the waveguides, the closer the splitting coefficient in the directional coupler to 50:50, the deeper the destructive interference at the minima at the output of the device. To extract the splitting coefficient from the experimental data, we first found the extinction ratio (ER):

\[ ER = \frac{T_{\max}}{T_{\min}}, \]  

where \( T_{\max} \) and \( T_{\min} \) are maximum and minimum transmitted through the MZI optical power close to 1550 nm wavelength.

![Figure 3 (a, b). (a) The measured transmission spectra of MZIs without long delay lines at a fixed interaction length of 10 μm and different gaps; (b) Contour map of the splitting ratio depending on interaction length and gap at a specific wavelength of 1550 nm.](image)

The output power ratio of the device (\( \rho \)) is directly related to the coefficient ER:

\[ \rho = \frac{(ER - 1)^2}{(ER + 1)^2}, \]  

where \( \rho \) is the ratio of the light power transmitted from the first port to port 3 (\( S_{31} \)), to the light power transmitted to port 2 (\( S_{21} \)).
Finally using equation (3) under the assumption that the losses inside the directional couplers are the same for different outputs, we can find power going to the upper port 3 (Figure 1 (e)):

\[ S_{31} = \frac{p}{1 + p}, \]  

(4)

The extracted from experimental data \( S_{ij} \) for various parameters of the directional coupler used in the fabricated 2D array of MZIs are shown in Figure 3 (b). With small gaps and interaction lengths, approximately half of optical power goes to the third port, while with an increase in the gap the power remains in the lower waveguide, and for equal distribution, the interaction length should be increased.

### 3.2. Study of Mach-Zehnder interferometer with a delay line

Using the measurement method described above and based on the data obtained from the study of directional coupler gap and interaction length, we fabricated 2D array of MZIs with delay lines. Figure 4 (b, c) shows the measured MZI transmission spectra and enlarged transmission spectra for one of the fabricated devices with a long delay line for QRNG application. We achieved the extinction ratio in a range of 0.76-13.86 dB and system coupling losses of 28-44 dB, depending on the parameters of directional couplers. Our numerical estimation and preliminary experimental results show that for the efficient operation of the QRNG it is necessary to have input/output losses < 33 dB, and extinction ratio (ER) > 3 dB.

![Figure 4 (a-c). (a) Optical micrograph of the MZI with long spiral delay line; (b) The measured transmission spectra of MZI together with focusing grating couplers; (c) Enlarged transmission spectrum of MZI.](image)

For several of the fabricated devices, losses were experimentally estimated to be around 30 dB with ER to be approximately equal to 3.14 dB, which makes it possible to use it in a QRNG and other applications of quantum communication technologies.

### 4. Conclusion

We performed both numerical and experimental studies of the splitting coefficient of a directional coupler made rib silicon nitride waveguides. Using obtained data, we fabricated silicon nitride on-chip MZI with two directional couplers and a spiral delay line integrated into one of the arms. Total optical losses of 30 dB and an extinction ratio of \( \approx 3.14 \) dB were achieved, which makes it possible to use it in a QRNG and other applications of quantum communication technologies. Further work will be devoted
to testing this MZI in a QRNG. Additional research will focus on improving the coupling efficiency.

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