A Zero-injection Initialization Numerical Method for Ill-conditioned Power Flow

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Abstract. Regarding the initial value sensitivity issues commonly found in Newton-like power flow calculation algorithms, a zero-injection initialization numerical method for ill-conditioned power flow is proposed in this paper, which improves the convergence of power flow calculation in ill-conditioned power systems. Based on the initial power flow provided by this method, the unbalanced active power of each bus (the slack bus excluded) relates to the net injection of active power of that bus and the unbalanced reactive power of each PQ bus relates to the net injection of reactive power of that bus, thus avoiding significant power unbalance. The method put forward herein can provide a reasonable initial value for the power flow calculation so that difficulty in power flow convergence caused by the unreasonable initial power flow is effectively avoided. The Newton’s method and the optimal multiplier method are implemented respectively on a small-scale ill-conditioned test system to verify the feasibility of the proposed zero-injection initialization numerical method.

1. Introduction

The methodology of power flow calculation, a commonly used analysing and calculation tool in power systems, is the numerical solution of a system of nonlinear equations. The computer algorithms for power flow calculation of complex power systems are basically iterative algorithms, and an initial guess for the power flow solution is required to start the iteration. The quality of this initial guess, or initial value of power flow calculation, is a key factor that decides the convergence of the power flow calculation. Commonly used power flow calculation algorithms in power systems, such as the Newton’s method, the PQ decomposition method, and the optimal multiplier method, all have a relatively high requirement on the initial value.

With the rapid growth of the power load and the implementation of the power transmission from West China to East China and regional power grid interconnection strategies, the scale of the power grid is expanding, and the operating conditions are complicated and changeable. Operating personnel often encounter situations where the power flow calculation is difficult to converge before they could obtain a reasonable operating condition of the system.

Small impedance branches and localized heavy loads are the most common ill-conditioned situations in power flow calculation. The small negative reactance branches in three-winding transformers, as well as the small impedance branches that simulate the closed switches, are important
factors that contribute to ill-conditioned power flow calculation in power systems. In large-scale power systems, a great number of three-winding transformers are utilized, which brings about multiple small-impedance branches, and localized heavy loads appear every now and then. The power flow calculation is often severely ill-conditioned, which makes it difficult to converge.

References [1,2] compare the optimal multiplier method, the tensor method and the self-adaptive Levenberg-Marquardt (LM) method, and the results show that the self-adaptive LM method has better robustness and numerical stability. Based on the continuation method, [3] transforms the problem of solving the power flow equations into an initial value problem of differential equations and uses the Runge-Kutta method to solve it. With the utilization of multi-continuation and multi-start techniques, it expands the convergence range of the solution and reduces the dependence on the initial value, whereas the algorithm is complex and involves massive calculation. Nevertheless, a proper initial value is beneficial to the improvement of the convergence of the algorithm. With regards to the convergence problem in the power flow calculation of a system with small impedance branches, a new Block Newton power flow calculation method is proposed in [4] on the basis of grid partitioning. It divides the power grid into several small sub-networks with selected contact lines, establishes the power flow equations for each sub-network, and converts the system of high-dimensional linear equations from the traditional Newton’s method to multiple systems of low-dimensional linear equations by means of linear transformation, joint solution and back substitution. Although this method can effectively accelerate the calculation speed, it has little effect on convergence. Reference [5] proposes the variable Jacobian Newton method, which improves the convergence of the power flow calculation by modifying the Jacobian matrix.

It is an important way to improve the convergence of power flow calculation by providing an initial power flow solution with smaller deviation. As the most commonly used algorithm initialization method in the power flow calculation of power systems, smooth start method can provide a proper initial value for a well-conditioned power system and has a wide range of applications. However, if applied to an ill-conditioned power system, the initial solution is usually diverged, and the calculation is difficult to converge. Reference [6] proposes a sufficient condition for the convergence of Newton-like power flow calculation algorithms, and defines an index to evaluate the initial value, which could be used to judge the convergence corresponding to the initial value. Based on the property that the power flow equations are a system of multivariable nonlinear quadratic algebraic equations in the Cartesian coordinate system, [7] proposes a distribution network power flow calculation method featuring accelerated convergence. First of all, the proposed method uses the Taylor series expansion to convert the power flow equations into a quadratic matrix equation; an approximate power flow calculation result is derived directly on this basis; then the approximate power flow result is used as the initial value for the iteration of the power flow calculation, and hence the iterative process is modified. Since the proposed method has better initial value for iterations, the convergence is better. References [8-10] compare and analyse several methods that choose proper initial values to enhance the convergence of the Newton’s method. Reference [11] analyses the influence of small impedance branches on the Newton’s method and proposes a small impedance branch zero-power method for selecting the initial voltage value, which effectively solves the power flow convergence problem in power systems with small impedance branches. Since the definition of the small impedance branch is qualitative, the convergence of the algorithm is affected by the selection criteria that qualifies a branch as a small impedance branch. In addition, if the small impedance branch forms a ring, and there exists a non-standard transformation ratio in the ring, or when the small impedance branch is connected to the constant voltage control node (i.e., PV buses or the slack bus), and the voltage settings are inconsistent, it is difficult to set the power flow of each small impedance branch to zero. Based on actual distribution network, [12] compress and simplify the topology of the distribution network, propose a modified fast decoupling power flow calculation algorithm to accelerate the process, and perform a quantitative analysis of the relationship between branch compensation ratio and convergence, focusing on distribution network branches with large R/X ratios.

This paper proposes an algorithm initialization method to enhance the convergence of power flow calculation in power systems, which solves the power flow calculation initialization issues in ill-conditioned power systems, and effectively avoids the difficulty in power flow calculation.
convergence caused by improper initial power flow solutions. Simulations of a small-scale ill-
conditioned test system with two separate Newton-like algorithms verify the correctness and the
effectiveness of the proposed method.

2. Zero-injection initialization method

2.1. Basic principle

The methodology of power flow calculation in power systems is the numerical solution of a system of
nonlinear equations. Commonly used power flow calculation algorithms in power systems, such as the
Newton’s method, the PQ decomposition method, and the optimal multiplier method, are all rooted in
iteration. The rationality of the iterative initial value is an important factor affecting the convergence
of power flow calculation in power systems.

The network equation of the AC power system can be described as:

\[ \dot{Y} \dot{U} = \dot{I} \]  

(1)

where \( \dot{Y} \) denotes the node admittance matrix, \( \dot{U} \) denotes the complex vector of the node voltage, and \( \dot{I} \) denotes the complex vector of the node injection current.

Group the complex vectors of voltages and currents according to the node type, we can get:

\[ \begin{bmatrix} \dot{Y}_{vv} & \dot{Y}_{vq} \\ \dot{Y}_{qv} & \dot{Y}_{qq} \end{bmatrix} \begin{bmatrix} \dot{U}_v \\ \dot{U}_q \end{bmatrix} = \begin{bmatrix} \dot{I}_v \\ \dot{I}_q \end{bmatrix} \]  

(2)

where the subscript \( v \) denotes the values corresponding to the PV buses and the slack bus, and the
subscript \( q \) denotes the values corresponding to the PQ buses.

From (2), we can get:

\[ \dot{Y}_{qv} \dot{U}_v + \dot{Y}_{qq} \dot{U}_q = \dot{I}_q \]  

(3)

Set \( \dot{I}_q = 0 \), and we can get:

\[ \dot{Y}_{qq} \dot{U}_q = -\dot{Y}_{qv} \dot{U}_v \]  

(4)

The voltage amplitude of each PV bus and the slack bus takes the set value, and the phases of the
corresponding voltages are set to zero, and hence we get a constant \( \dot{U}_v \). Solve the above system of
complex linear equation systems and get the voltage phasor of each PQ bus, which could be used to
initialize the power flow calculation.

If we ignore the real part of the admittance matrix, the system of complex linear equations is
replaced by:

\[ B_{qq} \dot{U}_q = -B_{qv} \dot{U}_v \]  

(5)

where \( B_{qq} \) denotes the imaginary part of \( \dot{Y}_{qq} \); and \( B_{qv} \) denotes the imaginary part of \( \dot{Y}_{qv} \).

Since the phase of the voltage of each PV bus and the slack bus is taken as zero, \( \dot{U}_v \) is a real vector.
Therefore, \( \dot{U}_q \) is also a real vector, and hence we get:

\[ B_{qq} V_q = -B_{qv} V_v \]  

(6)

where \( V_q \) is the real number description when the phase of \( \dot{U}_q \) is zero, representing the vector formed
by the voltage amplitude of the PQ buses; \( V_v \) is the real number description when the phase of \( \dot{U}_v \) is
zero, representing the vector formed by the voltage amplitude of the PV buses and the slack bus.

Solve (6) to obtain the voltage amplitude of each PQ bus and take the phase of the voltage of each
bus as zero, which could be used to initialize the power flow calculation.
2.2 Physical interpretation

Newton-like power flow calculation algorithms are relatively sensitive to the initial value, so it is expected that the initial value is located within a small range near the power flow solution. The smaller the distance between the initial value and the power flow solution, the better the convergence of the algorithm. The norm of the right-hand term of the power flow correction equation actually reflects the distance between the initial value and the power flow solution. For a small impedance branch, a small deviation between the voltage amplitude of both sides of the branch may give rise to a large current, resulting in a large power flow (reactive power flow, especially). The resultant considerable amount of imbalance of power between both sides of the branch affects the convergence of power flow calculation.

The initial power flow value provided by the zero-injection initialization method is applicable to the case where the PV buses and the slack bus are taken as voltage sources with identical phase, the amplitude of which are the corresponding set values, and the power injection of all PQ buses is zero. According to the initial power flow value provided by the zero-injection initialization method, the active power imbalance of each bus (the slack bus excluded) corresponds to the net active power injection of the node, and the reactive power imbalance of each PQ bus corresponds to the net reactive power injection of the node, thus avoiding huge unbalanced power. It can provide a reasonable initial value, thereby improving the convergence of the power flow calculation.

3. Case study

In order to test the effect of the method proposed in this paper, Newton’s method in polar coordinate system and the optimal multiplier method simulation programs are developed in C++. In the simulation, the base power is taken as 100MVA, and the convergence tolerance is taken as 10^-5p.u. The simulation environment is Microsoft Visual Studio 2015, and the computer hardware is configured as Intel® Core™ i7-9750MQ 2.6GHz CPU, 16GB RAM, 64-bit OS. The simulation program is performed on the ill-conditioned 6-node test system as described in [8]. The test results are as follows.

To verify the effectiveness of the algorithm, the ill-conditioned 6-node test system in [8] is selected, and the Newton’s method in polar coordinate system and the optimal multiplier method are used respectively to compare the impact of the smooth start method and the zero-injection initialization method as proposed in this paper for power flow calculation. The 6-node test system is illustrated in figure 1, where two power sources supply power to two loads through a three-winding transformer. The equivalent reactance of the medium-voltage winding of the three-winding transformer is a small impedance branch (x=0.000001). Line parameters and node data are shown in figure 2. The data in the figure and in the text below are all per unit values if not otherwise indicated.

![Figure 1. The 6-node test system diagram.](image)
Figure 2. The equivalent circuit diagram of the 6-node test system diagram.

Table 1 and table 2 depict the variation of the error to the number of iterations using Newton’s method in polar coordinate system with the smooth start method and the zero-injection initialization method, respectively. As shown in table 1, the error curve of the smooth start method oscillates up and down, without exhibiting a clear direction, and it fails to converge even after 50 iterations. The reason is that the initial value of the smooth start method has poor adaptability to ill-conditioned power flows. In contrast, the zero-injection initialization method, since it is able to provide a better initial value for the power flow calculation of ill-conditioned power flows, reduces the error to zero after the fourth iteration, as shown in table 2, which means it can improve the convergence reliability.

Table 1. The Iteration result of Newton’s method in polar coordinate system with the smooth start method.

| Iteration # | 0 (Initial value) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------|-------------------|---|---|---|---|---|---|---|---|---|----|
| Error       | 4707.58           | 1364.42 | 148.24 | 3.43 | 2.66 | 3.25 | 3.58 | 12.33 | 22.65 | 5.44 | 262.44 |
| Iteration # | 11                | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Error       | 801.24            | 178.65 | 14.53 | 8.69 | 3.68 | 724.65 | 186.32 | 2.35 | 76.58 | 61.33 |
| Iteration # | 21                | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Error       | 16.74             | 3.23 | 6.59 | 13.27 | 7.82 | 32.66 | 22.36 | 59.62 | 78.23 | 11.59 |
| Iteration # | 31                | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Error       | 11.00             | 8.65 | 68.75 | 18.65 | 206.87 | 66.36 | 368.56 | 78.63 | 1856.32 | 486.38 |
| Iteration # | 41                | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| Error       | 112.73            | 14.59 | 13.55 | 12.57 | 23.66 | 48.77 | 696.58 | 231.26 | 5.77 | Don't converge |

Table 2. The Iteration result of Newton’s method in polar coordinate system with the zero-injection initialization method.

| Iteration # | 0 (Initial value) | 1 | 2 | 3 | 4 |
|-------------|-------------------|---|---|---|---|
| Error       | 5.00              | 0.66 | 0.13 | 0.06 | 0 |

Next, optimal multiplier method is applied to the power flow calculation process of the ill-conditioned test system, and we compare simulation result using the smooth start method and the zero-
injection initialization method respectively, as shown in table 3 and table 4. Since the zero-injection initialization method can provide a better initial value for the power flow calculation (the initial error as 5), the rate of convergence is higher, and the convergence is better. As depicted in table 3, the convergent power flow solution is obtained after four iterations. The traditional smooth start method provides a poor initial solution (the initial error as 4707.582443), and it takes 18 iterations to converge. The comparison in the case of the optimal multiplier method once again proves that the zero-injection initialization method proposed in this paper can improve the convergence of the power flow calculation.

Table 3. The Iteration result of the optimal multiplier method with the smooth start method.

| Iteration # | 0 (Initial value) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------------|-------------------|---|---|---|---|---|---|---|---|---|
| Error       | 4707.58           | 14.42 | 10.24 | 3.14 | 2.96 | 2.86 | 2.18 | 2.13 | 1.95 | 1.94 |

| Iteration # | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|-------------|----|----|----|----|----|----|----|----|----|
| Error       | 1.84 | 1.44 | 1.24 | 0.53 | 0.24 | 0.15 | 0.09 | 0.003 | 0 |

Table 4. The Iteration result of the optimal multiplier method with the zero-injection initialization method.

| Iteration # | 0 (Initial value) | 1 | 2 | 3 | 4 |
|-------------|-------------------|---|---|---|---|
| Error       | 5.00              | 4.06 | 2.26 | 0.04 | 0 |

4. Conclusion

With a focus on the sensitivity of the initial value of Newton-like power flow calculation algorithms, this paper proposes a zero-injection initialization method suitable for power flow calculation in power systems. Through the comparative analysis of the power flow calculation iterations of the 6-node test system using both the zero-injection initialization method as well as the smooth start method respectively, the following conclusions are drawn:

(1) Compared with the traditional smooth-start method, the zero-injection initialization method can effectively reduce the deviation of the initial power flow solution.

(2) Compared with the traditional smooth-start method, in both the Newton’s method and the optimal multiplier power flow calculation method, the zero-injection initialization method can improve the convergence of power system power flow calculation.

Therefore, the zero-injection initialization method proposed in this paper can improve the power flow calculation process of power systems and proves effectiveness in solving the power flow calculation difficulties caused by small impedance branches. A new power flow algorithm is hence proposed.

5. References

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