The $D_s(2317)$ and $D_s(2463)$ Mesons as Scalar and Axial-Vector Chiralons in the Covariant Level-Classification Scheme

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(Received December 10, 2021)

The new narrow mesons observed recently in the final states $D_s^+\pi^0$ and $D_s^{∗+}\pi^0$ are pointed out to be naturally assigned as the ground-state scalar and axial-vector chiralons in the $(c\bar{s})$ system, which would newly appear in the covariant hadron-classification scheme proposed a few years ago.

§1. Introduction

(Covariant Classification Scheme and Chiral states/Chiralons) A few years ago we have proposed a covariant level-classification scheme\(^1\) for hadrons, unifying the seemingly contradictory two, non-relativistic and extremely relativistic, viewpoints. (Its essential points are reviewed in our review articles\(^2\).) Here the framework is manifestly Lorentz-covariant and the space for the static symmetry is extended from that of non-relativistic (NR) scheme to

\[ SU(6)_{SF} \otimes SU(2)_{\rho} \otimes O(3)_{L}, \]

where a new additional $SU(2)$-space for the $\rho$-spin ($\rho$- and $\sigma$- spin being Pauli-matrices in the decomposition of Dirac $\gamma$ matrices: $\gamma \equiv \sigma \otimes \rho$) is introduced for covariant description\(^*\) of hadron spin-wave function (WF).

The spin WF for the quark-antiquark meson systems are generally given by the Bargmann-Wigner (BW) spinors, and are represented as the bi-Dirac spinors $W^\beta_\alpha = u^\alpha_\alpha v^\beta_\beta : \alpha = (\rho_3, \sigma_3), \beta = (\bar{\rho}_3, \bar{\sigma}_3)$, where $\alpha(\beta)$ denotes the suffices of Dirac spinors of quarks(anti-quarks) represented by the eigenvalues of $\rho$-spin and $\sigma$-spin, and $\bar{\rho}_3 \equiv -\rho_3^T$. In the light-quark (LL) meson system the states with $(\rho_3, \bar{\rho}_3) =$

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\(^*\) It is to be noted that in our scheme the squared-mass spectra are globally $\tilde{U}(12)$-symmetric\(^1\) and the mass spectra themselves are able to be reconciled with the broken chiral symmetry.
They are decomposed into the pseudo-scalars, as quarks, respectively, as equations. For the HL-mesons we have the two physical solutions: choose the BW spinors, which are defined as solutions of the (local) Klein-Gordon time eigen-functions we choose the covariant, 4-dimensional Yukawa oscillator function.

In the heavy-light (HL) meson system the states with $\rho_3, \bar{\rho}_3 = (+, +)$ and $(+, -)$ are expected to be realized, reflecting the physical situation that the HL meson system has the non-relativistic $SU(6)_8$ spin symmetry (the relativistic, chiral symmetry) concerning the constituent Heavy quarks (Light quarks), and is predicted to exist in the ideal limit, leading to the chiral symmetric global structure of squared-mass spectra. The squared-mass operator is assumed to contain no light-quark Dirac matrices and are assumed to satisfy the master Klein-Gordon equation of Yukawa-type (in the non-relativistic scheme, while the states with the other values of $(\rho_3, \bar{\rho}_3)$ will be expected to appear newly in the covariant scheme due to the chiral symmetry.

\begin{align}
\Phi_{A}^B(x,y) & \sim \psi_{Q, A}(x)\tilde{\psi}^{q, B}(y) \\
A & = (\alpha, a), \ B = (\beta, b); \ \alpha, \beta = (1 \sim 4); \ a = (c \ or \ b), \ b = (u, d, s),
\end{align}

and are assumed to satisfy the master Klein-Gordon equation of Yukawa-type. The squared-mass operator is assumed to contain no light-quark Dirac matrices $\gamma^{(q)}$ in the ideal limit, leading to the chiral symmetric global structure of squared-mass spectra. The WF is separated into the two parts, the one of plane-wave center of mass motion and the other of internal WF: The internal WF with definite total momentum and the other of internal WF: The internal WF with definite total momentum.

\begin{align}
U_\alpha^\beta(P) & \equiv u_\alpha^{(Q)}(P)\bar{v}^\beta_{(q)}(P); \ C_\alpha^\beta(P) & \equiv u_\alpha^{(Q)}(P)\bar{v}^\beta_{(q)}(-P),
\end{align}

As is evidently seen from Eq. (3), through the chiral transformation on light antiquarks $\bar{v}(P)\gamma_5 = \tilde{v}(-P)$, the former is changed into the latter as $U(P)\gamma_5 = C(P)$. They are decomposed into the pseudo-scalars/vectors, and scalars/axial-vectors, respectively, as

\begin{align}
U_\alpha^\beta(v) & = 1/2\sqrt{2} \ (1 - iv \cdot \gamma) \ [i\gamma_5 P_s(P) + i\tilde{\gamma}_\mu V_\mu(P)], \\
C_\alpha^\beta(v) & = 1/2\sqrt{2} \ (1 - iv \cdot \gamma) \ [S(P) + i\gamma_5 \tilde{\gamma}_\mu A_\mu(P)] , \ (P_\mu \tilde{\gamma}_\mu = 0, \ v_\mu \equiv P_\mu/M).
\end{align}

Internal space-time WF/Yukawa oscillators As the complete set of space-time eigen-functions we choose the covariant, 4-dimensional Yukawa oscillator function.

The name of Paulon/Chiralon is reflecting that the covariant spin WF of the state with $(\rho_3, \bar{\rho}_3) = (+, +)$ is equivalent to that of the non-relativistic Pauli-WF and the states appear also in the non-relativistic scheme, while the states with the other values of $(\rho_3, \bar{\rho}_3)$ will be expected to appear newly in the covariant scheme due to the chiral symmetry.
tions. By imposing the freezing relative-time condition they become effectively the conventional, 3-dimensional oscillators:

$$\langle P_\mu r_\mu \rangle = \langle P_\mu p_\mu \rangle = 0 \Rightarrow O(3, 1)_L \approx O(3)_L.$$  (5)

§2. Mass spectra for low-lying $D$ and $D_s$-mesons

Since of the static symmetry (1) the global mass spectra are given by

$$M_N^2 = M_0^2 + N \Omega, \quad N \equiv 2n + L,$$  (6)

leading to phenomenologically well-known Regge trajectories. The masses of ground state mesons, $P_s$, $V_\mu$, $S$ and $A_\mu$ are degenerate in the ideal limit, and they are expected to split with each others between chiral partners (spin partners) by the spontaneous breaking of the chiral symmetry (the perturbative QCD spin-spin interaction) as

$$M_0(0^-/1^-) \lesssim M_0(0^+/1^+) < M_1(L = 1).$$  (7)

Actually, in §3, we shall apply the chiral symmetric Yukawa type interaction in non-derivative form, $L_{ND} = Eq. (15)$, to treat the interaction of the light quarks inside of HL mesons with the $\sigma$-meson nonet $s = s^c \lambda^c/\sqrt{2}$. In spontaneous chiral symmetry breaking, $s$ takes the vacuum expectation value $\langle s \rangle \equiv \text{diag}\{a, a, b\}$, which induce the splittings between chiral partners. Because of the HQS, the universal relation,

$$\Delta M^\chi = M_0(0^+) - M_0(0^-) = M_0(1^+) - M_0(1^-),$$  (8)

is expected to be valid within the same light-quark-flavor mesons; and the relation of those between the different light-quark configurations as $\Delta M^\chi(c \bar{n})/\Delta M^\chi(c \bar{s}) = a/b$.

In SU(3) linear $\sigma$ model\textsuperscript{13,14}, $a$ and $b$ are related with pion and kaon decay constants as $a \equiv f_\pi/\sqrt{2}$, $b \equiv f_K/\sqrt{2}$. Through the experimental values of $\Delta M^\chi(c \bar{s}) = 350\text{MeV}/c^2$ and $f_\pi/f_K$ (which gives $a/b = 1/1.44$), we predict $\Delta M^\chi(c \bar{n}) = 240\text{MeV}/c^2$. In Fig. 1 (a) and (b) we show, respectively, the low-lying $D$ meson$^*$ and $D_s$ meson mass spectra, presently known and/or predicted through the above relations.

§3. Decay properties of $D_s$-mesons

The observed properties of $D_s$-mesons to be examined are as follows\textsuperscript{9-11}:

$$D_s(0^+; 2.32) \rightarrow D_s(0^-; 1.97) + \pi^0 \quad \text{observed},$$
$$D_s(1^+; 2.46) \rightarrow D_s(1^-; 2.11) + \pi^0 \quad \text{observed}.$$  (9)

$^*$ As shown in Fig. 1(a), we predict the existence of $D_s^0(2110)$. New experimental data of $D\pi$ mass spectra by Belle\textsuperscript{7} show a peak structure with very wide width, which is identified as a single resonance $D_s^0(2308)$. We consider the possibility\textsuperscript{15} that this peak structure is explained by the interference of two resonant states, $D_s^0(2110)$ and the $P$ wave state with slightly-higher mass $D_s^*(j_q = 1/2)$, both of which are expected to have wide widths of a few hundreds MeV.
considered to occur by the mixing of intermediate \( \eta \) meson with \( \pi \) meson and Eq. (8) we get the relation

\[
R(0^+) = \frac{Br(D_{s0}^+(2.32) \rightarrow D_s^* \gamma)}{Br(D_{s0}^+(2.32) \rightarrow D_s \pi^0)} < 0.078 \quad \text{(CLEO)}.
\]  
(10)

\[
R(1^+) = \frac{Br(D_{s1}^+(2.46) \rightarrow D_s \gamma)}{Br(D_{s1}^+(2.46) \rightarrow D_s^* \pi^0)} = 0.47 \pm 0.10 \quad \text{(Belle)}.
\]  
(11)

\[
\Gamma_T[D_s(0^+;2.32)], \quad \Gamma_T[D_s(1^+;2.46)] < 7\text{MeV}.
\]  
(12)

The observed processes (9) are iso-spin violating and considered to occur by the mixing of intermediate \( \eta \) meson with \( \pi \)-meson. From this picture and Eq. (8) we get the relation (5),

\[
\Gamma(D_s^+(0^+) \rightarrow D_s(0^-) \pi^0) = \Gamma(D_s^+(1^+) \rightarrow D_s(1^-) \pi^0),
\]  
(13)

which is consistent with the property (12). We can estimate phenomenologically the value of mixing parameter \( \sin \theta \), by using the experimental branching ratio (12) of \( D_n(c \bar{n}) \) meson to the iso-spin violating decay channel as

\[
\left( \sin \theta \right)^2_{\text{exp}} \approx \frac{Br(D_s^{*+} \rightarrow D_s^{+} \pi^0)(M^{2}_{D_s^{*+}}/q^3)}{Br(D_s^{+} \rightarrow D_s^{*} \gamma)} \frac{2Br(D^{+} \rightarrow D^{+} \pi^0)(M^{2}_{D^{+}}/q^3)}{Br(D^{*+} \rightarrow D^{*+} \pi^0)(M^{2}_{D^{*+}}/q^3)}
\]
\[
= (0.9 \pm 0.4) \cdot 10^{-3}.
\]  
(14)
This seems to be of reasonable order of magnitude as due to the virtual EM-interaction. In order to estimate the absolute magnitude of the width (13) in relation with those of the other HL-mesons, we shall set up the chiral symmetric interaction Lagrangian in the framework of covariant oscillator quark model (COQM)\(^{16}\) as

\[
S_Y = \int d^4x_1 d^4x_2 \mathcal{L}(x_1, x_2) \equiv \int d^4X \mathcal{L}_I(X), \quad \mathcal{L} = \mathcal{L}^{ND} + \mathcal{L}^{AX} \tag{15}\]

\[
\mathcal{L}^{ND} = g_{ND} \langle \Phi(x_1, x_2) M(x_2) \bar{\Phi}(x_1, x_2) \rangle,
\]

\[
\mathcal{L}^{AX} = g_{AX} \langle \Phi(x_1, x_2)(\bar{D}_2\mu + \bar{D}_2\mu) + i\sigma_{\mu\nu}(\bar{D}_2\mu - \bar{D}_2\mu))\rangle \bar{D}_2\mu M(x_2) \bar{\Phi}(x_1, x_2) \rangle.
\]

\[
M \equiv s - i\gamma_5 \phi (s \equiv s^0 \lambda^0 / \sqrt{2}), \quad \phi \equiv \phi^a \lambda^a / \sqrt{2},
\]

\[
\Phi \propto (1 - iv \cdot \gamma)(i\gamma_5 D + i\gamma_\mu D_\mu + D_0 + i\gamma_\mu D_\mu \gamma^5),
\]

\[
\bar{\Phi} \equiv \gamma_4 \Phi^4 \gamma_4, \quad D = (\sqrt{2 M_D} D^0, \sqrt{2 M_D} D^+, \sqrt{2 M_D} D^+_D) \text{ etc.}, \tag{16}
\]

where only the Yukawa-type interaction of the scalar (s) and pseudo-scalar (\(\phi\)) nonets with the light quarks in the HL-meson is taken into account.

The interaction (15) consists of the two terms:

Firstly the \(g_{ND}\) term (Yukawa interaction in non-derivative form) gives dominant (compared to the \(g_{AX}\) term) contribution to the (quark-) spin flip processes. In spontaneous breaking of chiral symmetry, \(s = s^0 \lambda^0 / \sqrt{2}\) takes the vacuum expectation value which induces the mass-splittings between chiral partners through the equation \(\Delta M^X(c\bar{u}) = 2g_{ND}a\) and \(\Delta M^X(c\bar{s}) = 2g_{ND}b\) (see Eq. (8)), as explained in §2.

Secondly the \(g_{AX}\) term in the interaction (15) corresponds to the extended PCAC term, and concerns dominantly (compared to the \(g_{ND}\) term) to the spin-flip processes. The formula of the relevant pionic decay amplitudes and widths are given in Table I.

\[
\text{Table I. Formula of pionic decay amplitudes and widths. The invariant(helicity) amplitudes for respective processes are given first(second). The quantities in the columns are defined as: } M(M') \text{ is the mass of the initial(final) } D_{n(s)} \text{ meson; } q_0 \equiv P - P'; \quad |q| = M' \omega_3 \text{ (} M' \omega_3 \text{) is momentum (energy) of the final } D_{n(s)} \text{ meson in the CM system; } m_1(m_2) \text{ constituent quark(antiquark) mass.}
\]

The \(g_{ND}\) is fixed, from the experimental value of \(\Delta M^X(c\bar{s}) = 2g_{ND}b = 350\text{MeV}\), with the value \(g_{ND} = 1.848\), while the \(g_{AX}\) is fixed, from the experimental decay
width $\Gamma(D^{*+} \to D^0 \pi^+) = (96 \pm 23) \times 0.68\text{keV}$, with $g_{AX} = 5.10\text{GeV}^{-2}$. By using these values, we can predict the absolute values of the relevant pionic decay widths as

$$\Gamma(D^{\chi}_{n,0} \to D_n \pi) = \Gamma(D^{\chi}_{n,1} \to D^*_n \pi) = 158\text{MeV},$$  \hspace{1cm} (17)

$$\Gamma(D^{\chi}_{s,0} \to D_s \pi^0) = \Gamma(D^{\chi}_{s,1} \to D^*_s \pi^0) = 155 \pm 70\text{keV},$$  \hspace{1cm} (18)

where, in deriving Eq. (18), the estimated value Eq. (14) is used. The value of width (18) is consistent with the experiment (12).

*(Radiative decay)* In order to treat systematically all the radiative transitions between the HL-mesons we shall set up the basic EM-interaction Lagrangian in the framework of COQM\(^{16}\), as

$$S^{EM}_I = \int d^4x_1 d^4x_2 \sum_{j=1,2} j_{i,\mu}(x_1, x_2) A_\mu(x_i) = \int d^4x \sum_{i} j_{i,\mu}(X) A_\mu(X),$$  \hspace{1cm} (19)

$$j_{i,\mu}(x_1, x_2) = -ie_i\left( (m_1 + m_2)/m_i \right) \langle \Phi_U (\partial_{\mu} - igM\sigma_{\mu\nu}\partial^\nu) \Phi_U \rangle,$$

$$\Phi_U = \Phi_U (-i \not{v} \cdot \gamma), \quad \Phi_U = \Phi_U (-i \not{v} \cdot \gamma),$$

where $\Phi_U$ is the unitary correspondent of $\Phi$, so defined as $\langle \Phi_U \Phi_U \rangle \to \langle \Phi^{\dagger} \Phi \rangle$ at the rest frame. Here it is to be noted that our effective current $j_{i,\mu}(X)$ is obtained through the “minimal substitution” of $(\partial_{\mu} \to \partial_{i,\mu} - ie_i A_\mu(x_i))$, and accordingly it is conserved in the ideal limit.

Our effective current has also another remarkable feature due to the covariant nature of our scheme. The spin-current interaction (the second term in Eq. (19)) leads to the Hamiltonian

$$H^{(i) \text{spin}} = j_{\mu}^{(i) \text{spin}} A_\mu = \mu^{(i)} \sigma^{(i)} \cdot B + d^{(i)} \rho_1^{(i)} \sigma^{(i)} \cdot E, \quad \mu^{(i)} = d^{(i)} = e_i/2m_i$$  \hspace{1cm} (20)

This shows that our Hamiltonian contains the interaction through the “intrinsic electric dipole” $d\rho_1 \sigma$ as well as the one through the magnetic dipole $\mu \sigma$. The “intrinsic dipole” gives contributions only for the transitions between chiralons and Paulons, while does none for the other transitions.

From the effective currents $j_{i,\mu}$ in Eq. (19), we can derive the formula of the relevant radiative decay amplitudes and widths, which are given in Table II.

By using Table II we can predict the widths for all the radiative spin-flip transitions between ground state $D_s$ mesons. The results are given in Table III, where for reference, the width for the transition of $D_n$ meson, $D^0_n(1^-) \to D^+_n(0^-)\gamma$ is given in comparison with experiments. There, we have also shown the predicted values by the other chiral model.

From the results in Table III we see that our model gives the much larger widths for the transitions, (c) and (d), from chiralons to Paulons, compared to the other chiral model (, reflecting the above mentioned feature (20) of our currents,) while does the width of almost the same amount for transitions, (a) (and (b)), from Paulons(chiralons) to Paulons(chiralons). This difference is considered to come from
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| Processes/widths | invariant amplitude $M$/helicity amplitude $\mathcal{M}_{h_0,h_1}$ |
|------------------|---------------------------------------------------------------|
| $D_{s0}^+(\epsilon_\mu(P)) \to D_{s0}^+(P')\gamma(\eta_\mu(q))$ | $\mathcal{M}_{h_0,h_1} = \frac{1}{2} \left( \frac{\sigma_{12}}{2\alpha_{12}} \right)^2 (M + M') + (M + M')^2 \epsilon_{\mu\rho\sigma\eta_\mu} \eta_\mu(\eta_\mu)' v_\alpha v_\beta$ |
| $\Gamma = \frac{\alpha_0 q}{2 M_{D_{s0}^+}^2} |M + M|^2$ | $\mathcal{M}_{h_0,h_1} = -i \frac{d}{2} \left( \frac{\sigma_{12}}{2\alpha_{12}} + \frac{\sigma_{13}}{2\alpha_{13}} \right) (M + M') + (M + M')^2 \epsilon_{\mu\rho\sigma\eta_\mu} \eta_\mu(\eta_\mu)' v_\alpha v_\beta$ |

Table II. Formula of radiative decay amplitudes and widths. $e_1(-e_2)$ is the charge of the first(second) constituent. In the case of $D_{s0}^+ = c\bar{s}$, $e_1(e_2) = \frac{2}{3}(\frac{2}{3}) \cdots$. $d \equiv (m_1 + m_2)$. 

For other quantities, see, the caption in Table I.

| Processes | $P/\chi \to P/\chi$ | $\Gamma$(keV) ours | $\Gamma$(keV) others$^{17}$ |
|-----------|---------------------|-----------------|-----------------|
| (a) $D_s(1^+) \to D_s(0^-)\gamma$ | $P \to P$ | 0.33 | 0.43 |
| (b) $D_s(1^+) \to D_s(0^-)\gamma$ | $\chi \to \chi$ | 0.26 | 0.43 |
| (c) $D_s(1^+) \to D_s(1^-)\gamma$ | $\chi \to P$ | 21 | 1.74 |
| (d) $D_s(1^+) \to D_s(0^-)\gamma$ | $\chi \to P$ | 93 | 5.08 |

Table III. $\gamma$-decay widths for spin-flip transitions between the ground state $D_s$ mesons. $\alpha = 1/137.036$. The constituent quark masses are fixed with the values, $m_u = m_d \equiv m_0 = M_{\chi}/2$, $m_s = M_{\phi}/2$, and $m_c = M_{J/\psi}/2$.

The different identification of the relevant mesons in the two cases: The narrow $D_s$ mesons are assigned as the conventional $P$-wave excited states in the other model$^{17}$, while they are the $S$-wave chiral states other than the $P$-wave Pauli-states in our scheme.

(Branching ratios between radiative to pionic decay widths) From the predicted values of pionic (Eq. (18)) and radiative (Table III) decay widths we obtain the ratios between them as follows:

$$R(0^+) = 0.14^{+0.11}_{-0.05}, \quad R(1^+) = 0.6^{+0.5}_{-0.2},$$

which seems to be consistent with the experiments Eqs. (10) and (11).

§4. Concluding Remarks

- The $D_s(2317)$ and $D_s(2463)$ mesons are shown consistently assigned as the chiralons with $J^P = 0^+$ and $1^+$ in the $(c\bar{s})$ ground states.
- The decay width of $(D_s^0(1^+) \to D_s^*(1^-)\pi)$ is predicted as $\Gamma \simeq 130$ MeV, and the radiative decay widths of chiral states into Pauli-states are predicted to be remarkably larger than those estimated in other works. These are to be checked experimentally.
Further experimental search for chiralons is desirable.

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