IMPORTANCE OF SEMI INCLUSIVE DIS PROCESSES IN DETERMINING FRAGMENTATION FUNCTIONS

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Abstract

A NLO QCD analysis of the HERMES and COMPASS data on pion multiplicities is presented. Sets of pion fragmentation functions are extracted from fits to the data and compared with those obtained from other groups before these data were available. The consistency between HERMES and COMPASS data is discussed. We point out a possible inconsistency between the HERMES \([x, z]\) and \([Q^2, z]\) presentations of their data.

In the absence of charged current neutrino data, the experiments on polarized inclusive deep inelastic lepton-nucleon scattering (DIS) yield information only on the sum of quark and anti-quark parton densities \(\Delta q + \Delta \bar{q}\) and the polarized gluon density \(\Delta G\). In order to extract separately \(\Delta q\) and \(\Delta \bar{q}\) other reactions are needed. One possibility is to use the polarized semi-inclusive lepton-nucleon processes (SIDIS) \(l + N \rightarrow l' + h + X\), where \(h\) is a detected hadron (pion, kaon, etc) in the final state. In these processes new physical quantities appear - the fragmentation functions \(D_{h q, \bar{q}}(z, Q^2)\) which describe the fragmentation of quarks and antiquarks into hadrons. Due to the different fragmentation of quarks and anti-quarks, the polarized parton densities \(\Delta q\) and \(\Delta \bar{q}\) can be determined separately from a combined QCD analysis of the data on inclusive and semi-inclusive asymmetries. The key role of the fragmentation functions (FFs) for the correct determination of sea quark parton densities \(\Delta \bar{q}\) was discussed in \([1]\). There are different sources to extract the fragmentation functions themselves: The semi-inclusive \(e^+ e^-\) annihilation data, single-inclusive production of a hadron \(h\) at a high transverse momentum \(p_T\) in hadron-hadron collisions, unpolarized semi-inclusive DIS processes. It is important to mention that the data on hadron multiplicities in unpolarized SIDIS processes are crucial for a reliable determination of FFs, because only then one can separate \(D_{q h}^h(z, Q^2)\) from \(D_{\bar{q} h}^h(z, Q^2)\). Such data have been used only by the DSS group in their global analysis \([2]\). As a result, the properties of the extracted set of FFs significantly differ, especially in the kaon sector, from those of the other sets of FFs \([3]\). Unfortunately, the new properties of the DSS FFs are based on the unpublished HERMES’05 SIDIS data on hadron multiplicities which are not confirmed by the final HERMES data \([4]\). It turns out that not only the DSS FFs, but all other sets of pion and kaon FFs are NOT in agreement with the recent HERMES and COMPASS data \([5]\) on hadron multiplicities.

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In this talk we present our results on new pion fragmentation functions extracted from a NLO QCD fit to the HERMES and COMPASS (the first ref. in [5]) data on the pion multiplicities. While COMPASS reports data only on a deuteron target, HERMES presents data on both the proton and deuteron targets.

The multiplicity \( M_{\pi p(d)}(x, Q^2, z) \) of pions using a proton (deuteron) target are defined as the number of pions produced, normalized to the number of DIS events, and can be expressed in terms of the semi-inclusive cross section \( \sigma_{\pi p(d)}^{\pi} \) and the inclusive cross section \( \sigma_{p(d)}^{DIS} \):

\[
M_{\pi p(d)}(x, Q^2, z) = \frac{d^3 N_{\pi p(d)}(x, Q^2, z)/dx dQ^2 dz}{d^2 N_{DIS}^{p(d)}(x, Q^2)/dx dQ^2} = \frac{d^3 \sigma_{\pi p(d)}^{\pi}(x, Q^2, z)/dx dQ^2 dz}{d^2 \sigma_{p(d)}^{DIS}(x, Q^2)/dx dQ^2} = \frac{(1 + (1 - y)^2) 2 x F_{1p(d)}^h(x, Q^2) + 2(1 - y) x F_{2p(d)}^h(x, Q^2)}{(1 + (1 - y)^2) 2 x F_{1p(d)}^{hL}(x, Q^2) + 2(1 - y) F_{2p(d)}^{hL}(x, Q^2)}.
\]

In Eq. (1) \( F_{1h}, F_{2h} \) and \( F_{1L}, F_{2L} \) are the semi-inclusive and the usual nucleon structure function respectively, which are expressed in terms of the unpolarized parton densities and the fragmentation functions \( (F_{1h}, F_{2h}) \), and by the unpolarized parton densities \( (F_{1L}, F_{2L}) \).

Let’s start our discussion with the results of the fit to COMPASS deuteron data. In our fit we have used the \([y, x(Q^2), z] \) presentation of the data, where \( y = Q^2/2MEx \) is the fractional energy of the virtual photon, and \( M \) and \( E \) are the mass of the nucleon and the energy of the muon beam, respectively. The data on the multiplicities are distributed in five \( y \)-bins as functions of \( z \) at different fixed values of \((x, Q^2)\). The total number of the data points is 4834.

![Figure 1: Comparison of our NLO QCD results for COMPASS \( \pi^+ \) multiplicities with the data. The multiplicities computed with the DSS FFs are also shown.](image)

The total number of the data points is 4834.
Figure 2: Comparison of our NLO QCD results for COMPASS $\pi^-$ multiplicities with the data. The multiplicities computed with the DSS FFs are also shown.

The total number of the $\pi^+$ and $\pi^-$ data points is 398, 199 for $\pi^+$ and 199 for $\pi^-$ multiplicities. The errors used are quadratic combinations of the statistical and systematic errors. The number of free parameters, attached to the input parametrizations of the pion FFs $[D_{u}^{\pi^+}(z), D_{\bar{u}}^{\pi^+}(z), D_{g}^{\pi^+}(z)]$ at $Q^2 = 1 \text{ GeV}^2$ and determined from the fit, is 12. The assumption that all unfavored pion FFs are equal is used. For the unpolarized parton densities we use the NLO MRST’02 set of PDFs [6]. The charm contribution to the multiplicities is not taken into account. For the value of $\chi^2/DOF$ corresponding to the best fit to the data we obtain 283.12/386 = 0.73. An excellent description of the COMPASS pion data is achieved. The quality of the fit is illustrated for the $y_3$-bin [0.2-0.3] (see Fig. 1 for $\pi^+$ and Fig. 2 for $\pi^-$ multiplicities). In the figures are presented also the multiplicities at the COMPASS kinematics calculated using the DSS FFs (blue curves). The extracted pion FFs will be presented later and compared to those obtained from our fit to the HERMES data, as well as to some of the FFs sets available at present. Here we would like only to mention that it is obvious that the COMPASS data are in disagreement with the DSS FFs.

Let us discuss now our results on the pion FFs extracted from a NLO QCD fit to the HERMES proton and deuteron data on pion multiplicities, corrected for exclusive vector meson production [4]. In our analysis we have used the $[x, z]$ as well as the $[Q^2, z]$ presentation of the data. The pion multiplicities are given for 4 $z$-bins [0.2-0.3; 0.3-0.4; 0.4-0.6; 0.6-0.8] as functions of $x$ for the $[x, z]$ or functions of $Q^2$ for the $[Q^2, z]$ presentation. The total number of the $\pi^+$ and $\pi^-$ data points is 144. It turned out that we can not find a reasonable fit to the HERMES $[x, z]$ data. Also, there is a strong indication that the HERMES $[x, z]$ and COMPASS data are not consistent. We observe a big discrepancy between the values of the HERMES data on pion multiplicities and multiplicities at the
Figure 3: Comparison of HERMES \([x, z]\) proton data on \(\pi^+\) (left) and \(\pi^-\) multiplicities (right) with the multiplicities at the same kinematic points calculated by our FFs extracted from the COMPASS data (blue curves) and from HERMES \([Q^2, z]\) data (red curves).

Figure 4: Comparison of HERMES \([x, z]\) deuteron data on \(\pi^+\) (left) and \(\pi^-\) multiplicities (right) with the multiplicities at the same kinematic points calculated by our FFs extracted from the COMPASS data (blue curves) and from HERMES \([Q^2, z]\) data (red curves).
Figure 5: Comparison of HERMES $[Q^2, z]$ proton data on $\pi^+$ (left) and $\pi^-$ multiplicities (right) with the best fit curves. The blue curves correspond to the multiplicities at the same kinematic points calculated using our FFs extracted from the COMPASS data.

Figure 6: Comparison of HERMES $[Q^2, z]$ deuteron data on $\pi^+$ (left) and $\pi^-$ multiplicities (right) with the best fit curves. The blue curves correspond to the multiplicities at the same kinematic points calculated using our FFs extracted from the COMPASS data.
same kinematic points computed with our FFs extracted from the COMPASS data (see blue curves in Fig. 3 for proton and Fig. 4 for deuteron data).

We were very surprised to find that the situation is dramatically changed if the HERMES data on pion multiplicities in \([Q^2, z]\) presentation are used in the QCD analysis. In this case a reasonable fit to the data is achieved, \(\chi^2/DOF = 151.73/132 = 1.15\). The errors used in the fit are quadratic combinations of the statistical and point-to-point systematic errors. As in the COMPASS case: a) isospin symmetry for FFs is imposed, b) we assume that all unfavored pion FFs are equal and c) the same parametrizations for the input FFs are used in the analysis. We find that the description of the proton data (the mean value of \(\chi^2\) per point is equal to 0.96 for \(\pi^+\) and 0.70 for \(\pi^-\) multiplicities) is better than that of the deuteron data (where the mean value of \(\chi^2\) per point is equal to 1.25 for \(\pi^+\) and 1.31 for \(\pi^-\) multiplicities). The quality of the fit to the data is illustrated in Fig. 5 (for the proton target) and Fig. 6 (for the deuteron target).

Using the extracted FFs from the HERMES data on multiplicities in the \([Q^2, z]\) presentation we have calculated the multiplicities at the kinematic points for the \([x, z]\) presentation. The obtained value for \(\chi^2\) is huge, 2093.3 for 144 experimental points. The results are shown (red curves) in Fig. 3 for a proton and in Fig. 4 for a deuteron target. As seen from the figures, the discrepancy is very large for both the proton and deuteron targets for the first \(z\)-bin \([0.2-0.3]\), as well as at lowest \(x\), for all \(z\)-bins. It follows from this observation that the \([x, z]\) and \([Q^2, z]\) presentation of the HERMES data are not consistent and that the use of different presentations of the same data leads to different physical results. A further study of this unusual situation is urgently needed.

The extracted pion FFs from the fit to COMPASS data (blue curves) and from the fit to HERMES data on pion multiplicities (red curves) are presented in Fig. 7, and compared to those determined by DSS \([2]\) and HKNS (the 2nd reference in \([3]\)) in Fig. 8. Due to the visible difference in the \(z\) region \([0.4-0.6]\) between the favored fragmentation functions \(D^+_{\pi}\) extracted from HERMES and COMPASS data, and the large difference between the corresponding gluon FFs, the blue curves in Fig. 5 and Fig. 6 corresponding to the multiplicities at the HERMES \([Q^2, z]\) data points calculated by the FFs (COMPASS), lie systematically lower than the data points for the same \(z\)-bin. Combined fits to the

![Figure 7](image-url): Our FFs extracted from the fit to COMPASS data (blue curves) and HERMES \((Q^2, z)\) data (red curves).
COMPASS and HERMES $[Q^2, z]$ data on pion multiplicities will answer the important question if the discrepancy between the two data sets, shown in Figs. 5 and 6, will be removed, or more generally, if the HERMES $[Q^2, z]$ and COMPASS data are or are not consistent.

One can see from Fig. 8 that the new sets of pion FFs for the quarks are close to that of DSS. The differences, however, between $D_{g}^{\pi^+}$ corresponding to the different sets, are large. Also, for the DSS set the favored fragmentation function $D_{d}^{\pi^+}$ is larger than $D_{u}^{\pi^+}$ because in their analysis a violation of isospin symmetry was allowed. This is the main reason that the values of the multiplicities calculated by the DSS FFs for the COMPASS kinematics (blue curves in Figs 1 and 2) are systematically larger than the experimental values. The situation is the same for the HERMES data.

In conclusion, new sets of pion FFs are determined from the fits to the recent HERMES and COMPASS data on pion multiplicities. They differ from those of DSS and HKNS obtained before these data were available. There is a strong indication that the $[x, z]$ and $[Q^2, z]$ presentations of the HERMES data on the pion multiplicities are not equivalent and lead to different physical results, which suggests that there might be something wrong with the extraction of the data presentations from the measured experimental values. We find also that the COMPASS and HERMES $[x, z]$ data are not consistent. The situation about the consistency between the COMPASS and HERMES $[Q^2, z]$ data looks much better. Here the discrepancy is mainly for the third $z$-bin for the $\pi^+$ and for the second and third $z$-bins for the $\pi^-$ multiplicities. So, the important questions as to the consistency
between COMPASS and HERMES \([Q^2, z]\) data will depend on the results of a combined fit to the data, which is under way.

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