Recently, there has been a great deal of interest in \( p \)-wave superconductivity [1–26]. Besides UPt\(_3\), the first and most likely candidate \( p \)-wave superconductors are the ferromagnetic superconductors UGe\(_2\), UCoGe and URhGe, which exhibit long-range ferromagnetism well above the superconducting transition temperature \( T_c \), and the same electrons participate in the ferromagnetism and the superconductivity [1–7]. In URhGe, measurements of the temperature \( T \) dependence of the upper critical induction \( B_{c2}(T) \) in the three crystal axis directions were found to fit the Scharnberg–Klemm theory of the \( p \)-wave polar state with completely broken symmetry (CBS) [3, 8], with a single-component \( p_z \)-pairing state only along the crystal \( a \)-axis. Subsequent experiments found a re-entrant superconducting phase at much higher magnetic field \( H \) strengths, violating the conventional Pauli limit \( B_p = 1.85T_c \) (T/K) by a factor of 20. \( B_{c2} \) in UCoGe also violates \( B_p \) by a factor of 15+, though its anisotropy suggests that if the superconductivity were \( p \)-wave, it would be more likely to have an axial state form, as do the chiral ABM and chiral SK states [27–30]. Second, there has been an even greater interest in \( Sr_2RuO_4 \), since the Knight shift measurements for \( H \) parallel and perpendicular to the layers all showed no temperature \( T \) dependence below \( T_c \), suggestive of a parallel-spin state [11, 12]. However, \( B_{c2} \) experiments on that material appear to be strongly Pauli limited for \( H \) parallel to the layers [13–19], and scanning tunneling microscopy (STM) experiments showed strong evidence for a nodeless gap [20]. Third, there has been much interest recently in topological insulators in the hope that they might become chiral \( p \)-wave superconductors as a result of doping, applied pressure or proximity coupling [21–26]. The identification of the orbital symmetry of the order parameter (OP) in UCoGe, \( Sr_2RuO_4 \) and topological superconductors would be aided by accurate calculations of \( B_{c2}(\theta, \phi, T) \) for the ABM and SK states.
Previously, we generalized the microscopic calculation of $B_{c2}(T)$ for the $p$-wave polar state pinned to a crystal lattice direction to extend its validity to a superconductor with a dominant ellipsoidal FS and $B = B \sin(\theta \cos\phi, \sin\theta \sin\phi, \cos\theta) = V \times A$ in an arbitrary direction with respect to the crystal lattice, in order to provide a sound theoretical basis for a more sensitive probe of the actual OP in orthorhombic materials such as URhGe. Here we use the same technique to construct a theory of the full angular dependence of $B_{c2}(\theta, \phi, T)$ for the ABM and SK states in order to identify the symmetry of the OP in UCoGe, Sr$_2$RuO$_4$, Cu$_2$Bi$_2$Se$_3$ and other candidate materials. Since UCoGe is orthorhombic, the ellipsoidal FS model is the best that can be made without additional features such as magnetic pairing fluctuation effects and $B$ dependencies of the pairing interactions [31, 32], etc. For tetragonal Sr$_2$RuO$_4$, the lack of any detectable ferromagnetism strongly suggests weak coupling interactions, though there are three barrel shaped FSs. Although one could envision a scenario in which one FS dominated $B_{c2}(90^\circ, \phi, T)$ and another dominated $B_{c2}(0^\circ, \phi, T)$, since the latter is of primary interest, it suffices to consider only one FS. Moreover, as the $k_z$ dispersion of those bands is sufficient to avoid dimensional crossover effects in $B_{c2}(90^\circ, \phi, T)$ measurements [14], [15], [33–35], an ellipsoid of uniaxial anisotropy is sufficient to examine $B_{c2}$ measurements for all $B$ directions with high accuracy [33, 36]. As anticipated earlier, for a parallel-spin pairing interaction of the form $V(\hat{k}, \hat{k}^\prime) = 3V_0(\hat{k} \cdot \hat{k}^\prime + \hat{k}^\prime \cdot \hat{k})$, one would expect $B_{c2}(\theta, \phi, T)$ to be given by the SK state [9, 27]. Although a favorite pair state for Sr$_2$RuO$_4$ has the form $\hat{\xi}(\hat{k} + i\hat{k})$, where the $d$-vector $\hat{z}$ corresponds to the antiparallel-spin state in the lattice representation, we shall here assume that the spins are parallel [27], and will include Pauli limiting effects subsequently [36]. Here we present detailed calculations of the $B_{c2}(\theta, \phi, T)$ for both the ABM and SK states on a single ellipsoidal FS.

We assume weak coupling for a clean homogeneous type-II parallel-spin $p$-wave superconductor with effective Hamiltonian [9, 27],

$$H = \sum_{k,\sigma=\pm} c_{k,\sigma}^\dagger \{ \epsilon(k - eA) - \mu \} c_{k,\sigma} + \frac{1}{2} \sum_{k,\sigma=\pm} c_{k,\sigma}^\dagger c_{k,\sigma}^\dagger V(\hat{k}, \hat{k}^\prime) c_{k,\sigma} c_{-k,\sigma},$$

$$V(\hat{k}, \hat{k}^\prime) = \frac{3}{2} V_0 \sum_{\sigma=\pm} f_{\sigma}(\hat{k}) f_{\sigma}^\dagger(\hat{k}^\prime),$$

where $c_{k,\sigma}$ annihilates an electron with spin $\sigma$ and wave vector $k$, $\epsilon(k) = -\frac{1}{2} \sum_{i=1}^{3} k_i^2 / m_i$, we assume parallel-spin pairing with $f_{\sigma}(\hat{k}) = (\hat{k}_1 + i\sigma^\prime \hat{k}_2)$ from the degenerate $\Gamma_7$ and $\Gamma_4$ tetragonal point group representations [10], $e$ is the electronic charge, $\mu$ is the chemical potential, the unit wave vectors $\hat{k}$, $\hat{k}'$ were previously defined on an ellipsoidal FS [9], and we set $h = k_B = 1$.

For non-ferromagnetic candidate $p$-wave superconductors, $B_{c2} = \mu_0 H_{c2}$, where $H_{c2}$ is the upper critical field. After performing the Klemm–Clem (KC) transformations [37] that map the ellipsoidal FS onto a spherical one and then rotate and isotropically scale the transformed induction to the new $\hat{z}$ axis direction, the transformed linear gap equation becomes

$$\widetilde{\Delta}(\tilde{\mathbf{R}}, \tilde{\mathbf{k}}) = T \sum_{\alpha_1} \frac{N(0)}{2} \int d\Omega \tilde{\mathbf{V}}(\tilde{\mathbf{k}}, \tilde{\mathbf{k}}^\prime) \times \int_0^\infty d\xi \xi^2 \exp[-2|g| A \xi^2 \mu^2 / \hbar \tilde{\mathbf{k}} \cdot \nabla \tilde{\mathbf{R}} \cdot \nabla] \widetilde{\Delta}(\tilde{\mathbf{R}}, \tilde{\mathbf{k}}^\prime),$$

where $\Delta$ is the transformed $\Delta$ amplitude without the gauge phases [9], $N(0) = m k_B / (2 \pi^2)$ is the density of states per spin at $\mu$ for an

Figure 1. (a) Reduced $b_{c2} \equiv 2 \mu B_{c2} V^2 / (2 \pi T_c)^2$ versus $t = T/T_c$ for the chiral ABM state (equation (9)) at $\theta$ values from $0^\circ$ (H||\hat{e}, bottom) to $90^\circ$ (H\perp\hat{e}, top), in increments of $10^\circ$ for a spherical FS. (b) The same curves normalized to have the same slopes at $T_c$. 

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**Note:** The equations and figures in the text are not directly transcribed here due to the formatting constraints. The key points are captured in the narrative text, and additional details and derivations are referenced back to the original source for readers who require a deeper understanding.
effectively isotropic metal with a geometric mean effective mass \( m = (m_1 m_2 m_3)^{1/3} \), effective Fermi wave vector \( k_F = \sqrt{2m \gamma} \), effective Fermi velocity \( v_F = k_F m \), and \( \Pi(\mathbf{R}) = -i \alpha \mathbf{V}_0 + 2e \mathbf{A}(\mathbf{R}) \), where \( \alpha(\theta, \phi) = \sqrt{m_1 m_2} \cos^2 \theta + \gamma^2(\phi) \sin^2 \theta \), \( m_i = m / m \), and the ellipsoidal anisotropy function \([9]\). The KC transformations change \( V(\mathbf{k}, \mathbf{k'}) \) in equation (2) to

\[
\tilde{V}(\mathbf{k}, \mathbf{k'}) = \frac{3}{2} \chi_0 \sum_{n} \tilde{f}_n(\mathbf{k}) \tilde{f}^* n(\mathbf{k'})
\]

where \( \tilde{f}_n(\mathbf{k}) = \hat{k}_1 + i \alpha(\hat{k}_2 \cos \theta' + \hat{k}_3 \sin \theta') \), \( \cos \theta' = \sqrt{m_1 m_2} \cos \theta / \alpha \), etc \([9]\). From the form of \( \tilde{V}(\mathbf{k}, \mathbf{k'}) \), \( \mathbf{D}(\mathbf{k}, \mathbf{k'}) = \sum_{n} \mathbf{D}_n(\mathbf{k}) \tilde{f}^* n(\mathbf{k'}) \), we expand the \( \mathbf{D}_n(\mathbf{R}) \) in terms of the harmonic oscillator eigenfunctions \( |n(\mathbf{R})\rangle \), \( \mathbf{D}_n(\mathbf{R}) = \sum_{n=0}^{\infty} a_n^{(z)} |n(\mathbf{R})\rangle \), perform the integrals over the \( \hat{k}_i \) variables in the linearized gap equation, and obtain these double recursion relations for the \( a_n^{(z)} \),

\[
a_n^{(z)} = \left( \frac{1}{2} \left( 1 + \cos^2 \theta' \right) a_{n+2}^{(z)} + \frac{1}{2} \sin^2 \theta' a_{n-2}^{(z)} \right) a_n^{(z+1)}
+ \left( \frac{1}{2} \sin^2 \theta' a_{n+2}^{(z)} + \frac{1}{2} \left( 1 + \cos^2 \theta' \right) a_{n-2}^{(z)} \right) \alpha_n
+ \left( \frac{1}{2} \sin^2 \theta' a_{n+2}^{(z)} + \frac{1}{2} \left( 1 + \cos^2 \theta' \right) a_{n-2}^{(z)} \right) \alpha_n
+ \left( \frac{1}{2} \sin^2 \theta' a_{n+2}^{(z)} \right) \alpha_n^{(p)},
\]

where

\[
\alpha_n^{(p,z)} = \pi T \sum_{m=0}^{\infty} \int_{0}^{\pi} d\theta_k \sin \theta_k \left( \cos^2 \theta_k \left( \frac{3}{2} \sin^2 \theta_k \right) \right)
\times \int_{0}^{\pi} d\theta_k e^{-2i \Delta_m \eta_k} L_m(\eta_k),
\]

\[
\beta_n = -\pi T \sum_{m=0}^{\infty} \int_{0}^{\pi} d\theta_k \frac{3}{2} \sin \theta_k \int_{0}^{\pi} d\theta_k e^{-2i \Delta_m \eta_k} L_m(\eta_k)
\times [(n+1)(n+2)]^{-1/2},
\]

\[
\eta_k = eB(\theta, \phi) \nu \hat{z} \sin \theta \hat{z},
\]

\[
t = T/T_c, T_c = (2e^2 \omega_0/\pi) \exp(-1/N(0)V_0), \omega_0 \text{ is a cutoff frequency, } C = 0.5772, \text{ and } L_m(z) \text{ and } L_m^{(')}(z) \text{ are a Laguerre and an associated Laguerre polynomial, respectively \([9, 27]\).}
\]

For the chiral ABM state, \( \mathbf{D}(\mathbf{R}) = \sum_{n=0}^{\infty} a_n^{(z)} |n(\mathbf{R})\rangle \), the decoupled \( a_n^{(z)} \) each satisfy \( a_n^{(z)} D_n = \Gamma_n a_n^{(z)} + \Gamma_{n-2} a_n^{(z-2)} \), where \( \Gamma_n = \frac{1}{4} \sin^2 \theta' \beta_n \) and \( D_n = 1 - \frac{1}{2} (1 + \cos^2 \theta') \alpha_n^{(z)} - \frac{1}{2} \sin^2 \theta' \alpha_n^{(p,z)} \). Solving this recursion relation, we obtain the continued fraction expression from which \( b_{c2}(\theta, \phi) \) for the ABM state is obtained numerically,

\[
D_0 = \frac{\Gamma_0}{D_2 - \Gamma_2} = 0.
\]

As for the polar/CBS state \([9]\), one iteration is accurate to a few percent, but three or four iterations are needed for the accuracy necessary to observe the interesting effects.

The results for \( b_{c2}(\theta, t) \) for a parallel-spin superconductor in the p-wave ABM state with a dominant spherical \( \gamma^2(\phi) = 1 \text{ FS} \) are shown in figure 1. In figure 1(a), the curves for \( \theta = 0^\circ (B) \) (nodal direction) to \( 90^\circ (B \perp \hat{c}) \) (antinodal direction) are shown in increments of \( 10^\circ \). The result for the nodal direction \( \theta = 0^\circ \) was obtained previously \([27]\). Just below \( T_c \), \( b_{c2}(\theta, \phi, t) \propto m^2 \sin^2 \theta \gamma^2(\phi) \sin^2 \theta \), where the factor 2 arises from the ABM OP anisotropy. In order to identify which part of the overall \( b_{c2}(\theta, t) \) anisotropy is attributable solely to the
OP, in figure 1(b), those figure 1(a) results scaled to have the same slope at $t = 1$ are presented. Nothing unusual is evident from these spherical FS curves, and they are smooth and increase monotonically with increasing $\theta$.

However, we also studied the role of ellipsoidal (or uniaxial) FS anisotropy. In figure 2, we chose fixed FS anisotropy values $\gamma^2(\phi)$ ranging from 0.1 to 1.5 and plotted $b_{22}(\theta, t)/b_{22}(0, t)$ in figures 2 (a) and (b) at $t = 0$ and $\frac{1}{2}$, respectively. The solid curves are evaluated from equation (9). The dashed curves are the conventional ‘effective mass’ anisotropy $b_{22}(\theta, t)$ forms obtained by fitting the calculated $b_{22}(0^\circ, t)$ and $b_{22}(90^\circ, t)$.

$$b_{22}(\theta, t) = \left[ \cos^2 \theta / b_{22}(90^\circ, t) + \sin^2 \theta / b_{22}(0^\circ, t) \right]^{1/2}$$

We note that $b_{22}(\theta, t)$ exhibits an unusual $\theta$ dependence, with a peak at $\theta' = \gamma^2(\phi) < \frac{1}{2}$, where $0^\circ < \theta' < 90^\circ$, and by reflection symmetry about $90^\circ$, also for $90^\circ < \theta' < 180^\circ$, that is distinctly different than the conventional $b_{22}$ maxima at $\theta = 0^\circ$ or $90^\circ$.

Such anomalous double peaks at unconventional $\theta$ values were predicted earlier for the polar state pinned to the lattice [9]. However, in that case, the anomalous double peaks were predicted to occur for $\theta = \gamma^2(\phi) > 3$, with maximal $\lambda(t)$ values for finite $t$. Since the $\gamma^2(\phi) < \frac{1}{2}$, anomalous behavior is unlikely to be relevant to either Sr$_2$RuO$_4$ or UCoGe, for which $\gamma^2 \gg 1$, for brevity, the $\lambda(t)$ curve defining the lower limit of the range of $\theta'$ for $\lambda(t) < \gamma^2(\phi) < \frac{1}{2}$ will be presented elsewhere.

The much more interesting chiral axial $p$-wave state is the SK state. We note that it is chiral as long as $\Delta^{\pm}_n \Delta^{\pm}_n$ or $a^{\pm}_n \neq a^{\pm}_n$ for at least one relevant $n$ value [9]. It is easy to see that for $\theta = 0$, equation (5) reduces for $a^{\pm}_n \neq 0$ to $[1 - \alpha^{\pm}_n][1 - \alpha^{\pm}_n] = \beta^2$, which for $a^{\pm}_n \neq 0$ is the expression for the SK state $b_{22}$ with $B$ in the nodal direction [27], whereas for $\theta = \pi/2$, it reduces for $a^{\pm}_n \neq a^{\pm}_n$ to $\alpha^{\pm}_n = 1$, the expression for the SK state $b_{22}$ with $B$ in the antinodal (polar state) direction [27].

However, for a general $\theta'$, $a^{\pm}_n \neq a^{\pm}_n$, equation (5) is a double recursion relation in the $6$ harmonic oscillator amplitudes, $a^{(1)}_n$, $a^{(2)}_n$, and $a^{(3)}_n$, which requires further analysis to write the exact solution. We first write $\Psi^{(1)}_n = \frac{1}{2}(a^{(1)}_n \pm a^{(2)}_n)$, $\Delta^{(1)}_n = 1 - \alpha^{(1)}_n$, $D^{(1)}_n = 1 - \alpha^{(1)}_n \cos^2 \theta' - \alpha^{(1)}_n \sin^2 \theta'$, and construct $\phi^{(1)}_n = \cos \theta' \Delta^{(1)}_n \Psi^{(1)}_n \pm D^{(1)}_n \Psi^{(1)}_n$. After letting $n \rightarrow n + 2$ in the expression for $\phi^{(1)}_n$, we obtain two equations for $\Psi^{(1)}_n$ and $\Psi^{(1)}_{n+2}$ in terms of $\Psi^{(1)}_n$ and $\Psi^{(1)}_{n+2}$. Using these equations to eliminate $\Psi^{(1)}_n$ and $\Psi^{(1)}_{n+2}$ in favor of $\Psi^{(2)}_n$ and $\Psi^{(2)}_{n+2}$, letting $n \rightarrow n + 2$ in the expression for $\Psi^{(2)}_{n+2}$, and equating that with the other expression for $\Psi^{(2)}_{n+2}$, we obtain the simple recursion relation for the $\Psi^{(2)}_n$: $A_n \Psi^{(2)}_{n+2} + B_n \Psi^{(1)}_{n+4} + C_n \Psi^{(2)}_{n+4} = 0$, the solution of which may be expressed in the continued fraction equation.

$$B_0 = \frac{A_n C_n}{B_1 + B_3 C_n} = 0,$$

where $A_n = E_n - 3\beta_n [\cos^2 \theta' D^{(1)}_{n+2} - D^{(1)}_{n+2}]$, $B_n = E_n^{(1)} - B_n^{(1)}$, $C_n = E_n^{(1)} - B_n^{(1)}$, $B_n^{(1)} = E_n^{(1)} - B_n^{(1)}$, $E_n^{(1)} = D^{(1)}_{n+2} + D^{(1)}_{n+2}$, $E_n^{(1)} = D^{(1)}_{n+2} - D^{(1)}_{n+2}$. As for the polar/CBS state and the ABM state, one iteration is accurate to a few percent, but three or four iterations are necessary to display the most important features of this work. We also eliminated $\Psi^{(1)}_n$ and $\Psi^{(1)}_{n+2}$ in favor of $\Psi^{(2)}_n$ and $\Psi^{(2)}_{n+2}$, but the $b_{22}(\theta, \phi, t)$
values calculated from the resulting continued fraction equation were always lower than those calculated from equation (11).

In figure 3(a), we plotted the reduced $b_{c2}(t)$ for the nodal and antinodal directions of the ABM, SK and polar/CBS states, along with that (curve (4)) of a conventional $s$-wave superconductor without any Pauli limiting effects [38], all for a spherical FS. The antinodal directions of the polar and SK states both have $b_{c2}(t)$ curves described by curve (1), and the nodal direction of the SK state $b_{c2}(t)$ follows curve (2), as found previously [27]. Curve (3) is the new $b_{c2}(t)$ curve for the antinodal direction of the ABM state. Curves (5) and (6) describe the planar nodal polar/CBS state direction and the nodal direction of the ABM state, as also found previously [8].

We note that the SK state $b_{c2}(\theta, \phi, t)$ is larger for all field directions than is the ABM state $b_{c2}(\theta, \phi, t)$, as the second chiral component of the OP allows for the state to be superconducting at larger applied field strengths. In figure 3(b), the $t$ dependence of $b_{c2}(\theta, t)$ is illustrated for $\theta = 0^\circ (B \parallel \hat{c})$ (bottom) to $\theta = 90^\circ (B \perp \hat{c})$ (top), in increments of $10^\circ$. Surprisingly, the curves for $\theta = 0^\circ$, $10^\circ$, $20^\circ$, $30^\circ$ and $40^\circ$ are remarkably close to one another, and appear to cross at finite $t$ values! This is an indication of a chiral to non-chiral transition for $\theta \geq 40^\circ$ at various $t$ values, as the vortices just below $b_{c2}$ appear to lock onto the nodal direction for $\theta \leq 40^\circ$, but for $\theta > 40^\circ$ unlock from that direction, and favor the non-chiral antinodal (or polar) state direction. Similar behavior was predicted recently for the vortex structure in the mixed state of a chiral ABM state model of Sr$_2$RuO$_4$ [19].

To investigate this surprising feature in more detail, in figure 4 we show the $\theta$ dependence of $b_{c2}(\theta, \phi, t)$ at the effective mass anisotropy values $\gamma^2(\phi) = 0.1, 0.5, 1$, and $2$, at $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$.

In every case, there is a kink in $b_{c2}(\theta)$ at $\theta = \theta^*$, which we interpret as evidence for a first-order phase transition from a chiral to a non-chiral state. Although these
kinks are easiest to see for small $p^2$ values, and Sr$_2$RuO$_4$ has $p^2 > 10^3$, our high-accuracy solutions of equation (11) allow us to determine $\theta^* (\gamma(\phi), t)$ with great precision. In the inset to figure 3(b), we plotted $\theta^*$ in degrees versus $\log_{10}[\gamma(\phi)]$ from $-3$ to $3$ at the reduced $t$ values $0$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Thus, if Sr$_2$RuO$_4$ were a chiral $p$-wave parallel-spin superconductor as often purported, then one ought to observe a first-order chiral to non-chiral transition for $\theta = 90^\circ$, nearly parallel to the layers.

It is therefore quite interesting to note that some evidence for this sort of behavior may have already been observed in earlier and very recent $H_{c2}(T)$ measurement on Sr$_2$RuO$_4$ [17, 18]. However, a cautionary note is that $b_{c2}(90^\circ, \phi, t)$ appears to be strongly Pauli limited [13–16], and more details of such and other fits using this FS model will soon become available [36].

With regards to the ferromagnetic superconductor UCeGe, the ferromagnetism in the $c$-axis direction allows for an axial-type parallel-spin $p$-wave pairing interaction, most likely mediated by ferromagnetic exchange interactions, in the $ab$ plane. However, at large applied fields along the $b$-axis direction, not only does $B_{c2}(0)$ exceed the Pauli limit by a factor of at least 15, but the very strange behavior of $B_{c2}(T)$, including preliminary evidence for an S-shaped curve, strongly suggests something akin to a re-entrant superconducting phase overlapping the low-field phase, which would be similar to the two phases of URhGe. Fitting such behavior will require significant modifications to the theory, such as by including ferromagnetic fluctuations [31], field-dependent interactions [32], different FS shapes [39, 40], and two ferromagnetically split FSs, which modifications are currently under study [41]. Although it was originally thought that Cu$_2$Bi$_2$Se$_3$ might be a topological $p$-wave superconductor [21–25], recent STM measurements strongly suggest it is a classic $s$-wave bulk superconductor [26]. Hence, an axial $p$-wave topological superconductor is presently elusive, though this theory could help to identify a future candidate material.

In summary, we have studied the two most-common versions of an axially symmetric $p$-wave pair state, the ABM and SK states. For all induction $B$ directions and reduced temperatures $t = T/T_c$, the SK state upper critical induction $B_{c2}(\theta, \phi, t)$ exceeds that of the ABM state. Surprisingly, for $0 \leq \theta \leq \theta^*$, the only $\theta$-dependence of $B_{c2}(\theta, \phi, t)$ arises from effective mass anisotropy, but then $B_{c2}(\theta)$ exhibits a kink at $\theta^* [\gamma(\phi)]$. Hence, there appear to be two basic states evident in $B_{c2}(\theta, \phi, t)$: the nodal, chiral SK state for $\theta^* < \theta < \theta^*$ and $180^\circ - \theta^* < \theta < 180^\circ$, and the antinodal, non-chiral polar state for $\theta^* < \theta < 180^\circ - \theta^*$. With the uniaxial effective mass anisotropy of Sr$_2$RuO$_4$, $\theta^* = 90^\circ$, so the anti-nodal, non-chiral polar state region would be very narrow. This might provide a possible explanation of the first-order transition seen for $\theta = 90^\circ$ below 0.8 K $= 0.53 T_c$ [17, 18], although it could also be due to a Fulde–Ferrell–Larkin–Ovchinnikov transition, which typically occurs below $0.55 T_c$ [42]. In our calculations, this first-order chiral-to-non-chiral transition should occur for all $T < T_c$.

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