Sugawara form for AdS superstring

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Abstract

We show that the stress-energy tensor for a superstring in the AdS$_5 \times S^5$ background is written in a supersymmetric generalized “Sugawara” form. It is the “supertrace” of the square of the right-invariant current which is the Noether current satisfying the flatness condition. The Wess-Zumino term is taken into account through the supersymmetric gauge connection in the right-invariant currents, therefore the obtained stress-energy tensor is $\kappa$ invariant. The integrability of the AdS superstring provides an infinite number of the conserved “local” currents which are supertraces of the $n$-th power of the right-invariant current. For even $n$ the “local” current reduces to terms proportional to the Virasoro constraint and the $\kappa$ symmetry constraint, while for odd $n$ it reduces to a term proportional to the $\kappa$ symmetry constraint.

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1 Introduction

The conjectured duality between the type IIB superstring theory on the AdS$_5 \times S^5$ space (AdS superstring) and $D = 4$, $\mathcal{N} = 4$ Yang-Mills theory \cite{1,2,3} has been driven not only studies of variety of background theories but also studies of basic aspects such as integrability. The approach of the pp-wave background superstring theory \cite{4} was explored by Berenstein, Maldacena and Nastase \cite{5} and developed in, for example \cite{6,7}. For further development Mandal, Suryanarayan and Wadia pointed out the relevance with the integrability \cite{8}, and Bethe anzatz approach was explored by Minahan and Zarembo \cite{9} and in for example \cite{10,11,12}. The integrability is a powerful aspect expected in the large N QCD \cite{13} and shown to exist in the IIB superstring theory on the AdS$_5 \times S^5$ space by Bena, Polchinski and Roiban \cite{14}. The integrability provides hidden symmetry generated by an infinite number of conserved “non-local” charges \cite{15,16} as well as an infinite number of conserved “local” charges \cite{17} which are related by a spectral parameter at different points. Related aspects on the integrability of the AdS superstring were discussed in \cite{18}.

Recently the conformal symmetry of AdS superstrings was conjectured due to the $\kappa$ symmetry \cite{19}. The classical conformal symmetry of the AdS superstring theory also leads to an infinite number of conserved Virasoro operators. The naive questions are how the conformal generator is related to the infinite number of conserved “local” currents, and how many independent conserved currents exist. For principal chiral models the stress-energy tensor is written by trace of the square of the conserved flat current; for reviews see refs. \cite{20,8}. For the AdS superstring theory the Wess-Zumino term and the $\kappa$ symmetry make a difference. Recently issues related to the integrability and the conformal symmetry of the AdS superstring theory have been discussed \cite{21,22,23}. In this paper we will obtain the expression of the conformal generator, which is the stress-energy tensor relating to the lowest spin “local” current, and we calculate the higher spin “local” currents to clarify independent components.

The AdS space contains the Ramond/Ramond flux which causes difficulty of the standard Neveu-Schwarz-Ramond (NSR) formulation of the superstring theory. The AdS superstring was described in the Green-Schwarz (GS) formalism by Metsaev and Tseytlin based on the coset PSU(2,2$|$4)/[SO(4,1)$\times$SO(5)] \cite{24}. Later Roiban and Siegel reformulate it in terms of the unconstrained GL(4$|$4) supermatrix coordinate based on an alternative coset GL(4$|$4)/[Sp(4)$\times$GL(1)]$^2$ \cite{25}. In this formalism the local Lorentz is gauged, and it turns out that this treatment is essential for separation into $+/-$ modes (right/left moving modes) easier. Furthermore the fermionic constraint including the first class and second class is necessary for separation of the fermionic modes into $+/-$ modes. As the
first step toward the CFT formulation of the AdS superstring, the affine Sugawara construction \[26\], the Virasoro algebra and the algebra of currents carrying the space-time indices are also listed.

The organization of this paper is the following; in the next section the notation is introduced. In section 3 we analyze the superparticle in the \(\text{AdS}_5 \times S^5\) space, and the relation between the reparametrization constraint and the conserved right invariant (RI) current is given. In section 4 we analyze the superstring in the \(\text{AdS}_5 \times S^5\) space, and the infinite number of conserved currents are presented both from the conformal point of view and from the integrability point of view. We show that the stress-energy tensor is written by the “supertrace” of the square of the RI current as the lowest spin “local” current. Then we calculate higher spin “local” currents to clarify independent components of the “local” currents.

2 GL(4\(4)\) covariant coset

We review the Roiban-Siegel formulation of the \(\text{AdS}_5 \times S^5\) coset \[25\] and follow the notation in \[29\]. The coset \(\text{GL}(4|4)/[\text{GL}(1) \times \text{Sp}(4)]\) is used instead of \(\text{PSU}(2,2|4)/[\text{SO}(4,1) \times \text{SO}(5)]\) for the linear realization of the global symmetry after Wick rotations and introducing the auxiliary variables. A coset element \(Z_M^A\) is an unconstrained matrix defined on a world-volume carrying indices \(M = (m, \bar{m})\), \(A = (a, \bar{a})\) with \(m, \bar{m}, a, \bar{a} = 1, \ldots, 4\). The left invariant (LI) current, \(L^L\), is invariant under the left action \(Z_M^A \to \Lambda_M^N Z_N^A\) with a global parameter \(\text{GL}(4|4) \ni \Lambda\)

\[(J^L)_A^B = (Z^{-1} dZ)_A^B . \tag{2.1}\]

The LI current satisfies the flatness condition by definition

\[dJ^L = - J^L J^L . \tag{2.2}\]

The right invariant (RI) current, \(J^R\), is invariant under the right action \(Z_M^A \to Z_M^B \Lambda_B^A\) with a local parameter \([\text{Sp}(4) \otimes \text{GL}(1)]^2 \ni \lambda\)

\[(J^R)_M^N = (DZ \Lambda^{-1})_M^N , \quad (DZ)_M^A \equiv dZ_M^A + Z_M^B A_B^A \tag{2.3}\]

with

\[A \to \lambda A \lambda^{-1} + (d\lambda) \lambda^{-1} , \tag{2.4}\]

and

\[dJ^R = J^R J^R + Z( dA - AA) Z^{-1} . \tag{2.5}\]
Originally $A$ is bosonic $[\text{Sp}(4) \otimes \text{GL}(1)]^2 \ni A$, but we will show that the fermionic constraint i.e. $\kappa$ symmetry gives fermionic components of $A$.

The conjugate momenta are introduced

$$\{Z^A_M, \Pi_B^N\} = (-)^A \delta^A_B \delta^{\alpha N}_M$$

as the graded Poisson bracket and $\{q, p\} = -(-)^{qp}\{p, q\}$. There are also two types of differential operators; the global symmetry generator (left action generator), $G^N_M$, and the supercovariant derivatives (right action generator), $D^B_A$,

$$G^N_M = Z^A_M \Pi^N_A, \quad D^B_A = \Pi^M_A Z^B_M .$$

In our coset approach $8 \times 8 = 64$ variables for $Z^A_M$ are introduced and auxiliary variables are eliminated by the following constraints corresponding to the stability group $[\text{Sp}(4) \times \text{GL}(1)]^2$,

$$(D)_{(ab)} = (\bar{D})_{(\bar{a}\bar{b})} = \text{tr} \ D = \text{tr} \ \bar{D} \equiv 0 ,$$

where the bosonic components are denoted by boldfaced characters as $D_{ab} \equiv D_{ab}$ and $\bar{D}_{\bar{a}\bar{b}} \equiv D_{\bar{a}\bar{b}}$ of (2.7). The number of the coset constraints is $10 + 10 + 1 + 1 = 22$, so the number of the coset parameters is $64 - 22 = 42$ where 10 bosons and 32 fermions. The $[\text{Sp}(4)]^2$ invariant metric is anti-symmetric and a matrix is decomposed into trace part, anti-symmetric-traceless part and the symmetric part, denoted by

$$M_{ab} = -\frac{1}{4} \Omega^{ac} M^c_{(ab)} + M^c_{(ab)} + M_{(ab)} \equiv -\frac{1}{4} \Omega \text{ tr} M + \langle \text{M} \rangle + \langle \text{M} \rangle ,$$

with $M_{(ab)} = \frac{1}{2}(M_{ab} + M_{ba})$, and similar notation for the barred sector.

Both $G^N_M$ and $D^B_A$ in (2.7) satisfy $\text{GL}(4|4)$ algebra. If we focus on the AdS superalgebra part, the global symmetry generators $G^N_M$ satisfies the global AdS superalgebra

$$\{Q_{\alpha\alpha}, Q_{\beta\beta}\} = -2 \left[\tau_{3AB} P_{\alpha\beta} + \epsilon_{AB} M_{\alpha\beta}\right]$$

$$Q_{1\alpha} = G_{m\tilde{m}} + G_{\tilde{m}m}$$

$$Q_{2\alpha} = G_{m\tilde{m}} - G_{\tilde{m}m}$$

$$P_{\alpha\beta} = G_{(mn)} \Omega_{\tilde{m}\tilde{n}} - G_{(\tilde{m}\tilde{n})} \Omega_{mn} \cdots \text{total momentum}$$

$$M_{\alpha\beta} = -G_{(mn)} \Omega_{\tilde{m}\tilde{n}} + G_{(\tilde{m}\tilde{n})} \Omega_{mn} \cdots \text{total Lorentz}.$$
supersymmetry algebra is given by

\[
\{ d_{A\alpha}, d_{B,\beta} \} = 2 \left[ \tau_{3AB} \tilde{\alpha}_{\alpha \beta} + \epsilon_{AB} m_{\alpha \beta} \right]
\]  

\[ 
\begin{align*}
&d_{1\alpha} = D_{\dot{a}a} + \bar{D}_{\dot{a}a} \\
&d_{2\alpha} = D_{\dot{a}a} - \bar{D}_{\dot{a}a} \\
&\tilde{p}_{\alpha \beta} = D_{(ab)} \Omega_{\dot{a}b} - \bar{D}_{\langle \dot{a}b \rangle} \Omega_{\dot{a}b} \cdot \cdot \cdot \text{local LI momentum} \\
m_{\alpha \beta} = -D_{(ab)} \Omega_{\dot{a}b} + \bar{D}_{\langle \dot{a}b \rangle} \Omega_{\dot{a}b} \cdot \cdot \cdot \text{local Lorentz}
\end{align*}
\]

In our coset approach the local Lorentz generator is a constraint (2.8), so the local supercovariant derivative \( d_{A\alpha} \)'s can be separated into;

\[ 
\begin{align*}
&\{ d_{1\alpha}, d_{2\beta} \} = 2 m_{\alpha \beta} \equiv 0, \\
&\{ d_{1\alpha}, d_{1\beta} \} = 2 \bar{p}_{\alpha \beta}, \\
&\{ d_{2\alpha}, d_{2\beta} \} = -2 \bar{p}_{\alpha \beta}
\end{align*}
\]  

Although the global superalgebra can not be separated into irreducible algebras in the AdS background, the local superalgebra can be separated into irreducible sets on the GL(4|4) covariant coset approach. This property allows simpler description of the AdS superstring as the flat case at least in the classical mechanics level.

### 3 AdS Superparticle

We begin with the action for a superparticle in the AdS_5 × S^5

\[
S = \int d\tau \frac{1}{2e} \left\{ -J_r^{(ab)} J_r_{(ab)} + \tilde{J}_r^{(ab)} \tilde{J}_r_{(ab)} \right\}.
\]  

(3.1)

Here we omit the upper-subscript \( L \) for the LI currents and their components are denoted as

\[
(J^L_\mu)_A^B = \begin{pmatrix} J_{\mu,a}^b & \tilde{J}_{\mu,a}^b \\ \tilde{J}_{\mu,a}^b & \tilde{J}_{\mu,a}^b \end{pmatrix}.
\]  

(3.2)

From the definition of the canonical conjugates, \( \Pi_A^M = \delta S/\delta \partial_\tau Z_A^M(\cdot)^A \), we have the following primary constraints \[29\]

\[
A_P = \frac{1}{2} \text{tr} \left[ (D)^2 - (\tilde{D})^2 \right] = 0, \quad D_{\dot{a}b} = \tilde{D}_{\dot{a}b} = 0
\]  

(3.3)

with

\[
D_A^B = \begin{pmatrix} D_{a}^b & D_{a}^b \\ \tilde{D}_{\dot{a}}^\dot{b} & \tilde{D}_{\dot{a}}^\dot{b} \end{pmatrix}.
\]  

(3.4)
The Hamiltonian is chosen as
\[ H = -\mathcal{A}_p = \frac{1}{2} \text{tr} \left[ \langle \mathbf{D} \rangle^2 - \langle \bar{\mathbf{D}} \rangle^2 \right] \]
(3.5)
and the \( \tau \)-derivative is determined by the Poisson bracket with \( H \), \( \partial_\tau \mathcal{O} = \{ \mathcal{O}, H \} \). The fact that a half of the fermionic constraints is second class requires the Dirac bracket in general. Fortunately the Dirac bracket with the Hamiltonian is equal to its Poisson bracket because the fermionic constrains are \( H \) invariant.

The LI current is calculated as
\[ J^L_\tau = Z^{-1} \partial_\tau Z = \left( \begin{array}{cc} \langle \mathbf{D} \rangle & 0 \\ 0 & \langle \bar{\mathbf{D}} \rangle \end{array} \right) , \quad \partial_\tau J^L = 0 \]  
(3.6)
The RI current, generating the global \( \text{GL}(4|4) \) symmetry, is given as
\[ J^R_\tau \equiv Z \Pi = Z \left( J^L_\tau + A_\tau \right) Z^{-1} , \quad A_\tau = \left( \begin{array}{cc} \mathbf{D} - \frac{1}{4} \Omega \text{tr} \mathbf{D} & D \\ \bar{\mathbf{D}} & \mathbf{D} - \frac{1}{4} \Omega \text{tr} \bar{\mathbf{D}} \end{array} \right) \]  
(3.7)
Although the stability group does not contain fermionic components originally, the fermionic components of the gauge connection \( A \) in (3.7) is induced. It is noted that “\( A \)” is the gauge connection distinguishing from the reparametrization constraint “\( \mathcal{A} \)”. The RI current is conserved, since the Hamiltonian is written by LI currents which are manifestly global symmetry invariant
\[ \partial_\tau J^R = 0 \]  
(3.8)
The \( \kappa \) symmetry generators are half of the fermionic constraints by projecting out with the null vector as
\[ \mathcal{B}_{P_a}^b = \langle \mathbf{D} \rangle^b_a D^b_b + D^a_a \langle \bar{\mathbf{D}} \rangle^b_b , \quad \bar{\mathcal{B}}_{P\bar{a}}^b = \langle \bar{\mathbf{D}} \rangle^b_{\bar{a}} \bar{D}^b_b + \bar{D}^a_{\bar{a}} \langle \mathbf{D} \rangle^b_a \]  
(3.9)
If we construct the closed algebra including these \( \kappa \) generators with keeping the bilinear of the fermionic constraints, the \( \tau \)-reparametrization constraint, \( \mathcal{A}_P \), is modified to
\[ \tilde{\mathcal{A}}_P = \frac{1}{2} \text{tr} \left[ \langle \mathbf{D} \rangle^2 - \langle \bar{\mathbf{D}} \rangle^2 + 2D\bar{D} \right] \]  
(3.10)
This expression appears in the Poisson bracket of \( \mathcal{B} \) with \( \bar{\mathcal{B}} \), when we keep the bilinear of fermionic constrains. The RR flux is responsible for the last term “\( D\bar{D} \)”. The term which is bilinear of the constraints does not change the Poisson bracket since its bracket with an arbitrary variable gives terms proportional to the constraints which are zero on the constrained surface. In another word \( \mathcal{A}_P \) has an ambiguity of bilinear of the constraints,
and the \( \kappa \) invariance fixes it. On the original coset constrained surface \( \mathfrak{a} \) it is also rewritten as
\[
\tilde{A}_P = \frac{1}{2} \text{Str} \left[ D_A^R \right]^2 = \frac{1}{2} \text{Str} \left[ J^R \right]^2 .
\] (3.11)
This is zero-mode contribution of the classical Virasoro constraint for a superstring in the \( \text{AdS}_5 \times S^5 \) background.

4 AdS Superstring

4.1 Conserved currents

We take the action for a superstring in the \( \text{AdS}_5 \times S^5 \) given by
\[
S = \int d^2 \sigma \frac{1}{2} \left\{ -\sqrt{-g} g^{\mu \nu} (J^{(ab)}_{\mu} J_{\nu,(ab)} - J^{(ab)}_{\mu} J_{\nu,(\bar{a}b)}) + \frac{k}{2} \epsilon^{\mu \nu} (E^{1/2} j^a_{\mu} j_{\nu,ab} - E^{-1/2} \bar{j}^{\bar{a}b}_{\mu} j_{\nu,ab}) \right\}
\] (4.1)
where "\( k \)" represents the WZ term contribution with \( k = 1 \) and \( E = \text{sdet} Z_{M^4} \). The consistent \( \tau \) and \( \sigma \) reparametrization generators \( \mathfrak{a} \) are
\[
A_{\perp} = A_{0,\perp} + k \text{tr} \left[ -E^{1/4} F j_\sigma + E^{-1/4} \bar{F} j_\sigma \right]
\]
\[
A_{||} = A_{0||} + k \text{tr} \left[ E^{-1/4} \bar{F} j_\sigma - E^{1/4} F j_\sigma \right]
\] (4.2)
with the following primary constraints
\[
A_{0,\perp} = \frac{1}{2} \text{tr} \left[ (\langle D \rangle^2 + \langle J_\sigma \rangle^2) - (\langle \bar{D} \rangle^2 + \langle \bar{J}_\sigma \rangle^2) \right] = 0
\]
\[
A_{0||} = \text{tr} \left[ \langle D \rangle \langle J_\sigma \rangle - \langle \bar{D} \rangle \langle \bar{J}_\sigma \rangle \right] = 0
\] (4.3)
\[
F_{ab} = E^{1/4} D_{ab} + \frac{k}{2} E^{-1/4} (\bar{j}_{ba}) = 0
\]
\[
\bar{F}_{\bar{a}b} = E^{-1/4} \bar{D}_{\bar{a}b} + \frac{k}{2} E^{1/4} (j_{b\bar{a}}) = 0
\] (4.4)
Their Poisson brackets are
\[
\{ A_{\perp} (\sigma), A_{\perp} (\sigma') \} = 2 A_{||} (\sigma) \delta (\sigma - \sigma') + \partial_\sigma A_{||} (\sigma) \delta (\sigma - \sigma')
\]
\[
\{ A_{\perp} (\sigma), A_{||} (\sigma') \} = 2 A_{\perp} (\sigma) \delta (\sigma - \sigma') + \partial_\sigma A_{\perp} (\sigma) \delta (\sigma - \sigma')
\]
\[
\{ A_{||} (\sigma), A_{||} (\sigma') \} = 2 A_{||} (\sigma) \delta (\sigma - \sigma') + \partial_\sigma A_{||} (\sigma) \delta (\sigma - \sigma')
\] (4.5)

The Hamiltonian is chosen as
\[
\mathcal{H} = -\int d\sigma A_{\perp}
\]
\[
= -\int d\sigma \text{tr} \left[ \frac{1}{2} \left\{ \langle D \rangle^2 + \langle J_\sigma \rangle^2 - \langle \bar{D} \rangle^2 - \langle \bar{J}_\sigma \rangle^2 \right\} + \left( k E^{-1/2} \bar{D} j_{\sigma} - k E^{1/2} D j_\sigma + j_{\sigma} \bar{j}_{\sigma} \right) \right]
\] .

7
From now on $E = 1$ gauge is taken using the local GL(1) invariance. The global GL(1) symmetry is broken by the WZ term.

Using the Hamiltonian in (4.6) the $\tau$-derivative of $A_\perp$ and $A_\parallel$ are given as

$$\partial_\tau A_\perp = \partial_\sigma A_\parallel, \quad \partial_\tau A_\parallel = \partial_\sigma A_\perp.$$  \hspace{1cm} (4.7)

Although the coset parameter $Z_M^A$ does not satisfy the world-sheet free wave equation, it is essential to introduce the world-sheet lightcone coordinate

$$\sigma^\pm = \tau \pm \sigma, \quad \partial^\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma).$$  \hspace{1cm} (4.8)

The differential equations (4.7) are rewritten as

$$\partial^\pm A_\pm = 0, \quad \partial_\tau A_\perp = 2 D \nu + \frac{1}{2} \Omega_{\nu} \nu^\nu, \quad \partial_\tau A_\parallel = -\frac{1}{2} \Omega_{\nu} \nu^\nu + 2 D \nu + \frac{1}{4} \Omega_{\nu} \nu^\nu.$$  \hspace{1cm} (4.9)

so the infinite number of the conserved currents are

$$\partial^\pm \left[ \int d\sigma \ f(\sigma^\pm) A_\pm \right] = 0,$$  \hspace{1cm} (4.10)

with an arbitrary function $f$. Then there exist infinite number of conserved charges

$$\partial^\pm \left[ \int d\sigma \ f(\sigma^\pm) A_\pm \right] = 0.$$  \hspace{1cm} (4.11)

On the other hand the integrability of the superstring will provide the infinite number of “local” charges as well as the “non-local” charges written down in [30]. The LI currents is given by

$$J^L_{\tau} = \left(\begin{array}{cc} \langle D \rangle - k \vec{j}_\sigma \\ -k \vec{j}_\sigma \langle \bar{D} \rangle \end{array}\right) = \left(\begin{array}{cc} \langle D \rangle - 2 D \nu - 2 \bar{D} \nu + \frac{1}{2} \Omega_{\nu} \nu^\nu \\ 2 \bar{D} \nu + \frac{1}{2} \Omega_{\nu} \nu^\nu \end{array}\right) \approx \left(\begin{array}{cc} \langle D \rangle \\ 2 \bar{D} \nu \end{array}\right) \langle \bar{D} \rangle$$  \hspace{1cm} (4.12)

where the $\tau$ component is determined by (4.6). The LI currents satisfy the flatness condition but does not satisfy the conservation law. The RI currents are obtained in [30] as

$$J^R_{\tau} = Z D Z^{-1} = Z (J^L_{\tau} + A_\tau) Z^{-1}, \quad J^R_{\sigma} = Z (J^L_{\sigma} + A_\sigma) Z^{-1}, \quad J^L_{\sigma} + A_\sigma = \left(\begin{array}{cc} \langle J_\sigma \rangle + \frac{1}{2} \vec{j}_\sigma \\ F + \frac{1}{2} \vec{j}_\sigma \end{array}\right) \langle \bar{J}_\sigma \rangle$$  \hspace{1cm} (4.12)

where the gauge connection $A_\mu$ is

$$A_\tau = \left(\begin{array}{cc} \langle D \rangle - \frac{1}{4} \Omega_{\nu} \nu^\nu D \nu - \bar{D} \\ - \frac{1}{4} \Omega_{\nu} \nu^\nu \end{array}\right), \quad A_\sigma = \left(\begin{array}{cc} -\langle J_\sigma \rangle + \frac{1}{4} \Omega_{\nu} \nu^\nu J_\sigma \\ F + \frac{1}{2} \vec{j}_\sigma \end{array}\right) \langle \bar{J}_\sigma \rangle.$$
The fermionic components of $A_\mu$ appear again. In this paper the fermionic constraints, $F$ and $\bar{F}$, in the fermionic components of $A_\sigma$ are kept while they were absent in our previous paper [30] depending on the treatment of the constraint bilinear terms. Then the integrability of the superstring leads to the current conservation and the flatness condition for the RI current:

$$\partial_\tau J^R_\tau = \partial_\sigma J^R_\sigma , \quad \partial_\tau J^R_\sigma - \partial_\sigma J^R_\tau = 2 [J^R_\tau , J^R_\sigma] . \quad (4.13)$$

They are rewritten as

$$\partial_- J^R_+ = [J^R_-, J^R_+] , \quad \partial_+ J^R_- = [J^R_+, J^R_-] , \quad J^R_\pm = J^R_\tau \pm J^R_\sigma . \quad (4.14)$$

Taking the supertrace, denoting “Str”, leads to the infinite number of conserved “local” currents because $J^R_\mu$ are supermatrices,

$$\partial_- \text{Str} [(J^R_+)^n] = 0 , \quad \partial_+ \text{Str} [(J^R_-)^n] = 0 , \quad n = 1, 2, \cdots . \quad (4.15)$$

It gives the infinite number of conserved “local” charges

$$\partial_\tau \left[ \int d\sigma \ f(\sigma^+) \text{Str}(J^R_+)^n \right] = 0 , \quad \partial_\tau \left[ \int d\sigma \ f(\sigma^-) \text{Str}(J^R_-)^n \right] = 0 . \quad (4.16)$$

In this way classical 2-dimensional conformal symmetry and integrability of AdS superstring lead to two infinite sets of conserved currents, (4.10) and (4.15). In next sections the relation between them is examined.

### 4.2 Stress-energy tensor ($n = 2$)

The “+/−” (right/left moving) modes of the RI currents on the original coset constrained space (2.8) are written as

$$J^R_\pm = Z \left( \begin{array}{cc} \langle \mathbf{D}_\pm \rangle & D \pm \langle F + \frac{1}{2} j_\sigma \rangle \\ \bar{D} \pm \langle F + \frac{1}{2} \bar{j}_\sigma \rangle & \langle \bar{D}_\pm \rangle \end{array} \right) Z^{-1} = Z \left( \begin{array}{cc} \langle \mathbf{D}_\pm \rangle & d_\pm + \frac{1}{2} j_\pm \\ \pm (d_\pm - \frac{1}{2} j_\pm) & \langle \bar{D}_\pm \rangle \end{array} \right) Z^{-1} \quad (4.17)$$

with

$$\mathbf{D}_\pm = D \pm J_\sigma , \quad \bar{\mathbf{D}}_\pm = \bar{D} \pm \bar{J}_\sigma , \quad d_\pm = F \pm \bar{F} , \quad j_\pm = j_\tau \pm j_\sigma = -\bar{j}_\sigma \pm j_\sigma \quad (4.18)$$

carrying the LI currents indices, $AB$. This is supertraceless, Str$J^R_\pm = 0$, so $n = 1$ case of (4.15) gives just trivial equation.
Let us look at the $n = 2$ case of (4.15), $\text{Str} \left[ (J_R^2)^2 \right]$. Then the “+” sector is written as

$$
\frac{1}{2} \text{Str} \left[ (J_R^2)^2 \right] = \frac{1}{2} \text{Str} \left[ \left( \begin{array}{cc}
\langle D_+ \rangle & d_+ + \frac{1}{2} j_+
\end{array} \right)^2 \begin{array}{c}
d_+ - \frac{1}{2} j_+
\langle D_+ \rangle
\end{array} \right]
= \frac{1}{2} \text{tr} \left[ (\langle D_+ \rangle)^2 - (\langle D_+ \rangle)^2 + 2(d_+ + \frac{1}{2} j_+)(d_+ - \frac{1}{2} j_+) \right]
= \text{tr} \left[ \frac{1}{2} \left( (\langle D_+ \rangle)^2 - (\langle D_+ \rangle)^2 + j_+ d_+ \right) \right].
$$

(4.19)

The “−” sector is

$$
\frac{1}{2} \text{Str} \left[ (J_R^2)^2 \right] = \text{tr} \left[ \frac{1}{2} \left( (\langle D_- \rangle)^2 - (\langle D_- \rangle)^2 - j_- d_- \right) \right].
$$

(4.20)

On the other hand the conformal symmetry generator $A_\pm$ is rewritten from the relation (4.2) and (4.18) as

$$
A_\pm = \text{tr} \left[ \frac{1}{2} \left( (\langle D_\pm \rangle)^2 - (\langle D_\pm \rangle)^2 \right) \pm j_\pm d_\pm \right] = \frac{1}{2} \text{Str} \left[ (J_\pm^R)^2 \right].
$$

(4.21)

If we take care of the square of the fermionic constraints, the closure of the first class constraint set including the $\kappa$ symmetry generators,

$$
B_\pm = \langle D_\pm \rangle d_\pm + d_\pm \langle D_\pm \rangle
$$

(4.22)

determines the ambiguity of bilinear of the constraints as

$$
\tilde{A}_\pm = \text{tr} \left[ \frac{1}{2} \left( (\langle D_\pm \rangle)^2 - (\langle D_\pm \rangle)^2 \right) \pm (\frac{1}{2} d_\pm + j_\pm) d_\pm \right] = A_\pm + \text{tr} F \bar{F}
$$

(4.23)

obtained in [29] as a generator of the $\mathcal{A}BCD$ constraint set known to exist for a superstring in a flat space [27, 28]. Then the stress-energy tensor is

$$
T_{\pm\pm} \equiv \tilde{A}_\pm \approx A_\pm = \text{Str} J_\pm^R J_\pm^R.
$$

(4.24)

This is $\kappa$ symmetric stress-energy tensor in a supersymmetric generalization of Sugawara form.

### 4.3 Supercovariant derivative algebra

Existence of the conformal invariance should present the irreducible coset components of supercovariant derivatives [29];

$$
\langle D_\pm \rangle = \langle D \rangle \pm \langle J_\sigma \rangle,
\langle \bar{D}_\pm \rangle = \langle \bar{D} \rangle \pm \langle \bar{J}_\sigma \rangle
$$

$$
d_\pm = F \pm \bar{F} = (D \pm \frac{1}{2} j_\sigma) \pm (\bar{D} \pm \frac{1}{2} \bar{j}_\sigma).
$$

10
On the constraint surface (2.28) and (4.4) the $+/ -$ sector supercovariant derivatives are separated as

\[
\{ \langle \mathbf{D}_+ \rangle_{ab}(\sigma), \langle \mathbf{D}_- \rangle_{cd}(\sigma') \} = 2\Omega_{c|\hat{b}}(\mathbf{D})_{a|\hat{d}}\delta(\sigma - \sigma') \equiv 0
\]
\[
\{ \{ \mathbf{D}_+ \rangle_{ab}(\sigma), \langle \mathbf{D}_- \rangle_{cd}(\sigma') \} = \Omega_{c|\hat{b}}\hat{d}\delta(\sigma - \sigma') \equiv 0
\]
\[
\{ d_{+,ab}(\sigma), d_{-,cd}(\sigma') \} = 2[\Omega_{ac}(\mathbf{D})_{bd} + \Omega_{bd}(\mathbf{D})_{ac}]\delta(\sigma - \sigma') \equiv 0
\]

with analogous relation for the barred sector, $\langle \hat{\mathbf{D}} \rangle_ \pm$.

The “$+$” sector supercovariant derivative algebra is

\[
\{ \langle \mathbf{D}_+ \rangle_{ab}(\sigma), \langle \mathbf{D}_+ \rangle_{cd}(\sigma') \} = 2\Omega_{c|\hat{b}}(\mathbf{D})_{a|\hat{d}}\delta(\sigma - \sigma') + 4\Omega_{c|\hat{b}}(\mathbf{J}_\sigma)_{a|\hat{d}}\delta(\sigma - \sigma')
\]
\[
\{ d_{+,ab}(\sigma), d_{+,cd}(\sigma') \} = 2[\Omega_{bd}(\mathbf{D})_{ac} - \Omega_{ac}(\mathbf{D})_{bd}]\delta(\sigma - \sigma')
\]
\[
\{ \langle \mathbf{D}_+ \rangle_{ab}(\sigma), \langle \mathbf{D}_+ \rangle_{cd}(\sigma') \} = \Omega_{c|\hat{b}}(\hat{d} + 2\hat{j}_+)_{a|\hat{d}}\delta(\sigma - \sigma') \approx 2\Omega_{c|\hat{b}}\hat{d}\omega_{+,a}\delta(\sigma - \sigma')
\]
\[
\{ d_{+,ab}(\sigma), \omega_{+,cd}(\sigma') \} = -2\Omega_{bd}\Omega_{ac}\delta(\sigma - \sigma')2[-\Omega_{bd}(\mathbf{J}_\sigma)_{ac} - \Omega_{ac}(\mathbf{J}_\sigma)_{bd}]\delta(\sigma - \sigma')
\]
\[
\equiv -2\nabla_{bd;ac}\delta(\sigma - \sigma')
\]

(4.25)

\[
\{ \langle \mathbf{D}_+ \rangle_{ab}(\sigma), \omega_{+,cd}(\sigma') \} = \Omega_{c|\hat{b}}\omega_{-,a}\delta(\sigma - \sigma')
\]
\[
\{ \omega_{+,ab}(\sigma), \omega_{+,cd}(\sigma') \} = 0
\]

where

\[
\omega_{\pm} = j_{\pm} = -\hat{j}_\sigma \pm j_\sigma
\]

(4.26)

This is comparable with the flat case where the non-local term, $\partial_\sigma\delta(\sigma - \sigma')$, is replaced by the local Lorentz ( $[\mathrm{Sp}(4)]^2$ ) covariant non-local term, $\nabla_\sigma\delta(\sigma - \sigma')$. For the fifth Poisson bracket, \{ $\langle \mathbf{D}_+ \rangle$, $\omega$ \}, it is zero for the flat case but it is not for the AdS case. For a superstring in a flat space the consistency of the $\kappa$ symmetry constraint requires the first class constraint set, namely “ABCD” constraint, which are bilinear of the supercovariant derivatives (27) [28]. For the AdS case the situation is completely the same, despite of this anomalous term (29).

4.4 “Local” currents ($n \geq 3$)

Next let us look at $n \geq 3$ cases of the infinite number of conserved “local” current (1.15). For simplicity we focus on the “$+$” sector and replace “$+$” by “$^+$”, as $J_+ \rightarrow \hat{J}$. The first three powers of the RI current, $(J^R)^n$ with $n = 1, 2, 3$, are listed as below:

\[
[Z^{-1}\hat{j}^R Z]_{AB} = \begin{pmatrix}
\langle \hat{\mathbf{D}} \rangle_{ab} & (\hat{d} + \frac{1}{j})_{ab} \\
\pm(\hat{d} - \frac{1}{j})_{ba} & \langle \hat{\mathbf{D}} \rangle_{ab}
\end{pmatrix}
\]

(4.27)
\[
\left[Z^{-1}(\hat{J}R)^2Z\right]_{AB} = -\frac{1}{4} \left( \Omega_{ab} \text{ tr}(\langle \hat{D} \rangle^2 + \hat{j} \hat{d}) \right) - \Omega_{\hat{a}\hat{b}} \text{ tr}(\langle \hat{D} \rangle^2 - \hat{j} \hat{d}) \right)
+ \left( \langle \hat{d}^2 - \frac{1}{4} \hat{j}^2 \rangle_{(ab)} + \langle \hat{j} \hat{d} \rangle_{(ab)} \right) \hat{B}_{ab} + \frac{1}{2} \left( \langle \hat{D} \rangle \hat{j} + \hat{j} \langle \hat{D} \rangle \right)_{\hat{a}\hat{b}}
- \left( \langle \hat{d}^2 - \frac{1}{4} \hat{j}^2 \rangle_{(ab)} \right)_{\hat{a}\hat{b}}
\] (4.28)

\[
\left[Z^{-1}(\hat{J}R)^3Z\right]_{AB} = -\frac{1}{4} \left( \Omega_{ab} \text{ tr}[\hat{B}_j - (\langle \hat{D} \rangle \hat{d})_j] \right) - \Omega_{\hat{a}\hat{b}} \text{ tr}[\hat{B}_j - (\langle \hat{D} \rangle \hat{d})_j] \right)
- \frac{1}{4} \text{ tr}(\langle \hat{D} \rangle^2 - \langle \hat{D} \rangle \hat{j}) \hat{D} - \hat{j} \hat{d} \hat{D} - \hat{B}_j \right)_{(ab)}
\] (4.29)

In this computation 5-dimensional \(\gamma\)-matrix relations are used, for example \(V^{(ab)}U_{(bc)} + U^{(ab)} V_{(bc)} = \frac{1}{2} \delta^{a}_{c} \text{ tr} VU\) for bosonic vectors \(V, U\).

The conserved “local” current with \(n = 3\) becomes
\[
\text{Str}(\hat{J}R)^3 = \text{ tr} \left[ 2 \hat{B}_j - (\langle \hat{D} \rangle \hat{d})_j - (\langle \hat{D} \rangle \hat{d})_j \right] = \text{ tr} (\hat{B}_j) \] (4.30)

where \(\hat{B}\) is the \(\kappa\) generating constraint \(\text{M}2\). The conserved “local” current with \(n = 4\) becomes
\[
\text{Str}(\hat{J}R)^4 = -\frac{1}{2} \text{ tr} \left( \langle \hat{D} \rangle^2 + \langle \hat{D} \rangle \right) \hat{A} + (\cdots) \text{ tr} (\hat{B}_j) . \] (4.31)

The conserved “local” current with \(n = 5, 6\) are given as; \(\text{Str}(\hat{J}R)^5 = (\hat{B}\) dependent terms), \(\text{Str}(\hat{J}R)^6 = (\hat{A}\) and \(\hat{B}\) dependent terms). In general for even \(n = 2m\) its bosonic part is given as
\[
\text{Str}(\hat{J}R)^{2m} |_{\text{bosonic}} = \left( \text{ tr}(\langle \hat{D} \rangle^2) \right)^m - \left( \text{ tr}(\langle \hat{D} \rangle \hat{j}) \right)^m
= \text{ tr} \left( \langle \hat{D} \rangle^2 - \langle \hat{D} \rangle \hat{j} \right) \left\{ \left( \text{ tr}(\langle \hat{D} \rangle^2) \right)^{m-1} + \cdots + \left( \text{ tr}(\langle \hat{D} \rangle \hat{j}) \right)^{m-1} \right\}
\Rightarrow (\cdots) \hat{A} + (\cdots) \text{ tr} (\hat{B}_j) \] (4.32)

where the last equality is guaranteed by the \(\kappa\) invariance. It is also pointed out that the conserved supertraces of multilinearrs in the currents factorize in traces of lower number.
of currents and that for an even number of currents one of the factors is the stress tensor in \[13\]. For odd \( n = 2m + 1 \) its bosonic part is given as

\[
\text{Str}(J^R)^{2m+1} |_{\text{bosonic}} = 0 \Rightarrow (\cdots) \text{tr} (\hat{B}j)
\] (4.33)

where the possible fermionic variable dependence is a term proportional to \( \hat{B} \) guaranteed by the \( \kappa \) invariance.

In this way, after taking supertrace the even \( n \)-th power of \( J^R \) reduces terms proportional to \( \mathcal{A} \) and \( \mathcal{B} \), and the odd \( n \)-th power of \( J^R \) reduces a term proportional to \( \mathcal{B} \) only. In this paper \( CD \) constraints in the \( ABCD \) first class constraint set are not introduced for simpler argument, and set to zero because they are bilinears of constraints.

5 Conclusion and discussions

We obtained the expression of the conserved “local” currents derived from the integrability of a superstring in the \( \text{AdS}_5 \times \text{S}^5 \) background. The infinite number of the conserved “local” currents are written by the supertrace of the \( n \)-th power of the RI currents. The lowest nontrivial case, \( n = 2 \), is nothing but the stress-energy tensor which is also Virasoro constraint, \( \text{Str}(J^R)^2 \) in \[4.19\] and \[4.20\]. For even \( n \) the “local” current reduces to terms proportional to the Virasoro constraint and the \( \kappa \) symmetry constraint. For odd \( n \) it reduces to a term proportional to the \( \kappa \) symmetry constraint. In another word the integrability reduces to the \( \mathcal{AB}(CD) \) first class constraint set where \( \mathcal{A} \) is the Virasoro generator and \( \mathcal{B} \) is the \( \kappa \) symmetry generator. The \( ABCD \) first class constraint set is the local symmetry generator of superstrings both on the flat space and on the AdS space. It is natural that the physical degrees of freedom of a superstring is common locally, independently of flat or AdS backgrounds.

It seems that the combination of the \( B_\pm j_\pm \) in \[4.30\] plays the role of the world-sheet supersymmetry operator in a sense of the grading of the conformal generator. However it is not straightforward to construct the worldsheet supersymmetry operator. As in the flat case where the lightcone gauge makes the relation between the GS fermion and the NSR fermion more transparent, the \( \kappa \) gauge fixing will be a clue to make a connection to the world-sheet supersymmetry. We leave this problem in addition to the quantization problem for future investigations.

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References

[1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] arXiv:hep-th/9711200.

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428 (1998) 105 arXiv:hep-th/9802109.

[3] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253 arXiv:hep-th/9802150.

[4] R. R. Metsaev, “Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background,” Nucl. Phys. B 625 (2002) 70 arXiv:hep-th/0112044; R. R. Metsaev and A. A. Tseytlin, “Exactly solvable model of superstring in plane wave Ramond-Ramond background,” Phys. Rev. D 65 (2002) 126004 arXiv:hep-th/0202109.

[5] D. Berenstein, J. M. Maldacena and H. Nastase, “Strings in flat space and pp waves from N = 4 super Yang Mills,” JHEP 0204 (2002) 013 arXiv:hep-th/0202021.

[6] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “A semi-classical limit of the gauge/string correspondence,” Nucl. Phys. B 636 (2002) 99 arXiv:hep-th/0204051.

[7] S. Frolov and A. A. Tseytlin, “Semiclassical quantization of rotating superstring in AdS5 × S5,” JHEP 0206 (2002) 007 arXiv:hep-th/0204226.

[8] G. Mandal, N. V. Suryanarayana and S. R. Wadia, “Aspects of semiclassical strings in AdS5,” Phys. Lett. B 543 (2002) 81 arXiv:hep-th/0206103.

[9] J. A. Minahan and K. Zarembo, “The Bethe-ansatz for N = 4 super Yang-Mills,” JHEP 0303 (2003) 013 arXiv:hep-th/0212208.

[10] N. Beisert, “The complete one-loop dilatation operator of N = 4 super Yang-Mills theory,” Nucl. Phys. B 676 (2004) 3 arXiv:hep-th/0307015.

[11] L. Dolan, C. R. Nappi and E. Witten, “A relation between approaches to integrability in superconformal Yang-Mills theory,” JHEP 0310 (2003) 017 arXiv:hep-th/0308089; “Yangian symmetry in D = 4 superconformal Yang-Mills theory,” Contributed to 3rd International Symposium on Quantum Theory and Symmetries (QTS3), Cincinnati, Ohio, 10-14 Sep 2003. arXiv:hep-th/0401243.
[12] N. Beisert, S. Frolov, M. Staudacher and A. A. Tseytlin, “Precision spectroscopy of AdS/CFT,” JHEP 0310 (2003) 037 arXiv:hep-th/0308117.

[13] L.N. Lipatov, ‘High-energy asymptotics of multicolor QCD and exactly solvable lattice models,” JETP Lett. 59 (1994) 596 arXiv:hep-th/9311037; G. Korchemsky and L. Faddeev, ‘High-energy QCD as a completely integrable model,” Phys. Lett. B342 (1995) 311 arXiv:hep-th/9404173.

[14] I. Bena, J. Polchinski and R. Roiban, “Hidden symmetries of the AdS5 x S5 superstring,” Phys. Rev. D 69 (2004) 046002 arXiv:hep-th/0305116.

[15] M. Lüscher, “Quantum nonlocal charges and absence of particle production in the two-dimensional nonlinear sigma model,” Nucl. Phys. B 135 (1978) 1;
M. Lüscher and K. Pohlmeyer, “Scattering of massless lumps and nonlocal charges in the two-dimensional classical nonlinear sigma model,” Nucl. Phys. B 137 (1978) 46.

[16] E. Brezin, C. Itzykson, J. Zinn-Justin and J. B. Zuber, “Remarks about the existence of nonlocal charges in two-dimensional models,” Phys. Lett. B 82 (1979) 442.

[17] A. M. Polyakov, “Hidden symmetry of the two-dimensional chiral fields,” Phys. Lett. B72 (1977) 224;
Y.Y. Goldschmidt and E. Witten, “Conservation laws in some two-dimensional models,” Phys. Lett. B91 (1980) 392.

[18] B. C. Vallilo, “Flat currents in the classical AdS(5) x S**5 pure spinor superstring,” JHEP 0403 (2004) 037 arXiv:hep-th/0307018; J. Engquist, “Higher conserved charges and integrability for spinning strings in AdS(5) x S**5,” JHEP 0404 (2004) 002 arXiv:hep-th/0402092; I. J. Swanson, “On the integrability of string theory in AdS(5) x S**5,” arXiv:hep-th/0405172 “Quantum string integrability and AdS/CFT,” Nucl. Phys. B 709 (2005) 443 arXiv:hep-th/0410282; “Superstring holography and integrability in AdS(5) x S**5,” arXiv:hep-th/0505028
N. Berkovits, “BRST cohomology and nonlocal conserved charges,” JHEP 0502 (2005) 060 arXiv:hep-th/0409159; “Quantum consistency of the superstring in AdS(5) x S**5 background,” JHEP 0503 (2005) 041 arXiv:hep-th/0411170; G. Arutyunov and S. Frolov, “Integrable Hamiltonian for classical strings on AdS(5) x S**5,” JHEP 0502 (2005) 059 arXiv:hep-th/0411089; A. Mikhailov, “Anomalous dimension and local charges,” arXiv:hep-th/0411178; A. Das, J. Maharana, A. Melikyan and M. Sato, “The algebra of transition matrices for the AdS(5) x S**5 superstring,” JHEP 0412 (2004) 055 arXiv:hep-th/0411200; L. F. Alday, G. Arutyunov and A. A. Tseytlin, “On integrability of classical superstrings in AdS(5) x S**5,” JHEP 0507 (2005) 002 arXiv:hep-th/0502240; C. A. S. Young, “Non-local
charges, Z(m) gradings and coset space actions,” arXiv:hep-th/0503008; B. Chen, Y. L. He, P. Zhang and X. C. Song, “Flat currents of the Green-Schwarz super-strings in AdS(5) x S**1 and AdS(3) x S**3 backgrounds,” Phys. Rev. D 71 (2005) 086007 [arXiv:hep-th/0503089]; G. Arutyunov and M. Zamaklar, “Linking Baecklund and monodromy charges for strings on AdS(5) x S**5,” JHEP 0507 (2005) 026 [arXiv:hep-th/0504144]; T. McLoughlin and I. J. Swanson, “Open string integrability and AdS/CFT,” Nucl. Phys. B 723 (2005) 132 [arXiv:hep-th/0504203]; N. Drukker and B. Fiol, “On the integrability of Wilson loops in AdS(5) x S**5: Some periodic ansatze,” arXiv:hep-th/0506058; J. Plefka, “Spinning strings and integrable spin chains in the AdS/CFT correspondence,” arXiv:hep-th/0507136; S. Schafer-Nameki, M. Zamaklar and K. Zarembo, “Quantum corrections to spinning strings in AdS(5) x S**5 and Bethe ansatz: A comparative study,” arXiv:hep-th/0507189; T. G. Erler and N. Mann, “Integrable open spin chains and the doubling trick in N = 2 SYM with fundamental matter,” arXiv:hep-th/0508064; L. F. Alday, G. Arutyunov and S. Frolov, “New integrable system of 2dim fermions from strings on AdS(5) x S**5,” arXiv:hep-th/0508140; A. Das, A. Melikyan and M. Sato, “The algebra of flat currents for the string on AdS(5) x S**5 in the light-cone gauge,” arXiv:hep-th/0508183; N. Mann and J. Polchinski, “Bethe ansatz for a quantum supercoset sigma model,” Phys. Rev. D 72 (2005) 086002 [arXiv:hep-th/0508232].

[19] A.M. Polyakov, “Conformal fixed points of unidentified gauge theories”, Mod. Phys. Lett. A19 (2004) 1649, [hep-th/0405106].

[20] J. M. Evans, N. J. MacKay and M. Hassan, “Conserved charges and supersymmetry in principal chiral models,” arXiv:hep-th/9711140; J. M. Evans, M. Hassan, N. J. MacKay and A. J. Mountain, “Local conserved charges in principal chiral models,” Nucl. Phys. B 561 (1999) 385 [arXiv:hep-th/9902008]; “Conserved charges and supersymmetry in principal chiral and WZW models,” Nucl. Phys. B 580 (2000) 605 [arXiv:hep-th/0001222].

[21] N. Mann and J. Polchinski, “Finite density states in integrable conformal field theories,” arXiv:hep-th/0408162.

[22] J. Maharana, “NSR string in AdS(3) backgrounds: Nonlocal charges,” arXiv:hep-th/0501162.

[23] L. F. Alday, G. Arutyunov and A. A. Tseytlin, “On integrability of classical super-strings in AdS(5) x S**5,” arXiv:hep-th/0502240.

[24] R. R. Metsaev and A. A. Tseytlin, “Type IIB superstring action in AdS_5 x S_5 background,” Nucl. Phys. B 533 (1998) 109 [arXiv:hep-th/9805028].
[25] R. Roiban and W. Siegel, “Superstrings on AdS(5) x S(5) supertwistor space,” JHEP 0011, 024 (2000) arXiv:hep-th/0010104.

[26] M. B. Halpern, “Recent progress in irrational conformal field theory,” arXiv:hep-th/9309087;
M. B. Halpern, E. Kiritsis, N. A. Obers and K. Clubok, “Irrational conformal field theory,” Phys. Rept. 265 (1996) 1 arXiv:hep-th/9501144.

[27] W. Siegel, ‘Classical Superstring Mechanics,” Nucl. Phys. B263 (1985) 93; “The Superparticle Revisited,” Phys. Lett. B203 (1988) 79; “Introduction To String Field Theory,” Adv. Ser. Math. Phys. 8 (1988) 1.

[28] F. Essler, E. Laenen, W. Siegel and J.P. Yamron, “Brst Operator For The First Ilk Superparticle,” Phys. Lett. B254 (1991) 411;
F. Essler, M. Hatsuda, E. Laenen, W. Siegel and J.P. Yamron, “Covariant Quantization Of The First Ilk Superparticle,” Nucl. Phys. B364 (1991) 67.

[29] M. Hatsuda and K. Kamimura, “Classical AdS superstring mechanics,” Nucl. Phys. B611 (2001) 77, arXiv:hep-th/0106202.

[30] M. Hatsuda and K. Yoshida, “Classical integrability and super Yangian of superstring on AdS(5) x S**5,” to appear Adv. Theor. Math. Phys., arXiv:hep-th/0407044