Flat Model for Representing Contiguous UTM Coordinates over Iraq Territory

Baqir H. Sayed*, Loay E. George
Remote Sensing and GIS Department, College of Science, University of Baghdad, Baghdad, Iraq

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Abstract
The process of converting coordinates is, still, considered an important and difficult issue due to the way of conversion from geographic ellipsoidal system to the projected flat system. The most common method uses contiguous UTM system as one of the most accurate systems in the conversion process, but the users of the system face problems related to contiguity, especially at the large areas that lie within more than one zone. The aim of the present research is to solve the problem related to the multiple zones coverage found in the Iraqi territory using a mathematical model based on the use of Taylor series. The most accurate conversion equation used in this paper was based on the 4th order polynomial of two variables. The calculation of equations’ coefficients was performed using least square criterion for the coordinate’s values, i.e., either latitude, longitude) or East (E), North (N) coordinates. The two basic determinations, for the forward and backward, were applied. In the first stages, the conversion of the coordinates from Longitude/Latitude to East/ North was determined. Then, the second conversion stage was determined, i.e., the coordinates conversion from East, North to Longitude, Latitude). For each phase, a spatial accuracy assessment was conducted. The results showed that the adopted mathematical model was successful to accomplish the conversion process. A very small error average of about 3 cm at east and less than 5 cm at north was reached using the 4th order polynomial equations.

Keywords: Geographic Coordinate System, UTM, Polynomial Representation, Taylor Series
The Universal Transverse Mercator (UTM) is a projection coordinate system. It is a type of plane coordinate system, also called Cartesian coordinate system [1]. The position of a point in the rectangular coordinate system is defined by its distance from the x and y axes. The two distance values are the X and Y coordinates of the point, with the use of a measurement unit such as meters, kilometers, etc. [2, 3].

A position on the Earth is given by the UTM zone number and the easting and northing planar coordinate pair in that zone. The origin point of each UTM zone is the intersection of the equator and the zone's central meridian [4]. To avoid dealing with negative numbers, the central meridian of each zone is defined to coincide with 500,000 meters East. In any zone, a point that has an easting of 400,000 meters is about 100 km west of the central meridian [5]. For most such points, the true distance would be slightly more than 100 km, as measured on the surface of the Earth, due to the distortion of the projection. UTM eastings range from about 167,000 meters to 833,000 meters at the equator [6, 7, 8].

The UTM system covers almost every surface of the earth. Only polar areas latitudes that are higher than 84° North and 80° South are excluded. The UTM system divides the Earth surface into 60 zones, each with 6 degrees. In addition, the zone is numbered west to east from 1 to 60, starting at 180° West longitude [9, 10]. We cannot imagine the flattening of the Earth's surface without converting it to a plane form. The UTM system is the designer to provide a tool of representing each point on the Earth using a set of flat (X, Y or Easting, Northing) coordinates [11]. The advantages of representing locations on the Earth with flat coordinates are the easy planar mapping, easily derived spatial information (e.g., distances, angles, areas, etc.) from the location coordinates, and the ability to calculate the coordinates of a point based on spatial information. In UTM, all measurements are achieved in meters. Therefore, the calculation of distances, directions, and areas can be performed much more conveniently in comparison to the geographic coordinate system [12].

2. Contiguous Problem related to UTM

The UTM system, like any other system, has some problems when converting coordinates, but these problems are less than those found in other conversion systems. The principle of this system is based on the division of the world into zones, but each chip has certain characteristics that do not apply to other zones. Most states contain more than one chip and here lies the problem, so that when moving from one chip to another the results of the conversion would be unrealistic [13, 14]. For example, the United States is located within ten UTM zones and the fact that there are many tight UTM zones can produce troubles. For instance, Philadelphia city in Pennsylvania is east of the city of Pittsburgh. If the Eastings of centroids representing the two cities are compared, the result would be that the Easting of Philadelphia (almost 486 km) is less than that of Pittsburgh (almost 586 km). Despite that the two cities are both located in the state of Pennsylvania, they are existing in two various zones of the UTM system; Philadelphia is nearer to the Zone 18 than Pittsburgh which is located at the Zone 17.

over the Iraqi territory, the same type of problem exists, where Iraq is located in three zones (37, 38 and 39) within the UTM system. The zone 37 covers parts of western Iraq, the zone 38 covers the middle area which represents most of Iraq, and the zone 39 covers a small part of the city of Basra in east-south Iraq. Figure 1 illustrates the zones 37, 38 and 39 covering the territory of Iraq.
3. Theoretical Framework

In this section, the mathematical framework of the two phases of conversion, i.e., the forward and the backward, is provided.

3.1 Forward Representation

The coordinate value of East and North (X and Y) can be represented mathematically as

\[ X = F(\varphi, \lambda) , \quad Y = G(\varphi, \lambda) \]  

(1)

These functions are continuous over most of the regions of globe, except the poles. Thus, the Taylor series [15] of both functions near the point \((\varphi_0, \lambda_0)\) are

\[
X = F(\varphi_0, \lambda_0) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\partial F^{m+n}(\varphi, \lambda)}{\partial^m \varphi \partial^n \lambda} \bigg|_{\varphi=\varphi_0, \lambda=\lambda_0} (\varphi - \varphi_0)^m (\lambda - \lambda_0)^n \frac{m!}{n!},
\]

where \(n + m > 0\)  

(2)

\[
Y = G(\varphi_0, \lambda_0) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\partial G^{m+n}(\varphi, \lambda)}{\partial^m \varphi \partial^n \lambda} \bigg|_{\varphi=\varphi_0, \lambda=\lambda_0} (\varphi - \varphi_0)^m (\lambda - \lambda_0)^n \frac{m!}{n!},
\]

where \(n + m > 0\)  

(3)

In the case that the neighborhood of \((\varphi, \lambda)\) is bounded to an area extended to few degrees far from \(\varphi_0, \lambda_0\) then the values of \(m\) and \(n\) can be bounded to be small, i.e., less than 3 or 4. In such case, the above equations can be approximated to be:

\[
X = \sum_{i=0}^{\text{order}} \sum_{j=0}^{i} a_{n(i)+j}(\Delta \varphi)^{i-j}(\Delta \lambda)^j
\]

(4a)

\[
Y = \sum_{i=0}^{\text{order}} \sum_{j=0}^{i} b_{n(i)+j}(\Delta \varphi)^{i-j}(\Delta \lambda)^j
\]

(4b)

where, \(n(i) = \frac{1}{2} i(i + 1)\), \(\Delta \varphi = \varphi - \varphi_0\), \(\Delta \lambda = \lambda - \lambda_0\)

For the 2nd Order Polynomials

\[
X = a_0 + a_1 \Delta \varphi + a_2 \Delta \lambda + a_3 (\Delta \varphi)^2 + a_4 \Delta \varphi \Delta \lambda + a_5 (\Delta \lambda)^2
\]

(5a)

\[
Y = b_0 + b_1 \Delta \varphi + b_2 \Delta \lambda + b_3 (\Delta \varphi)^2 + b_4 \Delta \varphi \Delta \lambda + b_5 (\Delta \lambda)^2
\]

(5b)
For the 3rd Order Polynomials

\[ X = a_0 + a_1 \Delta \varphi + a_2 \Delta \lambda + a_3 \Delta (\Delta \varphi) + a_4 \Delta \varphi \Delta \lambda + a_5 (\Delta \lambda)^2 + a_6 (\Delta \varphi)^3 + a_7 (\Delta \varphi)^2 \Delta \lambda + a_8 \Delta \varphi (\Delta \lambda)^2 + a_9 (\Delta \lambda)^3 \]  
(6a)

\[ Y = b_0 + b_1 \Delta \varphi + b_2 \Delta \lambda + b_3 \Delta (\Delta \varphi) + b_4 \Delta \varphi \Delta \lambda + b_5 (\Delta \lambda)^2 + a_6 (\Delta \varphi)^3 + a_7 (\Delta \varphi)^2 \Delta \lambda + a_8 \Delta \varphi (\Delta \lambda)^2 + a_9 (\Delta \lambda)^3 \]  
(6b)

For the 4th Order Polynomials

\[ X = a_0 + a_1 \Delta \varphi + a_2 \Delta \lambda + a_3 (\Delta \varphi)^2 + a_4 \Delta \varphi \Delta \lambda + a_5 (\Delta \lambda)^2 + a_6 (\Delta \varphi)^3 + a_7 (\Delta \varphi)^2 \Delta \lambda + a_8 \Delta \varphi (\Delta \lambda)^2 + a_9 (\Delta \lambda)^3 + a_{10} (\Delta \varphi)^4 + a_{11} (\Delta \varphi)^3 \Delta \lambda + a_{12} (\Delta \varphi)^2 (\Delta \lambda)^2 + a_{13} \Delta \varphi (\Delta \lambda)^3 + a_{14} (\Delta \lambda)^4 
\]  
(7a)

\[ Y = b_0 + b_1 \Delta \varphi + b_2 \Delta \lambda + b_3 (\Delta \varphi)^2 + b_4 \Delta \varphi \Delta \lambda + b_5 (\Delta \lambda)^2 + a_6 (\Delta \varphi)^3 + a_7 (\Delta \varphi)^2 \Delta \lambda + a_8 \Delta \varphi (\Delta \lambda)^2 + a_9 (\Delta \lambda)^3 + b_{10} (\Delta \varphi)^4 + b_{11} (\Delta \varphi)^3 \Delta \lambda + b_{12} (\Delta \varphi)^2 (\Delta \lambda)^2 + b_{13} \Delta \varphi (\Delta \lambda)^3 + b_{14} (\Delta \lambda)^4 \]  
(7b)

3.2 Backward Representation

Also, the inverse mapping equations can be approximately written in the form:

\[ \varphi = H(X, Y), \quad \lambda = Q(X, Y) \]  
(8)

Also, the equation is continuous in the region of interest (i.e., the Iraqi territory), so the Taylor series [16, 17] for both equations no. 2 and 3 can be written as in equations 9 and 10 below:

\[ \varphi = H(X_o, Y_o) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\partial H(X, Y)}{\partial^{m+q} X \partial^{n+q} Y} \bigg|_{X_o, Y_o} \frac{(X - X_o)^m (Y - Y_o)^n}{m! n!} \]  
where, \( n + m > 0 \)  
(9)

\[ \lambda = Q(X_o, Y_o) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\partial Q^{m+n}(X, Y)}{\partial^{m+q} X \partial^{n+q} Y} \bigg|_{X_o, \lambda_o} \frac{(X - X_o)^m (Y - Y_o)^n}{m! n!} \]  
where, \( n + m > 0 \)  
(10)

The same mathematical manipulation can be performed to rewrite the above equations in terms of 2D polynomials:

\[ \varphi = \sum_{i=0}^{Order} \sum_{j=0}^{i} c_{n(i)+j} (\Delta X)^{i-j} (\Delta Y)^j \]  
(11a)

\[ \lambda = \sum_{i=0}^{Order} \sum_{j=0}^{i} d_{n(i)+j} (\Delta X)^{i-j} (\Delta Y)^j \]  
(11b)

where, \( n(i) = \frac{1}{2} i(i + 1) \), \( \Delta X = x - x_o \), \( \Delta Y = y - y_o \)

For the 2nd Order Polynomials

\[ \varphi = c_0 + c_1 \Delta x + c_2 \Delta y + c_3 (\Delta x)^2 + c_4 \Delta x \Delta y + c_5 (\Delta y)^2 \]  
(12a)

\[ \lambda = d_0 + d_1 \Delta x + d_2 \Delta y + d_3 (\Delta x)^2 + d_4 \Delta x \Delta y + d_5 (\Delta y)^2 \]  
(12b)

For the 3rd Order Polynomial

\[ \varphi = c_0 + c_1 \Delta x + c_2 \Delta y + c_3 (\Delta x)^2 + c_4 \Delta x \Delta y + c_5 (\Delta y)^2 + c_6 (\Delta x)^3 + c_7 (\Delta x)^2 \Delta y + c_8 \Delta x \Delta (\Delta y)^2 + c_9 (\Delta y)^3 \]  
(13a)

\[ \lambda = d_0 + d_1 \Delta x + d_2 \Delta y + d_3 (\Delta x)^2 + d_4 \Delta x \Delta y + d_5 (\Delta y)^2 + d_6 (\Delta x)^3 + d_7 (\Delta x)^2 \Delta y + d_8 \Delta x (\Delta y)^2 + d_9 (\Delta y)^3 \]  
(13b)
For the 4th Order Polynomials
\[ \varphi = c_0 + c_1 \Delta x + c_2 \Delta y + c_3 (\Delta x)^2 + c_4 \Delta x \Delta y + c_5 (\Delta y)^2 + c_6 (\Delta x)^3 + c_7 (\Delta x)^2 \Delta y + c_8 \Delta x (\Delta y)^2 + c_9 (\Delta y)^3 + c_{10} (\Delta x)^4 + c_{11} (\Delta x)^3 \Delta y + c_{12} (\Delta x)^2 (\Delta y)^2 + c_{13} \Delta x (\Delta y)^3 + c_{14} (\Delta y)^4 \] (14a)

\[ \lambda = d_0 + d_1 \Delta x + d_2 \Delta y + d_3 (\Delta x)^2 + d_4 \Delta x \Delta y + d_5 (\Delta y)^2 + d_6 (\Delta x)^3 + d_7 (\Delta x)^2 \Delta y + d_8 \Delta x (\Delta y)^2 + d_9 (\Delta y)^3 + d_{10} (\Delta x)^4 + d_{11} (\Delta x)^3 \Delta y + d_{12} (\Delta x)^2 (\Delta y)^2 + d_{13} \Delta x (\Delta y)^3 + d_{14} (\Delta y)^4 \] (14b)

4. Results and data sets
4.1 Datasets
The datasets represent grid points of the coordinates of longitude, latitude covering the Iraqi territory and some adjacent areas; so that the distance between each two points in the grid is 0.5 degrees. These coordinates are then converted to X, Y coordinates using UTM system conversion software in the conversion process, but we will determine this program on zone 38 and calculate X, Y coordinates values, as shown in Table-1 and Figure-3.

4.2. Results
4.2.1 For forward representation
This step represents the calculation rate of error for the first test at X east and Y north using the 2nd, 3rd, and 4th order polynomial equations, depending on normalization conversion process.
1. Tables-(1 and 2) represent the values of coefficients rate error, respectively, at X east and y north using the 2nd order polynomial equations 5a and 5b. Also, the error distributions could be drawn as shown in Figures-(4 and 5).

| Table 1-Values of coefficients using the 2nd order polynomial equations (5a & 5b) |
|---|---|---|
| a₀ | 0.056233205 | b₀ | 0.000481156 |
| a₁ | 10.67302884 | b₁ | -0.010988732 |
| a₂ | 0.093370086 | b₂ | 1.002562804 |
| a₃ | -0.097019663 | b₃ | 0.232457867 |
| a₄ | -4.023212612 | b₄ | -0.005665115 |
| a₅ | 0.012356132 | b₅ | 0.001717335 |
Table 2-The error rate values using the 2nd order polynomial equations (5a & 5b)

|                  | rate error at X easting (m) | rate error at Y northing (m) |
|------------------|-----------------------------|-----------------------------|
| Maximum          | 479.237                     | 303.906                     |
| Minimum          | 0.639                       | 0.0794                      |
| Average          | 111.704                     | 57.995                      |

![Figure 3](image1.png)  ![Figure 4](image2.png)

**Figure 3**-present the distribution of error values at X easting using a 2nd order polynomials  
**Figure 4**- present the distribution of error values at Y northing using a 2nd order polynomials

2. Tables-(3 and 4) represent the value of coefficients and rate error, respectively, at X east and y north using the 3rd order polynomial equations (6a and 6b. Also, the error distributions were drawn as shown in Figures-(6 and 7).

Table 3-Values of coefficients using the 3rd order polynomial equations (6a & 6b)

|      | Coefficients | Coefficients |
|------|--------------|--------------|
|      | a₀           | b₀           | 0.000474652 |
| a₁   | 10.67656151  | b₁           | -0.010679744 |
| a₂   | 0.090270214  | b₂           | 1.002351623  |
| a₃   | -0.025197051 | b₃           | 0.235597899  |
| a₄   | -3.979903972 | b₄           | -0.005187345 |
| a₅   | 0.039522455  | b₅           | 0.002347984  |
| a₆   | 0.477670176  | b₆           | -0.008588967 |
| a₇   | 0.133050544  | b₇           | 0.121444681  |
| a₈   | -1.730962083 | b₈           | 0.005931998  |
| a₉   | 0.004883848  | b₉           | 0.001189638  |
Table 4-The error rate values using the 3rd order polynomial equations (6a & 6b)

|                  | rate error at X easting (m) | rate error at Y northing (m) |
|------------------|-----------------------------|-----------------------------|
| Maximum          | 21.369                      | 25.593                      |
| Minimum          | 0.027                       | 0.025                       |
| Average          | 3.609                       | 4.604                       |

Calculate the error value at X easting using a 3rd order polynomial:

Calculate the error value at Y northing using a 3rd order polynomial:

Figure 5-present the distribution of error values at X easting using a 3rd order polynomials

Figure 6-present the distribution of error values at Y northing using a 3rd order polynomials

3. Tables-(5 and 6) represent the values of coefficients and rate error, respectively, at X east and Y north using the 4th order polynomial equations (7a and 7b. In addition, the error distributions are shown in Figures-(8 and 9).

Table 5-Values of coefficients using the 4th order polynomial equations (7a & 7b)

| Coefficients | Coefficients |
|--------------|--------------|
| a₀           | 0.056057752  | b₀           | 0.000474249 |
| a₁           | 10.67676232  | b₁           | -0.010706578|
| a₂           | 0.090342133  | b₂           | 1.002359723 |
| a₃           | -0.029358049 | b₃           | 0.235608589 |
| a₄           | -3.976320625 | b₄           | -0.00550418 |
| a₅           | 0.040289721  | b₅           | 0.002570736 |
Table 6- The error rate values using the 4th order polynomial equations (7a & 7b)

|          | rate error at X easting (m) | rate error at Y northing (m) |
|----------|-----------------------------|-----------------------------|
| Maximum  | 0.154                       | 0.286                       |
| Minimum  | 0.000                       | 0.002                       |
| Average  | 0.025                       | 0.0498                      |

**Figure 7** - present the distribution of error values at X easting using a 4th order polynomial.

**Figure 8** - present the distribution of error values at Y northing using a 4th order polynomials.
4.2.2 For Backward Representation

This step represents the calculation of the rate of error for the first test at Longitude and Latitude using the 2nd, 3rd, and 4th order polynomials equations depending on the normalization conversion process.

3. Tables- (7 and 8) represent the values of coefficients and rate error, respectively, at longitude and latitude using the 2nd order polynomial equations (5c and 5d). Also, we could draw the distributions of error as shown in Figures- (10 and 11).

Table 7-Values of coefficients using the 2nd order polynomial equations (5c & 5d)

| Coefficients | Coefficients |
|--------------|--------------|
| \( c_0 \)    | \( d_0 \)    |
| \( c_1 \)    | \( d_1 \)    |
| \( c_2 \)    | \( d_2 \)    |
| \( c_3 \)    | \( d_3 \)    |
| \( c_4 \)    | \( d_4 \)    |
| \( c_5 \)    | \( d_5 \)    |

| Values      | Coefficients |
|-------------|--------------|
| \( c_0 \)   | -0.005277382 |
| \( c_1 \)   | 0.093634296  |
| \( c_2 \)   | -0.010933052 |
| \( c_3 \)   | 0.000192554  |
| \( c_4 \)   | 0.035602483  |
| \( c_5 \)   | -0.002324351 |

Table 8-The error rate values using the 2nd order polynomial equations (5c & 5d)

| Coefficients | Coefficients |
|--------------|--------------|
| Coefficients | Coefficients |
| \( c_0 \)    | \( d_0 \)    |
| \( c_1 \)    | \( d_1 \)    |
| \( c_2 \)    | \( d_2 \)    |
| \( c_3 \)    | \( d_3 \)    |
| \( c_4 \)    | \( d_4 \)    |
| \( c_5 \)    | \( d_5 \)    |

| Rate Error at Longitude (sec) | Rate Error at Latitude (sec) |
|-------------------------------|-------------------------------|
| Maximum                       | 37.16646                      |
| Minimum                       | 0.006929                      |
| Average                       | 8.603299                      |

4. Tables-(9 and 10) represent the values of coefficients and rate error, respectively, at longitude and latitude using the 3rd order polynomial equations (6c and 6d). Also, we could draw the distributions of error as shown in Figures-(12 and 13).
Table 9-Values of coefficients using the 3rd order polynomial equations (6c & 6d)

| Coefficients | Coefficients |
|--------------|--------------|
| $c_0$        | -0.005244783 |
| $c_1$        | 0.093622695  |
| $c_2$        | -0.010410525 |
| $c_3$        | 9.98601E-05  |
| $c_4$        | 0.034756186  |
| $c_5$        | -0.008846928 |
| $c_6$        | -9.6549E-05  |
| $c_7$        | 0.00024294   |
| $c_8$        | 0.02920124   |
| $c_9$        | -0.000340033 |
| $d_0$        | -0.00053587  |
| $d_1$        | 0.001232322  |
| $d_2$        | 0.997474459  |
| $d_3$        | -0.002055162 |
| $d_4$        | -0.001557791 |
| $d_5$        | -0.002855796 |
| $d_6$        | -7.26E-06    |
| $d_7$        | -0.002572616 |
| $d_8$        | 0.000388714  |
| $d_9$        | 8.01967E-05  |

Table 10-The error rate values using the 3rd order polynomial equations (6c & 6d)

|                    | rate error at Longitude (sec) | rate error at Latitude (sec) |
|--------------------|-------------------------------|-------------------------------|
| Maximum            | 1.57353                       | 0.609209                     |
| Minimum            | 0.001294                      | 0.000471                     |
| Average            | 0.334704                      | 0.124751                     |

6. Tables-(11 and 12) represent the values of coefficients and rate error, respectively, at the longitude and latitude using the 4th order polynomial equations (7c and 7d). Also, we could draw the distributions of error as shown in Figures-(14 and 15).
Table 11-Values of coefficients using the 4th order polynomial equations (7c & 7d)

|     | Coefficients | Coefficients |
|-----|--------------|--------------|
| \(c_0\) | -0.0052448 | \(d_0\) | -0.000535607 |
| \(c_1\) | 0.093626764 | \(d_1\) | 0.001230629 |
| \(c_2\) | -0.010379846 | \(d_2\) | 0.997480429 |
| \(c_3\) | 9.43143E-05 | \(d_3\) | -0.002058185 |
| \(c_4\) | 0.034719336 | \(d_4\) | 0.001541364 |
| \(c_5\) | -0.008476756 | \(d_5\) | -0.002716049 |
| \(c_6\) | -0.000104558 | \(d_6\) | -4.9682E-06 |
| \(c_7\) | 0.000145654 | \(d_7\) | -0.002572951 |
| \(c_8\) | 0.028262383 | \(d_8\) | 0.000585766 |
| \(c_9\) | -0.004602165 | \(d_9\) | -0.000538666 |
| \(c_{10}\) | -6.20839E-07 | \(d_{10}\) | 3.54477E-06 |
| \(c_{11}\) | -0.000147522 | \(d_{11}\) | -1.34117E-05 |
| \(c_{12}\) | 0.000218654 | \(d_{12}\) | -0.000989218 |
| \(c_{13}\) | 0.015117426 | \(d_{13}\) | 0.000108676 |
| \(c_{14}\) | 0.000652888 | \(d_{14}\) | 0.000217122 |

Table 12-The error rate values using the 4th order polynomial equations (7c & 7d)

|     | rate error at Longitude (sec) | rate error at Latitude (sec) |
|-----|-------------------------------|-------------------------------|
| Maximum | 0.094228                     | 0.038256                     |
| Minimum | 0.000124                     | 2.36E-05                     |
| Average | 0.018406                     | 0.005913                     |

Figure 13-present the distribution of error values at Longitude using a 4th order polynomials

Figure 14-present the distribution of error values at Latitude using a 4th order polynomials
5. Conclusions

When the first test was used, the attained results held a very high error rate, leading to the failure of the conversion process. But, when the second test of the normalization method was used, the rate of error was gradually reduced to the lowest ratio, using the fourth-order polynomials equation. This resulted in the success of the conversion process and produced results that were near to idealism, as described in the following:

Using the second-order polynomial equation, the result of the error average values was approximately 120 meters at X east and about 50 meters at Y north. The average error value decreased when the third-order polynomial equation was used, where the average error values were less than 4 meters at the X east and about 5 meters at the Y north. The final conversion process was successful, causing a very large drop of average error values when the fourth order polynomial equations were used, where the average error values were less than 3 cm at X east and about 5 cm at Y north. These results are very good and we can depend on this model for the conversion process of X east and Y north to longitude and latitude for the Iraqi territory, due to the high precision of this model.

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