Noise-induced finite-time disentanglement in two-atomic system

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We discuss the influence of a noisy environment on entangled states of two atoms and show that all such states disentangle in finite time.

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I. INTRODUCTION

Entanglement of quantum states is the most non-classical feature of quantum systems. It shows up when the system consists of two (or more) subsystems and the total state cannot be written as a product state. This notion can be generalized to mixed states, and mixed state is non-separable or entangled if a corresponding density matrix cannot be expressed as convex combination of tensor products of density matrices of subsystems [2, 3]. Pure entangled states as superpositions of multiparticle states, are fragile with respect to noise resulting from interaction with environment. So to control the effects of noise, it is important to understand details of the process of disentanglement i.e. to analyse how entanglement can be destroyed by this interaction.

In the paper, we describe the model of two two-level atoms interacting with thermal bath at formally infinite temperature. When atoms are separated by large distance, we can assume that the atoms are located inside two independent baths. In such a case time evolution of two-atomic system is given, in the Markovian approximation, by the ergodic dynamical semi-group i.e. a trace preserving semi-group of completely positive operators which have the maximally mixed state \( \frac{1}{d} \mathbb{I}_d \) invariant. Density matrix corresponding to the state of the system satisfies the master equation which right hand side is given by the Lindblad generator of the semi-group. Since two atoms are largely separated, the dipol-dipol interaction and the photon exchanges between atoms are negligible, so in our model the generator is parametrized only by the dissipation rate \( \Gamma \).

From the general properties of this kind of evolution it follows that the entanglement as a function of time always decreases to zero, and that dynamics needs only a finite time to disentangle any initially entangled state. Thus for all times \( t \) greater then some \( t_d(\rho) \), the states \( \rho(t) \) are separable. This finite-time effect should be compared with asymptotic noise-induced decoherence effects (see also [2] [3]). We calculate time evolution of some class of initial density matrices and obtain analytic expression for its entanglement. We also find formulas for \( t_d \) in the cases of pure initial states and some mixed initial states. In that examples, the time of disentanglement is the increasing function of the initial entanglement. Another interesting aspect of dissipative evolution studied in the paper is connected with nonlocal properties of quantum states. Nonlocality of quantum theory manifests by violation of Bell inequalities, and in the case of two two-level atoms can be quantified by some parameter ranging from 0 for local states to 1 for states maximally violating some Bell inequality. Our dynamics enables also to consider evolution of this parameter. In particular, we show that the time after which all nonlocal properties of quantum state is lost is much shorter then the time of disentanglement.

II. TIME EVOLUTION IN A NOISY ENVIRONMENT

A. Two-level atom in a noisy environment

Time evolution of a density matrix of two-level atom \( A \) inside the bath with finite temperature \( T \) can be described by the following master equation

\[
\frac{d\rho}{dt} = -\frac{1}{2} \Gamma \{ [\sigma_\uparrow, \rho \sigma_\downarrow] + [\sigma_\downarrow, \rho \sigma_\uparrow] \} + \frac{1}{2} \Gamma \{ [\sigma_\downarrow, \rho \sigma_\uparrow] + [\sigma_\uparrow, \rho \sigma_\downarrow] \}
\]

(II.1)

where

\[
\sigma_\pm = \frac{1}{2} (\sigma_1 \pm i \sigma_2)
\]

and we identify ground state \( |0\rangle \) and excited state \( |1\rangle \) of the atom with vectors \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) in \( \mathcal{H}_A = \mathbb{C}^2 \). Moreover,

\[
\Gamma_\uparrow = \gamma_0 n(\omega_0), \quad \Gamma_\downarrow = \gamma_0 (1 + n(\omega_0))
\]

(II.2)

where

\[
n(\omega_0) = \frac{1}{e^{\beta \omega_0} - 1}, \quad \beta = \frac{1}{T}
\]

\( \gamma_0 \) is a spontaneous emission rate of the atom and \( \omega_0 \) is the frequency of the transition \( |0\rangle \rightarrow |1\rangle \). Since

\[
\frac{\Gamma_\downarrow}{\Gamma_\uparrow} = e^{\beta \omega_0} \rightarrow 1
\]

when \( \beta \rightarrow 0 \), for very high temperature \( (T \rightarrow \infty) \) we can assume that [4]

\[
\Gamma_\uparrow = \Gamma_\downarrow = \Gamma
\]
In this case (II.1) reduces to
\[
\frac{dp}{dt} = L_\Gamma \rho = \Gamma (\sigma_+ \rho \sigma_- + \sigma_- \rho \sigma_+ - \rho) \tag{II.3}
\]
and \(L_\Gamma\) generates the semi-group
\[
T_t = e^{L_\Gamma t}
\]
which is ergodic i.e. \(\{T_t\}_{t \geq 0}\) has a unique asymptotic state which is maximally mixed state \(\frac{\mathbb{I}}{d}\) in \(\mathbb{C}^2\). So the relaxation process described by (II.3) which models the evolution of two-level atom inside the bath with very high (infinite) temperature, brings all initial states of the atom into the state with maximal entropy. In the other words, the semi-group generated by (II.3) describes open quantum system (two level atom) interacting with noisy environment (a bath with very high temperature).

B. Two independent two-level atoms

In the case of two separated two-level atoms \(A\) and \(B\) located inside independent baths with formally infinite temperatures, the generator of the corresponding semi-group is given by the following generalization of (II.1)
\[
\frac{d\rho^{AB}}{dt} = L_\Gamma^{AB} \rho^{AB} = \Gamma (\sigma_+^A \rho^{AB} \sigma_-^A + \sigma_-^A \rho^{AB} \sigma_+^A + \sigma_+^B \rho^{AB} \sigma_-^B + \sigma_-^B \rho^{AB} \sigma_+^B - 2 \rho^{AB}) \tag{II.4}
\]
where \(\rho^{AB}\) is the state of the compound system \(AB\), described by the Hilbert space \(\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^4\) and the algebra of observables
\[
\mathfrak{A}_{AB} = M_{2 \times 2}(\mathbb{C}) \otimes M_{2 \times 2}(\mathbb{C}) \simeq M_{4 \times 4}(\mathbb{C})
\]
and
\[
\sigma_+^A = \sigma_+ \otimes \mathbb{I}, \quad \sigma_-^A = \mathbb{I} \otimes \sigma_-
\]
The semi-group generated by \(L_\Gamma^{AB}\) is also ergodic with unique asymptotic state \(\frac{\mathbb{I}}{2} \otimes \mathbb{I}\) in \( \mathbb{C}^4 \). To simplify the discussion of the evolution of two-atomic system, let us introduce the basis of so called collective states in the Hilbert space \( \mathcal{H}_{AB} = \mathbb{C}^4 \) [3]. If
\[
f_1 = |1\rangle \otimes |1\rangle, \ f_2 = |1\rangle \otimes |0\rangle, \ f_3 = |0\rangle \otimes |1\rangle, \ f_4 = |0\rangle \otimes |0\rangle
\]
then this basis containing excited state, ground state and symmetric and antisymmetric combination of the product states, is defined as follows
\[
|e\rangle = f_1, \ |g\rangle = f_4, \ |s\rangle = \frac{1}{\sqrt{2}} (f_2 + f_3), \ |a\rangle = \frac{1}{\sqrt{2}} (f_2 - f_3) \tag{II.5}
\]
From the master equation (II.4) it follows that the matrix elements with respect to the basis \(|e\rangle, |s\rangle, |a\rangle, |g\rangle\) of the state \(\rho\) satisfy
\[
\frac{d\rho_{aa}}{dt} = -2 \Gamma (\rho_{aa} + (\rho_{ee} + \rho_{gg}))
\]
\[
\frac{d\rho_{ss}}{dt} = -2 \Gamma (\rho_{ss} + (\rho_{ee} + \rho_{gg}))
\]
\[
\frac{d\rho_{ee}}{dt} = -2 \Gamma (\rho_{ee} + (\rho_{ss} + \rho_{aa}))
\]
\[
\frac{d\rho_{gg}}{dt} = -2 \Gamma (\rho_{gg} + (\rho_{ss} + \rho_{aa}))
\]
\[
\frac{d\rho_{as}}{dt} = -2 \Gamma (\rho_{as} - \Gamma (\rho_{ee} + \Gamma (\rho_{ss} + \rho_{aa}))
\]
\[
\frac{d\rho_{sa}}{dt} = -2 \Gamma (\rho_{ae} - \Gamma (\rho_{ee} + \Gamma (\rho_{ss} + \rho_{aa}))
\]
\[
\frac{d\rho_{eg}}{dt} = -2 \Gamma (\rho_{eg} + \Gamma (\rho_{ee} + \Gamma (\rho_{ss} + \rho_{aa}))
\]
Notice that if the initial state belongs to the class of density matrices of the form
\[
\rho = \begin{pmatrix}
\rho_{ee} & 0 & 0 & \rho_{eg} \\
0 & \rho_{ss} & \rho_{as} & 0 \\
0 & \rho_{as} & \rho_{aa} & 0 \\
\rho_{ge} & 0 & 0 & \rho_{gg}
\end{pmatrix} \tag{II.7}
\]
then \(\rho(t)\) given by the solution of the equations (II.6) also belongs to that class. In this case one finds that
\[
\rho_{aa}(t) = \frac{1}{4} + \frac{1}{2} e^{-2 \Gamma t} (\rho_{aa} - \rho_{ss}) + \frac{1}{2} e^{-4 \Gamma t} (\rho_{aa} + \rho_{ss} - \frac{1}{2})
\]
\[
\rho_{ss}(t) = \frac{1}{4} - \frac{1}{2} e^{-2 \Gamma t} (\rho_{aa} - \rho_{ss}) + \frac{1}{2} e^{-4 \Gamma t} (\rho_{aa} + \rho_{ss} - \frac{1}{2})
\]
\[
\rho_{ee}(t) = \frac{1}{4} + \frac{1}{2} e^{-2 \Gamma t} (\rho_{ee} - \rho_{gg}) + \frac{1}{2} e^{-4 \Gamma t} (\rho_{ee} + \rho_{gg} - \frac{1}{2})
\]
\[
\rho_{gg}(t) = \frac{1}{4} - \frac{1}{2} e^{-2 \Gamma t} (\rho_{ee} - \rho_{gg}) + \frac{1}{2} e^{-4 \Gamma t} (\rho_{ee} + \rho_{gg} - \frac{1}{2})
\]
\[
\rho_{as}(t) = e^{-2 \Gamma t} \rho_{as}
\]

III. ENTANGLEMENT IN TWO-ATOMIC SYSTEMS

A. Measure of entanglement

In the case when subsystems of the total system are described by two-dimensional Hilbert spaces, the natural measure of the amount of entanglement a given quantum state
contains i.e. the entanglement of formation \[ C(\rho) = \min \sum_k \lambda_k E(P_k) \] (III.1)
where the minimum is taken over all possible decompositions \( \rho = \sum_k \lambda_k P_k \) (III.2)
and
\[ E(P) = -\text{tr}[(\text{tr}_A P) \log_2 (\text{tr}_A P)] \] (III.3)
can be analytically computed as a function of another quantity \( C(\rho) \) called concurrence, which also can be taken as a measure of entanglement [7, 8]. \( C(\rho) \) is defined as follows
\[ C(\rho) = \max \{ 0, 2p_{\max}(\rho) - \text{tr} \hat{\rho} \} \] (III.4)
where \( p_{\max}(\rho) \) denotes the maximal eigenvalue of \( \rho \) and
\[ \hat{\rho} = (\sigma_2 \otimes \sigma_2) \rho (\sigma_2 \otimes \sigma_2) \] (III.5)
with
\[ \rho = (\sigma_2 \otimes \sigma_2) \rho (\sigma_2 \otimes \sigma_2) \] (III.6)
The value of the number \( C(\rho) \) varies from 0 for separable states, to 1 for maximally entangled pure states. Consider now the class (II.7) of density matrices. With respect to the canonical basis \( f_1, f_2, f_3, f_4 \), the matrices (II.7) have also the same form i.e.
\[ \rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \]

One can check that for this class
\[ C(\rho) = \max \{ 0, C_1, C_2 \} \] (III.7)
where
\[ C_1 = 2 (|\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}), \quad C_2 = 2 (|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}) \] (III.8)
or
\[ C_1 = 2 |\rho_{eg}| - \sqrt{(\rho_{aa} + \rho_{ss})^2 - (\rho_{as} + \rho_{sa})^2} \] (III.9)
when we use the matrix elements with respect to the collective basis. In the special case when \( \rho_{14} = 0 \), \( C_1 \) cannot be positive, so
\[ C = \max \{ 0, C_2 \} \]
Similarly, when \( \rho_{23} = 0 \), then
\[ C = \max \{ 0, C_1 \} \]

### B. Evolution of entanglement

Suppose that the initial state of the two-atomic system belongs to the class (II.7). Since the evolution given by the master equation (II.8) leaves this class invariant, to compute entanglement at time \( t \) we can use formulas (III.9) and (II.8). So we have
\[ C(\rho(t)) = \max \{ 0, C_1(t), C_2(t) \} \]
where
\[ C_1(t) = 2e^{-2\Gamma} |\rho_{eg}| - \sqrt{e^{-4\Gamma} (\rho_{aa} + \rho_{ss} - \frac{1}{2}) + \frac{1}{2}} - e^{-4\Gamma} (\rho_{as} + \rho_{sa})^2 \] (III.10)
and
\[ C_2(t) = e^{-2\Gamma} \sqrt{(\rho_{aa} - \rho_{ss})^2 - (\rho_{as} - \rho_{sa})^2} - \frac{1}{2} \sqrt{1 + e^{-4\Gamma} (-1 + 2\rho_{ee} + 2\rho_{gg})^2 + 4e^{-4\Gamma} \left[ \rho_{ee} + \rho_{gg} - \frac{1}{2} - (\rho_{ee} - \rho_{gg})^2 \right]} \] (III.11)

As an example, consider the following initial state
\[ \rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \] (III.12)
The initial entanglement is equal to
\[ C(\rho) = 2 |\rho_{23}| = \sqrt{(\rho_{ss} - \rho_{aa})^2 - (\rho_{as} - \rho_{sa})^2} \]
so
\[ C(\rho(t)) = \max \{ 0, e^{-2\Gamma} C(\rho) - \frac{1}{2} (1 - e^{-4\Gamma}) \} \] (III.13)
From (III.13) we see that there is the time \( t_d(\rho) \) after which \( C(\rho(t)) \) becomes equal to 0. We may call \( t_d(\rho) \) the time of disentanglement of a given initial state \( \rho \). For initial state (III.12)

\[
t_d(\rho) = \frac{1}{2\Gamma} \ln \left[ C(\rho) + \sqrt{1 + C(\rho)^2} \right] \quad \text{(III.14)}
\]

As we show in the next section, the appearance of finite time of disentanglement for all initially entangled states, is the characteristic feature of the dynamics governed by the master equation (II.4).

\section{IV. TIME OF DISENTANGLEMENT}

The evolution of states given by the semi-group generated by \( L_{AB}^{\Gamma} \) has the following important properties:

(i) it is local i.e. if \( \rho \) is separable, then \( \rho(t) \) is also separable for all \( t \geq 0 \),

(ii) every initial state \( \rho \) evolves to a maximally mixed state \( \frac{1}{d} \mathbf{1} \).

Since maximally mixed state is separable and there is a neighbourhood of this state which contains only separable states, for an arbitrary state \( \rho \) the set

\[
S_\rho = \{ t \in [0, \infty) : \rho(t) \text{ is separable} \} \quad \text{(IV.1)}
\]

is always non-empty. Moreover, if

\[
E_\rho = \{ t \in [0, \infty) : \rho(t) \text{ is entangled} \} \quad \text{(IV.2)}
\]

then \( S_\rho \) and \( E_\rho \) are disjoint and \( S_\rho \cup E_\rho = [0, \infty) \). Notice that for every \( t_1 \in E_\rho \) and every \( t_2 \in S_\rho \) we have \( t_1 < t_2 \), so \( S_\rho \) is bounded from below. Now we can define the time of disentanglement \( t_d(\rho) \) of a state \( \rho \) of two-atomic system as follows

\[
t_d(\rho) = \inf \{ S_\rho \} \quad \text{(IV.3)}
\]

Since the set of separable states is compact, \( S_\rho \) is closed and \( t_d(\rho) \in S_\rho \). Therefore \( t_d(\rho) \) may be also defined as the smallest time for which \( \rho(t) \) is separable. From the above discussion it is clear that for every initial state of the system, there exist finite time of disentanglement (which may be equal to 0 for separable initial states). Consider now some explicit examples.

1. Pure initial states.

Since all pure entangled states \( \Psi \) with concurrence \( C(\Psi) = c \) are locally equivalent to the state given by the vector (see e.g. [10])

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1 + \sqrt{1 - c^2}}, 0, 0, \sqrt{1 - \sqrt{1 - c^2}} \end{pmatrix} \quad \text{(IV.4)}
\]

and our dynamics is local, it is enough to consider (IV.4) as initial state. The corresponding density matrix

\[
P_\Phi = \frac{1}{2} \begin{pmatrix} 1 + \sqrt{1 - c^2} & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c & 0 & 0 & 1 - \sqrt{1 - c^2} \end{pmatrix} \quad \text{(IV.5)}
\]

evolves into

\[
P_\Phi(t) = \frac{1}{4} \begin{pmatrix} 1 + e^{-4\Gamma t} + 2\sqrt{1 - c^2}e^{-2\Gamma t} & 0 & 0 & 2ce^{-2\Gamma t} \\ 0 & 1 - e^{-4\Gamma t} & 0 & 0 \\ 0 & 0 & 1 - e^{-4\Gamma t} & 0 \\ 2ce^{-2\Gamma t} & 0 & 0 & 1 + e^{-4\Gamma t} - 2\sqrt{1 - c^2}e^{-2\Gamma t} \end{pmatrix} \quad \text{(IV.6)}
\]

One can easily check that

\[
t_d(P_\Phi) = \frac{1}{2\Gamma} \ln \left[ c + \sqrt{1 + c^2} \right] \quad \text{(IV.7)}
\]

This finite time of disentanglement should be contrasted with infinite time needed for decoherence process (see [2, 3] for discussion of different models). For pure initial states, we can introduce a decoherence rate \( \lambda(P) \) which tells us how fast given pure initial state becomes mixed during the evolution [3]. If as a measure of mixedness we take the linear entropy

\[
S_{\text{lin}}(\rho) = 1 - \text{tr} \rho^2
\]

then

\[
\lambda(P) = \frac{1}{2} \frac{dS_{\text{lin}}(P(t))}{dt} \bigg|_{t=0} \quad \text{(IV.8)}
\]

In the case of state (IV.5) one obtains

\[
\lambda(P_\Phi) = 2\Gamma \quad \text{(IV.9)}
\]

We see also that off-diagonal elements of \( P_\Phi(t) \) vanish asymptotically with that rate.

Another characteristic time of our evolution is connected with so called quantum non-locality i.e. the possibility of violating Bell inequalities in quantum states [11, 12]. It is known
that all pure entangled states violate Bell inequalities \[13\], but this is not true for mixed states. In the case of two two-level systems, there is an effective criterion for violating Bell inequalities in mixed states \[14, 15\]. For any density matrix \(\rho\), take the real 3 \(\times\) 3 matrix

\[
T_\rho = (t_{nm}), \quad t_{nm} = \text{tr}(\rho \sigma_n \otimes \sigma_m) \quad n, m = 1, 2, 3
\]

Define also real symmetric matrix

\[
U_\rho = T_\rho^T T_\rho \quad (IV.10)
\]

with eigenvalues \(u_1, u_2, u_3\). Then \(\rho\) violates some Bell inequality if and only if \[14\]

\[
m(\rho) > 1 \quad (IV.11)
\]

where

\[
m(\rho) = \max_{j < k} (u_j + u_k) \quad (IV.12)
\]

As a measure of nonlocality we may introduce a function

\[
n(\rho) = \max \{0, m(\rho) - 1\}
\]

Initial state (IV.5) violates Bell inequality since

\[
m(P_\Phi) = 1 + c^2 \quad (IV.13)
\]

On the other hand,

\[
m(P_\Phi(t)) = e^{-8\Gamma t} + c^2 e^{-4\Gamma t} \quad (IV.14)
\]

decreases to 0, so there exists the time \(t_{\text{loc}}\) after which (IV.14) is smaller than 1 and nonlocality of initial state is lost. We see that this locality time for the state (IV.5) equals

\[
t_{\text{loc}}(P_\Phi) = \frac{1}{4\Gamma} \ln \left[\frac{c^2 + \sqrt{4 + c^4}}{2}\right] \quad (IV.15)
\]

and is always smaller then the time of disentanglement \(t_d(\Phi_\Phi)\) (see FIG. 1).

We see that the evolution of pure initial states has the following remarkable properties:

(a) in the time interval \([0, t_{\text{loc}}]\) the states (IV.6) are entangled and violate Bell inequalities,

(b) for \(t \in [t_{\text{loc}}, t_d]\), \(P_\Phi(t)\) are still entangled but do not violate any Bell inequality,

(c) for all \(t \geq t_d\) the states (IV.6) are separable, although decoherence process takes infinite time.

2. Some mixed initial states.

(i) Consider the class of Werner states \[16\]

\[
W_\pm = (1 - p) \frac{I_4}{4} + p |\Psi_\pm \rangle \langle \Psi_\pm| \quad (IV.16)
\]

where

\[
\Psi_\pm = \frac{1}{\sqrt{2}} \left[ |0 \rangle \otimes |0 \rangle \pm |1 \rangle \otimes |1 \rangle \right]
\]

are maximally entangled pure states. It is known that \(W_\pm\) are entangled for \(p > 1/3\) and \(C(W_\pm) = \frac{3p - 1}{2}\). During the time evolution \(W_\pm\) become

\[
W_\pm(t) = \frac{1}{4} \begin{pmatrix}
1 + p e^{-4\Gamma t} & 0 & 0 & \pm 2p e^{-2\Gamma t} \\
0 & 1 - p e^{-4\Gamma t} & 0 & 0 \\
0 & 0 & 1 - p e^{-4\Gamma t} & 0 \\
\pm 2p e^{-2\Gamma t} & 0 & 0 & 1 + p e^{-4\Gamma t}
\end{pmatrix} \quad (IV.17)
\]

and

\[
C(W_\pm(t)) = \max \left\{0, p \left( e^{-2\Gamma t} + \frac{1}{2} e^{-4\Gamma t} \right) - \frac{1}{2} \right\} \quad (IV.18)
\]

One finds that

\[
t_{\text{loc}}(W_\pm) = \frac{1}{2\Gamma} \ln \left[ p + \sqrt{p(1 + p)} \right], \quad p > \frac{1}{3} \quad (IV.19)
\]

On the other hand, not all Werner states which are entangled, violate Bell inequalities. Nonlocal properties have only those states \(W_\pm\) with \(p > \frac{1}{\sqrt{2}} \[14\]. This nonlocality is lost when \(t > t_{\text{loc}}(W_\pm)\)

\[
t_{\text{loc}}(W_\pm) = \frac{1}{4\Gamma} \ln 2p^2, \quad p > \frac{1}{\sqrt{2}} \quad (IV.20)
\]

But even in the interval \(\frac{1}{\sqrt{2}} < p \leq 1\), this time is much smaller then time of disentanglement (FIG. 2)
(ii) Similar computation can be done for the class of maximally entangled mixed states [17], which have maximal value of entanglement for a given degree of impurity measured by linear entropy

\[
\rho_{\text{MEMS}} = \begin{pmatrix}
g(c) & 0 & 0 & c/2 \\
0 & 1 - 2g(c) & 0 & 0 \\
0 & 0 & 0 & 0 \\
c/2 & 0 & 0 & g(c)
\end{pmatrix}
\]  

(IV.21)

Direct calculations show that

\[
t_d(\rho_{\text{MEMS}}) = \begin{cases}
\frac{1}{4\Gamma} \ln \left[ \frac{5}{2} + 2c^2 + \frac{1}{16} \sqrt{(36c^2 + 10)^2 - 36} \right], & c \in [0, 2/3] \\
\frac{1}{4\Gamma} \ln \left[ 1 - 2c + 4c^2 + 2\sqrt{2c(1 - 2c + 2c^2)} \right], & c \in [2/3, 1]
\end{cases}
\]  

(IV.22)

One can also check that \(\rho_{\text{MEMS}}\) violates Bell inequality when \(c > \frac{1}{\sqrt{2}}\) and for such values of initial entanglement, the locality time equals

\[
t_{\text{loc}}(\rho_{\text{MEMS}}) = \frac{1}{4\Gamma} \ln 2c^2, \hspace{1cm} c > \frac{1}{\sqrt{2}}
\]  

(IV.23)

As in the previous cases, this time is always smaller than the time of disentanglement (FIG. 3).

We have shown that interaction of two-atomic system with a noisy environment, modeled by the master equation (II.4) leads to the disentanglement of initially entangled states in finite time. This time of disentanglement is (at least in examples considered above) the increasing function of initial entanglement - more entangled is the initial state, the longer period of time is needed to disentangle it. On the other hand, if the initial entangled state can violate Bell inequalities, the period of time in which it still have nonlocal properties, is much shorter then the duration of the process of disentanglement.

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