Some problems of determination of the spin structure of the scattering amplitude and experiments at NICA

O. V. Selyugin

JINR, Bogoliubov Laboratory of Theoretical Physics, 141980 Dubna, Moscow region, Russia

Abstract. The existing experimental data are examined under different assumptions about the structure of the scattering amplitude of the proton-proton and proton-antiproton elastic scattering at high energy to obtain the value of $\rho(s, t)$, the ratio of the real to imaginary part of the scattering amplitude in the Coulomb-hadron interference region. It is shown that the deviation of $\rho(s, t)$ obtained from the experimental data of the proton-antiproton scattering at $3.8 < P_L < 6.2$ GeV/c from the dispersion analysis is concern in all examined assumptions.

1 Introduction

The measure of the $s$-dependence of the total cross sections $\sigma_{\text{tot}}(s)$ and the value of $\rho(s, t)$, the ratio of the real to imaginary part of the elastic scattering amplitude are very important as they are connected to each other by the integral dispersion relations [1]. The validity of this relation can be check up at LHC energies. The deviation can point out the existence of the fundamental length at TeV order. But for such a conclusion we should know with high accuracy the lower energy data as well. At these energies we have many contributions to the hadron scattering amplitudes coming from the exchange of different regions. Now we cannot exactly calculate all contributions and find their energy dependence. However, a great amount of the experimental material allows us to make full phenomenological analysis and obtain the size and form of the different parts of the hadron scattering amplitude. The difficulty is that we do not know the energy dependence of these amplitudes and individual contributions of the asymptotic non-dying spin-flip amplitudes. As was noted in [2], the spin-dependent part of the interaction in $pp$ scattering is stronger than expected and a good fit to the data in the Regge model requires an enormous number of poles.

In the general case, the determination of the total cross sections depends on the parameters of the elastic scattering amplitude: $\sigma_{\text{tot}}$, $\rho(s, t)$, the Coulomb-nuclear interference phase $\varphi_{\text{cn}}(s, t)$ and the elastic slope $B(s, t)$. For the definition of new effects at small angles and especially in the region of the diffraction minimum, one must know the effects of the Coulomb-nuclear interference with sufficiently high accuracy. The Coulomb-nuclear phase was calculated in the entire diffraction domain taking into account the form factors of the nucleons [3-5].
There was a significant difference between the experimental measurement of $\rho$, the ratio of the real part to the imaginary part of the scattering amplitude, between the UA4 and UA4/2 collaborations at $\sqrt{s} = 541$ GeV. A more careful analysis [6,7] shows that there is no contradiction between the measurements of UA4 and UA4/2. Now the present model gives for this energy $\rho(\sqrt{s} = 541$ GeV, $t = 0) = 0.163$, so practically the same as in the previous phenomenological analysis.

There are many experimental data on the elastic proton-proton and proton-antiproton scattering at non-high energies (at $10 < p_L < 100$ (GeV/c). However, the extracted sizes of $\rho(s, t = 0)$ contradict each other in the different experiments and give the bad $\chi^2$ in the different models trying to describe the $s$-dependence of $\rho(s, t = 0)$ (see, for example, the results of the COMPETE Collaboration [8,9]). A more careful analysis of these experimental data give in some cases an essentially different value of $\rho(s, t = 0)$. For example, our analysis of the experimental data, which take into account the uncertainty of the total cross sections, gave the new sizes of $\rho(s, t = 0)$, which differ from original experimental data by 25% on average. For example, for $P_L = 19.23$ GeV/c the experimental work gave $\rho(s, t = 0) = -0.25 \pm 0.03$ and for $P_L = 38.01$ GeV/c $\rho(s, t = 0) = -0.17 \pm 0.03$. Our analysis gave for these values: $\rho(s, t = 0) = -0.32 \pm 0.08$ and $\rho(s, t = 0) = -0.12 \pm 0.03$, respectively. This picture was confirmed by the independent analyses of the experimental data [10] $52 < p_L < 400$ (GeV/c) of Fajardo [11] and our. Both new analyses coincide with each other but differed from original experimental data.

Of course, we have plenty of experimental data in the domain of small momentum transfer at low energies $3 < p_L < 12$ GeV/c). With pity, most of these experimental data gave the large errors of the experimental data. At these energies, we have many contributions to the hadron-spin-flip amplitudes from different region exchanges. Now we cannot exactly calculate all contributions and find their energy dependence. But a great amount of the experimental material allows us to make complete phenomenological analysis and find the size and form of different parts of the hadron scattering amplitude. The difficulty is that we do not know the energy dependence of such amplitudes and individual contributions of the asymptotic non-dying spin-flip amplitudes.

The data on the proton-proton elastic scattering at $3.7 < p_L < 6.2$ (GeV/c) [17,18] are most interesting. These experimental data have the high accuracies and give the extracted value of the $\rho(s)$ with high precision. The new data of $\rho(s)$ essentially differ from the theoretical analysis by the dispersion relations [12]. Hence, for the first time, a more careful analysis of the original experimental data is require, to take into account the different assumptions and corrections to the scattering amplitude.

**2 Coulomb phase factor with hadron form-factor**

The differential cross sections of the nucleon-nucleon elastic scattering can be written as the sum of the different spiral scattering amplitudes:

$$\frac{d\sigma}{dt} = 2\pi \frac{s}{s^2} \left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2\right).$$

Every amplitude $\phi_i(s, t)$, including the electromagnetic and hadronic forces, can be expressed as

$$\phi(s, t) = F_C \exp(i\alpha\varphi(s, t)) + F_N(s, t),$$

with

$$\varphi(s, t) = \varphi(t)_C - \varphi(s, t)_{CN},$$
where $\varphi(t)_{C}$ appears in the second Born approximation of the pure Coulomb amplitude, and the term $\varphi_{CN}$ is defined by the Coulomb-hadron interference.

If the hadron amplitude is chosen in the standard Gaussian form $F_{N}(s, t) = h_{nf}(s) \exp(-B(s)q^{2}/2)$, we can get a standard phase, which is used in most experimental works,

$$
\varphi(s, t) = \mp \ln \left( -B(s)t/2 + \gamma \right),
$$

where $-t = q^{2}$, $B(s)/2$ is the slope of the nuclear amplitude, $\gamma$ is the Euler constant, and the upper (lower) sign corresponds to the scattering of particles with the same (opposite) charges.

The influence of the electromagnetic form factor of scattered particles on $\varphi_{CN}$ in the framework of the eikonal approach was examined by Cahn [13]. He derived for $t \to 0$ the eikonal analogue [14] and obtained the corrections

$$
\varphi_{CN}(s, t) = \mp \left[ \gamma + \ln \left( B(s)|t|/2 \right) + \ln \left( 1 + 8/(B(s)A^{2}) \right) 
+ (4|t|/A^{2}) \ln (4|t|/A^{2}) + 2|t|/A^{2} \right],
$$

where $A$ is a constant entering into the dipole form factor. The calculations of the phase factor beyond the limit $t \to 0$ carried out in [3,4,5].

The impact of the spin of scattered particles was analyzed in [15,16] by using the eikonal approach for the scattering amplitude. Using the helicity formalism for high energy hadron scattering in [16] it was shown that at small angles, all the helicity amplitudes have the same $\varphi(s, t)$.

3 Impact of the CNI phase

Let us determine the hadronic and electromagnetic spin-non-flip amplitudes as

$$
F_{hf}^{h}(s, t) = \left[ \phi_{1}^{h}(s, t) + \phi_{2}^{h}(s, t) \right]/2;
\quad F_{hf}^{e}(s, t) = \left[ \phi_{1}^{em}(s, t) + \phi_{3}^{em}(s, t) \right]/2;
$$

and spin-flip amplitudes as

$$
F_{hf}^{s}(s, t) = \phi_{5}^{h}(s, t);
\quad F_{hf}^{e}(s, t) = \phi_{5}^{em}(s, t).
$$

Equation (6,7) was applied at high energies and at small momentum transfer, with the following usual assumptions for hadron spin-flip amplitudes: $\phi_{1} = \phi_{3}$, $\phi_{2} = \phi_{4} = 0$; the slopes of the hadronic spin-flip and spin-non-flip amplitudes are equal.

Let us make a new fit of the experimental data of the proton-antiproton scattering [17,18] at low energies with different approximations of the Coulomb-hadron interference phase factor. First, we used a simple form of the phase, Eq. 4. The obtained sizes of $\rho(s, t = 0)$ are shown in Table 1. The results are distributed near the sizes of $\rho(s)$ extracted during the experiments. In two cases they are only slightly above $\rho_{exper.}$ (at $P_{L} = 4.066$, 5.94 GeV/c); in three cases they are more above $\rho_{exper.}$ (at $P_{L} = 5.72$, 6.23 GeV/c); and only in one case they lie low ((at $P_{L} = 3.7$ GeV/c). If we take a more complicated phase, Eq. 5, the results of the fitting procedure will be practically the same (see Table 1). At last, if we use our phase, taking into account the two photon approximation and the dipole form factor, the new fitting procedure give the different sizes of $\rho(s)$ (the last coulomb of Table 1). Now the results lie above $\rho_{exper.}$ for all examined energies. Hence, the difference with the predictions of the dispersion analysis only increases.
4 Table 1

Proton-antiproton scattering (the phase dependence)

| $P_L$ (Gev/c) | N  | $\rho_{\text{exp.}}$ | $\rho_{\varphi}(\text{Born})$ | $\rho_{\varphi}(\text{Cahn})$ | $\rho_{\varphi}(\text{our})$ |
|----------------|-----|----------------------|-------------------------------|-------------------------------|-------------------------------|
| 3.702          | 34  | $+0.018 \pm 0.03$    | $+0.0077 \pm 0.02$            | $+0.0078 \pm 0.08$           | $+0.028 \pm 0.08$            |
| 4.066          | 34  | $-0.015 \pm 0.03$    | $+0.0377 \pm 0.02$            | $+0.0378 \pm 0.08$           | $+0.0324 \pm 0.08$           |
| 5.603          | 215 | $-0.047 \pm 0.03$    | $+0.035 \pm 0.02$             | $+0.036 \pm 0.08$            | $-0.0017 \pm 0.08$           |
| 5.724          | 115 | $-0.051 \pm 0.03$    | $+0.0139 \pm 0.02$            | $+0.014 \pm 0.08$            | $-0.0088 \pm 0.08$           |
| 5.941          | 140 | $-0.063 \pm 0.03$    | $-0.0003 \pm 0.02$            | $-0.004 \pm 0.08$            | $-0.0055 \pm 0.08$           |
| 6.234          | 34  | $-0.06 \pm 0.03$     | $+0.0162 \pm 0.02$            | $+0.0162 \pm 0.08$           | $-0.0216 \pm 0.08$           |

4 Impact of the spin-flip contribution

Usually, one makes the assumptions that the imaginary and real parts of the spin-non-flip amplitude have an exponential behavior with the same slope, and the imaginary and real parts of the spin-flip amplitudes, without the kinetic factor $\sqrt{|t|}$ [19], are proportional to the corresponding parts of the non-flip amplitude. For example, in [20] the spin-flip amplitude was chosen in the form

$$F_{fl}^h = \sqrt{-t/m_p} h_{sf} F_{nf}^h.$$  

That is not so as regards the $t$ dependence shown in Ref. [21], where $F_{fl}^h$ is multiplied dependent on $t$. Moreover, one mostly takes the energy independence of the ratio of the spin-flip parts to the spin-non-flip parts of the scattering amplitude. All this is our theoretical uncertainty [22,23].

According to the standard opinion, the hadron spin-flip amplitude is connected with the quark exchange between the scattering hadrons, and at large energy and small angles it can be neglected. Some models, which take into account the non-perturbative effects, lead to the non-dying hadron spin-flip amplitude [24]. Another complicated question is related to the difference in phases of the spin-non-flip and spin-flip amplitude.

In [25], it was shown that the analysis of the low energy experimental data does not reveal the impact of the contribution of the spin-flip amplitude on the extracted value of $\rho(s, t)$. Our opinion is that this result must be checked up, as at low energies the size of the spin-flip amplitude determined by the second region contributions has to be sufficiently large. In this work, we examine the simple model of the spin-flip amplitude and try to find its impact on the determination of $\rho(s, t)$ from the low energy data of proton-proton scattering. Hence, we take the spin-non-flip amplitude in the simplest exponential form

$$F_{nf}^h = h_{nf} [1 + \rho(s, t = 0)] e^{Bt/2},$$  

and the spin-flip amplitude in the form of eq. (8) The differential cross section in this case will be

$$\frac{d\sigma}{dt} = 2\pi \left[ |\phi_{nf}|^2 + 2|\phi_{sf}|^2 \right],$$  

where the amplitudes $\phi_{nf}$ and $\phi_{sf}$ include the corresponding electromagnetic parts and the Coulomb-hadron phase factors.
The results of our new fits of the proton-antiproton experimental data at $P_L = 3.76.2$ GeV/c are presented in Table 2. The changes of $\sum_i \chi^2$ after including the contribution of the spin-flip amplitude is reflected as the coefficient

$$R\sum_i \chi^2 = \frac{\sum_i \chi^2 \text{ without } sf. - \sum_i \chi^2 \text{ with } sf.}{\sum_i \chi^2 \text{ without } sf.}.$$  

(11)

| $P_L$(GeV/c) | N  | $\rho_{\text{exp.}}$ | $R\chi^2$ | $\rho_{\text{model}}$ | $h_{\text{spin-flip}}$ |
|--------------|----|----------------------|----------|-----------------------|-------------------------|
| 3.702        | 34 | $+0.018 \pm 0.03$    | 8%       | $+0.057 \pm 0.02$    | 49.8 $\pm$ 1.4         |
| 4.066        | 34 | $-0.015 \pm 0.03$    | 25%      | $+0.052 \pm 0.009$   | 48.9 $\pm$ 0.7         |
| 5.603        | 215| $-0.047 \pm 0.03$    | 3.5%     | $+0.014 \pm 0.005$   | 35.6 $\pm$ 4.4         |
| 5.724        | 115| $-0.051 \pm 0.03$    | 6.5%     | $+0.023 \pm 0.004$   | 38.2 $\pm$ 4.5         |
| 5.941        | 140| $-0.063 \pm 0.03$    | 4.5%     | $+0.007 \pm 0.003$   | 43.2 $\pm$ 4.4         |
| 6.234        | 34 | $-0.06 \pm 0.03$     | 1%       | $-0.016 \pm 0.001$   | 3.5 $\pm$ 0.3          |

We again obtain the sizes of $\rho$ near zero and most part positive. So the results do not come to the sizes are predicted by the dispersion analysis of [12]. Except the last energy the contribution of the spin-flip amplitude is filing and impact on the sizes of $\rho$. Most remarkable is that the obtained size of the constant of the spin-flip amplitude coincides for practically all examined energies, except the last. These constants have a larger size and sufficiently small errors. It is shown that in the careful analysis of the size of $\rho$ we need to take into account the contribution of the spin-flip amplitude to the differential cross section at small angles, at least at low energies. Of course, a more accurate analysis with the examined different $t$ dependence of the spin-flip amplitudes is needed.

5 Conclusion

The future new data from LHC experiments will give the possibility to carry out the new analysis of the dispersion relation which can open a new effects, for example, a fundamental length order TeV. However, for such analysis one also needs the knowledge of the low energy data with high accuracy. Now the existing forward-scattering data at $P_L = 440$ (GeV) of the size $\rho(s, t)$ contradict each other. The $\rho(s, t)$ - data of the proton-antiproton scattering at $P_L = 3.76.2$ (GeV) contradict the old dispersion analysis.

The present analysis, which includes the contributions of the spin-flip amplitudes, also shows a large contradiction between the extracted value of $\rho(s, t)$ and the predictions from the analysis based on the dispersion relations. However, our opinion is that it an additional analysis is needed which will include additional corrections connected with the possible oscillation in the scattering amplitude and with the $t$-dependence of the spin-flip scattering amplitude. We hope that the forward experiments at NICA can give valuable information for the improvement of our theoretical understanding of the strong hadrons interaction. This is especially true for the experiment at NICA with polarized beams.
The author would like to thank J.R. Cudell for helpful discussions, gratefully acknowledges the financial support from FRNS and would like to thank the University of Liège where part of this work was done.

References

1. A. Martin, Zeitschr.Phys. C15, (1982) 185
2. M. Sawamoto, S.Wakaizumi, Proc. Theor. Phys. 62, (1979) 1293
3. O. V. Selyugin, Mod. Phys. Lett. A9, (1994) 1207
4. O. V. Selyugin, Mod. Phys. Lett. A14, (1999) 223
5. O. V. Selyugin, Phys. Rev. D 60 (1999) 074028
6. O. Selyugin, Sov. J. Nucl. Phys. 55, (1992) 466
7. O. Selyugin, Phys. Lett. B333, (1994) 245
8. J.R. Cudell et al. [COMPETE Collaboration], Phys. Rev. D 65 (2002) 074024 [arXiv:hep-ph/0107219]; Phys. Rev. Lett. 89 (2002) 201801 [arXiv:hep-ph/0206172]
9. J.R. Cudell, V. Ezhela, K. Kang, S. Lugovsky and N. Tkachenko, Phys. Rev. D 61 (2000) 034019 [Erratum-ibid. D 63 (2001) 059901] [arXiv:hep-ph/9908218]
10. D. Gross, et al., Phys. Rev. Lett. 41, (1978) 217; A. A. Kuznetzov et al., preprint JINR P1-80-376, Dubna (1980)
11. L. A. Fajardo et al., Phys. Rev., D24, (1981) 46
12. P. Kroll, W. Schweiger, Nucl. Phys. A503, (1089) 865
13. R. Cahn, Zeitschr. fur Phys. C 15, (1982) 253
14. G. B. West, D. R. Yennie, Phis. Rev. 172, (1968) 1414
15. L. I. Lapidus, Particles & Nuclei 9, (1978) 84
16. N. H. Buttimore, E. Gotsman, E. Leader, Phys. Rev. D 35, (1987) 407
17. S. Trokenheim, Fermilab-PhD-1995-40, (1995)
18. Durham HepData Project, M. R. Whalley, http://durpdg.dur.ac.uk/hepdata/reac.html
19. K. R. Schubert, In Landolt-Bronstein, New Series, v.1/9a, (1979)
20. N. H. Buttimore et al., Phys.Rev. D 59 (1999) 114010
21. N. Akchurin, N. H. Buttimore, A. Penzo, Phys.Rev. D 51, (1995) 3944
22. C. Bourrely, J. Soffer, hep-ph/9611234
23. J.-R. Cudell, E. Predazzi, and O.V. Selyugin, Eur. Phys. J. A 21 (2004); hep-ph/0401040
24. A.V. Martin, and E. Predazzi, Phys. Rev. D 66, (2002) 034029
25. B.Z. Kopeliovich and B.G. Zakharov, Phys.lett. B 156 (1989); M. Anselmino and S. Forte, Phys. Rev. Lett. 71, (1993) 223 ; A.E. Dorokhov, N.I. Kochelev and Yu.A. Zubov, Int. Jour. Mod. Phys. A8, (1993) 603; N. Akchurin, S.V. Goloskokov, O.V. Selyugin, Int. J. Mod. Phys. A 14, (1999) 252; J. R. Cudell, E. Predazzi, O. V. Selyugin, Particles & Nuclei, 36(7), (2004) 132
25. V.A. Okorokov, in Proceedings “XII Advanced Research Workshop on High Energy Spin physics”, Dubna, September 3-7 (2007), Dubna, (2008) 117