Kinematic analysis and simulation of human upper limb rehabilitation training mechanism

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Abstract. A rehabilitation training mechanism of human upper limb is proposed, it consists of the 3-RRS parallel mechanism and the crank-rocker mechanism. First, their degrees of freedom are analyzed. Then, their kinematics are analyzed by analytical method. Finally, the 3-RRS parallel mechanism and the crank-rocker mechanism are simulated with the software UG and SolidWorks respectively, the results of simulation show that the movement path of the handle on the moving platform of the 3-RRS parallel mechanism coincides with the trajectory of the human arm, it can be used for the rehabilitation training of the human shoulder joint. For the crank-rocker mechanism, the center angles of movement path of the rocker and the handle on the connecting-rod are within the range of motion angles of human elbow joint and wrist joint, it can be used for the rehabilitation training of the elbow and wrist joints. They will all produce soft impulses at the beginning and end points, and the soft impulses will increase as the rotating speed of the driving links increases, so their driving links must rotate at a lower speed.

1. Introduction
At present, China cerebrovascular disease exists four high, that is high prevalence, high recurrence rate, high morbidity and mortality [1]. The human body trends to leave behavioral dysfunction after cerebrovascular disease, then need rehabilitation exercise training to patient [2]. The existing manual therapy methods have many drawbacks, human upper limb rehabilitation mechanisms designed at home and abroad have the disadvantages of complex structures, high cost and poor joint target [3, 4]. Human upper limb rehabilitation training mechanisms can be divided into the exoskeleton structure and the towel structure [5]. Considering that the human shoulder joint is directly connected with the trunk of human body, the installation of mechanical rehabilitation mechanisms is easy to cause injuries to other parts of the human body, increasing the body burden, and the working space will also be affected by the trunk of human body. So, the towel structure should be adopted for the rehabilitation training of the human shoulder joint. However, the elbow joint and wrist joint of the human upper limb are far away from the trunk of human body, and the movement space is large, which can facilitate the installation of mechanical rehabilitation mechanisms. Therefore, exoskeleton structure is considered.

2. Description of the human upper limb rehabilitation training mechanism
Considering the motion forms of the upper limb shoulder joint, and combining with the parallel mechanism with few degrees of freedom researched at home and abroad [6-12], the 3-RRS parallel
mechanism is used to achieve the rehabilitation of human upper limb shoulder joint. To achieve the elbow and wrist joints rehabilitation, the crank-rocker mechanism is used. The three-dimensional model of the human upper limb rehabilitation training mechanism is shown in Figure 1.

The 3-RRS parallel mechanism can be called the 3-RRS shoulder joint rehabilitation mechanism, its kinematic diagram is shown in Figure 2. It consists of the static platform \( A_1A_2A_3 \) (fixed base or frame), the moving platform \( C_1C_2C_3 \), and three branch chains RRS \( (A_iB_iC_i, \ i=1,2,3) \). Each branch chain is composed of two revolute pairs \( A_i, B_i \) and a spherical pair \( C_i \), the axes of the two revolute pairs \( A_i, B_i \) in the branch chain are parallel to each other and perpendicular to the \( oA_1 \) [6]. According to the actual needs of rehabilitation, a handle is installed in the center of the moving platform and a handle chute is opened on the moving platform.

When the 3-RRS shoulder joint rehabilitation mechanism is used for the human shoulder joint rehabilitation, the static platform must be tilted at angle \( \lambda \) relative to the horizontal position, the human body standing by the handle chute side of moving platform, fingers are holding the handle, the handle not only moves up and down with the moving platform, but also moves along the handle chute, its synthetic motion is similar to the motion of the shoulder joint.

![crank-rocker mechanism](image1)

**Figure 1.** The three-dimensional model of the human upper limb rehabilitation training mechanism.

The crank-rocker mechanism can be called the elbow and wrist joints rehabilitation mechanism, it is placed on the moving platform of the 3-RRS shoulder joint rehabilitation mechanism. In order to make it suitable for the human left and right hands, the lengths of the rocker and crank need to be set to adjustable, and handles are installed on both ends of the connecting-rod (coupler). When this mechanism is used for the elbow and wrist joints rehabilitation, the forearm of the human body passes through the sleeve on the rocker, the wrist joint passes around the revolute pair connecting the rocker and the connecting-rod, and holds the handle on the connecting-rod with the palm, so that the elbow and wrist joints will move with the movement of the rocker, thus realizing the rehabilitation training of the elbow and wrist joints.

![3-RRS parallel mechanism](image2)

**Figure 2.** The kinematic diagram of the 3-RRS shoulder joint rehabilitation mechanism.

3. **Analysis of degree of freedom**

According to the Formula of Kutzbach-Grubler [13], the degree of freedom of the 3-RRS shoulder joint rehabilitation mechanism is:

\[
M = 6(n - g - 1) + \sum_{i=1}^{3} f_i = 6 \times (8 - 9 - 1) + 15 = 3
\]  

(1)

where, \( n \) is the number of links \( (n=8) \), \( g \) is the number of kinematic pairs \( (g=9, \ it\ contains\ six\ revolute\ pairs\ and\ three\ spherical\ pairs) \), \( f_i \) is the degree of freedom of the kinematic pair \( i \). Take links \( A_1B_1, A_2B_2 \) and \( A_3B_3 \) as drivers.

The degree of freedom of the crank-Lirocker mechanism is one, take the crank as the driver.
4. Kinematic analysis and simulation of the 3-RRS shoulder joint rehabilitation mechanism

4.1. Kinematic analysis of the 3-RRS shoulder joint rehabilitation mechanism

In Figure 2, suppose the static platform \( A_1A_2A_3 \) and the moving platform \( C_1C_2C_3 \) are equilateral triangles, the geometric centers of the two platforms are \( o \) and \( O \) respectively, \( l_{io} = l_{oc} = r \) \((i=1,2,3)\).

The fixed coordinate system \( oxyz \) and the moving coordinate system \( OXYZ \) are established, the axes of \( x \) and \( X \) are parallel to \( A_2A_3 \) and \( C_2C_3 \) respectively, the axes of \( y \) and \( Y \) are along \( oA_1 \) and \( OC_1 \) respectively, the axes of \( z \) and \( Z \) are perpendicular to the static platform \( A_1A_2A_3 \) and the moving platform \( C_1C_2C_3 \) respectively [6].

In the branch chain \( A_iB_iC_i \), since the axes of the two revolute pairs \( A_i \), \( B_i \) are parallel to each other, the motion of spherical pair \( C_i \) can only move in the motion plane of \( A_i \) and \( B_i \) [6]. According to the cosine rule, the equation of the angle \( \theta_i \) between the chain \( A_iB_i \) of branch chain \( A_iB_iC_i \) and the static platform \( A_1A_2A_3 \) can be derived in triangle \( A_iB_iC_i \) and triangle \( oA_iC_i \),

\[
\theta_i = \pi - \arccos \left( \frac{r^2 + l_i^2 - L_i^2}{2rl_i} \right) - \arccos \left( \frac{l_{io}^2 + l_i^2 - l_{io}^2}{2l_i l_{io}} \right)
\]

where, \( l_i \) is the length of \( A_iC_i \), \( L_i \) is the length of \( oC_i \), \( l_{io} \) is the length of the chain \( A_iB_i \), \( l_{io} \) is the length of the chain \( B_iC_i \), \( l_i \) and \( L_i \) can be calculated according to the following vector equations established by geometric relations.

\[
\overrightarrow{AC}_i = \overrightarrow{oA}_i + \overrightarrow{A_iC}_i
\]

\[
\overrightarrow{OC}_i = \overrightarrow{oA}_i + \overrightarrow{A_iC}_i
\]

In the fixed coordinate system \( oxyz \), suppose the coordinate of point \( O \) is \((x_o, y_o, z_o)\), where \( x_o = 0 \), \( y_o = 0 \). Solving the above equation of the angle \( \theta_i \), obtain that

\[
\theta_i = \pi - \arccos \left( \frac{l_{io}^2 + z_o^2 - l_{io}^2}{2l_{io} z_o} \right)
\]

(2)

If \( z_o \) is given, the angles \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \) can be calculated. Differentiating \( \theta_i \) with respect to time \( t \) will result in velocity \( \dot{\theta}_i \). Differentiating \( \dot{\theta}_i \) with respect to time \( t \) will result in acceleration \( \ddot{\theta}_i \).

4.2. Simulation of the 3-RRS shoulder joint rehabilitation mechanism

4.2.1. Determination of main dimensions. In Figure 3, suppose the rotate angle of shoulder joint is \( \angle NJM \), the length of the upper limb is \( JM \) or \( JN \), which is 560mm. If the rotate angle of shoulder joint reaches the maximum 90°, the distance of \( MN \) is \( \sqrt{2}JM = 792mm \).

![Figure 3](image-url)

(a) The movement path of the palm
(b) Shoulder joint

**Figure 3.** The kinematic dimension relation of the shoulder joint rehabilitation mechanism.

From the 3-RRS shoulder joint rehabilitation mechanism, we known the maximum distance of \( MN \) is \( A_iB_i + B_iC_i \). If \( A_iB_i = B_iC_i \), so

\[
A_iB_i = B_iC_i = \frac{\sqrt{2}}{2} JM = 396mm
\]

(3)
4.2.2. Simulation. The 3-RRS shoulder joint rehabilitation mechanism is simulated with the software UG, the movement path of the geometric center $O$ of the moving platform is shown in Figure 4(a).

In Figure 3, to ensure that the palm moves in an arc $MN$, the handle at the center of the moving platform must move along the handle chute. Suppose the velocity of the moving platform is $v$, when the moving platform moves downward from the top position, the movement of the palm must satisfy the following equations.

$$z = 792 - ut$$

$$\left( y + 396 \right)^2 + \left( z - 396 \right)^2 = 560^2$$

If $v = \frac{792}{30} \text{mm/s}$, the displacement $y$ of the handle along the handle chute is

$$y = \sqrt{-697t^2 + 20907.7t + 156800} - 396$$

(4)

When the handle simultaneously moves along the handle chute with the displacement $y$, the movement path of the handle on the moving platform is shown in Figure 4(b). It shows that the movement path is able to coincide with the trajectory of the human arm.

![Figure 4. The simulation of the 3-RRS shoulder joint rehabilitation mechanism.](image)

The displacement curves of the handle in the $y$ and $z$ directions are shown in Figure 5.

![Figure 5. The displacement curves of the handle.](image)

(a) in the $y$ direction (b) in the $z$ direction

(a) The velocity curve of the handle (b) The acceleration curve of the handle

![Figure 6. The velocity curve and acceleration curve of the handle.](image)
The velocity curve and acceleration curve of the handle are shown in Figure 6. Generally, the velocity of human upper limbs does not exceed 700mm/s, it can be seen from Figure 6(a) that the maximum velocity is 37.9mm/s, it meets the requirement. From Figure 6(b), we can see that the acceleration curve will change abruptly at the beginning and end points, that would produce soft impulses. However, since the velocity is usually low, it can be used for the rehabilitation training of the human upper limb. In actual rehabilitation process, we should adjust the input speed according to the degree of damage to the human upper limbs, to get the best rehabilitation training intensity.

5. Kinematic analysis and simulation of the elbow and wrist joints rehabilitation mechanism

5.1. Kinematic analysis of the elbow and wrist joints rehabilitation mechanism

To achieve the elbow and wrist joints rehabilitation, the crank-rocker mechanism as shown in Figure 7 is used. The link AB is the crank, the link CD is the rocker, the lengths of the four links are \( l_1, l_2, l_3 \) and \( l_4 \). The distance between the handle E on the connecting-rod and the revolute pair C is \( l_5 \). Considering that the reciprocating stroke of crank-rocker mechanism used for rehabilitation training of elbow and wrist joints is working stroke, the crank-rocker mechanism should have no quick-return characteristics. It must satisfy the following equations.

\[
\begin{align*}
l_1 &= l_5 \sin \frac{\psi}{2} \\
l_1^2 + l_3^2 &= l_2^2 + l_4^2 \\
\cos \gamma_{\text{min}} &= \frac{l_1^2 + l_3^2 - (l_4 - l_2)^2}{2l_1l_3}
\end{align*}
\]

where, \( \psi \) is the angular stroke of the rocker, \( \gamma_{\text{min}} \) is the minimum transmission angle.

According to the structure and motion size of human upper limb, let \( l_3=240mm, \ \psi = 90^\circ, \ \gamma_{\text{min}} = 42^\circ, \ l_5=50mm \). Then \( l_1=169.7mm, l_2=489mm, l_4=517.6mm \).

![Figure 7. The crank-rocker mechanism.](image)

![Figure 8. The movement paths of the revolute pair C and the handle E on the connecting-rod.](image)

To achieve rehabilitation training of the elbow and wrist joints, it is necessary to do kinematics analysis of the revolute pair C and the handle E on the connecting-rod. If the crank AB rotates at constant speed \( \omega_1 \), the following equations can be derived [14].

\[
\begin{align*}
\phi_1 &= 2 \arctan \left( \frac{B + \sqrt{A^2 + B^2 - C^2}}{A - C} \right) \\
\phi_2 &= \arctan \left( \frac{B + l_3 \sin \phi_1}{A + l_3 \cos \phi_1} \right) \\
\omega_3 &= \omega_1 \cdot \frac{l_1 \sin(\phi_1 - \phi_2)}{l_5 \sin(\phi_1 - \phi_2)} \\
\omega_2 &= -\omega_1 \cdot \frac{l_1 \sin(\phi_1 - \phi_2)}{l_2 \sin(\phi_2 - \phi_3)}
\end{align*}
\]
\[ \begin{align*} 
\varepsilon_1 &= \frac{I_2 \omega_1^2 + I_1 \omega_1^2 \cos(\varphi_1 - \varphi_2) - I_2 \omega_2^2 \cos(\varphi_1 - \varphi_2)}{l_1 \sin(\varphi_1 - \varphi_2)} \\
\varepsilon_2 &= \frac{I_2 \omega_1^2 - I_1 \omega_1^2 \cos(\varphi_1 - \varphi_2) - I_2 \omega_2^2 \cos(\varphi_2 - \varphi_1)}{l_1 \sin(\varphi_1 - \varphi_2)} \\
\nu_c &= I_1 \omega_1 \\
\nu_e &= \sqrt{\nu_{1e}^2 + \nu_{2e}^2 - 2 \nu_{1e} \nu_{2e} \cos(180^\circ - \varphi_1 + \varphi_2)} \\

\end{align*} \]

(10)

where, \( A = l_4 - l_1 \cos \varphi_1 \), \( B = -l_1 \sin \varphi_1 \), \( C = \frac{\hat{A}^2 + \hat{B}^2 + l_2^2 - l_3^2}{2l_1} \), \( \nu_{1e} = I_1 \omega_1 \), \( \nu_{2e} = (l_2 - l_3) \omega_2 \).

Shown in Figure 7, \( \varphi_1, \varphi_2 \) and \( \varphi_3 \) are the angular displacements of links \( AB, BC \) and \( CD \), respectively. \( \omega_2 \) and \( \omega_3 \) are the angular velocities of links \( BC \) and \( CD \) respectively. \( \varepsilon_2 \) and \( \varepsilon_3 \) are the angular accelerations of links \( BC \) and \( CD \) respectively. \( \nu_c, \nu_k \) and \( \nu_b \) are the velocities at points \( C, E \) and \( B \) respectively. \( \nu_{1e} \) is the relative velocity of point \( E \) relative to point \( B \).

Differentiating \( \nu_c \) with respect to time \( t \) will result in acceleration of the revolute pair \( C \). Differentiating \( \nu_k \) with respect to time \( t \) will result in acceleration of the handle \( E \) on the connecting-rod.

5.2. Simulation of the elbow and wrist joints rehabilitation mechanism

The elbow and wrist joints rehabilitation mechanism are simulated with the software SolidWorks. The movement paths of the revolute pair \( C \) on the rocker and the handle \( E \) on the connecting-rod are shown in Figure 8. The center angles of their movement paths are 90° and 93° respectively, which are within the range of motion angles of human elbow joint and wrist joint. It can be used for the rehabilitation training of the human elbow joint and wrist joint.

![Figure 9](image1.png)

Figure 9. The velocity curve and acceleration curve of the revolute pair \( C \).

![Figure 10](image2.png)

Figure 10. The velocity curve and acceleration curve of the handle \( E \) on the connecting-rod.

When the crank \( AB \) rotates at a constant speed 10r/min, the velocity curve and acceleration curve of the revolute pair \( C \) are shown in Figure 9, the velocity curve and acceleration curve of the handle \( E \) on the connecting-rod are shown in Figure 10. We can see that their velocity curves and acceleration curves are smooth continuous curves, which will produce soft impulses at the beginning and end.
points, and the soft impulses will be greater as the speed of the crank $AB$ increases. So lower speed of the crank is more suitable for rehabilitation training of elbow and wrist joints.

6. Conclusions

(1) The human upper limb rehabilitation training mechanism is proposed, it consists of the 3-RRS parallel mechanism and the crank rocker mechanism.

(2) For the 3-RRS shoulder joint rehabilitation mechanism, the movement path of the handle on the moving platform coincides with the trajectory of the human arm. It can be used for the rehabilitation training of the human shoulder joint.

(3) For the elbow and wrist joints rehabilitation mechanism, the center angles of movement path of the revolute pair $C$ and the handle $E$ on the connecting rod are within the range of motion angles of human elbow joint and wrist joint. It can be used for the rehabilitation training of the elbow and wrist joints.

(4) The 3-RRS shoulder joint rehabilitation mechanism, the elbow and wrist joints rehabilitation mechanism will all produce soft impulses at the beginning and end points, and the soft impulses will increase as the rotating speed of the driving links increases. So, the driving links must rotate at a lower speed, and we should adjust the rotating speed of the driving links according to the degree of damage to the human upper limb, to get the best rehabilitation training intensity.

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