ON THE DURATION OF LONG GRBS: EFFECTS OF BLACK HOLE SPIN

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ABSTRACT

In the frame of the collapsar model for long gamma-ray bursts (GRBs), we investigate the formation of a torus around a spinning black hole (BH), and we check what rotational properties a progenitor star must have in order to sustain torus accretion over relatively long activity periods. We also study the time evolution of the BH spin parameter. We take into account the coupling between BH mass, its spin parameter, and the critical specific angular momentum of accreting gas needed for the torus to form. The large BH spin reduces the critical angular momentum which in turn can increase the GRB duration with respect to the Schwarzschild BH case. We quantify this effect and estimate the GRB durations in three cases: when a hyperaccreting torus operates or a BH spins very fast or both. We show under what conditions a given progenitor star produces a burst that can last as short as several seconds and as long as several hundred of seconds. Our models indicate that it is possible for a single collapse to produce three kinds of jets: (1) a very short, lasting between a fraction of a second and a few seconds, “precursor” jet, powered only by a hyperaccreting torus before the BH spins up, (2) an “early” jet, lasting several tens of seconds and powered by both hyperaccretion and BH rotation, and (3) a “late” jet, powered only by the spinning BH.

Subject headings: accretion, accretion disks — black hole physics — gamma rays: bursts

1. INTRODUCTION

The commonly accepted mechanism for a long gamma-ray burst (GRB) production invokes a collapsar scenario (Woosley 1993; Paczyński 1998; MacFadyen & Woosley 1999). In this model, the material from the collapsing star feeds the accretion disk, then the accretion energy is transferred to the jet, which in turn produces gamma rays at some distance from the central engine. Therefore, the whole event cannot last much longer than the existence of a rotationally supported torus in the collapsar center. Within the collapsar model the jet can also be produced by a rotating black hole (BH) which can be spun up by the accreting torus material.

Among the most plausible mechanisms of energy extraction from the accretion flow are the neutrino-neutrino annihilation (Mochkovitch et al. 1993) or the magnetic fields (e.g., Blandford & Payne 1982; Contopoulos 1995; Proga et al. 2003). The neutrino cooling (e.g., Popham et al. 1999; Di Matteo et al. 2002; Janiuk et al. 2004) is effective only if the accretion rate is large ($m \gtrsim 0.01 M_\odot$ s$^{-1}$). Also, a large BH spin ($A_{Kerr} \gtrsim 0.9$) is thought to be a necessary condition for the jet launching; for $A_{Kerr} \sim 0.9$, about 1% of the accreted rest-mass energy is emitted back as a Poynting jet (Blandford & Znajek 1977; McKinney 2005).

On the other hand, the rotationally supported torus may form only when a substantial amount of specific angular momentum is carried in the material. In our recent article (Janiuk & Proga 2008, hereafter Paper I) we studied the problem of whether the collapsing star envelope contained enough specific angular momentum in order to support the formation of the torus. This condition was parameterized by the so-called critical specific angular momentum which in case of a nonrotating BH depends only on its mass. In the present work, we take into account also the BH rotation and the coupling between the specific angular momentum of the accreting material, the BH mass, and its spin.

We show that as in Paper I, during the collapse the amount of rotating material, which was initially available for the torus for-
The normalization, $l_0$, of this dependence is defined with respect to the critical specific angular momentum, $l_{\text{crit}}$, for the seed BH,

$$l_{\text{crit}}(M, A) = \frac{2GM}{c} \sqrt{2 - A + 2\sqrt{1 - A}},$$ \hspace{1cm} (3)

so that $l_0 = x l_{\text{crit}}$, where $x$ is a free parameter. In equation (3), $M$ is the initial BH mass (iron core mass) and $A \equiv A_{\text{Kerr}}$ is its initial dimensionless spin parameter (see Appendix A).

In model D, we assume that the specific angular momentum depends on the polar angle, as well as on the radius in the envelope, as

$$l_{\text{spec}} = l_0 g(r) f(\theta),$$ \hspace{1cm} (4)

where

$$g(r) f(\theta) = \sqrt{\frac{r}{r_{\text{in}}}} \sin^2 \theta,$$ \hspace{1cm} (5)

$r_{\text{in}}$ is the inner radius of the envelope, and $l_0$ is given below equation (3).

The model D corresponds to a constant ratio between the centrifugal and gravitational forces. Note that the strong increase of $l_{\text{spec}}$ with radius will lead to a very fast rotation at large radii. Therefore, a cutoff may be required at some maximum value, $l_{\text{max}}$ (§ 3.2).

The normalization of the models is chosen such that the specific angular momentum is always equal to the critical value at $\theta = 90^\circ$ (and at $r = r_{\text{in}}$ if the model depends on radius). In § 3 we present the results of our calculations considering a range of $x$.

Initially, the mass of the BH is given by the mass of the iron core of the star, $M = M_{\text{core}}$. The initial conditions for the torus formation in the collapsar are such that only a fraction of the envelope material carries a specific angular momentum larger than the (initial) critical value. As shown in equation (3), $l_{\text{crit}}$ is defined by the mass of the BH, $M$, and its spin, $A_{\text{Kerr}}$. However, as the collapse proceeds, the mass of the BH will increase and also its spin will change (increase or decrease, depending on the accretion scenario). Therefore, the critical specific angular momentum will be changing as well.

To compute the mass of the part of the envelope that has specific angular momentum large enough to form a torus around a given BH and to estimate the time duration of the GRB powered by accretion, we need to know the BH mass and spin. We assume that at each step of the evolution the BH grows by accreting a mass $\Delta m^k$,

$$M^k = M^{k-1} + \Delta m^k,$$ \hspace{1cm} (6)

and that the BH angular momentum changes as

$$J^k = J^{k-1} + \Delta J^k.$$ \hspace{1cm} (7)

Here, the increment of mass of the BH is

$$\Delta m^k = 2\pi \int_{r_{\text{in}}}^{r_{\text{f}}} \int_0^\pi \rho(r, \theta) r^2 \sin \theta d\theta dr,$$ \hspace{1cm} (8)

and the accreted angular momentum is

$$\Delta J^k = 2\pi \int_{r_{\text{in}}}^{r_{\text{f}}} \int_0^\pi \min[l(r, \theta), l_{\text{crit}}(M, A)] \rho(r, \theta) r^2 \sin \theta d\theta dr.$$ \hspace{1cm} (9)

In the above equation we take into account the fact that the angular momentum larger than $l_{\text{crit}}$ is not accreted onto the BH, but transported outward. In this way we provide the physical condition for the spin parameter, which must always be $A_{\text{Kerr}} \leq 1.0$. (However, we do not specify any particular mechanism responsible for the angular momentum transport.)

The new spin parameter will then be equal to

$$A^k = \frac{c J^k}{G(M^k)^2},$$ \hspace{1cm} (10)

In the next step of the iteration, both the new parameter and new mass of the BH will affect the critical specific angular momentum. Now, depending on the accretion scenario, the part of the envelope material determined by the new $l_{\text{crit}}$ will accrete onto the BH.

We consider here three possible accretion scenarios:

1. The accretion onto the BH proceeds at the same rate both from the torus and from the gas close to the poles (uniform accretion).
2. The envelope material with $l < l_{\text{crit}}$ falls on the BH first. Thus, until the polar funnel is evacuated completely, only this gas contributes to the BH mass. After that, the material with $l > l_{\text{crit}}$ accretes.
3. The accretion proceeds only through the torus, and only this material contributes to the BH growth. In this case the rest of the envelope material is kept aside until the torus is accreted.

The above iterative procedure and the accretion scenarios were described in Paper I and illustrated in Figure 2 there. The main modification in the present work is the nonzero spin parameter of the BH, which leads to a different initial condition for $l_{\text{crit}}$ and more complex evolution of the collapsar. Now $l_{\text{crit}}$ is coupled to both the BH mass $M$ and the spin parameter $A_{\text{Kerr}}$.

Because of the increasing BH mass and its changing spin, the critical angular momentum also changes. We always stop the calculations when there is no material with $l > l_{\text{crit}}$, i.e., able to form a torus. However, in a real situation the GRB prompt phase will be stopped earlier, if the free-fall timescale is too large, or the accretion rate is too small, to be adequate to power the GRB ($\dot{m} = 0.01 - 1.0 M_\odot s^{-1}$). Also, in the present model it is important that the BH spin parameter is large during the GRB emission.

The duration of the GRB is estimated as the ratio between the mass accreted through the torus and the accretion rate $\dot{m}$,

$$T_{\text{GRB}} = \frac{M_{\text{accer}}}{\dot{m}},$$ \hspace{1cm} (11)

assuming that the GRB prompt emission is equal to the duration of the torus replenishment. The accretion rate, $\dot{m}$, depends on time, and we determine it instantaneously during the iterations by the free-fall velocity of gas in the torus. Finally, we impose the conditions for a minimum accretion rate and the minimum spin parameter. We then estimate the GRB duration as the ratio between the total mass accreted in the torus and the mean accretion rate.

3. RESULTS

3.1. Models with the Specific Angular Momentum Dependent on $\theta$

In this section we present the results for model A of the specific angular momentum distribution in the collapsing star and for accretion scenarios 1, 2, and 3. The models are hereafter...
labeled as A1, A2, and A3. In this model, \( \lambda_{\text{spec}} \) does not depend on radius, but only on the polar angle, \( \theta \). The normalization of this distribution, \( x = \lambda_{\text{spec}} / \lambda_{\text{crit}} \), is a free parameter of our model, and the results are presented as either a function of \( x \) or for some chosen, exemplary values of \( x \).

First, we study how much mass can be accreted onto the BH during the collapse, both in total and through the rotating torus, as long as such a torus exists. This, in the first approximation, will give an estimate of the GRB duration, because it is proportional to the amount of material which is available for accretion.

Figure 1 shows the mass accreted onto the BH, as a function of \( x \). The left panel of this figure shows the scenario A1, in which the mass accretes uniformly. The thick solid line is for the total accreted mass, and the thinner line is for the fraction of mass which has been accreted through the torus. The single dashed line shows the scenario A3, in which the mass accretes only through the torus. We see that in the model A3, the accreted mass can be larger than in the model A1 (for \( x \leq 8 \)), although the accretion in the model A3 proceeds only through the torus. This is because in model A3, the BH spin can only increase, which in turn lowers the value of \( \lambda_{\text{crit}} \), making the condition for \( \lambda_{\text{spec}} > \lambda_{\text{crit}} \) easier to be satisfied. Although at the same time the growing BH mass makes \( \lambda_{\text{crit}} \) increase, for small \( x \) this effect is less than the effect of the BH spin. In model A1, the BH spin decreases (see below), so both the decreasing spin and increasing BH mass affect \( \lambda_{\text{crit}} \) in the same way.

The right panel of Figure 1 shows the scenario A2, in which the matter accretes onto the BH first from the poles and then through the torus. In this scenario, for \( x < 7 \), there is no torus accretion. This is because the condition for \( \lambda_{\text{spec}} > \lambda_{\text{crit}} \) is never satisfied after the polar material has accreted onto the BH. The total mass accreted is that from the poles. Only for \( x > 7 \), some fraction of the envelope material is still capable of forming the torus and accretes through it (thin line). This mass adds to the total accreted mass (thick line).

All the results shown here are for a rotating BH (the initial Kerr parameter was assumed \( A_0 = 0.85 \), and in all the models we had \( A_{\text{Kerr}} > 0 \) throughout the collapse; see below). In general, the mass accreted onto the spinning BH was larger than in the case of the nonrotating BH studied in Paper I. For instance, for \( x = 10 \), it is about \( 15.5 \) and \( 15 \) \( M_\odot \) (model A1, total accreted mass), and 14 and 12 \( M_\odot \) (model A3), for rotating and nonrotating BHs, respectively.

The above results can be understood when we study the evolution of the critical specific angular momentum during the collapse, as shown in Figure 2. The figure shows \( \lambda_{\text{crit}} \) as a function of radius. For the scenario A1, the accretion is uniform and the result depends only weakly on \( x \). However, for small \( x \) the calculations were stopped earlier, when the torus ceased to exist. For scenario A3, the results depend on \( x \). In scenario A3, some mass is accreted onto the BH. Because the BH is less massive, the increase of \( \lambda_{\text{crit}} \) is slower. Obviously, the opposite is true in scenario A2 shown in the right panel of Figure 2 (only the first phase of polar accretion is shown, for clarity).

Of course, these results are affected by the large, and changing, Kerr parameter, \( A_{\text{Kerr}} \). Because the term under the square root in equation (3) is a decreasing function of \( A_{\text{Kerr}} \), the models with a rotating BH always result in a smaller critical angular momentum than for the nonrotating BH. Therefore, the conditions for the torus existence in the former models can be satisfied more easily, and one could expect that the GRB prompt emission can last longer than we found in Paper I.

Figure 3 shows the evolution of \( A_{\text{Kerr}} \) during the collapse, for scenarios A1 and A3 in the left panel and for scenario A2 in the right panel. In the uniform accretion scenario A1, the BH first spins up and then spins down. Here, the spin evolution depends strongly on \( x \). In scenario A2, the BH spins down during the polar accretion phase, and the larger \( x \) is, the smaller the final spin is. Then, during the second phase of the model A2, i.e., during the torus accretion, the BH spins up very quickly, up to \( A_{\text{Kerr}} = 0.9999 \). The latter is not shown in the right panel of Figure 3 for clarity.

In the left panel, the dashed lines mark results for the torus accretion scenario A3, where the final spin is always \( A_{\text{Kerr}} = 0.9999 \) and only initially, very weakly, depends on \( x \).
We note here that the BH spin never reaches $A_{\text{Kerr}} = 1.0$ and only approaches this value asymptotically. The result of $A_{\text{Kerr}} = 1.0$ would be unphysical, while the limit of $A_{\text{Kerr}} \leq 1.0$ is naturally provided by our model, in which only the specific angular momentum of $l_{\text{spec}} \leq l_{\text{crit}}$ contributes to the BH spin.

The evolution of the BH spin is summarized again in Figure 4. Here we plot the final value of $A_{\text{end}}$ as a function of $x$. As Figure 4 shows, $A_{\text{end}}$ can be less than the initial value of $A_0 = 0.85$ for models A1 and A2. In other words, the effective spin down of the BH is possible either for the uniform accretion, A1, but with a small normalization parameter $x$, or in the two-stage accretion, A2, but when the normalization $x$ is so small that the rotating torus is unable to form. In other models, the BH is either effectively spun up, to $A_{\text{Kerr}} = 0.9999$ (model A3), or the final spin does not differ much from the initial one (model A1, large $x$).

We checked that these results only very weakly depend on the assumed $A_0$. For $A_0 = 0.75$ and 0.95, the final distribution of $A_{\text{end}}$ with $x$ is also very close to that for $A_0 = 0.85$. Interestingly, this means that in the case of an initially rapidly spinning BH with $A_0 = 0.95$, the object is always effectively spun down by the uniform accretion.

### 3.2. Models with the Specific Angular Momentum Dependent on $r$ and $\theta$

Now, we investigate how the total accreted mass and, in consequence, the duration of the GRB will be affected if $l_{\text{spec}}$ in the

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**Fig. 2.**—Critical specific angular momentum during the collapse as a function of radius $r_\ast$ (the current inner radius of the envelope as it keeps accreting onto BH). *Left*: Uniform accretion scenario (A1; *solid lines*) and the torus accretion scenario (model A3; *dashed lines*), for a range of normalizations of the specific angular momentum, $x = 1.5, 2.0, 3.0, 5.0, \text{ and } 7.0$, marked on the right for each line. *Right*: Accretion scenario A2 (only phase 1), for the same normalizations $x$.

**Fig. 3.**—Same as Fig. 2, but for the BH spin parameter during the collapse.
collapsing star is given by a fixed ratio of the centrifugal to the gravitational force. We discuss here the model \( D \) for the \( I_{\text{spec}} \) distribution, and the three accretion scenarios are referred to as \( D_1 \), \( D_2 \), and \( D_3 \).

In this model, the specific angular momentum is a strong function of radius. Therefore, in a realistic situation we must have a maximum value of the specific angular momentum, \( I_{\text{max}} \). Here we adopt a moderate value of \( I_{\text{max}} = 10^{17} \text{ cm}^2 \text{ s}^{-1} \), following MacFadyen & Woosley (1999) and Proga et al. (2003). Figure 5 shows the mass accreted onto the BH in three accretion scenarios (\( D_1 \) and \( D_3 \) in the left panel and \( D_2 \) in the right panel). Contrary to what was found in Paper I, the accreted mass is constant with \( x \) only for the torus accretion scenario \( D_3 \), while in models \( D_1 \) and \( D_2 \) it depends on \( x \). This is because the critical specific angular momentum depends now not only on the BH mass but also on the spin parameter. The BH spin is changing during the collapse in models \( D_1 \) and \( D_2 \), because the accreting material is not only that from the torus (i.e., \( I_{\text{spec}} > I_{\text{crit}} \), which does not
influence the BH spin), but also that from the poles. The rate of change of the BH spin strongly depends on $x$, as we show in Figures 6 and 7.

Figure 6 shows the BH spin parameter, $A_{\text{Kerr}}$, as a function of radius during the collapse, and Figure 7 shows the final spin $A_{\text{Kerr}}^{\text{end}}$. When the torus accretes, similarly to model A3, in the model D3 the BH is spinning up to $A_{\text{Kerr}} = 0.9999$. However, in the uniform accretion model D1, the BH is effectively spun down for most normalizations, i.e., $x < 0.7$. For very small $x$, it is even possible for the BH to spin down almost completely at the end of the collapse. The same is true for the first phase of the scenario D2, i.e., the polar accretion. The existence of a torus in the second phase is possible only for $x > 0.7$, and in this case, the BH finally spins up to $A_{\text{Kerr}} = 0.9999$. For $x \geq 0.7$ in models D1 and D2, the BH spin slightly fluctuates. This is because of the density profile in the accreting envelope, which is not perfectly smooth, but consists of layers, in which various heavy elements are dominant. In models D, the specific angular momentum is a function of radius, which makes the angular momentum accreted onto the BH much more sensitive to the position of the current shell than in the case of models A. As a result, for some layers the BH may accrete more mass than the angular momentum, and $A_{\text{Kerr}}$ decreases.

**Fig. 6.** BH spin parameter during the collapse as a function of radius $r_k$ (the current inner radius of the envelope as it keeps accreting onto BH). Left: Uniform accretion scenario (D1; solid lines) and the torus accretion scenario (model D3; dashed lines), for a range of normalizations of the specific angular momentum, $x = 0.05, 0.5, 0.7, \text{and } 0.9$, marked on the right for each line. Right: Accretion scenario D2, for the same normalizations $x$.

**Fig. 7.** Final BH spin parameter after the collapse. Left: Models of the uniform accretion (D1; solid line) and torus accretion (D3; dashed line), as a function of the initial normalization of the specific angular momentum. Right: Accretion scenario D2, with first phase of polar accretion (dashed line) and second phase of torus accretion (solid line).
while for some other layers the BH obtains more angular momentum than mass, and $A_{Kerr}$ increases (see eqs. [8] and [9]). This is not the case for the model D3, because here the angular momentum that contributes to the BH spin is always given by $\dot{l}_{\text{crit}}$.

### 3.3. Duration of a GRB

In the first approximation, the duration of a GRB may be proportional to the mass accreted via the torus during the collapse (as shown in Figs. 1 and 5). However, as the accretion rate is not constant, the torus accretion will depend also on the accretion rate (see eq. [11]).

Figure 8 shows the instantaneous accretion rate during the collapse as a function of radius $r_i$ (the current inner radius of the envelope as it keeps accreting onto BH). The plots show two models and three exemplary values of the normalization parameter $x$. Left: Plots of model A with $x = 7.0, 5.0$, and 1.5 (marked by numbers), where solid lines mark accretion scenario 1 and dashed lines mark scenario 3. Right: Plots of model D with $x = 0.7, 0.5$, and 0.05 (marked by numbers), where solid lines mark scenario 1 and dashed lines mark scenario 3.

Another limitation for an efficient central engine will be the minimum spin of the BH. Here we assume a conservative value of $A_{\text{min}} = 0.9$, to provide the energy source for the jet (McKinney 2005). Figure 9 shows the duration of the central engine activity as a function of $x$, for both models A and D and for all three accretion scenarios. The plots account for the central engine activity time, when both assumptions are satisfied, i.e., the accretion rate must be larger than $\dot{m}_{\text{min}}$ and the BH spin must be larger than $A_{\text{min}}$.

As Figure 9 shows, the torus accretion scenario, marked with a dashed line, leads to the longest duration of a GRB: up to 50 s in model A3 and up to 130 s in model D3. In this scenario, the BH spin is always larger than our minimum value, and in most cases, the BH was spun up to $A_{Kerr} = 0.9999$. Therefore, in practice, what determines the GRB duration in this case is the accretion rate. Consequently, the model A3, in which the accretion rate is larger, results in shorter GRBs than the model D3.

The uniform accretion scenario leads to a shorter GRB than in the case of torus accretion. Now, also the condition for a minimum BH spin is important, because for some models the BH has not spun up or has been spun down, below $A_{Kerr} = 0.9$. In the model A1, for very small $x$, no GRB was produced. Moreover, in model D1 the GRBs occurred only for $x > 0.7$ (neither the spin nor the accretion rate conditions were satisfied for smaller $x$), and the longest GRB duration was $T \approx 100$ s.

The shortest GRBs were produced by scenario 2, i.e., the two-phase accretion (obviously, only in these models which had the second phase with a torus accretion). In models A2 and D2, the activity of the GRB central engine was never longer than 50 s.

Figure 9 shows the duration of a GRB resulting from the assumption of a minimum accretion rate and the minimum BH spin. However, as we discuss below in § 4, these two conditions refer to the two various mechanisms of powering the jet, which is emitting the gamma rays. The time $T_{\text{GRB}}$ is different, if we consider only one of these mechanisms, i.e., impose only one of the above conditions. For instance, if we took into account only the
condition for a minimum BH spin, in model A3 the GRB was up to 4 times longer than that presented in the left panel of Figure 9. Also, the model D3 produced long GRBs powered only by the BH spin, for which the minimum accretion rate condition was not satisfied (the models with $x < 0.3$). The very long GRB durations for the spin condition result from the fact that at the end of the collapse, the BH is still spinning fast, while the accretion rate is very small and the mass accreted through the torus is large. On the other hand, taking into account only the condition for the accretion rate, regardless of the BH spin, led to somewhat shorter (sometimes even 2 times shorter) GRBs than these presented in Figure 9. This comes from the fact that the largest accretion rate, leading to a shorter GRB, is always at the beginning of the collapse, when the BH has not yet spun up enough.

4. DISCUSSION AND CONCLUSIONS

In this article we studied the collapsar model for long GRBs, powered by accretion onto a spinning BH, which formed from the core of a massive, rotating Wolf-Rayet star. To describe the rotation of the stellar interior, we adopted two different analytical functions, accounting for either a differential rotation (models A1, A2, A3) or a constant ratio between the gravitational and centrifugal forces (models D1, D2, D3). This study is an important test for the rotation models of the GRB progenitor stars (e.g., Heger et al. 2005; Yoon et al. 2006; Detmers et al. 2008).

To describe how the accretion proceeds during the collapse, we adopted three different scenarios: (1) uniform accretion, (2) two-phase accretion, first from the poles and then from the torus, and (3) only torus accretion. The accretion onto the BH is in our approach a homologous process, in which the subsequent shells of the envelope add their mass to the central object. The angular momentum is also accreted, but the limit for it is the critical angular momentum, to prevent the BH from spinning with $A_{\text{Kerr}} \geq 1.0$. In this sense, we assume that the whole angular momentum with $I > I_{\text{crit}}$, i.e., in the torus, is transported outward. We do not invoke any particular mechanism of transport (i.e., the viscosity), and the momentum is taken out by a negligibly small amount of mass (e.g., Pringle 1981). This simplified approach describes well a more realistic situation, in which the matter with small and large angular momentum can be mixed. Therefore, some parts of the gas with large $l_{\text{spec}}$ might reach the BH, while some other parts might be blown out with the polar outflow.

We focused on the evolution of the BH spin during the collapse. The large BH spin is important for GRB production in two ways: first, to power the jet emission via the Blandford-Znajek (BZ) mechanism, and second, because it alters the condition for the torus formation, i.e., the critical specific angular momentum. We found that the spin of the BH strongly depends on both the model of the $l_{\text{spec}}$ distribution and the accretion scenario.

In the torus accretion (i.e., either the second phase of scenario 2, models A2 and D2, or scenario 3, models A3 and D3), the accreting material has specific angular momentum always $l_{\text{spec}} \geq l_{\text{crit}}$. This angular momentum must be transported outward before reaching the BH, so that the gas which is changing the BH spin has a specific angular momentum equal to $l_{\text{crit}}$. Nevertheless, it is enough to spin up the BH to the maximal rotation, $A_{\text{Kerr}} = 0.9999$, which happens in most cases at the very beginning of the collapse. The polar accretion, i.e., the first phase of scenario 2 (models A2 and D2), leads only to the BH spin-down in all the models. The uniform accretion scenario is the most complex. In the model A1 it leads only to a temporary increase of the BH spin, while during the accretion of the outer shells, the BH is spinning down. In the model D1, the BH spin first decreases, while later during the collapse it may increase, provided the stellar envelope contains enough $l_{\text{spec}}$.

We found that in model A1, the final BH spin after the collapse is always about $A_{\text{end}} \sim 0.85$, and it does not depend on the normalization of the specific angular momentum contained in the stellar envelope, i.e., on $x$. However, the pattern of the BH spin evolution is very sensitive to this parameter. Therefore, for small values of $x$ it may happen that even for a short time during the collapse, the BH never reaches a spin $A_{\text{Kerr}} > 0.9$, which we
consider necessary to power the jet with the BZ mechanism. However, in the same models, the torus does exist and the accretion rate in the torus is large enough to power the jet via neutrino annihilation. This might lead to a relatively short lived (less than ~7–8 s) GRB central engine without a very rapidly spinning BH. On the other hand, for \( A_{\text{Kerr}} > 0.9 \), the stage of a rapidly spinning BH begins very shortly after the collapse has started and lasts much longer after the accretion rate in the torus has dropped below \( \dot{m} = 0.01 \, M_\odot \, \text{s}^{-1} \). For instance, a GRB powered by the BZ mechanism may last almost \( \sim 120 \) s, while that powered by neutrino annihilation (concurrent with the spinning BH) lasted only \( \sim 40 \) s. A very short time required for the BH to spin up, while the collapse proceeds, is of the order of \( \sim 1.5 \) s.

In model D1 the situation is different. Here we do not find any models with only the neutrino-powered bursts, i.e., with a large accretion rate but not accompanied by a rapidly spinning BH. In other words, whenever there exists a torus with a large accretion rate, the BH is spun up to \( A_{\text{Kerr}} > 0.9 \), and the timescale for this spin up is a fraction of a second (\( \sim 0.15 \) s). Similarly to model A1, the stage of a large BH spin can last much longer, after the accretion rate has dropped below \( \dot{m} = 0.01 \, M_\odot \, \text{s}^{-1} \). For instance, the BZ-powered burst lasting \( \sim 430 \) s is accompanied by a \( \sim 100 \) s burst powered by both BZ and neutrino mechanisms.

Observationally, this behavior may have led to three kinds of jets. The first is a very short, lasting between a fraction of a second and a few seconds, “precursor” jet, powered by only the neutrino annihilation, before the BH spins up. The second is an “early” jet, lasting several tens of seconds and powered by both neutrino and BZ mechanisms. The third is a “late” jet, powered by only the spinning BH via the BZ mechanism. In our models, we can have the GRB jets with all three components, as well as the “orphan precursor” jets, as the BH fails to spin up.

The precursors have been detected by Ginga, BeppoSAX, BATSE, INTEGRAL, and Swift in some GRBs (e.g., Murakami et al. 1991; Piro et al. 2005; Lazzati 2005; Romano et al. 2006; McBreen et al. 2006). These GRB precursors are an important observational test for their theoretical models (e.g., Ramirez-Ruiz et al. 2002; Umeda et al. 2005; Morsony et al. 2007; Wang & Meszaros 2007). For instance, in the sample of BATSE bursts, studied by Lazzati (2005), about 20% of the bursts had a precursor, which was characterized by a nonthermal spectrum and contained less than 1% of the total counts. The main GRB in these events was delayed with respect to the precursor by 10–200 s. As argued by Morsony et al. (2007), who in the two-dimensional numerical MHD simulations identified three distinct phases during the jet propagation, this large gap in the emission might be a selection effect. Because of different opening angles of these three jets, some observers located at large viewing angles may see a “dead” phase, i.e., the break in the emission, related to the second jet. Another explanation of the gap between the precursor and the main jet could be the development of the instabilities in the hyperaccreting disk (Perna et al. 2006; Janiuk et al. 2007), possibly combined with the viewing angle effects.

We therefore conclude that in the present model, the “dead” phase would refer to an “early” jet, which is powered by both neutrino and BZ mechanisms and can be collimated to a much narrower angle than the “late” jet. For the viewing angle larger than the “early” jet but smaller than the “late” jet opening angle, the observer should see the precursor, followed by a gap in the emission on the order of 40–150 s, and then see the “main” GRB. We also notice that recently, the observation of the bright, long GRB 080319B (Racusin et al. 2008) seems to have confirmed that the jet’s opening angle may vary, indicating the two types of jets.

Finally, comparing our current models with the results presented for a nonrotating BH (Paper I), we notice that the GRB durations are similar in the case of model A1, i.e., \( \sim 40 \) s versus \( \sim 50 \) s for the Schwarzschild and Kerr BH main jet, respectively. In model D1, the discrepancy is more pronounced, namely, \( \sim 50 \) s versus \( \sim 100 \) s, respectively. On the other hand, in the current work the model D1 produces GRBs powered by neutrino annihilation only for a very narrow range of parameters (i.e., \( x \)).

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**APPENDIX A**

**SPIN EVOLUTION**

Spacetime around astrophysical BHs is described by the Kerr metric with two parameters, the mass-energy \( M \) and the angular momentum \( J \). Below, we use dimensionless angular momentum \( A \equiv J/GM^2 \) and dimensionless radius \( \tilde{r} = r c^2 / GM \). The change of the BH parameters due to accretion of rest mass \( dm \) is given by (Moderski & Sikora 1996; see also Moderski et al. 1998)

\[
\begin{align*}
  c^2 dM &= e \, dm, \quad (A1) \\
  dJ &= l \, dm, \quad (A2)
\end{align*}
\]

where \( e \) and \( l \) are the specific energy and angular momentum of accreted matter, respectively. Combination of equations (A1) and (A2) gives the evolution equation for the BH spin,

\[
\frac{dA}{d \ln M} = \frac{l}{\varepsilon} - 2A, \quad (A3)
\]

where dimensionless quantities \( \tilde{l} \equiv cl/GM \) and \( \varepsilon \equiv e/c^2 \) can be functions of \( A \). Thus, whether spin increases or decreases depends on the sign of the expression \( \tilde{l} - 2A\tilde{\varepsilon} \).
We consider accretion from a geometrically thick disk as an example. In such a case the inner edge of the accretion disk is located at the marginally bound orbit, \( r_{mb} \), and

\[
\bar{r}_{mb} = 2 \pm A + 2(1 \mp A)^{1/2}, \quad A = \pm \bar{r}_{mb}^{1/2} \left( 2 - \bar{r}_{mb}^{1/2} \right), \tag{A4}
\]

where upper signs are for direct accretion, while lower signs are for retrograde accretion. The solution of equation (A3) is (Abramowicz & Lasota 1980)

\[
\bar{r}_{mb} = \frac{1}{2}, \tag{A5}
\]

\[
\bar{\ell}_{mb} = 2\bar{r}_{mb}, \tag{A6}
\]

where \( \bar{r}_{mb,0} \) is the initial marginally bound orbit. From equations (A7) and (A4) we obtain

\[
A = \begin{cases} 
\pm \frac{\bar{r}_{mb,0} M_0}{M} \left( 2 - \frac{\bar{r}_{mb,0} M_0}{M} \right), & \frac{M}{M_0} \leq \sqrt{\frac{\bar{r}_{mb,0}}{M_0}}, \\
\pm 1, & \frac{M}{M_0} \geq \sqrt{\frac{\bar{r}_{mb,0}}{M_0}},
\end{cases} \tag{A8}
\]

where for a given \( A_0 \), the value of \( \bar{r}_{mb,0} \) can be calculated from equation (A4). Equation (A8) together with the solution of equation (A1),

\[
M = M_0 + m, \tag{A9}
\]

describe the evolution of \( A \) as a function of the amount of the accreted rest mass \( m \).

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