A novel scheme for parametrizing the chemical freeze-out surface in Heavy Ion Collision Experiments

Sumana Bhattacharyya, Deepak Biswas, Sanjay K. Ghosh, Rajarshi Ray, and Pracheta Singha
Department of Physics, Center for Astroparticle Physics & Space Science, Bose Institute, EN-80, Sector-5, Bidhan Nagar, Kolkata-700091, India

We introduce a new prescription for obtaining the chemical freeze-out parameters in the heavy-ion collision experiments using the Hadron Resonance Gas model. The scheme is found to reliably estimate the freeze-out parameters and predict the hadron ratios, which themselves were never used in the parametrisation procedure.

PACS numbers: 12.38.Mh, 21.65.Mn, 24.10.Pa, 25.75.-q
Keywords: Heavy Ion collision, Chemical freeze-out, Hadron Resonance Gas model

Introduction: — Strongly interacting matter is expected to exhibit a rich phase structure under extreme conditions of temperature and density. Exotic phases with quasi-free quarks and gluons may have existed at high temperatures in the very early universe [1]. Even today, in the core of compact stars with high baryon densities, various exotic phases like color superconductivity, color superfluidity, may be present [2].

Direct signatures of these phases can only be accessed in experiments with relativistic nuclear collisions that are being pursued at CERN (France/Switzerland) and BNL (USA), and also to be carried out at GSI (Germany) and JINR (Russia). In the canonical picture of heavy-ion collision (HIC), the high density fireball formed, is expected to thermализze rapidly, expand out fast and then cool down quickly. As the system expands, the inter-particle distances increase, and subsequently all thermal and chemical interactions freeze-out. Finally the detected strongly interacting particles are the hadrons and their resonances which may be in chemical equilibrium [3, 4].

In a pioneering work [5] using the Hadron Resonance Gas (HRG) model, it was argued that the freeze-out surface may be universally characterized by the average energy per hadron to have a value of 1 GeV. Subsequently there has been a huge interest in studying the properties of strongly interacting matter using HRG model [6–39]. This model has successfully described hadron yields from AGS to LHC energies [8–10, 12–16, 19–23, 40–45]. In this context the discussions of multi-strange enhancement [46] and saturation of strangeness [47] in a quark-gluon phase came up. Some authors also considered possible under-saturation of strangeness in the observed spectrum [48], [10]. Bulk properties of hadronic matter have also been studied in this model [12, 13, 19–23]. Moreover, role of various undiscovered resonance states in determining the freeze-out surface has been investigated [47]. There has been recent attempts to address the freeze-out conditions even from the first principle lattice QCD [50, 51]. The general perception from all these studies is that at freeze-out the hadrons are in thermodynamic equilibrium.

Here we propose a novel fitting procedure for extraction of thermal parameters from experimental hadron yields with the ideal HRG model and show that the ratios of the yields are very well reproduced for experiments over a wide range of collision energies.

HRG Model: — The grand canonical partition function of the hadron resonance gas is given by,

$$\ln Z_{\text{ideal}} = \sum_i \ln Z_i^{\text{ideal}},$$

(1)

The sum runs over all hadrons and resonances. The thermodynamic potential for $i$th species is given as,

$$\ln Z_i^{\text{ideal}} = \pm \frac{V g_i}{(2\pi)^3} \int d^3p \ln[1\exp(-(E_i-\mu_i)/T)]$$

(2)

where the upper sign is for baryons and lower for mesons. Here $V$ is the volume, $T$ is the temperature, and for the $i$th species of hadron, $g_i$, $E_i$ and $\mu_i$ are respectively the degeneracy factor, energy and mass, while $\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S$ is the chemical potential, with $B_i$, $Q_i$ and $S_i$ denoting the baryon number, electric charge and strangeness respectively. Here $\mu_B$, $\mu_Q$ and $\mu_S$ are the baryon, electric and strangeness chemical potentials respectively. For a thermalized system the number density $n_i$ can be calculated from partition function, which is given as,

$$n_i(T, \mu_B, \mu_Q, \mu_S) = \frac{g_i}{(2\pi)^3} \int \exp[(E_i-\mu_i)/T] + 1.$$

(3)

The thermal parameters $(T, \mu_B, \mu_Q, \mu_S)$ may then be obtained by fitting experimental hadron yields to the model parametrization via the relation between the rapidity density for $i$th detected hadron to the corresponding number density in the HRG model [22],

$$\frac{dN_i}{dy}|_{\text{Det}} = \frac{dV}{dy} n_i^{\text{Total}}|_{\text{Det}}$$

(4)
where the subscript Det denotes the detected hadrons. Here,
\[ n_i^{Tot} = n_i(T, \mu_B, \mu_Q, \mu_S) + \sum_j n_j(T, \mu_B, \mu_Q, \mu_S) \times \text{Branch Ratio}(j \rightarrow i) \]  (5)
where the summation is over the heavier resonances \( j \) that decay to the \( i \)th hadron. Usually the systematics due to the volume factor is removed by considering hadron yield ratios. It is a standard practice to fit temperature \( T \) and the baryon chemical potential \( \mu_B \) using \( \chi^2 \) analysis with the available particle yield ratios, while the charge chemical potential \( \mu_Q \) and the strange chemical potential \( \mu_S \) are determined by imposing the constraints \[ \sum_i n_i(T, \mu_B, \mu_Q, \mu_S)Q_i = r \]  (6)
and
\[ \sum_i n_i(T, \mu_B, \mu_Q, \mu_S)S_i = 0 \]  (7)
where \( r \) is net-charge to net-baryon number ratio of the colliding nuclei. For example, in \( \text{Au} + \text{Au} \) collisions \( r = N_p/(N_p + N_n) = 0.4 \), with \( N_p \) and \( N_n \) denoting the number of protons and neutrons in the colliding nuclei. Here we consider a different approach rather than the \( \chi^2 \) analysis to obtain \( T \) and \( \mu_B \). Since there are four independent freeze-out parameters, we need two more independent and unique relations, along with the two constraint relations given above. This can be obtained from the conserved quantities associated with strong interactions. Accordingly, we introduce the two independent quantities as the net baryon number normalized to the total baryon number and the net baryon number normalized to the total hadron yield for the detected hadrons. The total hadron yield may be considered as a close measure for hadronic entropy \[ 52 \]. For these quantities we form the two equations,
\[ \frac{\sum_i \text{Det} B_i \frac{dN_i}{d\eta}}{\sum_i \text{Det} |B_i| \frac{dN_i}{d\eta}} = \frac{\sum_i \text{Det} B_i n_i^{Tot}}{\sum_i \text{Det} |B_i| n_i^{Tot}} \]  (8)
\[ \frac{\sum_i \text{Det} \frac{dN_i}{d\eta}}{\sum_i \text{Det} \frac{dN_i}{d\eta}} = \frac{\sum_i \text{Det} B_i n_i^{Tot}}{\sum_i \text{Det} n_i^{Tot}} \]  (9)
The ratios on the left hand side consists of the rapidity density of hadron yields measured in the HIC experiments and those on the right are the number densities calculated in the HRG model. The sum runs only over the identified hadrons for which the yield data are available. The equations are clearly unique and independent of each other if a sufficient number of identified hadrons are involved.

Data Analysis: – We have used AGS \[ 53–61 \], SPS \[ 62–71 \], RHIC \[ 72–87 \] and LHC \[ 88–91 \] data for our analysis. STAR BES data has been used following \[ 44 \], \[ 92 \] and \[ 93 \]. In the present study we have only taken mid-rapidity data for the most central collisions.

In our HRG spectrum we have used all hadrons up to 2 GeV which are confirmed with known degrees of freedom. The masses and branching ratios used are as given by the Particle Data Group \[ 94 \]. But data are available for only a few hadrons at various collision energies. The identified hadrons used to obtain the freeze-out parameters are, \( \pi^\pm \) (139.57 MeV), \( k^\pm \) (493.68 MeV), \( p, \bar{p} \) (938.27 MeV), \( \Lambda, \bar{\Lambda} \) (1115.68 MeV), \( \Xi, \bar{\Xi} \) (1321.71 MeV). All those hadrons reported at one collision energy may not be available in another. For example, we could not find the \( \Lambda \) yield at LHC. At this energy we assumed \( \bar{\Lambda} \) yield to be same as that reported for \( \Lambda \). Similarly, we did not use \( \Omega \) data for any parametrization, as the individual yields of \( \Omega^+ \) and \( \Omega^- \) are not available for most of the \( \sqrt{s} \). Also, \( \phi \) (1019.46 MeV) has been excluded from the fitting as it is already included in the model through its strong decay channel to kaon. For the lower AGS energies we could not find any anti-baryon data. There we have used \( \Lambda \) to proton ratio and total baryon to total hadron yield as a substitution of Eq. (8) and Eq. (9).

The equations Eq. (6–7, 8–9), are highly non-linear and are solved numerically using the Broyden’s method with a convergence criteria of \( 10^{-6} \) or better. We had to tune the initial conditions accordingly for different \( \sqrt{s} \) to achieve desired convergence accuracy. The variances of the fitted parameters were obtained by extracting the freeze-out parameters at the extremum values of the hadron yields given by the experimental variances.

Freeze-out Parameters: – The freeze-out parameters are depicted in Fig. 1. The general behavior as well as the quantitative estimates are commensurate with those in the existing literature. The variation of the freeze-out temperature with the center of mass energy \( \sqrt{s} \) is shown in Fig. 1a. As expected, the temperature increases with increasing \( \sqrt{s} \) and approaches a saturation \[ 3 \], except at the LHC energy where the temperature is lower. This is probably due to the lower yield of protons at LHC \[ 91 \].

In Fig. 1b the various chemical potentials are shown as functions of \( \sqrt{s} \). The baryon chemical potential \( \mu_B \) decreases with increasing \( \sqrt{s} \), which is usually understood as follows. For low collision energies a significant amount of baryons may be deposited in the collision region (baryon stopping). On the other hand at high collision energies the colliding baryons may almost pass through each other and get deposited outside the collision region. Similarly the electric charge chemical potential \( \mu_Q \) should have also followed the same trend as the colliding nuclei only consisted of positively charged protons. However \( \mu_Q \) remains negative throughout, ap-
proaching zero for increasing $\sqrt{s}$. Here, the neutrons in the colliding nuclei (lead or gold), being more abundant than the protons, induce an isospin dominance in favor of $\pi^-$ than $\pi^+$. The pions being the lightest charged particles, dictates the sign of $\mu_Q$. On the other hand, strangeness production is expected to be dominant at higher baryon densities due to the possible redistribution of Fermi momentum among larger degrees of freedom lowering the Fermi energy [9,5]. Though it is not clear whether this picture should hold in the HIC scenario, the fitted strangeness chemical potential $\mu_S$, does indeed show such a behavior.

Hadron Yield Ratios: – With the freeze-out parameters obtained, we now discuss the various predicted hadron yield ratios. Though the hadrons yields were used in the analysis, none of their individual ratios were part of the equations solved, and are therefore quite independent predictions from the model. The only exception is the use of the single ratio $\Lambda/p$ for the lower AGS energies. From the experimental data the variations in the yield ratios are obtained from those of the individual yields using standard error propagation method [9]. We have consid-
ered both systematic and statistical errors and the total error for a particular yield was obtained in quadrature. Here we discuss some important representative hadron ratios. The predictions of other hadron yields also came out satisfactorily and will be presented elsewhere.

In Fig. 2 the $\pi^-/\pi^+$ ratio is shown as a function of $\sqrt{s}$. As discussed earlier, this ratio is greater than 1 for low $\sqrt{s}$ and approaches 1 at higher collision energies. Rather than the +ve charge of the protons the higher neutron abundance seems to push for the isospin asymmetry in favor of $\pi^-$. The data are well reproduced by our analysis.

Similarly the $p/\pi^+$ and $\bar{p}/\pi^-$ variation with $\sqrt{s}$ is also well reproduced as shown in Fig. 3. At lower $\sqrt{s}$, $p > \bar{p}$ and $\pi^- > \pi^+$, while at higher $\sqrt{s}$ the corresponding
particles and antiparticles become equal. This explains the variations shown.

The $k/\pi$ ratio is considered to be an important observable for strangeness enhancement in high energy collisions. A 'horn' in the $k^+/\pi^+$ ratio was originally suggested as a signature of QGP 97–100. Several authors have tried to explain the behavior of $k^+/\pi^+$ and $k^-/\pi^-$ using different approaches (see 101 and references therein). The comparison between the experimental data for these ratios and the corresponding predictions from our model analysis is shown in Fig. 4. We find that irrespective of the underlying physical mechanism that gives rise to the horn, which is beyond the scope of the HRG model, the experimental ratios and model predictions agree quite well.

In Fig. 5 the ratios $\Lambda/p$ and $\Xi^-/p$ are shown. The agreement for $\Lambda/p$ is reasonable except for a slight downshift of the predicted results as compared to the experimental data. Consideration of possible uncertainties in contribution from weak decays may remove this discrepancy 80. Such uncertainties are not included in our analysis. The model predictions for $\Xi^-/p$ is found to agree well with the experimental data, as shown in Fig. 5a.

Finally we present some ratios for the $\phi$, and $\Omega$ particles whose yield data were never used in the analysis. The $\phi/\pi^+$ ratio is shown in Fig. 6. Since $\phi$ has no net charge of any kind, it is dependent only on the temperature. The predicted $\phi/\pi^+$ plot closely resembles the temperature plot. Again prediction from our model agrees reasonably with the experimental data.

The predictions for the $\Omega/p$ ratios are shown in Fig. 7. The experimental data are available at only a few $\sqrt{s}$. The model predictions seem to agree quite well.

Summary and outlook: — To summarize, we have used an ideal HRG model to investigate the possible chemical equilibration of hadrons at freeze-out in the HIC experi-

FIG. 4. Variation of $k^+/\pi^+$ and $k^-/\pi^-$ with $\sqrt{s}$

FIG. 5. Ratios of yields of strange baryons to proton as a function of $\sqrt{s}$.

FIG. 6. Variation of $\phi/\pi^+$ with $\sqrt{s}$
ments from the reported hadron multiplicity data in the
central rapidity bins for the most central collisions in a
wide range of center of mass energies. Instead of us-
ing the χ2 analysis we have simply equated the baryonic
charges, as well as baryon to entropy ratios between the
HRG model and experimental data to obtain the freeze-
out parameters. We could satisfactorily solve the equa-
tions and generate the most reasonable predictions of the
various hadron ratios. Even the predictions for multi-
strange hadrons like Ω, whose yields were never used in
the parametrization, are predicted satisfactorily. There-
fore this simple elegant formalism may be an useful sub-
stitution for the χ2 analysis.

This work is funded by UGC, CSIR and DST of
the Government of India. DB thanks Sabita Das and
Sandeep Chatterjee for discussion about BES data. We
thank Subhasis Samanta for various useful discussions.

[1] E. W. Kolb and M. S. Turner, Front. Phys. 69, 1 (1990).
[2] K. Rajagopal and F. Wilczek, “The condensed matter
physics of qcd,” in At The Frontier of Particle Physics,
pp. 2061–2151.
[3] R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965).
[4] R. Hagedorn and J. Rafelski, Phys. Lett. 97B, 136 (1980).
[5] J. Cleymans and K. Redlich, Phys. Rev. Lett. 81, 5284 (1998).
[6] D. H. Rischke, M. I. Gorenstein, H. Stoecker, and
W. Greiner, Z. Phys. C51, 485 (1991).
[7] J. Cleymans, M. I. Gorenstein, J. Stalnacke, and
E. Suhonen, Phys. Scripta 48, 277 (1993).
[8] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B344, 43 (1995).
[9] J. Cleymans, D. Elliott, H. Satz, and
R. L. Thews, Z. Phys. C74, 319 (1997).
[10] G. D. Yen, M. I. Gorenstein, W. Greiner, and S.-N. Yang, Phys. Rev. C56, 2210 (1997).
[11] J. Cleymans, D. Elliott, H. Satz, and
R. L. Thews, Z. Phys. C74, 319 (1997).
[12] G. D. Yen, M. I. Gorenstein, W. Greiner, and S.-N. Yang, Phys. Rev. C56, 2210 (1997).
[13] J. Cleymans, D. Elliott, H. Satz, and
R. L. Thews, Z. Phys. C74, 319 (1997).
[14] J. Cleymans, D. Elliott, H. Satz, and
R. L. Thews, Z. Phys. C74, 319 (1997).
[15] P. Braun-Munzinger, J. Stachel, and
J. Cleymans, Phys. Lett. B465, 15 (1999).
[16] J. Cleymans and K. Redlich, Phys. Rev. C60, 054908 (1999).
[17] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[18] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[19] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[20] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[21] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[22] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
[23] J. Cleymans, D. Magestro, K. Redlich, and
J. Stachel, Phys. Lett. B518, 41 (2001).
