SYNCHROTRON MODEL FOR THE INFRARED, OPTICAL, AND X-RAY EMISSION OF THE CRAB PULSAR

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ABSTRACT

We develop a model for the infrared, optical, and soft X-ray emission of the Crab pulsar in terms of anisotropic synchrotron emission by relativistic particles in an outer gap scenario with a single energy distribution $N(\gamma) \propto \gamma^{-2}$. It is shown that such a distribution is naturally produced in an efficient pair cascade and that the energy of the primary particles is limited by synchrotron radiation to $\gamma \sim 10^7$. It is further shown that this synchrotron model is able to reproduce the spectral shape between the infrared and soft X-rays and also the corresponding luminosities. In particular, the long-standing problem of the rapid spectral decline toward infrared frequencies is understandable as emission at very small pitch angles from low-energy particles with $\gamma \sim 10^2$. Finally, we show that the scaling of our synchrotron model explains the observed correlation between the X-ray luminosity and the spin-down luminosity of the neutron star $L_X \sim 10^{-3}L_{sd}$ found by Becker & Trümper.

Subject headings: pulsars: individual (PSR 0531+21) — radiation mechanisms: nonthermal — stars: neutron — X-rays: stars

1. INTRODUCTION

The Crab pulsar has a continuous spectrum from the optical to X-rays and γ-rays with different power laws (Lyne & Graham-Smith 1990). The spectral index $\alpha$, defined as $I_\nu \propto \nu^{-\alpha}$, varies from $\alpha = 1.1$ in the γ-region via 0.7 in the hard X-ray region to 0.5 at soft X-rays (Toor & Seward 1977) and zero at optical frequencies. In the far-infrared region the spectrum is inverted $\alpha = -2$ and cuts off sharply toward lower frequencies. This behavior is not accompanied by dramatic pulse profile changes, as one would expect from saturation or self-absorption effects. Self-absorption should first influence the peak intensity. Although Penny (1982) claims to have measured about 20% peak depression in the near-infrared, the situation is ambiguous. Absorption outside the pulsar environment would not influence the pulse form, but the nature of such an absorber is not clear. The conventionally used optically thin synchrotron radiation theory yields rising spectra if the particle spectrum has a low-energy cutoff, with a spectral index of $-1/3$, if the angle-integrated single particle emissivity is used and $-2/3$, if the angular dependence is retained, e.g., when the emission is spread over angles becoming comparable to the pulsar emission cone angle. The infrared spectrum measured by Middleditch, Pennypacker, & Burns (1983) therefore is still awaiting an explanation. It is one of the purposes of this paper to show that the theory of optically thin synchrotron radiation at very small pitch angles gives a possible solution to this problem. Epstein & Petrosian (1973) applied small pitch-angle synchrotron radiation to the Crab optical pulsations, but the detailed spectral features were calculated only for active galactic nuclei.

2. OUTER GAP SCENARIO AND PARTICLE ENERGY SPECTRUM

We suggest that the infrared, optical, and ultraviolet emission (and also the X-ray and γ-ray emission) can be explained in terms of an outer magnetospheric gap model (Cheng, Ho, & Rudermann 1986). Models for the γ-ray emission from outer gaps have been published by Romani & Yadigaroglu (1995) and Chiang & Romani (1994). This idea is also supported by the different widths of pulse profiles in the radio (inner magnetosphere) and the higher frequency range. The magnetic field strength close to the light cylinder of the Crab pulsar is $B \approx 10^6$ G.

In such a scenario, particles are accelerated to high Lorentz factors $\gamma_p \sim 10^7$ by large potential drops along the magnetic field lines, usually thought to be limited by curvature radiation. The electric field strength in the gap of a pulsar rotating with angular velocity $\Omega = 2\pi/P$ has been calculated by Cheng et al. (1986) to be

$$E_0 = \frac{\Omega Bd^2}{R_c c} = 6.3 \times 10^3 \left(\frac{a}{10^7 \text{ cm}}\right)^2 \left(\frac{R_c}{10^8 \text{ cm}}\right)^{-1} \times \left(\frac{P}{33 \text{ ms}}\right)^{-1} \left(\frac{B}{10^6 \text{ G}}\right) \text{ statvolt cm}^{-1},$$

where $R_c$ is the curvature radius of the field lines in the gap region and $a$ is the dimension of the gap perpendicular to the magnetic field $B$. The rate at which particles gain energy in such an electric field is

$$P_{\text{gain}} = ceE_0,$$

and the energy loss rate due to curvature radiation is

$$P_{\text{el}} = \frac{2 e^2 c}{3 R_c^2} \gamma^4 = 4.6 \times 10^{-9} R_c^{-2} \gamma^4 \text{ ergs s}^{-1}.$$
where \( \sin \Psi \approx \Psi \) has been used.

Since we are interested in the acceleration process close to the light cylinder radius \( R_{lc} \), we now calculate the accelerating electric field there. Let \( r \) and \( \theta \) be the radial distance from the neutron star center and the angle between the magnetic moment and the radius vector to a point in the magnetosphere, respectively. The angle between the magnetic moment and the rotation axis is called \( \alpha \). Further, let \( x = r/R_{ns} \), where \( R_{ns} \approx 10^6 \) cm is the radius of the neutron star. The radius of the light cylinder \( R_{lc} \) is given by \( \Omega R_{lc} = c \), so that

\[
x_{lc} = \frac{cP}{2\pi R_{ns}}.
\]

(6)

In a purely dipolar magnetic field the relation

\[
\sin^2 \frac{\theta}{r} = \frac{\sin^2 \theta_0}{R_{ns}}
\]

holds for every field line, where \( \theta_0 \) defines its footpoint at the neutron star surface. The curvature radius of a magnetic field line is known to be

\[
R_c = \frac{R_{ns}}{\sin \theta_0} x^{1/2}.
\]

(8)

In an aligned rotator the last closed field line would be characterized by \( \theta = \pi/2 \) at the light cylinder radius, whereas in the nonaligned case it is given by \( \theta = \pi/2 - \alpha \). It is then easily verified with equation (7) that

\[
\sin \theta_0 = x_{lc}^{1/2} \cos \alpha.
\]

(9)

We therefore find for the curvature radius at the light cylinder

\[
R_c = \frac{R_{ns} x_{lc}}{\cos \alpha} = \frac{R_{lc}}{\cos \alpha}.
\]

(10)

The accelerating electric field near the light cylinder is then

\[
E_0 = \frac{\alpha^2}{R_{lc}^2} \cos \alpha B = \delta^2 \cos \alpha B,
\]

(11)

which defines \( \delta \) as the ratio of the gap thickness to the light cylinder radius, in the following assumed to be of order 0.1.

One can now estimate the equilibrium Lorentz factor for which energy gains and synchrotron losses are equal as

\[
\gamma_p = 9.5 \times 10^6 \cos^{1/2} \alpha \left( \frac{\delta}{0.1} \right) \left( \frac{B}{10^6 \text{ G}} \right)^{-1/2} \left( \frac{\Psi}{10^{-5}} \right)^{-1}.
\]

(12)

These particles are called primary in the following. By inverse Compton scattering they can produce \( \gamma \)-rays with energies up to 5 TeV. We use a pitch angle of order \( \Psi \sim 10^{-3} \), which in the following sections is shown to be required to reproduce the infrared and optical spectrum, as well as the luminosity in this band.

Owing to interactions of these \( \gamma \)-rays with photons, a pair creation cascade is ignited, which can produce several generations of positrons and electrons of equal amount and a certain multiplicity \( M \) compared to the Goldreich-Julian density. The initial momentum of the created pairs will be parallel to that of the primary particle, i.e., the pairs will also have a pitch angle \( \Psi \approx 10^{-3} \). In typical cascade models the pair density exceeds the density of primaries by a factor of \( 10^3 \)–\( 10^5 \) (Melrose 1995). Numerical simulations of such a cascade showed that a power-law energy distribution \( N(\gamma) = N_0 \gamma^{-s} \) will be produced, with a typical index of \( s \approx 2 \). The first cascade considerations were performed by Daugherty & Harding (1982) for an inner gap scenario, but if the cascade is possible in the outer gap (which is commonly assumed), the energy spectrum of the secondaries should not be too different. The spectrum cuts off below \( \gamma \lesssim 10^2 \) owing to the energy threshold of the interacting photons to produce a pair.

This particle energy spectrum can also be found by the following simple argument. Consider a primary particle with energy \( \gamma_p m_e c^2 \) cascading into \( M(\gamma) \) particles, each with an energy \( \gamma m_e c^2 \). If the cascade is very efficient, energy conservation leads to

\[
M(\gamma) = \gamma_p \sim \frac{10^7}{\gamma}.
\]

(13)

Since \( M(\gamma) \) is the number of particles around Lorentz factor \( \gamma \), the differential number density, or energy spectrum, is then

\[
N(\gamma) \propto \frac{M(\gamma)}{\gamma} \propto \frac{1}{\gamma^2}.
\]

(14)

The initial momentum of the primary particle (parallel to the local magnetic field) defines also the momentum of the \( \gamma \)-rays emitted by inverse Compton scattering with ambient photons. The \( \gamma \)-rays eventually leave the gap because they are not influenced by the magnetic field and proceed to created pairs by photon-photon interactions. The subsequent generations of pairs are then no longer affected by the electric field of the gap. So it seems reasonable to assume that via creation all particles have the same pitch angle with a subsequent evolution of the pitch-angle distribution due to motion along the magnetic field lines and radiative losses. Of course the emission of photons will influence the particle energy spectrum, giving a steepening after a loss timescale \( t_{syn} \), affecting first the high-energy part.

3. SYNCHROTRON EMISSION AT VERY SMALL PITCH ANGLES

Since the relativistic particles of the pair cascade are created almost in the direction of the magnetic field lines, one has to be careful to use the correct validity range of the approximations made in synchrotron radiation theory. In Epstein (1973) the synchrotron emission process by particles with very small pitch angles is discussed in detail. The emissivity at very low pitch angles (\( \Psi \ll 1/\gamma \)) of a single particle is given by

\[
e(\theta, \gamma) = \frac{\pi \varepsilon^2 \Psi^2 v^3}{v_B c} \left( 1 - \frac{v}{\gamma v_B} + \frac{v^2}{2 \gamma^2 v_B^2} \right) \delta \left( \gamma - \frac{2 \gamma v_B}{1 + \theta^2 \gamma^2} \right),
\]

(15)

where \( v_B = eB/2\pi mc = 2.80 \times 10^{12} B_4 \) Hz is the nonrelativistic gyrofrequency and \( \theta \) is the angle between the magnetic field \( B(B_4 \colon B \text{ in units of } 10^6 \text{ G}) \) and the direction of emission. Since we restrict our discussion to small pitch and emission angles, we set \( \Psi \approx \Psi \) and \( \sin \theta \approx \theta \). This formula has to be applied when angles \( \theta \lesssim 1/\gamma \) are resolved.
in the observations, i.e., in the case of a pulsar, when the pulse width $\Delta \phi$ becomes comparable to the emission cone angle of the particles, $1/\gamma \gtrsim 2\pi\Delta \phi$. Since the maximum of the emission is in the forward direction ($\theta = 0$), the emission is dominated by the field lines that point toward the observer at each phase of the pulse. The degree of circular and linear polarization is given by

$$\rho_c = \mu \frac{1 - \theta^2 \gamma^2}{1 + \theta^2 \gamma^2}$$

and

$$\rho_l = \frac{2\theta^2 \gamma^2}{1 + \theta^2 \gamma^2},$$

where $\mu = \pm 1$ is the sign of the charge.

When the typical angle of emission $\theta$ is small compared to the pulse width, it is useful to integrate the emissivity over all angles to get the total emission spectrum of a single particle

$$e_c(\gamma) = 2\pi \int e_c \sin \theta \, d\theta = \frac{2\pi^2 e^2 \gamma^2}{c} v \left(1 - \frac{v}{\gamma v_b} + \frac{v^2}{2\gamma^2 v_b^2}\right)$$

up to $v \lesssim 2\gamma v_b$. The energy loss rate of a single particle is obtained by integrating the total emission over all frequencies

$$I_c = -\frac{dE}{dt} = \frac{8\pi^2 e^2 \gamma^2 \Psi^2}{3c},$$

which is also the well-known result at large pitch angles, used in equation (4).

We now calculate the emission from a power-law distribution of relativistic particles, $N(\gamma) = N_0 \gamma^{-s}$, in the direction along the magnetic field $\theta = 0$ according to

$$I_c \propto \int e_c(\gamma) N(\gamma) \, d\gamma = \frac{\pi e^2 \gamma^2 N_0}{2\gamma v_b c} v^3$$

$$\times \int_0^\infty \left(1 - \frac{v}{2\gamma v_b}\right) \, dy$$

$$= \frac{4N_0 e^2 \gamma^2 v_b}{c} \left(\frac{v}{2\gamma v_b}\right)^{4-s}.$$  

This result shows that the spectrum rises very steeply; in fact, $I_c \propto v^2$ for $s = 2$. Therefore the sharp rise in the spectrum of the Crab pulsar at infrared frequencies can be explained as synchrotron emission at very small pitch angles. The Lorentz factors in the case of the Crab then have to be of the order of $\gamma \lesssim 1/\Delta \phi \sim 10^2$ so that this description applies. This is consistent with the requirement that $2\gamma v_b$ lies in the infrared part of the spectrum.

In case of somewhat higher Lorentz factors, the single-particle beam angle becomes smaller than the phase angle of the pulse, and the total (angle-averaged) power spectrum has to be applied. To find the spectrum from a power-law distribution, the monochromatic approximation

$$e_c \approx -\frac{dE}{dt} \delta(v - 2\gamma v_b)$$

is used for simplicity. This is a good approximation since the emission is sharply peaked at $2\gamma v_b$, and one may just put all the emission at this frequency. As a result we then find

$$I_c \propto \int e_c N(\gamma) \, dy = \frac{4\pi^2 e^2 N_0 v_b^2 \Psi^2}{3c} \left(\frac{v}{2\gamma v_b}\right)^{2-s}.$$  

In the case of $s = 2$, this gives the observed flat, $\alpha \approx 0$, optical/UV spectrum of the Crab pulsar, with $\gamma \sim 10^2$--$10^3$.

4. THE CRAB PULSAR SPECTRUM FROM THE INFRARED TO X-RAYS

We consider now the injection of a power-law distribution of a relativistic electron-positron plasma $N(\gamma) = N_0 \gamma^{-s}$, which is produced in the cascading process, along the magnetic field lines. We interpret the infrared and optical emission of the Crab pulsar with its fast rise toward the optical as synchrotron emission from relativistic particles with very small pitch angles ($\gamma \ll 1/\Psi$). As shown in the previous section, we expect a spectral index of $\alpha = -2$ in the infrared part and $\alpha = 0$ in the optical part, if the pitch angle is of order $\Psi \sim 10^{-3}$ for Lorentz factors of $\sim 10^2$--$10^3$.

Although the circular polarization will be at its maximum when looking along the field lines, in the cascade process an equal amount of electrons and positrons will be produced, giving a net zero circular polarization. On the other hand, the linear polarization becomes maximal for $\theta = 1/\gamma$ for both species and is zero for $\theta = 0$. If the emission cone of the pulsar is not cut right through the center by the line of sight but is slightly offset, then one expects a polarization angle swing.

The soft X-ray emission in this model comes from the particles with larger pitch angles ($\Psi \gg 1/\gamma$), radiating at a typical frequency

$$v_c = \frac{3}{2\gamma v_b} \Psi^2 \gamma^2.$$  

The spectrum in this large pitch-angle limit is given by

$$I_c \propto v^{-\alpha(1-s/2)},$$

which yields the observed $I_c \propto v^{-0.5}$ at soft X-rays for $s = 2$. Therefore, a single power-law distribution of relativistic electrons and positrons can produce the complete spectrum from infrared to X-ray frequencies.

5. SIMPLE LUMINOSITY ESTIMATES FOR THE OPTICAL AND X-RAY EMISSION

We now estimate the energy radiated by the pulsar in the optical and in the X-ray band simply as the number of particles times the energy radiated by one particle. Thus we have

$$L = M\nu \nu_0 V P_{\text{syn}},$$

with

$$P_{\text{syn}} = 1.6 \times 10^{-15} \Psi^2 \gamma^2 B^2 \text{ ergs}^{-1}$$

being the energy radiated by a single particle. $M$ is the multiplicity of the relativistic particle density within the
volume $V$ as compared to the Goldreich-Julian density

$$n _ {GJ} = 6.9 \times 10^{-2} BP^{-1} \text{ cm}^{-3} ,$$

(28)

where $P$ is the period of the pulsar in seconds, $x = R/R _ {ns}$ is the distance in units of the neutron star radius. The magnetic dipole field falls off as $B = B _ {0}/x ^ {3}$, with $B _ {0}$ being the field strength at the surface of the neutron star. Since we assume the radiation to occur close to the light cylinder, we set

$$x = x _ {lc} = 4774P .$$

(29)

The volume is estimated as a spherical shell within a fraction $f$ of the light cylinder radius

$$V = 4 \pi R _ {lc} ^ {2} \times fR _ {lc} = 1.4 \times 10^{30} fP ^ {3} \text{ cm} ^ {3} .$$

(30)

With $M = \gamma p / \gamma$ we then find that

$$L = 1.5 \times 10^{44} \gamma p \gamma ^ {2} B ^ {3} P ^ {2} \text{ ergs s} ^ {-1} .$$

(31)

The frequency of the optical emission (at small pitch angles) is given by

$$v _ {opt} = 2.8 \times 10^{6} \gamma _ {opt} B \text{ Hz} ,$$

(32)

so that, by eliminating $\gamma$, we get

$$L _ {opt} = 4 \times 10^{31} \cos ^ {1/2} \alpha \left( \frac{\delta}{0.1} \right) \left( \frac{f}{0.1} \right) \left( \frac{\Psi _ {p}}{10^{-3}} \right)^{-1} \times \left( \frac{\Psi _ {opt}}{10^{-3}} \right)^{2} \left( \frac{v _ {opt}}{10^{15} \text{ Hz}} \right) \left( \frac{B _ {0}}{10^{2.5} \text{ G}} \right)^{3/2} \times \left( \frac{P}{33 \text{ ms}} \right)^{-s/2} \text{ ergs s} ^ {-1} .$$

(33)

The parameter values chosen for the Crab pulsar are consistent with the considerations of the spectral shape and the cascading process, i.e., the particles with $\gamma \sim 10^{2} - 10^{4}$ have pitch angles $\Psi _ {opt} \sim 10^{-3}$. This crude estimate gives the right order of magnitude of the Crab pulsar’s optical luminosity.

In the X-ray regime where the large pitch-angle approximation for the frequency applies

$$v _ {x} = 2.8 \times 10^{6} \Psi _ {x} \gamma ^ {2} \text{ Hz} ,$$

(34)

we then find

$$L _ {x} = 6 \times 10^{35} \cos ^ {1/2} \alpha \left( \frac{\delta}{0.1} \right) \left( \frac{f}{0.1} \right) \left( \frac{\Psi _ {p}}{10^{-3}} \right)^{-1} \times \left( \frac{\Psi _ {x}}{10^{-1}} \right)^{3/2} \left( \frac{v _ {x}}{10^{17} \text{ Hz}} \right) \left( \frac{B _ {0}}{10^{12.5} \text{ G}} \right)^{2} \times \left( \frac{P}{33 \text{ ms}} \right)^{-4} \text{ ergs s} ^ {-1} .$$

(35)

In order to get the right X-ray luminosity, a pitch angle of order $\Psi _ {x} \sim 0.1$ is required. To emit at soft X-ray frequencies, again the Lorentz factors have to be in the range $\gamma \sim 10^{2} - 10^{4}$.

This result can now be compared with the spin-down luminosity of the Crab pulsar

$$L _ {sd} = 3.2 \times 10^{38} \left( \frac{B _ {0}}{10^{12.5} \text{ G}} \right) ^ {2} \left( \frac{P}{33 \text{ ms}} \right)^{-4} \text{ ergs s} ^ {-1} .$$

(36)

The luminosity radiated in X-rays by the synchrotron mechanism shows the same dependence on the magnetic field strength and the period as the rotational energy loss of the pulsar. We can therefore write

$$L _ {x} \approx 2 \times 10^{3} \cos ^ {1/2} \alpha \left( \frac{\delta}{0.1} \right) \left( \frac{f}{0.1} \right) \left( \frac{\Psi _ {p}}{10^{-3}} \right)^{-1} \times \left( \frac{\Psi _ {x}}{10^{-1}} \right)^{3/2} \left( \frac{v _ {x}}{10^{15} \text{ Hz}} \right)^{1/2} \text{ ergs s} ^ {-1} .$$

(37)

Since this relation is now independent of $B _ {0}$ and $P$, it should apply to any pulsar, not only the Crab pulsar. The observations of X-ray-selected pulsars by Becker & Trümper (1997) have revealed such a dependence in the form $L _ {x} \sim 10^{-3} L _ {sd}$. The theoretically found proportionality between the X-ray luminosity and the spin-down luminosity strongly supports our assumption that synchrotron emission is the dominant radiation process. On the basis of the model presented here, this observation implies that the parameter combination $\cos ^ {1/2} \alpha \delta ^ {1/2} \Psi _ {p} ^ {1/2} \Psi _ {x} ^ {1/2}$ has about the same value for every pulsar in their list.

The luminosity estimates also contain the correct spectral dependencies, because $I _ {x} \propto L _ {opt} / v \propto const$ for the optical and $I _ {x} \propto L _ {opt} / v \propto v^{-1/2}$ for the X-ray emission.

6. DISCUSSION AND CONCLUSIONS

The high-frequency emission of the Crab pulsar is a challenge for the theoretical description. Earlier investigations put the location of the infrared and optical pulses close to the light cylinder radius (Smith et al. 1988) and led us to the idea that these distant regions of the magnetosphere may be responsible for the emission up to the X-ray and $\gamma$-ray range. First we were able to show that within an outer gap description and a standard cascade model we expect a particle energy distribution $N(\gamma) \propto \gamma^{-2}$, being the basis for the investigations of the radiative properties of the Crab pulsar. The energy of the primary particles accelerated in the electric field of the gap is shown to be limited by synchrotron radiation and not curvature radiation to $\sim 10^{17} m _ {e} c ^ {2}$. This holds for pitch angles $\Psi \sim 10^{-4}$.

A detailed inspection of an anisotropic synchrotron model led to the result that the emission of particles with small pitch angles with this single energy distribution reproduces the key properties of the spectrum and luminosity from the infrared up to soft X-rays. This model explains all relevant features, including the long-standing problem of the increase of the infrared spectrum $I _ {x} \propto v^{2}$, the flat spectrum $I _ {x} \propto v^{0}$ in the optical and UV range, and the X-ray spectrum $I _ {x} \propto v^{-0.5}$. The only difference concerns the pitch angle of the emitting particles and the (energy-dependent) multiplicity $M = M(\gamma)$. The infrared/optical/UV part of the spectrum is emitted by particles with Lorentz factors $\gamma \sim 10^{2} - 10^{4}$ and pitch angle $\Psi \sim 10^{-3}$, corresponding to the pitch angle of the primary particles accelerated in the electric field of the gap. The X-rays are radiated by particles in the same energy range but with a pitch angle $\Psi \sim 0.1$. It is not clear what the fundamental physical scenario is, which gives the required values for the pitch angle.

The X-ray emission from particles with the smaller pitch angles, involving higher energy particles, which are smaller in number, would not be seen owing to the smaller power radiated. It is dominated by particles with the larger pitch angle. On the other hand, for pulsars with smaller magnetic
field strength at the light cylinder, the emission at larger pitch angles might be seen in the optical band. In that case, the $\nu^{-3/2}$-spectrum is also expected there, then accompanied by a luminosity relation similar to that for the X-rays.

From simple luminosity arguments we find that this model is able to yield the desired energy output in the optical as well as in the X-ray part of the spectrum. We also find that the observed proportionality of the X-ray flux and the spin-down luminosity is a consequence of the synchrotron radiation model for any pulsar that develops an efficient pair cascade.

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