The chiral symmetry restoration phase transition in baryon spectrum

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Abstract

It is shown that in the large $N_c$ limit the light baryon spectrum exhibits the chiral restoration phase transition at high enough excitation energy. Such a phase transition is evidenced by the systematical parity doublets observed in the upper part of $N$ and $\Delta$ spectra.

At low temperature and density the almost perfect $SU(2)_L \times SU(2)_R$ global chiral symmetry of the QCD Lagrangian in the $u, d$ sector is realized in the hidden Nambu-Goldstone mode. At higher temperatures or and nuclear densities the chiral symmetry restoration occurs, the phenomenon which is well established in the former case on the lattice [1] (there are little doubts that it also will happen at high density). Much efforts are being made to study the chiral restoration phase transition both theoretically and experimentally in heavy ion collisions. It has been recently speculated [2] that actually the upper part of $N$ and $\Delta$ spectra exhibits the chiral restoration phase transition which is evidenced by the systematical parity doublets there. The aim of the present note is to give a general proof that indeed there must happen the chiral symmetry restoration at some baryon excitation energy.

The $SU(2)_L \times SU(2)_R$ global chiral symmetry is equivalent to the independent vector and axial rotations in the isospin space. The axial transformation mixes states with different spatial parities. Hence, if this symmetry of the QCD Lagrangian were intact in the vacuum, one would observe parity degeneracy of all hadron states with otherwise the same quantum numbers. This is however not so and it was a reason for suggestion in the early days of QCD that the chiral symmetry of the QCD Lagrangian is broken down to the vectorial subgroup $SU(2)_V$ by the QCD vacuum, which reflects a conservation of the vector current (baryon number). That this is so is directly evidenced by the nonzero value of the quark condensate

$$<\bar{\psi}\psi>\simeq -(240-250 MeV)^3,$$

(1)

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which represents the order parameter associated with the chiral symmetry breaking. The nonzero value of the quark condensate directly shows that the vacuum state is not chiral-invariant.

Physically the nonzero value of the quark condensate implies that the energy of the state which contains an admixture of “particle-hole” excitations (real vacuum of QCD) is below the energy of the vacuum for a free Dirac field, in which case all the negative-energy levels are filled in and all the positive-energy levels are free. This can happen only due to some nonperturbative attractive Lorentz scalar gluonic interactions between quarks which pairs the left quarks and the right antiquarks (and vice versa) in the vacuum. In perturbation theory to any order the structure of the trivial Dirac vacuum persists. Such a situation is typical in many-fermion systems (compare, e.g., with the theory of superconductivity) and implies that there must appear quasiparticles with dynamical masses.

The physical mechanism for the chiral symmetry restoration at high temperatures and/or nuclear densities is rather simple. In the former case it happens because the thermal excitations of quarks and antiquarks from the vacuum lead to the Pauli blocking of the levels which are necessary for the formation of the condensate. In the latter case these levels are occupied by the valence quarks of the high density quark (nuclear) matter.

For the illustration of these phenomena we will use the Nambu and Jona-Lasinio model, that adequately reflects the underlying chiral symmetry of QCD and exhibits the chiral symmetry breaking (restoration) phase transition [3].

Any scalar gluonic interaction between current quarks, which is responsible for the chiral symmetry breaking in QCD (which is generally nonlocal), in the local approximation is given by the 4-fermion operator $(\bar{\psi}\psi)^2$. Because of the underlying chiral invariance in QCD this interaction should be necessarily accompanied by the interaction $(\bar{\psi}i\gamma_5\vec{\tau}\psi)^2$ with the same strength. Thus any generic Hamiltonian density in the local approximation is given by (contains as a part) the NJL interaction model (for simplicity we restrict discussion to the u,d flavour sector and to the chiral limit):

$$ H = -G \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right]. $$

(2)

If the strength of the effective interaction $G$ exceeds some critical level (which happens when the strong coupling constant $\alpha_s(q)$ is big enough), then the nonlinear gap equation

$$ M = -2G \langle \bar{\psi}\psi \rangle , $$

(3)

$$ \langle \bar{\psi}\psi \rangle = -\frac{N_c}{\pi^2} \int_0^{\Lambda_N} dp dp^2 \frac{M}{\sqrt{(p^2 + M^2)}} $$

(4)

admits a nontrivial solution $\langle \bar{\psi}\psi \rangle \neq 0$, which means that the initial vacuum becomes rearranged and instead of the Wigner-Weyl mode of chiral symmetry one obtains the Nambu-Goldstone one. Thus the appearance of the quark condensate is equivalent to the appearance
of the gap in the spectrum of elementary excitations in QCD (i.e. of the constituent mass $M$). Hence the constituent quark is a quasiparticle in the Bogoliubov sense, i.e. is a coherent superposition of the bare particle-hole excitations. In terms of the noninteracting constituent quarks the vacuum is again trivial, but contains a gap $2M$. The treatment of the scalar interaction between bare quarks in the vacuum in the Hartree-Fock (mean field) approximation, from which the gap equation (3)-(4) is obtained, is equivalent to the vacuum of noninteracting constituent quarks. In the vacuum state the second term of (2) does not contribute to the constituent quark self-energy in the mean field approximation. Contributions beyond the mean field approximation (i.e. constituent quark self energy due to pion and sigma loops, which are higher order effects in the $1/N_c$ expansion, see, e.g., [4]) do not violate significantly the qualitative picture of the vacuum in the mean field approximation.

Consider the chiral symmetry restoration in the vacuum at high temperatures. At zero temperature the vacuum condensate is given by (4). At finite temperature it is affected and given by

$$<\bar{\psi}\psi> = -\frac{N_c}{\pi^2} \int_0^{\Lambda} d|p|p^2 \frac{M}{E_p} [1 - n_+(p) - n_-(p)]$$

(5)

where $E_p = \sqrt{(p^2 + M^2)}$ and $n_{+-}(p)$ is the Fermi-Dirac distribution function for quarks and antiquarks at vanishing chemical potential,

$$n_{+-}(p) = \frac{1}{1 + e^{E_p/T}}.$$ (6)

In the latter equation $T$ is the temperature. At some critical temperature, $T_c$, the nontrivial gap solution disappears and the chiral symmetry becomes restored. Physically this is because the thermal excitations of quarks and antiquarks lead to the Pauli blocking of the levels which are necessary for the formation of the condensate. The formal (mathematical) reason is that the quark distribution function $n(p)$ is pushed out from the $p = 0$ point and becomes broad and thus affects the gap equation so that the self-consistent (gap) solution vanishes.

Consider baryons, i.e. the systems that contain valence quarks on the top of the vacuum. The valence quarks interact to each other and also to the quarks and antiquarks of the vacuum. In this case the eq. (3) should be modified and the distribution function of valence quarks should be incorporated (we still assume that the chemical potential is small so that we can neglect it):

$$<\bar{\psi}\psi> = -\frac{N_c}{\pi^2} \int_0^{\Lambda} d|p|p^2 \frac{M}{E_p} [1 - n_v(p) - n_+(p) - n_-(p)].$$

(7)

If the number of colors $N_c$ increases, the valence quarks in baryons become denser and denser because the size of baryons is governed by $\Lambda_{QCD}$ which remains fixed. In this case the Hartree-Fock picture becomes exact and the system of large $N_c$ valence quarks can be
described by the sum of identical quarks moving in the self-consistent field of other \( N_c - 1 \) quarks \([3]\). In the large \( N_c \) limit the strong decay of baryons vanishes, which means that the system of a large amount of valence quarks is in thermal equilibrium. This system can obviously be ascribed some temperature \( T \). The valence quarks interact not only to each other but also to quarks and antiquarks of the vacuum, which is due to fluctuations of valence quark into other one plus quark-antiquark pair (Fig. 1). The diagram of Fig. 1 is not supressed in the large \( N_c \) limit and represents an effective meson exchange between valence quarks in baryons \([4]\). Because of intensive interaction of the valence quarks with the quarks and antiquarks of the vacuum, the valence quarks in baryons are in thermal equilibrium with the vacuum (the vacuum plays a role of a thermostat or vice versa). This means that the same temperature must be ascribed to both the valence quarks and the vacuum. In particular, the same temperature persists in all distribution functions in eq. (6).

The large \( N_c \) system of valence quarks is an open system. Only together with the quarks and antiquarks (and gluons) of the vacuum it becomes a closed one. Hence the state of the system of valence quarks is necessarily a mixed one and the apparatus of quantum statistics should be used. A measurable physical baryon with its complete set of quantum numbers, represents a pure (coherent) state of a closed system (valence quarks plus vacuum). The mixed state of a system of valence quarks can be obviously presented as a superposition of all possible pure states with the same energy.

Consider now the ground state baryons. In this case all valence quarks occupy the same ground state orbital \( 0s \), which means that the temperature of both valence quarks and vacuum is \( T = 0 \). Since only one level is partially blocked by the valence quarks, the quark condensate in the present state is practically the same as the one of the true vacuum (the true vacuum is the vacuum with no valence quarks on the top of it). This is equivalent to the obvious statement that the small chemical potential does not perturb much the true vacuum at zero temperature.\(^1\)

In the excited baryon there is a coherent superposition of one, two, three,... valence quarks that are in the excited states. Such a pure state of valence quarks plus vacuum is described by a set of quantum numbers, in particular by its spin. If one considers a system of valence quarks only (which is an open one), such a system in the excited state contains an incoherent superposition of one, two, three,... valence quarks that are in the excited states and the temperature of this system is above zero. This system is described by the corresponding distribution function \( n_v \). Such a system, with the fixed temperature and energy,

\(^1\)Note that while these graphs contribute to baryon mass at the order \( N_c \), their contribution to \( N-\Delta \) mass splitting appears only at the order \( N_c^{-1} \).

\(^2\)One should not mix it with the chiral restoration at high nuclear (quark) densities at \( N_c = 3 \). In the latter case all the low levels are occupied by valence quarks.
can be expanded into the set of baryons with the same energy and all allowed spins $1/2$, $3/2$, $5/2$, ... Since the excited system of valence quarks lives in the thermal equilibrium with its vacuum, the temperature of the vacuum is also above zero. Physically this means that the true vacuum becomes strongly perturbed by the excitation of the baryon. At some critical excitation energy of the baryon (i.e. at some critical temperature) its vacuum undergoes the chiral restoration phase transition. In the example considered above it follows from the fact that at some temperature the self-consistent nontrivial solution of eqs. (3) and (7) vanishes. Around this excitation energy (temperature) there must appear systematical approximate parity doublets with all possible spins.

The argument above is robust and general and does not rely on NJL model, which is used only for illustration. The only important element is that when the number of colors increases, the valence quarks in baryons can be ascribed a temperature and this system lives in thermal equilibrium with its vacuum.

How close to reality is the large $N_c$ picture of baryons? It has been shown that in the large $N_c$ limit the $SU(6)$ symmetry of baryons becomes exact (to be precise, the low-energy observables such as baryon masses, magnetic moments and axial coupling constants are described by the corresponding $SU(6)$ symmetrical wave functions). As it is well known the $SU(6)$ symmetry works reasonably well for all these observables. For instance, the $SU(6)$ predictions for baryon magnetic moments are satisfied at the level 15-20%. The $N$-$\Delta$ splitting is of the order 25-30% of the baryon mass. This implies that one can expect predictions of the large $N_c$ limit to be correct at the level 20-30%. This confidence level is enough to anticipate that the arguments above for the chiral symmetry restoration at large $N_c$ will survive in the real world with $N_c = 3$.

How about mesons? In the latter case even in the large $N_c$ limit the meson still consists of one valence quark and antiquark, so the thermodynamical description cannot be used here for valence degrees of freedom. In addition in the present case there are no diagrams in the large $N_c$ limit, similar to that one of Fig. 1. This implies that there is no mutual impact of the valence degrees of freedom and of the quarks and antiquarks of the vacuum. Hence the meson spectra should not show the chiral symmetry restoration and parity doublets.

The still poorly mapped upper part of the light baryon spectrum exhibits remarkable parity doublet patterns. To these belong $N(2220), \frac{9}{2}^+ - N(2250), \frac{9}{2}^-, N(1990), \frac{7}{2}^+ - N(2190), \frac{7}{2}^-, N(2000), \frac{5}{2}^+ - N(2200), \frac{5}{2}^-, N(1900), \frac{3}{2}^+ - N(2080), \frac{3}{2}^-, N(2100), \frac{7}{2}^+ - N(2090), \frac{7}{2}^-, \Delta(2300), \frac{9}{2}^+ - \Delta(2400), \frac{9}{2}^-, \Delta(1950), \frac{7}{2}^+ - \Delta(2200), \frac{7}{2}^-, \Delta(1905), \frac{5}{2}^+ - \Delta(1930), \frac{5}{2}^-, \Delta(1920), \frac{3}{2}^+ - \Delta(1940), \frac{3}{2}^-, \Delta(1910), \frac{1}{2}^+ - \Delta(1900), \frac{1}{2}^-$. The splittings within the parity partners are typically within the 5% of the baryon mass. This value should be given a large uncertainty range because of the experimental uncertainties for the baryon masses of the order 100 MeV. Only a couple of states in this part of the spectrum do not have their parity partners so far observed. The low energy part of the spectrum, on the other hand, does not show this property of the parity doubling. The increasing amount of the near parity doublets in the high energy sector is an evidence of the chiral symmetry
If chiral and deconfinement phase transitions coincide, the conclusion should be that the highly excited baryons with masses above some critical value (where the phase transition is completed) should not exist because deconfinement phase transition should be dual to a very extensive string breaking at big separations of colour sources (colour screening). Whether this point corresponds to approximately 2.5 GeV or higher should be answered by future experiments on high baryon excitations. The phase transition can be rather broad because of the explicit chiral symmetry breaking by the nonzero value of current quark masses.

There is a couple of the well confirmed states $N(2600), \frac{11}{2}^-$ and $\Delta(2420), \frac{11}{2}^-$, in which case the parity partners are absent [8]. Thus it will be rather important to try to find them experimentally.
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Figure captions

Fig.1 An effective meson exchange diagram which represents a thermodynamical exchange between valence quarks in baryons and the quarks and antiquarks of the vacuum.
