Pionic fusion in light-ion systems
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RIJKSUNIVERSITEIT GRONINGEN

Pionic fusion in light-ion systems

Proefschrift

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To my dear husband
and my parents
## CONTENTS

| Section | Title | Page |
|---------|-------|------|
| 4.9     | Pedestal subtraction | 53   |
| 4.10    | Data stream | 54   |
| 5       | Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data | 57   |
| 5.1     | Introduction | 57   |
| 5.2     | Particle identification | 58   |
| 5.2.1   | Photon identification | 58   |
|         | The Plastic Ball and the Inner Shell time | 58   |
|         | Plastic Ball pulse shape | 59   |
| 5.2.2   | Ion identification | 59   |
|         | Ion Time Of Flight | 60   |
|         | Ion energy deposition | 61   |
| 5.3     | Calibration | 66   |
| 5.3.1   | Time calibration | 66   |
|         | Time calibration of the Plastic Ball and the Inner Shell | 66   |
|         | Calibration of the ion Time Of Flight | 67   |
| 5.3.2   | Energy calibration | 68   |
|         | Momentum calibration in the Heavy Ion array | 68   |
|         | Gain matching of the light output for the Heavy Ion detector | 71   |
|         | Energy calibration of Inner Shell and Plastic Ball backward hemisphere | 75   |
|         | Energy calibration of Plastic Ball forward hemisphere | 78   |
| 5.4     | Candidate selection | 78   |
| 5.4.1   | Clustering of the Plastic Ball detectors | 78   |
|         | Position reconstruction | 81   |
| 5.4.2   | Pion reconstruction methods | 82   |
| 5.4.3   | Presort | 84   |
| 5.4.4   | Final selections | 85   |
|         | Momentum distribution of $^{10}\text{B}$ | 85   |
|         | Opening angle distribution of two photons | 89   |
| 5.4.5   | Energy reconstruction of two decay photons | 91   |
| 5.5     | Data contamination and background events | 91   |
| 5.6     | Acceptance corrections | 94   |
|         | The Big-Bite Spectrometer and the Heavy Ion detector | 94   |
|         | The Plastic Ball and its Inner Shell | 97   |
| 5.7     | Data normalisation and the experimental cross section | 98   |
| 6       | Results | 101  |
| 6.1     | Introduction | 101  |
| 6.2     | Reconstructed $\pi^0$ energy | 101  |
| 6.3     | Two photon energies | 102  |
| 6.4     | Reconstructed $\pi^0$ mass | 104  |
| 6.5     | Pion angular and momentum distributions | 106  |
| 6.6     | Differential cross section | 106  |
| 7       | Discussion and outlook | 117  |
| CONTENTS     |     |
|-------------|-----|
| 8  Summary  | 125 |
| Nederlandse Samenvatting | 127 |
| Publications | 129 |
| Acknowledgements | 130 |
| Bibliography   | 133 |
1. General introduction

The pion plays an important role in nuclear physics. In order to explain the strong force to combine nucleons in a nucleus, Yukawa proposed that the pion acts as the mediator of the nucleon-nucleon (NN) interaction \([1]\). The long range part of the strong force is expressed as \(F(r) = \frac{\mu}{r^2}\), where the term \(\mu\) is given as \(\mu = \frac{m_\pi}{2}\) using the pion mass. The pion is also responsible for the tensor interaction between two nucleons, which plays a significant role in explaining the nuclear structure. There are many extensive calculations to explain the findings of few-body systems taking into account the interaction mediated by the pion. The calculated results agree with the experiments very well \([2, 3]\) and demonstrate a prominent role of the tensor force in the few-body system. Almost half of the attraction is caused by the tensor force, and hence the pion exchange interaction. Shell model calculations show that one-third to a half of single particle spin-orbit splittings are explained by the presence of the tensor force. It means that pion exchange forces are important for explaining the shell structure of nuclei. In addition, the recent variational calculations demonstrate the importance of the three-body interaction, which originates from the pion \([4]\).

From another point of view, it has been understood that the absorption of low-energy pions, in the process \(^2\text{H}(\pi^+, ^1\text{H})^1\text{H}\), is dominantly a two-nucleon process. The pion is scattered by one of the target nucleons before being absorbed by the second. In case of reactions including heavier nuclei like \(\text{A}(\pi^+, ^2\text{H})\text{B}\) and \(\text{A}(\pi^+, \text{B}_1\text{B}_2)\), where \(\text{B}_1\) and \(\text{B}_2\) are two nuclei, most absorption events still include in the final state a pair of nucleons. Therefore, theoretical estimates of those kind of reactions \([5]\) introduce an effective \(\pi\)-two-nucleon absorption operator which has been adjusted to fit the real deuteron case. Since this absorption operator is believed to be of quite short range, it was hoped that such experiments might show evidence for short range correlations in nuclei. It has been realized that such reactions can be carried out more economically and more flexibly in the inverse charge-symmetric channel which is involving the pion production process.

The production of pions near the threshold is an extremely coherent process since almost all available energy must be concentrated in the meson field \([6, 7, 8, 9, 10]\). Pion production near threshold occurs at large momentum transfers and therefore is a powerful and highly selective tool to study short range phenomena in the entrance channel. Although the corresponding cross sections are relatively small \([11, 12, 13, 14, 15, 16, 17]\), the characteristic kinematical and spectroscopic features of those reactions may make such processes a particularly suited tool to investigate the pion interaction with nuclear matter and many body effects as well as collective properties of nuclei. Therefore e.g. short range correlations and the role of virtual excitations of nucleon resonances in the nuclear medium can be investigated in much greater detail.

1.1 Introduction to the pion production process

In collisions between two free nucleons, a pion can be produced when the centre-of-mass energy of the nucleon-nucleon system exceeds the pion mass \(m_{\pi^0}\) (\(m_{\pi^\pm}\)) \([18]\). Since \(m_{\pi^0}\)
Figure 1.1: Elementary \( NN \rightarrow NN\pi \) transitions mediated by meson-exchange: a) s-wave process, b) non-resonant p-wave process and c) resonant p-wave process with \( \Delta \)-excitation.

\( (m_{\pi}^\pm) \) is approximately 135 MeV (140 MeV), we need to provide a laboratory energy of about 280 MeV (290 MeV) to exceed the threshold energy for an incoming nucleon hitting a nucleon at rest and producing a neutral (charged) pion. Figure 1.1 shows the Feynman diagrams for producing a pion in the free NN collision.

In addition to pion production in the free NN collision, pions have been produced in nuclear reactions at collision energies per nucleon which are considerably below the threshold energy in the free NN system \([11, 12, 19, 13, 14, 15, 16, 17]\). In collisions between nuclei, pion production is governed by the energy conservation principle: the centre-of-mass energy has to exceed \( m_{\pi} \). Since the energy necessary for pion production is shared among many nucleons, a pion can be produced in collisions between two nuclei at beam energies per nucleon that are more than an order of magnitude lower than the pion production threshold in the free NN collision. This range of energy is called the sub-threshold energy which is the energy/nucleon below 280 (290) MeV for neutral (charged) pion production. The coherent threshold of the pion production is the minimum energy required to produce a pion in the nucleus-nucleus collision. Since production of a single pion demands a significant fraction of the available energy, if the available energy is close to the coherent threshold of the pion production, a highly coherent mechanism is required. Pion production at subthreshold energy is dominated by collective behaviour and is not accessible in the free NN collision. A pair of nucleons requires an extra amount of energy in addition to the beam kinetic energy to overcome the threshold. The coherent threshold energy (in the laboratory system) of the pion production process in the nucleus-nucleus collision is calculated as follows:

\[
T_{thr} = \frac{m_{\pi}^2 + 2(A_1 + A_2)m_N m_{\pi}}{2A_1 m_N},
\]

(1.1)

where \( m_N \) and \( m_{\pi} \) stand for the nucleon and pion mass, respectively. \( A_1 \) and \( A_2 \) are the number of nucleons in the target and projectile nuclei, respectively. The coherent threshold energy/nucleon for pion production in collisions between two equal nuclei as a function of their mass number is shown in Fig. 1.2 by the solid curve. The dashed curve in Fig. 1.2 represents the Coulomb energy that two nuclei need to overcome and get
1.1. Introduction to the pion production process

![Graph showing the coherent threshold energy for pion production in collisions between two equal nuclei as a function of their mass number. The dashed curve is the laboratory Coulomb energy.]

Figure 1.2: The solid curve has been obtained using Eq. 1.1 divided by the mass number A and shows the coherent threshold energy for pion production in collisions between two equal nuclei as a function of their mass number. The dashed curve is the laboratory Coulomb energy.

close enough to fuse. For nuclei with a mass number larger than Oxygen, the threshold is below 20 MeV/nucleon. It comes close to the Coulomb barrier (the dashed curve in Fig. 1.2), which would eventually provide a natural cutoff for the process. At these low beam energies, the centre-of-mass energy in the nucleon-nucleon frame is not sufficient to produce a pion.

1.1.1 Inclusive and exclusive pion production

In case of a nuclear reaction around the pion production threshold, the reaction mechanism is dominated by thermalisation processes: the kinetic energy of the entrance channel is distributed among many degrees of freedom. This gives rise to characteristic final states with many outgoing reaction products. The inclusive pion production reaction may be described as

$$A_1 + A_2 \rightarrow \pi + X_1 + X_2 + \ldots + X_n,$$

(1.2)

where $A_1$ and $A_2$ denote the target and the projectile, respectively, and $X_i$ ($1 \leq i \leq n$) denotes the $i$th unobserved nuclear fragment (Fig. 1.3-(a)).
The exclusive pion production reaction may be described as
\[ A_1 + A_2 \rightarrow \pi + B_1 + B_2 + \ldots + B_n, \tag{1.3} \]
where \( B_i \) \((1 \leq i \leq n)\) denotes the \( i \)th measured nuclear fragment. Near the kinematical limit, the exclusive measurement is confined to detecting only a few final-state particles.

### 1.1.2 Pionic fusion

In the extreme limit of the two-body final state, the excess energy is used to produce a pion. The produced pion is emitted and the colliding nuclei fuse to form a united nucleus with a specific bound state. This scheme may be expressed as
\[ A_1 + A_2 \rightarrow \pi + B(J,I), \tag{1.4} \]
where \( B(J,I) \) denotes the (bound) united nucleus with spin and isospin quantum numbers \( J \) and \( I \), respectively. This exclusive pion production reaction is often referred to as the pionic fusion process and is depicted schematically in Fig. 1.3-(b). Obviously the two reaction products have to carry away the total free energy. The centre-of-mass kinetic energy \( T_{CM} \) in the incident channel is transferred to the two-body exit channel including the pion energy: \( T_{CM} = T_{CM}^{ex} + Q_{A_1,A_2,B} + m_\pi c^2 \), where \( Q \) denotes the Q-value of the complete fusion process. Here, the pion production reaction is referred to as pionic fusion for two reasons. First, the total free energy of the entrance channel is converted into the pion production energy. Second, the two nuclei undergo a “cold” fusion process: they have to melt completely to form the specific state in the final nucleus which is called a fusion product. The pionic fusion reaction is highly coherent. It is expected that there is no “spectator” nucleon. Momentum conservation determines the kinematical distributions of the pion and the fusion product (see Section 3.6).

It is expected that in the inclusive pion production, the increase of the phase space of the subsystem \( X \) will influence the energy dependence of the pion spectra. Figure 1.3-(c) shows the energy balance in inclusive pion production when the unobserved system consists of two excited fragments: \( X_1 \) and \( X_2 \). The total initial free energy \( T_{CM} \) is distributed over the pion total energy \( E_\pi \) (sum of the pion kinetic energy \( T_\pi \) and the pion mass \( m_\pi \)), the excitation energy \( E_X \) of the two fragments \( X_1 \) and \( X_2 \) \((E_{X_1}+E_{X_2})\) and the kinetic energy \( T_{12} \) of the final channel relative motion. The fastest pion is associated with the exclusive pion production in a complete fusion of the nuclei.

The cross section of the pionic fusion reaction is determined by the properties of the transition amplitude which contains the details on the reaction dynamics and the structure of the nuclear fragments. The complete microscopic calculation of such a process is a theoretical challenge which involves a correct description of the nuclear wave functions, the elementary pion-nucleon interaction and the deexcitation of the final pion-nucleus system. The sensitivity to properties of intermediate baryonic resonances makes the pionic fusion process an important and decisive tool to understand the underlying reaction dynamics.
1.1. Introduction to the pion production process

(a)

Figure 1.3: Schematic representation of inclusive pion production (a) and pionic fusion (b) in a nucleus-nucleus collision. (c): schematic representation of the double differential cross section for the inclusive pion production and exclusive pion production near the kinematical limit. For the explanation of all the symbols used, see the text.
1.2 The current status of the research

The major question about pionic fusion at subthreshold energies is how to understand the source of extra energy for a pair of nucleons to add to the beam kinetic energy in order to overcome the pion production threshold. Theoretical attempts to answer this question are scarce. The main difficulty is to identify the appropriate reaction mechanism. The complications related to the pion production mechanism stem already from the basic NNπ interaction. The lowest order single-vertex Born diagram produces a p-wave pion which is kinematically suppressed near threshold. The s-wave contribution comes only from the pion re-scattering involving higher order diagrams. The description of the s-wave re-scattering mechanism based on the work of Ref. [20] significantly under-predicts the measured cross sections. Since pions are pseudoscalars and a s-wave does not carry negative parity, the production of an s-wave pion is suppressed. On the other hand, calculations using Chiral Perturbation theory also encounter some difficulties related to the kinematics of the intermediate particle, and still require further studies [21]. At present, the most successful description [22] utilises the phenomenological isovector axial current of the NN system. Changes in this picture related to both dynamics and kinematics of the pion production mechanism can be expected in case of many nucleons being involved.

Theoretical attempts to understand the production mechanism range from the “Nucleon-Nucleon Single-Collision (NNSC)” picture [23, 24, 25], where the intrinsic (Fermi) motion of the nucleons in the initial states of the two colliding nuclei provides the necessary kinetic energy, to collective processes such as the mean-field approach [26], decay of the compound nucleus [27], pionic bremsstrahlung [28, 29], dynamical phase-space calculations [30], microscopic reaction model including intermediate baryon resonance excitation [7] and the clustering correlations of the colliding nuclei [8].

The earliest suggestion to understand this process dates back to Fermi. It exploits the fact that the initial intrinsic nucleonic motion offers the necessary extra boost for the elementary nucleon-nucleon pion production process. This original idea has been reconsidered in an NNSC model (shown schematically in Fig. 1.4-(a) and -(b) for NN→NNπ and NN→Dπ channels, respectively) for a quantitative explanation of the pion production at beam energies both below [25] and above threshold [31]. However, with decreasing beam energy this model becomes insufficient to explain the production rates as well as the shapes of the pion spectra. The required high relative energy among colliding nucleons...
1.2. The current status of the research

might originate from many-body correlations or cooperative multiple collisions. In fact, pion production very close to the absolute threshold may require that more and more nucleons in the projectile and target completely stop in the overall centre-of-mass system and their relative kinetic energy (except for a small Q-value effect) converts into the total energy of the pion. In competition with a single NN collision mechanism (NNSC), a collective process which involves the co-operation of many nucleons up to the formation of a compound nucleus is shown in Fig. 1.4-c and -d. Also multi-step processes with intermediate nucleon resonances might act as “energy storage” to concentrate the required meson-production energy.

In the mean-field approach, the dynamics of the collision is described by the time-dependent Hartree-Fock theory, and the pion production is treated in a first-order approximation by the so-called one-nucleon or two-nucleon mechanisms in the microscopic time-dependent orbits. Theoretical approaches that utilise mean field treatments \[32, 8, 33\] generally underestimate the total cross section significantly.

The mechanism of pionic bremsstrahlung is even more collective in nature as it only involves the ground-state densities of the two colliding nuclei. However, this depends sensitively on the assumed parameters related to the deceleration of the nuclei during the collision. The cross sections are qualitatively described, but severely underestimated, in a microscopic reaction model including intermediate baryon resonance excitation \[7, 34\].

In addition, the nuclear structure and especially coherent features play a determining role in heavier systems. This emphasises the importance of clustering correlations \[8\], since their high momentum components in the final nucleus may cooperate with the relative motion in the incident channel to facilitate the coherent process of pion production. Near the absolute threshold, the mechanism of “pionic fusion” in which two nuclei as a whole coherently interact with each other and fuse to form a compound nucleus, converting all the available free energy into a pion, may become the only possible production mechanism.

The above mentioned approaches neglect the effect of pion absorption which is reasonable for light nuclear systems. It can be concluded that the production mechanism of the pionic fusion reaction has not yet been settled. This unsatisfactory situation reflects the current poor understanding of the relevant many-body processes, and raises the question, “what kinds of mechanisms govern the transfer of energy to the meson channel?” In order to achieve a breakthrough in understanding the pion-production mechanism and to guide the theory, systematic data for simple systems with well-defined initial and final state configurations are mandatory. The study of pion production near the absolute threshold should thus provide a unique means of discovering whether or not cooperative mechanisms play a role in nuclear reactions.

Due to the low cross section (in the order of nb), pionic fusion experiments are not easily performed. It is the experimental challenge for the pionic fusion measurement to explore suitable target-projectile combinations in order to limit the number of possible intermediate states. Only a very small amount of data is available for pionic fusion mainly in light systems (see Table 1.1). The reported experimental data in the literature are mostly associated with the total cross section of charged pion production, and rarely complete angular distributions have been measured. The latest reported results of the pionic fusion experiments are the results of the \(^{12}\text{C}(^{12}\text{C},^{24}\text{Mg})\pi^0\) \[17\], \(^{12}\text{C}(^{12}\text{C},^{24}\text{Na})\pi^+\) \[17\] and \(^3\text{He}(^{4}\text{He},^{6}\text{Li})\pi^0\) \[12\] experiments. Only the fusion products in those experiments have been measured. In a model restricting the pion-nucleon interaction to the single-nucleon
Chapter 1: General introduction

Born term [32], the measured cross section of $208 \pm 38$ pb for the $^{12}$C($^{12}$C,$^{24}$Mg)$\pi^0$ reaction is underestimated by a factor of 10! The cause of this discrepancy is difficult to analyse since the existing data do not clearly separate the structure of the final state. Furthermore, the aim of the $^2$H($^4$He,$^6$Li)$\pi^0$ experiment was to use the pionic fusion of a deuteron and an $\alpha$ particle as a probe particularly sensitive to the cluster structure of the isobaric analogue state, at 3.56 MeV excitation energy in $^6$Li, of the $^6$He ground state.

This situation calls for reactions with well-defined final states and completely measured data set including the angular distribution of cross sections, which provides sensitivity to the relevant multipolarities involved in the pion production process. The main goal of the present work is to measure the angular distribution of the pionic fusion cross section in order to study systematically the effect of clustering and the influence of increased complexity in the clustered systems. In addition, in order to study the mass dependence of the pionic fusion cross section, we measured the total cross section of the pionic fusion reaction. The latter was compared with the predicted results of two different existing models [8, 32].

This work has two unique aspects which are not included in the previous pionic fusion experiments. First, the used experimental setup provided us the angular distribution of the pionic fusion cross section by measuring all the reaction products in overdetermined kinematics. Second, in the examined pionic fusion reactions with well-defined initial and final state configurations, simple cluster systems are involved. Hence, more complex clustered system demand more complicated theoretical work to model the reaction.

1.3 Pionic fusion experiments at KVI

In order to gain insight into the genuine quantum many-body problem and to provide clear test cases for theory, well-defined and simple reactions of subthreshold pion production need to be addressed. The necessary restrictions on relevant quantum numbers (spin, isospin) will provide the selectivity for particular processes. In the pionic fusion, detection of neutral pions has several advantages over charged pion detection. As an example, neutral pions decay with a probability of 98.8% and with a mean life time of $(8.4 \pm 0.6) \times 10^{-17}$ s, still in the target, into two $\gamma$-rays, which are detected with a multi-segmented $\gamma$-detector with a large solid angle.

Two pionic fusion experiments have been performed at KVI using the AGOR [41, 42] facility. The first experiment used a $^3$He beam with an energy of 85.3 MeV/A on a 130 mg/cm$^2$ liquid $^4$He target [43]. The final state is $^7$Be+$\pi^0$ and the beam current was on average about 0.3 nA. In the second experiment we used a 59.1 A MeV $^4$He beam on a 2 mg/cm$^2$ solid $^6$Li target. The final state in this case is $^{10}$B$^*$+$\pi^0$ and the beam current was on average 3 nA. Since we believe that with energies close to the coherent threshold the results are more sensitive to the internal dynamics of the reaction, both experiments were carried out at only about 10 MeV above the coherent threshold energy of pion production in the centre-of-mass system ($T_{CM} - T_{thr.CM} \approx 10$ MeV where $T_{CM}$ and $T_{thr.CM}$ are the beam kinetic energy and the threshold energy of the pionic fusion, respectively, in the centre-of-mass system). We identified the reaction by measuring the fused system and the produced $\pi^0$ with large acceptance. In this thesis, the results of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction will be presented and discussed together with the results of the $^4$He($^3$He,$\pi^0$)$^7$Be
1.3. Pionic fusion experiments at KVI

Table 1.5: Energy level scheme of $^{10}$B from the TUNL evaluation [44]. The thick arrows indicate the levels in $^{10}$B which are allowed to be formed in the pionic fusion. The numbers and the symbols in the figure from the left to the right are the excitation energy, the spin-parity $J^p$, the isospin $I$, the main decay channel and the lifetime or width, respectively.

For the $^4$He+$^6$Li→$^{10}$B$^+$$+\pi^0$ reaction, starting with an isospin $I=0$ entrance channel, the nature of the pion-production process necessitates a spin and isospin change by one unit, thus leaving the final nucleus in an $I=1$ state. Therefore, according to the rule of isospin conservation, the ground state of $^{10}$B is not excited. Figure 1.5 shows the energy levels of $^{10}$B from the TUNL evaluation [44]. The states in $^{10}$B in the pionic fusion are shown by the thick arrows. The first allowed excited state is located at $E=1.7402$ MeV ($J^p=0^+$ and $I=1$). This state decays by $\gamma$ emission. According to the available energy above the threshold (10 MeV), there are 5 more states in $^{10}$B to be possibly formed. Those states mainly decay by particle i.e. proton, deuteron, $\alpha$ and neutron emission. The angular distributions of decay particles were not fully covered by our experimental setup. Therefore, the pionic fusion events leading to those $^{10}$B states were not recorded in the
The unbound $I = 1$ level at $E=5.1639$ MeV is experimentally known to emit a $\gamma$-ray in competition with $\alpha$ particles \cite{ref45} ($\Gamma_\gamma \approx 1.5\pm0.4$ eV and $\Gamma_\alpha \approx 0.27\pm0.15$ eV). Since the $\alpha$ particles are not measured in the present experimental setup, only a part of the pionic fusion events leading to this $^{10}$B state at $E=5.1639$ MeV has been measured in the experiment. In conclusion, the $^{10}$B states that contribute to the pionic fusion reaction of $^4$He+$^6$Li$\rightarrow^{10}$B$^*+\pi^0$ in the experiment are the states at $E=1.7402$ MeV ($J^p = 0^+, I = 1$) and partly (85%) at $E=5.1639$ MeV ($J^p = 2^+, I = 1$).

1.4 Outline of this thesis

In Chapter 2 the theoretical approaches to predict the cross section of the pionic fusion reactions are introduced. Up to now, there is no theoretical prediction available for the $^6$Li$(^4$He,$\pi^0)^{10}$B$^*$ reaction, therefore, in Chapter 2 the predicted results for other pionic fusion reactions will be discussed. Numerical calculations of the $^6$Li$(^4$He,$\pi^0)^{10}$B$^*$ reaction kinematics using the Monte-Carlo techniques are introduced in Chapter 3. Chapter 4 is devoted to a description of the experimental setup, which is common for the $^6$Li$(^4$He,$\pi^0)^{10}$B$^*$ and $^4$He$(^3$He,$\pi^0)^{7}$Be experiments. The produced pion and the fusion product were measured using the Plastic Ball and the Big-Bite Spectrometer, respectively. The measurements were performed by requiring a coincidence between the $\gamma$-rays detected with the Plastic Ball and the fusion product detected with the Big-Bite Spectrometer. Furthermore, the experimental setup was such that for both processes the measured events were kinematically over determined. This experimental advantage was used to reduce the background.

In the succeeding Chapter 5 the analysis procedure of the $^6$Li$(^4$He,$\pi^0)^{10}$B$^*$ data is outlined. The experimental results and the measured cross section are presented in Chapter 6 and compared with the theoretical models described in Chapter 2 and numerical calculations described in Chapter 3. In Chapter 7, a discussion of the obtained results together with an outlook to future work are presented. Finally, in Chapter 8 a summary of this work is given.
### 1.4. Outline of this thesis

Table 1.1: Experimentally known cross sections for the \( A_2(A_1, \pi)B(I, J) \) reaction. \( T_{lab} \) and \( T_{thr} \) are the beam kinetic energy and the threshold energy of pionic fusion, respectively, in the laboratory system. Consequently, \( T_{CM} \) and \( T_{thr,CM} \) are the beam kinetic energy and the threshold energy of pionic fusion, respectively, in the centre-of-mass system. In order to calculate \( T_{thr} \), Eq. 1.1 has been used.

| Measured reaction | \( J^P \) | \( T_{lab} - T_{thr} \) [MeV] | \( T_{CM} - T_{thr,CM} \) [MeV] | \( d\sigma/d\Omega \) [nb/sr] | Forward |
|-------------------|----------|-------------------------------|-------------------------------|-----------------------------|---------|
| \( ^2\text{H}(^3\text{He},\pi^+)^6\text{Li} \) \[12\] | 1+:g.s. | 8.09                          | 1.2                           | \( \sigma_{tot}:228\pm6+70 \) | \( \sigma_{tot}:141\pm12+42 \) |
|                   | 0+:3.56 [MeV] | 10.44                        | 1.9                           |                             |         |
| \( ^3\text{He}(^3\text{He},\pi^+)^6\text{Li} \) \[13\] | 1+:g.s. | <1                            | 16+1.6                        |                             |         |
|                   | 3+:2.18   | 8.76                          | 15.44                         | 24+1.7                      |         |
|                   | 0+:3.56   | 8.76                          | 15.44                         | 43+4                        |         |
| \( ^4\text{He}(^3\text{He},\pi^+)\text{Li} \) \[14\] | 3/2−+1/2− | 18.89                        | 12.82                         | 25                          |         |
|                   | 7/2−,4.65 | 32.89                        | 20.72                         | 30                          |         |
| \( ^6\text{Li}(^2\text{H},\pi^-)^6\text{B} \) \[35\] | 2+:0.78  | 11.59                        | 86.21                         | 0.52±0.12                   |         |
|                   |          | 411.59                       | 301.51                        | 0.075±0.03                  |         |
|                   | 3+:2.32  | 11.59                        | 86.21                         | 0.71±0.14                   |         |
|                   |          | 411.59                       | 301.51                        | 0.085±0.03                  |         |
| \( ^6\text{Li}(^3\text{He},\pi^+)^9\text{Be} \) \[15\] | 3/2−,g.s. | 48.26                        | 55.98                         | <0.003                      |         |
| \( ^6\text{Li}(^3\text{He},\pi^-)^7\text{C} \) \[36, 37\] |          | 698.26                       | 446.22                        | <0.01                       |         |
| \( ^7\text{Li}(^3\text{He},\pi^+)\text{Be} \) \[16\] | 0+:g.s.  | 33.51                        | 37.43                         | \( \sigma_{tot}:0.47\pm0.06 \) | \( \sigma_{tot}:0.37\pm0.061 \) |
|                   | 2+:3.37  | 33.51                        | 37.43                         | \( \sigma_{tot}:0.47\pm0.06 \) | \( \sigma_{tot}:0.37\pm0.061 \) |
| \( ^7\text{Li}(^3\text{He},\pi^-)^7\text{C} \) \[16\] | 0+:g.s.  | 33.51                        | 37.43                         | \( \sigma_{tot}:0.47\pm0.06 \) | \( \sigma_{tot}:0.37\pm0.061 \) |
|                   | 2+:3.36  | 33.51                        | 37.43                         | \( \sigma_{tot}:0.47\pm0.06 \) | \( \sigma_{tot}:0.37\pm0.061 \) |
| \( ^{10}\text{B}(^3\text{He},\pi^+)^{13}\text{C} \) \[15\] | 1/2+,g.s. | 76.96                        | 82.4                          | \( \sigma_{tot}:0.028 \) | \( \sigma_{tot}:0.028 \) |
|                   |          | 99.96                        | 99.9                          | \( \sigma_{tot}:0.028 \) | \( \sigma_{tot}:0.028 \) |
| \( ^{12}\text{C}(^3\text{He},\pi^-)^{15}\text{N} \) \[16, 38, 39\] | 1/2−,g.s. | 59.13                        | 18.52                         | \( \sigma_{tot}:0.102\pm0.007 \) | \( \sigma_{tot}:0.115\pm0.005 \) |
| \( ^{12}\text{C}(^3\text{He},\pi^-)^{15}\text{F} \) \[16\] | 1/2−,g.s. | 59.13                        | 45.15                         | \( \sigma_{tot}:0.041 \) | \( \sigma_{tot}:0.041 \) |
| \( ^{12}\text{C}(^{12}\text{C},^{24}\text{Mg})\pi^0 \) \[17\] | g.s.     | 32                            | 6                             | \( \sigma_{tot}:0.208\pm0.038 \) | \( \sigma_{tot}:0.208\pm0.038 \) |
| \( ^{12}\text{C}(^{12}\text{C},^{24}\text{Na})\pi^+ \) \[17\] | g.s.     | 11                            | 6                             | \( \sigma_{tot}:0.182\pm0.084 \) | \( \sigma_{tot}:0.182\pm0.084 \) |
| \( ^{181}\text{Ta}(^3\text{He},\pi^-)^{185}\text{Os} \) \[11\] | g.s.     | 29.85                        | 26.34                         | <100                        | <100    |
| \( ^{208}\text{Pb}(^3\text{He},\pi^-)\text{At} \) \[40\] | g.s.     | <87.93                       | <91.79                        | 1< \( \sigma_{tot} <10 \) | 1< \( \sigma_{tot} <10 \) |
2. Theory

2.1 Introduction

The angular distribution of the pionic fusion cross sections contains the full dynamic information on the pion production and the pion re-scattering. Therefore, the differential cross sections provide sensitivity to the relevant multipolarities involved in the pion production process. The anisotropic nature of the angular distribution, the forward-backward asymmetry with a minimum in the vicinity of $90^\circ$ and its dependence on the pion kinetic energy and target mass contain information on the stopping of the projectile and the pion re-absorption, and could hint to $s$-wave and $p$-wave contributions in the pionic fusion. These facts suggest that the measurement of the pion angular distribution is a powerful tool to disentangle the contribution of the various processes. In this chapter, the scattering theory will be introduced and used to obtain the analytic form of the angular distribution. In addition, we will review some theoretical models and their predictions for the pionic fusion reaction.

2.2 Scattering theory

In quantum mechanics, the scattering of two particles is treated as an interaction of a wave packet with a local interaction potential. Starting with a wave packet $|\phi_E\rangle$ moving towards a localised interaction potential, the dynamics of the wave packet before the interaction is given by solutions of the Schrödinger equation

$$(E - H_0)|\phi_E\rangle = 0, \quad (2.1)$$

where $H_0 = -\frac{1}{2m}\Delta^2$ is the free Hamiltonian operator. A set of natural units with $\hbar = c = 1$ has been used in this thesis. As the wave packet approaches the potential region, it feels the interaction potential $H_{int}$ and the evolution in time of the modified wave packet will be given by the general scattering solutions of the time-dependent Schrödinger equation

$$(E - H_0 - H_{int})|\Psi_{\pm}^E\rangle = 0, \quad (2.2)$$

with $H = H_0 + H_{int}$. The behaviour of the wave packet before and after the interaction is constrained by boundary conditions and ± solutions indicate “in” and “out” boundary conditions. Using $H = H_0 + H_{int}$, Eq. 2.2 can be expressed as an inhomogeneous equation for $H_{int} \neq 0$ and a homogeneous equation for $H_{int} = 0$, with $|\Psi_{\pm}\rangle \rightarrow |\phi\rangle$ in the latter case. The scattering solution of Eq. 2.2 is then given by

$$|\Psi_{\pm}^E\rangle = |\phi_E\rangle + (E - H_0)^{-1}H_{int}|\Psi_{\pm}^E\rangle. \quad (2.3)$$

Applying the operator $(E - H_0)$ to the left side of Eq. 2.3 and by using Eq. 2.1, we again obtain Eq. 2.2. The inverse operator $(E - H_0)^{-1}$ does not exist for all the energies and
Figure 2.1: The \( T \) operator is depicted by the series of diagrams.

The \( T \) operator is depicted by the series of diagrams.

\[
\begin{align*}
\text{T} &= + + + + + + + + \ldots
\end{align*}
\]

has a singularity for specific energies. To fix this problem, \( i\epsilon \) (\( 0 < \epsilon \to 0, \epsilon \in \mathbb{R} \)) as a small imaginary term is added to \((E - H_0)\), after which it does have an inverse

\[
|\Psi^\pm_E\rangle = |\phi_E\rangle + (E \pm i\epsilon - H_0)^{-1}H_{\text{int}}|\Psi^\pm_E\rangle.
\]  

(2.4)

This leads us to the central operator, the Green’s function in the scattering theory, the resolvent operator to the Hamiltonian

\[
G(z) \equiv \frac{1}{z - H}.
\]  

(2.5)

Here, \( z = E \pm i\epsilon \) is a complex value which is not in the spectrum of \( H \) (a real value). We can approach the spectrum of \( H \) from \( G^+(E) \) and \( G^-(E) \) by taking the proper limit of \( \epsilon \to 0 \). The resolvent operator \( G(z) \) can be expressed using \( H, H_0 \) and \( G_0(z) \)

\[
G(z) = G_0(z) + G_0(z)H_{\text{int}}G(z),
\]  

(2.6)

where \( G_0(z) \equiv \frac{1}{z - H_0} \). This operator indicates that the final wave functions can be derived in an iterative process. Eq. 2.3 can be rewritten as a free part corresponding to a freely moving particle and a scattering part

\[
|\Psi^\pm_E\rangle = |\phi_E\rangle + G_0^\pm H_{\text{int}}|\Psi^\pm_E\rangle.
\]  

(2.7)

This integral equation is called the Lippmann-Schwinger equation (LSE). The transition matrix is defined by

\[
T_{if} = \langle \phi_E | H_{\text{int}} | \Psi^+_E \rangle.
\]  

(2.8)

Multiplying Eq. 2.7 on the left by \( \langle \phi_E | H_{\text{int}} \) and applying Eq. 2.8, we obtain the operator Lippmann-Schwinger equation

\[
T = H_{\text{int}} + H_{\text{int}}G_0^\pm T.
\]  

(2.9)

The transition operator \( T \) can be represented by the series of diagrams that are shown in Fig. 2.1, where the dashed lines represent the interaction potential \( H_{\text{int}} \).

### 2.3 Experimental determination of the reaction rate

The reaction rate measured in scattering experiments yields information about the dynamics of the interaction between projectile and target. The most important quantity for the description and interpretation of these reactions is the cross section \( \sigma \), which is a measure of the probability of a reaction between the two colliding particles.

In the reaction of two nuclei \( A_1 \) and \( A_2 \) as target and projectile, respectively, the total
reaction rate $\dot{N}$ will be given as

$$\dot{N} = \Phi_{A_2} \cdot N_{A_1} \cdot \sigma_{A_1}, \quad (2.10)$$

where $N_{A_1}$ and $\sigma_{A_1}$ are the number of scattering centres of the target within the beam area and the cross-sectional area of each scattering centre in the target, respectively. $\Phi_{A_2}$ is the number of projectiles hitting the target per unit time per unit area and is called flux,

$$\Phi_{A_2} = n_{A_2} \cdot v_{A_2}. \quad (2.11)$$

$n_{A_2}$ and $v_{A_2}$ are the projectile density and velocity, respectively.

### 2.4 Cross section evaluation from scattering theory

The “Fermi Golden Rule” expresses the reaction rate $W$ per target particle and per beam particle in the form:

$$W = 2\pi |T_{fi}|^2 \cdot \rho(\dot{E}), \quad (2.12)$$

where $\rho(\dot{E})$ is the density of final states in the energy interval $\dot{E}$ and $\dot{E} + d\dot{E}$. $T_{fi}$ is the transition matrix element given by

$$T_{fi} = \langle \Psi_f | H_{int} | \phi_i \rangle = \int \Psi_f^* H_{int} \phi_i dV, \quad (2.13)$$

where the outgoing particle is scattered into a volume $V$. We also know, however, from Eqs. 2.10 and 2.11 that:

$$W = \frac{\dot{N}}{N_{A_1} \cdot N_{A_2}} = \frac{\sigma \cdot v_{A_2}}{V}, \quad (2.14)$$

where $N_{A_2}$ is the number of projectile particles and $V = \frac{N_{A_2}}{n_{A_2}}$ is the volume occupied by the projectile particles. Hence, the cross section is

$$\sigma = \frac{2\pi}{v_{A_2}} |T_{fi}|^2 \cdot \rho(\dot{E}) \cdot V. \quad (2.15)$$

Therefore, the differential cross section for the scattering of the particle into a solid angle element $d\Omega$ is:

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{v_{A_2}} |T_{fi}|^2 \cdot \rho(\dot{E}) \cdot \frac{V^2}{4\pi}. \quad (2.16)$$

In the pionic fusion process of

$$A_1 + A_2 \rightarrow B(J, I) + \pi \quad (2.17)$$

the pion production is directly associated with the complete fusion of the initial nuclei $A_1$ and $A_2$ to a unified nucleus $B$ with spin and isospin quantum numbers $J$ and $I$, respectively. The free energy in the entrance channel, viz. the kinetic energy of the relative motion is completely transferred to the two-body exit channel including the pion field. The differential cross section in the CM system after simplification of Eq. 2.16 is
Chapter 2: Theory

given by

\[ \frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2} \frac{E_B E_{A_1} E_{A_2} k_f}{(E_{A_1} + E_{A_2})^2 k_i} \sum_{M_{f,v}} |T_{M_{f,v}}|^2, \]

where \( E_{A_1}, E_{A_2} \) and \( E_B \) are the relativistic energies of the colliding nuclei \( A_1 \) and \( A_2 \) and the fused nucleus \( B \), respectively. The symbols \( k_i \) and \( k_f \) denote the relative momenta in the entrance and the exit channels, respectively. The transition matrix is defined by

\[ T_{M_{f,v}} = \langle k_f; J_f M_f | H_{\text{int}} | k_i; J_i M_i \rangle. \]

\( H_{\text{int}} \) represents the pion production operator. \( | k_i; J_i M_i \rangle \) is the initial scattering state of the \( A_1 + A_2 \) system with channel spin \( J_i \) and its projection \( M_i \), and \( \langle k_f; J_f M_f | \) is the final bound state of \( B \) with total spin \( J_f \) and its projection \( M_f \).

2.5 Theoretical models of pionic fusion

In the introductory sections, we formulated the cross section of a pionic fusion reaction which is described in terms of the transition matrix and the associated kinematic factors. Here, we will review some theoretical models and their predictions for the pionic fusion reaction. There is no unified approach for the construction of the models for which the transition matrix is calculated. The main difficulty is to find the appropriate reaction mechanism. Two types of models are presented. The first type is the so-called independent nucleon-nucleon (NN) model. In the second type, the collective behaviour of nucleons to produce a pion is considered.

2.5.1 Independent Nucleon-Nucleon (NN) models

As a first step, it is natural to consider pion production from nucleus-nucleus collisions in terms of NN collisions yielding a free pion. In the approaching nuclei, the nucleons are initially bound in their (shell model) orbits in the nuclear mean field. Collisions between the nucleons of projectile and target will then take place within the nuclear medium, with some of the final states blocked by other nucleons (the Pauli exclusion principle). When the nuclear densities start to overlap, the initial nucleon distributions will rapidly be altered by the interactions. Therefore, the process can be much more complicated than the collisions between free nucleons leading to pion production. In the following, we discuss the predictions of two models for pion production calculated from the sequence of NN collisions by using a typical interaction model.

Fermi-gas model

The earliest suggestion to understand the pionic fusion process dates back to Fermi. He suggested an elementary NN interaction in the pionic fusion reaction. It exploits the fact that the initial intrinsic nucleonic motion offers the necessary extra boost for the elementary NN pion production process if one takes into account the coupling of the relative momentum between the projectile and target and the Fermi momenta of the nucleons \( P_F \). In other words, because of the Fermi motion of the nucleons in the projectile and target nucleus, some of the NN collisions would exceed the threshold value of 280
2.5. Theoretical models of pionic fusion

Theoretical models of pionic fusion

Figure 2.2: Maximum energies available for two colliding nucleons in the Fermi gas approach [33]. The two sets of curves are calculated under the assumption of anti-parallel momenta (the dashed curve) and momenta anti-parallel within $30^\circ$ (the solid curve).

(or 290) MeV for neutral (or charged) pion production and lead to pion production at subthreshold energy. In the simplest picture [33], the maximum energy between a nucleon from the projectile and one from the target can be written as

$$E_{\text{max}} = \frac{(P_F + P/2)^2}{M_N}. \quad (2.20)$$

Here, $P$ is the projectile momentum per nucleon and $M_N$ is the nucleon mass. In Eq. 2.20 non-relativistic kinematics are used as an appropriate approximation in the energy range below 90 MeV/nucleon. If $E_{\text{max}}$ exceeds $m_\pi$, a pion can be produced in such a collision. This original idea was reconsidered in an “NN Single Collision (NNSC)” model (shown schematically in Fig. 1.4-(a) and -(b), for NN→NN\(\pi\) and NN→D\(\pi\) channels, respectively) for a quantitative explanation of the pion production at beam energies both below [25] and above threshold [33]. However, according to Eq. 2.20, with decreasing beam energy this model will become insufficient to explain the production rates as well as the shapes of the pion spectra. To illustrate the coupling effect between the beam energy and Fermi momentum, Fig. 2.2 shows $E_{\text{max}}$, in the laboratory frame, under the assumptions that only nucleons with anti-parallel momenta (dashed curve) and nucleons with their momenta anti-parallel within a cone angle of $30^\circ$ (solid curve) contribute to the production of a pion. One should note that overall energy conservation for this process implies that near threshold the two colliding nucleons carry a large fraction of the total centre-of-mass energy. Consider, for example, the $^{12}\text{C}(^{12}\text{C},\pi^0)^{24}\text{Mg}$ reaction at $E_{\text{lab}}/A = 30$ MeV. From
Chapter 2: Theory

A schematic representation of pionic fusion of two nuclei in the sudden approximation [32].

![Figure 2.3: Schematic representation of pionic fusion of two nuclei in the sudden approximation.](image)

Fig. 2.2 we can easily understand that we reach $E_{max} \approx 160$ MeV when two colliding nucleons have anti-parallel momenta, while the overall centre-of-mass energy is 180 MeV. Nearly 90% of the centre-of-mass energy is then carried by the two colliding nucleons and the other 22 nucleons have to “conspire” coherently for this process to happen!

### Sudden Overlap model

The Sudden Overlap model [32] aims to describe the pionic fusion reaction at an excitation energy which is only a few MeV above the absolute threshold. This model considers the cross section in the Born approximation, assuming that pion production occurs through coupling to a single nucleon. All possible further re-scattering processes of the pion are ignored as higher order processes [20]. A schematic picture of the reaction is shown in Fig. 2.3 demonstrating the pionic fusion of two nuclei $A_1$ and $A_2$.

Many-body nuclear mean field parameters suddenly change when the reaction proceeds from the initial to the final stages. This will be referred to as the sudden approximation. The three-dimensional harmonic oscillator shell model is used to describe the structure of the incoming and outgoing nuclei ($| k_i; J_i M_i \rangle$ and $\langle k_f; J_f M_f |$ in Eq. 2.19). In addition, a plane wave solution is assumed for the pion wave function. On the single-nucleon level, a phenomenological Hamiltonian density has been used for the pion-nucleon interaction [21]

$$H_{int} = \frac{g_{\pi N}}{2f_{\pi}} \bar{\psi} \bar{\sigma} \cdot \nabla \bar{\tau} \cdot \bar{\pi} \psi + 4\pi \frac{\lambda_1}{m_\pi} \bar{\psi} \bar{\pi} \cdot \bar{\pi} \psi + 4\pi \frac{\lambda_2}{m_\pi} \bar{\psi} \bar{\tau} \cdot \bar{\pi} \times \partial \bar{\pi} \psi, \quad (2.21)$$

where

- $\psi, \bar{\psi}$: nucleon fields,
- $\bar{\pi}$: pion field,
- $\bar{\tau}$: isospin operator,
- $\bar{\sigma}$: spin operator,
2.5. Theoretical models of pionic fusion

- $g_A$: axial coupling constant,
- $f_\pi$: pion decay constant = 93 MeV,
- $\lambda_1$: coupling constant $\sim 0.005$,
- $\lambda_2$: coupling constant $\sim 0.05$.

The first term, which is called the impulse or Born term, is responsible for single pion production in a $p$-wave. The second and third are $s$-wave terms, and require an additional interaction to absorb the extra pion created in the four-point vertex [20]. It has been experimentally observed that in the pionic fusion reactions, the pion is predominantly produced in the $p$-wave [13, 17]. Therefore, the second and third terms are neglected in this model.

The model has been applied to reproduce the experimental cross section of the pionic fusion reaction $^3\text{He}(^3\text{He},\pi^+)^6\text{Li}$ [13]. The results are shown in Fig. 2.9. The solid and thick-dashed curves are the calculated results of the current model for the $^3\text{He}(^3\text{He},\pi^+)^6\text{Li}$ reaction leading to the $1^+$ ground state and the $3^+$ excited state in $^6\text{Li}$, respectively. The data points in this figure are the experimental results [13] leading to the $1^+$ ground state (squares) and the excitation of the ground state to the $3^+$ state in $^6\text{Li}$ (triangles). As can be seen, the general trend of the calculated and measured results is the same. However, the model predictions are lower than the experimental cross sections by approximately 40%. A reason for this low estimation could be attributed to the sensitivity of the model parameters in the shell model and the choices of the harmonic oscillator approximations. In addition, the sudden approximation seems to have another shortcoming in the framework of the modelling since the slow change of the nuclear mean field in the nuclear reaction process is not considered.

Near the absolute threshold, the pion momentum $k_\pi$ is small compared to the momentum of incoming nucleons. Therefore, $k_\pi$ can be ignored in polynomial expansions that are used to construct the wave functions. The obtained $p$-wave cross section in this approximation (low energy approximation) is proportional to $(k_\pi/m_\pi)^3$. The typical value for $k_\pi$ in this approximation is 50 MeV giving $(k_\pi/m_\pi)^3 \approx 0.05$. Using this model, the general behaviour of the pionic fusion cross section $A+A \rightarrow 2A + \pi$ has been studied as a function of the mass number of the initial nucleus “A”. The results are shown in Fig. 2.4. Although there are fluctuations, Fig. 2.4 clearly shows that the overall trend of a decreasing cross section with increasing mass in the mass range $A \in [1, 20]$ is well reproduced. However, the cross section magnitude remains in the $pb$ region for nuclei as large as Oxygen ($\approx 0.2$ pb). In Fig. 2.4, the full circles are the results calculated in this model. The empty circles are the measured results by the previous experiments [17, 13]. In case of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions, target and projectile are not the same. However, the target and projectile masses are rather close. Therefore, the results of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ experiments are also shown in Fig. 2.4 by cross and square, respectively. As can be seen, the total measured cross sections of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions confirm the mass dependence of the pionic fusion that has been predicted by the Sudden Overlap model.
Figure 2.4: The general behaviour of the pionic fusion cross section \( A + A \rightarrow \pi + 2A(J,I) \) versus the mass number of initial nucleus “A”. \( k_\pi \) and \( m_\pi \) are the pion momentum and mass, respectively. The full circles are the calculated results from the Sudden Overlap model which is described in Ref. [32]. The empty circles are the results measured by the previous experiments [17, 13]. The cross and empty square are the results of the KVI measurements for the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) and \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) reactions, respectively, where “A” is the average of projectile and target mass.

2.5.2 Models involving collective processes

The most interesting aspect of pion production at energies close to the absolute threshold is its high sensitivity to collective or coherent mechanisms in the production process. A precise definition of coherence or collectivity is useful only with a specific model in mind. We adopt here the operational definition that coherent or collective production is a mechanism whereby energy is extracted from the relative motion by slowing down the projectile and/or target as a whole and converted into one degree of freedom, the pion. Within this definition, a large fraction of the nucleons have to interact with a common “phase”. Following this definition, thermal production is not a collective process, although it involves pooling of the kinetic energy of many nucleons. Note that even at the absolute threshold one expects a small, but nonzero, non collective contribution from thermal production.

Microscopic Reaction model

The Microscopic Reaction model developed by Harzheim et al. [7] is based on the concept that the pionic fusion in nuclear collisions is essentially a many body process. In this model, the free energy of the entrance channel is transferred to the final channel through
the coupling of the relative motion to the internal excitation of $N^*$-resonances. The process is divided into three phases: (i) In the first stage the nuclear relative motion is decelerated by the interaction of nucleons belonging to the target and the projectile. This interaction can be thought of as being mediated by the exchange of mesons. Although the most likely evolution of the system will in general be towards thermalisation, i.e. the distribution of the free energy over many degrees of freedom, in the extreme limit of the pionic fusion reaction the total free energy has to be completely transferred from the entrance to the exit channel, i.e. from one degree of freedom to another one essentially avoiding any heating processes. (ii) This process occurs via an intermediate nuclear excitation, baryonic resonances, which may serve as a coherent energy storage. Especially the $\Delta$-resonance may offer an efficient reaction path to keep the free energy concentrated in a few degrees of freedom. (iii) Finally, such a nuclear $N^*$-resonance readily decays by emitting a meson and thus provides a natural mechanism for the coupling to the meson field of the exit channel. The corresponding transition operator for this three-step process is given by

$$T = \left( \sum_{k \in A_1} L^\dagger(k) \right) G \left( \sum_{i \in A_1, j \in A_2} V^{NN}(i, j) \right) + \left( \sum_{k \in A_2} L^\dagger(k) \right) G \left( \sum_{i \in A_2, j \in A_1} V^{NN}(i, j) \right), \quad (2.22)$$

which accounts both for the target- and projectile-emissions. Here, $V^{NN}(i, j)$ is the NN interaction between two nucleons belonging to the target and projectile. $L^\dagger$ ($L$) stands for the annihilation (creation) of a meson on one of the baryons. When the interaction $V^{NN}$ is dominated by pion exchange, the NN potential is written as

$$V^{NN}(i, j) = L^\dagger_{\pi NN^*}(j) g L_{\pi NN}(i), \quad (2.23)$$

with the pion exchange propagator $g$. The propagator $G = (E - H^{eff}(E))^{-1}$ for the intermediate $B$-baryon system is given in a spectral representation by

$$G = \sum_\mu |B^*_\mu(E)\rangle [E - \lambda_\mu(E)]^{-1} |B^*_\mu(E)\rangle, \quad (2.24)$$

where the nuclear eigenmodes $|B^*_\mu(E)\rangle$ of the internally excited nucleons are defined by

$$H^{eff}(E) |B^*_\mu(E)\rangle = \lambda_\mu(E) |B^*_\mu(E)\rangle. \quad (2.25)$$

Here, $H^{eff}(E)$ is the effective (energy dependent) Hamiltonian operator that describes the intermediate nuclear $\Delta$-excitation denoted by $B^*_\mu$. Thus the transition amplitude is given by

$$T_{fi} = \langle B_f, \pi \mid T \mid (A_1, A_2)_i \rangle = \sum_\mu \langle B_f, \pi \mid \sum_k L^\dagger(k) \mid B_f, \pi \rangle [z - \lambda_{\mu}(z)]^{-1} \langle B_f, \pi \mid W_{12} \mid (A_1, A_2)_i \rangle, \quad (2.26)$$

where

$$W_{12} = \left( \sum_{i \in A_1, j \in A_2} + \sum_{i \in A_2, j \in A_1} \right) V(i, j)$$
Figure 2.5: Schematic representation of the projectile- (a) and target-emission (b) contribution to the pionic fusion process according to the Microscopic Reaction model [7]. The box labelled as “G” stands for the propagation of the intermediate $B^*_\Delta$ system. “S” denotes the spectroscopic factor for the entrance channel decomposition of the final nuclear system.

is the nucleus-nucleus interaction. Consequently the transition amplitude depends on three factors: (i) the excitation strength for the transition from the entrance channel to the intermediate $B^*$-system; (ii) the propagation of this system and (iii) the de-excitation strength of the latter into the final nucleus channel. The schematic layout of the projectile and target emission contribution to the pionic fusion process in this model is shown in Fig. 2.5-(a) and -(b), respectively. According to this model, the dominant contribution to the transition amplitude is found to be due to the excitation of intermediate $\Delta$-nuclear states, which together with the form factors determine the energy dependence of the process. However, the cross section depends sensitively on the spectroscopic properties of the participating nuclei as well as the spectroscopic amplitudes for the initial channel decomposition of the final nuclear (bound) state.

The differential cross section has been calculated for the elementary process $p + p \rightarrow d + \pi^+$ as well as $p + d \rightarrow ^3\text{He} + \pi^0$ and $^3\text{He} + ^3\text{He} \rightarrow ^6\text{Li} + \pi^+$ reactions. Figure 2.6-(a) and -(b) shows the calculated cross section for the $p + p \rightarrow d + \pi^+$ and $p + d \rightarrow ^3\text{He} + \pi^0$ reactions, respectively, compared with the measured one. This model predicts how the target and projectile in the $^3\text{He} + ^3\text{He} \rightarrow ^6\text{Li} + \pi^+$ reaction contribute to the angular distribution (shown in Fig. 2.7). As can be seen, in general, the model can describe the pionic fusion. However, the calculated cross section underestimates the measured cross section at low energies in case of the $p + p \rightarrow d + \pi^+$ reaction and at backward angles in case of the $p + d \rightarrow ^3\text{He} + \pi^0$ reaction. In addition, in case of the $^3\text{He} + ^3\text{He} \rightarrow ^6\text{Li} + \pi^+$ reaction, the calculated cross section overestimates the measured cross section. It should be mentioned that in this model first of all the s-wave contribution (see Fig. 1.1) of pionic fusion has been neglected. In this treatment all intermediate $\Delta N$-states contribute to the amplitude with the same weight.
2.5. Theoretical models of pionic fusion

Figure 2.6: (a): The calculated cross section (curves) of the $p + p \rightarrow d + \pi^+$ reaction compared with the experimental results when $T_{lab} = 295, 560$ and $810$ MeV (squares, triangles and circles, respectively). (b): The calculated cross section (curve) of the $p + d \rightarrow ^3\text{He} + \pi^0$ reaction compared with the experimental results with $T_{lab} = 400$ MeV.

Figure 2.7: Comparison of calculated (the solid curve) and experimental differential cross sections (data points) for the $^3\text{He} + ^3\text{He} \rightarrow ^6\text{Li}(\text{g.s.}) + \pi^+$ reaction at $T_{lab} = 400$ MeV. The dashed (dotted) curve represents the target (projectile) emission contribution. Experimental data are taken from [10, 46, 47].
which is not in agreement with what has been found in $\pi - d$ scattering. Therefore, we expect that if a more realistic description of the wave functions is used in the calculations, the theoretical cross sections become more realistic.

It should be noted that the target (projectile) contribution in the pion production which is shown in Fig. 2.7 by the dashed (solid) curve show asymmetric and backward (forward) peaked angular distributions for the $^3\text{He} + ^3\text{He} \rightarrow ^6\text{Li} + \pi^+$ reaction. One possible explanation for the backward (forward) peaked behaviour of the angular distribution is that since the target (projectile) moves to the backward (forward) direction in the centre-of-mass system, the pion produced by the target (projectile) will dominantly move to the backward (forward) direction. In Chapter 7, the anisotropic behaviour of the angular distributions measured in previous pionic fusion experiments are discussed in more detail.

\section*{Semi-empirical model}

Germond et al. [48] have built a model to describe the coherent production of pions in the $A(^3\text{He},\pi^+)B$ processes leading to discrete states of the final nucleus $B$, where $A$ and $B$ are the target nucleus and the fusion product, respectively (see Fig. 2.8). The incident $^3\text{He}$ particle picks up a proton from nucleus $A$ to leave some core of nucleons $C$. The produced $^4\text{He}$ particle then combines with $C$ to form the final nuclear state $B$. The model called “Semi-empirical model” uses the input experimental data from the $^1\text{H}(^3\text{He},\pi^+)^4\text{He}$ reaction and a cluster decomposition of the state $B$. The basic three-nucleon-transfer reaction is then driven by the $^1\text{H}(^3\text{He},\pi^+)^4\text{He}$ sub-amplitude as shown in Fig. 2.8. The reaction is therefore considered to be $^1\text{H}C(^3\text{He},\pi^+)^4\text{He}C$, where $C$ is a spectator nucleus, being $^2\text{H}$ and $^3\text{H}$ in the $^3\text{He}(^3\text{He},\pi^+)^6\text{Li}$ and $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ cases, respectively.

The transition matrix is written as

$$T_{BA} = K_{BA} \langle k_\pi k_B s_B^2 | H_{\text{int}} | k_\tau k_A s_A^2 s_A^2 \rangle,$$

(2.27)

where

$$K_{BA} = -\frac{1}{(2\pi)^2} \left( \frac{E_\pi E_B E_\tau E_A}{(E_\pi + E_B)^2} \right)^{1/2}.$$

(2.28)
2.5. Theoretical models of pionic fusion

$k_\pi$, $k_\tau$, $k_A$ and $k_B$ stand for momentum in the $\pi - B$ and $^3$He$-A$ systems, momentum of the target and the produced nucleus, respectively. The symbols $E_\pi$, $E_B$, $E_\tau$ and $E_A$ denote the energy of $\pi$, $B$, $^3$He and $A$ nuclei, respectively. $s_A$, $s_\tau (= 1/2)$ and $s_B$ are the spin of the initial nucleus $A$, that of the $^3$He and the combined nucleus, respectively. For the “basic” amplitude $1$He($^3$He,$\pi^+)^4$He there are two independent terms, both of which couple to the $S=1$ combination of the $^3$He and proton spins. In the non-relativistic Galilean-covariant approach, the related transition matrix for the intermediate reaction can be written as

$$T_{4He,^1H} = \langle k_\pi k_\alpha | H_{\text{int}} | k_\tau k_p s^2_\pi s^2_p \rangle = \langle k_\pi k_\alpha | H_{\text{int}}^{(1)} | k_\tau k_p \rangle S_{\pi m_p}^{k_r m_\tau} + \langle k_\pi k_\alpha | H_{\text{int}}^{(2)} | k_\tau k_p \rangle S_{\pi m_\alpha}^{k_r m_\tau},$$

(2.29)

and equivalently for the transition amplitude $T_{BA}$ defined in Eq. 2.27. There is a linear relationship between the $^3$He$-A$ and $^3$He$-p$ transition matrices:

$$\langle k_\pi, p_B | H_{\text{int}, \pi A \rightarrow \pi B} | k_\tau, p_A \rangle = X_\alpha X_p \int \Phi^*_B(K - Q, \eta) \times \langle k_\pi, K + p_B - (1 - m_p/m_A)p_A | H_{\text{int}, \pi \pi \rightarrow \pi \pi A} | k_\tau, K + m_p p_A/m_A \rangle \times \Phi_A(K, \eta) dK d\eta.$$

Here, $\eta$ represents the internal variables of the cluster $C$ and $K$ the motion of the proton relative to it. The $X_p$ and $X_\alpha$ are spectroscopic factors reflecting the number of contributing protons and $\alpha$ particles in the nuclei $A$ and $B$. In this model, $\Phi^*_B(K - Q, \eta)$ and $\Phi_A(K, \eta)$ are chosen as follows:

$$\Phi^*_B(K - Q, \eta) \sim K^{n_B} \exp\left(-\frac{K^2}{2\beta^2}\right),$$

(2.30)

$$\Phi_A(K, \eta) \sim \exp\left(-\frac{K^2}{2\alpha^2}\right).$$

(2.31)

This model has been applied for the $^4$He($^3$He,$\pi^+)^7$Li and $^3$He($^3$He,$\pi^+)^6$Li reactions. The result of the calculation for the $^4$He($^3$He,$\pi^+)^7$Li reaction is shown in Fig. 2.13 by the thick-solid curve. Experimentally, cross sections of the $^4$He($^3$He,$\pi^+)^7$Li reaction leading to the $s_{Li} = \frac{3}{2}^-$ and $\frac{1}{2}^-$ doublet ($L_{Li} = 1$) have been measured (data points in Fig. 2.13) [14]. The calculation in the Semi-empirical model does not agree with the measured results of the $^4$He($^3$He,$\pi^+)^7$Li reaction. In case of the $^3$He($^3$He,$\pi^+)^6$Li reaction, the angular distribution (except for the most forward angles) is not reproduced. This situation is shown in Fig. 2.9 by the dashed and dotted curves when the ground state ($1^+$) and the excited state ($3^+$) in $^6$Li is formed. The major problem of this model is to assign the proper wave function for the compound fusion product, since the wave function obtained using the harmonic oscillator or the Wood-Saxon potential descriptions are rather different. The strong clustering correlations of the fusion product components are not taken into account in this model. Another problem of this model is that, as can be seen in Fig. 2.8, only one nucleon from the target nucleus contributes to the pion production. At low energies, close enough to the coherent threshold of pionic fusion, almost all the nucleons in the target and projectile need to contribute coherently to the reaction. Therefore this model is not suitable for the reaction at low available energies.
Figure 2.9: Angular distributions of the cross sections for the $^3\text{He}(^3\text{He},\pi^+)^6\text{Li}(\text{g.s.,2.18 MeV})$ reaction at the beam kinetic energy $E_{\text{lab}} = 280$ MeV. For the theoretical calculations, the two different methods, the Semi-empirical [48] and the Sudden Overlap [32] models, are applied. The solid and thick-dashed curves are the results obtained on the basis of Sudden Overlap calculations leading to the $1^+$ ground state and the $3^+$ state in $^6\text{Li}$, respectively. The dashed and dotted curves are the results of the Semi-empirical model in forming the $1^+$ ground state and the $3^+$ excited state in $^6\text{Li}$, respectively. The data points are the experimental results leading to the ground state (squares) and the $3^+$ state in $^6\text{Li}$ (triangles) [13, 49].

Model based on clustering correlations

Kajino et al. [8] proposed an interacting cluster model. Correlated microscopic cluster model wave functions are adopted for $\langle k_f; J_f M_f |$ in Eq. 2.19
2.5. Theoretical models of pionic fusion

Figure 2.10: Schematic representation of the pionic fusion mechanism with the strong clustering correlations taken from the model described in Ref. [8]. "H\text{int}" represents the pion production operator and "C" denotes the clustering correlations. The Pauli exclusion principle is taken into account in the Resonating Group Method (RGM).

\[ \langle \zeta_{A_1}\zeta_{A_2}r \mid k_f; J_f M_f \rangle = \frac{a_1!a_2!}{b!} S_{A_1 A_2} \Phi_{a_1}(\zeta_{A_1})\Phi_{a_2}(\zeta_{A_2}) \]
\[ \otimes i^l Y^L(\hat{r})]_{M\chi_{JL}}^{s_{A_1 A_2}}(r), \] (2.32)

where \( S_{A_1 A_2} \) is the antisymmetrizer of nucleons and \( \Phi_{a_1}(\zeta_{A_1}) \) and \( \Phi_{a_2}(\zeta_{A_2}) \) are the internal wave functions of \( A_1 \) and \( A_2 \) in Eq. 2.17, respectively. \( a_1, a_2 \) and \( b \) are the number of nucleons in \( A_1, A_2 \) and \( B \), respectively. \( \chi_{JL}(r) \) is the inter-cluster relative wave function which is a solution of the Resonating Group Method (RGM) equation of motion [50]

\[ \int [H_{RGM}(r, \hat{r}) + iW_D(\hat{r}-\hat{r})] \chi_{JL}(\hat{r})d\hat{r} = E \int N_{RGM}(r, \hat{r})\chi_{JL}(\hat{r})d\hat{r}. \] (2.33)

\( N_{RGM}(r, \hat{r}) \) is the normalisation kernel. \( H_{RGM}(r, \hat{r}) \) is the Hamiltonian kernel and consists of two parts: the kinetic energy of the nucleons with the CM energy subtracted and the interaction energy. For the central part of the nucleon-nucleon effective interaction, the modified Hasegawa-Nagata formulation has been used. For the spin-orbit part of the interaction, the Nagata force has been used. In addition to the real interaction, \( iW_D(r) \) as local imaginary potential between two clusters has been applied. \( iW_D(r) \) has the same radial dependence as the real part of the nuclear direct folding potential. The initial scattering wave function is constructed by superimposing the cluster wave function.
Chapter 2: Theory

Figure 2.11: Four different terms of the transition matrix for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction in the model based on clustering correlation (upper part) and the contribution of each term in the angular distribution (lower part) [8].

Eq. 2.32 as

$$\langle \zeta_{A_1} \zeta_{A_2} r \mid k_i; J_i M_i \rangle = \sum_{LJ} \sqrt{4\pi} \sqrt{2L + 1} (LOp_i; J_i M_i \mid J_f M_f) \langle \zeta_{A_1} \zeta_{A_2} r \mid p_f; J_f M_f \rangle \quad (2.34)$$

In this case, the inter-cluster relative wave function is the solution of Eq. 2.33 satisfying the scattering boundary condition.

This model has been applied to the experimentally studied pionic fusion reaction $^4\text{He}(^4\text{He},\pi^+)^7\text{Li}$. In this model, for the pion production operator $H_{int}$, only the contribution of a single nucleon process is taken into account as

$$H_{int} = -\frac{\sqrt{4\pi f}}{m_\pi} \int \sum_{N=1}^7 \delta(r_\pi - r_N) \sigma_N \cdot \left[ 1 + \frac{\omega}{tM_N} \right] \nabla_{\pi C} - \frac{\omega}{M_N} \nabla_{N C} \tau_N \Phi(r_\pi) dr_\pi. \quad (2.35)$$

Here, $\sigma_N$ and $\tau_N$ are spin and isospin operators of nucleons, respectively. $\Phi(r_\pi)$ is the pion field and $r_\pi$ and $r_N$ are the coordinates of pion and nucleon, respectively. $\omega$ is the
2.5. Theoretical models of pionic fusion

The predicted pionic fusion cross section for the $A(^3\text{He},\pi^+)B$ reaction. The calculations were performed using the shell-model (empty circles) and cluster-model wave functions (full circles) [8].

![Figure 2.12: The predicted pionic fusion cross section for the $A(^3\text{He},\pi^+)B$ reaction. The calculations were performed using the shell-model (empty circles) and cluster-model wave functions (full circles) [8].](image)

relativistic energy of the pion and the coupling constant is taken to be $f^2=0.08$. It was noticed that some correlations responsible for the two-nucleon process i.e., the $p$-wave process (see Fig. 1.1) are included automatically in the cluster wave functions. The remaining two-body terms, i.e., the $s$-wave coupling and $\Delta$-isobar intermediate coupling are neglected in this model. The schematic representation of the model is shown in Fig. 2.10. The transition matrix can be decomposed into four different terms which are schematically shown in Fig. 2.11. Each contribution of four different terms to the cross section is shown in the lower part of Fig. 2.11. As can be seen in Fig. 2.11, the contribution of the reaction type “a” is the most dominant. According to this model, it is concluded that in the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction a pion is created from the “$^3\text{He}$-$^3\text{H}$” side. The result of the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction calculation using clustering correlations is shown in Fig. 2.13 and compared with the results of Semi-empirical calculations and microscopic reaction model.
Chapter 2: Theory

Figure 2.13: Differential cross sections for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}(3/2^+ + 1/2^+)$ reaction leading to the excitation of the ground state doublet $(3/2^-, 1/2^-)$, at the beam kinetic energy of $E_{\text{lab}}=266.4 \text{ MeV}$, obtained from three different methods discussed in Refs. [8, 48, 10]. The thick line refers to the results of the semi-empirical calculation. The solid and dotted curves are the results based on clustering correlations with $W_D = -25$ and 0 MeV, respectively. The dashed curve depicts the result of microscopic calculations. The experimental results reported by Bimbot et al. [14] are shown as the ORSAY data.

The target-mass dependence of the $A(^3\text{He},\pi^+)B$ has been investigated by Kajino et al. [8], assuming that the process is dominated by a mechanism similar to those explained in Fig. 2.11-(a). The general trend of the target-mass dependence of the cross section at $\theta_{\text{c.m.}}=$
2.5. Theoretical models of pionic fusion

Figure 2.14: Minimum number of target nucleons required in a $A(^{14}\text{N},\pi)B$ reaction to produce a pion with kinetic energy of 0, 10, 50, 100 and 250 MeV (the dashed curves from bottom to top) in the laboratory. The full circles indicate the target mass and beam energies for which pion production has been observed experimentally. This figure is quoted from Ref. [51].

$0^\circ$ and $E_{lab}/\text{nucleon}=88.8$ MeV is shown in Fig. 2.12. The empty circles are the result of using shell-model wave functions when no clustering correlation was taken into account. The full circles are the results obtained with the strong clustering correlation. According to this model, the inter-cluster relative motion receives the whole momentum transfer. By increasing the momentum transfer, the momentum mismatch for the pionic fusion reaction will increase due to the fact that the overlapping of the inter-cluster wave functions of the initial and final states becomes smaller. Therefore, the cross section will decrease. The decreasing of the cross section for the reaction leading to $^{15}\text{N}$ ($^{12}\text{C}(^{3}\text{He},\pi^+)^{15}\text{N}$) compared to the reaction leading to $^7\text{Li}$ ($^4\text{He}(^{3}\text{He},\pi^+)^7\text{Li}$) is obvious, since the momentum transfer is quite different, 4.3 and 3.3 fm$^{-1}$, respectively. The cross section decreases gradually with increasing mass of the final nucleus. Some structures due to shell effects (e.g. at $A=20$ and 40) are visible.

In addition, there is the overall trend of a decreasing cross section with increasing the target mass. As the target mass increases, at the same beam kinetic energy the number of “active” target nucleons has to increase to provide the required centre-of-mass energy for pion production. This fact makes the coherence of the reaction more complicated.
Therefore, the cross section will decrease. Moreover, pion production close to the coherent threshold which is shown in Eq. 1.1 requires that more nucleons in the projectile and target are completely stopped in the overall centre-of-mass system, and that their relative energy is converted into the pion mass. This is illustrated in Fig. 2.14. As the beam kinetic energy decreases, the number of active nucleons has to increase.
3. The Monte-Carlo modelling of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction

3.1 Introduction

The determination of experimental cross sections requires a careful examination of the experimental limitations of the detector acceptances. The kinematical phase-space populations have to be calculated and integrated over the same phase-space of the accepted experimental data. It is difficult to perform analytic calculations since the boundaries of the experimental setup and the detector acceptances are difficult to express analytically. Therefore, we decided to approach the problem by Monte-Carlo simulation of the relevant processes. The numerically implemented model involves different steps as schematically depicted in Fig. 3.1.

Sections 3.2 and 3.3 discuss the computational technique used to calculate the two-body phase-space for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. This part is referred to in the figure as “2-body event generator”. The output of the two-body event generator which is shown in the figure as “4-momenta and weight (WT)” was used for the further steps of the numerical calculation. The next step would be to select only events in a certain kinematical range. Since we have analysed the data in the full angular and energy ranges of the produced $\pi^0$ and $^{10}\text{B}$, this step was not applied in the numerical calculation. In addition, although the solid angle of the experimental setup is large (see Chapter 4), its limited ac-

![Figure 3.1](image-url)
ceptance (see Section 5.6) suggests the introduction of the boundaries of the experimental setup in this step of the calculation. This part is referred to in Fig. 3.1 as “Kinematical constraints and geometrical boundaries”. Next, in Section 3.5 a description is given how the generated Monte-Carlo events can be integrated over the phase-space of the experimental setup (“Monte-Carlo integral” in Fig. 3.1). In the last step, the kinematical phase-space populations integrated over the same phase-space as the phase-space of the accepted experimental data have been determined and used to correct the cross section for the detector acceptances (“Acceptance of the experimental setup” in Fig. 3.1). This part will be discussed in Chapter 5. Furthermore, the “Model implementation” and “Final weights” will be discussed in Section 3.4, where it will be explained how a model can be implemented and imposed on a generated Monte-Carlo event. Since up to now there is no theoretical prediction available for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction, the processes indicated in the dashed squares in Fig. 3.1 were not included in the calculations. In Section 3.6, the results of the present numerical calculations will be discussed.

### 3.2 Two-body phase-space generator

A program including a Monte-Carlo event generator needs to be developed which can produce the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ events in a random sequence. An obvious approach would be to pick at random the momenta and directions for each of two outgoing particles ($\pi^0$ and $^{10}\text{B}$), and to check whether energy and momentum conservation for the generated particles is obeyed. Since in this simplistic approach the number of accepted events is extremely small, and therefore a long CPU time is used to produce kinematically allowed $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ events, this method is not efficient. Instead of this method, we used GENBOD, an n-body phase-space generator program [52], which employs a more efficient approach to generate kinematically allowed events. This part is referred as “2-body event generator” in Fig. 3.1. Note that the n-body event generator (GENBOD) always picks a kinematically allowed event, and therefore is extremely efficient in producing statistics for Monte-Carlo simulations. Here, the method as used by GENBOD to integrate the phase-space element is explained.

For the reaction of $p_a + p_b = P \rightarrow p_1 + ... + p_n$, the cross section can be calculated as:

$$\sigma = \frac{1}{F} \int |A_{p_j}|^2 \delta^4(\sum_{i=1}^n p_i - P) \prod_{j=1}^n \delta(p_j^2 - m_j^2) d^4p_j,$$

(3.1)

where $F$ denotes the flux factor proportional to the momentum of the associated particles and the dynamics of the reaction is given by the elements of the transition matrix $A(p_j) = \langle p_1...p_n|A|p_a,p_b\rangle$. $p_a$ and $p_b$ are the 4-momenta of the interacting particles and $p_1$, ..., $p_n$ are the 4-momenta of the produced particles. The first $\delta$ function serves to conserve 4-momentum and the second one is to force all outgoing particles on their mass shells. The integration is performed over the 4-momenta of the produced particles. By putting $|A_{p_j}|^2 = 1$ and $F = 1$, the resulting integral is called the phase-space integral and can be
3.2. Two-body phase-space generator

expressed as:

\[ R_n(P; m_1...m_n) = \int \delta^4(\sum_{i=1}^n p_i - P) \prod_{j=1}^n \delta(p_j^2 - m_j^2) d^4p_j. \]  

(3.2)

Splitting the first \( \delta \) function

\[ \delta^4(\sum_{j=1}^n p_j - P) = \int \delta^4(\sum_{j=l+1}^n p_j - \sum_{j=l+1}^n p_j - P_l) \delta^4(\sum_{j=1}^l p_j - P_l) d^4P_l, \]  

(3.3)

with \( P_l \) the 4-momentum of the \( l \)-body subsystem, one can derive the following important splitting relation

\[ R_n(P; m_1...m_n) = \int \int R_{n-l+1}(P; M_l, m_{l+1}...m_n) R_l(P_l; m_1...m_l) dM_l^2. \]  

(3.4)

Here \( M_l \) represents the invariant mass of the system of \( l \) particles with the 4-momentum \( P_l \). The conditions of the integration range for \( M_l^2 \) are

\[ \left( \sum_{i=1}^l m_i \right)^2 \leq M_l^2 \leq (M_n - \sum_{i=l+1}^n m_i)^2. \]  

(3.5)

Applying repeatedly the splitting relation, Eq. 3.4, and transforming \( dM^2 \) to \( 2MdM \), the final form of the phase-space integral is derived

\[ R_n = \int dM_{n-1}... \int dM_2 \prod_{i=1}^{n-1} 2M_i R_2(M_{i+1}; M_i, m_{i+1}), \]  

(3.6)

where the 2-body phase-space factor can be calculated as

\[ R_2(M_{i+1}; M_i, m_{i+1}) = \frac{2\pi}{M_{i+1}} \sqrt{M_{i+1}^2 + \left( \frac{M_i^2 - m_{i+1}^2}{M_{i+1}} \right)^2 - 2(M_i^2 + m_{i+1}^2)}. \]  

(3.7)

Eq. 3.7 is the basic expression for the phase-space integral. By employing the splitting relation Eq. 3.4, the \( n \)-body phase-space integral is replaced by a product of \( n-2 \) two-body decays. The sequence of the allowed two-body decays is sketched in Fig. 3.2-(a). \( M_n \) is the invariant mass of the system of \( n \) particles in their centre-of-mass frame. The two-body phase-space factor is applied at the first vertex \((A)\), to generate the particle with mass \( m_n \). The remaining system of \( n-1 \) particles has the invariant mass \( M_{n-1} \). The integrations in Eq. 3.6 run over all possible masses \( M_{n-1} \). Next, the two-body factor is applied at \( B \), producing the particle with mass \( m_{n-1} \) integrating over all possible masses \( M_{n-2} \) and all possible \( M_{n-1} \). This is then repeated \( n-2 \) times. Figure 3.2-(b) shows the same algorithm for a two-body phase-space integral to generate the \(^6\text{Li}(^4\text{He,}\pi^0)^{10}\text{B}^*\) reaction.

Each \( M_i \) is chosen to be independent of other vertices, within the condition defined by
Figure 3.2: (a): The schematic representation of the algorithm used by the Monte-Carlo event generator to evaluate the phase-space integral of an $n$-body system. This integral can be treated as a sequential decay, where only the two-body phase-space factors need to be applied at vertices A, B, etc. $M_n$ generates $m_n$ and $M_{n-1}$. $M_{n-1}$ generates $m_{n-1}$ and $M_{n-2}$ and this process continues to produce $m_1$. (b): the same algorithm for a two-body phase-space integral to generate the $^6\text{Li}({}^4\text{He},\pi^0)^{10}\text{B}^*$ reaction.

Eq. 3.5. This condition is less restrictive than the physical limit given by

$$M_{j-1} + m_j \leq M_j \leq M_{j+1} - m_{j+1}, \quad (3.8)$$

and therefore leads to many non-physical events making the integration less efficient and time-consuming. The above condition implies interdependence of the integration boundaries. To stay independent of each other, the masses $M_j$ can then follow this equation

$$M_j = r_j (M_n - \sum_{i=j+1}^{n} m_i) + \sum_{i=1}^{j} m_j, \quad (3.9)$$

where $r_j$ is a random number between zero and one, satisfying the restrictive condition, Eq. 3.8, if the random numbers are in ascending order

$$0 < r_1 < ... < r_j < r_{j+1} < ... < r_{n-2} < 1. \quad (3.10)$$

For a system of $n$ particles, there are $3n$ observables, but the four equations resulting from the energy and momentum conservation reduce the number of independent observables to $3n-4$. Up to now only $n-2$ variables have been used as is shown in Eq. 3.10. The remaining $2n-2$ variables are the two angular variables created at each vertex. They are chosen in the rest frame of each invariant mass $M_j$ at which $R_2$ is evaluated in such a way that an isotropic angular distribution is generated. This is obtained by choosing polar and azimuthal angles in the rest frame uniformly in $0 \leq \phi \leq 2\pi$ and $0 \leq \cos(2\theta) \leq 1$. To obtain a correct description of the event, one has to successively Lorentz-transform each momentum into the rest frame of the group of particles preceding it each time the $R_2$ is evaluated.
3.3 Phase-space weighting

Since GENBOD produces kinematically allowed events, the conservation laws are respected. However, it should be mentioned that the $n$-body phase-space program for $n > 2$ does not produce events according to the expected phase-space population, which is the consequence of the efficiency of this event generator. Each event has an associated phase-space weight proportional to the value of the phase-space integral. To account for the phase-space weight in each generated event, the corresponding phase-space density ($WT$) is calculated by GENBOD as follows and should be applied by the user:

$$WT = W(M_n, m_j) = \prod_{i=2}^{n-1} M_i R_2(M_{i+1}; M_i, m_{i+1}) R_2(M_2; m_1, m_2).$$

(3.11)

In case of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction, only two-body events are generated. Since a two-body phase-space is isotropic in angular space (angles in the rest frame of the vertex are isotropic in $\cos(\theta)$ and $\phi$), GENBOD is efficient to produce two-body events according to the expected phase-space distribution. Therefore, the phase-space density ($WT$) is intrinsically applied for every $^6$Li($^4$He,$\pi^0$)$^{10}$B* event.

3.4 Phase-space and model implementation

According to Eq. 3.1, cross sections depend not only on the phase-space factors but also on the squared amplitude of the matrix element $|A|^2$. Therefore, the dependence of the cross section $\sigma$ on some kinematical parameters (for instance the centre-of-mass angle or invariant mass) can be obtained as

$$\frac{d\sigma}{dx} = \frac{1}{F} \int |A|^2 dR.$$

(3.12)

In the Monte-Carlo method, instead of the integration, a discrete sum will be evaluated:

$$\frac{\Delta\sigma}{\Delta x} = \frac{1}{F} \lim_{\Delta x \to 0} \frac{1}{\Delta x} \sum_i |A_i|^2 R_i.$$

(3.13)

It is seen that the possibly complicated integration of Eq. 3.12 is replaced by a simple sum in Eq. 3.13.

The value of the phase-space integral is calculated by GENBOD but in addition we need to know the matrix element squared $|A_i|^2$ of every evaluated event. Since $|A_i|^2$ contains the reaction dynamics, this should be obtained from the theoretical calculations. Because, up to now, there is no theoretical prediction available for the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction, we were not able to produce the $^6$Li($^4$He,$\pi^0$)$^{10}$B* events by using the calculations on the basis of a theoretical model. The purpose of the Monte-Carlo simulation in this work is to study the kinematics of the reaction and to determine the detector response for every generated event. In this way we were able to calculate the acceptance of the used detector setup. The first application will be discussed in Section 3.6. For more information about the second application see Section 5.6.
3.5 Monte-Carlo integration over the experimental setup

The next step after the phase-space event generation is to treat these events in exactly the same way as the experimental data. The simulation was limited to the geometrical acceptance and the energy thresholds of the experiment. Furthermore, as shown on the lower side of Fig. 3.1 (“Kinematical constraints”), kinematically allowed events are checked whether they satisfy certain threshold conditions. For these purposes, the detector simulation package GEANT3 [53] was employed. Using GEANT3, the generated events obtained by GENBOD were tracked through the simulated experimental setup. The same selection criteria that were used on the variables of the measured data, have been applied on the simulated data. The simulated and the experimental data were analysed in an identical way and the experimental acceptance was imposed on the generated events (see Section 5.6). In Fig. 3.1, these steps are shown as “Monte-Carlo integral” and “Acceptance of the experimental setup”. More than 150,000 events from a total of 1,000,000 generated events were accepted by the simulated experimental setup.
3.6 Kinematical results for the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction

In this section, the kinematical results for the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction obtained by the Monte-Carlo event generator are discussed. The results of the simulation including the complete geometry of the experimental setup are compared with the measured results and discussed in Chapters 5 and 6. Furthermore, the efficiencies of the detector setup obtained by the Monte-Carlo simulation are discussed in Section 5.6.

Figure 3.3 presents the angular distribution of the products from the pionic fusion reaction of $^6$Li($^4$He,$\pi^0$)$^{10}$B*. In Fig. 3.3-(a), the maximum vertical and horizontal angles of $^{10}$B represented in the laboratory coordinate system are $2^\circ$ and $2.2^\circ$, respectively. The vertical and horizontal angles are defined as the projection of the angle in the y-z plane and x-z plane, respectively, where the positive z-direction is the beam direction. The acceptances of the Big-Bite Spectrometer (BBS), which was used to detect $^{10}$B (see Section 4.7), in the horizontal and the vertical directions are $1.9^\circ$ and $4^\circ$, respectively. The dashed line represents the BBS window. More than 86% of $^{10}$B particles enter the Big-Bite Spectrometer (for more details see Section 5.6). Figure 3.3-(b) shows the locus of $\theta_{^{10}B} \ versus \ \theta_{\pi^0}$, i.e. the correlation between the polar angles of $^{10}$B and $\pi^0$.

Figure 3.4-(a) and -(b) represents the energy-angle locus of $^{10}$B and $\pi^0$, respectively. The energy range of produced $\pi^0$ in the phase-space simulation is $3 \text{ MeV} < E_{\pi^0} < 18 \text{ MeV}$ but $^{10}$B nuclei cover a higher range of energy: $87 \text{ MeV} < E_{^{10}B} < 102 \text{ MeV}$.

Figure 3.5-(a) and -(b) shows the momentum distribution of the produced $^{10}$B and $\pi^0$, respectively, in the phase-space simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction. The hatched area in both figures corresponds to the remaining events after applying the acceptance of the BBS window for $^{10}$B. After applying the geometrical acceptance of the full detector setup, events in the first peak in the hatch-area shown in Fig. 3.5-(a) and the second peak in the hatched area shown in Fig. 3.5-(a) will be diminished (see Section 5.4.4).
Figure 3.5: (a) and (b) show the momentum distribution of the produced $^{10}$B and the produced $\pi^0$, respectively, in the phase-space simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction. The hatched area in both figures corresponds to the remaining events after applying the acceptance of the BBS window for $^{10}$B.

Figure 3.6: (a) and (b) represent the polar angle distributions of the produced $\pi^0$ in the laboratory system and the centre-of-mass system, respectively, from the phase-space simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction. Events in the hatched area in both panels are the remaining events after applying the acceptance of the BBS window for $^{10}$B.

In addition, the polar angle distributions of the produced $\pi^0$ in the laboratory and in the centre-of-mass systems from the phase-space simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction are presented in Fig. 3.6-(a) and -(b), respectively. Events in the hatched area in both panels of Fig. 3.6 are the remaining events after applying the acceptance of the BBS window for $^{10}$B.
4. Experimental setup

4.1 Introduction

The study of the pionic fusion at subthreshold energies requires to efficiently measure a very small cross section with a few tens of nano barn. In a novel experimental approach, we study the pionic fusion in the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction by exclusive pion production in overdetermined kinematics. This goal required a suitable combination of detector systems that allow us to measure an emitted pion and a residual nucleus in coincidence. Requiring this unique experimental condition, one can guarantee clean and truly exclusive data providing the angular distribution of pions. We studied this exclusive two-body reaction, with unprecedented accuracy by configuring and exploiting a detector combination at the AGOR cyclotron. This experimental setup allows to measure an emitted pion as well as the resulting fusion product. For the detection of low-momentum neutral pions, we used the spherically symmetric Plastic Ball detector (PB). Since the neutral pion is not

Figure 4.1: A cross-section view of the experimental setup showing the Plastic Ball together with its Inner Shell, the Big Bite Spectrometer including its magnets (Q1, Q2 and D) and the Heavy Ion detector.
Figure 4.2: Operating diagram for AGOR. Dots in the plane of energy/nucleon (E/A) and charge/nucleon (Q/A) represent the different beams accelerated by AGOR so far. The lines show a simplistic approximation of the real operating limits.

stable (mean life = $(8.4 \pm 0.6) \cdot 10^{-17}$ s) and immediately decays to two photons with a probability of 98.8%, the Plastic Ball detector is used to detect two photons. In order to improve the detector response and efficiency for photons, the Plastic Ball is equipped inside the hollow sphere with the Inner Shell (IS). Also for the detection and identification of ions, a specially designed array of Heavy Ion detectors (HI) has been used. This array was placed in the vacuum chamber just in front of the nominal focal plane of the Big-Bite Spectrometer (BBS) [54]. A cross-section view of the used experimental setup is shown in figure 4.1.

4.2 The KVI facility

The heart of the KVI facility is the super conducting cyclotron Accelerateur Groningen-ORsay, AGOR [41, 42], which has been constructed and built in collaboration with IPN Orsay, France. The cyclotron magnet is built with super conducting coils that can produce magnetic fields up to 4 T. Due to the super conducting coils, a compact design was possible such that the three-sector cyclotron has a pole diameter of 1.88 m with accelerating electrodes placed within the pole valleys. The maximum energy for the acceleration of
Figure 4.3: Floor plan of the cyclotron vault and the experimental areas. Shown are the AGOR cyclotron, the beam line with analysing magnets, and in the top-right area the Plastic Ball and the Big Bite Spectrometer including the Heavy Ion detector.
Chapter 4: Experimental setup

heavy ions depends on the charge/mass ratio, \( Q/A \). Figure 4.2 shows the operational diagram from which one can obtain the available energies for different \( Q/A \) ratios. The maximum energy for \( Q/A = 1/2 \) is about 95 MeV/u and the energy of the \( ^4\text{He} \) beam in the present experiment was 59.1 MeV/u. An axial beam line is used to guide ions into the cyclotron. There are three external ion sources in use. A multi-cusp source, capable of producing hydrogen and helium ions, was used in our experiment. The beam current was monitored using a well shielded Faraday Cup (FC). The typical range of the beam current used for the experiment was from 1 to 4 nA depending on fluctuations in the intensity of the source as well as the efficiency of the data acquisition and capability of the setup to function properly under high rates. In addition to the multi-cusp source, the facility is equipped with an ECR ion source, which is used to produce different highly-ionised heavy-ion beams and with a polarised ion source that can deliver polarised proton and deuteron beams. Figure 4.3 shows the floor plan of the facility with the beam lines and the experimental setups. The Plastic Ball, the Inner Shell, the Heavy Ion detector and the BBS are placed at the end of the so-called s-line. While this is the first experiment at KVI in which the Inner Shell and the Heavy Ion detector were used, the Plastic Ball has been a part of many experimental studies concerned mainly with hadronic interactions in the few-body systems. The BBS is positioned on the right side of the Plastic Ball (PB). The BBS and the Plastic Ball have been used in the coincidence mode.

4.3 The \( ^6\text{Li} \) target

In this experiment we have used a solid \( ^6\text{Li} \) target that was prepared in the KVI target laboratory and was positioned at the centre of the Plastic Ball. The \( ^6\text{Li} \) target was made in a cuboid geometry with a thickness of 37.45 \( \mu \)m and 16 mm wide by 12 mm high, much larger than the beam spot which is a few mm in diameter. The nominal thickness of the \( ^6\text{Li} \) target was 2 mg/cm\(^2\). To prevent oxidation, the \( ^6\text{Li} \) target was kept in an argon atmosphere during the preparation process and the transport to the vacuum chamber. Since the pressure of 1 mbar is the starting point for the oxidation, the pressure of the target was held below 0.001 mbar. Section 5.4.4 will explain that during the experiment some parts of the \( ^6\text{Li} \) target were oxidised. The positioning of the beam spot at the centre of the target was controlled by monitoring the fluorescent ZnS target with an IR camera. Also an empty frame was used to check for background originating from the beam-halo hitting the target cell material. For the calibration purposes, another \( ^6\text{Li} \) target with a nominal thickness of 4 mg/cm\(^2\) and a \( ^{12}\text{C} \) target with a nominal thickness of 0.2 mg/cm\(^2\) were used. The targets were mounted on a target ladder. The vertical position of the target was adjusted by using a remote control (see Fig. 4.4).

4.4 The Plastic Ball Detector

The Plastic Ball [55] which was initially built for light-fragment detection in heavy-ion collisions at the Bevalac in Berkeley, is a highly segmented 4\( \pi \) detector system consisting of E-\( \Delta E \) telescopes (see Fig. 4.5-(a) and -(b)) configured in a hollow ball structure (see Fig. 4.5-(c)). The Plastic Ball is split into two hemispheres, namely the forward and the backward hemispheres, which are placed in such a way that the equatorial plane is
perpendicular to the beam direction. A structure supporting both hemispheres is mounted on rails allowing movement of the hemispheres and an easy access to the target chamber placed inside (Fig. 4.5-(c)). In its original setup the solid angle coverage of the detector was 97% of $4\pi$. However, for the present experiment, a rearrangement of the detector setup was required to accommodate the Big-Bite Spectrometer and the Heavy Ion detector at forward angles. In addition, a number of detector modules have been taken out from the top of the forward hemisphere where the connections for vacuum pipes and cooling liquid for the liquid Helium target were placed. After all those rearrangements, 552 detector modules covering 77% of $4\pi$ solid angle and polar angles between $50^\circ$ and $160^\circ$ were used: 340 on the backward hemisphere covering polar angles between $90^\circ$ and $160^\circ$ and 212 on the forward hemisphere covering polar angles between $50^\circ$ and $90^\circ$ (see Fig. 4.6).

The Plastic Ball detector module

The geometrical shape of the detector modules is a triangular prism. By putting 11 different shapes together, it is possible to entirely fill up a hollow ball structure. This geometrical design was taken from the Crystal Ball detector [56]. The outer and inner radii of the sphere are 614 mm and 254 mm, respectively, and modules are arranged in 11 rings. Looking from the target, each module has a thin layer of scintillating material in front of the thick layer, which forms a $E - \Delta E$ phoswich detector module and covers a solid angle of about 17.45 msr. Choosing scintillating materials with different signal decay times, it is possible to glue both scintillators together and collect the scintillating light with one photomultiplier. The main reason for using a $E - \Delta E$ phoswich telescope is a good particle identification.
Chapter 4: Experimental setup

Figure 4.5: Sketches and photographs showing the different sets of used detectors in the experiment. Panels (a) and (b) show an individual Plastic Ball module. Panel (c) represents the complete Plastic Ball viewed from the downstream side. The hole is pointed to the BBS entrance window and is the position of the target equipments. Panels (d) and (e) correspond to the individual Inner Shell module (hexagonal type H1) and panel (f) shows the complete hemisphere of the Inner Shell detector when it is mounted inside the Plastic Ball. In panel (a), the backward and the forward hemispheres of the Plastic Ball are separated. In panels (g), (h) and (i) the individual sensors and the complete array of the Heavy Ion detector are presented, respectively.
4.4. The Plastic Ball Detector

Figure 4.6: The schematic layout of the Inner Shell detector on top of the Plastic Ball scheme. Triangles correspond to the individual Plastic Ball detector modules and hexagons and pentagons are individual Inner Shell detector modules that are positioned in the backward acceptance of the setup. H1, H2, H3 and P represent the three different hexagonal geometries and pentagonal geometry of the Inner Shell detector modules, respectively.

As shown in Fig. 4.5-(a) and -(b), the modules are built from a 4 mm thin layer of CaF$_2$ crystal measuring the energy loss ($\Delta E$) of the detected particles and a 356 mm thick plastic scintillator equivalent to about 1.2 radiation lengths measuring the residual energy ($E$) deposited in the detector module. The plastic scintillator acts as a light-guide for the CaF$_2$ crystal. The photomultiplier (2202 AMPERE) is mounted directly to a light-guide which is glued to the plastic scintillator. Characteristic decay times are 1 $\mu$s for the CaF$_2$ and 10 ns for the plastic scintillator. The common signal with different decay times can be separated by pulse-shape analysis in the way that the charge collected at the anode of the phototube is integrated within two different time windows (narrow- and wide-gate windows). In our case, the gates of the narrow and wide components were 133 ns and 944 ns, respectively, which were responsible to integrate the produced light output in the plastic scintillator ($E$) and the total produced light output in the detector module ($E + \Delta E$), respectively. According to the Monte-Carlo simulations, neutral particles like photons have a low probability (in case of photons this possibility is < 5%) to deposit energy in the CaF$_2$ due to the fact that the CaF$_2$ is very thin. Consequently, while plotting $E + \Delta E$ as a function of $E$, photons appear on a line at 45$^\circ$, while charged particles appear in regions well separated from that line (see Fig. 5.3). In high-energy high-multiplicity measurements [57], the Plastic Ball was found to have excellent particle identification properties. In pionic fusion measurements, the possibility to discriminate between photons and charged particles coming from the cosmic muon spectrum and other sources is a condition required for a successful measurement.
Chapter 4: Experimental setup

4.5 The Inner Shell detector

The depth of the Plastic Ball modules is equivalent to about one radiation length which is marginal for an efficient detection of photons with energies up to 120 MeV. In order to improve the detector response for photons, the Plastic Ball is equipped inside the hollow sphere of 50.8 cm diameter with a hemisphere of CsI(Tl) detectors, the Inner Shell, acting as active converter with a thickness of 5 cm equivalent to about 2.7 radiation lengths (Fig. 4.5-(f)). The individual 64 CsI(Tl) detectors have three different hexagonal and one pentagonal cross section, corresponding to the substructure of the Plastic Ball, in order to cover the backward acceptance of $2\pi$ steradian (Fig. 4.5-(d) and -(e)). Every Inner Shell hexagonal-shaped module is positioned in front of (as seen from the target) 6 Plastic Ball modules. The inner and outer radii of the Inner Shell are 147.5 mm and 212.5 mm, respectively. CsI(Tl) is a slow scintillator with average decay time of about 1 $\mu$s for photons. Since the decay time of the CsI(Tl) is in the same range as for the CaF$_2$, to integrate the light output of the Inner Shell modules, an integration gate with the width of 944 ns has been applied. The photomultiplier (Hamamatsu R7400U) is glued to the CsI(Tl) scintillator.

In order to study the response of the Inner Shell modules for photons, two measurements have been performed. The first measurement used 4.4 MeV photons originating from an AmBe source. The energy resolution for 4.4 MeV photons was found around 30% FWHM (Fig. 4.7-(b)). The second measurement was done by the SCIONIX company which delivered the Inner Shell detector. They used 662 keV photons originating from a $^{137}$Cs source and found the energy resolution of 26.7% FWHM (Fig. 4.7-(a)). The granularity of the Inner Shell setup is about 99 msr. According to the Monte-Carlo simulations, charged particles in the range of energies that were produced in our experiment, deposit all their energy in the Inner Shell. As result, mainly photons can reach the Plastic Ball

Figure 4.7: The energy deposition of photons in one Inner Shell module. (a): The deposited energy of 662 keV photons originating from a $^{137}$Cs source. (b): The deposited energy of 4.4 MeV photons originating from an AmBe source.
4.6. The Plastic Ball and the Inner Shell data acquisition

Figure 4.8: Schematic diagram of the detector electronics readout for the Plastic Ball, the Inner Shell and the Heavy Ion detectors. The Time Of Flight of 93.13 MeV $^{10}$B, from the target to the Heavy Ion detector is 178 ns, while in the cases of 115.06 MeV $^7$Be and 130 MeV $^{12}$C, the Time Of Flight is 135 ns and 166 ns, respectively. The time-zero is the reaction time in the target, see Fig. 4.11.

detector modules that are positioned in the backward acceptance of the setup. During the experiment, the Plastic Ball and the Inner Shell detectors were covered from the external light. The schematic layout of the Inner Shell modules on top of the Plastic Ball detector scheme is shown in Fig. 4.6. Triangles, H1, H2, H3 and P are representing the Plastic Ball modules, three different hexagonal geometries of the Inner Shell modules and one pentagonal geometry of the Inner Shell modules, respectively. The region which is marked as “Target equipments” represents the position of the target equipments. Therefore, there was no Plastic Ball module positioned in that region.
4.6 The Plastic Ball and the Inner Shell data acquisition

The readout of the Plastic Ball and the Inner Shell scintillators is performed via standard CAMAC and NIM modules and some dedicated electronic modules. The schematic diagram is shown in Fig. 4.8. The photomultiplier signal is split into two signals. One signal is sent through long cables (equivalent to 555 ns delay). This signal goes to a second splitter. In case of the Plastic Ball, the two signals obtained from the second splitter are connected to the two charge-integrating ADCs (Analog-to-Digital Converter modules: LeCroy 2282B, system processor: 2280, 12 bits resolution, range: 1024 channel, dynamic range: \(-1000\) pC). To separate the fast scintillator component from the total signal, different integration gates have been applied. In case of the Inner Shell detector, no pulse-shape analysis was planned and modules consist of only one scintillator material. Therefore after the second splitter, only one copy of the signal was used and connected to the ADC (LeCroy 2282B, system processor: 2280).

The other signal from the first splitter was sent to the Ball-Boxes (BB). These dedicated electronic modules were originally constructed for high-multiplicity measurements and to achieve sensitivity to hit-clusters on the trigger level. Each electronic module can accept up to eight input signals which are fed to leading-edge discriminators. In our experiment, the initial signals were fed into the Ball Box modules in clusters of seven (mainly), six and five neighbouring detectors. The digital output of every channel was sent to start the Plastic Ball and the Inner Shell TDCs (module: LeCroy 4298, controller: 4291A 32 drift chamber TDC) for every individual detector. Moreover, every module produces a multiplicity signal which is proportional to the number of scintillators hit. These multiplicity signals are further discriminated and ORed to make the signal “Plastic Ball OR”. The “Plastic Ball OR” signal was used to produce the Plastic Ball trigger in the single and coincidence modes, which will be explained in Section 4.10. In this experiment, the Plastic Ball trigger in the single mode was applied to generate gates for the ADCs of the Plastic Ball and the Inner Shell and to produce a common stop for the Plastic Ball and the Inner Shell TDCs. A VME AXP computer was used to initialise and read out the data using a CAMAC branch driver (CSE CBD 8210). The high-voltage was supplied to the photomultipliers using three LeCroy High Voltage 1440 mainframes providing negative high-voltages to the individual photomultiplier tubes.

4.7 The Big-Bite Spectrometer

The spectrometer is a QQD-type magnetic spectrometer with a K-value of 430 MeV, a solid angle up to 13 msr and the momentum-bite acceptance up to 25% (i.e. particles with a magnetic rigidity deviating less than 12.5% from the nominal rigidity are accepted in the spectrometer). There are three possible operation modes which differ in the positions of the quadrupole doublet with respect to the scattering chamber. The closer the doublet is to the scattering chamber, the larger the solid angle and the smaller the momentum acceptance. For this experiment, the spectrometer was used in the mode called “B” which has the momentum-bite acceptance of 19% (see Table 4.1). The region between the target and the entrance window of the BBS was held in a vacuum with the pressure of less than \(10^{-5}\) mbar. The design of the entrance region of the spectrometer allows enough free space around the scattering chamber for the user to set up large detection systems for the
measurement of decay products in coincidence with ejectiles measured in the focal-plane detection system. In addition, the magnets are designed such that the high-momentum side of the spectrometer can be used either as a beam-dump area or to allow a free passage for the direct beam from the cyclotron into the well-shielded external beam dump. These design features and the three different operation modes make this spectrometer a unique instrument that can be used for many different experiments in nuclear physics.

### 4.8 The Heavy Ion detector

For the detection and identification of ions with $1 \leq Z \leq 10$ and with energies of 7 - 43 AMeV, a specially designed array of Heavy Ion detectors has been used. This array was placed in the vacuum chamber in front of the nominal focal plane of the Big-Bite Spectrometer. The Heavy Ion detector array consists of 60 CsI(Tl)/Plastic phoswich scintillators in a segmented layer. Among the 60 modules, 58 of them were successfully used in the experiment. Scintillators are arranged in two parallel rows, 30 of them are positioned in the upper row with their photomultipliers pointing upwards and 30 of them are positioned in the lower row with their photomultipliers pointing downwards (see Fig. 4.5-(i)). Each of the phoswich detectors has a cuboid geometry consisting of CsI(Tl) crystal with a size of 4 cm $\times$ 1.3 cm $\times$ 1.3 cm (slow signal component) and a 80 µm Polystyrene based thin plastic scintillator (fast signal component) on top of the CsI(Tl) crystal. Plastic scintillators measure the energy loss of ions ($\Delta E$), while the CsI(Tl) scintillators measure the residual energy deposition ($E$) in the module. Looking from the target in the beam direction, every module is oriented vertically such that its dimensions in the horizontal and the vertical directions are 1.3 cm and 4 cm, respectively. The modules are positioned such that the angle of incidence of the particles is nearly perpendicular to the front face of the modules. The distance between the front face of the module and the focal plane is almost identical for all modules.

In order to keep the complete array thin, photomultipliers are oriented vertically perpendicular to the particle entrance. The vertical size of the array, the number of modules
in the array and the position of the array with respect to the BBS magnets were specially designed in order to cover the total momentum range of the fusion products that enter the Big-Bite Spectrometer in mode “B” and to achieve the desired energy, momentum and position resolutions. In a commissioning experiment, a 14.3 AMeV $^{12}$C beam on $^{197}$Au was used. Using the peak from elastic scattering at $\theta_{\text{lab}} = 14^\circ \pm 2^\circ$, energy resolutions of 1.7% FWHM and 1.9% FWHM from the wide- and narrow-gate integrations, respectively, were found (see Fig. 4.9). The kinematic broadening and target straggling contribute 370 keV to the measured widths.

The Heavy Ion detector data acquisition

The photomultiplier (Hamamatsu R647, gain $10^6$, 2.5 ns rise time) is 71±2 mm long with a standard 13 mm diameter and is mounted directly to a light-guide which is vertically glued to the horizontal surface of the scintillator (Figs. 4.5-(g), 4.5-(h) and 4.10). Every detector module produces a composed signal which can be separated into two components. The photomultiplier signal is preamplified before it is sent to the readout electronics. The readout of the Heavy Ion scintillators is performed via standard TAPS modules [59]. The schematic diagram is shown in Fig. 4.8. The photomultiplier signal after amplification is split into two signals. One signal is sent through long cables (equivalent to 375 ns delay) and fed to charge-to-digital converters, QDCs (TAPS module: GANELEC 1612F, controller: GANELEC 1302 type A-2, range: 4096 channel, dynamic range: 320 pC). Since the typical decay times are 3 ns for the plastic and 1100 ns for the CsI(Tl) scintillators, it is possible to separate the fast and the slow scintillator components for the pulse shape analysis by applying different integration gates to the electric signal.
4.9 Pedestal subtraction

The other signal from the splitter was used as input for the Leading-Edge Discriminator (LED: TAPS 1600). There are two outputs from the discriminator. One is sent to the RDV, gate and delay generator (TAPS 8/16 A), to generate the gates. The module RDV 8/16 is a variable gate and delay generator housed in a CAMAC unit. This unit is specially suited to generate narrow and wide integration gates for the QDC 1612F. The output of the RDV is split into two copies. One copy goes to the QDCs to be used for the integration of the signal and another copy provides the stop for the Heavy Ion TDCs (GANELEC 812F) for every individual detector. Another output of the discriminator is ORed to make the signal “Heavy Ion OR”. There are two copies of the “Heavy Ion OR”: one copy was directly connected to the trigger box and produced the Heavy Ion trigger in the single mode and the second copy was sent to the coincidence box to check the coincidence of the Heavy Ion and the Plastic Ball signals. The output of the Heavy Ion trigger in the coincidence mode was used to start the Heavy Ion TDCs in the common mode. We have applied a narrow gate of 117 ns to obtain the response of the plastic scintillator component ($\Delta E$) and a wide gate of 1115 ns to charge-integrate the complete signal ($E + \Delta E$). In this case, the wide gate overlaps with the narrow gate. One computer was used for the Plastic Ball data acquisition and the Heavy Ion data acquisition. The high voltage was supplied to the photomultipliers using a CAEN High Voltage mainframe operating for positive high voltages.

4.9 Pedestal subtraction

Charge-integrating digitising electronics usually produces a non-zero offset value even when no signal is present. This is caused by a small calibrated DC offset at the QDC input and is called the pedestal. The pedestal value of the QDC output is an effective offset. In order to correct for this offset, the pedestal values were measured and recorded periodically during
the data acquisition process from a "pedestal run", when no signals were generated at the inputs of the QDCs. During the data acquisition, these pedestals were subtracted from their respective QDC channels, in the event-by-event mode.

### 4.10 Data stream

In the data stream, all the digitised data from the three different sources, namely, the Plastic Ball detector, the Inner Shell detector and the Heavy Ion detector have been recorded. Since the data acquisition is not fast enough to record all events that the detectors register, and because many events stem from processes which are not interesting for this work, events were selected on-line. This selection was implemented by the trigger subsystem. In addition, all triggers were down-scaled so that the dead-time caused by the readout electronics at the typical beam current was on average 49%. Each recorded event represents a reaction that satisfied the predefined triggering conditions. Table 4.2 lists the type of information recorded for each detector system per event. In Table 4.3, the definitions of the triggers used in the present experiment are listed. The goal of the pionic fusion experiments was to focus on the coherence of the pion production at energies just a few MeV above the coherent threshold. In order to exclude background events from the other channels leading to pion production, we measured particle coincidences with ions in the momentum range expected for $^{10}\text{B}$. The Plastic Ball, the Inner Shell and the Heavy Ion detector were able to collect data independently (single mode) or in coincidence. The condition for the Plastic Ball trigger in the single mode was that the number of hits ($N_{PB,ALL}$) in the plastic or the CaF$_2$ scintillators must be larger than 0. To trigger the Inner Shell events in the single mode, the same condition has been applied ($N_{IS} > 0$). For the time calibration, the Plastic Ball-single and the Inner Shell-single triggers were used. The condition for the Heavy Ion detector trigger in the single mode was that the number of hits ($N_{HI}$) in the plastic or CsI(Tl) scintillators must be larger than 0. The Heavy Ion-single trigger was used to calibrate time and energy of the Heavy Ion detector. To study the $^{6}\text{Li}(^{4}\text{He},\pi^0)^{10}\text{B}^*$ reaction, the main trigger was made by ANDing the Heavy Ion-single trigger and the “Plastic Ball OR” ($N_{PB,BW} > 0$) trigger. The timing scheme

| detector          | source                      | information                              |
|-------------------|-----------------------------|------------------------------------------|
| Plastic Ball      | phoswich scintillator (CaF$_2$+plastic) | time energy deposition (narrow-gate integration) energy deposition (wide-gate integration) |
| Inner Shell       | scintillator (CsI(Tl))      | time energy deposition (wide-gate integration) |
| Heavy Ion         | phoswich scintillator (CsI(Tl)+plastic) | time energy deposition (narrow-gate integration) energy deposition (wide-gate integration) momentum of ion using module position |
4.10. Data stream

Table 4.3: The rates of the data stream recorded at a typical beam current of 3 nA and 49% dead-time. $N_{PB,ALL}$ ($N_{PB,BW}$) is the number of the Plastic Ball modules which have deposited energy (in the backward acceptance) for one recorded event.

| Trigger                             | description                     | rate (Hz) | downscaling |
|-------------------------------------|---------------------------------|-----------|-------------|
| $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}$ | $N_{PB,BW} > 0 \land N_{HI} > 0$ | 180       | 1           |
| Plastic Ball single                 | $N_{PB,ALL} > 0$                | 4         | $2^9$       |
| Inner Shell single                  | $N_{IS} > 0$                    | 17        | $2^9$       |
| Heavy Ion single                    | $N_{HI} > 0$                    | 14        | $2^7$       |

for the coincidence mode is shown in Fig. 4.11.

Since the major fraction of coincidences in the forward hemisphere of the Plastic Ball is caused by charged particles, this part of the detector was not included into the “Plastic Ball OR” trigger. In this way, pions which emitted both photons to the forward direction of the setup, were not recorded in the coincidence mode. According to the Monte-Carlo simulation, at the beam energy that was used in our experiment, only 1.8% of the produced pions emit both photons in the forward acceptance. To determine properly the measured cross section, the corresponding acceptance was taken into account. It was not necessary to include the Inner Shell into the main trigger since almost all the emitted photons that moved to the backward direction and hit the Inner Shell, entered the Plastic Ball as well and already were included in the trigger. In this way, those photons that move to the forward direction, where the target equipments are located, were not included in the trigger condition. The corresponding correction was taken into account to calculate the cross section (see Section 5.6).

In addition to data readout, a scaler readout was performed every second. The latter was used for the on-line monitoring and later for checking and normalization. Scaler events are composed of a number of scaler channels that are periodically read out. After each read out, or every second, all scaler channels are reset. In our case, in addition to the beam current, scalers (Lecroy 4434) recorded the number of different triggers before and after down-scaling and dead-time in order to correct for the dead-time.
Figure 4.11: Timing scheme of the experiment in the coincidence mode. The time origin is the reaction time in the target. The gray area in the “PB, IS TDC start” signal indicates the time jitter caused by generating the OR from many individual signals in the Ball Boxes.
5. Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

5.1 Introduction

In this chapter, we discuss the analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction at $T_{\text{beam}} = 236.4$ MeV. The basic steps of the analysis procedure are schematically illustrated in Fig. 5.1. Each of the elements in Fig. 5.1 is discussed separately.

In the first part, referred to in the figure as “Identification”, different techniques and available information are used to identify photons in the Plastic Ball and the Inner Shell part of the setup as well as ions in the Heavy Ion detector which is positioned in the focal plane of the Big-Bite Spectrometer. The method of identification is explained in Section 5.2. The “Calibration” part, which is followed in Section 5.3, involves the calibration of the time information of the Plastic Ball, the Inner Shell and the Heavy Ion detectors. In addition, the energy calibration of the Plastic Ball and the Inner Shell, and the gain matching of the Heavy Ion detector modules as well as the momentum calibration of the Heavy Ion detector array are included in this section. The following Section 5.4 discusses the selection of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ events. This includes the presorting of the data (“Pre-sort”), the reconstruction of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ kinematics (“Event reconstruction”) and

![Figure 5.1](image)

**Figure 5.1**: A schematic representation of the analysis method used to obtain $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ events and the corresponding observables. For a complete description see the text.
Chapter 5: Analysis of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ data

5.2 Particle identification

5.2.1 Photon identification

The Plastic Ball and the Inner Shell time

As was explained in Chapter 4, recording of the timing information by the TDCs (Time-to-Digital Converters) of the Plastic Ball and the Inner Shell detectors starts when the individual detector is hit and commonly stops when the single trigger is issued. The time difference between start and stop for one detector module depends on the difference in the wire lengths which are travelled by one or the other signal sent to the TDCs or to the trigger box. Figure 5.2-(a) and (b) shows the recorded time summed over all modules for the Plastic Ball detector and the Inner Shell detector, respectively. For events contained in the sharp peaks ($t = 610 \pm 60$ ns), the signals which start and stop the TDC are correlated in time. Events outside the sharp peaks are associated with random coincidences.
5.2. Particle identification

**Figure 5.3:** The calibrated Plastic Ball plus Inner Shell pulse shape and identification of photons. The spectrum is summed over all modules. Regions labelled “p” and “γ” represent protons and photons, respectively.

The Plastic Ball pulse shape

Figure 5.3 shows the two-dimensional spectrum (pulse shape spectrum) for the particle identification of protons and photons. The vertical axis shows the sum of the Inner Shell, the CaF$_2$ and the plastic light output using the wide-gate integration of the Plastic Ball and the Inner Shell signals within 944 ns. The horizontal axis shows the sum of the Inner Shell and the plastic signals using the narrow-gate integration with the gates of 944 ns and 133 ns width, respectively. The regions labelled “p” and “γ” represent the associated events with protons and photons, respectively. Photon events are populated around the diagonal line. The events associated with charged particles which are mainly protons, are well separated from the locus of photon events.

5.2.2 Ion identification

A two-dimensional spectrum was produced using the wide- and narrow-gate integration of the signal for a Heavy Ion detector module in coincidence with two photons in the Plastic Ball (Fig. 5.4). The horizontal and the vertical axes represent wide- and narrow-gate integrations, respectively, in one Heavy Ion detector module. Using the QDC modules, the result of the narrow-gate integration has been amplified by a factor of 5. The narrow-gate integration is supposed to represent the light output of the 80 μm thick plastic scintillator,
while the wide-gate integration shows the total light output of the plastic and the CsI(Tl) scintillators. Every peak in the pulse shape spectrum corresponds to one ion species. As can be seen, using the pulse shape techniques each peak is well separated. To identify ions, different tools were employed as explained in the following sections.

**Ion Time Of Flight**

Using the TDCs of the Heavy Ion detector modules, the time difference between ion and photon detections was recorded. This time difference is proportional to the Time Of Flight (TOF) of ions from the target to the Heavy Ion detector while passing through the Big-Bite Spectrometer. The Time Of Flight is calculated as:

$$ TOF = \frac{mL}{qB\rho}, $$

with $q$ being the charge, $m$ the particle mass, $L$ the length passed by the ion from the target into the Heavy Ion detector (typ. 7.5 m), $B$ the magnetic field and $\rho$ the radius of the trajectory in the dipole magnetic field. As can be seen in Fig. 5.5-(a), using Time Of Flight information most of the produced ions have been identified. $^{10}$B ions, produced in the pionic fusion reaction, as well as $^{6}$Li and $^{4}$He, produced in the elastic interaction
5.2. Particle identification

![Calibrated Time Of Flight](image)

Figure 5.5: Calibrated Time Of Flight of ions from the target to the Heavy Ion detector by passing through the Big-Bite Spectrometer. The original recorded time is the time difference between the ion and the photon detection which is proportional to the Time Of Flight of the ion from the target to the Heavy Ion detector. Using the kinematical calculations and the magnetic field of the Big-Bite Spectrometer, the original recorded time difference is converted into the Time Of Flight.

...of $^4$He with $^6$Li, and $^2$H have the same m/q ratio (Fig. 5.5-(b)). Therefore, the Time Of Flight information is not sufficient to identify the peak for the $^{10}$B events among the peaks in Fig. 5.4. In order to obtain a unique identification of $^{10}$B, other available information is required.

**Ion energy deposition**

The momentum acceptance strongly suppresses the elastically scattered $^4$He and $^6$Li in the focal plane of the Big-Bite Spectrometer. However, we aimed at a unique identification of the fusion product to suppress background caused by incomplete detection of photons from neutral pion decay. We used all the available information like the light output of the scintillators for different produced particles. Due to the high ionisation of charged particles inside the scintillators, the scintillator response to the energy deposition of the charged particles is not linear. In fact, a fraction of the deposited energy inside the scintillators is not converted into the light output. This effect is called “quenching”. Horn et al. [60, 61] have measured the response of plastic and CsI(Tl) scintillators for a variety of nuclear species between $^1$H and $^{12}$C as a function of atomic number, mass number and energy (up to 37 MeV/nucleon for $^{12}$C ions) and have studied the strong quenching effects. Figure 5.6 shows the result of their calibration for the CsI(Tl) scintillators. Data-points
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

Figure 5.6: Calibration curves for the CsI(Tl) scintillators obtained in the measurement done by Horn et al. [60]. The circles are the data points resulting from their experiment while the curves are the result of a least-squares fit by using Eq.(5.2).

are the experimental results which have been fitted to the light output using the following expression:

$$L = a_0 + a_1(E - a_2AZ^2)ln\left(\frac{E + a_2AZ^2}{a_2AZ^2}\right),$$  \hspace{1cm} (5.2)

where $a_0$, $a_1$ and $a_2$ are the electronics offset factor, electronics gain factor and the quenching factor, respectively. $E$, $A$ and $Z$ are the ion energy, the ion mass and the ion charge, respectively. Calibration factors from Horn’s experiment were used as a tool to identify different ions in the Heavy Ion detector. We compared the relative light output of the Heavy Ion detector for different ions with the relative light output of the Horn measurement.

In case of the Heavy Ion detector, using the narrow-gate integration, mainly the plastic light output is integrated. However, in the narrow-gate integration there is always some contribution of the light output from the CsI(Tl) scintillator. Also the result of the wide-gate integration consists of the total light output of the module which includes the plastic
and the CsI(Tl) light output. Therefore, the first step of the ion identification was to obtain the pure plastic and the pure CsI(Tl) light output of the Heavy Ion detector. For that purpose the results of the Horn calibration of the light output were used to determine the scaling factors of the plastic light output to the total contribution and the CsI(Tl) light output to the total contribution as follows:

\[
L_{CsI(Tl)}(Hi_{det}) = L_{total}(Hi_{det}) - \frac{L_{pl}(Horn)}{L_{CsI(Tl)}(Horn) + L_{plastic}(Horn)} \times L_{total}(Hi_{det}),
\]

(5.3)

\[
L_{plastic}(Hi_{det}) = L_{total}(Hi_{det}) - \frac{L_{CsI(Tl)}(Horn)}{L_{CsI(Tl)}(Horn) + L_{plastic}(Horn)} \times L_{total}(Hi_{det}),
\]

(5.4)

where \(L_{CsI(Tl)}(Hi_{det})\) and \(L_{CsI(Tl)}(Horn)\) are the light output of the CsI(Tl) scintillators of the Heavy Ion detector module and the Horn detector, respectively. \(L_{plastic}(Hi_{det})\) and \(L_{plastic}(Horn)\) are the light output of the plastic scintillators of the Heavy Ion detector module and the Horn detector, respectively. \(L_{total}(Hi_{det})\) indicates the total light output of the Heavy Ion detector module using the wide-gate integration.

The detector light output as a function of particle charge for the Horn measurement and the present measurement is depicted in Fig. 5.7 left and right panels, respectively. The error bars in Fig. 5.7-(b) and -(d) are the standard deviations of the peaks in the pulse shape spectrum of the Heavy Ion detector (Fig. 5.4). The error bars in Fig. 5.7-(c) originate from the uncertainty in the determination of \(a_0, a_1\) and \(a_2\) in Eq. 5.2. Fig. 5.7-(a) and -(b) shows the pure light output of the plastic scintillator from our data (b) and from the Horn data (a) for particles which hit the Heavy Ion detector. The full circle and full square in the Fig. 5.7-(b) are the results for the “peak number 1” and “peak number 2” in the pulse shape spectrum of the Heavy Ion detector, when assuming the “peak number 1” and “peak number 2” are \(^{10}\text{B}\) and \(^{6}\text{Li}\), respectively. Furthermore, the empty circle and empty square were obtained assuming that the “peak number 2” and “peak number 1” in Fig. 5.4 are \(^{10}\text{B}\) and \(^{6}\text{Li}\), respectively. In general, the relative behaviour of the plastic light output for ions in Fig. 5.7-(a) and -(b) is the same.

In order to have a direct comparison with the Horn measurement, the plastic light output difference of ions and protons which is normalised by the plastic light output for ions is depicted in Fig. 5.8. As can be seen, the relative behaviour of the plastic light output in both measurements for \(^3\text{He}\), \(^7\text{Li}\) and \(^7\text{Be}\) is the same with an uncertainty which is less than 2%. In case of \(^2\text{H}\) the uncertainty is 6%. Still we can not distinguish \(^{10}\text{B}\) among “peak number 1” and “peak number 2” in Fig. 5.4. The reason is that by assuming that the “peak number 1” or “peak number 2” is \(^{10}\text{B}\), almost the same relative light output from the plastic scintillator will be obtained (full and empty circles in Fig. 5.8). Figure 5.7-(c) and (d) shows the pure CsI(Tl) light output of our data (d) and that of Horn (c). In Fig. 5.7-(c), the light output of CsI(Tl) scintillator from the Horn measurement for \(^{10}\text{B}\) and \(^{6}\text{Li}\) (cross and empty square, respectively) are shown. Figure 5.7-(d) represents the CsI(Tl) light output of our measurement assuming that the “peak number 1” and “peak number 2” in Fig. 5.4 are \(^{10}\text{B}\) and \(^{6}\text{Li}\) (full circle and empty square), respectively. In addition, the empty circle and full square show the results when assuming that the “peak number 2” and “peak number 1” in Fig. 5.4 are \(^{10}\text{B}\) and \(^{6}\text{Li}\), respectively. The star in Fig. 5.7-(b) and -(d) shows the plastic and CsI(Tl) light output of the Heavy ion detector in case the “peak
Chapter 5: Analysis of the $^6\text{Li}({^4}\text{He},\pi^0)^{10}\text{B}^*$ data

Figure 5.7: Detector light output and identification of the fusion product. (a) and (c): The plastic and CsI(Tl) light output of the produced ions in the $^6\text{Li}({^4}\text{He},\pi^0)^{10}\text{B}^*$ reaction using Horn calibration factors [60, 61]. (b) and (d): The plastic and CsI(Tl) light output of the produced ions using the AGOR facility.

As was mentioned in chapter 1, another pionic fusion experiment using the same experimental setup with an 85.3 AMeV $^3\text{He}$ beam on a 130 mg/cm$^2$ liquid $^4\text{He}$ target has been performed at KVI [43]. Since Time Of Flight information was enough to identify all the produced ions in that experiment, we decided to use that data as a useful tool to have...
5.2. Particle identification

Figure 5.8: The light output of the plastic scintillator for ions divided by the light output for protons plotted as function of ion charge.

one more proof in the identification of ions. In order to compare the light output of detectors in two different reactions, the particle energy should be the same in both reactions. This means that the same settings of the BBS magnetic field in both reactions should be compared. The measured data related to the BBS magnetic fields of 0.9690 Tm and 0.9783 Tm, corresponding to \(^{6}\text{Li}(^{4}\text{He},\pi^{0})^{10}\text{B}^{*}\) and \(^{4}\text{He}(^{3}\text{He},\pi^{0})^{7}\text{Be}\) reactions, respectively, were selected. There is a 1% difference between the chosen magnetic field strengths of the two reactions, which is equivalent to a shift by about one Heavy Ion detector unit when comparing the \(^{6}\text{Li}(^{4}\text{He},\pi^{0})^{10}\text{B}^{*}\) reaction with the \(^{4}\text{He}(^{3}\text{He},\pi^{0})^{7}\text{Be}\) reaction (Fig. 5.13). In comparison of the two reactions this detector shift was taken into account. \(^{1}\text{H}\) and \(^{3}\text{He}\) from the \(^{6}\text{Li}(^{4}\text{He},\pi^{0})^{10}\text{B}^{*}\) reaction were selected as reference particles and in order to have the same channel difference between \(^{1}\text{H}\) and \(^{3}\text{He}\) in two reactions, a gain factor for every Heavy Ion detector module was calculated. Also to shift the channel number of the \(^{3}\text{He}\) particle in the \(^{6}\text{Li}(^{4}\text{He},\pi^{0})^{10}\text{B}^{*}\) reaction into the same channel number of that particle in the \(^{4}\text{He}(^{3}\text{He},\pi^{0})^{7}\text{Be}\) reaction, an offset factor for every Heavy Ion detector module was calculated. Figure 5.9 shows the result of the gain-matched light output of the \(^{6}\text{Li}(^{4}\text{He},\pi^{0})^{10}\text{B}^{*}\) experiment in comparison to the light output of the \(^{4}\text{He}(^{3}\text{He},\pi^{0})^{7}\text{Be}\) experiment. As result, the peak positions of the known particles like \(^{2}\text{H}\), \(^{7}\text{Li}\) and \(^{7}\text{Be}\) in both datasets are in agreement for all the Heavy Ion detector modules. The gain-matched peak
Figure 5.9: (a) (b): The empty markers are the light output of the Heavy Ion detector modules for different ions after the wide-gate (narrow-gate) integration for the $^4$He($^3$He,$\pi^0$)$^7$Be reaction. The full markers are the light output of the Heavy Ion detector modules after the wide-gate (narrow-gate) integration for different ions in the $^6$Li($^4$He,$\pi^0$)$^{10}$B reaction when the ion peak position is adjusted in gain to the positions of the same ion in the $^4$He($^3$He,$\pi^0$)$^7$Be reaction.

position of the particle which is called peak number 2 in Fig. 5.4 for the $^6$Li($^4$He,$\pi^0$)$^{10}$B reaction, covers almost the same region as $^6$Li in the $^4$He($^3$He,$\pi^0$)$^7$Be reaction. Large fluctuations for $^6$Li are due to the very low statistics and the difficulty in determining the peak position in the $^4$He($^3$He,$\pi^0$)$^7$Be reaction.

Summarising, we conclude that we have confirmed the unique identification of $^{10}$B by three different methods.

5.3 Calibration

The calibration of the data is a process in which the recorded (measured) values are identified and associated with physics processes that cause them. By knowing this relation we can actually give a physical meaning to the data stream.

5.3.1 Time calibration

Time calibration of the Plastic Ball and the Inner Shell

The first step in the calibration of the TDC information was a time matching of all the Plastic Ball detectors together. We have used the Plastic Ball-single trigger for the time matching of the Plastic Ball modules. Furthermore, using 600 ns time difference due to the wire lengths which are travelled by one or the other copy of the same signal, the
5.3. Calibration

Figure 5.10: Calibration of the ion Time Of Flight from the target to the Heavy Ion detector while passing the BBS. (a) and (b) show the calibrated Time Of Flight measured by the upper and lower row of the HI detector, respectively. The curves are the result of fitting Eq. 5.1 to the Time Of Flight of $^3$He and $^7$Li ions.

Plastic Ball TDCs were calibrated. For the time calibration of the Inner Shell modules the same method was applied. The calibrated time signals summed over all the Plastic Ball detectors and all the Inner Shell detectors were already shown in Fig. 5.2.

Calibration of the ion Time Of Flight

In the Time Of Flight information of the Heavy Ion detector, there are two self-triggered peaks originating from the Heavy Ion signals in the single and coincidence modes of the data acquisition (see Fig. 4.8). The relative time of the TDCs was calibrated using these
self-triggered peaks separated by a fixed time delay. A reference detector was chosen and, in order to have the same number of channels between the two triggered peaks in every detector and in the reference detector, a gain factor was calculated. In addition, the first triggered peak for all the detectors was shifted into the same channel of that peak in the reference detector. In this way for every detector the gain and offset factors were calculated. As we expect and as can be seen in Fig. 5.10, with increasing particle momentum the Time Of Flight of all particles decreases. That is because by increasing the particle energy, the particle not only becomes faster but also bends less in the magnetic field and the trajectory length of the particle from the target to the Heavy Ion detector decreases. As result according to Eq 5.1 the particle Time Of Flight decreases. Moreover, by increasing the \(m/q\) ratio, in general, the Time Of Flight of different ions with the same momentum increases. By fitting Eq. 5.1 to the calibrated Time Of Flight for every ion, it was found that by increasing the \(m/q\) ratio the slope of the fitted curves increases correspondingly (see Table 5.1). The fluctuations with respect to the fitted curves are in the order of a few ns.

| Table 5.1: The slope of the fitted curves to the ion Time Of Flight measured by the upper row of the Heavy Ion detector. |
|---|---|---|---|---|---|---|
| Ion | \(^1\)H | \(^3\)He | \(^7\)Be | \(^2\)H | \(^10\)B | \(^6\)Li |
| A/Z | 1 | 1.5 | 1.75 | 2 | 2 | 2.33 |
| Slope | -0.40 | -0.58 | -0.56 | -0.71 | -0.9 | -0.99 | -1. |
| ± | ±0.03 | ±0.03 | ±0.04 | ±0.03 | ±0.04 | ±0.04 |

### 5.3.2 Energy calibration

#### Momentum calibration in the Heavy Ion array

Since the Heavy Ion module position gives us the momentum of the detected ion, we have a useful observable to calculate the ion energy. The energy calibration of the individual Heavy Ion detectors is not necessary. In the magnetic field, charged particles move in a bent trajectory. The radius of the trajectory, \(\rho\), is linearly related to the momentum of the particle, \(p = mv\) by

\[
B\rho = \frac{mv}{q},
\]

where \(B\), \(q\) and \(m\) are magnetic field strength, charge and particle mass, respectively. The quantity \(B\rho\) is called magnetic rigidity. The deviation of the rigidity of a particle with respect to the particle travelling along the central ray of the spectrometer with trajectory radius \(\rho_0\) is defined as

\[
\Delta p/p_{\text{central}} = \frac{B\rho - B\rho_0}{B\rho_0}
\]

By adjusting the quadrupole- and the dipole-field strengths, particles can be positioned and bent in a certain direction in the Big-Bite Spectrometer (Fig. 4.1). During the experiment, the magnetic field was set such that \(^{10}\)B with \(E = 93\) MeV hits the middle of the Heavy Ion array. This setting is called the nominal setting of the Big-Bite Spectrometer for the present experiment and the corresponding values are shown in Table 5.2.
5.3. Calibration

Figure 5.11: Schematic relation between the rigidity and the detector position in the Heavy Ion detector array. The numbering sequence of detectors from 1-30 and 31-60 is indicated. The low- and high-momentum sides of the array are depicted in the figure.

Table 5.2: The experimental values of the Big-Bite Spectrometer for the nominal rigidity setting. Using these values, $^{10}$B with $E = 93$ MeV hits the centre of the Heavy Ion detector array.

| Description                  | Value     |
|------------------------------|-----------|
| current of the dipole        | 222.576 A |
| current of the first quadrupole | 117.270 A |
| current of the second quadrupole | 139.644 A |
| rigidity                     | 0.8809 Tm |

Suppose we have a group of the same kind of particles with different energies and apply dipole and quadrupole magnetic fields in the specific magnet setting of the Big-Bite Spectrometer. If the particles pass through the magnetic field, particles with different energies will hit Heavy Ion detector modules in different positions of the array.

Equivalently, if we have a group of the same kind of particles with the same energies and apply BBS magnetic fields in different settings, by changing the magnetic field the momentum of the particle will change and they will be detected at different detector position. In one setting of the Big-Bite Spectrometer detector ‘x’ will be hit, while in another setting, detector ‘y’ will detect the particle. Since the particle energy is the same in both settings, the energy deposition in detectors ‘x’ and ‘y’ should be the same. We used this fact for the momentum calibration of the Heavy Ion detector array also for the gain matching of the Heavy Ion detectors.

In commissioning runs for the Heavy Ion detectors, we have performed a test experiment with various projectiles ($^4$He, $^{16}$O, $^{12}$C, $^{20}$Ne) at laboratory energy of 14.3 AMeV on $^{197}$Au and $^{12}$C targets and measured the elastic peak. The aim was to study the response to the different ion species and the particle identification capability. As result, the elastic peak was mainly found in one detector module with a small contribution of the neighbour-
Figure 5.12: Heavy Ion detectors in the Heavy Ion detector array which are associated with the energy deposition of one particle from the elastic interaction.

Figure 5.13: Momentum shift measured by detector position in the Heavy Ion array.

ing detectors (Fig. 5.12). The horizontal position resolution was about 21 mm FWHM which is equivalent to 1.6 detector units, where the horizontal detector bin size is 13 mm. The horizontal position resolution is mainly determined by the acceptance of the Big-Bite Spectrometer (see Section 5.6). The mean detector position has been correlated with a momentum shift introduced by the magnetic field setting with respect to the setting for the central ray. Figure 5.13 shows the relation between the momentum shift in % and the
5.3. Calibration

mean detector position. As can be seen, a good linearity with the slope of 0.625%/detector unit has been obtained.

Gain matching of the light output for the Heavy Ion detector

During the pionic fusion experiment, data were taken with the nominal setting of the BBS ($\Delta p/p_{\text{central}} = 0\%$). A small amount of data was taken with the momentum deviations $\Delta p/p_{\text{central}}$ of -10%, -5%, 5% and 10% with respect to the central momentum $p_{\text{central}}$ of the nominal setting. These data were used to study the response of the Heavy Ion detector array to various ion masses, energies and momenta in order to calibrate the array. The negative value of the momentum or rigidity deviations implies the stronger magnetic field of the Big-Bite Spectrometer. After the experiment these data were used for the calibration. As is shown in Fig. 5.13, by changing the momentum of the particle by 0.625% with respect to the central momentum, the hit Heavy Ion detector will be shifted by one unit. Equivalently, the rigidity change of a particle at 0% and 5% rigidity settings, is equivalent to a shift by 8 detector units. Therefore in different settings, there are “equivalent detectors” with the same light output, e.g. 'HIdet', 'HIdet + 8', 'HIdet + 16' and 'HIdet + 24' with $\Delta p/p_{\text{central}} = -10\%$, -5\%, 0\% and 5\%, respectively. For the gain matching of the Heavy Ion detectors, four detectors were chosen as “equivalent detectors”. In addition, the 'HIdet + 16' in the $\Delta p/p_{\text{central}} = 0\%$ setting, positioned around the middle of the array, was chosen as reference detector. $^3\text{He}$ and $^{10}\text{B}$ were chosen as reference particles.

In the first step of the gain matching procedure, by calculating a gain factor for every “equivalent detector”, the same channel difference as found in the reference detector between the $^3\text{He}$ peak position and the $^{10}\text{B}$ peak position, was obtained

$$
\text{gain}(\text{HIdet}) = \frac{L_{^3\text{He},\text{rig}0}(\text{ref}) - L_{^{10}\text{B},\text{rig}0}(\text{ref})}{L_{^3\text{He},\text{rig}-10}(\text{HIdet}) - L_{^{10}\text{B},\text{rig}-10}(\text{HIdet})}, \quad (5.7)
$$

$$
\text{gain}(\text{HIdet} + 8) = \frac{L_{^3\text{He},\text{rig}0}(\text{ref}) - L_{^{10}\text{B},\text{rig}0}(\text{ref})}{L_{^3\text{He},\text{rig}-5}(\text{HIdet} + 8) - L_{^{10}\text{B},\text{rig}-5}(\text{HIdet} + 8)}, \quad (5.8)
$$

$$
\text{gain}(\text{HIdet} + 24) = \frac{L_{^3\text{He},\text{rig}0}(\text{ref}) - L_{^{10}\text{B},\text{rig}0}(\text{ref})}{L_{^3\text{He},\text{rig}+5}(\text{HIdet} + 24) - L_{^{10}\text{B},\text{rig}+5}(\text{HIdet} + 24)}, \quad (5.9)
$$

where 'ref' indicates the reference detector. $L_{^3\text{He},\text{rig}0}(\text{ref})$ indicates the light output of the reference detector which has been measured for the $^3\text{He}$ particle in the central momentum (rigidity) $p_{\text{central}}$ of the nominal setting. Furthermore, to have the $^3\text{He}$ peak in the same position as the $^3\text{He}$ peak in the reference detector, an offset for every “equivalent detector” was calculated

$$
\text{offset}(\text{HIdet}) = L_{^3\text{He},\text{rig}0}(\text{ref}) - \text{gain}(\text{HIdet}) \times L_{^3\text{He},\text{rig}-10}(\text{HIdet}), \quad (5.10)
$$

$$
\text{offset}(\text{HIdet} + 8) = L_{^3\text{He},\text{rig}0}(\text{ref}) - \text{gain}(\text{HIdet} + 8) \times L_{^3\text{He},\text{rig}-5}(\text{HIdet} + 8), \quad (5.11)
$$

$$
\text{offset}(\text{HIdet} + 24) = L_{^3\text{He},\text{rig}0}(\text{ref}) - \text{gain}(\text{HIdet} + 24) \times L_{^3\text{He},\text{rig}+5}(\text{HIdet} + 24). \quad (5.12)
$$

The second step was to use the derived gain factors and the offsets to calculate the calibrated channel number of all other particles in all the BBS magnet settings for the
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

“equivalent detector” as follows:

$$gL_{j,k}(i) = L_{j,k}(i) \times \text{gain}(i) + \text{offset}(i).$$  \hspace{1cm} (5.13)

$gL_{j,k}(i)$ and $L_{j,k}(i)$ are the light output of the detector “$i$” after and before the gain matching, respectively, when particle “$j$” in the rigidity deviation setting of “$k$” hits the detector “$i$”.

The third step was to calculate the gain factors and the offsets for the rest of the detectors. These detectors are called “middet” and are positioned between every two “equivalent detector”. The gain-matched “equivalent detector” were used to gain match the rest of the detectors. The light output of a detector is proportional to the particle momentum as follows:

$$L \propto E = \frac{p^2}{2m},$$ \hspace{1cm} (5.14)

where

$$\frac{p^2}{2m} \propto \text{(horizontal module position in the Heavy Ion detector array)}^2.$$ \hspace{1cm} (5.15)

$L$ is the detector light output. $E$, $p$ and $m$ are the particle kinetic energy, particle momentum and the particle mass, respectively. According to Eq. 5.14 and 5.15, by moving from a Heavy Ion detector module to the neighbouring one in the array, the light output does not change linearly. The calibrated light output of the four “equivalent detectors” for the reference particles in the central rigidity $p_{\text{central}}$ of the nominal setting were fitted to equation $y = ax^2 + b$. By using the resulting fitting coefficients $a$ and $b$ and the proper detector number $x$ (which is a number proportional to the horizontal module position in the Heavy Ion detector array), the gain-matched light output of the rest of the detectors in the array for $^3\text{He}$ and $^{10}\text{B}$ was calculated. Finally, the gain and the offset factors for the rest of the detectors were obtained

$$\text{gain(middet)} = \frac{gL_{^3\text{He},\text{rig0}}(\text{middet}) - gL_{^{10}\text{B},\text{rig0}}(\text{middet})}{L_{^3\text{He},\text{rig0}}(\text{middet}) - L_{^{10}\text{B},\text{rig0}}(\text{middet})},$$ \hspace{1cm} (5.16)

$$\text{offset(middet)} = gL_{^3\text{He},\text{rig0}}(\text{middet}) - \text{gain(middet)} \times L_{^3\text{He},\text{rig0}}(\text{middet}).$$ \hspace{1cm} (5.17)

These two values allowed the gain matching by shifting all the ions for the specific detector into the correct channel number. These steps were applied for the modules in the upper row of the Heavy Ion detector array (detector 1-30) and for the gain matching of the total light output (plastic+CsI(Tl)) and the plastic light output separately. The gain matching of the modules positioned in the lower row (detector 31-60) was achieved by comparison to modules that have the same horizontal position but are placed in the upper row. A gain and an offset factor were calculated to move the $^3\text{He}$ and $^{10}\text{B}$ peak positions to the same channel numbers as the $^3\text{He}$ and $^{10}\text{B}$ peak positions in the upper module.

The calibrated total and plastic light output of the Heavy Ion detectors for all particles in all the measured rigidities before and after the gain matching are shown in Figs. 5.14 and 5.15, respectively. The graphs in the left column depict the peak position of the particles before the gain matching. The graphs in the right column are representing the calibrated peak positions. From the top to the bottom of Figs. 5.14 and 5.15, the results
5.3. Calibration

Figure 5.14: Gain matching of the ion light output from the wide-gate integration in the different rigidity deviations with respect to the central ray. Left (Right) figures correspond to the ion peak position in the spectrum of the wide-gate integration before (after) gain matching of the detectors. Different markers are representing different ions. The rigidity deviations from the central ray from the top to the bottom graphs are -10%, -5%, 0%, 5% and 10%, respectively.
Figure 5.15: Gain matching of the ion light output from the narrow-gate integration in the different rigidity deviations with respect to the central ray. Left (Right) figures correspond to the ion peak position in the spectrum of the narrow-gate integration before (after) gain matching of the detectors. Different markers are representing different ions. The rigidity deviations from the central ray from the top to the bottom graphs are -10%, -5%, 0%, 5% and 10%, respectively.
5.3. Calibration

of the measurements using the momentum (rigidity) deviations $\Delta p/p_{central} = -10\%, -5\%, 0\%, 5\%$ and $10\%$ are shown. Furthermore, different symbols correspond to the different particles. The points which are completely off correspond to the detector modules which were not properly operating (two detector modules) and were not used for the further analysis.

As can be seen, after the gain matching using the two particles ($^3\text{He}$ and $^{10}\text{B}$) a smooth behaviour of the light output in the Heavy Ion detector array for all the other particles is obtained. This implies the existence of a systematic correlation between the modules of the Heavy Ion array. According to Eq 5.14, since $p$ is proportional to the horizontal position of the ion in the Big-Bite Spectrometer or the x-axis in Figs. 5.14 and 5.15, the slope of the lines is proportional to the particle mass. Thus, the positive slope of the lines is expected and implies that the light output of the modules positioned on the higher momentum-side of the Heavy Ion detector array is higher than the light output of the modules positioned on the lower momentum-side. In addition, by decreasing the magnetic field strength, in general, the light output decreases which is the result of the lower particle energy. This result is shown on the right side of Figs. 5.14 and 5.15. A detailed analysis of the dependence of the detector light output on particle type and detector material, including the quenching effect, is in progress and will be reported in another thesis [43].

Energy calibration of the Inner Shell and the Plastic Ball backward hemisphere

To convert the light output measured by the ADCs into energy units for the Inner Shell and the Plastic Ball modules, the signal from high energy cosmic muons was exploited. For the calibration only tracks of cosmic muons that pass the centre of the Plastic Ball were chosen. The track of a muon in the cosmic data was chosen such that the muon passes...
Chapter 5: Analysis of the $^6$Li($^4$He,$\pi^0$)$^{10}$B\(^*$\) data

one Plastic Ball module, the Inner Shell module behind it and the corresponding Plastic Ball module opposite to the first one (Fig. 5.16). In addition, in order to exclude random coincidences and those of cosmic muon events which don’t pass through three different detectors longitudinally, one more condition in the track selection has been applied. The condition was that a cosmic muon event is selected only if the closest 12 neighbours of the mentioned Plastic Ball modules in the track have not given a signal. The same condition for the closest 6 neighbours of the Inner Shell module in the track was applied. This sharp track definition for the Plastic Ball and the Inner Shell modules generated the clearest cosmic peak. A Monte-Carlo simulation for 1 GeV muons was performed to simulate minimum ionising particles. Muons were produced on the surface of a virtual hemisphere, centred in the centre of the Plastic Ball detector, with radius of 1500 cm. Muon tracks in the Plastic Ball and in the Inner Shell modules were selected under the same conditions as applied for the measured data. The calibration was performed by matching the measured peak positions to the peak positions from the simulation.

Due to the zenith angle distribution of cosmic muons, some of the almost horizontally oriented Plastic Ball modules, (about 20% of the total), and the Inner Shell modules that are positioned behind them, reveal a rather broad cosmic peak. Because of the almost cubic geometry, the energy deposition of the cosmic muons in the Inner Shell modules is independent of the angle of incidence. Even if the tracks are not well defined, cosmic muons deposit almost the same amount of energy with a clear peak. Therefore, to calibrate the horizontally oriented Inner Shell modules, vertical cosmic muons were used. For the horizontally positioned Plastic Ball modules, a cross calibration with respect to the neighbouring modules using photons in the pionic fusion data was done.

Figure 5.17-(a) and -(b) shows the energy deposition of cosmic muons inside the Plastic Ball module (plastic plus CaF\(_2\) scintillators) and inside the plastic scintillator, respectively, summed over all the modules. The measured and calibrated (solid curve) energy deposition of the cosmic muons in the Plastic Ball modules shows peaks around 74 MeV and 72 MeV with the resolution of 38% FWHM and 40% FWHM for the total deposited energy and for the energy deposition in the plastic scintillators, respectively. The muon peaks in the simulated results shown by the dotted curves are at the same positions as the experimental data but the peaks have different widths compared to the measured peaks. This difference in the peak widths is the result of the incomplete shower collections inside the scintillators which have not yet been taken into account accurately enough in the simulation. In particular, threshold effects for light production by low energy particles and light absorption at the module boundaries were not modelled in all details. In order to match the energy resolution of the detectors for the simulated and measured cosmic muons and correct for the incomplete shower collection, the simulated muon energy depositions inside the plastic plus CaF\(_2\), plastic and CsI(Tl) scintillators were multiplied by three different factors as Gaussian random deviates. The simulated results of the cosmic muon energy deposition after the correction for the absorption in the module, is shown in Fig. 5.17 by the dashed curves. It should be mentioned that the obtained factors work well to match the energy resolution of the detectors for the simulated and measured photons from $\pi^0$ decay in the $^6$Li($^4$He,$\pi^0$)$^{10}$B\(^*$\) reaction. Therefore, to simulate the $^6$Li($^4$He,$\pi^0$)$^{10}$B\(^*$\) reaction and the detector responses, the same factors have been employed.

Using cosmic muons, we have calibrated the total light output of the Plastic Ball modules. Because the cosmic muons deposit a very low energy in the CaF\(_2\) scintillator
5.3. Calibration

Figure 5.17: Cosmic muon energy deposition in a complete Plastic Ball module (a) and in the plastic scintillator only (b), summed over all modules. Solid curves are representing the measured results. The dotted and dashed curves show the simulated results before and after applying the smearing effects on the energy resolution, respectively.

Figure 5.18: Cosmic muon energy deposition in the CsI(Tl) scintillators, summed over all modules. The solid curve is representing the measured result. The dotted and dashed curves show the simulated results before and after applying the smearing effects on the energy resolution, respectively.
(less than 3% of the total deposition), in the pulse shape of the Plastic Ball (Fig. 5.3),

Women are found around the diagonal line. In Fig. 5.17-(b), there is still a difference

between the peak position of the simulated and the measured muon energy depositions

(solid and dotted curves) inside the plastic scintillators. The reason is that the same

calibration factors which were obtained for the total light output, were used to calibrate

the plastic light output. In this way the deposited energy of the cosmic muons inside

the CaF₂ scintillator was not considered. Since for the further analysis only the total

light output of the photons is used, an exact calibration for the plastic light output is not

necessary.

Figure 5.18 represents the measured (solid curve) and the simulated (dashed and dotted

curves) energy deposition of cosmic muons inside the Inner Shell modules, summed over

all the modules. The calibrated measured (solid curve) energy deposition of the cosmic

muons in the Inner Shell module shows a peak around 25 MeV, with the resolution of 38%

FWHM. The dotted and dashed curves are the simulated results before and after applying

the smearing effects on the energy resolution, respectively.

Energy calibration of the Plastic Ball forward hemisphere

For the gain matching of the Plastic Ball modules in the forward direction (\(\theta_{\text{module}} < 90^\circ\)),

high energy protons from \(^6\text{Li}\) breakup in the pionic fusion experiment were used. According

to the Monte-Carlo simulations, protons with energies less than 32 MeV deposit all their

energy inside the CaF₂. At energies above 32 MeV, protons punch through the CaF₂

and enter the plastic module (Fig. 5.19-(a)). For all the forward Plastic Ball modules,

the punch-through point can be clearly identified in the pulse shape spectrum (Fig. 5.3).

That point was used to gain match all the Plastic Ball modules positioned in the forward

direction. After the gain matching, the same method as described above for the calibration

of the backward direction with cosmic muons was used to calibrate one Plastic Ball module

in the forward direction. The calibrated proton punch through point in the pulse shape

spectrum of that module was used to calibrate the rest of the modules including the

horizontally oriented detectors in the forward direction. It should be mentioned that due

to the fact that protons with energies less than 120 MeV deposit all their energy inside the

Inner Shell detectors (Fig. 5.19-(b)) and can not reach the Plastic Ball detectors, which

are placed behind the Inner Shell, it was not possible to use the proton punch-through

point to gain match the backward part of the Plastic Ball.

5.4 Candidate selection

5.4.1 Clustering of the Plastic Ball detectors

According to Monte-Carlo simulations, the produced photons from \(\pi^0\) decay do not only

deposit energy into one single Plastic Ball module, but will share some energy with the

neighbouring modules. In order to determine the full deposited energy, we need to form a

cluster of relevant detectors and determine the total signal in the cluster. A cluster is the

largest possible group of adjacent Plastic Ball modules in which an energy greater than

the ADC threshold was registered. A module is hit whenever its ADC recorded values

above the pedestal. The hits in the modules forming a single cluster must be coincident
5.4. Candidate selection

Figure 5.19: (a): The simulated proton energy deposition in the CaF$_2$ scintillator as a function of energy deposition in the plastic scintillator of the Plastic Ball module. (b): The simulated proton energy deposition in the CsI(Tl) scintillator as a function of energy deposition in the CaF$_2$ plus plastic scintillator of the Plastic Ball module.

in time. The time window used for this cut ranges from -60 ns to 60 ns. A cluster-finding routine has been employed which scans over all the Plastic Ball modules in order to find the possible candidates. In the first step, if the energy deposition in a Plastic Ball module is larger than the hardware threshold or a suitably defined software energy threshold, a cluster including that module, called the seed module, will be formed. In the next step, the direct neighbours of the seed module will be checked and if they are also hit, they will be recognised as a part of that cluster. In case a hit module is not the direct neighbour of the seed module, that module will be part of another cluster. The cluster-finding routine takes care that in one event every hit module belongs to only one cluster. During the experiment and data analysis, the energy threshold for all the Plastic Ball modules was set just above the pedestal (for more information about the pedestal, see Section 4.9).

Sometimes, energy deposition is caused by a part of an electromagnetic shower which has split off from the main cluster. Then a group of crystals is identified by mistake as a separate cluster even though it should be part of the main cluster. This effect is caused by fluctuations of the electromagnetic shower. If the energy deposited in an individual module is too low and the amount of produced light is insufficient to be detected, such a fluctuation may form a small and low energy cluster close to, but separated, from the main cluster. This wrongly identified cluster is called a “split-off”.

The clustering routine was optimised to remove the split-offs. In the second step of the clustering, the routine not only checks the direct neighbours of a hit module but also checks the direct neighbours of the neighbours. The new routine passes the intermediate modules with no deposited energy up to two detector units and collects the next hit module into the same cluster. In order to check the optimised clustering routine, single photon events with $E_\gamma =$87 MeV produced in a Monte-Carlo simulation were analysed. Figure 5.20-(a) shows the multiplicity of the Plastic Ball clusters for a single photon, before
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

Figure 5.20: (a): The Plastic Ball (PB) cluster multiplicity using the energy deposition of a single photon with $E_\gamma = 87$ MeV produced in the Monte-Carlo simulation. The solid and dashed curves are the results before and after improving the clustering method, respectively. (b): The Plastic Ball detector multiplicity per cluster using the energy deposition of one of the two photons originating from $\pi^0$ decay in the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. The solid and dashed curves are the measured results before and after the optimisation of the clustering method, respectively. The dashed-dotted and dotted curves are the simulated results using the unmodified and modified clustering methods, respectively. The measured and simulated results have been normalised at multiplicity one for the unmodified clustering method.

(solid curve) and after (dashed curve) improving the clustering method. The chance of producing split-offs for an event using the unmodified clustering method has been studied and is 34% (multiplicity $\geq 2$ shown by the solid curve in Fig. 5.20-(a)). However, after the modification of the clustering method, the chance of producing split-offs, is reduced to less than 3% (multiplicity $\geq 2$ shown by the dashed curve in Fig. 5.20-(a)). Figure 5.20-(b) represents the Plastic Ball module multiplicity in a formed cluster, when the cluster is hit by one of the two photons from $\pi^0$ decay in the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. The solid and dashed curves are the measured results before and after the modification of the clustering method, respectively. For the multiplicity $\geq 2$, the dashed curve falls on top of the solid curve. The dashed-dotted and dotted curves are the simulated results using the unmodified and modified clustering methods, respectively. The simulated results of the module multiplicity in one cluster cover the same region as the measured results. After the clustering modification, the multiplicity of the Plastic Ball modules in one cluster has increased. As can be seen, the number of events with two or more detector units per Plastic Ball cluster increases. Consequently, the number of events with one detector unit per Plastic Ball cluster decreases. In fact, while the unmodified clustering routine produces a Plastic Ball cluster with more than two modules in less than 7% of events, this value increases to 15% after the modification of the clustering method. This is the result...
of including the split-offs into the main cluster.

**Position reconstruction**

Photons from $\pi^0$ decay hit the Inner Shell and the Plastic Ball detectors. Since the Plastic Ball is a highly segmented detector system, the position information of the Plastic Ball modules has been used to reconstruct the particle position. In order to estimate the direction of the incident particle, three different methods have been investigated. In the first method, the module in the cluster which has maximum energy deposition determines the position of the cluster. Therefore, the rest of the hit modules inside the cluster does not contribute to the cluster position determination. In the second method, the position of the cluster is simply the centre of gravity of the shower:

$$\theta_{\text{calc.}} = \frac{\sum_i w_i \theta_i}{\sum_i w_i}, \quad (5.18)$$

$$\phi_{\text{calc.}} = \frac{\sum_i w_i \phi_i}{\sum_i w_i}, \quad (5.19)$$

where $\theta_i$ and $\phi_i$ are the polar and the azimuthal angles of the $i$th individual Plastic Ball module in the cluster, respectively, and the weight factors $w_i$ are taken as the energy $E_i$ deposited in that module. Equivalently, the weights may be viewed as the fraction of the total shower energy in module $i$, $w_i = E_i/E_T$ with $E_T = \sum_i E_i$. In such a linear way of energy weighting, all the modules inside a cluster equally contribute to the cluster position determination.

In the third method, instead of the weights which are linearly proportional to the deposited energy in a module, logarithmic weights are used as given by the following expression [62]:

$$w_i = \max(0, [W_0 + \ln(E_i/E_T)]). \quad (5.20)$$

$W_0$ is a free dimensionless parameter and can be determined from the Monte-Carlo simulations. In case of Plastic Ball, $W_0$ was calculated and is 3.9. The motivation for this expression with logarithmic weights of the deposited energy in the module is to take into account the exponential falloff of the shower energy distribution.

In order to compare different methods, the response of the Plastic Ball for a single photon has been studied in the Monte-Carlo simulations. In our case, the results of all three methods for the single photon agree within the detector resolution. Figure 5.21 shows the simulated results of the reconstructed position in the Plastic Ball when two photons from $\pi^0$ decay in the pionic fusion reaction of $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ were used. Graphs (a) and (b) in Fig. 5.21 are the reconstructed photon polar and azimuthal angles plotted as a function of the incident photon polar and azimuthal angles, respectively. Graphs (c) and (d) in Fig. 5.21 show the projected results of the (a) and (b) panels, respectively, along the diagonal lines. As can be noticed, the resolution ($\sigma$) of the angle determination for the polar and the azimuthal angles is $7^\circ$ and $6^\circ$, respectively, which is about equal to the opening angle covered by one detector unit in the Plastic Ball. In Fig. 5.21, the first method of the position reconstruction has been used. Since the resulting angular resolutions of all the three reconstruction methods are the same, the first method was
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

5.4.2 Pion reconstruction methods

In the reaction of interest ($^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$), the produced neutral pions immediately decay in the target into two photons with large opening angle as expected from two-
5.4. Candidate selection

photon decay of $\pi^0$’s moving with small velocity in the laboratory. In the experiment, the polar and azimuthal angles of both photons from the $\pi^0$ decay, the energy deposition of both photons in the Plastic Ball and in the Inner Shell detectors, as well as the energy and the momentum of $^{10}$B using the Heavy Ion detector have been measured. Also, in order to achieve a clean selection of particles, the timing information of all detected particles was recorded during the experiment. The two-photon invariant mass is calculated as follows:

$$M_{\gamma\gamma} = \sqrt{2E_{\gamma\text{high}}E_{\gamma\text{low}}(1 - \cos(\alpha))},$$  \hspace{1cm} (5.21)

where $E_{\gamma\text{high}}$ and $E_{\gamma\text{low}}$ correspond to the energies of the photon with the higher and the lower energy, respectively, and $\alpha$ is the opening angle of the two photons. Also

$$\cos(\alpha) = \sin\theta_{\text{high}} \times \sin\theta_{\text{low}} \times \cos\phi_{\text{high}} \times \cos\phi_{\text{low}} + \sin\theta_{\text{high}} \times \sin\theta_{\text{low}} \times \sin\phi_{\text{high}} \times \sin\phi_{\text{low}} + \cos\theta_{\text{high}} \times \cos\theta_{\text{low}},$$

in which $\theta_{\text{high}}(\theta_{\text{low}})$ and $\phi_{\text{high}}(\phi_{\text{low}})$ are polar and azimuthal angles of the photon with the higher (lower) energy. In order to reconstruct the two-photon invariant mass, only two measured observables among $E_{\gamma\text{high}}, E_{\gamma\text{low}}$ and $\alpha$ are needed, since on basis of kinematical constraints the rest of the needed information can be calculated. Three different methods for the invariant mass reconstruction can be applied.

- Method 1: The measured two-photon opening angle, the measured $^{10}$B momentum to calculate the $\pi^0$ energy, and the measured energy of the photon with the higher energy from the $\pi^0$ decay are used. By subtracting the energy of the photon with the higher energy from the pion energy, the energy of the photon with the lower energy produced in the $\pi^0$ decay is calculated:

\[
\begin{align*}
\alpha &: \text{from the measured angles} \\
E_{\gamma\text{high}} &: \text{the measured energy by the Plastic Ball and the Inner Shell detectors} \\
E_{^{10}\text{B}} &: \text{the measured energy of } ^{10}\text{B using the Heavy Ion detector} \\
E_{\pi^0} &= E_{\text{total}} - E_{^{10}\text{B}}, \\
E_{\gamma\text{low}} &= E_{\pi^0} - E_{\gamma\text{high}}
\end{align*}
\]

where $E_{\text{total}} = T_{\text{beam}} + M_{^4\text{He}} + M_{^6\text{Li}}$ is the total available energy. $E_{\pi^0}$ and $T_{\text{beam}}$ are the pion total energy and the kinetic energy of the $^4\text{He}$ beam (236.4 MeV), respectively.

- Method 2: The measured two-photon opening angle and the measured $^{10}$B momentum are used to calculate the $\pi^0$ energy. Using kinematical constraints and the energy of $\pi^0$, the energy of the photon with the higher energy is reconstructed. By subtracting the energy of the photon with the higher energy from the pion energy, the energy of the photon with the lower energy is obtained:

\[
\begin{align*}
\alpha &: \text{from the measured angles} \\
E_{^{10}\text{B}} &: \text{the measured energy of } ^{10}\text{B using the Heavy Ion detector} \\
E_{\pi^0} &= E_{\text{total}} - E_{^{10}\text{B}}.
\end{align*}
\]
The kinematical constraints are:
\[
\begin{align*}
\vec{p}_{x\gamma_{\text{high}}} + \vec{p}_{x\gamma_{\text{low}}} + \vec{p}_{x10B} &= 0 \\
\vec{p}_{y\gamma_{\text{high}}} + \vec{p}_{y\gamma_{\text{low}}} + \vec{p}_{y10B} &= 0 \\
\vec{p}_{z\gamma_{\text{high}}} + \vec{p}_{z\gamma_{\text{low}}} + \vec{p}_{z10B} &= \vec{p}_{\text{beam}} \\
|\vec{p}_{x10B}|^2 + |\vec{p}_{y10B}|^2 + |\vec{p}_{z10B}|^2 &= (E_{\text{total}} - E_{\gamma_{\text{high}}} - E_{\gamma_{\text{low}}})^2 - M_{10B}^2.
\end{align*}
\]

After simplification of the kinematical conditions, the photon energies are obtained:
\[
\begin{align*}
E_{\gamma_{\text{high}}} &= \frac{p_{\text{beam}}^2 + M_{10B}^2 - E_{\text{total}}^2 - 2E_{\gamma_{\text{low}}}(p_{\text{beam}}\cos\theta_{\gamma_{\text{low}}} - E_{\text{total}})}{2(p_{\text{beam}}\cos\theta_{\gamma_{\text{high}}} - E_{\text{total}}) + 2E_{\gamma_{\text{high}}}(1 - \cos\alpha)} \\
E_{\gamma_{\text{low}}} &= E_{\pi^0} - E_{\gamma_{\text{high}}},
\end{align*}
\]
where $M_{10B}$ is the $^{10}$B mass.

- Method 3: The measured two-photon opening angle and the measured energy of the photon with the higher energy are used. In this method, the measured $^{10}$B momentum is not used. In order to reconstruct the energy of the photon with the lower energy and the momentum of $^{10}$B, kinematical constraints are applied.

\[
\begin{align*}
\alpha &: \text{ from the measured angles}, \\
E_{\gamma_{\text{high}}} &: \text{ the energy measured by the Plastic Ball and the Inner Shell detectors},
\end{align*}
\]
\[
\begin{align*}
E_{\gamma_{\text{low}}} &= \frac{p_{\text{beam}}^2 + M_{10B}^2 - E_{\text{total}}^2 - 2E_{\gamma_{\text{low}}}(p_{\text{beam}}\cos\theta_{\gamma_{\text{low}}} - E_{\text{total}})}{2(p_{\text{beam}}\cos\theta_{\gamma_{\text{high}}} - E_{\text{total}}) + 2E_{\gamma_{\text{high}}}(1 - \cos\alpha)} \\
E_{\pi^0} &= E_{\gamma_{\text{high}}} + E_{\gamma_{\text{low}}} \\
E_{10B} &= E_{\text{total}} - E_{\pi^0}
\end{align*}
\]

In our experimental setup all particles are measured in over-determined kinematics, but with limited resolution. Since the two-photon opening angle, the $^{10}$B momentum and the energy of the photon with higher energy deposition were the best constrained observables in our experiment, we decided to use Method 1 for the pion invariant mass reconstruction.

### 5.4.3 Presort

In the experiment various triggers were used (see Chapter 4). The $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ events represent only a very small subset of the total amount of data which were stored because the main trigger was in many cases generated by other reactions. An example is elastically scattered $^6\text{Li}$ particles leading to the Heavy Ion trigger condition, in coincidence with a cosmic ray detected by the Plastic Ball. As can be seen in Fig. 5.1, the raw data are presorted to limit the data set to a reasonable size for further analysis. The presorting algorithm for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ event selection reduces the data set by approximately 98.2% by selecting two or more photon tracks in the Plastic Ball. After presorting and final particle selection, which will be explained in Section 5.4.4, the pion track was defined such that at least two separate Plastic Ball clusters in coincidence with $^{10}$B should be detected.
5.4. Candidate selection

![Graphs showing ion Time Of Flight versus signal for different ions.](image)

**Figure 5.22:** (a): The ion Time Of Flight versus the Heavy Ion wide-gate signal. (b): The ion Time Of Flight versus the Heavy Ion narrow-gate signal

5.4.4 Final selections

After presorting the data, final selections to obtain background-free events were applied. In order to select photons and exclude charged particles in the Plastic Ball, only events around the diagonal line in the pulse shape spectrum of the Plastic Ball modules were selected. In the Plastic Ball and the Inner Shell time distribution, only events found in the sharp peaks (regions between two dashed lines) in Fig. 5.2 were selected. In case of the Heavy Ion detector, events which are associated with $^{10}$B in the pulse shape spectrum (Fig. 5.4) and the Time Of Flight spectrum (Fig. 5.5) of the Heavy Ion detector have been selected. This selection cut was performed on the 2-dimensional histograms of the Time Of Flight versus the result of the wide-gate integration as well as the Time Of Flight versus the result of the narrow-gate integration (Fig. 5.22).

In addition to the above selections and in order to isolate pionic fusion events which are cleanly separated from the background events, more selections on the momentum distribution of $^{10}$B and the opening angle distribution of two photons have been applied and will be discussed in the following sections.

**Momentum distribution of $^{10}$B**

Figure 5.23 shows the Heavy Ion detectors that are hit by ions in the $^{10}$B peak which are in coincidence with two photons and therefore are $^{10}$B candidates, obtained from Method 1 (a) and Method 2 (b) of the $\pi^0$ invariant mass reconstruction. As can be seen, the general behaviour is like a two-peak structure related to the top and bottom rows of the Heavy Ion detector array (see Fig. 5.11 and Fig. 4.5-(i)). Due to the kinematical constraints that are applied in Method 2, peaks in Fig. 5.23-(b) are well separated. In order to exclude background events from the momentum distribution, the same region of the momentum which has been obtained using Method 2, was selected in Method
Chapter 5: Analysis of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ data

Figure 5.23: The counts measured in the Heavy Ion detector modules hit by ions in coincidence with two photons. Panel (a) shows the result when using the first method of the pion invariant mass reconstruction and the results shown in panel (b) correspond to the second method of the pion invariant mass reconstruction.

1. The selected regions for events in the upper and lower row of the array are shown in Fig. 5.24 as the region between two dashed and two dotted lines, respectively. In Fig. 5.24, events in the two windows represented by the solid line are not associated with $^{10}$B. These events are called additional events and correspond to the momentum larger than 1370 MeV/c. The additional events do not produce the two-photon invariant mass with similar distribution that is expected from the phase-space simulation. In fact, events between the two dashed lines or between the two dotted lines but outside the windows in Fig. 5.24 are associated with the two-photon invariant mass distribution which is in a very good agreement with the one from the phase-space simulation (see Section 6.4). In order to find the source of the additional events, various possibilities have been studied. First,
5.4. Candidate selection

Figure 5.24: The same description as Fig. 5.23. Events in the region between the two dashed (dotted) lines are the accepted events as $^{10}$B in the upper (lower) row of the Heavy Ion array by Method 2 of the $\pi^0$ invariant mass reconstruction. The two windows represented by the solid line show the events which are originated from the random coincidences. These events are excluded from the further analysis.

it should be noticed that in the pionic fusion reaction of $^6$Li($^4$He,$\pi^0$)$^{10}$B* with about 10 MeV available energy above the coherent threshold in the centre-of-mass system, only a few states of $^{10}$B, defined by the conservation of isospin (see Fig. 1.5) are allowed to be excited. A Monte-Carlo simulation was performed in order to study the momentum distribution of $^{10}$B when it is produced in the excited state. The dashed and dotted curves in Fig. 5.25-(a) show the phase-space simulated $^{10}$B momentum distribution when $^{10}$B is produced in the state at 1.7402 MeV ($J^P, I:0^+,1$) and the examined state at 5.1639 MeV ($J^P, I:2^+,1$). In addition, the solid curve shows the measured $^{10}$B momentum distribution.
Figure 5.25: (a): The solid curve shows the momentum distribution of $^{10}$B resulting from the measured data from the lower row of the Heavy Ion detector array. The dashed curve shows the momentum distribution of $^{10}$B from the phase-space simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* reaction when $^{10}$B is produced in the state at 1.7402 MeV. The dotted curve is the result of the phase-space simulation when $^{10}$B is produced in the examined state at 5.1639 MeV. The simulated results were obtained using the pure $^6$Li target. (b): The dotted curve is the phase-space simulation of the $^{10}$B momentum distribution when the target is not pure $^6$Li but has about 17% ± 3% $^{16}$O as contamination. The data cover the same region as the phase-space simulation using the pure $^6$Li target, but are in better agreement with the simulation when 17% $^{16}$O as target contamination is used in the phase-space simulation.

obtained from the lower row of the Heavy Ion array, when events with momentum larger than 1370 MeV/c are excluded. As can be seen, the simulated momentum distribution of $^{10}$B produced in the examined state at 5.1639 MeV covers the same region as the result for producing the 1.7402 MeV state and the experimental result when events with momentum larger than 1370 MeV/c are excluded. Therefore, the additional events do not correspond to the produced $^{10}$B in the higher excited states. Furthermore, $^{10}$B particles originating from $^{16}$O($^4$He,$\pi^0$)$^{10}$B+X reaction (pionic fusion of $^4$He with the existing $^{16}$O in the $^6$Li target) are not accepted by the BBS window (see Section 5.5) and therefore can not explain the additional events. Additional events are not associated with $^4$He particles originating from the elastic interaction of $^4$He and $^6$Li due to the fact that the acceptance of the BBS window suppresses the $^4$He particles. The coincidence of the neighbouring Heavy Ion detector modules corresponding to both regions of the momentum ($p_{\text{noB}} \leq 1370$ MeV/c and $p_{\text{noB}} > 1370$ MeV/c) has been checked in order to see if the additional events are the result of an incomplete energy deposition inside the relevant detectors. No coincidence between the detector groups in two regions of the momentum was found (see the next chapter). Since the simulated results do not show that those events are $^{10}$B, the conclusion was that the additional events are random coincidences. Those additional events were not
5.4. Candidate selection

included in the further analysis. The azimuthal asymmetry in the number of accepted $^{10}$B events can be explained by the asymmetric acceptance of the Plastic Ball due to target equipments (see Fig 4.6) in the following way. Missing modules in the forward region near $\phi=90^\circ$ will cause a reduced number of two-photon coincidences with the backward photon near $-90^\circ$. These photons are correlated with $^{10}$B moving preferentially to the upper part of the Heavy Ion detector array.

In Fig. 5.25-(b) the solid curve shows the measured $^{10}$B momentum distribution using the lower part of the Heavy Ion detector array and the dashed curve corresponds to the result of the phase-space simulation. The measured momentum distribution covers the same region as the result of the phase-space simulation but the peak is shifted to the lower momentum region by less than 2%. As was mentioned in Section 4.3, to prevent oxidation, the $^6$Li target was kept in an argon atmosphere during the preparation process and the transport to the vacuum chamber. However, by comparing the simulated results of the reaction with the pure and the oxidised $^6$Li target in the phase-space simulation, it was found that about $17\pm3\%$ of the target nuclei were $^{16}$O. Using this combination for the target in the simulation, the simulated momentum distribution is shifted to lower momentum in the same way as the result of the experimental data (Fig. 5.25-(b) dotted curve).

Opening angle distribution of two photons

The opening angle of two photons is calculated as follows:

$$\theta_{\gamma\gamma} = \cos(\sin\theta_{\gamma\text{high}} \times \sin\theta_{\gamma\text{low}} \times \cos\phi_{\gamma\text{high}} \times \cos\phi_{\gamma\text{low}}$$

$$+ \sin\theta_{\gamma\text{high}} \times \sin\theta_{\gamma\text{low}} \times \sin\phi_{\gamma\text{high}} \times \sin\phi_{\gamma\text{low}}$$

$$+ \cos\theta_{\gamma\text{high}} \times \cos\theta_{\gamma\text{low}}).$$

In Fig. 5.26, the solid curve shows the opening angle distribution of two separated clusters as the positions of two photons in the present data. Since in our experiment the available energy above the coherent threshold of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction was 10 MeV, the produced pions have small kinetic energy ($<9.4$ MeV). Therefore, due to the kinematical constraints, the two photons are produced with a large opening angle.

The dashed curve in Fig. 5.26 shows the opening angle distribution of the two photons obtained from the Monte-Carlo simulation. There appear two peak structures. The peak in the lower region of the opening angle distribution is the result of producing “split-offs” after clustering of the Plastic Ball. As was mentioned in Section 5.4.1, after improving the clustering of the Plastic Ball, the number of split-offs was reduced to less than 3%, however, the number of different combinations made by the split-offs (either split-offs with split-offs or split-offs with the main clusters) for the pair of clusters is about 20% of the total number of combinations. Therefore, there is still a visible increase of the yield in the region of small opening angles. The second peak shown by the dashed curve is mainly related to the combination of the main clusters and is located around $150^\circ$.

The simulated distribution shown by the dashed curve has different peak position and width compared to the measured distribution. This can be caused by the limited position resolution in the Plastic Ball scintillators for low energy deposition which has not yet been taken into account in the simulation. We call the simulated and observed resolutions
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

Figure 5.26: The solid line shows the opening angle distribution of two photons in the data. The dashed and dotted curves are the opening angle distribution from the Monte-Carlo simulation using the ideal and the real resolution of the angle determination for the Plastic Ball, respectively. The simulated results are down scaled. For all the results, the improved clustering method for the Plastic Ball has been used.

Figure 5.27: Panel (a) shows the difference of the incident and the reconstructed polar angles and panel (b) indicates the difference of the incident and the reconstructed azimuthal angles for a single photon produced in the Monte-Carlo simulation. The solid and dashed curves are the results when using the ideal and the real resolutions of the angle determination, respectively.
of the angle determination the “ideal resolution” and “real resolution”, respectively. It should be noted that Fig. 5.21 shows the ideal resolution. To determine and apply the real resolution in the simulation, single photons with $E_\gamma=69$ MeV were analysed in the Monte-Carlo simulation. The ideal resolutions of the angle determination for the polar and azimuthal angles of the single photons are $5.6^\circ$ and $6.1^\circ$ ($\sigma$), respectively (Fig. 5.27-solid curve). Two different factors for the photon polar and azimuthal angles (which were reconstructed according to the Plastic Ball module angles) were calculated by the same method that was explained in Section 5.3.2, to match the angular resolution of the detectors for the simulated photons. After applying those factors (the smearing effects), resolutions of the angle determination for the polar and azimuthal angles were changed to $26.8^\circ$ and $27.2^\circ$ ($\sigma$), respectively. Figure 5.27 shows the simulated difference of the incident and the reconstructed polar (a) and azimuthal (b) angles for single photons. The solid and dashed curves are the results considering the ideal and the real resolution of the angle determination, respectively. Using the real resolutions of the angle determination in the simulation, the measured distribution of the opening angle are reproduced (Fig. 5.26-dotted curve). For further analysis, only the pairs of clusters with opening angle larger than $94^\circ$ have been selected.

5.4.5 Energy reconstruction of two decay photons

For the reconstruction of the pion invariant mass the measured photon energies are used. In this section, we verify that the reconstructed energy distribution can be understood on basis of phase-space Monte-Carlo simulations. According to the Monte-Carlo simulations, photons from the $\pi^0$ decay moving in the backward direction of the setup deposit part of their energy in the Inner Shell modules. However, due to the low efficiency of the Plastic Ball for photon detection (see Section 5.6), backward photons do not deposit all the rest of their energy into the Plastic Ball. Furthermore, in case of the forward photons, the energy deposition is only a fraction of their original energy. In order to reconstruct the original photon energies from the deposited energy, a Monte-Carlo simulation of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction has been performed. The same normalisation factor which was obtained from the simulation was used to reconstruct the measured photon energies. The reconstructed energy of the photon with the higher deposition in the Plastic Ball module cluster or in the Inner Shell module plus the Plastic Ball module cluster behind is shown in Fig. 5.28. The solid and dashed curves show the result of the data and the simulation, respectively, summed over all the modules. The simulated yield is downscaled. There is a good agreement in the peak position, shape of the distribution and the covered region of the distribution between the measured and the simulated results.

5.5 Data contamination and background events

As was explained in Section 5.4.4, during the experiment the $^6$Li target was contaminated by about $17\%\pm3\%$ Oxygen due to oxidation. Therefore one source of contamination can be the interaction of $^4$He with $^{16}$O. Using the Monte-Carlo simulation, the reaction of $^{16}$O($^4$He,$\pi^0$)$^{10}$B+$X$ was studied. “$X$” denotes the third particle that must be produced in this reaction. Figure 5.29 shows the momentum distribution of the produced $^{10}$B without taking the acceptance of the Big-Bite Spectrometer into account. The solid
Figure 5.28: The reconstructed energy spectrum of photons with the higher energy deposition in the Plastic Ball module cluster or in the Inner Shell module plus the Plastic Ball module cluster behind. The solid and dashed curves are the measured and the simulated results, respectively.

Figure 5.29: The momentum distribution of the produced $^{10}$B. The solid and dashed curves represent the momentum distribution of $^{10}$B resulting from the $^{16}$O($^4$He,$\pi^0$)$^{10}$B+X reaction when 20% and 10% of the target are $^{16}$O, respectively. The histogram indicated by the solid curve is down scaled by a factor 2. The dotted curve is the result of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction on a pure $^6$Li target.
and dashed curves are presenting the momentum distribution of $^{10}$B resulting from the $^{16}$O($^4$He,$\pi^0$)$^{10}$B+$X$ reaction when 20\% and 10\% of the target is $^{16}$O, respectively. In this study, the nominal thickness of the target is the same as the used $^6$Li target during the experiment (2 mg/cm$^2$). The dotted line is the result of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction on a pure $^6$Li target. By applying the acceptance of the Big-Bite Spectrometer, it is shown that independent of the $^{16}$O percentage in the target, only less than 0.12\% of the produced $^{10}$B from the $^{16}$O($^4$He,$\pi^0$)$^{10}$B+$X$ reaction can enter the BBS window, but the acceptance of the Heavy Ion detector for those events is very small and negligible. Therefore the corresponding events were not registered during the experiment.

A large fraction of the data includes other reaction channels. Using the Heavy Ion detector, in addition to $^{10}$B some particles from the other reaction channels have been measured. Different peaks in the Heavy Ion pulse shape spectrum (Fig. 5.4) represent different particles. The width of every peak is related to the detector response, kinematical broadening and straggling in the target. Events outside the main peaks are related to random coincidences between Plastic Ball and Heavy Ion detectors and can be rejected by suitable windows on the ion Time Of Flight as shown in Fig. 5.22. Figure 5.30-(a) shows the projection of the line including $^2$H and $^1$H peaks in the Heavy Ion pulse shape spectrum on the x-axis. Figure 5.30-(b) displays the projection of the line including $^{10}$B and $^3$He. In the Heavy Ion pulse shape spectrum (Fig. 5.4), the minimum ratio of the peak to background is about 4 and this occurs for the lowest deposited energies. With increasing energy, this ratio will increase to more than 20. In case of the $^{10}$B peak, the peak to background ratio is higher than 50, which makes the selection rather clean.
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

5.6 Acceptance corrections

Due to the systematic limitations of the experimental setup, not all the produced events can be detected. The acceptance provides us an account of events that will not be detected either due to the limited phase-space coverage of the apparatus, that is e.g. if the detector is not placed or functioning properly (geometrical acceptance) or due to the inefficiency. The efficiency of the measurement gives us an estimate how often a reaction will be registered with our experimental apparatus, that is e.g. when an event is registered in one part of the setup (like the Inner Shell) and not in the other part (like the Plastic Ball) or when a particle is not detected because of some detection threshold. In order to obtain reliable cross section results, correction factors for the acceptance have to be applied. The event generator GENBOD \[52\] was applied to produce kinematically allowed events and study the acceptance of the detectors for the reaction of $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$. The details of the detector setup were simulated by the detector-simulation program GEANT3 \[53\]. As was mentioned in Section 3.5, by combining the event generator and the detector-simulation program, simulated data were obtained and analysed with similar cuts and thresholds as experimental data.

The Big-Bite Spectrometer and the Heavy Ion detector

Figure 3.3-(a) shows the scatter plot of the simulated vertical and horizontal angle of incidence of $^{10}\text{B}$ in the Big-Bite Spectrometer. The vertical and the horizontal angles are defined as the projection of the angle in the y-z plane and the x-z plane, respectively, where the positive z-direction is the beam direction. The dashed line is the window of the BBS entrance. The vertical and the horizontal acceptances of the Big-Bite Spectrometer for $^{10}\text{B}$, defined as the ratio of the accepted $^{10}\text{B}$ particles at the BBS window over the number of produced $^{10}\text{B}$ particles, are 100% and 86%, respectively.

To relate the measured quantities in the Heavy Ion detector to parameters of the reactions at the target, one has to transport the particles through the spectrometer and this procedure is called ray-tracing. To connect the parameters at the target, referred as the subscript “t”, and at the Heavy Ion detector, subscripted by “h”, one uses a transfer matrix whose elements are the coefficients of the Taylor expansion to describe deviations from the central reference trajectory

$$\alpha_t = \sum_{\mu,\nu,\lambda,\eta} (\alpha | x^\mu \theta^\nu y^\lambda \phi^\eta) x_h^\mu \theta_h^\nu y_h^\lambda \phi_h^\eta,$$

(5.22)

The target variables are denoted by $\alpha$ and can be $\theta$, $\phi$ or $\delta$. The coordinates $x$ and $\theta$ are the distance and angle in the horizontal direction and similarly $y$ and $\phi$ for the vertical direction. $\delta$ is the deviation of the rigidity $B\rho$ of the particle with respect to a particle travelling along the central ray of the spectrometer. The ray-tracing method was used to trace $^{10}\text{B}$ inside the Big-Bite Spectrometer and to calculate the Heavy Ion acceptance (see Fig. 5.11). As origin of $^{10}\text{B}$ in the target a point slightly off centre was chosen, displaced by $x_0$ horizontally and $y_0$ vertically. This takes care of the finite size of the beam spot on the target. Figure 5.31 shows the position of $^{10}\text{B}$ in the Heavy Ion detector for different energies and different vertical angles $\phi = -4.02^\circ$, $-1.9^\circ$, $0^\circ$, $1.9^\circ$ and $4.02^\circ$ at the entrance of the Big-Bite Spectrometer. Figure 5.31-(a) shows the results when $^{10}\text{B}$ originates from a
5.6. Acceptance corrections

10^B moving in upward direction at the BBS entrance

(a) 4.02°

(b) -1.9°

10^B moving in downward direction at the BBS entrance

(a) -4.02°

(b) -1.9°

Figure 5.31: (a) ((b)): The position of a 10^B ion at the Heavy Ion detector if 10^B originates from a position in the target at x_0=0.1 cm and y_0=0.5 cm (y_0=-0.5 cm) and moves in the upward (downward) direction when entering the Big-Bite Spectrometer. Different symbols show different energies of 10^B and different points with the same symbol are representing different chosen rays for tracing. The vertical and horizontal axes correspond to the vertical and horizontal positions of 10^B ions with respect to the beam direction. Numbers 4.02°, 1.9° and 0° (-4.02°, -1.9° and 0°) are the vertical angles of 10^B at the BBS entrance.
Figure 5.32: (a): The $^{10}$B horizontal position in the Heavy Ion detector array as a function of the $^{10}$B horizontal angle at the BBS entrance. In this plot, the $^{10}$B vertical angle at the BBS entrance is $2.01^\circ$. (b): The $^{10}$B horizontal position in the Heavy Ion detector array as a function of the $^{10}$B positive vertical angle at the BBS entrance. The $^{10}$B horizontal angle at the BBS entrance is $0.95^\circ$. (c): The $^{10}$B vertical position in the Heavy Ion detector array as a function of the $^{10}$B horizontal angle at the BBS entrance. The $^{10}$B vertical angle at the BBS entrance is $2.01^\circ$. (d): The $^{10}$B vertical position in the Heavy Ion detector array as a function of the $^{10}$B positive vertical angle at the BBS entrance. The $^{10}$B horizontal angle at the BBS entrance is $0.95^\circ$. The original position of $^{10}$B in the target has been chosen at $x_0=0.1$ cm and $y_0=0.5$ cm. The position in the target at $x_0=0.1$ cm and $y_0=0.5$ cm, while shown in panel (b) are the results when $x_0=0.1$ cm and $y_0=-0.5$ cm. In case $\phi=0^\circ$ and $\phi=\pm1.9^\circ$, the results associated with $\theta=0^\circ$, $\theta=\pm0.95^\circ$ and $\theta=\pm1.89^\circ$ are presented. In case $\phi=\pm4.02^\circ$, the results associated with $\theta=0^\circ$ and $\theta=\pm0.95^\circ$ are shown. Only $^{10}$B ions with energies larger than 99 MeV and the vertical angle $\phi \geq 4.02^\circ$ (70 mrad) or $\phi \leq -4.02^\circ$ at the BBS entrance can go out of
5.6. Acceptance corrections

the region where the Heavy Ion detector is located. Since according to the kinematical constraints it is not possible for the produced \( ^{10}\text{B} \) to have a vertical angle larger than \( 2.2^\circ \) at the BBS entrance (Fig. 3.3-(a)), all the produced \( ^{10}\text{B} \) particles that enter the Big-Bite Spectrometer can be detected by the Heavy Ion detector. Therefore the acceptance of the Heavy Ion detector array for the \( ^{10}\text{B} \) detection is 100%.

Figure 5.32 represents the location of \( ^{10}\text{B} \) particles in the Heavy Ion detector array when they have different horizontal and positive vertical angles at the BBS entrance. Different symbols correspond to different energies of \( ^{10}\text{B} \) at the BBS entrance. Arrows represent the boundaries of the Heavy Ion array. The lower horizontal boundary of the array (-19.5 cm) is not in the range of the upper plots. For a constant \( ^{10}\text{B} \) energy and by changing the horizontal angle of \( ^{10}\text{B} \) at the BBS entrance from the minimum (-1.9°) to the maximum (1.9°) horizontal acceptance of the BBS window, the horizontal position of \( ^{10}\text{B} \) ions in the Heavy Ion detector array will shift by almost 1.85 cm which is equivalent to 1.4 detector units of the Heavy Ion detector (Fig. 5.32-(a)). For a constant \( ^{10}\text{B} \) energy and a constant horizontal angle of \( ^{10}\text{B} \) at the BBS entrance, the horizontal position of \( ^{10}\text{B} \) ions in the Heavy Ion detector array is not dependent on the vertical angle at the BBS entrance (Fig. 5.32-(b)). This observation implies that the BBS resolution for the momentum determination is good.

For a constant \( ^{10}\text{B} \) energy at the BBS entrance, the vertical position of \( ^{10}\text{B} \) particles depends on the horizontal and the vertical angles at the BBS entrance (Fig. 5.32-(c) and -(d), respectively). As can be noticed from Fig. 5.32-(c), by changing the horizontal angle of \( ^{10}\text{B} \) at the BBS entrance, \( ^{10}\text{B} \) ions do not move out of the vertical boundaries of the Heavy Ion array. In addition, for \( E_{10}\text{B} \leq 90 \text{ MeV} \), \( ^{10}\text{B} \) particles with positive (negative) vertical angles in the BBS entrance hit the lower (higher) part of the Heavy Ion detector. If \( E_{10}\text{B} > 90 \text{ MeV} \), \( ^{10}\text{B} \) can be detected either in the lower or in the higher part of the Heavy Ion detector array (Fig. 5.32-(d)). Therefore, it was difficult to use only the vertical position of \( ^{10}\text{B} \) in the array in order to reconstruct the vertical \( ^{10}\text{B} \) angle in the BBS entrance. It should be mentioned, however, that for the analysis other measured observables together with the kinematical constraints were sufficient to reconstruct the \( ^{10}\text{B} \) angular distribution at the BBS entrance.

The Plastic Ball and its Inner Shell

The total acceptances of the Plastic Ball and the Inner Shell for the detection of at least one photon originating from \( \pi^0 \) decay are 44.1% and 56.2%, respectively, mainly due to the geometrical acceptances of the Plastic Ball and the Inner Shell.

By applying similar cuts and thresholds as used for the experimental data, the total acceptance of the Plastic Ball or the Plastic Ball plus Inner Shell for the pion detection including the reconstruction efficiency was obtained as follows:

\[
\text{acceptance} = \frac{\text{number of successfully reconstructed events, per bin (all cuts included)}}{\text{all thrown events, per bin}}
\]  

(5.23)

In addition, a reconstruction efficiency of 57% was obtained as follows:

\[
\frac{\text{number of successfully reconstructed events, per bin (all cuts included)}}{\text{all accepted events by the geometry, per bin}}
\]  

(5.24)
Chapter 5: Analysis of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ data

The overall acceptance of the Plastic Ball together with its Inner Shell for the $\pi^0$ detection was calculated and is 14.1%. In Fig. 5.33-(a) and -(b), the acceptance of the Plastic Ball together with its Inner Shell as function of $\Phi_{\pi^0_{\text{lab}}}$ and $\theta_{\pi^0_{\text{lab}}}$ (the pion azimuthal and polar angles, in the laboratory system), respectively, is depicted. $\Phi_{\pi^0_{\text{lab}}}$ has almost a flat acceptance which varies between 12% and 15%. The fitted line to the $\Phi_{\pi^0_{\text{lab}}}$ acceptance has a reduced $\chi^2=1$. The fluctuations correspond to the geometrical acceptance of the Plastic Ball detector. In our analysis, only two photons that both hit the Plastic Ball modules were selected. Due to the existence of the target equipments at around $\Phi_{\pi^0_{\text{lab}}}=90^\circ$, there is no Plastic Ball module in that region. Therefore, photons that move to that direction are not selected for the analysis, even though they deposit energy in the Inner Shell module which is in their track. As result, a drop of acceptance can be seen in Fig. 5.33-(a) in the region $0^\circ < \Phi_{\pi^0_{\text{lab}}} < 180^\circ$, where the upper hemisphere of the Plastic Ball is located (Fig. 4.6). For $\theta_{\pi^0_{\text{lab}}}$ we see a large variation in acceptance. Variations, especially the drop in acceptance for forward angles ($\theta_{\pi^0_{\text{lab}}} < 90^\circ$), are caused by solid angle effects due to the fact that the Plastic Ball coverage limits the acceptance for the photon polar angles $50^\circ < \theta_{\gamma_{\text{lab}}} < 160^\circ$. Another reason for the drop in the acceptance for the most forward angles is the escape of the electromagnetic showers from the edges of the scintillators of the Plastic Ball and the Inner Shell. In addition, due to the existence of the Inner Shell, the efficiency of photon detection in the backward part of the geometry is higher than in the forward part.

5.7 Data normalisation and the experimental cross section

The differential cross section has been determined as a function of $\theta_{\pi^0_{\text{cm}}}$. In order to obtain the absolute scale of the distribution, the number of produced pions must be normalised. In the normalisation procedure, experimental parameters like the target thickness, the number of incident $^4\text{He}$ particles and the acceptance of the experimental setup were taken.
5.7. Data normalisation and the experimental cross section

into account. The number of incident particles in the beam was calculated using the total charge collected by the Faraday Cup. In addition, the bin size of the measured values for the angles has been entered into the normalisation. The differential cross section was calculated as follows:

\[
\frac{d\sigma}{d\Omega} = \frac{N_{\pi^0}}{N_i \times N_t \times LT \times a_{PB+IS} \times a_{BBS} \times a_{HI} \times \epsilon_{rec} \times \Delta\Omega},
\]

(5.25)

where

- \(N_{\pi^0}\): number of detected neutral pions in every angle bin,
- \(N_i\): total number of \(^4\)He particle projectiles,
- \(N_t\): number of \(^6\)Li particles per cm\(^2\) in the target,
- \(LT\): electronic live-time,
- \(a_{PB+IS}\): acceptance of the Plastic Ball and the Inner Shell (backward acceptance) or the Plastic Ball only per angle (forward acceptance) for the pion detection: \(\frac{N_{\text{out}}}{N_{\pi^0}}\),
- \(a_{BBS}\): acceptance of the BBS for the \(^{10}\)B detection,
- \(a_{HI}\): acceptance of the Heavy Ion detector array for the \(^{10}\)B detection,
- \(\epsilon_{rec}\): efficiency of the pion reconstruction,
- \(\Delta\Omega\): bin size of the solid angle for a Plastic Ball module at a particular scattering angle.

The number of incoming \(^4\)He particles was calculated from the measurement of the total integrated charge in the Faraday Cup (FC). The relative accuracy of this method was estimated to be 8\%. The measurement of the integrated charge was corrected for the dead-time of the data acquisition electronics (49\%) resulting in \(1.9 \times 10^6\) nC which is equivalent to \(N_i = 6.05 \times 10^{15}\). Using the nominal thickness of the \(^6\)Li target (2 mg/cm\(^2\)), \(N_t\) was calculated as \(1.74 \times 10^{20}\) cm\(^{-2}\). In general a differential cross section is reported in b/sr. \(N_{\pi^0}\) in our experiment is 101. By normalisation, the experimentally measured cross section integrated over the whole acceptance of the detector setup can be obtained and is reported in the next chapter. A summary of all applied corrections to calculate the cross section is listed in Table 5.3.
Table 5.3: Summary of all applied corrections for detector acceptances and efficiencies together with the measured values which have been used to calculate the cross section.

| Correction                                | Value                  |
|-------------------------------------------|------------------------|
| Plastic Ball acceptance for detection of at least one photon from $\pi^0$ decay | 44.1%                  |
| Inner Shell acceptance for detection of at least one photon from $\pi^0$ decay | 56.2%                  |
| BBS acceptance for the $^{10}$B detection | 86%                    |
| Heavy Ion detector acceptance for the $^{10}$B detection | 100%                   |
| PB+IS acceptance for the $\pi^0$ detection | 14.1%                  |
| BBS+HI detector acceptance for the $^{10}$B detection | 86%                    |
| Reconstruction efficiency                | 57%                    |
| Electronics dead-time                    | 49%                    |
| Total number of $^4$He in the beam corrected for the dead-time | 6.05 $\cdot 10^{15}$ |
| Total accumulated charge by the Faraday Cup corrected for the dead-time | 1.9 $\cdot 10^6$ nC   |
| Nominal target thickness                 | 2 mg/cm$^2$            |
| Number of $^6$Li particles per cm$^2$ in the target | 1.74 $\cdot 10^{20}$ |
| Bin size of the polar angle              | 10°                    |
| Total number of measured $\pi^0$         | 101                    |
6. Results

6.1 Introduction

The main objective of this work is to measure the differential cross sections of the pionic fusion in a wide angular range for the $^6$Li($^4$He, $\pi^0$)$^{10}$B$^*$ reaction. In the previous chapter, the analysis procedure that led us to the final set of data was introduced. In this chapter the final results of this thesis are presented. In Sections 6.2-6.5, a few general remarks about the extraction of the $\pi^0$ signal from the data compared with the results obtained from the Monte-Carlo calculations will be given. The Monte-Carlo simulations are based on phase-space distributions and do not contain any specific reaction dynamics (Chapter 3). In Section 6.6, the physics inferred from the differential cross section of the $^6$Li($^4$He,$\pi^0$)$^{10}$B$^*$ reaction will be discussed.

6.2 Reconstructed $\pi^0$ energy

The $\pi^0$ energy can be reconstructed using the measured observables in the two different approaches. The first one is simply to add the energy of two photons which were measured by the ADCs during the experiment

$$E_{\pi^0} = E_{\gamma_{\text{low}}} + E_{\gamma_{\text{high}}},$$

(6.1)

where $E_{\gamma_{\text{low}}}$ and $E_{\gamma_{\text{high}}}$ are the energy of the photon with the lower and the higher energy, respectively. Since we have achieved a measurement in over-determined kinematics under the condition with the present experimental setup, we have another useful observable to calculate the $\pi^0$ energy. The module position of the Heavy Ion detector gives us the momentum (and therefore the energy) of the fusion product. According to Method 1 of the $\pi^0$ invariant mass reconstruction which was explained in Section 5.4.2, in the second approach the measured energy of $^{10}$B ions was used to reconstruct the total $\pi^0$ energy

$$E_{\pi^0} = E_{\text{total}} - E_{^{10}\text{B}}.$$  

(6.2)

$E_{\text{total}} = T_{\text{beam}} + M_{^4\text{He}} + M_{^6\text{Li}}$ is the total available energy and $T_{\text{beam}} = 236.4$ MeV is the beam kinetic energy. The angle-integrated $\pi^0$ energy distribution is displayed in Fig. 6.1. The solid curve represents the measured result. The dashed and dotted curves are the results of the phase-space simulation when the pure and the oxidised $^6$Li target, respectively, were used (see Sections 4.3 and 5.4.4). Using the pure $^6$Li target, the simulated result (see the dashed curve in Fig. 6.1) covers the same range of the energy as the result of the data, but is shifted to lower energies by about 2%. However, after including the oxidised $^6$Li target into the simulation, the simulated $\pi^0$ energy is in good agreement with the measured one (see the dotted curve in Fig. 6.1). This effect is caused by the different densities of the pure and the oxidised target. It should be noted that in this reaction, we observe the clear $\pi^0$ energy spectrum indicating that pions are emitted with the total
energy higher than 140 MeV. This corresponds to more than 50% of the total energy available in the centre-of-mass frame and implies that more than half of the available centre-of-mass energy is transferred to produce a pion. The sharp cut off in the lowest \( \pi^0 \) energy range is due to the results of the kinematical constraints.

### 6.3 Two photon energies

For the energy of the photon with a high energy deposition, the values measured by the ADCs were used. According to Method 1 of the \( \pi^0 \) invariant mass reconstruction, the \( \pi^0 \) energy was used to reconstruct the energy of the photon with a low energy deposition

\[
E_{\gamma\text{low}} = E_{\pi^0} - E_{\gamma\text{high}}.
\]  

Figure 6.2-(a) shows the simulated results of the initial and the reconstructed photon energies. The initial photon energy is the simulated energy of the photon in the laboratory system right after the \( \pi^0 \) decay in the \( ^6\text{Li} \) target. The dashed-dotted and dotted curves are the initial energies of the photon with high and low energies, respectively. The solid and dashed curves are the reconstructed energies of the photon with high and low energies, re-
Figure 6.2: (a): The dashed-dotted and dotted curves are the energy spectra of initial photons with high and low energies, respectively. The solid and dashed curves are the reconstructed distributions of photons with high and low energies, respectively. (b): The measured photon energies for the photon with the high (solid curve) and the low (dashed curve) energy. Labels “initial” and “reconstructed” indicate the initial and reconstructed energy distributions.
spectively. Due to the incomplete shower collection in the Plastic Ball which was explained in Section 5.3.2, the width of the energy peak for the reconstructed photon energies is not the same as the original width. In Fig. 6.2-(b), the solid and dashed curves represent the distributions of reconstructed measured photon energies for the photon with the high and the low energy, respectively. The obvious cutoff at energies about 75 MeV is caused by the effect of the $E_{\pi^0}$ cutoff (which can be seen in Fig. 6.1 as well) in Eq. 6.3 ($E_{\gamma_{\text{low}}} > 0$, therefore, $E_{\gamma} > E_{\gamma_{\text{high}}}$). A good agreement between the simulated and measured results in the widths as well as the peak positions has been observed.

### 6.4 Reconstructed $\pi^0$ mass

The identification of neutral pions is usually based on the invariant mass analysis of two-photon events. The observables needed for the identification of $\pi^0$ through the invariant mass analysis are two photon energies $E_{\gamma_{\text{high}}}$ and $E_{\gamma_{\text{low}}}$ and the opening angle $\theta_{\gamma\gamma}$ between them:

$$M_{\gamma\gamma} = \sqrt{2E_{\gamma_{\text{high}}}E_{\gamma_{\text{low}}}(1 - \cos(\theta_{\gamma\gamma}))}. \quad (6.4)$$

Figure 6.3 shows the two-photon invariant mass distribution. The solid and dashed curves are the results of the measurement and the Monte-Carlo simulation, respectively.
6.4. Reconstructed $\pi^0$ mass

Figure 6.4: (a): The polar angle distribution of the produced $\pi^0$ in the laboratory system. (b): The polar angle distribution of the produced $\pi^0$ in the centre-of-mass system. (c): The momentum distribution of the produced $\pi^0$ in the laboratory system. (d): The momentum distribution of the produced $\pi^0$ in the centre-of-mass system. The solid and dashed curves represent the measured and the simulated results, respectively.

A very good agreement is observed, giving a high confidence in the data reconstruction. The tail structure in the low region of the invariant mass distribution originates from the detector energy resolution. In fact, events in the tail are associated with the situation that $E_{\gamma\text{high}} > 110$ MeV and $E_{\gamma\text{low}} < 40$ MeV. The peak resolution of the invariant mass is directly due to the results of the experimental angular and energy resolutions. In addition, the sharp cut around 150 MeV is caused by the kinematical limits imposed by the energy.
conservation.

6.5 Pion angular and momentum distributions

The polar angle distributions of the produced \( \pi^0 \) in the laboratory and the centre-of-mass system are shown in Fig. 6.4-(a) and -(b), respectively. Figure 6.4-(c) and -(d) represents the momentum distributions of the produced \( \pi^0 \) in the laboratory and the centre-of-mass system, respectively. The solid and dashed curves are the measured and the simulated results, respectively. As can be noticed, the measured angular and momentum distributions are well reproduced by the simulation. The original \( \pi^0 \) angular and momentum distributions are shown in Figs. 3.6-(b) and 3.5-(b), respectively. It should be noted that the original \( \pi^0 \) angular and momentum distributions display double peak structures when the BBS acceptance is introduced into the simulation (hatched area in Figs. 3.6-(b) and 3.5-(b)). However, since no Plastic Ball module is positioned in the region \( 0^\circ \leq \theta \leq 50^\circ \), by applying the full geometrical acceptances of the detector setup most of the highest energetic pions moving into the forward direction are not accepted. The momentum of those pions is associated with the momentum in the region of the second peak in Fig. 3.5-(b).

As result only one peak out of every double-peak structure from Figs. 3.6-(b) and 3.5-(b) will remain.

6.6 Differential cross section

Section 5.7 and Table 5.3 explain how the cross section of the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction is corrected for the detector efficiencies, target thickness and the beam intensity. Figure 6.5 shows the differential cross sections as a function of \( \pi^0 \) polar angle in the centre-of-mass system integrated over the covered azimuthal angle. Full circles are the results of the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) measurement at KVI at the incident energy of \( T_{\text{beam/nucleon}}=59.1 \text{ MeV} \). Empty circles are the preliminary results of the KVI measurement for the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) reaction at the incident energy of \( T_{\text{beam/nucleon}}=85.3 \text{ MeV} \). Triangles are the observed data from the ORSAY experiment [14] for the \( ^4\text{He}(^3\text{He},\pi^+)^7\text{Li} \) reaction at the incident energy of \( T_{\text{beam/nucleon}}=88.8 \text{ MeV} \). The curves represent the results of the theoretical predictions [8]. The thin black curve is the result obtained using the cluster model wave function of \(^7\text{Be}\) in the \(^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) reaction. The thick grey and black curves are the results calculated with the use of, respectively, the cluster model and the shell model wave functions of \(^7\text{Li}\) from the \(^4\text{He}(^3\text{He},\pi^+)\)\(^7\text{Li} \) reaction. The results shown by the solid curves were obtained when the adopted strength of the imaginary part of the optical potential in the entrance channel, \( W_D \), is \(-25 \text{ MeV} \) (see Chapter 2), while the dashed curves are the results when \( W_D=0 \text{ MeV} \). The theoretical predictions have been given for the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) and \( ^4\text{He}(^3\text{He},\pi^+)\)\(^7\text{Li} \) cross sections at \( T_{\text{beam/nucleon}}=88.8 \text{ MeV} \). Up to now, there is no theoretical calculation available for the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction. Calculations for the other reactions are shown to guide the eyes and to give an indication how and where the results should be expected. Only the statistical uncertainties of the measured results are displayed in Fig. 6.5. The systematic error includes uncertainties in the measurement of the beam current (8%), the live time of the data acquisition electronics (12%), and the determination of the detector acceptances (2%), summing up to \( \pm 15\% \).
6.6. Differential cross section

Figure 6.5: Angular distribution of the cross sections for the various pionic fusion reactions at subthreshold energies. The full and empty circles are the results of the KVI experiments for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions at the incident energies of $T_{\text{beam}}/\text{nucleon}=59.1$ and 85.3 MeV, respectively. The triangles are the observed result from the ORSAY experiment for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction at the incident energy of $T_{\text{beam}}/\text{nucleon}=88.8$ MeV. The thin black curve is the calculated result obtained using the cluster model wave functions for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction. The thick grey and black curves are the calculated results obtained with the use of, respectively, the cluster model and the shell model wave functions for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction. All the solid curves were obtained with the adopted strength of the imaginary part of the optical potential in the entrance channel $W_D=-25$ MeV, while the dashed curves are the results with $W_D=0$ MeV. The theoretical predictions for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ cross sections have been performed at $T_{\text{beam}}/\text{nucleon}=88.8$ MeV.
In the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experiment, with an angular binning of $10^\circ$ in the centre-of-mass system, we have not measured any pion at $\theta_{c.m.}=10^\circ$, $30^\circ$, $50^\circ$ and $170^\circ$. Moreover, at $\theta_{c.m.}=20^\circ$ and $40^\circ$ only one pion has been measured. Therefore, the statistical uncertainties are of the same size as the cross sections at these two angles. The Plastic Ball covers the laboratory polar angles between $50^\circ$ and $160^\circ$ and since the $^{10}\text{B}$ mass is about 67 times bigger than the $\pi^0$ mass, these angles are converted to almost the same angles in the centre-of-mass frame. This experimental constraint is the reason why we did not measure pions in the most forward and backward directions.

First of all it should be noted that the general behaviour of the measured differential cross sections of both reactions, $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$, is as predicted for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reactions by taking strong clustering correlations into account [8]. The measured angular distribution of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction in particular in the backward direction is well reproduced by using the cluster model. It is impossible for us to draw the same conclusion for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction since there is no measurement in the backward direction. It should be noted that the incident wave in the entrance channel is attenuated due to absorption by the local imaginary potential $W_D$. In case of the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction, it was found that in the cluster model the local imaginary potential plays a significant role at the most backward angles (see the grey solid and dashed curves). Therefore, employing the correct imaginary potential may improve the calculation for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction.

There is a remarkable disagreement between the cross sections for the two reactions $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$, although the target, the projectile and the beam kinetic energies are the same. It implies that the total pionic fusion cross section is not only defined by the available energy and the number of available nucleons, but also the isospin affects the strength of the pionic fusion cross section. The ratio of a factor of 2 between the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ cross sections is explained by charge symmetry.

For a phenomenological analysis of the angular distributions, the following expression for the differential cross section has been assumed:

$$
\frac{d\sigma}{d\Omega}_{c.m.} = \sum_{n=0}^{i} a_n P_n(\cos\theta_{c.m.}),
$$

where $P_n(\cos\theta_{c.m.})$ are the Legendre Polynomials and $\theta_{c.m.}$ is the centre-of-mass angle of $\pi^0$. We call $a_n$ the Legendre coefficients. Employing Eq. 6.5, we fitted the calculated results of the clustering model for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction (the solid curve in Fig. 6.5). There are two reasons for employing Eq. 6.5 in the fitting. The first aim was to study which different contributions of the Legendre polynomials are responsible to reproduce the asymmetric behaviour of the angular distribution and produce the same shape of the differential cross section as the prediction of the cluster model. Furthermore, it would be interesting to explore which component of the Legendre Polynomial expansion plays the most important role. By comparing the fitted results of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation and the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ measurement, one may compare the structure of the angular distributions, which are influenced by the clustering correlations, and draw conclusions on the importance of clustering correlations in case of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. Since the measured differential cross sections at $\theta_{c.m.}=20^\circ$ and $40^\circ$ are associated with only
one measured pion and the statistical uncertainties are too large, those points may not determine the shape of the cross section at forward angles in a confident way. For comparison of the sensitivity, we decided to exclude those two points from the polynomial fit and extrapolate the fit in order to cover the full angular range. Extrapolation is needed to calculate the total cross section. Thus the second aim of fitting the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation was to understand how the cross section data at forward angles influence the obtained fit parameters and the total cross section.

### Table 6.1: Legendre coefficients obtained from the fits to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculated results based on the clustering correlations. The fitted curves are shown in Fig. 6.6-(a). The fits were performed in the full $\pi^0$ angular range. $i$ indicates the order of the Legendre polynomials which have been fitted to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation. The $\chi^2$ is normalised to the number of degrees of freedom.

| $i$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $\chi^2$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| 3   | 3.78  | 1.77  | 4.42  | 2.87  |       |       |       |       |       |       | 0.65    |
| 4   | 3.77  | 1.77  | 4.34  | 2.86  | 0.18  |       |       |       |       |       | 0.92    |
| 5   | 3.77  | 1.69  | 4.37  | 2.44  | 0.09  | 0.99  |       |       |       |       | 0.21    |
| 6   | 3.76  | 1.69  | 4.33  | 2.44  | -0.05 | 0.96  | 0.35  |       |       |       | 0.25    |
| 7   | 3.77  | 1.69  | 4.33  | 2.43  | -0.05 | 0.89  | 0.34  | 0.16  |       |       | 0.24    |
| 8   | 3.77  | 1.69  | 4.33  | 2.43  | -0.03 | 0.89  | 0.39  | 0.17  | -0.12 |       | 0.36    |
| 9   | 3.77  | 1.69  | 4.33  | 2.42  | -0.03 | 0.86  | 0.39  | 0.08  | -0.14 | 0.2    | 0.75    |

### Table 6.2: Legendre coefficients obtained from the fits to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculated results based on the clustering correlations. The fitted curves are shown in Fig. 6.6-(b). The fits were performed in the $-1 \leq \cos\theta_{\pi^0}^{c.m.} \leq 0.5$ angular range. $i$ indicates the order of the Legendre polynomials which have been fitted to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation. The $\chi^2$ is normalised to the number of degrees of freedom.

| $i$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $\chi^2$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| 3   | 4.28  | 2.98  | 5.58  | 3.32  |       |       |       |       |       |       | 1.46    |
| 4   | 3.02  | -0.27 | 1.8   | 0.05  | -1.77 |       |       |       |       |       | 0.29    |
| 5   | 2.93  | -0.53 | 1.47  | -0.23 | -1.96 | -0.08 |       |       |       |       | 0.41    |
| 6   | 1.25  | -4.99 | -4.32 | -5.71 | -5.95 | -2.32 | -0.79 |       |       |       | 2.54    |
| 7   | 8.71  | 15.09 | 22.5  | 22.86 | 15.07 | 10.9 | 5.58  | 1.9   |       |       | 2.08    |
| 8   | 16.44 | 36.11 | 51.18 | 50.37 | 39.77 | 27.97 | 15.11 | 5.97  | 1.07  |       | 17.33   |
| 9   | 26.44 | 63.54 | 89.30 | 90.77 | 75.15 | 54.06 | 31.26 | 14.1  | 4.18  | 0.72  | 73.84   |

Figure 6.6 shows the results obtained by fitting Eq. 6.5 to the model result for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction (solid curve), when assuming $i$ in Eq. 6.5 is equal to 3, 4,..., 9. In Fig. 6.6, the differential cross section as a function of $\cos\theta_{\pi^0}^{c.m.}$ is displayed. It should be noted that the range of the vertical logarithmic scale of the graph is different from that used in Fig. 6.5. The results shown in Fig. 6.6-(a) have been obtained when the full $\pi^0$ angular range of the calculated cross section has been fitted. The results shown in Fig. 6.6-(b) correspond to the fits in the restricted angular range of $-1 \leq \cos\theta_{\pi^0}^{c.m.} \leq 0.5$. An average value of the experimental (statistical) errors of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction
Chapter 6: Results

Figure 6.6: The results obtained by fitting the Legendre polynomials from Eq. 6.5 to the angular distribution of the cross sections for the $^4\text{He}^3(\text{He},\pi^n)\text{Be}$ reaction (the solid curve) calculated in the cluster model, when assuming $i$ in Eq. 6.5 is 3, 4, ..., 9. (a) and (b) correspond to the fit results for the full and restricted $\pi^n$ angular range, $-1 \leq \cos\theta_{\text{c.m.}} \leq 1$ and $-1 \leq \cos\theta_{\text{c.m.}} \leq 0.5$, respectively. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits are listed in Tables 6.1 and 6.2, for the fits to the full and restricted $\pi^n$ angular range, respectively.
(the errors for the empty circles in Fig. 6.5) was employed to calculate the $\chi^2$ per degree of freedom of the fits. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits are listed in Tables 6.1 and 6.2, for the fits to the full ($-1 \leq \cos \theta_{c.m.}^{\pi^0} \leq 1$) and restricted ($-1 \leq \cos \theta_{c.m.}^{\pi^0} \leq 0.5$) $\pi^0$ angular range, respectively.

Table 6.3: Legendre coefficients from the fits to the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experimental data. The fitted curves are shown in Fig. 6.7. $i$ indicates the order of the Legendre polynomials which have been fitted to the experimental data.

| $i$=3 | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $\chi^2$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $i$=4 | 1.17  | 1.46  | 1.60  | 1.59  | 0.32  |       |       |       |       |       | 0.3    |
| $i$=5 | 0.25  | -0.98 | -1.47 | -1.13 | -1.46 | -0.82 |       |       |       |       | 0.36   |
| $i$=6 | 0.60  | -0.05 | -0.25 | 0.03  | -0.6  | -0.35 | 0.18  |       |       |       | 0.45   |
| $i$=7 | 3.78  | 8.53  | 11.21 | 11.47 | 8.44  | 5.4   | 2.92  | 0.9   |       |       | 0.6    |
| $i$=8 | -8.7  | -25.3 | -35.9 | -31.2 | -21.9 | -12.3 | -5.4  | -1.8  |       |       | 0.9    |
| $i$=9 | -34.1 | -94.9 | -131.4| -137.6| -119.4| -86.1 | -51.3 | -24.7 | -8.8  | -1.7  | 1.8    |

By changing the $\pi^0$ angular range in the fit from the full to the restricted range, the results of the polynomial fit to the $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ calculation change more drastically for higher-order polynomials. The lowest order polynomial with a $\chi^2$ close to one is found for $i=3$ in both the full and the restricted angular range. For higher-order polynomials, the $\chi^2$ increases far above one in the restricted angular range, which gains much less confidence in the obtained parameters. It is concluded that the most stable fit corresponds to the fit up to the third polynomial order ($i=3$). In Fig. 6.6, the results of the polynomial fit up to the third order are shown by the dashed curves. This fit is not only the most stable fit but also the best lowest-order polynomial fit to the clustering calculation. When the $\pi^0$ angular range changes, the rest of the fits change very much in the forward direction.

Figure 6.7 shows the same procedure for the experimental $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ results. In addition, for reasons of comparison the results produced for the $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ reaction by the clustering calculations (the solid curve), the two first polynomial fits ($i=3,4$) to the full angular range of that calculation (the upper dashed and dashed-dotted curves, respectively) and the measured $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ results (empty circles) are displayed in Fig. 6.7. The last two experimental points with large error bars were excluded in the fit. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits to the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experimental results are listed in Table 6.3.

It is noted that the two polynomial fits with $i=3$ and 4 follow the same trend as the two fits to the theory. The fit with $i=3$ has a good overlap with all the data points. We conclude that the fit with $i=3$ is the best fit to explain the $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ experimental results in the $-1 \leq \cos \theta_{c.m.}^{\pi^0} \leq 0.5$ range. The result of the fit with $i=3$ is shown as the dashed-dotted curve in Fig. 6.8. In case of the $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ reaction, the contribution of $P_2(\cos \theta_{c.m.}^{\pi^0})$ is the largest to reproduce the theoretical curve as well as the experimental result. In case of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction, the $P_3(\cos \theta_{c.m.}^{\pi^0})$ contribution is the largest. However, still the $P_2(\cos \theta_{c.m.}^{\pi^0})$ and $P_3(\cos \theta_{c.m.}^{\pi^0})$ contributions are comparable.

Figure 6.8 shows the final results of the Legendre polynomial fits with $i=3$. The dashed-dotted and dashed curves are the results of the polynomial fit with Eq. 6.5 to the
Chapter 6: Results

Figure 6.7: The results of fitting the Legendre polynomials to the measured angular distribution of the $^6$Li($^4$He,$\pi^0$)$^{10}$B* cross sections (full circles), when assuming $i$ in Eq. 6.5 is 3, 4, ..., 9. The fitting procedure has been applied to part of the $\pi^0$ angular range $-1 \leq \cos(\theta_{c.m.}) \leq 0.5$. The solid curve, upper dashed and upper dashed-dotted curves are the $^4$He($^3$He,$\pi^0$)$^7$Be results predicted by the clustering calculations, the polynomial fit with $i=3$ and the polynomial fit with $i=4$ to that calculation, respectively. The empty circles are the $^4$He($^3$He,$\pi^0$)$^7$Be experimental results. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits to the $^6$Li($^4$He,$\pi^0$)$^{10}$B* experimental results are listed in Table 6.3.
6.6. Differential cross section

\begin{equation}
\frac{d\sigma}{d\Omega} = 4.876 P_0 + 3.280 P_1 + 5.013 P_2 + 0.193 P_3, \chi^2 = 6.95
\end{equation}

\begin{equation}
\frac{d\sigma}{d\Omega} = 3.775 P_0 + 1.766 P_1 + 4.415 P_2 + 2.869 P_3, \chi^2 = 0.65
\end{equation}

\begin{equation}
\frac{d\sigma}{d\Omega} = 0.942 P_0 + 0.854 P_1 + 0.899 P_2 + 1.020 P_3, \chi^2 = 0.26
\end{equation}

**Figure 6.8:** Angular distribution of the pionic fusion reaction cross sections at subthreshold energies. The full and empty circles are the results of the KVI experiments for the \(^6\)Li\((^4\text{He},\pi^0)^{10}\text{B}^*\) and \(^4\text{He}(^3\text{He},\pi^0)^7\text{Be}\) reactions, respectively. The solid curve is the calculated result obtained using the cluster model wave function of \(^7\text{Be}\) for the \(^4\text{He}(^3\text{He},\pi^0)^7\text{Be}\) reaction, when the local imaginary potential \(W_D\) in the entrance channel is set to -25 MeV. The dashed-dotted and dashed curves are the results of the polynomial fit to the \(^4\text{He}(^3\text{He},\pi^0)^7\text{Be}\) experimental results and to the \(^4\text{He}(^3\text{He},\pi^0)^7\text{Be}\) calculated results of the cluster model, respectively. The dotted curve is the fitted result to the \(^6\)Li\((^4\text{He},\pi^0)^{10}\text{B}^*\) experimental data.
$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ experimental result and calculated result of the cluster model, respectively. The dotted curve is the resulting fit to the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experimental data. The fits to the experimental data were performed in the angular range of $-1 \leq \cos\theta_{c.m.}^0 \leq 0.5$, while the fit on the theoretical prediction of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction was performed in the angular range of $-1 \leq \cos\theta_{c.m.}^0 \leq 1$.

By integrating the fitted curves, the total cross sections for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions were obtained. The obtained Legendre coefficients ($a_n$) and the total cross sections are listed in Table 6.4. The variance in the fit parameters $a_n$ is the diagonal entry in the covariance matrix so that the errors that are shown in Table 6.4 are the square root of that number. It should be noted that the calculated total cross section of 47.4 nb for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction from the polynomial fit using the full angular range is in good agreement with 47.3 nb from the cluster model.

Table 6.4: Legendre coefficients and the total cross section obtained from the fitting of Eq. 6.5 to the present measured results and to the calculations of the cluster model for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction. The uncertainties are purely statistical. “pf” and “cc” indicate “polynomial fit” and “cluster calculation”, respectively.

| Reaction | $T_{\text{beam}}$ [MeV] | $a_0$ [nb/sr] | $a_1$ [nb/sr] | $a_2$ [nb/sr] | $a_3$ [nb/sr] | $\sigma_{\text{tot}}$ [nb] |
|----------|----------------|--------------|--------------|--------------|--------------|----------------|
| $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ | $T_{\text{beam}}=236.4$ KVI measurement | 0.94±0.38 | 0.85±1.11 | 0.90±0.27 | 1.02±0.32 | 11.8±1.2 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{\text{beam}}=256$ KVI measurement | 4.88±2.22 | 3.28±4.32 | 5.01±3.15 | 0.19±1.93 | 61.3±2.3 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{\text{beam}}=266.4$ Cluster calculation Restricted angular range | 4.28±0.32 | 2.98±0.76 | 5.58±2.16 | 3.32±2.36 | using pf 53.7 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{\text{beam}}=266.4$ Cluster calculation Full angular range | 3.78±0.38 | 1.77±1.11 | 4.42±0.27 | 2.87±0.32 | using pf 47.4 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{\text{beam}}=266.4$ Cluster calculation | using cc 47.3 |

The total cross section of the neutral pion production for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions follows the same trend as was predicted by the Sudden Overlap model. The results have already been shown in Fig. 2.4 by cross and square, respectively. Using the polynomial fits, the extrapolated cross section of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction at $\theta_{c.m.}=0^\circ$ is 3.5 nb/sr and is lower than the predicted value by the cluster model [8] for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction. This confirms the decreasing trend of the cross section with increasing mass, which has been predicted by the cluster model. This model predicts...
values of 11.4 and 2.3 nb/sr for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ and the $^{16}\text{O}(^3\text{He},\pi^+)^{19}\text{F}$ reactions, respectively. For the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction, the extrapolated cross section at $\theta_{c.m.}=0^\circ$ is about 12 nb/sr (again using the polynomial fit). Using the polynomial fit, the total measured cross section of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction was found to be $61.3\pm2.3$ nb. This value is comparable with the predicted result of the cluster model. In this model assuming $W_D=-25$ MeV, the total cross section of 47.3 nb was obtained.

The ratios of the Legendre coefficients are presented in Table 6.5. An asymmetry $(a_1/a_0)$ is obtained over the angular range in all four fits. The ratio $a_2/a_0$ is close to unity in all cases except for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation when the polynomial fit is applied in the restricted angular range $-1 \leq \cos\theta_{c.m.} \leq 0.5$ (1.30). Furthermore, it should be noted that in case of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ measurement, $a_3/a_0$ is much smaller (0.04) compared to the other measurements or calculations.

In the next chapter, the consequences of the presented analysis will be discussed and conclusions will be drawn.

**Table 6.5:** Comparison of ratios of the Legendre coefficients from the fits to the calculation and to the experimental data.

| Reaction | $T_{beam}$ [MeV] | $a_1/a_0$ | $a_2/a_0$ | $a_3/a_0$ |
|----------|-----------------|----------|----------|----------|
| $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ | $T_{beam}=236.4$ KVI measurement | 0.91 | 0.95 | 1.08 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{beam}=256$ KVI measurement | 0.67 | 1.03 | 0.04 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{beam}=266.4$ Cluster calculation Restricted angular range | 0.70 | 1.30 | 0.78 |
| $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ | $T_{beam}=266.4$ Cluster calculation Full angular range | 0.47 | 1.17 | 0.76 |
7. Discussion and outlook

In this work, we have compared two data sets for the pionic fusion reactions with well-defined initial and final state configurations. For the first time, we have measured almost the full pion angular distribution from pionic fusion reactions with a projectile heavier than $^1$H. We would like to mention with special emphasis that pionic fusion is a very rare process presenting a tiny fraction of the total pion production cross section. The observation of rather complete angular distributions of the extremely small cross sections of about $10^{-8}$ barn is remarkable. Figure 7.1 compares the results from the present experiment with the previous pionic fusion experimental results (that we found in the literature) in which the pion angular distributions have been measured. The presented angular distributions are sorted by the projectile mass from panels (a) to (j). Different symbols in every panel of the figure represent either a different beam kinetic energy or a different observed final state of the fusion product. The two lowest panels represent the KVI data. A number of remarks can be made:

(i) The measured angular distribution of different pion production experiments is anisotropic. Furthermore, from the inclusive experiments of the $^{12}$C($^{12}$C,$\pi^0$)$X$ [$^{70}$] and $^{12}$C($^{12}$C,$\pi^+$)$X$ [$^{71}$, $^{72}$] reactions, it has been observed that the anisotropy is getting more pronounced with increasing pion kinetic energy.

In fact, pion production from nucleus-nucleus collisions requires more and more “co-operative” interactions among nucleons in target and projectile the closer one gets to the coherent threshold. It has been observed that by increasing the energy the cross section increases. The cross section of the $^1$H($^1$H,$\pi^+$)$^2$H reaction as the simplest pionic fusion reaction at $T_{\text{beam}}=560$ MeV is about 50 times larger than the one at $T_{\text{beam}}=295$ MeV. This is shown in Fig. 7.1-(a). The beam kinetic energies $T_{\text{beam}}=560$ and 295 MeV in the laboratory system are equivalent to available centre-of-mass energies ($T_{\text{CM}}-T_{\text{thr.CM}}$) of 123.1 and 3.5 MeV, respectively, in the centre-of-mass system. As is shown in Fig. 7.1-(c), the cross section of the $^3$He($^1$H,$\pi^+$)$^4$He reaction increases by almost a factor of 2 when the beam kinetic energy changes from 178 MeV ($T_{\text{CM}}-T_{\text{thr.CM}}=10.9$ MeV) to 198 MeV ($T_{\text{CM}}-T_{\text{thr.CM}}=25.6$ MeV). In addition, it is shown in Table 1.1 that the cross section of the $^3$He($^3$He,$\pi^+$)$^6$Li and $^4$He($^3$He,$\pi^+$)$^7$Li reactions in the most forward angles increases when the beam kinetic energy increases.

It has been observed, however, that the increasing trend of the cross section will change when the beam kinetic energy is high enough compared to the coherent threshold energy. As an example, we again refer to the $^1$H($^1$H,$\pi^+$)$^2$H reaction in Fig. 7.1-(a). By increasing the beam kinetic energy from 560 to 810 MeV (or the available centre-of-mass energy from 123.1 to 230.1 MeV), the total cross section decreases (the full squares). In case of the $^3$He($^1$H,$\pi^+$)$^4$He reaction at 800 MeV beam kinetic energy (equivalent to $T_{\text{CM}}-T_{\text{thr.CM}}=437.9$ MeV) the cross section is considerably lower than the one obtained at 198 MeV ($T_{\text{CM}}-T_{\text{thr.CM}}=25.6$ MeV). Furthermore, the observations of the $^4$He($^3$He,$\pi^+$)$^7$Li reaction indicate that the cross section decreases when the beam kinetic energy increases from 266 ($T_{\text{CM}}-T_{\text{thr.CM}}=12.8$ MeV) to 348 MeV ($T_{\text{CM}}-T_{\text{thr.CM}}=58.4$ MeV) (Fig. 7.1-(h)). It is depicted in Table 1.1 that for the $^6$Li($^2$H,$\pi^-$)$^8$B reaction by increasing the
Figure 7.1: The measured pion angular distributions in the pionic fusion experiments, which were found in the literature, in comparison with the results of the KVI experiments. The results of the KVI experiments are shown in the lowest row of the figure. \( T_{\text{beam}} \) indicates the beam kinetic energy (in MeV) and the numbers inside the bracket are the available energies in the centre-of-mass system (in MeV). The used references are [46, 63, 64, 65, 66, 67, 68, 13, 49, 69, 14].
beam kinetic energy from 300 to 600 MeV ($T_{CM} - T_{thr,CM}$=86.21 to 301.51 MeV) the cross section in the most forward angles decreases by a factor of 6. One can draw the conclusion that by increasing the available centre-of-mass energy to a value higher than about 300 MeV which is the required energy to form the $\Delta$-resonance, more exit channels are available and therefore, the chance of pion production by pion fusion decreases.

(ii) Due to the symmetric target-projectile combination in the $^{1}$H($^{1}$H,$\pi^{+}$)$^{2}$H reaction, the $\pi^{+}$ angular distribution is forward-backward symmetric. It has also been shown that in the inclusive reaction of $^{12}$C($^{12}$C,$\pi^{0}$)X [70] the distribution is forward-backward symmetric since the target and projectile are the same. In the existing experimental results of the pionic fusion reaction with the same projectile and target, $^{12}$C($^{12}$C,$^{24}$Mg)$\pi^{0}$, $^{12}$C($^{12}$C,$^{24}$Na)$\pi^{+}$ and $^{3}$He($^{3}$He,$\pi^{+}$)$^{6}$Li reactions, either only the total cross section or a small part of the angular distributions are reported. Therefore, no conclusion about the possible symmetric behaviour of the pion angular distribution can be drawn. It can be noticed from Fig. 7.1 that in pionic fusion reactions with different target and projectile, the pion angular distribution is forward-backward asymmetric in the nucleus-nucleus centre-of-mass system and is dominantly forward peaked. The asymmetric behaviour increases mainly with increasing mass difference of target and projectile. The graphs (b), (c) and (d) in Fig. 7.1 show the asymmetric and forward peaked angular distributions for the $^{3}$H($^{1}$H,$\pi^{0}$)$^{4}$He, $^{3}$He($^{1}$H,$\pi^{+}$)$^{4}$He and $^{4}$He($^{1}$H,$\pi^{+}$)$^{5}$He reactions, respectively.

One possible explanation for the forward peaked behaviour of the angular distribution depicted in graphs (a)-(d) in Fig. 7.1 is that pion production is dominated by $\Delta$-excitation in the proton which is moving to the forward direction. The empty and full markers in graph (c) are the results of the $^{3}$He($^{1}$H,$\pi^{+}$)$^{4}$He measurements from ORSAY [69] and Indiana-Texas groups [66, 67], respectively. The two groups have used almost the same beam kinetic energies (180 and 190 MeV by the ORSAY group and 178 and 198 MeV by the Indiana and Texas groups), however, their results are not consistent. The angular distributions obtained by the ORSAY group (empty markers) seem to be flat compared with the one measured by the other groups. Furthermore, the $\pi^{+}$ angular distribution from the two reactions, $^{91}$Zr($^{1}$H,$\pi^{+}$)$^{91}$Zr and $^{208}$Pb($^{1}$H,$\pi^{+}$)$^{208}$Pb depicted in graphs (e) and (f) are anisotropic but not forward peaked. Therefore, these two distributions are not consistent with our explanations about the forward peaked behaviour of the distribution.

By moving to the pionic fusion reactions with a projectile heavier than proton, the $\pi$ angular distribution is still forward peaked as can be seen in the measured results of the $^{3}$He($^{3}$He,$\pi^{+}$)$^{6}$Li reaction (graphs (g) in Fig. 7.1). The possible explanation would be similar to the one for the reactions with $^{1}$H projectiles. Here, the pion can be produced from the $^{3}$He which moves to the forward (backward) direction causing a forward (backward) peaked angular distribution. The $\pi^{+}$ and $\pi^{0}$ angular distributions from the $^{4}$He($^{3}$He,$\pi^{+}$)$^{7}$Li (graph (h)) and $^{4}$He($^{3}$He,$\pi^{0}$)$^{7}$Be (graph (i)) experiments are not consistent in the forward direction since the latter is not forward peaked. In case of the $^{4}$He($^{3}$He,$\pi^{0}$)$^{7}$Be experiment, where the target and projectile are almost the same, the angular distribution is nearly symmetric around 90° (Fig. 7.1-(i)). We observed that the forward-backward asymmetric behaviour is stronger in case of the $^{6}$Li($^{3}$He,$\pi^{0}$)$^{10}$B reaction compared to the $^{4}$He($^{3}$He,$\pi^{0}$)$^{7}$Be reaction, which corresponds to the larger projectile-target mass difference of the former reaction.

(iii) The distributions exhibit a minimum at around 90°. According to the Microscopic reaction model, which is described in Chapter 2, the angle of this minimum depends on
how much target and projectile contribute to the pion production process. The conclusion of this model for the $^3\text{He}(^3\text{He},\pi^+)^6\text{Li}$ reaction is depicted in Fig. 2.7. In case of target contribution, the minimum of the angular distribution is found at $\theta_{\text{c.m.}}^\pi < 90^\circ$. Following this description, $\pi^0$ in the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction may dominantly be emitted from the $^6\text{Li}$ target side.

(iv) By increasing the target plus projectile mass, in general the total cross section decreases. It should be mentioned that different experiments used different centre-of-mass energy above the coherent pionic fusion threshold. Therefore, it is difficult on basis of the different experimental results to formulate the behaviour of the cross section in terms of the available number of nucleons. From the theoretical point of view, the Sudden Overlap model and the model based on clustering correlations predict that the overall trend is a decreasing cross section with increasing mass. According to the Sudden Overlap model, by increasing the momentum of the participating nucleon in the reaction, the calculated total cross section increases sharply. For a given available energy, by increasing the number of available nucleons the amount of the available energy per nucleon decreases, therefore the cross section decreases (Fig. 2.4). Results of the model based on the clustering correlations show that for the shell model wave functions the cross section decreases gradually as the mass of the final nucleus becomes larger up to the end of each shell. The cross section decreases as well when the cluster model wave function is used, but the slope is much steeper than for the shell model results (full circles in Fig. 2.12).

We observe that, up to now, the model based on strong clustering correlations has been the most successful one in describing the magnitude of the cross section together with the behaviour of the angular distributions for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions. Furthermore, we concluded that the clustering correlations are important for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction (see Chapter 6). Here we will discuss the next possible steps of future theoretical work that is required to understand the pionic fusion reaction of $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ based on the model with clustering correlations.

Different experiments have shown that the $^6\text{Li}$ nucleus exhibits an $\alpha d$ cluster structure [73]. In addition, different studies show that in case of the $^6\text{Li}$ nucleus, the cluster model works better than the shell model [74, 75]. Therefore, in the simplest picture, the entrance and exit channels can be seen as an “$\alpha\alpha d$” cluster system. The $\alpha$-deuteron cluster structure of the ground state of $^6\text{Li}$ has been investigated [76] by solving the coupled Schrödinger equation for the relative motion between the clusters. The equation has been derived by using the Resonating Group Method and taking into account the $D$-state contribution in the deuteron cluster. This method can successfully describe experimental data like $\alpha\alpha$ scattering [77]. In this approach of modelling the many-nucleon system, an effective inter-nucleon force has been used. In the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction in addition to the NN interaction, the cluster subsystems interact with each other as a whole. The pion production operator will then depend much on the relative movement of the clusters as has been investigated by the models based on clustering correlations [8].

The proposed pionic fusion processes for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction are schematically shown in Fig. 7.2. The first possible process for pion production is that the pion is emitted from the deuteron which is transformed into a “quasi deuteron” with $I=1$ and $J=0$ as has been suggested for the modelling of the $^2\text{H}(^4\text{He},^6\text{Li})\pi^0$ reaction [12]. The pion may be re-scattered on one (or both) of the $\alpha$ particle(s) which retains its identity (Fig. 7.2-(a)). The second and the third possibilities are that the pion is emitted from one of the $\alpha$
Figure 7.2: Schematic representation of the possible reaction mechanisms for the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction, taking the strong clustering correlations into account. The dashed square represents the pion production operator \((H_{\text{int}})\) and "C" denotes the clustering correlations. The graph (a) ((b)) represents pion production from the target emission when the deuteron (α) particle as a target sub-cluster produces the pion. After the pion is emitted, it can be re-scattered by the projectile or another sub-cluster of the target. (c) depicts pion production from the projectile side and pion re-scattering from the target side.

particles which either belongs to the target or the projectile and then is re-scattered by the other α particle and (or) the “quasi deuteron” (Fig. 7.2-(b) and -(c)). Subsequently the “quasi deuteron” and the two α particles form \(^{10}\text{B}^*\).

The differential cross section of the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction should be exhausted by the type (a) reaction mechanism, because of the isospin \( I=1 \) of the quasi-deuteron. This is in fact consistent with the explanation of the target emission and the related measured minimum of the angular distribution which occurs at angles lower than 90°. On the other hand, the \( \pi^0 \) angular distribution is expected to be mainly backward peaked since in the entrance channel the deuteron moves to the backward direction. However, as was shown by the polynomial fits to the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) angular distribution and the comparison with the fits to the calculated results of the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be}^* \) reaction (Chapter 6, Section 6.6), the measured \( \pi^0 \) angular distribution from the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction is forward peaked (see dotted curve in Fig. 6.8). One possible explanation could be that still the second and the third reaction mechanisms contribute and effectively change the angular distributions from the backward to a forward peaked distribution. Using the calculation based on clustering correlations for the \( ^4\text{He}(^3\text{He},\pi^+)\text{Li} \) reaction, it was found that the \( ^4\text{He} \) contribution in the \( \pi^+ \) production is negligible (Fig. 2.11). However, our results imply a larger probability of the \( ^4\text{He} \) contributions to the pion production.

In order to formulate the reaction mechanisms shown in Fig. 7.2, the entrance and exit channel wave functions together with the pion production operator need to be specified.
Following the method involving strong clustering correlations, Eq. 2.32 can be written as

\[
\langle \zeta_\alpha \zeta_\alpha \zeta_d | k_f; J_f M_f \rangle = \left[ \frac{4!4!2!}{10!} \right]^{1/2} S_{a\alpha d}(\Phi_\alpha(\zeta_\alpha)\Phi_\alpha(\zeta_\alpha)\Phi_d(\zeta_d)) \\
\otimes i^L \gamma^L(\hat{r}) \right]_M^J \chi_{JL}(r),
\]

(7.1)

where \( S_{a\alpha d} \) is the antisymmetrizer of nucleons and \( \Phi_\alpha(\zeta_\alpha) \) and \( \Phi_d(\zeta_d) \) are the internal wave functions of \( \alpha \) and deuteron, respectively. They can be assumed to have the highest spatial symmetries (0s)\(^4\) [71] and (0s)\(^2\), for \( \alpha \) and deuteron, respectively, with Gaussian radial dependence. The internal wave functions for the deuteron and \( \alpha \) particles using harmonic oscillator momentum-space wave functions can be written as

\[
\Phi(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_A) = A \prod_{i=1}^{A} (\beta \sqrt{\pi})^{-3/2} \exp\left\{ -\frac{1}{2\beta^2} (\vec{p}_i - \vec{p}_A)^2 \right\}.
\]

(7.2)

\( \vec{p}_i \) and \( \vec{p} \) are the nucleon and nucleus momentum, respectively, and \( A \) is 2 in case of deuteron and 4 in case of \( \alpha \) particle. \( \beta \) is the oscillator parameter which should be adopted for \( \alpha \) and deuteron separately. This parameter should satisfy the variational stability conditions. The observed charge radii and binding energies need to be reproduced well with the obtained wave function. The inter-cluster relative wave function \( \chi_{JL}(r) \) needs to be determined variationally by solving the RGM equation of motion Eq. 2.33. The many-body pion production Hamiltonian can be written as

\[
H = \sum_{i=1}^{10} t_i - T_G + \sum_{k<l} v_{kl} + iW_D(r),
\]

(7.3)

where \( t_i \) and \( T_G \) represent the kinetic energy of the \( i \)th nucleon and the centre-of-mass energy, respectively. The two-body interaction \( v_{kl} \) may consist of three different parts: the central part, the spin-orbit nuclear force and the Coulomb force. Hasegawa [76] showed that in the ground state of \(^{6}\text{Li}\), realistic two-nucleon potentials need to be employed in order to be able to properly reproduce the large intrinsic energy of an \( \alpha \) cluster and to obtain the reasonable values of the total and the relative binding energies. In case of the central part of the nuclear force, different approaches, e.g., the Volkov force [78], the modified Hasegawa-Nagata force, the modified Brink-Boeker force [79], and modified Wildermuth-Tang force [80] need to be examined.

The local imaginary potential \( iW_D(r) \) takes care of the mutual interaction between the three clusters. The model based on clustering correlations [8] shows that the local imaginary potential changes the pion angular distribution in the backward angles. Therefore, the anisotropy of the angular distribution can be also dependent on this potential. In addition, the total cross section is influenced by this interaction. The initial scattering wave can be constructed by superposition of the cluster wave functions from Eq. 7.1 as was reported for Eq. 2.34, since the clustering representations of \(^{6}\text{Li}\) indicate that the clustering correlations exist already before the interaction. Furthermore, the model based on clustering correlations has successfully employed the cluster interactions in the entrance channel which gives confidence in pionic fusion cross sections calculated using these clustering correlations.
To even better guide the theory and to obtain a systematic study of the anisotropic behaviour of the $\pi$ angular distribution, more pionic fusion experiments [81, 82] need to be performed. A systematic study of the target-projectile combinations in different mass regions is essential. The reaction $^6\text{Li}(^6\text{Li},\pi^0)^{12}\text{C}^*$, i.e. a measurement of the pion, the fused nucleus and its photon decay was already proposed earlier [81] but could not be carried out because of difficulties in producing the $^6\text{Li}$ beam at that time. In this reaction the target-projectile combination is symmetric and the same clusters as those in the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ ($\alpha$ and $d$) and $^3\text{He}(^3\text{He},\pi^0)^7\text{Be}$ experiments are involved in the pionic fusion process but the cluster structure is more complicated ($\alpha\alpha dd$) and, therefore, the analysis would be even more decisive for the underlying reaction dynamics.

The $^6\text{Li}(^6\text{Li},\pi^0)^{12}\text{C}^*$ reaction should lead from the $I=0$ ground state to the 15.1 MeV $I=1$ excited state, which is well separated from the ground state. In fact, the excited state of $^{12}\text{C}$ can be detected by the $M1\gamma$ decay to the ground state. In order to measure the pion, the fused nucleus and its photon decay, a combination with an excellent photon detector is required. The proposed experimental setup in Ref. [81] for this reaction offers ideal conditions for the detection of all final state particles. By choosing a beam energy of about 260 MeV, the available energy above the coherent threshold in the centre-of-mass frame will be about 7 MeV. This choice would produce pion momenta $k_\pi \approx 40$ MeV/c and would allow a comparison with the $^{12}\text{C}(^{12}\text{C},^{24}\text{Mg})\pi^0$ data from reference [17].
8. Summary

The pion plays a significant role in modelling the nuclear force and explaining nuclear structure and the interaction between nucleons. In this work, we have presented an almost complete data set for the pionic fusion reaction $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ with well-defined initial and final state configurations. The results were compared with the results of previous pionic fusion experiments and a parallel experiment $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ at KVI which is the subject of another thesis [43]. Both experiments were carried out at only about 10 MeV above the coherent threshold energy of pion production in the centre-of-mass system.

We identified the reaction by detecting the fused system and the produced $\pi^0$ with large acceptance. Neutral pions decay with a 98.8% probability and on a fast time scale of $8.4 \times 10^{-17}$ s, still in the target, into two $\gamma$-rays, which were detected in the Plastic Ball with a large solid-angle. In order to improve the detector response for photons, the Inner Shell detector was included in the Plastic Ball. The Plastic Ball has been employed in several experimental studies at KVI, mainly investigating hadronic interactions in few-body systems. It was brought from CERN to KVI in order to measure the hadronic interactions in few-body systems as well as the pionic fusion reactions. With an acceptance higher than 80% the fused systems were detected by the Big-Bite Spectrometer together with the Heavy Ion detector. The Inner Shell and the Heavy Ion detector were specially designed for the pionic fusion experiments at KVI. The commissioning and the calibration of these three different sets of detectors are described in this thesis.

A detailed explanation is presented for the analysis procedure of the measured reactions consisting of the energy and time calibration, the background reduction and the detailed simulation of the complete experimental setup including the real experimental resolutions in order to achieve a unique identification of the fusion product $^{10}\text{B}$ and the produced neutral pion.

For the first time, an almost complete pion angular distribution from pionic fusion reactions with a projectile heavier than $^1\text{H}$ has been measured. Angular distributions have been observed for a total cross section of about $10^{-8}$ barn.

The model based on strong clustering correlations [8] can explain the measured $\pi^0$ angular distribution of the $^4\text{He}(^4\text{He},\pi^0)^7\text{Be}$ reaction in the backward direction but still needs to be improved. In particular, all of the correlations responsible for the two-nucleon processes (e.g. $s$-wave coupling and $\Delta$-isobar intermediate coupling) need to be included in the calculations.

The rather specialised theoretical work that is required to completely explain the pionic fusion reaction of $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ still needs to be performed. According to the results of this work and the discussions presented in Chapters 6 and 7, there are hints of strong clustering correlations in the entrance and exit channels of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. Therefore, the proposed theoretical steps in Chapter 7 are based on the model with clustering correlations. A proposed pionic fusion process for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction is schematically shown in Fig. 8.2. In the simplest picture, the entrance and exit channels can be seen as “$\alpha\alpha d$”. In this sketch the pion is emitted from the deuteron which is transformed into a “quasi deuteron” with isospin $I=1$ and spin $J=0$. The pion is then
Figure 8.1: Schematic representation of a predicted reaction mechanism for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction, taking the strong clustering correlations into account. The dashed square represents the pion production operator ($H_{\text{int}}$) and “C” denotes the clustering correlations.

It was found that in case of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reactions, the total pionic fusion cross section decreases by increasing the target plus projectile mass. The measured total cross section is in agreement with the extrapolation based on the existing models [8, 32] to the examined reaction.

To even better guide the theory and to have a systematic study of the anisotropic behaviour of the $\pi$ angular distribution, more pionic fusion experiments need to be performed. A systematic study of target-projectile combinations in different mass regions is essential. We have demonstrated that the detector response in our experimental setup is accurately reproduced by the Monte-Carlo simulations. Therefore, the existing experimental setup with improved photon detection could be recommended to measure a complete set of pionic fusion reactions including the highly desired reaction $^6\text{Li}(^6\text{Li},\pi^0)^{12}\text{C}^*$. The more complicated cluster structure in this case would yield even higher sensitivity to the underlying reaction dynamics.
Nederlandse Samenvatting

Het pi-meson (oftewel pion) speelt een belangrijke rol in het modelleren van de sterke wisselwerking, het verklaren van kernstructuur en de wisselwerking tussen nucleonen. De productie van pionen nabij de drempel energie is een zeer coherent proces dat plaatsvindt bij grote impulsoverdracht. Het meten van een dergelijk proces is daarom een krachtig en zeer selectieve methode om de verschijnselen op korte afstanden en collectieve processen in kernen te bestuderen. Om de collectieve eigenschappen van kernen te bestuderen, is een vrijwel complete verzameling data van de pionische samensmelting (fusiereactie) \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) gemeten, met goed gedefinieerde begin- en eind-configuraties. Hierbij smelten een \(^4\text{He} \) kern (uit de versnelde deeltjes bundel) en een \(^6\text{Li} \) kern (uit het trefplaatje) samen tot het fusie-product \(^{10}\text{B} \) waarbij het overschot aan energie bijna volledig wordt overgedragen aan een neutraal pion (\( \pi^0 \)). (Het symbool * betekent hierbij dat de kern \(^{10}\text{B} \) in een aangeslagen toestand mag optreden.)

Een unieke experimentele opstelling is gebruikt om alle reactie producten te meten in overgedetermineerde kinematica en om zo een volledige en zuivere hoekverdeling van de werkzame doorsned te verkrijgen. In dit proefschrift worden de resultaten van het \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) experiment besproken in de context van de voorgaande pionische fusie experimenten, evenals een parallel experiment, te weten \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \), wat op het KVI werd verricht en onderwerp is van een ander proefschrift.

We identificeren de reactie door het meten van de samengesmolten kern en de geproduceerde \( \pi^0 \) met een grote acceptantie. Neutrale pionen vervallen met een 98.8% waarschijnlijkheid en op een tijdsschaal van \( 8.4 \times 10^{-17} \) s, nog steeds in het trefplaatje, in twee gamma deeltjes. Deze werden gemeten in de Plastic Ball, een detectorsysteem met een grote openingshoek. Om de respons van de detector te verbeteren is de binnenste detector (Inner Shell) in de Plastic Ball geplaatst. De samengesmolten kernen werden gemeten met een acceptantie hoger dan 80% in de zware ionen detector die geplaatst was in het brandvlak van de Big-Bite Spectrometer (BBS).

De analyse procedure van de gemeten reacties bestaat uit energie en tijd calibraties, reconstructie van de positie van gemeten deeltjes, identificatie van de deeltjes en onderdrukking van de achtergrond. Om een feilloze identificatie van het samensmeltproducts \(^{10}\text{B} \) en de geproduceerde neutrale pion mogelijk te maken, is een gedetailleerde simulatie uitgevoerd van de complexe experimentele opstelling, inclusief de werkelijke experimentele gevoeligheden.

Het is voor het eerst dat een bijna volledige hoekverdeling van een pionische fusie reactie met een projectiel zwaarder dan het proton is gemeten. We willen benadrukken dat pionische fusie een zeer zeldzaam proces is dat maar een klein deel uitmaakt van de volledige werkzame doorsnede voor pion productie. Hoekverdelingen zijn gemeten voor
Figuur 8.2: Schematische representatie van een voorgesteld mechanisme voor de $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reactie, waarbij rekening wordt gehouden met de (waargenomen) sterke clustering correlaties. Het voorgestelde reactie mechanisme wordt ondersteund door de gemeten hoekverdelingen van de werkzame doorsnede van pionische samensmelt producten zoals gepresenteerd in dit proefschrift. De ononderbroken lijnen stellen kerndeeltjes voor (nucleonen) en de onderbroken lijnen stellen kerndeeltjes voor (nucleonen) en de onderbroken lijnen tonen de koppeling door pion uitwisseling. “$H_{\text{int}}$” en “C” geven respectievelijk de pion productie operator en de clustering correlaties aan.

Een totale werkzame doorsnede van ongeveer $10^{-8}$ barn.

Het theoretische werk dat nodig is om de pionische fusie reactie $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ volledig te beschrijven moet nog worden uitgevoerd. Uit de resultaten van dit werk lijken aanwijzingen te komen voor sterke clustering correlaties in de ingangs- en uitgangs-kanalen van de $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reactie. Een voorgesteld proces voor deze pionische fusie reactie is schematisch weergegeven in figuur 8.2 en wordt ondersteund door de gemeten hoekverdelingen. In het eenvoudigste geval kunnen de ingangs- en uitgangs-kanalen beschouwd worden als alpha-alpha-d cluster structuur. Daarbij betekent alpha de sterk gebonden $^4\text{He}$ kern, en $^6\text{Li}$ kan beschouwd worden als een cluster van alpha en deuteron (d). In deze schets wordt het pion uitgezonden vanuit het deuteron, die daarbij getransformeerd wordt in een "quasi-deuteron"met isospin $I=1$ en spin $J=0$. Het pion wordt vervolgens herverstrooid door een of beide van de alpha deeltjes, welke hun identiteit behouden.

Uit de resultaten van de $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ en $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reacties blijkt dat de totale werkzame doorsnede voor pionische fusie afneemt met toenemende massa van de samensmeltenende kernen. De gemeten totale werkzame doorsnede is in overeenstemming met extrapolaties van bestaande modellen naar de onderzochte reacties.

Zowel voor de ontwikkeling van de theorie als voor een systematische studie van de anisotropie van de hoekverdeling in pionische fusie-reacties is het noodzakelijk meer pionische samensmelten experimenten uit te voeren. Daarbij is een zorgvuldige keuze van de mogelijke cluster-structuren in de betrokken kernen essentieel.
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