A Global Analysis of Light and Charge Yields in Liquid Xenon

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We present an updated model of light and charge yields from nuclear recoils in liquid xenon with a simultaneously constrained parameter set. A global analysis is performed using measurements of electron and photon yields compiled from all available historical data, as well as measurements of the ratio of the two. These data sweep over energies from 1 - 300 keV and external applied electric fields from 0 - 4060 V/cm. The model is constrained by constructing global cost functions and using a gradient descent minimizer, a simulated annealing algorithm, and a Markov Chain Monte Carlo approach to optimize and find confidence intervals on all free parameters in the model. This analysis contrasts with previous work in that we do not unnecessarily exclude data sets nor impose artificially conservative assumptions, do not use spline functions, and reduce the number of parameters used in NEST v0.98. We report our results and the calculated best-fit charge and light yields. These quantities are crucial to understanding the response of liquid xenon detectors in the energy regime important for rare event searches such as the direct detection of dark matter particles.

Keywords: Dark matter, coherent neutrino scattering, noble liquids, dual phase detectors

I. INTRODUCTION

Liquid xenon is currently of great interest as an active medium in the detection and measurement of ionizing radiation. Applications under study include research in direct dark matter detection, neutrino physics, nuclear non-proliferation, and medical imaging [1, 2]. Due to the wide application of the technique, it is important to develop a detector-independent understanding of how xenon responds to incident radiation.

Of particular interest to particle physics applications is the ability to discriminate between electronic recoils (ER) resulting from $\gamma$ and $\beta$ radiation and nuclear recoils (NR) produced by massive neutral particles. This is often accomplished in dual-phase time projection chambers (TPCs) by measuring both the scintillation signal produced by excited xenon molecules and the charge signal produced by ionization of the xenon atoms [3, 4]. The ratio of these two signals allows particle-type discrimination at low energies, allowing separation of NR from ER.

The Noble Element Simulation Technique (NEST) incorporates a semi-empirical physical model of the generation of photons and electrons in liquid xenon [5–7]. In this work, our focus is restricted to improving the model of nuclear recoils at the energies of interest to direct dark matter searches and coherent neutrino scattering experiments. We begin by explaining the physical interpretation and parameterization of our model, then constrain the model using a plethora of published experimental data. We end with a discussion of our results and their application in understanding the yields of NR interactions in the liquid.

II. THE NUCLEAR RECOIL MODEL

A. Theoretical Framework

The model used in NEST is constructed from a simple physical picture of the process of a particle depositing energy in liquid xenon, sketched in Fig. 1. An energy deposit $E_0$ in the medium is distributed between two measurable channels: formation of excitons and formation of electron-ion pairs. Some energy is additionally lost as unmeasurable dissipation of heat. The process conserves energy according to Platzman’s equation for rare gases [8], to which a quenching factor $L$ is applied to account for the energy lost to atomic motion rather than the de-
tectable electronic channels:

$$E_0 = \frac{N_{ex} E_{ex} + N_i E_i + N_i E_e}{L}$$  

(1)

The quantities $E_{ex}$ and $E_i$ are the energies required to create a single exciton or ion respectively, and $E_e$ is the average energy imparted to electrons that do not generate further excitations. The quenching factor $L$ is given by Lindhard’s theory as described in [9] and [10], with

$$L = \frac{k g(\epsilon)}{1 + k g(\epsilon)}$$  

(2)

where $k$ is a proportionality constant between the electronic stopping power and the velocity of the recoiling nucleus. The quantity $g(\epsilon)$ is proportional to the ratio of electronic stopping power to nuclear stopping power. It is a function of the energy deposited, usually converted to the dimensionless quantity $\epsilon$ with

$$\epsilon = 11.5(E_0/\text{keV})Z^{-7/3}$$  

(3)

In these terms, $g(\epsilon)$ is given in [11] by

$$g(\epsilon) = 3e^{0.15} + 0.7e^{0.6} + \epsilon$$  

(4)

With the assumptions $E_{ex} \approx E_i + E_e$ with $E_e$ very small, the above simplifies to

$$E_0 = \frac{(N_{ex} + N_i)W}{L}$$  

(5)

where $W$ is the average energy required to produce a quantum (either an exciton or ion) in the liquid. This quantity is an average that includes the energy lost to sub-excitation electrons, and may be higher than the actual energy required to produce quanta. However, from the equation we see that the number of quanta produced for a given energy deposition is governed by the ratio $W/L$, so any systematic shift in $W$ can be offset by a corresponding shift in $L$ in the fit. We treat the exciton-to-ion ratio $N_{ex}/N_i$ as a field- and energy-dependent parameter (described in [11]), enabling the above equation to be solved to obtain the number of excitons and ions created by a given energy deposition.

While excitons lead directly to the emission of scintillation photons, the electron-ion pairs undergo further division via electron-ion recombination. The probability of recombination $r$ is calculated using the Thomas-Imel box model [12], which gives

$$r = 1 - \frac{\ln(1 + N_i \zeta)}{N_i \zeta}$$  

(6)

The quantity $\zeta$ is parameterized with a power law dependence on applied electric field and fit to data. Free electrons that recombine with ions add to the scintillation light signal, while electrons that escape become the collected charge signal.

A final quenching is applied to the light signal to account for Penning effects, in which two excitons can interact to produce one exciton and one photon [13]. This quenching is parametrized by

$$f_i = \frac{1}{1 + \eta \epsilon^\lambda}$$  

(7)

where $\eta$ and $\lambda$ are left as free parameters. This expression is given by Birk’s Saturation Law, with $\eta \epsilon^\lambda$ proportional to the electronic stopping power. The result of this is an increased quenching effect with increasing energy, due to higher ionization density along the track of the recoiling Xe atom. Biexcitonic quenching in which excitons interact and produce an electron was also implemented and found to result in a worse fit to the data, so it is not included in the model.

The final expressions for number of electrons $n_e$ and number of photons $n_\gamma$ produced by an energy deposition $E_0$ are

$$n_e = L \times \frac{E_0}{W} \left( \frac{1}{1 + N_{ex}/N_i} \right) (1 - r)$$  

(8)

$$n_\gamma = L \times \left( \frac{E_0}{W} \right) \left[ 1 - \left( \frac{1}{1 + N_{ex}/N_i} \right) (1 - r) \right]$$  

(9)

B. Parameterizing The Model

To fit the model to data, the quantities $N_{ex}/N_i$ and $\zeta$ are parameterized to account for dependence of yields on applied electric field and energy. Each is treated as a power law function of an externally applied electric field $F$, and the exciton-to-ion ratio is given an exponential dependence on energy:

$$N_{ex}/N_i = \alpha F^{-\zeta} (1 - e^{-\beta F})$$  

(10)

$$\zeta = \gamma F^{-\delta}$$  

(11)

The parameters $\alpha, \zeta, \beta, \gamma,$ and $\delta$ are free parameters in the fit. Following [13], we fix $W = 13.7$ eV.

The functional forms of the above parameterizations are selected based on trends observed in the data combined with qualitative physical arguments. The power law field-dependence of $N_{ex}/N_i$ is chosen based on an observed trend in best-fits at different applied fields, and can be interpreted as electric field dependence in geminate recombination (immediate recombination of electrons with their parent ions), which occurs on a much shorter timescale than Thomas-Imel recombination. A variety of parameterizations were studied for energy dependence, and a function with increasing energy toward an asymptote proved to be the best fit. The power-law parameterization of $\zeta$ follows from Dahl’s arguments in [13], from which we expect a field-dependence on $\delta$ of
The downward trend of both field dependencies represents the increased probability of electrons being extracted away from their parent ions with increasing applied electric field.

Three additional parameters are allowed to float in the fit: $\eta$ and $\lambda$ from Eq. 7 and the Lindhard $k$ in Eq. 2.

Finally, we define a nuisance parameter, $F_0$. This is a small non-zero number that can be used as an $F$ value in Eqs. 10 and 11 to calculate yields with no applied electric fields. Its implementation allows the model to be completely continuous in field, improving on previous work that treated zero-field as a special case to be separately fit to data. Physically it can be argued to represent small fluctuations in the local field when no external drift field is applied to the liquid. It is allowed to float with the constraint that it must be greater than zero. We expect it to be close to zero.

The recent results from the SCENE collaboration suggest that a similar approach may be useful for modeling argon [16].

III. CONSTRAINING THE MODEL

The model is constrained using three categories of data sets that each consist of multiple measurements of yields in liquid xenon. The absolute NR charge yield, $Q_y$, is constrained using twelve measurements across different energies at a range of electric fields [15,17,23]. The photon yield is constrained with an additional seven measurements [21,24,29] of the parameter $\mathcal{L}_{\text{eff}}$, defined as the zero-field scintillation yield for nuclear recoils relative to that of the 122 keV $\gamma$-ray from $^{57}$Co. Finally, we constrain the ratio of charge to light, $n_c/n_\gamma$ using measurements at several fields from [15]. In total, there are 329 data points. In the singular case of [21], uncertainty regarding the threshold motivates the exclusion of the two lowest-energy points, for which no overlap in uncertainty exists between the lowest energy measurements and the remainder of the world’s data. Thus 327 data points are used to constrain the model.

In the case of measurements of the charge yield, this work follows the implicit assumption in each of the references that the electron extraction efficiency was 100%. The extraction field in each experiment can be compared to the saturation point measured by Gushchin et al. [30,31]. Lower extraction efficiencies would result in reported values being underestimates for $Q_y$, a systematic normalization uncertainty that has not been considered in this work.

Other sources of systematic uncertainties in the measurements are inconsistently reported, making a full treatment of all sources of error prohibitively difficult. The global analysis is intended to average over these effects to extract true absolute yields.

Employing an overabundance of caution, three independent fitting techniques are used in order to obtain best-fit values for the parameters, explained below. All three are found to be in agreement, increasing our confidence in the fit.

A. Gradient Descent Fits

The absolute best-fit values are obtained using a scan across the nine-dimensional parameter space combined with a gradient descent algorithm, seeking to minimize a global $\chi^2$ function of the form

$$\chi^2 = \sum_i \frac{(x_i - x_{\text{model}})^2}{\sigma^2_i}$$  \hspace{1cm} (12)

where $x \in \{Q_y, \mathcal{L}_{\text{eff}}, \frac{n_c}{n_\gamma}\}$ and $\sigma$ is the average of the upper and lower uncertainty on the measurement. This value is calculated across a coarsely grained grid covering a large region of parameter space, and the point with the lowest $\chi^2$ is then used as the initial point of the gradient descent minimizer. While this method is guaranteed to yield a global minimum, it is extremely computationally expensive, as the number of necessary calculations in the scan scales as the power of the number of free parameters.

B. Simulated Annealing

To facilitate experimentation with different models and constraints, we perform a maximum likelihood optimization using simulated annealing [32]. This technique allows the program to make random jumps in the parameter space drawn from a given proposal distribution. Steps with a higher likelihood are accepted automatically, and steps with a lower likelihood are accepted with a probability that decreases with step number $t$. The functional form used in this fit was

$$P(\text{accept}) = \exp \left( \frac{\log \mathcal{L}_{\text{proposed}} - \log \mathcal{L}_{\text{current}}}{T(t)} \right)$$  \hspace{1cm} (13)

where $\mathcal{L}$ is the likelihood and $T(t)$ is a decreasing linear function of the step number. This technique allows the fitter to initially explore a wide range of the parameter space and escape local extrema, settling later in a region very near the global maximum likelihood.

The likelihood function used in this algorithm assumed each data point behaved according to a split gaussian distribution, to take into account asymmetric errors in the data set:

$$\mathcal{L}(\vec{\theta}|x) = \prod_i \sqrt{\frac{2}{\pi(\sigma_+ + \sigma_-)}} \exp \left( \frac{-(x_i - x_{\text{model}})^2}{2\sigma^2_{x_i}} \right)$$  \hspace{1cm} (14)

In the above, $\sigma_{x_i}$ are the upper and lower uncertainties on the measurements, $x_i$ is the measured quantity, and $x_{\text{model}}$ is that calculated from the given model using the set of parameters $\vec{\theta}$. 

\[ \text{O (0.1)} \]
While this method is not guaranteed to locate the absolute optimal value, tuning of $T(t)$ and the proposal distribution allows the fit to get very close. The simulated annealing fitter enjoys a factor of $\sim 20$ improvement in speed over the gradient descent fit, allowing this algorithm to serve as a test bed for alternative models to be incorporated into NEST, as well as rapid incorporation of new data into the fit.

C. Metropolis-Hastings Markov Chain Monte Carlo (MCMC)

A Metropolis-Hastings MCMC technique was used to extend the capability of the simulated annealing fitter to extract variances and covariances on the free parameters in our model. The Metropolis-Hastings algorithm \cite{37} works in a manner similar to simulated annealing, but the probability of accepting a step with lower likelihood is given by the ratio

$$P(\text{accept}) = \frac{L(\bar{\theta}|x)_{\text{proposed}}}{L(\bar{\theta}|x)_{\text{current}}}$$

(15)

The values of all the parameters are recorded at each step. As the random walk progresses, it builds a sample of $L(\theta|x)$, revealing the structure of the underlying posterior probability distribution. By histogramming the values of any given subset of parameters, one marginalizes over the remaining variables, enabling quick numerical estimates of variances and covariances. For a large number of steps, these estimates can be made arbitrarily close to the true values.

To minimize the number of steps needed to obtain a fair sample of $L(\theta|x)$, the proposal distribution is carefully chosen and tuned. We use a multivariate normal, which has the form

$$P(\bar{\theta}) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp \left( -\frac{(\bar{\theta} - \bar{\theta}_0)^T \Sigma^{-1}(\bar{\theta} - \bar{\theta}_0)}{2} \right)$$

(16)

where $\Sigma$ is the covariance matrix, $\bar{\theta}$ and $\bar{\theta}_0$ are vectors containing proposed and current values of the free parameters, respectively, and $n$ is the number of dimensions. Initially, $\Sigma$ is assumed to have no off-diagonal elements, and the values are tuned by hand to mitigate random walk behavior. The covariance matrix of the samples obtained is then calculated and fed back into the proposal distribution to produce the final set of samples for analysis. At each stage, the autocorrelation of each parameter as a function of step length is studied to ensure that the resulting sample set contains enough points to wash out correlations. In the final run, we obtain autocorrelation lengths of approximately 1000 steps in all nine parameters, and use a data set of $3 \times 10^6$ samples.

| Parameter | Best Fit | 68% conf. |
|-----------|----------|-----------|
| $\alpha$  | 1.240    | +0.079    |
|           |          | -0.073    |
| $\zeta$   | 0.0472   | +0.0088   |
|           |          | -0.0073   |
| $\beta$   | 239      | +28       |
|           |          | -8.8      |
| $\gamma$  | 0.01385  | +0.00058  |
|           |          | -0.00073  |
| $\delta$  | 0.0620   | +0.0056   |
|           |          | -0.0064   |
| $k$       | 0.1394   | +0.0032   |
|           |          | -0.0026   |
| $\eta$    | 3.3      | +5.3      |
|           |          | -0.06     |
| $\lambda$ | 1.14     | +0.45     |
|           |          | -0.09     |
| $C$       | 0.00555  | +0.00025  |
|           |          | -0.00025  |

IV. ADDING FLUCTUATIONS TO THE MODEL

It has been widely observed that the measured widths of yields in liquid xenon are broader than the expectation from Poisson statistics \cite{33, 34}. This is true for both light and charge yields, and in both the ER and NR samples. It has further been shown in \cite{15} by subtracting detector effects that the variance in the yields is caused primarily by fluctuations in the recombination process. Consequently, combining the charge and light signals as in Eq. 5 results in a combined energy scale that has a superior resolution than that of either signal alone \cite{10}. In earlier versions of NEST, the broadening of fluctuations in electron-ion recombination was implemented as a complicated function of energy and field that had no physical justification.

Dobi has provided an improved treatment of this subject \cite{35}, and his approach has been implemented in NEST to model these fluctuations. Dobi finds a quadratic relationship between the variance (in units of quanta) and the number of ions produced in that event. This yields an effective Fano factor, $F$, which is proportional to the number of ions produced. The variance derived from Poisson statistics is multiplied by $F$, so that the recombination fluctuations have a width given by

$$\sigma_R^2 = n_e \times F = (1 - r) N_i \times F = (1 - r) C N_i^2$$

(17)

where the multiplicative factor, $C$, is a final, tenth free parameter. A fit to data from \cite{15} yields a value $C = \ldots$
FIG. 2. The calculated variance in the ratio of electrons to photons, converted to a function of recombination fraction as done in [15], plotted with the data contained therein. These data are used to constrain the fluctuation model described in Sec. IV. Note, we use the variable $r'$ because the conversion in [15] assumes an exciton-to-ion ratio of $N_{ex}/N_i = 0.06$. The exciton-to-ion ratio in this work is different, so $r'$ does not represent the recombination fraction $r$ presented in this work.

0.0056, given in Table I. The data used and the fit are shown in Fig. 2.

The fluctuations are modeled using a Poisson distribution, modified such that the variance can be tuned in software to fit the data. Physically, a binomial distribution is more appropriate for the application, but the computational expense is an impediment to efficient simulation. While a Gaussian distribution could also have been used, it was decided against due to the non-zero probability of calculating negative quanta and the need to produce an integer number of quanta.

Similar results can be derived by assuming that ions recombine with electrons freed from other atoms with a small probability of re-encounter [36].

V. RESULTS

The best fits values and corresponding confidence intervals constructed from the MCMC are given in Table I. The mean values are obtained from the gradient descent fitter, while confidence intervals are obtained by marginalizing the MCMC sample set in each parameter and measuring 34% of the sample space on either side of the mean. In fitting, the $\chi^2$/d.o.f. is calculated to allow comparison between models. This is simply the sum defined in section III divided by (327 points - 9 free parameters - 1) = 317 degrees of freedom. The best-fit gives $\chi^2$/d.o.f. = 1.33.

We note that the most tightly constrained parameter is the Lindhard $k$ factor. Earlier estimates place this value at $k = 0.166$ or $k = 0.110$ [10]. The best fit obtained here lies in between these two, and the precision is greatly improved.

The overall scaling of $N_{ex}/N_i$ is of $O(1)$ and the scaling of $\varsigma$ is $O(0.01)$, in agreement with work in [15] and [10].

The Penning quenching parameters, $\eta$ and $\lambda$, can also be compared to previous work. The parameter $\eta$ is found by Bezrukov et al. to be $k_B \times 0.166 = 3.55$ [14], where $k_B$...
is called Birk’s constant and is calculated in [13]. This result is well within our confidence interval. The parameter \( \lambda \) is quoted to be 1/2, significantly lower than our best fit. However, we are in agreement with the energy dependence of the stopping power at high energies calculated by SRIM, shown in Fig 4.11 of [15]. Because this

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FIG. 5. Relative scintillation yield \( S_n \) at different values of external electric field. This is calculated by dividing \( n_e \) at the given field by \( n_e(F_0) \). Measurements taken from [17]. All measurements are at 56.5 keV, but are artificially offset for clarity.

![Graph showing relative scintillation yield](image)

FIG. 6. All charge yield data included in the fit, plotted with best fit results from this work. Each color represents a different applied electric field. These data are at 100 V/cm (red) [17], 270 V/cm (dark green) [17], 530 V/cm (blue) [23], 730 V/cm (purple) [18,20], 1000 V/cm (gold) [22], 2025 V/cm (black) [17], 3400 V/cm (green band) [22], 3900 V/cm (light purple) [22], and 4000 V/cm (light purple) [21].

![Graph showing charge yield data](image)

Recent work suggests a significantly lower nuclear recoil quenching [41], which in this model would require a smaller \( L \) and therefore a significantly smaller \( k \). To maintain the global consistency across light and charge yields, this result can be brought into agreement by additionally assuming a smaller \( W \), allowing the same number of quanta to be produced in Eq. 5. This is shown in Fig. 11 discussed in greater detail in the following section.

A. Absolute Scintillation and Electron Yields

One of the most important goals of this work is to produce a model that predicts absolute yields, rather than remaining within the traditional paradigm of relative yields. While relative quantities remain critical to our understanding, the need to project the performance of future detectors from first principles necessitates accurate predictions of absolute fundamental quanta. With this goal in mind, the best-fit model is implemented in the NEST code and used to calculate absolute yields.

The light yield of liquid xenon is defined as

\[
L_y = \mathcal{L}_{eff} \frac{S_n}{S_e} \frac{\gamma}{E_0} = \frac{n_e}{E_0}
\]  

(18)

where \( S_n \) and \( S_e \) are the scintillation reduction factors due to applied fields for nuclear recoils and electronic re-
coils, and \( n_{\gamma}(^{57}\text{Co}) \) is the yield from the 122 keV \( \gamma \)-ray with no applied electric field. To compare with measurements in the literature, we calculate \( L_y \) at \( F = F_0 \) \((S_n = S_e = 1)\) and plot it against the measurements of \( L_{\text{eff}} \) used in the fit. In our calculations, \( n_{\gamma}(^{57}\text{Co}) \) is assumed to be 63 photons / keV \([6]\) and the \( L_{\text{eff}} \) data are scaled to give absolute yields. The resulting \( L_y \) as a function of incident energy is shown in Fig. 4 with all measurements used in this work. Charge yield is calculated simply as

\[
Q_y = \frac{n_e}{E_0} \quad (19)
\]

Figure 6 shows the NEST calculations vs. energy with all measurements used in the fit. The field dependence of this quantity is emphasized in Fig. 7. Finally, taking the calculated ratio \( n_e/n_{\gamma} \) allows direct comparison to the measurements in \([15]\). This is shown in Fig. 3. In the course of our experimentation, we tested allowing \( W \) and \( n_{\gamma}(^{57}\text{Co}) \) to float and did not find a significant improvement in the fit. The best-fit model can be used to predict yields in current and future experiments.

The uncertainty band in Fig. 4 shows the statistical uncertainty calculated using the MCMC sample set. Each of the \( 3 \times 10^6 \) points contains a set \( \vec{\theta} \) of the nine free parameters, corresponding to some unique incarnation of our model. At each energy, the value of interest \((L_y \text{ or } Q_y)\) is calculated for all points and placed into a histogram. The statistical uncertainty is then taken as the standard deviation of this histogram. This technique naturally incorporates all correlations between parameters. The width is constrained simultaneously by all light and charge data in the fit, and is further constrained by the choice of model, resulting in small uncertainties compared to the apparent spread in the data. Uncertainties on charge yield are comparable, but are not shown for the sake of clarity.

The best-fit model can be modified within the framework presented as new data becomes available, such as the in-situ calibrations from the LUX experiment \([38]\).

VI. DISCUSSION OF ALTERNATIVE MODELS

The deposition of energy in liquid xenon is a complicated process, and there are many theories that seek to calculate observable quantities such as charge and light yields from first principles. Several of these are described in detail by Bezrukov et al. in \([14]\), in which the authors compare several models for the distribution of energy into nuclear and electronic excitations. The models presented can act as alternatives to Lindhard quenching, which is expected to fail in liquid xenon at low energies \([39]\). In addition, recent work by Mu et al. \([40, 41]\) suggests a new alternative via an extrapolation of the electronic stopping power in gaseous xenon measured in \([42]\). In this section, we discuss how these models compare when using the global approach taken in this work, and show that the model described in Sec. IA is the best fit to existing data.

Bezrukov et al. study the effect of different calculations of the nuclear stopping power \( s_n \), which they define to be proportional to the probability that a recoiling xenon atom will scatter elastically from another xenon atom. This is used in conjunction with the electronic stopping power \( s_e \) to determine the fraction of energy a recoiling xenon atom loses to observable electronic excitation in
FIG. 10. Measurements of $Q_y$ at a drift field of 730 V/cm, plotted with alternative models of nuclear stopping power from [14]. This field was chosen as an example due to the existence of multiple analyses. The solid lines are the best fit (blue), Ziegler et al. (red), and Lenz-Jensen (green).

the detector,

$$L = \frac{s_e}{s_n + s_e}$$  \hspace{2cm} (20)

The Thomas-Fermi model, used by Lindhard in the treatment adopted in Sec. II A, is compared to two alternative models of $s_n$; that due to Ziegler et al. and that due to Lenz and Jensen. We incorporate both into our simulated annealing fitter, replacing the free parameter $k$ from Lindhard’s model with an overall scaling of the quantity $L$. This allows the resulting curve to shift vertically to fit the world’s data. These alternative models are used to produce the $L_y$ and $Q_y$ curves in Figs. 9 and 10. The best-fit $\chi^2$/d.o.f values obtained using the simulated annealing fitter were 2.88 and 4.32 respectively, compared to 1.33 in the current work.

In addition, Bezrukov et al. offer two possible low-energy correction factors corresponding to enhancement or suppression of the electronic stopping power. This affects the number of total quanta generated at low energies. To test these, we remove the energy dependence on the exciton-to-ion ratio and introduce these corrections into the Lindhard factor. These corrections are alternative ways of expressing an energy-dependent yield, with the current energy dependence of $N_{ex}/N_i$ corresponding to a suppression of the light yield. We find a best-fit $\chi^2$/d.o.f. = 2.43 for the Bezrukov enhancement model, and $\chi^2$/d.o.f. = 1.84 for suppression.

The calculations due to Mu et al. predict a quenching factor $L$ that is significantly smaller than previous work [40, 41]. In the global context, this results in fewer quanta produced for a given energy deposition. However, from Eq. 5 we see that this can be reconciled by assuming a smaller $W$. We demonstrate these effects in Fig. 11. Mu et al. favor the value $W = 9.76$ eV, for which they cite the experimental measurement in [43]. We find reasonable agreement with the global fit by assuming $W = 8.3$ eV (approximately 2 $\sigma$ below the measured $W$ in [43]), and we can bring the Lindhard model into agreement with this smaller $W$ by assuming $k = 0.07$. It is possible that the value 13.7 eV for $W$, higher than the xenon scintillation energy of 7 eV, already includes the effect of some heat, and we allow for the possibility of this case. However, in the present work we do not find a significant change in $W$ when it is allowed to float in our fit, so we retain $W = 13.7$ eV.

VII. SUMMARY

We have presented here a model of light and charge yields from nuclear recoils in liquid xenon, constrained simultaneously using measurements of both quantities. This approach incorporates an anti-correlation between the two and helps break degeneracies between quantities that can independently affect one or the other. We are able to obtain a better constrained mean for the semi-empirical NEST model, and find that it provides a better fit than many alternatives suggested in the literature. These results could be used to predict yields and extrapolate to low energies, allowing lowered thresholds in analyses of data from liquid xenon experiments.
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