QCD corrections to polarization of $J/\psi$ and $\Upsilon$ at Tevatron and LHC

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In this work, we present more details of the calculation on the next to leading order (NLO) QCD corrections to polarization of direct $J/\psi$ production via color singlet at Tevatron and LHC, together with the results for $\Upsilon$ for the first time. Our results show that the $J/\psi$ polarization status drastically changes from transverse polarization dominant at leading order (LO) into longitudinal polarization dominant in the whole range of the transverse momentum $p_t$ of $J/\psi$ when the NLO corrections are counted. For $\Upsilon$ production, the $p_t$ distribution of the polarization status behaves almost the same as that for $J/\psi$ except that the NLO result is transverse polarization at small $p_t$ range. Although the theoretical evaluation predicts a larger longitudinal polarization than the measured value at Tevatron, it may provide a solution towards the previous large discrepancy for $J/\psi$ and $\Upsilon$ polarization between theoretical prediction and experimental measurement, and suggests that the next important step is to calculate the NLO corrections to hadron production of color octet state $J/\psi^{(8)}$ and $\Upsilon^{(8)}$. Our calculations are performed in two ways, namely we do and do not analytically sum over the polarizations, and then check them with each other.

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I. INTRODUCTION

The study of $J/\psi$ production on various experiments is a very interesting topic since its discovery in 1974. It is a good place to probe both perturbative and nonperturbative aspects of QCD dynamics. To describe the huge discrepancy of the high-$p_t$ $J/\psi$ production between the theoretical calculation based on color singlet mechanism[1] and the experimental measurement by the CDF collaboration at the Tevatron[2], color-octet mechanism[3] was proposed based on the non-relativistic QCD(NRQCD)[4]. The factorization formalism of NRQCD provides a theoretical framework to the treatment of heavy-quarkonium production. It allows consistent theoretical prediction to be made and to be improved systematically in the QCD coupling constant $\alpha_s$ and the heavy-quark relative velocity $v$. The color singlet mechanism is straightforward from the perturbative QCD, but the color-octet mechanism depends on nonperturbative universal NRQCD matrix elements. So various efforts have been made to confirm this mechanism, or to fix the magnitudes of the universal NRQCD matrix elements. Although it seems to show qualitative agreements with experimental data, there are certain difficulties in the quantitative estimate in NRQCD for $J/\psi$ photoproduction at the DESY ep collider HERA[5-7, 8, 9, 10], $J/\psi(\psi')$ and $\Upsilon$ polarization of hadron production at the Fermilab Tevatron, and $J/\psi$ production in B-factories. A review of the situation could be found in Ref. [11].

Without NLO corrections, it is difficult to obtain agreement between the experimental results and leading order theoretical predictions for $J/\psi$ production. There are a few examples shown that NLO corrections are quite large. It was found that the current experimental results on inelastic $J/\psi$ photoproduction[12, 13] are adequately described by the color singlet channel alone once higher-order QCD corrections are included[7, 8]. Although ref. [14] found that the DELPHI[15] data evidence favor the NRQCD formalism for $J/\psi$ production $\gamma + \gamma \rightarrow J/\psi + X$, rather than the color-singlet model. And it was also found in ref. [16] that the QCD higher-order process $\gamma + \gamma \rightarrow J/\psi + c + \bar{c}$ gives the same order and even larger contribution at high $p_t$ than the leading order color singlet processes. In ref. [17], the NLO process $c + g \rightarrow J/\psi + c$ where the initial c quark is the intrinsic c quark from proton at Tevatron, gives larger contribution at high $p_t$ than the leading order color singlet processes. The large discrepancies found in the single and double charmonium production in $e^+e^-$ annihilation at B factories between LO theoretical predictions[18, 19, 20] and experimental results[21, 22] were studied in many work. It seems that they may be resolved by including higher order correction: NLO QCD and relativistic corrections [18, 23, 24, 25, 26, 27, 28].

Based on NRQCD, the LO calculation predicts a sizable transverse polarization for $J/\psi$ production at high $p_t$ at Tevatron[29, 30, 31] while the measurement at Fermilab Tevatron[32] gives slight longitudinal polarized result. In a recent paper[33], the measurement on polarization of $\Upsilon$ production at Tevatron is presented and the NRQCD prediction[34] is not coincide with it. Beyond the NRQCD framework, there is a try by using s-channel treatment to $J/\psi$ hadron production in the work of ref[35], which gives longitudinal polarization. Within the NRQCD framework, to calculate higher order corrections is an important step towards the solution of such puzzles. Recently, NLO QCD corrections to $J/\psi$ hadron production have been calculated in ref[36]. The results show that the total cross section is boosted by a factor of about 2 and the $J/\psi$ transverse momentum $p_t$ distribution is enhanced more and more as $p_t$ becomes larger. A real correction process $g + g \rightarrow J/\psi + c + \bar{c}$ at NLO, which
is not included in the ref. 36, was calculated in 16, 37. It gives sizable contribution to $p_t$ distribution of $J/\psi$ at high $p_t$ region, and it alone gives almost unpolarized result. Therefore it is very interesting to know the result of $J/\psi$ polarization when NLO QCD corrections are included. In a recent Letter 38, we presented a calculation on the NLO QCD corrections to the $J/\psi$ polarization in hadron production at Tevatron and LHC. In this paper, we give more details of the calculation, and the results for $\Upsilon$ polarization for the first time. The results show that the polarizations of $J/\psi$ and $\Upsilon$ are drastically changed from more transverse polarization at LO into more longitudinal polarization at NLO. Meanwhile, our results for total cross section and transverse momentum distribution is consistent with ref. 38. In this calculation, we use our Feynman Diagram Calculation package (FDC) 39 with newly added part of a complete set of method to calculate tensor and scalar integrals in dimensional regularization, which was used in our previous work 24, 25.

This paper is organized as follows. In Sec. II, we give the LO cross section for the process. The calculation of NLO QCD corrections are described in Sec. III. In Sec. IV, it presents the formula in final integration to obtain the transverse momentum distribution of $J/\psi$ production. Sec. V, is devoted to the description about the calculation of $J/\psi$ polarization. The color factor treatment for all the calculated processes are given in Sec. VI. In Sec. VII, treatment of $\Upsilon$ is given. The numerical results are presented in Sec. VIII. Finally, The conclusion and discussion are given in Sec. IX.

II. THE LO CROSS SECTION OF $J/\psi$ HADRONPRODUCTION

The related Feynman diagrams which contribute to the LO amplitude of the partonic process $g(p_1) + g(p_2) \rightarrow J/\psi(p_3) + g(p_4)$ are shown in Fig. 1 while the others can be obtained by permuting the places of gluons.

In the nonrelativistic limit, we can use the NRQCD factorization formalism to obtain the partonic differential cross section in $n = 4 - 2\epsilon$ dimension as

$$
\frac{d\hat{\sigma}^B}{d\hat{t}} = \frac{5\pi\alpha_s^2 [R_s(0)]^2 [s^2(s - 1)^2 + \hat{t}^2(\hat{t} - 1)^2 + \hat{u}^2(\hat{u} - 1)^2]}{144m_c^2s^2(s - 1)^2(\hat{t} - 1)^2(\hat{u} - 1)^2} + \mathcal{O}(\epsilon),
$$

where $\hat{s} = \frac{(p_1 + p_2)^2}{4m_c^2}$, $\hat{t} = \frac{(p_1 - p_3)^2}{4m_c^2}$, $\hat{u} = \frac{(p_1 - p_4)^2}{4m_c^2}$, and $R_s(0)$ is the radial wave function at the origin of $J/\psi$ and the reasonable approximation $M_{J/\psi} = 2m_c$ is taken.

The LO total cross section is obtained by convoluting the partonic cross section with the parton distribution function (PDF) $G_g(x, \mu_f)$ in the proton:

$$
\sigma^B = \int dx_1 dx_2 G_g(x_1, \mu_f) G_g(x_2, \mu_f) \hat{\sigma}^B,
$$

where $\mu_f$ is the factorization scale. In the following $\hat{\sigma}$ represents the corresponding partonic cross section.

III. THE NLO CROSS SECTION OF $J/\psi$ HADRONPRODUCTION

The NLO contributions to the process can be written as a sum of two parts: one is the virtual correction which arises from loop diagrams, the other is the real correction caused by radiation of a real gluon, or a gluon splitting into a light quark-antiquark pair, or a light (anti)quark splitting into a light (anti) quark and a gluon.

![Leading order Feynman diagrams for $g+g \rightarrow J/\psi+g$. The other five diagrams can be obtained by permutation the places of gluons.](image)

A. Virtual corrections

There are UV, IR and Coulomb singularities in the calculation of the virtual corrections. UV-divergences existing in the self-energy and triangle diagrams are removed by the renormalization of the QCD gauge coupling constant, the charm quark mass, charm quark and gluon fields. Here we adopt renormalization scheme used in ref. 40. For the charm quark mass, charm quark and gluon fields, the renormalization constant $Z_m$, $Z_2$ and $Z_g$ are determined in the on-mass-shell(OS) scheme while for the QCD gauge coupling constant, $Z_g$ is fixed in the...
modified-minimal-subtraction(\textit{MS}) scheme:
\[
\begin{align*}
\delta Z^{\text{OS}}_m &= -3C_F \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi\mu_r^2}{m_c^2} + \frac{4}{3} \right], \\
\delta Z^{\text{OS}}_2 &= -C_F \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3 \ln \frac{4\pi\mu_r^2}{m_c^2} + 4 \right], \\
\delta Z^{\text{OS}}_3 &= \frac{\alpha_s}{4\pi} \left[ (\beta_0 - 2C_A) \left( \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \right], \\
\delta Z^{\text{MS}}_g &= \frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) \right].
\end{align*}
\]

where \(\gamma_E\) is Euler’s constant, \(\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f\) is the one-loop coefficient of the QCD beta function and \(n_f = 3\). In SU(3)_c, color factors are given by \(T_F = \frac{1}{2}, C_F = \frac{4}{3}, C_A = 3\). And \(\mu_r\) is the renormalization scale.

After having fixed the renormalization scheme, there are 129 NLO diagrams in total, including counter-term diagrams. They are shown in Fig. 2 and divided into 8 groups. Diagrams of group (e) that has a virtual gluon line connected with the quark pair lead to Coulomb singularity, which can be isolated by introducing a small relative velocity \(v = |\vec{p}_v - \vec{p}_l|\). The corresponding contribution is also of \(\mathcal{O}(\alpha_s)\) and can be mapped into the \(c\bar{c}\) wave function.

\[
\sigma = |R_s(0)|^2 \sigma(0) \left( 1 + \frac{\alpha_s}{\pi} C_F \frac{\pi^2}{v} + \frac{\alpha_s}{\pi} C + \mathcal{O}(\alpha_s^2) \right)
\Rightarrow
|R_s^{\text{ren}}(0)|^2 \sigma(0) \left[ 1 + \frac{\alpha_s}{\pi} C + \mathcal{O}(\alpha_s^2) \right].
\]

The Passarino-Veltman reduction [41] is adopted in the tensor decomposition when it’s Gram determinant is nonzero. Otherwise, It is to do the integration directly with Feynman parametrization for two-point tensor case, and to write the Lorentz structure with independent external momentums and apply the Passarino-Veltman reduction again for other cases. In the calculation of scalar integral, we first try to decompose the scalar integral into several lower-point ones when it’s Gram determinant is zero, if it fails, then to do the integration directly with Feynman parametrization just like the treatment to scalar integral with nonzero Gram determinants. Above procedure, including both reduction and integration, are done by FDC automatically.

In our calculation, there are total 86 scalar integrals in total:

- 65 of the total 86 integrals, can be found in Ref. [7] after including the permutation of \(s, t\) and \(u\). But the explicit results for the three Coulomb singular five-point scalar integrals is not available in Ref. [7].

- The remaining 21 integrals are not listed in Ref. [7]. 12 of them can be reduced to combination of some lower-point scalar integrals and needn’t to be integrated directly.

- Another 6 of them can be expressed by the following two integrals, \(C(p_1, p_3, m_c, m_c, m_c)\) and \(D(p_1, p_4, p_3 + p_4, 0, m_c, m_c, m_c)\), through permutation of \(s, t\) and \(u\), where \(A, B, C, D, E\) are defined exactly the same as in Ref. [7]. They can be written into a linear combination of another two scalar integrals as:

\[
\begin{align*}
&\frac{1}{2} C(-p_3/2, -p_3/2 + p_1, 0, m_c, m_c) \\
&+ \frac{1}{2} C(p_3/2, -p_3/2 + p_1, 0, m_c, m_c), \quad \text{(6)}
\end{align*}
\]

\[
\begin{align*}
&\frac{1}{2} D(p_3/2, p_3/2 - p_2, -p_3/2 - p_4, m_c, m_c, m_c) \\
&+ \frac{1}{2} D(-p_3/2, p_3/2 - p_2, -p_3/2 - p_4, m_c, m_c, m_c).
\end{align*}
\]

But in our calculation, they are calculated independently, and above relationship can be used to check all three scalar integrals.

- The remaining 3 scalar integrals can be expressed by \(B(p_1, m_c, m_c)\) through the permutation of \(s, t\) and \(u\).

More details about these 86 scalar integrals can be found at FDC homepage [50].

By adding all diagrams together, the virtual corrections to the differential cross section can be expressed as

\[
\frac{d\sigma^V}{dt} \propto 2\text{Re}(M^B M^V^*),
\]
where $M^B$ is the amplitude at LO, and $M^V$ is the renormalized amplitude at NLO. $M^V$ is UV and Coulomb finite, but it still contains the IR divergences:

$$M^V_{IR} = \left[ \frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \left( \frac{4\pi\mu^2}{s_{12}} \right)^\epsilon \left( \frac{A^V_2}{\epsilon^2} + \frac{A^V_1}{\epsilon} \right) \right] M^B,$$

with

$$A^V_2 = -\frac{9}{2}, \quad A^V_1 = -\frac{3}{2} \ln \left( \frac{s}{\mu^2} \right) + \ln \left( \frac{s}{\mu^2} \right) - \frac{1}{2} n_f - \frac{33}{4}.$$

And the total cross section of virtual contribution could be written as

$$\sigma^V = \int d\xi_1 d\xi_2 G_\sigma(x_1, \mu_f) G_\sigma(x_2, \mu_f) \sigma^V.$$

### B. Real corrections

The real corrections arise from four parton level subprocesses:

\begin{align}
&g(p_1) + g(p_2) \rightarrow J/\psi(p_3) + g(p_4) + g(p_5), \quad (11) \\
&g(p_1) + g(p_2) \rightarrow J/\psi(p_3) + g(p_4) + \bar{q}(p_5), \quad (12) \\
&q(p_1) + q(p_2) \rightarrow J/\psi(p_3) + g(p_4) + q(p_5), \quad (13) \\
&q(p_1) + g(p_2) \rightarrow J/\psi(p_3) + c(p_4) + \bar{c}(p_5), \quad (14)
\end{align}

We have neglected the contribution from a real correction subprocess $q\bar{q} \rightarrow J/\psi gg$, which is IR finite and tiny (it only contributes about 0.002% at $p_t=3$ GeV and 0.05% at $p_t=50$ GeV to the differential cross section). And Feynman diagrams for above processes are shown in Fig. 3 and Fig. 4. The phase space integration of above processes (except $gg \rightarrow J/\psi + c\bar{c}$) generates IR singularities, which are either soft or collinear and can be conveniently isolated by slicing the phase space into different regions. We use the two-cutoff phase space slicing method \[12\] to introduce two small cutoffs to decompose the phase space into three parts.

Real gluon emission brings soft singularities. A small soft cutoff $\delta_s$ is used to divide the phase space into two regions according to that the emitted gluon is soft or hard. Then another small cutoff $\delta_c$ is used to decompose the hard region into collinear and noncollinear regions. Then the cross section of real correction processes can be written as

$$\sigma^R = \sigma^S + \sigma^{HC} + \sigma^{HC}.$$

The hard noncollinear part $\sigma^{HC}$ is IR finite and can be numerically computed using standard Monte-Carlo integration techniques. The subprocess $gg \rightarrow J/\psi + c\bar{c}$ consists of only hard noncollinear part.

#### 1. soft

It is easy to find that soft singularities caused by emitting soft gluons from the charm quark-antiquark pair in the S-wave color singlet $J/\psi$ are canceled by each other. Therefore only the real gluon emission subprocess in Eq. (11), where there could be a soft gluon emitted from the external gluons, contains soft singularities.
Suppose \( p_5 \) is the momentum of the emitted gluon. If we define the Mandelstam invariants as \( s_{ij} = (p_i + p_j)^2 \) and \( t_{ij} = (p_i - p_j)^2 \), the soft region is defined in term of the energy of \( p_5 \) in the \( p_1 + p_2 \) rest frame by \( 0 \leq E_5 \leq \delta_s \sqrt{s}/2 \). In this region, soft approximation can be made and the matrix element squared can be factorized as

\[
[M_R]^2_{\text{soft}} \simeq -4\pi\alpha_s\mu^2 R \sum_{i,j=1,2,4} \frac{-p_i \cdot p_j}{(p_i \cdot p_5)(p_j \cdot p_5)} M^0_{ij},
\]

with

\[
M^0_{ij} = \left[T^a(i)M^{B}_{b_1...b_4}\right]^{a} \left[T^a(j)M^{B}_{b_1...b_4}\right]^{a},
\]

and

\[
T^a(j) = i f_{ab_1b_2},
\]

where \( M^{B}_{b_1...b_4} \) is the color connected Born matrix element.

Meanwhile, if we parametrize the emitted gluon’s n-dimension momentum in the \( p_1 + p_2 \) rest frame as

\[
p_5 = E_5(1, \ldots, \sin \theta_1 \cos \theta_2, \cos \theta_1),
\]

the three-body phase space in the soft limit can also be factorized as

\[
d\Gamma_3|_{\text{soft}} = d\Gamma_2 \left[\left(\frac{4\pi}{s_{12}}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \frac{1}{2(2\pi)^2}\right] dS,
\]

with

\[
dS = \frac{1}{\pi} \left(\frac{4}{s_{12}}\right)^{-\epsilon} \int_0^{\delta_s \sqrt{s}/2} dE_5 E_5^{1-2\epsilon} \sin^{1-2\epsilon} \theta_1 \, d\theta_1 \times \sin^{-2\epsilon} \theta_2 \, d\theta_2,
\]

as given in ref. [42]. After analytical integration over the soft gluon phase space, the parton level cross section in soft region can be expressed as

\[
\hat{\sigma}^S = \hat{\sigma}^B \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2 R}{s_{12}}\right)^\epsilon \right] \left(\frac{A^S_{2}}{\epsilon^2} + \frac{A^S_{1}}{\epsilon} + A^S_{0}\right)
\]

with

\[
A^S_{2} = 9, \quad A^S_{1} = 3 \left[\ln \left(\frac{s-1}{-t}\right) + \ln \left(\frac{s-1}{-\bar{u}}\right)\right] - 18 \ln \delta_s,
\]

and

\[
A^S_{0} = 18 \ln^2 \delta_s - 6 \ln \delta_s \left[\ln \left(\frac{s-1}{-t}\right) + \ln \left(\frac{s-1}{-\bar{u}}\right)\right] + \frac{3}{2} \left[\ln^2 \left(\frac{s-1}{-t}\right) + \ln^2 \left(\frac{s-1}{-\bar{u}}\right)\right] + \frac{3}{2} \left[Li_2 \left(\frac{-t}{s-1}\right) + Li_2 \left(\frac{-\bar{u}}{s-1}\right)\right],
\]

2. hard collinear

The hard collinear regions of the phase space are those where any invariant \((s_{ij} \text{ or } t_{ij})\) becomes smaller in magnitude than \(\delta_s s_{12}\). It is treated according to whether the singularities are from initial or final state emitting or splitting in the origin. Subprocess in Eq. (12) contains final state collinear singularities, and subprocess in Eq. (13) contains initial state collinear singularities while subprocess in Eq. (11) contains both.

a. final state collinear For subprocesses in Eq. (11) and (13), the final state collinear region is defined by \( 0 \leq s_{45} \leq \delta_s s_{12} \). As a consequence of the factorization derivation [43, 44], the squared matrix element factorizes into the product of a splitting kernel and the LO squared matrix element.

\[
|M_R|^2_{\text{coll}} \simeq 4\pi\alpha_s\mu^2 R \left(\frac{2}{s_{45}}\right) P_{4\gamma}(z, \epsilon) |M^{B}|^2,
\]

where \( 4' \) denotes the parton which splits into parton 4 and 5 collinear pair and \( P_{ij}(z, \epsilon) \) are the unregulated \((z < 1)\) splitting functions in \( n = 4 - 2\epsilon \) dimensions related to the usual Altarelli-Parisi splitting kernels [45] with \( z \) denoting the fraction of the momentum of parton \( 4' \) carried by parton 4. For \( z < 1 \) the \( n \)-dimensional unregulated splitting functions are written as \( P_{ij}(z, \epsilon) = P_{ij}(z) + \epsilon P'_{ij}(z) \) with

\[
P_{qq}(z) = C_F \frac{1 + z^2}{1 - z},
\]

\[
P'_{qq}(z) = -C_F(1 - z),
\]

\[
P_{gg}(z) = 6 \left[\frac{z}{1 - z} + \frac{1 - z}{z} + z(1 - z)\right],
\]

\[
P'_{gg}(z) = 0,
\]

\[
P_{gq}(z) = \frac{1}{2} \left[z^2 + (1 - z)^2\right],
\]

\[
P'_{gq}(z) = -z(1 - z).
\]

Meanwhile, the three-body phase space in the collinear limit can also be factorized as [12]:

\[
d\Gamma_3|_{\text{coll}} = d\Gamma_2 \frac{(4\pi)^\epsilon}{16\pi^2 \Gamma(1-\epsilon)} d\omega ds_{45} [s_{45} z(1 - z)]^{-\epsilon}.
\]

Hence after integrations of \( z \) and \( s_{45} \), the parton level cross section in hard final state collinear region can be expressed as

\[
\hat{\sigma}^{HC} = \hat{\sigma}^B \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2 R}{s_{12}}\right)^\epsilon \right] \times \left(A_{1}^{gg} + A_{0}^{gg} + A_{0}^{qg} + A_{0}^{gq}\right).
\]
where $A_1$ and $A_0$ are
\[ A_1^{gg} = 3 (11/6 + 2 \ln \delta_s) \]
\[ A_0^{gg} = 3 \left[ 67/18 - \pi^2/3 - 2\ln 2 \delta_s - \ln \delta_c (11/6 + 2 \ln \delta_s) \right] \]
\[ A_1^{gq} = -n_f/3 \]
\[ A_0^{gq} = n_f/(3 \ln \delta_c - 5/3) \],
(29)
for subprocesses in Eq. (11) and (12), and
\[ \delta_s = \frac{s_{12}}{s_{12} + s_{45} - M_{j/\psi}^2} \approx \frac{\hat{s}}{\hat{s} - 1} \delta_s. \]
(30)
Thus the total cross section for real correction processes in hard final state collinear region can be written as:
\[ \sigma_{f}^{HC} = \int dz_1 dz_2 G_g(z_1, \mu_f) G_g(z_2, \mu_f) \hat{\sigma}_{f}^{HC}. \]
(31)

b. initial state collinear For subprocess in Eq. (13), the hard initial state collinear region is defined by $0 \leq -t_{25} \leq \delta_c s_{12}$. However for subprocess in Eq. (11), the hard initial state collinear region is defined if any of the following conditions is satisfied $0 \leq -t_{ij} \leq \delta_c s_{12}$, with $i = 1, 2$ and $j = 3, 4$. For convenience, suppose that $2$ and $5$ are the partons involved in the splitting $2 \to 2' + 5$ while $2'$ denotes an internal gluon. Following the similar way as in the final state collinear case, the squared matrix element can be written as
\[ |M_{Rji} |^2_{\text{coll}} \approx 4\pi \alpha_s \mu_{t}^{2\epsilon} \frac{2}{-t_{25}} P_{2'2}(z, \epsilon) |M_{B} |^2, \]
(32)
where $z$ denotes the fraction of parton $2$’s momentum carried by parton $2'$ with parton $5$ taking a fraction $(1 - z)$. And the three-body phase space in the collinear limit can also be factorized as:
\[ d\Gamma_3 |_{\text{coll}} = d\Gamma_2 \frac{(4\pi)^\epsilon}{16\pi^2 \Gamma(1 - \epsilon)} dz dt_{25} [1 - (1 - z) t_{25}]^{-\epsilon}. \]
(33)
The $t_{25}$ integration yields
\[ \int_{0}^{\delta_c s_{12}} (-t_{25})^{-1-\epsilon} = \frac{1}{\epsilon} (\delta_c s_{12})^{-\epsilon}. \]
(34)
If we write the total cross section of LO as
\[ d\sigma^B = dx_1 dx_2 G_g(x_1) G_g(x_2) d\sigma^B, \]
(35)
where $G_g(x_i)$ is the bare PDF. And using above results, the three-body cross section in the hard initial state collinear region can be written as [42]
\[ d\sigma_{f}^{HC} = G_g(x_1) G_g(y) dy d\hat{\sigma}^B (s_{121}, t_{13}, t_{14}) \]
\[ \times \left[ \frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \left( \frac{4\pi \mu_{t}^2}{s_{12}} \right)^\epsilon \right] \]
\[ \times \left( -\frac{1}{\epsilon} \right) \delta_c \hat{\sigma}^B P_{2'2}(z, \epsilon) dz (1 - z)^{-\epsilon} \]
\[ \times \delta(yz - x_2) dx_1 dx_2. \]
(36)

Notice that a factor of $1/z$ has been absorbed into the flux factor for the two-body subprocess, and the delta function used here ensures that the fraction of hadron’s momentum carried by $2'$ is $x_2$. And one more thing that need to be care is, $s_{12}$ here is related to the square of the overall hadronic squared center-of-mass energy $S$ by $s_{12} = x_1 y S$, but in the LO process the relation is $s_{12} = x_1 x_2 S$. From now on, we take the latter definition, so that the replacement $s_{12} \rightarrow y s_{12}/x_2$ should be made. After the $y$ integration we have
\[ d\sigma_{f}^{HC} = G_g(x_1) G_g(z_2, z_1/\epsilon) d\hat{\sigma}^B \left[ \frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \left( \frac{4\pi \mu_{t}^2}{s_{12}} \right)^\epsilon \right] \]
\[ \times \left( -\frac{1}{\epsilon} \right) \delta_c \hat{\sigma}^B P_{2'2}(z, \epsilon) dz \left( \frac{1 - z}{z} \right)^{-\epsilon} dx_1 dx_2. \]
(37)

When all possible two-to-three subprocesses are considered, there will be several contributions, corresponding to a sum over all possible parton 2. It can be $2 = g$ followed by $g \to gg$ or $2 = q(\overline{q})$ followed by $q(\overline{q}) \to q(\overline{q})$.g. The collinear singularity must be factorized and absorbed into the redefinition of the PDF, which is in general called mass factorization [40]. Here we adopt a scale dependent PDF using the modified minimal subtraction (MS) convention given by [42].
\[ G_g(x, \mu_f) = G_g(x) + \left( -\frac{1}{\epsilon} \right) \left[ \frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \left( \frac{4\pi \mu_{t}^2}{\mu_f^2} \right)^\epsilon \right] \]
\[ \times \int_1^{1/\epsilon} dz \frac{1}{z} P_{bb'}(z) G_{b'}(x/z). \]
(38)

Use this definition to replace $G_g(x_2)$ in the LO expression [35] and combine the result with the hard initial collinear contribution [37], then the resulting $\mathcal{O}(\alpha_s)$ expression for the hard initial collinear contribution is [42]
\[ d\sigma_{f}^{HC} = d\hat{\sigma}^B \left[ \frac{\alpha_s}{2\pi} \Gamma(1-\epsilon) \left( \frac{4\pi \mu_{t}^2}{s_{12}} \right)^\epsilon \right] \]
\[ \times \left\{ G_g(x_1, \mu_f) G_g(x_2, \mu_f) + A_1^{nc}(g \to gg) + A_0^{nc}(g \to gg) \right\} dx_1 dx_2, \]
(39)
with
\[ G_{c}(x, \mu_f) = \sum_{c} \int_{x}^{1/\epsilon} dz G_{c'}(x/y, \mu_f) \tilde{P}_{c'}(y), \]
(40)
and
\[ \tilde{P}_{ij}(y) = P_{ij}(y) \ln \left( \frac{y}{1 - y} \frac{\mu_{t}^2}{\mu_f^2} \right) - P_{ij}(y). \]
(41)
The soft collinear factors $A_1^{nc}$ result from the mismatch in the $z$ integrations. They are given by $A_0^{nc} = \ldots$
The total cross section of NLO QCD correction is the initial gluons, thus the cross section of hard initial collinear may happen to either of the initial hadrons, while for subprocess in Eq. (11), initial collinear can come from either initial hadrons, while for subprocess in Eq. (13), the light quark (antiquark) with $A_s$ and $A_{sc}$ add. Thus the cross section of all real corrections becomes

$$\sigma^i_{HC} = \sigma^i_{add} + \int \delta^i_{HC} G_g(x_1, \mu_f) G_g(x_2, \mu_f) dx_1 dx_2,$$

with

$$\sigma^i_{HC} = \int \delta^i_{HC} G_g(x_1, \mu_f) G_g(x_2, \mu_f) dx_1 dx_2,$$

and

$$\delta^i_{HC} = 2\delta^B \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi \mu_r^2}{s_{12}} \right)^\epsilon \right] \times \left[ A^c_i(g \to gg) + A^c_i(g \to gg) \right].$$

C. Cross section of all NLO contributions

The cross section of real correction processes in hard noncollinear regions could be written as

$$\sigma^{HC} = \int \left[ \delta^{HC}(gg \to J/\psi + gg) + \sum_{q=u,d,s,c} \delta^{HC}(gg \to J/\psi + q\bar{q}) \right] dx_1 dx_2 G_g(x_1, \mu_f) G_g(x_2, \mu_f) + \sum_{\alpha=u,d,s,\pi,\bar{d},\bar{\pi}} \delta^{HC}(g\alpha \to J/\psi + g\alpha) \left[ G_g(x_1, \mu_f) G_\alpha(x_2, \mu_f) + (x_1 \leftrightarrow x_2) \right] dx_1 dx_2,$$

Thus the cross section of all real corrections becomes

$$\sigma^R = \sigma^{HC}_{add} + \sigma^{HC}_{add} + \int \left( \delta^{S} + \delta^{HC} + \delta^i_{HC} \right) \times G_g(x_1, \mu_f) G_g(x_2, \mu_f) dx_1 dx_2.$$ (46)

And the total cross section of NLO QCD correction is

$$\sigma^{NLO} = \sigma^{HC}_{add} + \sigma^{HC}_{add} + \sigma^V_{add},$$ (47)

with

$$\sigma^V_{add} = \int \left( \delta^B + \delta^V + \delta^S + \delta_f^{HC} + \delta_i^{HC} \right) \times G_g(x_1, \mu_f) G_g(x_2, \mu_f) dx_1 dx_2.$$ (48)

It is easy to find that there is no IR singularities in above expression, for $2A^V_2 + A^S_2 = 0$ and $2A^V_1 + A^L_1 + A^L_{gg} + A^L_{g\bar{q}} + 2A^c_i(g \to gg) = 0$. The apparent logarithmic $\delta_s$ and $\delta_t$ dependent terms also cancel after numerically integration over the phase space.

IV. TRANSVERSE MOMENTUM DISTRIBUTION

To obtain the transverse momentum distribution of $J/\psi$, a transformation for integration variable $(dx_1 dt \to dp_t dy)$ is introduced. Thus we have

$$\sigma = \int dx_1 dx_2 dt G_g(x_1, \mu_f) G_g(x_2, \mu_f) \frac{d\sigma}{dt},$$

$$= \int J dx_1 dp_t dy G_g(x_1, \mu_f) G_g(x_2, \mu_f) \frac{d\sigma}{dp_t},$$ (49)

and

$$\frac{d\sigma}{dp_t} = \int J dx_1 dy G_g(x_1, \mu_f) G_g(x_2, \mu_f) \frac{d\sigma}{dx_1},$$ (50)

with

$$p_1 = x_1 \frac{\sqrt{S}}{2} (1, 0, 0, 1), \quad p_2 = x_2 \frac{\sqrt{S}}{2} (1, 0, 0, -1),$$

$$m_t = \sqrt{M^2_{J/\psi} + p_t^2}, \quad p_3 = (m_t \cosh y, p_t, 0, m_t \sinh y),$$

$$x_t = 2m_t \sqrt{s}, \quad \tau = \frac{m_t^2 - M^2_{J/\psi}}{\sqrt{s}},$$ (51)

$$J = \frac{4x_1 x_2 p_t}{2x_1 - x_t e^y}, \quad x_2 = \frac{2\tau + x_1 x_t e^{-y}}{2x_1 - x_t e^y},$$

$$x_1|_{\text{min}} = \frac{2\tau + x_t e^y}{2 - x_t e^{-y}},$$

where $\sqrt{s}$ is the center-of-mass energy of $p\bar{p}(p)$ at Tevatron or LHC, $m_t$ is the invariant mass of all the final
state particles except \( J/\psi \), and \( y \) and \( p_t \) are the rapidity and transverse momentum of \( J/\psi \) in the laboratory frame respectively.

V. POLARIZATION

The polarization factor \( \alpha \) is defined as:

\[
\alpha(p_t) = \frac{d\sigma_T/dp_t - 2d\sigma_L/dp_t}{d\sigma_T/dp_t + 2d\sigma_L/dp_t} \tag{52}
\]

It represents the measurement of \( J/\psi \) polarization as function of \( J/\psi \) transverse momentum \( p_t \) when calculated at each point in \( p_t \) distribution. To calculate \( \alpha(p_t) \), the polarization of \( J/\psi \) must be explicitly retained in the calculation. The partonic differential cross section for a polarized \( J/\psi \) could be expressed as:

\[
\frac{d\sigma_T}{dt} = a \epsilon(\lambda) \cdot \epsilon^*(\lambda) + \sum_{i,j=1,2} a_{ij} p_i \cdot \epsilon(\lambda) p_j \cdot \epsilon^*(\lambda), \tag{53}
\]

where \( \lambda = T_1, T_2, L \). \( \epsilon(T_1), \epsilon(T_2) \), \( \epsilon(L) \) are the two transverse polarization vectors and the longitudinal polarization one of \( J/\psi \), and the polarization of all the other particles are summed over in \( n \)-dimensions. It causes a more difficult tensor reduction path than that with all the polarizations being summed over in the calculation of virtual corrections. It is found that \( a \) and \( a_{ij} \) are finite when the virtual corrections and real corrections are summed up. Therefore there is no difference in the differential cross section \( d\sigma_T/dt \) whether the polarization of \( J/\psi \) is summed over in 4 or \( n \) dimensions. Thus we can just treat the polarization vectors of \( J/\psi \) in 4-dimension, and also the spin average factor goes back to 4-dimension.

To make a cross check, we carry out another calculation. Namely, we calculate the differential cross section \( \sigma_{HC}^{add} \) and \( \sigma^{V^+} \) with the the polarizations of all particles being summed up analytically. The results are numerically compared with that obtained without summing up the polarization of \( J/\psi \). Moreover, to check gauge invariance, in the expression we explicitly keep the gluon polarization vector and then replace it by its 4-momentum in the final numerical calculation. Definitely the result must be zero and our results confirm it. To calculate \( \sigma_{HC}^{add} \), only numerical computation is carried out and we only sum over the physical polarizations of the gluons to avoid involving diagrams which contain external ghosts lines.

VI. COLOR FACTOR

There is just one color factor \( d_{c_1c_2c_4} \) for the LO process in amplitude level with \( c_1, c_2 \) and \( c_4 \) being the color indices of the three gluons in the process. And it is the same for the virtual correction process that just one color factor \( d_{c_1c_2c_4} \) appears in amplitude level. For other processes, color factors are orthogonalized and normalized. There are three color factors in amplitude level for real correction process \( g + g \rightarrow J/\psi + g + g \)

\[
\frac{1}{\sqrt{5}} \text{Tr} \left[ T^{c_4c_1} T^{c_5c_2} - T^{c_4c_2} T^{c_5c_1} \right],
\]

\[
\frac{1}{\sqrt{5}} \text{Tr} \left[ T^{c_4c_5} T^{c_1c_2} - T^{c_4c_2} T^{c_5c_1} \right],
\]

\[
\frac{1}{\sqrt{5}} \text{Tr} \left[ T^{c_4c_1} T^{c_2c_5} - T^{c_4c_5} T^{c_2c_1} \right],
\]

where \( c_i \) are the color indices of the external gluons. For \( g + g \rightarrow J/\psi \) almost the same color factor as above. For \( g + g \rightarrow J/\psi + q + \bar{q} \), there is one color factor

\[
\frac{\sqrt{3}}{6\sqrt{5}} \left[ 3(T^{c_1c_2} + T^{c_2c_1})c_{c_4c_5} - \delta_{c_4c_5} \delta_{c_1c_2} \right], \tag{55}
\]

where \( c_1, c_2 \) and \( c_4, c_5 \) are the color indices of the external gluons and quark, anti quarks respectively. And \( g + q \rightarrow J/\psi + g + g \) has almost the same color factor as above. For \( g + g \rightarrow J/\psi + c + \bar{c} \), there are three color factors

\[
\frac{1}{2\sqrt{66}} \left[ 6(T^{c_2c_1})c_{c_4c_5} + \delta_{c_4c_5} \delta_{c_1c_2} \right],
\]

\[
\frac{1}{2\sqrt{858}} \left[ 4(T^{c_2c_1})c_{c_4c_5} - 22(T^{c_1c_2})c_{c_4c_5} - 3\delta_{c_4c_5} \delta_{c_1c_2} \right],
\]

\[
\frac{3\sqrt{26}}{52\sqrt{19}} \left[ 4(T^{c_1c_2})c_{c_4c_5} + 4(T^{c_2c_1})c_{c_4c_5} - 3\delta_{c_4c_5} \delta_{c_1c_2} \right],
\]

where \( c_1, c_2 \) and \( c_4, c_5 \) are the color indices of the external gluons and \( c \) quark, anti \( c \) quarks respectively.

VII. TREATMENT OF \( \Upsilon \)

The production mechanism of \( \Upsilon \) at Tevatron and LHC is much similar to that of \( J/\psi \) except that, color octet states contribute much less in \( \Upsilon \) production according to the experimental data and LO theoretical predictions. We can apply the results of above calculation to the case of \( \Upsilon \) by doing the substitutions:

\[
m_c \leftrightarrow m_b
\]

\[
M_{J/\psi} \leftrightarrow M_{\Upsilon}
\]

\[
R_s(0)^{J/\psi} \leftrightarrow R_s(0)^{\Upsilon}
\]

\[
n_f = 3 \leftrightarrow n_f = 4
\]

Note that charm quark is treated as light quark as an approximation. It is not coincide with the definition of CTEQ6M PDFs used in the calculation. The mass of heavy quark is not zero in the definition of CTEQ6M PDFs. This approximation can cause a small uncertainty.

VIII. NUMERICAL RESULT

In our numerical calculations, the CTEQ6L1 and CTEQ6M PDFs \[47\], and the corresponding fitted value
for $\alpha_s(M_Z) = 0.130$ and $\alpha_s(M_Z) = 0.118$, are used for LO and NLO predictions respectively. At NLO, we use $\alpha_s$ in two-loop formula as

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)} - \frac{\beta_1 \ln(\mu^2/\Lambda_{QCD}^2)}{\beta_0^2 \ln^2(\mu^2/\Lambda_{QCD}^2)},$$

where $\beta_1 = 34C_A^2/3 - 4(C_F + 5C_A/3)T_F n_f$ is two loop coefficient of the QCD beta function. For the heavy quark mass and the wave function at the origin, $m_c = 1.5$ GeV and $|R_c(0)|^2 = 0.810$ GeV$^3$ are used for $J/\psi$, and $m_b = 4.75$ GeV and $|R_b(0)|^2 = 0.479$ GeV$^3$ are used for $\Upsilon$. To choose the renormalization scale $\mu_r$ and the factorization scale $\mu_f$ in the calculations is an important issue and it causes the uncertainties for the calculation. We choose $\mu = \mu_r = \mu_f = \mu_0 = \sqrt{(2m_c)^2 + p_t^2}$ as the default choice in the calculation with $m_Q$ being $m_c$ and $m_b$ for $J/\psi$ and $\Upsilon$ respectively. The center-of-mass energies are chosen as 1.98 TeV at Tevatron and 14 TeV at LHC. The two phase space cutoffs $\delta_s$ and $\delta_c$ are chosen as $\delta_s = 10^{-3}$ and $\delta_c = \delta_s/50$ as default choice. To check the independence of the final results on the two cutoffs, different values of $\delta_s$ and $\delta_c$ are used, where $\delta_s$ can be as small as $\delta_s = 10^{-5}$. And the invariance is observed within the error tolerance of less than one percent.

It is known that the perturbative expansion cannot be applicable to the regions with small transverse momentum and large rapidity of $J/\psi$ or $\Upsilon$. Therefore, $p_t > 3$ GeV are used for all the calculations. For rapidity cut at Tevatron, we choose the same cut condition as the experiments at Tevatron [32, 33]: $|y| < 0.6$ for $J/\psi$ and $|y| < 1.8$ for $\Upsilon$. To follow the same cut condition used in Ref. [39], we choose $|y| < 3$ for all calculation at LHC and another calculation of $J/\psi$ production at Tevatron. All the cut conditions are explicitly expressed for each result.

The dependences of the total cross section at the renormalization scale $\mu_r$ and factorization scale $\mu_f$ are presented in Fig. 5. Since the contribution from the subprocess $gg \to J/\psi c\bar{c}$ is less than 10% of the total result at NLO in the whole region of $\mu$, it gives almost same plot as the Fig. 3 in ref. [39] which does not included the contribution. The results show that the NLO QCD corrections boost the total cross section by a factor of about 2 at the default choice of the scales $\mu = \mu_0 = \sqrt{(2m_c)^2 + p_t^2}$. one can find that the scale dependence at NLO is not improved for $J/\psi$.

In Figs. 6 and 7 the $p_t$ distribution of $J/\psi$ and $\Upsilon$ is shown. It is easy to see that the contribution of NLO correction becomes larger as $p_t$ increases, and in high $p_t$ region, the NLO prediction is 2-3 order of magnitude larger than the LO one. As already known, the contribution from subprocesses $gg \to J/\psi c\bar{c}$ or $gg \to \Upsilon b\bar{b}$, which is also of $\mathcal{O}(\alpha_s^2)$, is large at high $p_t$ region. In order to compare with results in ref. [39] and also to see how large is the contribution, result excluding this contribution is shown in the figures as NLO'. And we could see from the figures that the contribution from $gg \to \Upsilon b\bar{b}$ in $\Upsilon$ production is less than that from $gg \to J/\psi c\bar{c}$ in $J/\psi$ case.

The $p_t$ distribution of $J/\psi$ and $\Upsilon$ polarization factor $\alpha$ is presented in Fig. 8 and Fig. 10. We can see in the figures that $\alpha$ is always positive and becomes closer to 1 as $p_t$ increases at LO, and this figure means that the transverse polarization is more than the longitudinal one and even becomes dominant in high $p_t$ region. But there is dramatical change when the NLO QCD corrections are taken into account. For $J/\psi$, $\alpha$ is always negative and becomes closer to -0.5 as $p_t$ increases, this new figure indicates that the longitudinal polarization is always more.
than the transverse one and even becomes dominant in high $p_t$ region. Meanwhile the $J/\psi$ polarization in subprocess $gg \to J/\psi c\bar{c}$ is near zero. By including contribution of this subprocess, the total result shown in the left diagram of Fig. 9 is closer to the experimental result.

For $\Upsilon$, $\alpha$ varies from positive to negative and becomes closer to -0.6 as $p_t$ increases, this indicates that the longitudinal polarization becomes more and more, and even becomes dominant in high $p_t$ region. The $\Upsilon$ polarization of subprocess $gg \to \Upsilon b\bar{b}$ is also near zero. But from the total result in Fig. 10 we can see that this subprocess contributes less than the corresponding one in case of $J/\psi$. Also, we find that the contribution from light quarks affects the $p_t$ distribution of polarization less than 10%.

When compare the figures for $J/\psi$ with those for $\Upsilon$, we can see that they are very similar to each other except that $\alpha$ is higher and even becomes positive in lower $p_t$ region for $\Upsilon$. It could be understand by extending the curves for $J/\psi$ to lower $p_t$, because for a certain $p_t$ value in $\Upsilon$ production, it corresponds a lower $p_t$ in $J/\psi$ production by just considering the energy scale.

By comparing the experimental measurements for $J/\psi$ and $\Upsilon$ at Tevatron with above results, we could see that, although NLO corrections can boost the transverse momentum distribution of $J/\psi$ and $\Upsilon$ very much, it is still an order of magnitude smaller than the experimental data. The color octet channels are still needed to explain the $p_t$ distribution. Thus the NLO prediction for the polarization of direct $J/\psi$ and $\Upsilon$ via color singlet channel could not be used to compare with experimental data.

We can write contribution of each channel as

$$\sigma^i = C_i \left( C_0 \frac{1}{\epsilon} + C_1 \frac{1}{\epsilon^2} + C_2 \right)$$

where the overall $\epsilon$ dependent factor

$$C_i = \frac{1}{(1-\epsilon)^2} \sum \frac{4\pi \mu_i^2}{(2m_c)^2} e^{-\gamma E},$$

and the term $1/(1-\epsilon)^2$ is from the gluon spin average factor $1/(n-2)$. When all the contributions are summed up, we have $\sum C_i = 0$ and $\sum C_i' = 0$. Thus $C_i$ comes back to 1 and we have our result as $\sum C_i^i$. In Table I, $C_0^i$ is given. It should be careful that the $A_0^\mu(g \to gg)$ term has been put into the $gg \to J/\psi + gg$ channel even if it contains a term proportional to the number of active flavors $n_f$.

| $i$ | process | $C_0^i(10^2\text{nb})$ | $C_0^i/\sigma^{J/\psi}$ fraction |
|-----|---------|-----------------|--------------------------------|
| 1   | $gg \to J/\psi g$ | 0.4061±0.0006 | 0.2174 (0.1056) |
| 2   | $gg \to J/\psi gg$ | 2.47±0.04 | 1.32 (0.64) |
| 3   | $gg \to J/\psi q\bar{q}$ | 0.133±0.001 | 0.071 (0.035) |
| 4   | $gg \to J/\psi q\bar{q}$ | 0.582±0.001 | 0.312 (0.152) |
| 5   | $gg \to J/\psi c\bar{c}$ | 0.2583±0.0003 | 0.1382 (0.0672) |
| $\sum$ | $pp \to J/\psi + X$ | 3.84±0.04 | 2.06 (1.00) |

TABLE I: lists of contributions from each channel to the NLO total cross section of $J/\psi$ hadronproduction at Tevatron in the region $p_t > 3$ GeV and $|y_{J/\psi}| < 3$. We have set $\mu_r = \mu_f = \mu_0$. Corresponding result for $\sigma^{J/\psi}$ is $1.8682 \times 10^2$ nb.

**IX. CONCLUSION AND DISCUSSION**

We have calculated the NLO QCD corrections to the $J/\psi$ and $\Upsilon$ hadronproduction at Tevatron and LHC. Dimensional regularization is applied to deal with the UV
and IR singularities in the calculation, and the Coulomb singularity is isolated by a small relative velocity \( v \) between the quark pair in the meson and absorbed into the bound state wave function. To deal with the soft and collinear singularities in the real corrections, the two-cutoff phase space slicing method is used. By summing over all the contributions, a result which is UV, IR and Coulomb finite is obtained.

Numerically, we obtain a \( K \) factor of total cross section (ratio of NLO to LO) of about 2 for \( J/\psi \). The transverse momentum distributions of \( J/\psi \) and \( \Upsilon \) are presented and they show that the NLO corrections increase the differential cross sections more as \( p_t \) becomes larger and eventually can enhance it by 2 or 3 orders in magnitude at \( p_t = 50 \) GeV. It confirms the calculation by Campbell, Maltoni and Tramontano \[16, 57\]. The real correction subprocesses \( gg \rightarrow J/\psi c\bar{c} \) and \( gg \rightarrow \Upsilon b\bar{b} \) are also calculated and the results are in agreement with those of Ref. \[16, 57\].

The NLO contributions to \( J/\psi \) polarization is studied and our results indicate that the \( J/\psi \) polarization is dramatically changed from more transverse polarization at LO into more longitudinal polarization at NLO. All the results can be directly applied to \( \psi' \) production by multiplying a factor \( \langle \mathcal{O}_{\psi'}^n \rangle / \langle \mathcal{O}_n^\psi \rangle \). The NLO contributions to \( \Upsilon \) polarization is also studied and presented for the first time. Our results indicates that at NLO, the polarization of \( \Upsilon \) decreases gradually from near 0.2 to -0.6 as \( p_t \) increases from 3 GeV to 50 GeV. Namely, the \( p_t \) distribution of the polarization status behaves almost the same as that for \( J/\psi \) except that the NLO result is also transverse polarization at small \( p_t \) range. Since the fact that contribution via color-octet states is much less in \( \Upsilon \) production than that in \( J/\psi \) case, our new result for \( \Upsilon \) polarization plays an important role in understanding the experimental data. And even though our calculation results in a more longitudinal polarization state than the recent experimental result for \( J/\psi \) \[32\] and \( \Upsilon \).
at Tevatron, it raises a hope to solve the large discrepancy between LO theoretical predication and experimental measurement on $J/\psi$ and $\Upsilon$ polarization, and suggests that the next important step is to calculate the NLO corrections to hadron production of color octet state $J/\psi^{(8)}$ and $\Upsilon^{(8)}$. By re-fixing the color-octet matrix elements, we will see what an involvement of the NLO QCD corrections can induce for the polarization of $J/\psi$ and $\Upsilon$.

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