Hairpin Windings: Sensitivity Analysis and Guidelines to Reduce AC Losses

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Abstract— Nowadays, electrification in the transportation sector is one of the most viable solutions to reduce CO₂ emissions and meet fuel economy requirements. Being the electrical machine one of the most important players in this electrification trend, extensive research is currently being dedicated to the improvement of their efficiency and power density. In automotive applications, hairpin technologies are widely spreading due to their potential in reducing costs and life cycles in a mass production perspective, as well as in increasing the torque capabilities of machines. However, several challenges need to be addressed before the complete replacement of random windings with hairpins can take place. Of these challenges, the loss produced during high frequency operations is one of the most limiting. This paper aims at studying and investigating high frequency (AC) losses for different slot geometries and conductor cross sections, which in turn involve the analysis of different slots-per-pole-per-phase / layers-per-slot combinations. In addition, the effects on the AC losses of reducing the slot fill factor are studied, either by removing the closest conductors to the slot opening or by reducing the hairpin legs’ height. Analytical and numerical models are employed to investigate these concepts.

Keywords— AC losses, analytical model, electrical machines, winding, high frequency, automotive, hairpin, random, segmented hairpin, manufacturing, mass production.

I. INTRODUCTION

Due to the ever-more stringent emission and efficiency requirements, there is currently a wide interest in the research and development of more electric vehicles. Governments and transportation industries are pushing their research programs to realize hybrid and pure electric powertrains for both automotive and aerospace, and in general for all transport applications. For example, it is required that all new cars in Europe will emit only 81 g/km of CO₂ by 2025 [1].

When designing such more electrified systems, special attention is given to the power density of components, as all transport applications are volume and weight sensitive. The US Department of Energy has recently announced technical targets for light duty electric vehicles. In terms of power density, a target of 33 kW/L for a 100 kW traction drive system has to be reached by 2025 [2].

One of the key elements in hybrid and electric powertrains is the electrical machine. Of the techniques implemented to enhance the power density of electrical machines, the design for high operating speeds is one of the most effective [3]. However, this results in higher operating fundamental frequencies, which in turn means increased iron and ohmic losses as well as more elevated structural issues. Additionally, the relative distribution of these losses highly depends on the type of converter used to supply the machine [4]. As a typical electric/hybrid-electric vehicle propulsion motor can operate with fundamental frequencies higher than 1 kHz, these high-frequency challenges cannot be neglected. Referring to the ohmic losses, while at low frequency operation the current distribution within the armature conductors is usually uniform, at high frequency skin and proximity effects can occur if no suitable precautions are taken, thus leading to higher conductors’ resistances and losses [5]. To mitigate these phenomena, Litz wires are often used. These consist of bundles comprising very thin strands and present a very low AC-to-DC loss ratio even at frequencies higher than 1 kHz. However, they also have some disadvantages, such as complex shaping and impregnation, low fill factor and high manufacturing costs [6]. Hence, their use is only justified in very high frequency and no cost-sensitive applications. Therefore, most commonly, random-wound windings are employed. This winding topology represents a good trade-off between costs and performance. Also in this case the conductors can be subdivided in parallel strands, but their cross section is higher than those of a Litz wire. This kind of winding is a very competitive solution in automotive where a low number of turns is typically chosen to achieve low voltages and high currents [7].

This level of flexibility cannot be achieved when using bar-wound windings (namely the hairpin winding) [8]. Due to their pre-formed nature, the subdivision in strands cannot be flexibly applied, the number of turns is always even and the maximum number of conductors per slot is limited. The stress on insulation and conductive materials can be excessive when bending and twisting operations are not properly performed [9], [10]. Therefore, only a few configurations are feasible and can be manufactured [11], [12]. However, hairpin windings achieve a higher fill factor compared to the winding types mentioned above, thus obtaining higher current density and peak torque. In addition, in a series production context, a fully automated manufacturing process is possible, potentially reducing the associated costs [13]. Hence, this winding technology is currently seeing an ever-increasing interest for transport applications.

Nevertheless, the main bottleneck for the hairpin winding widespread is represented by the elevated AC losses at high frequency operation [14]. Current research efforts include new winding concepts and methodologies to overcome their high frequency challenges, as well as advanced models for AC losses prediction. Contrarily to random windings, where the main error in modeling AC losses comes from the unknown strands position [15], in hairpin windings a more
accurate AC losses estimation is possible thanks to the a-priori knowledge of the conductors’ position.

A review of analytical models to calculate AC losses in solid rectangular wires inside the slot is presented in [16]. A 2D analytical model able to calculate AC losses is presented in [17]. In [18], a comparison between random and hairpin windings is carried out always in terms of AC losses. In [19], innovative winding patterns that can significantly reduce AC losses are introduced. Always in [19], modifications to the classical analytical models are proposed for predicting AC losses in such new winding concepts.

However, a sensitivity study on how the number of slots-per-pole-per-phase, the number of layers-per-slots and the slot fill factor impact the AC losses in hairpin windings is missing in literature. These parameters have a direct impact on the slot geometry and on the bars’ cross sections, which are thus also evaluated. In addition, a 1D analytical model is first described and then validated against finite-element (FE) evaluations, highlighting inaccuracies under some geometrical conditions at very high frequencies (>1 kHz). Therefore, the FE method is used for the sensitivity analysis performed in this work.

II. ANALYTICAL MODEL

In hairpin windings, above a certain frequency level, the current distribution within the conductors becomes uneven and, thus, their actual (AC) resistance increases compared to the case when the current is uniformly distributed (DC). Therefore, AC ohmic losses can be much higher than DC ones, depending on the severity of skin and proximity effects. In addition, current flows can occur between conductors due to the non-linear impact of such phenomena along a stator slot. While with a perfect transposition these circulating currents can be nulled, skin and proximity effects cannot be neglected. Hence, their accurate estimation is necessary.

A. Domain discretization

To such purposes, a 1D model can be used leveraging on the discretization of the domain under study. This allows to achieve an acceptable accuracy while minimizing the calculation effort. The domain under analysis is shown in Fig. 1 and consists of a single slot comprising \( k \) conductors (hairpin legs). This domain can be analyzed using the discretization layer approach, where each conductor is located in a layer. The assumed slot geometry is rectangular with parallel sides, as usually designed when hairpins are used. The magnetic field produced by the current flowing in the conductors is considered parallel to their larger dimension \( W_{ch} \), thus the field component parallel to the shorter conductors’ dimension \( h \), is neglected.

\[
H_{wk} = \int_{y_{k-1}}^{y_k} - I_k w_{ck} \, dr + I_{k-1}
\]

\[
l_{k-1} = \sum_{j=0}^{k-1} l_z
\]

Manipulating (1) leads to the partial differential equation provided in (3) and, applying the Faraday’s law to the circuit \( \Upsilon_k \) of Fig 1, it is relatively straightforward to obtain (4), where the phasorial assumption is exploited thanks to the hypotheses mentioned above. In (4), \( \omega \) is the supply electric pulsation, while \( \mu \) and \( \sigma \) are the magnetic permeability and the electric conductivity of the conductive material, respectively.

\[
\frac{\delta H}{\delta r} = - \frac{I_k w_{ck}}{w_{sk}}
\]

\[
\frac{\delta I}{\delta t} = - \mu \sigma \frac{\delta H}{\delta t} = - j \omega \mu \sigma H
\]

The combination of (3) and (4) results in the final expression in (5), whose general solution is provided in (6). Here, \( p \) is as defined in (7), where \( \delta \) is the skin depth reported in (8).

\[
\frac{\delta^2 H}{\delta r^2} + \frac{j \omega \mu}{w_{ck}} = 0
\]

\[
H = A_1 e^{-pr} + A_2 e^{pr}
\]

\[
p = \frac{1 + j}{\delta}
\]

\[
\delta = \frac{w_{ck} 2}{\sqrt{w_{ck} \omega \mu \sigma}}
\]

The constant terms \( A_1, A_2 \) are found through the boundary conditions (9) and (10), where \( h \) is the height of the considered layer.

\[
H = \frac{I_{k-1}}{w_{sk}} ; \quad r = 0
\]

\[
H = \frac{I_k}{w_{sk}} ; \quad r = h
\]

The current density in the \( k \)-th layer is finally obtained as in (11), whereas the associated losses are determined as in (12).
\[ J_k = - \frac{l_k p \cosh(p r) - l_{k-1} p \cosh(p h - p r)}{w_k \sinh(ph)}; r \in [0, h] \]  
\[ P_k = \int_0^{\infty} \frac{l_k^2}{\sigma} \, dv = lck \, w_{ck} \int_0^h \frac{l_k^2}{\sigma} \, dr; \quad r \in [0, h] \]

This final expression represents Ohm’s law that is used to calculate the losses in each layer for all the considered case studies. These are presented in the next subsection.

C. Case studies

The investigated case studies relate to a stator design exercise for a motor intended for traction applications, with peak power of 115 kW and a maximum operating speed of 12000 rpm. Hairpin windings are used to fill the stator slots. The stator yoke material is NO20 SiFe lamination, whereas the hairpin material is pure copper. In particular, three different slot topologies are considered, corresponding to a number of slots-per-pole-per-phase \( q \) equal to 2, 3 and 4. The stator inner diameter, the outer diameter and the slot height are kept unchanged and the slot fill factor is imposed theoretically equal to 1 in first instance, meaning that the whole slot is filled with copper. Under such conditions, varying \( q \) implies varying the width of the various hairpin legs. In addition, the sensitivity study involves changing the number of layers \( k \) from 2 to 8, obviously excluding the odd numbers. Considering a number of slots-per-pole-per-phase higher than 4 or a number of layers greater than 8 would reduce the bar width to such a value that challenges in terms of mechanical vibrations, bending and twisting can occur at the end winding regions. A summary of the slot and conductors’ dimensions is provided in Table I for all the envisioned combinations.

The analytical model presented in the Section II.B is used to compare AC losses among all these configurations. In addition, a FE model is built to validate the analytical results. A sketch of the three stator angular sectors comprising 2, 3 and 4 slots-per-pole-per-phase is shown in Fig. 2. The conductors are modelled as solid to take skin and proximity effects into account. The same magneto motive force is applied to have a fair comparison among all cases. Time harmonic simulations are used to evaluate the AC losses and compare them against the analytical findings. The comparative exercise is the focus of the next subsection.

![FIG. 2. Stator angular sectors built and analyzed in the FE models for a) \( q=4 \); b) \( q=3 \); c) \( q=2 \).](image)

D. Analytical vs. FE Results

Aiming to highlight the limits of the 1D analytical model, the comparison between analytical and FE models is shown in Fig. 3 for the various combinations resulting from changing \( q \) and \( k \). In particular, Figures 3a, 3b and 3c respectively report the studies for \( q=4, q=3 \) and \( q=2 \) (varying \( k \) for each study), which are relative to the stator portions shown in Fig. 2. The analysis is carried out at several frequency values, ranging from 0 Hz to 3 kHz. It is reasonable to predict that the higher the ratio between slot width and slot height, the more accurate the analytical model will be, being the hypothesis of neglecting the field along the direction parallel to \( h \) very close the real case. The case study with \( q=4 \) (Fig. 3a) registers the best accuracy for the analytical model, with a maximum error of 7% obtained at 3 kHz and with \( k=2 \). It can be also noticed that the match is excellent up to 2.5 kHz for any \( k \) value. With \( q=3 \) (Fig. 3b) the discrepancies between the methods increase and become more and more evident above 1 kHz. The lowest accuracy is achieved for the \( q=2 \) case (Fig. 3c), with a maximum error of 45% registered at 3 kHz and with \( k=2 \). An acceptable match is achieved below 500 Hz, while above such frequency value the analytical method is rather inaccurate. In general, it can be noticed that the lower the number of layers, the lower the accuracy of the 1D analytical model. Taking as example the case study \( q=2 \), the error at 3 kHz goes from 18% to 45% ranging from 8 to 2 layers.

It can be concluded that, after a certain frequency range, the analytical model is not able to accurately predict the AC losses, due to the complexity of the phenomena. Under specific geometrical conditions, such as \( q=2 \) and/or \( k=2 \), the analytical prediction inaccuracy is even more emphasized. Therefore, for the sake of the sensitivity analysis discussed in this work, the FE model is used to extract useful design recommendations and guidelines that will be provided in the next section.

### TABLE I

| Slot and Conductor Geometrical Parameters | Value(mm) |
|------------------------------------------|-----------|
| Description                              | \( q=2 \) | \( q=3 \) | \( q=4 \) |
| Slot height                               | 24        | 24        | 24        |
| Slot width                                | 8.8       | 5.8       | 4.3       |
| Copper height(8layer)                     | 3         | 3         | 3         |
| Copper width(8layer)                      | 8.8       | 5.8       | 4.3       |
| Copper height(6layer)                     | 4         | 4         | 4         |
| Copper width(6layer)                      | 8.8       | 5.8       | 4.3       |
| Copper height(4layer)                     | 6         | 6         | 6         |
| Copper width(4layer)                      | 8.8       | 5.8       | 4.3       |
| Copper height(2layer)                     | 12        | 12        | 12        |
| Copper width(2layer)                      | 8.8       | 5.8       | 4.3       |

III. Influence of the Number of Layer to the Total AC Losses

Both the analytical and FE findings reported in Section II have proven that the losses depend both on the number of slots-per-pole-per-phase (and therefore on the slot geometry) and on the number of layers (and therefore on the bar geometry). Once the stator lamination is designed according to the design requirements, the only degree of freedom to reduce copper losses is the winding configuration. When using hairpin windings, while a perfect transposition ensures that no currents will be circulating among the conductors, the
losses due to skin and proximity effects can have a disruptive role at high frequency operation. In this context, the number of layers per-slot can help mitigating these phenomena. In Table II, III and IV, the influence of the number of layers on the total losses is evaluated for different \( q \) values, ranging from 0 Hz to 3 kHz. The minimum loss values are shown and underlined in red. The results of Table II, III and IV highlight that, for any value of the number of slots-per-pole-per-phase, the AC losses can be reduced by reducing the number of layers. This is partly contradictory with what is usually found in literature, i.e., the losses can be reduced by decreasing the conductors’ height. In fact, the tables inform that the latter statement is true at relatively low frequency, whereas the increase in frequency suggests reducing the number of layers-per-slot. The reason behind such conclusion is that at high frequency the proximity effect is much higher than the skin one. This concept can be seen in Fig. 4, where a current density maps for the cases with 2 and 8 conductors are shown and compared, at 100 Hz (Fig. 4a) and 3 kHz (Fig. 4b).

**TABLE II**

| Frequency (Hz) | 2 Layer | 4 Layer | 6 Layer | 8 Layer |
|----------------|---------|---------|---------|---------|
| 100            | 0.332   | 0.172   | 0.124   | 0.107   |
| 500            | 1.028   | 1.485   | 1.000   | 0.685   |
| 1000           | 1.443   | 2.763   | 2.840   | 2.115   |
| 1500           | 1.809   | 3.444   | 4.500   | 3.984   |
| 2000           | 2.434   | 3.923   | 5.725   | 5.849   |
| 2500           | 2.429   | 4.337   | 6.617   | 7.505   |
| 3000           | 2.700   | 4.727   | 7.300   | 8.903   |

**TABLE III**

| Frequency (Hz) | 2 Layer | 4 Layer | 6 Layer | 8 Layer |
|----------------|---------|---------|---------|---------|
| 100            | 0.336   | 0.175   | 0.128   | 0.108   |
| 500            | 1.073   | 1.542   | 1.069   | 0.908   |
| 1000           | 1.570   | 2.894   | 3.023   | 2.268   |
| 1500           | 2.014   | 3.646   | 4.773   | 4.228   |
| 2000           | 2.411   | 4.195   | 6.074   | 6.169   |
| 2500           | 2.774   | 4.678   | 7.035   | 7.890   |
| 3000           | 3.111   | 5.135   | 7.785   | 9.345   |

**TABLE IV**

| Frequency (Hz) | 2 Layer | 4 Layer | 6 Layer | 8 Layer |
|----------------|---------|---------|---------|---------|
| 100            | 0.353   | 0.193   | 0.144   | 0.127   |
| 500            | 1.311   | 1.786   | 1.308   | 0.955   |
| 1000           | 2.082   | 3.394   | 3.512   | 2.797   |
| 1500           | 2.205   | 4.395   | 5.483   | 5.008   |
| 2000           | 2.725   | 5.187   | 7.002   | 7.185   |
| 2500           | 4.031   | 5.912   | 8.183   | 9.132   |
| 3000           | 4.605   | 6.606   | 9.153   | 10.805  |

Fig. 4. Current density distribution for stators with \( q=2, \) \( k=8 \) and \( k=2 \) at a) 100 Hz and b) 3 kHz.

In particular, increasing the number of layers results in a reduction of the conductor height. In turn, being the skin depth dependent on the conductor dimensions (see (8)), the frequency at which the skin effect becomes evident increases with the reduction of the bar height, as proved in Table V.
Therefore, this strategy is effective in terms of $P_{\text{skin}}$ reduction up to a maximum frequency limit. Above this frequency limit, $P_{\text{pin}}$ becomes the highest contributor to $P_{\text{AC}}$.

| Winding Configuration | Bar height (mm) | Frequency where skin effect begins (Hz) |
|-----------------------|-----------------|----------------------------------------|
| 2 layers              | 12              | 30                                     |
| 4 layers              | 6               | 122                                    |
| 6 layers              | 4               | 274                                    |
| 8 layers              | 3               | 488                                    |

However, it must be kept in mind that the current frequency limitations deriving from the power electronics is rather lower than 3 kHz, so such a value is considered in this sensitivity study just for carrying out general design observations.

The analysis presented here suggests that the number of conductors within the slot, besides defining the voltage and magneto motive force levels of the machine at hand, can play an importance role in reducing the AC losses in hairpin windings. In light of this and of the results presented in Table II, III and IV, depending on the operating frequency range, a suitable choice of the number of layers can improve the overall performance of electrical machines equipping hairpin conductors.

IV. INFLUENCE OF THE FILL FACTOR TO THE TOTAL AC LOSSES

At winding design stage, the trend is to maximize the slot fill factor in order to reduce the DC losses and increase the current density. In the case where the current density constraint is satisfied also by a lower slot fill factor, two different solutions could be adopted:

1) When the voltage value constrains the number of turns, the height of the bars can be reduced and the conductors are moved farer from the slot opening, as in its proximity the high frequency phenomena are more evident. This method is schematized in Fig. 5, where basically the fill factor is reduced from 1 to 0.5, always maintaining the same slot height. For simplicity, only the case with $q=4$ and $k=4$ is presented, as similar conclusions can be drawn for the cases where different values of $q$ and $k$ are considered.

![Fig. 5. Stator with $q=4$ and $k=4$ with different slot fill factors: a) 1; b) 0.90; c) 0.8; d) 0.7; e) 0.6; f) 0.5.](image)

2) When the voltage value does not constrain the number of turns, the closest conductors to the slot opening can be removed, keeping unaltered the hairpin legs dimensions.

This method is schematized in Fig. 6, where 2 conductors are removed from the benchmark case study, i.e., $k=4$.

![Fig. 6. Stator with $q=4$ and $k=4$ with fill factor equal to 0.5 (2 conductors closest to the slot opening zone removed).](image)

In Fig. 7, the influence of the fill factor on the copper losses for frequency values ranging from 0 Hz to 3 kHz is reported keeping the same magneto motive force used in the previous analyses. This study shows that, within a certain frequency range, reducing the fill factor can lead to a significant reduction of the AC losses, whereas this benefit decreases at very high frequencies. In Fig. 8, the percentage loss reduction achieved using lower fill factor values than 1 is illustrated.

Considering the best scenario, i.e. fill factor equal to 0.5, from 200 Hz to 1.3 kHz the loss reduction is always larger than 40%, and reaches $\sim$70% at $\sim$500 Hz.

In Fig. 9, it is shown how the removal of the closest conductors to the slot opening could be an effective strategy to reduce AC losses. While this interesting result can be used as a design guideline in hairpin windings, the negative effects of such technique on the torque capability need to be also considered.

![Fig. 7. AC losses for the case study with $q=4$ and $k=4$, with variable slot fill factors.](image)

![Fig. 8. Percentage loss reduction with respect to the case with fill factor equal to 1.](image)

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The focus of this work was to predict the high frequency losses in hairpin windings and to provide some design recommendations and strategies aimed at minimizing them. A 1D analytical model was presented and implemented with the aim of predicting the high frequency copper losses. However, under specific geometrical and operating conditions (i.e., with low number of slots-per-pole-per-phase and with very high operating frequencies), the model was deemed to be rather inaccurate (as opposed to a finite-element approach) for investigating the AC losses’ sensitivity to some design parameters, which is the main objective of this work.

In particular, taking as case study a stator designed for an automotive application, the variation of losses against the following design parameters was analysed:

1) The number of slots-per-pole-per-phase \( q \), which in turn results in a modification of the conductors’ width. The findings showed that a too low \( q \) value produces higher losses, although in general its variation does not lead to significant benefits.

2) The number of layers-per-slot \( k \), which in turn results in a modification of the conductors’ height. Although the findings showed that increasing \( k \) can lead to a significant AC loss reduction up to a certain frequency range, i.e., \( \approx 500 \) Hz, a lower number of layers can be a very effective means to reduce losses at high frequency values.

3) The fill factor, which in turn means either reducing the number of conductors within the slot or decreasing their height. Both strategies showed a significant effectiveness. However, the fill factor reduction should be carefully evaluated against the torque capabilities of the machine.

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