Axion and neutrino red-giant bounds updated with geometric distance determinations

Francesco Capozzi and Georg Raffelt
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
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The brightness of the tip of the red-giant branch (TRGB) allows one to constrain novel energy losses that would lead to a larger core mass at helium ignition and thus to a brighter TRGB than predicted by standard stellar models. The required absolute TRGB calibration depends on reliable distances to the observed ensembles of stars. Motivated by its role as a rung in the cosmic distance ladder, the TRGB was recently recalibrated with geometric distance determinations of the Magellanic Clouds based on detached eclipsing binaries (DEBs). Moreover, we revise previous TRGB calibrations of the galactic globular clusters M5 and ω Centauri with recent kinematical distance determinations based on Gaia DR2 data. All of these TRGB calibrations have similar uncertainties and they agree with each other and with recent dedicated stellar models. We thus find an updated constraint on the axion-electron coupling of $g_{ae} < 1.6 \times 10^{-13}$ (95% CL) and $\mu_\nu < 1.5 \times 10^{-12} \mu_B$ (95% CL) on a possible neutrino dipole moment. The reduced observational errors imply that stellar evolution theory and bolometric corrections begin to dominate the overall uncertainties.

I. INTRODUCTION

The evolution of a low-mass star as it ascends the red-giant branch (RGB) is driven by the growing mass and shrinking size of its degenerate core until helium ignites and the core quickly expands [1]. The abrupt transition to a much dimmer helium-burning star on the horizontal branch (HB) leaves a distinct discontinuity at the tip of the red-giant branch (TRGB). It has been used for several fundamental applications besides, of course, for testing stellar-evolution theory.

Our main interest is to use the TRGB as a particle-physics laboratory in the sense that the emission of new low-mass particles, notably axions or neutrinos with anomalous magnetic dipole moments, would provide additional cooling of the helium core, thus increase the core mass before helium ignites, and therefore lead to a brighter TRGB. Comparing the modified stellar models with empirical calibrations provides constraints on e.g. the axion-electron interaction strength $g_{ae}$ or the anomalous neutrino dipole moment $\mu_\nu$ [2–12].

Traditionally these studies relied on globular-cluster stars and the main uncertainty derived from the adopted distances. So our reconsideration is motivated, in part, by recent kinematical distance determinations for several galactic globular clusters based on Gaia DR2 data [13]. This method is geometrical and does not rely, for example, on the HB brightness. The reported distances typically agree very well with the traditional ones [14], but have much smaller uncertainties.

A second motivation is the availability of new theoretical reference models explicitly for the purpose of TRGB calibration, including detailed error estimates [15]. These authors find that for stellar parameters appropriate for the globular cluster M5, their calibration agrees perfectly with earlier models dedicated to M5 [8] after one corrects for the treatment of screening of nuclear reaction rates relevant for conditions on the RGB.

Another empirical TRGB calibration uses red giants in the haloes of galaxies as these also represent an old population of stars. In our own galaxy, eventually Gaia parallaxes should provide such a calibration, but current results are not yet competitive [16]. However, the TRGBs of halo red giants in other galaxies have long been used as standard candles for distance determinations. One still needs a distance anchor, for example the Large Magellanic Cloud (LMC). Recently a precise LMC distance determination based on 20 detached eclipsing binaries (DEBs) has become available [18] and has been used for a detailed TRGB calibration and determination of the Hubble constant [19, 20].

So another motivation for reconsidering the red-giant particle bounds is to use extragalactic TRGB calibrations for the first time and specifically those recent ones that are based on geometric distance indicators. Besides the LMC [19, 20], this includes the Small Magellanic Cloud (SMC) by the same authors [20], and the galaxy NGC 4258 (M106) where a water megamaser provides a geometric distance estimate [17].

Each of these efforts provides the absolute I-band brightness $M_I^{TRGB}$ of the TRGB at a reference color, here taken to be $(V - I)^{TRGB} = 1.8$ mag. The true variation of $M_I^{TRGB}$ with $(V - I)^{TRGB}$ or with metallicity is somewhat debated. The empirical color dependence of Ref. [17] and the theoretical one of Ref. [15] agree that the slope is very small in the relevant range around $(V - I)^{TRGB} = 1.8$ and the LMC and SMC calibration of Refs. [19, 20] assumed a vanishing slope as an input assumption. Indeed, the attraction of using the I-band TRGB brightness as a standard candle is precisely its weak, if any, dependence on color or metallicity. In this sense $M_I^{TRGB}$ can be seen as the TRGB brightness, or it

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1 See e.g. the Introduction of Ref. [17] for a historical review and their Table 7 for TRGB calibrations up to the year 2016.

2 See e.g. Fig. 13 of Ref. [17] for a compilation of previous findings and Ref. [15] for a recent theoretical appraisal.
can be seen as the zero point at \((V - I)_{\text{TRGB}} = 1.8\) if a non-vanishing slope is considered.

Our first result, summarised in Fig. 1 and discussed in Sec. II, is a compilation of those recent TRGB calibrations that are based on direct geometric distance indicators. Moreover, in Sec. III we compare the theoretical reference models of Serenelli et al. [15], which henceforth we will refer to as S17,3 with the earlier ones of Viaux et al. [8], henceforth V13, and the uncertainties identified by these groups. These theoretical calibrations are also shown in Fig. 1, where in one case a Gaussian distribution of errors is assumed, in the other a maximum range of uncertainty. The stated zero points fortuitously are the same after the models of V13 have been corrected for the screening issue pointed out by S17. Note that the errors on what we call theoretical predictions include a contribution from the empirical bolometric correction (BC), which is essential to compare with observational data. To distinguish this uncertainty from stellar evolution theory we show in Fig. 1 theoretical error bars with (upper) and without (lower) the error of the BC.

We continue in Sec. IV with deriving limits on neutrino dipole moments and the axion-electron coupling by comparing the empirical calibrations of Sec. II with the modified theoretical models of V13 that were derived for the globular cluster M5, but equally apply to the other cases at the reference color \((V - I)_{\text{TRGB}} = 1.8\) mag. We finally wrap up in Sec. V with a discussion and summary.

II. EMPIRICAL TRGB CALIBRATIONS

A. Large Magellanic Cloud (LMC)

The LMC is a favored target to calibrate the TRGB because there are many observations and many distance indicators. Motivated by the role of the TRGB for the cosmic distance ladder, the most recent calibration was performed by Freedman et al. (2019, 2020) [19, 20] which henceforth we refer to as F19 and F20. They used stars outside the LMC bar, with data taken from OGLE-III for the I and V bands and from 2MASS for the JHK bands. The I-band TRGB in the LMC is spread over a range of colors for which F20 used \((V - I)_{\text{TRGB}} = 1.6 - 2.2\) (see the white box in the upper-right panel of their Fig. 5) which corresponds to the true color range 1.8 - 2.4. Moreover, they assumed that \(M_I^{\text{TRGB}}\) would not depend on \((V - I)_{\text{TRGB}}\) across this range.

With the usual technique of searching for the edge in the luminosity function (see their Section 3.1), F20 found \(\mu_{\text{LMC}}^{\text{TRGB}} = 14.595 \pm 0.023\). The correction for absorption was adopted as \(A_I = 0.160 \pm 0.02\), so for the true \(\mu_{\text{LMC}}^{\text{TRGB}}\) they found \((14.595 \pm 0.023) - (0.160 \pm 0.02) = 14.435 \pm 0.031\) after adding the errors in quadrature.

For the distance they used the geometric determination of Pietrzyński et al. (2019) [18] based on 20 DEBs. These authors found the true distance modulus to be \(\mu_{\text{LMC}} = 18.477 \pm 0.004_{\text{stat}} \pm 0.026_{\text{sys}} = 18.477 \pm 0.026\) where the final error is dominated by systematics. Subtracting this from the extinction-corrected \(\mu_{\text{TRGB}}^{\text{LMC}}\), F20 found

\[
M_I^{\text{TRGB}} = -4.047 \pm 0.022_{\text{stat}} \pm 0.039_{\text{sys}} = -4.047 \pm 0.045.
\] (1)

This result applies at the color \((V - I)_{\text{TRGB}} = 1.8\) mag, which fortuitously agrees with the globular cluster M5 that we consider in Sec. II E below.

An earlier study by Jang and Lee (2017) [17], who are co-authors of F20, was also based on the OGLE-III

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3 We will frequently refer to the following papers:

V13 — Viaux et al. (2013) [8].
S17 — Serenelli et al. (2017) [15].
F19 — Freedman et al. (2019) [19].
F20 — Freedman et al. (2020) [20].
Y19 — Yuan et al. (2019) [21].

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**FIG. 1.** Summary of TRGB calibrations, showing 68% and 95% confidence intervals (see Sec. II). For the theoretical prediction of Serenelli et al. [15] we show instead a maximum uncertainty (see Sec. III). The upper part of each theoretical error bar includes the contribution of the bolometric correction, whereas the lower part only includes uncertainties from stellar evolution theory. The cases marked “update” are our updates of previous results. The gray-shaded cases are only shown for illustration and comparison.
data and led to the true $I_{\text{TRGB}}^{\text{LMC}} = 14.490 \pm 0.082$ which is dominated by the adopted extinction error of $\pm 0.07$. With the previous DEB distance of 18.494 $\pm 0.049$ this leads to $M_I^{\text{TRGB}} = -4.004 \pm 0.096$. Within errors, this agrees with Eq. (1), which however has a much smaller error primarily due to a more precise distance and more detailed treatment of extinction.

However, other authors have found somewhat different results. Motivated by the question of the true value of the Hubble constant $H_0$, Yuan et al. [21], henceforth Y19, pointed out that in F19 the Magellanic Clouds Photometric Survey (MCPS, [22]) of the SMC was used to constrain the extinction of the TRGB in the LMC, whereas the OGLE-III data was adopted for the LMC itself. However, when applying MCPS data to SMC and OGLE-III data to the LMC one needs to take into account some inconsistency between the two data sets, mostly connected to the significant blending effect in MCPS. According to Y19 not taking into account this inconsistency introduces a bias in the TRGB zero point.

The final calibration obtained by Y19 is $M_I^{\text{TRGB}}_{F814W} = -3.97 \pm 0.046$, where $F814W$ is the Hubble Space Telescope equivalent of the $I$-band filter, so the TRGB is 0.08 mag dimmer than found in F19. Correspondingly Y19 found $H_0 = 72.4 \pm 2.0$ km s$^{-1}$ Mpc$^{-1}$, which is 2.6 km s$^{-1}$ Mpc$^{-1}$ larger than found by F19 and more consistent with the one based on Cepheid distances.

On the other hand, F20 have provided some critique of Y19 (the Appendix in F20) and have updated their TRGB calibration without using MCPS data for evaluating the LMC extinction. Their final zero point $M_I^{\text{TRGB}} = -4.047$ is basically unchanged with respect to F19.

For deriving particle bounds we will use the result of F20, which is a conservative choice in the sense that the Y19 calibration, compared with theoretical models, would provide more restrictive limits. However, we notice that details of the analysis have a significant impact both on the Hubble tension debate and on particle bounds. As the question of LMC extinction is perhaps not entirely settled, it is much more important to consider other TRGB calibrations that are not anchored to the LMC, even if the LMC calibration is nominally the one with the smallest error as seen in Fig. 1.

### B. Small Magellanic Cloud (SMC)

F20 also considered the SMC using a DEB-based distance determination. After locating the edge in the luminosity function and applying an extinction correction, they found

$$M_I^{\text{TRGB}} = -4.09 \pm 0.03_{\text{stat}} \pm 0.05_{\text{sys}}$$

$$= -4.09 \pm 0.06. \quad (2)$$

F20 assume that the $I$-band TRGB has no slope in terms of $(V - I)$ color and use this assumption in their edge-finding algorithm. The stars in the white box in the CMD in their Fig. 6 were used with an average $(V - I)$ of around 1.6. Both the empirical color dependence of Jang and Lee (2017) [17] and the theoretical one of S17 (2017) [15], shown below in our Eq. (13), is very weak, so interpreting the SMC calibration at our fiducial color of $(V - I)^{\text{TRGB}} = 1.8$ introduces a shift in the dim direction of at most 0.01 mag that we ignore.

### C. Galaxy NGC 4258 (M106)

Jang and Lee (2017) [17] also considered the galaxy NGC 4258 (M106) that hosts a water megamaser that can be used as a geometric distance indicator. They used the latest distance determination by Riess et al. (2016) [23] who found $d_{N4258} = 7.54 \pm 0.17_{\text{stat}} \pm 0.10_{\text{sys}}$ Mpc, corresponding to a distance modulus of $\mu_{N4258} = 29.387 \pm 0.057$ mag. Overall, these authors found $M_I^{\text{TRGB}} = -4.023 \pm 0.073$, where the error derives from adding the entries in their Table 4 in quadrature. This calibration pertains to $(V - I)^{\text{TRGB}} = 1.5$. To translate it to our fiducial color $(V - I)^{\text{TRGB}} = 1.8$, for consistency we use the color dependence provided in the last line of their Table 7 which implies a dimming by 0.006 mag, so finally

$$M_I^{\text{TRGB}} = -4.017 \pm 0.073 \quad (3)$$

is the zero-point calibration relevant for our comparison.

### D. Globular Cluster ω Centauri (NGC 5139)

The most luminous globular cluster in our galaxy is ω Centauri. Its TRGB was calibrated by Bellazzini, Ferraro and Pancino (2001) [24] and it was used to constrain novel energy losses of red giants by Arceo-D´ıaz et al. (2015) [11]. The $I$-band TRGB was found to be $I_{\omega\text{Cen}} = 9.84 \pm 0.04$ mag by searching for the corresponding edge in the luminosity function [24]. This result must be corrected for the amount of extinction in the $I$ band for which Bellazzini et al. used $A_I = 1.76 (0.13 \pm 0.02)$ mag, so the true $I$-band brightness is $I_{\omega\text{Cen}} - A_I = 9.61 \pm 0.05$ mag.

A distance of 5.360 $\pm 0.300$ kpc was determined with the detached eclipsing binary OGLE GC17 by Thompson et al. (2001) [25]. The corresponding distance modulus is 13.65 $\pm 0.11$ and fixes the absolute brightness to be $M_I^{\text{TRGB}} = -4.04 \pm 0.12$ mag.\footnote{We use the linearized mapping between distance and distance modulus, as done in Bellazzini et al. [24]. Adopting the logarithmic mapping would lead to a distance modulus of 13.656 $\pm 0.121$.}

This calibration is at the color $(V - I)^{\text{TRGB}} = 1.5$. S17’s theoretical calibration shown in Eq. (13) suggests that the brightness difference between this and our fiducial color $(V - I)^{\text{TRGB}} = 1.8$ is $\delta M_I^{\text{TRGB}} = 0.01256$ mag, so the zero point would be dimmer by this amount. The empirical color dependence found by Jang and Lee (2017)
shown in the last line of their Table 7 and as a red line in their Fig. 13 suggests that this difference would be only 0.00609 mag. This small shift makes no practical difference as it is within the rounding error of the main calibration. So henceforth we ignore this small color dependence of the calibration.

The canonical distance of ω Centauri in the catalog of Harris [14], based on the HB brightness, is 5.2 kpc without a specified uncertainty. It is perfectly consistent with the DEB distance cited earlier.

However, recently distances to selected galactic globular clusters were determined kinematically based on Gaia DR2 data by Baumgardt et al. (2019) [13] (see their Table 3). For ω Centauri they found 5.24 ± 0.05 kpc, consistent with both distance determinations mentioned earlier, but with a much smaller uncertainty. The corresponding distance modulus is 13.597 ± 0.021 and provides

\[ M_I^{TRGB} = -3.99 ± 0.05 \text{ mag} \]  \hspace{1cm} (4)

as a zero-point calibration at \((V - I)^{TRGB} = 1.50\), but within rounding errors also pertains to our fiducial color \((V - I)^{TRGB} = 1.80\). The main uncertainty now derives from the \(I\)-band apparent brightness and no longer from the distance.

E. Globular Cluster M5 (NGC 5904)

V13 studied the upper RGB in the globular cluster M5 in order to constrain novel energy losses in the degenerate cores before helium ignition. Their theoretical zero-point prediction is discussed in Sec. IIIB below. Their philosophy was to identify the brightest RG and determine the statistical offset to the true TRGB. For the brightest RGB star they report

\[ I_1 = 10.329 ± 0.023 , \]  \hspace{1cm} (5)

where they account for various sources of observational errors (photometry, crowding, etc). The color of this star is approximately \((V - I)_1 ≈ 1.8 \text{ mag}\). The second-brightest star is 0.034 mag dimmer, and the third-brightest one 0.057 mag dimmer than the second. Therefore, even using the second-brightest star would not vastly change the final result.

To estimate the difference between \(I_1\) and the true TRGB, V13 used the observed RGB population to estimate the evolutionary speed along the upper RGB. With a Monte Carlo simulation they generated random realisations of the RGB with the same underlying distribution and in this way found the statistical distribution for the brightness difference \(Δ_{\text{tip}} ≥ 0\) between the brightest star and the true TRGB. A good analytic fit function is given in their Eq. (4) as

\[ p(Δ_{\text{tip}}) = \frac{e^{-Δ_{\text{tip}}/λ}}{λ} \frac{1 + a Δ_{\text{tip}}^{-1/3}}{1 + a λ^{-1/3} Γ_{2/3}} , \]  \hspace{1cm} (6)

with the parameters \(a = 2.30\) and \(λ = 0.068\). This function is normalised to unity. It provides an average of \(⟨Δ_{\text{tip}}⟩ = 0.048 \text{ mag}\) and an rms variation of 0.058 mag. Notice that this distribution is only defined for positive \(Δ_{\text{tip}}\), it peaks at \(Δ_{\text{tip}} = 0\), and is very asymmetric relative to the average, so the rms variation is not a Gaussian error. Notice also that \(⟨Δ_{\text{tip}}⟩\) is similar to the brightness differences between the three brightest stars, so this results passes a simple sanity check. Conversely, one could have used a typical brightness difference among the few brightest red giants as a simple estimate for \(Δ_{\text{tip}}\).

If the core mass at helium ignition is increased by novel energy losses, the HB also becomes brighter because zero-age HB stars will have larger core masses. Therefore, V13 avoided distance determinations based on the HB brightness such as the canonical distance of 7.5 kpc reported in the catalog of Harris [14]. Instead, they used the distance modulus 14.45 ± 0.11 mag (7.6 ± 0.39 kpc) of Layden et al. (2005) [26] based on main-sequence fitting. It was the distance uncertainty that contributed the main error to the TRGB calibration, not the statistical offset between the brightest star and the true TRGB.

We now use the kinematical distance of Baumgardt et al. [13], shown in their Table 3, of \(d_{M5} = 7.58 ± 0.14 \text{ kpc}\) (distance modulus 14.398 ± 0.040) which is similar to the Harris distance and also consistent with that from main-sequence fitting. So the brightest RGB in M5 is now found to have an absolute \(I\)-band brightness of

\[ M_I^{1st} = -4.069 ± 0.046 , \]  \hspace{1cm} (7)

where we have added the errors in quadrature. The dominant error still derives from the distance. We show the corresponding Gaussian brightness distribution as a thin blue line in Fig. 2.

To include the statistical difference to the true TRGB
we convolve a Gaussian based on Eq. (7) with Eq. (6) and find the distribution shown as a thick orange line in Fig. 2. This asymmetric distribution provides

\[
M_I^{TRGB} = \begin{cases} 
-4.117 & \text{average (mean),} \\
-4.108 & \text{median,} \\
-4.096 & \text{best fit (max. of distr.),}
\end{cases}
\]  

which differ only minimally from each other. The rms variation from the mean is

\[
\sigma_{rms}^{TRGB} = 0.074 \text{ mag,}
\]

which however is not a Gaussian error, but gives us a feeling for the width of the distribution. Finally we find the confidence intervals (see also Fig. 2)

\[
M_I^{TRGB} \in [-4.038, -4.164] \text{ at } 68.27\% \text{ CL,} \\
M_I^{TRGB} \in [-3.983, -4.273] \text{ at } 95.45\% \text{ CL.}
\]

They are chosen such that their endpoints are at equal height of the distribution function, i.e., we consider “contours” that include a given integrated probability.

**F. Compound Globular Cluster**

As an overall consistency check, F20 considered a selection of 11 galactic globular clusters and produced compound CMDs in the \(JHK\) bands using 2MASS data. The relative distances were linked to the average HBs and/or RR Lyrae stars. The absolute distance was anchored to 47 Tuc based on DEB distances. While the result agrees with the other calibrations, it is the most uncertain of their cases and also more uncertain than the ones based on \(\omega\) Centauri and M5, so we will not use it. Moreover, it involves a distance ladder to the observed stars, not a direct geometric determination.

Using individual globular clusters suffers from the paucity of stars on the upper RGB and thus requires an estimate of \(\Delta_{tip}\) between the brightest stars and the true TRGB as discussed in Sec. II E and also requires an identification of the one brightest RGB star as opposed to an AGB contamination. In this sense, using a compound CMD is attractive but requires excellent relative distances and suffers from the spread of metallicities.

In principle, one could use the list of globular clusters with good kinematical distances of Baumgardt et al. (2019) and repeat the exercise of Freedman et al. (2020). Also, recently a list of 50 globular clusters was used to constrain axions and neutrino dipole moments [12], however again relying on HB brightness distances.

Likewise, the bolometric TRGB brightness determinations in many globular clusters from the near infrared photometry of Ferraro et al. (1999) [27] and Valenti et al. [28–31] are very interesting, but they did not provide an explicit \(I\)-band TRGB calibration and the uncertainties, especially those from the distances, are not clearly laid out. We find it too difficult to post-process these results for our present purpose where the uncertainty of \(M_I^{TRGB}\) is crucial, not only its zero point. The photometry itself does not seem to be the limiting issue, but the distances are, so a clear discussion of the different sources of uncertainty for the final result is mandatory to be able to compete with the calibrations shown in Fig. 1.

**G. Hubble Constant**

One main motivation for the TRGB calibration is to establish one rung in the cosmic distance ladder to determine the Hubble constant. On the basis of their LMC calibration, shown as our Eq. (1), F20 found \(H_0 = 69.6 \pm 0.8_{\text{stat}} \pm 1.7_{\text{sys}} = 69.6 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}\). The TRGB calibration was used to calibrate a sample of SNe Ia which then take us to cosmological distances. We may compare this result with the cosmological determination of \(H_0 = 67.4 \pm 0.5\) found by the Planck Collaboration [32]. The two values agree at the 1.1\(\sigma\) level, meaning that they agree, but the CMB determination has a much smaller uncertainty.

The Cepheid-based value of \(H_0 = 74.03 \pm 1.42\) [33] is around 1.9\(\sigma\) larger than the TRGB-based one but still compatible. However, the Planck and the Cepheid values differ by around 4.4\(\sigma\), a discrepancy debated in the literature as the Hubble tension between local and large-scale \(H_0\) calibrations. We will not pursue this topic here and simply note that there is no tangible Hubble tension between the Planck and TRGB results.

For illustration we can therefore turn these results around and ask: beginning with \(H_0\) found by Planck, which TRGB calibration would we get after climbing down the cosmic distance ladder? The direct and inverted LMC-based zero points of F20 are

\[
H_0 = 69.6 \pm 1.24 + 32.05 \ (4.047 + M_I^{TRGB}) , \\
M_I^{TRGB} = -4.047 \pm 0.039 + 0.0312 \ (H_0 - 69.6) ,
\]

where here and henceforth \(H_0\) is understood in units of km s\(^{-1}\) Mpc\(^{-1}\). These expressions are the linearised versions of mapping between distance and distance modulus and we note that \(H_0\) plays the dimensional role of an inverse distance. A larger \(H_0\) implies a smaller distance to a galaxy of fixed redshift and thus a dimmer TRGB.

In order to explain our stated uncertainties we note that \(M_I^{TRGB} \leftrightarrow H_0\) through SNe Ia accrues an error even if the input information were exact. Starting from Eq. (1) F20 find \(H_0 = 69.6 \pm 1.9\), but if we propagate only the error of 0.045 mag through Eq. (11a) without SNe Ia error we find only \(\pm 1.44\), so the difference \(\sqrt{1.9^2 - 1.44^2} = 1.24\) must come from the transition through SNe Ia. Inserting this result means that an uncertainty of 1.24 in \(H_0\) translates into one of 0.0386 mag in \(M_I^{TRGB}\). The cosmological determination by the Planck Collaboration [32] is \(H_0 = 67.4 \pm 0.5\), implying

\[
M_I^{TRGB} = -4.116 \pm 0.040 .
\]
Of course, this calibration depends on the assumption that the local $H_0$ is identical with the large-scale value. Therefore, we will not use this result to derive particle bounds and only show it for illustration.

H. Summary of Calibrations

We next summarise these results in Fig. 1, showing for each case the 68% and 95% confidence intervals. The LMC calibration was stated in Eq. (1), the SMC one in Eq. (2), the one from the galaxy NGC 4258 in Eq. (3), that from the globular cluster $\omega$ Centauri in Eq. (4), that from the globular cluster M5 in Eq. (10), and that from the CMB-implied Hubble constant in Eq. (12).

For comparison we also show the LMC calibration of Y19 in comparison to F20 (see Sect. II A for a discussion) and our CMB calibration that we backward-engineered from F20’s $H_0$ determination.

We also anticipate our update of the theoretical calibration by V13 that will be given in Eq. (15), whereas the one by S17 in Eq. (14). The main difference between these cases is the treatment of error, where S17 gave a maximum error.

All of the empirical results agree well with each other. Ignoring the CMB calibration which is only shown for comparison and illustration, the simple average of the remaining 5 empirical cases is $\langle M_I^{TRGB} \rangle = -4.052 \pm 0.046$ where the error represents the rms variation between the different cases. In other words, the scatter of the calibrations is roughly commensurate with the stated uncertainties, i.e. the agreement is not “too good to be true.” The weighted average is $\langle M_I^{TRGB} \rangle_w = -4.044 \pm 0.025$ where here the uncertainty is the variance of the mean, roughly $1/\sqrt{5}$ of a typical error of the individual measurements. For the asymmetric distribution of M5 we have here used the mean and rms variance as if it were a Gaussian distribution.

However, we are not convinced that one should combine these different results as if they were statistically independent because there can be common systematics, for example in the DEB distance determinations, in the kinematic distance determinations of globular clusters, in the treatment of extinction or photometric transformations, and others. It would be challenging to identify how different uncertainties might correlate. Therefore, we consider these averages only in point of illustration, showing that everything looks perfectly consistent. Moreover, the arithmetic and weighted averages are practically identical with each other and with the LMC calibration.

III. THEORETICAL TRGB CALIBRATION

A. Serenelli et al. (2017) – S17

S17 [15] have recently provided a new theoretical TRGB calibration, with a focus on various uncertainties that could affect the result. In particular from the comparison of two independent stellar evolution codes they derive a set of physical and numerical ingredients, which both minimize the results of the two codes and provide state-of-the-art inputs appropriate for low-mass stellar models. They recommend such models as a reference for further applications of the TRGB in astrophysics. In the color range $1.40 < (V - I)^{TRGB} < 2.40$ they find

$$M_I^{TRGB} = -4.090 + 0.017 \text{Col} + 0.036 \text{Col}^2,$$  (13)

where $\text{Col} = [(V - I)^{TRGB} - 1.4]$. At $(V - I)^{TRGB} = 1.8$, relevant for the empirical M5 and LMC calibrations, the zero-point is $M_I^{TRGB} = -4.077$ mag. Here the slope is $dM_I^{TRGB}/d(V-I)^{TRGB} = 0.046$, so the color dependence is very small.

Concerning uncertainties, these authors discuss in their Sec. 5.1 a list of input parameters that are varied between extreme assumptions. Taking them to add coherently in one and the other direction, they find a full width of the predicted $M_I^{TRGB}$ range of 0.25–0.30 mag. In their Sec. 6 they consider specifically the color relevant for the globular cluster M5 and recommend an error of $\pm 0.12$ mag, which we interpret as a maximum range.

However, this does not include the uncertainty of the bolometric correction (BC) for which they use MARCS [34] which is compared to other BCs in their Fig. 8. At $(V - I)^{TRGB} = 1.8$ the spread between different BCs is around 0.1 mag. S17 explicitly find that for M5 the predicted $M_I^{TRGB}$ becomes brighter by 0.07 mag if one uses the Worthey and Lee [35] BC instead of MARCS. In the spirit of adding theoretical uncertainties coherently we add 0.04 mag to their recommended half-width error of $\pm 0.12$. Therefore, we interpret S17’s prediction for the zero-point calibration at $(V - I)^{TRGB} = 1.8$ as

$$M_I^{TRGB} = -4.08 \pm (0.12_{\text{models}} + 0.04_{\text{BC}})_{\text{max}},$$  (14)

where we interpret the uncertainty as a maximum range.

B. Viaux et al. (2013) – V13

A similar study was performed earlier by V13 [8] specifically for the globular cluster M5. For their fiducial case, they give in their Eq. (6) $M_I^{TRGB} = -4.03$ mag, using the Worthey and Lee BCs. S17 find a value of $-4.14$ mag using the same BC, different from the benchmark MARCS assumed in the previous section (see also the discussion in their Sec. 6). These authors argue that V13 should have used in their numerical calculations the intermediate screening regime in nuclear reaction rates instead of
the Salpeter formulation of weak screening. This modification would make the V13 models brighter by 0.09 mag, so their zero point should be $M_{\text{TRGB}}^I = -4.12$ mag, very close to S17’s result. Thus for common input physics one finds a robust prediction despite many small differences in detail and despite using different codes.

Concerning the uncertainty of the prediction, V13 also listed a large number of possible input variations in their Table 4 as well as the assumed BC uncertainty given in their Eq. (9). Some of these ranges produce an asymmetric effect, notably the last two lines in Table 4 (equations of state and mass loss). This asymmetry shifts the prediction by 0.039 mag in the dim direction. Apart from the screening prescription, this shift is the missing bit of difference to the models of S17 that was left unexplained in their Sec. 6. So after correcting for the treatment of screening, apparently there are no unexplained differences between the predictions.

Concerning the formal error, V13 proposed to combine the systematic errors in quadrature and to assume a top-hat distribution for each individual one, leading to an rms error of $\pm 0.039$ from the uncertainties in Table 4. The BC error of $\pm 0.08$, assumed to represent a maximum range with a top-hat distribution, corresponds to an rms uncertainty of $\pm 0.08/\sqrt{3} = \pm 0.046$. Combining these errors in quadrature and after applying the screening correction, V13’s updated calibration is

$$M_{\text{TRGB}}^I = -4.08 \pm 0.06_{\text{rms}}. \quad (15)$$

Although the two groups made some different choices in detail, their zero points fortuitously coincide.\(^5\)

This calibration agrees very well with the empirical results. Therefore, in order to derive limits on axions or to provide a rung in the cosmic distance ladder, the main question is how to deal with systematic uncertainties, where the two groups have adopted different philosophies. However, we note that a top-hat distribution of half-width $\sigma$ has an rms width of $\sigma/\sqrt{3}$, so a maximum error of $\pm 0.16$ corresponds to an rms error of $\pm 0.09$. In other words, the formal uncertainties of the two calibrations are not as different as it may seem. The maximum error of S17 nominally corresponds to a 2.7σ error of V13.

C. Comparing with Empirical Calibrations

To compare the theoretical results with the empirical calibration we ask for the allowed range of a possible mismatch $\Delta M_{\text{TRGB}}^I = M_{\text{TRGB}}^I - M_{\text{TRGB}}^I$ between the calibration and prediction. For both theoretical studies this is $\Delta M_{\text{TRGB}}^I = 0.03$ mag.

The uncertainty of this offset depends on how to treat the uncertainties of the theoretical prediction. V13 have argued that one should combine many sources of systematic errors in quadrature and interpret the final result as a Gaussian uncertainty. We have already combined statistical and systematic uncertainties of the empirical result in quadrature and have interpreted the combined error as a Gaussian uncertainty. Combining the errors of the LMC calibration and of the theoretical prediction in quadrature, the offset is constrained by

$$\Delta M_{\text{TRGB}}^I = 0.03 \pm 0.08 \text{ mag} \quad (16)$$

where the error is dominated by that of the prediction.

If instead one combines the stellar-evolution errors to provide a possible maximum range in the spirit of S17, it is not completely clear on how to combine the empirical and theoretical errors. We simply note that the maximum range of $\pm 0.16$ mag is much larger than the LMC calibration error, so one could essentially neglect the latter. In this case the maximum allowed mismatch between theory and calibration of $\pm 0.16$ mag corresponds roughly to the 95% confidence range of Eq. (16). Therefore, the practical difference of the two philosophies becomes small because any substantial conclusion based on Eq. (16) would be based on a 95% confidence interval, so the nominal uncertainties are not very different.

IV. PARTICLE BOUNDS

A. Neutrino Dipole Moment

1. Predicted Brightness Increase

In the context of the Standard Model neutrino emission from the degenerate helium core is dominated by plasmon decay, $\gamma_{\text{pl}} \rightarrow \nu \bar{\nu}$, a process that can not be tested in the laboratory. The calculated emissivity is thought to be precise on the 5% level. In their Sec. 5.11, V13 showed that changing standard neutrino emission by $\pm 5\%$ changes $M_{\text{TRGB}}^I$ by $\pm 0.013$ mag. If we denote with $F_\nu$ a fudge factor multiplying standard neutrino emission, the constraint Eq. (16) can be interpreted as $F_\nu = 0.87 \pm 0.17$. In other words, standard neutrino emission is confirmed to occur with the expected rate, but the test is not very precise.

If the core of a red giant before helium ignition suffers energy losses in addition to the usual neutrino emission, a larger core mass is required so that also the brightness $M_{\text{TRGB}}^I$ increases. Viaux et al. (2013) [8, 9] specifically considered non-standard energy losses caused by neutrinos and axions and studied the impact on the TRGB for stellar parameters appropriate for the globular cluster M5. The color $(V-I)_{\text{TRGB}} \approx 1.8$ is very similar to the reference value for the LMC calibration Eq. (1) that we will use. Therefore, the modifications caused by particle emission of Viaux et al. carry over to the present case without further modification.

\(^5\) Notice in particular that V13 used the BC of Worthey and Lee whereas S17 used MARCS, so the exact coincidence of the two $M_{\text{TRGB}}^I$ zero points is indeed coincidental.
where the anomalous shift and the uncertainty are neutrino emission is the updated prediction of $V_{13}$ in the presence of anomalous terms of $\alpha$ correction changes and also its uncertainty.

The second case is the emission of axions that are assumed to have a direct Yukawa coupling $g_{ae}$ with electrons. In low-mass red-giant cores, they are primarily emitted by bremsstrahlung in electron-nucleon collisions. Therefore, the radial distribution of axion or neutrino energy losses is different. Standard neutrino cooling pushes the helium ignition point from the center of the core to some non-vanishing radius; explicit examples were shown by Raffelt and Weiss in their Fig. 2 [5]. Non-standard cooling enhances this effect, but for different cooling channels in quantitatively different ways so that the neutrino and axion cases are often treated separately. In view of the interesting range we use the parameter $g_{13} = g_{ae}/10^{-13}$. Another parameter is the axionic fine-structure constant $\alpha_{ae} = g_{ae}^2/4\pi$, often expressed in terms of $\alpha_{26} = \alpha/10^{-26} = g_{13}^2/4\pi$.

After adjusting the zero point according to the critique by Si17 discussed in Sec. III B, leading to Eq. (15), the updated prediction of V13 in the presence of anomalous neutrino emission is

$$M_I^{\text{Theory}} = -4.08 - \delta M_\mu \pm \sigma_\mu, \quad (17)$$

where the anomalous shift and the uncertainty are

$$\delta M_\mu = 0.23 \left(\sqrt{\mu_{12}^2 + 0.80^2} - 0.80 - 0.18\mu_{12}^{1.5}\right), \quad (18a)$$

$$\sigma_\mu = \sqrt{0.039^2 + (0.046 + 0.0075\mu_{12})^2}. \quad (18b)$$

For $\mu_{12} = 0$ the shift $\delta M_\mu$ vanishes whereas the error estimate is the same as in Eq. (15). The modification of the uncertainty with $\mu_{12}$ arises because with increasing brightness, also the color and therefore the bolometric correction changes and also its uncertainty.

2. LMC Calibration

We now compare our main case, the LMC calibration of Eq. (1), with this theoretical prediction and find

$$\Delta M_I^{TRGB} = M_I^{LMC} - M_I^{\text{Theory}} = 0.03 + \delta M_\mu \pm \bar{\sigma}_\mu \quad (19)$$

where the overall uncertainty derives from combining $\sigma_\mu$ and that of the LMC calibration in quadrature

$$\bar{\sigma}_\mu = \sqrt{0.045^2 + 0.039^2 + (0.046 + 0.0075\mu_{12})^2}, \quad (20)$$

which at vanishing $\mu_{12}$ is $\pm 0.075$ mag. Assuming Gaussian errors, these results define a distribution function

$$\frac{1}{\sqrt{2\pi} \bar{\sigma}_\mu} \exp \left[-\frac{(\Delta M_I^{TRGB} - 0.03 - \delta M_\mu)^2}{2\bar{\sigma}_\mu^2}\right], \quad (21)$$

which, for fixed $\mu$, can be interpreted as the probability distribution of possible $\Delta M_I^{TRGB}$ values based on the measurements and theory.

On the other hand, if the only uncertainty derives from $\mu$, the true $\Delta M_I^{TRGB} = 0$ and the remaining expression gives us the probability distribution for $\mu$ after normalising its integral $\int_0^\infty d\mu$ to unity. Integrating instead up to limiting values of $\mu$ that encompass 68.27% and 95.45% of the probability we find

$$\mu_{12} < 0.77 (1.50) \text{ at } 68\% (95\%) \text{ CL} \quad (22)$$

for our new limits. If instead of the error of the LMC calibration we would use half the error, as suggested by a combination of all 5 empirical calibrations, these bounds would be $\mu_{12} < 0.69 (1.34)$ instead, which is not very different in any practical sense. Evidently the main uncertainty now derives from theoretical models, not the observational calibration.

If instead we use the maximum-range error $\pm 0.16$ mag for the theoretical prediction of Eq. (14) the observational error is negligible by comparison, but to be conservative we may argue that $\Delta M_I^{TRGB} \lesssim 0.20$ mag is a reasonable requirement. It implies $\mu_{12} \lesssim 1.8$, not much worse than the 95% CL limit of Eq. (22).

If instead we use the previous theoretical calibration by V13 that was 0.09 mag dimmer, so the zero point was $-3.99$ mag instead of $-4.08$ mag, provides the limits $\mu_{12} < 1.2 (2.1)$ at 68% (95%) CL.

3. Globular Cluster M5

While we do not use the M5 calibration for our main result, we reconsider it here to compare with the previous results of V13. In that case there was a small discrepancy between theory and observations such that a little bit of extra cooling was suggested at less than $2\sigma$ significance. The corresponding limit was therefore relatively poor, i.e., $\mu_{12} < 2.6 (4.5) \times 10^{-12} \mu_B$ at 68% (95%) CL. Including the shift in the prediction of 0.09 mag in the bright direction as discussed in Sec. III B, these constraints change to $\mu_{12} < 1.9 (3.5) \times 10^{-12} \mu_B$, so the change of the limit is not very dramatic, but the small hint for extra cooling disappears.

If we next turn to the modification engendered by the changed distance and improved distance uncertainty, we need to be more careful because it becomes necessary to include the asymmetry of the probability distribution that can be gleaned from Fig. 2. V13 instead used a Gaussian distribution in view of the overall error being strongly dominated by the distance, but this is no longer
the case. Including this effect, we find for the M5 bounds $\mu_{12} < 1.2$ (2.3) at 68% (95%) CL. This is the least restrictive bound that one finds from any of the individual calibrations shown in Fig. 1 as one can easily glean from that figure because M5 favors a somewhat bright TRGB, thus allowing a bit more extra cooling than the other calibrations.

B. Axions

Next we repeat the LMC exercise for the case of axions for which similar expressions for the brightness shift and its uncertainty apply [9]. Actually it turns out that in terms of $g_{13}$ these results fortuitously are numerically so similar to the dipole ones in terms of $\mu_{12}$ that one could use these parameters almost interchangeably. Specifically we find

$$g_{13} < 0.82 (1.60) \text{ at } 68\% \text{ (95\%) CL}$$

for our new axion limits based on the LMC calibration.

V. DISCUSSION AND SUMMARY

In the first part of our paper, we have collected recent extragalactic TRGB calibrations that used geometric distance indicators. Moreover, we have re-calibrated the TRGB using the globular clusters $\omega$ Centauri and M5 using the recent kinematical distance determinations of Baumgardt et al. [13]. Overall we show a list of five local TRGB calibrations that all agree within errors, but also show a scatter commensurate with the stated errors. Combining them would require a better understanding of possible correlations between them, but we have noted that a weighted average agrees very well with the recent LMC calibration of F19 and F20, except with an error roughly a factor of 2 smaller.

Improving the empirical TRGB calibration would be interesting in the context of the debate about the Hubble tension. The TRGB-implied $H_0$ value is between the one derived from cosmological data and the local one based on Cepheid distances, being statistically compatible with either one. A refined TRGB calibration might help to clarify this situation. For example, one could use the recent kinematical globular cluster distances to re-calibrate the TRGB using the CMD of many globular clusters, not just the two we have specifically used.

Particle bounds, on the other hand, derive from a comparison of the empirical calibrations and theoretical predictions with or without novel cooling channels. Given the precision of the empirical results, the uncertainty of the stellar models has become the dominant source of uncertainty. To arrive at our limits, we have closely followed the earlier approach of V13 in that we have combined a large number of systematic issues in quadrature and have interpreted the overall uncertainty as a Gaussian error. Conversely, S17 have preferred to state maximum errors by taking input uncertainties to extremes and adding their effect coherently. Particle bounds based on this approach are roughly comparable to a 2–2.5$\sigma$ error of the former approach, so any substantial conclusion would be similar. Still, the bounds and their formal significance depend on how one deals with systematic effects (“the devil is in the tails”).

Concerning particle emission, we have reconsidered the red-giant bounds on neutrino dipole moments and the axion-electron coupling strength. The main new ingredients, relative to the earlier studies of Viaux et al. [8, 9], are a correction of the theoretical prediction to account for the treatment of nuclear reaction rates and new TRGB calibrations based on geometrical distance indicators. Our final results use a recent TRGB calibration based on the LMC [19, 20], but an average of the five local calibrations provides a similar result.

A previous analysis by Viaux et al. [8, 9], based on the globular cluster M5, was interpreted as providing a hint for extra cooling [36]. From Fig. 1 one can glean that even with the new distance, M5 prefers a somewhat bright TRGB, whereas the zero point of the old prediction of Viaux et al. was $-3.99$ mag, causing a mild tension. While the old and new results agree within the stated uncertainties, there is no longer any tension and no hint for any inconsistency, but by itself M5 still provides the least restrictive limit of the cases shown in Fig. 1.

We have noted that red-giant bounds on neutrino dipole moments $\mu_\nu$ and on the axion-electron coupling $g_{ae}$ are numerically the same in terms of the parameters $\mu_{12} = \mu_\nu/10^{-12}$ $\mu_B$ and $g_{13} = g_{ae}/10^{-13}$ unless one worries about second-digit precision, so we can summarise previous results in terms of either quantity even if the authors considered only one of them. Previous bounds are $\mu_{12} < 2$ [3], $\mu_{12} < 1$ [7], $\mu_{12} < 4.5$ (95% CL) [8], $g_{13} < 4$ (95% CL) [10], $\mu_{12} < 2.6$ [11], and $\mu_{12} < 2.2$ [12], to be compared with our new bound $\mu_{12} < 1.5$ (95% CL).

Compared with previous results, our bound is the most constraining in terms of specified CL. However, it is also clear that these bounds have not changed very much in around 30 years, reflecting the good agreement between standard stellar evolution theory and observations. What has changed, however, is a more systematic assessment of uncertainties and that the limiting factor now has become stellar evolution theory together with bolometric corrections and no longer the observational data and distance determinations.

We have also seen that TRGB calibrations based on globular clusters are competitive with extragalactic methods if one uses recent kinematical distance determinations. Using more globular clusters might help to sharpen the TRGB-based $H_0$ determination and in this way contribute to the debate about the Hubble tension.
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