On modelling and adaptive control of a linear smart beam model interacting with fluid

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Abstract. This paper deals with modelling and control of Euler-Bernoulli smart beam interacting with a fluid medium. Several distributed piezo-patches (actuators and/or sensors) are bonded on the surface of the target beam. To model the vibrating beam properly, the effect of the piezo-patches and the hydrodynamic loads should be taken into account carefully. The partial differential equation PDE for the target oscillating beam is derived considering the piezo-actuators as input controls. Fluid forces are decomposed into two components: 1) hydrodynamic forces due to the beam oscillations, and 2) external (disturbance) hydrodynamic loads independent of beam motion. Then the PDE is discretized using the Galerkin approach to obtain standard multi-modal equations. An adaptive approximation control structure is proposed to suppress the beam vibration. The controller consists of a proportional-derivative PD control plus an adaptive approximation compensator AAC with guaranteed stability. A simply supported beam with 2 piezo-patches interacting with fluid is simulated. The disturbance hydrodynamic force that excites the beam vibration is assumed as a harmonic force with 50 Hz frequency and 1 N amplitude. The results prove the efficacy of the proposed control architecture.

1. Introduction

Modelling, design and motion regulation of fluid-flexible structures witness much attention since they are used in many life applications such as underwater robotic systems, space vehicles or resonant beams for measurement purposes [1-3]. These structures would be destabilized if undesired disturbance loads are applied. To this end, active vibration attenuation is a powerful tool to resolve this dilemma. By bonding piezo-patches (actuators and sensors) on the surface of the vibrating beam, the oscillations of the dynamic system can be suppressed. However, a control structure is required to stabilize the fluid-beam oscillations [4]. Consequently, this paper is focused on modelling and adaptive control of a flexible beam immersed in a fluid under modelling uncertainty. Designing an appropriate control system requires proper modelling for the target dynamic system. Two key points should be considered in modelling of a smart beam-fluid system: 1) fluid hydrodynamics and 2) piezo-electric patches. Fortunately, the dynamics of piezo-patches integrated with the beam can be neglected due to the slight effect on vibrations characteristics of the system [5]. On the other hand, if the beam is imposed to oscillate with fluid vibrations then the hydrodynamic forces can be decomposed into two main forces: 1) hydrodynamic forces dependent of beam motion that can be represented by inertia, damping and stiffness forces and 2) external hydrodynamic forces independent of beam oscillations and these forces are of random signals and are determined experimentally [6]. For more details on fluid-structure interaction, see [1-3, 6-14].
For simplicity, linear modelling for the coupled beam-fluid system is considered and this eases the task of the controller; however, the parameters of the target system are assumed unknown. Therefore, conventional control structures such as conventional PID or optimal control etc. (see e.g. [15] and the references therein) may not be useful for this type of system. Two control techniques are strong to deal with the uncertainty: adaptive control and robust bounded control, see e.g. [16-18] for more details. In general, two strategies of adaptive control are available: regressor and approximation-based control. The former is a physics-based control that is difficult to deal with disturbances if exist, while approximation-based control is a powerful tool to treat complex systems, the reader is referred to [15, 18-23] for more information.

In view of the above, this work is concerned with modelling and adaptive approximation control of a linear smart beam model interacting with fluid. The control architecture consists of three main terms: PD term, adaptive approximation compensator term and a robust sliding term for reducing the modelling error if exist. A simply supported beam provided with two piezo-patches is simulated. The smart beam system is immersed in a fluid media. The vibration source is assumed as a harmonic force that excites the coupled fluid-beam system. In effect, this disturbance results from the external fluid hydrodynamic forces that are independent of beam vibrations. The results show the strength of the proposed controller under uncertain modelling and impulse disturbances.

The remainder of the paper is structured as follows. Section 2 describes the dynamic modelling of the smart beam-fluid system while section 3 introduces the proposed control architecture. Simulation results and discussions are evaluated in Section 4. Section 5 concludes.

2. Dynamic modelling of a coupled beam-fluid system

Below modelling of smart beam immersed in a fluid is presented, see Figure 1 for a depiction of beam-fluid model. For modelling purposes, the following points are assumed [4, 6]:

(i) A linear fluid-structure model is considered, i.e. small deflections are imposed.

(ii) The hydrodynamic forces are decomposed into two terms: forces dependent on beam oscillations and external disturbance forces free from beam vibrations.

(iii) A sufficient number of piezo-sensors are available such that modal amplitudes are measurable.

\[ E_b I_b \frac{\partial^4 w}{\partial x^4} + \rho_b A_b \frac{\partial^2 W}{\partial t^2} = f(x,t) - \frac{\partial^2 T_p}{\partial x^2} \tag{1} \]
\[ T_p = Dv_a(t) \frac{\partial^2}{\partial x^2} [H(x - x_j) - H(x - x_j)] \]  
\[ f(x,t) = f_m(x,t) + f_e(x,t) \]
where \( E_b, I_b, A_b, \) and \( \rho_b \) are Young's modulus, moment of inertia, cross-sectional area, and density of the beam respectively. \( T_p \) is the moment per unit length exerted by the piezo-actuator, \( D \) is a constant depending on properties of the regular beam and smart materials [23]. \( H(.) \) is a Heaviside step function of the beam displacement. On the other hand, \( f(x,t) \) denotes to the hydrodynamic forces that composed of two terms: \( f_m(x,t) \) denoting to the hydrodynamic force per unit length due to the beam motion and \( f_e(x,t) \) referring to an external hydrodynamic force independent of beam motion.

Remark 1. As aforementioned, the overall hydrodynamic fluid forces are partitioned into two terms; \( f_m + f_e \), where \( f_m \) is a function of acceleration and velocity of the beam oscillations (the concept of added mass and damping), while \( f_e \) is experimentally determined. In effect, this work assumes the vibration source results from \( f_e \).

In order transfer the PDF of Equation (1) into multi-modal ODEs, the Galerkin approach is used, hence
\[ w(x,t) = \sum_{j=1}^{N} \phi_j(x) q_j(t) \]
where \( \phi_j(x) \) is the mode shape, \( q_j(t) \) is the modal amplitude and \( N \) is the number of the mode shapes.

Substituting Equation (4) into Equation (1), multiplying by an arbitrary \( \phi_j(x) \), integrating along the beam length and using the orthogonal conditions for a simply supported beam with some manipulations, we obtain
\[ m_j \ddot{q}_j + b_j \dot{q}_j + k_j q_j - g_{mj}(x,t) - g_{dj}(x,t) = u_j, j = 1,2,3,...,N \]  
where
\[ m_j = \left( \frac{\rho_b A_b}{2} \right), b_j = (2\zeta_j \omega_j) m_j, -1 \leq \zeta_j \leq 1, k_j = \frac{E_b I_b}{l_b}, w_j = k_j / m_j, u_j = -\sum_{i=1}^{N} \mu_{ji} v_{ai}(t), \]
\[ \mu_{ji} = D(\phi_j(x_{0}) - \phi_j(x_{0})), g_{mj}(x,t) = -\int_0^l f_m(x,t) \phi_j(x) dx, g_{dj}(x,t) = -\int_0^l f_e(x,t) \phi_j(x) dx, \lambda_j = \frac{j\pi}{l_b} \]
where \( l_b \) referring to length of the beam and \( N_\text{a} \) is the number of piezo-actuators. In a matrix form, Equation (5) is expressed as
\[ M \ddot{q} + B \dot{q} + K q + g_m + g_e = u \]  
\[
\begin{pmatrix}
q_1 \\
\vdots \\
q_N
\end{pmatrix} \in \mathbb{R}^N, M = \begin{bmatrix}
m_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & m_N
\end{bmatrix} \in \mathbb{R}^{N \times N}, B = \begin{bmatrix}
b_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & b_N
\end{bmatrix}, K = \begin{bmatrix}
k_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & k_N
\end{bmatrix} \in \mathbb{R}^{N \times N}, \]
\[ G_m = \begin{bmatrix}
-g_{mi} \\
\vdots \\
g_{mN}
\end{bmatrix} \in \mathbb{R}^N, g_e = \begin{bmatrix}
-g_{ei} \\
\vdots \\
g_{eN}
\end{bmatrix} \in \mathbb{R}^N, u = \begin{bmatrix}
u_{1} \\
\vdots \\
u_{N}
\end{bmatrix} = -\begin{bmatrix}
\mu_{11} & \cdots & \mu_{1N_a} \\
\vdots & \ddots & \vdots \\
\mu_{N_1} & \cdots & \mu_{N_N_a}
\end{bmatrix} \begin{bmatrix}
v_{a1} \\
\vdots \\
v_{aN_a}
\end{bmatrix} \in \mathbb{R}^{N_a} \]
3. Control design

The objective of the controller is to regulate the beam oscillations due to external disturbances. The proposed control law consists of three terms: PD term, an adaptive approximation compensator term, and a robust sliding term for compensation of modelling errors if exist. Thus, the controller form can be represented as [18-23]

\[
\begin{align}
\mathbf{u} &= \hat{\mathbf{a}} - \mathbf{K}_p \mathbf{e} - \mathbf{K}_d \hat{\mathbf{e}} - \gamma \text{sgn} \hat{\mathbf{e}} \\
\dot{\mathbf{e}} &= \mathbf{q} - \mathbf{q}_d, \quad \hat{\mathbf{a}} = \hat{\mathbf{M}} \hat{\mathbf{q}} + \hat{\mathbf{B}} \dot{\mathbf{q}} + \hat{\mathbf{K}} \dot{\mathbf{q}} + \hat{\mathbf{g}}_m + \hat{\mathbf{g}}_e, \quad \hat{\mathbf{a}} = \hat{\psi}^T \mathbf{0} \tag{7a}
\end{align}
\]

where (:) refers to estimation, \( \mathbf{K}_j \in R^{N \times N} \) and \( \gamma \in R^{N \times N} \) are diagonal positive definite feedback gain matrices, \( \mathbf{q}_d \in R^N \) is the desired reference vector, \( \hat{\mathbf{q}} \in R^{N \times N} \) and \( \mathbf{0} \in R^{N \times \beta} \) are the weighting coefficient and the basis function matrix/vector, and \( \beta \) is the number of basis function. Equating Equation (7a) to Equation (6)

\[
\begin{align}
\hat{\mathbf{M}} \hat{\mathbf{q}} + \hat{\mathbf{B}} \dot{\mathbf{q}} + \hat{\mathbf{K}} \dot{\mathbf{q}} + \hat{\mathbf{g}}_m + \hat{\mathbf{g}}_e &= \hat{\mathbf{M}} \dot{\mathbf{q}} + \hat{\mathbf{B}} \ddot{\mathbf{q}} + \hat{\mathbf{K}} \ddot{\mathbf{q}} + \hat{\mathbf{g}}_m + \hat{\mathbf{g}}_e - \mathbf{K}_p \mathbf{e} - \mathbf{K}_d \dot{\mathbf{e}} - \gamma \text{sgn} \hat{\mathbf{e}} \tag{8}
\end{align}
\]

Adding \((-\hat{\mathbf{M}} \dot{\mathbf{q}} - \hat{\mathbf{B}} \ddot{\mathbf{q}} - \mathbf{\mu})\) to the above equation to obtain

\[
\begin{align}
\mathbf{M} \ddot{\mathbf{e}} + (\mathbf{B} + \mathbf{K}_d) \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} + \gamma \text{sgn} \hat{\mathbf{e}} &= -\mathbf{\tilde{a}} + \mathbf{e} \tag{9}
\end{align}
\]

where \(\mathbf{e} \in R^N\) is the modelling error. Equation (9) represents a closed-loop dynamics for the beam-fluid system.

As we see from Equation (7), the unknown matrix \(\hat{\mathbf{a}}\) is required to be updated with guaranteed stability. To this end, consider the following non-negative function along Equation (9)

\[
\begin{align}
V &= \frac{1}{2} \mathbf{e}^T \mathbf{M} \mathbf{e} + \frac{1}{2} \mathbf{e}^T \mathbf{K}_p \mathbf{e} + \frac{1}{2} tr (\hat{\psi}^T \mathbf{F}^{-1} \hat{\psi}) \tag{10}
\end{align}
\]

Taking time derivative of Equation (10)

\[
\begin{align}
\dot{V} &= \dot{\mathbf{e}}^T (-(\mathbf{B} + \mathbf{K}_d) \dot{\mathbf{e}} - \mathbf{K}_p \mathbf{e} - \gamma \text{sgn} \hat{\mathbf{e}} - \mathbf{\tilde{a}} + \mathbf{e}) + \dot{\mathbf{e}}^T \mathbf{K}_p \mathbf{e} - tr (\hat{\psi}^T \mathbf{F}^{-1} \hat{\psi}), \hat{\mathbf{M}} = \mathbf{0} \tag{11}
\end{align}
\]

By manipulating Equation (11), we can obtain

\[
\begin{align}
\dot{V} &= -\dot{\mathbf{e}}^T (\mathbf{B} + \mathbf{K}_d) \dot{\mathbf{e}} + \dot{\mathbf{e}}^T \mathbf{e} - \dot{\mathbf{e}}^T \gamma \text{sgn} \hat{\mathbf{e}} - tr (\hat{\psi}^T (\mathbf{0} \dot{\mathbf{e}}^T + \mathbf{F}^{-1} \dot{\mathbf{e}})) \tag{12}
\end{align}
\]

To stabilize Equation (12), the appropriate update law is chosen as

\[
\hat{\psi} = -\mathbf{F}^{-1} \mathbf{0} \dot{\mathbf{e}}^T \tag{13}
\]

where \(\mathbf{F} \in R^{N \times N} \) is an adaptation gain matrix.

Equation (12) becomes

\[
\dot{V} = -\dot{\mathbf{e}}^T (\mathbf{B} + \mathbf{K}_d) \dot{\mathbf{e}} + \dot{\mathbf{e}}^T \mathbf{e} - \sum_{i} \gamma_i | \dot{\mathbf{e}}_i | \tag{14}
\]

Choosing \(\gamma_i \geq \dot{\mathbf{e}}_i + \delta_i\), where \(\delta_i\) is a positive constant results in

\[
\dot{V} = -\dot{\mathbf{e}}^T (\mathbf{B} + \mathbf{K}_d) \dot{\mathbf{e}} - \sum_{i} \gamma_i | \dot{\mathbf{e}}_i | \tag{15}
\]

Equation (15) is stable according to Lyapunov theory.

4. Results and discussions

To prove the efficacy of the proposed controller, a pinned-pinned beam with a couple of piezo-patches installed on its surface. The dimension and the properties of piezo-materials, beam and fluid are listed in Table 1. The oscillations source that irritates the smart beam motion is considered as a sinusoidal disturbance force with a frequency of 50 Hz with 1 N amplitude. This disturbance is developed due to the hydrodynamic fluid loading that is free of beam motion. Below, we will not investigate the
configurations of the beam mode shapes and the frequency response for the coupled beam-fluid system, for more details see [4]. What is important here is the effectiveness of the proposed controller while the coupled beam-fluid system under harmonic oscillations.

Table 1. Properties of beam, piezo-materials, and fluid

| Component          | Properties                                      |
|--------------------|--------------------------------------------------|
| Beam               | \( \rho_b = 8000 \text{kg/m}^3 \), \( I_b = 0.4 \text{m} \), \( E_b = 190 \times 10^3 \text{MPa} \), \( A_b = 0.04 \times 0.001 \text{m}^2 \), \( b_1 = 007 \text{N} \), \( b_2 = 0.03 \text{N.s/m} \) |
| Piezo-material     | \( l_p = 0.08 \text{m} \), \( A_p = 0.035 \times 0.0004 \text{m}^2 \), \( E_p = 70 \times 10^3 \text{MPa} \) |
| Fluid (FC-72)      | \( c_m = 2.1 \), \( c_v / \mu = 2280 \), \( \mu = 0.4 \times 10^{-6} \text{m}^2/\text{s} \), \( \rho_f = 1.68 \times 10^3 \text{kg/m}^3 \) |

The suggested control structure is applied to the target beam system with the following control gains:

\[
K_p = 300 \mathbf{1}_2, \quad K_d = 100 \mathbf{1}_2, \quad \psi = 20I_{22}, \quad \beta = 11, \quad N = 2, \quad F = I_{22}.
\]

For proper tuning of control gains, a private strategy suggested in [25] was adopted. The key idea is to set values for the target gains gradually from zero to a value at which the system instability occurs and then the gain value is halved for safety. Besides, two important points should be noted:

1. The modelling error is neglected and hence the robust sliding term is nullified.
2. The Chebyshev polynomials are used as approximators for the adaptive approximation compensator.

The initial conditions for the beam oscillations and the weighting coefficient matrix are imposed zero. Investigating Figures 2 and 3 shows that the proposed controller strongly damps out the oscillations. Two experiments are tested: 1) beam oscillations without a controller, and 2) beam oscillations with the application of the proposed control law. It should be mentioned that the piezo-actuators would have limit value for the output voltage and hence a bounded control is preferred to be implemented that is lost in this work. Besides, we have simulated 2 piezo-actuators/sensors for the first two mode shapes and this can ease the calculations. In practice, the number of actuators could be not equal to the investigated mode of shapes and hence pseudo-inverse matrix should be used, see [26] for more details.

![Figure 2](image_url)  
Figure 2. Modal coordinates for the coupled smart beam-fluid system considering the first two modes.
Figure 3. The input control for the piezo-actuators.

5. Conclusions
This work supposes adaptive approximation control with integrated PD for motion regulation of the beam-fluid system. The analysis is confined to the linear behaviour of the beam-fluid system. Despite the results show that the proposed controller is a powerful strategy for oscillation damping of flexible structures, the following points should be noted:

1. The nonlinear model for flexible structures should be considered.
2. The fluid hydrodynamic forces are evaluated based on the concept of added mass and damping, however, more complex theory should be investigated considering different fluid mediums.
3. The current controller is applied to a simple beam. The behaviour of the suggested controller should be tested on plate and shells.

References
[1] Naik T, Longmire E K and Mantell S C 2003 Dynamic response of a cantilever in liquid near a solid wall Sensors and Actuators A: Physical, 103, p 240-254
[2] Lin W and Qiao N 2008 Vibration and stability of an axially moving beam immersed in fluid International Journal of Solids and Structures, 45, p 1445-1457
[3] Fadaee M and Talebitooti M 2020 Active vibration control of carbon nanotube-reinforced composite beam submerged in fluid using magnetostrictive layers, Mechanics Based Design of Structures and Machines, published online
[4] Wagg D and Neild S 2010 Nonlinear vibration with control (SMIA, Springer Verlag)
[5] Zhang W, Meng G and Li H 2006 Adaptive vibration control of micro-cantilever beam with piezoelectric actuator in MEMS Int J Adv Manuf Technol 28, p321–327
[6] Faltinsen O M 1990 Sea loads on ships and offshore structures (Cambridge University Press)
[7] Xing J T, Price W G, Pomfret M J, and Yam L H 1997 Natural vibration of a beam-water interaction system Journal of Sound and Vibration 199, p 491–512
[8] Sader J E 1998 Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope Journal of Applied Physics 84, p 64–76
[9] Zhao S, Xing J T and Price W G 2002 Natural vibration of a flexible beam–water coupled system with a concentrated mass attached at the free end of the beam Proceedings of the Institution of
Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment 216, 145–54

[10] Lin W and Qiao N 2008 Vibration and stability of an axially moving beam immersed in fluid. *International Journal of Solids and Structures* 45, p 1445–57

[11] Shabani R, Hatami H, Golzar F. G, Tariverdilo S and Rezazadeh G 2013 Coupled vibration of a cantilever micro-beam submerged in a bounded incompressible fluid domain *Acta Mechanica* 224, p 841–50

[12] Ni Q, Li M, Tang M, and Wang L 2014 Free vibration and stability of a cantilever beam attached to an axially moving base immersed in fluid *Journal of Sound and Vibration* 333, 2543–55

[13] Eftekhari S and Jafari A A 2014 A mixed modal-differential quadrature method for free and forced vibration of beams in contact with fluid *Meccanica* 49, p 535–64

[14] Rezaiee-Pajand M, Kazemiyan M S and Aftabi S A 2018 Solving coupled beam-fluid interaction by DTM *Ocean Engineering* 167, p 380–96

[15] Al-Shuka H F N, Song R 2018 Hybrid regressor and approximation-based adaptive control of piezoelectric flexible beams. 2018 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC), Xi'an, China, p 330-334

[16] Farrell J A and Polycarpou M M 2006 *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches* (John Wiley, USA)

[17] Ioannou P and Fidan B 2006 *Adaptive control tutorial* (SIAM, USA)

[18] Huang A-C, Chien M-C 2010 *Adaptive control of robot manipulators: a unified regressor-free approach* (World Scientific Publishing Co. Pte. Ltd. Singapore)

[19] Al-Shuka H F N, Corves B, Zhu W-H 2013 Function approximation technique-based adaptive virtual decomposition control for a serial-chain manipulator *Robotica* 32, p 375-399

[20] Al-Shuka H F N 2018 On local approximation-based adaptive control with applications to robotic manipulators and biped robots *Int. J. Dynam. Control* 6, p 339–353

[21] Al-Shuka H F N, Song R 2018 Hybrid regressor and approximation-based adaptive control of robotic manipulators with contact-free motion 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC), Xi'an, China, p 325-329

[22] Al-Shuka H F N and Song R 2019 Decentralized adaptive partitioned approximation control of high degrees-of-freedom robotic manipulators considering three actuator control modes. *International Journal of Dynamics and Control*, p 744-757

[23] Al-Shuka H F N 2021 FAT-Based Adaptive Backstepping Control of an Electromechanical System with an Unknown Input Coefficient *FME Transactions* 49, p 113-120

[24] Dowell E H 1975 *Aeroelastic plates and shells* (Noordhoff International Publishing)

[25] Zhu W-H. 2010 *Virtual decomposition control: Toward Hyper Degrees of Freedom Robots* (Springer Verlag Berlin Heidelberg)

[26] Preumont A 2018 *Vibration control of active structures* (4th Edition, Springer International Publishing AG)