Inertial Effects on Berry’s Phase of Neutrino Oscillations

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Abstract

The Berry phase of mixed states, as neutrino oscillations, is calculated in a accelerating and rotating reference frame. It turns out to be depending on the vacuum mixing angle, the mass–squared difference and on the coupling between the momentum of the neutrino and the spinorial connection. Berry’s phase for solar neutrinos and its geometrical aspects are also discussed.

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1 Introduction

Particle interferometry has been recognized as a tool to carry out delicate measurement of physical quantities that can be related to the difference in phase of the interfering beams. Besides, the observation of quantum mechanical phases provides accurate informations about quantities determining the phase shift (as for example, the flux of a magnetic field) and allows to test the quantum behaviour of systems. A remarkable contribution in this topic has been given by Berry in his pioneering paper. In his work [1], he showed that a quantum state acquires an additional phase (Berry’s phase) $\beta^g$, which is related to the geometry of parameter space, besides the normal dynamical phase $\phi$. He considered a non–degenerate quantum system in an initial eigenstate of a Hamiltonian which varies adiabatically through a circuit $C$ in the parametric space. The state evolves under the Schrödinger equation remaining in each instant an eigenstate of the Hamiltonian.

Aharonov and Anandan [2] reformulated and generalized Berry’s result by disregarding the parameter space and considering an arbitrary, not necessarily adiabatic, cyclic evolution in the projective Hilbert space $\mathcal{P}(\mathcal{H})$, which is the space of one–dimensional subspace (called also rays) of an appropriate Hilbert space $\mathcal{H}$ (see also [3]).

Nevertheless, in all formulations, Berry’s results are based on non–relativistic quantum mechanics and thus are not covariant. The covariant generalization of Berry phase was derived in Ref. [4] where, by using the proper time method [5], the authors showed that

$$\beta^g = i \oint_C < \tilde{\Psi}(\lambda) | \frac{\partial}{\partial \lambda_a} | \Psi(\lambda) > d\lambda_a$$

$$= \phi + i \oint_C < \Psi(\lambda) | \frac{\partial}{\partial \lambda_a} | \Psi(\lambda) > d\lambda_a,$$

where $\lambda_a$ are the evolution parameters depending on the proper time, the state $\tilde{\Psi}(\lambda)$ satisfies the cyclic condition, $|\tilde{\Psi}(\Lambda + \lambda) >= |\tilde{\Psi}(\lambda) >$, and $|\Psi(\Lambda + \lambda) >= e^{i\phi} |\Psi(\lambda) >$.

On the experimental side, a series of striking experiments have also been carried out and their results provide direct or indirect evidences for the existence of such a geometric phase [2].

The aim of this paper is to derive the geometrical phase attributed to the cyclic evolution of a mixed state, as for example, neutrino oscillations. This result is carried out in an non–inertial reference frame, i.e. for an accelerating and rotating observer. As well known, the effects of the acceleration and the angular velocity are relevant in interferometry experiments. In fact, by using an accelerated neutron interferometer, Bonse and Wroblewski derived the predicted phase shift of quantum systems [6]. Because of the validity of the equivalence principle, one expects that this effect occurs also in a gravitational field, as has been verified in Ref. [8]. Besides, Mashhoon has derived a coupling of neutron spin to the rotation of a non–inertial reference frame [9] from an extension of the hypothesis of locality. In Ref. [10], the neutron Sagnac effect has been found using an angular velocity of about 30 times that of Earth. The spin–rotation
and the spin–acceleration contributions to the helicity precession of fermions has been calculated in [11], and recently, the inertial effects on neutrino oscillations have been analyzed in [12].

We find that the geometrical phase, when evaluated in a non–inertial reference frame, does depend on the parameters characterizing the physics of neutrino oscillations, i.e. the vacuum mixing angle and the mass–squared difference, and the ones characterizing the background geometry, the acceleration and angular velocity in our case. Besides, a dependence on the energy of neutrino also appear. The quantum phase shift induced by the non trivial topology of the background space–time has, as we will see, interesting geometrical consequences in connection to the curvature of the parameter space of states.

The layout of this paper is the following. In Sect. 2, we review the main features of neutrino evolution in an accelerating and rotating frame (see Ref. [12]). The Berry phase in such a frame is calculated in Sect. 3. Discussion and Conclusions are drawn in Sect. 4.

2 Neutrino Evolution in a Non-Inertial Frame

In Ref. [13], the neutrino phase is generalized in the following way

\[ |\psi(\lambda)\rangle = \sum_j U_{fj} e^{i \int_{\lambda_0}^{\lambda} P_{\text{null}} d\lambda'} |\nu_j\rangle, \tag{2.1} \]

where flavor and mass indices are indicated by \( f \) and by Latin letters, respectively. \( U_{fj} \) are the matrix elements transforming flavor and mass bases, \( P \) is the four momentum operator generating space–time translation of the eigenstates and \( p_{\text{null}} = dx^\mu / d\lambda \) is the tangent vector to the neutrino worldline \( x^\mu \), parameterized by \( \lambda \); \( x^\mu = (x^0, \vec{x}) \) are the local coordinates for the observer at the origin. We use the natural units.

The momentum operator \( P_\mu \), used to calculate the phase of neutrino oscillations, is derived from the mass shell condition

\[ (P_\mu + A_{G\mu} \gamma^5)(P_\mu + A_{G\mu} \gamma^5) = -M_f^2, \tag{2.2} \]

where \( A_{G\mu} \) are related to the spinorial connections appearing into the covariant Dirac equation in curved space–time [14]. \( M_f^2 \) is the vacuum mass matrix in flavor base

\[ M_f^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \tag{2.3} \]

\( \theta \) is the vacuum mixing angle. Ignoring terms of the order \( \mathcal{O}(A_G^2) \) and \( \mathcal{O}(A_G M_f) \), one gets that, for relativistic neutrinos moving along generic trajectories parameterized by \( \lambda \), the column vector of flavor amplitude

\[ \chi(\lambda) = \begin{pmatrix} <\nu_e|\psi(\lambda)> \\ <\nu_\mu|\psi(\lambda)> \end{pmatrix}. \tag{2.4} \]
satisfies the equation
\[ i \frac{d\chi}{d\lambda} = \left( \frac{M_f^2}{2} + p \cdot A_G \gamma^5 \right) \chi. \] (2.5)

In deriving (2.5), one uses the relation \( P^0 = p^0 \) and \( P^i \approx p^i \) [13].

In a frame with acceleration \( \vec{a} \) and angular velocity \( \vec{\omega} \), the components of \( A_G^\mu \) are [12]
\[ A_0^0 = 0, \quad \vec{A}_G = \sqrt{-g} \frac{1}{2} \left\{ \frac{2}{c^2} [\vec{a} \wedge (\vec{x} \wedge \vec{\omega})] \right\}, \] (2.6)

so that Eq. (2.5) becomes
\[ i \frac{d}{d\lambda} \left( \begin{array}{c} a_e \\ a_\mu \end{array} \right) = T \left( \begin{array}{c} a_e \\ a_\mu \end{array} \right), \] (2.7)

\( \sqrt{-g} \) is the determinant of the metric tensor of the accelerating and rotating frame [15]. \( a_f \equiv \psi_f(\lambda) >, f = e, \mu \) and the matrix \( T \) is defined as
\[ T = \begin{bmatrix} -(\Delta/2) \cos 2\theta & (\Delta/2) \sin 2\theta - \vec{p} \cdot \vec{A}_G \\ (\Delta/2) \sin 2\theta - \vec{p} \cdot \vec{A}_G & (\Delta/2) \cos 2\theta \end{bmatrix}. \] (2.8)

up to the \((m_1^2 + m_2^2)/2\) term, proportional to the identity matrix. Here \( \Delta \equiv (m_2^2 - m_1^2)/2 \). We restrict to flavors \( e, \mu \), but this analysis works also for different neutrino flavors. In order to determine the mass eigenstates \( |\nu_1\rangle \) and \( |\nu_2\rangle \), one has to diagonalize the matrix \( T \). Using the standard procedure, one writes the mass eigenstates as a superposition of flavor eigenstates
\[ |\nu_1\rangle = \cos \tilde{\theta} |\nu_e\rangle + \sin \tilde{\theta} |\nu_\mu\rangle, \]
\[ |\nu_2\rangle = \sin \tilde{\theta} |\nu_e\rangle - \cos \tilde{\theta} |\nu_\mu\rangle, \] (2.9)

where the mixing angle \( \tilde{\theta} \) is defined in terms of the vacuum mixing angle \( \theta \)
\[ \tan 2\tilde{\theta} = \frac{\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G}{\Delta \cos 2\theta}. \] (2.10)

We note that \( \tilde{\theta} \to \theta \) as \( \vec{A}_G \to 0 \) (i.e. \( \vec{a} \to 0, \vec{\omega} \to 0 \)). The eigenvalues of the matrix (2.8) are
\[ \tau_{1,2} = \pm \sqrt{\frac{\Delta^2}{4} \cos 2\theta + \left[ \frac{\Delta}{2} \sin 2\theta - (\vec{p} \cdot \vec{A}_G) \right]^2}. \] (2.11)

In the base of the mass eigenstates, we have \( |\psi(\lambda)\rangle = a_1(\lambda) |\nu_1\rangle + a_2(\lambda) |\nu_2\rangle \), so that Eq. (2.7) assumes the form
\[ i \frac{d}{d\lambda} \left( \begin{array}{c} a_1 \\ a_2 \end{array} \right) = \left( \begin{array}{cc} \tau_1 & 0 \\ 0 & \tau_2 \end{array} \right) \left( \begin{array}{c} a_1 \\ a_2 \end{array} \right), \] (2.12)
where \( a_i = \langle \nu_1 | \psi(\lambda) >, i = 1, 2, \) and

\[
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix} = \tilde{U} \begin{pmatrix} a_\epsilon \\ a_\mu \end{pmatrix}, \quad \tilde{U} = \begin{pmatrix} \cos \tilde{\theta} & \sin \tilde{\theta} \\ -\sin \tilde{\theta} & \cos \tilde{\theta} \end{pmatrix}.
\]

(2.13)

We have used the adiabatic condition \( d\tilde{\theta}/d\lambda \approx 0 \) in order that (2.12) is a diagonal matrix, i.e., we are neglecting the variations of acceleration and angular velocity, with respect to the affine parameter \( \lambda \), in comparing to their magnitude. Eq. (2.12) implies

\[
a_k(\lambda) = a_k(\lambda_0) e^{\alpha_k(\lambda)}, \quad \alpha_k(\lambda) \equiv -i \int_{\lambda_0}^{\lambda} \tau_k d\lambda'.
\]

(2.14)

with \( k = 1, 2 \). For the initial condition \( |\psi(\lambda_0) >= |\nu_\epsilon >, \) the state \( |\psi(\lambda) > \) is

\[
|\psi(\lambda) > = [\cos \theta_0 \cos \tilde{\theta} e^{-i\tau_1 \lambda} + \sin \theta_0 \sin \tilde{\theta} e^{-i\tau_2 \lambda}] |\nu_\epsilon >
\]

\[
+ [ -\cos \theta_0 \sin \tilde{\theta} e^{-i\tau_1 \lambda} + \sin \theta_0 \cos \tilde{\theta} e^{-i\tau_2 \lambda}] |\nu_\mu >, \tag{2.15}
\]

where \( \theta_0 = \tilde{\theta}(\lambda_0) \) and the adiabaticity condition has been used in (2.14). Therefore, accelerating and rotating observers will experience a flavor oscillation of neutrinos. For example, the probability to observe an electron neutrino is readily calculated from (2.15) and it is given by

\[
| < \nu_\epsilon | \psi(\lambda) > |^2 = \cos^2(\theta_0 + \tilde{\theta}) \sin^2 \alpha + \cos^2(\theta_0 - \tilde{\theta}) \cos^2 \alpha. \tag{2.16}
\]

3 Inertial Effects on Berry’s Phase

In this Section, the Berry phase is calculated in an accelerating and rotating frame. For convenience, let us invert Eq. (2.9)

\[
|\nu_\epsilon > = \cos \tilde{\theta} |\nu_1 > + \sin \tilde{\theta} |\nu_2 >, \tag{3.1}
\]

\[
|\nu_\mu > = -\sin \tilde{\theta} |\nu_1 > + \cos \tilde{\theta} |\nu_2 >.
\]

Repeating the previous analysis for the state \( |\nu_\epsilon(\lambda) >, \) we see that Eq. (2.7) can be read, formally, as a Schrödinger equation, the matrix \( T \) playing the role of the effective Hamiltonian. Then, the neutrino state \( |\nu_\epsilon > \) evolves according to the relation

\[
|\nu_\epsilon(\lambda) > = e^{-iT\lambda} |\nu_\epsilon > = e^{-i\tau_1 \lambda} \cos \tilde{\theta} |\nu_1 > + e^{-i\tau_2 \lambda} \sin \tilde{\theta} |\nu_2 >, \tag{3.2}
\]

being \( \tau_1 \) and \( \tau_2 = -\tau_1, \) defined in (2.11), the eigenvalues of \( T \) corresponding to the eigenkets \( |\nu_1 > \) and \( |\nu_2 >, \) respectively (see Eq. (2.12)). In a cyclic evolution, i.e. from \( \lambda \) to \( \Lambda + \lambda, \) the final state \( |\nu_\epsilon(\Lambda + \lambda) > \) and the initial state \( |\nu_\epsilon(\lambda) > \) (see Eq. (3.2)) do coincide, apart a phase factor, provided that \( \Lambda = -2\pi/(\tau_1 - \tau_2) \)

\[
|\nu_\epsilon(\Lambda + \lambda) > = e^{i\phi}|\nu_\epsilon(\lambda) >, \quad \phi = \frac{2\pi \tau_1}{\tau_1 - \tau_2} = \pi, \tag{3.3}
\]
Inserting Eq. (3.3) into Eq. (1.1) (for \(a = 1\), i.e. the evolution parameter does coincide with the affine parameter), the Berry phase of an electron neutrino moving along a cyclic path can be written as

\[
\beta_e^g = \phi + i \int_0^A <\nu_e(\lambda)|\frac{\partial}{\partial \lambda}|\nu_e(\lambda)>,
\]

(3.4)

where \(\phi\) is the dynamical phase evaluated in Eq. (3.3). An explicit calculation of (3.4) yields the result

\[
\beta_e^g = 2\pi \sin^2 \tilde{\theta}
\]

(3.5)

\[
= \pi \left[ 1 - \frac{\Delta \cos 2\theta}{\sqrt{\Delta^2 \cos^2 2\theta + (\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G)^2}} \right],
\]

where Eqs. (2.11) and (3.1) have been used. In similar way we get the Berry phase of muon neutrino

\[
\beta_\mu^g = 2\pi \cos^2 \tilde{\theta}
\]

(3.6)

\[
= \pi \left[ 1 + \frac{\Delta \cos 2\theta}{\sqrt{\Delta^2 \cos^2 2\theta + (\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G)^2}} \right],
\]

As arises from Eqs. (3.3) and (3.6), \(\beta_{e,\mu}^g\) depend on the vacuum mixing angle, on the mass–squared difference and energy of neutrinos, and on the parameters characterizing, through the spinorial connection, the background geometry: the acceleration and angular velocity. Besides, the quantum shift phase induces also a dependence on the energy of neutrinos. Note that \(\beta_e^g + \beta_\mu^g = 2\pi\).

A direct or indirect evidence of the Berry phases (3.3) and (3.4) is very difficult to achieve and how a quantum interferometry experiment could reveal it goes beyond the aim this paper. We just analyze some interesting consequences of the above results.

- If the condition

\[
\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G = 0
\]

(3.7)

holds, then Eqs. (2.11), (3.3) and (3.6) imply \(\tilde{\theta} = n\pi/2\), \(n = 0, 1, 2, \ldots\), so that

\[
\beta_e^g = 0 \quad \text{and} \quad \beta_\mu^g = 2\pi.
\]

(3.8)

Furthermore, in the rotating frame with angular velocity \(\omega\) and acceleration zero, Eq. (3.7) reduces to the form

\[
|m_2^2 - m_1^2| \approx \frac{4E_\nu \omega}{\sin 2\theta}
\]

(3.9)

where \(E_\nu\) is the energy of ultrarelativistic neutrinos, \(E_\nu \approx |\vec{p}|\). Let us analyze Eq. (3.9) for solar neutrinos and in the case in which the observer is comoving with
the Earth, i.e. its angular velocity is $\omega \sim 7 \cdot 10^{-5}\text{rad/sec}$. Results of the mass–
squared difference, calculated by using (3.9) for typical values of the solar neutrino
energies and vacuum mixing angle, are reported in the Table. The agreement
with the experimental data comes from neutrinos with energy varying in the range
$10 \div 60\text{MeV}$. In this range in fact, we find a mass–squared difference of the order
$10^{-12} \div 10^{-10}\text{eV}^2$ for vacuum mixing angle $10^{-1} \leq \sin 2\theta \leq 1$ (see also [12]).

At the present, there is a strong evidence in favor of oscillations of neutrinos and
of their non–zero masses. Such results have been found in different experiments:
solar neutrino experiments [16], atmospheric neutrino experiments [17], and the
accelerator LSND experiment [18]. Recent reports indicate that the best fit in
favor of solar neutrino oscillations are obtained for the following case [19]

$$|m_2^2 - m_1^2| \simeq (0.5 \div 1.1) \cdot 10^{-10}\text{eV}^2, \quad \sin^2 2\theta \simeq 0.67 \div 1,$$

in very good agreement with our results summarized in the Table.

- If the momentum of the neutrino $\vec{p}$ is perpendicular to the spinorial field $\vec{A}_G$, then the
coupling term vanishes and Eq. (3.5) and (3.6) reduce to

$$\beta_{e}^g = 2\pi \sin^2 \theta, \quad \beta_{\mu}^g = 2\pi \cos^2 \theta,$$

i.e. the mixing angle $\tilde{\theta}$ coincides with the vacuum mixing angle $\theta$. Of course, when
the angular velocity and the acceleration of the frame are zero, we recover trivially
Eq. (3.10) and results of Ref. [20], obtained for Minkowskian background. Eqs.
(3.5) and (3.6) represent a generalization of the results obtained in Ref. [20].

- For ultra-relativistic neutrinos and high values of the acceleration and angular velocity,
so that the coupling term is much more greater than the mass–squared difference,
the Berry phase (3.5) and (3.6) assume the following values: $\beta_{e}^g \approx \pi$ and $\beta_{\mu}^g \approx \pi$.

Extension of these results can be done, invoking the equivalence principle, to station-
ary gravitational fields, last ones responsible of the quantum phase shift. The previous
analysis applies also to quantum system involving boson mixing, as the oscillations of
$K^0 - \bar{K}^0$.

4 Discussion and Conclusions

In this paper, the covariant generalization of the Berry phase proposed in Ref. [4] has been
applied to problems involving mixed states, as neutrino oscillations. We have calculated
the Berry phase of the electric and muon neutrinos in a cyclic evolution with respect
to an accelerating and rotating reference frame and we have found that these phases do
depend on the vacuum mixing angle, the mass–squared difference and on the coupling
between the momentum of neutrinos and the spinorial connection.
The condition of no-mixing expressed by Eq. (3.7), which diagonalizes the mixing matrix (2.8), implies that the Berry phase of the electron and muon neutrinos assume the values equal to 0 or $2\pi$, respectively. Such a condition has been analyzed for observers comoving with the Earth, so that $\omega \sim 10^{-5}\, \text{rad/sec}$ and $|\vec{a}| = 0$. In this particular case, the numerical values of the mass–squared difference, calculated by using Eq. (3.9) (see Table), are compared with ones of the recent experimental data, showing a very good agreement. It is worth to note that the inertial effects on the Berry phase, hence on the neutrino oscillations, induce a dependence on the orientation of the rotating observer (the direction of the angular velocity) with respect to the direction of the neutrino momenta. It implies hence a dependence on the zenith angle recently discussed in the framework of the atmospheric neutrino flux [19].

A direct or indirect measurement of inertial (or gravitational) effects on the Berry phases related to the neutrino particles is at the moment very difficult, considering also the fact that the values of the mass–squared difference of neutrinos and the vacuum mixing angle are till now open issues. Nevertheless, the Berry phases (3.5) and (3.6) have interesting implications from the geometrical point of view. As already said in the Introduction, the Berry phase is attributed to the holonomy in the parameter space. It appears when a quantum state is parallelly transported in the parameters space around a close path. The final quantum state is rotated, with respect to the initial one, by an angle which can be related to the connection (the Berry connection) $A = \langle \bar{\Psi} | d | \bar{\Psi} \rangle = A_{a} d\lambda^{a}$ appearing in Eq. (1.1), being $d = (\partial/\partial \lambda_{a}) d\lambda^{a}$ the exterior derivative in the parameter space. Besides, one can also define a curvature (the Berry curvature) as the field strength $F = dA = (d < \bar{\Psi} |) \wedge (d| \bar{\Psi} >)$.

The existence of this phase, associated with the cyclic evolution, is universal in the sense that it is the same for the infinite number of possible motions along the curves in the Hilbert space, which project to a given closed curve in the projective Hilbert space of the rays, and for the possible (effective) Hamiltonians, which rule the evolution of the state along these curves.

In a Minkowski space–time, the Berry phase is related only to the vacuum mixing angle: The quantum vector state is rotated of the angle $\theta$ when parallelly transported along a close curve, so that, as discussed above, it can be associated to the curvature in the space of states (see Eq. (3.10) and Ref. [20]).

In a non–inertial reference frame, new geometrical features arise, as one can see from Eqs. (3.5) and (3.6):

a) The parallel transport along a close curve contains also the contribution due to the spinorial connection $A_{\mu}^{\alpha}$, which induces a quantum shift phase, hence an additional rotation in the parameter space of the quantum vector state. The total rotation is given by the (mixing) angle $\tilde{\theta}$, so that the Berry curvature turns out to be related both the vacuum mixing angle and to the spinorial connection, i.e. to the non trivial geometry of the background space–time.

b) Due to the coupling term, $\vec{p} \cdot \vec{A}_{C}$, the geometrical phases (3.5) and (3.6) show a
dependence on the energy of the neutrinos, loosing in such a way the character of universality: different values of the neutrino energies correspond to different Berry’s phase.

Such a new geometrical setting follows from the generalization of the quantum mechanical phase \(2.1\) in which, as suggested by Stodolsky \[24\], the metric field appears in the definition of the scalar product. The dependence of the geometrical phase on the energy of particles and on the parameters characterizing the background geometry is a common aspect of the behaviour of a quantum particle in a certain class of four dimensional stationary space–time, as tubular matter source, slowly moving mass current and spinning cosmic string \[21, 22, 23\]. In the last case, for example, it is shown that the gravitational geometrical phase of a quantum particle with energy \(E\) moving around a spinning cosmic string (with angular velocity \(J\)) is \(\sim JE\) \[23\].

As final comment, we stress that the previous results holds also for stationary gravitational field, as an obvious consequence of the validity of the equivalence principle. Then, the understanding of the issues concerning the influence of the gravity on the Berry phase are of interest in view of theoretical arguments which try to construct general relativity as a gauge theory and, in particular, to formulate the symmetric second–rank tensor field in the framework of the unified gauge approach. In such a way, the gravitational field should play the role of electromagnetic field, in strict analogy with the Bohm–Aharonov effect, and its theoretical (and phenomenological) consequence could help for a better understanding of the quantum nature of particles evolving in non trivial geometries.

Table: Estimation of \(|m_2^2 - m_1^2|\) given by \((3.9)\) as function of \(E_\nu\), \(\sin 2\theta\), and fixed value of \(\omega \sim 7 \cdot 10^{-5}\) rad/sec.

| \(E_\nu\) (MeV) | \(\sin 2\theta\) | \(|m_2^2 - m_1^2|\) (eV²) |
|------------|-------------|----------------|
| 1          | 1           | \(10^{-13}\)   |
| 10         | 1           | \(10^{-12}\)   |
| 10         | 10⁻¹       | \(10^{-12}\)   |
| 50 ÷ 60    | 1           | \(10^{-10}\)   |

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