Lifting scheme-based matched wavelet design for effective characterisation of different types of voltage sag

Akanksha Aggarwal | Manish Kumar Saini

Electrical Engineering Department, D.C.R. University of Science & Technology, Murthal, Haryana, India

Correspondence
Manish Kumar Saini, Electrical Engineering Department, D.C.R. University of Science & Technology, Murthal, Haryana, India.
Email: manishkumar.ee@dcrustm.org

Abstract
This study presents a novel scheme for designing biorthogonal wavelets matched to the voltage sag (VS) signals. Instead of using standard wavelets, matched wavelets designed in this study can extract more distinguishing features for better characterisation of sag signals, although this study proposes only the designing process of matched wavelets. The key method used for designing matched wavelets is the lifting scheme, which has never been reported in the literature for constructing matched wavelets for PQ signals. Besides using the lifting scheme, the proposed work preprocesses sag signals to make input signal of lifting scheme rich of sag-related information. During pre-processing, 1-D sag signal is first converted to 2-D form for capturing information of every small transition (which may be overlooked in 1-D form) and then made 1-D by ring projection to implement 1-D lifting scheme. In this way, biorthogonal wavelets have been well matched to every type of VS signal collected through the experimental setup simulating different line faults, induction motor starting, transformer energisation and heavy load starting. Also, the characteristics of designed wavelets are evaluated in a MATLAB environment using frequency response and pole-zero mapping in \( z \)-plane. Higher PSNR values during signal reconstruction proved outperformance of newly designed matched wavelets.

1 | INTRODUCTION

Power quality disturbances (PQDs) are causing big nuisance mainly for big industries like petrochemical, textiles, paper, pharmaceutical, steel, fertiliser industry to name a few. The more worrisome fact is that the events causing these PQDs are inescapable because many electronic equipment, for example, solid-state switching devices, electronic loads, uninterruptible power supplies, industrial plant converters, flexible alternating current transmission system (FACTS) devices and so forth, have become an integral part of today’s industrial world [1]. Therefore, the field of power quality (PQ) analysis has witnessed the interest of a large community of researchers [2–6]. Among many PQDs like sag, swell, harmonics and inter-harmonics, transients, flickers, notch and so forth, voltage sag (VS) is the most frequent disturbance (approximately 92%–98% of total PQDs) and also causes severe economic loss due to degradation in product quality and production halts [7]. During an occurrence of VS, it would be the worst situation for both the utility and consumer if the reason responsible for its occurrence could not be detected. It will cause a delay in the application of appropriate mitigation devices and in preventing its further occurrences. So detection and recognition of causes responsible for generating VS are as much important as detection of its occurrence. Consequently, the attention of researchers is diverting more towards addressing the issue of recognition of sag-causing events like short-circuit faults, transformer energisation, startup of induction motor, connection of heavy three-phase load and so forth [8, 9]. All these events cause a momentary increase in current that results in sag at point of common coupling (PCC).

For detection of causes responsible for generating VS, several signal processing algorithms have been employed in the literature, for example, independent component analysis [10], empirical mode decomposition and Hilbert transform [11], discrete wavelet transform (DWT) [12–14], fractionally delayed wavelet transform [15, 16], S-transform [17, 18], wavelet transform (WT) with spectral and statistical analysis [19] and variational mode decomposition [20]. The domain of signal processing techniques employed for spectral analysis of PQ signals...
is being ruled by wavelets in one or another form because wavelets can capture all information of signal, for example, trends, breakdown points, discontinuities in higher derivatives and self-similarity [21], which cannot be revealed by other signal processing techniques proposed by other research works. Moreover, WT gives the freedom to select the most suitable basis function for the given signal unlike the unique basis used in all other signal processing techniques. In PQ literature, wavelet was first used by Ribeiro in 1993 for analysis of non-stationary PQ signals [22]. Since then, it has been a favourite technique of researchers for PQ signals due to its excellent multiresolution analysis (MRA) properties [23]. But most of the researchers have proposed the application of only standard available wavelets like Daubechies, Morlet, Coiflet, Haar, Symlet, Meyer and so forth irrespective of the fact that the wavelet is correlating well with the signal or not correlating. To the best of the authors’ knowledge, there are only a handful of research works in the PQ literature that deal with the matched wavelet [24, 25]. In [24], the authors matched wavelet spectrum amplitudes to the transient signal using closed-form expressions. In [25], the authors used the concept of fractional Brownian motion for designing statistically matched wavelet for PQD signals. Thus, the domain of matched wavelets is still unexplored in the field of PQ analysis.

Though it is well known that WT is nothing but extracting the correlation between the signal and scaled and translated versions of wavelet. Therefore, wavelet transform can obviously achieve the best results if wavelet is matched to the signal of interest. Utilising this fact, there is a need to exploit the power of matched wavelets in PQ analysis. However, the concept of matched wavelet is not new in the signal processing literature. Many researchers have put forward new wavelet functions that can characterise a signal in the best possible manner [24, 26–28]. Basically, there are two approaches followed for constructing signal-adapted or matched wavelets. The first approach uses the available wavelets and modifies them in order to embed the desired features in the wavelet [26, 29]. The second approach designs new wavelet from the scratch, so it is a more complex method. In direction to ‘adapt’ (i.e. a simpler method), many authors have proposed several design techniques like minimisation of signal projection on detail subspace [30], minimisation of the difference between desired and reference signals [31], matching energy spectrum of the available wavelet with the signal [32] and lifting scheme [33]. Lifting scheme outperforms other methods owing to its multiple advantages. It allows to design the wavelet filter while implementing WT in simple filtering steps (known as lifting steps) [34]. At any step, the process can be easily inverted just by reversing the sign of operations (+/-) and proceeding in the reverse direction. Lifting scheme works entirely in the spatial domain and does not depend upon Fourier transform; therefore, it facilitates to transform the finite length signal without introducing artefacts at the boundaries [34]. In the present work, VS signal used for constructing matched wavelet is also of finite length; it proves the suitability of lifting scheme for constructing matched wavelet in this work. Hitherto in the PQ literature, lifting scheme has been only used for implementing DWT in small filtering steps [35]. However, it has never been used for constructing the wavelets matched to the PQ signals.

Therefore, addressing these research gaps found in the literature, the present work proposes the use of the lifting scheme to construct matched wavelets for VS signals observed during different sag-causing events. The purpose of these matched wavelets is a more effective analysis and characterisation of VS signals leading to the extraction of highly distinguishing features. This study presently focuses only upon the design of matched wavelets for sag signals as more efficient tools to be used further for implementing DWT. The key contributions of this study can be specified as follows:

(i) Matched biorthogonal wavelets are designed for different types of VS signals using the lifting scheme.

(ii) Signal transition information during VS is intensified by using 2-D representation of VS signals, and this intensified information (after being converted to 1-D form by ring-projection technique to simplify further processing) is used as the input signal to lifting scheme in place of original sag signals.

(iii) A filterbank of matched wavelets is developed for time-frequency analysis of all possible VS signals found in case of a three-phase fault, three phase-to-ground fault, double phase fault, double phase-to-ground fault, single phase-to-ground fault, induction motor starting, transformer energisation and connection of heavy loads in the circuit.

(iv) Stability of designed wavelets is tested through the frequency response and pole-zero plots. The time-frequency analysis capability of the designed wavelets is checked through peak signal-to-noise ratio (PSNR) obtained after signal reconstruction with newly designed matched filters.

All these contributions are elucidated in the subsequent sections of this work. Section 2 elucidates all stages of the proposed algorithm. Section 3 posits the theoretical background of the concepts utilised in this work. Section 4 presents experimental results obtained by the proposed method with their analytical discussion followed by the conclusion in Section 5.

2 | PROPOSED METHODOLOGY

This section presents the proposed method for designing biorthogonal wavelets matched to VS signals in case of different underlying causes. The designing process proposed in this work is represented in Figure 1. The proposed method is mainly based upon lifting scheme and how the VS signal is pre-processed before being given as input to the lifting scheme. By combining the pre-processing stages and lifting scheme, there are overall five steps, that is, segmentation, 2-D representation, ring projection, design of biorthogonal matched wavelet using lifting scheme and construction of a filterbank comprising matched wavelets for every type of VS signal observed during different VS events. Step-wise detailed discussion is presented in the following subsections:
2.1 | Segmentation

Different events that cause VS are replicated through the experimental setup, and three-phase voltage signals are acquired from PCC, that is, the bus between fault and consumer endpoint. Voltage signals are stored in normalised form (i.e. per unit values). Normalised three-phase voltage signals are segmented into single phases. Besides important information, these voltage signals also exhibit redundant information before and after the event occurrence. Therefore, to make wavelet designing process more effective and embed only sag-related information in designed wavelet, only the relevant segment is taken out from the recorded voltage signal. For segmentation, WT-based adaptive threshold technique is followed [36]. After segmentation of event duration, one pre-event and post-event cycles are also included for extracting complete information about transitions and voltage recovery during the sag-causing event.

2.2 | 2-D representation of voltage waveforms

As the segmented voltage signals are in 1-D form, there are chances of missing the transition period of VS signals during analysis. Therefore, 1-D voltage signals obtained after segmentation are converted into 2-D representation for improving the detectability of any sudden transitions. The transformation from 1-D to 2-D representation is carried out by creating a 2-D voltage matrix whose rows correspond to the repeated patterns of segmented PQ waveform. Every value in this waveform is now considered as the pixel of the voltage image that corresponds to each sampling point of the waveform. Thus, all pixel intensities are scaled and plotted in form of a grayscale image to have the 2-D representation of 1-D voltage signal.

2.3 | Ring projection

For designing the matched wavelet using the lifting scheme, it is better to have event-related information in 1-D form, as the 2-D implementation of lifting scheme requires both horizontal and vertical operations. As a result, the application of the lifting scheme on 2-D signal becomes computationally very intensive [37, 38]. Therefore, ring projection is applied on 2-D grayscale voltage images obtained in the previous stage to first convert them into 1-D form. In this method, the Cartesian frame of the image is first converted into the polar frame. Then, circular rings are projected over the image and all the pixel values obtained by rotating the angular phasor from 0 to 360° with the resolution of 1° are summed up for each circular ring. The mathematical expression for summing up all the projected pixel intensities is given in Equation (5). This summation gives one scalar value for each circular ring. Thus, a total of $q$ values are obtained corresponding to $q$ circular projections and form a 1-D signal.
1-D signal is taken as representative of VS signal in the process of wavelet designing through the lifting scheme.

### 2.4 Lifting scheme for matched wavelet design

Since all information about VS event is embedded in the ring projected 1-D pattern, the 1-D ring projected pattern is, therefore, given as input to the lazy wavelet system. In the first step of lifting, the input signal is splitted into even-indexed and odd-indexed streams using the lazy filters. Then, odd-indexed values are predicted using their neighbours (i.e. even-indexed values) through predict filter $P(z)$. This step is known as ‘predict’ step. The difference between actual odd-indexed values and predicted odd-indexed values is regarded as detail subband $d_n[z]$. For achieving good prediction of odd-indexed values from its exact nearest even-indexed values, 2-tap predict filter is designed accordingly by minimising the energy of detail subband $d_n[z]$ using the least-square method. This step gives two coefficients of 2-tap predict filter, which are further used for updating decomposition highpass filter and reconstruction lowpass filter of standard biorthogonal 5/3 wavelet system. In this manner, decomposition highpass filter and reconstruction lowpass filter are adapted to the input signal.

Then, detailed coefficients obtained in ‘predict’ step are given as input to the update filter $U(z)$ to further update even-indexed values as shown in Equation (1). The updated even-indexed values are considered as approximation coefficients $a_n[z]$. Then, the approximation subband $a_n[z]$ is upsampled by 2 as per the reconstruction algorithm. The upsampled signal is passed through the new reconstruction lowpass filter. This filtered signal should closely resemble the input signal. By applying the least square method on their difference, coefficients of the update filter $U(z)$ are computed. Using these filter coefficients, decomposition lowpass and reconstruction highpass filter of biorthogonal 5/3 wavelet system are modified. In this manner, both lowpass and highpass filters of biorthogonal 5/3 wavelet system are adapted to the VS signal. The decomposition lowpass and highpass filter recursively form the matched biorthogonal wavelet for particular input VS signal.

### 2.5 Wavelet filterbank

By following the procedure explained in previous stages, biorthogonal matched wavelet is designed for all three single-phase voltage signals found in every category of VS. Since eight categories of VS (corresponding to eight sag-causing events, i.e. three-phase fault, three-phase-to-ground fault, double phase fault, double phase-to-ground fault, single phase-to-ground fault, induction motor startup, transformer energisation and heavy load starting) are considered in this work, there are 24 types of matched wavelets designed for all three phases of different types of VS signals. The set of all these designed matched wavelets forms matched wavelet filterbank. From this matched wavelet filterbank, only the wavelet matched to a particular type of input VS signal can best analyse and characterise that specific VS signal. This stage completes the proposed methodology for designing a complete filterbank having matched wavelets for different VS signals for their MRA.

### 3 THEORETICAL PRELIMINARIES

This section gives a concise description of various techniques used in the proposed work for designing matched wavelets. The key techniques are 2-D representation of 1-D voltage signals, ring projection technique and lifting scheme used to construct the matched biorthogonal wavelets. These techniques are briefed in the following sub-sections.

#### 3.1 2-D representation

The acquired voltage signals are 1-D waveforms. The discontinuity or any anomaly in the waveform can be better detected and analysed in its 2-D representation. Therefore, 1-D voltage signals are converted into 2-D representation $v(x,y)$, that is, image form, for improving the detectability of any abnormal features that can be overlooked while using 1-D waveform [39]. The 2-D representation of 1-D voltage signal is formed by creating a 2-D voltage matrix in which matrix rows are created by the repeated patterns of segmented VS data. In this manner, sag event information gets accentuated by $n$ times where $n$ is the number of rows in the image over which sag pattern is repeated as illustrated in the following equation:

$$I_v = \begin{bmatrix} v(1, 2, \ldots, L) \\ \vdots \\ \vdots \\ v(1, 2, \ldots, L) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ v(1, 2, \ldots, L) \end{bmatrix}_{\infty L}$$

In Equation (1), $v$ corresponds to the segmented waveform of any single-phase voltage signal, say phase-A. Here, $L$ is the number of samples in the segmented voltage signal. Thus, $v$ is repeated over $n$ rows to compose a 2-D matrix, $I_v$, which is referred to as voltage matrix for phase-A. Every value in this matrix is now considered as a pixel of the voltage image. Thus, all the pixel intensities are plotted in form of a grayscale image to have 2-D representation of 1-D voltage signal. Figure 2 shows the grayscale image formed by repeating the voltage pattern over multiple rows. The 2-D representation enables to exploit the benefits of introducing redundancy in this voltage matrix by having enhanced disturbance features. The 2-D representation transforms the time-varying sinusoidal voltage signal in a form similar to a longitudinal wave having some rarefactions and compressions. The compressions in the image reflect higher magnitude areas and rarefactions reflect lower magnitude areas. Transitions in the gray image of the normal voltage signal are regular and smooth. However, a drastic change appears in the
gray image at the time of any disturbance or transition from the normal waveform as shown in Figure 2. Thus, 2-D representation makes it easy to visualise the short discontinuity in the voltage waveforms.

3.2 Ring projection

Tang [40] first proposed the method of ring projection for converting 2-D patterns to 1-D patterns, thus, reducing the large pattern into a smaller dimension for feature extraction. Ring projection is utilised in this work for transforming 2-D voltage image into 1-D voltage information by projecting the pixels along circles. In this technique, the grayscale voltage image \(v(x,y)\) is like a 2-D density function representing planar distribution of mass. It other words, it can be understood as a homogeneous distribution over the complete region of the image. From this uniform distribution, centroid \(v(x_0, y_0)\) is derived in this region and the origin of the reference plane is shifted to this centroid by using the following equation [41]:

\[
R = \max\left|N(x_0, y_0) - v(x_0, y_0)\right|
\] (2)

In Equation (2), \(N(x_0, y_0) - v(x_0, y_0)\) refers to the Euclidean distance in two points, \(N\) and \(v\), in the plane. Now, the Cartesian frame being used as the original reference is converted to the polar frame using the below equations [41]:

\[
\begin{align*}
\rho &= r \cos \theta \\
\phi &= r \sin \theta
\end{align*}
\] (3)

Thus, the voltage image \(v(x_0, y_0)\) can now be represented in the polar frame as shown below [41]:

\[
v(x_0, y_0) = v(r \cos \theta, r \sin \theta)
\] (4)

where \(r \in [0, \infty)\) and \(\theta \in (0, 2\pi]\). With fixed \(r \in [0, R]\), pixel values projected along the circle of radius \(r\), with \(\theta\) varying from 0 to \(2\pi\), are summed for that particular radius as illustrated by the following integral [41]:

\[
f(r) = \int_{0}^{2\pi} v(r \cos \theta, r \sin \theta) d\theta
\] (5)

The value of \(f(r)\) gives total mass distributed along the circular ring of radius \(r\). The number of values to be summed in this integral is equal to the number of angles on which the projected image value is taken. This procedure is also illustrated in Figure 3. Value of \(f(r)\) is termed as ring projected value as it returns a single value for a particular \(r\) by summatiing all the pixels projected along a ring of specific radius \(r\). Here, radius decides the number of circles projected on the image. The single-variate function so obtained in the above steps, that is, \(f(r)\) is now used as input signal in place of VS signal.

3.3 Lifting scheme

Lifting scheme was introduced by Sweldens [34] for dividing conventional WT into a limited number of simple filtering stages and for constructing new adapted wavelet from the conventional wavelet. The design offered by lifting scheme is very modular, therefore, it makes every stage invertible, and thus provides perfect reconstruction at every stage. Utilising the benefit of lifting scheme, it has been used for constructing signal-matched biorthogonal wavelet in this work. Lifting scheme has three basic steps listed as follows: (i) Split (ii) predict and (iii) update. All three steps are shown in Figure 4 and described below:

During splitting, the input signal is partitioned into even-indexed and odd-indexed values by passing through a lazy wavelet system as shown in Figure 4. Here, the ring projected 1-D signal is regarded as coarser version of input VS signal [42]. Lazy wavelet structure has the following filters: Lowpass filter \(h_0 [n] = [1, 0]\) with \(h_0(0)\) as centre and highpass filter \(h_1 [n] = [1, 0]\) with \(h_1(1)\) as the centre. These filters can be represented in z-domain as \(H_0 (\zeta) = 1\) and \(H_1 (\zeta) = \zeta\). This lazy wavelet filterbank is named so because it does nothing except dividing the signal into two parts: Even-indexed \((r, n)\) and odd-indexed \((s, n)\) as shown in Figure 4(a).

During predict step, even-indexed values are passed through the predict filter, \(P(\zeta)\), and the filtered output is subtracted from odd-indexed values to get lower subband signal, denoted as \(d_{-1}[n]\) in Figure 4(a). The mathematical expression for this lower subband signal is written as below [34]:

\[
d_{-1}[n] = v[2n + 1] - v[2n] * p[n],
\] (6)
Here, ∗ represents convolution operation. The predict filter should be chosen carefully in such a manner that odd-indexed values are accurately estimated by using its adjacent past and future even-indexed values. For fulfilling this condition, 2-tap predict filter $P(\zeta)$ is chosen as in Equation (7):

$$P(\zeta) = p(0) + p(1) \zeta$$

Even length of $P(\zeta)$ ensures that the odd sample is predicted by using an equal number of past and future even values. By using the chosen expression of $P(\zeta)$ in Equation (7), the lower subband signal $d_{-1}[n]$ can be rewritten as follows:

$$d_{-1}[n] = v[2n + 1] - p(0)v[2n] - p(1)v[2n + 2]$$

Thus, odd sample $v[2n + 1]$ is estimated using its just before and after even samples. For better perception, Figure 5 presents even-indexed values, which are being used to estimate odd-indexed values using the 2-tap predict filter $P(\zeta)$. Since $d_{-1}$ presents the difference between odd samples and predicted odd samples, it is regarded as the detail subband that should have minimum energy. Thus, by minimising the energy of the detail subband $d_{-1}$ with the least square criterion, coefficients of predict filter $p[n]$ are computed as in the below equation:

$$p = \min_p \| d_{-1} \|^2_2 = \min_p \| n_0 - Cp \|^2_2$$

where $C$ is convolution matrix having even and shifted even-indexed values of $n$, and $p$ refers to the vector representation of predict filter $P(\zeta)$. Since the action of predict filter resembles the highpass filter, predict filter is used to modify the decomposition highpass filter and also the reconstruction low-pass filter as per Equations (10) and (11) [43]:

$$H_1(\zeta) = H_0^0(\zeta) - H_0^0(\zeta) P(\zeta^2)$$

$$G_0(\zeta) = G_0^0(\zeta) + G_1^0(\zeta) P(\zeta^2)$$

Here, $H_0^0(\zeta)$ is the older decomposition lowpass filter. $G_0^0(\zeta)$ is the older reconstruction highpass filter. Here, a node in the superscript shows the older filters that have been modified to get new decomposition highpass filter $H_1(\zeta)$ and new reconstruction lowpass filter $G_0(\zeta)$. This completes the predict
step where two filters of the biorthogonal matched filterbank have been constructed from standard 5/3 biorthogonal filterbank.

For the update step, the lower subband signal $d_{-1}(n)$ is passed through the update filter $U(z)$ and the filtered output is added to the even-indexed samples to get the upper subband signal $a_{-1}[n]$ as shown in Figure 4 and expressed below [34]:

$$a_{-1}[n] = v[2n] + d_{-1}[n]*u[n]$$ (12)

where $u[n]$ shows the time-domain representation of $U(z)$. Like predict filter, the update filter is to be chosen in such a manner that values in the upper branch are updated using the adjacent neighbours only. While considering this requirement, the update filter is expressed as in Equation (13):

$$U(z) = u(0) + u(1)z^{-1}$$ (13)

This expression of 2-tap update filter $U(z)$ ensures that the even-indexed values are being updated from the equal number of past and future adjacent values only. Using the selected update filter, expression of approximation subband can be rewritten as follows:

$$a_{-1}[n] = v[2n] + u(0)d_{-1}[n] + u(1)d_{-1}[n-1]$$ (14)

After replacing the value of the detail subband $d_{-1}[n]$ in Equation (14), we get

$$a_{-1}[n] = v[2n] - u(0)p(0)v[2n] + u(0)v[2n+1] - u(0)p(1)v[2n+2] - u(1)p(0)v[2n-2] + u(1)v[2n-1] - u(1)p(1)v[2n]$$ (15)

Rearranging the above equation, approximation subband can be expressed as follows:

$$a_{-1}[n] = [-(u(0)p(1))v[2n+2] + [u(0)]v[2n+1] + [1 - u(0)p(0) - u(1)p(1)]v[2n] + [u(1)p(2n-1) + [u(1)p(0)]v[2n-2]$$ (16)

This equation proves that the even-indexed coefficients $v[2n]$ are updated using the adjacent values ranging from $v[2n+2]$ to $v[2n-2]$. This is better illustrated in Figure 5. In Figure 5, $v[2n]$ is updated using the neighbouring detail subband values, which collectively include four neighbouring samples of $v[2n]$. Then, the upper subband signal $a_{-1}[n]$ is upsampled by 2 and passed through reconstruction lowpass filter $g[n]$, which is modified in the update step. The signal thus obtained in the upper branch of reconstruction phase should closely approximate to the input signal $r[n]$. Using this fact, coefficients of the update filter $u[n]$ are computed by solving the below equation with the help of least square method:

$$a = \arg \min_u \sum_u (r[n] - v[n])^2$$ (17)

As the decomposition lowpass filter $H_0(z)$ and reconstruction highpass filter $G_1(z)$ can be expressed in terms of the update filter, these filters have been modified by using the above-obtained update filter $U(z)$ as in Equations (18) and (19) [43]:

$$H_0(z) = H_0^0(z) + H_1^0(z)U(z^2)$$ (18)

$$G_1(z) = G_1^0(z) - G_0^0(z)U(z^2)$$ (19)

**4 | EXPERIMENTAL RESULTS AND DISCUSSION**

Different stages of the proposed scheme, as shown in Figure 1, are implemented for designing the wavelets matched to different VS signals. Here, the following eight sag-causing events have been taken cognizance of: Line faults (e.g. three-phase fault, three-phase-to-ground fault, double-phase-to-ground fault, double-phase fault, single-phase-to-ground fault), induction motor starting, transformer energisation and connection of heavy load. Among all these events, short-circuit faults cause the most severe VS and, therefore, demand more concern [44]. In case of short-circuit faults, the single phase-to-ground fault is the most common fault and the three-phase fault is the most severe fault [8]. Therefore, the three-phase fault has only been chosen for illustration of the output of various stages of the proposed method considering the brevity of work.

Every event/cause of VS generates different disturbances in all three phases of the voltage signal. So all types of voltage signals observed during the above-mentioned events have been collected from the experimental setup whose schematic diagram is shown in Figure 6. Parts A, B and C are used for generating VS by transformer energisation, by induction motor and heavy load starting and by line faults, respectively. VS data have been collected at 415 V bus with the sampling frequency of 20 kHz by varying the following parameters: Fault duration, fault resistance, load resistance and motor ratings. First, the single-phase voltage signals are segmented from the acquired three-phase voltage signal collected during three-phase fault as shown in Figures 7 and 8. As observed from Figure 8, all three-segmented phases have a symmetrical effect of VS. In all three phases, voltage drops equally to a lower value in sag duration. However, the voltage varies among the three phases during the recovery period. This variation shows that every phase signal needs to have their own matched wavelet for better MRA. All single-phase voltage signals, presently in 1-D form, are converted to 2-D forms, which are represented in Figure 9 as grayscale images. During the three-phase fault, sudden drop and recovery of all three single-phase voltages are clearly visible in their grayscale images. Thus, 2-D representations show more
pronounced effect of any sudden transition/change in waveform and are able to give more effective VS information for consequent stages of the proposed algorithm.

The 2-D representations having magnified signal information are transformed into 1-D patterns shown in Figure 10 by applying the ring projection technique. As it is evident from Figure 10 that no ring projected 1-D pattern is same, there is a certain difference even between two phases that were looking almost similarly affected by VS event. This observation proves the unique feature of these projected patterns. Thus, even a minor deviation in different phase voltages, which is not noticeable in the initial 1-D voltage signal form, gets embedded in the projected pattern to accurately represent the signal information during wavelet designing.

Using the ring projected patterns obtained as above, signal matched wavelets are designed using the lifting scheme for all types of VS signals. Biorthogonal 5/3 wavelet is used as the basic wavelet in lifting procedure. Here, alike biorthogonal 5/3 wavelet, the lowpass filters are of length five and highpass filters are of length three. All the lowpass and highpass filter coefficients obtained through the lifting scheme are tabulated in Table 1 along with the coefficients of predict filter \(h[n]\) and update filter \(u[n]\). Here, \(h[n]\) signifies decomposition lowpass filter, whereas \(b[n]\) signifies decomposition highpass filter. Using the filter coefficients of decomposition lowpass and highpass filters, matched wavelet for all the respective cases are obtained through the iterative method. Figure 11 presents the matched wavelets designed for all three single-phase VS signals found during the three-phase fault. It should be observed that every matched wavelet is different because its VS signal is also somewhat different and signal information is perfectly embedded into the matched wavelets. Thus, these matched wavelets are considered to be perfect in capturing fault-related changes in their own voltage pattern on account of resemblance between the two.

The characteristics of matched highpass filter are judged by their normalised frequency response presented in Figure 12. Magnitude response of matched filter of all three phases is maximally flat, which indicates good filter characteristics. Moreover, the phase response of the matched filter proves that the designed filter exhibits a linear phase response. The linear phase response is achieved on the grounds of biorthogonal wavelet chosen for designing the matched wavelets. The characteristics of matched filters are also checked from the stability point of view by their z-domain pole-zero plots as given in Figure 13. Every matched filter has all poles inside the unit circle, which validates the stability of designed filters. Thus, the designed matched wavelets are stable functions and can be efficiently used for characterising the VS signals. The optimality of matched wavelets is also validated through PSNR [25]. In signal reconstruction, PSNR gives the measure of how accurately the signal is reconstructed after passing through decomposition and reconstruction filterbank. Matched wavelets result in
better signal reconstruction. Consequently, the matched wavelet facilitates lesser reconstruction error and thus higher PSNR as compared to the standard biorthogonal 5/3 wavelet. PSNR

| TABLE 1 | Coefficients of decomposition filters designed for voltage sag (VS) signal during the three-phase fault |
|---------|--------------------------------------------------|
| Phase A | $p[n]$   | [0.875 0.119] |
|          | $a[n]$   | [0.558 0.047] |
|          | $h_0[n]$ | [$-0.067 0.558 0.517 -0.047 0.041$] |
|          | $h_1[n]$ | [$-0.119 1 -0.876$] |
| Phase B | $P(z)$   | [0.872 0.135] |
|          | $U(z)$   | [0.552 0.051] |
|          | $h_0[n]$ | [$-0.075 0.552 0.525 0.051 0.045$] |
|          | $h_1[n]$ | [$-0.135 1 -0.872$] |
| Phase C | $P(z)$   | [0.604 0.401] |
|          | $U(z)$   | [0.661 0.167] |
|          | $h_0[n]$ | [$-0.265 0.661 0.667 -0.167 0.101$] |
|          | $h_1[n]$ | [$-0.401 1 -0.604$] |

values obtained with matched and standard wavelets are tabulated in Table 2 from which it is evident that matched wavelet produces higher PSNR as compared to standard biorthogonal 5/3 wavelet in every case of VS. These results prove matched wavelets designed in this work more efficient as compared to standard wavelets.

Biorthogonal matched wavelets are, thus, designed for VS signals observed during all eight sag-causing events. This collection of biorthogonal matched wavelets creates a filterbank that consists of optimal wavelets for all types of single-phase VS signals. This filterbank may be utilised for extracting highly productive features from VS signals so that the underlying cause of VS can be accurately identified.

5 | CONCLUSION

This work proposed the design of matched biorthogonal wavelets for VS signals observed during eight different sag-causing events. The matched wavelets help in extracting more informative and distinguishing features of sag signals as
### TABLE 2 Values of PSNR obtained with standard 5/3 biorthogonal wavelet and matched wavelet

| VS causes                      | Phase A |               | Phase B |               | Phase C |               |
|-------------------------------|---------|---------------|---------|---------------|---------|---------------|
|                               | Standard wavelet | Matched wavelet | Standard wavelet | Matched wavelet | Standard wavelet | Matched wavelet |
| Three-phase fault             | 22.6510 | 22.7720       | 22.5889 | 22.6310       | 22.6651 | 22.6902       |
| Three phase-to-ground fault   | 25.6802 | 25.9327       | 25.7759 | 25.8825       | 28.3938 | 28.4179       |
| Double phase-to-ground fault  | 26.5982 | 26.9646       | 27.2828 | 27.4257       | 34.4067 | 40.3248       |
| Double phase fault            | 32.8295 | 33.2716       | 35.6306 | 38.8470       | 35.4783 | 38.8524       |
| Single phase-to-ground fault  | 36.0884 | 46.0353       | 35.5989 | 81.1909       | 34.6594 | 46.0370       |
| Induction motor starting      | 35.7206 | 43.0277       | 37.7984 | 38.7985       | 35.2994 | 45.9928       |
| Transformer energisation      | 21.9674 | 23.1369       | 26.0923 | 28.1588       | 22.8394 | 23.3971       |
| Heavy load starting           | 34.1385 | 45.7722       | 33.6380 | 39.8985       | 34.5775 | 45.0702       |

**FIGURE 11** Wavelets matched to VS signals during the three-phase fault (a) phase A, (b) phase B, and (c) phase C

**FIGURE 12** Frequency response of high pass filter designed for VS signal during the three-phase fault (a) phase A, (b) phase B, and (c) phase C

**FIGURE 13** Pole-zero plots of high pass filter designed for VS signal during the three-phase fault (a) phase A, (b) phase B, and (c) phase C
compared to standard available wavelets. For constructing new biorthogonal wavelets matched to the sag signals, the lifting scheme was used to which ring projected 1-D signals were given as input. These 1-D signals were made rich of signal information on account of 2-D representation, which magnified the sudden transitions in sag signal. The proposed scheme produced a filterbank of matched wavelets for every type of possible VS signals in case of different VS causes. Stability of all designed wavelets was assured by their maximally flat magnitude response and linear phase response. Pole-zero plots were also used to verify the stability of designed wavelets. The outperformance of all these matched wavelets over standard wavelets is reflected by higher values of PSNR. So the matched wavelets designed in this work can be used as a more efficient basis function while performing MRA of VS signals for recognition of sag-causing events.

REFERENCES

1. Dugan, R., et al.: Electrical Power Systems Quality, 2nd ed., Tata McGraw Hill, New Delhi (2008)
2. Saini, M.K., Kapoor, R.: Classification of power quality events—a review. Int. J. Electr. Power Energy Syst. 43(1), 11–19 (2012)
3. Mahela, O.P., Shaik, A.G., Gupta, N.: A critical review of detection and classification of power quality events. Renewable Sustainable Energy Rev. 41, 495–505 (2015)
4. Kohlhar, S., et al.: A comprehensive overview on signal processing and artificial intelligence techniques applications in classification of power quality disturbances. Renewable Sustainable Energy Rev. 51, 1650–1665 (2015)
5. Ray, P., Badumuru, G.K., Mohanty, B.K.: A comprehensive review on soft computing and signal processing techniques in feature extraction and classification of power quality problems. J. Renewable Sustainable Energy 10(2), 1–14 (2018)
6. Singh, U., Singh, S.N.: Detection and classification of power quality disturbances based on time–frequency-scale transform. IET Sci. Meas. Technol. 11(6), 802–810 (2017)
7. Bendre, A., et al.: Are voltage sags destroying equipment?. IEEE Ind. Appl. Mag. 12(4), 12–21 (2006)
8. Bollen, M.H.: Understanding Power Quality Problems: Voltage Sags and Interruptions, IEEE Press, New York (2000)
9. Bollen, M.H., Gu, I.Y.: Signal Processing of Power Quality Disturbances. John Wiley & Sons, Hoboken, NJ (2006)
10. Nagata, E.A., et al.: Real-time voltage sag detection and classification for power quality diagnostics. Measurement 164, 108907 (2020)
11. Manjula, M., Mishra, S., Sarma, A.: Empirical mode decomposition with Hilbert transform for classification of voltage sag causes using probabilistic neural network. Int. J. Electr. Power Energy Syst. 44(1), 597–605 (2013)
12. Erersisti, H., et al.: Optimal feature selection for classification of the power quality events using wavelet transform and least squares support vector machines. Int. J. Electr. Power Energy Syst. 49, 95–103 (2013)
13. Erersisti, B., et al.: A new embedded power quality event classification system based on the wavelet transform. Int. Trans. Electr. Energy Syst. 28(9), 1–15 (2018)
14. Ismail, H., Zakaria, Z., Hamzah, N.: Classification of voltage sag using multi-resolution analysis and support vector machine. J. Clean Energy Technol. 4(3), 183–186 (2016)
15. Saini, M., Aggarwal, A.: Fractionally delayed Legendre wavelet transform based detection and optimal features based classification of voltage sag causes. Renewable Sustainable Energy 11(1), 1–22 (2019)
16. Saini, M.K., et al.: Recognition of voltage sag causes using fractionally delayed biorthogonal wavelet. Trans. Inst. Meas. Control 41(10), 2851–2863 (2019)
17. Erersisti, H., et al.: Automatic recognition system of underlying causes of power quality disturbances based on S-transform and extreme learning machine. Int. J. Electr. Power Energy Syst. 61, 553–562 (2014)
18. Erersisti, H., et al.: Automatic recognition system of underlying causes of power quality events based on S-transform and extreme learning machine. Int. J. Electr. Power Energy Syst. 61, 553–562 (2014)
19. Gural, S., et al.: The search for optimal feature set in power quality event classification.Expert Syst. Appl. 36(7), 10266–10273 (2009)
20. Mishra, M., Panigrahi, R.R.: Advanced signal processing and machine learning techniques for voltage sag causes detection in an electric power system. Int. Trans. Electr. Energy Syst. 30(1), 1–16 (2020)
21. Ece, D.G., Gerek, O.N.: Power quality event detection using joint 2-D-wavelet subspaces. IEEE Trans. Instrum. Meas. 53(4), 1040–1046 (2004)
22. Ribeiro, P.: Wavelets: A new approach to analyze power system distortions. In: EPRI-Power Quality Steering Committee, Baltimore, MD (1993)
23. Barros, J., Diego, R.L., De Apraiz, M.: Applications of wavelets in electric power quality: Voltage events. Electr. Power Syst. Res. 88, 130–136 (2012)
24. Chapa, J.O., Rao, R.M.: Algorithms for designing wavelets to match a specified signal. IEEE Trans. Signal Process. 48(12), 3395–3406 (2000)
25. Kapoor, R., Gupta, R.: Statistically matched wavelet-based method for detection of power quality events. Int. J. Electron. 98(1), 109–127 (2011)
26. Shark, L.K., Yu, C.: Design of matched wavelets based on generalized Mexican-hat function. Signal Process. 86(7), 1451–1469 (2006)
27. Salmonpour, A., Brown, L.J., Shoemaker, J.K.: Spike detection in human muscle sympathetic nerve activity using a matched wavelet approach. J. Neurosci. Methods 193(2), 343–355 (2010)
28. Duddik, J.M., et al.: A matched dual-tree wavelet denoising for tri-axial swallowing vibrations. Biomed. Signal Process. Control 27, 112–121 (2016)
29. Sweldens, W.: The lifting scheme: A construction of second generation wavelets. SIAM J. Math. Anal. 29(2), 511–546 (1998)
30. Gogna, A., Gade, S.H., Gupta, A.: Design of signal-matched critically sampled fir rational filterbank. In: 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Brisbane, Australia, pp. 3856–3860 (2015)
31. Gillis, J.M., Alsharceef, S.M., Morsi, W.G.: Nonintrusive load monitoring using wavelet design and machine learning. IEEE Trans. Smart Grid 7(1), 320–328 (2015)
32. Raughuever, M.R., Chapa, J.O.: Object detection through matched wavelet transform. In: Wavelet Applications III, SPIE, Orlando, Florida, pp. 45–50 (1996)
33. Zhang, X., et al.: Design of IIR orthogonal wavelet filter banks using lifting scheme. IEEE Trans. Signal Process. 54(7), 2616–2624 (2006)
34. Sweldens, W.: Lifting scheme: A new philosophy in biorthogonal wavelet constructions. In: SPIE’s 1995 International Symposium on Optical Science, Engineering, and Instrumentation, San Diego, CA, pp. 68–79 (1995)
35. Yilmaz, A.S., et al.: Application of lifting based wavelet transforms to characterize power quality events. Energy Convers. Manage. 48(1), 112–123 (2007)
36. Andrade, L.C., Oleskovicz, M., Fernandes, R.A.: Adaptive threshold based detection and optimal features based classification of voltage sag causes. Renewable Sustainable Energy Rev. 51, 1650–1663 (2015)
42. Ansari, N., Gupta, A.: Image reconstruction using matched wavelet estimated from data sensed compressively using partial canonical identity matrix. IEEE Trans. Image Process. 26(8), 3680–3695 (2017)

43. Sweldens, W.: The lifting scheme: A custom-design construction of biorthogonal wavelets. Appl. Comput. Harmon. Anal. 3(2), 186–200 (1996)

44. Styvaktakis, E., Bollen, M.H.: Signatures of voltage dips: Transformer saturation and multistage dips. IEEE Trans. Power Delivery 18(1), 265–270 (2003)

How to cite this article: Aggarwal A, Saini MK. Lifting scheme-based matched wavelet design for effective characterisation of different types of voltage sag. IET Sci Meas Technol. 2021;15:364–375. https://doi.org/10.1049/smt2.12037