Using the 2dF galaxy redshift survey to detect gravitationally-lensed quasars

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Accepted Received; in original form 2000 June 5

ABSTRACT

Galaxy redshift surveys can be used to detect gravitationally-lensed quasars if the spectra obtained are searched for the quasars’ emission lines. Previous investigations of this possibility have used simple models to show that the 2 degree Field (2dF) redshift survey could yield several tens of new lenses, and that the larger Sloan Digital Sky Survey should contain an order of magnitude more. However the particular selection effects of the samples were not included in these calculations, limiting the robustness of the predictions; thus a more detailed simulation of the 2dF survey was undertaken here. The use of an isophotal magnitude limit reduces both the depth of the sample and the expected number of lenses, but more important is the Automatic Plate Measuring survey’s star-galaxy separation algorithm, used to generate the 2dF input catalogue. It is found that most quasar lenses are classed as merged stars, with only the few lenses with low-redshift deflectors likely to be classified as galaxies. Explicit inclusion of these selection effects implies that the 2dF survey should contain 10 lenses on average. The largest remaining uncertainty is the lack of knowledge of the ease with which any underlying quasars can be extracted from the survey spectra.

Key words: gravitational lensing – galaxies: surveys – methods: data analysis.

1 INTRODUCTION

Gravitationally-lensed quasars are extremely valuable, and so a great deal of effort is expended searching for new lens systems. Most lens surveys are based on high-resolution re-observation of known quasars, but lensed quasars can also be found galaxy redshift surveys (GRSs) if the spectra obtained are searched for the presence of quasar emission lines. As well as being efficient, this form of lens search is complementary to the conventional surveys, being primarily sensitive to low-redshift deflectors – the galaxies that would enter the GRS a priori. This is important as the proximity of the lens galaxy can facilitate a number of unique observations, as exemplified by Q 2237+0305 (Huchra et al. 1985), the only quasar lens yet found in a GRS.

A more detailed discussion of these points is contained in Mortlock & Webster (2000b), one of the two previous investigations of quasar lensing in redshift surveys. An ‘initial, order of magnitude estimate’ of the number of lenses expected in future GRSs by Kochanek (1992) suggested there would be one lens per \( \sim 10^5 \) galaxy spectra, but this was shown to be an under-estimate by up to an order of magnitude, as the combined light of the quasar images and the lens galaxy must be included in the calculation (Mortlock & Webster 2000b). Both these these results were obtained for an idealised GRS, characterised only by the number of spectra obtained and a survey’s magnitude limit. It is important to test the robustness of these simple predictions, which can be done most profitably by simulating a real survey in detail. The two obvious candidates are the 2 degree Field (2dF) galaxy survey (e.g. Colless 1999; Folkes et al. 1999) and the Sloan Digital Sky Survey (SDSS; e.g. Szalay 1998; Loveday & Pier 1998). The former was chosen primarily as it subject
to a number of complex, surface brightness-related selection effects, whereas the SDSS is much closer to the ideal galaxy sample modelled in Mortlock & Webster (2000b). Furthermore, the 2dF survey will be completed first, offering the most immediate opportunity to discover new quasar lenses in a galaxy survey.

The selection criteria of the 2dF GRS and its parent survey are carefully defined in Section 4. The main aim of these selection criteria is to exclude stars (Section 3) whilst maximising the completeness of the galaxy sample (Section 3), although care was also taken to reject unwanted merged images (Section 3). In order to determine the number of lenses in the survey, the same selection effects must be applied to a simulated lens population, and the spectral sensitivity of the lens search must also be included (Section 3). The results of the this calculation and the relative importance of the various survey characteristics are given in Section 3. The main conclusions are summarised in Section 3.

2 THE 2DF GALAXY REDSHIFT SURVEY

The 2dF instrument (Section 2.1) is used for a number of projects. The largest is the eponymous galaxy survey (Section 2.2), the properties of which are, however, determined mainly by its parent sample (Section 2.1).

2.1 The 2dF instrument

The 2dF instrument (Taylor 1994; Cannon 1995) is a wide-field, multi-fibre spectroscopic survey facility for the Anglo-Australian Telescope (AAT). Corrective optics give aberration-free images over the full 2 deg-diameter field of view, but the main innovation is the automatically-configured 400-fibre spectrograph in the prime focus plane of the telescope. The positional accuracy of 0.1 arcsec is required because the fibres have an effective angular radius of only $\theta_i = 1$ arcsec. This aperture size is comparable to the seeing at the site – which varies between 1 arcsec and 5 arcsec, with a median value of $\sim 2$ arcsec (Ryan & Wood 1995) – and so the spectra obtained are inevitably of variable quality.

2.2 The 2dF survey

The 2dF GRS (e.g. Colless 1999; Folkes et al. 1999) is the largest single project that is proposed for the 2dF instrument. The survey will require 100 nights of observing time to obtain the redshifts of $N_{\text{tot}} = 2.5 \times 10^5$ objects, about 95 per cent of which are expected to be galaxies. The spectrograph being used has a resolution of 9 $\AA$ from $\sim 3700 \AA$ to $\sim 8100 \AA$, and 45-minute integrations should yield spectra with a signal-to-noise ratio of $\sim 20$ per pixel at the survey magnitude limit of $m' \approx 19.5$.

In the absence of any other selection effects, the survey limit, together with $N_{\text{tot}}$, would completely characterise the survey, and it could then be simulated using standard techniques. However, the catalogue of galaxy candidates is taken from the Automatic Plate Measuring (APM) galaxy survey, which is subject to a number of complex, surface brightness-related selection effects.

2.3 The APM galaxy survey

The APM system (e.g. Kibblewhite et al. 1984), combines a high-speed microdensitometer with a dedicated computer system capable of on-line image detection. Used to analyse United Kingdom Schmidt Telescope (UKST) photographic plates, it can provide spatial information on a pixel scale of $\sim 0.5$ arcsec, and relative photometry over several orders of magnitude in flux. Scanned at this resolution, a typical UKST plate represents $\sim 3$ GB of information, but the image detection algorithms allow it to be stored as a $\sim 3$ MB list of image parameters with minimal information loss. The APM software defines an image as any connected set of 16 or more pixels which are all above the surface brightness limit. This limit varies between $\mu_{\text{lim}} = 24 \text{ mag arcsec}^{-2}$ and $\mu_{\text{lim}} = 25 \text{ mag arcsec}^{-2}$. The minimum image area of $\sim 4$ arcsec$^2$ then implies an isophotal magnitude limit of $m' \approx 22$. For each image, 15 parameters (defined in Maddox et al. 1990a) are calculated from the sky-subtracted plate densities; these comprise the standard output of the APM system.

The APM galaxy survey (Maddox et al. 1990a; Maddox, Efstathiou & Sutherland 1990b; Dalton et al. 1994; Maddox, Efstathiou & Sutherland 1996; Dalton et al. 1997) is a unique sample of $2 \times 10^6$ galaxies to a magnitude limit of $m' = 20.5$. It was compiled from APM scans of 185 contiguous UKST survey plates, and the galaxy sample was thus selected from $2 \times 10^7$ images brighter than $m' \approx 22$ spread over a total area of 4300 deg$^2$.

2.3.1 Star-galaxy separation

Given that only a third of objects brighter than $m \approx 20.5$ are galaxies, one of the most important aspects of the APM galaxy survey is the star-galaxy separation technique. In Mortlock & Webster (2000b), image ellipticity was used as a generic method of discriminating between point-like images and extended sources – this was useful as a simple estimate, but more sophisticated methods are required to construct a galaxy catalogue from real two-dimensional data. Most star-galaxy separation techniques (e.g. Jarvis & Tyson 1981; Godwin, Metcalfe & Peach 1983; Maddox et al. 1990a) are based on two facts: a galaxy has a lower peak surface brightness than a star of the same magnitude; and the surface brightness profile of a star is determined purely by observational effects. Stars and galaxies inhabit different regions of the phase-space defined by magnitude, area, peak surface brightness, etc., and scatter plots of the data can be used to separate the two populations.

The APM galaxy sample was defined in a more complex manner, using the areal profiles of each object as the main diagnostic. Amongst the image parameters provided by the APM is the area of each image above eight plate density levels, $A_i$, where $1 \leq i \leq 8$. The plate density is proportional to (logarithmic) surface brightness fainter than $\sim 20$ mag arcsec$^{-2}$ (Maddox et al. 1990a), so the levels can be approximated by
\[ \mu_i = \mu_{\text{lim}} + i \Delta \mu, \]

where \( \mu_{\text{lim}} \simeq 24.5 \text{ mag arcsec}^{-2} \) and \( \Delta \mu \simeq 0.75 \text{ mag arcsec}^{-2} \). This approximation slightly underestimates \( \mu_i \), due to saturation effects, and also ignores plate-to-plate variations. The random errors in the \( A_i \) were found to be well approximated by the Poisson error in the number of pixels above the relevant surface brightness. Given the APM scale of 0.5 arcsec per pixel, \( \Delta A_i \simeq 2\sqrt{A_i} \) if \( A_i \) is expressed in arcsec\(^2\).

The areal profile of an image can be thought of as a plot of area versus surface brightness, which is similar to a conventional surface brightness profile. As shown in Maddox et al. (1990a), stars have a characteristic areal profile at any given magnitude, which is distinct from the more varied profiles of galaxies. However, this differentiation breaks down for \( m' \gtrsim 20 \) (as there is limited information for such faint images) and for \( m' \lesssim 17 \) (as the stars appear extended due to saturation effects).

As a source of extra information, the peak surface brightness of each image, \( \mu_{\text{peak}} \), and the radius of gyration, \( \theta_G \), were also used, giving a total of 10 parameters for the star-galaxy separation. The radius of gyration is defined as

\[ \theta_G = \sqrt{\langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle}, \]

where the expectation values are given by surface brightness-weighted integrals over the area of the image for which \( \mu \leq \mu_{\text{lim}} \). The uncertainties in \( \mu_{\text{peak}} \) and \( \theta_G \) are not given in Maddox et al. (1990a), but are estimated to be \( \sim 10 \) per cent. As with the \( A_i \), a distinct stellar locus is apparent for both \( \mu_{\text{peak}} \) and \( \theta_G \) as a function of magnitude — the stars have higher peak surface brightnesses and smaller radii of gyration than the galaxies.

The final image classification statistic used by Maddox et al. (1990a) combines all 10 image parameters in a \( \chi^2 \)-like formulation. For an object of isophotal magnitude \( m' \), it is defined as

\[ \psi_{\text{APM}} = 2000 \log \left\{ \frac{1}{10} \left( \sum_{i=1}^{8} \left[ \frac{A_i - A_{i,s}(m')}{\sqrt{4A_{i,s}(m')}} \right]^2 + \left( \frac{\theta_G - \theta_{G,s}(m')}{\sqrt{2\theta_{G,s}(m')}} \right)^2 \right) \right\}, \]

where the \( A_i \) are in units of arcsec\(^2\), \( \theta_G \) is in units of arcsec, and \( f_{\text{peak}} \) is the peak surface brightness in linear units, defined to match the APM plate density. The 's' subscript denotes the parameter value of an isolated star of the same isophotal magnitude as the image in question. Each term is normalised by the expected uncertainty in the stellar locus, as in a \( \chi^2 \) formulation. Note that the definition of \( \psi_{\text{APM}} \) given in Maddox et al. (1990a) does not include the overall normalisation (i.e. the '1/10' term); it is included here to ensure that stellar locus is centred on \( \psi_{\text{APM}} = 0 \). The prefactor is changed from 1000 to 2000 to compensate for this.

Fig. 3 shows a plot of \( \psi_{\text{APM}} \) as a function of isophotal magnitude for stars, galaxies, and mergers, and is directly comparable to Fig. 10 of Maddox et al. (1990a). Some of the details differ (e.g. the exact position and width of the stellar locus and absence of saturation effects in the simulation), but the generic features are the same. The stars (See Section 4) are centred on \( \psi_{\text{APM}} = 0 \), and their spread is independent of the point-spread function (PSF) due to the normalisations in equation (3). The galaxies (See Section 4) lie along a 'ridge' with \( \psi_{\text{APM}} \gtrsim 1000 \), but are less distinct from the stars at fainter magnitudes. However \( \psi_{\text{APM}} \) cannot be used to separate merged images from extended objects; further specialised statistics are required.

### 2.3.2 Merged objects

Images separated by \( \lesssim 5 \) arcsec are registered as single objects by the APM software, and tend to have \( \psi_{\text{APM}} \gtrsim 1000 \), as illustrated in Fig. 3. If this was the only star-galaxy separation implemented, most lensed quasars would be included in the APM galaxy survey. However Maddox et al. (1990a) used the radius of gyration and a saturation parameter to detect (and, in general, reject) merged images.

The radius of gyration, \( \theta_G \), as defined in equation (2) is a weighted measure of the extent of an image, and is evaluated on a pixel-by-pixel basis by the APM software. If an image is assumed to be perfectly elliptical, an approximation to the radius of gyration is given by

\[ \theta'_G = \left[ \frac{\sum_{i=1}^{7} f_i A_i (A_i - A_{i+1})}{2\pi \sqrt{1 - c^2} \sum_{i=1}^{7} f_i (A_i - A_{i+1})} \right]^{1/2}, \]

where \( c \) is the overall measured ellipticity of the image and \( f_i \) the surface brightness of the \( i \)th level in linear units (Maddox et al. 1990a). For isolated images, \( \theta_G \simeq \theta'_G \), with some scatter due to random noise; for multiple images \( \theta_G > \theta'_G \), as dumbbell-shaped image pairs have higher second order moments than isolated images with the same areal profile. From this Maddox et al. (1990a) define
likely to be a merged pair of images or a higher multiple.

...tion. If the same isophotal magnitude as the (merged) object in question is much greater than unity the object is quite bright, but all other stellar mergers were rejected. This was done by making a cut ‘by eye’ in the $k_{\text{APM}}$-$\mu_{\text{APM}}$ plane, which is shown as the dashed line in Fig. 3. Note also that the normalisation of both $k_{\text{APM}}$ and $\mu_{\text{APM}}$ is such that there is no intrinsic magnitude dependence in the merger classification.

3 STARS

As the APM image classification scheme is defined relative to the stellar population, stars represent a control sample. Misclassified stars are also the primary source of contamination in the galaxy catalogue, as they represent such a large fraction of sources over the magnitude range of the survey.

The model of the stellar population used was described in Mortlock & Webster (2000b). Briefly, the number counts are taken from Bahcall & Soneira (1980) and the images are assumed to have a Moffat (1969) PSF, defined by

\[
 f_{\text{see}}(\theta) = \frac{(\eta - 1)(2^{1/\eta} - 1)}{\pi(\theta_{\ast}/2)^2} \left[ 1 + (2^{1/\eta} - 1) \frac{\theta^2}{(\theta_{\ast}/2)^2} \right]^{-\eta},
\]

where $\theta_{\ast}$ is the full width at half-maximum of the seeing disc and $\theta$ is angular position on the sky. As discussed in Mortlock & Webster (2000b), two values of $\eta$ are used: $\eta = 2.5$, the default value assumed by the IMAGE REDUCTION AND ANALYSIS FACILITY software (Tody 1986); and $\eta \to \infty$, a Gaussian PSF. It is important to note that this PSF is formally defined for an object of a given total, as opposed to isophotal magnitude. An iterative procedure is required to find the total magnitude, $m$, which corresponds to a given isophotal magnitude $m'$. The random errors due to the finite signal-to-noise of the APM pixels are not explicitly simulated here; only the approximately Gaussian errors discussed in Section 2.3 are included.

Fig. 3 shows the stellar locus as the ‘ridge’ with $\psi_{\text{APM}} \simeq 0$. The spread is independent of the PSF as $\psi_{\text{APM}}$ is normalised relative to the spread of the stellar images. In reality the stellar locus is not straight and has non-Gaussian outliers (c.f. Fig. 10 of Maddox et al. 1990a), but the most important features are present.

Isolated stars have an even simpler distribution on the $k_{\text{APM}}$-$\mu_{\text{APM}}$ plot (Fig. 3) used to distinguish between mergers and isolated objects. Their distribution in $k_{\text{APM}}$ is simply a Gaussian with spread determined by the assumed error in the radius of gyration discussed in Section 2.3. The use of $A_7$ and $A_8$ in the definition of $\mu_{\text{APM}}$ [equation (4)] means that the $\mu_{\text{APM}}$ distribution is marginally magnitude dependent, but is also close to a Gaussian for stars with $\mu_{\text{peak}} \simeq \mu_{\ast}$.

4 GALAXIES

The main purpose of the morphological classification criteria of the APM survey is to generate a reasonably complete and contamination-free sample of galaxies. The properties of the galaxy sample are determined by the intrinsic galaxy population (Section 4.1) and their observed surface brightness profiles (Section 4.2). The latter dominates both the APM classifications (Section 4.3) and the 2dF fibre magnitudes (Section 4.4), but the two combine to determine the redshift distribution of the survey galaxies (Section 4.5).

4.1 Populations

The model of the local galaxy population is described in detail in Mortlock & Webster (2000b), and thus only outlined here. Three galaxy types are considered: spirals (S) and two classes of elliptical galaxies (E and S0), the relative numbers of which are given in Postman & Geller (1984).
Each population is assumed to follow the Schechter (1976) luminosity functions detailed in Efstathiou, Ellis & Peterson (1988). (Thus $M_L = -19.4 \pm 0.1$ for all three types.) In the calculation of the galaxies’ lensing properties (Sections 6 and 7) they are modelled as isothermal spheres (e.g. Turner, Ostriker & Gott 1984; Binney & Tremaine 1987), with the standard Faber-Jackson (1976) and Tully-Fisher (1977) relations used to convert luminosities to velocity dispersions.

### 4.2 Surface brightness profiles

In the APM survey, galaxies are distinguished from stars primarily by their surface brightness profiles. In particular, the ellipticity or elongation of images is not used in the object classification and so all galaxies can be assumed to have circular isophotes without loss of generality.

In the absence of atmospheric seeing, the surface brightness of galaxies is given by their intrinsic (radial) surface brightness profiles. Spirals are assumed to follow a Freeman (1970) profile, given by

$$f_S(\theta) = \frac{F}{2\pi(0.596 \theta_g)^2} e^{-\theta/(0.596 \theta_g)},$$

(8)

the two types of ellipticals are modelled by a de Vaucouleurs (1948) profile, given by

$$f_E(\theta) = 296.7 \frac{F}{\pi \theta_g^2} e^{-7(\theta/\theta_g)^{1/4}},$$

(9)

where $F$ is the total flux of the galaxy and $\theta = |\theta|$ (where $\theta$ is the position on the sky relative to the centre of the galaxy). Here $\theta_g = R_g/d_\Delta(0, z_g)$, where $d_\Delta$ is the angular diameter distance to the galaxy and $R_g$ its intrinsic half-light or effective radius. This scales with the galaxy’s velocity dispersion as

$$R_g = R_{g*} \left( \frac{\sigma_g}{\sigma_{g*}} \right)^{u_g},$$

(10)

where $R_{g*} = (3 \pm 1)$ kpc and $u_g = 4 \pm 1$ for Es and S0s (Kormendy & Djorgovski 1989) and $R_{g*} = (4 \pm 1)$ kpc and $u_g = 3 \pm 1$ for spirals (de Vaucouleurs & Pence 1978).

The observed surface brightness profile of a galaxy, $f_S(\theta)$, can be found by convolving the intrinsic profile with the PSF. Even if the PSF is also rotationally-symmetric,

$$f_S(\theta) = \int_0^\infty \int_0^{2\pi} f_{S\text{ PSF}}(\theta') f_{\text{sec}} \left( \sqrt{\theta^2 + \theta'^2 - 2 \theta \theta' \cos \theta'_{\theta}} \right) \theta' d\theta'_{\theta} d\theta',$$

involves a two-dimensional integral, which must be evaluated numerically. This is time-consuming, and so an approximation was sought.

The analytic surface brightness profile used is

$$f_S(\theta) = f_S(0) e^{(\theta/\theta'_g)^{1/2}} e^{-\left(\sqrt{\theta^2 + \theta'^2}/\theta'_g\right)^{1/2}} \frac{\xi F}{2\pi \theta'_g \Gamma(2/\xi) 1 - P[2/\xi, (\theta/\theta'_g)^{\xi}]} e^{-\left(\sqrt{\theta^2 + \theta'^2}/\theta'_g\right)^{1/2}} \Gamma(z),$$

(12)

where $\Gamma(z)$ is the Gamma function and $P(z, x)$ is the incomplete Gamma function. Unfortunately $\theta'_g$ can only be found by solving

$$0 = 2P \left[ 2/\xi, \left( \sqrt{\theta^2 + \theta'^2}/\theta'_g \right)^{1/2} \right] - P \left[ 2/\xi, (\theta'/\theta'_g)^{1/2} \right] - 1$$

numerically, but, more importantly, $f_S(\theta)$ can be inverted, giving

$$\theta(f) = \sqrt{\theta'_g^2 \left[ (\theta/\theta'_g)^{1/2} \right] + \ln \left( f_S(0)/f \right)^{2/\xi} - \theta'^2},$$

(14)

provided $f \leq f_S(0)$. The profile can also be integrated, to give the flux within a specified angle as

$$F_S(< \theta) = P \left[ 2/\xi, \left( \sqrt{\theta^2 + \theta'^2}/\theta'_g \right)^{1/2} \right] - P \left[ 2/\xi, (\theta'/\theta'_g)^{1/2} \right] \frac{1 - P[2/\xi, (\theta'/\theta'_g)^{1/2}]}{1 - P[2/\xi, (\theta'/\theta'_g)^{1/2}]}.$$

(15)

Each of the three parameters plays a clear role in the shape of $f_S(\theta)$. The core width, $\theta'_g$, is determined by the mainly by the seeing, but has some dependence on the form of the PSF. The overall angular scale of the profile, given by $\theta'_g$, is purely a function of the galaxy’s distance and half-light radius. For $\theta \gg \theta'_g$ the surface brightness falls off as $\exp[\theta/(\theta'_g)^{-\xi}]$, assuming that the faint wings of the galaxy are broader than those of the PSF. Hence $\xi \simeq 1$ for spirals and $\xi = 1/4$ for ellipticals, and, if $\theta'_g = 0$, equation (12) reduces to $f_E$ and $f_S$. For small $\theta$ equation (12) tends to $f_S \propto \exp[-(\theta^2/20\theta'_g)^{1/2}]$, and so the profile has a Gaussian core only if $\xi \simeq 1$ (i.e., for spirals). Fig. 3 shows several examples of observed galaxies’ profiles, comparing the numerically integrated curves (dashed lines) to the relevant form of equation (12) for both ellipticals (a) and spirals (b). The agreement for the latter type is excellent, with relative errors of no more than 1 per cent for the profiles shown. The average error for the elliptical fits is $\sim 1$ per cent, and peaks at $\sim 5$ per cent for most of the profiles shown.

Both the approximate depth of the redshift survey and the importance of the use of isophotal magnitudes can be gauged by plotting the magnitude of a given galaxy as a function of redshift (i.e., a Hubble diagram). This is shown in Fig. 3 for both an $M_* = -19.4$ elliptical and an $M_* = -19.4$ spiral, with the 2dF fibre magnitude (defined in Section 4.4) also included. The $m(z)$ curve is the same for the elliptical and the spiral as the luminosity evolution of each type is assumed to exactly cancel their K-corrections (See Mortlock & Webster 2000b), but the redshift dependence of the isophotal magnitude is somewhat different. For moderate redshifts, the isophotal flux of spirals is actually greater than that of ellipticals, despite the fact that the latter have more peaked surface brightness distributions. This comes about as the average flux of the spirals over the region registered by the APM is higher. However at high redshifts $m' \to \infty$ for all galaxies as they are so faint that their peak surface brightness is lower than the APM limit of 24.5 mag arcsec$^{-2}$. $\xi$ This is not formally the case for a Moffat (1969) PSF, although it only becomes relevant on very large angular scales. $\parallel$ An alternative profile with

$$f_S(\theta) = f_S(0) e^{(\theta/\theta'_g)^{1/2}} - \left[ \left( \frac{\theta^2 + \theta'^2}{\theta'_g^2} \right)^{1/2} / \theta'_g \right]^{1/2},$$

(16)

would have a Gaussian core for all $\xi$ with the same properties as equation (12) for large $\theta$, but cannot be integrated analytically.
This occurs at a higher redshift for ellipticals, as it is determined mainly by the central peak of the surface brightness. Over the redshift range of the 2dF survey (see Section 4.5) \(m' - m \gtrsim 0.5\) for most galaxies, and so the redshift coverage of the survey is comparable to that of a GRS with a total limiting magnitude of \(m_{\text{lim}} \simeq 19\).

4.3 APM classifications

The observed surface brightness profile of a galaxy determines its classification as either a stellar or non-stellar image. Predictably, most all the brighter galaxies have \(\psi_{\text{APM}} \gtrsim 1000\), and are classified as non-stellar, as shown in Fig. 1. It is only fainter than the 2dF survey limit of \(m' = 19.5\) that the galactic and stellar loci begin to merge. The galaxies also occupy a well-defined locus on the \(k_{\text{APM}} - \mu_{\text{APM}}\) plot, as shown in Fig. 2. Isolated galaxies are monolithic and hence have \(k_{\text{APM}} \simeq 0\), but the cover a wider range in \(\mu_{\text{APM}}\). Fainter and more distant galaxies become progressively more stellar, and so \(\mu_{\text{APM}} \to 1\), the canonical stellar value. Galaxies with \(\mu_{\text{APM}} \geq 0.8\) are in fact classified as stellar mergers, and rejected from the galaxy sample. Overall, however, it is clear from Figs. 1 and 2 that very few real galaxies with \(m' \leq 19.5\) are lost from the 2dF catalogue, and a Monte Carlo simulation suggests the APM galaxy survey has a completeness of \(C \sim 95\) per cent, which is comparable to the values estimated by Maddox et al. (1990a).

4.4 Fibre magnitudes

In general, the flux from a survey galaxy that actually enters the 2dF spectrograph is considerably less than the isophotal (or total) flux, as the 2dF instrument’s optical fibres are only 1 arcsec in effective radius. The fibre magnitude of a galaxy of total magnitude \(m\) is given by

\[
m_f = m + 2.5 \log \left( \frac{F_g(\leq \theta_f)}{F_g(< \infty)} \right),
\]

where \(F_g(< \theta)\) is given in equation (13). The \(z\)-dependence of \(m_f\) is shown in Fig. 2 for both an \(M_e\) elliptical and an \(M_s\) spiral galaxy. The higher central surface brightness of the elliptical results in a much higher fraction of its flux entering the 2dF fibre (dot-dashed line), but the PSF dominates the observed surface brightness of both types by \(z \simeq 0.5\), and so there is little difference in \(m_f\). On average \(m_f - m' \simeq 2\) over most of the 2dF survey’s redshift range (Section 4.5), which increases the survey’s potential sensitivity to lensed quasars by \(\sim 2\) mag. However this is somewhat offset by an analogous reduction in the fibre flux of any lensed quasars behind the survey galaxies; this is discussed further in Section 6.3.
Lensing in the 2dF redshift survey

Figure 4. The total magnitude, $m$ (solid line), APM isophotal magnitude, $m'$ (dashed line), and the fibre magnitude, $m_f$ (dot-dashed line), of an $M_*$ elliptical galaxy (a) and an $M_*$ spiral galaxy (b), shown as a function of redshift, $z$. It is assumed that the passive luminosity evolution of the galaxies cancels out their $K$-corrections. The horizontal dashed line is the isophotal magnitude limit of the 2dF survey, $m'_{\text{lim}} = 19.5$.

Figure 5. The expected redshift distribution, $dP_g/dz$, of the galaxies in the 2dF GRS. The solid lines show the distributions generated if the correct isophotal magnitude limit of $m'_{\text{lim}} = 19.5$ is used; the dashed lines show the distributions implied by a total magnitude limit of $m_{\text{lim}} = 19.5$. In both cases the broader distribution assumes that the $K$-corrections are exactly cancelled by passive luminosity evolution of the galaxies; the narrower distribution assumes no luminosity evolution, and uses the $K$-corrections measured by Kinney et al. (1996).

5 MERGED OBJECTS

Images separated by $\lesssim 5$ arcsec are treated as single objects by the APM software, and thus Maddox et al. (1990a) introduced the $k_{\text{APM}}-\mu_{\text{APM}}$ plot to distinguish merged images from isolated stars and galaxies. The ideas that underpin this method were explained in Section 2.3; here the populations of mergers shown in Figs. 1 and 2 are defined.

A close pair of images of specified types are defined by their angular separation, $\Delta \theta$, and their total magnitudes $m_1$ and $m_2$. For chance alignments, the magnitudes are uncorrelated, and so drawn at random from the respective populations, and the distribution of angular separation is given by $dP_g/d\Delta \theta \propto \Delta \theta$ between 0 and $\Delta \theta_{\text{max}} \approx 5$ arcsec. The more complex separation distributions of binary stars and physically-associated galaxies are ignored here, as it is only the region of phase space inhabited by merged images that is of interest.

Merged image pairs are shown in Figs. 1 and 2 as open symbols. Star-star mergers are shown as open circles; star-galaxy mergers as open squares and galaxy-galaxy mergers as open triangles. All three populations have $\psi_{\text{APM}} \gtrsim 1000$, and lie a significant distant from the stellar locus in Fig. 1. Whilst they also tend to be slightly below the galactic population, they would be treated as extended objects in the APM survey if not for the use of the $k_{\text{APM}}-\mu_{\text{APM}}$ plot (Fig. 2). Not only are the mergers quite well separated from the monolithic objects (i.e. single galaxies and stars with $k_{\text{APM}} \approx 0$), but the stellar images (either isolated stars or stellar mergers) are quite distinct from those with strong extended components. This separation allowed Maddox et al. (1990a) to reject most stellar mergers from the galaxy

** Despite the fact that isophotal magnitudes are the ‘natural’ choice for simulations of the APM survey, the separate isophotal magnitudes of two merged objects are somewhat ill-defined. The total flux of an image pair is simply the sum of the two components’ total fluxes, but their combined isophotal flux cannot be determined from their individual isophotal fluxes.
sample, whilst retaining most of the galactic mergers. Unfortunately, many gravitational lenses would also have been rejected, as discussed below.

6 LENSED QUASARS

The determination of the number of lenses that enter the 2dF GRS splits neatly into two parts: a reasonably standard calculation of the lens population; and the application of the survey selection effects.

The lens calculation uses the same models and methods discussed in Mortlock & Webster (2000b): the deflectors are assumed to be a non-evolving population of isothermal spheres (with the possibility of both a core radius and ellipticity accounted for in Section 5); the quasar population is chosen to match the number counts of Boyle, Shanks & Peterson (1988) and the redshift distribution of the Large Bright Quasar Survey (Hewett, Foltz & Chaffee 1995); and conventional methods are used to evaluate the lensing probability (e.g. Kochanek 1996).

More complex, and more interesting, are the effects of the various APM selection effects on the lens statistics. To enter the 2dF GRS, a lensed quasar must satisfy the following conditions (c.f. Mortlock & Webster 2000b):

(i) The composite object defined by the quasar images and lens galaxy must have an isophotal magnitude $m' < m_{lim} = 19.5$. This is not a simple function of galaxy and quasar magnitudes, but must be computed numerically for each lens. Fortunately the quasar images are within the bright (i.e. $\mu \ll \mu_{lim}$) ’core’ of the galaxy in many cases, and their total – rather than isophotal – flux can be added to the isophotal flux of the galaxy.

(ii) The quasar must be bright enough to for its spectroscopic signature to be present in the composite spectrum obtained during the 2dF GRS. Following Kochanek (1992), the quasar is assumed to be detectable if the fibre magnitudes of the quasar images and galaxy satisfy $m_{t,qi} - m_{t,gi} \leq \Delta m_{iq} \simeq 2$. The explicit calculation of the fibre magnitudes is discussed in Sections 4.4 and 6.1.

(iii) The composite object must be classed as a galaxy according to the APM star-galaxy separation scheme. This condition is taken to be satisfied if $m_g - m_k \leq \Delta m_{go}$, where the value of $\Delta m_{go}$ is estimated in Section 5.3. Note that $m_q$ is the total, lensed magnitude of all the quasar images.

The small aperture size tends to result in a greater sensitivity to the more centralised quasar images (Section 5.3), but the use of isophotal magnitudes decreases the effective depth of the survey, and the rejection of multiple point sources also tends to reduce the number of lenses (Section 6.2). The relative importance of these competing effects cannot be judged a priori, but must be determined numerically, as discussed in Section 6.1.

6.1 Fibre magnitudes

Fig. 6 shows that only a small fraction of the light from most galaxies enters the small 2dF optical fibres. However, much of the light from a lensed quasar can also fall outside the fibre, both due to seeing effects and the fact that the quasar images are generally somewhat offset from the centre of the fibre.

The flux from a lensed quasar which enters the fibre is calculated by integrating the surface brightness of each quasar image over the fibre, and summing the resultant fibre fluxes. Thus the fibre magnitude, $m_{f,i}$, of a lensed quasar with source position $\beta$ is

$$m_{f,i} = m_q - 2.5 \log \left( \frac{\sum_j \mu_{\beta_j} |I(\theta_j(\beta))|}{\sum_j \mu_{\beta_j}} \right),$$

where $I(\theta)$ is the fraction of the light from an image at $\theta$ which enters the fibre. The image positions, $\theta_i$, and magnifications, $\mu_i$, are functions only of source position, $\beta$, and are regarded as inputs to this calculation. Performing the two-dimensional integrals over the aperture can be time-consuming, but, for simple PSFs [including equation (3)], the problem can be reduced to a one-dimensional integral (Mortlock 1999).

The dependence of $I$ on both $\theta_i$ and $\theta_q$ is shown in Fig. 6. The solid line in each plot shows the step function obtained if the PSF is ignored. The other four curves show that an appreciable portion of the flux from images up to $\sim 2$ arcsec from the centre of the fibre is registered in the worst seeing to be expected at the AAT. This effect is more important for PSFs with more pronounced wings, as can be seen by comparing the results for the ‘default’ Moffat (1969) profile ($\eta = 2.5$) shown in Fig. 6 (a) with those for Gaussian seeing ($\eta \to \infty$) shown in (b).

Irrespective of the details of the PSF, Fig. 6 illustrates how the positions of the quasar images can influence $m_{f,i}$ and thus the chance of a lens being detected in the GRS. Obviously the $\theta_{is}$ are determined by $\beta$, but the types of image configurations produced depend on the mass distribution of the deflector. Several representative examples of this are shown in Fig. 5, in which the surface brightness of a galaxy-quasar composite is shown for a number of plausible lens models. The galaxy is an $M_*$ elliptical at a redshift of 0.1, and has $m_{eq} = 17.9$ and $m_{eq} = 18.5$, ignoring the quasar light. In all panels the quasar is slightly mis-aligned with the galaxy, and has an unlensed magnitude of 22. The assumption of 1 arcsec seeing means that the quasar images are sufficiently smeared that they have only a gross influence on the surface brightness contours, and often remain unresolved.

The default mass model, a singular isothermal sphere (as used for most of the calculations discussed in Section 5.3), is shown in (a). The two images lie just outside the fibre, but the PSF is sufficiently broad that the fibre flux of the quasar is three times its unlensed flux.

In (b) the galaxy is no longer singular – the density flattens off within $r_c \simeq 100$ pc of its centre, resulting in the formation of an extra image and generally higher magnifications. This results in an increased fibre flux, but, for reasonable core radii, the effect is only minimal – just 10 per cent in the example shown. Note that there is no significant change in the image separation for small $r_c$ – use of a self-consistent dynamical normalisation (Kochanek 1996; Mortlock & Webster 2000a) cancels out the reduced depth of the galaxy’s central potential well. For very large cores, the image separation actually increases, and hardly any of the quasar’s light would enter the fibre. As shown in Fig. 5 (a) this reduces the lensing probability to zero. The proba-
bility of lensing by a spiral galaxy has a similar dependence on the core radius, but for very different reasons. The dynamical normalisation of spirals is taken from their rotation curves, which are essentially unaffected by the core structure. Hence increasing $r_c$ results in reduced image separations, and an even greater increase in the quasar’s fibre flux. However the shrinking of the radial caustic dominates the statistics, again reducing the lensing probability to zero for large cores.

More important than the core structure of the deflector is its ellipticity. A mass profile of ellipticity of $\epsilon$ can be modelled adequately by applying an external shear of magnitude $\gamma_0 = \epsilon/3$ (e.g. Kassiola & Kovner 1993), and Fig. 7 (c) shows a singular lens with $\gamma_0 = 0.2$, representing a galaxy of slightly greater than average ellipticity (e.g. Keeton, Kochanek & Seljak 1997). The source position was chosen so that it lies inside the lens’s tangential caustic, and the extra pair of images contribute significantly to the detectability of the lens. The total magnification of the source is $t \approx 16$, and the quasar’s fibre magnitude is $m = 20$. Extra magnification is a generic feature of quadrupole lenses (e.g. Kochanek & Blandford 1987; Keeton et al. 1997), and the inclusion of ellipticity in the calculation can increase the expected number of lenses by up to 50 per cent [Fig. 7 (b)].

Finally, Fig. 7 (d) shows the combined effect of a core radius and finite ellipticity relative to the singular model, the magnifications are slightly increased and there is an extra image, which increases the quasar’s fibre flux by a further 20 per cent. It can also been seen that, in rough terms, the effects of core radius and deflector ellipticity decouple, and it is the shape of the lens that dominates the lens statistics.

Note that in all four of the above examples the overall magnification is high enough to ensure the lensed quasar’s fibre flux is greater than its total unlensed flux would have been. Nonetheless, for most lenses more than half the light of the lensed quasar images falls outside the fibre – typically $m_{f,s} - m_q \lesssim 1$. This is a significant loss, but the effect is still much smaller than it is for galaxies, which have $m_{f,g} - m_g \simeq 2$ on average (Section 6.4). Thus the small diameter of the 2dF fibres is favourable for spectroscopic lens searches.

6.2 APM classifications

The APM classification of a lensed quasar as stellar, merged or non-stellar is a function of its surface brightness, and hence sensitive to both the properties of the lens galaxy and the positions and magnifications of the quasar images. The number of independent variables that ‘define’ a given lens is too high to facilitate an analytic investigation, and so Monte Carlo simulations of a large number of source-deflector pairs were generated. For each lens, the three APM image classification parameters ($\psi_{APM}$, $k_{APM}$ and $\mu_{APM}$, as described in Section 2.3) were calculated, and the simulated lenses are plotted as filled circles in Figs. 8 and 9.

As expected, almost all the lenses have $\psi_{APM} \gtrsim 1000$, indicating that their surface brightness profiles are significantly different from those of stars. However the population of lenses is quite similar to the merger population discussed in Section 2.2. In particular, two-image lenses with high-redshift deflectors are almost indistinguishable from binary stars. Hence the rejection of star-star mergers from the APM galaxy survey on the basis of their location in $k_{APM}$-$\mu_{APM}$ space also results in many lenses being lost from the 2dF GRS. In fact the only lenses that remain in significant numbers are those with bright (and hence nearby) lens galaxies. Almost all lenses with $m_g - m_q \lesssim -1$ are morphologically classified as galaxies, and those with $m_g - m_q \gtrsim 1$ are flagged as stellar mergers. In the calculation that follows, this is simplified to the requirement that all lenses satisfy $m_g - m_q \leq \Delta m_{2dF} = 0 \pm 1.$

7 RESULTS

The aim of this calculation is essentially to find one value: $N_1$, the number of lensed quasars expected in the 2dF GRS.
Figure 7. Simulated images of a $\sigma_*=220$ km s$^{-1}$ elliptical galaxy (at $z_g=0.1$ and with $m_g=17.9$) lensing a $m_q=22$ quasar at $z_q=2$. The galaxy has $m'\simeq18.5$ in each of the panels, which are labelled with the fibre magnitudes of the galaxy, $m_{fg}$, and the quasar, $m_{fq}$. In all cases the quasar is off axis by $\sim0.2$ arcsec, and its images are shown by the black disks, the areas of which are proportional to their magnifications. The heavy circle is the 2dF fibre, and the contours are uniformly spaced in logarithmic surface brightness. In (a) the galaxy is a singular isothermal sphere; in (b) it has a core radius of $r_c=100$ pc; in (c) there is an external shear of $\gamma_0=0.2$ (equivalent to an ellipticity of $e\simeq0.6$); and in (d) the galaxy has both a core radius of $r_c=100$ pc and an external shear of $\gamma_0=0.2$. In all cases the seeing is $\theta_s=1$ arcsec, and it is assumed that the night sky has zero surface brightness.

This is given by integrating the lensing probability of individual galaxies over the entire galaxy population, subject to the selection effects described in Section 6. In Section 7.1, the dependence of $N_l$ on the survey design is investigated; the variation with lens model is discussed in Section 7.2.

7.1 Survey design

The generic calculation of lensing in redshift surveys presented in Mortlock & Webster (2000b) implies that $\sim10$ lenses would be discovered in a survey of $N_{tot}=2.5\times10^5$ galaxies that is complete to a total magnitude limit of
m = 19.5. Implicit in this calculation were the assumptions that there are no surface brightness-related selection effects in the survey, and also that the apertures used to gather the survey spectra are larger than the typical angular scales of both the galaxies and lensed images. These assumptions are dropped here, and Fig. 8 shows how \( N_1 \) varies with various aspects of the survey design; the generic model can be reproduced by setting \( \mu_{\text{lim}} \to \infty \) and \( \theta_t \to \infty \). In this limiting case, both the fibre magnitude, \( m_f \), and its isophotal magnitude, \( m' \), of an object approach its total magnitude, \( m \).

Fig. 8(a) shows how \( N_1 \) increases with \( \mu_{\text{lim}} \), the isophotal limit of the survey. Most galaxies with \( m \simeq 19.5 \) have observed peak surface brightnesses close to 22 mag arcsec\(^{-2} \), and so the properties of the redshift survey (and \( N_1 \) in particular) do not vary with \( \mu_{\text{lim}} \) if the isophotal limit is either much smaller than or much greater than this critical value. The increase of \( N_1 \) with \( \mu_{\text{lim}} \) is simply due to the increased mean redshift of the survey galaxy population, as described in Section 4.5. However for \( \mu_{\text{lim}} \gtrsim 30 \) the isophotal and total magnitudes of all the objects considered here are essentially the same, and \( N_1 \) flattens off.

Interestingly, in the limiting case of \( \mu_{\text{lim}} \to \infty \), \( N_1 \) is twice the value obtained in Mortlock & Webster (2000b). This is due to the finite aperture size, as shown in Fig. 8(b). Taking \( \theta_t \to \infty \) halves \( N_1 \), as the increase in the fibre flux of the survey galaxies is much greater than that of the more centralised quasar images. However, \( N_1 \) does not increase for \( \theta_t \lesssim 1 \) arcsec, as there are few lenses with image separations less than \( \sim 2 \) arcsec. This implies that the 2dF fibre size is close to optimal for a lens search: any smaller and the integration times required to obtain reasonable spectra of \( m \simeq 19 \) galaxies would become prohibitively long; any larger and fewer lenses would be found.

Also of interest is the variation of \( N_1 \) with the seeing, which is illustrated by the five lines in each panel of Figs. 8 and 9. In most cases there is little difference between \( \theta_t = 0 \) arcsec and \( \theta_t = 1 \) arcsec, primarily because \( \theta_s \lesssim \theta_t \) in this range. However if \( \theta_s \gtrsim \theta_t \) the number of lenses decreases with the seeing, because so much of the flux from even centred quasar images misses the fibres, as illustrated in Fig. 8. (The survey galaxies are extended intrinsically, and so the relative reduction in fibre flux is lower.) The one exception to this qualitative explanation is shown in Fig. 8 (b), in which the number of lenses decreases rapidly with decreasing fibre size when \( \theta_t = 0 \). This case can be understood geometrically, as the quasar images must lie inside the 2dF fibres to be registered at all (there being no ‘spillage’ from the PSF), whereas most low-redshift galaxies produce image separations slightly larger than the fibre diameter.

Fig. 8(c) shows an exponential increase in \( N_1 \) with \( \Delta m_{\text{qg}} \), which reflects the steepness of the quasar luminosity function. For \( \Delta m_{\text{qg}} > 2 \) the change in \( \Delta m_{\text{qg}} \) corresponds almost directly to the increase in the effective depth of the ‘lens survey’. It is also clear that \( N_1 \) is more sensitive to \( \Delta m_{\text{qg}} \) than it is to any other model parameter. If a detailed examination of the 2dF spectra revealed that \( \Delta m_{\text{qg}} \simeq 2 \) was an underestimate of the quasars’ ‘spectral prominence’, the expected number of lenses could easily be doubled.

As shown in Fig. 8(d), the \( \Delta m_{\text{qg}} \) dependence is similar, but less pronounced than the dependence on \( \Delta m_{\text{qg}} \) discussed above. This is because a high value of \( \Delta m_{\text{qg}} \) only results in the ‘lens survey’ being more complete to a given depth, whereas increasing \( \Delta m_{\text{qg}} \) probes fainter magnitudes and thus larger numbers of quasars. Combined with the fact that \( \Delta m_{\text{qg}} \) is reliably constrained to be close to 0 (from Section 5.2), the uncertainties in the star-galaxy separation properties of the APM survey do not flow through to the calculation of \( N_1 \).

7.2 Lens model

The most significant uncertainties in the calculation of \( N_1 \) are related to the observational parameters discussed in Section 7.1, but the results do vary with the lens model, and the effects of a finite core radius and ellipticity were explored.

Fig. 8(a) shows how \( N_1 \) varies with canonical core radius, \( r_{c*} \). The core radius is assumed to vary with velocity dispersion as \( r_c = r_{c*}(\sigma/\sigma_*)^3 \), and \( r_{c*} \) is taken to be the same for each galaxy type. The fall-off of \( N_1 \) with core radius is very pronounced, as explained in Section 7.1: for large \( r_{c*} \) the spirals’ cross-sections become zero and the images formed by Es and S0s completely miss the 2dF fibres. So the assumption of a singular lens model overestimates \( N_1 \) by at most 30 per cent, given that \( r_{c*} < 100 \) pc (Kochanek 1996).

More important is the ellipticity of the lens galaxies, which are typically inferred to be as high as \( \epsilon \simeq 0.6 \) from lensing studies (e.g. Keeton et al. 1997). This was investigated by the application of an external shear of magnitude \( \gamma_0 = \epsilon/3 \), with all galaxies assumed to have the same projected shape – a more realistic distribution would simply smooth the curves shown in Fig. 8(b). As discussed in Section 7.1 and shown in Fig. 8(c), the extra images that can be produced by quadrupole lenses almost always lie inside the 2dF fibres, and so the greater magnification bias can significantly increase \( N_1 \). Given that \( \gamma_0 \gtrsim 0.3 \) is unrealistic for the dark matter-dominated galaxy halos, the use of a circularly-symmetric lens model underestimates the number of lenses by at most 50 per cent.

Overall, the uncertainty in the deflectors’ mass distributions does not greatly affect the estimates of \( N_1 \) as the two effects described above tend to cancel out. The most realistic model would have a small (\( \lesssim 50 \) pc) core radius and an ellipticity of between 0.3 and 0.6 (i.e. a shear of between 0.1 and 0.2) which would result in only slightly more lenses than the default singular model. The calculation using singular isothermal spheres also provides a robust lower limit of \( N_1 \simeq 10 \).

8 CONCLUSIONS

Kochanek (1992) and Mortlock & Webster (2000b) showed that appreciable numbers of lensed quasars could be discovered in galaxy surveys, but a number of potentially important selection effects and biases were ignored in both calculations. These generic results have been refined here to obtain a more realistic estimate of the number of lenses, \( N_1 \), expected in the 2dF GRS.

In particular, the importance of a finite collection area for the redshift survey spectra and a finite surface brightness limit were investigated. The 2dF optical fibres are unusually small (an angular radius of only 1 arcsec), which
The expected number of lenses, $N_l$, in a GRS of the same design as the 2dF survey, as a function of limiting surface brightness, $\mu_{\text{lim}}$ in (a), fibre radius, $\theta_f$ in (b), $\Delta m_{f,qg}$ in (c) and $\Delta m_{g,qg}$ in (d). The vertical dashed lines indicate the values appropriate to the 2dF GRS: $\mu_{\text{lim}} = 24.5$, $\theta_f = 1$ arcsec, $\Delta m_{f,qg} = 2$ and $\Delta m_{g,qg} = 0$. The lens model used was the singular isothermal sphere with no external shear. The results are shown for several values of the atmospheric seeing: $\theta_s = 0$ arcsec (solid lines); $\theta_s = 1$ arcsec (dashed lines); $\theta_s = 2$ arcsec (dot-dashed lines); $\theta_s = 3$ arcsec (dotted lines); and $\theta_s = 4$ arcsec (dot-dot-dot-dashed lines).

Figure 8. The expected number of lenses, $N_l$, in a GRS of the same design as the 2dF survey, as a function of limiting surface brightness, $\mu_{\text{lim}}$, fibre radius, $\theta_f$, $\Delta m_{f,qg}$ and $\Delta m_{g,qg}$.

approximately doubles the number of lenses, as so much of the light from the extended galaxies misses the fibres. Conversely, the use of isophotal rather than total magnitudes reduces the effective depth of the survey (by $\sim 0.5$ mag in the case of 2dF) which reduces the depth of the 'lens survey' by a similar amount.

The 2dF input catalogue was determined by the APM star-galaxy separation algorithm, and so this was applied to a simulated population of quasar lenses, with the result that most were classified as multiple stellar images. Nonetheless, any lens in which the deflector galaxy is at least as bright as the (magnified) quasar images should enter the APM galaxy survey and hence the 2dF GRS. Importantly, lenses with nearby deflectors (e.g. Q 2237+0305) are especially valuable, as discussed in Mortlock & Webster (2000b).

Whilst each of the above selection effects can change $N_l$ by a factor of several, they tend to cancel each other out. The generic estimate in Mortlock & Webster (2000b) implied that between ten (if only lenses with bright deflectors were included) and fifty (if lenses with high-redshift deflectors were not rejected from the survey) new quasar lenses could be discovered in the 2dF GRS. Clearly it is the former figure which is most relevant, and thus the prior prediction of $\sim 10$ low-redshift lenses is confirmed by the more detailed simulations presented here. The random uncertainty in this value is mainly due to shot noise, but the variability of the atmospheric seeing at the AAT site is also an issue. The most important systematic error is the quality of the survey spectra (characterised by $\Delta m_{f,qg}$), as it is not clear how efficiently galaxy spectra can be searched for the presence of quasar emission features.

Irrespective of the above uncertainties, it is clear that redshift surveys, and the 2dF GRS in particular, are potential sources of large numbers of lensed quasars. The next step then is to implement a systematic search for quasars' emission lines amongst the 2dF galaxy spectra. A notable success in this field was the spectroscopic discovery by Warren et al. (1996) of a lensed emission line galaxy. In a related project, Willis et al. (2000) have developed a more rigorous technique, using template-subtraction to search for lensed emission line galaxies in a 2dF sample of field ellipticals.

Unfortunately these methods are probably not applica-
Figure 9. The expected number of lenses in the 2dF GRS, $N_1$, as a function of the canonical core radius of the lens galaxies, $r_{c*}$, in (a) and the external shear, $\gamma_0$, in (b). The calculation is performed assuming $\Delta m_{\text{qg}} = 2$ and $\Delta m_{\text{gq}} = 0$ in all cases. The results are shown for several values of the atmospheric seeing: $\theta_s = 0$ arcsec (solid lines); $\theta_s = 1$ arcsec (dashed lines); $\theta_s = 2$ arcsec (dot-dashed lines); $\theta_s = 3$ arcsec (dotted lines); and $\theta_s = 4$ arcsec (dot-dot-dot-dashed lines).

ble to the 2dF spectra as the target galaxies are not as homogeneous as those in the above sample. The most obvious choice of search technique is principal components analysis (e.g. Murtagh & Hecht 1987), as it is already being used to analyse the 2dF galaxy spectra (Glazebrook, Offer & Deeley et al. 1999; Folkes et al. 1999). Further, an important part of the existing analysis software is the flagging of ‘unusual’ spectra††, and it is a subset of these that will become the lens candidates. Even if there are thousands of ‘unusual’ objects, it would be feasible to search for the presence of quasar emission features by eye, which would probably yield several hundred genuine lens candidates, most of which would then be easily rejected on morphological grounds. The completeness of the final candidate sample may well be quite low, but its efficiency (i.e. the fraction of lens candidates which are real) should be very high relative to conventional lens surveys (Mortlock & Webster 2000b). The discovery of even one lens in this manner would not only show the worth of such searches, but be valuable in its own right.

ACKNOWLEDGMENTS

Matthew Colless and Steve Maddox provided a great deal of insight into the workings of the 2dF redshift survey and the APM survey, respectively. The comments of the anonymous referee also resulted in some significant improvements to this paper. DJM was supported by an Australian Postgraduate Award.

†† At the time of writing, about $10^5$ survey spectra had been obtained, reduced, and processed. No lenses or lens candidates have been reported, but little effort has been put into the analysis of irregular objects thus far. It is not clear whether the pipeline reduction algorithms will be sufficient to generate a useful list of lens candidates; this should become more apparent as the survey nears completion.

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