Rotated ansatz for approximate counterdiabatic driving

Glen Bigan Mbeng\textsuperscript{1} and Wolfgang Lechner\textsuperscript{1,2}
\textsuperscript{1}Universität Innsbruck, Technikerstraße 21 a, A-6020 Innsbruck, Austria
\textsuperscript{2}Parity Quantum Computing GmbH, A-6020 Innsbruck, Austria
(Dated: July 11, 2022)

Approximate counterdiabatic (CD) protocols are a powerful tool to enhance quantum adiabatic processes that allow to reliably manipulate quantum systems on short time scales. However, implementing CD protocols entails the introduction of additional control fields in the Hamiltonian, often associated with highly non-local multi-body interactions. Here, we introduce a novel variational rotated ansatz (RA) to systematically generate experimentally accessible approximate CD protocols. We numerically benchmark our approach on state preparation and adiabatic quantum computing algorithms, and find that using RA protocols significantly enhances their performances.

Introduction– Adiabatic processes are a fundamental building block of quantum science and technology, with applications ranging from heat engines in thermodynamics to quantum state preparation and quantum computation [1–4]. They rely on the adiabatic theorem to manipulate quantum states by varying the system Hamiltonian at a rate proportional to the square gap between its instantaneous energy eigenvalues. However, despite the robustness against timing errors and thermal fluctuations, the efficiency of this approach is ultimately limited by the long runtimes associated with small gaps in the energy spectrum. Therefore, developing and understanding shortcuts to adiabaticity (STA) [5–7], that mimic adiabatic dynamics with faster drivings is key to the progress of quantum technologies [8–13].

Counterdiabatic (CD) drivings are promising STA strategies that aim at suppressing transitions between the energy levels with appropriately engineered CD Hamiltonians [14–18]. The exact CD Hamiltonian can mimic adiabatic dynamics using fast protocols with unit fidelity. However, the implementation of exact CD terms for many-body systems is experimentally challenging because it requires a priori knowledge of the eigenstates of the Hamiltonian, and it generally involves highly non-local interactions, impossible to realize on the current hardware [19–21].

A possible strategy to generate alternative, physically feasible shortcuts is using physical rotations and moving to an alternative Schrödinger picture, where the non-local Hamiltonian CD terms vanish [22, 23]. On the one hand, Refs. [24–29] used an approach based on Lie Algebras to compute necessary physical rotation or few-level systems. On the other hand, Refs. [30–33] developed problem-specific methods to generate physical rotation for specific many-body Hamiltonians. However, a systematic approach to determining the physical unitary transformations in many-body systems is not available. Recently, Ref. [31] introduced a variational approach to finding approximate CD driving Hamiltonians for many-body systems. This method has been recently applied both theoretically [34–40] and experimentally [41, 42] to speed up adiabatic processes in quantum spin systems. Our work aims at connecting the idea of multiple Schrödinger pictures presented in Ref. [22] with the variational approach of Ref. [31].

Here, we introduce the rotated ansatz method to simultaneously determine optimal approximate CD terms and the associated physical rotation. The resulting protocol, which can be computed efficiently for various many-body systems, does not require additional terms in the Hamiltonian. After introducing the novel variational approach to determine RA protocols, we study the efficacy of the protocols generated for (i) a two-level system, (ii) a non-integrable Ising chain and (iii) a fully programmable quantum annealing architecture [43]. We find significant performance enhancements for the protocols in all three cases.

Rotated ansatz.– We consider the parametric Hamiltonian $\tilde{H}_0(\lambda)$, which depends on the real control parameter $\lambda$, with instantaneous eigenvalues and eigenvectors $\epsilon_m(\lambda)$ and $|\epsilon_m(\lambda)\rangle$. Given an initial value $\lambda_i$ and a final value $\lambda_f$, we focus on the task of driving the system from $|\epsilon_m(\lambda_i)\rangle$ to $|\epsilon_m(\lambda_f)\rangle$ in a time $\tau$. According to the adiabatic theorem, if $\lambda(t)$ ramps sufficiently slowly from $\lambda(0) = \lambda_i$ to $\lambda(\tau) = \lambda_f$, the unassisted (UA) driving Hamiltonian $\tilde{H}_{UA}(t) = \tilde{H}_0(\lambda(t))$ implements the desired transformation [44, 45]. However, in practical cases, the finite coherent evolution time $\tau$ introduces diabatic transitions, leading to a fidelity loss.

The CD method aims at suppressing diabatic transitions by adding auxiliary terms to the driving Hamiltonian:

$$\tilde{H}_{CD}(t) = \tilde{H}_0(\lambda(t)) + \lambda \dot{\tilde{A}}.$$  \hspace{1cm} (1)

When $\dot{\tilde{A}}$ coincides with the (exact) adiabatic gauge potential (AGP) [16, 18]:

$$\dot{\tilde{A}}^{AGP} = i\hbar \sum_{m \neq l} \frac{|\epsilon_m(\lambda)\rangle \langle \partial_t \tilde{H}_0(\lambda) | \epsilon_l(\lambda)\rangle - |\epsilon_l(\lambda)\rangle \langle \partial_t \tilde{H}_0(\lambda) | \epsilon_m(\lambda)\rangle}{\epsilon_l(\lambda) - \epsilon_m(\lambda)},$$ \hspace{1cm} (2)

the Schrödinger dynamics $i\hbar \partial_t |\psi(t)\rangle = \tilde{H}_{CD} |\psi(t)\rangle$ results in a perfect driving, that evolves $|\psi(0)\rangle = |\epsilon(\lambda_i)\rangle$ into $|\psi(\tau)\rangle = |\epsilon(\lambda_f)\rangle$.

The direct computation of the exact AGP in non-integrable many-body systems has an exponential complexity cost due to the lack of knowledge of spectral information [46]. Moreover, the exact AGP can be highly
non-local and, thus, hard to implement in experiments involving many-body systems. We use the least action principle introduced in Ref. [31] to overcome these limitations. The approach is based on the observation that the exact AGP of Eq. (2) minimizes the following action [18, 31]:

\[ S(\hat{A}) = \text{Tr}[\hat{G}^2] , \quad \hat{G}_\lambda = \partial_\lambda \hat{H}_0 - \frac{i}{\hbar} [\hat{H}_0, \hat{A}] . \]  

(3)

Therefore, we can find approximate AGPs by minimizing the action over a specific variational ansatz for \( \hat{A} \). In particular, restricting the ansatz to the subspace of Hamiltonian terms available in the lab generates experimentally accessible AGPs for many-body systems.

As the AGP is approximated, the resulting CD protocol is usually associated with a finite final fidelity. In this case, improving the effectiveness of the CD driving requires additional Hamiltonian terms and control fields. However, we can alternatively implement the CD driving in a rotating frame. Following Refs. [22, 28], we introduce a unitary transformation \(|\tilde{\psi}(t)\rangle = \tilde{R}(t)|\psi(t)\rangle\). In the new frame, the Schrödinger equation describing the CD driving reads

\[ \frac{d}{dt} |\tilde{\psi}(t)\rangle = \tilde{H}_{RA}(t)|\tilde{\psi}(t)\rangle , \]  

(4)

where \( \tilde{H}_{RA}(t) = \tilde{R}\tilde{H}_{CD}\tilde{R}^\dagger - i\hbar \dot{\tilde{R}}\tilde{\lambda} \). Using Eq. (1), we express \( \dot{\tilde{A}} \) as function of \( \tilde{H}_{RA} \) and \( \tilde{R} \):

\[ \dot{\tilde{\lambda}} \tilde{A} = \tilde{H}_{CD} - \tilde{H}_0 = \tilde{R}^\dagger \left( \tilde{H}_{RA} + i\hbar \dot{\tilde{R}}\tilde{\lambda} \right) \tilde{R} - \tilde{H}_0 . \]  

(5)

Next, we simultaneously determine which \( \tilde{H}_{RA}(t) \) and \( \tilde{R}(t) \) result in an optimal approximate AGP \( \tilde{A} \). Since Eq. (5) simultaneously depends on \( \tilde{R} \) and \( \dot{\tilde{R}} \), we cannot yet apply a time-local least action principle. To solve this issue, we introduce the auxiliary potential \( \tilde{\mathcal{K}} = \tilde{H}_{RA} + i\hbar \ddot{\lambda} \dot{\tilde{R}} - \tilde{H}_0 \). Furthermore, we simplify the algebra by considering only abelian transformations \( \tilde{R}(t) = e^{-i\tilde{\lambda} \tilde{Q}(t)} \), where the generator \( \tilde{Q}(t) \) commutes at different times ([\( \tilde{Q}(t), \tilde{Q}(t') \)] = 0). We then get

\[ \dot{\tilde{\lambda}} \tilde{A} = e^{i\tilde{\lambda}} \left( \tilde{H}_0 + \tilde{\mathcal{K}} \right) e^{-i\tilde{\lambda}} \tilde{R} - \tilde{H}_0 \]  

\[ \tilde{H}_{RA} = \tilde{H}_0 + \tilde{\mathcal{K}} + \tilde{\mathcal{Q}} . \]  

(6)

(7)

We interpret Eq. (6) as a variational rotated ansatz (RA) for the AGP, which fundamentally differs from previous proposals [31, 35, 36]. Our ansatz depends on the derivative \( \dot{\lambda} \), however, we can still find the optimal values of the RA auxiliary potentials \( \tilde{\mathcal{K}} \) and \( \tilde{\mathcal{Q}} \), minimizing the action \( S(\tilde{A}) \) over the subspace of allowed physical operators. Then, Eq. (7) guarantees that the resulting Hamiltonian \( \tilde{H}_{RA} \) also belongs to the subspace of allowed operators.

The Hamiltonian \( \tilde{H}_{RA}(t) \) drives the rotated state \( |\tilde{\psi}(t)\rangle \). In particular, if time boundary conditions

\[ |\langle \epsilon(\lambda)|\tilde{\psi}(t)\rangle| = |\langle \epsilon(\lambda)|\psi(t)\rangle| \quad \text{for} \ t = 0, \tau \]  

(8)

hold, \( \tilde{H}_{RA}(t) \) implements an alternative approximate STA. In the Supplementary Material, we show that we can enforce such boundary conditions by using smooth ramps such that \( \lambda(0) = \lambda(\tau) = 0 \). In this case, the new terms appearing in Eq. (7) enhance the performance of the RA driving with respect to the conventional unassisted driving.

In most applications, we must rely on a numerical method to minimize the action and find \( \tilde{H}_{RA}(t) \) at each time. In the Supplementary Material, we describe a sequential local minimization procedure to carry out this task. The bottleneck of the algorithm is the evaluation of the action. However, in the following, we explicitly show that the complexity cost of computing the RA action is polynomial in the system size \( N \), for various relevant Hamiltonians.

Applications—We validate the RA approach on three problems: a two-level system, a quantum Ising chain, and quantum annealing. For each problem, we simultaneously compare the unassisted (UA) driving \( \tilde{H}_0(t) \), the rotated ansatz (RA) \( \tilde{H}_{RA}(t) \) and the purely imaginary local CD driving [31, 34, 47, 48]:

\[ \tilde{H}_{local-CD}(t) = \tilde{H}_0(\lambda) + \dot{\lambda} \tilde{A} , \quad \tilde{A} = \sum_{j=1}^{N} \alpha_j(\lambda) \hat{\sigma}_j^\dagger , \]  

(9)

where the coefficients \( \alpha_j(\lambda) \) minimize the action. The three protocols do not require additional non-local control fields. However, the local CD protocol which was extensively studied in Refs. [31, 34], introduces new local control fields.

In all examples, we use natural units and set \( \hbar = 1 \). We, then, consider the following smooth ramp [31, 34–36, 47] \( \lambda(t) = \sin^2 \left( \frac{\pi t}{2} \right) \), going from \( \lambda_j = 0 \) to \( \lambda_j = 1 \). Here, the derivatives \( \dot{\lambda} \) and \( \ddot{\lambda} \) vanish at the beginning and end of the protocol, ensuring ensuring the validity of Eq. (8). We initialize the system in \( \tilde{H}_0(\lambda_j) \)'s ground state and calculate the fidelity between the time-evolved state \( |\tilde{\psi}(t)\rangle \) and the instantaneous ground state \( |\epsilon_0(\lambda)\rangle \) of \( \tilde{H}_0(\lambda) \):

\[ F(t) = |\langle \epsilon_0(\lambda)|\psi(t)\rangle|^2 . \]  

(10)

The final fidelity \( 0 \leq F(\tau) \leq 1 \) measures the probability of successfully preparing \( \tilde{H}_0(\lambda_j) \)'s ground state. To account for the physical rotation \( \tilde{R}(t) \), for the RA protocol we also compute the fidelity between \( |\psi(t)\rangle \) and the rotated ground state \( |\epsilon_0(\lambda)\rangle = \tilde{R}(t)|\epsilon_0(\lambda)\rangle \):

\[ \tilde{F}(t) = |\langle \epsilon_0(\lambda)|\tilde{\psi}(t)\rangle|^2 = |\langle \epsilon_0(\lambda)|\tilde{R}^\dagger(t)|\psi(t)\rangle|^2 . \]  

(11)

The boundary conditions in Eq. (8) imply that \( \tilde{F}(0) = F(0) = 1 \) and \( \tilde{F}(\tau) = F(\tau) \). However, for intermediate times, we can have \( \tilde{F}(t) \neq F(t) \).
(i) **Two-level system (Bell state).** To illustrate the RA approach in a simple context, we consider the following two spin system \([35, 49]\):

\[
\hat{H}_{\text{two-spins}} = -h(t) (\hat{\sigma}_1^x + \hat{\sigma}_2^x) + J(t) (\hat{\sigma}_1^y \hat{\sigma}_2^y + \hat{\sigma}_1^z \hat{\sigma}_2^z),
\]

where \(J(t)\) and \(h(t)\) are the control fields. Eq. (12) describes a two-level system where \(\hat{H}(t)\) only couples the states \(|\uparrow\uparrow\rangle\) and \(|\downarrow\downarrow\rangle\). This observation allowed Refs. [35, 49] to engineer an effective CD driving for this problem based on fast oscillating fields. Here, we use the system’s two-level nature to analytically compute the RA driving Hamiltonian.

We focus on the UA protocol \(J_0(t) = -1, h_0(t) = 5(1 - \lambda)\), which aims at preparing the maximally entangled Bell state \(\psi_{\beta}(\lambda_f) = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}\). We parametrize the auxiliary potentials as \(\hat{Q} = -\gamma (\hat{\sigma}_1^z + \hat{\sigma}_2^z)\) and \(\hat{K} = \beta (\hat{\sigma}_1^y \hat{\sigma}_2^y + \hat{\sigma}_1^z \hat{\sigma}_2^z)\). In the Supplementary Material we compute and minimize the action for the two spin problem. The variational minimization leads to the following optimal functions:

\[
\beta(t) = \frac{1}{\lambda} \sqrt{J_0^2 + \varphi_0^2 - J_0}, \gamma(t) = \frac{1}{4} \arctan \left( \frac{\varphi_0}{J_0} \right),
\]

with \(\varphi_0(t) = \frac{J_0 h_0 - h h_0}{J_0^2 + 4 \lambda^2}\). The resulting analytical expression for the RA driving functions \(J_{RA}(t) = J(t) + \gamma(t), h_{RA}(t) = h(t) - \beta(t)\), are specific feature of two level systems. In particular, this RA driving implements the exact AGP, which prepares the bell state with unitary fidelity \(F_{RA}(\tau) = 1\) for any protocol duration \(\tau\). In the Supplementary Material, we verified that our sequential minimization algorithm correctly reproduces these analytical results.

(ii) **Non-integrable spin chain.** Next, we use the RA approach to improve the state preparation protocols in a non-integrable quantum many-body systems. We consider a chain of \(N\) spin-\(\frac{1}{2}\), with the following driving Hamiltonian:

\[
\hat{H}_{\text{spin-chain}} = -\sum_{j=1}^{N} (J(t) \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + b(t) \hat{\sigma}_j^y + h(t) \hat{\sigma}_j^z),
\]

where we assume periodic boundary conditions \((\hat{\sigma}_N^x \equiv \hat{\sigma}_1^x)\). Equation (14) describes the nearest neighbor quantum Ising chain with transverse and longitudinal fields. It can be experimentally realized with superconducting qubits [50], cold atoms [51], and trapped ions [52].

We consider the unassisted driving \(J_0(t) = \lambda, b_0(t) = \lambda/5\) and \(h_0(t) = 1 - \lambda/2\). This path connects the initial product state \(|\psi_{\beta_0}(\lambda_i)\rangle = |+\rangle^{\otimes N}\) with the non-trivial groundstate of Eq. (14) at \(h_f = 1/2, J_f = 1, b_f = 1/5\). We parameterize the auxiliary potentials as \(\hat{Q} = \gamma \sum_j \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \phi \sum_j \hat{\sigma}_j^y\) and \(\hat{K} = \beta \sum_j \hat{\sigma}_j^z\). Then, the RA driving functions read \(J_{RA} = J_0 - \gamma, b_{RA} = b_0 - \phi\) and \(h_{RA} = h_0 - \beta\).

Due to the translational symmetry and the short range of the interaction, the ratio \(S/(N2^N)\) is independent of system size \(N\) (for \(N \geq 4\)), as we prove in the Supplementary Material. Thus, we can readily compute \(S\) by considering a system of \(N = 4\) spins. We provide the resulting analytical expression of the action in the Supplementary Material.

We benchmark the RA method on a protocol of duration \(\tau = 1\). Using the sequential action minimization we find the optimal functions \(J^{UA}(t), b^{UA}(t)\) and \(h^{UA}(t)\) [Fig. 1(b)], which do not depend on \(N\). In Fig. 1(c), we compare instantaneous ground state fidelity of the UA (dashed black line), the local CD (dotted red line) and the RA (solid blue line) protocols, on a chain of \(N = 8\) spins. The diabatic transitions in the UA dynamics gradually reduce the instantaneous fidelity \(F_{UA}(\tau)\) (black dashed line), leading to the final value \(F_{UA}(\tau) = 8\%\). The local CD driving enhances the fidelity at all times \(F_{local-CD}(t) \geq F_{UA}(t)\), leading to a greater final fidelity \(F_{local-CD}(\tau) = 18\%\). The RA’s fidelity \(F_{RA}(\tau)\) is lower than the UA’s fidelity for intermediate times. However, the RA protocol actually boosts the final fidelity up to \(F_{RA}(\tau) = 36\%\). Indeed, the implemented RA driving, instantaneously tracks the rotated ground state with a fidelity \(F_{RA}(t)\) (light blue dot-dashed line) which is greater than those achieved by the UA and the local CD drivings.

(iii) **Quantum annealing.** Quantum annealing
 FIG. 2. Quantum annealing on the LHZ architecture. (a) and (b) control fields for the UA and RA protocols. (c) ground state fidelity $F(t)$ for the UA, local CD and RA protocols, and rotated ground state fidelity $	ilde{F}(t)$ for the RA protocol. The data refers to a system of $N = 6$ spins with random couplings $J_j \in [-1,1]$ and protocol’s duration $\tau = 1$. The inset of (c) shows a cartoon LHZ architecture.

(QA)[53–55] is a strategy to solve classical combinatorial optimization problems on quantum devices. Here, we consider the minimization of a generic quadratic unconstrained binary optimization (QUBO) cost function $f(s) = \sum_{\mu<\nu} J_{\mu\nu}s_\mu s_\nu$, which depends on the $n$ binary variables $s_1, \ldots, s_n = \pm 1$. We adopt the LHZ architecture [43] to translate the long-range QUBO problem into an experimentally accessible short-ranged spin model, where the physical spins $\hat{\sigma}_\mu^x, \hat{\sigma}_\mu^y, \hat{\sigma}_\mu^z$ represent the relative configuration of two $s_\nu, s_\mu \to \hat{\sigma}_j^z$. The resulting LHZ quantum annealing Hamiltonian reads

$$\hat{H}_{\text{LHZ}} = -A(t) \sum_{k=1}^N J_k \hat{\sigma}_k^x - B(t) \sum_{k=1}^N \hat{\sigma}_k^z - C(t) \sum_{l=1}^L \hat{H}_{\text{CD}}$$

where $N = n(n-1)/2$ is the number of physical qubits and $L = (n-1)(n-2)/2$ is the number of 4-body constraints $\hat{H}_{\text{CD}} = \hat{\sigma}_{(i,j)}^x \hat{\sigma}_{(i,j)}^y \hat{\sigma}_{(i,j)}^z \hat{\sigma}_{(i,j)}^z$ required to implement the parity mapping. The inset of Fig. 2 shows the LHZ architecture for $n = 4$ (and $N = 6$ qubits). Following Ref. [43], we consider the UA strategy $A_0(t) = \lambda$, $B_0(t) = 1 - \lambda$, $C_0(t) = C_f \lambda$ [see Fig. 2(a)]. Here, we the final constraints to be $C_f = 3$, which ensure that the final ground state encodes the solution of the considered QUBO problem [43, 57].

We parametrize the auxiliary potentials as $\hat{Q} = -\gamma \sum_k J_k \hat{\sigma}_k^z - \phi \sum_l \hat{H}_{\text{CD}}$ and $\hat{K} = -\beta \sum_k \hat{\sigma}_k^x$. Then, the RA driving functions read $A_{\text{RA}}(t) = A_0 + \gamma$, $C_{\text{RA}} = C_0 + \phi$ and $B_{\text{RA}} = B_0 + \beta$. The action for this problem depends on $N$ and the specific disorder implementation. In the Supplementary Material we compute the analytical expression of the action and show that its numerical evaluation requires a linear time in the number of qubits $N$. Furthermore, we show in the Supplementary Material that similar results hold for conventional quadratic quantum annealing architectures.

We benchmark the RA approach on a spin-glass problem with uniform random couplings $J_k \in [-1,1]$. We first analyze a single problem instance of $N = 6$ qubits. Figure 2(b) shows the optimal RA control fields obtained by minimizing the action. In Fig. 2(c), we compare the instantaneous ground state fidelity of the UA, the local CD, and the RA protocols on the same problem instance. Despite the disorder, we find a behavior similar to what we observed for the Ising chain. The UA fidelity of $F_{\text{UA}}(\tau) = 4\%$, a local CD fidelity of $F_{\text{local-CD}}(\tau) = 8\%$ and $F_{\text{RA}}(\tau) = 18\%$. The RA tracks the rotated ground states with higher fidelity than the UA and local CD protocols. This fact is particularly surprising because the inhomogeneous local CD driving uses $N = 6$ additional control fields and does not outperform the RA driving.

Finally, we investigate the finite-size scaling of the protocols’ relative performance by considering the relative improvement, which we define as the ratio between the protocol’s and the baseline UA final fidelities $F(\tau)/F_{\text{QA}}(\tau)$. Figure 3 shows the relative improvement of the RA and local CD, averaged over an ensemble of 100 random problems with different qubits numbers $N = 4, 6, 10, 15$. Although
the limited data points prevent us from identifying scaling forms for the degree of improvement, our numerical results suggest that the features observed for \( N = 6 \) also hold for larger systems. In particular, the relative performance of the RA driving increases with the number of qubits. In the Supplementary Material, we examine RA protocols for traditional commercially available quadratic annealing architectures [58], obtaining comparable results.

**Conclusion and outlook.**– We have extended the least action formalism of CD drivings to include physical rotations of the driving Hamiltonian. The emerging rotated ansatz approach generates alternative variational STAs, simultaneously optimizing the rotation’s generator \( Q \) and the auxiliary potential \( K \). The resulting RA protocols do not require additional control fields and can be readily implemented on analog NISQ devices.

For small instances of spin systems, the RA approach systematically generates experimentally feasible protocols that considerably outperform the unassisted and local CD protocols. Moreover, our scaling analysis suggests that the enhancement persists in larger systems, where RA protocols may aid state preparation and quantum optimization.

The variational RA approach differs from the optimal control and machine learning methods for schedule optimization considered in Refs. [59, 60], because we can efficiently compute the optimal RA protocol, without having access to a quantum device [34, 61, 62]. However, it may be fruitful to investigate the use of optimal control and machine learning techniques to further improve the RA protocol’s fidelity. For example, future investigations can use optimal control to select the best initial prior for the variational scheme, as proposed in Ref. [59].

The extension of the RA approach to include such non-abelian transformations is also an appealing future research direction. Indeed, the unitary transformations that relate fast-forward to counterdiabatic protocols are generally complex and beyond the presented abelian case. We, therefore, expect that developing variational methods to approximate non-abelian will lead to a more significant enhancement of the protocols’ performance.

**Acknowledgements**– We thank A. Hartmann, J. Wurtz and C. Dlaska for valuable discussions. This work was supported by the Austrian Science Fund (FWF) through a START grant under Project No. Y1067-N27 and the SFB BeyondC Project No. F7108-N38, and the European Union’s Horizon 2020 research and innovation program under grant agreement No. 817482.

[1] M. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).

[2] S. Vinjanampathy and J. Anders, Quantum thermodynamics, *Contemporary Physics* **57**, 545 (2016).

[3] T. Albash and D. A. Lidar, Adiabatic quantum computation, *Rev. Mod. Phys.* **90**, 015002 (2018).

[4] J. L. Bohn, A. M. Rey, and J. Ye, Cold molecules: Progress in quantum engineering of chemistry and quantum matter, *Science* **357**, 1002 (2017).

[5] X. Chen, A. Ruschhaupt, S. Schmidt, A. del Campo, D. Guéry-Odelin, and J. G. Muga, Fast optimal frictionless atom cooling in harmonic traps: Shortcut to adiabaticity, *Phys. Rev. Lett.* **104**, 063002 (2010).

[6] D. Guéry-Odelin, A. Ruschhaupt, A. Kiely, E. Torrontegui, S. Martínez-Garaot, and J. G. Muga, Shortcuts to adiabaticity: Concepts, methods, and applications, *Rev. Mod. Phys.* **91**, 045001 (2019).

[7] A. del Campo and K. Kim, Focus on shortcuts to adiabaticity, *New Journal of Physics* **21**, 050201 (2019).

[8] M. G. Bason, M. Viteau, N. Malossi, P. Huillery, E. Arimondo, D. Ciampini, R. Fazio, V. Giovannetti, R. Mannella, and O. Morsch, High-fidelity quantum driving, *Nature Physics* **8**, 147 (2012).

[9] A. C. Santos and M. S. Sarandy, Superadiabatic controlled evolutions and universal quantum computation, *Scientific Reports* **5**, 15775 (2015).

[10] B. Zhou, A. Buksc, H. Ribeiro, C. Yale, F. Hermanos, P. Jerger, A. Auer, G. Burkard, A. Clerk, and D. Awschalom, Accelerated quantum control using superadiabatic dynamics in a solid-state lambda system, *Nature Physics* **13**, 330 (2017).

[11] Y.-X. Du, X.-X. Yue, Z.-T. Liang, J.-Z. Li, H. Yan, and S.-L. Zhu, Geometric atom interferometry with shortcuts to adiabaticity, *Phys. Rev. A* **95**, 043608 (2017).

[12] S. Deng, A. Chenu, P. Diao, F. Li, S. Yu, I. Coulanny, A. del Campo, and H. Wu, Superadiabatic quantum friction suppression in finite-time thermodynamics, *Science Advances* **4**, eaar5909 (2018).

[13] A. Vepsäläinen, S. Danilin, and G. S. Paraoanu, Superadiabatic population transfer in a three-level superconducting circuit, *Science Advances* **5**, eaau5999 (2019).

[14] M. Demirplak and S. A. Rice, Adiabatic population transfer with control fields, *The Journal of Physical Chemistry A* **107**, 9937 (2003).

[15] M. Demirplak and S. A. Rice, Assisted adiabatic passage revisited, *The Journal of Physical Chemistry B* **109**, 6838 (2005), pMID: 16851769.

[16] M. V. Berry, Transitionless quantum driving, *Journal of Physics A: Mathematical and Theoretical* **42**, 365303 (2009).

[17] J. G. Muga, X. Chen, S. Ibáñez, I. Lizuain, and A. Ruschhaupt, Transitionless quantum drivings for the harmonic oscillator, *Journal of Physics B: Atomic, Molecular and Optical Physics* **43**, 085509 (2010).

[18] M. Kolodrubetz, D. Sels, P. Mehta, and A. Polkovnikov, Geometry and non-adiabatic response in quantum and classical systems, *Physics Reports* **697**, 1 (2017).

[19] A. del Campo, M. M. Rams, and W. H. Zurek, Assisted finite-rate adiabatic passage across a quantum critical point: Exact solution for the quantum ising model, *Phys. Rev. Lett.* **109**, 115703 (2012).

[20] K. Takahashi, Transitionless quantum driving for spin systems, *Phys. Rev. E* **87**, 062117 (2013).
[21] B. Damski, Counterdiabatic driving of the quantum ising model, Journal of Statistical Mechanics: Theory and Experiment 2014, P12010 (2014).
[22] S. Ibáñez, X. Chen, E. Torrontegui, J. G. Muga, and A. Ruschhaupt, Multiple schrödinger pictures and dynamics in shortcuts to adiabaticity, Phys. Rev. Lett. 109, 100403 (2012).
[23] S. Ibáñez, Y.-C. Li, X. Chen, and J. G. Muga, Pulse design without the rotating-wave approximation, Phys. Rev. A 92, 062136 (2015).
[24] S. Martínez-Garaot, E. Torrontegui, X. Chen, and J. G. Muga, Shortcuts to adiabaticity in three-level systems using lie transforms, Phys. Rev. A 89, 053408 (2014).
[25] E. Torrontegui, S. Martínez-Garaot, and J. G. Muga, Hamiltonian engineering via invariants and dynamical algebra, Phys. Rev. A 89, 043408 (2014).
[26] Y.-H. Kang, Y.-H. Chen, Z.-C. Shi, B.-H. Huang, J. Song, and Y. Xia, Pulse design for multilevel systems by utilizing lie transforms, Phys. Rev. A 97, 033407 (2018).
[27] U. Güngördü, Y. Wan, M. A. Fashi, and M. Nakahara, Dynamical invariants for quantum control of four-level systems, Phys. Rev. A 86, 062312 (2012).
[28] K. Takahashi, Unitary deformations of counterdiabatic driving, Phys. Rev. A 91, 042115 (2015).
[29] M. Bulok, D. Sels, and A. Polkovnikov, Geometric speed limit of accessible many-body state preparation, Phys. Rev. X 9, 011034 (2019).
[30] S. Deffner, C. Jarzynski, and A. del Campo, Classical and quantum shortcuts to adiabaticity for scale-invariant driving, Phys. Rev. X 4, 021013 (2014).
[31] D. Sels and A. Polkovnikov, Minimizing irreversible losses in quantum systems by local counterdiabatic driving, Proceedings of the National Academy of Sciences 114, E3909 (2017).
[32] R. R. Agundez, C. D. Hill, L. C. L. Hollenberg, S. Rogge, and M. Blaabjerg, Superadiabatic quantum state transfer in spin chains, Phys. Rev. A 95, 012317 (2017).
[33] A. del Campo, Shortcuts to adiabaticity by counterdiabatic driving, Phys. Rev. Lett. 111, 100502 (2013).
[34] A. Hartmann and W. Lechner, Rapid counter-diabatic sweeps in lattice gauge adiabatic quantum computing, New Journal of Physics 21, 043025 (2019).
[35] P. W. Claeys, M. Pandey, D. Sels, and A. Polkovnikov, Floquet-engineering counterdiabatic protocols in quantum many-body systems, Phys. Rev. Lett. 123, 090602 (2019).
[36] G. Passarelli, V. Cataudella, R. Fazio, and P. Lucignano, Counterdiabatic driving in the quantum annealing of the p-spin model: A variational approach, Phys. Rev. Research 2, 013283 (2020).
[37] A. Hartmann, V. Mukherjee, W. Niedenzu, and W. Lechner, Many-body quantum heat engines with shortcuts to adiabaticity, Phys. Rev. Research 2, 023145 (2020).
[38] A. Hartmann, V. Mukherjee, G. B. Mbeng, W. Niedenzu, and W. Lechner, Multi-spin counter-diabatic driving in many-body quantum Otto refrigerators, Quantum 4, 377 (2020).
[39] T. Villazon, P. W. Claeys, A. Polkovnikov, and A. Chandran, Shortcuts to dynamic polarization, Phys. Rev. B 103, 075118 (2021).
[40] A. Hartmann, G. B. Mbeng, and W. Lechner, Polynomial scaling enhancement in the ground-state preparation of ising spin models via counterdiabatic driving, Phys. Rev. A 105, 022614 (2022).
[41] H. Zhou, Y. Ji, X. Nie, X. Yang, X. Chen, J. Bian, and X. Peng, Experimental realization of shortcuts to adiabaticity in a nonintegrable spin chain by local counter-diabatic driving, Phys. Rev. Applied 13, 044059 (2020).
[42] N. N. Hegade, K. Paul, Y. Ding, M. Sanz, F. Albarrán-Arigaño, E. Solano, and X. Chen, Shortcuts to adiabaticity in digitized adiabatic quantum computing, Phys. Rev. Applied 15, 024038 (2021).
[43] W. Lechner, P. Hauke, and P. Zoller, A quantum annealing architecture with all-to-all connectivity from local interactions, Sci. Adv. 1, 1500838 (2015).
[44] A. Messiah, Quantum mechanics (Courier Corporation, 2014).
[45] S. Jansen, M.-B. Ruskai, and R. Seiler, Bounds for the adiabatic approximation with applications to quantum computation, Journal of Mathematical Physics 48, 102111 (2007).
[46] T. S. Cubitt, D. Perez-Garcia, and M. M. Wolf, Undecidability of the spectral gap, Nature 528, 207 (2015).
[47] L. Prieling, A. Hartmann, Y. Yamashiro, K. Nishimura, W. Lechner, and H. Nishimori, Two-parameter counter-diabatic driving in quantum annealing, Phys. Rev. Research 3, 013227 (2021).
[48] The AGP of models with time-reversal symmetry (real symmetric Hamiltonians) are purely imaginary [31]. Since we consider only time-reversal symmetric Hamiltonians, Eq. (11) describes the most general local approximate AGP.
[49] F. Petzioi, B. Dive, F. Mintert, and S. Wimberger, Fast adiabatic evolution by oscillating initial hamiltonians, Phys. Rev. A 98, 043436 (2018).
[50] R. Barends, A. Shabani, L. Lamata, J. Kelly, A. Mezzacapo, U. Las Heras, R. Babbush, A. G. Fowler, B. Campbell, Y. Chen, et al., Digitized adiabatic quantum computing with a superconducting circuit, Nature 534, 222 (2016).
[51] J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Quantum simulation of antiferromagnetic spin chains in an optical lattice, Nature 472, 307 (2011).
[52] P. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt, and C. F. Roos, Quasiparticle engineering and entanglement propagation in a quantum many-body system, Nature 511, 202 (2014).
[53] T. Kadowaki and H. Nishimori, Quantum annealing in the transverse ising model, Phys. Rev. E 58, 5355 (1998).
[54] G. E. Santoro, R. Martonák, E. Tosatti, and R. Car, Theory of quantum annealing of an ising spin glass, Science 295, 2427 (2002).
[55] P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, and W. D. Oliver, Perspectives of quantum annealing: methods and implementations, Reports on Progress in Physics 83, 054401 (2020).
[56] A. Lucas, Ising formulations of many np problems, Frontiers in Physics 2, 10.3389/fphy.2014.000045 (2014).
[57] M. Lanthaler and W. Lechner, Minimal constraints in the parity formulation of optimization problems, New Journal of Physics 23, 083039 (2021).
[58] M. W. Johnson, M. H. S. Amin, S. Gildert, T. Lanting, F. Hamze, N. Dickson, R. Harris, A. J. Berkley, J. Johansson, P. Bunyk, E. M. Chapple, C. Enderud, J. P. Hilton, K. Karimi, E. Ladizinsky, N. Ladizinsky, T. Oh, I. Perminov, C. Rich, M. C. Thom, E. Tolkacheva, C. J. S. Truncik, S. Uchaikin, J. Wang, B. Wilson, and
G. Rose, Quantum annealing with manufactured spins, Nature 473, 194 (2011).

[59] I. Čepaitė, A. Polkovnikov, A. J. Daley, and C. W. Duncan, Counterdiabatic optimised local driving (2022), arXiv:2203.01948 [quant-ph].

[60] J. Yao, L. Lin, and M. Bukov, Reinforcement learning for many-body ground-state preparation inspired by counterdiabatic driving, Phys. Rev. X 11, 031070 (2021).

[61] Y. Susa and H. Nishimori, Variational optimization of the quantum annealing schedule for the lechner-hauke-zoller scheme, Phys. Rev. A 103, 022619 (2021).

[62] B. M. Henson, D. K. Shin, K. F. Thomas, J. A. Ross, M. R. Hush, S. S. Hodgman, and A. G. Truscott, Approaching the adiabatic timescale with machine learning, Proceedings of the National Academy of Sciences 115, 13216 (2018).
I. LIMITATIONS OF THE VARIATIONAL COUNTERDIABATIC APPROACH

This section highlights some limitations of the traditional variational CD approach [1] (without frame rotations). We consider the case where the two Hamiltonians $\hat{H}_a$ and $\hat{H}_b$ generate the space of allowed physical operators [2, 3]. Without any further assumption on the form of $\hat{H}_a$ and $\hat{H}_b$, we consider the parametric Hamiltonian

$$\hat{H}_0(t) = A_0(\lambda)\hat{H}_a + B_0(\lambda)\hat{H}_b,$$

where $A_0(\lambda)$ and $B_0(\lambda)$ are arbitrary real schedule functions, describing the original UA driving. Within the space of allowed operators, the most general parametrization for the variational AGP reads

$$\hat{A} = \alpha_a(\lambda)\hat{H}_a + \alpha_b(\lambda)\hat{H}_b,$$

where $\alpha_a(\lambda)$ and $\alpha_b(\lambda)$ are variational parameters. The CD Hamiltonian is

$$\hat{H}_{CD}(t) = \hat{H}_0(\lambda) + \dot{\lambda}\hat{A} = A_{CD}(t)\hat{H}_a + B_{CD}(t)\hat{H}_b,$$

where $A_{CD}(t) = A_0(t) + \dot{\lambda}\alpha_a(t)$ and $B_{CD}(t) = B_0(t) + \dot{\lambda}\alpha_b(t)$ are the control fields of the CD protocol. Inserting Eq. (S.1) and Eq. (S.2) into Eq. (S.3) of the main text, we get the following expression for the action

$$S(\lambda) = \text{Tr} \left( \frac{\dot{\lambda}}{\hbar} \right) = \text{Tr} \left( (\partial_{\lambda} A_0\hat{H}_a + \partial_{\lambda} B_0\hat{H}_b - i[A_0\hat{H}_a + B_0\hat{H}_b, \alpha_a\hat{H}_a + \alpha_b\hat{H}_b]^2) \right)$$

$$= \text{Tr} \left( (\partial_{\lambda} \hat{H}_0)^2 \right) + \text{Tr} \left( [\hat{H}_a, \hat{H}_b]^2 \right) (A_0\alpha_b - B_0\alpha_a)^2.$$

(S.4)

Eq. (S.4) implies that, the trivial choice $\alpha_a(\lambda) = \alpha_b(\lambda) = 0$ is a global minimum of the action. Hence, the traditional CD variational approach, without frame rotations, returns the trivial result $A_{CD}(t) = A_0(t)$ and $B_{CD}(t) = B_0(t)$, which gives no performance improvement. However, as we showed in the main text, the RA method can overcome this performance limitation.

II. BOUNDARY CONDITIONS FOR THE ROTATED PROTOCOLS

In this section, we show that by choosing a ramp $\lambda(t)$ such that $\dot{\lambda}(0) = \dot{\lambda}(\tau) = 0$ we can enforce the time boundary condition, required by RA protocols.

We recall that for the RA driving to result in an alternative STA, we need the time boundary condition, given in Eq. (S.5) of the main text to hold for the initial and final times $t = 0, \tau$. Namely, we must require

$$|\langle \epsilon(\lambda)|e^{-\frac{i}{\hbar}\hat{Q}}|\psi(t)\rangle| = |\langle \epsilon(\lambda)|\psi(t)\rangle| \quad \text{for} \ t = 0, \tau$$

(S.5)

We now consider the specific case of $\lambda \to 0$ for $t = 0, \tau$. We scale the action by the non negative (time-dependent) factor $\lambda^2$. The scaled action $\bar{S}(\hat{A})$ is

$$\bar{S}(\hat{A}) = \lambda^2 S(\hat{A}) = \left( (\lambda \partial_{\lambda} \hat{H}_0 - \frac{i}{\hbar}[\hat{H}_0, \hat{A}]^2) \right)$$

$$= \left( (\lambda \partial_{\lambda} \hat{H}_0 - \frac{i}{\hbar}[\hat{H}_0, e^{\frac{i}{\hbar}\hat{Q}}(\hat{H}_0 + \hat{K})e^{-\frac{i}{\hbar}\hat{Q}}]^2) \right).$$

(S.6)

For $\lambda^2 > 0$, the optimal auxiliary potentials also minimize $\bar{S}(\hat{A})$. Assuming that $\partial_{\lambda} \hat{H}_0$ is bounded, we get

$$\lim_{\lambda \to 0} \bar{S}(\hat{A}) = \frac{1}{\hbar^2} \left( [\hat{H}_0, e^{\frac{i}{\hbar}\hat{Q}}(\hat{H}_0 + \hat{K})e^{-\frac{i}{\hbar}\hat{Q}}]^2 \right).$$

(S.7)
In particular, if \( \dot{\lambda} = 0 \) for \( t = 0, \tau \), by taking \( \hat{K} \) and \( \hat{Q} \) such that
\[
e^{-i\frac{\pi}{\hbar} \hat{Q}} = \hat{1}, \quad \text{and} \quad \hat{K} = \hat{0}
\]
we simultaneously minimize \( \tilde{\mathcal{S}}(\hat{A}) \) and satisfy Eq. (S.5). Hence, we can enforce the time boundary condition by choosing \( \lambda(t) \) such that \( \dot{\lambda}(0) = \dot{\lambda}(\tau) = 0 \).

### III. SEQUENTIAL LOCAL OPTIMIZATION OF THE ACTION

In the following, we describe the numerical algorithm we use to optimize the RA action and compute the RA driving protocols.

To simplify the description of the algorithm, we explicitly consider the case where the two Hamiltonians \( \hat{H}_a \) and \( \hat{H}_b \) generate the space of allowed physical operators [2, 3]. Without any further assumption on the form of \( \hat{H}_a \) and \( \hat{H}_b \), we consider the parametric Hamiltonian
\[
\hat{H}_0(t) = A_0(\lambda)\hat{H}_a + B_0(\lambda)\hat{H}_b,
\]
where \( A_0(\lambda) \) and \( B_0(\lambda) \) are arbitrary real schedule functions, describing the original UA driving. Within the space of allowed operators, we parameterize the auxiliary RA potentials as
\[
\hat{Q} = \gamma \hat{H}_a, \quad \hat{K} = \beta \hat{H}_b
\]
where \( \beta(t) \) and \( \gamma(t) \) are variational parameters. The corresponding AGP and RA driving Hamiltonian read
\[
\dot{\lambda} \hat{A} = e^{i\frac{\pi}{\hbar} \hat{Q}} (\hat{H}_0 + \hat{K}) e^{-i\frac{\pi}{\hbar} \hat{Q}} - \hat{H}_0 = (B_0 + \beta)e^{i\gamma_0 \hat{H}_a} \hat{H}_b e^{-i\gamma_0 \hat{H}_a} - B_0 \hat{H}_b = 0
\]
\[
\hat{H}_{RA} = \hat{H}_0 + \hat{K} + \hat{Q} = A_{RA}(t)\hat{H}_a + B_{RA}(t)\hat{H}_b
\]
where \( A_{RA}(t) = A_0(t) + \gamma(t) \) and \( B_{RA}(t) = B_0(t) + \beta(t) \) are the control fields of the RA protocol. The expression of the AGP leads to the following action
\[
\mathcal{S}(\hat{A}) = \text{Tr} \left( \hat{G}_{\lambda}^2 \right) = \text{Tr} \left( (\partial_\lambda A_0 \hat{H}_a + \partial_\lambda B_0 \hat{H}_b - i[A_0 \hat{H}_a + B_0 \hat{H}_b, (B_0 + \beta)e^{i\gamma_0 \hat{H}_a} \hat{H}_b e^{-i\gamma_0 \hat{H}_a} - B_0 \hat{H}_b])^2 \right).
\]

We observe that the trivial choice \( \beta(t) = \gamma(t) = 0 \) is generally not a global minimum of the action. Hence, to find the functions \( \gamma(t) \) and \( \beta(t) \), we must explicitly minimize Eq. (S.14).

Since \( A_{RA}(t) = A_0(t) + \gamma(t) \), we are specifically looking for differentiable functions and we are not necessarily interested in finding the global optimum of the action for each \( t \in [0, \tau] \). We, therefore, use a sequential local optimization algorithm to compute the functions in the interval \( t \in [0, \tau] \). We first consider the discrete time problem of finding \( \beta^{(m)} = \beta(t^{(m)}) \) and \( \gamma^{(m)} = \gamma(t^{(m)}) \), for \( t^{(m)} = \frac{m-1}{M} \tau \) and \( m = 1, \ldots, M+1 \). Here the integer \( M \) indicates the number of discrete points considered. Since we are ultimately interested in generating smooth differentiable functions, we can assume that \( \beta^{(m)} \) and \( \gamma^{(m)} \) are close to \( \beta^{(m+1)} \) and \( \gamma^{(m+1)} \) respectively. This observation suggests to use sequential local optimizations to solve the discrete problem. Starting from given initial values, eg. \( \gamma^{(0)} = 0, \beta^{(0)} = 0 \), we use the BFGS algorithm [4] to minimize the action at time \( 0 \), which returns the optimal values \( \beta^{(1)} \) and \( \gamma^{(1)} \). Then, we proceed iteratively, using \( \beta^{(m)} \) and \( \gamma^{(m)} \) as initial guess to find \( \beta^{(m+1)}, \gamma^{(m+1)} \). Finally, we interpolate the resulting \( M \) points to get the functions \( \beta(t) \) and \( \gamma(t) \), which we, then, use to obtain the new schedules \( A_{RA}(t) \) and \( B_{RA}(t) \).

The algorithm’s generalization to multiple variational parameters case is straightforward. The computation of RA protocols \( A_{RA}(t) \) and \( B_{RA}(t) \) does not require spectral information and can run on a classical CPUs. The algorithm’s bottleneck is the evaluation of the action, which can still be exponentially hard, as it involves the multiplication of large matrices. In particular, in a system of \( N \) spins, the matrix multiplication’s complexity scales exponentially with \( N \). However, as reported in the main text, for various physically relevant Hamiltonians, the complexity of computing RA action is only polynomial in the system size \( N \).
IV. ROTATED ANSATZ PROTOCOL FOR A TWO-LEVEL SYSTEM

This section provides additional details on the two-spin system studied in the main text. Refs. [1, 5] used the two-level nature of the system to compute an analytical expression for the adiabatic gauge potential. Here, we use the same problem to illustrate the RA approach. We use natural units and set \( \hbar = 1 \), we indicate with \( \dot{x}(t) \) the time derivative of a given function \( x(t) \).

The parametric Hamiltonian of the two-spin system considered in Eq. (22) of the main text is
\[
\hat{H}_0(\lambda) = h_0(\lambda)\hat{H}_a + J_0(\lambda)\hat{H}_b ,
\]
where
\[
\hat{H}_a = -\hat{\sigma}_1^z - \hat{\sigma}_2^z , \quad \hat{H}_b = \hat{\sigma}_1^x \hat{\sigma}_2^x + \hat{\sigma}_1^y \hat{\sigma}_2^y
\]  
(16)
generate the space of allowed physical operators.

We parametrize the auxiliary potentials as
\[
\hat{Q} = \gamma \hat{H}_a
\]  
(17)
\[
\hat{K} = \beta \hat{H}_b
\]  
(18)
where \( \beta(t) \) and \( \gamma(t) \) are variational parameters. The corresponding RA driving Hamiltonian reads
\[
\hat{H}_{\text{RA}} = \hat{H}_0 + \hat{K} + \hat{Q} = h_{\text{RA}}(t)\hat{H}_a + J_{\text{RA}}(t)\hat{H}_b .
\]  
(19)
where \( h_{\text{RA}}(t) = h_0(t) + \gamma(t) \) and \( J_{\text{RA}}(t) = J_0(t) + \beta(t) \) are the control fields of the RA protocol. To obtain the optimal values of \( \beta \) and \( \gamma \), we must minimize the RA action.

A. RA adiabatic potential and RA action

To simplify the computation of the RA Adiabatic gauge potential, we exploit the two-level nature of the system. Indeed, Eq. (S.15) describes a two-level system where \( \hat{H}(t) \) only couples the states \( |\uparrow\uparrow\rangle \) and \( |\downarrow\downarrow\rangle \). We, therefore, introduce the operators
\[
\hat{s}^x = \frac{1}{4}(\hat{\sigma}_1^x + \hat{\sigma}_2^x) , \quad \hat{s}^y = \frac{1}{4}(\hat{\sigma}_1^y - \hat{\sigma}_2^y) , \quad \hat{s}^z = \frac{1}{4}(\hat{\sigma}_1^z + \hat{\sigma}_2^z), \quad \hat{o} = \frac{1}{2}(\hat{\sigma}_1^x \hat{\sigma}_2^x + \hat{\sigma}_1^y \hat{\sigma}_2^y) + \hat{\sigma}_1^z \hat{\sigma}_2^z
\]  
(20)
The operators \( \hat{s}^x, \hat{s}^y \) and \( \hat{s}^z \) obey spin algebra, and \( \hat{o} \) is such that \( \hat{o} \hat{s}^\nu = \hat{s}^\nu \hat{o} = \hat{s}^\nu \) for \( \nu = x, y, z \). In particular, in the subspace generated by \( \{ |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle \} \), the operators \( \hat{s}^x, \hat{s}^y \) and \( \hat{s}^z \) are represented by spin-1/2 matrices, while \( \hat{o} \) is the identity matrix.

Using these operators, the two Hamiltonian terms read \( \hat{H}_a = -4\hat{s}^z \) and \( \hat{H}_b = 2\hat{s}^x + \hat{o} \). Inserting these expressions in the definition of the RA adiabatic gauge potential, we get
\[
\hat{\lambda}\hat{A} = e^{i\hat{Q}}(\hat{H}_0 + \hat{K})e^{-i\hat{Q}} - \hat{H}_0 = 2(\beta + J_0)e^{i4\gamma}\hat{s}^x e^{i4\gamma} \hat{s}^z + \beta \hat{o} - 2J_0\hat{s}^x
\]
\[
\beta \hat{o} + 2(\beta + J_0)(\cos(4\gamma)\hat{s}^x - \sin(4\gamma)\hat{s}^y) - 2J_0\hat{s}^x
\]
\[
= \beta \hat{o} + 4\alpha_{xx}\hat{s}^x + 4\alpha_{xy}\hat{s}^y
\]
\[
= \hat{o} + \alpha_{xx}\hat{s}^x + \alpha_{xy}\hat{s}^y + \alpha_{yy}\hat{s}^y + \alpha_{xy}(\hat{\sigma}_1^x \hat{\sigma}_2^x + \hat{\sigma}_1^y \hat{\sigma}_2^y)
\]  
(21)
where we introduced the AGP coefficients
\[
\alpha_{xx} = -\alpha_{yy} = \frac{1}{2}(\beta + J_0) \cos(4\gamma) - J_0 , \quad \alpha_{xy} = -\frac{1}{2}(\beta + J_0) \sin(4\gamma) .
\]  
(22)
Eq. (S.21) shows that the frame rotation introduces in the AGP a new term proportional to \( \hat{\sigma}_1^x \hat{\sigma}_2^x + \hat{\sigma}_1^y \hat{\sigma}_2^y \). We also compute the operator \( \hat{G} \), obtaining
\[
\hat{\lambda}\hat{G} = \hat{H}_0 + \frac{i}{\hbar} \left[ \hat{\lambda}\hat{A}, \hat{H}_0 \right]
\]
\[
= \hat{J}_0(2\hat{s}^x + \hat{o}) - 4\hat{h}_0\hat{s}^z + i [4\alpha_{xx}\hat{s}^x + 4\alpha_{xy}\hat{s}^y, 2J_0\hat{s}^x - 4\hat{h}_0\hat{s}^z]
\]
\[
= \hat{J}_0\hat{o} + 2(\hat{J}_0 + 8\hat{h}_0\alpha_{xy})\hat{s}^z - 16\hat{h}_0\alpha_{xx}\hat{s}^y + 4(2J_0\alpha_{xy} - \hat{h}_0)\hat{s}^z .
\]  
(23)
Finally, we insert Eq.(S.23) into the definition of the action, leading to
\[
S = \text{Tr} \left( \hat{G}^2 \right) = 6\hat{J}_0^2 + 2(\hat{J}_0 + 8\hat{h}_0\alpha_{xy})^2 + 27\hat{h}_0^2\alpha_{xx}^2 + 2^4(2J_0\alpha_{xy} - \hat{h}_0)^2
\]  
(24)
FIG. 1. Two-level system. (a) and (b) optimal value of the time-dependent parameter $\gamma(t)$ and $\beta(t)$ respectively. The data was obtained with the sequential local optimization algorithm’s results (blue dots) and the exact analytical solutions of Eq. (S.27) (orange line). (c) and (d) control fields for the UA and RA protocols. (e) ground state fidelity $F(t)$ for the UA, local CD and RA protocols, and rotated ground state fidelity $\tilde{F}(t)$ for the RA protocol. The inset of (e) shows the two-spin system. The data refers to protocols of total duration $\tau = 1$

B. RA action minimization and RA protocols

To minimize the action we use the equations $\frac{\partial S}{\partial \alpha_{xx}} = \frac{\partial S}{\partial \alpha_{xy}} = 0$. The quadratic function is minimized at the point

$$\begin{align*}
\alpha_{xx} &= 0 \\
\alpha_{xy} &= -\frac{1}{2} \varphi_0 \\
\cos(4\gamma) &= \frac{J_0 h_0 - J_0 h_0}{(J_0 + \beta)(4h_0^2 + J_0^2)} \\
\sin(4\gamma) &= \frac{\dot{J}_0 h_0 - J_0 \dot{h}_0}{J_0 + \beta}
\end{align*}$$

where

$$\varphi_0 = \frac{J_0 h_0 - J_0 h_0}{4h_0^2 + J_0^2}. $$

If we assume $J_0 + \beta > 0$ and $4\gamma \in (-\frac{\pi}{2}, \frac{\pi}{2})$, we can write

$$\beta = \sqrt{J_0^2 + \varphi_0^2} - J_0, \quad \gamma = \frac{1}{4} \arctan\left(\frac{\varphi_0}{J_0}\right).$$

By substitution Eq. (S.27), we get that the optimal RA

$$\frac{\partial S}{\partial \alpha_{xx}} = \frac{\partial S}{\partial \alpha_{xy}} = 0 \implies \dot{A} = -\frac{1}{2} \varphi_0 (\hat{\sigma}_1^x \hat{\sigma}_2^y + \hat{\sigma}_1^y \hat{\sigma}_2^x) + \left(\sqrt{J_0^2 + \varphi_0^2} - J_0\right) \dot{\omega}. $$

This result is compatible, up to an irrelevant term proportional to $\dot{\omega}$, with the well-known exact AGP $\dot{A}_{exact} = -2\varphi_0 \dot{s}_y$ for two level systems [1]. Hence, the RA Hamiltonian for the two spin problem implements an exact counterdiabatic protocol in the rotated frame, which tracks the instantaneous rotated ground state $|\tilde{\epsilon}_0(\lambda)\rangle$ with perfect unit $\tilde{F}(t) = \left|\langle \tilde{\epsilon}_0(\lambda) | \psi(t) \rangle\right|^2 = 1$.

The analytical expressions in Eq. (S.27) are a specific feature of two level systems. We Eq. (S.27) to benchmark the numerical sequential local optimization algorithm on the two spin problem. As in the main text we start from
the UA protocol \( J_0(t) = -1, h_0(t) = 5(1 - \lambda) \), of total duration \( \tau = 1 \). In Fig. 1a and Fig. 1b we compare the
M = 100 sequential local optimization data and the analytical expression for \( \beta(t) \) and \( \gamma(t) \). The results show that
the sequential local optimization points agree with the analytical expression. In Fig. 1c and Fig. 1d we show the
original UA control fields and the resulting RA control fields respectively. In Fig. 1e we compare the instantaneous
ground state’s fidelity of an UA and a RA protocol of duration \( \tau = 1 \). The diabatic transitions in the UA dynamics
monotonically decreases the fidelity \( F_{\text{UA}}(t) \) (black dashed line) up to the final value \( F_{\text{UA}}(\tau) = 0.66 \). The RA protocol’s
fidelity \( F_{\text{RA}}(t) \) (solid blue line), has a non monotonic time dependence. In particular, although for intermediate times
\( F_{\text{RA}}(t) \) is lower \( F_{\text{UA}}(t) \), the RA protocol enhances the final fidelity up to \( F_{\text{RA}}(\tau) = 1 \). Indeed, the implemented RA
driving, tracks the instantaneous rotated ground state \( |\tilde{\epsilon}_0(\lambda)\rangle \) with perfect fidelity \( F_{\text{RA}}(t) = 1 \) (light blue dot-dashed line).

V. ROTATED ANSATZ PROTOCOL FOR THE QUANTUM ISING CHAIN

This section provides additional details on the transnational invariant quantum spin chain system studied in the
main text (see Eq. (??)). Refs. [1] used the least action principle to construct approximate local CD drivings for the
spin chain. Here, we use the least action principle to design an approximate RA protocol. We use natural units and
set \( \hbar = 1 \), we indicate with \( \dot{x}(t) \) the time derivative of a given function \( x(t) \).

The parametric Hamiltonian of the spin chain considered in Eq. (??) of the main text is
\[
\hat{H}_{\text{spin-chain}} = J_0(\lambda)\hat{H}_a + h_0(\lambda)\hat{H}_b + b_0(\lambda)\hat{H}_c, \\
\] (S.29)

with
\[
\hat{H}_a = -\sum_{j=1}^{N} \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x, \quad \hat{H}_b = -\sum_{j=1}^{N} \hat{\sigma}_j^z, \quad \hat{H}_c = -\sum_{j=1}^{N} \hat{\sigma}_j^z. \\
\] (S.30)

We parametrize the auxiliary potentials as
\[
\hat{Q} = \gamma \hat{H}_a + \phi \hat{H}_c, \\
\hat{K} = \beta \hat{H}_b \\
\] (S.31)
(S.32)

where \( \beta(t), \gamma(t) \) and \( \phi(t) \) are variational parameters. The corresponding RA driving Hamiltonian reads
\[
\hat{H}_{\text{RA}} = \hat{H}_0 + \hat{K} + \hat{Q} = J_{\text{RA}}(t)\hat{H}_a + h_{\text{RA}}(t)\hat{H}_b + b_{\text{RA}}(t)\hat{H}_c \\
\] (S.33)

where \( J_{\text{RA}}(t) = J_0(t) + \gamma(t), b_{\text{RA}}(t) = b_0(t) + \phi(t) \) and \( h_{\text{RA}}(t) = h_0(t) + \beta(t) \) are the control fields of the RA protocol.
To obtain the optimal values of \( \beta, \phi \) and \( \gamma \), we must minimize the RA action.

A. RA adiabatic potential and RA action

We consider systems of \( N \geq 4 \) spins. Using Pauli’s matrices algebra to compute the RA adiabatic gauge potential, we get
\[
\dot{\lambda} \hat{A} = e^{i\hat{Q}}(\hat{H}_0 + \hat{K})e^{-i\hat{Q}} - \hat{H}_0 = -(J_0 + \beta) \sum_{j=1}^{N} e^{i\gamma(\hat{\sigma}_{j-1}^x \hat{\sigma}_j^y + \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z) + i\phi \hat{\sigma}_j^z} e^{-i\gamma(\hat{\sigma}_{j-1}^x \hat{\sigma}_j^y + \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z) - i\phi \hat{\sigma}_j^z} + b_0 \sum_{j=1}^{N} \hat{\sigma}_j^z \\
= \alpha_x \sum_{j=1}^{N} \hat{\sigma}_j^x + \alpha_y \sum_{j=1}^{N} \hat{\sigma}_j^y + \alpha_{xz} \sum_{j=1}^{N} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^z + \hat{\sigma}_j^z \hat{\sigma}_{j+1}^x) + \alpha_{yz} \sum_{j=1}^{N} (\hat{\sigma}_j^y \hat{\sigma}_{j+1}^z + \hat{\sigma}_j^z \hat{\sigma}_{j+1}^y) \label{eq:adia_potential}
\]

\[(S.34)\]

where we introduced the coefficients
\[
\alpha_x = h_0 - \frac{h_0 + \beta}{2} \cos 2\phi \cos(4\gamma + 1), \quad \alpha_y = \frac{h_0 + \beta}{2} \sin 2\phi \cos(4\gamma + 1), \quad \alpha_{xz} = \frac{h_0 + \beta}{2} \sin 2\phi \sin 4\gamma, \\
\alpha_{yz} = \frac{h_0 + \beta}{2} \cos 2\phi \sin(4\gamma - 1), \quad \alpha_{xyz} = -\frac{h_0 + \beta}{2} \cos 2\phi \cos(4\gamma - 1). \\
\] (S.35)
(S.36)
This RA adiabatic gauge potential explicitly contains terms proportional to \( \sum_j \hat{\sigma}_j^y \), \( \sum_j (\hat{\sigma}_j^z \hat{\sigma}_{j+1}^y + \ldots) \), and \( \sum_j \hat{\sigma}_{j-1}^z \hat{\sigma}_{j+1}^y \), which are not available in the original parametric Hamiltonian.

Finally, for \( N \geq 4 \), the action reads

\[
\frac{\mathcal{S}(\hat{A})}{N^2} = \frac{\text{Tr}(\hat{G}^2)}{N^2} = (4b_0^2 + 8J_0^2) \alpha_x^2 + (4b_0^2 + 8J_0^2) \alpha_y^2 + \alpha_x^2 (8b_0^2 + 8h_0^2 + 32J_0^2) + \alpha_y^2 (8b_0^2 + 32h_0^2 + 32J_0^2) + h_0^2 \alpha_x^2 + h_0^2 \alpha_y^2 + h_0^2 \alpha_{xx}^2 + h_0^2 \alpha_{yy}^2 + h_0^2 \alpha_{xy}^2 + (8b_0^2 + 16J_0^2) \alpha_x + 32b_0J_0 \alpha_{xx} + 16J_0^2 \alpha_{xx} + 32b_0J_0 \alpha_{xy} + 32b_0J_0 \alpha_{yx} + (8h_0J_0 - 8h_0J_0) \alpha_yz + (8h_0J_0 - 8h_0J_0) \alpha_yz + b_0^2 + h_0^2 + J_0^2 .
\]  

Eq. (S.37) reveals that, due to the Hamiltonian’s translation symmetry and locality, the ratio \( \frac{\mathcal{S}^{\text{RA}}(\hat{Q}, \hat{K})}{N^2} \) is not \( N \) dependent. This implies that the optimal RA coupling functions \( J^{\text{RA}}(t), b^{\text{RA}}(t) \) and \( h^{\text{RA}}(t) \) are also \( N \) independent, and that the RA protocol can be computed for arbitrarily large systems (with \( N \to \infty \)).

B. RA action minimization and RA protocols

The numerical minimization of the action Eq. (S.37) with respect to \( \beta, \gamma \) and \( \phi \) leads to the results presented in the main text.

VI. ROTATED ANSATZ PROTOCOL FOR QUANTUM ANNEALING

In this section, we compute the RA action for transverse field quantum annealing [6] for a system of \( N \frac{1}{2} \)-spins.

We can write the parametric Hamiltonian for transverse field quantum annealing as

\[
\hat{H}_0(\lambda) = \hat{H}_z(\lambda) - \sum_{j=1}^{N} h_{0,j}(\lambda) \hat{\sigma}_j^z
\]  

where \( h_{0,j}(\lambda) \) are local control fields of the parametric Hamiltonian and \( \hat{H}_z(\lambda) \) is a \( \lambda \)-depend operator, which is diagonal in the computational \( z \)-basis. Here, and in the following, we say that an operator \( \hat{D}_z \) is diagonal if \([\hat{D}_z, \hat{\sigma}_j^z] = 0\) for \( j = 1, 2, \ldots, N \).

We parameterize the auxiliary potentials as

\[
\hat{Q} = \hat{Q}_z
\]

\[
\hat{K}(\beta) = -\sum_{j=1}^{N} \beta_j \hat{\sigma}_j^z .
\]

where \( \beta_j(t) \), for \( j = 1, 2, \ldots, N \), are \( N \) real time-dependent variational parameters and \( \hat{Q}_z(t) \) is an operator diagonal in the computational \( z \)-basis. We make no further assumption on \( \hat{Q}_z(t) \). The corresponding RA driving Hamiltonian reads

\[
\hat{H}_{RA} = \hat{H}_0 + \hat{K} + \hat{Q} = \hat{H}_{RA,z}(t) - \sum_{j=1}^{N} h_{RA,j}(t) \hat{\sigma}_j^z
\]  

where \( h_{RA,j}(t) = h_{0,j}(t) + \beta_j(t) \) are local control fields of the RA protocol and \( \hat{H}_{RA,z}(t) = \hat{H}_z(t) + \hat{Q}_z(t) \) is the RA time dependent diagonal Hamiltonian. To obtain the optimal values of \( \beta_j \) and \( \gamma_j \), we must minimize the RA with respect to the optimal values of \( \beta_j(t) \) and \( \hat{H}_{RA,z}(t) \) we must minimize the RA action with respect to the variational parameters.
A. RA adiabatic potential and RA action

To compute the action, we rely on a convenient decomposition of diagonal operators. Let \( \hat{D}_z \) be an arbitrary diagonal operator. We define

\[
\hat{D}_z^{[j]} = \frac{1}{2} \hat{\sigma}_z^j \hat{D}_z + i \frac{1}{4} \left( \hat{\sigma}_z^j \hat{D}_z \hat{\sigma}_z^j - \hat{\sigma}_z^j \hat{D}_z \hat{\sigma}_z^j \right) \quad \text{for } j = 1, 2, \ldots, N \tag{S.42}
\]

\[
\hat{D}_z^{[-j]} = \frac{1}{2} \hat{D}_z + \frac{1}{2} \hat{\sigma}_z^j \hat{D}_z \hat{\sigma}_z^j \quad \text{for } j = 1, 2, \ldots, N . \tag{S.43}
\]

The operators \( \hat{D}_z^{[j]} \) and \( \hat{D}_z^{[-j]} \) then satisfy properties

(i) \( \hat{D}_z = \hat{D}_z^{[j]} \hat{\sigma}_z^j + \hat{D}_z^{[-j]} \) for \( j = 1, 2, \ldots, N \) \quad (desired decomposition)

(ii) \( [\hat{D}_z^{[\pm j]}, \hat{\sigma}_z^k] = 0 \) for \( k = 1, 2, \ldots, N \) \quad (\( \hat{D}_z^{[\pm j]} \) are diagonal operators)

(iii) \( [\hat{D}_z^{[\pm j]}, \hat{\sigma}_z^y] = \hat{D}_z^{[\pm j]}, \hat{\sigma}_z^y] = \hat{D}_z^{[\pm j]}, \hat{\sigma}_z^y] = 0 \) \quad (\( \hat{D}_z^{[\pm j]} \) do not involve \( \hat{\sigma}_z^y \))

The properties (i),(ii) and (iii) additionally imply the following identities

(iv) \( [\hat{D}_z, \hat{\sigma}_z^y] = i 2 \hat{D}_z^{[j]} \hat{\sigma}_z^y \) and \( [\hat{D}_z, \hat{\sigma}_z^y] = -i 2 \hat{D}_z^{[j]} \hat{\sigma}_z^y \).

(v) \( \left( \cos \hat{D}_z \right)^k = - \sin \left( \hat{D}_z \right)^k \) and \( \left( \sin \hat{D}_z \right)^k = \sin \left( \hat{D}_z \right)^k \cos \left( \hat{D}_z \right)^k \)

(vi) \( \hat{D}_z^{[jk]} = \hat{D}_z^{[kj]} \) and \( \hat{D}_z^{[j]k]} = 0 \).

The rationale behind the decomposition is to explicitly factor out the \( \hat{\sigma}_z^y \) from the diagonal operator. Using (i)-(iv), the RA for the gauge potential (defined in Eq. (??) of the main text) reads

\[
\hat{\lambda} \tilde{A} = e^{\pm \hat{Q}} \left( \hat{H}_0 + \hat{K} \right) e^{\mp \hat{Q}} - \hat{H}_0 = - \sum_{j=1}^{N} \left( h_{0,j} + \beta_j \right) e^{i \hat{Q}_z \hat{\sigma}_z^j} e^{-i \hat{Q}_z^y} + \sum_{j=1}^{N} h_{0,j} \hat{\sigma}_z^j \tag{S.44}
\]

\[
= - \sum_{j=1}^{N} \hat{\beta}_j \cos \left( 2 \hat{Q}_z^{[j]} \right) \hat{\sigma}_z^y + \sum_{j=1}^{N} \hat{\beta}_j \sin \left( 2 \hat{Q}_z^{[j]} \right) \hat{\sigma}_z^y + \sum_{j=1}^{N} h_{0,j} \hat{\sigma}_z^j , \tag{S.45}
\]

where we introduced the variables \( \hat{\beta}_j(t) = \beta_j(t) + h_{0,j}(t) \). Similarly, using (iv) and (v), the operator \( \hat{G} \) reads

\[
\hat{\lambda} \hat{G} = \hat{H}_0 + \frac{i}{\hbar} \left[ \hat{\lambda} \tilde{A}, \hat{H}_0 \right]
\]

\[
= \hat{H}_z - \sum_{j=1}^{N} \partial_{0,j} \hat{\sigma}_z^j + i \left[ \sum_{j=1}^{N} \partial_{0,j} \hat{\sigma}_z^j + \sum_{j=1}^{N} \hat{\beta}_j \left( \cos \left( 2 \hat{Q}_z^{[j]} \right) \hat{\sigma}_z^y - \sin \left( 2 \hat{Q}_z^{[j]} \right) \hat{\sigma}_z^y \right) \right] \tag{S.46}
\]
Finally, we use properties (i), (v) and (vi) to compute the RA action

\[ S = \text{Tr}(\hat{G}^2) \]

\[ = \text{Tr} \left( \hat{H}_z - 2 \sum_{j=1}^{N} \beta_j h_{0,j} \sin \left( 2\hat{Q}_z[j] \right) \sigma_j^x \right)^2 + \sum_{j=1}^{N} \left( h_{0,j} + 2\tilde{\beta}_j \sin \left( 2\hat{Q}_z[j] \right) \hat{H}_z[j] \right)^2 \]

\[ + 4 \sum_{j=1}^{N} \left( h_{0,j} \hat{H}_z[j] - \tilde{\beta}_j \cos \left( 2\hat{Q}_z[j] \right) \hat{H}_z[j] \right)^2 + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} \tilde{\beta}_j^2 h_{0,k}^2 \sin^2 \left( 2\hat{Q}_z[j][k] \right) \sin^2 \left( 2\hat{Q}_z[j][-k] \right) \]

\[ + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} \beta_j^2 h_{0,k}^2 \sin^2 \left( 2\hat{Q}_z[j][k] \right) \cos^2 \left( 2\hat{Q}_z[j][-k] \right) \]

\[ + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} h_{0,j} \beta_j h_{0,k} \tilde{\beta}_k \sin^2 \left( 2\hat{Q}_z[j][k] \right) \cos \left( 2\hat{Q}_z[j][-k] \right) \cos \left( 2\hat{Q}_z[k][-j] \right) \]

\[ = \text{Tr} \left( \hat{H}_z^2 - 4 \sum_{j=1}^{N} \beta_j h_{0,j} \hat{H}_z[j] \sin \left( 2\hat{Q}_z[j] \right) + 8 \sum_{j=1}^{N} \sum_{k=1}^{j-1} \beta_j h_{0,j} \beta_h h_{0,k} \sin^2 \left( 2\hat{Q}_z[j][k] \right) \cos \left( 2\hat{Q}_z[j][-k] \right) \cos \left( 2\hat{Q}_z[k][-j] \right) \right) \]

\[ + 4 \sum_{j=1}^{N} \tilde{\beta}_j^2 h_{0,j}^2 \sin^2 \left( 2\hat{Q}_z[j] \right) + 4 \sum_{j=1}^{N} h_{0,j} \tilde{\beta}_j \hat{H}_z[j] \sin \left( 2\hat{Q}_z[j] \right) + 4 \sum_{j=1}^{N} \tilde{\beta}_j^2 \left( \hat{H}_z[j] \right)^2 \sin^2 \left( 2\hat{Q}_z[j] \right) \]

\[ + 4 \sum_{j=1}^{N} \left( \hat{H}_z[j] \right)^2 \left( \tilde{\beta}_j^2 \cos^2 \left( 2\hat{Q}_z[j] \right) - 2h_{0,j} \beta_j \cos \left( 2\hat{Q}_z[j] \right) + h_{0,j}^2 \right) + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} \tilde{\beta}_j^2 h_{0,k}^2 \sin^2 \left( 2\hat{Q}_z[j][k] \right) \]

\[ + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} h_{0,j} \beta_j h_{0,k} \tilde{\beta}_k \sin^2 \left( 2\hat{Q}_z[j][k] \right) \cos \left( 2\hat{Q}_z[j][-k] \right) \cos \left( 2\hat{Q}_z[k][-j] \right) \]  

(S.47)

After using property (vi) and the identity \(1 - \cos X = 2 \sin^2 X\) to rearrange the terms in the sum, the final expression for the action reads

\[ S = \text{Tr} \left( \hat{H}_z^2 + \sum_{j=1}^{N} h_{0,j}^2 + 4 \sum_{j=1}^{N} \beta_j^2 \left( \hat{H}_z[j] \right)^2 + 16 \sum_{j=1}^{N} h_{0,j} \beta_j \left( \hat{H}_z[j] \right)^2 \sin^2 \left( \hat{Q}_z[j] \right) + 4 \sum_{j=1}^{N} h_{0,j}^2 (\beta_j + h_{0,j})^2 \sin^2 \left( 2\hat{Q}_z[j] \right) \right) \]

\[ - 4 \sum_{j=1}^{N} (\beta_j + h_{0,j}) \left( h_{0,j} \hat{H}_z[j] - h_{0,j} \hat{H}_z[j] \right) \sin \left( 2\hat{Q}_z[j] \right) + 4 \sum_{j=1}^{N} \sum_{k=1}^{N} (\beta_j + h_{0,j})^2 h_{0,k}^2 \sin^2 \left( 2\hat{Q}_z[j][k] \right) \]

\[ - \sum_{j=1}^{N} \sum_{k=1}^{N} h_{0,j} h_{0,k} (\beta_j + h_{0,j})(\beta_k + h_{0,k}) \sin^2 \left( 2\hat{Q}_z[j][k] \right) \cos \left( 2\hat{Q}_z[j][-k] \right) \cos \left( 2\hat{Q}_z[k][-j] \right) \]  

(S.48)

**B. RA action minimization and RA protocols**

Eq. (S.48) is the starting point to compute the RA action in various models. However, a numerical evaluation of the action involves computing traces of various \(2^N \times 2^N\) diagonal matrices. To appreciate the complexity of evaluating
the action we use the identity $e^{iX} = \cos X + i\sin X$ into Eq. (S.48), which leads to

$$S = \Re \text{Tr} \left( \dot{\hat{H}}_z^2 + \sum_{j=1}^{N} \hat{h}_0^2 \right) + 4 \sum_{j=1}^{N} \beta_j^2 \left( \dot{\hat{H}}_z^2 \right)^2 + 8 \sum_{j=1}^{N} h_{0,j} \beta_j \left( \dot{\hat{H}}_z \right)^2 \left( 1 - \exp \left( i2\dot{Q}_z \right) \right)$$

$$+ 2 \sum_{j=1}^{N} h_{0,j}(\beta_j + h_{0,j})^2 \left( 1 - \exp \left( i4\dot{Q}_z \right) \right) + 4 \sum_{j=1}^{N} (\beta_j + h_{0,j}) \left( h_{0,j}\dot{H}_z - h_{0,j}\dot{Q}_z \right) \exp \left( i2\dot{Q}_z \right)$$

$$+ 2 \sum_{j=1}^{N} \sum_{k=1}^{N} (\beta_j + h_{0,j})^2 h_{0,k}^2 \left( 1 - \exp \left( i4\dot{Q}_z \right) \right)$$

$$- \sum_{j=1}^{N} \sum_{j=1}^{N} h_{0,j} h_{0,k} (\beta_j + h_{0,j})(\beta_k + h_{0,k}) \sum_{s,u,v=\pm1} s \exp \left( i2 \left( 1 + s \right) \dot{Q}_z^{[j]k[u]} + u \dot{Q}_z^{[j]k[-v]} + v \dot{Q}_z^{[j]k[-u]} \right) \right),$$

(S.49)

where $\Re(x)$ indicates the real part of the complex number $x$.

There are at least two relevant cases for which the RA action (S.49) can be computed efficiently:

(a) quadratic Hamiltonians: $\hat{H}_z$ and $\dot{Q}_z$ contain only single-spin and two-spin terms.

(b) finite range Hamiltonians: both in $\hat{H}_z$ and in $\dot{Q}_z$ each spin interacts with at most $\ell$ other spins, with $\ell$ being an $N$ independent natural number.

The quadratic Hamiltonian realized on DWave annealers [7] belongs to (a), while the Hamiltonian associated with parity (or LHZ) architectures [8] belongs to (b). In the following, we will discuss the two cases separately.

1. Quadratic architectures

The relevant Hamiltonian terms for the QUBO annealing schedules are

$$\hat{H}_p = -\frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} J_{jk} \sigma_j^x \sigma_k^x - \sum_{j=1}^{N} J_{j0} \sigma_j^z, \quad \hat{H}_x = -\sum_{j=1}^{N} \sigma_j^z,$$

(S.50)

where the couplings $J_{jk}$, for $j, k = 0, 1, \ldots, N$ encode the optimization problem. Without loss of generality we assume that $J_{jk} = J_{kj}$ and $J_{j0} = 0$. The parametric QA Hamiltonian for the QUBO problem is

$$\hat{H}_{\text{QUBO}} = A_0(\lambda)\hat{H}_p + B_0(\lambda)\hat{H}_x,$$

(S.51)

where $A_0$ and $B_0$ are the QA control fields. We parametrize the auxiliary fields as $\dot{Q} = \gamma \hat{H}_p$ and $\dot{K} = \beta \hat{H}_x$. Then, the corresponding RA driving Hamiltonian reads

$$\hat{H}_{\text{RA}} = \hat{H}_0 + \dot{K} + \dot{Q} = A_{\text{RA}}(t)\hat{H}_p + B_{\text{RA}}(t)\hat{H}_x,$$

(S.52)

where $A_{\text{RA}}(t) = A_0(t) + \gamma(t)$ and $B_{\text{RA}}(t) = B_0(t) + \beta(t)$ are the control fields of the RA protocol.

To compute the RA action, we first substitute $\dot{Q}_z = \gamma(t)\hat{H}_p$, $h_{0,j}(t) = B_0(t)$, $\hat{H}_z(t) = A_0(t)\hat{H}_p$ and $\beta_j = \beta(t)$ in Eq. (S.49). The final expression of the RA action for the QUBO problem reads

$$S(\beta, \gamma) = 2^N \left( \frac{1}{2} A_0^2 \sum_{j=0}^{N} \sum_{k=0}^{N} J_{jk}^2 + N B_0^2 + 4 \beta^2 A_0^2 \sum_{j=0}^{N} \sum_{k=0}^{N} J_{jk}^2 + 8 \beta^2 B_0(\beta + B_0) \sum_{j=1}^{N} \sum_{k=0}^{N} J_{jk}^2 \left( 1 - f_j(\gamma) \right)$$

$$+ 16 \beta^2 B_0(\beta + B_0) \sum_{j=1}^{N} \sum_{k=0}^{N} \sum_{l=0}^{k-1} J_{jk} J_{jl} \tan(2\gamma J_{jk}) \tan(2\gamma J_{jl}) f_j(\gamma) + 2 B_0^2(\beta + B_0)^2 \sum_{j=1}^{N} \left( 1 - f_j(2\gamma) \right)$$

$$- 4(\beta + B_0)(B_0 A_0 - B_0 A_0) \sum_{j=0}^{N} \sum_{k=0}^{N} J_{jk} \tan(2J_{jk} \gamma) f_j(\gamma) + 4(\beta + B_0)^2 B_0^2 \sum_{j=1}^{N} \sum_{k=1}^{N} \sin^2(2J_{jk} \gamma)$$

$$+ 4(\beta + B_0)^2 B_0^2 \sum_{j=1}^{N} \sum_{k=1}^{N} \tan^2(2J_{jk} \gamma) \left( g_{+,jk}(\gamma) + g_{-,jk}(\gamma) \right) \right),$$

(S.53)
where we defined the functions \( f_j(\gamma) = \prod_{m=0}^{N} \cos(2J_{jm}\gamma) \) and \( g_{\pm,jk}(\gamma) = \prod_{m=0}^{N} \cos(2J_{jm}\gamma \pm 2J_{km}\gamma) \), and we used the following identity \( \frac{1}{2\pi} \text{Tr} \left( e^{i\sum_{m=1}^{N} \theta_m \sigma_m^z} \right) = \prod_{m=1}^{N} \cos(\theta_m) \).

Using Eq. (S.53), the computational complexity of evaluating the RA action is as \( \mathcal{O}(N^3) \). Thus, computing RA protocol numerically (with a sequential local optimization) will only result in a polynomial overhead in the number of qubits \( N \).

We benchmark the RA approach on a spin-glass problem with uniform random couplings \( J_{jk} \in [-1,1] \) (for \( j \neq k \)). We start from the UA protocol, shown in Fig. 2(a). We first analyze a single problem instance of \( N = 8 \) qubits. Figure 2(b) shows the optimal RA control fields obtained by minimizing the action in Eq. (S.53). In Fig. 2(c), we compare the instantaneous ground state’s fidelity of the UA, the local CD, and the RA protocols on the same problem instance. We find that the RA tracks the rotated ground states with higher fidelity than the UA and local CD drivings. Fig. 2(d) shows the relative improvement of the RA and local CD, averaged over an ensemble of 100 random problems with different qubits numbers \( N = 3, 4, 5, \ldots, 15 \). The data suggest that the features observed for \( N = 6 \) hold for larger systems. As observed in Ref. [9], the local CD driving shows no scaling advantage over the UA driving. However, the RA protocol provides a scaling advantage over the UA protocol.

2. Parity architectures

The relevant Hamiltonian terms for the LHZ annealing schedules are

\[
\hat{H}_p = -\sum_{k=1}^{N} J_k \hat{\sigma}_k^z, \quad \hat{H}_c = -\sum_{l} \hat{H}_{H,l} \quad \hat{H}_x = -\sum_{j=1}^{N} \hat{\sigma}_j^x, \quad (S.54)
\]
where the couplings $J_j$, for $j = 0, 1, \ldots, N$ encode the optimization problem. The parametric QA Hamiltonian for the LHZ driving is

$$\hat{H}_{\text{LHZ}} = A_0(\lambda)\hat{H}_p + B_0(\lambda)\hat{H}_x + C_0(t)\hat{H}_c,$$

(S.55)

Where $A_0, B_0$ and $C_0$ are the QA control fields. We parametrize the auxiliary fields as $\hat{Q} = \gamma\hat{H}_p + \phi\hat{H}_c$ and $\hat{K} = \beta\hat{H}_x$. Then, the corresponding RA driving Hamiltonian reads

$$\hat{H}_{\text{RA}} = \hat{H}_0 + \hat{K} + \hat{Q} = A_{\text{RA}}(t)\hat{H}_p + B_{\text{RA}}(t)\hat{H}_x + C_{\text{RA}}(t)\hat{H}_c,$$

(S.56)

where $A_{\text{RA}}(t) = A_0(t) + \gamma(t)$, $C_{\text{RA}}(t) = C_0(t) + \gamma(t)$ and $B_{\text{RA}}(t) = B_0(t) + \beta(t)$ are the control fields of the RA protocol.

To compute the RA action, we first substitute $\hat{Q}_z = \gamma(t)\hat{H}_p + \phi\hat{H}_c$, $h_{0,j}(t) = B_0(t)$, $H_z(t) = A_0(t)\hat{H}_p + C_0(t)\hat{H}_c$ and $\beta_j(t) = \beta(t)$ in Eq. (S.49). The final expression of the RA action for the QUBO problem reads

$$S(\beta, \gamma, \phi) = 2^N \left( \tilde{A}_0^2 \sum_{\mu=1}^{N} J_{\mu}^2 + N \tilde{B}_0^2 + L\tilde{C}_0^2 + 4 \left( B_0^2 + (\beta + B_0)^2 \right) \sum_{\mu=1}^{N} \left( A_0^2 J_{\mu}^2 + C_0^2 \left( L[\mu] - L[\mu](L[\mu] - 1) \tan^2(2\phi) \right) \right) \right) \cos^{L[\mu]}(2\phi)$$

$$- 8B_0(\beta + B_0) \sum_{\mu=1}^{N} \cos(2J_{\mu}\gamma) \left( A_0^2 J_{\mu}^2 + C_0^2 \left( L[\mu] - L[\mu](L[\mu] - 1) \tan^2(2\phi) \right) \right) \cos^{L[\mu]}(2\phi)$$

$$+ 16A_0B_0C_0(\beta + B_0) \sum_{\mu=1}^{N} J_{\mu}L[\mu] \sin(2J_{\mu}\gamma) \tan(2\phi) \cos^{L[\mu]}(2\phi)$$

$$+ 2(\beta + B_0)^2 B_0^2 \sum_{\mu=1}^{N} \left( 1 - \cos(4J_{\mu}\gamma) \cos^{L[\mu]}(4\phi) \right)$$

$$- 4 \left( B_0\tilde{C}_0 - \tilde{B}_0C_0 \right) (\beta + B_0) \sum_{\mu=1}^{N} L[\mu] \cos(2J_{\mu}\gamma) \tan(2\phi) \cos^{L[\mu]}(2\phi)$$

$$- 4 \left( B_0\tilde{A}_0 - \tilde{B}_0A_0 \right) (\beta + B_0) \sum_{\mu=1}^{N} J_{\mu} \sin(2J_{\mu}\gamma) \cos^{L[\mu]}(2\phi) + (\beta + B_0)^2 B_0^2 \sum_{\langle\mu, \nu\rangle} \left( 1 - \cos^{L[\mu][\nu]}(4\phi) \right)$$

$$+ 2B_0^2(\beta + B_0)^2 \sum_{\langle\mu, \nu\rangle} \cos(2J_{\mu}\gamma + 2J_{\nu}\gamma) \left( 1 - \cos^{L[\mu][\nu]}(4\phi) \right) \cos^{L[\mu][\nu]}(4\phi) \right)$$

(S.57)

Here $L$ is the total number of constraint terms, $L[\mu]$ is the number of constraint terms involving site $\mu$, $L[\mu][\nu]$ is the number of constraint terms involving both sites $\mu$ and $\nu$, and $L[\mu][\nu]$ is the number of constraint terms involving site $\mu$ but not site $\nu$. Consequently, natural numbers $L$, $L[\mu]$ and $L[\mu][\nu]$ depend only on the architecture, not on the problem instance. The notation $\sum_{\langle\mu, \nu\rangle}$ indicates a sum over all pairs of interacting spins in the LHZ architecture. To obtain Eq. (S.57), we used the identity

$$\frac{1}{2^N} \text{Tr} \left( e^{i\sum_{t=1}^{T} \theta_t H_{\mu}^{[\mu]}(t)} \right) = \prod_{l \in \tilde{C}_\mu} \cos \theta_l ,$$

(S.58)

where $\tilde{C}_\mu$ is the set of constrains involving the $\tilde{\sigma}_z^{\mu}$.

Using Eq. (S.57), the computational complexity of computing the RA action for the LHZ architecture is $O(N)$. Thus, computing RA protocol numerically (with a sequential local optimization) will only result in a polynomial overhead in the number of qubits $N$.

The numerical minimization of the action given in Eq. (S.57), for a spin-glass problem leads to the results presented in the main text.

[1] D. Sels and A. Polkovnikov, Minimizing irreversible losses in quantum systems by local counterdiabatic driving, Proceedings of the National Academy of Sciences 114, E3009 (2017).
[2] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, A quantum adiabatic evolution algorithm applied to random instances of an np-complete problem, Science 292, 472 (2001).
[3] T. Albash and D. A. Lidar, Adiabatic quantum computation, Rev. Mod. Phys. 90, 015002 (2018).
[4] J. Nocedal and S. J. Wright, Numerical Optimization, 2nd ed. (Springer, New York, NY, USA, 2006).
[5] F. Petiziol, B. Dive, F. Mintert, and S. Wimberger, Fast adiabatic evolution by oscillating initial hamiltonians, Phys. Rev. A 98, 043436 (2018).
[6] P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, and W. D. Oliver, Perspectives of quantum annealing: methods and implementations, Reports on Progress in Physics 83, 054401 (2020).
[7] M. W. Johnson, M. H. S. Amin, S. Gildert, T. Lanting, F. Hamze, N. Dickson, R. Harris, A. J. Berkley, J. Johansson, P. Bunyk, E. M. Chapple, C. Enderud, J. P. Hilton, K. Karimi, E. Ladizinsky, N. Ladizinsky, T. Oh, I. Perminov, C. Rich, M. C. Thom, E. Tolkacheva, C. J. S. Truncik, S. Uchaikin, J. Wang, B. Wilson, and G. Rose, Quantum annealing with manufactured spins, Nature 473, 194 (2011).
[8] W. Lechner, P. Hauke, and P. Zoller, A quantum annealing architecture with all-to-all connectivity from local interactions, Sci. Adv. 1, 1500838 (2015).
[9] A. Hartmann, G. B. Mbeng, and W. Lechner, Polynomial scaling enhancement in the ground-state preparation of ising spin models via counterdiabatic driving, Phys. Rev. A 105, 022614 (2022).