Advances in simulations of generic black-hole binaries

Manuela Campanelli¹, Carlos O Lousto¹, Bruno C Mundim¹, Hiroyuki Nakano¹, Yosef Zlochower¹ and Hans-Peter Bischof²

¹ Center for Computational Relativity and Gravitation, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623, USA
² Center for Computational Relativity and Gravitation, Department of Computer Science, Rochester Institute of Technology, Rochester, NY 14623, USA

E-mail: manuela@astro.rit.edu, lousto@astro.rit.edu, bcmisma@astro.rit.edu, nakano@astro.rit.edu, yosef@astro.rit.edu and hpb@cs.rit.edu

Received 30 November 2009, in final form 25 January 2010
Published 6 April 2010
Online at stacks.iop.org/CQG/27/084034

Abstract
We review some of the recent dramatic developments in the fully nonlinear simulation of generic, highly-precessing, black-hole binaries, and introduce a new approach for generating hybrid post-Newtonian/numerical waveforms for these challenging systems.

PACS numbers: 04.25.Dm, 04.25.Nx, 04.30.Db, 04.70.Bw

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The field of numerical relativity (NR) has progressed at a remarkable pace since the breakthroughs of 2005 [1–3] with the first successful fully nonlinear dynamical numerical simulation of the inspiral, merger and ringdown of an orbiting black-hole binary (BHB) system. In particular, the ‘moving-punctures’ approach, developed independently by the NR groups at NASA/GSFC and at RIT, has now become the most widely used method in the field and was successfully applied to evolve generic BHBs. This approach regularizes a singular term in spacetime metric and allows the black holes (BHs) to move across the computational domain. Previous methods used special coordinate conditions that kept the black holes fixed in space, which introduced severe coordinate distortions that caused orbiting-black-hole-binary simulations to crash. Recently, the generalized harmonic approach method, first developed by Pretorius [1], has also been successfully applied to accurately evolve generic BHBs for tens of orbits with the use of pseudospectral codes [4, 5].

Since then, BHB physics has rapidly matured into a critical tool for gravitational wave (GW) data analysis and astrophysics. Recent developments include studies of the orbital dynamics of spinning BHBs [6–12], calculations of recoil velocities from the merger of unequal mass BHBs [13–15] and the surprising discovery that very large recoils can be acquired by the remnant of the merger of two spinning BHs [9, 16–31], empirical models...
relating the final mass and spin of the remnant to the spins of the individual BHs [32–39] and comparisons of waveforms and orbital dynamics of BHB inspirals with post-Newtonian (PN) predictions [40–47].

One of the important applications of NR is the generation of waveform to assist GW astronomers in their search and analysis of GWs from the data collected by ground-based interferometers, such as LIGO [48] and VIRGO [49], and future space-based missions, such as LISA [50]. BHBs are particularly promising sources, with the final merger event producing a strong burst of GWs at a luminosity of \( L_{GW} \sim 10^{22} L_\odot \), greater than the combined luminosity of all stars in the observable universe. The central goal of the field has been to develop the theoretical techniques, and perform the numerical simulations, needed to explore the highly dynamical regions and thus generate GW signals from a representative sample of the full BHB parameter space. Accurate waveforms are important to extract physical information about the binary system, such as the masses of the components, BH spins and orientation. With advanced LIGO scheduled to start taking data in 2014–2015, there is a great urgency to develop these techniques in short order. To achieve these goals, the numerical relativity and data analysis communities formed a large collaboration, known as NINJA, to generate, analyze and develop matched filtering techniques for generic BHB waveforms. A wide range of currently available gravitational-waveform signals were injected into a simulated data set, designed to mimic the response of the initial LIGO and Virgo gravitational-wave detectors, and the efficiency of current search methods in detecting and measuring their parameters was successfully tested [51, 52]. The next step will be a more detailed study of the sensitivity of current search pipelines to BHB waveforms in real data.

In order to create effective templates for GW data analysis, we need to cover the 7D parameter space of possible BHB configurations, including arbitrary mass ratios (1d) \( q = m_1/m_2 \) and arbitrary orientation and magnitudes of the individual BH spins (6d), in an efficient way. There are two important challenges here. The first challenge is to adapt the numerical techniques developed for similar-mass, low-spin BHBs to tackle BHBs with extreme mass ratios, i.e. \( q < 1/10 \) (see [19, 53, 54]) and, independently, the highly spinning regime. In the latter regime, the binaries will precess strongly during the final stages of inspiral and merger, leading to large recoils and modulations in the waveform. These two regions are numerically highly demanding due to the high resolution required for accurate simulations. A second challenge is to efficiently generate the waveforms numerically. Ideally one would like to have a bank of templates with millions of waveforms, but the computational expense of each individual simulation makes this unrealistic.

At RIT, we have been particularly interested in studying spinning BHBs and the effects of spin on the orbital dynamics, waveforms and remnant BHs. In 2006 the RIT group began a series of analyses of spinning BHBs, with the goal of evolving a truly generic binary. Our studies began with the ‘orbital hangup’ configurations [6], where the spins are aligned or counter-aligned with the orbital angular momentum, and display dramatic differences in the orbital dynamics, see figure 1. In this study we were also able to provide strong evidence that the merger of two BHs will produce a submaximal remnant (i.e. cosmic censorship is obeyed). We then analyzed spin–orbit effects [7] and found that they were too weak near merger to force a binary to remain in a corotational state. Afterward, we analyzed spin precession and spin flips [8]. With this experience, we were able to begin evolving ‘generic’ binaries, that is, binaries with unequal and unaligned spins, and mass ratios differing from 1 : 1 [17].

Remarkably, we found for a generic binary that the gravitational recoil out of the orbital plane,

\[^3\] This luminosity estimate is independent of the binary mass and takes into account that 3–10% of the total mass \( M \) of the binary is radiated over a time interval of \( \sim 100 M \) [39].
which is a function of the in-plane spin, was potentially much larger than any in-plane recoil. In fact, the measured recoil for our ‘generic’ configuration was actually as large as the largest predicted in-plane recoil (which assumed maximal spins perpendicular to the orbital plane) [16, 30] (see figure 2 for a plot of the radiated power per unit solid angle for a ‘generic’ BHB). Based on these results, we were able to predict a recoil of thousands of km s$^{-1}$ for equal-mass, equal and anti-aligned spins (with spins entirely in the orbital plane). Based on our suggested configuration, the authors of [21] evolved a binary with a recoil of 2500 km s$^{-1}$. However, our prediction indicated that the recoil can vary sinusoidally with the angle that the spins make with respect to the initial linear momentum of each hole. After completing a study of these superkick configurations with various spin angles, we were able to show that the maximum recoil was, in fact, much closer to 4000 km s$^{-1}$. Later on, we evolved a set of challenging superkick configurations, with spins $S_i / m_H^2 = 0.92$, where $m_H$ is the horizon mass, and found a recoil of 3300 km s$^{-1}$ [55].

The cost of running a numerical simulation for many orbits, and in particular the cost of running a simulation with high spins and mass ratios that differ significantly from 1 : 1, means that we need to use hybrid analytic/numerical waveforms to model the full inspiral waveform. Combining post-Newtonian [56] (or effective-one-body [57]) waveforms and full numerical waveforms seems to be an ideal solution to this problem, but the modeling of even relatively distant (from an NR point of view) BHBs using PN is still an unsolved problem because the PN equations of motion are only known up to 3.5PN order, which as we show in section 2 is not accurate enough to evolve close, highly precessing, BHBs.

2. Inspiral and merger of generic black-hole binaries

In [58], we compared the numerical relativity (NR) and post-Newtonian (PN) waveforms of a generic BHB, i.e. a binary with unequal masses and unequal, non-aligned, precessing spins.
Comparisons of numerical simulations with post-Newtonian ones have several benefits aside from the theoretical verification of PN. From a practical point of view, one can directly propose a phenomenological description and thus make predictions in regions of the parameter space still not explored by numerical simulations. From the theoretical point of view, an important application is to have a calibration of the post-Newtonian error in the last stages of the binary merger.

To derive the PN gravitational waveforms, we start from the calculation for the orbital motion of binaries in the post-Newtonian approach. Here we use the ADM-TT gauge, which is the closest to our quasi-isotropic numerical initial data coordinates. We use the PN equations of motion (EOM) based on [59–61]. The Hamiltonian is given in [59], with the additional terms, i.e. the next-to-leading-order gravitational spin–orbit and spin–spin couplings provided by [60, 61], and the radiation-reaction force given in [59]. The Hamiltonian we use here is given by

$$ H = H_{O,\text{Newt}} + H_{O,1\text{PN}} + H_{O,2\text{PN}} + H_{O,3\text{PN}} + H_{SO,1.5\text{PN}} + H_{SO,2.5\text{PN}} + H_{SS,2\text{PN}} + H_{S_1S_2,3\text{PN}}, $$

(1)

where the subscripts O, SO and SS denote the pure orbital (non-spinning) part, spin–orbit coupling and spin–spin coupling, respectively, and Newt, 1PN, 1.5PN, etc refer to the perturbative order in the post-Newtonian approach. The $H_{S_1S_2,3\text{PN}}$ component of the Hamiltonian was recently derived in [62]. We should note that Porto and Rothstein also derived higher order spin–spin interactions using effective field theory techniques [63–66]. We obtain the conservative part of the orbital and spin EOMs from this Hamiltonian using the standard techniques of the Hamiltonian formulation. For the dissipative part, we use the non-spinning radiation reaction results up to 3.5PN, as well as the leading spin–orbit and spin–spin
coupling to the radiation reaction [59]. Although, not used here, higher order corrections to the spin-dependent radiation reaction terms were derived in [67–70] and can be applied to our method to improve the prediction for the BH trajectories (and hence the waveform). This PN evolution is used both to produce very low eccentricity orbital parameters at \( r \approx 11 M \) (the initial orbital separation for the NR simulations) from an initial orbital separation of 50 \( M \), and to evolve the orbit from \( r \approx 11 M \). We use these same parameters at \( r \approx 11 M \) to generate the initial data for our NR simulations. The initial binary configuration at \( r = 50 M \) had the mass ratio \( q = m_1/m_2 = 0.8 \), \( \tilde{S}_1/m_1^2 = (-0.2, -0.14, 0.32) \) and \( \tilde{S}_2/m_2^2 = (-0.09, 0.48, 0.35) \).

We then construct a hybrid PN waveform from the orbital motion by using the following procedure. First we use the 1PN accurate waveforms derived by Wagoner and Will [71] (WW waveforms) for a generic orbit. By using these waveforms, we can introduce effects due to the black-hole spins, including the precession of the orbital plane. On the other hand, Blanchet et al [72] recently obtained the 3PN waveforms (B waveforms) for non-spinning circular black-hole spins, including the precession of the orbital plane. On the other hand, Blanchet et al [72] recently obtained the 3PN waveforms (B waveforms) for a generic orbit. By using these waveforms, we can introduce effects due to the binary configuration, i.e. an actual location of the PN particle, we do not have any time shift or phase modification other than this retardation of the signal. Note that other methods, which are not based on the particle locations, have freedom in choosing a phase factor.

To compare PN and numerical waveforms, we need to determine the time translation \( \delta t \) between the numerical time and the corresponding point on the PN trajectory. That is to say, the time it takes for the signal to reach the extraction sphere (\( r \approx 11 M \)).

For the NR simulations we calculate the Weyl scalar \( \psi_4 \) and then convert the \((\ell, m)\) modes of \( \psi_4 \) into \((\ell, m)\) modes of \( h = h_+ - i h_\times \).

To compare PN and numerical waveforms, we need to determine the time translation \( \delta t \) between the numerical time and the corresponding point on the PN trajectory. That is to say, the time it takes for the signal to reach the extraction sphere (\( r = 100 M \) in our numerical simulation). We determine this by finding the time translation near \( \delta t = 100 M \) that maximizes the agreement of the early time waveforms in the \((\ell = 2, m = \pm 2)\), \((\ell = 2, m = \pm 1)\) and \((\ell = 3, m = \pm 3)\) simultaneously. We find \( \delta t \sim 112 M \) in good agreement with the expectation for our observer at \( r = 100 M \). Since our PN waveforms are given uniquely by a binary configuration, i.e. an actual location of the PN particle, we do not have any time shift or phase modification other than this retardation of the signal. Note that other methods, which are not based on the particle locations, have freedom in choosing a phase factor.

To quantitatively compare the modes of the PN waveforms with the numerical waveforms, we define the overlap or matching criterion, for the real and imaginary parts of each mode as

\[
M_{\ell m}^{3/3} = \frac{\langle h_{\ell m}^{\text{Num},3/3}, h_{\ell m}^{\text{PN},3/3} \rangle}{\sqrt{\langle h_{\ell m}^{\text{Num},3/3}, h_{\ell m}^{\text{Num},3/3} \rangle \langle h_{\ell m}^{\text{PN},3/3}, h_{\ell m}^{\text{PN},3/3} \rangle}},
\]

where \( h_{\ell m}^{3/3} \) are defined by the real and imaginary parts of the waveform mode \( h_{\ell m} \), respectively, and the inner product is calculated by \( \langle f, g \rangle = \int_0^T f(t)g(t) \, dt \). Hence, \( M_{\ell m}^{3/3} = 1 \) indicates that the given PN and numerical mode agree. We analyzed the long-term generic waveform produced by the merger of unequal mass, unequal spins, precessing black holes, and found a good initial agreement of waveforms for the first six cycles, with overlaps of over 98% for the \((\ell = 2, m = \pm 2)\) modes, over 90% for the \((\ell = 2, m = \pm 1)\) modes and over 90% for the \((\ell = 3, m = \pm 3)\) modes. The agreement degrades as we approach the more dynamical region of the late merger and plunge.

While our approach appears promising, there are some remaining issues. The PN gravitational waveforms used here do not include direct spin effects (spin contribution to
the waveform arises only through its effect on the orbital motion). Recently, direct spin effects on the waveform were analyzed in [70].

In figure 3 we show the \((\ell = 3, m = 3)\) mode of \(\psi_4\). A comparison of the PN and NR waveforms shows that there are significant errors in the 2.5PN approximate waveform that are significantly reduced by going to 3.5PN. However, it appears that still higher order corrections are needed in order to accurately model the waveform using PN at an orbital radius of \(r = 11M\). In figure 4 we show the orbital separation versus time. Here, as well, higher order PN correction are important.

2.1. Hybrid waveforms

To obtain a continuous and differentiable hybrid PN/NR waveform, we use a smoothing function for transition from the purely PN to purely NR parts of the waveform of the form

\[ h = (1 - F(x))h^{PN} + F(x)h^{Num}, \]  

(3)

where, for example, we can use a simple polynomial

\[ F(x) = x^3(6x^2 - 15x + 10). \]  

(4)

This guarantees the \(C^2\) behavior at \(F(x) = 0\) and 1. In [73] the authors chose \(F(x) = x\), which creates a discontinuity in the derivatives of the waveforms, especially \(\psi_4\), and also an amplitude scaling factor to correct the amplitudes. Note that here we do not have any free parameters (we allow the time translation, here \(\delta t \sim 112M\), to vary by \(\sim 5\%)\) about the retardation time (\(T_{ret} \sim 109M\)) of the observer location).

Figure 5 shows the hybrid waveform generated by the NR and PN waveforms for the binary discussed in the above section. Here, we use a half wavelength for the smoothing interval which starts at \(t = 226.78875M\), and the time translation \(\delta t = 112.64625M\) is considered.
Figure 4. The orbital separation versus time for the ‘generic’ binary configuration using the full numerical trajectories, as well as the trajectories derived from 2.5PN and 3.5PN EOMs. Note the much better agreement of the 3.5PN trajectories, and that 3.5PN captures the eccentricity of this configuration much better than 2.5PN, indicating that higher order PN terms are important to orbital dynamics.

Figure 5. The real part of the $\ell = 2, m = 2$ mode of the hybrid waveform. This is created by matching the NR waveform to the waveform derived from 3.5PN EOMs.

3. Discussion

The remarkable progress in both analytic and fully nonlinear numerical simulations of BHBs has made it possible to accurately model the inspiral waveform for a generic black-hole binary
by combining both post-Newtonian waveforms from large separations and smoothly attaching this waveform to the corresponding fully nonlinear waveform produced by the binary during the late-inspiral. We provide an example of one such hybrid waveform. Our waveform is available for download from http://ccrg.rit.edu/downloads/waveforms.

We found that 3.5PN produces a markedly better predicted waveform than 2.5PN, but there were still significant errors in the 3.5PN waveform for separations $r < 11M$. However, numerical simulations can start with larger separations (e.g. the 16-orbit simulation described in [4]) and there is significant progress in computing higher order PN corrections. Hence, we expect that highly accurate hybrid waveforms for generic binaries will soon be feasible.

Acknowledgments

We gratefully acknowledge NSF for financial support from grants PHY-0722315, PHY-0655303, PHY-0714388, PHY-0722703, DMS-0820923, and PHY-0929114; and NASA for financial support from grants NASA 07-ATFP07-0158 and HST-AR-11763. Computational resources were provided by Ranger cluster at TACC (Teragrid allocations TG-PHY080040N and TG-PHY060027N) and by NewHorizons at RIT.

References

[1] Pretorius F 2005 Evolution of binary black hole spacetimes Phys. Rev. Lett. 95 121101
[2] Campanelli M, Lousto C O, Marronetti P and Zlochower Y 2006 Accurate evolutions of orbiting black-hole binaries without excision Phys. Rev. Lett. 96 111101
[3] Baker J G, Centrella J, Choi D I, Koppitz M and van Meter J 2006 Gravitational wave extraction from an inspiraling configuration of merging black holes Phys. Rev. Lett. 96 111102
[4] Scheel M A et al 2009 High-accuracy waveforms for binary black hole inspiral, merger, and ringdown Phys. Rev. D 79 024003
[5] Szilagyi B, Lindblom L and Scheel M A 2009 Simulations of binary black hole mergers using spectral methods Phys. Rev. D 80 124010
[6] Campanelli M, Lousto C O and Zlochower Y 2006 Spinning-black-hole binaries: the orbital hang up Phys. Rev. D 74 044015
[7] Campanelli M, Lousto C O and Zlochower Y 2006 Spin–orbit interactions in black-hole binaries Phys. Rev. D 74 084023
[8] Campanelli M, Lousto C O, Zlochower Y, Krishnan B and Merritt D 2007 Spin flips and precession in black-hole-binary mergers Phys. Rev. D 75 064030
[9] Herrmann F, Hinder I, Shoemaker D M, Laguna P and Matzner R A 2007 Binary black holes: spin dynamics and gravitational recoil Phys. Rev. D 76 084032
[10] Marronetti P et al 2007 Binary black holes on a budget: simulations using workstations Class. Quantum Grav. 24 S43–58
[11] Marronetti P, Tichy W, Brugmann B, Gonzalez J and Sperhake U 2008 High-spin binary black hole mergers Phys. Rev. D 77 064010
[12] Berti E et al 2007 Inspiral, merger and ringdown of unequal mass black hole binaries: a multipolar analysis Phys. Rev. D 76 064034
[13] Herrmann F, Shoemaker D and Laguna P 2006 Unequal-mass binary black hole inspirals AIP Conf. Proc. 873 89–93
[14] Baker J G et al 2006 Getting a kick out of numerical relativity Astrophys. J. 653 L93–6
[15] González J A, Sperhake U, Brugmann B, Hannam M and Husa S 2007 Total recoil: the maximum kick from nonspinning black-hole binary inspiral Phys. Rev. Lett. 98 091101
[16] Herrmann F, Hinder I, Shoemaker D, Laguna P and Matzner R A 2007 Gravitational recoil from spinning binary black hole mergers Astrophys. J. 661 430–6
[17] Campanelli M, Lousto C O, Zlochower Y and Merritt D 2007 Large merger recoils and spin flips from generic black-hole binaries Astrophys. J. 659 L5–8
[18] Campanelli M, Lousto C O, Zlochower Y and Merritt D 2007 Maximum gravitational recoil Phys. Rev. Lett. 98 231102
[19] Lousto C O and Zlochower Y 2009 Modeling gravitational recoil from precessing highly spinning unequal-mass black-hole binaries Phys. Rev. D 79 064018
[20] Pollney D et al 2007 Recoil velocities from equal-mass binary black-hole mergers: a systematic investigation of spin–orbit aligned configurations Phys. Rev. D 76 124002
[21] González J A, Hannam M D, Sperhake U, Brugmann B and Husa S 2007 Supermassive kicks for spinning black holes Phys. Rev. Lett. 98 231101
[22] Brugmann B, González J A, Hannam M, Husa S and Sperhake U 2008 Exploring black hole superkicks Phys. Rev. D 77 124047
[23] Choi Dae Il, Lousto C O and Zlochower Y 2009 Modeling kicks from the merger of non-precessing black-hole binaries Gen. Rel. Grav. 41 525–39
[24] Boyle L, Kesden M and Nissanke S 2008 Binary black hole merger: symmetry and the spin expansion Phys. Rev. Lett. 100 151101
[25] Boyle Latham and Kesden Michael 2008 The spin expansion for binary black hole merger: new predictions and future directions Phys. Rev. D 78 024017
[26] Buenanano A, Kidder L E and Lehner L 2008 Estimating the final spin of a binary black hole coalescence Phys. Rev. D 77 064004
[27] Gesen M 2008 Can binary mergers produce maximally spinning black holes? Phys. Rev. D 78 084030
[28] Barausse E and Rezzolla L 2009 Predicting the direction of the final spin from the coalescence of two black holes Astrophys. J. Lett. 704 L40–4
[29] Rezzolla L 2009 Modelling the final state from binary black-hole coalescences Class. Quantum Grav. 26 094023
[30] Lousto C O, Campanelli M and Zlochower Y 2009 Remnant masses, spins and recoils from the merger of generic black-hole binaries arXiv:0904.3541 [gr-qc]
[31] Buenanano A, Cook G B and Pretorius F 2007 Inspiral, merger and ring-down of equal-mass black-hole binaries Phys. Rev. D 75 124018
[32] Baker J G, van Meter J R, McWilliams S T, Centrella J and Kelly B J 2007 Consistency of post-Newtonian waveforms with numerical relativity Phys. Rev. Lett. 99 181101
[33] Pan Yi et al 2007 A data-analysis driven comparison of analytic and numerical coalescing binary waveforms: nonspinning case Phys. Rev. D 77 024014
[34] Buenanano A et al 2007 Toward faithful templates for non-spinning binary black holes using the effective-one-body approach Phys. Rev. D 77 104049
[35] Hannam M, Husa S, Brugmann B and Gonzalez J A 2008 Where post-Newtonian and numerical-relativity waveforms meet Phys. Rev. D 77 044020
[36] Harman M, Husa S, Brugmann B, Brasil da Costa R and Gonzalez J A 2008 Comparison between numerical-relativity and post-Newtonian waveforms from spinning binaries: the orbital hang-up case Phys. Rev. D 78 104007
[37] Gopakumar A, Hannam M, Husa S and Brugmann B 2008 Comparison between numerical relativity and a new class of post-Newtonian gravitational-wave phase evolutions: the non-spinning equal-mass case Phys. Rev. D 78 064026
[38] Hinder I, Herrmann F, Laguna P and Shoemaker D 2008 Comparisons of eccentric binary black hole simulations with post-Newtonian models arXiv:0806.1037 [gr-qc]
[39] Abramović A A et al 1992 Ligo: the laser interferometer gravitational wave observatory Science 256 325
[40] Accernese F et al 2004 Status of VIRGO Class. Quantum Grav. 21 S385–94
[41] Danzmann K et al 1993 Lisa (laser interferometer space antenna), proposal for a gravitational wave detector in space Preprint, Max Planck Instüt für Quantenoptik, MPQ 177 (unpublished)
[42] Aylott B et al 2009 Status of NINJA: the numerical INjection Analysis project Class. Quantum Grav. 26 114008
[52] Aylott B et al 2009 Testing gravitational-wave searches with numerical relativity waveforms: results from the first Numerical INjection Analysis (NINJA) project Class. Quantum Grav. 26 165008

[53] Gonzalez J A, Sperhake U and Brugmann B 2009 Black-hole binary simulations: the mass ratio 10:1 Phys. Rev. D 79 124006

[54] Lousto C O, Nakano H, Zlochower Y and Campanelli M 2010 Intermediate mass ratio black hole binaries: numerical relativity meets perturbation theory arXiv:1001.2316 [gr-qc]

[55] Dain S, Lousto C O and Zlochower Y 2008 Extra-large remnant recoil velocities and spins from near-extremal-bowen-york-spin black-hole binaries Phys. Rev. D 78 024039

[56] Blanchet L 2002 Gravitational radiation from post-Newtonian sources and inspiralling compact binaries Living Rev. Relativ. 5 3

[57] Buonanno A and Damour T 1999 Effective one-body approach to general relativistic two-body dynamics Phys. Rev. D 59 084006

[58] Campanelli M, Lousto C O, Nakano H and Zlochower Y 2009 Comparison of numerical and post-Newtonian waveforms for generic precessing black-hole binaries Phys. Rev. D 79 084010

[59] Buonanno A, Chen Y and Damour T 2006 Transition from inspiral to plunge in precessing binaries of spinning black holes Phys. Rev. D 74 104005

[60] Damour T, Jaranowski P and Schafer G 2008 Hamiltonian of two spinning compact bodies with next-to-leading order gravitational spin-orbit coupling Phys. Rev. D 77 064032

[61] Steinhoff J, Hergt S and Schafer G 2008 On the next-to-leading order gravitational spin(1)-spin(2) dynamics Phys. Rev. D 77 081501

[62] Steinhoff J, Hergt S and Schafer G 2008 Spin-squared Hamiltonian of next-to-leading order gravitational interaction Phys. Rev. D 78 101503

[63] Porto R A and Rothstein I Z 2006 The hyperfine Einstein Infeld-Hoffmann potential Phys. Rev. Lett. 97 021101

[64] Porto R A and Rothstein I Z et al 2007 Comment on ‘On the next-to-leading order gravitational spin(1)-spin(2) dynamics’ by J Steinhoff (arXiv:0712.2032 [gr-qc])

[65] Porto R A and Rothstein I Z 2008 Spin(1)spin(2) effects in the motion of inspiralling compact binaries at third order in the post-Newtonian expansion Phys. Rev. D 78 044012

[66] Porto R A and Rothstein I Z 2008 Next to leading order spin(1)spin(1) effects in the motion of inspiralling compact binaries Phys. Rev. D 78 044013

[67] Mikoczi B, Vasuth M and Gergely L A 2005 Self-interaction spin effects in inspiralling compact binaries Phys. Rev. D 71 124043

[68] Blanchet L, Buonanno A and Faye G 2006 Higher-order spin effects in the dynamics of compact binaries: II. Radiation field Phys. Rev. D 74 104034

[69] Racine E, Buonanno A and Kidder L E 2009 Recoil velocity at 2PN order for spinning black hole binaries Phys. Rev. D 80 044010

[70] Arun K G, Buonanno A, Faye G and Ochsner E 2009 Higher-order spin effects in the amplitude and phase of gravitational waveforms emitted by inspiraling compact binaries: ready-to-use gravitational waveforms Phys. Rev. D 79 104023

[71] Wagoner R V and Will C M 1976 Post-Newtonian gravitational radiation from orbiting point masses Astrophys. J. 210 764–75

[72] Blanchet L, Faye G, Iyer B R and Sinha S 2008 The third post-Newtonian gravitational wave polarisations and associated spherical harmonic modes for inspiraling compact binaries in quasi-circular orbits Class. Quantum Grav. 25 165003

[73] Ajith P et al 2007 Phenomenological template family for black-hole coalescence waveforms Class. Quantum Grav. 24 S689–700