Theoretical Foundations of Water-Conducting Belt Design for Wide Coverage Sprinklers Using Numerical Simulation

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Abstract. Improvement of wide-coverage sprinkler technical level requires provision of high irrigation productivity and quality with the minimum cost of construction, rational use of water and energy. It is necessary to maintain the required flow-pressure characteristics regardless of sprinkler external operating conditions and the length of the water supply belt. The article discusses the possibility of mathematical modeling application for a wide sprinkler design. The studies showed that during the calculation it is possible to get by with the finite equations without resorting to the differential equations of fluid motion theory with variable mass. The obtained dependences served as the basis for the development of water-conducting belt calculation methods for a wide-sprinkler.

A numerical simulation of the water supply belt operation of the KASKAD sprinkler is presented with standard values for the pipeline diameters and the variable length of the sprinkler. Numerically obtained graphs of the flow rate and the required pressure dependences on water supply belt length for two design options: with a sector end sprinkler and an end long-range sprinkler operating from a booster pump. By eliminating the length of the water supply belt for the considered two design options, they designed the flow-discharge characteristics of the sprinkler. They presented an example of a constructive solution selection for the water-conducting belt of a sprinkler according to flow-pressure characteristics.

Keywords: Sprinkler, water supply belt, console, hydraulic module, sprinkler nozzle, flow rate, pressure, flow-rate characteristics.

1. Introduction
Currently, irrigation in Russia is carried out on the area of 4.3 million hectares. About 70% of this area is watered using wide coverage sprinklers.

The most important direction of modern domestic sprinkling equipment competitiveness increase is the widespread use of resource-saving technologies, i.e. rational use of water, energy and financial resources.

The improvement of wide coverage sprinklers is directly related to the water supply belt design, the optimally selected parameters of which will significantly reduce the material consumption and sprinkler cost, and will increase the irrigation efficiency and quality.

The aim of the research is to develop a methodology for the hydraulic calculation of the water supply belt for wide-angle circular sprinklers, which makes it possible to determine the parameters of the pipeline necessary to maintain the required flow-pressure characteristics regardless of sprinkler external operating conditions and its length.
The metal consumption and the significant cost of sprinkling equipment do not allow the necessary full-fledged field studies to be carried out in full, and mathematical modeling is the main economically feasible design method [1-3].

2. Research methodology

The motion of a fluid with weight change along the path was first considered by Hinds [4] and Favre [5]. Later, the ideas by Hinds and Favre were detailed in the studies [6, 7, 8, 9, 10, 11] concerning the hydraulic calculation of drip irrigation systems. The Hazen – Williams formula [8] was used in all works to determine the pressure losses along the length, which gives significant errors in the calculations during continuous liquid sampling.

Let's consider the process of uneven fluid selection in the water-conducting belt of a circular sprinkler. When you solve this problem, the model of continuous flow rate selection [4] is traditionally used according to the linear law [12]:

$$Q_2 = Q_s \frac{s}{L_M}$$  \hspace{1cm} (1)

where $Q_2$ is the selected flow rate;
$s$ is the current value of the polar radius.

$Q_s$, $L_M$ – flow in the last branch and the physical length of the sprinkler, respectively;

Following I.M. Konovalov [6, 13], you can get the differential equation of the set fluid motion with a variable mass in a water-conducting belt of constant cross-section with flow selection:

$$\frac{a'_0 (a_2 - 2)Q_s}{gw^2} \cdot \frac{dQ_s}{ds} + \frac{dH}{ds} + \frac{Q^2}{K^2} = 0,$$  \hspace{1cm} (2)

where $a'_0$ – the Boussinesq coefficient;
$a_2$ – the coefficient of flow detachment;
$Q, K$ – flow rate and flow module in an arbitrary flow section, respectively;
$H$ – hydrodynamic pressure;
$\omega$ – the living cross-section area of the stream;
$g$ – the acceleration of gravity.

Integration of the equation (2) with the condition (1) in polar coordinates gives the equation of the piezometric line for the water-conducting belt of the sprinkler[12,13]:

$$H = -L_M^2 \frac{s}{K^2} \left(1 - \frac{2}{3} \bar{s}^2 + \frac{1}{5} \bar{s}^4\right)Q_M^2 + a'_0 \frac{a_2 - 2}{2gw^2} \bar{s}^2 \left(2 - \bar{s}^2\right)Q_M^2,$$  \hspace{1cm} (3)

Where $\bar{s} = s / L_M$ – the relative coordinate;
$Q_M$ – the sprinkler consumption.

It follows from (3) that the total pressure loss along the water supply belt length:

$$H_{полн} = \left(\frac{8}{15} \frac{L_M}{K^2} - a'_0 \frac{a_2 - 2}{2gw^2}\right)Q_M^2.$$

When they solve using this method, it is assumed that:
- the selection of the flow rate $Q_s$ along the length of the water supply belt occurs continuously, which is not true, since the flow modulus $K$ and the disconnection coefficient of the flow $a_2$ vary along the length of the water supply belt, and the selection of the flow $Q_2$ occurs discretely [4, 5];
- the flow modulus $K$ and the flow disconnection coefficient $a_2$ are independent of the coordinate $s$;
In the work by A.A. Fedorets [13], this drawback is partially eliminated by introduction an empirical
discrete parameter KD into the formula (4), which depends on the frequency of tap placement $\gamma' = \frac{L_{OT}}{L_M}$,
where $L_{OT}$ - the distance between the taps.
Thus, the author [13] proposed the empirical relationship:
$$K_D = 1 + 1.7\gamma^{1.04}.$$ (5)
The flow disconnect coefficient $a_2$ is in the range $0 \leq a_2 < 1$. A.A. Fedorets [13] obtained the empirical formula for the coefficient $a_2$:
$$a_2 = \frac{0.00112 \cdot Re^{0.6422}}{e^{0.11 Re^{10^{-4}}}}.$$ (6)
where Re is the Reynolds number, determined by the diameter of the water supply belt and the total flow rate $Q_M$.
The mathematical model (3) - (6), obtained on the basis of the differential equation of the set fluid motion with a variable mass, gives a fairly good agreement with the experimental study results in the works by A.A. Fedorets [13]. However, the application scope of this model is limited to specific experimental conditions. In theoretical studies, A.A. Fedorets does not take into account the material of the pipes.
Let’s develop a model of discrete water withdrawal for the water supply belt of wide coverage sprinklers from hydraulic representation directly, without differential equation use.
The circular action sprinkler should provide the irrigation rate of $m_{Pi}, \text{m}^3/\text{ha}$ [14]:
$$m_{Pi} = \frac{h_{OC}}{10} \equiv h_{OC},$$ (7)
$h_{OC}$ – precipitation layer, mm.
With a uniform location of the branches at the distance $L_{OT}$ from each other along the length of the water supply belt, the calculated irrigation area will be the following:
$$S_0 = \pi \left( L_M^2 - \frac{L_{OT}^2}{4} \right) \cdot 10^{-4}, \text{ha},$$ (8)
and the necessary flow rate of a sprinkler can be described by the following expression [14]:
$$Q_M = \frac{25h_{OC}}{9 \cdot 24T_0} S_0, \text{m}^3/\text{s},$$ (9)
where $T_0$ is the operating time of the sprinkler, days.
The number of bends in the water supply belt is defined as follows:
$$N = INT \left( \frac{L_M}{L_{OT}} \right).$$ (10)
Each of the bends must ensure the irrigation norm $h_{OC}$ for the respective watered ring with the area of:
$$S_i = 2\pi L_{OT}^2 i, \text{m}^2; \quad i = 1, 2, \ldots, N.$$ (11)
Then the flow rate of each bend should be the following:
$$Q_i = \frac{h_{OC}}{24 \cdot 3600T_0} S_i, \text{m}^3/\text{s}.$$ (12)
Accordingly, for the rates of sections between branches, we have:

\[
\begin{align*}
Q_{TP}^{(i)} &= Q_{0}, \\
Q_{TP}^{(i)} &= Q_{TP}^{(i-1)} - Q_{i-1}, \quad i = 2, 3, \ldots, N.
\end{align*}
\]  

(13)

Knowing the inner diameter \(d_{in}\) of the water supply belt, one can find the average water velocity in each section:

\[
V_i = \frac{Q_{TP}^{(i)}}{\omega / 1000}, \text{ m/s,}
\]

(14)

where \(\omega = \frac{\pi d_{in}^2}{4}\).

Let's note that for economic reasons, the average speeds in the areas of the water supply belt should not exceed 3 m/s [5].

According to the research by A.A. Fedorets [13] when the condition:

\[
\frac{L_{OT}}{d_{in} h_{in}} > 7 - 10,
\]

(15)

is performed the pressure loss on the "mass mixing" and the pressure drop difference can be neglected and the pressure loss between the bends can be calculated by the uniform motion formulas.

where \(d_{opt}\) – the inner diameter of the bend,

To determine the pressure loss in the sections of the water supply belt between the bends, we will use the approved method to calculate the pressure loss along the flow length for new steel pipes by F.A. Shevelev [15,17]:

\[
\begin{align*}
\theta_i &= 0.889 \left(1 + \frac{0.684}{V_i}\right)^{0.226}; \quad i = 1, 2, \ldots, N.
\end{align*}
\]  

(16)

We take into account local pressure losses in the pipeline by adding 5% of the pressure losses along the length, which is consistent with the data by A.A. Fedorets [13]:

\[
h_{tp}^{(i)} = 1.05h_{i}^{(i)}.
\]

(17)

Then the total pressure loss in the pipeline will be the following:

\[
h_{tp} = \sum_{i=1}^{N} h_{tp}^{(i)}.
\]

(18)

Knowing the geodetic head and setting the free head at the last bend, you can determine the required head on the sprinkler hydrant. The computation algorithm (7) - (18) is easily implemented in any algorithmic language.

Using the presented algorithm, let's simulate the operation of "CASCADE" sprinklers [2, 3, 14] with the following initial data:

- a step at the beginning of the water supply belt (the standard pipe diameter - Ø159 mm, its length - 59.5 m) between the first 10 nozzles, makes 5.95 m;
- in the rest of the water supply belt, the step between the nozzles is 2.975 m;
- at the end of the sprinkler, the console is 24 m (12 m pipe, Ø133 mm; 12 m pipe Ø102 mm);
- at the end of the console they install:
  1) a sector sprinkler nozzle with the action range \(R_{nas} = 2.975\) m with the diameter of the nozzle orifice \(d_{nas} = 12\) mm (1st option);
  2) a long-range apparatus operating from a booster pump with the range of \(R_{DDA} = 25\) m (option 2);
- the diameter of the nozzle at the last bend of the console \(d_{N} = 12\) mm;
installation height of irrigation nozzles \( z_{nas} = 1.8 \) m;

- the height difference between the entry point to a sprinkler and the top point of the farm pipeline (geodetic pressure) makes 4.5 m;

- the sprinkler length varies from 180 m to 500 m;

- the values of the sprinkler hydraulic module (the ratio of flow to irrigation area) were taken equal to 0.81; 0.93; 1.0; 1.05.

After numerical simulation, we plotted (Fig. 1, 2) the dependences of the pressure on the hydrant (required pressures) and the sprinkler costs, depending on the length of the water supply belt (sprinkler length).

The maximum values of the sprinkler length (respectively, the maximum required pressure on the hydrant) were optimized from the condition of water average velocity limitation in the water supply pipeline \( V \leq 3.5 \) m/s, otherwise the hydraulic resistance of the water supply belt increases sharply and its diameter needs to be increased.

Calculations show that the longest sprinkler length at \( z_{nas} = 1.8 \) m is achieved with the hydraulic module \( q = 0.81 \) and makes:

\[
\begin{align*}
L_{m1} &= 499,891 \text{ m for the 1-st option;} \\
L_{m2} &= 477,866 \text{ m for the 2nd option.}
\end{align*}
\]

Accordingly, the costs and the required pressure on the sprinkler hydrant are the following:

\[
\begin{align*}
H_1 &= 39,297 \text{ m;} \\
H_2 &= 38,719 \text{ m;}
\end{align*}
\]

and the irrigation radius and flow rate in both cases \( R_{or} = 502,866 \) m and \( Q = 64,349 \) l/s.

\[\text{Figure 1. Dependence of the pressure } N \text{ on the hydrant and flow rate } Q \text{ on the sprinkler length (1-st option): 1-} q=0.81; 2- q=0.93; 3- q=1.0; 4- q=1.05\]
On the other hand, it is possible to check the results of numerical determination of the sprinkler maximum length by accepting the maximum average speed in the water supply belt at the beginning of the pipeline to the first sprinkler nozzle bend equal to \( V_{\text{max}} = 3.5 \text{ m/s} \), then the flow rate \( Q_{\text{DM}} = 64.349 \text{ l/s} \) will be established in the initial section (pipe \( \varnothing 159 \text{ mm} \), wall thickness \( e = 3 \text{ mm} \)).

Thus, the irrigated area will be:

\[
S_{\text{op}} = \frac{Q_{\text{DM}}}{q} = 794432 \text{ m}^2.
\]

And the irrigated area radius will be the following:

\[
R_{\text{op}} = \frac{S_{\text{op}}}{\pi} = 502,866 \text{ m}.
\]

![Figure 2](image_url)

**Figure 2.** Dependence of the pressure \( N \) on the hydrant and flow rate \( Q \) on the sprinkler length (2nd option) 1-\( q=0.81 \); 2-\( q=0.93 \); 3-\( q=1.0 \); 4-\( q=1.05 \)

According to the source data

\[
R_{\text{op}} = L_{\text{M1}} + R_{\text{DA}} = L_{\text{M2}} + R_{\text{DAA}} = 502,866 \text{ m},
\]

Hence:

\[
L_{\text{M1}} = R_{\text{op}} - R_{\text{DA}} = 499,891 \text{ m};
\]

\[
L_{\text{M2}} = R_{\text{op}} - R_{\text{DAA}} = 477,866 \text{ m},
\]

which confirms the results of numerical modeling.

The difference in sprinkler length is:

\[
L_{\text{M1}} - L_{\text{M2}} = 499,891 - 477,866 = 22,025 \text{ m}.
\]
Since the structural mass coefficient of the sprinkler (the ratio of the sprinkler total mass to the water supply belt mass) [15],[17] is lower for the 2nd option, respectively, \( H_2 < H_1 \), then the 2nd option is preferable from the point of view of metal consumption.

Excluding the sprinkler length from the dependencies shown on Fig. 1, 2 for the same values of the hydraulic module, we obtain the flow-pressure characteristics of the sprinkler at different values of the hydraulic module (Fig. 3, 4).

**Figure 3.** Flow-pressure characteristics of the sprinkler (I-st option):
1-\( q=0,81 \); 2-\( q=0,93 \); 3-\( q=1,0 \); 4-\( q=1,05 \)

**Figure 4.** Flow-pressure characteristics of the sprinkler (II-nd option)
1-\( q=0,81 \); 2-\( q=0,93 \); 3-\( q=1,0 \); 4-\( q=1,05 \)
Let's consider an example of sprinkler parameter selection according to flow-pressure characteristics. Let it be required to determine the pressure of circular action sprinkler for irrigation of a site with the area of $S_o = 57$ ha.

Let's set the hydraulic module of the sprinkler, for example, $q = 0.81$, and determine the necessary flow rate of the sprinkler:

$$Q_m = q \cdot S_o = 0.81 \cdot 57 \approx 46 \text{ l/s}.$$ 

Based on the flow rate $Q_m = 46$ l/s according to the graphs 3, 4 we find the required pressure on the hydrant $N = 21$ m for the I-st variant and $N = 20.5$ m for the II-nd variant. According to Fig. 1, 2 the parameters $Q_m = 46$ l/s ($N = 21$ m) and $Q_m = 46$ l/s ($N = 20.5$ m) correspond to two sprinkler lengths $L_m \approx 420$ m (1-st option) and $L_m \approx 400$ m (2-nd option). Direct calculations provide the following:

$$L_m = 423.025 \text{ m (1-st option)} \text{ and } L_m = 401 \text{ m (2-nd option)}.$$ 

3. Research results

This calculation technique was used in the design of sprinklers "Kuban-LK1M", "CASCADE", Figure 5, Table 1.

![Wide coverage sprinkler "Cascade"](image-url)
Table 1. The main specifications of the "Kuban-LK1M" (CASCADE) and "CASCADE" sprinkler

| Indicator name          | "Kuban-LK1M" (CASCADE) | "CASCADE" |
|-------------------------|------------------------|-----------|
| Designation             | Wheel, multi-support, electrified, circular movement, with water intake from a closed irrigation network or from a well |
| Machinedrive            | Electromechanical, reversible, individual for each trolley, from a gear motor |
| Powersupply             | External three-phase network with dead-earthed neutral and the rated voltage of 380 V, 50 Hz |
| The speed of the last truck, adjustable within, m/s, (m/min) | 1.6x10^-3(0,1) – 30,0x10^-3(1,8) |
| The distance from the surface of the earth to the lower belt of metal structure, m | 2,7 | 2,8 – 2,9 |
| Typeofwaterpipe         | Sectional, truss |
| Diameters of the water supply pipeline, mm | 159 | 219, 203, 168, 159, 133 |
| Diameters of console pipes, mm | 133 | 133, 114, 108, 89 |
| Spanlengths, m          | 48,7 | 48,7; 53,7; 59,5; 65,25 |
| Maximum machine length, m (with the console of 31 m) | 518 | 553 |
| The diameters of the fixed support riser, mm | 168, 203 | 168; 203; 219; 244,5; 273 |
| Wheelbase               | 3700 | 3700, 4200 |
| Water consumption at zero total slope, l/s | up to 90 |
| The water pressure at the entrance to the machine: MPa with the length of 500 m | 0,43 |
| Wheels of self-propelled carts: | pneumatic, chamber, two wheels on each cart |
| Gearmotors              | UMC |
| Wheel reducers of self-propelled carts, type | - control panel on a fixed support; - remote control panel of an external irrigation system |
| Locationof operational controls | - in motion with watering; - in motion without watering (driving) option |
| Machine operating modes | continuous, start-stop (software) |
| GSM control             | - in motion with watering; - in motion without watering (driving) option |
| Machine driving modes   | - control panel on a fixed support; - remote control panel of an external irrigation system |
| Average rain intensity, mm/min, no more than | 0,66 |
| Irrigation rate per passage (within regulation), m /ha | 95-600 |
| The minimum time for a full revolution of the machine, h, with the maximum length of 500 m | 32,8 |

4. Conclusion
Completed field tests confirm the adequacy of the mathematical model. Comparison of field study results with the calculated ones according to the proposed method gives a good agreement and indicates the correctness of the adopted method and the reliability of the obtained dependencies.
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