Validity of Quasi-Degenerate Neutrino Mass Models and their Predictions on Baryogenesis

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Abstract

Quasi-degenerate neutrino mass models (QDN) which can explain the current data on neutrino masses and mixings, are studied. In the first part, we study the effect of CP-phases on QDN mass matrix ($m_{LL}$) obeying $\mu - \tau$ symmetry in normal hierarchical (QD-NH) and inverted hierarchical (QD-IH) patterns. The numerical predictions are consistent with observed data on (i) solar mixing angle ($\theta_{12}$) which lies below tri-bimaximal (TBM) value, (ii) absolute neutrino masses consistent with $0\nu\beta\beta$ decay mass parameter ($m_{ee}$) and (iii) cosmological upper bound $\sum m_i$. $m_{LL}$ is parameterized using only two unknown parameters ($\epsilon, \eta$) within $\mu - \tau$ symmetry. The second part deals with the estimation of observed baryon asymmetry of the universe (BAU) where we consider the Majorana CP violating phases ($\alpha, \beta$) and the Dirac neutrino mass matrix ($m_{LR}$). $m_{LR}$ is taken as either the charged lepton or the up quark mass matrix. $\alpha, \beta$ is derived from the heavy right-handed Majorana mass matrix $M_{RR}$. $M_{RR}$ is generated from $m_{LL}$ and $m_{LR}$ through inversion of Type-I seesaw formula. The predictions for BAU are nearly consistent with observations for flavoured thermal leptogenesis scenario for Type-IA in both QD-NH and QD-IH models. We also observe some enhancement effects in flavour leptogenesis compared to non-flavour leptogenesis by a magnitude of order one. In non-thermal leptogenesis QD-NH Type-IA is the only model consistent with observed data on baryon asymmetry. QD-NH model appears to be more favourable than those of QD-IH. The predicted inflaton mass needed to produce the BAU is found to be $M_\phi \sim 10^{10}$ GeV corresponding to the reheating temperature $T_R = 10^6$ GeV. The present analysis shows that the three absolute neutrino masses may exhibit quasi-degenerate pattern in nature.

Keywords: QDN models, absolute neutrino masses, leptogenesis.

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1 Introduction

The presently available tightest cosmological upper bound of the sum of three absolute neutrino masses, has come down to the lowest value, $\sum_i m_i \leq 0.28$ eV [1], and a larger value of neutrino mass $m_i \geq 0.1$ eV in Quasi-Degenerate Neutrino (QDN) mass models, has therefore to be ruled out. Furthermore, the upper bound value of neutrino mass parameter $|m_{ee}| \leq 0.27$ eV appeared in the neutrinoless double beta decay ($0\nu\beta\beta$) experiments [2], also disfavours larger values of neutrino mass eigenvalues with same CP-parity. Investigations on QDN models in normal hierarchical (QD-NH) and inverted hierarchical (QD-IH) patterns of the three absolute neutrino masses, require a detailed numerical analysis to check whether such QDN models can really accommodate lower values of absolute neutrino masses $|m_i| \leq 0.09$ eV which are consistent with the above cosmological bound [1]. In the next step, the QDN models are again applied for the prediction of baryon asymmetry ($\eta_B$) of the universe [3]. In order to estimate the observed baryon asymmetry $\eta_B = (6.5^{+0.4}_{-0.3}) 10^{-10}$ [3] from a given neutrino mass model, one usually starts with a suitable texture of light Majorana neutrino mass matrix ($m_{LL}$) and then relates it with the heavy Majorana neutrinos matrix ($M_{RR}$) and the Dirac neutrino mass matrix ($m_{LR}$) through the inversion of Type-I seesaw mechanism in an elegant way. Since the structure of Yukawa matrix for Dirac neutrino is not known, we consider the texture of Dirac neutrino mass matrix $m_{LR}$ as either the charged lepton mass matrix or up quark mass matrix, as allowed by SO(10) GUT models for phenomenological analysis.

In many of the theoretical calculations on leptogenesis, the complex CP violating phases are usually derived from the Majorana phases appearing in PMNS leptonic mixing matrix $U_{PMNS}$ which diagonalizes $m_{LL}$. Hence in such approach $m_{LL}$ is no longer taken as the starting point. However, in the present work, we consider a different route for the origin of complex CP violating phases which are derived from $M_{RR}$ in the estimation of baryon asymmetry of the universe. This theoretical possibility is the main part of the present investigations.

We first introduce a general classification of QDN models based on their CP-parity patterns in the three absolute neutrino masses, and then parameterize the mass matrices using only two unknown parameters ($\epsilon, \eta$), which can reproduce correct predictions on neutrino oscillation mass parameters and mixing angles, consistent with the latest observational data. Though such parameterization is intuitive, it is quite realistic for phenomenological
analysis. The estimations of baryon asymmetry of the universe in the light of thermal and non-thermal leptogenesis, may thus serve as an additional criteria to discriminate the correct pattern of neutrino mass models and also to shed light on the structure of unknown Dirac neutrino mass matrix.

The paper is organised as follows. In section 2 we parameterize the neutrino mass matrix. In section 3, numerical analysis and predictions on baryon asymmetry are outlined. Finally in section 4 we conclude with a summary. In the Appendix we briefly summarize the formalism for estimating the lepton asymmetry in thermal leptogenesis through out-of-equilibrium decay of the heavy right-handed Majorana neutrinos. This is followed by flavoured thermal leptogenesis and non-thermal leptogenesis.

2 Parameterization and computations

2.1 Parameterizations of neutrino mass matrix

A general $\mu$-$\tau$ symmetric neutrino mass matrix [4,5] with its four unknown independent matrix elements, requires at least four independent equations for realistic numerical solution,

$$m_{LLL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \tag{1}$$

The three mass eigenvalues $m_i$ and solar mixing angle $\theta_{12}$, are given by

$$m_1 = m_{11} - \sqrt{2} \tan \theta_{12} m_{12},$$
$$m_2 = m_{11} + \sqrt{2} \cot \theta_{12} m_{12},$$
$$m_3 = m_{22} - m_{23}. \tag{2}$$

The observed mass-squared differences are calculated as

$$\Delta m_{12}^2 = m_2^2 - m_1^2 > 0, \quad \Delta m_{32}^2 = |m_3^2 - m_2^2|. \tag{3}$$

In the basis where charged lepton mass matrix is diagonal, we have the
leptonic mixing matrix, $U_{PMNS} = U$, where

$$U_{PMNS} = \begin{pmatrix}
\cos \theta_{12} & -\sin \theta_{12} & 0 \\
\frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & -1 \sqrt{\frac{1}{2}} \\
\frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & 1 \sqrt{\frac{1}{2}}
\end{pmatrix}. \quad (4)$$

The neutrino mass parameter $m_{ee}$ in $0\nu\beta\beta$ decay and the sum of the absolute neutrino masses in WMAP cosmological bound $\sum_i m_i$, are given respectively by,

$$m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|, \quad m_{\text{cosmos}} = m_1 + m_2 + m_3. \quad (5)$$

A general classification for three-fold quasi-degenerate neutrino mass models [5] with respect to Majorana CP-phases in their three mass eigenvalues, is adopted here. Diagonalization of left-handed Majorana neutrino mass matrix $m_{LL}$ in eq.(1) is given by $m_{LL} = UDU^T$, where $U$ is the diagonalising matrix in eq.(4) and $\text{Diag}=D(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta})$ is the diagonal matrix with two unknown Majorana phases ($\alpha, \beta$). In the basis where charged lepton mass matrix is diagonal, the leptonic mixing matrix is given by $U = U_{PMNS}$ [6]. We then adopt the following classification according to their CP-parity patterns in the mass eigenvalues $m_i$ namely Type IA: $(+-+)$ for $D=\text{Diag}(m_1, -m_2, m_3)$; Type IB:$(+++)\quad$ for $D=\text{Diag}(m_1, m_2, m_3)$ and Type-IC: for $(++)\quad$ for $D=\text{Diag}(m_1, m_2, -m_3)$ respectively. We now introduce the following parameterization for $\mu$-$\tau$ symmetric neutrino mass matrices $m_{LL}$ which satisfy the above classifications [4,5] and present a detailed numerical analysis.

### 2.2 Numerical analysis and predictions

For detailed numerical analysis we first choose the light Majorana neutrino mass matrix $m_{LL}$ presented in section 2.1. These mass matrices which obey $\mu - \tau$ symmetry [4], have the ability to deviate the solar mixing angle from that of TBM [7]. Next we estimate the numerical values of the three absolute neutrino masses. As discussed before, we need to introduce the neutrino mass scale $m_3$ as input parameter in addition to the observed data [8] on solar and atmospheric neutrino mass-squared differences ($\Delta m^2_{21}$ and $|\Delta m^2_{32}|$). The three input parameters are fixed at the stage of predictions of neutrino mass parameters and mixing angles given in Tables 2 and 3. These results are consistent with the recent data on neutrino oscillation parameters [8]. For numerical analysis we use the best-fit values of the global
neutrino oscillation observational data \cite{8} on solar and atmospheric neutrino mass-squared differences. $\Delta m^2_{12} = (m^2_2 - m^2_1) = 7.60 \times 10^{-5} eV^2$ and $|\Delta m^2_{32}| = |m^2_3 - m^2_2| = 2.40 \times 10^{-3} eV^2$, and define the following parameters \( \rho = \frac{|\Delta m^2_{32}|}{m^2_3} \) and \( \psi = \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \), where \( m_3 \) is the input quantity allowed by the latest cosmological bound. For QDN in normal hierarchy (QD-NH) pattern, the other two mass eigenvalues are estimated from,

\[
m_2 = m_3 \sqrt{1 - \rho}; m_1 = m_3 \sqrt{1 - \rho (1 + \psi)} \tag{6}
\]

and for QDN in inverted hierarchy (IH-QD) the mass eigenvalues are extracted from,

\[
m_2 = m_3 \sqrt{1 + \rho}; m_1 = m_3 \sqrt{1 + \rho (1 - \psi)} \tag{7}
\]

For suitable input value of \( m_3 \), one can estimate the numerical values of \( m_1 \) and \( m_2 \) for both QD-NH and QD-IH cases, using the observational values of \( |\Delta m^2_{32}| \) and \( \Delta m^2_{21} \). Table-1 gives the calculated numerical values for both models \cite{9}.

In the next step we parameterize the mass matrix in eq.(1) into three different types \cite{5,10}:

**Type IA** with \( \text{D}=\text{Diag}(m_1, -m_2, m_3) \). The mass matrix of this type can be parameterized using two parameters \((\epsilon, \eta)\):

\[
m_{LL} = \begin{pmatrix}
\epsilon - 2\eta & -\epsilon \eta & -\epsilon \\
-\epsilon \eta & \frac{1}{2} - d\eta & -\frac{1}{2} - \eta \\
-\epsilon & -\frac{1}{2} - \eta & \frac{1}{2} - d\eta
\end{pmatrix} m_3. \tag{8}
\]

This predicts the solar mixing angle,

\[
\tan 2\theta_{12} = -\frac{2\epsilon \sqrt{2}}{1 + (d - 1)\frac{\eta}{\epsilon}}. \tag{9}
\]
When we choose the constant parameters \( c = d = 1.0 \), we set for the tri-bimaximal mixings (TBM) \( \tan^2 \theta_{12} = -2\sqrt{2} \) (which is \( \tan^2 \theta_{12} = 0.50 \)) and the values of \( \epsilon \) and \( \eta \) are calculated for both QD-NH and QD-IH models, by using the values of observational data [8] in these two eigenvalue expressions:

\[
m_1 = (2\epsilon - 2\eta)m_3 \quad \text{and} \quad m_2 = (-\epsilon - 2\eta)m_3 \quad \text{obtained for TBM solution in eq.}(8).
\]

The results are given in Table-2 for \( \tan^2 \theta_{12} = 0.50 \). The solar mixing angle can be further lowered by taking the values \( c < 1 \) and \( d > 1 \), while retaining the earlier values of \( \epsilon \) and \( \eta \) extracted for TBM solution. For \( \tan^2 \theta_{12} = 0.45 \), case the results are shown in Table-3.

**Type-IB with \( D = \text{Diag} (m_1, m_2, m_3) \):** This class of quasi-degenerate mass pattern is given by the mass matrix,

\[
m_{LL} = \begin{pmatrix}
1 - \epsilon - 2\eta & \epsilon & \epsilon \\
\epsilon & 1 - d\eta & -\eta \\
\epsilon & -\eta & 1 - d\eta
\end{pmatrix} m_3.
\]

This predicts the solar mixing angle,

\[
\tan^2 \theta_{12} = \frac{2c\sqrt{2}}{1 + (1 - d)^2 \epsilon^2}.
\]

| Different parameters | QD-NH Type-IA | QD-NH Type-IB | QD-IH Type-IA | QD-IH Type-IB |
|----------------------|--------------|---------------|--------------|---------------|
| \( c \)              | 1.0          | 1.0           | 1.0          | 1.0           |
| \( d \)              | 1.0          | 1.0           | 1.0          | 1.0           |
| \( m_3 \)            | 0.10         | 0.10          | 0.08         | 0.08          |
| \( \epsilon \)       | 0.57972      | 0.0015        | 0.78004      | 0.00169       |
| \( \eta \)           | 0.14602      | 0.0649        | 0.19628      | -0.08546      |
| \( m_1 \) (eV)        | 0.08674      | 0.08675       | 0.09340      | 0.09340       |
| \( m_2 \) (eV)        | -0.08717     | 0.08717       | -0.09380     | 0.09380       |
| \( m_3 \) (eV)        | 0.10         | 0.10          | 0.08         | 0.08          |
| \( \sum |m_i| \) eV    | 0.27         | 0.274         | 0.267        | 0.274         |
| \( \Delta m^2_{21} \) eV^2 | 7.6 \times 10^{-5} | 7.6 \times 10^{-5} | 7.6 \times 10^{-5} | 7.6 \times 10^{-5} |
| \( |\Delta m^2_{23}| \) eV^2 | 2.2 \times 10^{-3} | 2.4 \times 10^{-3} | 2.4 \times 10^{-3} | 2.4 \times 10^{-3} |
| \( \tan^2 \theta_{12} \) | 0.50         | 0.50          | 0.50         | 0.50          |
| \( |m_{ee}| \) eV     | 0.08688      | 0.0869        | 0.09354      | 0.09354       |

Table 2: Predictions for \( \tan^2 \theta_{12} = 0.50 \) and other parameters consistent with observations.
Different QD-NH QD-IH

| parameters          | Type-IA | Type-IB | Type-IA | Type-IB |
|---------------------|---------|---------|---------|---------|
| c                   | 0.868   | 0.945   | 0.868   | 0.96    |
| d                   | 1.025   | 0.998   | 1.0     | 1.002   |
| $m_3$               | 0.10    | 0.10    | 0.08    | 0.08    |
| $\epsilon$         | 0.6616  | 0.00145 | 0.88762 | 0.00169 |
| $\eta$             | 0.1655  | 0.06483 | 0.22317 | -0.08546|
| $m_1$ (eV)          | 0.0876  | 0.08676 | 0.09392 | 0.09341 |
| $m_2$ (eV)          | -0.0880 | 0.08717 | -0.09432| 0.09381 |
| $m_3$ (eV)          | 0.0996  | 0.10002 | 0.08    | 0.080014|
| $\Sigma |m_i|eV$          | 0.274   | 0.274   | 0.268   | 0.267   |
| $\Delta m_{21}^2 eV^2$ | $7.7 \times 10^{-5}$ | $7.3 \times 10^{-5}$ | $7.6 \times 10^{-5}$ | $7.4 \times 10^{-5}$ |
| $|\Delta m_{23}^2| eV^2$ | $2.2 \times 10^{-3}$ | $2.4 \times 10^{-3}$ | $2.4 \times 10^{-3}$ | $2.4 \times 10^{-3}$ |
| $\tan^2 \theta_{12}$ | 0.45   | 0.45    | 0.45    | 0.45    |
| $|m_{ee}| eV$        | 0.0877  | 0.08688 | 0.09403 | 0.09354 |

Table 3: Predictions for $\tan^2 \theta_{12} = 0.45$ and other parameters consistent with observations.

which gives the TBM solar mixing angle with the input values $c = 1$ and $d = 1$. When $\epsilon = 0$, $\eta = 0$, this leads to $m_{LL}^{diag} = diag(1,1,1)m_3$. Like in Type-IA, here $\epsilon$ and $\eta$ values are computed for QD-NH and QD-IH, by using observational data [8] in the corresponding two eigenvalue expressions: $m_1 = (1 - 2\epsilon - 2\eta)m_3$ and $m_2 = (1 + \epsilon - 2\eta)m_3$ for TBM solution in eq.(10).

**Type-IC** with $D = \text{Diag}(m_1, m_2, -m_3)$: It is not necessary to treat this model separately as it is similar to Type-IB except with the interchange of two matrix elements $(m_{22})$ and $(m_{23})$ in the mass matrix in eq.(10), and this effectively imparts an additional odd CP-parity on the third mass eigenvalue $m_3$ in Type-IC. Such change does not alter the predictions of Type-IB. Tables 2 and 3 present our numerical results for $\tan^2 \theta_{12} = 0.50$ and $\tan^2 \theta_{12} = 0.45$ respectively which are consistent with cosmological bound.

## 3 Predictions of baryon asymmetry

We now apply the above Quasi-degenerate neutrino mass matrices with the values of the parameters already fixed in section 2, in the calculation of baryon asymmetry of the universe. This gives the second stage for the dis-
Table 4: Heavy right-handed Majorana neutrino mass $M_j$ for QDN with normal and inverted ordering mode for $\tan^2 \theta_{12} = 0.45$, using neutrino mass matrices given in section 2. The entry (m,n) indicates the type of Dirac neutrino mass matrix, as explained in the text.

| Type   | (m,n) | $M_1$ (GeV) | $M_2$ (GeV) | $M_3$ (GeV) |
|--------|-------|-------------|-------------|-------------|
| NH-IA  | (6,2) | $4.8659 \times 10^8$ | $-3.5068 \times 10^{12}$ | $9.1256 \times 10^{14}$ |
|        | (8,4) | $3.9414 \times 10^6$ | $-3.9774 \times 10^{10}$ | $6.0097 \times 10^{13}$ |
| NH-IB  | (6,2) | $1.8117 \times 10^8$ | $2.6219 \times 10^{12}$ | $3.2528 \times 10^{14}$ |
|        | (8,4) | $1.4994 \times 10^6$ | $2.1238 \times 10^{10}$ | $3.2527 \times 10^{14}$ |
| IH-IA  | (6,2) | $4.5568 \times 10^8$ | $-2.8771 \times 10^{12}$ | $1.3273 \times 10^{14}$ |
|        | (8,4) | $3.6910 \times 10^6$ | $-2.3241 \times 10^{10}$ | $1.8289 \times 10^{14}$ |
| IH-IB  | (6,2) | $1.7153 \times 10^8$ | $2.8234 \times 10^{12}$ | $3.5098 \times 10^{14}$ |
|        | (8,4) | $41.393 \times 10^6$ | $2.4809 \times 10^{10}$ | $3.8091 \times 10^{14}$ |

For the numerical calculation of baryon asymmetry, we refer to all the relevant expressions given in the Appendix. First we translate mass matrices $m_{LL}$ via the inversion of the seesaw formula [11], $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis for $U_R$ where $M_{RR}^{diag} = U_R^T M_{RR} U_R = diag(M_1, M_2, M_3)$, with real and positive mass eigenvalues. We then transform diagonal form of Dirac neutrino mass matrix, $m_{LR} = diag(\lambda^m, \lambda^n, 1)\nu$ to the $U_R$ basis: $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$ where $Q = diag(1, e^{i\alpha}, e^{i\beta})$ is the complex matrix containing CP-violating Majorana phases introduced by hand. Here $\lambda$ is the Wolfenstein parameter and the choice (m,n) in $m_{LR}$ gives the type of Dirac neutrino mass matrix. The value of the vev is taken as $v = 174$ GeV.

At the moment we consider phenomenologically two possible forms of Dirac neutrino mass matrices such as (i) (m,n) = (6,2) for the charged-lepton type mass matrix, and (ii) (8,4) for up-quark type mass matrix. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = \frac{h_{LR}}{\nu}$ which enters in the expression of CP-asymmetry $\epsilon$ in (14) and (23) given in the Appendix. The new Yukawa coupling matrix $h$ also becomes a complex quantity, and hence the term $Im(h^\dagger h)_{1j}$ appearing in lepton asymmetry $\epsilon_1$, gives a non-zero contribution. A straightforward simplification shows that $(h^\dagger h)_{1j} = (Q_{22}^*)^2 Q_{22}^2 R_2 + (Q_{11}^*)^2 Q_{33}^2 R_2$ where $R_{2,3}$ are real parameters. After inserting the values of phases, the above expression leads to $Im(h^\dagger h)_{1j} = -[R_2 \sin 2(\alpha - \beta) + R_3 \sin 2\alpha]$ which imparts a non-zero CP asymmetry for
Table 5: Values of CP asymmetry $\epsilon_1$ and baryon asymmetry ($\eta_{1B}, \eta_{3B}$) for all quasi-degenerate models, with $\tan^2 \theta_{12} = 0.45$, using neutrino mass matrices given in the text. The entry (m,n) in $m_{LR}$ indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrices (6,2) or up quark mass matrix (8,4) as explained in the text.

| Type  | (m,n) | $(h^h)_{11}$ | $\epsilon_1$ | $\eta_{1B}$ | $\eta_{3B}$ |
|-------|-------|-------------|-------------|-------------|-------------|
| NH-IA | (6,2) | $3.73 \times 10^{-6}$ | $1.92 \times 10^{-7}$ | $9.07 \times 10^{-12}$ | $2.11 \times 10^{-11}$ |
|       | (8,4) | $3.03 \times 10^{-8}$ | $1.55 \times 10^{-9}$ | $7.32 \times 10^{-14}$ | $1.71 \times 10^{-13}$ |
| NH-IB | (6,2) | $5.31 \times 10^{-7}$ | $1.12 \times 10^{-14}$ | $1.41 \times 10^{-18}$ | $5.67 \times 10^{-13}$ |
|       | (8,4) | $4.30 \times 10^{-9}$ | $8.87 \times 10^{-17}$ | $1.12 \times 10^{-20}$ | $4.71 \times 10^{-15}$ |
| IH-IA | (6,2) | $3.77 \times 10^{-6}$ | $1.94 \times 10^{-7}$ | $8.50 \times 10^{-12}$ | $1.98 \times 10^{-11}$ |
|       | (8,4) | $3.05 \times 10^{-8}$ | $1.57 \times 10^{-9}$ | $6.88 \times 10^{-14}$ | $1.60 \times 10^{-13}$ |
| IH-IB | (6,2) | $5.31 \times 10^{-7}$ | $9.75 \times 10^{-15}$ | $1.15 \times 10^{-18}$ | $5.95 \times 10^{-13}$ |
|       | (8,4) | $4.30 \times 10^{-9}$ | $7.80 \times 10^{-17}$ | $9.17 \times 10^{-21}$ | $4.76 \times 10^{-15}$ |

In our numerical estimation of lepton asymmetry, we chose some arbitrary values of $\alpha$ and $\beta$ other than $\frac{\pi}{2}$ and 0. For example, light neutrino masses $(m_1, m_2, m_3)$ of Type-IA model, leads to $M_{RR}^{\text{diag}} = \text{diag}(M_1, -M_2, M_3)$, and we thus fix the Majorana phase $Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) = \text{diag}(1, e^{i(\pi/2+\pi/4)}, e^{i\pi/4})$ for $\alpha = (\pi/2 + \pi/4)$ and $\beta = \pi/4$. The extra phase $\pi/2$ in $\alpha$ absorb the negative sign before heavy Majorana mass. In our search programmes such a choice of the phase leads to highest numerical estimation of lepton CP asymmetry.

In Table 4 we give numerical prediction on three heavy right-handed Majorana neutrino masses from these neutrino mass models under consideration for the case of $\tan^2 \theta_{12} = 0.45$. The three heavy right-handed Majorana mass matrices which are constructed through the inversion of Type-I seesaw mechanism, for two choices of diagonal Dirac-neutrino mass matrix discussed before. The corresponding baryon asymmetries $\eta_B$ are estimated for both non-flavour $\eta_{1B}$ and flavour $\eta_{3B}$ leptogenesis respectively in Table 5. As expected, there is an enhancement in baryon asymmetry by a magnitude of order one (approximately) when flavour dynamics is included (see Table 5 in Type-IA model with charged lepton mass matrix) [12,13]. Type-IA with charged lepton Dirac neutrino mass matrix, is the only model sensitive and
Table 6: Theoretical bound on reheating temperature $T_R$ and inflaton masses $M_\phi$ in non-thermal leptogenesis, calculated using data from Table 5, for all neutrino mass models with $\tan^2 \theta_{12} = 0.45$.

| Type   | (m,n) | $T_R^{min} < T_R \leq T_R^{max}$ (GeV) | $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ (GeV) |
|--------|-------|--------------------------------------|-----------------------------------------------|
| NH-IA  | (6,2) | $5.51 \times 10^6 < T_R \leq 4.87 \times 10^5$ | $9.73 \times 10^9 < M_\phi \leq 8.59 \times 10^{10}$ |
|        | (8,4) | $2.21 \times 10^2 < T_R \leq 3.94 \times 10^4$ | $7.88 \times 10^6 < M_\phi \leq 1.40 \times 10^9$ |
| NH-IB  | (6,2) | $3.7 \times 10^8 < T_R \leq 1.85 \times 10^6$ | $3.70 \times 10^8 < M_\phi \leq 1.89 \times 10^4$ |
|        | (8,4) | $2.96 \times 10^{13} < T_R \leq 1.49 \times 10^4$ | $2.99 \times 10^6 < M_\phi \leq 1.52 \times 10^{-2}$ |
| IH-IA  | (6,2) | $5.12 \times 10^9 < T_R \leq 4.56 \times 10^6$ | $9.11 \times 10^8 < M_\phi \leq 8.11 \times 10^4$ |
|        | (8,4) | $5.12 \times 10^5 < T_R \leq 3.69 \times 10^4$ | $7.38 \times 10^6 < M_\phi \leq 5.32 \times 10^5$ |
| IH-IB  | (6,2) | $3.84 \times 10^{12} < T_R \leq 1.72 \times 10^6$ | $3.44 \times 10^8 < M_\phi \leq 1.54 \times 10^4$ |
|        | (8,4) | $3.88 \times 10^{12} < T_R \leq 1.39 \times 10^4$ | $2.79 \times 10^6 < M_\phi \leq 9.99 \times 10^{-3}$ |

consistent with data on observed baryon asymmetry.

In case of non-thermal leptogenesis, the lightest right-handed Majorana neutrino mass $M_1$ from Table 4 and the CP asymmetry $\epsilon_1$ from Table 5, for all the neutrino mass models, are used in the calculation of the bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ in Table 6. The baryon asymmetry $Y_B = \frac{n_B}{s} = 8.7 \times 10^{-11}$ is taken as input value from WMAP observational data. Only those neutrino mass models which simultaneously satisfy the two constraints, $T_R^{max} > T_R^{min}$ and $M_\phi^{max} > M_\phi^{min}$, could survive in the non-thermal leptogenesis scenario. The predicted inflaton mass as $M_\phi \sim 10^{11}$ GeV for reheating temperature $T_R = 10^6$ GeV are needed to produce the observed baryon asymmetry of the universe [14,15]. From Table 6, the neutrino mass models with $(m,n)$ which are compatible with $M_\phi \sim (10^{10} - 10^{13})$ GeV and $T_R \approx (10^6 - 10^7)$ GeV are listed as NH-IA (6,2) and IH-IA (6,2) only. Again in order to avoid gravitino problem [8] in supersymmetric models, one has the bound on reheating temperature, $T_R \approx (10^6 - 10^7)$ GeV. This implies that Type-IA where Dirac neutrino mass matrix is taken as charged lepton mass matrix is the only model consistent with the observed baryon asymmetry. These findings nearly agree with flavoured thermal leptogenesis for Type NH-IA (6,2) model in Table 5. This result is also consistent with quasi-degenerate in inverted hierachical (IA) models for charged lepton mass matrix (6,2) [15].
4 Summary and Conclusions

We have studied the effects of Majorana CP phases on the prediction of absolute neutrino masses in three types of QDN models having both normal and inverted hierarchical patterns within $\mu-\tau$ symmetry. These predictions are consistent with data on the mass squared difference derived from various oscillation experiments, and from the upper bound on absolute neutrino masses in $0\nu\beta\beta$ decay as well as cosmological upper bound of $\sum_i m_i \leq 0.28$ eV. It has been found that QDN models with $m_i \leq 0.09$ eV, are still far from discrimination and hence the quasi-degenerate pattern is not yet ruled out. The prediction on solar mixing angle is also found to be lower than TBM value viz, $\tan^2 \theta_{12} = 0.45$ which coincides with the best-fit in the neutrino oscillation data [8].

In the next stage, left-handed Majorana neutrino mass matrices $m_{LL}$ have been employed to estimate the baryon asymmetry of the universe, in both thermal and non-thermal leptogenesis scenario (Tables 5-6). We use the CP violating Majorana phases derived from right-handed Majorana mass matrix, and also Dirac neutrino mass matrix as either charged lepton mass matrix or up-quark mass matrix. We also observe some enhancement effects in flavour leptogenesis [20] compared to non-flavour leptogenesis by a magnitude of order one. The predicted inflaton mass needed to produce the observed baryon asymmetry of the universe is found to be $M_\phi \sim 10^{10}$ GeV corresponding to the reheating temperature $T_R = 10^6$ GeV [14,15]. The analysis shows that quasi-degenerate model in normal hierarchical pattern (NH-IA) with charged lepton Dirac mass matrix (6,2), appears to be a favourable choice in nature. The quasi-degenerate model in inverted hierarchy (IH-IA) with the charged lepton Dirac mass matrix (6,2), is not completely ruled out. The results presented in this article are new and have subtle hints in the discrimination of neutrino mass models. This could establish the quasi-degenerate neutrinos as natural physical neutrinos in the neutrino oscillation experiments.

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Appendix

Thermal leptogenesis

The light left-handed Majorana neutrino mass matrix $m_{LL}$ and heavy right-handed Majorana mass matrix $M_{RR}$ are related through the canonical seesaw formula (known as Type I) \[11\] in a simple way:

$$m_{LL} = -m_{LR}M_{RR}m_{LR}^T$$ \[12\]

where $m_{LR}$ is the Dirac neutrino mass matrix. For our calculation of lepton asymmetry, we consider the model \[16,17\] where the asymmetry decay of the lightest of the heavy right-handed Majorana neutrinos, is assumed. The physical Majorana neutrino $N_R$ decays into two modes: $N_R \rightarrow l_L + \phi$, $N_R \rightarrow \bar{l}_L + \phi$ where $l_L$ is the lepton and $\bar{l}_L$ is the antilepton; $\phi$ and $\bar{\phi}$ are the Higgs and anti-Higgs particles respectively. The branching ratio for these two decay modes are likely to be different. The CP asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of the lightest of heavy right-handed Majorana neutrino $N_R$ is defined by \[16,18\]

$$\epsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$ \[13\]

Here $\Gamma = \Gamma(N_i \rightarrow l_L\phi)$ and $\bar{\Gamma} = \Gamma(N_i \rightarrow \bar{l}_L\bar{\phi})$ are the decay rates. A perturbative calculation from the interference between tree level and vertex plus self energy diagrams, gives \[19\] the lepton asymmetry $\epsilon_i$ for non-SUSY case as

$$\epsilon_i = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{ii}} \sum_{j=2,3} Im[(h^\dagger h)_y]^2 [f\left(\frac{M_j^2}{M_i^2}\right)g\left(\frac{M_j^2}{M_i^2}\right)]$$ \[14\]

where $f(x)$ and $g(x)$ represent the contributions from vertex and self energy corrections respectively, $f(x) = \sqrt{x(-1 + (x + 1)ln(1 + \frac{1}{x}))}$, $g(x) = \frac{\sqrt{x}}{x-1}$. For hierarchical right-handed neutrino masses where $x$ is large, we have the approximation \[16\], $f(x) + g(x) \cong \frac{3}{2\sqrt{x}}$. This simplifies to

$$\epsilon_i = -\frac{3}{16\pi} \frac{Im[(h^\dagger h)_{12}^2 M_1}{(h^\dagger h)_{11}} + \frac{Im[(h^\dagger h)_{13}^2 M_1}{(h^\dagger h)_{11}} M_3}$$ \[15\]

where $h = \frac{m_{LR}}{\nu}$ is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of $M_{RR}$ and $\nu = 174$ GeV is the vev of the standard model.
In term of light Majorana neutrino mass matrix $m_{LL}$ the above expression can be simplified to

$$
\epsilon_1 = -\frac{3}{16\pi} \frac{M_1}{(h^\dagger h)_{11}} Im[(h^\dagger m_{LL}h^*)_{11}] \quad (16)
$$

For quasi-degenerate spectrum of the heavy right-handed Majorana neutrino masses, i.e., for $M_1 \approx M_2 < M_3$ the asymmetry is largely enhanced by a resonance factor and in such situation, the lepton asymmetry is modified [21] to

$$
\epsilon_1 = \frac{1}{8\pi} \frac{Im[(h^\dagger h)^2_{12}]}{(h^\dagger h)_{11}} R \quad (17)
$$

where $R = \frac{M_1^2(M_2^2-M_3^2)}{(M_2^2-M_1^2)^2+\Gamma_2^2M_1^4}$ and $\Gamma_2 = \frac{(h^\dagger h)_{22}M_2}{8\pi}$. It can be noted that in case of SUSY, the functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x\ln(1 + \frac{1}{x})}$ and $g(x) = \frac{2\sqrt{x}}{x-1}$, for large $x$ one can have $f(x) + g(x) \approx 3/\sqrt{x}$. Therefore the factor $3/8$ will appear in place of $3/16$ in the expression of CP asymmetry [20] in eq. (15). The CP asymmetry parameter $\epsilon_i$ is related to leptonic asymmetric parameter $Y_L$ as

$$
Y_L \approx \frac{n_L - \bar{n}_L}{s} = \sum_i \frac{\epsilon_i \kappa_i}{g_{*i}} \quad (18)
$$

where $n_L$ is the lepton number density, $\bar{n}_L$ is the anti-lepton number density, $s$ is the entropy density, $\kappa_i$ is the dilution factor for the CP asymmetry $\epsilon_i$, and $g_{*i}$ is the effective number of degrees of freedom at temperature $T = M_1$. The baryon asymmetry $Y_B$ produced through the sphaleron transition of $Y_L$ while the quantum number B-L remains conserved, is given by [22]

$$
Y_B = \frac{n_B}{s} = CY_{B-L} = CY_L \quad (19)
$$

where $C = \frac{8N_F+4N_H}{22N_F+13N_H}$, $N_F$ is the number of fermion families and $N_H$ is the number of Higgs doublets. Since $s = 7.04n_\gamma$ the baryon number density over photon number density $n_\gamma$ corresponds to the observed baryon asymmetry of the universe [23],

$$
\eta_B^{SM} = \frac{n_B}{n_\gamma}^{SM} \approx d\kappa_1\epsilon_1 \quad (20)
$$

where $d \approx 0.98 \times 10^{-2}$ is used in the present calculation. In case of MSSM, there is no major numerical change with respect to the non-supersymmetric
case in the estimation of baryon asymmetry. One expects approximate enhancement factor of about $\sqrt{2} \sqrt{2}$ for strong (weak) washout regime [20].

In the expression for baryon-to-photon ratio in eq. (20), $\kappa_1$ describes the washout factor of the lepton asymmetry due to various lepton number violating processes. This efficiency factor (also known as dilution factor) mainly depends on the effective neutrino mass

$$\bar{m}_1 = \frac{(h^* h)_{11} \nu^2}{M_1}$$

(21)

where $\nu$ is the electroweak vev, $\nu = 174$ GeV. For $10^{-2} eV < \bar{m}_1 < 10^3 eV$, the washout factor $\kappa_1$ can be well approximately by [19,24]

$$\kappa_1(\bar{m}_1) = 0.3 \left[ \frac{10^{-3}}{\bar{m}_1} \right] \left[ \log \frac{\bar{m}_1}{10^{-3}} \right]^{-0.6}$$

(22)

The value of $\kappa_1$ is valid only for the given range of $\bar{m}$ [25].

**Flavoured thermal leptogenesis**

It is inevitable to include the flavour effects in thermal leptogenesis [26] and study its effects on the enhancement in baryon asymmetry over the single flavour approximation. In the flavour basis the equation for lepton asymmetry in $N_1 \rightarrow l_\alpha \phi$ decay where $\alpha = (e, \mu, \tau)$ becomes

$$\epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^* h)_{11}} \left[ \sum_{j=2,3} Im[h^*_\alpha(h^j h)_{1j} h_{\alpha j}] g(x_j) + \sum_j Im[h^*_\alpha(h^j h)_{1j} h_{\alpha j}] \frac{1}{(1 - x_j)^j} \right]$$

(23)

Here we have $x_j = \frac{M_i^2}{M_i^2}$ and $g(x_j) \sim \frac{3}{2} \frac{1}{2 \sqrt{x_j}}$. The efficiency factor for the out-of-equilibrium situation is given by $\kappa_\alpha = \frac{m_\ast}{m_{\alpha\alpha}}$. Here $\frac{8\pi H_\nu^2}{M_1^2} \sim 1.1 \times 10^{-3}$ eV, and $m_{\alpha\alpha} = \frac{h^\dagger_{\alpha \alpha} h_{\alpha \alpha}}{M_1} \nu^2$. This leads to the baryon asymmetry of the universe,

$$\eta_{BB} = \frac{\eta_B}{\eta_\gamma} \sim 10^{-2} \sum_\alpha \epsilon_{\alpha\alpha} \kappa_\alpha \sim 10^{-2} m_\ast \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\bar{m}_{\alpha\alpha}}$$

(24)

For single flavour case, the second term in $\epsilon_{\alpha\alpha}$ vanishes when summed over all flavours. Thus

$$\epsilon_1 \equiv \sum_\alpha \epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^* h)_{11}} \sum_j Im[(h^j h)_{1j}^2] g(x_j).$$

(25)
This leads to baryon asymmetry,

$$
\eta_{1B} \approx 10^{-2} m_\epsilon \frac{\eta_1}{m} = 10^{-2} \kappa_1 \epsilon_1
$$

(26)

where $\epsilon_1 = \sum \epsilon_{\alpha \alpha}$ and $\tilde{m} = \sum \tilde{m}_{\alpha \alpha}$.

**Non-thermal leptogenesis**

We now focus our attention on the non-thermal leptogenesis scenario [27] where the right-handed neutrinos are produced through the direct non-thermal decay of the inflaton $\phi$. We follow the standard procedure [28] where non-thermal leptogenesis and baryon asymmetry in the universe had been studied in different neutrino mass models whereby some mass models were excluded using bound from below and from above on inflaton mass and reheating temperature after inflation. We start with the inflaton decay rate given by

$$
\Gamma_\phi = \Gamma(\phi \to N_i N_i) \approx \frac{|\lambda_i|^2}{4\pi M_\phi}
$$

(27)

where $\lambda_i$ are the Yukawa coupling constants for the interaction of three heavy right-handed neutrinos $N_i$ with the inflaton $\phi$ of mass $M_\phi$. The reheating temperature after inflation is given by the expression,

$$
T_R = \left(\frac{45}{2\pi^2 g_*}\right)^\frac{1}{4} (\Gamma_\phi M_p)^\frac{1}{2}
$$

(28)

where $M_p \approx 2.4 \times 10^{18}$ Gev is the reduced Plank mass [29] and $g_*$ is the effective number of relativistic degrees of freedom at reheating temperature. For SM we have $g_* = 106.75$, and for MSSM $g_* = 228.75$. If the inflaton dominantly couples to $N_i$, the branching ratio of this decay process is taken as $\text{BR} \sim 1$, and the produced baryon asymmetry of the universe can be calculated by the following relation [30],

$$
Y_B = \frac{n_B}{s} = CY_L = C \frac{3}{2} \frac{T_R}{M_\phi} \epsilon_1
$$

(29)

where $Y_L$ is the lepton asymmetry generated by CP-violating out-of-equilibrium decays of heavy neutrino $N_1$ and $T_R$ is the reheating temperature. The fraction $C$ has the value $C=-28/79$ for SM and $C=-8/15$ in the
MSSM. The observed baryon asymmetry measured in WMAP data, \( \eta_B = \frac{n_B}{n_\gamma} = 6.5 \times 10^{-10} \) [3], where \( s = 7.04 \ n_\gamma \) is related to \( Y_B = n_B/s = 8.6 \times 10^{-11} \) in eq.(29). From eq.(29) the connection between \( T_B \) and \( M_\phi \) is expressed as,

\[
T_R = \left( \frac{2Y_B}{3C\epsilon_1} \right) M_\phi
\]  

(30)

The above expression is supplemented by two more boundary conditions [28]: (i) lower bound on inflaton mass, \( M_\phi > 2M_1 \) coming from allowed kinematics of inflaton decay to two right-handed Majorana neutrinos \( N_1 \), and (ii) an upper bound for the reheating temperature, \( T_R \leq 0.01M_1 \) coming from out-of-thermal equilibrium decay of \( N_1 \). Using the observed central value of the baryon asymmetry \( Y_B \) and theoretical prediction of CP asymmetry \( \epsilon_1 \) in eq.(30), one can establish the relation between \( T_R \) and \( M_\phi \) for each neutrino mass model. The lightest right-handed neutrino mass \( M_1 \) and the CP asymmetry \( \epsilon_1 \) neutrino mass models are used in the calculation of theoretical bounds: \( T_R^{\text{min}} < T_R \leq T_R^{\text{max}} \) and \( M_\phi^{\text{min}} < M_\phi \leq M_\phi^{\text{max}} \) following eq.(19) along with other two boundary conditions cited above. Only those models which satisfy the constraints \( T_R^{\text{max}} > T_R^{\text{min}} \) and \( M_\phi^{\text{min}} < M_\phi^{\text{max}} \) simultaneously can survive in the non-thermal leptogenesis.

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