The simplest *strange* three-body halo

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Abstract

The recently developed method to solve the Faddeev equations in coordinate space is used to study the weakly bound halo nucleus $^3\Lambda H(\Lambda+n+p)$. The long distances are treated carefully to achieve convergence and high accuracy. We use several sets of two-body interactions which reproduce the deuteron properties and provide the low-energy $\Lambda$-nucleon scattering data close to that of two of Nijmegen potentials. We show that the details of the potentials are unimportant unless the accuracy of the hypertriton binding energy is required to be better than 50 keV. We find that the most significant parameter of the $\Lambda$-nucleon interaction, the singlet s-wave scattering length, must be within 10% of 1.85 fm, when the $\Lambda$-separation energy from the deuteron is about 130 keV. Other details of the $\Lambda$-nucleon interaction are less important for the hypertriton structure. The scattering length and effective range are computed for scattering of a $\Lambda$-particle on a deuteron. The folding model reducing the three-body problem to a two-body problem is investigated in this context and found to be inadequate.
1 Introduction

The last decade has witnessed a rapid development following the discovery of nuclear halos [1]. The most carefully studied examples are $^{11}\text{Li}$ and $^{6}\text{He}$ [2]. The characteristic features of halos are very small particle separation energies and very large spatial extension [3, 4]. Simple few-body models are used to describe these features and the details of the interactions are not essential even in accurate computations of halo properties [3, 4].

It is interesting to extend the studies of "ordinary" nuclear halos to systems with non-vanishing strangeness. The halo perspective should then be kept and the general properties of halos extracted. The lightest nucleus with strangeness is the hypertriton ($^{3}_{\Lambda}\text{H}$). It consists of a neutron, a proton and a $\Lambda$-particle. The system may roughly be described as a deuteron and a $\Lambda$-particle bound in a state with the binding energy $B_{\Lambda} = 0.13 \pm 0.05 \text{ MeV}$ [7, 8]. The deuteron binding energy is $B_{d} = 2.224575 \text{ MeV}$ and the total three-body binding energy is then $B = 2.35 \text{ MeV}$. The spin and the parity $J_{d}^{\pi} = 1^{+}$ and $J_{\Lambda}^{\pi} = \frac{1}{2}^{+}$, respectively of the deuteron and the $\Lambda$-particle, are combined with the corresponding quantum numbers of the relative state into the total $J^{\pi} = \frac{1}{2}^{+}$ [7]. If the deuteron remains undisturbed within the hypertriton the total orbital angular momentum can only take the values $L = 0$ or 2. In any case, relative s-states are probably the all-dominating components.

The hypertriton or the approximate $\Lambda$-deuteron two-body system is very weakly bound and qualify as the simplest nuclear halo system [4] with finite strangeness. The spatial extension of the system is easily estimated by the root mean square radius $< r^{2} >^{1/2} \approx \frac{\hbar}{\sqrt{4\mu B_{\Lambda}}} = 10.2 \text{ fm}$, where the reduced mass $\mu = m_{\Lambda}m_{d}/(m_{\Lambda} + m_{d}) = 0.760m_{N}$ is about 3/4 of the nucleon mass $m_{N}c^{2} = 939 \text{ MeV}$. Thus the $\Lambda$-deuteron radius is 5 times larger than the root mean square radius $R_{d} = 1.971 \text{ fm}$ of the deuteron. The system appears to be very simple, but it is close to the $\Lambda$-dripline and therefore very sensitive to the strength of the $\Lambda$-nucleon interaction, which in this way therefore is constrained by the measured hypertriton properties.

The hypertriton offers a mixture of simplicity, sensitivity and fundamental interest in connection with the interactions between nucleons and strange particles. This combination have attracted considerable (theoretical) attention over the last three decades. The interest is reflected in a number of investigations where different techniques and models are used, e.g. variational models [10, 11], the hyperspherical method [12, 13], Faddeev calculations [14, 15] and simpler $\Lambda$-deuteron two-body models [16]. Virtually all available interactions have been used in these investigations including very sophisticated microscopic interactions. Unfortunately the results also exhibit considerable variation for example from the (observed) bound three-body structure to the (wrong) unbound $\Lambda$-deuteron system [17].

It is also very tempting to use the hypertriton to fine-tune the $\Lambda$-nucleon
two-body interaction. However, this certainly requires very accurate knowledge of both the interactions between all contributing partial waves of the subsystem and all contributing couplings to other systems for example like the Σ-nucleon-nucleon system. In addition the rather delicate numerical three-body problem also must be mastered to a high accuracy.

A convenient method to solve the quantum mechanical three-body problem was recently formulated [17] and tested on various systems for example on the numerically difficult problem of the occurrence of Efimov states in nuclei [18]. It is well-known [5, 6] that one of the difficult three-body problems is to give an accurate description of a system where one binary subsystem has a loosely bound state and the third particle is weakly bound to the other two particles. The characteristic size of the Λ-deuteron system and the related small binding energy requires a rather accurate treatment of the wave function to large distances up to about 50 fm [6]. This is only barely achieved in the best of the previous investigations.

Another interesting observation in this connection is that the large extension of the system, which implies high sensitivity to the two-body interactions, at the same time provides the simplification that only low-energy properties of the Λ-nucleon interaction is really needed in the description [5, 6]. These properties may be expressed in terms of scattering lengths and effective ranges of various relative states [19]. Thus extraction of other very detailed properties of the interaction from the properties of the hypertriton might turn out to be very difficult.

The purpose of this paper is two-fold. First we want to develop, refine and test the new method to solve the three-body problem [17]. An accurate determination of the hypertriton structure, including its binding energy, is a challenging problem due to the bound subsystem and the weak binding energy [6]. The present application is an investigation of a “strange” nuclear halo system. In this connection it is worth noting that the previous elaborate and complicated Faddeev calculations were obtained by using different methods for example by working in momentum space. Results from the present new method is desirable to get an independent comparison. In addition to the bound state investigation we use the occasion to apply the method in a strict three-body computation of scattering of a Λ-particle on a deuteron which subsequently is compared to two-body scattering calculations. Secondly, we want to investigate whether the low-energy properties of the Λ-nucleon interaction suffice for the desired accurate description of the hypertriton structure. In this connection we need to study the effects of the nucleon-nucleon interaction, as reflected in the deuteron properties, on the hypertriton structure.

This paper may be considered as part of a series discussing the general properties of three-body halo systems. They all deal with weakly bound and spatially extended three-body systems compared to the length and energy of the two-body interactions. In the first of these papers [5] we discussed Borromean systems where no subsystem is bound [6]. In the second [6] we extended the discussion
to general three-body systems especially those where two and three-body asymptotics are mixed. In the third [19] we discussed the effects of the finite spins of the particles. In the fourth [20] we discuss fragmentation interactions of halo nuclei in the sudden approximation and especially the effects of final state interactions.

In this paper dealing with a strange halo, we give in section 2 after the introduction, a brief sketch of the method. In section 3 we introduce and parametrize the all-dominating set of input parameters, i.e. the two-body potentials. In section 4 we describe and discuss the numerical results and in particular the strict three-body scattering computations compared to the two-body approximation to this three-body problem. Finally, in section 5 we offer a summary and the conclusions.

2 Method

The method is described in previous publications, see for example [6, 19]. Therefore we shall here only give a very brief sketch, which should be sufficient to define the quantities of interest and the notation used in the following sections. We consider a system of three particles labeled by \( i = 1, 2, 3 \) with corresponding masses \( m_i \), coordinates \( \mathbf{r}_i \) and momenta \( \mathbf{p}_i \). They interact via the two-body potentials \( V_{ij} \). We shall use the Jacobi coordinates, which apart from mass factors, are defined as the relative coordinates between two of the particles (\( \mathbf{x} \)) and between their center of mass and the third particle (\( \mathbf{y} \)). The explicit definitions and the corresponding three sets of hyperspherical coordinates \( (\rho, \alpha, \Omega_x, \Omega_y) \) can be found in [2, 5, 6]. Here \( \rho = \sqrt{x^2 + y^2} \) is the generalized radial coordinate and \( \alpha \), in the interval \([0, \pi/2]\), defines the relative size of \( \mathbf{x} \) and \( \mathbf{y} \), where \( \Omega_x \) and \( \Omega_y \) are the angles describing the directions of \( \mathbf{x} \) and \( \mathbf{y} \). The volume element is given by \( \rho^5 d\Omega d\rho \) where \( d\Omega = \sin^2 \alpha \cos^2 \alpha d\omega_2 d\Omega_y \).

The total wavefunction \( \Psi \) of the three-body system is given as a sum of three components \( \psi^{(i)} \) each expressed in terms of one of the three different sets of Jacobi coordinates:

\[
\Psi = \sum_{i=1}^{3} \psi^{(i)}(\mathbf{x}_i, \mathbf{y}_i). \tag{1}
\]

These wavefunctions satisfy the three Faddeev equations

\[
(T - E)\psi^{(i)} + V_{jk}(\psi^{(i)} + \psi^{(j)} + \psi^{(k)}) = 0, \tag{2}
\]

where \( E \) is the total energy, \( T \) is the kinetic energy operator and \( \{i, j, k\} \) is a cyclic permutation of \( \{1, 2, 3\} \). Each component \( \psi^{(i)} \) is now for each \( \rho \) expanded in a complete set of generalized angular functions \( \Phi^{(i)}_n(\rho, \Omega_i) \).

\[
\psi^{(i)} = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho) \Phi^{(i)}_n(\rho, \Omega_i), \tag{3}
\]
where the phase space factor $\rho^{-5/2}$ is extracted. The angular functions are now chosen for each $\rho$ as the eigenfunctions of the angular part of the Faddeev equations

$$\frac{\hbar^2}{2m\rho^2} \hat{\Lambda}^2 \Phi_n^{(i)} + V_{jk}(\Phi_n^{(i)} + \Phi_n^{(j)} + \Phi_n^{(k)}) \equiv \frac{\hbar^2}{2m\rho^2} \lambda_n(\rho) \Phi_n^{(i)},$$  \hspace{1cm} (4)

where we choose $m = m_N$ and $\hat{\Lambda}^2$ is the $\rho$-independent part of the kinetic energy operator defined by

$$T \equiv T_\rho + \frac{\hbar^2}{2m\rho^2} \hat{\Lambda}^2, \quad T_\rho = -\frac{\hbar^2}{2m} \left( \rho^{-5/2} \frac{\partial^2}{\partial \rho^2} \rho^{5/2} - \frac{1}{\rho^2} \right).$$  \hspace{1cm} (5)

Explicitly the generalized angular momentum operator $\hat{\Lambda}^2$ is given by

$$\hat{\Lambda}^2 = -\frac{1}{\sin \alpha \cos \alpha} \frac{\partial^2}{\partial \alpha^2} \sin \alpha \cos \alpha + \frac{l_x^2}{\sin^2 \alpha} + \frac{l_y^2}{\cos^2 \alpha} - 4$$  \hspace{1cm} (6)

in terms of the orbital angular momentum operators $l_x^2$ and $l_y^2$ related to the $x$ and $y$ degrees of freedom.

The radial expansion coefficients $f_n(\rho)$ are solutions to the coupled set of differential equations

$$\left( -\frac{d^2}{d\rho^2} - \frac{2mE}{\hbar^2} \rho^{-5/2} \lambda_n(\rho) + 15 \right) f_n + \sum_{n'} \left( -2P_{nn'} \frac{d}{d\rho} - Q_{nn'} \right) f_{n'} = 0,$$  \hspace{1cm} (7)

where the functions $P$ and $Q$ are defined by

$$P_{nn'}(\rho) \equiv \sum_{i,j=1}^{3} \int d\Omega \Phi_n^{(i)}(\rho, \Omega) \frac{\partial}{\partial \rho} \Phi_n^{(j)}(\rho, \Omega),$$  \hspace{1cm} (8)

$$Q_{nn'}(\rho) \equiv \sum_{i,j=1}^{3} \int d\Omega \Phi_n^{(i)}(\rho, \Omega) \frac{\partial^2}{\partial \rho^2} \Phi_n^{(j)}(\rho, \Omega).$$  \hspace{1cm} (9)

The asymptotic large-distance behavior of the radial potentials can be calculated in our case when one of the two-body subsystems has a bound state with the binding energy $B_d$ and the corresponding wavefunction $\phi(r)$. The result is obtained from the behavior of the individual quantities

$$\lambda_1 \to -4 - \frac{2mB_d\rho^2}{\hbar^2} + \langle \phi | 1 + 3r \frac{\partial}{\partial r} + r^2 \frac{\partial^2}{\partial r^2} | \phi \rangle,$$  \hspace{1cm} (10)

$$Q_{11} \to -\frac{1}{4\rho^2} + \frac{1}{\rho^2} \langle \phi | 1 + 3r \frac{\partial}{\partial r} + r^2 \frac{\partial^2}{\partial r^2} | \phi \rangle,$$  \hspace{1cm} (11)

where the index 1 refers to the lowest $\lambda$-value.
Since the diagonal terms of $P$ vanish, i.e. $P_{nn} = 0$, the diagonal effective radial potential corresponding to the lowest $\lambda$ is then given by

$$\frac{1}{\rho^2}(\lambda_1 + \frac{15}{4}) - Q_{11} \to -\frac{2mB_d}{\hbar^2} + O(\rho^{-3}) .$$

(12)

The diagonal effective potential for higher lying $\lambda$-values, $(\lambda_n + 15/4)/\rho^2 - Q_{nn}$, instead approaches zero for $\rho \to \infty$ at least as fast as $1/\rho^3$, see [8, 17]. The radial couplings arise from the non-diagonal parts of $P$ and $Q$. They also approach zero for $\rho \to \infty$ at least as fast as $1/\rho^3$.

3 Two-body potentials

The present three-body calculation explores how far it is possible to understand the hypertriton as a three-body system from its constituent particles and their mutual (phenomenological) interactions. The low-energy properties of the interactions and the interactions at distances much larger than the particle sizes are decisive due to the relatively weak binding of the system. We assume that the particles maintain their identity and their intrinsic structure only enter via their spins and parities. The requirements for the phenomenological two-body interactions are then first that they accurately reproduce the low-energy properties of the two-body subsystems. Secondly, the interactions must simultaneously be simple enough to allow a flexible and accurate treatment of the three-body problem. The latter requirement will be met by proper parametrization.

3.1 The nucleon-nucleon interaction

The main component of the relevant part of the nucleon-nucleon force corresponds to the quantum numbers of the deuteron, i.e. the triplet $s$ and $d$-states with the total angular momentum and parity $J^\pi = 1^+$. However, in the three-body system also two-body relative $p$-states are possible, since corresponding relative $p$-states of the last particle can couple to give the proper total angular momentum and parity. Higher orbital angular momenta are negligible. The total isospin of the hypertriton is zero and the nucleon-nucleon relative state then also must have vanishing isospin.

We parametrize the isospin zero part of the nucleon-nucleon interaction with central, spin-spin, tensor and spin-orbit terms as

$$V_{NN} = V_c(r) + V_{ss}(r)s_{N1} \cdot s_{N2} + V_T(r)\hat{S}_{12} + V_{so}(r)l_{NN} \cdot s_{NN} ,$$

(13)

where $s_{N1}$ and $s_{N1}$ are the spins of the nucleons, $s_{NN} \equiv s_{N1} + s_{N2}$, $l_{NN}$ is the relative orbital angular momentum and $\hat{S}_{12}$ is the usual tensor operator. It is
sometimes convenient to use a “central” potential including the spin-spin part, i.e.

\[ V_c^{(2s+1)} = V_c(r) + V_{ss}(r) \langle s|s_{N1} \cdot s_{N2}|s \rangle , \]

which then depends on the total spin \( s \) of the two-body system. To test the sensitivity of our results we want to vary the radial shapes of the interactions while keeping the deuteron properties and the low-energy scattering properties. We choose the radial shapes, \( V_c^{(1)}(r) \), \( V_c^{(3)}(r) \), \( V_T(r) \), \( V_{so}(r) \), of each of the terms to be either a gaussian \( V_{\text{exp}}(\frac{-r^2}{b^2}) \), an exponential \( V_{\text{exp}}(-\frac{r}{b}) \) or a Yukawa function \( V_{\text{br}}^{-1}\text{exp}(\frac{-r}{b}) \).

The low-energy scattering data are available as scattering lengths \( a \) and effective ranges \( r_e \) of the different relative states. We also have detailed experimental information about the deuteron [21], i.e. binding energy \( B_d = 2.224575(9) \) MeV, root mean square radius \( R_d = -\langle r^2 \rangle^{1/2} = 1.971(6) \) fm, admixture of the d-state \( P_d \sim 4\%-6\% \), the asymptotic ratio of the d to s-wave component \( \eta_d = 0.0256(4) \) and electric quadrupole moment \( Q_d = 0.2859(3) \) fm².

In the hypertriton wave function the deuteron component is essential and reproducing the deuteron properties is therefore expected to be more important than reproducing scattering data. As the s-wave by far is the most dominating component we shall first explore this question by using only the central part of the interaction and the corresponding deuteron wave function without d-state admixture. We then adjust the range and strength of the radial two-body potential to reproduce the measured values of \( B_d \) and \( R_d \). We also varied \( R_d \) to see the influence of the deuteron size on the hypertriton properties. The resulting parameters for these interactions labeled 1 and 2 can be found in the upper part of table 1 for gaussian, and exponential radial shapes. The many digits in the parameters are necessary to obtain sufficient accuracy on the computed quantities.

We now also include the d-wave admixture in the deuteron. For each radial shape we have then four ranges and four strengths to adjust to the four singlet and triplet s-state low energy parameters [22], binding energy, root mean square radius, d-wave admixture, quadrupole momentum and the asymptotic d- to s-wave ratio. The parameters for these interactions resulting from this consistent treatment are labeled with an additional C and given in the middle part of table 1.

In addition we constructed two more sets of potentials which provide slightly different deuteron properties in order to check the sensitivity of the resulting hypertriton structure to these differences. Those potentials are listed in the lower part of table 1.

The various properties of the deuteron for each of these nucleon-nucleon interactions are shown in table 2. Even for the simplest central interactions we obtain scattering lengths and effective ranges for the triplet state in rather close agreement with the measured values. The more complicated potentials from the middle part of table 1 also reproduce these scattering data rather well. The
quadrupole moment, the root mean square radius and the asymptotic d to s-wave ratio are reproduced as well although with minor deviations. These quantities are nicely determined and some of the remaining small deviations are in fact rather difficult to remove due to strong model correlations between the different data. The lower part of table 2 shows the properties of the additional sets of potentials. We shall use these slightly different properties of the deuteron to study various specific correlation with the hypertriton structure.

3.2 The Λ-nucleon interaction

The Λ-nucleon relative wave function within the hypertriton is dominated by s-wave components. In the computations we shall also allow p and d-state contributions, but we shall neglect the contributions from the higher orbital angular momenta. The parametrization of the interaction are again given by the form

\[ V_{AN}^{(l)} = V_c^{(l)}(r) + V_{ss}(r) s_A \cdot s_N + V_T(r) \hat{S}_{12} + V_{so}(r) l_{AN} \cdot s_{AN}, \]  

where \( s_A, s_N, \) and \( l_{AN} \), respectively are the spin of the Λ-particle, spin of the nucleon and the Λ-nucleon relative orbital angular momentum. The total spin is \( s_{AN} \equiv s_A + s_N \) and \( l = 0,1 \). The central part may have an orbital angular momentum dependence indicated by the superscript \( l \). This is to simplify simultaneous reproduction of all the different p-wave scattering lengths. We shall also use a parametrization without explicit \( l \)-dependence.

The low-energy properties of the interaction is not directly available from scattering experiments. However, it is known that the Λ-nucleon interaction does not bind the two-body system and the binding energy of the hypertriton can therefore be used to constrain models of this interaction. The role of the hypertriton and the Λ-nucleon interaction for nuclear systems with finite strangeness is in this way analogous to the role of the deuteron and the corresponding nucleon-nucleon interaction for ordinary nuclei.

Several models are available for the Λ-nucleon interaction. We have selected two different models developed by the Nijmegen group. They are one-boson exchange potentials obtained by using SU(3) symmetry and simultaneous fits to all nucleon-nucleon and Λ-nucleon data \([23, 24, 25]\). We shall here focus on the models called Nijmegen F and Nijmegen SC \([23, 24]\). The computed scattering lengths and effective ranges for s-states and the scattering lengths for the p-states are shown in table 3. Using the triplet p-state, this provides 7 constraints on our parametrization. For convenience we do not use the singlet p-state in the fits, since it is determined (as the s-waves) by the central and spin-spin parts of the interaction and the corresponding contributions to the hypertriton wave function are small. The low-energy parameters of the Nijmegen models F and SC are reproduced by the interactions labeled 1, 3 and 5, 6 respectively.
As for the nucleon-nucleon case we determine strengths and ranges of the potentials by adjusting to the model values of the s-wave scattering lengths and effective ranges and triplet p-wave scattering lengths given at the top of table 3. The remaining degrees of freedom are constrained by choosing the same ranges for some of the terms. These choices are rather arbitrary, but they influence mostly the small p-wave part of the Λ-nucleon relative wavefunction. The resulting different sets of interaction parameters (label 1) are collected in table 4.

These interactions all overbind the hypertriton as we shall see later. The binding energy is the decisive quantity determining the structure of the halo and we must therefore adjust some of the interaction parameters to reproduce the measured $B_\Lambda$. This can most efficiently be achieved by changing the scattering length of the dominating component in the wave function, i.e. the s-waves. For the gaussian potential (G1) we then simply reduce the strengths of the central s-wave potential by 10% resulting in the interactions G1r. For exponential and Yukawa shapes we then readjusted the parameters of the potentials to reproduce these new slightly altered low-energy scattering data. The resulting parameters for these interactions (Y1r and E1r) are also shown in table 4.

As an alternative choice of the interaction parametrization we do not allow now l-dependence in the central potential leaving 8 parameters and 7 constraints. The eighth parameter is constraint by additional fit to the $^3P_2$ effective range (labels 3 and 5). We use both the Nijmegen potentials, although mainly Nijmegen SC, and reproduce the scattering data with different radial shapes. Again the hypertriton binding energy turns out to be too large. We reduce the strength of (all partial waves of) the central potential by 10%. The parameters for the resulting interactions (labeled G5r, E5r and Y5r) are given in table 4. To study the dependence on scattering lengths we varied the singlet and triplet s-wave parameters independently. The new sets are labeled ”ra” and ”rb” and the corresponding interaction parameters for gaussian, exponential and Yukawa shapes are also given in table 4 (labeled G5ra, G5rb, E5ra, E5rb, Y5ra and Y5rb).

Finally, we have collected the scattering lengths and effective ranges in table 3 for all these Λ-nucleon interactions.

4 Numerical results

The method requires a priori specification of the quantum numbers of the contributing components in all three Jacobi set of coordinates. A complete basis is obtained by including all possible values of these quantum numbers consistent with the generalized Pauli principle, parity and the rules for coupling angular momenta. The isospin conservation requires that the two-nucleon system is in an isospin zero state, i.e. a deuteron-like state. Only very few of the lowest orbital angular momenta contribute to the accuracy we need. However, we shall include
as many as necessary to obtain the required accuracy.

In the Jacobi coordinate system, where \( x \) refers to the relative coordinate between the two nucleons, we need to include orbital angular momenta \( l_x = 0, 2 \) with the corresponding spin values \( s_x = 0 \). The intermediate value \( l_x = 1 \) (and the related \( s_x = 1 \)) is not allowed in the two-body system due to parity conservation, but the third particle can compensate by being in a relative \( p \)-state. In both the other two Jacobi coordinate systems, where \( x \) refers to the relative \( \Lambda \)-nucleon system, we expect again dominating components with \((l_x, l_y) = (0, 0)\) and admixture of components with \((l_x, l_y) = (0, 2), (2, 0), (1, 1)\). The \( d \)-wave components can be expected to be similar to the \( d \)-wave admixture in the deuteron. The resulting set of quantum numbers in our computations are given in table 5. A number of additional channels have also been included to test the convergence. Their contributions always turned out to be negligible in the present context.

4.1 Convergence and accuracy

The large extension or the weak binding of the system relative to a free deuteron and \( \Lambda \)-particle demands an accurate treatment. This means that the wave function must be calculated to large distances and small components must be included both in the angular Faddeev equations and in the coupled set of radial equations. These convergence problems are essential as emphasized previously [12, 13]. The large distances are handled by analytical extrapolation of the numerically obtained \( \lambda \)-values. This improves significantly both accuracy and speed of the computation.

The separation energy \( B_\Lambda \) is determined with a relative accuracy of about \( 10^{-3} \) already by including the lowest three values. This is roughly unchanged when the interactions result in both somewhat weaker and stronger binding energy. This relative accuracy is sufficient for our purpose and in the calculations we therefore always included the three lowest \( \lambda \)-values in the radial set of equations.

We first carry out a number of calculations using various combinations of N-N and \( \Lambda \)-N interactions. They are parametrized to fit some of the deuteron properties and the low-energy scattering data obtained in the two Nijmegen models. The results are shown in table 6. Different radial shapes lead to differences in binding energy of up to 340 keV out of the total energy of about 0.31 to 0.65 MeV. The N-N interactions which best fit the deuteron properties (label C) exhibit less binding and less deviations resulting from different radial shapes. All the interactions considered in the upper part of table 6 produce a significant overbinding of the hypertriton. The Nijmegen F model is always closest to the measured value. Since the properties depend on the size and binding energy of the three-body system, we adjusted the interactions to reproduce the hypertriton binding energy. The resulting reduced potentials in table 4 are from now on used
The significance of different components in the wave functions can be seen in the middle part of table 3. The contribution from the p-states amounts to \( \approx 10 \) keV compared to the value of \( B_\Lambda \approx 60-157 \) keV. One exception is the Yukawa interaction in the \( \Lambda-N \) interaction where the p-states contribute up to 40 keV. These p-state contributions are not allowed in the isolated deuteron wave function, but in the hypertriton they are allowed in combination with the negative parity components in the relative \( \Lambda \)-deuteron wave function.

The contribution from the d-state in the \( \Lambda \)-nucleon channel (13 and 14 in table 3) amounts to about 10-25 keV for most interactions and to 55-70 keV for the nucleon-nucleon interactions obtained by coupled channel analysis, where the tensor interaction effectively is stronger. This d-wave contribution is mediated by the d-wave in the deuteron and therefore only present when this admixture is included. With d-wave admixture in the deuteron the binding energy of the hypertriton is larger first due to the mentioned d-wave contribution in the \( \Lambda-N \) channel and secondly due to the deformation causing larger s-wave contribution.

The different radial shapes also marginally influence the contributions. The wave function moves with decreasing binding energy towards larger distances where the effective three-body potential is determined by the scattering lengths of the two-body potentials. Thus the detailed radial shapes should then be unimportant. However, different radial shapes of the potentials always add a small uncertainty. In the present case of \( B_\Lambda \approx 130 \) keV and a 2.2 MeV bound binary subsystem this uncertainty arises almost entirely from the \( \Lambda-N \) potential. The Yukawa potentials seems to provide up to about 50 keV more binding than the exponentials which in turn exceed the binding in gaussian potentials by about other 50 keV. The Yukawa potentials diverge at short distance which is unphysical and rather extreme. The more reasonable finite potentials like gaussians or exponentials lead to binding energies deviating by less than 40 keV. The deviations due to different radial shapes are significantly diminished when the consistent analysis of the scattering data are used to constrain the interactions. The resulting variations in binding energy then amounts to less than 66 keV between different shapes including Yukawa functions.

The sensitivity against variation of the radial shapes of the potentials is investigated by introducing a repulsive core in the central \( \Lambda-N \) potential. We simply use a linear combination of two gaussians where the shortest range corresponds to \( b = 0.53 \) MeV with a strength of 550 MeV. The longer-range gaussian is then used in the usual adjustment procedure where low-energy s-wave scattering data are reproduced as described before. We only include central and spin-spin terms. The resulting repulsive central potential at \( r = 0 \) is then \( \sim 400 \) MeV. If the low-energy properties of all partial waves should remain unchanged the l-dependent potential for each orbital angular momentum must be individually adjusted. We only do this for s and p-waves, see table 4. The effect of this repulsion is of course the tendency for the wavefunction to decrease or vanish at short distances.
between the two particles.

In an expansion on a basis, as we have used in the present work, this requires more basis states with sufficient nodes at short distances. Furthermore, the emphasis is somewhat shifted towards higher angular momenta needed to describe the exclusion of the wavefunction from the repulsive region. Both effects increase the difficulties in achieving high accuracy and convergence in the numerical computations. The typical results are a decrease of $B_A$ from 490 keV to 420 keV where we already included an energy gain of 22 keV by adjusting the p-wave scattering to resemble the result for the case without a repulsive core. For $B_A$ around 130 keV the total decrease in binding energy is only 17 keV and the gain by p-wave adjustment is reduced to about 17 keV, see table 7. The uncertainty related to a repulsive core is then less than the uncertainty due to different radial shapes.

Finally, we note that the mentioned total uncertainties of around 50 keV are extremely small compared to the central values of 10-100 MeV of the two-body potentials holding the system together. Thus the dependence on the details of the potentials is indeed remarkably small. On the other hand the demand for high accuracy at some point necessarily brings in some more details of the potentials. In the present case this turns out to be at around 50 keV. Improved accuracy can be achieved by an adjustment to the s-wave phase shifts over a range of (low) energies. Then the choice of radial shape would be much less important. The reason is simply that the decisive low-energy properties deviate less over the contributing range of energies.

4.2 Two-body interactions and the hypertriton structure

The structure of the hypertriton described as a three-body system is determined by the N-N and the Λ-N interactions. Since the deuteron is a main component in this structure the most important parts of the N-N interactions are those determining the deuteron structure. The all-decisive property of the deuteron is the binding energy and less importantly also the root mean square radius $R_d$ and the d-wave admixture. We show $B_A$ in table 6 for different sizes and identical binding energies of the deuteron. This Λ-separation energy decreases about 15 keV when $R_d$ increases from 1.904 fm to 1.971 fm. The variation of the radial shape of the N-N interaction only causes insignificant changes when both $B_d$ and $R_d$ are fixed. The inherent uncertainty in these calculations is therefore only a few keV, since the N-N interaction and the deuteron properties both are very well known.

The sensitivity to the Λ-N interaction is higher because the system is close to the threshold for binding the Λ-particle. Then only a little change of the potential will change the size and binding dramatically. Since the interaction is less known it is of considerable interest to determine the decisive degrees of
freedom. This will provide information about which constraints the hypertriton properties can place on the interaction. The numerical examples in table 6 show that different interactions with the same radial shape and the same s and p low-energy scattering data lead to the same hypertriton properties. Different radial shapes (i.e., gaussian and exponential) and identical scattering data produce a variation of $B_\Lambda$ of about 50 keV.

The same radial shape and a 10% bigger singlet s-wave scattering length (ra) increases $B_\Lambda$ by about 50-100 keV, respectively for gaussian and exponential shapes. A 10% decrease of the triplet s-wave scattering length (rb) decreases the $B_\Lambda$ only by about 15-30 keV, for both gaussian and exponential shapes of the interaction. At least the dependence on the singlet scattering length is significant in the present context. Thus, accurate knowledge of the binding energy constrains the s-wave scattering lengths and especially the singlet s-wave. The present accuracy on $B_\Lambda$ allows the claim of about 10% deviation from the value of the singlet s-wave scattering length for the interaction Fr. The original two potentials, Nijmegen F and Nijmegen SC, certainly both overbind the hypertriton. Their singlet s-wave scattering lengths are both too large.

The separation energy $B_\Lambda$ has so far been in focus, but the root mean square radius is another important and strongly correlated quantity. It is shown in table 6 for the different interactions. In these tables we also show the mean distance of the $\Lambda$-particle from the center of mass of the deuteron under the assumption that the deuteron is unperturbed in the hypertriton. The precise definition is given by

$$<r^2_\Lambda d> = \frac{m_N(m_\Lambda + m_d)}{m_\Lambda m_d}(<\rho^2> - 2R^2_d).$$

This distance is much larger than the size of the deuteron and the $\Lambda$-particle is therefore on average far outside the deuteron. It is easy to verify that the product $B_\Lambda \cdot <r^2_\Lambda d>$ remains approximately constant. The reason for this two-body halo dependence is the small separation energy from a “particle” asymptotically approaching the deuteron.

We are now in a position to show the realistic structure of the hypertriton. First, we recall that the angular eigenvalues $\lambda$ are closely related to the diagonal part of the effective radial potentials, $(\lambda_n + 15/4)/\rho^2 - Q_{nn}$. The $\lambda$-spectrum is therefore decisive for the spatial structure of the system. An example including the five lowest values is shown in fig.1 for the GC1 and the G5r interactions respectively for N-N and $\Lambda$-N. The behavior is typical and other realistic potentials like the combination GC1 and Y5r or GC1 and E5r would differ by about the thickness of the lines already after $\sim 3$ fm (see fig.1a).

At $\rho = 0$, we have the hyperspherical spectrum $\lambda = K(K + 4)$, where K is a non-negative even integer. The odd numbers are excluded here for parity reasons. The lowest $\lambda$ starts at zero, has a flat minimum and then bends over and diverges at large distances parabolically as $\lambda = -2B_d m_N \rho^2/\hbar^2$. The other $\lambda$-functions reestablish the hyperspherical spectrum asymptotically at large $\rho$. 
Therefore they all approach finite constants at large \( \rho \). In particular the lowest level originating from \( \lambda = 12 \) is not far from the lowest \( \lambda \)-value at small distance close to the attractive pocket. This level, which approaches zero for \( \rho \rightarrow \infty \), is responsible for the amount of relative p-states in the subsystems.

The radial wave functions corresponding to the three lowest \( \lambda \)-values are shown in fig.2. By far the most dominating component is that of the lowest \( \lambda \), which at large distances describes a \( \Lambda \)-particle bound to a deuteron in the ground state. The main part of the probability is found between 2 fm and 10 fm. The other components are very small, but they are essential to achieve the accuracy we want.

A more visual impression of the hypertriton structure is obtained from fig.3, where we plot the probability distribution in the plane defined by the three particles. We use the system of coordinates where the center of mass is in the middle of the figure, the largest principal moment of inertia is horizontal and the \( \Lambda \)-particle is in the right half. For simplicity, only s-waves are included in the computation and we used G2 N-N and G6r \( \Lambda \)-N interactions (see tables 1 and 4). The probability distribution reaches its maximum at the point where the hyperradius is equal to 6.20 fm, the hyperangle is equal to 12.27 and the angle between \( x \) and \( y \) is \( \pi/2 \). At this point the distance between the nucleons is 1.86 fm and the distance between their center of mass and the \( \Lambda \)-particle is 7.65 fm.

### 4.3 Scattering of a \( \Lambda \)-particle on a deuteron

The properties of the hypertriton are also accessible through information obtained by scattering of \( \Lambda \)-particles on deuteron. The \( \Lambda \)-N interaction could be further constrained by such scattering measurements. If such information should become available and subsequently utilized the corresponding three-body scattering problem must be solved. Our method, sketched in section 2, allows computation of bound states as well as scattering problems. Both types of computations are three-body calculations and in particular the scattering problem does not have to rely on two-body or any other approximation.

The procedure is analogous to the bound state problem. The angular eigenvalues \( \lambda \) are first computed for each value of \( \rho \). Then the coupled set of radial equations are solved with the proper boundary conditions, i.e. exponentially decreasing radial functions \( f_n \) except for the function corresponding to the lowest \( \lambda \)-value, which asymptotically describes the deuteron in its ground state. This function is at large distances turning into a phase shifted sine-function in the variable \( k_y \) or equivalently \( k \rho \). When the energy of the \( \Lambda \)-particle relative to the deuteron approaches zero we may in the usual way derive the scattering length \( a_\rho \) and effective range \( r_\rho \) from these phase shifts.

These quantities are then related to the \( \rho \) degree of freedom as indicated by the notation. They must then be transformed to the coordinates describing
the relative distance between the deuteron and the Λ-particle. The resulting scattering length and effective range are then given by

\[ a_{\Lambda-d} = a_{\rho} \sqrt{\frac{m_N (m_{\Lambda} + m_d)}{m_{\Lambda} m_d}}, \quad r_{\Lambda-d} = r_{\rho} \sqrt{\frac{m_N (m_{\Lambda} + m_d)}{m_{\Lambda} m_d}}. \] (17)

The hypertriton has angular momentum \( \frac{1}{2} \), which means that the deuteron spin of 1 couples to the spin \( \frac{1}{2} \) of the Λ-particle. Thus the relative Λ-deuteron state or equivalently the scattering channel has spin \( \frac{1}{2} \).

In the present case we aimed directly at computation of the scattering length and effective range by using zero energy from the beginning. Then the lowest radial wave function is a straight line outside the short-range potential. The intersection with the \( \rho \)-axis gives the scattering length \( a_{\rho}(2S) \). The effective range \( r_{\Lambda-d}(2S) \) is obtained by direct integration of the expression

\[ r_{\Lambda-d}(2S) = 2 \sqrt{\frac{m_N (m_{\Lambda} + m_d)}{m_{\Lambda} m_d}} \int_0^{\rho_{\text{max}}} \left( f_0^2 - \tilde{f}_0^2 \right) d\rho, \] (18)

where \( f_0 \), corresponding to the lowest \( \lambda \)-value, is the radial wave function normalized to unity at the origin. The related wave function \( \tilde{f}_0 \) is the radial solution without any interaction. It is identical to \( f_0 \) for distances larger than \( \rho_{\text{max}} \) or equivalently outside the short-range potential.

Using three \( \lambda \)-values as for the bound state calculation, we obtain a scattering length accurate up to about 1\% and an effective range accurate up to about 3\%. Both quantities are given in the table along with other properties of the hypertriton for the various cases considered. The large value of \( a_{\Lambda-d}(2S) \) is a natural consequence of the small Λ-separation energy \( B_{\Lambda} \). The connection is approximately given by

\[ B_{\Lambda} = \frac{\hbar^2}{2 \mu a_{\Lambda-d}^2(2S)} \frac{1}{1 + r_{\Lambda-d}(2S)/a_{\Lambda-d}(2S)}, \] (19)

which numerically is fulfilled. The effective ranges are in all cases about 4.0 fm \( \pm \) 0.8 fm, i.e. as usual of the same order as the range of the effective potentials.

4.4 Two-body approximations to the three-body problem

Reduction of a three-body problem to an effective two-body problem might be a computational advantage. It is tempting to try such a reduction for the hypertriton which approximately can be considered to be a Λ-particle and a deuteron. This case appears to be very well suited for a two-body description as also attempted previously [16]. The first step of the procedure is obviously to construct the Λ-deuteron effective potential by folding the deuteron wave function with the
Λ-N interaction. The remaining steps are then of two-body nature and therefore computationally much simpler.

The interaction between the Λ-particle and the deuteron is given by

\[ V_{\Lambda d}(r_{\Lambda d}) = \int dr_3 \psi_d^*(r_d) (V_{\Lambda N}(r_\Lambda - r_{N1}) + V_{\Lambda N}(r_\Lambda - r_{N2})) \psi_d(r_d), \]  

(20)

where \( \psi_d \) is the deuteron wave function obtained by solving the Schrödinger equation with the nucleon-nucleon potential. The solution \( \psi_{\Lambda d} \) to the Schrödinger equation with this effective two-body potential then gives an approximation to the original three-body problem, i.e. the product wave function \( \psi_{\Lambda d}(r_{\Lambda d})\psi_d(r_d) \).

The large-distance asymptotic behavior of the correct three-body wave function describes a Λ-particle bound by \( B_{\Lambda} \) to a deuteron in the ground state. The relative coordinate \( r_{\Lambda d} \equiv y \sqrt{m_N (m_\Lambda + m_d)/(m_\Lambda m_d)} \) is then, apart from the mass factor, identical to \( \rho (\approx y) \) at large distance. After correcting for this scale factor on the coordinate, the radial potential in the three-body computation should therefore be directly comparable to the two-body potential.

Fig.4 shows this comparison between the potential corresponding to the lowest \( \lambda \)-value and the folding result where for simplicity only s-waves in the Λ-nucleon channel were included. The differences at short distance obviously arise from the spatial extension of the deuteron causing the coordinates to differ. The differences at large distance are also significant. The folding potential necessarily falls off exponentially in accordance with the exponential behavior of the deuteron wave function and the short-range Λ-nucleon potential. On the other hand the three-body potential decreases faster than \( \rho^{-3} \), but presumably still much slower than exponentially. The reason is that the three-body potential also must account for polarization effects where the deuteron wave function in the hypertriton is distorted away from its ground state shape. Still asymptotically, but not at intermediate and short distances, the potentials both correspond to the deuteron in its ground state.

The hypertriton is in this two-body approximation always unbound, but scattering in the two-body approximation can still be studied and compared with the full three-body results. With a binding energy \( B_{\Lambda} \approx 130 \text{ keV} \) the two-body approximation leads to the scattering length and effective range around 4.8 fm and 3.7 fm respectively. Through eq. (19) this corresponds to a two-body virtual state at around 680 keV, i.e. about 800 keV above the correct energy value. Therefore we conclude that the folding model cannot be used to estimate the properties of loosely bound systems like the hypertriton. The three-body calculations are then necessary to obtain detailed information.

On the other hand an effective two-body model might sometimes be useful in qualitative understanding and in preliminary estimates. We can perhaps get some insight by specifying that the folding approximation in eq. (20) basically assumes a specific form for the total three-body wave function. The resulting energy is therefore only an upper bound and the hypertriton comes out unbound.
by about 500 keV. Clearly the potential must be more attractive than the folding potential if the separation should be reproduced.

Increasing the depth and/or the range parameter until the measured binding energy is reached then provide a two-body model for the hypertriton. The distance between \( \Lambda \) and the deuteron, measured for example as the root mean square radius, is then in this model too small due to the too attractive potential needed to compensate for the inflexible assumption of the wave function. The smaller the \( \Lambda \)-deuteron separation energy the more extended the system and the better is the two-body model. However, processes involving the intrinsic structure with one neutron and one proton can not be described.

An analogous example is the popular \( ^{11}\text{Li} \) described as a three-body system consisting of \( ^{9}\text{Li} \) surrounded by two neutrons. Again the intrinsic structure can not be described in the model. The distances between these three particles are of comparable size and detailed and accurate information therefore require three-body calculations. Contrary to the hypertriton with one bound binary subsystem, the Borromean system \( ^{11}\text{Li} \) would be progressively worse described for decreasing two-neutron separation energy.

5 Summary and conclusion

The general properties of nuclear three-body halo systems have been discussed in a recent series of papers. The characteristic features of these bound states are the weak binding and the large spatial extension. The hypertriton certainly qualifies as a halo system due to the relatively weak interactions between the \( \Lambda \)-particle and the nucleons. Thus strangeness is involved and this system is potentially able to provide information of a very different type from that of the usual halo nuclei.

To relate any available experimental information with the basic quantities determining the hypertriton structure, it is necessary to perform accurate and reliable computations. We solve the coordinate space Faddeev equations by use of a recently developed method where the angular part first is obtained for any given average distance of the particles. The large distance behavior is known analytically and used to improve accuracy and speed of the calculations. The coupled radial equations are solved numerically afterwards. The essential large-distance behavior is in this way carefully treated as required by the small binding and the related large extension.

We first sketch the method and then we parametrize the crucial input for the calculations, i.e. the two-body interactions. We use potentials with central, spin-spin, spin-orbit and tensor terms. To study the sensitivity we parametrize each of these with gaussian, exponential and Yukawa radial shapes and compare the results. For the nucleon-nucleon part we adjust the parameters to constraints
obtained from the measured deuteron properties and the s-wave scattering lengths and effective ranges. The $\Lambda$-nucleon interaction is much less known. We use the low-energy parameters provided by two potentials from the Nijmegen group obtained by SU(3) symmetry and fits to all available nucleon-nucleon and $\Lambda$-nucleon data. We adjust the parameters to reproduce the corresponding s-wave scattering lengths and effective ranges and the p-wave scattering lengths. We compare results where d-wave coupling is neglected in the two-body scattering analysis used for the parameter extraction.

The dependence of the hypertriton properties on the various choices are investigated in some detail. First the convergence and accuracy of the computations are investigated. Three radial equations suffice to an accuracy of 1 part in 1000. The sensitivity to the nucleon-nucleon interaction is rather low provided the deuteron properties are maintained. Both $\Lambda$-nucleon potential models overbind the hypertriton by up to half an MeV. Since the binding energy is the crucial quantity determining the size of the system, we reduced the attractive strength of the $\Lambda$-nucleon central potential by 10%. The hypertriton energy is then roughly correct. For a larger binding energy the $\Lambda$-particle is on average closer to the deuteron and the hypertriton structure is more sensitive to the details of the two-body potentials. Most effects would then be amplified. The sensitivities are more realistic when the measured binding energy is used as starting point in the investigations.

The p-wave contribution to the binding energy is small but visible. A typical value is 10 keV. It adds a negative parity component to the relative neutron-proton wave function. In the three-body system this is compensated by another p-wave in the relative $\Lambda$-deuteron wave function. The contribution to the binding energy from the d-wave in the $\Lambda$-nucleon relative state is also small but visible with a typical value of about 20 keV.

The most essential part of the $\Lambda$-nucleon interaction corresponds to the singlet s-wave. A 10% change of this scattering length changes the binding energy by about 70% around the $\Lambda$-separation energy $B_\Lambda \approx 130$ keV. Changing towards a weaker potential easily then results in an unbound system. For comparison, a change of 10% of the triplet s-wave scattering length changes $B_\Lambda$ by about 15 keV. The other scattering lengths are less important in this connection. To obtain the correct binding energy $B_\Lambda$ we must therefore have a singlet scattering length within 10% from the value 1.85 fm.

The uncertainty arising from the different choices of radial shapes of the $\Lambda$-nucleon potential amounts to 50 keV. A repulsive core of up to 550 MeV introduces an additional inaccuracy of about 15 keV for a $\Lambda$-separation energy of 130 keV. These uncertainties are significant when compared to the $\Lambda$-separation energy of about 130 keV, but extremely small compared to the strengths of the two-body potentials. This uncertainty is in fact also very small compared to 2.35 MeV which is the total hypertriton binding energy including the deuteron part. Thus the chosen low-energy scattering data determines the hypertriton structure.
and its binding energy to an accuracy of about 50 keV.

The origin of the differences due to the radial shapes evidently must arise from finer details of the potentials. The differences would be reduced by adjusting to phase shifts in an energy range extending beyond that reproduced by the scattering lengths and the effective ranges. This corresponds to a better description of the higher-energy or off-shell behavior. The extreme low-energy scattering properties we used in the fits can only predict structures to the accuracy 50 keV specified here. The lower the binding energy the better can the properties be predicted by the extreme low-energy scattering behavior.

The three-body scattering problem, where a Λ-particle is scattered on a deuteron, must also contain information about the hypertriton structure or the two-body interactions describing this three-body system. Using our method we carried out strict three-body calculations of scattering length and effective range. The scattering length is closely related to the size of $\lambda$ and the effective range is of the order of the range of the effective three-body radial potential.

We also tested the two-body approximation obtained by folding the deuteron wave function and the Λ-nucleon interaction. We computed binding energies and scattering properties. The approximation is inadequate for the details and the accuracy required in the present investigation.

In conclusion, the hypertriton is a challenging system where a careful treatment is necessary to connect measured values with the basic interaction involving strangeness. The halo structure makes the large distances important and the numerical work difficult. The details of the interactions enter rather weakly in the hypertriton structure which predominantly is determined by a few of the scattering lengths. Thus further constraints on the interaction would certainly require considerably higher accuracy both in measurements and in calculations.

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Figure Captions

Figure 1 The angular eigenvalue spectrum $\lambda$ as function of $\rho$ corresponding to the interactions GC1 and G5r, respectively for the N-N and $\Lambda$-N interactions. The five lowest $\lambda$-values are exhibited. a) Short-distance behavior of the four lowest $\lambda$-values for different $\Lambda$-N interaction radial shapes.

Figure 2 The components of the wave function for the lowest three $\lambda$-values corresponding to the calculation in figure 1.

Figure 3 The probability distribution for the hypertriton in the plane drawn through the three particles. The system of coordinates is defined by the following constraints: i) the center of mass is in the middle of the figure; ii) the principal moment of inertia is along horizontal axes; iii) the $\Lambda$-particle is in the right half of the figure. Only s-waves are included in the computations. The G2 and G6r interactions, respectively for the N-N and the $\Lambda$-N subsystems, are used (see tables 1 and 4). Regions with higher z values are darker.

Figure 4 The folding potential $V_{\Lambda d}$ (dot-dashed curve) compared to the effective three-body radial potential $V_{eff}$ (solid curve) for the interactions GC1 and G6r. The coordinate used is $\rho$ and the mass corrected distance between the $\Lambda$-particle and the deuteron $y = r_{\Lambda d}\sqrt{(m_\Lambda m_d)/m_N(m_\Lambda + m_d)}$. 

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Table 1: Parameters of the nucleon-nucleon (N-N) interaction. Strengths \( V \) in MeV and ranges \( r \) in fm of the singlet and triplet central (or central \((c)\) and spin-spin \((ss)\)), tensor \((t)\) and spin-orbit \((so)\) parts are listed. The radial shapes are gaussian \((G)\), exponential \((E)\) and Yukawa \((Y)\), respectively. The upper part of the table refers to a deuteron in the \( ^3S_1 \) state; the lower part refers to a deuteron in the coupled \( ^3S_1-^3D_1 \) two-nucleon state.

| N-N | \( V^{(3)}_c \) | \( r^{(3)}_c \) | \( V^{(1)}_c \) | \( r^{(1)}_c \) | \( V_t \) | \( r_t \) | \( V_{so} \) | \( r_{so} \) |
|-----|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|
| G1  | -79.852        | 1.3971      | 0              | -           | 0              | -           | 0              | -           |
| E1  | -221.048       | 0.6129      | 0              | -           | 0              | -           | 0              | -           |
| G2  | -61.263        | 1.6383      | 0              | -           | 0              | -           | 0              | -           |
| E2  | -164.994       | 0.7291      | 0              | -           | 0              | -           | 0              | -           |
| GC1 | -49.768        | 1.658       | -31.051        | 1.8164      | -8.636         | 2.1786      | 110.347        | 1.4838      |
| EC1 | -127.489       | 0.7997      | -105.072       | 0.7270      | -27.216        | 0.6552      | 322.983        | 0.7309      |

| N-N | \( V_t \) | \( r_t \) | \( V_{ss} \) | \( r_{ss} \) | \( V_t \) | \( r_t \) | \( V_{so} \) | \( r_{so} \) |
|-----|-------|--------|-------------|-----------|-------|--------|-------------|-----------|
| G3  | -55.714 | 1.6184 | -72.250     | 0.9712    | -14.33 | 0.59   | 89.0       | 1.9       |
| Y3  | -30.408 | 1.8013 | -6.879      | 2.6214    | -1.951 | 0.9703 | 68.6       | 1.9701    |

Table 2: Properties of the deuteron for the various interactions in table 1. The columns contain the root mean square radius \(<r^2>^{1/2}\), the \(d\)-state probability \(P_d\), the electric quadrupole momentum \(Q_d\), the asymptotic \(d\) to \(s\)-wave ratio \(\eta_d\), and the \(s\)-wave scattering lengths \(a\) and effective ranges \(r_e\). The deuteron binding energy is \(B_d = 2.224575\) MeV. The first row shows the experimental values. Note that in our sign convention a negative scattering length corresponds to a bound state.

| N-N | \(<r^2>^{1/2}\) (fm) | \(P_d\) (%) | \(Q_d\) (fm²) | \(\eta_d\) | \(a(^3S_1)\) (fm) | \(r_e(^3S_1)\) (fm) | \(a(^1S_0)\) (fm) | \(r_e(^1S_0)\) (fm) |
|-----|-----------------|------------|---------------|----------|-----------------|------------------|-----------------|------------------|
| exp[21,22] | 1.971(6) | 4-6 | 0.2859(3) | 0.0256(4) | -5.4194(20) | 1.759(5) | 23.748(10) | 2.75(5) |
| G1  | 1.904           | 0          | 0             | 0        | -5.324         | 1.620         | -               | -                |
| E1  | 1.904           | 0          | 0             | 0        | -5.330         | 1.598         | -               | -                |
| G2  | 1.971           | 0          | 0             | 0        | -5.485         | 1.836         | -               | -                |
| E2  | 1.971           | 0          | 0             | 0        | -5.485         | 1.799         | -               | -                |
| GC1 | 1.967           | 5.66       | 0.273         | 0.0233   | -5.419         | 1.93          | 23.748         | 2.75             |
| EC1 | 1.971           | 5.63       | 0.271         | 0.0227   | -5.394         | 1.87          | 23.748         | 2.75             |
| G3  | 1.904           | 4.07       | 0.24          | 0.0266   | -4.615         | 1.425         | 23.748         | 2.75             |
| Y3  | 2.039           | 4.07       | 0.35          | 0.0371   | -4.397         | 1.563         | 23.748         | 2.75             |
Table 3: Scattering lengths and effective ranges in fm for the two selected A-nucleon (Λ-N) models [23, 24]. The rows labeled Fr and SCr represent the values obtained by a 10% reduction of the s-wave central strength of the gaussian interactions, which otherwise reproduces the values for these quantities obtained from the two models. The rows labeled ra and rb represent variations where either the singlet S or the triplet S scattering lengths are changed from the values of Fr and SCr by 10%, respectively up or down. Finally, the two last rows represent the s-wave low-energy data of the models SC and SCr respectively, reproduced by using gaussian interactions with only central and spin-spin terms, with (label cp) and without repulsive core, see table 4.

| Λ-N   | \(a^{(1S)}_0\) | \(r_e^{(1S)}_0\) | \(a^{(3S)}_1\) | \(r_e^{(3S)}_1\) | \(a^{(3P)}_0\) | \(a^{(3P)}_1\) | \(a^{(3P)}_2\) |
|-------|----------------|----------------|---------------|---------------|-------------|-------------|-------------|
| Fr    | 2.29           | 3.17           | 1.88          | 3.36          | 0.047       | -0.114      | 0.02        | -0.188      |
| Fr    | 1.85           | 3.41           | 1.52          | 3.64          | 0.047       | -0.114      | 0.02        | -0.188      |
| G1    | 2.30           | 3.17           | 1.90          | 3.35          | 0.111       | -0.114      | 0.020       | -0.118      |
| Y1    | 2.31           | 3.21           | 1.90          | 3.40          | 0.107       | -0.099      | 0.016       | -0.103      |
| G3    | 2.29           | 3.16           | 1.88          | 3.36          | 0.421       | -0.114      | 0.020       | -0.686      |
| G1r   | 1.86           | 3.40           | 1.54          | 3.63          | 0.111       | -0.114      | 0.020       | -0.118      |
| E1r   | 1.86           | 3.47           | 1.53          | 3.69          | 0.150       | -0.108      | 0.018       | -0.178      |
| Y1r   | 1.89           | 3.68           | 1.55          | 3.85          | 0.115       | -0.097      | 0.019       | -0.100      |
| SC    | 2.78           | 2.88           | 1.41          | 3.11          | 0.062       | -0.096      | 0.061       | -0.20       |
| SCr   | 2.27           | 3.10           | 1.15          | 3.36          | 0.41        | -0.068      | 0.086       | -0.42       |
| G5    | 2.78           | 2.90           | 1.41          | 3.11          | 0.437       | -0.096      | 0.061       | -0.448      |
| E5    | 2.78           | 2.88           | 1.41          | 3.57          | 0.442       | -0.128      | 0.061       | -0.551      |
| Y5    | 2.78           | 2.88           | 1.41          | 3.42          | 0.432       | -0.059      | 0.062       | -0.525      |
| G5r   | 2.27           | 3.10           | 1.15          | 3.36          | 0.405       | -0.068      | 0.086       | -0.415      |
| E5r   | 2.27           | 3.10           | 1.15          | 3.54          | 0.407       | -0.075      | 0.085       | -0.451      |
| Y5r   | 2.27           | 3.10           | 1.16          | 3.85          | 0.397       | -0.066      | 0.085       | -0.494      |
| G5ra  | 2.50           | 3.11           | 1.15          | 3.36          | 0.451       | -0.067      | 0.085       | -0.416      |
| E5ra  | 2.49           | 3.10           | 1.15          | 3.48          | 0.446       | -0.072      | 0.086       | -0.442      |
| Y5ra  | 2.49           | 3.10           | 1.15          | 3.57          | 0.428       | -0.065      | 0.086       | -0.457      |
| G5rb  | 2.27           | 3.11           | 1.04          | 3.36          | 0.408       | -0.064      | 0.087       | -0.366      |
| E5rb  | 2.27           | 3.10           | 1.04          | 3.64          | 0.407       | -0.067      | 0.086       | -0.413      |
| Y5rb  | 2.27           | 3.13           | 1.05          | 3.87          | 0.399       | -0.053      | 0.087       | -0.450      |
| G6, G6cp | 2.78        | 2.88           | 1.41          | 3.11          | 0.434       | -0.230      | -0.230      | -0.230      |
| G6r, G6rcp | 2.27        | 3.10           | 1.15          | 3.36          | 0.407       | -0.201      | -0.201      | -0.201      |
Table 4: Parameters of the Λ-nucleon interaction (Λ-N). Strengths $V$ in MeV and ranges $r$ in fm of the central (c), spin-spin (ss), tensor (t) and spin-orbit (so) parts are listed. The central part $V_c^{(l=0)}(r)$ depends on the angular momentum $l = 0, 1$. The radial shapes are gaussian (G), exponential (E) and Yukawa (Y), respectively. The corresponding interactions reproduce the scattering lengths of the Nijmegen F model [23] (labels 1 and 3), the Nijmegen SC model [24] (labels 5 and 6), the reduced Nijmegen models (label r) and the modified potentials (label ra and rb) corresponding to Fra, Frb, SCa and SCb in table 3, see also the text. The interactions with labels 3 and 5 all include d-wave coupling in the analysis. Label c refer to interactions with a fictitious repulsive term in the central ($l=0$) part with parameters $V_{cr} = 550$ MeV and $r_{cr} = 0.53$ fm. Label p refers to interactions reproducing the same sets of low-energy data (including p-wave scattering lengths) as the interactions labeled G6 and G6r, respectively (see table 3).

| Λ-N   | $V_c^{(l=0)}$ | $r_c^{(l=0)}$ | $V_c^{(l=1)}$ | $r_c^{(l=1)}$ | $V_{ss}$ | $r_{ss}$ | $V_t$ | $r_t$ | $V_{so}$ | $r_{so}$ |
|--------|---------------|---------------|---------------|---------------|----------|----------|-------|-------|----------|----------|
| G1     | -24.354       | 1.4986        | -5.413        | 1.4986        | 1.931    | 1.5998   | 1.884 | 1.4986 | -2.347   | 1.4986   |
| Y1     | -50.833       | 0.8726        | -10.010       | 0.8726        | 2.529    | 1.0764   | 3.296 | 0.8726 | -4.109   | 0.8726   |
| G3     | -24.272       | 1.4972        | -24.272       | 1.4972        | 2.210    | 1.5486   | 2.053 | 1.7278 | -3.644   | 2.0488   |
| G1r    | -21.919       | 1.4986        | -5.413        | 1.4986        | 1.931    | 1.5998   | 1.884 | 1.4986 | -2.347   | 1.4986   |
| E1r    | -86.859       | 0.5678        | -28.426       | 0.5710        | 6.199    | 0.6503   | 7.696 | 0.5710 | -14.828  | 0.5710   |
| Y1r    | -44.093       | 0.8912        | -10.010       | 0.8726        | 1.977    | 1.2141   | 3.296 | 0.8726 | -4.109   | 0.8726   |
| G5     | -28.280       | 1.361         | -28.280       | 1.361         | 4.838    | 1.8221   | 4.364 | 1.4401 | -8.445   | 1.5902   |
| E5     | -97.431       | 0.5446        | -97.431       | 0.5446        | 25.420   | 0.6049   | 15.604 | 0.5740 | -4.964   | 1.0270   |
| Y5     | -56.291       | 0.8119        | -56.291       | 0.8119        | 4.684    | 1.4193   | 2.347 | 1.1822 | -2.214   | 1.7113   |
| G5r    | -25.452       | 1.361         | -25.452       | 1.361         | 4.838    | 1.8221   | 4.364 | 1.4401 | -8.445   | 1.5902   |
| E5r    | -98.351       | 0.5218        | -98.351       | 0.5218        | 18.896   | 0.6812   | 16.122 | 0.5532 | -2.565   | 1.2230   |
| Y5r    | -50.654       | 0.8145        | -50.654       | 0.8145        | 9.725    | 1.0234   | 0.922 | 1.5597 | -2.733   | 1.6423   |
| G5ra   | -25.185       | 1.3759        | -25.185       | 1.3759        | 5.122    | 1.8748   | 3.745 | 1.4837 | -9.056   | 1.5667   |
| E5ra   | -98.693       | 0.5236        | -98.693       | 0.5236        | 19.195   | 0.7137   | 16.780 | 0.5465 | -2.302   | 1.2634   |
| Y5ra   | -53.974       | 0.7977        | -53.974       | 0.7977        | 8.172    | 1.1573   | 0.318 | 2.2190 | -5.410   | 1.3073   |
| G5rb   | -25.506       | 1.3388        | -25.506       | 1.3388        | 5.443    | 1.8284   | 4.618 | 1.4062 | -8.935   | 1.5383   |
| E5rb   | -96.507       | 0.5172        | -96.507       | 0.5172        | 22.100   | 0.6752   | 17.087 | 0.5391 | -2.390   | 1.2215   |
| Y5rb   | -51.068       | 0.8006        | -51.068       | 0.8006        | 10.151   | 1.0598   | 0.444 | 1.9396 | -5.425   | 1.3061   |
| G6     | -28.644       | 1.3588        | -28.644       | 1.3588        | 4.694    | 1.8178   | 0    | -     | -        | -        |
| G6c    | -152.876      | 0.9853        | -152.876      | 0.9853        | 5.284    | 1.7492   | 0    | -     | -        | -        |
| G6cp   | -152.876      | 0.9853        | -236.971      | 0.8468        | 5.284    | 1.7492   | 0    | -     | -        | -        |
| G6r    | -25.723       | 1.3614        | -25.723       | 1.3614        | 4.574    | 1.8368   | 0    | -     | -        | -        |
| G6rc   | -151.130      | 0.9729        | -151.130      | 0.9729        | 5.094    | 1.7750   | 0    | -     | -        | -        |
| G6rep  | -151.130      | 0.9729        | -319.318      | 0.7722        | 5.094    | 1.7750   | 0    | -     | -        | -        |
Table 5: The contributing three-body channel quantum numbers for the hypertriton ground state. The upper part of the table refers to the Jacobi coordinates with the Λ-particle as spectator, while the lower part refers to the other two Jacobi systems where one of the nucleons is the spectator. The quantity $t_x$ is the total isospin of the two-particle system connected by the relative coordinate $x$.

| Channel | Config. | $l_x$ | $l_y$ | $L$ | $s_x$ | $S$ | $t_x$ |
|---------|---------|------|------|-----|------|-----|------|
| 1       | $^3S_1$ | 0    | 0    | 0   | 1    | 1/2 | 0    |
| 2       | $^1P_1$ | 1    | 1    | 0   | 0    | 1/2 | 0    |
| 3       | $^1P_1$ | 1    | 1    | 1   | 0    | 1/2 | 0    |
| 4       | $^3D_{1,2}$ | 2  | 0    | 2   | 1    | 3/2 | 0    |
| 5       | $^1S_0$ | 0    | 0    | 0   | 0    | 1/2 | 1/2  |
| 6       | $^3S_1$ | 0    | 0    | 0   | 1    | 1/2 | 1/2  |
| 7       | $^1P_1$ | 1    | 1    | 0   | 0    | 1/2 | 1/2  |
| 8       | $^1P_1$ | 1    | 1    | 1   | 0    | 1/2 | 1/2  |
| 9       | $^3P_j$ | 1    | 1    | 0   | 1    | 1/2 | 1/2  |
| 10      | $^3P_j$ | 1    | 1    | 1   | 1    | 1/2 | 1/2  |
| 11      | $^3P_j$ | 1    | 1    | 1   | 1    | 3/2 | 1/2  |
| 12      | $^3P_j$ | 1    | 1    | 2   | 1    | 3/2 | 1/2  |
| 13      | $^3S_1$ | 0    | 2    | 2   | 1    | 3/2 | 1/2  |
| 14      | $^3D_{1,2}$ | 2  | 0    | 2   | 1    | 3/2 | 1/2  |
Table 6: The $^3\Lambda H$ binding energy in MeV relative to the deuteron for different two-body interactions. The $\Lambda$-N interactions of the upper part (double separation line) reproduce the low-energy data of the F and SC Nijmegen models, while those of the lower part refer to the reduced ones (Fr and SCr), see tables 1-4. Labels $\alpha$ and $\beta$ indicate which channels of table 5 are included: "all except 13 and 14", and "only 1,4,5,6", respectively. The $\beta$ combination excludes all p- and d-states in the $\Lambda$-N subsystem. No greek labels means that all channels are included. The $^3\Lambda H$ and the $\Lambda$-deuteron rms radii, and the $\Lambda$-d $^2S$- scattering length (negative in our notation) and effective range are also showed. All lengths are in fm.

| N-N | $^3\Lambda$-N | $B_\Lambda$ | $<r^2>^{1/2}$ | $<r_{2d}^2>^{1/2}$ | $a_{\Lambda-d}(^2S)$ | $r_{\Lambda-d}(^2S)$ |
|-----|---------------|-------------|----------------|-------------------|---------------------|---------------------|
| G3  | G1            | 0.31        | 4.03           | 7.72              | -11.5               | 3.2                 |
| G3  | Y1            | 0.53        | 3.06           | 5.52              | -9.0                | 2.8                 |
| Y3  | G1            | 0.39        | 3.96           | 7.48              | -10.8               | 3.4                 |
| Y3  | Y1            | 0.65        | 3.21           | 5.75              | -8.9                | 3.2                 |
| GC1 | G3            | 0.22        | 4.53           | 8.79              | -13.0               | 3.3                 |
| EC1 | G3            | 0.23        | 4.36           | 8.41              | -12.5               | 3.2                 |
| GC1 | G5            | 0.37        | 3.71           | 6.95              | -10.5               | 3.1                 |
| GC1 | E5            | 0.44        | 3.44           | 6.35              | -9.7                | 2.9                 |
| GC1 | Y5            | 0.51        | 3.26           | 5.92              | -9.2                | 2.8                 |
| EC1 | G5            | 0.40        | 3.58           | 6.67              | -10.2               | 3.0                 |
| G1  | G1r           | 0.064       | 7.29           | 14.76             | -22.6               | 3.1                 |
| G1  | G1r ($\alpha$)| 0.063       | 7.32           | 14.81             | -22.7               | 3.1                 |
| G1  | G1r ($\beta$) | 0.055       | 7.62           | 15.44             | -24.1               | 3.1                 |
| G1  | Y1r           | 0.158       | 5.15           | 10.19             | -14.9               | 3.0                 |
| G1  | Y1r ($\alpha$)| 0.157       | 5.17           | 10.23             | -14.9               | 3.0                 |
| G1  | Y1r ($\beta$) | 0.117       | 5.40           | 10.72             | -15.2               | 3.0                 |
| E1  | E1r           | 0.114       | 5.86           | 11.72             | -17.2               | 3.0                 |
| E1  | E1r ($\alpha$)| 0.113       | 5.89           | 11.77             | -17.3               | 3.0                 |
| E1  | E1r ($\beta$) | 0.101       | 6.14           | 12.32             | -18.2               | 3.0                 |
| G2  | G1r           | 0.046       | 8.03           | 16.28             | -26.1               | 3.3                 |
| E2  | G1r           | 0.049       | 7.87           | 15.95             | -25.4               | 3.2                 |
| G3  | G1r           | 0.091       | 6.55           | 13.14             | -19.3               | 4.8                 |
| G3  | E1r           | 0.145       | 5.37           | 10.66             | -15.7               | 4.6                 |
| G3  | Y1r           | 0.172       | 5.01           | 9.89              | -15.6               | 4.9                 |
| GC1 | G5r           | 0.108       | 5.88           | 11.73             | -17.8               | 3.3                 |
| GC1 | E5r           | 0.161       | 5.09           | 10.01             | -14.9               | 3.2                 |
| GC1 | Y5r           | 0.174       | 4.95           | 9.71              | -14.4               | 3.2                 |
| GC1 | G5ra          | 0.166       | 5.06           | 9.96              | -14.8               | 3.3                 |
| GC1 | E5ra          | 0.235       | 4.41           | 8.53              | -12.7               | 3.1                 |
| GC1 | Y5ra          | 0.247       | 4.33           | 8.37              | -12.5               | 3.2                 |
| GC1 | G5rb          | 0.084       | 6.25           | 12.52             | -19.3               | 3.3                 |
| GC1 | E5rb          | 0.130       | 5.48           | 10.87             | -16.3               | 3.2                 |
| GC1 | Y5rb          | 0.149       | 5.23           | 10.32             | -15.3               | 3.2                 |
| EC1 | G5ra          | 0.176       | 4.87           | 9.53              | -14.1               | 3.3                 |
| EC1 | G5rb          | 0.100       | 5.96           | 11.89             | -18.1               | 3.2                 |
Table 7: The hypertriton binding energy in MeV relative to the deuteron for different two-body interactions. The N-N interaction is always the one labeled GC1, see table 1. The Λ-N interactions all reproduce the s-wave low-energy data of the Nijmegen models SC and SCr, see table 4. The interactions labeled G6cp and G6rcp reproduce the same s-wave low-energy data and p-wave scattering lengths as the interactions labeled G6 and G6r. The values with the (a) include the energy gain obtained adjusting the p-wave scattering lengths to resemble the result for the case without a repulsive core. The first column indicates the partial waves included in the computations.

| partial waves | Λ-N | B_Λ | Λ-N | B_Λ | Λ-N | B_Λ |
|---------------|-----|-----|-----|-----|-----|-----|
| s             | G6  | 0.442 | G6c | 0.303 | G6cp | 0.303 |
| s,p           | G6  | 0.484 | G6c | 0.365 | G6cp | 0.387 |
| s,p,d         | G6  | 0.490 | G6c | 0.396 | G6cp | 0.418^a |
| s,p,d,f       | G6  | 0.491 | G6c | 0.397 | G6cp | 0.419^a |
| s,p,d,f,g     | G6  | 0.491 | G6c | 0.397 | G6cp | 0.419^a |
| s             | G6r | 0.137 | G6rc | 0.083 | G6rcp | 0.083 |
| s,p           | G6r | 0.166 | G6rc | 0.119 | G6rcp | 0.136 |
| s,p,d         | G6r | 0.170 | G6rc | 0.125 | G6rcp | 0.152^a |
| s,p,d,f       | G6r | 0.170 | G6rc | 0.126 | G6rcp | 0.153^a |
| s,p,d,f,g     | G6r | 0.170 | G6rc | 0.126 | G6rcp | 0.153^a |
$f_n(\rho)$

- **gaussian**
- **exponential**
- **yukawa**

$\rho$ (fm)
