On “soft” physics at the CERN Large Hadron Collider

Andrei A. Arkhipov

State Research Center “Institute for High Energy Physics”
142280 Protvino, Moscow Region, Russia

Abstract

Three tightly inter-related topics have been discussed: the $pp(p\bar{p})$ total cross section; the single diffraction dissociation cross section; the $p(\bar{p})d$ total cross section and the defect of the total cross section in scattering from deuteron.

Keywords: Froissart theorem, total cross section, single diffraction dissociation, three-body forces

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“To reach great heights, one must possess great depth.”

1 Introduction: Global Features of the Total Cross Sections

I would like to discuss here some aspects of the so called “soft” physics, which is known as the physics of long-range strong interactions. More precisely, I will concentrate on three deeply inter-related topics where I have personal contribution to theoretical basis of our fundamental understanding the hadronic interactions:

• the $pp(p\bar{p})$ total cross section

• the single diffraction dissociation cross section

• the $p(\bar{p})d$ total cross section and the defect of the total cross section in scattering from nuclei.

Most of the material that I present here is taken from my works over the years. Some details may be found in recent article [1] and references therein.

The $pp$ total cross section is probably one that will measured at the LHC in the first. Fig. 1 shows the sets of data points for the $pp$ and $p\bar{p}$ total cross sections measured up to now.

Some time ago I have investigated whether a unified formula, similar to the Planck formula for black body radiation, can be obtained which simultaneously described hadron
Figure 1: The world data set on the $pp$ and $p\bar{p}$ total cross sections: the experimental (accelerator) data (270 points) on the proton-proton total cross-section (A); the experimental (accelerator) data (444 points) on the proton-antiproton total cross-section (B); the full experimental data (277 points) on the proton-proton total cross-section including the cosmic ray data (C); combined experimental data on the $pp$ and $p\bar{p}$ total cross sections (D). All presented data extracted from [2].
total cross sections in the whole range of energies from the most low energy to the most high one. It is remarkable that such formula can really be attainable, and here, I am going to show the results of our studies. But, first of all, let me say a few words instead of introduction.

From Fig. 1 it follows that the $pp$ and $p\bar{p}$ total cross sections rapidly increase at approaching the elastic scattering threshold. The large cross section near to the elastic scattering threshold was explained by the greatest physicist – Nobel Prize winner of the year 1967 Hans Albrecht Bethe without any additional assumption but only through a straightforward application of nonrelativistic quantum mechanics providing the cross sections of just the right magnitude. Bethe has proved that the cross section for the elastic scattering of slow nucleons is large due to the cross section is inversely proportional to nucleon velocity \[3\]

$$
\sigma_{tot}(s) = \sigma_{el}(s) \sim \frac{\sigma_0}{v} = \sigma_0 \sqrt{\frac{s}{s - 4m_p^2}}, \quad s \to 4m_p^2. \tag{1}
$$

Next, it has been revealed itself the dip structure in the proton-proton total cross-section at $\sqrt{s} \approx 2$ GeV compared to the proton-antiproton total cross-section where such structure is absent. At last, the increase of the $pp$ total cross section has been discovered at the CERN ISR \[4\] and then the effect of rising $p\bar{p}$ total cross sections was confirmed at the Fermilab accelerator \[5\] and CERN $Sp\bar{p}S$ \[6\].

Although nowadays we have in the framework of local quantum field theory a gauge model of strong interactions formulated in terms of the known QCD Lagrangian, its relations to the “soft” hadronic physics are far from desired. In spite of almost 40 years after the formulation of QCD we still cannot obtain from the QCD Lagrangian the answer to the question why all the hadronic total cross-sections grow with energy. We cannot predict total cross-sections in an absolute way starting from the fundamental QCD Lagrangian as well mainly because the effective QCD running coupling is not small and thus we cannot use perturbation theory. It is well known, e.g., that nonperturbative contributions to the gluon propagator influence the behaviour of “soft” hadronic processes and the knowledge of the infrared behaviour of QCD is certainly needed to describe the “soft” hadronic physics in the framework of QCD. Unfortunately, today we don’t know the whole picture of the infrared behaviour of QCD, we have some fragments of this picture though (see e.g. Ref. \[7\]). The understanding of the “soft” physics is of high interest because it has an intrinsically fundamental nature.

The behaviour of hadronic total cross sections at high energies is a wide and much discussed topic in high-energy physics community; see e.g. the proceedings of famous Blois Workshops. At present time there are a lot of different models which provide different energy dependencies of hadronic total cross sections at high energies and different predictions for a range of the LHC energies.

All different phenomenological models can conditionally be separated into two groups in according to two forms of strong interaction dynamics used: $t$-channel form and $s$-channel one. The fundamental quantity in the $t$-channel form of strong interaction dynamics is some set of Regge trajectories:

$$
t - \text{channel form} \quad \iff \quad \alpha_R(t),
$$
where subscript $R$ enumerates different Regge trajectories which are the poles in the t-channel partial wave amplitudes for the given process. The first group contains the Regge-type models with power-like, $s^{\alpha_P(0) - 1}$, behaviour of hadronic total cross sections. Here $\alpha_P(0)$ is an intercept of the supercritical Pomeron trajectory: $\alpha_P(0) - 1 = \Delta << 1$, $\Delta > 0$ ($\Delta = 0.0808$) is responsible for the growth of hadronic cross sections with energy. There are a lot of people who works with such a type of Regge-pole models; see excellent review articles and numerous conferences talks presented by Prof. Peter Landshoff from Cambridge (England) \[8\] and references therein.

Some part of scientific community works in the field of s-channel form of strong interaction dynamics. The fundamental quantity here is an effective interaction radius of fundamental forces:

\[
s - \text{channel form} \iff R_\alpha(s),
\]

where subscript $\alpha$ enumerates different types of hadrons and fundamental forces acting between them. The s-channel form of dynamics allows one to create a physically transparent and visual geometric picture of strong interactions for hadrons. It should be mentioned here the founders of geometric (impact) picture for strong interactions: the great theoretical physicist – Nobel Prize winner for the year 2005 Roy Glauber \[9\] and the greatest theoretical physicist – Nobel Prize winner for the year 1957 C.N. Yang \[10\]. I’d like to emphasize the attractive features of this form of strong interaction dynamics:

- universality (existence of pion with $m_\pi \neq 0$)

\[
R_\alpha(s) \sim \frac{r_\alpha}{m_\pi} \ln \frac{s}{s_0}, \quad s \to \infty
\]

- compatibility with the general principles of relativistic quantum theory.

My personal preference is in favour of the s-channel form of strong interaction dynamics. This is, first of all, related to the fact that the Regge phenomenology with the super-critical Pomeron exchange breaks down the fundamental principles of relativistic quantum theory such as unitarity, and this fact is often overlooked. In our opinion only this pathology of the super-critical Pomeron model is enough to reject the model from consideration. Moreover, accurate and complete analysis of experimental data on hadron total cross sections has shown that the super-critical Pomeron model is disfavoured from statistical point of view \[11\] \[12\], and experimental results from HERA \[13\] lead us to the same conclusion: The soft Pomeron phenomenology developed cannot incorporate the HERA data on structure function $F_2$ at small $x$ and data on total $\gamma^* p$ cross section from $F_2$ measurements as a function of $W^2$ for different $Q^2$. At last, Regge phenomenology without serious modifications cannot be applied to the description of experimental data on single diffractive dissociation cross sections in $p\bar{p}$ collisions; see e.g. discussion in \[14\].

2 Global Description of the $pp$ and $p\bar{p}$ Total Cross Sections

As mentioned above, we have tried to derive a possibly simple theoretical formula which would describe the global structure of $pp$ and $p\bar{p}$ total cross-sections in the whole range
Figure 2: The global description of the $pp$ and $p\bar{p}$ total cross sections: the full experimental (accelerator) data on the proton-proton total cross-section (A); the full experimental data on the proton-antiproton total cross-section (B); the proton-proton total cross-section at low energies (C); the full experimental data on proton-proton total cross-section including the cosmic ray data (D).
of energies available up today making such derivation in a maximally model independent way using general structures and general theorems in Quantum Field Theory. The fit to the experimental data with the formula was made, and it was shown that there is a very good correspondence of the theoretical formula to the existing experimental data obtained at the accelerators. Moreover, it turned out that there is a very good correspondence of the formula to all existing cosmic ray experimental data as well: The predicted values for \( \sigma_{\text{tot}}^{pp} \) obtained from description of all existing accelerators data are completely compatible with the values obtained from cosmic ray experiments \([15]\). The global description of the proton-proton and antiproton-proton total cross sections is shown in Fig. 2.

The theoretical formula describing the global properties of proton-proton and proton-antiproton total cross sections has the following quite a simple form \([16]\):

\[
\sigma_{\text{tot}}(s) = [1 + \chi(s)]\sigma_{\text{tot}}^a(s). \tag{2}
\]

That structure has been appeared as consistency condition to fulfil the unitarity requirements in two-particle and three-particle sectors simultaneously. In according to this structure the total cross section is represented in a factorized form. The first factor is responsible for the behavior of total cross section at low energies with the universal energy dependence at elastic threshold, it has a complicated resonance structure, and \( \chi(s) \) tends to zero at \( s \to \infty \). The other factor describes high energy asymptotic of total cross section, and it has the universal energy dependence predicted by the general theorems in Quantum Field Theory. For this factor one obtains

\[
\sigma_{\text{tot}}^a(s) = 2\pi B_{\text{el}}(s) + 4\pi(1 - \beta)R_0^2(s) = 2\pi B_{\text{el}}(s)[1 + 2\gamma(1 - \beta)], \tag{3}
\]

where \( B_{\text{el}}(s) \) is the slope of diffraction cone in forward elastic scattering processes

\[
B_{\text{el}}(s) = \left[ \frac{d}{dt} \ln \left( \frac{d\sigma_{\text{el}}}{dt}(s,t) \right) \right]_{t=0},
\]

\( R_2(s) \) – is the effective radius of two-nucleon forces related to the slope \( B_{\text{el}}(s) \) of diffraction cone by Equality \( B_{\text{el}}(s) = R_2^2(s)/2 \), \( B_{\text{sd}}(s) \) is the slope of diffraction cone for inclusive diffraction dissociation processes at a special value of missing mass

\[
B_{\text{sd}}(s) = \left[ \frac{d}{dt} \ln \left( \frac{d\sigma_{\text{sd}}}{dt}(s,t,M^2_X) \right) \right]_{M^2_X=2m_p^2} \bigg|_{t=0},
\]

\( R_0(s) \) is the effective radius of three-nucleon forces related to the slope \( B_{\text{sd}}(s) \) in the same way \( B_{\text{sd}}(s) = R_3^2(s)/2 \) as the effective radius of two-nucleon forces is related to the slope \( B_{\text{el}}(s) \) of diffraction cone in elastic scattering processes, \( \beta \) is slowly energy dependent dimensionless quantity from interval \( 0 \leq \beta \leq 1/4 \)

\[
\beta = \frac{x^2_{\text{inel}}}{4(1 + x^2_{\text{inel}})}, \quad x^2_{\text{inel}} = \frac{R_0^2(s)}{R_2^2(s)} = \frac{2B_{\text{sd}}(s)}{R_2^2(s)},
\]

\( R_d \) characterizes the internucleon distance where a two-nucleon bound state – the deuteron has arising. \( \beta \) tends to \( 1/4 \) at \( s \to \infty \) and \( \beta \ll 1 \) up to LHC energies, and \( \gamma = B_{\text{sd}}(s)/B_{\text{el}}(s) = R_0^2(s)/R_2^2(s) \) obviously. From the Froissart bound it follows \( \gamma < 2 \).
Figure 3: The total proton-antiproton cross sections versus $\sqrt{s}$ compared with formula (5). Solid line represents our fit to the data.

Using formula (2), we have made the global fit [10] to the experimental data on $pp$ and $p\bar{p}$ total cross sections shown above. The fitting procedure has been performed applying the following experimentally established and theoretically justified parameterizations

$$
\frac{d\sigma_{el}}{dt}(s,t) = \frac{d\sigma_{el}}{dt}(s,0) \exp[B_{el}(s)t], \quad \frac{2s}{\pi} \frac{d\sigma_{sd}}{dtdM_{X}^{2}}(s,t,M_{X}^{2}) = A(s,M_{X}^{2}) \exp[b(s,M_{X}^{2})t].
$$

(4)

At the first step, we have made a weighted fit to the experimental data on the proton-antiproton total cross sections in the range $\sqrt{s} > 10\, GeV$. The data were fitted with the function of the form predicted by the Froissart bound in the spirit of our approach. As mentioned above, a careful analysis of the experimental data and comparative study of the known characteristic parameterizations have revealed that statistically a “Froissart-like” type parameterization for proton-proton and proton-antiproton total cross sections is strongly favoured [11, 12]. So

$$
\sigma_{t}^{a} = a_0 + a_2 \ln^2(\sqrt{s}/\sqrt{s_0})
$$

(5)

where $a_0, a_2, \sqrt{s_0}$ are free parameters. We accounted for the experimental errors $\delta x_i$ (statistical and systematic errors added in quadrature) by fitting to the experimental points with the weight $w_i = 1/(\delta x_i)^2$. Our fit yielded

$$
a_0 = (42.0479 \pm 0.1086)\, mb, \quad a_2 = (1.7548 \pm 0.0828)\, mb,
$$

$$
\sqrt{s_0} = (20.74 \pm 1.21)\, GeV.
$$

The fit result is shown in Fig. 3. After that we have made a weighted fit to the experimental data on the slope of diffraction cone in elastic $p\bar{p}$ scattering. The experimental
Figure 4: Slope $B_{el}$ of diffraction cone in $p\bar{p}$ elastic scattering. Solid line represents our fit to the data.

points and the references, where they have been extracted from, are listed in [17]. The fitted function of the form

$$B_{el} = b_0 + b_2 \ln^2(\sqrt{s}/20.74),$$

which is also suggested by the asymptotic theorems of local quantum field theory, has been used. The value $\sqrt{s_0}$ was fixed at $\sqrt{s_0} = 20.74$ GeV from previous fit to the $p\bar{p}$ total cross sections data. Our fit yielded

$$b_0 = (11.92 \pm 0.15)\text{GeV}^{-2}, \quad b_2 = (0.3036 \pm 0.0185)\text{GeV}^{-2}.$$  

The fitting curve is shown in Fig. 4. From these fits in accordance with formula (3) one obtains

$$R_0^2(s)|_{\beta<<1} = \left[5.267 + 0.4137 \ln^2 \sqrt{s}/20.74\right] \text{(GeV}^{-2}).$$

At the final stage to build a global (weighted) fit to the all data on proton-antiproton total cross sections we have to choose an appropriate parameterization for the function $\chi(s)$ in R.H.S. of Eq. (2). In fact, we have for the function $\chi(s)$ the theoretical expression in the form

$$\chi(s) = \frac{C}{\kappa(s)R_0^2(s)},$$

where

$$\kappa^4(s) = \frac{1}{2\pi} \int_a^b dx \sqrt{(x^2 - a^2)(b^2 - x^2)[(a + b)^2 - x^2]}, \quad a = 2m_p, \quad b = \sqrt{2s + m_p^2 - m_p}.$$ 

It can be proved that $\kappa(s)$ has the following asymptotics.

$$\kappa(s) \sim \sqrt{s}, \quad s \to \infty; \quad \kappa(s) \sim \sqrt{s - 4m_p^2}, \quad s \to 4m_p^2.$$
Here, among other things, we have reproduced from the first principles the above mentioned Bethe’s result. On the other side, at high energy a Regge type asymptotic corresponding to secondary Reggeons exchange has been arisen from the first principles as well.

For simplicity, the global fit to the all data on proton-antiproton total cross sections was made with the function $\chi_{\bar{p}p}(s)$ of the form

$$\chi_{\bar{p}p}(s) = \frac{c}{\sqrt{s-4m_p^2R_0^3}s} \left( 1 + \frac{d_1}{\sqrt{s}} + \frac{d_2}{s} + \frac{d_3}{s^{3/2}} \right),$$

where $m_p$ is the proton mass, and $c, d_1, d_2, d_3$ are free parameters. The function $\chi_{\bar{p}p}(s)$ in Eq. (6) contained the preasymptotic terms as well needed to describe the region of middle (intermediate) energies – this is a price that we pay for simplicity. Our fit yielded

$$d_1 = (-12.12 \pm 1.023)\text{GeV}, \quad d_2 = (89.98 \pm 15.67)\text{GeV}^2,$$

$$d_3 = (-110.51 \pm 21.60)\text{GeV}^3, \quad c = (6.655 \pm 1.834)\text{GeV}^{-2}.$$

As seen, the experimental data on proton-proton total cross sections display a more complex structure at low energies than the proton-antiproton ones. We can describe this complex structure keeping the quantity $\sigma_{tot}^a(s)$ unchanged in Eq. (2) and taking the following expression for $\chi_{pp}(s)$

$$\chi_{pp}(s) = \left( \frac{c_1}{\sqrt{s-4m_p^2N^3R_0^3}} - \frac{c_2}{\sqrt{s-s_{thr}R_0^3}} \right) (1 + d(s)) + \text{Res}(s),$$

$$d(s) = \sum_{k=1}^{8} \frac{d_k}{s^{k/2}}, \quad \text{Res}(s) = \sum_{i=1}^{N} \frac{C^i_Rsl_R^iR_0^i}{\sqrt{s(s-4m_p^2N)}((s-s_i^R)^2 + s_i^R\Gamma_i^R)^2},$$

Our fit yielded

$$c_1 = (192.85 \pm 1.68)\text{GeV}^{-2}, \quad c_2 = (186.02 \pm 1.67)\text{GeV}^{-2},$$

$$s_{thr} = (3.5283 \pm 0.0052)\text{GeV}^2.$$

For the numerical values of the parameters $d_i$ and $C^i_R$ see original paper [16] and [18].

It should especially be emphasized that the global description of the proton-proton total cross-section revealed the new “threshold” $s_{thr} = 3.5283 \text{GeV}^2$, which is near the elastic scattering threshold. The position of the new “threshold” has been determined by the fit with a high accuracy. Note, all available experimental data on the proton-proton total cross-section lie above this “threshold”. One could imagine an appearance of the new “threshold” as a manifestation of a new unknown particle:

$$\sqrt{s_{thr}} = 2m_p + m_L, \quad m_L = 1.833 \text{MeV}.$$

This particle was named [18] as $L$-particle from the word lightest. Of course, the natural questions have been arisen. What is the physical nature and dynamical origin of $L$-particle? Could $L$-particle be related to the experimentally observed diproton resonances
spectrum? ..., and all that. Some discussing that questions of fundamental importance may be found in our articles [19, 20].

At the LHC we predict

$$\sigma_{pp}^{\text{tot}}(\sqrt{s} = 14 \text{ TeV}) = 116.53 \pm 3.52 \text{ mb}.$$  \hspace{1cm} (9)

Theoretical uncertainty in (20) has been calculated by one deviation in parameter $a_2$ in Eq. (5). It should to be compared to the best even though very crude estimate based on

Pomeron Physics and QCD [8]

$$\sigma^{LHC} = 125 \pm 35 \text{ mb},$$  \hspace{1cm} (10)

presented by Peter Landshoff at the Conference ”Diffraction 2008” [8].

### 3 On Single Diffractive Dissociation Cross Section

Concerning the single diffractive dissociation cross section, we have found the bound (like Froissart bound!)

$$\sigma_{tot}^{sd}(s) < \text{Const}, \quad s \to \infty,$$

and a good theoretically justified parameterization for the total single diffractive dissociation cross section looks as follows [14]

$$\sigma_{tot}^{sd}(s) = A_0 + A_2 \ln^2(\sqrt{s}/\sqrt{s_0})/R^2_0(s),$$  \hspace{1cm} (11)

where $A_0, A_2$ are free parameters to be found from the fit to the experimental data on $\sigma_{tot}^{sd}$. The fit yielded

$$A_0 = 28.05 \pm 0.66 \text{ mbGeV}^{-2}, \quad A_2 = 4.99 \pm 0.57 \text{ mbGeV}^{-2}.$$  \hspace{1cm}

The fit result is shown in Fig. 5 [21]. As seen, the fitting curve, as in the previous fit [14], goes excellently over the experimental points of the CDF group at Fermilab [22]. The experimental data points for the total single diffraction dissociation cross sections have been extracted from different papers and collected in [21]; see references therein.

One important note should be taken here. The main point of our approach is that the fundamental three-body forces are responsible for the dynamics of particle production processes of inclusive type. In fact, we have found a formula expressing one-particle inclusive cross section through the amplitude of three-body forces. Our consideration revealed several fundamental properties of one-particle inclusive cross-sections in the region of diffraction dissociation. In particular, it was shown that the slope of the diffraction cone in single diffraction dissociation process $pp \to pX$ is related to the effective radius of three-nucleon forces in the same way as the slope of the diffraction cone in elastic scattering process $pp \to pp$ is related to the effective radius of two-nucleon forces. As was demonstrated above, the effective radii of two- and three-nucleon forces, which are the characteristics of elastic and inelastic interactions of two nucleons, define the structure of the total cross-sections in a simple and physically clear form.
Some time ago many high energy physicists suggested that the increase of total cross-sections was due to the increase of single diffraction dissociation cross sections. Now we understand that this suggestion is wrong and, moreover, we know why this is wrong. We have established that the phenomenon of exceedingly moderate energy dependence of single diffraction dissociation cross-sections on $s$ discovered by CDF at Fermilab is due to effect of screening of three-body forces by two-body ones in regime of unitarity saturation of two- and three-nucleon forces at Fermilab Tevatron energies. In this context, the CDF data on single diffraction dissociation cross sections represent the significant experimental result which has to be tested at the LHC. At the LHC we predict

$$\sigma_{sD}^{sd}\text{tot}(\sqrt{s} = 14 \text{ TeV}) = 10.51 \pm 1.06 \text{ mb} \tag{12}$$

Here theoretical uncertainty in (12) has also been calculated by one deviation in parameter $A_2$ in Eq. (11).

4 Total Cross Section in Scattering from Deuteron

Being inspired with the success in global description of the proton-proton and proton-antiproton total cross sections we have attempted to carry out the similar global description for the proton-deuteron and antiproton-deuteron total cross sections.

It is well known that the total cross section in scattering from deuteron can be expressed by the formula

$$\sigma_{tot}^d = \sigma_{tot}^p + \sigma_{tot}^n - \delta\sigma_{tot}^d, \tag{13}$$

where $\sigma_{tot}^p$, $\sigma_{tot}^n$, $\sigma_{tot}^d$ are the total cross sections in the scattering of the incident particle from the deuteron, proton, and neutron, and $\delta\sigma_{tot}^d$ is called the defect of the total cross section in scattering from the deuteron.
In the framework of the diffraction theory, Glauber obtained an elegant expression for the defect of the total cross section in scattering from the deuteron,

\[ \delta\sigma_{\text{tot}}^d = \delta\sigma_G^d = \frac{\sigma_{\text{tot}}^p \cdot \sigma_{\text{tot}}^n}{4\pi} \cdot \frac{1}{\langle r^2 \rangle_d} \]  

which is called the Glauber correction. In formula (14), \( \langle r^{-2} \rangle_d \) denotes the average inverse square internucleon distance in the deuteron.

Glauber found an attractive physical interpretation of the correction that he obtained, and showed that it is related to configurations in the deuteron in which one nucleon is in the shadow of another nucleon, and describes the eclipse effect well known from data of astronomical observations on decrease in luminosity of binary stars during an eclipse. For this reason, this correction is often called the shadowing correction or the screening effect. Moreover, it is necessary to note the remarkable fact that formula (14) can be obtained from extremely simple, almost semiclassical considerations presented by Glauber in the introduction to his famous article [23].

However, it is quite clear that the Glauber correction and diffraction formalism proposed by Glauber to derive this result may theoretically be justified and understood only in the context of nonrelativistic quantum mechanics. At high energies where the processes of particle production are possible we have to apply the relativistic quantum theory. Besides, the experiments carried out at the accelerators have also shown that the Glauber correction (14), although yielding a correct order of magnitude, results in an underestimated value for the defect of the total cross section in scattering from the deuteron.

We have considered the problem of scattering from the deuteron in a detail using dynamical equations of the Bethe–Salpeter formalism for a system of three particles in quantum field theory. The first, and very important, circumstance revealed was related to the fact that consistent consideration of the three-body problem in the framework of relativistic quantum theory necessitates that the dynamics of the three-particle system should include, along with pair (two-particle) interactions, the fundamental three-particle forces as well which cannot be expressed in terms of pair interactions. It was established that fundamental three-particle forces are related to specific inelastic interactions in two-particle subsystems and determine the dynamics of special inelastic processes of interaction of two particles known as one-particle inclusive reactions. Making quite general assumptions, it was possible to calculate the contribution of three-particle forces to the total cross section in scattering from the deuteron and obtain a very simple and elegant formula with a clear physical interpretation for the defect of the total cross section in scattering from the deuteron. We have found that the defect of the total cross section in scattering from the deuteron can be represented as the sum of two terms

\[ \delta\sigma_{\text{tot}}^d = \delta\sigma_{\text{el}}^d + \delta\sigma_{\text{inel}}^d, \]  

where the quantity \( \delta\sigma_{\text{el}}^d \) was called the elastic defect, and \( \delta\sigma_{\text{inel}}^d \) the inelastic defect. For these quantities the following representations have been derived:

\[ \delta\sigma_{\text{el}}^d = 2 a_{\text{el}}(x_{\text{el}}^2) \sigma_{\text{el}}^N, \quad \delta\sigma_{\text{inel}}^d = 2 a_{\text{inel}}(x_{\text{inel}}^2) \sigma_{\text{sd}}^N, \]  

(16)
It was assumed in our considerations that for both elastic and inelastic interactions of the incident hadron with nucleons of the deuteron, the proton and the neutron are dynamically indistinguishable; i.e., corresponding dynamic characteristics of the proton and the neutron are similar. For example, \( \sigma_{el}^p = \sigma_{el}^n = \sigma_{el}^N \), \( B_{el}^p = B_{el}^n = B_{el}^N \) and so on. This proposition is quite justified if interactions occur at sufficiently high energies. It is clear that at very low energies, it is necessary to take into account that the proton and the neutron have different masses and electric charge; however, formula (16) admits a natural modification for this case.

It is natural to call the functions \( a_{el} \) and \( a_{inel} \) in the right-hand side of formula (16) elastic and inelastic deuteron structure functions, respectively. These functions have clear physical meaning; see details in [1]. It was remarkable that we succeeded in obtaining extremely simple formulas for the structure functions \( a_{el} \) and \( a_{inel} \) which have the following form

\[
a_{el}(x^2) = \frac{x^2}{1 + x^2}, \quad a_{inel}(x^2) = \frac{x^2}{(1 + x^2)^{3/2}}.
\]

(17)

Obviously, these formulas display new fundamental scaling regularities in processes of interaction of composite nuclear systems.

As seen, the structure functions \( a_{el} \) and \( a_{inel} \) have a quite different behavior: \( a_{el} \) is a monotonically increasing function of the argument in the semiinfinite interval \( 0 \leq x^2 < \infty \), and the domain of its values is bounded by the interval \( 0 \leq a_{el} \leq 1 \). The function \( a_{inel} \) first increases, reaches its maximum for \( x^2 = 2 \), and then decreases, vanishing at infinity; in this case, the domain of its values lies in the interval \( 0 \leq a_{inel} \leq 2/3\sqrt{3} \). Of course, the difference between the behaviors of the structure functions \( a_{el} \) and \( a_{inel} \) results in far-reaching physical consequences. For example, at ultrahigh energies, corresponding to \( x^2 \to \infty \), we find that the inelastic defect vanishes and the elastic defect tends to two times the value of the total elastic cross section for scattering on the nucleon, whereas the total cross section for scattering on the deuteron approaches two times the value of the nucleon total absorption cross section. Therefore, at ultrahigh energies, \( A \) dependence of total cross sections for scattering on nuclei should be recovered, with the difference that the fundamental quantity in front of \( A \) is the nucleon total absorption cross section, rather than the total cross section for scattering on the nucleon,

\[
\sigma_{total}^A = A \sigma_{inel}^N, \quad s \to \infty.
\]

(18)

A very interesting aspect of our consideration, that the inelastic defect in the total cross section for scattering on the deuteron is a manifestation of fundamental three-body forces. Clearly, the Glauber formula appears as a special case, if the inelastic defect is neglected, and for the elastic structure function, the following approximation (valid for \( x^2 \ll 1 \)) is used: \( a_{el}(x^2) \approx x^2 \), where we have to take into account that \( \sigma_{el}^N \approx (\sigma_{el}^N)^2 / 16\pi B_{el} \).

Of course, of special interest to us was the comparison of the theoretical results we obtained with available experimental data on total cross sections for the scattering of protons and antiprotons on deuterons. Figures 6 and 7 show the results of this comparison. We used the global description of \( pp \) and \( p\bar{p} \) total cross sections and cross sections for
diffraction dissociation, taking into account the most recent experimental data obtained by the CDF collaboration at FNAL. It should be added that the comparison with experimental data on total cross sections for the scattering of protons and antiprotons on deuterons was carried out in two stages. At the first stage, theoretical calculations were compared with experimental data on total cross sections for the scattering of antiprotons on deuterons under the assumption that $R_d^2$ is the only free parameter whose value should be determined by fitting experimental data. As a result of statistical analysis, the following value of $R_d^2$ was obtained: $R_d^2 = 66.61 \pm 1.16 \text{ GeV}^{-2}$. The latest experimental measurements of the deuteron matter radius indicate that $r_{d,m} = 1.963(4) \text{ fm}$ [24], which yields $r_{d,m}^2 = 3.853 \text{ fm}^2 = 98.96 \text{ GeV}^{-2}$. The value of $R_d^2$ obtained by us satisfies the relation $R_d^2 = 2/3 r_{d,m}^2$. Besides, in data analysis we have used a simplified assumption as $\sigma_{tot}^{\bar{p}d} = \sigma_{tot}^{pp}$. The results of theoretical calculations are shown in Fig. 6 up to Tevatron energies (FNAL). At the second stage, experimental data on total cross sections for the scattering of protons on deuterons were compared with theoretical calculations in which the value of $R_d^2$ was taken as equal to that obtained at the first stage from analysis of data on $\bar{p}d$ total cross sections. In other words, the curve in Fig. 7 corresponds to theoretical calculations carried out using formulas (13), (15), (16) and (17) without any free parameter. In this figure, the results of theoretical calculations are also shown up to Tevatron energies. As in the previous fit we supposed $\sigma_{tot}^{pN} = \sigma_{tot}^{pp}$ and $\sigma_{tot}^{pp}$ was taken from our global description of proton-proton total cross sections. We have also assumed that $B_{el}^{pN} = B_{el}^{pp} \equiv B_{el}$. As can be seen, Figs. 6 and 7 show quite a remarkable correspondence of the theory to the experimental data even though the resonance region requires a more careful consideration because simplified assumptions we used cannot be justified at low energies. Nevertheless, this is a remarkable fact that the dip structure of the proton-proton total cross section at low energies manifests itself in the proton-deuteron total cross section too.
Figure 7: The proton-deuteron total cross-section compared with the theory without any free parameters. The experimental data points extracted from [2].

At the LHC we predict

\[ \sigma_{d_{\text{tot}}}^{d}(\sqrt{s} = 14 \text{ TeV}) = 206.86 \text{ mb}. \] (19)

5 \hspace{1cm} \textbf{On the Defect of the Total Cross Section in Scattering from Deuteron}

Figure 8 shows the results of our theoretical calculations of elastic and inelastic defects of the total cross section for the scattering of (anti)protons on deuterons in the energy range 10 - 2000 GeV. It follows from these calculations that the value of the elastic defect is about 10% of the total nucleon-nucleon cross section, and the value of the inelastic defect is about 10% of that of elastic defect, i.e., approximately 1% of the total nucleon-nucleon cross section. Figuratively speaking, while the elastic defect represents a fine structure, the inelastic defect should be attributed to a hyperfine structure in the total cross sections for scattering on the deuteron. In our approach, the inelastic defect is related to the manifestation of fundamental three-body forces; therefore, in this sense, three-body forces play the role of fine tuning in the dynamics of the relativistic three-particle system. We should give credit to experimentalists who have created experimental setups capable of achieving the precision of measurement that makes it possible to discriminate between inelastic defects in total cross sections for the scattering of particles at high energies. Along these lines, we believe that further experimental high precision measurements of proton-deuteron total cross sections at the LHC would also be extremely desirable.

As was already noted above, the maximum value of the inelastic defect is reached at 

\[ x_{\text{inel}}^{2} = 2 \left( x_{\text{inel}}^{2} \equiv R_{0}^{2}/R_{d}^{2} \right). \]

In other words, the energy at which the inelastic defect reaches its maximum value is determined from the equation \( R_{0}^{2}(s_{\text{max}}) = 2R_{d}^{2} \). Calculations based on our analysis of available experimental data yield \( \sqrt{s_{\text{max}}} = 9.01 \cdot 10^{8} \text{ GeV} = 901 \text{ PeV} \). Obviously, such energies cannot be achieved at either existing or designed accelerators.
However, manifestations of this effect can be sought in phenomena observed in ultrahigh energy cosmic rays. This is the subject of a separate investigation; see, in particular, [25]. We note, however, that $s_{\text{max}}$ has a clear physical meaning, in that it separates two energy regions: the energy region $s < s_{\text{max}}$, in which the effective radius of three-particle forces does not exceed the deuteron size, or, more precisely, $1/2 R_0^2(s) < R_d^2$, and the energy region $s > s_{\text{max}}$, in which the effective radius of three-particle forces becomes larger than the deuteron size, $1/2 R_0^2(s) > R_d^2$. It should be especially underlined that the unitarity requirement results in the suppression of the inelastic defect at ultrahigh energies in such a way that only the elastic part of the total defect remains at asymptotically infinitely high energies. The existence of $s_{\text{max}}$, at which the inelastic defect begins to be suppressed, seems to us a very important characteristic of fundamental dynamics. Figure 9 shows the inelastic defect in the region of the maximum, calculated theoretically by us.

Our comparison of the theory with experimental data on total nucleon-deuteron cross sections shows that, in the description of particle scattering on the deuteron at high energies, it is sufficient to take into account only nucleonic degrees of freedom in the deuteron. A loosely bound two-nucleon system, the deuteron looks as though the clusterization of quarks into nucleons is not destroyed, even when nucleons come close to each other. Nucleons that are close to each other in the deuteron do not lose their individuality and, therefore, it is not necessary to introduce unspecified six-quark configurations in the deuteron. The structure for the defect of the total cross section for scattering on the deuteron we obtained corresponds to this pattern. The general formalism of quantum field theory admits representation of the particle scattering dynamics on a composite system in terms of the fundamental dynamics of particle scattering on isolated constituents and the structure of the composite system. Probably one of the most pleasant findings was that comparison with experimental data on proton-deuteron and antiproton-deuteron to-
Figure 9: The three-body forces contribution (inelastic screening) to the (anti)proton-deuteron total cross-section calculated with the theory in the range up to Planck scale.

Total cross sections at high energies already showed very good agreement of the theory with experiment. As a matter of fact, the experimental measurement of the proton-deuteron total cross section at the LHC might be as a crucial test to discriminate different models for the proton-proton total cross section proposed.

At the LHC we predict
\[ \delta \sigma_{\text{tot}}^d (\sqrt{s} = 14 \text{ TeV}) = 26.19 \text{ mb}, \quad \delta \sigma_{\text{el}}^d = 23.88 \text{ mb}, \quad \delta \sigma_{\text{inel}}^d = 2.31 \text{ mb}. \] (20)

6 “Soft” Physics Observables at the LHC

Our predictions at the LHC regarding the “soft” physics observables here discussed are collected in Table 1.

| \( \sqrt{s}(\text{TeV}) \) | \( \sigma_{\text{el}}^p (\text{mb}) \) | \( B_{\text{el}}(\text{GeV}^{-2}) \) | \( \sigma_{\text{el}}^d (\text{mb}) \) | \( \sigma_{\text{el}}^e (\text{mb}) \) | \( \sigma_{\text{el}} (\text{mb}) \) | \( \delta \sigma_{\text{el}}^d (\text{mb}) \) | \( \delta \sigma_{\text{el}}^e (\text{mb}) \) | \( \delta \sigma_{\text{inel}}^d (\text{mb}) \) |
|---|---|---|---|---|---|---|---|---|
| 1.80 | 77.01 | 17.97 | 10.86 | 9.44 | 140.75 | 13.27 | 11.82 | 1.45 |
| 7.00 | 101.52 | 22.21 | 23.71 | 10.22 | 182.05 | 20.99 | 18.97 | 2.02 |
| 10.00 | 109.03 | 23.51 | 25.84 | 10.38 | 194.51 | 23.55 | 21.38 | 2.17 |
| 14.00 | 116.53 | 24.81 | 27.97 | 10.51 | 206.86 | 26.19 | 23.88 | 2.31 |

Table 1: Theoretical predictions for the “soft” physics observables at the LHC.

7 Discussion

The sequence of our investigations, with rather cumbersome derivations and complicated tiresome computations, can be traced following the references cited and references therein. Here, I would only like to tell you a little bit about our long way to the Everest.
As a starting base the Quantum Field Theory with local fields satisfying the standard set of axioms has been chosen. We have built, in the first, the constructive Bethe-Salpeter formalism in any \( n \)-particle sector \( (n \geq 2) \) without having the use of perturbation theory. In fact, we have restricted to two-particle and three-particle sectors in detail. It turned out in relativistic quantum theory the dynamics of three-particle system contained with a necessity the fundamental three-body forces. Actually, the fundamental three-body forces take place in any multiparticle system where the number of particles is greater than two. The three-body forces represent the defect of total three-particle interaction over the sum of two-body forces and describe the true three-body interactions. Three-body forces are an inherent connected part of total three-particle interaction which cannot be reduced to the sum of pair interactions.

The Great Froissart Theorem was as a Guiding Star in our studies. The Froissart bound for the total cross section of two-body reaction \( a + b \rightarrow a + b \) can be written in the form \([26]\)

\[
\sigma_{ab}^{\text{tot}}(s) < 4\pi R_2^2(s).
\] (21)

Here the quantity \( R_2(s) \) has a strong mathematical definition with a clear and transparent physical meaning; see details in Ref. \([26]\) and references therein

\[
R_2(s) \overset{\text{def}}{=} \frac{L}{|q|} = \frac{2\sqrt{s} \ln \tilde{P}_2(s)}{\sqrt{2\epsilon(s)\lambda(s,m_a^2,m_b^2)}} = \frac{\ln \tilde{P}_2(s)}{\sqrt{t_0}} \quad (22)
\]

\[
\simeq \frac{9}{4\sqrt{t_0}} \ln(s/s_0) = \frac{9}{8m_\pi} \ln(s/s_0), \quad s \gg s_0, \quad (t_0 \equiv 4m_\pi^2). \quad (23)
\]

The pion mass \( m_\pi \) in R.H.S. of Eq. \((23)\) appears from the nearest \( t \)-channel threshold, \( s_0 \) is a determinative scale usually extracted from a fit to experimental data. The quantity \( R_2(s) \) is named as the effective radius of two-body forces, and it is simply related with the experimentally measurable quantity which is the slope of diffraction cone \( B_{el}(s) \) in elastic forward scattering for the two-body reaction

\[
B_{el}(s) = \frac{1}{2}R_2^2(s). \quad (24)
\]

It should especially be emphasized that the quantity \( R_2(s) \) accumulates all information concerning polynomial boundedness and analyticity of the two-body reaction amplitude in a topological product of complex \( s \)-plane with the cuts \( (s_{thr} \leq s \leq \infty, u_{thr} \leq u \leq \infty) \) except for possible fixed poles and circle \(|t| \leq t_0\) in complex \( t \)-plane, where \( s, t, u \) are Mandelstam variables. That analyticity is proved in the framework of axiomatic Quantum Field Theory, and this is enough to save and extend the fundamental Froissart result previously obtained at a more restricted Mandelstam analyticity in the framework of the analytic \( S \)-matrix theory. The cornerstone in that extension has to be referred to Harry Lehmann \([27]\) who proved that two-body elastic scattering amplitude is analytic function of \( \cos \theta \), regular inside an ellipse in complex \( \cos \theta \)-plane with center at the origin. The fundamental Jost-Lehmann-Dyson representation – brilliant quintessence of general principles in the theory of quantized fields – especially Dyson’s theorem for a representation of causal commutators in local Quantum Field Theory \([28, 29, 30]\) and not more have been used by Harry Lehmann. From the fundamental result of Harry Lehmann it follows
that the partial wave expansions which define physical scattering amplitudes continue to converge for complex values of the scattering angle, and define uniquely the amplitudes appearing in the nonphysical region of non-forward dispersion relations. In fact, expansions converge for all values of momentum transfer for which dispersion relations have been proved.

The Froissart bound represents a physically tangible consequence from abstract mathematical structures given by general axioms in the theory of quantized fields. That is why, the Froissart bound is often considered as intrinsic property of the theory of quantized fields.

In our opinion, the bound (21) represents the most rigorous mathematical formulation of the holographic principle [31] which is widely discussed in the recent literature. Thus the holographic principle has been incorporated in the general scheme of axiomatic Quantum Field Theory and resulted from the general principles of the theory of quantized fields [26].

From the Froissart bound in the case of the two-body forces saturated unitarity one obtains

\[ \sigma_{ab}^{\text{tot}}(s) = 4\pi R_2^2(s) \simeq C_{ab} \ln^2(s/s_0), \quad s \to \infty, \]  

(25) where

\[ C_{ab} = \frac{4\pi \cdot 81}{64 m^2_{\pi}} = \frac{15.9}{m^2_{\pi}} \simeq 339 \text{ mb}. \]  

(26)

Certainly, the value 339 mb for the constant \( C_{ab} \) is too large to fit to available experimental data, and this is a week place of the general theory. However, we cannot exclude that the two-body forces may not saturate unitarity in the range of reachable energies at now working accelerators. On the other hand, it is quite clear that Eq. (21) really represents the bound only, and we have to find the physical arguments to compare the general theory with experiment. Actually, we have found an elegant way for structurization of the constant \( C_{ab} \) in R.H.S. of Eq. (25) if we have taken into account not only two-particle but three-particle unitarity as well.

The Froissart bound in any \( n \)-particle sector \((n \geq 2)\) can be written in the following form:

\[ \text{Im} \mathcal{F}_n(s; \cos \omega = 1) < J_n(s) S_{D-1}[R_n(s)]^{D-1}, \]  

(27) where \( \mathcal{F}_n(s; \cos \omega) \) is the amplitude of \( n \)-body forces, \( \cos \omega = \mathbf{e}' \cdot \mathbf{e}, \) \( \mathbf{e} \) and \( \mathbf{e}' \) are two unit vectors on \((D - 1)\)-dimensional sphere \( S_{D-1} \), which characterize the initial and final states of \( n \)-particle system, dimensionality \( D \) of multidimensional space is related to the number of particles \( n \) by the equation \( D = 3n - 3 \), \( J_n(s) \sim s^{n/2} \) is \( n \)-particle flux, \( S_{D-1} = 2\pi^{D/2}/\Gamma(D/2) \) is a surface of \((D - 1)\)-dimensional unit sphere, \( R_n(s) \sim \ln(s/s'_0) \) is the effective radius of \( n \)-body forces; see details in [26].

As mentioned above, the structure given by Eq. (2) has been appeared as consistency condition to fulfil the unitarity requirements in two-particle and three-particle sectors simultaneously. It is a non-trivial fact that the constant in R.H.S. of Eq. (3), staying in front of effective radius of two-nucleon forces, is 4 times smaller than the constant in the Froissart bound. But this is too small to correspond to the experimental data if we use the experimental data on \( B_{el}(s) \). The second term in R.H.S. of Eq. (3) fills an emerged gap.
It is interesting to note that in the case of the two-body forces saturated the Froissart bound, taking into account that $\sigma_{el} = \sigma_{el}^2 / 16\pi B_{el}$ ($\rho_{el} = 0$), and $B_{el} = R_{2}^2 / 2$, one obtains

$$\sigma_{el}^t (s) = 4\pi R_{2}^2 (s) \Rightarrow \sigma_{el}^t (s) = \frac{1}{2} \sigma_{el}^t (s), \quad s \to \infty. \quad (28)$$

Thus we come to the following statement: \textit{The two-body forces saturated the Froissart bound saturate the Pumplin bound as well.}

Of course, following the general scheme of the local quantum field theory, we must not forget about the crossing

$$\sigma_{tot}^{pp}(s) = \sigma^{(+)}(s) + \sigma^{(-)}(s), \quad \sigma_{tot}^{pp}(s) = \sigma^{(+)}(s) - \sigma^{(-)}(s),$$

$$\sigma^{(+)}(s) = \frac{1}{2} \{ \sigma_{tot}^{pp}(s) + \sigma_{tot}^{pp}(s) \}, \quad \sigma^{(-)}(s) = \frac{1}{2} \{ \sigma_{tot}^{pp}(s) - \sigma_{tot}^{pp}(s) \}.$$

8 Conclusion

As was demonstrated above, the effective radii of two- and three-body forces being the characteristics of elastic and inelastic interactions in two-body subsystems have been combined in a special form determining the nontrivial dynamical structure for the total cross section clearly confirmed by the experimental data. The further experimental confirmation of this dynamical structure for the total cross section at the CERN Large Hadron Collider is a good task.

It would be very important to experimentally investigate the “soft” physics by the measurements of all above mentioned observables simultaneously at one and the same device which the CERN Large Hadron Collider is.

We believe that further experimental high precision measurements of proton-deuteron (in general proton-nucleus) total cross sections at the LHC would also be extremely desirable.

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