Reciprocating and Screw Compressor semi-empirical models for establishing minimum energy performance standards

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Abstract. The efficiency bar for a Minimum Equipment Performance Standard (MEPS) generally aims to minimize energy consumption and life cycle cost of a given chiller type and size category serving a typical load profile. Compressor type has a significant chiller performance impact. Performance of screw and reciprocating compressors is expressed in terms of pressure ratio and speed for a given refrigerant and suction density. Isentropic efficiency for a screw compressor is strongly affected by under- and over-compression (UOC) processes. The theoretical simple physical UOC model involves a compressor-specific (but sometimes unknown) volume index parameter and the real gas properties of the refrigerant used. Isentropic efficiency is estimated by the UOC model and a bi-cubic, used to account for flow, friction and electrical losses. The unknown volume index, a smoothing parameter (to flatten the UOC model peak) and bi-cubic coefficients are identified by curve fitting to minimize an appropriate residual norm. Chiller performance maps are produced for each compressor type by selecting optimized sub-cooling and condenser fan speed options in a generic component-based chiller model. SEER is the sum of hourly load (from a typical building in the climate of interest) and specific power for the same hourly conditions. An empirical UAE cooling load model, scalable to any equipment capacity, is used to establish proposed UAE MEPS. Annual electricity use and cost, determined from SEER and annual cooling load, and chiller component cost data are used to find optimal chiller designs and perform life-cycle cost comparison between screw and reciprocating compressor-based chillers. This process may be applied to any climate/load model in order to establish optimized MEPS for any country and/or region.

1. Introduction

Energy efficiency in buildings is a widely researched topic. Demand side management outperforms other energy, cost and emission reduction measures [6], but is it really true in developing countries where no MEPS and rating systems are practiced? United Arab Emirates (UAE) has hot and humid weather conditions for most of the year, which translates to cooling requirement of 40-55% of annual electricity consumption [3, 10]. Executive Affairs Authority in Abu-Dhabi is motivated to establish minimum energy performance standards by exploring various chiller design options appropriate to UAE conditions, development of various conventional and innovative design configurations are tested. Flexible heat pump models with component based sub-models for evaporator, compressor and condensers are developed so various design configurations can be extensively tested and compared for given boundary conditions. Compressors are the most intensive energy consuming components in chiller systems, hence evaluation of various compressor types is considered as a potential design option. This paper compares screw and reciprocating compressor semi-empirical models and their use in establishing MEPS for chillers.

Literature related to heat pump and components models for chiller systems is extensively reviewed by Armstrong et al., Zakula et al., Qureshi et al., Javed et al, Tauha et al. [1, 2, 5, 13, 19, 22], so literature review for this paper will specially focus on screw and reciprocating compressor modeling and applications. Compressors can be broadly classified into positive displacement and dynamic compressor types [8]. Positive displacement compressors, where compression is achieved through reduction of compression chamber volume, types include reciprocating, screw, scroll and rotary
compressors. Dynamic compression is achieved through kinetic energy transfer; centrifugal compressor is the most common type [14]. Development of Compressor maps is central to compressor modelling, such maps may involve physics based, empirical or semi-empirical models [1, 18].

Sjoholm presents some of the earlier work related to variable volume ratio and capacity control in screw compressors [20,9]. Le at al. developed screw chiller compressor model based on simple efficiency loss calculation [16, 17]. Fujiwara et al. developed empirical relation for flow and heat transfer characteristics of screw compressor; furthermore it models the effects of oil on gas leakage and cooling [11, 12]. Stosic et al. developed a thermodynamic performance simulation model which accounts for detailed modelling of oil injection in a screw compressor [21]. Reciprocating compressors are also extensively described in literature, both simple and detailed models. The Air Conditioning and Refrigeration Institute developed a bi quadratic empirical model to represent mass flow rate and compressor power for fixed speed compressors, but any interpolation with this model leads to erroneous results and the model cannot be used for variable speed applications [7]. Models proposed by Popovic et al., Jahning et al. and Armstrong et al. are valid over wide range of conditions and are based on volumetric efficiency which takes into account actual thermodynamic process for compression [18, 15, 1].

Literature reviewed readily describes under and over-compression for screw compressors in terms of isentropic work (power law) functions, but not well described, in terms of a polynomial function of reasonably low order. This paper demonstrates a semi-empirical screw compressor model adjustment for under-over compression losses. The adjusted model represent screw compressor performance with six bi-quadratic parameters, and two new parameters volume index and smoothing band. The Physical UOC model is derived, which evaluates effective volume index of the compressor under investigation. Holistic objective of the research work is to demonstrate the importance of accurate compressor models in establishing MEPS, air and evaporatively cooled chillers are modeled and compared for screw and reciprocating compressor designs.

2. Screw and Reciprocating Compressor Model

A complete representation of compressor performance requires two sub-models to compute refrigerant flow rate and input power given compressor speed and pressure ratio. The models are based on the fact that flow losses and internal power dissipation are more-or-less proportional to refrigerant flow rate and compressor work with an additional loss component proportional to departure from the built-in volume index. Thus we can expect to get by with low-order models to empirically represent deviations from the idealized models of (1) flow proportional to speed and (2) constant isentropic efficiency. Model coefficients are estimated from measured performance observed over the range of pressure ratio and mass flow rate encountered in chiller operation.

2.1. Screw Compressor Model without Over-Under Compression

Capacity and isentropic efficiency models are extensively described in Javed et al., 2014. Data required for such model training and analysis may be obtained from compressor selection tools[1]. A common isentropic model formulates compressor isentropic efficiency as a bi-cubic relation [19] involving pressure ratio(\(\pi\)), and compressor speed(\(f\)) shown in figure 1, coefficients in Table 1. Bi-cubic regression models are of the form

\[ y = \sum_i C_i g_i(\bar{X}) + \varepsilon \]

[1] www.bitzer.de/websoftware
where, $y$ is the dependent variable ($\eta_{\text{isen}}$), $\bar{X} = \bar{X}(f, \pi)$ the vector of independent variables, $g_i(\bar{X})$ is the $i^{th}$ bi-cubic term, $C_i$ is the regression coefficient associated with $g_i(X)$ and $\varepsilon$ is the error or residual.

Figure 1 : Isentropic Efficiency Map for range of Pressure ratios and compressor speed

Table 1 : Bi-cubic relation for $\eta_{\text{isen}}$ producing RMS=0.02431 for points shown in Figure 1

| Coefficient | Term      | Value  | t-stat |
|-------------|-----------|--------|--------|
| C1          | $f^3$     | 4.74E-08 | 4.4284 |
| C2          | $f^3$     | 2.32E-02 | 37.489 |
| C3          | $f^2$     | -1.59E-05 | -3.9376 |
| C4          | $f^2$     | -2.75E-01 | -43.762 |
| C5          | $\pi f^2$ | -2.79E-06 | -7.6101 |
| C6          | $\pi f$   | 5.57E-06  | 0.4527 |
| C7          | $\pi f$   | 9.76E-04  | 8.3714 |
| C8          | $f$       | 4.96E-04  | 0.9961 |
| C9          | $f$       | 9.64E-01  | 45.707 |
| C10         | Constant  | -5.96E-01 | -22.130 |

It is apparent that even with an increase of model order from bi-quadratic to bi-cubic the general polynomial model does not well represent isentropic efficiency of a screw compressor

2.2. Screw Compressor Over-Under compression Model

Under- and over-compression (UOC) processes affect the isentropic efficiency of a screw compressor. The theoretical UOC model involves a compressor-specific (but sometimes unknown) volume index parameter and the real gas properties of the refrigerant used. Isentropic efficiency is estimated by a quotient of the UOC model and a bi-cubic, used to account for flow, friction and electrical losses. A smoothing parameter is used to flatten the UOC model peak. The unknown volume index and bi-cubic coefficients may be identified by curve fitting to minimize root mean square error (RMSE) and coefficient of variance (COV).

Isentropic work done before the compression pocket reaches the discharge port is given by
\[ \dot{W}_{rev} = \dot{m} P_{suc} v_{suc} \left( \frac{\pi_{ext}}{\pi_{vi}} \right)^{-\frac{n}{n-1}} - 1 \]

\( \dot{m}, P_{suc}, v_{suc} \) = Mass flow rate of refrigerant; Suction Pressure and suction specific volume
\( \pi_{ext} \) = Actual pressure ratio, without UOC

The exponent \( n \) is given in Armstrong et al. [1] (real gas properties), where \( v_{lo} \) is inlet and outlet specific volume.

\[ n = \frac{P_{suc} v_{suc}}{RT_{suc}} \]

First we consider an ideal compressor where the only compressor losses were from under or over compression. The work done when \( \pi_{vi} > \pi_{ext} \) (over-compression) would be

\[ \dot{W}_{ov} = \dot{m} P_{suc} v_{suc} \left( \frac{\pi_{ext}}{\pi_{vi}} \right)^{-\frac{n}{n-1}} - 1 \]

Conversely the work done when \( \pi_{vi} < \pi_{ext} \) (under compression) would be

\[ \dot{W}_{uc} = \dot{m} P_{suc} v_{suc} \left( \frac{\pi_{ext}}{\pi_{vi}} \right)^{-\frac{n}{n-1}} \left( \frac{\pi_{vi}}{\pi_{ext}} \right)^{-\frac{n}{n-1}} - 1 \]

Thus the isentropic efficiency of a hypothetical compressor that suffers only UOC losses is

\[ \eta_{UOC} = \frac{\dot{W}_{rev}}{\dot{W}_{UOC}} \quad \text{where} \quad \dot{W}_{UOC} = W_{OC} \quad \pi_{vi} > \pi_{ext} ; \quad \dot{W}_{UOC} = W_{UC} \quad \pi_{vi} < \pi_{ext} \]

The theoretical isentropic efficiency incorporating under and over compression is shown in figure 2. Design pressure ratio (\( \pi_{vi} \)) if unknown, can be obtained empirically by minimizing RMSE \{\eta_{isen}-\eta_{UOC}(\pi_{vi},f)*bi-quad(\pi_{ext},f)} \ 0.0177 and COV. Design pressure ratio of 1.7 is obtained. The Matlab Smoothing function (moving average) is applied to the UOC model with a span value selected to minimize RMSE and COV.

![Figure 2: Theoretical Isentropic Efficiency (\( \eta_{UOC} \)) Map for Screw compressor, accounting only for UOC losses](image-url)
2.3. Screw Compressor Model Adjusted for Over-Under compression

A bi-Cubic relation in compressor speed \(f\) and external pressure ratio \(\pi_{\text{ext}}\) is developed by normalizing UOC curve to isentropic efficiency of the compressor with its UOC losses hypothetically removed (Figure 1). The model for isentropic efficiency considering all the other losses is therefore of the following form, shown in Figure 3, coefficients in table 2.

\[
y = \frac{\eta_{\text{isen}}}{\eta_{\text{UOC}}} = \eta_{\text{other}} + \varepsilon
\]

where \(\eta_{\text{isen}}\) is the test data \(\eta_{\text{other}} = f(\pi_{\text{ext}}, f)\) and \(\varepsilon\) is the error or residual.

![Figure 3: Adjusted Isentropic Efficiency Map with UOC losses for Screw Compressor](image)

Table 2: Bi-cubic relation for \(\eta_{\text{isen}}/\eta_{\text{UOC}}\) producing RMS=0.01912 for points shown in Figure 3

| Coefficient | Term | Value       | t-stat  |
|-------------|------|-------------|---------|
| C1          | \(f^3\) | 6.27E-06    | 7.1979  |
| C2          | \(\pi^2\) | -3.98E-03  | 13.479  |
| C3          | \(f^3\) \(\pi\) | -2.11E-05 | -6.3965 |
| C4          | \(\pi^2\) | 5.67E-02    | -15.433 |
| C5          | \(\pi f^3\) | -3.41E-06  | -12.083 |
| C6          | \(\pi^2 f\) | -5.93E-05  | -6.7629 |
| C7          | \(\pi f\) | 1.62E-03    | 17.448  |
| C8          | \(f\) | 2.24E-04    | 0.5460  |
| C9          | \(\pi\) | 1.95E-01    | 13.173  |
| C10         | Constant | 2.58E-01 | 12.362  |

2.4. Reciprocating compressor Model

Data used for capacity and isentropic model of a representative reciprocating compressor were generated from the compressor selection tool \(^2\), used by Armstrong et al., 2009(a). To characterize the

\(^2\) http://www.carlylecompressor.com/
compressor performance evaluation is done at 500 points, by generating the following data set, Table 3. The sizing tool output includes compressor input power, mass flow rate. Compressor inlet state is known, and compressor outlet enthalpy is calculated using model presented in Armstrong et al., 2009a. Performance is evaluated over four grids of operating conditions resulting in 5x5x5x4=500 unique conditions.

Table 3: Grid values of the four variables used to span the full range of operating conditions

| Shaft Speed (RPM) | Condensing Temp (°F) | Evaporating Temp (°F) | Evaporator Superheat (°R) |
|-------------------|----------------------|-----------------------|--------------------------|
| 900               | 80                   | 30                    | 0                        |
| 1100              | 90                   | 35                    | 5                        |
| 1300              | 100                  | 40                    | 10                       |
| 1525              | 110                  | 45                    | 20                       |
| 1750              | 130                  | 50                    | 30                       |

The compressor isentropic model is a bi-quadratic relation (Qureshi 2013) involving pressure ratio($\pi$), and compressor speed($f$) shown in figure 4, (coefficients in table 4).

Figure 4: Reciprocating Compressor Isentropic Efficiency Map for range of Pressure ratios and compressor speed

All 500 selection tool results are shown based on the input value shown in Table3.

Table 4: Bi-quadratic relation for $\eta_{\text{is}}$ producing RMS = 0.01042 for points shown in Figure 6

| Coefficient | Term | Value         | t-stat |
|-------------|------|---------------|--------|
| $C_1$       | Constant | 1.204E 00 | 81.13  |
| $C_2$       |       | 8.664E-02    | 15.85  |
| $C_3$       | $f$  | -4.166E-02   | -38.64 |
| $C_4$       | $f^2$ | -8.195E-03   | -10.50 |
| $C_5$       | $\pi f$ | 3.595E-04  | -2.77  |
| $C_6$       | $f^2$ | 6.993E-04    | 30.40  |
3. Application to Establishing Chillers MEPS

Heat pump products, like other energy-intensive capital equipment, compete in the market on both first cost (price) and operating cost. Although life-cycle cost (LCC) may be estimated, the average buyer is influenced more by first cost. To reduce the impact of this irrational behavior and simultaneously transform markets toward higher fleet efficiency, many countries adopt minimum equipment performance standards (MEPS) for heat pump and A/C equipment. The efficiency bar generally aims to minimize LCC of a given machine type and size category serving a typical load profile. Measures that could be cost-effective in UAE have not been thoroughly researched. Moreover available high performance chillers are rarely used because of subsidized electricity rates and weak price elasticity. One possible solution is to establish minimum energy performance standards by exploring various heat pump design options appropriate to UAE conditions. Trying out different compressors types is proposed as one such design option. Each heat pump component is modeled from first principles and/or empirical relations known to be valid over a wide range of conditions, capacity fractions and loads. Optimized air-cooled heat pump designs were evaluated, from an extension of previous work [13]. The component-based models are used to produce tabular performance maps. An empirical UAE cooling load model was adapted to produce hourly loads, scalable to any building, from TMY weather data. Together with a given chiller performance map, the cooling load model can be used to estimate annual electricity use.

Decoupling the evaporator from the rest of the system leaves two refrigerant side boundary conditions i.e. capacity and evaporating temperature and dry bulb/wet bulb temperature as the third boundary condition for air cooled /evaporatively-cooled heat pump respectively. In this paper, a comparison is made between screw compressor (air-cooled and evaporatively-cooled) and reciprocating compressor (air-cooled and evaporatively-cooled) heat pump designs [5].

The compressor and condenser represent a sub-system with capacity $Q_c$, evaporating temperature, $T_e$, and outdoor wet bulb temperature, $T_w$, as boundary conditions. Figure 5 and 6 map the performance of (a) air-cooled chiller (b) evaporatively-cooled chiller for screw and reciprocating compressor respectively. Figure 5 (a) shows that specific power has minimum values between 0.5-0.6 Capacity Fraction (CF). Hence the chiller is cycled for low Capacity Fraction (CF) over annual energy runs.

![Figure 5: Comparison (a) Air-cooled and (b) Evaporatively-cooled chiller performance map--(Screw Compressor)](image-url)
Figure 6: Comparison (a) Air-cooled and (b) Evaporatively-cooled chiller performance map—(Reciprocating Compressor)

The SEER comparison for air-cooled and evaporatively-cooled screw and reciprocating chillers is presented in table 5. In case of air cooled chillers percentage savings of 36.4% is reported with reciprocating compressors, whereas in case of evaporatively-cooled chillers savings are over 59.7% compared to respective base case.

|                      | Air Cooled chiller | Evaporatively-Cooled Chiller |
|----------------------|--------------------|-----------------------------|
| Screw Compressor (SEER) | 3.3506             | 3.9632                      |
| Reciprocating Compressor (SEER) | 4.5727             | 6.3309                      |
| SEER Percent (%) Increase | 36.4               | 59.7                        |

4. Conclusion

The isentropic efficiency of positive displacement compressors with outlet check valves (reciprocating and rolling-piston compressors) is typically represented by a low order polynomial in external pressure ratio and shaft speed. But the isentropic efficiency of a fixed-port compressor (e.g. screw or scroll compressor) is not well modelled by a polynomial in pressure ratio and speed even with higher polynomial order. The reason for this difficulty is the under- and over-compression losses which, although readily described in terms of isentropic work (power law) functions, are not well described, even for a fixed speed compressor, in terms of a polynomial function of reasonably low order. A model that combines the isentropic efficiency of a hypothetical compressor with no UOC losses, and the isentropic efficiency of a hypothetical compressor with only UOC losses, is found to represent screw compressor performance with only two new parameters, volume index and smoothing band, in addition to the ten bi-cubic parameters. The expressions for under- and over-compression specific work are functions of external pressure ratio and both reduce to isentropic work when external pressure ratio equals the design pressure ratio, $\pi_{vis}$.
that corresponds to the effective volume index of the machine in question. A moving average of the two isentropic efficiency functions reflects the fact that discharge ($\pi_{vi} > \pi_{ext}$) or back flow ($\pi_{vi} < \pi_{ext}$) at the discharge port is not instantaneous. The UOC model therefore incorporates a moving average window width as well as the volume-index parameter, $\pi_{vi}$.

The annual energy use of identical chillers, one with a typical $\pi_{vi} = 1.7$ screw compressor and the other with a reciprocating compressor of the same capacity, is simulated using the aggregate cooling load model of Abu Dhabi [3]. The reciprocating chiller is found to use from 35-60% less energy than the screw chiller.

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6. References
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