General treatment of anomalies in $(1,0)$ and $(1,1)$ two-dimensional super-gravity

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ABSTRACT

In this paper we discuss the interplay among (super-)coordinate, Weyl and Lorentz anomaly both in chiral and non-chiral super-gravity represented by (1,0) and (1,1) two-dimensional models. It is shown that for this purpose two regularization dependent parameters are needed in the effective action. We discuss in full generality the regularization ambiguities of the induced effective action and recover the corresponding general form of the anomalous Ward Identities. Finally, we explain the difference between chiral and non-chiral super-gravity models in terms of the free parameters and establish relation between these two models by projecting (1,1) into (1,0) super-symmetry.
1. Introduction

Anomalies are one of the central topic in modern quantum field theories and can be considered from a variety of perspectives ranging from the construction of quantum consistent phenomenological models of particle interactions to geometrical formulation of gauge theory.

One of the most intriguing aspect of the problem is the similarity between the anomaly pattern in two-dimensional abelian gauge theories and bi-dimensional fermionic quantum gravity [1,2,3]. The Schwinger model quantum ambiguities can be conveniently described in the effective action by a single free parameter interpolating between vector and axial symmetry preserving regularization schemes, and shifting anomaly from the gauge to the axial current [4]. In fermionic quantum gravity gauge and axial symmetries are replaced by Lorentz and Weyl invariance, so it is tempting to transfer the above scheme to the gravitational case as well. However, a third local symmetry, i.e. general covariance, has to be taken into account requiring a more general formulation of the problem. Just recently we proposed a general regularization scheme where all the three gravitational symmetries (Lorentz, Weyl, and general coordinate transformations) are treated on the same footing at the quantum level [5]. The formal analogy with the Schwinger model can be recovered under special conditions showing that the gravitational models contains more information due to more symmetries.

Renewed interest in 2-dimensional induced super-gravity, mainly in connection with strings theories and super-Liouville models [6], makes it important to address the problem of anomalies in this case [7]. In general chiral and non-chiral models
show a different anomalous behavior in the sense that the former always posses anomalous Ward Identities [1], while the latter allows the shifting of anomalies from one current to another, by properly choosing the regularization scheme. In order to discuss both physical situations we shall consider \((1,0)\) super-gravity as an example of chiral model, and \((1,1)\) super-gravity as a non-chiral model. In ordinary gravity it is possible to consider both cases on the same footing [5], while in the super-symmetric case such a simplified approach is not possible because, as shown later, projection of higher-N supersymmetric model to a lower one is a complex procedure that involves projections of covariant derivatives and cannot be described by simple use of a generalized chiral projector \(P_{\beta} = \frac{1}{2}(1 + \beta\gamma^5)\). Therefore, we shall separately consider the two models. The paper is planned as follows.

In Sect.2 we shall construct the most general form of the induced effective action for \((1,0)\) super-gravity in terms of arbitrarily weighted local counterterms, and study the interplay among various quantum symmetries. Though we start with a number of arbitrary parameters the result boils down to two parameters necessary to describe super-gravitational anomalies in their full generality, as long as the super-connection is the main ingredient in constructing the effective action.

In Sect.3 we shall establish connection between chiral and non-chiral super-gravity models by projecting \((1,1)\) super-fields in terms of \((1,0)\) super-fields.

Sect.4 is devoted to a brief summary of the results.
2. (1, 0) Induced super-gravity

Superfield formalism for (1, 0) supergravity is already known [9],[10],[11], and is described in terms of unconstrained prepotential super-fields some of which are gauge degrees of freedom. These redundant (gauge) degrees of freedom can be removed by an algebraic gauge choice \( H^+_L = 0 \), thus constraining parameter super-fields \( K^L \) of super-symmetry transformation. After this procedure one is left with four \textit{unconstrained prepotential super-fields} \( H^{(1,0)}_L, H^{(1,0)}_+, H^{(1,0)}_L \) and \( H^{(1,0)+}_L \) relevant for the symmetries whose anomalies we want to investigate. Their \textit{linearized} super-symmetry transformations are

\[
\begin{align*}
\delta H^{(1,0)}_L &= -D_L K^- , \\
\delta S^{(1,0)} &= \frac{1}{2} (\partial_- K^- + \partial_+ K^+) + \Lambda \\
\delta H^{(1,0)}_+ &= -\partial_- K^+ , \\
\delta L^{(1,0)} &= \frac{1}{2} (\partial_- K^- - \partial_+ K^+) + K
\end{align*}
\]

where \( K^+, K^-, K \) and \( \Lambda \) are coordinate, Lorentz and Weyl parameter super-fields. \( S^{(1,0)} \) and \( L^{(1,0)} \) are Weyl and Lorentz super-field compensators defined,

\* We shall use the following notation : + and - refer to light-cone components of bosonic quantities, while L and R labels left and right chirality components of fermionic quantities. Interior product between bosons is defined as

\[
A^a B^b = \frac{1}{2} (A_+ A_- + A_- B_+) .
\]

and for fermions is

\[
\psi^\alpha \psi^\alpha = 2i \psi_L \psi_R
\]

The two kind of indices are related by \( LL \equiv + \), and \( RR \equiv - \).

Finally, our notation can be translated into the language of ref.(4) according to the substitutions

\[
\begin{align*}
L &\rightarrow + \\
R &\rightarrow - \\
- &\rightarrow = \\
+ &\rightarrow \neq
\end{align*}
\]
at the linearized level, as

\[ S^{(1,0)} = - \left( H_L^{(1,0) -} + \frac{1}{2} H_-^{(1,0) -} \right) \]
\[ L^{(1,0)} = \left( H_L^{(1,0) -} - \frac{1}{2} H_-^{(1,0) -} \right) \]

(2.2)

and \( D_L = \partial_{\theta^L} + i \theta^L \partial_+ \) is the ordinary super-symmetric derivative.

Variation of a generic action with respect to independent prepotentials gives

\[ \delta I = \int d^2 x d\theta^L \left( \delta H_L^{(1,0) -} J_{--}^{(1,0)} + \delta H_-^{(1,0) +} J_{+L}^{(1,0)} + \delta H_L^{(1,0) L} J_{+R}^{(1,0)} + \delta H_-^{(1,0) -} J_{-R}^{(1,0)} \right) \]

(2.3)

where \( J_{--}^{(1,0)} , J_{+L}^{(1,0)} , J_{-R}^{(1,0)} \) and \( \tilde{J}_{-R}^{(1,0)} \) are the corresponding super-currents. If the action (2.3) is required to be invariant under linearized super-symmetry transformations (2.1), then one finds the classical conservation laws under:

i) general-super-coordinate invariance

\[ \partial_- J_{+L}^{(1,0)} + \frac{1}{2} \partial_+ J_{+L}^{(1,0)} = 0 \]
\[ D_L J_{--}^{(1,0)} + \partial_- \tilde{J}_{-R}^{(1,0)} = 0 ; \]

(2.4)

ii) super-Weyl invariance

\[ \tilde{J}_{-R}^{(1,0)} + \frac{1}{2} J_{-R}^{(1,0)} = 0 ; \]

(2.5)

iii) super-Lorentz invariance

\[ \tilde{J}_{-R}^{(1,0)} - \frac{1}{2} J_{-R}^{(1,0)} = 0 . \]

(2.6)

Whenever Lorentz invariance is assumed to be preserved at the quantum level, one can further gauge away Lorentz compensator \( L^{(1,0)} \) thus constraining the parameter super-field \( K \). However, regularization of induced effective action in quantum
field theories is an intrinsically ambiguous procedure and there is no \textit{a priori} reason to privilege one symmetry among the others. Regularization ambiguities manifest themselves in the effective action of the anomalous models as arbitrary parameters interpolating among different regularization schemes, preserving one or the other, or none of the classical symmetries \cite{1,4,3}. With this remark in mind we construct the \textit{general form of the effective action} for \((1,0)\) induced super-gravity:

\[
I_{\text{eff}} = \frac{1}{96\pi} \int d^2x \, d\theta^L \left[ A D_L H_L^{(1,0)} - \frac{1}{\sqrt{2}} \partial^+ H_L^{(1,0)} + B D_L H_-^{(1,0)} + \frac{1}{\sqrt{2}} \partial^3 H_-^{(1,0)} + c H_L^{(1,0)} - \nabla^2 H_-^{(1,0)} + d S^{(1,0)} \partial^2 H_L^{(1,0)} + e S^{(1,0)} \partial^3 D_L H_-^{(1,0)} + f S^{(1,0)} \partial_- D_L S^{(1,0)} + g L^{(1,0)} \partial^2 H_L^{(1,0)} + m L^{(1,0)} \partial^3 D_L H_-^{(1,0)} + n L^{(1,0)} \partial_- D_L L^{(1,0)} + v L^{(1,0)} \partial^- D_L S^{(1,0)} \right] \tag{2.7}
\]

where \(\nabla^2\) is the covariant D’Alembertian, and the coefficients \(A\) and \(B\) of the \textit{non-local} part of (2.7) are \textit{uniquely} fixed by the contributions from the matter part of the classical action.\* These numbers have been perturbatively computed, for instance, in super-string theories \cite{8,12}. All the other coefficients are regularization dependent and can be fixed according to which symmetry one wishes to preserve at the quantum level (hopefully all of the classical symmetries).

From the effective action (7) and the symmetry transformations (1), the following possible anomalies for the individual symmetries can be derived:

\* From now on, we shall suppress the global numerical factor \(1/96\pi\) in front of the effective action.
\[ \partial_- J_{+L}^{(1,0)} + \frac{1}{2} \partial_+ J_R^{(1,0)} = \left( \frac{m - e}{2} - 2B \right) \partial^2_D L_+ H^{(1,0)+} + \left( \frac{g - d}{2} + c \right) \partial_- \nabla^2 H^{(1,0)-}_L + \left( \frac{v}{2} + e - f \right) \nabla^2 D_L S + \left( m + n - \frac{v}{2} \right) \nabla^2 D_L L \]

\[ D_L J_-^{(1,0)} + \partial_- \tilde{J}^{(1,0)}_R = \left( 2iA - \frac{g + d}{2} \right) \partial^2_D H^{(1,0)-}_L + \left( c - \frac{m + e}{2} \right) \nabla^2 D_L H^{(1,0)+}_L + \left( d - f - \frac{v}{2} \right) \partial^2_D L S + \left( g - n - \frac{v}{2} \right) \partial^2_D L L \]

These general results can be simplified by reducing a number of free parameters through symmetry requirements. Our request is that the arbitrary coefficients should be chosen in a way compatible with the maximum symmetry one can obtain at the quantum level. As a first step we require super-coordinate quantum invariance which fixes parameters in the following way

\[ \begin{align*}
    d &= c + 2iA & m &= c + 2B & v &= 2(B + iA) \\
    e &= c - 2B & g &= 2iA - c \\
    f &= c + iA - B & n &= iA - B - c
\end{align*} \]

but induces Weyl and Lorentz anomalies

\[ \begin{align*}
    \tilde{J}^{(1,0)}_R - \frac{1}{2} J^{(1,0)}_R &= g \partial^2 H^{(1,0)-}_L + m \partial_+ D_L H^{(1,0)+}_L + 2n \partial_- D_L L + v \partial_- D_L S \\
    \tilde{J}^{(1,0)}_R + \frac{1}{2} J^{(1,0)}_R &= d \partial^2 H^{(1,0)-}_L + c \partial_+ D_L H^{(1,0)+}_L + 2f \partial_- D_L S + v \partial_- D_L L
\end{align*} \]

with Lorentz connection super-fields given by

\[ \begin{align*}
    \omega^{(1,0)}_L &= -D_L (S^{(1,0)} + L^{(1,0)}) - \partial_- H^{(1,0)-}_L \\
    \omega^{(1,0)}_- &= \partial_- (S^{(1,0)} - L^{(1,0)}) + \partial_+ H^{(1,0)+}_L \\
    \omega^{(1,0)}_+ &= -i D_L \omega^{(1,0)}_L
\end{align*} \]
and

$$\sum^{(1,0)L} = (D_L \omega^{(1,0)}_+ - \partial_- \omega^{(1,0)}_-).$$  \hfill (2.12)

The corresponding effective action is found, from (2.7) and (2.9), to be

$$I^{(1,0)}_{\text{eff}} = \int d^2x \, d\theta^L \left[ A \Sigma^{(1,0)L} \frac{1}{\sqrt{2}} D_L \Sigma^{(1,0)L} + (B + iA) \omega^{(1,0)}_- \frac{1}{\sqrt{2}} \partial_+ D_L \omega^{(1,0)}_- 
+ (c - 2iA) \omega^{(1,0)}_L \omega^{(1,0)}_- \right].$$  \hfill (2.13)

Notice the appearance of arbitrary coefficient \(c\) which is let free by the requirement of super-coordinate quantum invariance, and can be further used to eliminate a local piece of the effective action (2.13). Imposing the above symmetry requirements have drastically reduced the number of free parameters.

It is suggestive from (2.13) that the construction of the effective action in terms of super-vierbeins was not the best choice since it resulted in a number of arbitrary parameters. It is better to work with the spin connection [13] (2.11) that leave only one parameter in the effective action (when coordinate invariance is assumed). However, in this case Lorentz and Weyl anomalies are detached from the gravitational anomaly, in the sense that the free parameter influences only the former. We would like to present a unique treatment of all three anomalies. To do so one has to design a way of breaking simultaneously general covariance and, as shown above, a single parameter will not suffice. Since all the super-vierbeins transform under general coordinate transformation, contrary to Weyl and Lorentz symmetry, presence of all of them in a covariant expression is necessary to guarantee covariance. Therefore, absence of at least one of them will spoil general covariance. We shall introduce a second parameter \(b\) in front of some super-vierbeins in a way
to obtain generalized form of the super-connections as

\[
\tilde{\omega}_L^{(1,0)} = -bD_L(S^{(1,0)} + L^{(1,0)}) - \partial_- H_L^{(1,0)} - b\partial_-(S^{(1,0)} - L^{(1,0)}) + \partial_+ H_-^{(1,0)},
\]

\[
\tilde{\omega}_-^{(1,0)} = b\partial_- (\tilde{S}^{(1,0)} - \tilde{L}^{(1,0)}) + \partial_+ H_-^{(1,0)},
\]

\[
\tilde{\omega}_+^{(1,0)} = -iD_L \tilde{\omega}_L^{(1,0)}
\]

with their super-transformations given by

\[
\delta \tilde{\omega}_L^{(1,0)} = (1 - b)\partial_- D_L K^- - bD_L(\Lambda + K)
\]

\[
\delta \tilde{\omega}_-^{(1,0)} = -(1 - b)\nabla^2 K^+ + b\partial_- (\Lambda - K)
\]

\[
\delta \Sigma^{(1,0) L} = -(1 - b)\partial_- D_L (\partial_+ K^+ + \partial_- K^-) + 2b\partial_- D_L \Lambda.
\]

From (2.15) one can see that the choice \(b = 1\) restores general covariance (linearized super-connections do not transform under general coordinate transformations) while \(b \neq 1\) spoils it.

Starting from the effective action (2.13), where one substitutes \(\omega\) with \(\tilde{\omega}\) thus introducing dependence on the second parameter, one can find conservation laws for the symmetries (2.4-6) in a generalized forms as

\[
\partial_- J_L^{(1,0)} + \frac{1}{2}\partial_+ J_R^{(1,0)} = (1 - b) \left[ -2B\partial_+ \Sigma^{(1,0) L} - (c + 2B)\nabla^2 \tilde{\omega}_L^{(1,0)} \right]
\]

\[
D_L J_{-\cdots}^{(1,0)} + \partial_- \tilde{J}_R^{(1,0)} = (1 - b) \left[ c\partial_- \Sigma^{(1,0) L} + (c - 2iA)\partial_-^2 \tilde{\omega}_L^{(1,0)} \right]
\]

\[
\frac{1}{2} J_R^{(1,0)} + \tilde{J}_R^{(1,0)} = b \left[ (c - 2B)\Sigma^{(1,0) L} - 2(B + iA)\partial_- \tilde{\omega}_L^{(1,0)} \right]
\]

\[
\frac{1}{2} J_R^{(1,0)} - \tilde{J}_R^{(1,0)} = -b \left[ (c + 2B)\Sigma^{(1,0) L} + 2(c - B - iA)\partial_- \tilde{\omega}_L^{(1,0)} \right]
\]

In this way we set up a unique treatment of all quantum symmetries present in the (1, 0) model. Two independent parameters are needed to achieve this goal.
Assignment of the parameter $b$ interpolates between super-gravitational and super-Lorentz and Weyl anomalies. Parameter $c$ has no role in doing that, and it can only be used to determine the form of the Lorentz and Weyl anomaly but not to remove them. The choice $b = 1$ gives previous results described in (2.10-13), and in this case we are still free to choose the value of the second parameter $c$ as to eliminate local piece in (2.13). However, the super-connection dependence (and therefore Lorentz non-invariance) persists in the non-local piece unless $A = iB$. Absence of the Lorentz anomaly can be satisfied only if such condition is fulfilled \[9,8\]. This is to be expected since $(1,0)$ super-gravity is actually a chiral model where Lorentz anomalies are usually present. Therefore, the above mentioned relation between coefficients of the non-local piece is not a priori guaranteed and it depends on the matter contribution to the effective action. Matter coupling to $(1,0)$ super-gravity is given by the action \[9\]

\[
I_{\text{matt.}}^{(1,0)} = \int d^2x \, d\theta^L (E^{(1,0)})^{-1} \nabla_L \Phi^{(1,0)} \nabla_- \Phi^{(1,0)}
\]

(2.17)

where $\Phi^{(1,0)}$ is a scalar matter super-field, $(E^{(1,0)})^{-1}$ is a super-determinant and $\nabla_L$, $\nabla_-$ are covariant derivatives. Linearized couplings of the matter to prepotential super-fields, relevant for calculating non-local pieces of the effective action, are determined from (2.16) as

\[
H_L^{(1,0)} = \left( \partial_- \Phi^{(1,0)} \right)^2
\]

\[
H_-^{(1,0)} + D_L \Phi^{(1,0)} \partial_+ \Phi^{(1,0)}
\]

(2.18)

Now, it is possible to show, by projecting (2.18) in components, that the non-local part of the effective action in $H_-^{(0,1)}$ receives both contributions from the
scalar and fermionic components of the matter super-field, while the one in $H_{L}^{(1,0)-}$ receives contribution only from the scalar component. This produces different coefficients for various non-local terms ($A = 1$, $iB = 1 + 1/2$). This situation is slightly different from the non-super-symmetric case where only fermion contribution are considered, and where $A = 0$, $iB = 1/2$. The difference is due to super-symmetry that introduces super-partners. What is however surprising is that $A$ receives contribution only from the scalar component of the super-field due to the type of coupling. As a result one has Lorentz (and Weyl) anomaly in this model. As conjectured, this was to be expected since we are considering a chiral super-symmetric theory.

Alternatively, we could have eliminated Lorentz and Weyl anomalies through a different choice of parameter $b = 0$, thus introducing a gravitational anomaly. Again, the parameter $c$ can only influence the actual form of the super-gravitational anomaly but cannot eliminate it.

3. $(1, 1)$ Induced super-gravity

To gain insight about the difference between chiral and non-chiral super-gravity models we further consider $(1, 1)$ super-gravity as a non-chiral model. In this case [14] independent prepotential super-fields are $H_{L}^{(1,1)-}$, $H_{R}^{(1,1)+}$, $H_{-}^{(1,1)-}$ and $H_{+}^{(1,1)+}$, whose symmetry variations are quite similar to (2.1) with appropriate chirality adjustments (i.e. adding R-chirality pieces).

The $(1, 1)$ super-gravity effective action, can be constructed on the basis of
\[ I_{\text{eff}}^{(1,1)} = \int d^2 x \, d\theta^L d\bar{\theta}^R \left[ \mathbf{A} \bar{R}^{(1,1)} + \left( c - 2iA \right) \bar{\omega}_L^{(1,1)} \right] \]

where \( R \) is the Ricci scalar super-field given in terms of prepotential super-fields as

\[ \bar{R}^{(1,1)} = i \left( D_L \bar{\omega}^{(1,1)}_R + D_R \bar{\omega}^{(1,1)}_L \right) \]

\[ \bar{\omega}^{(1,1)}_R = bD_R (S - L) + \partial_+ H_R^{(1,1)} + \partial_- H_L^{(1,1)} \]

\[ \bar{\omega}^{(1,1)}_L = -bD_L (S + L) - \partial_- H_L^{(1,1)} - \partial_+ H_R^{(1,1)} \]

We have again introduced two arbitrary parameters in order to treat anomalies in full generality. In order to extract conservation laws from (3.1) the following variation is needed

\[ \delta \bar{R}^{(1,1)} = i(b - 1)D_L D_R (\partial_+ K^+ + \partial_- K^-) + 2ib \nabla^2 \Lambda \]

and we find anomaly relations

\[ D_R J_{+L}^{(1,1)} + \frac{1}{2} \partial_+ J^{(1,1)} = (1 - b) \left[ 2iA \partial_+ \bar{R}^{(1,1)} + i(c - 2iA) \partial_+ D_R \bar{\omega}^{(1,1)}_L \right] \]

\[ D_L J_{-R}^{(1,1)} - \partial_- \bar{J}^{(1,1)} = (1 - b) \left[ 2iA \partial_- \bar{R}^{(1,1)} - i(c - 2iA) \partial_- D_L \bar{\omega}^{(1,1)}_R \right] \]

\[ \frac{1}{2} J^{(1,1)} + \bar{J}^{(1,1)} = -b \left[ (c + 2iA) \bar{R}^{(1,1)} \right] \]

\[ \frac{1}{2} J^{(1,1)} - \bar{J}^{(1,1)} = b(c - 2iA) \left[ \bar{R}^{(1,1)} - 2iD_R \bar{\omega}^{(1,1)}_L \right] \]

As long as the parameter \( b \) is concerned the discussion from (1, 0) super-gravity repeats itself in (1, 1) as well. However, the role of the parameter \( c \) is slightly
different. It is still there to further shift between Lorentz and Weyl anomaly. But, one can have more symmetry in the \((1,1)\) than in \((1,0)\) super-gravity, due to the absence of the non-local piece in \((2.13)\) in terms of super-connection \((\mathbf{B} = -i\mathbf{A}; \text{see } (3.13))\). So, the parameter \(c\) can be chosen in such a way to eliminate either Lorentz or Weyl anomaly. This has led to the over-simplified popular belief that, in non-chiral models Lorentz anomaly is always absent, and one can shift the residual anomaly from the trace to the divergence of the energy-momentum tensor, and vice-versa. Our result shows that: it is possible to have Lorentz anomaly \((c = 2i\mathbf{A})\) even in this case, still preserving general covariance \((b = 1)\) at the expense of the trace anomaly, although contrary is usually preferred but not necessary. Comparing \((3.4)\) to \((2.16)\) confirms that the Lorentz anomaly is a genuine chiral effect which cannot be disposed of by any choice of parameter \(c\) in \((1,0)\) super-gravity.

It would be instructive to establish relation between the two models considered in this work. In the non-super-symmetric case it is possible to shift from the chiral \((\beta = \pm 1)\) and the non-chiral \((\beta = 0)\) models by introduction of a parameter in the chiral projector \(P_{\beta} \equiv \frac{1}{2}(1 + \beta\gamma^5)\) and both models can be treated on the same footing [5]. In the super-symmetric case it is not possible to obtain such a simple simultaneous description of both models due to the different structure of the super-fields in \((1,0)\) and \((1,1)\) super-gravity, so one has to treat the two models separately as we did. However, we can still establish relations between these models. In order to do so we shall use decomposition of \((1,1)\) into \((1,0)\) super-field which is the analogue of the usual projection of super-field components. The essence of such an approach is to eliminate super-symmetric parameter correspond-
ing to additional super-symmetry (in this case $\theta_R$) by appropriate gauge choice of the super-covariant derivative [17]. In this process new $(1,0)$ super-fields appear corresponding to “matter” gravitino and its trace of the reduced super-symmetry. Linearized decomposition of relevant prepotential super-fields are:

\[
\begin{align*}
H^{(1,1)-}_L \big|_{\theta^R=0} &= H^{(1,0)-}_L \\
D_R H^{(1,1)-}_L \big|_{\theta^R=0} &= -2i\Psi^{(1,0)R}_L \\
H^{(1,1)+}_R \big|_{\theta^R=0} &= 0 \\
D_R H^{(1,1)+}_R \big|_{\theta^R=0} &= -iH^{(1,0)+}_- \\
H^{(1,1)-}_- \big|_{\theta^R=0} &= H^{(1,0)-}_- \\
D_R H^{(1,1)-}_- \big|_{\theta^R=0} &= 2i\Psi^{(1,0)R}_- \\
H^{(1,1)L}_L \big|_{\theta^R=0} &= H^{(1,0)L}_L \\
D_R H^{(1,1)L}_L \big|_{\theta^R=0} &= 0
\end{align*}
\]

(3.5)

In order to decompose $(1,1)$ effective action (3.1) in terms of $(1,0)$ super-fields we need decompositions of the Ricci scalar which is given in terms of prepotentials by

\[
\bar{R}^{(1,1)} = i \left( \partial_+ D_L H^{(1,1)+}_R - \partial_- D_R H^{(1,1)-}_L \right) + 2ib D_L D_R S^{(1,1)}.
\]

(3.6)

and, with the help of (3.5) its decomposition is

\[
\begin{align*}
\bar{R}^{(1,1)} \big|_{\theta^R=0} &= -2(\partial_- \Psi^{(1,0)R}_L - b D_L \Psi^{(1,0)R}_-) \\
D_R \bar{R}^{(1,1)} \big|_{\theta^R=0} &= \bar{\Sigma}^{(1,0)L}.
\end{align*}
\]

(3.7)

With the above projections one finds decomposition of $(1,1)$ effective action (3.1)
\[ J_{\text{eff}}^{(1,1)} = \int d^2 x d\theta^L \left[ A \bar{\Sigma}^{(1,0)L} + 4i A \bar{\Psi}_L^{(1,0)R} \right] \]

Comparing (3.8) with (2.13) shows that (3.8) is the effective action of (1, 0) super-gravity in the case \( A = iB \), plus additional terms due to the "matter" gravitino and its trace. These additional terms are there because the effective action (3.8) differs from (2.13) by the fact that, although written in terms of (1, 0) super-fields, it actually has larger (1, 1) super-symmetry. The anomaly equations (3.4) are decomposed as

\[
\begin{align*}
&i \partial_- J_{+L}^{(1,1)} \bigg|_{\theta^R=0} + \frac{1}{2} \partial_- D_R J_{+L}^{(1,1)} \bigg|_{\theta^R=0} = (1 - b) \left[ 2i A \partial_+ \bar{\Sigma}^{(1,0)L} - (c - 2i A) \nabla^2 \bar{\omega}_L^{(1,0)} \right] \\
&D_L D_R J_{-R}^{(1,1)} \bigg|_{\theta^R=0} - \partial_- D_R J_{-R}^{(1,1)} \bigg|_{\theta^R=0} = (b - 1) \left[ 2i A \partial_- \bar{\Sigma}^{(1,0)L} + (c - 2i A) \partial_- D_L \bar{\omega}_-^{(1,0)} \right] \\
&\frac{1}{2} D_R J_{-R}^{(1,1)} \bigg|_{\theta^R=0} + D_R \bar{J}_{-R}^{(1,1)} \bigg|_{\theta^R=0} = b(c + 2i A) \bar{\Sigma}^{(1,0)L} \\
&\frac{1}{2} D_R J_{-R}^{(1,1)} \bigg|_{\theta^R=0} - D_R \bar{J}_{-R}^{(1,1)} \bigg|_{\theta^R=0} = b(c - 2i A) \left( \bar{\Sigma}^{(1,0)L} + 2 \partial_- \bar{\omega}_-^{(1,0)} \right)
\end{align*}
\]

Relation to (3.4) is obtained through the following identification of super-current

\[
\begin{align*}
&D_L J_{-R}^{(1,1)} \bigg|_{\theta^R=0} - \partial_- \bar{J}_{-R}^{(1,1)} \bigg|_{\theta^R=0} = 4i A (b - 1) \left[ \partial_+ \bar{\Psi}_L^{(1,0)R} - b \partial_- D_L \bar{\Psi}_-^{(1,0)R} \right] \\
&\bar{J}_{-R}^{(1,1)} \bigg|_{\theta^R=0} = 2i A b \left[ \partial_- \bar{\Psi}_L^{(1,0)R} - b D_L \bar{\Psi}_-^{(1,0)R} \right]
\end{align*}
\]

Relation to (3.4) is obtained through the following identification of super-current
Further comment is needed in order to explain the presence of only one parameter $A$ in (3.1) and (3.8). Prepotentials $H_L^{(1,1)-}$ and $H_R^{(1,1)+}$ appear symmetrically and one would expect the coefficients of the corresponding non-local pieces in the $(1,1)$ effective action to be the same. We check this point by considering the coupling of matter to $(1,1)$ super-gravity [14],

$$I_{\text{matt.}}^{(1,1)} = \int d^2x \, d\theta^L d\theta^R (E^{(1,1)})^{-1} \nabla_L \Phi^{(1,1)} \nabla_R \Phi^{(1,1)}$$  \hspace{1cm} (3.12)$$

from which we extract linearized interactions as

$$H_L^{(1,1)-} \partial_- \Phi^{(1,1)} D_R \Phi^{(1,1)}$$ \hspace{1cm} (3.13)

$$H_R^{(1,1)+} \partial_+ \Phi^{(1,1)} D_L \Phi^{(1,1)}.$$

Projecting out component couplings (by applying $D_L$ and $D_R$ super-symmetric derivatives to (17)) one finds the same contributions to non-local pieces of the effective action ($A = iB = 3/2$). In order to get the appropriate $(1,0)$ matter couplings we decompose (3.13) with the help of

$$\Phi^{(1,1)} \bigg|_{\theta^R=0} = \Phi^{(1,0)}, \quad \text{and} \quad D_R \Phi^{(1,1)} \bigg|_{\theta^R=0} = \lambda_R^{(1,0)}.$$  \hspace{1cm} (3.14)
and obtain

\[
H^{(1,0)}_L - \left[ \left( \partial_- \Phi^{(1,0)}(1,0) \right)^2 + i\lambda_R \partial_- \lambda_R^{(1,0)} \right]
\]

\[
H^{(1,0)} + \partial_+ \Phi^{(1,0)} D_L \Phi^{(1,0)}
\]  

(3.15)

Now, we can understand the possible absence of Lorentz anomaly in this case since the above projection introduces extra right-handed fermion super-field in addition to the scalar super-field described in Sect.(2). This additional contribution, whose presence is due to the larger (1, 1) super-symmetry, compensate for the missing chirality.

4. Summary

We have described the super-gravitational, Weyl and Lorentz anomalies, on the same, most general footing, both in (1, 0) (chiral) and (1, 1) (non-chiral) super-gravity. It is shown that, in order to do so, we need two arbitrary regularization dependent parameters similarly to the situation already found in the non-super-symmetric gravity model [5]. Super-symmetry introduces an additional freedom represented by the parameters in the non-local piece of the effective action. This occurs because matter super-fields contain more ingredients which are not present in non-super-symmetric models. Nevertheless, (1, 0) chiral model has in general genuine Lorentz anomaly.

In this work we have considered only super-gravitational anomalies, however it would be interesting to include super-Schwinger model anomalies as well. Preliminary investigation of the super-Schwinger model has been done in the case of (1, 1) super-symmetry [18] showing the same arbitrariness in terms of a single
parameter as in the non-super-symmetric case, together with some new features related to the breakdown of the duality relation between axial and vector gauge currents. From this point of view, it would particularly interesting to investigate (2,2) super-gravity where the axial gauge field is a member of the gravitational super-multiplet. Therefore one would expect the same parameter to describe both gravitational and gauge anomalies. Decomposing the gravitational super-multiplet to lower super-symmetry realizations would give new insight into these models since, so far, Schwinger and gravitational anomalies have been treated separately in terms of independent parameters. This problem is now under investigation.

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