The phenomenon of nonstationary nutation in the system of two-level quantum wells of a periodic superlattice with strong magnetic quantization

H S Nikoghosyan¹, V F Manukyan¹ and G H Nikoghosyan²

¹ Faculty of Natural Science and Mathematics, Shirak State University, 4 Paruyr Sevak Street, Gyumri 3126, Armenia
² Department of Physics, Yerevan State University, 1 Alex Manoogian Street, Yerevan 0025, Armenia

E-mail: mvardan_1972@mail.ru

Abstract. We investigate the superposition of nonstationary nutation processes that arose during the interaction of two-level systems of quantum wells of a superlattice with two exciting coherent optical pulses with various resonance detunings. According to the model under consideration, dipole transitions in two-level systems of quantum wells under the action of exciting pulses occur between the Landau levels of the main mini-band of the superlattice conduction band. This gives a direct opportunity to study the controlled variation of the course of optical nutation processes, by changing the intensity of an external quantizing magnetic field.

1. Introduction
The effects of coherence play a significant role in the propagation of radiation in a resonant medium. They have practical applications for measuring short relaxation times, dipole moments of transitions, and other parameters of the media [1, 2]. With a particular relevance studies the course of coherence processes in nanoscale semiconductor environments, where quantization in certain directions is determined by spatial constraints and external force fields. However, the situation is complicated by the fact that the relaxation times in semiconductors are determined by fast electrons and electron-phonon collisions and are very short [3]. In layered nanoscale structures exists a direct possibility by the way of selecting the elastic parameters of semiconductor layers, due to the spatial limitation and localization of the vibrational modes of the gratings, to achieve suppression of electron-phonon interactions. So, the phenomenological constants of the damping processes, which limit the duration of the nutation of the Bloch vectors of two-level systems, oscillations of the difference between the populations of the medium and amplitude modulation of the radiation fields of laser pulses at the Rabi frequency $\Omega_R$, will be determined rather by the processes of spontaneous decay of quantum well levels. Below is the task of identifying the resulting response of such a medium to the exciting laser pulses in the first approximation without taking into account relaxation processes. A particular interest is being caused by the consideration of a complex process of interaction between quantum systems of a medium simultaneously with two pulses of coherent radiation, interpreted as a geometric overlay of pseudospins and torques, and a subsequent analysis of the resulting nutation process.

We investigate the superposition of non-stationary nutation processes that arose during the interaction of two-level systems of quantum wells of a superlattice with two exciting coherent optical
pulses with various resonance detunings $\Delta \omega_1 = \omega_1 - \omega_{21}, \Delta \omega_2 = \omega_2 - \omega_{21}$, where $\omega_{21}$ is the transition frequency of a two-level system, $\omega_{1,2}$ are the frequency of incident waves. In optical experiments, the system is usually brought into a state of coherent macroscopic excitation, when an arbitrary operator for bosons is replaced by a coherent semiclassical amplitude. Maxwell fields are coherent amplitudes for the photons system. So the light field interacting with electrons in the studied systems is represented as $\hat{\mathbf{R}}(i) / 2 (i) e^{i \omega t}$. Here, $\mathbf{E}(t)$ is the amplitude of the external field, $\mathbf{h} \chi / 2 = d$ the dipole moment of the transition.

2. Electrodipole Pseudospin Vector

In the semiclassical theory of the interaction of two-level systems and radiation, taking into account only the classical dipole interaction (neglecting quantum correlations) is considering a vector operator $\hat{\mathbf{R}}$ with components $\hat{R}_j$ considered as a pseudospin operator for a set of two-level systems [6] satisfying the equation

$$\frac{d}{dt} \langle \hat{\mathbf{R}} \rangle = \left[ \hat{\mathbf{\Omega}}(\hat{\mathbf{R}}) \right]_\times.$$  \hspace{1cm} (1)

The components $\hat{R}_j$ are represented as $\hat{R}_j = \sum_i \hat{r}_{ji}$, where $j = 1, 2, 3$, and the summation is performed over all two-level systems, which are numbered by index $i$. $\hat{r}_{ji}$ - two-row matrices of spin Pauli operators. (1) describes the motion of a precessing electric dipole pseudospin vector under the action of a torque $\hat{\mathbf{\Omega}} \times \langle \hat{\mathbf{R}} \rangle$, where $\hat{\mathbf{\Omega}} = -\hat{\mathbf{E}}_{\text{eff}}$ - the angular velocity of the precession,

$$\hat{E}_{\text{eff}} = \hat{x} E_{\text{eff}1} + \hat{y} E_{\text{eff}2} + \hat{z} E_{\text{eff}3}, \ E_{\text{eff}1} = \hat{e}_1 \hat{E}(\vec{q}_0, t) / \hbar, E_{\text{eff}2} = \hat{e}_2 \hat{E}(\vec{q}_0, t) / \hbar, E_{\text{eff}3} = -\omega_{21} - \omega_1$$

is the component of the effective electric field in the laboratory coordinate system $(\hat{x}, \hat{y}, \hat{z})$. Below is considering the case when the excitation is carried out by a quasi-resonant field with circular polarization in the $(\hat{x}, \hat{y})$ plane, $\mathbf{E} = ((\hat{x} + i \hat{y}) / \sqrt{2}) \mathbf{e}_{\text{eff}}(t) e^{i \omega t - i \varphi_0} + \text{c.c.}$, where $\omega \sim \omega_{21}$.

3. Overlying the effects of non-stationary nutation

In our formulation of the problem, a pair of size-quantized levels of the potential wells of a superlattice in a strong magnetic field interact with pulses $\hat{E}_1(\vec{q}_1, t)$ and $\hat{E}_2(\vec{q}_2, t)$ coherent radiation with circular polarization in the plane $(\hat{x}, \hat{y})$ that satisfies the necessary conditions for observing non-stationary nutation. Electrodipole interactions, in the absence of phase modulation, are described by a system of vector equations, where, in the first approximation, small relaxation terms are neglected in comparison with derivatives.
\[
\frac{d}{dt} \langle \hat{R}_i \rangle = \left[ \bar{\Omega}_i \langle \hat{R}_i \rangle \right], \quad \frac{d}{dt} \langle \hat{R}_2 \rangle = \left[ \bar{\Omega}_2 \langle \hat{R}_2 \rangle \right],
\] (2)

where \( \langle R_i \rangle, i = 1, 2 \) are the electric dipole pseudospin vectors,
\[
\bar{\Omega}_i = -\left( \tilde{e}_i \tilde{E}_i \left( \tilde{q}_{i0}, t \right) / \hbar, \tilde{e}_i \tilde{E}_i \left( \tilde{q}_{i0}, t \right) / \hbar, -\omega_{_{22}} \right)
\]
are the angular velocities of the precession. The fixed third component \( \omega_{_{23}} \) of the vectors \( \bar{\Omega}_i \) of angular velocities characterizes a two-level system, and the transverse components \( \bar{\Omega}_{1i}, \bar{\Omega}_{2i} \) are determined by the electric vectors of the acting radiations.

So, in the considered model, the overlapping of semiclassical electric dipole interactions of vectors \( \bar{\Omega}_i \) can be considered two-dimensional, obeying the rules of vector addition on the transverse plane (1, 2).

It is easy to verify that the steady-state process of interaction of a set of two-level systems with incident pulses is represented by the precession of the total pseudospin electric dipole vector \( \langle \hat{R}_0 \rangle = \langle \hat{R}_1 \rangle + \langle \hat{R}_2 \rangle \) around the resulting torque with an angular velocity \( \bar{\Omega}_0 = \bar{\Omega}_1 + \bar{\Omega}_2 \), in accordance with the vectorial equation of motion
\[
\frac{d}{dt} \langle \hat{R}_0 \rangle = \left[ \bar{\Omega}_0 \langle \hat{R}_0 \rangle \right].
\] (3)

This can be verified by averaging over time both sides of equation (3). Due to the precession of the vectors \( \langle \hat{R}_1 \rangle \) and \( \langle \hat{R}_2 \rangle \) around \( \bar{\Omega}_1 \) and \( \bar{\Omega}_2 \) one has
\[
\left[ \left[ \bar{\Omega}_1 \langle \hat{R}_1 \rangle \right] \right] = \text{const.}, \quad \left[ \left[ \bar{\Omega}_2 \langle \hat{R}_2 \rangle \right] \right] = \text{const.}
\]

and
\[
\left[ \left[ \bar{\Omega}_0 \langle \hat{R}_0 \rangle \right] \right], \approx \left[ \left[ \bar{\Omega}_1 \langle \hat{R}_1 \rangle \right] \right], \approx 0, \quad \langle \sin (\bar{\Omega}_1, \langle \hat{R}_1 \rangle) \rangle, \approx \langle \sin (\bar{\Omega}_2, \langle \hat{R}_2 \rangle) \rangle, \approx 0.
\]

As a result, equation (3) is reduced to a form that is equivalent to a timed summation of equations (2). This confirms the vector representation (3) for the superposition of optical nutation processes caused by coherent pulses \( \tilde{E}_1 \) and \( \tilde{E}_2 \). Here, the pseudospin vector \( \langle \hat{R}_0 \rangle \) displays the state of the medium from two-level systems, which is formed as a result of interactions with pulses \( \tilde{E}_1 \) and \( \tilde{E}_2 \). So, the physical picture of the complex process of superposition of precessing pseudospin vectors is simplified and replaced by a model of interaction of two-level systems with linear superposition of incident harmonic pulses with frequencies \( \omega_1 \) and \( \omega_2 \) representing an almost harmonic resultant oscillation with an average frequency \( \omega_m = (\omega_1 + \omega_2) / 2 \), that is in the same frequency range.

Now the problem is reduced to solving the equation of motion of the pseudospin (3), where in the laboratory system, in the coordinate representation, for the angular velocity, keeping only terms with real time dependencies, we have
\[
\bar{\Omega}_0 = \bar{\Omega}_0 \left( -\sqrt{2} e\hbar^{-1} \text{Re} \left( \mu_{21} \right) e(t) \cos \omega_m t, \sqrt{2} e\hbar^{-1} \text{Re} \left( \mu_{21} \right) e(t) \sin \omega_m t, \omega_{_{22}} \right).
\] (4)

Here
\[
e(t) = 2e\omega_0 \cos [(\omega_1 - \omega_2) / 2t] = 2e\omega_0 \cos [(\Delta \omega_1 - \Delta \omega_2) / 2t]
\]
is the slowly varying amplitude of the resultant impulse. Below we have to reveal the behavior of the polarization of the medium \( \langle \hat{P} \rangle \), which is expressed by the operators \( \langle \hat{R}_{01} \rangle \) and \( \langle \hat{R}_{02} \rangle \), as well as the population differences of the two-level systems of the superlattice \( \langle \hat{R}_{03} \rangle \). In the absence of exciting pulses, the two-level systems are in the ground state
\[
\langle \hat{R}_{01} (t = 0) \rangle = \langle \hat{R}_{02} (t = 0) \rangle = 0, \langle \hat{R}_{03} (t = 0) \rangle = m.
\] (5)

Here \( m \leq N / 2 \), where \( N \) is the total number of two-tier systems. If the pseudospin vector \( \langle \hat{R}_0 \rangle \) is initially deviated from the \( \hat{z} \) axis, then it precesses around the \( \hat{z} \) with frequency \( 2\omega_{_{22}} \hat{z} \) and in the
absence of excitation pulses. Simultaneous switching on $t=0$ of coherent impulses in case of detuning resonance $\Delta \omega \rightarrow 0$ in quasi-resonant fields leads to precession of the pseudospin vector $\langle \hat{R}_0 \rangle$ around $\hat{\Omega}_0$ with a frequency $|\Omega_0|$, which, in turn, due to the action of the electric fields of the incident radiation of circular polarization, rotates around the axis $\hat{z}$ with frequency $\omega_m$. Difficult movement $\langle \hat{R}_0 \rangle$ correspond to the variation of the components $\langle \hat{R}_0 \rangle$. In order to simplify the physical picture, we move into a coordinate system rotating around the axis $\hat{z}$ with a frequency $\omega_m$. The result vector equation (3) is transformed into

$$\frac{d}{dt}\langle \hat{R}_{01} \rangle = -(\omega_{21} - \omega_m)\langle \hat{R}_{02} \rangle,$$

$$\frac{d}{dt}\langle \hat{R}_{02} \rangle = (\omega_{21} - \omega_m)\langle \hat{R}_{01} \rangle + \frac{\sqrt{2}e \text{Re}(\mu_{21})\varepsilon(t)}{\hbar}\langle \hat{R}_{03} \rangle,$$

$$\frac{d}{dt}\langle \hat{R}_{03} \rangle = -\frac{\sqrt{2}e \text{Re}(\mu_{21})\varepsilon(t)}{\hbar}\langle \hat{R}_{02} \rangle,$$

where $\langle \hat{R}_{01} \rangle, \langle \hat{R}_{02} \rangle, \langle \hat{R}_{03} \rangle$ are the components of the pseudospin vector in the new coordinate system.

Let us consider the parametric dependences of the course of the resulting optical nutation processes on the magnetic field strength and the magnitudes of the resonance detunings. We analyze the following special cases.

**a.** $\omega_1 = \omega_2 = \omega_{21}$. Linear superposition of pulses is represented as a harmonic signal with a duration $\delta t = t_2 - t_1$, of frequency $\omega_{21}$ and constant amplitude $2\varepsilon_0$. The unknown components of the pseudospin $\langle \hat{R}_{01} \rangle, \langle \hat{R}_{02} \rangle, \langle \hat{R}_{03} \rangle$ in a rotating coordinate system are determined by integrating system (6). Under the condition (5) one find $\langle \hat{R}_{02} \rangle = A \sin(\Xi t), \langle \hat{R}_{03} \rangle = A \cos(\Xi t)$, where $A = m, \Xi = 2\sqrt{2} \text{Re}(\mu_{21})\varepsilon_0 / \hbar$. Here it is assumed that the discrete quantization of the electronic levels of the potential wells of the superlattice is due to size quantization along the growth axis $z$ of the superlattice and Landau quantization in the plane of the superlattice layers in the field $H \parallel z$. According to the model under consideration, dipole transitions in two-level systems of quantum wells under the action of exciting pulses occur between the Landau levels of the main mini-band of the superlattice conduction band. This gives a direct opportunity to study the controlled variation of the course of optical nutation processes by changing the intensity of an external quantizing magnetic field. For the simplest model of a semiconductor with a straight gap, isotropic non-degenerate energy bands with a parabolic dispersion law, the component of the matrix element $\mu_{21}$ is represented as

$$\mu_{21} = V^{-1} \left(\frac{\hbar c(n+1)}{2eH}\right)^{1/2}(2eH)^{1/2} \delta_{k_+ k_+} \delta_{j+ j+} \delta_{j+ j+} \nu = \{n, k_x, k_z, j\} - \text{the quantum numbers of the electron,}\ n - \text{the quantum number of the magnetic oscillator,} \ j - \text{the number of the miniband where the nonequivalent values} k_z \text{ vary within the Brillouin zone } -\pi/d \leq k_z < \pi/d, \text{ where the} d - \text{ is period of the superlattice.}$$
Figure 1. The dependence of the inversion $2\langle \rho_{03}(t,0) \rangle / N_1$ at time for $\omega_1 = \omega_2 = \omega_m \neq \omega_{21}$.

The graphs of left panel are plotted for the fixed value of $H = 10^5$ Oe and different values of $\Delta \omega$: $1 - \Delta \omega = 0; 2 - \Delta \omega = 0, 2\Xi; 3 - \Delta \omega = \Xi; 4 - \Delta \omega = 1, 2\Xi; 5 - \Delta \omega = 2\Xi; 6 - \Delta \omega = 2, 2\Xi$.

The graphs of right panel are plotted for the fixed value of $\Delta \omega = 0, 2\Xi$ and different values of $H$: $1 - H = 10^5$ Oe; $2 - H = 5 \cdot 10^5$ Oe; $3 - H = 10^6$ Oe, for a $\Xi = 7,3927 \cdot 10^9$ s$^{-1}$.

c. $\omega_1 \neq \omega_2, \varepsilon(t) = 2\varepsilon' \cos \left[ t(\omega_1 - \omega_2) / 2 \right], \langle \rho_{03} \rangle = \Delta \omega A \left[ (\Delta \omega)^2 + B^2(t) \right]^{-1/2} \cos \theta(t) + K, 
\langle \hat{\rho}_{02} \rangle = A \sin \theta(t), \langle \hat{\rho}_{03} \rangle = A \cdot B(t) \left[ (\Delta \omega)^2 + B^2(t) \right]^{-1/2} \cos \theta(t) + L, 

where $B(t) = \sqrt{2} \varepsilon \Re(\mu_{21} \varepsilon(t)/h) = b \cos (\omega_0 t), b = 2 \sqrt{2} \varepsilon \Re(\mu_{21}) \varepsilon / h; \omega_0 = (\omega_1 - \omega_2) / 2,$

$\theta(t) = \Delta \omega \sqrt{1 + k^2 / \omega_0^2} \left[ E(\omega_0 t, q) + C_n \right], k^2 = b^2 / (\Delta \omega)^2, q^2 = b^2 / (\Delta \omega)^2 + b^2, C_n = \pi \omega_0 / \Delta \omega \sqrt{1 + k^2}$.

Here $E(\omega_0 t, q)$ – is the elliptic integral of the second kind, $n = 0, \pm 1, \pm 2, \ldots$ [7]. $K, L$ – are the integration constants. From the requirements of the initial conditions (5) one has

$K = -\frac{\Delta \omega}{\sqrt{\left( (\Delta \omega)^2 + b^2 \right)}}, L = m - \frac{Ab}{\sqrt{\left( (\Delta \omega)^2 + b^2 \right)}}, A = \frac{bn}{\sqrt{\left( (\Delta \omega)^2 + b^2 \right)}}.$
You can make sure that with such a choice of constants $A, K, L$ the obligatory condition
\[
\frac{d}{dt} \left( \langle \hat{p}_{01} \rangle^2 + \langle \hat{p}_{02} \rangle^2 + \langle \hat{p}_{03} \rangle^2 \right) = 0, \langle \hat{p}_{01} \rangle^2 + \langle \hat{p}_{02} \rangle^2 + \langle \hat{p}_{03} \rangle^2 = \text{const}
\]
is fulfilled.

Figure 3. The dependence of the inversion $2\langle \hat{p}_{03} (t, \Delta \omega) \rangle / N_1$ at time for $\omega_1 = \omega_{21} - 3\Delta \omega$, $\omega_2 = \omega_{21} + \Delta \omega$. The graphs of left/right panels are plotted for the fixed value of $\Delta \omega = 0, 2\Xi / \Delta \omega = 2, 2\Xi$. The full/dashed curves correspond to $H = 10^6 \text{Oe} / H = 10^6 \text{Oe}$.

One can use the well-known Brewer-Schumaker frequency Stark-switching technique for an experimental study of the superposition of non-stationary nutation processes, [8]. In this case, the test medium (superlattice) placed in the Stark cell must interact with the excitation radiation of two lasers with fixed emission frequencies. Variations of $\Delta \omega_1, \Delta \omega_2$ detuning can be achieved by changing the magnitude of the periodically switched on Stark field, allowing not only to observe consecutive optical nutation signals, but also to vary the $\Delta \omega_1$ and $\Delta \omega_2$ detuning values, and to follow the process of superimposing the motion of precessing pseudo-dipoles. It is also possible to use lasers with tunable frequencies, but to fix the magnitude of the pulsating Stark field and to observe the process of imposing nutation with the aid of Stark switching.

4. Conclusion
The work revealed the evolution of the characteristics of two-level systems during the process of non-stationary nutation, due to the interaction with a pair of ultrashort pulses in a periodic superlattice with magnetic quantization. The variations of population inversion are demonstrated for different values of the detuning of the resonance and the magnetic field strength.

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