THE TURBULENT DYNAMO IN HIGHLY COMPRESSIBLE SUPersonic PLASMAS

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ABSTRACT

The turbulent dynamo may explain the origin of cosmic magnetism. While the exponential amplification of magnetic fields has been studied for incompressible gases, little is known about dynamo action in highly compressible, supersonic plasmas, such as the interstellar medium of galaxies and the early universe. Here we perform the first quantitative comparison of theoretical models of the dynamo growth rate and saturation level with three-dimensional magnetohydrodynamical simulations of supersonic turbulence with grid resolutions of up to 10243 cells. We obtain numerical convergence and find that dynamo action occurs for both low and high magnetic Prandtl numbers $Pm = v/\eta = 0.1$–10 (the ratio of viscous to magnetic dissipation), which had so far only been seen for $Pm \gtrless 1$ in supersonic turbulence. We measure the critical magnetic Reynolds number, $Rm_{\text{crit}} = 129^{+44}_{-33}$, showing that the compressible dynamo is almost as efficient as in incompressible gas. Considering the physical conditions of the present and early universe, we conclude that magnetic fields need to be taken into account during structure formation from the early to the present cosmic ages, because they suppress gas fragmentation and drive powerful jets and outflows, both greatly affecting the initial mass function of stars.

Key words: dynamo – galaxies: ISM – ISM: clouds – magnetic fields – magnetohydrodynamics (MHD) – turbulence

Online-only material: animation, color figures

1. INTRODUCTION

Magnetic field amplification via the turbulent dynamo is believed to be the main cause of cosmic magnetism. The turbulent dynamo is important for the formation of the large-scale structure of the universe (Ryu et al. 2008), in clusters of galaxies (Subramanian et al. 2006) and in the formation of the first cosmological objects (Schleicher et al. 2010). It determines the growth of magnetic energy during solar convection (Cattaneo & Hughes 2001; Moll et al. 2011; Pietarila Graham et al. 2010), in the interior of planets (Roberts & Glatzmaier 2000), and in liquid metal experiments on Earth (Monchaux et al. 2007). It may further explain the far-infrared–radio correlation in spiral galaxies (Schleicher & Beck 2013). After the turbulent dynamo has amplified tiny seeds of the magnetic field, which can be generated during inflation, the electroweak or the QCD phase transition (Grasso & Rubinstein 2001), the large-scale dynamo kicks in and generates the large-scale magnetic fields that we observe in planets, stars, and galaxies today (Beck et al. 1996; Brandenburg & Subramanian 2005).

The properties of the turbulent dynamo strongly depend on the magnetic Prandtl number, $Pm = v/\eta$, defined as the ratio of viscosity $v$ to magnetic diffusivity $\eta$ (Schekochihin et al. 2004). On large cosmological scales and in the interstellar medium, we typically have $Pm \gg 1$, while for the interior of stars and planets, the case with $Pm \ll 1$ is more relevant (Schekochihin et al. 2007). Numerical simulations, on the other hand, are typically restricted to $Pm \sim 1$, because of limited numerical resolution. Simulations by Iskakov et al. (2007) have clearly demonstrated that the turbulent dynamo operates for $Pm \lesssim 1$ in incompressible gases, even though an asymptotic scaling relation has not been confirmed. While the bulk of previous work was dedicated to exploring the turbulent dynamo in the incompressible regime (Brandenburg et al. 2012), most astrophysical systems show signs of high compressibility. This is particularly true during the formation of the first cosmological objects (Latif et al. 2014), in the interstellar medium of galaxies (Larson 1981) and in the intergalactic medium (Iapichino et al. 2013). The compressibility of the plasma can be characterized in terms of the sonic Mach number $M = V/c_s$, the ratio of the turbulent velocity $V$, and the sound speed $c_s$. The Mach number typically exceeds unity by far in all of these systems, which is a hallmark of highly compressible, supersonic turbulence.

In the framework of the Kazantsev model (Kazantsev 1968), Schober et al. (2012a) derived analytical dynamo solutions for the limiting cases $Pm \to \infty$ and $Pm \to 0$, considering different scaling relations of the turbulence, while Bovino et al. (2013) derived a numerical solution of the Kazantsev equation for finite values of $Pm$. These studies strongly suggest that the turbulent dynamo operates for different values of $Pm$, as long as the magnetic Reynolds number, $Rm = VL/\eta$, is sufficiently high, where $L$ is the characteristic size of the large-scale turbulent structures.

However, a central restriction of the Kazantsev framework is the assumption of an incompressible velocity field, for which a separation into solenoidal and compressible parts is not necessary. The distinction between solenoidal and compressible modes, however, may be essential for highly compressible, supersonic turbulence. Furthermore, the Kazantsev framework assumes that the turbulence is $\delta$-correlated in time, which is not appropriate for real turbulence. The resulting uncertainties introduced by that assumption, however, are only a few percent (Schekochihin & Kulsrud 2001; Kleerorin et al. 2002; Bhat & Subramanian 2014), while the assumption of incompressibility is a severe limitation. Ultimately, the full nonlinear solution through three-dimensional (3D) simulations is needed to determine the behavior of the growth rates under more realistic conditions.

We note that the turbulent dynamo has also been studied in the context of so-called shell models (Frick et al. 2006, and
In this Letter, we present the first investigation of the turbulent dynamo and its dependence on the magnetic Prandtl number in the highly compressible, supersonic regime. For this purpose, we consider supersonic turbulence with Mach numbers ranging from $M = 3.9$ to $11$, and magnetic Prandtl numbers between $Pm = 0.1$ and $10$. The results are compared with the predictions from the Kazantsev model. Section 2 defines the numerical methods used in the simulations, Section 3 summarizes current dynamo theories, Sections 4 and 5 present our results and conclusions.

2. NUMERICAL SIMULATIONS

We use a modified version of the FLASH code (Fryxell et al. 2000; v4) to integrate the 3D, compressible, magnetohydrodynamical (MHD) equations, including viscous and resistive dissipation terms,

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot \left( \rho \mathbf{v} \otimes \mathbf{v} - \frac{1}{4\pi} \mathbf{B} \otimes \mathbf{B} \right) + \nabla p_{tot} = \nabla \cdot (2\nu \mathbf{S}) + \mathbf{F},$$

$$\frac{\partial}{\partial t} \mathbf{E} + \nabla \cdot \left[ (E + p_{tot}) \mathbf{v} - \frac{1}{4\pi} (\mathbf{B} \cdot \mathbf{v}) \mathbf{B} \right] = \nabla \cdot \left[ 2\nu \mathbf{v} \otimes \mathbf{S} + \frac{1}{4\pi} \mathbf{B} \times (\eta \nabla \times \mathbf{B}) \right],$$

$$\frac{\partial}{\partial t} \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\nabla \cdot \mathbf{B} = 0.$$

In these equations, $\rho$, $\mathbf{v}$, $p_{tot} \equiv p_{th} + (1/8\pi) |\mathbf{B}|^2$, $\mathbf{B}$, and $E = \rho e_{int} + (1/2) |\mathbf{v}|^2 + (1/8\pi) |\mathbf{B}|^2$ denote plasma density, velocity, pressure (thermal plus magnetic), magnetic field, and energy density (internal plus kinetic, plus magnetic), respectively. Physical shear viscosity is included via the traceless rate of strain tensor, $S_{ij} = (1/2)(\partial_i u_j + \partial_j u_i) - (1/3)\delta_{ij} \nabla \cdot \mathbf{v}$ in the momentum Equation (2), and controlled by the kinematic viscosity, $\nu$. Physical diffusion of $\mathbf{B}$ is controlled by the magnetic resistivity $\eta$ in the induction Equation (4). To solve the MHD equations, we use the positive-definite second-order accurate HLL3R Riemann scheme, capable of handling strong shocks (Waagan et al. 2011).

The MHD equations are closed with an isothermal equation of state, $p_{th} = c_s^2 \rho$.

To drive turbulence with a given Mach number $M$, we apply a divergence-free large-scale forcing term $\mathbf{F}$ as a source term in the momentum Equation (2). The forcing is modeled with a stochastic Ornstein–Uhlenbeck process (Federrath et al. 2010), such that $\mathbf{F}$ varies smoothly in space and time with an autocorrelation equal to the eddy-turnover time, $T = L/(2Mc_s)$ on the largest scales, $L/2$ in our periodic simulation domain of side length $L$.

The efficiency of magnetic field amplification depends on the growth rate, which in turn depends on the driving mode, the Mach number, the Reynolds numbers $Re$ and $Rm$, and the Prandtl number, $Pm$ (Federrath et al. 2011; Schober et al. 2012a; Bovino et al. 2013; Schleicher et al. 2013). We run most of our simulations until saturation of the magnetic field is reached. Given the Reynolds numbers achievable in state-of-the-art simulations, this can take several hundred crossing times. Saturation occurs when the Lorentz force induces a back reaction of the magnetic field strong enough to counteract the turbulent twisting, stretching and folding of the field (Brandenburg & Subramanian 2005). We determine the saturation levels by measuring the ratio of magnetic to kinetic energy, $(E_{mag}/E_{kin})_{sat}$.

Here we study the dependence of the turbulent dynamo on $Pm$, which is accomplished by varying the physical viscosity and resistivity. Table 1 provides a complete list of all simulations and key parameters. To test convergence, we run simulations with $N_{res}^3 = 128^3$–$1024^3$ grid points.

3. DYNAMO THEORY

Theories for the turbulent dynamo are based on the Kazantsev model (Kazantsev 1968; Brandenburg & Subramanian 2005),

$$- \kappa_{int}(\ell) \frac{d^2 \psi(\ell)}{d\ell^2} + U(\ell) \psi(\ell) = -\Gamma \psi(\ell),$$

which assumes zero helicity, $\delta$-correlation in time, and does not take into account the mixture of solenoidal-to-compressible modes in the turbulent velocity field. These limitations are related to the fact that the Kazantsev equation was historically only applied to incompressible turbulence, while we apply it here to highly compressible, supersonic turbulence.

The similarity of Equation (6) with the quantum-mechanical Schrödinger equation allows us to solve it both numerically and analytically, which requires an assumption for the scaling of the turbulent velocity correlations. Numerical simulations of turbulence find a power-law scaling within the inertial range ($\ell_\nu < \ell < L$),

$$\delta v(\ell) \propto \ell^\vartheta,$$

where $\ell_\nu$ and $L$ are the viscous and integral scale, respectively. The exponent $\vartheta$ varies from $1/3$ for incompressible, non-interactive Kolmogorov turbulence up to $1/2$ for highly compressible, supersonic Burgers turbulence. Numerical simulations of mildly supersonic turbulence with Mach numbers $M \approx 2$–$7$ find $\vartheta \approx 0.37$–$0.47$ (Boldyrev et al. 2002; Kowal & Lazarian 2010; Federrath et al. 2010). Highly supersonic turbulence with $M > 15$ asymptotically approaches the Burgers limit, $\vartheta = 0.5$ (Federrath 2013). Observations of interstellar clouds indicate a comparable velocity scaling with $\vartheta \approx 0.38$–$0.5$ (Larson 1981; Heyer & Brun 2004; Roman-Duval et al. 2011). Given this range of exponents, we investigate how the theoretical results depend on $\vartheta$, by studying cases with $\vartheta = 0.35, 0.40, and 0.45$.

Using the Wentzel–Kramers–Brillouin (WKB) approximation we obtain an analytical solution of the Kazantsev equation, which depends on the velocity scaling exponent $\vartheta$. Results for $Pm \gg 1$ and $Pm \ll 1$ have been reported in Schober et al. (2012a, 2012b). More recently, Bovino et al. (2013) applied a Numerov scheme to solve Equation (6) numerically for $Pm \approx 0.1$–$10$, the regime currently accessible in dynamo simulations. The dependence on the velocity correlation exponent $\vartheta$ forms the main extension of the original, incompressible Kazantsev equation into the compressible regime (note that...
Rogachevskii & Kleerom 1997 have followed a similar approach for mildly compressible, low-Mach number turbulence. However, the generalizations by Schober et al. (2012a, 2012b) still do not account for variations in the solenoidal-to-compressible mode mixture that is excited in supersonic turbulence.

4. RESULTS AND DISCUSSION

To get a visual impression of the differences in the magnetic field structure between low-Pm and high-Pm dynamo action, we plot magnetic energy slices in Figure 1. By definition, magnetic dissipation is much stronger in low-Pm compared to high-Pm turbulence (for Re = const, as in our numerical experiments), but we find that the dynamo operates in both cases. This is the first time that dynamo action is confirmed in low-Pm, highly compressible, supersonic plasma.

We now determine the dynamo growth rate as a function of Pm for fixed Re = 1600 and as a function of Re for fixed Pm = 10, in order to compare the analytical and numerical solutions of the Kazantsev equation with the MHD simulations. Depending on Pm and Re, we find exponential magnetic energy growth over more than six orders of magnitude for simulations in which the dynamo is operational. We determine both the exponential growth rate $\Gamma$ and the saturation level ($E_{\text{mag}}/E_{\text{kin}}$) sat.

The measurements are listed in Table 1 and plotted in Figure 2.

In the left-hand panel of Figure 2, we see that $\Gamma$ first increases strongly with Pm for Pm $\lesssim 1$. For Pm $\gtrsim 1$ it keeps increasing, but more slowly. The theoretical models by Schober et al. (2012a) and Bovino et al. (2013) both predict an increasing growth rate with Pm. The purely analytical solution of the Kazantsev equation (6) by Schober et al., using the WKB approximation, yields power laws for Re $\gg$ Rm RT, while the numerical solution of Equation (6), using the Numerov method by Bovino et al., yields a sharp cutoff when Pm $\lesssim 1$, closer to the results of the 3D MHD simulations. The agreement of the theoretical prediction with the MHD simulations is excellent for Pm $\gtrsim 1$, while for Pm $\lesssim 1$ they only agree qualitatively. The discrepancy arises because the theoretical models assume zero helicity, $\delta$-correlation of the turbulence in time, and currently do not distinguish different mixtures of solenoidal and compressible modes in the turbulent velocity field. Finite time correlations, however, do not seem to change the Kazantsev result significantly (Bhat & Subramanian 2014) and our simulations have zero helicity. Thus, the missing distinction between solenoidal and compressible modes may be the main cause of the discrepancy, because the dynamo is primarily driven by solenoidal modes and the amount of vorticity strongly depends on the driving and Mach number of the turbulence (Mee & Brandenburg 2006; Federrath et al. 2011).

The saturation level as a function of Pm is shown in the bottom left-hand panel of Figure 2. It increases with Pm similar to the growth rate and is also well converged with increasing numerical resolution. We currently do not have a theoretical model to predict the dynamo saturation level, but it may be possible to develop one based on an effective magnetic diffusivity, which limits the growth of the magnetic field when the back reaction through the Lorentz force prevents turbulence from further stretching, twisting and folding the field (Subramanian 1999; Brandenburg & Subramanian 2005). However, we currently lack a model that applies to the highly compressible regime of MHD turbulence and that covers the dependence on Pm, although we provide a simple model for the dependence of ($E_{\text{mag}}/E_{\text{kin}}$) sat on Re below.

Finally, the right-hand panels of Figure 2 show the growth rate and saturation level as a function of Re. Similar to the dependence on Pm, we find a nonlinear increase in $\Gamma$ with Re, which is qualitatively reproduced with the numerical solution by Bovino et al. (2013). However, the critical Reynolds number for dynamo action is much lower in the MHD simulations than...
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Figure 1. Magnetic energy slices through our simulations with grid resolutions of $1024^3$ points. The magnetic field grows more slowly for magnetic Prandtl numbers of $P_m = 0.1$ (left-hand panel) compared to $P_m = 10$ (right-hand panel), but we find dynamo action in both cases, for the first time in highly compressible, supersonic plasmas.

(An animation and a color version of this figure are available in the online journal.)

Figure 2. Left panels: dynamo growth rate $\Gamma$ (top panel) and saturation level $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}}$ (bottom panel) as a function of $P_m$ for fixed $Re = 1600$. Resolution studies with $256^3$, $512^3$, and $1024^3$ grid cells demonstrate convergence, tested for the extreme cases $P_m = 0.1$ and 10. Theoretical predictions for $\Gamma$ by Schober et al. (2012a) and Bovino et al. (2013) are plotted with different line styles for a typical range of the turbulent scaling exponent $\vartheta = 0.35$ (dotted), 0.40 (solid), and 0.45 (dashed). Right panels: same as left panels, but $\Gamma$ and $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}}$ are shown as a function of $Re$ for fixed $P_m = 10$. The dot-dashed line is a fit to the simulations, yielding a constant saturation level of $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}} = 0.05 \pm 0.01$ for $Re > R_{\text{crit}} = R_{\text{mcrit}}/P_m = 12.9$ and the triple-dot-dashed line shows the result of Subramanian’s modified model for the saturation level (Subramanian 1999).

(A color version of this figure is available in the online journal.)

Predicted by the theoretical model, which may have the same reasons as the discrepancy found for the dependence on $P_m$, i.e., the lack of dependence on the actual turbulent mode mixture in the theoretical models.

In order to determine the critical magnetic Reynolds number for dynamo action, we perform fits with

$$\Gamma = \beta [\ln(P_m) + \ln(Re)] - \gamma,$$  

predicted by the theoretical model, which may have the same reasons as the discrepancy found for the dependence on $P_m$, i.e., the lack of dependence on the actual turbulent mode mixture in the theoretical models.
using the fit parameters $\beta$ and $\gamma$, which are related to the
critical magnetic Reynolds number $R_{\text{m crit}} = \exp(\gamma/\beta)$. From
the fits with Equation (8) to all our simulations, we find that
dynamo action is suppressed for $R_m < R_{\text{m crit}} = 129^{+43}_{-31}$ in
highly compressible, supersonic MHD turbulence. Our result is
significantly higher than the critical magnetic Reynolds number
measured in simulations of subsonic, incompressible MHD
turbulence by Haugen et al. (2004a), who find $R_{\text{m crit}} \sim 20-40$
for $P_m \gtrsim 1$, and higher than in mildly compressible simulations,
where $R_{\text{m crit}} \sim 50$ for $P_m = 5$ and $M \sim 2$ (Haugen et al. 2004b).
The reason for the higher $R_{\text{m crit}}$ compared to
incompressible turbulence is the more sheet-like than vortex-like
structure of supersonic turbulence (Boldyrev 2002; Schmidt
et al. 2008) and the reduced fraction of solenoidal modes (Mee &
Brandenburg 2006; Federrath et al. 2010, 2011). The difference
with the theoretical models lies primarily in $R_{\text{m crit}}$. Bovino et al.
(2013) predicted a much higher $R_{\text{m crit}} \sim 4100$ for $\vartheta = 0.45$,
while fits to their theoretical model yield $\beta = 0.11-0.19$, in
agreement with the range found in the MHD simulations
($\beta = 0.141 \pm 0.004$). This demonstrates that the discrepancy
between the MHD simulations and the Kazantsev model is
primarily in the predicted $R_{\text{m crit}}$ value, while the qualitative
behavior (determined by the $\beta$ parameter) is covered by the
theoretical dynamo models.

The saturation level shown in the bottom right-hand panel of
Figure 2 is consistent with a constant level of $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}} = 0.05 \pm 0.01$ for $Re > Re_{\text{crit}} \equiv R_{\text{m crit}}/P_m = 12.9$ in
highly compressible, supersonic turbulence with Mach numbers $M \sim 10$, typical for molecular clouds in the Milky Way. Given our
measurement of $R_{\text{m crit}} = 129$, we can compute Subramanian’s
theoretical prediction (Subramanian 1999) for the saturation
level, $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}} = (3/2)(L/V)\tau^{-1} R_{\text{m crit}}^{-1} \sim 0.01$, which is
significantly smaller than our simulation result, assuming that $\tau = T = L/V$ is the turbulent crossing time on the
largest scales of the system. However, Subramanian notes that the
timescale $\tau$ is an “unknown model parameter.” Thus, a more
appropriate timescale for saturation may be the eddy timescale on
the viscous scale, $t_e = LRe^{-(2/3)}$ for a given turbulent velocity
scaling following Equation (7), because this is where the field saturates first. We find $\tau(t_e) = t_e/v(t_e) = TRe^{-(2/3)}$ and with $Re = Re_{\text{crit}} = 12.9^{+43}_{-31}$, we obtain $(E_{\text{mag}}/E_{\text{kin}})_{\text{sat}} = 0.035 \pm 0.005$ for a typical range of the
velocity scaling exponent $\vartheta = 0.4 \pm 0.1$, from molecular cloud
observations and simulations of supersonic turbulence ( Larson
1981; Heyer & Brunt 2004; Roman-Duval et al. 2011). The
saturation level of our 3D MHD simulations thus agrees within
the uncertainties with our modified version of Subramanian’s
model. We note that the dependence on $P_m$ (see the bottom left-
hand panel of Figure 2) is, however, not included in the current
model and requires further theoretical development.

To support our conclusions, we show magnetic energy power
spectra in Figure 3. They are qualitatively consistent with the
incompressible dynamo studies by Mason et al. (2011) and
Bhat & Subramanian (2013). We clearly see that the power
spectra for $P_m = 0.1$ dissipate on larger scales (lower $k$)
unlike $P_m = 10$ spectra, consistent with the theoretical
expectation by a factor of $(10/0.1)^{3/2} \sim 22-27$ for our
relevant $\vartheta \sim 0.4-0.5$. Nevertheless, even for $P_m = 0.1$, we
see the dynamo-characteristic increase in magnetic energy over
all scales. The magnetic spectra roughly follow the Kazantsev
spectrum ($\sim k^{3/2}$) on large scales (Kazantsev 1968; Bhat &
Subramanian 2014) in the $P_m = 10$ case, but we would expect
the same to hold in the $P_m = 0.1$ case, if our simulations had
larger scale separation. The final spectrum for $P_m = 10$ has
just reached saturation on small scales (approaching the kinetic
energy spectrum at high $k$), but continues to grow on larger
scales during the nonlinear dynamo phase. The $P_m = 0.1$ runs
did not have enough time to reach saturation yet (see Figure 2),
but we expect a qualitatively similar behavior in the nonlinear
dynamo phase also for models with $P_m < 1$. We emphasize that
the kinetic energy spectra shown in Figure 3 and the saturation
levels plotted in the bottom panels of Figure 2 take into account
the variations in the density field, i.e., $E_{\text{kin}} = (1/2)\rho v^2$,
because—unlike incompressible turbulence—the density varies
by several orders of magnitude in our highly compressible,
supersonic turbulence simulations (for a recent analysis of the
typical density structures and probability density functions, see
Federrath 2013).

5. CONCLUSIONS

We presented the first quantitative comparison of theoretical
models of the turbulent dynamo with 3D simulations of super-
sonic MHD turbulence. We find that the dynamo operates at low
and high magnetic Prandtl numbers, but is significantly more
efficient for $P_m > 1$ than for $P_m < 1$. We measure a critical
magnetic Reynolds number for dynamo action, $R_{\text{m crit}} = 129^{+43}_{-31}$
in highly compressible, supersonic turbulence, which is a factor
of $\sim 3$ times higher than found in studies of subsonic and
incompressible turbulence. $R_{\text{m crit}}$ is, however, still several
orders of magnitude lower than the magnetic Reynolds number
in stars, planets, and in the interstellar medium of galaxies in
the present and early universe, allowing for efficient turbulent
dynamo action in all of these environments. This has important
consequences for the star formation rate and for the initial mass
function of stars, because magnetic fields suppress gas fragmen-
tation and lead to powerful protostellar jets and outflows (see
Krumholz et al. 2014; Padoan et al. 2014; Offner et al. 2014;
Federrath et al. 2014, and references therein). We conclude that
magnetic fields need to be taken into account during structure
formation in the present and early universe.
We thank R. Banerjee and R. Klessen for stimulating discussions on the turbulent dynamo and the anonymous referee for useful comments. C.F. acknowledges funding provided by the Australian Research Council’s Discovery Projects (grants DP110102191, DP130102078, DP150104329). D.R.G.S., J.S. and S.B. acknowledge funding via the DFG priority program 1573 “The Physics of the Interstellar Medium” (grants SCHL 1964/1-1, BO 4113/1-2). We gratefully acknowledge the Jülich Supercomputing Centre (grant hhd20), the Leibniz Rechenzentrum, and the Gauss Centre for Supercomputing (grant pr89mu), and the Australian National Computing Infrastructure (grant ek9). The software used in this work was in part developed by the DOE-supported Flash Center for Computational Science at the University of Chicago.

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