Stabilizing wave attenuation effects in turning process

Chigbogu Godwin Ozoegwu*

Department of Mechanical Engineering, Nnamdi Azikiwe University, PMB 5025, Awka, Nigeria

(Received 13 September 2013; accepted 17 December 2013)

Efforts have been made and continue to be made to understand the self-excited vibrations of machine tools called regenerative chatter. Theory of stabilizing wave attenuation effects in machining process presented in another work is expanded in this work to postulate the behaviour of turning stability lobes to changes in material, process and structural parameters. The wave attenuation theory simply stated that rise in attenuating forces suppresses chatter instability. Analysis in this work showed that rise in tool natural frequency \(\omega_n\), damping ratio \(\xi\) or feed speed \(v\) causes a rise in attenuating forces thus suppressing chatter. As expected from the wave attenuation theory harder workpiece materials with higher cutting coefficients \(C\) will better resist wave attenuation, and thus exhibit more chatter instability. Numerical verification of postulations is given.

**Keywords:** turning process; wave attenuation; chatter; turning point critical force; stability

1. Introduction

Regenerative vibrations are the self-excited vibrations in machine tools that are referred to as secondary chatter in literature (Wiercigroch & Budak, 2001). Regenerative chatter is a much more detrimental problem to the machining industry than the primary chatter phenomena that stem from frictional, thermo-mechanical and mode-coupling effects. Some of the unwanted effects of regenerative chatter are noise pollution in the work environment, poor surface quality, aggravated tool wear, reduced productivity and waste of materials and energy. Two of the most important reviews of machine tool chatter research (Quintana & Ciurana, 2011; Siddhpura & Paurobally, 2012) noted that regenerative chatter is not yet fully understood even though it has received most attention of all chatter phenomena studied over a century. Regenerative chatter is caused by Regenerative effects which are effects of waviness on a machined surface that are in turn caused by response of the machine tool system to disturbances. The waves of consecutive turns are normally out of phase causing cutting force variation that excites the tool. If cutting process parameters are stable, then cutting force variation decays because phase difference of consecutive turns die out with time. The opposite is the case when cutting process parameters constitute an unstable combination. Before regenerative chatter was distinguished and understood as stemming from self-excitation effects it was originally thought by Arnold (1946) as stemming from negative damping. Some of the standard early works (Merritt, 1965; Tobias, 1961; Tobias & Fishwick, 1958) on regenerative chatter considered a single degree of freedom orthogonal turning process vibrating in...
the feed direction only. Analysis of non-linear regenerative chatter was introduced by Hanna and Tobias (1974). It is known that the so-called Hopf bifurcation (lose of stability) in turning operation occurs when a pair of complex conjugate characteristic roots crosses from the left-half plane to right-half plane of the complex plane. More recently non-linear analysis has been strengthened by the experiment of Shi and Tobias (1984) and analysis of Stepan and Kalmar-Nagy (1997) by revealing that Hopf bifurcation of turning operation is subcritical. In addition to chatter frequencies of Hopf bifurcation type, damped natural frequency can also be spotted in the frequency response spectrum of high-speed turning tool due to the effects of tool-workpiece contact loss. Damped natural frequency has been experimentally confirmed to exist in the frequency response spectrum of high-speed milling (Insperger, Stepan, Bayly, & Mann, 2003). Experimental data of chatter and other vibrations in the turning and milling processes have been characterized using multi-scale methods (Litak, Syta, & Rusinek, 2011; Sen, Litak, Syta, & Rusinek, 2013) and flicker-noise spectroscopy (Litak, Polyakov, Timashev, & Rusinek, 2013). The theory of stabilizing wave attenuation effects in machining process was presented by Ozoegwu and Omenyi (2013). The theory posits that some of the force of interaction between the tool and workpiece acts normal to surface waves, thus depressing or flattening them in a way as to attenuate the chatter—causing regenerative effects. The attenuating force is either compressive when there is a continuous cutting edge-workpiece engagement or impulsive when the active pass of the cutting edge is highly interrupted as in high-speed machining. The theory further posits that any variable of the attenuating force that varies in a way as to increase the amplitude of the attenuating force will suppress chatter instability. In Ozoegwu and Omenyi (2013), the wave attenuation theory was able to explain why chatter stability rises with rise in feed, fall in number of teeth and fall in hardness of workpiece material. The theory of stabilizing wave attenuation effects is being expanded in this work to cover the turning process.

2. The cutting force, dynamics and stability of regenerative turning

Figure 1 represents an orthogonal turning process in which the cutting edge of the tool is perpendicular to the feed motion. In the physical configuration the workpiece is clamped in the chuck of a rotating spindle while the tool is fed in the workpiece. Regenerative chatter is the self-excited vibration in machining due to the effects of waviness (regenerative effects) on a machined surface resulting from disturbed dynamic interaction between the tool and the workpiece. The present turn of the workpiece has waviness that is not in phase with those of the last turn resulting chip thickness variation, hence cutting force variation that powers vibration which subsequently builds up to chatter if cutting parameter combination is unfavourable. As shown in Figure 1, a tool with modal parameters; $m$ the mass, $c$ the equivalent viscous damping coefficient and $k$ the stiffness is fed into the workpiece at a speed $v$ in $x-$ direction. The response of the tool $x(t)$ to the regenerative cutting force variation is seen from Figure 1 to be governed by the differential equation

$$m\ddot{x}(t) + c[\dot{x}(t) - vt] + k[x(t) - vt] + F_x(t) = 0$$  \hspace{1cm} (1)

The cutting force $F_x(t)$ has the empirical form (Insperger, 2002; Stepan, Szalai, & Inspeger, 2004; Tlusty, 2000);

$$F_x(t) = Cwf_a^2$$  \hspace{1cm} (2)
where \( C \) is the cutting coefficient, \( w \) is the depth of cut, \( f_a \) is the actual feed rate defined as difference of present and one period delayed position of tool; \( f_a = x(t) - x(t - \tau) \) and \( \gamma \) is the feed exponent of the cutting force that has typical values 1, .8 and 3/4. With the use of Equation (2), Equation (1) is re-arranged to give

\[
m\ddot{x} + c\dot{x} + kx = cv + kvt - Cw(x - x_\tau)^\gamma.
\]

(3)

where \( x(t) = x \) and \( x(t - \tau) = x_\tau \). The response of the tool is a linear combination of prescribed feed motion \( vt \), static deflection \( x_\tau(t) = -(Cwf^{\gamma})/k = -[Cw(vt)^{\gamma}]/k \) and perturbation \( z(t) \) such that Equation (3) becomes.

\[
m\ddot{z} + c\dot{z} + kz = Cw(vt)^{\gamma} - Cw[v\tau + (z - z_\tau)]^{\gamma}.
\]

(4)

The linearized Taylor’s series expansion Equation (4) about \( vt \) gives

\[
m\ddot{z} + c\dot{z} + kz = -hw(z - z_\tau).
\]

(5)

where \( h = C\gamma(vt)^{\gamma-1} \). Recognizing that the natural frequency and damping ratio of the tool are given in terms of modal parameters \( k, m \) and \( c \), respectively as \( \omega_n = \sqrt{k/m} \) and \( \xi = c/2\sqrt{mk} \) gives the modal form

\[
\ddot{z} + 2\xi_\omega_n\dot{z} + \left(\omega_n^2 + \frac{hw}{m}\right)z = \frac{hw}{m}z_\tau.
\]

(6)

Equation (6) is the linear dynamical model for the regenerative turning tool. With the substitutions \( y_1 = z \) and \( y_2 = \dot{z} \) made, Equation (6) could be put in state differential equation form

\[
y = Ay + By_\tau
\]

(7)

where \( A = \begin{bmatrix} 0 & 1 \\ -(\omega_n^2 + \frac{hw}{m}) & -2\xi_\omega_n \end{bmatrix}, B = \begin{bmatrix} \frac{hw}{m} & 0 \\ 0 & 0 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, y_\tau = \begin{bmatrix} y_{1,\tau} \\ y_{2,\tau} \end{bmatrix} \) and \( y_{i,\tau} = y_i(t - \tau) \) for \( i = 1 \) and 2. A trial solution of form \( y(t) = Ke^{\lambda t} \) put in Equation (7) to give the characteristic equation

Figure 1. Mechanical model of orthogonal turning.
On the stability lobes of turning process the pair of maximum-magnitude eigenvalues of Equation (8) are on the imaginary axis; \( \lambda = \pm i\omega \) given the pair of equations

\[
-\omega^2 + \omega_n^2 + \frac{hw}{m} (1 - \cos \omega \tau) = 0
\]

\[
2\xi \omega_n \omega + \frac{hw}{m} \sin \omega \tau = 0.
\]

These are solved simultaneously to give the critical parameter combinations

\[
\Omega_c = \frac{60}{\tau_c} = \frac{30\omega}{j\pi - \tan^{-1}\left(\frac{\omega - \omega_n}{2\xi \omega_n \omega}\right)}, \quad j = 1, 2, 3,
\]

\[
w_c = \frac{m}{2h} \left\{ \frac{(\omega^2 - \omega_n^2)^2 + 4\xi^2 \omega_n^2 \omega^2}{\omega^2 - \omega_n^2} \right\}.
\]

The critical speed \( \Omega_c \) and critical depth of cut \( w_c \) constitute the pair of stability equations for turning process. The details of deriving Equations (11) and (12) from Equations (9) and (10) are seen in other works (Ozoegwu, 2011; Ozoegwu & Omenyi, 2012).

3. The stabilizing wave attenuation effects in turning process

What follows is geared towards establishing the wave attenuation theory for orthogonal turning process. The cutting force model used in this work is seen from Figure 1 to be normal to the machined surface waves and thus, it is reasonably true that the magnitude of attenuating force will be proportional to cutting force \( F_x(t) \) in the manner; \( |F_{\alpha r}| = \alpha |F_x(t)|^\beta \) where \( \alpha \) and \( \beta \) are both positive real numbers. Equations (11) and (12) that jointly describe the stability lobes of turning process also define the critical cutting force \( F_c \). Being that \( \alpha, \beta \geq 0 \) the values \( \alpha = 1 \) and \( \beta = 1 \) can be adopted without loss of generality such that \( F_c \) becomes used as a measure of attenuating force on a stability lobe.

This is possible because sign of the partial derivative \( \frac{\partial}{\partial \varepsilon} |F_{\alpha r}| = \alpha \beta |F_x(t)|^{\beta-1} \frac{\partial}{\partial \varepsilon} |F_x(t)| \) is solely determined by the sign of the partial derivative \( \frac{\partial}{\partial \varepsilon} |F_x(t)| \) where \( \varepsilon \) is a parameter of \( F_{\alpha r} \). Since a turning process on stability lobe is stable in the Liapunov sense, then the critical cutting force \( F_c \) is seen from Equation (2) to be given by

\[
F_c = C \gamma^\gamma w_c \tau_c^\gamma
\]

where in light the specific force variation \( h = C \gamma (\nu \tau)^{\gamma-1} \) and Equations (11) and (12), \( \tau_c \) and \( w_c \) are as given as

\[
\tau_c = \frac{60}{\Omega_c} = \frac{2}{\omega} \left\{ j\pi - \tan^{-1}\left(\frac{\omega - \omega_n}{2\xi \omega_n \omega}\right) \right\}
\]

\[
w_c = \frac{mv^{1-\gamma}}{2C\gamma^{1-\gamma}} \left\{ \frac{(\omega^2 - \omega_n^2)^2 + 4\xi^2 \omega_n^2 \omega^2}{\omega^2 - \omega_n^2} \right\}
\]

It is of interest here to investigate the effect of variation of the passive parameters \( \omega_n \) and \( \xi \) and the prescription parameter \( \nu \) on \( F_c \) and interpret the result in light of the wave
attenuation theory. The most mathematically amenable is the critical cutting force at the turning points of the stability lobes. The stability lobes generated using Equation (14) are normally given as \( w_c \) as a function of \( \Omega_c \). The turning point (minimum point) for the \( j \)th lobe is derived in details in (Ozoegwu, Omenyi, Achebe & Uzoh, 2012) the non-dimensionalized cutting parameter space to be located at

\[
\frac{\Omega_c}{\Omega_c^*} = \frac{\pi \Omega_c^*}{30 \omega_n} = \frac{\pi \sqrt{2 \xi + 1}}{\left\{ j\pi - \tan^{-1}\left( \frac{1}{\sqrt{2 \xi + 1}} \right) \right\}} \\
W_c = \frac{w_c h}{m\omega_n^2} = 2\xi (\xi + 1)
\]  
(15)

It is clear from Equations (14) and (15) in light of the specific force variation \( h = C_{\gamma}(v\tau)^{\gamma-1} \) that the critical cutting parameters at the turning point of \( j \)th lobe is at

\[
\frac{\dot{F}_{ct}}{F_{ct}} = \frac{2}{\pi \sqrt{2 \xi + 1}} \left\{ j\pi - \tan^{-1}\left( \frac{1}{\sqrt{2 \xi + 1}} \right) \right\}
\]  
(16)

\[
w_{ct} = \frac{2^{2-\gamma} m\omega_n^\gamma}{\gamma C_{\omega_n}^{\gamma-1}} \left( \xi^2 + \xi \right)^{\gamma-1} \left( 2\xi + 1 \right)^{-\gamma} \left\{ j\pi - \tan^{-1}\left( \frac{1}{\sqrt{2 \xi + 1}} \right) \right\}^{(1-\gamma)}
\]  
(17)

When Equations (16) and (17) are inserted in Equation (13) the turning point critical force reads

\[
F_{ct} = \frac{4mv\omega_n}{\gamma} \left( \xi^2 + \xi \right)^{\gamma-\frac{1}{2}} \left\{ j\pi - \tan^{-1}\left( \frac{1}{\sqrt{2 \xi + 1}} \right) \right\}
\]  
(18)

It is seen from Equation (18) that the magnitude of the turning point critical force \( F_{ct} \) will rise with rise in either feed \( v \) or natural frequency \( \omega_n \) since both partial derivatives; \( \frac{\partial F_{ct}}{\partial \xi} \) and \( \frac{\partial F_{ct}}{\partial \omega_n} \) are positive. This causes a rise in the attenuating forces which by the wave attenuation theory means more flattening of the regenerative effects and suppression of chatter instability. This point is verified by generating stability charts of 15 lobes \((j = 1 \text{ to } 15)\) for a system with \( m = 0.0431 \text{ kg}, \gamma = 0.75, C = 3.5 \times 10^7 \text{ Nm}^{-7/4} \) and damping ratio \( \zeta = 0.02 \). The natural frequencies \( \omega_n = 1000, 3000, 5700 \text{ and } 7000 \text{ rad s}^{-1} \) are considered at fixed feed \( v = 0.0025 \text{ ms}^{-1} \) in generating Figure 2. The feed speeds \( = 0.008333, 0.0025, 0.08 \text{ and } 0.6 \text{ ms}^{-1} \) are considered for a turning tool with natural frequency \( \omega_n = 5700 \text{ rad s}^{-1} \) in generating Figure 3.

The effect of variation of damping ratio \( \zeta \) on the turning point critical force \( F_{ct} \) is not obvious from Equation (18) except on further investigation. This further investigation involves determining and checking the sign of the derivative \( \frac{\partial F_{ct}}{\partial \zeta} \). The derivative is

\[
\frac{\partial F_{ct}}{\partial \zeta} = \frac{4mv\omega_n}{\gamma(2\xi + 1)} \left\{ \frac{\xi^2 + \xi}{2\xi + 2} + \frac{3\xi^2 + 3\xi + 1}{\sqrt{2\xi + 1}} \left[ j\pi - \tan^{-1}\left( \frac{1}{\sqrt{2 \xi + 1}} \right) \right] \right\}
\]  
(19)

Since machine tools usually have low damping ratio that lie in the range \( \zeta \approx 0.005 - 0.02 \) (Insperger, 2002), the derivative \( \frac{\partial F_{ct}}{\partial \zeta} \) as given in Equation (19) is positive meaning that \( F_{ct} \) rises with rise in damping ratio \( \zeta \). Based on the wave attenuation theory a rise in \( \zeta \) will suppress chatter being that rise in \( F_{ct} \) which is a measure of the attenuating force improves flattening of the waves. To verify this point damping ratios \( \zeta = 0.005, 0.01, 0.015 \text{ and } 0.02 \) are considered at fixed natural frequency \( \omega_n = 5700 \text{ rad s}^{-1} \) and feed speed \( v = 0.0025 \text{ ms}^{-1} \) in generating Figure 4. It is visible from Figure 4 that rise in \( \zeta \) only improves the domain of delay independent stability of the turning stability chart. This
Figure 2. Stability charts of turning tools with same $m = .0431$, $\gamma = .75$, $C = 3.5 \times 10^7 \text{Nm}^{-7/4}$, $\xi = .02$ and $v = .0025 \text{ms}^{-1}$ when natural frequencies are (a) 1000 rpm, (b) 3000 rpm, (c) 5700 rpm and (d) 7000 rpm. It is seen that in conformity with the wave attenuation theory that the chatter instability (union of all domain above the stability lobes) is suppressed with rise in natural frequency.

Figure 3. Stability charts of a turning tool with $m = .0431$, $\gamma = .75$, $C = 3.5 \times 10^7 \text{Nm}^{-7/4}$, $\xi = .02$ and $n = 5700 \text{rpm}$ when feed speeds are (a) .0008333 ms$^{-1}$, (b) .0025 ms$^{-1}$, (c) .08 ms$^{-1}$, (d) .6 ms$^{-1}$. It is seen that in conformity with the wave attenuation theory that the chatter instability (union of all domain above the stability lobes) is suppressed with rise in feed speed.
Figure 4. Stability charts of turning processes with $\omega_n = 5700$, $v = 0.0025 \text{ ms}^{-1}$ and $C = 3.5 \times 10^7 \text{ Nm}^{-7/4}$ and damping ratios (a) .005, (b) .01, (c) $\zeta = .015$ and (d) $\zeta = .02$. The domain of delay-independent stability (the domain in which a fixed depth of cut line does not cross any of the lobes) rises with rise in $\zeta$.

Figure 5. Stability charts of turning with $m = .0431$, $\gamma = .75$, $\zeta = .02$ and $v = 0.0025 \text{ ms}^{-1}$ and $\omega_n = 5700$ for the cutting coefficients (a) $C = 3.5 \times 10^7 \text{ Nm}^{-7/4}$ (b) $C = 2.0667 \times 10^8 \text{ Nm}^{-7/4}$ (c) $C = 3.7834 \times 10^8 \text{ Nm}^{-7/4}$ (d) $C = 5.5 \times 10^8 \text{ Nm}^{-7/4}$. It is seen that size of stable domain falls with rise in $C$ suggesting that stabilizing effects of wave attenuation is resisted more at higher $C$. 
point was noted in Ozoegwu et al. (2012). One major consequence of the wave attenuation theory pointed in Ozoegwu and Omenyi (2013) is that the waves of harder and more heat resistant materials will better resist the attenuating forces, and thus enhance chatter instability. Thus, wave attenuation effect is expected to favor chatter stability in cutting of some light materials like composites and magnesium but this favor will considerably diminish when machining materials of hard machinability like stainless steel and inconel. Numerical verification of this point is presented in Figure 5 for a turning tool with parameters; \( m = 0.0431 \text{ kg} \), \( \gamma = 0.75 \), \( \zeta = 0.02 \) and \( v = 0.0025 \text{ ms}^{-1} \) and \( \omega_n = 5700 \text{ rad s}^{-1} \) machining four workpiece materials with different cutting coefficients \( C \).

4. Conclusion

The theory of stabilizing wave attenuation effects is developed for the turning process. The theory generally states that some of the force of interaction between the tool and workpiece is flattening on regenerative waves thus attenuating and depopulating the chatter –causing regenerative effects. The flattening force is called the attenuating force. In this work a measure for the attenuating force in turning process is established as a function of tool modal parameters; natural frequency \( \omega_n \), damping ratio \( \zeta \) and turning process parameter; feed speed \( v \). It is shown that the attenuating force rises in response to rise of each of \( \omega_n \), \( \zeta \), \( v \). This by the wave attenuation theory means more flattening of the regenerative effects and suppression of chatter instability. One major consequence of the wave attenuation theory is noted; the waves of harder and more heat-resistant materials will better resist the attenuating forces thus retaining chatter instability. All the above points deriving from the wave attenuation are verified numerically.

References

Arnold, R. N. (1946). The Mechanism of tool vibration in the cutting of steel. Proceedings of institution of mechanical engineers, 54, 261–284.

Hanna, N., & Tobias, S. (1974). A theory of nonlinear regenerative chatter. ASME Journal of Engineering for Industry, 96, 247–255.

Insperger, T. (2002). Stability analysis of periodic delay-differential equations modelling machine tool chatter. Budapest: Budapest University of Technology and Economics.

Insperger, T., Stepan, G., Bayly, P. V., & Mann, B. P. (2003). Multiple chatter frequencies in milling processes. Journal of Sound and Vibration, 262, 333–345.

Litak, G., Polyakov, Y. S., Timashev, S. F., & Rusinek, R. (2013). Dynamics of stainless steel turning: Analysis by flicker-noise spectroscopy. Physica A: Statistical Mechanics and its Applications, 392, 6052–6063.

Litak, G., Syta, A., & Rusinek, R. (2011). Dynamical changes during composite milling: Recurrence and multiscale entropy analysis. The International Journal of Advanced Manufacturing Technology, 56, 445–453.

Merritt, H. E. (1965). Theory of self-excited machine-tool chatter: Contribution to machine-tool chatter research – 1. Journal of Engineering for Industry, 87, 447–454.

Ozoegwu, C. G. (2011). Chatter of plastic milling CNC machine. Awka: Nnamdi Azikiwe University Awka.

Ozoegwu, C. G., & Omenyi, S. N. (2012). Stability characterization of a turning process. Journal of Engineering and Applied Sciences, 8, 68–75.

Ozoegwu, C. G., & Omenyi, S. N. (2013). Wave attenuation effects on the chatter instability of end-milling. Noise Control Engineering Journal, 61, 436–444.

Ozoegwu, C. G., Omenyi, S. N., Achebe, C. H., & Uzoh, C. F. (2012). Effect of modal parameters on both delay-independent and global stability of turning process. Journal of Mechanical Engineering and Automation, 2, 159–168.
Quintana, G., & Ciurana, J. (2011). Chatter in machining processes: A review. *International Journal of Machine tools and manufacture, 51*, 363–376.

Sen, A. K., Litak, G., Syta, A., & Rusinek, R. (2013). Intermittency and multiscale dynamics in milling of fiber reinforced composites. *Meccanica, 48*, 738–789.

Shi, H. M., & Tobias, S. A. (1984). Theory of finite amplitude machine tool instability. *International Journal of Machine Tool Design and Research, 24*, 45–69.

Siddhpura, M., & Paurobally, R. (2012). A review of chatter vibration research in turning. *International Journal of Machine tools and manufacture, 61*, 27–47.

Stepan, G., & Kalmar-Nagy, T. (1997). *Nonlinear regenerative machine tool vibrations*. Proceedings of DETC’97 1997 ASME Design Engineering Technical Conferences, Sacramento, CA, September 14–17.

Stepan, G., Szalai, R., & Inspeger, T. (2004). This is a chapter. In R. Radons & R. Neugebauer (Eds.), *Nonlinear dynamics of production systems: Nonlinear dynamics of high-speed milling subjected to regenerative effect* (pp. 111–127). Weinheim: Wiley-VCH.

Tlusty, J. (2000). *Manufacturing processes and equipment*. Abingdon: Prentice Hall, New Jersey.

Tobias, S. A. (1961). Machine tool vibration research. *International Journal of Machine Tool Design and Research, 1*, 1–14.

Tobias, S. A., & Fishwick, W. (1958). The chatter of lathe tools under orthogonal cutting conditions. *Transactions of ASME, 80*, 1079–1088.

Wiercigroch, M., & Budak, E. (2001). Sources of nonlinearities, chatter generation and suppression in metal cutting. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 359*, 663–693.