Constraining the mass of the graviton using coalescing black-hole binaries

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We study how well the mass of the graviton can be constrained from gravitational-wave (GW) observations of coalescing binary black holes. Whereas the previous investigations employed post-Newtonian (PN) templates describing only the inspiral part of the signal, the recent progress in analytical and numerical relativity has provided analytical waveform templates coherently describing the inspiral-merger-ringdown (IMR) signals. We show that a search for binary black holes employing IMR templates will be able to constrain the mass of the graviton much more accurately (∼ an order of magnitude) than a search employing PN templates. The best expected bound from GW observatories (λg > 7.8 × 10^12 km from Adv. LIGO, λg > 7.1 × 10^14 km from Einstein Telescope, and λg > 5.9 × 10^12 km from LISA) are several orders-of-magnitude better than the best available model-independent bound (λg > 2.8 × 10^12 km, from Solar system tests).

I. INTRODUCTION AND SUMMARY

General relativity (GR) has proven to be very successful in predicting and explaining a variety of gravitational phenomena. Over the last century, basic ingredients of GR have been tested in many different ways and in many different settings. In GR, gravitational interactions propagate with the speed of light, which means that the hypothetical quantum particle of gravity, the “graviton”, has no rest mass. A non-zero graviton mass mg would produce several interesting effects: for e.g., it will cause the gravitational potential to take the Yukawa form, effectively cutting off gravitational interactions at distances greater than the Compton wavelength λg = h/mg c of the graviton. Indeed, the absence of such effects in the solar system has provided a lower bound on λg (hence an upper bound on mg). But a graviton mass with corresponding Compton wavelength much larger than the size of the solar system has not yet been ruled out in a model-independent way.

Also, if the graviton has non-zero mass, the gravitational waves (GWs) will have extra degrees of freedom (such as longitudinal modes), and will travel with a frequency-dependent speed, different from the speed of light.

Whereas the current experimental bounds on theories of gravity are from the weak-field limit, GW observations provide excellent test beds in the dynamical, strong-field regime (see [2] and [3] for recent reviews). Indeed, these are exciting times for the word-wide GW community. The Initial LIGO [4] detectors have completed their fifth science run, at design sensitivity. The Virgo [5] and GEO 600 detectors ran concurrently with LIGO for part of that run. Although a direct detection of GWs is yet to be made, a number of interesting astrophysical upper limits are constructed based on this data (see e.g., [6, 7]). After the ongoing sixth science run at a further improved sensitivity, the LIGO-Virgo detectors will undergo a commissioning break with the target of achieving much improved sensitivities with Advanced LIGO and Advanced Virgo. Implementing a number of advanced technologies including the use of squeezed light, the GEO 600 is already undergoing commissioning work for the high-frequency detector configuration GEO-HF [8]. Also, the design study for a third generation detector, called the Einstein Telescope (ET) is ongoing in Europe. With the ever closer approach of the era in which GW observations become routine, it is interesting to see and update what information could be gathered about the Universe from these observations.

In this paper we investigate the possible bounds that can be put on theories with an effective mass in the propagation of GWs (massive-graviton theory, in short) from GW observations of coalescing binary black holes (BBHs). In massive-graviton theories, the speed of propagation of GWs depends on the wavelength. If the GW emission is accompanied by electromagnetic (EM) emission (such as the case of a core-collapse supernova or a white-dwarf binary), a bound on the mass of the graviton can be placed by comparing the time of arrival of the GW signal with the EM signal [10], or, by correlating the EM and GW signals [11–13]. But, since many of these sources are not very well understood in terms of their emission mechanisms and the delay between EM and GW emissions, this could introduce significant uncertainties in such measurements.

Coalescing BBHs provide us with the potential to constrain the mass of the graviton without relying on the presence of an EM counterpart. In the case of BBHs, since the frequency of gravitational radiation sweeps from lower to higher frequencies, the speed of the emitted gravitons will vary, from lower speeds to higher speeds. This will cause a distortion in the observed phasing of the GWs. Since BBHs can be accurately modelled using analytical/numerical solutions of Einstein’s equations, any deviation from GR can be parametrized and measured (see, e.g., [3] for a recent discussion on such a general framework). A framework for testing this possibility by measuring the distortion of GWs was originally developed by Will [10]. Will found a dispersive effect that appeared as an additional term in the post-Newtonian (PN) expansion of the GW phase, and showed that a bound on the mass of the graviton can be placed from the GW observations by applying appropriate matched filters.

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It has to be mentioned that so far there is no complete theory of gravity in which the graviton can have a mass. Constructing such a theory in a ghost-free manner has proven to be nontrivial; see, e.g., [9] and references therein.
Will’s original work was performed using restricted PN waveforms describing the inspiral stage of non-spinning coalescing compact binaries, the phase of which was expanded to 1.5PN order. Recent work has elaborated on this by incorporating more accurate detector models, and by including more physical effects such as effects rising from the spin angular momentum of the compact objects, from the eccentricity of the orbit, and from the inclusion of higher harmonics rising from the contribution of the higher multipoles [14–20].

Since the PN formalism has enabled us to compute accurate waveforms from the inspiral stage of non-spinning coalescing binaries, this work has involved numerical simulations of compact binaries in order to develop accurate detector models and to include physical effects that arise from the spin angular momentum of the compact objects, the eccentricity of the orbit, and from the inclusion of higher harmonics rising from the contribution of the higher multipoles.

In this paper, we estimate the bounds that can be placed on the mass of graviton that can be placed from observations of equal-mass binaries located at distances such that they produce optimal SNRs of 10 in the Adv. LIGO (black traces) and ET (grey traces) detectors using their smallest low-frequency cutoffs (10 Hz and 1 Hz, respectively). Middle panels show the same bounds from binaries located at 1 Gpc, and the bottom panels show the optimal SNR produced by these binaries. Horizontal axes report the total mass of the binary. Solid and dashed lines correspond to IMR and restricted 3.5PN waveforms, respectively. Right. Same plots for the case of binaries located at 3 Gpc detected in the LISA detector.

FIG. 1. Left. Top panels show the lower bound on the Compton wavelength $\lambda_g$ of the graviton that can be placed from observations of equal-mass binaries located at distances such that they produce optimal SNRs of 10 in the Adv. LIGO (black traces) and ET (grey traces) detectors using their smallest low-frequency cutoffs (10 Hz and 1 Hz, respectively). Middle panels show the same bounds from binaries located at 1 Gpc, and the bottom panels show the optimal SNR produced by these binaries. Horizontal axes report the total mass of the binary. Solid and dashed lines correspond to IMR and restricted 3.5PN waveforms, respectively. Right. Same plots for the case of binaries located at 3 Gpc detected in the LISA detector.

If stable black holes exist in such theory, then we might expect that the major effect comes from the propagation, since the length scales involved in the propagation are typically much larger than the length scales involved in the wave generation. However, it is difficult to address this properly in the absence of a complete theory of massive graviton.
are neglecting spins and higher harmonics in the (general relativistic) signal model, this introduces another set of systematic errors; but these can be circumvented by including these effects in the signal model. We remind the reader that this paper only tries to quantify the noise-limited statistical errors in the observations.

The main findings of the paper are summarized below (Section I A). The following sections present the details of the analysis. Section II briefly reviews the effect of massive graviton on the dispersion of GWs, and summarizes the existing bounds on the graviton mass. In Section III we compute the expected upper bounds that can be placed on the mass of the graviton using the observations of IMR signals. In that section, we review the signal and detector models used, provide the details of the computation and present a discussion of the results and the limitations of this work.

A. Summary of results

An executive summary of results is presented in Fig. I for the case of ground-based detectors Adv. LIGO and ET as well as the space-borne detector LISA. For ground-based detectors, the binary is assumed to be located optimally oriented at 1 Gpc, and for LISA, the binary is located at 3 Gpc. For the case of Adv. LIGO (with low-frequency cutoff, $f_{\text{low}} = 10$ Hz), the best bound ($\lambda_g > 7.8 \times 10^{15}$ km $\sim 2.5$ pc; $m_g < 1.6 \times 10^{-23}$ eV) using IMR templates is obtained from the observation of binaries with total mass $M \simeq 360 M_\odot$. This is significantly better than the best bound obtained using restricted 3.5PN templates ($\lambda_g > 7.9 \times 10^{12}$ km for $M \simeq 18 M_\odot$). For ET (with $f_{\text{low}} = 1$ Hz), the best bound using IMR templates ($\lambda_g > 7.1 \times 10^{14}$ km $\simeq 23$ pc; $1.7 \times 10^{-24}$ eV) is obtained from binaries with $M \simeq 3000 M_\odot$, while the best bound for PN templates ($\lambda_g > 1 \times 10^{14}$ km) is obtained for binaries with $M \simeq 65 M_\odot$. For LISA observation of supermassive black-hole (BH) binaries, the best bound ($\lambda_g > 5.9 \times 10^{17}$ km $\sim 19$ kpc; $2.1 \times 10^{-22}$ eV) using IMR templates is obtained from binaries with $M \simeq 4.8 \times 10^9 M_\odot$, while the best bound using PN templates ($\lambda_g > 6.3 \times 10^{16}$ km) is obtained from binaries with $M \simeq 1.9 \times 10^9 M_\odot$. In summary, the best bounds using IMR templates are roughly an order of magnitude better than the best bounds using restricted PN templates. The improvement is partly due to the higher SNR and partly due to the extra information harnessed from the post-inspiral stages.

The best expected bound from ground-based observatories is over two orders-of-magnitude better than the best available model-independent bound ($\lambda_g > 2.8 \times 10^{12}$ km) given by monitoring the orbit of Mars (see Sec. IIB). Additionally, LISA will be able to improve on those bounds by several orders of magnitude. Most importantly, GW observations will provide the first constraints from the highly dynamical, strong-field regime of gravity. Table I summarizes the bounds that can be placed with future GW observations employing different signal models / analysis methods.

| Signal model / method | Adv. LIGO | ET | LISA |
|----------------------|-----------|----|------|
| 1.5PN inspiral        | 0.6 (20)  | 0.7 (2 $\times$ 10$^5$) | 0.5 (10$^7$) |
| 1.5PN inspiral, more accurate detector model | [14] | | |
| 3.5PN inspiral, no spin [this paper] | 0.8 (18)  | 0.6 (1.9 $\times$ 10$^6$) |
| 3.5PN inspiral, higher harmonics, no spin [20] | 0.7 (60)  | 0.5 (2 $\times$ 10$^6$) |
| 3.5PN inspiral, 2PN spin, no precession [15] | 0.5 (2 $\times$ 10$^6$) |
| 2PN inspiral, spin precession [18] | 0.7 (2 $\times$ 10$^7$) |
| 2PN inspiral, simple spin precession, eccentricity [13] | 0.4 (10$^7$) |
| IMR, no spin [this paper] | 8 (360)  | 70 (3000)  | 6 (4.8 $\times$ 10$^7$) |
| Measuring phase of arrival of diff. harmonics from eccentric binaries [16] | 10$^{-3}$ (2.8) |
| Correlating GW and EM observations from white dwarf binaries [12] | 9 $\times$ 10$^{-2}$ (10$^7$) |

TABLE I. Expected bounds on $\lambda_g$ (in units of 10$^{13}$ km for the case of Adv. LIGO and ET, and 10$^{17}$ km for LISA) to one significant digit using future GW observations employing different signal models / analysis methods. Total mass (in $M_\odot$) giving the bound are shown in brackets. It should be noted that different investigations have used slightly different source- and detector models, and hence, are not strictly comparable.

II. DISPERSION OF GRAVITATIONAL WAVES AND THE EXISTING BOUNDS ON THE MASS OF THE GRAVITON

A. Dispersion of gravitational waves

Effect of low frequency cutoff—The best bound that can be placed with Adv. LIGO noise spectrum with low-frequency cutoff, $f_{\text{low}} = 10$ Hz will be $\sim 25$% better than the same obtained with $f_{\text{low}} = 20$ Hz. For ET, the best bounds using a configuration with $f_{\text{low}} = 1$ Hz is $\sim 70$% better than the same obtained with $f_{\text{low}} = 10$ Hz. We hope that this information can contribute to weighing the scientific case of different configurations of advanced detectors.

Detectability of the signals by GR-based templates:—If we assume the constraints on the graviton mass given by solar system tests, the “mismatch” between the signal and GR-based templates is unacceptably high (7–50%). However, if the mass of the graviton is a factor of three smaller than the solar system limit, then the mismatch is negligible. This provides a rough estimate of the detectability of the deformed signals (due to the graviton mass) using GR-based templates.
a speed $v_g$ different from the speed of light, given by

$$v_g^2 = 1 - m_g^2/E_g^2 = 1 - \left(\frac{\lambda_g}{c}\right)^2,$$

(2.1)

where $m_g$ is the graviton rest mass, $E_g$ its rest energy, $\lambda_g \equiv m_g^{-1}$ its Compton wavelength, and $f$ is the frequency of the gravitational radiation. Consider two gravitons with frequency $f_e$ and $f_e'$ emitted in the time interval $\Delta t_e$ at the source. The dispersion relation given in Eq. (2.1) will change this time interval to

$$\Delta t = (1 + Z) \left[ \Delta t_e + \frac{D}{2\lambda_g^2} \left( \frac{1}{f_e^2} - \frac{1}{f_e'^2} \right) \right],$$

(2.2)

when observed from a distance $D$. The distance parameter $D$ is related to the luminosity distance $D_L$ by

$$D = D_L \left[ 1 + \frac{2 + Z}{2} \left( 1 + Z + \sqrt{1 + Z} \right) \right],$$

(2.3)

where $Z$ is the cosmological redshift. Such a change in the arrival time will lead to a distortion of the observed phasing of the GW signal at the detector, as described by Eq. (3.2).

### B. Existing bounds on the mass of the graviton

The most stringent available bound on the mass of the graviton (which does not rely on specific massive-graviton theories) comes from the effect of a massive graviton field on the static (Newtonian) gravitational potential, changing it from $M/r$ to the Yukawa form $M r^{-1} e^{-r/\lambda_g}$. If this were the case, Kepler’s third law would be violated since the gravitational force would no longer follow an inverse-square law. The absence of such an effect in the solar system thus provides an upper limit on the graviton mass. The most stringent bound comes from the orbit of Mars which limits the mass of the graviton such that $\lambda_g > 2.8 \times 10^{12}$ km. \[35\].

Another, slightly less sensitive, bound is given by the binary pulsar observations. If the graviton had mass, the orbits of binary pulsars would decay at a slightly faster rate than predicted by GR, due to additional energy loss from the leading order massive graviton terms in the power radiated. Combining the observations of PSR B1913+16 and PSR B1534+12, Ref. [36] obtained the 90% confidence bound $\lambda_g > 1.6 \times 10^{10}$ km.

A number of model dependent, albeit more sensitive bounds have been constructed by invoking assumptions such as specific massive-graviton theories and specific distributions of dark matter. For a summary of such results, we refer the reader the recent review [1].

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4 Throughout the rest of this paper, we use geometric units: $G = c = \hbar = 1$. 

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![FIG. 2. Optimal SNR (lower panel) and the lower bound on the Compton wavelength $\lambda_g$ of graviton (upper panel) for IMR waveforms detected at 1 Gpc by the Adv. LIGO detector. Black and grey curves correspond to low-frequency cutoffs 10 Hz and 20 Hz, respectively, while solid, dashed, and dotted curves correspond to mass ratios $\eta = 1/4$, 2/9 and 4/25.](image)
sponding phase by $\phi_0$, and $\mathcal{L}^\prime(f, f_2, \sigma) \equiv \frac{1}{2\pi} (f - f_2)^2 + \sigma^2$ is a Lorentzian function with width $\sigma$ centered around the frequency $f_2$.

For these waveforms, the amplitude of the “inspiral” part is described by the amplitude of the PN inspiral waveform and the amplitude of the ringdown portion is modelled as a Lorentzian, which agrees with the quasi-normal mode ringing of a perturbed BH from BH perturbation theory. The merger amplitude is empirically estimated from the numerical-relativity simulations. The frequencies $f_1$ and $f_2$ correspond to the transition points between the inspiral-merger and merger-ringdown, and $f_3$ is a convenient cutoff frequency such that the signal power above this frequency is negligible. The normalization constants $w_m$ and $w_r$ make $A(f)$ continuous across the transition frequencies $f_1$ and $f_2$.

The phase of the waveform is written as an expansion in terms of different powers of the Fourier frequency $f$ (analogous to the phasing expression of the PN inspiral waveform computed using the stationary-phase approximation). But the phenomenological phase parameters $\psi_k$ are tuned so that the analytical templates have the best overlaps with the complete IMR waveforms. Thus, $\psi_k$ are different from the corresponding coefficients describing PN inspiral waveforms. This also restricts the validity of these waveforms to the range of GW frequency $f \gtrsim 2 \times 10^{-3}/M$. In this paper, we only consider binaries with total mass $M \gtrsim 2 \times 10^{-3}/f_{\text{low}}$, where $f_{\text{low}}$ is the low-frequency cutoff of the detector sensitivity.

The phenomenological parameters $\psi_k$ and $\mu_k \equiv \{f_1, f_2, \sigma, f_3\}$ are written in terms of the physical parameters $M$ and $\eta$ of the binary as:

$$\psi_k = \psi_k^0 + x_k^{(10)} \eta + x_k^{(20)} \eta^2 + x_k^{(30)} \eta^3 + \pi M \mu_k = \mu_k^0 + y_k^{(10)} \eta + y_k^{(20)} \eta^2 + y_k^{(30)} \eta^3,$$ (3.2)

where the coefficients $x_k$ and $y_k$ are tabulated in Table III.

The effect of a massive graviton is that it will create a distortion in the observed phasing of GWs, which can be written as [10]

$$\Psi_{\text{eff}}(f) = \Psi(f) - \beta f^{-1} + \phi_0 + \tau_0 f,$$ (3.3)

where $\Psi(f)$ is given by Eq. (3.1). The terms involving $\tau_0$ and $\phi_0$ will only result in a redefinition of the measured arrival time $\tilde{t}_0$ and the phase offset $\tilde{\phi}_0$, and hence can not be independently measured. But the term involving $\beta \equiv \pi D/\lambda_\gamma (1 + Z)$ will produce an observable effect. In general, $\beta$ will have non-zero correlation with other parameters describing the waveform, which will limit our ability to estimate $\beta$ (see Sec. III C).

For this calculation, we have assumed that the wave generation is given correctly by GR. At least for the inspiral portion of the signal, it is reasonable to expect that corrections to GR will be of the order $\eta/\lambda_\gamma$, where $\eta$ is the size of the binary system [10]. Assuming the existing bounds on the graviton mass ($\lambda_\gamma > 10^{-2}$ km), it can be shown that, for the binaries we are interested in, the correction is negligible. But, if the BHs in the true massive-graviton theory is significantly different from the BHs in GR, this could affect the wave generation in the merger-ringdown stages. Since we do not have a complete massive graviton theory, we are unable to address this issue at the moment.

| $\psi_k$ | $x^{(10)}$ | $x^{(20)}$ | $x^{(30)}$ |
|-------|--------|--------|--------|
| $\psi_2$ | $\frac{3715}{756}$ | -920.9 | 6742 | -1.34 $\times 10^4$ |
| $\psi_3$ | -1.254 $\times 10^5$ | -1.214 $\times 10^3$ | 2.386 $\times 10^5$ |
| $\psi_4$ | -1.254 $\times 10^5$ | -1.214 $\times 10^3$ | 2.386 $\times 10^5$ |
| $\psi_5$ | 0 | 8.989 $\times 10^5$ | 5.981 $\times 10^6$ | -1.128 $\times 10^7$ |
| $\psi_6$ | 0 | 8.696 $\times 10^5$ | -5.383 $\times 10^6$ | 1.089 $\times 10^7$ |

**TABLE II.** Coefficients describing the analytical IMR waveforms (see Eq. (3.2)) in the non-spinning limit.

![Fig. 3.](image)

**FIG. 3.** Optimal SNR (lower panel) and the lower bound on the Compton wavelength $\lambda_\gamma$ of graviton (upper panel) for IMR waveforms detected at 1 Gpc by the ET detector. Black, dark grey and light grey curves correspond to low-frequency cutoffs 1 Hz, 5 Hz, and 10 Hz, respectively, while solid, dashed, and dotted curves correspond to mass ratios $\eta = 1/4$, 2/9 and 4/25.

**B. Detector models**

Assuming that the detector noise is zero-mean stationary Gaussian, the noise characteristics are completely determined by its (one-sided) power spectral density (PSD) $S_h(f)$.

An analytical fit to the expected noise PSD of Adv. LIGO is given in terms of a dimensionless frequency $x = f/f_0$ [38]:

$$S_h(f) = 10^{-49} \left[ x^{-4.14} - 5x^{-2} + 111 \left( \frac{1 - x^2 + x^4/2}{1 + x^2/2} \right) \right],$$ (3.4)

where $f_0 = 215$ Hz. For Adv. LIGO, we perform our studies assuming two values for the low-frequency cutoff, $f_{\text{low}} = 10$ Hz and 20 Hz.

For the ET, the design (including the topology) is not com-
parameters, the frequency cutoff of the sensitivity, and three different values for $f_{\text{low}}$: 1 Hz, 5 Hz and 10 Hz.

For LISA, we use the total effective non-sky-averaged spectral density, neglecting the signal modulation due to the orbital motion of the detector $[15]$, $S_h(f) = \min\left\{\frac{S_{\text{NSA}}}{\exp}\left(-\kappa T^{-1} dN/df\right), S_h^{\text{gal}}(f) + S_h^{\text{ex-gal}}(f)\right\}$, (3.6)

where $dN/df = 2 \times 10^{-13} x^{1/3} \text{Hz}^{-1}$ is the number density of galactic white-dwarf binaries, $\kappa \approx 4.5$ is the average number of frequency bins that are lost when each galactic binary is fitted out, and $T$ is the observation time. The instrumental contributions are given by,

$$S_h^{\text{NSA}}(f) = \left[9.18 \times 10^{-52} x^4 + 1.59 \times 10^{-41} + 9.18 \times 10^{-38} x^2\right],$$ (3.7)

where $f_0 = 1$ Hz. The galactic/extra-galactic white-dwarf confusion-noise contributions are given by,

$$S_h^{\text{gal}}(f) = 2.1 \times 10^{-45} x^{-7/3}, S_h^{\text{ex-gal}}(f) = 4.2 \times 10^{-47} x^{-7/3}.$$ (3.8)

We start the integration at $f_{\text{low}} = 10^{-4}$ Hz. Binaries that we consider in this paper would spend less than $T \approx 5/256 \eta M^{5/3} (\pi f_{\text{low}})^{8/3} \approx 8$ months in the detection band.

C. Computing error bounds using Fisher information matrix

Searches for GWs from coalescing compact binaries make use of the optimal technique of matched filtering, which involves cross-correlating the data $d$ with a number of theoretical templates $h(\theta^a)$ of the signal waveforms:

$$\rho \equiv (d | \hat{h}) = 4 \Re \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{d(f)}{S_h(f)} \tilde{h}^*(f) \, df,$$ (3.9)

where $\rho$ is the SNR, tildes denote Fourier transforms, $S_h(f)$ is the one-sided PSD of the detector noise, $f_{\text{low}}$ the low-frequency cutoff of the sensitivity, and $\hat{h} \equiv h/(|\hat{h}|)^{1/2}$.

Owing to the intrinsic randomness of the detector noise, the parameters $\theta^a$ estimated from the search (parameters of the template giving the maximum SNR) will be, in general, different from the “actual” parameters $\theta^a$ of the source. If the noise is stationary Gaussian and if the templates are faithful representations of the true signals, then the errors $\delta \theta^a \equiv \theta^a - \bar{\theta}^a$ will be distributed according to a multivariate Gaussian distribution with zero mean, whose spread can be quantified by the elements of the variance-covariance matrix $\Sigma_{ab}$. Specifically, the rms error in estimating the parameter $\theta^a$ is given by

$$\Delta \theta^a = \sqrt{\Sigma_{aa}}.$$ A relation between $\Sigma_{ab}$ and the signal is available through the Cramér-Rao inequality $[32, 33]$, which states that

$$\Sigma \geq \Gamma^{-1},$$ (3.10)

where $\Gamma$ is the Fisher Information matrix, given by

$$\Gamma_{ab} \equiv \left(\frac{\partial h}{\partial \theta^a} \frac{\partial h}{\partial \theta^b}\right),$$ (3.11)

$h$ being the signal waveform, where the inner product is defined in Eq. (3.9). Thus, a lower bound on the expected errors is given by: $\Delta \theta^a = \sqrt{(\Gamma^{-1})_{aa}}$.

The parameters used in the Fisher-matrix calculation are $\theta^a = \{\ln C, \phi_0, f_0, \ln M, \ln \eta, \beta\}$. The matrix elements are computed by analytically computing the derivatives and numerically evaluating the inner products. The Fisher matrix is then numerically inverted to yield the variance-covariance matrix. The resultant Fisher matrix is well-conditioned throughout the parameter space we consider except at the highest total masses, where less and less of the signal is in the detectors’ sensitive bands.

D. Results and discussion

An executive summary of results is presented in Fig. 1 and Section 1A. For ground-based detectors, the binary is assumed to be optimally located and optimally oriented at 1 Gpc, and for LISA, the binary is located at 3 Gpc. For the ground-based detectors, we recompute bounds on the mass of the graviton for various combinations of mass ratio and low-frequency cutoff (see Figs. 2 and 3). For the LISA detector, we duplicate the calculations for various mass-ratio combinations with a single low-frequency cutoff (see Fig. 4).
For the ground-based detectors, one can easily see the effect of different low-frequency cutoffs by looking at the SNR panels of Figs. 2 and 3. These figures show that at the low-mass end, the mass ratio has a stronger effect on the SNR than the low-frequency cutoff. This makes intuitive sense since most of the power of the signal lies above the highest low-frequency cutoff in that regime, thus the variation in low-frequency cutoffs doesn’t significantly affect the SNR. As the signal moves toward higher masses, the bandwidth of the waveform moves out of the detectors’ sensitive bands and the SNRs rapidly decrease.

It should be noted that the mass range giving larger bounds on \( \lambda_g \) need not correspond to the mass range giving the best SNR. This bound is sensitive to more than just the power of the signal present in the detectors. Additional degeneracy-breaking information is present at lower frequencies for a given mass signal. Using a smaller lower-frequency cutoff includes this information and thus improves the bound we could place on the mass of the graviton.

The best bounds for each combination of mass ratio and low-frequency cutoff are summarized in Table III. The best bound that can be placed with Adv. LIGO noise spectrum with low-frequency cutoff, \( f_{\text{low}} \equiv 10 \text{ Hz} \) will be \( \sim 25\% \) better than the same obtained with \( f_{\text{low}} = 20 \text{ Hz} \). Similarly, for ET, the best bounds using a configuration with \( f_{\text{low}} = 1 \text{ Hz} \) is \( \sim 70\% \) better than the same obtained with \( f_{\text{low}} = 10 \text{ Hz} \). We hope that this information can contribute to weighing the scientific case of different configurations of advanced detectors.

| \( f_{\text{low}} \) | \( \eta = 1/4 \) | \( \eta = 2/9 \) | \( \eta = 4/25 \) |
|-----------------|----------------|----------------|----------------|
| Adv. LIGO       |                 |                 |                 |
| 10 Hz           | 7.8 (360)       | 7.0 (400)       | 6.2 (360)       |
| 20 Hz           | 6.2 (220)       | 5.6 (220)       | 4.9 (200)       |
| ET              |                 |                 |                 |
| 1 Hz            | 71 (3000)       | 64 (3000)       | 56 (3000)       |
| 5 Hz            | 52 (800)        | 47 (800)        | 41 (730)        |
| 10 Hz           | 41 (440)        | 37 (440)        | 32 (400)        |
| LISA            |                 |                 |                 |
| \( 10^{-4} \text{ Hz} \) | 5.9 (4.8 \times 10^{17}) | 5.4 (4.8 \times 10^{17}) | 4.7 (4.3 \times 10^{17}) |

TABLE III. Best bounds on \( \lambda_g \) (in units of \( 10^{13} \text{ km} \) for the case of Adv. LIGO and ET, and \( 10^{17} \text{ km} \) for LISA) obtained from various combinations of mass ratio \( \eta \) and low-frequency cutoff \( f_{\text{low}} \). Total mass values (in \( M_\odot \)) giving the best bound are shown in brackets.

Another concern is the detectability of signals deformed by the propagation effect of the massive graviton by GR-based templates. In order to address this question (indeed partially), we have computed the fitting factor (fraction of optimal SNR recovered by a suboptimal template family) \([51]\) of the GR-based IMR templates with signals from equal-mass binaries (in the mass range \( 20-2000 M_\odot \), located at 1Gpc) in Adv. LIGO, assuming different values for the graviton mass. If we assume the constraints on the graviton mass given by solar system tests (\( \lambda_g = 2.8 \times 10^{12} \text{ km} \)), the fitting factor is unacceptably low (\( 0.5-0.93 \)). However, if the mass of the graviton is a factor of three smaller than the solar system limit, then the fitting factors are greater than 0.97. This dramatic change is due to the fact that the deformation of the observed signal is proportional to the square of the Compton wavelength (see Eq. 3.3).

E. Limitations of this work

Due to the lack of a complete massive graviton theory, we have considered only the propagation effects of a massive graviton, and have assumed that the wave generation is correctly given by GR. If BHs in the true massive-graviton theory are significantly different from the BHs in GR, this could affect our estimates. Additionally, we have considered only the leading-harmonic gravitational waveforms (in GR) produced by non-spinning BH binaries. For the case of binaries with high mass ratios, a considerable fraction of the emitted energy is deposited in the higher harmonics (see, e.g., \([52]\)). Also, the current understanding is that most of the BHs in nature may be spinning, with possibly high spin magnitudes (see, e.g., \([53–55]\)). This imperfect description of the model waveforms can introduce systematic errors in the estimated parameters (see, e.g., \([56]\)), including in the massive graviton bounds, which this paper does not try to address.

In this paper, the error bounds are estimated by means of the Fisher information matrix approach. This approach has a number of limitations (see e.g., \([57\) and \([58]\) for a discussion), including the fact that this is valid only in the limit of large SNRs, does not include various priors in the parameters (such as \( \eta \in (0,0.25) \)), nor the known issues involved in real data analysis pipelines. These limitations are better addressed by techniques like Markov-Chain Monte-Carlo methods.

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