Effects of Grain Alignment with Magnetic Fields on Grain Growth and the Structure of Dust Aggregates

Thiemo Hoang

1 Korea Astronomy and Space Science Institute, Daejeon 34055, Republic of Korea; thiemhoang@kasi.re.kr
2 Korea University of Science and Technology, 217 Gajeong-ro, Yusong-gu, Daejeon, 34113, Republic of Korea

Received 2021 September 28; revised 2022 February 1; accepted 2022 February 9; published 2022 March 30

Abstract

Dust grains drift through the interstellar medium and are aligned with the magnetic field. Here we study the effect of grain alignment and motion on grain growth in molecular clouds (MCs). We first discuss the characteristic timescales of alignment of the grain axis of maximum inertia ($a_1$) with its angular momentum ($J$, i.e., internal alignment) and alignment of $J$ with the magnetic field ($B$; i.e., external alignment). We determine the maximum grain size with efficient internal ($a_{\text{max,al}}$) and external ($a_{\text{max,el}}$) alignment for composite grains. For the MC density of $n_H \sim 10^3 - 10^8$ cm$^{-3}$, we find that external alignment can occur for very large grains, but internal alignment only occurs for grains smaller than $a_{\text{max,al}} \sim 2$ μm. The presence of iron clusters within dust grains or suprathermal rotation increases $a_{\text{max,el}}$ to $\sim 10 - 50$ μm. We then study the growth of aligned grains drifting through the gas. Due to the motion of aligned grains across the magnetic field, gas accretion would increase the grain elongation rather than decrease, as expected from the growth of randomly oriented grains. Coagulation by grain collisions also increases grain elongation, leading to the increase of elongation with the grain size. The coagulation of aligned grains forms dust aggregates that contain elongated binaries comprising a pair of grains with parallel short axes. Grains within dust aggregates in 67P/Churyumov–Gerasimenko obtained by Rosetta have the grain elongation increasing with the grain radius, implying that such dust aggregates might form from aligned grains.

Unified Astronomy Thesaurus concepts: Astrophysical dust processes (99); Interstellar dust (836); Interstellar dust processes (838); Interstellar magnetic fields (845); Interplanetary grains (823); Starlight polarization (1571)

1. Introduction

Dust is an essential component of the interstellar medium (ISM) and plays an important role in numerous astrophysical processes, including star and planet formation, gas heating and cooling, and formation of water and complex molecules. Dust is mainly formed in the envelope of evolved stars and ejecta of core-collapse supernovae. Upon its injection into the ISM, dust is reprocessed due to gas–dust physical properties and grain motion. Grain motion due to gas accretion and grain coagulation are important for grain coagulation and shattering by grain–grain collisions (Hirashita & Yan 2009; Hirashita et al. 2021).

Previous studies on grain growth usually ignore the effect of magnetic fields and assume spherical or randomly oriented grains. Nevertheless, dust grains in general have nonspherical shapes (e.g., elongated shapes) and are systematically aligned with the magnetic field, as revealed by starlight polarization (Hall 1949; Hiltner 1949) and polarization of thermal dust emission (e.g., Planck Collaboration et al. 2015b). Moreover, modern theory based on radiative torques (RATs; Lazarian & Hoang 2007a; Hoang & Lazarian 2008) implies that grain alignment with the magnetic field is efficient in most astrophysical environments, from the ISM to dense MCs (Hoang & Lazarian 2014; Hoang et al. 2021), where grain growth can occur efficiently. Indeed, multiwavelength polarization observations reveal that grains are still aligned at a high visual extinction of $A_V > 20$ within dense clouds (DCs; Wang et al. 2017; Vaillancourt et al. 2020). Therefore, grain growth naturally involves aligned grains instead of randomly oriented grains. Because grains tend to align with their shortest axes perpendicular to the local magnetic field (Andersson et al. 2015; Lazarian et al. 2015), grain growth from collisions between aligned grains would be dramatically different from randomly oriented grains and would leave fundamental features in the large composite particles resulting from grain collisions.

Another interesting property of grain motion in the magnetized turbulent ISM is that grain acceleration by MHD turbulence is mostly perpendicular to the magnetic field because gyroresonance tends to increase the pitch angle between the grain motion and the magnetic field (Yan & Lazarian 2003; Yan et al. 2004). In star-forming clouds, ambipolar diffusion also induces grain motion perpendicular to
the ambient magnetic field. The direction of the grain’s motion is therefore parallel to the grain’s long axis, and both are perpendicular to the magnetic field. This fundamental property may affect the shape and internal structure of dust aggregates formed by gas accretion and grain–grain collisions because the collisional cross section becomes anisotropic. The main goal of this paper is to study the effect of grain alignment and motion with respect to the magnetic field on the grain shape and structure and discuss the implications for observational constraints of dust physics.

The paper is organized as follows. In Section 2, we review the leading processes that induce the grain motion perpendicular to the magnetic field. In Section 3, we discuss the characteristic timescales relevant in grain alignment and identify the range of grain sizes that have efficient alignment with the magnetic field for the different grain magnetic properties. In Sections 4 and 5, we study the grain growth due to gas accretion and grain–grain collisions for aligned grains, respectively. A discussion and summary of our main results are presented in Sections 6 and 7, respectively.

2. Grain Motion Perpendicular to the Magnetic Field

We first discuss the main processes inducing grain motion in the direction perpendicular to the magnetic field, including gyroresonance acceleration, ambipolar diffusion, and shocks (see also Lazarian 2020).

2.1. Gyroresonance Acceleration by MHD Turbulence

It is found that MHD turbulence efficiently accelerates charged grains (Yan et al. 2004; Hoang et al. 2012). Figure 1 shows a schematic illustration of grain acceleration by gyroresonance leading to grain motion perpendicular to the mean magnetic field. The velocity of grains due to MHD acceleration is found to change with environment (Hoang et al. 2012).

For both silicate and carbonaceous grains in MCs, the grain velocity is approximately equal to \( v_d \sim 0.4–1 \text{ km s}^{-1} \) for a grain radius \( >0.05 \mu m \), and \( v_d \) decreases rapidly for smaller grains. For DCs, the grain velocity is smaller, \( v_d \sim 0.3–1 \text{ km s}^{-1} \). In the cold neutral medium, the grain velocity can reach \( v_d \sim 1 \text{ km s}^{-1} \) (Yan et al. 2004). Note that, for a gas of temperature \( T_{\text{gas}} \), the thermal velocity of hydrogen atoms of mass \( m_H \) is \( v_T = (2k_B T_{\text{gas}}/m_H)^{1/2} \approx 0.4T_1 \text{ km s}^{-1} \), where \( T_1 = T_{\text{gas}}/10 \text{ K} \). Thus, MHD turbulence can induce subsonic to supersonic motion of grains across the interstellar magnetic field, depending on the grain size and the gas properties.

2.2. Ambipolar Diffusion

In star-forming magnetized MCs, such as dense cores or prestellar cores, charged grains are tied to the magnetic field, but neutral gas can cross the field and experiences gravitational collapse to form a protostar. The gas–dust collisions will then drag dust together with the neutral gas. Such an ambipolar diffusion effect results in the drift of grains in the direction perpendicular to the magnetic field. Gas accretion is then anisotropic, similar to gyroresonance acceleration. The ambipolar diffusion velocity is \( v_d \sim 0.2–0.3 \text{ km s}^{-1} \) for grains of size \( a \sim 0.01–0.1 \mu m \) (Roberge et al. 1995). Thus, ambipolar diffusion can induce subsonic drift of charged grains relative to the neutral gas.

2.3. Shocks in Magnetized Media

Shocks are present in dense, magnetized MCs due to young stellar outflows and supernovae. The shock can induce the grain motion perpendicular to the magnetic field. The idea is that charged grains gyrate around the magnetic field and are tied to the field. Neutral gas–dust drag can then induce the grain motion across the magnetic field. The drift velocity of dust relative to the neutral gas depends on the shock speed and magnetic field strength and can reach supersonic motion (Hoang & Tram 2019).

Note that the efficiency of the aforementioned acceleration mechanisms depends on the grain charge. In general, grain charged is induced by collisions with electrons and ions in the gas, a photoelectric effect by UV photons, and ionization by cosmic rays (e.g., Hoang et al. 2012).

3. Grain Alignment with Magnetic Fields

The alignment of an irregular grain with the magnetic field in general can be divided into two stages: (i) the alignment of the grain axis of the maximum inertia moment (shortest axis) parallel to its angular momentum, \( \mathbf{J} \) (internal alignment), and (ii) the alignment of \( \mathbf{J} \) with the magnetic field (external alignment; see Lazarian 2007; Lazarian et al. 2015 for recent reviews). When the internal alignment occurs with the grain’s shortest axis perpendicular to \( \mathbf{J} \), it is called “wrong internal alignment.” In this section, we review the main processes involved in grain alignment in MCs and show that submicron-sized grains of ordinary paramagnetic (PM) material or micron-sized grains with iron inclusions can be efficiently aligned with the grain shortest axis parallel to the magnetic field.

3.1. Grain Model and Assumptions

A grain of irregular shape is described by the principal axes \( \hat{a}_1, \hat{a}_2, \hat{a}_3 \). Let \( I_1 > I_2 \geq I_3 \) be the principal moments of inertia along the principal axes, respectively. For convenience of numerical estimates, throughout this paper, we assume an oblate spheroidal grain, such that the principal moments of inertia along the principal axes are \( I_1 > I_2 = I_3 \). Let us denote \( I_\parallel = I_1 \) and \( I_\perp = I_2 \) for simplicity. The length of the symmetry (semiminor) axis is denoted by \( c \), and the lengths of the
The Astrophysical Journal, 928:102 (16pp), 2022 April 1

Hoang

3.2. Grain Rotational Damping

The grain rotation can be damped by various processes, including sticky collisions of gas species followed by evaporation and infrared emission (see, e.g., Hoang et al. 2021). The rotational damping time due to gas collisions is given by

$$\tau_{\text{gas}} = \frac{3}{4\sqrt{\pi}} \frac{I_i}{1.2n_H m_H v T_{\text{a}1}} \approx 8.3 \times 10^{4} \rho a_{-5} \left( \frac{1}{n_{\text{H}1}^{1/2} T_{\text{a}1}^{1/2}} \right) \text{yr},$$

where $a_{-5} = a/(10^{-5} \text{cm})$, $\rho = \rho_f / (3 \text{ g cm}^{-3})$, and $T_{\text{a}1}$ is the geometrical factor of order unity (Roberge et al. 1993; Hoang & Lazarian 2009b). The gas density is normalized to its typical value of a dense MC with $n_f = n_{\text{H}1}/(10^{3} \text{ cm}^{-3})$. For dense MCs, the damping by infrared emission is subdominant and disregarded in this study for simplicity.

In addition to rotational damping, grains can also be spun up due to evaporation of molecules from the grain surface. In thermal equilibrium, the grain rotational kinetic energy is equal to the gas thermal energy. We can then define the thermal angular velocity as $\Omega_T = (kT_{\text{gas}}/I_{\parallel})^{1/2} \approx 7.4 \times 10^{4} \rho a_{-5}^{-3/2} T_{\text{a}1}^{1/2} \text{rad s}^{-1}$ and the thermal angular momentum as $J_T = I_{\parallel} \Omega_T = (I_{\parallel} kT_{\text{gas}})^{1/2}$.

3.3. Grain Magnetic Properties

In order to interact and align with the ambient magnetic field, dust grains must have magnetic susceptibility. Amorphous silicate grains or dust grains with iron inclusions are PM material due to the existence of unpaired electrons. Atomic nuclei within the grain can also have nuclear paramagnetism due to unpaired protons and nucleons. We first describe the magnetic susceptibility in this section.

3.3.1. Magnetic Susceptibility from Electron Spins

An atom with unpaired electrons with the angular momentum $J$ has the magnetic moment $\mu_{p} = -g_e e / (2m_e) \mathbf{J} = \gamma_e \mathbf{J}$, where $e$ is the elementary charge, $g_e \approx 2$ is the electron $g$-factor, and $\gamma_e = -g_e e / (2m_e c) = 4\pi/m_e c$ is the gyromagnetic ratio of electron spins.

The magnitude of the atomic magnetic moment is then given by

$$\mu_p = g_e \mu_B \sqrt{J(J+1)} = \rho \mu_B,$$

where $\mu_B = e h / 2m_e c$ is the Bohr magneton, and $\rho = g_e \sqrt{J(J+1)}$, with $J$ the angular momentum quantum number of electrons in the outer partially filled shell (see Draine 1996 and references therein).

Let $f_p$ be the fraction of atoms that are PM (e.g., Fe) in the silicate component. The number density of PM atoms in the dust grain is then $n_p = f_p n$, with $n$ the total number density of atoms.

The zero-frequency susceptibility $\chi(0)$ of a PM material at dust temperature $T_d$ is described by Curie’s law,

$$\chi(0) = \frac{n_p \mu_p^2}{3 k_B T_d},$$

which corresponds to

$$\chi(0) \approx 0.06 f_p n_{23} \rho_{-5}^{2} \left( \frac{10 \text{ K}}{T_d} \right).$$
where \( n_{23} = n/10^{-3} \text{ cm}^{-3} \) and \( \delta = p/5.5 \). For a silicate of MgFeSiO\(_4\) structure that accommodates 100% of Fe abundance, since \( X(\text{Si}) \approx X(\text{Fe}) \), one has \( f_p = 1/7 \) and \( J = 5/2 \) (see Hoang & Lazarian 2016). However, a lower fraction of Fe in silicate is expected, and the value of \( f_p \) is then lower. Here we take \( f_p = 0.01 \) for a typical value.\(^{3}\)

The presence of iron clusters within the composite grain makes the dust become superparamagnetic (SPM) material because the iron clusters act as giant magnetic moments. Let \( N_{\text{cl}} \) be the number of Fe atoms per cluster and \( \phi_{\text{Fe}} \) be the volume filling factor of iron clusters in the composite grain. The zero-frequency magnetic susceptibility is enhanced significantly to Hoang & Lazarian (2016),

\[
\chi_{\text{sp}}(0) \approx 0.052 N_{\text{cl}} \phi_{\text{Fe}} 2 \left( \frac{10 \text{ K}}{T_d} \right),
\]

which is about a factor \( N_{\text{cl}} \) greater than the \( \chi(0) \) of ordinary PM material (Equation 6). The possible value of \( N_{\text{cl}} \) spans \( \sim 20 \) to \( 10^{2} \) (Jones & Spitzer 1967), and \( \phi_{\text{Fe}} \approx 0.3 \) if the Fe abundance is 100% in iron clusters (see, e.g., Hoang & Lazarian 2016). For the following calculations, we consider two values of \( \phi_{\text{Fe}} \) (i.e., ordinary PM grains) and \( \phi_{\text{Fe}} = 0.03 \) (e.g., Draine & Lazarian 1999) for SPM grains.

The imaginary part of the complex magnetic susceptibility is a function of frequency and usually represented as \( \chi_{\text{i}}(\omega) = \omega K(\omega) \), where \( K(\omega) \) is the function obtained from solving the magnetization dynamics equation.

For PM material, \( K(\omega) \) can be described by the critically damped solution (Draine & Lazarian 1999)

\[
K(\omega) = \frac{\chi(0) \tau_d}{1 + (\omega \tau_d/2)^2} \approx 1.7 \times 10^{-13} \delta \left( \frac{10 \text{ K}}{T_d} \right) \left[ 1 + (\omega \tau_d/2)^2 \right]^{-1} \text{ s}, \tag{8}
\]

where \( \tau_d \approx 2.9 \times 10^{-12}/f_p \text{ s} \) is the electron spin–spin relaxation time. The susceptibility \( K(\omega) \) is constant for low frequency but decreases rapidly at a high frequency of \( \omega > 2/\tau_d \approx 10^{12} f_p \text{ rad s}^{-1} \). So, for large grains with a lower rotation frequency, \( K \) is independent of \( \omega \).

Similarly, for SPM material, \( K_{\text{sp}}(\omega) \) is given by (see Hoang & Lazarian 2016)

\[
K_{\text{sp}}(\omega) = \frac{\chi_{\text{sp}}(0) \tau_{\text{sp}}}{1 + (\omega \tau_{\text{sp}}/2)^2} \approx 2.6 \times 10^{-11} N_{\text{cl}} \phi_{\text{Fe}} 2 \exp \left( N_{\text{cl}} T_{\text{act}}/T_d \right) \frac{\tau_d}{1 + (\omega \tau_{\text{sp}}/2)^2} \text{ s}, \tag{9}
\]

where \( \tau_{\text{sp}} \) is the timescale of thermally activated remagnetization, given by

\[
\tau_{\text{sp}} \approx \nu_0^{-1} \exp \left( N_{\text{cl}} T_{\text{act}}/T_d \right), \tag{10}
\]

where experiments give \( \nu_0 \approx 10^9 \text{ s}^{-1} \) and \( T_{\text{act}} \approx 0.011 \text{ K} \) (see Morrish 2001). The susceptibility \( K_{\text{sp}}(\omega) \) is constant for low frequency but decreases rapidly at high frequency for \( \omega > 2/\tau_{\text{sp}} \approx 2\nu_0 \approx 10^9 \text{ rad s}^{-1} \).

### 3.3.2. Magnetic Susceptibility from Nuclear Spins

A nucleus with unpaired nucleons can also induce magnetism due to the intrinsic magnetic moment of protons and neutrons (see, e.g., Lazarian & Draine 1999, hereafter LD99).

Let \( \mu_n \) be the magnetic moment of a nucleus in the dust grain. The zero-frequency magnetic susceptibility of the grain can also be described by Curie’s law,

\[
\chi_{\text{i}}(0) = \frac{n_n \mu_n^2}{3kT_d},
\]

\[
\approx 4.5 \times 10^{-10} \left( \frac{\mu_n}{2.79 \mu_N} \right)^2 \left( \frac{n_n}{10^{22} \text{ cm}^{-3}} \right) \left( \frac{10 \text{ K}}{T_d} \right), \tag{11}
\]

where \( n_n \) is the number density of nuclei in the composite grain that has the magnetic moment \( \mu_n \), and \( \mu_N = e\hbar/2m_p c = 5.05 \times 10^{-26} \text{ erg G}^{-1} \) is the nuclear magneton with \( m_p \) the proton mass.

As seen in LD99, the hydrogen nucleus (proton) has the largest magnetic moment of \( \mu_n = 2.79 \mu_N \), whereas heavy elements like Si, Mg, and Fe have a much lower magnetic moment. Therefore, as in LD99, we consider the nuclear magnetism induced by hydrogen nuclei only and take \( n_n = 10^{22} \text{ cm}^{-3} \) (i.e., the hydrogen fraction is 10% of the dust) as an example.

The frequency-dependence nuclear susceptibility is given by

\[
K_{\text{n}}(\omega) = \frac{\chi_{\text{n}}(0) \tau_n}{1 + (\omega \tau_n/2)^2} \approx 4.5 \times 10^{-14} \frac{\chi_{\text{n}}(0)}{4.5 \times 10^{-10}} \left( 10^{-4} \text{ s} \right)^{-1} \times \frac{1}{1 + (\omega \tau_n/2)^2} \text{ s}, \tag{12}
\]

where \( \tau_n \) is the time of nuclear spin–spin relaxation, which is given by

\[
\tau_n^{-1} = \tau_{\text{n,e}}^{-1} + \tau_{\text{n,m}}^{-1}, \tag{13}
\]

where \( \tau_{\text{n,e}} \approx g_e \hbar / (3.8 n_n \mu_n^2) \approx 2.3 \times 10^{-4}(3.1/g_e^2)(10^{22} \text{ cm}^{-3}/n_n) \text{ s} \), with \( n_n \) the number density of unpaired electrons in the dust, \( g_e = \mu_e/\mu_N \), and \( \tau_{\text{n,m}} = \hbar / (3.8 n_n g_n^2 \mu_n^2) \approx 0.16 \tau_{\text{n,e}}(n_e/n_n) \) (see LD99 for more details).

Similar to electron susceptibility, the nuclear susceptibility \( K_{\text{n}}(\omega) \) is constant for low frequency but decreases rapidly at frequency \( \omega > 1/\tau_n \approx 10^4 \text{ rad s}^{-1} \). However, nuclear magnetism is suppressed at a much lower frequency than the case of electron magnetism (see Equations 8 and 9).

### 3.3.3. Grain Magnetic Moment

A rotating PM grain can thus acquire a magnetic moment thanks to the Barnett effect (see Barnett 1915; Landau & Lifshitz 1969) and the rotation of its charged body.

---

\(^{3}\) For the amorphous silicate of structure Mg\(_{1.3}\)Fe\(_{0.7}\)Si\(_{0.3}\)O\(_{3.6}\) that accommodates 30% of Fe in the Astrodust model (Draine & Hensley 2021), \( f_p = 0.3/(1.3 + 0.3 + 1 + 3.6) \approx 0.05 \). The mass density of amorphous silicate is \( \rho_{\text{sil}} = 3.41 \text{ g cm}^{-3} \), so \( n = 3.41/\mu = 9.5 \times 10^{22} \approx 10^{23} \text{ cm}^{-3} \), where the mean mass per atom \( m = (1.3m_{\text{Mg}} + 0.3m_{\text{Fe}} + m_n + 3.6m_{\text{O}})/6.2 \).
(Martin 1972; Dolginov & Mytrophanov 1976). The Barnett effect, which is shown to be much stronger than the latter, induces a magnetic moment proportional to the grain angular velocity,

$$\mu_{\text{Bar}} = \frac{\chi(0) V}{\gamma_{e}} \Omega,$$

where $$V = 4\pi sa^3/3$$ is the grain volume, $$\chi(0)$$ is the magnetic susceptibility of the grain at rest (i.e., zero-frequency susceptibility), and the gyromagnetic ratio $$\gamma_e = \gamma_n = -g_a\mu_B/h$$ for electron spins and $$\gamma_n = g_a\mu_N/h$$ for nuclear spins.

### 3.4. Internal Alignment

Internal alignment of the grain axis of maximum inertia with the angular momentum is caused by the dissipation of the grain rotational energy due to internal relaxation effects, including Barnett, nuclear, and inelastic relaxation (Purcell 1979; Lazarian & Draine 1999). Here we provide the basic formulae for references.

#### 3.4.1. Barnett, Super-Barnett, and Nuclear Relaxation

Consider a grain rotating with $$J$$ and $$\Omega$$ not aligned with the axis of maximum inertia $$\hat{a}_i$$. In the grain’s body frame, the tip of $$J$$ and $$\Omega$$ precess around $$\hat{a}_i$$ with the angular rate $$\omega = (h - 1)\Omega_1 = (h - 1)/\cos \theta / \|$$ (e.g., Purcell 1979; Hoang et al. 2010). The grain instantaneous magnetization by the Barnett effect, $$M = \mu_{\text{Bar}}/V = (\chi(0)/\gamma_e) \Omega$$ (see Equation 14), has a component perpendicular to $$\hat{a}_i$$, which is rotating with respect to $$\hat{a}_i$$ at an angular rate $$\omega$$ (see Figure 2). The rotating magnetization component has some lag behind the grain material and induces the dissipation of the grain rotational energy, leading to the internal alignment of $$\hat{a}_i$$ with $$\Omega$$ and $$J$$ (Purcell 1979).

The characteristic timescale for Barnett relaxation (BR) is given by (Purcell 1979; Roberge et al. 1993; see Appendix B for the derivation)

$$\tau_{\text{BR}} = \frac{\gamma_{n}^2 h^3}{VK(\omega) h^2 (h - 1) J^2},$$

$$\approx 0.5 \bar{\rho}^{2} a_{5} f(\dot{\omega}) \left( \frac{J_d}{J} \right)^2 \left( 1 + \frac{\omega_{nr}^2}{\mu_n^2} \right)^2 \text{yr},$$

where $$\dot{s} = s/0.5, f(\delta) = \delta [(1 + \delta^2)/2]^2$$, and $$J_d = \sqrt{k_B T_d/(h - 1)}$$ is the dust thermal angular momentum (see also Hoang & Lazarian 2014).

For SPM grains, the relaxation time by the Barnett effect (so-called super-BR) is significantly reduced by a factor $$N_{cl}$$:

$$\tau_{\text{BR,sp}} = \frac{\gamma_{n}^2 h^3}{VK_{sp}(\omega) h^2 (h - 1) J^2},$$

$$\approx 0.16 \bar{\rho}^{2} a_{5} f(\delta) \left( \frac{1}{N_{cl} \phi_{sp,-2}} \right) \left( \frac{J_d}{J} \right)^2 \times \left[ 1 + \left( \frac{\omega_{nr}^2}{\mu_n^2} \right)^2 \exp \left( \frac{N_{cl} T_{act}}{T_d} \right) \right] \text{yr},$$

where $$\phi_{sp,-2} = \phi_{sp}/0.01$$, which is a factor of $$\sim N_{cl}$$ smaller than the BR time for ordinary PM material (Equation 15).

Nuclear magnetism can also induce internal relaxation as a Barnett effect for electron spins (Lazarian & Draine 1999). The nuclear relaxation (NR) time is given by

$$\tau_{\text{NR}} = \frac{\gamma_{n}^2 h^3}{VK_{n}(\omega) h^2 (h - 1) J^2},$$

$$\approx 125 \bar{\rho}^{2} a_{5} f(\delta) \left( \frac{n_e}{n_{nr}} \right) \left( \frac{J_d}{J} \right) \left( \frac{g_0}{3.1} \right)^2 \left( \frac{2.79 \mu_n^2}{\mu_n^2} \right)^2 \times \left[ 1 + \left( \frac{\omega_{nr}^2}{2} \right)^2 \right] \text{s},$$

which increases rapidly with the grain size as $$a^7$$ and angular momentum as $$J^2$$, as does the BR (Equation 15). Also, the relaxation time rapidly increases with the frequency at $$\omega > 2/\tau_{nr}$$. The total rate of internal relaxation by Barnett, super-Barnett, and nuclear relaxation for a composite grain is given by

$$\tau_{\text{NR}}^{-1} = \tau_{\text{BR}}^{-1} + \tau_{\text{BR,sp}}^{-1} + \tau_{\text{NR}}^{-1}.\tag{18}$$

In Figure 3, we plot the internal relaxation times for both ordinary PM and SPM grains with the different levels of iron inclusions for two cases of the grain thermal ($$J/J_d = 1$$; left panel) and suprathermal ($$J/J_d = 100$$; right panel) rotation for the DC condition (see Table 1 for the chosen parameters). The Barnett time increases with the grain size as implied by Equation (15), but the NR time increases with the grain size only for large grains of $$a > 0.1 \mu m$$. For the PM grains of size $$a < 0.1 \mu m$$, NR changes the trend, and $$\tau_{\text{NR}}$$ increases with a decrease in the size due to the suppression of the nuclear susceptibility at a high angular rate at $$\omega > 2/\tau_{nr}$$. Thus, the NR time is much shorter than BR for grains larger than $$a > 0.03 \mu m$$. A similar trend is observed for suprathermal rotation, but the NR time changes its trend at a larger size of $$a > 0.05 \mu m$$, and BR is more efficient than NR at $$a < 0.1 \mu m$$. For SPM grains (dotted lines), the BR time becomes shorter than NR for $$N_{cl} > 10^5$$ or a small grain size of $$a < 0.05$$ (left panel) or 0.2 (right panel) $$\mu m$$. In all cases, the internal relaxation time can be longer than the gas damping time for the largest grains of $$a > 1$$ and 10 $$\mu m$$ for thermal and suprathermal rotation, respectively.

It is worth noting that Purcell (1979) realized that dust grains are not ideal solids and exhibit viscous properties. As a result, the alternating stress caused by the precession of $$\Omega$$ with $$\hat{a}_i$$ lags behind the grain material and induces the dissipation of the grain rotational energy into the heat, resulting in the internal alignment of $$\Omega$$ with $$J$$ with $$\hat{a}_i$$. However, the characteristic inelastic timescale is uncertain for dust grains due to its unknown modulus of rigidity and elastic $$Q$$ parameter. Thus, this effect is disregarded in the present paper.

#### 3.4.2. Maximum Size for Internal Alignment

As shown in Equations (15)–(17), the relaxation time increases rapidly with the grain size as $$a^7$$. Thus, for sufficiently large grains, the internal relaxation can be slower than the gas randomization (see Figure 3), for which the internal alignment ceases. Thus, it is important to estimate the maximum size at which internal relaxation is still efficient to maintain efficient internal alignment.

We first consider grains with iron inclusions that have the most efficient BR. From Equations (3) and (16), it is
straightforward to obtain the ratio of the BR time to the gas damping:

\[
\frac{\tau_{BR,sp}}{\tau_{gas}} = \frac{\gamma^2 \eta^2 4 \sqrt{\pi} 1.2 n_H m v_{T,sp} \Gamma}{3 \pi^2 (h - 1) J^2} \times (1 + (\omega_{sp}/2)^2)^2 \eta_{sp} \frac{\eta_{sp}}{\eta},
\]

\[
\approx 10^{-6} \frac{n_H T_{sp}^{3/2} a_{sp}^{3/2}}{n H_2} \left(1 + (\omega_{sp}/2)^2\right)^2 \eta^2 H_{\text{damp}} \exp\left(N_{cl} T_{act}/T_d\right)
\]

(19)

Equation (19) implies the steep increase with the grain size as \(a\), so that sufficiently large grains may not have internal alignment. Let \(a_{\text{max,al}}\) be the maximum grain size for internal alignment between \(\vec{a}\) and \(\vec{J}\). The maximum size for internal alignment by BR can be determined by \(\tau_{BR,sp}/\tau_{gas} = 1\), yielding

\[
a_{\text{max,al}}(BR) = 1.0 h^{1/3} \left(\frac{N_{cl} \phi_{sp,2}}{n H_2 T_{sp}^{3/2}}\right)^{1/6} \exp\left(N_{cl} T_{act}/T_d\right) \left(\frac{J}{J_d}\right)^{1/3},
\]

(20)

which implies \(a_{\text{max,al}}(BR) \approx 1.8 \mu m\) for \(N_{cl} \sim 10^4\) and \(J/J_d = 1\) and \(n_5 = 1\) (i.e., mild thermal and suprathermal rotation enhanced the values of \(a_{\text{max,al}}\), which differs from Equation (20) by a factor of [\exp(N_{cl} T_{act}/T_d)]^{1/6} / 1.4 = 4.5 for \(N_{cl} = 10^4\). The value of \(a_{\text{max,al}}(BR)\) increases with the rotation rate as \(J^{1/3}\) but decreases with the gas density as \(n_H^{-1/6}\).

Similarly, one can find the maximum size for internal alignment induced by NR using Equations (3) and (17),

\[
a_{\text{max,al}}(NR) < 1.41 h^{1/3} \left(\frac{n_H}{n_5 (3.1)^2}{n H_2 T_{sp}^{3/2}}\right)^{1/6} \left(1 + (\omega_{sp}/2)^2\right)^{1/3}\left(\frac{J}{J_d}\right)^{1/3} \mu m,
\]

(21)



**Table 1**

| Parameters               | MC                  | DC                  |
|--------------------------|---------------------|---------------------|
| Gas density, \(n_H\) (cm\(^{-3}\)) | \(10^3\)            | \(10^7\)            |
| Gas temperature, \(T_{gas}\) (K)      | 15                  | 15                  |
| Dust temperature, \(T_d\) (K)          | 15                  | 15                  |
| Magnetic field strength, \(B\) (\(\mu G\)) | 10                  | 200                 |
| Fraction of Fe atoms in silicate, \(f_p\)                  | 0.01                | 0.01                |
| Volume filling factor of Fe clusters, \(\phi_{sp}\)                  | 0.03                | 0.03                |
| Nuclear proton density, \(n_p\) (cm\(^{-3}\)) | \(10^{12}\)         | \(10^{22}\)         |
| Gas thermal speed, \(v_T\) (cm s\(^{-1}\)) | 0.5                 | 0.41                |
| Grain drift velocity, \(v_d\) (km s\(^{-1}\)) | 0.5                 | 0.5                 |

Figure 3. Internal relaxation timescales due to BR for PM grains (grains without iron clusters), NR, and BR for SPM grains, assuming grain thermal (\(J/J_d = 1\); left panel) and suprathermal (\(J/J_d = 100\); right panel) rotation. The gas damping for the DCs is also overplotted for comparison. Suprathermal rotation reduces the timescale of BR, but the minimum of NR time is shifted to a larger size, from \(a \sim 0.08 \mu m\) for thermal rotation to \(a \sim 0.5 \mu m\).

3.5. External Alignment

External alignment of the grain angular momentum \(\vec{J}\) with the magnetic field \(\vec{B}\) can be induced by Davis–Greenstein magnetic relaxation (Davis & Greenstein 1951), RATs (Dolginov & Mitrofanov 1976; Draine & Weingartner 1996;
Lazarian & Hoang 2007a; Hoang & Lazarian 2008), and mechanical torques (METs; Lazarian & Hoang 2007b; Hoang et al. 2018). In the following, we review the basic processes and formulae for reference (see Lazarian et al. 2015 for a review).

3.5.1. Larmor Precession

The interaction of the grain magnetic moment (Equation 14) with the external static magnetic field governed by the torque component $\langle dJ/dt \rangle = \sin \xi \dot{\phi}/d\hat{\phi} = -\mu_{\text{Bar}} \times B \dot{\phi} B \sin \xi \dot{\phi}$ causes the regular precession of the grain angular momentum around the magnetic field direction (see Figure 5). The rate of such a Larmor precession, denoted by $\tau_{\text{Lar}}$, is given by

$$\tau_{\text{Lar}} = \frac{2\pi}{|d\phi/dt|} = \frac{2\pi I_\| \Omega}{|\mu_{\text{Bar}}| B} = \frac{2\pi|\gamma_0| I_\|}{\chi(0)VB},$$

$$= 0.84 \frac{\rho a^2}{B} \text{yr},$$

where $\hat{B} = B/5\mu$G and $\chi = \chi(0)/10^{-4}$ are the normalized magnetic field and magnetic susceptibility, respectively.

For SPM grains, the Larmor timescale, $\tau_{\text{Lar,sp}}$, is given by the same formula, but $\chi(0)$ is replaced by $\chi_{\text{sp}}$ (Equation 7).

Above, for simplicity, we have assumed that internal relaxation is efficient so that the grain is spinning along the axis of maximum inertia with $J = I\| \Omega$. This assumption usually holds for dust grains in MCs, since internal relaxation is much faster than Larmor precession.

3.5.2. Davis–Greenstein Relaxation: PM and SPM Material

Following Davis & Greenstein (1951), a rotating PM grain with angular momentum $J$ subject to an external magnetic field experiences magnetic dissipation of the grain rotational energy due to the existence of the rotating magnetization with respect to the grain body. This Davis–Greenstein magnetic relaxation eventually leads to the alignment of $J$ with the magnetic field (see Figure 5). The characteristic time of the magnetic relaxation is given by

$$\tau_{\text{DG}} = \frac{I_\|}{V K(\omega)B^2} = \frac{2\rho a^2}{5K(\omega)B^2},$$

$$\simeq 2.4 \times 10^6 \rho a^2 B^{-2} K^{-1} \text{yr},$$

where $K(\omega)$ is given by Equation 8 and $K_{\text{sp}}(\omega)$ (Equation 9) into Equation (23), one obtains the timescales of alignment by magnetic relaxation for PM and SPM material.

3.5.3. RAT Alignment

The interaction of anisotropic radiation with irregular grains induces RATs (Dolginov & Mitrofanov 1976; Draine & Weingartner 1996). The RATs cause the grain precession around the radiation direction ($k$; see Figure 5). Grains can be spun up by RATs to angular velocities above the thermal value, so-called suprathermal rotation. Due to grain suprathermal rotation, efficient internal relaxation (Barnett and nuclear effects) quickly induces the alignment of the grain axis of maximum inertia with its angular momentum. Modern theory based on RATs (Lazarian & Hoang 2007a; Hoang & Lazarian 2008; Hoang & Lazarian 2016) implies grain alignment...
alignment with its angular momentum parallel to the magnetic field, resulting in the alignment of the shortest axis with the magnetic field when the Larmor precession is faster than the radiative precession. A detailed discussion of RAT alignment can be found in the review by Lazarian et al. (2015); here we provide some essential formulae for reference.

The radiative precession time around the radiation direction ($\hat{k}$) by RATs is given by (Lazarani & Hoang 2007a; Hoang & Lazarani 2014)

$$\tau_k = \frac{2\pi}{|d\phi/dt|} = \frac{2\pi I}{\gamma_{\text{rad}} \lambda_{\text{eff}} \Omega_{\text{R}}},$$

$$\simeq 731 \frac{1}{2} \frac{2}{3} \left( \frac{1.2 \mu m}{\gamma_1 \Omega_{\text{R}}} \right) \left( \frac{1}{J} \left( \frac{F}{\lambda} \right) \right) \text{yr},$$

where $\Omega_{\text{R}} = Q_{\text{R}}/10^{-2}$, with $Q_{\text{R}}$ the third component of RATs that induces the grain precession around $k$ (see Table 1 in Lazarani et al., 2015). Comparing $\tau_k$ to the Larmor precession time, it can be seen that the Larmor precession is always faster than the radiation precession. Thus, the magnetic field is the axis of grain alignment (see Figure 5).

The time required for RATs to spin up grains to the suprathermal rotation threshold, which is considered the RAT alignment time, is given by

$$\tau_{\text{RAT}} \equiv \frac{3 \gamma_{\text{T}}}{\Gamma} = \frac{3 \tau_{\text{gas}}}{J_{\text{RAT}}/J_{\text{T}}},$$

$$\simeq 2 \times 10^3 \left( \frac{\gamma_{\text{T}}}{\gamma_{\text{gas}}} \right) \frac{1}{2} \text{yr},$$

where $\Gamma$ is the RAT magnitude (Draine & Weingartner 1996; Lazarani & Hoang 2007a), and $J_{\text{RAT}} = \Gamma \tau_{\text{gas}}$ is the maximum angular momentum spun up by RATs. For grains of $a = a_{\text{align}}$ with $J_{\text{RAT}} = 3 \gamma_{\text{T}}$, the alignment timescale is comparable to $\gamma_{\text{gas}}$, and for $a > a_{\text{align}}$, the RAT alignment is much shorter than the gas alignment timescale.

Numerical simulations show that grains can be efficiently aligned by RATs when they can rotate suprathermally (Hoang & Lazarani 2008; Hoang & Lazarani 2016). Taking the suprathermal condition that corresponds to the maximum grain angular momentum spun up by RATs, $J_{\text{RAT}} = 3 \gamma_{\text{T}}$, one can determine the minimum size of the grains that can be efficiently aligned by RATs. For a starless MC, Hoang et al. (2021) derived the minimum effective size of the aligned grains by the interstellar radiation field (ISRF) as follows:

$$a_{\text{align}} \approx \frac{1.2 \mu m T_{\text{gas}}}{\gamma_{\text{L}} \lambda_{\text{eff}}^2 \Sigma_{\text{gas}}} \frac{1}{4} \left( \frac{15 \mu m k^2}{4 \rho} \right)^{1/7} \left( \frac{\gamma_{\text{T}}}{\gamma_{\text{gas}}} \right)^{2/7} \lambda \left( \frac{1.2 \mu m}{\Sigma_{\text{gas}}} \right)^{1/7} \mu m,$$

where $U = u_{\text{rad}}/u_{\text{MMP83}}$ is the ratio of the radiation energy density $u_{\text{rad}}$ relative to that of the local ISRF in the solar neighborhood, $u_{\text{MMP83}}$ (Mathis et al. 1983); $\lambda$ is the mean wavelength of the local radiation field; $\gamma_{\text{T}} = 15/0.1$, with $\gamma$ the anisotropy of the radiation field; and the damping due to infrared emission has been disregarded for starless DCs.

Equation (26) implies $a_{\text{align}} \sim 0.055 \, \mu m$, and $0.4 \, \mu m$ for a DC of $n_{\text{H}} = 10^3 \, cm^{-3}$, and $10^3 \, cm^{-3}$ exposed to the local radiation field of $\gamma = 0.1$, $U = 1$, and $\lambda = 1.2 \, \mu m$. Accounting for the attenuation of the ISRF toward the MC center, calculations in Hoang et al. (2021) find that, at $U_{\nu} \sim 20$, the grain alignment size increases to $a_{\text{align}} \sim 0.3 \, \mu m$, and $1.5 \, \mu m$ for $n_{\text{H}} \sim 10^2, 10^3$, and $10^4 \, cm^{-3}$. Therefore, the grain coagulation to micron sizes in dense MCs essentially involves aligned grains. For grains smaller than $a_{\text{align}}$, the coagulation can first occur for random grains and then become aligned by RATs when becoming larger than $a_{\text{align}}$.

3.5.4. MET Alignment

Grains drifting through the gas experience METs. As shown in Lazarian & Hoang (2007b) and Hoang et al. (2018), METs can efficiently align grains with the magnetic field, in analogy to RATs; i.e., the grain’s shortest axis is parallel to the magnetic field when internal relaxation is efficient. For inefficient internal relaxation, grain alignment can occur with the long axis parallel to the magnetic field. However, the efficiency of METs is uncertain due to their complicated dependence on the grain shapes (Hoang et al. 2018). Further study of METs is required to have an accurate assessment (Reissl et al. 2022). One expects that the RAT alignment can work in a wider range of environments than METs due to the ubiquity of radiation fields. However, METs may be important in DCs and the midplane of protoplanetary disks (PPDs), where the radiation field is weak and grain drift is important.

3.5.5. Maximum Grain Size for External Alignment with the Magnetic Field

Figure 6 shows the timescales versus the grain size for various physical processes involved in external alignment, including Larmor precession and Davis-Greenstein (D-G) magnetic relaxation for ordinary PM and SPM material, radiation precession, and gas damping for the MCs (left panel) and DCs (right panel) with the physical parameters shown in Table 1. The radiation precession time by RATs (Equation 24) is calculated for the typical values of $\gamma = 0.3$, $\lambda = 1.2 \, \mu m$, and $U = 1$.

The Larmor precession of SPM grains is faster than the radiation precession, which implies that the magnetic field is an axis of grain alignment instead of the radiation. The timescale for D-G relaxation for PM grains is longer than the gas damping time. However, SPM grains have a D-G relaxation time much shorter than the gas damping time such that the joint action of D-G relaxation and RATs can make the perfect alignment of $\mathbf{J}$ with $\mathbf{B}$ (Hoang & Lazarani 2016).

We now estimate the maximum size of grains that still have external alignment with the magnetic field. Using Equations (3) and (22), one can write the ratio of the Larmor precession to gas damping time for SPM grains as

$$\frac{\tau_{\text{Lar,sp}}}{\tau_{\text{gas}}} \approx \frac{2\pi I_{\text{gas}}}{\lambda_{\text{L}}} \left( \frac{1.2 \sqrt{\pi} n_{\text{H}} m_{\text{H}} v_{\text{T}}^2 \Gamma_{\text{d}}}{L_{\text{sp}}(0) B} \right) \left( \frac{5}{n_{\text{H}} T_{\text{L}}^2 \beta_{\text{L}}^2 \text{a}_{\text{sp}} \text{B}^3} \text{N}_{\text{L}} \phi_{\text{L}} \right),$$

where $n_{\text{H}}$, $m_{\text{H}}$, and $v_{\text{T}}$ are the number density, mass, and thermal speed of hydrogen, respectively; $T_{\text{L}}$ is the local radiation temperature; $\phi_{\text{L}}$ is the effective size of the radiation field; $\beta_{\text{L}}$ is the anisotropy of the radiation field; and the damping due to infrared emission has been disregarded for starless DCs.
where \( n_s = n_{\text{H}}/10^5 \text{ cm}^{-3} \), \( \phi_{\text{sp}, -2} = \phi_{\text{sp}}/0.01 \), and \( B_3 = B/1000 \ \mu\text{G} \). Above, we have assumed \( K(\omega) = \chi_{\text{d}}(\omega)/\omega \approx \chi(0) \), which is valid for large grains of \( \omega \tau_3 \ll 1 \).

The maximum size for the grain alignment with \( \mathbf{J} \) aligned with the magnetic field \( (B) \), denoted by \( a_{\text{max,JB}} \), is then determined by \( \tau_{\text{Lar},sp}/\tau_{\text{gas}} = 1 \), yielding

\[
a_{\text{max,JB}} = 1.4 \times 10^3 \left( \frac{N_{\text{cl}}\phi_{\text{sp}, -2}B_3}{n_s T_1^{1/2} \Gamma_{||}} \right) \text{ cm}, \tag{28}
\]

which implies that VLGs of 1000 cm can still be aligned with the magnetic field in DCs of \( n_{\text{H}} \sim 10^5 \text{ cm}^{-3} \). However, in the region of high density, \( n_{\text{H}} \sim 10^{12} \text{ cm}^{-3} \), like the midplane of PPDs, only grains up to \( a_{\text{max,JB}} \sim 1 \mu\text{m} \) can be aligned with the magnetic field, assuming \( N_{\text{cl}} \sim 10^5 \).

In summary, our main results in this section suggest that composite dust grains of size \( a \sim [a_{\text{align}} - a_{\text{max,JB}}] \) have efficient internal and external alignment with the magnetic field, whereas VLGs of sizes \( a \sim [a_{\text{max,J}} - a_{\text{max,JB}}] \) have inefficient internal alignment but efficient external alignment with the magnetic field. Even in protostellar cores of high density, \( n_{\text{H}} \sim 10^5 - 10^6 \text{ cm}^{-3} \), composite grains with iron inclusions can have both efficient internal and external alignment up to \( a_{\text{max,J}} \sim 10 \mu\text{m} \) due to internal relaxation, leading to the alignment of the grain’s shortest axis with the angular momentum and then with the magnetic field. However, VLGs of size \( a \sim [-10 \mu\text{m} - 1 \text{cm}] \) have efficient external alignment but inefficient internal alignment, which may induce the alignment with their shortest axes perpendicular to the angular momentum and magnetic field, (i.e., wrong alignment; Hoang & Lazarian 2009a).

### 4. Grain Growth by Gas Accretion to Drifting Aligned Grains

In this section, we discuss the effect of grain alignment with the magnetic field on grain growth from submicron to micron sizes in MCs by gas accretion and grain–grain collisions.

Suppose that a randomly oriented, tiny grain of size \( a < a_{\text{align}} \) drifts through the interstellar gas with velocity \( v_d \) and the gas includes H, He, and heavy (i.e., metal) elements (X) with \( X = \text{C}, \text{O, Mg, Si, Fe, etc.} \). In MCs, the accretion of H leads to the formation of \( \text{H}_2 \) and a water-ice mantle, and part of the \( \text{H}_2 \) molecules rapidly desorb from the grain surface. Accretion of metal elements and the ice mantle buildup gradually increase the grain size to \( a > a_{\text{align}} \). Subsequently, gas accretion acts on the grain that is aligned with the shortest axis parallel to the magnetic field.

#### 4.1. Nonaligned Grains

We first consider the accretion of the gas onto a tiny, randomly oriented grain. Since the drift velocity of tiny grains by MHD turbulence is small (subsonic), one can ignore the drift. Due to randomization by gas collision, the gas accretion to the grain leads to an isotropic increase in the grain size. The accretion rate of heavy (metal) atoms of element \( i \) to the grain is then given by the same formulae as the gas accretion onto a spherical grain,

\[
\frac{dn_{\text{gi}}}{dt} = \sum_i m_i n_i \langle \mathbf{v}_i \rangle \pi a^2 S_i, \tag{29}
\]

where \( S_i \) is the sticking coefficient, \( \langle \mathbf{v}_i \rangle = (8kT_{\text{gas}}/\pi m_i)^{1/2} \approx 0.46T_1^{1/2}A_i^{-1/2} \text{ km s}^{-1} \), where \( T_1 = T_{\text{gas}}/10 \text{ K} \) is the mean thermal speed; \( m_i \approx A_i m_{\text{H}} \), with \( A_i \) the atomic mass number; and the summation is carried over all heavy elements (X).

The increase in the grain size by gas accretion is

\[
\frac{da}{dt} = \frac{\sum_i m_i n_i \langle \mathbf{v}_i \rangle \pi a^2 S_i}{4\pi a^2 \rho} = \frac{m_X n_X \langle \mathbf{v}_X \rangle \pi a^2 S_X}{4\pi a^2 \rho}, \tag{30}
\]

The accretion time to increase the grain size by a thickness of \( a \) is

\[
\tau_{\text{acc}} = \frac{a}{da/dt} = \frac{4a \rho}{\rho_X \langle \mathbf{v}_X \rangle S_X} = \frac{4a \rho}{X_{\text{gas}} \langle \mathbf{v}_X \rangle S_X} \approx 1.6 \times 10^7 \left( \frac{X_{\text{gas}}}{20} \right)^{1/2} \left( \frac{\rho a^{-5}}{n_3 T_1^{1/2} X^{-2} S_X} \right) \text{ yr}, \tag{31}
\]
where the ratio of metal to gas mass density, \(X = \rho_X / \rho_{\text{gas}}\),
where \(\rho_X = \mu m_X n_X\) and \(\rho_{\text{gas}} = \mu m_H n_H\), with \(\mu = 1.4\) the mean molecular weight of the gas with 10% mass from He, and \(AX\) is
the mean atomic mass of metals, which is taken to be \(AX = 20\) in
our numerical calculations.

Equation (31) reveals that the grain can grow to twice its
original size in \(\sim 30\) Myr, assuming the sticking coefficient
\(S_X = 0.5\) and \(n_T = 10^3 \text{ cm}^{-3}\). However, grain growth by gas
accretion is constrained by the metal budget in the gas and
destruction processes such as shattering and rotational disrup-
tion by RATs (Hoang et al. 2019).

\[ F_x = \frac{1}{4} n_X \langle v \rangle_X (e^{-\Delta t} + \sqrt{\pi} s_d [1 + \text{erf}(s_d)]), \]  
\( \langle v \rangle_X \)
where \(s_d = \frac{v_d}{c_x} = 1.1 T_1^{-1/2} \left( \frac{v_d}{0.1 \text{ km s}^{-1}} \right) \left( \frac{A_X}{20} \right)^{1/2}, \)
with \(v_{T,X} = (2kT_{\text{gas}}/A_X m_H)^{1/2}\) being the thermal velocity of the
metal \(X\) in the gas. The first term in Equation (32) describes the
reduction effect from trailing collisions due to grain motion,
while the second term describes the head-on collisions.
The collision rates along the perpendicular direction to the
grain motion are
\[ F_y = \frac{1}{4} n_X \langle v \rangle_X, \]
\( \langle v \rangle_X \)
and
\[ F_z = \frac{1}{4} n_X \langle v \rangle_X. \]
The ratio of the collision rates along the two parallel and
perpendicular directions to the grain motion is
\[ \frac{F_x}{F_y} = \frac{F_x}{F_z} = (e^{-\Delta t} + \sqrt{\pi} s_d [1 + \text{erf}(s_d)]). \]

For \(s_d = 0\), one has \(F_x/F_y = 1\), which is the case of isotropic
gas accretion. For \(s_d \gg 1\), the second term of Equation (36)
dominates such that \(F_x/F_y \to 2\sqrt{\pi} s_d\), which implies that gas
accretion is mostly along the direction of the grain motion. At the
drift parameter of \(s_d = 1\), the head-on collision rate is about
four times greater than the collision rate along the magnetic
field (see Figure 7, red line).

\[ \frac{\Delta c/a}{\Delta a/a} = 0.1 \quad \frac{\Delta c/a}{\Delta a/a} = 0.2 \quad \frac{\Delta c/a}{\Delta a/a} = 0.3 \quad \frac{\Delta c/a}{\Delta a/a} = 0.5 \]

Figure 7. Ratio of the collision rate along the grain motion to its perpendicular
direction \((F_x/F_y)\) and the grain elongation by gas accretion as functions of the
dimensionless parameter of the grain drift relative to the metals, \(s_d = v_d/c_x\)
(Equation 33), when the length of the grain’s semiminor axis is increased by
\(\Delta c/a = 0.1–0.5\). Grains can become highly elongated due to gas accretion
when they move at \(s_d \gtrsim 1\) through the gas. The shaded area marks the possible
range of grain drift enabled by MHD turbulence in MCs.

The rate of increase in the grain mass by accretion of metals
along the direction of the grain motion is
\[ \frac{dm_x}{dt} = A_X m_H F_z \pi a X X \]
\( \pi a X X \)
where \(S_X\) is the sticking coefficient of metals.
The rate of the increase in the grain mass by accretion along
the direction perpendicular to the direction of the grain motion is
\[ \frac{dm_y}{dt} = A_X m_H F_x \pi a X X = \frac{1}{4} A_X m_H S_X X \pi a^2 \langle v \rangle_X. \]
\( \pi a^2 \langle v \rangle_X. \)
Assuming that the initial grain shape is slightly elongated with \(a \approx c\), the new axial ratio of the grain after time \(\Delta t\) is then
\[ \frac{a_{\text{new}}}{c_{\text{new}}} = \frac{a + \Delta a}{c + \Delta c} \approx \frac{1 + \Delta a/a}{1 + \Delta c/a}. \]
\( \frac{a + \Delta a}{c + \Delta c} \)
For \(\Delta c/a \ll 1\), then, \(1/(1 + \Delta c/a) \approx 1 - \Delta c/a\). Thus,
\[ \frac{a_{\text{new}}}{c_{\text{new}}} = \frac{1 + \Delta a/a}{\left(1 - \frac{\Delta c}{a} \right)} = 1 + \frac{\Delta c}{a} \left( \frac{F_x}{F_z} - 1 \right). \]
\( \frac{1 + \Delta a/a}{\left(1 - \frac{\Delta c}{a} \right)} \)
It can be seen that for \(v_d = 0\), the axial ratio is kept constant.
For \(v_d > 0\), the axial ratio increases because \(F_x/F_z > 1\). By the
time the gas accretion increases the length of the grain’s
semiminor axis by \(\Delta c = a\), the axial ratio is increased to
\(a_{\text{new}}/c_{\text{new}} = (F_x/F_z)\).

Figure 7 shows the increase of the grain elongation due to
accretion of metals with an average mass number \(AX = 20\) with
the drift parameter at four epochs determined by the increase in
the length of the grain’s semiminor axis of \(\Delta c/a = 0.1–0.5\).
Table 2: Timescales of Physical Processes Involved in Grain Alignment and Growth

| Definitions and Physical Processes | Typical Values |
|-----------------------------------|-----------------|
| Oblate spheroidal grain shape and axial ratio | $a \times a \times c$, $s = c/a$ |
| Gas damping time, $\tau_{\text{gas}}$ (yr) | $8.3 \times 10^3 \frac{n_{\text{H}} a_s}{m_{\text{H}}}$ |
| BR (PM), $\tau_{\text{BR}}$ (yr) | $0.5 \frac{n_{\text{H}}}{n_{\text{H}} + m_{\text{g}}} \frac{a_s}{d_f} \left( v_d \right)^2 \leq 1 + \left( \frac{2 c}{c_f} \right)^2$ |
| NR, $\tau_{\text{NR}}$ (s) | $125 \frac{n_{\text{H}}}{n_{\text{H}} + m_{\text{g}}} \frac{a_s}{d_f} \left( v_d \right)^2 \left( \frac{\tau_{\text{g}}}{n_{\text{H}}} \right) \left( \frac{2 c}{c_f} \right)^2 \left( 1 + \left( \frac{2 c}{c_f} \right)^2 \right)^2$ |
| BR (SPM), $\tau_{\text{BR}_{\text{op}}}$ (yr) | $3.2 \frac{n_{\text{H}}}{n_{\text{H}} + m_{\text{g}}} \frac{a_s}{d_f} \left( v_d \right)^2 \left( \frac{\tau_{\text{g}}}{n_{\text{H}}} \right) \left( 1 + \left( \frac{2 c}{c_f} \right)^2 \right)^2$ |
| D-G relaxation, $\tau_{\text{D-G}}$ (yr) | $2.4 \times 10^6 \frac{n_{\text{H}} a_s}{m_{\text{H}} \gamma} \left( 10^{16} \frac{T_a}{K} \right)$ |
| Larmor precession, $\tau_{\text{L}}$ (yr) | $8.4 \frac{\gamma - 1}{\gamma} \frac{B}{a_{\text{crit}}}$ |
| Radiative precession, $\tau_{\text{R}}$ (yr) | $731 \frac{\gamma}{12^{1/2}} \left[ \frac{1.2 \mu_{\text{H}}}{\gamma - 1} \right] \frac{g_{\text{f}}}{g_{\text{f}}^{1/2}} \left( \frac{J_{\text{f}}}{J_{\text{f}}} \right)$ |
| Gas accretion time, $\tau_{\text{acc}}$ (yr) | $1.2 \times 10^6 \frac{n_{\text{H}} a_s}{m_{\text{H}} \gamma} \left( \frac{1}{10^{15} \frac{T_a}{K}} \right)^{2}$ |
| Grain–grain collision, $\tau_{\text{gg}}$ (yr) | $7.6 \times 10^4 \frac{a_{\text{crit}}}{g_{\text{f}}^{1/2}} \frac{1}{\left( \frac{v_{\text{new}}}{v_{\text{crit}}} \right)^{1/2}}$ |
| External alignment (RAT) time, $\tau_{\text{RAT}}$ (yr) | $1.4 \times 10^7 \frac{\gamma_{\text{crit}}}{\gamma} \frac{m_{\text{g}}}{m_{\text{H}}}$ |
| Maximum size of magnetic alignment ($J$ with $B$), $a_{\text{max},BR}$ (cm) | $1.0 \times 10^{-1/3} \left[ \frac{\gamma_{\text{crit}}}{\gamma} \right]^{1/6} \left( \frac{J_{\text{f}}}{J_{\text{f}}} \right)^{-1/3} \frac{m_{\text{g}}}{m_{\text{H}}}^{1/2} \left( 1 + \frac{1}{1 + \omega_{\gamma}} \frac{1}{2} \right)$ |
| Maximum size of internal alignment ($a_1$ with $J$), $a_{\text{max},a}$ ($\mu$m) | $\exp \left[ - \frac{\gamma_{\text{crit}}}{\gamma} \right]^{1/6} \left( \frac{J_{\text{f}}}{J_{\text{f}}} \right)^{1/3}$ |

Notes. $s = s/0.5$, $a_s = a/(10^{-3} \text{ cm})$, $f(\tilde{i}) = \tilde{i} \left[ (1 + \tilde{i}^2) / 2 \right]^2$, $n_1 = n_{\text{H}}/(10^{-3} \text{ cm}^{-3})$, $T_1 = T_{\text{gas}}/10^4 \text{ K}$, $\gamma = \gamma(0)/10^{-4}$.

Transonic and supersonic grains (i.e., $s_d \gtrsim 1$) can become highly elongated, with an axial ratio of $a_{\text{new}}/c_{\text{new}} > 2$. For the silicate grains accelerated by MHD turbulence in MCs with $v_d \approx 0.5 \text{ km s}^{-1}$ (see Table 1), which corresponds to $s_d \approx 5.5$ (see Equation 33), Figure 7 implies an elongation of $a_{\text{new}}/c_{\text{new}} \approx 3$ and 5 by the time $\Delta c/a = 0.1$ and 0.2, respectively. It is worth noting that the realistic elongation of an individual grain grown from gas accretion is affected by the metal abundance in the gas, grain shattering, and rotational disruption.

5. Grain Coagulation from Drifting Aligned Grains

In this section, we first estimate the timescales required for grain growth by grain collisions and compare with the characteristic timescales of grain alignment to see the relevance of the latter process. We then discuss the expected shape and structure of dust aggregates formed from the coagulation from aligned grains.

5.1. Grain Coagulation by Collisions

The mean time between two successive collisions of two equal-sized grains is given by

$$\tau_{\text{gg}} = \frac{1}{\pi a n_g v_d} = \frac{4 \mu_a M_{s/d}}{3 n_{\text{H}} m_{\text{H}} v_d} \simeq 7.6 \times 10^4 \frac{\rho a_{\text{crit}}}{g_{\text{f}}^{1/2}} \left( \frac{v_d}{1 \text{ km s}^{-1}} \right)^{-1} \text{ yr},$$

where $n_g$ is the number density of dust grains, $M_{s/d} = \mu_a m_{\text{H}} m_{\text{g}} = 100$ with $m_{\text{g}} = 4 \pi a^2 \rho / 3$ is the gas-to-dust mass ratio, and we have assumed the single-grain size distribution.

The shattering threshold is $v_{\text{shat}} \sim 2.7 \text{ km s}^{-1}$ for silicate and $1.2 \text{ km s}^{-1}$ for graphite grains (Jones et al. 1996; Yan et al. 2004). At high velocities, shock waves are produced inside the grains and can shatter them into smaller fragments. Grain velocities achieved by MHD turbulence in MCs and DCs are $v_d < v_{\text{shat}}$. Therefore, grain shattering does not occur in MCs and DCs. However, whether grains of $v < v_{\text{shat}}$ stick upon collision is unclear, and the presence of ice mantles is expected to enhance sticking collisions due to its larger threshold $v_{\text{crit}}$. Therefore, in this paper, we assume that sticky collisions and grain coagulation occur whenever $v < v_{\text{shat}}$.

Table 2 summarizes the characteristic timescales involved in grain alignment and growth. The accretion and grain–grain collision time are much longer than the internal alignment time (BR) and external alignment processes (Larmor precession and RAT alignment). As shown, the alignment time by RATs is at most $\tau_{\text{RAT}} \sim \tau_{\text{gas}}$, which is equal to the time required to collide with the gas of the same mass as the dust grain. Since the dust mass is $\sim 1\%$ of the gas mass, the grain collision time is 100 times longer. Note that the collision time for a large grain size $a$ with a tiny grain is shorter due to its higher density. However, such collisions do not significantly randomize the orientation of the large grain because of its lower mass. Therefore, we can assume that aligned grains can be rapidly realigned before hitting another grain.

5.2. Nonaligned–Nonaligned Grains: Stick and Disalign

Consider first the collisions between randomly oriented grains. In this scenario, grain collisions are similar to what is studied previously. Figure 8 illustrates the grain coagulation by collisions of nonaligned grains moving perpendicular to the mean magnetic field. After the first collision, grains stick and form an elongated binary. The newly formed grain is rapidly randomized by gas collisions and then experiences the next
random collision with the high probability along the direction with maximum cross section. Therefore, coagulation of nonaligned grains leads to a dust aggregate of low elongation.

5.3. Aligned–Nonaligned Grains: Stick and Align

We now consider coagulation by collisions between an aligned grain with another nonaligned grain. Figure 9 illustrates the coagulation of one aligned grain with nonaligned grains. After sticking collisions, the monomers stick to become a binary (stage 1). If the collision results in a sudden disalignment so that the shortest axis deviates significantly from the angular momentum that is parallel to the magnetic field, then internal relaxation rapidly brings the grain axis to be aligned with the magnetic field, provided that the particle size is smaller than $a_{\text{max,df}}$ (see Figure 4). Note that the alignment timescale is shorter than the grain collision time $\tau_{\text{gg}}$ (see Table 2). Thus, the resulting elongated binary can be rapidly realigned with the long axis perpendicular to the magnetic field (stage 2). The subsequent collision occurs along the long axis, resulting in a more elongated particle (stage 3), and the realignment occurs due to internal relaxation and RAT alignment (stage 4). A dust aggregate is finally formed, with a larger elongation than in the case of growth from nonaligned grains (see Figure 8). Moreover, in the RAT paradigm, the nonaligned grain is smaller than the aligned grain, so the collision would not result in a significant deviation of the net grain angular momentum from the magnetic field.

5.4. Aligned–Aligned Grains: Stick and Align

Figure 10 illustrates the coagulation from collisions between two aligned grains. After the first collision, a binary is formed and comprises a pair of aligned grains with parallel minor axes (short arrows). The minor axis of the binary (long arrow) is not necessary parallel to the grain’s axes and makes a small angle (stage 1). However, internal relaxation rapidly brings the binary axis to be aligned with the magnetic field (stage 2). The first binary continues to collide with another aligned grain and coagulates, but the minor axes of the binary are slightly tilted from the third grain’s axis (stage 3). The resulting particle has the shortest axis tilted from the magnetic field and the angular momentum. The internal relaxation again acts to bring the internal alignment on a timescale of $\tau_{\text{BR}}$ (stage 4). An aligned grain moving along the magnetic field can collide with the large grain and forms a dust particle (stage 5); subsequently, relaxation processes bring it to be aligned with the magnetic field (stage 6). A dust aggregate is finally formed. The shortest axis of the dust aggregate is tilted by some small angle with the short axes of the first binary. Thus, the coagulation from aligned grains leads to the formation of composite dust aggregates that contain elongated binaries made of oblate spheroids with parallel short axes inherited from grain alignment with the magnetic field. The shortest axis of the dust aggregate is aligned along the magnetic field.

Figure 11 presents the coagulation of grains with wrong alignment with the long axis parallel to the magnetic field. The process is similar to the coagulation of aligned grains in
6. Discussion

6.1. Grain Shape and Structures from Grain Growth of Aligned Grains

We have studied the effect of grain alignment and motion across the magnetic field on grain growth by accretion of metals and grain–grain collisions. For grains aligned with the axis of maximum inertia (e.g., shortest axis) parallel to the magnetic field, the flux of gas species arriving along the direction of grain motion is increased significantly compared to the particle flux in the perpendicular direction (see Equation 36). As a result of gas accretion, the shape of grains becomes oblate spheroidal due to fast rotation of the grain around the magnetic field. In addition, the grain elongation by gas accretion increases over time, and grains become highly elongated if they move supersonically through the metals (see Figure 7). Interestingly, even with a low drift velocity of \( v_d \sim 0.2 \text{ km s}^{-1} \), the dust grain drifts with \( s_d \sim 2 \), i.e., supersonically relative to metals due to the high atomic mass of metals (see Equation 33). However, the ultimate elongation of interstellar grains by gas accretion is also constrained by the metallicity available in the cloud and destruction processes such as grain shattering and rotational disruption by RATs (Hoang et al. 2019).

Coagulation from sticking collisions between aligned grains first produces a binary comprising two aligned grains of oblate shapes with parallel short axes. The binary grains can be rapidly aligned by efficient internal relaxation and RATs and then continue to collide with another aligned grain, forming a bigger particle with aligned axes. The grain growth process continues and eventually forms a micron-sized composite dust aggregate (Dominik & Tielens 1997; Ormel et al. 2009).

Therefore, the dust aggregate contains elongated binaries of two aligned particles with different sizes and orientations, forming a hierarchy structure.

Due to the rapid decrease in the efficiency of internal relaxation by the Barnett effect with the grain size, as \( a^7 \) (see Equation 15), there exists a maximum grain size that still has an efficient internal alignment, \( a_{\text{max,el}} \). Thus, the largest size of an aligned binary within the dust aggregate is described by \( a_{\text{max,el}} \), which depends on the dust magnetic properties, grain rotation rate, and gas density of the environment when grain growth occurs. As shown in Figure 4, for DCs of \( n_H \sim 10^3 \text{ cm}^{-3} \), the dust of ordinary PM material has \( a_{\text{max,el}} \sim 0.5 \text{ \( \mu \)m} \) for thermal rotation (\( J/J_d = 1 \)) and \( 2 \mu \text{m} \) for suprathermal rotation (\( J/J_d = 100 \)), but it can increase to \( 5–10 \mu \text{m} \) for SPM grains with \( N_H \sim 10^4 \text{ iron atoms per cluster} \). For the typical protostellar core with \( n_H \sim 10^7 \text{ cm}^{-3} \), only grains of sizes \( a < a_{\text{max,el}} \sim 2 \mu \text{m} \) can have internal alignment. For the interior of PPDs of \( n_H \sim 10^{10} \text{ cm}^{-3} \), the maximum size of the internally aligned grains is only \( a_{\text{max,el}} \lesssim 0.7 \mu \text{m} \), as implied by Equation (20). For grains larger than \( a_{\text{max,el}} \), internal relaxation is not efficient to align the grain’s shortest axis with the magnetic field, but it may induce the wrong alignment with the shortest axis perpendicular to the magnetic field. In this case, dust aggregates may contain the aligned grains with parallel long axes due to grain–grain collisions.

Note that in the diffuse ISM, large grains of highly elongated shapes and structures are found to be disrupted by centrifugal stress due to grain fast rotation by RATs (Hoang et al. 2019; Hoang 2019; see Hoang 2020 for a review). This process removes highly elongated shapes and leaves behind the less extreme grain shapes.

6.2. Implications for Polarization Observations toward Star-forming Regions

Our study implies that grain growth from aligned grains has the elongation increasing from the diffuse ISM to MCs because the denser region has a higher growth rate, and the elongation can be large, \( \epsilon > 3 \) (see Figure 12). Therefore, the polarization cross-section efficiency (or intrinsic polarization efficiency), which is determined by the grain elongation, would increase toward denser environments as long as the grains are still...
aligned. Assuming the perfect alignment of grains with the magnetic field, the maximum level of thermal dust polarization would increase and can be larger than the maximum level of 19.8% measured by the Planck satellite (Planck Collaboration et al. 2015a). Interestingly, high-resolution polarimetric observations of protostellar disks with ALMA report a maximum polarization level of 30%–40% (Kwon et al. 2019; see Table 1 in Gouellec et al. 2020), which reveal that dust grains in these environments are more elongated than interstellar dust and consistent with our present prediction.

Observations of dust polarization in far-infrared and submillimeter wavelengths usually reveal the decrease of dust polarization with increasing the visual extinction or column gas density. One popular explanation is that grains become less elongated toward dense regions due to grain growth (e.g., Juárez et al. 2017). Such an explanation is expected for grain growth by gas accretion on randomly oriented grains because of the isotropic Brownian motion of thermal gas. However, our present study suggests that anisotropic gas accretion to aligned grains makes the grain more elongated. Therefore, the origin of the polarization hole toward dense starless clouds is most likely caused by the loss of grain alignment due to attenuation of the radiation field and enhanced gas damping, as shown in Hoang et al. (2021), or magnetic field tangling.

6.3. Coagulation of Aligned Grains in PPDs and Implications for Cometary Dust

Comets form out of dust, ice, and gas beyond the snow line in PPDs; thus, cometary dust aggregates contain valuable information about the coagulation process. However, where the growth from submicron-sized grains to micron-sized dust aggregates starts, in PPDs or the early phase of MCs, is uncertain.

Suppose that grain coagulation to micron size begins in MCs. In this case, it involves grains aligned with the magnetic field and thus produces an elongated binary comprising two aligned grains’ parallel short axes embedded within a dust aggregate. The maximum size of the aligned grains, $a_{\text{max},\alpha J}$, could reach several microns (see Figure 4). This fundamental structure represents the earliest phase of grain growth where the environment is still not dense enough and grains can still be aligned. The elongated binary with axially aligned grains would be imprinted in cometary dust particles. Moreover, the elongation of dust grains resulting from gas accretion and coagulation should increase with the grain size. Since aligned grains require PM and SPM inclusions, the aligned binary and elongated grains would contain a high level of iron inclusions.

Suppose that grain growth mainly occurs in the disk midplane of PPDs. In this case, only submicron grains of $a < a_{\text{max},\alpha J} \lesssim 0.7 \, \mu m$ can have internal alignment (see Equation 20), while larger grains with wrong internal alignment will lead to a dust aggregate made of aligned grains with parallel long axes. The orientation of the alignment axis in dust aggregates thus reveals unique information about where dust growth occurs.

Although the radiation is weak, grains in PPDs may be spun up by METs (Lazarian & Hoang 2007b; Hoang et al. 2018; Tatsuuma & Kataoka 2021) and rotate with high angular velocities. Thus, inelastic rotation may be efficient to induce internal alignment for millimeter-sized grains (B. T. Draine, private communication). However, METs can both spin up and spin down the grain.

Carbonaceous grains are not likely to be aligned with the magnetic field due to their diamagnetic property (Hoang & Lazarian 2016). Although carbonaceous grains can still drift across the magnetic field as silicate grains, due to their random orientation, their shapes resulting from gas accretion are radically different from silicate grains. If stardust is shattered upon being injected into the ISM (e.g., by shocks) and regrows by gas accretion, then carbonaceous grains would have a smaller elongation than silicate grains if these two populations are separate. As a result, the fundamental carbonaceous grains’ imprint in cometary dust would be less elongated than that of silicates. Carbonaceous grains would produce dust aggregates consisting of elongated grains with random axes. The identification of such a random orientation of elongated carbonaceous grains within dust aggregates would be direct evidence for nonalignment of carbonaceous dust.

Observations of scattered light from cometary dust reveal circular polarization (CP; Kolokolova et al. 2016). The alignment of cometary dust grains is suggested as a potential mechanism to produce CP (Hoang & Lazarian 2014). Scattering of sunlight by dust aggregates containing aligned grains would induce CP, in analogy to aligned grains in the gas. If this is the case, CP can be produced as long as dust aggregates are lifted from the nucleus.

Finally, the Rosetta mission analyzed dust from 67P/Churyumov–Gerasimenko and found that cometary dust particles are aggregates of smaller, elongated grains of submicron sizes (Bentley et al. 2016). Figure 12 shows the variation of the fine grain elongation with the radius of dust grains within dust aggregates using data from Bentley et al. (2016). The observed elongation appears to increase with radius, which can be described by a slope of 0.3 (solid line). This result reveals that grain growth is not isotropic, as expected from Brownian motion or randomly oriented grains. Nevertheless, it is consistent with grain growth from gas accretion and grain coagulation involving aligned grains. Further studies on the fundamental aligned structures and elongation of aligned grains within cometary and interplanetary dust aggregates are essential to test this scenario of grain growth.

7. Summary

We study the effect of grain alignment with the interstellar magnetic field on grain growth due to gas accretion and grain–grain collisions. The main results are summarized as follows.

1. Charged grains in the ISM can be accelerated by gyroresonance with MHD turbulence, ambipolar diffusion, or shocks and move in the direction perpendicular to the magnetic field.
2. In dense MCs of $n_H > 10^3 \text{ cm}^{-3}$, PM grains can have efficient internal alignment by BR with the shortest axis parallel to the magnetic field for grain sizes of $a \lesssim a_{\text{max},\alpha J} \sim 0.5–2 \, \mu m$ for the grain angular momentum of $J_a/J_d \sim 1–100$. The inclusion of iron clusters can shift the range of internal alignment grains to $a_{\text{max},\alpha J} \sim 10–50 \, \mu m$.
3. Grain alignment by RATs for grains of $a < a_{\text{max},\alpha J}$ occurs with the shortest axis parallel to the magnetic field, so the long axis is thus parallel to the direction of grain motion. The grain alignment with the magnetic field occurs on a timescale shorter than the grain growth time.
by gas accretion and coagulation; therefore, grain alignment should be considered for grain growth in MCs.

4. Fast rotation of the grain along the shortest axis (also magnetic field) leads to an oblate spheroidal shape for large grains that are grown by gas accretion to aligned elongated grains. The grain elongation by gas accretion increases over time if the grain moves supersonically with respect to the metals.

5. Coagulation from aligned grains first produces a binary of aligned grains with parallel short axes and then composite particles of oblate spheroid shapes due to the fast rotation around the magnetic field. The dust aggregates would contain a number of elongated binaries comprising two aligned grains with parallel short axes.

6. Elongation of grains formed from coagulation of aligned grains should be larger than that formed from nonaligned grains. This is the opposite of grain growth in the case of randomly oriented grains that makes the grain less elongated because the newly formed outer layer is isotropic. Due to their nonalignment with the magnetic field, carbonaceous grains would have lower elongation than silicate grains or grains with iron inclusions.

7. The orientation of elongated grains within a dust aggregate of cometary dust would provide information about where grain growth begins. Parallel short axes of aligned grains imply that dust aggregates are formed in MCs or the surface layer of PPDs where grains are aligned. A random orientation of elongated grains implies that grain growth begins in very dense, shielded regions where grains are not aligned due to high density and a lack of a radiation field.

8. Grains within dust aggregates in 67P/Churyumov–Gerasimenko studied by Rosetta have the elongation increasing with the grain radius, which implies that such dust aggregates might form by coagulation of aligned grains.

We thank the referee for a constructive report and Bruce T. Draine, Ludmilla Kolokolova, and Hiroshi Kimura for various comments. T.H. acknowledges support by the National Research Foundation of Korea (NRF) grants funded by the Korean government (MSIT) through the Mid-career Research Program (2019R1A2C1087045).

Appendix A

Flux of Gas Accretion onto a Drifting Aligned Grain

Let \( \xi \parallel B \). We assume that an aligned dust grain drifts across the magnetic field with the drift velocity \( v_d = v_y x \). We consider the collision of the grain with a gas species \( i \) with mass \( m_i \) and number density \( n_i \). Let \( s_d = v_d / \Omega_T, \) where \( \Omega_T = (2k T_{\text{gas}}/m_i)^{1/2} \) is the thermal velocity of the gas species \( i \).

The velocity distribution of the gas species in the lab frame is Maxwellian,

\[
 f(v_x, v_y, v_z) dv = f^* f f^* f^* f^* \frac{Z^3}{\exp \left( \frac{-m_i v_y^2}{2k T_{\text{gas}}} \right)} dv_x dv_y dv_z,
\]

where \( Z = (m_i/2\pi k T_{\text{gas}})^{1/2} \), and \( \int f^* f dv = 1 \).

In the grain’s reference frame, the gas velocity is \( v'_x = v_x - v_d, \) \( v'_y = v_y, \) \( v'_z = v_z \).

The number flux of the gas species colliding with the upper surface of the grain along the \( z \)-direction is given by

\[
 F_z = \int_{-\infty}^{\infty} f_x dv_x \int_{-\infty}^{\infty} f_y dv_y \int_{-\infty}^{\infty} n_i v_z f_z dv_z = n_i \int_{0}^{\infty} Z v_z e^{-m_i v_x^2/2k T_{\text{gas}}} dv_x
 = Z n_i (2k T_{\text{gas}}/m_i) \int_{0}^{\infty} Z e^{-v_x^2/2} dv_x / 2
 = \frac{1}{2} n_i (2k T_{\text{gas}}/m_i)^{1/2} \int_{0}^{\infty} v_x dv_x
 = \frac{1}{4} n_i \langle v_y \rangle,
\]

where \( x = v_x^2/(2k T_{\text{gas}}/m_i) \), the lower limit is taken to be zero in order for the gas to hit the grain surface, and \( \langle v_y \rangle = (8k T_{\text{gas}}/m_i)^{1/2} \) is the mean speed defined by \( \langle v_y \rangle = \int_{0}^{\infty} v_x v_z^2 e^{-m_i v_x^2/2k T_{\text{gas}}} dv_x dv_z \).

Similarly, the collision rate by the gas onto one upper surface along the \( y \)-direction is given by

\[
 F_y = \int_{-\infty}^{\infty} f_x dv_x \int_{-\infty}^{\infty} f_y dv_y \int_{-\infty}^{\infty} n_x f_x dv_x = \frac{1}{4} n_i \langle v_y \rangle.
\]

Appendix B

Derivation of BR Time

For the pedagogical purpose here, we provide the detailed derivation of BR for an oblate spheroidal grain.

In the grain’s body frame, the tips of \( J \) and \( \Omega \) precess around \( \tilde{a}_4 \) with the angular rate \( \omega = (h - 1) \Omega_4 \) (e.g., Purcell 1979; Hoang et al. 2010; see Figure 2). The rotational energy of the
where

\[ J_1^2 = J_2^2 + J_3^2 = J^2 \sin^2 \theta. \]

The basic idea of this dissipation process is that the instantaneous magnetization is along

\[ M = \chi(0)/\gamma_0 = \chi(0)H_{BE}, \]

with \( H_{BE} \equiv \Omega/\gamma_0 \) being the equivalent Barnett field. In the body frame, this magnetization has a component perpendicular to \( \mathbf{a}_3 \), which is seen rotating with respect to the axis of maximum inertia at an angular rate of \( \omega \) (see Figure 2). This rotating magnetization induces the dissipation of rotational energy at a rate

\[
\frac{dE_{\text{rot}}}{dt} = H_{\text{Be,rot}}^2 V \chi \omega \left( \frac{\Omega \sin \theta_1}{\gamma_e} \right)^2 V K(\omega) \omega^2,
\]

\[
\frac{dE_{\text{rot}}}{dt} = \frac{\Omega^2 \sin^2 \theta_1}{\gamma_e^2} V K(\omega)(h - 1)^2 \Omega_0^2 \sin^2 \theta.
\]

The Barnett dissipation reduces the rotation of the rotational energy as

\[
\frac{dE_{\text{rot}}}{dt} = \frac{J^2}{J_1^2} (h - 1) \sin \theta \cos \theta.
\]

Using \( dE_{\text{rot}}/dt = -dE_{\text{Bat}}/dt \) and Equations (B8) and (B9), one obtains

\[
\frac{d\theta}{dt} = -\frac{J^2}{\Omega_0^2 \gamma_e^2} V K(\omega) h^2 (h - 1) \sin \theta \cos \theta,
\]

which corresponds to

\[
\frac{d\tan \theta}{\tan \theta} = -\frac{V K(\omega) h^2 (h - 1) J^2}{\Omega_0^2 \gamma_e^2} dt,
\]

\[
\tan \theta = C e^{-t/\tau_{\text{BR}}},
\]

where \( \tau_{\text{BR}} \) is the characteristic timescale for BR.

\[ \tau_{\text{BR}} = \frac{\gamma_e^2 J_1^2}{VK(\omega) h^2 (h - 1) J^2}. \]