Nuclear response for the Skyrme effective interaction with zero-range tensor terms. III. Neutron matter and neutrino propagation

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The formalism of the linear response for the Skyrme energy density functional including tensor terms derived in articles [1, 2] for nuclear matter is applied here to the case of pure neutron matter. As in article [2] we present analytical results for the response function in all channels, the Landau parameters and the odd-power sum rules. Special emphasis is given to the inverse energy weighted sum rule because it can be used to detect non physical instabilities. Typical examples are discussed and numerical results shown. Moreover, as a direct application, neutrino propagation in neutron matter is investigated through its neutrino mean free path at zero temperature. This quantity turns out to be very sensitive to the tensor terms of the Skyrme energy density functional.

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I. INTRODUCTION

In a recent series of articles [1, 2], hereafter denoted respectively article I and II, the contribution of the zero-range tensor terms in the Skyrme effective interaction has been analyzed in the context of the Linear Response theory. The first result from these articles is that the tensor terms have very sizable effects on the response functions. Another important result is that the inverse energy weighted sum rule can be used as a tool of diagnosis for instabilities. These two articles were devoted to Symmetric Nuclear Matter (SNM) only. Since the construction of an Energy Density Functional (EDF) reliable for both symmetric matter and neutron matter is of fundamental importance, we present here the response functions and some associated sum rules for Pure Neutron Matter (PNM) with the same approach. The interest of the present study is related to spin susceptibilities and ferromagnetic finite size instabilities in neutron matter. Moreover, we use these results to study the impact of the tensor terms on the determination of the neutrino mean free path in PNM. This is a quantity of intrinsic importance since the cooling of a neutron star core in its first moments is governed by neutrino emission and therefore by their mean free path trough dense matter. Some previous studies using non-relativistic approaches have revealed some very interesting features of the mean free path properties but they all neglected the possible tensor contribution. Since the neutrino mean free path is directly related to the response functions which are themselves affected by the tensor, it is worthwhile to determine precisely the induced modifications.

The article is organized as follows: in the first part devoted to the linear response theory approach, we present explicit expressions for the spin response functions, the Landau parameters and the sum rules. Since the technical approach follows closely those of the previous articles, this part mainly contains figures and discussion, formula being written in appendices. The second part deals with the problem of neutrino mean free path. We first give explicit expression of this quantity in presence of tensor interaction then we show the influence of the parameterizations of the Skyrme functional.

II. LINEAR RESPONSE APPROACH TO NEUTRON MATTER

A. Response function

Following article II, the starting point for the determination of the response functions is the Skyrme energy functional. Since in neutron matter the isospin is no longer a relevant quantum number and isovector and isoscalar densities are equal, it is convenient to define new coupling constants \( C_x^z = C_0^z + C_1^z \), where \( z = \rho, \tau, \Delta \rho, \ldots \) in such a way that the Energy Density Functional can be written as

\[
E = \int \mathcal{E} \, d^3r
\]
The expressions of the coupling constants as functions of the parameters of the Skyrme interaction can be found in article I. From this expression, it is straightforward to obtain the residual interaction (see table in Appendix A) by taking the second derivative of the EDF with respect to the density.

The Random Phase Approximation (RPA) response function in each channel \((\alpha) = (S,M) \equiv \text{(spin, projection of the spin)}\) (see Appendix B) is then obtained by solving the Bethe-Salpeter equations for the correlated Green functions \(G_{\text{RPA}}^{(\alpha)}\). Finally, from these response functions, we can easily obtain the Landau parameters (see Appendix C). The quantities of interest are not directly the response functions \(S^{(\alpha)}(q, \omega)\), also called the dynamical structure functions by some authors, which are defined at zero temperature by

\[
S^{(\alpha)}(q, \omega) = -\frac{1}{\pi} \text{Im} \chi^{(\alpha)}(q, \omega). \tag{3}
\]

From now on we choose the direction of \(q\) along the \(z\) axis, as done in article I and II. We show this function in Fig. 1 for two different values of the transferred momentum \((q = 0.05 \text{ fm}^{-1} \text{ and } q = 0.5 \text{ fm}^{-1})\) and two different densities \((\rho = 0.08 \text{ fm}^{-3} \text{ and } \rho = 0.16 \text{ fm}^{-3})\) as a function of the energy \(\omega\). As in article I and II we use a system of natural units so that \(\hbar = c = 1\). We consider one interaction without tensor, i.e., SLy5, and one with tensor, T16 (see article of Lesinski et al. for the definition of the TLJ parameterizations). Among the several TLJ possibilities the choice of T16 is motivated by the study of the neutrino mean free path (see later). In order to illustrate the effect of the interaction and of the RPA correlations we plot in each panel the corresponding Fermi gas (FG) and Hartree-Fock results (i.e., uncorrelated response functions). A first effect of the interaction clearly appears at the Hartree-Fock level where the mean field is responsible for the dressing of the bare neutron mass giving a density-dependent effective one. The difference between the Fermi gas and Hartree-Fock structure functions increases with the density. With the RPA correlations, the difference between the \(S = 0\) and \(S = 1\) channel of the p-h interaction are reflected in the corresponding response functions. Let us focus on the \(S=1\) channel, particularly important for the neutrino mean free path. The \((S = 1, M = 0)\) and \((S = 1, M = 1)\) structure functions practically coincide, as it is expected for the SLy5 force, while they clearly differ in the presence of tensor interaction. Concerning the \(S = 1\) for low \(q\) (as illustrated in the figure for \(q = 0.05 \text{ fm}^{-1}\)) a spin zero-sound mode appears. It stands out above the p-h continuum. The existence of this spin collective mode, called magnon, makes harder the excitation of the system, hence, correspondingly the p-h continuum response is depleted. This anti-ferromagnetic behaviour disappears when \(q\) increases. For high \(q\) another kind of divergence may appear. As illustrated in Fig. 2 the enhancement of the response function may become dra-
FIG. 2: (Colors online) The response functions $S^{(a)}(q, \omega)$ calculated for the Skyrme tensor parameterization T16 [27], for the channel $S = 1$, $M = 0$ only. The transfer momentum is $q = 1.051$ fm$^{-1}$ and the density of the system is $\rho = 0.16$ fm$^{-3}$. 

$\rho=0.16$ fm$^{-3}$  
$q=1.051$ fm$^{-1}$  
$S=1$ M=0

FIG. 3: (Color on line) IEWSR (in MeV$^{-1}$) as function of the transferred momentum $q$ (in fm$^{-1}$) for the T16 tensor parameterization and for SNM (panel (a)) and for PNM (panel (b)). The horizontal dashed-dotted line is placed at $\rho = 0.16$ fm$^{-3}$ just to guide the eye.

FIG. 4: (Color on line) Critical densities (in fm$^{-3}$) as functions of the transferred momentum $q$ (in fm$^{-1}$) for the T16 tensor parameterization and for SNM (panel (a)) and for PNM (panel (b)). The horizontal dashed-dotted line is placed at $\rho = 0.16$ fm$^{-3}$ just to guide the eye.

FIG. 5: (Color on line) For the two $S = 1$ channels in PNM, critical densities (in fm$^{-3}$) are plotted as functions of the transferred momentum $q$ (in fm$^{-1}$) for the TJ$\bar{J}$ family of Skyrme forces. The horizontal dashed-dotted line is placed at $\rho = 0.16$ fm$^{-3}$ just to guide the eye.

B. Sum rules and moments of the strength function

Following the notations of article II, we can calculate the most relevant odd-power sum rules for PNM, in particular the $M_1$ Energy Weighted Sum Rule (EWSR), the $M_3$ Cubic Energy Weighted Sum Rule (CEWSR) and the $M_{-1}$ Inverse Energy Weighted Sum Rule (IEWSR) defined as

$$M_k^{(\alpha)}(q) = \int_0^\infty d\omega \, \omega^k \, S^{(\alpha)}(q, \omega).$$

As stated previously, all the expressions given below are derived for the general Skyrme EDF given by Eq. 49 in which all the coupling constants could be considered as independent ones from the others. We refer to article II for a detailed discussion on their derivation.
The EWSR in each channel reads

\[ M_{1}^{(0,0)} = \frac{q^2}{2m^*} \left( 1 - W_2^{(0)} m^* \rho \right) , \]  
\[ M_{1}^{(1,0)} = \frac{q^2}{2m^*} \left[ 1 - (W_2^{(1)} + 2CF)m^* \rho \right] , \]  
\[ M_{1}^{(1,\pm 1)} = \frac{q^2}{2m^*} \left( 1 - W_2^{(1)} m^* \rho \right) . \]  

Taking into account the \( W_2^{(0)} \) value as well as the neutron effective mass \( m^* \), defined as

\[ \frac{m}{m^*} = 1 + 2m(C_0^T + C_1^T)\rho , \]  
the \( M_{1}^{(0,0)} \) EWSR reduces to the free value \( q^2/2m \), as it should. As in article II for the case of a Skyrme force, these \( M_1 \) moments can be obtained from the double commutator method \([36, 37]\).

For the CEWSR we have

\[ M_{3}^{(0,0)} = q^4 \frac{k_F^2}{2m^{*3}} \left[ 1 - m^* \rho W_2^{(0)} \right]^2 \times \left\{ \frac{k^2}{2} + k^2 m^* \rho W_2^{(0)} + \frac{m^* \rho}{2k_F^2} \left[ W_2^{(0)} + 2k_F^2 W_2^{(0)} \right] \right\} , \]  
\[ M_{3}^{(1,0)} = q^4 \rho^2 [CF]^2 \frac{k_F^2}{5tm^*} \left\{ 2m^* \rho \left[ W_2^{(1)} + 2CF \right] - 1 \right\} + q^4 \frac{k_F^2}{2m^{*3}} \left\{ m^* \rho \left[ W_2^{(1)} + 2CF \right] - 1 \right\}^2 \times \left\{ \frac{k^2}{8} + k^2 m^* \rho CF + k^2 m^* \rho W_2^{(1)} + \frac{m^* \rho}{2k_F^2} \left[ W_2^{(1)} + 4q^2 CF + 2k_F^2 W_2^{(1)} \right] \right\} , \]  
\[ M_{3}^{(1,\pm 1)} = q^4 \rho^2 [CF]^2 \frac{k_F^2}{10m^*} \left\{ 2m^* \rho W_2^{(1)} - 1 \right\} + q^4 \frac{k_F^2}{2m^{*3}} \left[ m^* \rho W_2^{(1)} - 1 \right]^2 \times \left\{ \frac{k^2}{8} + k^2 m^* \rho CF + k^2 m^* \rho W_2^{(1)} + \frac{m^* \rho}{2k_F^2} \left[ W_2^{(1)} + 2k_F^2 W_2^{(1)} \right] \right\} . \]
And finally for the IEWSR we have

\[ M_{-1}^{(0,0)} = f(k) \frac{3m^*}{2k_F} \left\{ -24k^2 \left[ m^* \rho \mathcal{C} \nabla J \right]^2 + \frac{f(k) \left[ 1 - 3 \left( k^2 - 1 \right) f(k) \right]}{4 - m^* \rho \left[ 1 - 3 \left( k^2 - 1 \right) f(k) \right]} W_2^{(1)} - C^F \right\} - \frac{3}{10} \left[ m^* \rho f(k) (k^2 - 1) \right]^2 W_2^{(1)} \left[ k_F m^* \left( W_1^{(1)} + 4q^2 \mathcal{C}^F \right) - \frac{3}{2} m^* \rho \left( 4k^2 C^F + (k^2 - 1) W_2^{(1)} \right) \right]^{-1}, \]  

\[ M_{-1}^{(1,0)} = f(k) \frac{3m^*}{2k_F} \left\{ -12 \left[ m^* \rho \mathcal{C} \nabla J \right]^2 + \frac{k^2 f(k) \left[ 1 + 3f(k)(1 - k^2) \right]}{4 - m^* \rho \left[ 1 + 3f(k)(1 - k^2) \right]} W_2^{(0)} \right\} + \left[ 1 + \frac{1}{4} m^* \rho \left( 3W_2^{(1)} + C^F \right) \right]^2 - \frac{3}{10} \left[ m^* \rho f(k)(1 - k^2) \right]^2 \left( 5 \left[ C^F \right]^2 + 2C^F W_2^{(1)} + \left[ W_2^{(1)} \right]^2 \right) \] 

\[ + f(k) \frac{k_F m^*}{2\pi^2} \left[ W_1^{(1)} + \frac{3}{2} m^* \rho (1 - k^2) \left[ W_2^{(1)} + C^F \right] \right] + \frac{1}{8} m^* \rho^2 \left( \left[ C^F \right]^2 (k^2 - 9) - 8k^2 C^F W_2^{(1)} - (3 + 13k^2) \left[ W_2^{(1)} \right]^2 \right) \right\}^{-1}. \]  

where the \( W_i^{(S)} \) coefficients are given in Appendix B.

As in article II we define the function \( f(k) \) as

\[ f(k) = \frac{1}{2} \left[ 1 + \frac{1}{2k} (1 - k^2) \ln \left( \frac{k + 1}{k - 1} \right) \right], \]  

while \( \rho \) and \( k \) are now defined for PNM with Fermi momentum \( k_F \), hence

\[ \rho = \frac{1}{3\pi^2} k_F^2, \] 

\[ k = \frac{q}{2k_F}. \]

As already illustrated in the previous section, the main effect of the tensor terms is in the \( S = 1 \) channel where one can even observe a divergence at zero energy, but finite transferred momentum. As explained in detail in the article II, the IEWSR, \( M_{-1} \), can be used to detect these poles. As an example, on Fig. 6 we plotted the IEWSR for T16 obtained from the analytic expansion of the response function (see Eqs. (12)-(14)) and from the direct numeric integration. We observe on this figure that in the channels \( S = 1, M = 0 \) and \( S = 1, M = 1 \) the IEWSR is violated. This indicates the presence of a pole in the response function as shown for example for \( S^{1,0}(q, \omega) \) on figure 2 for \( \rho = 0.16 \text{ fm}^{-3} \) and transferred momentum \( q = 1.051 \text{ fm}^{-1} \).

This connection between the pole (when it does exist) observed in the response function and the pole observed in the \( M_{-1} \) sum rule has been discussed in article II and we refer to it for a more detailed discussion on this point. It is thus possible to determine in a systematic way the critical densities at which a pole occurs for a given momentum \( q \) from Eqs. (12)-(14). On Fig. 4 we display such critical densities, \( \rho_c \), with respect to the transferred momentum for the \( T16 \) interaction. On the left panel we first show the position of the poles of the response function for SNM for each \( (S, M, I) \) channel. On the right panel we then show the position of the poles for PNM for each \( (S, M) \) channel.

Even if we exclude the case of spinodal instability which is not present in PNM, one can see that the presence and the location of the poles depends strongly on the system under analysis: for a given interaction, the critical densities are very different for PNM and SNM.
Similarly we show in Fig. 5 the critical densities for the tensor parameterizations that we use in the following section to study neutrino free path.

Following article II, we show for completeness in Fig. 5 the critical densities for the Skyrme EDF previously analyzed in article II for SNM, but in this case for PNM. We observe that SkP behaves very differently from the other Skyrme forces presented in this article. It presents a first instability in the \( S = 0 \) channel at \( \rho \approx 0.16 \text{ fm}^{-3} \) and a second one at higher density \( \rho \approx 0.45 \text{ fm}^{-3} \) due to the presence of a pole in the effective mass defined in Eq. (5).

### III. Neutrino Mean Free Path

The aim of this section is to investigate the effect of the choice of the parameters of the tensor terms on the neutrino mean free path in neutron matter. In this article we restrict ourselves to the academic case of neutron matter at zero temperature (the generalisation at finite temperature \( T \) is in progress). This, of course, implies some restriction on the application of our approach to neutron stars studies. At first stages of the cooling process of the neutron stars, modeled as asymmetric nuclear matter, the high temperatures involved allow charged current reactions; then, when the temperature decreases, neutral currents dominate. Since we consider the case \( T = 0 \) and the pure neutron matter, we will take into account only neutral currents for the determination of the neutrino mean free path. This quantity is defined as

\[
\lambda = (\sigma \rho)^{-1},
\]

where \( \sigma \) is the total cross section for the neutral current reaction \( \nu + n \rightarrow \nu' + n' \). In the absence of tensor (and spin-orbit) interaction this total cross section is obtained by integrating the double differential cross-section per neutron

\[
\frac{d^2\sigma(E_{\nu})}{d\Omega_{\nu} d\omega} = \frac{G_F^2 E_{\nu}^2}{16\pi^2} \left[ (1 + \cos \theta) R^V(q, \omega) + g_A^2 (3 - \cos \theta) R^A(q, \omega) \right],
\]

where \( G_F \) is the weak coupling constant, \( g_A = 1.255 \) is the axial charge of the nucleon, \( E_{\nu} \) and \( E_{\nu}' \) are the incoming and outgoing neutrino energies, \( \theta \) the scattering angle between the incoming and outgoing neutrino momenta and \( \omega = E_{\nu} - E_{\nu}' \) and \( \mathbf{q} = \mathbf{k} - \mathbf{k}' \) the energy and the momentum transfer in the reaction. The response functions \( R^V(q, \omega) \) and \( R^A(q, \omega) \) describe the response of the system to density fluctuations \( (S = 0) \) and spin fluctuations \( (S = 1) \) related to the coupling of neutrino to vector and axial currents of the neutron. These response functions are defined as

\[
R^V(q, \omega) = -\frac{1}{\pi \rho} \text{Im} \chi^{(S=0)}(q, \omega),
\]

\[
R^A(q, \omega) = -\frac{1}{\pi \rho} \text{Im} \chi^{(S=1)}(q, \omega).
\]

When tensor forces are considered the spin response is splitted into two components, i.e. the spin longitudinal response

\[
R^V_L(q, \omega) = -\frac{1}{\pi \rho} \text{Im} \chi^{(S=1, M=0)}(q, \omega)
\]

and the spin transverse response

\[
R^A_T(q, \omega) = -\frac{1}{\pi \rho} \text{Im} \chi^{(S=1, M=\pm 1)}(q, \omega).
\]

These responses can be considerably different one of the other and compared to the case without tensor interaction. This has important consequences on the neutrino cross sections and the neutrino mean free path.

![FIG. 7: (Color online) Axial spin transverse (AT), axial spin longitudinal (AL) and vector (V) contributions to the cross section per neutron for the reaction \( \nu + n \rightarrow \nu' + n' \) in the neutron matter. Three different densities as well three different interactions are considered. The Fermi gas result is also shown.](image)

In the presence of tensor interaction the double differential cross-section per neutron for neutral current reaction is given by

\[
\frac{d^2\sigma(E_{\nu})}{d\Omega_{\nu} d\omega} = \frac{G_F^2 E_{\nu}^2}{16\pi^2} \left\{ (1 + \cos \theta) R^V + g_A^2 \left[ 2 \left( E_{\nu}' \cos \theta - E_{\nu} \right) (E_{\nu}' - E_{\nu} \cos \theta) \right] + 1 - \cos \theta \right\} R^L_L
\]

\[
+ g_A^2 \left[ \frac{E_{\nu} E_{\nu}'}{q^2} \sin^2 \theta + 1 - \cos \theta \right] R^T_T .
\]

This expression reduces to Eq. (17) when \( R^L_L = R^A_T = R^A \) as one can easily observe remembering that for neutral current

\[
q^2 = (\mathbf{k} - \mathbf{k}')^2 = k^2 + k'^2 - 2 \mathbf{k} \cdot \mathbf{k}' = E_{\nu}^2 + E_{\nu}'^2 - 2 E_{\nu} E_{\nu}' \cos \theta.
\]

\[
q = \frac{E_{\nu} - E_{\nu}'}{\sin \theta} = \frac{(k - k')^2}{\sin \theta},
\]

\[
(\cos \theta)^2 = \left( \frac{E_{\nu} - E_{\nu}'}{E_{\nu} + E_{\nu}'} \right)^2 /
\]

\[
\sin \theta = \left( 1 - \left( \frac{E_{\nu} - E_{\nu}'}{E_{\nu} + E_{\nu}'} \right)^2 \right)^{1/2} = \left( 1 - \frac{(k - k')^2}{(E_{\nu} + E_{\nu}')^2} \right)^{1/2}.
\]
In Eq. (21), as well as in Eq. (17), we neglect corrections of order $\hbar^2$ from weak magnetism and other effects $^{39}$ like the finite size of the nucleon or nucleon excitations $^{40}$. A generalization of Eq. (21) taking into account of all these effects can be found for example in Ref. $^{38}$. As already stressed in Ref. $^{38}$ (and illustrated in Ref. $^{41}$ for charged current reaction) the cross section is dominated by the spin transverse response $R_{1t}$. In Fig. 7 we present the relative axial spin transverse, axial spin longitudinal and vector contributions to the neutral current cross section. We consider four different cases. The first is the Fermi gas. In this case $R_{1t} = R_{1s} = R^A$ so the difference between the three contributions is only due to the kinematical factors and the coupling constants multiplying the response functions. Second we consider the case when the response functions are calculated with the SLy5 force. In this case the coupling constants $C^F = C^V = 0$, purely related with the tensor part of the interaction, do not contribute. A possible difference between $R_{1t}^A$ and $R_{1s}^A$ is due to the spin orbit contribution and according to the Eqs. (35) and (37) given in Appendix B the $\chi^4$ factor reduces these differences at low momentum transfer. Finally we consider the T13 and T16 parameterizations of tensor contributions. The results for three different densities, $\rho = 0.08, 0.16$ and $0.24$ fm$^{-3}$ are also shown on Fig. 7. As it clearly appears all the cross sections are dominated by the spin transverse response but this contribution may vary between $\sim 60\%$ and $\sim 90\%$ depending on the interactions and densities considered. It reflects the possible quenching, enhancement or divergence of the nuclear response functions. This behavior was already illustrated in the section 4.1 in connection with the Fig. 1. To complete our discussion we plot in Fig. 8 the spin transverse response for $q = 0.05$ fm$^{-1}$ and $q = 0.5$ fm$^{-1}$, for $\rho = 0.16$ fm$^{-3}$ and $0.24$ fm$^{-3}$ and for the tensor forces T13, T43 and T63. A collective spin zero sound characterizes the $S = 1, M = \pm 1$ response for T13 at $q = 0.05$ fm$^{-1}$ for $\rho = 0.16$ fm$^{-3}$ and $0.24$ fm$^{-3}$. A similar behaviour, with the corresponding quenching of the p-h continuum characterizes the $S = 1, M = \pm 1$ responses for SLy5 and T16 as shown on Fig. 9 for $\rho = 0.08$ fm$^{-3}$ and $0.16$ fm$^{-3}$. The T63 force on the other hand is characterized by an enhancement of the response at low $\omega$ for $\rho = 0.16$ fm$^{-3}$. At $q = 0.5$ fm$^{-1}$ this enhancement seems to be critical. At $\rho = 0.24$ fm$^{-3}$ the enhancement of the T16 response no longer holds. In this case this response is suppressed with respect to the corresponding Hartree-Fock response. An enhancement with respect to the HF and FG case for small $\omega$ characterizes for this density the T43 force.

These different behaviors affect obviously the neutrino mean free path. We have calculated it for T11-T16 and T13-T63 chains of parameterizations of Skyrme tensor interactions for a neutrino energy $5$ MeV $< E_{\nu} < 40$ MeV and for three values of densities, i.e. $\rho = 0.08, 0.16$ and $0.24$ fm$^{-3}$. Note that when a spin zero-sound collective mode appears one must in principle include it in the calculation of the neutrino mean free path. For this mode the response function reduces to a delta distribution. Nevertheless, as already observed in $^{21}$, the collective mode itself gives little scattering, its contribution is negligible when calculated in the Landau approximation. It was also observed in $^{23}$ that for all the Skyrme forces there considered, this magnon rapidly disappears with the temperature because of a strong Landau damping. Hence we do not explicitly compute the spin zero sound contribution. Its effect is on the other hand present as a suppression of the corresponding p-h continuum and has a consequent on the cross section.

The results for the neutrino mean free path are reported in Fig. 9 for the T11-T16 chain and in Fig. 10 for the T13-T63 chain. In each figure we include the Fermi gas result and the calculation with the SLy5 force which does not have tensor terms. From Figs. 9 and 10 clearly emerges the crucial dependence of the neutrino mean free path on the parameterizations of the Skyrme tensor terms. For $\rho = 0.08$ fm$^{-3}$ the different parameterizations give quite similar results and higher with respect to the non interacting case. Already at $\rho = 0.16$ fm$^{-3}$ the spread becomes important. For some cases $\lambda$ is higher than the Fermi gas one, for other is lower. In many cases it is lower than the corresponding SLy5 result. Also the behavior of $\lambda$ with the density is not so trivial, as already appears in the three panels of Figs. 9 and 10. This is clearly shown on Fig. 11 where the ratio $\lambda/\lambda_{\text{FG}}$ is plotted. For example for T13 this quantity stays quite con-
In this article we have calculated the RPA response function for pure neutron matter considering Skyrme energy density functionals including tensor terms. This article parallels with [1, 2] where similar calculations were performed for the symmetric nuclear matter. As in previous articles divergences and instabilities of the response [2, 42], in particular in the $S = 1$ channel, are discussed in connection with the sum rules.

We applied our results to the study of the neutrino mean free path. The advantage of the present framework is that it allows to describe nuclear (and neutron) matter equation of state and the neutrino mean free path simultaneously, hence in a self-consistent way. Obviously, before to achieve reliable description of neutrino transport phenomena in neutron stars, the calculations performed here must be generalized at finite temperature and for asymmetric nuclear matter. Nonetheless, already at this oversimplified level (pure neutron matter at zero temperature) we have shown the strong dependence of the neutrino mean free path on the tensor term parameterizations. It represents an important reason, among others, to an accurate treatment of Skyrme functionals including tensor contribution.

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Appendix A: Particle-hole matrix elements in presence of a zero range tensor interaction.

Following the notation adopted in article I and II, we give in Table I the values of the particle-hole residual interaction for the tensor part of the functional in terms of the $B^x$, with $x = \Delta s, F, \ldots$, coefficients of the functional. This particular notation has been already discussed in article II and we refer to it for detailed explanations.

TABLE I: Contribution of the EDF tensor part to the residual interaction in terms of the $B^x$ coupling constants. For the sake of simplicity we have introduced $K_{ij} = [(k_{12})_i(k_{12})_j]$, where $(k_{12})_M$ is defined in Eq.(9) of article I. The term $\delta_{S_0} \delta_{S_1}$ is implicit everywhere.

| $M'$ = 1 | $M'$ = 0 | $M'$ = −1 |
|----------|----------|----------|
| $M$ = 1  | −2$q^2 (B^T + 4B^\Delta s)$ + 4 $B^T K_{0,0} - 4 (2B^T + B^F) K_{1,1}$ | −4 $B^F K_{1,0}$ | −4 $B^F K_{1,1}$ |
| $M$ = 0  | 4 $B^F K_{0,1}$ | −2$q^2 (B^T - 4B^s + 4B^\Delta s + B^F)$ + 4 $(B^T + B^F) K_{0,0} - 8B^F K_{1,1}$ | 4 $B^F K_{1,0}$ |
| $M$ = −1 | −4$B^F K_{1,1}$ | −4$B^F K_{1,0}$ | −2$q^2 (B^T + 4B^\Delta s)$ + 4 $B^F K_{0,0} - 4 (2B^T + B^F) K_{1,1}$ |

Appendix B: Response functions

This appendix contains the explicit expressions for the response functions for pure neutron matter. Since the isospin is no longer a relevant quantum number, each channel is denoted as $(1, M)$ for $S = 1$ or only $(0)$ for $S = 0$.

We have

- for the $S = 0$ channel

\[
\frac{\chi_{HF}}{\chi_{RP}}(0) = 1 - \tilde{W}_{1}^{(0)} \chi_0 + W_{2}^{(0)} \left( \frac{2}{2} \chi_0 - 2k_F^2 \chi_2 \right) + [W_{2}^{(0)}]^2 k_F^2 \left[ 2 - \chi_0 \chi_4 + \left( \frac{m^* \omega}{k_F^2} \right)^2 \chi_0 - \frac{m^* \omega^2}{6 \pi^2 k_F^2 q^2} \chi_0 \right] + 2 \chi_0 \left( \frac{m^* \omega}{q} \right)^2 \frac{W_{2}^{(0)}}{1 - \frac{m^* k_F^2}{3 \pi^2 \omega} W_{2}^{(0)}}, \tag{B1}
\]

- for the $S = 1$ channels

\[
\frac{\chi_{HF}}{\chi_{RP}}(1) = \left( 1 + \frac{k_F^3 m^* C^F}{3 \pi^2} \right)^2 + \tilde{W}_{1}^{(1)} \chi_0 + W_{2}^{(1)} \left[ \frac{2 C^F k_F^3 m^*}{3 \pi^2} \chi_0 - \frac{2k_F^2}{k_F^2 \chi_2 + 2k_F^3 m^* C^F}{3 \pi^2} \chi_0 - \frac{2k_F^3 m^* C^F}{3 \pi^2} \chi_0 \right] + [W_{2}^{(1)}]^2 \left[ k_F^4 \chi_2 - k_F^2 \chi_0 \chi_4 + m^* \omega^2 \chi_0 - \frac{k_F^3 m^* q^2}{6 \pi^2} \chi_0 \right] + \frac{2 m^* \omega^2 (W_{2}^{(1)} + 2 C^F)}{q^2} \left[ 1 + \frac{k_F^3 m^*}{3 \pi^2} \chi(1.0) - W_{2}^{(1)} - 2 C^F \right] \chi_0, \tag{B2}
\]
and

\[
\frac{\chi_{HF}}{\chi_{RPA}} = \left[ 1 - C_F^* \frac{m^* k_F^5}{6 \pi^2} \right]^2 - \tilde{W}_1^{(1, \pm 1)} \chi_0 \\
\quad + \left[ W_2^{(1)} + C_F \right] \left\{ \frac{q^2}{2} \chi_0 + \left[ 1 - C_F^* \frac{m^* k_F^5}{3 \pi^2} \right] - 2k_F^2 \chi_2 - C_F^* \frac{m^* k_F^5}{3 \pi^2} (\chi_0 - \chi_2) \right\} \\
\quad + \left[ W_2^{(1)} + C_F \right] k_F^4 \chi_2 - \chi_0 \chi_4 + \left( \frac{m^* \omega}{k_F^2} \right)^2 \chi_0 \chi_2 \frac{m^*}{2 \pi^2} q^2 \chi_0 \\
\quad + 2 \chi_0 \left( \frac{m^* \omega}{q} \right)^2 \frac{W_2^{(1)} \left[ 1 + \frac{m^* k_F^5}{3 \pi^2} X^{(1, \pm 1)} / 2 \right]}{1 - \frac{m^* k_F^5}{3 \pi^2} \left[ W_2^{(1)} - \frac{X^{(1, \pm 1)}}{2} \right]}. \tag{B3}
\]

The coefficients \( W_{i=1,2}^{(S)} \) are defined as

\[
\frac{1}{2} W_1^{(0)} = 2( C_0^{s,0} + C_1^{s,0} ) + (2 + \gamma)(1 + \gamma)(C_0^{s\gamma,0} + C_1^{s\gamma,0}) \rho_1 - \left( 2C_0^{s\Delta,0} + 2C_1^{s\Delta,0} + \frac{1}{2} C_0^T + \frac{1}{2} C_1^T \right) q^2, \\
\frac{1}{2} W_1^{(1)} = 2( C_0^{s,0} + C_1^{s,0} \rho_1^3 ) + C_1^{s,0} C_1^{s\gamma,0} - \left( C_0^{s\Delta,0} + 2C_1^{s\Delta,0} + \frac{1}{2} C_0^T + \frac{1}{2} C_1^T \right) q^2, \\
\frac{1}{2} W_2^{(0)} = C_0^T + C_1^T, \\
\frac{1}{2} W_2^{(1)} = C_0^T + C_1^T. \tag{B4}
\]

We have also defined the \( \tilde{W}_1^{(S,M)} \) and \( X^{(1,M)} \) coefficients as

\[
\tilde{W}_1^{(0,0)} = W_1^{(0)} + 4q^4 \left[ C^{\nabla J} \right] \frac{2}{1 + \frac{q^2}{\beta_2 - \beta_3} \left[ W_2^{(1)} - C_F \right]}, \tag{B5}
\]

\[
\tilde{W}_1^{(1,0)} = - \left[ W_1^{(1)} + 4q^2 C^{\nabla s} \right] + C_F \left[ q^2 - 4 \left( \frac{m^* \omega}{q} \right)^2 \right] + m^* \rho \left[ C_F^2 \left[ 2k_F^2 + \frac{1}{2} q^2 - \frac{2}{k_F^2} \left( \frac{m^* \omega}{q} \right)^2 \right] \right], \tag{B6}
\]

\[
\tilde{W}_1^{(1, \pm 1)} = W_1^{(1)} + 2q^4 \left[ C^{\nabla J} \right] \frac{2}{1 + \frac{q^2}{\beta_2 - \beta_3} \left[ W_2^{(0)} \right]} - 2C_F \left( \frac{m^* \omega}{q} \right)^2 \\
\quad + \left[ C_F^2 \right] \left[ \frac{1}{2} q^2 m^* \rho + \frac{1}{16} \left[ q^2 - 4 \left( \frac{m^* \omega}{q} \right)^2 \right] \chi_0 - \frac{1}{2} k_F^2 \left[ q^2 + 4 \left( \frac{m^* \omega}{q} \right)^2 \right] \chi_2 + k_F^2 \chi_4 \right]. \tag{B7}
\]

\[
X^{(1,0)} = 2q^2 \left[ C_F^2 \right] \frac{2}{1 + \frac{q^2}{\beta_2 - \beta_3} \left[ W_2^{(1)} + 3C_F \right]}, \tag{B8}
\]

\[
X^{(1, \pm 1)} = 2q^2 \left[ C_F^2 \right] \frac{2}{1 + \frac{q^2}{\beta_2 - \beta_3} W_2^{(1)}}. \tag{B9}
\]

**Appendix C: Landau approximation**

Since we have no isospin quantum number, the \( ph \) interaction is reduced to three terms. As done in article II we take the limit \( q \to 0 \) and \( q_1,2 \to k_F \) of the second functional derivative and we obtain

\[
V_{ph}^{\text{Landau}}(k_F, k_F) = \frac{1}{2} W_1^{(0)} + \frac{1}{2} W_1^{(1)} \mathbf{\sigma}_a \cdot \mathbf{\sigma}_b + \frac{1}{2} \left[ W_2^{(0)} + W_2^{(1)} \right] \mathbf{\sigma}_a \cdot \mathbf{\sigma}_b \left[ 2k_F^2 \left[ 1 - \cos \theta \right] + \frac{2}{3} \frac{k_F^2 C_F}{k_F^2 + \frac{3}{k_F^2} S_{ab}} \right]. \tag{C1}
\]
where the symbols $S_{ab}$ has been defined in refs. \[43\] \[44\]. The various $W_{i,L}^{(S)}$ coefficients with $i = 1, 2$ can be easily calculated from the $W^{(S)}$ coefficients defined in Eq. \[43\] taking the limit of $W_{i}^{(S)}$ for $q \to 0$.

So the Landau parameters can be written as

\[
N_0^{-1} F_0 = \frac{1}{2} W_{1,L}^{(0)} + k_F^2 W_{2,L}^{(0)},
\]
\[
N_0^{-1} F_1 = -k_F^2 W_{2,L}^{(0)},
\]
\[
N_0^{-1} G_0 = \frac{1}{2} W_{1,L}^{(1)} + k_F^2 W_{2,L}^{(1)} + \frac{3}{2} k_F^2 C^F,
\]
\[
N_0^{-1} G_1 = -k_F^2 W_{2,L}^{(1)} + \frac{3}{2} k_F^2 C^F,
\]
\[
N_0^{-1} H_0 = \frac{1}{2} k_F^2 C^F,
\]

where $N_0^{-1} = \frac{2m^2}{g_{a} a_{N}}$ is the normalization factor and $g = 2$ is the degeneracy in PNM.

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