The Solar Neutrino Problem in the Light of a Violation of the Equivalence Principle

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We have found that long-wavelength neutrino oscillations induced by a tiny breakdown of the weak equivalence principle of general relativity can provide a viable solution to the solar neutrino problem.

1. INTRODUCTION

Neutrinos have had, since their childhood in the early 30’s, profound consequences on our understanding of the forces of nature. In the past they led to the discovery of neutral currents and provided the first indication in favour of the standard model of electroweak interaction. They may be today at the very heart of yet another breakthrough in our perceptions of the physical world.

Today the results coming from solar neutrino experiments \cite{1-4} as well as from atmospheric neutrino experiments \cite{5} are difficult to be understood without admitting neutrino flavour conversion. Nevertheless the dynamics underlying such conversion is yet to be established and in particular does not have to be a priori related to the electroweak force.

The interesting idea that gravitational forces may induce neutrino mixing and flavour oscillations, if the weak equivalence principle of general relativity is violated, was proposed about a decade ago \cite{6,7}, and thereafter, many works have been performed on this subject \cite{8}.

Many authors have investigated the possibility of solving the solar neutrino problem (SNP) by such gravitationally induced neutrino oscillations \cite{9-11}, generally finding it necessary, in this context, to invoke the MSW like resonance \cite{7} since they conclude that it is impossible that this type of long-wavelength vacuum oscillation could explain the specific energy dependence of the data \cite{9,10}. Nevertheless we demonstrate that all the recent solar neutrino data coming from gallium, chlorine and water Cherenkov detectors can be well accounted for by long-wavelength neutrino oscillations induced by a violation of the equivalence principle (VEP).

2. THE VEP FORMALISM

We assume that neutrinos of different types will suffer different time delay due to the weak, static gravitational field in the space on their way from the Sun to the Earth. Their motion in this gravitational field can be appropriately described by the parameterized post-Newtonian formalism with a different parameter for each neutrino type. Neutrinos that are weak interaction eigenstates and neutrinos that are gravity eigenstates will be related by a unitary transformation that can be parameterized, assuming only two neutrino flavours, by a single parameter, the mixing angle $\theta_G$ which can lead to flavour oscillation \cite{6}.

In this work we assume oscillations only between two species of neutrinos, which are degenerate in mass, either between active and active ($\nu_e \leftrightarrow \nu_\mu, \nu_\tau$) or active and sterile ($\nu_e \leftrightarrow \nu_s, \nu_s$ being an electroweak singlet) neutrinos.

The evolution equation for neutrino flavours $\alpha$ and $\beta$ propagating through the gravitational potential $\phi(r)$ in the absence of matter can be found in Ref. \cite{12}. In the case we take $\phi$ to be a constant, this can be analytically solved to give the

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survival probability of $\nu_e$ produced in the Sun after travelling the distance $L$ to the Earth

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_G \sin^2 \frac{\pi L}{\lambda},$$

(1)

where the oscillation wavelength $\lambda$ for a neutrino with energy $E$ is given by

$$\lambda = \left[ \frac{\pi \text{ km}}{5.07} \right] \left[ \frac{10^{-15}}{|\phi \Delta \gamma|} \right] \left[ \frac{\text{MeV}}{E} \right],$$

(2)

which in contrast to the wavelength for mass induced neutrino oscillations in vacuum, is inversely proportional to the neutrino energy. Here $\Delta \gamma$ is the quantity which measures the magnitude of VEP.

3. ANALYSIS

We will discuss here our analysis and results for active to active conversion. The same analysis for the $\nu_e \rightarrow \nu_e$ channel can be found in Ref. [12], given similar results.

In order to examine the observed solar neutrino rates in the VEP framework we have calculated the theoretical predictions for gallium, chlorine and Super-Kamiokande (SK) water Cherenkov solar neutrino experiments, as a function of the two VEP parameters ($\sin^2 2\theta_G$ and $|\phi \Delta \gamma|$), using the solar neutrino fluxes predicted by the Standard Solar Model by Bahcall and Pinsonneault (BP98) [13] taking into account the eccentricity of the Earth orbit around the Sun.

We do a $\chi^2$ analysis to fit these parameters and an extra normalization factor $f_B$ for the $^8\text{B}$ neutrino flux, to the most recent experimental results coming from Homestake [1], R\text{C}l = 2.56 ± 0.21 SNU, GALLEX [3] and SAGE [2] combined $R_{\text{Ga}} = 72.5 \pm 5.5$ SNU and SK [4] $R_{\text{SK}} = 0.475 \pm 0.015$ normalized to BP98. We use the same definition of the $\chi^2$ function to be minimized as in Ref. [4], except that our theoretical estimations were computed by convoluting the survival probability given in Eq. (1) with the absorption cross sections [13], the neutrino-electron elastic scattering cross section with radiative corrections [16] and the solar neutrino flux corresponding to each reaction, $pp$, $pep$, $^7\text{Be}$, $^8\text{B}$, $^{13}\text{N}$ and $^{15}\text{O}$; other minor sources were neglected.

We present in Fig. 1 (a) the allowed region determined only by the rates with free $f_B$, for fixed $^8\text{B}$ flux ($f_B = 1$) the allowed region is similar. In Ref. [12] one can find a table which gives more details on best fitted parameters as well as the $\chi^2_{\text{min}}$ values for fixed and free $f_B$. We found for $f_B = 1$ that $\chi^2_{\text{min}} = 1.49$ for 3-2=1 degree of freedom and for $f_B = 0.81$ that $\chi^2_{\text{min}} = 0.32$ for 3-3=0 degree of freedom.

We then perform a spectral shape analysis fitting the $^8\text{B}$ spectrum measured by SK [4] using the following $\chi^2$ definition:

$$\chi^2 = \sum_i \left[ \frac{S_{\text{obs}}(E_i) - f_B S_{\text{theo}}(E_i)}{\sigma_i} \right]^2,$$  

(3)

where the sum is performed over all the 18 experimental points $S_{\text{obs}}(E_i)$ normalized by BP98 prediction for the recoil-electron energy $E_i$, $\sigma_i$ is the total experimental error and $S_{\text{theo}}$ is our theoretical prediction that was calculated using the BP98 $^8\text{B}$ differential flux, the $\nu-e$ scattering cross section [16], the survival probability as given by Eq. (1) taking into account the eccentricity as we did for the rates, the experimental energy resolution as in Ref. [17] and the detection efficiency as...
a step function with threshold $E_{\text{th}} = 5.5$ MeV.

After the $\chi^2$ minimization with $f_B = 0.8$ we have obtained $\chi^2_{\text{min}} = 15.8$ for 18-3 = 15 degree of freedom. The best fitted parameters that also can be found in Ref. [12] permit us to compute the allowed region displayed in Fig. 1 (b).

Finally, we perform a combined fit of the rates and the spectrum obtaining the allowed region presented in Fig. 1 (c). The combined allowed region is essentially the same as the one obtained by the rates alone. In all cases presented in Figs. 1 (a)-(c) we have two isolated islands of 90% C.L. allowed regions. See Ref. [12] for a table with the best fitted parameters for this global fit as well as for the fitted values corresponding to the local minimum in these islands. Note that only the upper corner of the Fig. 1 (c), for $|\phi \Delta \gamma| > 2 \times 10^{-23}$ and maximal mixing in the $\nu_e \rightarrow \nu_\mu$ channel can be excluded by CCFR [18]. Moreover, there are no restrictions in the range of parameters we have considered in the case of $\nu_e \rightarrow \nu_\tau, \nu_s$ oscillations.

4. DISCUSSIONS AND CONCLUSIONS

Let’s finally remark that, in contrast to the usual vacuum oscillation solution to the SNP, in this VEP scenario no strong seasonal effect is expected in any of the present or future experiments, even the ones that will be sensitive to $^7$Be neutrinos [18,21]. Contrary to the usual vacuum oscillation case, the oscillation length for the low energy pp and $^7$Be neutrinos are very large, comparable to or only a few times smaller than the Sun-Earth distance, so that the effect of the eccentricity in the oscillation probability is small. On the other hand, for higher energy neutrinos relevant for SK, the effect of the eccentricity in the probability could be large, but it is averaged out after the integration over a certain neutrino energy range. These observations are confirmed by Fig. 4 of Ref. [12].

We have found a new solution to the SNP which is comparable in quality of the fit to the other suggested ones and can, in principle, be discriminated from them in the near future. In fact a very-long-baseline neutrino experiment in a $\mu$-collider [21] could directly probe the entire parameter region where this solution was found.

ACKNOWLEDGMENTS

We thank P. Krastev, E. Lisi, G. Matsas, H. Minakata, M. Smy, P. de Holanda and GEFAN for valuable discussions and comments. H.N. thanks W. Haxton and B. Balantekin and the Institute for Nuclear Theory at the University of Washington for their hospitality and the Department of Energy for partial support during the final stage of this work. This work was supported by the Brazilian funding agencies FAPESP and CNPq.

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