NUMERICAL INVESTIGATION OF CATTANEO-CHRISTOV HEAT FLUX IN CNT SUSPENDED NANOFUID FLOW OVER A STRETCHING POROUS SURFACE WITH SUCTION AND INJECTION

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Abstract. The present study analyzes the heat energy transfer in nano fluids flow through the porous stretching surface. Cattaneo-Christov heat flux model is employed to study the heat energy transfer. Darcy law is used to discuss the flow characteristics over the different types of permeable sheets with suction and injection. Nanofluids is considered as water based single-walled carbon nanotubes (SWNTs) and multi-walled carbon nanotubes (MWNTs) nanofluids. A comparative study for SWCNT and MWCNT is also made. Governing equations are transformed into set of ordinary differential equations using similarity transformations. The computational results are obtained by using Runge-Kutta fourth order method along with shooting technique. Numerical and graphical results are presented to discuss the effects of various physical parameters on velocity profile, temperature profile, Nusselt number, Sherwood number and skin friction coefficient for different type of nanoparticles for suction and injection cases. Stream lines and isotherms are also plotted for three different cases viz. permeable sheet with suction, impermeable sheet and permeable sheet with injection. A comparative analysis with existing results is tabulated which validate that the numerical results of present study have good correlation with existing results. The outcomes of the results show that skin friction coefficient is more for SWCNT in comparison of MWCNT and the boundary layer thickness is maximum for permeable stretching sheet with suction parameter.

1. Introduction. The study of exchange of thermal energy between two objects or physical systems is very important in physical transport problems and other science and engineering problems. The temperature of the systems and properties of the intervening medium play important role in rate of heat transfer. Conduction, convection and radiation are the three fundamental modes of heat transfer which are defined by classical Fouriers law. Maxwell-Cattaneos model \cite{10} is the extended form of classical Fouriers law in which thermal relaxation time is taken into

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consideration. In this model, Oldroyd's upper-convected derivative in order to preserve the material-invariant formulation is employed. This model is now known as Cattaneo-Christov heat flux model. Straughan [35] extended for natural convection in horizontal layer of incompressible Newtonian fluid. He concluded that convection cells go narrow with increasing the Cattaneo number. Ciarletta and Straughan [11] solved for unique solution of the Cattaneo-Christov equations and discussed the structural stability. Tibullo and Zampoli [39] investigated the Cattaneo-Christov flux model for incompressible fluid and discussed the uniqueness result. Ciarletta et al. [12] explored the Christov-Morro theory for non-isothermal diffusion. Han et al. [17] studied viscoelastic fluid flow with Cattaneo-Christov heat flux model and used homotopy analysis method (HAM) also validated the results with Finite Difference Method (FDM) results. Haddad [15] presented Cattaneo-Christov model to study the thermal instabilities in fluid saturating a porous medium while Mustafa [27] analyzed the thermal relaxation of fluid flow over rotating frame with Cattaneo-Christov heat flux model. Recently, Khan and Khan [24] numerically investigated the Cattaneo-Christov heat flux in three-dimensional flow and heat transfer of burgers fluid; Hayat et al. [20] studied the Cattaneo-Christov heat flux in MHD flow of Oldroyd-B fluid with homogeneous-heterogeneous reactions; Khan et al. [25] further extended the chemical processes on 3D Burgers fluid utilizing Cattaneo-Christov double-diffusion; Salahuddin et al. [32] carried out the Cattaneo-Christov heat flux model in MHD flow of Williamson fluid over a stretching sheet with variable thickness; Liu et al. [26] reported fractional Cattaneo-Christov flux anomalous convection diffusion and wave coupling transport of cells on comb frame.

The study of heat transfer and heat flux in nanofluids flow has recently attracted more and more attention due to much enhancement of thermal conductivity of base fluids. Nanofluid flows has also wide range of science and technology applications such are: biomedical and processing system engineering such as biological organisms on their primary cellular level, snapping shrimps and super-hydrophobic beetle wings, drug delivery, photodynamic therapy, use of charged polymers for lubrication, the lotus effect for self-cleaning surfaces, molecular motors, neuro electronic interfaces, membranes for filtering on size or charge (e.g., for desalination) and nanoporous materials for size exclusion chromatography, cancer diagnosis and therapy, surgery, in vivo therapy, passive selective transport in aquaporin and active transport in the ion channels, neuro electronic interfaces, cell repair machines, protein engineering, shedding new light on cells molecular motors like kinesis and charge-based filtration in the kidney basal membrane etc. CNT nanofluids are the nanofluids where carbon nanotubes are suspended in base fluids, an interesting review on thermal features of carbon nanotubes-based nanofluids has been presented by Murshed et al. [34]. They concluded that superior thermal properties enhance with increasing the concentration of CNT as well as fluid temperature. The future scope of this field in terms of “ionanofluids” is also reported. Another interesting work on thermal conductivity of CNT nanofluids are presented by Estellé et al. [14].

Motivated from the huge application and utilities of CNT nanofluids, many investigations [16, 1, 31, 9, 38, 2, 3] on CNT nanofluids with various flow regimes, various approximations and various numerical approaches, are incorporated in literature.

In boundary layer theory, the boundary layer flow over different flow geometries has many engineering and industrial applications in which the boundary layer flow over stretching sheets is mainly applicable in extrusion of polymer sheet from a die. The combined study of nanofluids and boundary layer flow over stretching sheets
are important in the industrial manufacturing processes. Some interesting combined works [18, 4, 19, 28, 13] are recently reported to discuss the convective heat transfer, effect of variable thermal conductivity and thermal radiation, effects of homogeneous and heterogeneous reactions, Stagnation Point Flow, natural convection and magnetic field effects. In boundary layer theory, suction and injection are also key parameters to influence the flow. With the effects of suction and injection in study of nanofluids flow are presented in Refs. [33, 29, 36]. In this direction, some more generalized work with non-Newtonian nanofluids like Williamson nanofluid, Carreau nanofluid, eyring-powell nanofluid and Casson nanofluid have been reported in Refs.[6, 7, 8, 30].

In spite of the considerable importance of the Cattaneo-Christov heat flux model in physical and engineering applications, current model presents the magnetic field effects on the flow of Cattanneo-Christov heat flux model for water based CNT suspended nanofluid over a porous stretching sheet. Because according to the authors knowledge idea of Cattaneo-Christov heat flux model for water based CNT suspended nanofluid is not explored so far for porous stretching sheet. The flow equations are modeled first time in literature transformed into ordinary differential equations using similarity transformations. The numerical solutions are computed using shooting technique and compared with the existing results in literature for the special case of pure fluid flow and found to be in good agreement.

2. Mathematical formulation of physical model. We consider the two-dimensional nanofluid flow over a porous stretching sheet with water based nanofluids encompassing single- and multi-wall CNTs. The flow is presumed to be laminar, steady, and incompressible. The base fluid and the CNTs are expected to be in updraft equilibrium. Sheet is assumed to be stretched with the different velocity $U_w$, $V_w$ along the x-axis and axis, respectively. We consider a constant ambient temperature $T_\infty$. Further new heat model named as Cattanneo-Christov heat flux model is employed to analyze the heat transfer phenomena. The x-axis is taken along the sheet and y-axis is chosen normal to it. The geometry of boundary layer flow regime is depicted in Fig. 1.

![Figure 1. Schematic representation of SWCNT and MWCNT nanofluids flow over porous stretching Surface.](image)

With the above assumptions, the governing equations for boundary layer flow i.e. continuity, momentum and energy equations can be written as follows:
\[
\n\nabla \cdot \mathbf{V} = 0, \quad (2.1)
\]
\[
V(\nabla \cdot \mathbf{V}) = V \nabla^2 V - \frac{V_{nf}}{k} \mathbf{V}, \quad (2.2)
\]
\[
\rho_{nf} (c_p)_{nf} \mathbf{V} \cdot \nabla T = -\nabla \cdot \mathbf{q}, \quad (2.3)
\]

where \( \mathbf{V} = (u, v) \) is velocity vector, \( T \) is the temperature of the fluid. And \( \mathbf{q} \) is the heat flux for Cattaneo-Christov flux model which is expressed as Salahuddin et al.[32]:

\[
\mathbf{q} + \lambda_2 (\mathbf{V} \cdot \nabla \mathbf{q} - \mathbf{q} \cdot \nabla \mathbf{V} + (\nabla \cdot \mathbf{V}) \mathbf{q}) = -K_{nf} \nabla T, \quad (2.4)
\]

where \( \lambda_2 \) is the thermal relaxation time. Eliminating \( \mathbf{q} \) from Eqs.(2.3) and (2.4), we get

\[
\left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + \lambda_2 \left( u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial y} + 2uv \frac{\partial^2 T}{\partial x \partial y} + u^2 \frac{\partial^2 T}{\partial x^2} + v^2 \frac{\partial^2 T}{\partial y^2} \right)
\]  
\[
= \frac{K_{nf}}{\rho_{nf} (c_p)_{nf}} \frac{\partial^2 T}{\partial y^2}, \quad (2.5)
\]

Further, \( \rho_{nf} \) is the effective density, \( \mu_{nf} \) is the effective dynamic viscosity, \((c_p)_{nf}\) is the heat capacitance, \( \alpha_{nf} \) is the effective thermal diffusibility, and \( k_{nf} \) is the effective thermal conductivity of the nanofluid, which are defined as (Ref. [37]):

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{CNT}, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^2}, \quad \alpha_{nf} = \frac{k_{nf}}{(c_p)_{nf}}, \quad (2.6a)
\]
\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi (\rho C_p)_{CNT}, \quad (2.6b)
\]
\[
k_{nf} = 1 + \frac{1}{3} \frac{k_{CNT} \phi r_f}{k_f (1 - \phi) r_{CNT}}, \quad (2.6c)
\]

where \( \mu_f \) is the viscosity of base fluid, \( \phi \) is the nanoparticles fraction, \((\rho C_p)_f\) is the effective heat capacity of a fluid, \((\rho C_p)_{CNT}\) is the effective heat capacity of a carbon nanotubes, \( k_f \) and \( k_{CNT} \) are the thermal conductivities of the base fluid and carbon nanotubes, respectively, \( \rho_f \) and \( \rho_{CNT} \) are the thermal conductivities of the base fluid and carbon nanotubes, respectively.

The following boundary conditions are employed:

\[
u = cx, \quad v = 0, \quad T = T_w, \quad \text{at} \ y = 0 \quad (2.7a)
\]
\[
u \to 0, \quad T \to T_\infty, \quad \text{at} \ y = 0, \quad (2.7b)
\]

where \( T \) and \( T_w \) are the ambient and wall fluid temperature respectively.

The partial differential equations (PDE) which govern the fluid flow, are nonlinear and is also difficult to obtain the solution. Therefore, we use transformations, called similarity transformations to transform the partial differential equations into ordinary differential equations. The following similarity transformations are introduced:

\[
\eta = \sqrt{\frac{\alpha}{v_f^2}} y, \quad u = ax f'(\eta), \quad v = -\sqrt{\alpha v_f^2} f'(\eta), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}. \quad (2.8)
\]
Making use of Eqs. (2.6-2.8) in Eqs.(2.1-2.5), we have
\begin{equation}
\frac{f'''}{f} + (1 - \varphi)^{2.5} \left[ (1 - \varphi + \varphi \frac{\rho_{\text{CNT}}}{\rho_f}) \{ f f'' - f'^2 \} \right] - N f' = 0, \tag{2.9}
\end{equation}
\begin{equation}
\frac{k_n f}{k_f} \theta'' + Pr \left( 1 - \varphi + \varphi \frac{\rho_e}{\rho_f} \right) \left[ (f'') - \gamma (f f' + f^2) \theta'' \right] = 0 \tag{2.10}
\end{equation}
\begin{equation}
f(0) = S, \quad f'(0) = 1, \quad f'(\infty) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0, \tag{2.11}
\end{equation}
where, \( N = v_f / ak_f \) (porosity parameter), \( Pr = (\mu c_p_f) / k_f \) is the Prandtl number, \( \gamma = a \lambda_2 \) is the non-dimensional thermal relaxation time and \( S = c/a \) (Suction /Injection parameter). The skin friction coefficient (\( c_f \)) and Nusselt number (\( Nu_x \)) which are defined as:
\begin{equation}
c_f = \frac{\mu_n f}{\rho_f U_w} \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad Nu_x = \frac{-x K_{nf}}{k_f (T_f - T_\infty)} \left( \frac{\partial T}{\partial y} \right)_{y=0}. \tag{2.12}
\end{equation}

Using transformations (2.8), Eq.(2.12) is obtained as:
\begin{equation}
(Re_x^{1/2} c_f) = \frac{f''(0)}{(1 - \phi)^{2.5}}, \quad (Re_x^{-1/2} Nu_x) = -\frac{k_n f}{k_f} \theta'(0). \tag{2.13}
\end{equation}

3. Numerical scheme. The nonlinear ordinary differential equations (2.9)-(2.10) subjected to the boundary conditions (11) have been solved numerically using an efficient Runge-Kutta fourth order method along with shooting technique. The asymptotic boundary conditions given by Eq. (2.11) were replaced by using a value of 15 for the similarity variable \( \eta_{\text{max}} \). The choice of \( \eta_{\text{max}} = 15 \) and the step size \( \Delta \eta = 0.001 \), ensured that all numerical solutions approach the asymptotic values correctly. For validating of the proposed scheme, a comparison for the Nusselt number with the literature has been shown in Table 2 for both active and passive control of \( \phi \) in the special case when \( \phi = 0 \). Therefore, the applied numerical scheme is very accurate.

4. Results and discussion. In this section, the graphical discussion of the numerical results for velocity, temperature, skin friction coefficients, Nusselt number, stream lines and Isotherms are expressed with respect to certain changes in the physical parameters through illustrations (Figs.2-9). The thermophysical properties of different base fluid and CNTs are tabulated in Table 1.

Figs. 2(a-c) represent the changes in the fluid velocity profiles with respect to different values of solid nanoparticles volume fraction (\( \phi = 0, 0.1, 0.2 \)) and porosity parameter (\( N = 0, 1, 2 \)). Three cases on the basis of suction and injection parameter (a) \( S < 0 \) i.e. permeable sheet with suction (b) \( S = 0 \) i.e. impermeable sheet (no suction and injection) (c) \( S > 0 \) permeable sheet with injection on velocity profile are illustrated through the Figs.2(a-c) respectively. It is noticed that the asymptotic nature of boundary layer starts closer (\( \eta \approx 1 \)) to origin in the case of permeable sheet with injection (Case c) while it starts with \( \eta \approx 3 \) in the case of permeable sheet with suction (Case a). It is also revealed that the velocity profile and also boundary layer thickness enhances with increment in the magnitude of solid volume fraction of nanoparticles. However the velocity profile and also boundary layer thickness diminishes with increasing the porosity parameter for all the three cases of suction, injection and impermeable.

The effects of solid nanoparticles volume fraction (\( \phi = 0, 0.1, 0.2 \)) and porosity parameter (\( N = 0, 1, 2 \)) on Temperature profile are presented in Figs. 3(a-c).
Three cases (a) $S < 0$ i.e. permeable sheet with suction (b) $S = 0$ i.e. impermeable sheet (no suction and injection) (c) $S > 0$ permeable sheet with injection on velocity profile are taken into discussion. It is pointed out that the asymptotic nature of thermal boundary layer starts with $\eta \approx 2$, $\eta \approx 1.5$ and $\eta \approx 1$ for permeable sheet with suction, impermeable sheet and permeable sheet with injection respectively. It is observed that with the increase in solid nanoparticles volume fraction temperature profile increases and thermal boundary layer thickness also increases. It is also noticed that thermal boundary layer thickness expands with increasing the permeability of the stretching surface. The effects of thermal relaxation time ($\gamma = 0, 0.2, 0.4$) on temperature profile are depicted through the Figs. 4(a-c). It is incurred that temperature profile decreases with the rise in thermal relaxation time however thermal boundary layer increases with an increase in thermal relaxation time. The behavior of relaxation time on temperature profile for all three cases (permeable sheet with suction, impermeable sheet and permeable sheet with injection) are same.

Variations of skin friction coefficient for different values of porosity parameter ($N = 0, 0.3, 0.6$) are presented in Figs. 5(a-c). The effects of suction parameter (for $S < 0$, $S = 0$ and $S > 0$) on skin friction coefficient are also discussed. It is pointed out that the skin friction coefficient is more ($Re_x^{1/2}c_f \approx 0.7$) for $S > 0$ at $\phi = 0$ while it is least ($Re_x^{1/2}c_f \approx 1.6$) for $S < 0$ at $\phi = 0$. It is seen that with the increase in porosity parameter ($N$) skin friction coefficient increases for SWCNT as well as for MWCNT. A comparative observation for skin friction coefficient between SWCNT and MWCNT is also made and it is revealed that density and thermal conductivity of SWCNT are greater as compared to the MWCNT which is concluded that skin friction coefficient for SWCNT is more than that of MWCNT.

The impacts of porosity parameter ($N = 0, 1, 2$) on Nusselt number at thermal relaxation time ($\gamma = 0.1$) are shown in Figs.6(a-c). The Nusselt number for SWCNT and MWCNT are also computed for three cases ($S < 0$, $S = 0$ and $S > 0$). The Nusselt number is maximum ($Re_x^{-1/2}Nu_x \approx 2.5$) at $\phi = 0$ for $S > 0$ however it is minimum ($Re_x^{-1/2}Nu_x \approx 1$) for $S < 0$ at $\phi = 0$. This physically interpreted that Nusselt number for permeable stretching sheet with suction is more and tiniest for permeable stretching sheet with injection. It is noticed that with the increase in porosity parameter, Nusselt number diminishes for SWCNT as well as for MWCNT. It is also observed that Nusselt number is more for SWCNT as compared with MWCNT. The influences of thermal relaxation time ($\gamma = 0, 0.2, 0.4$) on Nusselt number at $N = 3$ are depicted through the illustrations Figs.7(a-c). It is observed that the higher values of thermal relaxation time raise the Nusselt number for SWCNT as well as for MWCNT. The behavior of thermal relaxation time on Nusselt number for permeable sheet with suction, impermeable sheet and permeable sheet with injection is same.

Stream lines are plotted in Figs.8(a-c) at $\phi = 0.2, \gamma = 1, N = 0.5$ for three cases of stretching sheet (a) $S < 0$ i.e. permeable sheet with suction (b) $S = 0$ i.e. impermeable sheet (no suction and injection) (c) $S > 0$ permeable sheet with injection. It is shown that streamlines are similar to boundary layer for impermeable sheet while it goes to shift right for permeable sheet with suction and it goes to shift left for permeable sheet with injection. The contour lines at equal temperature i.e. Isotherms are sketched for at $\phi = 0.2, \gamma = 1, N = 0.5$ for three cases of stretching sheet (a) $S < 0$ i.e. permeable sheet with suction (b) $S = 0$ i.e. impermeable sheet
It is revealed that the curvature of isotherms reduces from case (a) to case (c) i.e. it is largest for $S < 0$ and least for $S > 0$.

Table 2 shows the numerical values of skin friction for different values of volume fraction. These values are compared with the numerical values obtained by Salahuddin et al. [32] and Noreen et al. [5]. It is observed that the numerical values of present model are very closer to the numerical values of existing results. Table 3 depicts the numerical values of Nusselt number for different values of Prandtl number. Same values are compared with the numerical values of Khan et al. [21], Khan & Pop. [23], Wang [40] and Kandasamy et al. [22]. A good correlation is noticed among the present results and existing results. From both table, it is clear that the present model is valid.

### Table 1: Thermophysical properties of different base fluid and CNTs.

| Physical properties | Base fluid | Nanoparticles |
|---------------------|------------|---------------|
|         | Water     | SWCNT | MWCNT |
| $\rho$ (kg/m$^3$)   | 997        | 2,600 | 1,600 |
| $c_p$(J/kg K)       | 4,179      | 425   | 796   |
| $k$(W/m K)          | 0.613      | 6,600 | 3,000 |
| $r$ (nm)            | 0.1        | 10    | 10    |

### Table 2: Comparison of results for the skin friction for pure fluid ($\phi = 0$).

| N    | Present results | Salahuddin et al. [32] | Noreen et al. [5] |
|------|----------------|-------------------------|------------------|
| 0.0  | -1.11703       | -1.11701                | -1.11703         |
| 0.5  | -1.41321       | -1.41318                | -1.41321         |
| 1    | -2.44849       | -2.44842                | -2.44849         |
| 5    | -3.31653       | -3.31656                | -3.31653         |
| 100  | -10.04978      | -10.04971               | -10.04978        |
| 500  | -22.38313      | -22.38383               | -22.38313        |
| 1000 | -31.63849      | -31.63856               | -31.63869        |

### Table 3: Comparison of results for the Nusselt number for pure fluid ($\phi = 0$) with $N = 0$ and $\gamma = 0$.

| Pr   | Present results | Khan et al. [21] | Khan & Pop. [23] | Wang [40] | Kandasamy et al. [22] |
|------|----------------|------------------|------------------|-----------|-----------------------|
| 0.07 | 0.0664         | 0.0664           | 0.0664           | 0.0655    | 0.0662                |
| 0.20 | 0.1692         | 0.1692           | 0.1692           | 0.1692    | 0.1692                |
| 0.70 | 0.4538         | 0.4538           | 0.4538           | 0.4538    | 0.4538                |
| 2    | 0.9113         | 0.9113           | 0.9114           | 0.9115    | 0.9115                |
| 7    | 1.8953         | 1.8953           | 1.8953           | 1.8953    | 1.8952                |
| 20   | 3.3538         | 3.3538           | 3.3538           | 3.3538    | 2.0952                |
| 70   | 6.4621         | 6.4623           | 6.4622           | 6.4623    | –                     |

5. **Conclusions.** The numerical study of heat flux on CNT nanofluids flow over stretching porous surface with suction and injection is discussed in above section with help of computational illustrations and tabular solutions. Cattanneo-Christov Heat Flux model is employed for better outcomes. The key features of discussion are summarized as:

1. The boundary layer goes to thicker with increasing the magnitude of solid volume fraction of nanoparticles however it goes to thinner with more porous surface.
2. The boundary layer is thicker for permeable stretching sheet with suction and thinner with injection.

3. The thermal boundary layer thickness elaborates with nanoparticles volume fraction and also with permeable parameter and thermal relaxation time.

4. The asymptotic nature of thermal boundary layer increases with increasing the magnitude of suction parameter from negative value to zero and from zero to positive value.
5. Skin friction coefficient enhances with increasing the porosity of stretching sheet.
6. Nusselt number rises with increasing the thermal relaxation time and reduces with increasing the porosity of the surface.
7. Skin friction coefficient and Nusselt number are maximum for +ve value of suction parameter and least for -ve value of suction parameter.
8. Skin friction coefficient and Nusselt number are more for SWCNT as compared to MWCNT.
9. Streamlines moves right for -ve value of suction parameter however it moves left for +ve value of suction parameter.
10. Curvature of Isotherms reduces from -ve value of suction parameter to +value of suction parameter.

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