Prospects for time-dependent mixing and $CP$-violation measurements at Belle II

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The Belle II experiment is under construction at the KEK laboratory in Japan. Belle II will study $e^+e^-$ collisions at or near the $\Upsilon(4S)$ resonance with the goal of collecting 50 ab$^{-1}$ of data, which is a large increase over that recorded by the Belle and BaBar experiments. This data will provide a large sample of charm meson decays. In this report we present the expected sensitivity of Belle II for measuring time-dependent mixing and $CP$ violation in the $D^0-\bar{D}^0$ system. We focus on measurements of $D^0 \rightarrow K^+\pi^-$, $D^0 \rightarrow K^+\pi^-\pi^0$, and $D^0 \rightarrow K^0_\Lambda\pi^+\pi^-$ decays.

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1. Introduction

The Belle experiment at the KEK laboratory in Japan is now being upgraded to the Belle II experiment. The goal of Belle II is to collect a data set corresponding to 50 times what Belle obtained. The experiment is designed to search for new physics in $B$ and $D$ meson decays, and in $\tau$ lepton decays. Within this broad physics program, measuring mixing and $CP$ violation in the neutral charm system is an important goal. The first evidence for $D^0\bar{D}^0$ mixing was obtained in 2007 by the Belle [1] and BaBar [2] experiments, with subsequent observations made by LHCb [3] and CDF [4]. However, the current precision on the mixing parameters $x = \Delta m / \Gamma$ and $y = \Delta \Gamma / (2 \Gamma)$, where $\Delta m$ and $\Delta \Gamma$ are the differences in masses and decay widths between the two mass eigenstates and $\Gamma$ is the mean decay width, is insufficient to determine whether $y > x$ or even whether $x \neq 0$ [5]. To establish this, higher statistics measurements are needed. $CP$ violation in the $D^0\bar{D}^0$ system has not yet been observed. The predicted rate within the Standard Model (SM) is very small, and an observation of $CP$ violation by Belle II could indicate new physics.

There are several advantages of performing these measurements at an $e^+e^-$ experiment such as Belle II rather than at a hadron machine: lower backgrounds, higher trigger efficiency, excellent $\gamma$ and $\pi^0$ reconstruction (and thus $\eta$, $\eta'$, and $\rho^+$ reconstruction), high flavor-tagging efficiency with low dilution, and numerous control samples with which to study systematics. In this report we present Belle II prospects for measuring the mixing parameters $x$ and $y$, and $CP$-violating parameters $|q/p|$ and $\text{Arg}(q/p) = \phi$, where $q$ and $p$ are the complex coefficients relating the $D^0\bar{D}^0$ flavor eigenstates with the mass eigenstates. We focus on three benchmark decay modes: $D^0 \to K^+ \pi^-$, $D^0 \to K^0 \pi^+ \pi^-$, and $D^0 \to K^0_s \pi^+ \pi^- \pi^-$ [6]. In all cases the flavor of the neutral $D$ is determined by requiring that it originate from a $D^{*+} \to D^0 \pi^+$ or $D^{*-} \to \bar{D}^0 \pi^-$ decay; the charge of the $\pi^+$ identifies the $D^0$ or $\bar{D}^0$ flavor. All results given are preliminary.

2. Decay-time resolution

Measurements of time-dependent mixing and $CP$ violation depend on measuring decay times precisely. Due to an improved vertex detector, the decay time resolution of Belle II should be superior to that of Belle and BaBar. Whereas Belle used a four-layer silicon-strip detector, Belle II will use four layers of silicon strips plus two layers of silicon pixels, as shown in Fig. 1. Some parameters of the vertex detector are listed in Table 1. The pixel layers will lie only 14 mm and 22 mm from the interaction point (IP), and the smallest pixel size will be $55 \times 50 \mu$m$^2$.

Figure 2 (left) shows the resolution on the impact parameter of tracks with respect to the IP as a function of track momentum, as obtained from Monte Carlo (MC) simulation. Also shown are corresponding results from BaBar. The figure indicates that the Belle II resolution will be approximately half that of BaBar. Figure 2 (right) plots the residuals of decay time for a large sample of MC $D^{*+} \to D^0 \pi^+$, $D^0 \to K^+ K^-$ decays. The RMS of this distribution is 140 fs, which is almost half the decay time resolution of BaBar (270 fs). Similar results have been obtained for prompt $D^0 \to K^+ K^-$ decays, and also for $D^0 \to \pi^+ \pi^-$ decays.

3. Mixing and $CP$ violation in $D^0 \to K^+ \pi^-$

To study the sensitivity of Belle II to parameters $x$, $y$, $|q/p|$, and $\phi$ in “wrong-sign” $D^0 \to
Figure 1: Left: layers of the Belle II vertex detector, which consists of two layers of silicon pixels and four layers of silicon strips. Right: a cutaway view of the vertex detector.

| Layer | Type | Distance from IP (mm) | Size or pitch (µm) | Number of pixels or strips |
|-------|------|----------------------|--------------------|---------------------------|
| 1     | pixels | 14                  | (55, 60) × 50      | 3072000                   |
| 2     | pixels | 22                  | (70, 85) × 50      | 4608000                   |
| 3     | strips | 38                  | 50 (φ), 160 (z)    | 768 (φ), 768 (z)          |
| 4     | strips | 80                  | 50, 75 (φ), 240 (z)| 768 (φ), 512 (z)          |
| 5     | strips | 104                 | 50, 75 (φ), 240 (z)| 768 (φ), 512 (z)          |
| 6     | strips | 135                 | 50, 75 (φ), 240 (z)| 768 (φ), 512 (z)          |

Table 1: Layers and segmentation of the Belle II vertex detector.

$K^+\pi^-$ decays, we perform a “toy” MC study as follows. We generate an ensemble of 1000 MC experiments, with each experiment consisting of a sample of $D^0$ decays and a separate sample of $\bar{D}^0$ decays. The number of events in each sample corresponds to 5 ab$^{-1}$, 20 ab$^{-1}$, and 50 ab$^{-1}$ of data. For the full dataset (50 ab$^{-1}$), there are a total of 438600 $D^0 \to K^\pm \pi^\mp$ decays. The decay times are obtained by sampling from the following probability density functions (PDFs):

$$
\frac{dN}{dt} (D^0) = e^{-\Gamma t} \left\{ R_D + \left| \frac{p}{q} \right| \sqrt{R_D (y' \cos \phi - x' \sin \phi)} (\Gamma t) + \frac{q}{p} \left( \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right) \right\} \quad (3.1)
$$

$$
\frac{dN}{dt} (\bar{D}^0) = e^{-\Gamma t} \left\{ \bar{R}_D + \left| \frac{p}{q} \right| \sqrt{\bar{R}_D (y' \cos \phi + x' \sin \phi)} (\Gamma t) + \frac{q}{p} \left( \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right) \right\} \quad (3.2)
$$

where $x' = x \cos \delta + y \sin \delta$, $y' = -x \sin \delta + y \cos \delta$, and $\delta$ is the strong phase difference between $D^0 \to K^- \pi^+$ and $\bar{D}^0 \to K^- \pi^+$ amplitudes. This strong phase difference can be measured at BESIII using $CP$-tagged $D_{CP} \to K^- \pi^+$ decays [7]. The parameter $R_D$, $\bar{R}_D$ is the ratio of amplitudes squared $|\langle \overline{A} (D^0 \to K^+ \pi^-)/\overline{A} (D^0 \to K^- \pi^+) \rangle^2|/|\langle \overline{A} (\bar{D}^0 \to K^- \pi^+)/\overline{A} (\bar{D}^0 \to K^- \pi^-) \rangle^2|$. After generation, the decay times are smeared by the expected decay time resolution of 140 fs, and the resulting $D^0$ and $\bar{D}^0$ time distributions are simultaneously fitted. Backgrounds are not yet included in this study; a first look at backgrounds indicates that when they are included, the fitted errors increase by only a small amount. The PDFs used for the fit are the convolution of Eqs. (3.1) and (3.2) with a Gaussian resolution function; this results in products of exponential functions and
Figure 2: Left: Belle II track impact parameter with respect to the IP as a function of track momentum, as obtained from MC simulation. Right: Belle II residuals of decay vertex position, from MC simulation of $D^{+} \rightarrow D^{0}\pi^+$, $D^{0} \rightarrow K^{+}K^{-}$ decays. The RMS of this distribution is 140 fs.

Error functions. The preliminary results are listed in Table 2. The final Belle II sensitivity for $|q/p|$ is < 0.1%, and that for $\phi$ is 6°. This precision is a significant improvement over that which Belle and BaBar achieved.

| Parameter            | 5 ab$^{-1}$ | 20 ab$^{-1}$ | 50 ab$^{-1}$ |
|----------------------|-------------|--------------|--------------|
| $\delta x'$ (%)      | 0.37        | 0.23         | 0.15         |
| $\delta y'$ (%)      | 0.26        | 0.17         | 0.10         |
| $\delta |q/p| (%)      | 0.20        | 0.09         | 0.05         |
| $\delta \phi$ (°)    | 16          | 9.2          | 5.7          |

Table 2: Preliminary results of a toy MC study of $D^{0} \rightarrow K^{+}\pi^{-}\pi^{0}$ decays: uncertainty on “rotated” mixing parameters $x'$ and $y'$, and on CP-violating parameters $|q/p|$ and $\phi$, for three values of integrated luminosity.

4. Mixing in $D^{0} \rightarrow K^{+}\pi^{-}\pi^{0}$

We study mixing in $D^{0} \rightarrow K^{+}\pi^{-}\pi^{0}$ decays using an MC simulation based on EVTGEN [8]. For this study we generate an ensemble of 10 experiments, with each experiment consisting of 225000 $D^{0} \rightarrow K^{+}\pi^{-}\pi^{0}$ decays corresponding to 50 ab$^{-1}$ of Belle II data. The generated decay times are smeared by a resolution of 140 fs, and the decay model used to generate and fit the Dalitz plot is the isobar model used by BaBar [9]. Possible $CP$ violation and backgrounds are neglected in this phase of the study. More details are given in Ref. [10].

The mixing parameters are $x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$ and $y'' = -x \sin \delta_{K\pi\pi} + y \cos \delta_{K\pi\pi}$, where $\delta_{K\pi\pi}$ is the strong phase difference between $D^{0} \rightarrow K^{-}\pi^{+}\pi^{0}$ and $\bar{D}^{0} \rightarrow K^{-}\pi^{+}\pi^{0}$ amplitudes evaluated at $m_{\pi\pi} = M_{\rho}$. For this study the strong phase $\delta_{K\pi\pi}$ is fixed to 10°; and the fitted parameters are $x$ and $y$. The results are shown in Fig. 3. The input values are $x = 0.0258$ and $y = 0.0039$, and the RMS of the distributions of residuals are $\delta x = 0.057\%$ and $\delta y = 0.049\%$. This precision is approximately one order of magnitude better than that achieved by BaBar [9]. The projections of a typical fit (one experiment in the ensemble) are shown in Fig. 4. In practice, to extract values for
$x$ and $y$ requires knowledge of the strong phase $\delta_{K\pi\pi}$; this can in principle be measured at BESIII using CP-tagged $D_{CP}\rightarrow K^{-}\pi^{+}\pi^{0}$ decays.

![Figure 3: Left: preliminary results of fitting an ensemble of 10 experiments, with each experiment corresponding to 50 ab$^{-1}$ of data [10]. Middle and right: projections of the left-most plot. The RMS values of these distributions are $\delta x = 0.057\%$ and $\delta y = 0.049\%$.](image)

![Figure 4: Projections of the fit to $D^{0}\rightarrow K^{0}\pi^{+}\pi^{-}$ events for one typical experiment in the ensemble [10].](image)

### 5. Mixing and CP violation in $D^{0}\rightarrow K_{S}^{0}\pi^{+}\pi^{-}$

Another method for measuring mixing and CP violation is by fitting the time-dependent Dalitz plot for $D^{0}\rightarrow K_{S}^{0}\pi^{+}\pi^{-}$ decays. One calculates the observables $m_{+} \equiv (P_{K_{S}}^{0} + P_{\pi^{+}})^{2}$ and $m_{-} \equiv (P_{K_{S}}^{0} + P_{\pi^{-}})^{2}$ and performs an unbinned maximum likelihood fit to $m_{+}, m_{-}$, and the decay time $t$. To fit for CP-violating parameters $|q/p|$ and $\phi$ (in addition to $x$ and $y$) requires dividing the data sample into $D^{0}$ and $\bar{D}^{0}$ decays and fitting the two subsamples simultaneously.

The time-dependent PDFs are

$$
\frac{dN}{dt}(D^{0}) = \frac{e^{-\Gamma t}}{2} \left\{ \left| \alpha f \right|^{2} + \left| \frac{q}{p} \right|^{2} \left| \overline{\alpha f} \right|^{2} \right\} \cosh(yt) + \left( \left| \alpha f \right|^{2} - \left| \frac{q}{p} \right|^{2} \left| \overline{\alpha f} \right|^{2} \right) \cos(xt)
+ 2\text{Re}\left( \frac{q}{p} \alpha f \overline{\alpha f}^{*} \right) \sinh(yt) - 2\text{Im}\left( \frac{q}{p} \alpha f \overline{\alpha f}^{*} \right) \sin(xt) \right\},
$$

and

$$
\frac{dN}{dt}(\bar{D}^{0}) = \frac{e^{-\Gamma t}}{2} \left\{ \left| \overline{\alpha f} \right|^{2} + \left| \frac{p}{q} \right|^{2} \left| \alpha f \right|^{2} \right\} \cosh(yt) + \left( \left| \overline{\alpha f} \right|^{2} - \left| \frac{p}{q} \right|^{2} \left| \alpha f \right|^{2} \right) \cos(xt)
+ 2\text{Re}\left( \frac{p}{q} \alpha f \overline{\alpha f}^{*} \right) \sinh(yt) - 2\text{Im}\left( \frac{p}{q} \alpha f \overline{\alpha f}^{*} \right) \sin(xt) \right\},
$$

(5.1)
where $A_f$ ($\bar{A}_f$) is the decay amplitude for $D^0 \to f$ ($\bar{D}^0 \to f$) decays. These amplitudes must be modeled, and the parameters of the model must also be fitted or else taken from a previous measurement. For a Belle analysis of $D^0 \to K^0_S\pi^+\pi^-$ decays using 976 fb$^{-1}$ of data [11], the decay model used consisted of 14 intermediate resonances modeled by isobars, plus $K\pi$ and $\pi\pi$ $S$-waves. The magnitudes and phases of the isobars, and parameters of the $S$-waves, were obtained from a separate time-independent fit to the data.

An important advantage of this measurement is that the fitted PDFs (Eqs. 5.1 and 5.2) depend on $x$ and $y$ without being “rotated” by an unknown strong phase difference. The results obtained by Belle [11] are listed in Table 3 along with the precision estimated for Belle II. This precision is obtained as follows. The statistical errors of the Belle measurement ($\sigma_{\text{stat}}$) are scaled by the square root of the ratio of luminosities. The systematic errors are divided into two categories: “reducible” errors ($\sigma_{\text{syst}}$) that should decrease with luminosity such as errors due to background modeling as determined from control samples; and “irreducible” errors ($\sigma_{\text{irred}}$) that do not decrease with more data such as uncertainty in decay time resolution due to detector misalignment. The total error is calculated as $\sigma_{\text{Belle II}} = \sqrt{(\sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2) \cdot (L_{\text{Belle}}/50 \text{ ab}^{-1}) + \sigma_{\text{irred}}^2}$. These errors are conservative, as the simple scaling used does not account for the improved decay time resolution of the Belle II vertex detector as compared to that of Belle (see Section 2).

|          | $\sigma(x)$ | $\sigma(y)$ | $|q/p|$ | $\sigma(\phi)$ |
|----------|-------------|-------------|--------|----------------|
| Belle $\sigma_{\text{stat}}$ | 0.19        | 0.15        | 0.155  | 10.7           |
| Belle $\sigma_{\text{syst}}$ | 0.06        | 0.06        | 0.054  | 4.5            |
| Belle $\sigma_{\text{irred}}$ | 0.11        | 0.04        | 0.069  | 3.8            |
| Belle II $\sigma_{\text{tot}}$ | 0.11        | 0.05        | 0.073  | 4.1            |

Table 3: Precision obtained for $x$, $y$, $|q/p|$, and $\phi$ from a Belle analysis of $D^0 \to K^0_S\pi^+\pi^-$ decays [11], and the precision expected for Belle II as obtained by scaling the Belle errors (see text).

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