Black holes in higher curvature theory and third law of thermodynamics

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Abstract. We derive the black hole solutions in higher curvature gravitational theories and
discuss their spacetime structures. In our analysis Lovelock theory is mainly investigated, which
includes cosmological constant, Einstein-Hilbert action, and Gauss-Bonnet term as its lower
order terms. Among the solutions, there are solutions which may become extreme solutions
with zero temperature through physical processes. This is a counterexample of the third law of
black hole thermodynamics. We also discuss solutions with product space which are extensions
of Nariai and Bertotti-Robinson solutions.

1. Introduction
Recently a large number of studies have focused on the question of higher dimensions. One
of the reasons for this is the fascinating new picture of our Universe called the braneworld
universe. Because the fundamental scale could be about TeV in this scenario, these models
suggest that creation of tiny black holes in the upcoming large hadron collider will be possible.
The braneworld model is an underlying fundamental theory, superstring/M-theory. Although
superstring/M-theory has been highly elaborated, it is not enough to understand black hole
physics in the string context. Hence at present to take string effects perturbatively into classical
gravity is one approach to the study of quantum gravity effects.

With the restriction that the tension of a string be large as compared to the energy scale of
other variables, i.e., in the $\alpha'$-expansion, the Gauss-Bonnet terms appear as the first curvature
correction terms to general relativity. Though many complicated combinations may appear
for the higher order corrections, here we will adopt Lovelock gravitational theory expecting
some aspects of higher curvature corrections are obtained. Lovelock gravitational theory is the
most general theory which gives the equation of motion with up to the second order derivative.
$n$-dimensional Lagrangean is described as

$$\mathcal{L} = \sum_{p=0}^{N} \frac{\alpha_p}{(n-2p)(n-2)} \mathcal{L}_p,$$

(1)

$$\mathcal{L}_p = 2^{-p} \delta_{\rho_1\sigma_1...\rho_p\sigma_p} R^{\rho_1\sigma_1...\mu_1\nu_1...\rho_p\sigma_p} R^{\rho_p\sigma_p}_{\mu_1\nu_1}...R^{\rho_p\sigma_p}_{\mu_p\nu_p}.$$  

(2)

In this theory we investigate black hole solutions and spacetimes with product metric which are
Nariai and Bertotti-Robinson types [1].
2. Black hole solutions and violation of the third law of black hole thermodynamics

In the first part we investigate black hole solutions in Lovelock gravity. We assume the static spacetime with the following line element

\[ ds^2 = -f(r)e^{-2\delta(r)}dt^2 + f^{-1}(r)dr^2 + R(r)^2d\Omega^2_{n-2}, \]  

(3)

where \( d\Omega^2_{n-2} = \gamma_{ij}dx^idx^j \) is the metric of the \((n-2)\)-dimensional maximally symmetric spacetime with curvature \( k = 1, 0, -1 \).

For the black hole solution, the metric function \( R \) is not constant, and we take the gauge \( R(r) = r \). Then the gravitational equation gives \( \delta \equiv 0 \) and an algebraic equation of metric function

\[ U(F) := \sum_{p=0}^{N} \alpha_p F^p = \frac{\mu}{r^{n-1}}, \]

(4)

where \( F(r) \) is defined as

\[ f(r) = k - r^2 F(r), \]

(5)

and \( \mu \) is the mass of the black hole. Although Eq. (4) cannot be solved analytically except for some simple cases, we have developed a technique to find the spacetime structure of solutions, i.e., singularity, horizons, and asymptotic structures.

As some examples we study solutions in Gauss-Bonnet gravity (\( \alpha_p = 0 \), where \( p \geq 3 \)) [2] and gM-theory models (\( \alpha_0 = \alpha_2 = \alpha_3 = \alpha_q = 0 \), where \( q \geq 5 \)). We find that there are solutions which can be extreme black hole with zero temperature by throwing into suitable amount of matters into black holes. This is the violation of the third law of black hole thermodynamics.

3. Solutions with product space

In the second part we investigate solutions with product metric. By decomposing into 2-dim. Riemannian manifold and \((n-2)\)-dimensional Euclidean submanifold with maximal symmetry, the metric is given by Eq. (3) with \( R \equiv B^2 = constant \) and \( \delta \equiv 0 \).

The equation of motion becomes an algebraic equation of single metric function as

\[ V(\tilde{B}^2) := \sum_{p=1}^{N} \alpha_p (k\tilde{B}^2)^p = -\alpha_0, \]

(6)

where

\[ \zeta_i := \frac{\sum_{p=0}^{N} \alpha_p(n-2)_{2p+2}(kB_i^{-2})^p}{\sum_{p=0}^{N} \alpha_p(n-2)_{2p}(kB_i^{-2})^{p-1}}, \]

(7)

and \( \tilde{B} := B^{-1}, f = 1 - \zeta_i r^2 \). We obtain the way to analyze such equation and to classify the solution into Nariai, anti-Nariai, Bertotti-Robindon, and Plebanski-Hachyan solutions. These solutions are related to the throat portion of the extreme black hole solutions in the same system by certain coordinate transformations.

References

[1] Torii T, in preparation
[2] Torii T and Maeda H 2005 Phys. Rev. D 71 124002