Abstract. Many technologies of the agro-industrial complex are associated with the need for a quality blending of two or more components. These include the processes of obtaining biodiesel fuel based on vegetable oils, obtaining aqueous emulsions of fats in the production of mixed fodders. In this process, the most effective is the use of a hydrodynamic mixer, the main design parameters of which are determined in the process of investigation. The design of a nozzle with a variable cross-sectional area is calculated, which allows to control the mixing process. The obtained mathematical dependences quite fully characterize the features of the work and the requirements for the design of the ejector installed in the hydrodynamic mixer to create cavitation with the required intensity.

From the point of view of the most important factors of modern agro-industrial production [1] (renewability, environmental friendliness, cost), one of the most important problems is the problem of obtaining biofuel based on renewable raw materials [2].

The production of biofuel is several stages, the main of which is the esterification reaction based on the process of qualitative mixing of components [3, 4].

The hydrodynamic mixer is designed for mixing drip liquids in the production process, for example, biodiesel [5].

The purpose of mixing is homogenization, consisting in reducing the concentration or temperature gradient, or both simultaneously, in a mixture.

The most promising way to create new mixing equipment can be the use of such a physical phenomenon as hydrodynamic cavitation, which has a number of significant advantages: lower specific energy consumption and material consumption, simplicity of mixer design, continuous operation, high productivity, regulation of cavitation-cumulative impact. As a hydrodynamic cavitation mixer it is possible to use jet devices - ejectors [6, 7].

In the ejector there is a mixing and energy exchange of two flows of liquids under different pressures to form a mixed flow under intermediate pressure. This is caused by the following circumstances:

- the jet of liquid flowing out of the nozzle of the ejector opens and takes the form of a cone expanding in the direction from the nozzle cutoff;

* Corresponding author: dmitriyrudou@gmail.com
- the pressure in the jet behind the nozzle cut is reduced by 20-30 times and cavitation occurs in the jet [8];

- the working stream in the form of a jet of liquid flowing out of the nozzle, pushes the ejected stream before it and directs it into the mixing chamber of the ejector.

The scheme of the hydrodynamic mixer containing the ejector is shown in Fig. 1.

![Fig. 1. The design scheme of a hydrodynamic mixer: 1 - mixer; 2 - suction line; 3 - volumetric pump; 4 - electric motor; 5 - pressure line; 6 - nozzle; 7 - storage chamber; 8 - mixing chamber; 9 - slit nozzle; 10 - reflector; 11 - cone; 12 - hole; 13 - housing; 14 - drain line; 15 - hydrolime.]

The flow of the main component of the mixture by the pump 3, which is driven by the electric motor 4 and the suction line 2 connected to the tank 1, is fed through the pressure line 5 to the nozzle 6. From the nozzle 6, the flow is directed to a receiving chamber 7, into which an additional component of the mixture along the hydrolysis line 15 from the container 1. Capacity 1, conditionally, consists of several tanks of smaller volumes, in each of which are both components and the finished mixture. In the receiving chamber 7, the mixing process is started and continues in the mixing chamber 8, terminated in the slit nozzle 9 formed by the end face of the mixing chamber 8 and the reflector 10. The reflector 10 is closed by a cone 11 and attached to it. In the cone there are holes 12 for passage of the mixture. The cone 11 is closed by the body 13, from the cavity of which the mixture on the drain line 14 enters the container of the finished mixture 1.

With reference to the scheme in Fig. 1, the quantities characterizing the functioning of the ejector of the hydrodynamic mixer are indicated:

- \( p_c, V_c \) – pressure and flow rate of the main component of the mixture (working stream) in the pressure line;
- \( p_1, V_1 \) – pressure and flow velocity of the working flow in section A-A, the nozzle section;
- \( p_2, V_2 \) – pressure and speed of the flow of the additional component of the mixture (ejected flow) in section B-B, the entrance to the mixing chamber;
- \( p_3, V_3 \) – pressure and flow velocity of the mixture of components in the cross section BB, the exit from the mixing chamber and the entrance to the reflector;
- \( p_4, V_4 \) – The pressure and flow rate of the mixture in the tank;
- \( F_1, F_2, F_3, F_4 \) – area of: the cross-section of the nozzle, the entrance to the mixing chamber, the exit from the mixing chamber and the section of the slit nozzle.

On the basis of the impulse theorem for a cylindrical mixing chamber, it is possible to form an equation for the cross sections B-B and C-C [9, 10]:

\[
M_1 V_1 + M_2 V_2 - (M_1 + M_2) V_3 = F_3 (p_3 - p_1 + \Delta p_{\infty}),
\] (1)
where \( M_1 \) and \( M_2 \) – working and ejected working fluid flows masses; 
\( \Delta p_{\text{sc}} \) – pressure loss to overcome the resistance of the mixing chamber.

In the formulation of equation (1), the following assumptions were made:
- the blurring of the working jet does not occur immediately after the exit from the nozzle, so the speed \( V_1 \) in section V-B does not change;
- The process of mixing the working and ejected flows ends at the cross section BB, In this case, the total flow rate \( V_3 \) over the cross section of the mixing chamber;
- the pressure in the section B-B is constant, therefore \( p_1 = p_2 \);
- The values of the densities of the working, ejected and mixed flows differ, but insignificantly, i.e. \( \rho_1 \approx \rho_2 \approx \rho_3 \approx \rho \).

The Bernoulli equation for an ejected flow flowing along a ring with a cross-sectional area \( F_2-F_1 \) from section AA to section B-B [11] has the form:

\[
p_a + \rho \frac{V_a^2}{2} = p_2 + \rho \frac{V_2^2}{2} + \zeta_{\text{sc}} \rho \frac{V_2^2}{2},
\]

(2)

where \( \zeta_{\text{sc}} \) – coefficient of resistance of the entrance to the mixing chamber.

As \( V_a = 0; \rho_1 \approx \rho_2 \) then from equation (2) we obtain

\[
p_1 = p_a - \rho \frac{V_2^2}{2} (1 + \zeta_{\text{sc}}),
\]

(3)

where \( p_a \) – atmosphere pressure.

Expressing the mass and velocity of the flows through their corresponding costs and cross-sectional areas

\[
M_1 = \rho_1 Q_1; \quad M_2 = \rho_2 Q_2; \quad M_1 + M_2 = \rho Q_1 (1 + q),
\]

(4)

\[
V_1 = \frac{Q_1}{F_1}; \quad V_2 = \frac{Q q}{F_1 (m-1)}; \quad V_3 = \frac{Q_1 (1 + q)}{m F_1},
\]

(5)

where \( m = \frac{F_2}{F_1}, \quad q = \frac{Q_2}{Q_1} \), the pressure loss in the mixing chamber can be determined as follows

\[
\Delta p_{\text{sc}} = \zeta_{\text{sc}} \rho \frac{V_2^2}{2} = \zeta_{\text{sc}} \rho \frac{Q_1^2 q^2}{2 F_1^2 (m-1)^2},
\]

(6)

Substituting expressions (3), (4), (5) and (6) into equation (1) and after corresponding transformations, an equation was obtained for determining the excess pressure \( p_3' = p_3 - p_a \) at the outlet from the mixing chamber

\[
p_3' = \rho \frac{Q_1^2}{m F_1^2} \left[ 1 + \frac{q^2}{m-1} - \frac{(1 + q)^2}{m} - \frac{m q^2}{2 (m-1)^2} (1 + \zeta_{\text{sc}} + \zeta_{\text{sc}}') \right],
\]

(7)

The flow rate of the working fluid through the nozzle, as a cylindrical nozzle, can be represented by the dependence [12]
\[ Q_i = \mu F_i \sqrt{\frac{2}{\rho}(p_c - p_a)}, \]  

(8)

where \( \mu \) – coefficient of flow.

From the dependence (8) we obtain

\[ p_c - p_a = \rho \frac{Q_i^2}{2 \mu^2 F_i^2}, \]  

(9)

To determine the excess pressure in front of the nozzle \( p'_c = p_c - p_a \) we compose the Bernoulli equation for the work flow in section A-A

\[ p_c + \rho \frac{V_c^2}{2} = p_i + \rho \frac{V_i^2}{2} + \zeta_c \rho \frac{V_i^2}{2}, \]  

(10)

where \( \zeta_c \) – nozzle resistance coefficient.

Using the dependences (3) and (5) and substituting them into equation (10), \( \frac{V_c^2}{2} \ll \frac{V_i^2}{2} \) neglected, we obtain

\[ p_c - p_a = \rho \frac{Q_i^2}{2 F_i^2} \left[ (1 + \zeta_c) - \frac{q^2}{(m-1)^2} (1 + \zeta_m) \right], \]  

(11)

Equating the right-hand sides of (8) and (11) determined the flow coefficient

\[ \mu = \frac{1}{\sqrt{\left(1 + \zeta_c\right) - \frac{q^2}{(m-1)^2} (1 + \zeta_m)}}. \]  

(12)

The obtained dependences (8) and (12) make it possible to represent the excess pressure equation at the entrance to the reflector (7) in the relative geometric parameters of the ejector. After a number of transformations, we obtain

\[ p'_s = \frac{p'_c}{m} \frac{1 + \frac{q^2}{m-1} - \left(1 + \frac{q}{m}\right)^2 - m \frac{q^2}{2(m-1)^2} \left(1 + \zeta_m + \zeta_k\right)}{\left(1 + \zeta_c\right) - \frac{q^2}{(m-1)^2} (1 + \zeta_m)}, \]  

(13)

\[ p_s = f\left(p'_s; q, m, h, \zeta_c, \zeta_m, \zeta_k, \zeta_k, \zeta_m, \zeta_c, \zeta_k, \zeta_m\right). \]  

(14)

The arguments of this function are interrelated, but the degree of their influence on the pressure value at the outlet of the mixing chamber of the ejector is different. The hydrodynamic mixer functions in a closed system, the inflow of energy into which is supplied from the pump drive motor.

The excess pressure in front of the nozzle, according to (8), is a function

\[ p'_c = f(Q_i), \]  

(15)
In [13, 14] it is indicated that between the parameters of the ejector \( m \) and \( q \) there exists a dependence, \( q=f(m) \), the character of which does not depend on other arguments of the function (14). To obtain one of \( q=f(m) \), let us consider, for example, the work of an ejector under the condition

\[
p'_3 = 0; \quad \zeta_c = \zeta_{ax} = \zeta_{sc} = \zeta_{\phi} = 0,
\]

We substitute these values (4.16) into equation (13) and solve it with respect to \( q \), whence we obtain

\[
q = \frac{m-1}{2m- m^2 -2} \left[ 2(m-1) - m\sqrt{2(m-1)} \right],
\]

The graph of function (17) (Figure 2) shows that with increasing \( m \) the value of \( q \) increases and, conversely, with increasing \( q \), the value of \( m \) should also increase. At the output of the mixing chamber \( p'_3 = \text{const} \).

![Fig. 2. Dependence of the coefficient of flows on the ejection coefficient.](image)

The values of the resistance coefficients of the ejector elements \( \zeta_c, \zeta_{ax}, \zeta_{sc} \) and \( \zeta_{\phi} \) depend on many factors [15]: working fluid, pressures at different points of the ejector, parameters \( m, q \), etc.

So we can assume that the effect of the resistances of the ejector elements \( p_s \) will be reduced due to excess kinetic energy of the working flow.

The considered features of the operation of the ejector of a hydrodynamic mixer make it possible to construct in dimensionless units the graphs of functions \( p'_e = f(Q) \) and \( p_s = f(Q) \) for different values of \( m \) is shown in Figure 3.

1. \( m=4 \); 2. \( m=5 \); 3. \( m=6 \); 4. \( m=8 \); 5. \( m=11 \)

![Fig. 3. Dependence of the pressure before the nozzles and on the ejector output from the working flow and the ejection coefficient.](image)
In order to reduce the influence of the flow with the flow rate $Q_1$ on the pressure values $p'_c$, on which the intensity of cavitation depends, measured by the vacuum value in the ejector chamber, depends, the nozzle cross-section must vary according to a given law.

The pressure value $p'_c$ depends on the viscosity of the fluid in the flow $Q_1$ and on the flow rate $Q_1$. Consequently, the same dynamic mixer can be used in installations for obtaining biodiesel with different capacities, while the intensity of cavitation in the hydrodynamic mixer remains unchanged.

From the analysis of the curves in Figure 3 it follows that for different flow values $Q_1$ through the nozzle the pressure values $p'_c$ and $p_s$ will vary uniquely with $m$.

It is also necessary to take into account the possible change in the ejection factor $q$ and the related change in the values $p_s$, (figure 4).

![Fig. 4. Effect of the ejection coefficient on the ejector outlet pressure.](image)

Analysis of the curves in Figures 3.7 and 3.8 shows that the pressure $p'_c = \text{const}$ along with changing $Q_1$ and $q$ can be obtained by changing the design parameter $m$. In this case, the pressure $p_s$ will change according to a certain law, which will allow to eliminate the decrease in the intensity of cavitation in the mixer.

Thus, the ejector must satisfy the requirements

$$m_i = f(Q_i); \quad p'_c = f(m); \quad p_s = f(m). \quad (18)$$

Since $m = F_2/F_1$, it can be changed due to the areas of the nozzle or mixing chamber. It is most simple to constructively change the parameter $m$ due to the area of the nozzle section $F_1$. A schematic diagram of a hydrodynamic mixer with such ejector is shown in Figure 5.

The flow of the main component of the mixture by the pump 3, which is driven by the electric motor 4 and the suction line 2 connected to the tank 1, is fed through the pressure line 5 into the body 6 of the nozzle 9 of the ejector. Capacity 1, conditionally consists of several containers of smaller volumes, in each of which are both components and the finished mixture. In the housing 6 there is a piston 7, equipped with a needle with a profiled end and a spring 10. As the pressure of the main component of the mixture increases, the piston 7 moves to the left and the passage section of the nozzle increases, and vice versa, if the pressure decreases.

From the nozzle 9, a receiving chamber 11 is directed to the stream, into which an additional component of the mixture from the tank 1 along the hydrolysis line 13 is also fed. The housing 6 of the nozzle 9 is connected to the receiving chamber 11 and the following mixing chamber 12. At the output of the mixing chamber 12 in the cone 14, a reflector 15 is fixed (Figure 5). The end of the mixing chamber 11 and the reflector 15 form a slit nozzle 16. The cone 14 is
closed by the housing 17, from the cavity of which the mixture through the drain line 18 enters the container of the finished mixture 1.

Fig. 5. Hydrodynamic mixer (the nozzle section of the ejector is adjustable): 1 - capacity; 2 - suction line; 3 - pump; 4 - electric motor; 5 - pressure chamber 6 - nozzle housing; 7 - piston; 8 - needle; 9 - nozzle; 10 - spring; 11 - receiving chamber; 12 - mixing chamber; 13 - hydrol ine; 14 - cone; 15 - reflector; 16 - slit nozzle; 17 - housing; 18 - drain line

The mixing process starts in the receiving chamber 11, where cavitation occurs, continues in the mixing chamber 12 and terminates in the slit nozzle 16.

The displacement of the needle relative to the nozzle under the influence of the force of the spring and the pressure of the working flow on the piston causes a change in the cross-sectional area of the nozzle (Figure 3.11):

\[ F_i = F_k - \int_{0}^{l} dF_u, \]  

(19)

where \( F_k \) – sectional area of the body nozzle; \( F_u \) – cross-sectional area of the needle on the nozzle section; \( l_u \) – moving the needle.

Fig. 6. Nozzle with variable cross-sectional area.

The force of the spring is balanced by the force generated by the pressure \( p_c^f \) on the piston (Figure 6), then the spring stiffness is

\[ C_u = \frac{\Delta p_c^f F_{II}}{l_u}, \]  

(20)
where $\Delta p'_c = p'_{c_{\text{max}}} - p'_{c_{\text{min}}}$ – change in pressure before the nozzle for the full stroke of the needle; 
$F_P$ – piston area.

The pressure $p'_{c_{\text{min}}}$ и the corresponding value of the parameter $m$ at $q_{\text{max}}$ should provide a pressure $p_s$, that is sufficient to maintain the required vacuum in the ejector storage chamber. The pressure $p'_{c_{\text{max}}}$ and the parameter $m$ must be such that for $q=0$ the conditions are satisfied

$$
\begin{align*}
 p_c &= \text{const} \\
 p &= \text{const} \\
 \rho \mu Q &= \text{var}.
\end{align*}
$$

These conditions determine the type of dependence

$$
F_u = f(l_u).
$$

Therefore, the parameter $m$ is equal to

$$
m = \frac{F_2}{F_k - \int_0^l \left[F_u(l_u)\right] d}. \quad (23)
$$

The derived equations and obtained dependences quite fully characterize the features of the work and the requirement for the design of the ejector, which must be installed in a hydrodynamic mixer to create cavitation in it with the required intensity.

The obtained dependences were used in determining the design parameters of the ejectors of a hydrodynamic mixer intended for laboratory research.

The volumetric pump - gear, was driven by an electric motor with a power of 3 kW at a speed of 3000 rpm, the pump was supplied with 59 l/min.

The pressure drop across the uncontrolled ejector nozzle is 0.6 MPa, because The pump safety valve is adjusted to a pressure of 1 MPa [12].

The design of the nozzle is made with a variable cross-section and is shown in Figure 7.

![Fig. 7. Nozzle with adjustable cross-section: 1 - flange; 2 - pipe; 3 - screw; 4 - spring; 5 - glass; 6 - piston; 7 - sealing ring; 8 - needle; 9 - sleeve; 10 - nozzle; 11 - hydrolne](image_url)
On the working hydroline 5, the working flow enters the sleeve 6 (Figures 1 and 5). Inside the sleeve, the piston 6 moves with the needle 8. The piston rests against the glass 5, on which the spring 4 is clad. Preliminary compression of the spring is regulated by the screw 3. The spring allows, when moving the needle, to change the pressure before the nozzle by 0.1 MPa.

Summary

Near the end of the mixing chamber, a reflector is installed so that there is a gap between the end face and the edge of the reflector with a circumferential passage area equal to the cross section of the mixing chamber. The cylindrical cavity inside the reflector serves to destabilize the hydrodynamic force of the mixture flow acting at the bottom of the cavity. The instability of the value of the hydrodynamic force can increase due to cavitation if the pressure near the inner edge of the nozzle drops to values lower than the atmospheric. This is facilitated by the beveling of the edge of the reflector at an angle and the deflection of the flow of the mixture leaving the mixing chamber and approaching the reflector by an angle of about 35 degrees [12]. Instability of the hydrodynamic force will cause pressure fluctuations in the nozzle body with subsequent longitudinal movements of the needle and changes in the values of the parameters of the ejector – m and q. The intensity of cavitation should increase both in the intake chamber of the ejector and at the slit nozzle. To verify the above assumptions, it is necessary to carry out experimental studies.

References

1. Information on: http://en.wikipedia.org/wiki/Biodiesel_by_region
2. Information on: http://www.wilson.edu/wilson/uploadedfiles/offices/academic_affairs/fulton_ctr/Biodiesel%20Safety%20Manual.pdf/
3. Information on: http://www.caseih.com/northamerica/AboutUs/Pages/ biodiesel
4. Information on: http://www.deere.com/en_US/rg/servicesupport/ faqs/index.html
5. A.V. Gromakov, VAND, Mixer for biofuel production, 2, 31-34 (2010)
6. I.L. Knunyants, SE, Brief Chemical Encyclopedia, 3, 1112 (1964)
7. B.P. Nikolski, Chemistry, Reference book of the chemist. Main properties of organic and inorganic compounds, 2, 1168 (1964)
8. R. Rid, J. Prausnits, T. Sherwood, Chemistry, Properties of gases and liquids: A Guidebook, 3, 592 (1982)
9. P.P. Korolev, CIAM, Investigation of the operation of a liquid ejector, 149, 37 (1948)
10. B.S. Stefanovski, RIJIT, Elements of the theory of an ejector with a cylindrical mixing chamber, 21, 177-217 (1958)
11. Y.L. Kirillovski, L.G. Podvidz, VIGM, Workflow and the basics of calculating jet pumps, 26, 96-135 (1960)
12. D.N. Popov, Mechanical engineering, Dynamics and regulation of hydraulic and pneumatic systems, 464 (1987)
13. M.H. Karapetyants, S.I. Drakin, VS, The structure of matter, 2, 312 (1970)
14. G. Taylor, MIR, Fundamentals of organic chemistry for students of non-chemical specialties, 384 (1989)
15. V.P. Bogdanovich, VNIPTIMES, Technique of engineering calculation of ejectors with variable parameters, 116-122 (1983)