A method of simulated motion tracking control based on Lyapunov vector field

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Abstract. For the problem of irregular formation in aircraft cluster planning, this paper proposes a simulated motion tracking control of cluster planning. The simulated motion is the expected motion trajectory for one aircraft provided by the planning system. Using Lyapunov vector field guidance and omnidirectional speed control method, this paper realizes the tracking control of the simulated motion for the aircraft. The simulation results demonstrate that the proposed method can realize the tracking control of the simulated motion and achieve the expected effect, which lays a foundation for the cluster planning.

1. Introduction

In recent years, with the continuous improvement of unmanned aerial vehicle technology, the unmanned aerial vehicle cluster technology has shown a rapid development trend. Those cluster technologies simulated biological behavior such as fish, birds, ant colonies and bee colonies have developed rapidly [1-5], which stems from humans' longing for nature. At the same time, the cooperative control has become more and more active in the field of unmanned aerial vehicles [6].

In general, multi-aircraft cooperative control technology can be divided into two major categories, one of which is cluster consistency control and another is SimAnimals. The anterior one belongs to the control problem which is converted cluster control problem into a linear system. Then the relatively mature matrix theory and modern control methods are utilized to achieve the control of the cluster, however, this kind of control method is relatively weak for the flexibility of the cluster. Although some scholars are carrying out research in this area, Compared to animal groups such as flocks and bee colonies, simulation of animal cluster control is more flexible for cluster movement and more suitable for practical applications, but the research of simulating animal cluster movement is still a frontier topics and the research results are not many. Some new enlightenment need to be obtained through interdisciplinary research.

For the cluster control problems, the simulated motion tracking control method for the planning system is researched and the simulated motion tracking control method based on Lyapunov vector field is proposed. This method can realize the stable tracking of the desired aircraft heading and speed given by the planning system. It enables the cluster aircrafts to fly in formation according to the planned results.

The structure of this paper is as follows: the first subsection gives the research background and conclusion. The second subsection deduces the Lyapunov vector field. The third subsection deduces the omnidirectional velocity regulation law. The fourth subsection gives the mathematical simulation results. The fifth subsection gives the paper in conclusion.
2. Lyapunov Vector Field Guidance Method

2.1. Guidance Vector Field Construction

Assume that the equation of motion of the simulated motion motion is

\[
\begin{align*}
\dot{x}_e &= V_e \cos \psi_f \\
\dot{y}_e &= V_e \sin \psi_f
\end{align*}
\]

(1)

Where, \([x_e, y_e]\) is the position coordinates of the simulated motion in the plane. \(V_e\) is The desired speed of the simulated motion. \(\psi_f\) is expected heading of the simulated motion.

As shown in Figure 1, the tracking transition region is set to be \(r\). The heading moving toward simulated motions is perpendicular to moving direction of the simulated motion when the transition region is outside, so that the time to reach the desired straight line can be shortened. So the desired heading control law is

\[
\psi_d = \psi_f - \rho \psi_e
\]

(2)

Where, \(\rho\) is judgment parameter which determines that the aircraft is in which side of the line that the simulated motion is on. \(\psi_e\) is the term of adjustment heading which provides the right heading \(\psi_d\) for aircraft.

When the aircraft is in the transition zone, the heading angle needs to be adjusted according to the distance from the aircraft to the line where the simulated motion is located, so that the aircraft gradually approaches the desired straight line. So we have

\[
\psi_d = \psi_f - \rho \psi_e \left(\frac{d}{r}\right)^k
\]

(3)

Where, \(k\) is the control parameter. \(d\) is the distance with the symbol \(\pm\) from the aircraft to the desired straight line. So it can be judged the aircraft on which side of the line based on the distance \(d\) and then the value \(\rho\) can be obtained and the judgment rule is

\[
\begin{aligned}
\rho &= 1, \quad d \geq 0 \\
\rho &= -1, \quad d < 0
\end{aligned}
\]

(4)

So far, the design processes of tracking control are completed.

Figure 1. Tracking control schematic.
2.2. Guidance Vector Field Construction

The heading rate of aircraft is described by the following first-order dynamics

$$\dot{\psi} = \alpha \left( \psi_d - \psi \right)$$  \hspace{1cm} (5)

Where, $\psi$ is the heading variable of the aircraft. $\psi_c$ is the control input of the aircraft. Then the following theorem exists.

**Theorem 1**: As shown in figure 1, for the case where the aircraft is outside the transition area, which is $[|d| \geq r]$. And with the heading rate dynamics equation, heading command, if $\alpha > 0$, heading error is $\dot{\psi} \rightarrow 0$ asymptotically.

**Proof**: Outside the transition area, $\psi_d$ is constant. So let

$$\psi_d = \psi_c$$  \hspace{1cm} (6)

Then make the heading error is

$$\dot{\psi} = \psi_d - \psi$$  \hspace{1cm} (7)

Its Derivative is

$$\dot{\dot{\psi}} = -\dot{\psi} = -\alpha \dot{\psi}$$  \hspace{1cm} (8)

Lyapunov function is selected

$$V = \frac{1}{2} \dot{\psi}^2$$  \hspace{1cm} (9)

Derived from the above formula, we have

$$\dot{V} = \psi \ddot{\psi}$$  \hspace{1cm} (10)

And then

$$\dot{V} = -\alpha \dot{\psi}^2$$  \hspace{1cm} (11)

Due to $\alpha > 0$, so $\dot{V} \leq 0$. The heading command vector field is Lyapunov stable.

Considering the situation within the transition region, the following theorem also exists.

**Theorem 2**: As shown in figure 1, for the case where the aircraft is in the transition area, which is $[|d| < r]$. And with the heading rate dynamics equation, heading command, if $\alpha > 0$, heading error is $\dot{\psi} \rightarrow 0$ asymptotically.

**Proof**: When the aircraft enters the transition zone, it is desirable for the aircraft to transition from the adjusted heading $\psi_c$ toward the desired heading $\psi_f$ at this time.

Lyapunov function is selected

$$V = \frac{1}{2} \dot{\psi}^2$$  \hspace{1cm} (12)

Its Derivative is

$$\dot{V} = \dot{\psi} \ddot{\psi} = \dot{\psi} (\dot{\psi}_d - \dot{\psi})$$

$$= \dot{\psi} \left[ \psi_d - \alpha (\psi_c - \psi) \right]$$

$$= \psi \left[ -\frac{\psi_c k V_c \sin \psi}{r^k} (\rho d)^{k-1} - \alpha (\psi_c - \psi) \right]$$  \hspace{1cm} (13)

Let

$$\psi_c = \psi_d - \nu$$  \hspace{1cm} (14)

So

$$\dot{V} = \dot{\psi} \left[ -\frac{\psi_c k V_c \sin \psi}{r^k} (\rho d)^{k-1} - \alpha \dot{\psi} + \alpha \nu \right]$$  \hspace{1cm} (15)

Then let

$$\nu = -\frac{\psi_c k V_c \sin \psi}{r^k} (\rho d)^{k-1}$$  \hspace{1cm} (16)

Then we have
\[ \dot{\psi} = -\alpha \hat{\psi}^2 \]

\[ \psi_e = \psi_d - \left( -\frac{\psi_e k V_e \sin \psi}{r^k} \right) \]  \hspace{1cm} (17)

For any \( \alpha > 0 \), the derivative of the Lyapunov function is negative. Therefore, if the vector field (3) satisfies the heading error is \( \hat{\psi} \to 0 \) asymptotically in the transition region.

In addition, it is necessary to give a description that when \( \hat{\psi} \to 0 \), then \( d \to 0 \), which can ensure when the heading error trends to zero, the aircraft can converge to the desired route. So, there is a following theorem.

**Theorem 3:** As shown in figure 1, for the case where the aircraft is in the transition area, which is \( |d| < r \). And with the heading rate dynamics equation, heading command, if \( \alpha > 0 \) and heading error is \( \hat{\psi} \to 0 \) asymptotically, then the distance from the aircraft to the desired route is \( d \to 0 \).

**Proof:** A coordinate system in the direction of the simulated motion is created, so we can get \( \psi_f = 0 \).

Lyapunov function is selected with \( d \) and \( \hat{\psi} \), namely

\[ V(d, \hat{\psi}) = \frac{1}{2} d^2 + \frac{1}{2} \hat{\psi}^2 \]  \hspace{1cm} (18)

Its Derivative is

\[ \dot{V}(\hat{\psi}) = d \dot{d} + \dot{\psi} \hat{\psi} = d V_e \sin \psi + \hat{\psi} \left( -\frac{\psi_e k V_e \sin \psi}{r^k} \right) - \alpha (\psi_e - \psi) \]

\[ = d V_e \sin (\psi_e - \hat{\psi}) - \alpha \psi_e \hat{\psi}^2 \]

\[ = d V_e \sin \left( -\psi_e \left( \frac{\rho d}{r} \right)^k \right) - \alpha \psi_e \hat{\psi}^2 \]  \hspace{1cm} (19)

When it is satisfied that

\[ |\hat{\psi}| < \psi_e \left( \frac{\rho d}{r} \right)^k \]  \hspace{1cm} (20)

Then \( \dot{V}(\hat{\psi}) \) is negative.

Thus the set of headings \( \psi \) is the following invariant set \( R \), and the expression is

\[ R = \left\{ \psi \left| -2 \psi_e \left( \frac{\rho d}{r} \right)^k < \psi < 0, 0 < d < r \right. \right\} \]  \hspace{1cm} (21)

\( \dot{d} \) is estimated at the boundary \( R \).

For the case \( \psi = -2 \psi_e \left( \frac{\rho d}{r} \right)^k \)

\[ \dot{d} = V_e \sin \left( -2 \psi_e \left( \frac{\rho d}{r} \right)^k \right) < 0 \]  \hspace{1cm} (22)

For the case \( \psi = 0 \)

\[ \dot{d} = V_e \sin (0) = 0 \]  \hspace{1cm} (23)

For the case \( \psi = 0 \), derivative \( \dot{d} \) is not negative. However, since the collection is an open set, \( \psi \neq 0 \).

While \( \psi < 0 \), \( d < 0 \). Because the collection is an invariant set, so If the condition in equation is satisfied, it meets the negative requirement that is the vector field is Lyapunov stable.

3. **Omnidirectional Speed Control Method**

The formula for calculating distance \( d \) with sign is
\[ d = \frac{y - \tan \psi_d x - y_0^e - \tan \psi_d x_0^e}{\sqrt{1 + \tan^2 \psi_d}} \] (24)

Where, \((x_0^e, y_0^e)\) is the initial value for simulated motion. And the linear equation of the simulated motion is

\[ y = \tan \psi_d x + y_0^e - \tan \psi_d x_0^e \] (25)

where, \([x, y]\) is the position coordinates of the aircraft in the plane.

After heading adjustment is realized, it is also necessary to adjust the speed according to the distance between the aircraft and the simulated motion, so that the tracking purpose can be achieved. Adjustment rule is

\[ V_{\text{true}} = k_v \frac{2}{\pi} \arctan (\rho_d d) \] (26)

Where, the parameter \(\rho_1\) is a adjustment term, and \(d\) is the distance from the line of simulated motion to the coordinate of aircraft, and the adjustment rules are as follows

\[
\rho_1 = \begin{cases} 
\text{sign}(x-x_c), & 0 \leq \psi_d \leq \frac{\pi}{4} \\
\text{sign}(y-y_c), & \frac{\pi}{4} < \psi_d \leq \frac{3\pi}{4} \\
-\text{sign}(x-x_c), & \frac{3\pi}{4} < \psi_d \leq \frac{3\pi}{2} \\
-\text{sign}(y-y_c), & \frac{3\pi}{2} < \psi_d \leq \frac{7\pi}{4} \\
\text{sign}(x-x_c), & \frac{7\pi}{4} < \psi_d \leq 2\pi 
\end{cases} \] (27)

The distance \(d\) without sign is calculated as follows.

\[ d = \sqrt{(x-x_c)^2 + (y-y_c)^2} \] (28)

where, \([x_c, y_c]\) is the position coordinates of the simulated motion in the plane.

4. Simulation

The initial position of the aircraft is set \((0,20)\). The initial heading of the aircraft is 0 deg. The initial speed of the aircraft is 50m/s. The initial position of the simulated motion is \((60,0)\). The initial heading of the simulated motion is 20 deg. The initial speed of the simulated motion is 80m/s. The time of simulation is 10s. The results of simulation are listed as follows.

**Figure 2.** Simulated motion tracking control.

**Figure 3.** Speed adjustment control.
Figure 4. Distance between the aircraft and the simulated motion.

Mathematical simulation results analysis as follows:

1) Figure 2 shows when the direction and speed of simulated motion are given, the aircraft can complete to track the simulated motion at different locations.

2) Figure 3 shows the aircraft adjusts the speed depending upon the distance from the simulated motion to the aircraft in the process of tracking the simulated motion. When the distance is long, the aircraft is accelerated. When the distance is short, the aircraft is decelerated until it is consistent with the speed of the simulated motion.

3) Figure 4 shows that the distance between the aircraft and the simulated motion is continuously reduced and finally remains at zero during the simulated motion tracked by the aircraft.

In summary, the tracking control method of simulated motion proposed by this paper can realize the tracking a given simulated motion by path planning system for the aircraft.

5. Simulation

Aiming at the problems in cluster planning, this paper proposes a simulated motion tracking control method, which can ensure that the aircraft tracks the track produced by planning system, which can lay a foundation for subsequent formation flight control.

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