On the plane wave scattering from a circular cylinder with core or coating made of ENZ and DNZ mediums

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Abstract

The phase profiles of the field components and the scattering widths of a circular cylinder with coating or core made of epsilon-near-zero (ENZ) and double-near-zero (DNZ) mediums for normally incident plane wave are investigated. It was found that the axial components generally had a uniform phase inside a bare ENZ or DNZ cylinder for both the TE and TM polarization states in the cylinders with small as well large radii. The azimuthal components could have a slowly varying phase only for large cylinders. When the ENZ or DNZ mediums were used as cores or coatings, the uniformity of the phases decreased. Generally, DNZ mediums have more slowly varying phases than the ENZ mediums, and the TE polarization has a more uniform phase than the TM polarization. The scattering widths were found to be the smallest for DNZ-coated dielectric cylinder and the largest for dielectric-coated ENZ or DNZ cylinder. This study can provide a guideline for the thickness of the near-zero index medium in device design since different thicknesses are needed for achieving uniform phase inside the ENZ or DNZ medium for a particular component of the electric and magnetic fields in the three configurations studied in this paper.

1. Introduction

The optical metamaterials can be divided into two broad categories with respect to the real part of the values of the refractive index: (i) negative refractive index and (ii) near-zero index materials. The enabling property of the first type of metamaterials is the negative phase velocity of the plane waves propagating in the negative index mediums [1]. This property results in several fascinating applications such as superlens [2], photonic band gaps with averaged zero refractive index [3], the reversal of Snell’s law [4], perfect absorber [5], reduction of the size of patch antennas [6], and invisible cloaking [7], to name a few.

The enabling property of the second type of metamaterials, that is, near-zero index materials, is the spatially uniform phase of the electromagnetic field within the materials. This property has inspired several applications such as tunneling of electromagnetic energy through subwavelength channels and bends [8], super-reflection and cloaking [9], total transmission and total reflection even in the presence of defects [10], and tailoring the phase of the radiation pattern of arbitrary sources for canonical geometries [11]. The effective use of near-zero index materials may give the desired phase distribution in the scattered fields from various geometries such as planar slabs, cylindrical shells, lenses with various exit faces [11], and wide-angle transmission [12]. To exploit the properties of the near-zero-index mediums, the imaginary part of the refractive index must be small. However, the metamaterials with real part of the refractive index near zero, but high imaginary part (either due to the negative real part of the permittivity or permeability) also offers many interesting properties like photonic tunneling [13] and photonic spin Hall effect [14].

The near-zero index mediums can be either epsilon-near-zero (ENZ), mu-near-zero (MNZ), or double-near-zero (DNZ) mediums. The ENZ mediums can be realized by electrically small inclusions of metallic particles in a host dielectric material and DNZ mediums can be realized as a photonic crystal tuned so that a
Dirac-like cone dispersion is obtained at the \( \Gamma \)-point in the photonic band structure \([15, 16]\) under appropriate conditions \([17, 18]\).

The applications of near-zero index mediums require the spatial uniformity of the phase inside the metamaterial that can give rise to the interesting properties of the scattered fields. Several researchers have examined the phase of fields in some near-zero index mediums such as \([11]\) (ENZ coated cylinder, ENZ slab, ENZ lens) and \([19]\) (ENZ coated air cavity). All the devices employing near-zero index of the medium engineer the phase-front of the transmitted or reflected waves. The phase front of the transmitted light has the same shape as the output surface since the phase of the wave is the same everywhere inside the near-zero medium. Therefore, for the device-design, it is very important to investigate the phase inside the near-zero medium. However, no reports exist of systematic study of the phase of the fields inside ENZ, MNZ, and DNZ mediums. Therefore, we set out to investigate the phases of the fields inside the core–shell cylinders of the near-zero mediums. Our goal is to find the conditions required to have a spatially uniform phase inside the near-zero index mediums, since it may not always be the case throughout the near-zero index medium. This is very essential if we want to design the devices utilizing near-zero index mediums. We chose the cylindrical object for this purpose because most of the applications of these mediums are on-chip applications where the light propagation is confined to a plane and the different optical components are cylindrical in nature. Moreover, the study of scattering from objects coated with metamaterials is important in its own right, e.g., in radar design, communication and detection problems. The coatings are often used to control the scattering behavior such as radar cross section (RCS) reduction, RCS enhancement and the focusing of the scattered field etc \([20]\). For instance, the planewave scattering by a conducting cylinder coated with metamaterials is studied by Shen and Li \([21]\), the scattering by an eccentric coated cylinder by Musharaf \([22]\), an array of parallel concentric coated cylinders are considered by Henin et al \([23]\), transparency and maximization of the scattering were achieved using metamaterial-coated conducting cylinder by Irci and Ertürk \([24]\), and multilayered coated cylinder with double negative metamaterials was considered by Yao et al \([25]\).

The plan of the paper is as follow: in section 2, theoretical formulation of the problem is given briefly for both the TE and TM cases with both the core and shell having arbitrary permeability and permittivity. Numerical results are reported and discussed in section 3. The concluding remarks are presented in section 4. An \( \exp(-i\omega t) \) time dependency is assumed and is suppressed throughout the analysis, with \( \omega \) being the angular frequency, \( t \) being the time, and \( i = \sqrt{-1} \). The free-space permittivity and permeability are denoted by \( \varepsilon_0 \) and \( \mu_0 \), respectively.

### 2. Theoretical formulation

Let us consider the boundary-value problem, shown schematically in figure 1. A linearly polarized planewave propagating in the xy plane is normally incident upon a coated cylinder. The background medium is assumed to be occupied by an isotropic dielectric-magnetic medium with permittivity \( \varepsilon_3 = \varepsilon_0 \varepsilon_{31} \) and permeability \( \mu_3 = \mu_0 \mu_{31} \). The core cylinder with radius \( a \) is occupied by an isotropic dielectric-magnetic medium with relative permittivity \( \varepsilon_2 = \varepsilon_0 \varepsilon_{23} \) and relative permeability \( \mu_2 = \mu_0 \mu_{23} \). The core is coated by a layer of isotropic dielectric-magnetic medium with relative permittivity \( \varepsilon_1 = \varepsilon_0 \varepsilon_{13} \) and relative permeability \( \mu_1 = \mu_0 \mu_{13} \), and the thickness of the coating layer is \( d \). Since we only consider the normal incidence, the transverse electric (TE) and
the transverse magnetic (TM) polarized planewaves decouple from each other and are considered separately in the following subsections.

2.1. TE polarization

Let us begin with the TE-polarized incident planewave with its magnetic field along the $z$ axis and electric field in the $xy$ plane. The magnitude of the magnetic field phasor of the incident wave in the cylindrical coordinates $(\rho, \phi, z)$ can be expressed as [26]

$$H_z^{inc} = E_0 \sum_{n=-\infty}^{\infty} i^n J_n(k_\ell \rho) e^{in\phi}, \quad \rho > a + d,$$

where $J_n$ is the Bessel function of order $n$, and $E_0 = 1$ is the amplitude of the incident wave, and

$$k_\ell = \omega \sqrt{\varepsilon_\ell \varepsilon_r}, \quad \ell \in \{1, 2, 3\}.$$  

Using Maxwell equations, the $\phi$ component of the electric field phasor can be found as

$$E^{inc}_\phi = \frac{k_1}{k_0 \varepsilon_1} E_0 \sum_{n=-\infty}^{\infty} i^n J_n'(k_1 \rho) e^{in\phi}, \quad \rho > a + d,$$

where $'$ represents the derivative with respect to argument. The components of the field phasors for the scattered field can be written as

$$H^s_\rho = E_0 \sum_{n=-\infty}^{\infty} A_n i^n H_n^{(1)}(k_\rho) e^{in\phi}$$

$$E^s_\phi = \frac{k_0}{k_0 \varepsilon_1} E_0 \sum_{n=-\infty}^{\infty} A_n i^n H_n^{(1)'}(k_\rho) e^{in\phi},$$

where $A_n$ are the unknown coefficients and $H_n^{(1)}$ are the Hankel functions of the first kind. In the coating region, the electric and magnetic field phasors consisting of the outgoing and incoming waves can be expressed in terms of unknown coefficients $B_n$ and $C_n$ as

$$H^s_\rho = E_0 \sum_{n=-\infty}^{\infty} i^n [B_n H_n^{(2)}(k_\rho) + C_n H_n^{(1)}(k_\rho)] e^{in\phi}$$

$$E^s_\phi = \frac{k_0}{k_0 \varepsilon_1} E_0 \sum_{n=-\infty}^{\infty} i^n [B_n H_n^{(2)'}(k_\rho) + C_n H_n^{(1)'}(k_\rho)] e^{in\phi},$$

when $a < \rho < a + d$, where $H_n^{(2)}$ are Hankel functions of the second kind. The components of the field phasors inside the core region can be written as

$$H^s_\rho = E_0 \sum_{n=-\infty}^{\infty} i^n D_n J_n(k_\rho) e^{in\phi}$$

$$E^s_\phi = \frac{k_0}{k_0 \varepsilon_1} E_0 \sum_{n=-\infty}^{\infty} i^n D_n J_n'(k_\rho) e^{in\phi},$$

where $D_n$ are the unknown coefficients.

The use of the standard boundary conditions at the interfaces $\rho = a$ and $\rho = a + d$ results in a system of four algebraic equations. These equations can be rearranged to find the unknown coefficients as

$$\begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix} = W^{-1} \begin{bmatrix} -J_n(k_b) \\ 0 \\ -J_n'(k_b) \\ 0 \end{bmatrix},$$

where

$$W = \begin{bmatrix} H_n^{(1)}(k_b) & -H_n^{(2)}(k_b) & -H_n^{(1)}(k_0) & 0 \\ 0 & H_n^{(2)}(k_0) & H_n^{(1)}(k_0) & -J_n(k_b) \\ \frac{k_0}{\varepsilon_1} H_n^{(1)'}(k_b) & \frac{k_0}{\varepsilon_2} & -\frac{k_0}{\varepsilon_2} & 0 \\ 0 & \frac{k_0}{\varepsilon_2} & \frac{k_0}{\varepsilon_3} & -\frac{k_0}{\varepsilon_3} \end{bmatrix}. $$

3
2.2. TM polarization

Let us now consider the TM-polarized incident planewave with its magnetic field entirely in the xy plane and the electric field along the z axis. The magnitude of the electric field phasor and the \( \phi \) component of the magnetic field phasor of the incident planewave can be written as

\[
\begin{align*}
E_x^{inc} &= E_0 \sum_{n=-\infty}^{\infty} i^n J_n(k_0 \rho) e^{i\phi} \\
H_\phi^{inc} &= -\frac{k_0}{\omega \mu_1} E_0 \sum_{n=-\infty}^{\infty} i^n n J_n(k_0 \rho) e^{i\phi} 
\end{align*}
\]

where \( E_0 = 1 \) is the magnitude of the incident wave. The components of the scattered field phasors can be written as

\[
\begin{align*}
E_x^s &= E_0 \sum_{n=-\infty}^{\infty} A_n i^n J_n^{(1)}(k_\rho) e^{i\phi} \\
H_\phi^s &= -\frac{k_0}{\omega \mu_2} E_0 \sum_{n=-\infty}^{\infty} A_n i^n n J_n^{(1)}(k_\rho) e^{i\phi} 
\end{align*}
\]

where \( A_n \) are unknown coefficients. The components of the field phasors in the coating region can similarly be written using unknown coefficients \( B_n \) and \( C_n \) as

\[
\begin{align*}
E_x^c &= E_0 \sum_{n=-\infty}^{\infty} i^n [B_n J_n^{(2)}(k_\rho) + C_n J_n^{(3)}(k_\rho)] e^{i\phi} \\
H_\phi^c &= -\frac{k_0}{\omega \mu_3} E_0 \sum_{n=-\infty}^{\infty} i^n [B_n J_n^{(2)'}(k_\rho) + C_n J_n^{(3)'}(k_\rho)] e^{i\phi} 
\end{align*}
\]

when \( a < \rho < a + d \), and the components in the core can be written as

\[
\begin{align*}
E_x^c &= E_0 \sum_{n=-\infty}^{\infty} i^n D_n J_n(k_\rho) e^{i\phi} \\
H_\phi^c &= -\frac{k_0}{\omega \mu_1} E_0 \sum_{n=-\infty}^{\infty} i^n D_n J_n'(k_\rho) e^{i\phi} 
\end{align*}
\]

where \( D_n \) are unknown coefficients. To find the unknown coefficients, the continuity of the tangential components of the fields are imposed at the interfaces at \( \rho = a \) and \( \rho = a + d \) to get

\[
\begin{bmatrix}
A_n \\
B_n \\
C_n \\
D_n
\end{bmatrix} = W^{-1}
\begin{bmatrix}
-J_n(k_1 b) \\
0 \\
-k_0 J_n(k_3 b) \\
0
\end{bmatrix}
\]

where

\[
W =
\begin{bmatrix}
J_n^{(1)}(k_1 b) & -J_n^{(2)}(k_1 b) & -J_n^{(1)}(k_2 b) & 0 \\
0 & J_n^{(2)}(k_2 a) & J_n^{(1)}(k_2 a) & -J_n(k_3 a) \\
\frac{k_1}{\mu_1} J_n^{(1)}(k_1 b) & \frac{k_2}{\mu_2} J_n^{(1)}(k_2 b) & -\frac{k_3}{\mu_3} J_n(k_3 a) & 0 \\
0 & \frac{k_2}{\mu_2} J_n^{(2)}(k_2 a) & \frac{k_3}{\mu_3} J_n^{(2)}(k_3 a) & -\frac{k_1}{\mu_1} J_n(k_1 a)
\end{bmatrix}
\]

3. Numerical results

We will now present and discuss numerical results showing the phases of the fields inside the coated cylinder for various combinations of ENZ and DNZ mediums in the core and coating. The results for replacing ENZ mediums with MNZ are not presented here as it simply interchanges the results of the TE and TM cases for the corresponding results of the ENZ medium. For this purpose, the system of equations (7) and (13) were implemented in Matlab to find the unknown coefficients and then compute fields inside the core, coating, and outside of the coated cylinder using equations (1)–(6) for the TE and equations (9)–(12) for the TM case. Also, the scattering widths defined by [26]
The programs for both the TE and TM cases were verified by computing \( \sigma_{\text{TE, TM}} \) and compared with figure 3 of [25] for an air cavity coated with dielectric material. There was an excellent agreement for both the TE and TM polarizations confirming the accuracy of the numerical implementation.

Our goal of the manuscript is twofold: study the phases inside the near-zero index mediums to see the effect of thicknesses as a guide for designing the devices and the scattering properties of the core–shell cylinders made of near-zero mediums. Therefore, in the following sections, we present the results for the phases and scattering widths for the three possible cases: (i) when the cylinder is made of only a near zero medium (ENZ or MNZ, or DNZ cylinder), (ii) when the near-zero medium is made up the core and coated by a dielectric-magnetic medium, and (iii) when the near-zero medium coats a core of dielectric medium. Let us now present the results for bare cylinders before presenting the results for coated cylinders.

### 3.1. ENZ cylinder

Figure 2 shows the phase of the fields inside and outside an ENZ cylinder for an incident planewave of TE polarization state. An electrically small as well as a large cylinder is considered, when \( \mu_{\text{r1}} = \mu_{\text{r2}} = \mu_{\text{r3}} = 1, \) \( \epsilon_{\text{r1}} = \epsilon_{\text{r2}} = 1, \) and \( \epsilon_{\text{r3}} = 0.001. \) The figure shows that the phase of the electric field is not uniform when \( a = 0.4 \lambda_0; \) however, the phase of \( H_z \) is uniform almost throughout the cylinder when \( a = 0.4 \lambda_0 \) and \( 5 \lambda_0 \) except in a small region on left where the planewave was incident. The phase of \( E_y \), however, changes abruptly midway, though the rate of change of the phase inside the bigger cylinder is much slower than outside. Therefore, it can be concluded that the phase of \( H_z \) is almost uniform inside the ENZ cylinder if the cylinder radius is much larger than the operating wavelength, but the phase of \( E_y \) changes inside the ENZ cylinder for TE polarized incident planewave.

To see the effect of the polarization of the incident wave on the phase inside the ENZ cylinder, the phases of the fields are given in figure 3 for the TM polarized incident planewave. The figure shows that the phase of \( E_z \) is almost uniform inside the cylinder when its radius is either \( 0.4 \lambda_0 \) or \( 5 \lambda_0. \) The phase of \( H_y \) has an abrupt change inside the cylinder near the left side where the planewave was incident for \( a = 5 \lambda_0 \) but the phase is not uniform.
inside the cylinder with radius $a = 0.4\lambda_0$. A comparison of figures 2 and 3 show that the $z$ components (axial components) of the fields are almost uniform inside the ENZ cylinder when the cylinder is large but the $\phi$ components (azimuthal components) are almost uniform only for the larger radius of the cylinder.

The scattering widths for several electrically small ENZ cylinder and a large ENZ cylinder are shown in figure 4. For large cylinder, the scattering widths have similar variations for both the polarization states but for the small cylinders, the scattering widths depend upon the polarization state. For both the small and the large cylinders, forward scattering widths ($\phi = 0$) is much larger than the rest of the space for both polarization states. The small cylinder has larger backscattering widths for the TE polarized waves than the TM polarized waves.
3.2. DNZ cylinder

Let us now consider a DNZ cylinder with \( m_1 = m_2 = 1, \epsilon_1 = \epsilon_2 = 1, \) and \( \epsilon_3 = \mu_3 = 0.001. \) The phase of the fields inside a DNZ cylinder are shown in figures 5 and 6 for TE and TM polarizations, respectively. For the TE case, figure 5 shows that the phase of \( H_z \) is uniform inside the smaller as well as larger DNZ cylinder. Similarly, for the TM case, figure 6 shows that the phase of \( E_z \) is uniform inside the smaller as well as larger DNZ cylinder. This is in line with the results for the ENZ cylinder. Therefore, for both the ENZ and the DNZ cylinder, the azimuthal components (\( \phi \) components) of the fields does not have a uniform phase when the cylinder was small and has abrupt change in the phase when the cylinder is large, though the phase change, in general, is much slower than the outside and the axial components has uniform phase inside the small as well large cylinder. The
scattering widths for the DNZ cylinders were computed (not shown in the paper) for the TE and TM polarized incident plane waves. The scattering widths for the DNZ cylinder were found to be similar in magnitude as for the ENZ cylinder of both the smaller as well as larger radius.

3.3. ENZ cylinder coated with dielectric medium
Now, let us consider an ENZ cylinder coated with dielectric material. The phases of the fields for the TE and TM polarized incident waves are given in figures 7 and 8, respectively, where $\mu_1 = \mu_2 = \mu_3 = 1$, $\epsilon_1 = 1$, $\epsilon_2 = 4$, and $\epsilon_3 = 0.001$. Both figures confirm the conclusion drawn from ENZ cylinder that $H_z$ has a uniform phase inside both the smaller as well larger ENZ cylinder coated with the dielectric material for the TE case; however, $E_z$ has uniform phase only inside the larger ENZ cylinder for the TM case. The azimuthal components of the fields...
for both cases do not have a uniform phase inside the smaller coated ENZ cylinders, but the phase varies slowly inside the larger cylinder. The scattering widths for the TE and TM cases for smaller as well larger ENZ cores are shown in figure 9. This figure shows that the forward scattering width is much larger than the scattering widths in other directions. A comparison of scattering widths of dielectric-coated ENZ cylinder (figure 9) with those of the bare ENZ cylinder (figure 4) shows that the scattering widths have increased due to the presence of the dielectric coating around the ENZ cylinder.

3.4. DNZ cylinder coated with dielectric or dielectric-magnetic material
To see the effect of the coating on DNZ cylinder, the phases of the fields are presented for a DNZ cylinder coated with dielectric material in figures 10 and 11 for the TE and TM polarized incident waves, respectively, when $\mu_{11} = \mu_{12} = \mu_{13} = 1$, $\varepsilon_{11} = 1$, $\varepsilon_{12} = 4$, and $\varepsilon_{13} = 0.001$. Inside the core, phase of the $H_z$ and $E_z$ field components is observed to be uniform for both the smaller as well larger cores. These results combined with the earlier observation regarding bare DNZ cylinder show that the axial components of the fields have uniform phases inside the DNZ medium for the smaller as well as larger size. However, the azimuthal components do not have uniform phase inside the small core DNZ cylinder and have an abrupt change in the phase inside the large DNZ cylinder, either coated (figures 10 and 11) or bare (figures 5 and 6). The scattering widths for DNZ cores coated with the dielectric material were found to be similar as for the ENZ cores coated with a dielectric medium. We also computed the results for the coating made of dielectric-magnetic medium for the TE and TM polarization states when $\varepsilon_{11} = \mu_{11} = 1$, $\mu_{12} = 2$, $\varepsilon_{12} = 5$, $\varepsilon_{13} = \mu_{13} = 0.001$ (not shown) and found that the same conclusions hold true as in the foregoing paragraph about the phases of the axial and azimuthal components of the fields. Let us note that the value of the relative permeability used here is a representative value. However, with the rapid progress in the design of the magnetic materials, it could soon be possible to design mediums with desired magnetic properties at optical frequencies [27].

3.5. Dielectric cylinder coated with an ENZ material
To see the phase of the fields in the ENZ medium coating a dielectric cylinder, the phase profiles are presented in figures 12 and 13 for the TE and TM polarization states, respectively, when $\mu_{11} = \mu_{12} = \mu_{13} = 1$, $\varepsilon_{11} = 1$, $\varepsilon_{12} = 0.001$, and $\varepsilon_{13} = 4$. For the TE incidence, the axial component has slowly varying phase when the coating thickness is small or large. The azimuthal component does not have a uniform phase for small thickness but has a slowly varying phase when the thickness is large. For the TM case, none of the field components has a uniform phase when the coating thickness is small and has slowly varying phase when the thickness is large, though the change in phase is much more near the interface and on the side of the incident plane wave.
The scattering widths for the dielectric cylinder coated with the ENZ medium in figure 14 show that the scattering widths depend strongly on the direction for the TE case. A comparison of this figure with the scattering widths from bare ENZ cylinder and dielectric-coated ENZ cylinder shows that the scattering widths are smaller, in general, when the dielectric cylinder is coated with ENZ medium than for either the bare ENZ cylinder (Figure 4) or coated ENZ cylinder (figure 9).

3.6. Dielectric cylinder coated with a DNZ medium

The phases of the fields for a dielectric cylinder coated with a DNZ medium were computed for the TE and TM polarization states but the results are presented only for the TE case in figure 15 when \( \mu_3 = 1, \mu_3 = 1, \varepsilon_{3} = 1 \),
For the TE excitation, the phase of the axial component is uniform inside the DNZ medium for both the small as well larger coatings. However, the azimuthal component had a slowly varying phase only inside the larger coating for the same polarization state. For the TM polarization state, the axial component had a uniform phase inside the larger coating but not inside the smaller coating. For the azimuthal component, the phase is not completely uniform inside the coating of any size for both the TE and TM cases.

The scattering widths for the dielectric cylinder coated with a DNZ medium are presented in figure 16. The figure shows that the scattering widths for the TE case show stronger dependence upon the direction. A comparison of these scattering widths with the scattering widths for the bare DNZ cylinder and DNZ cylinder coated with a dielectric medium shows that the scattering width of DNZ-coated dielectric cylinder is smaller.
than either the bare DNZ cylinder or dielectric-coated DNZ cylinder. A comparison of figure 16 with figure 14 shows that scattering width for DNZ-coated dielectric cylinders is smaller than the ENZ-coated dielectric cylinders as well.

To understand the effect of losses, we computed the phase profiles for a dielectric cylinder coated by the DNZ medium. For small losses, the phase profiles were unaffected. As the losses increased, the phase profiles begin to show more variations inside the DNZ medium. A representative profile is presented in figure 17 for the TE case when \( 0.1i \) is added to the relative permittivity and permeability of the DNZ medium. A comparison of
this figure with figure 15 for the lossless case shows that the profiles have begun to change. The similar effect of the losses was seen for bare ENZ and DNZ cylinders and dielectric-coated ENZ and DNZ cylinder that the small losses had a negligible effect on the phase but the large losses changed the phase profiles. The same conclusions were drawn for the TM polarization state that the phase uniformity tends to disappear with the increase in the losses.

4. Concluding remarks

The boundary-value problem of scattering from coated circular cylinder made of epsilon-near-zero (ENZ) and double-near-zero (DNZ) were solved and analyzed to gain insight into the scattering properties and the phase
uniformity inside the near-zero index mediums. When the uncoated cylinder of ENZ and DNZ mediums were considered, it was found that (i) the phase of the axial components of the fields inside the ENZ and DNZ mediums was nearly uniform for the smaller as well larger radii of the cylinders for both the TE and the TM incidence, and (ii) the azimuthal components was not uniform when the cylinders were small for both the ENZ and DNZ mediums and changed slowly inside the large ENZ and DNZ cylinders for both the TE and the TM incidence. Furthermore, the scattering widths for the DNZ cylinder were almost the same as for the ENZ cylinder for both the TE and TM incidence. Therefore, ENZ and DNZ cylinders have similar scattering characteristics.

When the ENZ cylinder was coated with a dielectric medium, the axial component had uniform phase inside both the small and large core for the TE incidence only and the axial components was not uniform inside the small-core coated ENZ cylinder for the TM incidence. The azimuthal components were again not uniform inside the small cores for both the TE and TM cases. Replacing the ENZ core with the DNZ core and making the coating either dielectric or dielectric-magnetic, the axial components of both the TE and TM cases were uniform inside the DNZ core for the smaller as well as larger radius. The azimuthal components were again found to have slowly varying phase only inside the larger cores. Therefore, the coated near-zero cylinders have better phase uniformity for the DNZ core than the ENZ core.

If the ENZ medium is used in the coating on a dielectric cylinder, the phase for the axial components is uniform only for the TE case for both the smaller as well as larger coating, but the azimuthal components are not uniform inside both the smaller and larger coating. For the TM case, the phase is somewhat uniform for larger coating for the axial component and the phase is varying inside both the smaller and larger coating for the azimuthal component. When the ENZ coating is replaced by the DNZ coating, the uniformity of phase improves slightly but the general conclusions of the ENZ coating hold. Parenthetically, we note that the improvement of the phase uniformity due to the DNZ medium could be due to the better impedance match between DNZ medium and its surrounding than the ENZ medium. It was seen that the scattering widths of the ENZ or DNZ cylinder coated with a dielectric medium were generally the highest. The scattering widths can be reduced, in general, by coating a dielectric cylinder with either the ENZ or DNZ medium. Furthermore, the scattering widths for the DNZ-coated dielectric cylinders were found to be the smallest.

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References

[1] Veselago V G 1968 The electrodynamics of substances with simultaneously negative values of ε and μ Sov. Phys. Usp. 10 509
[2] Pendry J B and Smith D R 2006 The quest for the superlens Scientific Am. 295 60–7
[3] Li J, Zhou L, Chan C T and Sheng P 2003 Photonic band gap from a stack of positive and negative index materials Phys. Rev. Lett. 90 085901
[4] Smith D R, Padilla W J, Vier D C, Nasse N, Syrus C and Schultz S 2000 Composite medium with simultaneously negative permeability and permittivity Phys. Rev. Lett. 84 1148
[5] Zhang X and Wu Y 2017 Scheme for achieving coherent perfect absorption by anisotropic metamaterials Opt. Express 25 4860–74
[6] Xu W, Li W, Yao H Y, Yeo T S and Wu Q 2005 Left-handed material effects on waves modes and resonant frequencies: filled waveguide structures and substrate-loaded patch antennas J. Electromagn. Waves Appl. 19 2033–47
[7] Cho A 2006 Voila! Cloak of invisibility unveiled Science 314 403
[8] Silveirinha M and Engheta N 2006 Tunneling of electromagnetic energy through subwavelength channels and bends using ε-near-zero materials Phys. Rev. Lett. 97 157403
[9] Hao J, Yan W and Qiu M 2010 Super-reflection and cloaking based on zero index metamaterial Appl. Phys. Lett. 96 101109
[10] Nguyen V C, Chen L and Halterman K 2010 Total transmission and total reflection by zero index metamaterials Phys. Rev. Lett. 105 233908
[11] Alù A, Silveirinha M G, Saldanrin A and Engheta N 2007 Epsilon-near-zero metamaterials and electromagnetic sources: tailoring the radiation phase pattern Phys. Rev. B 75 155410
[12] Yang R, Yang P, Chen Y, Li J and Lei Z 2018 Wide-angle transmissions of electromagnetic fields through the sandwiched transparent epsilon-near-zero metamaterial screen Opt. Lett. 43 35–8
[13] Alù A and Engheta N 2003 Pairing an epsilon-negative slab with a mu-negative slab: resonance, tunneling and transparency IEEE Trans. Antennas Propagat. 51 2558–71
[14] Guo Z, Jiang H, Long Y, Yu K, Ren J, Xue C and Chen H 2017 Photonic spin Hall effect in waveguides composed of two types of single-negative metamaterials Sci. Rep. 7 7742
[15] Huang X, Lai Y, Hang Z H, Zheng H and Chan C T 2011 Dirac cones induced by accidental degeneracy in photonic crystals and zero-refractive-index materials Nat. Mat. 10 582–6
[16] Li Y, Wu Y, Chen X and Mei J 2013 Selection rule for Dirac-like points in two-dimensional dielectric photonic crystals Opt. Express 21 7699–711
[17] Ashraf M W and Faryad M 2015 Dirac-like cone dispersion in two-dimensional core–shell dielectric photonic crystals J. Nanophoton. 9 093057
[18] Ashraf M W and Faryad M 2016 On the mapping of Dirac-like cone dispersion in dielectric photonic crystals to an effective zero-index medium J. Opt. Soc. Am. B 33 1008–13
[19] Ziolkowski R W 2004 Propagation in and scattering from a matched metamaterial having a zero index of refraction Phys. Rev. E 70 04668
[20] Arslanagic S, Ziolkowski R W and Breinbjerg O 2006 Excitation of an electrically small metamaterial-coated cylinder by an arbitrarily located line source Microwave Opt. Technol. Lett. 48 2598–606
[21] Shen Z and Li C 2003 Electromagnetic scattering by a conducting cylinder coated with metamaterials Prog. Electromagn. Res. 42 91–105
[22] Mushref M A 2007 Closed solution to electromagnetic scattering of a plane wave by an eccentric cylinder coated with metamaterials Opt. Commun. 270 441–6
[23] Henin B H, Al Shkarky M H and Elsherbeni A Z 2007 Scattering of obliquely incident plane wave by an array of parallel concentric metamaterial cylinders Prog. Electromagn. Res. 77 285–307
[24] Irci E and Ertürk V B 2007 Achieving transparency and maximizing scattering with metamaterial-coated conducting cylinders Phys. Rev. E 76 056603
[25] Yao H-Y, Li L-W, Qiu C-W, Wu Q and Chen Z-N 2007 Scattering properties of electromagnetic waves in a multilayered cylinder filled with double negative and positive materials Radio Sci. 42 RS2006
[26] Balanis C A 2012 Advanced Engineering Electromagnetics (New York: Wiley)
[27] Linden S, Enkrich C, Dolling G, Klein M W, Zhou J, Koshny T, Soukoulis C M, Burger S, Schmidt F and Wegner M 2006 Photonic metamaterials: magnetism at optical frequencies IEEE J. Sel. Topics Quantum Electron. 12 1097–105