Search for Sub-eV Sterile Neutrinos in the Precision Multiple Baselines Reactor Antineutrino Oscillation Experiments

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Abstract

According to their different effects on neutrino oscillations, the unitarity violation in the MNSP matrix can be classified into direct unitarity violation and indirect unitarity violation which are induced by the existence of the light and the heavy sterile neutrinos respectively. Of which sub-eV sterile neutrinos are of most interesting. We study in this paper the possibility of searching for sub-eV sterile neutrinos in the precision reactor antineutrino oscillation experiments with three different baselines at around 500 m, 2 km and 60 km. We find that the antineutrino survival probabilities obtained in the reactor experiments are sensitive only to the direct unitarity violation and offer very concentrated sensitivity to the two parameters $\theta_{14}$ and $\Delta m_{41}^2$. If such light sterile neutrinos do exist, the active-sterile mixing angle $\theta_{14}$ could be acquired by the combined rate analysis at all the three baselines and the squared mass difference $\Delta m_{41}^2$ could be obtained by taking the Fourier transformation to the $L/E$ spectrum. Of course, for such measurements to succeed, both the high energy resolution and the large statistics are essentially important.

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I. DIRECT AND INDIRECT UNITARITY VIOLATION IN THE LEPTON FLAVOR MIXING MATRIX

Besides the three known active neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$, there may exist additional sterile neutrinos which do not directly take part in the weak interactions except those induced by mixing with the active neutrinos [1]. In the presence of $n$ generations of sterile neutrinos, the $3 \times 3$ Maki-Nakagawa-Sakata-Pontecorvo (MNSP) matrix [2] is the submatrix of the full $(3+n) \times (3+n)$ unitary mixing matrix. If there is small mixing between the active and sterile neutrinos, the MNSP matrix must be slightly non-unitary. According to the different effects on neutrino oscillations, the unitarity violation in the MNSP matrix can be classified into two categories: direct unitarity violation and indirect unitarity violation [3].

- The indirect unitarity violation is brought by the existence of heavy sterile neutrinos, which themselves are too massive to be kinematically produced in the neutrino oscillation experiments. The heavy right-handed sterile neutrinos are natural ingredients of the canonical type-I seesaw mechanism [4] and some other seesaw models [5].

- The direct unitarity violation is caused by the existence of light sterile neutrinos which are able to participate in neutrino oscillations as their active partners. The sterile neutrinos with masses $m \sim O(1)\text{ eV}$ are proposed to explain the LSND [6], MiniBooNE [7], reactor antineutrino [8] and Gallium [9] anomalies. Furthermore, current cosmological observations [10] still allow the existence of sub-eV sterile neutrinos.

To study their different effects on neutrino oscillations, we consider in a special $(3+1+1)$ framework where 1 light sterile neutrino $\nu_s$ and 1 heavy right-handed neutrino $\nu_N$ are added to the standard 3 active neutrinos framework $^1$. In the $(3+1+1)$ scenario, the full picture of the neutrino

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$^1$ The reason why we consider the $(3+1+1)$ scenario is that on one hand, since the masses of heavy sterile neutrinos are not concerned in neutrino oscillation probabilities, no matter how many species of heavy sterile neutrinos there are, the effects of the indirect unitarity violation on neutrino oscillations can always be effectively parametrized by three additional complex mixing parameters [11], same as the case of just one heavy neutrino where three additional independent complex parameters $V_{e5}$, $V_{\mu5}$ and $V_{\tau5}$ are introduced in; on the other hand, current cosmological observations favored the existence of at most one species of light sterile neutrino.
mixing should be described by a $5 \times 5$ unitary matrix $V$

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s \\
\nu_N
\end{pmatrix} =
\begin{pmatrix}
V_{e1} & V_{e2} & V_{e3} & V_{e4} & V_{e5} \\
V_{\mu 1} & V_{\mu 2} & V_{\mu 3} & V_{\mu 4} & V_{\mu 5} \\
V_{\tau 1} & V_{\tau 2} & V_{\tau 3} & V_{\tau 4} & V_{\tau 5} \\
V_{s1} & V_{s2} & V_{s3} & V_{s4} & V_{s5} \\
V_{N1} & V_{N2} & V_{N3} & V_{N4} & V_{N5}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4 \\
\nu_5
\end{pmatrix},
$$

where $\nu_4$ and $\nu_5$ are corresponding mass eigenstates of the light and the heavy sterile neutrinos.

Here we restrict us to the typical neutrino oscillation process $\nu_\alpha \rightarrow \nu_\beta$ where both the production of $\nu_\alpha$ and the detection of $\nu_\beta$ are via the charged-current interaction. Then the neutrino oscillation probability in vacuum can be written as $[11, 12]$

$$
P \left( \nu_\alpha \rightarrow \nu_\beta \right) = \frac{1}{\left( \sum_{i=1,2,3,4} |V_{\alpha i}|^2 \right) \left( \sum_{i=1,2,3,4} |V_{\beta i}|^2 \right)} \sum_{i=1,2,3,4} V_{\alpha i}^* V_{\beta i}^2 \left\{ \sum_{i>j} Re \left[ V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}^* \right] \sin^2 \Delta_{ji} \pm 2 \sum_{i>j} Im \left[ V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}^* \right] \sin 2\Delta_{ji} \right\}$$

$$
= \frac{1}{\left( 1 - |V_{55}|^2 \right)^2} \left\{ \left| \delta_{\alpha\beta} - V_{55}^* V_{55} \right|^2 - 4 \sum_{j>i} Re \left[ V_{\alpha 1} V_{\beta 2} V_{\alpha 2}^* V_{\beta 1}^* \right] \sin^2 \Delta_{21} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 1} V_{\beta 2} V_{\alpha 2}^* V_{\beta 1}^* \right] \sin 2\Delta_{21} \right\}
$$

$$
- 4 \sum_{j>i} Re \left[ V_{\alpha 1} V_{\beta 3} V_{\alpha 3}^* V_{\beta 1}^* \right] \sin^2 \Delta_{31} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 1} V_{\beta 3} V_{\alpha 3}^* V_{\beta 1}^* \right] \sin 2\Delta_{31}
$$

$$
- 4 \sum_{j>i} Re \left[ V_{\alpha 2} V_{\beta 3} V_{\alpha 3}^* V_{\beta 2}^* \right] \sin^2 \Delta_{32} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 2} V_{\beta 3} V_{\alpha 3}^* V_{\beta 2}^* \right] \sin 2\Delta_{32}
$$

$$
- 4 \sum_{j>i} Re \left[ V_{\alpha 1} V_{\beta 4} V_{\alpha 4}^* V_{\beta 1}^* \right] \sin^2 \Delta_{41} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 1} V_{\beta 4} V_{\alpha 4}^* V_{\beta 1}^* \right] \sin 2\Delta_{41}
$$

$$
- 4 \sum_{j>i} Re \left[ V_{\alpha 2} V_{\beta 4} V_{\alpha 4}^* V_{\beta 2}^* \right] \sin^2 \Delta_{42} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 2} V_{\beta 4} V_{\alpha 4}^* V_{\beta 2}^* \right] \sin 2\Delta_{42}
$$

$$
- 4 \sum_{j>i} Re \left[ V_{\alpha 3} V_{\beta 4} V_{\alpha 4}^* V_{\beta 3}^* \right] \sin^2 \Delta_{43} \pm 2 \sum_{j>i} Im \left[ V_{\alpha 3} V_{\beta 4} V_{\alpha 4}^* V_{\beta 3}^* \right] \sin 2\Delta_{43} \right\},
$$

which in general consists of six CP-conserving oscillatory terms and six CP-violating oscillatory terms. Here $\Delta_{ji} \simeq 1.27 \Delta m_{ji}^2 L/E$ with $\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$ is the neutrino mass-squared difference in eV$^2$, $L$ is the baseline from the source to the detector in meters and $E$ is the neutrino or antineutrino energy in MeV. The Greek letters $\alpha, \beta$ are the flavor indices $e, \mu, \tau$, while the Latin letters $i, j$ are the mass indices. Note that the indices $i, j$ in Eq. (2) run over only the light neutrinos which can be kinematically produced in neutrino oscillation experiments and the normalization factor $1/\left( \sum_{i=1,2,3,4} |V_{\alpha i}|^2 \right) \left( \sum_{i=1,2,3,4} |V_{\beta i}|^2 \right)$ ensures that the total rate $P(W \rightarrow \bar{\nu}_\alpha \nu_\alpha) \equiv \sum_i |A(W \rightarrow \bar{\nu}_\alpha \nu_i)|^2 = 1$ (at the source) and $P(\nu_\beta W \rightarrow l_\beta) \equiv \sum_i |A(\nu_i W \rightarrow l_\beta)|^2 = 1$ (at the detector).

The possible effects of both the direct and the indirect unitarity violation in neutrino oscillation experiments have been discussed in many previous papers. For example, in the presence of heavy
sterile neutrinos, the oscillation probabilities have the property \( P (\nu_\alpha \rightarrow \nu_\beta) \neq \delta_{\alpha\beta} \) in the limit \( L \rightarrow 0 \) which is well known as the “zero-distance” effect [13]. We can clearly see from Eq. (2) that such effect will not take place if there exist only light sterile neutrinos. Therefore it would be a definite signal of the indirect unitarity violation if the “zero-distance” effect can be observed in future neutrino oscillation experiments. To obtain the best sensitivities to certain parameters (mixing angles or CP-violating phases) of the direct or indirect unitarity violation, plenty of works have been done to find the optimum setups by choosing appropriate neutrino source, oscillation channels and baselines or by proceeding a combined analysis of the data from different baselines where the matter effect may play quite different roles [14].

However, in this paper, we focus on the reactor antineutrino oscillation experiment where only CP-conversing terms are involved in the electron antineutrino survival probability

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = \frac{1}{(1 - |V_{e5}|^2)^2} \left\{ (1 - |V_{e5}|^2)^2 - 4|V_{e1}|^2|V_{e2}|^2 \sin^2 \Delta_{21} - 4|V_{e1}|^2|V_{e3}|^2 \sin^2 \Delta_{31} - 4|V_{e2}|^2|V_{e3}|^2 \sin^2 \Delta_{32} - 4|V_{e1}|^2|V_{e4}|^2 \sin^2 \Delta_{41} - 4|V_{e2}|^2|V_{e4}|^2 \sin^2 \Delta_{42} - 4|V_{e3}|^2|V_{e4}|^2 \sin^2 \Delta_{43} \right\} .
\]

The standard formula for 3 active neutrinos can be easily reproduced by simply choosing \(|V_{e4}| = |V_{e5}| = 0\) in Eq. (3). For the \((3+1)\) or \((3+1)\) scenario where only one light or heavy sterile neutrino is added, the corresponding survival probabilities can be obtained by taking \(|V_{e5}| = 0\) or \(|V_{e4}| = 0\) respectively.

An elegant parametrization has been proposed to parametrize the full unitary mixing matrix [15]. In the \((3+1+1)\) scenario, the \(5 \times 5\) matrix \(V\) in Eq. (1) can be decomposed as

\[
V = \begin{pmatrix} 1 & 0 \\ 0 & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix},
\]

in which \(V_0\) and \(A\) are \(3 \times 3\) matrices, \(U_0\) and \(B\) are \(2 \times 2\) matrices, \(R\) is a \(3 \times 2\) matrix, \(S\) is a \(2 \times 3\) matrix and \(0\) and \(1\) stand respectively for the zero and identity matrices. These matrices are parametrized as

\[
\begin{pmatrix} 1 & 0 \\ 0 & U_0 \end{pmatrix} = O_{23} O_{13} O_{12}, \quad \begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix} = O_{45}, \quad \begin{pmatrix} A & R \\ S & B \end{pmatrix} = O_{35} O_{25} O_{15} O_{34} O_{24} O_{14},
\]

(5)
where ten $O_{ij}$ (for $1 \leq i < j \leq 5$) are two-dimensional rotation matrices in the five-dimensional complex space whose explicit expressions can be found in Ref. [15]. One can easily see from this parametrization that the matrices $V_0$ and $U_0$ are unitary while $A$, $B$, $R$, $S$ are not. The production $AV_0$ can be regarded as the effective $3 \times 3$ MNSP matrix in this $(3+1+1)$ scenario which is in general non-unitary.

An apparent advantage of this parametrization is that all the five moduli $|V_{ei}|$ that are involved in Eq. (3) have very concise expressions:

\begin{align*}
|V_{e1}| &= c_{12}c_{13}c_{14}c_{15}, \\
|V_{e2}| &= s_{12}c_{13}c_{14}c_{15}, \\
|V_{e3}| &= s_{13}c_{14}c_{15}, \\
|V_{e4}| &= s_{14}c_{15}, \\
|V_{e5}| &= s_{15},
\end{align*}

(6)

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ with $ij = 12, 13, 14$ and 15. Here $\theta_{14}$ stands for the mixing between the light sterile neutrino and the active neutrinos while $\theta_{15}$ stands for the mixing between the heavy sterile neutrino and the active neutrinos. We can clearly find in Eq. (6) that if $\theta_{14} = 0$ then we have $|V_{e4}| = 0$, and $|V_{e5}| = 0$ can be easily obtained by taking $\theta_{15} = 0$. The survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ is then given by

\begin{align*}
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - P_{21} - P_{31} - P_{32} - P_{41} - P_{42} - P_{43} \\
&= 1 - 4s_{12}^2c_{12}^2s_{13}^2c_{13}^2c_{14}^2 \sin^2 \Delta_{21} - 4c_{12}^2s_{13}^2c_{13}^2c_{14}^2 \sin^2 \Delta_{31} - 4s_{12}^2s_{13}^2c_{13}^2c_{14}^2 \sin^2 \Delta_{32} \\
&\quad - 4c_{12}^2s_{13}^2c_{13}^2c_{14}^2 \sin^2 \Delta_{41} - 4s_{12}^2s_{13}^2c_{14}^2 \sin^2 \Delta_{42} - 4s_{13}^2s_{14}^2c_{14}^2 \sin^2 \Delta_{43}.
\end{align*}

(7)

One may immediately find that the mixing angle $\theta_{15}$ is not shown in the electron antineutrino survival probability which implies that the reactor experiment is almost insensitive to the indirect unitarity violation induced by the heavy sterile neutrinos. An exception is that the indirect unitarity violation of the PMNS matrix will result in corrections to the cross sections of both the charged current and the neutral current interactions [12]. However, precise calculation of the reactor antineutrino spectrum, exact value of the detector efficiency and accurate absolute energy scale calibration in the detectors are required for probing this minor effect. In this paper we will only discuss the antineutrino survival probability and focus on the direct unitarity violation effects in the reactor experiments induced by sub-eV sterile neutrinos. Therefore the following discussions are simply carried out in the $(3+1)$ scenario.
Before ending this section, it is worth to mention that in the \((3+1+1)\) scenario, altogether 14 new independent mass and mixing parameters are introduced\(^2\), but only two of them \((\theta_{14} \text{ and } \Delta m_{41}^2)\) are included in the electron antineutrino survival probabilities. And we will show in the next section that the reactor experiment can provide definite signals for each of them.

II. SEARCH FOR THE SUB-eV STERILE NEUTRINOS

Now we focus on the reactor antineutrino oscillation experiment with three different baselines: \(L_1 = 500\) m, \(L_2 = 2\) km and \(L_3 = 60\) km. The six oscillatory terms in Eq. (7) may behave very different at the three different baselines, which provide the opportunity to distinguish the unitarity violation parameters from the standard ones. The combination of the DayaBay experiment\(^{16}\) and the upcoming JUNO experiment\(^{17}\) is just of this type so is the RENO experiment\(^{18}\) combined with the proposed RENO-50 reactor experiment\(^{19}\). Studying from two aspects: the rate analysis and the spectral analysis, we are going to discuss the sensitivities of this kind of reactor experiment to the parameters \(\theta_{14} \text{ and } \Delta m_{41}^2\) in detail. In the following, \(\theta_{12} = 33.65^\circ\), \(\theta_{13} = 8.9^\circ\), \(\Delta m_{21}^2 = 7.6 \times 10^{-5}\) eV\(^2\), and \(|\Delta m_{31}^2| = 2.4 \times 10^{-3}\) eV\(^2\) are chose as default unless otherwise specified.

A. Rate Analysis

For a reactor neutrino experiment, the observed neutrino spectrum at a baseline \(L\), \(F(L/E)\) in the \(L/E\) space can be written as\(^{20}\)

\[
F(L/E) = \phi(E)\sigma(E)P(\bar{\nu}_e \rightarrow \bar{\nu}_e)\frac{E^2}{L},
\]

where \(E\) is the electron antineutrino (\(\bar{\nu}_e\)) energy, \(\phi(E)\) is the flux of \(\bar{\nu}_e\) from the reactor and \(\sigma(E)\) is the interaction cross section of \(\bar{\nu}_e\) with matter. The \(\bar{\nu}_e\) flux \(\phi(E)\) from the reactor can be parametrized as\(^{21}\)

\[
\phi(E) = 0.58 \exp(0.870 - 0.160E - 0.091E^2) + 0.30 \exp(0.896 - 0.239E - 0.0981E^2) \\
+ 0.07 \exp(0.976 - 0.162E - 0.0790E^2) + 0.05 \exp(0.793 - 0.080 - 0.1085E^2),
\]

2 These 14 additional parameters consist of 7 mixing angles (of which \(\theta_{14}, \theta_{24} \text{ and } \theta_{34}\) describe the mixing between three active neutrinos and the light sterile neutrino, \(\theta_{15}, \theta_{25} \text{ and } \theta_{35}\) describe the mixing between three active neutrinos and the heavy sterile neutrino, and \(\theta_{45}\) describes the mixing between the light and the heavy sterile neutrinos as one can clearly see in Eq. (5)), 5 phases and 2 sterile neutrino masses.
FIG. 1: Reactor antineutrino spectra in the $L/E$ space for no oscillation (dashed line), the standard 3 active neutrinos case (dash-dotted line) and the (3+1) scenario with $\Delta m^2_{41} = 0.3$ eV$^2$ and $|V_{e4}|^2 = 0.01$ (solid line) at the baselines of 500 m, 2 km and 60 km.

where four exponential terms are contributions from isotopes $^{235}$U, $^{239}$Pu, $^{238}$U and $^{241}$Pu in the reactor fuel, respectively. The leading-order expression for the cross section \cite{22} of inverse-$\beta$ decay ($\bar{\nu}_e + p \rightarrow e^+ + n$) is $\sigma^{(0)} = 0.0952 \times 10^{-42}$cm$^2$($E_{\nu}^{(0)}p_e^{(0)}/$MeV$^2$), where $E_{\nu}^{(0)} = E_{\nu} - (M_n - M_p)$ is the positron energy when neutron recoil energy is neglected, and $p_e^{(0)}$ is the positron momentum.

Taking the baseline $L$ to be 500 m, 2 km and 60 km respectively, the observed neutrino spectra in the $L/E$ space are shown in Fig. 1 where the solid line stands for the (3+1) scenario with $\Delta m^2_{41} = 0.3$ eV$^2$ and $|V_{e4}|^2 = 0.01$, the dash-dotted line for the standard 3 active neutrinos case and the dashed line is the no oscillation spectrum for comparison. With current reactor energy resolution, the oscillatory frequencies of $P_{41}$, $P_{42}$ and $P_{43}$ at the baseline $L_3 = 60$ km are rather high, thus their oscillatory behaviors are highly suppressed and only the averaged spectrum can
The total number of events observed in the detector can be calculated by integrating the antineutrino flux over the energy. Fig. 2 shows the total event ratio which is the ratio of the total energy-integrated events to the no oscillation expectation as a function of $\Delta m_{41}^2$ at the three different baselines. In this figure, the solid line stands for the $(3+1)$ scenario with $|V_{e4}|^2 = 0.01$, the dash-dotted line for the $(3+1)$ scenario with $|V_{e4}|^2 = 0.02$ and the dotted line for the standard 3 active neutrinos case. One can see that the total event ratio is sensitive only to very small $\Delta m_{41}^2$. The reason is that if $\Delta_{ji}$ is large, $\sin^2 \Delta_{ji}$ oscillates very fast with the varying of $E$, and therefore is fully averaged when integrated over the energy. We can see from Fig. 2, if $\Delta m_{41}^2 > 0.05 \text{ eV}^2$, the total event ratio is almost independent of $\Delta m_{41}^2$ at all the three baselines while still sensitive to the sterile-active mixing angle $\theta_{14}$.

Compare with the standard 3 active neutrinos case, the existence of additional light sterile neutrinos will in generally lead to additional depression of the total event ratio and can mimic the
$\Delta m_{41}^2 = 0.3 \text{ eV}^2$

Total event rate @ $L = 500 \text{ m}$

Total event rate @ $L = 2000 \text{ m}$

Total event rate @ $L = 60000 \text{ m}$

FIG. 3: Contour lines of the total event ratio in the $\theta_{13}$-$\theta_{14}$ plane at the baselines of 500 m, 2 km and 60 km. Although we have typically set $\Delta m_{41}^2 = 0.3 \text{ eV}^2$ in plotting these contour figures, the results are almost exactly the same for any $\Delta m_{41}^2 > 0.05 \text{ eV}^2$.

signal of $\theta_{13}$ if it is extracted from the rate analysis at a single baseline [23]. To see this point more clearly, Fig. 3 shows the contour lines of the the total event ratio in the $\theta_{13}$-$\theta_{14}$ plane at the three different baselines. Instead of a definite value of $\theta_{13}$, the measured total event ratio at any single baseline gives only the possible ranges of $\theta_{13}$ and $\theta_{14}$ together with the relation between these two mixing angles.

However, this situation can be basically changed for the multiple baselines reactor experiment, where there are usually detectors at the near site playing the role of calibrator and $\theta_{13}$ is determined by comparing the event rates at the near and far baselines. Fig. 4 shows the contour lines of the relative total event ratio in the $\theta_{13}$-$\theta_{14}$ plane at the baselines of 2 km and 60 km, where the total event ratios at these two baselines are normalized by that at the baseline of $L_1 = 500 \text{ m}$. We can find that the relative event rate at $L_2 = 2 \text{ km}$ can determine the true value of $\theta_{13}$ independently of $\theta_{14}$. On the other hand, the relative event rate at $L_3 = 60 \text{ km}$ is jointly determined by the values of $\theta_{13}$ and $\theta_{14}$. It implies that if the relative event rate at the third baseline around 60 km can be precisely measured in the upcoming JUNO experiment, together with the already obtained $\theta_{13}$, we are able to draw information on the active-sterile mixing angle $\theta_{14}$. Note that, although we have typically set $\Delta m_{41}^2 = 0.3 \text{ eV}^2$ in plotting Fig. 4, the conclusion keeps unchanged for any $\Delta m_{41}^2 > 0.05 \text{ eV}^2$.

Above results are theoretically understandable as the result of the different behaviors of the
\[ \Delta m_{41}^2 = 0.3 \text{ eV}^2 \]

\[
\frac{R(L = 2000 \text{ m})}{R(L = 500 \text{ m})} \quad \text{and} \quad \frac{R(L = 60000 \text{ m})}{R(L = 500 \text{ m})}
\]

**FIG. 4:** Contour lines of the relative total event ratio in the \( \theta_{13}-\theta_{14} \) plane at the baselines of 2 km and 60 km. Although we have typically set \( \Delta m_{41}^2 = 0.3 \text{ eV}^2 \) in plotting these contour figures, the results are almost exactly the same for any \( \Delta m_{41}^2 > 0.05 \text{eV}^2 \).

six oscillatory terms in Eq. (7) at different baselines. Here we focus on the case of \( \Delta m_{41}^2 > 0.05 \text{eV}^2 \) which means \( \sin^2 \Delta_{41}, \sin^2 \Delta_{42} \) and \( \sin^2 \Delta_{43} \) are fully averaged (\( \approx 1/2 \)) at all the three baselines. At the near site \( L_1 = 500 \text{ m} \), the energy-averaged antineutrino survival probability can be approximately written as

\[
P(L = 500 \text{ m}) \approx 1 - 2s_{14}^2c_{14} = c_{14}^4 + s_{14}^4, \tag{10}
\]

where the three terms \( P_{21}, P_{31} \) and \( P_{32} \) are neglected because of the smallness of \( \sin^2 \Delta_{21}, s_{13}^2 \sin^2 \Delta_{31} \) and \( s_{13}^2 \sin^2 \Delta_{32} \) at this baseline. At the baseline \( L_2 = 2 \text{ km} \), the survival probability can be approximately written as

\[
P(L = 2000 \text{ m}) \approx 1 - 2s_{14}^2c_{14} - c_{14}^4 \sin^2 2\theta_{13} \left[ \sin^2 \Delta_{31} \right]_{2000 \text{ m}}
\]

\[
= c_{14}^4 \left( 1 - \sin^2 2\theta_{13} \left[ \sin^2 \Delta_{31} \right]_{2000 \text{ m}} \right) + s_{14}^4, \tag{11}
\]

where \( \left[ \sin^2 \Delta_{31} \right]_{2000 \text{ m}} \) stands for the energy-averaged value of \( \sin^2 \Delta_{31} \) at \( L_2 = 2 \text{ km} \) and terms proportional to \( \sin^2 \Delta_{21} \) are safely neglected. Then the relative event rate at \( L_2 = 2 \text{ km} \) can be estimated by

\[
\frac{P(L = 2000 \text{ m})}{P(L = 500 \text{ m})} \approx 1 - (1 - s_{14}^4) \sin^2 2\theta_{13} \left[ \sin^2 \Delta_{31} \right]_{2000 \text{ m}} + O(s_{14}^8). \tag{12}
\]
The leading terms that are dependent of $\theta_{14}$ in Eq.(12) are proportional to $s_{14}^4$ and are further suppressed by the small factor $\sin^2 2\theta_{13}$. This clearly explained that the estimate of $\theta_{13}$ by the combined rate analysis at the two baselines $L_1 = 500$ m and $L_2 = 2$ km is nearly independent of the value of $\theta_{14}$ as long as $\Delta m_{21}^2 > 0.05\text{eV}^2$ is satisfied.

At the baseline of $L_3 = 60$ km, we can infer from the third plot of Fig. 2 that all the $\sin^2 \Delta_{ji}$ terms with $\Delta m_{ji}^2 > 10^{-3} \text{eV}^2$ are fully averaged out. Therefore $P_{21}$ is the dominate oscillatory term at this baseline and the energy-averaged electron antineutrino survival probability should be approximately written as

$$P(L = 60000 \text{ m}) \approx 1 - 2s_{13}^2 c_{13} c_{14}^4 - 2s_{14}^2 c_{14}^2 - c_{13}^4 c_{14}^4 \sin^2 2\theta_{12} [\sin^2 \Delta_{21}]_{60000\text{m}}$$

$$= c_{13}^4 c_{14}^4 (1 - \sin^2 2\theta_{12} [\sin^2 \Delta_{21}]_{60000\text{m}}) + s_{13}^4 c_{14}^4 + s_{14}^4,$$

(13)

where $[\sin^2 \Delta_{21}]_{60000\text{m}}$ is the energy-averaged value of $\sin^2 \Delta_{21}$ at $L_3 = 60$ km. Then we have

$$\frac{P(L = 60000 \text{ m})}{P(L = 500 \text{ m})} \approx c_{13}^4 [1 - (1 - s_{14}^4) \sin^2 2\theta_{12} [\sin^2 \Delta_{21}]_{60000\text{m}}]$$

$$+ s_{13}^4 + \frac{1}{2} \sin^2 2\theta_{13} s_{14}^4 + \mathcal{O}(s_{14}^8).$$

(14)

Figure 5 shows this relative event ratio $P(L = 60000 \text{ m})/P(L = 500 \text{ m})$ as a function of $\theta_{14}$. Provided the relative event ratio can be precisely measured to the level of $\mathcal{O}(10^{-4})$ in future precision reactor experiments, together with the already measured $\theta_{13}$, it should be possible to determine the active-sterile mixing angle $\theta_{14}$ or give an upper limit on it.
FIG. 6: Big picture of the Fourier sine (FST) and cosine (FCT) transformation spectra at the baselines of 500 m and 2 km.

B. Spectral Analysis

One can see from Eq. (3) that three new oscillatory terms $P_{41}$, $P_{42}$ and $P_{43}$ are included in due to the existence of one light sterile neutrino $\nu_4$. A direct measurement of the oscillatory behaviors of these three terms will certainly provide the direct evidence of the existence of the light sterile neutrinos. However the amplitudes of all these three oscillations are small (proportional to $s_{14}^2$). It has been found that comparing to a normal $L/E$ spectrum analysis, the Fourier analysis naturally separates the mass hierarchy information from uncertainties of the reactor neutrino spectra and
other mixing parameters, which is critical for very small oscillations. We will show in the following,
by applying the Fourier transformation to the $L/E$ spectrum of neutrinos detected at the detectors,
it is possible to obtain the information of $\Delta m^2_{41}$.

The frequency spectrum can be obtained by the following Fourier sine transformation (FST) and Fourier cosine transformation (FCT):

$$FST(\omega) = \int_{t_{\min}}^{t_{\max}} F(t) \sin(\omega t) dt, \quad (15)$$

$$FCT(\omega) = \int_{t_{\min}}^{t_{\max}} F(t) \cos(\omega t) dt, \quad (16)$$

where $\omega$ is the frequency, we set $\omega = \Delta m^2_{ji}$ just to be the squared mass differences and $t = L/2.54E$ is the viable in $L/E$ space. In this convention, we can easily read the value of the corresponding $\Delta m^2_{ji}$ from the FST or FCT spectra.

Figure 6 shows the big picture of the FST and FCT spectra at $L_1 = 500$ m and $L_2 = 2$ km. The main waves in these figures mirror the feature of the antineutrino spectrum from the reactor but have nothing to do with the oscillations. Between each two adjacent main waves, there are only very small fluctuations. If $\Delta m^2_{4i}$ (for $i = 1, 2, 3$) just lie in these flat frequency regions, it is possible to find the signals of $P_{41}$, $P_{42}$, $P_{43}$ oscillations by searching the FST and FCT spectra. We choose three typical value of $\Delta m^2_{41} = 0.1$ eV$^2$, 0.3 eV$^2$ and 0.5 eV$^2$ and show in Fig. 7 the corresponding FST and FCT spectra at the three different baselines.

It has been pointed out that whether the information of $\Delta m^2_{41}$ can be extracted from the spectra depend strongly on the energy resolution and the statistics. The simulation in Ref. [24] suggests that the energy resolution $\delta E/E$ should be better than $0.68\pi/\Delta_{ji}$ so that the corresponding high frequency oscillatory behavior of $P_{ji}$ is not completely suppressed. Taking $\Delta m^2_{41} \sim 0.3$ eV$^2$ and the neutrino energy $E \sim 4$ MeV, we can than give a estimate of the required lowest energy resolutions: 4.49% at the baseline $L_1 = 500$ m, 1.12% at $L_2 = 2$ km and at 0.04% $L_3 = 60$ km. Note that the larger $\Delta m^2_{41}$ we are aiming and the longer baseline we have chosen, the higher energy resolution are required.

Of course, the most optimistic situation is that the FST and FCT spectra of $P_{41}$, $P_{42}$ and $P_{43}$ can be observed at two or more different baselines, therefore these different measurements can be cross-checked with each other. Nevertheless, we have to say the most promising way is to measure the $\Delta m^2_{41}$ with the frequency spectra from the near detector at short baselines (e.g. 500 m or shorter), for the near detectors can provide large statistics as well as require relative low energy resolution for this kind of measurement.
FIG. 7: Fourier sine (FST) and cosine (FCT) transformation spectra at the baselines of 500 m, 2 km and 60 km for 3 active neutrinos case (dotted line) and (3+1) case with $|V_{e4}|^2 = 0.01$ (solid line) or $|V_{e4}|^2 = 0.02$ (dash-dotted line).
FIG. 8: Fourier sine (FST) and cosine (FCT) transformation spectra at the baseline of 60 km for only 3 active neutrinos case (dotted line) and (3+1) case with $|V_{e4}|^2 = 0.01$ (solid line) or $|V_{e4}|^2 = 0.02$ (dash-ditted line).

C. On the Neutrino Mass Hierarchy

As we have mentioned above, the existence of one sub-eV sterile neutrino $\nu_4$ will add three new oscillation components $P_{41}$, $P_{42}$ and $P_{43}$ in $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ correspond to three oscillation frequencies which are proportional to $\Delta_{41}$, $\Delta_{42}$ and $\Delta_{43}$, respectively. Their relative amplitude (oscillation intensity) is $c^{2}_{12}c^{2}_{13} : s^{2}_{12}s^{2}_{13} : s^{2}_{13} \approx 28 : 13 : 1$. Depend on the mass hierarchy of three active neutrinos (i.e., the sign of $\Delta m^2_{31}$), there are two possible ordering of these three new squared mass differences:

- Normal hierarchy (NH) with $\Delta m^2_{31} > 0$, then we have $\Delta m^2_{43} < \Delta m^2_{42} < \Delta m^2_{41}$;

- Inverted hierarchy (IH) with $\Delta m^2_{31} < 0$, then we have $\Delta m^2_{42} < \Delta m^2_{41} < \Delta m^2_{43}$.

It means the ordering of $\Delta m^2_{41}$, $\Delta m^2_{42}$ and $\Delta m^2_{43}$ is just an indication of the mass hierarchy of three active neutrinos (or the sign of $\Delta m^2_{31}$). Figure 8 shows the frequency spectra of the three new oscillation components $P_{41}$, $P_{42}$ and $P_{43}$, in which the main waves are the superposed frequency spectra of $P_{41}$ and $P_{42}$ and the oscillatory term $P_{43}$ modulates the spectra with a small fluctuation at a fixed distance about $2.4 \times 10^{-3}$ eV$^2$ away from the main waves. If the frequency spectrum of
$P_{43}$ lies at lower frequency than the spectra of $P_{41}$ and $P_{42}$, one can then conclude that $\Delta m_{31}^2 > 0$. On the contrary, if the spectrum of $P_{43}$ lies at higher frequency than that of $P_{41}$ and $P_{42}$, then we must have $\Delta m_{31}^2 < 0$.

Although such a measurement is theoretically feasible, it is in practice challenging. Firstly, a relative long baseline is needed so as the spectrum of $P_{43}$ can be separated from the main spectra of $P_{41}$ and $P_{42}$. We find that the minimum baseline is 20 km for this propose, as shown in Fig. 9. Meanwhile extremely high energy resolution and large statistic are required so as the spectra of these three high-frequency oscillatory terms are not smeared out and the small amplitude fluctuation of $P_{43}$ can be observed. The longer the baseline is, the higher experimental requirements of energy resolution and statistic are required. Therefore, it should be considered only as a complementary to the measurement by the analysis of the frequency spectra of three standard oscillatory terms $P_{21}$, $P_{31}$ and $P_{32}$ [20, 24, 25].
III. SUMMARY

Even though there have been many positive hints of the possible existence of sterile neutrinos and small unitarity violation in the MNSP matrix from both the theoretical and the experimental sides, there is no theoretical constraint on the mass of these particles. It is one of the important jobs to determine or constrain the number of sterile neutrinos and their mass and mixing properties in future precision experiments. The existence of sterile neutrinos can produce various kinds of effects on neutrino oscillations depending on the properties of the sterile neutrinos (e.g., the scale of the sterile neutrino mass, the magnitude and the structure of the active-sterile mixing) as well as the configurations of the experiments (e.g., the oscillation channel, the energy spectrum of the neutrino flux, the baseline $L$, whether the matter effect need to be taken into account).

In this paper we studied the possibility of searching for sub-eV sterile neutrinos in the precision reactor antineutrino oscillation experiments with three different baselines at around 500 m, 2 km and 60 km. The strategy of placing functionally identical detectors at different baselines and carrying out a combined analysis can offer a “clean” measurement of the electron antineutrino survival probabilities which is CP-phases independent as well as neutrino flux independent. We found that the active-sterile mixing angle $\theta_{14}$ could be determined or constrained by the precision measurement of the relative event ratio $P(L = 60000 \text{ m})/P(L = 500 \text{ m})$, provided $\theta_{13}$, $\theta_{12}$ and $\Delta m^2_{21}$ were well determined. The squared mass difference $\Delta m^2_{41}$ could be obtained from the Fourier transformation to the $L/E$ spectrum at the near detector. Surely, for such measurements to succeed, both the high energy resolution and the large statistics are essentially important.

We underline that the antineutrino survival probabilities obtained in reactor experiments are sensitive only to the direct unitarity violation which is induced by the existence of light sterile neutrinos but independent of the indirect unitarity violation parameters. More specifically, the reactor experiments offer very concentrated sensitivity only to two of the direct unitarity violation parameters $\theta_{14}$ and $\Delta m^2_{41}$. This means if any signals of unitarity violation are observed, we can then draw some definite informations on the mass and mixing properties of the light sterile neutrinos. On the contrary, if no observable effect of the unitarity violation are found in the reactor experiments, strong constraints on $\theta_{14}$ and $\Delta m^2_{41}$ should be obtained without the possibilities of cancelations between different unitarity violation parameters. Also, accurate informations of the pure standard mass and mixing parameters are also crucial for determining the unitarity violation parameters. The global analysis of various oscillation experiments are highly required for the complete determination of the full mass and mixing pattern of the active and sterile neutrinos.\textsuperscript{26}
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