Abstract

Ernst Mach (1838-1916) suggested that the origin of gravitational interaction could depend on the presence of all masses in the universe. A corresponding hypothesis of Sciama (1953) on the gravitational constant, \( c^2/G = \sum m_i/r_i \), is linked to Dicke’s (1957) proposal of an electromagnetic origin of gravitation, a precursor of scalar-tensor-theories. In this an equivalent description in terms of a variable speed of light (VSL) is given, and the agreement with the four classical tests of general relativity is shown. Moreover, VSL opens the possibility to write the total energy of a particle as \( E = mc^2 \); this necessarily leads to the proportionality of inertial and gravitating mass, the equivalence principle. Furthermore, a formula for \( c \) depending on the mass distribution is given that reproduces Newton’s law of gravitation.\(^1\) This mass distribution allows to calculate a slightly variable term that corresponds to the ‘constant’ \( G \). The present proposal may also supply an alternative explanation to the flatness problem and the horizon problem in cosmology.

1 Introduction

1.1 Overview

After a brief motivation given in 1.2 I shall outline in sec. 2 how Mach’s principle can be realized as a quantitative statement, following Sciama (1953). Before Sciama’s theory is related to a variable speed of light, in sec. 3 general considerations on time and length scales and a brief historical perspective of VSL theories are given. A very similar approach of describing gravity as an electromagnetic effect has been outlined in Dicke’s (1957) paper, a precursor of the so-called scalar-tensor theory. Using similar arguments as Will (1986), sec. 4 repeats how the respective experimental tests of general relativity (GR) can alternatively be described by a variable \( c \). While this is merely another viewpoint of classical physics, Sec. 5 describes as a new proposal how Newton’s law of gravitation can be directly deduced from a variable \( c \). While this general approach has interesting consequences for the equivalence principle, in sec. 6 an explicit formula for \( c \) depending on mass distributions is given that is in accordance with Mach’s principle. In sec. 7, consequences are discussed with respect to observational data.

1.2 Open questions in gravitational physics

While the theory of general relativity (GR), at the time of its discovery, could predict the last observation incompatible with Newtonian gravity, the mercury perihelion, new riddles have shown up in gravitational physics in the past decades. Since Zwicky’s observation of ‘missing mass’ in galaxy clusters an overwhelming evidence for ‘dark matter’ has been collected, in particular the flat rotation curves of galaxies. Since 1998, the relatively faint high-redshift supernovae are commonly explained...
as a manifestation of a new form of matter called ‘dark energy’. As Aguirre et al. (2001) comment, these new discoveries have been achieved “at the expense of simplicity”; it is quite disappointing that no counterpart to these forms of matter has been found in the laboratories yet. Thus in the view of recent data the following comment of Binney and Tremaine (1994), p. 635, is more than justified:

“It is worth remembering that all of the discussion so far has been based on the premise that Newtonian gravity and general relativity are correct on large scales. In fact, there is little or no direct evidence that conventional theories of gravity are correct on scales much larger than a parsec or so. Newtonian gravity works extremely well on scales of $\sim 10^{14}$ cm (the solar system). (...) It is principally the elegance of general relativity and its success in solar system tests that lead us to the bold extrapolation that the gravitational interaction has the form $GM/r^2$ on the scales $10^{21} - 10^{26}$ cm...”

In the meantime, the bold extrapolation seems to have encountered new observational problems. The detection of an anomalous acceleration from the Pioneer missions (Anderson, Laing, Lau, Liu, Nieto, and Turyshev 2002; Turyshev et al. 2005) indicate that Newtonian gravity may not even be correct at scales within the solar system. Interestingly, this anomaly occurred at the same dynamic scale - about $10^{-9}m/s^2$ - as many phenomena indicating dark matter.

Furthermore, the scale $10^{-9}m/s^2$ is the scale that most of the precision measurements of $G$ approach - it is still discussed if the discrepancies between the $G$ measurements in the last decade (Gundlach and Merkowitz 2000; Uzan 2003) have a systematic reason.

Besides the distance law, there is another lack of experimental data regarding the mass dependence. The Cavendish torsion balance uses masses of about 1kg to determine the mass of the earth, since there is no precise independent geological measure. The two-body treatment of Newtonian gravity tells us that the relative acceleration $a_{12}$ is proportional to $m_1 + m_2$. It may well be that simple form is just an approximation valid for $m_1 >> m_2$. A slightly different mass dependence$^2$ would remain undetected even by the most precise emphemers and double pulsar data, since in most cases just the product $GM$ is measured.

In summary, there is quite a big gap between the common belief in the universality of Newton’s law of gravitation and the experimental evidence supporting it.$^3$ So far there is no evidence at all that it still works for accelerations below $10^{-9}m/s^2$.

### 2 Mach’s principle

Besides the observational facts above, Newtonian gravity had to face theoretical problems, too. In his famous example of the rotating bucket filled with water, Newton deduced the existence of an absolute, nonrotating space from the observation of the curved surface the water forms. Ernst Mach criticized this ‘non observable’ concept of absolute space as follows, suggesting that the water was rotating with respect to masses at large distance:

“No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several... [miles]... thick.’

Mach’s principle is commonly known as follows: The reason for inertia is that a mass is accelerated with respect to all other masses in the universe, and therefore gravitational interaction would be impossible without the distant masses in the universe.

A possible solution to the ‘rotating bucket’ part of the problem where the distant masses instead of absolute space define the physical framework has been given in a brilliant paper by Lynden-Bell and Katz (1995).

However, from Mach’s principle it has been further deduced that the numerical value of the Gravitational $G$ constant must be determined by the mass distribution in the universe, while in Newton’s theory it is just an arbitrary constant. Since the square of the speed of light times the radius of the universe divided by the mass of the universe is approximately

$G$: See the interesting measurements by Holding et al. (1986), Ander et. al. (1989) and Zumberge et. al. (1991).

$^3$: A more detailed discussion is given in Unzicker (2007c).
equal to G, such speculations were first raised by Dirac (1938), Sciama (1953) and Dicke (1957).

2.1 Sciama’s implementation of Mach’s principle.

Since Mach never gave a quantitative statement of his idea, it has been implemented in various manners; the reader interested in an overview is referred to Bondi (1952), Barbour and Pfister (eds.) (1995) and Graneau and Graneau (2003). An idea of a quantitative statement proposes the following functional dependence of Newton’s constant $G$ (Sciama 1953; Unzicker 2003):

$$\frac{c^2}{G} = \sum_i \frac{m_i}{r_i} = \Phi$$

(1)

As in Sciama’s proposal, (1) may be included in the definition of the gravitational potential, thus yielding

$$\varphi = -G \sum_i \frac{m_i}{r_i} = -G \Phi = -c^2.$$  (2)

Alternatively, one can apply the hypothesis (1) directly to Newton’s law. The potential instead can then be brought to the form

$$\varphi = -c^2 \ln \sum_i \frac{m_i}{r_i},$$  (3)

from which follows Newton’s law

$$-\nabla \varphi = -c^2 \sum_i \frac{m_i \dot{r}_i}{\sum_j m_j r_j} =: -G(m_i, r_i) \sum_i \frac{m_i \dot{r}_i}{r_j}.$$  (4)

This was already developed by Unzicker (2003), though I do not support any more the further considerations following there. I prefer instead, as (2) suggests, to relate the (necessarily varying) gravitational potential to a variable $c$. It seems that Sciama did not consider this possibility; an explicit form will be given in sec. 6 which has however similarities with (3). For systematic reasons however, we should analyze the consequences of a variable $c$ at first.

3  A spacetime with variable $c$.

3.1 Early attempts of Einstein.

Before further relating $c$ to the gravitational potential it is worth remembering that a possible influence of the speed of light was already considered by Einstein in the years 1907-1911. Contrarily to first intuition, this does not contradict the special theory of relativity (SR). ‘Constancy’ of $c$, as far as SR is concerned, refers to a limiting velocity with respect to Lorentz transformations in one spacetime point; it does not mean $c$ has to be a constant function in spacetime. Ranada (2004) collected several citations of Einstein that put into evidence that the principle of constancy (over spacetime) of the speed of light is not a necessary consequence of the principle of relativity.$^4$

Considering variable atomic length and frequency standards $\lambda$ and $f$ $^5$ does nothing else but influence our measurements of distances, i.e. the metric of spacetime. A variable metric however manifests as curvature.$^6$ It will be discussed in sec. 4, how the arising curvature can be brought in agreement with the known tests of GR.

3.2 Electromagnetic and Brans-Dicke-theories

Shortly after GR was generally accepted, Wilson (1921) reconsidered Einstein’s early attempts and deduced Newton’s law from a spacetime with a variable refractive index depending on the gravitational field. This and related proposals like Rosen’s (1940) remained quite unknown until Dicke’s paper (1957) attracted much attention with the statement that gravitation could be of electromagnetic origin. While the second term in Dicke’s index of refraction (eqn. 5 there)

$$\epsilon = 1 + \frac{2GM}{rc^2}$$  (5)

is related to the gravitational potential of the sun, Dicke was the first to raise the speculation on the first term having ‘its origin in the remainder of the matter in the universe’. While this implementation of Mach’s principle was fascinating, in the following, more technical development of the theory (Brans and Dicke 1961), a new parameter $\omega$ was introduced that later failed to agree with a reasonable experimental value (Reasenberg 1979). Without Mach’s principle, theories with a variable refractive index, and therefore with a variable $c$, were developed as ‘polarisable vacuum models’ of GR.

$^4$Further comments are given in Unzicker (2007b).

$^5$Therefore $\lambda f = c$ is variable, too.

$^6$A textbook example herefore is given by Feynman et al. (1963), chap. 42.
In particular, Puthoff (2002) showed a far-reaching agreement with all experimental tests of GR known so far.

### 3.3 Recent VSL theories

Recently, Rañada (2004) considered a VSL in the context of the Pioneer anomalous acceleration. As an alternative to inflation models in cosmology, variable speed of light (VSL) theories were developed by Moffat (1993; 2002) and Magueijo (2000; 2003). These attempts had to face some criticism that claimed that considering any variability of \( c \) is just a trivial and obsolete transformation. Indeed, this criticism would be correct if spacetime were flat. A review article by Magueijo (2003) that focusses on cosmological aspects gives a detailed answer to that wrong argument (sec. 2). Unfortunately, a lot of controversial discussion on VSL theories recently arose. Some clarifying comments on this topic were given by Ellis and Uzan (2003). Since the present proposal refers to old established ideas of Einstein, Dirac, Sciama and Dicke, I shall not like to enter the actual discussion.

### 3.4 Time and Length scales

The only reasonable way to define time and length scales is by means of the frequency \( f \) and wave length \( \lambda \) of a certain atomic transition, as it is done by the CODATA units. Thus by definition, \( c = f \lambda = 299792458 \text{ m/s} \) has a fixed value with respect to the time and length scales that locally however may vary.\(^7\) To analyze such situations in the following, we get from the definition,

\[
dc = \lambda df + f d\lambda,
\]

and

\[
\frac{dc}{c} = \frac{df}{f} + \frac{d\lambda}{\lambda},
\]

using the product rule. Since we are interested in analyzing first-order effects, we will use (7) for finite differences \( \delta \) as well.

### 3.5 Spatial variation of \( c \).

How can we reasonably speak of a variation of \( c \)? Let’s start from eqn. (7). According to GR, \( \lambda \) is lower in the gravitational field, seen from outside, i.e. at a large distance from the field mass. Equally, frequency scales are lowered, and therefore, time scales raised. That means time runs slower, if we again observe from outside.

According to (7), we must assume \( c \) to be lowered by the double amount in the vicinity of masses, if we perform measurements at large distances. This point of view is not that common because we usually do not consider the value of \( c \) at a distance, though we do consider \( \lambda \) and \( f \) (gravitational redshift!). For a detailed and elucidating discussion of this viewpoint in the context of the radar echo delay, see Will (1986), p. 111 ff.

The lowered \( c \) in the vicinity of masses is sometimes called ‘non-proper speed of light’, whereas the locally measured ‘proper’ speed of light is a universal constant (Ranada 2004).

### 4 Variable speed of light and tests of GR.

#### 4.1 Notation

In order to fulfill (7) while analyzing variations of \( c \), we are seeking a simple notation. In first order, all GR effects involve the relative change \( \frac{\delta x}{x} \) of the quantity \( x \) of the amount \( -\frac{GM}{r^2} \). \( \delta x = x_g - x_0 \), where \( x_0 \) is the locally measured quantity and \( x_g \) the quantity in the gravitational field, measured from the observer outside. We shall abbreviate a lowering in the gravitational field as \( \downarrow \), e.g. the well-known slower running clocks with \( \frac{\delta f}{f} = -\frac{GM}{r^2} \) as \( f \downarrow \), and equally the shortening of rods \( \frac{\delta \lambda}{\lambda} = -\frac{GM}{r^2} \) as \( \lambda \downarrow \). \( \downarrow \downarrow \) stands for the double amount \( -\frac{2GM}{r^2} \), and \( \uparrow \) indicates a relative increase, etc.\(^8\) Eqn. (7) has to be fulfilled everywhere. No change is indicated as \( x \rightarrow \). Only two hypotheses are needed for describing the following experiments:

- In the gravitational field \( c \downarrow\downarrow = f \downarrow + \lambda \downarrow \) holds.
- During propagation, the frequency \( f \) does not change.

\(^7\)Of course, such variation cannot be detected unless we use spacetime curvature as indirect measurement.

\(^8\)Equivalently, \( \downarrow \) stands for a factor \( 1 - \frac{GM}{r^2} \). For factors close to 1, \( (1 - \frac{GM}{rc^2})^2 \approx 1 - \frac{2GM}{rc^2} \) holds, corresponding to \( \downarrow \).
4.2 The Hafele-Keating experiment.

In 1972, two atomic clocks were transported in aircrafts surrounding the earth eastwards and westwards in an experiment by Hafele and Keating. Besides the SR effect of moving clocks that could be eliminated by the two flight directions, the results showed an impressive confirmation of the first-order general relativistic time delay. We briefly describe the result by \( f \downarrow \), since the slower rate at which clocks run in a gravitational field was measured directly. The accuracy was very much improved by Vessot et. al. (1980). From this experiment alone it does not follow yet it does not keep in mind, so to speak, that its origin was a cesium transition - in that case we would not note a redshift at the photons arrival - but it maintains its frequency. Since this happens at a space position outside the gravitational field where \( c \) is higher by the double amount (see (7)), the photon has to adjust its \( \lambda \), and raise it with respect to the value ‘at home’. Since the adjustment to \( c \) overcompensates the originally lower \( \lambda \), we detect the photon as gravitationally redshifted.

4.3 Gravitational redshift and the Pound-Rebka experiment.

Photons leaving the gravitational field of the earth show a slight decrease in frequency at distances of about 20 m. This high-precision experiment became possible after the recoil-free emission of photons due to the Mössbauer effect (Pound and Rebka 1960; Pound and Snider 1965). Since the experiment consists in absorption of photons \( E = hf \), this is a true frequency measurement. Again, the frequency change \( f \downarrow \) was verified. For this point of view it is important to remember the assumption that the photon does not change its frequency while propagating in space, but had a lowered rate \( f \) already at the emission.

The same effect, but measured as an optical wavelength shift, was verified analyzing the emission spectra of the sun (Snider 1972). The situation at the emission is:

\[
c \downarrow\downarrow = f \downarrow + \lambda \downarrow \quad (8)
\]

During propagation \( f = \text{const.} \) holds, and the local \( c \) at arrival remains per definition unchanged. Thus at the time of detection,

\[
c \to = f \downarrow + \lambda \uparrow \quad (9)
\]

holds, what indeed is observed. In plain words, one can imagine the process as follows: consider a photon travelling from its home star with huge gravitation to the earth (with approximately zero gravity). While travelling through different regions in space, it keeps its (lowered) frequency. The photon does not keep in mind, so to speak, that its origin was a cesium transition - in that case we would not note a redshift at the photons arrival - but it maintains its frequency. Since this happens at a space position outside the gravitational field where \( c \) is higher by the double amount (see (7)), the photon has to adjust its \( \lambda \), and raise it with respect to the value ‘at home’. Since the adjustment to \( c \) overcompensates the originally lower \( \lambda \), we detect the photon as gravitationally redshifted.

4.4 Radar echo delay and light deflection.

The above results, linked to eqn. (7), already suggest a relative change of the speed of light of the amount \( c \downarrow \downarrow \) in the gravitational field. During propagation, \( f \) again does not change. The first idea of measuring this effect directly was launched by Shapiro (1964). Data of spectacular precision that agreed on the level of \( 10^{-3} \) with the theoretical prediction were collected by the Viking lander missions (Reasenberg 1979). Recently, the Cassini mission delivered a still better agreement (Williams, Turyshhev, and Boggs 2004).

Measuring the radar echo delay is only possible because the photon maintains its frequency while travelling. Since \( c \) is lowered by the double amount in the vicinity of the star \( c \downarrow\downarrow \), it has to shorten its wave length accordingly, and while bypassing the star

\[
c \downarrow\downarrow = \lambda \downarrow\downarrow \quad (10)
\]

holds. There, the photon’s \( \lambda \) is even shorter than \( \lambda \) of a photon produced at the star by the same atomic transition (see above, \( \lambda \downarrow \)). Naturally, the property which the photon has to leave at home for sure, is ‘its’ \( c \), otherwise it wouldn’t be considered as light any more. As it is outlined by Will (1986), p. 111, too, interpreting the radar echo delay as a local modification of \( c \) is an uncommon, but correct interpretation of GR. All the tests are in agreement with (7).

A lower \( c \) in the vicinity of masses creates light deflection just as if one observes the bending of light rays towards the thicker optical medium. Quantitatively, light deflection is equivalent to the radar echo delay. A detailed calculation for the total deflection
\[ \Delta \phi \text{ yields } \delta \phi = \frac{4GM}{rc^2}. \] 

(11)

Again the bending of light appears as curvature, therefore it is justified to describe GR by a spatial variation of \( c \), as it was already pointed out by Einstein 1911:

‘From the proposition which has just been proved, that the velocity of light in the gravitational field is a function of the place, we may easily infer, by means of Huyghens’s principle, that light-rays propagated across a gravitational field undergo deflexion’.

In the same article, Einstein obtained only the half value for the light deflection, because he considered \( c \) and \( f \) as subject to variation, but not \( \lambda \).

Independently from describing GR by a VSL theory, in a given spacetime point there is only one reasonable \( c \), from whatever moving system one tries to measure. This, but not more is the content of SR, which is not affected by expressing GR by a variable \( c \).

4.5 Measuring masses and other quantities

Not only time and length but every quantity is measured in units we have to wonder about, once we allow changes of the measuring rods. An interesting question is how to measure masses. Consider a large field mass \( M \) with gravitational field and at large distance a test mass \( m \). By Newton’s 3rd law

\[ F_M = -F_m \] 

(12)

holds, thus the ‘force measured at a distance’ is always of the same amount as the ‘local force’. However, in

\[ a_M M = -a_m m \]  

(13)

\( a_m \) is measured with the unchanged scales, and \( a_M \) with the scales inside the gravitational field. The measurement of accelerations depends on time and length scales, and the unit \( \frac{m}{s} \) corresponds to a three times lower value for the acceleration in gravitational fields, or a \( \downarrow \downarrow \downarrow \) in the above notation. To remember, this holds if we observe the behavior of an accelerated object at a large distance, i.e. from outside of the gravitational field. Consequently, the measurement of masses is influenced as well. Due to (13), masses are proportional to reciprocal accelerations and \( m \uparrow \uparrow \uparrow \) holds.

Similar considerations lead to \( v \downarrow \downarrow \) for velocities (as \( c \)), and \( l \rightarrow \) for the angular momentum. This last observation will turn out useful for keeping Planck’s constant \( h \) spatially constant, since it has units \( kg \frac{m^2}{s} \).

Table I gives an overview on the change of quantities. One \( \downarrow \) abbreviates a relative decrease of the quantity \( \frac{\delta x}{x} = -\frac{GM}{rc^2} \) in the gravitational field, or a factor \( (1 - \frac{GM}{rc^2}) \).

| Quantity   | symbol | unit     | rel. change |
|------------|--------|----------|-------------|
| Frequency  | \( f \) | \( \frac{1}{s} \) | \( \downarrow \) |
| Time       | \( t \) | \( s \)   | \( \uparrow \) |
| Length     | \( \lambda \) | \( m \)   | \( \downarrow \) |
| Velocity   | \( v \) | \( \frac{m}{s} \) | \( \downarrow \downarrow \downarrow \) |
| Acceleration | \( a \) | \( \frac{m}{kg} \) | \( \uparrow \uparrow \uparrow \) |
| Mass       | \( m \) | \( kg \)  | \( \uparrow \uparrow \uparrow \) |
| Force      | \( F \) | \( N \)   | \( \rightarrow \) |
| Pot. energy | \( E_p \) | \( Nm \)  | \( \downarrow \) |
| Ang. mom.  | \( l \) | \( kgm^2 \) | \( \rightarrow \) |

Table I. Overview on the change of quantities.

4.6 Advance of the perihelion of Mercury

In the Kepler problem, the Lagrangian is given by

\[ L = \frac{m}{2}(\dot{r}^2 + r^2 \dot{\phi}^2) + \frac{GMm}{r}. \] 

(14)

which after introducing the angular momentum \( l = mr^2 \dot{\phi} \) transforms to

\[ L = \frac{1}{2}mr^2 + \frac{l^2}{2mr^2} + \frac{GMm}{r}. \] 

(15)

Now we have to consider the relative change of the quantities as given above. \( m \uparrow \uparrow \uparrow \) however refers to a test particle at rest. The kinetic energy causes an additional, special relativistic mass increase \( \uparrow \) (see sec. 5.1 and 5.2 below in detail), thus in eq.(15), \( m \uparrow \uparrow \uparrow \uparrow \uparrow \) holds, while \( r \downarrow \) and \( l \rightarrow \). Hence, the middle term changes as \( \downarrow \downarrow \) in total, which means that \( m \) is

\[ ^9 \text{In agreement, Dicke (1957), p.366, and Puthoff (2002), eqns. 8-13, with different arguments.} \]
can be described by a spatial variation of the orbit. Since time and length measurement effects of GR are effectively multiplied by a factor \((1 - \frac{2GM}{rc^2})^{10}\) The secular shift \(-\frac{GMl}{rc^2}\) represents the well-known correction which leads to the secular shift

\[
\Delta \phi = \frac{6\pi GM}{A(1 - \epsilon^2)c^2},
\]

\(A\) being the semimajor axis and \(\epsilon\) the eccentricity of the orbit.

5 Newton’s law from a variable\(c\).

Since time and length measurement effects of GR can be described by a spatial variation of \(c\), one is tempted to try a description of all gravitational phenomena in the same framework. However, \(c \downarrow\downarrow\) (see sect. 4.1) requires

\[
\delta(c^2) = 2c\delta c = -\frac{4GM}{r}.
\]

This differs by a factor 4 from Sciama’s original idea and suggests a Newtonian gravitational potential of the form

\[
\varphi_{\text{Newton}} = \frac{1}{4}c^2.
\]

5.1 Kinetic energy.

As it is well-known from special relativity (SR), the mass increase due to relativistic velocities is, applying Taylor series to the square root,

\[
\delta m = m(v) - m_0 = m_0\left(\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right) \approx m_0\left(\frac{1}{2} \frac{v^2}{c^2}\right).
\]

If we apply conventional energy conservation \(E_k = E_p\) and start at \(r = \infty\), \(v = 0\), with \(\frac{1}{2}v^2 = \frac{GM}{r}\) follows

\[
\frac{\delta m}{m} = \frac{GM}{rc^2},
\]

for the relative mass increase due to kinetic energy, corresponding to \(\uparrow\).

5.2 Energy conservation.

A quite interesting consequence of the foregoing calculation arises if we consider the quantity \(E = mc^2\) of a test particle moving into a gravitational field. Recall that time and length measurements influence all quantities, thus \(\frac{\delta v}{c^2} = -\frac{4GM}{rc^2}\) or \(c^2 \downarrow\downarrow\downarrow\). This is partly compensated by \(m \uparrow\uparrow\uparrow\) as outlined above. There is one relative increase of the amount \(\frac{GM}{rc^2}\) left - the part in eqn. (20) due to the increase in kinetic energy!

This means, the total energy \(E = mc^2\) of a test particle appears as a conserved quantity during the motion in a gravitational field. In first order, we may write

\[
\delta(mc^2) = c^2\delta m + m\delta(c^2) = 0.
\]

Roughly speaking, the left term describes the increase in kinetic energy due to the relativistic \(\delta m\) and the right term the decrease of potential energy in a region with smaller \(c^2\). To be quantitative, however, the change in \(m\) contributes to the change in potential energy such that \(E_p \downarrow\). The change in the first term, \(m\delta(c^2)\) amounts to \(-\frac{4GM}{r^2}\), four times as much as needed to compensate \(dE_k\). Three parts of it are compensated by an increase of \(m\), \(\uparrow\uparrow\uparrow\) due to mass scales, the remaining \(\downarrow\downarrow\downarrow\downarrow\downarrow\delta d(c^2)\) is converted into kinetic energy:

\[
m^2c^2 \rightarrow m(c^2 \downarrow\downarrow\downarrow\downarrow\downarrow) + c^2(m \uparrow\uparrow\uparrow) + E_k = E_p + E_k
\]

In summary, there is nothing like a gravitational energy, just \(E = mc^2\). Kinetic energy is related to a change of \(m\), and potential energy to a change of \(c^2\). As it should be, photons do not behave differently, since they conserve their energy \(hf\) as well during propagation.

5.3 Foundation of the equivalence principle.

While GR provides most elegantly a formalism that incorporates the equality of inertial and gravitating masses, the question as to the deeper reason of this outstanding property of matter (‘I was utterly astonished about its validity’, Einstein 1930) is still open. Why does the elementary property of matter, inertial resistance to accelerations, act at the same time as a ‘charge’ of a particular interaction? Or, equivalently: why does kinetic energy show the
same proportionality to $m$ as potential energy in a gravitational field?  

If one supposes that the total energy of a particle can always be written as $E = mc^2$, the differentiation in eqn. (21) shows that both terms have to be proportional to the test mass $m$.

One fourth of the first term describes the (relativistic) increase of kinetic energy,\footnote{\textsuperscript{11}$\frac{3}{4}$ of it are due to the mass increase $m$.} which is usually approximated as $\frac{1}{2}mv^2$. The remainder plus the right term is the change in potential energy. Since $dm$ is proportional to $m$, both terms are obviously proportional to the mass $m$ of the test particle. There is no reason any more to wonder why inertial and gravitating mass are of the same nature. Describing gravity with a varying speed of light leads to the equivalence principle as a necessary consequence. The deeper reason for this is that gravitation is strictly speaking not an interaction between particles but just a reaction on a changing $c$.

6 Dependence of $c$ on the mass distribution in the universe

6.1 Test masses and field masses.

While dealing with the equivalence principle, there is a subtle difference between test and field masses. The above derivation holds for the former only; all spectacular tests of the EP, starting from Einstein to recently proposed satellite missions, deal with test masses.

An entirely new question is how matter does influence $c$ in its vicinity. Taking the proposal $\varphi = \frac{1}{2}c^2$ seriously, there are no more masses creating fields, because once you give up Newton’s $F = \frac{G M m}{r^2}$, the force does not need to depend on a product of masses. The modification of $c$ could depend instead on quantities which are approximately correlated to mass like the baryon number. Eötvös-like experiments could not detect that, but there could be material dependencies in torsion balance experiments.

6.2 General considerations on the functional dependence of $c$.

We are therefore seeking a Machian formula which describes explicitly the dependence of $c$. The most general form would be $c(m_i, \vec{r}_i, \vec{v}_i, \vec{a}_i, t)$, $i$ indicating a single mass point in the universe. Here I do not give detailed reasons why I exclude anisotropy and an explicit dependence on $v_i, a_i$ and $t$.\textsuperscript{12} The resulting hypothesis

$$c^2 = c_0^2 f(m_i, r_i),$$

$c_0$ being the speed of light in some preferred coordinate system of the universe, has however to face the problem how to make the argument dimensionless. Again, the only acceptable hypothesis seems to measure $m_i$ and $r_i$ either in multiples of $m_p, r_p$ of elementary particles (here protons) or as fraction of the respective quantities of the universe $m_u, r_u$. In both (or considering combinations, even four) cases, simple algebraic relations of $m_i$ and $r_i$ yield a very large or very small number. For reasons that will become clear later, I prefer the choice $m_p, r_p$. To avoid that $f(m_i, r_i)$ in (23) is of the order $10^{40}$, one has to introduce a function like $\ln$.

6.3 Proposal for the explicit dependence of $c$.

Obviously, the $m_i$ should enter in an additive way, and the influence of each mass $i$ should decay with $r_i$. For these reasons, $\ln \sum \frac{m_i}{r_i}$ is likely to be a component of eqn. (23), as one could suspect immediately from Sciama’s original proposal (1). Taking into account that $c$ has to decrease in the vicinity of masses, one of the most simple possibilities\footnote{This would be unusual, but not a priori senseless. Barbour (2002) recently gave an interesting proposal for $G$ depending on $v_i$.} is the formula

$$c^2(m_i, r_i) = \frac{c_0^2}{\ln \sum \frac{m_i}{r_i}} = : \frac{c_0^2}{\ln \Sigma}. \quad (24)$$

6.4 Recovery of Newton’s law

For the acceleration of a test mass

$$|a| = \frac{1}{4} \frac{\nabla c^2}{c_0^2} = \frac{c_0^2}{4(ln \Sigma)^2} \sum \frac{m_i}{r_i} \quad (25)$$

\textsuperscript{12}The alternative $c^2 = -c_0^2 \ln \Sigma$, while yielding similar results, appears less natural since it requires $m_i$ instead of $m_p$.\footnote{\textsuperscript{13}The alternative $c^2 = -c_0^2 \ln \Sigma$, while yielding similar results, appears less natural since it requires $m_i$ instead of $m_p$.}
holds, yielding the inverse-square law. The gravitational ‘constant’ can be expressed as

\[ G = \frac{c^2}{4(\ln \Sigma)^2} \sum \frac{m_i}{r_i} = \frac{c^2}{4 \ln \Sigma} \sum \frac{m_i}{r_i}. \]  

(26)

which differs by a numerical factor \(4 \ln \Sigma\) from Sciama’s proposal. It should be noted that the term \(\sum m_i\) contained in \(G\) is approximately constant. The contributions from earth, sun, and milky way, \(9.4 \cdot 10^{17}, 1.33 \cdot 10^{19}\), and \(7.5 \cdot 10^{20}\) (in \(kg/m\)), are minute compared to the distant masses in the universe that approximately\(^{14}\) amount to \(1.3 \cdot 10^{27}\). This was first observed by Sciama (1953), p. 39. Thus slight variations due to motions in the solar system are far below the accuracy of current absolute \(G\) measurements (\(\Delta G/G = 1.5 \cdot 10^{-4}\)).

6.5 Gravitational potential.

The gravitational \(\varphi\) potential of classical mechanics arises in first order from eqn. (24). Taking the example \(\varphi_s = \frac{GM_s}{r_s}\) of the sun, and let \(\Sigma^r\) be the sum as defined in (24) without the sun, then, \(\Delta\) denoting a difference,

\[ \Delta c^2 = \frac{c^2}{\ln \Sigma} \frac{1}{\ln \Sigma} - \frac{1}{\ln \Sigma} = \frac{c^2}{\ln \Sigma} \frac{\ln \Sigma - \ln \Sigma^r}{\ln \Sigma} = \frac{c^2}{\ln \Sigma} \ln (1 + \alpha), \]

holds, whereby \(\alpha\) is defined as \(\frac{M_s}{\sum \frac{M_i}{r_i}}\). The Taylor series of the \(\ln\), after applying (26), yields in first approximation

\[ \Delta c^2 = \frac{c^2}{\ln \Sigma} \alpha = \frac{4GM_s}{R_o} = 2c\Delta c =: 4\varphi_1, \]

\(\varphi_1\) being the first-order approximation of the gravitational potential of the sun.

Though the present proposal differs from Sciama’s by a numerical factor, the comment on \(G\) given on page 39 in the 1953 paper still applies:

‘... then, local phenomena are strongly coupled to the universe as a whole, but

owing to the small effect of local irregularities this coupling is practically constant over the distances and times available to observation. Because of this constancy, local phenomena appear to be isolated from the rest of the universe.’

6.6 Taking away the mystery from Mach

There is another satisfactory aspect of a gravitational potential of the form \(\frac{1}{r} c^2\). A Machian dependence of Newton’s constant \(G\) which is determined by the distribution of all masses in the universe is in a certain sense beautiful, but after all somewhat mysterious. How does a free falling apple feel the mass distribution of the universe? According to Sciama’s interpretation, its acceleration is much slower because there are so many galaxies besides the milky way.

On the other hand, assumed that the gravitational potential is just \(\frac{1}{r} c^2\), it has been our belief in the universality of Newton’s law and the somewhat naive extrapolation of the potential to \(\sum \frac{M_i}{r_i}\) that created so much surprise while rediscovering eqn. (1).

Despite the fact that the potential \(\sum \frac{M_i}{r_i}\) is a purely mathematical tool which cannot be observed directly, we have internalized it so much that \(G\) seems to depend on the mass distribution. In fact, if gravity depends just on \(c\) instead, \(G\) turns out as kind of an artifact.

7 Discussion of consequences and observational facts

In the meantime, the observational status of cosmology has improved dramatically. Besides the unresolved problems sketched briefly in sec. 1.2, the increasing precision in measuring the mass of the universe has revealed further riddles.

7.1 Visible matter and flatness

The approximate coincidence \(\frac{c^2}{\sigma} \approx \frac{M_s}{r_s}\) is usually expressed as critical density of the universe

\[ \rho_c = \frac{3H_o^2}{8\pi G}. \]

(29)
A universe with $\rho_0 > \rho_c$ is called closed, and open in the case $\rho_c > \rho_0$. An analysis of the evolution of the universe tells that if $\Omega := \frac{\rho}{\rho_c}$ is about the order of 1 at present, it must have been as close as $10^{-60}$ to 1 at primordial times. For that, many cosmologists believe $\Omega$ has to be precisely 1, but a cogent theoretical reason is still missing. Moreover, observations clearly indicate that the fraction of visible matter, $\Omega_v$, is about 0.01, with a large uncertainty.

From (26) and the assumption of an homogeneous universe, elementary integration over a spherical volume yields $\sum \frac{m_i}{r_i} \approx \frac{3m_u}{2r_u}$, and therefore

$$m_u \approx \frac{c^2 r_u}{6G \ln \Sigma}$$

holds. If one assumes $r_p = 1.2 \cdot 10^{-15} m$, $m_p = 1.67 \cdot 10^{-27} kg$ (the proton values), and taking the recent measurement of $H_0^{-1} = 13.7 \text{ Gyr}$ that leads to $r_u = 1.3 \cdot 10^{26} m$, $m_u$ is in approximate agreement with the observations, and corresponds to the fraction of visible matter $\Omega_v = 0.004$. Moreover, there is no more reason to wonder about the approximate coincidence (29), since it is a consequence of the variable speed of light (24), if one uses $\rho = \frac{3m}{4\pi r^3}$ and $r_u H_0 \approx c$.

Newtonian gravity deduced from a variable speed of light can thus give an alternative explanation for the flatness problem and may not need dark forms of matter (DM, DE) to interprete cosmological data.

### 7.2 The horizon problem

If we consider the sum in (24) for cosmological time scales, we know very little about its time evolution. In particular, we know about $r_i(t)$ only the momentary Hubble expansion $\dot{r}_i(13.7 \text{ Gyr})$.

It is tempting however to speculate about an increasing number of $m_i$ dropping into the horizon, thus lowering $c$ during the cosmological evolution. In this picture, the big bang could happen at a horizon of the size of an elementary particle ($\Sigma = 1$) with infinite $c$, the logarithmic dependence causing however a very rapid decrease to the almost constant actual value. A decreasing speed of light was recently considered by Magueijo (2000) as an alternative scenario to inflation.

### 7.3 Time evolution of $G$.

The increase of the number of $m_i$ is also necessary for compensating the increase of the $r_i$. It is at the moment not excluded that $G$ in (26) could meet the quite restrictive observational evidence for $G \approx 0$ (Uzan 2003).

### 7.4 Temporal evolution of $c$.

In sec. 4, a spatial variation of $c$ was considered describing effects that occurred while $\dot{c} \approx 0$. As it can be deduced from Maxwell’s equations, in this case a photon keeps its frequency $f$ unchanged. On the other hand, if we assume $\dot{c} \lt 0$ during cosmological evolution in an almost isotropic universe with vanishing gradients of $c$, Maxwell’s equations require a constant $\lambda$ during wave propagation, in the foregoing notation $c \downarrow = f \downarrow \lambda \rightarrow$. Since a photon keeps its original $\lambda$, the wavelength appears longer with respect to the measuring rods that evolve according to $c \downarrow = f \downarrow \lambda \downarrow$. Dicke (1957), p. 374, suspected this result to be related to the cosmological redshift.

### 8 Outlook

Galileo’s $F = mg$, a formula that today could enter the elementary schools, has been a quite remarkable discovery, though describing effects on earth only. Newton’s generalizing of this formula and describing celestial mechanics of the solar system was even more difficult.

I fear that the hierarchy of structures earth - solar system - galaxy - cosmos may require a corresponding hierarchy of physical laws that cannot be substituted by introducing new parameters, data fitting and numerical simulations.

Due to satellite technique, improved telescopes and the computer-induced revolution in image processing we are collecting data of fantastic precision. At the moment it is unlikely that our theoretical understanding on the galactic or cosmologic scale can match the amount of data.

Though the range of the universe we know about since the discovery of GR has increased dramatically, we usually extrapolate conventional theories of gravity to these scales. Physicists have learned

\footnote{Of course, I do not distinguish between an electromagnetic $c_{EM}$ and a spacetime $c_{ST}$, see Ellis and Uzan (2003).}
a lot from quantum mechanics, but the historical aspect of the lesson was that the 19th-century extrapolation of classical mechanics over 10 orders of magnitude to the atomic level was a quite childish attempt.

Under this aspect, the amount of research in cosmology which is done nowadays on the base of an untested extrapolation over 14 orders of magnitude is a quite remarkable phenomenon.

The wonderful observational data of the present should not lead us to neglect the theoretical efforts of scientists in the past. Sometimes one can learn more from the ‘blunders’ of deep thinkers like Einstein, Lord Kelvin, Mach or Sciama than from actual theories in fashion.

Since it deals with the very basics, it may be to early to try to test the present proposal with sophisticated data. It is merely an attempt to open new roads than a complete solution to the huge number of observational riddles.\textsuperscript{16}

Thus I consider sec. 7 as minor and preliminary results; besides the reproduction of GR and Newton, the more satisfactory aspect of this proposal seems the motivation for the equivalence principle, the implementation of Mach’s principle and the ‘removal’ of the constant $G$.

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References

Aguirre, A., C. P. Burges, A. Friedland, and D. Nolte (2001). Astrophysical constraints on modifying gravity at large distances. \textit{arXiv hep-ph/0105083}.

Ander et. al., M. (1989). Test of Newton’s inverse-square-law in the Greenland ice cap. \textit{Physical Review Letters} 62(6), 985–988.

Anderson, J. D., P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev (2002, April). Study of the anomalous acceleration of Pioneer 10 and 11. \textit{Physical Review D} 65(8), 082004–+.\textsuperscript{11}

Barbour, J. (2002). Scale-invariant gravity: particle dynamics. \textit{arXiv gr-qc/0211021}.

Barbour, J. and Pfister (eds.) (1995). \textit{Mach’s Principle}. Boston: Birkhäuser.

Binney, J. and S. Tremaine (1994). \textit{Galactic Dynamics}. Princeton.

Bondi, H. (1952). \textit{Cosmology}. Cambridge: University Press.

Brans, C. and R. H. Dicke (1961). Mach’s principle and a relativistic theory of gravitation. \textit{Physical Review} 124, 925–935.

Dicke, R. H. (1957). Gravitation without a principle of equivalence. \textit{Review of modern Physics} 129(3), 363–376.

Dirac, P. A. M. (1938). A new basis for cosmology. \textit{Proc. Roy. Soc. London A} 165, 199–208.

Einstein, A. (1907).

Einstein, A. (1911). On the influence of gravitation on the propagation of light. \textit{Annalen der Physik} 35(German org: Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes), 35.

Einstein, A. (1930). \textit{Mein Weltbild}. Piper.

Ellis, G. F. R. and J.-P. Uzan (2003). ‘c’ is the speed of light, isn’t it? \textit{arXiv gr-qc/0305099}.

Feynman, R., L. R.B., and M. Sands (1963). \textit{The Feynman Lectures in Physics} (2nd ed.), Volume II. California Institute of Technology.

Graneau, P. and N. Graneau (2003). Machian inertia and the isotropic universe. \textit{General Relativity and Gravitation} 35(5), 751–770.

Gundlach, J. and M. Merkowitz (2000). Measurement of Newton’ s constant using a torsion balance with angular acceleration feedback. \textit{arXiv gr-qc/0006013}.

Hafele, J. C. and R. Keating (1972). \textit{Science} 177, 168.

Holding, S., F. Stacey, and G. Tuck (1986). Gravity in mines - an investigation of Newton’s law. \textit{Physical Review D} 33, 3487–3494.

Lynden-Bell, D. and J. Katz (1995). Classical mechanics without absolute space. \textit{arXiv astro-ph/9509158}.

\textsuperscript{16}An excellent overview on the observations modified gravity has to match gives Aguirre et al. (2001).
Magueijo, J. (2000). Covariant and locally lorentz-invariant varying speed of light theories. arXiv gr-qc/0007036.

Magueijo, J. (2003). New variable speed of light theories: an overview. arXiv astro-ph/0305457.

Moffat, J. (2002). Variable speed of light cosmology: An alternative to inflation. arXiv hep-th/0208122.

Moffat, J. W. (1993). Superluminary Universe: a Possible Solution to the Initial Value Problem in Cosmology. International Journal of Modern Physics D 2, 351–365.

Pound, R. V. and S. A. Rebka (1960). Apparent weight of photons. Physical Review Letters 4, 337–340.

Pound, R. V. and J. L. Snider (1965). Effect of gravity on gamma radiation. Physical Review 140B, 788.

Puthoff, H. (2002). Polarizable-vacuum (PV) representation of general relativity. Found. Phys. 32, 927–943, arXiv: gr-qc/9909037.

Ranada, A. (2004). The Pioneer anomaly as an effect of the dynamics of time. arXiv gr-qc/0403013.

Reasenberg, R. D. (1979). Viking relativity experiment: Verification of signal retardation by solar gravity. Astrophys. J. Lett. 234, L219–L221.

Rosen, N. (1940). General relativity and flat space. I. +II. Physical Review 57, 147–153.

Sciama, D. W. (1953). On the origin of inertia. Monthly Notices of the Royal Astronomical Society 113, 34–42.

Shapiro, I. I. (1964). Fourth test of general relativity. Physical Review Letters 13(26), 789–792.

Snider, J. L. (1972). New measurement of the solar gravitational redshift. Physical Review Letters 28, 853–855.

Turyshev, S. G., N. N. Nieto, and J. D. Anderson (2005). The Pioneer anomaly and its implications. arXiv gr-qc/0510081.

Unzicker, A. (2003a). A Look at the Abandoned Contributions to Cosmology of Dirac, Sciama and Dicke. ArXiv: 0708.3518.

Unzicker, A. (2007b). The VSL Discussion: What Does Variable Speed of Light Mean and Should we be Allowed to Think About? ArXiv: 0708.2927.

Unzicker, A. (2007c, February). Why do we Still Believe in Newton’s Law? Facts, Myths and Methods in Gravitational Physics. ArXiv: gr-qc/0702009.

Uzan, J.-P. (2003). The fundamental constants and their variation: observational and theoretical status, hep-ph/0205340. Reviews of modern physics 75, 403–455.

Vessot et. al., R. F. C. (1980). Test of relativistic gravitation with a space-borne hydrogen maser. Physical Review Letters 45(26), 2081–2084.

Will, C. (1986). Was Einstein Right? New York: Basic Books.

Williams, J. G., S. G. Turyshev, and D. H. Boggs (2004). Progress in lunar laser ranging tests of relativistic gravity. arXiv gr-qc/0411113.

Wilson, H. A. (1921). An electromagnetic theory of gravitation. Physical Review 17, 54–59.

Zumberge et. al., M. (1991). Submarine measurement of the Newtonian gravitational constant. Physical Review Letters 67(22), 3051–3054.