Non-equilibrium Spacetime Thermodynamics, Entanglement viscosity and KSS bound

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Abstract. We propose a dual lower dimensional description of the vacuum state associated to a strongly coupled CFT living on Rindler wedge slice close to the horizon hypersurface. From this field theory, with a linear response approach, we show the possibility to derive an entanglement horizon viscosity via a holographic Kubo formula in terms of a two-point function of the stress tensor of matter fields in the bulk. The entanglement viscosity over entropy density ratio come out to satisfy the universal Kovtun-Son-Starinets (KSS) value $1/4\pi$ in four dimensions, suggesting the universal ratio may be a fundamental property of quantum entanglement.

1. Introduction

In the past decade, there has been significant interest in the idea of gravity as the thermodynamics of a quantum theory associated to some underlying microscopic structure of spacetime [1, 2, 3, 4, 5, 6]. A fundamental hint in this direction is given by Jacobson [1], who derive the Einstein equation as an equilibrium equation of state for a spacetime thermal system built on the thermodynamical properties of the Minkowski vacuum. The key point in the derivation is the possibility to characterize the Minkowski vacuum perceived by an uniformly accelerated observer as a canonical thermal state (Unruh effect), together with the idea that the local acceleration horizon, associated with the former observer, can be analogous to a tiny piece of a black hole event horizon, thereby allowing for an area-entropy proportionality assumption.

In this context, the coupling between geometry and matter is effectively provided by demanding a local equilibrium Clausius relation, $TdS = \delta Q$, as the main tool to relate the variation of entropy of the vacuum fields $dS$ (horizon area deformation) to the perturbative effects of a local flux of matter energy $\delta Q$ through the acceleration horizon.

More recently, in the attempt to understand whether and how the thermodynamical derivation of the Einstein equation could be generalized to allow for the higher curvature terms expected by the effective field theory, it has been realized that the thermodynamical prescription requires a generalization of the local equilibrium condition to a more general entropy balance law. The new local equilibrium condition, $TdS = \delta Q + \delta N$ (generalized Clausius relation), can account for some extra irreversible entropy production terms due to the source contributions for the horizon area/entropy evolution which are quadratic in expansion and shear of the null geodesic bundle comprising the horizon.

Following the approach in [3, 4], one can separate the equilibrium and the non-equilibrium features associated with the gravitational dynamics, in relation with the activation and the
propagation of the gravitational degrees of freedom entailed in the definition of the entropy functional, via the area-entropy proportionality relation. A deep investigation in this line has been carried on in a series of papers extending the thermodynamical derivation of the gravity field equations from General Relativity to generalized Brans-Dicke theories [5, 7]. These works show that the contributions leading to nonequilibrium spacetime thermodynamics can be actually related to the presence of heat fluxes associated to the purely gravitational/internal degrees of freedom of the theory. In particular, the irreversible interpretation for extra entropy contributions defined in terms of purely geometrical quantities, with relation to the notion of horizon viscosity, provided further hints toward the possible presence of a consistent underlying microscopic theory. The query for a viable description for the underlying microscopic theory is then a fundamental clue in this sense.

The fluctuation-dissipation theorem links viscous dissipation to fluctuations of a thermal equilibrium state. A natural attempt to interpret the viscous dissipation rate of a horizon would rely on the study the quantized gravitational fluctuations of the horizon shear, as already developed many years ago [9]. However, alternatively, one can take a different approach and associate the properties of vacuum fluctuations to the notion of quantum entanglement.

2. Horizon Shear Viscosity via Entanglement

In analogy to the black hole case, the equations governing the dynamics of a local Rindler causal horizon can be associated to those of a viscous fluid, according to the membrane paradigm approach [10, 11, 12]. This endows the local horizon system with some apparent hydrodynamic transport coefficients such as viscosities. In general, the relationship between the dynamics of a fluid and the dynamics of any black hole event horizon is just an analogy. In order for hydrodynamics to be a valid description, the characteristic wavelength and time scale of perturbations to the system must be much larger than the microscopic scale set by a correlation length (or mean free path). This basic criterion cannot be fulfilled even in the familiar example of a spherically symmetric Schwarzschild horizon. However, for the scale free Rindler horizon, the hydrodynamic limit exists.

The thermodynamic properties of the Rindler wedge are apparently encoded into a “pre-holographic” lower dimensional description associated with the horizon boundary. Namely, when a quantum state is restricted to a sub-region of the spacetime (in this case Minkowski vacuum state in the Rindler wedge), quantum fluctuations of this state have a dual, thermal description associated with the horizon boundary. Furthermore, an effective description of the large-scale dynamics of this vacuum thermal state is always provided by hydrodynamics. To this end, one can develop a simple Kubo-like formula for the viscosity induced on the horizon in terms of a two point stress-energy tensor correlation function for the quantum fields in the Rindler wedge [13]. In this line, one could expect to find a holographic “entanglement viscosity”. This quantity, when cut off, should scale exactly with the entanglement entropy.

To test this hypothesis, we propose a microscopic Kubo-like formula for the shear viscosity associated with the fluid lower dimensional description of the vacuum thermal state, characterized by a conserved stress tensor operator [13],

$$\langle \mathcal{T}^{(d+1)}_{\mu \nu} \rangle = \int_{\ell_c}^{\infty} d\xi \langle \mathcal{T}^{(R)}_{\mu \nu} \rangle = \int_{\ell_c}^{\infty} d\xi \kappa \xi \langle \mathcal{T}^\mu_\nu \rangle,$$

defined, by ansatz, as the radial integral of the bulk Minkowski vacuum stress energy tensor.

Now, in four spacetime dimensions the bulk energy density for a scalar field has the Planckian form

$$\epsilon(\xi) = \frac{\pi^2 T^4}{30} = \frac{1}{480\pi^2 \xi^4}.$$

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From our ansatz (1), we find an energy density that appropriately scales like the area of the horizon boundary \[14\]
\[\epsilon^{2+1} = \frac{\kappa}{960\pi^2 \ell_c^2}.\] (3)

Using the Gibbs relation \(\epsilon + P = sT\), and equation of state \(\epsilon = 3P\) for the massless bulk scalar field, we find the entropy density \(s\) obeys
\[s = \frac{2\pi^3}{45} T^3 = \frac{1}{180\pi \xi^3}.\] (4)

Integrating over \(\xi\) from \(\ell_c\) to \(\infty\) to find the effective area entropy yields
\[s = \frac{1}{360\pi \ell_c^2},\] (5)
which agrees with standard results in the literature for the brick wall/entanglement entropy \([14, 15, 16]\).

If we apply the formalism of viscous hydrodynamics to this system, the shear viscosity should be given by the Kubo formula
\[\chi = \lim_{k^0 \to 0} \frac{1}{k^0} ImG_R(k^0, \vec{k} = 0)\] (6)
in terms of the effective stress tensor of the lower dimensional theory associated with the horizon,
\[\eta = \lim_{\omega \to 0} \frac{1}{\omega} \int_{\ell_c}^{\infty} d\xi' \int_{\ell_c}^{\infty} d\xi \int d\tau d^d x e^{i\omega \tau} \theta(\tau) \langle [T_{xy}^{d+1}(\tau, x, y, \xi), T_{xy}^{d+1}(0)] \rangle,\] (7)
where \(\omega\) is a Rindler frequency. Using our ansatz that the lower dimensional densities are radial integrals of the bulk matter stress-tensor, we arrive at the following formula
\[\eta = \lim_{\omega \to 0} \frac{1}{\omega} \int_{\ell_c}^{\infty} d\xi' \int_{\ell_c}^{\infty} d\xi \int d\tau d^d x e^{i\omega \tau} \theta(\tau) \kappa^2 \xi \xi' \langle [T_{xy}(\tau, x, y, \xi), T_{xy}(0, \xi')] \rangle.\] (8)
which ends up to be a function of the correlator of the bulk stress energy tensor for the matter fields in the wedge.

As a test case of our viscosity formula, we consider the thermal state to consist of a free, minimally coupled scalar field in a four dimensional Rindler spacetime and we eventually get
\[\eta = \frac{1}{1440\pi^2 \ell_c^4},\] (9)
which, as expected, is divergent in the limit \(\ell_c \to 0\) and scales in \(\ell_c\) as a \(2 + 1\) quantity. Remarkably, the ratio of our shear viscosity to the entanglement entropy density is exactly the KSS ratio \([17]\). This suggests that the KSS ratio may be a fundamental holographic property of spacetime, rather than just of the AdS black hole solutions. The saturation of the universal KSS ratio for the Rindler acceleration horizon, in flat Minkowski spacetime, is indeed an interesting result, since gravity is absent. This is not in contrast with the ADS/CFT correspondence results \([18, 19]\), where the KSS ratio, though apparently rooted in gravitational physics, curiously does not depend on the Newton constant.
3. Discussion

Our result suggests that the $1/4\pi$ ratio might be a fundamental property of quantum entanglement and its associated holography. It also provides support for the hypothesis that semi-classical gravity on macroscopic scales is induced or emergent as an effective theory of some lower dimensional, strongly coupled quantum system with a large number of degrees of freedom. In this picture, the $1/4\pi$ ratio is saturated in gauge theories with a Einstein gravity dual because 1) they have an area (BH) entropy and 2) in the large $N$ limit the number of degrees of freedom diverges and gravity is turned off as the Newton constant goes zero.

In our formula for the viscosity, we consider the thermal state to consist of a free, minimally coupled scalar field in a four dimensional Rindler spacetime. One apparent problem with this choice is that the shear viscosity in an free field theory is typically ill-defined. In physical terms, shear viscosity measures the rate of transverse momentum diffusion between the elements of a fluid. Although the quasi-particle description in kinetic theory is not a good one in a strongly coupled system, we can gain some guidance by thinking of shear viscosity as a diffusion process. One can show that $\eta \sim \epsilon l_{mfp}$, where $l_{mfp}$ is the mean free path of the fluid. Since in a free field theory the mean free path diverges, $\eta$ diverges as well. On the other hand, in our case the equivalence principle implies a field theory in Rindler space can be thought of as being in a constant gravitational field. Imposing a UV cutoff on this system seems to introduce gravitational dynamics. If the cutoff is placed near the Planck length (as we suspect) the gravitational dynamics is strongly coupled there. The idea is that the dominant effect in the relaxation of the vacuum thermal state is the strongly coupled gravitational interaction. This also seems to explain how there can be universality in the result for $\eta$.

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