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Offset Risk Minimization for Open-loop Optimal Control of Oil Reservoirs

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Abstract: Simulation studies of oil field water flooding have demonstrated a significant potential of optimal control technology to improve industrial practices. However, real-life applications are challenged by unknown geological factors that make reservoir models highly uncertain. To minimize the associated financial risks, the oil literature has used ensemble-based methods to manipulate the net present value (NPV) distribution by optimizing sample estimated risk measures. In general, such methods successfully reduce overall risk. However, as this paper demonstrates, ensemble-based control strategies may result in individual profit outcomes that perform worse than real-life dominating strategies. This poses significant financial risks to oil companies whose main concern is to avoid unacceptable low profits. To remedy this, this paper proposes offset risk minimization. Unlike existing methodology, the offset method uses the NPV offset distribution to minimize risk relative to a competing reference strategy. Open-loop simulations of a 3D two-phase synthetic reservoir demonstrate the potential of offset risk minimization to significantly improve the worst case profit offset relative to real-life best practices. The results suggest that it may be more relevant to consider the NPV offset distribution than the NPV distribution when minimizing risk in production optimization.

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1. INTRODUCTION

Industrial strategies of oil field water flooding rely on reactive control to shut in producer wells as they become unprofitable. To enhance production, the oil literature has proposed optimal control technology, including nonlinear model predictive control (NMPC). The use of NMPC is referred to as closed-loop reservoir management (CLRM) (Janssen et al., 2009). The goal of CLRM is to determine the optimal operating profile that maximizes a key performance indicator (KPI) over the reservoir life-cycle, e.g., the cumulative oil recovery or a financial measure such as the net present value (NPV). CLRM consists of 1) an optimizer that uses the reservoir model to determine the optimal operating profile by solving a constrained open-loop optimization problem and 2) a state estimator for history matching to update the reservoir model as new data becomes available. This paper focuses on the optimizer, i.e. feedback and state-estimation is not considered. In the oil literature, this open-loop optimal control problem is referred to as life-cycle production optimization. The problem corresponds to computing the a priori optimal operating profile before the oil recovery process has begun and feedback becomes available. While simulation studies have demonstrated a significant potential of production optimization to increase overall profit, real-life applications are challenged by a wide range of uncertainties tied to reservoir simulation. To address the challenges of uncertainty, the oil literature has considered ensemble-based methods. Such methods represent the uncertainty by approximating the continuous NPV distribution by a finite number of possible outcomes, i.e., by an ensemble of realizations. To minimize risk, the ensemble members are combined to form a sample estimated risk measure that is optimized over the reservoir life-cycle. Popular ensemble-based methods include robust optimization (RO) (Van Essen et al. (2009)), mean-variance optimization (MVO) (Bailey et al. (2005), Capolei et al. (2015b)) and conditional value-at-risk optimization (CVaRO) (Capolei et al. (2015a), Siraj et al. (2015), Codas et al. (2016)). Such methods have proven to reduce overall risk relative to real-life dominating strategies of reactive control. However, ensemble-based control strategies may still result in individual profit outcomes that perform worse than reactive control. For reservoir asset managers whose primary concern is profit loss, this poses a significant risk of unacceptable low profit realizations. Therefore, despite overall lower risk, oil companies may be inclined to discard ensemble-based methodology. To meet this challenge, this paper proposes offset risk minimization. The offset approach seeks to determine the control strategy that minimizes the risk of performing worse than a competing reference strategy. To this end, the method maximizes the worst-case outcome of the NPV offset distribution. As opposed to methods of the oil literature, the offset approach mitigates the risk of low profit realizations relative to the

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comparing reference strategy. In this way, the risk of profit loss relative to industrial standards is minimized. Using an ensemble of 100 realizations of a 3D synthetic reservoir, open-loop simulations demonstrate the potential of offset risk minimization to significantly increase the offset worst-case scenario relative to reactive control. Compared to the conventional use of the NPV distribution, the results suggest that the NPV offset distribution may be more relevant for risk mitigation in life-cycle production optimization. The paper is organized as follows. In section 2, life-cycle production optimization under uncertainty is formulated as a risk minimization problem. Section 3 introduces offset risk minimization. Numerical results are presented in Section 4 and conclusions are made in Section 5.

2. LIFE-CYCLE PRODUCTION OPTIMIZATION UNDER UNCERTAINTY

Oil recovery by water flooding uses injection wells to dynamically inject water into the reservoir to displace hydrocarbons towards a set of production wells. The well injection strategy is referred to as the operating profile, \( u \). The goal of life-cycle production optimization is to determine the operating profile that maximizes profit, \( \psi \), over the reservoir life by solving the optimal control problem (Brouwer and Jansen, 2004; Sarma et al., 2005; Nævdal et al., 2006; Foss and Jensen, 2011; Völcker et al., 2011; Capolei et al., 2013):

\[
\max_{u \in \mathcal{U}} \psi(u; \theta). \tag{1}
\]

Here \( \mathcal{U} \) expresses linear decision constraints and \( \theta \subset \mathbb{R}^m \) represents geological, petrophysical and economical model parameters. In this paper, profit is given by the cumulative profit over the reservoir life by solving the optimal control problems (Van Essen et al., 2009) refers to the ensemble-based method that maximizes the life-cycle sample estimated expected return, i.e.,

\[
\mathcal{R}_{RO} := \frac{1}{n_d} \sum_{i=1}^{n_d} \tilde{\psi}^{(i)}. \tag{6}
\]

As a drawback, the expected profit is a risk neutral measure (Capolei et al., 2015a). As such, RO does not directly account for important risk indicators such as the lowest profit outcome.

2.1 Risk mitigation by ensemble-based methods

The inaccessible geographical location of oil fields severely limits the amount of available geological data. Consequently, reservoir model parameters such as permeability, porosity and initial states are often highly uncertainty. The control strategy that solves (1) therefore imposes significant risks of profit loss and becomes unreliable for practical purposes. To reduce the financial risks of model discrepancies with real-life reservoirs, the oil literature has proposed ensemble-based production optimization. Ensemble-based methods represent geological uncertainty by a discrete set of equiprobable model realizations

\[
\theta_{n_d} = \{\theta^1, \theta^2, ..., \theta^{n_d}\} = \{\theta^i\}_{i=1}^{n_d}. \tag{3}
\]

The ensemble (3) is used to approximate the continuous NPV probability distribution by the related finite set of profit outcomes

\[
\psi_{n_d} = \{\psi^i\}_{i=1}^{n_d}, \quad \psi^i = \psi(u; \theta^i), \quad 1 \leq i \leq n_d. \tag{4}
\]

To minimize risk, the idea is to manipulate the discrete NPV profit distribution (4) by formulating an appropriate optimal control problem. To this end, it is customary to use a risk measure \( \mathcal{R} : \psi_{n_d} \rightarrow \mathbb{R} \) to replace the overall profit distribution and quantify risk in terms of the scalar objective, \( \mathcal{R}(\psi) : \)

\[
\min_{u \in \mathcal{U}} \mathcal{R}(\psi(u; \theta_{n_d})). \tag{5}
\]

Figure 1 illustrates the key features of ensemble-based production optimization.

2.2 Specific risk measures and ensemble-based methods

Risk measures quantify the stochastic profit, \( \psi \), by a numerical value, \( \mathcal{R}(\psi) \), which serves as a surrogate for the overall profit distribution. The quantification of risk allows for fast and efficient decision-making. In particular, risk assessment of two scenarios, \( \psi' \) and \( \psi'' \), reduces to comparing the values \( \mathcal{R}(\psi') \) and \( \mathcal{R}(\psi'') \). However, the quality of the risk assessment heavily depends on the properties of the risk measure in question. The following briefly discusses the risk measures and related ensemble-based method used in this paper, Capolei et al. (2015a) provide a detailed overview of risk quantification in production optimization.

Robust optimization (RO) (Van Essen et al., 2009) refers to the ensemble-based method that maximizes the life-cycle sample estimated expected return, i.e.,

\[
\mathcal{R}_{RO} := \frac{1}{n_d} \sum_{i=1}^{n_d} \tilde{\psi}^{(i)}. \tag{6}
\]

As a drawback, the expected profit is a risk neutral measure (Capolei et al., 2015a). As such, RO does not directly account for important risk indicators such as the lowest profit outcome.

Worst-case optimization (WCO) (Alhuthali et al., 2010) focuses solely on maximizing the lowest profit outcome, i.e.,

\[
\mathcal{R}_{WCO} := -\min_{\theta^*} \psi(u; \theta^*) = -\tilde{\psi}. \tag{7}
\]

Here \( \tilde{\psi} \) denotes the lowest profit realization associated with the ensemble, i.e., \( \tilde{\psi} \leq \psi^i, \quad 1 \leq i \leq n_d \). The restriction to a single profit outcome implies that the measure is blind
to features of the NPV probability distribution. Consequently, the risk quantification may be too conservative and expected return may be compromised.

**Conditional value-at-risk optimization (CVaRO)** maximizes the sample estimated average of the $\alpha \cdot 100\%$ lowest outcomes, $\{\psi^i\}_{i=1}^{n_\psi}$, i.e.

$$R_{\text{CVaRO},\alpha} := -\frac{1}{n_\psi} \sum_{i=1}^{n_\psi} \psi^i, \quad \alpha \in (0,1).$$

Here $\psi^i$ denotes the $i$th lowest profit outcome associated with the ensemble. By design, CVaRO$_\alpha$ accounts for the entire $\alpha$-tail of the profit distribution. When $\alpha$ increases, CVaRO$_\alpha$ includes more of the profit distribution. As such, emphasis is gradually moved towards promoting expected return. In particular, for $n_\psi := n_d$, CVaRO$_\alpha$ reduces to RO (6). On the other hand, when $\alpha$ decreases, CVaRO$_\alpha$ includes only low profit realizations and in the extreme case of $n_\psi := 1$, CVaRO$_\alpha$ reduces to WCO (7).

As a drawback, CVaRO$_\alpha(\psi(u, \theta))$ is non-differentiable with respect to the controls, $u$, for any $\alpha \neq 1$ (Christiansen et al., 2016). The non-differentiability may interfere with the optimization procedure (5). This potentially leads to suboptimal solutions. As a way to overcome this issue, Rockafellar and Uryasev (2002) and Rockafellar and Royset (2010) show that the minimization problem (5) is equivalent to the following smooth problem

$$\min_{c \in \mathbb{R}, w \in \mathcal{U}, y \in \mathbb{R}^{n_d}} -c + \frac{1}{\alpha \cdot n_d} \sum_{i=1}^{n_d} y_i,$$

$$s.t. \quad y_i \geq c - \psi(u, \theta^i), \quad i = 1, \ldots, n_d,$$

$$y_i \geq 0, \quad i = 1, \ldots, n_d.$$

This paper uses the formulation (9) to minimize CVaRO$_\alpha$, whenever $n_\psi \neq 1$.

### 3. OFFSET RISK MITIGATION

Simulations studies have demonstrated the potential of ensemble-based methodology to reduce overall risk of profit loss relative to real-life dominating practices such as reactive control. However, the conventional ensemble-based methods and the associated risk measures take no precautions to avoid profit loss relative to a competing strategy. Consequently, despite overall lower risk, ensemble-based control strategies may lead to individual profit outcomes that perform significantly worse than a given competing industrial control strategy, $u_{ref}$. Such unacceptable low profit outcomes impose risks of profit loss. Overall, the risk may be small compared to the gains that stand to be made. However, to oil companies, risk of profit loss outweighs potential profit gains. Consequently, ensemble-based methods may be considered too risky relative to conventional reactive control. To meet this challenge, this paper proposes **offset risk minimization** as a mean to reduce risk of profit loss relative to industrial standards.

#### 3.1 The profit offset distribution

Unlike ensemble-based methods that rely on the NPV profit distribution, the offset approach uses the profit offset distribution $\psi_{\text{off}} = \{\psi_{\text{off}}^i\}_{i=1}^{n_d}$, where

$$\psi_{\text{off}}^i(u; \theta^i) = \psi(u; \theta^i) - \psi(u_{ref}; \theta^i), \quad 1 \leq i \leq n_d.$$

Here $u_{ref}$ denotes a competing reference strategy. For a given control strategy, $u$, the profit offset distribution provides a complete picture of the risk profile relative to the industrial reference case. The offset distribution therefore provides management with a tool for assessing new methodology relative to existing practices. In this regard, two distributions are of special interest: the tail profit offset distribution,

$$\{\psi_{\text{off}}(u; \theta^i) | \psi_{\text{off}} < 0\},$$

and the upper tail profit offset distribution,

$$\{\psi_{\text{off}}(u; \theta^i) | \psi_{\text{off}} \geq 0\}.$$  

These distributions represent, respectively, the distribution of the profit loss and the profit gain with respect to the reference profit.

#### 3.2 Offset risk minimization

Offset risk minimization seeks to determine the operating profile, $u$, that minimizes the risk of performing worse than a competing reference strategy. As opposed to the conventional approach of minimizing risk of the profit distribution (5), the offset approach minimizes risk of the profit offset distribution

$$\min_{u \in \mathcal{U}} R(\psi_{\text{off}}(u; \theta_{na})).$$
In this way, the risk of profit loss relative to industrial standards is minimized. Fig. 2 illustrates the idea of offset risk minimization.

3.3 Worst case offset risk minimization

In oil reservoir management, new methodology is typically not judged based on the chance of increased expected return, but rather on the risk of performing worse than existing practices. In particular, reservoir asset managers primarily focus on risks of low profit realizations. Therefore, this paper uses the offset approach with the worst-case risk measure (7) to maximize the worst profit outcome of the offset distribution:

$$
\max_{u \in \mathcal{U}} \min_{\theta} \left( \psi_{\text{off}}(u, \theta) \right) = \max_{u \in \mathcal{U}} \left( \min_{i=1, \ldots, n_d} \left( \psi_{\text{off}}(u, \theta^i) \right) \right).
$$

(14)

The optimization problem (14) is non-smooth. However, for $n_a = 1$, the numerical solution of (14) is equivalent to the solution of the smooth constrained optimization problem

$$
\min_{u \in \mathcal{U}} \left[ - \inf_{i=1, \ldots, n_d} \left( \psi_{\text{off}}(u, \theta^i) \right) \right] = \min_{u \in \mathcal{U}} \left[ \text{CVaR}_\alpha (\psi_{\text{off}}(u, \theta)) \right],
$$

(15)

Consequently, the maximization of the worst case offset profit can be regarded as an offset profit CVaR minimization problem that can be solved by (9).

4. NUMERICAL RESULTS

The following case study demonstrates the potential of offset risk minimization to reduce the risks of low profit realizations relative to academic and industrial best practices. Firstly, the offset approach is used to maximize the worst-case offset profit relative to reactive control over the reservoir life-cycle. Secondly, the offset approach is compared to RO, WCO and CVaRO to illustrate the main benefits of offset risk mitigation relative to conventional ensemble-based methods.

4.1 Reservoir model description

The numerical simulations use the standard version of the Egg model (Jansen et al., 2014). This model has been used in a number of publications as a benchmark to test optimal control methodologies (Van Essen et al., 2009). The Egg model is a synthetic reservoir model consisting of $60 \times 60 \times 7 = 25,200$ grid cells of which 18,553 cells are active. The reservoir is produced for 3,600 days under water flooding conditions. It contains eight water injectors and four producers, which are completed in all seven layers. The bilps of the producer wells are kept fixed at 395 bar and the water injection rates are subject to control with a sample time of 90 days. The water injection rates are bound to be in the interval $[0, 79.5]$ m$^3$/day. Fig. 3 shows the well setup. Model uncertainty is represented by an ensemble of 100 permeability realizations. Table 1 provides petrophysical and economical simulation parameters. Reservoir fluid flow is simulated using a two phase (oil and water) immiscible flow model with zero capillary pressure and incompressible fluids and rocks.

4.2 Numerical optimization method

The optimization problem (13) is solved using a gradient based optimization algorithm provided by MATLABs optimization toolbox (MATLAB, 2014). Given an iterate of the optimizer, $\psi(u, \theta)$ is computed by solving the flow equations using MRST (Lie et al., 2012). The gradient, $\nabla_u \psi$ is computed by the adjoint method (Jorgensen, 2007; Vöicker et al., 2011; Capolei et al., 2012a,b; Jansen, 2011; Sarma et al., 2005; Suwartadi et al., 2012). An optimal solution is reported if the KKT conditions are satisfied to within a relative and absolute tolerance of $10^{-6}$. The current best but non-optimal iterate is returned in cases for which the optimization algorithm uses more than 400 iterations, the relative change in the cost function is less than $10^{-4}$, or the relative change in the step size is less than $10^{-10}$. These stopping criteria are independent, i.e. when one of the criteria is satisfied, the optimizer stops. Furthermore, the cost function is normalized to improve convergence. The normalization consists of dividing by $10^6$ such that the objective function is appropriately scaled.

![Permeability field of the ensemble and well setup for the case study.](image-url)

Table 1. Petro-physical and economical model parameters

| Description | Value | Unit |
|-------------|-------|------|
| $h$ | 4 | m |
| $\Delta x, \Delta y$ | 8 | m |
| $\phi$ | 0.2 | - |
| $c_o$ | $1.0 \times 10^{-10}$ | Pa$^{-1}$ |
| $\sigma_r$ | 0 | - |
| $c_w$ | $1.0 \times 10^{-10}$ | Pa$^{-1}$ |
| $\mu_o$ | 5 | 10$^{-1}$ Pa$^{-1}$ |
| $\mu_w$ | 1 | 0$^{-1}$ Pa$^{-1}$ |
| $k_{ro}$ | 0.8 | - |
| $k_{rw}$ | 0.75 | - |
| $n_o$ | 3.0 | - |
| $n_w$ | 4.0 | - |
| $S_{w0}$ | 0.1 | - |
| $S_{w1}$ | 0.2 | - |
| $p_c$ | 0 | Pa |
| $p_{I_b}$ | 40 | 10$^6$ Pa |
| $S_{w1}$ | 0.1 | - |
| $p_{bhp}$ | 39.5 | 10$^6$ Pa |
| $q_{w1, min}$ | 0 | m$^3$/day |
| $q_{w1, max}$ | 79.5 | m$^3$/day |
| $r_{well}$ | 0.1 | m |
| $T_c$ | 3600 | day |
| $N$ | 40 | - |
| $r_o$ | 126 | USD/m$^2$ |
| $r_{w1}$ | 19 | USD/m$^2$ |
| $r_{w1}$ | 6 | USD/m$^2$ |
| $d$ | 0 | - |
4.3 Worst-case offset risk minimization

Fig. 4 compares the profit offset realizations associated with 1) the worst case offset optimization strategy (offset w.c. opt), 2) the worst case optimization strategy (w.c. opt), 3) the CVaR_{20\%} optimization strategy (c.s. 20\%), and 4) the robust optimization strategy (RO). All strategies produce realizations that perform worse than reactive control. However, as indicated by the 5th percentile, the worst case offset optimization strategy manages to significantly reduce both the number of negative offset realizations and the amount of potential profit loss compared to the ensemble-based strategies. In this way, the worst case offset optimization solution represents the strategy that reduces risk of profit loss relative to reactive control to the largest extent. Fig. 5 confirms these observations. In particular, for low risk levels, \( \alpha < 0.2 \), all control strategies risk to perform worse than reactive control. Nevertheless, the offset worst case optimization strategy offers the lowest risk. As a minor drawback, the low risk of profit loss comes at the price of overall lowest expected return.

Table 2 quantifies the above observations by comparing key performance indicators for the profit offset worst case optimization strategy and the ensemble-based methods. The first column compares expected returns, \( E_\theta(\psi_{off}) \) and the second column compares the worst case offset profit outcomes, \( \inf(\psi_{off}) \). The results confirm that the worst case optimization strategy offers the lowest potential profit loss at the cost of the lowest expected return. The fourth to the sixth column report the probability of negative offset profits, \( \beta = \text{Prob}[\psi_{off} < 0] \), the average offset profit of the negative offsets profits, \( E_\theta[\psi_{off}|\psi_{off} < 0] \), and the average offset profit of the positive offsets profits, \( E_\theta[\psi_{off}|\psi_{off} \geq 0] \). The results show that the offset worst case optimization strategy has a mere 8\% chance of yielding a negative offset profit of -0.22 mio USD, but a 92\% chance of yielding positive offset profits with an average value of 1.10 mio USD. This implies that the offset worst case optimization strategy provides 1) the lowest risk of profit loss and 2) at the same time, has a high probability (92\%) of outperforming reactive control. The price to be paid is that the offset worst case optimization provides the lowest average positive offset profit. This implies that the offset worst case optimization stands to improve reactive control by the smallest amount on average.

5. CONCLUSION

This paper has introduced and investigated offset risk minimization for life-cycle production optimization under geological uncertainty. Using 100 realizations of a 3D synthetic reservoir, open-loop simulations have demonstrated the potential of offset risk minimization to reduce the risk of low profit outcomes relative to the industrial standards of reactive control. To illustrate benefits over conventional risk mitigation methods, the offset approach was compared to a representative selection of ensemble-based strategies.

Based on the numerical results, the following conclusion can be made:

- Among the strategies considered in this paper, the offset worst case optimization strategy offers the lowest risk of profit loss relative to reactive control.
- Compared to the ensemble-based strategies, the worst case offset optimization strategy manages to significantly reduce both the number of negative offset realizations and the amount of potential profit loss.
- The low risk of profit loss comes at the price of overall lowest expected return.
- Due to oil companies main concern of avoiding unacceptable low profits, the results suggest that it may be more relevant to consider the NPV offset distribution than the NPV distribution when minimizing risk in production optimization.

As a minor drawback, the offset worst case optimization strategy could not ensure zero probability of yielding lower profit realizations than the reactive strategy. This is most
likely because reactive control relies on feedback. Future work seeks to explore the benefits of combining the offset risk minimization procedure with feedback using a receding horizon implementation of combined data assimilation and optimization.

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