Probabilistic distributions of $M/L$ values for ultra-faint dwarf spheroidal galaxies: stochastic samplings of the IMF

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ABSTRACT
We explore the ranges and distributions which will result for the intrinsic stellar Mass to Light ratios ($M/L$) of single stellar populations, at fixed IMF, age and metallicity, from the discrete stochastic sampling of a probabilistic IMF. As the total mass of a certain stellar population tends to infinity, the corresponding $M/L$ values quickly converge to fixed numbers associated to the particulars of the IMF, age, metallicity and star formation histories in question. When going to small stellar populations however, a natural inherent spread will appear for the $M/L$ values, which will become probabilistic quantities. For the recently discovered ultra-faint local dwarf spheroidal galaxies, with total luminosities dropping below $10^3L_V/L_\odot$, it is important to assess the amplitude of the probabilistic spread in inherent $M/L$ values mentioned above. The total baryonic masses of these systems are usually estimated from their observed luminosities, and the assumption of a fixed, deterministic $M/L$ value, suitable for the infinite population limit of the assumed ages and metallicities of the stellar populations in question. This total baryonic masses are crucial for the testing and calibrating of structure formation scenarios, as the local ultra-faint dwarf spheroidals represent the most extreme galactic scales known. Also subject to reliable $M/L$ values, is the use of these systems as possible discriminants between dark matter and modified gravity theories. By simulating large collections of stellar populations, each consisting of a particular collection of individual stars, we compute statistical distributions for the resulting $M/L$ values. We find that for total numbers of stars in the range of what is observed for the local ultra-faint dwarf spheroidals, the inherent $M/L$ values of stellar populations can be expected to vary by factors of upwards of 3, interestingly, systematically skewed towards higher values than what is obtained for the corresponding infinite stellar population limit $M/L$. This can serve to explain part of the spread in reported baryonic masses for these systems, which also appear as shifted systematically towards high dark to baryonic mass ratios at fixed stellar velocity dispersions, when going to the ultra-faint limit.

Key words: stars: statistics — galaxies: stellar content — stars: luminosity function, mass function

1 INTRODUCTION

In determining the inherent mass to light, $M/L$, ratios of stellar populations, it is customary to treat the distribution of stellar masses through a probability function, the IMF. When dealing with a large galaxy, or a large stellar population in general, it is justified to think of the IMF as a densely sampled probability function. In practice, this translates into models where the IMF appears as a series of weighting factors to be applied to the heavily mass dependent stellar luminosities, when calculating the overall $M/L$ ratios for stellar populations.

In going to small star formation events, individual star forming regions, fractions of small galaxies or the stellar populations of the local ultra-faint dwarf spheroidals recently available to detailed observation however, the standard assumption of stellar populations, that the IMF is a densely sampled probabilistic distribution function, breaks down. If we are dealing with a small star formation event, calculating the resulting stellar $M/L$ will require the explicit inclusion of the probabilistic nature of the IMF, in a regime where this function is being only poorly and discretely sampled.

The consequences of this change in regime are amplified by the strong power law character of the IMF, further
compounded by the heavily mass weighted dependence of stellar luminosities. In a sample of large stellar populations of fixed total mass, the actual number of giant stars for any one will vary by only a fraction of a percent, due to the intrinsic variance associated with the probabilistic sampling of the IMF. The resulting variance in the intrinsic \( M/L \) values will be correspondingly small, and is hence customarily ignored. If the total mass of stars is of up to a few thousand \( M_\odot \) however, the intrinsic probabilistic variance of a standard IMF will lead one to expect only a few giant stars. It is clear that the resulting intrinsic variance to be expected in the final \( M/L \) values resulting from such a population will be of a large factor, with distributions heavily dependent on both the total mass and age e.g. Cerviño & Luridiana (2006), Carigi & Hernandez (2008). Both the stochastic effects of IMF sampling for very low total mass populations, and correlations between intrinsic IMF and total star formation masses have been shown to be important by Weidner & Kroupa (2004), Weidner & Kroupa (2006) and Koeppen et al. (2007), for star formation episodes resulting in a few thousand stars, with of order of \( 10^4 M_\odot \) in total gas mass involved, considering typical efficiency factors of a few percent.

The statistical variance in the light output of a stellar population has been studied previously e.g. Tonry & Schneider (1988), Cerviño & Valls-Gabaud (2003). The most important practical application of which has been the development of the surface density fluctuation method of distance determination in galaxies, Tonry & Schneider (1988). Similarly, such studies lead to the realization that the intrinsic variances in populations of even thousands of stars, in certain observed bands or emission lines, can lead to relevant effects. The chemical consequences of a stochastic star formation event have recently been studied by Koeppen et al. (2007). Also, Carigi & Hernandez (2008) showed that the intrinsic variance present in populations of even more than a few thousand stars, can be significant towards explaining part of the scatter in abundance ratios observed within the classical local dwarf spheroidals. Similarly, Cescutti (2008) applied precisely such ideas to explain the spread in neutron capture elements for low metallicity stars in the Solar Neighbourhood. Using results of population synthesis codes which assume the IMF has been densely sampled, and which yield a unique answer for certain observed properties of a stellar population of fixed mass and metallicity, can lead to the over-interpretation of observations if one is not careful e.g., Cerviño & Valls-Gabaud (2003).

The recently discovered and studied ultra-faint local dwarf spheroidal galaxies (e.g. Belokurov et al. 2006, Simon & Geha 2007, Geha et al. 2009, Martin et al. 2009), with total luminosities of the order of \( 10^2 \) in solar units and below, and total dynamical \( M/L \) values \( > 10^3 \), represent a population of objects where the intrinsic variance in the stellar \( M/L \) ratios, at fixed age and metallicity, might become important. To date, the inferences of total baryonic masses for these objects have been performed in the standard way, through the use of \( M/L \) ratios thought of as deterministic parameters, as appropriate for finite stellar populations, e.g. Angus (2008) or McGaugh & Wolf (2010). These total baryonic masses become important in determining the baryonic to dark matter ratios of the systems in question (e.g. Sanchez-Salcedo et al. 2006 or Walker et al. 2009), the calibrating and testing of structure formation models at the smallest galactic scales (e.g. Strigari et al. 2010), or the testing of modified gravity theories, where gravity is assumed to couple exclusively to the observable baryonic mass e.g. Hernandez et al. (2010), Iorio (2010). Given however the negligible dynamical relevance of the stellar components (under standard gravity), the dark matter to total mass ratios would remain unchanged at values very close to 1.

In this paper we calculate the range in stellar \( M/L \) values for small stellar populations of fixed input parameters, which result from the intrinsic statistical variations of a discretely and poorly sampled standard IMF. Throughout this paper we shall use \( M/L \) to denote total intrinsic stellar mass to light ratios, and not in connection with dynamical mass to light ratios, which of course are completely robust to the considerations explored here. As we shall see, the low mass weighted character of the IMF will result in a slow drift in the mean \( M/L \) values of stellar populations, as the total stellar mass goes down. More important will be the appearance of a large spread in the resulting \( M/L \) values, which will shift qualitatively from being the deterministic proportionality factors which they are for large stellar populations, to becoming probabilistic entities with broad distributions skewed towards high values. As the total mass diminishes the stochastic effects on the \( M/L \) ratios appropriate for star formation events will increase. Therefore, it is the regime of the ultra-faint dwarf spheroidals, very small systems with old ages, where the effects being explored will be largest.

We simulate stellar populations having various total masses by sampling directly an assumed IMF, this produces a discrete collection of stars, which is then used together with a detailed isochrone library to produce distributions of \( M/L \) values for collections of stellar populations having various fixed total masses. We explore the resulting distributions for \( M/L \) ratios as functions of the metallicity, stellar age and the total mass, by directly keeping track of each individual star formed. Repeating the process a large number of times yields a distribution of \( M/L \) values resulting from the same input parameters.

The paper is organized as follows: The construction of discrete IMF realizations and their use in constructing stochastic distributions of \( M/L \) values for simple stellar populations of fixed input parameters is described in section (2), section (3) gives our results for total stellar masses in the ranges of the observed ultra-faint dwarf spheroidals, and section (4) presents our conclusions.

## 2 Constructing Statistical \( M/L \) Values

In order to explore the range of intrinsic \( M/L \) values which small stellar populations will present, we begin by setting up a discrete IMF. We assume a fixed underlying probabilistic IMF, from which stochastic samplings will be constructed, discrete collections of individual stars. We take the IMF of Larson (1998):

\[
dN/d\log m \propto (1 + m/m_\star)^{-1.35},
\]

where a choice of \( m_\star = 0.4 M_\odot \) adequately serves to reproduce a present day solar neighbourhood IMF, with a mean mass close to 1 \( M_\odot \), e.g. Hernandez & Ferrara (2001). We
have taken lower and upper mass bounds of 0.09 \( M_\odot \) and 20 \( M_\odot \) for the above IMF, fixed throughout this study. Although the details of our study will be slightly sensitive to the choice of this function, the trends we describe and our conclusions are generic to any IMF found in the literature, where the probability of picking a certain mass strongly decreases with stellar mass. These discrete IMFs will only tend to the infinite mass limit for very large total stellar masses. Even for total stellar masses of a few thousand \( M_\odot \), which one could naively assume to constitute ‘statistical samples’, the distribution of stars above 1\( M_\odot \) will be dominated by shot noise effects, e.g. see Carigi & Hernandez (2008). This leads to an effective upper stellar mass which decreases as the total mass of a stellar population goes down e.g. Massey (1998), Weidner & Kroupa (2004). This in turn leads to substantial scope for variations in the resulting intrinsic \( M/L \) values, as the light output of a stellar population is heavily dominated by the giants, while the total mass is a much more robust quantity, anchored on the integral of the main sequence. We start by picking a value for the total stellar mass of a single stellar population, and then proceed to randomly pick stars out of the fixed underlying probabilistic IMF, until the chosen total stellar mass has been reached. This produces a collection of individual stars, a particular discrete IMF.

Next, we use an extensive isochrone library which carefully interpolates directly on the output of stellar evolutionary codes at fixed stellar phase, having close to 250 masses between a lower mass bound of 0.15\( M_\odot \) and the tip of the RGB. This was prepared using the stellar evolutionary codes of the Padova group (Girardi et al. 2002), for use in the probabilistic parameter inference study for globular clusters of Hernandez & Valls-Gabaud (2008). With this at hand, we then assign to each of the individual stars selected its corresponding \( M_V \) value. By then adding the corresponding luminosities, we calculate the total \( V \) band luminosity of a particular realisation of the fixed underlying IMF, at a given age and metallicity.

We have set to zero the luminosity of all stars outside of the mass range of the isochrones. This introduces a slight error, but one which will not affect our results significantly, as the integrated luminosity of stars between our lower IMF limit of 0.09\( M_\odot \) and our lower isochrone mass limit of 0.15\( M_\odot \) is only a very minor contribution to the total light output of a stellar population. Beyond our upper isochrone limit at the tip of the RGB, the number of bright sources is small, and its exclusion does not significantly alter the total light budget. A further small error is introduced by having also ignored stellar mass loss throughout, i.e., the initial mass of the total stellar population is divided by the total \( V \) band luminosity within the isochrone range, to obtain the final \( M/L \) value of a particular IMF realization, at a given total mass, age and metallicity.

Depending on the chosen value for the total stellar mass, and on the assigned age of the stellar population, the number of stars selected varies from a few hundred to upwards of 70,000, for the range of models presented here, for each individual discrete IMF realization. Each of these stars is then assigned a \( V \) band luminosity, as described above. The whole process is then repeated 2000 times, changing the random seed of the simulation, to construct a distribution of 2000

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Distributions of \( M/L \) values for 2000 statistical realizations of single stellar populations, all at a fixed age of 10.5 Gyr, all at a fixed metallicity of \([Z]=-3.0\), for different total stellar masses of \(4 \times 10^4\), \(1 \times 10^5\), \(3 \times 10^5\), \(1 \times 10^6\), and \(5 \times 10^6\) solar masses, from the most to the least peaked respectively.}
\end{figure}

\section*{3 RESULTING DISTRIBUTIONS OF \( M/L \) VALUES}

Here we present results for the experiment described in the previous section. We begin by taking a fixed age of 10.5 Gyr and a fixed metallicity of \([Z]=-3\) in solar units, parameters as appropriate for the old local dwarf spheroidals e.g. Hernandez et al. (2000), Kirby et al. (2008), Sand et al. (2009), Geha et al. (2009), Sand et al.(2010). We then simulate 2000 single stellar populations having always a fixed total mass of \(5 \times 10^2 M_\odot \). Each results in a discrete collection of between 650 and 850 stars, all of which are individually assigned a luminosity from the appropriate isochrone.

A binned distribution for the resulting \( M/L \) values for this case is given by the thick solid curve in figure (1). A very broad distribution presenting evident fluctuations is apparent, extending to values of \( M/L = 11 \). This is natural if one considers that the small samples of only several hundred stars from which these \( M/L \) values have been calculated, reproduce the underlying probabilistic IMF only for low mass stars, shot noise dominates the distribution already for masses above 1\( M_\odot \). These low mass stars in turn, typically have very low luminosities, hence resulting preferentially in \( M/L \) values higher than the infinite population limit for the parameters used, of slightly below 3.5. Occasionally, an IMF realization with over average numbers of comparatively massive stars appears, resulting in the extension seen towards lower than average \( M/L \) values.

As we increase the total mass of this stellar population to \(1 \times 10^3 M_\odot \), we obtain the thin solid curve for the cor-
responding distribution of intrinsic $M/L$ values, still at the same fixed age, metallicity and IMF. In this case, we see that the fluctuations start to disappear, as a smoother distribution results. Still, the thin solid curve of this case shows that the fluctuations start to disappear, as a smoother distribution remains skewed towards higher than average values, for the same reasons as mentioned above. We can see that at these numbers of stars, we can still expect to find $M/L$ values ranging from 2 to 7, even though $1 \times 10^3$ might ordinarily be considered an 'statistical number' of stars. As it also happens when considering the intrinsic spread in the chemical enrichment properties of a stellar population (Carigi & Hernandez 2008), the heavily low mass weighted nature of the IMF, plus the strongly top heavy nature of the light output or enrichment properties of stars, allows for wide distributions in the intrinsic properties of stellar populations, at fixed input parameters.

In going to total stellar masses of $3 \times 10^5 M_\odot$, $1 \times 10^4 M_\odot$, and $4 \times 10^4 M_\odot$, we obtain the corresponding distributions of intrinsic $M/L$ values, given by the long dashed, short dashed and dotted curves in figure (1). We see that these distributions tend towards the infinite population limit, $(M/L)_\infty$ with mean values which clearly converge quite rapidly. The inherent spread of the distributions however, takes longer to tend to zero, and even for quite large numbers of stars of upwards of 60,000 contained in each of the 2000 simulations with results given by the dotted curve, a noticeable width to the distribution is still evident.

To better appreciate the manner in which the resulting distributions tend to the infinite population limit, we present figure (2), where the average values of the $M/L$ ratios, $< M/L >$, for stochastic realizations of the IMF are given, as a function of the total stellar mass, for three different stellar ages of 10.5 Gyr, 5 Gyr and 2 Gyr, from top to bottom, respectively.

As seen from figure (1), the mean values for the distributions of $M/L$ ratios quickly tend to the infinite population limit, $(M/L)_\infty$, also as expected, this occurs at smaller total stellar masses for the younger populations. The drift towards larger $M/L$ values as the total stellar mass goes down, for the mean of the distributions is also evident. This effect is fairly important in the case of the oldest populations, and probably accounts for at least part of the trends seen for the dark to baryonic matter ratios to increase in the ultra-faint dwarf spherioiads. Corresponding plots at different metallicities are practically indistinguishable, until one reaches metallicities of upwards of $[Z] = -1$, values no longer relevant for the local dwarf spheroidal galaxies.

The corresponding plot for the standard deviations, $\sigma$, of the distributions of $M/L$ ratios is given in figure (3). The three curves correspond to the same cases of figure (2), again, ages of 10.5 Gyr, 5 Gyr and 2 Gyr, from top to bottom, respectively. This time we see that although the dispersion in the distribution of $M/L$ values goes down as the total mass of the stellar populations increases, this happens at a much slower rate than what characterises the trend for $< M/L >$ with total stellar masses. We see that for the oldest age, even for stellar populations well into the thousands of $M_\odot$, dispersions of more than 1 are to be expected. Since the corresponding $< M/L >$ values are of around 4, variations in $M/L$ by factors of 2 and above will be frequent. Also, notice that since the distributions are heavily skewed, the mean and the dispersion offer only crude approximate descriptions, with higher than average values being the norm.

The resulting $< M/L > - (M/L)_\infty$ and $\sigma$ values can be very accurately described by the following fitting functions:
< M/L > = \frac{1}{\sigma} \ln \frac{M_{\text{tot}}}{L}\), \quad \sigma = BM_{\text{tot}}^{-1/2},

(2)

with the constants \((A, B)\) in eq. (2) having values (524.72, 48.53), (180.56, 22.16), and (74.84, 10.02) for stellar ages of 10.5 Gyr, 5 Gyr and 2 Gyr, respectively, for metallicities below \([Z] = -1\) in solar units. In the above equations \(M_{\text{tot}}\) is the fixed total stellar mass of a set of IMF realizations.

Finally, we present in figure (4) an estimate of the factor by which one can expect the intrinsic \(M/L\) values of small stellar populations to vary, for metallicities and ages suitable for the local ultra-faint dwarf spheroidals. The factor \(F\) is defined as \((< M/L > + 1.5\sigma)/(< M/L > - 1.5\sigma)\), to better account for the strongly skewed distributions present. We see again this factors tending rapidly to \(F = 1\) as one exceeds total stellar masses of \(3 \times 10^3 M_\odot\), but reaching quite high values of more than 6 for total stellar masses below \(3 \times 10^3 M_\odot\) in the range of parameters inferred for the systems in question. For comparison, the following of the local ultra-faint dSphs discovered to date have inferred total stellar masses of order \(10^3 M_\odot\) and below: Boötes II, Ursa Major II, Willman I, and Coma Berenices, with Segue I coming in at a mere \(600 M_\odot\) in stars, as reported in the compilation of Misgeld & Hilker (2011).

As can be seen from figure (4), the differences between the results for ages of 10.5 Gyr and 5 Gyr are much smaller than between results for 5 Gyr and 2 Gyr. In fact, the inherent convergence of the isochrones at large ages implies that results for any stellar ages beyond 10.5 Gyr, will be scarcely distinguishable from the curves shown for 10.5 Gyr. Thus, the results given for 10.5 Gyr ages are suitable for the directly inferred ages of some of these systems, e.g. the values of between 12 and 13 Gyr obtained by Sand et al. (2009) for the Hercules system, or by Sand et al. (2010) for Leo IV. Also, although the trends presented will be qualitatively the same for \(M/L\) ratios in other bands, the amplitude of the effect presented will grow towards bluer bands, and decrease towards redder ones, as the relative contribution of the different evolutionary phases changes to include a smaller or larger fraction of the stars.

4 CONCLUSIONS

After calculating directly statistical distributions for the inherent \(M/L\) ratios of small stellar populations, we find that in the low mass range of the local ultra-faint dwarf spheroidals, assigning a \(M/L\) ratio to a stellar population changes from the deterministic problem of finding the value which corresponds to an infinite population having a required metallicity, age and star formation history, to an entirely probabilistic situation. Indeed, below total stellar masses of \(3 \times 10^3 M_\odot\), the \(M/L\) distributions become so broad, that the probabilistic nature of the problem becomes the dominant ingredient, relegating age and metallicity to a secondary role in establishing the intrinsic \(M/L\) ratio of a stellar population. This is particularly relevant to the study of the local ultra-faint dwarf spheroidals, as determining their baryonic masses through assigning fixed, standard \(M/L\) values, can easily lead to significant error. This is particularly delicate as the \(M/L\) distributions which result are far from symmetric about \((M/L)_{\infty}\), being heavily and systematically skewed towards higher values.

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![Figure 4. Expected factors over which the intrinsic M/L ratios of single stellar populations are expected to vary, at fixed age and metallicity, as a function of total stellar mass in solar units, for three different ages of 10.5 Gyr, 5 Gyr and 2 Gyr.](image-url)
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