On the Degeneration of Turbulence at High Reynolds Numbers

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Turbulent fluctuations in a fluid wind down at a certain rate once stirring has stopped. The role of the most basic parameter in fluid mechanics, the Reynolds number, in setting this decay rate is not generally known. This paper concerns the high-Reynolds-number limit of the process. In a wind-tunnel experiment that reached higher Reynolds numbers than ever before and covered more than two decades in the Reynolds number \(10^4 < Re = UM/\nu < 5 \times 10^6\), we measured the decay rate with the unprecedented precision of about 2%. Here \(U\) is the mean speed of the flow, \(M\) the forcing scale, and \(\nu\) the kinematic viscosity of the fluid. We observed that the decay was Reynolds number independent, which contradicts some models and supports others.

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Turbulence dissipates kinetic energy, and the easiest way to see this is by letting turbulence decay freely, by removing the agitation that initially set the fluid in motion. In so doing, one observes qualitatively that the fluid comes again to rest. The rate at which that happens is the subject at hand, and underlies general turbulence phenomena and modeling. To simplify the problem, the statistics of the turbulent fluctuations are often arranged to be spatially homogeneous and isotropic [1, 2]. Even under these conditions, it is not possible quantitatively to predict the decay rate; it is even difficult precisely to measure the decay rate [3]. Yet empirical constraints on the properties of decaying turbulence are what is needed to advance the knowledge of the subject.

The physics that control the decay are not fully understood, to the extent that the role of the most basic parameter in fluid mechanics, the Reynolds number, is not clear. The rate of decay in the limit of large Reynolds numbers, in particular, cannot be inferred from previous data. Toward low Reynolds numbers there is a transition to a more rapid decay [4].

A dominant theoretical framework predicts that the decay rate depends on the large-scale structure of the flow, and not on the Reynolds number once turbulence is fully developed [2, 5, 7]. As the Reynolds numbers diverge to infinity, the scale at which energy is dissipated grows arbitrarily small, but these scales continue to dissipate energy as quickly as it is transferred to them by the large scales. This picture is compatible with an emerging consensus that the initial structure of the turbulence sets its decay rate, even when the flow is homogeneous [8, 11].

There is also, however, a persistent line of thinking that an elegant and fully self-similar decay emerges in the limit of high Reynolds number [8, 9, 12, 13]. In this description, turbulence tends to become statistically similar to itself, when appropriately rescaled, even as it decays. Observation of a tendency toward slower decay with increasing Reynolds numbers would support this view [8, 12, 17, 18]. Our data, however, do not show this tendency.

The issue is of practical consequence because decaying turbulence is used to benchmark turbulence models [1, 19, 21]. Not only this, but both simulations and experiments can be performed easily at low Reynolds numbers, say \(Re < 10^3\); how well can their findings describe the high-Reynolds number flows that exist in nature, where \(Re > 10^6\)? Our experiment bridges this gap, and also contributes to a basic understanding of the nature of turbulence. The objectives are similar to those in other recent programs in fluid mechanics, where asymptotic scaling behavior was sought in other types of flows [22, 23].

According to dimensional reasoning, it is useful to think of physical laws in terms of dimensionless numbers [24]. In turbulence, these numbers include the Reynolds number and a family of others that describe the initial and boundary conditions (BCs) of the flow. Some set of these numbers might control the decay of the turbulence. To isolate Reynolds number effects in our experiment, we held fixed the other numbers. That is, we fixed the BCs so that the large-scale structure of the flow was approximately fixed. We changed the Reynolds number by varying the viscosity of the fluid [23]. The ability to do this was almost unique among turbulence decay experiments, and made it possible both to cover a wider range and to reach higher Reynolds numbers than ever before. In previous experiments, changes to the BCs and to the Reynolds number were often coupled, which confounded Reynolds number effects with those arising from changes in the large-scale structure of the flow.

Our experiment was based on a tradition established by the early pioneers of using wind tunnels with grids in them as instruments to discover empirically how turbulence decays [22, 26]. Grid turbulence can also be thought of as the canonical wake flow. Recent work has focused on the turbulence behind grids with novel geometries and on the flow very close to the grids where unstable mean gradients energize the turbulence [11, 29, 31].

We performed the experiments in the Variable Density Turbulence Tunnel (the VDTT) [25]. The VDTT circulated both air and pressurized sulfur-hexafluoride. The
Reynolds number was adjusted by changing the pressure of the gas, which changes its kinematic viscosity. Turbulence was produced at the upstream end of the 8.8 m long upper test section by a bi-planar grid of crossed bars with square cross section. The mesh spacing, \( M \), of the grid was 0.18 m, and the projected area of the grid was 40\% of the cross section of the tunnel. A linear traverse positioned probes at 50 logarithmically spaced distances, \( x \), between 1.5 m and 8.3 m downstream of the grid. A Galilean transformation converts the distances from the grid into the time over which the fluctuations decayed, so that \( t = x/U \). Here \( U \) is the mean speed of the flow down the tunnel, which was about 4.2 m/s for most experiments.

We used hot-wire probes to obtain long traces of the component of the velocity aligned with the mean flow. At each of the several distances from the grid we acquired 5 minutes, or of the order of \( 10^4 \) integral lengths, of data. We used classical hot wires produced by Dantec Dynamics from both 1.25 mm lengths of 5 micron wire, dubbed the P11 probe, and from 450 micron lengths of 2.5 micron diameter wire, dubbed the Mini probe, as well as the new NSTAP probe developed at Princeton that is just 60 microns long [32, 33]. The probes were approximately at the centerline of the tunnel, with the P11 downstream of a grid bar, the Mini behind the edge of a grid bar, and the NSTAP between grid bars. The small differences between the results given by different probes do not affect the conclusions of this paper, and will be the subject of future detailed report.

Figure 1a shows the normalized turbulent kinetic energy as a function of the time since the flow passed through the grid for several Reynolds numbers. The variance of the velocity, \( u^2 \), is proportional to the total kinetic energy in the turbulent fluctuations, and we refer in this paper to \( u^2 \) as the kinetic energy itself. This is because the total mass of the fluid is fixed, because grid turbulence is nearly isotropic, and because the residual anisotropy in the fluctuation amplitudes decays much more slowly than their energy [2, 11, 18, 34, 35]. The solid curve, the master curve, was calculated by taking the mean of 99 decay curves accumulated by all three probes at all Reynolds numbers. The decay curves are nearly identical to each other, even though their Reynolds numbers span more than two orders of magnitude.

In Fig 1a, we offset time by \( t_0 \), the so-called virtual origin, which represents the time it takes for the turbulence to organize itself after the flow has passed through the grid. In practice, the virtual origin is obtained by performing a three-parameter power-law fit of the data to

\[
\frac{u^2}{U^2} = C \left( \frac{t - t_0}{M} \right)^n
\]

(1)

using a nonlinear least-squares algorithm [36], where \( n \) quantifies the decay rate that is the subject of this paper.

![Figure 1](image.png)

**FIG. 1:** (a) The turbulence kinetic energy, \( u^2 \), decays approximately as a power law function of time, \( t \), for 7 representative Reynolds numbers. The data are shifted vertically for clarity. The values of \( Re \times 10^{-3} \) for each curve were 26, 54, 140, 410, 820, 1700, 3200, and 4800 from bottom to top. The lowest curve, the master curve, is the mean of 99 such curves acquired at different Reynolds numbers. We normalized the kinetic energy by that in the mean flow, \( U^2 \), and time by \( M/U \). Time was offset by \( t_0 \), the virtual origin. (b) We found the virtual origin by fitting each decay curve to a power law as described in the text. Over the full range of Reynolds numbers, the virtual origin fluctuated with a standard deviation of less than 10\% of its mean value of \( t_0 U/M = 3.66 \pm 0.03 \).

From this formula it is clear why the origin is ‘virtual,’ since it corresponds to an unphysical time of divergent energy. It is well-known that the uncertainties in \( t_0 \) and \( n \) are coupled, leading to unreliable estimates of these parameters [3]. We describe two ways to avoid this difficulty below, though the conclusion of the paper is not sensitive to the method of analysis.

Figure 1b shows that the virtual origin, to a first approximation, was \( Re \) independent. Indeed, the position of the virtual origin is connected to the development of the wakes behind the square bars that compose the grid, and these wakes are themselves nearly \( Re \) independent in this regime [41]. The wake does not show the drag crisis, for example, characteristic of circular cylinders. Because of this, we assigned a fixed value to the virtual origin.

With the virtual origin fixed to its mean value, \( t_0 U/M = 3.66 \), we could measure the decay exponents
data, for which the mean decay rate was

\[ n = -1.18 \pm 0.02. \]

The 95% confidence interval for each value of \( n \) (±0.9%) is given approximately by the size of the symbols. Various additional data were drawn from the literature as follows: \( \heartsuit \): Mohamed and LaRue [3], \( \diamondsuit \): White et al. [37], \( \bigodot \): Lavoie et al. [9], \( \bigtriangledown \): Antonia et al. [38], \( \bigstar \): Krogstad and Davidson [32], \( \triangle \): Thormann and Meneveau [11], \( \bigcirc \): Bewley et al. [39], \( \bigtriangleup \): Kistler and Vrebalovich [40]. In some of these experiments, variations in conditions were made deliberately to elicit changes in the decay rate. The inset of Fig. 2 follow. The scatter in \( n \) probably arose both because of uncertainty in the measurements, or because of variation in the initial and boundary conditions between experiments. Note that Mohamed and LaRue [3] constrained the virtual origin in their analysis of collected data to be equal to zero. For classical grids, where the virtual origin is positive, this has the effect of depressing the decay exponents toward more negative values. Among the highest-Reynolds-number experiments, Bewley et al. [39] and Kistler and Vrebalovich (KV) [40] observed particularly slow decays. In computer simulations [17, 42, 43], the decay slowed down slightly with increasing \( Re \), but with similar reach in \( Re \) and scatter in the exponents as in previous experiments. The results from some novel decay experiments are not shown because the form of the decay was not a power law or because the decay exponents fell out of the range of the plot. Collectively, the previous data leave open the question of how the decay rate behaves in the limit of large Reynolds numbers. The present study was the first to revisit the high Reynolds numbers of KV, and not only reached higher but did so in a better controlled flow.

Because the Reynolds number seemed not to be a governing parameter in the decay, we sought an expla-
nation for the decay rate in terms of the large-scale structure of the flow. One way to derive predictions for the decay rate is to consider the evolution equation for the kinetic energy in freely decaying turbulence: \((3/2)du^2/dt = -\epsilon\). In the classical description, the dissipation rate, \(\epsilon\), is independent of \(Re\). To produce the smooth curve, we took the median values of \(L/M\) and \(u^2/U^2\) data falling within logarithmically spaced bins in \(u^2/U^2\). The relationships posited by well-known theories are indicated by straight lines. Our data are consistent with Saffman’s theory, with slope \(-1/3^d\).

In order to integrate the energy equation, some relationship between the energy, \(u^2\), and the correlation length, \(L\), must be derived. Typically, this relationship is a power law, \(L \sim (u^2)^m\), with \(m = -1/2, -1/3,\) and \(-1/5\), for each of the self-similar, Saffman, and Kolmogorov theories, respectively [2, 7, 12]. Yet other exponents could be supported by different arguments [2, 7]. Integration of the energy equation then yields predictions for the decay law, \(u^2 \sim t^n\), with \(n = -1, -6/5\) and \(-10/7\), for the three theories, respectively, and \(n = 2/(2m - 1)\) generally [34]. Fortunately, it is possible to look directly for a relationship between \(u^2\) and \(L\), without the ambiguity of determining a virtual origin, since the virtual origin must be a property of the flow and hence be the same for all of its statistics.

Figure 3 shows the correlation length, \(L\), calculated from the scale, \(r\), at which the velocity correlation function dropped below \(1/e\). The velocity correlation function was \(f(r) = \langle u(x)u(x+r) \rangle_x/u^2\), which could be calculated from our time series of velocity in the usual way through Taylor’s hypothesis [45]. Grey symbols mark our data, whereas the solid lines denote the various theoretical predictions. Neither Kolmogorov’s nor the self-similar decay agrees with our data, whereas Saffman’s prediction is consistent with our data for large \(u^2/U^2\).

We attribute the deviations from Saffman’s prediction at large times to confinement of the flow by the boundaries of the tunnel. Where the data are approximately straight, a power-law fit yields \(m = -0.348\), so that the theory predicts \(n = 2/(2m - 1) = -1.18\), which is consistent to the third digit with our direct measurement of the decay exponent, \(n\).

In summary, we observed the decay rate of turbulence to be independent of the Reynolds number, and to proceed in a way consistent with Saffman’s prediction [5]. Different initial and boundary conditions might decay differently and even with different Reynolds number dependencies, but our results indicate that the self-similar decay is probably not the generic high-Reynolds number decay. We attribute the residual non-monotonic Reynolds number dependencies in our data, if there were any, to small Reynolds-number variation in the initial conditions and not to an effect of the Reynolds number on the physics of the decay process.

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