Can conventional forces really explain the anomalous acceleration of Pioneer 10/11?

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Abstract

A conventional explanation of the correlation between the Pioneer 10/11 anomalous acceleration and spin-rate change is given. First, the rotational Doppler shift analysis is improved. Finally, a relation between the radio beam reaction force and the spin-rate change is established. Computations are found in good agreement with observational data. The relevance of our result to the main Pioneer 10/11 anomalous acceleration is emphasized. Our analysis leads us to conclude that the latter may not be merely artificial.

1 Introduction

First published in 1998, results from an almost twenty years study of radiometric data from Pioneer 10/11, Galileo and Ulysses spacecraft have been continuously reported by Anderson et al. [1]. They indicate an apparent anomalous, constant, acceleration acting on the spacecraft with magnitude $a_P = (8.74 \pm 1.33) \times 10^{-8}$ cm s$^{-2}$, directed towards the Sun, to within the accuracy of the Pioneers’ antennas. Besides, an independent analysis of radio Doppler tracking data from the Pioneer 10 spacecraft for the time period 1987-1994 confirms the previous observations [2].
Also, the possibility that there exists an error in the JPL’s ODP program have been removed by using an independent program, CHASMP.

Now, a number of potential causes have been ruled out by the Anderson et al. team, namely gravity from Kuiper belt, gravity from the Galaxy, spacecraft "gas leaks", anisotropic heat (coming from the RTGs) reflection off of the back of the spacecraft high-gain antennae (Katz’s proposal [3], see Anderson et al. [4]), radiation of the power of the main-bus electrical systems from the rear of the craft (Murphy’s proposal [5], see Anderson et al. [6]), errors in the planetary ephemeris, and errors in the accepted values of the Earth’s orientation, precession, and mutation, as well as nongravitational effects such as solar radiation pressure, precessional attitude-control maneuvers and a possible nonisotropic thermal radiation due to the Pu$^{238}$ radioactive thermal generators. Indeed, according to the authors, none of these effects may explain the apparent acceleration and some are 3 orders of magnitude or more too small. So, they conclude that there is an unmodeled acceleration towards the Sun of $(8.09 \pm 0.20) \times 10^{-8}$ cm s$^{-2}$ for Pioneer 10 and $(8.56 \pm 0.15) \times 10^{-8}$ cm s$^{-2}$ for Pioneer 11.

In a further study Anderson et al. [7], observed that the difference of the spin-rate history for the Pioneers 10 and 11 explains the small difference of magnitude of the anomalous acceleration for Pioneer 10 and for Pioneer 11. The crucial point is that, removing the spin-rate change contribution, one is left with an anomalous acceleration of the same amount with a great accuracy ($(7.84 \pm 0.01) \times 10^{-8}$ cm s$^{-2}$ instead of $(8.74 \pm 1.33) \times 10^{-8}$ cm s$^{-2}$) during a very long time interval (almost 20 years) for both Pioneer 10/11 to explain. In view of the latter point, it is clear that a conventional explanation of the Pioneer 10/11 anomalous acceleration versus spin-rate change would at the same time clarify and emphasize the possible importance of the main Pioneer anomaly for fundamental physics. Finally, the Pioneer 10/11 anomaly should deserve more serious attention both on the theoretical and
observational grounds.

2 Study of the Pioneer 10/11 anomalous acceleration versus spin-rate change

2.1 Reformulation of the new correlation

From the study of the Pioneer 10/11 anomalous acceleration, $a_P$, Anderson et al. [7] have found a correlation between $a_P$ and the rotational acceleration, $\ddot{\theta}$, of both spinning spacecraft. They expressed this as follows

$$a_P = a_P(0) - \kappa \ddot{\theta},$$  \hspace{1cm} (1)

where $\kappa$ is a constant with unit of length, $a_P = a_P(\ddot{\theta})$ and $a_P(0) \simeq 8 \times 10^{-8}$ cm s$^{-2}$ are respectively the Pioneer anomalous acceleration with and without any spin-rate change, and $\ddot{\theta}$ is the rotational acceleration derived from the best fit to the data [7]. The overall study is based on the observation of discrepancies between the frequency of the re-transmitted signal observed by the DSN (Deep Space Network) antennas, $\nu_{obs}(t)$, and the predicted frequency of that signal, $\nu_{model}(t)$. The observed two-way anomalous effect is expressed (in the first order in $v/c$) as

$$[\nu_{obs}(t) - \nu_{model}(t)]_{DNS} = -2 \nu_0 \frac{a_P}{c} t,$$  \hspace{1cm} (2)

where $\nu_0$ is the reference downlink frequency and $\lambda_0 = c/\nu_0 = 13.06$ cm is the corresponding wave-length. Combining both relations (1) and (2) yields

$$[\nu_{obs}(t) - \nu_{model}(t)]_{DNS} = 2 \nu_0 \frac{\Delta v}{c} + 2 \frac{\kappa}{\lambda_0} \Delta \dot{\theta},$$  \hspace{1cm} (3)

where $\Delta v = -a_P(0) t$ and $\Delta \dot{\theta} = \ddot{\theta} t$ denotes respectively the variations of the radial velocity and the spin-rate of the spacecraft. Clearly, relation (3) above suggests that beside the familiar Doppler effect connected to the linear velocity, a less commonly known frequency shift that is connected to the rotational velocity may be
acting. The latter effect is known indeed both on the theoretical [8, 9, 10] and observational [11, 12, 13, 14] grounds being referred to as the rotational Doppler effect (RDE). According to the authors, the RDE due to the spin component of the beam has already been taken into account in [7] (sec. III-E) talking about circular polarisation rather than a spin eigenstate. Nevertheless, the authors describe the part of the RDE due to spin, as circularly polarised electromagnetic (EM) radiation has a spin of $\hbar$ per photon. Now, a closer look at their eq. (15) reveals an inconsistency in their modelling of the RDE.

### 2.2 The RDE contribution

Let us consider the influence of RDE on the frequency received by DSN in detail. In the first order in $v/c$ we have

$$\nu_{\text{observedDSN}} = \left(1 - \frac{v}{c}\right) (\nu_{\text{sentPioneer}} \pm \nu_R), \quad (4)$$

where $\nu_R = \dot{\theta}/2\pi$ is the rotation frequency of the Pioneer spacecraft, and $\pm$ is for the sign of the downlink signal circular polarization. The frequency sent by Pioneer is first shifted by $\nu_R$ into the non-rotating, but co-flying with the Pioneer frame, and then shifted by linear Doppler effect. Pioneer communication system converts the transmitted frequency:

$$\nu_{\text{sentPioneer}} = f \nu_{\text{observedPioneer}}, \quad (5)$$

where $f = 240/221$ is the frequency turnaround ratio (see. [7], sec. II D and sec. III E). Now,

$$\nu_{\text{observedPioneer}} = \left(1 - \frac{v}{c}\right) \nu_{\text{sentDSN}} \pm \nu_R, \quad (6)$$

where the frequency sent by DSN is first shifted by linear Doppler effect in the co-flying, non-rotating frame, and then by RDE. The $\pm$ sign is for the uplink signal circular polarization helicity. Note that the $\pm$ sign in eqs. 4 and 6 could be different,
depending on down- and uplink signal polarizations, respectively. Hence, in the first order in $v/c$ we have

$$\nu_{\text{observed DSN}} = \left(1 - \frac{2v}{c}\right)\nu_0 \pm \left(1 - \frac{v}{c}\right)\nu_R(1 \pm f),$$ \hspace{1cm} (7)

and $\nu_0 = f\nu_{\text{sent DSN}}$ is the downlink frequency. As Anderson et al. [7] argue, that the rotation of the spacecraft always increases the radio frequency, we take $+$ sign instead of both $\pm$’s. As one can see, the additional term $(1 + f) \left(1 - \frac{v}{c}\right)\nu_R$ that is involved by the RDE is just missing in eq. (15) of [7]. The comparison with relation (3) shows that this gives a contribution to $\kappa$ approximately equal to $\frac{1 + f}{2}\frac{\lambda_0}{2\pi}$. As this amounts to only about 1/15 of the whole, we still need to explain the remainder. Besides, since the beam does not possess helical phase fronts, the orbital angular momentum in the EM beam is zero. This leads us to search for a dynamical explanation of relation (1) that may compete with the RDE (kinematical effect).

2.3 The radio beam dynamical contribution

Radio beam reaction force have been already discussed in [7] (sec. VIII-A). The authors concluded that this would yield a substantial contribution $a_{rp} = (1.10 \pm 0.11) \times 10^{-8}$ cm s$^{-2}$ to $a_P$ (according to us this result should be doubled). Also, the spin-rate change produced by the torque of radiant power directed against the rotation have been investigated in [7] (sec. VIII-B). However, in both cases the authors did not try to formulate from the above considerations the possible link between $a_P$ and $\ddot{\theta}$. Now, it is proved [15] that emitted or absorbed photons can convert their angular momenta into a torque applied to a solid. As one knows, these photons may convert at the same time their momenta into a force applied to a solid. Let us emphasize that although the radiation pressure is proportional to the emitted power, the torque is not. Since the spacecraft transmits continuously their signals
to the Earth even without receiving any, this provides a continuous contribution to the dynamical effect we are looking for. A rough estimate shows that this is by far the major contribution. Let us show how one can explain in this way the correlation between the different values of $a_P$ and the spin-rate data of Pioneer 10/11 (whatever the value of the torque). Let $\dot{\theta}_0$ be the spin-rate change of the spacecraft due to other causes than the absorption or emission of EM radiation at the communication frequencies. Hence, there will be an additional contribution to the spin-rate change related to the interaction with the photons, $\Delta \ddot{\theta} = \ddot{\theta}(t) - \dot{\theta}_0$, given by

$$J \Delta \ddot{\theta} = \Delta \dot{N} \hbar,$$

(8)

where $J$ is the moment of inertia along the spin axis of the spacecraft and $\Delta \dot{N}$ denotes the number difference between absorbed and emitted photons per unit of time. Recall that the angular momentum of a photon of an EM wave is $\pm \hbar$ with the sign depending on the helicity of the EM wave, namely right or left circularly polarized. Also, a photon of frequency $\nu = \omega/2\pi$ carries a momentum $\hbar \omega/c$. Moreover, a photon emitted at an angle $\theta$ with respect to the spin axis transfers to the spacecraft a momentum $(\hbar \omega/c) (1 + \cos \theta)$ after reflexion on the parabolic dish. As the computation made in [7] (eq. (37), sec. VIII-B) manifestly only accounts for the case $\theta = 0$ that is a momentum transfer $2 \hbar \omega/c$ per photon, substracting the latter yields a remainder of momentum transfer to the spacecraft the magnitude of which amounts to $p(\theta) = (\hbar \omega/c) (1 - \cos \theta)$ per photon emitted at an angle $\theta$. So, the spacecraft will be subject to an acceleration in excess $\Delta a_P$ given by

$$M \Delta a_P = \int_0^{\Omega_{beam}} (p(\theta) + p(-\theta)) \frac{\Delta \dot{N}}{\Omega_{beam}} d\Omega = \Delta \dot{N} \frac{\hbar \omega \Omega_{beam}}{c 2\pi},$$

(9)

where $M$ is the mass of the spacecraft, $\Omega = 2\pi (1 - \cos \theta)$ the solid angle subtended by the cone with half-aperture $\theta$ and $\Omega_{beam}$ denotes the total solid angle of the conical beam within which the photons are emitted. Because of the way the high-gain antenna of the spacecraft is directed with respect to the Earth, clearly $\Delta a_P$ is
in the opposite direction to $a_P(0)$ and consequently substracts to it. Thus, one gets a net apparent anomalous acceleration $a_P = a_P(\ddot{\theta}_0) - \Delta a_P$ which yields combining relations (8) and (9)

$$a_P = a_P(0) - \Omega_{\text{beam}} \frac{\nu J}{M} \ddot{\theta}. \quad (10)$$

### 2.4 Both contributions

So, the comparison of relation (10) above with (1) yields on account of the RDE term

$$\kappa = \frac{1}{\lambda_0} \frac{J}{M} \Omega_{\text{beam}} + \frac{1 + f}{2} \frac{\lambda_0}{2\pi} \approx \frac{2\pi}{\lambda_0} \frac{J}{M} (1 - \cos \Theta) + \frac{1 + f}{2} \frac{\lambda_0}{2\pi}, \quad (11)$$

where $\Theta$ denotes the angle between the antenna axis (spin axis) and the first diffraction minimum of the conical beam. The nominal working values of Pioneer 10 used by Anderson et al. are $M = 251.883 \text{ kg}$, $J \approx 588.3 \text{ kg m}^2$ and $\Theta \approx 4^\circ$ (see [7], sec. VIII-A). These yield $\kappa \simeq 30 \text{ cm}$ in good agreement with $\kappa = 30.7 \pm 0.6 \text{ cm}$ found in [7]. Thus, it seems that one aspect of the Pioneer anomaly has found a conventional explanation which can be checked in the laboratory. As regards the constant part $a_P(0)$ of the anomalous acceleration (the main Pioneer 10/11 anomalous acceleration), it still remains to explain.

### 3 Conclusion

We have provided a conventional and quantitative explanation to the small difference of magnitude of the anomalous acceleration observed between the Pioneers 10 and 11. As observed by Anderson et al. [7], we find that this is related to the difference of spin-rate history for both spacecraft. Now, these authors have shown that when the spin-rate change contribution is removed, one is left with an anomalous acceleration $a_P(0)$ of the same amount with a great accuracy during a very long
time interval for both Pioneer 10/11 to explain. Since the Pioneers 10 and 11 are remote spacecraft moving in opposite directions from the Sun, this makes difficult to understand such a persistent constant anomalous acceleration in terms of conventional physics. Hence, the conventional explanation we suggest for the Pioneer 10/11 anomalous acceleration versus spin-rate change points out in favour of the possibility that something new of physical interest may be responsible of the main Pioneer anomaly at the expense of any possible internal cause (see [16], for the revival of such a possibility).

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