Compactifications of the $\mathcal{N} = 2^*$ flow

Alex Buchel

Michigan Center for Theoretical Physics
Randall Laboratory of Physics, The University of Michigan
Ann Arbor, MI 48109-1120

Abstract

In hep-th/0004063 Pilch and Warner (PW) constructed $\mathcal{N} = 2$ supersymmetric RG flow corresponding to the mass deformation of the $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory. In this paper we present exact deformations of PW flow when the gauge theory 3-space is compactified on $S^3$. We consider also the case with the gauge theory world-volume being $dS_4$ instead of $R^{3,1}$. The solution is constructed in five-dimensional gauged supergravity and is further uplifted to 10d.

February 2003
1 Introduction

Probably the most intriguing aspect of the gauge theory/string theory duality [1] (see [2] for a review) is the fact that it provides a dynamical principle for the non-perturbative definition of string theory in the asymptotically Anti de Sitter spacetime, where there is no notion of an $S$-matrix. The best understood example of this duality is for the $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang Mill theory. Given the original correspondence [1], new examples can be constructed by deforming the gauge theory by relevant operators. By now there is an extensive literature on such, renormalization group (RG) flow deformations [2]. In [3] it was suggested that the duality can be extended to cases when one deforms the gauge theory space-time. Furthermore, in [4, 5] it was suggested that gauge theories on nondynamical de Sitter backgrounds might be relevant for understanding string theory in backgrounds with cosmological horizons. Unfortunately it is difficult to use space-time deformations of [3–5] for developing a detailed gauge/string theory duality map. The main problem stems from the fact that the examples considered there typically involve gauge theory with not well understood ultraviolet properties. It seems desirable to construct nontrivial examples of such deformations for “simpler” gauge theories in the UV.

Probably the simplest candidate is to consider space-time deformations of the massive $\mathcal{N} = 4$ RG flow. In this paper we discuss how to construct such deformations for the $\mathcal{N} = 2^*$ RG flow of Pilch and Warner [6].

We should emphasize that though we concentrate on the flow [6], the construction presented here can be applied to other RG flows. In particular, in Appendix we construct the $S^3$ deformation of the recent non-supersymmetric ($\mathcal{N} = 0^*$) flow [7] in five dimensional gauged supergravity. While supergravity flow [7] is actually singular, our deformation is completely smooth. We also comment on the physical reason for the singularity of this $\mathcal{N} = 0^*$ flow.

The paper is organized as follows. In the next section we review the Pilch-Warner RG flow in five dimensions, and discuss its $S^3$ and $dS_4$ deformations. In section 3 we discussed the details of the 10d uplift of the deformations. We conclude in section 4.
2 $\mathcal{N} = 2^*$ RG flow and its deformations in five dimensions

2.1 The gauge theory story

In the language of four-dimensional $\mathcal{N} = 1$ supersymmetry, the mass deformed $\mathcal{N} = 4$ $SU(N)$ Yang-Mills theory ($\mathcal{N} = 2^*$) in $R^{3,1}$ consists of a vector multiplet $V$, an adjoint chiral superfield $\Phi$ related by $\mathcal{N} = 2$ supersymmetry to the gauge field, and two additional adjoint chiral multiplets $Q$ and $\tilde{Q}$ which form the $\mathcal{N} = 2$ hypermultiplet. In addition to the usual gauge-invariant kinetic terms for these fields, the theory has additional interactions and hypermultiplet mass term summarized in the superpotential

$$W = \frac{2\sqrt{2}}{g_{YM}^2} \text{Tr}([Q, \tilde{Q}]\Phi) + \frac{m}{g_{YM}^2} (\text{Tr} Q^2 + \text{Tr} \tilde{Q}^2).$$

When $m = 0$ the gauge theory is superconformal with $g_{YM}$ characterizing an exactly marginal deformation. The theory has classical $3(N-1)$ complex dimensional moduli space. This moduli space is protected by supersymmetry against (non)-perturbative quantum corrections. With $m \neq 0$, the $\mathcal{N} = 4$ supersymmetry is softly broken to $\mathcal{N} = 2$. This mass deformation lifts $\{Q, \tilde{Q}\}$ hypermultiplet moduli directions, leaving the $(N-1)$ complex dimensional Coulomb branch of the $\mathcal{N} = 2$ $SU(N)$ Yang-Mill theory, parameterized by expectation values of the adjoint scalar

$$\Phi = \text{diag}(a_1, a_2, \cdots, a_N), \quad \sum_i a_i = 0,$$

in the Cartan subalgebra of the gauge group. For generic values of the moduli $a_i$ the gauge symmetry is broken to that of the Cartan subalgebra $U(1)^{N-1}$, up to the permutation of individual $U(1)$ factors. Additionally, the superpotential (1) induces the RG flow of the gauge coupling. While from the gauge theory perspective it is straightforward to study this $\mathcal{N} = 2^*$ gauge theory at any point on the Coulomb branch [8], the PW supergravity flow [6] corresponds to a particular Coulomb branch vacuum. More specifically, matching the probe computation in gauge theory and the dual PW supergravity flow it was argued in [9] that the appropriate Coulomb branch vacuum corresponds to a linear distribution of the vevs (2) as

$$a_i \in [-a_0, a_0], \quad a_0^2 = \frac{m^2 g_{YM}^2 N}{\pi},$$

The classical Kähler potential is normalized $(2/g_{YM}^2) \text{Tr}[\Phi \Phi + \tilde{Q} Q + \tilde{Q} \tilde{Q}].$
with (continuous in the large $N$ limit) linear number density
\[ \rho(a) = \frac{2}{m^2 g_s^2 T_{YM}} \sqrt{a_0^2 - a^2}, \quad \int_{-a_0}^{a_0} da \rho(a) = N. \] (4)

Unfortunately, the extension of the $N = 2^*$ gauge/gravity correspondence of [6, 9, 10] for vacua other than (4) is not known.

In [9, 10] the dynamics of the gauge theory on the D3 brane probe in the PW background was studied in details. It was shown in [9] that the probe has one complex dimensional moduli space, with bulk induced metric precisely equal to the metric on the appropriate one complex dimensional submanifold of the $SU(N + 1)\, \mathcal{N} = 2^*$ Donagi-Witten theory Coulomb branch. This one dimensional submanifold is parameterized by the expectation value $u$ of the $U(1)$ complex scalar on the Coulomb branch of the theory where $SU(N + 1) \to U(1) \times SU(N)_{PW}$, and the $PW$ subscript denotes that the $SU(N)$ factor is in the Pilch-Warner vacuum (4). As $u$ coincides with any of the $a_i$ of the PW vacuum, the moduli space metric diverges, signaling the appearance of the additional massless states. Identical divergence is observed [9, 10] for the probe D3-brane at the enhancon singularity of the PW background. Away from the singularity locus, $u = a \in [-a_0, a_0]$, the gauge theory computation of the probe moduli space metric is 1-loop exact. This is due to the suppression of instanton corrections in the large $N$ limit [9, 11] of $N = 2$ gauge theories.

Consider now the $R^{3,1} \to R \times S^3$ or $R^{3,1} \to dS_4$ deformations of the $N = 2^*$ gauge theory. Both deformations introduce a new scale, let’s call it $\mu$, to the model — the $S^3$ scale in the former case and the Hubble parameter in the latter. Depending on the ratio $\frac{\mu}{m}$ we expect an interesting interplay between the strongly coupled $N = 2^*$ IR dynamics and the IR curvature induced cutoff. For one reason, we expect that for the sufficiently high $\mu$ the number density distribution $\rho(a)$ should be just a $\delta$-function at zero. In what follows we present and indication for this phase transition while postponing the detailed analysis for the future.

### 2.2 PW RG flow

The gauge theory RG flow induced by the superpotential (1) corresponds to five dimensional gauged SUGRA flow induced by scalars $\alpha \equiv \ln \rho$ and $\chi$. The effective 5d action is
\[ S = \int d^5 \xi \sqrt{-g} \left( \frac{1}{4} R - 3(\partial \alpha)^2 - (\partial \chi)^2 - \mathcal{P} \right), \] (5)
where the potential $\mathcal{P}$ is\(^2\)

$$\mathcal{P} = \frac{1}{48} \left( \frac{\partial W}{\partial \alpha} \right)^2 + \frac{1}{16} \left( \frac{\partial W}{\partial \chi} \right)^2 - \frac{1}{3} W^2, \quad (6)$$

with the superpotential

$$W = -\frac{1}{\rho^2} - \frac{1}{2} \rho^4 \cosh(2\chi). \quad (7)$$

The PW geometry [6] has the flow metric

$$ds_5^2 = e^{2A} \left( -dt^2 + d\bar{x}^2 \right) + dr^2. \quad (8)$$

The scalar equations of motion and the Einstein equations can be reduced to the first order equations

$$\frac{d\alpha}{dr} = \frac{1}{12} \frac{\partial W}{\partial \alpha}, \quad \frac{d\chi}{dr} = \frac{1}{4} \frac{\partial W}{\partial \chi}, \quad \frac{dA}{dr} = -\frac{1}{3} W. \quad (9)$$

### 2.2.1 Asymptotics of the PW flow

Given the explicit solution of the flow equations (9) in [6] is it easy to extract the UV/IR asymptotics. In the ultraviolet, $r \to +\infty$, we find

$$UV:\quad \rho \to 1, \quad \chi \to 0, \quad A \to \frac{1}{2} r, \quad (10)$$

while in the infrared, $r \to 0$

$$IR:\quad \rho \to 0, \quad \chi \to +\infty, \quad A \to -\frac{8}{3} \chi. \quad (11)$$

### 2.3 Deformations of the PW flow

Unlike the PW flow, the deformed flows break the supersymmetry and are given by second order equations. From (5) we have Einstein equations

$$\frac{1}{4} R_{\mu\nu} = 3 \partial_\mu \alpha \partial_\nu \alpha + \partial_\mu \chi \partial_\nu \chi + \frac{1}{3} g_{\mu\nu} \mathcal{P}, \quad (12)$$

\(^2\)We set the 5d gauged SUGRA coupling to one. This corresponds to setting $S^5$ radius $L = 2$.\]
plus the scalar equations

\begin{align*}
0 &= 6 \sqrt{-g} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu \alpha) - \frac{\partial P}{\partial \alpha}, \\
0 &= 2 \sqrt{-g} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu \chi) - \frac{\partial P}{\partial \chi}.
\end{align*}

We consider two deformations of the flow metric (8):

\begin{align*}
(a) : \quad ds_3^2 &= e^{2A} (-dt^2 + e^{2B} dS_3^2) + dr^2, \\
(b) : \quad ds_3^2 &= e^{2A} (-dt^2 + \cosh^2 t dS_3^2) + dr^2.
\end{align*}

In the first case from (12),(13) we find

\begin{align*}
0 &= \alpha'' + (4A' + 3B') \alpha' - \frac{1}{6} \frac{\partial P}{\partial \alpha}, \\
0 &= \chi'' + (4A' + 3B') \chi' - \frac{1}{2} \frac{\partial P}{\partial \chi}, \\
0 &= B'' + 4A'B' + 3 (B')^2 - 2e^{-2A-2B}, \\
\frac{1}{4} A'' + (A')^2 + \frac{3}{4} A'B' &= -\frac{1}{3} P, \\
- A'' - (A')^2 - \frac{3}{2} A'B' - \frac{3}{4} B'' - \frac{3}{4} (B')^2 &= 3 (\alpha')^2 + (\chi')^2 + \frac{1}{3} P,
\end{align*}

while in case (b) we find

\begin{align*}
0 &= \alpha'' + 4A' \alpha' - \frac{1}{6} \frac{\partial P}{\partial \alpha}, \\
0 &= \chi'' + 4A' \chi' - \frac{1}{2} \frac{\partial P}{\partial \chi}, \\
\frac{1}{4} A'' + (A')^2 - \frac{3}{4} e^{-2A} &= -\frac{1}{3} P, \\
- A'' - (A')^2 &= 3 (\alpha')^2 + (\chi')^2 + \frac{1}{3} P.
\end{align*}

It is easy to check that above equations are consistent. Thus for the deformed flows we could use the same scalars as in the PW case.
2.3.1 Asymptotics of the $S^3$ deformation

The flow equations are given by (15). The nonsingular in the IR flows are represented by a two parameter $\{\rho_0 > 0, \chi_0\}$ Taylor series expansion\(^3\)

$$
e^A = 1 + \left( \sum_{i=1}^{\infty} \alpha_i r^{2i} \right),$$

$$
e^B = r \left( 1 + \sum_{i=1}^{\infty} b_i r^{2i} \right),$$

$$\rho = \rho_0 + \left( \sum_{i=1}^{\infty} \rho_i r^{2i} \right),$$

$$\chi = \chi_0 + \left( \sum_{i=1}^{\infty} \chi_i r^{2i} \right),$$

with the first terms being

$$a_1 = \frac{1}{24} \rho_0^{-4} + \frac{1}{12} \rho_0^2 \cosh(2\chi_0) - \frac{1}{96} \rho_0^6 \sinh^2(2\chi_0),$$

$$b_1 = -\frac{1}{36} \rho_0^{-4} - \frac{1}{18} \rho_0^2 \cosh(2\chi_0) + \frac{1}{144} \rho_0^8 \sinh^2(2\chi_0),$$

$$\rho_1 = \frac{1}{48} \rho_0^{-3} - \frac{1}{48} \rho_0^3 \cosh(2\chi_0) + \frac{1}{96} \rho_0^9 \sinh^2(2\chi_0),$$

$$\chi_1 = -\frac{1}{16} \rho_0^2 \sinh(2\chi_0) + \frac{1}{128} \rho_0^8 \sinh(4\chi_0).$$

(17)

We expect that for an appropriate choice of $\{\rho_0, \chi_0\}$ we recover the UV asymptotics (10). It is tempting to identify the 2 dimensionless parameters of the regular in the IR flow with the ratio of $m/\mu$ of the gauge theory (the $\chi_0$ parameter), and the $\rho_0$ parameter as a characteristic of the brane distribution (similar to the enhancon scale $a_0$ in (4)) in the IR. Notice, that unlike PW flow, where $\chi \to +\infty$ in the IR, here it is consistent to choose\(^4\) $\chi_0 = 0$. In fact $\chi(r) \equiv 0$ is a solution to (15)\(^5\).

The nonsingular flows that asymptote to (10) would have a well defined (finite) mass, being a function of $\{\rho_0, \chi_0\}$, characterizing phases of the model\(^6\).

\(^3\)Without loss of generality we set $A|_{r=0} = 0$. This corresponds to rescaling the time coordinate in (14).

\(^4\)We would like to interpret $\chi_0 = 0$ flow as a supergravity dual to the $N = 2^*$ flow induced by the $N = 4$ scalar expectation values. Typically, scalar expectation value does not give rise to an RG flow. Since these scalars are conformal (and thus couple to the $S^3$ curvature), given them an expectation value would induce a flow.

\(^5\)Also, $\chi(r) \equiv 0$ and $\rho(r) \equiv 1$ is a trivial solution corresponding to the global $AdS_5$.

\(^6\)The details of the phase structure will be discussed elsewhere.
2.3.2 Asymptotics of the $dS_4$ deformation

The flow equations are given by (16). The nonsingular in the IR flows are represented by a two parameter $\{\rho_0 > 0, \chi_0\}$ Taylor series expansion

$$e^A = r \left( 1 + \sum_{i=1}^{\infty} a_i \, r^{2i} \right),$$
$$\rho = \rho_0 + \left( \sum_{i=1}^{\infty} \rho_i \, r^{2i} \right),$$
$$\chi = \chi_0 + \left( \sum_{i=1}^{\infty} \chi_i \, r^{2i} \right),$$

with the first terms being

$$a_1 = \frac{1}{72} \rho_0^{-4} + \frac{1}{36} \rho_0^2 \cosh(2\chi_0) - \frac{1}{288} \rho_0^8 \sinh^2(2\chi_0),$$
$$\rho_1 = \frac{1}{60} \rho_0^{-3} - \frac{1}{60} \rho_0^3 \cosh(2\chi_0) + \frac{1}{120} \rho_0^9 \sinh^2(2\chi_0),$$
$$\chi_1 = -\frac{1}{20} \rho_0^2 \sinh(2\chi_0) + \frac{1}{160} \rho_0^8 \sinh(4\chi_0).$$

As in the case of the $S^3$ deformation it is also consistent here to choose $\chi(r) \equiv 0$.

3 The ten-dimensional solutions

3.1 Type IIB SUGRA equations of motion

We use mostly positive convention for the signature $(- + \cdots +)$ and $\epsilon_{1\ldots10} = +1$. The type IIB equations consist of [12]:

- The Einstein equations:

$$R_{MN} = T^{(1)}_{MN} + T^{(3)}_{MN} + T^{(5)}_{MN},$$

where the energy momentum tensors of the dilaton/axion field, $\mathcal{B}$, the three index antisymmetric tensor field, $F_{(3)}$, and the self-dual five-index tensor field, $F_{(5)}$, are given by

$$T^{(1)}_{MN} = P_M P_N^* + P_N P_M^*,$$
$$T^{(3)}_{MN} = \frac{1}{8} (G^{PQ}_M G^{*}_{PQN} + G^{*PQ}_M G_{PQN} - \frac{1}{6} g_{MN} G^{PQR} G^{*}_{PQR}),$$
$$T^{(5)}_{MN} = \frac{1}{6} F^{PQRS}_M F_{PQRSN}. $$
In the unitary gauge $\mathcal{B}$ is a complex scalar field and
\[ P_M = f^2 \partial_M \mathcal{B}, \quad Q_M = f^2 \text{Im}(\mathcal{B} \partial_M \mathcal{B}^*), \tag{25} \]
with
\[ f = \frac{1}{(1 - BB^*)^{1/2}}, \tag{26} \]
while the antisymmetric tensor field $G_{(3)}$ is given by
\[ G_{(3)} = f(F_{(3)} - \mathcal{B}F_{(3)}^*). \tag{27} \]

- The Maxwell equations:
\[ (\nabla^P - iQ^P)G_{MNP} = P^P G_{MNP}^* - \frac{2}{3} i F_{MNPQR}G^{PQR}. \tag{28} \]

- The dilaton equation:
\[ (\nabla^M - 2iQ^M)P_M = -\frac{1}{24} G^{PQR}G_{PQR}. \tag{29} \]

- The self-dual equation:
\[ F_{(5)} = \ast F_{(5)}. \tag{30} \]

In addition, $F_{(3)}$ and $F_{(5)}$ satisfy Bianchi identities which follow from the definition of those field strengths in terms of their potentials:
\[ F_{(3)} = dA_{(2)}, \tag{31} \]
\[ F_{(5)} = dA_{(4)} - \frac{1}{8} \text{Im}(A_{(2)} \wedge F_{(3)}^*). \tag{31} \]

For the 10d uplift of the RG flows in the 5d gauged SUGRA the metric ansatz and the dilaton is basically determined by group theoretical properties of the $d = 5$ $\mathcal{N} = 8$ scalars, and thus must be the same for both the deformed and original PW flows. Specifically, we assume [6] that the 10d Einstein frame metric is
\[ ds_{10}^2 = \Omega^2 ds_5^2 + 4 \left(\frac{cX_1X_2}{\rho^3}\right)^{1/4} \left(c^{-1}d\theta^2 + \rho^6 \cos^2 \theta \left(\frac{\sigma_1^2}{cX_2} + \frac{\sigma_2^2 + \sigma_3^2}{X_1}\right) + \sin^2 \theta \frac{d\phi^2}{X_2}\right), \tag{32} \]
where $ds_5^2$ is either the original PW flow metric (8) or its deformations (14), $c \equiv \cosh(2\chi)$. The warp factor is given by
\[ \Omega^2 = \frac{(cX_1X_2)^{1/4}}{\rho}, \tag{33} \]
and the two functions $X_i$ are defined by
\begin{align*}
X_1(r, \theta) &= \cos^2 \theta + \rho(r)^6 \cosh(2\chi(r)) \sin^2 \theta, \\
X_2(r, \theta) &= \cosh(2\chi(r)) \cos^2 \theta + \rho(r)^6 \sin^2 \theta .
\end{align*}

(34)

As usual, $\sigma_i$ are the $SU(2)$ left-invariant forms normalized so that $d\sigma_i = 2\sigma_j \wedge \sigma_k$. For the dilaton/axion we have
\begin{align*}
f &= \frac{1}{2} \left( \left( \frac{cX_1}{X_2} \right)^{1/4} + \left( \frac{cX_1}{X_2} \right)^{-1/4} \right), \\
 fB &= \frac{1}{2} \left( \left( \frac{cX_1}{X_2} \right)^{1/4} - \left( \frac{cX_1}{X_2} \right)^{-1/4} \right) e^{2i\phi} .
\end{align*}

(35)

The consistent truncation ansatz does not specify the (3-) 5-form fluxes. As in [6] we assume the most general ansatz allowed by the global symmetries of the background
\begin{equation}
A_{(2)} = e^{i\phi} (a_1(r, \theta) \ d\theta \wedge \sigma_1 + a_2(r, \theta) \ \sigma_2 \wedge \sigma_3 + a_3(r, \theta) \ \sigma_1 \wedge d\phi + a_4(r, \theta) \ d\theta \wedge d\phi) ,
\end{equation}

(36)

where $a_i(r, \theta)$ are arbitrary complex functions. For the 5-form flux we assume
\begin{align*}
(a) & : \quad F_5 = \mathcal{F} + \star \mathcal{F}, \quad \mathcal{F} = dt \wedge \text{vol}_{S^4} \wedge d\omega , \\
(b) & : \quad F_5 = \mathcal{F} + \star \mathcal{F}, \quad \mathcal{F} = \cosh^3 t \ dt \wedge \text{vol}_{S^3} \wedge d\omega ,
\end{align*}

(37)

where $\omega(r, \theta)$ is an arbitrary function.

We will do all the computation in the natural orthonormal frame given by
\begin{equation}
e^1 \propto dt, \quad e^2 \propto dr, \quad e^3 \propto \tilde{\sigma}_1, \quad e^4 \propto \tilde{\sigma}_2, \quad e^5 \propto \tilde{\sigma}_3, \\
e^6 \propto d\theta, \quad e^7 \propto \sigma_1, \quad e^8 \propto \sigma_2, \quad e^9 \propto \sigma_3, \quad e^{10} \propto d\phi ,
\end{equation}

(38)

where $\tilde{\sigma}_i$ are again $SU(2)$ left-invariant one forms, such that the round $S^3$ metric of unit radius is $(dS^3)^2 = \sum \tilde{\sigma}_i^2$.

As in the PW case, examination of the Einstein equations reveals that 2-form potential functions $a_i$ have the following properties: $a_4 \equiv 0$, $a_1, a_2$ are pure imaginary, and $a_3$ is real.

### 3.2 Lift of $S^3$ deformation

Explicitly computing Ricci tensor with above ansatz, we find nonvanishing components $R_{11}, R_{22}, R_{33} = R_{44} = R_{55}, R_{66}, R_{77}, R_{88} = R_{99}, R_{1010}, R_{26} = R_{62}$. Given the 5d flow equations (15), we find relations
\begin{equation}
R_{77} + R_{88} = 2R_{11} , \\
R_{11} + R_{33} = 0 .
\end{equation}

(39)
The 3-form energy-momentum tensor has nontrivial components $T^{(3)}_{11} = -T^{(3)}_{33} = -T^{(3)}_{44} = -T^{(3)}_{55} = T^{(3)}_{77} = T^{(3)}_{88} = T^{(3)}_{99} = T^{(3)}_{1010}$, $T^{(3)}_{22} = T^{(3)}_{26} = T^{(3)}_{62}$. The nonvanishing components of the dilaton/axion energy-momentum tensor are $T^{(1)}_{22} = T^{(1)}_{66} = T^{(1)}_{1010} = T^{(1)}_{26} = T^{(1)}_{62}$.

Finally, the 5-form energy-momentum tensor has nonvanishing components $T^{(5)}_{11} = -T^{(5)}_{33} = -T^{(5)}_{44} = -T^{(5)}_{55} = T^{(5)}_{77} = T^{(5)}_{88} = T^{(5)}_{99} = T^{(5)}_{1010} = A^2_1 + A^2_2$, $T^{(5)}_{22} = T^{(5)}_{66} = T^{(5)}_{26} = T^{(5)}_{62} = 2 A_1 A_2$, (40)

where $A_1 \propto \frac{\partial \omega}{\partial r}$, $A_2 \propto \frac{\partial \omega}{\partial \theta}$. (41)

Besides Einstein equations, we have nontrivial 5-form Bianchi identity, dilaton/axion equation (29), and 4 equations from the Maxwell equation (28) for components $\{MN\} = \{27, 67, 710, 89\}$.

As in [6] we find the following consistency checks on the metric and dilaton/axion ansatz$^7$:

$$T^{(3)}_{1010} - T^{(3)}_{11} = \frac{e^{-2i\phi}}{24} G^{MN} G^{MN} ,$$

$$R_{1010} - R_{11} = 2|P_{10}|^2 - e^{-2i\phi} \left( \nabla^M - 2iQ^M \right) P_M .$$

Next combination is

$$R_{1010} - R_{77} - 2|P_{10}|^2 = T^{(3)}_{1010} - T^{(3)}_{77} .$$

As in [6], we find that (43) (and the linearized solution of all equations in the UV) is satisfied provided$^8$

$$a_1 = -i 4 \tanh(2\chi) \cos \theta ,$$

$$a_2 = i 4 \rho^6 \sinh(2\chi) \frac{X_1}{X_1} \sin \theta \cos^2 \theta ,$$

$$a_3 = -4 \sinh(2\chi) \frac{X_2}{X_1} \sin \theta \cos^2 \theta .$$

Finally, from the $\{MN\} = \{11, 22\}$ Einstein equations we find

$$\frac{\partial \omega}{\partial \theta} = -\frac{3}{2} e^{4A + 3B} (\ln \rho)' \sin 2\theta ,$$

$$\frac{\partial \omega}{\partial r} = \frac{1}{8} e^{4A + 3B} \frac{1}{\rho^4} \left( -\rho^{12} \sin^2(2\chi) \sin^2 \theta + 2 \rho^6 \cosh(2\chi) (1 + \sin^2 \theta) + 2 \cos^2 \theta \right).$$

$^7$There is a typo in the second equation in (42) in [6] (eq.(4.3)).

$^8$Note that there is a sign typo for $a_3$ in the corresponding equations in [6], (eq.(4.8)).
We explicitly verified that supplementing the metric and the dilaton/axion ansatz of the previous section with (44), (45), and the 5d flow equations (15), all the equations of 10d type IIB supergravity are satisfied.

3.3 Lift of \(dS_4\) deformation

In this case the analysis are similar to those in the previous section. Thus we present only the results. First, we find the same complex functions \(a_i\), specifying the 2-form potential (36)

\[
\begin{align*}
    a_1 &= -i \frac{4}{3} \tanh(2\chi) \cos \theta , \\
    a_2 &= i \frac{4}{3} \frac{\rho^6 \sinh(2\chi)}{X_1} \sin \theta \cos^2 \theta , \\
    a_3 &= -4 \frac{\sinh(2\chi)}{X_2} \sin \theta \cos^2 \theta .
\end{align*}
\] (46)

Second, the \(\omega\) in the 5-form potential (37) is

\[
\begin{align*}
    \frac{\partial \omega}{\partial \theta} &= -\frac{3}{2} e^{4A} (\ln \rho)' \sin 2\theta , \\
    \frac{\partial \omega}{\partial r} &= \frac{1}{8} e^{4A} \frac{1}{\rho^3} \left( -\rho^{12} \sinh^2(2\chi) \sin^2 \theta + 2\rho^6 \cosh(2\chi)(1 + \sin^2 \theta) + 2 \cos^2 \theta \right) .
\end{align*}
\] (47)

4 Conclusion

In this paper we observed that certain 5d gauged supergravity flows on the background \(R^{3,1} \times R_+\) can be deformed to flows on backgrounds \(S^3 \times R \times R_+\) or \(dS_4 \times R_+\) with the same 5d scalars. If the 10 dimensional lift of the original backgrounds is known, this implies that deformed flows can be uplifted to ten dimensions as well. We explicitly demonstrated this for the \(\mathcal{N} = 2\) PW flow, constructing for the first time massive RG flow with asymptotically global \(AdS_5\) geometry. We hope that study of these backgrounds would help develop gauge/gravity dictionary for gauge theories in curved space-time, including \(dS_4\) deformations which might be relevant for understanding strings in backgrounds with cosmological horizons [4, 5].

Acknowledgments

It is a pleasure to thank Ofer Aharony for very interesting and stimulating discussions. I would like to thank the Weizmann Institute of Science, University of Pennsylvania
for hospitality during part of this work. I would also like to thank the Aspen Center for Physics (2001 workshop) for hospitality, where this work started. I would like to thank Nick Evans for the e-mail correspondence concerning potential singularity of RG flow in [7].

Appendix

In [7], Babington, Crooks and Evans (BCE) proposed a supergravity dual to $\mathcal{N} = 0^*$ gauge theory. This gauge theory is obtained by giving the same mass to all four Weyl fermions in the $\mathcal{N} = 4$ SYM theory. Note that this $\mathcal{N} = 4$ mass term completely breaks the supersymmetry, hence the name for the deformation. In the infrared the $\mathcal{N} = 0^*$ gauge theory is expected to confine with a mass gap in the spectrum. The mass gap in the gauge theory spectrum in particular would imply that the dual, non-supersymmetric, supergravity background of [7] is nonetheless stable. The $\mathcal{N} = 0^*$ dual supergravity solution is constructed first in 5d gauged supergravity and then was uplifted to the full ten dimensional solution [7]. The authors turned on only the 5d scalar, called $\lambda$, dual to the $SO(4)$ invariant fermion mass term. They argued, based on the D3 brane probe computation, that the six $\mathcal{N} = 4$ scalars have positive radiatively induced $(mass)^2$, that thus, naively expected $9$, expectation value for these scalars is not induced. In other words turning on $\lambda$ alone is consistent, and the confined $\mathcal{N} = 0^*$ vacuum is at the origin of the $\mathcal{N} = 4$ moduli space.

If this would be the case, the supergravity solution [7] should have been nonsingular. We believe the solution of [7] has a naked singularity in the interior [13]. This in particular is reflected in the fact that the dilaton of [7] is singular in the infrared, $|\lambda| \to \infty$, for $\alpha = \pi/2$. The technical problem appears to be attributed to the geodesic incompleteness of geometry [7] for their choice of a radial coordinate, and thus incorrect identification of what is the infrared part of the geometry. From the physics perspective, we suspect that the probe computation in [7] is not a reliable tool to address the issue of radiatively induced scalar masses, due to the large nonperturbative corrections (fractional instantons) in theories with less than eight supercharges\(^{11}\). In a sense solu-

---

\(^9\)Note that this is precisely what is happening for the $\mathcal{N} = 2^*$ flow of [6]: the fermion mass term induces the negative $(mass)^2$ term for scalars causing the D3 branes, originally at the origin, to 'spread' up in an enhancon configuration (4).

\(^{10}\)For notations see [7].

\(^{11}\)This is briefly discussed in [11].
tion [7] is akin to a singular Klebanov-Tseytlin (KT) geometry [14], where unphysical requirement of unbroken chiral symmetry led to a singularity of the dual supergravity solution. In [7], we believe this requirement is a zero expectation value for scalars. The analog of the Klebanov-Strassler (KS) [15] resolution of the KT singularity in the BCE case is likely to be the inclusion of the additional 5d gauged supergravity scalar\textsuperscript{12} corresponding to $SO(4)$ invariant 'distribution' of D3-branes. We expect the resulting nonsingular geometry will be similar to the polarized-branes solution of Polchinski and Strassler [16].

While KT solution is unphysical in its original form, it is easy to turn it to a physical one by deforming the theory in such a way that the spontaneously broken chiral symmetry in the IR gets restored. This can done by either considering sufficiently hot thermal state of the gauge theory [17,18], or with an $S^3$ [3] or $dS_4$ [4] deformation of the gauge theory space-time. In the remaining of this section we argue that the prescription [3, 4] resolves BCE singularity. The latter is a reflection of the expectation that the infrared cutoff due to the $S^3$ scale or the Hubble parameter in the $dS_4$ deformation would stabilize the $\mathcal{N} = 4$ scalar masses for the $\mathcal{N} = 0^*$ flow, resulting, as proposed in [7], in the vacuum at the origin of the $\mathcal{N} = 4$ moduli space. We demonstrate this in the $S^3$ deformed 5d gauged supergravity solution of BCE\textsuperscript{13}.

The 5d solution of BCE comes from the effective action

$$S = \int d^5\xi \sqrt{-g} \left( \frac{1}{4} R - \frac{1}{2} (\partial \lambda)^2 - \mathcal{P} \right),$$

where the potential $\mathcal{P}$ is

$$\mathcal{P} = -\frac{3}{2} \left( 1 + \cosh^2 \lambda \right),$$

and the flow metric

$$ds_5^2 = e^{2A} (-dt^2 + d\bar{x}^2) + dr^2.$$  

For the $R^3,1 \rightarrow R \times S^3$ deformation\textsuperscript{14} (14), the equations of motion are

$$0 = \lambda'' + (4A' + 3B') \lambda - \frac{\partial \mathcal{P}}{\partial \lambda},$$

$$0 = B'' + 4A'B' + 3 (B')^2 - 2e^{-2A-2B},$$

$$\frac{1}{4} A'' + (A')^2 + \frac{3}{4} A'B' = -\frac{1}{3} \mathcal{P},$$

$$A'' - (A')^2 - \frac{3}{2} A'B' - \frac{3}{4} B'' - \frac{3}{4} (B')^2 = \frac{1}{2} (\lambda')^2 + \frac{1}{3} \mathcal{P}. \quad (51)$$

\textsuperscript{12}Playing the role similar to $\rho$ for the $\mathcal{N} = 2^*$ PW flow (7).

\textsuperscript{13}There is a nonsingular lift of these deformed 5d solutions.

\textsuperscript{14}The $R^3,1 \rightarrow dS_4$ deformation can be analyzed in a similar fashion.
Though we can’t find an exact analytical solution of (51), we can still argue that the corresponding backgrounds are nonsingular, provided we choose infrared boundary condition $B|_{r=0} = 0$. In the infrared ($r \to 0$) these nonsingular solutions form a one-parameter $\{\lambda_0\}$ family of Taylor series expansions$^{15}$

\begin{align*}
e^A &= 1 + \left( \sum_{i=1}^{\infty} \alpha_i r^{2i} \right), \\
e^B &= r \left( 1 + \sum_{i=1}^{\infty} b_i r^{2i} \right), \\
\rho &= \lambda_0 + \left( \sum_{i=1}^{\infty} \lambda_i r^{2i} \right),
\end{align*}

(52)

with the first terms being

\begin{align*}
a_1 &= \frac{3}{8} + \frac{1}{8} \cosh(2\lambda_0), \\
b_1 &= -\frac{1}{4} - \frac{1}{12} \cosh(2\lambda_0), \\
\lambda_1 &= -\frac{3}{16} \sinh(2\lambda_0).
\end{align*}

(53)

Here $\lambda_0 = 0$, corresponds to global AdS$_5$ solution. Numerical analysis of (51) with boundary condition (52) suggests that for arbitrary value of $\lambda_0$, in the ultraviolet ($r \to \infty$) we obtain UV asymptotics of [7]

\begin{align*}
B &\to 1, \\ A &\to \frac{1}{2} r, \\ \lambda &\propto e^{-r}.
\end{align*}

(54)

References

[1] J. M. Maldacena, “The large $N$ limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].

[2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].

[3] A. Buchel and A. A. Tseytlin, “Curved space resolution of singularity of fractional D3-branes on conifold,” Phys. Rev. D 65, 085019 (2002) [arXiv:hep-th/0111017].

$^{15}$By rescaling the time coordinate in (14) we can always set $A|_{r=0} = 0$. 

15
[4] A. Buchel, “Gauge / gravity correspondence in accelerating universe,” Phys. Rev. D 65, 125015 (2002) [arXiv:hep-th/0203041].

[5] A. Buchel, P. Langfelder and J. Walcher, “On time-dependent backgrounds in supergravity and string theory,” Phys. Rev. D 67, 024011 (2003) [arXiv:hep-th/0207214].

[6] K. Pilch and N. P. Warner, “N = 2 supersymmetric RG flows and the IIB dilaton,” Nucl. Phys. B 594, 209 (2001) [arXiv:hep-th/0004063].

[7] J. Babington, D. E. Crooks and N. Evans, “A stable supergravity dual of non-supersymmetric glue,” arXiv:hep-th/0210068.

[8] R. Donagi and E. Witten, “Supersymmetric Yang-Mills Theory And Integrable Systems,” Nucl. Phys. B 460, 299 (1996) [arXiv:hep-th/9510101].

[9] A. Buchel, A. W. Peet and J. Polchinski, “Gauge dual and noncommutative extension of an N = 2 supergravity solution,” Phys. Rev. D 63, 044009 (2001) [arXiv:hep-th/0008076].

[10] N. Evans, C. V. Johnson and M. Petrini, “The enhancon and N = 2 gauge theory/gravity RG flows,” JHEP 0010, 022 (2000) [arXiv:hep-th/0008081].

[11] A. Buchel, “Comments on fractional instantons in N = 2 gauge theories,” Phys. Lett. B 514, 417 (2001) [arXiv:hep-th/0101056].

[12] J. H. Schwarz, ”Covariant Field Equations of Chiral N = 2 D = 10 Supergravity,” Nucl. Phys. B226 (1983) 269.

[13] A. Buchel, unpublished; N. Evans, private correspondence.

[14] I. R. Klebanov and A. A. Tseytlin, “Gravity duals of supersymmetric SU(N) x SU(N+M) gauge theories,” Nucl. Phys. B 578, 123 (2000) [arXiv:hep-th/0002159].

[15] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities,” JHEP 0008, 052 (2000) [arXiv:hep-th/0007191].

[16] J. Polchinski and M. J. Strassler, “The string dual of a confining four-dimensional gauge theory,” arXiv:hep-th/0003136.
[17] A. Buchel, “Finite temperature resolution of the Klebanov-Tseytlin singularity,” Nucl. Phys. B 600, 219 (2001) [arXiv:hep-th/0011146].

[18] S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, “Restoration of chiral symmetry: A supergravity perspective,” JHEP 0105 (2001) 028 [arXiv:hep-th/0102172].