Radiative gravitino decays from R-parity violation

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ABSTRACT

We study radiative gravitino decay within the framework of R-violating supersymmetry. For trilinear R-violating couplings that involve the third generation of fermions, or for light gravitinos, we find that the radiative loop-decay $\tilde{G} \rightarrow \gamma \nu$ dominates over the tree-level ones for a wide set of parameters. We calculate the gravitino decay width and study its implications for cosmology and collider physics. Slow-decaying gravitinos are good dark matter candidates, for a range of parameters that would also predict observable R-violating signatures in colliders. In general the branching ratios are very dependent on the relative hierarchies of R-violating operators, and may provide relevant information on the flavour structure of the underlying fundamental theory.
1 Introduction

The possibility of R-violating supersymmetry [1, 2] has been extensively studied, as an alternative scenario to the Minimal Supersymmetric Standard Model (MSSM) [3]. Indeed, the symmetries of the Standard Model, in addition to the Yukawa couplings that generate fermion masses, allow for additional dimension-four couplings of the form

$$\lambda L_i L_j \bar{E}_k + \lambda' L_i Q_j \bar{D}_k + \lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$$

(1.1)

where the \(L(Q)\) are the left-handed lepton (quark) doublet superfields, and the \(\bar{E} (\bar{D}, \bar{U})\) are the corresponding left-handed singlet fields. Amongst these so-called R-violating couplings, the first two violate lepton number, while the third violates baryon number, and the symmetries of the theory imply that there are 45 operators in total — 9, 27, and 9 respectively for the three terms, due to \(SU(2)\) and \(SU(3)\) invariance.

The stricter bounds on R-violating operators come from proton stability. However, it is by now well-understood that R-parity [4] is not the only symmetry that can guarantee proton stability; baryon or lepton parities [5, 6] can have the same effect. In fact, it is crucial to exclude the simultaneous presence of only certain products of \(LQD\) and \(UDD\) couplings [7]. In addition to proton stability, most operators are subject to experimental constraints from the non-observation of modifications to Standard Model processes, or of possible exotic processes [8].

Going beyond proton stability and parameter constraints, one of the reasons that R-violating supersymmetry has not seemed very appealing, is the notion that in these models the lightest supersymmetric particle (LSP) is not stable, and therefore one has to search elsewhere for dark matter.

However, how absolute is this statement? Is it possible that the lightest supersymmetric particle decays so slowly that \textit{cosmologically it is almost stable}, while at the same time there is at least one coupling large enough to lead to observable collider signatures for R-violation? In this letter we shall take this to mean that at least one coupling is larger than approximately \(10^{-6}\) for sparticles of 100 GeV mass. This could for instance happen, if the LSP is a light gravitino whose R-parity violating decays to conventional particles are for some reason very suppressed. Scenarios with light gravitinos are well-motivated — and one could even expect masses as low as \(10^{-5}\) eV [10].

Tree-level R-violating gravitino decays have been studied in [11], where gravitinos were found to be able to decay before the present epoch for large values of the R-violating couplings and gravitino masses, but not earlier than the start of big-bang nucleosynthesis (BBN). It was concluded that the considered scenario would upset the standard BBN scenario, and did not seem to constitute a natural solution for the cosmological gravitino problem. Two-body gravitino decays to photon and neutrino, for bilinear R-parity violation, were studied in [12] — with the decays arising from the mixing of neutralinos with neutrinos — and it was concluded that gravitinos of less than 1 GeV mass can be

1Additional constraints can be obtained from a possible detection of photon or lepton plus missing energy signatures from NLSP decays to a gravitino LSP [9].

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the dark matter and at the same time consistent with neutrino mass generation from bilinear R-violation. Recently, in [13], these decays were revisited, and in models where R-parity breaking is tied to B-L breaking, very small R-violating couplings were predicted, in conjunction with a photon flux that can explain the apparent EGRET excess in the extragalactic diffuse gamma-ray flux [14] for \( \sim 10 \) GeV gravitinos. In addition, radiative neutralino decays in R-violating schemes have been studied in [2, 15].

Here we will focus on radiative gravitino decays

\[ \tilde{G} \rightarrow \gamma \nu, \]  

in schemes with explicitly broken R-violation from trilinear terms, and will compare them to the tree-level three-body decays that are expected from the same terms. Which processes will dominate, and what will be the cosmological and collider signatures depends on the flavour structure of the R-violating operators, which we will also discuss.

In principle one may imagine that gravitinos are almost-stable and could “act” as dark matter in the following cases:

- Suppose the only relevant operators are the \( \lambda' U_3 D_j D_k \). In the three-body gravitino decays discussed in [11] there would inevitably be a top-quark in the final state. If the gravitino is lighter than the top quark — even if heavier than all other fermions — then it is stable with respect to tree-body decays, up to mixing effects\(^2\).

- This brings us to the next step: Suppose that all dominant operators involve third-generation fermions, and lead to bottom-quark or tau final states. For a gravitino below the \( \sim 1 \) GeV scale, tree-level decays will again be very suppressed.

- Finally, for “super-light” gravitinos, as in [10], gravitinos are essentially stable under the 3-body decays!

In either of the above cases — classified according to the possible range of gravitino masses for the suppression of three-body decays — gravitinos have a very large lifetime, that can exceed the age of the universe. In the case of a \( \lambda' U_3 D_j D_k \) operator, tree-level decays can proceed only via quark mixing (which in some models could be very suppressed). For \( LLE \) or \( LQD \) operators, the radiative decay of the gravitino can still dominate over the tree-body modes. In particular for the cases where third-generation fermions are involved, where both the loop factor of the radiative mode, and the phase-space suppression of the tree-level diagrams are larger.

In what follows, we will calculate the amplitude for radiative gravitino decays and discuss the effects of the underlying flavour structure in Section\(^2\), leading to a comparison between radiative and tree-level gravitino decays in Section\(^3\). We outline the related cosmological implications for dark matter, photon spectra and big-bang nucleosynthesis, and discuss possible collider phenomenology from the decays of the next-to-lightest supersymmetric particle (NLSP) in Section\(^4\) that could help to further distinguish between the different scenarios of slowly decaying gravitinos, before we Conclude.

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\(^2\)Note that for an operator of the form \( \lambda' L_i Q_3 \bar{D}_k \) this argument does not hold, since, when we pass from superfields to component fields the \( L_i Q_3 \) part can become \( \ell_i t \) or \( \nu_i b \).
2 Radiative gravitino decays

There are three main structures of diagrams that generate the decay (1.2), as shown in Fig. 1. Note that the class of operators that can be involved are of the form $LL_j \bar{E}_j$ or $LQ_j \bar{D}_j$. In the latter case the neutrino is coupled to down-type quark to preserve SU(2) invariance. The common $j$ index appears since the same fermion flavour should couple to both the gravitino and the photon. In Fig. 2 we show the corresponding three-body decay induced by the same operators.

![Figure 1: Basic set of Feynman diagrams for radiative gravitino decay, shown for (s)lepton loops. Arrows denote flow of lepton number for left-chiral fields.](image1)

![Figure 2: Three-body decay of gravitino via R-parity violating coupling.](image2)
The $R$-parity violating coupling for incoming left-chiral fields can be parametrised as

$$\bar{\ell}'\ell\nu : -i\lambda P_{L},$$

(2.1)

with $\lambda$ a dimensionless quantity, and $P_{L/R} = \frac{1}{2}(1 \mp \gamma_{5})$. The photonic coupling to sparticles is given by

$$\tilde{\ell}^{+}\tilde{\ell}^{-}A^{\nu} : +i\epsilon(q_{+} - q_{-})^{\nu},$$

(2.2)

with $q_{-}$ and $q_{+}$ the incoming slepton momenta. The gravitino couplings are (see [16, 17]):

$$\tilde{G}^{\mu}\bar{\ell}\ell : \frac{i}{\sqrt{2}M}P_{R}\gamma^{\mu}\bar{\ell},$$

$$\tilde{G}^{\mu}\bar{\ell}A^{\nu} : -\frac{ie}{\sqrt{2}M}P_{R}\gamma^{\mu}\gamma^{\nu},$$

(2.3)

with $M = (8\pi G_{N})^{-1/2} = 2.4 \times 10^{18}$ GeV the reduced Planck mass.

In order to have a non-zero contribution, there must be an even number of $\gamma$ matrices between the projection operators at the vertices where the gravitino and the neutrino couple. Thus, we immediately see that the amplitude is proportional to the lepton mass, $m_{\ell}$ — or a down-type quark mass, for the $LQ\bar{D}$ case. This amounts to a mass term insertion, as is required in order to get the correct helicity combinations at each vertex. In addition to the diagrams shown in Fig. 1 there is a corresponding set of diagrams with reversed fermion flow and right-handed (s)leptons.

### 2.1 Feynman amplitudes and gauge invariance

Extracting one lepton mass factor, the total amplitude for the Feynman diagrams shown in Fig. 1 can be written as

$$\mathcal{M} = -\frac{ie\lambda m_{\ell}}{16\sqrt{2}\pi^{2}M}m_{\tilde{G}}\mathcal{F}(m_{\tilde{G}}, m_{\tilde{\ell}}, m_{\ell}).$$

(2.4)

Using the equations of motion of the gravitino [16]:

$$\gamma_{\mu}\bar{\psi}^{\mu}(p) = 0, \quad p_{\mu}\bar{\psi}^{\mu}(p) = 0, \quad (\not{\partial} - m_{\tilde{G}})\bar{\psi}^{\mu}(p) = 0,$$

(2.5)

and of the neutrino: $\not{\partial}u(p-k) = k\not{u}(p-k)$ (neglecting the neutrino mass), one can simplify the matrix elements considerably. The dimensionless quantity $\mathcal{F}$ can thus be written as

$$\mathcal{F} = \bar{u}(p-k)[a k_{\mu}\ell + b k_{\mu}(\epsilon \cdot p) + c \epsilon_{\mu}]\bar{\psi}^{\mu}(p).$$

(2.6)

The constraint of gauge invariance imposes a relation on $a$, $b$ and $c$, and we are left with two gauge-independent structures. This is different from the case of $\tilde{\chi}_{0} \rightarrow \gamma\nu$ [2,15] where there is only one amplitude.

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3For outgoing left-chiral fields, this becomes $+i\lambda P_{R}$. 
Using the kinematical relation, \((p - k)^2 = 0\), or \(2p \cdot k = m_G^2\), it is convenient to choose the two amplitudes as follows:

\[
F = \bar{u}(p - k)P_R [c_1 A^{(1)}_\mu + c_2 A^{(2)}_\mu] \bar{\psi}\mu(p),
\]

where \(c_1\) and \(c_2\) are dimensionless coefficients, the overall factor \(P_R\) arises from the coupling to the left-handed neutrino, and the structures \(A^{(1)}_\mu\) and \(A^{(2)}_\mu\) are given by

\[
A^{(1)}_\mu = k_\mu [2(\epsilon \cdot p) - \epsilon m_G]/m_G^3,
\]

\[
A^{(2)}_\mu = [2k_\mu(\epsilon \cdot p) - \epsilon_\mu m_G^2]/m_G^3.
\]

The integrals from diagrams (a) and (b) will be logarithmically divergent and effectively regularised by the contributions of diagram (c). Thus, we see that \(F\) can be expressed in terms of two three-point functions, and finite differences of two-point functions. These terms must separately be gauge independent:

\[
F = \bar{u}(p - k) \left[ X_0 C^{(a)}_0 + X_0 C^{(b)}_0 + Y^{(1)} \Delta B^{(1)}_0 + Y^{(2)} \Delta B^{(2)}_0 \right] \bar{\psi}\mu(p),
\]

where the coefficients \(X\) and \(Y\) must be of the forms given in (2.8), and the \(C_0\) are three-point functions corresponding to diagrams (a) and (b), whereas the \(\Delta B_0\) are finite differences of two-point functions. In the notation of LoopTools [18, 19], we have

\[
C^{(a)}_0 = C_0(m_G^2, 0, 0, m_\ell^2, m_\ell^2, m_\ell^2),
\]

\[
C^{(b)}_0 = C_0(m_G^2, 0, 0, m_\ell^2, m_\ell^2, m_\ell^2),
\]

\[
\Delta B^{(1)}_0 = 2B_0(m_G^2, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2),
\]

\[
\Delta B^{(2)}_0 = B_0(m_G^2, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2).
\]

Adding the contributions of the three diagrams, we find that the coefficients of the amplitude are given by:

\[
c_1 = 2[(m_G^2 - m_\ell^2 + m_\ell^2)C^{(a)}_0 + 2B_0(m_G^2, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2)],
\]

\[
c_2 = 2[m_\ell^2 C^{(a)}_0 + m_\ell^2 C^{(b)}_0 + B_0(m_G^2, m_\ell^2, m_\ell^2) - B_0(0, m_\ell^2, m_\ell^2)].
\]

These are both UV-finite, and have the structures given in eqs. (2.9) and (2.10).

### 2.2 Flavour considerations

Before comparing the radiative and the three-body decays, we would like to comment on the dependence of our results on flavour physics. The relative magnitude of radiative gravitino decays as compared to the tree-level ones depends on the flavour structure of the R-violating operators — as we shall see, for higher generations the radiative decay becomes larger and the same is true for the suppression of the tree-level diagrams. This picture is not unknown to us: the Yukawa couplings that generate fermion masses also have clear
hierarchies, and progressively lead to higher masses as we pass from the first to the third generation of quarks and leptons.

Given that the same fermion fields that enter into R-violating operators, also enter in the Yukawa mass terms, one may try to directly link R-violating hierarchies to those of fermion masses [20, 21]. This is done using models with family symmetries. Fermion generation charges are chosen in such a way that only the third generation mass terms have a zero charge (and thus are allowed when the symmetry is exact). The rest of the masses are generated at a higher order by the spontaneous breaking of this symmetry by the vacuum expectation values of singlet fields, and are suppressed by the heavy mass scales of the theory [22, 23]. If $R$ parity is violated in such models, couplings with different family charges will also appear with different powers of the family symmetry-breaking parameter, and thus with different magnitudes.

In general, for models appearing in the literature, the relative flavour charges and thus the mass matrices, are determined by the GUT multiplet structure: particles in the same GUT multiplet have the same charge. Nevertheless, in all cases, the observed fermion hierarchies require smaller charges for the operators of the higher generations — typically zero for the top Yukawa mass terms, but also for the bottom and tau in supersymmetric models with large $\tan \beta$. This implies that it is reasonable to expect that R-violating operators that contain fields of the third generation may also be larger. Whether this happens will depend on whether fermion hierarchies can be directly linked to the R-violating ones, or whether extra singlet fields with non-zero flavour charge are involved, thus reversing the hierarchies with respect to those of the masses [21]. In this latter case, tree-level gravitino decays are more likely to dominate, unless gravitinos are very light.

Moreover, one should appropriately take into account mixing effects. Indeed, even in the case of one dominant operator in the basis of current eigenstates, mixing effects will induce non-zero coefficients for related operators in the basis of mass eigenstates. These will be suppressed by the mixing parameters with respect to the dominant operator, but will be non-zero, and this may affect phenomenological and cosmological predictions. We should also keep in mind that experiments only provide information on the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix $V_{\text{CKM}} = V_u^L V_d^L$, and that one can construct theoretical models where the left quark mixing is either in the up sector, or in the down sector, or both. Similarly, the Maki–Nakagawa–Sakata lepton mixing comes from the product of those of charged leptons and neutrinos; with the additional complication that, for the latter, we have both Dirac and Majorana mass terms.

Without entering into detailed model building, we generically observe the following:

(i) The right-handed quark mixing (relevant for $\bar{U}$ and $\bar{D}$) is essentially not constrained by the data. Therefore, for a model with left-right asymmetric mass matrices, one could also imagine a manifestation with minimal mixing in the right-handed sector.

The fact that there are strict bounds on some operators, implies that mixing effects may in given models generate additional bounds on couplings that at a first glance look less constrained. This has been analysed in detail in [21], where it was shown that in theories with strong correlations between operators (such as left-right symmetric models), the effects can be particularly significant.
in which case a $\bar{U}_3 \bar{D}_j$ operator would be the only relevant one, and the gravitino would be essentially stable!

(ii) For the left quark mixing (relevant for $Q$), we know the values from $V_{CKM}$, where for instance the 2-3 mixing is a factor of $\approx 0.04$. Thus, a coupling $\lambda' L_3 Q_3 \bar{D}_3$ also implies the coupling $0.04 \lambda' L_3 Q_2 \bar{D}_3$.

(iii) The left lepton mixing (relevant for $L$) is constrained by the lepton data, giving large 1-2 and 2-3 mixing, and small 1-3.

We see that there are in principle several flavour choices that can lead to significant effects from the decays under discussion, and one could in fact reverse the argument: the study of the relative magnitude of radiative versus tree-level violating decays, may yield relevant information for the flavour structure of the underlying theory.

3 Radiative versus three-body decays

The decay rate for the radiative decay $\tilde{G} \rightarrow \gamma \nu$ is given by

$$\Gamma = \frac{1}{16\pi m_{\tilde{G}}} \frac{1}{|\mathcal{M}|^2}, \quad (3.1)$$

where $|\mathcal{M}|^2$ denotes the absolute square of the Feynman amplitude, averaged over gravitino spin states, and summed over photon polarisations. In terms of the decomposition (2.4), the decay rate can be given by

$$\Gamma = \frac{\alpha'^2 m_{\tilde{G}}^2 m_\tilde{G}^2}{2048 \pi^4 M^2} \overline{|\mathcal{F}|^2}, \quad (3.2)$$

with

$$\overline{|\mathcal{F}|^2} = \frac{1}{4} \sum_{\text{spin, pol.}} |\bar{u}(p-k)P_R[c_1 \mathcal{A}^{(1)} + c_2 \mathcal{A}^{(2)}] \bar{\psi}^\mu(p)|^2$$

$$= \frac{3}{4} |c_1|^2 + \frac{2}{6} |c_2|^2 - \frac{1}{6} \text{Re}(c_1^* c_2), \quad (3.3)$$

where the averaging over gravitino spin states involves

$$P_{\mu\nu}(p) \equiv \sum_{\text{spin}} \bar{\psi}_\mu(p) \bar{\psi}_\nu(p)$$

$$= - (\not{p} - m_{\tilde{G}}) \left\{ \left( g_{\mu \nu} - \frac{p_{\mu} p_{\nu}}{m_{\tilde{G}}^2} \right) - \frac{1}{3} \left( g_{\mu \alpha} - \frac{p_{\mu} p_{\alpha}}{m_{\tilde{G}}^2} \right) \left( g_{\nu \beta} - \frac{p_{\nu} p_{\beta}}{m_{\tilde{G}}^2} \right) \gamma^\alpha \gamma^\beta \right\}. \quad (3.4)$$

The diagrams with reversed fermion flow, and right-handed s(fermions) in the loop are found to have the same structure, differing only in the sfermion loop mass. The decay rate for $\tilde{G} \rightarrow \gamma \bar{\nu}$ is identical, and will provide a factor of two in the total rate.
In Fig. 3 we compare the resulting lifetime of the gravitino for radiative versus tree-level decays resulting from trilinear R-violating terms. We fix $m_{\tilde{\ell}} = 200$ GeV and $\lambda = 0.001$. The interested reader should rescale the results to suit his or her preferred value of $\lambda$, using $\tau \propto 1/\lambda^2$. The black lines denote radiative decays for $(s)\text{tau}$ and $(s)\text{muon}$ loops, while the green lines represent the results for the corresponding three-body decays, with intermediate stau and smuon, taken from \cite{11}.

![Figure 3: Gravitino lifetime versus mass for two-body (black) and three-body decays (green). Also shown is the area excluded by a universe age of 13.7 Gyr (blue), and the kinematical threshold for the two-tau final state of the three-body decay (red line).](image)

We see that the radiative decays can easily dominate for gravitino masses below 30 GeV. While the three-body decay involving an intermediate stau hits the kinematical threshold at $2m_{\tau}$, the radiative dominance is still present for decays involving an intermediate smuon, where there is no threshold at the mass scales shown. For a selectron intermediary (not shown) the conclusions are the same, with radiative dominance below a gravitino mass of around 2 GeV. This behaviour is controlled by the mass dependence of the decay width: for the three-body decay $\Gamma_{\tilde{G}} \propto m_{\tilde{G}}^7$, while for the radiative decay $\Gamma_{\tilde{G}} \propto m_{\tilde{G}}$ at low gravitino masses. This can be understood from Eq. (2.11), where we see that the leading term in $c_1$ and $c_2$, for the limit of slepton masses much larger than the gravitino and lepton masses, $2m_{\tilde{l}}^2C_0$, is essentially independent of mass. The physical interpretation of this is that the gravitational coupling compensates for the high loop-mass by its increasing strength for higher masses. Because of the helicity structure of the couplings, the two-body decay width is also heavily dependent on the fermion mass, $\propto m_{l}^2$ at low gravitino masses. Thus dominant third generation couplings, which imply a tau or bottom quark, give significantly shorter lifetimes for the radiative decay.
To constitute a realistic dark matter candidate the gravitino lifetime should exceed the age of the universe. This lower boundary is shown as a blue area in Fig. 3. For the R-parity violating couplings considered here this only allows us to exclude gravitino masses above $\sim 150$ GeV. However, we shall see in the next Section that the photon flux from the gravitino decays can provide us with a much stronger bound.

In Fig. 4 we show the dependence of the gravitino lifetime on the stau mass for the sum of radiative and three-body decays, and again for a fixed $\lambda_{233} = 0.001$. As expected the gravitino mass where the three-body decay takes over depends on the stau mass. The insensitivity of the lifetime to the slepton mass for light gravitinos can again be understood from the dominant terms in $c_1$ and $c_2$.

![Figure 4: Gravitino lifetime versus mass for different slepton masses.](image)

4 Cosmological implications and collider signatures

From the above we see that the mass and lifetime of the gravitino in principle allow it to be dark matter. Whether this could be the case for a specific model depends on the predicted gravitino density. If the universe has gone through a period of inflation, the primordial gravitino abundance is erased, and gravitinos are created either thermally or from the decays of the NLSP\(^5\). This implies that the abundance of gravitinos is very sensitive to the reheating temperature of the universe — which is an open question — and can well be in the correct range for gravitinos to be dark matter, or a component of it.

\(^5\)With non-zero R-parity violation the NLSP production mechanism no longer applies.
In MSSM dark matter considerations, the allowed parameter space is severely con-strained by possible effects of late NLSP decays on the light element abundances, as pre-dicted by Big Bang Nucleosynthesis [24]. However, as has been remarked by several au-thors [12, 13, 25], in models where the NLSP decays via R-violating couplings, the BBN bounds are easily satisfied.

With regards to baryogenesis and leptogenesis it has been observed that severe con-straints on R-parity violating interactions can be derived from cosmological arguments [26]: a pre-existing baryon asymmetry would be erased if the interactions are strong enough to come to equilibrium at the time of the electroweak phase transition. However, it has been argued that this need not be the case in various schemes where flavour effects in leptogenesis/baryogenesis are considered [27]. In our analysis therefore we consider larger R-violating couplings than those used in e.g. [13].

Our final cosmological consideration is whether the presence of photons in the final state is compatible with measurements on the extragalactic diffuse photon background, and may even be reconciled with an apparent excess in the EGRET data, as proposed in [13]. This will in fact give the strictest bounds on R-violating couplings, for the case of radiative decays.

The photon flux from gravitino decays is described by a red-shifted monochromatic line, which is given by [13]:

\[
F(E) = E^2 \frac{dJ}{dE} = \text{BR}(\tilde{G} \rightarrow \gamma \nu) \times C_\gamma (1 + \kappa x^3)^{-1/2} \times 5/2 \theta(1 - x),
\]

where

\[
x = \frac{2E}{m_{\tilde{G}}}, \quad C_\gamma = \frac{\Omega_{\tilde{G}} \rho_c}{8\pi \tau_{\tilde{G} \rightarrow \gamma \nu} H_0 \Omega M^{1/2}} \quad \text{and} \quad \kappa = \frac{\Omega_\Lambda}{\Omega M}.
\]

With current values for the cosmological parameters [28] we arrive at [3]

\[
C_\gamma = 1.06 \times 10^{-6} \left( \frac{10^{27}s}{\tau_{\tilde{G} \rightarrow \gamma \nu}} \right) \text{GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \quad \text{and} \quad \kappa \simeq 3.
\]

For comparison, the original EGRET analysis [29] gave a power law description of the extragalactic flux as

\[
E^2 \frac{dJ}{dE} = 1.37 \times 10^{-6} \left( \frac{1 \text{GeV}}{E} \right)^{0.1} \text{GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1},
\]

in the energy range 30 MeV to 100 GeV. The non-observation of an excess with monochromatic origin can be used to restrict the gravitino mass and lifetime.

In Fig. [3] we show the maximum allowed value of the R-violating coupling \(\lambda\) for a range of gravitino masses. We require that the gravitinos can be dark matter, with a lifetime of at

\[^6\text{Note that there is an order of magnitude difference between our value, and that of [13]. We have learnt from the authors that this is due to a misprint in their paper.}\]

\[^7\text{Similar considerations are directly applicable to } \lambda'.\]
least 10 times the current age of the universe, and that their radiative decays are consistent with the photon spectrum measured by EGRET, extrapolating the power law behaviour up to 400 GeV. The result is a simple linear dependence on the log of the gravitino mass for the range where the radiative decays dominate. For higher gravitino masses, where the three-body decays become important, the bound relaxes as the branching ratio to photons decreases dramatically. Eventually the bound on the lifetime, for gravitinos to be dark matter, becomes dominant and the maximum allowed value decreases rapidly towards the slepton mass threshold. The extra features seen at the very highest masses are due to threshold effects for the three-body decays.

![Figure 5: Maximum value of R-violating coupling $\lambda$ versus gravitino mass.](image)

We find that the allowed couplings are small for radiative decays in an intermediate mass range, but can be significantly larger when tree-level decays dominate or for very light gravitino masses. In all cases the couplings can be sufficiently large to lead to interesting expectations for collider phenomenology.

It is interesting to note that the strict upper bounds on the couplings derived from the photon spectrum, naturally bring us to the range that is relevant for the generation of neutrino masses [2, 8, 30] compatible with neutrino data. Moreover, the power law excess claimed for the 2-10 GeV photon energy range in the more recent analysis of the EGRET data [14] could easily be explained by decaying gravitino dark matter with a mass of around 10 GeV and R-violating couplings of $\mathcal{O}(10^{-4})$.

Moving on to the collider signatures, these will be the standard signatures of R-violating supersymmetry, with multi-lepton or multi-jet events and the possibility of explicit lepton number violation at the final state. Given the magnitude of R-violating couplings required from the photon spectrum and for gravitino dark matter, MSSM production of sparticle pairs followed by R-violating decays would be expected. However, for the parameter space
and flavour structure where tree-body decays dominate, single superparticle productions may still occur along the lines previously proposed in the literature [1, 2, 31].

The specific decay modes depend on the NLSP and the branching ratios of R-violating versus R-conserving modes. If the NLSP is the neutralino it will decay to three fermions, with diagrams similar to those of the tree-level decay of the gravitino, see Fig. 2. If instead the NLSP is a slepton — e.g. \( \tilde{\nu} \) or even \( \tilde{\tau}_R \) — its decay will dominantly proceed through a direct decay to two fermions, for the cases with large R-violating couplings involving its flavour, or through an intermediary neutralino to four fermions (for small R-violating couplings). The specific branching ratios depend on the supersymmetric parameter space and the magnitude of the R-violating coupling, and have been extensively studied [1, 2, 31, 32].

| NLSP     | \( LL\tilde{E} \)                        | \( LQ\tilde{D} \)                      | \( \tilde{U}\tilde{D}\tilde{D} \) |
|----------|------------------------------------------|----------------------------------------|------------------------------------|
| \( \tilde{\chi}^0_1 \) | \( \ell_1^+\ell_1^-\nu \)               | \( q_j\tilde{q}_k\ell_1^\pm(q_j\tilde{q}_k\nu) \) | \( q_iq_j\tilde{q}_k(\tilde{q}_i\tilde{q}_j\tilde{q}_k) \) |
| \( \tilde{\nu} \)   | \( \ell_j^+\ell_j^-\nu \)               | \( q_j\tilde{q}_k \)                  | \( \nu q_iq_j\tilde{q}_k(\tilde{q}_i\tilde{q}_j\tilde{q}_k) \) |
| \( \tilde{\tau}_R \) | \( \ell_i^+\ell_i^-\nu \)               | \( q_j\tilde{q}_k \)                  | \( \tau q_iq_j\tilde{q}_k(\tilde{q}_i\tilde{q}_j\tilde{q}_k) \) |

Table 1: NSLP R-violating decays.

The choice of NLSP in the above table has been made according to the principle that, for universal sparticle masses at a high scale, the running of couplings to low energies implies an increase in masses due to gauge interactions and a decrease due to Yukawa couplings. Thus, the neutralino and the sneutrino, having no electric charge, and the right-handed stau, due to the lack of weak interactions and a large Yukawa coupling in supersymmetric models with large \( \tan\beta \), are the most obvious candidates.

5 Conclusions

We have studied radiative gravitino decays to a photon and a neutrino, generated from trilinear R-violating operators. We calculated the decay rate, and compared it to tree-level decays of the gravitino to three fermions, which arise from the same operators. There is a wide region of parameters for which the loop suppression of the radiative mode is less effective than the phase-space suppression of the tree-level one, particularly for light gravitino masses and R-violating operators that involve the third generation of fermions.

Whether enhanced gravitino decays can be expected, depends on the flavour structure of the R-violating operators. Without entering into details of model building, it is nevertheless possible to make simple analogies, and link R-violating couplings to those that generate Yukawa mass terms for the fermions. Mixing effects can also be important, since tree-level decays that are disallowed by phase-space considerations for certain operator flavours could
nevertheless proceed through quark mixing effects, suppressed by the mixing factors. For a dominant operator $U_3 \tilde{D}_i \tilde{D}_j$, the absence of the radiative mode and the top quark in the vertex, imply that the gravitino can be essentially stable in models with small right-handed quark mixing.

We find that the gravitino lifetime is typically longer than the age of the universe due to suppression by Planck scale and loop effects, meaning that gravitinos can constitute dark matter. The R-violating couplings and sparticle masses in such scenarios have values that can give rise to distinct signals in colliders, and the R-violating couplings are in general higher than those expected in bilinear R-violating schemes [13].

However, for the parameter space where radiative decays dominate, photon spectra further constrain the magnitude of R-violating couplings, particularly for $L_{1,2}L_3\tilde{E}_3$ and $L_{1,2,3}Q_3\tilde{D}_3$ operators. In fact the presence of photons in the final state of R-violating gravitino decays is compatible with an excess seen in the EGRET data on extragalactic diffuse gamma-rays, as proposed in [13]. Moreover, the range of couplings allowed by these considerations is compatible with the one required in order to generate acceptable neutrino mass patterns.

Finally, the decays of the next-to-lightest supersymmetric particle can still proceed via the R-violating vertexes at significant rates, and are thus easily accommodated within the framework of Big Bang Nucleosynthesis.

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