

Lévy robotics

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Two the most common tasks for autonomous mobile robots is to explore the environment and locate a target. Targets could range from sources of chemical contamination to people needing assistance in a disaster area. From the very beginning, the quest for most efficient search algorithms was strongly influenced by behavioral science and ecology, where researchers try to figure out the strategies used by leaving beings, from bacteria to mammals. Since then, bio-inspired random search algorithms remain one the most important directions in autonomous robotics. Recently a new wave arrived bringing a specific type of random walks as a universal search strategy exploited by immune cells, insects, mussels, albatrosses, sharks, deers, and a dozen of other animals including humans. These Lévy walks combine two key features, the ability of walkers to spread anomalously fast while moving with a finite velocity. The latter is especially valuable in the context of robotics because it respects the reality autonomous robots live in. There is already an impressive body of publications on Lévy robotics; yet research in this field is unfolding further, constantly bringing new results, ideas, hypothesis and speculations. In this mini-review we survey the current state of the field, list latest advances, discuss the prevailing trends, and outline further perspectives.

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INTRODUCTION

The problem of target search by mobile autonomous robots is a research topic which remains hot for already several decades. The performance of the robot (or, equivalently, the efficiency of the corresponding algorithm) can be quantified, depending on the context, by using either the time it takes the robot to find a specific target or by the number of targets robot finds for a fixed time (‘foraging’ [1]). There are two ultimate alternatives of search, that are random and deterministic searches [2]. No information is available about the location of targets in the case of a pure random search – until the robot simply hits it. When performing a pure deterministic search, the robot can ‘sense’ the location of targets from a distance, by mounted sensors, and just moves towards it. However, there is a continuous spectrum of strategies in between these two extremes [4]. In planar environments where the distribution of targets is unknown a priori or changes over time randomized search strategies were suggested to be more efficient [5].

We can assume that the robot moves with a constant velocity. This simple, intuitive, and, in fact, legit assumption allows us to handle, in one go, such important issues as energy cost of the movement and continuity of the motion. Then an optimal search strategies can be defined as that minimizing the mean time needed for a robot to hit a target. This condition is identical to minimization of the expenditure of energy along the way. In the case of random search, optimization could only be achieved by tuning the parameters of robot’s motion.

Animals perform random search almost every day of their lives. Often they do it with no knowledge of the environment and a limited sensory range. However, they were given thousands of years to optimize their search (a proper name in this context is ‘foraging’) strategies [6]. They also pay an energy price for their motion, their movements are continuous and usually performed with some more or less constant velocity (at least, during the search phase). Shortly, they are quite similar to robots in this respect. In this situation the idea of learning from nature is beneficial.

During the last two decades, it has been noticed that the foraging trajectories of many animals, ranging from honeybees [7] to sharks [8] and human gatherers [9], appear to be very similar. Namely, they all look like periods of localized diffusive-like search activity altered with ballistic relocations to a new spot [10]. Such intermittent trajectories are a trademark of the so-called ‘Lévy walks’ (LW) [11], which, in fact, have no characteristic length scale [12]. Indeed, the statistical analysis of the experimental trajectories uncovered the main common feature: the length distribution of ‘ballistic relocations’ is well approximated by a power-law distribution $\varphi(l) \propto l^{-1-\gamma}$.
with the exponent $1 < \gamma < 2$. There is only a one-step distance from this observation to the concept of Lévy walks, namely a constant (by absolute value) velocity of motion, $\upsilon$, during each relocation step \cite{13} (generalizations of the original model with non-constant but always finite velocities are collected in Ref. \cite{11}). Viswanathan et al. \cite{14} has considered a Lévy walker which is wandering over an infinite plane and constantly searching for targets, randomly distributed in space with a uniform density. The set-up falls in the category of what described above as random search. Viswanathan et al. considered two possible scenarios, with targets destroyed after being found (destructive case) or becoming temporally depleted (non-destructive case). The main finding was that the Lévy walk with exponent $\gamma = 2$ turned to be most efficient, as compared to other LWs and simple ballistic motion. This result greatly promoted LWs as an optimal search strategy and generated a whole wave of research activity. We will not survey here the still growing body of publications (at the moment about a hundred) on Lévy foraging and search; we refer the interested reader to the available reviews and monographs. It is noteworthy, however, that the overall attitude in these fields has changed from the enthusiasms to a bona fide skepticism and a sober critical view on the LW-oriented interpretations of the collected statistical data; see a recent work \cite{15} and a series of commenting publications initiated by it.

It is not a surprise then that the wave of studies on Lévy foraging and animal search strategies has attracted attention of the researchers working in the field of robotics. We have noticed two trends in Lévy robotics, which are complementary to one another. First one goes along the line “learn from the Nature” we discussed above and deals with the development of new LW-inspired search algorithms for autonomous mobile robots \cite{17,19,21,23}. Second, a relatively recent one, aims at the understanding of how Lévy-walk motion patterns emerge from combinations of different external factors and also used as a test to probe theoretical assumptions on animal strategies \cite{26}.

In this small review we survey the filed of Lévy robotics as it stands by now. To make it self-sufficient, we start with Section 1 introducing the LW concept. In Section 2 we discuss some important (in our opinion) publications on the subject. In the concluding section we outline some perspectives and discuss potentially interesting problems for us, LW theoreticians, motivated by what we have learned from the publications.

LÉVY WALK SUMMARY

The definition of a simplest Lévy walk model on a two-dimensional plane is very close to the original formulation of the random walk given more than a hundred years ago by Pearson \cite{16}. A walker chooses a random direction and a random time $\tau$ and walks with a constant speed $\upsilon$ in the selected direction. After the time has elapsed a new random direction and a new random time are picked and the process repeats. Importantly, the durations of walks are distributed according to a power-law density:

$$\psi(\tau) = \frac{1}{\tau_0} \left(1 + \frac{\tau}{\tau_0}\right)^{-1-\gamma}, \gamma > 0$$

(1)

Particular details of this distribution are not as important, but its slowly decaying power-law tail is very central in determining the dispersal process on long time scales. Different values of $\gamma$ correspond to different regimes of the dispersal. Such for $\gamma > 2$ the mean squared step length [calculated simply as $\langle \upsilon^2 \tau^2 \rangle$] is finite and the Cen-
tral Limit Theorem CLT predicts normal Gaussian diffusion as an outcome of such a walk. Situation changes when \( \gamma \) drops below 2. The mean squared step size becomes infinite and the CLT breaks down. Instead, there is a generalized CLT saying that in this case the spatial distribution of walkers should look like a Lévy distribution (hence the name of the walk). The hallmark of the Lévy distribution are slowly decaying power-law tails which clearly set it apart from the Gaussian profile with super-exponentially fast vanishing tails, see Fig. 1. The tails, however, do not spread to infinity, but are cut off by the ballistic front – at any moment of time \( t \) there can be no particles beyond the line \( r_{bal} = \upsilon t \) (which often is referred to as a ‘light cone’). As more quantitative information we mention that the density of particles in the bulk of the distribution follows the scaling \( x \propto t^{1/\gamma} \) (one can understand that as the characteristic scale of the particle cloud) and the mean squared displacement, a standard measure of how far are particles from the starting point, behaves as \( \langle r^2 \rangle \propto t^{3-\gamma} \). For \( \gamma < 1 \) even more dramatic things happen in that the density of particles looks more like a well instead of a hump and exhibits ballistic scaling for the whole distribution. As currently there are not so many known real world examples of the ballistic regime, we will not consider it further, but they are worthy of being remembered as a principal possibility in the context of robotics.

**LÉVY WALKS IN AUTONOMOUS ROBOTICS**

A first idea to combine Lévy walks with chemotaxis in order to obtain a search algorithm for an autonomous agent to find a source of chemical contamination in a turbulent aquatic environment, was proposed by Pasternak et al. [17]. It is not a typical search task because the searcher should scan a constantly changing chemical field and follow plumes in order to find their origin. In the computational studies, a virtual AUV (Autonomous Underwater Vehicle), floating in a virtual two-dimensional river-like turbulent flow, contaminated from a point-like source, was used. Events of unidirectional motion, characterized by a power-law distribution of their lengths and a wrapped Cauchy distribution of their direction angles, were intermingled with short re-orientation events. During the latter the vehicle was randomly choosing a new movement direction along the local concentration upstream flow. This strategy somehow corresponds to a Lévy walk in a flow-oriented reference frame. When compared to other strategies, based on Brownian walk, simple Lévy walk, correlated Brownian walk and a brute-force zig-zag scanning, Lévy-taxis outperformed all of them, both in terms of detection success rate and detection speed.

Another searching strategy for a mobile robot, a sequence of Lévy walks alternated with taxis events, was proposed by Nurzaman et al. [18]. In computer simulations, the robot task was to locate a loudspeaker by using the information on the local sound intensity obtained from a robot-mounted microphone. The loudspeaker was stationary and the robot’s speed \( \upsilon \) was constant. The robot orientation was defined by the angle \( \theta \). The robot dynamics was governed by three stochastic equations,

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{y}(t) \\
\dot{\theta}(t)
\end{bmatrix} = A(t) \begin{bmatrix}
\upsilon \cos \theta(t) \\
\upsilon \sin \theta(t) \\
0
\end{bmatrix} + [1 - A(t)] \begin{bmatrix}
0 \\
0 \\
\varepsilon \phi(t)
\end{bmatrix} \tag{2}
\]

where the Cartesian coordinates \( x(t) \) and \( y(t) \) specify the position of the robot at time \( t \). Activity \( A(t) \) is a dichotomous function switching between 1 and 0 so that the robot is either moving forward with velocity \( \upsilon \) (activity is “1”) or is choosing randomly a new direction of motion (activity is “0”). When the duration of a single 1-event is distributed according to a power-law, see Fig. 2(a), the robot performs a two-dimensional version of the Lévy walk with rests shown on Fig. 1(a). Alternatively, a stochastic sonotaxis strategy by using which the robot tried to locate and move towards the loudspeaker was probed. However, neither of the two strategies was able to accomplish the task when used alone. The sonotaxis turned out to be effective in a close vicinity of the speaker only, and did not work when the sound gradient was small, see Fig. 2(a). The Lévy walk did not care about the sound intensity by default and produced unbiased wandering only, Fig. 2(b). The combination of the two solved the problem: the Lévy walk first brought the robot to the area where the sound-intensity gradient was high enough and from there the sonotaxis strategy was able to lead the robot to the loudspeaker.
Its assigned region only. However, this situation may be divided into two equal volumes and each submarine scouts independently.

A simple divide-and-conquer strategy, when the tank is divided into two equal volumes, and each submarine scouts its assigned region only. However, this situation may change when the number of AUVs is larger than two so that communication between searchers could be beneficial. Group Lévy foraging with an artificial pheromone communication between robots was studied recently by Fujisawa and Dobata [22]. Each robot had a tank filled with a “pheromone” (alcohol) which was sprayed around by a micropump. Rovers also carried alcohol and touch sensors and their motion was controlled by a program which took into account the local pheromone concentration. The swarm foraging efficiency peaked when the robots were programmed beforehand to perform a Lévy walk in the absence of the communication. Multi-robot underwater exploration and target location were studied with a swarm of Lévy-swimming AUVs by Sutantyo et al. [23], see Fig. 3 (a). Interaction between the robots was introduced by using a modification of the Firefly Optimization, an algorithm popular in the field of particle swarm optimization [24]. The “attractiveness” of each AUV was defined by the time since the robot last found a target; it increased every time a target was located and then slowly decayed. The task was for each searcher to find all the targets. The results of the experiments showed that the interaction decreases the averaged search time substantially, see Fig. 3 (b).

Finally, an attempt to get insight into the machinery causing the emergence of Lévy walk-like patterns in the motion of different biological species was made recently by Fricke et al. [25]. Inspired by the results obtained for immune T-cells [26], researchers from the University of New Mexico and Santa Fe Institute used six small rovers, equipped with ultrasound sensors, compasses, and cameras. This navigation set enabled each robot to find patches of resources distributed over 2-d area. Tunable adaptive algorithms based on five different search strategies were tested. It turned out that the algorithm using correlated random walks, in which correlations between consequent step angles of a rover depend on the target last observed by the rover, produces Lévy-like motion patterns.

**PERSPECTIVES**

Here we want to share our feeling concerning the ongoing activity in the field of Lévy robotics. But let us start from animals.

Any organism, even a bacteria, is a much more intellectual being than a point-like particle driven by a finite-length algorithm. From another point of view, “a wandering albatross does not care about math”, as it was perfectly noted by Travis [27]. It is naïve to think that the albatross utilizes LWs when preying, by independently drawing a length of the next flight from a power-tailed probability distribution. Motion of a living organism is a product of a complex multi-layered activity of informational circuits which are constantly processing exter-
nal signals and generating internal signals to control the organism’s motion. An anomalous dispersal pattern appears as the result of this activity and not as a result of copycatting of some mathematical models.

Therefore, the current activity looks to us a bit as a ‘cargo cult’ of LWs. Let us next put the problem upside-down: If there are so many organisms, while performing on so different time-range scales, produce motional patterns bearing one common and peculiar feature – would it not be more reasonable to understand first what is behind of it?

Maybe (of course, it is a hope at the moment) there is some simple mechanism, a kind of a sensory - locomotion loop which is responsible for the appearance of the LW-like patterns. The autonomous mobile robotics serves a perfect test-bed to validate (or refute) any hypothesis. This path is already taken – but very recently – in Refs. [26, 27]; we do believe that it deserves more active research. There are some results from model simulations which suggest that LW-like patterns can be generated by a bacteria during a chemotactic activity because the bacteria is driven by a simple nonlinear circuit appearing due to chemotaxis-signaling pathway [30, 31]. Recent experimental observations demonstrated that swarming bacteria E. coli also migrate "by Lévy walk" – again, because they chemically sense each other [32]. To speculate further, we may think of the following conjecture:

Any white noise (Gaussian, shot noise, etc) when being 'filtered' through a system of a few nonlinear coupled differential equations and then used as an input to locomotion gears produces a motional pattern which could qualify (in some region of parameters) for a 'Lévy walk' (with tunable exponent $\gamma$) on a certain time scale.

This speculation (in case one was able to figure out a system of equations, a 'sensory loop') can be checked in vitro, with a mobile robot, wheeled, legged etc. Next, a tunable sensing can also be introduced so that the robot is not only kicked by a spatially homogeneous noise but can sense a target, though dimly. Then the activity of the robot can be continuously tuned from the task of locating a target to free-range exploration; see Fig. 4.

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