Breaking and Fixing Unlinkability of the Key Agreement Protocol for 2nd Gen EMV Payments

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Abstract—To address privacy problems with the EMV standard, EMVco proposed a Blinded Diffie-Hellman key establishment protocol. We point out that active attackers were not previously accounted for in the privacy requirements of this proposed protocol, despite the fact that an active attacker can compromise unlinkability. Here, we adopt a strong definition of unlinkability that does account for active attackers and propose an enhancement of the protocol proposed by EMVco where we make use of Verheul certificates. We prove that our protocol does satisfy strong unlinkability, while preserving authentication.

Index Terms—unlinkability, authentication, protocols

I. INTRODUCTION

The majority of payment cards and terminals use the EMV standard [1], the set of protocols developed by the union of payment processing companies Europay, Mastercard and Visa to execute financial operations. The initial purpose of EMV, introduced in 1996, was to support the replacement of mag-stripe cards with integrated circuit cards that are harder to copy. The EMV standard now supports contactless cards, that require no card holder action to be involved in the EMV session with any capable device. The nature of contactless cards allows an active attacker to easily interact with the card without the card holder realising, making privacy properties harder to enforce.

In this paper, we address privacy vulnerabilities in payment cards with a particular focus on the unlinkability of payments. The EMV standard trivially does not satisfy privacy properties such as anonymity and unlinkability. This is due to the current EMV standard transferring the card number in cleartext during a transaction. Hence transaction data allows us to link transactions made with the same card and effortlessly track card holders. The fact that no actual payments need to be made ease the task of the adversary when tracking a contactless card as it is ready to present its identity to any device.

In 2011 EMVCo launched the development of the new version of the standard, the EMV 2nd Gen. To protect the card against tracking, EMVCo proposed the use of secret channels. A secret channel is a symmetric key that the card and the terminal establish at the start of each session and use to encrypt further communications. A channel establishment procedure is based on Diffie-Hellman key agreement with a twist: the card uses a freshly blinded static certified public key instead of an ephemeral public key. Hence, the name of the proposed protocol, Blinded Diffie-Hellman (BDH) [2].

The BDH protocol is meant to satisfy the official requirements for channel establishment from the architecture overview of the EMV 2nd Gen [3]:

- Use elliptic-curve based cryptography (ECC).
- Computational resources of the card are respected.
- An attacker who passively eavesdrops on communications cannot identify a particular card.

Several authors published a security proof for the Blinded Diffie-Hellman protocol [4, 5] and established that a passive eavesdropper, that only listens to transmitted messages, cannot reidentify a card, therefore BDH satisfies the above requirements. Brzuska, Smart, Warinschi, and Watson [4] named this property of BDH “external unlinkability”. Since active attackers are not taken into account by external unlinkability, external unlinkability is too weak for contactless payment systems. As mentioned above, active attackers must be accounted for in the threat model, since it is easy for an attacker to initiate sessions with contactless cards using devices, such as smartphones, that need not be official terminals.

In this paper we propose to strengthen the definition of unlinkability, formalised as a process equivalence in the applied π-calculus, and introduce an enhancement of the BDH protocol satisfying this strong notion of unlinkability. Our enhancement of BDH uses anonymous credentials to hide the card’s identity. This is a minor upgrade that preserves the initial goals of BDH; notably, it respects the limited computing power of smart cards.

The contributions of the work are as follows.

- A new definition for unlinkability suitable for EMV payments that accounts for active attackers.
- An attack on Blinded Diffie-Hellman, in the form initially proposed by EMVco.
- An improved proposal for the Blinded Diffie-Hellman protocol by integrating Verheul certificates.
- Modelling Verheul certificates in applied π-calculus and a proof of the unlinkability of our improved BDH.

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The paper is organised as follows. Section II is devoted to previous work on EMV security and privacy issues. In Section III, we present the original Blinded Diffie-Hellman and illustrate why there are attacks on unlinkability in the presence of an active attacker. In Section IV, we provide background on the applied \( \pi \)-calculus. Sections V and VI contain the main contributions of the paper: a new definition of unlinkability for EMV payments, an enhanced version of BDH that includes blinded certificates, and a proof of the unlinkability of this enhanced version. Section VII confirms that authentication properties are preserved by our enhanced protocol. Section VIII concludes the paper and presents directions for future research.

II. RELATED WORK

Much related work on EMV is concerned with authentication and secrecy problems essential for avoiding fraudulent payments, rather than privacy issues such as protecting the identity of card holders. The recent work of Basin, Sasse, and Toro-Pozo \[6\] contains an overview of attacks on EMV that can lead to fraudulent transactions, e.g., criminals can make high-value purchases using a contactless Visa card without knowing the PIN. Contactless specific relay attacks may be mitigated by using distance-bounding techniques \[7\], e.g., Boureauau et al. verified Mastercard’s relay-resistant EMV protocol PayPass-RRP \[8\].

Related work has explored anonymous credentials systems that may be used to enhance privacy, since they allow credentials to be verified without disclosing the identity of a smart card holder; although such mechanisms have not been explored in the context of EMV payments. Idemix \[9\] and U-Prove \[10\] are general-purpose examples of such systems. However, they barely fit the context of this paper since Idemix requires a large key size, therefore implementation on smartcards is rather slow \[11\] and it is straightforward to link transactions in U-Prove when the same credentials (in our case, the card’s identity) are used twice. A more suitable anonymous credential system that we propose to employ in this work to improve the Blinded Diffie-Hellman protocol is the self-blindable attribute certificates due to Verheul \[12\]. Verheul certificates use elliptic curve cryptography, aligning with stated requirements of 2nd Gen EMV, and have been demonstrated to be efficiently implementable on smart cards \[13\].

Arapinis et al. \[14\] proposed to express unlinkability as an equivalence problem; specifically, they defined strong unlinkability using bisimulation. Hirschi, Baelde and Delaune \[15\] weakened this definition by redefining unlinkability as a trace equivalence problem for which they develop tool support for obtaining proofs of unlinkability. However, in general, using trace equivalence may lead to missing attacks as pointed out in \[16\] where Horne, Mauw, and Smith study ePassport protocols and revisit bisimilarity-based strong unlinkability definitions.

Finally, we mention works on symbolic methods for analyzing Diffie-Hellman (DH) groups. The general case requires both exponentiation and the group operation to be modelled and a straightforward approach may lead to the unification problem in a field \[17\] which is undecidable. Tools like Tamarin \[18\] or ProVerif \[19\] use prime order group abstractions to facilitate verification. Cremers and Jackson investigate in detail the subtleties of modelling DH groups in automated tools and propose improved models in \[20\].

III. BLINDING DIFFIE-HELLMAN AND EXTERNAL UNLINKABILITY

In this motivating section, we introduce the Blinded Diffie-Hellman protocol from the original EMVCo request for comments \[2\] and highlight unlinkability issues of the BDH protocol.

A. The Blinded Diffie-Hellman protocol

To present the BDH protocol we define the syntax of messages in Fig. 1.

\[
M, N := g \quad \text{DH group generator (constant)}
\]

\[
| x \quad \text{variable} |
\]

\[
| M \cdot N \quad \text{multiplication} |
\]

\[
| \phi(M, N) \quad \text{scalar multiplication} |
\]

\[
| \{M, N\} \quad \text{pair} |
\]

\[
| h(M) \quad \text{hash (for key derivation)} |
\]

\[
| \text{pk}(M) \quad \text{public key} |
\]

\[
| \text{sig}(M, N) \quad \text{signature} |
\]

\[
| \{M\}_N \quad \text{symmetric encryption} |
\]

\[
| \text{dec}(M, N) \quad \text{symmetric decryption} |
\]

\[
| \text{check}(M, N) \quad \text{check signature} |
\]

\[
| \text{fst}(M) \quad \text{get first} |
\]

\[
| \text{snd}(M) \quad \text{get second} |
\]

\[
| \text{auth} \quad \text{authenticate} |
\]

Fig. 1. Blinded Diffie-Hellman syntax.

The syntax for messages includes abstractions for the operators of elliptic curve arithmetic that protocols in this paper employ, enabling us to represent protocols symbolically. We leave the details of the underlying cryptographic operators and their domain parameters as a footnote. The exact signing mechanism modelled by \( \text{sig}(M, N) \) is not specified by EMVCo. Other operators are standard for symmetric encryption, pairs, and hash functions.

The Blinded Diffie-Hellman protocol is presented in Fig. 2.

There are two honest agents in the system that actively participate in the execution of the protocol: the card \( C \) and the terminal \( T \). The payment system holds a secret key \( s \) and acts as a certification authority. The private key \( c \), the public

\[1\] The public parameters are as follows: the elliptic curve \( E(F_q) \) over a known finite field \( F_q \); the order \( q \) of \( E \); the generator \( g \) of \( E \); the key derivation function \( \text{pk} \); the public key of the payment system \( \text{pk}(s) \) needed for the verification of the certificate. We employ (left) group action notation \( \phi : F_q \times G \to G \) for group operations: we write \( \phi(r, Q) \) for the element \( Q \) added \( r \) times and call \( \phi \) scalar multiplication. The symbol \( \cdot \) denotes multiplication between two scalars (field elements). All freshly generated values are picked uniformly at random from \( F_q \). The secret key \( k \) is an element of \( F_q \) and the corresponding public key is of the form \( \phi(k, g) \). Blinding of the element \( Q \) uses a fresh scalar \( \alpha \) and internally works as a scalar multiplication: \( \phi(\alpha, Q) \).
key $\phi(c, g)$ and the certificate $\langle \phi(c, g), \text{sig}(\phi(c, g), s) \rangle$ are permanently embedded in the card when it is manufactured. The card can only be issued by the bank in cooperation with payment systems like Amex, Visa, etc. The terminal, in contrast to the card, can be manufactured by anyone. To verify the legitimacy of the card, the terminal uses a public key of the payment system that is available on the system’s website.

The card starts the communication by sending its public key $\phi(c, g)$ blindered with a fresh scalar $a$ to the terminal. In response, the terminal sends ephemeral public key $\phi(t, g)$ to the card. This is enough to establish a common secret key $k_c = h(\phi(a \cdot c, \phi(t, g)), \langle \phi(c, g), \text{sig}(\phi(c, g), s) \rangle)$. Finally, the terminal verifies the received certificate by checking the signature against the public key of the payment system $\phi(c, g)$, checks that $\phi(c, g)$ blindered with $a$ coincides with the first message received from the card, and only then continues with the transaction.

**B. Blinded Diffie-Hellman is not unlinkable**

In order to verify that blinding the card’s public key protects against eavesdroppers external to the execution, the property of external unlinkability was introduced \cite{4}. In an externally unlinkable payment system, an attacker observing a message exchange between a card and a terminal cannot link that card’s current session with a previous session from the same card. In the real world, anyone could build a device imitating the terminal, for instance, an app on a smartphone supporting NFC. Such a device need not be certified or connected to any bank. Taking this into account, there is a straightforward attack on the BDH protocol presented in Fig. 2.

1) A malicious terminal establishes a key with an honest card, then successfully decrypts the message $z_2$ and obtains the card’s public key $\phi(c, g)$.

2) Another terminal operated by the attacker runs a new session with the same card to obtain again the card’s public key $\phi(c, g)$; and hence recognises the card.

This attack however would not be considered to be an attack on external unlinkability, due to the fact that, at the second step, the attacker activates the card. Since it is easy to activate a contactless card, e.g., while the card is in the holder’s pocket, external unlinkability is too weak. This compels us to adopt a stronger notion of unlinkability which can be used to discover the above attack formally.

The above attack suggests that any network of malicious powerful terminal-like devices unrelated to any payment system may track selected contactless cards in real-time without the cardholder being aware simply by starting sessions with the card in the cardholder’s pocket. Thus we propose to view unlinkability as a *property of the card* in a hostile environment. The attack also highlights the main flaw of the BDH protocol – the ability of the terminal to obtain the card’s public key which serves as the card’s identity.

To address the unlinkability vulnerability highlighted above, we propose to modify the Blinded Diffie-Hellman protocol in such a way that the signature may also be blinded and hence the public key need never be revealed to a terminal in order to check the signature. We will present and verify our enhanced version of BDH in Section VI. However, first, we dedicate the next two sections to the machinery required to formally specify and verify that our proposal is unlinkable.

**IV. APPLIED $\pi$-CALCULUS AND QUASI-OPEN BISIMILARITY**

This section contains background on a state-of-the-art formulation of the applied $\pi$-calculus \cite{21}, a language for modelling concurrent processes and their interactions. The calculus is presented in a reduced form that is just enough for the purpose of the paper. We start with the syntax and move towards the definition of an equivalence relation on processes that we use to express the unlinkability definition in Section VII.

**A. Syntax, notation, conventions**

The syntax of processes is presented in Fig. 3.

$$P, Q ::= \begin{array}{l}
0 \quad \text{deadlock} \\
M(N).P \quad \text{send} \\
M(y).P \quad \text{receive} \\
v.x.P \quad \text{new} \\
\parallel \quad \text{parallel} \\
!P \quad \text{replication} \\
\text{if } M = N \text{ then } P \quad \text{match}
\end{array}$$

Fig. 3. A syntax for processes in applied $\pi$-calculus processes.

Processes are used to capture the behaviour of a system, and, in particular, a behaviour of honest parties during the execution of a protocol. Processes can output and consume messages. To do that they use *channels*, e.g. $\overline{M}(N)$ means that the message $N$ is sent out on the channel $M$. Messages can be defined with respect to any message language subject to any equational theory, axiomatizing the properties of the cryptographic functions. We write $M =_E N$ for equality.
modulus an equational theory \( E \). Examples of a message language and an equational theory could be found in Fig. [1] and Fig. [3] respectively.

Variables in processes may be \textit{bound} by new name binders or inputs: specifically \( \nu x.P \) and \( M(x).P \) bind \( x \) in the scope \( P \). In other words, the variable \( x \) becomes local to the process \( P \). If a variable is not bound, it is a \textit{free} variable. We denote by \( \text{fv}(T) \) the set of free variables in a process or a message term \( T \).

The processes \( P \) and \( Q \) in \( P | Q \) run concurrently. The replication \( !P \) is an infinite parallel composition of \( P \) with itself. Finally, the process \( if \ M = N \ then \ P \) can behave as \( P \) whenever \( M = E \ N \).

A \textit{substitution} is a function from a finite set of variables to message terms. We use vector notation to indicate the list of variables \( \vec{x} \) or messages \( \vec{M} \). Whenever \( \vec{x} \) is involved in set-theoretic operations we treat \( \vec{x} \) as the set of variables in \( \vec{x} \). We use \( \sigma \), \( \rho \) and \( \theta \) to refer to substitutions and write \( x \sigma \) for \( x \) applied to the variable \( x \). The result of applying the substitution \( \sigma \) to the process \( P \) in the replacement of all free occurrences of \( x \) in \( P \) with \( xx \). We write \( P \sigma \) for the resulting process. When \( \sigma \) is given explicitly, we write \( \sigma = \{M_x \} \). Substitutions must avoid capture of bound variables: if a bound variable \( x \) in the process \( P \) occurs in the range of \( \sigma \), it must be renamed to avoid a name clash. The renaming of bound variables is a standard \( \alpha \)-conversion. For instance, to compute \( a(x).\pi((\{x, y\} )) \{z(x)\} \{x(x)\} \} \) we apply \( \alpha \)-conversion first and get \( a(z).\pi((\{z, y\} )) \{z(x)\} \{x(x)\} \} \) where \( z \) is chosen fresh for \( h(x) \) and then apply the substitution to obtain the result \( a(z).\pi((\{z, h(x)\} )) \). We generalize the concept of a variable not belonging to some set of variables in the following definition.

\textbf{Definition 1.} (fresh, #) The set of variables \( \vec{x} \) is fresh for the set of variables \( \vec{y} \) if \( \vec{x} \cap \vec{y} = \emptyset \); \( \vec{x} \) is fresh for a term \( P \) if \( \vec{x} \) is fresh for \( \text{fv}(P) \); \( \vec{x} \) is fresh for a substitution \( \sigma \) whenever \( \vec{x} \) is fresh for \( \text{dom}(\sigma) \) and fresh for \( \text{fv}(y \sigma) \) for any \( y \) fresh for \( \vec{x} \). Notation: \( \vec{x} \not\# \vec{y} \) if \( \vec{x} \not\# \vec{y} \).

That is, fresh variables never appear in the set of free variables or the domain of the substitution.

Throughout the paper we use several conventions. We do not distinguish between \( \nu x_1.\nu x_2.P \) and \( \nu x_2.\nu x_1.P \) and typically write \( \nu x_1.x_2.P \). The symbol \( \stackrel{\triangleq}{=} \) is used to define a process. For readability purposes we introduce the following abbreviations.

\[
\begin{align*}
\text{let } x &= M \text{ in } P & P \triangleq P[M/x] \\
\text{let } (x_1, x_2) &= M \text{ in } P & P \triangleq P[fst(M),snd(M)/x_1,x_2]
\end{align*}
\]

As an example of the introduced syntax, below we give the formal specification for the roles in the BDH protocol presented in Fig. [2].

\[
\begin{align*}
C_{dfc}(s, c, ch) &\triangleq \nu a.\overline{ch}((\phi(a, c, g))). \\
ch(y). &\text{let } k_e := h(\phi(a \cdot c, y)) \text{ in } \\
&\text{let } cert := (\phi(c, g), sig(\phi(c, g), s)) \text{ in } \\
&\overline{ch}(\{(a, c, g), cert\})_L
\end{align*}
\]

\[
T_{dfc}(s, ch) \triangleq \nu t.ch(z_1). \\
ch(z_2). &\text{let } k_t := h(\phi(t, z_1)) \text{ in } \\
&\text{let } \langle m_1, m_2 \rangle := \\
&\{\text{fst}(\text{dec}(z_2, k_t)), \text{snd}(\text{dec}(z_2, k_t))\} \text{ in } \\
&\text{if } \text{snd}(m_1) = \text{check}(\text{snd}(m_2), pk(s)) \text{ then } \\
&\text{if } \phi(fst(m_1), \text{snd}(m_1)) = z_1 \text{ then } \overline{ch}(\text{auth})
\]

The card role process is parametrised by the secret key \( s \) of the payment system, the secret key \( c \) of the card and the session channel \( ch \). The terminal role is parametrised only by \( s \) and \( ch \). The action \( \overline{ch}(\text{auth}) \) is an event used to indicate at what point the terminal believes it has authenticated the card.

\textbf{B. Semantics}

We present the state of a process as an extended process \( \nu \vec{x}.(\sigma | P) \). The syntax for extended processes is given in Fig. [5]. An extended process comprises private nonces \( \vec{x} \), messages already sent on the network \( \sigma \) and the future actions \( P \). For example, the extended process \( \nu s.\{\{\text{pk}(s).M/u_1,u_2\} | a(z)\} \) is composed of the fresh private secret key \( s \), the sent messages \( \text{pk}(s) \) and \( M \), and the input action \( a(z) \), that is not executed yet. Notice that to list the messages sent we use the substitution \( \sigma = \{M_1,\ldots,M_n / u_1,\ldots,u_n\} \), meaning that the message \( M_i \) is available through the “alias” variable \( u_i \). When a substitution serves as a ledger of sent messages, we refer to it as a frame. We require extended processes \( \nu \vec{x}.(\sigma | P) \) to be in normal form, i.e., to satisfy the restriction that the variables in \( \text{dom}(\sigma) \) are fresh for \( \vec{x} \), \( \text{fv}(P) \) and \( \text{fv}(y \sigma) \), for all variables \( y \). That is, \( \sigma \) is idempotent, and substitutions are fully applied to \( P \). We follow the convention that operational rules are defined directly on extended processes in normal form. This avoids numerous complications caused by the structural congruence in the original definition of bisimilarity for the applied \( \tau \)-calculus.

An extended process \( \nu \vec{x}.(\sigma | P) \) may make a transition to a new state by executing an action available in \( P \). We present transitions as labelled arrows. The syntax for labels is presented in Fig. [5]. To describe the transition rules we define the bound names of the transition label such that \( \text{bn}(\pi) = \{x\} \) only if \( \pi = M(x) \) and \( \text{bn}(\pi) = \emptyset \) otherwise and the names such that \( \text{u}(M N) = \text{fv}(M) \cup \text{fv}(N) \) and \( \text{u}(M(x)) = \text{fv}(M) \cup \{x\} \). Finally, we present the transition rules in Fig. [4]. The label of a transition represents an action
that the process took to arrive at a new state. In our reduced version of the applied \(\pi\)-calculus those actions are either input or output: specifically \(MN\) denotes the input of message \(N\) on channel \(M\) and \(MN(z)\) denotes an output on channel \(M\) of a message bound to the variable \(z\).

C. Equivalence notion

In Section \[\Box\] we define the unlinkability of the payment system as an equivalence notion: if the system behaves like the ideal unlinked system, then it is unlinkable. In this subsection we formally define the exact equivalence notion for (extended) processes that we use throughout the paper.

An equivalence captures both static and dynamic parts of processes’ behaviour: no distinction is made for processes if they output the same sequence of messages so far and if they can match each other’s actions. That is, we require such relation to be bisimilarity. The exact type of bisimilarity is important though, and there are many notions of bisimilarity [22]. We consider a bisimilarity that is also a congruence, because we wish the unlinkability property to hold in a larger context, that is an EMV transaction, not only for the key establishment.

We start with a standard definition of static equivalence and will make our way to a bisimilarity congruence through a series of definitions.

**Definition 2.** (static equivalence) Two extended processes \(\nu\tilde{x}.(\sigma \mid P)\) and \(\nu\tilde{y}.(\theta \mid Q)\) are statically equivalent whenever for all messages \(M\) and \(N\) such that \(\tilde{x}, \tilde{y} \# M, N\), we have \(M\sigma =_E N\sigma\) if and only if \(M\theta =_E N\theta\).

To ensure that our notion of bisimilarity is a congruence relation we require our bisimulation to be an open relation. A relation is open if it is preserved under substitutions fresh for the domain of the frame of the extended process, as stated formally below. By introducing the freshness condition we give our attacker the capacity to influence messages bound to free variables without the access to the outputs recorded in the frame – they may only be used as a part of the input.

**Definition 3.** (open relation) A relation over extended processes \(R\) is open whenever, if \(A = \nu\tilde{x}.(\sigma \mid P)\) and \(B = \nu\tilde{y}.(\theta \mid Q)\) and \(A \mathbin{\not\equiv} B\) then, for all \(\rho\) such that \(\text{dom}(\sigma)\) is fresh for \(\rho\), we have \(A\rho \mathbin{\not\equiv} B\rho\).

The precise technical name for the notion of bisimilarity restricted to open relations is quas-open bisimilarity.

**Definition 4.** (quasi-open bisimilarity) An open symmetric relation between extended processes \(R\) is a quasi-open bisimulation whenever, if \(A \mathbin{\not\equiv} B\) then the following hold:

- \(A\) and \(B\) are statically equivalent.
- \(A\mathbin{\not\equiv}A'\) there exists \(B'\) such that \(B \mathbin{\not\equiv} B'\) and \((A\mathbin{\not\equiv}A') \mathbin{\not\equiv} (B\mathbin{\not\equiv}B')\).

Processes \(P\) and \(Q\) are quasi-open bisimilar, written \(P \sim Q\), whenever \(P \mathbin{\not\equiv} Q\) for some quasi-open bisimulation \(R\).

The bisimilarity-based approach takes into account the fact that an attacker can make decisions during the execution of a protocol. Moreover, in comparison to familiar trace equivalence, for checking which tools like DeepSec [23] may help, bisimilarity is a safer option since trace equivalence is coarser. Spelled out, this means that if a privacy property is defined using bisimilarity and it holds, then it holds when the bisimilarity is replaced by trace equivalence in the definition of the property. The opposite is not true [16]. In Section \[\Box\] we define our unlinkability property by using quasi-open bisimilarity — the coarsest notion of bisimilarity for the applied \(\pi\)-calculus that is a congruence.

D. Describing attacks as modal logic formulas

To conclude the background section we describe a succinct way of expressing attacks on bisimilarity. We will use a minimal fragment of a modal logic [24, 25], sufficient for the purpose of the paper. The syntax for formulae is very concise.

\[
\psi ::= \begin{array}{c}
\neg \psi \quad \text{equality} \\
\psi \land \psi \quad \text{conjunction} \\
\psi \lor \psi \quad \text{disjunction} \\
\psi \to \psi \quad \text{implication} \\
\psi_1 \leftrightarrow \psi_2 \quad \text{equivalence} \\
\psi \land \psi \quad \text{conjunction} \\
\psi \lor \psi \quad \text{disjunction} \\
\psi \to \psi \quad \text{implication} \\
\psi_1 \leftrightarrow \psi_2 \quad \text{equivalence}
\end{array}
\]

\(\text{dom}(\psi)\) is an invariant property of a bisimulation, meaning that any pairs of processes not satisfying this property can be safely removed from a bisimulation.
The semantics of our minimal modal logic is as follows.

\[ \nu \tilde{\sigma} (\sigma | P) \models M = N \iff M \sigma =_{E} N \sigma, \not \exists M \neq N \]
\[ A \models \langle \pi \rangle \psi \iff \exists B \text{ s.t. } A \xrightarrow{\pi} B \text{ and } B \models \psi \]

If there is a formula \( \psi \) that is satisfied by one process, but is not satisfied by the other, e.g. \( A \models \psi \), but \( B \not\models \psi \), then we know that \( A \not\equiv B \) holds. The converse does not hold unless we take a larger modal logic, but this fragment suffices for the current protocol. The formula \( \psi \) captures the strategy of an attacker for distinguishing two processes. Such distinguishing strategy is a trace of transitions that the process \( A \) can make, but the process \( B \) may fail to match, followed by a test \( M = N \) demonstrating the violation of static equivalence. We will use this modal logic approach to formally present our previously mentioned attack on the BDH protocol in the proof of Theorem 1 in the next section.

V. UNLINKABILITY

In this section we introduce a formal definition of unlinkability as a process equivalence, and show that the BDH protocol in Fig. 2 does not satisfy this definition.

A. Definition of unlinkability

Perhaps the most straightforward way to design unlinkable payments is to introduce cards that immediately expire after one use. Such cards can never participate in a purchase more than once and payments are undoubtedly unlinkable. We say that if the "real world" system where cards are used multiple times, is indistinguishable by an attacker from an idealised unlinkable world in which cards are disposed of after each use, then unlinkability of payments is achieved.

Let \( C(s, ch, c) \) be the card process scheme parametrised by the payment system’s secret key \( s \), communication channel \( ch \) and the card’s secret key \( c \). It can be instantiated as, for instance, \( C_{\text{rfc}}(s, ch, c) \) or \( C_{\text{impl}}(s, ch, c) \). Then we have the following.

**Definition 5. (unlinkability)** A card process scheme \( C \) is unlinkable whenever

\[ \nu s. \text{out}(pk(s)) . \nu c. \text{vch} . \text{card}(ch) . C(s, ch, c) \]
\[ \sim \]
\[ \nu s. \text{out}(pk(s)) . \nu c. \text{vch} . \text{card}(ch) . C(s, ch, c) \]

The process on the left of the above relation models the idealised world where a card participates in no more than one transaction. This process starts by creating the secret key of the payment system \( s \). Then the public key \( pk(s) \) of the payment system is made available via the output on the public channel \( out \). Each newly manufactured card \( c \) is allowed to participate in the execution of the payment protocol just once. The process on the right of the above relation models the more realistic situation where each card \( c \) may participate in several runs of the protocol. In both cases, the card uses a newly created session channel \( ch \) that is output on the public channel \( card \).

If the idealised situation is equivalent to the real world one, where the equivalence we employ is quasi-open bisimilarity (Def. 4), we say that the payment system satisfies unlinkability.

In summary, the process scheme on the left of the equation above is the specification and on the right is the implementation. If we can prove that the equivalence problem holds for a particular protocol then that protocol complies with the specification. This is similar to the pattern for specifying unlinkability introduced in related work [14], with the key difference being that only cards need be accounted for, since the only information shared with the terminal is the public key of the payment system.

B. An attack on the unlinkability of the BDH protocol

In order to complete our specification of the BDH protocol in applied \( \pi \)-calculus, we introduce the equational theory \( E_0 \) in Fig. 6. The first three equations capture the interaction between field arithmetic and scalar multiplication followed by standard destructors: projections, decryption, and signature check.

\[ M \cdot N = E_0 N \cdot M \]
\[ \langle M \cdot N \rangle \cdot K = E_0 M \cdot \langle N \cdot K \rangle \]
\[ \phi(M \cdot N, K) = E_0 \phi(M, \phi(N, K)) \]
\[ \text{fst} \langle \{M, N\} \rangle = E_0 M \]
\[ \text{snd} \langle \{M, N\} \rangle = E_0 N \]
\[ \text{dec} \langle M \rangle_K, K = E_0 M \]
\[ \text{check} \langle \text{sig}(M, K), pk(K) \rangle = E_0 M \]

Fig. 6. Equational theory \( E_0 \) for the Blinded Diffie-Hellman protocol.

With respect to the equational theory \( E_0 \) and the formal definition of unlinkability in Def. 5 we can prove that the Blinded Diffie-Hellman protocol from Fig. 2 is not unlinkable.

**Theorem 1.** \( C_{\text{rfc}}(s, c, ch) \) violates unlinkability.

**Proof.** To describe the attack on unlinkability of the BDH protocol we follow the modal logic formula notation described in Section IV-D. Consider the following processes, where \( C_{\text{rfc}} \) is as defined in Section IV-A.

\[ R_{\text{RFC}}^{\text{spec}} \triangleq \nu s. \text{out}(pk(s)) . \nu c. \text{vch} . \text{card}(ch) . C_{\text{rfc}} \]
\[ R_{\text{RFC}}^{\text{impl}} \triangleq \nu s. \text{out}(pk(s)) . \nu c. \text{vch} . \text{card}(ch) . C_{\text{rfc}} \]

To show that \( R_{\text{RFC}}^{\text{spec}} \not\equiv R_{\text{RFC}}^{\text{impl}} \) we present a formula that is satisfied by \( R_{\text{RFC}}^{\text{impl}} \), but not by \( R_{\text{RFC}}^{\text{spec}} \). Let the formula \( \psi \) be as follows.

\[ \langle \text{out}(pk_1) \rangle \]
\[ \langle \text{card}(u_1) \rangle \langle \text{ch}_1(v_1) \rangle \langle u_1 \phi(y_1, g) \rangle \langle \text{ch}_1(w_1) \rangle \]
\[ \langle \text{card}(u_2) \rangle \langle \text{ch}_2(v_2) \rangle \langle u_2 \phi(y_2, g) \rangle \langle \text{ch}_2(w_2) \rangle \]
\[ \langle \text{snd} \langle \text{dec}(w_1, h(\phi(y_1, v_1))) \rangle \rangle = \]
\[ \langle \text{snd} \langle \text{dec}(w_2, h(\phi(y_2, v_2))) \rangle \rangle \]

The above formula describes two sessions of the BDH protocol, which, for \( R_{\text{RFC}}^{\text{impl}} \), can be with the same card, say \( c_1 \). The equality test at the end of \( \psi \) compares the certificates obtained from each session to each other, which the terminal can decrypt in both sessions. This certificate can be the same for both sessions of \( R_{\text{RFC}}^{\text{impl}} \) involving the same card, since it
is bound to the card’s identity \(c_1\). Therefore \(RFC_{\text{imp}} \models \psi\). In contrast, \(RFC_{\text{spec}} \not\models \psi\) since every session is with a new card and hence the equality test never holds, since the certificates will always differ.

In the next section, we present our enhanced BDH protocol that satisfies unlinkability.

VI. MAKING BLINDED DIFFIE–HELLMAN TRULY UNLINKABLE

In this section, we propose our fix to the BDH protocol proposed by EMVCo. This fix makes use of a certification scheme with certificates invariant under blinding. We present an existing instance of such a certification scheme, the Verheul certification scheme, and, finally, we prove that our fix indeed makes the Blinded Diffie-Hellman protocol unlinkable [2].

A. Blinded Diffie-Hellman with blinded certificates

Recall from Section [11-13] and Theorem [1] that the reason behind the failure of unlinkability of the BDH protocol proposed by EMVCo is that the card gives away its static certificate and its blinding factor. While this allows an honest terminal to authenticate the card, the public key of the card ultimately obtained by the terminal may be used to track the card in the future. We demonstrate in this section that authentication can still be performed without disclosing the public key or the signature. In order to achieve this, we specify more precisely the signature scheme (initially unspecified by EMVCo) used for certificate verification. In particular, we require that blinding and signing operations must commute. In this case, the signature can be blinded with the same nonce as the card’s public key at the beginning of the session and later checked against the public key of the payment system directly in its blinded form. As a result, only the blinded version of the card’s public key is ever revealed.

The equational theory \(E\) for the improved protocol is the equational theory \(E_0\) in Fig. 6 extended with the property expressed in Fig. 7 which permits scalar multiplication and signing to commute.

\[
\phi(M, \text{sig}(N, K)) = E \text{ sig}(\phi(M, N), K)
\]

Fig. 7. Equation for blinding extending the equational theory in Fig. 6

It now follows from the equational theory \(E\) that the check of the signature, blinded with some blinding factor, returns the message, blinded with the same factor. This property of signatures is used by the terminal when authenticating the card in our proposed update of the BDH protocol. The updated BDH protocol is presented informally in Fig. 8 and the corresponding formal \(\pi\)-calculus specification of the two roles involved is presented below.

\[
\begin{align*}
\phi(s, c) &\leftarrow (\phi(c, g), \text{sig}(\phi(c, g), s)) \\
\text{fresh } a \quad \text{ fresh } t \\
z_1 &\leftarrow \phi(a, \phi(c, g)) \\
\phi(t, g) \quad \text{kc} \leftarrow h(\phi(a \cdot c, \phi(t, g))) \\
\{\langle \phi(a, \phi(c, g)), \phi(a, \text{sigt}(\phi(c, g), s)) \rangle \}_\text{kc} \\
\langle m_1, m_2 \rangle &\leftarrow \text{dec}(z_2, \text{kt}) \\
\text{verify}(\langle m_1, m_2 \rangle, \text{pk}(s)) \quad m_1 = z_1
\end{align*}
\]

Fig. 8. The Fixed BDH protocol.

Our version differs from the original proposal in message \(z_2\) sent by the card to the terminal, i.e. now only the (encrypted) blinded certificate is transferred. At no point in the protocol, can the terminal unblind the card’s public key since the blinding factor \(a\) is never revealed to any terminal. An existing instantiation of a signature scheme satisfying the blinding condition is described in detail in the next subsection.

B. Verheul self-blindable certificates

As an example of a certification scheme satisfying the blinding condition, we present the Verheul certification scheme [12] in the context of card-terminal communication. We list the public parameters of the system below.

- \(G = E(F_p)\) elliptic curve over a known finite field \(F_p\).
- \(q \in \mathbb{N}\) order of \(G\).
- \(g \in G\) generator of \(G\).
- \(h\) key-derivation/hash function.
- \(v \in E(F_p)\) system’s generator.
- \(\text{pk}(s) = \phi(s, v)\) public key of the payment system.
- \(c : E(F_p) \times E(F_p) \rightarrow F_p^*\) fixed bilinear map.
Recall that the map $\epsilon$ is bilinear if
\[ \epsilon(\phi(\alpha, x), \phi(\beta, y)) = \epsilon(x, y) \cdot \alpha \cdot \beta. \]

Let $s$ and $c$ be the secret keys of the payment system and the card respectively. The certificate issued by the payment system on the card’s public key $\phi(c, g)$ is the pair $(\phi(c, g), \text{sig}(\phi(c, g), s))$, where the signature on the card’s public key is, by definition, $\text{sig}(\phi(c, g), s) := \phi(s, \phi(c, g))$. To verify the certificate $(\phi(c, g), \text{sig}(\phi(c, g), s))$ the terminal checks the following equality
\[ \epsilon(\phi(c, g), \text{pk}(s)) = \epsilon(\text{sig}(\phi(c, g), s), \nu). \]

Thus, the terminal utilises $\text{pk}(s), \nu$ and $\epsilon$ from the above list of public parameters. This equality indeed holds since
\[ \epsilon(\phi(c, g), \text{pk}(s)) = \epsilon(\phi(c, g), \phi(s, \nu)) = \epsilon(g, \nu) \cdot \epsilon(s, \phi(c, g)), \nu) = \epsilon(g, \nu) \cdot \epsilon(s, \phi(c, g)). \]

Notice that Verheul certificates respect blinding. Let $a$ be the fresh blinding scalar, then the certificate $(\phi(a, \phi(c, g)), (a, \text{sig}(\phi(c, g), s)))$ is valid if and only if the certificate $(\phi(c, g), \text{sig}(\phi(c, g), s))$ is valid.

The Verheul scheme is a practical solution since it respects the limited computational resources of a smart card. It has been implemented on smart cards \cite{13} by Batina et al. with on-card computation times around 1.5 seconds.

C. Self-blindable certificates bring unlinkability in BDH

In this section we present a detailed proof of unlinkability of the Fixed BDH protocol that will illustrate the importance of the chosen equivalence relation (quasi-open bisimilarity, Def. \[\ref{def:specify}.\]) We define $\text{FIX}_{\text{spec}}$ and $\text{FIX}_{\text{impl}}$ as
\[ \text{FIX}_{\text{spec}} \triangleq \nu \cdot \text{out}(\text{pk}(s)). \nu \cdot \text{vch.card}(\text{ch}), \text{cardfix}(s, c, ch) \]
\[ \text{FIX}_{\text{impl}} \triangleq \nu \cdot \text{out}(\text{pk}(s)). \nu \cdot \text{vch.card}(\text{ch}), \text{cardfix}(s, c, ch) \]

The Fixed Blinded Diffie-Hellman protocol is unlinkable as established by the following theorem.

**Theorem 2.** $\text{cardfix}(s, c, ch)$ satisfies unlinkability.

**Proof.** By Def. \[\ref{def:specify} of unlinkability, we must show that $\text{FIX}_{\text{spec}} \sim \text{FIX}_{\text{impl}}$. Therefore we shall provide a quasi-open bisimulation relation $\mathcal{R}$ such that $\text{FIX}_{\text{spec}} \mathcal{R} \text{FIX}_{\text{impl}}$.

To define such $\mathcal{R}$ we have to introduce some notation. Let $L, D \in \mathbb{N}$ be the number of sessions and the number of cards in the system, respectively. We use indices $l \in \{1, \ldots, L\}$ and $d \in \{1, \ldots, D\}$ to track sessions and cards.

Define $m^d(a, y)$ as the encrypted blinded certificate:
\[ m^d(a, y) := \{ (\phi(a, \phi(c_d, g)), \phi(a, \text{sig}(\phi(c_d, g), s))) \}_{\phi(a, c_d, g)} \]

Define a partition $\mathcal{P} \triangleq \{(\alpha, \beta, \gamma, \delta)\}$ of the set of all sessions $\{1, \ldots, L\}$, where $\alpha$ is the set of sessions in which the channel is created, but no message has been sent; $\beta$ is the set of sessions in which the blinded public key has been sent but the response has not been received; $\gamma$ is the set of all sessions in which the response has been received but the encrypted blinded certificate has not been sent; $\delta$ is the set of all sessions in which the encrypted blinded certificate has been sent.

Define a partition $\Omega \triangleq \{\{c, \ldots, c\}\}$ of the set of all sessions $\{1, \ldots, L\}$, where $\zeta$ is the set of all sessions with the card $d$.

Let $Y_l := (Y_1, \ldots, Y_L)$ be the list of inputs, where $Y_l$ is the input in session $l$. Recall that $Y_l$ can refer to messages already output on the network (the last line in Fig. \[\ref{fig:impl}.\]) Let $K := |\beta \cup \gamma \cup \delta|$ be the number of started sessions. Since we consider processes up to $\alpha$-conversion and permutation of names (aka. equivalence), we assume that $a_l$ is the blinding factor in session $l$.

Finally, we define the following process subterms, which correspond to the elements of the partition $\Psi$.
\[ \mathcal{E}^d(ch) \triangleq \nu.a.\text{ch}(\phi(a, \phi(c_d, g))), \mathcal{F}^d(ch, a) \]
\[ \mathcal{G}^d(ch, a) \triangleq \text{ch}(y), \mathcal{H}^d(ch, a, y) \]
\[ \mathcal{J}^d(ch, a, y) \triangleq \text{c}(m^d(a, y)) \]
\[ \mathcal{K}^d \triangleq \emptyset \]

The bisimulation relation $\mathcal{R}$ is defined as the least symmetric open relation satisfying the constraint in Fig. \[\ref{fig:impl}.\] Spelt out, we pair the reachable states of $\text{FIX}_{\text{spec}}$ and $\text{FIX}_{\text{impl}}$ based on the number of sessions and the respective stages of the card in a session. Notice that $\text{FIX}_{\text{spec}} \mathcal{R} \text{FIX}_{\text{impl}}$ by the definition of $\mathcal{R}$.

To prove that $\mathcal{R}$ is indeed a quasi-open bisimulation, according to Def. \[\ref{def:specify} \]we must demonstrate
1) (bisimulation) Whenever $A \mathcal{R} B$, and $A \xrightarrow{\alpha} A'$, there exists $B'$ such that $B \xrightarrow{\alpha} B'$ and $A' \mathcal{R} B'$.
2) (openness) $\mathcal{R}$ is closed under the application of a substitution fresh for the domain of the frame of any of the related states.
3) (static equivalence) Whenever $A \mathcal{R} B$, $A$ is statically equivalent to $B$.

Since $\mathcal{R}$ is by definition a symmetric relation, we provide proof only for the cases when the left-side process starts first. Below we present the exhaustive list of cases for the defining conditions of the relation $\mathcal{R}$ in Fig. \[\ref{fig:impl}.\] Proof trees justifying each transition can be found in Appendix B. Openness and static equivalence are discussed separately.

**Case 1.** $\text{FIX}_{\text{spec}} \mathcal{R} \text{FIX}_{\text{impl}}, \text{out}(pk_s)$. The process $\text{FIX}_{\text{spec}}$ can do the transition $\text{out}(pk_s)$ to the state $\text{FIX}_{\text{spec}}(\emptyset)$. There is a state $\text{FIX}_{\text{impl}}(\emptyset)$ to which the process $\text{FIX}_{\text{impl}}$ can do the transition $\text{out}(pk_s)$. By the definition of $\mathcal{R}$ we have $\text{FIX}_{\text{spec}}(\emptyset) \mathcal{R} \text{FIX}_{\text{impl}}(\emptyset)$.

**Case 2.** $\text{FIX}_{\text{spec}}(\vec{Y}) \mathcal{R} \text{FIX}_{\text{impl}}(\vec{Y}), \text{cardfix}(u_{L+1})$. The process $\text{FIX}_{\text{spec}}(\vec{Y})$ can do the transition $\text{cardfix}(u_{L+1})$ to the state $\text{CHR}_{\text{spec}} \triangleq \text{FIX}_{\text{spec}}(\text{cardfix}(u_{L+1}), \beta, \gamma, \delta)(\vec{Y})$. In the process $\text{FIX}_{\text{impl}}(\vec{Y})$ either some card $d$ starts a new session and the resulting state is $\text{CHR}_{\text{impl}} \triangleq \text{FIX}_{\text{impl}}(\text{cardfix}(u_{L+1}), \beta, \gamma, \delta, \ldots, \zeta \cup (L+1), \ldots)(\vec{Y})$ or
\[
\begin{align*}
\text{FIX}^\Psi_{\text{spec}} \equiv & \nu s, c_1, \ldots, c_L, ch_1, \ldots, ch_L, \\
& a_{1t}, \ldots, a_{1K}, \sigma \\
| C_1 | \cdots | C_L \\
| \text{lvc Chern}. card(ch), C_{\text{fix}}(s, c, ch) \rangle
\end{align*}
\]

\(\exists \text{C}_l \equiv \{ \begin{cases} 
\text{C}_l \left( \begin{array}{c}
\text{ch}_l, a_l \\
\text{ch}_l, a_l, Y_l \sigma \\
\text{ch}_l, a_l, Y_l \theta \\
\text{if l belong to \alpha} \\
\text{if l belong to \beta} \\
\text{if l belong to \gamma} \\
\text{if l belong to \delta} \\
\text{if l belong to \zeta^d \cap \alpha} \\
\text{if l belong to \zeta^d \cap \beta} \\
\text{if l belong to \zeta^d \cap \gamma} \\
\text{if l belong to \zeta^d \cap \delta} \\
\end{array} \right. 
\end{cases}\}

\begin{align*}
\text{pk}_s, \sigma = & \text{pk}(s) \\
u_s, \sigma = & \text{ch}_l \\
v_l, \sigma = & \sigma(a_l, \phi(c_l, g)) \\
w_l, \sigma = & m^d(a_l, Y_l \sigma) \\
pk_k, \theta = & \text{pk}(s) \\
u_k, \theta = & \text{ch}_l \\
v_l, \theta = & \sigma(a_l, \phi(c_l, g)) \\
w_l, \theta = & m^d(a_l, Y_l \theta) \\
\text{pk}_q, a_i, u_i, v_i # & \{ \text{card}, s \} \cup \{ c_l, ch_1, a_l | l \in \{ 1, \ldots, L \} \} \\
Y_i # & \{ s, c_l, ch_1, a_l | l \in \{ 1, \ldots, L \} \} \\
fv(Y_i) \cap & \{ \{ v_i | i \in \alpha \} \cup \{ w_i | i \in \alpha \cup \beta \cup \gamma \cup \{ l \} \} = \emptyset
\end{align*}

\[\Psi : = \{ \alpha, \beta, \gamma, \delta \}, \quad \Omega : = \{ \zeta^1, \ldots, \zeta^d \} \] are partitions of \(\{ 1, \ldots, L \} \)

\[
K : = | \beta \cup \gamma \cup \delta | \\
= l_1, \ldots, l_K \in \beta \cup \gamma \cup \delta
\]

\[\text{pk}_s, u_i, v_i \text{ and } \{ \text{card}, s \} \cup \{ c_l, ch_1, a_l | l \in \{ 1, \ldots, L \} \} \]

\[\text{fv}(Y_i) \cap \{ \{ v_i | i \in \alpha \} \cup \{ w_i | i \in \alpha \cup \beta \cup \gamma \cup \{ l \} \} = \emptyset
\]

Fig. 9. Defining conditions for the bisimulation relation \(\mathcal{R}\).

The new card is created and the resulting state is
\(\text{CH}_\text{norm} \equiv \text{FIX}_\text{spec}^\Psi(a_{0L+1}, \beta, \gamma, \delta, \Omega, \{ \{ l \} \}) \) in both cases by the definition of \(\mathcal{R}\) we have \(\text{CH}_\text{spec} \equiv \text{CH}_\text{norm} \) and \(\text{CH}_\text{spec} \equiv \text{CH}_\text{norm} \).

Case 3. \(\text{FIX}_\text{spec}^\Psi(\tilde{Y}) \equiv \text{FIX}_\text{impl}(\tilde{Y}), \text{fv}(v_i), \) and \(l \in \alpha\). The process \(\text{FIX}_\text{spec}^\Psi(\tilde{Y})\) can do the transition \(\text{fv}(v_i)\) to the state \(\text{APK}_\text{spec} \equiv \text{FIX}_\text{impl}^\Psi(a_{1L+1}, \beta, \gamma, \delta, \Omega, \{ \{ l \} \}) \) to which the process \(\text{FIX}_\text{impl}(\tilde{Y})\) can do the transition \(\text{fv}(v_i)\). By the definition of \(\mathcal{R}\) we have \(\text{APK}_\text{spec} \equiv \text{APK}_\text{impl} \).

Case 4. \(\text{FIX}_\text{spec}^\Psi(\tilde{Y}) \equiv \text{FIX}_\text{impl}(\tilde{Y}), u_i Y_i, \) and \(l \in \beta\). Let \(\chi_l(\tilde{Y}, M)\) be the list of message terms obtained from \(\tilde{Y}\) by the replacement of \(l\)th entry in \(\tilde{Y}\) with \(M\). The process \(\text{FIX}_\text{spec}^\Psi(\tilde{Y})\) can do the transition \(u_i Y_i\) to the state \(\text{IN}_\text{spec} \equiv \text{FIX}_\text{impl}^\Psi(a_{1L+1}, \beta, \gamma, \delta, \Omega, \{ \{ l \} \}) \) to which the process \(\text{FIX}_\text{impl}(\tilde{Y})\) can do the transition \(u_i Y_i\). By the definition of \(\mathcal{R}\) we have \(\text{IN}_\text{spec} \equiv \text{IN}_\text{impl} \).

Case 5. \(\text{FIX}_\text{spec}^\Psi(\tilde{Y}) \equiv \text{FIX}_\text{impl}^\Psi(\tilde{Y}), \text{fv}(v_i), \) and \(l \in \beta\). The process \(\text{FIX}_\text{spec}^\Psi(\tilde{Y})\) can do the transition \(\text{fv}(v_i)\) to the state \(\text{CRT}_\text{spec} \equiv \text{FIX}_\text{impl}^\Psi(a_{1L+1}, \beta, \gamma, \delta, \Omega, \{ \{ l \} \}) \) to which the process \(\text{FIX}_\text{impl}(\tilde{Y})\) can do a transition \(\text{fv}(v_i)\). By the definition of \(\mathcal{R}\) we have \(\text{CRT}_\text{spec} \equiv \text{CRT}_\text{impl} \).

Openness. \(\mathcal{R}\), by definition, is open: whenever \(A \mathcal{R} B\), then \(A \rho \mathcal{R} B\rho\) for any \(\rho\) fresh for the domain of \(A\). No such substitution \(\rho\) introduce transitions not considered above. Indeed, since \(\text{fv}(\text{FIX}_\text{spec}) = \text{fv}(\text{FIX}_\text{impl}) = \{\text{out}, \text{card}\}\), the substitution \(\rho\) may only affect \(\text{out}, \text{card}\) and free variables in the input of \(Y_l\). Therefore it is straightforward to modify proof trees in Appendix \[\square\] the transition label \(\text{out}(\text{pk}_s)\) is replaced by \(\text{out}(\text{pk}_s, \text{card}(\text{pk}_s))\). Then, freshness conditions remain untouched up to the renaming of variables directly affected by \(\rho\).

Static equivalence. To conclude, we prove that \(A\) is statically equivalent to \(B\) whenever \(A \mathcal{R} B\). There is nothing to prove in the case of \(\text{FIX}_\text{spec} \equiv \text{FIX}_\text{impl}\) since frames in that case are empty. The proof for the case \(\text{FIX}_\text{spec} : = \text{FIX}_\text{impl}\) is presented separately in Lemma \[\square\]

To prove that \(\text{FIX}_\text{spec}^\Psi(\tilde{Y})\) is statically equivalent to \(\text{FIX}_\text{impl}^\Psi(\tilde{Y})\) we introduce the notions of the normal form of a message term and the normalisation of a frame with respect to the equational theory \(E\).

Definition 6. \(m\)-irreducible, \(\phi\)-irreducible. A message term \(M\) is \(m\)-irreducible if there are no such \(M_1, M_2\), s.t. \(M = E M_1 \cdot M_2\); it is \(\phi\)-irreducible if there are no such \(M_1, M_2\), s.t. \(M = E \phi(M_1, M_2)\).

A subterm \(N\) of \(M\) is an immediate \(m\)-factor if it is \(m\)-irreducible and there is a message term \(K\), s.t. \(N \cdot K = M\).

The normal form \(M\) of a message term \(M\) with respect to \(E\) captures the least complex (up to \(E\)) expression of \(M\) and is defined below. We do not require the normal form of a message term to be unique.

- \(M = g\) or \(M\) is a variable, then \(M\) is unique.
- \(M = M_1 \cdot M_2\), then \(M\) is unique.

\[\text{M} = \text{M}_1 \cdot \text{M}_2\]
• $M = \phi(M_1, M_2)$, then $M \downarrow = \phi(M_1 \downarrow, M_2 \downarrow)$ if $M_2 \downarrow$ is $\phi$-irreducible. Otherwise $M \downarrow = \phi(M_1 \downarrow, M_2^{\prime} \downarrow, M_2^{\prime\prime} \downarrow)$, where $M_2 = E \phi(M_2^{\prime}, M_2^{\prime\prime})$ and $M_2^{\prime} \downarrow$ is $\phi$-irreducible.
• $M = \langle M_1, M_2 \rangle$, then $M \downarrow = \langle M_1 \downarrow, M_2 \downarrow \rangle$.
• $M = \text{h}(M_1)$ or $M = \text{pK}(M_1)$, then $M \downarrow = \text{h}(M_1 \downarrow)$ or $M \downarrow = \text{pK}(M_1 \downarrow)$ respectively.
• $M = \sigma(\text{sig}(M_1, M_2))$, then $M \downarrow = \sigma(\text{sig}(M_1 \downarrow, M_2 \downarrow))$ if $M_2 \downarrow$ is $\phi$-irreducible. Otherwise $M \downarrow = \sigma(\text{sig}(M_1 \downarrow, \text{sig}(M_2^{\prime}, M_2^{\prime\prime})))$, where $M_1 = \phi(M_1^{\prime}, M_1^{\prime\prime})$ and $M_2^{\prime \downarrow}$ is $\phi$-irreducible.
• $M = \text{fst}(\langle M_1, M_2 \rangle)$ or $M = \text{snd}(\langle M_1, M_2 \rangle)$ then $M \downarrow = M_1 \downarrow$ or $M \downarrow = M_2 \downarrow$ respectively.
• $M = \text{dec}(\langle M_1 \downarrow, M_2 \downarrow \rangle)$, then $M \downarrow = M_1 \downarrow$.
• $M = \text{check}(\text{sig}(M_1, M_2), \text{pK}(M_2))$, then $M \downarrow = M_1 \downarrow$.

Definition 7. (recipe) For an extended process $\nu \bar{x}.(\sigma \mid P)$ and a message term $N$ we say that the message $M$ is a recipe for $N$ under $\sigma$ if $M \neq \bar{x}$ and $M \downarrow = N$.

We say that the recipe $M$ is non-trivial if $\text{fv}(M_\downarrow) \cap \text{dom}(\sigma) \neq \emptyset$. For an extended process $\nu \bar{x}.(\sigma \mid P)$, the normalisation of $\sigma$ is a frame with recipes allowed in the domain. The normalisation is constructed following the procedure described below.

1) $u_\sigma = M$ for any $u \in \text{dom}(\sigma)$ is replaced by $u_\sigma = M \downarrow$.
2) If $u_\sigma = K_1 \cdot K_2$ and there is a recipe $M_1$ for an immediate $m$-factor $K_1$, then $M_1 \sigma$ is added to the normalisation.
3) If $u_\sigma = \{K_1, K_2\}$, then $u_\sigma$ is replaced by $\text{fst}(u_\sigma) = K_1$ and $\text{snd}(u_\sigma) = K_2$.
4) If $u_\sigma = \{K_1\}$ and there is a recipe $M_2$ for $K_2$, then $u_\sigma$ is replaced by $\text{dec}(u_\sigma, M_2) \sigma = K_1$.
5) If $u_\sigma = \text{sig}(N_1, N_2)$ and there is a recipe $M_2$ for $N_2$, then $u_\sigma$ is replaced by $\text{check}(u_\sigma, \text{pK}(M_2)) \sigma = N_1$.
6) If $u_\sigma = \text{sig}(N_1, N_2)$ and there is a recipe $M_2$ for $\text{pK}(N_2)$, then $\text{check}(u_\sigma, M_2) \sigma = N_1$ is added to the normalisation.

To illustrate the introduced notion of normalisation consider the extended process $\nu \bar{x}.(\sigma \mid P)$, where $\bar{x} = \{s, a, b\}$ and $\sigma = \{\phi(s) \cdot \text{h}(a), \phi(a) \cdot \text{sig}(a, x, s), \phi(x) \cdot \text{check}(u_\sigma, \text{pK}(s)) \}$. Then the normalisation of $\sigma$ is given below.

- $pk_s \sigma = \text{pK}(s)$
- $\text{dec}(u_1, u_2) \sigma = \text{h}(a)$
- $u_1 \sigma = b$
- $\text{fst}(\text{check}(u_3, pk_a)) \sigma = a$
- $\text{snd}(\text{check}(u_3, pk_a)) \sigma = x$
- $u_3 \sigma = \text{sig}(a, x, s)$

Essentially, we saturate $\sigma$ with normal forms of messages that have a recipe under $\sigma$. By doing that we remove as much complexity as possible from messages in the range of $\sigma$ without affecting static equivalence: $M$ is a recipe under $\sigma$ if and only if $M$ is a recipe under the normalisation of $\sigma$. We conclude the proof of Theorem [2] with the following.

Lemma 3. $\text{FD}^{\Psi}_{\text{esp}}(\bar{Y})$ is statically equivalent to $\text{FD}^{\Psi, \Omega}_{\text{impl}}(\bar{Y})$.

Proof. Considering the definition of $\mathcal{R}$ in Fig. [9] let $\nu \bar{x}.(\sigma \mid P) \equiv \text{FD}^{\Psi}_{\text{esp}}(\bar{Y})$ and $\nu \bar{y}.(\theta \mid Q) \equiv \text{FD}^{\Psi, \Omega}_{\text{impl}}(\bar{Y})$. We aim to show that $\nu \bar{x}.(\sigma \mid P)$ is statically equivalent to $\nu \bar{y}.(\theta \mid Q)$. Since $\bar{x}$ is always a superset of $\bar{y}$, we prove that for all messages $M$ and $N$ s.t. $\bar{y} \neq M \cdot N$, we have $M \sigma = N \sigma$ if and only if $M \theta = N \theta$.

Recall the definition of $m^d(a, g)$:

$$m^d(a, g) \equiv (\{\phi(a, \phi(c_d, g)), \phi(a, \text{sig}(\phi(c_d, g), s))\}, \text{h}(\phi(a, c_d, g)))$$

Since it is sufficient to consider normalisations when proving static equivalence, we present the normalisations of $\sigma$ and $\theta$ with respect to $E$ in Fig. [10].

$$\begin{align*}
pk_s \sigma &= \text{pK}(s) \\
u_1 \sigma &= \text{ch}_{1} \text{if } l \in \{1, \ldots, L\} \\
u_l \sigma &= \phi(a_1 \cdot c_1, g) \text{if } l \in \beta \cup \gamma \cup \delta \\
\text{fs}t(\text{dec}(\text{check}(u_1, \text{h}(\phi(T_1, v_1)))) \sigma) &= \phi(a_1 \cdot c_1, g) \\
\text{snd}(\text{dec}(\text{check}(u_1, \text{h}(\phi(T_1, v_1)))) \sigma) &= \phi(a_1 \cdot c_1, \text{sig}(g, s)) \\
\text{check}(\text{check}(\text{check}(u_2, \text{h}(\phi(T_1, v_1)))) \cdot \text{pK}(s)) \sigma &= \phi(a_1 \cdot c_1, g) \\
\text{if } l \in \delta \text{ and } Y_l \neq \phi(T_1, g) \\
w_1 \sigma &= m^d(a_l, Y_1 \sigma)
\end{align*}$$

Fig. 10. The normalisations of $\sigma$ and $\theta$ for a fixed partitions $\{\alpha, \beta, \gamma, \delta\}$, $\{\zeta_1, \ldots, \zeta_d\}$ of the set of all sessions $\{1, \ldots, L\}$.

We prove static equivalence by induction on the structure of the normal form of $\mathcal{R}$ exploring all cases allowed by the grammar in Fig. [1]. We present proofs starting from the equation under the frame $\sigma$. The argument for the converse case is the same. From now on $M, M_k, N, N_k$ are always fresh for $\bar{x}$.

Case 1. $N \sigma = g \cdot g$. 
Case 1.1. $N = g$. If $M$ is a recipe for $g$, then $M = g$, since there is no non-trivial recipe for $g$ under the normalisation of $\sigma$. Then we have $g \sigma = g \cdot g \sigma$ if and only if $g \theta = g \cdot g \theta$ as required.

Case 1.2. $N \neq g$. There is nothing to prove in this case, since there is no non-trivial recipe for $g$ under the normalisation of $\sigma$.

Case 2. $N \sigma = z \cdot z$, $z$ is a variable.
Case 2.1. $N = z$. If $M$ is a recipe for $z$, then $M = z$, since there is no non-trivial recipe for $z$ under the normalisation of $\sigma$. Then we have $z \sigma = z \cdot z \sigma$ if and only if $z \theta = z \cdot z \theta$ as required.
Case 2.2. $N\sigma = E \ ch_t$. Since $N$ is fresh for $x$, $N = u_t$. There is unique recipe $M = u_t$ for $ch_t$ and we have $u_t \sigma = E \ u_t \sigma$ if and only if $u_t \theta = _E u_t \theta$ as required.

Case 3. $N \sigma = K_1 \cdot K_2$.

Any message term in the normalisation of $\sigma$ is m-irreducible, hence no message is an immediate m-factor of another message in the normalisation of $\sigma$. Therefore there is only one case to consider.

Case 3.1. $N = N_1 \cdot \cdots \cdot N_k$, that is $N \sigma$ is generated by m-factors which have a recipe under the normalisation of $\sigma$: $N \sigma = N_1^\sigma \cdot \cdots \cdot N_k^\sigma$. By the induction hypothesis suppose that for all recipes $M_i$ for an m-factor $N_i \sigma$ of $N \sigma$, we have $M_i \sigma = E \ N_i \sigma$ if and only if $M_i \theta = _E N_i \theta$, $i \in \{1, \ldots, k\}$.

By applying multiplication, we have $M_i^\theta \cdot \cdots \cdot M_k^\theta = (M_1^\theta \cdot \cdots \cdot M_k^\theta) \sigma = E \ (N_1^\sigma \cdot \cdots \cdot N_k^\sigma) \sigma = N_1^\sigma \cdot \cdots \cdot N_k^\sigma \sigma$ as required, and $N_1 \sigma$ is an m-factor of $N \sigma$.

Case 4. $N \sigma = \phi(K_1, K_2)$.

Let us define

$$V_1 := v_1, V_2 := \text{fst}(\text{dec}(w_1, h(\phi(T_1, v_1)))),$$

$$V_3 := \text{snd}(\text{dec}(w_1, h(\phi(T_1, v_1)))),$$

$$V_4 := \text{check}(\text{dec}(w_1, h(\phi(T_1, v_1)))) \cdot pk_x.$$ 

Case 4.1. $N \sigma = \phi(a_1 \cdot c_1, g)$ and $Y_1 = \phi(T_1, g)$. Since $N$ is fresh for $x$, $N \in \{V_1, V_2, V_3\}$. Let $M$ be a recipe for $\phi(a_1 \cdot c_1, g)$, then $M \in \{V_1, V_2, V_3\}$ and we have $M \sigma = E \ N \sigma$ if and only if $M \theta = _E N \theta$ for any $N$ and $M$ as required.

If $Y_1 \neq \phi(T_1, g)$, $N = V_1$, there is unique recipe $M_1 = V_1$ and the argument is the same.

Case 4.2. $N \sigma = \phi(a_1 \cdot c_1, \text{sig}(g, s))$ and $Y_1 = \phi(T_1, g)$. Since $N$ is fresh for $x$, $N = V_3$ and there is unique recipe $M = V_3$ for $\phi(a_1 \cdot c_1, g)$, and we have $V_3 \sigma = E \ V_3 \sigma$ if and only if $V_3 \theta = _E V_3 \theta$ as required. If $Y_1 \neq \phi(T_1, g)$, there is no recipe for $\phi(a_1 \cdot c_1, \text{sig}(g, s))$ and there is nothing to prove.

Case 4.3. $N = \phi(N_1, N_2)$, $N_2 \in \{V_1, V_2, V_3, V_4\}$ and $Y_1 = \phi(T_1, g)$. By the induction hypothesis suppose that for all recipes $N_1 \sigma$ for $N_1 \sigma$, we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$, then multiply $N_2$ by a scalar $M_1$ and obtain $\phi(M_1, N_2 \sigma) = \phi(M_1, N_2 \theta) = E \ \phi(N_1, N_2 \theta) = \phi(N_1 \sigma, N_2 \sigma)$ as required. In case $Y_1 \neq \phi(T_1, g)$, $N_2 = V_1$ and the argument is the same.

Case 4.4. $N = \text{sig}(\cdots \text{sig}(N_1, N_2), \cdots, N_k), N_1 \in \{V_1, V_2, V_3, V_4\}$ and $Y_1 = \phi(T_1, g)$. By the induction hypothesis suppose that for all recipes $M_i$ for $N_i \sigma$, we have $M_i \sigma = E \ N_i \sigma$ if and only if $M_i \theta = _E N_i \theta$, $i \in \{2, \ldots, k\}$.

By applying the signature operation to $N_1 \sigma$ we have

$$\text{sig}(\cdots \text{sig}(N_1, M_2), \cdots, N_k \theta) =$$

$$\text{sig}(\cdots \text{sig}(N_1 \theta, M_2 \theta), \cdots, M_k \theta) = _E$$

$$\text{sig}(\cdots \text{sig}(N_1, N_2 \theta), \cdots, N_k \theta) =$$

$$\text{sig}(\cdots \text{sig}(N_1, N_2), \cdots, N_k \theta)$$

as required. In case $Y_1 \neq \phi(T_1, g)$, $N_1 = V_1$ and the argument is the same.

Case 4.5. $N = \phi(N_1, N_2)$. By the induction hypothesis we suppose that for all recipes $M_1, M_2$ for $N_1 \sigma, N_2 \sigma$ respectively, we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$ and $M_2 \sigma = E \ N_2 \sigma$ if and only if $M_2 \theta = _E N_2 \theta$. By applying scalar multiplication, we have $\phi(M_1, M_2 \theta) = \phi(M_1, M_2) \theta = _E \phi(N_1, N_2 \theta) = \phi(N_1 \theta, N_2 \theta)$ as required.

Case 5. $N \sigma = \{K_1, K_2\}$.

Since no pair is contained in the normalisation of $\sigma$, there is only one case to consider.

Case 5.1. $N = \{N_1, N_2\}$. To construct a pair $\langle N_1, N_2 \rangle = \{N_1 \sigma, N_2 \sigma\}$ is to construct it element-wise. By the induction hypothesis suppose that for all recipes $M_1, M_2$ for $N_1, N_2 \sigma$ respectively, we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$, and $M_2 \sigma = E \ N_2 \sigma$ if and only if $M_2 \theta = _E N_2 \theta$.

By applying the pair operation, we have $\{M_1 \theta, M_2 \theta\} = \{M_1, M_2 \theta\} = \{M_1, M_2\} \theta = \langle N_1 \theta, N_2 \theta \rangle$ as required.

Case 6. $N \sigma = h(K_1)$.

Since no message term in the normalisation of $\sigma$ is a hash, there is only one case to consider.

Case 6.1. $N = h(K_1)$. To construct a hash $h(N_1 \sigma) = h(K_1)\sigma$ is to hash a message that has a recipe. By the induction hypothesis suppose that for all recipes $M_1$ for $N_1 \sigma$, we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$. By applying the hash function, we have $h(M_1 \theta) = h(M_1) \theta = _E h(N_1 \theta)$ as required.

Case 7. $N \sigma = \text{pk}(K_1)$.

Let $N = \text{pk}(s \cdot K_1)$. Then $N = \text{pk}(s), \text{pk}(s \cdot K_1)$, since $N$ is fresh for $x$. There is a unique recipe $M = \text{pk}(s)$ and we have $\text{pk}(s \cdot K_1) = \text{pk}(s \cdot K_1)$ if and only if $\text{pk}(s) \theta = _E \text{pk}(s) \theta$ as required.

Case 7.2. $N = \text{pk}(N_1)$. To construct a public key $\text{pk}(N_1) \sigma = \text{pk}(N_1) \sigma$ is to derive it from a secret key that has a recipe. By the induction hypothesis suppose that for all recipes $M_1$ for $N_1 \sigma$, we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$. By applying the public key operation, we have $\text{pk}(M_1 \theta) = \text{pk}(M_1) \theta = _E \text{pk}(N_1) \theta = \text{pk}(N_1 \theta)$ as required.

Case 8. $N \sigma = \text{sig}(K_1, K_2)$.

Since no signed message term is contained in the normalised frame, there is only one case to consider.

Case 8.1. $N = \text{sig}(N_1, N_2)$. To construct a signature is to sign a message with a message term playing a role of a signing key. By the induction hypothesis we suppose that for all recipes $M_1$ for $\text{sig}(N_1, N_2) \sigma$ we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$. By applying the signature operation, we have $\text{sig}(M_1 \theta, M_2 \theta) = \text{sig}(M_1, M_2 \theta) = _E \text{sig}(N_1, N_2) = \text{sig}(N_1 \theta, N_2 \theta)$ as required.

Case 11. $N = \{N_1, N_2\}$. To construct an encryption is to encrypt a message with a message term, playing a role of a key. By the induction hypothesis we suppose that for all recipes $M_1$ for $\text{sig}(N_1, N_2) \sigma$ we have $M_1 \sigma = E \ N_1 \sigma$ if and only if $M_1 \theta = _E N_1 \theta$. By applying encryption, we have $\{M_1 \theta, M_2 \theta\} = \{M_1, M_2 \theta\} = _E \{N_1 \theta, N_2 \theta\}$ as required.
Case 9.2. \( N \sigma = E \cdot m_1(a_1, Y_1 \sigma) \). Since \( N \) is fresh for \( \bar{x} \), \( N = w_1 \). There is unique recipe \( M = w_1 \) for \( m_1(a_1, Y_1 \sigma) \) and we have \( w_1 \sigma = w_1 \sigma \) if and only if \( w_1 \theta = w_1 \theta \) as required.

**VII. Unlinkable Authentication for BDH**

The twofold aim of our BDH protocol is to guarantee unlinkability of the card, while allowing the terminal to authenticate the card. In this paper we emphasise unlinkability, since this is the more novel of the two requirements. Indeed, ProVerif, and other tools, can be used to automatically confirm our target authentication property – injective agreement – holds for both BDH protocols in this paper.

The process scheme below specifies the behaviour of honest terminals and honest cards. The attacker is the implicit environment that interacts with these honest participants.

\[
\text{SYS} \triangleq \nu s. \left( \begin{array}{l} \nu \nu \! c. \nu \! \text{card}(ch) \! C(s, ch, c) \left|_{\text{out}(\nu \! k(s)). \nu \text{term}(ch). T(s, ch) } \right. \\
\end{array} \right)
\]

In the above, the processes \( C \) and \( T \) can be instantiated with \( C_{\text{rfc}} \) and \( T_{\text{rfc}} \) or with \( C_{\text{fix}} \) and \( T_{\text{fix}} \) to obtain \( \text{SYS}_{\text{rfc}} \) and \( \text{SYS}_{\text{fix}} \), respectively. Notice a fresh channel for each run is advertised on channels card or term. These allow the messages associated with a run to be uniquely identified in the formulation of injective agreement below.

The following injective agreement property is standard \([26],[27].\) Agreement here means that when a terminal thinks it has authenticated a card, an honest card really executed the protocol while exchanging the same messages as the terminal. Injectivity strengthens agreement by ensuring that every successfully authenticating run of a terminal corresponds to a separate run of a card.

**Definition 8.** A process \( \text{SYS} \) satisfies injective agreement whenever for every trace \( t \) \( \models (\pi_0) \ldots (\pi_n) \) true there exists an injective function \( f: \mathbb{N} \rightarrow \mathbb{N} \) and for every, \( 0 < a \leq n \) such that \( \pi_a = \bar{ch}_{\bar{a}}(w) \) and \( \text{SYS} \models (\pi_0) \ldots (\pi_n)(w = \text{auth}) \) we have the following:

- for some \( 0 \leq i < j < k < a \), we have the following \( \pi_i = \bar{ch}_{\bar{i}}(u_i), \pi_j = \bar{ch}_{\bar{j}} M_j, \) and \( \pi_k = \bar{ch}_{\bar{k}} v_k; \)
- for \( 0 \leq f(a) < i' < j' < k' < a \), s.t. \( \pi_{f(a)} = \text{card}(ch_c). \) we have \( \pi_{i'} = ch_c M_i, \pi_{j'} = \bar{ch}_{\bar{j}}(u_j), \) and \( \pi_{k'} = ch_c M_k; \)
- for all \( \ell \in \{i, j, k\} \) \( \text{SYS} \models (\pi_0) \ldots (\pi_n)(u_{\ell} = M_{\ell}); \)

We can now verify that our target functional property holds. The proof is conducted in ProVerif [see Appendix A], with respect to an extension of the standard Diffie-Hellman theory for ProVerif, which approximates the equations for multiplication with the equation \( \phi(a, \phi(b, g)) = \phi(b, \phi(a, g)). \)

We had to extend that standard theory further with an equation \( \phi(a, \phi(b, \phi(c, g))) = \phi(b, \phi(a, \phi(c, g))) \), so that blinding factors are treated correctly.

**Theorem 4.** Both \( \text{SYS}_{\text{rfc}} \) and \( \text{SYS}_{\text{fix}} \) satisfy injective agreement.

Despite ProVerif requiring an approximation of the Diffie-Hellman theory, we find this proof to be sufficient, since authentication already held for the BDH protocol of EMVCo, and we simply aim to show that our proposed fix does not inadvertently break authentication. This contrasts, to our thorough proof of unlinkability (Theorem 2), which takes equations in Fig. 6 and Fig. 7 fully into account.

**VIII. Conclusion**

In this paper, we have investigated the Blinded Diffie-Hellman key agreement protocol in Fig. 4 proposed by EMVCo to introduce encryption in EMV payments. Although BDH indeed introduces a way to establish a symmetric key between the card and the terminal, we have shown that the privacy of the cardholder is not protected. In particular, in Theorem 1 we have shown that the presence of an active adversary leads to the failure of BDH to be unlinkable.

In our proposal for improving the protocol in Fig. 8, we use a signature scheme that respects blinding, e.g., Verheul signatures. To prove that our Fixed BDH protocol meets the privacy requirements, we have introduced a suitable definition of unlinkability (see Def. 5) and modelled our proposed protocol in the applied \( \pi \)-calculus in Section VI-A. The main contributions of the paper are as follows.

- The new unlinkability definition, tailored to real-world contactless payments (Def. 5).
- The new Fixed BDH protocol (Fig. 8).
- The proof of unlinkability of the Fixed BDH protocol in Theorem 2.

The proof of Theorem 2 is significant for three reasons. Firstly, the use of a state-of-the art approach to bisimilarity facilitates proving that the property holds for unboundedly many sessions, where the main challenge is to define the relation in Fig. 9 after which we apply the method to show that the relation is a quasi-open bisimulation. Secondly, as part of this proof we solved an instance of a static equivalence problem in Lemma 3 in the presence of a rich equational theory that is known to be hard in general. Thirdly, the facts that our notion of bisimilarity is a congruence and there is no shared secret key in this protocol, enabled us to solve the problem for a smaller process consisting of cards only, and then know that unlinkability extends to a larger system featuring the terminal.

We acknowledge that unlinkability and other privacy issues might arise later in a transaction but we focus on the key establishment and authentication of the card by the terminal, since that initial part of a future ePayment protocol should really be unlinkable. EMVCo are still in the process of revising the protocol for the 2nd Gen standard \([28].\) As awareness of these privacy issues is growing, we expect EMVCo and banks to take seriously the possibility of making contactless payments unlinkable. Future work includes drawing insight from our proofs to improve methods for unlinkability checking in the presence of Diffie-Hellman equational theories and blinding factors, so that we may be prepared to tackle larger protocols with similar requirements.
APPENDIX

A. Blinded Diffie-Hellman is an authentication protocol

The set of queries in the ProVerif code below are all evaluated as true, meaning that BDH satisfies injective agreement [26, 27].

```verbatim
free cout, card, term: channel.

type key.
type sskey.
type spkey.
type point.
type scalar.

fun smult(scalar, point): point.
fun h(point): key.
fun smult(scalar, point): point.
type scalar.

fun sign(point, sskey): bitstring.
fun h(point): key.
fun smult(scalar, point): point.

check(sign(m, k), pk(k)) = m.
```

[26] C. Cremers and D. Jackson, “Prime, order please! revisiting small subgroup and invalid curve attacks on protocols using diffie-hellman,” in 2019 IEEE 22nd Computer Security Foundations Symposium (CSF), 2019. doi: 10.1109/CSF.2019.00013 pp. 78–7815.
[27] M. Abadi and C. Fournet, “Mobile values, new names, and security communication,” SIAM J. Comput., vol. 36, no. 3, p. 104–115, Jan. 2001. doi: 10.1137/S0097539703436023
query k: key, z2: bitstring, y: point;
inj-event(terminalTerm(k, z2)) ==>
(inj-event(rec2(y)) ==> inj-event(snd2(y))).

query k: key, z2: bitstring;
inj-event(terminalTerm(k, z2)) ==>
inj-event(cardTerm(k, z2)).

let C(s: sskey, ch: channel, c: scalar) =
new a: scalar;
event snd1(smult(a, smult(c, R))); out(ch, smult(a, smult(c, R))); in(ch, y: point);
event rec2(y);
let k = h(smult(a, smult(c, y))) in
let cert = (smult(c, R), sign(smult(c, R), s)) in
event cardTerm(k, enc(((a, smult(c, R)), cert), k)); out(ch, enc(((a, smult(c, R)), cert), k))).

let T(s: sskey, ch: channel) =
new t: scalar;
in(ch, z1: point);
event rec1(z1);
event snd2(smult(t, R)); out(ch, smult(t, R));
in(ch, z2: bitstring);
let k = h(smult(t, z1)) in
let (m1: point, m2: point) = dec(z2, k) in
if (m1 = check(m2, pk(s))) && (m1 = z1) then
event terminalTerm(k, z2).

(*populate system with cards*)
let PopCard(s: sskey)=
new c: scalar;
!(new chc: channel;
out(card, chc);
C(s, chc, c)).

(*populate system with terminals*)
let PopTerminal(s: sskey)=
new cht: channel;
out(term, cht);
T(s, cht).

process
new s: sskey;
out(cout, pk(s));
!PopCard(s) | !PopTerminal(s)

B. Proof trees for transitions in Theorem 2

By Rule\(\text{\textsuperscript{n}}\) below we assume \(n\) applications of the transition rule Rule\(\text{\textsuperscript{4}}\) from Fig.\(\text{\textsuperscript{4}}\). In case of \(n\) consecutive applications of rules Par-l, Par-r we write Par\(\text{\textsuperscript{\alpha}}\). Notice that \(\alpha\)-conversion is often used: in particular when the rule Extrusion is applied.

We define \(\chi_{\text{\textsuperscript{1}}}(\vec{Y}, M)\) as the list of message terms obtained by the replacement of \(l\)th entry in \(\vec{Y}\) with \(M\). In Case 5, \(\sigma', \theta'\) are the frames accumulated at the point of input of \(Y_l\). In the proof trees presented below we use the following abbreviations

\[ S \triangleq \text{ve.ve.\text{card}(ch).C_{\text{fix}}(s, ch, c)} \]
\[ I \triangleq \text{ve.ve.\text{card}(ch).C_{\text{fix}}(s, ch, c)} \]
Fig. 11. Case 1. Transition $\text{FIX}_{\text{spec}} \xrightarrow{\text{out}(pk_s)} \nu_{\text{S}}\left(\left\{pk_s\right\}_{pk_s}\right) | I$.

Fig. 12. Case 1. Transition $\text{FIX}_{\text{impl}} \xrightarrow{\text{out}(pk_s)} \nu_{\text{S}}\left(\left\{pk_s\right\}_{pk_s}\right) | I$.

Fig. 13. Case 2. Transition $\text{FIX}_{\text{spec}}^\phi(\bar{Y}) \xrightarrow{\text{card}(u_{L+1})} \nu_{s,c_1,\cdots,c_{L+1},\bar{Y}}(\text{S}) \left(\langle Y_1, \cdots, Y_L, \varnothing \rangle\right)$. 

\[pk_s \# \text{out}, s, \! IS \text{ out} \Rightarrow \text{Out} \]
\[\text{out}(pk_s) \xrightarrow{\text{out}(pk_s)} \nu_{\text{S}}\left(\left\{pk_s\right\}_{pk_s}\right) | IS \text{ s \# out, pk_s} \]
\[\text{FIX}_{\text{spec}} \xrightarrow{\text{out}(pk_s)} \nu_{\text{S}}\left(\left\{pk_s\right\}_{pk_s}\right) | IS \text{ Res} \]

\[\text{card} \sigma = \text{card} \]
\[
\begin{align*}
&\text{Fig. 17. Case 3. Transition } F I X^\Psi_\text{impl}(\tilde{Y}) \xrightarrow{u_1Y} F I X^\Psi_\text{impl}(\chi_1(\tilde{Y}), Y_1), \; i \in \alpha. \\
&\text{Fig. 18. Case 4. Transition } F I X^\Psi_\text{spec}(\tilde{Y}) \xrightarrow{u_1Y} F I X^\Psi_\text{impl}((\alpha, \beta, \gamma, \delta), \Omega_1(\tilde{Y}, Y_1)), \; l \in \beta. \\
&\text{Fig. 19. Case 4. Transition } F I X^\Psi_\text{spec}(\tilde{Y}) \xrightarrow{u_1Y} F I X^\Psi_\text{impl}((\alpha, \beta, \gamma, \delta), \Omega_1(\tilde{Y}, Y_1)), \; \text{if there is a card at the stage } \mathcal{F}. \\
&\text{Fig. 20. Case 5. Transition } F I X^\Psi_\text{impl}(\tilde{Y}) \xrightarrow{u_1Y} F I X^\Psi_\text{impl}((\alpha, \beta, \gamma), \Omega_{11}(\tilde{Y})), \; l \in \gamma. \\
&\text{Fig. 21. Case 5. Transition } F I X^\Psi_\text{impl}(\tilde{Y}) \xrightarrow{u_1Y} F I X^\Psi_\text{impl}((\alpha, \beta, \gamma), \Omega_{11}(\tilde{Y})), \; l \in \gamma.
\end{align*}
\]