Z-SCALING AT RHIC

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Abstract

The concept of z-scaling reflecting the general regularities of high-$p_T$ particle production is reviewed. Properties of data z-presentation are discussed. New data on high-$p_T$ particle spectra obtained at the RHIC are analyzed in the framework of z-presentation. It was shown that these experimental data confirm z-scaling. Predictions of strange particle spectra are considered to be useful for understanding of strangeness origin in mesons and baryons and search for new physics phenomena at the RHIC.

Submitted to ”Physics of Particles and Nuclei, Letters”
1. Introduction

Search for scaling regularities in high energy particle collisions is always to be a subject of intense investigations [1]-[10]. Commissioning of the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL) gives new possibilities to perform experimental investigations in a new physics domain. The RHIC is a next generation of a proton-proton colliders after ISR designed to accelerate protons at center of mass energy range $\sqrt{s} = 50 - 500$ GeV aimed to clarify origin of proton’s spin and discover a new state of nuclear matter, Quark Gluon Plasma.

High energy of colliding particles and high transverse momentum of produced particles are most suitable for precise QCD test of production processes with hard probes like high-$p_T$ hadrons, direct photons and jets. Therefore, search for general regularities of high-$p_T$ single inclusive particle spectra of hadron–hadron and hadron–nucleus collisions are of interest to establish complementary restrictions for theory.

The universal phenomenological description (z-scaling) of high-$p_T$ particle production cross sections in inclusive reactions is developed in [11, 12]. The approach is based on properties of particle structure, their constituent interaction and particle formation such as locality, self-similarity and fractality. The scaling function \( \psi \) and scaling variable \( z \) are expressed via experimental quantities such as the inclusive cross section \( Ed^3/dp^3 \) and the multiplicity density of charged particles \( dN/d\eta \). Data \( z \)-presentation is found to reveal symmetry properties (energy and angular independence, A-and F-dependence, power law). The properties of \( \psi \) at high \( z \) are assumed to be relevant to the structure of space-time at small scales [13, 14, 15]. The function \( \psi(z) \) is interpreted as the probability density to produce a particle with a formation length \( z \).

In the report we present the results of analysis of new data on high-$p_T$ particle spectra obtained at the RHIC. The obtained results are compared with other ones based on the data obtained at lower collision energy $\sqrt{s}$. The results are considered as a new confirmation of z-scaling at the RHIC.

2. Z-scaling

The idea of z-scaling is based on the assumptions [6] that gross feature of inclusive particle distribution of the process (1) at high energies can be described in terms of the corresponding kinematic characteristics

$$M_1 + M_2 \rightarrow m_1 + X$$

(1)

of the constituent subprocess written in the symbolic form (2)

$$(x_1M_1) + (x_2M_2) \rightarrow m_1 + (x_1M_1 + x_2M_2 + m_2)$$

(2)

satisfying the condition

$$x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2.$$  

(3)

The equation is the expression of locality of hadron interaction at constituent level. The \( x_1 \) and \( x_2 \) are fractions of the incoming momenta \( P_1 \) and \( P_2 \) of the colliding objects with the masses \( M_1 \) and \( M_2 \). They determine the minimum energy, which is necessary for production of the secondary particle with the mass \( m_1 \) and the four-momentum \( p \). The parameter \( m_2 \) is introduced to satisfy the internal conservation laws (for baryon number, isospin, strangeness, and so on).
The equation (3) reflects minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, the quantity $\Omega$ is introduced. It is chosen in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \quad (4)$$

where $m$ is a mass constant and $\delta_1$ and $\delta_2$ are factors relating to the anomalous fractal dimensions of the colliding objects. The fractions $x_1$ and $x_2$ are determined to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling the condition (3)

$$d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \quad (5)$$

The fractions $x_1$ and $x_2$ are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

Self-similarity is a scale-invariant property connected with dropping of certain dimensional quantities out of physical picture of the interactions. It means that dimensionless quantities for the description of physical processes are used. The scaling function $\psi(z)$ depends in a self-similar manner on the single dimensionless variable $z$. It is expressed via the invariant cross section $E d^3\sigma / d\rho^3$ as follows

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)|_{\eta=0} J} E \frac{d^3\sigma}{d\rho^3} \quad (6)$$

Here, $s$ is the center-of-mass collision energy squared, $\sigma_{in}$ is the inelastic cross section, $J$ is the corresponding Jacobian. The factor $J$ is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles.

The function $\psi(z)$ is normalized as follows

$$\int_0^\infty \psi(z) dz = 1. \quad (7)$$

The relation allows us to interpret the function $\psi(z)$ as a probability density to produce a particle with the corresponding value of the variable $z$.

Principle of fractality states that variables used in the description of the process diverge in terms of the resolution. This property is characteristic for the scaling variable

$$z = z_0 \Omega^{-1}, \quad (8)$$

where

$$z_0 = \sqrt{\hat{s}_\perp/(dN/d\eta)}. \quad (9)$$

The variable $z$ has character of a fractal measure. For the given production process (1), its finite part $z_0$ is the ratio of the transverse energy released in the binary collision of constituents (2) and the average multiplicity density $dN/d\eta|_{\eta=0}$. The divergent part $\Omega^{-1}$ describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents relative number of all initial configurations containing the constituents which carry fractions $x_1$ and $x_2$ of the incoming momenta. The $\delta_1$ and $\delta_2$ are the anomalous fractal dimensions of the colliding objects (hadrons or nuclei). The momentum fractions $x_1$ and $x_2$ are determined in a way to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of the fractal measure $z$ with respect
to all possible sub-processes \[^2\] subjected to the condition \[^3\]. The variable \( z \) was interpreted as a particle formation length.

The scaling function of high-\( p_T \) particle production, as shown below, is described by the power law, \( \psi(z) \sim z^{-\beta} \). Both quantities, \( \psi \) and \( z \), are scale dependent. Therefore we consider the high energy hadron-hadron interactions as interactions of fractals. In the asymptotic region the internal structure of particles, interactions of their constituents and mechanism of real particle formation manifest self-similarity over a wide scale range.

3. Z-scaling before RHIC

It was established \[^{11,12}\] that of data \( z \)-presentation reveals the properties such as the energy and angular scaling, the power law, \( A \)- and \( F \)-dependencies of the scaling function \( \psi(z) \). Numerous experimental data on high-\( p_T \) particle spectra obtained at U70, ISR, SpS and Tevatron used in the analysis \[^{11,12}\] are compatible each others in \( z \)-presentation and give us a good reference frame for future analysis of RHIC data.

Let us remind some properties of \( p_T \)-presentation. The first one is the strong dependence of the cross section on energy \( \sqrt{s} \). The second feature is a tendency that the difference between particle yields increases with the transverse momentum \( p_T \) and the energy \( \sqrt{s} \). The third one is a non-exponential behavior of the spectra at \( p_T > 4 \text{ GeV/c} \). The energy independence of data \( z \)-presentation means that the scaling function \( \psi(z) \) has the same shape for different \( \sqrt{s} \) over a wide \( p_T \) range.

Figure 1(a) shows the dependence of the cross section of \( \pi^+ \)-meson production in \( p-p \) interactions on transverse momentum \( p_T \) at \( \sqrt{s} = 11.5 - 53 \text{ GeV} \) in a central rapidity range. The data cover a wide transverse momentum range, \( p_T = 0.2 - 10 \text{ GeV/c} \).

Figure 1(b) demonstrates \( z \)-presentation of the same data sets. One can see that the scaling function \( \psi(z) \) demonstrates independence on collision energy \( \sqrt{s} \) over a wide energy and transverse momentum range at \( \theta_{\text{cms}} \simeq 90^0 \).

As seen from Figure 1(b) the scaling function reveals a linear \( z \)-dependence on the log-log scale at high-\( z \). It corresponds to the power law, \( \psi(z) \sim z^{-\beta} \). The value of the slope parameter \( \beta \) is independent of the energy \( \sqrt{s} \) over a wide range of high transverse momentum. This is considered as indication that the mechanism of particle formation reveals self-similar and fractal properties.

The \( p_T \)-presentation demonstrates a strong angular dependence as well. Figure 1(c) shows the dependence of the cross section of \( \pi^0 \)-meson production in \( p-p \) collisions on transverse momentum at \( \sqrt{s} = 53 \text{ GeV} \) and the center of mass angle \( \theta_{\text{cms}} = (5 - 90)^0 \).

The angular independence of data \( z \)-presentation means that the scaling function \( \psi(z) \) has the same shape for different values of an angle \( \theta_{\text{cms}} \) of produced particle over a wide \( p_T \) and \( \sqrt{s} \) range. Figure 1(b) demonstrates \( z \)-presentation of the same data sets and experimental confirmation of the angular scaling of \( \psi(z) \).

The \( z \)-presentation of data gives indication on \( F \)-independence of the scaling function \[^{16}\]. The property means that the scaling function \( \psi(z) \) for different species of produced hadrons (\( \pi^{\pm,0}, K^{\pm}, \bar{p} \)) at high-\( z \) is described by the power law, \( \psi(z) \sim z^{-\beta} \), and the slope parameter \( \beta \) is independent of flavor content of produced hadrons. Figure 1(e) illustrates the \( F \)-independence of \( \psi(z) \) at high-\( z \) for hadron production in \( p-Bc \) collisions. For comparison of different data sets the transformation \( z \rightarrow (\alpha_A \alpha_F) \cdot z \), \( \psi \rightarrow (\alpha_A \alpha_F)^{-1} \cdot \psi \) of the scaling variable \( z \) and the scaling function \( \psi \) have been used. The parameters \( \alpha_A \) and \( \alpha_F \) are indepen-
dent of energy $\sqrt{s}$ and momentum $p_T$. The property is considered as universality of particle formation mechanism over a wide range of small scales. We assume that it relates to a structure of space-time itself.

4. **Z-scaling at RHIC**

Recently the STAR and PHENIX Collaborations presented new data on inclusive high-$p_T$ particle spectra measured at the RHIC in $p-p$ collisions at $\sqrt{s} = 200$ GeV. In the section the data are compared with other ones and used as the experimental test of $z$-scaling.

4.1 **Charged hadrons**

The high-$p_T$ spectra of charged hadrons produced in $p-p$ and $Au-Au$ collisions at energy $\sqrt{s} = 200$ GeV within $|\eta| < 0.5$ were measured by the STAR Collaboration [17]. The results are presented in Figure 2(a). The $p_T$-distribution of charged hadrons produced in $Au-Au$ collisions were measured at different centralities. The shape of the cross section drastically changes as centrality increases. The spectrum for $p-p$ collisions is similar to the spectrum observed in the peripheral $Au-Au$ collisions. The STAR data [17] for $p-p$ collisions correspond to non-single diffraction cross section. Other experimental data correspond to inelastic cross section. Therefore in the analysis the multiplicity particle density $dN/d\eta$ for non-single diffraction interaction for STAR data were used. The RHIC data and other ones for $p-p$ collisions obtained at the U70 [18], Tevatron [19, 20] and ISR [21] are shown in Figure 2(b). The charged hadron spectra were measured over a wide kinematic range $\sqrt{s} = 11.5 - 200$ GeV and $p_T = 0.5 - 9.5$ GeV/c. The strong energy dependence and the power behavior of particle $p_T$-spectrum are found to be clearly. The energy independence of data $z$-presentation shown in Figure 3(c) is confirmed. Verification of the asymptotic behavior of $\psi$ at $\sqrt{s} = 200$ GeV and reach of value of $z$ up to 30 and more are of interest.

4.2 **$\pi^0$-mesons**

The PHENIX Collaboration published the new data [25] on the inclusive spectrum of $\pi^0$-mesons produced in $p-p$ collisions in the central rapidity range at RHIC energy $\sqrt{s} = 200$ GeV. The transverse momenta of $\pi^0$-mesons were measured up to 13 GeV/c. The $p_T$- and $z$-presentations of data for $\pi^0$-meson spectra obtained at ISR [26, 27, 28, 29, 30] and RHIC [25] are shown in Figures 3(a) and 3(b). One can see that $p_T$-spectra of $\pi^0$-meson production reveal the properties similar to that found for charged hadrons. The new data [25] on $\pi^0$-meson inclusive cross sections obtained at the RHIC as seen from Figure 3(b) are in a good agreement with our earlier results [11]. Thus we can conclude that the available experimental data on high-$p_T$ $\pi^0$-meson production in $p-p$ collisions confirm the property of the energy independence of $\psi(z)$ in $z$-presentation.

4.3 **$\eta$-mesons**

New data on $\eta$-meson spectra in $p-p$ collisions at $\sqrt{s} = 200$ GeV in the range $p_T = 1.2 - 8.5$ GeV/c are presented by the PHENIX Collaboration in [23]. The $\eta/\pi^0$ ratio is found to be $0.54 \pm 0.05$ in the range $p_T = 3.5 - 9$ GeV/c. The value is in agreement with existing data. We compare the data with other ones obtained at $\sqrt{s} = 30., 31.6, 38.8, 53.$ and 63. GeV [24, 36]. Data $p_T$- and $z$-presentations are shown in Figures 4(a) and 4(b). As seen from Figure 4(b) the results of our new analysis confirm the energy independence of the scaling function for $\eta$-meson production in $p-p$ collisions over a wide $\sqrt{s}$ and $p_T$ range. Note that new result on
the $\eta/\pi^0$ ratio indicates on flavour independence of the scaling function at high-$z$.

4.4 Λ and \(\bar{\Lambda}\) hyperons

Here we analyze the new data obtained by the STAR Collaboration \cite{31} on $p_T$-spectra of neutral strange particles ($K^0_S, \Lambda, \bar{\Lambda}$) produced in $p - p$ collisions at $\sqrt{s} = 200$ GeV. The transverse momentum spectra are shown in Figure 5(a). We have not any other data to compare with the STAR data and construct the scaling function. Therefore the $F$-dependence of $z$-presentation was used to determine the scaling function for $\Lambda$ and $\bar{\Lambda}$. The experimental data (see Figure 3) on inclusive cross section of $\pi^0$-mesons produced in $p - p$ collisions at $\sqrt{s} = 23 - 200$ GeV are used to construct the asymptotics for $\psi(z)$. The dashed line shown in Figure 5(b) is the fit of the data. As seen from Figure 3(b) and Figure 5(b) the scaling function is described by the power law, $\psi(z) \sim z^{-\beta}$, on the log-log scale at high-$z$. The transformation of the variable $z$ and the scaling function $\psi$ for $\Lambda$ (Fig.5(b)) and $\bar{\Lambda}$ in the form $z \rightarrow \alpha_F \cdot z, \ \psi \rightarrow \alpha_F^{-1} \cdot \psi$ was used for coincidence of the asymptotics for $\Lambda, \bar{\Lambda}$ and $\pi^0$. Note that the scaling function $\psi(z)$ for $\Lambda$ reveals different behavior at low- and high-$z$ ranges. It is valid for $\bar{\Lambda}$ as well. The parameterizations of $\psi(z)$ for $\Lambda$ and $\bar{\Lambda}$ were used to predict particle spectra (see Figures 5(c) and 5(d)) at $\sqrt{s} = 63$, 200 and 500 GeV at high-$p_T$.

4.5 $\phi$-mesons

Recently the STAR Collaboration presented the new data \cite{32} on spectra of $\phi$-mesons produced in $Au - Au$ and non-singly-diffractive $p - p$ collisions at energy $\sqrt{s} = 200$ GeV. The decay mode $\phi \rightarrow K^+K^-$ was used to reconstruct $\phi$-mesons up to $p_T = 3.7$ GeV/c.

The cross sections as a function of the difference of the transverse mass $m_t$ and the mass of $\phi$-meson $m_\phi$ are shown in Figure 6(a). The data can be used to test $z$-scaling and verify models of strange particle formation. The $p_T$-distribution of $\phi$-mesons produced in $Au - Au$ collisions measured at different centralities reveals exponential behavior. The slope of spectra changes with the centrality. The spectrum for $p - p$ collisions at high-$p_T$ indicates on power behavior. The scaling functions for $\phi$ and the asymptotics for $\pi^0$ are shown in Figure 6(b). The $F$-dependence of data $z$-presentation was used to predict $\phi$-meson spectra at $\sqrt{s} = 41.6, 63, 200$ and 630 GeV and $\theta_{cms} = 90^\circ$ at high-$p_T$. Note also that $p - p$ spectra of $\phi$-mesons is important to study the nuclear modification factor $R_{AA}$ over a wide $p_T$-range and obtain direct information about the dense nuclear matter at hadron formation.

4.6 $\Xi^-$ and $\Xi^+$ hyperons

The STAR Time Projection Chamber (TPC) provides excellent tracking of charged particles with good momentum resolution \cite{33}. Present statistics are sufficient to reconstruct $\Xi^-$ and $\Xi^+$ hyperons over a wide $p_T$-range \cite{34}. The decay mode $\Xi^- \rightarrow \Lambda \pi^-$ was used to reconstruct $\Xi^-$ up to $p_T = 4$ GeV/c. Mid-rapidity transverse momentum spectra for $\Xi^-$ and $\Xi^+$ from $p - p$ at energy $\sqrt{s} = 200$ GeV and $|y| < 0.75$ are shown in Figure 7(a).

The flavor independence of $\psi$ at high-$z$ for different pieces allows us to construct the scaling function of $\Xi^-$ (see Fig. 7(b)) and $\Xi^+$ over a wide $z$-range using the asymptotics for $\pi^0$-mesons and to predict inclusive cross sections of $\Xi^-$ and $\Xi^+$ hyperon production at high-$p_T$. The transverse momentum spectra for $\Xi^-$ and $\Xi^+$ are shown in Figure 7(c) and Figure 7(d), respectively.

As shown in \cite{35} the PYTHIA simulation of $\Xi^-$ spectrum in $p - p$ collisions at $\sqrt{s} = 200$ GeV does not reproduce the STAR data. Therefore our predictions can be used to tune various PYTHIA parameters. To study azimuthal correlation of strange and charged particles and strange tagging jet production in $p - p$ collisions statistics should be gained. Moreover so-
phisticated algorithms could essentially decrease background and increase efficiency of strange particles reconstruction as well.

4.7 \(K^0\)-mesons

Here we compare the STAR data [31] for \(K^0\)-meson cross sections with data for \(K^+\)-mesons obtained at the U70 [18], Tevatron [19, 20] and ISR [21] at lower energies 11.5, 19.4, 23.8, 27.4, 38.8 and 53 GeV. The data \(p_T\)- and \(z\)-presentations are shown in Figures 8(a) and 8(b), respectively. As seen from Figure 8(a) the energy dependence of the cross section enhances with \(p_T\). The shape of the scaling function for \(K^0\)-mesons coincides with similar one for \(K^+\)-mesons in the range \(z = 0.2 - 3.0\). It gives evidence that mechanism of neutral and charged strange \(K\)-meson formation is the same one and it reveals property of self-similarity.

4.8 \(\pi^\pm\)-mesons

The PHENIX Collaboration presented in [22] the new data on inclusive cross section of identified hadrons (\(\pi^\pm, K^\pm, p, \bar{p}\)) produced in \(p-p\) collisions at \(\sqrt{s} = 200\) GeV in the central rapidity range. Transverse momentum of particles is measured up to 2.2 GeV/c. Data \(p_T\)- and \(z\)-presentations for \(\pi^\pm\)-mesons are shown in Figure 9. We compare the data with another ones obtained at the U70 [18], Tevatron [19, 20] and ISR [21] at lower energies \(\sqrt{s} = 11.5 - 53\) GeV. As seen from Figure 9(b) the scaling function corresponding to data [22] is in good agreement with our results obtained previously [11].

5. Conclusion

Analysis of new experimental data on high-\(p_T\) hadrons (\(h^\pm, \pi^0, \eta, \Lambda, \bar{\Lambda}, \phi, \Xi^-, \Xi^+, K^0, \pi^\pm\)) produced in \(p-p\) collisions at the RHIC in the framework of data \(z\)-presentation was performed.

The scaling function \(\psi(z)\) and scaling variable \(z\) are expressed via the experimental quantities, the invariant inclusive cross section \(E d^3\sigma/dp^3\) and the multiplicity density of charged particles \(\rho(s, \eta)\). The scaling function \(\psi\) is interpreted as a probability density to produce a particle with the formation length \(z\).

The general regularities of high-\(p_T\) particle production described by \(z\)-scaling were found to be valid in the new kinematical range accessible at the RHIC. Using the properties of \(z\)-scaling predictions of spectra of strange particles (\(\Lambda, \bar{\Lambda}, \phi, \Xi^-, \Xi^+, K^0, \pi^\pm\)) produced in \(p-p\) collisions at RHIC energies in high-\(p_T\) range were made. The obtained results are considered to be useful for understanding of strangeness origin in meson and baryons. New evidence that mechanism of particle formation reveals self-similar and fractal properties at high-\(p_T\) range was obtained.

Thus we conclude that new data obtained at RHIC confirm the general concept of \(z\)-scaling. The further inquiry and search of violation of the scaling can give information on new physics phenomena in high energy hadron collisions and determine domain of applicability of the strong interaction theory.

Acknowledgments. The author would like to thank I.Zborovský, Yu.Panebratsev, O.Rogachevski and A.Kechechyan for collaboration and numerous fruitful and stimulating discussions of the problem.

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Figure 1. The $p_T$-(a,c) and $z$-presentations of experimental data on inclusive cross section of particles produced in $p – p$ collisions. The energy (b), angular (d) scaling and $F$-dependence (e) of the function $\psi(z)$. Experimental data are taken from [17, 18, 19, 20, 21].
Figure 2. (a) Experimental data [17] on the inclusive cross sections of charged hadrons produced in $Au - Au$ and $p - p$ collisions at $\sqrt{s_{nn}} = 200$ GeV and $\theta_{\text{cms}} \simeq 90^\circ$ as a functions of the transverse momentum $p_T$. Data (b) $p_T$- and (c) $z$-presentations of data taken from [18, 19, 20, 21] and [17].
Figure 3. The dependence of the inclusive cross section of $\pi^0$-meson production on the transverse momentum $p_T$ in $p-p$ collisions at $\sqrt{s} = 30, 53, 62$ and 200 GeV and the angle $\theta_{cm}$ of 90°. The experimental data are taken from [26, 27, 28, 29, 30] and [25]. (b) The corresponding scaling function $\psi(z)$.

Figure 4. (a) The inclusive cross section of $\eta$-mesons produced in $p-p$ collisions in the central rapidity range as a function of the transverse momentum $p_T$ at $\sqrt{s} = 30 - 63$ GeV and 200 GeV. Experimental data are taken from [24, 36] and [23]. (b) The corresponding scaling function $\psi(z)$. 
Figure 5. (a) The inclusive cross sections of hadrons $K^0_S, \Lambda, \bar{\Lambda}$ produced in $p - p$ collisions at $\sqrt{s} = 200$ GeV in central rapidity range as a functions of the transverse momentum $p_T$. Experimental data are taken from [31]. (b) $Z$-presentation of $\Lambda$ and $\pi^0$ experimental data on cross sections. Predictions of the inclusive spectra for $\Lambda$ (c) and $\bar{\Lambda}$ (d) production in $p - p$ collisions at $\sqrt{s} = 63, 200, 500$ GeV and $\theta_{cm} \simeq 90^0$. 

STAR Preliminary
Figure 6. (a) The inclusive cross sections of $\phi$-mesons produced in $Au - Au$ and $p - p$ collisions at $\sqrt{s_{NN}} = 200$ GeV in central rapidity range as a functions of $m_t - m_\phi$. Experimental data are taken from [32]. (b) $Z$-presentation of $\phi$ and $\pi^0$ experimental data on cross sections. (c) Predictions of the inclusive spectra for $\phi$-meson production in $p - p$ collisions at $\sqrt{s} = 41.6, 63, 200, 500$ GeV and $\theta_{cms} \approx 90^0$. 
Figure 7. (a) The inclusive cross sections of $\Xi^-$ and $\Xi^+$ hyperons produced in $p - p$ at $\sqrt{s} = 200$ GeV in central rapidity range as a functions of the transverse momentum $p_T$. Experimental data are taken from [34]. (b) $Z$-presentation of $\Xi^-$ and $\pi^0$ experimental data on cross sections. Predictions of the inclusive spectra for $\Xi^-$ (c) and $\Xi^+$ (d) production in $p - p$ collisions at $\sqrt{s} = 63, 200, 500$ GeV and $\theta_{cms} \approx 90^\circ$. 
Figure 8. (a) The inclusive cross sections of $K^+$- and $K_S^0$-mesons produced in $p - p$ collisions in the central rapidity range as a function of the transverse momentum at $\sqrt{s} = 11.5 - 53$ GeV and 200 GeV. Experimental data are taken from [18, 19, 20, 21] and [31]. (b) The corresponding scaling function $\psi(z)$.

Figure 9. (a) The inclusive cross section of $\pi^+$-mesons produced in $p - p$ collisions in the central rapidity range as a function of the transverse momentum $p_T$ at $\sqrt{s} = 11.5 - 53$ GeV and 200 GeV. Experimental data are taken from [18, 19, 20, 21] and [22]. (b) The corresponding scaling function $\psi(z)$.