Decoupling Structure of the Principal Sigma Model-Maxwell Interactions

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Abstract

The principal sigma model and Abelian gauge fields coupling is studied. By expressing the first-order formulation of the gauge field equations an implicit on-shell scalar-gauge field decoupling structure is revealed. It is also shown that due to this decoupling structure the scalars of the theory belong to the pure sigma model and the gauge fields sector consists of a number of coupled Maxwell theories with currents partially induced by the scalars.

1 Introduction

The scalar sectors of the supersymmetric field theories especially the supergravity theories which are the theories that govern the massless sector low energy background coupling of the relative superstring theories [1] can be formulated as principal sigma models or non-linear sigma models whose target spaces are group manifolds or coset spaces. In particular a great majority of the supergravity scalar sectors are constructed as symmetric space sigma models [2, 3, 4, 5, 6, 7, 8]. When the Abelian (Maxwell) vector multiplets are coupled to the graviton multiplets in these theories the scalar sector which
has the non-linear sigma model interaction with in itself is coupled to the
Abelian gauge fields through a kinetic term in the Lagrangian [9, 10, 11, 12].

In this work, we will focus on the gauge field equations of the principal
sigma model and the Abelian gauge field couplings mentioned above. Bearing
in mind that the non-linear sigma model can be obtained from the principal
sigma model by imposing extra restrictions for the sake of generality we
will consider the generic form of the principal sigma model as the non-linear
interaction of the scalars. We will simply show that the gauge field equations
can be locally integrated so that they can be expressed as first-order equations
containing arbitrary locally exact differential forms. The first-order form of
the gauge field equations will be used to show that there exists a one-sided
decoupling between the scalars and the gauge fields in a sense that the scalar
field equations do not contain the gauge fields in them whereas the scalars
enter as sources in the gauge field equations. Thus the scalar solution space of
the coupled theory coincides with the general solution space of the pure sigma
model. Furthermore we will also discuss that this hidden on-shell scalar-
matter decoupling results in a number of coupled Maxwell theories with
sources whose currents contain the general solutions of the principal sigma
model which is completely decoupled from the Maxwell sector. Therefore we
will show that when the general solutions of the pure principal sigma model
are obtained and when one fixes the sector of the solution space of the coupled
theory by fixing the field dependence or the independence of the locally exact
differential forms appearing in the first-order gauge field equations one may
determine the currents of the coupled Maxwell fields. In this respect one may
solve the gauge fields from the Maxwell sector field equations. Consequently
the solution space of the scalar-matter coupling can be entirely generated
by the general solutions of the pure principal sigma model and the arbitrary
choice of the locally exact differential forms. This fact is a consequence of
the solution methodology which is based on the implicit on-shell decoupling
between the matter fields and the scalar sector which provides current sources
to the former.
2 Hidden Decoupling Structure of the Gauge Fields and Their Sources

In a $D$-dimensional spacetime $M$ the Lagrangian which gives the inhomogeneous Maxwell equations can be given as

$$\mathcal{L} = -\frac{1}{2} dA \wedge *dA - A \wedge *J,$$  \hspace{1cm} (2.1)

where $A$ is the $U(1)$ electromagnetic gauge potential one-form and $F = dA$ is the field strength of it. Also in the units where the speed of light is unity the current one-form in a local coordinate basis $\{dt, dx^a\}$ is

$$J = -\rho dt + J^a dx^a,$$  \hspace{1cm} (2.2)

where in the temporal component $\rho$ is the charge density and the spatial components $J_a$ are the current densities. The Lagrangian (2.1) defines a media in which the charge density and the currents are not influenced by the electromagnetic field. The current one-form is predetermined and static that is to say although it acts as a source for the electromagnetic field it does not interact with it dynamically. From (2.1) the inhomogeneous Maxwell equations read

$$d * F = - * J.$$  \hspace{1cm} (2.3)

In this section we will consider the coupling of $N$ $U(1)$ gauge field one-forms $A^i$ to the principal sigma model whose target space is a group manifold $G$. The sigma model Lagrangian can be given as

$$\mathcal{L} = \frac{1}{2} \text{tr}(\star dg^{-1} \wedge dg).$$  \hspace{1cm} (2.4)

Here we take a differentiable map

$$h : M \longrightarrow G,$$  \hspace{1cm} (2.5)

we also consider a representation $f$ of $G$ in $Gl(N, \mathbb{R})$

$$f : G \longrightarrow Gl(N, \mathbb{R}),$$  \hspace{1cm} (2.6)

which may be taken as a differentiable homomorphism. Then the map $g$ can be given as

$$g = f \circ h : M \longrightarrow Gl(N, \mathbb{R}),$$  \hspace{1cm} (2.7)
which is a matrix-valued function on $M$

$$g(p) = \begin{pmatrix} \varphi^{11}(p) & \varphi^{12}(p) & \cdots \\ \varphi^{21}(p) & \varphi^{22}(p) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad (2.8)$$

$\forall p \in M$. Due to the presence of the scalar fields $\{\varphi^{ij}\}$ the theory can be considered to be a scalar field theory. In $(2.4)$ we have a matrix multiplication with the wedge product used between the components and the trace is over the representation chosen in $(2.6)$. Depending on the restrictions on the sigma model and the nature of $G$ the scalars in $(2.8)$ can be all independent or not. This model also covers the non-linear (coset) sigma models $[13, 14]$ and in particular the symmetric space sigma models $[2, 3, 4]$ which shape the scalar sectors of the supergravities thus the low energy effective string theories. The construction of the Lagrangian of the symmetric space sigma model in which an internal metric substitutes the map $g$ can be found in $[5, 6, 7]$.

By generalizing the supersymmetric coupling of the supergravity matter multiplets with the graviton multiplets $[9, 10, 11, 12, 15]$ we can write down the coupling of $NU(1)$ gauge field one-forms $A^i$ to the principal sigma model whose target space is a group manifold as

$$\mathcal{L}_{tot} = \frac{1}{2} tr(*dg^{-1} \wedge dg) - \frac{1}{2} * F^T g \wedge F, \quad (2.9)$$

where we define the column vector $F$ whose components are $F^i$ thus the coupling term can be explicitly written as

$$-\frac{1}{2} * F^T g \wedge F = -\frac{1}{2} g^{ij} * F_i \wedge F^j. \quad (2.10)$$

Now if we vary the Lagrangian $(2.9)$ with respect to $A^i$ we find the corresponding field equations as

$$d(T^i_k * F_i) = 0, \quad (2.11)$$

where

$$T^i_k = g^{ij}_k + (g^T)^i_k. \quad (2.12)$$

The expression $(2.11)$ defines a closed form. Since locally any closed form is equal to an exact form we can integrate $(2.11)$ to write

$$T^i_k * F_i = dC_k, \quad (2.13)$$
where \( \{C_k\} \) are arbitrary \( N (D - 3) \)-forms on \( M \). They may be chosen to depend on the fields \( \{\varphi^{ij}, A^i\} \) or they may be fixed. The former case is the most general one. The general solutions of the field equations of the Lagrangian (2.9) must satisfy (2.13) on-shell in which one can freely change the set of \( (D - 3) \)-forms \( \{C_k\} \) which may be functions of \( \{\varphi^{ij}, A^i\} \) or not. To generate the entire solution space one can first choose a set of field-dependent or field-independent \( \{C_k\} \) and solve the system of field equations of (2.9) to find the corresponding solutions then one can repeat this procedure for a different set of \( \{C_k\} \). In this respect the field-independent or the fixed \( (D - 3) \)-forms \( \{C_k\} \) can be considered as the integration constants coming from the reduction of the degree of the gauge field equations which are second-order differential equations. Now we will follow the methodology described above to show that there exists an on-shell decoupling between the scalars and the gauge fields. Let us first take a look at the more restricted case of field-independent \( \{C_k\} \) and let us fix a set of field-independent \( \{C_k\} \). In this case if we take the partial derivative of both sides of (2.13) with respect to the scalar fields \( \{\varphi^{ml}\} \) we immediately see that

\[
\frac{\partial T^i_k}{\partial \varphi^{ml}} F_i = 0, \tag{2.14}
\]

since the right hand side of (2.13) is a fixed \( (D - 2) \)-form and it does not depend on the scalar fields \( \{\varphi^{ml}\} \). These conditions must be satisfied on-shell by a sector of the solution space of \( \{\varphi^{ml}, A^i\} \) which is restricted to the condition of choosing fixed \( \{C_k\} \). After fixing the set of \( (D - 3) \)-forms \( \{C_k\} \) if we use (2.13) in (2.9) we can obtain the on-shell Lagrangian as

\[
\mathcal{L}_{\text{tot}} = \frac{1}{2} \text{tr} (\ast dg^{-1} \wedge dg) - \frac{1}{2} dC_j \wedge F^j. \tag{2.15}
\]

Since we have made use of the gauge field equations varying this on-shell Lagrangian with respect to \( A^i \) yields an identity. On the other hand by varying the above Lagrangian with respect to the scalars to find the solutions which satisfy (2.13) for a chosen fixed set of \( \{C_k\} \) we find that the scalar field equations are the same ones which can be obtained directly from the pure sigma model Lagrangian (2.4) since the coupling part in (2.15) does not depend on the scalars. This result can also be obtained by directly varying (2.9) and by using the conditions (2.14) which are satisfied by the sector of the general solutions of the theory which we have restricted ourselves in
by fixing \(\{C_k\}\). The same result would be obtained if one chooses another field-independent set of \(\{C_k\}\). Thus we observe that the scalar solutions of a sub-sector of the scalar-gauge field theory defined by the coupling Lagrangian (2.9) are the same with the general solution space of the pure principal sigma model. This is a consequence of the local integration given in (2.13) and fixing the arbitrary \(\{C_k\}\).

On the other hand a similar result with a minor difference can also be derived for the rest of the solution space of the theory. If we consider the most general case of field-dependent differential forms \(\{C_k(\varphi^{ml}, A^i)\}\) and use (2.13) in the Lagrangian (2.9) we get

\[
\mathcal{L}_{tot} = \frac{1}{2} tr(\ast dg^{-1} \wedge dg) - \frac{1}{2} dC_j(\varphi^{ml}, A^i) \wedge F^j. \tag{2.16}
\]

This on-shell Lagrangian gives us the same decoupling conditions discussed above for the restricted sub-sector case. To see this we should realize that the second term in the Lagrangian (2.16) which is written in an on-shell form is a closed differential form. Locally any closed differential form is an exact one. Thus the second term in the Lagrangian (2.16) can be written as an exact differential form as

\[
- \frac{1}{2} dC_j(\varphi^{ml}, A^i) \wedge F^j = dB(\varphi^{ml}, A^i). \tag{2.17}
\]

In fact we may simply calculate \(B(\varphi^{ml}, A^i)\) as

\[
B(\varphi^{ml}, A^i) = - \frac{1}{2} C_j(\varphi^{ml}, A^i) \wedge F^j. \tag{2.18}
\]

Thus the Lagrangian (2.16) becomes

\[
\mathcal{L}_{tot} = \frac{1}{2} tr(\ast dg^{-1} \wedge dg) + d(- \frac{1}{2} C_j(\varphi^{ml}, A^i) \wedge F^j). \tag{2.19}
\]

If one varies the above Lagrangian one immediately sees that the second term does not contribute to the field equations as by using the Stoke’s theorem we have

\[
\int_M d\delta B(\varphi^{ml}, A^i) = \int_{\partial M} \delta B(\varphi^{ml}, A^i). \tag{2.20}
\]

If \(M\) does not posses a boundary the right hand side of the above equation is automatically zero whereas if it has a boundary then the usual variation
principles demand that the variation of the fields on the boundary are chosen to be zero in which case again the right hand side of (2.20) becomes zero. Therefore we conclude that the on-shell Lagrangian (2.16) which is responsible for the most general structure of the solution space gives us the scalar field equations which are the same with the pure sigma model field equations since the coupling part in (2.16) does not contribute to the scalar field equations at all as we have proven as an on-shell condition above. Also like we have already encountered for the non-field-dependent \( \{C_k\} \) sub-sector case varying (2.16) does not give any information (which may also mean an identity) about the gauge fields \( A^i \) since we have already made use of their field equations in writing it.

In summary, we have shown that the solutions of the theory must obey (2.13) for arbitrary right hand sides. If one determines the right hand sides in (2.13) one restricts him or herself to a layer of solutions. In this case (2.13) which are the descendants of the gauge field equations become on-shell conditions. We have proven that if we use these first-order field equations which are on-shell conditions back in the general Lagrangian (2.9) then the scalars of this particular layer of solutions do not have gauge fields in their field equations. Therefore the scalars of this particular layer belong to the general solutions of the pure principal sigma model. By running the right hand sides in (2.13) over the entire local field-dependent exact \( (D-2) \)-forms one can generalize this result to the whole solution space. Thus in this way we have proven that the entire scalar solutions of the coupled theory coincide with the pure sigma model solution space. Now from (2.13) we can write down the gauge field strengths as

\[
F_l = (-1)^s (T^{-1})^k_l \ast dC_k(\varphi^{mn}, A^i),
\]

where \( s \) is the signature of the spacetime. We observe that if we consider the sector of the solution space generated by the field-independent \( \{C_k\} \) then after obtaining the general solutions of the pure principal sigma model and after choosing a fixed set \( \{C_k\} \) one can use these in (2.21) to find the corresponding \( U(1) \) gauge field strengths. Also for the more general field-dependent case of \( \{C_k(\varphi^{ml}, A^i)\} \) one again obtains the general solutions of the pure principal sigma model then one inserts these solutions in (2.21) to solve for the gauge fields. We can say that in general we have a partial decoupling between the scalars and the gauge fields. The scalars are not affected by the presence of the gauge fields on the other hand as we will show next they act as sources for the gauge fields. Now after multiplying by the
Hodge star operator if we take the exterior derivative of both sides of (2.21) we obtain

\[ d \ast F_i = (d T^{-1})^k_l \wedge d C_k(\varphi^{mn}, A^i). \tag{2.22} \]

When we compare this result with the inhomogeneous Maxwell equations (2.3) we observe that the current one-forms become

\[ J_l = (-1)^{(D+s)} \ast ((d T^{-1})^k_l \wedge d C_k(\varphi^{mn}, A^i)). \tag{2.23} \]

One can furthermore verify that the currents in (2.23) obey the current conservation law

\[ d \ast J_l = 0, \tag{2.24} \]

which guarantees that the equations (2.22) have solutions. In a local moving co-frame field \( \{e^\alpha\} \) on the spacetime \( M \) if we introduce the components of the one-forms \((d T^{-1})^k_l\), the \((D-2)\)-forms \(d C_k(\varphi^{ml}, A^i)\), and the one-forms \(J_l\) as

\[ \begin{align*}
(d T^{-1})^k_l &= (T^{-1})^k_l e^\alpha, \\
(d C_k(\varphi^{ml}, A^i)) &= \frac{1}{(D-2)!} (C_k)_{\alpha_1 \cdots \alpha_{(D-2)}} e^{\alpha_1 \cdots \alpha_{(D-2)}}, \\
J_l &= J_{l\beta} e^\beta,
\end{align*} \tag{2.25} \]

from (2.23) we can calculate the components of the current one-forms as

\[ J_{l\beta} = \frac{(-1)^s \sqrt{|\text{det} H|}}{(D-2)!} (T^{-1})^k_l (C_k)_{\alpha_1 \cdots \alpha_{(D-2)}} H^{\alpha_1 \beta_1} \cdots H^{\alpha_{(D-2)} \beta_{(D-2)}} \varepsilon_{\beta_1 \cdots \beta_{(D-2)}}, \tag{2.26} \]

where we have introduced the metric \( H \) on \( M \) and the Levi-Civita symbol \( \varepsilon \). In (2.26) \( \alpha, \beta, \alpha_i, \beta_j = 1, \cdots, D \). Also for \( \alpha_i, i = 1, \cdots, D-2 \) and for \( \beta_j, j = 1, \cdots, D-1 \).

If one cancels an exterior derivative (performs integration) on both sides of (2.22) one finds

\[ \ast F_i = (T^{-1})^k_l d C_k(\varphi^{mn}, A^i). \tag{2.27} \]

Now if we compare this with the first-order equations (2.13) originating from (2.9) we see that our model is a sub-sector of the one defined in (2.27) with
the choice of \( dC_l'(\varphi^{mn}, A^i) = 0 \). One may also inspect which scalar-gauge field coupling kinetic term would result in a first-order formulation of gauge fields in the form (2.27).

We should finally state that by using the local first-order formulation (2.13) of the gauge field equations of (2.9) we have shown that there exists a decoupling between the \( U(1) \) gauge fields and the scalar fields of the theory. The scalars are proven to be the general solutions of the pure principal sigma model and they generate current sources for the gauge fields as can be explicitly seen in (2.22). For the sub-sector of the solution space which is generated by the field-independent \( \{ C_k \} \) we end up with a decoupled set of \( N \) non-interacting Maxwell theories with prescribed and known currents whose sources are predetermined by the principal sigma model scalar fields. These currents interact with each other via the sigma model which is completely decoupled from the Maxwell sector and they define a medium which does not interact with the Maxwell fields. For this sub-sector the \( N \) decoupled and non-interacting Maxwell theories can alternatively be formulated by the Lagrangian

\[
\mathcal{L}' = \sum_{i=1}^{N} \left( -\frac{1}{2} dA^i \wedge * dA^i + A^i \wedge (dT^{-1})^k_i \wedge dC_k \right). \tag{2.28}
\]

We see that for a single scalar field the above Lagrangian drops to be the ordinary Maxwell Lagrangian with a known current one-form. We also realize that the Maxwell fields in this restricted case are coupled to each other only by means of integration constants. Each of these decoupled Maxwell theories is an embedding into the ordinary Maxwell theory with known sources. We should remark that (2.28) must only be used to derive the field equations of the gauge fields which belong to a restricted sector of the solution space generated by fixing \( \{ C_k \} \) and in this case the field equations of the scalars must again be derived from (2.21). On the other hand for the most general solution space elements generated by choosing field-dependent \( \{ C_k(\varphi^{nl}, A^i) \} \) we can again adopt the general solutions of the pure sigma model as the general scalar solutions of our theory however in this case we have \( N \) coupled Maxwell theories whose potentials may enter in the currents. For the sub-sector generated by \( \{ C_k(\varphi^{ml}) \} \) which are not dependent on the gauge fields we have a similar situation of decoupled Maxwell theories with prescribed currents discussed above.

\(^1\)By integrating the field equations to first-order and by choosing \( dC_l' = 0 \).
Finally before concluding we should summarize the local general solution methodology of the principal sigma model and \( U(1) \) gauge fields coupling defined in (2.9). The method has three steps first find the general scalar field solutions of the pure principal sigma model, secondly define field-dependent or field-independent \( \{ C_k \} \) and use the general scalar solutions in them then insert these in (2.21) to solve the corresponding gauge field strengths. By combining the general pure principal sigma model solutions with a different set of \( \{ C_k \} \) each time one can generate the entire set of solutions. Such a solution methodology of the general solution space of (2.9) is a consequence of the first-order formulation of the gauge field equations in (2.13) and the on-shell decoupling between the scalars and the gauge fields which we have described in detail for both the restricted sector of the solution space and for the entire solution space in its most general generation process. From the scalars point of view there is a complete decoupling so that the scalars of the coupled theory coincide with the pure sigma model solution space. However the gauge fields are not decoupled from the scalars since we have shown that the scalars act in the current part of the gauge field equations.

3 Conclusion

By integrating the gauge field equations of the principal sigma model and Abelian gauge field coupling Lagrangian we have expressed these equations in a first-order form. Then we have shown that when these first-order and on-shell expressions which contain local \((D - 3)\)-forms in them are used in the scalar-matter Lagrangian one reveals an implicit decoupling between the scalars and the matter gauge fields. We have proven that this decoupling structure exists for the entire solution space. Therefore it is shown that the scalar solutions of the coupled theory are the general solutions of the pure principal sigma model. We should state here that a similar result is derived in [16] for the heterotic string. However in that work the decoupling occurs due to the existence of a dilatonic field and one derives the decoupling structure of the coset scalars from the field equation of the dilaton. On the other hand in the present work we prove that such a decoupling exists for a generic principal sigma model with an arbitrary number of Abelian gauge field couplings.

We have also mentioned that the on-shell hidden scalar-matter decoupling mentioned above generates a theory composed of the pure sigma model and a
number of coupled Maxwell theories with sources induced by the scalars. In particular we have discriminated a sub-sector of the general theory generated by the field-independent integration constants. This sub-sector contains a number of separate and dynamically non-interacting Maxwell theories whose current sources are drawn from the general scalar solutions of the pure principal sigma model and the integration constants of the first-order formulation.

In this work, we prove that the scalars of the coupled theory are not affected by the presence of the $U(1)$ gauge fields however they take role in the currents of the gauge fields. Thus one may call such a coupling between the scalars and the gauge fields a one-sided or a one and a half coupling. We also see that for the restricted sub-sector of the theory which is obtained by fixing the integration constants the currents of the electromagnetic fields interact with in each other via the sigma model but they do not interact with the corresponding gauge fields. Thus for this sub-sector the currents form up a subset of the general currents of the prescribed current-Maxwell theory. In this sub-sector the scalar-gauge field decoupling induces another decoupling scheme also among the gauge fields and one obtains $N$ non-interacting Maxwell theories whose sources come from the pure principal sigma model. For each $U(1)$ gauge field we have an embedded sector of the general predetermined current-Maxwell theory which has the most general form of currents. In our case these currents are restricted to the solutions of the pure principal sigma model. The $N U(1)$ gauge fields in this case probe each other only by the presence of the integration constants however this does not correspond to a dynamic coupling. The coupling among the gauge fields occur only at the level of determining the integration constants from the boundary conditions. Since we have a static and unchanged current structure which emerges from the pure principal sigma model where the currents interact with each other and since these currents are not affected by the presence of the gauge fields this sub-sector of the general theory defines a theory of a static media with $N$ non-dynamically interacting Maxwell fields. One can inspect the other sectors of the general theory based on various choice of field dependent $\{C_k\}$ which define dynamic media with current-gauge field interactions. For example one can study the sectors that result in gauge field equations that define dynamic media with specially defined conducting properties [17]. The analysis of this work can furthermore be extended on another scalar-gauge field coupling which might have broader dynamic media sub-sectors namely on the gauged sigma model.

In conclusion, we may state that the decoupling structure studied in this
work is an important remark on the solutions of the scalar-gauge field interactions which form up a basic sector of the Bosonic dynamics of the supergravities and the effective string theories. Therefore the scalar-gauge field decoupling revealed here points out a simplification in seeking solutions to these theories.

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