Non-Axiomatic Term Logic: 
A Computational Theory of Cognitive Symbolic Reasoning

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Abstract

This paper presents Non-Axiomatic Term Logic (NATL) as a theoretical computational framework of humanlike symbolic reasoning in artificial intelligence. NATL unites a discrete syntactic system inspired from Aristotle’s term logic and a continuous semantic system based on the modern idea of distributed representations, or embeddings. This paper positions the proposed approach in the phylogeny and the literature of logic, and explains the framework. As it is yet no more than a theory and it requires much further elaboration to implement it, no quantitative evaluation is presented. Instead, qualitative analyses of arguments using NATL, some applications to possible cognitive science/robotics-related research, and remaining issues towards a machinery implementation are discussed.

Keywords: argumentation, creativity, neuro-symbolic reasoning, artificial general intelligence

1. Introduction

In his book on argument, Toulmin (1958) states that there are at least four positions in the way of thinking about logic. That is,

1. Psychological position: Logic is the laws of thought within individual minds.
2. Sociological position: Logic is the habits and practices of inference developed in the course of social evolution and passed on by parents and teachers from one generation to another.
3. Technological position: Logic is a technology, the rules of a craft, to argue soundly.
4. Mathematical position: Logic is about the validity of the form of argument rather than the content of argument.

From a phylogenetic point of view of logic along human history, we would say that there were the stages of (1) and (2) in prehistory, followed by the stage of (3) (traditional logic), and then the stage of (4) in modern times (mathematical logic). Therefore, the four positions of (1) to (4) are not independent and contradictory viewpoints, but have a relationship. In fact, until now, it is well known that propositional logic and predicate logic developed from the point of (4) have been used for many subjects of artificial intelligence related to (1) to (3) (for example, (Litman and Allen, 1987)). However, according to Wittgenstein, “In constructing symbolic logic, Frege, Peano and Russell always had their eye on its application to mathematics alone, and they never gave any thought on the representation of real states of affairs.” (Janik and Toulmin, 1973)

This paper proposes a kind of symbolic logic and a framework of reasoning based on it for artificial intelligence. This study temporarily abandons the position of (4) and returns to (1) and (2). In short, this paper is neither about predicate nor propositional logic. Therefore, the proposed framework does not belong to the category of mathematical logic. However, since it is a framework for artificial intelligence, it is necessary to be formalized to the extent that it can be implemented on a computer. Thus, it is formal logic. Artificial intelligence realized by this framework will eventually acquire (socially learn) the logic belonging to (3) and (4) as a culture, just as elementary school students learn mathematics. In the process leading up

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1. This manuscript is an author-translation (to be proofread) of the original paper in Japanese, which has been submitted to “Transactions of the Japanese Society for Artificial Intelligence.”
2. Intuitively, the stage of (1) seems to precede the stage of (2), but if the view that society (culture) evolved human beings (Henrich, 2016) is correct, (1) and (2) might be interwoven, and it is better to say that (2) may precede (1).
3. Since it abandons the position of (4), it cannot directly take the place of the contribution that mathematical logic has made to mathematics.
to that stage, the main target of this research is “rules that are not always correct but have a certain degree of usefulness in specific situations,” as exemplified by (Ichisugi et al., 2020).

The framework of Non-Axiomatic Term Logic (NAL) proposed in this paper is inspired by Non-Axiomatic Logic (NAL) (Wang, 1994, 2013). Term logic goes back to the syllogism of Aristotle. It does not have discrimination between predicates and individuals, and expresses a sentence by connecting two terms by a copula. It is included in the category of traditional logic, which is virtually eliminated by predicate logic invented by Frege in modern times.

Nevertheless, a small number of researchers who are attracted by the simplicity and ease of intuitive understanding of term logic occasionally proposed original logic frameworks and reasoning methods to replace predicate logic (Sommers, 1982; Morita, Nishihara, and Emura, 1987; Nishihara, Morita, and Saito, 1994; Wang, 1994; Goertzel et al., 2008; Moss, 2010). The starting point of this research is the assumption that term logic is useful for modeling the aforementioned daily reasoning of human beings by fusing with recent information processing technologies based on deep learning. However, this research is different from the above-mentioned previous studies, and the proposal in this paper is not an extension of those studies.

Being based on term logic as a discrete system, which seems to have a high affinity with human thinking, as the skeleton, and being based on semantic representation as a continuous system in a multidimensional space implicitly and empirically constructed from data, as the flesh and blood, we aim to realize an AI system that performs human-like daily reasoning, creative symbol processing, and cultural learning through symbols.

Such an AI system, if we only consider its performance on a specific task, will probably be inferior to a task-specifically trained system based on deep learning alone and with large amounts of data. Especially in tasks that can be defined independently of the presence of humans, the difference is expected to become more pronounced. However, it may establish its own usefulness in situations that require similarity and affinity with human thought, such as interpretation of a story or conversation; in situations where the process and basis of inference must be presented to others; in situations where human errors must be predicted; and in situations where a very small number of ad-hoc knowledge and rules given symbolically must be operated on the spot.

The structure of this paper is as follows. First, §2 is used to sort out the relationship between term logic and predicate logic, and sort out then the positions and plans of each of the studies so far that have tried to develop term logic. §3 discusses the outline of Wang’s Non-Axiomatic Logic and its issues. Next, in §4 and §5, we present our Non-Axiomatic Term Logic and its knowledge representation language, based on the issues presented in §3. To show the usefulness of Non-Axiomatic Term Logic partially, the qualitative case analysis of arguments using Non-Axiomatic Term Logic is shown in §6. To demonstrate the promise of the concept, the possible development of research in connection with cognitive science and robotics will be discussed in §7. Finally, in §8, we discuss the prospects and future issues of computational implementation, and in §9, we summarize this paper.

The main contributions of this paper are two-fold:

1. Based on the premise of integrating a discrete syntactic system and a continuous semantic system, we propose “Term Representation Language” (TRL) that has higher expressive power than conventional term logic. As “Non-Axiomatic Term Logic” (NAL), we formally define five types of reasoning that can be performed on three classes of knowledge represented by using TRL.

2. By applying NATL to the three arguments collected from related literature and analyzing them, we have provided a qualitative proof of the formal descriptive power and explanatory power of NATL.

2. Term logic, Predicate logic, and Non-Axiomatic Logic

Consider sentences (propositions) that have one of the following four patterns $A, I, E, O$:

$A$: all $X$ are $Y$.

4. “There was chocolate in the cupboard yesterday, then, there should be chocolate today as well.”
Roughly speaking, the syllogism that goes back to Aristotle distinguishes between valid and invalid inferences of the type that concludes one proposition from two propositions. Here, the symbols $X$ and $Y$ that appear in the propositions are called terms (names) and represent specific categories and properties such as “human”, “animal”, and “immortality”. For example, from the two propositions $A_1$: “All humans are mammals” and $A_2$: “All mammals are animals”, we conclude $A_3$: “All humans are animals”.

Predicate logic is also a tool that makes it possible to formally describe the internal structure of propositions and the different content between propositions. For example, the proposition “All humans are mammals”,

$$(\forall x)(\text{Human}(x) \Rightarrow \text{Mammal}(x))$$

can be described using the predicates $\text{Human}$ and $\text{Mammal}$. In predicate logic, “existences” (or individuals) which are specified based on set theory, “predicates” which can define various relations by taking any number of variables, and “quantifiers” ($\forall$ and $\exists$) which allow a more flexible description of scope, make it possible to strictly express various propositions, especially mathematical definitions and theorems that require multiple quantification, without natural language, and not be confined to the four sentence patterns between two terms that traditional logic has analyzed.

The price is that when expressing a proposition with predicate logic, a transformation that is not always intuitive is required. In predicate logic, we must always go back to individual beings and describe the proposition as the relationship between those beings. In the previous example, when describing the relationship between the concepts of human and mammal, we need a translation such as “For all $x$, if it is a human, it is a mammal.”

Sommers (Sommers, 1982; Sommers and Englebretsen, 2000) dislikes such unnaturalness and proposes a framework that can formally express logical relationships in natural language more naturally without using individual variables in predicate logic. Independently of this, Morita, Nishihara, and Emura (1987), emphasized the fact that predicate logic is extensional and term logic is intensional, and aimed to construct a logical system for knowledge representation in which intensional descriptions are directly possible. Nishihara, Morita, and Saito (1994) introduced the handling of verbs into this to enhance its expressiveness. Moss (2010) is trying some improvements to Nishihara et al.’s proposal.

The main interest of these term logic researchers seems to be in the semantic representation of natural language. On the other hand, they are also interested in what they present has soundness and completeness as a logical system. In other words, while dealing with natural language, they take the position of axiomism.

On the contrary, Wang (1994, 2013) started from the consideration of what intelligence is, acknowledged the usefulness of mathematical logic, and insisted on the importance of non-axiomism for reasoning as a general-purpose intelligence. In other words, the axiomatic approach, which assumes the existence of complete knowledge and resources, cannot serve as a model of intelligence. From this perspective, he focuses on the usefulness of the properties of term-logic for knowledge representation in realizing useful inferences.

Non-Axiomatic Logic (referred to as NAL) proposed by Wang is designed with the realization of a general-purpose reasoning system in mind and does not focus on semantic representation of natural language.

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5. This explanation is more traditionally logical than Aristotle’s own. See (Łukasiewicz, 1951; Englebretsen, 1996) etc. for the difference between Aristotle’s syllogism and traditional logic.

6. Morita et al. point out that Montague semantics (Thomason, 1974) became very complicated because it introduced the concepts of implication and higher orders on top of the predicate logic system, and Munemiya (1996) points out that Montague semantics is not capable of handling “concepts” (i.e., powerless for statements using gerunds and infinitives) in the first place because of its extensionalism based on predicate logic.

7. Wang defines intelligence as “the ability for a system to adapt to its environment and to work with insufficient knowledge and resources”.

8. NAL itself is described in mathematical logic.

9. Wang (2013) argues that, just as the laws and principles of the world are different in physics and biology, computer science and (general-purpose) artificial intelligence are also based on their different laws and principles.
sentences as Sommers, Morita, Nishihara, and others have done. In NAL, inference rules are given, but axioms (absolute propositions) do not exist. Even if reasoning is based on the same finite knowledge source, the results obtained can vary and can be updated due to the limited resource of time. In addition, “weak inference rules” other than deduction are provided, and the possibility of drawing invalid conclusions is allowed from the beginning. It does not follow propositional logic either. The entailment relation between propositions in NAL \((P \Rightarrow Q)\) is defined as the ability to derive the proposition \(Q\) from the proposition \(P\) (that is, \(P \vdash Q\)), and not equivalent to \(\neg P \lor Q\). The concept of “truth” in NAL is what is reasonable in view of past experience. Therefore, meaningless propositions such as “If a man is not a man, then I am a god” are true in propositional logic, but not in NAL.

3. Non-Axiomatic Logic

This section explains the outline of Non-Axiomatic Logic (NAL) (Wang, 2013) from the aspects of syntax, semantics, and inference rules, and then discusses the issues of NAL.

3.1 Syntax

A proposition “\(S\) is \(P\)” is expressed by joining the terms \(S\) and \(P\) with the copula \(\rightarrow\) in the form \(S \rightarrow P\). This form of proposition is called a statement. Based on this statement syntax, many extensions have been made to enhance the expressiveness of the NAL language (Narses). Only those that are relevant to the discussion in this paper are explained below.

As mentioned above, when statement \(P\) implies statement \(Q\), it is written as \(P \Rightarrow Q\). \(P \Rightarrow Q\) is also a (higher-order) statement, and the statement is also a term.

When expressing the relationship of three or more terms, it is expressed like \(U \times V \rightarrow R\). For example, “water resolves salt” is expressed as \(\text{water} \times \text{salt} \rightarrow \text{resolve}\). \(U \times V\) is treated as one term (product term) and \(R\) is called a relational term.

3.2 Semantics (experience-grounded semantics)

Based on the idea that copula \(\rightarrow\) expresses the inheritance relationship (hyponymy) between \(S\) and \(P\), the semantics (true or false) of a statement is defined by a set of extensions (lower terms) and intensions (upper terms) of a term. That is, given that \(S \rightarrow P\) is always true, and the extension set \(T^E\) of a term \(T\) is \(T^E = \{x| (x \in V_K) \land (x \rightarrow T)\}\) and intension set \(T^I\) is \(T^I = \{x| (x \in V_K) \land (T \rightarrow x)\}\), then the truth of \(S \rightarrow P\) is said to be equivalent to whether \(S^E \subseteq P^E\) and \(P^I \subseteq S^I\). In other words, it is true if \(|S^E - P^E| + |P^I - S^I| = 0\). In the sense that each statement is an observation (experience), this is called experience-grounded semantics.

However, the above definition of truth and falsehood can only work in an ideal situation (a situation in which there is no uncertainty) called inheritance logic by Wang. Since the premise of NAL is uncertainty, the above equivalence relation is generalized to incorporate uncertainty. However, the details of this generalization are not related to this paper directly, so the explanation is omitted.

3.3 Inference rules

The main reasoning in NAL is based on syllogistic rules that lead to one conclusion from two premises, but there are also rules that take only one statement, such as negation and conversion, and rules for structural transformations between product terms and relation terms. Among the syllogistic rules in NAL, only three

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10. By the influence rule of conversion, the \(P \rightarrow S\) can be inferred from \(S \rightarrow P\) in NAL. It still remain doubt that whether \(\rightarrow\) can represents inheritance in \(P \rightarrow S\). In traditional logic, it is reasonable to apply conversion for type \(I, E\) of statements whereas it is not suitable for other two types of statement.
rules, deduction, induction, and abduction, are taken up here. That is,

- **deduction**: \( \{ S \rightarrow M, M \rightarrow P \} \vdash S \rightarrow P \),
- **induction**: \( \{ S \rightarrow M, S \rightarrow P \} \vdash M \rightarrow P \),
- **abduction**: \( \{ M \rightarrow P, S \rightarrow P \} \vdash S \rightarrow M \).

Induction and abduction are forms in which the premises and conclusions of deduction are interchanged. For each rule, Wang presents a truth-value function that gives the truth-value of the conclusion statement from the truth-values of the two premise statements.

Induction is a generalization of cases. Consider an example from Task 16 Basic Induction of bAbi tasks (Weston, 2015): A: Lily is a swan; B: Lily is white; C: Greg is a swan. For these three input sentences, “white” should be the answer for the question “What color is Greg?” The inference required from the sentences A and B is “a swan is white.” With the correspondences of \( S \): Lily, \( M \): a swan, \( P \): white, this is the induction of the knowledge \( M \rightarrow P \). And if we deduce “Greg is white” from sentence C and the last inferred sentence, we get the expected answer.

Abduction is another form of reasoning that Peirce found for “obtaining knowledge from experience,” and its essence is the logic of interpretation (explanation) (Arima, 2014). Although it can easily lead to erroneous inference results, it contributes to the flexibility and creativity of human thinking. An example of abduction shown by Peirce is the following (Arima, 2014):

- **Rule**: All the beans from this bag are white.
- **Result**: The beans here are white.
- **Hypothesis**: The beans here are from this bag.

With the correspondences of \( M \): beans from this bag, \( P \): white, \( S \): beans here, this case is exactly in the form of \( \{ M \Rightarrow P, S \Rightarrow P \} \vdash S \Rightarrow M \).

These three rules hold even if the inheritance copula \( \rightarrow \) is replaced with the implication copula \( \Rightarrow \). The explanation of abduction is often given as an example of causal (implication) relationships between situations. For example, with the causal knowledge that “a tree sways when the wind blows”, the result “a tree sways” is observed and the cause “the wind blows” is inferred. At first glance, the inference form of \( \{ M \Rightarrow P, S \Rightarrow P \} \vdash S \Rightarrow M \) does not seem to apply well to this example, but when the \( S \Rightarrow \) part is contextualized, it becomes \( \{ M \Rightarrow P, P \Rightarrow \} \vdash M \). It can be seen that the cause is inferred (interpreted) from the observed results in a given context.

### 3.4 Issues of Non-Axiomatic Logic

Non-Axiomatic Logic presented by Wang is intuitively concise and easy to describe intensional knowledge (the judgment of “what is an individual entity and what is a predicate” required when applying predicate logic is not so obvious), and it has a framework for non-deductive reasoning such as abduction. It has features that are considered desirable for modeling the purpose of this research, “inferences that humans make on a daily basis based on rules that are not always correct but have certain usefulness in specific situations”. Also, unlike natural logic (van Benthem, 2008; MacCartney and Manning, 2009), it is a general model of symbolic reasoning and not just for natural language. Natural logic aims to make inferences by using a surface form of natural language directly.

On the other hand, we can point out some issues inherent in NAL. This section presents these issues and approaches to them.

#### 3.4.1 Assignment of Latent Space Semantic Representations to Terms

The reasoning process of NAL can be seen as the process of substituting or replacing one term with another ((Wang, 2013) p.22). This substitution process in NAL is guided by the superficial match of terms or symbols.
and knowledge of similarity between symbols ($S \leftrightarrow T$). Although it can ensure accuracy, there is a problem in obtaining practical robustness, which is a common topic of symbol processing AI in the past. It is also not easy to properly design a function that evaluates the similarity between symbols by fully considering the characteristics of the problem domain.

In recent years, the semantic relationship/similarity between symbols has been investigated by “embedding” as the positional relationship in the space, which maps a set of symbols to points in a multidimensional continuous space (latent space) under specific constraints. By embedding, it has become possible to perform natural language processing and knowledge graph processing tasks with high accuracy and robustness (for example, (Mikolov et al., 2013; Bordes et al., 2013)). For NAL, it is expected to increase the accuracy and robustness of inference by adding embedded representations to each term. There are also theorem proving researches based on the same idea (for example, (Arabshahi et al., 2021)).

3.4.2 Training of reasoners based on reinforcement learning

The algorithm of the reasoning system NARS proposed by Wang based on NAL is very naive, and it is difficult to eliminate useless inference and obtain significant inference results within a valid time. Inference in NAL is an iterative process of selecting statements to be processed and rules to be applied, which is a kind of tacit intellectual skill, and training the reasoner by reinforcement learning seems to be effective. The above-mentioned latent space semantic representations classified by terms and those aggregated by graph convolution might be valuable to represent as state space in reinforcement learning.

3.4.3 The continuity and prototype hypothesis of copula

In NAL, in addition to the inherited copula and implied copula, eight types of copula were introduced mainly for the requirement of expressing ability. From an engineering point of view, there is a concern that the system has become complicated, while from a scientific point of view, there is a suspicion that this alone is sufficient.

For example, $S \rightarrow P$ means $S$ is an individual case (that has a proper noun) of $P$, that is, $S$ has no sub-terms (for example, Tweety $\rightarrow$ bird). However, in the case of nonliving things, it is often natural to think that an individual with a name has individual subspecies under it (for example, a particular piece of music has many arrangements, and they also have individual performances, and further there is a compilation of recordings of the performance, etc.). While Wang’s explanation that $\rightarrow$ is necessary is understandable, the request for a fuzzy distinction that defines a difference in static and definite quality on both sides after the line is drawn, seems to impair the succinctness of term logic by introducing something similar to the distinction between predicate and existence in predicate logic, which Wang criticized.

Meanwhile, when emphasizing inference ability based on empirical meaning, it is not obvious whether the conditional inclusive relations such as “If one likes pasta, the one likes spaghetti” and the causal relationship like “If the wind blows, the tree shakes” can be treated collectively with the same implicational copula. There are other questions about copulas. In the above-mentioned conversion rule, the copula of $P \rightarrow S$ converted from $S \rightarrow P$ may have better be called the associative, representative, or heterogeneous copula, rather than the inheritance copula. In (Wang, 2013), the inheritance copula is explained (implicitly) like the $A$-type copula in traditional logic, while $I, E, O$-type copulas are not discussed, but $I, E, O$-type copulas would be also necessary. In the first place, in human daily reasoning, there might be a continuous and diverse quantification recognition between “all” and “some”, such as “most”, “a lot”, “a little”, and so on.

Taken together, it seems better to think of copula as a family/prototype category (Wittgenstein, 2009; Taylor, 2012) in a continuous space. The approach is that the copulas given in a prior manner are kept minimally and other nuances (for example, $\circ \rightarrow$ ) are learned empirically, that is, copulas as described in §3.4.1 take continuous representations, which are learned empirically, and the differences of nuances of copulas according to the situations are handed over to the reasoner (the distinction may be made in some situations and not in others). However, a major challenge is how to design the truth functions with regard to copulas. It is no longer feasible to design the truth functions in a top-down, detailed manner by hand.
3.4.4 Separation and Coordination of Reasoning and Language Processing

It may be desirable from the point of view of proof of principle and versatility of logic/reasoning systems to represent the meaning of natural language sentences analyzed by morpheme units in a logical language, and to be able to make reasonable inferences on that basis. However, this is not always necessary. Pierce’s example shown in §3.3 demonstrates that it is possible to make inferences without decomposing the sentence structure until each word becomes an individual term. What is necessary is recognition of similarities and relationships between terms that express meanings and concepts with appropriate granularity according to the situation. In the past, the only way to do this was to unify formal symbols while decomposing sentences at the smallest granularity, but this is no longer the case.

It seems that humans can make inferences such as those shown in Peirce’s example above, even if they do not necessarily express the situation verbally. Nor does it appear that symbolic reasoning is performed each time for comprehension and response in our daily language use, which seems greatly reflexive and automatic. Looking at the development of children, we can see that humans do not become able to speak after being able to think logically, nor are they able to think logically just because they can speak fluently.

On the other hand, human beings are also able to make more reasonable inferences by logically thinking about the meaning of expressions and verbalizing complicated situations based on the needs at necessary granularity. A reasonable approach would be to regard the human’s “reasoning system” and “language system” as separate and mutually dependent. Each system individually works effectively to a certain extent, but mutual utilization and coordination are the key to being more efficient and effective, i.e., intelligent.

Recent deep learning technology seems to have already modeled a large part of the above-mentioned reflexive and automatic language ability at a level that can sufficiently approximate humans. The approach we should take in this research is to consider this as a “language system” and to consider the realization of the remaining “reasoning system”.

It may not be impossible to express grammatical structures finely with NAL product terms and relational terms, but it would require the introduction of complex mechanisms. If the processing of grammatical elements can be left to the “language system”, the construction of the “reasoning system” can be kept simple.

The effectiveness of a similar idea has already been demonstrated by Nye et al. (2021), but in a limited problem setting. As we will see later, the concept of this research is to expand this to various symbolic thinking of human beings such as hypothesis inference, argumentation, metaphor/analogy, heuristic problem solving, and so forth, and try to handle them uniformly on the formal framework of Non-Axiomatic Term Logic.

4. Term Representation Language

In response to the issues in NAL presented in the previous section, we present Non-Axiomatic Term Logic proposed in this paper in this and the next sections. In this section, we first define Term Representation Language used by Non-Axiomatic Term Logic.

Term Representation Language (hereafter, TRL) is a description model of human cognitive internal representations that use symbols to represent recognition and knowledge. Non-Axiomatic Term Logic, shown in the next section, expresses the process of reasoning using knowledge representations explicitly structured by TRL. Here we present the syntax of TRL.

Nye et al. (2021) uses a system that returns intuitive verbal responses (so-called System 1 (Kahneman, 2011)) and a system that logically filters the results (System 2) to show that it is possible to generate linguistically natural and consistent stories. System 1 uses GPT-3 (Brown et al., 2020). When GPT-3 is given the following question that is used to test the functioning of human System 1 and System 2, “A ball and a bat cost $1.10. The bat costs one dollar more than the ball. How much does the ball cost?”, it returns the same “wrong answer” (10 cents) as many humans do in a time limit (the correct answer is 5 cents) (Nye et al., 2021). Cognitive linguists such as Taylor consider linguistic knowledge to be “knowledge about the appropriate use of language”, and “knowing the meaning of a word” is equal to “knowing the usage of the word” in the particular language (Taylor, 2012). In this sense, this test confirmed that GPT-3 could understand “the meaning of words” exactly as well as humans do. The present paper also takes the standpoint of cognitive linguistics with respect to language. The “reflective and automatic language ability” mentioned in this paper refers to such kind of linguistic ability. Meanwhile, GPT-3 has failed this test. It is because this test is a problem that cannot be solved by language usage alone. The focus of our research is also on that part.
Roughly speaking, TRL is based on the notation of statements by Narses, which is the representation language of NAL (Wang, 2013), and it introduces a proposition notation that takes an arbitrary number of terms, similar to predicate logic.

4.1 Term

A term represents an immediate perception of an experience (for example, one apple in front of you, or one apple falling from a tree), an episodic memory of that experience, a semantic memory extracted and abstracted from an accumulation of episodic memories (they can be called concepts, such as “apple” and “falling” in the general sense), which emerges in consciousness as a unity through human psychological processing. The symbols $T_1, T_2, T_3, \ldots$ are used to denote arbitrary terms.

When assuming a specific concept associated with a certain English word as the referent of a term, italicized English words such as *sheep*, *human* are used to improve readability. To distinguish the objects whose concepts (types) are instantiated as tokens, use terms like $human_1, human_2$.

4.2 Basic term and composed term

A term treated as the minimum unit of recognition is called a basic term. For example, *sheep*, *human*, $human_1$, $human_2$ mentioned above are basic terms. Besides, whether or not a concept can be expressed in one word by natural language has nothing to do with the singularity of the concept (Hofstadter and Sander, 2013). Even if a concept or an object is expressed by multiple words, they can be hyphenated like *polar-bear*, *getting-wet* and used as a single basic term.

Symbols $B_1, B_2, \ldots$ are generally used to denote basic terms. The compound terms, statement terms, and linkage terms to be described below are collectively called composed terms. A basic term is a typical unit that constitutes compound terms and statement terms.

4.3 Compound term

A composed term that expresses a relationship between multiple terms or a secondary recognition of a single term is called a compound term. The elements (element terms) that make up a compound term are not limited to basic terms, but can be all kinds of terms. Let $R$ denote the term representing these concepts of recognition and relationship, and let $C$ denote the compound term $C: (R, T_1, T_2, \ldots, T_n)$. Here $T_1, T_2, \ldots$ are the element terms of $C$. The number of elements in the compound term, $n$, depends on $R$. In NAL, a three-term relation is expressed as a statement $U \times V \rightarrow R$ using the product and relational terms (see §3.1), but in TRL, it is expressed as a compound term $C: (R, U, V)$.

A “situation” expressed with a general verb in a natural language can be expressed as compound terms like $(rotate, Earth)$, $(eat, sheep, grass)$ in the same way as predicate argument structures. The conjunctions and disjunctions can be expressed as $(and, T_1, T_2, \ldots)$. Aggregates with specific relationships, such as couples, can also be expressed as $(couple, husband_1, wife_2)$.

4.4 Statement term

A statement term $S$ is a composed term of two arbitrary terms connected by a copula. That is, $S: (T_1 \rightarrow T_2)$, where $T_1$ is called the left term of $S$ and $T_2$ the right term. A statement term corresponds roughly to a statement in NAL and represents the categorization of a term. The most basic copula expresses an is-a relationship, such as “Socrates is a man”. However, there are several variations of the is-a relation, that is, the negation of is-a (for example, “Socrates is not a woman”), equivalence ($T_1 = T_2$), the four sentence patterns of A/I/E/O in traditional logic, and so on. All these types are the derivatives from the is-a relationship. TRL does not specify how many types of copulas exist.

4.5 Linkage term
### Term Representation Language

| Term | Non-Axiomatic Term Logic |
|------|--------------------------|
| Basic term | \(C\) Thing term |
| Composed term | \(S\) Logic term |
| Statement term | \(S\) Linkage term |
| Linkage term | \(L\) |
| Variable term | |

Table 1: Term Types and Correspondences between TRL and NATL

A linkage term \(L\) corresponds to what is called a higher-order statement or implication statement in NATL. Like a statement term, it is a composed term that takes two terms as element terms. That is, \(L : (T_1 \Rightarrow T_2)\). Typical element terms of a linkage term are statement terms and compound terms that represent situations. The copulas used in the linkage terms represent rules, causal relationships, etc., and can have many different kinds, but TRL does not specify how many kinds there are.

#### 4.6 Variable term

In order to express knowledge that is not tied to specific individual terms, variable terms are introduced. Variable terms are represented by lowercase letters \(x, y, \ldots\). For example, the knowledge that “if \(x\) is human, then \(x\) likes narratives” can be expressed as linkage term \(L_1\) as

\[
L_1 : S_1 : (x \rightarrow \text{human}) \Rightarrow C_1 : (\text{likes}, x, \text{narratives}).
\]

Here, \(x\) in \(S_1\) and \(x\) in \(C_1\) are bound as the same term.

### 5. Non-Axiomatic Term Logic

In this section, Non-Axiomatic Term Logic (NATL) is presented based on TRL introduced in §4. Like NATL, NATL is oriented towards proof-theoretic semantics rather than model-theoretic semantics.

#### 5.1 Classes of terms and distinction of thing/logical terms

\(S\) denotes the class of statement terms, \(L\) denotes the class of linkage terms, and \(C\) denotes the class of compound terms and basic terms.\(^{12}\)

A term of class \(C\) is called a thing term, and a term of classes \(S\) and \(L\) together is called a logic term. Table 1 shows the distinction of terms and correspondences to the term classes introduced in this section.

#### 5.2 Reasoning and logic in NATL

Reasoning based on NATL draws conclusions from prior knowledge by performing substitution operations between one logical term and another term (thing term or logical term). Substitution operations are allowed when two terms can be unified, including when the two terms are identical. Here, we assume soft unification based on embedding vectors, as (Arabshahi et al., 2021) does, rather than hard unification by exact string matching.

Reasoning in NATL as a combination of classes of two terms is divided into the following five types. That is, \(S \cdot S, S \cdot C, S \cdot L, L \cdot C, L \cdot L\). The reasoning in these five types are all derived based on the substitution operations by common terms that can be unified, as illustrated later.

\(^{12}\) A basic term can be regarded as a special composed term with one element term and one empty relational term having no semantic effect.
In the remaining one possible combination type $C \cdot C$ between $S$, $L$, $C$, we cannot define an operation that is generalizable over the type.  
13 Meanwhile, narratives and argumentation conveyed in natural language often present a series of events, i.e., compound terms. The essence of “reasoning” in NATL is the complementary connection between this series of compound terms, that is, the chain of thing terms $C \cdot C$, using logical terms. And the “logic” in NATL, from the psychological, social and technological standpoints mentioned at the beginning of this paper is what defines how these connections should be.  
14 In other words, NATL is a theory that argues that inferences that humans make in daily life can be completely described by five types of symbolic operations on three classes of knowledge representations. Although whether this claim is valid needs to be verified in the future, the next section §6 will illustrate its promise.

As also mentioned in §3.4.4, we present the theoretical position of NATL on the relationship between explicit, deliberative reasoning and implicit, reflective reasoning. We believe that when the “logic” regarding the connections between situations becomes habitual within an individual, inferences will be made reflexively and quickly without derivation of explicit inference paths by logic terms. And when that logic becomes socially customary, it ceases to be verbalized (This is related to the implicity of warrants, which is covered in the next section, §6). NATL’s position is that inference based on large-scale generative language models (for example, (Bhagavatula et al., 2020)) exactly catches that phenomenon.

On the other hand, in order to gain the understanding of others about logic that is not socially customary or shared, it is necessary to derive and present an explicit inference path using logic terms. In such cases, NATL predicts that there is a strong correlation between the perceived quality of the “explanation” actually presented based on the derived inference path and the explicitly explained inference path (logic terms) for the unshared part. In this “explanation”, if the part that has been fully habitual is shown in detail, NATL expects that it is negatively evaluated as redundant. Thus, NATL has a certain degree of falsifiability as a theory of how human reasoning and logic should be.

5.3 Reasoning types

In the following examples, we assume that the copula of statement terms represents the inheritance relationship (is-a) and the copula of linkage terms represents the entailment relationship. In general, the validity of the conclusion obtained should be different depending on the meaning of the premise copulas, and here we show possible types of inference operations regardless of the validity of conclusion.

5.3.1 $S \cdot S \rightarrow S$

Taking two statements to make an inference is the basic form of a syllogism. The conclusion is also a statement term.

The deductive example given at the beginning of §2 looks like this:

$S_1 : \text{human} \rightarrow \text{animal}$

$S_2 : \text{animal} \rightarrow \text{mammal}$

$S_3 : \text{human} \rightarrow \text{mammal}$

We also show examples of induction and abduction given in §3.3.

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13. To be precise, a meaningful operation cannot be found from the perspective of reasoning. For example, from what is represented by using the format of $U \times V \rightarrow R$ in NATL corresponding to the what is represented by using the format of $C : (R, U, V)$ in NATL. From the correspondence, $C_1 : (R_1, U_1, V_1)$ and $C_2 : (R_2, U_2, V_2)$ are transformed to $S_1 : (U_1, V_1) \rightarrow R_1$ and $S_2 : (U_2, V_2) \rightarrow R_2$, and then we can define a formal operation that leads to the conclusion $S_1, S_2 \vdash C_3 : ((U_2, V_2), U_1, V_1)$ respectively, whereas the meaning of $C_3$ obtained by this operation is unclear.

14. Consider, for example, a common narrative such as “Ann likes pandas. She goes to the zoo every day.” This narrative can be expressed as $C_1 : (\text{likes}, \text{Ann, pandas}), C_2 : (\text{goes-to}, \text{she, the-zoo, everyday})$ as a series of thing terms. From the thing terms $C_1, C_2$, related, various inferences can be made such as why to go to the zoo and where to be in the zoo. NATL considers that it is realized by the connection of thing term $C_1, C_2$ and using logic terms (statement terms and linkage terms) presented by generalized knowledge related to “the relationship between pandas and the zoo” and “the action of people who are in the zoo”.
\(S_4: \) Lily → swan
\(S_5: \) Lily → white
\(S_6: \) swan → white

\(S_7: \) these-beans → white
\(S_8: \) beans-from-the-bag → white
\(S_9: \) these-beans → beans-from-the-bag

There is no formal or theoretical reason why \(S_6\) and \(S_9\) should not be backwards (for example, \(S'_6: \) white → swan). If \(S_6\) is preferred over \(S'_6\), it may be due to ad-hoc criteria built into the reasoner in advance (cognitive bias in humans), or tendencies that the reasoner has acquired through experience.

5.3.2 \(S \cdot C \rightarrow C\)

An element term of the given compound term is replaced according to the given statement term.

\(S_1: \) polar-bear → white
\(C_1: \) (likes, John, white)
\(C_2: \) (likes, John, polar-bear)

Although reverse inferences such as the following are also allowed, generally the certainty of inference results is expected to be low.

\(S_1: \) polar-bear → white
\(C_2: \) (likes, John, polar-bear)
\(C_1: \) (likes, John, white)

5.3.3 \(S \cdot L \rightarrow C / S / L\)

In the case that one of the two element terms of the linkage term and the statement term are unifiable, the remaining element terms of the linkage term are taken as the conclusion. In this case, the category of the consequent terms depends on the given linkage item (\(/ \) in \(C / S / L\) means exclusive disjunction). Also, if there are unified variable terms, their bindings are preserved in the conclusion.

\(S_1: \) polar-bear → white
\(L_1: \) (x → white) ⇒ (likes, John, x)
\(C_2: \) (likes, John, polar-bear)
\(C_1: \) (likes, John, white)

\(L_1\) means “If there are \(x\) which are white, John likes \(x\)”, and the above example is equivalent to the content of \(\{S_1, C_1\} \vdash C_2\) in §5.3.2.

In this way, some compound terms can be translated to linkage terms by using variable terms. However, as shown by \(L_2\), it is not always possible to translate arbitrary linkage terms into compound terms. Therefore, we need both the \(S \cdot C\) and \(S \cdot L\) types of reasoning.

\(S_2: \) weather-of-today → bad
\(L_2: \) (weather-of-the-day → bad) ⇒ no-school
\(B_1: \) no-school

\(L_2\) in this example represents the rule of “If the weather is bad, the school will be closed”. Although \(\text{weather-of-today}\) and \(\text{weather-of-the-day}\) are not exactly the same concept, the premise here is to allow the unification of them in an appropriate inferential context. When it is allowed is determined empirically by the reasoner described later in §8.3.
5.3.4 $C \cdot L \rightarrow C/S/L$

When one of the two element terms of a linkage term and the thing term are unifiable, the remaining element terms of the linkage items are taken as a conclusion. In this case, the class of the consequent terms depend on the given linkage terms.

\[ C_1 : (\text{likes, John, polar-bear}) \]
\[ L_1 : (\text{likes, x, polar-bear}) \Rightarrow (\text{likes, x, penguin}) \]
\[ C_2 : (\text{likes, John, penguin}) \]

5.3.5 $L \cdot L \rightarrow L$

Like $S \cdot S \rightarrow S$, two remaining element terms construct a new linkage term via the two element terms that are unifiable. Here we only give a deductive example.

\[ L_1 : (\text{likes, x, polar-bear}) \Rightarrow (\text{likes, x, penguin}) \]
\[ L_2 : (\text{likes, y, penguin}) \Rightarrow (\text{likes, y, dolphin}) \]
\[ L_3 : (\text{likes, x, polar-bear}) \Rightarrow (\text{likes, x, dolphin}) \]

6. Elucidating the Internal Structures of Arguments

This section attempts to demonstrate the validity and effectiveness of NATL through qualitative analysis of arguments. Sperber, who proposed the relevance theory (Sperber and Wilson, 1986) closely related to abduction, also argues with Mercier, that argumentation is the primary function of reasoning (or human reason) (Mercier and Sperber, 2017). Since NATL is a theory that attempts to deal with human daily reasoning, it must be able to represent and construct the content of human daily arguments within the framework of the theory.

Toulmin referred to in §1 also presents a diagrammatic pattern called the “Toulmin model” (Fig. 1) for the structure of demonstrative argumentation commonly used in society (Toulmin, 1958). Here, D, datum, is the basis of an argument, C is the claim of the argument, and W is the warrant that enables the claim to be drawn from the datum. (Toulmin’s model also includes other elements such as backing B to support W and rebuttal R to limit the claim.)

Consider the following argumentation (Toulmin, 1958): D: Harry was born in Bermuda, C: Harry is a British subject, W: A person born in Bermuda is a British subject. As the NATL inference \( \{ S_D, L_W \} \vdash S_C \), it is formalized as follows:

\[ S_D : \text{Harry} \rightarrow \text{born-in-Bermuda} \]
\[ L_W : (x \rightarrow \text{born-in-Bermuda}) \Rightarrow (x \rightarrow \text{British-subject}) \]
\[ S_C : \text{Harry} \rightarrow \text{British-subject} \]

In this way, Toulmin’s argumentation scheme is also based on syllogism, but since W is not an absolute rule (truth), and thus deviates from the mathematical position (4) (see §1). (The rule mentioned in §1 (see footnote 4) also falls under this warrant.) In addition, Toulmin argues that the rationale as W is generally implied. Habernal and Gurevych (2017), who collected web-based argument data by the general public and analyzed them based on the Toulmin model, also excluded warrants from their “modified Toulmin model” they created for the annotation of argument texts, because they did not find any cases that could be clearly identified as warrants.
On the other hand, Habernal et al. proposes the Argument Reasoning Comprehension Task (ARCT) (Habernal et al., 2018; Niven and Kao, 2019) that recognizes the appropriate warrant of the argument with two alternatives. However, the warrant here is not limited to regular ones such as $L_W$ above, but is defined more broadly as “A claim that supports an argument and whose conclusion changes when the claim is reversed.” Therefore, it is not obvious how the warrant in ARCT is positioned in the logical structure (inference process) in the argumentation. Below, we show that NATL can explain these inference processes and clarify the position of warrants using three examples from (Habernal et al., 2018; Niven and Kao, 2019). At the same time, we also confirm that the basic framework described in §5 is insufficient to fully realize the argumentation and the reasoning behind it.

### 6.1 Example 1: Should I bring an umbrella because it will rain?

The first example consists of the following three sentences ((Niven and Kao, 2019), p.1). D: It is raining. C: You should take an umbrella. W: It is bad to get wet. In this example, if W sets to “It is good to get wet.”, the conclusion will be “You should not take an umbrella.”

This can be translated in TRL as $S_D, C_C, S_W$ as follows. $C_C$ is shown last because sentence C is the conclusion

- $S_D$: weather-of-the-day $\rightarrow$ raining
- $S_W$: getting-wet $\rightarrow$ bad
- $C_C$: (take, You, umbrella)

Obviously, $C_C$ cannot be derived directly from $S_D$ and $S_W$. All what can be assumed from what is explicitly shown here, including warrant $S_W$, is the existence of the rule (wisdom of life) below.

$L_0$: (and, (weather-of-the-day $\rightarrow$ raining), (getting-wet $\rightarrow$ bad)) $\Rightarrow$ (take, one, umbrella)

What is stated in $L_0$ is not wrong in common sense, and it would not be surprising if such rules were commonly shared by people. However, for people who do not know what rain is or what an umbrella is, this alone is not an enough explanation. Similarly, the mere existence of this rule does not explain why, when $W$ becomes $W'$: “It is good to get wet.”, the consequence of the rule should be $C'$: “You should not take an umbrella.”. So, only by these rules, the argument consisting of the three sentences $D$, $C$, and $W$ can not be completely understood or explained.

NATL can explain the reason why $L_0$ or $S_D$ $\Rightarrow$ $C_C$ can be claimed with the following three pieces of knowledge $L_1, L_2, L_3$. $L_1$ is common sense knowledge about general behavior of people, $L_2$ is common knowledge about rain, and $L_3$ is common knowledge about umbrellas.

- $L_1$: (causal-and, $x$, bad) $\Rightarrow$ (avoid, people, $x$)
- $L_2$: (weather-of-the-day $\rightarrow$ raining) $\Rightarrow$ getting-wet
- $L_3$: (have, $x$, umbrella) $\Rightarrow$ (avoid, $x$, getting-wet)

Then, the following reasoning can be made on the premise of $\{S_D, S_W, L_1, L_2, L_3\}$.

- $\{S_D, L_2\} \vdash B_1$ : getting-wet
- $\{S_W, B_1\} \vdash B_2$ : bad
- $\{B_1, B_2\} \vdash C_1$ : (causal-and, $B_1$, $B_2$)
- $\{C_1, L_1\} \vdash C_2$ : (avoid, people, getting-wet)
- $\{C_2, L_3\} \vdash C_3$ : (have, people, umbrella)

In this context, $C_3$ can be considered synonymous with $C_C$. If we accept this, we can explain $C_C$ from $S_D$. However, the step $\{B_1, B_2\} \vdash C_1$ has not been explained so far. It introduced a conjunction with a causal reading.

While the step of $\{S_D, L_2\} \vdash B_1$ is a forward deduction, the $\{C_2, L_3\} \vdash C_3$ step is a backward abduction. The fact that forward and reverse reasoning can coexist in this way is a major feature of NATL shared with NAL.
In this explanation, the meaning of ‘should’ in sentence C has disappeared (at least in appearance). The translation method from natural language to TRL is not given in detail in this paper, but it is one of the issues that cannot be ignored in order to demonstrate the technical feasibility of the proposal in the future, and thus the outlook will discussed in §8.4. Though the rules $L_1$, $L_2$, $L_3$ are also neatly chained, it is difficult to operate a perfectly consistent symbol system throughout the knowledge database in an actual system, and a mechanism is needed to overcome this difficulty. Future research is required to clarify whether the mechanism of soft unification based on semantic vectors alone can overcome this problem.

Even though $L_1$, $L_2$, and $L_3$ are deeper (more fundamental) knowledge than $L_0$, it is possible to ask a further question, for example, “Why do we get wet when it rains?” In this sense, the above explanation does not fully explain everything. For all causal knowledge, we can keep asking “why” as much as we want, but for much knowledge (for example, the rain is water and we get wet when we touch the water), we learn it directly in our interactions with the world, and naturally come to understand without asking why. This is a problem of embodiment, which is outside the theoretical scope of NATL because it goes into issues before cognition.

### 6.2 Example 2: Is marijuana a gateway drug?

The second example consists of the following three sentences ((Habernal et al., 2018), Figure 1). That is, D: Milk isn’t a gateway drug even though most people drink it as children. C: Marijuana is not a gateway drug. W: Milk is similar to marijuana.

It can be translated into TRL as $C \rightarrow D$, $C \rightarrow W$, $S \rightarrow C$. (For simplicity, we assume that the anaphora of ‘it’ in D has been resolved. We also omit “as children”.) Here, the inheritance negation copula $\not\rightarrow$ and the similarity copula $\Rightarrow$ are introduced.

- $C_D$: (even-though, $C_1$ : (drink, most-people, milk), $S_1$ : (milk $\not\rightarrow$ gateway-drug))
- $S_W$: milk $\Rightarrow$ marijuana
- $S_C$: marijuana $\not\rightarrow$ gateway-drug

Sentence D could be expressed as a linkage term, but since it is not a conditional rule, it is treated as a compound term in the present analysis. Regarding how to think about such ambiguity, it is desirable to clarify a consistent standard from a cognitive science point of view rather than from an engineering point of view.

Coming back to the topic, the main purpose of this argument is to derive $S_C$ from $C_D$, specifically $S_1$ in $C_D$. This itself can be achieved with one step of $S \cdot S \rightarrow S$ type inference, namely, $\{S_1, S_W\} \vdash S_C$. However, as sentence W is not W’: “Marijuana is similar to milk.”, if taken literally, the orthodox syllogism cannot be applied (the direction of $S_W$ is reversed). A NATL framework is necessary to allow this logic. (If an argumentation supporting AI systems based on NATL is realized, it will be possible to accept this argumentation and then point out to the user with a diagrammatic visualization and explanation that it is better to argue as W’).

By the way, if $\{S_1, S_W\} \vdash S_C$ can draw the conclusion, what is the role of $C_1$? The role of $C_1$ is considered to be a social thing, not to directly reinforce $S_1$’s argument logically, but to enhance the authority of the advocate by showing that the advocate has high reasoning ability, thereby indirectly enhancing the legitimacy of the argument. In other words, the argumentation pattern of “A even though B” is strongly related to the logic at the sociological position and the technological position shown by Toulmin.

It has been observed that people unconsciously make statements that assert their competence in social interaction situations (Komuro and Funakoshi, 2022). $C_1$ is considered to be a manifestation of such interaction practice. This leads to Mercier and Sperber (2017)’s claim that human reason, which is the ability to make argumentative reasoning, is primarily a social ability.

Let us continue to consider the role of $C_1$ through analysis using NATL. Behind the provisory reference “even though most people drink it as children”, it can be assumed that there are two common sense knowledge pieces about the addictive nature and the category of gateway drugs. That is, “Addictive ones are popular” ($L_1$) and “Addictive ones are gateway drugs” ($L_2$).
Then, if \( C'_1 : \text{milk} \rightarrow \text{popular} \) is understood as an implication of \( C_1 \), \( S_4 \) contradicts \( S_1 \) as follows. A conclusion is drawn as follows. That is, \( C_1 (C'_1) \) negates \( S_1 \) and \( SC \) rather than supporting them.

\[
\{C'_1, L_1\} \vdash S_3 : \text{milk} \rightarrow \text{addictive} \\
\{S_3, L_2\} \vdash S_4 : \text{milk} \rightarrow \text{gateway-drug}
\]

But why does the arguer present the information that denies one’s own claim? The answer is the aforementioned social interaction practice. By suggesting the weaknesses of one’s logic in advance, it is an act of showing that one has the ability of careful reasoning, thereby increasing the authority of the arguer (rather than the validity of one’s logic itself). Of course, the above discussion is speculative based on the assumption that \( L_1, L_2 \) were represented in the arguer’s brain, but it has shown that the relationship between the components of the argument can be analyzed clearly within the framework of the formal theory of NATL.

### 6.3 Example 3: Is Google a Harmful Monopoly?

The third example uses the following three sentences ((Niven and Kao, 2019), Fig. 1). \(^1\)

D: People cannot choose not to use Google. C: Google is a harmful monopoly. W: Other search engines redirect to Google. These can be expressed as \( C_D, C_W, S_C: \)

\[
C_D: \ (\text{cannot-choose-not-to-use, people, Google}) \\
C_W: \ (\text{redirect-to, other-search-engines, Google}) \\
S_C: \ \text{Google} \rightarrow \text{harmful-monopoly}
\]

In this example, W is not the warrant in the original Toulmin model. Unlike the two examples we have seen so far, W in this example is not included in the reasoning path leading from D to C. The role of W is not to establish an argument starting from D, but to ensure the validity of D, which is the starting point in the first place.

First of all, considering the true warrant required to derive C from D first, it would be the knowledge to protect oneself in a society like \( L_0 \). That is, \( L_0: \text{“If } x \text{ cannot avoid using } y, \text{ then } y \text{ is dangerous for } x.\)"

\[
L_0: \ (\text{cannot-choose-not-to-use, } x, y) \Rightarrow (\text{is-harmful-for, } y, x)
\]

Since there is no physical necessity for \( y \) to be detrimental to \( x \) even though \( x \) must use \( y \), innocent human beings are indifferent and not alarmed by such situations. Because humans easily make category errors, they may over-apply their experience with physical tools to non-physical tools (services). However, it is a common social misfortune that the side with no choice is weaker, and in turn, is sacrificed to the interests of the stronger side which should have been just a tool. So, teaching knowledge such as \( L_0 \) will often be done. The body of reasoning for this argument is completed by deriving \( (\text{is-harmful-for, Google, people}) \) from \( C_D \) and \( L_0 \) while recognizing that it is almost synonymous with the term \( S_C \) in semantic content.

Next, let’s look at the relationship between W and D. To understand this relationship, the following three knowledge pieces may be necessary.

\[
L_1: \ (\text{and}, (use, x, y), (use, y, z)) \Rightarrow (use, x, z) \\
S_1: \ (\text{redirect-to}, x, y) \Rightarrow (use, x, y) \\
C_1: \ (\text{want}, people, (use, people, other-search-engines-than-Google})
\]

\( L_1 \) indicates that the transitive law can be established for the act of “use” (the transitive law is not established for verbs in general and must be known individually). \( S_1 \) indicates that the act of “redirect” is some kind of “use” (this would be an expert knowledge of computer networks). \( C_1 \) should be recognized as a premise of D, which means that “People are trying to use search engines other than Google.”.

---

\(^1\) For the sake of naturalness, we have inverted the claims of the original argument.
Often in communication, when we want to say, “I want to do A, but I can’t do A.”, we simply say, “I can’t do A.”. For example, when we want to open a door but we can’t open it, we simply say, “This door is too heavy to open.” The typical listener infers from this statement that the speaker wants to open the door. This is a kind of problem generally referred to as intention recognition, and the act of telling the speaker “I can’t do it” can be appropriately viewed as an interaction practice (“mechanism for understanding others’ actions” (Komuro and Funakoshi, 2022)), as mentioned earlier. It is not clear whether, in humans, such intention recognition is performed by symbolic reasoning, or whether it is processed in parallel with symbolic reasoning by the ability that has been provided in another way (In many cases, intention recognition is a multi-modal problem that relies extensively on information about the surrounding situation, so it seems likely that intention recognition itself is a distinct mechanism from symbolic reasoning.), but let us assume that $C_1$ is recognized from $C_D$ anyway. Here, $C_2 : (use, other-search-engines, Google)$ is understood from $S_1$ and $C_W$. Then, $C_1$ and $C_2$ are (softly) unified with the left term of $L_1$, resulting in $C_3 : (use, people, Google)$ from $L_1$. Since the result $C_3$ of the inference using $C_W$ is consistent with the original assertion $C_D$, $W$ comes to have the effect of increasing the probability of statement D.

7. Other Applications

In §6, we saw that NATL can reveal the structure of reasoning behind arguments. If this process can be implemented on a computer, it will contribute to the realization of artificial intelligence that supports discussions. This chapter presents three possible directions for other applications.

7.1 Solving math problems heuristically

The process of solving math problems (at least until it becomes habitualized and automated by repeating the same type of problem) seems to depend largely on conscious symbolic reasoning. Of course, spatial recognition ability is also essential for solving geometric problems, but combining these abilities with the representing and reasoning capabilities of NATL may lead to the creation of an AI that can solve arithmetic problems in the same way as humans do (and reproduce the same mistakes as humans). Here, we will try to explain the process of getting answers to elementary math problems.

As an actual example of basic math for elementary school students, we take up the problem: “Using the number line in the figure, think of a number that fits □.” The number axis is shown at the top of Fig. 2, (a), and □ is defined by the following equation $\frac{4}{10} = □$. Elementary school students are expected to deepen their geometric understanding of the relationship between the numerator and the denominator by finding the correspondence between two number sequences heuristically using the number line as a clue. It is not to mechanically apply the algebraic symbol operation taught in advance, that is, to perform the arithmetic operation of $\frac{4}{10} = \frac{5 \times 4}{10}$ to obtain the solution. Here, we use NATL to follow the process of an imaginary elementary school student given this problem until he or she derives the answer.

First, the pupil pays attention to and observe the number line Fig. 2 (a) according to the instructions of the problem, and notices that there are 11 bars (or tick marks) between the bar annotated with 0 and the bar annotated with 1. (It can be considered that this counting behavior is driven by a conditional reflex as a habitual act acquired through play from childhood.) The pupil already knows how to use a ruler, and by analogy with their experience and the fact that the number 10 is on the left side of the equation in question, the pupil comes up with an idea to arrange the numbers from 0 to 10 under the number line. The arrangement is shown as Fig. 2 (b), which are the basic terms handled by NATL. Since each placed term is a token symbol with a unique spatial position along with the corresponding number concept, we indicate that by noting 0, . . . , 10.

Returning to the equation, the pupil focus on the numbers 4 and 10 that form a group on the left side of the equation (for this problem, the concepts of equal sign and fraction have been taught, which is the premise)

16. NATL should be able to handle operations using algebraic symbolic manipulations, and this is an important problem (Fujisawa and Kanai, 2021) that general-purpose deep learning models are not good at, but we will leave the discussion of this to another article.
and connects them with the basic terms listed earlier. That is, \( S_1 : 4 \leftrightarrow \frac{4}{1}, S_2 : 10 \leftrightarrow 10 \) (The copula \( \leftrightarrow \) denotes the recognition that the tokens are different but the types are the same).

Also, from the symmetry of the role of the numerator/denominator in fractions and symmetry as the spatial arrangement of equations, the pupil recognizes the following statement terms: \( S_3 : 4 \sim 2 \), \( S_4 : 10 \sim 5 \). (Copula, \( \sim \), expresses that is not identical but has a certain correspondence here.) Then, \( \{S_2, S_4\} \vdash S_5 : 10 \sim 5 \) is inferred. (It is a future work to examine what kind of copulas the inference result has when two statement terms as premises have different copulas).

Inspired by the recognition of \( S_5 \), the pupil comes up with the idea of arranging different sequences under \( \frac{10}{1} \). This is the sequence \( 0, \ldots, 5 \) of (c). (The starting of (c) is 0 rather than 1, and the idea of arranging numbers starting from 0 at equal intervals in space is induced based on other factors rather than logic, such as aesthetic sense and Gestalt perception.\(^{17}\))

The pupil perceives \( S_6 : 4 \sim 2 \) by the perceptual grouping between rows (b) and (c). Then, \( \{S_1, S_6\} \vdash S_7 : 4 \sim 2, \{S_3, S_7\} \vdash 2 \sim \Box \) is inferred and the answer “The number in \( \Box \) is 2.” is obtained.

Since NATL allows soft matching of terms, the range of possible inferences is almost unlimited in the framework. In the previous example, the useless inference is made such as \( \{S_4, S_6\} \vdash S_8 : 10 \leftrightarrow 2 \). To eliminate this useless inference, the reasoning should be controlled appropriately in order to obtain a useful conclusion. A reasoner that can judge what should do at what time implicitly, and intelligently is needed. Also, without the implicit and spatial ideas used in the above-mentioned thinking process, it is impossible to obtain appropriate statements that serve as the components of reasoning. The NATL theory shown in this paper alone cannot fully explain the various cognitive thinking abilities of human beings. However, NATL can become a “skeleton” that can integrate these various cognitive abilities and enables the construction and explanation of symbolic reasoning and thinking processes.

### 7.2 Internal Representation Language of Multimodal AI Systems

Like mathematical language can express objects more clearly and concisely than natural language, TRL introduced in §4 might express the structure of cognition and knowledge in the subject (AI system) more clearly and concisely than natural language. In addition, those expressed by terms are not limited to those that can be expressed by natural language. Therefore, TRL itself is useful as an interface language between various cognitive modules that constitute a multi-modal AI system such as an intelligent robot. Since it is oriented toward term logic, there is no need to distinguish between predicate symbols and existential symbols. On the other hand, the introduction of compound terms allows us to describe the relationships between any number of terms as in predicate logic.

Even in tasks that mainly involve natural language processing, there are many cases where elements that are not explicitly verbalized are important. Understanding procedural documents is one example. Maeta, Yamakata, and Mori (2017) propose a method to extract the semantic structure (recipe flow graph) from a recipe cooking procedure as a directed acyclic graph. In the recipe flow graph, the ingredients, tools,

\(^{17}\) Schelling (1960) pointed out that “imagination such as aesthetic sense rather than logic” is important for the success in his designed coordination games (e.g., a game to coincide with each other in a map without communication) through his experiments.
1. Heat the oil in a two-handled pan. Add celery, green onion, and garlic, and fry for 1 minute.
2. Add broth, water, macaroni, and pepper, and simmer until the pasta is tender.
3. Sprinkle with chopped sage.

Figure 3: 3-step recipe example (Maeta, Yamakata, and Mori, 2017)

and cooking actions used for cooking are the vertices of the graph, and the relationships between them are expressed as edge labels.

A recipe flow graph expresses the order (dependency) of a procedure by using the fact that an action target or a product that causes a state change due to a certain action becomes an object or place for performing another action after that. In this case, the vertex of an action represents the product or the object whose state is changed by it. For example, the act of “heating” in the first step of Fig. 3 represents the resulting “hot two-handled pan with hot oil”, and it is related as the “direction (Dest)” of the following act of “adding”. In this way, the key to interpreting procedure documents is the appropriate handling of intermediate products and state changes that are not verbalized, but there is a limit to expressing the content only with verbalized elements. By using TRL, it is possible to uniformly and concisely express both verbalized and non-verbalized objects, so we can expect to achieve more accurate semantic analysis and situation understanding.

7.3 Understanding and generation of analogy and metaphor

Analogy maps a set of concepts in one domain to a set of concepts in another domain to facilitate problem solving in new situations (Holyoak and Thagard, 1995). Similarly, metaphor helps the understanding of abstract objects and new situations, and their verbalization (Lakoff and Johnson, 1980). Metaphor is also an important phenomenon from the perspective of deepening (emotional) ties and social interaction (Jang et al., 2017). Both are recognized as the sources of human creativity, but at least analogy is not necessarily an innate ability, and it is known that many parts are acquired step by step in the process of development (Holyoak and Thagard, 1995).

A compound term in NATL can be used as a set representation, and meta-information about the set itself, what the set represents, can be encoded in the semantic vector of the compound term’s relational term $R$.

With TRL, we can simply represent the mapping in analogy and metaphor in the form of a statement term $B_S \rightarrow B_T$ as a connection between concepts. Then, the individual mappings can be bundled into one compound term $(R, B_1^S \rightarrow B_1^T, B_2^S \rightarrow B_2^T, \ldots, B_n^S \rightarrow B_n^T)$. Based on this expression method, it is possible to realize a system that makes full use of analogies and metaphors on a NATL-based reasoner. The developmental acquisition of the use of analogies and metaphors in humans is similar to the need for appropriate training for reasoners.

Hofstadter and Sander (2013) goes further and claims that analogy is the root of human cognition and thinking. According to Hofstadter et al., recall of memory is based on the same mechanism as analogy. He then argues that the fundamental cognitive ability of categorization is the same as the ability to create analogies. Another name for term logic, the starting point of NATL, is categorical logic. If Hofstadter et al.’s assertion is correct, NATL should become the basis of the formal expression of thought.

Fauconnier and Turner (2002) proposed the conceptual blending theory as a universal mode of human creativity and discovery thinking including metaphor and analogy. The theory of conceptual blending is based

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18. For example, after the act of “cutting out a watermelon,” the cut fruit and skin remain, but they are often used and referred to separately in later processes as ingredients and containers, respectively. In the flow graph, two different objects must be represented by one state that is expressed as a result of the act of “hollowing out”.

19. For example, an example from (Jang et al., 2017), “He is the pointing gun. We are the bullets of his desire.”, will give an mapping structure like $(METAPHOR, he \rightarrow gun, we \rightarrow bullet)$. If it is simply “He is the pointing gun.”, you will generally get $(METAPHOR, he \rightarrow gun)$. The interpretation of “He will not be stingy with his bullets.” and the subsequent use of metaphors will change depending on which mapping structure is used as the context. For the vector semantic representation attached to the relational term $METAPHOR$, it is conceivable to encode information that marks metaphor and that represents the semantic frame used in the metaphor.
on the structure where the two input areas (mental spaces) are connected by various relationship mappings and merged into a new interpretation area and can explain the flexible thinking of human beings which is not constrained by the laws of physics and causality. Although there are not many studies that attempt to process blending computationally 20, our another goal is to realize the computational theory of thinking that integrates conceptual blending and NATL by representing mental spaces and mapping relationships based on TRL.

8. Prospects for Computer Implementation and Future Challenges

The reasoning based on NATL exemplified in S6 and §7 is premised on the ability of the flexible unification based on semantic similarity and the ability to select only appropriate unification. The realization of these abilities is a future research topic. This section describes the outlook and challenges for implementing these capabilities.

8.1 Semantic vector representations

As described in §3.4.1, by adding a semantic representations to a term as a point in the latent space in the form of multidimensional vector, information about the things and concepts to which the terms refer is retained. As mentioned in §3.4.3, copulas are given semantic representations as well as terms. We can define the function that gives the semantic vector of basic term $B$, compound term $C$, statement term $S$, linkage term $L$, and copula $c$ as follows ($R$ and $T_i$ follow the definition of §4. $T_{l/r}$ denotes the left/right terms). Functions $F()$ can be constructed as neural networks.

$$
\mu(B) = F_B(B) \\
\mu(C) = F_C(\mu(R), \mu(T_1), ..., \mu(T_n)) \\
\mu(S) = F_S(\mu(c), \mu(T_l), \mu(T_r)) \\
\mu(L) = F_L(\mu(c), \mu(T_l), \mu(T_r)) \\
\mu(c) = F_c(c)
$$

$F_B(B)$ encodes various information associated with basic term $B$ as a latent space representation. For example, if term $B$ is a symbol presented by a natural language, such as a term recognized for a particular word sequence, then a pre-trained language model such as BERT (Devlin et al., 2019) could be used to construct $F_B$. In the same way, if the term is associated with a particular object that is perceived visually or audibly, $F_B$ could be constructed as a function that encodes that sensory stimulus. Perhaps the basic implementation approach is to configure it as an autoencoder.

$F_c$ as a so-called embedded layer (Matrix representation as a lookup table that learns weights by error back propagation and performs a linear transformation from the sparse vector of the explicit input space to the dense vector of the latent space) learns the corresponding vector representation directly, which is a good starting point. However, in this case, instead of viewing copulas as derivative and open as discussed in §3.4.3, a closed system with a predefined finite number of copulas is constructed. Therefore, this approach just serves a starting point. Whether $F_c$ is constructed in advance through some kind of sub-tasks or or to acquire it from the initial random values at the same time in the training of reasoner described later is a subject for future study.

How to construct a vector representation that expresses the meaning of a copula is an important issue, and it will be essential to deepen scientific understanding of copulas based on qualitative considerations rather than just leaving it to engineering brute force. In the following, we will focus on “negation”, which is an important feature of natural language, and proceed with qualitative considerations.

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20. There are only 12 papers in the list collated by M.Turner, the co-author of (Fauconnier and Turner, 2002), from 2010 to 2019. [https://markturner.org/blending.html](https://markturner.org/blending.html)
Negation is an important factor for intelligence that enables humans to perform “thought experiments” (Todayama, 2014), but it is one of the problems that it is unclear to what degree it can be handled even in large-scale language models (Tashiro et al., 2022). In addition, there is a lot of research on technology to search large-scale knowledge graphs at high speed by embedding. Among them, there is also research that deals with search queries expressed in predicate logic that include negation (for example, (Zhang et al., 2021)).

In the former, negation as the words not and never are learned as “use of words” under self-supervised learning in the same way as other common words. Therefore, it is expected that the semantic expressions acquired by a language model can be used to some extent to deal with things that have a strong usage element, such as reasoning about antonyms. The discussion of (Nye et al., 2021) also suggests that there are many situations where this alone does not work well. The current findings (Tashiro et al., 2022) seem to confirm such a two-sided tendency.

In the latter case, things such as “Edison” and “light bulb” are used as nodes, and the relationship such as “invented” is expressed as the edge of the graph, the node is represented as a vector representation of the location in the latent space and the edge is represented as a vector representation of the orientation in the same space, allowing the search to be performed as an operation based on proximity in space (neighborhood search). Here, logical elements such as OR, AND, and negation themselves are not given embedded representations, but are fixedly incorporated into the processing system as special operations (for example, distribution inversion).

In the proposed NATL framework, copulas containing negative relations are given a special position different from general relational expressions such as “invented”, but like negative words in large-scale language models, it needs to be given a vector representation. In the simplest case, as mentioned above, it is conceivable to obtain the optimum vector representation for the task discretely with respect to the set of copulas by the “embedding layer”. However, for copulas with complementary relations, such as “A is B” and “A is not B”, the reasoner would work more effectively by applying a constraint that makes those copulas point to the opposite directions in the latent space.

Moving further away from negation and considering copulas in general, we can say that the essence of copulas is to combine (or separate) two elements from a specific point of view, beyond the overall similarities or differences. The most straightforward example of this is the mapping relationship that dynamically associates different things in metaphors and analogies, as discussed in §7.3. On the other hand, embedding in predicate logic-based knowledge graphs is static in nature, functioning by embedding similar things in similar positions and having the same relationship in the same direction. It does not fit into the cognitive processing of recognizing what were previously perceived as completely dissimilar and distant, and then in the next instant, recognizing them in close proximity. This discussion of copulas is reminiscent of wormholes, which are discussed as a theoretical entity in astrophysics. In other words, it is a warp of space. From this, although it is still only at the stage of fanciful conception, this suggests the possibility of attempting to model the dynamic deformation of a latent space according to the vector representation of the copula, using a mathematical framework that describes the distortion of the space by means of differential geometry.

8.2 Unification

Unification between basic terms \(B_i, B_j\) can be achieved by explicitly recognizing a new statement term \(B_i \leftrightarrow B_j\) when the similarity calculated based on the semantic vectors examined in the previous section exceeds a certain level.

For unification among the composed terms, if the number and order of elements are consistent among the terms to be unified, a relatively simple procedural algorithm can be used to determine whether unification is possible or not and to bind the variables. For example, the soft unification of (Arabshahi et al., 2021) is based on this assumption.

However, when trying to target more general and large domains, making such assumptions quickly becomes a stumbling block. For example, this assumption does not hold even if we only consider unifying the compound terms \((\text{gives}, X, Y, Z)\) and \((\text{gives-to}, U, V, W)\) recognized from two simple sentences, ‘X gives..."
Y Z' and ‘U gives V to W’. What should correspond to Y is not V but W. In addition, in §6.3, we assumed unification between \( \{\text{use, x, y}\} \) and \( \{\text{want, people, (use, people, other-search-engines)}\} \) in the inference process. As a general method for realizing such unification, we can consider an approach to generatively find the correspondence in unification in the form of seq2seq modeling.

### 8.3 Reasoner

There are at least two kinds of reasoning that NATL should handle. That is, problem-solving reasoning (§7.1), which leads to a new recognition (conclusion) as an answer to a given prerequisite knowledge and question, and explanatory reasoning (§6), which leads to a path connecting a given premise and conclusion. In any case, the NATL reasoner can be formulated as a function \( R \) that, given a term set \( E \) as the assumed knowledge and a term \( Q \) representing the question or conclusion as input, outputs a triplet \( (T_1, T_2, T_c) \) consisting of the two terms \( T_1, T_2 \in E \) used in the inference and a new consequent term. Namely, \( (T_1, T_2, T_c, t) = R(E, Q) \). Here, \( t \) is the confidence of the inference result and corresponds to the truth value in NAL. It seems to be the most natural way to express it as a subjective probability with a real-value between \([0, 1]\), but it is not necessary to limit it to that.

Whether or not the inference is completed is judged by whether or not the output \( T_c \) is the answer to the question, or whether or not the inference path between the prerequisite knowledge and the conclusion is completed. If the inference is not completed here, \( T_c \) is added to \( E \) with meta-information such as the information that it is an intermediate result in the inference and its confidence level, and then the inference (Calculation of \( R(E, Q) \)) of the next step is repeated.

The main process that the reasoner performs internally is to first select \( T_1 \) and \( T_2 \) to which the substitution operation is applied, and then select the direction of the substitution and the copula of the output term if necessary. To realize this process, an approach in which the function \( R \) is trained by supervised learning or reinforcement learning, as discussed in §3.4.2, seems promising. A major issue is how to prepare the training data set. The outlook for this point will be discussed below.

The operation of NATL is premised on the existence of a knowledge base (KB) of logic terms (It is also possible to simultaneously acquire knowledge and thinking skills, as in the development of a human child, but this is an issue for further study in the future). In order to perform meaningful verification and evaluation of a particular inference task toward the realization of a functioning reasoner, it is necessary to clarify that the given problem can be solved by operating the KB. We can collect a considerable amount of knowledge from existing knowledge graphs and ontologies such as FreeBase (Bollacker et al., 2008) and Yago (Suchanek, Kasneci, and Weikum, 2007) for statement terms, and from common sense KB collecting causal knowledge such as ATOMIC-20 \(^{20}\) (Hwang et al., 2021) for linkage terms, but how much the knowledge is required when combined with specific reasoning tasks is not clear.

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exists implicitly in the background of the arguments, such as “when it rains, you get wet”, “getting wet is unpleasant”, “people avoid unpleasant things”, and “if you carry an umbrella, you can avoid getting wet” by asking “Why?” to the workers. This annotation work is also expected to be more difficult than simple labeling of text, both in terms of the work itself and in controlling the level of granularity and quality of the output, but we believe that by developing a work environment and procedures that are constrained to follow the NATL format presented in §5, we can have better feasibility than the second approach of creating problems. In addition, through this annotation work, the descriptive power of NATL can be verified. In the future, we would like to proceed with the first and third approaches in parallel to demonstrate the theoretical validity and practical usefulness of NATL.

Other major considerations that need to be resolved in order to demonstrate the above proofs are (1) to determine the generation procedure of the statement term $B_i \leftrightarrow B_j$ as a unification of the basic terms described in §8.2 and the design choice that the conjunction of the two inference results introduced in §6.1 are done by the reasoner $R$ as one step of inference or in parallel by another module juxtaposed to the reasoner, and (2) to specify how to estimate confidence $t$.

### 8.4 Translation from natural language to term representation language

Finally, we give a perspective on how to convert knowledge described in natural language into TRL. Various approaches can be considered depending on the complexity of the linguistic data in the target domain. First of all, it is necessary to extract the chunks corresponding to basic terms (in the simplest cases, a word sequence as a span in a sentence). If they can be extracted, vector semantic expressions can be obtained using a pretrained language model as described in §8.1.

For such extraction, Open Information Extraction (Open IE) (Angeli, Premkumar, and Manning, 2015; Stanovsky et al., 2018) can be used. A simple linguistic expression (for example, “Mary went to the bathroom.”), such as the one that appears in the bAbi dataset mentioned in §8.3, may be of some use by combining ad-hoc modification rules with, for example, a pre-trained model published as part of AllenNLP (Gardner et al., 2017).21 Similarly, coreference resolution (Lee, He, and Zettlemoyer, 2018)22 can be used to identify the referent (antecedent) of pronouns such as “she”. After replacing the pronoun with its referent found by coreference resolution, Open IE can be performed.

However, it is not so easy to prepare sufficient modification rules even for a simple domain. For example, if the example sentence “She goes to the zoo everyday.” in footnote 14 is given to the learned model of Open IE described above, we obtain the output [ARG0: She] [V: goes] [ARG4: to the zoo everyday]. If we want to translate this into (goes-to, she, the-zoo, everyday), we need to reinclude the preposition “to” in the interval [ARG4:] to [V:] and modify the adverb “everyday” to separate it. If the correction using the rules is not tractable enough, it is necessary to prepare the training data from scratch to build an Open IE model.

In addition to recognizing the chunks corresponding to basic terms, it is necessary to correctly recognize the type of composed term (compound term, statement term, and conjunction term) that corresponds to the sentence. This could also be handled by rule processing based on keyword pattern matching while dealing with small domains where only plain expressions appear. If it becomes complicated and large scale, a classifier by supervised learning will be necessary.

For the following two tasks, recognition of basic terms in a sentence and recognition of the types of composed terms corresponding to a sentence, if training data prepared for both tasks and supervised learning is performed, it would be better to combine both tasks as a translation task, that is, fine-tuning a generative large-scale pre-trained language model such as GPT (Radford et al., 2019; Brown et al., 2020) or T5 (Xue et al., 2022) to handle the task in a seq2seq format.

### 9. Conclusion

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21. https://demo.allenai.org/open-information-extraction
22. https://demo.allenai.org/coreference-resolution
This paper has proposed Term Representation Language (TRL), a knowledge representation language inspired by term logic and Non-Axiomatic Term Logic (NA TL), a computational theory of cognitive symbolic reasoning, as the first step of research aiming at the realization of an AI system that performs daily reasoning, creative symbol processing, and cultural learning through symbols as humans do. The theory claims that five types of symbolic manipulation on knowledge representations in three classes can describe human reasoning and argumentation. It aims to explain the divers human thinking, including analogy and metaphor, on the basis of formal symbolic processing, which consists of two basic cognitive mechanisms: (1) To re-recognize multiple things as a single object, which have a certain relationship with each other while being separated temporally and spatially. (2) Recognize and connect further relationships between the two objects so grouped together.

We believe that the theoretical validity and practical usefulness of NATL has been proven to a certain extent through the qualitative analysis of exemplary arguments. Verification and revision of the theory through further case analysis, quantitative evaluation, and demonstration of feasibility and applicability through computer implementation are future tasks.

The name of Non-Axiomatic Term Logic owes its name to Non-Axiomatic Logic of Wang (1994, 2013). However, Toulmin also states the same thing (“Unfortunately an idealized logic, such as the mathematical model leads us to, cannot keep in serious contact with its practical application. A Rational demonstration is not a suitable subject for a timeless, axiomatic science” (Toulmin, 1958), p. 136), to whom this paper owes another major debt. Wang does not cite Toulmin’s work. The approaches of the two are not necessarily the same, one is from reasoning and the other from argumentation. However, the relevance between them is obvious. The importance of being axiom is not the only claim of Wang. Perhaps the same claim has been repeated by many ancestors.23

One of the major contributions of Discourse Representation Theory (Kamp and Reyle, 1993), which attempted to explain the phenomenon of anaphora using predicate logic, was, according to Krahmer (1998), to provide a means of formal representation that was visually appealing (p.35). Fauconnier and Turner (2002) point out that the invention of appropriate formal means of representation was important in the development of mathematics, and goes further to appeal that formality inseparable from meaning supports imagination. (“Like the warrior and the armor, meaning systems and formal systems are inseparable. . . . forms prompt largely unconscious and unnoticed constructions of the imagination.”., p.11).24 From this point of view, it can be said that NATL presents one idea of a formal representation means that is inseparable from meaning in order to reproduce human daily reasoning and thinking through a computer.

Mathematical logic promoted formalism and separated meanings from symbols, enabling rigorous discussions based on form alone. This research goes against this trend, and the submitted theory of NATL can no longer say anything only in form (The analysis presented in §6 follows a certain form, but the judgment of the validity of the results depends on the subjectivity of the reader so far). However, by integrating the form with objective and reproducible information processing technology of meaning (embedding), NATL tries to transcend the problem of semantic ambiguity and personality of descriptions in natural language that formalism attacked, and aims to make computers coexist in the same meaningful world as humans.

Reasoning by NATL presented in this paper is carried out based on various rules and causal knowledge expressed as linkage terms. However, it seems unreasonable to assume that all the necessary linkage terms for reasoning are explicitly retained in advance. For example, the causal rule beliefs shown in footnote 4 can be created infinitely different by changing the “yesterday” and “cupboard” parts. Of course, it is possible to assume the existence of rules in which these parts are abstracted and generalized. However, it is also reasonable to suppose that, as necessary for reasoning, linkage terms are regenerated from the accumulation of a large amount of past experience. This seems close to the idea of (Hofstadter and Sander, 2013), and large-scale generative language models such as GPT (Radford et al., 2019; Brown et al., 2020) seem to make this technically possible. On the other hand, the generation by those language models is daydreaming, drifting

23. For example, Toulmin, in his more recent book (Toulmin, 2001), criticizes modern Western rationalism from a similar idea (Return to Reason in the title of the book means a return from rationality to reasonableness). A similar criticism has been developed from the perspective of the history of Japanese thought by Nakano (Nakano, 2012), but he does not cite Toulmin.
24. Lakoff also argues that many important concepts in mathematics are figurative blending (Lakoff and Núñez, 2000).
through the ocean of generation like a ship that has lost its anchor. We hope that the NATL-based reasoner will globally assemble the parts produced by language models under local conditioning to create AI that can speak like a reasonable human. This is exactly the separation and coordination of reasoning and linguistic competence discussed in §3.4.4.

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