Time reflection and refraction in synthetic frequency dimension

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Abstract: We demonstrate that time reflection and refraction can be observed in a two-band model centered around a non-zero reference energy. Our model can be physically implemented as a system of two coupled dynamically-modulated ring resonators. © 2023 The Author(s)

The space-time duality inherent in Maxwell’s equations suggests that there are temporal analogs to many of the more familiar spatial phenomena. Previous theoretical works have demonstrated that at a temporal boundary created by changing the refractive index, time-refracted and time-reflected waves are generated [4, 5]. However, to observe time boundary effects experimentally for optical waves, the change in refractive index must be comparable to that of the material and occur on the time scale of the optical wave period (few femtoseconds). These requirements are difficult to satisfy in real materials. To date, time refraction has been experimentally achieved at optical frequencies by using epsilon-near-zero media [3, 7]. However, time reflection has not been achieved in the optical domain.

In this work, we propose using a two-band model to achieve time reflection and refraction [2]. The time boundary is implemented by changing the coupling constants in time. Using this approach, the time scale for the modulation is no longer controlled by the frequency of the waves themselves, but rather by the band gap in the two-band model. Consequently, time reflection and refraction effects can be observed in optical waves by applying a time boundary occurring on the microwave time scale, in contrast to previous works. We illustrate our approach by using a two-legged ladder model (Fig. 1a), as described by:

\[ H = \sum_m E_m \hat{a}_m^\dagger \hat{a}_m + \sum_m [C \hat{a}_m^\dagger \hat{b}_m + \text{h.c.}] + \sum_m [-C \hat{b}_m^\dagger \hat{b}_{m+1} + \text{h.c.}] + \sum_m [\kappa \hat{a}_m^\dagger \hat{b}_m + \text{h.c.}] \] (1)

Fig. 1. (a) Schematic of two-leg ladder lattice model. Blue and red spheres represent a and b lattice sites, respectively. (b) Band structure of Hamiltonian given by Eqn. 1. Blue indicates before the time boundary \((t < 0)\) with \(C/\kappa = 1\), while red indicates after the time boundary \((t > 0)\) with \(C/\kappa = -4\). (c) Temporal evolution of the system initialized in eigenstate centered around \(k_0 = \pi/3\), with time boundary at \(t = 0\).

To implement the time boundary in our system, we impose a sudden change on some of the coupling constants in Eqn. 1. As a numerical example, we consider a sudden change of \(C\) in Eqn. 1 from \(C = \kappa\) for \(t < 0\) to \(C = -4\kappa\) for \(t > 0\), creating a time boundary at \(t = 0\). The band structures corresponding to \(C = \kappa\) and \(C = -4\kappa\) are shown in Fig. 1b as blue and red lines, respectively. For each band structure, at a given \(k\), there are two allowed eigenstates with different energies that propagate with opposite-signed group velocities of the same magnitude. For \(t > 0\), we
thus expect that the system should be in the two states as indicated by the red dots in Fig. 1b. These two states are the time-refracted wave that has the same sign in its group velocity as the initial state, and the time-reflected wave that has the opposite sign. This is demonstrated numerically in Fig. 1c, where we initialize the state in a linear superposition of the eigenstates around the blue dot in Fig. 1b at \( k_0 = \pi / 3 \). Complete time reflection can also be achieved when the following condition is satisfied:

\[
H(t > 0) - E_0 = -[H(t < 0) - E_0]
\]

We now investigate the sharpness of the time boundary needed to observe time refraction and reflection. Using the same parameters as in Fig. 1c, we vary \( C(t) \) smoothly from \( C = \kappa \) at \( t = -10 / \kappa \) to \( C = -4 \kappa \) at \( t = +\infty \) using the functional form:

\[
C(t) = -\frac{\delta}{\pi} \arctan(\Omega t) - \alpha(\Omega)
\]

where \( \delta = 5 \kappa \) and \( \alpha(\Omega) \) is a constant offset to ensure that \( C(-10 / \kappa) = \kappa \). Using different values of \( \Omega \), we compare the temporal evolution of the initial state from Fig. 1c (Fig. 2a–d). The corresponding change in the value of \( C(t) \) is depicted in Fig. 2e–h. Fig. 2 indicates that the required switching rate for the time boundary in our system is on the order of \( \kappa \) in order to achieve a significant time refraction effect. The required switching rate thus can be far lower than the frequency of the optical wave.

![Fig. 2](image)

This system can be implemented in two dynamically coupled ring resonators [6]. The frequency modes of ring A correspond to the \( a \) lattice sites, while the modes of ring B are the \( b \) sites in the two-leg ladder Hamiltonian (Fig. 1a). We note that the switching rate (\( \Omega \) in Eqn. 3) needed to observe time reflection is on the order of the coupling constant \( \kappa \), which can be in the microwave frequency regime if the rings are implemented using optical fibers. Experimental values of \( \Omega_R = 2\pi \cdot 6 \text{ MHz} \) and \( \kappa \approx 0.08 \Omega_R = 3 \text{ MHz} \) have been realized for such a system [1].

References

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