QUARK DEGREES OF FREEDOM IN FINITE NUCLEI

KAZUO TSUSHIMA\textsuperscript{1}, KOICHI SAITO\textsuperscript{2} and ANTHONY W. THOMAS\textsuperscript{3}

\textsuperscript{1,3}Department of Physics and Mathematical Physics and Institute for Theoretical Physics
University of Adelaide, Adelaide, SA 5005, Australia
\textsuperscript{2}Physics Division, Tohoku College of Pharmacy, Sendai 981, Japan
E-mail: ktsushim@physics.adelaide.edu.au\textsuperscript{1}
saito@nucl.phys.tohoku.ac.jp\textsuperscript{2}
athomas@physics.adelaide.edu.au\textsuperscript{3}

Properties of finite nuclei are investigated based on relativistic Hartree equations which have been derived from a relativistic quark model of the structure of bound nucleons. Nucleons are assumed to interact through the (self-consistent) exchange of scalar ($\sigma$) and vector ($\omega$ and $\rho$) mesons at the quark level. The coupling constants and the mass of the $\sigma$-meson are determined from the properties of symmetric nuclear matter and the rms charge radius in $^{40}$Ca. Calculated properties of static, closed-shell nuclei, as well as symmetric nuclear matter are compared with experimental data and with the results of Quantum Hadrodynamics (QHD).

There is no doubt now that explicit quark degrees of freedom for nucleon structure are certainly required to understand deep-inelastic scattering at momentum transfers of several GeV\textsuperscript{1}. Furthermore, it has also proven possible to successfully describe the properties of nuclear matter by taking account of the quark structure of nucleons\textsuperscript{2,3}. Here, we address the question: ‘Are the quark degrees of freedom necessary to describe the properties of finite nuclei ?’, where the typical energy scale is a few tens of MeV. Corresponding to this question, we will report our recent work on the properties of finite nuclei (as well as symmetric nuclear matter) based on the quark meson coupling model (QMC)\textsuperscript{a}, whose original version was suggested by Guichon\textsuperscript{2}. The main feature of the QMC model is that nucleons in the nucleus (matter) are described by the non-overlapping MIT bag model within Born-Oppenheimer approximation.

In QMC, the Lagrangian density for mesons and these nucleons can be defined:

\begin{equation}
\mathcal{L} = \overline{\psi} [i\gamma \cdot \partial - M_N^*(\hat{\sigma}) - g_\omega \hat{\omega}^\mu \gamma_\mu] \psi + \mathcal{L}_{\text{mesons}}, \tag{1}
\end{equation}

\begin{equation}
\mathcal{L}_{\text{mesons}} = \frac{1}{2} (\partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - m_\sigma^2 \hat{\sigma}^2) - \frac{1}{2} \partial_\mu \hat{\omega}_\nu (\partial^\mu \hat{\omega}^\nu - \partial^\nu \hat{\omega}^\mu) + \frac{1}{2} m_\omega^2 \hat{\omega}^\mu \hat{\omega}_\mu, \tag{2}
\end{equation}

where, $\psi (M_N^*(\hat{\sigma}))$, $\hat{\sigma}$ ($m_\sigma$), $\hat{\omega}$ ($m_\omega$) are the field operators (masses) of the nucleon, $\sigma$-, and $\omega$-mesons, respectively, where the effective nucleon mass $M_N^*(\hat{\sigma})$ will be defined below. In mean field approximation, equations of motions for these fields are given by:

\begin{equation}
[i\gamma \cdot \partial - M_N^*(\sigma) - g_\omega \gamma_\mu \omega] \psi = 0, \tag{3}
\end{equation}

\textsuperscript{a}Recently, Blunden and Miller have also studied the properties of finite nuclei with QMC\textsuperscript{6}.
\[(\nabla^2_r + m^2_\sigma)\sigma(\vec{r}) = -\frac{\partial}{\partial \sigma} M^*_N(\sigma) \rho_\sigma(\vec{r}) = g_\sigma C(\sigma) \rho_\sigma(\vec{r}), \quad (4)\]

\[(\nabla^2_r + m^2_\omega)\omega(\vec{r}) = g_\omega \rho_B(\vec{r}). \quad (5)\]

On the right hand side of Eq (4), a new, and characteristic feature of QMC beyond QHD appears, namely, \(-\frac{\partial}{\partial \sigma} M^*_N(\sigma)\), or \(C(\sigma)\). These quantities are defined by,

\[\frac{\partial M^*_N}{\partial \sigma} = -3g^q_\sigma \int_{bag} d\vec{r} \bar{\psi}_q \psi_q \equiv -3g^q_\sigma S(\sigma) = -\frac{\partial}{\partial \sigma} [g_\sigma(\sigma) \sigma], \quad (6)\]

with the MIT bag model quantities

\[M^*_N(\sigma) = \frac{3\Omega(\sigma(\vec{r})) - z_0}{R_B^*} + \frac{4}{3}\pi(R_B^*)^3 B,\]

\[S(\sigma(\vec{r})) = \frac{\Omega/2 + m^*_q R_B^*(\Omega - 1)}{\Omega(\Omega - 1) + m^*_q R_B^*/2}, \quad \Omega = \sqrt{x^2 + (R_B^* m^*_q)^2},\]

\[m^*_q = m_q - g^q_\sigma(\vec{r}), \quad C(\sigma) = S(\sigma)/S(0), \quad g_\sigma = 3g^q_\sigma S(0). \quad (7)\]

Here, \(z_0, B, x\) and \(m_q\) are the parameters for the sum of the c.m. and gluon fluctuation effects, bag pressure, lowest eigenvalue and current quark mass, respectively. \(z_0\) and \(B\) are fixed by fitting the nucleon mass in free space, and assumed to be independent of density, or \(\sigma\)-meson field. The bag radius in-medium, \(R_B^*\), is obtained by the equilibrium condition \(dM^*_N(\sigma(\vec{r})) / dR_B|_{R_B=R_B^*} = 0\). The results reported in this article are obtained with the values, \(z_0 = 3.295, R_B = 0.8\) fm (in free space), \(B = (170\text{MeV})^4\) and \(m_q = 5\) MeV, respectively. At the hadron level, the entire information on the quark dynamics is condensed in \(C(\sigma)\) of Eq. (4). Furthermore, when this \(C(\sigma) = 1\), the equations of motions given by Eqs. (3), (4) and (5) are exactly identical to those derived from QHD. By solving these equations of motions at the hadron level, we can investigate the properties of finite nuclei.

We present the calculated results in the following. For realistic calculations of finite nuclei, contributions of the Coulomb force and the \(\rho\) meson are also included. The model parameters at the hadron level, i.e. coupling constants and mass of the sigma meson, \(m_\sigma\), are determined from the properties of symmetric nuclear matter (binding energy per nucleon of -15.7 MeV) and rms charge radius in \(^{40}\text{Ca}\) (3.48 fm), with the ratio \(g_\sigma/m_\sigma\) fixed so as to reproduce the symmetric nuclear matter properties (See Table I). Other parameters used for the calculations are, \(m_\omega = 783\) MeV, \(m_\rho = 770\) MeV and \(e^2/4\pi = 1/137.036\). The calculated binding energy with results of QHD are shown in Fig. 1. One of the successes of QMC is that the nuclear compressibility, \(K\), is well reproduced the experimentally required values 200 - 300 MeV, whereas QHD tends to overestimate it significantly.

In Figs. 2 and 3, we show the calculated charge density distributions for \(^{40}\text{Ca}\) and \(^{208}\text{Pb}\) with the results of QHD and experimental data. The charge density is calculated as a convolution of the point-proton density with the proton charge...
Table 1. Coupling constants and calculated properties for symmetric nuclear matter at normal nuclear density and finite nuclei. The effective nucleon mass, $M_N^*$, and the nuclear compressibility, $K$, are the values for symmetric nuclear matter at normal nuclear density $\rho_0 = 0.15$ fm$^{-3}$. $M_N^*$, $K$, and the sigma meson mass, $m_\sigma$, are quoted in MeV.

|          | symmetric nuclear matter | finite nuclei |
|----------|--------------------------|--------------|
| $M_N^*$  | $K$                      | $m_\sigma$  |
| QMC      | 754 280                  | 550 5.40     |
| QHD      | 504 565                  | 520 8.72     |
|          | $g_\sigma^2/4\pi$       | $g_\omega^2/4\pi$ |
| QMC      | 5.31                     | 15.2         |
| QHD      | 6.93                     | 15.2         |

![Figure 1. Calculated (binding energy)/nucleon by QMC (the solid line) and QHD (the dashed line).](image)

distribution. They are fairly well reproduced, and especially the QMC results for $^{40}$Ca are impressive. These quantities are not sensitive to the values of $R_B$ and $m_q$.

As a summary, we would like to stress the successful generalization of the QMC model to finite nuclei opens a tremendous number of opportunities for further work. For example, to investigate the Okamoto-Nolen-Schiffer anomaly, the nuclear EMC effect, super-allowed Fermi beta-decay, and so on, which could be resolved by the introduction of the quark degrees of freedom. Although there are a number of important ways in which this model could be extended, the present model can be applied to all the problems for which QHD has proven so attractive, with very little extra effort. Finally, our answer to the question: ‘Are the quark degrees of freedom necessary to describe the properties of finite nuclei ?’ is that, a quantitative investigation has just now started!

References
Figure 2. Charge density distribution for $^{40}\text{Ca}$ (for $m_q = 5$ MeV and $R_B = 0.8$ fm) compared with the experimental data and that of QHD.

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Figure 3. Charge density distribution for $^{208}\text{Pb}$ (for $m_q = 5$ MeV and $R_B = 0.8$ fm) compared with the experimental data and that of QHD.