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To cite this article: Fei Zhang and Yan Piao 2018 J. Phys.: Conf. Ser. 1004 012006

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Design of Restoration Method Based on Compressed Sensing and TwIST Algorithm

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Abstract. In order to improve the subjective and objective quality of degraded images at low sampling rates effectively, save storage space and reduce computational complexity at the same time, this paper proposes a joint restoration algorithm of compressed sensing and two step iterative threshold shrinkage (TwIST). The algorithm applies the TwIST algorithm which used in image restoration to the compressed sensing theory. Then, a small amount of sparse high-frequency information is obtained in frequency domain. The TwIST algorithm based on compressed sensing theory is used to accurately reconstruct the high frequency image. The experimental results show that the proposed algorithm achieves better subjective visual effects and objective quality of degraded images while accurately restoring degraded images.

1. Introduction

The famous mathematicians D. Donoho and E. Candes put forward the compressed sensing theory in 2016 [1-2]. Compressed sensing technology has made breakthrough progress in image reconstruction and encryption. However, the application of compressed sensing to the field of image restoration [3-4] is still in its infancy. Both signal reconstruction part of compressed sensing and image restoration need to solve the inverse problem [5]. In addition, some methods in the field of image restoration [6-9] are also the reconstruction methods of compressed sensing. Therefore, it is possible to apply compressed sensing technology to the field of image restoration. Ma Jianwei proposed a method based on Poisson Singular Integral (PSI) and Curvelet iterative hard threshold to reconstruct degraded images. The algorithm improves the subjective and objective effect of degraded image by using a small amount of observation data, but the algorithm does not consider the existence of noise.

The algorithm proposed in this paper obtains the sparse high-frequency signal and the non-sparse low-frequency signal by the wavelet transform of the degraded image. The algorithm uses compressed sensing to reconstruct sparse high-frequency signals and reconstruct the low-frequency signals by using the traditional restoration algorithm. Finally, the reconstructed high and low frequency signals are inverse transformed to obtain a restored image. Though the algorithm only extracted a small amount of high-frequency observations, it can not only improve the subjective and objective effects of degraded images but also reduces the storage space of data and reduces the computational cost.

2. Compressed sensing

Compressed sensing theory mainly includes three parts: sparse signal transformation, signal measurement and signal reconstruction. Figure 1 shows the theoretical framework of compressed sensing.
Sparse transformation
\[ X = \Theta \Psi^T X \]

Sparse representation of the signal

Measurement matrix
\[ Y = \Phi \Theta \]

Signal reconstruction
\[ \hat{X} = \text{argmin} \| X \|_1 \quad \text{s.t.} \quad Y = \Phi \hat{X} \]

Figure 1 Theory frame of compressed sensing

Suppose \( X = [X_1, X_2, ..., X_N]^T \) is a one-dimensional discrete signal, then any set of \( N \)-dimensional vectors can be represented by an orthogonal vector basis \( \{\Psi_i\}_{i=1}^N \) of \( N \times 1 \) dimensions. Any signal can be expressed as:

\[ X = \Psi \Theta \] (1)

Where \( \Theta \) is the transform coefficient. If there are only \( K(K \ll N) \) nonzero coefficients (or the value is large) in \( \Theta \) and all other coefficients are all zero (or the value is small), so the signal \( X \) is said to be \( K \)-sparse and \( \Psi \) is the sparse basis of the signal. Then, the transform coefficients can be linearly transformed with an observation base \( \Phi (M \times N \text{ dimension}, M \ll N) \) to obtain the observation set \( Y \) (\( M \times 1 \) dimension), which is:

\[ Y = \Phi \Theta = \Phi \Psi^T X \] (2)

Sparse signal transformation and signal measurement can also be expressed by the matrix \( A^{CS} \), then the signal is observed as:

\[ Y = A^{CS} X \quad (A^{CS} = \Phi \Psi^T) \] (3)

When \( A^{CS} \) satisfies the RIP criterion, that is to say, the observation matrix is not related to signal sparse basis. Then, the reconstruction of the original signal \( X \) from the observation set \( Y \) transforms into an optimization problem:

\[ \min \| \Psi^T X \|_1 : A^{CS} = \Phi \Psi^T X = Y \] (4)

3. Restoration method based on compressed sensing and TwIST algorithm

3.1. Traditional recovery methods

The key to image restoration is to establish a model of image degradation and restoration. Assuming that the original image is \( f(x, y) \), the degraded image is \( g(x, y) \), the restored image is \( \hat{f}(x, y) \), the random noise is \( n(x, y) \) and \( H[\cdot] \) is a function that combines all the degradation factors. Then, the image degradation and restoration model shown in Figure 2.

\[ f(x, y) \xrightarrow{\text{Degradation function}(H)} g(x, y) \xrightarrow{\text{Inverse filter}(H^{-1})} \hat{f}(x, y) \]

Figure 2 Model of image degradation and Restoration

As can be seen from the figure above, the degraded image can be expressed as:

\[ g = Hf + n \] (5)

The process of image restoration is the inverse filtering process. Then, the traditional image restoration model is expressed as:

\[ \hat{f} = H^{-1}[g] = f + H^{-1}[n] \] (6)

3.2. Image recovery based on Compressed Sensing

The study found that in the compressed sensing sparse transform, the frequency domain transform can decompose the image into high frequency part and low frequency part. However, the low-frequency
part contains most of the information of the image, and the overall coefficient is larger and less sparse. Therefore, it is not suitable for compressed sensing sampling. The high-frequency image contains the details of the image, and most of the coefficients are 0 or approximately 0. So it is sparse enough to apply to compressed sensing samples.

Therefore, for degraded images, this paper adopts wavelet transform to decompose the image into high-frequency signal and low-frequency signal. This paper uses the GPSR_BB algorithm to recover the low-frequency image and uses compressed sensing method to recover the high-frequency image. The Gaussian random measurement matrix is used to observe the high-frequency signals and use zero to fill the unobserved part. Then using the TwIST recovery algorithm based on compressed sensing to recover the degraded image. Specific algorithm flow shown in Figure 3.

Figure 3 Image Restoration Based on Compressed Sensing

3.2.1. Image restoration based on Compressed Sensing. The objective function of traditional image restoration is expressed as:

\[ J(x) = \frac{1}{2} \| g - Hx \|^2 + \lambda \varphi(x) \quad (7) \]

Where \( \varphi(x) \) is the normalization factor and \( \lambda \) is a regularization parameter, which is used to adjust the proportion of the two parts of the objective function. Through Bayes estimation, the optimization problem of \( j \) in (7) can be transformed into the problem of maximum a posteriori estimation[11]. It can be expressed as:

\[ \hat{x} = \arg \min_{x} \frac{1}{2} \| Hx - g \|^2 + \lambda \| x \|_1 \quad (8) \]

Let \( W \) be a two-dimensional orthogonal wavelet inverse transform matrix, and \( W^TW = I \). Then it performs a wavelet transform on \( x \) to get the coefficient vector \( \theta \). That is \( \theta = W^T x \). The recovery model of sparse coefficient is obtained by Bayesian-MAP estimation.

\[ \hat{\theta}_{MAP} = \arg \min_{\theta} \frac{1}{2} \| g - HW\theta \|^2 + \lambda \| \theta \|_1 \ \ (9) \]

The restored image formula is expressed as:

\[ \hat{x} = W\hat{\theta}_{MAP} \quad (10) \]

Under the compressed sensing theory, the optimal solution obtained by \( l_1 \) is expressed as follows

\[ \hat{x} = \arg \min_{x} \frac{1}{2} \| \Phi x - g \|^2 + \lambda \| \Psi^Tx \|_1 \quad (11) \]

In the above equation, \( \Phi \) is the observation matrix, \( \Psi^T \) is the transformation matrix.
Transform the formula (10) into the wavelet domain, the wavelet coefficient is $\hat{\theta}$ and connect to the maximal posterior estimation of it, the wavelet domain image reconstruction, based on the compressed sensing theory, is shown as follows:

$$\hat{\theta} = \arg\min_\theta \left\{ \frac{1}{2} \left\| g - \Phi \mathbf{H} \mathbf{W} \theta \right\|^2 \right\} + \lambda \| \theta \|$$

(12)

In the above equation, $\Phi$ is the observation matrix, $H$ is the degenerate function, $W$ is the orthogonal wavelet transform matrix, and $\theta$ is the wavelet coefficient.

From the analysis above, we can see that (12) is the equivalent formula of signal reconstruction algorithm and image restoration algorithm under the compressed sensing theory.

3.2.2 TwIST algorithm. In solving the problem of (12), The TwIST algorithm mainly uses the two previous estimated values to update the current value. The update process is given by (13) and (14).

$$x_0 = \Gamma_0(x_0)$$

(13)

$$x_{t+1} = (1 - \alpha) \cdot x_{t-1} + (\alpha - \beta) \cdot x_t + \beta \cdot \Gamma_\lambda(x_t)$$

(14)

In the formula, $x_0$ is the initial value, $\Gamma_0$ is a noise reduction processing function. $\alpha$, $\beta$ are the parameters for adjusting each iteration estimation value, $x_{t+1}, x_t, x_{t-1}$ are the values when estimate $t+1, t, t-1$ times, $x_{t+1}$ is jointly decided by $x_t$ and $x_{t-1}$. This is the so-called two-step iterative contraction.

Noise reduction processing function $\Gamma_\lambda(x_t)$ is defined as:

$$\Gamma_\lambda(x) = \Psi_\lambda(x + \Phi^T (g - \Phi x))$$

(15)

$\Psi_\lambda(x)$ is defined as:

$$\Psi_\lambda(x) = \sum_i T((x, \theta_i)) \cdot \theta_i$$

(16)

Where, $T_\lambda^T(x)$ represents the threshold processing operator. It is represented by (17) and (18):

Soft threshold:

$$T(x) = \text{sign}(x) \cdot \max(|x| - \text{th}, 0)$$

(17)

Hard Threshold:

$$T(x) = x \cdot \max(|x| - \text{th}, 0)$$

(18)

Where, $\text{th}$ represents the threshold.

Literature [7] made a detailed study on the convergence of TwIST algorithm. The convergence speed of the algorithm is decided by the parameters $\alpha$ and $\beta$. $\alpha$, $\beta$ is expressed as follows:

$$\alpha = \rho^2 + 1, \quad \beta = \frac{2\alpha}{\xi_m + \xi_i}$$

(19)

Where, $\rho = \frac{1 - \sqrt{k}}{1 + \sqrt{k}} < 1, k = \frac{\xi}{\xi_m}$. Here $\xi_m = 1$, under different circumstances, $\xi_i$ take a different value.

4. Experimental results and analysis
The experiment uses 256*256 size image, including Figure (A) Changchun University of Science and Technology’s Family Floor and Figure (B) Cameraman.
The test images (A) and (B) were degenerated by the uniform degenerate function \( L = 4 \). A Gaussian white noise with a variance of 2 was added. After extracting 60\% of the high frequency components of the degraded image in the wavelet domain, the proposed algorithm is compared with the proposed algorithm [7] and [9]. The simulation results are shown in Figure 5-6 and Table 4.1.

Observing Figure 5-6, at the subjective aspects, all the three restoration algorithms have improved the quality of the degraded image effectively. However, the algorithm proposed in this paper is obviously superior than the one proposed in [7] and [9] in color contrast. Only the extraction of 60\% of the amount of information cases of high-frequency part, there are some white spot appears in some detail part of the restoration algorithm in the literature [7] and [9]. While the more accurate algorithm in this paper has obtained of the restored image detail part. In order to show that the proposed algorithm is superior than the traditional one, we analyze the enlargement of Figure(A) and Figure (B), they are shown in Figure7 and Figure 8.
It can be seen from the analysis of Figure 7 and Figure 8 that in the case of obtaining 60% information amount of the high-frequency information, it can be seen from the result of the restoration shown in image (A) that the contour of the restored house and the texture of the tile have the approximation degree to the original image. It is obviously better than the literature [7], [9] restoration algorithm. It can be seen from the result of the image (B) restoration that the proposed algorithm outperforms the traditional algorithm in the restoration of the contour of the face, the clothes and the structure of the camera. In the case of extracting only 60% of the amount of information, the details of the original image are reconstructed precisely without any loss of detail.

The objective of this algorithm is to analyze the quality of the restoration effect from the peak signal-to-noise ratio (PSNR) and the feature similarity (FSIM).

**Table 4.1** Recovery performance comparison with different algorithm

|          | Figure A        | Figure B        |
|----------|-----------------|-----------------|
| PSNR (dB)| FSIM            | PSNR (dB)       | FSIM            |
| document [7] | 25.01 0.853     | 25.62 0.842     |
| document [9] | 25.12 0.861     | 25.21 0.824     |
| Our method       | 27.55 0.895     | 27.26 0.876     |

It can be concluded from Table 4.1 that under the same conditions, the image reconstructed by our algorithm is higher than the algorithms of [7] and [9]. The PSNR of this algorithm is 1-2dB higher than that of the literature [7] and literature [9]. In the aspect of FSIM, the proposed algorithm is 0.02-0.05 higher than that of the original algorithm [7] and [9], which shows that the structural similarity between the restored image and the original image is higher than the other two algorithms, and Image distortion is small.

5. Conclusion
In this paper, we propose a new TwIST image restoration algorithm based on compressed sensing in the traditional image restoration algorithm. The compressed sensing technology is applied to the field of image restoration, breaking the constraints of signal sampling by the Nyquist sampling law. Through the sparse decomposition and observation of degraded images, it achieves the subjective and objective effects that in a large extent to change the degradation of images can be observed with a small amount of data. At the same time it saved storage space and reduced the amount of computation. In this paper, compressed sensing technology is applied to the field of image restoration, which provides a new feasible idea and an effective solution with the use value for the future image restoration.

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