Data Assimilation to Estimate the Water Level of River

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Abstract. Data assimilation is an estimation method for stochastic dynamic system by combining the mathematical model with measurement data. Water level and velocity of river are stochastic dynamic system, and it is important to estimate the water level and velocity of river flow to reduce flood risk disaster. Here, we estimate the water level and velocity of river flow by using data assimilation specially Kalman filter and Ensemble Kalman filter. We define mathematical model of river flow, discretize and do simulation by Kalman filter and Ensemble Kalman filter. In data assimilation, we forecast the water level and velocity by using mathematical model and based on the measurement data, the correction of forecasting is made.

1. Introduction
Data assimilation is an estimation method for stochastic dynamic system. Data assimilation is combination between mathematical model and measurement data. Kalman filter, Extended Kalman filter, Ensemble Kalman filter and other modification of Kalman filter are data assimilation methods. Data assimilation has been applied to environmental problem such as the distribution of groundwater pollution [1], air pollution [2], the high of tidal [3], and also in guidance and control problem such as estimation and control of mobile robot [4], estimation on autonomous underwater vehicle [5]-[6]. Here, we applied the data assimilation method to estimate the water level of river. Some researchers also studied the river problems such as Estimation of Surabaya River Water Quality [7], Auto Floodgate Control [8], and the estimation of water level by using square root information filter [9]. The mathematical model of water flow in river is a shallow water problem and it is called the Saint Venant Equation [9]-[10]. The Saint Venant equation is non linear, so Kalman filter can’t be applied directly to estimate the water level of river. The equation must be linearized. For the non linear system, we can applied ensemble Kalman filter to estimate the state variable. Here, we estimate by using Kalman filter for linear system and the Ensemble Kalman filter for the original system. We compare the accuracy of those methods.

2. Mathematical Model of River Flow
The mathematical model of river flow is based on the Saint Venant Equation[9]-[10]

\[
\begin{align*}
B \frac{\partial}{\partial t} + A \frac{\partial u}{\partial x} + uB \frac{\partial h}{\partial x} &= q \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} &= g(S_o - S_f) - \frac{q}{A} u
\end{align*}
\]

(1)

The Saint Venant is a non linear continue system, so it is necessary to discretize respect to time \( t \) and position \( x \). Here, we use forward difference scheme for time and central difference scheme for position. The discrete system of river flow is
In this paper $h_i^k$ is the water level, $u_i^k$ is the velocities, $i = 1,2, \ldots, n$ where $k = 1, 2, 3, \ldots$. Suppose, the water level and the velocities of are three location which can be measured $i = 1, 5, 12$. Based on six data: $h_1^k, u_1^k, h_5^k, u_5^k, h_{12}^k, u_{12}^k$, it will be estimate the water level and the velocity for all position.

3. The Data Assimilation

Data assimilation method is an estimation method to estimate the state variable of stochastic dynamic system. Kalman filter is one of data assimilation method. Kalman filter can be applied to estimate the linear dynamic system. Some modifications of Kalman filter are proposed to be applied to the non linear system such as Extended Kalman Filter, Ensemble Kalman filter, Fuzzy Kalman filter and Unscented Kalman filter. In this paper, Kalman filter and Ensemble Kalman filter are applied to estimate the water lever and velocity of river flow.

Suppose is given the system and measurement equation

$$X_{k+1} = A_k X_k + B_k u_k + G_k W_k$$

$$Z_k = H_k X_k + V_k$$

where $W_k$ is Gaussian white noise of system, $W_k \sim N(0,Q)$, $V_k$ is Gaussian white noise of measurement, $V_k \sim N(0,R)$. $X_k$ is state variable, $u_k$ is an input. $Z_k$ is a measurement data.

Kalman filter algorithm [11] contains two step, prediction step (time update) and correction step (measurement update). Prediction step is determined based on mathematic model. It is predict the state variable for one time step ahead. The correction step make correction the prediction result based on measurement data. Kalman filter is applied in linear model and linear measurement.

The Ensemble Kalman filter is one of Kalman modification. The Ensemble Kalman filter [12] is applied to estimate the state variable of nonlinear dynamic stochastic system with linear measurement equation. In the ensemble Kalman filter method, the initial estimation is duplicated $N$ ensemble of initial estimation. The ensemble are generated by Gaussian normal distribution. The measurement data are also made $N$ ensemble. The Kalman filter and Ensemble Kalman filter algorithm are presented on figure 1-2.
4. Simulation Result and Discussion

The Saint Venant equation is nonlinear, so we must linearize the system before applying the Kalman filter and discretize respect to time $k$ and position $i$. In this simulation, the water level and velocity are estimated by using Kalman Filter and the Ensemble Kalman Filter. We divide the length of river into 12 grids and measure the water level and the velocity on three positions.

Figure 3-4 show the water level and velocity estimation on the 6th position by Kalman filter. Figure 5-6 show the water level and velocity estimation on the 6th position by Ensemble Kalman filter. The estimation result by Kalman filter seem difference with the real system and the estimation by the Ensemble Kalman filter has the pattern look like the pattern of the real system, but from figure 7-8, we know that the mean error of estimation by Kalman filter less than the mean error of estimation by the Ensemble Kalman filter.
Figure 3. The water level estimation by Kalman filter at the 6th position.

Figure 4. The velocity estimation by Kalman filter at the 6th position.

Figure 5. The water level estimation by Ensemble Kalman filter at the 6th position.
Figure 6. The velocity estimation by Ensemble Kalman filter at the $6^{th}$ position.

Figure 7. The error of water level estimation at the $6^{th}$ position.

Figure 8. The error of velocity estimation at the $6^{th}$ position.
Figure 9-10 show the estimation result at the 9th position by Kalman filter, and the Ensemble Kalman filter are compared by the real system. Seem that Kalman filter doesn’t give the good estimation, the water level and velocity estimation are constant (the green straight line). The Ensemble Kalman filter is still give the estimation for water level and the velocity, but the estimation result is not more accurate than the estimation result on the 6th position.

![Figure 9. The water level estimation at the 9th position.](image1)

![Figure 10. The velocity estimation at the 9th position.](image2)

The next simulation, we present the water level for all position at 90 time step. Figure 11-12 show that the patron of water level estimation and the velocity by using Kalman filter and the Ensemble Kalman filter for all position are almost same.
Figure 11. The water level estimation for all position.

Figure 12. The velocity estimation for all position.

Table 1. The mean error of estimation.

| Water level on the ith position | The Mean Error of estimation by Kalman Filter | The Mean Error of estimation by Ensemble Kalman Filter |
|--------------------------------|---------------------------------------------|------------------------------------------------------|
| Water level                   | -0.3343                                     | -0.6934                                              |
| Velocity                      | -0.2037                                     | -0.3083                                              |
| Water level on the 1st position| -0.0018                                     | 0.0015                                               |
| Water level on the 2nd position| -0.0029                                     | 0.2843                                               |
| Water level on the 3rd position| -0.0016                                     | -0.1814                                              |
| Water level on the 4th position| 0.0023                                      | 0.1071                                               |
| Water level on the 5th position| -8.0635e-004                                | -0.0786                                              |
| Water level on the 6th position| 2.3638e-004                                 | 0.0042                                               |
| Water level on the 7th position| -1.9803e-004                                | -0.0807                                              |
| Water level on the 8th position| -0.0106                                     | -0.0085                                              |
| Water level on the 9th position| 7.2343e-004                                 | 0.1508                                               |
| Water level on the 10th position| -0.0017                                     | 0.0244                                               |
| Water level on the 11th position| -0.0033                                     | 0.0115                                               |
| Water level on the 12th position| 0.3343                                      | 0.6934                                               |
Table 1 represents the mean error of water level estimation by using Kalman filter and the Ensemble Kalman filter. The Kalman filter estimation have less mean error than the Ensemble Kalman filter for almost all position, that mean for this simulation, Kalman filter give more accurate estimation than the Ensemble Kalman filter. But, from figure 9-10, Kalman filter is bad estimator for water level and velocity on the 9th position. In some position, Kalman filter can’t give estimation result, because we approximate the non linear system by using linear system.

5. Conclusion
From the discussion above and the simulation result we conclude that
1. The river flow can be present as the Saint Venant Equation, a non linear system
2. Kalman filter can be applied to estimate the water level and velocity of river by doing linearization of system
3. The Ensemble Kalman filter also can be applied to estimate the water level and velocity of river flow.
4. The Ensemble Kalman filter is better estimator than Kalman filter, because Kalman filter didn’t give the estimation on some position (the estimation result of water level and velocity are constant).

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