Induced spontaneous symmetry breaking chain

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Abstract – The article discusses a scenario based on the idea of induced spontaneous symmetry breaking. In this type of scenario, spontaneous symmetry breaking is assumed at some higher energy level, which leads to a chain of several subsequent induced symmetry violations at lower levels caused by small mixtures between the levels. We present a simple model in which the idea is realized by small mixing between the levels using scalar portals. In this approach, the large difference between energy scales, for example the Planck scale and the electroweak scale, occurs due to the product of several small factors proportional to the mixing coefficients. Dark matter fields can be formed in this scenario from matter fields on one or possibly several intermediate levels between the highest one and lowest one.

Introduction. – The Standard Model, quantum gauge field theory with spontaneous breaking of electroweak symmetry, reached its logical conclusion with the discovery of the Higgs boson at the LHC [1,2]. The successes of the SM in describing the processes of particle production and decay in terrestrial and space experiments are well known. However, the SM is unable to explain a number of observed phenomena in nature, such as baryon and lepton asymmetries in the Universe, dark energy and dark matter, small cosmological constant, mass and oscillations of neutrinos, etc.

There are some internal problems of the SM, such as the problem of the negative mass parameter $\mu^2 = -|\mu^2|$, artificially introduced into the theory, the hierarchy problem associated with this parameter and the problem of electroweak scale stabilization on a value of about 100 GeV. The negative parameter $\mu^2$ is necessary for the realization of the mechanism of spontaneous electroweak symmetry breaking (mechanism BEH [3–5]), but its origin as well as the magnitude of the corresponding EW scale of the order of 100 GeV remain unclear.

Before the launch of the LHC, there was the so-called No-Lose Theorem [6–9], which, based on an analysis of the behavior of the scattering amplitudes of longitudinal modes of massive electroweak SM bosons and the requirement of perturbative unitarity, asserted that only two situations were possible: either the existence of a sufficiently light Higgs boson with a mass smaller than $\sim 700$ GeV, or a change of regime and the appearance of new physics on a scale of the order of 1 TeV. The Higgs boson with a mass of about 125 GeV has been discovered, and, accordingly, the argument about the magnitude of the new scale is gone. Today we do not know the closest to the SM scale of new physics, and, of course, we do not know what this new physics is. The results of the search for manifestations of new physics at the LHC are still negative, although these negative results themselves are extremely important, narrowing the range of parameters or closing some possible variants for new physics.

We are well aware that there is at least one scale that is significantly larger than the electroweak scale. This is the Planck scale, $\sim 10^{19}$ GeV. Note that many extensions of the Standard Model predict the presence of other intermediate scales. Grand Unification models, their supersymmetric or string motivated generalizations, predict the scale $\sim 10^{15}$–$10^{16}$ GeV, on which the running electroweak and strong coupling constants become equal or close. In models of neutrino physics, in particular, based on the see-saw mechanism, the scale $\sim 10^{11}$–$10^{12}$ GeV appears. In supersymmetric models or in models with extra space-time dimensions, scales of the order of 10 TeV are quite admissible. However, the answer to the question of exactly what and how phase transitions occur as we move from large to smaller scales remains very unclear.

This short letter proposes a simple scenario based on the idea of induced spontaneous symmetry breaking. In this type of scenario, a chain of phase transitions can be realized. Only in the theory at the highest scale, it can be the Planck scale, there is a negative mass parameter $\mu^2$. Possibly the negative sign of this parameter is due to...
nonlinear effects of quantum gravity. At this highest scale, a phase transition occurs, and this transition through mixing causes subsequent transitions in a chain in which one link (i.e., theory at a given scale) connects to an adjacent link (i.e., theory at the nearest scale) through a portal with a rather small mixing. Small mixing parameters between the two nearest links (theories on the nearest scales) in the chain lead to the fact that the farther from the lowest level a certain link in the chain is located, the more weakly the hypothetical particles of the theory at this intermediate scale interact with the particles of the theory at the lowest level. In this scenario, the scale of the highest level, say the Plank scale $M_{Pl}$, is the scale of the lowest level, say the electroweak (EW) scale of the SM. In this way, the SM gets mass $m_{SM} = 2\lambda_{SM}v_{SM}^2$, and the parameter $\mu_{N-2}$ is induced being proportional to the mixing coefficient $k_{N-2}$ between levels $N-1$ and $N-2$ of the chain. This process continues and, as a result, the mixing among the scalar fields with the SM quartic coupling $\lambda_{SM}$ is the same as the one in the SM, as a result of the usual mechanism of spontaneous symmetry breaking BEH [3–5].

Suppose all coefficients $\mu_l^2$ except one are equal to zero, $\mu_1^2 = \mu_2^2 = \ldots = \mu_{N-1}^2 = 0$, and only the last coefficient is nonzero, $\mu_N^2 \neq 0$, and has a negative value, $\mu_N^2 = -|\mu_N^2|$. Let also all mixing parameters between the theories at the adjacent levels (links in the chain) be small, namely

$$\frac{k_{12}}{\lambda_1}, \frac{k_{12}}{\lambda_2}, \frac{k_{23}}{\lambda_2}, \frac{k_{23}}{\lambda_3}, \ldots, \frac{k_{N-1N}}{\lambda_{N-1}}, \frac{k_{N-1N}}{\lambda_N} \ll 1. \tag{2}$$

The exact formulas for the case of two generations as well as a proof of the diagonalization of the mass matrix $N \times N$ are given in the appendix. The boson $h_N$ at the last step of the chain acquires the mass

$$m_{N-1}^2 \simeq k_{N-1N} v_N^2 \simeq \frac{k_{N-1N}^2 m_N^2}{2\lambda_N}, \tag{4}$$

and the parameter $\mu_{N-2}$ is induced being proportional to the mixing coefficient $k_{N-2}$ between levels $N-1$ and $N-2$ of the chain. This process continues and, as a result, at the first step of the chain, the $h_1$-boson mass appears,

$$m_1^2 \simeq \frac{k_{12}}{2\lambda_1}, \frac{k_{23}}{2\lambda_2}, \ldots, \frac{k_{N-1N}}{2\lambda_N} m_N^2, \tag{5}$$

with the induced vacuum expectation value

$$v_1^2 \simeq \frac{k_{12}}{2\lambda_1}, \frac{k_{23}}{2\lambda_2}, \ldots, \frac{k_{N-1N}}{2\lambda_N} v_N^2. \tag{6}$$

As one can see, the vacuum expectation value $v_1$ and the Higgs mass $m_1$ can be significantly smaller than the scale $v_N$ and the mass $m_N$, respectively, due to the product of a number of factors proportional to small mixing coefficients. This observation could explain the hierarchy between a small scale of the order of $v_1$ and a very large scale of the order of $v_N$. The scalar field potentials before and after spontaneous symmetry breaking corresponding to the discussed scenario are illustrated in fig. 1.

If we match the smallest vacuum expectation value $v_1$ in the chain with the vacuum expectation value of the Standard Model $v_{SM} = v_{SM}$, the mass $m_1$ of the boson $h_1$ with the SM Higgs boson mass $M_{H_{SM}} = m_{H_{SM}}$, and the coupling $\lambda_1$ with the SM quartic coupling $\lambda_{SM} = \lambda_{SM}$, the SM relation must take place:

$$M_{H_{SM}}^2 = 2\lambda_{SM} v_{SM}^2. \tag{7}$$
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Fig. 1: Illustrative form of potentials before and after spontaneous symmetry breaking induced at the lower levels by symmetry breaking at the highest level. The lowest level corresponds to the smallest scale, and is designated conventionally as electroweak (“EW”). The uppermost level corresponds to the largest scale, and is conventionally designated as the Planck scale (“Planck”).

It is easy to see that, if we divide the left- and right-hand sides of eq. (5) by the left- and right-hand sides of eq. (6), correspondingly, then taking into account eq. (7) we get

$$2\lambda_N = \frac{m_N^2}{v_N^2},$$

which exactly matches eq. (3), as it should.

Suppose that the largest scale in the chain $v_N$ and the corresponding mass $m_N$ are of the order of the Planck scale $\sim 10^{19}$ GeV. Then the product of the factors

$$\varepsilon_{12} \cdot \varepsilon_{23} \cdot \ldots \cdot \varepsilon_{i_{i+1}} \cdot \ldots \cdot \varepsilon_{N-1N},$$

where $\varepsilon_{i_{i+1}} = \sqrt{\frac{k_{i_{i+1}}}{2\lambda_i}}$, should be about $10^{-17} - 10^{-16}$ assuming that the scale $v_1$ is of the order of the electroweak scale $\sim 10^2$ GeV.

The model (1) is renormalizable if the Lagrangians $L_{Fields(i)}$, $i = 1, \ldots, N$ involve the operators with dimension 4 or less. One may expect a serious hierarchy problem appearing in the model. Indeed, the correction to the mass squared of the scalar $h_1$ from the loop contribution of the scalar $h_i$ is proportional to

$$m_i^2 \cdot \log(m_i^2/m_1^2),$$

leading to a very large shift. However in the model (1) after the diagonalization the interaction vertex of the scalars $h_1$ and $h_i$ contains the coupling constant $\varepsilon_{12} \cdot \ldots \cdot \varepsilon_{i-1}$. Therefore, the loop contribution is proportional to

$$\frac{k_{12} \cdot \ldots \cdot k_{i-1i}}{2\lambda_2 \cdot \ldots \cdot 2\lambda_N} \cdot m_i^2 \cdot \log(m_i^2/m_1^2) \approx m_1^2 \cdot \log(m_i^2/m_1^2),$$

demonstrating that the little hierarchy problem does not show up in the model under consideration. The same argument also holds for the case with possible loop contributions to the scalar $h_1$ mass parameter related to some other fields from $L_{Fields(i)}$ when the potentially large correction is suppressed by the product of small mixing parameters.

**Conclusion.** – In this short letter, we discuss a scenario in which a spontaneous symmetry breaking or spontaneous phase transition occurs at some large energy scale. Due to a low mixing, such a phase transition leads to an induced symmetry breaking at some smaller induced scale, which, in turn, due to the next mixing leads to the next induced symmetry breaking at an even smaller scale, and so on. The result is a chain of induced spontaneous symmetry breaking scales. In such a scenario, the large difference between the largest and smallest scales is due to the product of several small factors proportional to the small mixing coefficients, but each of these factors may not be that small.
In each level of induced symmetry breaking there might be its own gauge theory. Obviously the interactions of particles at some level with particles at the lowest level will be smaller and smaller for higher and higher levels being proportional to smaller and smaller mixing coefficients. Once the interactions get small enough particles at the corresponding level may play the role of dark matter particles.

We give an example of a very simple model in which small mixing between the levels in a chain of spontaneous gauge symmetry breaking is due to scalar portals. As some speculation, an example is given for which the difference in the Planck scale and the electroweak scale can be estimated in terms of about nine or five induced spontaneous transitions, assuming the same order of small coefficients $10^{-2}$ and $10^{-4}$, respectively.

Obviously, having several levels more and more weakly interacting with the first level associated with the SM, dark matter might be easily accumulated at far enough levels with small enough couplings to the SM via such a multi-portal scenario.

We considered an example of a chain with mixing between the nearest neighbors, i.e., theories at the adjacent levels connected by mixing through scalar portals. Of course, one can think about more complex mixing not only with the nearest neighbors and not only through scalar portals. In this case, more complex chains with branching and more complex mixing arise.

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APPENDIX

Let the chain contain two levels only as given by eq. (A.1),

\[
L = (D_{(1)}^{(1)} H_1)^\dagger (D_{(1)}^{(1)} H_1) - \mu_1^2 H_1^\dagger H_1 - \lambda_1 (H_1^\dagger H_1)^2 \\
+ k_{12} (H_1^\dagger H_1) (H_2^\dagger H_2) + (D_{(2)}^{(1)} H_2)^\dagger (D_{(2)}^{(1)} H_2) \\
- \mu_2^2 H_2^\dagger H_2 - \lambda_2 (H_2^\dagger H_2)^2,
\]

where we set $\mu_1$ equal to zero ($\mu_1^2 = 0$). In the unitary gauge on both levels the quadratic form of the scalar fields reads as follows:

\[
2\lambda_1^2 \tilde{v}_1^2 \bar{h}_1^2 - 2k_{12} \tilde{v}_1 \tilde{v}_2 \bar{h}_1 \bar{h}_2 + 2\lambda_2 \tilde{v}_2^2 \bar{h}_2^2,
\]

where \[ v_1 = \sqrt{\frac{k_{12}}{2\lambda_1}} \tilde{v}_2, \quad v_2 = \sqrt{\frac{4\lambda_1 |\mu_2|}{4\lambda_1 \lambda_2 - k_{12}^2}} \]

and the mixing coefficient $k_{12}$ as well as the quartic couplings $\lambda_1$ and $\lambda_2$ are chosen to be positive. Rotation from the unphysical fields $h_1$ and $h_2$ to physical fields $\tilde{h}_1$ and $\tilde{h}_2$ with definite masses leads to the following expressions for the masses squared of eigenstates:

\[
m_{1,2}^2 = \lambda_2 + \frac{k_{12}}{2} \pm \lambda_2 \sqrt{\left(1 - \frac{k_{12}^2}{2\lambda_2^2}\right) + \frac{k_{12}^2}{2\lambda_2^2}} v_2^2.
\]

(A.4)

Assuming

\[
\frac{k_{12}}{\lambda_1} \frac{k_{12}}{\lambda_2} \ll 1,
\]

one gets the following values for the masses:

\[
m_2^2 = 2\lambda_2 v_2^2, \quad m_1^2 = k_{12} v_2^2
\]

(A.5)

in complete agreement with eqs. (3) and (4).

The mixing matrix corresponding to the above example with two levels looks as follows:

\[
\begin{pmatrix}
2\lambda_1 v_1^2 & -k_{12} v_1 v_2 \\
k_{12} v_1 v_2 & 2\lambda_2 v_2^2
\end{pmatrix}.
\]

This is easily generalized to the model given in (1). The mixing matrix has the following form:

\[
\begin{pmatrix}
x_{11} & x_{12} & 0 & 0 & 0 & \ldots & 0 \\
x_{21} & x_{22} & x_{23} & 0 & 0 & \ldots & 0 \\
0 & x_{32} & x_{33} & x_{34} & 0 & \ldots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & \ldots & x_{N-2N-1} \\
0 & 0 & 0 & 0 & 0 & \ldots & x_{NN-1} \\
0 & 0 & 0 & 0 & 0 & \ldots & x_{NN}
\end{pmatrix},
\]

where

\[
x_{11} = 2\lambda_1 v_1^2, \quad x_{12} = x_{21} = -k_{12} v_1 v_2, \quad x_{22} = \lambda_2 v_2^2, \quad x_{23} = x_{32} = -k_{23} v_2 v_3, \quad x_{33} = 2\lambda_3 v_3^2, \quad x_{34} = x_{43} = -k_{34} v_3 v_4, \quad x_{N-1N-1} = 2\lambda_{N-1} v_{N-1}^2, \quad x_{N-2N-1} = -k_{N-2N-1} v_{N-1} v_{N-2}, \quad x_{NN} = 2\lambda_N v_N^2, \quad x_{N-1N} = x_{NN-1} = -k_{N-1N} v_{N-1} v_N.
\]

Such a $N \times N$ matrix can be diagonalized using the product of rotation matrices that sequentially lead to a
diagonal form of the $2 \times 2$ matrix blocks:

\[
\begin{pmatrix}
\cos \theta_1 & \sin \theta_1 & 0 & 0 & 0 & \ldots & 0 \\
-\sin \theta_1 & \cos \theta_1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 & 0 & 0 & \ldots & 0 \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 & 0 & 0 & \ldots & 0 \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 0 & 0 & \ldots & 0 & \ldots & \ldots \\
0 & 0 & 0 & \ldots & 0 & \ldots & \ldots \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

where $\theta_i, i = 1, \ldots, N$ are the rotation angles. Under conditions (2) and (3) one gets expressions (4) and (5) for the mass eigenstates.

REFERENCES

[1] ATLAS Collaboration (Aad G. et al.), Phys. Lett. B, 716 (2012) 1 (arXiv:1207.7214 [hep-ex]).
[2] CMS Collaboration (Chatrchyan S. et al.), Phys. Lett. B, 716 (2012) 30 (arXiv:1207.7235 [hep-ex]).
[3] Higgs P. W., Phys. Rev. Lett., 13 (1964) 508.
[4] Englert F. and Brout R., Phys. Rev. Lett., 13 (1964) 321.
[5] Guralnik G. S., Hagen C. R. and Kibble T. W. B., Phys. Rev. Lett., 13 (1964) 585.
[6] Lee B. W., Quigg C. and Thacker H. B., Phys. Rev. Lett., 38 (1972) 883.
[7] Lee B. W., Quigg C. and Thacker H. B., Phys. Rev. D, 16 (1977) 1519.
[8] Chanowitz M. S., Universal W, Z Scattering Theorems and No Lose Corollary for the SSC, in Proceedings of the 23rd International Conference on High-Energy Physics, LBL-21973, August 1986.
[9] Dicus D. A. and Mathur V. S., Phys. Rev. D, 7 (1973) 3111.
[10] Silveira V. and Zee A., Phys. Lett. B, 161 (1985) 136.
[11] McDonald J., Phys. Rev. D, 50 (1994) 3637 (arXiv:hep-ph/90072143).
[12] Burgess C. P., Pospelov M. and Ter Veldhuis T., Nucl. Phys. B, 619 (2001) 799 (arXiv:hep-ph/0111335).
[13] Davoudiasl H., Kitano R., Li T. and Murayama H., Phys. Lett. B, 609 (2005) 117.
[14] Schabinger R. M. and Wells J. D., Phys. Rev. D, 72 (2005) 093007 (arXiv:hep-ph/0509209).
[15] Barbieri R., Gregoire T. and Hall L. J., Mirror world at the large hadron collider, arXiv:hep-ph/0509242 (2005).
[16] Patr B. and Wilczek F., Higgs-field portal into hidden sectors, arXiv:hep-ph/0506188 (2006).
[17] Strassler M. J. and Zurek K. M., Phys. Lett. B, 661 (2008) 263 (arXiv:hep-ph/0605193).
[18] Bowen M., Cui Y. and Wells J. D., JHEP, 03 (2007) 036 (arXiv:hep-ph/0701035).
[19] Barger V., Langacker P., McCaskey M., Ramsey-Musolf M. J. and Shaughnessy G., Phys. Rev. D, 77 (2008) 035005 (arXiv:0706.4311 [hep-ph]).
[20] Barger V., Langacker P., McCaskey M., Ramsey-Musolf M. J. and Shaughnessy G., Phys. Rev. D, 79 (2009) 015018 (arXiv:0811.0393 [hep-ph]).
[21] He X.-G., Li T., Li X.-Q., Tandean J. and Tsai H.-C., Phys. Rev. D, 79 (2009) 023521 (arXiv:0811.0658 [hep-ph]).
[22] Bock S., Lafaye R., Plehn T., Rauch M., Zerwas D. and Zerwas P. M., Phys. Lett. B, 694 (2011) 44 (arXiv:1007.2645 [hep-ph]).
[23] Englert C., Plehn T., Zerwas D. and Zerwas P. M., Phys. Lett. B, 703 (2011) 298 (arXiv:1106.3097 [hep-ph]).
[24] Lebedev O. and Lee H. M., Eur. Phys. J. C, 71 (2011) 1821 (arXiv:1105.2284 [hep-ph]).
[25] Mambrini Y., Phys. Rev. D, 84 (2011) 115017 (arXiv:1108.0671 [hep-ph]).
[26] Belanger G., Kannike K., Pukhov A. and Raidal M., JCAP, 01 (2013) 022 (arXiv:1211.1014 [hep-ph]).
[27] Cline J. M., Kainulainen K., Scott P. and Weniger C., Phys. Rev. D, 88 (2013) 055025; 92 (2015) 039906 (arXiv:1306.4710 [hep-ph]).
[28] Gabrielli E., Heikinheimo M., Kannike K., Racioppi A., Raidal M. and Spethmann C., Phys. Rev. D, 89 (2014) 015017 (arXiv:1309.6632 [hep-ph]).
[29] Falkowski A., Gross C. and Lebedev O., JHEP, 05 (2015) 057 (arXiv:1502.01361 [hep-ph]).
[30] Robens T. and Stefaniak T., Eur. Phys. J. C, 75 (2015) 104 (arXiv:1501.02234 [hep-ph]).
[31] Martín Lozano V., Moreno J. M. and Park C. B., JHEP, 08 (2015) 004 (arXiv:1501.03799 [hep-ph]).
[32] Casas J. A., Cerdeño D. G., Moreno J. M. and Quilis J., JHEP, 05 (2017) 036 (arXiv:1701.08134 [hep-ph]).
[33] The GAMBIT Collaboration (Athron P. et al.), Eur. Phys. J. C, 77 (2017) 568 (arXiv:1705.07931 [hep-ph]).
[34] Arcadi G., Dijoudi A. and Raidal M., Phys. Rep., 842 (2020) 1 (arXiv:1903.03616 [hep-ph]).
[35] Kannike K., Kohyama N. and Raidal M., Nucl. Phys. B, 968 (2021) 115441 (arXiv:2010.09718 [hep-ph]).
[36] Díaz Sáez B., Möhling K. and Stöckinger D., Two real scalar WIMP model in the assisted freeze-out scenario, arXiv:2103.17064 [hep-ph] (2021).
[37] Lebedev O., Prog. Part. Nucl. Phys., 120 (2021) 103881 (arXiv:2104.03342 [hep-ph]).