A strategy for a simultaneous measurement of $CP$ violation parameters related to the $CKM$ angle $\gamma$ in multiple $B$ meson decay channels

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Abstract

Several methods exist \cite{1,2,3,4,5} to measure $CP$ violation observables related to the $CKM$ angle $\gamma$. These observables are different for every $B$ meson decay channel. However, the information they contain on $\gamma$ is encoded in a similar way for all of them. This paper describes a strategy for a simultaneous measurement including several $B$ meson decay channels, while taking into account any possible correlations between them.
1 Introduction

The CKM angle $\gamma$ is currently the least constrained of the angles of the unitarity triangle. It is defined as

$$\gamma = \arg \left( \frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right).$$

Because $V_{ub}$ is the largest contribution to this phase, $\gamma$ is measured in decays that involve an interference between the transitions $b \to u$ and $b \to c$, such as $B^\pm \to D(\ast)K^\ast(\pm)$, $B^0 \to DK^{*0}$ or, eventually in the future, $B^\pm \to D(\ast)\pi^\pm$, where $D$ represents some admixture of the $D$ meson flavor eigenstates $D^0$ and $\overline{D}^0$, and the notation $(\ast)$ indicates a regular expression representing either one $\ast$ or none.

Several methods exist [1, 2, 3, 4, 5] to measure different CP violation observables that constrain the CKM angle $\gamma$. This paper presents an approach to simultaneously measure a reduced set of observables in multiple $B$ decay channels, independently of which of these methods is used for any $B$ decay considered.

2 Unified approach

2.1 The admixture coefficients $z^c_{\pm}$

Several $B$ meson decay channels produce admixtures of neutral $D$ mesons that involve $\gamma$. The notation for the $D$ meson states is $|D^0\rangle$ and $|\overline{D}^0\rangle$ for the flavor eigenstates and $|D^\pm\rangle$ for the $D$ meson produced in a $B$ decay. In this paper, $|D^+_c\rangle$ denotes the $D$ meson produced in a $B^+_c$ or $B^0_c$ decay, and $|D^-_c\rangle$ denotes the $D$ meson produced in a $B^-_c$ or $\overline{B}^0_c$ decay. A superscript $c$ is used to denote the $B$ decay channel. In general, one can write

$$|D^-_c\rangle \sim |D^0\rangle + z^c_- |\overline{D}^0\rangle, \quad |D^+_c\rangle \sim |\overline{D}^0\rangle + z^c_+ |D^0\rangle,$$

(1)

where $A_D = \langle f |H|D^0\rangle$, $A_{\overline{D}} = \langle f |H|\overline{D}^0\rangle$ and $A^c_\pm = \langle f |H|D^\pm_c\rangle$. The complex coefficients $z^c_\pm$ are generally specific for every $B$ decay channel $c$, and are typically expressed in either Cartesian or polar coordinates as

$$z^c_\pm = x^c_\pm + i y^c_\pm = r^c_\pm e^{i\delta^c_\pm e^{\pm i\gamma}},$$

(2)

where all the parameters with a subscript or superscript $c$ are specific for every $B$ decay channel. The total number of parameters in Cartesian coordinates for $N$ different $B$ decay channels is $4N$.

It should be noted that, by defining

$$z_c = r_c e^{i\delta_c},$$

(3)

we have

$$z^c_\pm = z_c e^{\pm i\gamma},$$

(4)

This reveals a clear channel invariant,

$$\frac{z^c_+}{z^c_-} = e^{2i\gamma} \quad \Rightarrow \quad \gamma = \frac{1}{2} \arg \left( \frac{z^c_+}{z^c_-} \right).$$

(5)
If the $z_{\pm}$ coefficients are both multiplied by any complex coefficient $\xi$, the result will contain the exact same information on $\gamma$. In particular, it is always possible to express the $z_{\pm}$ coefficients for channel $c$ as

$$z_{\pm}^c = \xi_c z_{\pm}^{DK},$$

where

$$\xi_c = \frac{z_c}{z^{DK}}.$$  

(6)

(7)

Notice that, by definition, from expressions (3) and (7), the $\xi_c$ coefficients do not depend on $\gamma$ so they are formally nuisance parameters.

In order to simplify the notation, this paper uses $z_{\pm} = z_{\pm}^{DK}$, where $DK$ refers to the $B^\pm \rightarrow DK^\pm$ decay mode, so that $z_{\pm}^c = \xi_c z_{\pm}$.

Performing a simultaneous fit for the Cartesian parameters using these $\xi_c$ coefficients reduces the number of independent parameters in the fit from $4N$ to only $2(N + 1)$ (4 for the $DK$ channel, and then only 2 for each of the rest of the channels). This is only one more parameter than a simultaneous fit for the polar coordinates, but it has the advantage that, with a Cartesian fit, the real and imaginary components of $z_{\pm}$ and $\xi_c$ are expected to exhibit Gaussian behavior.

### 2.2 The $\eta$ function

It is useful to define the $\eta$ function as

$$\eta(a, b, \kappa) = |a|^2 + |b|^2 + 2\kappa \text{Re}(a^*b),$$

(8)

where $a, b \in \mathbb{C}$ and $\kappa \in \mathbb{R}$. The $\kappa$ coefficient is called the coherence factor because it gives an idea of the fraction of coherent sum that contributes to $\eta$,

$$\eta(a, b, \kappa) = \kappa |a + b|^2 + (1 - \kappa) (|a|^2 + |b|^2).$$

(9)

In this paper, when the coherence factor argument is omitted it should be assumed that it is implicit. If one of the complex arguments is omitted, it should be assumed that it is 1. So,

$$\eta(a) = \eta(a, 1, \kappa).$$

(10)

The $\eta$ function has the following properties:

$$\eta(a, b, \kappa) = \eta(b, a, \kappa), \quad \text{Symmetric}$$

(11)

$$\eta(a, b, \kappa) = |a|^2 \eta\left(1, \frac{b}{a}, \kappa\right), \quad \text{Scaling}$$

(12)

### 2.3 Signal amplitude

The signal probability distribution function is proportional to the squared amplitude integrated over a certain region of the $B$ phase space,

$$p_{\pm}^c \sim \int dp_B |A_{\pm}^c|^2.$$  

(13)

If $A_c$ is the decay amplitude corresponding to a $b \rightarrow c$ transition and $A_u e^{\pm i\gamma}$ is the decay amplitude corresponding to a $b \rightarrow u$ transition, then

$$A_- \sim A_c A_D + A_u e^{-i\gamma} A_{\bar{D}},$$

(14)

$$A_+ \sim A_c A_D + A_u e^{+i\gamma} A_{\bar{D}}.$$  

(15)
It is important not to forget that \( A_c \) and \( A_u \) are different for each \( B \) decay channel, but to avoid cluttering the following notation with too many indices, I skip them for just a moment.

In the case of a 2-body \( B \) decay, such as \( B^\pm \to DK^\pm \), the \( B \) decay amplitudes \( A_c \) and \( A_u \) are constants, and one can write

\[
A_- \sim A_D + z_- A_D, \tag{16}
\]

\[
A_+ \sim A_D + z_+ A_D, \tag{17}
\]

where

\[
z_\mp = \frac{A_u}{A_c} e^{\pm i\gamma}. \tag{18}
\]

This implies that, for 2-body decays,

\[
p_- \sim \eta(A_D, z_- A_D), \tag{19}
\]

\[
p_+ \sim \eta(A_D, z_+ A_D). \tag{20}
\]

In the case of a multibody \( B \) decay with 3 or more particles in the final state, such as \( B^0 \to DK\pi \), the amplitude \( |A_\pm|^2 \) is usually integrated over some region in the \( B \) decay phase space (around the \( K^{*0} \) resonance for \( B^0 \to DK\pi \)).

By squaring the modulus of expressions (16) and (17) and by defining

\[
N_{\alpha\beta} = \int dP_B A^*_\alpha A_\beta, \tag{21}
\]

\[
X_{\alpha\beta} = \frac{N_{\alpha\beta}}{\sqrt{N_{\alpha\alpha} N_{\beta\beta}}}, \tag{22}
\]

one can write

\[
p_- \sim |A_D|^2 + \frac{N_{uu}}{N_{cc}} |A_D|^2 + 2 |X_{cu}| \operatorname{Re} \left( \sqrt{\frac{N_{uu}}{N_{cc}}} \frac{X_{cu}}{|X_{cu}|} e^{-i\gamma} A^*_D A_D \right), \tag{23}
\]

\[
p_+ \sim |A_D|^2 + \frac{N_{uu}}{N_{cc}} |A_D|^2 + 2 |X_{cu}| \operatorname{Re} \left( \sqrt{\frac{N_{uu}}{N_{cc}}} \frac{X_{cu}}{|X_{cu}|} e^{+i\gamma} A^*_D A_D \right). \tag{24}
\]

Notice that, because of the Cauchy-Schwarz inequality, \(|X_{\alpha\beta}| \leq 1\). Also, one can define

\[
\kappa = |X_{cu}|, \tag{25}
\]

\[
r = \sqrt{\frac{N_{uu}}{N_{cc}}}, \tag{26}
\]

\[
e^{i\delta} = \frac{X_{cu}}{|X_{cu}|}, \tag{27}
\]

\[
z = r e^{i\delta}, \tag{28}
\]

\[
z_\pm = z e^{\pm i\gamma}, \tag{29}
\]

so, adding back the channel index and using the formalism described in §2.1, the signal amplitude probability distribution can always be expressed as

\[
p_c^- \sim \eta(A_D, \xi_c z_- A_D, \kappa_c) = |A_D|^2 + |\xi_c z_- A_D|^2 + 2 \kappa_c \operatorname{Re} (\xi_c z_- A_D^* A_D), \tag{30}
\]

\[
p_c^+ \sim \eta(A_D, \xi_c z_+ A_D, \kappa_c) = |A_D|^2 + |\xi_c z_+ A_D|^2 + 2 \kappa_c \operatorname{Re} (\xi_c z_+ A_D^* A_D). \tag{31}
\]

It should be noted that these expressions describe the physics of the admixture that leads to the different \( CP \) observables used to measure \( \gamma \), but they are not specific to any method.
3 Specific equations for established methodologies

3.1 GLW equations

The GLW method uses states that are accessible from only one of the flavor eigenstates, such that $A_f^D = \langle f | D | H | D \rangle$, $A_f^{\bar{D}} = \langle f | \bar{D} | H | D \rangle$, but $\langle f | D | H | D \rangle = \langle f | H | D \rangle = 0$, and, on the other hand, states that are accessible from one of the CP eigenstates $| D^\pm \rangle$, such that $A_{CP}^\pm = \langle f_\pm | H | D^\pm \rangle$, but $\langle f_\pm | H | D^\pm \rangle = 0$.

The observables of interest are

$$R_{CP}^{\pm} = \frac{\Gamma (B^- \rightarrow D^\pm CP h^-) + \Gamma (B^+ \rightarrow D^\pm CP h^+)}{\Gamma (B^- \rightarrow D^0 h^-) + \Gamma (B^+ \rightarrow D^0 h^+)} = \frac{1}{2} \left| \frac{A_{CP}^\pm}{A_f^D} \right|^2 \frac{\eta (\pm \xi_c z_-) + \eta (\pm \xi_c z_+)}{2}$$  \hspace{1cm} (32)

$$A_{CP}^{\pm} = \frac{\Gamma (B^- \rightarrow D^\pm CP h^-) - \Gamma (B^+ \rightarrow D^\pm CP h^+)}{\Gamma (B^- \rightarrow D^\pm CP h^-) + \Gamma (B^+ \rightarrow D^\pm CP h^+)} = \frac{\eta (\pm \xi_c z_-) - \eta (\pm \xi_c z_+)}{\eta (\pm \xi_c z_-) + \eta (\pm \xi_c z_+)}$$  \hspace{1cm} (33)

3.2 ADS equations

The ADS method uses states that are accessible from both flavor eigenstates. With the convention $CP | D^0 \rangle = | \bar{D}^0 \rangle$ and assuming no direct CP violation in the $D$ decay, one can define the $\rho$ ratio of amplitudes as

$$\rho = \frac{A_f^D}{A_f^\bar{D}},$$  \hspace{1cm} (34)

and

$$\Gamma_{fav}^\pm = \Gamma (B^\pm \rightarrow D_{fav} h^\pm) = \left| A_{D}^{\pm} A_f^D \right|^2 \eta (1, \rho \xi_c z_{\pm}),$$  \hspace{1cm} (35)

$$\Gamma_{sup}^\pm = \Gamma (B^\pm \rightarrow D_{sup} h^\pm) = \left| A_{D}^{\pm} A_f^\bar{D} \right|^2 \eta (\rho, \xi_c z_{\pm}),$$  \hspace{1cm} (36)

where $D_{sup}$ and $D_{fav}$ refer to suppressed and favored decay modes of the produced $D$ meson.

The observables of interest are

$$R_{ADS}^{\pm} = \frac{\Gamma_{sup}^{\pm}}{\Gamma_{fav}^{\pm}} = \frac{\eta (\rho, \xi_c z_{\pm})}{\eta (1, \rho \xi_c z_{\pm})}.$$  \hspace{1cm} (37)

It is also frequent to use the observables

$$R_{ADS}^{c} = \frac{\Gamma_{sup}^{c} + \Gamma_{sup}^{\pm}}{\Gamma_{fav}^{c} + \Gamma_{fav}^{\pm}} = \frac{\eta (\rho, \xi_c z_-) + \eta (\rho, \xi_c z_+)}{\eta (1, \rho \xi_c z_-) + \eta (1, \rho \xi_c z_+)},$$  \hspace{1cm} (38)

$$A_{ADS}^{sup} = \frac{\Gamma_{sup}^{c} - \Gamma_{sup}^{\pm}}{\Gamma_{sup}^{c} + \Gamma_{sup}^{\pm}} = \frac{\eta (\rho, \xi_c z_-) - \eta (\rho, \xi_c z_+)}{\eta (\rho, \xi_c z_-) + \eta (\rho, \xi_c z_+)},$$  \hspace{1cm} (39)

$$A_{ADS}^{fav} = \frac{\Gamma_{fav}^{c} - \Gamma_{fav}^{\pm}}{\Gamma_{fav}^{c} + \Gamma_{fav}^{\pm}} = \frac{\eta (1, \rho \xi_c z_-) - \eta (1, \rho \xi_c z_+)}{\eta (1, \rho \xi_c z_-) + \eta (1, \rho \xi_c z_+)}.$$  \hspace{1cm} (40)

3.3 GGSZ equations

The GGSZ method uses 3 or more body decays to final states that can be accessed from any of $| D^0 \rangle$ or $| \bar{D}^0 \rangle$. As opposed to GLW or ADS, this method does not involve intermediate observables, and the goal is to fit for the CP observables, using equations \[30\]-\[31\] directly.
3.4 Time dependent equations

The time evolution of the amplitude of the $B_s \rightarrow D_s K$ decay is governed by the equation

$$A_{D_s^+}^{B_s}(t) = A_{D_s^+}^{B_s}[g_+(t) + \lambda_+g_-(t)], \quad (41)$$

where $t$ is the $B_s$ meson lifetime and $g_\pm(t)$ are the functions that describe the mixing of the $B$ meson flavor eigenstates. The $\lambda_\pm$ parameters can be expressed as

$$\lambda_\pm = \xi_c z_\pm e^{\pm i\phi_q}, \quad (42)$$

where the channel $c$ is $B_s \rightarrow D_s K$ in this case.

Unfortunately, current measurements of $\gamma$ in decays with $B$ meson mixing introduce intermediate parameters instead of targeting the $z_c^\pm$ observables themselves. In these cases, the squared amplitude of the time-dependent probability distribution function is expressed as

$$e^{-\Gamma t} \frac{1 + |\lambda_-|^2}{2} \left[ C_f \frac{1 - |\lambda_-|^2}{1 + |\lambda_-|^2} \cos(x\Gamma t) - \frac{2 \text{Re}(\lambda_-)}{1 + |\lambda_-|^2} \sinh(x\Gamma t) - \frac{2 \text{Im}(\lambda_-)}{1 + |\lambda_-|^2} \sin(y\Gamma t) \right], \quad (43)$$

and similarly for the conjugated amplitude, where $x$ and $y$ are parameters that describe the $B$ meson mixing and $\Gamma$ is its decay rate. This approach makes this kind of analysis observable based, but even in this case, it is possible to introduce the $C_f, A_{\Delta \Gamma}^f$ and $S_f$ parameters, and the similar parameters for the conjugate decay, in a simultaneous fit that shares the $z_\pm$ parameters with other $B$ decay channels and only introduces $\xi_c$ as a new coefficient.

4 Experimental considerations

There are several advantages in a simultaneous measurement, compared to a combination of standalone measurements:

- It allows to properly take into account the correlations introduced by crossfeed components that are signal in one channel but background in another.

- For $N$ $B$ decay channels, the number of parameters in the fit is reduced from $4N$ to $2N+2$ in Cartesian coordinates or $2N+1$ in polar coordinates. The minimum number of physical parameters is $2N+1$ and, introducing just one additional redundant parameter, all the observables exhibit Gaussian behavior.

- Systematic uncertainties can also be evaluated simultaneously, with $B$ decay channel correlations properly taken into account.
5 Sensitivity study

As a proof of concept, I have used \texttt{cfit} [6] to generate 1000 experiments of toy Monte Carlo signal events for the 4 $B$ decay channels $B^\pm \to D K^\pm$, $B^0 \to D K^{\ast 0}$, $B^\pm \to D^{\ast 0} K^\pm$ and $B^\pm \to D \pi^\pm$, and for the 2 $D$ decay channels $K_s \pi^+ \pi^-$ and $K_s K^+ K^-$, using world averages from UTfit [2]. Since there is no information on the admixture coefficients for $B^\pm \to D \pi^\pm$, and considering that, for this channel, they are expected to be one or der of magnitude smaller than for $B^\pm \to D K^\pm$, I used $\xi_{DK} = 0.08 + 0.06 i$. The size of the toy experiments has been estimated for 3 ifb of LHCb data, using world average branching fractions and reasonable guesses for the efficiencies.

I have used the GGSZ method to perform a fit to the $CP$ observables $z_\pm$ and $\xi_\epsilon$ presented in [2,3]. To ensure convergence, I have used a triple fit strategy, with the following fit steps:

1. Fit for $z_\pm$ to the $D K$ sample only.
2. Fix $z_\pm$ from the previous step and fit for $\xi_{DK^{\ast 0}}$, $\xi_{D^{\ast} K}$ and $\xi_{D \pi}$ to the whole sample.
3. Float all parameters using the previous fit results as initial values.

All the fits for the $z_\pm$ and $\xi_\epsilon$ parameters have been obtained using \texttt{cfit} [6].

A standalone fit to $B^\pm \to D K^\pm$ is used as reference, and several simultaneous fits are obtained by progressively adding more channels to the approach. The obtained statistical uncertainties and biases are summarised in table 1, where all collections include $B^\pm \to D K^\pm$ and the $K_s \pi^+ \pi^-$ and $K_s K^+ K^-$ $D$ decay channels are considered.

| collection | $\sigma_{c_x}$ | $\sigma_{c_y}$ | $\sigma_{c_\epsilon}$ | $\sigma_{c_\phi}$ | $\text{Re}(\sigma_{c_\xi_D})$ | $\text{Im}(\sigma_{c_\xi_D})$ | $\text{Re}(\sigma_{c_\xi_{DK^{\ast 0}}})$ | $\text{Im}(\sigma_{c_\xi_{DK^{\ast 0}}})$ | $\text{Re}(\sigma_{c_\xi_{D^{\ast} K}})$ | $\text{Im}(\sigma_{c_\xi_{D^{\ast} K}})$ | $\text{Re}(\sigma_{c_\xi_{D \pi}})$ | $\text{Im}(\sigma_{c_\xi_{D \pi}})$ |
|------------|----------------|----------------|----------------------|----------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $DK^\pm$   | 0.0172         | 0.0186         | 0.0210               | 0.0240               | —                          | —                          | —                          | —                          | —                          | —                          | —                          | —                          |
| $D \pi^\pm$| 0.0175         | 0.0185         | 0.0212               | 0.0245               | 0.0673                     | 0.0794                     | —                          | —                          | —                          | —                          | —                          | —                          |
| $DK^{\ast 0}$| 0.0149       | 0.0169         | 0.0211               | 0.0236               | —                          | —                          | 2.11                       | 2.26                       | —                          | —                          | —                          | —                          |
| $D \pi^{\ast 0}$| 0.0151      | 0.0170         | 0.0211               | 0.0235               | 0.0648                     | 0.0750                     | 2.09                       | 2.26                       | —                          | —                          | —                          | —                          |
| $D^{\ast} K^\pm$| 0.0146      | 0.0169         | 0.0211               | 0.0239               | —                          | —                          | —                          | —                          | 0.442                      | 0.426                      | —                          | —                          |
| all        | 0.0132         | 0.0159         | 0.0208               | 0.0233               | 0.0654                     | 0.0739                     | 2.02                       | 2.28                       | 0.429                      | 0.418                      | —                          | —                          |

| collection | $\delta x_\pm$ | $\delta y_\pm$ | $\delta x_\epsilon$ | $\delta y_\epsilon$ | $\text{Re}(\delta x_D)$ | $\text{Im}(\delta x_D)$ | $\text{Re}(\delta x_{DK^{\ast 0}})$ | $\text{Im}(\delta x_{DK^{\ast 0}})$ | $\text{Re}(\delta x_{D^{\ast} K})$ | $\text{Im}(\delta x_{D^{\ast} K})$ | $\text{Re}(\delta x_{D \pi})$ | $\text{Im}(\delta x_{D \pi})$ |
|------------|----------------|----------------|----------------------|----------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $DK^\pm$   | -0.00198       | -0.00209       | -0.00225             | -0.00178             | —                          | —                          | —                          | —                          | —                          | —                          | —                          | —                          |
| $D \pi^\pm$| -0.00207       | -0.00077       | -0.00154             | -0.00109             | 0.0133                      | 0.00917                     | —                          | —                          | —                          | —                          | —                          | —                          |
| $DK^{\ast 0}$| -0.00184      | -0.00141       | -0.00185             | -0.00117             | —                          | —                          | -0.149                      | -0.411                      | —                          | —                          | —                          | —                          |
| $D \pi^{\ast 0}$| -0.00162     | -0.00059       | -0.00154             | -0.00096             | 0.0137                      | 0.00782                     | -0.150                      | -0.448                      | —                          | —                          | —                          | —                          |
| $D^{\ast} K^\pm$| -0.00207     | 0.00018        | -0.00138             | -0.00072             | —                          | —                          | —                          | —                          | -0.00980                    | -0.0206                    | —                          | —                          |
| all        | -0.00165       | 0.00065        | -0.00143             | -0.00058             | 0.0144                      | 0.00739                     | -0.126                      | -0.394                      | 0.00078                     | -0.0094                    | —                          | —                          |

Table 1: Statistical uncertainties (upper part) and biases (lower part) of all the parameters involved in each fit. All collections include $B^\pm \to D K^\pm$ plus the indicated $B$ decay channels.

It should be noted that including $B^\pm \to D \pi^\pm$ does not improve the sensitivity on $\gamma$ with the used sample size. However, each other additional channel contributes significantly to a smaller uncertainty on the $(x, y)_{\pm}$ parameters. All the biases obtained on any parameter are at least an order of magnitude smaller than the statistical uncertainty on that parameter.
6 Conclusions

I have presented an approach to measure the CP parameters $z_\pm$ simultaneously in different $B$ decay channels. This approach allows to properly take into account the correlations introduced by crossfeed components that are signal in one channel but background in another, it significantly reduces the number of parameters in the fit and allows a common treatment of the systematic uncertainties.

A more detailed study with realistic backgrounds expected at LHCb will follow.

Acknowledgements

I wish to thank the Science and Technology Facilities Council (STFC, UK). I am very thankful to my colleagues at Cambridge, Valerie Gibson, Susan Haines and Chris Jones, for very interesting discussions.

References

[1] M. Gronau and D. London, *How to determine all the angles of the unitarity triangle from $B^0_d \to D K_S$ and $B^0_s \to D \phi$*, Phys. Lett. B 253 (1991) 483.

[2] M. Gronau and D. Wyler, *On determining a weak phase from CP asymmetries in charged $B$ decays*, Phys. Lett. B 265 (1991) 172.

[3] D. Atwood, I. Dunietz, and A. Soni, *Enhanced CP violation with $B \to K D^0 (\bar{D}^0)$ modes and extraction of the CKM angle $\gamma$*, Phys. Rev. Lett. 78 (1997) 3257, arXiv:hep-ph/9612433.

[4] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining $\gamma$ using $B^\pm \to D K^\mp$ with multibody $D$ decays*, Phys. Rev. D 68 (2003), no. 5 054018.

[5] R. Fleischer, *New strategies to obtain insights into CP violation through $B_{(s)} \to D_{(s)}^{\pm} K^{\mp}$, $D_{(s)}^{\star \pm} K^{\mp}$, $D_{(s)}^{\star \pm} \pi^{\mp}$, and $B_{(d)} \to D^{\pm} \pi^{\mp}$, $D^{\star \pm} \pi^{\mp}$, ... decays*, Nucl. Phys. B671 (2003) 459, arXiv:hep-ph/0304027.

[6] J. Garra Ticó, *The cfit library*, https://github.com/cfit.

[7] UTfit Collaboration, A. Bevan et al., Summer 2014 combination results. Updated results and plots available at https://www.utfit.org/foswiki/bin/view/UTfit.