Feynman’s Entropy and Decoherence Mechanism

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Abstract

If we reduce coherence in a given quantum system, the result is an increase in entropy. Does this necessarily convert this quantum system into a classical system? The answer to this question is No. The decrease of coherence means more uncertainty. This does not seem to make the system closer to classical system where there are no uncertainties. We examine the problem using two coupled harmonic oscillators where we make observations on one of them while the other oscillator is assumed to be unobservable or to be in Feynman’s rest of the universe. Our ignorance about the rest of the universe causes an increase in entropy. However, does the system act like a classical system? The answer is again No. When and how does this system appear like a classical system? It is shown that this paradox can be resolved only if measurements are taken along the normal coordinates. It is also shown that Feynman’s parton picture is one concrete physical example of this decoherence mechanism.
I. INTRODUCTION

According to Feynman, the adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing [1]. Feynman wrote many papers on different subjects of physics, but they are coming from one paper according to him. We are not able to combine all of his papers, but we can consider three of his papers published during the period 1969-72.

In this report, we would like to consider Feynman’s 1969 report on partons [2], the 1971 paper he published with his students on the quark model based on harmonic oscillators [3], and the chapter on density matrix in his 1972 book on statistical mechanics [4]. In these three different papers, Feynman deals with three distinct aspects of nature. We shall see whether Feynman was saying the same thing in these papers.

We approach this problem by developing a mathematical instrument which can support Feynman’s physical ideas spelled out in these seemingly different papers. We shall use the mathematics of two coupled harmonic oscillators [5]. The standard procedure for this two-oscillator system is to separate the Hamiltonian using normal coordinates. The transformation to the normal coordinate system becomes very simple if the two oscillators are identical. We shall use this simple mathematics to find a common ground for the above-mentioned articles written by Feynman.

First, let us look at Feynman’s book on statistical mechanics [4]. He makes the following statement about the density matrix. When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts - the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.

In order to see clearly what Feynman had in mind, we use the above-mentioned couples oscillators. One of the oscillators is the world in which we are interested with the other oscillator as the rest of the universe. There will be no effects on the first oscillator if the system is decoupled. Once coupled, we need a normal coordinate system in order separate the Hamiltonian. Then it is straightforward to write down the wave function of the system.

We shall then observe that the mathematics of this oscillator system is directly applicable to Lorentz-boosted harmonic oscillator wave functions, where one variable is the longitudinal coordinate and the other is the time variable. The system is uncoupled if the oscillator wave function is at rest, but the coupling becomes stronger as the oscillator is boosted to a high-speed Lorentz frame [6].

We shall then note that for two-body system, such as the hydrogen atom, there is a time-separation variable which is to be linearly mixed with the longitudinal space-separation variable. This space-separation variable is known as the Bohr radius, but we never talk about the time-separation variable in the present form of quantum mechanics, because this time-separation variable belongs to Feynman’s rest of the universe.

If we pretend not to know this time-separation variable, the entropy of the system will increase when the oscillator is boosted to a high-speed system [7]. Does this increase in entropy correspond to decoherence? Not necessarily. However, in 1969, Feynman observed the parton effect in which a rapidly moving hadron appears as a collection of incoherent partons [4]. This is the decoherence mechanism we like to discuss in this report.
In Sec. II, we review the quantum mechanics of coupled harmonic oscillators in which one of them corresponds to the world in which we do physics, and the other in the rest of the universe. In Sec. III, it is shown that the time-separation variable in a two-body bound state belongs to Feynman’s rest of the universe. It is shown also that Feynman’s oscillator formalism includes this time-separation variable. We review in Sec. IV Feynman’s parton picture. Finally, in Sec. V, we discuss why partons appear as incoherent particles.

II. COUPLED OSCILLATORS

Two coupled harmonic oscillators serve many different purposes in physics. It is well known that this oscillator problem can be formulated into a problem of a quadratic equation in two variables. To make a long story short, let us consider a system of two identical oscillators coupled together by a spring. The Hamiltonian is

\[ H = \frac{1}{2m} \left\{ p_1^2 + p_2^2 \right\} + \frac{1}{2} \left\{ K \left( x_1^2 + x_2^2 \right) + 2Cx_1x_2 \right\}. \]  

(1)

We are now ready to decouple this Hamiltonian by making the coordinate rotation:

\[ y_1 = \frac{1}{\sqrt{2}} (x_1 - x_2), \quad y_2 = \frac{1}{\sqrt{2}} (x_1 + x_2). \]  

(2)

In terms of this new set of variables, the Hamiltonian can be written as

\[ H = \frac{1}{2m} \left\{ \frac{1}{2} \left( e^{-\eta} y_1^2 + e^{\eta} y_2^2 \right) + K \right\}. \]  

(3)

with

\[ \exp(\eta) = \sqrt{\frac{K + C}{K - C}}. \]  

(4)

Thus \( \eta \) measures the strength of the coupling. If \( y_1 \) and \( y_2 \) are measured in units of \( (mK)^{1/4} \), the ground-state wave function of this oscillator system is

\[ \psi_\eta(x_1, x_2) = \left( \frac{1}{\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2} (e^{\eta} y_1^2 + e^{-\eta} y_2^2) \right\}. \]  

(5)

The wave function is separable in the \( y_1 \) and \( y_2 \) variables. However, for the variables \( x_1 \) and \( x_2 \), the story is quite different.

The key question is how quantum mechanical calculations in the world of the observed variable are affected when we average over the other variable. The \( x_2 \) space in this case corresponds to Feynman’s rest of the universe, if we only consider quantum mechanics in the \( x_1 \) space. As was discussed in the literature for several different purposes, the wave function of Eq.(5) can be expanded as

\[ \psi_\eta(x_1, x_2) = \frac{1}{\cosh \eta} \sum_k \left( \frac{\eta}{2} \right)^k \phi_k(x_1) \phi_k(x_2). \]  

(6)

The question then is what lessons we can learn from the situation in which we average over the \( x_2 \) variable.
In order to study this problem, we use the density matrix. From this wave function, we can construct the pure-state density matrix

\[
\rho(x_1, x_2; x'_1, x'_2) = \psi_\eta(x_1, x_2)\psi_\eta(x'_1, x'_2),
\]

(7)

If we are not able to make observations on the \( x_2 \), we should take the trace of the \( \rho \) matrix with respect to the \( x_2 \) variable. Then the resulting density matrix is

\[
\rho(x, x') = \int \psi_\eta(x, x_2) \{\psi_\eta(x', x_2)\}^* \, dx_2.
\]

(8)

We have simplicity replaced \( x_1 \) and \( x'_1 \) by \( x \) and \( x' \) respectively. If we perform the integral of Eq.(8), the result is

\[
\rho(x, x') = \left( \frac{1}{\cosh(\eta/2)} \right)^2 \sum_k \left( \tanh \frac{\eta}{2} \right)^{2k} \phi_k(x)\phi_k^*(x'),
\]

(9)

which leads to \( Tr(\rho) = 1 \). It is also straightforward to compute the integral for \( Tr(\rho^2) \). The calculation leads to

\[
Tr(\rho^2) = \left( \frac{1}{\cosh(\eta/2)} \right)^4 \sum_k \left( \tanh \frac{\eta}{2} \right)^{4k}.
\]

(10)

The sum of this series is \( 1/\cosh(\eta) \), which is smaller than one if the parameter \( \eta \) does not vanish.

This is of course due to the fact that we are averaging over the \( x_2 \) variable which we do not measure. The standard way to measure this ignorance is to calculate the entropy defined as

\[
S = -Tr(\rho \ln(\rho)),
\]

(11)

where \( S \) is measured in units of Boltzmann’s constant. If we use the density matrix given in Eq.(9), the entropy becomes

\[
S = 2 \left\{ \cosh^2 \left( \frac{\eta}{2} \right) \ln \left( \cosh \frac{\eta}{2} \right) - \sinh^2 \left( \frac{\eta}{2} \right) \ln \left( \sinh \frac{\eta}{2} \right) \right\}.
\]

(12)

This expression can be translated into a more familiar form if we use the notation

\[
\tanh \frac{\eta}{2} = \exp \left( -\frac{\hbar \omega}{kT} \right),
\]

(13)

where \( \omega \) is given in Eq.(10).

It is known in the literature that this rise in entropy and temperature causes the Wigner function to spread wide in phase space causing an increase of uncertainty [5]. Certainly, we cannot reach a classical limit by increasing the uncertainty. On the other hand, we are accustomed to think this entropy increase has something to do with decoherence, and we are also accustomed to think the lack of coherence has something to do with a classical limit. Are they compatible? We thus need a new vision in order to define precisely the word “decoherence.”
III. TIME-SEPARATION VARIABLE IN FEYNMAN’S REST OF THE UNIVERSE

Quantum field theory has been quite successful in terms of perturbation techniques in quantum electrodynamics. However, this formalism is basically based on the S matrix for scattering problems and useful only for physically processes where free a set of particles becomes another set of free particles after interaction. Quantum field theory does not address the question of localized probability distributions and their covariance under Lorentz transformations. The Schrödinger quantum mechanics of the hydrogen atom deals with localized probability distribution. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be a time separation between the two constituent particles.

Before 1964 \[10\], the hydrogen atom was used for illustrating bound states. These days, we use hadrons which are bound states of quarks. Let us use the simplest hadron consisting of two quarks bound together an attractive force, and consider their space-time positions \(x_a\) and \(x_b\), and use the variables

\[
X = \frac{(x_a + x_b)}{2}, \quad x = \frac{(x_a - x_b)}{2\sqrt{2}}. \tag{14}
\]

The four-vector \(X\) specifies where the hadron is located in space and time, while the variable \(x\) measures the space-time separation between the quarks. According to Einstein, this space-time separation contains a time-like component which actively participates as can be seen from

\[
\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}, \tag{15}
\]

when the hadron is boosted along the \(z\) direction. In terms of the light-cone variables defined as \[11\]

\[
u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}. \tag{16}
\]

The boost transformation of Eq.(15) takes the form

\[
u' = e^{\eta} \nu, \quad v' = e^{-\eta} v. \tag{17}
\]

The \(u\) variable becomes expanded while the \(v\) variable becomes contracted.

Does this time-separation variable exist when the hadron is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Indeed, this variable belongs to Feynman’s rest of the universe. In this report, we shall see the role of this time-separation variable in decoherence mechanism.

Also in the present form of quantum mechanics, there is an uncertainty relation between the time and energy variables. However, there are no known time-like excitations. Unlike Heisenberg’s uncertainty relation applicable to position and momentum, the time and energy separation variables are c-numbers, and we are not allowed to write down the commutation relation between them. Indeed, the time-energy uncertainty relation is a c-number uncertainty relation \[12\].
How does this space-time asymmetry fit into the world of covariance \[13\]. This question was studied in depth by the present author and his collaborators. The answer is that Wigner’s \(O(3)\)-like little group is not a Lorentz-invariant symmetry, but is a covariant symmetry \[14\]. It has been shown that the time-energy uncertainty applicable to the time-separation variable fits perfectly into the \(O(3)\)-like symmetry of massive relativistic particles \[6\].

The c-number time-energy uncertainty relation allows us to write down a time distribution function without excitations \[6\]. If we use Gaussian forms for both space and time distributions, we can start with the expression

\[
\left(\frac{1}{\pi}\right)^{1/2} \exp \left\{ -\frac{1}{2} \left( z^2 + t^2 \right) \right\}
\]

for the ground-state wave function. What do Feynman et al. say about this oscillator wave function?

In his classic 1971 paper \[3\], Feynman et al. start with the following Lorentz-invariant differential equation.

\[
\frac{1}{2} \left\{ x^\mu - \frac{\partial^2}{\partial x^\mu} \right\} \psi(x) = \lambda \psi(x). \tag{19}
\]

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. Feynman et al. insist on Lorentz-invariant solutions which are not normalizable. On the other hand, if we insist on normalization, the ground-state wave function takes the form of Eq.\(18\). It is then possible to construct a representation of the Poincaré group from the solutions of the above differential equation \[6\]. If the system is boosted, the wave function becomes

\[
\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp \left\{ -\frac{1}{2} \left( e^{-2\eta u^2} + e^{2\eta v^2} \right) \right\}. \tag{20}
\]

This wave function becomes Eq.\(18\) if \(\eta\) becomes zero. The transition from Eq.\(18\) to Eq.\(20\) is a squeeze transformation. The wave function of Eq.\(18\) is distributed within a circular region in the \(uv\) plane, and thus in the \(zt\) plane. On the other hand, the wave function of Eq.\(20\) is distributed in an elliptic region with the light-cone axes as the major and minor axes respectively. If \(\eta\) becomes very large, the wave function becomes concentrated along one of the light-cone axes. Indeed, the form given in Eq.\(20\) is a Lorentz-squeezed wave function.

It is interesting to note that the Lorentz-invariant differential equation of Eq.\(19\) contains the time-separation variable which belongs to Feynman’s rest of the universe. Furthermore, the wave function of Eq.\(18\) is identical to that of Eq.\(5\) for the coupled oscillator system, if the variables \(z\) and \(t\) are replaced \(x_1\) and \(x_2\) respectively. Thus the entropy increase due to the unobservable \(x_2\) variable is applicable to the unobserved time-separation variable \(t\).

IV. FEYNMAN’S PARTON PICTURE

It is a widely accepted view that hadrons are quantum bound states of quarks having localized probability distribution. As in all bound-state cases, this localization condition
is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [3,4]. However, this picture of bound states is applicable only to observers in the Lorentz frame in which the hadron is at rest. How would the hadrons appear to observers in other Lorentz frames? To answer this question, can we use the picture of Lorentz-squeezed hadrons discussed in Sec. III.

The radius of the proton is $10^{-5}$ of that of the hydrogen atom. Therefore, it is not unnatural to assume that the proton has a point charge in atomic physics. However, while carrying out experiments on electron scattering from proton targets, Hofstadter in 1955 observed that the proton charge is spread out [15]. In this experiment, an electron emits a virtual photon, which then interacts with the proton. If the proton consists of quarks distributed within a finite space-time region, the virtual photon will interact with quarks which carry fractional charges. The scattering amplitude will depend on the way in which quarks are distributed within the proton. The portion of the scattering amplitude which describes the interaction between the virtual photon and the proton is called the form factor.

Although there have been many attempts to explain this phenomenon within the framework of quantum field theory, it is quite natural to expect that the wave function in the quark model will describe the charge distribution. In high-energy experiments, we are dealing with the situation in which the momentum transfer in the scattering process is large. Indeed, the Lorentz-squeezed wave functions lead to the correct behavior of the hadronic form factor for large values of the momentum transfer [16].

Furthermore, in 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many “partons” whose properties do not appear to be quite different from those of the quarks [2]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

a. The picture is valid only for hadrons moving with velocity close to that of light.

b. The interaction time between the quarks becomes dilated, and partons behave as free independent particles.

c. The momentum distribution of partons becomes widespread as the hadron moves fast.

d. The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together.

In order to resolve this paradox, let us write down the momentum-energy wave function corresponding to Eq.(20). If the quarks have the four-momenta $p_a$ and $p_b$, we can construct two independent four-momentum variables [3]

$$P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b).$$

The four-momentum $P$ is the total four-momentum and is thus the hadronic four-momentum. $q$ measures the four-momentum separation between the quarks. Their light-cone variables are
\[
q_u = (q_0 - q_z)/\sqrt{2}, \quad q_v = (q_0 + q_z)/\sqrt{2}.
\]

The resulting momentum-energy wave function is
\[
\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp \left\{ -\frac{1}{2} \left( e^{-2\eta q_u^2} + e^{2\eta q_v^2} \right) \right\}.
\]

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [6,17,18].

When the hadron is at rest with \(\eta = 0\), both wave functions behave like those for the static bound state of quarks. As \(\eta\) increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the z-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function. The longitudinal momentum distribution becomes wide-spread as the hadron speed approaches the velocity of light. This is in contradiction with our expectation from nonrelativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a wide-spread distribution.

However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the momentum-energy width increase in the same direction as the hadron is boosted. This is of course an effect of Lorentz covariance. This indeed is the key to the resolution of the quark-parton paradox [6,17].

V. DECOHERENCE IN THE PARTON PICTURE

The most puzzling problem in the parton picture is that partons in the hadron appear as incoherent particles, while quarks are coherent when the hadron is at rest. Does this mean that the coherence is destroyed by the Lorentz boost? The answer is NO, and here is the resolution to this puzzle.

When the hadron is boosted, the hadronic matter becomes squeezed and becomes concentrated in the elliptic region along the positive light-cone axis. The length of the major axis becomes expanded by \(e^\eta\), and the minor axis is contracted by \(e^\eta\).

This means that the interaction time of the quarks among themselves become dilated. Because the wave function becomes wide-spread, the distance between one end of the harmonic oscillator well and the other end increases. This effect, first noted by Feynman [2], is universally observed in high-energy hadronic experiments. The period is oscillation is increases like \(e^\eta\).

On the other hand, the interaction time with the external signal, since it is moving in the direction opposite to the direction of the hadron, it travels along the negative light-cone axis. If the hadron contracts along the negative light-cone axis, the interaction time decreases by
$e^{-\eta}$. The ratio of the interaction time to the oscillator period becomes $e^{-2\eta}$. The energy of each proton coming out of the Fermilab accelerator is 900 GeV. This leads the ratio to $10^{-6}$. This is indeed a small number. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron.

Indeed, Feynman’s parton picture is one concrete physical example where the decoherence effect is observed. As for the entropy, the time-separation variable belongs to the rest of the universe. Because we are not able to observe this variable, the entropy increases as the hadron is boosted to exhibit the parton effect. The decoherence is thus accompanied by an entropy increase.

Let us go back to the coupled-oscillator system. The light-cone variables in Eq. (20) correspond to the normal coordinates in the coupled-oscillator system given in Eq. (2). According to Feynman’s parton picture, the decoherence mechanism is determined by the ratio of widths of the wave function along the two normal coordinates.
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