Robustness of Power Distribution System: A Comparative Study of Network and Performance Based Metrics

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ABSTRACT Improving the resilience of the power distribution networks is becoming a top priority for the utility companies. Robustness is a key part of resilience, and is often studied through failure-based analysis. Furthermore, with the increasingly dynamic nature of the grid, voltage fluctuation is an important factor to consider while assessing system robustness. Very few metrics exist for capturing robustness to voltage fluctuations, primarily due to their complex computation process. While several failure-based robustness metrics have been proposed in the literature, there is no strict consensus regarding the applicability of these metrics. Therefore, this paper addresses two key gaps in robustness analysis of power distribution networks. First, this paper presents a systematic study of different failure-based robustness metrics by comparing their similarity and dissimilarity in ranking critical nodes of the power distribution network. Secondly, the efficacy of these metrics in characterizing voltage fluctuations is evaluated by comparing their ranking to that of voltage influencing scores. From experimental results, it is shown that hybrid failure-based metrics can quantify voltage fluctuations to a reasonable extent. We highlight the major shortcomings of current metric formulations and discuss possible future research directions related to robustness characteristics and analysis.

INDEX TERMS Voltage fluctuations, robustness, critical nodes, resilience, distribution network.

I. INTRODUCTION

The power grid is a critical infrastructure that ensures the functionality and well-being of communities. Unfortunately, the power system is vulnerable to both natural hazards and man-made attacks, which in turn can result in severe social and economic disruptions [1]. Some of the bold examples include blackouts in the United States (2003) and India (2012), which affected millions and led to financial losses of more than a billion US dollars [2]. These and other such events highlight the need for research on the resilience and robustness of power systems against failure [3]. Robustness analysis typically deals with identifying key attributes of the system which are majorly responsible for the system’s resilience [4]. For network-based models where nodes represent physical assets like buses, these key attributes correspond to critical nodes whose removal/disruption leads to maximum impact on the system functionality. The knowledge of critical nodes could help us to take effective preemptive actions to mitigate the impact of disruption, thereby improving system resilience.

Several metrics are proposed in the literature to identify critical nodes which are referred as “failure based robustness” metrics in this paper. Failure-based metrics can be broadly classified into three classes, namely network-based, performance-based and hybrid [5]–[7]. Network theory-based metrics solely involve topological characteristics such as degree, betweenness, etc., whereas
performance-based metrics deal with the electrical properties of the network like power flow, and branch impedances [6], [8]. There are also several works in the literature that take a hybrid approach of fusing both these classes [9], [10]. This fusion leads to more effective metrics for quantifying a node’s role in determining the entire system’s robustness to failures caused by extreme natural events and/or cyber intrusions. Authors in [5], [7] have conducted a comparative study of several robustness metrics, but they are analyzed from the perspective of general complex networks. Although plenty of robustness metrics are defined for power distribution systems (PDS), there is no systematic study on their coherency and applicability. By coherency, we mean the similarity or dissimilarity (consistency) in rankings of critical nodes from various indices.

With the increasing penetration of renewable generation and electric vehicles, the distribution grid is witnessing significant new dynamics. Specifically, active consumers with rooftop photovoltaics and distributed generation are expected to alter their generation and usage patterns [11]. This in turn induces frequent power variations. The uncontrolled operations of DERs (Distributed energy resources) under this condition leads to voltage fluctuations in the PDS. Specifically, uneven PV penetration and unpredictable cloud coverage changes the pattern of daily loading profiles and the variability of power among feeders. As a result, these feeders may seem to peak at a different time of the day and suffer fast load fluctuations [12], [13]. This problem is further exacerbated by the unbalanced network at a distribution level which results in unbalanced voltage variations for each phase. The voltage fluctuations can have a detrimental impact on the connected devices and customer experience [12], [14]. Therefore, voltage fluctuations across the grid become an important factor to consider while assessing system robustness.

Typically, various voltage control devices, such as voltage regulators, and on-load tap changing transformers, are used to mitigate voltage fluctuations. However, this setup is slow and inadequate to deal with bi-directional power flows and the fast dynamics of PDS. This necessitates the development of computationally efficient and fast dynamic voltage control algorithms that can handle the dynamics of power/voltage variation [15]. The speed of response to impending voltage issues and the computational efficiency of these dynamic algorithms relies on the ability to select the optimal set of nodes for control that have the highest influence on the system voltage profile [16]. Therefore, there is a need to design a metric that can rank the nodes based on their voltage influence on other nodes of the PDS. In another way, an individualistic score is required for each node that determines how robust its voltage variations stand against the power fluctuations in other nodes of the PDS. Along with ensuring operational robustness by offering optimal control nodes, these node level scores can also be used for planning in terms of (1) determining hosting capacity for PDS with growing DERs installation [17], [18]; (2) fair incentivization of end-users by combining this score with other demand response objectives.

Recently, authors in [16] have used an information-theoretic approach to design a novel voltage influence score for ranking the nodes in a computationally efficient manner. Overall, very little attention has been devoted to defining node-level metrics for accurately quantifying voltage fluctuations, which necessitates the need for more metrics in this area. Hence, it is worthwhile to explore the potential of readily available failure-based robustness metrics to quantify voltage fluctuations.

Contributions: In this paper, for the first time, we conduct a comparative study of various failure-based robustness metrics to analyze their coherency and then investigate their efficacy in characterizing the impact of voltage fluctuations. In particular, this paper makes the following contributions to the literature.

- Study formulations of modern failure-based robustness metrics and voltage influencing indices.
- Investigate the coherency in rankings of critical nodes by different classes of failure-based robustness metrics.
- Analyze the expressiveness of failure-based robustness metrics in characterizing the impact of voltage fluctuations.
- Identify key challenges with existing approaches and provide potential research directions related to the design of novel robustness metrics for PDS.

The rest of the paper is organized as follows. Existing failure based robustness metrics are described in Section II, followed by voltage fluctuation metrics in Section III. Section IV presents a comparative study of these metrics. Section V discusses key challenges and future research directions. Final conclusions are provided in Section V.

II. ROBUSTNESS METRICS

The term “robustness” in this paper is primarily concerned with the drop in performance of a power grid when a disruption occurs [18]. Several metrics have been proposed in the literature to study and improve the system robustness against node/link failures [3], [6], [10], [18], [19]. These approaches can be divided into three classes, namely network (topology) based, performance (power flow, system dynamics) based and hybrid which combines topology with electrical properties of the network. Further, this section also describes a voltage variation metric, which would later be compared with the failure-based indices. Figure 1 depicts the taxonomy of robustness metrics studied in this work.

A. ROBUSTNESS TO FAILURES

The failure of the system corresponds to the loss of connectivity which further leads to the loss of power flow to the end users. Some of the widely used metrics to determine system robustness to failures include effective graph resistance, flow robustness, etc. More importantly, different nodes contribute differently to the overall robustness. Hence, some nodes could be more critical compared to others in terms of inducing cascading failures once they are affected. Therefore, it is
worthwhile to identify such critical nodes so that effective preventive actions and systematic system hardening can be carried out beforehand to mitigate the impact of extreme events. Relevant metrics from each of the three introduced earlier classes are summarized below.

1) NETWORK-BASED METRICS

- **Network efficiency (NEF):** It is the communication effectiveness of a networked system. That is,

  \[ E = \frac{1}{N(N-1)} \sum_{u \neq v} \frac{1}{d_{uv}}, \]  

  (1)

  is a measure of the network performance under the assumption that the efficiency for sending load (electricity, information, packets, whatsoever) between two nodes \( u \) and \( v \) is proportional to the reciprocal of their distance. Based on this definition, the robustness of a network can be defined as the drop in the efficiency when a node \( u \) is removed from the network, i.e.,

  \[ R_e(u) = \frac{E - E_u}{E}, \]  

  (2)

  where \( E_u \) is the score after removing node \( u \) from the network.

- **Betweenness centrality:** It quantifies how often a node \( u \) occurs in the paths linking other pairs of nodes. That is,

  \[ C_B(u) = \sum_{s=1}^{n} \sum_{t=1}^{n} \sigma_{st}(u) \sigma_{st}, s \neq t \neq u \in V. \]  

  (3)

  where, \( \sigma_{st} \) is the total number of shortest paths from node \( s \) to \( t \), and \( \sigma_{st}(u) \) is the number of those paths that pass through \( u \).

- **Effective node resistance (ENR):** It is related to the eigenvalues of the graph Laplacian matrix and corresponds to [20]

  \[ R(u) = \frac{2}{N-1} \sum_{u=1}^{N-1} \frac{1}{\lambda_u}, \]  

  (4)

  where \( \lambda \) are the non zero eigen values of graph Laplacian matrix. Recently, authors in [21] introduce the metric Effective Node Resistance by extending the notion of EGR from graph level to node level. Essentially, it is defined as the sum of all node-to-node effective resistances \( R_{uv} \) with all other nodes. It can be expressed as:

  \[ R_e(u) = \sum_{v \in V} R_{uv}. \]  

  (5)

  It has been shown that ENR correlates well with other standard robustness metrics against node and link failures [21].

2) PERFORMANCE-BASED METRICS

The performance of the network with respect to robustness is typically measured via loss of active power due to disruption. Some of the relevant metrics under this category are:

- **Electrical coupling connection degree (ECD):** For a power network with total \( N \) nodes, the electrical coupling connection degree \( D_u \) for the node \( u \) is defined as [22]:

  \[ D_u = \left| \frac{1}{\sum_{v=1, u \neq v}^{N} Z_{uv}^{equ}} \right| \]  

  (6)

  where, \( Z_{uv}^{equ} \) is the electrical distance, and can be described as the equivalent impedance between two nodes (similar to the concept of distance in graph theory). It is computed for each node through entries in the node impedance matrix as [22]:

  \[ Z_{uv}^{equ} = (Z_{uu} - Z_{uv}) - (Z_{uv} - Z_{vv}), \]  

  (7)

  where, \( Z_{uv} \) is the branch impedance between node pair \( u \) and \( v \). ECD can be used to rank the nodes based on electrical attribute. The shorter the electrical distance, greater the dependence will be. A node with a high electrical coupling connection degree has a high current transmission capability and a high electrical dependence on the other nodes. When this node fails, it will soon induce power flow changes among numerous nodes, resulting in a cascade of failure.

- **Active power flow loss (APL):** The power supplied (PS) is an essential measurement in DC power flow model. Its loss can be evaluated as [10]:

  \[ P_l = 1 - \frac{\sum_{u=1}^{N} P_u^d}{\sum_{u=1}^{N} P_u^i}, \]  

  (8)

  where \( P_u^i \) is the active power of the node \( u \) under normal operating conditions of the grid, and \( P_u^d \) is the power after failure.

- **Electrical node significance (ENS):** Typically in power grids, some nodes serve as hubs distributing a large amount of power while others distribute very little. A considerable quantity of power is exposed to the rest of the network when a link from one of the hub nodes fails. Redistributing this extra power to neighboring components gradually leads to more link overload failures, which can lead to a large-scale power outage. However, if a link connected to a less critical node fails, the power is instantly re-routed to surrounding components, and the disruption is usually mitigated. This implies that nodes have varied effects on the cascading failure robustness, and that this impact is dependent on the quantity of power distributed by the associated node. Therefore, electrical node significance is introduced in [9] to determine the impact of a node as:

  \[ \delta_u = \frac{P_u}{\sum_{v=1}^{N} P_v}. \]  

(9)
where, $P_u$ denotes the total power distributed by node $u$, and $N$ is the number of nodes in the network.

3) HYBRID METRICS
The unexpected behavior in the power system is related to both the topological (location of events, interconnection of components) and the operative state (flow distribution, demand level, etc.) of the system. Therefore, there is a growing interest in combining both the factors to obtain novel metrics that can better capture network robustness to failures [10]. Some of the hybrid metrics that have been found to work well are presented below:

- **Electrical betweenness (EBW):** The electrical betweenness centrality of a node $u$ in a network of $N$ nodes is defined as [23];

$$C_{EB}^E(u) = \sum_{s=1}^{n} \sum_{t=1}^{n} \frac{P_{st}(u)}{P_{st}}, s \neq t \neq u \in V.$$  \hspace{1cm} (10)

where, $P_{st}$ is the maximum power flowing in the shortest electrical path between nodes $s$ and $t$, and $P_{st}(u)$ is the maximum of inflow and outflow at bus $k$ within the shortest electrical path between nodes $s$ and $t$.

- **Electrical degree (EDG):** The power flowing in the adjacent links of the target node can be considered as electrical degree of the node and can be written as [23],

$$C_{ED}^E(u) = \sum_{u \sim v} P_{uv} \frac{N - 1}{N}$$  \hspace{1cm} (11)

where, $u \sim v$ indicates that node $u$ and $v$ are connected. $P_{uv}$ represents power flowing in line connected in between nodes $u$ and $v$.

- **Electrical node robustness (ENT):** It quantifies the ability of a node to resist cascades of link overload failures. It incorporates both flow dynamics and network topology. The electrical nodal robustness of a node $u$ (i.e. $R_n(u)$) can be expressed as [9],

$$R_n(u) = -\sum_{l=1}^{L} \alpha_l P_l \log P_l.$$  \hspace{1cm} (12)

where, $L$ refers to the out-degree of the corresponding node, $\alpha$ denotes the loading level of the line, and $P_l$ corresponds to normalised flow values on the out-going links given as,

$$P_l = \frac{f_l}{\sum_{l=1}^{L} f_l},$$  \hspace{1cm} (13)

where, $f_l$ refers to the power flow in line $l$. The formulation is similar to entropy because entropy of a load distribution of a node increases as flows over lines are distributed more homogeneously and the node out-degree increases. The loading level of the line has inverse relationship on robustness, therefore $\alpha$ is inversely proportional to loading level. Thus, higher the value of nodal robustness, the more robust the node is.

B. ROBUSTNESS TO VOLTAGE FLUCTUATIONS
Apart from the robustness metrics to node failures, it is also important to investigate metrics to quantify voltage variations since we are interested in comparing the efficacy of different failure-based metrics in characterizing robustness to voltage fluctuations. In this regard, one of the relevant metrics for voltage variations is the Voltage influencing score (VIS) which can be expressed as [16],

$$VIS(O, A) = \frac{1}{D(A, O)} - \frac{1}{D(S, O)},$$  \hspace{1cm} (14)

where, $D(A, O)$ is the statistical distance between the voltage change distribution at a target node $O$ due to aggregate effect of all actor nodes (actor node refers to node where power varies) and when actor node $A$ is solely present in the system. $D(S, O)$ is the statistical distance between source node and

**FIGURE 1.** Taxonomy of Robustness metrics for power distribution network.
observation node. The distance can be computed with any of the information theoretic metrics such as KL divergence, Frechet distance, among others. To provide an absolute value to the score, VIS is normalized with minimum and maximum values. As VIS is defined for a pair of nodes, the net influencing capacity of a particular node can be determined by averaging its score across all the other nodes of the network. The lower the distance, the more the actor node contributes to aggregate voltage change and consequently the more influencing the actor node is and vice-versa. The nodes with high VIS score are critical with respect to voltage fluctuations. Furthermore, VIS-based approach of determining the voltage influencer nodes is computationally efficient compared to other voltage sensitivity approaches that rely on load flow simulations.

### III. COMPARATIVE RESULTS AND DISCUSSION

The metrics that have been developed under the network and performance-based robustness analysis paradigms can be used to identify critical nodes for enhancing network resilience against failures. This section presents a comparative analysis of metrics introduced in the previous section. To this end, this study first investigates the coherency of these metrics in explaining the system’s robustness. Then, we analyze the efficacy of the methods in representing cross-domain metrics, i.e., leveraging failure-based robustness metrics to quantify voltage variations.

Table 1 reports the metrics studied in this work and their relationship with critical node ranking. Criticality ranking denotes the ranking of nodes based on the corresponding robustness metric scores. The lower the rank, the more important the node is with respect to network robustness. Depending upon the metric formulation, increasing metric values have a positive or negative impact on the ranking. For example, a higher value of the node betweenness score implies a better node criticality ranking, whereas the opposite effect can be seen in the case of ENR. Furthermore, this paper considers the IEEE-37 node network as a test system. This test network is chosen since it represents a typical unbalanced distribution system and is being used by many researchers for illustrating the efficiency of their proposed methods [13], [24]. This test network is also suitable for our application because it is a long radial distribution feeder with large voltage variations from end to end, making it susceptible to voltage fluctuations. Figure 2 depicts the modified IEEE 37-node test network. In addition, since the computation of metrics involves two different processes (i.e., graph theoretic and electrical measures), their experimental setups and settings are rather different, and therefore they are being discussed along with the results for each metric category in the forthcoming subsections.

#### A. NETWORK BASED METRICS

For network-based analysis, the IEEE-37 test network is abstracted as a directed weighted graph, with nodes representing buses (nodes) of the distribution system and links corresponding to physical connections between buses. The test network under this scenario is solely analyzed from the perspective of topological (i.e., network) features. Radial distribution networks like the IEEE-37 node network are not scale-free like other engineered networks, since their degree distribution follows a uniform distribution. We have selected three widely used effective metrics under this category, namely betweenness (BTW), network efficiency (NEF) and effective node resistance (ENR).

![IEEE 37-node test network.](image)

#### TABLE 1. Robustness metrics and their relationship with critical node ranking. Up-arrow denotes increasing value and down-arrow for decreasing value.

| Metric                          | Class        | Relation to criticality ranking |
|---------------------------------|--------------|--------------------------------|
| Betweenness (BTW) †             | Network      | †                              |
| Network efficiency (NEF) †      | Network      | †                              |
| Effective node resistance (ENR) † | Network      | †                              |
| Active power flow loss (APL) †  | Performance  | †                              |
| Electrical node coupling degree (ECD) † | Performance  | †                              |
| Electrical node significance (ENS) † | Performance  | †                              |
| Electrical betweenness (EBW) †  | Hybrid       | †                              |
| Electrical degree (EDG) †       | Hybrid       | †                              |
| Electrical node robustness (ENT) † | Hybrid       | †                              |
| Voltage influencing score (VIS) † | Hybrid       | †                              |

#### TABLE 2. Ranking of nodes in regard to network-based metrics.

| Metrics Rank | BTW | NEF | ENR |
|--------------|-----|-----|-----|
| Node         | Cₜ  | Node | H  |
| 1            | 14  | 9   | 0.067 |
| 2            | 9   | 0.047 | 28 | 0.056 | 14 | 0.635 |
| 3            | 18  | 0.029 | 14 | 0.052 | 28 | 0.734 |
| 4            | 28  | 0.021 | 18 | 0.026 | 18 | 0.796 |
| 5            | 30  | 0.004 | 30 | 0.009 | 30 | 0.958 |
| 6            | 7   | 0.0  | 22 | 0.004 | 22 | 0.969 |
| 7            | 8   | 0.0  | 34 | 0.003 | 34 | 0.981 |
| 8            | 12  | 0.0  | 17 | 0.003 | 17 | 0.982 |
| 9            | 17  | 0.0  | 36 | 0.003 | 17 | 0.983 |
| 10           | 22  | 0.0  | 31 | 0.002 | 31 | 0.987 |
| 11           | 26  | 0.0  | 7  | 0.002 | 7  | 0.991 |
| 12           | 27  | 0.0  | 8  | 0.002 | 8  | 0.991 |
| 13           | 31  | 0.0  | 26 | 0.001 | 26 | 0.997 |
| 14           | 34  | 0.0  | 27 | 0.001 | 27 | 0.997 |
| 15           | 36  | 0.0  | 12 | 0.001 | 12 | 0.998 |
Table 2 tabulates the criticality ranking of 15 actor nodes with the three selected metrics. Although power measurements are not utilized to determine the ranking in this case, they are provided for these 15 nodes to allow for a fair comparison to other approaches. Table 2 also includes metric values in addition to node numbers, denoted by “Node”. It can be seen that the betweenness values are non-zero only for the top 5 nodes, indicating that only the rank of the Top 5 nodes makes sense. All the remaining nodes can be rearranged in any manner, and the presented rank in Table 2 is merely one among several ways. Since none of the shortest paths pass through leaf nodes (nodes in the periphery of the distribution network), their betweenness scores are zero, making this metric less effective in distinguishing nodes. This shortcoming is addressed in both NEF and ENR. They provide differentiating values to different nodes as much as possible. However, nodes lying in the lower score band are less distinguishable compared to those in the upper band.

It is also evident from Table 2 that the rankings of Top-1 or Top-5 nodes are different with different metrics. For instance, BTW assigns the top position to node 14, whereas the other two metrics place node 9 in position one. Furthermore, the correlation between the rankings of NEF and ENR is high compared to that of BTW. However, it is interesting to note that although different metrics offer different ranks, there is a consistency in the Top-5 nodes when seen as a set rather than ranked entries, i.e., all three metrics assign nodes 9, 14, 18, 28, 30 in Top-5 positions. Thus, in applications where node selection has major computational and financial implications, i.e., high discriminatory scores are necessary, the priority of metrics usage should be ENR followed by NEF and BTW.

### B. PERFORMANCE BASED METRICS

The term “performance” in this category of metrics refers to the electrical attributes of the distribution network, such as power flow in lines, voltage quality, etc. The nominal voltage of the test system is 4.8 kV, and the base load is kept the same as reported in the IEEE PES distribution system subcommittee report. Furthermore, for better generalizability, different load scenarios are simulated by varying the base load at 15 actor nodes. The mean results are reported from over 100,000 different simulations.

Table 3 illustrates the ranking of 15 actor nodes for the three electrical-based robustness metrics, namely active power flow loss (APL), electrical coupling connection degree (ECD), and electrical node significance (ENS). It can be observed from the very first glance that the ECD has the largest discriminatory power compared to the other two. In comparison to network-based metrics, here the ranking among the three metrics is relatively less consistent, likely due to the diverse electrical attributes employed to compute these ranks. However, considering the Top-5 node set, APL and ENS appear to match to a greater extent compared to ECD. This is because, both APL and ENS involve node power in their formulations, unlike ECD, which deals with the branch impedances. Overall, the performance-based rankings are noticeably different than those of network one, except for a few nodes such as node 9, which is consistently present in the Top-5 nodes in all the cases. In a nutshell, one must experiment with different metrics before determining the relevant metric for a specific use case.

### C. HYBRID METRICS

Hybrid approaches consider both topological and electrical attributes of the network. The base loads, actor nodes, and other settings are kept the same as in the previous two cases.

Table 4 presents the ranking of nodes based on three powerful hybrid metrics, namely electrical betweenness (EBW), electrical degree (EDG), and electrical node robustness (ENT). Similar to the performance-based metric case, there is no consistency in ranking among the three approaches. However, there is noticeable consistency with respect to the Top-5 nodes set. In particular, the rankings of EBW and EDG have a high correlation compared to that of ENT, although all three involve power flows in their formulations. Another distinguishing characteristic of hybrid metrics is their substantially lower discriminatory power compared to performance and network-based methods. In fact, only the top nodes have distinct values, and the remaining bottom nodes have almost zero values in EBW and ENT. Specifically for ENT, most
of the nodes have zero values, either because of no outgoing links (as appears to be the case in leaf nodes) or because of a single outgoing link. Thus, although these hybrid approaches seem to be more elegant in their formulation due to the incorporation of both electrical and topological features, they are not very effective in identifying critical nodes, as evident via entries in Table 4.

Overall, when comparing the rankings of the three approaches, node 9 appears to be in the first position in a majority of cases, followed by node 14. For network-based metrics, it is the central position of nodes that makes them critical. As for hybrid cases, both power flow and central position make them strong candidates. On the other hand, node 36 lies in the bottommost position for most of the metrics, followed by 34. These nodes are leaf nodes whose loss would have a minimal impact on the electrical connectivity of the major part of the network.

D. Failure Based Robustness Metrics for Voltage Fluctuations

It is interesting to study whether failure-based robustness metrics can capture voltage variations. To this end, we must first rank the actor nodes based on their voltage influencing capacity (VIC). VIC refers to a node’s capability of inducing voltage variations in other nodes of the network, and it depends on the node’s position as well as its power variation. The experimental setup for this case study is kept the same as in the case of previous metrics. The only extra factor to consider here is the probability distribution of power changes with which different power change scenarios are simulated. This distribution is needed since we rely on [16] for determining the VIC of nodes, which essentially utilizes power change distribution and topological information such as shared path impedances. The change in real and reactive power at 15 actor nodes is modeled as a zero-mean Gaussian random variable. The Gaussian distribution is commonly used to validate statistical frameworks, and it has been considered a common assumption in many prior works related to distribution systems [25], [26]. Furthermore, a covariance matrix consisting of power change variances and co-variances is required which can be learned from the historical data as illustrated in [16]. In this regard, three different power change scenarios are considered with different means and variances as indicated below:

\[
\Delta S^1 \sim \mathcal{N}\left(\begin{bmatrix} 0.0 \end{bmatrix}, \begin{bmatrix} 1.5 & 0.05 \\ 0.05 & 0.25 \end{bmatrix}\right),
\]

\[
\Delta S^2 \sim \mathcal{N}\left(\begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} 3 & -0.1 \\ -0.1 & 0.5 \end{bmatrix}\right),
\]

\[
\Delta S^3 \sim \mathcal{N}\left(\begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} 4.5 & -0.2 \\ -0.2 & 0.75 \end{bmatrix}\right),
\]

(15)

where \(\Delta S\) signifies the power change vector across all actor nodes, \(A\) denotes the actor node set, and superscript over actor nodes, i.e., \([a, b, c]\) represent respective phases at which power is varying. Changes in power across different actor nodes can be correlated because of the geographical proximity of DERs (PV and wind turbines). The diagonal elements of covariance matrices represent the variance of change in real and reactive power at actor nodes, while off-diagonal elements reflect the covariance between real and reactive power change as shown in eqn. (15).

Table 5 depicts the ranking of 15 actor nodes for various scenarios. The voltage influence score is computed in a pairwise manner, with one node being an actor where power varies and the other being an observation node where voltage change is monitored [16]. For illustration, the rankings in Table 5 are shown for two observation nodes, 7 and 16. Furthermore, one can compute the mean of each actor node’s voltage influencing score across all observation nodes to determine its overall influencing capacity. The last column of Table 5 shows the average ranking of the actor nodes. It can be seen that certain nodes (9, 14, etc.) are consistently possessing high rank compared to others across different metrics. This is primarily due to their locations and associated power changes.

If we strictly compare the top positions of voltage influencing nodes with those of failure-based robustness metrics in tables 2, 3, 4, none of them seems to match. It is not even fair to make such a strict comparison since process of computing these two classes of metrics are quite different. In fact, it makes sense only to compare the hybrid robustness metrics with voltage influencing scores since both involve power flow and topological related characteristics, unlike network and performance-based metrics that only consider one of the two aspects at a time. EBW and EDO appear to match VIF ranking to a certain extent since they capture two of the Top-5 nodes, i.e., nodes 9 and 14. So, hybrid metrics can be safely employed in applications that require a set of critical nodes to monitor/control voltage fluctuations. There are numerous advantages to using these kinds of hybrid metrics for voltage fluctuations, including (1) light and easy computation; (2) no need to rely on computationally expensive simulations and other system states that are difficult to obtain.

| Node Rank | Obs Node 7 | Obs node-16 | All nodes |
|-----------|------------|-------------|-----------|
| 1         | 7          | 14          | 7         |
| 2         | 9          | 22          | 8         |
| 3         | 12         | 12          | 9         |
| 4         | 14         | 9           | 12        |
| 5         | 22         | 7           | 14        |
| 6         | 25         | 17          | 18        |
| 7         | 8          | 26          | 22        |
| 8         | 8          | 18          | 26        |
| 9         | 17         | 28          | 28        |
| 10        | 18         | 8           | 30        |
| 11        | 27         | 27          | 31        |
| 12        | 30         | 34          | 17        |
| 13        | 31         | 31          | 27        |
| 14        | 34         | 34          | 34        |
| 15        | 36         | 36          | 36        |
IV. CHALLENGES AND OPPORTUNITIES
The metrics studied in this work, reveal the importance of different factors in determining the system’s robustness and the advantage of fusing performance and network-based methodologies. However, to enable widespread adoption, there is scope for improvement in robustness metric design. This section identifies key challenges that lie ahead and suggests future research directions in effective metric formulation.

A. GENERIC FORMULATION
There are plenty of metrics developed for exploring various aspects of robustness in PDS. However, there is no clear consensus or analysis regarding their applicability, which ultimately leads users to experiment with different metrics before finding the most effective ones for their use case. In addition, most of the metrics are designed to work for a specific use case, rendering them useless for other applications. Thus, there is a pressing need to design robustness metrics with a generic formulation that can work for multiple applications.

One way to approach the generic paradigm is by incorporating all relevant aspects into the metric formulation itself, i.e., all the factors that contribute to the target objectives should be included in the metric formula. For example, in the ENT metric, the loading level of the line is explicitly added to the actual power flow to effectively account for the link overload failure. Similarly, the voltage influencing score of the studied node can be merged into the current ENT formulation to account for the voltage fluctuations. This will allow the metric to simultaneously account for two objectives, i.e., link overload failure and voltage fluctuations.

B. SCALABLE COMPUTATION
Existing methods for computing robustness metrics, especially those involving electrical characteristics, are computationally intensive and do not scale well with the network size. As a result, robustness analysis on large test networks such as the IEEE 8500 node feeder or 10477 bus system is difficult and time-consuming [27]–[29]. Furthermore, for every small change in network configuration, the process of computing the metric needs to be repeated without leveraging any past solutions. Thus, a more elegant framework is required that can tackle these issues.

Data-driven models could be a potential candidate to address some of the computational shortcomings of the existing approaches. Deep neural network-based models can be trained to estimate robustness scores at the node level as well as at the graph level. For example, some inspirations can be drawn from [30], where graph neural network is utilized to obtain critical nodes by training a node classifier. These models are easily scalable to larger networks since they only rely on the sub-graph or a relevant part of the network for any node/link level prediction rather than relying on entire network. Furthermore, this type of model can make predictions across networks of varying sizes. There are numerous other advantages compared to conventional approaches, such as the ability to capture a node’s non-linear relationships with the robustness of the entire network, high expressive power due to a large number of model parameters, etc.

C. HOLISTIC MODELING
The majority of current modeling frameworks for studying robustness analyze the electrical network in isolation, without taking into account its interdependency with other coexisting critical infrastructures such as water, transportation, etc. In addition, disruptive events disproportionately impact the low-income and socially vulnerable communities under-scoring the need for incorporating social equality via true assessment of community resilience. However, the robustness of complex systems is typically evaluated solely in terms of engineering attributes, such as network topology or electrical parameters, and tends to ignore social factors that are equally important. As a result, robustness evaluations of such decoupled and partially informed systems are sub-optimal. Thus, there is a need to develop a holistic modeling framework for accurate robustness assessment.

Stochastic hetero-functional graph theory (SHFGT), inspired by [18], [31] is one potential modeling framework to incorporate the above-discussed factors. SHFGT can effectively model complex interdependent systems, including electrical, power, and transport networks, via a set of graphs corresponding to different activities. Essentially, these graph-structured models leverage functionality as building blocks, unlike conventional frameworks that only describe physical attributes in terms of nodes. This allows them to efficiently integrate various aspects, including engineering and social robustness factors. Robustness assessment within this kind of modeling framework would be more realistic and thus enable one to take effective decisions in improving system resilience.

V. CONCLUSION
This paper presents a systematic study of different failure-based robustness metrics by comparing their similarity and dissimilarity in ranking critical nodes in a power distribution network. These metrics are broadly divided into three classes, namely network, performance, and hybrid. Then, the efficacy of these metrics in characterizing voltage fluctuations is accessed by comparing their rankings with that of voltage influencing scores. From experimental results, it appears that the hybrid metrics can express voltage fluctuations to a reasonable extent. However, there is still a lot of room for improvement in the metric formulation. We also discussed the key challenges and future research directions in this space.

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