χ_{cJ} polarizations at the Fermilab Tevatron

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Abstract

We propose the measurement of χ_{cJ} polarizations at high energy hadron colliders to study heavy quarkonium production mechanism. We find that the color-singlet model in the k_t factorization approach predicts very different behavior for χ_{cJ} polarizations at the Fermilab Tevatron compared with the NRQCD predictions in the collinear parton model. In the color-singlet k_t factorization approach, for both χ_{c1} and χ_{c2} productions, the helicity h = 0 states dominate over other helicity states at large p_T. These properties are very useful in distinguishing between the two production mechanisms which are related to the interesting issue of J/ψ and ψ' polarizations, and may provide a crucial test for the k_t factorization approach.

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Studies of heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. In recent years, heavy quarkonium production has attracted much attention from both theory and experiment. To explain the $J/\psi$ and $\psi'$ surplus problem of large transverse momentum production at Tevatron [1], the color-octet production mechanism was introduced for the description of heavy quarkonium production [2] based on the NRQCD factorization framework [3]. During the last few years, extensive studies have been performed for the test of this color-octet production mechanism. However, most recently the CDF collaboration have reported their preliminary measurements on the polarizations of the promptly produced charmonium states [4], which appear not to support the color-octet predictions that the directly produced $S$-wave quarkonia have transverse polarizations at large $p_T$ [5,6]. In [7,8], the authors considered the feeddown contributions from $\chi_c$ decays, and found the prompt $J/\psi$ polarization disagree with the CDF data at large $p_T$ by 3 standard deviations. This conflict shows that the heavy quarkonium production mechanism may be more complicated than we knew before, and further studies on heavy quarkonium production mechanisms other than the color-octet mechanism in the collinear parton model are still needed at present.

In [9], the authors studied $\chi_cJ$ hadroproduction at Tevatron in the $k_t$ factorization approach [10,11]. Their results show that only color-singlet contributions can reproduce the Tevatron data on $\chi_cJ$ production in the $k_t$ factorization approach, and the color-octet contributions disagree with the data. However, we note that previous studies in the collinear parton model have also given a good description for the data of $\chi_cJ$ production at Tevatron in the NRQCD approach including both color-singlet and color-octet contributions [12]. In this context, we have two different mechanisms, i.e., the color-singlet $k_t$ factorization approach and the NRQCD approach in the collinear parton model, of which both can successfully describe the Tevatron data on the large $p_T \chi_cJ$ production rates. So, it is quite urgent now to distinguish between these two mechanisms for understanding heavy quarkonium production at high energy hadron colliders. For this purpose, we propose here the measurements of $\chi_cJ$ polarizations at the Fermilab Tevatron. From our calculations we find that the color-singlet
model in the $k_t$ factorization approach predicts very different behavior for $\chi_{cJ}$ polarizations compared with the NRQCD predictions in the collinear parton model. In the color-singlet $k_t$ factorization approach, for both $\chi_{c1}$ and $\chi_{c2}$ productions, the helicity $h = 0$ states dominate over other helicity states at large transverse momentum. This novel property can provide a crucial test for this production mechanism. Furthermore, the $\chi_{cJ}$ polarization measurements can help to clarify the present conflict between the color-octet predictions and the experimental data on prompt $J/\psi$ and $\psi'$ polarizations at Tevatron.

The polarized cross section formulas for $\chi_{cJ}$ hadroproduction in the NRQCD approach have been calculated in [12,13,8] (including both color-singlet and color-octet processes). In this paper, we will calculate the color-singlet $\chi_{cJ}$ polarized cross sections in the $k_t$ factorization approach. We will not include the color-octet processes in this approach, because their contributions to $\chi_{cJ}$ production disagree with the Tevatron data in shape [3].

The $k_t$-factorization approach differs greatly from the conventional collinear approximation because it takes the non-vanishing transverse momenta of the scattering partons into account. The conventional gluon densities are replaced by the unintegrated gluon distributions which depend on the transverse momentum $k_t$. In the calculations, for every 4-momenta $k_i$ we make a Sudakov decomposition as

$$k_i = \alpha_i p_1 + \beta_i p_2 + \vec{k}_iT,$$

where $p_1$ and $p_2$ are the momenta of the incoming hadrons. In the high energy limit, we have $p_1^2 = 0$, $p_2^2 = 0$, and $2p_1 \cdot p_2 = s$, where $s$ is the c.m. energy squared. $\alpha_i$ and $\beta_i$ are the momentum fractions of $p_1$ and $p_2$ respectively. $k_iT$ is the transverse momentum, which satisfies

$$k_iT \cdot p_1 = 0, \quad k_iT \cdot p_2 = 0.$$

For the momenta of the incident gluons $q_1$ and $q_2$, we have the following decompositions [11],

$$q_1 = x_1 p_1 + q_1T, \quad q_2 = x_2 p_2 + q_2T.$$


That is to say, the longitudinal component of $q_1$ ($q_2$) is only in the direction of light-like vector $p_1$ ($p_2$).

Using the above defined Sudakov variables, we can express the polarized cross sections for $\chi_{cJ}$ hadroproduction as the following form,

$$
\frac{d\sigma}{d\alpha_\chi}(p\bar{p} \rightarrow \chi_{cJ}X) = \frac{1}{64 \times 16\pi} \frac{d\alpha_\chi}{\alpha_\chi} d^2q_1T d^2q_2T \frac{f(x_1; q_{1T}^2) f(x_2; q_{2T}^2)}{q_{1T}^2 q_{2T}^2} |A_0^{(\lambda)}(q_{1T}, q_{2T})|^2,
$$

where $\lambda$ denotes the helicity of $\chi_{cJ}$, and $\alpha_\chi$ is the momentum fraction of $p_1$ carried by $\chi_{cJ}$.

The $\chi_{cJ}$ transverse momentum $p_T$ comes from the sum of the transverse momenta of $q_1$ and $q_2$ as, $\vec{p}_T = \vec{q}_{1T} + \vec{q}_{2T}$. The amplitude squared $|A_0^{(\lambda)}|^2$ describes $\chi_{cJ}$ (with helicity $\lambda$) production in the gluon-gluon fusion processes $g + g \rightarrow \chi_{cJ}^{(\lambda)}$. To calculate these helicity amplitudes, we need the polarization sums for individual helicity levels of $\chi_{cJ}$. For $\chi_{c1}$, the longitudinal and transverse polarization sums can be written in the following covariant forms

$$
\sum_{\lambda=0} \epsilon_\alpha^{(\lambda)} \epsilon_\beta^{(\lambda)*} = P_{\alpha\beta}^L,
$$

$$
\sum_{|\lambda|=1} \epsilon_\alpha^{(\lambda)} \epsilon_\beta^{(\lambda)*} = P_{\alpha\beta}^T = P_{\alpha\beta} - P_{\alpha\beta}^L,
$$

where

$$
P_{\alpha\beta} = -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{p^2}.
$$

And in the laboratory frame (the helicity frame), $P_{\alpha\beta}^L$ is expressed as

$$
P_{\alpha\beta}^L = \frac{(p \cdot Q)^2}{(p \cdot Q)^2 - M^2 s} \left( \frac{p_\alpha}{M} - \frac{M}{p \cdot Q} Q_\alpha \right) \left( \frac{p_\beta}{M} - \frac{M}{p \cdot Q} Q_\beta \right),
$$

where $M = 2m_c$ is the mass of $\chi_{cJ}$, and $Q = p_1 + p_2$ is the sum of the initial hadron 4-momenta. For $\chi_{c2}$, the polarization sums for individual helicity levels ($\lambda = 0, 1, 2$) can also be expressed in terms of $P_{\alpha\beta}$, $P_{\alpha\beta}^T$, and $P_{\alpha\beta}^L$ [12]. With these polarization sums, we can calculate the production cross sections for individual helicity states of $\chi_{cJ}$, which are more involved and will be presented elsewhere. We have checked that these cross section formulas can numerically reproduce the results of [9] for the inclusive production rates of $\chi_{cJ}$ at Tevatron after summing up all helicity states contributions.
The $\chi_{cJ}$ polarizations can be measured by studying the photon’s angular distribution in the $\chi_{cJ}$ rest frame in the decay processes $\chi_{cJ} \to J/\psi \gamma$. These angular distributions have the following form,

$$\frac{d\Gamma(\chi_{cJ} \to J/\psi \gamma)}{d\cos \theta} \propto \frac{3}{2(3 + \alpha)}(1 + \alpha \cos^2 \theta),$$

(9)

where $\theta$ is the angle between the photon’s 3-momentum in $\chi_{cJ}$ rest frame and the $\chi_{cJ}$ 3-momentum in the laboratory frame. $\alpha$ is the polarization parameter (angular distribution parameter). For $\chi_{c1}$, $\alpha$ is defined as

$$\alpha = \frac{2 - 3\rho}{2 + \rho},$$

(10)

where

$$\rho = \frac{d\sigma(\chi_{c1}^{(|\lambda|=1)})}{d\sigma(\chi_{c1})}.$$  

(11)

For $\chi_{c2}$, the polarization parameter is

$$\alpha = -\frac{6 - 3\eta - 12\tau}{10 - \eta - 4\tau},$$

(12)

where

$$\eta = \frac{d\sigma(\chi_{c2}^{(|\lambda|=1)})}{d\sigma(\chi_{c2})}, \quad \tau = \frac{d\sigma(\chi_{c2}^{(|\lambda|=2)})}{d\sigma(\chi_{c2})}.$$  

(13)

For numerical calculations, we choose the unintegrated gluon distribution of [14] which can well fit the $F_2(x, Q^2)$ data over a wide range of $x$ and $Q^2$, and we set the scales $\mu^2$ for the strong coupling constant $\alpha_s(\mu^2)$ in the amplitude squared $|A_0|^2$ to be $q_{1T}^2$ for the interaction vertex associated with the incident gluon $q_1$, and $q_{2T}^2$ for the vertex associated with $q_2$ [15,16].

We first display in Fig. 1 the production ratio of $\chi_{c1}$ to $\chi_{c2}$ at the Tevatron as a function of $p_T$, $R = \sigma(\chi_{c1})/\sigma(\chi_{c2})$. The solid line is for the color-singlet prediction in the $k_t$ factorization approach, and the dotted-dashed line for the NRQCD prediction in the collinear parton model. For comparison, in this figure we also plot the results for other two cases in the collinear parton model: the color-singlet prediction as the dotted line and the color-octet prediction as the dashed line. However, we must note that neither the color-singlet
contributions nor the color-octet contributions alone can describe the Tevatron data on $\chi_{cJ}$ productions [13], and in this collinear parton model only the NRQCD predictions (including both the color-singlet and the color-octet contributions) can make sense to describe the Tevatron data on $\chi_{cJ}$ productions. From Fig. 1, we can see that the $R$ ratio increases as $p_T$ increases in the color-singlet model $k_t$ factorization approach, and its value approaches to 2.0 at large transverse momentum, which means that at large $p_T$ $\chi_{c1}$ production dominates over $\chi_{c2}$ production. In contrast, the NRQCD approach predicts the $R$ ratio to be much smaller, and its value approaches to 0.6 at large $p_T$. This is because, at large transverse momentum the color-octet gluon fragmentation (the $3{S}_1^{(8)}$ channel) dominates the $\chi_{cJ}$ productions in NRQCD, which leads to $\chi_{cJ}$ production rates as $\sigma(\chi_{c0}) : \sigma(\chi_{c1}) : \sigma(\chi_{c2}) = 1 : 3 : 5$ (consistent with our numerical calculations at large $p_T$). This figure shows that the difference on $R$ ratio between the color-singlet $k_t$ factorization approach and the NRQCD approach in the collinear parton model is distinctive at sufficiently large $p_T$.

We then study the $\chi_{cJ}$ polarizations at Tevatron. With the polarized cross section formulas, we can calculate the production rates for definite helicity states of $\chi_{cJ}$, and get the angular distribution parameter $\alpha$. From our numerical calculations, we find that the color-singlet $k_t$ factorization approach predicts very different behavior for $\chi_{cJ}$ polarizations at the Fermilab Tevatron compared with the NRQCD predictions in the collinear parton model. In the color-singlet $k_t$ factorization approach, for both $\chi_{c1}$ and $\chi_{c2}$ productions, the helicity $h = 0$ states dominate over other helicity states at large transverse momentum. This property has distinguished consequence to the decay angular distribution parameter $\alpha$ for $\chi_{cJ} \to J/\psi \gamma$ processes. These results are displayed in Figs. 2-4. For $\chi_{c1} \to J/\psi \gamma$, at large $p_T$ the color-singlet $k_t$ factorization approach predicts $\alpha$ around 0.8 while the NRQCD approach in the collinear parton model predicts $\alpha$ around 0.2. For $\chi_{c2} \to J/\psi \gamma$, the difference between these two mechanisms are more distinctive. The color-singlet $k_t$ factorization approach predicts $\alpha(\chi_{c2} \to J/\psi \gamma)$ to be negative (down to $-0.6$) at large $p_T$, while the NRQCD approach in the collinear parton model predicts $\alpha$ to be positive (around 0.3).

For the experimental measurement, it may be difficult to distinguish between $\chi_{c1}$ and $\chi_{c2}$
contributions in the observation of the photon’s angular distributions in the decay processes \( \chi_{cJ} \to J/\psi \gamma \). So, it may be more useful to give the angular distributions in \( \chi_{cJ} \to J/\psi \gamma \) with both \( \chi_{c1} \) and \( \chi_{c2} \) taken into account. We plot this result in Fig. 4. From this figure, we find that the color-singlet \( k_t \) factorization approach predicts \( \alpha \) for \( \chi_{cJ} \to J/\psi \gamma \) being about 0.5 while the NRQCD approach in the collinear parton model predicts \( \alpha \) around 0.25 at large transverse momentum. The difference between these two mechanisms is still distinctive.

Finally, we note that the polarized cross section formulas for \( \chi_{cJ} \) production can also be used to predict the polarization of \( J/\psi \) which comes from \( \chi_{cJ} \) feeddown decays. \( J/\psi \) polarization can be measured by the lepton’s angular distribution in the \( J/\psi \) rest frame in \( J/\psi \to \mu^+ \mu^- \) decay process. These distributions have a similar form to those for \( \chi_{cJ} \) decays,

\[
\frac{d\Gamma(J/\psi \to \mu^+ \mu^-)}{d \cos \theta} \propto \frac{3}{2(3 + \alpha)} (1 + \alpha \cos^2 \theta),
\]

where \( \theta \) is the angle between the 3-momentum of the lepton in \( J/\psi \) rest frame and the 3-momentum of \( J/\psi \) in the laboratory frame. \( \alpha \) is the polarization parameter, and is equal to

\[
\alpha = \frac{3\xi - 2}{2 - \xi},
\]

where \( \xi \) is the ratio of the transversely polarized to the total \( J/\psi \), which can be calculated by using the polarized cross sections for \( \chi_{cJ} \) production \[12\]. In Fig. 5 we give the \( J/\psi \) polarization from the feeddown contributions of \( \chi_{cJ} \) decays. Again, we find that the color-singlet \( k_t \) factorization approach predicts \( \alpha(J/\psi \to \mu^+ \mu^-) \) being about 0.5 while the NRQCD approach in the collinear parton model predicts \( \alpha \) around 0.25 at large \( p_T \).

At the Tevatron, the CDF collaboration has measured the inclusive production cross sections of \( \chi_{cJ} \) states, which contribute about 30% of prompt \( J/\psi \) production in a wide range of \( p_T \) \[1\]. The \( \chi_{cJ} \) states can be identified with a photon plus the \( J/\psi \) which decays into a muon pair. The production cross section of \( \chi_{cJ} \) is found to be comparable to that of \( J/\psi \), and is not small. Unfortunately, the present statistics of Tevatron Run I is not high enough to allow separate polarization measurements for this part of \( J/\psi \) (from \( \chi_c \)
decays) from the direct $J/\psi$ production and $\psi'$ decay's contribution [4]. However, with the upgrade Tevatron Run II, the luminosity will be increased by a factor of more than 8, we will have much more data for $J/\psi$ and $\chi_{cJ}$ production, so it will be feasible to distinguish between different contributions to prompt $J/\psi$ polarizations, and then to measure the $\chi_{cJ}$ polarizations.

In conclusion, in this paper we have calculated the $\chi_{cJ}$ polarizations at high energy hadron colliders in the color-singlet $k_t$ factorization approach and the NRQCD approach in the collinear parton model. We find the difference on the polarization parameters for $\chi_{cJ}$ and their decay products $J/\psi$ between these two approaches are distinctive at large transverse momentum. Therefore, $\chi_{cJ}$ polarizations can be used to study these two production mechanisms at hadron colliders and may provide important information on heavy quarkonium polarization mechanisms. Especially, the color-singlet $k_t$ factorization approach predicts that $\chi_{cJ}$ productions are dominated by the helicity $h = 0$ states at large transverse momentum. This unique property may provide a crucial test for this production mechanism.

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Figure Captions

FIG. 1. The production ratio of $\chi_{c1}$ to $\chi_{c2}$ as a function of $p_T$. The solid line is for the color-singlet $k_t$ factorization approach prediction, the dotted-dashed line for the NRQCD prediction in the collinear parton model, the dotted and dashed lines are respectively for the color-singlet and color-octet predictions alone in the collinear parton model.

FIG. 2. The polarization parameter $\alpha$ for $\chi_{c1} \rightarrow J/\psi \gamma$ as a function of $p_T$. The definitions of the curves are the same as in Fig. 1.

FIG. 3. The same as in Fig. 2 but for $\chi_{c2}$.

FIG. 4. The same as in Fig. 2 but for $\chi_{cJ} \rightarrow J/\psi \gamma$ with both $\chi_{c1}$ and $\chi_{c2}$ taken into account.

FIG. 5. The polarization parameter $\alpha$ for $J/\psi \rightarrow \mu^+\mu^-$ for $J/\psi$ coming from $\chi_{cJ}$ decays.
Fig. 1

\[ R = \frac{\sigma_{\chi_1}}{\sigma_{\chi_2}} \]

\[ P_T (\text{GeV}) \]
Fig. 2: \( \alpha(\chi_{c1} \rightarrow J/\psi \gamma) \) vs. \( P_T \) (GeV)
Fig. 3

\[ \alpha(\chi_{c2} \rightarrow J/\psi \gamma) \]

\[ P_T (\text{GeV}) \]
Fig. 4

$\alpha(\chi_{cJ} \rightarrow J/\psi \gamma)$ vs. $P_T$(GeV)
\( \alpha(J/\psi \rightarrow \chi_c^+) \)

Fig. 5