Masses and sigma terms of doubly charmed baryons up to $O(p^4)$ in manifestly Lorentz-invariant baryon chiral perturbation theory

De-Liang Yao

1Instituto de Física Corpuscular (centro mixto CSIC-UV), Institutos de Investigación de Paterna, Apartado 22085, 46071, Valencia, Spain

We calculate the masses and sigma terms of the doubly charmed baryons up to next-to-next-to-leading order (i.e., $O(p^4)$) in a covariant baryon chiral perturbation theory by using the extended-on-mass-shell renormalization scheme. Their expressions both in infinite and finite volumes are provided for chiral extrapolation in lattice QCD. As a first application, our chiral results of the masses are confronted with the existing lattice QCD data in the presence of finite volume corrections. Up to $O(p^4)$ all relevant low energy constants can be well determined. As a consequence, we obtain the physical values for the masses of $\Xi_{cc}$ and $\Omega_{cc}$ baryons by extrapolating to the physical limit. Our determination of the $\Xi_{cc}$ mass is consistent with the recent experimental value by LHCb collaboration, however, larger than the one by SELEX collaboration. In addition, we predict the pion-baryon and strangeness-baryon sigma terms, as well as the mass splitting between the $\Xi_{cc}$ and $\Omega_{cc}$ states. Their quark mass dependences are also discussed. The numerical procedure can be applied to the chiral results of $O(p^4)$ order, where more unknown constants are involved, when more data are available for unphysical pion masses.

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I. INTRODUCTION

A doubly charmed baryon termed as $\Xi_{cc}^+$, was first reported by SELEX collaboration [1] and its mass was observed to be $3519 \pm 2$ MeV [2]. Unfortunately, for very long time this state was not confirmed by any other experimental collaborations: FOCUS [3], Babar [4], Belle [5] or LHCb [6]. Very recently, renewed interest has been triggered in studying doubly charmed baryons due to the confirmation of the existence of the doubly charged state $\Xi_{cc}^{++}$ with a mass of $3621.4 \pm 0.78$ MeV by LHCb collaboration [7]. Relevant theoretical efforts have been accumulated rapidly, for instance, in the investigations of their magnetic moments [8], weak decays [9, 10], strong and radiative decays [11, 12], interactions with light states [13], etc.

The masses of the doubly charmed baryons are basic quantities classifying the baryon spectrum. Understanding the origin of the masses of ground-state baryons is one of the most important issues in hadron physics. Especially for the $\Xi_{cc}$ baryons, the difference between the reported values of the masses by SELEX and LHCb collaborations are abnormally large, which is in conflict with the fact that the isospin breaking effect should be small as it is proportional to the mass difference of the $u$ and $d$ quarks. More specifically, the isospin splitting in $\Xi_{cc}$ baryons is estimated to be $m(\Xi_{cc}^{++}) - m(\Xi_{cc}^+) = 1.41 \pm 0.12^{+0.76}_{-0.12}$ MeV [14], while the corresponding value calculated from experimental results is around 100 MeV. On the other hand, there is a multitude of the theoretical determinations using various methods such as relativistic quark model [15, 16] and effective potential [17]. Interestingly, they all tend to support the LHCb result rather than the SELEX one. On the side of lattice QCD (LQCD), calculations of the masses are performed by many collaborations [18–22], whereas only the result in Ref. [20] agrees with the SELEX value. Nevertheless, as pointed out in Ref. [22], the chiral extrapolation of the lattice data of Ref. [20], especially the datum at $M_\pi = 260$ MeV, using the next-to-leading-order (NLO) heavy baryon chiral perturbation theory would lead to a sizeable systematic uncertainty of the baryon mass in physical limit. Hence a more appropriate and higher-order extrapolating formula for the masses is required. To that end, we will calculate the masses of the doubly charmed baryons up to next-to-next-to-next-to-leading (N$^3$LO) within the framework of covariant baryon chiral perturbation theory (BChPT).

Chiral perturbation theory (ChPT) nowadays plays a prominent role in the study of modern hadronic physics at low energies. It has been intensively applied to calculate a multitude of physical quantities and extrapolate lattice QCD data to physical point, see e.g., Ref. [23]. Moreover, within ChPT, the finite volume corrections (FVCs) can be systematically obtained by discretizing the integrations involved in the loop contributions [24–26]. For baryon masses, calculations can be performed by using various subtraction methods such as heavy baryon (HB) approach [27–29], infrared regularization (IR) [30] and extended-on-mass-shell (EOMS) scheme [31–33]. Such methods are proposed to settle the power counting issue caused by the presence of non-vanishing baryon mass in the chiral limit, see Refs [34–36] for reviews. Nevertheless, the EOMS scheme is more appropriate for the extrapolation of LQCD data. This is because it respects the proper analytical properties when...
functions are relegated to Appendices A and B respectively.

II. Masses and Sigma Terms in BCHPT

A. Chiral effective Lagrangian

The chiral effective Lagrangian relevant for our calculation of the masses and sigma terms up to $\mathcal{O}(p^4)$ can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\psi}^{(1)} + \mathcal{L}_{\pi\psi}^{(2)} + \mathcal{L}_{\pi\psi}^{(4)},$$

(1)

where the numbers in the superscripts denote the chiral orders. The leading order (LO) chiral Lagrangian reads

$$\mathcal{L}_{\pi\psi}^{(1)} = \bar{\Psi} \left[ i D_\mu \gamma^\mu - m + \frac{g_A}{2} u \gamma^\mu \gamma_5 \right] \Psi,$$

(2)

where $g_A$ and $m$ are the axial coupling and the mass of the doubly charmed baryons in the chiral limit, respectively. According to SU(3) symmetry of light quarks, the doubly charmed baryons of spin-1/2 are compiled in the triplet

$$\Psi = (\Xi_{cc}^+, \Xi_{cc}^0, \Omega_{cc}^+)^T.$$  

(3)

The covariant derivative acting on the baryon fields is defined by

$$D_\mu = \partial_\mu + \frac{1}{2} (u^i \partial_\mu u + u \partial_\mu u^i),$$

$$u = \exp \left( i \lambda^a \phi^a \sqrt{2} F_0 \right),$$

(4)

where $F_0$ is the decay constant of the Goldstone bosons (GBs) in the chiral limit. The GBs are collected in the octet

$$\chi^a = \left( \begin{array}{c} \pi^0 + \frac{1}{\sqrt{2}} \eta \\ \pi^+ \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ K^+ \\ \pi^- \\ -\frac{1}{\sqrt{2}} \pi^0 - \frac{1}{\sqrt{6}} \eta \\ K^0 \\ -\frac{1}{\sqrt{2}} \eta \end{array} \right).$$

(5)

Here the $\lambda^a$ ($a = 1, \cdots, 8$) denote the Gell-Mann matrices and summation over repeated indices is implied. Furthermore, the so-called chiral vielbein $u_\mu$ is given by

$$u_\mu = i (u^i \partial_\mu u^i - u \partial_\mu u).$$

(6)

Analogous to the procedure in Ref. [55], the NLO Lagrangian is constructed in Ref. [56] and has the form

$$\mathcal{L}_{\pi\psi}^{(2)} = \bar{\Psi} \left[ c_1 (\chi^+) - \left( \frac{c_2}{8 m^2} \{u_\mu, u_\nu\} \{D^\mu, D^\nu\} + h.c. \right) 
- \left( \frac{c_3}{8 m^2} \{u_\mu, u_\nu\} \{D^\mu, D^\nu\} + h.c. \right) + \frac{c_4}{2} (u^2) 
+ \frac{c_5}{2} u^2 + \frac{i c_6}{4} \sigma^{\mu\nu} [u_\mu, u_\nu] + c_7 \chi^+ \right] \Psi.$$  

(7)
The corresponding traceless chiral operator \( \hat{\chi} \) contributing to the baryon two-point functions up to \( O(\text{GeV}) \) is given by

[\text{equation}]

where \( \chi_+ = u^\dagger \chi u + u^\dagger u \) with the mass matrix \( \chi = \text{diag}(M_1^2, M_2^2, 2M_3^2 - M_2^2) \). The corresponding traceless chiral operator \( \chi_+ \) is defined as \( \chi_+ = \chi - \frac{1}{4} \langle \chi_+ \rangle \). The low-energy constants \( c_i \) (\( i = 1, \ldots, 7 \)) are unknown parameters and have dimension GeV\(^{-1}\). At \( O(p^4) \), the following counter terms are needed,

[\text{equation}]

where \( e_i \) (\( i = 1, \ldots, 4 \)) are unknown LECs with mass dimension GeV\(^{-3}\).

### B. Self-energies of doubly charmed baryons

The one-particle irreducible Feynman diagrams contributing to the baryon two-point functions up to \( O(p^4) \) are displayed in Fig. 1.

At \( O(p^2) \), the tree level contribution corresponding to diagram (a) reads

[\text{equation}]

with the combination \( \hat{e}_1 = e_1 - \frac{1}{3} c_7 \). The tree contribution of \( O(p^4) \) is from diagram (b) and its explicit expression is

[\text{equation}]

with \( \hat{e}_1 = e_1 - \frac{1}{3} e_2 - \frac{1}{3} e_3 + \frac{1}{3} e_4 \) and \( \hat{e}_2 = e_2 - \frac{2}{3} e_4 \).

At \( O(p^3) \), the leading one-loop order, diagram (c) gives zero contribution, i.e. \( \Sigma_c^{(3)}(\hat{p}) = 0 \), while diagram (d) yields

[\text{equation}]

where summation over repeated indices is implied. The loop function \( G_D \), together with \( G_{E,L,R} \), appearing below, are defined in appendix A.

At \( O(p^4) \), the N\(^3\)LO loop contributions to the self-energy are

[\text{equation}]

where the unit vectors in the \( SU(3) \) flavour space are

[\text{equation}]

### C. The mass and the sigma term

The dressed propagator \( i S_B \) of the doubly charmed baryon is expressed as

[\text{equation}]

with the wave function renormalization constant

[\text{equation}]

The mass is defined as the pole at \( \hat{p} = m_B \),

[\text{equation}]

Using the self-energies calculated in the above section and truncating at \( O(p^4) \), one has

[\text{equation}]

FIG. 1: One-particle-irreducible diagrams. Dashed and solid lines represent pions and nucleons, respectively. Numbers in the squares mark the chiral orders of the vertices.
where the derivative is defined by
\[ \Sigma_d^{(3)}(\phi) = -\frac{g_A^2}{4F^2} \lambda_\alpha \lambda_\beta \frac{\partial}{\partial \phi} G_D(\phi, M_0, m) . \]

In Eq. (20), the UV divergences from loop contributions are subtracted using the modified minimal subtraction (MS) scheme and cancelled by the counter terms generated by the effective Lagrangian. Further, the finite PCB terms due to presence of the internal baryon propagators are absorbed in the LECs. To that end, one needs to perform the following substitutions of the LECs:
\[ X \rightarrow X + \frac{\beta_X m R}{16\pi^2 F^2} + \frac{\beta_X m}{16\pi^2 F^2}, \quad X \in \{m, \hat{e}_1, c_1\} , \]
\[ Y \rightarrow Y + \frac{\beta_Y R}{16\pi^2 F^2}, \quad Y \in \{\hat{e}_1, \hat{e}_2, c_3, c_4\} , \]
where the \( \beta \)-functions are all given in appendix [3]. Here \( R = 2/(d - 4) + \gamma_E - 1 - \ln(4\pi) \), with \( d \) the number of space-time dimensions and \( \gamma_E \) the Euler constant.

To be specific, one can organize the explicit expressions of the masses up to \( \mathcal{O}(p^4) \) as
\[ m_B = m + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} , \]
where the \( \mathcal{O}(p^2) \) contribution reads
\[ m_B^{(2)} = \sum_{\phi, \pi, K} C_{B,\phi}^{(2)} M^2_\phi , \]
with the coefficients \( C_{B,\phi}^{(a)} \) given in table [1]. The \( N^2\)LO corrections to the masses of doubly charged baryons are
\[ m_B^{(3)} = -\sum_{\phi, \pi, K, \eta} \frac{g_A^2}{64\pi^2 F^2} C_{B,\phi}^{(d)} \mathcal{H}_D(M_\phi) , \]
while the \( N^3\)LO ones read
\[ m_B^{(4)} = C_{B,\pi}^{(b)} M_{\pi}^4 + C_{B,\pi}^{(b)} M_{\pi} M_{K}^4 + C_{B,\pi}^{(b)} m_\pi M_{\pi} M_{K}^2 \]
\[ -\sum_{\phi, \pi, K, \eta} C_{B,\phi}^{(c)} \mathcal{H}_{E1}(M_\phi) \]
\[ +\sum_{\phi, \pi, K, \eta} \frac{g_A^2}{32\pi^2 F^2} C_{B,\phi}^{(e)} \mathcal{H}_{E2}(M_\phi) \]
\[ +\sum_{\phi, \pi, K, \eta} \frac{g_A^2}{32\pi^2 F^2} C_{B,\phi}^{(c)} \mathcal{H}_{E3}(M_\phi) \]
\[ -\sum_{\phi, \pi, K, \eta} \frac{g_A^2}{32\pi^2 F^2} C_{B,\phi}^{(f)} \mathcal{H}_F(M_\phi) \]
\[ -\sum_{\phi, \pi, K, \eta} \frac{g_A^2}{32\pi^2 F^2} C_{B,\phi}^{(w)} \mathcal{H}_{w}(M_\phi) . \]

All the relevant coefficients are listed in table [1]. In appendix [A], the expressions of the subtracted loop integrals are shown.

### Table I: Coefficients in the mass formulae: Eqs. (23), (24) and (25)

| \( \Sigma_f \) | \( \Xi \) | \( \Omega \)
|---|---|---|
| \( c_{B,\pi}^{(a)} \) | \( -2\delta_{17} \) | \( -2\delta_{17} \)
| \( c_{B,K}^{(b)} \) | \( -4c_3 \) | \( -4c_3 \)
| \( c_{B,\pi}^{(b)} \) | \( -4(\hat{e}_1 + \hat{e}_2 + 3e_3 + e_4) \) | \( -4(\hat{e}_1 - \hat{e}_2 + 3e_3 + e_4) \)
| \( c_{B,\pi}^{(b)} \) | \( -16(\hat{e}_1 + e_3) \) | \( -16(\hat{e}_1 + \hat{e}_2 + e_3 + e_4) \)
| \( c_{B,\pi K}^{(b)} \) | \( -8(\hat{e}_1 + \hat{e}_2 - 2e_3) \) | \( 16(\hat{e}_1 + e_3) \)
| \( c_{B,\pi}^{(d)} \) | \( 3 \) | \( 0 \)
| \( c_{B,K}^{(d)} \) | \( 2 \) | \( 4 \)
| \( c_{B,\eta}^{(d)} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \)
| \( c_{B,\pi}^{(e)} \) | \( 12(2\hat{e}_1 + c_7)M_{\pi}^2 + 0 \) | \( 12(2\hat{e}_1 + c_7)M_{\pi}^2 \)
| \( c_{B,K}^{(e)} \) | \( 8(4\hat{e}_1 + c_7)M_{K}^2 + 16(2\hat{e}_1 + c_7)M_{K}^2 \) | \( 8(2\hat{e}_1 + c_7)M_{K}^2 \)
| \( c_{B,\eta}^{(e)} \) | \( \frac{1}{6} \sum_{\pi, K, \eta} \left[ 4M_{\pi}^2 c_3 + c_7 M_{K}^2 \right] \) | \( \frac{1}{6} \sum_{\pi, K, \eta} \left[ 4M_{\pi}^2 c_3 + c_7 M_{K}^2 \right] \)
| \( c_{B,\pi}^{(f)} \) | \( 6\hat{e}_1 c_3 + c_5 \) | \( 6\hat{e}_1 \)
| \( c_{B,K}^{(f)} \) | \( 4(2\hat{e}_1 + c_3) \) | \( 8(2\hat{e}_1 + c_3) \)
| \( c_{B,\eta}^{(f)} \) | \( \frac{2}{3} \left[ 3c_3 + 2c_7 \right] \) | \( \frac{2}{3} \left[ 3c_3 + 2c_7 \right] \)
| \( c_{B,\pi}^{(w)} \) | \( 2(2\hat{e}_1 + c_7)M_{\pi}^2 \) | \( 2(2\hat{e}_1 + c_7)M_{\pi}^2 \)
| \( c_{B,K}^{(w)} \) | \( 4\hat{e}_1 M_{K}^2 + 4\hat{e}_1 M_{K}^2 + 3\hat{e}_1 M_{K}^2 + 3\hat{e}_1 M_{K}^2 \) | \( 8\hat{e}_1 M_{K}^2 + 4\hat{e}_1 M_{K}^2 \)
| \( c_{B,\eta}^{(w)} \) | \( \frac{1}{2}(2\hat{e}_1 M_{K}^2 + \hat{e}_1 M_{K}^2) - \frac{1}{2}(2\hat{e}_1 M_{K}^2 + \hat{e}_1 M_{K}^2) \) | \( \frac{1}{2}(2\hat{e}_1 M_{K}^2 + \hat{e}_1 M_{K}^2) - \frac{1}{2}(2\hat{e}_1 M_{K}^2 + \hat{e}_1 M_{K}^2) \)

The sigma terms can be obtained by applying the Hellmann-Feynman theorem to the masses,
\[ \sigma_{\pi B} = \frac{\partial m_B}{\partial m} , \quad \sigma_{\pi B} = \frac{\partial m_B}{\partial m} , \]
where \( \hat{m} = (m_u + m_d)/2 \). Here the up, down and strange quark masses are denoted by \( m_u, m_d \) and \( m_s \), respectively. In the isospin limit, i.e. \( m_u = m_d = \hat{m} \), the quark masses are simply related to the LO masses of the GBs through:
\[ M_{\pi}^2 = 2B_0\hat{m} , \quad M_{K}^2 = B_0(m + m_s) , \]
\[ M_{\eta}^2 = 2B_0(\hat{m} + m_s)/3 \]
with \( B_0 \) being a constant related to quark condensate. Therefore, in practice, the derivatives can be rewritten with respect to the GBs masses, instead of the quark masses.

### D. Finite volume corrections

On the lattice, simulations are performed for a system of interest enclosed in a finite box. The momentum is discretized and can only take values of \( 2\pi n/L \) with \( n \) a
vector of integers and \( L \) the side length of the hypercube. Consequently, an integration over spatial momenta in infinite volume corresponds to a summation over the momentum modes in finite volume. The difference caused by such a replacement is named as finite volume correction. Specifically, the finite volume correction for a given quantity \( Q \) is given by

\[
\delta_L[Q] = Q(L) - Q(\infty) ,
\]

where \( Q(L) \) and \( Q(\infty) \) are calculated in finite volume \( L^3 \) and infinite volume, respectively. In the so-call \( p \)-regime where \( M_0L \gg 1 \), ChPT provides a systematical tool to investigate finite-volume dependence of observables. To that end, one just needs to calculate the integrals stemming from loop diagrams in a finite box, while the temporal dimension can be treated as infinite since it is generally much larger than the spatial components in LQCD simulation for zero-temperature.

To obtain finite volume corrections to the masses of doubly charmed baryons, we choose to work in the rest frame of the baryons and follow the procedure demonstrated in Ref. \[31, 50\]. For the loop integral \( \mathcal{H}_D \), we obtain

\[
\delta_L[\mathcal{H}_D] = \int_0^1 dx \left\{ m \left( \frac{1}{2} + x \right) \delta_2(L, M_B^2) - \frac{m}{4} \left[ x^3 m^2 + (2 + x) M_B^2 \right] \delta_4(L, M_B^2) \right\} ,
\]

with \( M_B^2 = x^2 m^2 + (1 - x) M_0^2 - i0^+ \). Here the integration is performed over the Feynman parameter \( x \). Furthermore, the master function is given by

\[
\delta_r(L, \mathcal{M}^2) = \frac{2^{-1/2 - r} (\sqrt{\mathcal{M}})^{2 - 2r}}{\pi^{3/2} \Gamma(r)} \sum_{n=1}^{\infty} \text{Mul}(n) \left( L \sqrt{\mathcal{M}} \sqrt{n} \right) K_{3/2 - r}(L \sqrt{\mathcal{M}} \sqrt{n}) , \quad (30)
\]

where \( K_r(z) \) is the modified Bessel function of the second kind, and \( \text{Mul}(n) \) is multiplicity whose value up to \( n = 20 \) can be found in, e.g., Ref. \[30\]. Analogously, the FVCs for \( \mathcal{H}_{Ei} (i = 1, 2, 3) \) read

\[
\delta_L[\mathcal{H}_{E1}] = -\frac{1}{2} \delta_2(L, M_0^2) , \quad (31)
\]

\[
\delta_L[\mathcal{H}_{E2}] = -\frac{1}{2} m^2 \delta_2(L, M_0^2) , \quad (32)
\]

\[
\delta_L[\mathcal{H}_{E3}] = -\frac{1}{2} M_0^2 \delta_4(L, M_0^2) . \quad (33)
\]

There are no integrations over Feynman parameters in the above expressions since only one internal propagator is involved in each tadpole loop. The calculation of the FVCs corresponding to diagram (f) in Fig. 1 is more complicated because of the presence of three internal propagators. Nonetheless, the result can be obtained straightforwardly, which is

\[
\delta_L[\mathcal{H}_F] = - \int_0^1 dx \left\{ \delta_2(L, M_B^2) - \frac{1}{2} \left[ 3 m^2 (1 + x^2) + 2 M_B^2 \right] \delta_2(L, M_B^2) + \frac{3}{8} \left[ x^4 m^4 + 2 m^2 M_B^2 (2 + x^2) \right] \delta_4(L, M_B^2) \right\} , \quad (34)
\]
where \( \mathcal{M}_B \) is the same as the one in Eq. [29]. Lastly, the contribution due to the wave function renormalization is given by

\[
\delta_L[\mathcal{H}_{w.f.}] = \int_0^1 \frac{dx}{4} \left\{ 4x\delta_\perp(L, \mathcal{M}_B^2) - [m^2x(9x^2 - x - 6) + (1 + x)\mathcal{M}_B^2\delta_\parallel(L, \mathcal{M}_B^2) + 3m^2(x - 1)x \times (m^2x^3 + (2 + x)\mathcal{M}_B^2)\delta_\parallel(L, \mathcal{M}_B^2)] \right\}. \tag{35}
\]

In the end of this section, it is worth stressing that the calculations of FVCs are performed in four dimensions: a finite hypercube plus an infinite time interval. This is feasible due to the fact that \( Q(L) \) and \( Q(\infty) \) have the same ultraviolet property which guarantees that \( \delta_L[Q] \) is finite in four dimensions. Besides, as pointed out in Ref. [50], there are no PCB terms in \( \delta_L[Q] \) either, since the short-distance behaviours of \( Q(L) \) and \( Q(\infty) \) should be exactly identical. Thus, the quantities respecting power counting in finite volume can be easily obtained just by adding the FVCs to the corresponding EOMS-renormalized ones in infinite volume.

III. NUMERICAL RESULTS AND DISCUSSION

A. Properties of finite volume corrections

We compute the finite volume corrections given in section [41] as functions of the lattice size \( L \) with three different Goldstone masses \( \mathcal{M}_\phi = 0.2, 0.4 \) and 0.6 GeV. The baryon mass \( m \) is fixed to 3.6 GeV. The results are
shown in Fig. 2. From the figure, on the one hand, it is found that all the relevant FVCs decrease rapidly as \( L \) increases up to \( \sim 2 \) fm, behaving quite typically as the FVCs for the nucleons shown in Ref. \[50\]. The lattice QCD data used in our fit are obtained using lattice spaces ranging from 1.8 fm to 2.7 fm, which are in the vicinity of the turning point. The data corresponding to \( L = 1.8 \) fm might receive a larger FVC than the others. On the other hand, the smaller the Goldstone mass \( m \), the bigger the modules of the FVCs are. Therefore, contributions due to coupling of light pions dominate and for lattice data FVCs are larger when simulations are done with values of masses close to physical ones. Note that we checked that the influence of changing the baryon mass \( m \), e.g., in the range \([2.6, 4.6]\) GeV, is negligible.

The finite volume corrections \( \delta_L[H_F] \) and \( \delta_L[H_{\text{eff}}] \) are rather similar. Both of them are respectively larger than the other ones in Fig. 2. Nonetheless, for the FVCs to the masses in Eq. \[25\], there should exist sizeable cancellation between the two relevant terms in the last two rows, since their corresponding coefficients have opposite signs, as can be seen in table 1.

### B. Fit to lattice QCD data

We are now in the position to confront the chiral expression of doubly charmed baryons with lattice QCD determinations by explicitly including finite volume corrections. As already discussed in the introduction, it is interesting to study the lattice QCD data given in Ref. \[20\]. Unfortunately, in our theoretical formula, Eq. \[22\], there are too many unknown LECs, twelve in total: \( m, c_i \) \((i = 1, \ldots , 5.7)\), \( g_A \) and \( e_j \) \((j = 1, \ldots , 4)\). Hence, we start with mass formula just at \( \mathcal{O}(p^3) \) order where only four parameters, \( m, c_1, 7 \) and \( g_A \), are involved.

The lattice QCD data are obtained by numerical simulations with unphysical quark masses. The \( u-, d-\) and \( s-\) quark mass dependence can be always expressed in terms of the dependence on the leading-order masses of the Goldstone bosons shown in Eq. \[27\]. More specifically, the light \( u- \) or \( d-\) quark mass dependence is usually re-expressed as pion mass dependence. The \( s-\) quark mass dependence can be casted to the kaon mass in the limit of \( M_7^2(\alpha \hat{m}) \to 0 \), denoted as \( M_K^2 \). Then, with the help of Eq. \[27\], the pion- and strange-mass dependence of the kaon mass can be written as

\[
M_K = \sqrt{M_K^2 + M_\pi^2/2}, \quad M_K^2 = B_0 m_s. \tag{36}
\]

The data for the strange-doubly-charmed baryon \( \Omega_{cc} \) is obtained with a strange quark mass very close to the tuned value using physical kaon mass \[20\]. Therefore, as a good approximation, one can fix \( M_F^2 \) just by imposing the physical values of the pion and kaon masses: \( M_\pi^{\text{phy}} = 139 \) MeV and \( M_K^{\text{phy}} = 496 \) MeV. As for \( M_7 \), it is always obtained from pion and kaon masses through the Gell-Mann-Okubo mass relation: \( 3M_7^2 = 4M_K^2 - M_\pi^2 \). The masses of doubly charmed baryons also depend on the valence \( c-\) quark mass. In \( SU(3) \) chiral limit, all the light quark masses are zero and the baryon mass is equal to \( m \), i.e., the first term on the right hand side of Eq. \[22\]. It is thus reasonable to assume that only the chiral-limit baryon mass, \( m \), carries the information of the dependence on the \( c-\) quark mass. In line with heavy quark expansion, such a dependence can be expressed in the form of

\[
m = \tilde{m} + 2m_c + \alpha/m_c + \mathcal{O}(1/m_c^2), \tag{37}
\]

where \( \tilde{m} \) and \( \alpha \) are unknown constants. Since the QCD data of Ref. \[21\] are provided for various values of the \( c-\) quark mass, those two constants should be treated as fitting parameters, instead of \( m \).

In our fitting procedure, we employ \( F_\pi = 92.2 \) MeV, \( F_K = 112 \) MeV and \( F_\eta = 110 \) MeV as done in Ref. \[53\]. The pion mass dependences of the decay constants are not taken into account, since the caused differences are of higher orders - at least \( \mathcal{O}(p^5) \). Furthermore, the axial coupling constant is fixed to \( g_A = -0.2 \) \[54\]. The \( g_A \) here is related to a common coupling \( g \) involved in an effective Lagrangian respecting heavy quark-diquark symmetry \[57\], whose value can be further estimated by fitting to the \( D^{**} \) decay width. Finally, it is better to use the combination \( \hat{c}_1 \) rather than \( c_1 \) as a fitting parameter such that possible large correlation between \( c_1 \) and \( c_7 \) can be avoided. In summary, the fitting parameters in our fit at \( \mathcal{O}(p^3) \) order are \( \tilde{m}, \alpha, c_1 \) and \( c_7 \).
if we lessen the range of pion mass to uncertainties. All the data with masses of Ξ
is not feasible to decrease further the range as the data obtained regardless of FVCs. In Fig. 3 we plot
parameters dramatically when compared to the results in the inclusion of FVCs does not change the values of the
tions are collected in table II. As one can see from the best-fitted results of the parameters and their correla-
ations are dedicated to the fit using mass formula truncated at O(p^4). Extension to O(p^4) is straightforward. Nonetheless, similar to the case for
nucleon mass at O(p^4) [51], one has to replace the LO meson masses in m_B^{(2)} by their corresponding O(p^4) counterparts, which can be found, for instance, in Ref. [29]. Such a replacement generate O(p^4) contributions to m_B^{(4)}. The relevant LECs of L_i in O(p^4) Goldstone masses can be fixed to the empirical values given in Ref. [58]. We fitted to the lattice QCD data but no stable results can be achieved. The data set is not sufficient to pin down twelve fitting parameters.

C. Predictions

We can make predictions based on the fitted values of the parameters in table IV. In Fig. 3 the masses of the doubly charmed baryons are plotted as functions of m_c with M_\tau = M_\tau^\text{phys} and L \to \infty. In Ref. [29], three different values of lattice spacing are used in the simulations, which are denoted by β = 3.9, β = 4.05 and β = 4.2. The corresponding physical values of the charm quark mass are m_c^{\text{phys}}[β_1] = 0.598 GeV, m_c^{\text{phys}}[β_2] = 0.591 GeV and m_c^{\text{phys}}[β_3] = 0.598 GeV, respectively. We take the average as the central value of m_c^{\text{phys}} and the standard deviation as the error, which leads to m_c^{\text{phys}} = 0.596(4). Correspondingly, in Fig. 4 the vertical green slashed band corresponds to the physical region of m_c within its 1-σ standard deviation. In addition, the purple back-slashed band is obtained by varying the parameters within their 1-σ uncertainties given in table IV. Our predicted physical masses of the baryons are located in the overlaps of the two bands. In the top panel of Fig. 4 we also show

![FIG. 5: Quark mass dependences of the sigma terms for the doubly charmed baryons.](image)
the strangeness sigma terms of the Ξ
charmed baryon are comparable to those of the ground-
that the predicted strangeness sigma terms of the doubly
charged baryons up to \( \mathcal{O}(p^4) \) in a covariant
baryon chiral perturbation theory with Goldstone bosons
and the baryons as degrees of freedom. The masses at
complete one-loop order is renormalized by making use
of the EOMS scheme, which restores the correct power
counting while respecting the proper analytic structure.

As a consequence, we also obtained the pion-baryon and
strangeness-baryon sigma terms by applying Hellmann-
Feynman theorem to the obtained masses. In order to
make comparison with LQCD results in a more rigor-
ous manner, the finite volume corrections to the chiral
dynamics of the doubly charmed baryons. In Fig. 6 we show \( M^2_{\pi}(\bar{m}) \) dependence of \( \sigma_{\pi B} \) where the other quark masses are set to physical values. As expected, the values of \( \sigma_{\pi B} \) with \( B = \Xi_{cc}, \Omega_{cc} \) increase with \( M^2_{\pi} \). We checked also that the \( \sigma_{sB} \) are not sensitive to the variation of \( M^2_{\pi} \). Nonetheless, there exists strong \( M^2_{K}(\bar{m}) \) dependence for \( \sigma_{sB} \) as one can see from the last two plots in Fig. 6.

Another interesting quantity related to the light quarks
is the mass splitting between the \( \Xi_{cc} \) and \( \Omega_{cc} \). In Fig. 6 the \( M^2_{\pi} \) and \( M^2_{K} \) dependences of the mass splitting \( \Delta m \) are shown. It is found that \( \Delta m \) depends more strongly on \( M^2_{K} \) than \( M^2_{\pi} \). Furthermore, the different trends of \( \Delta m \) as the quark masses increase validate the fact that \( \Delta m \propto m_s - \bar{m} \).

At physical quark masses our prediction is \( \Delta m = 65.9 \pm 51.3 \text{ MeV} \), in agreement with the determination extrapolated by the Lattice QCD group of Ref. [19].

| \( B = \Xi_{cc} \) | \( B = \Omega_{cc} \) |
|---|---|
| \( m_B \) | 3.591 ± 0.067 GeV | 3.657 ± 0.100 GeV |
| \( \sigma_{\pi B} \) | 10.5 ± 3.4 MeV | 4.0 ± 2.8 MeV |
| \( \sigma_{sB} \) | 48.7 ± 34.7 MeV | 118.0 ± 76.1 MeV |

**TABLE III: Physical masses and sigma terms.**

**FIG. 6: Quark mass dependence of the mass splitting.**

of sigma terms on \( m_c \) is negligible. In other words, the
influence of the heavy \( c \) quark is almost absent for the
sigma terms. This observation verifies that the sigma
terms are more appropriate than the masses to explore
the chiral dynamics of the doubly charmed baryons.

**IV. SUMMARY**

We have calculated the masses and sigma terms of
the doubly charmed baryons up to \( \mathcal{O}(p^4) \) in a covariant
baryon chiral perturbation theory with Goldstone bosons
and the baryons as degrees of freedom. The masses at
complete one-loop order is renormalized by making use
of the EOMS scheme, which restores the correct power
counting while respecting the proper analytic structure.

As a consequence, we also obtained the pion-baryon and
strangeness-baryon sigma terms by applying Hellmann-
Feynman theorem to the obtained masses. In order to
make comparison with LQCD results in a more rigor-
ous manner, the finite volume corrections to the chiral
dynamics of the masses are derived systematically by dis-
cretizing the loop contributions. FVCs corresponding to
the relevant loop integrals are studied numerically and
typical behaviour when varying the lattice size \( L \) is ob-
served, namely, FVCs decrease rapidly as \( L \) increases up
to \( \sim 2 \text{ fm} \).

Using the mass formulae with FVCs, we investigated the
pion-mass and \( m_c \) dependences for the masses of dou-
by charmed baryons by performing fits to lattice QCD
data of Ref. [20]. It is found that more data, with respect
to more values of unphysical pion masses, are required

In Fig. 6, we show the mass dependence of the mass splitting \( \Delta m \) at physical quark masses. The grey bands represent the physical \( m_c \) region. Our predicted values of the sigma terms are inside the outermost grey bands, and the experimental values of the mass of \( \Xi_{cc} \) by LHCb [7] and SELEX [1]. Interestingly, it is found that our prediction is in good agreement with the LHCb determination. On the contrary, the SELEX value is just below the border of our predicted region. For easy reference, our predicted physical masses and sigma terms of the doubly charged baryon are compiled in Table III. Note that the predicted strangeness sigma terms of the doubly charged baryon are comparable to those of the ground-state octet baryons, see, e.g., in Ref. [59]. For instance, the strangeness sigma terms of the \( \Xi_{cc} \) and the nucleon, i.e., states without valence \( s \) quark in quark-model interpretation, turn out to be of the same order, i.e., tens of MeV. However, regarding the pion sigma terms, unlike the case for the nucleon that a large value of \( \sigma_{\pi N} \) \( \geq 50 \text{ MeV} \) was obtained [60,61], our predicted values of \( \sigma_{\pi \Xi_{cc}} \) and \( \sigma_{\pi \Omega_{cc}} \) are small.

Likewise, the \( m_c \)-dependence of sigma terms, at physical pion mass and in infinite volume, are shown in first line of Fig. 5. The grey bands are due to the variation of the fitted parameters within their uncertainties. The vertical green bands represent the physical \( m_c \) region. Our predicted values for the sigma terms are inside the overlaps. Unlike the masses in Fig. 4 which strongly depend on \( m_c \), one can notice from Fig. 6 that the dependence
to pin down the LECs appearing in the N^3LO formulae. Nevertheless, the LECs in the N^2LO mass expressions can be well determined. Based on the fitted values, we have extrapolated the baryon masses to the physical limit. We find that our result for \( m_{\Xi_{cc}} \) is in agreement with the latest experiment determination by LHCb collaboration within uncertainty. However, it is larger than the value by SELEX collaboration. Finally, we predict the sigma terms \( \sigma_{\pi B} \) and \( \sigma_{sB} \) with \( B \in \{ \Xi_{cc}, \Omega_{cc} \} \), as well as the mass splitting between \( \Xi_{cc} \) and \( \Omega_{cc} \) states. Their quark mass dependences are studied as well.

The masses calculated in the present work will be useful in the future investigation of observables like axial charge and scattering lengths, related to the doubly charmed baryons, within the framework of covariant BChPT, since they are basic quantities involved in expressions of almost all the others. The sigma terms are related to the potentials of the GBs scattering off the doubly charmed baryons, and hence can be implemented as an additional constraint when making prediction of exotic doubly charmed baryons based on unitarized potentials.

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Appendix A: Loop integrals

The loop functions involved in the self-energies in section 11B are defined as follows:

\[
G_D(\phi, M_\phi, m) = \frac{1}{i} \int \frac{dk}{2\pi^d} \frac{k[(k + \phi) - m]k}{[k^2 - M_\phi^2][(k + p)^2 - m^2]^2},
\]

\[
G_{E1}(\phi, M_\phi, m) = \frac{1}{i} \int \frac{dk}{2\pi^d} \frac{1}{k^2 - M_A^2},
\]

\[
G_{E2}(\phi, M_\phi, m) = \frac{1}{i} \int \frac{dk}{2\pi^d} \frac{1}{k^2 - M_A^2},
\]

\[
G_{E3}(\phi, M_\phi, m) = \frac{1}{i} \int \frac{dk}{2\pi^d} \frac{k^2}{k^2 - M_A^2},
\]

\[
G_F(\phi, M_\phi, m) = \frac{1}{i} \int \frac{dk}{2\pi^d} \frac{k[(k + \phi) - m]^2k}{[k^2 - M_\phi^2][(k + p)^2 - m^2]^2}. 
\]

The loop integral in the \( \mathcal{O}(p^3) \) mass formula \cite{24} is

\[
\mathcal{H}_D(M_\phi) = \frac{2M_\phi^2}{m} \left\{ \mathcal{T}_B + m^2 \left[ 1 - \mathcal{T}_B(m^2) \right] \right\}, \tag{A1}
\]

while the ones in the \( \mathcal{O}(p^4) \) mass formula \cite{25} read

\[
\mathcal{H}_{E1}(M_\phi) = \mathcal{T}_\phi, 
\]

\[
\mathcal{H}_{E2}(M_\phi) = \frac{1}{8} M_\phi^2 m^2 \left[ \mathcal{T}_\phi + 2 \mathcal{T}_B \right], 
\]

\[
\mathcal{H}_{E3}(M_\phi) = M_\phi^2 \mathcal{T}_\phi, 
\]

\[
\mathcal{H}_F(M_\phi) = \frac{1}{4m^2 - M_\phi^2} \left\{ 4M_\phi^2 \mathcal{T}_B + (M_\phi^2 - 12m^2) \mathcal{T}_\phi \right. 
\]

\[
- 2M_\phi^2 \left[ m^2 + (M_\phi^2 - 6m^2) \mathcal{T}_B(m^2) \right] \right\}, 
\]

\[
\mathcal{H}_{wf}(M_\phi) = \frac{1}{4m^2 - M_\phi^2} \left\{ (5M_\phi^2 - 12m^2) \mathcal{T}_\phi - 4M_\phi^2 \mathcal{T}_B \right. 
\]

\[
+ 4M_\phi^2 \left[ m^2 + (3m^2 - M_\phi^2) \mathcal{T}_B(m^2) \right] \right\}. \tag{A2}
\]

Above, the one-loop scalar integrals are defined by

\[
\mathcal{T}_\phi = -M_\phi^2 \ln \frac{M_\phi^2}{\mu^2}, 
\]

\[
\mathcal{T}_B(p^2) = -m^2 \ln \frac{m^2}{\mu^2}, 
\]

\[
= 1 - \ln \frac{m^2}{\mu^2} + \frac{M_\phi^2 - m^2 + p^2}{2p^2} \ln \frac{m^2}{M_\phi^2} 
\]

\[
+ \frac{p^2 - (M_\phi - m)^2}{p^2} \rho_\phi(p^2) \ln \frac{\rho_\phi(p^2) - 1}{\rho_\phi(p^2) + 1}, \tag{A3}
\]

with \( \mu \) the renormalization scale and

\[
\rho_\phi(p^2) = \sqrt{\frac{p^2 - (M_\phi + m)^2}{p^2 - (M_\phi - m)^2}}. \tag{A4}
\]

In our numerical calculation, \( \mu \) is set to 1 GeV.

Appendix B: \( \beta \) functions

In Eq. (21), the \( \beta \) functions involved in the cancellation of UV divergences are

\[
\beta_m = \frac{8}{3} g_A^2 m^2, 
\]

\[
\beta_{\epsilon_1} = -\frac{11}{36} g_A^2 + (8\epsilon_1 + 3c_7) g_A^2 m, 
\]

\[
\beta_{c_7} = -\frac{5}{12} g_A^2 - c_7 g_A^2 m, 
\]

\[
\beta_{c_8} = \frac{4(c_8 - 33c_4 + 2c_5) + 264\epsilon_1(1 + g_A^2)}{864} - 33c_2, 
\]

\[
\beta_{c_9} = -33c_3 - 26c_5 + 44c_7 + (15\epsilon_1 + 11c_7) g_A^2, 
\]

\[
\beta_{c_3} = \frac{1}{288} (120\epsilon_1 - 15c_2 - 13c_3 - 60c_4) 
\]
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