A stochastic optimization formulation for the transition from open pit to underground mining

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Abstract As open pit mining of a mineral deposit deepens, the cost of extraction may increase up to a threshold where transitioning to mining through underground methods is more profitable. This paper provides an approach to determine an optimal depth at which a mine should transition from open pit to underground mining, based on managing technical risk. The value of a set of candidate transition depths is calculated by optimizing the production schedules for each depth’s unique open pit and underground operations which provide yearly discounted cash flow projections. By considering the sum of the open pit and underground mining portion’s value, the most profitable candidate transition depth is identified. The optimization model presented is based on a stochastic integer program that integrates geological uncertainty and manages technical risk. The proposed approach is tested on a gold deposit. Results show the benefits of managing geological uncertainty in long-term strategic decision-making frameworks. Additionally, the stochastic result produces a 9% net present value increase over a similar deterministic formulation. The risk-managing stochastic framework also produces operational schedules that reduce a mining project’s susceptibility to geological risk. This work aims to approve on previous attempts to solve this problem by jointly considering geological uncertainty and describing the optimal transition depth effectively in 3-dimensions.
Keywords Mine production scheduling · Stochastic optimization · Stochastic mine planning

1 Introduction

The transition from open pit (OP) to underground (UG) methods requires a large capital cost for development and potential delays in production but can provide access to a large supply of reserves and subsequently extend a mine’s life. Additionally, an operating mine may benefit from such a transition because of the opportunity to utilize existing infrastructure and equipment, particularly when in a remote location. Optimization approaches towards the open pit to underground transition decision (or OP-UG) may commence with discretizing the space above and below ground into selective units. For surface mining, material is typically discretized into mining blocks, while underground material is frequently grouped into stopes of varying size depending on the mining method chosen. From there and through production scheduling optimization, the interaction between the OP and UG components can be modeled to realistically value the asset under study.

Historically, operations research efforts in mine planning have been focused on open pits as opposed to underground operations. Most commonly, the open pit planning process begins by determining the ultimate pit limits, and the industry standard is the nested implementation of the Lerchs–Grossman’s algorithm (Lerchs and Grossmann 1965; Whittle 1988, 1999). This algorithm utilizes a maximum closure concept to determine optimal pit limits, and a nested implementation facilitates economic discounting. For underground mine planning, optimization techniques are less advanced than those employed for open pit mines and heavily depend on the mining method used. In practice, long-term underground planning is divided into two phases: stope design and production sequencing. For stope design methods, the floating stope algorithm (Alford 1995) is the oldest computerized design tool available, although not an optimization algorithm. Mine optimization research has developed methods that schedule the extraction of discretized units in underground mines (e.g. Trout 1995; Nehring and Topal 2007) based on mixed integer programming (MIP) approaches. Nehring et al. (2009), Little and Topal (2011), and Musingwini (2016) extend MIP approaches to reduce the solution times by combining decision variables and also extend application. More recent are the efforts to develop geological risk-based optimization approaches for stope design and production sequencing; these have been shown to provide substantial advantages, including more reliable forecasts, increased metal production and higher cash flows (Bootsma et al. 2014; Carpentier et al. 2016).

Some of the world’s largest mines are expected to reach their ultimate pit in the next 15 years (Kjetland 2012). Despite the importance of the topic, there is no well-established algorithm to simultaneously generate an optimal mine plan that outlines the transition from open pit mining to underground (Fuentes and Caceres 2004) or approaches that can address the topic of technical risk management, similarly to approaches for open pit mining (e.g. Godoy and Dimitrakopoulos 2004; Montiel and Dimitrakopoulos 2015; Goodfellow and Dimitrakopoulos 2016; Montiel et al. 2016).
The first attempt to address the OP to UG transition was made by Popov (1971), while more recently, a movement towards applying optimization techniques has been made starting with Bakhtavar et al. (2008) who present a heuristic method that compares the economic value of mine blocks when extracted through OP versus their value when extracted by UG techniques. The method iterates progressively downwards through a deposit, concluding that the optimal transition is the depth reached when the value of a block mined by UG methods exceeds the corresponding OP mining value. A major drawback of this method is that it provides a transition depth only described in two dimensions, which is unrealistic from a practical standpoint. An effort is presented in Newman et al. (2013) where the transition depth is formulated as a longest-path network flow. Each path within the network has a unique extraction sequence, a transition depth and a corresponding net present value (NPV). A major limitation of this development is again that it amounts to a 2D solution of what is a 3D problem, as the orebody is discretized into horizontal strata for the above and below ground mining components. At the same time a worst-case bench-wise mining schedule is adopted for open pit production and a bottom-up schedule for the underground block caving component of the mine. These highly constrained mining bench-wise progressions have been demonstrated to be far from optimal (Whittle 1988) and are rarely implemented in practice. More realistic selective mining units and an optimized schedule can provide a more accurate representation of a mine’s value, and this is the approach taken by Dagdelen and Traore (2014) who further extend this OP to UG transition idea to the context of a mining complex. In this work, the authors investigate the transition decision at a currently operating open pit mine that exists within the context of a mining complex that is comprised of five producing open pits, four stockpiles and one processing plant. Dagdelen and Traore (2014) take an iterative approach by evaluating a set of selected transition depths through optimizing the life-of-mine production schedules of both the open pit and underground mines using mixed linear integer programming techniques. The authors begin by using Geovia’s Whittle software (Geovia 2012) to generate a series of pits which provide an ultimate pit contour. The crown pillar, a large portion of undisturbed host material that serves as protection between the lowest OP working and the highest UG levels, is located below the ultimate pit. The location of the ultimate pit and crown pillar provide a basis for the underground mine design. Optimized life-of-mine production schedules are then created to determine yearly cash flow and resulting NPV. This procedure is repeated for progressively deeper transition depths until the NPV observed in the current iteration is less than what was seen for a previously considered transition depth, at which point the authors conclude that the previously considered depth, with a higher NPV, is optimal.

All the above mentioned attempts to optimize the OP-UG transition depth fail to consider geological uncertainty, a major cause of failure in mining projects (Vallee 2000). Stochastic optimizers integrate and manage space dependent geological uncertainty (grades, material types, metal, and pertinent rock properties) in the scheduling process, based on its quantification with geostatistical or stochastic simulation methods (e.g. Goovaerts 1997; Soares et al. 2017; Zagayevskiy and Deutsch 2016). Such scheduling optimizers have been long shown to increase the net present value of an operation, while providing a schedule that defers risk and has a
high probability of meeting metal production and cash flow targets (Godoy 2003; Ramazan and Dimitrakopoulos 2005; Jewbali 2006; Kumral 2010; Albor and Dimitrakopoulos 2010; Goodfellow 2014; Montiel 2014; Gilani and Sattarvand 2016; and others). Implementing such frameworks is extremely valuable when making long-term strategic decisions because of their ability to accurately value assets.

In this paper, the financial viability of a set of candidate transition depths is evaluated in order to identify the most profitable transition depth. To generate an accurate projection of the yearly cash flows that each candidate transition depth is capable of generating, a yearly life-of-mine extraction schedule is produced for both the OP and UG components of the mine. A two-stage stochastic integer programming (SIP) formulation for production scheduling is presented, which is similar to the work developed by Ramazan and Dimitrakopoulos (2005, 2013). The proposed method improves upon previous developments related to the OP-UG transition problem by simultaneously incorporating geological uncertainty into the long-term decision-making while providing a transition depth described in three dimensions that can be implemented and understood by those who operate the mine.

In the following sections, the method of evaluating a set of pre-selected candidate transition depths to determine which is optimal is discussed. Then a stochastic integer programming formulation used to produce a long-term production schedule for each of the pre-selected candidate transition depths is presented. Finally, a field test of the proposed method is analyzed as the method is applied to a gold mine.

2 Method

2.1 The general set up: candidate transition depths

The method proposed herein to determine the transition depth from OP to UG mining is based on the discretization of the orebody space into different selective units and then accurately assessing the value of the OP and UG portions of the mine based on optimized yearly extraction sequences of these discretized units. More specifically, this leads to a set of several candidate transition depths being assessed in terms of value. The candidate depth that corresponds to the highest total discounted profit is then deemed optimal for the mine being considered. Stochastic integer programming (SIP) provides the required optimization framework to make an informed decision, as this optimizer considers stochastic representations of geological uncertainty while generating the OP and UG long-term production schedules that accurately predict discounted cash flows.

For each transition depth being considered, the OP optimization process begins by discretizing the OP orebody space into blocks, sized based on operational selectivity. Candidate transition depths can be primarily identified based on feasible crown pillar locations. A crown pillar envelope outlined by a geotechnical study delineates an area that the crown pillar can be safely located within. As the crown pillar location changes within this envelope, the extent of the OP and UG orebody also changes and the impact this has on yearly discounted cash flow can be investigated (Fig. 1). The year in which the transition is planned to occur varies
across the candidate transition depths. Since the orebodies vary in size across the candidate transition depths, it is logical to allocate more years of open pit production for transition depths with a larger OP orebody and vice versa. In addition to a unique transition year, each candidate transition depth corresponds to a unique ultimate open pit limit, crown pillar location and underground orebody domain, all of which are described in the three-dimensional space.

An optimization solution outlining a long-term schedule that maximizes NPV is produced separately for the OP and UG operations at each of the candidate transition depths considered. Once optimal extraction sequences for the open pit and underground portions have been derived for each depth, the value of transitioning at a certain depth can be determined by summing the economic value of the OP and UG components. From here, the combined NPVs at each depth can be compared to easily identify the most favorable transition decision. This process is outlined in Fig. 2.

2.2 Stochastic integer programming: mine scheduling optimization

The proposed stochastic integer program (SIP) aims to maximize discounted cash flow and minimize deviations from key production targets while producing an extraction schedule that abides by the relevant constraints. The OP optimization produces a long-term schedule that outlines a yearly extraction sequence of mining blocks, while UG optimization adopts the same two-stage stochastic programming approach for scheduling stope extraction. The formulation for both OP and UG scheduling are extremely similar, so only the OP formulation is shown. The only difference for the UG formulation is that stopes are being scheduled instead of blocks, and yearly metal is being constrained instead of yearly waste as seen in the OP formulation.
Fig. 2 Schematic representation of the proposed optimization approach. The approach begins with identifying a set of candidate transition depths, then evaluating the economic viability of each through optimized productions schedules that project cash flows under geological uncertainty. Comparisons can be made within the set of transition depths to determine the most profitable option.

2.3 Developing risk-management based life-of-mine plans: open pit optimization formulation

The objective function for the OP SIP model shown in Eq. (1) maximizes discounted cash flows and minimizes deviations from targets, and is similar to that presented by Ramazan and Dimitrakopoulos (2013). Part 1 of the objective function contains first-stage decision variables, \( b_i \), which govern what year a given block \( i \) is extracted within. These are scenario-independent decision variables and the metal content of each block is uncertain at the time this decision is made. The terms in Part 1 of Eq. (1) represent the profits generated as a result of extracting certain blocks in a year and these profits are appropriately discounted based on which period they are realized in.

Part 2 of Eq. (1) contains second-stage decision variables that are used to manage the uncertainty in the ore supply during the optimization. These recourse variables \( (d) \) are decision variables determined once the geological uncertainty associated with each scenario has been unveiled. At this time, the gap above or below the mine’s annual ore and waste targets is known on a scenario-dependent basis and these deviations are discouraged throughout the life-of-mine. This component of the objective function is important because it is reasonable to suggest that if a schedule markedly deviates from the yearly ore and waste targets, then it is unlikely that the projected NPV of the schedule will be realized throughout a mine’s life. Therefore, including these variables in the objective function and reducing deviations allows
the SIP to produce a practical and feasible schedule along with cash flow projections that have a high probability of being achieved once production commences.

The following notation is used to formulate the first-stage of the OP SIP objective function:

- \( i \) is the block identifier;
- \( t \) is a scheduling time period;
- \( b_i^t = \begin{cases} 1 & \text{Block } i \text{ is mined through OP in period } t; \\ 0 & \text{Otherwise} \end{cases} \)
- \( g_i^s \) grade of block \( i \) in orebody model \( s \);
- \( Rec \) is the mining and processing recovery of the operation;
- \( T_i \) is the weight of block \( i \);
- \( NR_i = T_i \times g_i^s \times Rec \times (Price - Selling\ Cost) \) is the net revenue generated by selling all the metal contained in block \( i \) in simulated orebody \( s \);
- \( MC_i \) is the cost of mining block \( i \);
- \( PC_i \) is the processing cost of block \( i \);
- \( EV_{i}^{fg} = \frac{NR_i - MC_i - PC_i}{Price - Selling\ Cost} \) if \( NR_i > PC_i \), is the economic value of a block \( i \);
- \( r \) is the discount rate;
- \( E\{V_i\} = \begin{cases} NR_i - MC_i - PC_i & \text{if } NR_i > PC_i \\ -MC_i & \text{if } NR_i \leq PC_i \end{cases} \) is the economic value of a block \( i \);
- \( S \) is the number of simulated orebody models;
- \( z \) is an identifier for the transition depth being considered;
- \( P_z \) is the number of production periods scheduled for candidate transition depth \( z \).

The following notation is used to formulate the second-stage of the OP SIP objective function:

- \( s \) is a simulated orebody model;
- \( w \) and \( o \) are target parameters, or type of production targets; \( w \) is for the waste target; \( o \) if for the ore production target;
- \( u \) is the maximum target (upper bound);
- \( l \) is the minimum target (lower bound);
- \( d_{su}^{to}, d_{su}^{lw} \) are the excessive amounts for the target parameters produced;
- \( d_{sl}^{to}, d_{sl}^{lw} \) are the deficient amounts for the target parameters produced;
- \( c_{u}^{to}, c_{u}^{lw}, c_{l}^{to}, c_{l}^{lw} \) are unit costs for \( d_{su}^{to}, d_{su}^{lw}, d_{sl}^{to}, d_{sl}^{lw} \) respectively in the optimization’s objective function.

**OP Objective function**

\[
\text{Max} \sum_{t=1}^{P_z} \sum_{i=1}^{N} E\{NPV_i\} \cdot b_i^t - \sum_{s=1}^{S} \sum_{t=1}^{P_z} \frac{1}{S} \left( c_{u}^{to} d_{su}^{to} + c_{l}^{to} d_{su}^{to} + c_{u}^{lw} d_{su}^{lw} + c_{l}^{lw} d_{su}^{lw} \right) 
\]

Part 1

\[
E\{(NPV_i)\} = \frac{E\{V_i\}}{(1+r)^t} 
\]

Part 2

\[
\text{Part 1} - \text{Part 2} \]
OP Constraints
The following notation is required for the constraints:

- $W_{tar}$ is the targeted amount of waste material to be mined in a given period;
- $O_{tar}$ is the targeted amount of ore material to be mined in a given period;
- $O_{si}$ is the ore tonnage of block $i$ in the orebody model $s$;
- $Q_{UG,tar}$ is the yearly metal production target during underground mining;
- $MCap_{min}$ is the minimum amount of material required to be mined in a given period;
- $MCap_{max}$ is the maximum amount of material that can possibly be mined in a given period;
- $l_i$ is the set of predecessor for block $i$.

**Scenario-Dependent**
Waste constraints for each time period $t$

$$
\sum_{i=1}^{N} W_{si} b^t_i - d^{g}_{su} + d^{g}_{sl} = W_{tar} \quad s = 1, 2, \ldots, S; \quad t = 1, 2, \ldots, P_z
$$

(2)

Processing constraints

$$
\sum_{i=1}^{N} O_{si} b^t_i - d^{o}_{su} + d^{o}_{sl} = O_{tar} \quad s = 1, 2, \ldots, S; \quad t = 1, 2, \ldots, P_z
$$

(3)

**Scenario-Independent**
Precedence constraints

$$
b^t_i - \sum_{k=1}^{l_i} b^k_h \leq 0 \quad i = 1, 2, \ldots, N; \quad t = 1, 2, \ldots, P_z; \quad h \in l_i
$$

(4)

Mining capacity constraints

$$
MCap_{min} \leq \sum_{i=1}^{N} T_i b^t_i \leq MCap_{max} \quad t = 1, 2, \ldots, P_z
$$

(5)

Reserve constraints

$$
\sum_{i=1}^{N} b^t_i \leq 1 \quad i = 1, 2, \ldots, N
$$

(6)

Constraints (2) and (3) are scenario-dependent constraints that quantify the magnitude of deviation within each scenario from the waste and ore targets based on first-stage decision variables ($b^t_i$). Constraints (4)–(6) contain only first-stage decision variables ($b^t_i$) and thus are scenario-independent. The precedence constraint (4) ensures that the optimizer mines the blocks overlying a specific block $i$ before it.
can be considered for extraction. The reserve constraint (6) prevents the optimizer from mining a single block \( i \) more than once.

The size of OP mine scheduling applications cause computational issues when using commercial solvers since it can take long periods of time to arrive at or near an optimal solution, if able to solve (Lamghari et al. 2014). In order to overcome these issues, metaheuristics can be used. These are algorithms which efficiently search the solution space and have the proven ability to find high quality solutions in relatively small amounts of time (Ferland et al. 2007; Lamghari and Dimitrakopoulos 2012; Lamghari et al. 2014). To be effective these algorithms must be specifically tailored to match the nature of the problem being solved. In the context of mine production scheduling, the tabu search algorithm is well suited, and a parallel implementation is utilized here to schedule the open pit portion of the deposit for each transition depth that is considered (Lamghari and Dimitrakopoulos 2012; Senecal 2015). For more details on tabu search, the reader is referred to the Appendix.

### 2.4 Developing risk-managing life-of-mine plans: underground optimization formulation

The UG scheduling formulation is very similar to the OP formulation. Both have objective functions which aim to maximize discounted profits, while minimizing deviations from key production targets. The UG objective function is similar to that proposed for the OP scheduling function in Eq. (1), except the binary decision variables can be represented using \( d_j \) which designates the period in which extraction-related activities occur for each stope \( j \). As well, recourse variables in the second portion of the objective function aim to limit deviations from the ore and metal targets, as opposed to the ore and waste targets in the OP objective function. Since UG mining methods have a higher level of selectivity than OP mining, waste is often not mined, but rather left in situ and only valuable material is produced. Therefore, it is more useful to constrain the amount of yearly metal produced in a UG optimization. Underground cost structure is viewed from a standpoint of cost per ton of material extracted. This standard figure contains expenses related to development, ventilation, drilling, blasting, extracting, backfilling and overhead. In terms of size and complexity, the UG scheduling model presented here is simpler than the OP model. The reduced size is due to only considering long-term extraction constraints and a small number of mining units that require scheduling. This allows for the schedule to be conveniently solved using IBM ILOG CPLEX 12.6 (IBM 2011), a commercially available software which relies on mathematical programming techniques to provide an exact solution.

**UG Constraints**

**Scenario-Dependent:**

Metal constraints for each time period \( t \)

\[
\sum_{j=1}^{M} g_{sj} d_j - d_{su}^m + d_{sl}^m = Q_{UG,\text{tar}} \quad s = 1, 2, \ldots, S; \quad t = 1, \ldots, P_z \quad (7)
\]
Processing constraints

\[ \sum_{j=1}^{M} O_{sj} d'_{ij} - d''_{vij} + d''_{sil} = O_{tar} \quad s = 1, 2, \ldots, S; \quad t = 1, \ldots, P_z \quad (8) \]

Scenario-Independent

Precedence constraints

\[ d'_{ij} - \sum_{k=1}^{l} d''_{hij} \leq 0 \quad j = 1, 2, \ldots, M; \quad t = 1, \ldots, P_z; \quad h \in l_j \quad (9) \]

Mining capacity constraints

\[ MCap_{min}^{UG} \leq \sum_{j=1}^{M} T_j d'_{ij} \leq MCap_{max}^{UG} \quad t = P_{OP} + 1, \ldots, P \quad (10) \]

Equations (7)–(10) show the constraints included in the UG SIP formulation. In Eq. (9), the set of predecessors for each stope \((l_j)\) is defined by considering the relevant geotechnical issues which constrain the sequencing optimization. These precedence relationships are created using the Enhanced Production Scheduler (EPS) software from Datamine (Datamine Software 2013). For the application presented in this paper, the precedence relationships implemented were passed along by industry-based collaborators who operate the mine. Once the optimization for both the OP and UG components is completed for each candidate transition depth, the optimal transition depth can then be identified as the depth \(z\) that leads to a maximum value of the expression below.

\[ NPV_{OP}^{z} + NPV_{UG}^{z} \quad z = 1, \ldots, D \quad (11) \]

3 Application at a gold deposit

In order to evaluate the benefits of the proposed method, it is applied to a gold deposit that has been altered to suit an OP-UG transition scenario. In this case study, the optimal transition depth from open pit to underground mining of a gold operation is investigated. The mine’s life begins with open pit mining and will transition to production through underground mining by implementing the underhand cut and fill method. Underground production is planned to commence immediately after open pit production ceases. On the mine site there is one mill processing stream with a fixed recovery curve. No stockpile is considered. A crown pillar envelope for the deposit is identified a priori along with four crown pillar locations within this envelope leading to four distinct candidate transition depths which are evaluated. The size of the crown pillar remains, although the location changes. Investigating the impact of the size of OP and UG mines on the dimensions of the crown pillar is a topic for future research. Each transition depth possesses a
unique above and below ground orebody, dictated by a varying crown pillar location in the vertical plane. The year in which the transition between mining methods occurs varies throughout the candidate transition depths to accommodate increased reserves in the OP or UG orebody as the location of the crown pillar shifts. It should be noted that the capital investment required to ramp up UG mining is not considered in the application presented, and can be integrated to the results of the approach presented; as expected, the related capital investment would have an impact on overall project NPV. The combined OP and UG mine life is 14 years for all candidate transition depths tested. The discrepancy in orebody size and reserves that can be accessed by OP and UG methods for each candidate transition depth along with the transition year is shown in Table 1. As the size of the OP deepens and the number of OP blocks increases, the amount of UG stopes within the accessible underground resource decreases. Despite the variation in the number of blocks and stopes in each OP and UG mine, the annual tonnage capacity remains the same. It is also important to note that the tonnage varies throughout the UG stopes targeted for production. A schematic of how the crown pillar location varies can be seen in Fig. 3. The relevant economic and technical parameters used to generate the optimization models are shown in Table 2.

### 3.1 Stochastic optimization results and risk analysis

The transition depth determined to be optimal for the proposed stochastic optimization framework is Transition Depth 2 (TD 2) as seen in Fig. 4. This transition depth can be described by having a crown pillar located at an elevation of 760ft, and access to 72,585 open pit blocks and 356 stopes. The optimal transition depth in this case study provides a 5% higher NPV than the next best candidate transition depth and a 13% NPV improvement over the least optimal depth. Such a large impact on the financial outcome of a mine confirms that in-depth analysis before making this type of long-term strategic decision is beneficial.

In order to evaluate the risk associated with stochastic decision making, a risk analysis is performed on the life-of-mine plans corresponding to the optimal transition depth stated above. Similar analysis has been done extensively on open pit case studies (Dimitrakopoulos et al. 2002; Godoy 2003; Jewbali 2006; Leite and Dimitrakopoulos 2014; Ramazan and Dimitrakopoulos 2005, 2013; Goodfellow 2014). To do so, a set of 20 simulated scenarios of the grades of the deposit are used.

### Table 1  Size of orebodies and life of mine length at each transition depth

| Transition Depth | Number of OP blocks | Number of UG stopes | Production years through OP | Production years through UG |
|------------------|---------------------|---------------------|-----------------------------|-----------------------------|
| 1                | 64,255              | 418                 | 7                           | 7                           |
| 2                | 72,585              | 356                 | 8                           | 6                           |
| 3                | 80,915              | 340                 | 9                           | 5                           |
| 4                | 89,245              | 311                 | 10                          | 4                           |
and passed through the long-term production schedule determined for the optimal transition depth, which in this case is Transition Depth 2. This process provides the yearly figures for mill production tonnages, metal production and cash flow projections for each simulation if the schedule was implemented and the grades within a given simulation were realized.

Figure 5 shows that the stochastic schedule produced for Transition Depth 2 has a high probability of meeting mill input tonnage targets on a yearly basis. The ability to meet targets translates into a high level of certainty with regards to realizing yearly cash flow projections once production commences; this is expanded upon later. Stochastic schedules perform well during risk analysis because the inherent geological variability within the deposit is captured within the simulations and then considered while making scheduling decisions in a stochastic framework.

Figure 5 there are large deviations from the target yearly ore production targets in period 7 and 8, before the Transition Depth 2 schedule shifts to underground production in period 9. This is because geological risk discounting (Ramazan and Dimitrakopoulos 2005, 2013) is utilized as a risk management technique during OP scheduling, which penalizes deviation from the production targets more heavily in

| Table 2 Economic and technical parameters |   |
|-------------------------------------------|---|
| Metal price                               | $900/oz |
| Crown pillar height                       | 60 ft   |
| Economic discount rate                    | 10%     |
| Processing cost/ton                       | $31.5   |
| OP mining cost/ton                        | $1.5    |
| UG mining cost/ton                        | $135    |
| OP mining rate                            | 18,500,000 t/year |
| UG mining rate                            | 350,000 t/year |
| OP mining recovery                        | 0.95    |
| UG mining recovery                        | 0.92    |
the early years of production. This is valuable in the capital-intensive mining sector to increase certainty within early year project revenue and potentially decrease the length of a project’s payback period. In addition to this, common long-term scheduling practices within the mining industry involve updating the schedule on a yearly basis as new information about the orebody is gathered, so the large deviations later in the open pit mine life are not a large cause for concern. After the transition is made to underground mining in year 9, a high penalty is incurred on deviations from ore targets to ensure that ore targets are met in the early years of the underground mine. This leads to a tight risk profile throughout the underground life

Fig. 4 Risk profile on NPV of stochastic schedules. Lines show the expected NPV for each transition depth while considering geological uncertainty. It should be noted that Transition Depth 1 makes the transition in year 7, Transition Depth 2 in year 8, Transition Depth 3 in year 9 and Transition Depth 4 in year 10. Transition Depth 2 is the most profitable decision of the set, with an expected NPV of $540 M

Fig. 5 Performance of stochastic schedule in meeting yearly ore targets
of the mine (periods 9–14). Figure 6 shows the stochastic schedule’s ability to produce metal at a steady rate throughout the entire life-of-mine.

### 3.2 Comparison to deterministic optimization result

To showcase the benefit of incorporating geological uncertainty into long-term strategic decision making, the SIP result is benchmarked against a deterministic optimization that uses the same formulation. The deterministic optimization process however receives an input of only a single orebody model containing estimated values for the grade of each block and stope. Yearly production scheduling decisions are made based on these definitive grade estimates, and from there yearly cash flows streams are projected. This procedure is followed for each of the four transition depths considered, as was done for the stochastic case. Geovia’s Whittle software (Geovia 2012) is used to schedule the open-pit portion of the mine, while an MIP is used for the underground scheduling. This underground scheduling utilizes the deterministic equivalent of the stochastic underground schedule formulation seen earlier. The projected yearly discounted cash flows can be seen and suggest that Transition Depth 2 (TD 2) is also optimal from a deterministic perspective (Fig. 7).

To assess the deterministic framework’s ability to manage geological uncertainty, risk analysis is performed on the deterministic schedule for the optimal transition depth 2. The 20 geological (grade) simulations mentioned earlier are passed through the deterministic schedule produced for Transition Depth 2, and the yearly cash projections based on each simulation are summarized in Fig. 8. The results are compared to identical analysis on the stochastic schedule, also for Transition Depth 2. The P50 (median) NPV of the simulations when passed through the stochastic schedule is 9% or $42 M higher than the P50 observed for the deterministic case. Further to that point, this analysis suggests that there is a 90%
chance that the deterministic schedule’s NPV falls below the NPV of the stochastic schedule.

In Fig. 8, the NPV projected by risk analysis is 5% below what the optimizer originally predicted. Along with this, there is a large variation in the yearly cash generated. Figure 8 also concludes that there is a 70% chance that once production commences, the realized NPV will be less than the original projection. Figure 8 shows that the P50 of the stochastic risk profiles for transition depth 2 are higher than the deterministic projected NPV and the P50 of the deterministic risk profiles by 4% and 9% respectively. This trend of increased value for the stochastic framework extends to other transition depths as well. Figure 9 shows that in addition to the stochastic schedule at the optimal transition depth (TD 2) generating a higher NPV than the optimal deterministic result, also TD 2, the next best transition depth in the stochastic case (TD 3) is $17 M or 3.4% lower than the optimal deterministic result.

The increased NPVs seen for the stochastic approach are due to the method’s ability to consider multiple stochastically generated scenarios of the mineral deposit, so as to manage geological (metal grade) uncertainty and local variability while making scheduling decisions. Overall, the stochastic scheduler is more informed and motivated to mine lower risk, high grade areas early in the mine life and defer extraction of lower grade and risky materials to later periods.

Figure 10 shows the magnitude of deviation from a predetermined yearly mill tonnage for the schedules produced by both the stochastic and deterministic optimizer at transition depth 2. Figure 10 shows the median (P50) of deviations from the yearly mill tonnage targets for the stochastic and deterministic schedules with respect to the 20 simulated orebody models. Throughout the entire life of the mine, the stochastic schedule limits these deviations while the deterministic

![Image](image-url)
schedule has no control over such risk. The deterministic schedule’s inability to meet yearly mill input tonnage is a cause for concern and suggests that the mine is unlikely to meet important targets once production commences if such a schedule is implemented.

Figure 11 shows a visual comparison between the stochastic and deterministic schedules produced for Transition Depth 2. The shading in Fig. 11 describes which

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**Fig. 8** Risk analysis of projected deterministic NPV. The impact of geological uncertainty on the deterministic schedule can be quantified through risk analysis. The NPV of the deterministic schedule falls from $520 M to $497 M as the impact of geological uncertainty is considered. The stochastic schedule remains robust to uncertainty with an NPV of $540 M, 9% or $43 M higher than the projected deterministic value when considering geological uncertainty in the cash flow projections.

**Fig. 9** Comparison of NPV at different transition depths
A stochastic optimization formulation for the transition…

period a mining block is scheduled to be extracted in. Overall, the stochastic schedule appears to be smoother and more mineable than the deterministic schedule, meaning that large groups of nearby blocks are scheduled to be extracted within the same period. As well, both cross-sections reveal that the stochastic schedule mines more material than the deterministic schedule produced by Geovia’s Whittle (Geovia 2012), resulting in a larger ultimate pit for the stochastic case. These differences stem from Whittle determining the ultimate pit before scheduling by

Fig. 10 Magnitude of deviation from yearly mill input tonnage target. Based on deterministic and stochastic schedules produced for Transition Depth 2 yearly ore tonnage projections can be made along with how these projections deviate from the yearly tonnage target. Show here is the difference in magnitude of deviations for a deterministic schedule created with no information regarding geological uncertainty

Fig. 11 Two cross-sectional views of the schedule obtained by the proposed SIP (left) and the deterministic schedule produced by Whittle (right) for Transition Depth 2. The colored regions indicate the period in which a group of material is scheduled for extraction. (Color figure online)
utilizing a single estimated orebody model containing smoothed grade values. In the stochastic case, the task of determining the ultimate pit contour is done while having knowledge of 20 geological simulations which provide detailed information on the high and low grade areas within the deposit. In this case the stochastic scheduler identifies profitable deep-lying high-grade material that cannot be captured using traditional deterministic methods.

4 Conclusions and future work

A new method for determining the optimal OP-UG transition depth is presented. The proposed method improves upon previously developed techniques by jointly taking a truly three-dimensional approach to determining the optimal OP-UG transition depth, through the optimization of extraction sequences for both OP and UG components while considering geological uncertainty and managing the related risk. The optimal transition decision is effectively described by a transition year, a three-dimensional optimal open pit contour, a crown pillar location and a clearly defined underground orebody. In the case study, it was determined that the second of four transition depths evaluated is optimal which involves transitioning to underground mining in period 9. Making the decision to transition at the second candidate transition depth evaluated results in a 13% increase in NPV over the worst-case decision, as predicted by the stochastic framework. Upon closer inspection through risk analysis procedures, the stochastic framework is shown to provide a more realistic valuation of both the OP and UG assets. In addition to this, the stochastic framework produces operationally implementable production schedules that lead to a 9% NPV increase and reduction in risk when compared to the deterministic result. It is shown that the yearly cash flow projections outlined by the deterministic optimizer for the underground mine life are unlikely to be met, resulting in misleading decision criteria. Overall, the proposed stochastic framework has proven to provide a robust approach to determining an optimal open pit to underground mining transition depth. Future studies should aim to improve on this method by considering more aspects of financial uncertainty such as inflation and mining costs.

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Appendix

In the presented work, a parallel implementation of tabu search is used to solve the large open pit mine scheduling problem in a reasonable time. This metaheuristic method takes advantage of the multi-core processing architecture in modern computers to effectively distribute tasks and find high quality solutions. Essentially, the algorithm perturbs an initial feasible production schedule by changing the yearly scheduling decision for a given block, then impact of these perturbations is evaluated and they are accepted based on their ability to increase the value of the solution. As the algorithm accepts perturbation and progresses through the solution space, it prohibits itself from repeatedly visiting the same solution by labeling these previously visited solutions as tabu (forbidden) for a certain amount of time. The tabu search procedure stops after a specified number of proposed perturbations have been evaluated which fail to improve the solution. In order to prevent the algorithm from getting trapped in a locally (as opposed to globally) optimal solution, a diversification strategy is included in the metaheuristic to generate new, unique starting solutions that can then be improved.

The specific implementation used in the work presented here is known as Parallel Independent tabu search (Senecal 2015) where the so termed master–slave (Hansen 1993) parallel algorithm design is used. In this scheme, a master thread delegates the task of performing tabu search to each available thread and provides them with a unique starting solution. These threads then operate independently to identify the best solution possible using tabu search. The solutions for each are then compared to identify the optimal solution. With this efficient implementation of tabu search, more instances of the algorithm can be run simultaneously to thoroughly cover the solution space in less time than a purely sequential and single threaded approach. More algorithmic details can be found in Lamghari and Dimitrakopoulos (2012).

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