POST-NEWTONIAN PARAMETERS FOR GENERAL BLACK HOLE AND SPHERICALLY SYMMETRIC $p$-BRANE SOLUTIONS

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Black hole $p$-brane solutions for a wide class of intersection rules are considered. The solutions are defined on a manifold which contains a product of $(n - 1)$ Ricci-flat “internal” spaces. The post-Newtonian parameters $\beta$ and $\gamma$ corresponding to a 4-dimensional section of the metric for general intersection rules are studied. It is shown that $\beta$ does not depend but $\gamma$ depends on $p$-brane intersections. For “block-orthogonal” intersection rules spherically symmetric solutions are considered, and explicit relations for post-Newtonian parameters are obtained. The bounds on parameters of solutions following from observational restrictions in the Solar system are presented.

1. Introduction

Multidimensional gravitational models (see, e.g., [1–5] and references therein) are rather popular at present. There exist two main approaches within the multidimensional paradigm: (i) extra dimensions are considered as real space-time dimensions; (ii) extra dimensions are considered as some imaginary (e.g. mirror, or virtual) reality, as a mathematical tool for generating the 4-metric and extra (dilatonic) scalar fields. In the second approach the 4-dimensional metric is a “real physical object” that should be extracted in some way from the multidimensional one.

In both cases we deal with exact solutions to multidimensional field equations with 4-dimensional sections which may describe some astrophysical objects (e.g. the Sun, neutron stars, etc.) and the motion of test bodies (e.g. Mercury). The 4-metric is usually considered to be a small “deformation” of the original Schwarzschild one and is governed by some parameters related to the post-Newtonian (PN) ones (for definition see Sec. 2). Applying the classical tests of general relativity: gravitational redshift, light deflection, perihelion advance and time delay, (see [6, 7]) along with the classical tests we can find the restrictions on the PN parameters.

In both the PN parameters and classical tests were considered (in the framework of the approach (ii)) for a metric generalizing the Schwarzschild solution to the case of several Ricci-flat internal spaces. Later on, Kramer’s more special 5-dimensional solution (also known as the Gross-Perry-Sorkin-Davidson-Owen one) was considered in [9–11] for the approach (i) and the classical tests (e.g. perihelion advance and geodetic precession) were generalized to situations in which the components of momentum and spin along the extra coordinate do not vanish.

In this paper we deal with 4-dimensional sections of multidimensional spherically symmetric and black hole solutions with generalized $p$-branes charged by fields of forms. In Sec. 2 black hole $p$-brane solutions for general intersection rules are considered and the PN parameters are calculated. In Sec. 3 more general spherically symmetric solutions are considered, but for more special “block-orthogonal” intersection rules, and explicit relations for PN parameters are obtained. In both sections the bounds on the solution gravitation parameter following from observational restrictions in the Solar system are considered.

2. Black hole $p$-brane solutions

2.1. Solutions

We consider a model governed by the action

$$S = \int d^D x \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \theta_a n_a! \exp[2 \lambda_a(\varphi)] (F^a)^2 \right\}$$  \hfill (2.1)$$

where $g = g_{MN}(x) dx^M \otimes dx^N$ is a D-dimensional metric, $\varphi = (\varphi^a) \in \mathbb{R}^l$ is a vector of scalar (dilatonic) fields, $(h_{\alpha\beta})$ is constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $\theta_a = \pm 1$, $F^a = dA^a = da \wedge F_{M_1 ... M_n} dz^{M_1} \wedge ... \wedge dz^{M_n}$ is an $n_a$-form ($n_a \geq 1$), $\lambda_a$ is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{a\alpha} \varphi^\alpha$, $\alpha = 1, \ldots, l$; we denote $|g| = |\det(g_{MN})|$ and $(F^a)^2 = F_{M_1 ... M_n}^a F_{N_1 ... N_{n_a}} g^{M_1 M_{N_{n_a}}} \ldots g^{M_n \cdot N_{n_a}}$, $a \in \Delta$. Here $\Delta$ is some finite set.

Let us consider a family of black-hole solutions to the field equations corresponding to the action (2.1).
These solutions are defined on the manifold
\[ M = (R_0, +\infty) \times (M_1 = S^{d_1}) \times (M_2 = \mathbb{R}) \times \ldots \times M_n, \]
and have the following form:
\[
g = \left( \prod_{s \in S} H_s^{2\delta_s d(I_s)/(D-2)} \right) \left\{ F^{-1} dR \otimes dR + R^2 d\Omega^2_\text{det} - \left( \prod_{s \in S} H_s^{-2h_s} \right) F dt \otimes dt + \sum_{i=1}^n \left( \prod_{s \in S} H_s^{-2\delta_i s} \right) g^I \right\}, \tag{2.3} \]
where
\[
F^a = \sum_{s \in S} \delta^a \delta^s F^s, \tag{2.5} \]
and the parameter \(h_s\) appearing in the solution satisfy the relations
\[ h_s = K_s^{-1}, \quad K_s = B_{ss}, \tag{2.12} \]
where
\[ B_{ss'} = \frac{d(I_s \cap I_{s'}) + d(I_s) d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{a\alpha} \lambda_{b\beta} h^{\alpha\beta}, \tag{2.13} \]
\[ s, s' \in S, \quad \text{with } (h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}. \]
Here we assume that
(i) \( B_{ss} \neq 0 \),
(ii) \( \det(B_{ss'}) \neq 0 \),
i.e., the matrix \((B_{ss'})\) is non-degenerate. Let
\[ (A_{ss'}) = \left(2B_{ss'}/B_{ss's'}\right). \tag{2.16} \]

Here some ordering in \( S \) is assumed.

The functions \( H_s = H_s(z) > 0 \), \( z = 2\mu/R^d \in (0, 1) \) obey the equations
\[
\frac{d}{dz} \left( \frac{1 - z}{H_s} \frac{d}{dz} H_s \right) = B_s \prod_{s' \in S} H_{s'}^{-A_{ss'}}, \tag{2.17} \]
equipped with the boundary conditions
\[
H_s(1-0) = H_{s0} \in (0, +\infty), \tag{2.18} \]
\[ H_s(+0) = 1, \tag{2.19} \]
\[ B_s = K_s \varepsilon_s Q_s^2/(2\mu)^2, \tag{2.20} \]
si.e., the matrix \((B_{ss'})\) is non-degenerate. Let
\[ \chi_s = (-\varepsilon[g])^{(1-\chi_s)/2} \varepsilon(I_{s0}) \theta_{ax}, \tag{2.21} \]
si.e., \( \varepsilon[g] \equiv \text{sign} \det(g_{MN}) \). More explicitly, \( (2.21) \) reads: \( \varepsilon_s = \varepsilon(I_{s0}) \delta_{ax} \) for \( \varepsilon_s \equiv \varepsilon(I_{s0}) \theta_{ax} \), for \( \varepsilon_s = -\varepsilon(g) \varepsilon(I_{s0}) \theta_{ax}, \)
\[ s \in S. \]

Eqs. \( (2.17) \) are equivalent to special Toda-type equations. The first boundary condition \( (2.18) \) guarantees the existence of a horizon at \( R^d = 2\mu \) for the metric \( (2.3) \). The second condition \( (2.19) \) ensures the asymptotic (at \( R \to +\infty \)) flatness of the \((2 + d_1)\)-section of the metric.

The solutions are valid if the following restrictions on the sets \( \Omega_{a,v} \) are imposed. (These restrictions guarantees the block-diagonal structure of the stress-energy tensor, for as the metric, and the existence of a \( \sigma \)-model representation \( (1.3) \)). We denote \( w_1 \equiv \{ i | i \in \{1, \ldots, n\}, \quad d_i = 1 \}, \)
and \( n_1 = |w_1| \) (i.e. \( n_1 \) is the number of 1-dimensional spaces among \( M_i, \quad i = 1, \ldots, n \)).
Restriction. Let 1a) \( n_1 \leq 1 \) or 1b) \( n_1 \geq 2 \) and for any \( a \in \Delta \), \( v \in \{ e, m \} \), \( i, j \in w_1 \), \( i < j \) and there are no \( I, J \in \Omega_{a,v} \) such that \( i \in I \), \( j \in J \) and \( I \setminus \{ i \} = J \setminus \{ j \} \).

This restriction is satisfied in the non-composite case: \(|\Omega_{a,v}| = 1\), (i.e. when there is only one \( p \)-brane for each value of the colour index \( a, a \in \Delta \)); The restriction forbids certain intersections of two \( p \)-branes with the same colour index for \( n_1 \geq 2 \).

The Hawking “temperature” corresponding to the solution is found to be

\[
T_H = \frac{d}{4\pi(2\mu)^{1/2}} \prod_{s \in S} H_{s_0}^{-h_s},
\]

(2.23)

where \( H_{s_0} \) are defined in (2.18)

“Block-orthogonal” black holes \([16]\). Let

\[
S = S_1 \cup \ldots \cup S_k,
\]

(2.24)

\( S_i \neq \emptyset \), \( i = 1, \ldots, k \), and

(iii) \( (U^s, U^{s'}) = 0 \)

(2.25)

for all \( s \in S_i \), \( s' \in S_j \), \( i \neq j \); \( i, j = 1, \ldots, k \). In \([14],[15]\) “block-orthogonal” solutions were obtained:

\[
H_s(z) = (1 + \bar{P}_s)z^{b_0^s},
\]

(2.26)

\( s \in S \), where

\[
b_0^s = 2 \sum_{s' \in S} A^{ss'},
\]

(2.27)

\[
(A^{ss'}) = (A_{ss'})^{-1}
\]

and \( \bar{P}_s = \bar{P}_{s'}, s, s' \in S_i \), \( i = 1, \ldots, k \).

Let \( (A_{ss'}) \) be a Cartan matrix for a finite-dimensional semisimple Lie algebra \( \mathcal{G} \). In this case all powers in \((2.26)\) are positive integers \([17]\).

\[
b_0^s = n_s \in \mathbb{N},
\]

(2.28)

and hence all functions \( H_s \) are polynomials, \( s \in S \).

Conjecture. Let \( (A_{ss'}) \) be a Cartan matrix for a semisimple finite-dimensional Lie algebra \( \mathcal{G} \). Then the solutions of Eqs. \((2.17)-(2.19)\) are polynomials

\[
H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k,
\]

(2.29)

where \( P_s^{(k)} \) are constants, \( k = 1, \ldots, n_s \), \( P_s^{(n_s)} \neq 0 \), \( s \in S \).

In \([12]\) this Conjecture was verified for simple Lie algebras \( \mathfrak{a}_m = sl(m + 1, \mathbb{C}) \), \( m \geq 1 \). In extremal case (\( \mu = +0 \)) an a analogue of this conjecture was suggested previously in \([18]\).

2.2. Post-Newtonian approximation

Let \( d_1 = 2 \). Here we consider the 4-dimensional section of the metric \((2.3)\), namely,

\[
g^{(4)} = U \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\Omega_2^2 - U_1 \left( 1 - 2\mu/R \right) dt \otimes dt \right\}
\]

(2.30)

in the range \( R > 2\mu \), where

\[
U = \prod_{s \in S} H_s^{2d(I_s)h_s/(D-2)},
\]

(2.31)

\[
U_1 = \prod_{s \in S} H_s^{-2h_s}.
\]

(2.32)

Let us imagine that some real astrophysical objects (e.g. stars) may be described (or approximated) by the 4-dimensional physical metric \((2.3)\), i.e. they are traces of extended multidimensional objects (charged \(p\)-brane black holes).

In the post-Newtonian approximation we restrict ourselves to the first two powers of \(1/R\), i.e.

\[
H_s = 1 + \frac{P_s}{R} + \frac{P_s^{(2)}}{R^2} + o\left( \frac{1}{R^3} \right),
\]

(2.33)

for \( R \to +\infty \), \( s \in S \).

Introducing a new radial variable \( \rho \) by the relation

\[
R = \rho \left( 1 + \frac{\mu}{2\rho} \right)^2,
\]

(2.34)

\( \rho > \mu/2 \), we may rewrite the metric \((2.30)\) in a 3-dimensional conformally flat form and calculate the post-Newtonian parameters \( \beta \) and \( \gamma \) (Eddington parameters) using the following standard relations:

\[
g^{(4)}_{00} = -(1 - 2V + 2\beta V^2) + O(V^3),
\]

(2.35)

\[
g^{(4)}_{ij} = \delta_{ij} (1 + 2\gamma V) + O(V^2),
\]

(2.36)

\( i, j = 1, 2, 3 \), where \( V = GM/\rho \) is Newton’s potential, \( G \) is the gravitational constant and \( M \) is the gravitational mass. From \((2.35)-(2.36)\) we deduce the formulae

\[
GM = \mu + \sum_{s \in S} h_s P_s \left( 1 - \frac{d(I_s)}{D-2} \right)
\]

(2.37)

and

\[
\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} h_s (P_s^2 + 2\mu P_s - 2P_s^{(2)} \left( 1 - \frac{d(I_s)}{D-2} \right)
\]

(2.38)

\[
\gamma - 1 = \frac{1}{GM} \sum_{s \in S} h_s P_s \left( 1 - \frac{d(I_s)}{D-2} \right).
\]

(2.39)

For special “block-orthogonal” solutions this relation coincides with that of \([10]\). For a more general
spherically symmetric case it was obtained in [15] (see next section).

The parameter $\beta$ is determined by the squared charges $Q_s^2$ of the $p$-branes (or, more correctly, by the charge densities) and the signature parameters $\varepsilon_s$.

Now, we will show that in the general case, as in [16], the following theorem is valid:

**Theorem.** The parameter $\beta$ does not depend on the dimensions of $p$-brane intersections.

**Proof.** From (2.17) we get in the zero order of a $\varepsilon$-decomposition:

$$P_s^2 + 2\mu P_s - 2P_s^{(2)} = -K_s\varepsilon_s Q_s^2,$$

(2.40)

$s \in S$. Hence,

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} (-\varepsilon_s)Q_s^2 \left(1 - \frac{d(I_s)}{D - 2}\right).$$

(2.41)

Thus $\beta$ depends on $Q_s/GM$, i.e., on the ratios of the charges and the mass. It does not depend on the intersections. The theorem is proved.

The parameter $\beta$ is obtained without knowledge of the general solution for $H_s$ and does not depend on the quasi-Cartan matrix and on $p$-brane intersections. The parameter $\gamma$ depends on the ratios $P_s/GM$, where $P_s$ are functions of $GM, Q_s$ and also $A = (A_{s,s'})$. Thus it depends on the intersections. The calculation of $\gamma$ needs an exact solution for the radial functions $H_s$.

For the most physically interesting $p$-brane solutions, $\varepsilon_s = -1$ and $d(I_s) < D - 2$ for all $s \in S$, which implies $\beta > 1$.

Using the relations (2.34) and (2.41), we can obtain bounds on $\beta$ and $\gamma$ which may be compared with the experimental data.

**Observational restrictions.** The observations in the Solar system give tight constraints on the Eddington parameters [2]:

$$\gamma = 1.000 \pm 0.002$$

(2.43)

$$\beta = 0.9998 \pm 0.0006.$$  

(2.44)

The first restriction is a result of the Viking time-delay experiment [13]. The second one follows from (2.43) and the analysis of the lunar laser ranging data. In this case a high precision test based on the calculation of the combination $(4\beta - \gamma - 3)$, appearing in the Nordtvedt effect [21], is used [22].

For small enough $p_s = P_s/GM$ and $P_s^{(2)} \sim P_s$ we get $GM \sim \mu$ and hence

$$\beta - 1 \sim \sum_{s \in S} h_s p_s \left(1 - \frac{d(I_s)}{D - 2}\right),$$

(2.45)

$$\gamma - 1 \sim - \sum_{s \in S} h_s p_s \left(1 - 2\frac{d(I_s)}{D - 2}\right).$$

(2.46)

i.e., $\beta - 1$ and $\gamma - 1$ are of the same order. Thus, for small enough $p_s$, it is possible to fit the Solar-system restrictions (2.43) and (2.44).

3. Spherically symmetric solutions in the block-orthogonal case

3.1. Solutions

Here we consider the spherically symmetric solutions in the block-orthogonal case (2.24)–(2.25).

The solutions for the metric and scalar fields may be written as follows:

$$g = \left(\prod_{s \in S} \hat{H}_s^{2n_s} d(I_s)\right)^{1/(D - 2)} \left(F^{b_s - 1} dR \otimes dR + R^2 F^{b_s} d^2\Omega - \left(\prod_{s \in S} \hat{H}_s^{2n_s} \nu_s^2\right) F^{b_s} dt \otimes dt + \sum_{i = 3}^n \left(\prod_{s \in S} \hat{H}_s^{-2n_s} \delta_{i,s}\right) F^{b_s} g^{ij}\right),$$

(3.1)

$$\varphi^\alpha = \sum_{s \in S} \eta_s \nu_s^2 \chi_s \lambda_s^\alpha \ln \hat{H}_s + \frac{1}{2} b_s^\alpha \ln F,$$  

(3.2)

where $F = 1 - 2\mu/R^2$,

$$\hat{H}_s = \hat{H}_s F^{(1 - b_s)/2},$$

(3.3)

$$\hat{H}_s = 1 + \hat{P}_s \left(\frac{1 - F^{b_s}}{2\mu b_s}\right).$$  

(3.4)

$s \in S$. Here

$$\sum_{s' \in S} (U_s^s, U_s^s) \eta_{s'} \nu_{s'}^2 = 1$$

(3.5)

for all $s \in S$. The parameters $b_s$ and $\hat{P}_s$ coincide within the blocks:

$$b_s = b_{s'}, \quad \hat{P}_s = \hat{P}_{s'},$$

(3.6)

$s, s' \in S_i, i = 1, \ldots, k$. The parameters $b_s, b^i, b^\alpha$ obey the relations

$$U^1(b) = -b^1 + \sum_{j=1}^n d_j b^j = 1,$$  

(3.7)

$$U^s(b) = \sum_{i \in I_s} d_i b^i - \chi_s \lambda_{s,s} b^\alpha = 1,$$  

(3.8)

$s \in S$, and

$$\sum_{s \in S} \eta_s \nu_s^2 (b_s^2 - 1) + h_{s,s} b_s^\alpha b^\alpha + \sum_{i = 2}^n d_i (b^i)^2$$

$$+ \frac{1}{d_1 - 1} \left(\sum_{i = 2}^n d_i b^i\right)^2 = \frac{d_1}{d_1 - 1}.$$  

(3.9)

The fields of forms in (2.25) are given by the relations

$$F^s = d\Phi^s \wedge \tau(I_s),$$

(3.10)

$$F^s = \exp[-2\lambda_{s,s}(\varphi)] \ast (d\Phi^s \wedge \tau(I_s)),$$  

(3.11)
where $* = *[g]$ is the Hodge operator and

$$\Phi^s = \nu_s H_s^s,$$  \hspace{1cm} (3.12)

$$\dot{H}_s = \left[1 - P_s \dot{H}_s^{-1} \left(1 - \frac{F_{bs}}{2\mu bs}\right)\right]^{-1},$$  \hspace{1cm} (3.13)

$s \in S$. Here

$$(P_s)^2 = -\varepsilon_s \eta_s \dot{P}_s(\dot{P}_s + 2b_s \mu),$$  \hspace{1cm} (3.14)

$s \in S$.

### 3.2. Post-Newtonian parameters

Let $d_1 = 2$. Consider the 4-dimensional section of the metric (3.1)

$$g^{(4)} = U \left\{ F^{b_1} \left( dR \otimes dR + F^{b_2} R^2 d\Omega_2^2 - U_2 F^{b_2} dt \otimes dt \right) \right\},$$  \hspace{1cm} (3.15)

where $F = 1 - 2\mu/R$ and

$$U = \prod_{s \in S} \dot{H}_s^{2\nu_s d(I_s)\nu_s^2/(D - 2)},$$  \hspace{1cm} (3.16)

$$U_2 = \prod_{s \in S} \dot{H}_s^{-2\eta_s \nu_s^2},$$  \hspace{1cm} (3.17)

$$R > 2\mu.$$

Introducing the radial variable $\rho$ by the relation (2.34), we rewrite the metric (3.15) in a 3-dimensional conformally-flat form:

$$g^{(3)} = U \left\{ -U_2 F^{b_2} dt \otimes dt + F^{b_1} \left( 1 + \frac{\mu}{2\rho^2} \right)^4 \delta_{ij} dx^i \otimes dx^j \right\},$$  \hspace{1cm} (3.18)

$$F = \left( 1 - \frac{\mu}{2\rho} \right)^2 \left( 1 + \frac{\mu}{2\rho} \right)^{-2},$$  \hspace{1cm} (3.19)

where $\rho^2 = |x|^2 = \delta_{ij} x^i x^j$ ($i, j = 1, 2, 3$).

From (3.18)–(3.19) and (2.33)–(2.36) we get for the gravitational mass:

$$GM = \mu b_2^2$$

$$+ \sum_{s \in S} \eta_s \nu_s^2 \dot{P}_s(\dot{P}_s + (b_s - 1)\mu) \left( 1 - \frac{d(I_s)}{D - 2} \right),$$  \hspace{1cm} (3.20)

and, for $GM \neq 0$,

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} \eta_s \nu_s^2 \dot{P}_s(\dot{P}_s + 2b_s \mu) \left( 1 - \frac{d(I_s)}{D - 2} \right),$$  \hspace{1cm} (3.21)

$$\gamma - 1 = \frac{1}{GM} \left[ \mu(\dot{b}^1 + \dot{b}^2 - 1) \right.$$  

$$\left. + \sum_{s \in S} \eta_s \nu_s^2 \dot{P}_s(\dot{P}_s + (b_s - 1)\mu) \left( 1 - 2 \frac{d(I_s)}{D - 2} \right) \right].$$  \hspace{1cm} (3.22)

As follows from (3.14), (3.21) and the inequalities $d(I_s) < D - 2$ (for all $s \in S$),

$$\beta > 1 \text{ if all } \varepsilon_s = -1,$$  \hspace{1cm} (3.23)

$$\beta < 1 \text{ if all } \varepsilon_s = +1.$$  \hspace{1cm} (3.24)

There exists a large variety of configurations with $\beta = 1$ when the relation $\varepsilon_s = \text{const}$ is broken. There also exist non-trivial $p$-brane configurations with $\gamma = 1$.

**Proposition.** Let the set of $p$-branes consist of several pairs of electric and magnetic branes. Let any such pair $(s, \bar{s} \in S)$ correspond to the same colour index, i.e. $a_s = a_{\bar{s}}$, and $\dot{P}_s = \dot{P}_{\bar{s}}$, $b_s = b_{\bar{s}}$, $\eta_s \nu_s^2 = \eta_{\bar{s}} \nu_{\bar{s}}^2$. Then for $b^1 + b^2 = 1$ we get

$$\gamma = 1.$$  \hspace{1cm} (3.25)

The Proposition can be readily proved using the relation $d(I_s) + d(I_{\bar{s}}) = D - 2$, following from (2.23).

For small enough $\dot{p}_s = \dot{P}_s/GM$, $b_s - 1$, $b^2 - 1$, $b^1$ ($i > 2$) of the same order we get $GM \sim \mu$ and hence

$$\beta - 1 \sim \sum_{s \in S} \eta_s \nu_s^2 \dot{p}_s \left( 1 - \frac{d(I_s)}{D - 2} \right),$$  \hspace{1cm} (3.26)

$$\gamma - 1 \sim b^1 - b^2 + 1$$

$$- \sum_{s \in S} \eta_s \nu_s^2 [\dot{p}_s + (b_s - 1)] \left( 1 + \frac{d(I_s)}{D - 2} \right),$$  \hspace{1cm} (3.27)

i.e., $\beta - 1$ and $\gamma - 1$ are of the same order. Thus for small enough $\dot{p}_s$, $b_s - 1$, $b^2 - 1$, $b^1$ ($i > 2$) it is possible to fit the Solar-system restrictions (2.43) and (2.44).

There also exists another opportunity to satisfy these restrictions.

**One-brane case.** Consider the special case of a single $p$-brane. In this case we have

$$\eta_s \nu_s^{-2} = d(I_s) \left( 1 - \frac{d(I_s)}{D - 2} \right) + \lambda^2.$$  \hspace{1cm} (3.28)

The relations (3.21), (3.22) and (3.28) imply that for large enough values of the dilatonic coupling constant squared $\lambda^2$ and $b^1 + b^2 = 1$ it is possible to “fine-tune” the parameters $\beta, \gamma$ near the point $(1, 1)$ even if the parameters $\dot{P}_s$ are large.

### 4. Conclusions

We have considered a family of black-hole solutions from [12] with intersecting $p$-branes with next to arbitrary intersection rules. The metric of the solutions contains $n - 1$ Ricci-flat “internal” space metrics. The solutions are defined up to a set of functions $H_s$ obeying a set of equations with certain boundary conditions. In [12] a conjecture on the polynomial structure of $H_s$ for intersections related to semisimple Lie algebras was suggested and proved for the $A_n$ series.
Here we have presented the post-Newtonian parameters $\beta$ and $\gamma$ corresponding to the 4-dimensional section of the metric. The parameter $\beta$ is written in terms of ratios of the physical parameters: the charges $Q_s$ and the mass $M$. Thus $\beta$ does not depend on $p$-brane intersections. Unlike that, the parameter $\gamma$ is intersection-dependent, and its calculation requires an exact solution for the radial functions $H_s$. We have considered the post-Newtonian parameters for spherically symmetric solutions in the block-orthogonal case and singled out the “corridors” for the $b$-parameters which follow from the Solar-system observational restrictions.

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