The Origin and Development of Fuzzy Geometric Programming

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\textbf{ABSTRACT}

Fuzzy Geometric Programming has been in discussion for 32 years since 1987. According to Cao, the author, he in the paper introduces its development process, aiming to promote this new branch, attracting scholars home and abroad to join in ranks of the research, and helping to solve the three conjectures of Fuzzy Geometric Programming proposed by Cao [Three guess of fuzzy geometric programming. Vol. 147. Springer; 2012. p. 591–594. (Advances in intelligent and soft computing)]. At present, many books and teaching materials of it have been translated into Persian with some published.

\textbf{KEYWORDS}

Geometric programming; development process; past; present and future; conjecture

1. Introduction

From introduction of its generation, application and achievements, it is confirmed that it has developed into a new branch of discipline. Firstly, we in this paper trace back the origin of fuzzy geometric programming, besides we introduce its present situation, and finally, according to its present situation, we look forward to its development prospect, and once again put forward three conjectures of fuzzy geometric programming, which provides a difficult and promising path for the development of this branch.

2. The Past of Geometric Programming

In 1987, B. Y. Cao proposed the fuzzy geometric programming (GP) theory in International Fuzzy Systems Association (IFSA) Congress for the first time [1,2]; its general form is

\begin{align*}
(GP) \quad \min \quad & \tilde{g}_0(x) \\
\text{s.t.} \quad & \tilde{g}_i(x) \lesssim 1 (1 \leq i \leq p'), \\
& \tilde{g}_i(x) \gtrsim 1 (p' + 1 \leq i \leq p), \\
& x > 0
\end{align*}
it is a reversed posynomial GP, where all of \( g_i(x)(0 \leq i \leq p) \) are posynomial functions of variable \( x \) with \( g_i(x) = \sum_{k=1}^{J_i} v_{ik}(x) \); here

\[
v_{ik}(x) = \begin{cases} 
 c_{ik} \prod_{l=1}^{m} x_l^{\gamma_{ikl}}, & (1 \leq k \leq J_i; 0 \leq i \leq p') \\
 c_{ik} \prod_{l=1}^{m} x_l^{-\gamma_{ikl}}, & (1 \leq k \leq J_i; p' + 1 \leq i \leq p) 
\end{cases}
\]  

(1)

is a monomial function of variable \( x \), variable \( x = (x_1, x_2, \ldots, x_n)^T \), and coefficients \( c_{ik} > 0 \), exponents \( \gamma_{ikl} \) \((1 \leq k \leq J_i, 0 \leq i \leq p, 1 \leq l \leq n)\) are arbitrary real numbers.

We have done a lot of work in the research of fuzzy GP [3–12], putting forward its original algorithm, and using fuzzy arithmetic geometric inequality and setting value theory to obtain its dual form and dual algorithm. Then, we proposed the geometric programming with a Fuzzy coefficient and Fuzzy variable and a multi-objective fuzzy geometric programming model [10,13–21]. At the same time, it is applied in power system, management optimization and decision-making [22–27]. Based on the published fuzzy GP, another monograph of Optimal Models and Methods with Fuzzy Quantities was published in Springer in 2010, in which he introduced the work, such as GP models and algorithms with fuzzy coefficients and fuzzy variables [28,29].

The author has been awarded the National Natural Science Foundation of China for three consecutive times: (1) Fuzzy generalized GP decision model and method, 2008–2010; (2) Universal fuzzy GP and optimization techniques in management, 2003–2005; (3) Theory and method of fuzzy GP in power system management, 1997–1999; and has achieved rich results. In 2002, Kluwer Academic Publishers published a series of fuzzy GP in the Applied Mathematics series. In 2005, Fuzzy GP was awarded the third prize of Guangdong Science and Technology Award. The American Mathematical Review MathSciNet? and the West German Mathematical Digest Zentralblatt MATH commented on its original research. In 1993, Liu Yingming, an academician of the Chinese Academy of Sciences, called this work an international frontier. In 2004, Academician Wang Zikun, an academician of the Chinese Academy of Sciences, wrote an article in the Science Bulletin, saying that the monograph was at a high level in academic value with his identification of fuzzy GP reaching the international advanced level. In 2007, Guo Bolin, another academician of the Chinese Academy of Sciences, who served as the director of the appraisal committee, identified this work as the advanced level of the international frontier, saying that it partially reached the leading level. In 2008, specialists in the international academic circle confirmed that the fuzzy GP proposed by the applicant is the seventh kind of optimization model [30].

While studying the theory and application of fuzzy GP, Cao and others are actively expanding in other direction.

In 2001, the author proposed the extension GP. In 2005, Luo Dang proposed the gray positive GP. In 2009, the author proposed the rough posynomial GP. In 2014, Cao’s doctoral student Zeinab Kheiri wrote the doctoral thesis with intuitionistic fuzzy posynomial GP [31–36].

In 2005, the author and his doctoral student Yang Jihui, in the IEEE Fuzzy System Annual Meeting held in the United States, first proposed the fuzzy relation GP, and its form was
described as follows:

$$\text{(PGPF)} \quad \min z(x) = \sum_{k=1}^{p} c_k \prod_{j=1}^{n} x_{\gamma kj}$$

s.t. $A \circ x = b$

where $A = (a_{ij})_{m \times n}$, $x = (x_j)_{n \times 1}$, $b = (b_i)_{m \times 1}$, $a_{ij}, x_j, b_i \in [0, 1]$, $c_k, \gamma_{kj} \in \mathbb{R}$, $c_k > 0$, $i \in I = \{1, 2, \ldots, m\}$, $j \in J = \{1, 2, \ldots, n\}$, $k \in K = \{1, 2, \ldots, p\}$, and for given $j \in J$, $\gamma_{kj} (k \in K)$ are either all non-positive real numbers or all non-negative real numbers. Without loss of generality, we assume that problem (2) satisfies the following inequalities:

$$1 \geq b_1 \geq b_2 \geq \cdots \geq b_m \geq 0.$$

Otherwise, rearrange the components of $b$ in decreasing order and adjust the rows of $A$ according to $b$ [37–45].

So far, research in this direction has begun to heat up.

3. The Present of Geometric Programming

From 30th July to 1st August 2016, Cao Bingyuan’s students, family and relatives gathered in the Applied Mathematics Conference Room on the 7th floor of the Computer Experiment Building of Guangzhou University to celebrate the 30 years of fuzzy geometry planning and 40 years of his teaching job, and published a conference proceeding by Chinese Science and Education Press in 2019. The book, containing the main articles of Cao and his students, has been published in the fuzzy GP for 30 years, reporting their projects, achievements and rewards. At present, the research on fuzzy relational linear programming and fuzzy relational GP is becoming a hot topic. Nearly 100 papers have been published in the magazine; recently, they have headed to some new research directions [46–49], and Cao’s book of fuzzy relationship programming as well as the book of Indian scholars’ fuzzy relational GP and its application will be published by Springer-Nature.

Now Cao’s colleagues and he are already preparing to collaborate with the team on optimized secured sharing of documents, which was proposed in Jana Wyżykowskiego University, Poland, on fuzzy relationships and their programming. At the same time, the three conjectures of fuzzy GP proposed in 2012 still remain to be solved. They are expanding our propaganda and taking strong measures to attract more scholars to discuss them. And they will use the 9th International Information and Engineering Conference of Kish Island to prepare for the establishment of the International Fuzzy Information and Engineering Association. We will use the EI collection and ISC conference proceedings and the magazine ‘Operational Management and Fuzzy Mathematics’ to exchange ideas. The excellently chosen papers in this direction will be published in the magazine Fuzzy Information and Engineering, which is included in the Web of Sciences.

At present, copies of works in Fuzzy geometric programming researches have been published by famous Springer publishing house, including essays on Fuzzy geometric programming, Fuzzy relational geometric programming and Fuzzy programming. Besides, China Science and Education Press and Fuzzy Information and Engineering Journal will strongly support the development in this direction. Two monographs [29,30] will be translated into Persian language for publication (Figure 1).
The world’s first high school fuzzy mathematics textbook ‘Fuzzy sets theory preliminary’ was officially published in February 2018 by the world-famous publishing houses Springer-Nature and China Science and Education Publishing House, which draws strong attention of scholars and readers from all over the world. In April this year, it was selected as the fourth of the eight most popular teaching materials of Springer-Nature, which exerts a great international influence (Figure 2).

4. The Future of Geometric Programming

For 32 years, we have witnessed the development of fuzzy GP. At present, its research has entered a critical phrase. More research scholars are involved in it and infiltration fields are constantly expanding, and its research needs more talents to participate in and support for. The three conjectures we have to solve are (i) the local optimal (satisfactory)
solution to fuzzy GP is still its global optimal (satisfactory) solution; (ii) after replacing the operator \((+,-)\) in (1) with other logical operators, the fuzzy relational still holds with its dual programming (2) established; (iii) confirmation of the existence of fuzzy GP taxonomy and identification. As for the above three, they will continue to organize the team to deal with them, strive to make a breakthrough in theory, further find the background in the application, and make arduous efforts to establish a branch of the fuzzy geometric programming.

The fuzzy GP will attract all of us to further research because many aspects remain untouched. In the basic field, we shall consider the following topics.

(i) Fuzzy reverse GP, including a GP problem with mixed sign-terms, is much more complex than the fuzzy convex (resp. concave) GP and fuzzy PGP; we want to continue to explore their properties.

(ii) Fuzzy fractionation, extension, gray, and rough GP still need to be studied.

(iii) GPs with intuitionistic fuzzy coefficients and fuzzy variables have yet to be further refined and expanded.

(iv) Further solve real-world problems paradox with fuzzy GP.

(v) Solving fuzzy relation GP.

(vi) Explore GP with discrete variables and coefficients.

(vii) Fuzzy GP in application in BitTorrent-like Peer-to-Peer file sharing system.

(viii) GP problem subject to max-product fuzzy relation inequalities. The local optimal solution of the GP problem subject to fuzzy relation inequalities is also its global optimal solution.

(ix) Study of fuzzy GP classification.

(x) The export of fuzzy GP’s genetic algorithm.
On 26 April 2019, the signing ceremony for the Persian copyright export of the monograph ‘Applied fuzzy mathematics and systems’ was jointly held by Shokoh-e-Sahel Publishing House and Scientific Publishing at the China Guest of Honor booth at the Iran International Book Fair. The translation project of this book has been included in the national ‘the belt and road initiative’ Silk Road Fund Plan applied by Science Press (Figure 3).

Colleagues are welcome to participate and work hard to establish a new branch of fuzzy GP.

5. Conclusion

The research on the theory and algorithm to Fuzzy geometric programming has achieved phased results, and the application background was first found in power systems. The monographs and applications of Fuzzy Optimization have been and are being disseminated in the world through Persian, English, Chinese languages, etc. As long as scholars in many other countries persist in their research and experiments, the three conjectures of Fuzzy geometric programming [50] could be overcome.

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