Establishment of a two-stage constitutive model based on dislocation density theory for as-forged SA508 Gr.3Cl.1

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Abstract
The high-temperature deformation behavior of the SA508 Gr.3Cl.1 steel was investigated by the uniaxial isothermal compression tests at the deformation temperature from 1223 K to 1473 K and strain rate from \(0.001 \text{s}^{-1}\) to \(1 \text{s}^{-1}\), which were carried out using Gleeble-1500 thermo-mechanical simulator. Based on the experimental data, the material parameters and active energy of hot deformation were determined according to the regression analysis method. The critical strain of dynamic recrystallization (DRX) was confirmed by simplified P-J method. Furthermore, the constitutive model for work hardening-dynamic recovery (WH-DRV) stage and DRX stage was established based on dislocation density theory and the kinetics of DRX. Finally, the flow stress calculated using the established model and experiment data was compared and the results showed that it is enough accurate to predict the flow stress during hot deformation for SA508 Gr.3Cl.1. Thus, the model will be beneficial for the accuracy of simulation by Finite Element Method (FEM) method.

1. Introduction

Because of its properties of high strength, high fracture toughness and high impact toughness, excellent welding performance, lower sensitivity of irradiation embrittlement, SA508 Gr.3Cl.1 steel is widely used for nuclear island equipment of PWR Nuclear Power Plant, such as reactor pressure vessels (PRV), steam generators (SG) and pressurizers (PRZ), etc. According to the data of the World Nuclear Association, Nuclear power currently provides about 13% of electrical power worldwide in 2013, and has emerged as a reliable baseload source of electricity. With the rapid development of nuclear power, the demands for SA508 Gr.3Cl.1 are increasing sharply [1–3]. However, most forgings made of SA508 Gr.3Cl.1 steel belong to heavy forgings characterized by large volume, heavy weight and complex shape. For these parts, FEM simulation method is widely used as an economic and effective method to predict the deformation behavior in their manufacturing process because the experimental cost of trial and error is very high. As we all known, the accuracy of the constitutive model has a great influence on the accuracy of prediction by numerical simulation method. So establishing a high-precision model is an important work for the effective use of FEM method.

For a long time, many constitutive models of materials were proposed and used to different materials. Johnson and Cook [4] proposed an empirical constitutive model considering deformation temperature and strain rate. Zerilli and Armstrong [5] put forward a constitutive model based on dislocation dynamics theory considering strain hardening, strain rate hardening and temperature softening. Follansbee and Kocks [6] established MTS constitutive model through Mechanical Threshold Stress based on the theory of thermally activated dislocation movement. Zohreh Akbari [7] established a relatively simple constitutive model for medium carbon microalloyed steel during hot deformation by revising Johnson-Cook (JC) model. Gong et al. [8–12] conducted comprehensive and innovative research on both phenomenological and physically-based constitutive models for 34CrNiMo6, 20CrMnTi and a nitrogen alloyed ultralow carbon stainless steel, low carbon bainitic steel, respectively. However, there is few related research for as-forged SA508 Gr.3Cl.1 steel. It is
necessary to comprehend the high temperature deformation behavior and establishing an effective constitutive model in order to improve the simulation accuracy by FEM.

According to the previous research, the selection of the model should be related to the high temperature deformation characteristics of material. According to the literature [13], the phenomenon of DRX is obvious for SA508 Gr.3Cl.1 steel when it deformed at high temperature with low strain rate. The stress-strain curve presented work hardness-dynamic recovery stage and dynamic recrystallization stage. So the Arrhenius equation cannot describe the complex changes in the stress-strain curve and the effect of microstructure on macroscopic mechanical behavior [14, 15]. And it is well known that plastic deformation of the metal is associated with lattice defects of metals, mainly with the generation, movement and interaction of dislocations. According to the above two points, the aim of this paper is to establish the constitutive model for SA508 Gr.3Cl.1 based on dislocation density theory and the kinetics of DRX for the two stage, respectively.

2. Hot deformation experiment

2.1. Chemical composition and initial microstructure
The as-forged SA508 Gr.3Cl.1 steel in this study is provided by Taiyuan Iron & Steel (Group) Co., Ltd. The chemical composition is given in table 1. The initial microstructure is showed in figure 1. and the average grain size is about 160 μm.

| C   | Mn  | S   | P   | Si  | Cr  | Ni  | Cu  | Mo  | V   | Al  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.19| 1.36| 0.001| 0.005| 0.19| 0.085| 0.74| 0.022| 0.5 | 0.002| 0.01|

Table 1. Chemical composition of SA508 Gr.3Cl.1 (wt%).

![Figure 1. Initial microstructure.](image)

2.2. Hot deformation experiments
The hot deformation experiments were conducted on a Gleeble-1500 thermo-simulator through the uniaxial isothermal compression tests at deformation temperatures of 1223, 1273, 1323, 1373, 1423, 1473 K, and strain rates of 0.001, 0.01, 0.1 and 1s⁻¹ with high reduction of 50%, respectively. The steel was machined into cylindrical specimens 8mm in diameter and 12mm in height. The processing condition of tested specimens is shown in figure 2. The tested specimens were heated to the temperature of 1473K at a rate of 10 K s⁻¹ by a thermo-coupled feedback-controlled AC current and isothermally held for 3 min before compress tests, and then the compression deformation is carried out according to the given experiment scheme. After the compression deformation, the specimens were immediately quenched in water. In order to minimize the friction between the specimen and the anvil during deformation, graphite spacers were used between the specimen and the anvil. The data for the true stress–strain curves were recorded automatically during deformation.
3. High temperature deformation behavior of SA508 Gr.3Cl.1

High temperature plastic deformation of metallic materials is a combining process of work hardening which was caused by multiplication and entanglement of the dislocation, and the work softening which was caused by DRV and DRX. The DRV and DRX are two typical kinds of softening mechanisms during hot deformation [15, 16]. Figure 3 showed that the flow stress for SA508 Gr.3Cl.1 at deformation temperature of 1223~1473 K, with interval of 50K and strain rate of 0.001, 0.01, 0.1, 1s⁻¹. It can be seen that the flow stress was closely dependent on the deformation temperature and strain rate. At the first stage of the deformation (from the beginning of

![Figure 2. Procedure of compression tests.](image)

![Figure 3. Stress–strain curves of the SA508 Gr.3Cl.1 steel at different condition.](image)
deformation to the critical strain $\varepsilon_c$, seen in figure 3(b)), the flow stress presented obvious WH characteristics. It sharply increased with the increase of the strain at all conditions investigated in this work. During this stage, the lattice distortion of metal caused by strain increase produces a great deal of multiplication and entanglement of dislocation. The WH contributes much more to the increase of the stress than the softening caused by atomic diffusion and dislocation annihilation. During the following deformation, the flow stress curve exhibited different trends at different conditions. Under the conditions of high temperature and low strain rate (taking 1323K, 0.01s$^{-1}$ as an example), a peak in the stress curve was observed. The DRX occurred and the increase rate of the flow stress decreased when the strain reaches the critical value $\varepsilon_c$, namely, the work hardness rate decreased with the increase of true strain. The typical grain morphology at this stage was shown in figure 4(a). A few recrystallized grains appeared at the grain boundary. During this stage, the dislocation density continued to increase with the increase of the strain, which can lead to the increase of recrystallization driven force. This promotes the continuous development of DRX. The high dislocation movement rate and the long deformation time increase the probability of dislocation annihilation, which can strength the softening effect. With the WH rate decreased to zero, the flow stress reached the peak value $\sigma_p$. Then, the softening rate caused by DRV and DRX exceeded the hardening rate and the flow stress declined due to the sustaining of DRX with the increase of the strain. Until the WH and the softening reached a balance, a steady-state plateau appeared in flow stress curve. The typical grain morphology at this stage was shown in Figure 4(b). More recrystallized grains appeared and the original grain boundary disappeared. However, the flow stress became a steady state after it reached the peak value under the conditions of low temperature and high strain rate (1223 K, 1 s$^{-1}$). With the increase of strain, the dislocation density increased. Dislocation stack, dislocation jog and dislocation forest formed due to the interactions among the dislocations, which hinder the movement of dislocations. At the same time, the DRX is inhibited because of the lower deformation temperature and higher deformation speed. Therefore, DRV is the main softening mechanism when the specimens deformed at low temperature with high strain rate and DRX is the dominant softening mechanism when it deformed at high temperature with low strain rate. Under the different deformation conditions, softening mechanism classification was shown in table 2.

Based on the above analysis, the two softening mechanisms should be deal with separately for the establishing the high temperature constitutive model for SA508 Gr.3Cl.1.

### 4. Establishment of constitutive model during hot deformation

#### 4.1. Determination of the hot deformation parameters

Generally, the hot deformation of metallic materials is a thermal-activation process. The Arrhenius equation was widely used to demonstrate the thermal deformation behavior of metals [17–19]. Thus, the flow stress is mainly
depends on the deformation temperature and strain rate.

\[
\dot{\varepsilon} = A_0 \sigma^\alpha \exp \left( -\frac{Q}{RT} \right) \quad (\alpha \sigma < 0.8) \\
\dot{\varepsilon} = A_2 \exp (\beta \sigma) \exp \left( -\frac{Q}{RT} \right) \quad (\alpha \sigma > 1.2) \\
\dot{\varepsilon} = A [\sinh (\alpha \sigma)]^b \exp \left( -\frac{Q}{RT} \right) \quad \text{(for all } \sigma) 
\]

where \( \dot{\varepsilon} \) is the strain rate (s\(^{-1}\)), \( R \) is the gas constant, 8.314 J mol\(^{-1}\) K\(^{-1}\), \( T \) is the deformation temperature (K), \( Q \) is the hot deformation activation energy (J mol\(^{-1}\)), \( \sigma \) is the flow stress (MPa), \( A_0, A_2, A, \alpha, \beta, n_0, n \) are the constants related to material and \( \alpha = \beta / n_0 \). The equation (1) is suitable for low stress. Equation (2) is preferred for high stress. And equation (3) can be widely used for all stress. Meanwhile, the effect of the strain rate and the deformation temperature on the hot deformation behavior of the materials can also be expressed as Zener-Hollomon parameter (Z) in an exponent-type equation.

\[
Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) 
\]

In order to obtain the constants in equations (1)–(3), taking logarithm on both sides of equations (1)–(3), then

\[
\ln \dot{\varepsilon} = n_1 \ln \sigma + \ln A_1 - \frac{Q}{RT} \quad (\alpha \sigma < 0.8) \\
\ln \dot{\varepsilon} = \beta \sigma + \ln A_2 - \frac{Q}{RT} \quad (\alpha \sigma > 1.2) \\
\ln \dot{\varepsilon} = n \ln [\sinh (\alpha \sigma)] + \ln A - \frac{Q}{RT} \quad \text{(for all } \sigma) 
\]

According to equations (5)–(7), \( \ln \dot{\varepsilon} \) is linear with \( \ln \sigma, \sigma \) and \( \ln [\sinh (\alpha \sigma)] \) when \( T \) is constant and \( \ln [\sinh (\alpha \sigma)] \) is linear with \( \frac{1}{T} \) when the \( \dot{\varepsilon} \) is constant. Therefore, \( \beta, n_1, n \) and \( Q \) can be derived by regression analysis of the experiment data in figure 3. The peak stress \( \sigma_p \) was used to substitute the \( \sigma \) in the equations (5)–(7) and \( \sigma_p \) can be read directly from the stress-strain curve in figure 3. The plots of \( \ln \dot{\varepsilon} - \ln \sigma_p, \ln \dot{\varepsilon} - \ln \sigma_p, \ln \dot{\varepsilon} - \ln [\sinh (\alpha \sigma)] \) and \( \ln [\sinh (\alpha \sigma)] - 1 / T \) at different temperature were drawn in figure 5. \( \beta \) and \( n_1 \) are the average slope of the linear fit line in figures 5(a) and (b). \( n_1 \) and \( \beta \) are 0.127 and 6.775, respectively. Therefore, the value of \( \alpha \) is 0.019. Analogously, the value of \( n \) is 5.050. The value of Q represents the difficulty degree of a metallic material to deform. The material is easier to deform when the Q value decreases. According to equation (7), the value of \( \frac{Q}{nRT} \) is the average slope of the fitting lines in figure 5(d), which is 5.4559. Then the value of Q is determined as 419315.99 J mol\(^{-1}\). Substituting the Q value into equation (4), \( Z \) for SA508 Gr.3Cl.1 is expressed as

\[
Z = \dot{\varepsilon} \exp \left( \frac{419315.99}{RT} \right) 
\]

4.2. The characteristic stress and strain of the hot deformation

The characteristic stress and strain of the hot deformation play important roles on describing the hot deformation behavior and establishing the constitutive model [20]. It include yield stress (\( \sigma_y \)), critical stress of DRX (\( \sigma_c \)), peak stress (\( \sigma_p \)), saturation stress (\( \sigma_{sat} \)), critical strain of DRX (\( \varepsilon_c \)), peak strain (\( \varepsilon_p \)) and steady stress (\( \sigma_s \)). \( \sigma_0, \sigma_p \) and \( \varepsilon_p \) at specific conditions can be obtained directly from the stress-strain curves. After the regression analysis of experimental data, the relationships between \( \ln \sigma_0, \ln \sigma_p, \ln \sigma_{sat}, \ln \varepsilon_p \) and \( Z \) were showed in figure 6. According to the results, \( \ln \sigma_0, \ln \sigma_p, \ln \sigma_{sat}, \ln \varepsilon_p \) were linear with \( \ln Z \). Then, \( \sigma_0, \sigma_p, \varepsilon_p \) can be expressed as the function of \( \ln Z \) in the form of power-law.

\[
\sigma_0 = 2.06857Z^{0.08527} \quad \sigma_p = 52.63150 \times \sinh^{-1}(0.00260Z^{0.18203}) \quad \varepsilon_p = 2.06857Z^{0.08527} 
\]

The critical strain is a key parameter for establishing the constitutive model. Recent years, many researchers proposed different mathematic model for confirming the onset of DRX. According to the difference of DRX and WH-DRV stage, Ryan [21] and Mecking [22] took the stress starting to deviate from linear relation in work hardness rate curve (\( \theta - \sigma \) curve, figure 7(a)) as the critical stress. Then the corresponding strain is the critical strain. Poliak and Jonas [23] took the true stress at the point of \( \frac{\partial \theta}{\partial \sigma} \) in the \( \frac{\partial \theta}{\partial \sigma} - \sigma \) curve, namely, the inflection of the \( \theta \) versus \( \sigma \) plot, as the critical stress for the onset of DRX (figure 7(b)). Then the corresponding strain is determined by the stress-strain curve, that is, P-J method. Najafizadeh and Jonas [24] simplified the P-J
method and determined the critical stress and strain of DRX by fitting the data of work hardening with third order polynomial. In this work, the P-J method was used.

Taking the deformation under the strain rate of $0.01 \text{ s}^{-1}$ and the deformation temperature of $1373K$ as an example to calculate $\sigma_c$ and $\varepsilon_c$. Taking the derivative of $\sigma$ with respect to $\varepsilon$, then $\theta$ is known (figure 7(a)). In order to obtain the inflection point of $\theta - \sigma$ curves, fitting the curves with a third order polynomial up to the peak stress was conducted (figure 8(a)). The result indicated that the fitting curve was consistent with the experimental data. The fitting result was given as

$$q_{ss} = -0.32598 \sigma^3 + 44.05203 \sigma^2 - 1992.9049 \sigma + 30306.8472$$

Taking first and second order differentiation of equation (12) with respect to $\sigma$, yield

$$-\frac{\partial \theta}{\partial \sigma} = 3*0.32598\sigma^2 - 2*44.05203\sigma + 1992.9049$$

$$-\frac{\partial^2 \theta}{\partial \sigma^2} = 6*0.32598\sigma - 2*44.05203$$

Then, the $-\frac{\partial \theta}{\partial \sigma}$ versus $\sigma$ plot was obtained (figure 7(b)). When $-\frac{\partial^2 \theta}{\partial \sigma^2} = 0$, the $-\frac{\partial \theta}{\partial \sigma}$ arrived the minimum value (red spot), the corresponding stress is critical stress. When $-\frac{\partial \theta}{\partial \sigma} = 0$, the $-\frac{\partial^2 \theta}{\partial \sigma^2}$ received the minimum value (red spot), the corresponding stress is critical stress, $\varepsilon_c = 0.097$. Moreover, the saturation stress $\sigma_{sat}$ and the steady-state stress $\sigma_s$ also can be gained by figure 8. The saturation stress is the intersection of the tangent extended curve with the line which corresponds to $\theta = 0$. In the same way, the characteristic stress and strain under other conditions were obtained. The relationships between $\sigma_{sat}$, $\sigma_{sat}$, $\varepsilon_c$ and the Z parameter were showed in figure 9. The $\varepsilon_c$ was linear with $\varepsilon_p$ (figure 10). They can be expressed

Figure 5. The relationships between strain rate, peak stress and T (a) $\ln \varepsilon - \ln \sigma_p$; (b) $\ln \varepsilon - \ln \sigma_p$; (c) $\ln \varepsilon - \ln [\sinh (\alpha \sigma_p)]$; (d) $\ln [\sinh (\alpha \sigma_p)] - 1/T$. 
in the form of power-law as followings.

\[ \sigma_{\text{sat}} = 52.63158 \times \sinh^{-1}(0.00159Z^{0.2106}) \]  
(15)

\[ \sigma_s = 52.63158 \times \sinh^{-1}(0.00145Z^{0.2015}) \]  
(16)

\[ \varepsilon_{\zeta} = 0.00037Z^{0.1734} \]  
(17)

\[ \varepsilon_{\varphi} = 0.00094Z^{0.1590} \]  
(18)

Figure 6. The relationships between ln \( \sigma_s \) (a) ln \{sinh (\alpha \sigma_s)\} (b) ln \( \varepsilon_{\sigma} \) (c) ln \( \varepsilon_{\varphi} \) and lnZ.

Figure 7. The third order polynomial of \( \theta \) (a) and \( \frac{\partial^2 \theta}{\partial \sigma^2} \) (b) versus \( \sigma \) plot at strain of 0.01 and deformation temperature of 1373 K.
4.3. Determination of the high temperature constitutive model

Different models correspond to different softening mechanisms. For the high temperature deformation process dominated with DRV softening mechanism, the constitutive model can be established based on dislocation density theory. For the deformation dominated with DRX softening mechanism, the process can be divided into WH-DRV stage and DRX stage with the critical strain $\varepsilon_c$. So it is necessary to build different constitutive models for the two stages.

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Figure 8. The plot of $\theta$ versus $\sigma$ at deformation temperature of 1373 K and strain rate of 0.01 s$^{-1}$.

Figure 9. The relationships between $\ln [\sinh (\alpha \sigma_a)]$ (a) $\ln [\sinh (\alpha \sigma_{at})]$ (b) $\ln \varepsilon_c$ (c) and $\ln Z$. 

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4.3.1. The constitutive model for the WH-DRV stage

During the hot deformation of the metals, the strengthening of WH and softening of DRV are always accompanying. The dislocation density is simultaneously affected by the multiplication of dislocation during WH and the annihilation of dislocation during DRV. And the result depended on the competition of the two processes. Mecking and Kocks proposed the work hardening dependent on the dislocation density is as follows [22, 25].

\[
\frac{d\rho}{d\varepsilon} = k_1 \sqrt{\rho} - k_2 \rho \tag{19}
\]

\[
\sigma = \alpha \mu b \sqrt{\rho} \tag{20}
\]

where \( \rho \) is the dislocation density, \( \varepsilon \) is the strain, \( \sigma \) is flow stress, \( \alpha \) is a material constant, \( \mu \) is the shear modulus, \( b \) is the Burgers vector and coefficients \( k_1 \) and \( k_2 \) represent the dislocation storage and annihilation rate. Estrin and Mecking [25, 26] modified the KM model and proposed the equation (21) by taking the assumption that the storage rate is constant.

\[
\frac{d\rho}{d\varepsilon} = D - \gamma \rho \tag{21}
\]

Where \( D \) represents the dislocation multiplication caused by work hardening during the deformation process. \( D \) is a constant with respect to strain.

Differentiation of equation (20) with respect to \( \sigma \), the following formula can be obtained.

\[
\frac{d\sigma}{d\rho} = \frac{1}{2\sqrt{\rho}} \alpha \mu b \tag{22}
\]

Combining equations (21) and (22), the work hardening rate \( \theta \) can be expressed as

\[
\theta \cdot \sigma = \frac{d\sigma}{d\varepsilon} \cdot \sigma = \frac{\alpha \mu b D}{2\sqrt{\rho}} - \frac{\alpha \mu b \gamma \sqrt{\rho}}{2} \tag{23}
\]

Combining equations (20) and (23),

\[
\theta \cdot \sigma = \frac{d\sigma}{d\varepsilon} \cdot \sigma = 0.5(\alpha^2 \mu^2 b^2 \cdot D - \gamma \sigma^2) \tag{24}
\]

It is supposed that the amount of dislocation is relatively constant when the deformation is under steady state. This means that the value of \( \theta \) will approach zero at last [16]. The flow stress when \( \theta = 0 \) is saturated stress \( \sigma_{sat} \). Thus, according to equation (24), \( \sigma_{sat} \) can be expressed as

\[
\sigma_{sat} = \alpha \mu b \sqrt{\frac{D}{\gamma}} \tag{25}
\]
According to (24), $de$ can be expressed as

$$
de = \frac{\sigma \cdot d\sigma}{0.5(\alpha^2\mu^2b^2 \cdot D - \gamma\sigma^2)} = \frac{d\sigma^2}{\alpha^2\mu^2b^2 \cdot D - \gamma\sigma^2}$$

(26)

Taking integration of both sides of equation (26), the flow stress during WH-DRV stage is given by

$$\sigma_{wh} = \sqrt{\sigma_{sat}^2 - (\sigma_{sat}^2 - \sigma_0^2) \exp(-\gamma\varepsilon)}$$

(27)

where $\sigma_0$ is the yield stress. Once the deformation condition is given, the value of $\sigma_{sat}$ and $\sigma_0$ is available according to equation (15). Then $\sigma_{wh}$ can be available as long as $\gamma$ is known. According to equation (27), $\gamma$ can be derived by

$$\gamma = -\frac{1}{\varepsilon} \ln \frac{\sigma_{sat}^2 - \sigma_0^2}{\sigma_{sat}^2 - \sigma_0^2}$$

(28)

Working hardening coefficient $\gamma$ can be obtained by substituting $\sigma_p$ at different deformation conditions into the formula (27), respectively. The relationship between $\ln \gamma$ and $\ln Z$ was plotted in figure 11. Then $\gamma$ is

$$\gamma = 157.43143Z^{-0.0835}$$

(29)

So the constitutive model for WH-DRV stage can be summarized as

$$\begin{cases}
\sigma_{wh} = \sqrt{\sigma_{sat}^2 - (\sigma_{sat}^2 - \sigma_0^2) \exp(-\gamma\varepsilon)} \\
\sigma_{sat} = 52.63158 \times \sinh^{-1}(0.00159Z^{0.21064}) \\
\sigma_0 = 2.06857Z^{0.08327} \\
\gamma = 157.43143Z^{-0.0835}
\end{cases}$$

(30)

4.3.2. The constitutive model for DRX stage

DRX is now recognized as an effective and feasible method for controlling the structure and properties of metals under industrial processing operations [7]. Generally, DRX is easy to occur at high temperature and low strain rate. The volume fraction of DRX is usually described as [27]

$$X_d = 1 - \exp \left[ -K_d \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)^{n_d} \right] \quad (\varepsilon \geq \varepsilon_c)$$

(30)

where $X_d$ is the volume fraction of DRX, $K_d$ and $n_d$ are material constants, and $\varepsilon_c$ and $\varepsilon_p$ are critical strain and peak strain, respectively. Meanwhile, the softening fraction caused by the DRX can be also estimated by [27]

$$X_d = \frac{\sigma_{wh} - \sigma}{\sigma_{sat} - \sigma_{sat}} \quad (\varepsilon \geq \varepsilon_c)$$

(31)
Then combining equations (30) and (31), the flow stress during DRX period can be expressed as

$$\sigma = \sigma_{wh} - (\sigma_{sat} - \sigma_0) \left\{ 1 - \exp \left[ -K_d \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)^{n_d} \right] \right\} \quad (\varepsilon \geq \varepsilon_c)$$

In equation (32), $\sigma_{wh}$, $\sigma_{sat}$, $\sigma_0$, $\varepsilon_c$ and $\varepsilon_p$ were obtained and there are only two undetermined parameters ($K_d$ and $n_d$). In order to obtain the value of $K_d$ and $n_d$, taking natural logarithm of equation (30) twice, gives

$$\ln [ - \ln (1 - X_d)] = \ln K_d + n_d \ln \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)$$

According to equation (33), $\ln [ - \ln (1 - X_d)]$ was linear with $\ln \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)$, so the constants $K_d$ and $n_d$ can be gained using the previous regression method (figure 12). The average value of $K_d$ and $n_d$ were 0.70608 and 2.0411, respectively. Finally, the constitutive model for DRX stage can be expressed as

$$\sigma = \sigma_{wh} - (\sigma_{sat} - \sigma_0) \left\{ 1 - \exp \left[ -0.70608 \times \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)^{2.0411} \right] \right\} \quad (\varepsilon \geq \varepsilon_c)$$

4.4. Comparison between the calculated data using the established model and experiment data

In order to illustrate the reliability of the established constitutive model, comparisons between the calculated flow stress using the established models and experiment data at different conditions were done and the result showed in figure 13. It indicated that the calculated data were in good agreement with the experiment data both during the WH-DRV and DRX stages under the processing conditions researched. Furthermore, for evaluating the accuracy of the constitutive model quantitatively, the standard statistical parameters such as correlation coefficient ($R$), the average absolute relative error (AARE) and root mean square error (MRSE) were calculated according to equations (35)–(37).

$$R = \frac{\sum_{i=1}^{n}(P_i - \bar{P})(\bar{P}_e - \bar{P})}{\sqrt{\sum_{i=1}^{n}(P_i - \bar{P})^2 \sum_{i=1}^{n}(\bar{P}_e - \bar{P})^2}}$$

$$\text{AARE} = \left( \frac{1}{n} \sum_{i=1}^{n} \frac{P_i - \bar{P}_e}{\bar{P}_e} \right) \times 100\%$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (P_i - \bar{P}_e)^2}$$

where $P_e$ is the experimental flow stress, $\bar{P}$ is the calculated flow stress, $n$ is the number of data point, $\bar{P}_e$ and $\bar{P}$ are the average value of $P_e$ and $\bar{P}_e$, respectively. $R$ is a statistical parameter used to characterize the degree of linear
correlation between the predicted and experimental data. R is between 0 \sim 1. AARE is an unbiased statistical parameter which can be used to accurately measure the accuracy of the constitutive equation by analyzing the relative errors between points. RMSE is used to evaluate the overall level of error. The calculated results show that R is 0.967, AARE is 3.89% and RMSE is 2.835. Therefore, the constitutive model established in this work can be used to predict the flow stress accurately at high temperature for the SA508 Gr.3Cl.1 steel.

5. Conclusions

(1) The high temperature deformation behavior for the SA508 Gr.3Cl.1 steel was analyzed by uniaxial isothermal compression tests at the deformation temperatures from 1223 K to 1473 K and strain rates from 0.001 s \^{-1} to 1 s \^{-1}. Two kinds of softening mechanisms during hot deformation for SA508 Gr.3Cl.1 are DRV and DRX. The flow stress for SA508 Gr.3Cl.1 presented classic DRX characteristics when $Z \geq 36.733$, while it showed DRV characteristics when $Z \leq 36.633$.

(2) The critical stress and strain of the DRX for SA508 Gr.3Cl.1 were obtained by solving the inflection of the fitted $\theta - \sigma$ curve with third order polynomial. The characteristic parameters of the investigated steel during hot deformation were determined by regression analysis of the hot compression experimental data. Finally, the high temperature constitutive model based on dislocation density theory for the WH-DRV stage and the DRX stage were established respectively. It can be expressed as following.

$$\sigma_{wh} = \sqrt{\sigma_{sat}^2 - (\sigma_{sat}^2 - \sigma_0^2) \exp(-\gamma \varepsilon)} \quad (\varepsilon \leq \varepsilon_c)$$

Figure 13. Comparison between calculated flow stress by model and experiment data at strain rate of (a) 0.001 s \^{-1}, (b) 0.01 s \^{-1}, (c) 0.1 s \^{-1}, (d) 1 s \^{-1}.
\[
\sigma = \sigma_{sw} - (\sigma_{sat} - \sigma_0) \left\{ 1 - \exp \left[-K_d \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_p} \right)^{n_d} \right] \right\} \quad (\varepsilon \geq \varepsilon_c)
\]

\[
\begin{align*}
\sigma_{sat} &= 0.00159Z^{0.21064} \\
\sigma_0 &= 2.06857Z^{0.08527} \\
\gamma &= 157.43143Z^{0.08549} \\
\sigma_{as} &= 0.00145Z^{0.20151} \\
Z &= \xi \exp \left( \frac{419315.99}{RT} \right) \\
K_d &= 2.0411 \\
n_d &= 0.70608
\end{align*}
\]

(3) The flow stress during hot temperature for SA508 Gr.3Cl.1 was predicted by using the established model in this work and the comparisons of the predicted and experimental data indicated that this high temperature constitutive model is enough accurate to predict the flow stress during hot temperature for SA508 Gr.3Cl.1.

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Data availability statement

All data included in this study are available upon request by contact with the corresponding author.

The conflict of interest disclosure

The authors declare that there is no conflict of interest regarding the publication of this paper.

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