ON FOSSIL DISK MODELS OF ANOMALOUS X-RAY PULSARS

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ABSTRACT

Currently, two competing models are invoked in order to explain the observable properties of Anomalous X-ray Pulsars (AXPs). One model assumes that AXP emission is powered by a strongly magnetized neutron star – i.e., a magnetar. Other groups have postulated that the unusually long spin periods associated with AXPs could, instead, be due to accretion. As there are severe observational constraints on any binary accretion model, fossil disk models have been suggested as a plausible alternative. Here we analyze fossil disk models of AXPs in some detail, and point out some of their inherent inconsistencies. For example, we find that, unless it has an exceptionally high magnetic field strength, a neutron star in a fossil disk cannot be observed as an AXP if the disk opacity is dominated by Kramers’ law. However, standard alpha-disk models show that a Kramers opacity must dominate for the case $\log B \gtrsim 12$, making it unlikely that a fossil disk scenario can successfully produce AXPs. Additionally, we find that in order to sufficiently spin down a neutron star in a fossil disk, an unusually efficient propeller torque must be used. Such torques are inconsistent with observations of other accreting systems – particularly High Mass X-ray Binaries. Thus, our analysis lends credence to the magnetar model of AXPs.

\textit{Subject headings:} accretion, accretion disks – pulsars: general – stars: neutron – X-rays: stars

1. INTRODUCTION

Anomalous X-ray Pulsars (AXPs) are a recently discovered subclass of X-ray pulsators sharing distinct properties that are markedly different from that of their high-mass X-ray binary (HMXB) and low-mass X-ray binary (LMXB) cousins (Mereghetti & Stella 1995; Van Paradijs, Taam, & Van den Heuvel 1995). In particular, the six known AXPs have an extremely narrow range of

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pulse periods, $\sim 6 - 12$ s. This is remarkable when compared to HMXBs, for example, with spin periods in the range $0.069 - \sim 10^4$ s. Additionally, AXPs have relatively low X-ray luminosities, $(L_x \sim 10^{34} - 10^{36}$ erg s$^{-1}$), no known optical companions and are undergoing stable spindown. They also have relatively soft spectra, fit by a combination of power-law and blackbody models. Finally, the neutron stars at the heart of AXPs are thought to be young, as evidenced by measured spindown ages in the range $10^3 - 10^5$ yr. This assumption is enhanced by the observation of several clear supernova remnant associations. A related class of objects (not addressed in this paper) are the so-called Soft Gamma Repeaters (SGRs) which share, in their quiescent state, similar spin periods, spindown ages, and luminosities. For a recent observational review of AXPs, see Mereghetti et al. (2002).

The major theoretical hurdle in understanding AXPs lies in discovering the source of their X-ray emission. Rotational power is clearly not responsible for their power as $E_{rot} = I\Omega \dot{\Omega}$ for typical values. There have been many attempts to model AXPs in recent years but all models can, more or less, be grouped into one of two categories. The first class of models assume that the source of AXPs are isolated neutron stars with magnetic field strengths in the range $10^{14} - 10^{15}$ Gauss – i.e. “magnetars” (Duncan & Thompson 1992). It was discovered that magnetars could account for the timing properties of both AXPs and SGRs by simply invoking standard magneto-dipole braking (Thompson & Duncan 1995, 1996). Furthermore, in this model, the source of the X-ray luminosity may be explained by rapid magnetic field decay (Thompson & Duncan 1996; Colpi, Geppert, & Page 2000). The second class of models used to explain the observable features of AXPs invoke some type of accretion. Such models do not require neutron stars with unusually strong magnetic fields; they are assumed to have properties consistent with ordinary radio pulsars, $B \lesssim 10^{13}$ G. In this case, the spindown torque is external and, supposedly, a natural consequence of accretion braking. It was quickly shown, however, that traditional accretion models lead to a number of problems. Standard Bondi accretion models (Bondi 1952) show that unless AXPs all lie near particularly dense regions of the Interstellar Medium (ISM), for example, near molecular clouds [e.g. Israel, Mereghetti, & Stella (1994)], it is unlikely that they are directly accreting from the ISM. Mereghetti & Stella (1995) suggested that AXPs may be members of very low-mass X-ray binaries, (VLMXBs) – a subclass of LMXBs with slightly higher magnetic field strengths, $\sim 10^{11}$ G. One possible example of a VLMXB is the X-ray pulsar, 4U1626 – 67. Verbunt, Wijers, & Burn (1990) had estimated that a low-mass donor star of about $0.02 M_\odot$ accreting onto an old neutron star may be responsible for its X-ray emission and, in fact, an optical identification has been made (Chakrabarty 1998) for this system. Although this scenario can not be ruled out for AXPs, there are severe observational restrictions that necessarily limit the possibility of detecting any potential optical companion they might have (Mereghetti, Israel & Stella 1998). For example, there have been no detected Doppler modulations in the X-ray frequency for any of the AXPs. Such modulations are expected, however, even in the case of accretion from a low-mass Helium star or a very low-mass white dwarf. Additionally, there seem to be clear observational differences between the AXPs and 4U 1626 – 67. For example, the LMXB has a much harder spectrum than any of the anomalous X-ray pulsars. Another discrepancy is that while 4U 1626 – 67 has undergone
clear intervals of both spinup and spindown (to be expected for an accreting source), this is not
the case for the AXPs which show persistent spindown only. The first non-binary accretion model
was suggested by Van Paradijs, Taam, & Van den Heuvel (1995). In this scenario, AXPs are
descendants of close HMXBs and are observed during the post Thorne-Żytkow/spiral-in phase. In
this case the source of the X-ray luminosity is a neutron star accreting from a residual disk formed
from the disrupted companion star. The apparent youth of AXPs is consistent with this model
as is their association with supernova remnants. Ghosh, Angelini, & White (1997) suggested that
this model can naturally explain the multi-phase spectra of AXPs. For example, disk accretion can
account for the power-law phase of the spectra whereas a spherically symmetric flow, emanating
from the companion’s envelope would account for the blackbody component. Of course, this model
naturally assumes the nascent neutron star can survive the common-envelope/spiral-in phase of its
evolution. It has been demonstrated, however, that this is not the case (Brown 1995; Chevalier
1993) as hypercritical accretion forces $\geq 1M_\odot$ onto the Thorne-Żytkow object, crushing it, forming
a black hole.

It is unknown whether or not all of the mass ejected from a core-collapse supernova can escape
the gravitational well of the embedded compact star. With this in mind, it is certainly plausible
that a significant amount of mass may fall back onto a young neutron star after the supernova (Lin,
Woosley, & Bodenheimer 1991; Chevalier 1989). Most recently, several groups have independently
proposed that fossil disks may interact with young neutron stars and that this interaction can result
in timing signatures similar to what is observed for AXPs (Chatterjee, Hernquist, & Narayan 2000;
Alpar 2001; Marsden et al. 2001). In this latest challenge to the magnetar hypothesis it is asserted
that neutron stars embedded in fossil disks have conventional field strengths, similar to those of
radio pulsars. Chatterjee, Hernquist, & Narayan (2000) (hereafter CHN) are the first to present a
detailed physical model of fossil disk accretion incorporating a time-dependent mass-transfer rate.
In this model, it is assumed that after a brief transient phase, mass-loss is self-similar, obeying a
power-law decay. The narrow range of observed spin periods can be explained by assuming the
inflow quickly becomes an advection-dominated accretion flow (ADAF). Alpar (2001) goes further
and posits that not only can AXPs (and possibly SGRs) be explained by fossil disk models but
varying fossil accretion rates can account for all “non-standard” young neutron stars that do not
appear to be radio pulsars (Kaspi 2000). Marsden et al. (2001) suggest that the environment
surrounding AXPs (and SGRs) are unusually dense, thereby facilitating the formation of a fossil
disk. This so-called “pushback” model has been criticized however (Duncan 2002) as the inferred
densities calculated by Marsden et al. (2001) very strongly depend on the AXP distance, a quantity
which is not known with accuracy. In any case, the unifying feature of all these fossil disk models
is that the torque causing the necessary rapid spindown is induced by the propeller-effect (Pringle
& Rees 1972; Illarionov & Sunyaev 1975; Fabian 1975). The propeller mechanism allows a rapidly
rotating neutron star to prevent the flow of material from accreting to its surface. The resulting
interaction between the neutron star magnetosphere and the accreting material somehow causes
the neutron star to lose angular momentum.
Here, we show that all fossil disk models carry inherent inconsistencies. We focus our analysis on the CHN model, as it is the only current model to provide a detailed physical picture of fossil disk accretion but our arguments apply generally. In §2, we show that the final spin period of a neutron star embedded in a fossil disk strongly depends on the disk opacity. We find that, if free-free and bound-free transitions dominate over scattering in the disk and $B \lesssim 3.7 \times 10^{13}$ Gauss, the neutron star can not be observed as an AXP. Employing a simple alpha-disk model (Shakura & Sunyaev 1973), we show that, in fact, a Kramers opacity must be assumed if one is to believe the fossil disk mass-transport rate of CHN.

In §3, we discuss the consequences of the propeller torque model employed by CHN. In order to spin the neutron star down to the observed AXP range in a time approximating its spindown age, CHN assumed an “efficient” propeller torque, allowing the ejected plasma to be spun up to corotation with the pulsar. Although the exact mechanism by which a neutron star loses angular momentum by the propeller effect is not well understood, we show that other “standard” torque mechanisms, in fact, yield the opposite result. As pointed out by Thompson (2002), if one, instead, assumes the plasma is ejected with the specific angular momentum of a particle in a Keplerian orbit at the magnetospheric boundary, one must then assume a dipole field several orders of magnitude greater than what is inferred for radio pulsars in order to yield similar results. In fact, we will show that such propeller mechanisms fail to produce AXPs. Of course, the reality of the magnetospheric interaction is a very complex magnetohydrodynamical problem so it is hard to say, a priori, which propeller model is inherently more accurate therefore it is necessary to treat all models as competitive and compare the results. In §4, we examine the effect of invoking various propeller torques to the (hopefully) well-understood HMXB, SMC X-1. We find the CHN “efficient” propeller suggests a spin period for SMC X-1 that deviates orders of magnitude from its observed period. In §5, we summarize our findings and discuss the possible implications.

2. FOSSIL DISK MODELS

It has long been considered that immediately following the formation of a neutron star in a core-collapse supernova, some fraction of the ejected plasma may be unable to escape the gravitational well of the star. Following Lin, Woosley, & Bodenheimer (1991) and Chevalier (1989), CHN suggest a fallback mass $\lesssim 0.1M_\odot$ is not unreasonable. At that point, it is unclear what fraction of this fallback mass will eventually form the fossil disk but it is suggested that a large amount can either be immediately ejected from or accreted onto the neutron star. Comparing this problem with that of the tidal disruption of stars by massive black holes, discussed earlier by Cannizzo, Lee, & Goodman (1990), CHN find that fallback matter can circularize into an accretion disk in (roughly) a local dynamical time. For their analytical model, CHN assume the dynamical timescale is $\sim 1$ ms but claim that the final outcome is insensitive to the exact numerical value. Furthermore, an initial disk mass, $M_0 = 0.006M_\odot$ is assumed. Later we will discuss the effects of varying the disk mass (as well as the initial spin period of the neutron star) but, for now, we note that as the observed
spin periods of AXPs are in a very narrow range, it is not likely possible to vary the initial disk mass much and find results consistent with the predictions of CHN. The newly born neutron star is assumed to have “canonical” properties with a magnetic field strength in the range $10^{12} < B < 10^{13}$ Gauss and an initial spin period, $P_0 \sim 15$ ms. Such properties agree with what is observed for the Crab pulsar (PSR 0531 − 21) although recent analyses have shown that this may not be the norm (Kaspi 2000; Alpar 2001).

Unlike the case of binary accretion, the fossil disk can not be replenished so accretion is necessarily a time-dependent phenomenon. Following Cannizzo, Lee, & Goodman (1990), CHN suggest that after the brief transient phase, the accretion rate declines self-similarly. In our analysis, we have devised a computer model that can exactly reproduce the analytical model of CHN with the added benefit that it is simple to vary any parameter and determine its influence on the final outcome. We discuss the results of the model in §2.1 and explore the consequences of varying one particular parameter (the opacity) in §2.2.

### 2.1. Self-Similar Fossil Disks

Following the arguments of Cannizzo, Lee, & Goodman (1990), CHN suggest that after a dynamical time the fossil disk loses mass self-similarly:

$$
\dot{M} = \begin{cases} 
\dot{M}_0, & 0 < t < t_D, \\
\dot{M}_0 \left(\frac{t}{t_D}\right)^{-\Gamma(\kappa)}, & t \geq t_D
\end{cases}
$$

Here, $t_D \sim 10^{-3}$ s, is the local dynamical time and $\Gamma > 1$ is a constant that directly depends on the disk opacity, $\kappa$ (see below). Not all of the mass lost from the fossil disk will be accreted onto the neutron star surface. Thus, in general, the neutron star accretion rate, $\dot{M}_x \leq \dot{M}$ and $\dot{M}_x = 0$ during the propeller phase. Of course, only the surface accretion rate gives rise to the observed X-ray luminosity, $L_x = GM_x \dot{M}_x / R_x$. Normalizing to the total initial disk mass, $\dot{M}_0$, it is determined from equation (1):

$$
\dot{M}_0 = \left(\frac{\Gamma - 1}{\Gamma}\right) \left(\frac{M_0}{t_D}\right)
$$

From Cannizzo, Lee, & Goodman (1990), we see the parameter $\Gamma(\kappa)$ directly depends on the disk opacity. For a standard parameterization, $\kappa(\rho, T) = \kappa_0 \rho^\lambda T^\nu$, where $\rho$ and $T$ are the local density and temperature respectively, one finds:

$$
\Gamma = \frac{38 + 18\lambda - 4\nu}{32 + 17\lambda - 2\nu}
$$

Thus, a standard Kramers opacity ($\lambda = 1, \nu = -7/2$) yields $\Gamma = 1.25$ whereas if the opacity is predominantly due to Thomson scattering ($\lambda = \nu = 0$), then $\Gamma = 19/16 \cong 1.188$. In their analytic model, CHN suggest $\Gamma = 7/6 \cong 1.167$. 
The subsequent evolution of the system can be divided into four phases depending on the relative strengths of several parameters. The magnetospheric boundary of a neutron star is often defined as the point where ram pressure of infalling matter is balanced by the pulsar’s magnetic dipole pressure [see Francischelli, Wijers, & Brown (2002) and references therein]. Hence, the flow of accreted matter is governed by the dipole field for any $r \leq R_m$. Here $R_m$ is the magnetospheric radius which, for a neutron star with $R_x = 10$ km and $M_x = 1.4M_\odot$, is determined to be:

$$R_m \approx 0.5 \left( \frac{B^4 R_{12}^{12}}{8GM_x M^2} \right)^{1/7} \approx 6.6 \times 10^7 B_{12}^{4/7} \left( \frac{\dot{M}}{M_{\text{Edd}}} \right)^{-2/7} \text{ cm} \quad (4)$$

In equation (4), $\dot{M}_{\text{Edd}} \approx 9.46 \times 10^{17} \text{ g s}^{-1}$ is the (hydrogen) Eddington accretion rate, $B_{12} = B/10^{12}$ Gauss, and the factor of $1/2$ comes from the assumption of a disk geometry. Regardless of whether the flow starts out in a disk or has a spherical geometry, gravitationally captured plasma can only reach the neutron star’s magnetosphere if it can fall along a closed field line. This is only permitted within the neutron star’s light cylinder, defined such that $R_{lc} = c/\Omega = cP/2\pi$. Thus, plasma can interact with the neutron star’s magnetosphere if and only if $R_m \leq R_{lc}$. If this is not the case, the neutron star evolves independently of the fossil disk and spins down by magnetodipole braking (i.e. emitter phase). Once the neutron star spins down sufficiently such that matter can couple to its magnetic field, the interaction’s effect on the overall spin evolution depends on the balance between centrifugal and gravitational accelerations. If the pulsar is initially rotating too fast, the neutron star will eject infalling plasma, propelling it away tangentially, while simultaneously losing angular momentum in the process. This “propeller” effect (Pringle & Rees 1972; Illarionov & Sunyaev 1975) can be parameterized by a fastness parameter $\omega_s \equiv \Omega/\Omega_K(R_m)$, where $\Omega_K(R_m) = (GM_x/R_m^3)^{1/2}$ is the Keplerian angular velocity at the magnetospheric boundary. If the fastness parameter greatly exceeds unity, the propeller mechanism is initiated. For both the propeller and emitter phases, no material can accrete to the neutron star surface and $L_x \propto \dot{M}_x \sim 0$. As discussed in the introduction, the way in which a propelling neutron star loses angular momentum is not well understood. However, it is safe to assume that some kind of propeller torque (see equation [6], §3) acts on the neutron star, spinning it down with time, and reducing the fastness parameter. When $\omega_s \lesssim 1$, surface accretion can take place and, provided the transfer rate is sub-Eddington, $\dot{M} = \dot{M}_x$. Contrary to what is expected in an X-ray binary, however, $\dot{M}$ steadily decreases in a fossil disk and an extended accretion phase never really occurs. Instead, a quasi-equilibrium period, which CHN dub a “tracking” phase, occurs, and it is during this period that the neutron star can be observed as an X-ray pulsar. Furthermore, they suggest that the tracking phase must be short lived as the accretion flow quickly becomes advection-dominated (ADAF). It is proposed that an ADAF transition occurs when the accretion luminosity falls to $\approx 10^{-2} L_{\text{Edd}} \approx 1.8 \times 10^{36} \text{ erg s}^{-1}$ (Chatterjee, Hernquist, & Narayan 2000; Narayan & Yi 1995). At this point, it is thought that most of the captured matter is ejected prior to reaching the neutron star’s surface and, once again,
\[ \dot{M}_x \sim 0. \] From equation (1), the ADAF transition time is given by

\[ t_{\text{ADAF}} \cong t_d \left( \frac{100 \dot{M}_0}{\dot{M}_{\text{Edd}}} \right)^{1/\Gamma} \tag{5} \]

Typically, \( t_{\text{ADAF}} \sim 20,000 - 40,000 \) yr. In sum, the neutron star can only be observed as a bright X-ray source during the tracking phase of its evolution, \( t_{\text{track}} \lesssim t \lesssim 2t_{\text{ADAF}} \).

To determine the spin evolution of the neutron star + fossil disk system, a spinup/spindown torque is needed. For \( R_x < R_m < R_{lc} \), CHN propose the following (Menou et al. 1999):

\[ \dot{J} = \dot{I} \Omega = 2 \dot{M} R_m^2 \Omega_K(R_m) \left[ 1 - \frac{\Omega}{\Omega_K(R_m)} \right] \tag{6} \]

Assuming an arbitrary value for \( \Gamma \), equations (1) and (6) can be combined to yield an analytic formula for \( \Omega(t) \), in terms of incomplete gamma functions. For the particular case \( \Gamma = 7/6 \), CHN find a solution using exponential integrals (see their eq. 4). Instead of using their analytic method, however, we solved the above equations numerically, using an Eulerian integration scheme. The results of our analysis (with \( \Gamma = 7/6 \)) is shown in Figure 1. Here we plot the spin evolution of the neutron star as a function of magnetic field strength until the approximate onset of the ADAF phase, \( 2t_{\text{ADAF}} \sim 38,000 \) yr. Initial conditions \( P_0 = 15 \) ms and \( \dot{M}_0 \sim 0.006 M_\odot \) are assumed. We see that a high magnetic field strength is needed in order for the neutron star to be observed as an AXP. For the above initial conditions, neutron stars with \( B_{12} \gtrsim 3.9 \) do not spin down efficiently and, consequently, are observed as radio pulsars. On the other hand, for \( B_{12} \lesssim 3.9 \), the neutron star, after a brief emitter phase, undergoes a rapid propeller spindown and enters the tracking phase of its evolution at \( t_{\text{track}} \sim 10^4 \) yr. Our numerical analysis agrees with the analytic model of CHN (see their figure 1).

The initial spin period of neutron stars is largely unknown and the long-accepted paradigm that assumed the Crab pulsar is the prototypical young neutron star is now being challenged (Kaspi 2000). Thus, it is important to investigate how the previous results depend on \( P_0 \). In fact, we find that the duration of the AXP phase does not strongly depend on the initial period. For example, assuming \( P_0 = 150 \) ms had little effect on the final evolutionary state of the system (pre-ADAF). For a particular case, consider a neutron star with \( B_{12} = 7.5 \) and \( P_0 = 150 \) ms. Our analysis has shown that it is able to spin down via the propeller effect to the AXP range (10.0 s) in a time \( t_{\text{ADAF}} \cong 4 \times 10^4 \) yr. The only deviation from what is seen in figure 1 is in the early evolution. In this case, we found a short-lived accretion phase can occur \( \sim 100 \) yr after birth. This is compensated by a somewhat steeper propeller cycle and a slightly shorter tracking phase. The overall evolution, however, is qualitatively the same as the 15 ms case and, as this case is representative of the general trend, we therefore conclude that the CHN model can accommodate a wide range of initial periods.

Another free parameter to be considered is the initial disk mass. Even assuming a fallback mass \( \lesssim 0.1 M_\odot \), it is unclear how much matter will circularize into an accretion disk. In the initial period following the supernova, much of this material may be accreted onto the neutron star, possibly
hypercritically. Additionally, as detailed modelling probably depends on the progenitor history, we make only a few qualitative remarks about assigning possible values to the disk mass. As CHN have shown, an AXP can be formed even if $M_0 \ll 0.1 M_\odot$. However, there still must be a sufficient amount of disk matter in order to penetrate the neutron star’s light cylinder. From equation (4), we see the magnetospheric radius increases as the accretion rate falls and if $\dot{M}$ falls below some critical value, matter will be unable to couple to the closed field lines and no (extended) propeller phase will take place. Our numerical simulations indicate that $M_0 \gtrsim 7.5 \times 10^{-4} M_\odot$ is needed for the neutron star to be observed as an AXP. A neutron star embedded in a fossil disk with significantly less mass than this, rarely leaves the emitter phase of its evolution and, hence, would be observed as a radio pulsar. The exact critical value strongly depends on the neutron star’s magnetic field strength; for $B_{12} \gtrsim 7.5$, even more disk matter is needed. The most important observational constraint in determining the initial disk mass is the narrow distribution of AXP spin periods. Deviations far from the CHN sample value of $0.006 M_\odot$, although often producing neutron stars with a tracking period, often yield a wider range of spin periods than what is currently observed. Thus, for our model, $M_0 = 0.006 M_\odot$ was consistently used.

2.2. The Effect of Varying Disk Opacity

One quantity that deserves special attention is the disk opacity, parameterized by $\Gamma(\kappa)$ in equation (3). We now show that if free-free and bound-free transitions dominate over electron scattering within the disk (i.e., if a Kramers opacity is assumed), then a canonical neutron star will never reach a tracking phase and, consequently, will not be observed as an AXP. We recall from equation (3) that $\Gamma_{es} = 19/16 = 1.1875$ and $\Gamma_{Kr} = 1.25$ so it is not immediately obvious that such numerically similar parameters could lead to completely different evolutionary scenarios. Figure 2, however, illustrates a strong correlation between $\Gamma$ and the overall spin evolution of the neutron star. Here we plot the final spin period of a neutron star embedded in a fossil disk as a function of $\Gamma(\kappa)$ for various magnetic field strengths. An efficient propeller torque given by equation (6) is assumed. As we have seen in section §2.1, the system’s evolution also depends on the neutron star’s magnetic field. In particular, for a given disk opacity, the field strength determines the extent of the propeller phase and, ultimately, whether or not a tracking phase can occur at all. This dependence is shown in figure 2 by comparing opacity-period evolution curves for $B_{12} = 2.5, 5.0, 7.5, 10.0$ and $40.0$. Although the shape and qualitative behavior are the same for the various field strengths, the overall range of final periods differs greatly. Generally, each curve shows essentially no dependence on opacity below some cutoff value. As $\Gamma$ increases past a value, $\Gamma_c$, however, there is a sharp drop in final spin period followed by a more gradual decline. These curves illustrate the fact that, depending on the strength of the field, a neutron star embedded in a disk with opacity parameter greater than some $\Gamma_c$, will not be able to spin down to the AXP range and, consequently, it will end its observable life as a radio pulsar. The decline in final spin period with increasing $\Gamma$, can be interpreted physically by noting from equation (1) that the overall mass-transport rate steeply decreases with time by a factor $\propto t^{-\Gamma}$ and for $\Gamma > \Gamma_c$, disk matter is depleted too quickly to
sustain an extended propeller phase. A secondary cause for the decline is the ADAF transition, and subsequent X-ray shut-off, must necessarily occur at earlier times for increased $\Gamma$. For $\Gamma = 7/6$, our results agree with the conclusions of CHN. As mentioned previously, in this case, the minimum field strength needed to support a tracking phase $\sim 3.9 \times 10^{12}$ G. Thus, in figure 2, $\Gamma_{\text{CHN}} < \Gamma_c$ for all fields except $B_{12} = 2.5$. If the fossil disk were to be dominated by electron scattering, we find that a stronger field is needed in order for the neutron star to enter a tracking period before the ADAF phase of its evolution. In this case, the minimum field required is $\sim 6.6 \times 10^{12}$ Gauss. For $B_{12} = 5.0$, $\Gamma_{\text{es}} > \Gamma_c$ and for $B_{12} = 7.5$, $\Gamma_{\text{es}} \sim \Gamma_c$. If the disk is dominated by a Kramers opacity, however, unusually strong magnetic fields are necessary in order for the neutron star to be observed as an AXP. In fact, $\Gamma_{\text{Kr}} \gg \Gamma_c$ for all field strengths below $\sim 3 \times 10^{13}$ Gauss. All neutron stars with magnetic fields below $B_{\text{Kr}} \sim 3.7 \times 10^{13}$ Gauss and otherwise similar initial conditions will remain a radio pulsar throughout its observable lifetime. Of course, the strength of this conclusion directly depends on the exact point at which the ADAF transition commences. As this quantity is not known with accuracy, the true value of $B_{\text{Kr}}$ may be either higher or lower.

We have seen that the fossil disk opacity is a key parameter in determining the ultimate evolution of the neutron star. In particular, we have shown that if a Kramers opacity dominates, it is unlikely (though not impossible) for an AXP to be observed whereas if electron scattering is more important, it is more likely that a “canonical” neutron star can become an AXP. In fact, we now show that, for $B_{12} \gtrsim 1$, a Kramers opacity must be assumed. In order to calculate the opacity we adopt the so-called alpha-prescription of Shakura & Sunyaev (1973), implicitly assuming that a thin-disk structure is valid for fossil disks. Such an assumption seems valid as it is probably reasonable to expect the fossil disk is approximately thin in the regime, $t_{\text{dyn}} \ll t < t_{\text{ADAF}}$. Furthermore, we assume the (kinematic) turbulent viscosity, $\nu$, can be parameterized according to the standard relation, $\nu \equiv \alpha c_s h$. Here, $c_s$ is the local (isothermal) sound speed and $h$ represents the disk’s vertical scale-height. It has been argued on physical grounds that the viscosity parameter, $\alpha < 1$ (Shakura & Sunyaev 1973). The thin-disk assumption is parameterized by the condition $h/r \sim c_s/v_\phi \ll 1$, where $r$ is the radial measure of the disk and $v_\phi(r) = r\Omega_k(r) = \sqrt{GM/r}$ represents the azimuthal flow within the disk. The general procedure for determining the time-evolution of a viscous disk undergoing mass-transfer variations has been extensively studied. Bath & Pringle (1981), for example, have shown that the time-dependent conservation equations lead to a non-linear diffusion equation for the surface density, $\Sigma(r,t)$. It has been shown, however, (Frank, King & Raine 1992), that if at each point in time, and for a given $\dot{M}(t)$, the timescale in which matter diffuses through the disk greatly exceeds the dynamical timescale, $\sim r/v_\phi(r)$, the disk may be considered to be approximately steady-state. Except at early times, this condition is generally valid for the fossil disk we are considering and, thus, in our analysis, we assume steady-state solutions may be used for each value of $t$. This approach is roughly equivalent to numerically integrating the diffusion equation of Bath & Pringle (1981).

We first assume an opacity dominated by bound-free transitions, $\kappa_{\text{Kr}} \cong 4 \times 10^{25} Z(1+X)\rho T^{-7/2}$ cm$^2$ g$^{-1}$, (Schwarzschild, M. 1958) where $X \approx 0$ and $Z \lesssim 1$ are the hydrogen and heavy element
mass fractions, respectively. Following the analysis of Frank, King & Raine (1992), we can then estimate the region within the disk where a Kramers-type opacity may be dominant.

$$\kappa_{Kr}(r, t) = 90[Z(1 + X)]^{1/2} \dot{M}_{17}^{-1/2}(t) r_{10}^{3/4} f^{-2}(r) \text{ cm}^2 \text{ g}^{-1}$$

Here $\dot{M}_{17} = \dot{M}/10^{17}$ g s$^{-1}$, and $r_{10} = r/10^{10}$ cm as usual and $f(r) \equiv [1 - (r_i/r)^{1/2}]^{1/4}$. Fortunately, the result is independent of $\alpha$ and, thus, does not depend on the poorly understood viscosity mechanism within the disk. The quantity, $r_i$ represents the inner edge of the disk and for neutron stars, $r_i = R_m$, given by equation (4). In our analysis, we ignore the perturbations brought about by edge effects and concentrate on the region, $r \gg r_i$, $f(r) \approx 1$. At temperatures $\gtrsim 10^4$ K, the other dominant source of opacity is electron scattering, $\kappa_{es} = 0.2(1 + X)$ cm$^2$ g$^{-1}$. From equation (7), we can estimate the region of the fossil disk where $\kappa_{Kr} \gtrsim \kappa_{es}$.

$$r_{Kr}(t) \gtrsim (3 \times 10^6) \left[ \frac{1 + X}{Z} \right]^{2/3} \dot{M}_{17}^{2/3}(t) \text{ cm}$$

A $1.4 M_\odot$ neutron star is assumed throughout our analysis. From equation (4), we estimate the magnetospheric boundary of the neutron star, $R_m$, as the point at which the disk truncates. We use this result along with equation (8) to determine the conditions necessary such that electron scattering may be ignored. I.e., we calculate the necessary conditions such that $R_m(t) \geq r_{Kr}(t)$.

$$\dot{M}_{17}(t) \lesssim 50 \left[ \frac{1 + X}{Z} \right]^{-7/10} B_{12}^{3/5}$$

Finally, combining the above relation with the mass-transfer equation, (1), we can estimate the time after which electron scattering may be completely ignored within the disk. From (1), (2), and (9), we find

$$t_{Kr} \gtrsim 20 \left[ \frac{1 + X}{Z} \right]^{14/25} B_{12}^{-12/25} \text{ yr}$$

For the CHN AXP-cutoff $B_{12} \sim 4$, this corresponds to a value of $\sim 10$ yr. With $B_{12} \sim 37$, $t_{Kr} \sim 3.5$ yr. In any case, we see that, regardless of the magnetic field strength, the fossil disk model of CHN must be Kramers-dominated throughout its evolution and $\Gamma = 1.25$ is the proper choice of exponents in equation (1). Coupling this with the preceding analysis and figure 2, we have now shown that, unless the neutron star has an extraordinarily high magnetic field, in excess of $\sim 3.7 \times 10^{13}$ Gauss, the CHN model fails to produce an anomalous X-ray pulsar.

### 3. PROPELLER TORQUES

In its most general form, the exact manner in which angular momentum is transferred between an accretion disk and a neutron star is a complex magnetohydrodynamical problem to which no simple analytic solution has been determined. More than two decades of numerical analyses have resulted in only minor modifications to the original model of Ghosh & Lamb (1979) [but see Wang...
It has been determined that when modelling disk accretion, it is important to include the effects of magnetic torques within the disk to the overall spin rate. Ghosh & Lamb were able to show that for slow rotators \( \Omega / \Omega_K(R_m) \equiv \omega_s \ll 1 \), magnetic coupling may enhance spin-up torques by as much as 40%. For \( \omega_s \lesssim 1 \), the opposite is true, and magnetic effects might actually oppose the spin-up. However, as we have seen, due to a time-dependent mass-transfer rate, a neutron star in a fossil disk never really has an extensive accretion period nor can it reach true equilibrium. Unfortunately, unlike the case for accretion, angular-momentum transfer during the propeller phase is not well modelled at all. At present, no ab initio theory exists to compute the torque from an accretion disk on a magnetized star. However, over time, several approximate schemes have been introduced, generally based on basic physical principles such as conservation laws (Pringle & Rees 1972; Illarionov & Sunyaev 1975; Fabian 1975). Additionally, Menou et al. (1999) have introduced the “efficient” propeller torque, given by equation (6). As several competing models exist, the best we can do is compare the models with each other and, most importantly, with observations. With this in mind, it is important to emphasize that while we can not really claim one model to be intrinsically “better” than another, we can reject a particular model if observations force our hand.

For a strong propeller, \( \omega_s \gg 1 \), and equation (6) can be written in the form:

\[
N_{\text{CHN}} = I \dot{\Omega} \cong -2 \dot{M} R_m^2 \Omega \tag{11}
\]

Thus, we see the CHN model assumes a rapidly rotating neutron star propels matter from the magnetospheric boundary with (twice) the angular velocity of the neutron star, thereby spinning it up to corotation. Historically, other propeller models that have been used are not quite so efficient. In fact, it has generally been assumed that the propelled matter is, instead, ejected from the magnetospheric boundary with the specific angular momentum of a particle in an escaping parabolic orbit.

\[
N_J = I \dot{\Omega} = -\dot{M} R_m v_{\text{esc}}(R_m) = -\sqrt{2} \dot{M} R_m^2 \Omega_K(R_m) \tag{12}
\]

The J-subscript in equation (12) indicates we have used angular momentum methods to estimate the propeller torque. An alternative approach is to employ energy conservation principles. Fabian (1975) has suggested that, over time, the rotational kinetic energy of the neutron star will be transmitted through shocks to the wind plasma falling near the magnetospheric boundary. Consequently, the infalling gas will heat up and be dispersed when it attains escape velocity. Thus, we find

\[
\dot{E} = I \dot{\Omega} = -\frac{1}{2} \dot{M} v_{\text{esc}}^2(R_m) = -G \dot{M} M_x / R_m.
\]

The propeller torque is therefore expressed as

\[
N_E = I \dot{\Omega} = -\frac{\dot{M} R_m^2 \Omega_K(R_m)}{\omega_s} \tag{13}
\]

where \( \omega_s \) is the fastness parameter. Upon comparison of equations (11) - (13), we find the relationship between the various proposed propeller torques:

\[
N_{\text{CHN}} = \sqrt{2} \omega_s N_J = 2 \omega_s^2 N_E \tag{14}
\]

In the evolution models just discussed here, we found that the fastness parameter varies in the range \( \sim 20 - 75 \). It is therefore not very surprising that the choice of propeller torques has a great
deal of influence on the overall evolution of the neutron star/fossil disk system. This is illustrated in figure 3. Here, we plot a period-time relationship for the various propeller models, keeping all other initial parameters fixed. We have assumed, as usual, an initial fossil disk mass of $M_\text{f} = 0.006 M_\odot$ and the initial field strength and spin period are $7.5 \times 10^{12}$ Gauss and 15 ms, respectively. In order to make our argument independent of the results of the preceding section, we have kept the opacity parameter fixed at $\Gamma(\kappa) = 7/6$, the original CHN value. Clearly, employing the efficient propeller torque of Chatterjee, Hernquist, & Narayan (2000) makes all the difference in determining the overall evolution. When the torque given by equation (11) is used, the neutron star, after a sharp propeller cycle reaches a tracking phase at $\sim 1.5 \times 10^4$ yr. It is then visible as a bright AXP until $2t_{\text{ADAF}} \sim 3.8 \times 10^4$ yr, at which point its final spin period is 13.5 sec. For both the energy and angular momentum propellers, the neutron star never reaches equilibrium and despite much higher fastness parameters ($\sim 70$), the neutron star’s final period never exceeds $\sim 0.2$ s. Note the early evolution for the energy and angular momentum propellers deviate slightly as the energy propeller, for these initial conditions, lacks an early emitter phase, and begins propelling matter near $t \sim 0$. At later times, the evolution traces the angular momentum propeller case exactly. Of course, if given sufficient time, we would see the angular momentum propeller force much higher spin periods but only at times $t \gg 2t_{\text{ADAF}}$. Thus, neither the angular momentum nor the energy propellers yield AXPs but, instead, ordinary radio pulsars.

We recall, from § 2.2, that the choice of $\Gamma(\kappa)$ strongly determines the evolution of a neutron star in a fossil disk provided the CHN torque is employed. In fact, we found that exceptional magnetic fields are required if one were to assume a physically realistic Kramers opacity. The exact value of $\Gamma$ is much less important to the overall evolution for the other propeller torques, however. Assuming a disk dominated by electron scattering, for example, does not change the final spin period for either the angular momentum or energy propellers. For a Kramers disk, ($\Gamma = 1.25$), we also see no early evolutionary changes in spin period. However, in this case, the onset of ADAF flow occurs much earlier ($t_{\text{ADAF}} \equiv 2650$ yr by equation [5]). For a Kramers disk, therefore, we find the final spin period for all three propeller torques is the same: about 0.13 s. As pointed out by Thompson (2002), increasing the magnetic field strength by an order of magnitude does allow the less efficient propeller torques to spin down the neutron star to longer periods, but we find that, regardless of the strength of $B$, the J-propeller and E-propeller can never produce an anomalous X-ray pulsar. For example, in order to spin down a neutron star in a fossil disk by the angular momentum propeller torque to a final period of $\sim 5$ s, it is necessary to increase the initial field strength to $\approx 9.8 \times 10^{13}$ G. In fact, even at such high fields, the neutron can never reach equilibrium. Instead, as $\omega_s \gtrsim 24$ (and is slowly decreasing) at $t \sim 2t_{\text{ADAF}}$, there will be no tracking period at all. Borrowing from the terminology of Chatterjee, Hernquist, & Narayan (2000), we dub such a neutron star a “dim propeller.” Spun down into the graveyard ($P_f \sim 5.3$ s), the neutron star is a weak emitter in both radio and X-rays. This trend is even more obvious for the case of the energy propeller. Here, in order to spin the neutron star down to a final period of 5.9 s, a magnetic field $\gtrsim 8.6 \times 10^{14}$ G is needed. Even ignoring the fact that such a field strength is already well into the so-called “magnetar” range, we find this neutron star, too, is destined to be a dim propeller.
Thus, we see the results of Chatterjee, Hernquist, & Narayan (2000), namely that AXPs can be produced from ordinary radio pulsars, hinges upon the condition of an efficient propeller torque. If one can show that a canonical neutron star can not spin propelled matter at the magnetospheric boundary to corotation, then it is a necessary consequence that fossil disk models, as outlined by Chatterjee, Hernquist, & Narayan (2000), cannot produce anomalous X-ray pulsars.

4. A COMPARISON OF VARIOUS PROPELLER TORQUES FOR SMC X-1

Initially discovered during a rocket flight (Price et al. 1971) and identified by Uhuru as a discrete X-ray source in the Small Magellanic Cloud (Leong et al. 1971), SMC X-1 is the most luminous and one of the most extensively studied of the X-ray pulsars. Its binary nature, implied by periodic X-ray eclipses (Schreier et al. 1972) was confirmed by the identification of an optical companion: B0I supergiant, Sk 160 (Webster et al. 1972; Liller 1973). The 3.89 day orbital period is decaying at a rate \( \frac{\dot{P}_{\text{orb}}}{P_{\text{orb}}} = (-3.36 \pm 0.02) \times 10^{-6} \text{ yr}^{-1} \) (Levine et al. 1993), presumably due to tidal interactions between the orbital motion and the primary’s rotation. The orbital elements of the system have been measured with the neutron star’s mass estimated at \( M_2 \approx 0.8 - 1.8 M_\odot \) and the primary mass and radius given by \( M_1 \approx 19 M_\odot \) and \( R_1 \approx 18 R_\odot \) respective (Prinini, Rappaport, & Joss 1977). For consistency, we will continue to assume \( M_2 \approx 1.4 M_\odot \) and \( R_2 \approx 10^6 \text{ cm} \). The orbit is circular \( [e < 0.00004, (Levine et al. 1993)] \) and Kepler’s third law suggests an orbital separation \( a \sim 30 R_\odot \). With an 0.71 s pulse period (Lucke et al. 1976), SMC X-1 is the only X-ray pulsar undergoing stable spinup at a rate \( \dot{P} \approx -1.2 \times 10^{-11} \text{ s s}^{-1} \) (Kunz et al. 1993; Kahabka & Li 1999). Spinup implies the neutron star is accreting and, indeed, there is evidence of an accretion disk around SMC X-1 (Van Paradijs & Zuiderwijk 1977; Tjemkes, Zuiderwijk, & Van Paradijs 1986). Its luminosity, \( L_\text{x} \), has been measured to vary from \( \sim 10^{37} \text{ erg s}^{-1} \) in the low-intensity state to the extremely high value of \( 5 \times 10^{38} \text{ erg s}^{-1} \sim 5 L_{\text{Edd}} \) in the high-intensity state. The presence of an accretion disk, excess X-ray luminosity and persistent spinup, together suggest mass-transfer may be due to atmospheric Roche-lobe overflow from the massive primary (Savonije 1979). Recently, an X-ray burst was discovered from SMC X-1 (Angelini, Stella, & White 1991), which was suggested to be due to instabilities in the accretion flow, i.e., a type II burst (Li & Van den Heuvel 1997). If, indeed, SMC X-1 is a member of the class of “bursting pulsars,” like the recently discovered transient X-ray pulsar, GRO J1744-28, then a diminished pulsar magnetic field might be the cause of such bursts. In fact, Li & Van den Heuvel (1997) have estimated a magnetic moment for SMC X-1 to be somewhat lower than normal, \( \mu = BR_2^3 \sim 10^{29} \text{ G cm}^3 \). This is consistent with claims that magnetic field decay is connected with binary accretion [see Francischelli, Wijers, & Brown (2002) for a discussion].

In order to test the various propeller torques of the preceding section, we apply a modified accretion model to the SMC X-1/Sk 160 system. Since the giant phase is only about \( \sim 10\% \) of the main sequence lifetime, it is important to consider the mass transfer mechanism for the progenitor system as well. Most likely, the progenitor to the supergiant primary is an 09 V star, with ZAMS
mass and radius, $\sim 20M_\odot$ and $\sim 10R_\odot$. Its main sequence lifetime can be estimated by the mass-luminosity relation, $L \propto M^{3.5}$, such that $\tau_{\text{ms}} \sim 10^{10} \text{ yr} (M/M_\odot)^{-2.5} \approx 5.6 \times 10^6 \text{ yr}$. We assume the progenitor lies within its Roche lobe throughout the hydrogen core-burning phase so mass transfer is, then, most likely due to a steady stellar wind, of order $\sim 10^{-7}M_\odot \text{ yr}^{-1}$ (Kudritzki & Puls 2000). Thus, the overall evolution of the system has two distinct parts: In the first part, an O9 V star transfers mass, by steady stellar wind, to a young neutron star with initial spin and field strength, $\sim 15 \text{ ms}$ and $\sim 10^{13} \text{ G}$. During this phase of the evolution, mass transfer is highly non-conservative as only a fraction of the mass emitted by the primary can be captured by the neutron star, $\dot{M}_2 = -f_c \dot{M}_1$. The non-conservative phase lasts the entire main-sequence lifetime of the primary. In the second part of the evolution, a B0I supergiant conservatively transfers mass via Roche-lobe overflow (while non-conservative wind-transfer continues) for a time $\sim 0.1\tau_{\text{MS}} \sim 6 \times 10^5 \text{ yr}$. Accretion-induced field decay is assumed for both parts of the evolution. The evolution code for such a model, while not much different from the original code for the fossil disk model described above, is almost identical to the analysis described in Francischelli, Wijers, & Brown (2002). Thus, for details of the input physics assumed, we refer the interested reader there. For the sake of brevity, here we simply show the results of our analysis in figure 4. We plot the neutron star spin period as a function of time for all three propeller models. Of course, we have no way of estimating how long SMC X-1 has been undergoing shell burning so the most we can do is compare the known spin period of 0.71 s to the range of predicted values calculated during the supergiant phase.

If an energy propeller torque is assumed we see that, after a brief emitter phase, the neutron star in SMC X-1 undergoes an extended propeller cycle. Weak propeller-torquing allows the neutron star to gradually spin down until it reaches a maximum period of about 30 s at $t \sim 3.1$ Myr. At this point, accretion spinup commences and continues until a renewed propeller-spindown at $t \sim 4.4$ Myr. This leads to a more gradual accretion cycle and, at the onset of hydrogen-shell burning, at 5.6 Myr, the neutron star is still in the graveyard at $P = 5.3$ s. Overall, during the main-sequence evolution, the radiation-driven wind can only transfer $3.6 \times 10^{-4}M_\odot$ to the neutron star surface. As soon as the primary overflows its Roche-lobe, however, heavy accretion ensues, recycling the neutron star back into observability, spinning it up and bringing down its magnetic field. After a total evolution time of 6.2 Myr, $P_1 = 0.99$ s. Employing the accretion-induced field-decay model of Francischelli, Wijers, & Brown (2002), the final magnetic field is calculated to be $B_f \sim 4 \times 10^{10} \text{ G}$. For the case of an angular momentum ($J$) propeller, propeller-spindown is much more rapid and occurs much earlier in the system’s evolution. As a result, the accretion phase also occurs much earlier and the pulsar spins back up to $\sim 2.2$ s at the end of the core-hydrogen burning phase. Once again, a rapid accretion cycle occurs during Roche-lobe overflow and the final spin period is 0.52 s. As the total accretion time is longer, we find the final magnetic field is necessarily lower for the angular momentum propeller and $B_f = 1.3 \times 10^{10} \text{ G}$. Finally, we look at the case of the efficient, CHN propeller torque. Here, propeller spindown is even more rapid than before and the neutron star reaches $P = 31.3$ s in only $\sim 3.3 \times 10^4 \text{ yr}$. Unlike what was observed for the other two propeller models, here there is no subsequent recycling and the neutron star remains, roughly, in spin equilibrium for the duration of the main-sequence evolution. Consequently, no
mass is accreted to the neutron star surface during this period. As soon as the primary begins to
overflow its Roche lobe, however, a new propeller cycle spins the neutron star down further into
the graveyard, reaching $P_{\text{max}} \sim 100$ s. Finally accretion is allowed, but the pulsar can never be
sufficiently recycled back into observability and $P_f = 15.9$ s. Altogether, only $9.7 \times 10^{-5} M_\odot$ is
accreted to the neutron star surface and, as a consequence of such weak accretion, $B_f \approx 1.2 \times 10^{12}
G$.

In conclusion, we see that both the energy and angular momentum propellers, for the given
initial conditions, yield reasonable spin histories for our model of SMC X-1. The magnetic field
evolution is not as accurate, however, as the heuristic field-decay model of Francischelli, Wijers, &
Brown (2002), while yielding results consistent with observations of LMXBs and relativistic binary
pulsars, seems to deviate by a factor or $\sim 2.5 - 10$ from the magnetic moment calculations of Li &
Van den Heuvel (1997). Although for the purpose of this paper, we are only interested in the spin
evolution of neutron stars for the various propeller torques, the larger subject of pulsar magnetic
field decay in general remains an open issue. The most important result of our analysis, is that
we have definitively shown that the CHN propeller torque model is completely inconsistent with
observations of SMC X-1. In particular, we have shown that such a propeller torque yields a neutron
star with a spin period somewhere in the range $15.9 - 100$ s, orders of magnitude different from the
observed period. On this basis, we reject the efficient propeller torque model and determine it is
extremely unlikely that a neutron star can propel matter to corotation. Since we have also shown
that less efficient (yet more physically plausible) propeller torques can not spin neutron stars in
fossil disks down into the AXP range, regardless of their magnetic field strength, we must reject
fossil disk models of anomalous X-ray pulsars.

5. DISCUSSION

Accretion from a fossil disk is the latest scenario that has been suggested as a plausible alternative
to ultramagnetized neutron stars, or magnetars, as a way of explaining the timing signatures
of anomalous X-ray pulsars. Although several groups have independently postulated fossil disk
accretion models, (Chatterjee, Hernquist, & Narayan 2000; Alpar 2001; Marsden et al. 2001) only
Chatterjee, Hernquist, & Narayan (2000), CHN, have provided a detailed, time-dependent mass-
transfer mechanism and, as a result, our analysis focusses on their model. In fact, we have shown
that the CHN model contains inherent inconsistencies and, ultimately, when measured against ob-
servations and realistic physical assumptions, fails. In section 2.2, for example, we have shown that
a realistic estimate of the disk opacity forces us to either assume unusually strong neutron star
magnetic fields, or abandon the CHN model altogether. Using the alpha-disk model of Shakura
& Sunyaev (1973), we have shown that a Kramers opacity must be assumed for any fossil disk
with a mass-transfer rate given by equation (1). This is in contrast with the analysis of CHN,
where a non-physical opacity parameter, $\Gamma(\kappa) = 7/6$, was assumed. We can interpret this result as
being due to the fact that the magnetospheric boundary increases with increasing field strength. If
the magnetic field, therefore, exceeds some minimum value, the inner edge of the fossil accretion disk necessarily truncates far enough away from the neutron star such that temperatures are low enough to ignore electron scattering. Furthermore, we have shown that a neutron star in a Kramers disk ($\Gamma_{\text{Kr}} = 1.25$), cannot spin down to the AXP range fast enough for $B_{12} \approx 1 - 30$. We have estimated the minimum pulsar field needed to give results consistent with CHN to be $\sim 3.7 \times 10^{13}$ G. Although seemingly unusual, there have been recent discoveries of radio pulsars with inferred field strengths in this range. Camilo et al. (2000) have reported the discovery of J1119 – 6127, a 1670-year old pulsar apparently centered in a previously uncatalogued supernova remnant, with an inferred field strength $B = 4.1 \times 10^{13}$ G. Additionally, the same group has reported the discovery of PSR J1814 – 1744 with $\tau_c = 85$ kyr and $B = 5.5 \times 10^{13}$ G. Note that such field strengths are quite close to the so-called magnetar critical field, $B_c \equiv m_e^2 c^3 / e \hbar = 4.4 \times 10^{13}$ G. It has been suggested that, at $B > B_c$, QED processes such as photon splitting may inhibit pair-production cascades near the magnetic poles and, neutron stars with field strengths in this range should be radio-quiet (Baring & Harding 1998). In fact, both J1119 – 6127 and J1814 – 1744 seem to be ordinary radio pulsars and have no discernible X-ray emission. Complicating matters further, we note that 1814-1744 has spin properties quite similar to the AXP, 1E 2259 + 586. Thus, it seems that the quantum demarcation line outlined above may be a bit fuzzy and it is not clear whether or not a neutron star in a fossil disk with $B_{12} \gtrsim 3.7$ can be observed as a radio pulsar or AXP.

In any case, the thesis of CHN, namely “ordinary” neutron stars can produce AXPs, seems a bit strained at this point.

Another apparent inconsistency with the CHN model is their assumed propeller torque, given by equation (6). We have seen that, for rapid rotators, this model assumes that the neutron star forces propelled material to be ejected from the magnetospheric boundary with the same angular velocity as the star itself – i.e. it must be spun up to corotation. After noting that such a propeller torque is somewhat inconsistent with historical precedent, we realize there is insufficient observational data on the propeller effect to immediately rule it out. The exact nature of the torque is, however, crucial in determining the overall evolution of neutron stars in fossil disks, as was pointed out in section 3. For example, if one assumes material is propelled from the magnetosphere with the angular velocity of a Kepler particle then an AXP will not be observed. Instead, the neutron star will either be observed as a radio pulsar, or in the case of an extremely high magnetic field ($B \gtrsim 10^{14} - 10^{15}$ G), a dim propeller. As a test, we applied the CHN propeller torque to the well-studied HMXB, SMC X-1 and compared it to known models of angular momentum transfer, namely the energy and angular momentum propeller torques. We found that, during the supergiant phase, the CHN model predicts a range of spin periods for SMC X-1 $\sim 2$ orders of magnitude different from the known period. It seems that the propeller torque employed by Chatterjee, Hernquist, & Narayan (2000) is simply too efficient to be consistent with observations. A more robust comparison of propeller torque models would be illuminating. For example, one could examine the evolutionary history of all HMXBs with known spin periods. As there is a wide range of spin periods observed for these systems (especially the Be HMXBs), it is unknown whether or not one propeller model can account for such variety. In any case, our preliminary work seems
to rule out the CHN propeller torque as physically unrealistic and, consequently, strikes a blow against fossil disk models in general.

Ultimately, only observations will definitively determine the true nature of anomalous X-ray pulsars. In particular, optical and/or infrared studies of the X-ray sources could help solve the mystery of AXPs once and for all. A direct detection of disk emission, for example, would strongly support the fossil disk hypothesis. Until now, the poor spatial resolution of X-ray telescopes have made precise positioning of AXPs difficult at best. However, the recently deployed Chandra observatory and Newton XMM mission, with their much improved positioning abilities, should remedy this problem and help facilitate optical identifications. In fact, the first possible optical counterpart to an AXP has recently been identified (Hulleman, Van Kerkwijk, & Kulkarni 2000). The proposed counterpart to 4U 0142 + 61 has a measured flux ratio, $L_{\text{opt}}/L_x \sim 10^{-3}$, an order of magnitude smaller than what has been predicted for fossil disk models (Perna, Hernquist, & Narayan 2000). Although this measurement seems to, initially, lend more credence to the magnetar hypothesis, it should be noted that disk models can not be definitively ruled out either as the emission model of Perna, Hernquist, & Narayan (2000) sensitively depends on some poorly understood parameters, such as disk size.

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Fig. 1.— Time evolution of the pulsar spin period as a function of magnetic field strength for a neutron star accreting from a fossil disk. Assumed initial conditions are $P_0 = 15$ ms, $M_0 = 0.006M_\odot$, and $\Gamma = 7/6$ (see text). The CHN “efficient” propeller torque, given by equation (6), is assumed. We find that a magnetic field strength in the range $\sim 4 - 10 \times 10^{12}$ G is needed for the neutron star to be observed as an AXP. For weaker field strengths, the neutron star cannot spin down efficiently and is observed as a radio pulsar. When $B_{12} \gtrsim 3.9$, a brief emitter phase is followed by an extensive propeller period. The neutron star is observed as an AXP during the “tracking” phase of its evolution, i.e. when $\omega_s \sim 1$. In our model (as in CHN) the neutron star never enters an extended accretion period. The AXP phase is assumed to exist, for the above initial conditions from the transition time between propeller and tracking phases, $t_{\text{track}} \sim 10^4$ yr, to the onset of ADAF, $2t_{\text{ADAF}} \sim 38,000$ yr.
Fig. 2.— Final spin period of a neutron star embedded in a fossil disk as a function of opacity parameter, $\Gamma(\kappa)$ for various magnetic field strengths. An efficient propeller torque is given by equation (6) and the ADAF transition time, $t_{\text{ADAF}} = t_{\text{ADAF}}(\Gamma)$ is given by equation (5). Five period-opacity curves are shown for neutron star fields ranging from $2.5 \times 10^{12}$ G to $4.0 \times 10^{13}$ G. All curves show the same general trend and the existence of a cutoff opacity parameter, $\Gamma_c$. For a given field strength, if $\Gamma \gtrsim \Gamma_c$, the neutron star can not sustain an extended propeller cycle and will, consequently, end its life as a radio pulsar. If electron scattering ($\Gamma_{es} = 1.1875$) dominates in the disk, then the neutron star can be observed as an AXP provided $B_{12} \gtrsim 6.6$. For a Kramers disk opacity $\Gamma_{Kr} = 1.25 \gg \Gamma_c$ for all $B_{12} < 30$. To observe an AXP in a Kramers disk, the minimum field strength necessary is $\sim 3.7 \times 10^{13}$ Gauss. In their analytic model, Chatterjee, Hernquist, & Narayan (2000) assumed $\Gamma = 7/6 \approx 1.167$. 
Fig. 3.— Spin evolution for a neutron star embedded in a fossil disk for three different propeller torques. Initial conditions for the various torques are otherwise fixed: $M_0 = 0.006 M_\odot$, $P_0 = 15$ ms, $B_0 = 7.5 \times 10^{12}$ Gauss, and $\Gamma(\kappa) = 7/6$. Employing a CHN propeller torque yields, after a sharp propeller phase, an AXP in the tracking phase near $t \sim 1.5 \times 10^4$ yr. The neutron star remains at quasi-equilibrium until $2t_{\text{ADAF}} \sim 3.8 \times 10^4$ yr with a final spin period, $13.5$ s. For both the energy (E) and angular momentum (J) propellers, the neutron star never leaves the propeller phase of its evolution and the final spin period is $\sim 0.2$ s. In both cases, the neutron star is observable as a radio pulsar. The early evolutionary deviations for the energy and angular momentum propellers are due to the lack of an emitter phase for the former. At later times, their spin evolution is consistent.
Fig. 4.— Spin evolution models for SMC X-1 assuming various propeller torque models. The progenitor system, assumed to be an O9 V star and a newly born neutron star, have the following initial properties: $M_1 = 20 M_\odot$, $R_1 = 10 R_\odot$, $B_0 = 10^{13}$ G, $P_0 = 15$ ms. Mass is transferred by a radiation-driven wind during the (non-conservative) main sequence phase of the system’s evolution. At $t = 5.6$ Myr, Hydrogen shell burning initiates in the primary, instigating atmospheric Roche-lobe overflow. If an energy propeller is assumed, the calculated spin period during the supergiant phase ranges from 5.3 s, at the onset of the heavy accretion cycle, to 0.99 s at $t = 6.2$ Myr. For the angular momentum propeller, $P_f$ ranges from 2.15 s to 0.52 s. Assuming the efficient propeller torque of Chatterjee, Hernquist, & Narayan (2000) yields a neutron star with a spin period ranging from 15.9 s to $\sim$ 100 s. The observed spin period of SMC X-1 $\approx 0.71$ s is illustrated for reference.