INSTANTONS IN THE QUANTUM FRAMEWORK OF 2D GRAVITY *

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We analyze the non–perturbative features of 2D quantum gravity defined by stochastic regularization of the unstable matrix model showing, first, that the WKB approximation of the well-defined quantum Fokker-Planck hamiltonian corresponds to the semiclassical eigenvalue density of the former. The double scaled potential exhibits an instanton–like behaviour, which is universal and scales, but whose interpretation in terms of pure gravity is still open.

1) INTRODUCTION

In the last year considerable effort has been devoted to solve the non–perturbative ambiguities in the possible definition of 2D quantum gravity through matrix model techniques. While the solution[1] for the $k = 3\,(k$ odd in general) multicritical model is better understood [2], the problem for the $k$ even models is much more complicated and it is still open whether the non–perturbative features of one–matrix models are related to pure gravity at all [3]. The difficulty is related to the instability of the matrix model with potentials unbounded from below, an old problem of euclidean gravity for which the two classical cures have been tried also here. The contour deformation [4] leads to a complex solution, presumably related to the complex Borel sum of the perturbative expansion, whose imaginary part is proportional to the non–perturbative ambiguity obtained from the linearized Painlevé equation and has been interpreted as an instanton effect [6]. The second method is stochastic regularization, where one converts the ill defined $D = 0$ problem into a well defined $D = 1$–like one, and so it is considered by many to be the only realistic hope for a consistent real solution, albeit suspect as a possible source of additional arbitrariness. So far, only numerical computations have been attempted [7], some of them related to the supersymmetry breaking[12], which is a very interesting aspect of the original proposal [5] directly related to the $D = 1$ properties which will not be discussed here.

Most of the numerical results rely on the decoupled N–fermion formulation, which turns out to be a property of potentials of degree $< 4$. We prove here also that a mean field may be introduced in the WKB approximation to decouple the system, so that the semiclassical limit of the Matrix Model is reproduced. This way, the $D = 1$ system becomes an ideal Fermi gas whose spectral density is expressed in terms of the Fokker–Planck potential.

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We are then able to investigate analytically the leading non-perturbative behaviour of the stochastic stabilized quantum gravity, also in the WKB approximation. We find that the double scaled (universal) stabilized Fokker–Planck potential exhibits in the $k - even$ case instanton-like effects, related to the non–Borel summability, and corresponding to the lifetime of a metastable state which is universal and characteristic of the non–perturbative contributions. On the other hand there are no such effects in the $k = 3$ multicritical potentials, as one should expect.

On the light of this result we comment the previous numerical attempts and discuss the leading effect of this tunnelling (tunnel) contribution of the $D = 1 - like$ spectrum in the free energy of the $D = 0$ model, which can have different interpretations.

2) WKB APPROXIMATION AND THE SEMICLASSICAL MATRIX MODEL

Let us consider the $D = 0$ hermitean one-matrix model defined by the partition function

$$ Z = \int D\phi e^{-\beta V(\phi)} $$

(1)

where $\phi$ is an $N \times N$ hermitean matrix, and $V(\phi)$ is a generic potential of degree $L$:

$$ V(\phi) = \sum_{n=2}^{L} g_n Tr(\phi^n) $$

(2)

This model can be understood in terms of the ground state of a $D = 1$ quantum mechanical system, whose Hamiltonian is the (positive semi-definite) Fokker–Planck Hamiltonian [11,8]

$$ H_{FP} = Tr(P^2) + W_{FP}; \quad P_{ij} = -i \frac{\partial}{\partial \phi_{ji}} $$

$$ W_{FP} = \frac{\beta^2}{4} \frac{\partial^2 V}{\partial \phi_{ij} \partial \phi_{ji}} - \frac{\beta}{2} \frac{\partial^2 V}{\partial \phi_{ij} \partial \phi_{ji}} $$

(3)

This means that the expectation value of an operator $Q(\phi)$ in the matrix model can be defined as the vacuum–expectation–value (VEV) of the quantum operator $Q(\hat{\phi})$

$$ \langle Q \rangle \equiv \langle 0 | Q(\hat{\phi}) | 0 \rangle = \int D\phi \Psi_0^2(\phi) Q(\phi) $$

$$ H_{FP}\Psi_0(\phi) = E_0\Psi_0(\phi) $$

(4)

where $E_0$ and $\Psi_0(\phi)$ are the energy and the wave function of the ground state. $E_0 = 0$ corresponds to the case when the potential is bounded from below, the matrix model is well defined, and $\Psi_0(\phi) = \exp(-\beta V(\phi)/2)$; while eq.4, with $E_0 > 0$, defines $\langle Q \rangle$ in terms of the true ground state wave function.

Following well known techniques [9], it is natural to make a change of variables to the eigenvalues $\lambda_i$ of the matrix $\phi$, by introducing the effective ground state wave function

$$ \Psi_{eff}^0(\{\lambda_i\}) = \prod_{i<j} (\lambda_i - \lambda_j) \Psi_0(\phi) $$

(5)

$\Psi_{eff}^0$ is totally antisymmetric, and it describes a gas of $N$ Fermi particles. In general, the Fokker–Planck potential, $W_{FP}$, can be split into its diagonal ($D$) and non–diagonal ($ND$) parts, which after diagonalization of the matrix read

$$ W_{FP} = W_{FP}^{(D)} + W_{FP}^{(ND)} $$

(6)

$$ W_{FP}^{(D)} = \frac{\beta^2}{4} \sum_{i=1}^{N} ((\lambda_i')^2 - 4X(g_2 + \sum_{n=3}^{L} n g_n \lambda_i^{-n-2})) $$

$$ W_{FP}^{(ND)} = -\frac{\beta}{2} \sum_{i,j=1}^{N} \left( \sum_{n=4}^{L} n g_n \sum_{s=0}^{n-4} \lambda_i^{s+1} \lambda_j^{n-3-s} \right) $$

Where $X = N/\beta = e^\gamma$ is related to the (bare) 2D cosmological constant in the usual way. Obviously the $ND$ piece does not vanish in general if $L \geq 4$, and the Fermi gas is not decoupled.
Nevertheless, in the semiclassical WKB approximation ($\beta \approx h^{-1} \to \infty$), a mean field approximation (à la Hartree–Fock) may be performed to decouple the system. We show below that

$$\text{Tr}(\phi^k)\text{Tr}(\phi^p) \approx N \left( \omega_k \text{Tr}(\phi^p) + \omega_p \text{Tr}(\phi^k) - N\omega_k\omega_p \right) + \cdots$$

(7)

where the normalization is fixed by

$$\langle \text{Tr}(\phi^k)\text{Tr}(\phi^p) \rangle_c \approx N^2(\omega_k\omega_p + O(1/N)) ;$$

$$\omega_k = \frac{1}{N} < \text{Tr}(\phi^k) >$$

(8)

is consistent with the semiclassical limit of the matrix model.

Under eq.7, the quantum mechanical system becomes an ideal Fermi gas of $N$ particles, with the hamiltonian

$$H_{FP} \approx \sum_{i=1}^{N} h^{FP}(\lambda_i)$$

$$h^{FP}(\lambda) = -\frac{\partial^2}{\partial \lambda^2} + \frac{\beta^2}{4}U_{FP}(\lambda)$$

(9)

$$U_{FP}(\lambda) = -4X \left( \sum_{i=1}^{L-2} \left( \sum_{j=i+2}^{L} jg_j \omega_{j-i-2} \right) \lambda^i \right) + g_2 - \frac{1}{2} \sum_{i=4}^{L} \sum_{j=2}^{i-2} \omega_{j-1} \omega_{i-j-1} + (V'(\lambda))^2$$

Notice that the potential $U_{FP}$ is bounded from below, and the one-fermion hamiltonian has a well-defined discrete spectrum, $h^{FP}\phi_n(\lambda) = e_n\phi_n(\lambda)$. All the relevant information about the $D = 1$ quantum mechanical model is contained in the density, $\rho(\lambda, e) = \langle \lambda | \delta (h^{FP} - e) | \lambda \rangle$, and its normalization fixes the Fermi energy, $e_F$,

$$N = \int d\lambda \int_{-\infty}^{e_F} de \rho(\lambda, e)$$

(10)

The energy integral of $\rho(\lambda, e)$ provides the quantum mechanical version of the semiclassical density of eigenvalues in the matrix model

$$u(\lambda) = \frac{1}{\beta} \int_{-\infty}^{e_F} de \rho(\lambda, e) ; X = \frac{N}{\beta} = \int d\lambda \; u(\lambda)$$

which, in the WKB approximation, reads

$$u^{WKB}(\lambda) = \frac{1}{2\pi} \sqrt{e_F - U_{FP}(\lambda)} \theta(e_F - U_{FP}(\lambda))$$

$$\frac{1}{\beta} \langle \text{Tr} Q(\phi) \rangle = \int d\lambda \; Q(\lambda) \; u^{WKB}(\lambda)$$

(11)

Let us go back to the $D = 0$ matrix model. We shall use the Schwinger–Dyson loop equations [10] to get a detailed expression for the semiclassical density of eigenvalues, and compare with the WKB approximation of the quantum mechanical system, eq.11. In the semiclassical (planar) limit, the generating function of monomial expectation values,

$$F(p) = \frac{1}{\beta} \langle \text{Tr} \frac{1}{p - \phi} \rangle_c$$

(12)

satisfies the Schwinger–Dyson equation

$$F(p)^2 - V'(p)F(p) + X \sum_{i=0}^{L-2} \left( \sum_{j=i+2}^{L} jg_j \omega_{j-i-2} \right) p^i = 0$$

(13)

where $\omega_k$ are the “constants of integration” of this loop equation, and have already been defined in eq.8. Therefore, eq.13 is solved as

$$F(p) = \int d\lambda \frac{u^{SC}(\lambda)}{p - \lambda}$$

(14)

$$= \frac{1}{2} \left( V'(p) - \sqrt{\Delta(p)} \right)$$

$$\Delta(p) = (V'(p))^2 - 4X \sum_{i=0}^{L-2} \left( \sum_{j=i+2}^{L} jg_j \omega_{j-i-2} \right) p^i$$

The “constants of integration”, $\omega_k$, are fixed by the condition that the imaginary part of $F(p)$ defines a proper semiclassical density of eigenvalues, $u^{SC}(\lambda)$. Obviously, $\Delta(p)$ is a polynomial in $p$ and all the branch cuts of eq.14 will be
squared root branch cuts. Therefore, $\Delta(p)$ has to satisfy the following constraints \([9,4]\): (i) $\Delta(p)$ must have only real zeros in the complex $p$–plane, and (ii) $\Delta(p)$ cannot have three consecutive odd degree zeroes. Under these conditions, the semiclassical density of eigenvalues is given by

$$u^{SC}(\lambda) = \frac{1}{\pi} \text{Im} F(\lambda)$$

$$= \frac{1}{2\pi} \sqrt{-\Delta(\lambda)} \theta(-\Delta(\lambda))$$ (15)

The square root branch cuts of $F(\lambda)$, i.e., the intervals between odd degree (real) zeros of $\Delta(\lambda)$, are the “bands” on which $u^{SC}(\lambda)$ has support.

Therefore, under the above mentioned restrictions, the comparison between eqs.(14), (15) and (9), (11) shows that the WKB limit of the Fokker–Planck hamiltonian, with the mean field approximation of eq.7, reproduces the semiclassical limit of the matrix model. Besides, the Fermi energy can be identified in terms of the $\omega_k$

$$\epsilon_F = g_2 + 3g_3\omega_1 +$$

$$+ \sum_{i=4}^{L} \left( \omega_{i-2} + \frac{1}{2} \sum_{j=2}^{i-2} (\omega_{j-1}\omega_{i-j-1}) \right) i g_i$$

This result particularizes for the $D = 0$ hermitean one–matrix model the general arguments about the stabilization of bottomless euclidean field theories \([13]\). It also agrees with, and generalizes, previous results obtained for the simplest potentials with $L = 3$ \([5,11,8]\) and $L = 4$ \([12]\), showing that the critical behaviour of the Fokker–Planck hamiltonian is precisely that of the matrix model.

Therefore, it is possible to describe the hermitean matrix model like an ideal Fermi gas of $N$ particles, whose one–fermion potential is fixed by the semiclassical density of eigenvalues

$$U_{FP}(\lambda) = \epsilon_F - [2\pi u^{SC}(\lambda)]^2$$ (17)

This relationship formally holds only when the above mentioned constraints on $\Delta = U_{FP} - \epsilon_F$ are satisfied, and $u^{SC}$ is well defined. Nevertheless, the quantum mechanical system, and $u^{WKB}$, is defined even when this is not the case. Such quantum mechanical configurations arise when the naive ground state wave function, $\Psi_0 = \exp(-\beta V/2)$, is not normalizable in the WKB approximation. Therefore, they are related to the true ground state with $E_0 > 0$.

3) CONTRIBUTION OF METASTABLE STATES AND INSTANTONS

Once we have shown that the semiclassical density of eigenvalues is just the WKB density of the Fokker–Planck hamiltonian, we can use the matrix model results to get information about the Fokker–Planck potential $U_{FP}$. In particular, we want to find the features of $U_{FP}$ that are directly related to the critical behaviour and, therefore, which will survive after the double scaling limit.

The main problem of the matrix model for pure gravity is that the susceptibility which should be determined by the Painlevé equation is only given as an asymptotic series, which is non-Borel summable and so the difference between any Borel sum (corresponding to different boundary conditions) and the hypothetical exact solution is proportional to $T^{-\frac{1}{2}} e^{-\frac{1}{4} \sqrt{5} T^{\frac{1}{2}}}$. The exponential factor is an instanton-like effect as expected from the large order behaviour of the series, and it has been analyzed in the semiclassical limit of the matrix model and interpreted as the imaginary part of the complex Borel sum \([6]\), related to the obstruction of real solutions \([4]\), with the conclusion that this matrix model does not define the sum over topologies\([15]\).

The first attempts to understand this problem in the stabilizing Fokker–Planck \([7]\) was the
computation of the $D = 0$ free energy, which was obtained differentiating with respect to a source term added to the Fokker–Planck potential and nothing related to the instability was observed in the numerical results. This motivates further the present attempt to investigate these non-perturbative aspects analytically.

Let us start by computing $\langle F(p) \rangle_c$, eqs.12, 14, using orthogonal polynomials [14] (we restrict ourselves to the case of even potentials):

$$\langle F(p) \rangle_c = \frac{1}{\beta} \langle \theta \frac{1}{p - \phi} \rangle = \frac{1}{\pi} \frac{1}{\beta} \int d\lambda e^{-\beta V(p - \lambda)} P^2_n \equiv \frac{1}{\beta} \sum_{n=0}^{N-1} \frac{1}{p \phi} \phi^n n e^{-\beta \phi} |n\rangle$$

$$\equiv \frac{1}{p \phi} \sum_{n=0}^{N-1} \sum_{j=0}^{N-1} \frac{1}{p \phi} \phi^j |n\rangle$$

$$= \frac{1}{p \phi} \sum_{n=0}^{N-1} \sum_{j=0}^{N-1} \frac{1}{p \phi} \phi^j \langle \hat{p}^j \rangle$$

$$= \frac{1}{p \phi} \sum_{n=0}^{N-1} \sum_{j=0}^{N-1} \frac{1}{p \phi} \phi^j \langle \hat{p}^j \rangle$$

Taking the imaginary part, we get the semiclassical density of eigenvalues [9]

$$u_{SC}(p) = \frac{Im\langle F(p) \rangle_c}{\pi} = \frac{1}{\pi} \int_0^\infty dx \frac{\theta(4R(x) - p^2)}{\sqrt{4R(x) - p^2}}$$

Next, we make the change of variables, $x = W(R(x))$, with $W(R)$ defined in the usual way in terms of the potential

$$W(R) = \int \frac{dz}{2\pi i} V'(z + \frac{R}{z})$$

and the final expression reads

$$u_{SC}(p) = \frac{1}{\pi} \int_0^{R(X)} dR \frac{W'(R)}{\sqrt{4R - p^2}} \theta(4R(X) - p^2) \equiv \frac{1}{2\pi} \sqrt{e_F - U_{FP}(p)} \theta(e_F - U_{FP}(p))$$

As a first result, we see that the semiclassical range for the eigenvalues is given by $p^2 \leq 4R(X)$.

Now we can obtain the critical behaviour of $u_{SC}(p)$ in the double scaling limit. For the k-model it corresponds to #1

$$W(R) = X_c (1 - \gamma(R_c - R)^k)$$

$$X = X_c \left(1 - \beta \frac{1}{4R(X)} \right)$$

$$R = R_c \left(1 - \beta \frac{1}{4R(X)} \right)$$

Therefore,

$$u_{SC}(p) = \frac{2k}{\pi 2e} \gamma X_c \int_0^{\frac{4R(X)}{p^2}} dy \left[\frac{4R(X) - p^2}{4R(X) - p^2} - \frac{1}{3} \left(4R(X) - p^2\right)^{2} \right]$$

This means that the critical behaviour of the Fokker–Planck potential is

$$U_{FP}(\lambda) - e_F = \left(\frac{\gamma X_c}{3}\right)^2 \left(\lambda^2 - 4R(X)\right) \left(6R_c - 2R(X) - \lambda^2\right)$$

$$\approx \left(\frac{\gamma X_c}{3}\right)^2 \left(\lambda^2 - 4R_c\right)^3 - \frac{12\beta^2}{2} \gamma^2 \left(\lambda^2 - 4R_c\right) T + \cdots$$

Now, the leading behaviour of the potential around $\lambda_0 = \pm \sqrt{4R_c}$ when $\beta \rightarrow \infty$ is

$$U_{FP}(\lambda) \approx \pm 16\beta^2 \gamma X_c^2 \left(\frac{4\gamma R_c^3 - T^2}{9}\right)$$

#1 Notice that $X = W(R)$ and, with these general definitions,

$$f(T) = \frac{1}{R_c} \left(\frac{T}{\gamma}\right)^{\gamma^2} + \cdots$$
where $z = \beta \tilde{z} (\lambda - \lambda_0)$. Therefore, the double scaling limit of $U_{FP}$ has two (symmetric) degenerated secondary minima at $z_m^2 = \pm \sqrt{T/(4\gamma R_c)}$, if $T > 0$, and one absolute minimum at $\lambda = 0$.

The perturbative expansion around $\lambda = 0$, the absolute minimum of the potential, corresponds to the semiclassical WKB expansion, which, as it is well known, reproduces the $1/N$ expansion of the matrix model[13]. In this expansion, the subdominant terms in eq.24 are crucial because the normalization condition of having $N \to \infty$ bound states depends also on $\beta$ (in fact, these terms ensure that the potential has bound states!). Nevertheless, there is also a non–vanishing contribution related to the secondary minima which implies the existence of a metastable state, corresponding precisely to the Fermi level, which decays due to barrier penetration, and whose non–perturbative lifetime remains finite in the double scaling limit. Such lifetime is proportional to the inverse of the imaginary part of the, non–perturbative, imaginary part of the energy $E_m^i$ of the lowest eigenvalue that becomes complex.

We assume that the potential has a secondary minimum at $q = q_m$, which obviously requires that the equation $U(q) = U(z_m)$ has, at least, another solution; we shall call $q_0$ the one that is closest to the minimum, assuming $q_0 < q_m$. Therefore, the would–be eigenvalue corresponding to the secondary minimum becomes complex, providing the dominant contribution to $\text{Im} \, \text{tr} (e^{-LH})$. Its imaginary part is given by an instanton, which is a solution of the euclidean equations of motion that starts from the secondary minimum at euclidean time $L = -\infty$, is reflected in $q_0$, and comes back to the minimum at time $L = +\infty$.

The euclidean equation of motion is

$$\frac{1}{2} \ddot{q}_c = \frac{\beta^2}{4} U'(q_c)$$

which has a conserved quantity

$$\dot{q}_c^2 - \beta^2 U(q_c) \equiv -\beta^2 U(q_m) \equiv -E_m^r$$

Therefore, the instanton action is

$$S[q_c] = \int_{-L/2}^{+L/2} dt \left( \frac{1}{4} \dot{q}_c^2(t) + \frac{\beta^2}{4} U(q(t)) \right)$$

$$= LE_m^r + \int_{-L/2}^{+L/2} dt \frac{1}{2} \dot{q}_c^2 \rightarrow LE_m^r + (29)$$

$$+ \int_{q_0}^{q_m} dq \sqrt{\beta^2 (U(q) - U(q_m))} \equiv LE_m^r + S_i$$

The integration around the saddle point corresponding to $q_c$ in the gaussian approximation provides the instanton contribution to the partition function. This calculation involves a Jacobian factor, which becomes complex because of the turning point $q_0$, and is proportional to $L$ because of translational invariance in the euclidean
time. The final result is

\[ Im \; tr(e^{-LH}) \propto L e^{-S[q]} = L e^{-L E^r_m - S_i} \]

\[ \propto Im \; exp(-L(E^r_m + i e^{-S_i})) \] (30)

Therefore, \( e^{-S_i} \) corresponds to the imaginary part of the eigenvalue whose real part is the value of the potential at \( q_m, E = E^r_m + i e^{-S_i} \), which corresponds to a metastable state

Let us particularize this formula to the case of pure gravity, eqs.(9,24), where

\[ U(q) = \beta^{-\frac{2}{5}} v(z) ; \quad v(z) = A z^3 - B T z \]

\[ q = \sqrt{4R_c} + \beta^{-\frac{2}{5}} z \] (31)

and \( E^r_m \) is just the semiclassical value of the Fermi energy. Then, \( S_i \) becomes finite in the double scaling limit (\( \beta \to \infty \)) and the result is

\[ S_i = \frac{12}{5} \sqrt{\frac{B^5}{27A^3}} T^{\frac{5}{2}} \] (32)

Therefore, the lifetime of this metastable state is

\[ \tau \propto e^{-S_i} ; \quad S_i = \frac{4\sqrt{6}}{5} X_c(\gamma R^2_c)^{-\frac{3}{4}} T^{\frac{13}{4}} \] (33)

As mentioned above, no related effect has been observed in the numerical computation [7]. This negative result could be related to the numerical precision. In [7] the the \( D = 0 \) specific heat is computed by introducing a source in the Fokker–Planck potential, \( U_{FP}(\lambda) + J \lambda^2 \). In this way, the derivative of the corresponding free energy with respect to the source provides \( < Tr \; \Phi^2 > \), which is related to the first derivative of the pure gravity free energy, \( < Tr \; \Phi^2 > \propto F' = - < P > \). The consistency of this method requires that the value of \( J \) is small enough to keep the phase structure of pure gravity. Implementing this for the generic even matrix model potential of fourth degree shows [16] that in the double scaling limit, \( \beta \to \infty \), this requires that the source scales, \( J = \beta^{-\frac{2}{5}} j \) and if this scaling is not explicitly performed, the constraint on the source \( J \) fixes the precision to be a function of \( T \) decreasing substantially at small values of \( T \). In particular, for the range explored in [7], to preserve the precision achieved at large \( T \) when \( T \) is small, one should increase the value of \( N \) with a factor of three. This is precisely the region where the instanton-like behaviour should be observed.

Let us finally consider eq.21 for the next multicritical model, \( k = 3 \). Notice that in general, eq.21 only gives the leading terms of \( u_{SC} \) in the double scaling limit, but it is exact when the canonical representative is chosen for the potential. When \( k = 3 \) this means a potential of degree 6. In such case, the result is

\[ u_{SC}(p) \propto (4R(X) - p^2)^{\frac{1}{2}} \left( (p^2 + R(X) - 5R_c)^2 + 5(R(X) - R_c)^2 \right) \] (34)

which means that, in this case, \( U_{FP}(\lambda) \) has only one absolute minimum at \( \lambda = 0 \). Therefore, no metastable states are present in the \( k = 3 \) model, and no imaginary part is expected. This, of course, agrees with the fact that the \( k = 3 \) model is already well defined as a matrix model.

We have been considering non-perturbative effects in the \( D = 1 \) like spectrum of the doubled scaled Fokker–Planck potential which stabilizes the matrix model, whose original Borel–nonsummability is reflected in the appearance of real instantons from the secondary minima of the potential. The problem is how to translate these results into the actual \( D = 0 \) problem. This requires the computation of the non-perturbative contributions to the \( D = 1 \) density, not yet completed, which leaves the interpretation of this instability open at present. On the other hand, one can clearly establish [16] that the tunneling scales properly, remaining finite in the limit and that it involves the instanton action in eq. 33.
4) CONCLUSIONS

We have analyzed the problem of the non-perturbative definition of 2D pure gravity in k-even matrix models with the Fokker–Planck stabilization, discussing other proposals. We have proved that the fermionic formulation, on which previous results were based, is only valid for \( k < 4 \), but that with a mean field in the WKB approximation the generic potential decouples and it is explicitly related to the density of states of the matrix model. We have shown then how to analyze analytically the critical behaviour of the corresponding Fokker–Planck potential performing explicitly the double scaling limit. It turns out that the scaled potential has secondary minima in the relevant \( k \) even case, which exhibit instanton-like behaviour corresponding to the Fermi level becoming complex and which could reflect the non-Borel sumability of the pure gravity series solution of the Painlevé equation. We have discussed how this result can be seen in one of the previous numerical computations and why is not seen in the other. The final interpretation of these instabilities in terms of sum over surfaces, or the pure gravity true vacuum, requires more work as mentioned at the end of last section, which is in progress.

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References

[1] E. Brezin, E. Marinari and G. Parisi, Phys. Lett. 242B (1990) 35
[2] S. Chaudhuri and J. D. Lykken, Fermilab–PUB–90/267–T
[3] J. Ambjørn, Talk at LATTICE–90, NBI–HE–90–76 (Nov. 90)
[4] F. David, Nucl. Phys. B 348 (1991) 507; P. G. Silvestrov and A. S. Yelkhovsky, Phys. Lett. 251B (1990) 525.
[5] E. Marinari and G. Parisi, Phys. Lett. 240B (1990) 375
[6] P. Ginsparg and J. Zinn–Justin, Phys. Lett. 255B (1991) 189.
[7] J. Ambjørn and J. Greensite, Phys. Lett. 254B (1991) 66
[8] J. Miramontes, J. Sanchez–Guillen and M. H. Vozmediano, Phys. Lett. 235B (1991) 38
[9] E. Brezin, C. Itzykson, G. Parisi and J. B. Zuber, Comm. Math. Phys. 59 (19978) 35
[10] F. David, Mod. Phys. Lett. A5 (1990) 1019
[11] J. Ambjørn, J. Greensite and S. Varsted, Phys. Lett. 249B (1990) 411
[12] J. Gonzalez and M. H. Vozmediano, Phys. Lett. 258 (1991) 55
[13] J. Greensite and M. B. Halpern, Nucl. Phys. B 242 (1984) 167
[14] D. J. Gross and A. A. Migdal, Nucl. Phys. B 340 (1990) 333; J. Jurkiewicz, Phys. Lett. 261B (1991) and K. Demeterfi, N. Deo, S. Jain and C. Tan BROWN–HET–764.
[15] J. Zinn–Justin, Quantum Field Theory and Critical Phenomena, Clarendon Press, Oxford 1990 and Proceedings 1990 Cargese Workshop, ed. O. Alvarez, E. Marinari and P. Widney
[16] J. Miramontes and J. Sanchez Guillen, PUPT-11269 and MAD/PH/66 (July 91)