Coulombic effect and renormalization in nuclear pairing

H. Nakada

Department of Physics,
Graduate School of Science, Chiba University
Yayoi-cho 1-33, Inage, Chiba 263-8522, Japan

M. Yamagami

Department of Computer Science and Engineering, University of Aizu
Aizu-Wakamatsu, Fukushima 965-8580, Japan

(Dated: February 15, 2011)

We investigate effects of the Coulomb force on the nuclear pairing properties by performing the Gogny Hartree-Fock-Bogolyubov calculations for the \( N = 20, 28, 50, 82 \) and 126 nuclei. The Coulomb force reduces the proton pair energy and the even-odd mass difference by about 25%, except for nuclei at and around the proton shell or subshell closure. We then propose a renormalization scheme via a reduction factor \( \gamma_p \) for the proton pairing channel. It is found that a single value of \( \gamma_p(= 0.90) \) well takes account of the Coulombic effect, for nuclei covering wide range of the mass number and the neutron excess including the nuclei around the shell or subshell closure.

PACS numbers: 21.60.Jz, 21.30.Fe, 21.10.Sf, 21.10.Dr

Introduction. The energy density functional (EDF) approach (or the mean field approach) provides us with a microscopic framework for describing the static and dynamical properties of atomic nuclei from the nucleonic degrees of freedom \( \frac{1}{1} \). Because of its numerical feasibility, the quasi-local EDF’s (i.e. the EDF’s represented by local densities and currents that include low-order derivatives, such as the Skyrme EDF) \( \frac{2}{2} \) have been applied to calculations covering wide range of the nuclear chart. With the coordinate space representation, the quasi-local EDF’s are suitable for describing various exotic deformations \( \frac{3}{3} \) and continuum effects \( \frac{4}{4} \) of neutron-rich nuclei.

The pairing correlations play a significant role in static and dynamic properties of nuclei at low energy \( \frac{5}{5} \). It is crucially important to construct the pairing channel of EDF (pair-EDF) reproducing the pairing properties across the nuclear chart \( \frac{10}{10} \). In the approaches using the quasi-local EDF’s, the pair-EDF is usually taken to be local, in order to keep the numerical feasibility. In most Skyrme EDF calculations so far, the energy density of the form \( \mathcal{H}_{\text{pair}}(\mathbf{r}) = A_\tau[\rho(\mathbf{r})] \kappa_\tau(\mathbf{r}) \kappa_\tau(\mathbf{r}) \) \( \tau = p, n \) has been adopted for the pair-EDF \( \frac{1}{1}, \frac{11}{11}, \frac{12}{12} \), where \( \rho = \rho_p + \rho_n \) is the isoscalar density of nucleons and \( \kappa_\tau \) the local pair density, by adjusting a few parameters in the function \( A_\tau[\rho] \). It is found that the strength parameter in \( A_\tau \) is substantially stronger than that in \( A_p \) to reproduce the observed pairing properties \( \frac{11}{11}, \frac{12}{12} \). Such asymmetry should originate in the dependence of the pairing on the neutron excess as well as in the Coulomb force which acts only on protons. To include effects of the neutron excess, \( \mathcal{H}_{\text{pair}}(\mathbf{r}) \) has been extended as \( \mathcal{H}_{\text{pair}}(\mathbf{r}) = B_\tau[\rho(\mathbf{r}), \rho_1(\mathbf{r})] \kappa_\tau(\mathbf{r}) \kappa_\tau(\mathbf{r}) \) in Refs. \( \frac{13}{13} \frac{14}{14} \), where \( \rho_1 = \rho_n - \rho_p \), though keeping the charge symmetry.

Although the Coulomb force is an important ingredient of the nuclear systems, the Coulomb force has not explicitly been included in the pair-EDF in most systematic calculations because of its non-local nature. It was reported that the proton pairing gaps are reduced by 20 – 30% if the Coulomb repulsion is treated self-consistently \( \frac{16}{16}, \frac{17}{17} \). It is not likely that the charge symmetric pair-EDF appropriately represents the Coulombic effect. An approximate method to take into account the Coulombic effect with keeping the local nature could be renormalizing the strength parameter of the proton pair-EDF as in Ref. \( 18 \). However, it is not obvious whether such a simple renormalization scheme works sufficiently well. Moreover, an appropriate value of the renormalization parameter and its dependence on \( Z \) and \( N \) have not been known.

In this paper we investigate Coulombic effect on the nuclear pairing by the self-consistent Hartree-Fock-Bogolyubov (HFB) calculations, particularly focusing on the renormalizability. A numerical method that is applicable to wide range of the nuclear chart with a finite-range interaction is required for this purpose. Note that the HFB theory with a finite-range interaction is practically identical to the approach with a non-local EDF. We employ the Gaussian expansion method \( \frac{19}{19}, \frac{20}{20} \), which is adaptable to drip-line nuclei even with finite-range interactions. For both the particle-hole (ph) and particle-
particle (pp) channels, we adopt the Gogny-D1S 21 plus Coulomb interaction with the center-of-mass correction. Although we restrict ourselves to the $N = 20, 28, 50, 82$ and 126 nuclei, assuming the spherical symmetry, they distribute over wide range of the mass number $(30 \leq A \leq 220)$ and the neutron excess $(-0.13 \leq (N-Z)/A \leq 0.36)$. It is numerically examined whether the Coulombic effect on the pairing can be incorporated by a renormalization factor for the proton pairing.

**Hamiltonian.** We here describe the EDF in terms of the effective Hamiltonian. The Hamiltonian for the HFB calculations consists of the nuclear part and the Coulomb interaction,

$$ H = H^N + V^C, \quad (1) $$

where $H^N = K + V^N - H^c.m.$ with the kinetic energy $K$, the effective nuclear interaction $V^N$, and the center-of-mass Hamiltonian $H^c.m.$ $V^N$ may include many-body forces, which are often simplified by a density-dependent two-body force. The HFB energy can be represented in the EDF form owing to Wick’s theorem, though including non-local terms in general. For $V^N$ we adopt the D1S parameter-set of the Gogny interaction in this paper, which can be employed without the energy cut-off for the pairing channel. Since the short-range $NN$ correlation hardly influences matrix elements of $V^C$, we use the bare Coulomb force for $V^C$. The spherical HFB calculations are implemented for the $N = 20, 28, 50, 82$ and 126 nuclei, by applying the Gaussian expansion method 11,20, with the basis functions given in Ref. 22. It is noted that the exchange term of $V^C$ is treated exactly, and that both one- and two-body terms of $H^c.m.$ are subtracted before iteration.

In correspondence to the expression of the HFB energy by the density matrix and the pairing tensor 23, we separate the Hamiltonian into the pp part $H_{pp}$ that gives the pairing tensor and the ph part $H_{ph}$. Each of them is comprised of the nuclear and the Coulomb parts;

$$ H_{ph} = H_{ph}^N + V_{ph}^C, \quad H_{pp} = H_{pp}^N + V_{pp}^C. \quad (2) $$

We consider the pairing between like nucleons as usual, neglecting the proton-neutron pairing, which is not important except $Z \approx N$ cases. $H_{pp}^N$ is then separable into the proton and neutron parts,

$$ H_{pp}^N = H_{pp}^p + H_{pp}^n. \quad (3) $$

The proton pairing should be subject to $H_{pp}^p + V_{pp}^C$. If the Hamiltonian contains only the zero-range interactions, we need only the local limit of the density matrix and the pairing tensor, which leads to a local or quasi-local EDF. However, the interactions have finite range in general, and it is not obvious whether and how the energy of nuclei can be approximated to sufficient precision by the local limit. In particular, whereas there have been validating arguments for $H_{ph}^2 22,24$, local approximation for $H_{pp}$ from which $\gamma^\text{pair}(r)$ is derived has not been well explored. In the HFB calculations of nuclei, we reasonably postulate that $H^N$ is isoscalar. Acting only on protons, $V^C$ breaks the charge symmetry. While the charge symmetry is broken at the Hartree-Fock level, there should also be difference between $H_{pp}$ for protons and for neutrons because of $V_{pp}^N$. $H_{pp}^N$ has often been determined so as to reproduce the observed pairing properties among nuclei 13,14,21,27,26; namely, by using only $H_{pp}^n$. To examine whether effects of $V_{pp}^C$ can be treated in a simple renormalization scheme, we define the following Hamiltonian,

$$ \tilde{H}(\gamma_p) = H_{ph}^N + V_{ph}^C + \gamma_p H_{pp}^p + H_{pp}^n = (H - V_{pp}^C) - (1 - \gamma_p) H_{pp}, \quad (4) $$

dropping $V_{pp}^C$ and introducing the renormalization parameter $\gamma_p$. While the charge symmetry in the pairing channel does not hold because of $V_{pp}^C$ in $H$, $-(1 - \gamma_p) H_{pp}$ gives the charge symmetry breaking in the pairing channel of $\tilde{H}(\gamma_p)$. Note that many HFB calculations so far have employed $\tilde{H}(1)$, by presuming the charge symmetry for $H_{pp}$. The central question here is whether or not we have

$$ \langle H \rangle_H \approx \langle \tilde{H}(\gamma_p) \rangle_{\tilde{H}(\gamma_p)}, \quad (5) $$

with an appropriate $\gamma_p$. We can then renormalize $H_{pp}^N$ via $\gamma_p$ to represent the Coulombic effect. The prescription for the Coulombic effect using $\gamma_p$ has been applied to $^{17}$Ne within a three-body model 27. Since we here carry out self-consistent HFB calculations, the HFB energy at the left-hand side (lhs) is evaluated by a calculation with the full Hamiltonian $H$, while the energy at the right-hand side (rhs) with $\tilde{H}(\gamma_p)$. These Hamiltonians are explicitly written as subscripts in Eq. 5. If the wave functions are similar, we have $(H_{ph}^N + V_{ph}^C + H_{pp}^n) \approx (H_{ph}^N + V_{ph}^C + H_{pp}^n)_{\tilde{H}(\gamma_p)}$ and Eq. 5 therefore indicates

$$ \langle H_{pp}^p + V_{pp}^C \rangle_H \approx \langle \gamma_p H_{pp}^p \rangle_{\tilde{H}(\gamma_p)}, \quad (6) $$

which is further reduced to

$$ \langle V_{pp}^C \rangle_H \approx -(1 - \gamma_p) \langle H_{pp}^p \rangle_{\tilde{H}(\gamma_p)}, \quad (7) $$

via $\langle H_{pp}^p \rangle_H \approx \langle H_{pp}^n \rangle_{\tilde{H}(\gamma_p)}$. The value of $\gamma_p$ may be determined for individual nucleus. However, for the renormalization scheme via $\gamma_p$ to be useful, $\gamma_p$ has to be fixed without referring the result of $H$ for individual nucleus.
It is hence desired that $\gamma_p$ is insensitive to nuclide or expressed by a simple function of $Z$ and $N$. In this work we consider the simplest case that $\gamma_p$ is a constant, with no $Z$ or $N$ dependence.

**Results.** Figures 1 and 2 depict the spherical HFB results of the pair energy (for $Z$ even-odd mass difference) and the even-odd mass difference (for $Z$ odd nuclei) in the $N = 20, 28, 50, 82$ and 126 isotones \[28]. Because the neutron pair energy vanishes in these calculations, the even-odd mass difference is straightforwardly connected to the proton pairing. While several neutron-rich $N = 20$ and 28 nuclei seem to be deformed in reality \[29], it is sufficiently meaningful to compare the results of $H$ and $\tilde{H}(\gamma_p)$ within the spherical HFB calculations. The pair energy $E_p^{\text{pair}} (= E_p)$ is defined by the energy contribution of $E_{pp}^C$ (i.e. $\langle \bar{H}_{pp} + V_{pp}^C \rangle_H$ or $\langle \gamma_p H_{pp} \rangle_H(\gamma_p)$), which is a simple and clear indicator to the pairing. The even-odd mass difference is defined by

$$\Delta_p(Z) = E(Z, N) - \frac{1}{2} \left[ E(Z - 1, N) + E(Z + 1, N) \right].$$

for $Z$ odd nuclei. The HFB energies of the $Z$ odd nuclei are calculated in the equal-filling approximation \[30], which has been shown to work well \[31]. As an observable corresponding to the data, $\Delta_p$ has clear physical meaning.

Let us first compare the $E_p^{\text{pair}}$ and $\Delta_p$ values without $V_{pp}^C$ (i.e. by $\tilde{H}(1)$, green triangles in Figs. 1 and 2) and those of the full Hamiltonian $H$ (red circles). Similar comparison was made in Ref. \[17], although they viewed the pairing gap of the canonical basis locating adjacent to the Fermi energy. It is found that, both for $E_p^{\text{pair}}$ and $\Delta_p$, the ratio of the value of $H$ to that of $\tilde{H}(1)$ is about 75% (Fig. 1 (b,d,f,h,j) and Fig. 2 (b,d,f,h,j)). This result seems consistent with those in Refs. \[16,17]. This ratio is almost stable for the nuclides under consideration except those in the vicinity of $^{34,42}$Si, $^{90}$Zr, $^{146,190}$Gd, $^{208}$Pb and $^{218}$U. $^{208}$Pb is a typical doubly-magic nucleus. The $^{90}$Zr and $^{146}$Gd nuclei are well-known as the proton-subshell-closed ones, reasonably having suppressed $E_p^{\text{pair}}$ and $\Delta_p$. Similar suppression takes place in $^{34,42}$Si, $^{190}$Gd and $^{218}$U in the spherical HFB calculation with the D1S interaction, whereas $^{42}$Si has been suggested to be deformed by experiments \[33]. Because of the subshell nature, the ground states of these nuclei lie around the boundary between the normal fluid and the superfluid phases. Hence the usually perturbative $V_{pp}^C$ affects $E_p^{\text{pair}}$ and $\Delta_p$ to exceptional extent. The same consequence was reported in Ref. \[16] for $^{90}$Zr.

We next apply the Hamiltonian $\tilde{H}(\gamma_p)$ to the self-consistent HFB calculations, adjusting $\gamma_p$ so as to reproduce $E_p^{\text{pair}}$ and $\Delta_p$ obtained from the full Hamiltonian $H$. We find that a single value $\gamma_p = 0.90$ satisfies Eq. (9) to good approximation over all the nuclei in this wide range of $A$ and $(N - Z)/A$, as is clear by comparing the blue squares with the red circles in Figs. 1 and 2. Remark that this is true even for the nuclei around $^{34,42}$Si, $^{90}$Zr, $^{146,190}$Gd, $^{208}$Pb and $^{218}$U, in which the Coulombic effect looks exceptionally strong. It has been confirmed that the difference in the HFB energies between $H$ and $\tilde{H}(0.90)$ is less than 0.1 MeV, indicating that Eq. (5) itself is fulfilled to good precision. As the wave functions of $\tilde{H}(0.90)$ resemble those of $H$, Eq. (7) with $\gamma_p = 0.90$ is good as well. Thus the full Hamiltonian $H$ is well approximated by the renormalized Hamiltonian $\tilde{H}(0.90)$ in
FIG. 2. (Color) Comparison of the even-odd mass difference $\Delta_p$ and of its ratio obtained from the HFB calculations for the (a,b) $N=20$, (c,d) $N=28$, (e,f) $N=50$, (g,h) $N=82$, and (i,j) $N=126$ isotones. See Fig. 1 for conventions. Experimental values taken from Ref. [32] are shown by black short-dashed lines in (a,c,e,g,i).

we have confirmed that Eq. (7), in which the lhs is the long-range Coulomb interaction while the rhs is a short-range nuclear interaction, is well fulfilled for wide range of mass region. This indicates that suppression of the pair correlation by such weak repulsion is not sensitive to interaction form. Therefore the renormalization of the pairing channel with $\gamma_p$ will plausibly be applicable to other interactions or pair-EDF’s including local pair-EDF’s. Moreover, while the value of $\gamma_p$ may somewhat depend on $H_N^{pp}$ or the pair-EDF, it should not largely deviate from 0.90 as far as the pairing has appropriate strength. Although charge-symmetric pair-EDF’s have been assumed in the usual Skyrme EDF approaches [1, 13–15], the charge symmetry in the pair-EDF should be broken because of $V_{C}^{pp}$. It is desirable to readjust the pair-EDF (with $\rho$ and $\rho_1$ dependence) by taking into account this Coulombic effect, for which the renormalization with $\gamma_p(\approx 0.90)$ will be useful.

Summary. We have investigated influence of the Coulomb interaction on the pairing channel in the spherical HFB calculations. Using the Gogny-D1S plus Coulomb interaction for the neutron-closed nuclei of $N=20, 28, 50, 82$ and 126, we have found that the Coulomb interaction reduces the pair energy and the even-odd mass difference by about 25%, except for nuclei around the proton-shell- or subshell-closed ones $^{34,42}$Si, $^{90,146,190}$Zr, $^{208}$Pb and $^{218}$U. Because of the non-local nature, explicit inclusion of the Coulomb force is not adaptable to the local or quasi-local EDF approaches. As a renormalization scheme, we have introduced a reduction factor for the proton pairing channel of the nuclear force (or the pair-EDF), and adjusted the factor to the results with the Coulomb interaction. It is found that the Coulombic effect is approximated with a single renormalization factor $\gamma_p (= 0.90)$ to good precision, all over the nuclei under consideration ranging $30 \leq A \leq 220$ and $-0.13 \leq (N - Z)/A \leq 0.36$, even including the shell- or subshell-closed nuclei.

In the present work we have numerically investigated the Coulombic effect on the pairing channel in the spherical HFB calculations. Using the Gogny-D1S plus Coulomb interaction for the neutron-closed nuclei of $N=20, 28, 50, 82$ and 126, we have found that the Coulomb interaction reduces the pair energy and the even-odd mass difference by about 25%, except for nuclei around the proton-shell- or subshell-closed ones $^{34,42}$Si, $^{90,146,190}$Zr, $^{208}$Pb and $^{218}$U. Because of the non-local nature, explicit inclusion of the Coulomb force is not adaptable to the local or quasi-local EDF approaches. As a renormalization scheme, we have introduced a reduction factor for the proton pairing channel of the nuclear force (or the pair-EDF), and adjusted the factor to the results with the Coulomb interaction. It is found that the Coulombic effect is approximated with a single renormalization factor $\gamma_p (= 0.90)$ to good precision, all over the nuclei under consideration ranging $30 \leq A \leq 220$ and $-0.13 \leq (N - Z)/A \leq 0.36$, even including the shell- or subshell-closed nuclei.

In the present work we have numerically investigated the Coulombic effect on the pairing channel within the HFB framework. However, we also display the experimental data of $\Delta_p$ for reference in Fig. 2, excluding those of the neutron-rich $N=20$ and 28 nuclei that have been indicated to be deformed. The HFB results of $H$ and $\hat{H}(0.90)$ are in better agreement with the data than those of $\hat{H}(1)$ for the $N=82$ and 126 nuclei, but not for the $N=20$ and 28 nuclei. This might imply a room to improve the pairing channel in the D1S interaction or influence of additional correlations.

In the present work all the calculations are implemented by using the Gogny-D1S interaction. However,
[1] M. Bender, P.-H. Heenen and P.-G. Reinhard, Rev. Mod. Phys. 75, 121 (2003).
[2] J. Dobaczewski, B. G. Carlsson and M. Kortelainen, J. Phys. G 37, 075106 (2010).
[3] M. Yamagami, K. Matsuyanagi and M. Matsuo, Nucl. Phys. A693, 579 (2001).
[4] M. Grasso, N. Sandulescu, Nguyen Van Giai and R. J. Liotta, Phys. Rev. C 64, 064321 (2001).
[5] M. Matsuo, Nucl. Phys. A696, 371 (2001).
[6] K. Mizuyama, M. Matsuo and Y. Serizawa, Phys. Rev. C 79, 024313 (2009).
[7] E. Khan, N. Sandulescu, M. Grasso and Nguyen Van Giai, Phys. Rev. C 66, 024309 (2002).
[8] D. M. Brink and R. A. Broglia, Nuclear Superfluidity (Cambridge University Press, Cambridge, 2005).
[9] D. J. Dean and M. Hjorth-Jensen, Rev. Mod. Phys. 75, 607 (2003).
[10] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn and J. Dechargé, Phys. Rev. C 53, 2809 (1996).
[11] F. Tondeur, S. Goriely, J. M. Pearson and M. Onsi, Phys. Rev. C 62, 024308 (2000).
[12] G. F. Bertsch, C. A. Bertulani, W. Nazarewicz, N. Schunck and M. V. Stoitsov, Phys. Rev. C 79, 034306 (2009).
[13] J. Margueron, H. Sagawa and K. Hagino, Phys. Rev. C 76, 064316 (2007); J. Margueron, H. Sagawa and K. Hagino, Phys. Rev. C 77, 054309 (2008).
[14] M. Yamagami and Y. R. Shimizu, Phys. Rev. C 77, 064319 (2008).
[15] M. Yamagami, Y. R. Shimizu and T. Nakatsukasa, Phys. Rev. C 80, 064301 (2009).
[16] M. Anguiano, J. L. Egido and L. M. Robledo, Nucl. Phys. A683, 227 (2001).
[17] T. Lesinski, T. Duquet, K. Bennaceur and J. Meyer, Eur. Phys. J. A 40, 121 (2009).
[18] N. Chamel, S. Goriely and J. M. Pearson Nucl. Phys. A812, 72 (2008).
[19] H. Nakada and M. Sato, Nucl. Phys. A699, 511 (2002); ibid. A714, 696 (2003).
[20] H. Nakada, Nucl. Phys. A764, 117 (2006); ibid. A801, 169 (2008).
[21] J. F. Berger, M. Girod and D. Gogny, Comp. Phys. Comm. 63, 365 (1991).
[22] H. Nakada, Nucl. Phys. A808, 47 (2008).
[23] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).
[24] J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).
[25] H. Nakada, Phys. Rev. C 78, 054301 (2008); ibid. 82, 029902(E) (2010).
[26] H. Nakada, Phys. Rev. C 81, 027301 (2010); ibid. 82, 029903(E) (2010).
[27] T. Oishi, K. Hagino and H. Sagawa, Phys. Rev. C 82, 024315 (2010).
[28] For the calculation of the $N = 126$ nuclei, we adopt the $\ell \leq 8$ single-particle space instead of $\ell \leq 7$ [22]. Influence of the $\ell = 8$ states is confirmed to be insignificant for the lighter nuclei.
[29] O. Sorlin and M.-G. Porquet, Prog. Part. Nucl. Phys. 61, 602 (2008).
[30] S. Perez-Martin and L.M. Robledo, Phys. Rev. C 78, 014304 (2008).
[31] N. Schunck et al., Phys. Rev. C 81, 024316 (2010).
[32] G. Audi, A.H. Wapstra and C. Thibault, Nucl. Phys. A729, 337 (2003).
[33] B. Bastin et al., Phys. Rev. Lett. 99, 022503 (2007).