Where does the $X(5568)$ structure come from?

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\textbf{A B S T R A C T}

We study the semi-exclusive production of $\pi^\pm B_s^0$ pairs in hadron colliders which is associated with the $X(5568)$ structure observed by the D0 Collaboration in 2016, but that was not confirmed by LHCb and CMS later. The reason of its appearance in the D0 and absence in LHCb and CMS is discussed in this letter. In a semi-exclusive process, one might miss the third particle which is produced together with the $\pi^\pm B_s^0$ simultaneously. In the three-body Dalitz plot, once the remaining region is narrow enough after the kinematic cuts, its reflection to another invariant mass distribution will accumulate a large number of events within a specific energy region. If there is an enhancement in the remaining region, it will make the reflection structure more pronounced. The precise line shape of the reflection will depend on the specific interaction form. A combined study of different cone cuts and the low-energy dynamics, e.g. the Landau singularity, demonstrates that the $X(5568)$ structure could come from this kinematic reflection. This conclusion can be checked by both searching for the enhancement in another invariant mass distribution, such as $B_s^0 B_s^0$, and the cone cut dependence of the $X(5568)$ mass. Such a combined study can be used to distinguish the effects of the triangle singularity from a genuine state. We also propose how to avoid this kinematic reflection in future experimental analysis.

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Numerous unexpected particles have been observed in recent years. They are exotic candidates since they cannot fit into model of either $qq$ mesons or $qqq$ baryons with $q$ a generic quark. In 2016, the D0 Collaboration reported a new state $X(5568)$ with four different valence quarks at 5567.8 $\pm$ 2.9 MeV in the $\pi^\pm B_s^0$ channel at $\sqrt{s} = 1.96$ TeV [1]. To suppress the background, the transverse momentum $p_T$ of the $\pi^\pm B_s^0$ system is required to be larger than 10 GeV, and the cone cut\textsuperscript{1} $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} < 0.3$ between the $B_s^0$ and $\pi^\pm$, with $\eta$ the pseudorapidity and $\phi$ the azimuthal angle is also imposed.

Based on the diquark-antidiquark picture, Refs. [2–9] calculated the masses of the potential tetraquark states using either QCD sum rules or the quark model and concluded that the $X(5568)$ can be understood as a tetraquark $[su][bd]$ state. However, an opposite conclusion was obtained in Refs. [10–15] in the same scenario, since the mass of the predicted tetraquark is higher than the observed mass of the $X(5568)$. To further confirm or exclude the tetraquark scenario, measuring other physical quantities in the relevant processes is proposed in Refs. [16–23], such as the decay width of the $X(5568)$, searching for its charmed partner and its neutral partner.

Since the $X(5568)$ is hundreds of MeV below the $B_s^0 \bar{K}$ threshold and is observed in the $B_s \pi$ channel, it could strongly couple to these two channels. One interpretation is that the $X(5568)$ could be a hadronic molecule [24] as an analogue of the $D\bar{K}$ hadronic molecule $D_s^*\bar{D}(2317)$. Although the $X(5568)$ structure from D0 could be described by a pole stemming from the $B_s \pi - B\bar{K}$ coupled channel interaction, its interpretation as a resonance is questionable due to the unusually large cutoff $\Lambda$ required to describe the experimental spectrum [25]. On the other hand, such a scenario was also questioned by the authors in Refs. [26,27], as the difference between the mass of $X(5568)$ and the $B\bar{K}$ threshold is too large and it is not easy to form such a deeply bound state. The difference is even larger than that between the mass of the $D_s^*(2317)$ and the $D\bar{K}$ threshold. It contradicts the expectation that the hyperfine splitting in the bottom sector should be smaller than that in the charm sector. Furthermore, the detailed calculations using the chiral unitary approach and lattice simulations [28–30] confirmed the inconsistency of both the $X(5568)$ and the $D_s^*(2317)$ as hadronic molecules. Even after enlarging the channel basis to the $B_s \pi$, $B_s^* \pi$, $B\bar{K}$ and $B^*\bar{K}$ [31] channels, the calculation still dis-

\textsuperscript{1} A cone cut is used to select relevant events within a given cone angle in the laboratory frame of the experiment.

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favors the $X(5568)$ to be a hadronic molecule. The near-threshold behavior also indicates that the structure might come from the triangle singularity in the meson loop as discussed in Ref. [32].

However, an alternative opinion [26,27] is that all these interpretations, such as tetraquark, hadronic molecule, threshold effect from the meson loop, and so on, cannot give a consistent explanation of the $X(5568)$ structure. Due to the inconsistency of the interpretations in both the tetraquark and hadronic molecular scenarios, the authors of Refs. [33,34] claim that the state might originate from a mixing of these two scenarios.

The analyses from both LHCb [35] and CMS [36] do not confirm the existence of the $X(5568)$ structure and set an upper limit on the production rate of the $X(5568)$ state in pp collisions. In the analysis of LHCb, they only impose the requirement of $p_T(B^0_s)$ being greater than 5 GeV, 10 GeV and 15 GeV but smaller than 50 GeV. In their baseline selection, CMS implements $p_T(B^0_s) > 10$ GeV and $p_T(\pi^\pm) > 0.5$ GeV cuts. They also investigate the effect of different $p_T(B^0_s\pi^\pm)$ cuts, finding no significant signal at the claimed mass. Separately, to investigate the effect of the cone cut, they also performed their analysis with the upper limit of the cone cut at 0.4, 0.3, 0.2 and 0.1, respectively, and claim that the cone cut cannot be used in the analysis since it can stimulate a peak shape and could enhance the significance of statistical fluctuations in the data.

No matter whether the structure exists or not, it has been attracting a lot of attention from both theoretical and experimental sides. In this letter, we explain why the $X(5568)$ structure is only observed by D0 and might have generated some effects from the kinematic reflection in the CMS analysis. If the structure really comes from the dynamics in the $\pi^+B^0_s$ channel, either from a genuine state or a singularity in the $\pi^+B^0_s$ [32] channel, the peak should be always there and stable no matter the cut is implemented or not. Its absence in the analysis of LHCb without cone cut has already indicated that it is not from the dynamics in the $\pi^+B^0_s$ channel. We demonstrate that it could be coming from a kinematic reflection. The key point is that, for the scattering from two particles to an n-body final state, there are $3n-4$ independent Mandelstam variables. On the $3n-4$ dimensional surface, once an enhancement in one dimension is cut by the experimental analysis, its projection to another dimension could lead to an accumulation of events within a specific energy region.

In the following, we use the semi-exclusive production of the $\pi^+B^0_s$ associated with the third particle, such as $B^0_s$, as an example to illustrate how the mechanism works. The scattering process of two particles to the n-body final state can be parameterized as a quasi-$j+1$-body process, with $j$ the number of the exclusive particles, if the dynamics only depends on the invariant mass of the other $n-j$ particles within the energy region of interest. Therefore, although the quasi-4-body final state, cf. Fig. 1, is used to illustrate the problem, the conclusion is general, because the other hard process can be viewed as a background contribution.

We only consider that all the final valence quarks and antiquarks come from the primary vertex. As a result, the incoming "1" and "2" particles could be baryon and antibaryon. Since there are $u$, $b$ quarks and $d$, $\bar{s}$ antiquarks in the final $\pi^+$ and $B^0_s$, the incoming baryon and antibaryon can be either $uud$ ($\Lambda_b$) and $d\bar{s}d$ ($\Sigma^+$), with a $\pi^+$ the third undetected particle or $uub$ ($\Lambda_b$) and $d\bar{s}b$ ($\Sigma^{*+}$), with a $B^0$ the third undetected particle. However, the widths of the light antibaryons $\Sigma^+$ and the exchanged $\Sigma^*$ are hundreds of MeV which cannot produce narrow structures even if the condition of the triangle singularity is satisfied. Consequently, only the double heavy baryon loop could give a significant peak structure. In what follows, we only consider the $\Sigma^{*+}(5955)$, $\Lambda^0_b(5920)$, $\Xi^0_b(5912)$ loop denoted in Fig. 1 as an example. The other double heavy baryon loops have a similar behavior such as the $\Xi^{*+}(5955)$, $\Lambda^0_b(5912)$, $\Xi^0_b(5912)$.
in Fig. 2, when \( m_3 \) has the proper mass, the singularity can happen within a specific region for the incoming energy. Since the energy of the semi-exclusive production varies in a large region, the larger the singularity region of the incoming invariant mass \( M_{abc} \), the more important the loop is. When the incoming \( M_{abc} \) is smaller than the threshold \( M_{abc}^{\text{min}} = m_1 + m_3 \) or larger than the upper limit \( M_{abc}^{\text{max}} \), the singularity condition cannot be satisfied. As shown in Fig. 2, the upper limit is the cross point of \( m_1 = m_{\text{min}} \) and \( m_3 = m_0 + m_c - m_2 \), which satisfies the equation

\[
(M_{abc}^{\text{max}})^2 = m_1^2 + m_2^2 - 2m_2(m_0 + m_c - m_2) = (m_1^2 + m_2^2 - M_{abc}^{\text{max}2})^2 - 4m_2^2(m_1^2 - m_2^2). 
\]

(1)

One can avoid the signal from the triangle singularity and its reflection by using the events outside the energy region \([M_{abc}^{\text{min}}, M_{abc}^{\text{max}}])\).

Our discussion in the following will be based on the factorization of the phase space integral of the full process \( p\bar{p} \to abc + \text{all} \) into two processes, i.e. the \( p\bar{p} \to M_{abc} + \text{all scattering process and the } M_{abc} \text{ decay to } a, b, c \),

\[
\frac{d\sigma(p\bar{p}\rightarrow abc+all)}{dM_{abc}}=\frac{2M_{abc}d\sigma(p\bar{p}\rightarrow M_{abc}+all)}{(2\pi)^4} \times d\Gamma(M_{abc}\rightarrow abc)\delta^4(p_{abc}-p_0-p_b-p_c)dM_{abc}.
\]

(2)

assuming that there is no interaction between “a”, “b”, “c” and the other inclusive particles. For a given \( M_{abc} \), the second process in Eq. (2) only depends on the first one through the implicit integration variable \( p_{abc} \) in \( d\sigma(p\bar{p}\rightarrow M_{abc}+all) \). The \( p_{abc} \) dependence can be obtained by using the event generator PYTHIA [42] and has been integrated out. We use the VEGAS program [43] to integrate the kinematic phase space generated by RAMBO [44] and the dynamic three-point loop [45]. The \( M_{\pi^+B_0^\pm} \) distributions with \( p_T(\pi^+B_0^\pm) > 10 \text{ GeV} \) and the cone cuts, \( \Delta R < 0.4 \), \( \Delta R < 0.3 \), \( \Delta R < 0.2 \), \( \Delta R < 0.1 \) are shown in Fig. 3. There are always clear peak structures near the \( X(5568) \) with and without cone cuts. When the cone cut becomes smaller, the peak structure will move to lower energy and vice versa. The invariant mass distributions in Fig. 3 should not be compared with the experimental data from

\footnote{The lower limit \( m_{\text{min}} \) of \( m_1 \) can be found in Ref. [40].}

D0, as the latter one also includes other contributions and a background subtraction was performed.

This kind of behavior can be easily understood by the cone cut influence on the Dalitz plot in Fig. 4. Once the cone cut is implemented, some of the events at the lower right-hand-side will be cut off. Even if there is no singularity enhancement in another dimension, the reflection of the narrow upper left corner to the \( M_{\pi^+B_0^\pm} \) invariant mass distribution could also be pronounced, if the cone cut is small enough, e.g. \( \Delta R < 0.1 \) of Fig. 3 in Ref. [36]. When the cone cut becomes smaller, the cut region will move to smaller \( M_{\pi^+B_0^\pm} \). Therefore, the reflection moves to lower energy. For a fixed \( |p_{abc}| \), the larger it is, the weaker the cone cut dependence of the \( X(5568) \) peak structure will be. That makes the cone cut dependence after integration smaller than that before integration. If the structure comes from a genuine state which can decay into \( \pi^+B_0^\pm \) or a singularity in the \( \pi^+B_0^\pm \) channel, the peak position should not depend on the cone cut.

The cone cut dependence of the mass of \( X(5568) \) is similar to what has been observed by the D0 Collaboration, see the supplemental material of Ref. [1]. This is an evidence that the \( X(5568) \) is not a genuine state but a kinematic reflection from other invariant mass distributions, such as \( B_0^\pm \bar{B}_0^\pm \), due to the third undetected particle which is produced associated with the \( \pi^+B_0^\pm \). In high energy hadron collision, since the gluon is dominant in the parton distribution functions of both \( p \) and \( \bar{p} \), the dynamics in \( pp \) and \( pp \) collisions should be similar. However, because the center-of-mass energy of LHCb and CMS is about four times as that of D0, both \( p_{abc} \) and \( M_{abc} \) distributions of the production of the double heavy baryons are much broader than that of D0. It makes that the signal from the kinematic reflection might be weakened by the large number of events at higher \( M_{\pi^+B_0^\pm} \). This is the reason why there is no \( X(5568) \) structure in the analyses of both LHCb and CMS. In addition, the positions of the maximum values of the \( B_0^\pm \pi^\pm \) distributions have much larger cone cut dependence in CMS than that in D0. It is because that they do not implement a \( p_T(\pi^+B_0^\pm) \) cut at the same time which is also the reason that the structure with the same cone cut in CMS is broader than that in D0. In this case, the lowest value of \( |p_{abc}| \) in CMS is smaller than that in D0. Thus, the \( M_{\pi^+B_0^\pm} \) invariant mass distribution in CMS is more sensitive to the cone cut.
One might expect that the narrow structure could also come from
the reflection of a resonance, such as the $M_{abc} \to \pi^+ \tau^+(5S)\to
\pi^+ \eta(588)^0 \eta(588)^0$ and $M_{abc} \to \pi^+ \rho_{12}^0 \to \pi^+ \eta(588)^0 \eta(588)^0$ processes. However, the enhancement is always there and stable in the $B_{u}^{0} \to \eta^{0} \eta^{0}$ and $B_{d}^{0} \to \eta^{0} \eta^{0}$ invariant mass distributions, if phase space allows. When $M_{abc}$ increases, the overlap between the resonance and the Dalitz plot (or its cut region) varies smoothly. Therefore, the kinematic reflection from a resonance would not show up as a pronounced structure. As a byproduct, one can distinguish a genuine state from the triangle singularity by looking at the $M_{abc}$ dependence of its reflection, i.e. a drastic change within a small $M_{abc}$ region means that there is a triangle singularity.

The quest of hunting for the true origin of the $X(5568)$ is important
due to the discrepancies among the different experiments and
between theoretical expectations and the measurement by D0. In this work, we have demonstrated that:

- The $X(5568)$ could be a kinematic reflection from the singularity in another dimension of the Dalitz plot, such as the singularity in the $B_{d}^{0} \to \eta^{0} \eta^{0}$ invariant mass distribution. If this is the case, the mass of the $X(5568)$ decreases when the cone cut becomes smaller, which is similar to the observation made by D0.
- Since the larger center-of-mass energy in LHCb and CMS leads to an accumulation of events at higher $M_{\pi^+ \eta^0}$, larger than that of D0, this narrow reflection structure could be diminished.
- Whether the cone cut dependence of the reflection is large or not is determined by the $p_T$ cut, i.e. a larger $p_T$ cut makes the reflection less sensitive to the cone cut.

Although all our arguments have been obtained from considering
the three exclusive particle process and the specific double heavy
baryon loop $\bar{B}_{h}^{+}(5955)\lambda_{q}^{(5920)}(\bar{B}_{d}^{0})$, the conclusions are more general. The final measurements in experiment should be the sum of all the possible reflections in the multi-dimension space.

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