Robust Radar Detection of a Mismatched Steering Vector Embedded in Compound Gaussian Clutter

Mai P. T. Nguyen and Iickho Song, Fellow, IEEE

Abstract—The problem of radar detection in compound Gaussian clutter when a radar signature is not completely known has not been considered yet and is addressed in this paper. We proposed a robust technique to detect, based on the generalized likelihood ratio test, a point-like target embedded in compound Gaussian clutter. Employing an array of antennas, we assume that the actual steering vector departs from the nominal one, but lies in a known interval. The detection is then secured by employing a semi-definite programming. It is confirmed via simulation that the proposed detector experiences a negligible detection loss compared to an adaptive normalized matched filter in a perfectly matched case, but outperforms in cases of mismatched signal. Remarkably, the proposed detector possesses constant false alarm rate with respect to the clutter covariance matrix.

Index Terms—Generalized likelihood ratio test, compound Gaussian clutter, semi-definite programming

I. INTRODUCTION

The problem of radar detection in Gaussian clutter has been addressed in the pioneering work [1]. Therein, the presence of a point-like target was sought in a single vector of the form $bs$, where $b$ was an unknown complex scalar accounting for the combined effect of a target’s reflectivity and channel propagation and $s$, representing the radar signature, was perfectly known. It was reported that the detector in [1] suffers a performance loss when the actual radar signature departs from its nominal one, for example cases of an imperfect array calibration.

To increase detection probability when a mismatch occurs, detecting $bs$ where $s$ is not completely known but lies in an assumed range has been proposed. Such a range where $s$ lies in could be possibly modelled as a known linear subspace or a cone with axis the nominal radar signature. Subspace detectors, based on the former approach, have been proposed in [2]–[9]. Coordinates of the signal to detect are unknown; detection is performed by computing energy of the measurement in the signal subspace [2]. However, there is no guidance on choosing an appropriate subspace to which a signal of interest belongs. The latter approach circumvents this drawback by assuming a nominal radar signature as axis of a cone to which a signal of interest belongs to [10]. Cone class based detectors have been proposed in [11]–[14]: In most cases, likelihood ratios are obtained by numerical methods, hence it is difficult to explain and investigate the detection nature and performance.

In the above mentioned research, the radar clutter is modelled as a Gaussian process, whereas in many circumstances, for instance under a low aspect angle, radar clutter is better characterized as a spherically invariant random process (SIRP) (compound Gaussian process) [15][16]. Briefly, SIRP is a Gaussian process $g(t)$ (called speckle) modulated by a temporally and spatially “more-slowly varying” non negative random process $s(t)$ (called texture), which is independent of $g(t)$ and represents the illumination patch’s reflectivity. Problems of detecting a perfectly known radar signature in compound Gaussian clutter have been addressed in [17]–[29], where detectors are called normalized matched filters. The problem of detecting in compound Gaussian clutter a mismatched signal, a possibility in some practical cases, has been not considered yet and is solved in this paper.

We addressed, based on the generalized likelihood ratio test (GLRT), the problem of detecting a point-like target embedded in compound Gaussian clutter, employing an uniform array of antennas. Here, the radar signature is the steering vector that, due to some reason, departs from the nominal one. The maximum likelihood estimate (MLE) of the unknown steering vector lying in a cone then leads to a fractional quadratically constrained quadratic optimization problem, which is not easy to solve [30]. We hence introduced a more specific constraint on the mismatched steering vector: phase shifting of the mismatched steering vector lying in a known range. A practical example demonstrating the rationale of this assumption is a case...
of inaccurate estimate of an arrival angle. The optimization problem associated to the mismatched steering vector estimate then can be transferred in a form solvable via a semi–definite programming (SDP) [31]. In case of a perfect match, the proposed detector provides a comparable detection probability with that of a normalized matched filter. In the presence of a mismatch, the proposed detector outperforms a normalized matched filter even with a slight mismatch. Additionally, with a maximum mismatch lying in the range designed numerical results showed a detection loss of around 3 dB. Factors that affect the proposed detector’s performance have been investigated. Remarkably, the proposed detector possess CFAR w.r.t the clutter covariance matrix.

The rest of the manuscript is organized as follows. The problem formulation is stated and the proposed detector is derived in Section II. Numerical results are represented in section III. Finally, conclusion is reported in section IV.

Notation: We adopt the notation of using boldface lower case and upper case for vectors and matrices, respectively. The transpose and complex conjugate transpose of a matrix are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. For a square matrix, $\text{tr}(\cdot)$, $|\cdot|$, and $\text{rank}(\cdot)$ respectively stand for its trace, determinant, and rank. $\text{diag}(A)$ denotes a vector whose $i$-th element is the $i$-th diagonal element of a matrix $A$; while $\text{diag}(a)$ denotes a diagonal matrix whose diagonal elements are elements of a vector $a$. $(\cdot)^{-1}$ represents the inverse of an invertible square matrix. $\odot$ denotes the Hadamard product. $A_{nm}$ denotes the element at $n$-th row, $m$-th column of a matrix $A$. $\mathbb{C}^{N\times N}$, $\mathbb{R}^{N\times N}$, and $\mathbb{H}^{N\times N}$ stand for the set of $N \times N$ complex, $N \times N$ real, and $N \times N$ Hermitian matrices, respectively. For any $A \in \mathbb{H}^{N\times N}$, $A \succeq 0$ means that $A$ is a positive semi-definite matrix. $\mathbb{R}^+$ is the set of non-negative real numbers. The real part of a complex scalar, vector, or matrix is represented by $\Re\{\cdot\}$. $|\cdot|$ is the Euclidean norm of a vector, and $|\cdot|$ denotes the modulus of a complex number. Finally, the letter $j$ represents the square root of $-1$ and $E[\cdot]$ denotes a statistical expectation.

II. PROBLEM FORMULATION AND PROPOSED DETECTOR

A. Problem Formulation

Consider a problem of detecting the presence of a point-like target using an uniform linear array of $N$ antennas. For the detection, reflection is collected at the cell under test (CUT) and surrounding range cells, in which data is assumed to compose of only noise and referred to as secondary data. Target’s return in an equivalent baseband form is represented as $\alpha p$, where $\alpha$ is a complex scalar accounting for the combined effect of a target’s reflection and channel propagation and $p$ is an $N \times 1$ steering vector departing from the nominal steering vector $s$, i.e., $s = [1, \exp(j\theta), \ldots, \exp(j(N - 1)\theta)]^T$ and $p = [1, \exp(j\phi), \ldots, \exp(j(N - 1)\phi)]^T$, where $(\theta - \phi)$ is unknown but $(\theta - \phi) \in [-\beta, \beta]$ with $\beta$ is a known quantity expressing the discrepancy of $p$ from $s$. Clutter at a range cell $c_t$ is modelled as a complex SIRP with index $t = 0$ indicating the CUT and $t = 1, 2, \ldots, K$ indicating surrounding range cells. From the definition of a complex SIRP [15], we have

$$c_t = s_t \cdot g_t, \quad t = 0, 1, 2, \ldots, K. \quad (1)$$

where $s_t$, the texture component of $c_t$, is a real non-negative random variable with some distribution $f_s(s_t)$ and $g_t$, the speckle component of $c_t$, is an $N \times 1$ vector of zero mean and multivariate complex normal distribution with normalized covariance matrix $M$, i.e., $\text{tr} \, M = N$. We also assume that $\{g_0, g_1, \ldots, g_K\}$ are independent identically distributed and possess the circular symmetric property. Note that $s_t$ and $g_t$ are independent, and we can consider $s_t$ as, as in this paper, unknown deterministic parameters [24]. This assumption then leads to the independence of clutter at all range cells.

The detection problem can now be stated as a problem of binary hypotheses

$$\begin{cases} H_0 : z = c_0, \\ H_1 : z = \alpha p + c_0, \end{cases} \quad (2)$$

where the null hypothesis $H_0$ and alternative hypothesis $H_1$ denote the cases of clutter-only and signal plus clutter, respectively, and $z$ denotes the equivalent baseband of received signal at the CUT. The pdf of the observed data $z$ can be expressed as

$$f_0(z) = \frac{1}{\pi N^2 s_0^2} \exp \left\{ -z^H \frac{1}{s_0^2} C^{-1} z \right\} \quad (3)$$
and

\[
f_1 (z) = \frac{1}{\pi^N s_0^{2N} |C|} \exp \left\{ - (z - \alpha p)^H \frac{1}{s_0} C^{-1} (z - \alpha p) \right\},
\]

where \( C = E[c_sc_s^H] \) the covariance matrix of radar clutter. It’s easy to see that \( C = E[s^2] \times M \). To maximise the detection probability given a predetermined false alarm rate, we employ the Neyman-Pearson criterion. Due to the ignorance of the clutter covariance \( C \), texture \( s_0 \), the steering vector \( p \), and \( \alpha \), we resort to a GLRT scheme, replacing these nuisance parameters with their MLEs under each hypothesis

\[
\max_p \max_{\alpha} \max_{s_0} \max_C f_1 (z) \quad \frac{H_1}{H_0} \geq G_2, \tag{5}
\]

where \( G_2 \) is a threshold set for a predetermined false alarm rate. For detection, the next logical step is to replace \( C \) with its MLE. However, it is proved in [20] that a closed-form of the MLE of covariance matrix \( C \) does not exist. Hence, we assumed known \( C \) in the following development and derive a detector. Later, \( C \) is replaced by some estimators and properties of the resulting detectors will be discussed.

**B. Detection with known structure of covariance matrix \( C \)**

MLEs of the texture components under each hypothesis are given as in [22]

\[
H_0 : s_0^2 = \frac{1}{N} z^H C^{-1} z, \tag{6}
\]

\[
H_1 : s_0^2 = \frac{1}{N} (z - \alpha p)^H C^{-1} (z - \alpha p). \tag{7}
\]

Direct substitution of the MLEs of \( s_0 \) into (5) leads to

\[
\min_{\alpha} \min_p \left\{ (z - \alpha p)^H C^{-1} (z - \alpha p) \right\}^{N} \quad \frac{H_1}{H_0} \geq G_1, \tag{8}
\]

where \( G_1 \) is a suitable modification of \( G_2 \). We then proceed by replacing \( \alpha \) with its MLE, which is [17]

\[
\alpha = \frac{p^H C^{-1} z}{p^H C^{-1} p}. \tag{9}
\]

into (8). After some manipulations, the likelihood ratio is recast as

\[
\max_p \frac{|z^H C^{-1} p|^2}{(z^H C^{-1} z) (p^H C^{-1} p)} \quad \frac{H_1}{H_0} \geq G, \tag{10}
\]

where \( G \) is a suitable modification of \( G_1 \). It is easy to see, by using the Schwatz’s inequality, that \( G \in [0,1] \). Also, the test in (10) does not change if we substitute \( C \) with \( M \). In addition, if \( p \) is known, (10) becomes the likelihood ratio of the detector proposed in [17],

\[
\frac{|z^H M^{-1} p|^2}{(z^H M^{-1} z) (p^H M^{-1} p)} \quad \frac{H_1}{H_0} \geq G. \tag{11}
\]

The detector in [17] (11) is referred to as the normalized matched filter (NMF) with known \( M \) and as an adaptive NMF (ANMF) with an estimated \( M \). It is worth noting that (11) was derived in [17] to detect a coherent pulse trains with the number of pulses goes to infinity, given that \( s_0 \) was a random variable of a well-behaved distribution.

Now, we have to solve the maximization problem in (10) w.r.t \( p \) before proceeding to a decision on a target’s presence. From the observation that the expression to be maximized depends only on \( \phi \), we rewrite (10) as

\[
\frac{1}{z^H C^{-1} z} \left[ \max_{\phi} F(\phi) \right] \quad \frac{H_1}{H_0} \geq G_2, \tag{12}
\]

where \( F(\phi) = \frac{p^H C^{-1} z z^H C^{-1} p}{p^H C^{-1} p} \). Note that \( F(\phi) \) is always non-negative since \( C^{-1} \) and \( z z^H C^{-1} \) are semi-definite. Expanding \( F(\phi) \) in terms of \( \phi \) and using \( \exp(-jk\phi) = (\cos \phi - j \sin \phi)^k \), we have

\[
F(\phi) = \frac{x_0 + 2\Re\sum_{k=1}^{N-1} x_k \exp(-jk\phi)}{y_0 + 2\Re\sum_{k=1}^{N-1} y_k \exp(-jk\phi)}, \tag{13}
\]

with

\[
x_0 = \text{tr} \left( C^{-1} z z^H C^{-1} \right), \tag{14}
\]

\[
y_0 = \text{tr} \left( C^{-1} \right), \tag{15}
\]

\[
x_k = \sum_{m-n=k} \left( C^{-1} z z^H C^{-1} \right)_{nm}, \tag{16}
\]

\[
y_k = \sum_{m-n=k} \left( C^{-1} \right)_{nm}, \tag{17}
\]

and \( k = 1, 2, \ldots, N - 1 \). Note that \( x_0 \) and \( y_0 \) are real since \( C^{-1} z z^H C^{-1} \) and \( C^{-1} \) are Hermitian.
observe that finding the maximum w.r.t. \( \phi \) of \( F(\phi) \) in (13) is not straightforward since the numerator and denominator are polynomials of at most \((N-1)\)-th degree in \( \cos \phi \) and \( \sin \phi \). We then solve the maximization here by a numerical method. Firstly, denote by \( t \) the maximum value of \( F(\phi) \), then \( t \) is the lowest upper bound of \( F(\phi) \), i.e., \( t \) is the solution of the optimization problem

\[
\begin{align*}
\text{minimize} & \quad t \\
\text{such that} & \quad g(\phi, t) \triangleq \exp(\sum_{k=1}^{N-1}(ty_k - x_k)\exp(-jk\phi)) \\
& \quad \phi \in [\theta - \beta, \theta + \beta].
\end{align*}
\]

where

\[
g(\phi, t) \triangleq t y_0 - x_0 + 2\Re\left\{ \sum_{k=1}^{N-1} (ty_k - x_k) \exp(-jk\phi) \right\}
\]

We have another observation that \( g(\phi, t) \) is a real non-negative trigonometric polynomial over the interval \([\theta - \beta, \theta + \beta]\), so coefficients of \( g(\phi, t) \) follow the following theorem [37].

**Definition II.1.** Let \( W_{DFT} \in \mathbb{C}^{M \times M} \) be the DFT matrix

\[
W_{DFT} = [w_0, w_1, \ldots, w_{M-1}],
\]

where \( w_k \equiv [1, \exp(-jk2\pi/M), \ldots, \exp(-j(M-1)k2\pi/M)]^T \).

We define \( W \) and \( W_1 \) as matrices composed of the first \( N \) and \( N-1 \) columns of \( W_{DFT} \), respectively.

**Theorem 1.** Let \( p(\phi) \) be a trigonometric polynomial in \( \phi \) with degree \((N-1)\) or less, and have the form

\[
p(\phi) = q_0 + 2\Re\left\{ \sum_{k=1}^{N-1} q_k \exp(-jk\phi) \right\},
\]

with \( q = [q_0, q_1, q_2, \ldots, q_{N-1}] \in \mathbb{R} \times \mathbb{C}^{N-1} \), \( p(\phi) \) is non-negative on \([\theta - \beta, \theta + \beta]\) if and only if there exist \( X_1 \in \mathbb{H}^{N \times N} \) and \( X_2 \in \mathbb{H}^{(N-1) \times (N-1)} \) so that \( q = W^H \left[ \text{diag} \left( W X_1 W^H \right) + d \circ \text{diag} \left( W_1 X_2 W_1^H \right) \right] \), where \( X_1 \succeq 0, X_2 \succeq 0, d \in \mathbb{M}^{N \times 1} \) has elements \( d_k = \cos(2\pi k/M - \theta) - \cos \beta \) for \( k = 0, 1, \ldots, M-1 \) and \( M \geq 2N - 1 \).

For a detailed explanation of the idea underlying the above theorem as well as its applications, interest readers may refer [31]. Applying the theorem and denoting \( y = [y_0, y_1, \ldots, y_{N-1}]^T \) and \( x = [x_0, x_1, \ldots, x_{N-1}]^T \), with \( x_i, y_i, i = 0, 1, \ldots, N-1 \) are computed as in (14)–(17), we have \( ty - x = W^H \left[ \text{diag} \left( W X_1 W^H \right) + d \circ \text{diag} \left( W_1 X_2 W_1^H \right) \right] \).

The minimization (18) is now recast as a SDP minimize \( t \)

\[
\begin{align*}
\text{s.t.} & \quad ty - x = W^H \left[ \text{diag} \left( W X_1 W^H \right) + d \circ \text{diag} \left( W_1 X_2 W_1^H \right) \right] \\
& \quad t \in \mathbb{R}^+ \\
& \quad X_1 \succeq 0, X_1 \in \mathbb{H}^{N \times N} \\
& \quad X_2 \succeq 0, X_2 \in \mathbb{H}^{(N-1) \times (N-1)}.
\end{align*}
\]

The SDP above can be solved efficiently using the interior point method. In passing, we note that this algorithm was applied in [38] to detect a point-like target in correlated Gaussian noise under an unknown direction of arrival.

Denote by \( t^* \) the optimal value attained from solving (19), the likelihood ratio test is written as follows

\[
\frac{1}{z^H C^{-1} z} t^* \geq \frac{1}{h_0} G_2.
\]

C. Detection with estimated structure of covariance matrix \( C \)

As mentioned in the previous subsection, the likelihood ratio (and its statistic) in (10) remains unchanged if we substitute \( C \) by \( M \). Hence, instead of estimating \( C \), we employ an estimate of \( M \). The MLE of \( M \) has been proved to uniquely exist and derived in [25], in which MLE of \( M \) satisfies the equation

\[
M_{MLE} = f(M_{MLE}),
\]

where

\[
f(M_{MLE}) = \frac{N}{K} \sum_{t=1}^{K} c_t c_t^H M_{MLE} c_t.
\]

Solution for the above equation uniquely exists but a closed-form for such solution does not exist [25]. Instead, MLE of \( M \) is computed by recursion computing [25], which is employed in this paper.

Now, we replace \( C \) in (10) by the MLE of \( M \) and assess the CFAR property of the resulting detector, called \( \theta \)-MLE detector, employing the likelihood ratio

\[
\frac{1}{z^H \hat{M} z} \geq \frac{1}{h_0} G,
\]

where \( \hat{t} \) is the optimal value attained from (19), in which \( y \) and \( x \) are computed with \( M_{MLE} \).
It is easily to see that the $\theta$–MLE detector have CFAR w.r.t texture components $\{s_0, s_1, s_2, \ldots, s_K\}$. This claim is easily proved based on the following arguments. Firstly, notice that the MLE of $M$ can be derived based on the relation

$$M_{MLE} = \frac{N}{K} \sum_{t=1}^{K} g_t M^{-1}_{MLE} g_t^H,$$  

(24)

which is independent of $\{s_0, s_1, s_2, \ldots, s_K\}$. In addition, the texture component $s_0$ embedded in $z$ has been cancelled out in the numerator and denominator of

$$\frac{|z^H M^{-1}_{MLE} p|^2}{(z^H M^{-1}_{MLE} z)} \frac{p^H M^{-1}_{MLE} p}{h_1} \geq \frac{p^H M^{-1}_{MLE} p}{h_0} G.$$  

(25)

For the simulation, we use an uniform linear array consisting of $N = 8$ antennas, assuming $\theta = \pi/3$ and $\beta = \pi/6$ (i.e. $\phi \in [\pi/6, \pi/2]$) and $K = 32$. As to the clutter, we assume that $s_0, s_1, \ldots, s_K$ follow the chi distribution, so $s_0^2, s_1^2, \ldots, s_K^2$ follow the chi-square distribution with degree of freedom $\nu = 3$ ([33]), i.e. $E[s_i^2] = 3$. The generation of $g_t$ follows the guide in [39]. Briefly, we firstly generate complex Gaussian random vectors $u_i$ of zero-mean and identity covariance matrix; next $g_t = Ru_i$, where $R$ is the Cholesky decomposition of $C$, i.e., $R R^H = C$ where $C_{nm} = \rho^{|n-m|}$ and $\rho$ the correlation efficient. Clutter return at each range cell is $c_t = s_t g_t$. Since it is difficult to derive closed-forms of detection ($P_d$) and false alarm ($P_{fa}$) probabilities, such quantities will be numerically analyzed through independent $10^2/P_d$ and $10^2/P_{fa}$ Monte Carlo trials, respectively. To lower the computational burden, we choose $P_{fa} = 10^{-3}$. We use the software CVX (http://cvxr.com/) to solve the semi–determinate problem (23) on a computer equipped with a 3.4 GHz Intel processor. Finally, the signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{|\alpha|^2 ||p||^2}{N \times E[s^2]}.$$  

(26)

which is $|\alpha|^2 ||p||^2/(N \times \nu)$ in our simulation.

### III. Numerical Results

In this section, via computer simulation we assess and compare performance of the $\theta$-MLE detector (23) with that of the ANMF [17], referred to as in the following as MLE-NMF since the unknown $M$ is replaced with its MLE.

$$\frac{|z^H M^{-1}_{MLE} p|^2}{(z^H M^{-1}_{MLE} z)} \frac{p^H M^{-1}_{MLE} p}{h_1} \geq h_0 G.$$  

(25)

For the simulation, we use an uniform linear array consisting of $N = 8$ antennas, assuming $\theta = \pi/3$ and $\beta = \pi/6$ (i.e. $\phi \in [\pi/6, \pi/2]$) and $K = 32$. As to the clutter, we assume that $s_0, s_1, \ldots, s_K$ follow the chi distribution, so $s_0^2, s_1^2, \ldots, s_K^2$ follow the chi-square distribution with degree of freedom $\nu = 3$ ([33]), i.e. $E[s_i^2] = 3$. The generation of $g_t$ follows the guide in [39]. Briefly, we firstly generate complex Gaussian random vectors $u_i$ of zero-mean and identity covariance matrix; next $g_t = Ru_i$, where $R$ is the Cholesky decomposition of $C$, i.e., $R R^H = C$ where $C_{nm} = \rho^{|n-m|}$ and $\rho$ the correlation efficient. Clutter return at each range cell is $c_t = s_t g_t$. Since it is difficult to derive closed-forms of detection ($P_d$) and false alarm ($P_{fa}$) probabilities, such quantities will be numerically analyzed through independent $10^2/P_d$ and $10^2/P_{fa}$ Monte Carlo trials, respectively. To lower the computational burden, we choose $P_{fa} = 10^{-3}$. We use the software CVX (http://cvxr.com/) to solve the semi–determinate problem (23) on a computer equipped

with a 3.4 GHz Intel processor. Finally, the signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{|\alpha|^2 ||p||^2}{N \times E[s^2]}.$$  

(26)

which is $|\alpha|^2 ||p||^2/(N \times \nu)$ in our simulation.

### A. Performance Assessment

We first investigate if the $\theta$-MLE detector has a CFAR property w.r.t structure of the clutter covariance matrix (i.e. $M$). Fig. 1 shows false alarm probabilities versus threshold of the $\theta$-MLE detector at varied degrees of correlation $\rho = 0.1, 0.4, 0.8, 0.9, 0.99, 0.999$. Here we used $5 \times 10^4$ Monte Carlo runs. It is observed that $\theta$-MLE possesses CFAR w.r.t all simulated degrees of correlation. Hence, $\theta$-MLE detector possesses CFAR w.r.t all the statistics of the clutter, a property that is also possessed by MLE-NMF [24]. From now on, $\rho = 0.4$ in all simulations.

In Fig. 2 we compare detection probabilities of the $\theta$-MLE with that of the MLE-NMF in case that the actual steering vector $p$ perfectly matches with the nominal one $s$, i.e. $\phi = \theta$. With incomplete knowledge of the actual steering vector, $\theta$-MLE suffers a detection loss w.r.t that of the MLE-NMF, defined as the horizontal displacement of the corresponding curves, of nearly 2dB. However, it is obvious and shown in Fig. 3 that even with a slight mismatch, i.e., $\theta - \phi = \pi/15$, $\theta$-MLE outperforms the MLE-NMF, especially in the high SNR region. Robustness of the $\theta$-MLE detector to
The mismatched signal is further demonstrated, in Fig. 4 in cases of more serious mismatches, i.e., $\theta - \phi = -\pi/10$, $0$, $\pi/15$, $\pi/6$, $\pi/24$. Loss in detection probabilities, in a comparison with perfectly matched case, of MLE-NMF is comparatively small when a mismatch lies in the designed interval of the $\theta$-MLE and becomes significant with a mismatch lying outside the designed interval, i.e., $|\theta - \phi| > \beta$, i.e. in case of $\theta - \phi = 5\pi/24$ (the designed $\beta = \pi/6$). Finally, influence on $\theta$-MLE’s detection performance of the size of secondary data is investigated in Fig. 5. Interestingly, $P_d$ of the proposed detector exhibits a little improvement with an increasing value of $K$, meaning that we do not need to collect more secondary data from the surrounding range cells to achieve better detection capacity. This property is also reported in the previous research [17]–[24] and is opposite to the results in case of homogeneous/partially homogeneous Gaussian noise [34].

IV. Conclusion

This paper has addressed the problem of detecting a mismatched signal embedded in compound Gaussian noise. Specifically, phase shifting of the actual steering vector departs from that of the nominal one but belongs to a known interval. The proposed detector is shown to be more robust to mismatched signals than the adaptive NMF, and even achieves reasonable detection probabilities when the signal to detect lying out of the designed interval. Remarkably, the $\theta$-MLE detector has CFAR w.r.t all statistic of noise. A drawback of the proposed detector is
that the likelihood ratio has no explicit form, for which it is difficult to gain a deeper insight into the performance of the detector. Another drawback is the complexity associated with the SDP. Though proposed scheme can detect a seriously mismatched signal, it does not include effects of possible interference, which might be a topic for a further research.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the paper.

REFERENCES

[1] E. J. Kelly, “An adaptive detection algorithm,” IEEE Trans. Aerosp. Electron. Syst., vol. 22, no. 1, pp. 115-127, Mar. 1986.
[2] L. L. Scharf and B. Friedlander, “Matched subspace detectors,” IEEE Trans. Signal Process., vol. 42, no. 8, pp. 2146-2157, Aug. 1994.
[3] S. Kraut, L. L. Scharf, and L. T. McWhorter, “Adaptive subspace detectors,” IEEE Trans. Signal Process., vol. 49, no. 1, pp. 1-16, Jan. 2001.
[4] O. Besson, L. L. Scharf, and S. Kraut, “Adaptive detection of a signal known only to lie on a line in a known subspace, when primary and secondary data are partially homogeneous,” IEEE Trans. Signal Process., vol. 54, no. 12, pp. 4698-4705, Dec. 2006.
[5] O. Besson, “Detection of a signal in linear subspace with bounded mismatch,” IEEE Trans. Aerosp. Electron. Syst., vol. 42, no. 2, pp. 1131-1138, Apr. 2006.
[6] F. Bandiera, O. Besson, D. Orlando, G. Ricci, and L. L. Scharf, “GLRT-based direction detectors in homogeneous noise and subspace interference,” IEEE Trans. Signal Process., vol. 55, no. 6, pp. 2386-2394, June 2007.
[7] F. Bandiera, A. D. Maio, A. S. Greco, and G. Ricci, “Adaptive radar detection of distributed targets in homogeneous and partially homogeneous noise plus subspace interference,” IEEE Trans. Signal Process., vol. 55, no. 4, pp. 1223-1237, Apr. 2007.
[8] A. Svensson and A. Jakobsson, “Adaptive detection of a partly known signal corrupted by strong interference,” IEEE Signal Process. Lett., vol. 18, no. 12, pp. 729-732, Dec. 2011.
[9] A. Aubry, A. D. Maio, D. Orlando, and M. Piezzo, “Adaptive detection of point-like targets in the presence of homogenous clutter and subspace interference,” IEEE Signal Process. Lett., vol. 21, no. 7, pp. 848-852, July 2014.
[10] S. Ramprashad, T. W. Parks, and R. Shenoy, “Signal modeling and detection using cone classes,” IEEE Trans. Signal Process., vol. 4, no. 2, pp. 329-338, Feb. 1996.
[11] A. D. Maio, “Robust adaptive radar detection in the presence of steering vector mismatches,” IEEE Trans. Aerosp. Electron. Syst., vol. 41, no. 4, pp. 1322-1337, Oct. 2005.
[12] F. Bandiera, A. D. Maio, and G. Ricci, “Adaptive CFAR radar detection with conic rejection,” IEEE Trans. Signal Process., vol. 55, no. 6, pp. 2533-2541, June 2007.
[13] F. Bandiera, D. Orlando, and G. Ricci, “CFAR detection strategies for distributed targets under conic constraints,” IEEE Trans. Signal Process., vol. 57, no. 9, pp. 3305-3316, Sep. 2009.
[14] A. D. Maio, S. D. Nicola, Y. Huang, S. Zhang, and A. Farina, “Adaptive detection and estimation in the presence of useful signal and interference mismatches,” IEEE Trans. Signal Process., vol. 57, no. 2, pp. 436-450, Feb. 2009.
[15] E. Conte and M. Longo, “Characterisation of radar clutter as a spherically invariant random process,” IEE Proc., vol. 134, no. 2, pp. 191-197, Apr. 1987.
[16] K. D. Ward, C. J. Baker, and S. Watt, “Maritime surveillance radar-Part I: Radar scattering from the ocean surface,” IEE Proc., vol. 137, no. 2, pp. 51-62, Apr. 1990.
[17] E. Conte, M. Lops, and G. Ricci, “Asymptotically optimum radar detection in compound-Gaussian clutter,” IEEE Trans. Aerosp. Electron. Syst., vol. 31, no. 2, pp. 617-625, Apr. 1995.