Viable and simplified semi-direct gauge mediation with the 4–1 model

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Abstract

We present a simple and phenomenologically acceptable extension of the 4–1 model of dynamical supersymmetry breaking, in which messengers and MSSM superfields are directly coupled to the hidden sector without participating in the supersymmetry breaking mechanism; although parametrically suppressed by a loop factor, gaugino masses turn out to be comparable to sfermion masses because of the presence of enhancing factors ultimately due to the different origin of the gaugino and the sfermion mass terms. We also describe what can be considered the simplest realization of the Higgs sector and how electroweak symmetry breaking can take place in this model. Finally, in the Appendix, we have listed a set of closed-form expressions for the computation of $A$–terms and soft squared masses of light scalars at the messenger scale in the presence of the most general form of "matter-messenger" couplings, extending in this way some results already known from the literature.

1 Introduction

Many models with gauge-mediated dynamically broken supersymmetry run into phenomenological problems, among which the most serious is represented by the gaugino screening [1]. It has been shown in [2] that it is possible to overcome this difficulty if the sources of the soft mass terms for sfermions and gauginos are different and, in particular, if sfermions are charged under an extra gauge symmetry which is broken at a very high scale and gauginos acquire a soft Majorana mass term through the coupling of the messengers to the hidden sector. This mechanism of mediation of supersymmetry

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1 The possibility of Dirac masses has been studied in [3]
breaking effects, providing a dynamical realization of tree-level gauge mediation [4], can be seen as a simplified and viable version of the semi-direct gauge mediation framework [5], where only messengers are directly coupled to the hidden sector.

In this paper, we want to show a simple extension of the 4–1 calculable model of dynamical supersymmetry breaking [6] which is based on the following scheme: the hidden sector is composed of the 4–1 model in which the strong SU(4) dynamics generates, at the non-perturbative level, a term in the superpotential whose main effect lies in the stabilization of the vacuum of the theory in a point in which both supersymmetry and gauge symmetry are broken spontaneously, making, in particular, the vacuum expectation value of the U(1) D–term non-vanishing; if the observable chiral superfields are charged under the U(1) factor, then a soft mass term is generated for the sfermions.

The outline of the paper is the following: in Section 2 we summarize and review the features of the 4–1 model which are necessary for the understanding of the rest of the paper; in Section 3 we describe how to couple the observable sector to the hidden sector, exploring the implications in the low-energy spectrum of the theory; in Section 4 we show one possible realization of the Higgs sector and comment about the breaking of the electroweak symmetry; in Appendix A we explain how it is possible to recover the MSSM at low energies and in Appendix B we list a set of closed-form expressions for the computation of $A$–terms and soft-squared mass terms for light fields in the most general setup of gauge mediation which includes matter-messenger couplings, extending some of the results already known in the literature [7, 8].

2 The 4–1 sector

We first review the 4–1 model, which is based on the SU(4) $\times$ U(1) gauge group and matter content as in Table 1.

For later convenience, we collect the $\chi$’s in a matrix $A$:

$$A = \begin{pmatrix}
0 & \chi_1 & \chi_2 & \chi_4 \\
-\chi_1 & 0 & \chi_3 & \chi_5 \\
-\chi_2 & -\chi_3 & 0 & \chi_6 \\
-\chi_4 & -\chi_5 & -\chi_6 & 0
\end{pmatrix}. \tag{1}$$

If we assume perturbativity of the U(1) dynamics at the scale $\Lambda$ at which the SU(4) gauge coupling $g_4$ becomes strong, then the superpotential can be written as the sum of a classical and a non-perturbative term:

$$W_{4-1} = W_{cl} + W_{np} \tag{2}$$

where

$$W_{cl} = hST_i F_i \tag{3}$$
$$S \mathbb{U}(4) \mathbb{U}(1)$$

| S | 1 | 4 |
|---|---|---|
| F | 4 | −3 |
| F | 4 | 1 |
| χ | 6 | 2 |

Table 1: matter content of the 4–1 model.

and

$$W_{np} = 2 \frac{\Lambda^5}{\sqrt{F_i F_j A_{ik} A_{lm} \epsilon_{jklm}}}$$

where $\epsilon_{jklm}$ is the totally antisymmetric tensor with $\epsilon^{1234} = 1$.

If we assume a hierarchy of the coupling constants $g_4 \gg g_1 \gg h$ (where $g_1$ is the $\mathbb{U}(1)$ gauge coupling constant) and $h \ll 1$, then the vacuum of the theory is calculable and can be found as a power expansion in the parameters $h/g_4$ and $h/g_1$. In particular, at the lowest order, the vacuum is located at

$$A = \begin{pmatrix} 0 & a \sqrt{2} & 0 & 0 \\ -\frac{a}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a}{\sqrt{2}} \\ 0 & 0 & -\frac{a}{\sqrt{2}} & 0 \end{pmatrix} \quad M \quad F = F = \begin{pmatrix} b \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad M \quad S = cM,$$

where

$$M \equiv \frac{\Lambda}{h^2} \gg \Lambda$$

$$c \equiv \sqrt{b^2 - \frac{a^2}{2}}$$

and, approximately, $a \approx 1.492$, $b \approx 1.102$. At the lowest order, furthermore, the non-zero vacuum expectation values of the F–terms are the following:

$$F_{\chi_1} = F_{\chi_6} = \frac{-\sqrt{2}}{a^2 b} F \quad F_{F_1} = F_{F_4} = \frac{ab^3 c - 1}{ab^2} F \quad F_S = b^2 F,$$

where

$$F = h^2 \Lambda^2.$$
particular, the $\mathcal{U}(1)$ D–term is

$$D_{\mathcal{U}(1)} = \frac{4b^2 - 2a^2 + a^6b^6 + 2a^3b^3\sqrt{4b^2 - 2a^2}}{2a^4b^4(6b^2 - a^2)} \frac{1}{g_1^2} \frac{F^2}{M^2} \equiv \frac{d}{g_1^2} \frac{F^2}{M^2},$$

where $d \approx 0.349$.

We can now couple the 4–1 model to the MSSM and messenger sectors.

### 3 Coupling the observable sector to the 4–1 model

The aim of this section is to show an extension of the 4–1 model, which from now on we will identify with the hidden sector, based on the gauge group $\mathcal{SU}(4) \times \mathcal{U}(1) \times \mathcal{G}_{SM}$, in which the scalar components of the would-be MSSM superfields, charged under the $\mathcal{U}(1)$ gauge group, gain a positive soft squared mass via the $\mathcal{U}(1)$ D–term coupling and in which it is possible to identify a set of messenger superfields which give mass to gauginos at the one-loop level, as in the standard gauge mediation framework [9].

Let us consider the spectrum in Table 2, where $j$ is a family index which runs from 1 to 3, $i$, which can take values 1, 2, ..., $n_m$, labels the messengers $\varphi, \tilde{\varphi}, \psi$ and $\tilde{\psi}$ within each family; moreover, for reasons that soon will become apparent, we require that one among the following $\mathcal{U}(1)$ charge assignments is realized: either

$$q_m = -2 - \sqrt{9y^2 - 12n_my} + \frac{4}{3}(4n_m^2 - 1)$$

$$\tilde{q}_m = -2 + \sqrt{9y^2 - 12n_my} + \frac{4}{3}(4n_m^2 - 1)$$

or

$$q_m = -2 + \sqrt{9y^2 - 12n_my} + \frac{4}{3}(4n_m^2 - 1)$$

$$\tilde{q}_m = -2 - \sqrt{9y^2 - 12n_my} + \frac{4}{3}(4n_m^2 - 1);$$

the superfields $q, l, \tilde{u}, \tilde{d}, \tilde{e}$ and $\tilde{n}$ are the light observable fields; the superpotential is:

$$W = W_{4\text{-}1} + W_{\text{mess}}$$

$$W_{\text{mess}} = \lambda_{i,j,k,l}^{1} S_{\varphi_i^k} S_{\tilde{\varphi}_j^l} + \lambda_{i,j,k,l}^{2} S_{\psi_i^k} S_{\tilde{\psi}_j^l}. \hspace{1cm}(13)$$

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2A convenient way to gain information on the vacuum structure of the model is the following: in the vacuum, F–terms, D–terms and scalars $\phi$ are not independent: $F^\dagger T_a F = \sum_b g_b^2 D_b \phi^i T_b T_a \phi$, where $T_a$ is any symmetry global/gauge symmetry generator, the index $b$ labels the gauge symmetry generators and $g_b$ is the gauge coupling constant of the group which the generator $T_b$ belongs to; it is easy to see that once the zeroth-order vacuum expectation values of $\phi$ and F–terms in the expansion in $h/g_1$ and $h/g_4$ are known, then the aforementioned relation yields the vacuum expectation values of the D–terms at the second-order in $h/g_1$ and $h/g_4$. 

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The U(1) charge assignment described in Table 2 (with one among eq. (11) and eq. (12) satisfied) is derived imposing the following conditions: family independence of the U(1) charges; all of the φ’s (˜φ’s) and all of the ψ’s (˜ψ’s) have the same U(1) charges; cancellation of all the anomalies; the superpotential in eq. (13) is allowed; the requirement that the charges of l, ˜e, q and ˜d (denoted with q_l, ˜q_e, q_m and ˜q_d respectively) are not independent: q_l + ˜q_e = q_m + ˜q_d (the reason for this requirement lies in the coupling to the Higgs).

\[
\begin{array}{cccc}
S U(4) & U(1) & G_{SM} \\
S & 1 & 4 & 1 \\
F & 4 & -3 & 1 \\
\bar{F} & 4 & -1 & 1 \\
\chi & 6 & 2 & 1 \\
\varphi_j^i & 1 & q_m & (3, 1, -1/3) \\
\tilde{\varphi}_j^i & 1 & \tilde{q}_m & (\bar{3}, 1, 1/3) \\
\psi_j^i & 1 & q_m & (1, \bar{2}, 1/2) \\
\tilde{\psi}_j^i & 1 & \tilde{q}_m & (1, 2, -1/2) \\
q_j & 1 & y & (3, 2, 1/6) \\
l_j & 1 & 4n_m - 3y & (1, 2, -1/2) \\
\tilde{u}_j & 1 & y & (3, 1, -2/3) \\
\tilde{d}_j & 1 & 4n_m - 3y & (\bar{3}, 1, 1/3) \\
\tilde{e}_j & 1 & y & (1, 1, 1) \\
\tilde{n}_j & 1 & 5y & (1, 1, 0) \\
\end{array}
\]

Table 2: spectrum of the theory (without the Higgs sector; the G_{SM} representations of the messengers φ, ˜φ, ψ and ˜ψ are chosen in such a way that it is possible to embed (φ, ψ) and (˜φ, ˜ψ) in complete SU(5)_{GUT} multiplets; q, l, ˜u, ˜d, ˜e and ˜n denote the MSSM superfield together with a right-handed neutrino ˜n.

We finally require that q_m and ˜q_m (as well as y) are rational \[10\].

The spectrum so defined does not destabilize the vacuum of the 4–1 model because messengers have a large supersymmetric mass term via the coupling in eq. (13), while light sfermions acquire a positive soft squared mass term via the U(1) D–term coupling provided 0 < y < 4/3n_m:

\[
\tilde{m}_{q,\tilde{q},\tilde{e}}^2 = yd \frac{F^2}{M^2}, \quad \tilde{m}_{l,d}^2 = (4n_m - 3y) d \frac{F^2}{M^2}, \quad \tilde{m}_{\tilde{n}}^2 = 5yd \frac{F^2}{M^2}, \quad (14)
\]

Gaugino masses arise at the one-loop level; at the messenger scale

\[
M_i(M_{mess}) = 3n_m \frac{b^2}{\sqrt{b^2 - \frac{\alpha_i(M_{mess})}{4\pi}}} \frac{F}{M^2}, \quad (15)
\]

since at the one-loop level the gaugino mass evaluated at a generic scale \(\mu\) is given by eq. (15) with the substitution \(\alpha_i(M_{mess}) \to \alpha_i(\mu)\), we find at the TeV scale the following
relation:

\[ M_3 \approx 0.13 \frac{n_m}{\sqrt{y}} \tilde{m}_q, \]  

(16)

where \( \tilde{m}_q \) is defined in eq. (14); the ratio between the gluino and \( \tilde{m}_q \) depends on the free parameters \( n_m \) and \( y \), which can be chosen in such a way to make \( \tilde{m}_q \) not hierarchically larger than the gluino mass.

In order to keep perturbativity of the gauge coupling constants up to the unification scale, a lower bound on the messenger masses should be imposed and, in the simple case in which messenger masses are almost degenerate, it is possible to estimate this value \[ \frac{M_{\text{mess}}}{M_{\text{GUT}}} \gtrsim e^{-\frac{20}{n_m}}. \]  

(17)

From eq. (16) and eq. (17), it is possible to deduce that, in general, a reduction of the hierarchy between gluino and sfermion masses can be obtained if the lower bound on the messenger mass is raised. To have an idea of the energy scales and of the parameters involved in the theory let us consider the following situation: \( n_m = 5, y = 4 \); in this case, it is found that

\[ M_3 \approx 0.33 \tilde{m}_q \]  

(18)

and that messenger masses have the following bound

\[ M_{\text{mess}} > 10^{12} \text{ GeV}; \]  

(19)

this implies that the Yukawa coupling \( h \) that appears in \( W_{4-1} \) is constrained by the following relation

\[ h = \frac{F}{M^2} = \frac{F}{M} \frac{1}{M} \lesssim 10^{-9} \frac{\tilde{m}}{1 \text{ TeV}}, \]  

(20)

which is compatible with the hypothesis \( h \ll 1 \).

It should be finally mentioned that it is possible to require the existence of messenger-matter couplings in the superpotential, which may be necessary to raise the Higgs boson mass via the generation of sizeable \( A \)-terms (see Appendix B); such a situation can be realized, for example, if we take \( y = 32/5 \) and \( n_m = 5 \); indeed, in this case \( q_m = -64/5 \) and terms which couple \( \varphi \) or \( \psi \) to \( q, \tilde{u} \) and \( \tilde{e} \) are allowed.

4 The Higgs sector and the Yukawa interactions

To complete the model, we have to introduce the Higgs fields and their couplings to the matter; since this is highly model-dependent, we present here what we consider the simplest realization of the Higgs sector.

We add to the spectrum in Table 2 four Higgs superfields in a vectorlike representation of the gauge group, see Table 3 and, at the renormalizable level, the following

\[ \frac{M_{\text{mess}}}{M_{\text{GUT}}} \gtrsim e^{-\frac{20}{n_m}}. \]  

(17)

3In eq. (20) we have assumed that \( M_{\text{mess}} \approx M \).
superpotential is allowed:

\[ W = W_{4-1} + W_{\text{mess}} + W_{\text{Yukawa}} + W_{\text{Higgs}} \]

\[ W_{\text{Yukawa}} = \lambda_{ij}^u h_u q_i \tilde{u}_j + \lambda_{ij}^d h_d q_i \tilde{d}_j + \lambda_{ij}^e h_d \tilde{e}_j. \]  

(21)

In writing eq. (21) we have required that \( h_u \) and \( h_d \) either have a component or can be identified with the MSSM up and down-type Higgs superfields.

\[ SU(4) \quad U(1) \quad G_{\text{SM}} \]

| S   | 1   | 4   | 1   |
|-----|-----|-----|-----|
| F   | 4   | -3  | 1   |
| \bar{F} | 4   | -1  | 1   |
| \chi | 6   | 2   | 1   |

\[ q_j \quad 1 \quad q_m \quad (3,1,\frac{-1}{3}) \]

\[ \bar{q}_j \quad 1 \quad \bar{q}_m \quad (\overline{3},1,\frac{1}{3}) \]

\[ l_j \quad 1 \quad 4n_m - 3y \quad (1,2,-\frac{1}{2}) \]

\[ u_j \quad 1 \quad y \quad (\overline{3},1,-\frac{2}{3}) \]

\[ d_j \quad 1 \quad 4n_m - 3y \quad (\overline{3},1,\frac{1}{3}) \]

\[ \bar{e}_j \quad 1 \quad y \quad (1,1,1) \]

\[ \bar{n}_j \quad 1 \quad 5y \quad (1,1,0) \]

\[ h_u \quad 1 \quad -2y \quad (1,2,\frac{1}{2}) \]

\[ h_d \quad 1 \quad -4n_m + 2y \quad (1,2,-\frac{1}{2}) \]

\[ \tilde{h}_u \quad 1 \quad 2y \quad (1,2,-\frac{1}{2}) \]

\[ \tilde{h}_d \quad 1 \quad 4n_m - 2y \quad (1,2,\frac{1}{2}) \]

Table 3: spectrum of the theory, with the Higgs sector

We are now in the position to describe two scenarios which lead to a phenomenologically acceptable pattern of electroweak symmetry breaking, without attempting neither to solve the \( \mu \)-problem nor to explain possible hierarchies of the parameters in the Higgs sector.

The simplest choice consists in the assumption that the Higgs superpotential is

\[ W_{\text{Higgs}} = \mu_u h_u \tilde{h}_u + \mu_d h_d \tilde{h}_d, \]  

(22)

where we consider the situation in which \( \mu_u \) and \( \mu_d \) are \( \mathcal{O}(\text{TeV}) \) and, as a consequence, in the end we find four Higgs superfields at the TeV scale. Since, at the messenger scale, the soft terms generated in the Higgs sector accidentally conserves the \( U(1) \) symmetry, then

\[ ^4 \text{Notice that all the interactions between the observable sector and light fields present in the spectrum of the hidden sector are suppressed by factors of } \mathcal{O}(1/M), \text{ so that they can be considered completely harmless for low-energy phenomenology.} \]
a $\mathbb{U}(1)$ preserving $B\mu$-term is radiatively generated below the scale $M$; in this context, then, it is possible to conclude as in [2]: only one out of the two pairs of doublets $(h_u, \tilde{h}_u)$ and $(h_d, \tilde{h}_d)$ acquires a non-vanishing vacuum expectation value; in order to have non-vanishing vacuum expectation values for $h_u$ and $h_d$, it is necessary to break the accidental $\mathbb{U}(1)$ symmetry and this can be done via small non-renormalizable interactions.

Alternatively, it is possible to have two Higgs superfields at the TeV scale if we consider the following tree-level Higgs superpotential

$$W_{\text{Higgs}} = \mu_u h_u \tilde{h}_u + \mu_d h_d \tilde{h}_d + W_{\text{non-ren}},$$

where $W_{\text{non-ren}}$ is the non-renormalizable part of the superpotential, in which terms like $(F\tilde{F})^n / (\Lambda^{2n-1}) \tilde{h}_u \tilde{h}_d$ and $S^n / \Lambda^{n-1} h_u h_d$ are included, generating $\mathbb{U}(1)$-breaking $B\mu$-term. Such a possibility is explored in Appendix A, where it is shown that the MSSM with two Higgs doublets can be recovered at low energies.

5 Summary

We have proposed a phenomenologically viable model, based on a simple extension of the 4–1 model, in which supersymmetry is broken dynamically and communication of the supersymmetry breaking effects in the observable sector is achieved via renormalizable non-SM gauge interactions [4].

In particular, we have realized a situation in which the tree-level generated soft sfermion masses are flavor independent, solving in this way the supersymmetric flavor problem; gaugino masses arise at the one-loop level, as in the minimal gauge mediation mechanism; because of the presence of various enhancing factors, the hierarchy between sfermions and gauginos is reduced in a remarkable and acceptable way.

In the situation described in the current paper, the Higgs sector is highly model-dependent; we have proposed what we think can be considered its simplest realization. In particular, in Appendix A, we explore the possibility to recover at low energies the MSSM.

In Appendix B, we list closed-form expressions for $A$–terms and soft squared masses generated at the messenger scale in models of gauge mediation with generic "matter-messenger" couplings.

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Appendix A: recovering the MSSM at low energies

The purpose of this Appendix is to show that there exists the possibility to recover the two-Higgs-doublet MSSM at low energies; since the Higgs sector is highly model dependent, in the specific framework that we are going to describe we ignore both the fine-tuning of the parameters and the hierarchies of the coupling constants which identify the Higgs sector.

Let us first consider the Higgs sector superpotential including non-renormalizable terms

\[ W_{\text{Higgs}} = \mu_u h_u \tilde{h}_u + \mu_d h_d \tilde{h}_d + X_1 \tilde{h}_u \tilde{h}_d + X_2 h_u h_d, \]  

(24)

where \( X_1 \) and \( X_2 \) can be considered as spurion fields which acquire both a scalar and an \( F \)-term vacuum expectation values \( (X_1 = \mu_1 + F_1 \theta^2 \) and \( X_2 = \mu_2 + F_2 \theta^2) \), and Kähler potential

\[ K_{\text{Higgs}} = h_u^\dagger h_u + h_d^\dagger h_d + \tilde{h}_u^\dagger \tilde{h}_u + \tilde{h}_d^\dagger \tilde{h}_d + \]

\[- \theta^2 \theta^2 \left( -m_1^2 h_u^\dagger h_u - m_2^2 h_d^\dagger h_d + m_1^2 \tilde{h}_u^\dagger \tilde{h}_u + m_2^2 \tilde{h}_d^\dagger \tilde{h}_d \right). \]

(25)

For reasons that will become apparent in a moment, we assume that \( \bar{\mu} \equiv \sqrt{\mu_u^2 + \mu_d^2} \) is much bigger than any other mass scale appearing in the Higgs sector and that \( m_1, m_2, \mu_u, \mu_2, F_1/\mu_1 \) and \( \sqrt{F_2} \) are of the same order of magnitude, namely the TeV scale. This setup is inspired by the model we have analyzed in Section 4, where \( m_1 \) and \( m_2 \) can be identified with the soft masses generated via tree-level gauge mediation and \( X_1 \) and \( X_2 \) can be identified, for example, with \( (F \theta)^n/ \Lambda^{2n-1} \) and \( S^n/\Lambda^{n-1} \) respectively, where \( \Lambda \) is an ultraviolet cutoff.

For later convenience, we define the following parameters

\[ X_i = \mu_i + \phi_i \]

\[ \tan \alpha = \frac{\mu_d}{\mu_1} \]

\[ \bar{\mu} = \sqrt{\mu_1^2 + \mu_d^2} \]

\[ h_h = \cos \alpha \tilde{h}_u + \sin \alpha h_d \]

\[ h_l = -\sin \alpha \tilde{h}_u + \cos \alpha h_d, \]

(26)

where \( \phi_i \)'s are spurion superfields acquiring only F-term vacuum expectation values. We write again the superpotential and the Kähler potential in terms of the new variables:

\[ W_{\text{Higgs}} = \mu_u h_u \left( \cos \alpha h_h - \sin \alpha h_l \right) + \bar{\mu} h_h \tilde{h}_d + \mu_2 h_u \left( \sin \alpha h_h + \cos \alpha h_l \right) + \]

\[ + \phi_1 \left( \cos \alpha h_h - \sin \alpha h_l \right) \tilde{h}_d + \phi_2 h_u \left( \sin \alpha h_h + \cos \alpha h_l \right) \]

(27)

\[ K_{\text{Higgs}} = h_u^\dagger h_u + h_d^\dagger h_d + \tilde{h}_u^\dagger \tilde{h}_u + \tilde{h}_d^\dagger \tilde{h}_d + \]

\[- \theta^2 \theta^2 \left( -m_1^2 h_u^\dagger h_u + m_2^2 \left( \cos \alpha h_h - \sin \alpha h_l \right)^\dagger \left( \cos \alpha h_h - \sin \alpha h_l \right) + \right. \]

\[ - m_2^2 \left( \sin \alpha h_h + \cos \alpha h_l \right)^\dagger \left( \sin \alpha h_h + \cos \alpha h_l \right) + m_2^2 \tilde{h}_d^\dagger \tilde{h}_d \right) \]

(28)
The assumptions that we have made allow us to integrate out the heavy fields $h_h$ and $h_d$ in a manifest supersymmetric way \[12\]. The effective low-energy superpotential and Kähler potential, neglecting more irrelevant operators, are:

\[
W_{\text{Higgs}}^{\text{eff}} = (\mu + \phi_\mu) h_u h_l
\]

\[
K_{\text{Higgs}}^{\text{eff}} = h_u^\dagger h_u + h_l^\dagger h_l - \tilde{m}_u^2 \theta^2 \bar{\theta}^2 h_u^\dagger h_u - \tilde{m}_l^2 \theta^2 \bar{\theta}^2 h_l^\dagger h_l
\]

where

\[
\mu = -\mu_u \sin \alpha + \mu_2 \cos \alpha
\]

\[
\phi_\mu = \frac{\phi_1}{\mu} (\mu_u \sin \alpha \cos \alpha + \mu_2 \sin \alpha^2) + \cos \alpha \phi_2
\]

\[
\tilde{m}_u^2 = -m_u^2
\]

\[
\tilde{m}_l^2 = m_l^2 \sin \alpha^2 - m_2^2 \cos \alpha^2 - \frac{F^2}{\mu^2} \sin \alpha^2.
\]

We have then recovered the MSSM at low energies.

**Appendix B: complete and explicit formulae for the radiatively generated $A$–terms and soft squared masses in theories with matter-messenger couplings**

The discovery of a 125 GeV Higgs-like particle has raised a problem in the MSSM because, in the absence of $A$–terms, the Higgs boson mass is bounded to be smaller than 118 GeV if sfermions are lighter than 2 TeV \[7\]; if, on the contrary, $A$–terms are introduced the limits on the Higgs boson mass in the MSSM can be relaxed.

In general models of gauge mediation, the presence of matter-messenger couplings has on one side the rôle to generate $A$–terms at the one-loop order, but, on the other side, may be dangerous because of the introduction of unwanted flavor-violating interactions which should be kept under control.

The goal of this section is to collect a set of useful closed-form formulae for the computation of soft squared masses and $A$–terms *at the messenger scale* in general models of gauge mediation with *the most generic* form of matter-messenger couplings; this section can be seen as an extension and a generalization of the results derived and listed in \[7\,8\].

Let us consider a superpotential containing the following terms:

\[
W \supset \frac{1}{2} \lambda_{ijk} \phi_i q_j q_k + \frac{1}{2} \bar{\lambda}_{ijk} \bar{\phi}_i \bar{q}_j \bar{q}_k + \frac{1}{6} \rho_{ijk} q_i q_j q_k
\]

where $i$, $j$ and $k$ parametrize both gauge and flavor indices, the messenger superfields, which are assumed to be coupled to a spurion field $X$ such that \( \langle X \rangle = M + F \theta^2 \), are denoted by $\phi$, the fields $q$ represent the observable fields and the following symmetry relations are valid:

\[
\lambda_{ijk} = \lambda_{ikj} \quad \bar{\lambda}_{ijk} = \bar{\lambda}_{ikj};
\]
moreover, the coefficients $\rho_{ij k}$ can be assumed to be completely symmetric in the indices $i$, $j$ and $k$.

For symmetry reasons it is convenient to write the above expression in the following way:

$$W \supset \frac{1}{6} \left( \lambda_{ijk}^1 \phi_i \phi_j \phi_k + \lambda_{ijk}^2 \phi_i \phi_j \phi_k + \lambda_{ijk}^3 \phi_i \phi_j \phi_k \right) +$$

$$+ \frac{1}{6} \left( \lambda_{ijk}^1 q_i \phi_j \phi_k + \lambda_{ijk}^2 q_i \phi_j \phi_k + \lambda_{ijk}^3 q_i \phi_j \phi_k \right) +$$

$$+ \frac{1}{6} \rho_{ijk} q_i q_j q_k,$$

where we assume the validity of the following relations:

$$\lambda_{ijk}^1 = \lambda_{jik}^2 = \lambda_{kji}^3 = \lambda_{ijk}$$

$$\lambda_{ijk}^1 = \lambda_{jik}^2 = \lambda_{kji}^3 = \lambda_{ijk}$$

(35)

After the integration of the messenger fields we are left with an effective theory in which soft supersymmetry breaking interactions appear, in particular:

$$V_{\text{soft}} \supset \left( \frac{m^2}{\Lambda} \right)_b q_i^a q_b + \left( \frac{1}{6} A_{abc} \bar{q}_a \bar{q}_b \right) + \text{h.c.}$$

(36)

For later convenience we write explicitly the anomalous dimensions of the fields, where the underscripts $+$ and $-$ refer respectively to the theory above and below the messenger scale:

$$\gamma_{+q_b} = \left( \frac{1}{16 \pi^2} \right) \left( \frac{1}{2} \rho_{aj k} \rho_{aj k} + \frac{1}{2} \lambda_{aj k} \lambda_{aj k} + \frac{1}{2} \lambda_{aj k} \lambda_{aj k} - 2 C_c(a) g_c^2 \delta_{ij} \right)$$

$$\gamma_{+\phi_b} = \left( \frac{1}{16 \pi^2} \right) \left( \frac{1}{2} \lambda_{aj k} \lambda_{aj k} + \frac{1}{2} \lambda_{aj k} \lambda_{aj k} + \frac{1}{2} \lambda_{aj k} \lambda_{aj k} - 2 C_c(a) g_c^2 \delta_{ij} \right)$$

$$\gamma_{+q_b} = \left( \frac{1}{16 \pi^2} \right) \left( \frac{1}{2} \lambda_{aj k} \rho_{aj k} + \frac{1}{2} \lambda_{aj k} \rho_{aj k} + \frac{1}{2} \lambda_{aj k} \rho_{aj k} - 2 C_c(a) \bar{g}_c^2 \delta_{ij} \right)$$

$$\gamma_{-\phi_b} = \gamma_{+\phi_b} = 0$$

$$\gamma_{q_b} = \gamma_{+q_b} = \gamma_{\phi_0}$$

$$\Delta \gamma_{\psi_0} = \gamma_{\psi_0} - \gamma_{\psi_0}$$

(37)

where $C_c(a)$ is the quadratic Casimir of the field $q_a$ or $\phi_a$ for the gauge group factor with coupling constant $g_c$ and $\psi$ denotes a generic messenger or observable field.

We are now in the position to write down both the $A$-terms and the soft squared masses in eq. (36) induced for the fields $q$ for the superpotential in eq. (31):

$$\frac{A_{abc}}{F/M} = - \left( \rho_{abc} \Delta \gamma_{q_b} + \rho_{adbc} \Delta \gamma_{q_b} + \rho_{adbc} \Delta \gamma_{q_b} \right).$$

(38)
For what concerns the soft squared masses, it is convenient to write down the following expression:

$$\frac{\tilde{m}^2}{|F/M|^2} \equiv \tilde{m}^2_1 - \tilde{m}^2_2 - \tilde{m}^2_3$$  \hspace{1cm} (39)$$

where

$$\begin{align*}
(\tilde{m}^2_1)_{q_i}^q &= \frac{1}{64\pi^2} \left[ \lambda^2_{\beta jbc} \left( \tilde{\lambda}^2_{\beta bc \gamma + q_i} + 2 \lambda^2_{\beta d \gamma + q_i} + 2 \tilde{\lambda}^2_{\beta c \gamma + q_i} + \tilde{\lambda}^2_{\beta c \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) + \\
&+ \lambda^2_{\beta d \gamma + q_i} + 2 \lambda^2_{\beta d \gamma + q_i} + 2 \tilde{\lambda}^2_{\beta c \gamma + q_i} + \tilde{\lambda}^2_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right] \\
&+ \lambda^3_{\beta jbc} \left( \tilde{\lambda}^3_{\beta bc \gamma + q_i} + 2 \lambda^3_{\beta d \gamma + q_i} + 2 \tilde{\lambda}^3_{\beta c \gamma + q_i} + \tilde{\lambda}^3_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \lambda^3_{\beta jbc} \left( \tilde{\lambda}^3_{\beta bc \gamma + q_i} + 2 \lambda^3_{\beta d \gamma + q_i} + 2 \tilde{\lambda}^3_{\beta c \gamma + q_i} + \tilde{\lambda}^3_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \tilde{\lambda}^3_{\beta jbc} \left( \tilde{\lambda}^3_{\beta bc \gamma + q_i} + 2 \tilde{\lambda}^3_{\beta c \gamma + q_i} + \tilde{\lambda}^3_{\beta d \gamma + q_i} + \tilde{\lambda}^3_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \tilde{\lambda}^1_{\beta jbc} \left( \tilde{\lambda}^1_{\beta bc \gamma + q_i} + 2 \tilde{\lambda}^1_{\beta c \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \tilde{\lambda}^1_{\beta jbc} \left( \tilde{\lambda}^1_{\beta bc \gamma + q_i} + 2 \tilde{\lambda}^1_{\beta c \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \lambda^1_{\beta jbc} \left( \tilde{\lambda}^1_{\beta bc \gamma + q_i} + 2 \tilde{\lambda}^1_{\beta c \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \rho_{\beta d} \gamma_{+ q_i} + \tilde{\lambda}^1_{\beta d \gamma + q_i} + \lambda^1_{\beta d \gamma + q_i} \right) \\
&+ \beta_{\gamma^i}^{\pm} = b_{\gamma^i}^{\pm} \frac{1}{16\pi^2} g_a^3 \Delta b_a \equiv b_{\gamma^i}^+ - b_{\gamma^i}^- \hspace{1cm} (42)$$
\end{align*}$$

and the superscripts $+$ and $-$ refer to the theory above and below the messenger scale respectively.

$$\begin{align*}
(\tilde{m}^2_3)_{q_i}^q &= \Delta_{\gamma^i}^{\gamma^i_{\pm}} - \gamma^{-q_i}_{-q_i} \Delta_{\gamma^i_{-q_i}} \hspace{1cm} (43)
\end{align*}$$

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