Bounds on New Physics from the New Data on Parity Violation in Atomic Cesium

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Abstract

We assume the latest experimental determination of the weak charge of atomic cesium and analyze its implications for possible new physics. We notice that the data would imply positive upper and lower bounds on the new physics contribution to the weak charge, $\delta_N Q_W$. The required new physics should be of a type not severely constrained by the high energy precision data. A simplest possibility would be new neutral vector bosons almost un-mixed to the $Z$ and with sizeable couplings to fermions. The lower positive bound would however forbid zero or negative $\delta_N Q_W$ and exclude at 99\% CL not only the standard model but also models with sequential $Z'$, in particular simple-minded towers of $Z$-like excitations from extra-dimensions. The bound would also imply an upper limit on the $Z'$ mass within the models allowed. Conclusions are also derived for models of four-fermion contact interactions.
In a recent paper \[1\] a new determination of the weak charge of atomic cesium has been reported. It takes advantage from a new measurement of the tensor transition probability for the $6S \rightarrow 7S$ transition in cesium and from improvement of the atomic structure calculations in light of new experimental tests. The value reported

$$Q_W(^{133}\text{Cs}) = -72.06 \pm (0.28)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

represents a considerable improvement with respect to the previous determination \[2, 3\]

$$Q_W(^{133}\text{Cs}) = -71.04 \pm (1.58)_{\text{expt}} \pm (0.88)_{\text{theor}}$$

In the following we assume the validity of the determination in eq. (1), verbatim et literatim. We note that the new measure of $Q_W$ is at a level ($\approx 0.6\%$) comparable to the precisions attained for electroweak observables in high energy experiments.

On the theoretical side, $Q_W$ can be expressed as \[4\]

$$Q_W = -72.84 \pm 0.13 - 102\epsilon_3^{\text{rad}} + \delta_N Q_W$$

including hadronic-loop uncertainty. We use here the variables $\epsilon_i$ (i=1,2,3) of ref. \[5\], which include the radiative corrections, in place of the set of variables $S$, $T$ and $U$ originally introduced in ref. \[6\]. In the above definition of $Q_W$ we have explicitly included only the Standard Model (SM) contribution to the radiative corrections. New physics (that is physics beyond the SM) contributions to $\epsilon_3$ are represented by the term $\delta_N Q_W$. Also, we have neglected a correction proportional to $\epsilon_1^{\text{rad}}$. In fact, as well known \[4\], due to the particular values of the number of neutrons ($N = 78$) and of protons ($Z = 55$) in cesium, the dependence on $\epsilon_1$ almost cancels out.

From the theoretical expression we see that $Q_W$ is particularly sensitive to new physics contributing to the parameter $\epsilon_3$. This kind of new physics is severely constrained by the high energy experiments. From a recent analysis \[7\], one has that the value of $\epsilon_3$ from the high energy data is

$$\epsilon_3^{\text{expt}} = (4.1 \pm 1.4) \times 10^{-3}$$

To estimate new physics contributions to this parameter one has to subtract the SM radiative corrections, which, for $m_{\text{top}} = 175 \text{ GeV}$ and for $m_H = 2$
70, 300, 1000 GeV, are given respectively by (we use a linear interpolation of the results of ref. 7)

\[m_H = 70 \text{ GeV} \quad \epsilon_3^\text{rad} = 4.925 \times 10^{-3}\]
\[m_H = 300 \text{ GeV} \quad \epsilon_3^\text{rad} = 6.115 \times 10^{-3}\]
\[m_H = 1000 \text{ GeV} \quad \epsilon_3^\text{rad} = 6.65 \times 10^{-3}\]  

Therefore new physics contributing to \(\epsilon_3\) cannot be larger than a few per mill. Since \(\epsilon_3\) appears in \(Q_W\) multiplied by a factor 102, this kind of new physics which contributes through \(\epsilon_3\) cannot contribute to \(Q_W\) for more than a few tenth. On the other side the discrepancy between the SM and the experimental data is given by (for a light Higgs)

\[Q_W^{\text{expt}} - Q_W^{\text{SM}} = 1.28 \pm 0.46\]  

where we have added in quadrature the uncertainties. In order to reproduce such an effect with new physics contributing to \(\epsilon_3\) we would need \(\epsilon_3 = (-12.5 \pm 4.5) \times 10^{-3}\), just an order of magnitude bigger than what possibly allowed from high energy. Therefore, if we believe that the latest experiment and the theoretical evaluations of the atomic structure effects are correct, we need new physics of a type which is not constrained by the high energy data.

Before discussing this item let us see which bounds the data put on \(\delta_N Q_W\).

The deviation from the data is a function of \(m_H\) given by

\[\Delta Q_W = Q_W^{\text{expt}} - Q_W(m_H) = 0.78 + 102\epsilon_3^\text{rad} - \delta_N Q_W\]  

with an uncertainty of 0.46. We thus get the following bounds

\[(0.78 + 102\epsilon_3^\text{rad}) - 0.46c \leq \delta_N Q_W \leq (0.78 + 102\epsilon_3^\text{rad}) + 0.46c\]  

where \(c\) specifies the confidence level, \((c = 1.96 \text{ for } 95\% \text{ CL and } c = 2.58 \text{ for } 99\% \text{ CL})\). Notice that \(\epsilon_3^\text{rad}\) increases with \(m_H\) and therefore the upper and lower bounds also increase with \(m_H\). For example, for the choice \(m_H = 70 \text{ GeV}\) (the direct experimental bound on \(m_H\) is larger than this value, but this is just to make an example of what is going on), we get

\[95\% \text{ CL}, \quad 0.38 \leq \delta_N Q_W \leq 2.18\]
\[99\% \text{ CL}, \quad 0.10 \leq \delta_N Q_W \leq 2.47\]
whereas for $m_H = 300 \text{ GeV}$ the bounds are

\begin{align*}
95\% \text{ CL}, \quad &0.50 \leq \delta_N Q_W \leq 2.30 \\
99\% \text{ CL}, \quad &0.22 \leq \delta_N Q_W \leq 2.59
\end{align*}

(10)

In particular we see that a negative or zero contribution to $Q_W$ from new physics is excluded at 99% CL. In view of the preceding observation this result gets stronger when increasing the Higgs mass. Therefore, if one assumes that the new result on atomic cesium is not due to some statistical fluctuation and that the theoretical errors have not been underestimated in some way, one would formally be lead to conclude that the SM is excluded at 99% CL.

Let us now look at models which, at least in principle, could give rise to a sizeable modification of $Q_W$. In ref. [8] it was pointed out that models involving extra neutral vector bosons coupled to ordinary fermions can do the job (there is an extensive literature on the phenomenology of additional $Z$-bosons [9]). In fact it was shown that the corrections to $Q_W$ are given in these models by

\begin{align*}
\delta_N Q_W &= 16 \left\{ \frac{1}{16} \left[ \left( 1 + 4 \frac{s_\theta^4}{c_\theta^2} \right) Z^2 - N \right] \Delta \rho_M \right. \\
&\quad + \left. \left[ (2Z + N) (a_\phi v'_u + a'_\phi v_u) + (Z + 2N) (a_\phi v'_d + a'_\phi v_d) \right] \xi \right. \\
&\quad + \left. \left[ (2Z + N) a'_\phi v'_u + (Z + 2N) a'_\phi v'_d \right] \frac{M^2_Z}{M^2_{Z'}} \right\} \\[5pt]
&\quad + \left[ (2Z + N) a'_\phi v'_u + (Z + 2N) a'_\phi v'_d \right] \frac{M^2_Z}{M^2_{Z'}} \xi
\end{align*}

(11)

where $\xi$ is the mixing angle, $a_f, v_f, a'_f, v'_f$ are the couplings of $Z$ and $Z'$ to fermions, and $\Delta \rho_M$ is an additional contribution to the $\rho$ parameter arising from the mixing [8]

\[ \Delta \rho_M = \sin^2 \xi \left( \frac{M^2_{Z'}}{M^2_Z} - 1 \right) \]  

(12)

The first term in $\delta_N Q_W$ arises from the modification of the $\rho$ parameter, which induces also a redefinition of $s_\theta^2$. The second is due to the $Z - Z'$ mixing, whereas the third one is due to the direct exchange of the $Z'$.

The high energy data at the $Z$ resonance strongly bound the first two terms which depend on the mixing, but they say nothing about the third contribution. Therefore, in order to get an insight about the order of magnitude of the effects due to this kind of new physics which is not mainly expressed in the mixing, we will concentrate only on the last term, since we
already know that the first two terms would be unable to give corrections to $Q_W$ of the right order of magnitude. For this reason we will assume for simplicity in the following calculations $\xi = 0$. In this case $\delta_N Q_W$ is completely fixed by the $Z'$ parameters: its couplings to the electron and to the up and down quark, and its mass.

We will discuss three classes of models: the left-right (LR) models, the extra-U(1) models, and the so-called sequential SM models (that is models with fermionic couplings just scaled from those of the SM). The relevant couplings are given in Table 1.

Table 1: Vector and axial-vector coupling constants for the determination of $\delta_N Q_W$ for the various models considered in the text. The SM couplings are for the sequential SM. The different extra-U(1) models are parameterized by the angle $\theta_2$, and in the table $c_2 = \cos \theta_2$, $s_2 = \sin \theta_2$. This angle takes a value between $-\pi/2$ and $+\pi/2$.

| SM          | Extra-U(1)                                      | LR                      |
|-------------|-------------------------------------------------|-------------------------|
| $a_e = \frac{1}{4}$ | $a_e' = \frac{1}{4}s_\theta \left(-\frac{2}{3}c_2 + \sqrt{\frac{5}{3}}s_2\right)$ | $a_e' = -\frac{1}{4}\sqrt{c_{2\theta}}$ |
| $v_u = \frac{1}{4} - \frac{2}{3}s_\theta^2$ | $v'_u = 0$                                       | $v'_u = \frac{1}{4} - \frac{2}{3}s_\theta^2 / \sqrt{c_{2\theta}}$ |
| $v_d = -\frac{1}{3} + \frac{1}{3}s_\theta^2$ | $v'_d = \frac{1}{4}s_\theta \left(c_2 + \sqrt{\frac{5}{3}}s_2\right)$ | $v'_d = \frac{1}{4} - \frac{1}{3}s_\theta^2 / \sqrt{c_{2\theta}}$ |

In the case of the LR model [10] we get a contribution

$$\delta_N Q_W = -\frac{M_{Z'}^2}{M_{Z'}^2} Q_W^{SM}$$

For this model one has a 95% lower bound on $M_{Z'}$ from Tevatron [11] given by $M_{Z'} \geq 630$ GeV implying, for $70 \leq m_H (GeV) \leq 1000$, $\delta_N Q_W \leq 1.54$. Therefore a LR model could explain the data allowing for a mass of the $Z'$ varying between the intersection from the 95% CL bounds deriving from eq. [10]

$$m_H = 70 \, GeV \quad 529 \leq M_{Z'} (GeV) \leq 1267$$

$$m_H = 300 \, GeV \quad 515 \leq M_{Z'} (GeV) \leq 1105$$

(14)
and the lower bound of 630 GeV.

In the case of the extra-U(1) models \[12\] the experimental bounds for the masses vary according to the values of the parameter $\theta_2$ (see Table 1), but in general they are about 600 GeV at 95% CL \[11, 13\]. From eq. (11) with $\xi = 0$ we can easily see that the models with $\theta_2$ in the interval $-0.66 \leq \theta_2 \text{(rad)} \leq 0.25$ give $\delta N Q W \leq 0$, and therefore they are excluded at the 99% CL. In particular the model known in the literature as the $\eta$ or $A$ model, which corresponds to $\theta_2 = 0$, is excluded.

The bounds in eqs. (9) and (10) at 95% CL can be translated into lower and upper bounds on $M_{Z'}$. The result is given in Fig. 1, where the bounds are plotted versus $\theta_2$. In looking at this figure one should also remember that the direct lower bound from Tevatron is about 600 GeV at 95% CL, leading to an exclusion region $-0.84 \leq \theta_2 \text{(rad)} \leq 0.43$ for $m_H = 70$ GeV and $-0.89 \leq \theta_2 \text{(rad)} \leq 0.48$ for $m_H = 300$ GeV.

The last possibility we consider is a sequential SM. In this case we assume that the couplings are the ones of the SM just scaled by a common factor $a$. Therefore we get

$$\delta N Q W = a^2 \frac{M_Z^2}{M_{Z'}^2} Q_{W}^{SM}$$

We see that no matter what the choice of $a$ is, the sign of the new physics contribution turns out to be negative. Therefore all this class of models are excluded at 99% CL.

This result can be trivially extended to certain models based on extra dimensions which have a tower of Kaluza-Klein resonances of the $Z$ all coupled as the $Z$, except for a factor $a = \sqrt{2}$ \[14\]. Taking also into account the modification of the Fermi constant $G_F$ due to the KK excitations of the $W$, the eq. (12) gets an additional positive factor $s_{\theta}^2$. The effect of having an infinite tower, if any, worsens the situation. For instance, in the case of one extra-dimension, one gets a further factor of $\pi^2/6$. Therefore the experimental result on atomic parity violation in atomic cesium excludes at a very high confidence level these simple-minded extra-dimension models of additional bosons $Z'$.

Another interesting possibility one can analyze is that of a four-fermion contact interaction, which could arise from different theoretical origins. Also this case has no visible effects at the $Z$ peak. We will follow the analysis and the notations of ref. \[15\]. In this situation it turns out to be convenient to
express the weak charge as

$$Q_w = -2 \left[ c_{1u}(2Z + N) + c_{1d}(Z + 2N) \right]$$

(16)

where $c_{1u,d}$ are products of vector and axial-vector couplings. We will consider models with a contact interaction given by

$$\mathcal{L} = \pm \frac{4\pi}{\Lambda^2} \bar{e} \Gamma_{\mu} e \bar{q} \Gamma_{\mu} q, \quad \Gamma_{\mu} = \frac{1}{2} \gamma_{\mu} (1 - \gamma_5)$$

(17)

This leads to a shift in the couplings given by

$$c_{1u,d} \rightarrow c_{1u,d} + \Delta C, \quad \Delta C = \pm \frac{\sqrt{2\pi}}{G_F \Lambda^2}$$

(18)

Since a variation of the couplings induces a variation of $Q_w$ of opposite sign, we see that the choice of the negative sign in the contact interaction is excluded. In the case of the positive sign, using the 95% CL bounds given in eq. (9) we get

$$11.8 \leq \Lambda (TeV) \leq 28.2$$

(19)

Let us now consider a contact interaction induced by lepto-quarks. Following again ref. [15], we take the case of so-called $SU(5)$-inspired lepto-quarks, leading to the interaction

$$\mathcal{L} = \frac{\eta_L^2}{2M_S^2} \bar{e}_L \gamma_{\mu} e_L \bar{u}_L \gamma^\mu u_L + \frac{\eta_R^2}{2M_S^2} \bar{e}_R \gamma_{\mu} e_R \bar{u}_R \gamma^\mu u_R$$

(20)

From the constraints on $\pi e_2/\pi \mu_2$ one expects $\eta_L \approx 0$ or $\eta_R \approx 0$. Only the coupling $c_{1u}$ has a shift

$$c_{1u} \rightarrow c_{1u} + \Delta C, \quad \Delta C = \pm \frac{\sqrt{2\eta_{L,R}^2}}{8G_F M_S^2}$$

(21)

It follows that the shift on $Q_w$ is negative for $\eta_R \neq 0$. Therefore only the left coupling is allowed ($\eta_R = 0$). In that case we get the bounds (again from eq. (9))

$$1.6 \leq \frac{M_S (TeV)}{\eta_L} \leq 3.9$$

(22)
If one assumes \( \eta_2^2 \approx 4\pi\alpha \), it follows

\[
0.48 \leq M_s(\text{TeV}) \leq 1.17
\]  

(23)

Constraints on four-Fermi contact interactions from low energy electroweak experiments have been also considered in [14].

In conclusion, we have assumed the validity of the new determination of the weak charge of atomic cesium, which represents a big improvement with respect to the previous result on this important quantity, and we have analyzed some of its theoretical implications. We have shown that the present data imply positive upper and lower bounds on the new physics contribution to the weak charge, \( \delta N Q_W \). It turns out from our analysis that new physics contributing to both high and low energy precision electroweak data cannot easily reproduce the experimental situation, due to the existing experimental constraints from the high energy precision data. In principle it is possible to explain the new data with new neutral vector bosons which are un-mixed (or almost so) with the \( Z \) and have a sizeable couplings to the fermions. In this way the \( Z' \) contributes only to the low energy physics, allowing, in principle, to explain the new data on parity violation, without contradicting the \( Z \)-physics results from LEP and SLD. The lower positive bound on such new physics effects to the weak charge would have striking consequences. In fact it would forbid zero or negative \( \delta N Q_W \) at more than 99% CL. That is, if one takes the data seriously, the SM and extra-vector boson models with \( \delta N Q_W < 0 \) would seem to be excluded at 99% CL. In particular, models with \( Z' \) couplings to fermions obtained via a simple scaling from the SM couplings (sequential SM) would be excluded. In fact, since \( Q_W < 0 \), they would give a negative contribution. Therefore, also simple-minded models from extra-dimensions describing a tower of \( Z \)-like excitations would be excluded at 99% CL. Also, the existence of the positive lower bound would imply for the allowed models an upper limit on the \( Z' \) mass. We have also analyzed the implications of the new data for possible four-fermion contact interactions of different origins.

Of course, one should avoid drawing general conclusions, since more data will be necessary and because only some typical models have been considered here. We can only repeat, with Gustave Flaubert, in his correspondence with Louis Bouilhet: Nous sommes un fil et nous voulons savoir la trame. Contentons-nous du tableau, c’est aussi bon.
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Figure 1: The 95% CL lower and upper bounds for $M_{Z'}$ for the extra-U(1) models versus $\theta_2$. The continuous and the dashed lines correspond to $m_H = 70 \text{ GeV}$ and $m_H = 300 \text{ GeV}$ respectively.