What is erroneous worked example and how is it looks like in social arithmetic?

I A Kurnia¹, E Retnowati ²

¹ Graduate School of Mathematics Education, Universitas Negeri Yogyakarta, Sleman, Indonesia
² Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Negeri Yogyakarta, Sleman, Indonesia

Email: ¹isokaamanah.2017@student.uny.ac.id, ²e.retno@uny.ac.id

Abstract. Erroneous worked example is a part of worked example with some part of the solution intentionally given wrong or incorrect. Through erroneous worked example, students are asked to learn from the example given, find errors, then correct it. This method is believed to help students avoiding mistakes in the future. One of the materials that considered to be difficult in grade VII is social arithmetic because it mostly given in the form of word problems and requires some initial concepts such as basic operation, percentage, currency, relevant equation, etc. Research studies have found that students often made mistakes in solving social arithmetic problems. Erroneous worked example might become an alternative to help students making less mistakes, but it is also very important to know how to design a good erroneous example in order to make it works well with students. This review study focused on erroneous worked example and types of errors occurred when students solving social arithmetic or word problems. Those types of error used to design an instructional example in social arithmetic field. If the design is good then erroneous worked example would help students to systemically evaluate their mistakes and improve their problem-solving skill.

1. Introduction

In the last few decades, education system in Indonesia has gone through a crucial process of paradigm transition from teacher-centered to student-centered approach. It is crucial because learning is no longer focused on learning tools (teacher-centered), but rather focuses on student activities constructing their own knowledge through classroom learning activities [1]. Student-centered learning is believed to raise students' self-confidence in carrying out learning tasks. During the learning process, the teacher can monitor the needs and interests of each student as individuals and in groups in order to maximize student involvement in learning activities [2].

Learning design must be adjusted to the learning materials. If there is no supporting information or prior knowledge provided, students who learn new material or usually called novice tend to have difficulty in understanding the material. This absence of initial knowledge usually found in learning with problem-solving method. Expert students who already have sufficient initial knowledge can use their knowledge to solve problems through problem-solving method [3].

Undoubtedly, problem-solving is one of the methods that is very frequently used in mathematics learning nowadays. In problem-solving, students are asked to solve a given problem using their initial knowledge without any hints, guidance, or example provided in the worksheets. Their learning
experience using problem-solving is quite good for students with enough prior knowledge, but how is it going with students with low prior knowledge?

Many research and studies have shown that the implementation of problem-solving is sometimes becoming not effective in some conditions, especially for novice students. Novice students with less prior knowledge may find difficulties in carrying out the materials without having sufficient information. Expertise students use their information and knowledge to solve problems and also tend to be able to solve more complex problems [3]. This issues are related to students’ cognitive system wrapped up as in cognitive load theory (CLT).

CLT was originally built by John Sweller as a follow-up from the previous studies focused on the limitation on using problem-solving for students with low prior knowledge or novice students. Three components in CLT namely intrinsic, extraneous, dan germane loads. The essence of intrinsic cognitive load is the complexity of the material, while extraneous load could be defined as all the activities that consume students’ working memory capacity but did not have the relevancy with learning goals. In the cognitive process, the germane load help students constructing new ideas and knowledge then store it to the long-term memory [4].

The point of those three components is how to design a learning experience with some level of complexity but can be easily understood by students, in order to minimize the extraneous load and give more space of working memory during the cognitive process. Creating instructional design that supports generative processing without burdening students with too many processes that possibly spend too much space of working memory is a big challenge for teachers. One of the methods recommended by CLT studies is worked example.

Worked example which consists of a problem given, step by step solution, and the final result has been used effectively to suppress the extraneous load process. This method believed to avoid using working memory excessively by searching for prior knowledge that students do not have yet. The greater the space of free working memory capacity, students could activate the germane load process easily. Therefore, students are able to build and store new knowledge in long-term memory, and then use that knowledge for problem-solving procedures [5]. To increase the effectiveness of worked example, a number of studies have evaluated the application of worked example and develop some innovations. One of the innovations from the worked example is placing errors in the solution steps presented, also known as erroneous worked examples [6].

Error is basically not a new issue because it could be happen to everyone in almost every activities. However, people may have different perception towards error, especially about talking error in class. Some teachers considered error as something that should never be mentioned during the learning process. Carol Dweck, a famous professor of psychology, stated that the teacher’s negative perception about errors would increase students’ failures as they misunderstand that they cannot learn from their mistakes. If error discussion happened in the class, students are likely improve their concepts to learn by their mistakes [7]. Talking errors in class are good to encourage students to be sensitive to errors in order to develop a positive learning environment [8].

Considering the importance of error discussion in class, students need to engage in a learning that supports error discussion, and erroneous worked example is a solution that fits the needs. Therefore, we need to know more about erroneous worked example. The details are discussed in this paper.

In Indonesia, students of grade VII has introduced to social arithmetic. This material contains a number of studies have evaluated the application of worked example and develop some innovations. One of the innovations from the worked example is placing errors in the solution steps presented, also known as erroneous worked examples [6].

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In order to make the erroneous example works successfully with students who learn social arithmetic, it is also very important to pay attention to how the erroneous example is constructed. In other words, designing erroneous example is about how to place errors in the example so students will notice the error and then correct it. The focus on this paper would be what is the erroneous worked example and how to design erroneous example in the social arithmetic field. To reach the discussion about how to design a good erroneous worked example in social arithmetic, it is necessary to review erroneous worked example generated by some experts in education and psychology. Definition, background, phase, etc are discussed in the next sub-chapter.

Designing learning instruments should be based on something to make it effective for students. In erroneous worked example, error placing is needed to make the example become an incorrect one. In order to design an erroneous worked example to learn decimals, the focus should be on the most common or constant errors and misconceptions made by students while solving decimal problems [9]. Therefore, in social arithmetic area, we can also use the results and studies of students’ error and misconceptions happened when students solve social arithmetic problems.

Since there might be limited sources to specifically discussed about social arithmetic material, this paper also reviewed the errors and misconceptions of students while solving word problems. Word problems in mathematics is a problem served in form of a story sentences and must be transformed into mathematics form or equations, and solved by numerical skills or computation [10]. Based on the description, social arithmetic problems are categorized as word problems too.

After reviewing what is erroneous worked example and common errors or misconceptions in social arithmetic, we can analyze some principals to design the erroneous worked example.

2. Erroneous Worked Example

Erroneous worked example is a step by step solution to a problem with one part of the steps there is an incorrect step or error. Studies of using erroneous worked example have shown that learning through mistakes in one or several steps can improve students’ thinking ability on mathematical concepts [11]. Based on the explanation, erroneous worked example is basically a part of worked example, but there are errors that are deliberately raised in the solution steps presented.

Learning through erroneous worked examples is expected to improve students’ precision and help students to be more focus to learn the example given then be able to solve another problem correctly. Learning through erroneous worked examples can help students increase their sensitivity in detecting errors so students can avoid the same possible error in the future [12]. While identifying errors and find out why the solution is error, erroneous worked example can also encourage students to explain the example to themselves then correct the error so it becomes the correct example [13].

The results of previous studies have shown that erroneous examples can facilitate mathematics learning [5]. Students who learn by comparing correct and incorrect examples when studying decimal numbers produce better understanding than students who only learn through the correct example. Expert opinion and research findings regarding the erroneous worked example above show that there is nothing wrong with students learning through the erroneous example, on one condition that students also knowing the correct one [14]. If students only learn the incorrect example without knowing how to do it in the right way, students may misunderstand and think that it was the correct one.

An advantage from learning through erroneous worked example is that this method can produce a deeper learning experience because students have to identify the errors parts then correct these errors [13]. Erroneous worked example can also help students to focus on each step of the solution more while looking for errors. This is very important on the learning process because when students learn using problem-solving approach, students who have studied the example must be able to understand it properly in order to have sufficient knowledge to solve the problem.

There are two phases of learning through worked example, the first one is the introductory phase then followed by the acquisition phase. Introduction phase (introductory) is the initial stage of learning where students learn the prerequisites material that should have been mastered before receiving new material. The purpose of this stage is to generate students’ prior knowledge that will be integrated into
new problems [15]. Acquisition phase is the main stage of learning through worked example. The prerequisite materials given in the previous stage are used in this stage. Students learn from some examples then asked to solve problem that is very similar to the examples. Through many examples given, students will get an overview of strategies that can be used to solve problems so that they are more familiar with the various possible problems that appear in the material.

Since erroneous worked example is a part of worked example, these two phases are also becoming the phases of learning with erroneous worked example. The additional steps are students have to look for errors and then correct them. Erroneous worked example consist of two stages: error detection and error correction. Therefore, when students learn through erroneous worked examples, students are not only asked to find errors but are also correct them so that it becomes the correct worked example [12].

3. Common mistakes in solving mathematics problems

There are some popular procedures to analyze students’ error in solving problems. One of them is developed by Anne Newman, and it became popular as Newman Error Analysis (NEA). There are five stages of NEA, each stages have different error characteristic that commonly happened in solving word problems. Table 1 below shows the stages, description of what students do during the stage, and characteristics of errors or mistakes that possibly occurred according to NEA procedure.

| Stages                  | Description                                                         | Characteristics                                           |
|-------------------------|---------------------------------------------------------------------|-----------------------------------------------------------|
| Reading (decoding)      | Students read and understand the word problems                      | • Not be able to understand the terms                      |
| Understanding (comprehension) | Students determine the purpose of the problem                      | • Not be able to write unknowns information                |
| Modeling (transformation) | Students choose methods, strategies, or formula to solve problems  | • Not be able to choose the right methods, strategies, or formula |
| Process Skills          | Students are solving the problems that have been modeled with rules, procedures, algorithm, etc | • Not be able to write a mathematical model based on the problem |
| Encoding                | Write the answers with full unit of attribute                       | • Error in calculation                                    |
|                         |                                                                     | • Error in rewrite what is asked                           |

These stages are in line with Ballew and Cunningham who classified four different abilities involved in solving word problems; ability to read problems, ability to set up the problem so the computation is ready to performed, ability to perform the necessary computation, and ability to integrate reading, interpreting, and computation to find the solution [16]. Moreover, there are also two common mistakes that occurred in solving mathematics problem in general, namely conceptual and procedural error which has the details as in table below [17].
Table 2. Type of error in solving mathematics problem

| Type of Error | Characteristics                                                                 |
|---------------|----------------------------------------------------------------------------------|
| Conceptual    | • Not be able to determine the proper formula, theorem, or definition             |
|               | • Using the formula, theorem, or definition in wrong condition                    |
|               | • Not be able to recognize what is known                                        |
|               | • Not include the theorem of formula used in the solution                        |
| Procedural    | • Not be able to solve the problems in the systematic way                        |
|               | • Not be able to do manipulation                                                  |

More specifically, a research has generated some type of errors in solving social arithmetic problems, which divided into three stages [18]. The details are described in the table below.

Table 3. Stages of error in solving social arithmetic problems

| Stages       | Characteristics                                                                 |
|--------------|----------------------------------------------------------------------------------|
| Error Stage I| • Reading/translating error                                                      |
|              | • Not be able to understand what is known and unknown in the problem             |
|              | • Unable to recognize what is known                                              |
|              | • Symbol error                                                                  |
|              | • Unable to transform the sentence into a mathematical model                     |
| Error Stage II| • Lack of prior knowledge                                                       |
|              | • Not be able to plan a solution                                                  |
|              | • Error in carry out the plan                                                    |
| Error Stage III| • Computation error                                                            |
|              | • Wrong conclusion                                                              |

Based on Table 1, Table 2, and Table 3 we can see that there are some similarity of the characteristics of error that commonly occurred. Reading error is originated from the student itself and it cannot be predicted, so it is difficult to design an erroneous example based on the possibility of reading error. The other characteristics that still possible to construct on the worked example design are errors in calculation, conceptual, choosing formula/method/theorem/concept, solution procedure, and making conclusion.

4. Instructional design of erroneous worked example in social arithmetic

There are many ways to classify, categorize, or describe students’ mistakes in solving arithmetic problems. In this paper, instructional design of erroneous worked example is using the classification as shown in table 2, conceptual and procedural errors. These classification is considered to be very simple to put possible errors in the worked example.

4.1. Erroneous worked example based on conceptual error

Conceptual errors can be expressed in various forms, including writing incorrect formulas, misusing concepts, applying incorrect formulas to a condition, or writing incorrect mathematical model. In social arithmetic, misconceptions can occur for example when students learn about discounts or sale.
The price of an item after getting a discount should be less than the original price. Thus, the final price of an item is the result of a subtraction the original price with the amount of discount. The concept will be incorrect if students add up the discount at original price and the final result will be bigger. What does it looks like? First, we have to make a problem related to discount.

Problem:
Latifah wants to buy a school bag as a birthday present for her sister. Her sister wanted a bag that has the color combination of black and red. Latifah entered a shop that sells various types of school bags. However, there are two bags that are black and red but has different brand, namely Eureka and Godel. The price of Eureka bag is Rp190,000.00 and the Godel brand costs Rp180,000.00. The store gives 15% discount for Eureka bags and 10% discount for Godel bags. Since both of them are meet the requirement, Latifah will buy the one with lowest price. Which bag should Latifah choose?

From the problem we get the situation that Latifah has to choose one of two different bags which have the lowest price. Since both of the brands offering discount, the goals is to calculate the final price of each bag using the concept of discount in social arithmetic. The main principal in discount concept is the price after discount always cheaper than the original price on the tag. If the price after discount is higher, than it must be something wrong on the concept. The example of correct and incorrect (erroneous) worked example is shows as in table below.

| Correct Worked Example | Erroneous Worked Example |
|------------------------|--------------------------|
| Price of Eureka bag = 190,000 (Disc 15%) | Price of Eureka bag = 190,000 (Disc 15%) |
| Price of Godel bag = 180,000 (Disc 10%) | Price of Godel bag = 180,000 (Disc 10%) |
| Discount of Eureka = $\frac{15}{100} \times 190,000 = 28,500$ | Discount of Eureka = $\frac{15}{100} \times 190,000 = 28,500$ |
| Discount of Godel = $\frac{10}{100} \times 180,000 = 18,000$ | Discount of Godel = $\frac{10}{100} \times 180,000 = 18,000$ |

*(the final price is determined by subtracting the original price with the amount of discount)* *(the final price is determined by adding the original price with the amount of discount)*

Final price of Eureka bag = 190,000-28,500 = 161,500
Final price of Godel bag = 180,000 - 18,000 = 162,000

After getting discount, Eureka bag get the lowest price, so Latifah should choose the Eureka bag as a birthday present to her sister.

As we can see in table 4 above, the conclusion will become incorrect if the concept is wrong. This should lead students to think that one mistake could bring them into another mistakes, so they have to be very careful in using concepts. There are a lot of topic discussed in social arithmetic, and the design could be constructed in other types of error.
4.2. Erroneous worked example based on procedural error

This type of errors are mostly occurred in form of calculation error. In general, procedural errors happened when the use of concepts, principle or formulas may be correct and suitable for the conditions, but the mistakes could appears when students working on the procedures or operation needed.

Take one submaterial of social arithmetic, profit and loss. In common selling and buying problems, there are two possible conditions. If the sales results are greater than the capital (modal), the seller would gain profits. Otherwise, if the sales are lower, then the seller have to gain some loss. If students have mastered the concepts very well and successfully avoiding conceptual mistakes, it is possible if they make mistakes in calculating some number operations. As in the previous subchapter, we have to prepare a problem related to profit and loss.

Problem:
Mrs. Eka is a mat maker in Sleman. She received order to make 20 mats for a traditional restaurant near the city. The cost of making one mat is Rp117.000. In order to complete the orders in time, Mrs. Eka gets help from three workers with get paid Rp50.000 for each. If Mrs. Eka sets the price Rp145.000 for each mat, what percentage of profit she will earn?

The problem gives information about the making of mat and the selling price. The goal is to determine the percentage of profit Mrs. Eka will get. The table below shows the example of instructional design of correct and erroneous worked example based on procedural error in solving this problem.

**Table 5. Correct and erroneous worked example based on procedural error**

| Correct Worked Example | Erroneous Worked Example |
|------------------------|--------------------------|
| Cost of making a mat = **117.000** | Cost of making a mat = 117.000 |
| Cost of making 20 mats = **20 × 117.000** = **2.340.000** | Cost of making 20 mats = 20 × 117.000 = 2.340.000 |
| Selling price for a mat = **145.000** | Selling price for a mat = 145.000 |
| Selling price for 20 mats = **20 × 145.000** = **2.900.000** | Selling price for 20 mats = 20 × 145.000 = 2.900.000 |
| Salary for 3 employees = **3 × 50.000** = **150.000** | Salary for 3 employees = 3 × 50.000 = 150.000 |
| Profit = Selling price – capital | Profit = Selling price – capital |
| ⇔ Selling price – (costs + salary) | ⇔ Selling price – (costs + salary) |
| ⇔ 2.900.000 – (2.340.000 + 150.000) | ⇔ 2.900.000 – (2.340.000 – 150.000) |
| ⇔ 2.900.000 – 2.490.000 | ⇔ 2.900.000 – 2.190.000 |
| ⇔ 410.000 | ⇔ 710.000 |
| Percentage of profit = \( \frac{Profit}{Capital} \times 100 \) | Percentage of profit = \( \frac{Profit}{Capital} \times 100 \) |
| ⇔ \( \frac{410.000}{2.490.000} \times 100 \) | ⇔ \( \frac{710.000}{2.490.000} \times 100 \) |
| ⇔ 16.47 (approximation) | ⇔ 32.42 (approximation) |
Therefore, Mrs. Eka gets 16.47% profit. Therefore, Mrs. Eka gets 32.42% profit.

When students learn through the erroneous example as shown in table 5, they might be aware of this type of error. If they mastered the concept and write it down correctly on the worksheets, it is still possible to make mistakes by simply put the wrong symbol of basic operation. Symbol error will lead them to another mistakes, such as the final result and conclusion at the end of the solution. Facing the students with some possible procedural errors will improve they precision to solve problems in the future. This is likely also improve their problem solving skills because they are already aware to some errors and mistakes that could be occurred in the solution.

The use of erroneous worked example is providing information to help students learn the material. In other words, worked example is same as lending teacher’s knowledge to students, but still provides an opportunity for students to learn it independently.

It’s not true that worked example is against student-centered. Students are allowed to use their own strategy to solve the problems even if it is not described in the examples. Students are allowed to create new alternative of using the concept as long as there’s no misconceptions. This method is basically has purpose to press the use of working memory. When students are exhausted, the next activities will be tough for students. If they are running out of interest and focus, it is likely they will fail to build knowledge and the learning will become meaningless.

Beyond the original worked example, erroneous worked example added another function that is to train students aware for the possible errors of mistakes that could happened. If they notice the patterns and the holes that mostly error in the example given, students should be more careful in those part of solution. Since error is very important to mentioned in the class during the learning process, erroneous worked example will be a perfect alternative to be used in the mathematics learning.

5. Conclusion
Erroneous worked example is one of the alternatives in learning mathematics by presenting examples (problems and step-by-step solution) by placing errors in a part of the solution. While identifying errors and find out why the solution is error, erroneous worked example can facilitate students self-explanation [13]. Students are expected to increase awareness of possible mistakes when solving similar problems in the future. To design an erroneous worked example, mistakes or errors that often occurred when students solve arithmetic social problems can be used as our guidelines. Some types of errors that can be used as guidelines are reading errors, conceptual errors, using formula errors, calculating errors, making a settlement plan error, and concluding errors. Erroneous worked example designed to encourage students to compare incorrect and correct solutions, improve their self explaining about why the solution is incorrect, and then finally lead them to solve the problem correctly [9]. If the erroneous example is well prepared, then it is likely the instruments could facilitate students in learning social arithmetic, help students to avoid mistakes, and improve their problem-solving skills.

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