Thermally induced gaplessness and Fermi arcs in a “s-wave” magnetic superconductor

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An electron system with pre-existing local moments and an effective electron-electron attraction can exhibit simultaneous magnetic and superconducting order. Increasing the magnetic coupling weakens pairing and the ground state loses superconductivity at a critical coupling. In the vicinity of the critical coupling magnetic order dramatically modifies the quasiparticle dispersion in the superconductor, creating low energy spectral weight and significant gap anisotropy in the notional ‘s-wave’ state. Using a Monte Carlo approach to the Hubbard-Kondo lattice problem we establish a thermal phase diagram, for varying magnetic coupling, that corresponds qualitatively to the borocarbide superconductors. In addition to the superconducting and magnetic transition temperatures, we identify two new thermal scales in this nominal s-wave system. These are associated, respectively, with crossover from gapped to gapless superconductivity, and from an anisotropic (nodal) ‘Fermi surface’ at low temperature, through a Fermi arc regime, to an isotropic Fermi surface at high temperature. Some of the spectral effects are already visible in the Ho and Er based borocaribdes, others can be readily tested.

The interplay of superconductivity and magnetism [1] is of fundamental interest in condensed matter. Prominent examples include the high- Tc cuprates [2], heavy fermions [3], and the iron based superconductors [4]. The superconductivity in the cuprates emerges on doping an antiferromagnetic insulator [2], in the iron-pnictide it emerges from a collinear antiferromagnet [4], in the iron chalcogenide from bicollinear antiferromagnets [5], while iron selenide superconductors are proximate to an antiferromagnetic insulator [6]. In many of these compounds superconductivity is dictated by off-site d-wave type pairing while the magnetic moments arise from electron-electron repulsion. Relatively less explored is the interplay of s-wave pairing and magnetic order in, e.g., the rare earth quaternary borocarbides (RTBC), a traditional phonon mediated superconductor [7–13].

The combination of magnetic and superconducting ordering tendencies lead to an unusual electronic state. Evidence of an unconventional superconducting gap in the RTBCs has been found in thermal conductivity [10, 14, 15], and ultrasonic attenuation experiments[16]. Direct evidence was recently provided by angle resolved photoemission spectroscopy (ARPES) [17], and point contact spectroscopy [11, 18] in YNi₂B₂C and LuNi₂B₂C - suggesting a superconducting gap with point nodes. HoNi₂B₂C shows gapless superconductivity at finite temperature [11] while ErNi₂B₂C has a gap structure which deviates significantly from the BCS prediction [19].

Considerable effort has been invested in analyzing the ground state of these materials [20–28], with the rare earth deGennes (DG) factor, S(S + 1), where S is the angular momentum, mimicking a changing magnetic coupling. These studies suggest the coexistence of non collinear magnetic order with superconductivity. The superconducting (SC) state becomes gapless at a critical magnetic coupling J_c, say, and at a still larger coupling, J_c ~ 2J_k, superconductivity is destroyed. At J > J_k the dispersion comprises of as many as eight branches (if the non magnetic SC had only two bands), the density of states exhibit additional van Hove singularities, and the low energy spectral weight maps out a non trivial “Fermi surface” in these superconductors [28].

However, there seems to be little work that addresses the simultaneous effect of magnetic and superconducting thermal fluctuations in these superconductors, particularly the effect of thermally induced magnetic disorder. The classic Abrikosov-Gorkov (AG) theory [29] describes the impact of random uncorrelated moments on the superconductor. It applies to the regime where the moment concentration η, electron-moment coupling J, and pairing gap Δ, satisfy ηJ²N(0) ≪ Δ, N(0) being the normal state density of states at the Fermi level. The theory predicts a window of gapless superconductivity, and finally the loss of SC order, on increasing ηJ². In case of RTBC the moments are on every site, so η = 1, the J and Δ are comparable, and the moments have an ordered low temperature state. The relatively large J means that magnetic effects cannot be treated perturbatively, while spatial correlation between the moments require a sophisticated ‘disorder averaging’.

We use a method that treats the pairing and magnetic effects on equal footing, and retains the spatial correlation between the moments when considering thermal disorder effects on the electrons. Our principal findings are the following. (i) We map out a thermal phase diagram that captures all the qualitative features of the RTBC family and predict two new thermal scales: T_g related to gap closure in the superconductor, and T_an related to the appearance of Fermi surface anisotropy. (ii) We demonstrate the realization of gapless superconductivity, as observed, for parameters corresponding to HoNi₂B₂C. (iii) At moderate magnetic coupling, the scattering from short range magnetic fluctuations leads to a strongly momentum dependent (non s-wave) gap in the superconductor. This provides an understanding of the experimental observations in YNi₂B₂C and LuNi₂B₂C, which reveal an anisotropic SC gap in ARPES measurements [17].

We study the attractive Hubbard model in two dimension on a square lattice in presence of Kondo like coupling [28]:

\[ H = H_0 - |U| \sum_i n_{i\uparrow} n_{i\downarrow} - J \sum_i S_i \sigma_i \] (1)
with, $H_0 = \sum_{ij,\sigma}(t_{ij} - \mu \delta_{ij})c^\dagger_{i\sigma}c_{j\sigma}$, where $t_{ij} = -t$ for nearest neighbor hopping and is zero otherwise. $\mathbf{S}$, is the core spin, arising from $f$ levels, for example, in a real material. $\sigma_i$ is the electron spin operator. $U$ is the attractive on-site interaction, giving rise to s-wave superconducting order, and $\mu$ is the chemical potential. The spin size $S$ dictates the de Gennes factor $\mathcal{S}(S+1)$ in the rare earths. We treat the $\mathbf{S}$ as classical spins, setting the size $|\mathbf{S}| = 1$, and vary $J$ to mimic the varying DG factor. We set $U = 3t$ (although in real materials it is likely to be smaller) due to system size limitations.

We solve this model by (i) decoupling the Hubbard interaction using an auxiliary pairing field and retaining only its static (zero Matsubara frequency) mode, (ii) generating the equilibrium configurations for the pairing field and local moments via Monte Carlo, and (iii) solving the electronic problem in the "disordered" but spatially correlated backgrounds by exact diagonalisation. The method is detailed in the Supplementary materials (SM).

We use a variety of spatial and spectral indicators to characterise the phases of the system. These include: (i) the pairing field structure factor, $S_A(q)$, and magnetic structure factor $S_m(q)$, (ii) the density of states (DOS), $N(\omega)$ and its value $N_0$ at the Fermi level, and (iii) the momentum dependent low energy spectral weight $A(k,0)$. The method for calculating these is discussed in the SM.

Fig.1 shows the thermal phase diagram obtained by our numerical calculations and it’s comparison with the experimental phase diagram of the RTBC family. The theory results correspond to $U = 3t$, and a filling of $n \sim 0.5$.

The ground state at this choice of parameters (see SM) has magnetic order at the boundary of a spiral ($q,\pi$) and an antiferromagnetic ($0,\pi$) state. In the ground state SC order survives upto $J \sim 0.75t$ and comprises of a gapped SC phase for $0 < J < 0.5t$ followed by a gapless regime for $0.5t < J < 0.75t$. For $J > 0.75t$ the ground state is a magnetic metal.

Fig.1(a) shows the thermal phase diagram as inferred from experimental measurements. There are four major phases: disordered moment-superconductor (DM-SC), antiferromagnetic-superconductor (AFM-SC), disordered moment-metal (DM-M), and antiferromagnetic-metal (AFM-M). The effective coupling between the core moments and itinerant electrons increase with spin size $S$, so the relevant magnetic interaction scale is $\sim \sqrt{S(S+1)}$. We use this as our $x$ axis, normalising by the value for Gd. There are two thermal scales: the superconducting transition, $T_c$, and the magnetic transition, $T_{AF}$.

With increasing DG factor members of the RTBC family shows the following behaviour in the ground state: (i) non magnetic superconductors (Lu), with no magnetic moments, (ii) (antiferro)magnetic superconductors (Tm, Er, Ho and Dy) and (iii) magnetic metal (Tb, Gd).

In our theory result, Fig.1(b), there are two main temperature scales: $T_c$ and $T_{AF}$ as in the experiments. There are, however, two additional thermal scales: $T_g$ and $T_{an}$, inferred from the quasiparticle spectra. $T_g$ indicates crossover from gapped to gapless superconductivity with increasing $T$. $T_{an}$ marks the onset of pronounced momentum dependence of the spectral weight at the Fermi level. This is inferred from the behaviour of the spectral function $A(k,0)$, computed from Green’s functions in the finite temperature backgrounds.

Fig.1 indicates that the qualitative features of RTBC physics, particularly the occurrence of the various phases vis-a-vis experiments, is reasonably captured by the theory. To be specific: (i) in Fig.1(b) the AFM-SC phase in the regime $0 < J < 0.6$ includes Tm, Er, Ho and Dy - as is the case with experiments. (ii) Tb and Gd (with large magnetic moments) are indeed AFM-M, with no SC order. This correspondence suggests that the most suitable members to observe gapless superconductivity are Er and Ho. Indeed, a gapless SC state at finite $T$ has already been reported in HoNi$_2$B$_2$C through point contact measurements [11]. In ErNi$_2$B$_2$C point contact spectroscopy [19] shows clear evidence of the SC gap behavior deviating from the BCS predictions. It was suggested that the gap behaviour can be described by a superconducting theory [20] that incorporates magnetic fluctuations.

The effect of magnetic fluctuations on RTBC superconductivity has been demonstrated also through inelastic light scattering [30], photoemission spectroscopy [31], thermal conductivity measurements [15], ultrasonic attenuation [16], ARPES [17] experiments on LuNi$_2$B$_2$C and YNi$_2$B$_2$C which are “non magnetic” members of the RTBC. While the absence of a finite magnetic moment in these materials rule out a competing magnetic long range order in the ground state, short range correlation among thermally induced ‘moments’ is still possible at high temperature. Our present work does not dis-
directly address these compounds, since they involve a ‘soft magnetic moment’ arising from Hubbard repulsion, but the high temperature effects, we guess, would be similar to what we observe here. We will address this separately.

Fig.1(b) however shows that our $T_0^c$ scale is too large compared to the $T_{AF}$ for Gd, an obvious inconsistency vis-a-vis the experiments. This is due to the choice $U = 3t$, forced by computational constraints. In the real RTBC the $U/t \lesssim 1$, but this involves large coherence lengths, difficult to access numerically. For a closer correspondence with experiments, in terms of absolute $T_c$ scales, the pairing interaction $U$ would have to be smaller. This would also require the $J$ to be reduced to ensure that the magnetism does not suppress the superconductivity. Attaining this within a fully microscopic approach is difficult and we plan to explore a Ginzburg-Landau scheme separately.

Fig.2 examines the thermal evolution of the single particle DOS at three magnetic couplings, $\tilde{J}$. At small $\tilde{J}$ is similar to that of a non magnetic $s$-wave superconductor. Increasing $T$ drives the gapped low $T$ state to a pseudogapped high $T$ state. At $\tilde{J}^2 \sim 1$ we observe the featureless low energy DOS of the magnetic metal, with gradual increase in the DOS with increasing temperature. At intermediate $\tilde{J}^2$, where $T_c$ and $T_{AF}$ are comparable, we have a gapless SC ground state (verified also via a Green’s function check [28]). Rise in temperature increases the low energy DOS as the system transits from a AFM-SC to AFM-M and then the DM-M.

In the final panel, Fig.2(d), we show the DOS at the Fermi level, $N_0$, as a function of temperature at different magnetic coupling. The weak $\tilde{J}$ regime shows vanishing $N_0$ over a finite $T$ window. At intermediate coupling there is a finite DOS at the Fermi level, giving rise to gapless SC, and a prominent $T$ dependence, while at large $\tilde{J}$ the $N_0$ is large but only weakly $T$ independent.

We next show the emergence of anisotropy in momentum dependence of the low energy spectral weight with lowering temperature, Fig.3, by plotting $A(k,0)$, the spin summed weight at $\omega = 0$. Both point contact spectroscopy as well as ARPES measurements carried out on rare earth borocarbide family gave evidence of considerable deviation of the gap structure from the naive expectation of a BCS superconductor. The evidence of “nodal” gap in members of the RTBC family has also been inferred from experiments [19]. A crude connection between the spectral weight to the momentum dependent gap is given by $A(k,0) \propto e^{-\Delta(k)/2k_BT}$, where $\Delta(k)$ is the momentum dependent gap.

In Fig.3 the top row shows the SC with weak magnetic coupling. There is no low energy spectral weight at the lowest temperature and the weight increases isotropically with increasing $T$. This suggests an essentially $k$ independent gap. Low temperature gives rise to a ‘ghost Fermi surface’ which gradually evolves into a well defined Fermi surface at $T \sim T_c$.

At intermediate coupling a very unusual gap structure emerges out of the competing orders. The low temperature state at this parameter point is a gapless superconductor with spectral weight at selected $k$-points of the Brillouin zone, decided by the magnetic wave vector of the underlying order, see the leftmost panel, $0.25T_c$. With increasing temperature, next panel, the pointlike structure has broadened into an arc due to magnetic fluctuations. In fact at $1.25T_c$, where the system is an antiferromagnetic metal, the Fermi surface, expectedly, is still different from the tight binding shape. In fact only at $T \sim 2.5T_c$, when it exits the magnetic phase, does it regain the the tight binding shape (extreme right of all panels). This should be visible in the Ho based RTBC, and in general in other members of the family which have a strongly momentum dependent magnetic susceptibility.

The bottom row corresponds to strong coupling where the system is a magnetic metal. The Fermi surface continues to be anisotropic but connected. This structure arises due to the modified band structure generated by the magnetic order. At $T\sim 0.3T_c$ the isotropy of the Fermi surface is restored as mag-
In conclusion, the present work is the first theoretical attempt to describe the thermal behavior of the entire borocarbide family within a single framework. We have mapped out the thermal phase diagram of this family based on the thermodynamic and quasiparticle signatures. Along with the transition temperatures, $T_c$ and $T_{AF}$, we identify two new thermal scales, $T_g$ and $T_{an}$, related, respectively, to the transition from gapped to gapless SC, and from anisotropic to isotropic SC gap structure. All this happens within a model where the attractive on site interaction tends to generate an isotropic s-wave gap, while magnetic order creates a nodal Fermi surface in the ground state, and a diffuse Fermi arc like structure at intermediate temperature below $T_c$. These results have a direct bearing on spectroscopy of the Ho and Er based borocaribdes.

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SUPPLEMENTARY MATERIALS

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MODEL AND METHOD

We study the attractive Hubbard model in two dimension on a square lattice in presence of Kondo-like coupling:

$$H = H_0 - |U| \sum_i n_{i\uparrow} n_{i\downarrow} - J \sum_i \mathbf{S}_i \cdot \mathbf{\sigma}_i$$  \hspace{1cm} (S1)

with, $H_0 = \sum_{\langle ij \rangle, \sigma} (t_{ij} - \mu \delta_{ij}) c_{i\sigma}^\dagger c_{j\sigma}$, where $t_{ij} = -t$ for nearest neighbor hopping and zero otherwise. $\mathbf{S}_i$ is the core spin, arising from the $f$ electrons in real material; $\mathbf{\sigma}_i$ is the electron spin operator.

Using a single channel Hubbard-Stratonovich decomposition we decompose the four fermion attractive interaction term into quadratic fermions in an arbitrary space-time fluctuating pairing field. For this we introduce the auxiliary complex scalar field $\Delta_i(\tau) = |\Delta_i(\tau)| \ e^{i\theta_i(\tau)}$. A complete treatment of the problem requires retaining both space and time dependence of $\Delta_i$ and can be addressed only through Quantum Monte Carlo technique. We however drop the $\tau$ dependence and retain the complete spatial dependence thereby rendering $\Delta_i$ classical. In terms of Matsubara frequency we have retained only the $\Omega = 0$ mode. This is a fair approximation at high temperature where the energy levels are well separated and only the $\Omega = 0$ mode is vital. The approximation thus becomes progressively accurate as $T \to \infty$. At $T \neq 0$ the approximation gives fairly accurate results and captures the thermal scales correctly. At $T = 0$ the results obtained are as good as that from the mean field theory [1].

For the magnetic order, the core spin $\mathbf{S}_i$ is treated as classical (with a fixed magnitude $S$). An approximation valid when $2S \gg 1$. The angular fluctuations of the spin are retained completely. The resulting effective Hamiltonian can thus be expressed as:

$$H_{eff} = H_0 + \sum_i (\Delta_i^\dagger \Delta_i^\gamma + H.c) - J \sum_i \mathbf{S}_i \cdot \mathbf{\sigma}_i$$ \hspace{1cm} (S2)

where, the last term correspond to the stiffness cost associated with the now classical pairing field. The configurations $\{\Delta_i, \mathbf{S}_i\}$ that need to be considered obey Boltzmann distribution, obtained by tracing over the electrons:

$$P\{\Delta_i, \mathbf{S}_i\} \propto Tr e^{-\beta H_{eff}}$$ \hspace{1cm} (S3)

which in turn is related to the free energy of the electrons in that configuration. For large and random $\{\Delta_i, \mathbf{S}_i\}$ the trace has to be taken numerically. For the finite temperature results presented in this paper we generate equilibrium configurations by using Metropolis algorithm for $\{\Delta_i, \mathbf{S}_i\}$ and estimate the update cost by diagonalizing the electron Hamiltonian $H_{eff}$ for every microscopic move. This numerically expensive method has been rendered tractable by applying travelling cluster approximation (TCA) wherein instead of diagonalizing the entire lattice for each attempted update a smaller cluster surrounding the update site is diagonalized [2].

Indicators

Using the equilibrium configurations obtained through simulated annealing we calculate the thermodynamic and quasiparticle indicators required to characterize the phases, viz. the pairing field structure factor ($S_{\Delta}(q)$), magnetic structure factor ($S_m(q)$), low energy weight distribution ($A(k, 0)$) and the electronic density of states ($N(\omega) = \sum_k A(k, \omega)$) (where $A(k, \omega)$ is the momentum resolved spectral function) defined as,

$$S_{\Delta}(k) = (1/N) \sum_{ij} \langle \Delta_i \Delta_j^\gamma \rangle e^{i\mathbf{q} \cdot \mathbf{r}}$$ \hspace{1cm} (S4)

$$S_m(k) = (1/N) \sum_{ij} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle e^{i\mathbf{q} \cdot \mathbf{r}}$$ \hspace{1cm} (S5)

$$A(k, \omega) = \sum_\sigma \langle | u_{\mathbf{n},\sigma}^k|^2 \delta(\omega - E_n) \rangle + \langle | v_{\mathbf{n},\sigma}^k|^2 \delta(\omega + E_n) \rangle$$ \hspace{1cm} (S6)

here, $\mathbf{S}_i = (S_i^\alpha \hat{x} + S_i^\beta \hat{y} + S_i^\gamma \hat{z})$, where,

$$S_i^\alpha = S \sin \alpha_i \cos \phi_i$$ \hspace{1cm} (S7)

$$S_i^\beta = S \sin \alpha_i \sin \phi_i$$ \hspace{1cm} (S8)

$$S_i^\gamma = S \cos \alpha_i$$ \hspace{1cm} (S9)

where, $\alpha_i$ and $\phi_i$ are the polar and azimuthal angles of the magnetic core spin, respectively. $u_{\mathbf{n},\sigma}^k$ and $v_{\mathbf{n},\sigma}^k$ are the Bogoluibov de Gennes (BdG) eigen vectors with the corresponding eigen value $E_n$. 


FIG. S1. Color online: Ground state n-J phase diagram at U=3t showing the phases (a) gapped superconductor, (b) gapless superconductor, (c) magnetic metal and (d) phase separation. The black dashed line corresponds to Jc (see text). The vertical dotted line represents the cross section at which the thermal data is presented in the main text.

GROUND STATE PHASE DIAGRAM AT U = 3t

To determine the ground state we use a variational scheme. We minimize the energy over a restricted family of \{\Delta_i, S_i\} configurations. We assume \Delta_\parallel = \Delta_0, a site independent real quantity and for the magnetic order we assume spiral configurations where the polar angle \(\alpha_i = \pi/2\) and the azimuthal angle \(\phi_i\) is periodic, with \(S_i^z = 0, S_i^x = \cos(q_i r_i)\) and \(S_i^y = \sin(q_i r_i)\). The allowed wave vectors \(\{q_x, q_y\}\) are of the form \(2\pi n/L, \) where \(n = 1, 2, 3, \ldots\) We minimize the energy over \(\{q_x, q_y\}\) and \(\Delta_0\) for a fixed \(\mu, J\) and \(U\). We obtain the optimized \(\{\Delta_0, q\}\) configuration for a fixed \(\mu\) and then evaluate the density \(n\) from it so as to obtain the phase diagram in the n-J space for any given U/t [3].

Fig. S1 shows the ground state n-J phase diagram at U=3t. In the absence of any superconducting order the ground state is characterized only by the magnetic ordering vector \(Q\) (where \(Q\) is the optimized \(\{q_x, q_y\}\)). The small J/t limit is governed by RKKY interaction with the order being dictated by the maxima of spin susceptibility. The magnetic state depends on \(\mu\) or filling \(n\). At low filling the magnetic state correspond to ferromagnetic order with \(Q = \{0, 0\}\). With increasing filling the magnetic state undergoes transition to a \(\{0, q\}\) state at intermediate filling, followed by an antiferromagnetic \(\{0, \pi\}\) state to a spiral \(\{q, \pi\}\) state and finally to a Neel antiferromagnet \(\{\pi, \pi\}\) at half filling \(n = 1\).

At \(U = 3t\) and \(J = 0\) there is the usual \(k \uparrow\) and \(-k \downarrow\) pairing. At a small finite J/t the superconducting state is weakly modified by the magnetic order. The pairing field \(\Delta_0\) undergoes suppression but superconducting state continues to be gapped up to a coupling, \(J_g\), say. The maximum \(J_g \sim 0.3t\) is obtained for a filling of \(n \sim 0.75\). For \(J > J_g\) superconducting state is significantly modified by the magnetic order. Along with the suppression in \(\Delta_0\) there is now emergence of subgap features in the DOS at the Fermi level. With a finite DOS at the Fermi level superconductivity is rendered gapless over the regime \(J_g < J < J_c\), where, \(J_c\) is the critical coupling beyond which superconductivity gives way to magnetic metal. \(J_c/t\) increases with increasing magnetic coupling and has its maxima of \(J_c \sim 0.8t\) for the filling of \(n \sim 0.7\). In the limit of large filling \(n \approx 1\) there is phase separation regime over most part except for the small \(J_c/t\) regime, where a gapped superconducting state survives in spite of an underlying \(\{\pi, \pi\}\) order. At and near half filling the DOS is always gapped at the Fermi level. Upto \(J \sim 0.3t\) there exists a superconducting gap at the Fermi level. For \(J > 0.3t\) the gap arises out of the \(\{\pi, \pi\}\) magnetic order.

The thermal evolution of the ground state phases were tracked using the superconducting and magnetic structure factors. We plot the peak amplitude of superconducting \(S_\Delta(0, 0)\) and magnetic \(S_m(q)\) structure factor in Fig. S2, at different magnetic couplings. The superconducting order gets progressively suppressed with increasing J/t. At the density regime we are in a non collinear \(\{q, \pi\}\) magnetic order is realized at all J/t. The magnitude of \(q\) shows only weak dependence on J/t.

ADDITIONAL ISSUES

The general validity of the approximations used here

The results presented in this paper are based on the attractive Hubbard model with Kondo like coupling. The interaction is decomposed in terms of auxiliary complex scalar field \(\Delta_i(\tau) = |\Delta_i(\tau)| e^{i\theta_i(\tau)}\) using Hubbard-Stratonovich decomposition which converts the four fermion term into quadratic fermions in an arbitrary space-time fluctuating pairing field. For the magnetic order, we have a quantum “spin” \(S\) with magnetic moment \(S_z\) coupled to the electrons. There are two main approximations involved in this calculation. We
discuss them pointwise. (a) In order to make the problem numerically tractable we have dropped the \( \tau \) dependence of the pairing field thereby treating it as classical. The spatial fluctuations of \( \Delta_i \) are retained which are essential to capture the finite temperature behavior. The approximation gets progressively accurate as \( T \to \infty \), is fairly accurate at \( T \neq 0 \) and akin to mean field theory at \( T=0 \). Since the present paper discusses about the finite temperature behavior of the system, this is a reasonable approximation and captures the thermal scales successfully. (b) \( S_i \) has been treated as classical spin but its angular fluctuations are retained at finite temperature. This approximation is valid when \( 2S \gg 1 \). In case of the family of rare earth borocarbides (RTBC), the 4f shells for the magnetic superconductor involves \( 2S \sim 3-5 \) thereby making the “classical” spin approximation reasonable. However, the behavior of low moment and nonmagnetic superconductors such as Tm and Lu can not be suitably captured with our present approximation.

**Survival of the effects to \( U/t \ll 1 \), and the various \( T \) scales that one expects then**

Owing to the numerical complexities it is difficult to study the thermal physics beyond a lattice size of \( 30\times30 \) within a reasonable computation time. The results presented in this paper corresponds to a typical interaction of \( U = 3t \). We consider it as a representative of the “weak” coupling regime. The real materials (RTBC) however has \( U \ll 1t \) and thus a suppressed pairing field. Consequently, both the superconducting gap and the thermal scales are strongly suppressed. Superconductivity in this case would be realized over a narrow window in the \( J-T \) phase diagram. However, it could still be classified into gapped and gapless regimes. There is a small but nonzero \( J_g \) and within the regime \( J_g < J < J_c \) there would be finite DOS at the Fermi level. We emphasize that the emergence of gapless superconductivity is not an artifact of strong interaction and is expected to be observable in the real materials, albeit in a narrow parameter window.

### Possible Landau-Ginzburg functional

In order to develop an insight on the system under consideration through a phenomenological Landau-Ginzburg (LG) theory we carry out a systematic expansion of the magnetic and superconducting order parameters. The resulting free energy functional takes the form,

\[
F_{eff} = F_\Delta + F_J + F_{\Delta, J},
\]

\[
F_\Delta = \sum_{ij} a_{ij} \Delta_i \Delta_j^* + \sum_{ijkl} b_{ijkl} \Delta_i \Delta_j^* \Delta_k \Delta_l^* + O(\Delta^6),
\]

\[
F_J = \sum_{ij} J_i^{(2)} S_i S_j + \sum_{ijkl} J_i^{(4)}(S_i S_j S_k S_l + ...) + ...,
\]

\[
F_{\Delta, J} = \sum_{ijkl} [c_{ijkl} \Delta_i \Delta_j^* S_k S_l + H.c] + ..., \tag{S10}
\]

where, \( a_{ij} \sim -\chi_{ij}^S + (1/U)\delta_{ij} \), \( \chi_{ij}^S \) being the nonlocal pairing susceptibility of the free Fermi system, and \( b_{ijkl} \) arises from a convolution of four free Fermi Green’s functions. \( J_i^{(2)} \sim -J^2 \chi_i^S \), \( \chi_i^S \) is the nonlocal spin susceptibility of the free-electron system, leading to the RKKY interaction and \( J_i^{(4)} \). \( c_{ijkl} \) involves a four Fermi cumulant. \( c_{ijkl} \) can be constructed again from a combination of four Green’s functions [3].

The terms above define a relatively low order classical field theory on a lattice. \( H_\Delta \) involves the first two terms in the superconducting LG theory, and \( H_J \) describes the leading interaction coupling magnetic moments. \( H_{\Delta, J} \) indicates how the two orders modify each other. All of this holds when \( \Delta_i \) and \( JS_i \) are \( \lesssim t \).

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