The Determination of Optimal Parameters of Fuzzy PI Sugeno Controller

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Abstract. Describe the procedure for determining by means of Matlab and Simulink optimal parameters of the fuzzy PI controller Sugeno, where some indicators of the quality of the transition process in a closed system control with this controller satisfies the specified conditions.

1. Introduction

Along with the rapid increase in the number of fuzzy proportional-integral-derivative (PID) controllers in the last 15 years have seen the emergence of fuzzy PID controllers Sugeno based on TS (Tagachi, Sugeno) fuzzy model, called the Sugeno fuzzy model. In this context there is the need to define such parameters tuning of fuzzy PID controllers Sugeno, which are adopted restrictions on the quality indicators of the transition process in a closed system.

This work is devoted to solution of this problem with respect to fuzzy PI controller using the software MATLAB [1] and ideas of the decomposition of identification described in [2]. Fuzzy PI controller (Figure 1) has two inputs: error of regulation $e_1(k) = y_0(k) - y(k)$ equal to the difference between the given $y_0(k)$ and the current $y(k)$ values of the output variation of the error on one clock cycle $e_2(k) = e_1(k) - e_1(k - 1)$, two vectors of parameters $a = (a_1, a_2, a_3, a_4)$ and $b = (b_1, b_2, b_3, b_4)$ and one output $u$.

![Figure 1. Diagram of PI fuzzy regulator Sugeno](image-url)
2. Structure of fuzzy PI controller Sugeno

Now let us write 4 basic production rules of a fuzzy PI controller Sugeno [3]

\[ R_1: \text{if } \hat{e}_1(k) \text{ is } P, \hat{e}_2(k) \text{ is } P, \text{ then } \Delta \hat{u}_1(k) = a_1 \hat{e}_1(k) + b_1 \hat{e}_2(k), \]
\[ R_2: \text{if } \hat{e}_1(k) \text{ is } P, \hat{e}_2(k) \text{ is } N, \text{ then } \Delta \hat{u}_2(k) = a_2 \hat{e}_1(k) + b_2 \hat{e}_2(k), \]
\[ R_3: \text{if } \hat{e}_1(k) \text{ is } N, \hat{e}_2(k) \text{ is } P, \text{ then } \Delta \hat{u}_3(k) = a_3 \hat{e}_1(k) + b_3 \hat{e}_2(k), \]
\[ R_4: \text{if } \hat{e}_1(k) \text{ is } N, \hat{e}_2(k) \text{ is } N, \text{ then } \Delta \hat{u}_4(k) = a_4 \hat{e}_1(k) + b_4 \hat{e}_2(k), \]

where \( P, N \) – fuzzy sets, describing the positive and negative values of the variables \( \hat{e}_1(k), \hat{e}_2(k) \); \( a = (a_1, a_2, a_3, a_4) \) and \( b = (b_1, b_2, b_3, b_4) \) – two vectors with nonnegative coefficients \( a_j, b_j, j = 1, 4 \).

As the procedure of defazification in PI fuzzy regulator Sugeno is used the method of center of gravity (centroid). The right part of the fuzzy PI controller, Sugeno-can be written as

\[ \Delta u(k) = K_p(\hat{e}_1, \hat{e}_2) \cdot \hat{e}_1(k) + K_i(\hat{e}_1, \hat{e}_2) \cdot \hat{e}_2(k), \]

where \( K_p, K_i \) – the proportional and integral coefficients.

To obtain accurate values of \( K_p \) and \( K_i \) should evaluate the fuzzy And operation in the four fuzzy rules (1), which divide the input space into 12 regions as shown in Figure 2.

Figure 2. Decomposition of the spaces errors \( \hat{e}_1, \hat{e}_2 \)

Applying to the input variables \( \hat{e}_1 \) and \( \hat{e}_2 \) belonging to the region ICN, \( N = 1, 2, ..., 12 \), operation And regulation (1), the output mechanism of the Sugeno, and the operation of defazifikatsii method of center of gravity for each ICN, we find formulas for the calculation of the coefficients \( K_p \) and \( K_i \), are given in table 1.
Table 1. Formulas for calculation of coefficients $K_p$ and $K_i$

| ICN | $K_p(\hat{e}_1, \hat{e}_2), K_i(\hat{e}_1, \hat{e}_2)$ |
|-----|--------------------------------------------------|
| 1, 3 | $\frac{(L-|\hat{e}_1(k)|)A_1 + (L+\hat{e}_2(k))A_2 + (L-\hat{e}_2(k))A_3}{2(2L-|\hat{e}_1(k)|)}$ |
| 2, 4 | $\frac{(L-|\hat{e}_2(k)|)B_1 + (L+\hat{e}_1(k))B_2 + (L-\hat{e}_1(k))B_3}{2(2L-|\hat{e}_1(k)|)}$ |
| 5, 9 | $\frac{(L+\hat{e}_2(k))C_1 + (L-\hat{e}_2(k))C_2}{2L}$ |
| 7, 11 | $\frac{(L+\hat{e}_1(k))D_1 + (L-\hat{e}_1(k))D_2}{2L}$ |
| 6, 8, 10, 12 | $E$ |

The coefficients $A_i$, $B_i$, $C_i$, $D_i$ from table 1, depending on the formulas for $K_p$ and $K_i$, are expressed through the coefficients $a_j$, $b_j$ in table 2.

Table 2. Connection settings $K_p$ and $K_i$ coefficients $a_i$ and $b_i$.

| ICN | $K_p$ | $K_i$ |
|-----|-------|-------|
|     | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ |
| 1   | $b_1 + b_4$ | $b_1$ | $b_2$ | $a_3 + a_4$ | $a_1$ | $a_2$ |
| 2   | $b_1 + b_2$ | $b_1$ | $b_3$ | $b_4$ | $a_1 + a_2$ | $a_3$ | $a_4$ |
|     | $B_1$ | $B_2$ | $B_3$ |
| 3   | $b_1 + b_3$ | $b_1$ | $b_2$ | $b_4$ | $a_1 + a_3$ | $a_2$ | $a_4$ |
| 4   | $b_1 + b_3$ | $b_1$ | $b_2$ | $b_3$ | $a_2 + a_4$ | $a_1$ | $a_3$ |
| 5   | $b_1$ | $b_2$ | $a_1$ | $a_2$ |
| 6   | $b_3$ | $b_4$ | $a_3$ |
| 7   | $D_1$ | $D_2$ | $D_1$ | $D_2$ |
| 8   | $b_1$ | $b_2$ | $a_1$ | $a_3$ |
| 9   | $b_3$ | $b_4$ | $a_2$ | $a_4$ |
| 10  | $b_1$ | $a_1$ |
| 11  | $b_2$ | $a_4$ |
| 12  | $b_2$ | $a_2$ |

3. Optimization of fuzzy PI coefficients

In the model of digital closed-loop control system (figure 3) drag two Gain block in the modeling window. Double click on the Gain block (or Gain1) open the settings window and set in the field Gain: Ke (or Ku for block Gain1) in the box Sample time: 0.1 and click OK.
Figure 3. The model of the control system with fuzzy PI controller

In the model window, drag the block optimization Check Step Response Characteristics block and the transfer function object Transfer Fcn, which will enter the expression of the transfer function

\[ W(s) = \frac{4.8}{50.5s^2 + 11.96s + 1}. \]  

(3)

In the model window you can also drag the two blocks To Workspace block multiplexer Mux signals and scopebox Scope with two inputs. Add to the model 15 Constant units and 7 units Display. In the window the parameters of the Constant and Constant1 will enter the names of the coefficients \( Ke = K_1 \), \( Kde = K_2 \), and \( Kdu = K_\Delta u \), and in the Windows of the blocks Constant2 and Constant3 – the names of the components of the vectors a and b, respectively. Make the necessary settings and connect the elements according to figure 3.

In the MATLAB function block will open a window where you can record a program in the language Matlab to calculate the control input variables \( u \) with \( \dot{e}_1 \) and \( \dot{e}_2 \) according to the formulas of tables 1 and 2.

At the command prompt, Matlab will enter the values of the coefficients \( Ke = 1 \); \( Kde = 3.125 \); \( Kdu=1.152 \) found by the method of Ziegler – Nichols [4], and start the modeling process of control system by clicking on Start simulation. The units Display will reflect the current values of the coefficients \( Ke \), \( Kde \) and \( Kdu \).

The simulation was obtained the graph of the transition process in window Scope (Figure 4) violating the restrictions or performance targets quality:

- the duration of the transition process is not more than 15,
- overshoot no more than 30 %,
- average modular relative error, calculated according to the formula

\[ \bar{e} = \frac{1}{N} \sum_{k=1}^{N} \left| y_0 - y(k) \right| / y_0 \cdot 100\% , \]

should be no higher than 5%.
To achieve the required quality regulation was initially used Gradient descent algorithm (Sequential Quadratic Programming) that apply to all the search parameters $Ke$, $Kde$, $Kdu$, $a$, $b$. It turned out that the transition indicators are very poorly responsive to changes in these parameters and the quality of regulation remains unacceptable (Figure 4). Therefore, it was applied a two-step decomposition method for the determination of parameters.

The first stage is splitting the controller parameters into groups of homogeneous parameters, such as related or multiplied by similar variables (error, change, or speed error, acceleration error, the fuzzy outputs of the regulator or the operator). In our case there are two homogeneous groups of parameters ($Ke$, $Kde$, $Kdu$) and ($a_1$, $a_2$, $a_3$, $a_4$, $b_1$, $b_2$, $b_3$, $b_4$). One search algorithm (in our case, Gradient descent (Sequential Quadratic Programming)) has consistently changed the parameters of each group to fulfill the conditions. This approach was also observed a very low rate of convergence to the solution.

Therefore proceed to the second stage of decomposition of the optimization problem, the essence of which lies in the selection and securing each group of parameters of the search algorithm, which provides a satisfactory convergence rate. Then sequentially each group of parameters is specified "your" optimization algorithm to fulfill the above conditions.

In table 3 shows the initial and optimal values of parameters in groups matching the search algorithms, as well as the start and end values of relative error $\bar{e}$.

### Table 3. The results of the decomposition method of parameter optimization

| The initial values of the parameters | The method (algorithm) optimization | The initial error $\bar{e}$ |
|--------------------------------------|-------------------------------------|---------------------------|
| $Ke=1$; $Kde=3.12$; $Kdu=1.152$     | Gradient descent (Interior-Point)   |                           |
| $a_1=1$; $a_2=1$; $a_3=1$; $a_4=1$; $b_1=1$; $b_2=1$; $b_3=1$; $b_4=1$; | Simplex search (Active-Set)         | 56.35%                    |
| The optimal values of the parameters |                                     |                           |
| $Ke=0.216$; $Kde=3.395$; $Kdu=0.191$. |                                     |                           |
| $a_1=0.947$; $a_2=0.71$; $a_3=0.75$; | Pattern search (Active-Set)         |                           |
| $a_4=1.29$; $b_1=1.25$; $b_2=0.957$; | Simplex search (Active-Set)         | 0.39%                     |
| $b_3=1.23$; $b_4=1.384$;            |                                     |                           |
Omitting the technical details of optimization in Matlab, we write the optimal values of the first group parameters $K_1 = K_e = 0.216; \ K_2 = K_{de} = 3.395; \ K_{du} = K_{du} = 0.191$, found by search method Pattern search (Active-Set) and the second group of parameters $a_1 = a_1 = 0.947; \ a_2 = a_2 = 0.708; \ a_3 = a_3 = 0.751; \ a_4 = a_4 = 1.293; \ b_1 = b_1 = 1.247; \ b_2 = b_2 = 0.957; \ b_3 = b_3 = 1.123; \ b_4 = b_4 = 1.384$, found by search the Simplex search method (Active-Set). The quality indicators of the transition (figure 5) are not above specified.

They have the following meanings:

- the duration of the transition process 15 s,
- overshoot 9.3 %,
- relative mean modular error $\bar{\epsilon} = 0.39\%$,

satisfies the accepted conditions.

4. Conclusion

Identified two groups of parameters of fuzzy PI controller, which are subject to optimization. Used decomposition optimization method, wherein for each parameter group is assigned a corresponding search algorithm. During sequential optimization of each group of parameters by "their" search algorithm has been found such parameters, in which all indicators of the quality of the transient process in the closed-loop control system with optimal fuzzy PI controller have been specified below.

References

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