A Game-Theoretic Approach to Covert Communications

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Abstract—This paper considers a game-theoretic formulation of the covert communications problem with finite blocklength, where the transmitter (Alice) can randomly vary her transmit power in different blocks, while the warden (Willie) can randomly vary his detection threshold in different blocks. In this two player game, the payoff for Alice is a combination of the coding rate to the receiver (Bob) and the detection error probability at Willie, while the payoff for Willie is the negative of his detection error probability. Nash equilibrium solutions to the game are obtained, and shown to be efficiently computable using linear programming. For less covert requirements, our game theoretic approach can achieve substantially higher coding rates than uniformly distributed transmit powers. An alternative scheme making use of a jammer is also considered, and a game between the jammer and Willie is formulated which can also be solved using linear programming.

I. INTRODUCTION

In covert communications, a transmitter (Alice) transmits to a receiver (Bob) in the presence of a warden (Willie). The aim is for the transmission to be such that the very presence of a transmission or non-transmission is difficult for Willie to distinguish between [1], [2]. Applications of covert communication include the prevention of knowledge of transmission for use as metadata or to maintain privacy, communication in the presence of authoritarian governments, and military communications where detection of transmissions can reveal one’s location to enemies [3].

In [1] it was shown that Alice can transmit $O(\sqrt{N})$ bits in $N$ channel uses covertly and reliably to Bob as $N \to \infty$. Covertness is defined in the sense that

$$\Pr_{FA} + \Pr_M \geq 1 - \epsilon$$

for any $\epsilon > 0$, (1)

with $\Pr_{FA}$ denoting the probability of false alarm and $\Pr_M$ the probability of missed detection. Further refinements of this result include [4]–[6]. Later, it was shown that in certain situations, it is possible to transmit $O(N)$ bits in $N$ channel uses as $N \to \infty$, such as when there is uncertainty in the receiver noise variance [7], or when there is an uninformed jammer [5].

The above results are asymptotic in that the results apply for $N \to \infty$. The case of finite $N$ has been considered in [8], where expressions for $\Pr_{FA}$ and $\Pr_M$ were derived, and the use of uniformly distributed transmission powers was also proposed as a way to improve performance over the use of constant powers. The current paper also considers the case of finite $N$. Instead of uniformly distributed transmission powers, we instead wish to find the “optimal” distribution of transmit powers. Note that if Alice knows the detection threshold that Willie uses, then such an optimal distribution can be found. On the other hand, Willie himself could also try to randomly vary his detection threshold to confuse Alice and potentially improve his detection performance. Due to the competing objectives for Alice and Willie, in this paper we will use game theory to model such interactions. We will formulate the situation as a two player game and show that Nash equilibrium solutions can be computed efficiently using linear programming.

For comparison, we also consider the case where there is a jammer [3], where we now allow the jamming powers to randomly vary while the transmission power remains constant. Here we formulate a two player game between the jammer and Willie, and similarly show that Nash equilibria can be computed using linear programming.

The paper is organized as follows. The system model is presented in Section II both for the scheme where the transmitter varies its power randomly, and the scheme where there is a jammer. Game-theoretic formulations are then presented in Section III. Numerical studies and comparisons are given in Section IV.

II. SYSTEM MODELS

In this paper we consider two different setups, one where the transmitter (Alice) randomly varies her transmit power [8], and one where there is a jammer who randomly varies its jamming power [3].

A. Transmitter Varying Transmit Power

A diagram of the system model is shown in Fig. 1. Let $x_k$ be the signal that is to be transmitted. The warden (Willie)

![Fig. 1. System model - Transmitter varying transmit power](image_url)
wishes to decide between two hypotheses:
\[ \mathcal{H}_0 : y_{w,k} = n_{w,k}, \quad k = 1, \ldots, N \]
\[ \mathcal{H}_1 : y_{w,k} = x_k + n_{w,k}, \quad k = 1, \ldots, N \]

based on collecting \( N \) observations, where \( y_{w,k} \) is the received signal by Willie at time \( k \), and \( n_{w,k} \sim \mathcal{CN}(0, \sigma^2_w) \) is complex Gaussian channel noise. Hypothesis \( \mathcal{H}_0 \) means that the transmitter (Alice) did not transmit to the receiver (Bob), while hypothesis \( \mathcal{H}_1 \) means that Alice transmitted. We assume that the coding blocklength is equal to \( N \). The received signals at Bob under the different hypotheses are:
\[ \mathcal{H}_0 : y_{b,k} = n_{b,k}, \quad k = 1, \ldots, N \]
\[ \mathcal{H}_1 : y_{b,k} = x_k + n_{b,k}, \quad k = 1, \ldots, N \]

where \( n_{b,k} \sim \mathcal{CN}(0, \sigma^2_b) \).

We assume Gaussian signalling such that \( x_k \sim \mathcal{CN}(0, P) \). The transmit power \( P \) varies between different blocks, but stays constant within each block of \( N \) time slots. We assume that Bob knows the (random) values of \( P \) used in each block via some shared secret between Alice and Bob, but that Willie only knows the distribution of \( P \).

Willie wants to detect transmissions of Alice. Optimal detection at Willie usually takes on the form of a likelihood ratio test \([9, 10]\). Given \( \mathcal{H}_0 \), we have \( y_{w,k} \sim \mathcal{CN}(0, \sigma^2_w) \), and given \( \mathcal{H}_1 \), we have \( y_{w,k} \sim \mathcal{CN}(0, P + \sigma^2_w) \) for \( k = 1, \ldots, N \). Then the likelihood ratio test can be easily shown to be equivalent to an energy detector which decides \( \mathcal{H}_1 \) if
\[ T \triangleq \frac{1}{N} \sum_{k=1}^{N} |y_{w,k}|^2 \]

exceeds a threshold \( t \), and decides \( \mathcal{H}_0 \) otherwise \([9]\). In covert communications, Alice wants to transmit to Bob while constraining the probability of being detected at Willie is sufficiently low \([2]\). One strategy for Alice to improve its performance (e.g. in terms of the transmission rate to Bob, or the detection probability at Willie) is by randomizing between a few different transmission powers, with the aim of confusing Willie. In \([8]\) the case of uniformly distributed \( P \) was considered and shown to outperform the use of constant \( P \). For the current paper we consider the problem of optimizing the distribution for \( P \). Suppose that \( P > 0 \) can take on a finite number of values
\[ P_1, P_2, \ldots, P_I, \]

and denote
\[ \pi_i^P \triangleq \mathbb{P}(P = P_i), \quad i = 1, \ldots, I. \]

Now if Willie uses a fixed detection threshold \( t \), then Alice can optimize her transmission power distribution for that particular threshold \([8]\). However, if Willie decides to randomize his detection threshold, he in turn could confuse Alice

\footnote{In game theoretic terminology this is equivalent to saying that Willie knows the mixed strategy that Alice will play.}

\footnote{For instance, one can pose a problem of maximizing the transmission rate to Bob while constraining the detection error probability for Willie.}

and possibly increase his detection performance. Due to the competing objectives of Alice and Willie, in this paper we will adopt a game-theoretic formulation of the situation, which will be presented in Section \( \text{III} \). We thus assume that \( t \) can take on values
\[ t_1, t_2, \ldots, t_M \]

with
\[ \pi_m^t \triangleq \mathbb{P}(t = t_m), \quad m = 1, \ldots, M. \]

The case where \( t \) can take on a continuum of values can be approximated by discretization of the real interval using a large number of discretization points.

The statistic \( T \) defined in (2) is equivalent to a scaled chi-squared distributed random variable with \( 2N \) degrees of freedom under both hypotheses, with scaling \( \frac{\sigma^2_w}{\sigma^2_b} \) under \( \mathcal{H}_0 \), and scaling \( \frac{P + \sigma^2_w}{2N} \) under \( \mathcal{H}_1 \) and transmit power \( P \). The likelihood functions of \( T \) under \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) are then
\[ f(T|\mathcal{H}_0) = \frac{T^{N-1}}{\Gamma(N)} \left( \frac{N}{\sigma^2_w} \right)^N \exp\left( -\frac{NT}{\sigma^2_w} \right) \]
\[ f(T|\mathcal{H}_1) = \frac{T^{N-1}}{\Gamma(N)} \sum_{i=1}^{I} \left( \frac{N}{P_i + \sigma^2_w} \right)^N \exp\left( -\frac{NT}{P_i + \sigma^2_w} \right) \pi_i^P \]

where \( \Gamma(\cdot) \) is the gamma function. Let \( \mathbb{P}_{FA} = \mathbb{P}(\text{decide } \mathcal{H}_1|\mathcal{H}_0) \) and \( \mathbb{P}_M = \mathbb{P}(\text{decide } \mathcal{H}_0|\mathcal{H}_1) \) denote the probability of false alarm and probability of missed detection respectively. We will say that the communication scheme is covert \([8]\) if
\[ \mathbb{P}_{FA} + \mathbb{P}_M \geq 1 - \epsilon \]

for some \( \epsilon > 0 \).

From the relation
\[ \int T^{N-1} \exp\left( -\frac{NT}{x} \right) dT = - \left( \frac{N}{x} \right)^{-N} \Gamma\left( N, \frac{NT}{x} \right) \]

where
\[ \Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt \]

is the incomplete gamma function, one can easily show that for given distributions of transmit powers \( \pi^P \) and detection thresholds \( \pi^t \), the probabilities of false alarm and missed detection are
\[ \mathbb{P}_{FA}(\pi^P, \pi^t) = \mathbb{P}(T > t|\pi^P, \pi^t, \mathcal{H}_0) = \sum_{m=1}^{M} \frac{\Gamma(N, \frac{Nt_m}{\sigma^2_w})}{\Gamma(N)} \pi_m^t \]
\[ \mathbb{P}_M(\pi^P, \pi^t) = \mathbb{P}(T < t|\pi^P, \pi^t, \mathcal{H}_1) \]
\[ = \sum_{m=1}^{M} \sum_{i=1}^{I} \left[ 1 - \frac{\Gamma(N, \frac{Nt_i}{P_i + \sigma^2_w})}{\Gamma(N)} \right] \pi_i^P \pi_m^t. \]

Note that the expression for \( \mathbb{P}_{FA}(\pi^P, \pi^t) \) does not actually depend on \( \pi^t \), but for notational consistency with Section \( \text{II-B} \) we will use \( \mathbb{P}_{FA}(\pi^P, \pi^t) \) rather than \( \mathbb{P}_{FA}(\pi^t) \).
B. Presence of a Jammer

A diagram of the system model for this scheme is shown in Fig. 2. In this scheme there is a jamming signal which is generated by a separate jammer. It is known [3] that by using a jammer with jamming powers unknown to Willie, the transmit powers of Alice do not need to go to zero (as the blocklength increases) in order to remain covert in the sense of [1]. In this paper, we will consider the optimization of the jamming power distribution.

Let $x_k$ again denote the signal which is to be transmitted. Willie now wishes to decide between the two hypotheses:

$$
\mathcal{H}_0 : y_{w,k} = n_{w,k} + j_k, \quad k = 1, \ldots, N
$$

$$
\mathcal{H}_1 : y_{w,k} = x_k + n_{w,k} + j_k, \quad k = 1, \ldots, N
$$

where $j_k \sim \mathcal{CN}(0, J)$ is the random jamming signal. The jamming signal power $J \geq 0$ varies randomly between different blocks, but stays constant within each block of $N$ time slots.

We assume Gaussian signalling such that $x_k \sim \mathcal{CN}(0, P)$, with $P$ now remaining constant at all times, so that the shared secret of transmit powers in Section II-A is not needed here.

The received signals at Bob under the two hypotheses are:

$$
\mathcal{H}_0 : y_{b,k} = n_{b,k} + j_k, \quad k = 1, \ldots, N
$$

$$
\mathcal{H}_1 : y_{b,k} = x_k + n_{b,k} + j_k, \quad k = 1, \ldots, N
$$

where $\alpha > 0$ can be used to model different distances between the jammer and Bob, and between the jammer and Willie. We assume a friendly jammer such that the jamming powers $J$ used in each block are known to Bob but unknown to Willie, which involves a shared secret between the jammer and Bob. The actual values of the random jamming signal $j_k$ are unknown to either Bob or Willie.

We suppose that $J$ can take on a finite number of values

$$J_1, J_2, \ldots, J_{I'},$$

and denote

$$\pi_i^t \triangleq \mathbb{P}(J = J_i), \quad i = 1, \ldots, I'.$$

We will also consider that the detection thresholds $t$ can take on values

$$t_1, \ldots, t_M$$

We will now remain constant at all times, so that the shared secret of transmit powers in Section II-A is not needed here.

For finite blocklengths, the channel coding rate from Alice to Bob in bits per channel use is approximated by (see [11], [12])

$$R \approx \text{log}_2(1 + \text{SNR}_b) - \frac{1}{N} \left(1 - \frac{1}{(\text{SNR}_b + 1)^2}\right) Q^{-1}(\delta) \ln(2)$$

(4)

where $\text{SNR}_b$ is the signal-to-noise ratio at Bob, $Q^{-1}(\cdot)$ is the inverse $Q$-function, and $\delta$ is the decoding error probability.

For future reference, define the function $\overline{R}(\cdot)$ by

$$\overline{R}(x) \triangleq \text{log}_2(1 + x) - \frac{1}{N} \left(1 - \frac{1}{(x + 1)^2}\right) Q^{-1}(\delta) \ln(2)$$

(5)

A. Transmitter varying transmit power

Here we consider posing the situation in Section II-A as a two player game between Alice and Willie, where we wish to find Nash equilibrium solutions to the game. It is well known that for finite games, mixed strategy Nash equilibria always exist. We now formulate a two player game where the players are Alice and Willie, with mixed strategies $\pi^t$ and $\pi^t$ respectively.

For transmit power $P$, the signal-to-noise ratio at Bob is $\text{SNR}_b = \frac{P}{\sigma_b^2}$. Alice wants to maximize the payoff

$$\sum_{i=1}^{I'} \overline{R} \left( \frac{P_i}{\sigma_b^2} \right) \pi_i^t + \beta (\mathbb{P}_{FA}(\pi^P, \pi^t) + \mathbb{P}_{M}(\pi^P, \pi^t))$$

(6)

where $\overline{R}(\cdot)$ is defined by (5) and the parameter $\beta > 0$ controls the tradeoff between the (approximate) expected channel coding rate at Bob and coverture at Willie. Smaller
values of $\beta$ will place more emphasis on achieving a large coding rate, while larger values of $\beta$ will have more emphasis on achieving higher detection error probabilities (i.e., be more covert). Willie on the other hand wants to minimize $\mathbb{P}_{FA}(\pi^P, \pi^t) + \mathbb{P}_M(\pi^P, \pi^t)$, so it has payoff

$$-(\mathbb{P}_{FA}(\pi^P, \pi^t) + \mathbb{P}_M(\pi^P, \pi^t)). \tag{7}$$

This game with payoffs (6) and (7) for Alice and Bob respectively is a non-zero-sum game. Nash equilibria to general non-zero-sum games can be found numerically using algorithms such as the Lemke-Howson algorithm. 

An alternative zero-sum game can also be posed, where Alice has payoff (6) and Willie has payoff $-\mathbb{P}_{FA}(\pi^P, \pi^t)$ for fixed $\pi^t$. The zero-sum game with payoffs (6) and (8), have the same Nash equilibria. Hence optimizing (8) over $\pi^t$ for Alice is the same as optimizing (7), and thus there is no incentive for Willie to deviate from $\pi^t$ in the zero-sum game. Hence $(\pi^{P*}, \pi^{t*})$ is also a Nash equilibrium to the zero-sum game with payoffs (6) and (8).

A similar argument can be used to show that Nash equilibria to the zero-sum game are also Nash equilibria to the non-zero-sum game.

One of the advantages of zero-sum games is that they can be solved efficiently using linear programming (note that the Lemke-Howson algorithm itself is similar to the simplex algorithm). A Nash equilibrium mixed strategy for Alice can be found by solving the linear program:

$$\min_{(\pi^t_m)_U} U$$

s.t. $\sum_{m=1}^{M} \left[ R\left( \frac{P_i}{\sigma_b^2} \right) \pi^t_m - \beta (\mathbb{P}_{FA}(\pi^P, \pi^t) + \mathbb{P}_M(\pi^P, \pi^t)) \right] \pi^t_m \leq U,$$

$$i = 1, \ldots, I,$$

$$\sum_{m=1}^{M} \pi^t_m = 1, \quad 0 \leq \pi^t_m \leq 1. \tag{10}$$

Another advantage of zero-sum games is that their Nash equilibria have nice “uniqueness” properties. We first give the following definition (see also [15, p.233]):

**Definition 1.** Two Nash equilibria $(\pi^P, \pi^t)$ and $(\pi'^P, \pi'^t)$ are:

(i) interchangeable if $(\pi^P, \pi^t)$ and $(\pi'^P, \pi'^t)$ are also Nash equilibria

(ii) equivalent if the payoffs from using the mixed strategy $(\pi^P, \pi^t)$ are the same as the payoffs from using the mixed strategy $(\pi'^P, \pi'^t)$.

The following is a standard result in game theory, see e.g. [15, p.232] for a proof.

**Theorem 2.** All Nash equilibria in zero-sum games are interchangeable and equivalent.

We have shown in Theorem 1 that our original game with payoffs (6) and (7) has the same Nash equilibria as the zero-sum game with payoffs (6) and (8). A Nash equilibrium to this zero-sum game can be found by solving the linear programs (9)-(10). By Theorem 2, this Nash equilibrium has performance as good any other Nash equilibria of the game. Hence there is no loss of performance in using the mixed strategies obtained by solving the linear programs (9)-(10).

### B. Presence of a Jammer

For a given jamming power $J$, the signal-to-noise ratio at Bob is now $\text{SNR}_b = \frac{P}{\sigma_b^2 + \alpha^2 J}$. We will formulate a two player game where the players are the jammer and Willie, with mixed strategies $\pi^J$ and $\pi^t$ respectively. The jammer wants to maximize the payoff

$$\sum_{i=1}^{I} R\left( \frac{P}{\sigma_b^2 + \alpha^2 J_i} \right) \pi^J_i + \beta' (\mathbb{P}_{FA}(\pi^J, \pi^t) + \mathbb{P}_M(\pi^J, \pi^t)),$$

where $R(.)$ is defined in (5) and $\beta' > 0$ controls the tradeoff between the coding rate at Bob and covertness at Willie. Willie on the other hand wants to minimize $\mathbb{P}_{FA}(\pi^J, \pi^t) + \mathbb{P}_M(\pi^J, \pi^t)$, so he has payoff

$$-(\mathbb{P}_{FA}(\pi^J, \pi^t) + \mathbb{P}_M(\pi^J, \pi^t)). \tag{12}$$
An alternative zero-sum game can be posed, where the jammer has payoff (11) and Willie has payoff

\[- \sum_{i=1}^{I'} \mathcal{R}\left(\frac{P}{\sigma_b^2 + \alpha^2 I_i}\right) \pi_i^J - \beta' (\mathbb{P}_{FA}(\pi^J, \pi^J) + \mathbb{P}_{M}(\pi^J, \pi^J)).\]

(13)

**Theorem 3.** The non-zero-sum game with payoffs (11) and (12), and the zero-sum game with payoffs (11) and (13), have the same Nash equilibria.

**Proof.** Similar to the proof of Theorem 1. \qed

A Nash equilibrium mixed strategy for the jammer can be found by solving the linear program:

\[
\max_{\{\pi^J_i\} U} \sum_{i=1}^{I'} \left[ \mathcal{R}\left(\frac{P}{\sigma_b^2 + \alpha^2 I_i}\right) \pi_i^J \right] + \beta' \left( \Gamma(N, \frac{N_{tm}}{\sigma_w^2 + J + I_i}) + 1 - \frac{\Gamma(N, \frac{N_{tm}}{P + \sigma_w^2 + J + I_i})}{\Gamma(N)} \right) \pi_i^J \geq U,
\]

\[
m = 1, \ldots, M,
\]

\[
\sum_{i=1}^{I'} \pi_i^J = 1, \quad 0 \leq \pi_i^J \leq 1
\]

while a Nash equilibrium mixed strategy for Willie can be found by solving the linear program:

\[
\min_{\{\pi^M_i\} U} \sum_{m=1}^{M} \left[ \mathcal{R}\left(\frac{P}{\sigma_b^2 + \alpha^2 I_i}\right) \pi_m^M \right] + \beta \left( \Gamma(N, \frac{N_{tm}}{\sigma_w^2 + J + I_i}) + 1 - \frac{\Gamma(N, \frac{N_{tm}}{P + \sigma_w^2 + J + I_i})}{\Gamma(N)} \right) \pi_m^M \leq U,
\]

\[
i = 1, \ldots, I',
\]

\[
\sum_{m=1}^{M} \pi_m^M = 1, \quad 0 \leq \pi_m^M \leq 1.
\]

Similar uniqueness properties of the Nash equilibria as discussed at the end of Section III-A will also hold here.

**IV. NUMERICAL STUDIES**

**A. Plots of probability distributions**

We first show some plots of the Nash equilibrium mixed strategies / probability distributions. In the case of the transmitter varying its transmit power, we use the following parameters: \(\sigma_b^2 = 0\) dB, \(\sigma_w^2 = 0\) dB, \(\delta = 0.1\), \(N = 200\), \(\beta = 6\). The transmit powers range from 0.02 mW to 10 mW in steps of 0.01 mW\(^4\) and the detection thresholds are discretized from 0 to 10 in steps of 0.01. Fig. 3 shows the transmit power distribution and Fig. 4 shows the threshold distribution. The transmit powers here are concentrated on two values, randomizing between the lowest (0.02 mW) and highest (10 mW) power levels. The detection thresholds of Willie are randomized between the two neighbouring values 1.02 and 1.03.

In the case of a jammer, we use the following parameters: \(\sigma_b^2 = 0\) dB, \(\sigma_w^2 = 0\) dB, \(\delta = 0.1\), \(P = 1\) mW, \(\alpha = 1\), \(N = 200\), \(\beta = 2\). The jamming powers range from 0 mW to 10 mW in steps of 0.01 mW, and the detection thresholds are discretized from 0 to 10 in steps of 0.01. Fig. 5 shows the jamming power distribution and Fig. 6 shows the threshold distribution. The jamming powers and detection thresholds are now concentrated on multiple values.

**B. Trade-off between rate and detection error probabilities**

Next we look at the trade-off between the expected coding rate per channel use and \(\mathbb{P}_{FA} + \mathbb{P}_{M}\), by finding Nash equilibria for different values of \(\beta\) and \(\beta'\). In the case of the transmitter varying its transmit power, we use the following parameters: \(\sigma_b^2 = 0\) dB, \(\sigma_w^2 = 0\) dB, \(\delta = 0.1\), \(N = 200\). The transmit powers range from 0.02 mW to 10 mW in steps of 0.01 mW\(^4\) and the detection thresholds are discretized from 0 to 10 in steps of 0.01. Fig. 7 shows plots for various block lengths \(N\). For lower values of \(\mathbb{P}_{FA} + \mathbb{P}_{M}\) (less covert), varying the transmit power seems to provide better performance than...

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\(^4\)We omit the transmit power of 0.01 mW, as this gives a negative rate in the expression (1) when \(N = 200\).
using a separate jammer, in achieving higher expected rates per channel use for the same value of $P_{FA} + P_M$. However, for larger values of $P_{FA} + P_M$ (more covert), we see from Fig. 7 that using a jammer provides better performance.

Interestingly, when the transmitter is varying its transmit power, the performance is not monotonic with $N$, but seems to be worse for both small and large values of $N$. For small $N$, this could be due to the finite blocklength correction in the second term of (4), while the poorer performance for large $N$ is due to the fact that Willie can achieve better detection when he can collect more observations, and is consistent with the result from Fig. 1 that the number of bits per channel use is $O(\sqrt{N}/N) = O(1/\sqrt{N})$ as $N \to \infty$.

On the other hand, when using a jammer, the performance appears to improve with $N$, though the improvement is slight when $N$ is large. The poorer performance for small $N$ could also be due to the finite blocklength correction in the second term of (4), with the performance not deteriorating for large $N$ now consistent with the result of [3], that when using a jammer the number of bits per channel use is $O(N/N) = O(1)$ as $N \to \infty$.

C. Comparison with uniformly distributed and constant powers

We will compare our approach with the case of uniformly distributed transmission powers that was proposed in [8]. We consider the case $N = 200$ with the parameters $\sigma_b^2 = 0$ dB, $\sigma_w^2 = 0$ dB, $\delta = 0.1$. Fig. 8 plots the trade-off between the expected rate per channel use and $P_{FA} + P_M$ for 1) our game-theoretic approach, 2) uniformly distributed powers, 3) constant powers. For uniformly distributed powers, we consider powers uniformly distributed among $(0.02 \text{ mW}, 0.03 \text{ mW}, \ldots, 0.01k \text{ mW})$ for different values of $k \in \mathbb{N}$, in each case searching for and using the detection threshold $t$ in $(0.01, 0.02, \ldots, 10)$ which minimizes $P_{FA} + P_M$. For constant powers, we consider different constant transmission powers $0.02 \text{ mW}, 0.03 \text{ mW}, \ldots$, and use in each case the detection threshold $t$ which minimizes $P_{FA} + P_M$. We see that for very strict covertness requirements (larger $P_{FA} + P_M$) all three approaches will give similar performance, but when the covertness requirement is less strict (smaller $P_{FA} + P_M$) our game-theoretic approach can achieve substantially higher rates.

V. CONCLUSION

We have studied a game-theoretic approach to the finite blocklength covert communications problem, where Alice can randomly vary her transmit power and Willie can randomly vary his detection threshold. For less covert requirements, our game theoretic approach can achieve substantially higher coding rates than uniformly distributed transmit powers. An alternative scheme using a jammer has also been considered, with the formulation of a game between the jammer and Willie. We have shown that Nash equilibria to these games can be efficiently obtained using linear programming.
Fig. 8. Expected rate per channel use vs. $P_{FA} + P_M$

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