We present an effective field theory of the $\Delta$-resonance as an interacting Weinberg's $(3/2,0) \oplus (0,3/2)$ field in the multi-spinor formalism. We derive its interactions with nucleons $N$, pions $\pi$ and photons $\gamma$, and compute the $\Delta$-resonance cross-sections in pion-nucleon scattering and pion photo-production. The theory contains only the physical spin-3/2 degrees of freedom. Thus, it is intrinsically consistent at the Hamiltonian level and, unlike the commonly used Rarita-Schwinger framework, does not require any additional ad hoc manipulation of couplings or propagators. The symmetries of hadronic physics select a unique operator for each coupling $N\pi\Delta$ and $\gamma\tau\Delta$. The proposed framework can be extended to also describe other higher-spin hadronic resonances.

I. INTRODUCTION

The $\Delta$-resonance \cite{1} is the most important baryon resonance. Its mass is close to the mass of the nucleon, and it has significant couplings to nucleons, pions and photons. Albeit not elementary, it is the lightest known particle with spin-3/2. Clearly, a systematic study of the properties of $\Delta$-resonance would require a solid theoretical understanding of the physics of massive spin-3/2 fields.

The Rarita-Schwinger (RS) framework \cite{2}, which is most commonly used to describe physical spin-3/2 particles, including the $\Delta$-resonance, describes these fields as vector-spinors that contain additional unphysical spin-1/2 degrees of freedom (d.o.f.). The free theory for the RS field is fully consistent as the unphysical spin-1/2 d.o.f. is eliminated due to a local symmetry of the free RS Lagrangian. However, the unphysical spin-1/2 d.o.f. can be excited in interacting theories. This leads to theoretical inconsistencies and phenomenological disasters \cite{3}. The difficulties in formulating the $\Delta$-resonance interactions in a Lagrangian description of the RS field are generic and are summarized, e.g., in Ref. \cite{5}. The presence of unphysical spin-1/2 d.o.f. has often been ignored in the computations of $\Delta$-resonance processes and the inconsistent interactions have been thus used \cite{6–21}.

The most common approach to avoid these problems has been to construct the RS field interactions so that the unphysical d.o.f. are not involved in physical processes. This is accomplished by demanding that the interaction terms of the RS field manifest the same symmetries as the free Lagrangian. The only known consistent way to remove the unphysical spin-1/2 d.o.f. is to embed the RS theory into a locally supersymmetric set-up \textit{i.e.}, supergravity \cite{22,25}. In supergravity, the aforementioned local symmetry of the free RS Lagrangian is the local supersymmetry, which allows for consistent interactions for the spin-3/2 gravitino \cite{26,27}, the superpartner of the spin-2 graviton. Certainly, the $\Delta$-resonance cannot be identified with the universally coupled fermionic quanta of gravity.

There have been several attempts to find consistent interactions of spin-3/2 fields in flat space-time. Early steps in this direction were taken in Ref. \cite{28}. The first consistent non-supersymmetric interactions for spin-3/2 fields were presented in Ref. \cite{29,31} and applied to the $\Delta$-resonance\cite{3}. However, there is no well-established theory behind these procedures. Instead, the consistent interaction terms are constructed using ad hoc methods which work for some parameter values but fail for others \cite{32–84}. Thus, it is unclear whether a potentially inconsistent non-supersymmetric RS starting point can lead to a consistent fundamental theory of massive spin-3/2 fields.

Covariant field theoretical formulation of spin-3/2 particle interactions is sorely needed for proper description of hadronic resonances, such as in chiral perturbation theory \cite{35,50}, covariant isobar models \cite{51,53}, coupled channels models \cite{54,57} and when studying nucleon scattering either non-relativistically \cite{58,59} or relativistically \cite{18,21,60,61}. The presence of the $\Delta$-resonance inside compact stars has been studied very recently \cite{66–68} and pion-nucleon scattering has also been recently considered in lattice QCD \cite{69}.

To overcome the difficulties associated with higher-spin fields, Weinberg suggested employing these fields in the Lorentz representations $(j,0) \oplus (0,j)$ \cite{70}, where the spin $j$ is arbitrary. These representations contain only the physical higher-spin d.o.f.. Thus, their interactions are not plagued by the issues related to non-physical components. However, this approach does not admit a Lagrangian description\cite{2} but allows for a consistent calculation of physical observables using interaction Hamiltonians. Unfortunately, the original formulation of this idea has never been applied to the phenomenology of higher-spin fields.

---

1 It was argued that inconsistent interactions linear in the RS field may be acceptable in the context of chiral perturbation theory because the unphysical spin-1/2 d.o.f. can be absorbed into pion-nucleon contact terms \cite{51}.

2 There were attempts \cite{71,73} to derive higher-derivative Lagrangians for these fields. These second-order Lagrangians contain ghosts and lead to pathological theories \cite{74}.
On the other hand, the effective field theory (EFT) re-formulation of Weinberg’s original idea using multi-spinor representations [74] has turned out to be simple and practical, providing a consistent description of generic massive higher-spin particle interactions below some cut-off scale Λ. This EFT has successfully been used to study generic higher-spin dark matter [74], production and decays of higher-spin particles at colliders [75] and higher-spin induced contributions to the anomalous lepton moments [76].

In this note, we formulate a theory of the ∆-resonance interacting with nucleons, pions and photons, and compute the observed physical processes mediated by the ∆-resonance. The theory is based on the multi-spinor EFT formalism applied to Weinberg’s representation (3/2, 0)⊕(0, 3/2). We treat the ∆-resonance as a generic massive spin-3/2 field without specifying how it arises from QCD and hadronic physics. As expected, in the multi-spinor formalism, the ∆ interactions with nucleons, NπΔ, appear already at the level of unbroken isospin symmetry. However, ∆ interactions with photons, γπΔ, require a minimal breaking of this symmetry, as the ∆ quadruplet components have different electric charges. After applying the hadronic symmetries respected by the pions, nucleons and photons to the interaction Hamiltonian, we find that, in contrast to the RS formulation, each relevant interaction is controlled by a single coupling constant.

We use the interacting theory to compute the cross-sections of physical processes πN → πN and γN → πN mediated by the ∆-resonance. To do so, we conventionally cut off the nucleon momenta from above by applying a nuclear form-factor such as the one of Ref. [77]. To compare our results with the existing ones for the RS field, we will refer to this description of a spin-3/2 field as the multi-spinor framework (MSF).

We describe the system of photons, pions, nucleons and ∆-baryons at low energies through a Lorentz-invariant EFT in which the ∆ is a field in the (3/2, 0)⊕(0, 3/2) representation of the Lorentz group, using the multi-spinor formalism introduced in Refs. [74][78]. From here on, we will refer to this description of a spin-3/2 field as the multi-spinor framework (MSF).

The internal global symmetry group of the current model is SU(2)×U(1)Y, corresponding to the nuclear isospin T and hypercharge Y. This symmetry is approximate, as it is broken explicitly by gauging the electromagnetic U(1)Q subgroup. In Table I we show the relevant fields, organized into irreps of the Lorentz and internal symmetry groups. The fields NL,R and ΔL,R correspond to the left- and right-handed components of the nucleon and ∆-baryon multiplets, respectively. FL,R are the left- and right-handed parts of the electromagnetic field strength tensor (FL,R)ab = σμνFμν and (FR)ab = ¯σμνFμν, where σμν = i (σaμbνσaμbν − σbμσaν) and σμν = i (σbμσaν − σaμσbν). We will focus on describing the tree level ∆-mediated πN → πN and γN → πN scattering. Since electromagnetic gauge interactions are not involved in these processes, we will not consider them. In the limit of a vanishing electromagnetic coupling constant, U(1)Q acts only on the photon field, and the SU(2)×U(1)Y symmetry becomes exact.

II. THEORETICAL FRAMEWORK

We describe the system of photons, pions, nucleons and ∆-baryons at low energies through a Lorentz-invariant EFT in which the ∆ is a field in the (3/2, 0)⊕(0, 3/2) representation of the Lorentz group, using the multi-spinor formalism introduced in Refs. [74][78]. From here on, we will refer to this description of a spin-3/2 field as the multi-spinor framework (MSF).

3 For a review of baryon spectroscopy see, e.g., Ref. [78] and for a review on EFTs in nuclear interactions see Ref. [79].
The relevant scales in this EFT, apart from the masses of the particles, are the pion decay constant $f_\pi \sim 100$ MeV, and the chiral symmetry breaking scale $\Lambda \approx 4\pi f_\pi \sim 1$ GeV. Wilson coefficients of higher-dimensional operators in the Lagrangian are suppressed by products of these two scales. The series in $1/f_\pi$ can be resummed using chiral perturbation theory \cite{GR1,GR2} by embedding the pion fields into a non-linear realization of the QCD chiral symmetry. This generates a tower of interactions with any number of pions at each order in $1/\Lambda$. Since we are interested in interactions with the lowest number of pions, we will not perform this resummation and keep only the first terms. We will write Wilson coefficients of order $1/(f^n \Lambda^m)$ as $c/\Lambda^{n+m}$, so the dimensional coefficients $c \sim (4\pi)^n$ may naturally contain factors of $4\pi$.

The lowest-dimensional interaction operators constructed out of the fields in Table I containing at least three of the particles, are the pion decay constant $\pi \sim 100$ MeV, and the chiral symmetry breaking scale $\Lambda$. Since $\pi$ is non-linear in the fields, the Hamiltonian can be written as

$$-\mathcal{H}_\Delta = \frac{c_{\pi L}}{\Lambda^3} \left[ \partial^a_\pi (N_R^a)^b_c \partial^b \pi A \Delta_L^{abc} \right] + \frac{c_{\pi R}}{\Lambda^3} \left[ \partial^a_\pi (N_L^a)^b_c \partial^b \pi A \Delta_R^{abc} \right] + h.c.,$$

where $A$ denotes SU(2) triplet indices, while SU(2) doublet and quadruplet indices are implicit, and $T_a$ are the isospin-1/2 to 3/2 transition matrices given by

$$T_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & 0 & 1 & 0 \\ 0 & -1 & 0 & \sqrt{3} \end{pmatrix},$$

$$T_2 = \frac{i}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 0 & 1 & 0 \\ 0 & 1 & 0 & -\sqrt{3} \end{pmatrix},$$

$$T_3 = \frac{2}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

The left- and right-handed nucleons are $N_{L,a} = (p_{La}, n_{La})^T$ and $N_{R,a} = (p_{R,a}^\dagger, n_{R,a}^\dagger)^T$, where $p$ is proton and $n$ is neutron. The left- and right-handed $\Delta$-fields are $\Delta_{L,a} = (\Delta_{L,a}^+ \Delta_{L,a}^- \Delta_{L,a}^0 \Delta_{L,a}^-)^T$ and $\Delta_{R,a} = (\Delta_{R,a}^+ \Delta_{R,a}^- \Delta_{R,a}^0 \Delta_{R,a}^-)^T$. Both $\Delta_{L,a}$ and $\Delta_{R,a}$ are totally symmetric in spinor indices. In the MSF, the $\Delta$-field has mass dimension 5/2 in contrast to the RS framework, where its mass dimension is 3/2.

To study the $\Delta$-resonance in $\gamma N \to \pi N$ scattering, we need to model the $\Delta N\gamma$ interactions. Since baryons, nucleons, and photons have isospin 3/2, 1/2 and 0, respectively, these interactions violate isospin symmetry. To construct the $\Delta N\gamma$ interaction terms, we introduce an isospin-1, $Y = 0$ spurion $S$ as

$$-\mathcal{H}_{\gamma N} = \frac{c_{\gamma L}}{\Lambda^3} F^{ab} (N_R^a)^c_b T_a (\Delta_L^{abc}) S_A + \frac{c_{\gamma R}}{\Lambda^3} F^{ab} (\Delta_R^{abc}) S_A T_a N_L S_A + h.c..$$

The $T_3 \neq 0$ components of $S$ are charged, and must vanish for $U(1)_Q$ to be preserved, so we will take $S_3 = 1$. Invariance under parity imposes further relations between the Wilson coefficients. Parity transformation acts as $\pi \leftrightarrow -\pi$, $N_L \leftrightarrow N_R$, $\Delta_L \leftrightarrow \Delta_R$, $F_L \leftrightarrow F_R$, Therefore,

$$c_{\pi L} = c_{\pi R}, \quad c_{FL} = c_{FR}, \quad c_{\gamma L} = c_{\gamma R},$$

$$a^{(k)} = a^{(k)*}, \quad d_F = d_F,$

implying that there is a single $\Delta N\pi$ interaction. With these relations, the interaction terms can be expressed using the parity eigenstates $\pi, N = N_L \oplus N_R, \Delta = \Delta_L \oplus \Delta_R, F$ and $\bar{F}$. However, we will not follow this path since these fields do not transform as irreps of the Lorentz group and do not fit well into the MSF.

To conclude, the relations \cite{GR1} imply that $\pi N \to \pi N$ and $\gamma N \to \pi N$ scatterings are controlled by two complex parameters $c_{\pi}$ and $c_{\gamma}$ with the interactions given by

$$-\mathcal{H}_{\pi N} = \frac{c_{\pi}}{\Lambda^3} \left[ \partial^a_\pi (N_R^a)^b_c \partial^b \pi A T_a (\Delta_L^{abc}) + \partial_{ab} (N_L^a)^b_c \partial^b \pi A T_a (\Delta_R^{abc}) + h.c., \right.$$

$$-\mathcal{H}_{\gamma N} = \frac{c_{\gamma}}{\Lambda^3} F^{ab} (N_L^a)^b_c T_a (\Delta_L^{abc}) + F_{ab} (N_L^a T_a (\Delta_R^{abc}) + h.c..$$

### III. RESULTS AND DISCUSSION

Using MSF, we compute the cross-sections for $\Delta$-baryon mediated $\pi N \to \pi N$ and $\gamma N \to \pi N$ scattering. For simplicity, we omit the details related to isospin and hence drop the isospin transition matrices from the interaction terms. The $\pi N\Delta$ and $\gamma N\Delta$ interactions then become

$$-\mathcal{H}_{\pi N\Delta} = \frac{1}{\Lambda^3} \left[ c_{\pi L} \left( \partial^a_\pi (N_R^a)^b_c \partial^b \pi \right) (\Delta_{L,a}^{abc}) + \partial_{ab} (N_L^a)^b_c \partial^b \pi (\Delta_{L,a}^{abc}) + h.c., \right.$$

$$-\mathcal{H}_{\gamma N\Delta} = \frac{1}{\Lambda^3} \left[ c_{\gamma L} \left( \partial^a_\pi (N_R^a)^b_c \partial^b \pi \right) (\Delta_{L,a}^{abc}) + c_{\gamma R} \left( \partial_{ab} (N_L^a)^b_c \partial^b \pi \right) (\Delta_{L,a}^{abc}) + h.c., \right.$$

The resonant Feynman diagrams contributing to the $\pi N \to \pi N$ and $\gamma N \to \pi N$ processes are displayed in the upper and lower parts of Fig. 1 respectively. There
The coefficients of ing functions of the masses, have been computed assuming its high energy limit given by

$$L_{\pi N}^{RS} = f \bar{N} \gamma_5 G^{\mu\nu} \partial_\mu \pi + h.c.,$$

$$L_{\gamma N}^{RS} = \bar{N} \left[ g_1 G^{\mu\nu} + g_2 \gamma_5 G^{\mu\nu} + g_3 \gamma_{\mu\nu} \rho G_{\rho\mu\nu} \right] F^{\mu\nu} + h.c.,$$

where $f, g_i, i = 1, ..., 4$ are dimensionful coupling constants, $G^{\mu\nu} \equiv \partial^\mu \Delta^- - \partial^\nu \Delta^-$ is the manifestly invariant RS field tensor and $G^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$ its dual. Here, the isospin-structure has been omitted.

The RS framework is an EFT with the $\pi N\Delta$ and $\gamma N\Delta$ interactions given by dimension-6 operators. Notably, the $\gamma N\Delta$ interactions presented in Eq. (11) depend on four free coupling constants, whereas the MSF contains a single $\gamma N\Delta$ coupling. The interactions for $\pi N\Delta$ in both frameworks involve two space-time derivatives. On the other hand, the $\gamma N\Delta$ interactions contain two derivatives in the RS but only a single derivative in the MSF. In both approaches, the vertices introduce additional momentum dependence.

The propagator for the $\Delta$-resonance, however, behaves quite differently. In the MSF (see Appendix [3]), it has at most three powers of momentum in the numerator, effectively contributing one power of momentum to the scattering amplitude. The propagator of the RS field, instead, takes the following form

$$S^{\mu\nu}(p) = \frac{p^\mu + m}{p^2 - m^2 + i\epsilon} \left[ -\eta^{\mu\nu} + \frac{1}{3} \gamma^{\mu\nu} \gamma^5 \right] + \frac{1}{3m} (\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m^2} p^\mu p^\nu.$$

Like in the MSF, the numerator of the RS propagator contains terms that are at most cubic in momentum, but the last two terms decouple when used with consistent interactions, yielding an energy dependence in the ultraviolet regime that is milder than in the MSF. The resulting $\pi N \rightarrow \pi N$ cross-section is given by

$$\sigma^{\pi N \rightarrow \pi N}_{RS} = \frac{f^4}{576 \pi s} \frac{s^6 \bar{P}(m_\Delta^2/s)}{(s - m_\Delta^2)^2 + m_\Delta^2 \Gamma_\Delta^2},$$

where

$$\bar{P}(x) = 1 + 8.2 x - 16 x^2 + 4.0 x^3 + 6.1 x^4 - 3.8 x^5 + 0.51 x^6 + 0.033 x^7.$$

with the coefficients evaluated for the same masses as for Eq. (8). In this case, the process $\gamma N \rightarrow \pi N$ depends on four coupling constants, which we set equal for simplicity, $g \equiv g_1 = g_2 = g_3 = g_4$. The analytic form of the $\gamma N \rightarrow

5 We have set $A = -1$ as needed for consistent RS interactions [31].
The cross-sections with the ones computed in the Rarita-Schwinger framework in which the \((1, 1/2) \oplus (1/2, 1)\) field also contains unphysical spin-1/2 components that must be excluded from the physical spectrum using additional symmetries. In the proposed multi-spinor formalism, the interaction terms follow solely from the isospin, Lorentz-, parity- and CP-invariance, and they cannot spoil the counting of degrees of freedom. Therefore, the theory is automatically consistent and can be used for physical computations without any additional assumption, like any other effective field theory in particle physics.

Using the proposed multi-spinor formalism, we derived the most general \(\pi N\Delta\) and \(\gamma N\Delta\) interaction terms and computed the resulting \(\pi N \to \Delta^* \to \pi N\) and \(\gamma N \to \Delta^* \to \pi N\) cross-sections. We compared these cross-sections with the ones computed in the Rarita-Schwinger framework and found that, formally, both approaches can reproduce the observed resonant behavior.

However, we argue that the multi-spinor framework provides a theoretically consistent and easily implementable effective framework for studying the \(\Delta\)-resonance. Since this field theory is formulated using physical degrees of freedom only, it provides a promising avenue for extending the model of the \(\Delta\)-resonance to include new interactions not considered in this work, and for constructing effective theories describing other hadronic higher-spin resonances.

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\textsuperscript{6} The RS formalism of Ref. \cite{29} was generalized to spins higher than 3/2 in Ref. \cite{31}.
Appendix A: Symmetric multi-spinor formalism

The multi-spinor notation, first presented in Ref. [74], is based on the well established two-component spinor formalism (see Refs. [84] [85]). Dotted indices (\(\hat{a}, \hat{b}, \ldots\)) and undotted indices (\(a, b, \ldots\)) transform in the (1/2, 0) and (0, 1/2) irrep of the Lorentz group, respectively. The indices are raised and lowered with antisymmetric \(\epsilon_{ab}\)- and \(\epsilon_{\hat{a}\hat{b}}\)-symbols with \(\epsilon_{12} = -\epsilon^{12} = 1\). We utilize the standard convention where dotted (undotted) indices are contracted in ascending (descending) order. The notable exception to this rule is the raising and lowering of indices, e.g. \(t^a = \epsilon^{ab}t_b\), \(t^\hat{a} = t_a\epsilon^{\hat{a}\hat{b}}\) so that \(t^\mu t_a = t_b\epsilon^{\hat{a}\hat{b}}t_a\). A pair consisting of a dotted and an undotted spinor index can be converted into a vector index \(\mu\), i.e., \(p_{\hat{a}a} = p_\mu\sigma^\mu_\hat{a}a\) or \(p_{\hat{a}a} = p_\mu\tilde{\sigma}^\mu_\hat{a}a\) and inversely \(p^\mu = \sigma^\mu_{\hat{a}a}t_\hat{a}\) or \(p^\mu = \tilde{\sigma}^\mu_{\hat{a}a}t_\hat{a}\)/2. In \(\sigma^\mu\), the \(\sigma^0\) is the identity matrix and \(\sigma^i\) with \(i = 1, 2, 3\) are the Pauli matrices; \(\tilde{\sigma}^\mu = (\sigma^0, -\sigma^i)\).

We use the bracket-notation around the index to represent a completely symmetric multi-spinor index. Objects in the \((j, 0)\) irrep are denoted by \(t_{(a)} \equiv t_{(a_1a_2...a_2)}\) and those in the \((0, j)\) irrep by \(t_{(a)} \equiv t_{(\hat{a}_1\hat{a}_2...\hat{a}_2)}\) where all the spinor indices are symmetrized. The objects in \((j, j)\) rep are denoted as \(t_{(a)(\hat{a})}\). For example, in the case of spin-3/2, the momentum \(p_{\hat{a}a}\) corresponds to the multi-spinor object

\[
p_{(a)(\hat{a})} \equiv \frac{1}{3!} [p_{a_1\hat{a}_1}p_{a_2\hat{a}_2}p_{a_3\hat{a}_3} + \text{permutations}].
\]

In a similar way, the \(\epsilon_{ab}, \epsilon^{ab}\) and \(\delta_a^a\) symbols are generalized to \(\epsilon_{(a)(b)}, \epsilon^{(a)(b)}\) and \(\delta_{(a)}^{(a)}\) symbols that can be used to raise and lower symmetric multi-spinor indices.

Appendix B: Feynman rules

Here we present the Feynman rules for \(\Delta\)-resonance propagators and vertices. All vertices are completely symmetric in the spinor indices. Below, \((a)\) and \((\hat{a})\) stand for three totally symmetric spinor indices.

\(\Delta\)-Propagators:

\[
\begin{align*}
\langle \hat{a} | \overset{\sigma}{\not{p}} \overset{\sigma}{\not{p}} | (a) \rangle &= i\frac{\hbar^2}{\sqrt{-m^2}} \epsilon_{\hat{a}a}, \\
\langle b | \overset{\sigma}{\not{p}} | (a) \rangle &= i\frac{m^2\alpha^2}{\sqrt{-m^2}} \epsilon_{b\hat{a}}, \\
\langle \hat{a} | \overset{\sigma}{\not{p}} | \hat{a} \rangle &= i\frac{m^2\alpha^2}{\sqrt{-m^2}} \epsilon_{\hat{a}\hat{a}}, \\
\langle b | \overset{\sigma}{\not{p}} | \hat{a} \rangle &= i\frac{m^2\alpha^2}{\sqrt{-m^2}} \epsilon_{b\hat{a}}.
\end{align*}
\]

External lines:

\[
\begin{align*}
\gamma_\nu(p_1) &\to \Delta^{abc}_\nu(p_2) = \frac{e}{\lambda^\nu} \epsilon^{abc}_\nu \gamma_\nu(p_2) \\
\gamma_\mu(p_1) &\to \Delta^{\hat{a}bc}_\mu(p_2) = \frac{e}{\lambda^\mu} \epsilon^{\hat{a}bc}_\mu \gamma_\mu(p_2) \\
\gamma_\nu(p_1) &\to \Delta^{abc}_\nu(p_2) = \frac{2e}{\lambda^\nu} \epsilon^{abc}_\nu \\
\gamma_\mu(p_1) &\to \Delta^{\hat{a}bc}_\mu(p_2) = \frac{2e}{\lambda^\mu} \epsilon^{\hat{a}bc}_\mu \\
\gamma_\nu(p_1) &\to \Delta^{\hat{a}bc}_\nu(p_2) = \frac{2e}{\lambda^\nu} \epsilon^{\hat{a}bc}_\nu \\
\gamma_\mu(p_1) &\to \Delta^{abc}_\mu(p_2) = \frac{2e}{\lambda^\mu} \epsilon^{abc}_\mu.
\end{align*}
\]

Completeness relations:

\[
\sum_{\sigma} u_{(a)}(p, \sigma) u^*_{(a)}(p, \sigma) = \delta^{(a)}_{(a)},
\]

\[
\sum_{\sigma} v_{(a)}(p, \sigma) v^*_{(a)}(p, \sigma) = \delta^{(a)}_{(a)},
\]

\[
\sum_{\sigma} u_{(a)}(p, \sigma) v^*_{(\hat{a})}(p, \sigma) = m^3 \delta^{(a)}_{(\hat{a})}.
\]

Interaction vertices:

\[
\pi(p_1) \to \Delta^{abc}_{\mu}(p_2) = \frac{i}{\lambda^\mu} \gamma^\mu \gamma^\nu \gamma^\rho \epsilon^{abc}_\mu \\
\pi(p_1) \to \Delta^{\hat{a}bc}_{\mu}(p_2) = \frac{i}{\lambda^\mu} \gamma^\mu \gamma^\nu \gamma^\rho \epsilon^{\hat{a}bc}_\mu.
\]
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