Evolving QCD

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We give an overview of the current theoretical status of proper time renormalization group flow equations applied to QCD. These equations give the evolution of coupling constants in an effective QCD Lagrangian as a function of an infra-red cut-off parameter. This parameter characterizes the resolution with which the system is looked at. Decreasing resolution transforms partonic quarks into constituent quarks and generates pion bound states. The evolution equations can also be applied to finite temperature and finite density QCD.

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Quarks and gluons are the ultimate building blocks of hadrons. Their interaction is governed by the QCD coupling which grows dynamically for low energies. With the increasing coupling gluons and quark-antiquark pairs multiply indefinitely. Therefore QCD forms new structures of dominant collective units in the infra-red. The renormalization group is the principal tool to investigate the formation of these new structures. Its importance in quantum field theory can only be compared to the Schrödinger equation in quantum mechanics. In this paper I use the term renormalization group in a wide sense including the evolution of systems of Lagrangians. It may well be that the degrees of freedom change under evolution. Such an approach is not so well researched yet, but the necessity of a theoretical framework for it - especially in QCD - is universally recognized. I will try to give a summary of our work using renormalization group techniques to handle quark degrees of freedom evolving into hadronic bound states.

The quark-hadron transition governs strong interaction physics when the resolution is lowered below 1 GeV. Hadronic physics is well researched up to a mass scale of 1.5 GeV. Its spectroscopy still lacks a clear identification of glueballs and hybrid states. Yukawa models like the sigma model are natural to describe hadron physics at low energies. In order to bridge the gap between quark and meson physics we choose a hybrid approach where the original action contains a four-quark interaction which is bosonized into sigma and pion fields. These mesonic degrees of freedom are not propagating fields in the ultraviolet, i.e., their kinetic term is zero. They couple to the quark fields and mimic the local four-fermion interaction. In principle all the mesonic couplings arising from the Fierz transformation of a local color exchange interaction should be taken into account. Mean field theory shows a basic ambiguity to the possible Fierz transformations. The RG-equations can overcome this problem. At the ultraviolet scale the meson-quark Yukawa coupling can be set to unity. There is only the mass parameter of the combined sigma and pion fields in the Wigner mode which sets the mass scale for the evolving system. The renormalized Yukawa coupling decreases strongly in the infra-red which makes the evolution infra-red stable. In the approximation with meson loops we have indications that the infra-red evolution has fixed point character. Reversely the divergence of the Yukawa coupling for large scales signals the unsatisfactory high energy behaviour of the hadronic theory. Above the 1 GeV scale the compositeness of hadronic objects becomes important and one has to choose a quark-gluon basis.

In Fig. 1 we summarize the traditional approach to hadronic and nuclear physics indicating the hierarchy between quark physics, nucleon-mesonic physics and nuclear physics. A division into these subfields is highly efficient when special topics in a field are researched, because then refined methods can be applied to get maximum insight into a detailed aspect. On the other hand it is refreshing to cross the borderlines of these fields and see how things are connected. Indeed modern high energy experiments in nuclear physics look at nuclei with high resolution in electron nucleus collisions and at high excitation energies in relativistic heavy ion collisions. They naturally relate nuclear physics to quark physics.

The renormalization group connects regions of different length scales and is the appropriate tool for such an interdisciplinary approach. It avoids the appearance of arbitrary cut-off functions when additional quantum corrections appear. Going beyond the large $N_c$ approximation in the Nambu Jona Lasinio (NJL) model one needs to include meson loops which
Figure 1. The traditional separation of quark physics, the physics of the nucleon/delta and nucleon-nucleon interaction and nuclear physics necessitates an additional cut-off in the Schwinger-Dyson approach. The unifying aspect of the renormalization group is shown in Fig. 2 and illustrated by the work here. Starting from the renormalization group in the vacuum, which only tests physics along the resolution axis, I will cover high temperature physics and high density investigations in the same theoretical framework.

Figure 2. The renormalization group unifies physics along the three different axis: physics at varying resolution, finite temperature and density

Renormalization group flow equations describe the average of an effective action and represent...
the continuum analogue of a block spin transformation. I have tried before to use the Monte Carlo renormalization group to extend the lattice gluon action of Wilson into the long distance region \[2, 3, 4\]. This approach introduces new collective variables into the lattice action, namely the color-dielectric fields. The resulting QCD action has a strong coupling which is modulated by the color-dielectric field and confines. It reproduces results obtained on the finer lattice with an accuracy of 10\%. A large numerical effort is needed to solve a set of overdetermined Schwinger-Dyson equations. Already at that time it was desirable to have an analytic scheme to follow the flow of couplings when high momenta larger than a coarse-graining scale $k$ are integrated out.

The flow equations proposed by Wegner and Houghton \[5\], Polchinsky \[6\], Wetterich and Berges \[7, 8\] present such a scheme. They are ultraviolet and infra-red finite through the introduction of a scale dependent cut-off function. We use renormalization group flow equations with a heat kernel cut-off \[10, 11, 12\] which we think are more practical. Moreover, they are so simple that they are ideally suited to learn and teach renormalization in field theory. No infinities appear at any time. All integrals can be done analytically and have a simple form. The solution of these RG-equations includes complicated summations of diagrams which are similar to solutions of a coupled set of Schwinger-Dyson equations. The heat kernel regularization method \[9\] preserves the symmetries of the theory and keeps the physical interpretation of the evolution equations particularly simple for phenomenological applications.

We choose as a starting action at the UV scale the NJL Lagrangian

$$S_{NJL} = \int d^4x [\bar{q} i\gamma_\mu \partial^\mu q + G \left\{ (\bar{q}q)^2 + (\bar{q}\vec{\tau}\gamma_5 q)^2 \right\}]$$ (1)

which is embedded into a linear $\sigma$-model with $Z_q = 1$, a vanishing wave function renormalization constant $Z_\Phi = 0$ and an effective potential $U(\Phi^2) = \frac{m^2\Phi^2}{2}$. We assume the linear $\sigma$-model to be a valid description of nature below scales of $1.5 \text{ GeV}$. In this region the gluon degrees of freedom are supposed to be already frozen out. Confinement is not dealt with correctly. The physics is governed by chiral symmetry, which is spontaneously broken in the infra-red. The Euclidian action of the linear $\sigma$-model in generalized form is given by

$$S[\Phi, \bar{q}, q] = \int d^4x \left( Z_q(\Phi^2)\bar{q}\gamma_\mu \partial^\mu q + \frac{1}{2} Z_\Phi(\Phi^2) \left( \partial_\mu \Phi^a \right) \left( \partial_\mu \Phi^b \right) + g(\Phi^2)\bar{q}Mq + U(\Phi^2) \right).$$ (2)

The quark and meson fields are $\bar{q}, q$ and $\Phi$, where $\Phi = (\sigma, \vec{\pi})$ is the $O(4)$- and $M = \sigma + i\vec{\tau}\vec{\pi}\gamma_5$ the chiral $SU(2)_L \otimes SU(2)_R$-representation of the meson fields. We consider this action to be preserved during the evolution and follow the generalized couplings and the effective potential during evolution by calculating the effective action $S_{eff}$ in a one-loop approximation.

$$S_{eff}[\Phi, \bar{q}, q] = S[\Phi, \bar{q}, q] - \frac{1}{2} \text{Tr} \log S_{qq} + \frac{1}{2} \text{Tr} \log \left( S_{\Phi^i, \Phi^j} - 2S_{\Phi^i, q}^{-1}S_{qq}^{-1}S_{q, \Phi^j} \right),$$ (3)

with

$$S_{qq}(x, y) = \left. \frac{\delta^2 S[\Phi, \bar{q}, q]}{\delta \bar{q}(x) \delta q(y)} \right|_{av},$$

$$S_{\Phi^i, \Phi^j}(x, y) = \left. \frac{\delta^2 S[\Phi, \bar{q}, q]}{\delta \Phi^i(x) \delta \Phi^j(y)} \right|_{av}.\) (4)
The first logarithm results from the fermion loop fluctuations, whereas the two terms in the second logarithm are the contributions of the bosonic and the mixed loop, respectively. The derivative of $S_{\text{eff}}$ with respect to the evolution scale can be computed after a cut-off function is introduced. For this purpose we represent the fluctuation determinants by Schwinger proper time integrals with the cut-off function $f(k^2\tau) = (1 + k^2\tau + \frac{1}{2}(k^2\tau)^2)e^{-k^2\tau}$.

$$\text{Tr} \log(A) = -\text{Tr} \int_0^\infty \frac{d\tau}{\tau} e^{-\tau A} f(k^2\tau). \quad (6)$$

The heat kernel cut-off function $f$ suppresses all fluctuations with momenta below the cut-off scale $k$. Going to $k \to 0$ means to include more and more infra-red modes. Finally, all modes are included, because of the limiting behaviour $f_{k \to 0} \to 1$. The ultraviolet region is left undisturbed due to the extra $\tau$ and $\tau^2$ terms in front of the exponential. Therefore we can extend the $\tau$-integration to the interval $[0, \infty]$. We further replace the masses and couplings of the classical action by the running masses and running couplings of the effective action $S_{\text{eff}}$. Generically, we obtain the following evolution equation for $S_{\text{eff}}$ which contains its second order derivatives $S''_{\text{eff}}$ indicated in Eqs.(4,5):

$$\frac{\partial S_{\text{eff}}}{\partial k} = -\frac{1}{2} \text{Tr} \int \frac{d\tau}{\tau} e^{-S''_{\text{eff}}(k)} \frac{\partial}{\partial k} f(k^2\tau). \quad (7)$$

This replacement of $S''$ by $S''_{\text{eff}}$ turns the one-loop equation into a renormalization group improved flow equation, which goes beyond the standard one-loop renormalization group running and includes higher loop terms successively into the proper time integral. Therefore it is capable to include nonperturbative physics in the strong coupling region. The evolution equations are obtained by comparing the derivative on $S_{\text{eff}}(k)$ with respect to $k$ with the formal expression of Eq. (2) which contains the generalized couplings and the effective potential as functions of $k$. The local potential terms are easy to evaluate and lead to nonlinear partial differential equations containing derivatives of the potential with respect to $\Phi^2$. The evolution of the other couplings can be obtained in a derivative expansion which is an expansion in derivatives (momentum) over mass. This expansion is well controlled because of two reasons. Firstly, we approach the infra-red, so momenta become always smaller, we need only few terms in the expansion. Secondly, the infra-red cut-off scale $k$ acts like a mass cut-off, so the expansion is well defined even if the excitations themselves are massless, like the quarks in the ultraviolet or the mesons in the infra-red region.

Let me give an overview of the results we have achieved so far. We have solved in local potential approximation the vacuum evolution, the finite temperature evolution and finite baryon density evolution [14]. There are strong similarities between the vacuum evolution as a function of resolution scale and as a function of temperature. Both show the transition from partons to constituent quarks which has also been phenomenologically checked in deep inelastic scattering at fixed energy $\sqrt{s}$ as a function of photon virtuality $Q^2$ see Ref. [13] and Fig. (2). Numerically, the transition temperature $2\pi T_c$ is about equal to the chiral symmetry breaking scale $k_{\chi_{sb}}$ where the quarks condense in the vacuum. Going from the ultraviolet to the infra-red the strong attractive quark-antiquark interaction leads to a condensation of quark pairs in the
vacuum. At the resolution $k_{\chi sb}$ the effective mass parameter of the mesons equals zero. The
critical exponents agree with the $0(4)$ critical exponents, but differ from the mean field values
\[14\].

In large $N_c$ approximation we have shown that the renormalization group flow equations are
identical to the gap equations of the NJL model \[15\]. Further it is possible to calculate the
eigenmode spectrum of the Euclidean Dirac operator \[16\] which previously was only known for
very small eigenvalues in random matrix theories.

The full equations possess extra complexity via the expansion of all running couplings and
wavefunction renormalization parameters in $\Phi^2$. As mentioned above the global form of the
meson potential $U(\Phi^2)$ changes from the parabolic shape in the ultraviolet to a mexican hat
shape in the infra-red. This evolution leads to a change of the minimum of the effective potential
with resolution $k$. We track all couplings in Taylor series around this running minimum with
terms up to 5th order in $\Phi^2$. This leads to 20 coupled non-linear differential equations. Including
meson loop terms the infra-red properties of the system depend only on the mass term in
the ultraviolet and no longer on the starting ultraviolet cut-off. We have a real fixed point
behaviour for the pion decay constant, meson quark coupling, constituent quark mass and
quark condensate. In comparison to NJL the flow equations have predictive power since only
one mass scale fixes all the other dimensionful parameters.

For finite density \[17\] the model has similar deficiencies as the conventional NJL model. It
overbinds and restores the chiral symmetry too early. We have improved the model by including
$\omega$-repulsion and plan to do this calculation including loop corrections.

The very successful chiral perturbation theory alone cannot connect to QCD in the ultraviolet.
Typically it starts to fail when the resolution scale is of the order of half the $\rho$ mass. Hybrid
models like the one presented here containing meson degrees of freedom and quarks extend
to larger momentum scales, which is a definite advantage. There are, however, two problems:
Firstly, the number of mesons in the theory should include also the vector mesons. Secondly,
the size of the scalar Yukawa couplings increases dramatically in the ultraviolet, therefore it
seems difficult to cross over to the asymptotically free gauge theory of QCD. For nuclei a theory
with quarks and nucleons \[18\] seems to be the most efficient way to describe the transition from
purely nucleonic matter to quark matter at high baryon density \[19\]. Our model like most of the
other hybrid models lacks a field theoretic mechanism to avoid the appearance of free quarks as
asymptotic states. A major progress would be to capture the smooth infra-red limit of lattice
QCD in a field theoretic continuum picture.

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