How Does Transverse MHD Wave-driven Turbulence Influence the Density Filling Factor in the Solar Corona?

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Abstract

It is well established that transverse MHD waves are ubiquitous in the solar corona. One of the possible mechanisms for heating both open (e.g., coronal holes) and closed (e.g., coronal loops) magnetic field regions of the solar corona is MHD wave-driven turbulence. In this work, we study the variation of the filling factor of overdense structures in the solar corona due to the generation of transverse MHD wave-driven turbulence. Using 3D MHD simulations, we estimate the density filling factor of an open magnetic structure by calculating the fraction of the volume occupied by the overdense plasma structures relative to the entire volume of the simulation domain. Next, we perform forward modeling and generate synthetic spectra of Fe XIII 10749 Å and 10800 Å density-sensitive line pairs using FoMo. Using the synthetic images, we again estimate the filling factors. The estimated filling factors obtained from both methods are in reasonable agreement. Also, our results match fairly well with the observations of filling factors in coronal holes and loops. Our results show that the generation of turbulence increases the filling factor of the solar corona.

Unified Astronomy Thesaurus concepts: Solar corona (1483); Magnetohydrodynamics (1964)

1. Introduction

The study of the magnetic and thermodynamic structure of solar corona is important, as it plays a key role in heating the corona up to temperatures of millions of kelvin. There are mainly two schools of thought for the coronal heating problem: dissipation of magnetohydrodynamic (MHD) waves in the solar atmosphere and magnetic reconnection. Evidence of MHD waves and their different modes have been reported for both ground- and space-based observations in open magnetic field regions (e.g., Banerjee et al. 2021). The MHD waves can dissipate energy and heat the solar corona as they propagate through the medium (Ruzmaikin & Berger 1998; Klimchuk 2006; Arregui 2015; Morton et al. 2015, and references therein). Due to the partial reflection of the high-frequency torsional Alfvén waves in the transition region, a significant amount of energy flux (∼10^3 W m^-2) is transferred to the corona (Srivastava et al. 2017). The other mechanism of coronal heating is the topological rearrangement of the magnetic field lines due to the motion of the magnetic footpoints leading to magnetic reconnection, which releases energy in the solar corona (Parker 1988; Peter et al. 2005; Thalmann et al. 2013, and references therein). Along with the wave dissipation and reconnection, another class of coronal heating mechanism that has drawn attention in the solar community over the last decade is wave-driven MHD turbulence (Einaudi et al. 1996; Matthaeus et al. 1999; Cranmer & Van Ballegooijen 2005; Rappazzo et al. 2008; Van Ballegooijen et al. 2011; Van der Holst et al. 2014). The wave-driven turbulence is either generated by counterpropagating waves (Dmitruk et al. 2001) or unidirectionally propagating transverse MHD waves in a transversely inhomogeneous medium (also called uniturbulence; Magyar et al. 2017, 2019; Van Doorsselaere et al. 2020).

The counterpropagating transverse MHD waves observed as quasi-periodic Doppler velocity shifts were reported by Tomczyk et al. (2007, 2008), Tomczyk & McIntosh (2009), and Morton et al. (2015, and references therein) using the CORonal Multi-channel Polarimeter (CoMP; Tomczyk et al. 2008). The estimated energy carried by the transverse waves was found to be 2–3 orders of magnitude less than that required for heating the solar corona (Tomczyk et al. 2007). This discrepancy could be due to the unresolved Doppler velocity shifts due to the line-of-sight superposition of different oscillating structures (De Moortel & Pascoe 2012; McIntosh & De Pontieu 2012; Pant et al. 2019). Thus the true wave energies are hidden in the nonthermal line widths (Pant et al. 2019). Therefore, a relation between the nonthermal line widths and rms wave amplitudes is given by Pant & Van Doorsselaere (2020) to correctly estimate the wave energy carried by the bulk Alfvén waves. However, using the relation of the energy density of the bulk Alfvén waves overestimates the true energy (Goossens et al. 2012). Also, Van Doorsselaere et al. (2014) has shown that the wave energies are reduced by several factors due to the inclusion of the filling factor. The study of the filling factors helps us to understand the overall density structure and how the overdense regions are spatially distributed in the medium. This gives an estimate of the density inhomogeneity of the medium (Hahn & Savin 2016). That said, kink waves (body waves) have a maximum wave energy inside the overdense plasma structures, while the surface waves (Alfvén waves) have energy concentrated at the boundaries of it (Van Doorsselaere et al. 2014). In this regard density inhomogeneity, and hence the study of the density filling factor, is imperative to estimate the true energy flux carried by the waves in the solar corona.

We define the density filling factor as the fraction of the overdense volume that is embedded within a low-density region. A medium with the density filling factor close to unity indicates that the medium is nearly homogeneously filled with overdense structures. The filling factor in open magnetic...
structures (e.g., coronal holes (CHs), coronal bright points, plumes, interplumes, and moss regions) has been calculated in the past (Warren et al. 2008; Tripathi et al. 2010; Hahn & Savin 2016). These regions are the open magnetic structures, where the magnetic field lines rise but never come back in the same horizontal plane within the region of interest (Sen & Mangalam 2019). Dere (2008) has estimated the filling factor of the coronal bright points in the quiet Sun (QS) regions using data from the EUV Imaging Spectrometer (EIS) on board Hinode (Culhane et al. 2007) and found it to vary from $4 \times 10^{-5}$ to 0.20 with a median value of 0.015. Verbeeck et al. (2014) have calculated the filling factors varying from 0.02 to 0.22 in active regions and from 0.02 to 0.18 in the quiet Sun regions. Hahn & Savin (2016) have inferred the filling factors using a “Two-slab density model” in the CHs, plume, interplume, and quiet Sun regions observed in EIS/Hinode data. They have found the filling factors varying between 0.10 and 0.20 in the interplume regions, varying between 0.004 and 0.02 in the plumes, and equal to 0.10 in the CHs and QS regions. The volumetric filling factor of the coronal loops can be calculated by emission measure (EM), plasma density of the medium and the apparent column depth. Apart from open magnetic field regions, Gupta et al. (2015) have calculated the filling factor along an entire coronal loop observed by EIS/Hinode and the Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) on board the Solar Dynamics Observatory, and found it to vary from 0.11 at the footpoint and to more than 1 at the loop top. Using the emission lines of EIS/Hinode, Tripathi et al. (2010) have estimated the filling factor for five different moss regions, which vary from 0.002 to 0.1 for Fe XII, 0.03 to 1.9 for Fe XIII, and 0.2 to 1.8 for Fe XIV lines. In spite of these observational advances in the estimations of the filling factors in the solar corona, our understanding of the physical mechanism responsible for the generation of filling factors and its variation with height is limited.

In this work, we infer the effects of transverse MHD wave-driven turbulence on the filling factor of the plasma using two different methods. First, by taking the ratio of the volume of overdense plasma columns with density of more than a threshold value to the entire volume of the simulation domain. Second, we generate the synthetic intensity images of Fe XII 10749 Å and 10800 Å density-sensitive emission lines by forward modeling using FoMo. We compare the estimated filling factors obtained from both methods and with the observations of solar coronal structures. Also, we investigate the variation of the filling factor with height due to the presence of turbulence. The rest of the paper is organized as follows. In Section 2, we discuss the simulation setup and the input parameters of the simulation. In Section 3, we describe the methods for estimating the filling factor from area estimation in 3.1, and from forward modeling in 3.2, and report the main findings of the work. In Section 4, we compare our estimated values with the observations, discuss the novelty of the work, and finally conclude our findings.

2. Simulation Setup

To understand the effect of the MHD wave-driven turbulence on the filling factor of an open field region of the solar corona (e.g., CHs), we use an ideal 3D MHD simulation using MPI-AMRVAC (Porth et al. 2014). A detailed description of the simulation setup is given in Pant et al. (2019). The simulation has a grid size of $128 \times 512 \times 512$, which spans 50 Mm $\times$ 5 Mm $\times$ 5 Mm in the spatial scale that solves the set of the following MHD equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla (\rho \mathbf{v} \mathbf{v}) - \nabla (p + B^2/2\mu_0) - \rho g = 0, \tag{2}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot [(\mathbf{v} E - B \mathbf{v} B)/\mu_0] + \nu (p + B^2/2\mu_0) - \rho g = 0, \tag{3}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \tag{4}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{5}
\]

where $\rho$ is the mass density, $\mathbf{B}$ is the magnetic field strength, $p$ is the gas pressure, $E$ is the total energy density, and $\mathbf{g}$ is the acceleration due to gravity that acts along the negative $z$-axis of the simulation setup. Here, the ideal gas law $p = \frac{\rho k T}{\mu m}$ is used in the simulation, and the energy density $E = \frac{p}{\gamma - 1} + \frac{\mu^2}{2} + \frac{B^2}{2\mu_0}$, where $T$ is the temperature, $\mu_0$ is the magnetic permeability in vacuum, $\mu = 0.6$ is the coronal abundance, $m_H$ is the proton mass, $\gamma = 5/3$ is the gas constant for the monoatomic gas, and $k_B$ is the Boltzmann constant. The solenoidal condition of the magnetic field ($\nabla \cdot \mathbf{B} = 0$) is satisfied by the Powell’s scheme.

The simulation represents an open field magnetic structure. The simulation box is implanted in the lower corona with the spatial dimension of $x \in [0, 50]$ Mm, $y \in [-2.5, 2.5]$ Mm, and $z \in [-2.5, 2.5]$ Mm with a spatial resolution of 0.59 Mm, 0.01 Mm, and 0.01 Mm along the $x$, $y$, and $z$ directions, respectively. Furthermore, density inhomogeneities are placed randomly along the $y$-$z$ plane according to

\[
\rho(x, y, z) = \left(\rho_0 + \sum_{i=0}^{50} A_i \exp(-[(y - y_i)^2 - (z - z_i)^2]/2\sigma_i^2)\right) \exp (-x/H(y, z)), \tag{6}
\]

where $\rho_0 = 2 \times 10^{-16}$ g cm$^{-3}$ is the background density, which is the typical density of the lower solar corona (~1.05 solar radius), $A_i$’s are the density amplitudes that are taken randomly from the uniform distribution of [0, 5]$\rho_0$, $\sigma_i$’s are the spatial extent of the density inhomogeneities that are drawn randomly from the uniform distribution [0, 250] km, and $H(y, z)$ is the scale height that increases with the temperature and has different values in different locations in the $y$-$z$ plane. The scale height in Equation (6) is determined by $H(y, z) = g_2/(\mu m_0 k T)$, where $g = 274 m s^{-2}$ is the acceleration due to gravity at the solar surface. In the initial condition of the simulation, the gas pressure is given by $p = p_0 \exp(-x/H(y, z))$, where $p_0 = \beta B_0^2/2$ and plasma-$\beta = 0.15$, and the background magnetic field $B_0 = B_0 \hat{x}$, with the magnetic field strength $B_0 = 5$ G, is used in the simulation. Therefore, the temperature $T$ is constant along the field lines at the initial state of the simulation. However, the evolution of the gas pressure and temperature is governed by the Equations (2) and (3).

After the initial setup of the simulation, the system is allowed to evolve for ~100 s before the implementation of any drivers so that the system comes to a pressure equilibrium state. In this stage, all of the boundaries are kept open such that any
generated MHD wave can leave the simulation domain. Once the system reaches the pressure equilibrium, the bottom boundary is excited by incorporating velocity drivers given by the following equations:

\[
v_x(t) = \sum_{i=1}^{10} U_i \sin(\omega_i t),
\]

\[
v_z(t) = \sum_{i=1}^{10} V_i \sin(\omega_i t).
\]

In this stage, the boundaries in the \(y-z\) directions are set to be periodic. Here, \(\omega_i\)'s are the angular frequencies that are chosen from the observed oscillation periods of transverse waves in the off-limb solar plumes, which follow a log-normal distribution in the range \(61-2097\) s with a mode value of \(121\) s (Thurgood et al. 2014). \(U_i\), \(V_i\) are the velocity amplitudes that are chosen randomly from the uniform distributions of \([-U_0, U_0]\), and \([-V_0, V_0]\), where \(U_0 = V_0 = 11/\sqrt{2}\) \(\text{km s}^{-1}\) (see Pant et al. 2019, for details). The total duration of the simulation is \(1000\) s with a \(20\) s cadence, which gives the 50 snapshots of the simulation. Incorporation of the transverse velocity drivers at the base of the simulation box generates transverse MHD waves that propagate along the vertical direction (+\(x\) direction). Propagation of these waves in the density-inhomogeneous medium leads to the generation of turbulence (Magyar et al. 2017; Pant et al. 2019). The top boundary of the simulation box is kept open so that a negligible amount of the pointing flux reflects back, and there are no counterpropagating waves. We estimate the temporal evolution of the average density at the open boundary at \(x = 50\) Mm between \(t = 0\) to \(880\) s, which has a maximum variation of \(4.68\%\). This implies that the mass flux through the open boundary is very small, and hence it does not much affect the density distribution. We also estimate the temporal evolution of the average density of the entire simulation domain and determine that the maximum change of the average density is \(0.367\%\) between \(t = 0\) to \(880\) s. The numerical mass diffusion in a simulation code depends on the resolution of the setup. The higher the resolution, the lower the numerical mass diffusion effect. The \(\rho_0\) gives the \(50\) snapshots of the simulation. Incorporation of the open boundary at \(t = 0\), when the system in equilibrium, there is a variation of \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) along the \(x\)-direction. This is due to the presence of the density scale height, the \(H(y, z)\) term, in the density inhomogeneity formulation in Equation (6). \(H(y, z)\) increases with the temperature and has different values at different locations in the \(y-z\) plane. The temperature inside the density-enhanced regions are lower compared to the background medium, and the temperature variation along the field lines is quite small in the pressure equilibrium state. Hence, the scale height of the overdense regions is smaller than background medium. Therefore, the variation of the density along the \(x\)-direction in the overdense regions is faster than in the low-density regions, and hence the \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) varies along the \(x\)-direction at \(t = 0\). The variation of the \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) along the \(x\)-direction is shown in Figure 2(a) for different times, when the system is in equilibrium (\(t = 0\)), and when the turbulence is present in the medium (\(t = 880\) s). Density \(\rho_0(x)\) in Equation (9) is prescribed in such a way that \(\rho_{\text{min}}(x) < \rho_0(x) < \rho_{\text{max}}(x)\), which is valid only if \(0 < \lambda < 1\). Here, \(\lambda\) is a free parameter that decides the area of coverage. We choose \(\lambda = 0.08, 0.1,\) and \(0.2\) for estimating the density filling factor of the medium. The variation of \(\rho_0(x)\) along the \(x\)-direction is shown in Figure 2(b) for different \(\lambda\). The larger the value of \(\lambda\), the larger the density threshold, \(\rho_0(x)\); (see Equation (9)). Hence, for a larger \(\lambda\), the density contours separate out a smaller area of the overdense regions from the background and hence the filling factor drops down. Similarly, for smaller values of \(\lambda\), the density filling factor increases. We estimate the value of \(\rho_0(x)\) for \(t = 0\) and find it to be decreasing with \(x\). This is the consequence of density stratification due to gravity. The area of the region in which the density is more than \(\rho_0(x)\) on the \(y-z\) plane is calculated by setting binary masks of density. For that, we create binary masks in the \(y-z\) plane and assign “1” to the pixels that have a density more than \(\rho_0\) and “0” to the pixels with a density less than \(\rho_0\). Then, we count the number of pixels that are assigned “1”. This gives the area of the region in which the density is more than \(\rho_0\) in units of pixel numbers of the simulation grid of the \(y-z\) plane. Finally, the filling factor, \(\phi_{\text{simu}}(x)\), in the simulation is given by

\[
\phi_{\text{simu}}(x) = \frac{\text{Area of the region whose density is more than } \rho_0(x)}{\text{Area of the } y-z \text{ plane}}.
\]

The variation of the filling factor along the \(x\)-direction is shown in Figure 3(a). Here, we calculate the cross-sectional area due to all of the plasma columns that increase with \(x\) when the system is in equilibrium (\(t = 0\)), and hence the density filling

3. Method and Results

We define the density filling factor as the fraction of the volume occupied by the overdense plasma structures with respect to the total volume of the region. The filling factor gives an estimation of the inhomogeneity of the medium. We have estimated the density filling factor by two different methods described in the following subsections.

3.1. Estimation of Filling Factor Using MHD Simulation

We calculate the density filling factor of the overdense regions obtained from the MHD simulation by taking the fraction of the area filled with plasma having a density greater than the chosen density threshold, \(\rho_0(x)\), to the area of the \(y-z\) plane. To find the area of the region that has a density of more than \(\rho_0(x)\) at each plane along \(x\), we define the contours equal to the density threshold \(\rho_0(x)\) given by

\[
\rho_0(x) = \rho_{\text{min}}(x) + \lambda_f (\rho_{\text{max}}(x) - \rho_{\text{min}}(x)),
\]

where \(\rho_{\text{min}}(x)\) and \(\rho_{\text{max}}(x)\) are the minimum and maximum densities, respectively. The density maps obtained from the simulation are shown in Figure 1 at \(x = 0\) and \(x = 50\) Mm for two different times, \(t = 0, 880\) s. The contours represent the density threshold, \(\rho_0(x)\), estimated from Equation (9). At \(t = 0\), when the system is in pressure equilibrium, there is a variation of \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) along the \(x\)-direction. This is due to the presence of the density scale height, the \(H(y, z)\) term, in the density inhomogeneity formulation in Equation (6). \(H(y, z)\) increases with the temperature and has different values at different locations in the \(y-z\) plane. The temperature inside the density-enhanced regions are lower compared to the background medium, and the temperature variation along the field lines is quite small in the pressure equilibrium state. Hence, the scale height of the overdense regions is smaller than background medium. Therefore, the variation of the density along the \(x\)-direction in the overdense regions is faster than in the low-density regions, and hence the \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) varies along the \(x\)-direction at \(t = 0\). The variation of the \(\rho_{\text{min}}(x)/\rho_{\text{max}}(x)\) along the \(x\)-direction is shown in Figure 2(a) for different times, when the system is in equilibrium (\(t = 0\)), and when the turbulence is present in the medium (\(t = 880\) s). Density \(\rho_0(x)\) in Equation (9) is prescribed in such a way that \(\rho_{\text{min}}(x) < \rho_0(x) < \rho_{\text{max}}(x)\), which is valid only if \(0 < \lambda < 1\). Here, \(\lambda\) is a free parameter that decides the area of coverage. We choose \(\lambda = 0.08, 0.1,\) and \(0.2\) for estimating the density filling factor of the medium. The variation of \(\rho_0(x)\) along the \(x\)-direction is shown in Figure 2(b) for different \(\lambda\). The larger the value of \(\lambda\), the larger the density threshold, \(\rho_0(x)\); (see Equation (9)). Hence, for a larger \(\lambda\), the density contours separate out a smaller area of the overdense regions from the background and hence the filling factor drops down. Similarly, for smaller values of \(\lambda\), the density filling factor increases. We estimate the value of \(\rho_0(x)\) for \(t = 0\) and find it to be decreasing with \(x\). This is the consequence of density stratification due to gravity. The area of the region in which the density is more than \(\rho_0(x)\) on the \(y-z\) plane is calculated by setting binary masks of density. For that, we create binary masks in the \(y-z\) plane and assign “1” to the pixels that have a density more than \(\rho_0\) and “0” to the pixels with a density less than \(\rho_0\). Then, we count the number of pixels that are assigned “1”. This gives the area of the region in which the density is more than \(\rho_0\) in units of pixel numbers of the simulation grid of the \(y-z\) plane. Finally, the filling factor, \(\phi_{\text{simu}}(x)\), in the simulation is given by

\[
\phi_{\text{simu}}(x) = \frac{\text{Area of the region whose density is more than } \rho_0(x)}{\text{Area of the } y-z \text{ plane}}.
\]
factor of the medium also increases by the same fraction. For the turbulent medium \((t = 880 \text{ s})\), the density filling factor increases with \(x\) more rapidly due to the generation of turbulence that leads to the mixing and distortion of the plasma column boundaries. This lead to a more homogeneous density variation, leading to the increase in the density filling factor. The temporal variation of the density filling factor for different heights and \(\lambda_f\) are shown in Figure 3(b). The variation of the filling factor with time is very small at the bottom boundary of the \(x\) plane \((x = 0)\) where the velocity drivers are incorporated. At \(x = 0\), the filling factor varies from 0.38 to 0.39 for \(\lambda_f = 0.08\), 0.34 to 0.36 for \(\lambda_f = 0.1\), and 0.23 to 0.25 for \(\lambda_f = 0.2\) with time, \(t = 0\) to \(880 \text{ s}\). Due to the wave-driven turbulence, the filling factor variation is greater with time at a higher height. At \(x = 50 \text{ Mm}\), the filling factor increases from 0.59 to 0.92 for \(\lambda_f = 0.08\), 0.55 to 0.80 for \(\lambda_f = 0.1\), and 0.48 to 0.72 for \(\lambda_f = 0.2\) with time, \(t = 0\) to \(880 \text{ s}\).

To estimate the temperature of the medium, we use the pressure, \(p\), the background density, \(\rho_0\), and the density contrast values (from Equation (6)) using the ideal gas law. This gives the initial temperature of the medium in the range \(\approx 0.4–4 \text{ MK}\). An example of the variation of the temperature distribution at \(x = 25 \text{ Mm}\) along the \(y-z\) plane is shown in Figure 4 for \(t = 0\) and \(880 \text{ s}\). This shows that the regions with higher densities have lower temperatures, and the regions with lower densities have higher temperatures. Figure 5 shows the temperature along the field lines \((x-z\) plane\) at \(y = 0\) for \(t = 0\) and \(880 \text{ s}\), which shows that the variation of the temperature along the field lines are very small in the pressure equilibrium state \((t = 0)\). Furthermore, when the turbulence is generated in the medium due to wave excitation, the overdense regions mix with the background, and

Figure 1. Top panel: variation of the density along the \(y-z\) plane when the system is in equilibrium \((t = 0)\) at (a) \(x = 0\) and (b) \(x = 50 \text{ Mm}\), obtained from the simulation using MPI-AMRVAC. The contours represent the density threshold, \(\rho_\text{th}(x)\), values for \(\lambda_f = 0.1\) given by Equation (9). Bottom panel: same as the top panel at (c) \(x = 0\) and (d) \(x = 50 \text{ Mm}\) when the turbulence is present in the medium \((t = 880 \text{ s})\).
hence the temperature distribution becomes more homogeneous in space (see Figures 4(b) and 5(b)). Figure 6 shows the comparison of the temperature distribution in the $y$–$z$ plane at different times at $x = 25$ Mm. This reflects the fact that the temperature distribution for $t = 880$ s is more homogeneous than at $t = 500$ and 0 s. We have estimated that the minimum and maximum temperatures of the medium at $t = 0$ are 0.452 MK and 4.36 MK, which increases to 0.812 MK and 4.41 MK at $t = 880$ s, respectively. One of the possible causes of this small increase of the temperature is the adiabatic compression and rarefaction of the medium due to the wave-driven turbulence. However, heat conduction and optically thin radiative loss terms are not incorporated in the simulation setup in Equations (1)–(5). Certainly, these terms play roles in establishing the coronal density structures, but, the main aim of this work is to investigate the effect of wave-driven turbulence only in the density inhomogeneities of the solar corona without accounting for conduction and radiative loss terms. We understand that this is the limitation of this study, but nevertheless, it is interesting because it shows that MHD turbulence alone is able to increase filling factors in the solar corona. A more detailed investigation incorporating the energy losses and transport can be performed.

3.2. Estimation of Filling Factor from Synthetic Spectra Using FoMo

To convert the physical variables obtained from the simulation (e.g., density, velocity, and energy density) into spectroscopic observables, e.g., specific intensity, we use forward modeling with FoMo (Antolin & Van Doorsselaere 2013; Van Doorsselaere et al. 2016). FoMo calculates the line-of-sight (LOS) integrated emission from the optically thin coronal plasma. Here, we define LOS = 0 and $\pi/2$ along the $+z$ and $-y$ directions, respectively. In this method, we generate synthetic spectra for the density-sensitive lines pairs, Fe XIX 10749 Å and 10800 Å (Landi et al. 2016) for two LOS directions, 0 and $\pi/2$, when the system is in pressure.
Figure 4. Variation of the temperature along the $y$–$z$ plane at $x = 25$ Mm when (a) the system is in equilibrium ($t = 0$), and (b) turbulence is present in the medium ($t = 880$ s), obtained from the simulation.

Figure 5. Variation of the temperature along the $x$–$z$ plane at $y = 0$ Mm when (a) the system is in equilibrium ($t = 0$), and (b) turbulence is present in the medium ($t = 880$ s), obtained from the simulation.
equilibrium \((t = 0)\), and when turbulence is present in the medium \((t = 880\) s; see Figure 7). Next, we degrade the generated synthetic images to the spatial resolution of 720 km along the \(x\) and \(y\) directions, which is comparable to the spatial resolution \((1^{st}\) per pixel) of the EIS (Culhane et al. 2007). The EIS degraded synthetic images are shown in Figure 8 for two different LOS directions and \(t\). Following Gupta et al. (2015), the EM can be calculated as

\[
EM_1 = \frac{I_{\text{EIS}}}{0.83A_eG(n_e, T)}.
\]

Here, \(G(n_e, T)\) is the contribution function that we calculate in a 3D grid of \((n_e, T)\) within the domain, \(n_e = 10^8-10^{11} \text{ cm}^{-3}\), and \(T = 5.5-6.95\) K for the density-sensitive lines Fe XIII 10749 Å and 10800 Å. For calculating \(G(n_e, T)\), we use \texttt{goft} \_\texttt{table} \_\texttt{pro} from FoMo-idi (Van Doorsselaere et al. 2016), and CHIANTI version 9 (Dere et al. 1997, 2019). \(I_{\text{EIS}}\) is the EIS degraded synthetic intensity for the Fe XIII 10749 Å line obtained from FoMo. The temperature, \(\log(T) = 6.25\) K, is taken as the peak formation temperature for the Fe XIII 10749 line, where the contribution function has the maximum value, and \(A_e = 0.6\) is the coronal abundance. Using the intensity ratio of the EIS degraded synthetic maps of Fe XIII 10749 Å and 10800 Å lines, and the intensity versus density calibration obtained from CHIANTI (see Figure 10), we calculate the electron density, \(n_e\), in the projected plane perpendicular to the LOS direction. The EM can be estimated by

\[
EM_2 = \int_{h_{\text{eff}}} n_e^2 dh,
\]

where \(h_{\text{eff}}\) is the LOS depth of the overdense plasma regions of the EIS degraded synthetic maps. In this case, the \(h_{\text{eff}}\) at each height, \(x\), is measured assuming the overdense plasma region has an azimuthal symmetry around the \(x\)-direction. Hence, the effective depth of the overdense plasma regions, \(h_{\text{eff}}\), is calculated by fitting a Gaussian to the intensity across the overdense plasma regions at different heights and taking the FWHM of the Gaussian fit. Thus, the plasma filling factor in the plane perpendicular to the LOS direction can be calculated by (Gupta et al. 2015)

\[
\phi = \frac{I_{\text{EIS}}}{0.83A_eG(n_e, T)n_e^2 h_{\text{eff}}}.
\]

The intensity and the Gaussian fit across the plasma columns at \(x = 25\) Mm are shown in Figure 9 for two different times and LOS directions. The variation of \(h_{\text{eff}}\) along the \(x\)-direction is shown in Figures 11(a) and (c) for LOS directions 0 and \(\pi/2\), respectively. For calculating the \(h_{\text{eff}}\), the FWHM of the intensity Gaussian fit mainly depends on two factors: (1) the extension of the Gaussian distribution along the \(y\)-direction, and (2) how much the Gaussian distribution is skewed due to the intensity enhancement in the synthetic spectra. For example, the horizontal spread (along the \(y\)-direction) of the Gaussian fit for the plasma column in Figure 8(b) is more at \(x = 0\) \((-2.5\) Mm \(\leq y \leq 1\) Mm) than at \(x = 10\) Mm \((-1.8\) Mm \(\leq y \leq 1\) Mm), and hence the FWHM of the Gaussian fit or \(h_{\text{eff}}\) drops from 3.2 Mm at \(x = 0\) to 2 Mm at \(x = 10\) Mm, which is reflected in Figure 11(a) for \(t = 880\) s. By contrast, the intensity across the black dashed line in the synthetic map of Figure 8(d) is more skewed at \(x = 25\) Mm than at \(x = 20\) Mm (where \(-1\) Mm \(\leq z \leq 1.5\) Mm for both cases). Therefore, the Gaussian fit becomes more skewed at \(x = 25\) Mm, and hence the \(h_{\text{eff}}\) decreases from 3.4 Mm at \(x = 20\) Mm to 2.5 Mm at \(x = 25\) Mm, which is shown in Figure 11(c). The variation of the filling factors obtained from the EIS degraded synthetic spectra are shown in Figures 11(b) and (d) for the LOS directions 0 and \(\pi/2\), respectively (along the black dashed line marked in Figure 8(a)–(d)). We find that the maximum filling factor value is \(\approx 0.8\) for the turbulent medium (for LOS = 0), whereas, for the equilibrium case, it reduces to \(\approx 0.2\). This indicates that the medium becomes more density homogeneous due to the development of turbulence. From Figures 11(b) and (d), we notice that the filling factor values are different for two LOS directions, and this is more significant when the turbulence is present in the medium. This implies that the density inhomogeneities are irregular and asymmetric around the \(x\)-direction.

4. Discussions and Conclusion

While estimating the density filling factor by taking the ratio of the area of the density-enhanced region to the area of the \(y-z\) plane (the method described in Section 3.1), we notice that the density filling factor increases with height for both \(t = 0\) and 880 s, as shown in Figure 11(a). However, the increase in the filling factor is larger for the latter case. In the pressure equilibrium state \((t = 0)\), the overdense plasma columns expand with height in the corona because the total pressure scale height inside the plasma columns that is due to the magnetic and plasma pressure is more than the pressure scale height of the background medium, and hence the background pressure drops more rapidly with height than the pressure inside the overdense plasma columns. This leads to the expansion of the overdense plasma columns along the transverse direction, and therefore the density filling factor of the medium increases with height. For \(t = 0\), we estimate that the total cross-sectional area due to all of the overdense plasma columns, and hence the density filling factor, increases along the \(x\)-direction (see Section 3.1 for the estimated values). When turbulence is present in the medium \((t = 880\) s), the plasma columns containing overdense regions disrupt and the density-enhanced regions start to mix with the background medium, causing a further increase of the density filling factor. This makes the medium more density
Figure 7. Top panel: intensity map of the synthetic images for LOS = 0 obtained by forward modeling using FoMo for Fe XIII 10749 Å when (a) the system is in pressure equilibrium ($t = 0$), and (b) there is turbulence in the medium ($t = 880$ s). Bottom panel: same as top panel with LOS direction $\pi/2$ for (c) $t = 0$ and (d) $t = 880$ s.
Figure 8. Top panel: EIS degraded intensity map for LOS = 0 for (a) $t = 0$ and (b) $t = 880$ s. Bottom panel: same as top panel with LOS = $\pi/2$ for (c) $t = 0$ and (d) $t = 880$ s. The dashed black lines represent the cut along the $x$-direction where we estimate the filling factors.
homogeneous. For $t = 880$ s, the area of the overdense regions, and hence the density filling factor, increases along the $x$-direction (see Section 3.1 for the estimated values). The variation of the density filling factor with height along coronal loops has been reported by Tripathi et al. (2009). They found the filling factor varies from 0.02 at the loop footpoints to 0.80 at a height of 40 Mm using the Fe XII spectral lines. For estimating the filling factor by forward modeling, the synthetic spectra that we generate are the LOS integrated intensity obtained from FoMo. The plasma column regions with enhanced densities that are located randomly along the $y$–$z$ plane and extend along the $x$-direction are the major sources of the plasma emission. We calculate the thickness of these plasma emitting regions ($h_{\text{eff}}$) by taking the FWHM of the intensity Gaussian fit across the plasma columns. When the cylindrical overdense plasma column structures distort due to turbulence, it is possible that some parts of the plasma emitting regions squeeze and some other parts expand along the transverse direction, and hence $h_{\text{eff}}$ decreases or increases accordingly. Therefore, the variation of $h_{\text{eff}}$ along the $x$-direction is more significant for the turbulent medium than for the equilibrium states, as shown in Figures 11(b) and (d). The presence of turbulence might be responsible for the generation of smaller substructures in the medium, or different substructures can mix with the background medium. This might also be a reason for the large variation of the filling factor along the $x$-direction for a turbulent medium. The LOS direction is also a crucial parameter for estimating the density filling factor from forward modeling using FoMo. Due to the presence of turbulence, the plasma columns become irregular in shape and non-axisymmetric around the axis. Therefore, we find that the values of the density filling factors along the black dashed lines in Figures 8(b) and (d) are different for LOS = 0 and $\pi/2$ (Figures 11(b) and (d)). However, we find that the filling factor along the $x$-direction for the turbulent medium ($t = 880$ s) is...
more than for the equilibrium state \((t = 0)\). It implies that the medium becomes more density homogeneous due to the presence of turbulence. The estimated filling factors we obtain from both methods are in good agreement with those of previous studies. For example, Hahn & Savin (2016) have estimated the filling factor for quiet Sun and interplume regions of coronal holes using a “two-density slab model,” where they have estimated the filling factors are \(\approx 10\% - 20\%\). From spectroscopic observations, the filling factor is found to be 0.10 for an active region loop (Warren et al. 2008), and 0.30 for a cooling loop in quiescent active regions (Landi et al. 2009). For a fan loop structure, Young et al. (2012) found the filling factor to be 0.03–0.30. The filling factor along an entire coronal loop is estimated by Gupta et al. (2015) using spectroscopic observations, and they found the filling factor increases along the loop length from \(\approx 0.11\) at the footpoint of the loop and exceeds 1 at the loop top. The authors claim that the estimation of filling factors using spectroscopic observations depends on good background subtraction and the LOS depth of the emitting plasma, which might be the cause of the errors in the above measurements at higher heights. In our study, the density inhomogeneities are incorporated as an initial condition in the simulation setup, which follows Gaussian distributions along the \(y-z\) plane (according to Equation (6)). Therefore, due to the emission from individual overdense plasma columns, the intensity distribution along the \(y-z\) plane follows a Gaussian distribution when the system is in equilibrium. But when the turbulence develops in the system, the overdense plasma columns disrupt and hence the intensity distribution does not remain like a Gaussian. That said, due to the LOS superposition of the overdense plasma columns, the intensity pattern across the plasma columns does not exactly follow a Gaussian distribution. However, even if there is no LOS superposition of the plasma columns, the intensity distribution across the plasma columns still may not be Gaussian. These effects might cause an error in fitting and hence in calculating \(h_{\text{eff}}\). In this study, we have calculated the filling factor by degrading the synthetic images to the resolution of EIS (1\" per pixel). We can also extend this work by degrading the synthetic spectra to higher-resolution images, for example those of the “Daniel K. Inouye Solar Telescope” (DKIST; Rast et al. 2021), which has an angular resolution of 0\".1. These higher-resolution synthetic maps will provide information about the finer structures and will allow us to investigate the density inhomogeneities of structures at smaller spatial scales of the medium. But, these are beyond the scope of this paper, and we plan to study them in the future.

The study of the density filling factor in the solar corona helps us to understand the overall density structure of the solar corona, and how different the density structure is from the homogeneous approximation. The effect of turbulence on the density filling factor is presented in our work and can provide...
scope with which to compare the solar coronal structures that will be observable with the further advancement of observations. Van Doorsselaere et al. (2014) provides an analytical relation in terms of the filling factor that gives the correction term of the energy flux of Alfvén waves. Hence, the estimation of the filling factor is crucial for estimating the energy flux in coronal waves, which plays an important role in coronal heating.

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