Supersymmetric higher spin theories

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Abstract

We revisit the higher spin extensions of the anti de Sitter algebra in four dimensions that incorporate internal symmetries and admit representations that contain fermions, classified long ago by Konstein and Vasiliev. We construct the $\text{dS}_4$, Euclidean and Kleinian version of these algebras, as well as the corresponding fully nonlinear Vasiliev type higher spin theories, in which the reality conditions we impose on the master fields play a crucial role. The $\mathcal{N} = 2$ supersymmetric higher spin theory in $\text{dS}_4$, on which we elaborate further, is included in this class of models. A subset of the Konstein–Vasiliev algebras are the minimal higher spin extensions of the $\text{AdS}_4$ superalgebra $\text{osp}(4|\mathcal{N})$ with $\mathcal{N} = 1, 2, 4 \mod 4$, whose R-symmetry can be realized using fermionic oscillators. We tensor these algebras with appropriate internal symmetry algebras, namely $\text{u}(n)$ for $\mathcal{N} = 2 \mod 4$ and $\text{so}(n)$ or $\text{usp}(n)$ for $\mathcal{N} = 1, 4 \mod 4$. We show that the $\mathcal{N} = 3 \mod 4$ higher spin algebras are isomorphic to those with $\mathcal{N} = 4 \mod 4$. We describe the fully nonlinear higher spin theories based on these algebras, including the coupling between the adjoint and twisted-adjoint master fields. We elaborate further on the $\mathcal{N} = 6$ model in $\text{AdS}_4$, and provide two equivalent descriptions one of which exhibits manifestly its relation to the $\mathcal{N} = 8$ model.

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1. Introduction

Minimal higher spin (HS) gravity in four dimensional spacetime describes a generalization of Einstein gravity by inclusion of a real scalar field and infinite set of even HS fields, each occurring once, in terms of nonlinear and consistent field equations proposed by Vasiliev...
[1, 2]. For reviews, see [3–5]. In recent years a strong motivation for exploring HS theories has been their holographic descriptions by means of remarkably simple boundary CFTs [6, 7], the weak–weak coupling nature of these descriptions, and the prospects of unraveling a highly symmetric phase of string theory.

While most investigations in HS theories have been in the context of bosonic models in four dimensions, efforts toward understanding the string/M theory origin of these theories are expected to involve supersymmetry. In fact, historically, a connection between massless HS fields and free boundary CFT seems to have been first proposed in 1988 by Bergshoeff et al in [8] where it was shown that the \( \mathfrak{osp}(8|4) \) invariant superconformal field theory [9], which arises in the quantization of the supermembrane in \( \text{AdS}_4 \times S^7 \) with its worldvolume wrapped round the boundary of \( \text{AdS}_4 \) [10–13], contains in its spectrum an infinite set of massless HS fields. These states correspond to the symmetric product of two \( \mathfrak{osp}(8|4) \) singletons and arrange themselves into an infinite tower of \( \mathfrak{osp}(8|4) \) supermultiplets with maximum spin \( s_{\text{max}} = 2, 4, 6, \ldots, \infty \), the lowest one being that of gauged \( \mathcal{N} = 8 \) supergravity in four dimensions (4D). The details of the underlying super HS theory were spelled out much later in [15, 16].

In the context of HS holography, the next appearance of supersymmetric boundary CFTs occurred in 2002 [6] where super HS theories in \( D = 4, 5, 7 \) were considered. These dimensions were motivated by \( \text{AdS}_5 \times S^5 \) vacuum of type IIB string, and the \( \text{AdS}_4 \times S^7 \) and the \( \text{AdS}_7 \times S^4 \) vacua of M theory. The following year, aspects of HS holography in the context of \( \mathcal{N} = 1 \) super HS theories in 4D were treated in [17, 18]. More recently, an \( \mathcal{N} = 6 \) supersymmetric HS theory in 4D and its connection with string theory has been investigated in [19].

In formulating the super HS theories in their own right, it is natural to start with the determination of a suitable HS superalgebra. Various HS superalgebras in 4D were proposed in [20–22]. However, once the importance of the HS spectrum being determined by the two-fold product of the singletons was understood [8, 23], it soon became clear that a sensible HS algebra that underlies a consistent description of HS interactions must have generators in one to one correspondence with the spin \( s \geq 1 \) part of the spectrum that follows from the two-singleton product. An alternative way to state this requirement is that the HS algebra must admit a massless unitary representation with the same spectra of spins as predicted by the structure of gauge fields originating from the HS algebra. This requirement was called the ‘admissibility condition’ in [23], and it was shown in [24] that only a subset of the HS algebras of [20–22] were actually admissible. A class of admissible HS superalgebras in 4D were subsequently constructed by Konstein and Vasiliev (KV) [23]. Further HS superalgebras in 4D [25], in 5D [26, 27] and 7D [6], and generically in \( D > 4 \) dimensions have also been constructed [28]. The construction of these algebras uses fermionic or vector oscillators, and in some cases they are extended by tensoring with suitable matrix and Clifford algebras.

Given a HS superalgebra in \( D \leq 4 \), the construction of the corresponding fully nonlinear HS theory proceeds uneventfully along the lines of the minimal Vasiliev system. In each case, it is important to impose suitable reality and projection conditions on the master fields, which depend on a set of commuting fermionic oscillators, and to ensure the correct spin-statistics for their component fields. In dimensions \( D > 4 \), however, the situation is different because certain properties of the fermionic oscillators that hold in \( D \leq 4 \) cease to hold. For example, in 5D, it is not clear how to construct an appropriate central term in curvature

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4 The fact that the symmetric product of two \( \mathfrak{so}(3, 2) \) singletons contains an infinite set of massless HS fields was discovered by Flato and Fronsdal [14].

5 The terminology of ‘HS superalgebra’ is used in a wider context in [23] to include algebras that contain fermionic generators but not necessarily contain the ordinary spacetime superalgebras as finite dimensional subalgebras.
constraints in a Cartan integrable fashion for the simple reason that the fermionic oscillators are four-component, while in 4D this is not a problem owing to the fact that the fermionic oscillators are two-component in that case. Nonetheless, HS field equations can still be written down at the linearized level for models based on HS superalgebras in certain cases; see [27] for $D = 5$ and [6] for $D = 7$. Employing suitable vector oscillators instead of fermionic oscillators, bosonic HS theories in arbitrary dimensions have been constructed [29], but supersymmetric extension of these models is apparently not known.

The purpose of this paper is to extend in various directions the results that have been obtained so far on fully nonlinear HS theories with fermions and supersymmetry in four dimensions [1, 2, 15, 16, 30, 25, 19] (see also the preamble to section 4). More specifically, we shall construct the dS$_4$, Euclidean and Kleinian version of KV algebras, and the corresponding fully nonlinear Vasiliev type HS theories, in which the reality conditions on the master fields play a crucial role. HS theories admitting dS$_4$ vacua provide a convenient framework for studying holography which is a considerably more difficult task in string theory. An important advantage of HS theories in this context is the extreme simplicity of the expected boundary conformal field theory. As for the Euclidean signature, in addition to being relevant to instanton-like solutions that play role in non-perturbative aspects of the theory, it also arises in AdS/CFT correspondence as the signature of the Euclideanized AdS that provides a convenient framework. The Kleinian signature is useful to consider as well because in this signature the consistency of the Vasiliev equations allows the relaxation of the standard spin-statistics conditions on the master fields. Thus one may speculate that HS gravities in Kleinian backgrounds can be used as holographic duals of three-dimensional conformal field theories in Lorentzian signature with non-standard spin-statistics.

In the case of HS theories based on KV algebras in AdS$_4$, the construction of the fully nonlinear HS field equations have been outlined in earlier works [31, 32] (see also [4]). Here, we spell out in detail such constructions, exhibiting the explicit form of the reality and projection conditions on the master fields, and the intertwining of the adjoint and quasi-adjoint representations in the resulting Vasiliev type fully nonlinear HS field equations.

A subset of the KV algebras are the minimal HS extensions of the AdS$_4$ superalgebra $osp(4|\mathcal{N})$ with $\mathcal{N} = 1, 2, 4 \mod 4$, whose R-symmetry can be realized using fermionic oscillators. We shall tensor these algebras with appropriate internal symmetry groups, namely $u(n)$ for $\mathcal{N} = 2 \mod 4$ and $so(n)$ or $usp(n)$ for $\mathcal{N} = 1, 4 \mod 4$. It turns out that the $\mathcal{N} = 3 \mod 4$ HS algebras are isomorphic to those with $\mathcal{N} = 4 \mod 4$. We shall describe the fully nonlinear HS theories based on these algebras for all $\mathcal{N}$, including the coupling between the adjoint and twisted-adjoint master fields. Given that the KV algebras have been conjectured [23] to form a complete list, it is natural to expect that the HS algebras underlying these supersymmetric models can be obtained by starting from a much larger KV algebra and imposing suitable projections, though this remains to be shown.

Finally, we shall exhibit the truncations of a model with $\mathcal{N}$ supersymmetry to that with $\mathcal{N} - 2$, and in particular obtain an $\mathcal{N} = 6$ HS theory from an $\mathcal{N} = 8$ HS theory, which provides an alternative description to that of [19], and makes its relation to the $\mathcal{N} = 8$ HS theory [15, 16] manifest. We shall elaborate further on these $\mathcal{N} = 6$ supersymmetric HS theories in AdS$_4$, as well as the $\mathcal{N} = 2$ supersymmetric HS theory in dS$_4$.

2. The bosonic models in four dimensions

In this section, we review the structure of Vasiliev’s equations in the case of bosonic models in AdS$_4$, including their $\hat{\Theta}$-deformations, HS gauge symmetries and the minimal projections,
in particular defining the type A and type B models. We shall then review their formulation in dS_4 as well as spacetimes with Euclidean, Kleinian signatures.

2.1. Vasiliev equations in AdS_4

The four dimensional minimal bosonic Vasiliev models in AdS_4 have perturbative spectra given by a real scalar, a graviton and a tower of Fronsdal tensors of even ranks greater than two, which form an irreducible and unitarizable representation of the minimal HS Lie algebra extension hs(4) of so(3, 2) \cong sp(4, \mathbb{R}). While the linearization only refers to algebraic structures within the enveloping algebra of sp(4, \mathbb{R}), the full theory is based on the finer structure provided by the associative \ast-product algebra generated by two Grassmann even \alpha, \beta \in \{1, \ldots, 4\}, obeying

\begin{equation}
[Y_{\alpha}, Y_{\beta}] = 2iC_{\alpha\beta} = -[Z_{\alpha}, Z_{\beta}]_\ast,
\end{equation}

and the reality conditions\(^6\)

\begin{equation}
Y_{\alpha} := (y_{\alpha}, \bar{y}_{\alpha}), \quad Z_{\alpha} := (z_{\alpha}, -\bar{z}_{\alpha}), \quad (y_{\alpha})^\dagger := \bar{y}_{\alpha}, \quad (z_{\alpha})^\dagger := \bar{z}_{\alpha}.
\end{equation}

The nonvanishing entries of the charge conjugation matrix \(C_{\alpha\beta}\) are \(C_{\alpha\beta} = \epsilon_{\alpha\beta}\) and \(C_{\alpha\beta} = \epsilon_{\alpha\beta}\). Letting \(x^\mu\) coordinatize a bosonic base manifold, which we refer to as \(X\)-space, and introducing anti-commuting line elements \(\text{d}x^\mu, \text{d}Z^\alpha\) and

\begin{equation}
\hat{d} := \text{d}x^\mu \partial_\mu + \text{d}Z^\alpha \partial_\alpha, \quad \partial_\alpha := \frac{i}{2}[Z_{\alpha}, \cdot],
\end{equation}

using conventions in which

\begin{equation}
\hat{d}(\hat{f} \ast \hat{g}) = (\hat{d}f) \ast \hat{g} + (-1)^{\text{deg}(f)} f \ast (\hat{d}g), \quad (\hat{d}f)^\dagger = \hat{d}((f)^\dagger),
\end{equation}

\begin{equation}
(f \ast \hat{g})^\dagger = (-1)^{\text{deg}(f)\text{deg}(\hat{g})} (\hat{g})^\dagger \ast (f)^\dagger,
\end{equation}

where \(\text{deg}\) denotes form degree, and writing \(\text{d}z^2 \equiv \text{d}x^\mu \ast \text{d}z^\alpha \ idem \text{d}z^2\) and

\begin{equation}
[f, \hat{g}]_{\text{tr}} \equiv \hat{f} \ast \hat{g} - (-1)^{\text{deg}(f)\text{deg}(\hat{g})} \hat{g} \ast \hat{f},
\end{equation}

\(\text{idem} \hat{\pi}, \) where the automorphism

\begin{equation}
\pi (f(y, \bar{y}; z, \bar{z})) := \hat{f}(-y, \bar{y}; -z, \bar{z}), \quad \pi \hat{d} = \hat{d}\pi,
\end{equation}

\(\text{idem} \hat{\pi}, \) the equations of motion for the minimal bosonic models read\(^2\)

\begin{equation}
\hat{\mathcal{F}} := \hat{d}\hat{A} + \hat{\mathcal{A}} \ast \hat{\mathcal{A}} = \frac{i}{4} (\text{d}z^2 \hat{\mathcal{V}} + \text{d}\bar{z}^2 \hat{\mathcal{V}})
\end{equation}

\begin{equation}
\hat{D}\hat{\Phi} := \hat{d}\hat{\Phi} + [\hat{\Delta}, \hat{\Phi}]_{\text{tr}} = 0
\end{equation}

where

(i) the master one form \(\hat{\Delta} = \text{d}x^\mu \hat{\Delta}_\mu(x, Y, Z) + \text{d}Z^\alpha \hat{\Delta}_\alpha(x, Y, Z)\) and master zero form \(\Phi = \Phi(x, Y, Z)\) are subject to the reality conditions

\begin{equation}
\hat{\Delta}^\dagger = -\hat{\Delta}, \quad \hat{\Phi}^\dagger = \pi (\hat{\Phi}),
\end{equation}

and minimal bosonic projections

\begin{equation}
\tau (\hat{\Delta}) = -\hat{\Delta}, \quad \hat{\Delta}^\dagger = -\hat{\Delta}, \quad \tau (\hat{\Phi}) = \pi (\hat{\Phi}),
\end{equation}

where the graded anti-automorphism

\begin{equation}
\tau (f(Y, Z)) := \hat{f}(\bar{Y}, -\bar{Z}), \quad \tau \hat{d} := \hat{d}\tau,
\end{equation}

\begin{equation}
\tau (\hat{f} \ast \hat{g}) := (-1)^{\text{deg}(f)\text{deg}(\hat{g})} \tau (\hat{g}) \ast \tau (\hat{f});
\end{equation}

\(^6\) The minus sign in the second equation is a convention which we have found to be convenient. As for the Hermitian conjugation of spinors, our conventions differ from those of \(^{19}\).
(ii) the deformations

\[ \hat{\mathcal{V}} := \sum_{p=0}^{\infty} v_p(\hat{\Phi} \star \hat{\kappa})^p, \quad \hat{\mathcal{V}} := (\hat{\mathcal{V}})^\dagger, \]  

(2.14)

where the Kleinians are defined by

\[ \hat{\kappa} \star \hat{f} \star \hat{\kappa} := \pi(\hat{f}), \quad \hat{\kappa} := (\hat{\kappa})^\dagger, \]  

(2.15)

and \( v_p = v_p(\hat{\Phi}, \hat{A}_u) \) are \( \tau \)-invariant complex valued functionals that are constant on shell, i.e. \( d v_p = 0 \) modulo the equations of motion; for non-trivial examples of such zero-form invariants, see [33–36].

Key features of the Vasiliev system are:

- Its universal Cartan integrability [37, 5], i.e. its compatibility with \( (\hat{\mathcal{A}})^2 = 0 \) on base manifolds of arbitrary dimension, which implies invariance under the gauge transformations

\[ \delta_2\hat{\mathcal{A}} = \hat{\mathcal{D}}\hat{\epsilon} := d + [\hat{\mathcal{A}}, \hat{\epsilon}], \quad \delta_2\hat{\Phi} = -[\hat{\epsilon}, \hat{\Phi}]_\pi, \]  

(2.16)

where \( \hat{\epsilon} = \hat{\epsilon}(x, Y, Z) \) is subject to the same kinematic constraints as \( \hat{\mathcal{A}} \). The closure relation \( [\delta_2 \hat{\epsilon}, \delta_2 \hat{\Phi}] = \delta_2[\hat{\epsilon}, \hat{\Phi}]_\pi \) defines the HS algebra \( \mathcal{H}^{\mathcal{A}}(4) \).

- Its equivalent formulation as the deformed oscillator system

\[ \hat{\mathcal{S}}_{(\alpha} \star \hat{\mathcal{S}}_{\beta)} = -2i\epsilon_{\alpha\beta}(1 - \hat{\mathcal{V}}), \quad \hat{\mathcal{S}}_{(\alpha} \star \hat{\mathcal{S}}_{\beta)} = -2i\epsilon_{\alpha\beta}(1 - \hat{\mathcal{V}}), \]  

\[ [\hat{\mathcal{S}}_u, \hat{\mathcal{S}}_u]_\pi = 0, \quad [\hat{\mathcal{S}}_u, \hat{\Phi}]_\pi = 0, \quad [\hat{\mathcal{S}}_u, \hat{\Phi}]_\pi = 0, \]  

(2.17)

(2.18)

where

\[ \hat{\mathcal{S}}_u := Z_u - 2i\hat{\mathcal{A}}_u, \]  

(2.19)

coupled to the Maurer–Cartan system

\[ \hat{\mathcal{F}}_{u\mu} = 0, \quad \hat{\mathcal{D}}_{u\mu} \hat{\Phi} = 0, \]  

\[ \hat{\mathcal{D}}_{u\mu} \hat{\mathcal{S}}_u := \partial_{\mu} \hat{\mathcal{S}}_u + [\hat{\mathcal{A}}_{\mu}, \hat{\mathcal{S}}_u]_\pi = 0. \]  

(2.20)

(2.21)

Splitting

\[ \hat{\mathcal{V}} = \hat{\mathcal{V}}_+ + \hat{\mathcal{V}}_-, \quad \hat{\mathcal{V}}_\pm = \sum_{p=0}^{\infty} \frac{1}{2} (1 \pm (-1)^p) v_p(\hat{\Phi} \star \hat{\kappa})^p, \]  

(2.22)

it follows from \( [\hat{\mathcal{S}}_u, \hat{\Phi} \star \hat{\kappa}]_\pi = 0 = [\hat{\mathcal{S}}_u, \hat{\Phi} \star \hat{\kappa}]_\pi \) that the perturbatively defined redefinition \( \hat{\mathcal{S}}_u := \sqrt{1 - \hat{\mathcal{V}}_+ \star \hat{\mathcal{S}}_u} \) is true for

\[ \hat{\mathcal{V}}_+ \to 0, \quad \hat{\mathcal{V}}_- \to (1 - \hat{\mathcal{V}}_+)_{\pi(-1)} \star \hat{\mathcal{V}}_. \]  

(2.23)

By perturbatively redefining \( \hat{\Phi} \), one may take

\[ \hat{\mathcal{V}} = \exp_\tau(i\hat{\Theta}) \star \hat{\Phi} \star \hat{\kappa}, \quad \hat{\Theta} = \sum_{p=0}^{\infty} \theta_p \hat{\Theta}^p, \quad \hat{\tilde{X}} := (\hat{\Phi} \star \hat{\kappa})^2 \]  

(2.24)

where \( \theta_p \) are \( \tau \)-invariant real valued zero-form invariants such that

\[ \hat{\Theta} = (\hat{\Theta})^\dagger = \tau (\hat{\Theta}), \quad \hat{\mathcal{D}}\hat{\Theta} := \hat{\mathcal{D}}\hat{\Theta} + [\hat{\mathcal{A}}, \hat{\Theta}]_\pi = 0, \]  

(2.25)

modulo the equations of motion.

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7 The universality plays a rôle in certain off-shell formulations [35, 36] and in HS geometries [33].

8 The quantity \( \hat{\tilde{X}} \) obeys \( D\hat{\tilde{X}} := d\hat{\tilde{X}} + [\hat{\mathcal{A}}, \hat{\tilde{X}}]_\pi = 0 \) and \( (\hat{\tilde{X}})^\dagger = \tau (\hat{\tilde{X}}) = \hat{\tilde{X}} \). One also has \( \hat{\tilde{X}} = \hat{\Phi} \star \hat{\tau} (\hat{\Phi}) = \hat{\Phi} \star \pi (\hat{\Phi}) = (\hat{\Phi} \star \hat{\kappa})^2 \).
The deformation $\hat{\mathcal{V}}$ cannot be simplified further by perturbatively defined master field redefinitions [2], which shows the key rôle played by the Kleinian operators. As perturbatively defined redefinitions need not be globally defined in moduli space, the classification of non-perturbatively inequivalent Vasiliev systems remains an interesting open problem.

The minimal bosonic models are consistent truncations of non-minimal bosonic models obtained by replacing the $\tau$-projection by the weaker bosonic projection and removing the $\tau$-invariance condition on $\theta_\rho$.

\begin{equation}
\tau^2(\hat{A}) = \pi \tilde{\tau}(\hat{A}) = \tilde{\hat{A}}, \quad \tau^2(\hat{\Phi}) = \pi \tilde{\tau}(\hat{\Phi}) = \tilde{\hat{\Phi}},
\end{equation}

and the parity map is the automorphism of the oscillator algebra defined by

\begin{equation}
P(A) = \tilde{A}, \quad P(\hat{\Phi}) = e^{2i\theta_0} \tilde{\hat{\Phi}},
\end{equation}

which assign intrinsic parity $e^{2i\theta_0}$ to the physical scalar. The parity invariant models may be minimal or non-minimal depending on whether the bosonic projection is imposed using $\tau$ or $\tau^2$.

### 2.2. De Sitter space, Euclidean and Kleinian signatures

Bosonic models in four dimensional spacetimes with different signatures and different signs of the cosmological constant can be obtained by complexification of (2.8) and (2.9) by keeping the bosonic projection while dropping all reality conditions, and in particular treating $\hat{\mathcal{V}}$ and $\hat{\mathcal{V}}$ as independent odd $\star$-functions, followed by imposition of suitably modified reality conditions; for a detailed construction of these models, see [38], and for their harmonic expansions and spectra, see [39]. The complexified HS algebra admits three distinct real forms, containing either $so(5), so(4, 1)$ and $so(3, 2)$. Each of the latter are compatible with two different spacetime signatures, leading to five distinct models in total. The reality conditions read

\begin{equation}
\hat{A}^\dagger = -\sigma(\hat{A}), \quad \hat{\Phi}^\dagger = \sigma(\pi(\hat{\Phi})),
\end{equation}

where the map $\sigma$ is given in table 1 and Hermitian conjugates of doublets are defined as follows for different Lorentz algebras:

\begin{equation}
su(2)_L \times su(2)_R : \quad (\rho^a)^\dagger = \gamma_a, \quad (\lambda^a)^\dagger = \tilde{\gamma}_a,
\end{equation}

\begin{equation}
su(2; C)_{\text{diag}} : \quad (\rho^a)^\dagger = \tilde{\gamma}^a, \quad (\lambda^a)^\dagger = \tilde{\tilde{\gamma}}^a.
\end{equation}

For example, replacing $\hat{\mathcal{V}}(\Phi \star \Phi)$ by $\hat{\Phi}(\hat{\Phi})$ where $\hat{\Phi}$ is an adjoint zero-form obeying $\hat{D} = \hat{D} = [\hat{A}, \hat{\Phi}]_L = 0$ yields a system without local perturbative degrees of freedom which can be brought to $\hat{F} = 0 = \hat{D}^2$ by means of a perturbatively defined field redefinition.

We are assuming standard spin-statistics such that bosonic projection is tantamount to setting half-inter spins to zero.

In Euclidean signature, where we have absorbed the isomorphism $\rho$ used in [38] into the symbol $\dagger$, one has $((\hat{f}^\dagger)) = \pi \tilde{\tau}(\hat{f})$ and the reality conditions are consistent in view of the bosonic projection.

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9 For example, replacing $\hat{\mathcal{V}}(\Phi \star \Phi)$ by $\hat{\Phi}(\hat{\Phi})$ where $\hat{\Phi}$ is an adjoint zero-form obeying $\hat{D} = \hat{D} = [\hat{A}, \hat{\Phi}]_L = 0$ yields a system without local perturbative degrees of freedom which can be brought to $\hat{F} = 0 = \hat{D}^2$ by means of a perturbatively defined field redefinition.

10 We are assuming standard spin-statistics such that bosonic projection is tantamount to setting half-inter spins to zero.

11 In Euclidean signature, where we have absorbed the isomorphism $\rho$ used in [38] into the symbol $\dagger$, one has $((\hat{f}^\dagger)) = \pi \tilde{\tau}(\hat{f})$ and the reality conditions are consistent in view of the bosonic projection.
The reality conditions in (2.31) define the HS algebras ho(p, 5 - p) ⊃ so(p, 5 - p), which only refer to the signatures of the isometry algebras, and their twisted adjoint representations, which also refer to the spacetime signatures. The signatures can be determined by using van der Waerden symbols to map \( Y_\alpha Y_\beta \) into \( M_{AB} \) obeying \( (M_{AB})^\dagger = \sigma(M_{AB}) \); for further details, see [38, 39]. Regarding the deformations, their reality conditions read

\[
\text{Signature (3, 1)}: \hat{\nu}^\dagger = \hat{\nu}, \quad \text{Signatures (4, 0) and (2, 2)}: \hat{\nu}^\dagger = \hat{\nu}, \quad \hat{\nu}^\dagger = \hat{\nu},
\]

and by re-defining \( \hat{\Phi} \), one may take

\[
\text{Signature (3, 1)}: \hat{\nu} := e^{i\theta} \hat{\Phi} \hat{\kappa}, \quad \hat{\nu} := (\nu)^\dagger = e^{-i\theta} \hat{\Phi} \hat{\kappa}
\]

(2.37)

\[
\text{Signature (4, 0) and (2, 2)}: \hat{\nu} := e^{i\theta} \hat{\Phi} \hat{\kappa}, \quad \hat{\nu} := (\nu)^\dagger = \sigma_0 e^{-i\theta} \hat{\Phi} \hat{\kappa}
\]

(2.38)

with \( \hat{\Theta} \) given by (2.24) and \( \sigma_0 = \pm 1 \). Note that the case of (3, 1) signature applies to dS\(_4\) and AdS\(_4\), and that the case of (4, 0) applies to S\(^4\) and H\(_4\), using (2.31) and table 1.

The parity assignments

\[
P(\hat{A}) = \hat{A}, \quad P(\hat{\Phi}) = e^{2i\theta_0} \hat{\Phi}, \quad \theta_0 = 0, \quad \frac{\pi}{2},
\]

(2.39)

with \( P \) defined as in (2.27) for all signatures, imply that the physical scalar has intrinsic parity \( e^{2i\theta_0} \) and that

\[
\text{(3, 1) signature: } \hat{\Theta} = \theta_0, \quad \text{(4, 0) and (2, 2) signature: } \hat{\Theta} = 0, \quad \sigma_0 = e^{2i\theta_0}.
\]

(2.40)

(2.41)

In non-Lorentzian signatures, one has the maximally parity violating

\[
\text{Chiral models ((4, 0) and (2, 2) signatures): } \hat{\nu} = \hat{\Phi} \hat{\kappa}, \quad \hat{\nu} = 0.
\]

(2.42)

### 3. Fermions and Yang–Mills symmetries

In this section, we shall begin by reviewing the KV construction of HS algebras [23], which involve the extension of the minimal HS algebras containing the AdS\(_4\) algebra, to include fermionic generators and internal symmetry. As mentioned earlier, the KV algebras
concluding fermionic generators are referred to as HS superalgebras even though not all of them contain the standard spacetime superalgebras as finite dimensional subalgebras. In [23], the massless unitary representations of these algebras, corresponding to suitable two-fold singleton products, were determined and, to first order in Weyl zero-form, the required modification of the HS field equations given in [40] were pointed out. Here, we shall first generalize the KV algebras to those which contain the de Sitter, Euclidean and Kleinian spacetime algebras in four dimensions. Next, we shall provide the Vasiliev type fully nonlinear HS field equations for all these HS algebras. As we shall see, the reality conditions on the master fields and the definition of the twisted adjoint representations play key roles in these equations. For a discussion of earlier works on the construction of these models, see the introduction.

3.1. Konstein–Vasiliev algebras in AdS4

Fermions and internal Yang–Mills symmetries can be introduced by tensoring the \((Y, Z)\)-oscillator algebra by suitable matrix and Clifford algebras. In the AdS4 case, Konstein and Vasiliev [23] have constructed three families of extended HS algebras. These algebras admit unitary representations given by squares of singletons and contain bosonic subalgebras given by the direct sums of \(sp(4; \mathbb{R}) \cong so(3, 2)\) and the Yang–Mills algebras \(u(m) \oplus u(n), o(m) \oplus o(n)\) and \(usp(m) \oplus usp(n)\), respectively, namely

\[
\begin{align*}
\text{hu}(m; n[4]) & : S_+ \otimes S_+ , \\
\text{ho}(m; n[4]) & : [S_+ \otimes S_-]_+, \\
\text{husp}(m; n[4]) & : [S_+ \otimes S_-]_+ ,
\end{align*}
\]

(3.1)

(3.2)

(3.3)

where \([\_]_\pm\) stand for symmetric and antisymmetric tensor products, respectively. More precisely, we have

\[
S_+ := (m, \text{Rac}) \oplus (n, \text{Di}), \quad S_- := (m, \text{Di}) \oplus (n, \text{Rac}).
\]

(3.4)

Using the standard notation \(D(E_0, s)\) for the discrete unitary representations of \(sp(4; \mathbb{R}) \cong so(3, 2)\), where \(E_0\) is the lowest energy and \(s\) is the spin of the lowest weight state. Di refers to the \(D(1, 1/2)\) and Rac refers to the \(D(1/2, 0)\) representations. Furthermore, \(m\) labels the fundamental representations of \(U(m)\) and \(USp(m)\) and a vector representation of \(so(m)\).

The singleton products decompose under the bosonic subalgebras as [23]

\[
\begin{align*}
\text{hu}(m; n[4]) & : (m^2 - 1, 1) \oplus (1, n^2 - 1) \oplus (1, 1) \oplus (1, 1) \oplus (m, \bar{n}) \oplus (\bar{m}, n) \\
\text{ho}(m; n[4]) & : (\frac{1}{2} m (m - 1), 1) \oplus \left( \frac{1}{2} n (n - 1) \right) \oplus (1, \frac{1}{2} m (m + 1) - 1, 1) \oplus \left( \frac{1}{2} n (n + 1) - 1 \right) \oplus (1, 1) \oplus (1, 1) \\
\text{husp}(m; n[4]) & : (\frac{1}{2} m (m + 1), 1) \oplus \left( \frac{1}{2} n (n + 1) \right) \oplus (1, \frac{1}{2} m (m - 1) - 1, 1) \oplus \left( \frac{1}{2} n (n - 1) - 1 \right) \oplus (1, 1) \oplus (1, 1)
\end{align*}
\]

(3.5)

where the dimensions of the representations are shown. The fields with spin \(s \geq \frac{1}{2}\) carry the massless representation \(D(s + 1; s)\) of the AdS4 algebra with lowest energy \(s + 1\), and the scalars carry \(D(1; 0)\) or \(D(2; 0)\) depending on their intrinsic parity. The realization of these spectra in terms of linearized fields subject to suitable boundary conditions will be described.
The algebras $(\mathfrak{h}u(m; n), \mathfrak{ho}(m; n), \mathfrak{husp}(m; n))$ and their adjoint and twisted adjoint representations have superalgebras based on HS algebras.

3.2. Full models in (anti) de Sitter space and diverse signatures

The construction of these models employs extended master fields $(\mathring{A}, \mathring{\Phi})$ valued in the direct product of the $(Y, Z)$-oscillator algebra and internal associative algebras given by

$$\text{AdS}_4, H_{2,2} : \text{Cliff}_1(\mathbb{C}) \otimes \text{Mat}_{m+n}(\mathbb{C}),$$

$$\text{dS}_4, H_4, S^4 : \text{Cliff}_2(\mathbb{C}) \otimes \text{Mat}_{m+n}(\mathbb{C}),$$

where $\text{Cliff}_k(\mathbb{C})$ denotes the Clifford algebra with $k$ fermionic generators, say $\theta^r, r = 1, \ldots, k$, obeying $[\theta^r, \theta^s] = 2\delta^{rs}$. In what follows, we shall suppress the matrix indices on the master fields $(\mathring{A}, \mathring{\Phi})$. These master fields

(i) are Grassmann even, i.e.

$$\varepsilon_{\ell}(\mathring{A}) = 0 = \varepsilon_{\ell}(\mathring{\Phi}),$$

where $\varepsilon_{\ell}$ counts the Grassmann parity of generators as well as component fields;

(ii) obey the spin-statistics conditions (except in the case of $H_{2,2}$; see table 2):

$$\pi \mathring{\pi} \pi_\theta(\mathring{A}) = \mathring{A}, \quad \pi \mathring{\pi} \pi_\theta(\mathring{\Phi}) = \mathring{\Phi},$$

where $\pi_\theta$ is the automorphism with non-trivial action

$$\pi_\theta(\theta^r) = -\theta^r;$$

(iii) belong to the graded matrix algebra defined by the projection

$$\pi_\theta \text{Ad}^{-1}_\Gamma(\mathring{A}) = \mathring{A}, \quad \pi_\theta \text{Ad}^{-1}_\Gamma(\mathring{\Phi}) = \mathring{\Phi}, \quad \Gamma = \begin{bmatrix} I_{m \times m} & 0 \\ 0 & -I_{n \times n} \end{bmatrix}.$$

The further imposition of reality and $\tau$-conditions displayed in table 2, yields three families of models based on the adjoint and twisted adjoint representations of extended HS algebras as follows:

12 The algebras $hu(m; n)(2, 3), \ ho(m; n)(2, 3)$ and $husp(m; n)(2, 3)$, respectively, are isomorphic to $hu(m; n)(4), \ ho(m; n)(4)$ and $husp(m; n)(4)$.
By its definition, the Hermitian conjugation obeys
\[ J \text{ (Y, Z)-oscillator algebras it acts as in equations (2.32)–(2.34); (ii) in Mat}_{m+n}(\mathbb{C}) it acts as standard matrix Hermitian conjugation; (iii) for the component fields we adopt the rule that if } \lambda \text{ is a Grassmann number then } ((\lambda)')^\dagger = \lambda \text{ (we shall comment on an alternative rule at the end of this subsection), and (iv) on the Clifford algebras as}
\[
\begin{align*}
\text{AdS}_4, H_{2,2} : (\theta^i)^\dagger & := \theta^i, \quad (3.12) \\
\text{dS}_4, H_4, S^4 : (\theta'^i)^\dagger & = \epsilon^i \theta'^i, \quad (3.13)
\end{align*}
\]
where the doublet structure, on which we shall elaborate further below, is introduced so that the Hermitian conjugation in the total \((Y, Z, \theta)\)-oscillator algebra, which need not square to the identity, is compatible with the reality conditions\(^{13}\);

- \( h_{0m; n|p; 5-p} \) and \( h_{\text{hsp}; n|p; 5-p} \): the master fields obey the same reality conditions as in the case of \( h_{0m; n|p; 5-p} \), and the additional projections

\[
\tau(\hat{A}) = -A, \quad \tau(\hat{\Phi}) = \bar{\pi}(\Phi),
\]
where the graded anti-automorphism\(^{14}\)

\[
\tau(\hat{f}(Y, Z, dZ, \theta)) := (\gamma \star \hat{f}(iY, -iZ, \theta)) \star \gamma^{-1})^T,
\]
with \((-)^T\) denoting transposition in \(\text{Mat}_{m+n}(\mathbb{C})\) and

\[
\text{ho}(m; n|p; 5-p) : \gamma = \begin{bmatrix} 1_{m \times m} & 0 \\ 0 & 1_{n \times n} \end{bmatrix},
\]
\[
\text{hsp}(m; n|p; 5-p) : \gamma = \begin{bmatrix} \epsilon_{m \times m} & 0 \\ 0 & \epsilon_{n \times n} \end{bmatrix},
\]
where \(\epsilon_{m \times m}\) denotes the constant symplectic matrix of rank \(m\).

In all cases, the Vasiliev equations take the form (2.8) and (2.9) with

\[
\hat{\nu} := \hat{V}(\hat{\Phi} \star \hat{k} \Gamma), \quad \hat{\nu} := \hat{V}(\hat{\Phi} \star \hat{k}),
\]
which may be simplified using perturbatively defined field redefinitions to obtain

\[
\text{Signature (3, 1)} : \hat{\nu} = \epsilon_{\bar{a}\bar{b}}^{\dagger} \star \hat{\Phi} \star \hat{k} \Gamma, \quad \hat{\nu} = \epsilon_{\bar{a}\bar{b}} \star \hat{\Phi} \star \hat{k}
\]
\[
\text{Signature (4, 0) and (2, 2)} : \hat{\nu} = \epsilon_{\bar{a}\bar{b}}^{\dagger} \star \hat{\Phi} \star \hat{k} \Gamma, \quad \hat{\nu} = \sigma_0 \epsilon_{\bar{a}\bar{b}} \star \hat{\Phi} \star \hat{k}
\]

with \(\sigma_0 = \pm 1\) and \(\bar{\Theta}\) given by (2.24) where \(\hat{X} := (\hat{\Phi} \star \hat{k})^2 = \hat{\Phi} \star \bar{\pi}(\Phi) = \hat{\Phi} \star \pi_{\bar{a}}(\Phi) = (\hat{\Phi} \star \hat{k} \Gamma)^2\). We recall that the case of (3, 1) signature applies to \(\text{dS}_4\) and \(\text{AdS}_4\), and that the case of (4, 0) applies to \(S^4\) and \(H_4\), using the rules for the action of \(\tau\) as describe above, and table 2.

In \(\text{AdS}_4\) and \(H_{2,2}\), the resulting set of dynamical Lorentz tensors and tensor-spinors is summarized in (3.5) with the understanding that \(s\) refers to Lorentz spin.

The \(\text{dS}_4\), \(H_4\) and \(S^4\) cases exhibit an additional doublet structure, viz.

\[
\tilde{\Lambda} = \sum_{\sigma \in \pm} (\tilde{\Lambda}_{\sigma} + \tilde{\Psi}_{\sigma} \theta^f) \star P_{\sigma}, \quad \tilde{\Phi} = \sum_{\sigma \in \pm} (\tilde{\Phi}_{\sigma} + \tilde{\varphi}_{\sigma} \theta^f) \star P_{\sigma},
\]

\(^{13}\) By its definition, the Hermitian conjugation obeys \(\hat{f} \star \hat{g}^\dagger = (-1)^{\deg(f) \deg(g)} \hat{g}^\dagger \star \hat{f}^\dagger\).

\(^{14}\) By its definition, one has \(\tau(f \star g) = (-1)^{\deg(f) \deg(g)} \tau(f) \star \tau(g)\).
where $P_{\sigma} := \frac{1}{2} (1 \pm \Gamma_{\sigma})$ with $\Gamma_{\sigma} := i \theta^{1} \bullet \theta^{3}$ obeying $(\Gamma_{\sigma})^{\dagger} = \Gamma_{\sigma} = -\tau (\Gamma_{\sigma})$ and $(\Gamma_{\sigma})^{3} = 1$, implying that in models based on $hu(m; n|5 - p, p)$ one has

$$\begin{align*}
\text{dS}_{4} : (\tilde{A}_{\sigma})^{\dagger} &= -\tau (\tilde{A}_{\sigma}), \\
(\tilde{\Psi}_{\sigma})^{\dagger} &= -e^{\tau} \tau (\tilde{\Psi}_{\sigma}), \\
(\tilde{\Phi}_{\sigma})^{\dagger} &= \tilde{\Phi}_{\sigma}, \\
(\tilde{\chi}_{\sigma})^{\dagger} &= e^{\tau} \tilde{\chi}_{\sigma}.
\end{align*}$$

(3.22)

$$\begin{align*}
H_{4} : (\tilde{A}_{\sigma})^{\dagger} &= -\tilde{\pi} (\tilde{A}_{\sigma}), \\
(\tilde{\Psi}_{\sigma})^{\dagger} &= -e^{\tau} \tilde{\Psi}_{\sigma}, \\
(\tilde{\Phi}_{\sigma})^{\dagger} &= \tilde{\Phi}_{\sigma}, \\
(\tilde{\chi}_{\sigma})^{\dagger} &= e^{\tau} \tilde{\chi}_{\sigma}.
\end{align*}$$

(3.23)

$$\begin{align*}
S^{4} : (\tilde{A}_{\sigma})^{\dagger} &= -\tilde{\pi} (\tilde{A}_{\sigma}), \\
(\tilde{\Psi}_{\sigma})^{\dagger} &= -e^{\tau} \tilde{\Psi}_{\sigma}, \\
(\tilde{\Phi}_{\sigma})^{\dagger} &= \tilde{\Phi}_{\sigma}, \\
(\tilde{\chi}_{\sigma})^{\dagger} &= e^{\tau} \tilde{\chi}_{\sigma}.
\end{align*}$$

(3.24)

(3.25)

(3.26)

while in models based on $ho(m; n|5 - p, p)$ and $husp(m; n|5 - p, p)$ one has the additional $\tau$ conditions

$$\begin{align*}
\tau (\tilde{A}_{\sigma}) &= -\tilde{\pi} (\tilde{A}_{\sigma}), \\
\tau (\tilde{\Psi}_{\sigma}) &= i \tilde{\Psi}_{\sigma}, \\
\tau (\tilde{\Phi}_{\sigma}) &= \tilde{\Phi}_{\sigma}, \\
\tau (\tilde{\chi}_{\sigma}) &= -i \tilde{\chi}_{\sigma}.
\end{align*}$$

(3.28)

(3.29)

Assigning parities as

$$P(\tilde{A}) = \tilde{A}, \quad P(\tilde{\Phi}) = e^{2i\theta_{0}} \tilde{\Phi} \bullet \Gamma,$$

(3.30)

with $P$ defined as in (2.27) for all signatures, parity invariance requires that

$$\tilde{\Theta} = \theta_{0} = 0, \quad \pi = \frac{\pi}{2}, \quad \sigma_{0} = e^{2i\theta_{0}},$$

(3.31)

which we refer to as the type A and type B models with fermions and Yang–Mills symmetries, respectively.

We conclude this section by emphasizing that, as noted earlier, we have adopted the rule according to which if $\lambda$ is a Grassmann number then $((\lambda)^{\dagger})^{\dagger} = \lambda$. In fact, there exists an alternative rule [41, 42] where $((\lambda)^{\dagger})^{\dagger} = (-1)^{e(\lambda)}$. With this rule, the reality conditions in Table 2 are consistent in the cases of $\text{dS}_{4}$, $\text{H}_{4}$ and $\text{S}^{4}$ with internal Clifford algebra $\text{Cliff}_{1}(\mathbb{C})$ (instead of $\text{Cliff}_{2}(\mathbb{C})$). More precisely, letting $\pi_{\Phi}$ denote the map that acts as $\pi_{\Phi}(F) = -F$ and $\pi_{\Phi}(B) = B$ on component fermions $F$ and bosons $B$, one has $((\tilde{A})^{\dagger})^{\dagger} = \pi_{\Phi}(\tilde{A}) = \pi_{\Phi}(\tilde{A})$ and $\text{idem} \tilde{\Phi}$, using $\epsilon_{f}(\tilde{A}) = 0 = \epsilon_{f}(\tilde{\Phi})$. Hence, in the cases of $\text{dS}_{4}$, $\text{H}_{4}$ and $\text{S}^{4}$ all entries in Table 2 remain unchanged and thus consistent. However, the HS algebras and spectrum will be different. Indeed, the resulting HS algebras cannot be cast into a standard $\mathbb{Z}_{2}$-graded Lie superalgebra but rather a generalized $\mathbb{Z}_{2} \times \mathbb{Z}_{2}$-graded Lie superalgebra. In particular, the supercharges transform as Lorentz spinors under the adjoint action of the Lorentz $\text{so}(3, 1)$ while the translations in $\text{so}(4, 1)/\text{so}(3, 1)$ act on the supercharges via anti-commutators. Thus, the supercharges do not transform as spinors under the isometry $\text{so}(4, 1)$ and the resulting $\text{dS}_{4}$ supergravity [43, 44] has been shown to harbor vector ghost upon standard quantization [44].

15 We thank M Vasiliev for bringing to our attention this alternative rule and the references where it is discussed.
3.3. Linearization and spectrum

In linearizing the HS field equations, we shall use the following realization of the star product:

\[ \hat{f}(Y, Z) \ast \hat{g}(Y, Z) = \int \frac{d\Lambda d\Sigma}{(2\pi)^{2}} e^{\Lambda z \tilde{z}} \hat{f}(Y + \Lambda, Z + \Sigma) \hat{g}(Y + \Sigma, Z - \Lambda). \] (3.32)

The contour of the integration is chosen such that the components of \( \Lambda = (\lambda, \tilde{\lambda}) \) and \( \Sigma = (\xi, \tilde{\xi}) \) are treated as independent two-component real variables. With this definition of the star product, the Kleinian operators \( \kappa \) and \( \tilde{\kappa} \) take the form

\[ \kappa = e^{i\omega z}, \quad \tilde{\kappa} = e^{-i\omega \tilde{z}}. \] (3.33)

Letting \((\Phi, W)\) be the fluctuations in \((\hat{\Phi}, \hat{A})|_{Z=0}\) around a spin 2 background \((h, \Lambda)\) obeying

\[ d\Omega + \Omega \ast \Omega = 0, \quad \Omega := e + \omega, \] (3.34)

\[ e := \frac{1}{2i} e^{a} y_{a} \ast \tilde{y}_{a}, \quad \omega := \frac{1}{4} (\omega_{a} \gamma_{a} \ast y_{a} + \omega_{a} \gamma_{a} \ast \tilde{y}_{a}), \] (3.35)

where \( e^{a} = \frac{1}{2} (\sigma_{a})^{\alpha} e_{\alpha} \) with \( \lambda^{2} > 0 \) for AdS_{4}, \( H_{2,2} \) and \( S^{2} \), and \( \lambda^{2} < 0 \) for dS_{4} and \( H_{4} \).

Further details are given in Appendix A. Assuming that \( \hat{\Phi} \) and \( \hat{A} \) are real analytic functions, the linearized equations of motion read

\[ \nabla W + [e, W]_{\ast} + \frac{i}{2} \left( e^{-i\theta} e^{a} e_{a} \beta \frac{\partial^{2}}{\partial y^{a} \partial \bar{y}^{\beta}} \Phi|_{\bar{y}=0} + e^{-i\theta} e_{a} e_{\beta} \frac{\partial^{2}}{\partial y^{a} \partial \bar{y}^{\beta}} \Phi|_{\bar{y}=0} \ast \Gamma \right) = 0, \] (3.36)

\[ \nabla \Phi + [e, \Phi]_{\ast} = 0, \] (3.37)

where \( \nabla = d + \text{ad}_{e} \) denotes the background Lorentz covariant derivative. As equations (3.34), (3.36) and (3.37) have been obtained by linearizing a nonlinear Cartan integrable system, it follows on general grounds that they are left invariant under linearized Cartan gauge symmetries, viz.

\[ \delta_{e} e = \nabla \epsilon_{e} + [e, \epsilon_{\Lambda}]_{\ast}, \quad \delta_{e} \omega = \nabla \epsilon_{\Lambda} + [e, \epsilon_{\bar{e}}]_{\ast}, \] (3.38)

\[ \delta_{e} W = \nabla \epsilon_{e} + [e, \epsilon_{\Lambda}]_{\ast} + [W, \epsilon_{\Lambda}]_{\ast} + [W, \epsilon_{\bar{e}}]_{\ast} - i \left( e^{-i\theta} e_{e} e_{a} \beta \frac{\partial^{2}}{\partial y^{a} \partial \bar{y}^{\beta}} \Phi|_{\bar{y}=0} + h.c. \right), \] (3.39)

\[ \delta_{e} \Phi = [\Phi, \epsilon_{\Lambda}]_{\ast} - [\Phi, \epsilon_{\bar{e}}]_{\ast}, \] (3.40)

as well as non-Abelian Killing symmetries with parameters \( \epsilon_{0} \) valued in the KV algebra at \( Z^{2} = 0 \) obeying

\[ \nabla \epsilon_{0} + [e, \epsilon_{0}]_{\ast} = 0, \] (3.41)

and acting on the fluctuation fields in accordance with the structure of quadratic terms in nonlinear system (see appendix B) which in the case of the Weyl zero-form amounts to

\[ \delta_{e} \Phi = -[\epsilon_{0}, \Phi]_{\ast}, \] (3.42)

while \( \delta_{e} W = [W, \lambda]_{\ast} + \text{trilinear terms} \); see appendix B. By means of algebraic Cartan integration, viz. \((\Omega, W, \Phi) = (\exp \tilde{T}^{\ast})(\Omega', W', \Phi')|_{W=0} = \Omega = 0\) where \( \tilde{T} \) is the generator of Cartan gauge transformations with finite gauge functions and \( \Phi' \) is a constant twisted-adjoint element, one has

\[ \Omega = L^{-1} \ast dL, \quad \Phi = L^{-1} \ast \Phi' \ast \tilde{\pi}(L), \] (3.43)
where $L$ is a background gauge function, and $W$ given algebraically in terms of $\Phi'$, $L$ and its own gauge functions. Likewise, the Killing parameters

$$\epsilon_0 = L^{-1} \star \epsilon'_0 \star L.$$ \hfill (3.44)

Hence, taking $\epsilon'_0$ and $\Phi'$ to belong to adjoint and twisted-adjoint representations of the KV algebra, respectively, and choosing a gauge function $L$, one obtains expansions of the dynamical fields and Killing parameters in terms of harmonic functions on the maximally symmetric background, which by construction obey the linerized field equation subject to boundary conditions that are consistent with their forming representations of the nonAbelian KV algebra. In particular, in AdS$_4$ and dS$_4$, this method can be applied using unitary twisted-adjoint representations for massless fields; for a treatment of dS$_4$, see [39], and for mixed-symmetry fields in diverse dimensions, see [45, 46].

The dynamical scalars in $\Phi_{y=Z=0}$ can be arranged into real scalars with definite intrinsic parities as follows:

$$\text{AdS}_4, \text{dS}_4 : \Phi_{y=Z=0} := \phi_+ + i\phi_- + (\phi_+ - i\phi_-) \star \Gamma,$$ \hfill (3.45)

$$H_{2,2}, S^4, H_4 : \Phi_{y=Z=0} := \phi_+ \star (1 + \Gamma) + \phi_- \star (1 - \Gamma),$$ \hfill (3.46)

where

$$(\phi_{\pm})^\dagger = \phi_{\pm}, \quad P(\phi_{\pm}) = \pm e^{2\i\theta_0} \phi_{\pm}.$$ \hfill (3.47)

In AdS$_4$, one may consider harmonic expansions in which the fields with $s \geq \frac{1}{2}$ carry lowest weight spaces $D(s+1; s)$ of $so(3, 2)$ with intrinsic parity $(-1)^s$ for bosons; the compact basis element with energy $\omega$ and spin $m$ corresponds to a harmonic Weyl zero-form $\Phi_{\omega}^{\alpha(1/2), \omega, m}(x^\mu)$ that is an eigenfunction of the AdS$_4$ Killing vectors corresponding to the energy operator and a compact spin generator. The notation $a(s)$ denotes $n$ symmetrized indices. Since the spin $s$ Weyl tensor of the spin $s$ gauge field is given by

$$C_{\alpha(2s)} = e^{-\i\theta_0} \Phi_{\alpha(2s)},$$ \hfill (3.48)

it follows that if one introduces the electro-magnetic characteristic $^{16}$

$$(-1)^s \Phi_{\alpha(1), b(1)} \Phi_{\alpha(1), b(1)} \begin{cases} > 0 & \text{magnetic} \\ < 0 & \text{electric} \end{cases}$$ \hfill (3.49)

\text{idem} $C_{\alpha(1), b(1)}$, then the characteristics of $C_{\alpha(1), b(1)}$ and $\Phi_{\alpha(1), b(1)}$ are the same for sufficiently small $\theta_0$ and opposite for $\theta_0$ sufficiently close to $\frac{\pi}{2}$ [47]. As for the scalars, unbroken HS symmetry requires $\phi_{\pm}$ to carry the lowest weight spaces $D(2, 0)_-$ and $D(1, 0)_+$ for $\theta_0 = 0$ and vice versa for $\theta_0 = \frac{\pi}{2}$.

Instead of using lowest-weight spaces, the linearized fields can be expanded in $\theta_0$-dependent bases induced by Fronsdal tensors with point-like magnetic sources at the Minkowski boundary of the Poincaré coordinate chart in AdS$_4$ [19]. The resulting linear solution space in the twisted-adjoint module, corresponding to unfolded bulk-to-boundary propagators with the aforementioned magnetic boundary conditions, is spanned by Weyl zero-forms $\Phi_{\hat{\mu}; \tilde{x}}(x^\mu)$ labeled by points $\tilde{x}$ in three-dimensional Minkowski spacetime and real polarization spinors $\chi_\alpha$. As found in [19], if $0 < \theta_0 < \frac{\pi}{2}$, this solution space, in general, is not left invariant by all Killing transformations, though supersymmetric HS models were constructed in which the solution space is preserved by finite dimensional AdS$_2$ superalgebras for $N = 1, 2, 3, 4, 6$. We shall comment on the general structure of these models in the next section.

$^{16}$ More generally, the generalized Petrov classes for $\Phi_{\alpha(1), b(1)}$ are labeled by ordered partitions $[n_1, \ldots, n_k]$ of $2s$ characterized as $\Phi_{\alpha(2s)} = n_1 \prod_{i=1}^{k} (\mu_i')^n$ where $\mu_i'$ are $k$ non-collinear polarization spinors.
4. Supersymmetric higher spin theories

The fully nonlinear Vasiliev type HS field equations known in 4D so far are those with $\mathcal{N} = 1, 2$ [1, 2, 30, 25], $\mathcal{N} = 4$ [25] and $\mathcal{N} = 8$ [15, 16] without internal symmetries, and all even $\mathcal{N}$ models with internal $u(n)$ symmetry [19]. In this section we describe a subset of the KV models which are based on HS extension of ordinary AdS$_4$ superalgebras. These are $\mathcal{N} = 1, 2, 4$ mod 4 models with suitable internal symmetries. Here we show that the minimal models with $\mathcal{N} = 3$ mod 4 automatically have $\mathcal{N} = 4$ mod 4. In the case of $\mathcal{N} = 1, 4$ mod 4 models in AdS$_4$, in addition to the non-minimal versions provided in [19], here we construct minimal $\mathcal{N} = 1, 4$ mod 4 models with internal $so(n)$ or $usp(n)$ symmetries; see table 4. As we noted in the Introduction, it is natural to expect a connection between the HS algebras involved in these models and a larger KV algebras supplemented with suitable truncation conditions, though this remains to be shown. In this section, considering the case of $\mathcal{N} = 6$ HS theory in AdS$_4$, we also present an alternative description to that given in [19] that makes its relation to the $\mathcal{N} = 8$ model [15, 16] manifest. Finally, we spell out some details of the minimal $\mathcal{N} = 2$ model in dS$_4$, which has the minimum amount of supersymmetry allowed in dS$_4$.

4.1. Even $\mathcal{N}$ in AdS$_4$

In the AdS$_4$ case, the extended models based on KV algebras are supersymmetric in the usual sense if $m = n$, i.e. if the underlying supersingleton contains equal number of bosons and fermions, in which case

\[
\text{any } m : ho(m; m|4) \supset osp(1|4) \ni Q_\alpha := \theta^i y_\alpha \otimes \sigma^i \otimes 1_{m \times m},
\]

\[
\text{odd } m : hu(m; m|4) \supset osp(2|4) \ni Q^\pm_\alpha := \theta^i y_\alpha \otimes (\sigma^i \pm i \sigma^2) \otimes 1_{m \times m},
\]

\[
\text{even } m : husp(m; m|4) \supset osp(4|4) \ni Q^I_\alpha := \theta^i y_\alpha \otimes \Sigma^I \otimes 1_{2 \times 2},
\]

where, in the last case, $\gamma = 1_{2 \times 2} \otimes i \sigma^2 \otimes 1_{2 \times 2}$ and $\Sigma^I = (\sigma^2 \otimes \sigma^I, \sigma^I \otimes 1_{2 \times 2}), I = 1, \ldots, 4$, are $so(4)$ gamma matrices, and if $m$ is even then $hu(m; m|4) \supset husp(m; m|4)$. Recall that $\gamma$ enters the definition of the graded anti-automorphism as in (3.15).

If $m = n = 2^k$ then

\[
\frac{1}{2} (1 + \pi_\theta \text{Ad}_{1^k}) (\text{Cliff}(\mathbb{C}) \otimes \text{Mat}_{m+n}(\mathbb{C})) \cong \text{Cliff}_{4^{2k}}(\mathbb{C}),
\]

\[
\mathcal{N} = 2(k + 1),
\]

with generators $\xi_i^I, i = 1, \ldots, \mathcal{N}$, obeying

\[
\{\xi^i, \xi^j\}_\ast = 2\delta^{ij}, \quad (\xi^i)^\dagger = \xi^i, \quad \tau(\xi^i) = \begin{cases} i\xi^i & \mathcal{N} = 4 \text{ mod 4}, \\ -i\xi^i & \mathcal{N} = 2 \text{ mod 4}, \end{cases}
\]

and one can identify

\[
\Gamma \cong \Gamma_\xi := \xi^{k+1} \cdot \ldots \cdot \xi^\mathcal{N}.
\]

Thus, the sequence of models based on the minimal HS extensions $shs^g(\mathcal{N}|4)$ of $osp(N|4)$ in which the master fields $(A, \Phi)$ are valued in the $(Y, Z, \xi)$-oscillator algebra and obey (see table 4)

- spin-statistics, viz.

\[
\epsilon_1(A, \Phi) = (0, 0), \quad \pi \bar{\pi}(A, \Phi) = (A, \Phi),
\]

where $\pi(\xi^i) := -\xi^i$;

- reality conditions

\[
(A)^\dagger = -\overline{A}, \quad (\Phi)^\dagger = \pi(\Phi) \ast \Gamma;
\]
is equivalent to sequences of models based on isomorphic KV algebras as follows [23]

\[ N = 4 \mod 4 : \tau(\widehat{A}) = -\widehat{A}, \quad \tau(\widehat{\Phi}) = \pi(\widehat{\Phi}) \star \Gamma; \]

(4.9)

- Vasiliev equations (2.8) and (2.9) with deformations as in (3.19) including the parity

invariant \( shs^E(N)[4] \) models of type A and type B defined by (3.30) and (3.31);

is equivalent to sequences of models based on isomorphic KV algebras as follows [23]

\( N = 2(k + 1) \):

\[ \text{shs}^E(N)[4] \cong \begin{cases} \text{hus}(4^k) & k = 0, 2, \ldots, \\ \text{husp}(4^k) & k = 1, 5, \ldots, \\ \text{hot}(4^k) & k = 3, 7, \ldots. \end{cases} \]

(4.10)

The chain of consistent truncations from \( N \to N - 2 \) can be made manifest by reformulating

the subsequence of \( \text{shs}^E(N)[4] \) models with \( N = 2 \mod 4 \) by introducing an additional

Clifford algebra with fermionic generators \( \eta', r = 1, 2 \), obeying

\[ [\eta', \eta'] := \delta^r, \quad \varepsilon_i(\eta') = 0, \quad (\eta')^i := \eta^i, \quad \tau(\eta') := i\eta', \]

and re-defining (see table 4)

\[ N = 2 \mod 4 : \tau(\xi^{i}) := i\xi^{i}, \quad \Gamma := \Gamma_\xi \star \Gamma_\eta, \quad \Gamma_\eta := \text{in}^1 \star \eta^2. \]

(4.12)

The master fields (\( \widehat{A}, \widehat{\Phi} \)) are now valued in the \( (Y, Z, \xi, \eta) \)-oscillator algebra and they obey the spin-statistics conditions

\[ \varepsilon_i(\widehat{A}, \widehat{\Phi}) = (0, 0), \quad \pi \pi \pi \pi_\eta_\eta (\widehat{A}, \widehat{\Phi}) = (\widehat{A}, \widehat{\Phi}), \]

(4.13)

where \( \pi_\eta(\eta') = -\eta' \); the reality conditions (4.8); the \( \tau \)-conditions (4.9); and the additional conditions

\[ [\Gamma_\eta, \widehat{A}] = 0 = [\Gamma_\eta, \widehat{\Phi}]. \]

(4.14)

The Vasiliev equations again take the form (2.8) and (2.9) with deformations as in (3.19), and

the parity invariant minimal \( N = 2 \mod 4 \) models of type A and type B are defined by (3.30)

and (3.31).

To illustrate the truncation from \( N = 4 \mod 4 \) to \( N = 2 \mod 4 \), let us consider the

\( \text{shs}^E(8)[4] \) model which was analyzed in detail in [15]17. The minimal \( N = 6 \) model,
given recently in [19], can be obtained by first splitting \( \xi^{i} = (\xi^i, \eta^i) \) where \( i = 1, \ldots, 8 \),

\( i = 1, \ldots, 6 \) and \( r = 1, 2 \) and then imposing the conditions (4.14), which eliminate the

gravitino supermultiplet and its HS analogues, resulting in the spectrum of the minimal \( N = 6 \)
model given in table 3. As shown above, this model can equivalently be formulated [19] using

only the fermionic \( \xi^i \) oscillators, provided one drops the \( \tau \) conditions; compare the second

and third rows of table 4. A similar procedure can be applied to truncate the minimal \( N = 2 \)
mod 4 models to those with \( N = 4 \mod 4 \).

4.2. Odd \( N \) in \( \text{AdS}_4 \)

The sequence of minimal \( N = 1 \mod 4 \) models, including the \( N = 1 \) model of [25],
based on the extended HS algebras \( \text{shs}^E(N)[4] \) with maximal finite-dimensional subalgebras

\( \text{osp}(N)[4] \), has master fields \( (\widehat{A}, \widehat{\Phi}) \) depending on \( (Y^2, Z^2, \xi^i, \eta) \), where \( (\xi^i, \eta), i = 1, \ldots, N \)

are fermionic generators of \( \text{Cliff}_{N+1}(\mathbb{C}) \), obeying spin-statistics, i.e.

\[ \varepsilon_i(\widehat{A}, \widehat{\Phi}) = (0, 0), \quad \pi \pi \pi \pi_\eta_\eta (\widehat{A}, \widehat{\Phi}) = (\widehat{A}, \widehat{\Phi}); \]

(4.15)

17 The truncations of this model to minimal models with \( N = 1, 2, 4 \) were described in [25]. Models with \( N = 1, 2, 4 \)

have also appeared recently in [19], which, however, are not minimal as they are all formulated using four Clifford

algebra generators and not imposing any \( \tau \) condition.
Table 3. The table contains the $so(3, 2) \times so(6)$ content of the minimal $\mathcal{N} = 6$ model in 4D arranged into $osp(6|4)$ supermultiplets labeled by $\ell$ with the entries referring to $so(6)$ irreps with the supergravity multiplet at $\ell = 0$. The spin 1 sector of the supergravity multiplets arises by gauging $R$-symmetry generators $T^I = \xi I$ and the central generator $\Gamma_Y$.

| $\ell \backslash s$ | 0     | $\frac{1}{2}$ | 1     | $\frac{3}{2}$ | 2     | $\frac{5}{2}$ | 3     | $\frac{7}{2}$ | 4     | $\frac{9}{2}$ | 5     | $\frac{11}{2}$ | 6     | $\ldots$ |
|-------------------|-------|---------------|-------|---------------|-------|---------------|-------|---------------|-------|---------------|-------|---------------|-------|---------|
| 0                 | $15 + 15$ | 20 + 6       | 15 + 1 | 6             | 1      | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
| 1                 | 1 + 1  | 6            | 15 + 1 | 20 + 6       | 15 + 1 | 6             | 1      | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
| 2                 | 1     | $\ldots$    | $\ldots$ | $\ldots$    | $\ldots$ | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
| 3                 | 1     | 6           | 15 + 1 | 20 + 6       | 15 + 1 | 6             | 1      | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
| 4                 | 1     | $\ldots$    | $\ldots$ | $\ldots$    | $\ldots$ | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
| $\ldots$          | $\ldots$ | $\ldots$    | $\ldots$ | $\ldots$    | $\ldots$ | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       | $\ldots$      |       |         |
the reality conditions \((4.8)\) where
\[
\mathcal{N} = 1 \mod 4 : \Gamma := i \xi^1 \cdots \xi^N \eta;
\] (4.16)
the \(\tau\)-conditions \((4.9)\) using
\[
\tau (\xi^i) := i \xi^i, \quad \tau (\eta) := -i \eta,
\] (4.17)
which implies \(\tau (\Gamma) = \Gamma\). The equations of motion are of the standard form \((2.8)\) and \((2.9)\) with deformations as in \((3.19)\) and parity invariant minimal models of type A and type B defined as in \((3.30)\) and \((3.31)\).

The minimal \(\mathcal{N} = 1\) model is based on the HS algebra \(shs(1|4)\) whose maximal finite-dimensional subalgebra is \(osp(1|4)\). The spectrum of the \(shs(1|4)\) gauge theory is given by the symmetric product of two \(osp(1|4)\) singlets, which decomposes into a tower of \(\mathcal{N} = 1\) massless supermultiplets with \(s_{max} = \ell + \frac{1}{2}\) and \(s_{max} = 2\ell + 2\) for \(\ell = 0, 1, 2, \ldots\), thus containing, in particular, a Wess-Zumino scalar multiplet and the supergravity multiplet.

In the case of \(\mathcal{N} = 3 \mod 4\), the analogue of the above construction requires fermionic oscillators \((\xi^i, \eta^i), i = 1, \ldots, \mathcal{N}, r = 1, 2, 3\), obeying \(\tau (\xi^i, \eta^i) = (i \xi^i, -i \eta^i)\) in order for there to exist the element
\[
\mathcal{N} = 3 \mod 4 : \Gamma := i \xi^1 \cdots \xi^N \eta^1 \eta^2 \eta^3, \quad \Gamma \Gamma = 1, \quad (\Gamma) = \Gamma = \Gamma.
\] (4.18)
The resulting minimal model, in which the master fields obeys \((4.15), (4.8)\) and \((4.9)\) together with the further projection
\[
\mathcal{N} = 3 \mod 4 : [T^{r\alpha}, \tilde{A}^r] = [T^{r\alpha}, \tilde{\Phi}^r], \quad T^{r\alpha} := \eta^r \eta^r,
\] (4.19)
which is consistent as \([T^{r\alpha}, \Gamma] = 0\), thus has \(\mathcal{N} + 1\) supersymmetries generated by \(Q^r_{\alpha} = \xi^1 Y_a\) and \(Q^{N+1} \eta = i n^1 n^2 n\eta Y_a\). Thus, \(\mathcal{N} = 3 \mod 4\) implies \(\mathcal{N} = 4 \mod 4\) in HS theories.

4.3. Internal symmetries in AdS₄

The minimal models in AdS₄ with \(\mathcal{N} = 1, 2, 4 \mod 4\) supersymmetries can be extended by internal symmetries without changing \(\mathcal{N}\) by first removing the reality and \(\tau\) conditions on the master fields and then tensoring them with an internal Matₙ(ℂ) algebra, after which the reality and \(\tau\) conditions can be reimposed with \(\dagger\) and \(\tau\) acting on matrices as in sections 4.1 and 4.2.

---

Table 4. The table displays the basic features of the three sequences of supersymmetric models in AdS₄ in accordance with \(\mathcal{N}\) standard supersymmetries: The second and third columns list the fermionic oscillators and corresponding \(\Gamma\) operators, respectively. In all models, the reality conditions read \((\tilde{A}, \tilde{\Phi}) = (-A, \pi \tilde{\Phi} \bullet \Gamma)\). The \(\tau\)-conditions, when imposed as indicated in the fourth column, read \(\tau (\tilde{A}, \tilde{\Phi}) = \gamma \bullet (-A, \pi \tilde{\Phi} \bullet \gamma) \bullet \gamma\) where \(\tau (\xi) = i \xi, \tau (\eta) = -i \eta, \gamma = 1\), and \(\gamma = 1\) for \(\mathcal{N} = 0\) and \(\gamma = \epsilon_\alpha \), for \(\mathcal{N} = 1\). Extending a given minimal model without increasing \(\mathcal{N}\) yields the internal symmetry algebras listed in the last column. The \(\mathcal{N} = 2\) mod 4 models admit two equivalent formulations; the one with additional fermionic oscillators requires a \(\tau\) condition and a further projection \([\Gamma^r, A^r] = 0 = [\Gamma^r, \tilde{\Phi}^r]\), where \(\Gamma^r := i n^1 n^2\).

| \(\mathcal{N}\) mod 4 | Fermionic oscillators | \(\Gamma\) | Projections | Internal symmetry algebra |
|-----------------|---------------------|---------|------------|--------------------------|
| 1               | \(\xi^i, \eta^i\)   | \(i \xi^1 \cdots \xi^N \eta\) | \(\tau\) | \(o(n)\) or \(usp(n)\)  |
| 2               | \(\xi^i, \eta^j\)   | \(i \xi^1 \cdots \xi^N \eta^j\) | \(\tau, \gamma\) | \(u(n)\)               |
| 3               | \(\xi^i\)           | \(i \xi^1 \cdots \xi^N\) | \(-\)  | \(o(n)\) or \(usp(n)\)  |
| 4               | \(\xi^i\)           | \(i \xi^1 \cdots \xi^N\) | \(\tau\) | \(o(n)\) or \(usp(n)\)  |
In the case of $\mathcal{N} = 1, 2, 4$ supersymmetric HS theories with internal symmetry, the Young tableaux refer to the symmetry properties of the matrix valued quantities in (4.20) and (4.22). In the case of $\mathfrak{u}(n)$ internal symmetry, the Yang tableau with crosses denote $\mathfrak{su}_n(C)$ matrices. The maximal finite dimensional supersubalgebra is given by the direct sum of $\mathfrak{osp}(\mathcal{N}/4)$ and the internal symmetry algebra.

| Supersymmetry | Internal | $s = 0 \mod 2$ | $s = 1 \mod 2$ | $s = \frac{1}{2} \mod 2$ | $s = \frac{3}{2} \mod 2$ |
|---------------|----------|----------------|----------------|--------------------------|--------------------------|
| $\mathcal{N} = 1$ | $\mathfrak{o}(n)$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ |
| $\mathfrak{u}(n)$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ |
| $\mathcal{N} = 2$ | $\mathfrak{u}(n)$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \end{array}$ |
| $\mathcal{N} = 4$ | $\mathfrak{o}(n)$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ |
| $\mathfrak{u}(n)$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ | $\begin{array}{c} \Box \oplus \Box \oplus \Box \oplus \Box \end{array}$ |

In the case of $\mathcal{N} = 1, 2, 4$, the spectra can be read off from the following expansions of the now matrix-valued Weyl zero-form:

$\mathcal{N} = 1: \Phi|_{y=0} = M(y) + \Gamma M'(y) + \xi \Psi(y) + \eta \Psi'(y)$,

(4.20)

$\mathcal{N} = 2: \Phi|_{y=0} = M(y) + \Gamma M'(y) + \xi^i \Psi_i(y)$,

(4.21)

$\mathcal{N} = 4: \Phi|_{y=0} = M(y) + \xi^i \star \xi^j M_{ij}(y) + \Gamma M'(y) + \xi^i \Psi_i(y) + \xi^i \star \xi^j \Psi_{ij}(y)$,

(4.22)

where $\Phi = \hat{\Phi}|_{z=0}$. The resulting spectra are given in table 5. In the case of $\mathcal{N} = 2$, for which there are two types of realizations as described in section 4.1, the corresponding spectra are the same. Furthermore, the reality properties of the fields shown in table 5 can be determined from (4.8). For example, in the scalar sector, $M^i|_{y=0} = M'|_{y=0}$ for $\mathcal{N} = 1, 2, 4$, and in the case of $\mathcal{N} = 4$, we have $M^i_{ij}|_{y=0} = \frac{1}{2} \epsilon_{ijkl} M^{kl}|_{y=0}$.

The spectrum analysis proceeds similarly for all $\mathcal{N} = 1, 2, 4$ mod 4 as well. In particular, in the case of $\mathcal{N} = 2$ mod 4, for which there are two types of realizations as described in section 4.1, the corresponding spectra are the same. For example, in the case of $\mathcal{N} = 6$, the spectrum is the one given in table 3, with every field is taken to be $u(n) \sim su(n) \oplus u(1)$ valued. The $u(1)$ part contains the $\mathcal{N} = 6$ supergravity multiplet at the lowest level.

4.4. $\mathcal{N} = 2$ in dS$_4$

In dS$_4$, the $\mathcal{N} = 2$ supersymmetric HS theory is based on the algebra $h\mathfrak{o}(1; 1|4, 1)$ contained in the general construction given above. In particular, the HS field equations have the standard form (2.8) and (2.9) with deformations as in (3.19), $\Gamma = \sigma_3$ and parity invariant minimal models of type A and type B defined as in (3.30) and (3.31).

As in dS$_4$ supergravity, $\mathcal{N} = 2$ is the smallest possible number of supersymmetries, with the rules for Hermitian conjugation of fermionic variables adopted in this paper. In what follows, we shall elaborate further on this case and provide an alternative description in terms

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18 We shall not consider the alternative reality condition considered by Vasiliev in [41, 42] and mentioned in section 3.2 above, which would allow a generalized $\mathcal{N} = 1$ supersymmetry algebra with $\mathbb{Z}_2 \times \mathbb{Z}_2$ grading.
of oscillators alone. The master fields belong to the associative algebra given by Mat\(_1\), \(C\) times the oscillator algebra generated by \((y^\alpha, \bar{y}^\dot{\alpha}, \bar{z}^\alpha, \theta^r)\) where the fermionic doublet obeys
\[
\{\theta^r, \theta^s\} = 2\delta^{rs}, \quad (\theta^r)^\dagger = e^{r\alpha} \theta^\alpha.
\]
The Hermitian conjugation is defined as usual on matrices, Grassmann numbers and twistor oscillators in accordance with \(SL(2, C)\) invariance. As a result
\[
(\hat{f})^\dagger \equiv \pi\theta(\hat{f}).
\]
This is consistent with imposing spin-statistics projection
\[
\epsilon_f(\hat{A}) = 0 = \epsilon_f(\hat{\Phi}), \quad \pi\pi\theta(\hat{A}, \hat{\Phi}) = (\hat{A}, \hat{\Phi}), \quad \Gamma\star(\hat{A}, \hat{\Phi})\star\Gamma = (\hat{A}, \hat{\Phi}),
\]
where \(\epsilon_f\) denotes the Grassmann parity and \(\Gamma = \sigma^3\). The reality conditions leading to the dS\(_4\) vacuum are
\[
(\hat{A}^\dagger, \hat{\Phi}^\dagger) = -\pi(\hat{A}, \hat{\Phi}), \quad \hat{\Phi}^\dagger = \hat{\Phi}\star\Gamma.
\]
In the ho\((1, 1|4, 1)\) model the master fields also obey the \(\tau\) condition
\[
\tau(\hat{A}) = -\hat{A}, \quad \tau(\hat{\Phi}) = \bar{\pi}(\hat{\Phi}),
\]
where \(\tau\) acts on \(M_{1,1}(C)\) by transposition. The supercharges of the \(N = 2\) dS\(_4\) supersymmetry algebra are realized as
\[
Q'_\alpha := y_\alpha \theta^r \sigma^1, \quad \bar{Q}'_{\dot{\alpha}} := \bar{y}_{\dot{\alpha}} \theta^r \sigma^1,
\]
obeying
\[
(Q'_\alpha)^\dagger = e^{r\alpha} Q'_\alpha,
\]
and
\[
\{Q'_\alpha, Q'_{\dot{\alpha}}\} = 2\delta^{r\alpha} M_{\alpha\beta} + 2i\epsilon_{\alpha\beta} T^{r\alpha}, \quad \{Q'_\alpha, Q'_{\dot{\alpha}}\} = 2\delta^{r\alpha} P_{\alpha\dot{\alpha}},
\]
where \(T^{r\alpha} := \frac{1}{2}[\theta^r, \theta^s]_\alpha = (T^{r\alpha})^\dagger\) is the so\((2)\)\(_R\) generator. This model can equivalently be realized in terms of master fields depending on \((Y^\alpha, \xi^i), i = 1, 2\) and where \(\xi^i\) are fermionic Clifford algebra generators obeying
\[
(\xi^i)^\dagger = \epsilon^{ij} \xi^j.
\]
The reality conditions read
\[
(\hat{A}^\dagger, \hat{\Phi}^\dagger) = (-\hat{A}, \hat{\Phi}\star\Gamma), \quad \Gamma = i\xi^1 \star \xi^2,
\]
while there are no \(\tau\) conditions. The supersymmetry charges read
\[
Q'_\alpha = \xi^i y_\alpha, \quad \bar{Q}'_{\dot{\alpha}} = \xi^i \bar{y}_{\dot{\alpha}}.
\]
The set of dynamical fields coincides with that of the \(N = 2\) model in AdS\(_4\) though the reality conditions on the fermions are modified. As mentioned in section 3.2, the \(N = 2\) de Sitter supergravity in 4D \([43, 44]\) contains a vector ghost \([44]\). It would be interesting to determine the corresponding situation in the HS version of the theory presented here.
5. Conclusions

The results on supersymmetric HS theories described here, old and new, are hoped to play a role in understanding their relation to string/M theory. Moreover, HS theories in dS4, Euclidean and Kleinian spacetimes provide fertile grounds for sharpening ideas in holography, in the case of de Sitter space providing a framework in which problems that are notoriously difficult to study in the usual string theory can now be addressed [48, 49]. Investigations on HS holography, attempts to make connection with string/M theory and the need to understand better the already existing interaction deformation in parity non-invariant HS theories are likely to motivate further generalizations of HS theories. It would be interesting, for example, to construct matter couplings systematically. Chern–Simons-quiver theories in 3D, which are holographically dual to the Freund–Rubin compactifications of M theory to AdS4 (see, for example, [50, 51]), in appropriate limits may be relevant for such constructions. In this context, the \( \mathcal{N} = 3 \) compactification of M theory on AdS4 \( \times N^{0|0} \) has been considered briefly in [25].

In the case of \( \mathcal{N} = 1 \) HS theories, the problem of constructing chiral matter couplings would be of great interest.

In analyzing certain aspects of a wide class of supersymmetric HS theories covered here, it may be useful to formulate them in superspace. Such a formulation is conceptually simple and mathematically manageable, given the universal Cartan integrable nature of Vasiliev equations. Indeed, starting with the standard formulation of Vasiliev equations in AdS4, they can be formulated in superspace simply by replacing the 4D spacetime with a \( D = (4|4\mathcal{N}) \) superspace with \( 4\mathcal{N} \) anti-commuting \( \theta \)-coordinates [37]. This introduces extra spinorial directions in the 1-forms as well as \( \theta \)-dependence in all component fields. On the other hand, there are also new constraints coming from projections of the differential form constraints in the new spinorial directions. As a result, each supermultiplet in the spectrum is described by a single constrained superfield, and we arrive at a superspace description of HS theory in AdS4 which is equivalent to the formulation in ordinary spacetime. This procedure has been performed in [37] for \( \mathcal{N} = 1 \) supersymmetric HS theory in AdS4.

In their current form, Vasiliev equations for HS theories in four dimensions contain an infinite set of free parameters of which a finite number show up at every new order in perturbation theory. All of these parameters break parity when they are non-vanishing, except the first parameter, denoted by \( \theta_0 \), which preserves parity when it takes the values 0 or \( \pi/2 \), for which the physical scalar has intrinsic parity +1 and −1, respectively. The parameter \( \theta_0 \) shows up already at the free level, where it can be redefined away, however, by a duality rotation that mixes electric and magnetic components of the Weyl tensors. At cubic order, \( \theta_0 \) remains the only free parameter, and it has been proposed in [52] and tested in [52, 53] that this deformation parameter corresponds to the ’t Hooft coupling of Chern–Simons theories in 3D coupled to singletons in such a way that free scalars go into free fermions at strong coupling and vice versa; for a recent review, see [54].

So far, the nature of the three-dimensional counterparts of the higher-order deformation parameters remains less clear. In [35, 36] it was found that the off-shell formulation based on generalized Hamiltonian actions requires the deformation function to be linear, hence containing only the \( \theta_0 \) parameter, albeit under certain extra assumptions that remain to be validated. On the other hand, as pointed out in [33], at the classical level, the set of deformations can be enriched even further by replacing each deformation parameter by a zero-form charge [55, 38]. The properties of the perturbative expansions of zero-form charges found in [34, 56] suggest that these new deformations could correspond to extensions of the generating function in three dimensions by additions of composite operators coupled to nonlinear sources. In particular, beyond \( \mathcal{N} = 8 \) such extensions would resolve some issues related to the absence
of an $\mathcal{N} = 8$ barrier for 4D HS supergravities with singlet gravitons and generally covariant weak field expansions with AdS$_4$ vacua, though the presence of the $\theta_0$ parameter for $\mathcal{N} \geq 8$ remains a puzzle in the context of holography since Chern–Simons-matter type CFTs do not exist in 3D with such supersymmetries.

The singletons play a key rôle in HS gravity in AdS$_4$, as they can be realized as unitary representations of the HS algebra using either the $Y^2$ oscillators or free fields in 3D. These two dual realizations carry over to other signatures with the key difference that they are no longer unitary. In particular, in dS$_4$ the massless bulk fields can be expanded harmonically in unitarizable representations consisting of states that are tensors of the maximal compact $\text{so}(4) \subset \text{so}(4, 1)$. These representations are not of lowest-weight type, but rather generalized Harish–Chandra modules: the analogue of the ground state is thus the smallest $\text{so}(4)$ irrep, from which the remainder of the representation space can be generated by acting with the coset $\text{so}(4, 1)/\text{so}(4)$. In fact, the $\text{so}(4)$ content of the unitarizable representation for a massless spin 1 field in dS$_4$ is exactly the same as the $\text{so}(3, 1)$ content of its twisted-adjoint representation [39]. It would thus be interesting to characterize group theoretically the free scalar and fermion fields on $\mathbb{S}^4$, thought of as a boundary of dS$_4$, and examine how the square of these representations, which are nonunitary, can be rearranged into the unitary irreps for the massless fields, that is, to generalize to dS$_4$ the Flato–Fronsdal formula.

In higher dimensions, in order for supersymmetric models to admit standard spacetime interpretations in AdS$_D$, one must have $D \leq 7$. Higher spin models with 32 supercharges in $D = 5, 7$ have been constructed at the linearized level, including their twisted adjoint representations in [27, 6]. These constructions rely on Grassmann even spinor oscillators $Y^2$ and Grassmann odd Clifford algebra generators. An interesting open problem is the construction of corresponding fully nonlinear models.

Breaking of HS symmetries is another clearly important problem. A mechanism for breaking of HS symmetries [57], which is well understood at the kinematic level in the case of bosonic models, involves Goldstone bosons as composite of a scalar and spin $(s - 2)$ field for giving mass to a spin $s$ field. Some aspects of this mechanism has been discussed in [17] for the $\mathcal{N} = 1$ HS theory, and it would be useful to explore this further. Given that there is no $\mathcal{N} = 8$ barrier in writing down fully nonlinear and consistent HS theories, it would also be interesting to determine whether in such models the symmetries can be broken down to $\mathcal{N} \leq 8$ supersymmetric HS gravity or ordinary gravity by a GPZ-like mechanism [57] or by the mechanism proposed in [19].

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Appendix A. Spinor conventions in different signatures

We use spinor conventions in which doublet indices are raised and lowered as follows:

\[ y^\alpha = \epsilon^{\alpha\beta} y_\beta, \quad y_\alpha = y^\beta \epsilon_{\beta\alpha}, \quad \epsilon^{\alpha\beta} \epsilon_{\gamma\delta} = 2 \delta^{\alpha\gamma} \delta_{\beta\delta}, \quad \epsilon^{\alpha\beta} \epsilon_{\alpha\beta} = \delta^\beta_{\beta}. \]  

(A.1)

The van der Waerden symbols $(\sigma^a)_{\alpha\beta} \equiv (\bar{\sigma}^a)_{\overline{\beta}\overline{\alpha}}$ obey

\[ (\sigma^a)_{\alpha} (\bar{\sigma}^b)_{\beta} = \eta^{ab} \delta^\alpha_\beta + (\sigma^{ab})_{\alpha} \beta, \quad (\bar{\sigma}^a)_{\bar{\beta}} (\sigma^b)_{\bar{\alpha}} = \eta^{ab} \delta^{\bar{\alpha}}_{\bar{\beta}} + (\bar{\sigma}^{ab})_{\bar{\beta}} \bar{\alpha}, \]  

(A.2)

\[ \frac{1}{2} \epsilon_{abcd} (\sigma^{cd})_{\alpha\beta} = \eta (\sigma_{ab})_{\alpha\beta}, \quad \frac{1}{2} \epsilon_{abcd} (\bar{\sigma}^{cd})_{\bar{\alpha}\bar{\beta}} = -\eta (\bar{\sigma}_{ab})_{\bar{\alpha}\bar{\beta}}. \]  

(A.3)
where $\eta = \sqrt{\det R_{ab}}$. The reality conditions on doublet variables are summarized in (2.32), (2.33) and (2.34) and for the van der Waerden symbols we use

$$
(\epsilon_{ab}, (\sigma^a)_{ab}, (\sigma^a)_{ab}) = \begin{cases} 
(\epsilon^{ab}, -(\sigma^a)_{ab}, (\sigma^{ab})_{ab}) & \text{for (4, 0) signature,} \\
(\epsilon_{ab}, (\sigma^a)_{ab}, (\sigma^{ab})_{ab}) & \text{for (3, 1) signature,} \\
(\epsilon_{ab}, (\sigma^a)_{ab}, (\sigma^{ab})_{ab}) & \text{for (2, 2) signature,}
\end{cases}
$$

(A.4)

corresponding to the following representations:

$$(4, 0) : (\sigma^a)_{ab} = (i, \sigma^a)_{ab}, \quad (\bar{\sigma}^a)_{ab} = (-i, \sigma^a)_{ab},$$

(A.5)

$$(3, 1) : (\sigma^a)_{ab} = (-i^2, -i\sigma^a)_{ab}, \quad (\bar{\sigma}^a)_{ab} = (-i^2, i\sigma^a)_{ab},$$

(A.6)

$$(2, 2) : (\bar{\sigma}^a)_{ab} = (1, \bar{\sigma}^a)_{ab}, \quad (\bar{\sigma}^a)_{ab} = (-1, \bar{\sigma}^a)_{ab},$$

(A.7)

where $\bar{\sigma}^a := (\sigma^1, i\sigma^2, \sigma^3)$. The real forms of $so(5; \mathbb{C})$ are defined by [38]

$$[M_{AB}, M_{CD}] = i\eta_{BC}M_{AD} + 3 \text{ more}, \quad (M_{AB})^\dagger = \sigma(M_{AB}).$$

(A.8)

where $\eta_{AB} = (\eta_{ab}; -\lambda^2)$. The commutation relations decompose into

$$(M_{ab}, M_{cd}) = 4i\eta_{[ab}M_{d|]} + 3 \text{ more}, \quad [M_{ab}, P_a] = 2i\eta_{[ab}P_a], \quad [P_a, P_b] = i\lambda^2 M_{ab}.$$  

(A.9)

The corresponding oscillator realization is taken to be

$$M_{ab} = -\frac{1}{8}[\sigma_{ab}\epsilon^{ab}y_\alpha y_\beta + (\bar{\sigma}_{ab})\epsilon^{ab}y_\alpha \bar{y}_\beta], \quad P_a = \frac{\lambda}{4}(\sigma_a)\epsilon^{ab}y_\alpha \bar{y}_\beta.$$  

(A.10)

The real form of the $so(5; \mathbb{C})$-valued connection $\Omega$ we choose as

$$\Omega = \frac{1}{4i}d\epsilon^{ab}[\omega_{ab}y_\alpha y_\beta + \bar{\omega}_{ab}y_\alpha \bar{y}_\beta + 2\epsilon_{ab}\epsilon^{cd}y_\alpha y_\beta],$$

(A.11)

where

$$\omega^{ab} = -\frac{1}{2}(\sigma_{ab})\epsilon^{cd} \epsilon^{\alpha\beta}, \quad \bar{\omega}_{ab} = -\frac{1}{2}(\bar{\sigma}_{ab})\epsilon^{cd} \epsilon^{\alpha\beta}, \quad \epsilon^{\alpha\beta} = \frac{1}{2}(\sigma_{ab})^{\alpha\beta} \epsilon^{ab}.$$  

(A.12)

Likewise, for the curvature $\mathcal{R} = d\Omega + \Omega \wedge \Omega$ one finds

$$\mathcal{R}_{ab} = d\omega_{ab} + \omega_{a\gamma} \wedge \omega_{\beta}^\gamma + e_{a\gamma} \wedge e_{\beta}^\gamma,$$

(A.13)

$$\bar{\mathcal{R}}_{ab} = d\bar{\omega}_{ab} + \bar{\omega}_{a\gamma} \wedge \bar{\omega}_{\beta}^\gamma + e_{a\gamma} \wedge e_{\beta}^\gamma,$$

(A.14)

$$\mathcal{R}_{ab} = d\bar{\omega}_{ab} + \omega_{a\gamma} \wedge \omega_{\beta}^\gamma + e_{a\gamma} \wedge e_{\beta}^\gamma,$$

(A.15)

and

$$\mathcal{R}^{ab} = d\omega^{ab} + \omega_{c}^{ab} \wedge \omega^{\alpha\beta} + \lambda^2 \epsilon^{ab} \wedge \epsilon^{\alpha\beta}, \quad \mathcal{R}^{ab} = d\epsilon^{ab} + \epsilon_{b}^{\beta} \wedge \epsilon^{ab}.$$  

(A.16)

**Appendix B. Linearized symmetries**

The generalized curvature constraint

$$dX^a + Q^a(X) = 0,$$

(B.1)

where $Q^a$ obey the Cartan integrability condition $Q^a \partial_a Q^b = 0$, is invariant under gauge transformations

$$\delta X^a = T^a(X, \epsilon) := \epsilon \partial^a - \epsilon^{ab} \partial_b Q^a.$$  

(B.2)

If $X^a$ is a background then so is $X^a(X, v) := (\exp \mathcal{T}) X^a$, where $\mathcal{T} := T^a(X, v) \partial_a$ and $v^a$ are finite gauge functions; cf normal coordinates. The locally defined moduli space thus consists
of the gauge orbits over the constant solutions, i.e. of elements $X^α(\overline{X}, \lambda)$ with $\overline{X}^α = \delta_{p_α}^α C^α$ where $p_α := \text{deg}(X^α)$ and $dC^α = 0$. Expanding

$$X^α = \overline{X}^α + x^α,$$

(B.3)
yields

$$dx^α + \epsilon^β_α \overline{\partial}_β \overline{Q}^α + \frac{1}{2} \epsilon^β_α x^γ \overline{\partial}_β \overline{\partial}_γ \overline{Q}^α + \cdots = 0,$$

$$\delta x^α = de_1^α - \epsilon^β_α \overline{\partial}_β \overline{Q}^α - \epsilon^β_0 \overline{x}^γ \overline{\partial}_β \overline{\partial}_γ \overline{Q}^α + \cdots.$$  

(B.4)

Thus, the linearized limit $dx^α + \epsilon^β_α \overline{\partial}_β \overline{Q}^α = 0$ is Cartan integrable with Abelian local symmetries

$$\delta ϵ^1_α x^α = de^1_α - ϵ^β_1 \overline{\partial}_β \overline{Q}^α,$$

(B.5)
as well as nonAbelian global symmetries

$$\delta ϵ^0_α x^α = - ϵ^β_0 \overline{x}^γ \overline{\partial}_β \overline{\partial}_γ \overline{Q}^α,$$

(B.6)

for parameters obeying the Cartan–Killing equation $de^0_α - ϵ^β_0 \overline{\partial}_β \overline{Q}^α = 0$.

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