Density of states and reflectionless tunneling in NS junction with a barrier

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The effect of the barrier on the proximity effect in normal-superconductor junction is analyzed. A general criterion for the barrier, though large, to be effectively transparent, is given. This criterion is applied to both the conductance of a disordered NS junction (reflectionless tunneling) and the density of states across it, showing that both phenomena stem from the same physical effect.

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At the interface between a superconductor and a normal metal Cooper pairs in the superconductor are being transformed to electrons and holes in the normal metal by the process of Andreev reflection \[ \langle \psi_k \psi_{k'} \rangle \]. This causes the pair amplitude \[ \langle \psi_k \psi_{k'} \rangle \] to be finite in the normal metal close to the superconductor (the proximity effect). The existence of a barrier at the interface reduces the probability of Andreev reflection, and therefore weakens the proximity effect.

The effect of the barrier on the conductance of an NS junction (reflectionless tunneling) and on the density of states (DOS) across the junction was studied intensively (see e.g. \[ \text{[2–8]} \]). It was found that the condition for the barrier not to be effective is that trajectories that reflect from the interface more than \( \Gamma \) times before electron-hole coherence is lost are dominant. We then show how this criterion is applied to both the conductance of a disordered NS junction (reflectionless tunneling) and the density of states across it, showing that both phenomena stem from the same physical effect.

\[ \frac{d^2}{dx^2} \Theta = -\frac{2i\epsilon}{D} \sin \Theta - \frac{2\Delta}{D} \cos \Theta + \frac{1}{2} \gamma \sin 2\Theta, \]  

(1)

where \( \Theta(\epsilon, x) \) is defined by the retarded Green functions: \( G_e = \cos \Theta, F_e = \sin \Theta \), and \( \gamma = e^2H^2W^2/3 = W^2/3l_B^2 \). After solving the equation, the local DOS is obtained from \( N(\epsilon, x) = N_0 \Re G_e(x), N_0 \) is the normal metal DOS at the Fermi energy.

We consider first the case \( H = 0 \). Applying the boundary conditions at \( +\infty \) (normal metal) and \( -\infty \) (superconductor) one obtains a general solution to Eq. (1) and

\[ \Theta(\epsilon, x) = \begin{cases} 
4 \tan^{-1}\{\tan\left[\frac{x}{2}\right]\} \exp[-x/\zeta_n(\epsilon)], & x > 0 \\
\Theta + 4 \tan^{-1}\{\tan\left[\frac{x}{2}\right]\} \exp[x/\zeta_n(\epsilon)], & x < 0.
\end{cases} \]  

(2)

Here \( \zeta_n = \sqrt{1/2}\xi_n \), where \( \xi_n = \sqrt{\hbar D_n/\epsilon} \) is the coherence length of electron-hole pairs in the normal metal,
which determines the energy dependent length scale of the proximity effect in the normal metal, and diverges at small energies ($D_0$ is the diffusion constant in the normal metal). $\zeta_n = \xi_n(1 - e^2/\Delta^2)^{-1/4}$ is the decay length for states with $\epsilon < \Delta$ in the superconductor. The constants $\psi_n$ and $\psi_s$ are to be determined by the two boundary conditions at the NS interface, and $\Theta = \tan(\Delta/\epsilon)$ is the value of $\Theta$ in a bulk superconductor.

One of the boundary conditions for the Usadel equation [11,12] is a consequence of current conservation, and is therefore independent of the existence and strength of the barrier at the interface. This boundary condition reads: $\sigma_d d/\sigma_n d/\sigma_s d/\sigma^4 d/\sigma^4 d(\Theta(\epsilon,0_-)) = \sigma_n d/\sigma_s d(\Theta(\epsilon,0_+))$, where $\sigma_n$ and $\sigma_s$ are the normal state conductances of the N and S metals. The second boundary condition depends on the transmission probability of the barrier, and in the limit of small transmission reads [3]:

$$l_n d/\sigma_n d\Theta(\epsilon,0_+) = \tilde{\Gamma} \sin(\Theta(\epsilon,0_+)-\Theta(\epsilon,0_-)).$$

(3)

Here $\tilde{\Gamma}$ is a measure of the transparency of the barrier, and is given by $\tilde{\Gamma} = \int \cos \varphi \Gamma_{\varphi}/(1 - \Gamma_{\varphi})$, where $\Gamma_{\varphi}$ is the angular dependent transmission probability of the barrier, $\varphi$ is the angle of incidence at the barrier, and the averaging is over the total solid angle. Using the above boundary conditions and defining: $\eta = \sigma_n \xi_s/\sigma_s \xi_n$, $\alpha = -(\epsilon_i/\sqrt{\Delta^2 - \epsilon^2})^{1/2}$, we obtain two equations for $\psi_s$ and $\psi_n$:

$$-2\beta \sin(\psi_n/2) = \tilde{\Gamma} \sin(\psi_n - \psi_s - \Theta),$$

$$\sin(\psi_s/2) = -\eta \alpha \sin(\psi_n/2).$$

(4)

(5)

From Eq. (4) and the definitions of $\beta$ and $\zeta_n$ we see that the important parameter of the problem is the ratio between $\tilde{\Gamma}$ and the parameter $l_n/\xi_n$ measuring the "dirtiness" of the normal metal [3].

In the limit of infinite barrier ($\tilde{\Gamma}/\alpha \approx 0$), $\psi_n = 0$, and using Eq. (3), $\psi_s = 0$, and the DOS on each side of the interface equals its bulk value (zero for the superconductor, $N_0$ for the normal metal, see Fig. [3] thick line). When the transmission of the barrier is small compared to the "dirtiness" ($\tilde{\Gamma}/\alpha < 1$), $\psi_n$ and $\psi_s$ are small, we obtain a large jump in the DOS across the barrier, and the deviation from the infinite result is of first order in $\tilde{\Gamma}/\alpha$ in the superconducting side, and of second order in $\tilde{\Gamma}/\alpha$ in the normal metal side (Fig. [3] dotted line). In the limit of $|\alpha|/\tilde{\Gamma} = 0$ Eq. (4) takes the form of $\psi_n = \psi_s + \Theta$, which is the boundary condition for the case of no barrier at the boundary, and since the second boundary condition does not depend on the barrier, the results for the local DOS in this case are equivalent to those obtained for the case of no barrier [10,11,12], even though $\Gamma < 1$ (Fig. [1] thin continuous line). For $|\alpha|/\tilde{\Gamma} < 1$ we have $\psi_n - \psi_s - \Theta < 1$, and the local DOS is close to the no barrier limit (Fig. [4] dashed line). This means that in this limit the barrier, though large, has only a small effect on the DOS in the system. Since $\xi_n$ is energy dependent and diverges at low energies, there is always an energy region where the limit $\tilde{\Gamma} > l_n/\xi_n$ is satisfied and the barrier is not effective. In the case $\tilde{\Gamma} > l_n/\xi_n$ the barrier is not effective for all energies smaller than $\Delta$.

In the next sections we show that the barrier is not effective in certain limits due to the coherence of electrons and holes in the normal metal. Therefore, it is expected that an applied magnetic field will destroy the effect. We first consider the effect of finite magnetic field at the Fermi energy. In the normal metal the Usadel equation can be solved exactly, and the solution is $\Theta = 2 \tan^{-1}[\tan(\psi_n/2) \exp(-\sqrt{\pi}x)]$. In order to solve the Usadel equation analytically in the superconductor we consider small magnetic fields, $H \ll \Phi_0/W \xi_s$, i.e. $\xi_H > \xi_s$, where $\Phi_0$ is the flux quantum and $\xi_s$ is defined by $H W \xi_H = \Phi_0$. We then use the boundary condition [4], and consider $\tilde{\Gamma} \ll l_n/\xi_n$. We see that the barrier is not effective for $\tilde{\Gamma} > l_n/\xi_n$, and for $\tilde{\Gamma} < l_n/\xi_n$ the effect is destroyed (the barrier becomes effective) [4].

With both finite magnetic field and finite energy the Usadel equation is not solved exactly in the normal metal. However, multiplying the equation by $d\Theta/dx$, integrating over $x$ and using the boundary condition at infinity, we obtain the first order equation:

$$d\Theta/dx = -[-4i\epsilon D/\sigma_n (1 - \cos \Theta) + 1/2 \gamma(1 - \cos 2\Theta)]^{1/2}.$$  

(6)

Using the boundary condition [4] we obtain an equation for the values of $\Theta$ at the interface on both sides:

$$l_n[-4i\epsilon D/\sigma_n (1 - \cos \Theta_n(0)) + 1/2 \gamma(1 - \cos 2\Theta_n)(0)]^{1/2} = \tilde{\Gamma} \sin(\Theta_n(0) - \Theta_n(0)).$$

(7)

To obtain analytical results we assume $\epsilon \ll \Delta$ and $H \ll \Phi_0/W \xi_s$, and then it can be shown that $\Theta_n(0) \approx \pi/2$. The condition for the barrier to be noneffective is:
where the energy and magnetic field destroy the effect, which is effective. The validity of the semiclassical treatment is discussed both in their paper and by Beenakker and van Houten. They discuss the phenomenon of reflectionless tunneling in a semiclassical model, in which the motion of the electrons and holes in the normal metal follows a deterministic trajectory which depends on the initial position and direction of the electron, but the barrier between the normal metal and the superconductor is treated quantum mechanically. The validity of the semiclassical treatment is discussed both in their paper and by Beenakker and van Houten.

van Wees et al. consider an N’NS system with a barrier at the NS interface. N’ is an electron reservoir. Assuming, at first, a totally reflecting barrier at the interface with the superconductor, an electron leaving the reservoir into the disordered normal metal at a given position and direction has a given trajectory in the normal metal till it will reach the reservoir again. This trajectory can not hit the barrier at all or hit the barrier any number of times N (see Fig. 2).

For a disordered normal metal of length d the transmission probability of electrons across it is $T = l_n/d$, and therefore, the fraction of trajectories which return to the reservoir after hitting the barrier N times is:

$$F(N) = \begin{cases} 1 - T & N = 0, \\ T^2(1 - T)^{N-1} & N \neq 0. \end{cases}$$

(8)

Considering a barrier with finite transmission probability $\tilde{\Gamma}$, at each reflection from the interface either normal or Andreev reflections are possible.

When $\epsilon = 0, H = 0$, the Andreev reflected hole retraces exactly the path of the incoming electron. Therefore, an incoming electron can return to the reservoir either as an electron at the end of the trajectory, or as a hole at the beginning of the trajectory (Fig. 2).

We define $r_{ee}(N)$ and $r_{hh}(N)$ as the amplitudes of electron and hole reflections in an N trajectory, respectively. The average contribution to the charge current of trajectories involving N reflections at the interface is $I(N) = 2|\Gamma_{ee}(N)|^2$ and therefore the conductance of the system is given by: $G(V \to 0, H = 0) = G_s \sum_{N=0}^{\infty} F(N) I(N)$, where $G_s$ is the Sharvin conductance of the interface, and for a system with a finite cross-section $G_s = 2e^2 n /h$, $n$ is the number of transverse channels. Considering first the case where $\epsilon = 0$, $H = 0$, we obtain a recurrence formula:

$$r_{ee}(N) = i(1 + r_{ee}(N - 1)),$$

$$(10)$$

Inserting this result into the conductance formula we obtain:

$$G(V \to 0, H = 0) = \begin{cases} \frac{2e^2}{n} \frac{1}{(1 + \frac{1}{2})} (\Gamma \ll T), \\ \frac{2e^2}{n} \frac{T^2}{\Gamma} (\Gamma \gg T). \end{cases}$$

(10)

This result differs from the result obtained by Beenakker et al. by a factor of 2 in the $\Gamma \gg T$ limit since they consider the case of short range disorder while we consider the case where the electrons and holes follow a classical trajectory, which prevails when the scattering potential varies slowly on the scale of a wavelength $\xi$. The strong dependence of the amplitude of Andreev reflection on the number of times the trajectory hits the interface, as manifested in Eq. (10), is due to the coherence of the incoming electron and reflected hole. Due to this coherence, the probability of Andreev reflection, $|\Gamma_{ee}(N)|$, approaches 1 for $N \gg \Gamma^{-1}$, and the probability of normal reflection, $|\Gamma_{hh}(N)|$ approaches zero. The phenomenon of reflectionless tunneling is a result of the fact that as the disorder is enhanced, the fraction of large N trajectories grows.

Reflectionless tunneling and density of states. Both reflectionless tunneling and the influence of the barrier...
on the DOS in an NS structure are related to the coherent transport of pairs from the superconductor across the barrier to the normal metal, which is given by the probability of an electron approaching the boundary to be Andreev reflected from it. The probability of Andreev reflection approaches 1 for trajectories with $N \gg \Gamma^{-1}$, so the barrier is not effective if large $N$ trajectories occur with high probability. Therefore, the general criterion for the barrier not to be effective is that trajectories that hit the barrier more than $\Gamma^{-1}$ times before losing the coherence between holes and electrons are dominant in the system. Using random walk theory it can be shown that the length of a trajectory between $N$ consecutive times it hits the barrier is of the order of $\tilde{L}_N \approx N^2 l_n$ \cite{17}, and therefore only when coherent trajectories with lengths larger than $L_\Gamma \equiv l_n/\Gamma^2$ occur with high probability will the barrier not be effective. This criterion can be applied to various cases, where different mechanisms limit the length of coherent trajectories.

In the case of reflectionless tunneling the length of the normal metal, $d$, is what limits the trajectories to lengths of order $d^2/l_n$ and therefore the barrier is not effective when $\Gamma \gg T$. In the case of the DOS in NS structure with semi-infinite normal metal, there is no limit to the length of the trajectories, and therefore, when $\epsilon = 0$ and $H = 0$, the barrier is not effective. However, at finite $\epsilon$ and $H$ the electron-hole coherence is limited. The equation for $r_{he}(2)$ is generalized \cite{3}: $r_{he}(2) = |r_{he}(1 + \exp(i\Delta \phi))/(1 + |r_{he}|^2 \exp(i\Delta \phi))|$. Here $\Delta \phi = 2\pi L/hv_F + 4\pi(HA/\Phi_0)$, the first part is due to the phase accumulated by an electron and a hole traversing a part of a trajectory of length $\tilde{L}$ between the two intersection points with the barrier, and the second part is due to the magnetic flux, $HA$, through the Andreev loop consisting of the part of the trajectory of length $\tilde{L}$ and closed by the superconductor. For electrons at $\epsilon = 0$, $H = 0$, $\Delta \phi = 0$, $r_{he}(2) = 2r_{he}/(1 + |r_{he}|^2)$ and the probability $|r_{he}(2)|^2$ has a maximum. In the same way $|r_{he}(N)|^2$ \cite{3} has a maximum when $\Delta \phi = 0$. When electron-hole coherence breaks ($\Delta \phi \approx 2\pi$) there is no enhancement in the Andreev reflection probability due to constructive interference, and the barrier becomes effective. At finite energies coherence will be maintained up to lengths of order $L_n = hv_F/\epsilon$, and therefore the DOS only at $\epsilon \ll \Gamma^2 h/\tau_n$ will be affected by the barrier ($L_\Gamma \ll L_n$). This is the same condition ($\Gamma \gg l_n/\xi_n$) which was obtained using the Usadel equation.

At $\epsilon = 0$, $H \neq 0$, it can be shown that in a slab of width $W$ a trajectory of length $l_n^2/\Gamma W l_n$ encloses one flux quantum and therefore the condition for the barrier not to be effective is $\Gamma \gg l_n/\xi_H$, again as was obtained using the Usadel equation.

Similar considerations lead to the result that reflectionless tunneling is destroyed at energies of the order of the Thouless energy, $hD_n/d^2$, and magnetic fields of the order of $H = \Phi_0/Wd$.

The criterion of hitting the barrier more than $\Gamma^{-1}$ times before losing the electron-hole coherence can also be applied to ballistic systems where the geometry allows returns to the interface, as an SININ double barrier system \cite{18}, or the system considered by Hekking and Nazarov \cite{19}.

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