Persistent current on a disordered mesoscopic ring with an embedded quantum dot

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We calculate the ensemble averaged persistent current on disordered mesoscopic rings with an embedded quantum dot. We model the quantum dot as a single resonance and use Random Matrix Theory to model the impurities in the ring. Using Efetov’s supersymmetry technique, we develop an analytical expression for the current. We find not only one, but two resonance peaks in the current. This is interpreted as quantum interference phenomenon.

INTRODUCTION

Given the rapid development of the technology in the fabrication and manipulation of semiconductor structures, there has been great interest in the study of different mesoscopic systems, specially involving quantum dots. Many atomic effects as the Kondo and Fano effects can also be observed in these systems and in controllable way. Those work deal mainly with the well known Aharonov-Bohm interferometer, which is basically a ring attached to at least two external leads. Then it is possible to measure, for instance, the interference pattern of the electronic wave function as a function of an external magnetic field, or the dephasing of this very function by the tunneling of the electron through the quantum dot, among many other features.

We are here basically interested in an isolated ring with an embedded quantum dot, and its effect on the persistent current. Some recent works have driven attention to this problem, especially in the Kondo regime. However none of them consider the disorder in the ring, and are solved numerically by means of the slave-boson mean-field theory. We propose a new approach to the problem, and also find some interesting typically quantum features of the current that were not observed before.

SUPERSYMMETRIC APPROACH

Following the IWZ model, we consider the simplest case and divide our system into two boxes: one of the quantum dot (QD) and the other of the ring (see Fig. 1). Only the ring has a disorder given by a random GOE Hamiltonian $H_{\text{GOE}}$ of rank $N$ and second momentum given by

$$H_{\mu\nu}H_{\mu'\nu'} = \frac{\lambda^2}{N} [\delta_{\mu\nu'}\delta_{\nu\mu'} + \delta_{\mu\mu'}\delta_{\nu\nu'}].$$

2$\lambda$ is the radius of the Wigner semicircle. The QD is assumed to have one single level $\epsilon_d$. We include the Aharonov-Bohm (AB) phase in the coupling term between the walls of the QD and the ring. Doing so, the coupling is given by

$$v(\phi) = v_L e^{i\phi} + v_R e^{-i\phi} = v_0 \cos(\phi).$$

Here $v_{L,R}$ are the coupling through the left and right side of the dot, which we can consider essentially the same. The AB phase is $\phi = \pi \Phi/\Phi_0$, with $\Phi_0 = hc/e$. In this way, the Hamiltonian of the system has dimension $(1 + N) \times (1 + N)$ and is given by

$$H = \begin{pmatrix} \epsilon & V \\ V^T & H_{\text{GOE}} \end{pmatrix}$$

$V$ is a $N$ dimensional vector with constant elements $v(\phi)$.

The persistent current in the ring can be written in terms of the eigenenergies $E_n$ of the system as

$$I(\phi) = -\frac{2\pi e}{\Phi_0} \sum_n \frac{\partial E_n}{\partial \phi},$$

with the condition that $\max E_n < E_F$, $E_F$ being the Fermi energy. It is more convenient though to write this
expression using Green functions. In doing so we find
\[
I(\phi) = -\frac{2\pi c}{\Phi_0} \int_0^{E_F} dE \sum_{n=1}^{\infty} \frac{\partial E_n}{\partial \phi} \delta(E' - \varepsilon_n) \\
= -\frac{i c}{\Phi_0} \int_0^{E_F} \text{Tr} \left\{ \frac{\partial H}{\partial \phi} [(E'^+ - H)^{-1} - (E'^- - H)^{-1}] \right\}
\]
(5)

\(E'^+\) and \(E'^-\) carry respectively a positive and a negative infinitesimal imaginary part and the limit is implicit.

By performing the average over the impurities, we can not keep \(E_F\) fixed, since it changes both with impurity realization and the magnetic flux. The correct way to overcome this difficulty was proposed in Ref. [11], and what we have to do is make an average over the number of electrons too. This approach is ideal for treating an ensemble of rings, since due to uncertainties in their shapes, it is clear that the number of electrons is not the same in every ring, but must fluctuate about an average value \(\bar{N}\) with an amplitude \(\delta N\). This is equivalent to say that the Fermi energy is allowed to vary in an interval \([E_1, E_2]\) of length \(S\) such that \(S = d\delta N\), with \(d\) the mean level space at the Fermi surface. Thus we have the following expression for the current:

\[
\mathcal{T}(\phi) = -\frac{c}{2\pi \Phi_0 \delta N} \int_S dE \int_0^{E_F} dE' \left[ \text{Tr} \left\{ \frac{\partial H}{\partial \phi} (E'^+ - H)^{-1} \right\} - \text{Tr} \left\{ (E'^- - H)^{-1} \right\} + \text{c.c.} \right].
\]
(6)

The bar denotes the average over the impurities. We see here that the current is expressed as a product of an advanced and a retarded Green function, which is essentially a two-point function.

It is useful to define a generating functional \(Z\) from which we can obtain the current as simple derivatives, and furthermore, we can write our Hamiltonian in a Gaussian form such the procedure of averaging can be easily performed. We follow here the standard steps in the way presented in Ref. [12]. This generating function must then have the form

\[
Z[j, \delta \phi] = \int d[\Psi] \exp \left\{ \frac{i}{\Phi_0} \int_0^{L} D\Psi L^{1/2} D\Psi^{1/2} \right\}
\]
(7)

where \(\Psi\) is a graded vector of dimension \(8(N+1)\) and has the form:

\[
\Psi^{T}_{\mu} = (S_{1\mu}, S^{*}_{1\mu}, \chi_{1\mu}, \chi^{*}_{1\mu}, S_{2\mu}, S^{*}_{2\mu}, \chi_{2\mu}, \chi^{*}_{2\mu}),
\]
(8)

and with the definitions:

\[
D = (E - \frac{\omega}{2} L - j k_2) \otimes 1_{N+1} + (e \otimes \lambda \otimes \sigma \otimes 1_N)
\]

\[
V = v_0 \cos(\phi + k_1 \delta \phi) \otimes e_1 \otimes \epsilon_1.
\]

Here \(E = (E'^+ + E'^-)/2, \omega = E'^- - E'^+\) and \(j\) and \(\delta \phi\) are source terms. \(L, k_1\) and \(k_2\) are \(8\times8\) diagonal matrix and given by \(L = \text{diag}(1_4, -1_4), k_1 = (1_2, -1_2, 0, 0), k_2 = (0, 0, 1_2, -1_2)\). Finally \(1_m\) represents the identity matrix of dimension \(m\times m\) and \(e_1 \otimes \epsilon_1\) is a constant matrix of dimension \(1 \times N\) and all elements equal 1.

Hence the averaged persistent current is written in terms of the functional as

\[
\mathcal{T}(\phi) = -\frac{c d}{8\pi S \Phi_0} \int_S dE \int_0^{E_F} dE' \left[ \frac{\partial^2 Z}{\partial \delta \phi \partial j} \right]_{\delta \phi = 0, j = 0} + \text{c.c.}
\]
(9)

We may the proceed and perform the ensemble average in a standard fashion followed by the Hubbard–Stratonovitch (HB) transformation. This yields for the generating functional

\[
\mathcal{Z} = \int d[\sigma] \exp\{\mathcal{L}[\sigma]\},
\]
(10)

where the Lagrangian is

\[
\mathcal{L}[\sigma] = -\frac{N}{4} \text{Trg} \sigma^2 - \frac{1}{2} \text{Trg} \ln \mathcal{D}
\]
(11)

and

\[
\mathcal{D} = \left( E - \frac{\omega}{2} L - j k_2 - V \right) \otimes \epsilon_1 \otimes \epsilon_1.
\]
(12)

\(\sigma\) is a \(8 \times 8\) graded matrix which appears as an auxiliary field to do the HB transformation. \(\text{Trg}\) denotes the graded trace, both over the \(8 \times 8\) graded space and also the \(N + 1\) degrees of freedom of the system. It is understood here that the right bottom block of the above matrix has dimension \(8N \times 8N\).

Since we are mostly interested in the limit \(N \rightarrow \infty\), we do the saddle point approximation to determine the main contribution for the exponent with \(\omega = j = \delta \phi = 0\).

It is useful to rewrite the logarithm of \(\mathcal{D}\) as

\[
\ln \mathcal{D} = \ln \left( \frac{\Lambda}{0} \right) + \ln \left( \frac{\Sigma_N^{-1} V^T}{1} \right).
\]
(13)

where \(\Lambda = E - \epsilon - \frac{\omega}{2} L - j k_2\) and \(\Sigma_N = (E - \lambda \sigma - \frac{\omega}{2} L - j k_2) \otimes 1_N\). Expanding the second term of Eq. (13) and taking its trace, we can rewrite it as

\[
\text{Trg} \ln \left( \frac{\Sigma_N^{-1} V^T}{1} \right) = \text{Trg} \ln [1 - \Lambda^{-1} V \Sigma_N^{-1} V^T]
\]
(14)
Now \( \text{trg} \) runs only over the graded space.

We obtain for the saddle point equation

\[
\sigma = \frac{\lambda}{E - \lambda \sigma} \left( 1 + \frac{1}{N} \left( \frac{E - \epsilon}{(E - \lambda \sigma) - N v_0^2 \cos^2(\phi)} \right) \right).
\]

(15)

We note here that \( N v_0^2 \) is of order of unity, and thus the second term in r.h.s. of Eq. (15) may be dropped in the limit \( N \to \infty \). We find then the diagonal solution for the saddle point equation given by

\[
\sigma_D = \frac{E}{2\lambda} - i\Delta L,
\]

(16)

where \( \Delta = \sqrt{1 - (E/2\lambda)^2} \). This is not however the only solution. There is actually a manifold of solutions, which can be parametrized by some group generators \( T \) (see Ref. [13] to see its actual form), and the general solution is

\[
\sigma_{sp} = T^{-1} \sigma_D T \equiv \frac{E}{2\lambda} - i\Delta Q .
\]

(17)

We proceed with separation of \( \sigma \) into the Goldstone modes \( \sigma_G \) and the massive modes \( \delta \sigma \), writing \( \sigma = \sigma_G + N^{-1/2}\delta \sigma \) and expanding the logarithm in function of \( \delta \sigma \). We first note that \( \omega, v_0^2 \sim N^{-1} \), and then make the shift \( \lambda \sigma_G \to \lambda \sigma_G - \frac{N}{2} L - jk_2 \), to find the effective Lagrangian

\[
\mathcal{L}_{el}[\sigma] = -\frac{N}{4} \text{trg}(1 - \sigma_G^2)\delta \sigma^2 + \frac{N\omega}{4\lambda} \text{trg}(\sigma_G L) + \frac{Nj}{2\lambda} \text{trg}(\sigma_G k_2) - \frac{1}{2} \text{trg} \ln \left[ 1 - \frac{N v_0^2}{\lambda(\epsilon - E)} \cos(\phi + k_1 \delta \phi) \sigma_G \cos(\phi + k_1 \delta \phi) \right].
\]

(18)

We have kept in the expansion only terms in first order in \( \omega \) and \( j \) and up to second order in \( \delta \sigma \). The linear terms in \( \delta \sigma \) coming from the logarithm cancel against those from \( \sigma^2 \). The integral over the massive modes is Gaussian and straightforward. Taking then the derivatives of the generating functional with respect to the sources \( j \) and \( \delta \phi \) yields

\[
\frac{\partial^2 Z}{\partial j \partial \delta \phi} |_{j=0=\delta \phi} = -\frac{N^2 v_0^2}{8\lambda^2} \sin(2\phi) \int d[\sigma_G] \exp \{ L_0 \}
\times \text{trg}[\sigma_G k_2] \text{trg} \left\{ \frac{k_1 \sigma_G + \sigma_G k_1}{(\epsilon - E) - (N v_0^2 \cos^2(\phi)/\lambda) \sigma_G} \right\} .
\]

(19)

where \( L_0 = (N\omega/4\lambda) \text{trg}[\sigma_G L] \).

Recalling Eq. (17), the second trace of Eq. (19) can be put into a more convenient manner as

\[
\text{trg} \left\{ \frac{k_1 \sigma_G + \sigma_G k_1}{E - \epsilon - \Gamma(\phi) \sigma_G} \right\} =
\frac{2\Delta^2 (E - \epsilon) \text{trg}(Qk_1)}{[E - \epsilon - (E/2\lambda)\Gamma(\phi)]^2 + (\Gamma(\phi)\Delta)^2}.
\]

(20)

Here, \( \Gamma(\phi) = N v_0^2 \cos^2(\phi)/\lambda \). The averaged persistent current is then

\[
I(\phi) = -\frac{cd}{4\pi\Phi_0(E_2 - E_1)} \frac{N^2 v_0^2 \Delta^2}{4\lambda^2} \sin(2\phi) \int d\omega \Re[f(\omega)]
\times \int dE \frac{(E - \epsilon)}{[E - \epsilon - (E/2\lambda)\Gamma(\phi)]^2 + (\Gamma(\phi)\Delta)^2}.
\]

(21)

**SUPERSYMMETRIC INTEGRATION**

The integral \( f(\omega) \) is done exactly with the parametrization for \( Q \) given in Ref. [13] and following the same steps. This is a lengthy and monotonous task, where we have to pay special attention to the anticommutative variables. At the end of the day, we come to the final result for the
We have used that the mean level spacing is given by
\[ \frac{1}{\text{meV}} \]
where \( d = 10^{-3} \), \( e_d = 1.0 \), all in units of \( E_F \). The energy laying in the meV. The electron effective mass is \( \frac{1}{2} \), double of the value found in the case without the QD. First, we should remember that the persistent current in a single isolated ring has a periodicity with the magnetic flux of \( \Phi_0 \) both for the case of an odd or even number of electrons. What differs the current in each case is just a shift of \( \Phi_0 / 2 \). Thus, due to the very symmetry of those plots, when we perform an average over the number of electrons, i.e., we sum the currents in both cases, we end up with a current with a periodicity of \( \Phi_0 / 2 \). This is valid for a ring with or without disorder. The conclusion we can derive here is that the very presence of the QD breaks down the symmetry of the current for an even or odd number of electrons. Such asymmetry was also observed for a clean ring in the Kondo regime, but now we can see that this feature is even more general.

Another remark to be done is about the coupling constant. For small values of this quantity (\( \Gamma_0 \leq 10^{-3} E_F \)), the current is basically a sine and it scales with \( \Gamma_0 \). In the first plot we observe that the periodicity of the current is \( \Phi_0 \), double of the value found in the case without the QD. First, we should remember that the persistent current in a single isolated ring has a periodicity with the magnetic flux of \( \Phi_0 \) both for the case of an odd or even number of electrons. What differs the current in each case is just a shift of \( \Phi_0 / 2 \). Thus, due to the very symmetry of those plots, when we perform an average over the number of electrons, i.e., we sum the currents in both cases, we end up with a current with a periodicity of \( \Phi_0 / 2 \). This is valid for a ring with or without disorder. The conclusion we can derive here is that the very presence of the QD breaks down the symmetry of the current for an even or odd number of electrons. Such asymmetry was also observed for a clean ring in the Kondo regime, but now we can see that this feature is even more general.

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where we have analyzed the variation of the current amplitude as function of the energy of QD level. We have observed a peak in the current for $\epsilon_d$ near the Fermi energy of the ring. Such result is expected if we note that, in the case without the QD, the main contribution for the total current comes from the current associated with the electron in the uppermost level. Thus, when the energy of the QD coincides with this energy, there is a peak in the current indicating the resonance between both levels. The second peak however is quite curious. It occurs at about half of the Fermi energy and is symmetric to the first, only with opposite signal. The explanation for this phenomenon is the following. The coupling between the energy levels in the ring and that of the QD gives then a width and a Lorentzian shape. Since the current is essentially a derivative of the energy with respect to the magnetic flux, we get that the individual level contribution for the total current is proportional to the difference $\epsilon - \epsilon_d$. Thus, when the QD level crosses down the Fermi energy, the contribution of the levels with higher energies than $\epsilon_d$ have opposite signal. For a given value of $\epsilon_d$, the total current vanish. Lowering $\epsilon_d$ even more, the current changes signal and increases until it reaches a maximum value and then decreases again as $\epsilon_d$ is much lower than the energy levels of the ring.

![FIG. 4: Current as a function of QD energy $\epsilon_d$ in units of $eE_F/\Phi_0$. The parameters are $E_1 = 0.99\ e\ E_2 = 1.01, d = 10^{-3}, \Gamma_0 = 0.1, \Phi = 0.3\Phi_0$.](image)

Unlike the results obtained in other works, this extra peak is neither related to the Fano effect nor to the Kondo effect. The Fano effect occurs when there is an interference between two paths through which the electron can pass, one with discrete levels, and the other a continuous band. To observe such effect, one may use one ring with a QD and two leads connected in opposite sides of the ring. Measuring the conductance we find peaks with asymmetric line shapes. On the other hand, Kondo effect is related to the fact that the QD behaves like a magnetic impurity in the ring, and to observe it, it is necessary that the net spin in the dot be different of zero. As a result of the correlations between the electrons in the QD and those of the conduction band, there is a resonance near the Fermi energy with works as an extra channel for the electron tunneling. Our model does not account for the electronic spin and therefore there is no way of interpreting our result as a Kondo resonance, furthermore the second peak lays far below the Fermi level. In conclusion, the second peak arises as a constructive interference phenomenon between the levels of the ring and is purely a manifestation of the quantum nature of the system.

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