Particle localization and the Notion of Einstein Causality *

Gerhard C. Hegerfeldt
Institut für Theoretische Physik, Universität Göttingen
Bunsenstr. 9, 37073 Göttingen, Germany

The notion of Einstein causality, i.e. the limiting role of the velocity of light in the transmission of signals, is discussed. It is pointed out that Nimtz and coworkers use the notion of signal velocity in a different sense from Einstein and that their experimental results are in full agreement with Einstein causality in its ordinary sense. We also show that under quite general assumptions instantaneous spreading of particle localization occurs in quantum theory, relativistic or not, with fields or without. We discuss if this affects Einstein causality.

Keywords: superluminal, signal, localization

1. Introduction

The notion of ‘Einstein causality’ refers to the limiting role of the velocity of light in the transmission of signals. Einstein’s principle of finite signal velocity is of fundamental importance for the foundations of physics, both in classical as well as in quantum physics. If signal velocities could be arbitrarily high this would either lead to the possibility of absolute clock synchronization and to a change of special relativity or to the possible existence of superluminal tachyons with their associated acausal effects [1]. Hence the name Einstein causality.

To be more precise, in this context a signal means the experimental creation of any sort of “disturbance” at some space point or small space region and the influence of this on a measuring device further away. For example, one could produce an electromagnetic pulse and then measure the field strength at some other point. The start time of the signal is the time when the experiment is set into motion, i.e., when the button is pressed. The arrival time of the signal is the first instance a measuring device can or does respond. The limiting role of the light velocity means that the corresponding time difference divided by the distance cannot exceed c.

Nimtz and coworkers [2] have reported superluminal signal velocities in tunneling experiments with microwaves. These experiments and their interpretation, advocated for example in the article of Nimtz et al. appearing in this issue, has given rise to considerable controversy [3]. It will be shown further below that the controversy is easily resolved by a careful analysis of the notions used by different authors. Nimtz and coworkers employ a definition of signal velocity which is different from the one Einstein had in mind. Using

---

*This article appeared in: Extensions of Quantum Theory, Eds. A. Horzela and E. Kapuscik, published by Apeiron, Montreal, 2001, p. 9-16.
the old definition it will be seen that the experimental results of Nimtz and coworkers, sophisticated as they are, do not contradict Einstein causality in the original sense but, rather, are in full agreement with it. Thus a conceptual confusion lies at the heart of the matter which explains a lot of the controversy.

Are there superluminal phenomena in the quantum realm? For a free nonrelativistic particle instantaneous spreading of the wave function is well known. If, at time $t = 0$, the wave function vanishes outside some finite region $V$ then the particle is localized in $V$ with probability 1. Instantaneous spreading implies that the probability of finding the particle arbitrarily far away from the initial region is nonzero for any $t > 0$. In a nonrelativistic theory, however, this superluminal propagation is of no great concern.

If the localization of a free relativistic particle is described by the Newton-Wigner position operator then instantaneous spreading also occurs, as noted in Refs. [4] and [5] (cf. also Ref. [6]). This also happens for a proposed photon position operator [7]. In 1974 the present author [8] showed that this phenomenon of instantaneous spreading is quite general for a free relativistic particle, irrespective of the particular notion of localization, be it in the sense of Newton-Wigner or others. Later an alternative proof of this result was given [9] and the result was extended to the center-of-mass motion of relativistic systems with possibly more than one particle [10]. Ruijsenaars and the author [11] then showed that instantaneous spreading occurs for quite general, relativistic or nonrelativistic, interactions. The main result of Ref. [11] was that this instantaneous spreading is mainly due to positivity of the energy plus translation invariance. More recently it was shown by the author [12] that translation invariance is also not needed. Hilbert space and positivity of the Hamiltonian (energy) suffices to ensure either instantaneous spreading or, alternatively, confinement in a fixed region for all times. Another extension was given by the author [13] for free relativistic particles and for relativistic systems which have exponentially bounded tails in their localization outside some region $V$. It was shown that the state spreads out to infinity faster than allowed by a probability flow with finite propagation speed. Probably the most astonishing part of our results is the fact that so little is needed to derive them. They hold with and without field theory and with and without relativity. Only Hilbert space and positivity of the energy is needed.

What do these results mean for Einstein causality? This will be discussed in the following where we concentrate on the role played by positivity of the energy for instantaneous spreading. We also briefly discuss Fermi’s two-atom model [14, 15]. But first we turn to the Nimtz controversy.

2. Resolution of the Nimtz controversy

Nimtz et al. [2] define in Section 2.2 of their paper in this issue what they mean by signal velocity and arrival time. Their definition is motivated by usage in modern engineering. In particular, their notion of arrival time is connected to the read-out time of the signal. However, Einstein had a different meaning in mind when he formulated his principle of the limiting role of the velocity of light for signal velocities, and this has been explained in the Introduction. Definitions are of course neither right nor wrong, but clearly the meaning of a statement as well as its truth depend on the definition of the notions employed in the formulation of the statement. So what do the Nimtz experiments have to say on the question of Einstein causality in its original sense? Are they compatible with it?

In these experiments, typically, a rapid sequence of microwave pulses is generated. Each pulse is split into two and sent over different paths of the same length to a receiver.
Calibration of the path length is achieved by displaying the two pulse sequences stroboscopically as still pictures on an oscillograph. Then a photonic tunnel barrier is inserted into one of the paths which attenuates the corresponding pulses and reshapes them. To compare tunneled and non-tunneled pulses the former are re-amplified to their original amplitude height at the receiving end and again displayed stroboscopically on the oscillograph. The effect is dramatic. Upon insertion of the tunnel barrier the still picture of the tunneled pulses makes a jump to earlier times, seemingly indicating that they are arriving earlier than the non-tunneled pulses. With the definition of signal velocity and arrival time used by Nimtz and coworkers this is indeed true.

To see, however, whether this has anything to do with superluminal signal velocities in the Einstein sense it is an eye opener to look at the tunneled pulses without amplification. Experimentally it has been verified by Nimtz and coworkers that the amplitudes of the tunneled pulses are always below the amplitude of the non-tunneled pulses [22]. In these experiments, the maxima as well as the half widths of the tunneled pulses are ahead of those of the non-tunneled pulses and therefore arrive earlier. This is graphically depicted in Fig. 1 by the pulses traveling from left to right. The figure is not to scale and and does not represent experimental curves, but is just for illustration.

For the signal velocity in the Einstein sense, however, the arrival time of the pulse maximum or the read-out time of the half width is not relevant since they are not used for clock synchronization. Relevant, rather, is the first possible response time of the measuring device, as explained in the Introduction. Now, since experimentally the tunneled pulses are always below the non-tunneled pulses in amplitude, any measuring device will respond first to the non-tunneled pulses and then to the tunneled ones, or at most simultaneously to both. Thus the limiting role of the speed of light as signal velocity in the sense of Einstein is not violated in the experiments.
What then is superluminal here? Let us consider the group and the phase velocity of light. Both are mathematical constructs useful for the description of electromagnetic phenomena. It is well known that both can be larger than $c$ \cite{16}, but this cannot be used for superluminal signals in the Einstein sense. Similarly, it has been shown in Ref. \cite{17} that in a somewhat idealized situation the tunneling pulse can be fully described within Maxwell theory by means of another mathematically introduced auxiliary phase-time velocity notion. Again, this auxiliary velocity cannot be used for superluminal signal transmission in the Einstein sense.

So it seems that the controversy about the interpretation of Nimtz’s experiments arises from an indiscriminate use of terminology. Terms like signal velocity and arrival time are used by Nimtz and coworkers in a sense different from that of Einstein. Using the notions in the original sense the experiments are fully compatible with Einstein causality as ordinarily understood.

3. Fermi’s two-atom model

To check the speed of light in quantum electrodynamics, Fermi \cite{14} considered two atoms, separated by a distance $R$ and with no photons initially present. One of the atoms was assumed to be in its ground state, the other in an excited state. The latter could then decay with the emission of a photon. Fermi calculated the excitation probability of the atom which had initially been in its ground state. Using standard approximations he found the excitation probability to be zero for $t < R/c$. In Ref. \cite{15} the following mathematical result was proved and applied to the Fermi problem.

**Theorem:** Let $H$ be a self-adjoint operator, positive or bounded from below, in a Hilbert space $\mathcal{H}$. For given $\psi_0 \in \mathcal{H}$ let $\psi_t, t \in \mathbb{R}$, be defined as

$$\psi_t = e^{-iHt}\psi_0 \ .$$

Let $A$ be a positive operator in $\mathcal{H}, A \geq 0$, and let $p_A(t)$ be defined as

$$p_A(t) = \langle \psi_t, A\psi_t \rangle \ .$$

Then either

$$p_A(t) \neq 0 \quad \text{for almost all } \ t$$

and the set of such $t$’s is dense and open, or

$$p_A(t) = 0 \quad \text{for all } \ t \ .$$

For the proof, which is based on an analyticity argument, both the positivity of $H$ and of $A$ are needed. Positivity means that all expectation values of the operator are nonnegative. Positivity of $H$ alone is not enough. If $A$ is not positive the theorem does not hold. In Eq. (3) one can replace $p_A(t)$ by

$$p_A(t) = \text{tr}Ae^{-iHt}\rho e^{iHt}$$

where $\rho$ is a positive trace-class operator.

If one takes for $\psi_0$ in the theorem the initial state considered by Fermi and for $A$ the operator describing the excitation probability, e.g. the projector onto the excited states,
then \( p_A(t) \) becomes the excitation probability, and the theorem states that this probability is immediately nonzero. Already in \([15]\) it was discussed how to avoid a possible conflict with causality, and this was continued in more detail for example in \([18, 19, 20, 21]\). The conclusion was that the immediate excitation could be understood in a field-theoretic context through vacuum fluctuations due to virtual photons. The part of the excitation due to the second atom behaves causally \([20, 21]\). Causality then holds for expectation values after the spontaneous part has been subtracted. This corresponds to the notion of weak causality, i.e. for expectation values, introduced in \([6]\), which contrasts to the notion of strong causality, i.e. causality for individual events, as discussed in \([18]\). Fermi seems to have had strong causality in mind.

4. Particle localization and spreading

Let us suppose that it makes sense to speak of the probability to find a particle at a given time inside a space region \( V \). This is a highly nontrivial assumption. In a quantum theory the probability to find a particle or system inside \( V \) should be given by the expectation of an operator, \( N(V) \) say. Since probabilities lie between 0 and 1, one must have

\[
0 \leq N(V) \leq 1. \tag{5}
\]

Now let us assume that the system, with state \( \psi_0 \) at \( t = 0 \), is strictly localized in a region \( V_0 \), i.e. with probability 1, so that \( \langle \psi_0, N(V_0)\psi_0 \rangle = 1 \) or, equivalently,

\[
\langle \psi_0, (1 - N(V_0))\psi_0 \rangle = 0. \tag{6}
\]

From Eq. (5) one has \( 1 - N(V_0) \geq 0 \) and hence the theorem can be applied, with

\[
A \equiv 1 - N(V_0). \tag{7}
\]

As a consequence one either has

\[
\langle \psi_t, N(V_0)\psi_t \rangle \equiv 1 \quad \text{for all} \quad t \tag{8}
\]

or

\[
\langle \psi_t, N(V_0)\psi_t \rangle < 1 \quad \text{for almost all} \quad t. \tag{9}
\]

The alternative in Eq. (8) means that the particle or system stays in \( V_0 \) for all times, as might happen for a bound state in an external potential.

Now, if the particle or system is strictly localized in \( V_0 \) at \( t = 0 \) it is, \textit{a fortiori}, also strictly localized in any larger region \( V \) containing \( V_0 \). If the boundaries of \( V \) and \( V_0 \) have a finite distance and if finite propagation speed would hold then the probability to find the system in \( V \) would also have to be 1 for sufficiently small times, e.g. \( 0 \leq t < \epsilon \). But then the theorem, with \( A \equiv 1 - N(V) \), states that the system stays in \( V \) for all times. Now we can make \( V \) smaller and smaller and let it approach \( V_0 \). Thus we conclude that if a particle or system is strictly localized in a region \( V_0 \) at time \( t = 0 \), then finite propagation speed implies that it stays in \( V_0 \) for all times and therefore prohibits motion to infinity. Or put conversely, if there exist particle states which are strictly localized in some finite region at \( t = 0 \) and later move towards infinity, then finite propagation speed cannot hold for localization of particles.
This can be formulated somewhat more strongly as follows. If at \( t = 0 \) a particle is strictly localized in a bounded region \( V_0 \) then, unless it remains in \( V_0 \) for all times, it cannot be strictly localized in a bounded region \( V \), however large, for any finite time interval thereafter, and the particle localization immediately develops infinite "tails". The spreading is over all space except possibly for "holes" which, if any, will persist for all times, by the same arguments as before. If the theory is translation invariant then there can be no holes, as shown in Ref. [11] under some mild spectrum conditions.

5. **Counterexample Dirac equation?**

At first sight the Dirac equation might seem to be a counterexample to our results on instantaneous spreading. Indeed, this wave equation is hyperbolic, implying finite propagation speed. For the localization operator \( N(V) \) one might take the characteristic function \( \chi_V(x) \), just as in the nonrelativistic case and in contrast to the Newton-Wigner operator. Then, for a wave function initially vanishing outside a finite region, i.e. of finite support, the localization does evolve with finite propagation speed! Doesn’t this contradict the results of the preceding section?

This example is instructive since it shows the importance of the positive-energy condition. The Dirac equation contains positive and negative energy states. Now, consider a solution of the Dirac equation which vanishes outside some finite region and make the additional assumption that it is composed of positive-energy solutions only. Then one gets a contradiction to our results and therefore the additional assumption must be wrong, i.e. a solution with finite support at some time must contain negative-energy contributions. This means that positive-energy solutions of the Dirac equation always have infinite support to begin with! This is phrased as a mathematical result for instance in the book of Thaller [23].

Thus the results of the preceding section do not apply if there are no strictly localized states in the theory! Strict localization of a state \( \psi \) in a region \( V \) means that \( \langle \psi, N(V)\psi \rangle = 1 \), and this gives

\[
0 = \langle \psi, (1 - N(V))\psi \rangle = \| (1 - N(V))^{1/2} \psi \|^2
\]

where the root exists by positivity of \( N(V) \). This implies

\[
N(V)\psi = \psi.
\]

Hence \( \psi \) is an eigenvector of \( N(V) \) for the eigenvalue 1 if \( \psi \) is strictly localized in \( V \), and vice versa. The eigenvalue 0 means strict localization outside \( V \).

The existence or nonexistence of strictly localized states depends on the form of \( N(V) \). For example, if one has a self-adjoint position operator \( \hat{X} \) with commuting components, then \( N(V) \) is a projection operator from the spectral decomposition of \( \hat{X} \) and thus has eigenvalues 1 and 0. Hence in this case there are strictly localized states for any region \( V \), and the result of the previous section implies instantaneous spreading.

This instantaneous spreading also occurs for position operators with self-adjoint but non-commuting components \( \hat{X}_i \). Each \( \hat{X}_i \) has a spectral decomposition whose projection operators give the localization operators for infinite slabs. Eigenvectors for the eigenvalue 1 represent states strictly localized in these slabs, and there is instantaneous spreading in this case, too.
To avoid instantaneous spreading one therefore has to consider localization operators \( N(V) \) which are not projectors, for example positive operator-valued measures. However, if one insists on arbitrary good localization, i.e. on tails which drop off arbitrarily fast, then one runs into our results in Ref. [13].

Discussion. Could instantaneous spreading be used for the transmission of signals if it occurred in the framework of relativistic one-particle quantum mechanics? Let us suppose that at time \( t = 0 \) one could prepare an ensemble of strictly localized (non-interacting) particles by laboratory means, e.g. photons in an oven. Then one could open a window and would observe some of them at time \( t = \varepsilon \) later on the moon. Or to better proceed by repetition, suppose one could successively prepare strictly localized individual particles in the laboratory. Preferably this should be done with different, distinguishable, particles in order to be sure when a detected particle was originally released. Such a signaling procedure would have very low efficiency but still could be used for synchronization of clocks or, for instance, for betting purposes.

Field-theoretic aspects of our results have been discussed in detail in Ref. [24]. Permanent infinite tails in field theory can be understood intuitively through clouds of virtual particles due to renormalization (‘dressed states’). Also, counters could be influenced by vacuum fluctuations.

References

[1] Cf., e.g., W. Rindler, *Introduction to Special Relativity*, 2nd Edition, Clarendon, Oxford 1991, p. 17 f. But see also O. M. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. 30, 718 (1962).

[2] G. Nimtz, A. Haibel, A. A. Stahlhofen and R.-M. Vetter, APEIRON, this issue. See also H. Aichmann, A. Haibel, W. Lennartz, G. Nimtz, and A. Spanoudaki, Proc. of the International Symposium on Quantum Theory and Symmetries, Goslar, 18-22 July 1999 (ISBN 9-8102-4237-9), 605-611 (2000); G. Nimtz, Ann. Phys. (Leipzig) 7, 618 (1998); A. Enders and G. Nimtz, Phys. Rev. E 48, 632 (1994), and references therein. For related experiments see R. Y. Ciao and A. M. Steinberg, *Progress in Optics* vol. XXXVII, edited by E. Wolf, Elsevier, Amsterdam (1997), p. 345; L. J. Wang, A. Kuzmich, and A. Dogariu, Nature 406, 277 (2000).

[3] Cf. e.g. H. Goenner, Ann. Phys. (Leipzig) 7, 774 (1998)

[4] W. Weidlich and K. Mitra, Nuovo Cim. 30, 385 (1963)

[5] G. N. Fleming, Phys. Rev. 139, B963 (1965)

[6] S. Schlieder, in: *Quanten und Felder*, H.P. Dür Ed., Vieweg, Braunschweig (1971), p. 145

[7] W. O. Amrein, Helv. Phys. Acta 42, 149 (1969)

[8] G. C. Hegerfeldt, Phys. Rev. D 10, 3320 (1974)

[9] B. Skagerstam, Int. J. Theor. Phys. 15, 213 (1976)

[10] J. F. Perez and I. F. Wilde, Phys. Rev. D 16, 315 (1977)

[11] G. C. Hegerfeldt and S. N. M. Ruijsenaars, Phys. Rev. D 22, 337 (1980)
[12] G. C. Hegerfeldt, in: *Irreversibility and Causality in Quantum Theory*, edited by A. Bohm, H.-D. Doebner and P. Kielanowski, Lecture Notes in Physics **504**, p. 238, Springer, Berlin (1998)

[13] G. C. Hegerfeldt, Phys. Rev. Lett. **54**, 2395 (1985)

[14] E. Fermi, Rev. Mod. Phys. **4**, 87 (1932); M. I. Shirokov, Yad. Fiz. **4**, 1077 (1966) [Sov. J. Nucl. Phys. **4**, 774 (1967)]; W. Heitler and S. T. Ma, Proc. R. Ir. Acs. **52**, 123 (1949); J. Hamilton, Proc. Phys. Soc. A **62**, 12 (1949); M. Fierz, Helv. Phys. Acta **23**, 731 (1950); B. Ferretti, In: *Old and new Problems in Elementary Particles*, edited by G. Puppi, Academic Press, New York (1968), p. 108; P. W. Milonni and P. L. Knight, Phys. Rev. A **10**, 1096 (1974); M. I. Shirokov, Sov. Phys. Usp. **21**, 345 (1978); M. H. Rubin, Phys. Rev. D **35**, 3836 (1987); A. K. Biswas, G. Compagno, G. M. Palma, R. Passante, and R. Persico, Phys. Rev. A **42**, 4291 (1990); A. Valentini, Phys. Lett. A **153**, 321 (1991)

[15] G. C. Hegerfeldt, Phys. Rev. Lett. **72**, 596 (1994)

[16] L. Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York (1960)

[17] Th. Emig, Diplomarbeit, Universität Köln (1995); Phys. Rev. E **54**, 5780 (1996)

[18] G. C. Hegerfeldt, in: *Nonlinear, deformed and irreversible quantum systems*, edited by H.-D. Doebner, V.K. Dobrev, and P. Nattermann, World Scientific (1995)

[19] D. Buchholz and J. Yngvason, Phys. Rev. Lett. **73**, 613 (1994)

[20] A. Labarbara and R. Passante, Phys. Lett. **206**, 1 (1994)

[21] P. W. Milonni, D. F. V. James, and H. Fearn, Phys. Rev. A **52**, 1525 (1995)

[22] H. Aichmann, private communication, June 1998

[23] B. Thaller, *The Dirac Equation*, Springer, Berlin (1992)

[24] G. C. Hegerfeldt, Ann. Phys. (Leipzig) **7**, 716 (1998)