Neutrino Interactions In
Color-Flavor-Locked Dense Matter

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Abstract

At high density, diquarks could condense in the vacuum with the QCD color spontaneously broken. Based on the observation that the symmetry breaking pattern involved in this phenomenon is essentially the same as that of the Pati-Salam model with broken electroweak–color \( SU(3) \) group, we determine the relevant electroweak interactions in the color-flavor locked (CFL) phase in high density QCD. We briefly comment on the possible implications on the cooling of neutron stars.

I. INTRODUCTION

Recent developments on high-density QCD \cite{1} suggest that diquarks condense in super-dense hadronic matter giving rise to a color-flavor locked state \cite{2}. Among the excitations on this broken phase vacuum are massive color gluons metamorphosed to vector mesons and integrally charged quarks behaving as baryons \cite{3,4}. Although such a state may not be

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formed in heavy-ion collisions, it may be relevant in the physics of compact stars such as the cooling of neutron stars.

The ultimate goal of studying the state at high density and at low temperature is to explore the astrophysical implication of this broken phase: What is particularly interesting is the role of neutrinos in the cooling of neutron stars. In preparation for such a study, we consider how matter in the color-flavor locked state responds to electroweak (EW) probes. In this talk, we exploit the observation that the symmetry breaking pattern in the color-flavor locked (CFL) state is basically the same as that of the broken color gauge theory, originally proposed by Pati and Salam [5] for a grand unification scheme, to study how neutrinos (or generally weak current) interact in the CFL matter.

II. GAUGE THEORETICAL MODEL FOR THE CFL PHASE

In the minimal Pati-Salam model, $SU(3)_{\text{color}}$ gauge group as well as the flavor group, $SU(2)_L \times U(1)_R$, are spontaneously broken. One of the features of this model is the integrally charged quarks (i.e., Han-Nambu quark model [6]) and the charged massive gluons, both of which are analogous to the excitations in the color-flavor locked QCD phase. The gauge bosons get mixed to form mass eigenstates: for example the neutral gauge bosons, photon $A$, weak boson $Z_0$, and a combination of gluon $\tilde{V}_0$ are a mixture of the original gauge bosons, $B$ of $U(1)_R$, $W_0$ of $SU(2)_L$ and a combination, $V_3 + V_8/\sqrt{3}$, of $SU(3)_{\text{color}}$. Consequently it is very natural to expect weak interactions from the colored objects. An interesting feature of this model is that not only photon [7] but also neutrinos (in general, weak current) can probe the color. Despite its intrinsic elegance, this model has not been considered as a relevant model for Nature since no experiments performed at low density have shown any evidence for broken $SU(3)_{\text{color}}$ gauge symmetry. However recent theoretical developments indicate the possibility of spontaneous symmetry breaking of $SU(3)_{\text{color}}$ in the hadronic matter at very high density and more intriguingly, the symmetry breaking pattern – when suitably reinterpreted – is identical to that of the minimal Pati-Salam model. This suggests that in the color-flavor locked phase, the Pati-Salam model could be exploited to infer the structure of electroweak interactions involving the colored objects. This does not mean that this is the only way of deriving the interaction forms. As discussed in [8], one can do a systematic weak-coupling calculations valid at high density taking into account the color-flavor locking and consequent non-perturbative effects. The point of this paper is that the symmetry breaking pattern shares its generic characteristic with other schemes already available in the literature for different reasons that allows a simple understanding of the electroweak couplings.

For our purpose we shall choose the gauge group to be $SU(2)_L \times U(1)_R \times SU(3)_{\text{color}}$ as in the minimal version of Pati-Salam model. One can easily extend it to $SU(3)_L$ or $SU(4)_L$ [5]. The basic structure remains the same. \footnote{Operationally the symmetry scheme can be extended to incorporate the left-right global symmetry of nature:}

\begin{equation}
(SU(2)_L \times U(1)_R \times SU(3)_{\text{color}})_{\text{local}} \times (SU(3)_L \times SU(3)_R)_{\text{global}}
\end{equation}

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which is responsible for the electro-weak symmetry breaking transforming as \(\{2\}, \{1\}\) for \(SU(2)_L, SU(3)_c\), there are color-flavored Higgs, \(\sigma\), which transform as \(\{2\} + \{2\}, \{3^*\}\) with nonzero vacuum expectation values for the color flavor diagonal elements:

\[
\langle \sigma_{aj} \rangle = \sigma \quad \text{for} \quad a = j \quad (a = 1, 2, 3, 4, \quad j = 1, 2, 3)
\]

\[
= 0 \quad \text{otherwise}
\]

where \(a = 1, 2\) and \(3, 4\) are the doublets of \(SU(2)_L\) and \(j\) is the color index. In fact, \(a\) are identical to the indices for global flavor symmetry of the strong interaction when Cabibbo mixing is understood. Hence the color-flavored Higgs transforms as a fundamental representation under the global flavor transformation. The charm quark with the flavor index \(a = 4\) is too heavy for consideration at high density, so we will be primarily concerned with \(SU(3)_f\). The flavor index \(a = 4\) will not figure in our discussion. We note that it is analogous to the color-flavor locked diquark condensate:

\[
\langle q^{a}_{L\alpha}(\vec{p})q^{b}_{L\beta}(\vec{-p}) \rangle = - \langle q^{a}_{R\alpha}(\vec{p})q^{b}_{R\beta}(\vec{-p}) \rangle = \kappa(p_F)\epsilon_{\alpha\beta\epsilon_{0I}^{ab}I},
\]

where \(\alpha, \beta = 1, 2\) are Weyl indices, \(a, b = 1, 2, 3\) flavor indices, and \(i, j = 1, 2, 3\) color indices. (We will specify the CFL phenomenon more precisely later.) For now we focus on the essential point. The diquark transforms under color as \(\{3^*\}\) or \(\{6\}\). One can see that \(\{3^*\}\) is equivalent to the color-flavored Higgs \(\sigma\). This observation leads us to propose that the symmetry breaking pattern in the CFL phase can be directly mapped to that in the Pati-Salam model. Of course we are not implying that the Pati-Salam model is effective in the zero-density regime.

Let us imagine that both electroweak and color-flavor locking symmetry breakings have taken place, with \(v\) representing the VEV for the former and \(\sigma\) the VEV for the latter. Denote the electroweak mixing angle (or Weinberg angle) by \(\theta_W\) and the QCD mixing angles by \(\beta\) and \(\delta\). In the standard procedure, the gauge covariant couplings of the gauge bosons to the Higgs scalars give rise to the mass terms for the gauge bosons. After diagonalizing the mass terms, we have a massless photon given by

\[
\bar{A} = \cos \beta (\sin \theta_W W_3 + \cos \theta_W B_0) + \sin \beta V_0
\]

where

\[
\tan \beta = \frac{2}{\sqrt{3}} \frac{g}{g_s} \sin \theta_W
\]

\[
\tan \theta_W = \frac{g'}{g}
\]

\[
\rightarrow (U(1)_{em})_{local} \times (SU(3)_{c+L+R})_{global}.
\]
and

\[ \tilde{Z} = Z - \frac{4}{\sqrt{3}} g_s g \frac{\sigma^2}{v^2} \cos \theta_W V_0 + \mathcal{O}(\sigma^2/v^2), \quad (9) \]

\[ \tilde{V}_0 = -\sin \beta (\sin \theta_W W_3 + \cos \theta_W B_0) + \cos \beta V_0 + \mathcal{O}(\sigma^2/v^2), \quad (10) \]

where

\[ Z = \cos \theta_W W_3 - \sin \theta_W B_0, \quad (11) \]

\[ V_0 = \sqrt{\frac{3}{4}} (V_3 + \frac{1}{\sqrt{3}} V_8). \quad (12) \]

Here \( g_s, g \) and \( g' \) are respectively the gauge coupling constants for \( SU(3)_c \), \( SU(2)_L \) and \( U(1) \). Now \( \tilde{W}^\pm \) and \( \tilde{V}^\pm \) are mixed states of \( W^\pm \) and \( V^\rho \pm \):

\[ \tilde{W}^\pm = \cos \delta W^\pm - \sin \delta V^\rho^\pm, \quad (13) \]

\[ \tilde{V}^\pm = \sin \delta W^\pm + \cos \delta V^\rho^\pm. \quad (14) \]

where

\[ \tan \delta = \frac{g}{g_s} \left( \frac{M_V}{M_W} \right)^2. \quad (15) \]

The other four colored gauge bosons remain unmixed:

\[ V_{K^*}, V_{K^{*0}}, \quad (16) \]

\[ V_{K^0}^0, V_{K^{*0}}^0, \quad (17) \]

The subscripts \( \rho, K^* \) etc. represent the Higgsed gluons with the corresponding quantum numbers \(^3\). The masses of the gauge bosons are given by

\[ M_V \sim g_s \sigma/\sqrt{2}, \quad (18) \]

\[ M_W \sim g v/\sqrt{2}. \quad (19) \]

Taking \( M_V \sim \) few 100 MeV, we expect that

\[ \frac{M_V}{M_W} \sim 10^{-3}. \quad (20) \]

We also expect that

\[ \frac{g_s}{g} \sim \frac{g_s}{g'} \sim 10. \]

The unit of electromagnetic charge is defined as \[ ]

\[ e^2 = \frac{g_s^2 g^2 \sin \theta_W}{g_s^2 + 4 g^2 \sin^2 \theta_W/3}. \quad (21) \]

\(^4\)We use \( \hat{e} \) to distinguish it from the zero-density charge \( e \).
The left-handed quarks and leptons are classified as \( \{2\}, \{3^*\} \) and \( \{2\}, \{1\} \) respectively. The right-handed quarks are classified as \( \{1\}, \{3^*\} \). Using eqs.(6) and (21), the electromagnetic charge can be obtained by

\[
\bar{Q}_{\text{em}} = Q_{f(\text{flavor})} + Q_{c(\text{color})} \\
Q_f = (I_3 + Y/2)_f, \\
Q_c = (I_3 + Y/2)_c
\]

which gives integer charges of the Han-Nambu type to the quarks:

\[
\begin{pmatrix}
\bar{Q}_u \\
\bar{Q}_d \\
\bar{Q}_s
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 1 \\
-1 & 0 & 0 \\
-1 & 0 & 0
\end{pmatrix}
\]

where the (three) columns represent the (three) colors.

In order to identify the charge states of the quarks, which due to the locking of color with flavor make up a nonet (an octet plus a singlet), with the baryons in the CFL phase, we have to take the tensor product of the color and the flavor \( \mathbb{I} \). One can identify the two \( \bar{Q} = 1 \) states with \( \bar{p} \) and \( \bar{\Sigma}^+ \), the two \( \bar{Q} = -1 \) states with \( \bar{\Sigma}^- \) and \( \bar{\Xi}^- \) and three \( \bar{Q} = 0 \) with \( \bar{n} \), \( \bar{\Sigma}^0 \) and \( \bar{\Xi}^0 \). The remaining \( \bar{Q} = 0 \) state is the singlet baryon which is assumed to be very massive and hence to decouple.

### III. NEUTRINO INTERACTIONS WITH CFL EXCITATIONS

Given the gauge bosons (6), (9) and (10), it is straightforward to read off their weak interaction vertices. We are interested in processes that probe the dense CFL phase. The electromagnetic interaction may not be useful for probing the dense phase because the coupling with the matter must remain strong and most likely lose its memory of the broken phase when observed by the outside detector. However neutrino interactions are very weak and can be a good probe.

#### A. Weak interaction mediated by colored gauge bosons

Consider the color-gluon annihilation into the lepton pair, \( l\bar{l} \), as an example of weak neutral current interactions:

\[
\bar{V}^+ + \bar{V}^- \rightarrow \bar{Z} \rightarrow l\bar{l}, \\
\bar{V}^+ + \bar{V}^- \rightarrow \bar{V}_0 \rightarrow l\bar{l}.
\]

The coupling at the \( \bar{V}\bar{V}\bar{Z} \) vertex in the process mediated by \( \bar{Z} \), eq.(23), is given by

\[5\text{The simplest mnemonic for this operation is to arbitrarily assign colors so that the first column corresponds to a flavor equivalent of } \bar{u}, \text{ the second column to } \bar{d} \text{ and the third column to } \bar{s}. \text{ One would obtain the nonet equivalent to the octet and singlet of mesons.}\]
\[ f \cos^2 \delta \frac{4}{\sqrt{3}} g_s g \frac{\sigma^2}{v^2} \cos \theta_W \sim \frac{4}{\sqrt{3}} g \cos^3 \theta_W \left( \frac{M_V}{M_W} \right)^2 \]  

(27)

which gives a suppression factor

\[ \sim \left( \frac{M_V}{M_W} \right)^2 \]  

(28)

compared to the conventional $\nu \bar{\nu}$ production. However if there are substantial amounts of gluon excitations confined in dense hadronic matter at nonzero $T$ before it cools down completely, it may overcome the suppression factor and affect the cooling process appreciably.

The suppression factor in the process mediated by $\tilde{V}_0$, eq.(26), due to the vertex $\tilde{V}_0 l \bar{l}$ is given by

\[ \sim \sin \beta \sim g/g_s. \]  

(29)

The propagator in the low energy limit $Q^2 \ll M_V^2$ is greater than in eq.(25), i.e.,

\[ \frac{1}{Q^2 - M_V^2} \sim \frac{1}{M_V^2}. \]  

(30)

However the amplitude for fusion is enhanced at the strong interaction vertex, $\tilde{V} \tilde{V} \tilde{V}_0$, by a factor of $f$, and we get the factor for the amplitude

\[ \sim Q_f g \frac{g}{g_s M_V^2} \]  

(31)

with $Q_f$ given by eq.(23). One can now see that the gluon fusion into the charged flavor $l \bar{l}$ pair is greater than the weak neutral current by a factor of $\sim (M_V/M_W)^2 \sim 10^6$ and hence comparable to photon mediated processes [7]. However this enhancement does not apply to gluon-mediated $\nu \bar{\nu}$ processes because $Q_f$ is zero for neutrino. In general, for the neutral current with neutrinos, the contribution from color-gluon mediated processes in the broken phase vanishes since the amplitude is proportional to $Q_f(\text{neutrino})$ which is $= 0$. We arrive at the same conclusion for $q \bar{q} \rightarrow \nu \bar{\nu}$.

The charged current weak interaction in the process mediated by $\tilde{V}_0$ is also comparable to the ordinary weak interaction strength for the neutrino-quark interaction in the low-energy limit. Consider the following processes in matter,

\[ q + l \rightarrow q' + \nu(\bar{\nu}), \]  

(32)

\[ q \rightarrow q' + l + \nu(\bar{\nu}). \]  

(33)

As in the gluon annihilation processes, there are two amplitudes that can be decomposed into three parts: quark gauge boson vertex, propagator, gauge boson-lepton-neutrino vertex,

\[ qq' \tilde{W}^\pm \rightarrow \tilde{W}^\pm \rightarrow l \nu \tilde{W}^\pm, \]  

(34)

\[ qq' \tilde{V}^\pm \rightarrow \tilde{V}^\pm \rightarrow l \nu \tilde{V}^\pm. \]  

(35)

In the low energy limit, eq.(34) gives the ordinary weak amplitude
\[ g_2 \mathcal{M} \gtrsim g^2 \mathcal{M}_W^2. \]  
\[ (36) \]

It is easy to see that the contribution of the color gauge-boson-mediated process, eq. (35), also gives an amplitude comparable to that of the $W^\pm$ mediated process,

\[ \sim g g_s \left( \frac{\mathcal{M}_W}{g_s} \right)^2 \frac{1}{\mathcal{M}_W^2} \sim \frac{g^2}{\mathcal{M}_W^2}. \]  
\[ (37) \]

It should be noted however that the quark decay mediated by $\tilde{V}_0$ in eq. (33) cannot take place because of the energy conservation: the quarks with different colors but with same flavor have the same mass. Therefore the neutrino production mediated by the color-changing weak current is limited to the process in eq. (32)

\[ q_r + \tilde{\nu} \rightarrow q_b + \nu \]  
\[ (38) \]

\[ q_b + e^+ \rightarrow q_r + \bar{\nu}. \]  
\[ (39) \]

To keep the system in a color-singlet state in the cooling process, these processes should occur equally to compensate the color change in each process. It implies that these processes depend on the abundance of positrons in the system. At finite temperature in the cooling period, it is expected that there will be a substantial amount of positrons as well as electrons as long as the temperature is not far below $\sim \mathcal{M}eV$. Of course the additional enhancement of the neutrino production due to the CFL phase depends on the abundance of positrons in the system which depends mainly on the temperature. If confined colored gluons are present in the matter in the CFL phase, the same amplitude can be obtained in eq. (35) when $qq'$ is replaced with $VV'$.

The result obtained above can be summarized as predicting an enhancement of the effective four-point coupling constant for the neutrino production process in the low energy limit. The enhancement due to the neutrino-color interaction is suppressed by factors of $e^{-\Delta/T}$ or $e^{-\mathcal{M}_V/T}$, since it depends on the unpaired excitations above gap which can participate into neutrino-color interaction. Hence for the cooling process at low temperatures as $\sim 10^9K$ it is not so effective. However during the early stage of proto-neutron star the temperature is expected to be high enough $\sim 20 - 50MeV$ to see the effect of the enhancement due to color excitations.

In the next section, the evolution of the effective coupling constant is used with the help of a renormalization group analysis to show that there is an additional enhancement of the coupling constant down to the low temperature.

**B. Effective four-point Fermi coupling constant**

In this subsection, we calculate how the weak coupling constant runs in dense matter.

In dense matter the gluons are screened since soft gluons can decay into particles and holes near the Fermi surface. The one-loop screening effect at zero temperature has been calculated in the literature \[13\]. In the high density limit, the one-loop vacuum polarization density is given, with $M^2 = N_f g_s^2 \mu^2 /(2\pi^2)$ and $V^\mu = (1, \bar{v}_F)$, $\bar{V}^\mu = (1, -\bar{v}_F)$, by
\[
\Pi_{ij}^{\mu \nu}(q) = -\frac{i M^2}{2 \delta_{ij}} \int \frac{d\Omega_{\vec{p}}}{4\pi} \left( -\frac{2 \vec{p} \cdot \vec{v}_F V^\mu V^\nu}{p \cdot v + i\epsilon \vec{p} \cdot \vec{v}_F} + g^{\mu\nu} - \frac{V^\mu V^\nu + \bar{V}^\mu V^\nu}{2} \right) + \mathcal{O}(1/\mu). \tag{40}
\]

The terms involving quark-anti-quark pair creation and gluon loops are suppressed by \(1/\mu\) and are ignored. At low energy, \(p_0 \ll |\vec{p}| \sim p_F\) for the gluon and hence the gluon propagators take the following form in the Landau gauge \([14]\):

\[
iD_{i\mu}^{j\nu}(p_0, \vec{p}) \simeq \delta_{ij} \frac{|\vec{p}|}{|\vec{p}|^3 + i\pi M^2 p_0} O^{(1)}_{\mu\nu} + \delta_{ij} \frac{1}{p_0^2 - |\vec{p}|^2 - M^2} O^{(2)}_{\mu\nu}. \tag{41}\]

The projectors for the magnetic and electric modes are respectively

\[
O^{(E)} = P^\perp + \frac{(u \cdot p)^2}{(u \cdot p)^2 - p^2} P^u, \quad O^{(M)} = -\frac{(u \cdot p)^2}{(u \cdot p)^2 - p^2} P^u, \tag{42}\]

with

\[
P^\perp_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad \text{and} \quad P^u_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} - \frac{p_\mu u_\nu + u_\mu p_\nu}{(u \cdot p)} + \frac{u_\mu u_\nu}{(u \cdot p)^2} p^2, \tag{43}\]

where \(u_\mu = (1, 0, 0, 0)\). At low energy, the electric gluons are Debye screened with a screening mass \(M\) and the magnetic modes are dynamically screened (or Landau damped) at \(p_0 \neq 0\) \([15]\).

As argued by Alford et al \([2]\), at high density with three light flavors, the \(SU(3)\) gauge symmetry is spontaneously broken by forming a color-flavor-locked diquark condensate, eq. \([3]\). Then, by Higgs mechanism all eight gluons get mass of order of \(g_s M\), which can be easily seen by calculating the one-loop vacuum polarization tensor with quarks with eq. \((5)\). Then, by Higgs mechanism all eight gluons get mass of order of \(g_s M\), which can be easily seen by calculating the one-loop vacuum polarization tensor with quarks with Majorana mass (or a gap) \(\Delta\) generated by the diquark condensate \([16, 19]\):

\[
\Pi_{ij}^{\mu \nu}(p) = -g_s^2 \frac{d^4 q}{(2\pi)^4} \text{tr} \left[ T^A \gamma^\mu q_\parallel \cdot \gamma + \Delta - T^A \gamma^\nu q_\parallel \cdot \gamma + \Delta \right] \frac{q_\parallel^2 + \Delta^2}{q_\parallel^2} \tag{44}\]

\[
\simeq 0.86 \frac{i N_f g_s^2 m^2}{3} \frac{1}{2 \pi^2} \delta_{ij} \left( g^{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \cdots \tag{45}\]

where \(q_\parallel^\mu = (q_0, \vec{q}_F \cdot \vec{v}_F)\) and the ellipses denote terms containing more powers of momentum. At low momentum all gluons get a dynamical mass, \(M_V \simeq 0.2 g_s M\) for \(N_f = 3\), independent of the gap, \(\Delta\), though the relevant scale for the dynamical mass generation is of order of \(\Delta\). Let us consider the weak decay of light quasi-quarks, described by the four-Fermi interaction:

\[
\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \sum_{\tilde{v}_F} \tilde{v}_L(x) \gamma^\mu \tilde{\psi}_L(\tilde{v}_F, x) \tilde{\nu}_L(x) \gamma_\mu \nu_L(x) \tag{46}\]

\[
= \frac{G_F}{\sqrt{2}} \sum_{\tilde{v}_F} \psi_L^\dagger(\tilde{v}_F, x) \psi_L(\tilde{v}_F, x) \tilde{\nu}_L(x) \tilde{\nu}_L(x) \tag{47}\]

where \(G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}\) is the Fermi constant and \(\psi\) denotes the quasi-quark near the Fermi surface, projected from the quark field \(\Psi\) as in \([20]\),

\[
\psi(\tilde{v}_F, x) = \frac{1 + \tilde{\alpha} \cdot \tilde{v}_F}{2} e^{-i\tilde{v}_F \cdot \tilde{x}} \tilde{\Psi}(x). \tag{48}\]
Since the four-Fermi interaction of quarks with opposite momenta are marginally relevant and gets substantially enhanced at low energy, it may have significant corrections to the couplings to quarks of those weakly interacting particles [20]:

\[
\delta L_{\nu q} = \frac{G_F}{\sqrt{2}} \psi_L^\dagger(\vec{v}_F, x) \psi_L(\vec{v}_F, x) \bar{\nu}_L(x) \gamma \nu_L(x) \\
\times \frac{ig_3}{2M_V} \delta_{tv;us} \int_y \left[ \bar{\psi}_t(\vec{v}_F, y) \gamma^0 \psi_s(\vec{v}_F, y) \bar{\psi}_u(-\vec{v}_F, y) \gamma^0 \psi_u(-\vec{v}_F, y) \right] \\
= \frac{4}{3} \frac{g_3}{2 \pi} \frac{G_F}{\sqrt{2}} \psi_L^\dagger(\vec{v}_F, x) \psi_L(\vec{v}_F, x) \bar{\nu}_L(x) \gamma \nu_L(x),
\]

where \( \vec{v}_F \) and \( \vec{v}_F' \) are summed over and \( g_3 \) is the value of the marginal four-quark coupling at the screening mass scale \( M \). In terms of the renormalization group (RG) equation at a scale \( E \)

\[
\frac{dG_F(t)}{dt} = 4 \frac{g_3(t)}{3} \frac{1}{2\pi} G_F(t),
\]

where \( t = \ln E \). The scale dependence of the marginal four-quark coupling in the color anti-triplet channel is calculated in [11,20]. At \( E \ll \mu \)

\[
\bar{g}_3(t) \simeq \frac{4\pi}{11} \alpha_s(t).
\]

Since \( \alpha_s(t) = 2\pi/(11t) \), we get

\[
G_F(E) \simeq G_F(\mu) \left( \frac{\mu}{E} \right) \frac{16\pi^3}{305}.
\]

Since the RG evolution stops at scales lower than the gap, the low energy effective Fermi coupling in dense matter is therefore

\[
G_F^{\text{eff}} = G_F \left( \frac{\mu}{\Delta} \right) \frac{16\pi^3}{305}.
\]

We emphasize that this enhancement applies equally to the \( \beta \) decay of quarks and other neutrino production processes described in the previous section.

### IV. NEUTRON-STAR COOLING PROCESSES

At asymptotic density and low temperature (\( T \neq 0 \)), the relevant excitations are quasi-quarks that are not Cooper-paired, and 17 Nambu-Goldstone bosons. All other massive particles, Higgsed gluons and other massive excitations [21] are expected to be out of thermal equilibrium and decoupled.\(^6\) Thus the main cooling processes must be the emission of weakly

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\(^6\)If density is not too high, that is, in the regime relevant for such compact stars as neutron stars, there may take place Goldstone boson condensation from the top-down point of view as from the bottom-up. There may also be excitations of generalized (bound-state) mesons discussed in [21] that become low-lying and hence participate in the cooling process. We cannot address these issues in this paper.
interacting light particles like neutrinos or other (weakly interacting) exotic light particles (e.g., axions) from the quasi-quarks and Nambu-Goldstone bosons in the thermal bath.

We can think of two processes. The first one is the weak decay of quasi-quarks considered in the previous subsection where the running weak coupling indicates a modest enhancement of the process. Since neutrinos interact weakly, it can effectively carry away the energy of quark matter. For the neutrino emissivity from quasi quarks, the so-called Urca process is relevant. The neutrino emissivity by the direct Urca process in quark matter, which is possible for most cases in quark matter, was calculated by Iwamoto [22]. For the CFL superconductor, we expect the calculation goes in parallel and the neutrino emissivity is

$$\epsilon_{\text{direct}} \propto \alpha_s \rho Y_e^{1/3} T^6,$$

(54)

where $\rho$ is the density, $T$ is the temperature of the quark matter, and $Y_e$ is the ratio between the electron and baryon density. On the other hand, the neutrino emissivity by the modified Urca process, which is the dominant process in the standard cooling of neutron stars [23], is suppressed by $(\Delta/\mu)^4$, since the pion coupling to quarks is given by $g_{qq\pi} \sim \Delta/\mu$ [24]. Thus, the neutrino emissivity by the modified Urca process in the CFL quark matter is greatly suppressed in the CFL quark matter, compared to normal quark matter. Furthermore, since most excitations in the CFL quark matter are gapped and frozen out, the CFL quark matter has a quite small heat capacity and cools down very slowly at temperatures lower than the gap [26].

The second process that appears to be important is the bremsstrahlung emission of neutrino pairs from massless colored Nambu-Goldstone bosons: $\phi \phi \to \phi \phi \nu \bar{\nu}$.

The relevant terms before the gauge boson mass matrix is diagonalized can be written as

$$\mathcal{L}^{\text{int}} = \cdots + \frac{g}{2} \phi^{a'}_i \phi^{a'}_j (\lambda^A)_{ij} V^A_\mu + \frac{g}{\cos \theta_W} Z_\nu \left( T_3 - \sin^2 \theta_W Q_f \right)_{ab} \bar{l}_a L^\gamma_{bL} + \cdots$$

(55)

where $\phi$’s are the scalar Nambu-Goldstone bosons and $l_L = (\nu, e)_L^T$ is the left-handed lepton isospin doublet. Then the coupling for $\phi \phi \nu \bar{\nu}$ is induced by the $Z$ exchange

$$(\phi \phi \bar{Z}) G_Z (\nu \bar{\nu} \bar{Z})$$

(56)

where $G_Z$ is the $Z$ propagator. This can be written as an effective vertex given by

$$\mathcal{L}^{\text{eff}} = \cdots + \frac{4}{\sqrt{3}} \frac{g_s g}{g^2 + g'^2} \cos \theta_W \frac{g}{v^2} \frac{1}{2} \frac{1}{M_W^2} \cos \theta_W \phi^{a'}_i \phi^{a'}_j \bar{\nu}_L \gamma^\mu \nu_L + \cdots$$

(57)

A similar result was given in eq.(27) for $V^+ V^- \nu \bar{\nu}$. The result (57) can be easily understood by noting that the Golstone bosons in eq.(57) are nothing but the longitudinal components of the massive gluons. This process is again suppressed by a factor of $\sim (M_V/M_W)^2$ compared to the conventional $\nu \bar{\nu}$ production.

V. DISCUSSION AND SUMMARY

In this talk, we argue that the symmetry pattern of the color-flavor-locked phase of QCD at high density in the presence of electroweak interactions is mapped to the Pati-Salam
model of grand unification. Then, we have shown that this is a simple way of deducing the electroweak coupling of the CFL degrees of freedom. We find that the neutrino interaction with matter in the color-flavor locked phase can be enhanced by additional gluon-mediated processes. It remains to be verified that one can arrive at the same result in the weak-coupling QCD calculation of the type performed in [8,21].

It is perhaps useful to further comment on the idea of mapping the EW responses of the CFL phase to the Pati-Salam model. The symmetry (breaking) pattern is presumably encoded in the effective potential of the scalars $\phi$ and $\sigma$, $V_{\text{eff}}(\phi, \sigma)$. At low density, the physical vacuum has minimum at $\langle \phi \rangle \neq 0$, $\langle \sigma \rangle = 0$ and defines the electro-weak gauge theory, i.e., the established Standard Model. At some high density, however, the effective potential could develop a VEV of $\sigma$, as suggested by model calculations [1]. One of the possibilities is the color-flavor-locked phase considered here. (Other possibilities discussed in the literature can also be addressed similarly.) There are two issues to be resolved in the symmetry breaking schemes with scalar particles. As in the electro-weak theory at zero density, there are massive Higgs particles for which the effective potential is yet to be derived or explained. Presumably the effect of density might be marginal for these excitations. As for the color-flavor symmetry breaking, while there is rather compelling renormalization-group-flow argument to suggest that the vacuum expectation value of $\sigma$ is non-zero at some (asymptotic) density, the explicit form of the effective potential (with density dependence) is yet to be derived.

The essential feature of the neutrino production (except for the fusion processes) is that the color changes as neutrinos (or anti-neutrinos) are produced by the processes mediated by the color gluon exchange. One might therefore think that such processes are forbidden due to the color singlet requirement of the system. However at finite temperature the presence of positron excitations in the system induces processes which preserve the color singlet status as explained in the text. It is noted that the neutrino-color interaction is suppressed at low temperature cooling stage of neutron star but is expected to be effective at the early stage of proto neutron star.

Together with the general enhancement of the effective four-point coupling constant in RG analysis, the enhancement of the neutrino production implies that the cooling process speeds up as the CFL phase sets in dense hadronic matter near the critical temperature. But, at temperature much below the critical temperature, the interaction of quasi-quarks and pions and kaons is extremely weak, suppressed by $\Delta/\mu$, and the CFL quark matter cools down extremely slowly.

For a realistic calculation of the cooling rate of compact stars, we need to also consider the neutrino propagation in the CFL matter before the neutrinos come out of the system. A recent study [27] suggests that the presence of the CFL phase can accelerate the cooling process because neutrino interactions with matter are reduced in the presence of a superconducting gap $\Delta$. However this result is subject to modification by the effect of additional interactions – not taken into account in this work – mediated by the colored gluons on the quark polarization. It would be interesting to see how the enhancement of the neutrino production correlates with the neutrino-medium interaction. This is one of the physically relevant questions on how the enhanced neutrino interaction could affect neutron-star (proto neutron star) cooling following supernova explosion. This issue is currently under investigation.
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