Mößbauer null redshift experiment II

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Abstract

Accurate limits for the violation of the Principle of Equivalence are found by comparing the redshifts of $^{57}$Fe in different chemical environments.

The Principle of Equivalence is the basis of gravitational theory. It states that a freely falling frame is locally equivalent to an inertial frame [1]. Its main consequences, that can be used to verify it, are [2, 3]:

Local Lorentz Invariance: Local non gravitational phenomena are independent of the reference frame velocity.

Local Position Invariance: Local non gravitational phenomena are independent of the origin of the reference frame.

The simplest test theory of the above statements refers to a single (composite) test body in a background gravitational field. It is defined by the Lagrangian [4]:

$$L = -m_Rc^2 + \frac{1}{2}m_Iv^2 - mBU(x) + O\left(\frac{v^4}{c^2}\right)$$

(1)
In this equation $x$ and $v$ are the center of mass coordinate and velocity of the test body, $U(x)$ its gravitational potential and $m_R$, $m_I$ and $m_P$ are parameters called the rest, inertial and passive gravitational masses of the test body, respectively.

The Principle of Equivalence imposes severe restrictions on the adjustable parameters of equation (1). Universality of Free Fall implies:

$$m_I = m_P,$$  \hfill (2)

while Local Position Invariance requires:

$$m_R = m_P$$  \hfill (3)

and finally, Local Lorentz Invariance demands:

$$m_R = m_I$$  \hfill (4)

These three equations can be used to verify (more accurately, to falsify) the Principle of Equivalence. In this letter we obtain bounds on the violation of Local Lorentz Invariance and Local Position Invariance from equations (3) and (4); testing for the existence of anomalous redshifts in a Mößbauer emitter-absorber experiment [5].

Consider an atomic system suffering an electromagnetic transition between two levels $E_0$ and $E_1$, with an energy difference $\Delta E = E_1 - E_0$. A change of position in a gravitational field produces a frequency shift, the gravitational redshift [4, 5]:

$$z_g = \frac{\omega - \omega'}{\omega} = (1 + \xi_P) \frac{\Delta U}{c^2}$$  \hfill (5)

with

$$\xi_P = \frac{[\delta m_P(E_2) - \delta m_P(E_1)]c^2}{\Delta E}$$  \hfill (6)

where $\delta m_P$ are the differences between the gravitational and rest masses of the upper and lower energy levels. The first term in (5) is the universal gravitational redshift predicted by the Principle of Equivalence, while the second term is an anomalous, composition dependent redshift [4, 5].

In a similar way, if the system is boosted with velocity $v$, its frequency will be shifted by the amount (the transverse Döppler shift) [4]:

$$z_i = -\frac{1}{2} \frac{v^2}{c^2} + \xi_I \frac{(-\frac{1}{2}v^2 + \mathbf{W} \cdot \mathbf{v})}{c^2}$$  \hfill (7)
where $W$ is the initial velocity of the atomic system with respect to a privileged ("absolute") reference frame, and

$$\xi_I = \frac{[\delta m_I(E_2) - \delta m_I(E_1)]c^2}{\Delta E}$$

(8)

where $\delta m_I$ are the differences between inertial and rest masses.

In a null redshift experiment, the redshifts of two different atomic systems, $A$ and $B$, are compared. Only the "anomalous" terms in equations (5) and (7) would contribute to the differential redshift:

$$\Delta z = [\xi_P(A) - \xi_P(B)]\frac{\Delta U}{c^2} + [\xi_I(A) - \xi_I(B)]\left(-\frac{1}{2}v^2 + W.v\right)$$

(9)

and so, a null redshift experiment is a test for the universality of the redshifts.

The Mößbauer null redshift experiments test the difference of redshift of the same nuclear species (in our case, $^{57}\text{Fe}$) in different chemical environments. At first sight one expects a null relative shift, since we are comparing the rates of two identical clocks. This is not so, however: in the resonant emission-absorption phenomenon, the transition occurs between two collective states of the crystal, of extremely big masses [6]. The mass differences $\delta m_P$ and $\delta m_I$ are functions of the chemical composition of the system having, in general, nonzero values.

Consider now a resonant emitter-absorber system moving with the Earth along orbit. Because of its eccentricity, the Sun gravitational potential at the laboratory will have a sinusoidal variation with the period of one year. The same seasonal periodicity but with a different phase is expected for the redshift due to the "æther wind" term in equation (7). This characteristic signature of the breakdown of the Principle of Equivalence, was used to seek for an anomalous redshift in the previous experiment [6]. The differential anomalous redshift would appear as a yearly fluctuation of the isomer shift between the two samples. Since the zero-velocity channel of the spectrometer is obtained from the calibrated difference between the isomer shifts of a standard emitter-absorber pair, a variable isomer shift would induce a sinusoidal variation of the zero-velocity channel of the spectrometer:

$$v(t) = v_0 + \Delta \xi_P\frac{\Delta U}{c^2} + \Delta \xi_I\left(-\frac{1}{2}v^2 + W.v\right)$$

(10)
In this experiment we have analyzed data taken over a period of nine years, to search for any such yearly periodic signal. The Mößbauer spectra were taken with a natural α-Fe foil 6 μm thick absorber in a conventional 512 channel, constant acceleration spectrometer, with transmission geometry [1]. The sources used over this time were nominally 25 mCi $^{57}$Co in Rh matrices. The laboratory temperature stability was of ±2 °C. The magnetically split spectra were fitted with a nonlinear least-squares program to sextets of lorentzians with equal linewidths for each component. The zero-velocity channel position was determined with respect to the centroid of the spectrum. The baseline was simulated with a second order polynomial to account for cosine smearing. The velocity calibration was obtained by means of the measured α-Fe hyperfine field splitting. Figure 1 shows the zero-velocity channel position $v(t)$ (in mm/s) as a function of time. Although the data were taken for calibration purposes, the absolute values obtained are not relevant, since we are interested only in their internal consistency.

The time dependence of the zero-velocity channel data was adjusted to equation (10) with a linear least-squares program, using equal weights. For $W$, the velocity of the Solar System derived from the Cosmic Microwave Background dipole was taken. All terms up to linear in the eccentricity of the Earth orbit were kept. The resulting parameter estimates are:

\[
\begin{align*}
v_0 &= (-3.9 \pm 0.2) \times 10^{-3} \text{ mm/s} \quad (11) \\
\Delta \xi_P &= (1.5 \pm 1.9) \times 10^{-5} \quad (12) \\
\Delta \xi_I &= (-0.7 \pm 2.7) \times 10^{-8} \quad (13)
\end{align*}
\]

A Kolmogorov-Smirnov test applied to the relative residuals show they are gaussian with probability $P = 0.86$. Different weighting schemes, based on statistical counting errors, were also tried, but the Kolmogorov-Smirnov test showed that the corresponding residuals are not gaussian. From this we conclude that the error is dominated by random fluctuations in the zero-velocity channel, possibly of mechanical or thermal origins.

Our results (11) to (13) can be used to set upper bounds for any violation of the Principle of Equivalence. Table I shows the 95% confidence limit upper bounds to $\Delta \xi_P$ and $\Delta \xi_I$, derived from (12) and (13), together with the results of other previous null redshift results. It can be seen that the present work reproduces the bounds on $\Delta \xi_P$ previously obtained. To our knowledge, the upper bound on $\Delta \xi_I$ is the first ever obtained, since the
Turner-Hill experiment \cite{9} is an absolute experiment \cite{4}, that puts a bound on $\xi_I$.

We can combine our results (12) and (13) to test the Universality of Free Fall. The difference of acceleration between the upper and lower level in different chemical environments will certainly be bound by the quantity:

$$\eta = \frac{\Delta E}{M}|(\Delta \xi_P - \Delta \xi_I)| \quad (14)$$

where $M$ is the mass of a $^{57}$Fe atom, much smaller than the mass of the collective states. From the upper bounds in table 1 we obtain:

$$|\eta| < 10^{-11} \quad (15)$$

which is comparable to the results of modern versions of the classical Eötvös experiment \cite{4, 5, 15, 16}.

We can also use the former results to find bounds for the scalar (or pseudoscalar) fields in theories having a pseudoscalar coupling with electromagnetism \cite{10, 11, 12}. In these theories, the coupling of the new field $\phi$, in the form of a Chern-Simmons term:

$$\mathcal{L}_I = \kappa \phi E \cdot B \quad (16)$$

is assumed. The Gauss law in such theories is modified in the form:

$$\nabla \cdot E = 4\pi(\rho + \kappa \nabla \phi \cdot B) \quad (17)$$

which implies the existence of a charge (pseudo)density originated by the magnetic field:

$$\delta \rho_B = \kappa \nabla \phi \cdot B \quad (18)$$

in the $\alpha$-Fe absorber. This charge density will contribute with $\delta v = \alpha \delta \rho/e$ (with $e$, the electron charge) to the isomer shift \cite{13}. Since the sample is not polarized, different domains have randomly oriented magnetization and an upper bound for the root mean square value of $\kappa \nabla \phi$ can be found from:

$$|\kappa \nabla \phi| < \frac{3 e \sigma(v_0)}{B \alpha} \quad (19)$$

Using the typical values for iron ($\alpha \sim 0.3$ a.u.-mm/s; $B = 330$ kgauss) \cite{14} and the estimate of $\sigma(v_0)$ from \cite{11} we find

$$|\kappa \nabla \phi| < 2 \times 10^5 \text{ cm}^{-1} \quad (20)$$
an enormous value, compared with the estimates coming from cosmology \[1, 2\]:

\[|\kappa \nabla \phi| < 10^{-28} \text{ cm}^{-1} \]  \hspace{1cm} (21)

Our result (20), however, is a strictly local bound, based only in laboratory, short time data. The factor \(\sim 10^{33}\) between equations (20) and (21) comes mainly from the factor \(\sim 10^{41}\) between the distances used to estimate the gradient: \(\sim 1 \text{ fm}\) and \(\sim 10^8 \text{ pc}\) respectively.

In conclusion: the present experiment is an accurate test of the Principle of Equivalence. Equations (12), (13) and (15) provide rigorous upper bounds on the adjustable parameters of Lagrangian (1), and in turn they provide accurate tests for the validity of the three main consequences of the Principle of Equivalence. Besides, being carried out in systems in a strong magnetic field, it provides a local, short time bound on the gradient of any (pseudo)scalar field coupling to electromagnetism through a Chern-Simmons-like term.

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Table 1: Bounds on the Principle of Equivalence violation parameters.

| Experiment                  | $\Delta \xi_P$  | $\Delta \xi_I$ | Reference |
|-----------------------------|------------------|-----------------|-----------|
| H-maser vs SCSO             | $2 \times 10^{-2}$ | —               | [7]       |
| Free neutrons vs $^{147}$Sm | $7 \times 10^{-3}$ | —               | [8]       |
| “Moving” vs “Rest” $^{57}$Fe| —                | $5 \times 10^{-5}$ | [9]       |
| Null Mößbauer               | $3.6 \times 10^{-5}$ | —               | [5]       |
| Null Mößbauer               | $3.8 \times 10^{-5}$ | $5.4 \times 10^{-8}$ | This paper |

Figure 1: Zero-velocity channel vs time