Charm Production and Parton Distributions

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Abstract
Recent accurate data on $F_2(x, Q)$ and on $F_2^c(x, Q)$ from HERA at small-$x$ require a more precise treatment of charm production in the global QCD analysis of parton distributions. We improve on existing global analyses by implementing the leptoproduction formalism of Aivazis et al. which represents a natural generalization of the conventional zero mass QCD parton framework to include heavy quark mass effects. We also perform analyses based on the fixed-flavor-number scheme, which is widely used in the literature, and demonstrate their uses and limitations. We discuss the implications of the improved treatment of heavy quark mass effect in practical applications of PQCD and compare our results with recent related works.

1 Introduction
The QCD parton picture, based on perturbative Quantum Chromodynamics (PQCD), has furnished a remarkably successful framework to interpret a wide range of high energy lepton-lepton, lepton-hadron, and hadron-hadron processes. Through global analysis of these processes, detailed information on the parton structure of hadrons, especially the nucleon, has been obtained. Existing global analyses have mostly been performed within the traditional zero-mass parton picture since, up to now, “heavy quark” partons (charm, bottom and top) have played a relatively minor role in physically measured quantities used in the analyses. With the advent of precise data on inclusive $F_2$ and the direct measurement of the charm component $F_2^c$ from HERA, this is no longer the case. The latter comprises about 25% of the inclusive structure function at small-$x$. It is now necessary to sharpen the formulation of the theory for heavy flavor production used in these global analyses.

The conventional QCD parton model is formulated in the zero-mass parton limit. Since the early 1980’s, when the parton picture became the essential tool in calculating all high energy processes in the Standard Model and in the search for “New Physics”, it has been generalized to the form of a master (factorization) formula

$$\sigma_{N \rightarrow X}(s, Q) = \sum_a f_n^a(x, \mu) \otimes \hat{\sigma}_{a \rightarrow X}(\hat{s}, Q, \mu)$$  \hspace{1cm} (1)
where \( N \) denotes one or two hadrons in the initial state; \( X \), a set of inclusive or semi-inclusive final states consisting of ordinary or new particles; “\( a \)”, the parton label; \( f_a^N(x, \mu) \), the parton distribution at the factorization scale \( \mu \); \( \hat{\sigma}_{a \to X} \), the perturbatively calculable partonic cross-section; and the parton label “\( a \)” is to be summed over all possible active parton species.

“Active” partons, according to this widely accepted credo, include all quanta which can participate effectively in the dynamics at the relevant energy scale \( Q \) (e.g., \( Q \) in deep inelastic scattering, \( M_W \) in \( W \) production, \( p_t \) in direct photon or jet production, ... ). Thus, for heavy particle production (\( W, Z, \text{Higgs}, \text{SUSY particles}... \)), \( a = \{g, u, d, s, c, b, (t)\} \). This general picture has been adopted by most global analysis work \( \cite{3, 4, 5, 6, 7} \), resulting in several generations of parton distributions which include all the parton flavors. The charm, bottom (and sometimes top) quark distributions all contribute to high energy processes above energy scales higher than the respective quark masses.

Viewed from this perspective, the QCD theory for heavy quark flavor production poses a special challenge. If a heavy quark, say charm \( c \), is produced as part of the final state \( X \), (i) should one treat this process literally like the production of other heavy particles (\( W, Z, \text{Higgs},..., \)) i.e., differentiate \( c \) from the other partons, and exclude it from the initial state (by restricting the sum over “\( a \)” \( \text{Eq. 1} \) to only the light quarks) \( \cite{8, 9} \); or, (ii) based on the very physical ideas behind the factorization formula, should one still count \( c \) among the initial state partons because (unlike the electroweak heavy particles) it certainly is an active participant in strong interaction dynamics, including its own production, provided the energy scale is high enough \( \cite{4, 10} \)? In principle, the two alternatives can be regarded as two different but equivalent schemes for organizing the perturbation series in PQCD. In practice, since the perturbation series is truncated after one or two terms, the effectiveness (\textit{i.e.,} accuracy) of the two approaches can be quite different in different kinematic regions.

Much of the recent specific literature on heavy quark production is based on the first approach ((i) above) \( \cite{11, 12} \), extended to next-to-leading order (NLO), including in the initial states only the light partons—\( \{g, u, d, s\} \) for charm production. These are referred to as fixed-flavor-number (FFN) calculations—3-flavor \((n_f = 3)\) for charm. This scheme is conceptually simple. However, when the ratio of energy scales \( Q/m_c \) becomes large, \textit{i.e.}, when the charm quark becomes relatively light compared to the prevalent energy scale, its reliability comes into question because of the presence of large \( \ln \frac{Q}{m_c} \) factors—both in terms kept and terms left out. For measurements at HERA (and for both charm and bottom production at the Tevatron), this consideration becomes increasingly relevant. In fact, tell-tale signs of the inadequacy of these calculations have been known for some time: (i) the calculated cross-sections have unacceptably large dependence on the renormalization and factorization scale \( \mu \); and (ii) the theoretical cross-sections, in most cases, fall well below the experimental values for all reasonable choices of \( \mu \).

On the other hand, as mentioned above, most work on global analysis of inclusive structure functions (which contain heavy quark final state contributions) use the second approach— but with a simplification: once a massive quark is turned on above its threshold, it is treated as massless, on the same footing as the other light flavors. This is an approximation which is strictly correct only in the asymptotic region \( Q \gg m_c \). In practice, this approximation makes

\footnote{For clarity, and for the specific applications of this paper, we shall focus on charm in most of our discussions. The same considerations apply to bottom quark production with the substitution \( c \to b \) everywhere.}
little difference when charm production only contributes a small fraction of the measured structure functions used in the global analysis. This is no longer the case.

A consistent formulation of PQCD with massive quarks, representing a natural implementation of the physical principles behind the general formula, Eq. [4], has been given by Aivazis et al. [10]. It reduces to the appropriate limits – FFN scheme near the threshold region \((Q \approx m_c)\), and the simple massless QCD formalism in the high energy limit \((Q \gg m_c)\) – and provides a unified description of charm production over the full energy range. This formalism has been applied to the analysis of charged current lepto-production of charm [13]. Some recent papers have attempted to compare this approach, referred to as the variable-flavor-number (VFN) scheme—because the active number of flavors depends on the scale, with the FFN scheme [14]. Results are not conclusive, since comparable parton distributions in the various schemes are not available in the literature for a consistent comparison.

In this paper, we apply the more complete variable-flavor-number formalism to the global analysis of parton distributions. To carry out a systematic comparison, we also perform equivalent new analyses in the FFN scheme using both \(n_f = 3\) and \(n_f = 4\). We compare these results with each other, and with those obtained previously in the zero-mass approximation (CTEQ4) [7]. We find that: (i) the more complete formalism gives the best fit to the global data, further confirming the robustness of the PQCD theory; (ii) the \(n_f = 3\) FFN scheme has difficulty accommodating the hadron collider data included in the global analysis, whereas the \(n_f = 4\) fit appears to be acceptable; and (iii) the recent precision data from HERA at small-\(x\) are also sensitive to the small differences among the various schemes and approximations. In the concluding section, we discuss the implications of the improved treatment of heavy quark mass effect in practical applications of PQCD and compare our results with recent related works.

### 2 Treatment of Heavy Quark Mass Effect and New Global Analysis

We will extend the global QCD analysis to incorporate heavy quark mass effects according to the formalism of Aivazis et al. [10]. In comparison to recent analyses using the massless approach, significant differences only arise in the charm contribution to deep inelastic structure functions at small-\(x\). Thus, we will focus on this aspect of the calculation in most of the following discussions.

To be consistent with the treatment of all the processes used in the NLO global analysis, the charm production contribution is calculated to order \(\alpha_s\)—the same as the light-flavor contributions. Thus, the partonic processes included in the calculation are, for charged current interactions:

\[
\begin{align*}
\alpha_s^0 : & \quad W^+(W^-) + s(\bar{s}) \rightarrow c(\bar{c}) \\
\alpha_s^1 : & \quad W^+(W^-) + g \rightarrow \bar{s}(s) + c(\bar{c})
\end{align*}
\]

In addition, the proper implementation of the VFN scheme requires a delicate cancellation between three inter-related terms. (See next section.) When the parton distributions near the threshold region are not precisely generated to match the scheme, the cancellation will fail. This hampers most of these references.
and, for neutral current interactions:

\[\alpha_0^s : \gamma^s(Z) + c(\bar{c}) \rightarrow c(\bar{c})\]

\[\alpha_1^s : \gamma^s(Z) + g \rightarrow c + \bar{c}\]  

(3)

As pointed out in Ref. [10], the flavor excitation \(\alpha_0^s\) term and the flavor creation (or gluon fusion) \(\alpha_1^s\) term cannot be simply added because there is an overlapping region where they represent the same physics. The formalism provides a consistent procedure to subtract out the contribution of the collinear configuration in the gluon fusion process which is already included in the DGLAP evolved quark distribution in the flavor excitation process. See Fig. 1 for an illustration. This procedure eliminates the double counting.

By keeping the quark (Overlap) Gluon Fusion

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{Charm production in NC deep inelastic scattering in PQCD: flavor excitation and gluon fusion production mechanisms and their overlap which has to be subtracted. The mark ‘′ on the internal quark line indicates it is on-mass-shell and collinear to the gluon.}
\end{figure}

mass \(m_c\) in the hard cross-sections for all terms, this formalism reduces to the gluon fusion results of the FFN scheme near threshold because the flavor excitation and subtraction terms cancel each other in that region. This represents the correct physics for that region. In the other limit, when \(Q \gg m_c\), the subtraction term cancels the mass singularity of the gluon fusion term, the theory is free from large logarithms of the FFN scheme, and the perturbation series provides a good approximation to the correct physics in that region as well. For details, see Ref. [10]. For our purposes, the charm quark distribution \(f_c(x, Q)\) is assumed to be zero at the threshold \(Q = m_c = 1.6\) GeV; and it is dynamically generated by QCD evolution to higher scales. This approach does allow the option of including the presence of intrinsic charm inside the nucleon at threshold, which is excluded in the FFN scheme by definition. This option is of particular interest in the dedicated study of \(F_2^c(x, Q)\); it will be explored elsewhere.

Before performing a new global fit, it is instructive to quantify the difference between the above treatment of the partonic processes for charm production and the approximation made in previous CTEQ global analyses. Thus, we first convolute the existing CTEQ4M distributions with the improved hard-scattering cross-section described above to calculate the structure functions, and compare the results with the experimental data which were used in the CTEQ4 analysis. As is known from previous studies, very little difference is

\[3\text{Not shown explicitly is the order } \alpha_s \text{ quark scattering process, e.g., } \gamma^s(Z) + c \rightarrow g + c \text{ for NC. Numerically, this process gives a contribution which is at least an order of } \alpha_s \text{ down from the ones shown because } c/g \propto \alpha_s.\]

\[4\text{This subtraction has its counter-part in the subtraction of the } \frac{1}{\epsilon} \text{ pole in the dimensional regularization of collinear singularities in massless PQCD.}\]

\[5\text{CTEQ notes for the CTEQ3 analysis, 1994 (unpublished).}\]
found for all the fixed-target experiments. However, the differences at small-$x$ for the HERA experiments are comparable to the current experimental errors; hence they do matter. This is illustrated for three $Q^2$ data bins in Fig. 2. The contrast originates from the fact that charm production comprises a few percent of the measured $F_2$ in the fixed-target energy range; but it rises to about 25% at small-$x$ for HERA. This is shown in Fig. 3. At the few percent level, a relatively large theoretical uncertainty on $F_2^c$ can be tolerated, since experimental errors themselves are in the few percent range. This is no longer the case when the fractional contribution rises to 25% with experimental errors in the few percent range. Of course, it will be even more important to have the theory formulated accurately in the study of measured $F_2^c$ itself.

These results clearly imply the need to perform new global analyses to account for the correct physics behind the recent measurements. Thus, we repeat the CTEQ4 global analysis, using the improved theory for heavy quark production. Charged current and neutral current DIS processes are treated consistently as described above. We find the overall $\chi^2$ for the global fit is improved from the previous best fit CTEQ4M—1293 vs. 1320 for 1297 data points, as shown in Table 1 where both the overall $\chi^2$ and its distribution among the DIS and D-Y data sets are presented. The small improvement in $\chi^2$ is spread over both the fixed-target and HERA DIS data sets. The comparison of this new fit to the small-$x$ data of H1 is included in Fig. 2. We shall refer to this new set of parton distributions as CTEQ4HQ.

\[\text{Figure 2: Comparison of H1 data in the small-}x\text{ region with calculations using CTEQ4M parton distributions in the (original) massless QCD scheme (solid line) and with massive hard cross-section formulas of Sect. 2 (dashed line). Also shown is the result of the new fit CTEQ4HQ.}\]

\[\chi^2 \text{ values are difficult to interpret; hence they are not explicitly presented. (See Ref. } \text{for discussions.)}\]
As expected, the deviation of CTEQ4HQ distributions from CTEQ4M are rather minor, and they are most noticeable at small-$x$. Fig. 4 shows the comparison of these parton distributions. Interestingly, the differences are more obvious for the light quarks than for the gluon and charm; and they consist of an increase in these distributions throughout the small-$x$ range (which is most visible in the plot). This is because the improved theory for heavy quark production reduces the predicted cross-section which has to be brought back to the experimental values by increased parton distributions. This can be done most efficiently by a small fractional increase in the light quark flavors. Of course, if the momentum sum rule is to be preserved, there has to be a slight decrease in the parton distributions at large $x$. This is indeed the case, even though it is not quite visible in this plot. Proportionally, this decrease can be very small, since the momentum sum-rule integral strongly suppresses differences at small-$x$.

3 Global Analyses in the Fixed-Flavor-Number Scheme

Since much of the current literature on heavy flavor production in recent years adopts the fixed-flavor-number scheme described in the introduction[8, 9, 12, 14], it is useful to have available up-to-date parton distributions in this scheme and to study its efficacy in describing existing global data. By comparing results from such an analysis with those obtained above,

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7Computer code for this and the following parton sets will be available at the CTEQ Web site [http://www.phys.psu.edu:80/~cteq].
one can also draw more definitive conclusions concerning the effectiveness of the FFN scheme versus the more complete variable-flavor-number-number formalism.

The FFN scheme is simpler to implement, since it includes only light partons \{g, u, d, s\} in the initial state. Thus, for neutral current deep inelastic scattering as an example, the only partonic process contributing (to order \(\alpha_s\)) is the gluon fusion mechanism \(\gamma^*(Z) + g \to c + \bar{c}\). There is no need for any subtraction, since the charm mass regulates the collinear singularity, and there is no double counting. Using this scheme consistently for all processes, one can perform a global analysis in the \(n_f = 3\) FFN scheme, and obtain an appropriate set of parton distributions. As in the previous section, the treatment of data sets and the global analysis procedures are the same as in the previous CTEQ4 analysis [7]. We shall call this the CTEQ4F3 set.

Although we have emphasized charm production due to its immediate relevance, the same physics issues apply to bottom quark production. At HERA, \(b\) production is quite negligible in the total \(F_2\). But at LEP, the Tevatron and LHC, it is not. For this reason, and due to the fact that \(m_b\) is genuinely “heavy” whereas \(m_c\) is really on the borderline, we also carry out a global analysis in the \(n_f = 4\) FFN scheme for completeness. For this case, charm is counted as an active parton and treated above threshold in the manner described in the previous section, but bottom is treated as a heavy quark in the spirit of the FFN scheme. The set of parton distributions obtained this way is called CTEQ4F4.

An overview of the comparison between these FFN scheme global fits to the ones described in the previous section is included in Table 1. We see that the overall \(\chi^2\) for the two FFN scheme fits are 1380 and 1349 (for \(n_f = 3, 4\) respectively) compared to 1293 for CTEQ4HQ. A substantial part of the difference is due to the HERA data (particularly the ZEUS data set). Other than that, the \(n_f = 4\) FFN fit (CTEQ4F4) is very close to both CTEQ4M and CTEQ4HQ. This is expected since the only difference lies in the treatment of the small
More revealing is the \( n_f = 3 \) FFN fit (CTEQ4F3) which shows some signs of difficulty in achieving a good fit. In addition to the increased \( \chi^2 \) for the HERA data, there are two possible problems with hadron collider data: (i) the \( \chi^2 \) on the CDF \( W \)-lepton asymmetry data \footnote{Since the publication of the previous CTEQ4 analysis, NMC has published results of their final data analysis \cite{17}. The differences between the old and new results are not significant. We did not replace the previous NMC data by the new ones in this analysis because, in order to demonstrate the small differences} increases slightly; and, not shown in this table but more significantly, (ii) this fit requires both the CDF and D0 inclusive jet production data sets \footnote{8} to be normalized down by 10\% for a reasonable fit. Because of the absence of \( c \) and \( b \) partons inside the nucleon in the \( n_f = 3 \) FFN scheme (by definition), the light quark distributions have to be substantially increased in order to compensate. This distorts the \( u \) and \( d \) distributions with respect to the standard parton sets. The \( d/u \) ratio is known to be important in determining the \( W \)-lepton asymmetry. The latter is also affected by the different treatment of charm parton. \footnote{9} Since both points (i) and (ii) concern high energies, they suggest the neglect of important physics at large energy scales in this scheme. At the \( M_W \) scale, it surely is not a good idea to treat the charm quark as a “heavy quark”. It may be argued that the inclusion of higher order \( (\alpha_s^2) \) terms could improve the agreement between theory and experiment. (We have not done this in order to keep the discussions clear, and to keep all calculations to the same order.) This may be true. But that will only postpone the same problems to higher energy ranges. In fact, as mentioned above, the \( M_W \) scale is already here with us; any temporary gain in using a scheme appropriate for the threshold region has to be weighed in that broader context.

With these provisos, the CTEQ4F3 and CTEQ4F4 do represent the most up to date parton distributions to be used with FFN scheme calculations. CTEQ4F3 is in the same scheme as GRV94\cite{9}. CTEQ4F3 gives better fits to the global data, and are comparable to the other CTEQ4 distributions for the purpose of studying scheme dependences. These distributions can be further improved with the inclusion of order \( \alpha_s^2 \) hard cross-sections and with more recent data\footnote{8}.
4 Concluding Remarks

Recent HERA data both on precision measurement of the inclusive $F_2(x, Q)$ and on $F_2^c(x, Q)$ require a more careful theoretical treatment of heavy quark production in PQCD. This entails, in turn, both a clearer definition of the perturbative scheme used and the careful choice of a scheme which is appropriate for the full energy range probed by the experiments in question. We have described the salient features of most of the schemes which have been used (explicitly or implicitly) in current literature, and made a systematic comparison in the context of the global QCD analysis of hard processes to extract the universal parton distributions.

The advantages of both the massless approximation to the variable-flavor-number scheme used by most previous global analyses (e.g., both CTEQ and MRS) and the FFN scheme used by previous heavy quark production calculations (as well as by the GRV parton sets) lie in their simplicity in implementation. They both have limitations which can no longer be totally neglected. The generalized variable-flavor-number formalism contains the right physics in the full energy range, including both the threshold and asymptotic regions, and is free of large logarithms (except those of the small-$x$ kind which we have not discussed). It is reassuring to see that the more complete theory does give a better description of the wide range of experimental data included in the global analysis; and, non-trivially, the less complete theory does fall short where it is expected to. This provides further confirmation of the soundness of the PQCD theory.

Since the completion of this study, a preprint on a new treatment of heavy quark production appeared[15]. This (MRRS) approach is also in the variable-flavor-number scheme and is the same in spirit to the formalism of Aivazis et al. which we adopt, hence it shares the characteristics mentioned in the previous paragraph. The new MRRS parton set is comparable to CTEQ4HQ. The differences in implementation of the basic ideas of the VFN scheme lie in the specific choice of the location of the charm threshold and in the detailed treatment of mass effects in the region just above threshold.

We note that, the increase in precision of experiments and in sophistication of the theory place more demand on users of the QCD parton formalism: for consistency, each set of new parton distributions can only be applied to hard scattering cross-sections calculated in the same scheme. When charm and bottom mass effects matter, one must be fully aware of which heavy quark mass scheme should be used in a given calculation – this is in addition to the familiar \(\overline{\text{MS}}\) and DIS schemes of the massless theory. For most lepton-hadron and hadron-hadron processes away from very small-$x$, data cannot distinguish among the various schemes studied in this paper, provided the parton distributions and hard scattering calculations are not mismatched. In these cases, the usual parton distributions, even if not precisely the true ones of nature, provide an effective description of the physics probed. In this sense, the continued use of existing standard parton distribution sets (such as CTEQ[7] and MRS[6]) for these processes is acceptable. However, for processes sensitive to initial or final state heavy quarks, it will be imperative to use the more complete theory, with matching parton distributions, if meaningful physical quantities are to be extracted. For among the various schemes, including the previous CTEQ4M analysis, we have to use the same data sets. We also anticipate new data on the CCFR measurement of the charge current structure functions in the near future.
these processes, current theory can still be improved to include higher order terms; and
detailed phenomenology is yet to be done when both experiment and theory mature.

Acknowledgments: We thank John Collins, Frederick Olness and Carl Schmidt for useful
discussions on heavy quark physics; Joey Huston, Steve Kuhlmann, Joseph Owens, Davison
Soper and Harry Weerts for collaboration on the previous CTEQ4 analysis on which the
current study is based; and Kuhlmann and Raymond Brock for detailed comments on the
manuscript.

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