Confinement and monopole condensation: some properties of the disorder parameter.

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1. Introduction

We consider a $U(1)$ gauge theory with a conserved magnetic current $j^M_\mu = \partial^\nu F^*_\mu\nu$. The corresponding magnetic $U(1)$ symmetry can either be Wigner or broken \`a la Higgs. In the first case (Wigner) the Hilbert space consists of superselected sectors with definite magnetic charge. In the second case (Higgs) at least one magnetically charged operator $\mu$ exists, with $\langle \mu \rangle \neq 0$. $\langle \mu \rangle$ is the order parameter. The free energy density (effective Lagrangean), is uniquely determined by symmetry and dimensional arguments\textsuperscript{2} to be

$$
\mathcal{L} = (D_\rho \langle \mu \rangle)^* D_\rho \langle \mu \rangle - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(\langle \mu \rangle)(1)
$$

Here $D_\rho = \partial_\rho - iq M A_\rho$ is the dual covariant derivative, $A_\rho$ the dual vector potential, $V(\langle \mu \rangle)$ the usual quartic potential. If $V$ has a non trivial minimum, $\langle \mu \rangle \neq 0$, the system is a dual superconductor. In the usual non compact formulation $j^M_\mu \equiv 0$ (Bianchi identities). On a lattice the theory is compact and a non zero magnetic current can be defined\textsuperscript{3}. In QCD $U(1)$ is defined by abelian projection\textsuperscript{4}. A magnetically charged operator $\mu$ has been constructed\textsuperscript{1} and $\langle \mu \rangle$ has been used as a disorder parameter for detecting dual superconductivity. The results of this investigation are the following\textsuperscript{5}: 1) The confined phase of quenched QCD has $\langle \mu \rangle \neq 0$, and behaves as a dual superconductor in all the abelian projections\textsuperscript{1}. 2) In the deconfined phase $\langle \mu \rangle = 0$ and superselected magnetic sectors exist. 3) In the vicinity of the transition $\langle \mu \rangle \propto (1 - T/T_c)^\delta$. $\delta$ is independent of the abelian projection, and is equal to the analogous index of the dual Polyakov line\textsuperscript{6}. 4) A similar behaviour is found in the presence of dynamical quarks. These results are obtained by properly performing the infinite volume limit, by use of finite size scaling techniques\textsuperscript{5,6}, and provide basic information on the dual structure of QCD. The questions we want to address here are the following

1) Does $\langle \mu \rangle \neq 0$ contradict the so called Elitzur’s theorem\textsuperscript{8}? 2) How precisely is the abelian projection definable on the lattice?

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2. The order parameter of a superconductor.

The ground state of a superconductor is a superposition of states with different charges\[3\], and the order parameter $\langle \varphi \rangle$ is the v.e.v. of a charged operator $\varphi$. **Theorem** (continuum version of ref.\[8\]) If $\varphi(x)$ is a local charged operator, in a gauge invariant formulation (no gauge fixing), $|0\rangle$ is gauge invariant and hence

$$
\langle 0|\varphi(x)|0\rangle = e^{i\Lambda(x)}\langle 0|\varphi(x)|0\rangle \quad \forall \Lambda(x)
$$

or $\langle 0|\varphi(x)|0\rangle = 0$. Does the existence of a superconductor violate gauge invariance? The answer is of course no. In the usual perturbative treatment of the Higgs phenomenon a gauge is fixed, e.g. the unitary gauge\[10\], and the theorem is eluded. In a gauge invariant formulation, like lattice, the way out is to define gauge invariant charged operators\[11\], $\tilde{\varphi}(x)$, as follows

$$
\tilde{\varphi}(x) = \varphi(x)e^{i(A,h)}
$$

where

$$(A,h) = \int d^4y A_\mu(y)h_\mu(y - x)$$

$\partial^\mu h_\mu(z) = \delta^4(z)$. Under a generic gauge transformation $U_\Lambda$, with $\Lambda(x) \to 0$ as $|x| \to \infty$, $(A,h) \to (A,h) - \Lambda(x)$, $\varphi(x) \to e^{i\Lambda(x)}\varphi(x)$ and hence $\tilde{\varphi}(x) \to \tilde{\varphi}(x)$. $\tilde{\varphi}(x)$ is gauge invariant. However, under a global transformation (with $\partial_\mu \Lambda = 0$) $A_\mu \to A_\mu$ and $\tilde{\varphi}(x) \to e^{i\Lambda}\tilde{\varphi}(x)$ like any charged operator. $\tilde{\varphi}(x)$ is charged and gauge invariant. It is non local, but by a judicious choice of $h_\mu$, it obeys cluster property

$$
G(x) \equiv \langle \tilde{\varphi}(x)\bar{\tilde{\varphi}}(0) \rangle \sim A e^{-M|x|} + |\langle \varphi \rangle|^2 \quad (3)
$$

and defines the order parameter $\langle \varphi \rangle$ by the asymptotic behaviour. Possible choices for $h_\mu$ depend on the dimension $d$ of its support. For $d = 1$, $h^\mu = \delta_0^\mu \theta(x_0 - y_0)\delta^3(\vec{x} - \vec{y})$ and $\exp(i(A,h) = \exp(i \int_{\infty}^{x_0} A_0(y_0, \vec{x})dy_0)$ is a parallel transport from $-\infty$ along time axis (Mandelstam string). The string can be put on any path $C$ going to infinity and $\tilde{\varphi}(x) = \varphi(x)\exp(i \int_C A_\mu dx^\mu)$. For $d = 3$, $h^\mu = (0, \frac{1}{4\pi |\vec{x} - \vec{y}|})$ (Dirac choice). For a lattice version of the Higgs model

$$
\mathcal{L} = \frac{\beta}{2} - a^2 \cos(d\theta - eA)
$$

it can be proved that

1) At sufficiently small $a$ (Coulomb phase) $|G(x)| \leq |x| \to \infty C \exp(-\rho|x|)$

2) At sufficiently large $a$ and small $e$ (Higgs phase) $|G(x)| \geq a^2$ as $|x| \to \infty$, provided $(h,h)$ is uniformly bounded in the limit $V \to \infty$. This is true for the Dirac choice of $h^\mu$ ($d = 3$) but not for the string ($d = 1$).

3) The Hilbert space enlarged by the correlators of $\tilde{\varphi}$ obeys Osterwalder-Schrader positivity, and admits a decomposition in superselected sectors in the Coulomb phase.

3. Dual superconductivity.

For compact $U(1)$ the operator $\mu$ which creates a monopole is defined as\[12\]

$$
\mu(x_0, \vec{x}) = e^\beta \sum_n \cos(\theta_n(x_0, \vec{n}) - \theta(x_0, \vec{n}))
$$

where $b_i(\vec{x} - \vec{y})$ is the vector potential at $\vec{y}$ of a monopole sitting at $\vec{x}$. Theorems

1) $\mu$ carries magnetic charge\[12\]. 2) $\mu$ is gauge invariant ($\theta_0i = F_0i$). 3) $\mu$ is Dirac
4. About the abelian projection.

The abelian projection is a gauge transformation which diagonalizes an operator $\Phi(x) = \sum \Phi^a T^a$ in the adjoint representation. In this gauge the generic link on the lattice $U_\mu(n)$ can be written in the form

$$U_\mu(n) = V_\nu(n)C_\mu(n)$$

where $C_\mu(n)$ is an exponent of the diagonal generators (photons), $V_\mu(n)$ of the off diagonal (charged) generators. $C_\mu$ is uniquely defined. Also the form

$$U_\mu(n) = C_\mu(n)V'_\nu(n)$$

is possible, with $V'_\mu = C^\dagger(n)V_\mu C_\mu(n)$, again a charged operator. For the plaquette $\Pi_{\mu\nu} = U_\mu(n)U_\nu(n + \hat{\mu})U^\dagger_\mu(n + \hat{\nu})U^\dagger_\nu(n)$ the abelian projection is not uniquely defined. It can be defined, e.g., as the projected part of $\Pi_{\mu\nu}$ by a procedure similar to that leading to eq.(1), or, as usually done, as the plaquette constructed with the abelian links $\Pi^0_{\mu\nu} = C_\mu(n)C_\nu(n + \hat{\mu})C^\dagger_\mu(n + \hat{\nu})C^\dagger_\nu(n)$. By use of eq.(7)

$$\Pi_{\mu\nu} =$$

$$V_\mu(n)V'_\nu(n + \hat{\mu})(V''_\mu)^\dagger(n + \hat{\nu})(V'''_\nu)^\dagger(n)\Pi^0_{\mu\nu}$$

The $V$’s in eq.(8) are separately charged, but their product contains at the exponent terms coming from the commutators in the Baker-haussdorf formula, which are $O(a^2)$ and belong to the diagonal subspace of the algebra. The abelian projection on the lattice is undefined by terms $O(a^2)$.

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