Wideband-RL-cancel circuit for the $E$-$J$ property measurement using the third-harmonic voltage method

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Abstract. A precise and generalisable non-destructive measurement technique is required for evaluation of critical current density $J_c$ and electric field $E$ vs. current density $J$ properties in large-area and long-length high-temperature superconducting films. We measured $E$-$J$ properties of a 2-inch φ Y-123 standard film for the calibration of our inductive measurement system using the third-harmonic voltage method. With adoption of a wideband-RL-cancel circuit, frequency normalised third-harmonic resistance contained noise less than 0.2 µΩs and power-law $E$-$J$ dependencies were in the relatively wide electric field range of $1.4 \times 10^{-6}$–$1.6 \times 10^{-4}$ V/m. Critical current density $J_c$ at $10^{-4}$ V/m on 7 different positions changed from an average of $2.839 \times 10^3$ A/m$^2$ by a maximum of ±0.52%. As the $J_c$ distribution was also included, the maximum error margin of $J_c$ became remarkably small in the $V_3$ inductive method. $E$-$J$ properties of a 20×20mm square Dy-123 film agreed well with the properties measured by another research group. $J_c$ at $10^{-4}$ V/m and $n$ value were $2.23 \times 10^3$ A/m$^2$ and 26.74 on the average of four different positions, with estimated systematic errors of −8.6% and +1.4%, respectively, by comparison of the averages of 16 positions measured by the same research group.

1. Introduction

Inductive measurement methods using the third-harmonic voltage, $V_3$, are among the most reliable methods for non-destructive measurement of the critical current density, $J_c$, of the recently developed high critical current and large-area high-temperature superconducting films [1-5]. In the general inductive methods, an a.c. magnetic field is applied to a superconducting film with a small drive coil up to the extent where a magnetic shield of the film collapses and the $V_3$ voltage is generated in the coil. Threshold coil current, $I_{th}$, is then measured, and $J_c$ can be obtained by a simple calculation. Mawatari et al. reported that this normal method includes a large systematic error at the time of definition of $I_{th}$ based on a constant $V_3$ voltage standard, if we consider the scaling law of the $V_3$ voltage [1]. It is possible to reduce the systematic error by applying a third-harmonic inductance $V_3/(I_0 \omega)$ [H] in which the $V_3$ voltage is divided by the coil current $I_0$ and the excitation frequency $f$ ($=\omega/2\pi$) as a definition of the threshold $I_{th}$, and thus it is also possible to measure even the $E$-$J$ properties and $n$ values [2, 3]. In our previous study, we evaluated the $E$-$J$ properties of a homogeneous Y-123 standard thin film using this scaling method, and we demonstrated a variable-RL-
cancel circuit to reduce a mixed third-harmonic voltage noise from a signal generator [4]. As a result, we successfully obtained the E-J properties by plotting the coil current \( I_0 \) dependences of \( V_3/(I_0 f) \) [Ωs] and defining the \( I_{th} \) at a constant criterion of \( V_3/(I_0 f) = 2 \mu\Omega s \). We also confirmed the noise-cancelling effect of the variable-RL-cancel circuit within the frequency range of 0.2–5 kHz. In addition, the error margin of the \( J_C \) by measuring 5 different positions in the standard film was estimated to a maximum of \( \pm 1.7\% \), and was successfully reduced to the same extent as the transport \( J_C \) measurement. However, the \( n \) values measured while changing frequency \( f \) were dependent on the instability of impedances in the RL-cancel circuit, and the error margin by the measured position became \( \pm 12.9\% \) at maximum [4].

In this study, we first improved the frequency characteristics of the RL-cancel circuit to perform measurement within a wide frequency range, and a precise calibration method was investigated using a homogeneous Y-123 standard film. The E-J properties were then measured using a Dy-123 sample film, and the measurement errors of the \( J_C \) and \( n \) values were estimated.

2. Experiments

2.1. Wideband-RL-cancel circuit

In our earlier work, we measured \( J_C \) non-destructively using a typical circuit to detect the \( V_3 \) voltage [5]. As we picked up the \( V_3 \) voltage directly from a drive coil without any noise cancellation, there was a large amount of harmonic voltage noise from the signal generator. To remove this harmonic noise, Yamazaki et al. proposed a technique involving the arrangement of a cancel coil on another standard superconducting film where \( J_C \) is much higher than the sample film and connecting this cancelling coil and drive coil with the series [2]. As another approach to achieve this noise cancellation, we proposed a variable-RL-cancel circuit [4] that can precisely emulate the impedance of the drive coil, \( Z_d \), using variable resistors and coils as shown in Figure 1. Two pairs of coils and resistors (\( L_{va} \) and \( R_{va} \), \( L_{vb} \) and \( R_{vb} \)) were connected to the drive coil in series, and then both impedances \( Z_a \) and \( Z_b \) of the cancel circuit were adjusted to the impedance \( Z_d \). As the third-harmonic voltage noises, \( V_{3\text{nois}} \) from the signal generator arose synchronously at both the drive coil and cancel circuit, so both voltages cancelled each other using an amplifier with differential inputs. Therefore, we detected the third-harmonic voltage signal, \( V_{3\text{sig}} \), at a sufficiently high S/N ratio.

In this study, we used an amplification circuit (with NJM4580DD op amp.), in which the phase difference in the I/O voltages and wave distortion rate were very small, to amplify the pickup voltage from the cancellation circuit over a wide frequency range. The phase difference and the distortion rate of this amplification circuit were less than 0.02° and 0.04%, respectively, in the range of 0.1–10 kHz. Moreover, to prevent the impedance of the cancellation circuit from changing due to the joule heating, heat was radiated by air-cooling. All the measurements were performed at the temperature of liquid nitrogen (77.3 K). The excitation frequency \( f \) was changed at seven points within the range of 0.2–20

![Figure 1. Schematic electric circuit with modified variable-RL-cancel circuit.](image-url)
kHz. In the excitation coil (Hayama Co.), the diameter of the lead wire was 50 µm, the turn number was 400, the inside diameter was 1.0 mm, the outside diameter was 4.2 mm, and the height was 1.0mm. The width on the edge of the bobbin was 0.15mm, and when the thickness of the Capton film was included, the distance of the coil and the sample film was kept to 0.2mm. The pressure holding the coil against the film was set to 0.27 MPa [4].

2.2. Y-123 standard film and Dy-123 sample film
A 2-inch φ YBa$_2$Cu$_3$O$_{7-δ}$ thin film (Theva Co.) was used as a standard film for calibration for the inductive measurements. A 330 nm thickness Y-123 film was developed on a CeO$_2$-buffered R-cut Al$_2$O$_3$ substrate by a thermal coevaporation method and coated with a layer of Au 300 nm thick for the protection. Half of the area of the standard film was protected further with the Capton film 50µm in thickness. In the inductive measurement, measurements were taken at seven points (#1–#7) in the vicinity of the film centre in straight lines along the diameter. On the other hand, the transport measurements were made three times in a different bridge at four places, B2–B5, in the vicinity of the film centre. We used four bridges 1 mm in length and of different widths: 24 µm (for B3), 44 µm (for B2 and B4) and 64 µm (for B5). After calibration, the $E$-$J$ property of a DyBa$_2$Cu$_3$O$_{7-δ}$ film of 20×20mm square (Theva Co.) was measured as a sample film for evaluation of the measurement error. The thickness of the Dy-123 film developed on a CeO$_2$-buffered R-cut Al$_2$O$_3$ substrate was 330 nm, and it was coated with a Teflon layer for chemical protection. The surface was covered with Capton film, inductive measurements were taken at four points (P1–P4).

2.3. Calibration method
The calibration using the Y-123 standard thin film was performed according to the following procedures. (i) The maximum value $2H_0$ of the magnetic field impressed on the film when the current of the coil equalled $I_0$ was calculated and the theoretical coil factor $k$, which was the proportion constant, was determined. In addition, the threshold magnetic field, $2H_{th0}$, where the applied magnetic field begins to penetrate through the film of thickness $d$ is equal to $J_C \times d$ from the critical current state model [6]. The theoretical threshold current $I_{th0}$ was supplied to the coil, and the following expression was obtained:

$$J_C \times d = 2H_{th0} = k I_{th0} = k' I_{th0}$$  (1)

Here, $I_{th0}$ is the experimental threshold coil current, and is measured as slightly larger than $I_{th0}$ by the third harmonic voltage method because of its sensitivity. In addition, $k'$ is the experimental coil factor for calibration. (ii) The $E$-$J$ properties of the electric field range of about $10^3$–$10^5$ V/m were measured by the transport method, and the fitting equation ($E = a J^n$) averaged by four measured points was obtained. (iii) We measured the third-harmonic voltage $V_3$ as the sweeping coil current $I_{th0}$, and experimental $I_{th0}$ was determined by the constant $V_3/(I_{th0} f)$ standard, which was sufficiently high as compared with the noise from the signal generator. (iv) The scaling characteristics of the normalised coil current $I_0/I_{th0}$ dependence of $V_3/(I_{th0} f)$ at a different frequency were compared, and the minimum definition standard for $I_{th0}$ was selected. (v) The average electric field $E$ induced in the film by the excitation coil is proportional to the film thickness, frequency, and the impressed magnetic field [1]. Therefore, $E$ is given by the following expression at $I_0=I_{th0}$, considering the relation in equation (1):

$$E \approx 2.041 \mu_0 d f k' \mu_0 d f k' I_{th0}$$  (2)

From equations (1) and (2) and the $E$-$J$ power law formula $E = a J^n$, the involution characteristics as in the next expression were derived as the relation between experimental $I_{th0}$ and frequency $f$.

$$f = p \cdot I_{th0}^{n-1}$$  (3)

Here, $p = a k'^{(n-1)} / 2 \mu_0 d^{n+1}$. Thus, the $n$ value of $E$-$J$ property can be easily obtained by plotting the involution characteristic of $f$-$I_{th0}$. (vi) The experimental $k'$ was derived from equations (1) and (2) as
inductive measurement and the \(E-J\) property formula at the transport measurement, and \(k'\) was given by the following expression:

\[
k' = \frac{d}{I_{th}} \cdot \left(2 \mu_0 fd^2 / a\right)^{\frac{1}{n-1}}
\]  (4)

Here, we chose the highest frequency \(f\) according to the induced electric field \(E\), which is proportional to \(f\) as shown in equation (2), near the electric field range of the transport \(E-J\). (vii) Using a calibrated \(k'\), the inductive \(E-J\) properties were obtained by calculating \(J_c\) and \(E\) from equations (1) and (2).

3. Experimental results

3.1. Calibration using Y-123 standard film

First, we analysed the distribution of the impressed magnetic field above the film from the excitation coil by FEM, and the theoretical coil constant \(k\) was calculated as \(1.09 \times 10^6 \text{m}^{-1}\). The \(E-J\) characteristics were obtained by the transport measurements at four bridges of Y-123 standard film within the electric field range of \(7 \times 10^{-4} – 1.2 \times 10^{-1} \text{V/m}\), and the relational expression \((E=6.65 \times 10^{227} f^{16.0})\) of the average by the bridges was obtained. Figure 2 shows the results of the inductive measurements in seven positions of a Y-123 standard film. As the frequency response of the variable-RL-cancel circuit was improved, the coil current dependences of \(V_3/(I_0 f)\) were measured successfully over a wide frequency range of 0.2–20 kHz. In addition, \(V_3/(I_0 f)\) became a clear curve with extremely low noise of about 0.2 \(\mu\Omega\). Figure 3 shows the normalised current \(I_0/I_{th}\) dependences of the \(V_3/(I_0 f)\) when the standard of the \(I_{th}\) definition was 1–20 \(\mu\Omega\). When the definition standard of \(I_{th}\) was selected at 10\(\mu\Omega\) or more, the curves under different frequencies were scaled almost as a single curve.

The relationship between frequency and the threshold coil current \(I_{th}\) became the involution characteristic of the \(n-1\) power as shown in equation (3), and then the \(n\) value became 31.96 on the average of seven positions. We choose 20 kHz as the calibration frequency at which the electric field was \(1.6 \times 10^4 \text{ V/m}\), and then the theoretical \(I_{th0}/\sqrt{2}\) was obtained from equations (1) and (4), and was estimated as 61.42 mA. On the other hand, the measured value of \(I_{th}/\sqrt{2}\) was 94.58 mA with application of the definition standard of \(I_{th}\) was 10\(\mu\Omega\) or more, the experimental coil factor \(k'\) by this calibration method was derived as \(7.10 \times 10^5 \text{m}^{-1}\) from equation (1), and \(k'\) was 64.94% of the theoretical coil constant \(k\).

![Figure 2](image1.png)  
**Figure 2.** Coil current dependences of the inductive \(V_3/(I_0 f)\) for Y-123 standard film.

![Figure 3](image2.png)  
**Figure 3.** Scaling properties of \(V_3/(I_0 f)\) vs. \(I_0/I_{th}\) curves. Criterion for the \(I_{th}\) was 1–20\(\mu\Omega\).

3.2. \(E-J\) property of the Y-123 standard film

Applying the experimental coil factor \(k'\) obtained by the calibration to equations (1) and (2), the \(E-J\) characteristic of Y-123 standard film was obtained and is shown in Figure 4. The \(E-J\) characteristic by the \(V_3\) inductive method was smoothly connected with that by the transport method. Moreover, due to
the wideband-RL-cancel circuit, the \(E-J\) properties were obtained by the inductive method over the relatively wide electric field range of \(1.4 \times 10^{-6} - 1.6 \times 10^{-4}\) V/m. The \(n\) value of 31.96 determined by the inductive method was slightly higher than the value of 26.04 obtained by the transport method in the lower electric field region. This cuspidate \(E-J\) property is generally observed even at the temperature of liquid nitrogen [7], and Fisher explained this phenomenon in the glass-liquid transition model [8].

\(J_c\) in the electric field \(E=1 \times 10^{-4}\) V/m by the \(V_3\) inductive method changed from the average value by a maximum of \(\pm 0.52\%\), while \(J_c\) in \(E=1 \times 10^{-3}\) V/m by the transport method changed by a maximum of \(\pm 2.32\%\), and the error margin of \(J_c\), which also included its distribution in the film, became extremely small in the inductive method. Moreover, the \(n\) value determined by the inductive method changed by \(\pm 2.29\%\) or less from the average, while that determined by the transport method changed by \(\pm 3.16\%\) or less. Before improvement of the RL-cancel circuit, the margins of measurement error of inductive \(J_c\) and \(n\) value were \(\pm 1.7\%\) and \(\pm 12.9\%\), respectively, similar to another Y-123 standard film of the same type [4]. The nonsystematic errors of \(J_c\) and \(n\) value were markedly reduced by stabilisation of the cancel circuit to prevent change in the internal impedances.

3.3. \(E-J\) properties of the Dy-123 sample film

The Dy-123 sample film was measured by the \(V_3\) inductive method at four positions (P1–P4) with using the experimental coil factor \(k'\). Figure 5 shows the coil current dependences of the \(V_3/(I_0 f)\) at position P2 of the Dy-123 sample. The \(E-J\) properties at all positions of P1–P4 are shown in Figure 6 when the threshold coil current \(I_{th}\) was defined based on 10 \(\mu\Omega\)s as well as the Y-123 standard film. Clear and low-noise frequency responses of the \(V_3/(I_0 f)\) vs. \(I_0\) curves were obtained as shown in Figure 5, and the plotted \(E-J\) data showed a good fit to the theoretical power law dependence over the wide frequency range of \(5.7 \times 10^{-7} - 7.3 \times 10^{-5}\) V/m as shown in Figure 6. \(J_c\) at \(E=1 \times 10^{-4}\) V/m showed a slightly larger distribution of \(2.189 \times 10^{10} - 2.291 \times 10^{10}\) A/m² and the average was \(2.23 \times 10^{10}\) A/m². The \(n\) values ranged within \(26.22 - 27.85\) and the average was \(26.74\). On the other hand, Yamasaki et al. also measured \(J_c\) and \(n\) values for the same Dy-123 sample film as shown in Table 1 and Figure 6; they obtained an inductive \(J_c\) value of \(2.44 \times 10^{10}\) A/m⁴ at \(E=1 \times 10^{-4}\) V/m and \(n\) value of 26.34 from the average of 16 positions. The systematic errors of the \(J_c\) and \(n\) value in our inductive measurement were estimated as \(-8.6\%\) and \(+1.4\%\), respectively, if we assume no systematic error in this group’s measurements.
4. Summary

As we improved the frequency response of the variable-RL-cancel circuit, the coil current dependence of the \( V_3/(I_0 f) \) was measured over the wide frequency range of 0.2–20 kHz and yielded clear curves with extremely low noise up to 0.2 \( \mu \Omega \)s. Using the wideband-RL-cancel circuit, we successfully obtained the \( E-J \) properties over the wide electric field ranges of \( 1.4 \times 10^{-6}–1.6 \times 10^{-4} \) V/m for the 2inch \( \phi \) Y-123 standard film and of \( 5.7 \times 10^{-7}–7.3 \times 10^{-5} \) V/m for the 20×20 mm square Dy-123 sample film. The critical current density \( J_C \) determined by the \( V_3 \) inductive method at \( E=1 \times 10^{-4} \) V/m changed from the average by a maximum of \( \pm 0.52\% \), while that determined by the transport method at \( E=1 \times 10^{-3} \) V/m changed by a maximum of \( \pm 2.32\% \), and the maximum margin of error of \( J_C \) became remarkably small in the \( V_3 \) inductive method. The \( E-J \) properties of the Dy-123 sample film agreed well with the results measured by another research group. \( J_C \) at \( 10^{-4} \) V/m and \( n \) value were \( 2.23 \times 10^{10} \) A/m\(^2\) and 26.74 as the average of four different positions, and each systematic errors were estimated as \(-8.6\%\) and \(+1.4\%\), respectively, with comparison of the averages of 16 positions measured by the same research group.

5. References

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