The Vector Meson And Heavy Meson Strong Interaction

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We calculate the coupling constants between the light vector mesons and heavy mesons within the framework of the light-cone QCD sum rule in the leading order of heavy quark effective theory. The sum rules are very stable with the variations of the Borel parameter and the continuum threshold. The extracted couplings will be useful in the study of the possible heavy meson molecular states. They may also helpful in the interpretation of the proximity of X(3872), Y(4260) and Z(4430) to the threshold of two charmed mesons through the couple-channel mechanism.

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I. INTRODUCTION

A number of hadronic states which can not be easily accommodated in the conventional quark model have been observed experimentally in recent years, such as X(3872), Y(4260) and Z(4430). Their masses are very close to the thresholds of DD*, D*D* and D*D1 respectively. It was speculated that the coupled-channel effect may play an important role because of the attraction between these D mesons. Alternatively, they were considered to be possible candidates of the heavy molecular states composed of two D mesons. These loosely bound states are formed by exchanging light mesons such as π, σ, ρ and ω etc. Up to now, the pion heavy meson strong interaction is relatively known due to chiral symmetry. However, the vector meson heavy meson strong interaction has not be extensively studied yet, which accounts for the relatively short distance interaction between two heavy mesons.

Heavy quark effective theory (HQET) [4] is a systematic approach to study the spectra and transition amplitudes of heavy hadrons. In HQET, the expansion is performed in terms of 1/mQ, where mQ is the mass of the heavy quark involved. The two states in a doublet share the same quantum number J, P, j respectively. It was speculated that the coupled-channel effect may play an important role because of the attraction between these two D mesons. Alternatively, they were considered to be possible candidates of the heavy molecular states composed of two D mesons. These loosely bound states are formed by exchanging light mesons such as π, σ, ρ and ω etc. Up to now, the pion heavy meson strong interaction is relatively known due to chiral symmetry. However, the vector meson heavy meson strong interaction has not been extensively studied yet, which accounts for the relatively short distance interaction between two heavy mesons.

In this work we use LCQSR to calculate the coupling constants between the light vector mesons and heavy mesons. The coupling between doublets $T_{ij}$ was calculated with LCQSR in full QCD in Ref. [7]. The couplings $\beta H_{ij}$ and $f H_{ij}$ were calculated in full QCD in Ref. [8]. Their values in the limit $m_Q \to \infty$ are also discussed in this paper. The coupling between doublets $T$ and $H$ are studied in the leading order of HQET in Ref. [9].

In this work we use LCQSR to calculate the coupling constants between three doublets $H, S, T$ and within the two doublets $H, S$. Due to the covariant derivative in the interpolating currents of $T$ doublet, the contribution from the 3-particle light-cone distribution amplitudes of the $\rho$ meson has to be included when dealing with the $\rho$ decay between doublets $T$ and $H(S)$. We work in HQET to differentiate the two states with the same $J^P$ value yet quite different decay widths. The interpolating currents $J_{ij}^{\alpha_1 \cdots \alpha_j}$ adopted in our work have been properly constructed in Ref. [10]. They satisfy

$$
\langle 0 | J_{ij}^{\alpha_1 \cdots \alpha_j} (0) | j', P', j'_i \rangle = f_{Pj} \delta_{jj'} \delta_{PP'} \delta_{ji} \eta^{\alpha_1 \cdots \alpha_j},
$$

$$
i(0) [ T \{ J_{ij}^{\alpha_1 \cdots \alpha_j} (x) J_{ij}^{\beta_1 \cdots \beta_j} (0) \} | 0 \} = \delta_{j_1 j_2} \delta_{P_1 P_2} \delta_{j_1 j_2} (-1)^i \int dt \delta (x - vt) \Pi_{Pj} (x),
$$

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in the limit \( m_Q \to \infty \). Here \( \eta^{\alpha_1 \cdots \alpha_j} \) is the polarization tensor for the spin \( j \) state, \( v \) is the velocity of the heavy quark, \( g^{\alpha \beta}_l = g^{\alpha \beta} - \nu^{\alpha} v^{\beta} \), \( S \) denotes symmetrizing the indices and subtracting the trace terms separately in the sets \( \{\alpha_1 \cdots \alpha_j\} \) and \( \{\beta_1 \cdots \beta_j\} \).

## II. Sum Rules for the \( \rho \) Coupling Constants

We shall perform the calculation to the leading order of HQET. According to Ref. [10], the interpolating currents for doublets \( H, S \) and \( T \) read as

\[
J^1_{0, -\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 q_v \tag{3}
\]
\[
J^{1\alpha}_{1, -\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_\alpha q_v \tag{4}
\]
\[
J^1_{0, +\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v q_v \tag{5}
\]
\[
J^{1\alpha}_{1, +\frac{1}{2}} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_\alpha q_v \tag{6}
\]
\[
J_{1, +\frac{1}{2}} = \sqrt{\frac{3}{4}} \bar{h}_v \gamma_5 (-i) \left\{ D^{\mu}_t - \frac{1}{3} \gamma_\alpha \bar{D}_\alpha \right\} q_v \tag{7}
\]
\[
J^{\alpha_1 \alpha_2}_{2, +\frac{1}{2}} = \sqrt{\frac{1}{8}} \bar{h}_v (-i) \left\{ \gamma^{\alpha_1} D^{\alpha_2} + \gamma^{\alpha_2} D^{\alpha_1} - \frac{2}{3} \gamma^{\alpha_1 \alpha_2} \bar{D}_I \right\} q_v \tag{8}
\]

where \( h_v \) is the heavy quark field in HQET, \( \gamma_\mu \equiv \gamma^\mu - \hat{v} \gamma^5 \mu \), \( D^{\mu}_t \equiv D^\mu - (D \cdot v) v^\mu \), \( g^{\mu \nu}_l \equiv g^{\mu \nu} - v^\mu v^\nu \), and \( v \) is the velocity of the heavy quark.

We consider the \( \rho \) decay of \( T_1 \) to \( H_1 \) to illustrate our calculation. Here the subscript of \( T(H) \) indicates the spin of the meson involved. Owing to the conservation of the angular momentum of light components in the limit \( m_Q \to \infty \), there are three independent \( \rho \) coupling constants between doublets \( T \) and \( H \). We denote them as \( g_{1T1H}^{2\rho} \), \( g_{1T1H}^{1\rho} \) and \( g_{1T1H}^{d2}\rho \), where \( s, p, d \cdots \) and the number following them indicates the orbital and total angular momentum \( (l, j_h) \) of the final \( \rho \) meson respectively. All of these three coupling constants appear in the decay process under consideration. The decay amplitude can now be written as

\[
\mathcal{M}(T_1 \to H_1 + \rho) = I_i \left\{ e^{\nu^* \nu} g_{1T1H}^{d1} + \left[ e^{\nu^* \nu} (e^* \cdot q_v) - \frac{1}{3} e^{\nu^* \nu} q_v^2 \right] g_{1T1H}^{d2}\rho \right. + \left. e^{\nu^* \nu} (\eta \cdot q_v) + e^{\nu^* \nu} (\eta \cdot q_v) \right\} g_{1T1H}^{d2}\rho \tag{9}
\]

where \( \eta, e^* \) and \( e^* \) denote the polarization vector of \( T_1, H_1 \) and \( \rho \) respectively, \( q \) is the momentum of the \( \rho \) meson, \( q^2 = m_\rho^2 \) and \( q_\nu^2 \equiv \sum \mu q^\mu - (q \cdot v) v^\mu \). \( I = 1, 1/\sqrt{2} \) for the charged and neutral \( \rho \) meson respectively. The vector notations in Levi-Civita tensor come from an index contraction between Levi-Civita tensor and the vectors, for example, \( e^{\nu^* \nu} \equiv \epsilon_{\mu \nu \rho \sigma} \eta^\mu e^\nu \epsilon^\rho_{\nu} \eta^\sigma \).

To obtain the sum rules for the coupling constants \( g_{1T1H}^{d1}\rho \), \( g_{1T1H}^{d1}\rho \) and \( g_{1T1H}^{d2}\rho \), we consider the correlation functions

\[
\int d^4 x e^{-ik \cdot x} \langle \rho(q)|T\{J_{1, -\frac{1}{2}}^1(0) J_{1, +\frac{1}{2}}^{1\alpha}(x)\}|0\rangle = I_i \left\{ e^{\alpha \beta \nu^* \nu} G_{1T1H}^{1\alpha}(\omega, \omega') + \left[ e^{\mu \beta \nu^* \nu} (q_v^* \cdot q_v) - \frac{1}{3} e^{\mu \beta \nu^* \nu} q_v^2 \right] G_{1T1H}^{1\rho}(\omega, \omega') \right. + \left. e^{\nu^* \nu} q_v^\beta + e^{\mu \beta \nu^* \nu} q_v^\alpha \right\} G_{1T1H}^{d2}\rho(\omega, \omega') \tag{10}
\]

where \( \omega \equiv 2v \cdot k, \omega' \equiv 2v \cdot (k - q) \). In the leading order of HQET, the heavy quark propagator reads as

\[
\langle 0|T\{h_v(0)h_v(x)\}|0\rangle = \frac{1 + \hat{v} t}{2} \int dt \delta^4(-x - vt). \tag{11}
\]
The correlation function can now be expressed as
\[
-\sqrt{\frac{3}{8}} \int dx e^{-ikx} \int_0^\infty dt \delta(-x-\nu) \text{Tr} \left\{ \gamma^\mu \left( \frac{1}{2} + \hat{u} \right) (-i\gamma_5)(D^\mu_\alpha - \frac{1}{3} \gamma^\alpha \hat{D}_\alpha)(\rho(q)|q(x)\bar{q}(0)|0) \right\}. \tag{12}
\]

It can be further calculated using the light cone wave functions of the \( \rho \) meson. To our approximation, we need the two and three-particle light-cone wave functions. Their definitions are collected in the Appendix B.

At the hadron level, the \( G' \)s in \([13]\) have the following pole terms
\[
G_{T_1H_1\rho}(\omega,\omega') = \frac{f_{-1/2}f_{+3/2}G_{T_1H_1\rho}}{(2\Lambda_{-1/2}-\omega')(2\Lambda_{+3/2}-\omega)} + \frac{c}{2\Lambda_{-1/2}-\omega'} + \frac{c'}{2\Lambda_{+3/2}-\omega'}, \tag{13}
\]
where \( \Lambda_{-1/2} \equiv m_H - m_Q \), \( \Lambda_{+3/2} \equiv m_T - m_Q \). \( f_{-1/2} \) etc is the overlap amplitudes of their interpolating currents with the heavy mesons.

\( G_{T_1H_1\rho}(\omega,\omega') \) can now be expressed by the \( \rho \) meson light-cone wave functions. After the Wick rotation and the double Borel transformation with \( \omega \) and \( \omega' \), the single-pole terms in \([13]\) are eliminated. We arrive at
\[
\sqrt{3g_{T_1H_1\rho}^1} f_{-1/2} f_{+3/2} e^{-\frac{\omega+\omega'}{2} + \frac{\Lambda_{+3/2}+\Lambda_{-1/2}}{4}} = -f_T^T m_{\rho p}^2 h_\parallel^1(\bar{u}_0) \frac{1}{T^2} + \frac{f_T^T m_{\rho p}^2 S_{-1,0}^1(\bar{u}_0) \frac{1}{T}}{24} + \frac{f_T^T m_{\rho p}^2 A_T(\bar{u}_0) \frac{1}{T}}{12} + \ldots \label{eq:14}
\]
\[ -f_\rho^T m_\rho^2 T^{-1,0}(u_0) \frac{1}{T} + f_\rho^T m_\rho^2 T^{-1,0}(u_0) \frac{1}{T} - f_\rho^T m_\rho^2 T^{-1,0}(u_0) \frac{1}{T} - \frac{1}{4} f_\rho m_\rho A^{[2]}(u_0) \frac{1}{T^2} \\
+ \frac{1}{4} f_\rho m_\rho A^{[1]}(\bar{u}_0) \frac{1}{T^2} - \frac{1}{4} f_\rho m_\rho A^{[1]}(\bar{u}_0) \frac{1}{T^2} - 4 f_\rho m_\rho C^{[4]}(\bar{u}_0) \frac{1}{T^2} + 2 f_\rho m_\rho C^{[3]}(\bar{u}_0) \frac{1}{T^2} \\
- 2 f_\rho m_\rho A^{[0,0]}(u_0) - \frac{1}{4} f_\rho m_\rho g^{[a]}_\perp(\bar{u}_0) - \frac{1}{8} f_\rho m_\rho g^{[a]}(\bar{u}_0) \\
+ \frac{1}{8} f_\rho m_\rho g^{[a]}_\perp(\bar{u}_0) \bar{u}_0 - f_\rho m_\rho g^{[a]}(\bar{u}_0) + f_\rho m_\rho A^{[0,0]}(u_0) + f_\rho m_\rho A^{[2]}(u_0) - f_\rho m_\rho A^{[1]}(u_0) \\
+ f_\rho m_\rho (u_0) \frac{1}{T^2} - 4 F_{\rho} m_\rho T^{-2,0}(u_0) \frac{1}{T^2} - 4 F_{\rho} m_\rho b^{[2,0]}(u_0) \frac{1}{T^2}, \] 

\[ \frac{\sqrt{3} g_{\rho T_{1} H_{1}} f_{\rho}}{f_{\rho} + \frac{3}{2} e^{-\frac{\lambda_{s}}{2}} + \frac{\lambda_{s}}{2}} = \frac{3}{16} f_\rho^T m_\rho^2 A T(\bar{u}_0) \frac{1}{T} - \frac{3}{16} f_\rho^T m_\rho^2 A T(\bar{u}_0) \bar{u}_0 \frac{1}{T} - \frac{3}{4} f_\rho^T \varphi_\perp(\bar{u}_0) T f_0(\omega_c) + \frac{3}{4} f_\rho^T \varphi_\perp(\bar{u}_0) \bar{u}_0 T f_0(\omega_c) \\
- 3 f_\rho^T m_\rho^2 T^{-1,0}(u_0) \frac{1}{T} - 3 f_\rho^T m_\rho^2 T^{-1,0}(u_0) \frac{1}{T} + \frac{3}{2} f_\rho m_\rho A^{[0,0]}(u_0) - \frac{3}{8} f_\rho m_\rho g^{[a]}_\perp(\bar{u}_0) + \frac{3}{8} f_\rho m_\rho g^{[a]}(\bar{u}_0) \bar{u}_0, \] 

where \( f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} x^k \) is the continuum subtraction factor, and \( \omega_c \) is the continuum threshold, \( u_0 = \frac{T}{T_1 + T_2} \), \( T = \frac{T_1 T_2}{T_1 + T_2} \), and \( \bar{u}_0 = 1 - u_0 \). \( T_1 \) and \( T_2 \) are the two Borel parameters. We have used the Borel transformation \( \tilde{g}_{\rho}^T e^{\alpha \omega} = \delta(\alpha - \frac{1}{T}) \) to obtain (14), (15), and (16). In the above expressions we have used the following functions \( F^{[a]}(u_0) \) and \( F^{[a,b]}(u_0) \). They are defined as:

\[ F^{[a]}(u_0) = \int_{0}^{u_0} \cdots \int_{0}^{x_3} \int_{0}^{x_2} F(x_1) dx_1 dx_2 \cdots dx_n, \] 

\[ F^{[a,0]}(u_0) = \int_{0}^{u_0} \int_{0}^{1-\alpha_2} F(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2, \] 

\[ F^{[1,0]}(u_0) = \int_{0}^{u_0} \frac{F(1 - u_0, \alpha, \alpha_2, \alpha_3)}{\alpha_2} d\alpha_2 - \int_{0}^{1-u_0} F(u_0, 1 - u_0, \alpha_2, \alpha_3) d\alpha_2, \] 

\[ F^{[2,0]}(u_0) = \int_{0}^{u_0} \frac{\partial [F(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)]}{\alpha_2} \bigg|_{\alpha_3 = u_0} - \int_{0}^{1-u_0} \frac{\partial [F(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)]}{\alpha_2} \bigg|_{\alpha_3 = u_0}, \] 

\[ F^{[-1,0]}(u_0) = \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 + \int_{0}^{u_0} \int_{0}^{1-\alpha_2} (u_0 - \alpha_2) F(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2, \] 

\[ F^{[-2,0]}(u_0) = \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} \int_{0}^{\alpha_2} F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 \] 

\[ + \frac{1}{2} \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} \int_{0}^{\alpha_2} F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 \] 

\[ + \frac{1}{2} \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} (u_0 - \alpha_2)^2 F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2, \] 

\[ F^{[-3,0]}(u_0) = \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} \int_{0}^{\alpha_2} \int_{0}^{\alpha_3} F(1 - \alpha_2 - x_1, \alpha_2, \alpha_3) dx_1 dx_2 d\alpha_3 d\alpha_2 \] 

\[ + \frac{1}{2} \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} \int_{0}^{\alpha_3} x F(1 - \alpha_2 - x, \alpha_2, \alpha_3) dx_1 dx_2 d\alpha_3 d\alpha_2 \] 

\[ + \frac{1}{2} \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} \alpha_2^3 F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 \] 

\[ + \frac{1}{6} \int_{0}^{u_0} \int_{0}^{u_0-\alpha_2} (u_0 - \alpha_2)^3 F(1 - \alpha_2, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2, \]
III. NUMERICAL ANALYSIS

In our numerical analysis, we need the mass parameters $\bar{\Lambda}$’s and $f$’s, the overlapping amplitudes of these interpolating currents. We adopt $\bar{\Lambda}_{-1/2}$ from Ref. [11]: $\bar{\Lambda}_{-1/2} = 0.5$ GeV, $f_{-1/2} = 0.25 \pm 0.04$ GeV$^{3/2}$. $\bar{\Lambda}_{+1/2}$, $f_{+1/2}$, $\bar{\Lambda}_{+3/2}$, and $f_{+3/2}$ are given in Ref. [12]:

\[ \begin{align*}
\bar{\Lambda}_{+1/2} &= 1.15 \text{ GeV}, \quad f_{+1/2} = -0.40 \pm 0.06 \text{ GeV}^{3/2}, \\
\bar{\Lambda}_{+3/2} &= 0.82 \text{ GeV}, \quad f_{+3/2} = 0.19 \pm 0.03 \text{ GeV}^{5/2}.
\end{align*} \]

The parameters appear in the distribution amplitudes of the $\rho$ meson take the values from Ref. [14]. We use the values at the scale $\mu = 1$ GeV in our calculation under the consideration that the heavy quark behaves almost as a spectator of the decay processes in our discussion in the leading order of HQET:

| $f_ρ[\text{MeV}]$ | $f_ρ^±[\text{MeV}]$ | $a_2^∥$ | $a_2^⊥$ | $ζ_3^∥$ | $ζ_3^⊥$ | $ω_3^∥$ | $ω_3^⊥$ | $ζ_4^∥$ | $ω_4^∥$ | $ζ_4^⊥$ | $ω_4^⊥$ |
|-------------------|-------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 216(3)            | 165(9)            | 0.15(7)| 0.14(6)| 0.030(10)| -0.09(3)| 0.15(5)| 0.55(25)| 0.07(3)| -0.03(1)| -0.03(5)| -0.08(5) |

We will work at the symmetry point, i.e., $T_1 = T_2 = 2T$, $u_0 = 1/2$. This comes from the observation that the mass differences between $H$, $S$ and $T$ are less than 0.4 GeV in the leading order of HQET. They are much smaller than the Borel parameter $T_1$, $T_2 \sim 3$ GeV used below. On the other hand, every reliable sum rule has a working window of the Borel parameter $T$ within which the sum rule is insensitive to the variation of $T$. So it is reasonable to choose a common point $T_1 = T_2$ at the overlapping region of $T_1$ and $T_2$. Furthermore, choosing $T_1 = T_2$ will enable us to subtract the continuum contribution cleanly while the asymmetric choice will lead to the continuum subtraction very difficult [15].

From the requirement that the pole contribution is larger than 60%, we get the upper bound of the Borel parameter. This leads to $T < 1.7$ GeV. The convergence requirement of the operator product expansion leads to the lower bound of the Borel parameter $T = 1.3$ GeV, starting from which the stability of the sum rule develops. The resulting sum rules are plotted in Fig[12] and [3] in the working interval 1.3 GeV $< T < 1.7$ GeV and $\omega_c = 2.8, 3.0, 3.2$ GeV.

![Figure 1](image.png)

**FIG. 1:** The sum rule for $g_{T,H,ρ}^0 f_{-1/2}^± f_{+3/2}$ with $\omega_c = 2.8, 3.0, 3.2$ GeV

Other $ρ$ coupling constants between $H$, $S$ and $T$ doublets can be calculated in the same way as $g_{T,H,ρ}$. Their definitions and the relevant correlators are given in Appendix A. Here we simply present the sum rules for these coupling constants:

\[
g_{H,T,ρ}^0 f_{-1/2}^2 = \frac{1}{2\sqrt{2}} e^{25/12} \left\{ -f_ρ m_ρ^3 A^{[1]}(\bar{u}_0) \frac{1}{T^2} - 8 f_ρ m_ρ^3 C^{[3]}(\bar{u}_0) \frac{1}{T^2} \right\}
\]
FIG. 2: The sum rule for $g_{T_{1}H_{1}f_{-1/2}^{1/2}}$ with $\omega_c = 2.8, 3.0, 3.2\text{GeV}$

FIG. 3: The sum rule for $g_{T_{1}H_{1}f_{-1/2}^{1/2}}$ with $\omega_c = 2.8, 3.0, 3.2\text{GeV}$

$$g_{T_{1}H_{1}f_{-1/2}^{1/2}} = \frac{1}{4\sqrt{2}} e^{2\Delta_{-1/2}} \left\{ f_{T_{1}} m_{T_{1}}^{2} h_{T_{1}}^{\parallel} (\tilde{u}_{0}) \frac{1}{T} + 4 f_{T_{1}} m_{T_{1}}^{2} \varphi_{T_{1}}^{\parallel} (\tilde{u}_{0}) \right\},$$

$$g_{S_{1}S_{1}f_{-1/2}^{1/2}} = \frac{1}{2\sqrt{2}} e^{2\Delta_{-1/2}} \left\{ -4 f_{T_{1}} m_{T_{1}}^{2} h_{T_{1}}^{\parallel} (\tilde{u}_{0}) \frac{1}{T} - f_{T_{1}} m_{T_{1}}^{2} A_{T_{1}}^{\parallel} (\tilde{u}_{0}) \frac{1}{T^2} - 8 f_{T_{1}} m_{T_{1}}^{2} C_{T_{1}}^{\parallel} (\tilde{u}_{0}) \right\},$$

$$g_{S_{1}S_{1}f_{-1/2}^{1/2}} = \frac{1}{8\sqrt{2}} e^{2\Delta_{-1/2}} \left\{ -f_{T_{1}} m_{T_{1}}^{2} A_{T_{1}}^{\parallel} (\tilde{u}_{0}) \frac{1}{T} + 4 f_{T_{1}} m_{T_{1}}^{2} \varphi_{T_{1}}^{\parallel} (\tilde{u}_{0}) \right\} - 2 f_{T_{1}} m_{T_{1}}^{2} g_{T_{1}}^{(a)} (\tilde{u}_{0}) \right\},$$

$$g_{S_{0}H_{1}f_{-1/2}^{1/2}} = \frac{1}{8\sqrt{2}} e^{2\Delta_{-1/2}} \left\{ -8 f_{T_{1}} m_{T_{1}}^{2} C_{T_{1}}^{\parallel} (\tilde{u}_{0}) + f_{T_{1}} m_{T_{1}}^{2} A_{T_{1}}^{\parallel} (\tilde{u}_{0}) \right\}.$$
\[
\begin{align*}
\sqrt{3g^2_{1,1,\rho}} f_T f_{-T} &+ e^{-\frac{5\rho}{2}} g^2_{1,1,\rho} f_T f_{-T} + e^{-\frac{5\rho}{2}} g^2_{1,1,\rho} f_T f_{-T}
\end{align*}
\]

\[
\begin{align*}
g^2_{1,1,\rho} f_T f_{-T} &+ e^{-\frac{5\rho}{2}} g^2_{1,1,\rho} f_T f_{-T} + e^{-\frac{5\rho}{2}} g^2_{1,1,\rho} f_T f_{-T}
\end{align*}
\]
Due to heavy quark symmetry, the $\rho$ coupling constants with the same $(l, j_h)$ between two doublets are not independent in the leading order of HQET. The values of these coupling constants multiplied by the decay constants of the initial and the final heavy mesons are:

\[
\begin{align*}
\bar{g}^\rho_{H_0 H_\rho} &= -g^\rho_{H_1 H_\rho} = -0.32 \pm 0.04 \text{ GeV}^2, \\
\bar{g}^\rho_{H_1 H_\rho} &= g^\rho_{H_1 H_\rho} = -0.46 \pm 0.01 \text{ GeV}^2, \\
\bar{g}^d_{S_0 H_\rho} &= -g^d_{S_1 H_\rho} = -g^d_{S_1 H_\rho} = -0.39 \pm 0.03 \text{ GeV}^3, \\
\bar{g}^d_{S_0 H_\rho} &= -g^d_{S_1 H_\rho} = -g^d_{S_1 H_\rho} = -0.38 \pm 0.06 \text{ GeV}, \\
\bar{g}^p_{S_0 S_\rho} &= -g^p_{S_1 S_\rho} = -0.32 \pm 0.03 \text{ GeV}^2, \\
\bar{g}^p_{S_0 S_\rho} &= -g^p_{S_1 S_\rho} = -0.45 \pm 0.02 \text{ GeV}, \\
\bar{g}^d_{T_1 H_\rho} &= -2\sqrt{\frac{2}{3}} g^d_{T_2 H_\rho} = -0.04 \pm 0.002 \text{ GeV}^4, \\
\bar{g}^p_{T_1 H_\rho} &= 2\sqrt{\frac{2}{3}} g^p_{T_2 H_\rho} = -0.47 \pm 0.06 \text{ GeV}^2, \\
\bar{g}^d_{T_1 H_\rho} &= \sqrt{\frac{3}{2}} g^d_{T_2 H_\rho} = \sqrt{6} g^d_{T_2 H_\rho} = -0.15 \pm 0.01 \text{ GeV}^2, \\
\bar{g}^p_{T_1 S_\rho} &= 2\sqrt{\frac{2}{3}} g^p_{T_2 S_\rho} = 0.27 \pm 0.02 \text{ GeV}^3, \\
\bar{g}^d_{T_2 S_\rho} &= -\sqrt{\frac{3}{2}} g^d_{T_2 S_\rho} = -\sqrt{6} g^d_{T_2 S_\rho} = -0.28 \pm 0.02 \text{ GeV}^3, \\
\bar{g}^d_{T_2 S_\rho} &= -\sqrt{\frac{3}{2}} g^d_{T_2 S_\rho} = -\sqrt{6} g^d_{T_2 S_\rho} = 0.31 \pm 0.04 \text{ GeV},
\end{align*}
\]

where $g^\rho_{H_0 H_\rho} = \tilde{g}^\rho_{H_0 H_\rho} f_\rho^{1/2}$ etc. The errors come from the variations of $T$ and $\omega_c$ in the working region and the central value corresponds to $T = 1.5\text{ GeV}$ and $\omega_c = 3.0\text{ GeV}$. The g's with their errors are

\[
\begin{align*}
\bar{g}^\rho_{H_0 H_\rho} &= -g^\rho_{H_1 H_\rho} = -5.1 \pm 0.6 \pm 1.3 \text{ GeV}^{-1}, \\
\bar{g}^d_{H_1 H_\rho} &= g^d_{H_1 H_\rho} = -7.4 \pm 0.2 \pm 1.8 \text{ GeV}^{-1}, \\
\bar{g}^d_{S_0 H_\rho} &= -g^d_{S_1 H_\rho} = -g^d_{S_1 H_\rho} = 3.9 \pm 0.3 \pm 1.0, \\
\bar{g}^d_{S_0 H_\rho} &= -g^d_{S_1 H_\rho} = -g^d_{S_1 H_\rho} = 3.8 \pm 0.6 \pm 0.9 \text{ GeV}^{-2}, \\
\bar{g}^p_{S_0 S_\rho} &= -g^p_{S_1 S_\rho} = -2.0 \pm 0.2 \pm 0.5 \text{ GeV}^{-1}, \\
\bar{g}^p_{S_0 S_\rho} &= -g^p_{S_1 S_\rho} = -2.8 \pm 0.2 \pm 0.7 \text{ GeV}^{-1}, \\
\bar{g}^d_{T_1 H_\rho} &= -2g^d_{T_1 H_\rho} = -2\sqrt{\frac{2}{3}} g^d_{T_2 H_\rho} = -0.8 \pm 0.05 \pm 0.2, \\
\bar{g}^d_{T_1 H_\rho} &= 2g^d_{T_1 H_\rho} = -2\sqrt{\frac{2}{3}} g^d_{T_2 H_\rho} = -9.9 \pm 1.3 \pm 2.5 \text{ GeV}^{-2}, \\
\bar{g}^d_{T_2 H_\rho} &= \sqrt{\frac{3}{2}} g^d_{T_2 H_\rho} = \sqrt{6} g^d_{T_2 H_\rho} = -3.1 \pm 0.1 \pm 0.8 \text{ GeV}^{-2},
\end{align*}
\]
\[ g_{T^1_{S_0 \rho}}^{p_1} = 2g_{T^1_{S_1 \rho}}^{p_1} = 2\sqrt{\frac{2}{3}} g_{T^0_{S_0 \rho}}^{p_0} = 3.5 \pm 0.3 \pm 0.9 \text{ GeV}^{-1}, \]
\[ g_{T^0_{S_1 \rho}}^{p_2} = -\sqrt{\frac{3}{2}} g_{T^0_{S_0 \rho}}^{p_0} = -\sqrt{6} g_{T^0_{S_1 \rho}}^{p_0} = -3.7 \pm 0.3 \pm 0.9 \text{ GeV}^{-1}, \]
\[ g_{T^0_{S_1 \rho}}^{f_2} = -\sqrt{\frac{3}{2}} g_{T^0_{S_0 \rho}}^{f_0} = -\sqrt{6} g_{T^0_{S_1 \rho}}^{f_0} = 4.1 \pm 0.5 \pm 1.0 \text{ GeV}^{-3}. \] (34)

The second error comes from the uncertainty of f’s. The above relations between coupling constants are consistent with the HQET leading order expectation.

Replacing the ρ meson parameters by those for the ω meson, one obtains the ω meson couplings with the heavy mesons:

\[ \tilde{g}_{T^0_{H_0 \omega}}^{p_0} = -\tilde{g}_{T^1_{H_1 \omega}}^{p_0} = -0.29 \pm 0.04 \text{ GeV}^2, \]
\[ \tilde{g}_{T^0_{H_0 \omega}}^{p_1} = \tilde{g}_{T^1_{H_1 \omega}}^{p_1} = -0.41 \pm 0.01 \text{ GeV}^2, \]
\[ \tilde{g}_{S_0 H_0 \omega}^{p_0} = \tilde{g}_{S_1 H_0 \omega}^{p_0} = -0.36 \pm 0.03 \text{ GeV}^3, \]
\[ \tilde{g}_{S_0 H_0 \omega}^{p_1} = \tilde{g}_{S_1 H_0 \omega}^{p_1} = -0.33 \pm 0.05 \text{ GeV}, \]
\[ \tilde{g}_{S_0 S_0 \omega}^{p_0} = \tilde{g}_{S_1 S_0 \omega}^{p_0} = -0.29 \pm 0.03 \text{ GeV}^2, \]
\[ \tilde{g}_{S_0 S_1 \omega}^{p_1} = -0.38 \pm 0.02 \text{ GeV}^2, \]
\[ \tilde{g}_{T^1_{H_1 \omega}}^{p_1} = -2\sqrt{\frac{2}{3}} \tilde{g}_{T^1_{H_1 \omega}}^{p_1} = -0.94 \pm 0.002 \text{ GeV}^4, \]
\[ \tilde{g}_{T^2_{H_1 \omega}}^{p_1} = -2\sqrt{\frac{2}{3}} \tilde{g}_{T^2_{H_1 \omega}}^{p_1} = -0.33 \pm 0.05 \text{ GeV}^2, \]
\[ \tilde{g}_{T^1_{H_1 \omega}}^{p_2} = \sqrt{\frac{3}{2}} \tilde{g}_{T^1_{H_1 \omega}}^{p_2} = -0.13 \pm 0.01 \text{ GeV}^2, \]
\[ \tilde{g}_{T^1_{S_1 \omega}}^{p_2} = 2\sqrt{\tilde{g}_{T^1_{S_1 \omega}}^{p_1} = 0.15 \pm 0.01 \text{ GeV}^3, \]
\[ \tilde{g}_{T^2_{S_1 \omega}}^{p_1} = -\sqrt{\frac{3}{2}} \tilde{g}_{T^2_{S_1 \omega}}^{p_2} = -0.16 \pm 0.01 \text{ GeV}^3, \]
\[ \tilde{g}_{T^1_{S_1 \omega}}^{p_1} = -\sqrt{\frac{3}{2}} \tilde{g}_{T^2_{S_1 \omega}}^{p_2} = 0.17 \pm 0.02 \text{ GeV}^2, \] (35)
\[ g_{T_1 S_1 \omega}^{f_2} = -\sqrt{\frac{3}{2}} g_{T_2 S_0 \omega}^{f_2} = -\sqrt{6} g_{T_2 S_1 \omega}^{f_2} = 3.6 \pm 0.4 \pm 0.9 \text{ GeV}^{-3}. \]  

**IV. CONCLUSION**

We have calculated the light vector meson couplings with heavy mesons in the leading order of HQET within the framework of LCQSR. The sum rules are stable with the variations of the Borel parameter and the continuum threshold. Some possible sources of the errors in our calculation include the inherent inaccuracy of LCQSR: the omission of the higher order terms in OPE, the choice of \( \omega_c \), the variation of the coupling constant with the Borel parameter \( T \) in the working interval and the approximation in the light-cone distribution amplitudes of the \( \rho \) meson. The uncertainty in \( f_s \)'s and \( \bar{\Lambda}'s \) also leads to errors.

The extracted vector meson heavy meson coupling constants may be helpful in the study of the interaction between two \( B(D) \) mesons. They may play an important role in the formation of these possible molecular candidates composed of two \( B(D) \) mesons. They may also play a role in the interpretation of the proximity of \( X(3872), Y(4260) \) and \( Z(4430) \) to the threshold of two charmed mesons through the couple-channel mechanism.

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**APPENDIX A: THE \( \rho \) DECAY AMPLITUDES OF HEAVY MESONS**

The definitions of the \( \rho \) coupling constants not presented in the text are

\[ \mathcal{M}(H_0 \to H_0 + \rho) = (e^* \cdot q_i) g_{H_0 H_0 \rho}^{p_0}, \]

\[ \mathcal{M}(H_1 \to H_0 + \rho) = e^{v^* q^v} g_{H_1 H_0 \rho}^{p_1}. \]  

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\[ M(H_1 \rightarrow H_1 + \rho) = (e^* \cdot q_1)(e^* \cdot \eta_1)g_{H_1 H_1 \rho}^{00} + \left[(e^* \cdot \eta_1)(e^* \cdot q_1) - (e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{H_1 H_1 \rho}^{11}, \]  
(33)

\[ M(S_0 \rightarrow S_0 + \rho) = (e^* \cdot q_1)g_{S_0 S_0 \rho}^{00}, \]  
(34)

\[ M(S_1 \rightarrow S_0 + \rho) = e^{\nu e^* v} g_{S_1 S_0 \rho}^{11}, \]  
(35)

\[ M(S_1 \rightarrow S_1 + \rho) = (e^* \cdot q_1)(e^* \cdot \eta_1)g_{S_1 S_1 \rho}^{00} + \left[(e^* \cdot \eta_1)(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{S_1 S_1 \rho}^{11}, \]  
(36)

\[ M(S_0 \rightarrow H_1 + \rho) = (e^* \cdot \epsilon_1^*)g_{H_1 S_1 \rho}^{11} + \left[(e^* \cdot q_1)(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{S_0 S_1 \rho}^{11}, \]  
(37)

\[ M(S_1 \rightarrow H_0 + \rho) = (e^* \cdot \eta_1)g_{H_1 S_1 \rho}^{11} + \left[(\eta \cdot q_1)(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \eta_1)(\eta \cdot q_1)\right]g_{S_1 S_1 \rho}^{11}, \]  
(38)

\[ M(S_1 \rightarrow H_1 + \rho) = e^{\nu e^* v} g_{S_1 H_1 \rho}^{11} + \left[e^{\nu e^* v}(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{S_1 S_1 \rho}^{11}, \]  
(39)

\[ M(T_1 \rightarrow H_0 + \rho) = (e^* \cdot \eta_1)g_{H_1 T_1 \rho}^{11} + \left[(\eta \cdot q_1)(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \eta_1)(\eta \cdot q_1)\right]g_{T_1 T_1 \rho}^{11}, \]  
(40)

\[ M(T_1 \rightarrow H_1 + \rho) = e^{\nu e^* v} g_{T_1 T_1 \rho}^{11} + \left[e^{\nu e^* v}(e^* \cdot q_1) - \frac{1}{3}(e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{T_1 T_1 \rho}^{11}, \]  
(41)

\[ M(T_1 \rightarrow S_1 + \rho) = \left[(e^* \cdot \eta_1)(e^* \cdot q_1) - (e^* \cdot \epsilon_1^*)(\eta \cdot q_1)\right]g_{T_1 S_1 \rho}^{11}, \]  
(42)

\[ M(T_1 \rightarrow S_0 + \rho) = \eta_{1\alpha 2\beta}(e^{\alpha e^* v} q_{1\alpha} q_{2\beta} + e^{\alpha e^* v} q_{1\alpha} q_{3\beta})g_{T_1 H_0 \rho}^{00}, \]  
(43)

\[ M(T_2 \rightarrow H_1 + \rho) = \eta_{1\alpha 2\beta}(e^{\alpha e^* v} q_{1\alpha} q_{2\beta} + e^{\alpha e^* v} q_{1\alpha} q_{3\beta})g_{T_2 H_1 \rho}^{00}, \]  
(44)

\[ M(T_2 \rightarrow S_0 + \rho) = \eta_{1\alpha 2\beta}(e^{\alpha e^* v} q_{1\alpha} q_{2\beta} + e^{\alpha e^* v} q_{1\alpha} q_{3\beta})g_{T_2 S_0 \rho}^{00}, \]  
(45)

\[ M(T_2 \rightarrow S_1 + \rho) = \eta_{1\alpha 2\beta}(e^{\alpha e^* v} q_{1\alpha} q_{2\beta} + e^{\alpha e^* v} q_{1\alpha} q_{3\beta})g_{T_2 S_1 \rho}^{00}, \]  
(46)

Note that these decay amplitudes may be organized in another way. For example, the tensor structure corresponding to \( g_{H_1 H_1 \rho}^{00} \) was defined as \((e^* \cdot v)(e^* \cdot \eta)\) in equation (28) of Ref. [8] rather than \((e^* \cdot q_1)(e^* \cdot \eta_1)\) in Eq. (33). Since we have \((e^* \cdot q_1) = -(q \cdot v)(e^* \cdot v)\), the essentially same sum rule as Eq. (24) of Ref. [8] can be obtained if we isolate the
tensor structure \((e^* \cdot v)(e^* \cdot \eta)\).

To derive sum rules for these coupling constants, we consider the following correlators:

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{0,-\frac{1}{2}}(0)J_{0,+\frac{1}{2}}(x)\}\rangle = (e^* \cdot q_1)G_{H_0}^{0}\langle \omega, \omega'\rangle,
\]

(A18)

\[
\int -ikx \langle \rho(q)T\{J_{0,+\frac{1}{2}}(0)J_{0,-\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}g_{H_1}^{0}\langle \omega, \omega'\rangle,
\]

(A19)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,-\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = g_{t}^{0}\langle e^* \cdot q_1\rangle G_{H_1}^{0}\langle \omega, \omega'\rangle + (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_1}^{1}\langle \omega, \omega'\rangle,
\]

(A20)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,-\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}G_{T_1}^{0}\langle \omega, \omega'\rangle,
\]

(A21)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{0,+\frac{1}{2}}(0)J_{0,+\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}G_{T_1}^{0}\langle \omega, \omega'\rangle,
\]

(A22)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = g_{t}^{0}\langle e^* \cdot q_1\rangle G_{T_1}^{0}\langle \omega, \omega'\rangle + (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_1}^{1}\langle \omega, \omega'\rangle,
\]

(A23)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}G_{T_1}^{0}\langle \omega, \omega'\rangle,
\]

(A24)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{0,+\frac{1}{2}}(0)J_{0,+\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}G_{T_1}^{0}\langle \omega, \omega'\rangle,
\]

(A25)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_2}^{1}\langle \omega, \omega'\rangle,
\]

(A26)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_2}^{1}\langle \omega, \omega'\rangle,
\]

(A27)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{1,+\frac{1}{2}}(0)J_{1,+\frac{1}{2}}(x)\}\rangle = e^{\alpha e^* v}G_{T_1}^{0}\langle \omega, \omega'\rangle,
\]

(A28)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{0,+\frac{1}{2}}(0)J_{0,+\frac{1}{2}}(x)\}\rangle = (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_2}^{1}\langle \omega, \omega'\rangle,
\]

(A29)

\[
\int d^4x e^{-ikx} \langle \rho(q)T\{J_{0,+\frac{1}{2}}(0)J_{0,+\frac{1}{2}}(x)\}\rangle = (e^* e^* q_1^2 + e^* e^* q_1^2)G_{T_2}^{1}\langle \omega, \omega'\rangle,
\]

(A30)
functions

\[ f_{\bar{q}} + \left\{ \begin{array}{l}
\alpha_1 \bar{q} e^* v q_t \\
\alpha_2 \bar{q} e^* v q_t
\end{array} \right. + \epsilon^{\alpha_1 \beta q}_v e_t^{\alpha_1} + \epsilon^{\alpha_2 \beta q}_v e_t^{\alpha_2} + \epsilon^{\alpha_1 \beta q}_e e_t^{\alpha_1} + \epsilon^{\alpha_2 \beta q}_e e_t^{\alpha_2} \right\} G_{T_{\bar{q}S_1}(\omega, \omega')}
\]

\[ - \frac{q_t^2}{2} \left\{ \begin{array}{l}
\alpha_1 \bar{q} e^* v q_t \\
\alpha_2 \bar{q} e^* v q_t
\end{array} \right. + \epsilon^{\alpha_1 \beta q}_v e_t^{\alpha_1} + \epsilon^{\alpha_2 \beta q}_v e_t^{\alpha_2} + \epsilon^{\alpha_1 \beta e}_v q_t^{\alpha_1} + \epsilon^{\alpha_2 \beta e}_v q_t^{\alpha_2} \right\} \right) G_{T_{\bar{q}S_1}(\omega, \omega')}
\]

(A33)

**APPENDIX B: THE \( \rho \) MESON LIGHT-CONE DISTRIBUTION AMPLITUDES**

The definitions of the distribution amplitudes used in the text read as

\[
\langle 0 | \bar{u}(z) \gamma_{\mu} d(\bar{z}) | \rho^-(P, \lambda) \rangle = f_{\rho} m_{\rho} \left[ \int_0^1 du e^{i \lambda p \cdot z} \phi_{\parallel}(u, \nu^2) + \epsilon_{\perp}^{(\lambda)} \int_0^1 du e^{i \lambda p \cdot z} g_{3}(u, \nu^2) \right],
\]

\[
\langle 0 | \bar{u}(z) \gamma_5 d(\bar{z}) | \rho^-(P, \lambda) \rangle = \frac{1}{2} f_{\rho} m_{\rho} \epsilon_{\mu}^{(\lambda)} \int_0^1 du e^{i \lambda p \cdot z} h_{3}(u, \nu^2),
\]

\[
\langle 0 | \bar{u}(z) \sigma_{\mu \nu} d(\bar{z}) | \rho^-(P, \lambda) \rangle = i f_{\rho}^{T} \left[ \epsilon_{\mu}^{(\lambda)} p_{\nu} - \epsilon_{\nu}^{(\lambda)} p_{\mu} \right] \int_0^1 du e^{i \lambda p \cdot z} \phi_{\perp}(u, \nu^2)
\]

\[
+ \left( p_{\mu} z_{\nu} - p_{\nu} z_{\mu} \right) \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} \int_0^1 du e^{i \lambda p \cdot z} h_{3}(u, \nu^2)
\]

\[
+ \frac{1}{2} \left( \epsilon_{\mu}^{(\lambda)} z_{\nu} - \epsilon_{\nu}^{(\lambda)} z_{\mu} \right) \frac{m_{\rho}^2}{p \cdot z} \int_0^1 du e^{i \lambda p \cdot z} h_{3}(u, \nu^2),
\]

\[
\langle 0 | \bar{u}(z) d(\bar{z}) | \rho^-(P, \lambda) \rangle = -i f_{\rho}^{T} e^{(\lambda) \cdot z} m_{\rho} \int_0^1 du e^{i \lambda p \cdot z} h_{3}(u, \nu^2).
\]

The vector and tensor decay constants \( f_{\rho} \) and \( f_{\rho}^{T} \) are defined as

\[
\langle 0 | \bar{u}(0) \gamma_{\mu} d(0) | \rho^-(P, \lambda) \rangle = f_{\rho} m_{\rho} \epsilon_{\mu}^{(\lambda)},
\]

\[
\langle 0 | \bar{u}(0) \sigma_{\mu \nu} d(0) | \rho^-(P, \lambda) \rangle = i f_{\rho}^{T} \left( \epsilon_{\mu}^{(\lambda)} P_{\nu} - \epsilon_{\nu}^{(\lambda)} P_{\mu} \right).
\]

The distribution amplitudes \( \phi_{\parallel} \) and \( \phi_{\perp} \) are of twist-2, \( g_{3}^{(\nu)} \), \( g_{3}^{(\lambda)} \), \( h_{3}^{(\sigma)} \) and \( h_{3}^{(t)} \) are twist-3 and \( g_{3}, h_{3} \) are twist-4. All functions \( \phi = \{ \phi_{\parallel}, \phi_{\perp}, g_{3}^{(\nu)}, g_{3}^{(\lambda)}, h_{3}^{(\sigma)}, h_{3}^{(t)}, g_{3}, h_{3} \} \) are normalized to satisfy \( \int_0^1 du \phi(u) = 1 \).

The 3-particle distribution amplitudes are defined as

\[
\langle 0 | \bar{u}(z) g G_{\mu \nu} \gamma_\alpha d(\bar{z}) | \rho^-(P, \lambda) \rangle = f_{\rho} m_{\rho} p_{\alpha} \left[ p_{\nu} \epsilon_{\perp}^{(\lambda)} - p_{\mu} \epsilon_{\perp}^{(\lambda)} \right] \mathcal{A}(v, p)
\]

\[
+ f_{\rho} m_{\rho} \frac{e^{(\lambda) \cdot z}}{p \cdot z} \left[ p_{\nu} g_{\alpha \nu} - p_{\mu} g_{\alpha \mu} \right] \Phi_{3}(v, p)
\]

\[
+ f_{\rho} m_{\rho} \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} p_{\alpha} \left[ p_{\mu} z_{\nu} - p_{\nu} z_{\mu} \right] \Phi_{3}(v, p)
\]

\[
\langle 0 | \bar{u}(z) g G_{\mu \nu} i \gamma_{\alpha} d(\bar{z}) | \rho^-(P, \lambda) \rangle = f_{\rho} m_{\rho} p_{\alpha} \left[ p_{\nu} \epsilon_{\perp}^{(\lambda)} - p_{\mu} \epsilon_{\perp}^{(\lambda)} \right] \mathcal{V}(v, p)
\]

\[
+ f_{\rho} m_{\rho} \frac{e^{(\lambda) \cdot z}}{p \cdot z} \left[ p_{\nu} g_{\alpha \nu} - p_{\mu} g_{\alpha \mu} \right] \Phi_{3}(v, p)
\]

\[
+ f_{\rho} m_{\rho} \frac{e^{(\lambda) \cdot z}}{(p \cdot z)^2} p_{\alpha} \left[ p_{\mu} z_{\nu} - p_{\nu} z_{\mu} \right] \Phi_{3}(v, p),
\]

\[
\langle 0 | \bar{u}(z) \sigma_{\alpha \beta} g G_{\mu \nu}(vz) d(\bar{z}) | \rho^-(P, \lambda) \rangle
\]
\[ \frac{f^T m^2}{2(p \cdot z)} [p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\nu g^\perp_{\alpha\mu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\mu g^\perp_{\alpha\nu}] T(v, pz) \]
\[ + f^T m^2 [p_\alpha e^{(\lambda)}_{\perp\beta\gamma} - p_\beta e^{(\lambda)}_{\perp\alpha\mu} - p_\alpha e^{(\lambda)}_{\perp\beta\mu} + p_\beta e^{(\lambda)}_{\perp\alpha\nu}] T^4_1(v, pz) \]
\[ + f^T m^2 [p_\mu p_\nu e^{(\lambda)}_{\perp\alpha\beta} - p_\alpha e^{(\lambda)}_{\perp\beta\gamma} - p_\nu e^{(\lambda)}_{\perp\alpha\mu} + p_\beta e^{(\lambda)}_{\perp\alpha\nu}] T^4_2(v, pz) \]
\[ + \frac{f^T m^2}{p \cdot z} [p_\alpha p_\mu e^{(\lambda)}_{\perp\beta\nu} - p_\beta p_\nu e^{(\lambda)}_{\perp\alpha\mu} - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta\mu} + p_\beta p_\mu e^{(\lambda)}_{\perp\alpha\nu}] T^4_3(v, pz) \]
\[ + \frac{f^T m^2}{p \cdot z} [p_\alpha p_\mu e^{(\lambda)}_{\perp\nu\beta} - p_\beta p_\nu e^{(\lambda)}_{\perp\mu\alpha} - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta\mu} + p_\beta p_\mu e^{(\lambda)}_{\perp\mu\alpha}] T^4_4(v, pz) \]
\[ + \ldots \quad (B9) \]

\[ (0|\bar{u}(z)gG_{\mu\nu}(vz)d(-z)|\rho^{-}(P, \lambda)) = \int f^T m^2 e^{(\lambda)}_{\perp\mu\nu} p_\nu - e^{(\lambda)}_{\perp\nu\mu} p_\nu] S(v, pz), \]
\[ (0|\bar{u}(z)i\bar{g}G_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^{-}(P, \lambda)) = \int f^T m^2 e^{(\lambda)}_{\perp\nu\mu} p_\mu - e^{(\lambda)}_{\perp\mu\nu} p_\mu] \tilde{S}(v, pz). \quad (B10) \]

where

\[ A(v, pz) = \int D\alpha e^{-ipz(\alpha_\mu - \alpha_\nu + \alpha_\sigma)} A(\alpha), \quad (B11) \]

etc. The integration measure is

\[ \int D\alpha = \int^1_0 d\alpha_d \int^1_0 d\alpha_v \int^1_0 d\alpha_\delta \left( 1 - \sum \alpha_i \right) \quad (B12) \]

The distribution amplitudes \( A, V \) and \( T \) are twist-3 and the others are twist-4.