Unruh response functions for scalar fields in de Sitter space

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We calculate the response functions of a freely falling Unruh detector in de Sitter space coupled to scalar fields of different coupling to the curvature, including the minimally coupled massless case. Although the responses differ strongly in the infrared as a consequence of the amplification of superhorizon modes, the energy levels of the detector are thermally populated.

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I. INTRODUCTION

It is expected that an observer, corresponding to an Unruh detector\(^{1,2}\) coupled to a scalar field, when freely falling in de Sitter space, will perceive radiation with a thermal spectrum of the de Sitter temperature \(T_H = H/(2\pi)\)\(^2,3\), where \(H\) denotes the Hubble parameter. It is the purpose of this article, to clarify in what sense this result is universal to scalar fields of different couplings to the de Sitter background and how the detector apprehends the differences.

Let us therefore refine what is meant by the observation of thermal radiation: At first order in perturbation theory, the detector response function is proportional to the Fourier transform of the

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scalar propagator \( w.r.t. \) the proper time of the detector, and it describes how many particles are absorbed and emitted per unit time. When being in equilibrium with the de Sitter background, the energy levels of the detector are thermally populated according to the temperature \( T_H \).

The easiest way to derive this is to note that the propagator for nonminimally coupled scalars has in the imaginary direction of its proper time \( t \) the periodicity \( t \rightarrow t + 2\pi i/T_H \). As we will point out, this however does not completely characterize the response function of the detector, which describes the number of particles detected per unit time. The rate turns out to depend on the scalar mass and on its coupling to the curvature, as was first shown in Ref. [6], circumventing the use of the scalar propagator.

The fact that in de Sitter space the invariance of the quantum vacuum becomes manifest when the scalar propagator only depends on the proper time separation along a geodesic has led to the practice of defining the de Sitter vacua through this quantity [7, 8, 9, 10, 11]. However, in the case of a massless scalar, which is minimally coupled to the curvature, this leads to a problem since the propagator is infrared divergent [10]. We argue that, when regulated by a cutoff, this divergence gives rise to a contribution which is irrelevant to the total detector response. In addition, we compare with the response functions of a detector immersed in a thermal bath in Minkowski space and discuss the situation in de Sitter space-time with dimension other than four.

\section{II. Unruh's Detector}

Unruh’s detector [1, 2, 4] corresponds to a heavy slowly moving particle along a trajectory \( x = x(t) \), which interacts with a scalar bath of particles as

\[
\mathcal{L}_{\text{Unruh}} = -\hbar \hat{m}(t) \Phi(x(t)) \, ,
\]

where \( \hbar \) is a coupling constant, \( t \) is the proper time and \( \hat{m}(t) \) represents the quantum operator describing the monopole interaction of the detector with the scalar bath \( \Phi = \Phi(x) \). Since the detector is assumed to be very heavy, it does not fluctuate in space, and hence the only time dependence is in \( \hat{m}(t) \). The response of Unruh’s detector can be derived as follows. One assumes that the state of the detector is specified by a set of energy eigenstates, \( \{ |E \rangle \} \), and that each absorption of a scalar quantum elevates the energy of the detector by \( \Delta E = E - E_0 \); the converse is true for each emission.
Consider now at first order perturbation theory the transition amplitude $\mathcal{M}$ from a state $|E_0\rangle \otimes |\varphi_0\rangle$ to a state $\langle E | \otimes \langle \varphi |$

$$\mathcal{M}(E_0 \to E; t_0, t_f) = h m_{E_0,E} \int_{t_0}^{t_f} dt_1 e^{-i(E-E_0)t_1} \langle \varphi | \Phi(x(t_1)) | \varphi_0 \rangle,$$  \hspace{1cm} (2)

where $m_{E_0,E} \equiv \langle E | \hat{m}(0) | E_0 \rangle$ ($\hat{m}(t_1) = e^{i\hat{H}t_1} \hat{m}(0)e^{-i\hat{H}t_1}$) is defined such that it does not include stimulated emission. In order to take account of the influence of initial conditions, for simplicity we assume that the interaction $\hat{m}(t_1)$ switches on at $t_1 = t_0$ as $\Theta(t_1 - t_0)$, where $\Theta(x) = 1$ when $x > 0$, and $\Theta(x) = 0$ when $x < 0$. More generally, one would expect that the interaction switches on adiabatically. Since we are not interested in studying in detail how the response function depends on the initial conditions and on how the interaction turns on, this rough treatment should suffice. Upon squaring $\mathcal{M}$ and making use of orthonormality and completeness of the scalar eigenstates $\{|\varphi\rangle\}$, and taking $t_0 \to -\infty$, we get for the transition probability per unit proper time,

$$\frac{dP(E_0 \to E)}{dt} = h^2|m_{E_0,E}|^2 \frac{dF(\Delta E)}{dt} \hspace{1cm} (\Delta E = E - E_0),$$  \hspace{1cm} (3)

where

$$\frac{dF(\Delta E)}{dt} = \int_{-\infty}^{\infty} d\Delta t e^{-i\Delta E \Delta t} iG^{<}(x(t + \Delta t/2); x(t - \Delta t/2)).$$  \hspace{1cm} (4)

is the response function per unit proper time. Here $iG^{<}(x_1; x_2) = \langle \varphi_0 | \Phi(x_2) \Phi(x_1) | \varphi_0 \rangle$ is the (positive frequency) coordinate space Wightman function, and we defined the time variables, $t \equiv (t_1 + t_2)/2$ and $\Delta t \equiv t_1 - t_2$. The expression (4) is real by construction, which can be formally shown by taking the complex conjugate of the integrand in (4) and then making use of the hermiticity property of the Wightman functions, $[iG^{<\rightarrow}(x_1, x_2)]^* = iG^{<\rightarrow}(x_2, x_1)$. We shall study the particle spectrum observed in an appropriate vacuum state $|\varphi_0 \rangle = |0 \rangle$, the precise nature of which will be specified later. It is generally accepted that it is most natural to consider (a freely-falling) Unruh detector moving on a geodesic, since in this case the detector does not see additional particles due to the Unruh effect, which would be present only because of the acceleration of the detector.

A nice property of Unruh’s detector is the separability of the transition probability (3) into a product of the selectivity function, which depends on the inner structure of the detector, which is completely specified by the operator $\hat{m}(t)$ and the coupling $h$, and the response function, which depends on the state of the scalar field only. It appears useful to rewrite Eq. (4) in terms of the
Wigner function as
\[ \frac{dF}{dt} = \int \frac{d^3k}{(2\pi)^3} iG^<(k^0 = \Delta E, \vec{k}, x(t)), \]
where the Wigner function is defined as the Fourier transform \textit{w.r.t.} the relative coordinate of the Wightman function,
\[ iG^<(k, x) = \int d^4re^{i\vec{k} \cdot \vec{r}}iG^<(x + r/2, x - r/2), \]
such that \( r^0 = x_1^0 - x_2^0 \) are proper times. This means that the response function of Unruh’s detector is completely insensitive to particle momenta (which is consistent with the assumption that the detector must be very massive), and it measures (absorbs) scalar particles of all possible momenta \( \vec{k} \); likewise, it isotropically emits particles of all momenta.

### III. THERMAL RESPONSE

In order to get an insight in how a certain distribution function of a scalar field is perceived by the detector, we consider the response function for the Bose-Einstein distribution. The Wightman function for a thermally excited scalar field has the well known form \[12\]
\[ iG_{\text{th}}^<(k) = 2\pi \text{sign}(k^0)\delta(k^2 + m^2_\phi) \frac{1}{e^{\beta k^0} - 1}, \]
where \( \beta = 1/T \) denotes the inverse temperature, and \( m_\phi \) the scalar mass. The response function \[13\] for \( d \) dimensions in flat space-time is easily calculated,
\[ \frac{dF_{\text{th}, d}(\Delta E)}{dt} = \frac{2^{2-d} \pi^{\frac{2-d}{2}}}{\Gamma\left(d-\frac{1}{2}\right)} \text{sign}(\Delta E) \Theta \left( (\Delta E)^2 - m^2_\phi \right) \frac{d-3}{2} \left( \frac{1}{e^{\beta \Delta E} - 1} \right), \]
and explicitly, for \( d = 4 \), one finds
\[ \frac{dF_{\text{th}, d=4}(\Delta E)}{dt} = \frac{\text{sign}(\Delta E) \Theta \left( (\Delta E)^2 - m^2_\phi \right) \sqrt{(\Delta E)^2 - m^2_\phi}}{2\pi} \frac{1}{e^{\beta \Delta E} - 1}. \]
Hence, the response function of Unruh’s detector to a scalar thermal state contains, apart from the Bose-Einstein distribution, an additional factor, which depends on the scalar particle mass and reduces to \( \Delta E/(2\pi) \) in the massless limit and \( d = 4 \). This is precisely the factor that arises for the conformal scalar vacuum \[19\] in de Sitter space \[2\].
IV. SCALAR FIELDS IN DE SITTER INFLATION

The Lagrangean for a massive real scalar field in a curved space-time background is given by

$$\sqrt{-g} \mathcal{L}_\Phi = -\frac{1}{2} \sqrt{-g} g^\mu\nu (\partial_\mu \Phi)(\partial_\nu \Phi) - \frac{1}{2} \sqrt{-g} (m_\phi^2 + \xi \mathcal{R}) \Phi^2,$$

(10)

where $g = \det[g_{\mu\nu}]$ denotes the determinant of the metric $g_{\mu\nu}$, $g^{\mu\nu}$ is the inverse of the metric, $m_\phi$ the scalar mass and $\mathcal{R}$ the curvature scalar. In conformal space-times, in which the metric is of the form

$$g_{\mu\nu} = a^2 \eta_{\mu\nu},$$

(11)

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the flat Minkowski metric and $a = a(\eta)$ the scale factor, the Lagrangean (10) reduces to

$$\sqrt{-g} \mathcal{L}_\Phi = -\frac{1}{2} a^2 \eta^{\mu\nu} (\partial_\mu \Phi)(\partial_\nu \Phi) - \frac{1}{2} a^4 (m_\phi^2 + \xi \mathcal{R}) \Phi^2.$$  

(12)

For example, in (a locally) de Sitter inflation $a = -1/(H\eta)$ ($\eta < 0$), $\mathcal{R} = 12H^2$ (in $D = 4$ space-time dimensions), while in radiation-matter era, $a = a_r \eta + a_m \eta^2$, where $H$ denotes the Hubble parameter, $\eta$ conformal time, $a_r$ and $a_m$ are constants.

The special cases of interest are:

- conformally coupled massless scalar field ($\xi = 1/6$, $m_\phi = 0$), for which a conformally rescaled scalar $\varphi \equiv a \Phi$ satisfies the simple differential equation ($\nabla$ denotes a spatial derivative)

$$\left(\partial_\eta^2 - \nabla^2\right) \varphi(x) = 0;$$

(13)

- and nearly minimally coupled light scalar ($|\xi| \ll 1$, $m_\phi \ll H$), which obeys

$$\left(\partial_\eta^2 - \nabla^2 - \frac{1}{a} \frac{d^2 a}{d\eta^2} + a^2 (m_\phi^2 + \xi \mathcal{R})\right) \varphi(x) = 0.$$

(14)

(in de Sitter inflation, $-a^{-1} d^2 a/d\eta^2 = -2/\eta^2 = -2a^2 H^2$);

- and the minimally coupled massless scalar ($\xi = 0$, $m_\phi = 0$), which satisfies the following differential equation,

$$\left(\partial_\eta^2 - \nabla^2 - \frac{1}{a} \frac{d^2 a}{d\eta^2}\right) \varphi(x) = 0.$$

(15)
A. Conformal vacuum in de Sitter space

We now calculate the response function (4) for a conformally coupled massless scalar field (13) ($\xi = 1/6$, $m_\phi = 0$), for which the de Sitter invariant Green function during inflation in $D = 4$ reads

$$iG_{\text{conf}}(y) = \frac{H^2}{4\pi^2} \frac{1}{y},$$

where $y$ denotes the de Sitter length function

$$y = \frac{\Delta x^2}{\eta_1 \eta_2} \equiv 4 \sin^2 \left(\frac{1}{2} H \ell\right),$$

which is related to the geodesic distance $\ell$ as indicated, and $\Delta x^2 = -(\eta_1 - \eta_2)^2 + \|\vec{x}_1 - \vec{x}_2\|^2$. For an observer moving along a geodesic, $y = y(x(t + \Delta t/2); x(t - \Delta t/2)) = -4 \sinh^2 \left( H \Delta t/2 \right)$.

The response function (4) for the conformal vacuum (16) can then be written as

$$\frac{dF_{\text{conf}}(\Delta E)}{dt} = -\frac{H}{4\pi^2} \int_{-\infty}^{\infty} du \, e^{-i\Delta E u/H} \frac{1}{4 \left[\sinh \left( u/2 \right) - i\epsilon \right]^2},$$

where the pole prescription corresponds to that of the Wightman function $iG^<$. This integral can be easily performed by contour integration. The (double) poles (which also correspond to the zeros of $y$) all lie on the imaginary axis, $u_n = Ht_n = 2i\pi n (n \in \mathbb{Z})$. For $E > E_0$ the contour of integration ought to be closed by a large circle below the real axis, such that the integral in (18) can be evaluated by summing the residues which lie (strictly) below the real axis, as illustrated in figure 1. The result is

$$\frac{dF_{\text{conf}}(\Delta E)}{dt} = -\frac{H}{4\pi^2} (-2\pi i) \sum_{n=-\infty}^{\infty} \frac{-i\Delta E}{H} e^{(2\pi i \Delta E/H) n} = \frac{\Delta E}{2\pi} \frac{1}{e^{(2\pi/\Delta E)} - 1},$$

which is identical to the response function (9) of the thermal Bose-Einstein distribution for massless scalars, confirming thus the well known result [2]. For $\Delta E < 0$, the contour should be closed above the real axis, such that the contributing poles are $n \geq 0$, also shown in figure 1. The result of integration is again given by Eq. (19).
FIG. 1: The integration contour for the Unruh’s detector response function in conformal vacuum. The solid (blue) contour corresponds to $\Delta E > 0$; the dashed (red) contour to $\Delta E < 0$.

B. Nearly minimally coupled light scalar

The Green function for a massive scalar field coupled to gravity as indicated by the Lagrangean (12) is given by the Chernikov-Tagirov [7] (Bunch-Davies [8]) vacuum

$$iG(y) = \frac{H^2}{4\pi^2} \Gamma\left(\frac{3}{2} - \nu\right) \Gamma\left(\frac{3}{2} + \nu\right) \, _2F_1\left(\frac{3}{2} - \nu, \frac{3}{2} + \nu, 2; 1 - \frac{y}{4}\right),$$

(20)

where

$$\nu = \sqrt{\left(\frac{3}{2}\right)^2 - \frac{m_\phi^2 + 12\xi H^2}{H^2}}. \quad (21)$$

The uniqueness of $iG(y)$ follows from the requirement that the lightcone singularity is of the Hadamard form.

When expanded in powers of

$$s \equiv \frac{3}{2} - \nu = \frac{m_\phi^2}{3H^2} + 4\xi + O\left(\left[(m_\phi^2/H^2) + 12\xi\right]^2\right), \quad |s| \ll 1, \quad (22)$$

the Green function for the Chernikov-Tagirov vacuum reduces to the following simple form [13]

$$iG(y; s) = \frac{H^2}{4\pi^2} \left\{ \frac{1}{y} - \frac{1}{2} \ln(y) + \frac{1}{2s} - 1 + \ln(2) + O(s) \right\}. \quad (23)$$

The nontrivial new integral comes from the term $iG_{m=0}^\infty \propto \ln(y)$, and its contribution to the response function yields the integral

$$\frac{dF_{\text{ln}}(\Delta E)}{dt} = -\frac{H}{8\pi^2} \int_{-\infty}^{\infty} du \, e^{-i\Delta Eu/H} \left[ \ln\left(4 \sinh^2\left(u/2\right)\right) + i\pi \text{sign}(u) \right], \quad (24)$$
where we broke the logarithm into the real and imaginary contributions, in accordance with the ε-prescription for \(iG^<\). The real part of the logarithm can be evaluated by breaking it into positive and negative \(u\) and then performing a partial integration (or, as it is more commonly done, by expanding the logarithm), while the imaginary part can be integrated trivially:

\[
\frac{dF_{\ln}(\Delta E)}{dt} = \frac{H^2}{4\pi^2\Delta E} \int_0^\infty du \sin \left( \frac{\Delta E}{H} u \right) \coth \left( \frac{u}{2} \right) - \frac{H^2}{4\pi\Delta E} \frac{1}{2\pi\Delta E e^{2\pi\Delta E/H} - 1},
\]

(25)

where in the last step we made use of Eq. (3.981.8) of Ref. [14].

The remaining integrals in the response function simply yield δ-function contributions,

\[
\frac{dF_\delta(\Delta E)}{dt} = \frac{H^2}{2\pi} \delta(\Delta E) \left( \frac{1}{2s} - 1 + \ln(2) + O(s) \right).
\]

(26)

Collecting all terms together, we get the response function for the nearly minimally coupled massive scalar:

\[
\frac{dF_{m\neq 0}(\Delta E)}{dt} = \frac{\Delta E}{2\pi} \left( 1 + \frac{H^2}{\Delta E^2} \frac{1}{e^{(2\pi/H)\Delta E} - 1} + \frac{H^2}{2\pi} \delta(\Delta E) \left( \frac{1}{2s} - 1 + \ln(2) + O(s) \right) \right).
\]

(27)

C. Minimally coupled massless scalar field

The Green function of a minimally coupled massless scalar field exhibits an infrared divergence, and the construction of a finite propagator necessarily breaks de Sitter invariance. For the purpose of calculating loop diagrams and using dimensional regularisation, one considers the propagator in \(d\) dimensions with appropriate counterterms to cancel the infrared divergence. For a detailed discussion, see Ref. [15]. In four dimensions, one obtains

\[
iG_{m=0}(x_1; x_2) = \frac{H^2}{4\pi^2} \left\{ \frac{1}{y} - \frac{1}{2} \ln(y) + \frac{1}{2} \ln \left( a(\eta_1)a(\eta_2) \right) - \frac{1}{4} + \ln(2) \right\},
\]

(28)

where the term \(\propto \ln \left( a(\eta_1)a(\eta_2) \right)\) breaks de Sitter invariance.

Yet, this does not imply, that there is no de Sitter-invariant vacuum, as pointed out in Ref. [17], where such an invariant state is explicitly constructed by quantizing the mode with zero momentum separately. However, singling out that mode does not render the propagator finite, as one sees when regulating the propagator with an infrared cutoff \(k^0\), such that one obtains

\[
iG_{m=0,k^0}(x_1; x_2) = \frac{H^2}{4\pi^2} \left\{ \frac{1}{y} - \frac{1}{2} \ln(y) + \frac{1}{2} \ln \left( a(\eta_1)a(\eta_2) \right) - \ln(k^0H) - \gamma_E + O(k^0) \right\},
\]

(29)
where $\gamma_E = 0.577215$ is Euler’s constant. This expression differs from the propagator \(^{(28)}^\), which we shall use for calculating the response, only by a constant.

The response function is easily reconstructed from the results of section \(\textbf{IV B}\)

\[
\frac{dF_{m_0=0}(\Delta E)}{dt} = \frac{\Delta E}{2\pi} \left( 1 + \frac{H^2}{\Delta E^2} \right) e^{\frac{\pi \Delta E}{\Delta E}} - 1 + \frac{H^2}{2\pi} \delta(\Delta E) \left( \ln(a) - \frac{1}{4} + \ln(2) \right),
\]

where $\ln(a) = Ht = N$ is the number of e-folds elapsed since the beginning of inflation, if we set the initial scale factor to be one. This contribution to the response function vanishes for all $\Delta E \neq 0$, such that under the assumption that its energy levels are not degenerate, the detector is insensitive to the breaking of de Sitter invariance by the propagator. Note also that, when integrated over $\Delta E$ around $\Delta E = 0$, the terms $\propto \delta(\Delta E)$ are subdominant, because they only give a finite contribution to the response, provided that the scale factor $a$ and the cutoff $k^0$ in \(\textbf{24}\) are finite and nonzero, while the remaining terms yield a divergence.

However, for a detector sensitive to time separations up to $\Delta t \leq \Delta_{\text{max}}$, $F_\delta$ in \(\textbf{26}\) becomes a smeared “$\delta$-function,” $dF_\delta(\Delta E; \Delta_{\text{max}})/dt \propto \sin \left( \Delta E \Delta_{\text{max}} / 2 \right) / (\Delta E / 2)$, such that the detector responds roughly up to the energies $\Delta E \leq 2\pi / \Delta_{\text{max}}$. Let us now study the question of finite-time measurements in more detail.

### D. Boundary terms through finite-time measurements

Strictly speaking, the propagators \(\textbf{16 23 28}\) describe the dynamics of the scalar field for $t \geq t_0$. Hence it is natural to consider the Unruh detector, which corresponds to the transition amplitude \(\textbf{2}\), which begins measuring at $t_0 = 0$ and ends at $t_f = t$. In this case Eqs. \(\textbf{3}\) and \(\textbf{4}\) generalize to

\[
\frac{dP(E_0 \rightarrow E, 0, t)}{dt} = h^2 |m_{E_0, E}|^2 \frac{dF(\Delta E, 0, t)}{dt} (\Delta E = E - E_0),
\]

with

\[
\frac{dF(\Delta E, 0, t)}{dt} = \int_{-t}^{t} d\Delta t e^{-i\Delta E \Delta t} iG^<(t + \Delta t / 2, \bar{x}; t - \Delta t / 2, \bar{x}) \Delta t,
\]

implying that the response function gets modified by the boundary (initial) effects. For example, the response function associated with the Hadamard (conformal) form, $iG_{\text{conf}} = H^2 / (4\pi^2 y)$, for an Unruh detector reads

\[
\frac{dF_{\text{conf}}(\Delta E, 0, t)}{dt} = \frac{\Delta E}{2\pi} e^{\frac{\pi \Delta E}{\Delta E}} - 1 + \frac{H^2}{2\pi^2} \sum_{n=1}^{\infty} ne^{-nHt} \frac{n \cos(\Delta Et) - (\Delta E / H) \sin(\Delta Et)}{(\Delta E / H)^2 + n^2}.
\]
Similar (though more technical) analysis can be performed for other contributions from the massless scalar propagator \( \text{(28)} \). Quite generically, boundary effects give rise to oscillatory contributions to the response function of Unruh’s detector. For \( \Delta E \sim H \), these terms become unimportant when \( t \gg H^{-1} \). In the ultraviolet, where \( \Delta E \gg H \), the oscillatory contributions become subdominant when \( t \gg \Delta E/H^2 \) – much more than a Hubble time. Since these effects are however not related to the intrinsic nature of the quantum fields in de Sitter space, we here do not study them further.

### E. Dimensions other than four

So far, we have calculated the response functions from the scalar propagator, which is motivated by the practice of defining vacua in de Sitter space through this quantity. However, there is a method due to Higuchi \[6\] using a basis of wave functions as starting point, which we generalize here to \( d \) dimensions. We define

\[
\nu = \sqrt{\left( \frac{d-1}{2} \right)^2 - \frac{m^2 + \xi R}{H^2}}, \tag{31}
\]

where the curvature is given by \( R = d(d-1)H^2 \). The scalar wave equation is (cf. Eq. \( 14 \)),

\[
\left[ \frac{\partial^2}{\partial \eta^2} + k^2 - \frac{\nu^2}{\eta^2} - \frac{1}{4} \right] \varphi_k(\eta) = 0 \tag{32}
\]

and has the properly normalized positive frequency solution

\[
\varphi_k(\eta) = \frac{1}{2} (-\pi \eta)^{\frac{d}{2}} e^{\pi |\nu| H^{(2)}(\nu)} (-\eta) \varphi_k(\eta). \tag{33}
\]

The transition probability for the detector in terms of these modes is then

\[
P(\Delta E) = \int \frac{dk}{(2\pi)^{d-1}} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{1}{a(t)a(t')} \varphi_k^*(t) \varphi_k(t') e^{-i\Delta E(t'-t)} \tag{34}
\]

\[
= \int \frac{dk}{(2\pi)^{d-1}} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{\pi}{4H} e^{\pi |\nu| - \frac{3}{2}H(t+t')} e^{i\Delta E(t-t')} H^{(2)}(\nu) H^{(2)}(\nu) \left( \frac{|k|}{H} e^{-Ht} \right) \left( \frac{|k|}{H} e^{-Ht'} \right). \tag{35}
\]

From this expression, one obtains for the response function

\[
\frac{d\mathcal{F}_d(\Delta E)}{dt} = \frac{H^{d-3} e^{\pi \Delta E/H}}{8\pi^{(d+1)/2} \Gamma((d-1)/2)} \left| \Gamma \left( \frac{(d-1)/2 + i\Delta E/H + \nu}{2} \right) \Gamma \left( \frac{(d-1)/2 + i\Delta E/H - \nu}{2} \right) \right|^2, \tag{36}
\]

which reduces for \( d = 4 \) to Higuchi’s result \[6\]

\[
\frac{d\mathcal{F}(\Delta E)}{dt} = \frac{H}{4\pi^3} e^{-\pi \Delta E/H} \left| \Gamma \left( \frac{3/2 + i\Delta E/H + \nu}{2} \right) \Gamma \left( \frac{3/2 + i\Delta E/H - \nu}{2} \right) \right|^2. \tag{37}
\]
For $\nu = 1/2$ this coincides with our result for the conformal case (18), and when expanded in $s = 3/2 - \nu$ with the nearly minimally coupled case (27).

It is however also interesting to derive some special responses from the scalar propagator, which is in $d$ dimensions [13]

$$iG_d(y) = \frac{\Gamma\left(d-1/2 + \nu\right) \Gamma\left(d-1/2 - \nu\right)}{(4\pi)^{d/2} \Gamma\left(d/2\right)} H^{d-2} \ _2F_1\left(d-1/2 + \nu, d-1/2 - \nu, d/2; 1 - \frac{y}{4}\right).$$  \hspace{1cm} (37)

In particular, for $d = 3$, the response function is exactly calculable for arbitrary $\nu$ from the propagator, because we can express the hypergeometric function in terms of the geodesic distance $\ell$, using Eq. (9.121.30) of Ref. [14], as

$$2F_1\left(1 + \nu, 1 - \nu, \frac{3}{2}; 1 - \frac{y}{4}\right) = \frac{\sin[\nu(\pi - H\ell) - i\nu(H\ell)]}{\nu\sin(H\ell)}. \hspace{1cm} (38)$$

With $\ell \to i\Delta t$, the response function (4) is then given by the integral

$$\frac{dF_3(\Delta E)}{dt} = \frac{\Gamma(\nu)\Gamma(1 - \nu)}{4i\pi^2} H \int_{-\infty}^{\infty} d\Delta t e^{-i\Delta E\Delta t} \frac{\sin(\pi\nu - i\nu H\Delta t)}{\sinh(H\Delta t)}, \hspace{1cm} (39)$$

with the poles of the integrand at $\Delta t_n = i\pi n/H, n \in \mathbb{Z}\setminus\{-1\}$. Note, that for odd dimensions, the analytic structure of the propagator is different than for even dimensions. There are additional poles, but there is no branch cut (except for special values of $\nu$). We perform the integration by closing the contour in the lower complex half plane. According to the $\epsilon$-prescription for the Wightman function $iG^<$, only the poles $n \leq -1$ contribute, and we obtain

$$\frac{dF_3(\Delta E)}{dt} = \frac{1}{2} \sinh\left(\frac{\pi \Delta E}{H}\right) \frac{1}{\cos(\pi\nu) + \cosh\left(\frac{\pi \Delta E}{H}\right) e^{2\pi\Delta E/H} - 1}. \hspace{1cm} (40)$$

Note that (40) applies not only when $m_\phi < H$, and $\nu$ is real, but also for $m_\phi > H$, when $\nu$ is imaginary. When $m_\phi \gg H, \Delta E$, the response is exponentially suppressed as $dF/dt \propto \exp(-\pi m_\phi/H)$, which is a consequence of an exponential suppression of scalar particle production in de Sitter space in the limit when $m_\phi \gg H$.

Note that for no value of the parameter $\nu$ the response (40) agrees with the thermal response function, which in three dimensions can be read off from Eq. (3),

$$\frac{dF_{3,\text{th}}(\Delta E)}{dt} = \frac{1}{2} \text{sign}(\Delta E) \Theta\left((\Delta E)^2 - m_\phi^2\right) \frac{1}{e^{2\pi\Delta E/H} - 1}. \hspace{1cm} (41)$$

In particular, for a conformal massless scalar, $\nu = 1/2$, and Eq. (40) yields the following ‘fermionic-like’ response function,

$$\frac{dF_{3,\text{conf}}(\Delta E)}{dt} = \frac{1}{2} \frac{1}{e^{2\pi\Delta E/H} + 1}. \hspace{1cm} (42)$$
This disagreement with the thermal case is not a special feature of odd dimensions. E.g. the conformal Green function in $d$ dimensions

$$iG_{\text{conf},d} = \frac{\Gamma\left(\frac{d}{2} - 1\right)}{4\pi^{d/2}} H^{d-2} y^{1-d/2}$$  \hfill (43)$$

leads for $d = 6$ to the response

$$\frac{dF_{6,\text{conf}}(\Delta E)}{dt} = \frac{H^3}{12\pi^2} \left(\frac{\Delta E^3}{H^3} + \frac{\Delta E}{H}\right) \frac{1}{e^{3\pi\Delta E/H} + 1},$$  \hfill (44)$$

while the flat-space thermal response is

$$\frac{dF_{6,\text{th}}}{dt} = \frac{1}{12\pi^2} \text{sign}(\Delta E) \Theta \left((\Delta E)^2 - m_{\phi}^2\right) \left(\Delta E^2 - m^2\right)^{3/2} \frac{1}{e^{\beta \Delta E} - 1}. \hfill (45)$$

Generally, for $d > 4$, the conformal response consists of the Planck factor times a polynomial involving different powers of $\Delta E$, therefore deviating from the thermal response, which involves only a single power of $\Delta E$.

For $d = 2$ conformal and minimally massless coupled case coincide and we find from (35)

$$\frac{dF_{1+1,\text{conf}}(\Delta E)}{dt} = \frac{1}{\Delta E e^{\beta \Delta E - 1}}, \quad (\Delta E \neq 0), \hfill (46)$$

in agreement with the flat-space thermal response.

Hence, we have found that an agreement with the thermal response occurs for conformal coupling only in $d = 2$ and $d = 4$, just as for an accelerated observer in flat space [19].

V. DETAILABLE, RESPONSE FUNCTIONS AND SPECTRA

Based on the assumption that the principle of detailed balance holds, which states that the absorption rate of the detector $R_a$ and the emission rate $R_e$ are equal,

$$R_a(E_0 \rightarrow E) = R_e(E \rightarrow E_0), \quad (\forall E_0, E), \hfill (47)$$

and on the fact that the transition probabilities per unit proper time are related as follows:

$$\frac{dP(E_0 \rightarrow E)}{dt} = e^{-\beta(E - E_0)} \frac{dP(E \rightarrow E_0)}{dt}, \hfill (48)$$

or, equivalently, the response function of the detector fulfills,

$$\frac{dF(\Delta E)}{dt} = e^{-\beta \Delta E} \frac{dF(-\Delta E)}{dt} \quad (\Delta E = E - E_0), \hfill (49)$$

or, equivalently, the response function of the detector fulfills,
one can infer that the detector is thermally populated, with the temperature given by $T = 1/\beta$, as follows (for a related discussion see Ref. [4]). Let us rewrite the principle of detailed balance (47) as
\[
n(E_0)\frac{dP(E_0 \rightarrow E)}{dt} (1 + n(\beta)) = n(E)\frac{dP(E \rightarrow E_0)}{dt} (1 + n(E_0)),
\]
where $n(E)$ and $n(E_0)$ denote the occupation numbers of detector states with energies $E$ and $E_0$, respectively. From this, it immediately follows
\[
n(E) = \frac{1}{e^{\beta(E-\mu)} - 1},
\]
such that the states of the detector are populated according to a chemical equilibrium at temperature $T = 1/\beta$ and chemical potential $\mu$.

In fact, any response which can be written as
\[
\frac{dF(\Delta E)}{dt} = g \frac{\Delta E}{2\pi} \frac{1}{e^{\beta\Delta E} - 1},
\]
with $g = g(\Delta E)$ being an even function of $\Delta E$, fulfills the relation (49). We have shown explicitly for different scalar fields in $d = 4$ (cf. Eqs. (19), (30) and (27)), that they are of the form (52),
\[
g_{conf} = 1,
\]
\[
g_{m=0} = 1 + \left(\frac{H}{\Delta E}\right)^2 + 2\pi H \delta(\Delta E) \left[Ht - \frac{1}{4} + \ln(2)\right],
\]
\[
g_{m\neq0} = 1 + \left(\frac{H}{\Delta E}\right)^2 + 2\pi H \delta(\Delta E) \left[\frac{1}{2s} - 1 + \ln(2)\right] + O(s),
\]
with $\beta_H = 1/T_H = 2\pi/H$. Moreover, the more general expressions (35), (36) and (40) also satisfy equation (49).

The relation (49) can also be viewed as a consequence of the periodicity of the Green function $iG^<$ in imaginary proper time, $t \rightarrow t + 2\pi i/\beta$, which is in turn a consequence of the same periodicity of the metric in Euclidean time [24]. An example where the periodicity of the metric however does not coincide with the Hawking temperature is a quasi-de Sitter space considered in Ref. [20], and hence cannot in general be used as an argument for the thermality of a scalar field.

\section{VI. DISCUSSION}

We found the response functions for different scalar fields to differ strongly in the infrared, where $\Delta E < H$. Moreover, they do not in general coincide with the response to an equilibrium
state in flat space. A disagreement with the thermal response does not yet imply that the
detector does not equilibrate with the de Sitter background. In fact, the energy levels of the
detector are thermally populated. Similar deviations from a Minkowski-space thermal response
are also known for accelerated detectors [19, 21]. The disagreement of the response functions
should be attributed to the fact, that for the conformally and the minimally coupled scalar field
the density of modes per frequency is different.

However, for fields which are massive or nonconformally coupled to the metric, the infrared
enhancement is a consequence of the amplification of superhorizon modes leading to cosmological
density perturbations, an effect which is absent in the conformally coupled massless case. This
makes the different fields clearly distinguishable by observables. The fact that physically distinct
situations such as thermally populated flat space and the different types of de Sitter-invariant
vacua result in the same thermal distribution function for the energy levels of the detector is due
to the insensitivity of this quantity to whether there is a mixed (thermal) state or a pure (de
Sitter-invariant) state and to how often the detector responds per unit time [25].

Finally, we want to point out that de Sitter invariance of the stress-energy tensor of the
conformal vacuum implies that the stress-energy tensor must be proportional to the metric, such
that $p_{\text{conf}} = -\rho_{\text{conf}}$ [22]. This is inconsistent with a thermal equation of state and leads us to the
investigation, whether the energy density is captured at higher orders in perturbation theory [23],
allowing for the pair creation operators of the scalar field Hamiltonian, which are not captured
by the first order perturbation expansion used here. While we argued in this paper that the
thermal state of the detector does not contain the full information about the quantum field, it
is yet remarkable, that at first order in perturbation theory the detector is insensitive to the
(regulated) stress-energy tensor, which is clearly nonthermal.

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[1] W. G. Unruh, “Notes On Black Hole Evaporation,” Phys. Rev. D 14 (1976) 870.
[2] N. D. Birrell and P. C. W. Davies, “Quantum Fields In Curved Space,” Cambridge University Press (1984).
[3] G. W. Gibbons and S. W. Hawking, “Cosmological Event Horizons, Thermodynamics, And Particle Creation,” Phys. Rev. D 15 (1977) 2738.
[4] M. Spradlin, A. Strominger and A. Volovich, “Les Houches lectures on de Sitter space,” published in “Les Houches 2001, Gravity, gauge theories and strings” 423-453 [arXiv:hep-th/0110007].
[5] R. Bousso, A. Maloney and A. Strominger, “Conformal vacua and entropy in de Sitter space,” Phys. Rev. D 65 (2002) 104039 [arXiv:hep-th/0112218].
[6] A. Higuchi, “Quantization Of Scalar And Vector Fields Inside The Cosmological Event Horizon And Its Application To Hawking Effect,” Class. Quant. Grav. 4 (1987) 721.
[7] N. A. Chernikov and E. A. Tagirov, “Quantum Theory Of Scalar Fields In De Sitter Space-Time,” Annales Poincare Phys. Theor. A 9 (1968) 109.
[8] T. S. Bunch and P. C. Davies, “Quantum Field Theory In De Sitter Space: Renormalization By Point Splitting,” Proc. Roy. Soc. Lond. A 360 (1978) 117.
[9] E. Mottola, “Particle Creation In De Sitter Space,” Phys. Rev. D 31 (1985) 754.
[10] Bruce Allen, “Vacuum States In De Sitter Space,” Phys. Rev. D 32 (1985) 3136.
[11] B. Allen and A. Folacci, “The Massless Minimally Coupled Scalar Field In De Sitter Space,” Phys. Rev. D 35 (1987) 3771.
[12] M. Le Bellac, “Thermal Field Theory,” Cambridge University Press (1996).
[13] T. Prokopec and E. Puchwein, “Photon mass generation during inflation: de Sitter invariant case,” JCAP 0404 (2004) 007 [arXiv:astro-ph/0312274].
[14] Izrail Solomonovich Gradshteyn, Iosif Moiseevich Ryzhik, *Table of integrals, series, and products*, 4th edition, Academic Press, New York (1965).
[15] V. K. Onemli and R. P. Woodard, “Quantum effects can render w ¡ -1 on cosmological scales,” arXiv:gr-qc/0406098.
[16] T. Prokopec and R. P. Woodard, “Dynamics of super-horizon photons during inflation with vacuum polarization,” Annals Phys. 312 (2004) 1 [arXiv:gr-qc/0310056].
[17] K. Kirsten and J. Garriga, “Massless Minimally Coupled Fields In De Sitter Space: O(4) Symmetric States Versus De Sitter Invariant Vacuum,” Phys. Rev. D 48 (1993) 567 [arXiv:gr-qc/9305013].

[18] N. C. Tsamis and R. P. Woodard, Class. Quant. Grav. 11 (1994) 2969.

[19] C. Gabriel, P. Spindel, S. Massar and R. Parentani, “Interacting charged particles in an electric field and the Unruh effect,” Phys. Rev. D 57 (1998) 6496 [arXiv:hep-th/9706030].

[20] R. H. Brandenberger and R. Kahn, “Hawking Radiation In An Inflationary Universe,” Phys. Lett. B 119 (1982) 75.

[21] R. Brout, S. Massar, R. Parentani and P. Spindel, “A Primer for black hole quantum physics,” Phys. Rept. 260 (1995) 329.

[22] R. H. Brandenberger, “An Alternate Derivation Of Radiation In An Inflationary Universe,” Phys. Lett. B 129 (1983) 397.

[23] B. Garbrecht and T. Prokopec, “Energy density in expanding universes as seen by Unruh’s detector,” arXiv:gr-qc/0406114.

[24] For more general de Sitter invariant states, the so called $\alpha$-vacua, one can show that [19] does not in general hold [5]. However, these states have a different ultraviolet structure than the Chernikov-Tagirov vacuum [20], which has the standard Hadamard lightcone singularity, and hence they are most likely unphysical.

[25] An example for the interpretation of the de Sitter invariant states as thermal can be found in Ref. [6], where scalar field quantization is performed in static coordinates. The mode functions are chosen to vanish beyond the horizon, where the static coordinates exhibit a coordinate singularity. Since the horizon distance is singled out, the mode functions in static coordinates violate spatial homogeneity. An Unruh detector placed at the origin of the static vacuum measures no particles. On the other hand, when the static vacuum is thermally populated, the response function is the same as for the de Sitter-invariant vacuum, and it is given by [30]. Note first that a thermally populated state in static coordinate does not correspond to a usual thermal equilibrium state, since spatial homogeneity is broken. In addition, the mode functions in the static and the de Sitter-invariant vacuum have different support. Indeed, the static mode functions vanish beyond the de Sitter horizon, while the de Sitter-invariant mode functions exhibit superhorizon correlations.