A phase plate for transmission electron microscopy using the Aharonov-Bohm effect

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Abstract. The Aharonov-Bohm effect provides a way to change the phase difference between scattered and direct electron waves in the transmission electron microscope by 90 degrees, and so to convert phase variation into amplitude information. In a thin ring of magnetic material a continuous loop of azimuthal flux provides the state of lowest magnetic energy (the vortex state). Permanently-magnetized rings can maintain this flux pattern in the presence of a strong axial field such as that of a microscope objective lens. The radial width required for such a ring is of the order of 50 nm and so the radial fraction of the scattered electrons that is intercepted can be very small.

1. Introduction
In cryo- and other transmission electron microscopy of biological specimens, there is a need to minimise the electron dose by increasing the image contrast for given beam current. It is known that this can be achieved by use of a phase plate, designed to change by 90 degrees the difference between the phases accumulated by scattered and direct electron waves in their transit from the specimen to the detector. We propose to achieve the required phase change by use of the Aharonov-Bohm effect, produced by a ring of magnetic flux set up in a thin magnetic ring inserted in the plane of the objective aperture. The ring dimensions are chosen so that scattered electrons pass outside the ring while direct electrons pass through it.

2. The Aharonov-Bohm effect
When an electron passes through a region in which the vector potential $A$ is not zero, the phase of its wave function changes by $\delta \theta = \frac{e}{\hbar} \int A \cdot dr$ along its path [1,2]. If parts of a coherent electron wave can arrive at a single point on the detector by traversing more than one path, each part $i$ accumulates a phase change $\delta \theta_i$ along its own path. The change in relative phase due to the different values of $\delta \theta$ for two paths is then

$$\Delta \theta = \delta \theta_2 - \delta \theta_1 = \frac{e}{\hbar} \int A \cdot d\mathbf{r} = \frac{e}{\hbar} \int B \cdot dS = \frac{e}{\hbar} \Phi$$

where $(-e)$ is the electronic charge and $\Phi$ is the flux enclosed between the two paths (figure 1). This effect has been demonstrated by electron holography [3], when it is necessary that the wave parts travelling by the different paths should arrive with similar amplitudes.
3. Use of thin magnetized ring

For application in a transmission electron microscope (TEM), we seek to produce a phase shift between the electrons that are scattered and those with zero net scattering, and this phase shift should be the same at all angles in azimuth around the axis. Thus we require a ring of magnetic flux directed azimuthally around the axis, and located where the direct and the scattered electrons are spatially separated. The most obvious place to locate it is in the region of the back focal plane, making use of the standard objective aperture holder (figure 2).

The azimuthal (or \(\phi\)-directed) magnetic flux needed is small by macroscopic standards – about \(10^{-15}\) Wb for a phase difference of \(\pi/2\). However, to be useful in biological imaging this azimuthal flux must be maintained in the presence of axial magnetic flux density of the order of 1T from the objective lens of the microscope. These two requirements appear to conflict since, in a macroscopic ring of a soft material such as permalloy, an axial field of this magnitude turns all spins to near-axial directions and randomizes the directions of the remaining transverse flux. However, JAC Bland suggested in 2006 [4] that the desired behaviour can be obtained in thin rings of material with greater coercivity. For such a ring, a continuous azimuthal loop of flux (the ‘vortex’ state) is the state of lowest magnetic energy, and has zero accompanying external field. If the width and thickness of the ring are sufficiently small, the ring may be induced into this state by suitable application of in-plane fields. The existence of the vortex state in rings of sub-micrometric widths has been found by experiment [5] and by simulation [6] as shown in figure 3. In this figure, the colours (visible online) in the rings show the local direction of magnetic field: red corresponds to +x; mid-green, +y; cyan, −x; mid-blue, −y; with continuous gradation between these. The results differ from those of macroscopic structures in that plots of in-
plane magnetisation $M_x$ against in-plane applied flux density $B_x$ show steps indicating that the vortex state is maintained over a substantial central range of $B_x$. The specific values at which switching occurs depend on the dimensions of the ring, but it appears that a range of the order of ± 0.1 T may be achievable.

If the axial thickness of the ring is less than its radial width, a further beneficial effect occurs. The spins resist alignment by an external field perpendicular to the plane of the film, unless that field is very strong. The planar surfaces of the ring thus have a confining effect (shape anisotropy).

If the ring consists of a material of sufficiently high coercivity, the vortex state of azimuthal flux can persist in applied axial flux density of the order of 1 T. Further simulation (figure 4) suggests that a ring of thickness of the order of 20 nm, as required for a phase plate, might tolerate an axial field of more than 1 T while maintaining the vortex of flux. Thus it appears that, in suitable material, a ring of flux can be maintained even in the presence of an axial field comparable with that of typical objective lenses. There is some variation of azimuthal flux density with applied axial field. However, the geometry of conversion of phase variation to amplitude information suggests that some variation in the phase difference from the nominal value of $\pi/2$ may be tolerable.

The minimum width needed for such a ring appears to be of the order of 50 nanometres. Thus there will be some interception of the scattered electrons over this range of radius, at the axial position of the objective aperture. However, this range of interception is very much smaller than those of some other forms of phase plate that have been suggested, such as an electrostatic lens. Hence the contrast transfer function can be maintained near its maximum, down to much lower spatial frequencies than are usable at present (figure 5).

In order to define suitable dimensions for the ring, details will be needed for the objective lens with which it is to be used. To check that the phase shift obtained is that intended, measurement will be needed; holography appears to provide the most suitable method.

Figure 4. (colour online) Simulation of $B_\phi$ vs $B_z$ for the ring of figure 3.

Figure 5. (colour online) Contrast transfer function as function of spatial frequency, with and without $\pi/2$ phase plate. Here $Q$ is the reduced spatial frequency, $q(C_s/\lambda)^{1/4}$, where $q$ is the spatial frequency, $C_s$ is the third-order spherical aberration coefficient and $\lambda$ is the electron wavelength; $B(Q)$ is half the contrast transfer function $B(q)$ as defined by Reimer [7] and $D$ is the reduced defocus, given by (physical defocus) / $(C_s\lambda)^{1/2}$. 

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4. Conclusion
Thin, permanently-magnetized rings have properties that are convenient for use as phase plates in TEMs. Detailed design will require some knowledge of the instrument into which each is to be fitted. It may be possible to design them to fit existing TEMs as well as new ones, with or without aberration correction.

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