Experimental Setup for Studying Thermosolutal Convection in Moist Air

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Abstract. The paper is concerned with designing an experimental scheme for studying convection of moist air under conditions closely approximating the atmospheric conditions. Detailed numerical estimates, necessary conditions and dimensionless gas-dynamic parameters characteristic of the problem of dry and moist air convection are presented and analyzed. The distinctive features of convection of dry and wet air at constant pressure caused by changes in the concentration of water vapor as a result of its evaporation (condensation) are discussed. The effect of water vapor concentration on the convective instability of moist air is analyzed in the context of solutal convection, characterized by the Rayleigh concentration number. The obtained numerical estimates are used to demonstrate the possibility of differentiating and comparing between the processes of convective heat and mass transfer in humid and dry air under laboratory conditions. In future, this will allow researchers to obtain experimental data on the influence of the water-vapor phase transition on the intensity of convective motion in moist air. The structure and detailed technical characteristics of the constructed experimental setup designed for studying convection in moist air in the temperature range from 10 to 30 C are described.

1. Introduction

Convection in a gaseous environment (air) is an internal motion caused by a disturbance of mechanical equilibrium in the system aimed at recovering the equilibrium state. The processes of complex convective heat and mass transfer observed in the atmosphere of the Earth require a physical and mathematical description in terms of gas dynamic models, which must be subjected to experimental verification. The study of convection in the atmosphere is of great importance for the development and improvement of methods for weather and climate forecasting. Convection has a considerable dynamic effect on large-scale atmospheric phenomena [1,2]. This problem has been the focus of a large number of studies [4–8], the results of which make it possible to relate various types of atmospheric circulation with changes in the weather.

In the water-water vapor system, the first-order phase transition plays an important role in convection of the atmospheric moist air [3], but in most cases this phenomenon is considered in terms of thermodynamics, i.e. the computation of the latent heat of phase transition as a result of condensation / evaporation and the evaluation of the effect of the associated released / absorbed heat on the behavior of the system. On the other hand, the Rayleigh numbers (thermal) $Ra$ for moist and dry air in the atmospheric temperature range (up to 50 C) are found to be close - the observation, which the researchers take into use when performing the experiments and calculations for the parameters of dry air, ignoring its humidity. Such a simplified approach is based on the fact that in the specified temperature range the volume fraction of water vapor in moist air does not exceed 15% and has proved to be valid only in the absence of phase transition. We consider it inappropriate to study heat and mass transfer in moist air solely in the framework of a thermal convection model including the elements of thermodynamics to take into account the caloric effects associated with the release / absorption of the latent heat of phase transition. We believe that this problem should also be studied in...
the context of solutal convection, since heating or cooling of moist air changes the mass percent composition of the dry air-water vapor mixture due to evaporation and condensation of moisture, respectively.

It should be noted that the laboratory study of thermosolutal convection and the construction of a model of the atmosphere seems to be insolvable problem, since the atmosphere is a substantially stratified system with the characteristic size of ten kilometers and the characteristic velocity of the air mass flow of ten meters per second. It is impossible to build a laboratory model for the description of the process which would satisfy all criteria for similarity of the real Earth’s atmosphere. Therefore, our study will concentrate only on a single aspect of the problem - the influence of the water-vapor phase transition on the intensity of convection in moist air.

The aim of this study is to design an experiment to investigate convection of moist air in conditions closely approximating the atmospheric conditions based on the analysis and numerical estimates of the dimensionless gas-dynamic parameters characterizing convection in dry and moist air. The distinguishing features of convection of dry and moist air caused by changes in the concentration of water vapor as a result of its evaporation/condensation are discussed.

2. Constitutive parameters

Planning of an experiment based on the approximate calculations is the most essential part of experimentalist’s work. In order to conduct an experiment correctly, it is necessary to choose similarity criteria (dimensionless quantities), which form the framework for constructing a laboratory model of the examined physical system ensuring maximum possible accuracy in the description of the main features of processes occurring in the atmosphere. In practice, qualitative estimates often turn to be inaccurate in the order of magnitude leading to incorrect result interpretation, which can significantly affect the final version of the model. Therefore, the researchers tried to obtain a more accurate numerical estimate of free convection [9–11] characterized by a dimensionless Rayleigh number. Since in the problem under study, heating and concentration variation in a mixture of two gases (dry air and water vapor) occur simultaneously, it is necessary to use two independent Rayleigh numbers, which take into account both of these mechanisms, namely the thermal $Ra_T$ and solutal $Ra_C$ Rayleigh numbers. Their sum corresponds to the effective Rayleigh number $Ra_E$:

$$Ra_T = \frac{g\kappa\rho^2 d^3 \Delta T}{\eta T_0^2}, \quad Ra_C = \frac{g\eta D}{\rho^2}, \quad Ra_E = Ra_T + Ra_C.$$  \hspace{1cm} (1)

where $d$ is the characteristic size of the cavity with moist air, $\Delta T$ is the temperature difference, $T_0$ is the average temperature value in terms of the cavity volume, $\kappa$ is the thermal conductivity, $C_p$ is the specific heat of the gas mixture, $\eta$ is the dynamic viscosity, $D$ is the kinematic diffusion coefficient for the gas mixture, $\rho$ is the average gas density, $\Delta \rho_c$ is the change in density caused by the change in the concentration composition (volume percent composition) of the mixture. In a non-isothermal system, local values of thermophysical parameters can vary significantly, therefore, a functional relation between the dimensionless quantities depends on which temperature and concentration is chosen as a governing factor. In the case when the medium-averaged temperature is taken as the governing factor, the temperature dependence of the parameters of a liquid or gas has inessential effect on the experimental results. The same observation applies to the concentration dependence. Therefore, the formulas for $Ra_T$ and $Ra_C$ involve the average values of temperature and concentration.

To find correct values of $Ra_T$, $Ra_C$, it is necessary to use the exact values of the thermophysical parameters of gas available in the reference literature [12]. The dependence of the thermophysical properties of gases on temperature and pressure has been well studied. For water vapor there exist few experimental skeleton tables [12], but in recent studies it is common practice to use the well-known exact interpolation formulas, rather than tabular values. In what follows, we will also use semi-empirical equations, which take into account the dependence of the parameters $C_p$, $\kappa$, $\eta$, $D$, $\rho$ on pressure, temperature and concentration pattern of the gaseous mixture of dry air and water vapor.

For pure gases and gas mixtures in the temperature range from 270 to 400 K and at pressures from $10^{-3}$ to 10, the Mendeleev-Clapeyron equation is fulfilled and the use of equations for an ideal gas has often proved its worth. In this case, the parameters of the state of the air and water vapor conditions (far from the phase transition) calculated within the framework of the classical theory of an ideal gas are accurate within 1-2%. With this in mind, for the density of moist air $\rho$ it is reasonable to use the
formula, which takes into account the variation in the partial pressure of water vapor \( P_1 \) and of dry air \( P_2 \) with temperature:

\[
\rho = \frac{P_1 M_1 + P_2 M_2}{k_b N_a},
\]

where \( M_1 = 0.018 \) kg / mol and \( M_2 = 0.029 \) kg / mol is the molar mass of water vapor and dry air, respectively, \( k_b = 1.38 \times 10^{-23} \) J / K is the Boltzmann constant, \( T \) is the gas mixture temperature (in degrees K), \( N_a = 6.02 \times 10^{23} \) mol\(^{-1}\) is the Avogadro number. The sum of partial pressures \( P_1 \) and \( P_2 \) is equal to the atmospheric pressure \( P_0 \) which is known from the experimental conditions:

\[
P_1 + P_2 = P_0.
\]

The partial pressure of saturated water vapor \( P_1 \) is a well-studied function of temperature [12, 13]:

\[
\log_{10} \left( \frac{P_1}{P_0} \right) = 10.79586 \left( 1 - \frac{273.16}{T} \right) - 5.02808 \cdot \log_{10} \left( \frac{T}{273.16} \right) + 1.50474 \cdot 10^{-4} \left( 1 - 10^{-8.26902 \left( \frac{T}{273.16} \right)} \right) + \\
0.42873 \cdot 10^{-3} \left( 10^{4.76955 \left( \frac{T}{273.16} \right) - 1} \right) - 2.2195983,
\]

where the value of \([ P_1 ]\) is measured in mbar. Thus, knowing the temperature of moist air \( T \) at the atmospheric pressure \( P_0 \), one can calculate the corresponding partial pressure of water vapor from (3), the partial pressure of dry air from (2.1) and the desired density from (2).

The kinematic diffusion coefficient \( D_{1,2} \) of a binary mixture was calculated using the Hirschfelder formula [18]:

\[
D_{1,2} = \frac{BT^{3/2}}{p \sigma_{1,2}^{6/3}(M_1+M_2)^{1/2}}
\]

\[
B = 0.00214 - 0.0000491(\frac{M_1+M_2}{M_1M_2})^{1/2},
\]

\[
\xi^{(1,1)} = 1.075(\frac{k_b T}{\epsilon_{1,2}})^{-0.01615} + 2 \left( \frac{10^6 k_b T}{\epsilon_{1,2}} \right)^{-0.74 \log_{10} \left( \frac{10^6 k_b T}{\epsilon_{1,2}} \right)},
\]

where \( B \) is the refined Wilke coefficient [19], \( D_{1,2} \) is measured in cm\(^2\) / s, the mixture pressure \([ p ]\) is measured in atm, \( \epsilon_{1,2} \) and \( \sigma_{1,2} \) are constant forces in the Lennard-Jones potential, describing the molecular interaction in gas. The constants of interactions \( \epsilon_{1,2} \) and \( \sigma_{1,2} \) were calculated by the method [14], using the tabulated values for the components of moist air: \( \epsilon_1 / k_b = 380 \) K, \( \sigma_1 = 0.265 \) nm and \( \epsilon_2 / k_b = 97.0 \) K, \( \sigma_2 = 0.3617 \) nm for molecules of water and dry air, respectively. The constant forces \( \epsilon_{1,2} \) and \( \sigma_{1,2} \) were calculated using the formulas for a mixture of polar (water vapor) and non-polar (dry air) gases:

\[
\sigma_{1,2} = 0.5(\sigma_1 + \sigma_2) \xi^{1/6} = 0.3103 \text{ nm},
\]

\[
\epsilon_{1,2} / k_b = (\epsilon_1 / k_b)^{0.5} (\sigma_1 / k_b) = 215.8 \text{ K},
\]

where \( \epsilon_{1,2} / k_b \) is the dimensionless Stockmayer parameter for a water molecule, and \( \alpha_{1,2} = \sigma_1 / \sigma_2 \) is the dimensionless polarizability of an air molecule. The parameter \( t_1^* \) characterizes the difference in the behavior of polar and non-polar substances. The relationship \( D_{1,2} \) calculated by formula (4) is described by the fitting function with accuracy of less than 0.5%:

\[
D_{1,2} = (p^{-1})(0.2177 + 1.5562 \cdot 10^{-3} T + 2.2813 \cdot 10^{-6} T^2).
\]

The approximate calculations were carried out on the assumption that the coefficient \( D_{1,2} \) does not depend on the relative concentration of the components, since the ratio of water vapor to dry air at any temperature realized in the experiment does not exceed 1/8. The validity of this statement can be demonstrated by comparing our results with the known data for other gas mixtures. Thus, according to experimental data [14] for a mixture of \( H_2 \) and \( CO_2 \), a change in the molar ratio from 3/1 to 1/1 leads to an increase in \( D_{1,2} \) by only 1.8%. It is obvious that in our case the dependence of \( D_{1,2} \) on the percentage ratio of the mixture components will be an order of magnitude less.

To calculate the dynamic viscosity, heat capacity, and thermal conductivity, we have developed semi-empirical formulas by approximating the data from the skeleton tables of the National Standard Reference System NSRSD 125-88 [20] for moist air. The calculation of these thermophysical parameters in the temperature range of 270-400 K and pressures of \( 10^3 - 10^4 \) is independent of pressure at a relative humidity of \( \varphi = 100% \) (the calculation error does not exceed 1%):
\[ C_p = 1785.60629 - 372.25007 t + 68.79781 t^2 - 6.57854 t^3 + 0.36963 t^4 - 0.012978 t^5 + 0.00029 t^6 - 4.22899 \times 10^{-6} t^7, \]  
\[ \kappa = -3.77888 \times 10^{-12} t^5 + 5.97899 \times 10^{-10} t^4 - 3.80594 \times 10^{-8} t^3 + 1.06486 \times 10^6 t^2 + 5.88323 \times 10^{-5} t + 0.02435, \]  
\[ \text{The temperature scale } [t] \text{ is Celsius degree (C), viscosity } [\eta] \text{ – Pa·s, heat capacity } [C_p] \text{ – J / (kg·K), thermal conductivity } [\kappa] \text{ – W / (m·K).} \]

The known thermophysical parameters of moist air (2-9) allow us to calculate the convective factor \( k_c \) of the Rayleigh number (1)

\[ \text{Ra}_E = k_c d^3 \Delta T. \]

It has been known that the onset of convection corresponds to the critical value of the Rayleigh number \( \text{Ra}^* = 1700 \pm 50 \) [17], which must be substituted into (9.1) in order to find the main parameters of the experimental setup - the characteristic size of the experimental cell \( d \) and the temperature difference \( \Delta T \) between the cooler and the heater. The order of magnitude of the quantities entering into (9-1) at \( \text{Ra}_E = \text{Ra}^* \) is \( d \sim 10^{-2} \text{ m}, \Delta T \sim 1 \text{ K.} \)

It is obvious that the characteristic scale of \( d \) and \( \Delta T \) makes it difficult to experimentally study convection in moist air and would necessitate searching for a compromise solution. Due to the cubic dependence \( \text{Ra}_E \sim d^3 \), even a slight (desirable) increase in the size of the experimental cell \( d \) leads to a sharp increase in \( \text{Ra}_E \), while a hypothetical decrease in the temperature difference \( \Delta T \) is limited by the ability of the experimental equipment to control and maintain constant temperatures at the cooling and heating units with acceptable accuracy (~0.1 K). To determine the optimal compromise values of \( d \) and \( \Delta T \), we constructed a table (see Table 1) and a graph (Fig. 1).

The final combination of the values for the parameters \( d \) and \( \Delta T \) will be determined after analyzing the potentials of the proposed experimental technique in the next section of the paper.

**Fig. 1.** The plot of the effective Rayleigh number versus temperature for cavity height \( d = 1 \text{ cm} \). The average temperature \( T_0 \) varies in the range from 15 to 30 C, since exactly in this temperature range it is easier to maintain a stable temperature difference \( \Delta T \).
**Table 1.** Desired cell size for the onset of convection

| $T_0$, C | $\Delta T=2$ C | $\Delta T=3$ C | $\Delta T=4$ C | $\Delta T=5$ C | $\Delta T=6$ C |
|---------|----------------|----------------|----------------|----------------|----------------|
| 15      | 17.7           | 15.3           | 13.8           | 12.8           | 12.0           |
| 20      | 17.6           | 15.2           | 13.7           | 12.7           | 11.9           |
| 25      | 17.4           | 15.0           | 13.5           | 12.5           | 11.7           |
| 30      | 17.0           | 14.6           | 13.2           | 12.2           | 11.4           |

### 3. Experimental setup

Let us select a suitable experimental technique for the observation and study of free convection in moist air. As far as we know, the most widely used standard methods for studying the phenomena of heat and mass transfer in transparent media are as follows: the shadow method, the interferometric method using the Fabry-Perrot scheme, and holographic interferometry [15, 16]. The use of classical shadow and interference methods is complicated by insufficient sensitivity at small temperature differences $\Delta T$ (which is desirable) and low sensitivity due to a small coherence length of the laser radiation (for the most widely used He-Ne laser, the coherence length is approximately 20 cm). For these reasons, we chose the method of holographic interferometry. This method allowed us to visualize the distribution of the heat and concentration fields in the objects of complicated configuration in real time. A significant advantage of the holographic interferometer is the lack of requirements to the coherence length of the laser radiation and the quality of optical components (lenses and mirrors).

To study convection in moist air, we have developed and constructed an experimental setup shown in Fig. 2.

![Fig. 2.](image-url) Schematic representation of the experimental setup: 1 - measuring cell filled with moist air; 2 - holographic plate; 3 - video camera; 4 - heat exchanger and cooler thermostat, 5 - heat exchanger and heater thermostat 6 - He-Ne laser

Fig. 2 elucidates the operation of the setup. The source of coherent radiation in the setup is a He-Ne laser with a wavelength of 632.8 nm. Using translucent mirror the laser beam is split into two beams (reference and object beams). The reference beam after it has been reflected several times from the system of mirrors falls on a holographic photographic plate, where it interferes with the object beam, which passes twice through the measuring cell with moist air. The interferometric pattern obtained on a holographic plate is recorded by a video camera for further image processing and interpretation of experimental observations.
However, the acquisition of useful information on convection in moist air is possible only in the case when the experimental setup has sufficient sensitivity. The sensitivity and resolution of the installation (Fig. 2) ultimately depends on whether the test medium (inhomogeneously heated moist air) filling the measuring cell is able to change significantly the difference in the optical path length (the optical path difference) between the object beam and the reference beam. The greater the path difference of the beam, the more interference fringes can be observed – the higher the resolution of the setup, and eventually, the more detailed picture of the gas-dynamic flow can be obtained. It has been found that to characterize the processes occurring in the object under study, it is necessary to obtain at least five interference fringes (i.e. ten alternations of dark and light bands).

So, the next step is to obtain numerical estimates, which will allow us to determine what dimensions the measuring cell should have in order to ensure the required resolution of the setup with the known $\Delta T$. To this end, we consider the dependence of the refractive index of light on two parameters: temperature and concentration of water vapor:

\[
n(T, C) = n_0 + \frac{\Delta n}{\Delta T} (T - T_0) + \frac{\Delta n}{\Delta C} (C - C_0),
\]

where $n_0$ is the refraction index at $T = T_0$ and $C = C_0$ is the concentration of water vapor at $T_0 = 20$ C. The relation between the refraction index of the mixture of gases and its density, and, consequently, its concentration is determined by the Lorentz-Lorentz equation [16]

\[
\frac{n^2 - 1}{n^2 + 2} = \sum_i \rho_i R_i,
\]

where $R_i$ is the specific refraction, $\rho_i$ is the specific density. For moist air equation (11) takes the following form:

\[
\frac{n^2 - 1}{n^2 + 2} = R_1 \rho_1 + R_2 \rho_2,
\]

in which $R_1$ and $R_2$ are the specific refraction of dry air and water vapor, $\rho_1$ and $\rho_2$ – are the specific densities of these gases, respectively.

To find the constant refractive index $R_1$ and $R_2$, we transform the expression (11):

\[
R_1 = \frac{n^2 - 1}{(n^2 + 2) \rho_1}.
\]

The first component can be calculated by the formula:

\[
(n1 - 1) \cdot 10^8 = 8340.78 + \left( \frac{240560}{130} \right) + \left( \frac{15994}{38.9} \right),
\]

\[
\rho_1 = \frac{348.328 \cdot P}{T} \left[ 1 + P \left( 57.90 \cdot 10^{-8} - \frac{0.94581 \cdot 10^{-8}}{T} + \frac{0.25844}{T} \right) \right].
\]

At the gas temperature of 150 C, pressure of 1013.25 mbar and $\lambda_{vac} = 632.8$ nm, the specific refraction of dry air is $1.48 \cdot 10^{-7}$. The second component is calculated by the formula [12,13]:

\[
(n2 - 1) \cdot 10^8 = 295.235 + 2.6422 \cdot 10^{-7} \lambda_{vac} - 0.032380 \lambda_{vac} + 0.004028 \lambda_{vac}.
\]

\[
\rho_2 = \frac{216.582 \cdot P}{T} \left[ 1 + P^2 \left( 1 + (3.7 \cdot 10^{-4}) \left( -2.37321 \cdot 10^{-3} + \frac{223366}{T} - \frac{710792}{T^2} + \frac{7.75141 \cdot 10^4}{T^3} \right) \right) \right].
\]

At the temperature of 200 C, pressure of 13.33 mbar, and $\lambda_{vac} = 468$ nm, the specific refraction of water vapor is $1.99 \cdot 10^{-7}$. The refraction index at arbitrary temperature and concentration of molecules in the gas mixture is corrected according to the Mendeleev-Clapeyron equation and (2). Table 2 shows the values of $dn/dT$ and $dn/dC$ in the temperature range from 15 to 30 C.

| $t$, C | $dn/dT \cdot 10^7$ | $dn/dC \cdot 10^7$ |
|--------|-------------------|-------------------|
| 10     | 9.8               | 5.9               |
| 15     | 9.4               | 5.8               |
| 20     | 9.1               | 5.7               |
| 25     | 8.8               | 5.6               |
| 30     | 8.5               | 5.5               |
As is evident from Table 2, the derivatives of the refraction index with respect to concentration and temperature have the same order of magnitude. Hence, when calculating the resulting value of \( n(T) \), in equation (10) it is necessary to take into account both these terms, since the contributions of both terms to the final sensitivity of the interferometer are equivalent. Therefore, the cell length \( L \), which is necessary for obtaining 5 interference fringes (as mentioned earlier) is calculated based on the relationship between the optical path difference \( L \Delta n \) and the number of wavelengths \( N \) by the formula

\[
L = \frac{N \cdot \lambda}{2 \Delta n} \approx 0.32 \text{ m},
\]

where \( \Delta n \) is a change in the refraction index of moist air caused by the temperature difference \( \Delta T \) between the cooler and the heater, \( \lambda \) is the He-Ne laser wavelength of 632.8 nm, \( N = 5 \) is the required number of interference fringes. The coefficient 2 in the denominator (18) implies that the laser beam passes the cell twice. Here, we again face the problem of finding a compromise solution associated with increasing \( \Delta T \) (in order to increase the number of interference fringes \( N \) and reduce the length of the installation \( L \)) and decreasing the height of the cell \( d \) (9-1). With the aim of determining the optimal compromise values for \( L, \Delta T \) and \( d \) we have constructed a table (see Table 3).

| \( T \), °C | \( \Delta T = 2 \) °C | \( \Delta T = 3 \) °C | \( \Delta T = 4 \) °C | \( \Delta T = 5 \) °C | \( \Delta T = 6 \) °C |
|---|---|---|---|---|---|
| 15 | 53.1 | 31.8 | 22.7 | 17.7 | 14.5 |
| 20 | 54.1 | 32.4 | 23.2 | 18.0 | 14.7 |
| 25 | 54.8 | 32.9 | 23.5 | 12.5 | 14.9 |
| 30 | 55.3 | 33.2 | 23.7 | 18.4 | 15.1 |

Summarizing the data of tables 1 and 3, we may conclude that for the experimental study of convection in moist air it is necessary that the experimental setup should have the following optimal parameters: maintained temperature difference \( \Delta T = 3 \) K, cavity height \( d = 15 \) mm, cavity length \( L = 32 \) cm.

These parameters were used to develop a computer model of the experimental setup with the aid of three-dimensional design program Kompas-3D. The laboratory model was constructed according to its computer version.

In addition to the visual study of convection using holograms, the experimental setup allows taking measurements of heat and mass transfer in the experimental cell by means of embedded thermocouples (not shown in Fig. 3).
4. Conclusion
A review of the specialized literature devoted to the numerical simulation of the thermophysical properties of moist air has been made. The numerical estimates of the effective Rayleigh number characterizing the thermal convection of moist air have been obtained. The numerical estimates substantiate the possibility of distinguishing and comparing the convective heat and mass transfer in wet and dry air under laboratory conditions. The numerical calculations have been performed to determine the optimal technical parameters of the laboratory experimental setup designed for studying convection by the method of holographic interferometry in moist air in the temperature range from 10 to 30°C.

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