High-Q states and Strong mode coupling in high-index dielectric resonators.

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Abstract. We study strong coupling between eigenmodes of a single subwavelength high-index dielectric resonator and analyze the mode transformation and Fano resonances by varying resonator’s aspect ratio. We demonstrate that the strong mode coupling is associated with the physics of bound states in the continuum when the radiative losses are almost suppressed due to the Friedrich-Wintgen scenario of destructive interference. We confirm our theoretical findings with microwave experiments by using a high-index cylindrical resonator with tunable aspect ratio.

Introduction
Optical resonators form the basis of lasers, optical sensors, switches and amplifiers. The most important characteristic of the resonator is Q factor, which characterize amplification degree of the electromagnetic field in the resonator. To date the record Q factor of optical resonators today is about $10^{11}$ [1]. It is achieved in spherical resonators with the whispering gallery mode. However, large sizes do not allow them to be installed into optical integrated circuits. A more attractive way to confine light is to use destructive interference in the regime of the strong mode coupling. This mechanism is related to the physics of bound states in the continuum (BICs) [2]. The BIC-inspired mechanism of light localization provides make possible realization of high-Q states in photonic crystal cavities and slabs [3, 4], coupled waveguide arrays [5, 6, 7], dielectric gratings [8], core-shell spherical particles [9], and dielectric resonators [10, 11]. Using this mechanism, we have shown that even a subwavelength dielectric resonator could demonstrate high Q factors.

Results
We consider a subwavelength dielectric cylindrical resonator with permittivity $\varepsilon_1 = 80$, radius $r$, and length $l$ placed in vacuum ($\varepsilon_2 = 1$), as shown in Fig. 1a, and analyze its spectrum by calculating the maps of the scattering cross-section ($C_{\text{ sca}}$), depending on the aspect ratio $r/l$. The spectra are calculated by using the CST Microwave Studio software and T-matrix
Figure 1. (a) TE- and TM-polarized waves incident on a dielectric cylindrical resonator with permittivity $\varepsilon_1 = 80$, radius $r$, and length $l$ placed in vacuum ($\varepsilon_2 = 1$). (b) Distribution of the electric field amplitude $|E|$ for the Fabry-Perot-like mode TM$_{1,1,1}$ (point A) and Mie-like mode TE$_{1,1,0}$ (point B).

computations [12, 13]. The electric field of the incident wave is assumed to be perpendicular to the axis of the cylinder (see Fig. 1a). To compare $C_{sca}$ for cylinders with different aspect ratios, we normalize $C_{sca}$ by the projected cross-section of the resonator, $S = 2rl$. The maps of the normalized $C_{sca}$ calculated for cylinders with different aspect ratio $r/l$ excited by TM and TE-polarized wave are shown in Figs. 2a and 2b, respectively. We denote the modes of a cylindrical resonator as $TE_{n,k,p}$ and $TM_{n,k,p}$, where $n$, $k$, $p$ are the indices denoting the azimuthal, radial, and axial wavenumbers, respectively. Generally speaking, distinguishing between $TE_{n,k,p}$ and $TM_{n,k,p}$ modes for a cylinder of a finite length is justified only for $n = 0$. For other cases, the polarization is hybrid [14]. In the case of arbitrary $n, k, p$ the mode polarization is mixed. Thus, under the terms TE or TM we further imply the dominant polarization of the modes.

Figure 2. Dependencies of the total scattering cross-section (SCS) of the cylinder $\sigma$ normalized to the projected cross-section $S = 2rl$ on the aspect ratio of the cylinder and size parameter $x = r\omega/c = 2\pi r/\lambda$ for TM and TE-polarized incident wave, respectively.
The low-frequency spectrum of the dielectric cylinder under consideration consists of three types of modes. The modes with the axial index \( p = 0 \) and azimuthal index \( n = 0, 1 \) demonstrate a small frequency shift with changing changing \( r/l \). They are formed mainly due to reflection from a side wall of the cylinder, and they could be associated with the Mie resonances of an infinite cylinder (see Figs. 1a and 3a). The modes with the indices \( p > 0 \) and \( n = 0,1 \) demonstrate a strong shift to higher frequencies with increasing aspect ratio \( r/l \). They are formed mainly due to reflection from the faces of the cylinder, and they could be associated with the Fabry-Perot modes (see Figs. 1a and 3a). The modes with the azimuthal index \( n = 2, 3, ... \) are formed due to the wave incident on the side wall of the cylinder at the angles bigger than the total internal reflection angle, which is about 6.4 degrees for \( \varepsilon_1 = 80 \). Therefore, they are close in nature to the whispering gallery modes (see Fig. 3a) and their high Q factor is explained by total internal reflection but not by destructive interference as we have for quasi-BIC. Properties of WGMs are well-studied (see, e.g., Refs. 15, 16, and 17) and further we focus on the Mie-like \((TE_{1,1,0})\) and Fabry-Perot-like \((TE_{1,1,1})\) modes. Their electric field distributions are shown in Fig. 1b.

Figure 3. Modes of a dielectric resonator and models of their coupling. (a) Classification of eigenmodes of a dielectric resonator. (b) FriedrichWintgen approach describing an open cylindrical resonator as a closed resonator and a radiation continuum. Eigenmodes of the resonator interact via the radiation continuum. (c) Non-Hermitian approach describing an open cylindrical resonator by a complex spectrum of eigenfrequencies. Eigenmodes of the resonator interact via perturbation \( \delta\varepsilon(r) \) responsible for change of resonator aspect ratio.
In quantum mechanics, in the simplest case, the system with light-matter interaction is described by a sum of Hamiltonian without interaction $\hat{H}_0$ and an interaction potential $\hat{V}$ (see, e.g., Ref. 18). The diagonal components of $\hat{V}$ are responsible for energy shift and the off-diagonal components are responsible for the coupling. The interaction results in a mixing of the light and matter states and in appearance of an avoided resonance crossing the characteristic feature of the strong coupling regime [19].

In electromagnetism, due the fact that a resonator is an open system, description of the interaction between the modes becomes more complicated. There are two main approaches describing the interaction between the modes in open system. The first one considers an open system (dielectric cylindrical resonator in our case) as a closed system with non-radiating modes $|\phi_a\rangle$ and $|\phi_b\rangle$ interacting with a continuum of the radiation modes outside of the resonator in accord with the Friedrich-Wintgen mechanism [22] (Fig. 3b). The difficulty of this method is to correctly define the basis of the non-radiating modes and their coupling constants with the radiation continuum. In the second approach, the resonator is primordially considered as an open non-Hermitian system, characterised by a complex eigenfrequency spectrum. In this approach, a small change of resonator shape could be described as a perturbation $\delta\varepsilon(r)$ playing a role of the interaction potential $\hat{V}$ between modes $|\phi_a\rangle$ and $|\phi_b\rangle$. In our case, a perturbation $\delta\varepsilon(r)$ is responsible for change of the aspect ratio of the cylindrical resonator (Fig. 3c). This method is well-developed for quantum mechanics and electrodynamics [20, 21]. It allows to find spectrum, eigenmodes, and interaction constants straightforwardly from the Maxwell’s equations.

For the cylindrical resonator, the strong coupling between the Mie-like and Fabry-Perot-like modes is clearly manifested in the map of the SCS as avoided resonance crossing points (Figs. 2a and 2b). The most pronounced regions of the avoided resonance crossing are marked by red ellipses in Fig. 2b. More detailed analysis shows that in the vicinity of the avoided resonance crossing, the Q factor of one the coupled mode becomes very high that corresponds to the appearance of a quasi-BIC. The dramatical increase of the Q factor is a result of destructive interference between the modes with similar radiation patterns in far field.

**Experimental results**

Finally, we perform the experimental study to demonstrate the existence of the avoided crossing regime between the $TE_{1,1,0}$ and $TM_{1,1,1}$ resonances in the microwave frequency range. In the experiment, the plastic cylindrical vessel filled with water is placed in the middle between two antennas. The aspect ratio of the cylindrical resonator is defined by the amount of water. The photo of the experimental setup is shown in Fig. 4a. The resonator is excited by TE polarized electromagnetic wave incident perpendicular to the cylinder axis $z$ (see Fig. 4a). The measured dependence of the SCS of the cylindrical resonator depending on its aspect ratio is shown in Fig. 4b. The results of the numerical simulations taking into account the losses in water are shown in Fig. 4c. One can see that the experimental positions of the resonances are in a good agreement with the real part of eigenfrequencies (marked by white circles) calculated using the resonant state expansion method. In spite of losses in water, which broaden the resonances, the avoided crossing regime between the $TE_{1,1,0}$ and $TM_{1,1,1}$ modes and suppression of SCS clearly manifest themselves for the aspect ratio in the range of $0.5 < r/l < 0.6$. Discrepancies between the measured and calculated maps of SCS could be explained by not perfect plane wave radiated by a horn antenna and parasitic scattering from the auxiliary equipment (holder of the resonator and plastic cylindrical vessel).
Figure 4. (a) Experimental setup for the measurement of SCS spectra of the cylindrical resonator filled with water depending on its aspect ratio $r/l$ and size parameter $x$. (b) Measured SCS map demonstrating the avoided crossing regime between TE$_{1,1,0}$ and TM$_{1,1,1}$ resonances. The circles are the real part of eigenfrequencies obtained from the resonant state expansion method for a dielectric cylinder with the permittivity $\varepsilon_1 = 80$ embedded in air ($\varepsilon_2 = 1$). (c) Calculated SCS map of the cylindrical resonator filled with water depending on the size parameter $x$ and aspect ratio $r/l$.

Conclusion
We have demonstrated that a subwavelength homogeneous dielectric resonator can support strongly interacting modes. We have confirmed our theoretical results in microwave experiment by using a cylindrical resonator filled with water. Our results open new horizons for active and passive optical nanodevices including efficient biosensors, low threshold nanolasers, perfect filters, waveguides, and nanoantennas.

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