Externally controlled local magnetic field in a conducting mesoscopic ring coupled to a quantum wire

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In the present work the possibility of regulating local magnetic field in a quantum ring is investigated theoretically. The ring is coupled to a quantum wire and subjected to an in-plane electric field. Under a finite bias voltage across the wire a net circulating current is established in the ring which produces a magnetic field at its centre. This magnetic field can be tuned externally by regulating the in-plane electric field, and thus, our present system can be utilized to control magnetic field at a specific region.

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I. INTRODUCTION

The phenomenon of voltage driven circular currents in conducting junctions with single or multiple loop substructures is a notable quantum effect in low-dimensional systems. Most of the studies involving electron transport through different bridge systems essentially focus on net current transfere$^{16,17}$, rather than analyzing current distribution among different branches$^{16,18}$ of the materials within the junction. In last few years, some interesting works have been done considering different molecular structures where distribution of currents in different arms has been analyzed providing the possibilities of getting voltage induced circular currents$^{19,20}$. These circular currents produce substantial magnetic fields at ring centre and can be exploited in many ways to explain several interesting quantum mechanical phenomena as well as to design quantum devices for future applications. Controlling of a single spin placed at or near the centre of a ring-shaped geometry by means of magnetic field associated with circular currents may be the most suitable application towards this direction, since proper spin regulation is extremely important in designing spin-based quantum devices$^{21,22}$. Nowadays people are highly focused in computing with single electron spin since it involves much lower power dissipation rather than traditional computing which is always charge based. In traditional computing, computable informations are encoded by electrical charge which has only a magnitude but no direction i.e., a scalar quantity. Thus, to encode binary logic bits 0 and 1 using electrical charge two different amounts of charge are required. Now, if the bit is switched the magnitude of the charge required to encode logic levels should be changed, and accordingly, a net current flow takes place. This net current flow certainly produces a power loss. On the other hand, if computable informations are encoded by using spin, which is a pseudovector, then the bits 0 and 1 can be described by up and down spin configurations, respectively. In this case the switching between two bits can be associated with the flipping of the spin without transferring any net charge and thus much lower power dissipation can be achieved. In order to design spin-based quantum devices proper regulation of magnetic field at a particular point is highly important. Very few attempts have been made so far to control magnetic field locally, and therefore, further studies are still required. In 2004, Lidar et al. have shown how to generate localized magnetic field by using some wires those are arranged parallelly$^{23}$. Later, in 2005 Pershin et al. have proposed another scheme to create magnetic field at a specific region taking a semi-conducting quantum ring by means of phase-locked infrared laser pulse$^{24}$.

In the present work we describe a new technique to control local magnetic field in a conducting ring. The ring is coupled to a quantum wire and subjected to an in-plane electric field. Under a finite bias voltage a net circulating current is established in the ring which produces a magnetic field at its centre. This magnetic field can be changed externally by tuning the in-plane electric field which is the main motivation behind this work.

![FIG. 1: (Color online). A quantum wire coupled to a quantum ring is attached to source and drain electrodes. An in-plane electric field $E$, perpendicular to the quantum wire, is applied to the ring.](image)
different compared to the circulating currents in conducting junctions where current is driven by applied voltage bias. In presence of a finite bias, a net circulating current can be obtained in a loop geometry even in the absence of any AB flux $\phi$.

We organize the paper as follows. In Section II we describe the model together with theoretical formulations for the calculations. Essential results are described in Section III. Finally, we summarize our findings in Section IV.

II. MODEL AND THEORETICAL FRAMEWORK

A. Tight-binding model

Our system comprises a quantum ring which is coupled to a quantum wire and subjected to an in-plane electric field $\mathcal{E}$ perpendicular to the wire. The wire is attached to two semi-infinite one-dimensional metal electrodes, usually known as source and drain. The schematic diagram of our system is presented in Fig. 1. In order to get a non-vanishing circulating current in the ring, we couple it to the wire through two vertical bonds. No net current will be obtained in the ring if it is connected to the wire via single vertical bond due to mutual cancellation of currents moving clockwise and anti-clockwise directions. We use a tight-binding framework to describe the model which is extremely suitable for describing electron transport through a conducting junction especially for the case where electron-electron interaction is not taken into account.

The single particle tight-binding Hamiltonian that involves the ring, wire and side-attached electrodes can be written as,

$$H = H_s + H_{\text{el}} + H_{\text{tn}}.$$  \hspace{1cm} (1)

where, $H_s$, $H_{\text{el}}$ and $H_{\text{tn}}$ correspond to different sub-Hamiltonians those are described as follows. The first term $H_s$ corresponds to the Hamiltonian of the conductor bridging two electrodes i.e., the ring with coupled quantum wire. Under nearest-neighbor hopping approximation the Hamiltonian $H_s$ gets the form,

$$H_s = \sum_i \epsilon_i^r c_i^r c_i^r + \sum_i t_r c_{i+1}^r c_i^r + \sum_i t_w c_i^w c_i^w$$

$$+ \sum_i \epsilon_i^w c_i^w c_i^w + \sum_i t_w c_{i+1}^w c_i^w + \sum_i \lambda\left[ c_i^r c_i^p + c_i^w c_i^q + c_i^r c_i^r + c_i^r c_i^r \right].$$ \hspace{1cm} (2)

where, $c_i^r$ and $c_i^w$ represent the creation and annihilation operators of an electron at $i$-th site of the ring. Similarly, for the wire these operators are represented by $c_i^{w\dagger}$ and $c_i^{w\dagger}$, respectively. $\epsilon_i^r$ is the on-site energy of an electron at $i$-th site of the ring, while it is $\epsilon_i^w$ for the side-attached wire. To describe the sizes of this ring and the wire we introduce the parameters $N$ and $M$ as shown in Fig. 1. In presence of an in-plane electric field, perpendicular to the wire, site energy of the ring becomes field dependent and it gets the form: $\epsilon_i^r = (eaN\mathcal{E}/2\pi)\cos[2\pi(i-1)/N]$, where $e$ corresponds to the electronic charge, $a$ represents the lattice spacing and $\mathcal{E}$ measures the strength of the in-plane electric field. The nearest-neighbor hopping integrals in the wire and ring are described by $t_w$ and $t_r$, respectively. The ring is again coupled to the wire through atomic sites $p$ and $q$, those are variables, where the coupling parameter is given by $\lambda$.

The second term in Eq.\hspace{1cm}1 corresponds to the Hamiltonian of one-dimensional source and drain electrodes. It is expressed as

$$H_{\text{el}} = H_g + H_{\text{tn}} = \sum_{\alpha=r,w} \left\{ \sum_n \epsilon_0 d_n^\dagger d_n + \sum_n t_0 \left[ d_n^\dagger d_{n+1} + \text{h.c.} \right] \right\},$$ \hspace{1cm} (3)

where, $\epsilon_0$ presents the on-site energy and $t_0$ gives the nearest-neighbor hopping strength in the electrodes. For a $n$-th site electron in these electrodes the creation and annihilation operators are described by $d_n^\dagger$ and $d_n$, respectively.

Finally, the third term in Eq.\hspace{1cm}1 denotes the coupling of the wire to the source and drain electrodes. Considering $\tau_g$ and $\tau_d$ are coupling integrals between the wire and side-attached electrodes, the Hamiltonian $H_{\text{tn}}$ can be written as,

$$H_{\text{tn}} = H_{\text{tr,wire}} + H_{\text{td,wire}} = \tau_g c^w d_0 + h.c. + \tau_d c^w d_{M+1} + h.c.$$ \hspace{1cm} (4)

B. Circular current in the ring

To determine circular current in our quantum system let us first consider the current distribution in a simple model where a mesoscopic ring is coupled to two electrodes (see Fig. 2). A net junction current $I_T$ passes between source and drain, where $I_1$ and $I_2$ are the currents propagating through upper and lower arms of the ring, respectively. For the current flowing in the counterclockwise direction we use positive sign, while it is negative for the other direction. Following the current distribution given in Fig. 2 the net circulating current in the ring is defined as,

$$I_c = \frac{1}{L} \left( I_1 L_1 + I_2 L_2 \right)$$ \hspace{1cm} (5)

where, $L = L_1 + L_2$. $L_1$ and $L_2$ are the arm lengths. This is the basic definition of circular current in any loop geometry attached to two electrodes. The relation, Eq.\hspace{1cm}4 immediately suggests that for a symmetrically connected ring no net circular current will appear since in this case
$L_1 = L_2$ and $I_1 = -I_2$. Now to calculate $I_c$ using Eq. 4 we have to determine currents in different segments of the ring geometry. We compute these currents using Green’s function formalism. At absolute zero temperature ($T=0$ K), current in a bond connecting the sites $i$ and $j$, where $j = i \pm 1$, can be expressed as,

$$I_{ij} = \frac{E_F + \frac{1}{4}}{E_F - \frac{1}{4}} \int J_{ij}(E) dE.$$  \hfill (6)

$J_{ij}(E)$ is the charge current density and it provides a net bond current upon integrating over a particular energy window. $E_F$ is the equilibrium Fermi energy and $V$ is the applied bias voltage between source and drain electrodes. To describe current density we impose corre-

![Diagram](image)

**FIG. 2:** (Color online). Schematic view of current distribution through a ring geometry coupled to two electrodes. The filled blue circles correspond to the positions of the atomic sites.

lation Green’s function $G^n$ and in terms of it, $J_{ij}(E)$ can be written as \cite{17}

$$J_{ij}(E) = \frac{4e}{\hbar} \text{Im} \left[ (H_c)_{ij} G^n_{ij} \right].$$  \hfill (7)

The correlation Green’s function $G^n$ is defined in the form: $G^n = G^r \Gamma_s G^a$, where $G^r$ and $G^a$ are the retarded and advanced Green’s functions, respectively, and they obey the relation $G^r = (G^a)^\dagger$. $\Gamma_s$ is the coupling matrix associated with the coupling of the conductor to the source electrode. Introducing the concept of contact self-energies $\Sigma_s$ and $\Sigma_d$ due to two semi-infinite one-dimensional electrodes we can write \cite{26},

$$G^r = (E - H_c - \Sigma_s - \Sigma_d)^{-1}$$  \hfill (8)

where, $E$ is the energy of an injecting electron. During evaluation of the correlation function $G^n$ we fix the occupation function of the source electrode to unity while for the drain electrode it becomes zero.

C. Magnetic field in the ring associated with circular current

Due to circular current $I_c$, a net magnetic field is established in the ring. At any point $\vec{r}$ inside the ring, we calculate the magnetic field by using Biot-Savart’s law which looks like \cite{21},

$$\vec{B}(\vec{r}) = \sum_{(i,j)} \frac{\mu_0}{4\pi} I_{ij} \frac{d\vec{r} \times (\vec{r} - \vec{r}')}{|(\vec{r} - \vec{r}')|^3},$$  \hfill (9)

where, $\vec{r}'$ is the position vector for the bond current element $I_{ij}d\vec{r}$ and $\mu_0$ is the magnetic constant.

Throughout the results discussed below (Sec. III), unless otherwise stated, we fix the electronic temperature to zero and assume that the entire voltage is dropped at the wire-to-electrode interfaces. The other common parameters are as follows: $\epsilon_i^s = \epsilon_i^w = 0 \forall i, t_r = t_w = \lambda = 1$, $\tau_s = \tau_0 = 1$, $\epsilon_0 = 0$ and $t_0 = 2$. The equilibrium Fermi energy $E_F$ is set at zero and choose $c = e = \hbar = 1$ for simplification. All the energies are scaled in unit of $t_w$.

III. NUMERICAL RESULTS AND DISCUSSION

Based on the above theoretical framework, in this Section, we present our numerical results.

To explore the possibility of controlling local magnetic field, associated with voltage induced circular current, by means of external agency i.e., in-plane electric field, without changing other system parameters, we start with the current distribution shown in Fig. 3. A net junction current $I_T$ passing through the source and drain electrodes is distributed among different branches of the bridge setup where distinct colored arrows correspond to different magnitudes of the bond currents. All these bond currents are computed when the bias voltage $V$ is fixed at 1 and the in-plane electric field $E$ is set equal to 0.5. Here, we consider a 18-site ring in which the upper arm contains 17 identical bonds while only a single bond exists in the lower arm, and, currents in all these bonds are flowing in a particular direction (anti-clockwise) associated with the voltage bias which result a non-zero circular current in the ring. Direction of these bond currents for a particular voltage bias strongly influenced by the way the ring is coupled to the wire. In the particular configuration shown in Fig. 3 two neighboring atomic sites of the ring are coupled to the wire such that the difference between the arm lengths becomes maximum. If this length difference gets reduced then oppositely rotating currents can be obtained depending on the voltage window, and eventually when the arm lengths become exactly equal currents with identical magnitude are available in the upper and lower arms of the ring those propagate in reverse directions irrespective of the choice of the voltage bias which provide a vanishing circular current. The phenomenon of voltage driven circular current in a
FIG. 3: (Color online). Current distribution in different segments of the bridge setup when the bias voltage $V$ is fixed at 1 and the in-plane electric field $\mathcal{E}$ is set equal to 0.5. Here we consider $N = 18$ and $M = 8$. The ring is coupled through the atomic sites $p$ and $q$ of the wire, where $p = 4$ and $q = 5$. The arrows correspond to the propagation directions of the currents in different branches, while the magnitudes of these currents are described by different colors of the arrows. A net circulating current $I_c$ is established in the ring.

FIG. 4: (Color online). (a) Circular current in the ring as a function of applied bias voltage for different values of in-plane electric field. (b) The associated magnetic field at the ring centre, in unit of $\mu_0/2\pi R$ where $R$ is the radius of the ring, as a function of voltage bias. Here we set $N = 60$, $M = 10$, $p = 5$ and $q = 6$. The red, green and blue lines correspond to $\mathcal{E} = 0.1$, 0.15 and 0.2, respectively.

Conducting loop can be explained in terms of current carrying states associated with the energy eigenvalues of the system. For a particular bias voltage, if only one resonating state lies within the voltage window then contribution to the circulating current comes from this state. On the other hand, if more resonating states those can carry currents in opposite directions appear within the voltage window, then all these states contribute and a resultant current is obtained. Of course, the sign of the net current depends upon the dominating states i.e., the states contribute more to the current than the others. So, in short, it can be emphasized that the nature of the circular current is significantly affected by the voltage bias as well as the geometric configurations. We verify these facts through our detailed exhaustive numerical calculations.

Following the above analysis let us now demonstrate the characteristic features of circular current and associated magnetic field as a function of voltage bias, and, the effect of in-plane electric field on these quantities. As representative example, in Fig. 4 we present the results for three distinct values of the in-plane electric field $\mathcal{E}$ considering a 60-site ring which is coupled to a 10-site wire according to the prescription given in Fig. 1. It is observed that the circular current developed in the ring increases gradually with the applied bias voltage keeping its sign unchanged, and eventually, it saturates when the maximum allowed energy window is reached corresponding to the applied voltage bias. For this setup i.e., when the difference between two arm lengths becomes maximum, currents in individual bonds of the ring always circulate in one particular direction irrespective of the voltage bias which provides a net larger current with the increment of the voltage window. For other configurations bond currents with opposite directions can be obtained in different voltage regimes, and thus, reduced amplitude together with phase reversal may be expected in circular current as stated earlier.
The effect of external in-plane electric field on circular current is really very interesting. For a typical bias voltage magnitude of the circular current gets decreased with the increase of in-plane electric field, which is clearly visible by comparing the curves shown in Fig. 4(a). In presence of the in-plane electric field $\mathcal{E}$, site energies of the ring are modified following the relationship illustrated in Section II, and since these on-site energies are distinct form each other, the ring can be treated as a disordered one which results reduced current amplitude with increasing the electric field strength.

Exactly similar nature is obtained in the variation of magnetic field, developed at the ring centre associated with the circular current, as a function of applied bias voltage. The results are shown in Fig. 4(b). We compute this magnetic field at the ring centre, using Biot-Savart’s law (Eq. 4), in unit of $\mu_0/2\pi R$, where $R$ represents the radius of the ring. The spectrum shown in Fig. 4(b) points out that the magnetic field at the ring centre can be modulated by means of external electric field.

To have a comprehensive understanding to the problem i.e., fine tuning of local magnetic field at the ring centre with the help of external electric field, let us finally focus on the spectra given in Fig. 5, where we present the variations of circular current and associated magnetic field as a function of in-plane electric field for a fixed bias voltage. Two different colored curves in each spectrum correspond to the results for two different bias voltages.

![Figure 5](image.png)

**FIG. 5:** (Color online). (a) Circular current in the ring as a function of in-plane electric field for different values of bias voltage. (b) The associated magnetic field at the ring centre, in unit of $\mu_0/2\pi R$ where $R$ is the radius of the ring, as a function of voltage bias. Here we set $N = 60$, $M = 10$, $p = 5$ and $q = 6$. The red and blue lines correspond to $V = 0.5$ and 1, respectively.

To conclude, in the present work, we have demonstrated one possible route of controlling local magnetic field at a particular point externally without disturbing the system parameters. Our system comprises a quantum ring which is directly coupled to a quantum wire under a specific configuration. The ring is subjected to an in-plane electric field and it is the key controlling parameter of our present study. In presence of a finite bias voltage across the two ends of the quantum wire, a net circulating current appears in the ring, and accordingly, a magnetic field is established at its centre. Changing the in-plane electric field and keeping all other parameters unchanged, the magnetic field at the ring centre can be adjusted in a tunable way. We have employed a simple tight-binding framework to illustrate the quantum system and numerically calculated all the results using Green’s function formalism. Our presented results undoubtedly suggest that the system can be utilized as a source of local magnetic field that can be controlled externally. We strongly believe that the present investigation provides a much simpler way of controlling local magnetic field than the conventional techniques.

Some valid approximations have been taken into account in this study. The first one is the zero temperature approximation. Here we have computed the results considering zero temperature limit, but these results are equally valid even at finite (low) temperatures since thermal broadening of the energy levels of the bridging system is much weaker than the broadening caused by the wire-to-electrode coupling. The other approximation is the consideration of non-interacting electron picture. In presence of electron-electron interaction, one might expect some interesting patterns, but all the physical pictures analyzed here will remain unchanged. Beside these we have also ignored the effects of electron dephasing, system impurities, etc. These issues will be addressed elsewhere in our future work.

Finally, it is important to note that during the numerical calculations we have taken some specific parameter values to compute the results, but all these physical pictures stated above are absolutely invariant for another choices of these parameter values, which essentially sug-
gests the robustness of our investigation.

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