Mathematical modeling of deformation of pre-stressed steel trusses taking into consideration the possibility of emergencies

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Abstract. An algorithm of finite-element modeling has been developed in a geometrically non-linear setting of steel plane trusses which are pre-stressed using high strength cables, in the event of emergency in the form of single cable breakage. The application of framework gravity forces, consecutive introduction of tie bars and their pre-stressing, application of running load and emergency impact have all been monitored. Herewith, the condition of static loading using the dynamic relaxation method has been simulated. A factor taken into consideration is non-linearity associated with the inclusion of new cables into the framework, analysing the possibility of their behaviour strictly under tension, and removal of a cable from the loading system in the event of its breakage. The problem has been solved using numerical integration based on the development of the Newmark method via deriving equations of equilibrium of a discretised structure in the deformation state during each step of integration. The operability of the suggested algorithm is illustrated by the example of a steel truss with a span of 60 m, including two cables. The dynamic behaviour of an object is considered at a sudden breakage of the pre-stressed cable.

Introduction

In recent years, much attention has been paid to solving problems of the survivability of building structures under impacts beyond normal facility operating conditions [1-3]. This is due to increasingly frequent emergencies which sometimes result in collapse of buildings and facilities or parts thereof. According to GOST 27751-2014, “Reliability of building structures and foundations. Basic provisions,” for construction systems of an elevated criticality rating, some design, organisational, and technical measures shall be provided for the protection of the life and health of people and the environment against hazardous consequences arising from accidents. To assess the efficiency of such measures, it is necessary to use computational schemes which permit estimating the behaviour of frameworks in the event of breakage of internal and external links. Papers [4–6] consider power approaches to the assessment of dynamic loading of structures in the event of possible local damage without any detailed study of their behaviour over time. In [7–13] more detailed studies have been performed in a dynamic setting of a construction facility during emergencies. Herewith, paper [11] assesses the influence of local breakage process conditions on the structural behaviour under such conditions. At the same time, issues of a theoretical analysis of pre-stressed frameworks during...
breakage of pre-stressed elements still require the development of approaches which take into consideration the deformed facility loading history efficiently.

The objective hereof is the development of a general methodology and algorithm for mathematical modeling of the behaviour of steel plane trusses pre-stressed by means of tie bars in an integrated structural design, in accordance with the chronology of the facility impacts in the form of pre-stresses, design loads, and emergency breakage of one tie bar.

**Problem solution method**

The steel truss loading has been analysed while taking into consideration the structural non-linearity associated with the cable behaviour under tension, the technological process of consecutive inclusion of cables into the framework, and rapid breakage of one cable in the event of an emergency. The calculation has been carried out in a geometrically non-linear setting. Numerical experiments illustrate that for a dynamic process, the accounting of the initial conditions obtained during solution of a problem in a static setting from pre-stresses and design loads may result in the loss of stability of differential equation numerical integration during the event of an emergency impact. Therefore, all truss loading stages have been considered as non-stationary. Herewith, the dynamic relaxation method has been applied up to the breakage of a cable, the static loading conditions being simulated using rapidly attenuated processes.

The cable pre-stressing is considered to be performed on the structure under conditions when a truss is installed on the facility. A system of \( l \) tie bars has been stipulated. The following basic impacts on the truss have been taken into consideration:

A). Application of gravity forces from the truss bars.

B). Inclusion of a tie bar \( T_i \) into the facility and application of constant forces \( P_i \) to the tie bar and the truss. Such forces correspond to the effect of a jack (Figure 1a).

\[ T_i \quad \bar{T}_i \quad P_1 \quad P_1 \]

\[ \bar{T}_i \quad F_i \quad T_i \quad F_i \]

\[ \bar{T}_i \quad H_1 \quad F_i \quad P_1 \]

**Figure 1.** Diagrams accounting for introduction and breakage of cables: a) application of forces \( P_i \), b) element setting \( H_i \), c) application of forces \( F_i \)

C). Introduction of a conditional element \( H_i \) of greater stiffness simulating an anchor (Figure 1b).

D). Removal of forces \( P_i \), which is equivalent to elimination of the jack.
E). Carrying out operations B, C, D for each of the remaining tie bars using forces $P_i$ and elements $H_i$ ($i=2, ..., i_o$).

F). Application of a running load.

G). Removal of element $H_i$, associated with tie bar being broken $i$, and application of forces $F_i$ (Fig. 1c), each of which is equal modulo and opposite in direction to the force with which such tie bar acted on the truss structure in the same point prior to the breakage.

In accordance with d’Alambert’s principle, let us put down a set of equations for dynamic equilibrium of a finite element model of the facility in the deformed state as

$$\{R(\{\dot{\delta}\}, t)\} + \{\Phi(\{\delta\}, \{\dot{\delta}\}, t)\} + \{L(\{\delta\}, \{\dot{\delta}\}, t)\} + \{P(t)\} + \{F(t)\} = 0,$$

where $\{R(\{\dot{\delta}\}, t)\}$, $\{\Phi(\{\delta\}, \{\dot{\delta}\}, t)\}$, $\{L(\{\delta\}, \{\dot{\delta}\}, t)\}$ are respectively vectors of finite elements normalised to the reaction nodes, inertia forces, and viscous resistance forces expressed as vectors of nodal displacements $\{\dot{\delta}\}$, velocities $\{\ddot{\delta}\}$ and accelerations $\{\dddot{\delta}\}$, and time $t$, $\{P(t)\}$, $\{F(t)\}$ are respectively $t$-dependent vector of external forces and vector taking into consideration forces $F_i$.

Cauchy problem has been solved for set of equations (1) at zero initial conditions. A numerical integration has been carried out based on Newmark method approach stipulating the consideration of constant values of acceleration at each problem solution step [14]. In accordance with the methodology of studying the geometrically non-linear problems detailed in [10], let us report the tangent stiffness matrix $[K_{\epsilon_t}]$ of the finite element model as

$$[K_{\epsilon_t}] = [K_{\epsilon_0}] + [K_{\epsilon\sigma}],$$

where $[K_{\epsilon_0}]$, $[K_{\epsilon\sigma}]$ are stiffness matrix and initial stress matrix created for the deformation state.

Based on matrices $[K_{\epsilon_t}]$ a respective tangent matrix $[K_{\tau}]$ can be formed for the finite-element model of the facility as a whole.

In accordance with the procedure of finite-element model [15], let us assume

$$\{\Phi\} = -[M][\dddot{\delta}], \{L\} = -[C][\dddot{\delta}],$$

where $[M]$, $[C]$ are the mass matrix and finite-element system damping matrix for the deformation state.

Assume that at each step $\Delta t$ of numerical integration a linear problem has been solved. For the initial time $t_{n+1}$ of some step $n$ the considered matrices are mass matrix $[M(t_{n+1})]$, damping matrix $[C(t_{n+1})]$ and tangent stiffness matrix $[K_{\tau}(t_{n+1})]$.

Take into consideration the damping using the Rayleigh formula [14]:

$$[C(t_{n+1})] = \alpha[M(t_{n+1})] + \beta[K_{\tau}(t_{n+1})]$$

where $\alpha$, $\beta$ are the prescribed coefficients of, respectively, inertial and structural damping.

Only energy losses due to the action of dry friction forces have been taken into consideration. Herewith, $\alpha = 0$, and the structural damping coefficient can be determined during the analysis of the beyond-design-basis impact using the expression

$$\beta = \frac{\xi}{\omega_1},$$

where $\xi$ is the attenuation coefficient, $\omega_1$ is the first natural-vibration frequency.

Vector $\{R\}$ for time moment $t_n$ of the end of the $n^{th}$ integration step has been approximately determined using the formula
\{ R(\{ \delta_n \}) \} = - \sum_{k=1}^{n} \{ K_\tau(t_{k+1}) \} \{ \Delta \delta_k \}, \tag{6}

where \{ \Delta \delta_k \} is the displacement increment vector for the \( k \)th step:
\[ \{ \Delta \delta_k \} = \{ \delta(t_i) \} - \{ \delta(t_{k+1}) \}. \tag{7} \]

In accordance with the Newmark method approach, we obtain [14]
\[ \{ \delta(t_n) \} = b_1 (\{ \delta(t_i) \} - \{ \delta(t_{n-1}) \}) - b_2 \{ \delta(t_{n-1}) \} - \{ \delta(t_{n-1}) \}, \tag{8} \]
\[ \{ \delta(t_i) \} = b_1 (\{ \delta(t_i) \} - \{ \delta(t_{n-1}) \}) - b_2 \{ \delta(t_{n-1}) \} - \{ \delta(t_{n-1}) \}, \tag{9} \]

where integration parameters are \( b_0 = 4/\Delta \tau^2; \ b_1 = 2/\Delta \tau; \ b_2 = 4/\Delta \tau. \)

Inserting functional connections (4), (6)–(9) into equality (1), we obtain the following set of linear algebraic equations solved at each integration step:
\[ (b_0 [M(t_{n+1})] + b_1 [C(t_{n+1})] + [K_\tau(t_{n+1})]) \{ \Delta \delta_n \} = \{ P(t_n) \} + \{ F(t_n) \} - \sum_{k=1}^{n} \{ K_\tau(t_{k+1}) \} \{ \Delta \delta_k \} + (b_2 [M(t_{n+1})] + [C(t_{n+1})]) \{ \delta(t_{n+1}) \} + [M(t_{n+1})] \{ \delta(t_{n+1}) \}. \tag{10} \]

Sample calculation
A double-support arch-type truss was considered (Figure 2). It was stipulated that pre-stressed cable \( T_1 \), and then non-pre-stressed cable \( T_2 \) have been introduced onto the truss bar system installed at the construction facility. The basic function of cable \( T_2 \) is a provision of safeguarding in case of a possible beyond-design-basis impact on cable \( T_1 \). The truss is detached from its plane over nodes.

![Figure 2. A truss with tie bars](image_url)

The bar material is steel with a yield strength of \( \sigma_0 = 325 \) MPa and Young’s modulus \( E = 2.06 \times 10^6 \) MPa. The bars are made of pipes according to GOST 32931-2015, “Steel shaped tubes for steel structures. Specifications.” For bars 1 and 2, the structural steel sections have been accepted with the following outer diameter and thickness (\( D \times t \)) values: 140×3.5 mm; 23, 31 – 140×3 mm; 24, 25, 29, 30 – 89×4 mm; 26, 27, and 28 – 89×3.5 mm; 3, 12 – 152×5.5 mm; 4, 5, 6, 7, 8, 9, 10, and 11 – 219×10 mm; 13 and 22 – 168×6 mm; 14, 15, 20, 21, 16, 17, 18, and 19 – 152×5.5 mm; 32, 33, 34, 35, 36, 37, 38, 39, 40, and 41 – 140×3 mm.

The tie bars have been made according to GOST 3081-80, “Two lay cable of LK-O type, design 6×19(1+9+9)+7×7(1+6).” The diameter of tie bar \( T_1 \) is 29.5 mm, that of \( T_2 \) is 35.5 mm. Breaking forces for cables \( T_1 \) and \( T_2 \) are equal to 527 kN and 786.5 kN, respectively. Young’s modulus for tie bars \( E_T = 1.47 \times 10^5 \) MPa (SP 16.13330.2017. Steel structures. The updated revision of SNiP II-23-81*).
The running load on the truss is represented by the system of forces \( Q_1 = 12.75 \text{ kN}, \ Q_2 = 2Q_1 \). During analysis of deformations of the facility in a dynamic setting the inertial factors corresponding to loads \( Q_1, Q_2 \) were added to the loading system as concentrated masses \( m_1, m_2 \). According to [16], for the steel structure in question, the assumed value was \( \xi = 0.02 \). For the condition of the behaviour of a truss without tie bar \( T_1 \) the value obtained by means of calculation \( f = 1.1 \text{ 1/s} \) was taken into consideration. Before emergency, for the dynamic relaxation stages, the conventional value \( \beta = 0.25 \) was accepted. The calculations showed the possibility of ensuring the numerical integration process stability in the suggested modification of the Newmark method for problems of such type.

Figures 3 and 4 illustrate the obtained curves of vertical displacement \( W_L \) of node \( L \) and tension forces \( N \) in the tie bars as a function of time at integration step \( \Delta t = 0.01 \). Herewith, prior to impact stage \( G \), the time intervals are conditional, and the static loading conditions have been simulated in such a way. Decreasing the integration step by factor of 10 did not result in any significant change of the experimental results. As a whole, it should be noted that in the dynamic process the force in cable \( T_2 \) did not exceed the breaking force, and the stresses in the bars did not exceed the material yield strength, which means the obtained serviceability of the structure based on the results of the emergency.

**Summary**

A structural design is presented for analysis of an emergency impact on a pre-stressed steel truss in the form of breakage of one of its cables. The research methodology supposes a description based on the dynamic relaxation of static effects due to pre-stresses and loading prior to an emergency. Taking into consideration the geometrically non-linear behaviour of the load-carrying system, and transformation of the structural design associated with adding and removing cables. A Newmark method modification has been developed, making it possible to solve non-linear problems of such type. The numerical experiments have been carried out for an arch truss with two cables, one of which has been pre-stressed and subjected to emergency impact, while the other is a safeguarding element. The use of dynamic realization allowed reproducing the truss behaviour following the cable breakage with the provision of stability of the numerical integration process. The calculations demonstrated the possibility of a detailed analysis of the current processes in the truss with assessment of preventing destruction of the safeguarding device.
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