An Action Principle for Biological Systems

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Abstract. In the analysis of physical systems, the forces and mechanics of all system changes as codified in the Newtonian laws can be redefined by the methods of Lagrange and Hamilton through an identification of the governing action principle as a more general framework for dynamics. For the living system, it is the dimensional and relational structure of its biologic continuum (both internal and external to the organism) that creates the signature informational metrics and course configurations for the action dynamics associated with any natural systems phenomena. From this dynamic information theoretic framework, an action functional can be also derived in accordance with the methods of Lagrange. The experiential process of acquiring information and translating it into actionable meaning for adaptive responses is the driving force for changes in the living system. The core axiomatic procedure of this adaptive process should include an innate action principle that can determine the system’s directional changes. This procedure for adaptive system reconciliation of divergences from steady state within the biocontinuum can be described by an information metric formulation of the process for actionable knowledge acquisition that incorporates the axiomatic inference of the Kullback’s Principle of Minimum Discrimination Information powered by the mechanics of survival replicator dynamics. This entropic driven trajectory naturally minimizes the biocontinuum information gradient differences like a least action principle and is an inference procedure for directional change. If the mathematical expression of this process is the Lagrangian integrand for adaptive changes within the biocontinuum, then it is also considered as an action functional for the living system.

1. Introduction
As noted by the imminent physicist Erwin Schrödinger, the application of traditional physical principles to living systems has had limited success secondary to the inherent complexity of these systems and the constraints in a detailed accounting and knowledge of the particulars of any single biologic entity [1]. Rather than considering the direct forces and mechanisms for change codified in the Newtonian laws of physics, the rederivation of the description of physical phenomena by Lagrange and Hamilton in the late 18th century emphasized the identification of the right action functional as a more general framework for the study of dynamics that is grounded in basic system energy exchanges [2]. The Lagrangian functional inherently includes the internal directing logic of a variational principle known as the least action principle. This form of analytics is really the mapping of the state information governed by the inherent system constraints and degrees of freedom that can be translated to the more familiar elements of force and matter. This Lagrangian procedure has since become a very powerful methodology for the investigations of complicated problems in modern areas of physics such as cosmology and quantum mechanics that typically resist analysis within more traditional simple coordinate systems.
The well-known physicist Paul Davies has proposed that the concept of information is at the heart of the computational description of Nature. For environmentally integrated open living systems, it is the sensory-experiential establishment of the dimensional structure of the materials, thermodynamics, and space-time within the whole of the biologic continuum that creates the information metric space and the configuration of the action dynamics of all biologic phenomena [3]. The overall information signature of this biocontinuum state then becomes the currency for the living system’s processes. As this information signal drives actions for change and system adaptation, the dynamic information processing as described by Lagrangian mechanics also serves as a general action functional for the biocontinuum of the living system. Furthermore, the extremum reconciliation of the biocontinuum steady state through the mechanism of a minimization of information divergence can be viewed as the action principle for living systems. In this paper, the derivation of such an action functional for living systems is described which contains an innate action principle based on the logical entropic mechanics of the Kullback’s Principle of Minimum Discrimination Information.

2. Methods

Beyond the matter and energy elements that comprise the physical and structural components of living system, the role that information (i.e. Maxwell’s Demon) plays in the coordination of the necessary complexity and autonomy of these systems is central to understanding their mechanics and dynamics. The methodology utilized for the derivation of biological action dynamics involves a practical description of the mechanisms of information discrimination and processing by the living system. The tools of Shannon and Kullback-Leibler information theory and the biologic mechanisms of replicator dynamics were employed in formulating this description in an information-theoretic framework.

The considered mathematical framework or biocontinuum for the analysis of action within the living system’s internal and external milieu has been termed the biocontinuum [3]. This biocontinuum is defined as a coherent information state space that includes everything that could have a potential interaction and information exchange with the life system processes which in turn can alter and adapt the conditions of the space. So that biocontinuum space includes all possible energy and material exchanges as well as any informational communiques originating from within or external to the usual considered boundaries of the organism. As a continuum, there is a no real distinction between the living system and its embedded environment since the considered space includes everything within the experiential realm of the life processes, including the organism’s own state. The inseparability of the open living system from its environment, especially with regards to exchanges in information, is a common notion in many modern theoretical constructs [4]. An information space, as generally defined by Gregory Newby, is a set of concepts and the relations between them that are contained in an information system [5]. Therefore, it is a space containing a set of systematically and logically interconnected pieces of information as a coherent whole and therefore defines the information state of the organism and its environment. Such a space then describes the range of all possible values or meanings an entity can have under a given set of rules and conditions.

The information in the biocontinuum space takes the form of Shannon information represented as the probability spectrum of possible conditions in which the points in a manifold of a 2-dimensional geometry can represent the mean $\mu$ and variance $\sigma$ of the probability distribution [6][7][8]. This formulation will allow for a geometric representation of the biocontinuum in which the contours of the space provide for the natural trajectory of action dynamics as driven by a general functional of the living system’s adaptation to changing conditions. The dimensions of a geometrical/topological measure of any physical phenomena and its properties is specified in terms of the number of points in some coordinate system. It is within such a dimensional construct that objects can be distinguished or differentiated and becomes materially known to the living system. Therefore, the dimensionality of the biocontinuum platform is determined by the sensory and measurement capacity of the observing system in terms of an information metrics signature. Even at the cellular level chemo-tactile facilities exist to
distinguish time, acceleration, and some rudimentary space dimensionality and may have a special type of aggregated sequential ordering in experiential determination of any dimensionality [9].

In the formulation of the functional, a nonequilibrium thermodynamic steady state is considered as the base state for all living systems with any deviations from this condition resulting from some force for change. This thermodynamic state for the whole of the biocontinuum (environment and organism) is broadly described in terms of traditional Shannon entropic-information metrics [3]. In order for any organism to survive and sustain the steady state condition, the ubiquitous process of acquiring experiential information (both internal and external) and translating it into actionable meaning for adaptive responses becomes the general action dynamic for all living system changes. Therefore, the core axiomatic structure of this experiential dynamic process should include an innate action principle that can determine the system directional changes without requiring a detailing of the particulars of the system. Adaptive changes within any complex living system can then be inferentially mapped by the information geometry of its experiential biocontinuum space and the resultant Lagrangian of that space.

2.1. Theory

The mathematical derivation of the concepts presented in this paper are based on the approach of Harper and Baez in their study and analysis of evolutionary dynamics [10][11][12]. These investigators have considered the integration of the Kullback-Leibler information divergence metric (KLD) into a replicator dynamic to function as an inference engine for informing the natural evolutionary behaviors of population distributions. They found that using the differentiating capability of the KLD and the mechanics of replicator dynamics within the background of the system fitness creates a natural inference engine with Bayesian learning competence. Bergstrom and others have also found a strong relationship between the concept of Darwinian fitness and Shannon-type information metrics such as mutual information [13]. The formulation presented herein is directed to the individual organism dynamics.

While the inherent mechanics of such an inference equation naturally reduces the KLD, the broader phenomenological and axiomatic rationale for this behaviour is grounded in the entropic gradient drive of the Kullback’s Principle of Minimum Discrimination Information [14][15]. This inference solution principle states that an introduction of any new information into a system will most likely result in a dispersal of system state that is as close to the original distribution as possible and with as small a gain in information as possible. This general inference procedure is based on the Principle of Maximal Entropy as described by Jaynes and has been successfully used as an alternative approach in the derivation of statistical mechanics and a variety of other physical phenomena [16]. This Maximal Entropy method posits that inferences are made based on limited information and as such should be drawn from the probability distribution that has the maximum entropy permitted by the available information. The inferred distribution is the one that is considered to represent the most conservative assignment of values in order not to draw conclusions unsubstantiated by the existing information and known constraints. The information dynamics for this reconciliation of state is then considered the mechanism that drives a change in a conclusion during the inference process. This process is also really the basis of the entropic drive for the physical dispersal of matter and energy to maximum disorder based on the logical statistical mechanisms of chance variations. In fact, this statistically derived entropic drive is the same as that used in traditional mathematical descriptions of entropy transitions in physical phenomena. Hence, all systems naturally evolve to reconcile the introduction of new conditions in a way that maximizes their entropy and minimizes their gain in order or information.

For the living system, it is the establishment of the dimensional structure and state of space-time through experienced sensory data that creates the information signature that is considered to be the meaningful form, and order of the biocontinuum and biological phenomena. Therefore, the entropic drive for resolution of divergences includes the entirety of the information signature characterizing the matter, energy and dynamic conditions of the biocontinuum. Kullback’s Principle of Minimum
Discrimination Information then operates like a least action principle for reconciliation of information divergence across the biocontinuum.

J. E. Shore and R. W. Johnson provided an axiomatic examination of Kullback’s Principle of Minimum Discrimination Information and determined that it is a logically consistent method of inference from new information [17]. The axioms employed were based on fundamental principles requiring a consistent probability model of inductive inference and reliable results regardless of the solution pathway. Their proofs based on these consistency axioms demonstrated there is only one probability distribution solution satisfying the introduction of new information constraints. The axioms of Shore and Johnson included:

1. Uniqueness: The result of the inference should be unique.
2. Invariance: The choice of coordinate system should not matter.
3. System Independence: It should not matter whether one accounts for independent information about independent systems separately in terms of different densities or together in terms of a joint density.
4. Subset Independence: It should not matter whether one treats an independent subset of system states in terms of a separate conditional density or in terms of the full system density.

A mathematical expression that includes replicator-like dynamics incorporating the Kullback’s Principle of Minimum Discrimination Information then serves as the axiomatic logic for the inference process with the intrinsic goal of predictions for adaptive actions to optimize survival. The central idea achieved by this inference and probability updating process is the simple notion that living organisms innately contain various “hypotheses” about actions concerning survival strategies [12]. These hypotheses are continuously tested using empirical evidences from sensed internal and environmental information. The biology of the living system adapts to the best hypothesis based on experienced observations within the biocontinuum and according to the axioms and the entropic gradient driven inference mechanisms of Kullback’s Principle of Minimum Discrimination Information processing as operated by replicator dynamics. In this way, the information entropic dynamic path along a biocontinuum geodesic trajectory naturally minimizes the distances between information points for least action and serves as an inference procedure for directional change. As variations in observed biocontinuum conditions drive actions for change and adaptation, this procedure also serves as a general action functional for the living system.

This methodology also determines how much information the organism essentially gains if it has a hypothesis about the state of the biocontinuum represented by the probability distribution $p$ and then observations through some sensory experience reveals the best hypothesis is $q$ [12]. It is typical to treat a hypothesis as a population distribution and the best hypothesis is determined by the fitness of that hypothesis to produce system stability. The adapted steady state difference in hypothesis and observation ($\Delta X$) is a measure of remaining uncertainty about the biocontinuum space state. We can determine the relative information and how the hypothesis probability distribution evolves and is updated in time according to the replicator dynamic. This updating with a reduction to minimization in the $\text{KLD}$ is the inferred gain in knowledge by the living system about the probable state of the biocontinuum that happens continuously over time until it reaches a steady state in the replicator dynamic. The $\text{KLD}$ provides an injection of drive to change the system state by releasing the pent up potential energy constrained in the organism’s nonequiliribium condition. For biochemical systems, store energy is also spent and dissipated in the adjustment process. As such, this divergence is a quantitative measure of the distance from the system steady state. Additionally, as the living system learns its environment and adjusts then the divergence approaches zero and there is a new system steady state to account for the new reality of the biocontinuum.

So the formulation described above relates the relative information of the biocontinuum to the system entropy and free energy and the drive of the Second Law of Thermodynamics. This pragmatically structured information-theoretic is also consistent with the free-energy principle of Karl Friston [18]. In
this physics-based approach, free-energy is a quantity founded in information theory that bounds the state of a system’s organization as it is adapted from incoming sensory data. For a nonequilibrium system to maintain its existence in that organized state then it must naturally minimize the free energy bounds of any incoming surprisals. The natural entropic drive of the Kullback Principle of Minimum Discrimination Information directs the minimization of those surprisals as the living system’s actions for adaptation utilize the sequestered energies in the nonequilibrium tensions.

2.2 Information Geometry and Biocontinuum Dynamics

The concept of employing the methods of information geometry to study biological phenomena and naturally driven dynamics is informed by the work of Ariel Catchita and Carlo Cafaro in their work describing entropic dynamics [8][19][20]. Geometric structures defined by mathematical models are commonly used to study the dynamics of physical systems. In fact, action functionals inferred by these mathematics and geometries define the system dynamics. Such mathematical constructs can also be used to understand how information changes and flows along functional gradients within living systems. Shun’ichi Amari formalized an approach to describe the information theoretic framework of systems by applying the methods of differential geometry to the field of probability theory in a new branch of mathematics called information geometry [7][8][21][22]. The idea is that for each information point in a geometry of an n-dimensional manifold space we can associate the defining parameters of some model describing a probability distribution. The points in a manifold of a 2-dimensional geometry can represent the mean µ and variance σ of the standard equation for a normal probability distribution. However, this is a special Riemannian space with metrics determined by the properties of the probability distributions that form the points on the space. So in information geometry the space of probability distributions for a statistical model are described by the points of this Riemannian manifold. The degree to which one probability distribution point can be distinguished from another is through a measure of the distance between the points. The unique geometric information spaces configured in this way then form a statistical manifold in which the Fisher-Rao information Riemannian metric provides a way to measure distances and angles of trajectory along the manifold. The KLD localizes the Fisher Information metric at its smallest distinguishable distance [9]. Therefore, KLD as a measure of information divergence quantifies this distinction in units of uncertainty. The Fisher information takes the role of assessing the geometric curvature of the manifold and the natural flow of the trajectory [22]. Knowledge acquisition concerning the system state occurs as the divergence changes [23].

In statistics, the hypothesis concerning the likely state of the living system is really defined by a probability distribution. The dynamic changes in information specifying the evolving hypothetical system state as obtained through experiential sensory information capture can then be studied by analyzing the information divergence and the gradient flows of the Fisher-Rao information metric on the space of statistical manifold. Shun’ichi Amari showed that maximum entropy distributions are precisely the ones with minimal interaction between their variables and most closely approach independence [7]. The Fisher metric is localized by the KLD [10][11]. The Kullback-Leibler formula provides a Lyapunov function whose energy for change is driven by the computational mechanics of the replicator equation and derived from constrained tensions in the nonequilibrium condition [10][24]. As a Lyapunov, a natural gradient for flow is formed based on the dynamics for processing new information. Marc Harper and John Baez have related this process to that of Bayesian inference with minimum cross-entropy using KL_D and replicator dynamics as the path for the inference [10][11][12]. So in information geometry the entropic dynamic path is along a geodesic and the trajectory of change is that which naturally minimizes the distances between points like a least action principle (see Figure 1) [18]. The trajectory naturally minimizes the length through the processing of the replicator equation [13]. This trajectory is also the same as the minimization of free energy in a system as it naturally moves to steady state in the context of its constraints [18].
3. Results
Presented below is a derived mathematical expression incorporating the natural entropy driven Kullback’s Principle of Minimum Discrimination Information and nonlinear system replicator dynamics that describes biologic information processing and can serve as an action functional for living systems. This information entropic dynamic path guided by the Kullback’s Principle of Minimum Discrimination Information along a geodesic trajectory naturally minimizes the distances between information points in the biocontinuum as a least action principle and serves as an inference procedure for directional system change. Expressions for the potential and kinetic information of the biocontinuum are then defined by:

\[ I(q, p) = \sum_{i} \ln \left( \frac{q_i}{p_i} \right) q_i = \sum_{i} [\ln (q_i) - \ln (q_i)] q_i \]

and

\[ \frac{d}{dt} I(q, p) = -\sum_{i} [f_i(P) - f(P)]q_i = \sum_{i} f_i(P)(p_i - q_i) \]

\[ Action = S = \int_{t_1}^{t_2} (KE - PE) dt \]

I(p,q) is the information state
q is the target state with a fixed probability distribution
p is the time dependent probabilities of current state
fi(P) is the fitness of each type i in the population with fitness being a survival likelihood or probability characteristic in the context of the environment
KE is kinetic energy, PE is potential energy
Action (S) is defined as the integral summation of the Lagrangian integrand defined as the difference between the kinetic and potentials naturally minimized by Kullback’s Principle of Minimum Discrimination Information within the trajectory of the dynamics.

Figure 1. The natural physical process for reconciliation of material and energy determined information divergences within the biocontinuum is grounded in the gradient flow formed by the entropic drive of the Kullback’s Principle of Minimum Discrimination Information and serves as an action principle and functional for living systems [3].
4. Discussion and Conclusions

Modern physics often utilizes Lagrangian mechanics to describe the paths and trajectories of dynamics based on the guidance of a standard least action principle for physical phenomena. At the root of this approach is the perspective that phenomenal dynamics are more simply represented by logic-based informational and geometric constructs rather than adopting the notions of unseen forces and universal laws. This observation has led Ariel Caticha and others to conclude that the laws of the physical sciences are not really laws of Nature but rather are rules for processing information about Nature [19][20][25]. The same concepts also holds true for chemical and biological systems [26]. Information can then be considered the real currency of action and the driving “force” for dynamic changes. However, the creation of information requires the discerning experience of an observing living system [3].

In this paper we describe the results of a derivation of a general action functional for biologic systems that contains an ordering action principle uniquely suited for complex biologic systems. The drive for a living system’s adaptive reconciliation of divergence from the open steady state condition within its biocontinuum can be described by a mathematical formulation of the experiential process for actionable knowledge acquisition. That defined perception-action process incorporates the axiomatic inference of the Kullback’s Principle of Minimum Discrimination Information as driven by natural entropic forces and is operationalized by replicator dynamics with the prime objective for the living system stability and survival. The Kullback’s Principle of Minimum Discrimination Information as the fundamental rule for information processing is then established as the action principle for the processes of the living system dynamics in the context of its information biocontinuum. An axiomatic examination of Kullback’s Principle has determined that it is a logically consistent method of inference with only one probability distribution solution satisfying the introduction of new information constraints into the biocontinuum and regardless of the solution pathway. If this mathematical expression of the experiential knowledge acquisition process is regarded as the Lagrangian integrand for adaptive change in the biocontinuum, then it can also be considered as the general action functional for the living system.

5. References

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