Truss structure optimization for two design variable elements using Genetic Algorithm with stress and failure probability constraints

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Abstract. This paper presents the results of a study on trusses which are need to satisfy optimal conditions, i.e. lowest cost possible with maximal performance. The trusses considered were statically indeterminate steel structures with multi-system of loading. The cost is represented by the material volume of the structure and the maximal performance is reflected by the high working stresses within allowable stress limits. The material strength was modeled as a random variable with a Log Normal distribution. The structures are also required to meet a failure probability of $P_f=10^{-3}$, which may occur locally within the elements as well as globally on the structure as a whole. The complexity of optimization problems depends in general on the number of the considered variables. The larger the number of variables considered, the more complicated becomes the solution process. Therefore, cases of single variable elements such multi variables ones were considered in this study. Optimization problems are usually solved applying iterative procedures, frequently resorting to mathematical programming. In these procedures the process usually converges to unreliable solutions; it even may completely bogged down with no solution. To circumvent this problem, iteration was carried out applying Genetic Algorithms where the process proceeds in a stochastic manner.

1. Introduction
Engineering Solutions are optimum solutions, i.e. the best out of all the existing solutions without violating applicable restrictions or constraints. The best solution is usually reflected by the lowest cost or price of the solution. In this study, where engineering structures are considered, the price of the structure is represented by the volume of materials used in the structure, while the allowable stress and the risk of failure (probability of collapse) of the elements become restrictions or constraints to be satisfied by the structure. Such structures are usually briefly known as the Optimum Structure.

The Optimum Structure can in general be obtained iteratively by applying mathematical programming, especially for nonlinear optimization problems. Iterative procedures in mathematical programming often converge at a miss optimum solution. This frequently is caused by the iteration process which seems to continually stick at a certain point, it even may not provide a solution at all. Therefore, the desired optimum solution is, in this study, searched by means of iteration processes implementing Genetic Algorithm (GA).

GA is not built based on mathematical concepts. The algorithm is derived following Darwin's theory of evolution [1]. It is based on the principle of survival of the fittest where strong individuals will
implement cross-over breeding with other strong surviving members of the population. The results of cross-over breeding are better offspring which will dominate the population of the next generation. Weak individuals perish and will no longer affect the fitness of emerging populations. Therefore, Genetic Algorithm does not require derivatives of various functions like those necessary in conventional mathematical programming methods. GA is a stochastic process, where random values are generated to represent the controlling variables of the problem defining functions. Moreover, beside cross-over, Genetic Algorithm also provide mutation mechanisms, which also take place in a random manner based on the probability of its occurrence. The advantage of GA is that it usually succeeds in providing reliable solutions, especially in cases of complicated nature where known conventional methods fail to provide solutions.

A preliminary assessment made in this study indicated that GA is very versatile in its application. The following cases demonstrate the versatility of GA in successfully solving difficult optimization problems. [2], applied Genetic Algorithm in optimizing the shape and size of axially symmetrical shells under various loading systems. The same author [3] also conducted an assessment of the optimum shapes of deterministic trusses. [4] conducted a study of optimized castellated beams which led to the achievement of beams with upright elliptical holes.

Based on the results of the above studies, where deterministic cases were considered, a probabilistic optimization of an indeterminate truss will be reviewed in this study.

2. Problem formulation

An indeterminate steel truss like that shown on figure 1, is considered in this study. The structure is supported at points A, C, and D. It is at one time loaded with PB = 15 kN at point B and at another time with PF = -15 kN at point F. The element cross-sections are to have T-shapes like that shown in Figure 2. Each cross-section consists of two independent design variables xi and xj and two dependent variables αixi and α jxj, where αi = 0.15 and αj = 0.20. The optimization problem may be represented by its mathematical form as,

$$\min_{x \in X} F(x)$$

Subject to,

$$|\sigma| \leq \sigma_{all}$$

$$P_{fk} \leq 10^{-3}; \quad k = 1, \ldots, n$$

$$\max \{P_{fk} | k = 1,2, \ldots, n\} \leq P_f \leq 1 - \prod_{k=1}^{n} (1 - P_{fk})$$

where:

F(x) = objective function
x = design variables
Lk = length of element k
σk = working stress in element k
σall = allowable stress
k = element number
P_{fk} = failure probability of k-th element
P_f = failure probability of global structure
n = number of elements

The structure is assumed to consist of BJ 37 steel with yield stress σy = 240 MPa, allowable stress σall = 160 MPa, and is required to have a safety factor of SF = 1.5 [5]. It is instructive to note the non-linearity of the objective function.
The cross-section of each truss elements are depicted as follows:

![Figure 2. Element Cross-Section.](image)

3. Genetic algorithm

The logical flow of GA is illustrated by the chart of figure 3. The iteration process is clearly depicted in the figure. The evolution of the structure takes place as the direct result of the iteration process. The increasing change of quality of the optimum solution is reflected by the increasing average value of the fitness of each successive generation. The initial generation (Generation 0), with a population of 50 individuals, was generated entirely at random, while the next generations consisting of 30 individuals, were generated using a roulette wheel which refers to the stiffness of the previous structural generation. Individuals, who probabilistically qualify, implement cross-over fertilization with other individuals, producing two offspring. Offspring with better fitness, i.e. higher fitness values, which will survive placing parents with less degrees of fitness. Individuals with new characteristics can also be obtained by the mutation process of certain individuals who satisfy the applicable probability criteria.

![Figure 3. Flowchart of genetic algorithm.](image)
4. Individual qualifications
Each individual represents a structure with varying design variables which originally was generated at random. Hence, there exists the possibility that the structure is not a feasible solution. In other words it does not meet the given safety requirements. Therefore, the structure will most likely collapse or fail. Individuals representing the collapsed structure will have a poor fitness value. The fitness value is given by the following equation (6),

\[ Fit = \frac{C}{\phi} \]  

where:
Fit = fitness of the individual
C = constant = 10^8

\[ \phi = F(x) + (r_1 \sum_{i=1}^{I} (g_i(x))^2 + r_2 \sum_{i=1}^{I} (h_i(x))^2) \]

where:
\( \phi \) = penalty function
\( g_i \) = constraint functions applied to elements
\( h_i \) = constraint functions applied to global structure
\( r_1, r_2 \) = constants of significance

Equation (7), which is a penalty function, is applied in its complete form to infeasible structures. The last term of equation (7), is a bracket function the value of which is applied to structures violating the constraints. For feasible structures, the bracket function is ignored. The constants \( r_1=104 \) and \( r_2=105 \) are large numbers deliberately chosen such that it may dominate the value of the function \( \phi \) in the event of constraint violation. The values of the constants \( r_1, r_2, \) and \( C \) are determined by trial and error. The constant \( C \) is used for the purpose of achieving its proper value which facilitates the handling of the structural stiffness.

5. Cross-over, fertilization, and mutation
Each individual representing a structure has its typical characteristics determined by the genes contained in its chromosomes. In the cases reviewed in this study, the genes are represented by the structural design variables \( x_i \) and \( x_j \) of each structural member. Since the structure considered has 10 members, the individual therefore carries a chromosome of 20 genes. These genes were originally generated as real numbers in a random mode. The chromosome may be represented by a single linear arrangement of the genes. To enable individuals to implement cross-over fertilization these real numerical genes need to be transformed into their binary equivalence. These binary genes, beside of representing their numerical values, they must also accommodate the desired precision of the design variables. This affects the necessary number of binary digits to represent the genes correctly. The length of these binary digits may be acquired by applying equation (8) given by [7],

\[ L_{bit} = 2 \log\{(U_b - L_b).10^\alpha + 1\} \]

where :
\( L_{bit} \) = number of binary digits
\( U_b \) = upper bound of design variable
\( L_b \) = lower bound of design variable
\( \alpha \) = value of precision

Hence the length of the chromosome becomes:

\[ L_{ch} = L_{bit} \cdot n_{gene} \]
where:
- $L_{ch}$ = length of binary chromosome
- $L_{bit}$ = number of binary digits
- $n_{gene}$ = number of genes in one chromosome

The transformation of the real numbered genes into their binary equivalence proceeds according a definite procedure. Knowing the necessary length of the binary string from equation (8), the real numbered variables are first transformed into decimal-substrings and then translated into their binary number representation. The decimal-substring may be obtained from equation (10) [7]:

$$\text{decimal - substring} = (real_x - L_b) \frac{2^{n-1}}{u_b-L_b}$$

where:
- decimal-substring = value to be translated into its binary equivalence
- $real_x$ = real value of randomly generated variable
- $n$ = length of the binary sequence $= L_{bit}$

The above obtained decimal-substrings are then translated into their binary number representation. These binary strings may then be manipulated to implement cross-over fertilization as well as genetic mutations.

In this study both design variables $x_i$ and $x_j$ are given the same lower bounds of $L_b = 10.00$ mm and upper bounds $U_b = 45.00$ mm. The accuracy assumed was $\alpha = 2$. Therefore applying the above equations the binary number of the genes has a string length of $L_{bit} = 12$ bits. Each element has two design variables and the whole structure has 10 elements. Total length of one chromosome thus becomes $L_{ch} = 12 \times 2 \times 10 = 240$ bits. The whole chromosome may be illustrated in a tabular format like that shown in figure 4.

| ELEMENT 1 | ELEMENT 2 | ELEMENT 3 | ELEMENT 4 | ... | ELEMENT 10 |
|-----------|-----------|-----------|-----------|-----|-----------|
| $x_i$     | $x_j$     | $x_i$     | $x_j$     | ... | $x_i$     |

**Figure 4.** Individual chromosome consisting of 20 genes.

Cross-over fertilization was implemented utilizing a randomly generated single point cross-over. This proceeds for every gene of the participating individuals. The genetic manipulation is further illustrated in the following example, where two individuals are performing as parents, producing 2 offspring.

Individual 1 : 0 1 1 | 1 0 1 0 0 1 0 0 1
Individual 2 : 1 0 1 | 0 1 1 0 0 1 1 0

Cross-over Point

Offspring 1 : 0 1 1 | 0 1 1 1 0 0 1 1 0
Offspring 2 : 1 0 1 | 1 0 1 0 0 1 0 0 1

The process of mutation is illustrated in the following example:

Individual gene before mutation 0 1 0 1 1 0 1 0 0 1 0

Individual gene after mutation 0 1 0 1 1 0 1 0 0 1 0

The location of the mutating bits are also generated in a random fashion.
6. Convergence criteria
The iterative process shown in the chart of figure 3 is terminated when a convergent condition is reached. The average individual stiffness and the average total material volume of the whole population are taken to be the determining parameters of the convergent conditions. The criteria are formulated as follows [8]:

\[
|\text{Fit}_{n+1} - \text{Fit}_n| \leq 0.001 \\
|V_{n+1} - V_n| \leq 0.001
\]

where
\begin{align*}
\text{Fit}_n &= \text{fitness average value from } n^{th} \text{ iteration} \\
V_n &= \text{fitness volume value from } n^{th} \text{ iteration}
\end{align*}

7. Reliability analysis
Reliability analysis was conducted to determine the probability of failure or the risk of the structure considered in this study. The main principle of a structure to be safe is a resistance capacity greater than, or at least equal to, every load system working on the structure. For risk calculations the independent random design variables were assumed to have a Log Normal distribution. It was applied to each element which eventually were united into a single truss structure. This final structure needs to have a resistance capacity meeting the failure criterion of \( P_f = 10^{-3} \). A 5% Lower Tail Criteria was applied on the average resistance of the steel material due to the nominal working load. The Coefficient of Variance was taken as \( \Omega_R = 0.15 \) and a Distribution Measure \( \xi_{fy} = 0.15 \) was used. For the calculation of the average load due to the nominal working load, a 40% Upper Tail Criteria, a Coefficient of Variance \( \Omega_L = 0.30 \) and Distribution Measure \( \xi_L = 0.30 \) were applied.

The Reliability of the structure may not be acquired directly. However, it may be obtained from equation (4), namely [9],

\[
1 - \prod_{k=1}^{n} (1 - P_{fk}) \geq P_f
\]

The left side of the inequality sign represents the structural reliability which must be larger than or equal to the Structural Probability of Failure.

8. Constraints
The function of constraints is to make certain that the structure does not fail during its service. The parameters considered for this purpose are working stresses and the failure risk of the structure. The working stress may not at any time exceed the allowable stress and the risk of failure may not be larger than that required. In other words the working stress \( \sigma \leq \sigma_{all} \) and the risk of failure satisfies \( P_{fk} \leq 10^{-3} \). These requirements are included in the Penalty Function of equation (7).

9. Results
A special program in MATLAB was written to simulate the iteration process of this study. The program was run several times to acquire the lowest final value of the objective function. It took longer to achieve a convergent result in the case of indeterminate trusses with two variables elements than in the case of determinate ones with single variables elements. In this study the program was run 10 times. The iteration process was terminated at 250-th cycle, where the element x-sections, the failure probability of elements as well as that of the global structure, the fitness and material volume have reached their convergent values. Iteration beyond 250 cycles did not contribute any improvement. Table 1 exhibits the characteristics of the final structure. It shows a global probability of failure of 0.00099. Note the much lower probabilities of failure of the individual elements.

It can be seen clearly from the table 1 that the probability structure maximum in iteration number 250th is 0.000996, in element number 1 whereas the average structural fitness is 34.617 as a result in MATLAB. Furthermore, total material volume, which is an optimum volume of truss structure, is about 88,662,668 mm³.
Table 1. The characteristics of the final optimal structure in iteration number 250th.

| Number of Element | Length of Element (mm) | Force(s) (N) | Area (mm²) | Stress (N/mm²) | Probability of Failure | Volume (mm³) |
|-------------------|------------------------|--------------|------------|----------------|-----------------------|--------------|
| 1                 | 4000                   | 15488.85     | 132.52     | 119.56         | 9.96E-04              | 530090.55    |
| 2                 | 4000                   | 6951.52      | 60.61      | 115.85         | 7.18E-04              | 240027.35    |
| 3                 | 4000                   | 1689.71      | 32.00      | 52.80          | 1.25E-04              | 128000.00    |
| 4                 | 5656.85                | -1194.80     | 32.00      | -37.34         | 1.73E-04              | 181019.34    |
| 5                 | 4000                   | 6106.66      | 60.01      | 101.77         | 1.71E-08              | 240027.35    |
| 6                 | 5656.85                | -7441.32     | 63.51      | -117.17        | 8.09E-06              | 359253.76    |
| 7                 | 5656.85                | 12577.08     | 112.63     | 111.67         | 4.86E-04              | 637121.42    |
| 8                 | 4000                   | -7203.63     | 60.38      | -119.30        | 9.74E-07              | 241536.53    |
| 9                 | 5656.85                | -2389.61     | 32.00      | -74.68         | 3.08E-16              | 181019.34    |
| 10                | 4000                   | 4416.96      | 37.52      | 117.71         | 8.48E-10              | 150099.15    |
| Total Volume      |                        |              |            |                |                      | 88662667.76  |

Figure 5 shows a schematic of the x-section area of the individual elements:

![Figure 5](image1)

**Figure 5.** Optimal x-section area for multiple design variable elements (in mm²).

Figure 6 exhibits the iteration history of the structural fitness. The blue line indicates the progress of the maximum fitness value, the red line represents that of the average value, and the green line shows that of the minimum value of the population.

![Figure 6](image2)

**Figure 6.** Iteration history of the structural fitness.

On the other hand, figure 7 shows the iteration history of the structural material volume after optimization process.
10. Concluding remarks
This study reveals that Genetic Algorithms may be applied to perform optimization of indeterminate trusses with multi-variables elements, provided the necessary number of constraints are available. In the case considered in this study elements with two design variables are assumed to satisfy the two stress and probability constraints. The optimization problem is a non-linear one since the objective and the probability are nonlinear functions. It is immediately obvious that to solve the problem by the usual mathematical programming method is not an attractive proposal. The stochastic process of Genetic Algorithm is not based on mathematical concepts. Results acquired from multiple runs of the Genetic Algorithm program usually converged to similar points, hence leading to the acceptance of the final structure as the reliably valid solution.

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