Radiatively non-isothermal Bondi accretion onto a massive black hole

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ABSTRACT
In this paper, we present a generalization of the classical Bondi accretion theory for the case of non-isothermal accretion processes onto a supermassive black hole (SMBH), including the effects of X-ray heating and the radiation force due to electron scattering and spectral lines. The radiation field is calculated by considering an optically thick, geometrically thin, standard accretion disk as the emitter of UV photons and a spherical central object as a source of X-ray emission. In the present analysis, the UV emission from the accretion disk is assumed to have an angular dependence, while the X-ray/central object radiation is assumed to be isotropic. The influence of both types of radiation is evaluated for different flux fractions of the X-ray and UV emissions with and without the effects of spectral line driving. We find that the radiation emitted near the SMBH interacts with the infalling matter and modifies the accretion dynamics. In the presence of line driving, a transition from pure accretion to outflows takes place regardless of whether or not the UV emission dominates over the X-ray emission. Estimated values of the accretion radius and accretion rate in terms of the classical Bondi values are also given. The results are useful for the construction of proper initial conditions for time-dependent hydrodynamical simulations of accretion flows onto SMBH at the centre of galaxies.

1 INTRODUCTION

The spherically symmetric Bondi solution (Bondi 1952) has become a reference model for interpreting the observations of mass accretion onto supermassive black holes (SMBHs) in the centre of galaxies. This is so because SMBHs at the centre of elliptical galaxies are likely to accrete primarily from the surrounding hot, quasi-spherical interstellar medium, suggesting that accretion rates can be simply estimated using Bondi accretion theory. These studies on accretion of matter were considered in the context of Newtonian gravity (Bondi & Hoyle 1944; Bondi 1952; Hoyle & Lyttleton 1939) and then generalized to curved spacetime in Michell (1972). Moreover, lines of research have been proposed considering quantum corrections to the general relativistic version of accretion (Yang 2015; Contreras et al. 2018).

In recent years, the accretion onto SMBHs has been the subject of intense research because of its role in the co-evolution of SMBHs and their host galaxies. In particular, the number of these studies down to the parsec scale in the centre of galaxies has significantly increased with the advances in instrumental and computational capabilities. On the other hand, fully analytical solutions on isothermal Bondi-like accretion including radiation pressure and the gravitational potential of the host galaxy have started to appear recently (Korol et al. 2016; Ciotti & Pellegrini 2017; Ciotti & Ziaeef Lorzad 2018, hereafter KCP16, CP17 and CZ18). Moreover, recent detailed numerical calculations on Bondi accretion (Ramírez-Velasquez et al. 2018), using state-of-the-art consistent Smoothed Particle Hydrodynamics (SPH) techniques (Gabbasov et al. 2017; Sigalotti et al. 2018), suggest that it would be possible to push these studies further to cover the sub-parsec scales in active galactic nuclei (AGNs), including the effects of radiation pressure due to spectral lines (Ramírez-Velasquez et al. 2016, 2017).

Accretion onto compact objects is now recognized to be a major influencer on the environment surrounding SMBHs in the centre of galaxies (e.g., Salpeter 1964; Fabian 1999; Barai 2008; Germain et al. 2009). In the study of the several
properties governing the evolution of the system, i.e., Bondi quantities such as the accretion rate and the solution of the hydrodynamical equations, a quite common assumption is to prescribe values of temperature, pressure and density at infinity and use them as the “true” boundary conditions for the solution of the equations, even when the problem at hand needs sub-parsec resolutions scale as, for instance, in the case of AGNs. However, such resolution is never achieved in observational and numerical studies (Pellegrini 2005), introducing under- and over-estimations of the physical quantities (CP17). The outflow phenomenon is believed to play a major role in the feedback processes invoked by modern cosmological models (i.e., Λ-Cold Dark Matter) to explain the possible relationship between the SMBH and its host galaxy (e.g., Magorrian et al. 1998; Gebhardt et al. 2000) as well as in the self-regulating growth of the SMBH. The problem of accretion onto a SMBH has been studied via hydrodynamical simulations of galaxy evolution (e.g., Ciotti & Ostriker 2001; Di Matteo et al. 2005; Li et al. 2007; Ostriker et al. 2010; Novak et al. 2011). For example, in numerical studies of galaxy formation, spatial resolution permits resolving scales from the kpc to the pc, while the sub-parsec scales of the Bondi radius are not resolved. This is why a prescribed sub-grid physics is often employed to solve this lack of resolution. With sufficiently high X-ray luminosities, the falling material will have the enough opacity, developing outflows that originate at sub-parsec scales (e.g., Ramírez et al. 2005, 2008a,b; Ramírez 2008c,d; Ramírez & Bautista 2011; Ramírez 2011; Ramírez & Tombris 2012; Ramírez 2013a; Ramírez 2013b; Ramírez, J. M. and J., García 2016; Pérez & Ramírez 2014). Therefore, calculations of the processes involving accretion onto SMBH have become of primary importance (e.g., Proga 2000; Proga et al. 2000; Proga 2003; Proga & Kallman 2004; Proga 2007; Ostriker et al. 2010). In order to provide a robust and reliable methodology for the production of initial conditions (ICs) for the numerical calculations of mass accretion on SMBHs, we have combined the geometric effects and assumptions employed by Proga (2007) (hereafter P07) to compute the radiation field from the disk and central object with the analytical solution procedure provided by KCP16 and CP17.

In particular, P07 reported axisymmetric, time-dependent hydrodynamical calculations of gas flow under the influence of the gravity of black holes in quasars by taking into account X-ray heating and the radiation force due to electron scattering and spectral lines. He found that for a SMBH with a mass of $10^8 M_\odot$ with an accretion luminosity of 0.6 times the Eddington luminosity, the flow settles into a steady state and has two components: an equatorial inflow and a bipolar inflow/outflow with the outflow leaving the system along the disk rotational axis and the inflow being a realization of a Bondi-like accretion flow. To calculate the radiation field an optically thin accretion disk was considered as a source of UV photons and a spherical central object as a source of X-rays. In contrast, KCP16 generalized the classical Bondi accretion theory, including the radiation pressure feedback due to electron scattering in the optically thin approximation and the effects of a general gravitational potential due to a host galaxy. In particular, they presented a full analytical discussion for the case of a Hernquist galaxy model. An extension of this analytical solution was reported by CP17 in terms of the Lambert-Euler $W$-function for isothermal accretion in Jaffe and Hernquist galaxies with a central SMBH. They found that the flow structure is sensitive to the shape of the mass profile of the host galaxy and that for the Jaffe and Hernquist galaxies the value of the critical accretion parameter can be calculated analytically.

In this paper we derive radiative, non-isothermal ($\gamma \neq 1$) solutions for use as initial conditions in numerical simulations of mass accretion flows onto massive compact objects. To do so we introduce a radiation force term due to a non-isothermal extended source in the momentum equation as in P07 and develop a semi-analytical solution for the non-isothermal accretion onto a SMBH at the centre of galaxies using a procedure similar to that developed by KCP16 and CP17. The effects of the gravitational potential due to the host galaxy are ignored here, and they will be left for a further analysis in this line. In Section 2, we introduce the mathematical methodology and the fundamental equations, while Section 3 contains the analysis of the results. Section 4 deals with a general analysis of the bias in the estimates of the Bondi radius and mass accretion rate and discusses the importance of using the semi-analytical solution as true initial conditions for numerical simulations of accretion flows. Finally, Section 6 contains the relevant conclusions.

2 GENERALIZED BONDII ACCRETION FOR A NON-ISOTHERMAL FLOW

Under the assumption of spherical symmetry, the classical Bondi solution describes a purely adiabatic accretion flow on a point mass, for the case of a gas at rest at infinity and free of self-gravity. Detailed numerical calculations of the classical Bondi accretion flow onto a stationary SMBH were performed by Ramírez-Velasquez et al. (2018), using a mathematically consistent SPH method. In real situations, however, the accretion flow is affected by the emission of radiation near the SMBH. The radiation emitted near the SMBH interacts with the inflowing matter and modifies the accretion dynamics. The radiation effects can be strong enough to stop the accretion and shut off the central object. Here we follow the recipe of P07 to model the radiation field from the disk and central object, where a radiation force term is added as a source in the momentum equation. In this formulation, the disk is assumed to be flat, Keplerian, geometrically thin and optically thick. The structure and evolution of a flow irradiated by a central compact object can be described by the continuity and momentum equations

\[
\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v},
\]

\[
\frac{dv}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \mathbf{F}^{\text{rad}},
\]

where $\rho$ is the mass density, $\mathbf{v}$ is the velocity vector, $p$ is the gas pressure, $\mathbf{g}$ is the gravitational acceleration due to the central object and $\mathbf{F}^{\text{rad}}$ is the total radiation force per unit mass. In equation (2), the gas pressure is related to the mass density by the equation

\[
p = \frac{k_B T}{\mu m_p} = p_{\infty} \tilde{\rho}^\gamma,
\]

where $k_B$ is the Boltzmann constant, $T$ is the gas temperature, $\mu$ is the mean molecular weight, $m_p$ is the proton mass,
where \( f \) is the internal optical thin gas is approximated by the relation

\[
\xi = \frac{\dot{M}}{c_s^2} \leq 1
\]

where \( \xi \) is the photoionization parameter. As in P07, the photoionization parameter is calculated as

\[
\xi = \frac{g_{\text{ion}}(\theta)}{m_{\text{e}} c_s^2}
\]

where \( g_{\text{ion}}(\theta) \) is the ionization correction factor. The parameter \( \xi \) is determined by the integral

\[
\tau_X = \int_0^\tau \kappa_X p d\tau,
\]

between the central source (\( r = 0 \)) and a point \( r \) in the accreting flow, where \( \kappa_X \) is the absorption coefficient. Here the attenuation of the X-rays is calculated using \( \kappa_X = 0.4 \) g\(^{-1}\) cm\(^2\) for all \( \xi \).

In terms of the normalized quantities

\[
x = \frac{r}{r_B}, \quad \xi = \frac{\dot{M}}{c_s^2} = \tilde{\nu}^{(\gamma-1)/2}, \quad \mathcal{M} = \frac{\nu}{c_s},
\]

where \( r_B = \frac{G M_B}{c_s^2} \), is the Bondi radius, \( \tilde{\nu} = \gamma\dot{M}_\infty/\rho_\infty \), \( G \) is the Newtonian gravitational constant, \( M_B \) is the mass of the black hole and \( \mathcal{M} \) is the Mach number, equation (2) reduces to the dimensionless generalized Bernoulli equation

\[
\tilde{\nu}^{(\gamma-1)} \left( \frac{\tilde{M}^2}{2} + \frac{1}{\gamma-1} \right) = \frac{1}{x^2} + \frac{\tilde{F}^{\text{rad}}(\theta)}{x} + \frac{1}{\gamma-1},
\]

for the steady-state, spherically symmetric, non-isothermal (\( \gamma > 1 \)), gravitational accretion, including the effects of radiation emission due to electron scattering and spectral discrete lines with appropriate boundary conditions at infinity, where

\[
\tilde{F}^{\text{rad}}(\theta) = \frac{F^{\text{rad}}(\theta)}{r_B},
\]

\[
\tilde{L}^{\text{Edd}} = \frac{L}{L^{\text{Edd}}}, \quad L^{\text{Edd}} = 4\pi G M_B m_p / \sigma T
\]

is the Eddington luminosity, \( \sigma T = 6.6524 \times 10^{-25} \) cm\(^2\) is the Thomson cross section and \( f^{\text{rad}}(\theta) \) is the radiative force parameter given by

\[
\tilde{f}^{\text{rad}}(\theta) = f_s + 2 \cos(\theta) f_{\text{disk}}[1 + \mathcal{M}(t)].
\]

Although the force multiplier depends on the gas temperature through the parameter \( k \), as shown by equation (17) of P07 based on detailed photoionization calculations performed with the XSTAR code (T. Kallman 2006, private communication), here we adopt the temperature-independent relation given by equation (12) and leave the corrections for the temperature effects for future numerical work in this line (for instance, when using an energy equation accompanied by the net heating and cooling function developed by Ramírez-Velasquez et al. 2016, 2017). Equation (13) must be solved coupled to the continuity equation (1), which in terms of the above normalized parameters can be written as

\[
x^2 \tilde{M} \tilde{\nu}^{(\gamma+1)/2} = \lambda,
\]

where

\[
\lambda = \frac{\dot{M}}{4\pi r_B \rho_\infty c_s}
\]

is the dimensionless accretion parameter that determines the accretion rate for given \( \dot{M}_B \) and boundary conditions. Using equation (16) to eliminate \( \tilde{\nu} \) from equation (13), the generalized Bondi problem reduces to solving the equation

\[
g(M) = \chi f(x), \quad \text{with} \quad \Lambda = \left[ \chi \tilde{f}^{\text{rad}}(\theta)^3 \right]^{2(1-\gamma)/(\gamma+1)},
\]
KCP16 and CP17, the new feature of the present model is the angular dependence of the radiative force parameter. Although equation (18) together with the definitions (19)-(22) look the same as those derived by KCP16 and CP17, there are some differences that are described in the following.

\[ \chi_{\text{rad}}(\theta) = 1 - \tau_{\text{rad}}^2 \lambda_{\text{ext}}, \]
\[ g(M) = \mathcal{M}^{2(1-\gamma)/(\gamma+1)} \left( \frac{M^2}{2} + \frac{1}{\gamma - 1} \right), \]
\[ f(x) = x^{\gamma - 1} \left( \frac{\chi_{\text{tot}}(\theta)}{2 + \frac{1}{\gamma - 1}} \right), \]
\[ \lambda_{\text{ext}} = \frac{1}{4} \left( \frac{2}{5 - 3\gamma} \right). \]

Although equation (18) together with the definitions (19)-(22) look the same as those derived by KCP16 and CP17, there are some differences that are described in the following.

Compared to the previous analyses performed by KCP16 and CP17, the new feature of the present model is the angular dependence of the UV emission of the accretion disk, while it is assumed that the X-ray/central object radiation does not change with θ. For an incident angle of θ = 0, the particles experience maximum intensity. As θ increases towards the equator, it follows from equation (15) that the radiation flux from the accretion disk becomes weaker until it vanishes for θ = π/2. Consequently, the ratio between the X-ray and the UV flux increases with increasing θ. Thus, for large θ pure gravitational accretion takes place until a critical angle θ_{\text{phase}} develops for which radiation dominates over gravity and outflows. In Fig. 1 we plot this feature of the solutions for three different flux fractions of the X-ray and UV emissions with and without the force multiplier. For instance, models M1, M2 and M3 have \( f_x = 0.5 \) and \( f_{\text{disk}} = 0.5 \), \( f_x = 0.2 \), and \( f_{\text{disk}} = 0.05 \) respectively, with \( M(t) = 0 \) so that we can evaluate the influence of both types of radiation in detail. Models M1, M2 and M3 are identical to models M1, M2 and M3, respectively, but with \( M(t) \neq 0 \) and a radially varying ionization parameter given by

\[ \xi = \frac{L}{n \pi r^2}, \]

where the total accretion luminosity is set to be \( L = 7.45 \times 10^{45} \text{ erg s}^{-1} \), which is appropriate for a SMBH of \( 10^6 M_\odot \) at a distance \( r = 10 \text{ pc} \), accreting at an efficiency of 8%. The gas density is \( n_H = 10^{15} \text{ cm}^{-3} \) and the optical depth is set to \( t = 0.3 \). The physical parameters employed for all these models are listed in Table 1. For all models we choose \( \theta = \pi/4 \) and \( \gamma = 1.1 \). The values of \( f_x \) and \( f_{\text{disk}} \) listed in Table 1 are the same employed by P07 and are guided by the observational results from Zheng et al. (1997) and Laor et al. (1997).

Models M1, M2 and M3 represent non-isothermal accretion models with only the effect of electron scattering as in KCP16, CP17 and CZ18. A look to Fig. 1 shows that for model M1, where the X-ray and UV emitters have each the same fraction of participation (50%) in the emission of radiation, there is no angle at which outflows are produced. In fact, model M1 represents a pure accretion process with a nearly cancellation of the force of gravity close to \( \theta = 0 \) and a radiative force parameter, \( \tau_{\text{rad}}(\theta) \), always below unity. In contrast, model M2 with \( M(t) \neq 0 \) and \( \theta_{\text{phase}} \approx 0.51 \) undergoes a transition from pure accretion to moderate outflow with \( \lambda_{\text{rad}}(\theta) \approx 1.1 \) at \( \theta \approx 0 \). Similarly, models M2 and M3 have values of \( \theta_{\text{phase}} \) for which there will be moderate outflows, while models M2 and M3 develop from moderate to powerful outflows. In what follows we shall explore in more detail these numerical solutions separately.

3 ANALYSIS OF THE RESULTS

From equation (18) it is clear that all physical quantities can be expressed in terms of the radial Mach number profile.

Figure 2 shows the radial Mach number profiles of the inflow (solid lines) and outflow (dotted lines) solutions of equation (18) for models M1, M2 and M3. The supersonic inflow (\( M > 1 \)) solution is depicted in orange, while the subsonic (\( M < 1 \)) solution is given by the blue lines. For model M1, X-ray heating is the strongest (i.e., \( f_x = f_{\text{disk}} = 0.5 \)). In this case, the solution predicts a transition from pure accretion to a moderate outflow (i.e., \( \theta > \theta_{\text{phase}} \)) with a critical radius \( x_{\text{crit}} \approx 0.0092 \), where the supersonic and subsonic flows match to produce a full transonic flow. For model M2, X-ray heating by the central object (\( f_x = 0.2 \)) is smaller than the UV emission from the accretion disk (\( f_{\text{disk}} = 0.8 \)) and we observe two important differences compared to model M2. First, the critical point occurs at a larger radius \( x_{\text{crit}} \approx 0.03425 \) and so the flow becomes supersonic at a distance from the SMBH about 4 times larger than for model M2. Second, at an inner radius of \( \approx 0.001 \), the inflow for model M2 reaches \( M \approx 8.1 \) against \( \approx 4.1 \) for model M2.

In Fig. 2, the supersonic outflow solution (\( \theta < \theta_{\text{phase}} \)) is depicted by the dotted blue lines, while the subsonic solutions are given by the dotted orange lines. Notice that for the M3 model, the winds are developed farther away from the centre, i.e., around \( x \approx 0.1 \) compared to model M2, where the thermal outflows are produced closer to the central object at \( x \approx 0.035 \). Therefore, at larger distances from the SMBH, i.e., at \( x \approx 3 \), faster winds are developed for model M3 (\( M(x = 3) \approx 6.2 \)) compared to model M2 for which \( M(x = 3) \approx 4.4 \). The outflow phase does not produce real solutions for Mach numbers beyond a certain radius and so there is no a transonic point for the outflows. In all plots, the subsonically and supersonically flows escaping from the
Table 1. Parameters of the generalized Bondi accretion models.

| Run  | Lines(α) | \( f_\star \) | \( f_{\text{disk}} \) | \( \gamma \) | \( \theta_{\text{acc}}^{\text{phase}} \) (rad) | \( \theta_{\text{out}}^{\text{phase}} \) (rad) | \( x_{\text{crit}} \) |
|------|-----------|---------------|----------------|--------|-----------------|-----------------|--------|
| M_1^* | yes       | 0.5           | 0.5            | 1.1    | 0.517465        | 1.1             | 0.9    | 0.0092 |
| M_2^* | yes       | 0.2           | 0.8            | 1.1    | 0.821151        | 1.1             | 0.9    | 0.03425 |
| M_3 | yes       | 0.05          | 0.95           | 1.1    | 0.886684        | 1.1             | 0.9    | 0.0464 |
| M_1 | no        | 0.5           | 0.5            | 1.1    | Imaginary       | 1.1             | 0.9    | 0.119  |
| M_2 | no        | 0.2           | 0.8            | 1.1    | 0.391333        | 1.1             | 0.9    | 0.00635 |
| M_3 | no        | 0.05          | 0.95           | 1.1    | 0.540618        | 1.1             | 0.9    | 0.014  |

(a) \( M(t) \neq 0 \) (yes) and \( M(t) = 0 \) (no).

The angles \( \theta_{\text{acc}}^{\text{phase}} \) and \( \theta_{\text{out}}^{\text{phase}} \) are given in terms of \( \theta_{\text{phase}} \), i.e., \( \theta_{\text{acc}}^{\text{phase}} = 1.1\theta_{\text{phase}} \) and \( \theta_{\text{out}}^{\text{phase}} = 0.9\theta_{\text{phase}} \).

Figure 2. Mach number of the inflow (solid lines) and outflow (dotted lines) solutions of equation (18) with \( M(t) \neq 0 \). The supersonic part of the inflow solutions are depicted in orange, while the subsonic part is given in blue. In contrast, the supersonic outflow solution is depicted in blue and the subsonic one in orange. The vertical lines in each frame mark the position of the sonic radius. When the radiation dominates, the gas will escape from the gravitational potential of the accretor. The upper, middle and bottom frames correspond to models with \( f_{\text{disk}} = 0.5, 0.8 \) and 0.95, respectively (see Table 1).

Figure 3. Density profiles of the radiative non-isothermal models with \( M(t) \neq 0 \). The vertical lines mark the position of the sonic radius for each model. As the UV emission becomes stronger, the accreting gas becomes supersonic at larger radial distances from the SMBH, while the density becomes higher for \( x \lesssim 0.5 \).

gravitational potential of the SMBH are indicated by the arrows. When only 5% of the heating from the central object contributes to the radiation factor, as in model M_3*, \( x_{\text{crit}} \approx 0.046 \) and mass accretion proceeds supersonically below this radius with \( M \approx 10 \) at \( x = 0.001 \), while both subsonic and supersonic outflows are produced for \( x \gtrsim 0.2 \). The numerical simulations of P07 for \( f_\star = f_{\text{disk}} = 0.5 \) (his run case A) shows that a strong X-ray heating can accelerate the outflow to maximum velocities of 700 km s\(^{-1}\) with the outflow collimation by the infall increasing with increasing radius. When \( f_\star = 0.2 \) and \( f_{\text{disk}} = 0.8 \), the line driving is seen to accelerate the outflow to higher velocities (up to 4000 km s\(^{-1}\)) while the outflow collimation becomes very strong for \( f_\star = 0.05 \) and \( f_{\text{disk}} = 0.95 \), i.e., when the X-ray heating is the smallest, which corresponds to our model M_1*.

In this case, the gas outflow is confined within a very narrow channel along the rotational axis of the accretion disk, while most of the computational volume is occupied by the inflow.

The density profiles for models M_1*, M_2* and M_3* are shown in Fig. 3. As the UV emission intensity from the accretion disk becomes stronger, the gas density close to the SMBH becomes higher. This is counter-intuitive because we would have expected the radiation to push the gas away from the centre, resulting in decreasing density close to the SMBH. This is in fact the case. As the radiation intensity increases, it pushes the critical radius farther.
SMBH increases by factors as large as occupations a larger central volume, the density close to the personic at larger radial distances. As the supersonic flow away from the SMBH and consequently the gas becomes su-

gravitational potential of the accretor. With no force multi-

dius. When the radiation dominates, the gas will escape from the subsonic part is given in blue. In contrast, the superson ic out-

personic part of the inflow solutions are depicted in orange, while

The vertical lines in each frame mark the position of the soni c ra-

Mach number of the inflow (solid lines) and outflow

dotted lines) solutions of equation (18) with \( M(t) = 0 \). The supersonic part of the inflow solutions are depicted in orange, while the subsonic part is given in blue. In contrast, the supersonic out-

solution is depicted in blue and the subsonic one in orange. The vertical lines in each frame mark the position of the sonic ra-

radius. When the radiation dominates, the gas will escape from the gravitational potential of the accretor. With no force multiplier, no winds are developed for \( f_* = f_{\text{disk}} = 0.5 \) (model \( M_1 \)).

away from the SMBH and consequently the gas becomes supersonic at larger radial distances. As the supersonic flow occupies a larger central volume, the density close to the SMBH increases by factors as large as \( \approx 10 \) when going from \( f_{\text{disk}} = 0.5 \) to \( f_{\text{disk}} = 0.95 \).

The Mach number profiles for models \( M_1, M_2 \) and \( M_3 \) with no force multiplier are shown in Fig. 4. In the absence of line driving and strong X-ray heating (model \( M_1 \)), no outflows are developed regardless of the angle of incidence. In this case, the critical point occurs at \( x \approx 0.12 \) and close to the SMBH at \( x = 0.001 \) the Mach number reaches values as high as \( \approx 250 \). As the UV emission intensity increases over the X-ray intensity (models \( M_2 \) and \( M_3 \)), the critical point and the outflows occur at smaller radii from the SMBH compared to models \( M_2^* \) and \( M_3^* \). On the other hand, at radii sufficiently far from the SMBH, i.e., at \( x = 3 \) the outflows become more supersonic than for models \( M_2^* \) and \( M_3^* \) as we may see by comparing Figs. 2 and 4. These results clearly show that radial velocities can differ from system to system depending on the details of the radiative processes dominating the source.

For our choice of \( \theta = \pi/4 \), Fig. 5 shows the density profiles for models \( M_1, M_2 \) and \( M_3 \). We see that model \( M_1 \) reaches a density as high as \( \approx 3000 \) at \( x = 0.001 \), which is about 30 times larger than for model \( M_1^* \). This result implies that when X-ray emission is strong enough, the accretion rate will also increase by the same factor and the accretion lifetime will decrease for systems with the same gas reservoirs. In contrast, as the UV emission intensity dominates over the X-ray emission, the effects of line driving are those of increasing the density close to the SMBH. In particular, comparing Figs. 3 and 5 we may see that the density at \( x = 0.001 \) is from 10 to 5 times higher for models \( M_2^* \) and \( M_3^* \) than for models \( M_2 \) and \( M_3 \).

4 ESTIMATED BONDI RADIUS AND MASS ACCRETION APPLICATIONS

An important application of the present analysis is to quan-
tify the differences between the true \( (r_B) \) and estimated \( (r_e) \) Bondi radius as well as the true \( (\dot{M}_{\text{rad}}) \) and estimated \( (\dot{M}_{\text{accreted}}) \) accretion rate when the instrumental resolution is limited or the numerical resolution is inadequate at the sub-parsec scales. Following KCP16 and CP17, these differences as a function of radial distance from the central source are defined by the relations

\[
\frac{r_e (\dot{M}_{\text{accreted}}) (x)}{r_B} = \frac{x^2 \dot{M}_{\text{accreted}}}{\chi_{\text{tot}}^2 (\theta)(\theta)} = \left( \frac{\chi_{\text{tot}}^2 (\theta)(\theta)}{\chi_{\text{tot}}^2 (\theta)(\theta)} \right)^{\frac{2(\gamma-1)/(\gamma+1)}{2}}.
\]

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Figure 6. Estimated Bondi radius for all models. Models with $M(t) \neq 0$ are represented by the solid lines, while models with $M(t) = 0$ are depicted by the dashed lines. For comparison, the black solid line displays the asymptotic behaviour for the pure gravitational case. In the limit when $x \to \infty$, $r_e \to r_B$.

Figure 7. Estimated accretion rate for all models. Models with $M(t) \neq 0$ are represented by the solid lines, while models with $M(t) = 0$ are depicted by the dashed lines. For comparison, the black solid line displays the asymptotic behaviour for the pure gravitational case.

\[ \dot{M}_{\text{acc}}(x) = \frac{1}{\chi_{\text{tot}}^{\text{rad}}(\theta)} \left[ \frac{r_e(\text{acc, out})(x)}{r_B} \right]^{(5-3\gamma)/(2(\gamma-1))} \]

for the accretion rate, where $\chi_{\text{tot}}^{\text{rad}}(\theta)$ is the angular-dependent radiative factor used in this work. In Fig. 6 we show the estimated values of the Bondi radius for all models. For comparison the solid black line is the estimated Bondi radius for the classical Bondi accretion problem. At small radii, i.e., at $x = 0.001$, model $M_1^*$ has an estimated Bondi radius which is $\approx 0.64r_B$ compared to model $M_2^*$, which has $r_e/r_B \approx 0.51$ when the UV emission from the accretion disk dominates the radiation field. These values are comparatively lower than those resulting for models $M_2$ and $M_3$ with no spectral line driving, which have $r_e/r_B \approx 0.68$ and $\approx 0.6$, respectively. Moreover, comparing the asymptotic behaviour of $r_e/r_B$ for the pure gravitational case (solid black line) given by $r_e(x)/r_B \sim x^{3(\gamma-1)/2}$ when $x \to 0^+$ (see equation (22) of KCP16), we find differences of $\approx 10\%-40\%$ between the classical and present generalized Bondi models. In addition, for models $M_1$, $M_2$ and $M_1^*$ the ratio $r_e/r_B \to 1$ at $x \approx 0.1$, while at larger radii all models have values of $r_e/r_B$ close to unity, as shown in Fig. 6 for $x = 3$.

The estimated accretion rates for all models are displayed in Fig. 7 as compared with the asymptotic behaviour of the pure gravitational accretion at small radii given by $\dot{M}_\text{Grav} \sim x^{-3(5-3\gamma)/4}$ (see equation (23) of KCP16). We may see that the radiative effects lead to an overestimation of the accretion rates. In particular at $x = 0.001$, models $M_1$ and $M_2$ with $M(t) = 0$ have accretion rates that are from 2 to 5 times larger than models $M_1^*$ and $M_2^*$. The differences between these models grow up to a factor of 10 at $x \approx 3$. The level of overestimation of the accretion rates compared to the classical Bondi problem is smaller when the effects of line driving are ignored.

Figure 8 shows the estimated values of the Bondi radius for models $M_1^*$, $M_2^*$ and $M_1^*$ for the inflow and outflow solutions. For the outflow solutions the limit of $r_e/r_B$ when $x \to \infty$ is no longer one, but deviates from it by about 10%. This occurs because $r_B$ looses its meaning for $\chi_{\text{tot}}^{\text{rad}}(\theta) < 0$, which is precisely the case for our wind solutions. For com-
pletteness, Fig. 9 displays the accretion rates as a function of the radial distance from the SMBH. We may see that the accretion rates associated with the outflows are close to those associated with the infalling material with the differences not being larger than about 3%.

A further important application of these semi-analytical models is the construction of proper initial conditions for studying the stability of the Bondi accretion flow (Foglizzo et al. 2005). In most simulations of accretion flows the accretor is assumed to be stationary and small-scale density and velocity gradients that develop near it, which are the result of non-linear amplification of numerical noise, may lead to different flow patterns as the resolution is increased. This noise arises because mutually repulsive pressure forces do not cancel in all directions simultaneously, giving rise to non-radial velocities whose magnitudes are larger near the accretor. On the other hand, the use of standard artificial viscosity with a constant coefficient leads to spurious angular momentum advection in the presence of vorticity. However, the intrinsic noise generated due to numerical effects is not sufficient to make the flow unstable at parsec scales, while it may become prominent in the proximity of the accretor at sub-parsec scales. On the other hand, the strength and type of the instability may also depend on the size of the accretor (Blondin & Raymer 2012), the Mach number and the polytropic index $\gamma$. Understanding how these parameters can influence noise amplification at sub-parsec scales will shed light on the instability production mechanisms and the accretion rate for long-stage accretion systems.

5 SUMMARY AND CONCLUSIONS

We have presented a generalization of the classical Bondi accretion theory for the case of non-isothermal accretion processes onto a supermassive black hole (SMBH), including the effects of X-ray heating and the radiation force due to electron scattering and spectral lines. The radiation field is modelled following the recipe of Proga (2007) (P07), where an optically thick, geometrically thin, standard accretion disk is considered as a source of UV photons and a spherical central object as a source of X-ray emission. A semi-analytical solution for the radiative non-isothermal accretion onto a SMBH at the centre of galaxies is obtained using a procedure similar to that developed by Korol et al. (2016) (KCP16). A novel feature of the present analysis is the angular dependence of the UV emission from the accretion disk, while the X-ray/central object radiation is assumed to be isotropic. The influence of both types of radiation is evaluated for different flux fractions of the X-ray/central object radiation ($f_x$) and the UV emission from the disk ($f_{disk}$) with and without the effects of spectral line driving for an incident angle $\theta = \pi/4$.

The main conclusions can be summarized as follows: (i) The ratio between the X-ray and the UV flux increases with increasing $\theta$. For $\theta = \pi/2$, the radiation flux from the accretion disk vanishes and so only the X-ray emission contributes to the radiation flux.

(ii) When the radiation force due to spectral lines does not contribute to the heating, pure gravitational accretion onto the SMBH occurs when the X-ray emission intensity is the strongest and has the same fraction of participation as the UV emission (i.e., $f_x = f_{disk} = 0.5$). As long as the UV emission dominates over the X-ray heating, a transition from pure accretion to moderate and then to powerful outflows occurs.

(iii) When the radiation force due to spectral lines is taken into account, a transition from pure accretion to outflows always takes place independently of the intensity of the X-ray emission. However, as the UV emission dominates over the X-ray heating, the angle of incidence for which the pure accretion/outflow transition occurs and the radial distance from the SMBH for which the inflow and the outflow becomes supersonic both increase.

(iv) As the UV emission becomes stronger, the gas density close to the centre becomes higher. The same trend also applies to the cases where line driving is not considered, except when the X-ray luminosity is strong enough to suppress the outflow. In this case, only pure accretion takes place and the central density achieves much higher values than the other models.

(v) For our generalized radiative, non-isothermal Bondi accretion model, we also provide the exact formula for the Bondi radius ratio $r_c/r_B$ as a function of the radial distance from the central accretor, the Mach number, the critical accretion parameter and the total radiation luminosity. The exact formula for the mass accretion bias $\dot{M}_e/M_B$ is also given in terms of $r_c/r_B$.

(vi) The estimated values of the Bondi radius $r_e$ are between $\approx 0.2$ and $\approx 0.68$ times the classical Bondi radius $r_B$ close to the accretor, while $r_e \rightarrow r_B$ at large distances from the accretor ($r \gtrsim 3r_B$).

(vii) The radiative effects produce an overestimation of the estimated accretion rates compared to the classical Bondi accretion rates. However, the level of overestimation is smaller when the effects of line driving are ignored.

(viii) Under the effects of line driving, the limit of $r_c/r_B$ at large distances from the central accretor is no longer one, but deviates from unity by about 10%. On the other hand, the accretion rates associated with the outflows are close to those associated with the accreting material with differences $\lesssim 3\%$.

We conclude by emphasizing that the present results are useful to model proper initial conditions for time-dependent simulations of accretion flows onto massive black holes at the centre of galaxies. A further important application of the present semi-analytical solutions concerns the stability of the Bondi accretion flow at sufficiently close distances from the accretor, where the stability of the flow is expected to be affected by the growth of small density and velocity fluctuations. As a further step in this line of research, we plan to include the additional effects of the gravitational potential of the host galaxy (Korol et al. 2016) for the cases of Hernquist and Jaffe galaxy models (Ciotti & Pellegrini 2017), which are applicable to early-type galaxies.

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Radiatively non-isothermal Bondi accretion onto a massive black hole

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