Dilaton, moduli and string/five-brane duality as seen from four dimensions.

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Abstract
A naive dimensional reduction of the $N = 1, D = 10$ supergravity theory that naturally arises in five-brane models is used to determine the rôle of two fields which are basic ingredients of string models: the dilaton and, among the moduli, the breathing mode. It is shown that, under the duality transformation that relates five-branes and strings, these two fields exchange the rôles of 10-dimensional dilaton and radius of the compact manifold. A description of this phenomenon in terms of the linear multiplets of the 4-dimensional supergravity is also presented.

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1 Introduction.

The growing body of circumstantial evidence that superstrings are dual to five-branes [1, 2, 3, 4] in the critical dimension \( d = 10 \) has led to think that one might tackle some of the problems raised in the string formulation by going over to the dual formulation. It was in particular suggested by Strominger [2] that the weakly coupled five-brane is a dual representation of the strongly coupled heterotic string.

Although a heterotic five-brane was constructed by Strominger as a soliton solution of the low-energy heterotic string field equations[2], to this date there is no complete covariant construction of the heterotic five-brane at the quantized level. If such a theory can be constructed, we know that it would yield a supergravity theory in \( d = 10 \) dimensions in the formulation with a seven-form field strength [6]. We can use this knowledge to extract some information on the properties of the fields involved at the four-dimensional level.

One particular sector where new input is most eagerly awaited consists of the dilaton and of the moduli fields. This set of fields, together with their supersymmetric partners, undoubtedly plays a key rôle in the vast problems of supersymmetry breaking, vanishing cosmological constant and, probably, strong CP. Any information on the behavior of these fields in the strongly interacting string regime is precious. In the weakly interacting regime, it turns out that the bulk of their properties was extracted by Witten [7] by using a naive dimensional reduction of 10-dimensional supergravity (in the formulation with a three-form field strength [8], dual to the one mentionned above) mimicking a compactification on a Calabi-Yau manifold. More than eight years later, and although a lot of information has been gathered on string models, this simple scheme of compactification still adequately describes the gross features of the dilaton and moduli properties.

In the case of the five-brane where a quantized version remains to be constructed, there is no other way open presently than to try a naive dimensional reduction. The duality between the two theories lets us hope that more information can be extracted this way than could be expected. In section 2, we undertake such a dimensional reduction, keeping in mind the string version and the fact that the fields that we consider are conjectured to be the same dilaton and moduli, only seen from the vantage point of a different string regime. We find that their rôle is actually exchanged when one goes to the dual formulation: one of the moduli takes over as the 10-dimensional dilaton whereas the dilaton of the string picture plays the
rôle of the radius of the 6-dimensional compact manifold. This can be put in perspective with an earlier observation that the string/five-brane duality interchanges the rôles of the $\sigma$-model loop expansion and of the quantum loop expansion.

In section 3, we discuss this duality from the point of view of 4-dimensional supergravity. We stress that moduli, being associated in the five-brane picture with antisymmetric tensors, are described by linear multiplets and we discuss the relevance of some properties of the geometrical structures associated with linear multiplets to some of the issues at stake here. Finally, section 4 discusses the possible implications of these results to the physics of the string dilaton and moduli.

2 Dilaton and moduli in Planck, string and five-brane units.

We start by recalling the situation in the string case which will constitute the backdrop of our discussion of the five-brane regime. The action describing the low-energy field theory limit of the heterotic string reads, if we specify to the boson fields:

$$S = \frac{1}{2} \int d^{10}x \sqrt{-g^{(10)}} \left( R^{(10)} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{12} e^{-\phi} H^2 - \frac{1}{4} e^{-\phi/2} F^2 + \ldots \right), \quad (1)$$

in agreement with 10-dimensional supergravity (we have set the Planck scale $M_{Pl}$ to 1). In eq.(1), $R^{(10)}$ is the 10-dimensional curvature, $\phi$ is the dilaton, $F^{MN}$ is the Yang-Mills field strength and $H_{MNP} = 3 \nabla_M (B_{NP})$ is the field strength of the antisymmetric tensor $B_{MN}$ which naturally appears at the $\sigma$-model level as a background field, through the term:

$$S = -\frac{1}{2\pi \alpha'} \int d^2 \sigma \frac{1}{2} e^{ab} \partial_a X^M \partial_b X^N B_{MN}. \quad (2)$$

The simple dimensional reduction adopted by Witten in amounts to choose for the ten-dimensional metric (in the following, greek letters $\mu, \nu, \ldots$ refer to 4-dimensional Lorentz indices while latin letters $I, J, \ldots$ refer to compact indices):\(^1\)

\[^1\]The second of these transformation laws ensures that $M_{Pl}$ is the same in 4 and 10 dimensions.
\[ g^{(10)}_{\mu\nu} = e^{-3\sigma} g_{\mu\nu}, \]  

(3)

where \( \int d^{6}y \sqrt{|g^{(0)}|} = M_{Pl}^{-6} \). The field \( e^{\sigma} \) plays the rôle of the radius of the compact manifold and \( \sigma \) is referred to as the breathing mode, the simplest example of a modulus. Correspondingly, in order to remain compatible with four-dimensional supersymmetry, one single massless mode is extracted from the part of the antisymmetric tensor with compact indices:

\[ B_{IJ} = \epsilon_{IJ} a_{2}(x), \quad (I, J) \in \{(4, 5), (6, 7), (8, 9)\} \]  

(4)

The dimensional reduction of the action (1) is easily seen to be:

\[
S = \frac{1}{2} \int d^{4}x \sqrt{-g} \left[ R - \frac{1}{2s^{2}} \partial^{\mu}s \partial_{\mu}s - \frac{3}{2t^{2}} \partial^{\mu}t \partial_{\mu}t \right. \\
\left. - \frac{3}{2t^{2}} \partial^{\mu}a_{2} \partial_{\mu}a_{2} - \frac{1}{12}s^{2}H^{2} - \frac{1}{4}sF^{2} \right].
\]  

(5)

with

\[ s = e^{-\phi/2}e^{3\sigma}, \quad t = e^{\phi/2}e^{\sigma}. \]  

(6)

Eq.(5) is easily seen to agree with the standard formulation of 4-dimensional supergravity [10] by performing a duality transformation on the antisymmetric tensor:

\[ s^{2} H^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma} \partial_{\sigma}a_{1}, \]  

(7)

and using a Kähler potential \( K(S, \bar{S}) = - \ln(S + \bar{S}) - 3 \ln(T + \bar{T}) \), where the complex scalar fields \( S \) and \( T \) are defined as \( S = s + ia_{1} \) and \( T = t + ia_{2} \). But the natural formulation is in terms of a linear supermultiplet [11], [12], [13] for describing both \( s \) and the invariant field strength \( H_{\mu\nu\rho} \) plus a chiral supermultiplet for \( T = t + ia_{2} \). We will come back to this point in the next section.

It is however somewhat more appropriate for discussing the rôle played by each of these fields to carry out the dimensional reduction in string units, i.e. to set \( M_{S} = 1 \) instead of \( M_{Pl} = 1 \) [14]. Using the metric that naturally arises in the string sigma-model, the 10-dimensional action reads:

\[
S = \frac{1}{2} \int d^{10}x \sqrt{-g^{(10)}(x)} e^{-2\phi} \left( R^{(10)} + 4(\partial\phi)^{2} - \frac{1}{12}H^{2} - \frac{1}{4}F^{2} + \ldots \right). \]  

(8)

The 10-dimensional dilaton thus naturally plays the rôle of the string loop expansion \( (\text{the } \ell^{th} \text{ order being proportional to } e^{2(\ell-1)\phi}) \).
In these string units, the dimensional reduction is even simpler than previously [14]. Writing

\[ \bar{g}^{(10)}_{IJ} = e^\sigma \bar{g}_IJ, \]

\[ \bar{g}^{(10)}_{\mu\nu} = \bar{g}_{\mu\nu}, \]

(9)

where \( \int d^6y \sqrt{\bar{g}^{(0)}} = M_S^{-6} \), one obtains for the 4-dimensional action in natural string units

\[ S = \frac{1}{2} \int d^4x \sqrt{-\bar{s}} \left[ \bar{R} - \frac{1}{2s^2} \partial^\mu s \partial_\mu s - \frac{3}{2t^2} \partial^\mu t \partial_\mu t \right. \]

\[ \left. - \frac{3}{2s^2} \partial^\mu a_1 \partial_\mu a_2 - \frac{1}{12} H^2 - \frac{1}{4} F^2 \right], \]

(10)

with this time (compare with eq. (6))

\[ s = e^{-2\phi} e^{3\sigma}, \quad t = e^\sigma. \]

(11)

The two 4-dimensional actions (9) and (10) are of course in complete agreement. One is obtained from the other by a Weyl rescaling:

\[ g_{\mu\nu} = s \bar{g}_{\mu\nu}, \]

(12)

But the natural string units of eqs. (10,11) make it more transparent that \( t \) is the radius of the compact manifold whereas \( s \) retains all the properties of the 10-dimensional dilaton and is the string loop expansion parameter as seen from 4 dimensions.

We now turn our attention to five-branes. In a canonical metric, the action for the 10-dimensional effective field theory reads [3]

\[ S = \frac{1}{2} \int d^{10}x \sqrt{-g^{(10)}} \left( R^{(10)} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2 \cdot 7!} e^\phi K^2 + \ldots \right), \]

(13)

where we have discarded for the moment Yang-Mills terms. In (13), \( K = dA \) is the curl of the 6-form which naturally appears as a background field at the sigma model level:

\[ S = -\frac{1}{2\pi \beta'} \int d^6 \sigma \frac{1}{6!} \epsilon^{abcdef} \partial_a X^M \partial_b X^N \partial_c X^P \partial_d X^Q \partial_e X^R \partial_f X^S A_{MNPQRS}. \]

(14)

\[ ^2 \text{We use here and throughout this paper the term “five-brane” in a loose sense since, strictly speaking, we are only dealing with 10-dimensional supergravity in the formulation which is the dual of the one that appears as the field theory limit of string theory.} \]
The action (13) is simply the dual version of the string action (1) if we interpret the 7-form as the dual of the 3-form encountered previously:

\[ K = e^{-\phi^*} H. \] (15)

Seen from the 4-dimensional point of view, this has the interesting consequence that the moduli are now connected through supersymmetry with an antisymmetric tensor and therefore described by linear multiplets whereas the dilaton now fits into a chiral supermultiplet. In order to see that, we will perform the same dimensional reduction as in the string case, that is using eq.(3). A general dimensional reduction of the 10-dimensional action has been performed in Ref.\[6\]. We define

\[ K_{\mu\nu IJKL} \simeq K_{\mu\nu}, \]
\[ K_{\mu IJKLMN} \simeq \partial_\mu a_1, \] (16)

where we have restricted our attention to the case of a single 3-form in order to parallel exactly the previous discussion (a single modulus was considered in eq.(4)). Then we obtain for the 4-dimensional action:

\[ S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2s^2} \partial^\mu s \partial_\mu s - \frac{3}{2t^2} \partial^\mu t \partial_\mu t - \frac{1}{2s^2} \partial^\mu a_1 \partial_\mu a_1 - \frac{1}{12} t^2 K^2 \right], \]

(17)

with \( s \) and \( t \) still given by eq.(6). Performing two inverse duality transformations, namely eq.(7) and

\[ t^2 K_{\mu\nu} = \sqrt{3} \epsilon_{\mu\nu\rho\sigma} \partial^\rho a_2 \] (18)

we find the same action as in eq.(4). In other words, the theory, to this order is still described on-shell\[4\] by the same Kähler potential \( K(S, \bar{S}) = - \ln(S + \bar{S}) - 3 \ln(T + \bar{T}) \).

However, just as in the string case, we expect to get more meaningful information by going over to natural five-brane units \( (M_B = 1) \). Indeed, using the metric which appears naturally in the five-brane sigma model, the 10-dimensional action now reads:

\[ S = \frac{1}{2} \int d^{10}x \sqrt{-\hat{g}^{(10)}} e^{2\phi/3} \left( \hat{R}^{(10)} - \frac{1}{2 \cdot 7!} K^2 + \ldots \right). \] (19)

\(^3\)Since we use duality transformations.
The loop expansion parameter is now given by $e^{2\phi/3}$. It has been noted by Duff and Lu \[3\] that, surprisingly enough, the dilaton field has no kinetic term in this frame. Of course, it is still a propagating degree of freedom as can be seen from the Einstein frame in eq.(13) but it is interesting to note that a similar phenomenon occurs in four dimensions: in no-scale models, the dilaton field associated with the flat directions has a vanishing kinetic term in a specific frame \[15\]. We are going to see that the comparison is not fortuitous.

Indeed the compactification goes as follows in five-brane units. We write (compare with eq. (9)):

\[
\hat{g}^{(10)}_{IJ} = e^\sigma \hat{g}^{(0)}_{IJ}
\]

with \(\int d^6y \sqrt{\hat{g}^{(0)}} = M_B^{-6}\), and use the definitions (11) to obtain the dimensionally reduced action:

\[
S = \frac{1}{2} \int d^4x \sqrt{-\hat{g}} \quad \hat{R} - \frac{1}{2s^2} \partial^\mu s \partial_\mu s
\]

\[
- \frac{1}{2s^2} \partial^\mu a_1 \partial_\mu a_1 - \frac{1}{12} t^2 K^2,
\]

where now \(s\) and \(t\) are given in five-brane units by (compare with eq.(11)):

\[
s = e^{3\sigma}, \quad t = e^\sigma e^{2\phi/3}.
\]

It is straightforward to show that such an action can also be derived from the action (17) through the rescaling

\[
g_{\mu\nu} = t \hat{g}_{\mu\nu}.
\]

Nevertheless, the derivation in five-brane units sheds new light on several aspects. First, one checks that the \(t\) fields has no kinetic term which is related to the fact that it is the scalar field connected with the no-scale structure of the theory (the factor 3 in the logarithmic term of the Kähler potential). Moreover, eq.(22) clearly shows that the \(t\) field is the one that retains the character of the 10-dimensional dilaton, as well as the five-brane loop expansion parameter. On the other hand, the \(s\) field is now the breathing mode associated with the compactification of the five-brane theory. It is also the five-brane sigma model coupling.\[4\] In other words, compared with

\[4\] From eq.(22), it seems that, both in the limit \(s \to 0\) and \(s \to \infty\), the \(\sigma\)-model is strongly coupled.
the string case, the rôle of the two scalar fields is interchanged \[8\]. Thus the duality \( \phi \rightarrow -\phi/3 \) between the string action (8) and the five-brane action (19) does not seem to be related to a \( S \rightarrow 1/S \) “duality” \[14\] but rather to an exchange of the rôles of \( S \) and \( T \). On the other hand, in the five-brane regime, since \( s \) is interpreted as the radius, a \( S \rightarrow 1/S \) “duality” – or more precisely an \( SL(2, Z) \) invariance – does seem plausible (whereas the \( SL(2, Z) \) invariance associated with the \( T \) field in the string formulation is no longer explicit in the five-brane picture).

It is interesting to note that adding in the action (19) a gauge term of the form:

\[
\delta S = \frac{1}{2} \int d^{10}x \sqrt{-\hat{g}^{(10)}} \left( -\frac{1}{4}F^2 \right)
\]

yields in four dimensions

\[
\delta S = 1/2 \int d^4x \sqrt{-g} \left( -\frac{1}{4}sF^2 \right) = 1/2 \int d^4x \sqrt{-g} \left( -\frac{1}{4}sF^2 \right),
\]

i.e. a standard gauge term in 4 dimensions (see eq.(5)). The absence of a factor \( e^{2\phi/3} \) in the 10-dimensional Lagrangian (or a factor \( t \) in the 4-dimensional Lagrangian in five-brane units) shows that such a term is only expected at the one-loop level. e will return to this in the next section.

3 Linear multiplets and duality transformations.

In \( N = 1 \) supergravity in 4 dimensions, the natural framework to deal with an antisymmetric tensor field is the linear supermultiplet \[11, 12, 13\]. In the weakly coupled string regime, this has proven to be useful to describe the couplings of the dilaton \( s \), the antisymmetric tensor field strength \( H_{\mu\nu\rho} \) and their fermionic partner \[17, 18, 19, 20\]. The moduli, on the other hand, are described by chiral supermultiplets since there is no four-dimensional antisymmetric tensor involved. One should note however that their pseudoscalar partners are provided by some of the components of the ten-dimensional antisymmetric tensor (see for example eq.(4)).

In the five-brane regime, the situation is reversed and the \( s \) field fits into a chiral supermultiplet whereas the moduli are now described by linear multiplets. The geometrical structure of multi-linear supermultiplets has some interesting features \[21\] that we will sketch below. But we will start by describing the duality transformations at the supergravity level.
In fact, in order to show the specificity of the duality transformation involved when going from the string regime to the five-brane regime, we will consider for the time being a slightly more general situation.

Consider a theory with a linear multiplet $L_1$, a chiral supermultiplet $T$ and some matter chiral supermultiplets which we will denote generically by $\Phi$, whose interactions are described by the action (we work in the Kähler superspace of Refs.\[22, 13, 23\])

$$S = -3 \int E \left[ a + L_1 V(\Phi, \bar{\Phi}) \right]$$

and a Kähler potential:

$$K = -\alpha \ln(T + \bar{T} + W(\Phi, \bar{\Phi})) + \beta \ln(L_1).$$

In our case, $\alpha = 3$ and $\beta = 1$ but, as said above, we will keep them general for the time being (assuming $\beta \neq 3$). Also in eq.(26), $E$ is the supervierbein determinant, $a$ is a real constant and $V(\Phi, \bar{\Phi}), W(\Phi, \bar{\Phi})$ are general real functions of the matter fields. Indeed, had we included a $T$ dependence in $V$, eq.(26) would be the most general Lagrangian that we can write since a Weyl rescaling can absorb terms of order $L_1^n, n \neq 1$ in the Kähler potential ($E$ has Weyl weight $-2$ and $L_1$ has Weyl weight $2$). Of course, eq.(27) is not the most general Kähler potential that one can write. Also, we did not write the superpotential terms but we are assuming here that $T$ does not appear in them.

We are now going to make a “dual” duality transformation which will replace $L_1$ by the chiral superfield $S$ and $T$ by the linear multiplet $L_2$. The method is standard and amounts to a Legendre transformation \[24, 12, 13\]. One starts with the action:

$$S = -3 \int E \left[ a + L_1 V(\Phi, \bar{\Phi}) + L_1 (S + \bar{S}) - L_2 Y \right]$$

with a Kähler potential:

$$K = -\alpha \ln(Y + W(\Phi, \bar{\Phi})) + \beta \ln(L_1).$$

$L_1$ and $Y$ being unconstrained real superfields, $L_2$ a linear superfield and $S$ a chiral one.

\[5\] In this section, $\int$ stands for $\int d^4x d^4\theta$. 

8
If we minimize with respect to the constrained superfields $S$ and $L_2$, we obtain:

\[
(D^\alpha D_\alpha - 8R) L_1 = 0 \quad , \quad (D^\alpha D_\beta - 8R) L_1 = 0 \\
Y = T + \bar{T} \quad , \quad D^\alpha T = 0,
\]

(30)

and we therefore recover the previous theory described by the action (29).

In order to get the dual theory, we can alternatively minimize with respect to the unconstrained superfields $L_1$ and $Y$. We obtain respectively:

\[
- \frac{1}{3} \frac{\partial K}{\partial L_1} [a + L_1(S + \bar{S} + V)] + S + \bar{S} + V = 0,
\]

\[
- \frac{1}{3} \frac{\partial K}{\partial Y} [a + L_1(S + \bar{S} + V)] - L_2 = 0.
\]

(31)

Using the explicit form for the Kähler potential, eq.(29), we can easily express $L_1$ and $Y$ in terms of $S$ and $L_2$ and we finally obtain the following action, dual to (26),

\[
S = -3 \int E \left[ a \frac{3 - \alpha}{3 - \beta} + L_2 W(\Phi, \bar{\Phi}) \right]
\]

(32)

with the Kähler potential:

\[
K = \alpha \ln(L_2) - \beta \ln(S + \bar{S} + V(\Phi, \bar{\Phi})) - \alpha \ln \alpha + \beta \ln \beta + (\alpha - \beta) \ln \frac{3 - \beta}{\alpha}.
\]

(33)

It is straightforward to check that this “dual” duality transformation can be inverted, provided $\alpha$ and $\beta$ are different from 3.

It is very interesting indeed that the case of the string/five-brane duality corresponds to a singular situation, i.e. $\alpha = 3$. We can see it from the fact that the action (29) is in this case scale invariant. Such an action has been developed in terms of component fields in Ref.[18], but, due to the singular properties of the limit $\alpha = 3$, the methods used there cannot be readily applied to this case. Moreover, as we have just said, the duality transformation cannot be inverted to get back to the string Lagrangian.

It might be that the “bare bone” Lagrangian of eq.(23) cannot, by itself, describe the low energy string Lagrangian if a dual theory such as the

\[\begin{array}{l}
\delta L_1 E = -\frac{1}{3} \frac{\partial K}{\partial L_1} E \delta L_1 \\
\delta Y E = -\frac{1}{3} \frac{\partial K}{\partial Y} E \delta Y
\end{array}\]

(34)

\[\begin{array}{l}
\delta L_2 L_2 = \frac{1}{3} \frac{\partial K}{\partial L_2} L_2 \delta L_1 \\
\delta Y L_2 = \frac{1}{3} \frac{\partial K}{\partial Y} L_2 \delta Y
\end{array}\]

(35)

In Kähler superspace where the Kähler invariance is implemented into the superspace structure, we have $\delta L_1 E = -\frac{1}{3} \frac{\partial K}{\partial L_1} E \delta L_1$ and $\delta Y E = -\frac{1}{3} \frac{\partial K}{\partial Y} E \delta Y$ (see for example ref.[15]). Similarly, because the constraint on $L_2$ involves the Kähler connection, $\delta L_2 L_2 = \frac{1}{3} \frac{\partial K}{\partial L_2} L_2 \delta L_1$ and $\delta Y L_2 = \frac{1}{3} \frac{\partial K}{\partial Y} L_2 \delta Y$ (on the other hand, $S$ being of chiral weight zero, its constraint does not involve the Kähler potential).
five-brane can be constructed. We actually do know that eq. (26) is not the complete story because the Green and Schwarz anomaly-cancelling mechanism \cite{25} has to be implemented at the 4-dimensional level. It has actually been shown that this is most easily done in the linear multiplet formulation \cite{17, 19}. The term linear in $L_1$ in (26) now depends on the $T$ field and we must couple $L_1$ to the Chern-Simons form \cite{26, 12, 13} through the constraint

\begin{equation}
(D_\alpha D_\alpha - 8R^1)L_1 = 2k \mathrm{tr}(W_\alpha W_\alpha) \equiv k (D_\alpha D_\alpha - 8R)\Omega,
\end{equation}

where we have restricted our attention to Yang-Mills Chern-Simons forms, described by the superfield $\Omega$ ($k$ is a normalisation constant).

We thus start instead of (28) with the action:

\begin{equation}
S = -3 \int E \left[ a + L_1 V(Y, \Phi, \bar{\Phi}) + (L_1 - \Omega)(S + \bar{S}) - L_2 Y \right]
\end{equation}

where the Kähler potential is still given by eq. (29) (with $\alpha = 3$ and $\beta = 1$) and we choose

\begin{equation}
V(Y, \Phi, \bar{\Phi}) = c \ln(Y + W(\Phi, \bar{\Phi})).
\end{equation}

Minimizing with respect to the constrained fields $L_2$ and $S$, one recovers the constraints for $Y$ in eq. (30) and the modified constraint of eq. (34) for $L_1$.

On the other hand, minimizing with respect to the unconstrained fields, one now obtains:

\begin{align}
S + \bar{S} &= \frac{a}{2L_1} - c \ln(Y + W) \\
L_2 &= \frac{1}{Y + W} \left( \frac{3a}{2} + cL_1 \right)
\end{align}

Thus the action reads:

\begin{equation}
S = -3 \int E \left[ L_2 W(\Phi, \bar{\Phi}) - cL_1(S + \bar{S}, L_2) \right] + 3 \int E \Omega(S + \bar{S})
\end{equation}

where $L_1(S + \bar{S}, L_2)$ is the solution of the equation

\begin{equation}
\frac{a}{2L_1} - \ln \left( \frac{3a}{2} + cL_1 \right) = S + \bar{S} - c \ln L_2.
\end{equation}

\footnote{Note that this is a one-loop effect.}
The second term in the action is nothing but the standard Yang-Mills kinetic term \[13\] which thus appears as a one-loop effect in this dual formulation (see the comments at the end of the previous section):

\[
S = -\frac{3}{4} k \int \frac{E}{R} STr(W^a W_a) - \frac{3}{4} k \int \frac{E}{R^i} STr(W_{a\dot{a}} W^{a\dot{a}}). \tag{40}
\]

From eq. (39), we see that \(L_1\) depends only on the combination \(S + \bar{S} - c \ln L_2\) which is nothing but the renormalised gauge coupling of the string theory \(1/g_{\text{loop}}^2\). In the limit of large \(c L_1\), one obtains for the first term of the action

\[
S = -3 \int E \left[ L_2 W(\Phi, \bar{\Phi}) - e^{-(S + \bar{S} - c \ln L_2)} \right] \tag{41}
\]

where the second term is of order \(e^{-1/g^2}\), whereas in the limit of small \(L_1\), one obtains

\[
S = -3 \int E \left[ L_2 W(\Phi, \bar{\Phi}) - \frac{a}{2} S + S - c \ln L_2 \right] \tag{42}
\]

where the second term is of order \(g^2\).

Let us finally make a few comments about the more realistic situation where one considers several moduli, and thus, in the dual formulation, several linear multiplets.

There is an interesting geometrical structure of the projective type involved with the superspace formulation of theories with several linear multiplets \(L^A, A = 1 \ldots N\) [21]. This is already apparent at the level of the duality transformation [13] which can be reformulated in terms of one given \(L^B\) and of the homogeneous variables \(\xi^A_B \equiv L^A/L^B, A \neq B\), which play a spectator role in the duality transformation. The procedure does not depend on the choice of \(L^B\) i.e. it is invariant under the transformations \(\xi^A_B \rightarrow \xi^C_B = \xi^B_C \xi^A_B\). This might indeed be of relevance for our discussion above since it is difficult to understand why the \(T\) field should play such a special role in the dual formulation. In fact, this projective structure can be implemented into the superspace structure by introducing derivatives covariant under the above transformations; \(\xi^A_B\) then satisfies the accordingly modified linear multiplet constraints [23].

4 Conclusions.

If the conjecture that weakly coupled five-brane theories form a dual representation of strongly coupled heterotic strings is verified, this might have
deep implications for long-standing problems such as the breaking of supersymmetry. Indeed both the dilaton and the moduli play an important rôle in this problem. Seen from the $N = 1, D = 4$ supergravity point of view, one might have to allow for more general couplings than the ones identified so far. This was hinted at in the last section by considering the duality transformation. Another example is the following: if moduli are components of linear supermultiplets in the dual formulation, how can they appear in the superpotential? Also, regarding supersymmetry breaking, the possibility of having a formulation where a $SL(2, \mathbb{Z})$ symmetry associated with the “string coupling” $S$ is manifest is certainly tantalizing [16].

Actually, while completing this work, we realized that, in a recent preprint [27], J. Schwarz and A. Sen have addressed some of the issues discussed here, precisely in the spirit of formulating the theory in a manifestly $SL(2, \mathbb{Z})$ invariant way. They perform a dimensional reduction of the dual formulation of 10-dimensional supergravity which is more complete than the one presented here where our main purpose was to unravel the rôle of the different fields. We refer the reader to their discussion of the $SL(2, \mathbb{Z})$ symmetry in the dual formulation which we have only briefly mentionned here.

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References

[1] M. Duff, Class. Quant. Grav. 5 (1988) 189.
[2] A. Strominger, Nucl. Phys. B343 (1990) 167.
[3] M. Duff and J. Lu, Nucl. Phys. B354 (1991) 129, 141; Nucl. Phys. B357 (1991) 534.
[4] C.G. Callan, J.A. Harvey and A. Strominger, Nucl. Phys. B359 (1991) 611; Nucl. Phys. B367 (1991) 60.
[5] J.A. Dixon, M.J. Duff and J.C. Plefka, Phys. Rev. Lett. 69 (1992) 3009.
[6] A. Chamseddine, Phys. Rev. D24 (1981) 3065.
[7] E. Witten, Phys. Lett. 155B (1985) 151.
[8] E.A. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuisen, Nucl. Phys. B195 (1982) 97; G.F. Chapline and N.S. Manton, Phys. Lett. B120 (1983) 105.
[9] C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B262 (1985) 593.
[10] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and A. van Nieuwenhuisen, Phys. Lett. 79B (1978) 231, Nucl. Phys. B147 (1979) 105; E. Witten and J. Bagger, Phys. Lett. 115B (1982) 202; J. Bagger, Nucl. Phys. B211 (1983) 302; E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Nucl. Phys. B212 (1983) 413.
[11] S. Ferrara, J. Wess and B. Zumino, Phys. Lett. 51B (1974) 239.
[12] S. Ferrara and M. Villasante, Phys. Lett. 186B (1987) 85.
[13] P. Binétruy, G. Girardi, R. Grimm and M. Müller, Phys. Lett. 195B (1987) 389.
[14] M. Dine and N. Seiberg, Phys. Rev. Lett. 55 (1985) 366.
[15] J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406.
[16] A. Font, L. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B249 (1990) 35; A. Sen, preprint TIFR-TH-92-41 (1992); J. Schwarz, preprint CALT-68-1815 (1992).

[17] S. Cecotti, S. Ferrara and M. Villasante, Int. J. of Mod. Phys. A2 (1987) 1839.

[18] P. Binétruy, G. Girardi and R. Grimm, Phys. Lett. 265B (1991) 111; P. Adamietz, P. Binétruy, G. Girardi and R. Grimm, preprint ENSLAPP-A-388/92 (July 1992) (to be published in Nuclear Physics).

[19] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1991) 145.

[20] M.K. Gaillard and T. Taylor, Nucl. Phys. B381 (1992) 577.

[21] P. Binétruy, G. Girardi and R. Grimm, in preparation.

[22] P. Binétruy, G. Girardi, R. Grimm and M. Müller, Phys. Lett. 189B (1987) 83.

[23] P. Binétruy, G. Girardi and R. Grimm, preprint LAPP-TH-275/90 (unpublished).

[24] U. Lindström and R. Roček, Nucl. Phys. B222 (1983) 285.

[25] M.B. Green and J.H. Schwarz, Phys. Lett. B149 (1984) 117.

[26] R. Grimm, Proceedings of “Supersymmetry”, p. 607, ed. by K. Dietz et al., Bonn 1984, Plenum Press (1985).

[27] J.H. Schwarz and A. Sen, Duality symmetric actions, preprint NSF-ITP-93-46, hep-th/9304154 (April 1993).