Abstract

I review work developing the idea that string is a composite of point-like entities called string bits. Old and new insights this picture brings into the nature of string theory are discussed. This paper summarizes my talk presented to the Strings96 conference at Santa Barbara, CA, 14-20 July 1996.
1 Introduction

Today I would like to describe work which attempts to develop a theory of string based on the idea that string is made of smaller entities[1, 2, 3, 4]. I have tried to encapsulate these ideas in a basic hypothesis, about the nature of string, which I believe will survive even if our particular efforts to implement it falter:

**Hypothesis:** String and related structures in $d + 2$ space-time dimensions are composites of point-like entities, **string bits**, which move in at most $d$ space dimensions.

I have emphasized in bold type the crucial aspects of the hypothesis. If it is true, it should be possible to formulate string theory in 4 dimensional space-time as a theory of a world with only 2 space dimensions. If string theory is the right description of nature, it would be a concrete realization of ‘t Hooft’s idea that the world is a hologram[5], an idea also vigorously pursued by L. Susskind[6].

It will be characteristic of an underlying theory, of the type envisaged above, that it will not in any obvious way describe gravity. How could it? The space or manifold on which the geometry of gravity could be defined is not completely present in the formulation. String bit dynamics must accomplish two rather remarkable feats:

- It must produce **at least** one new spatial dimension. Moreover, any new manufactured dimensions must behave as though they were on exactly the same footing as the spatial dimensions already present in the underlying theory.

- Having produced new dimensions, the dynamics must **induce gravity** in this higher dimensional space.

It is this last point that offers the possibility of avoiding information loss paradoxes. Since gravity will be an induced collective effect, there is no obstruction to insisting that the underlying theory be **unitary**. In effect, we have turned the problem around: rather than trying to establish that the theory of gravity is or is not consistent with unitarity, we ask whether gravitational phenomena can be induced
in a unitary theory. I hasten to add that there is, of course, no a priori guarantee of a positive outcome.

Having agreed to build an underlying theory of string with at least one missing dimension, we next must decide how much symmetry the bit dynamics should retain. There is no single answer. However for definiteness we shall provisionally retain a maximal symmetry consistent with our hypothesis. We shall assume there is only one missing space dimension. The space-time boost group should be a maximal subgroup, acting on the smaller space, of the Poincaré group. There are two possibilities. I suppose most physicists would choose the \( O(d, 1) \) subgroup of Poincaré \( O(d + 1, 1) \). However, an equally logical choice would be Galilei\((d, 1)\). We choose the latter for reasons of simplicity and expediency. I include expediency because Galilei invariant dynamics is just that of ordinary non-relativistic quantum mechanics. Making this choice allows us to tap all of the many methods and techniques developed by our condensed matter colleagues over the years to deal with diverse physical phenomena. These resources will aid the implementation of our hypothesis, which must effectively handle composite systems and collective excitations.

Finally, we come to supersymmetry. Once we choose Galilei as our boost group, we are forced to incorporate Galilei supersymmetry. It turns out that the maximal Galilei superalgebra is very close in structure to the Poincaré superalgebra that would be present in the higher dimensional theory. There are two types of supercharge: \( Q^A \), which is a square root of the Newtonian mass, and \( R^{\dot{A}} \), which is a square root of the Hamiltonian \( H \). The Newtonian mass of a system of string bits is just \( mM \), where \( m \) is the mass of a bit, and \( M \) is the number of bits. The maximal Galilei superalgebra is

\[
\{Q^A, Q^B\} = mM\delta^{AB}, \quad \{Q^A, R^{\dot{B}}\} = \frac{1}{2} \alpha^{AB} \cdot \mathbf{P},
\]

\[
\{R^{\dot{A}}, R^{\dot{B}}\} = \delta^{\dot{A}\dot{B}} H/2,
\]

where the components of \( \alpha \) are elements of a \( d \) dimensional Clifford algebra and \( \mathbf{P} \) is the total \( d \) dimensional momentum. Notice that this would become precisely the \( O(d + 1, 1) \) Poincaré superalgebra with the replacement of \( mM \) by \( P^+ \) and \( H \) by
$P^{-}$! I must concede here that the last of these equations, the closure of $R$ onto the Hamiltonian, is technically difficult to implement. In the models Bergman and I have so far considered, it fails for higher dimensional models: there are extra terms on the r.h.s. not proportional to the Kronecker delta. For $d = 1$, the supercharges have only one component each and the full algebra is satisfied by default. Studying this $d = 1$ “toy” model has led to some important insights into the behavior of superstring bit models. We still believe higher dimensional models with the maximal supersymmetry can be constructed.

2 Dynamics of String Bits

The dynamics we set up for string bits is guided by the way string dynamics works in light-cone gauge. Light-cone coordinates are introduced in $d + 2$ space-time dimensions by defining $x^{\pm} = (t \pm x^{d+1})/\sqrt{2}$ and by denoting the remaining $d$ “transverse” components $x$. The corresponding momentum components are $P^{\pm}$ and $P$. Notice that $P^{-}$ is conjugate to $x^{+}$ and $P^{+}$ is conjugate to $x^{-}$. The relativistic dispersion law for a single particle state with rest mass $\mu$ then reads

$$P^{-} = \frac{P^{2} + \mu^{2}}{2P^{+}}.$$  

Light-cone gauge for a free string is specified by identifying the time-like world sheet parameter $\tau$ with $x^{+}$ and choosing $\sigma$ so that $P^{+}$, the density of $P^{+}$ is uniform along the string. In this gauge the dynamics is precisely that of a non-relativistic string in $d$ dimensions.

The coordinate $x$ of a string bit has $d$ components—just like the transverse coordinates of the light-cone. It possesses a Newtonian mass $m$, and can carry momentum $p$. The new dimension is generated by considering objects made of a large variable number $M$ of bits. Then $mM$ can be interpreted as an effectively continuous $P^{+}$. There is a dynamical requirement for this interpretation to work. By Galilean invariance one can always separate the center of mass motion so that we automatically
have for any collection of particles

\[ H = \frac{\mathbf{P}^2}{2mM} + H_{\text{int}}, \]

where \( \mathbf{P} \) is the total momentum and \( H_{\text{int}} \) is the energy of internal motion. To achieve the relativistic dispersion law with \( P^+ = mM \), it must be the case that the excitation energies associated with the internal motion scale as \( 1/M \) for large \( M \). This behavior can be expected for a wide class of one dimensional composite systems, but it is by no means guaranteed.

3 Insights from the Composite Picture of String

3.1 String as a Polymer of String Bits

To get even a noninteracting string we must confront a many body bound state problem. The bits must interact with each other in a way that allows them bind into arbitrarily long polymers. It is not hard to set up such a dynamics if we imagine that the bits have somehow been ordered around a loop. Then a Hamiltonian

\[ H = \frac{1}{2m} \sum_{k=1}^{M} p_k^2 - \frac{1}{m} \sum_{k=1}^{M} V(x_{k+1} - x_k) \]

will predict polymer formation provided the potential \( -V \) is attractive enough to bind a pair of bits. If so the bond breaking energy \( B = O(1/m) \). The low energy internal excitations will correspond to vibrational frequencies \( \omega_n/m \). One can show that these frequencies scale as \( n/M \), so that a string tension \( T_0 \) can be defined by

\[ \lim_{M \to \infty} \frac{2\pi T_0 n}{M}. \]

This scaling law ensures the relativistic dispersion law. Notice that the energy to break a bond is huge compared to the excitation energies for large \( M \): the bits are effectively confined to polymers.

3.2 Issues of Stability

The dynamics of the polymer system we considered in the previous section was artificial in that the cyclic ordering of bits around a loop was preserved. Any realistic
many body dynamics would permit the bonding pattern to be rearranged. Such rearrangements would include ones in which the number of polymers changes. A polymer in its ground state could be unstable to decay into two smaller polymers. To address this issue compare the energy of an \( M \) bit polymer at rest in its ground state to the energy of a system of two polymers, one with \( M_1 < M \) and the other with \( M - M_1 \) bits, each at rest in their ground states. The ground state of each polymer is determined by the Hamiltonian of the previous section. By the uncertainty principle the internal kinetic energy of the two polymer system is less than that of the one polymer system, since the latter is more localized. Because there are exactly \( M \) bonds in both systems, and no non-nearest neighbor interactions, the two polymer system will be more tightly bound (since it has less internal kinetic energy), so its potential energy will also be less. Thus we conclude that

\[
E_G(M) > E_G(M_1) + E_G(M - M_1).
\]

Since the large \( M \) behavior of \( E_G \) is expected to have the asymptotic form

\[
E_G(M) \sim \alpha M + \frac{\gamma}{M} + O\left(\frac{1}{M^3}\right),
\]

we conclude that \( \gamma < 0 \). But \( 2\gamma \) is just the \((\text{mass})^2\) of the lowest string state, \textit{i.e.} it is a tachyon. This provides a clear understanding of the tachyonic instability of the bosonic string\[7\]. It seems unavoidable if the only bit dynamical variables are \( x \) and \( p \).

3.3 Bits Can Have Discrete Degrees of Freedom.

Of course bits can also occur in multiplets. For example, a bit could carry spin or “flavor” such as isospin \( 1/2 \). When such bits are bound into polymers, spin or isospin waves will naturally describe some of the low energy excitations. Isospin waves have precisely the energy and degeneracy patterns of a compactified coordinate\[8\]. Spin waves would provide the world sheet fields \( H^\mu, \Gamma^\mu \) of the NS, respectively R sectors of the spinning string. The contributions of spin or isospin wave fluctuations to the ground state energy of the polymer depend on the bit number. If \( M \) is even (NS
sector), the effective $\gamma$ is negative, but $\gamma$ is positive for odd $M$. That doesn’t really help with stability, since a polymer with $M$ odd can decay into two polymers, one of which must have even bit number. We know of course, that in the NSR formulation of superstring theory, the closed string tachyon can be consistently projected out of scattering amplitudes through the miracle of the GSO projection. We would prefer that a bit model for superstring not depend on such a delicate cancelation. However, the possibility of interpreting compactified dimensions as “flavor waves” remains an intriguing way to manufacture the extra dimensions required by critical superstring.

3.4 Statistics Waves

If string bits occur in multiplets which contain both fermions and bosons, fluctuations in statistics would naturally lead to what we call statistics waves\cite{3}. Unlike spin or isospin waves, the $\gamma$ parameter for statistics waves is positive regardless of whether $M$ is even or odd. Thus statistics fluctuations can stabilize large polymers. When bits are put in supersymmetry multiplets, these waves on large polymers are precisely described by the Green-Schwarz spinor valued world sheet fields. For superstring, the contribution to the energy of statistics fluctuations exactly cancels that of vibrational fluctuations, \textit{i.e.} $E_G = 0$ exactly. Thus all super polymers, large and small are marginally stable.

3.5 Interacting Polymers

To handle polymers that can interact with each other we must specify a complete many body dynamics, which doesn’t artificially suppress bond rearrangement. We give each bit an adjoint “color” degree of freedom by introducing a super-bit creation operator that is a matrix transforming in the adjoint representation of color $SU(N_c)$. Quite generally,

$$\phi^{\alpha_{\beta}}_{a_1 \cdots a_n}(x)$$

will create a bit at the position $x$, with flavor $f$, spin state specified by $a_1 \cdots a_n$ (bosons (fermions if $n$ is even (odd)), and color $\alpha, \beta$ where lower (upper) color indices transform in the $N_c$ ($\bar{N}_c$) representations of $SU(N_c)$. We can employ ‘t Hooft’s $1/N_c$
expansion[9] to isolate a limit ($N_c \to \infty$) in which the bits on a polymer maintain their cyclic order and polymers do not interact with each other[10]. The Hamiltonian is very complicated, but if we suppress indices, it has roughly the schematic form:

$$H = \int dx \frac{1}{2m} \text{Tr}[\nabla \phi^\dagger \cdot \nabla \phi] + \frac{1}{mN_c} \int dx dy V(x-y) \text{Tr}[\phi^\dagger(x,\phi(x))|\phi^\dagger(y),\phi(y)|].$$

At leading order in $1/N_c$ a state of the form

$$|\Psi(x_1, \ldots, x_M)\rangle = \int dx_1 \cdots dx_M \times \text{Tr}[\phi^\dagger(x_1) \cdots \phi^\dagger(x_M)]|0\rangle \Psi(x_1, \ldots, x_M),$$

(1)
can be shown to be an energy eigenstate with the properties of a noninteracting closed polymer. At order $1/N_c$ this Hamiltonian allows bond rearrangement in which a single closed polymer can transform into two closed polymers. The amplitude for such a process can be expected to have the same large $M$ scaling behavior as the discretized closed string vertex studied in [11]. Assume the two polymers in the final state have bit numbers $fM$ and $(1-f)M$. Then for the bosonic string this vertex was found to have the large $M$ behavior

$$\text{Vertex} \sim \frac{1}{N_c} \left( \frac{M}{m} \right)^{3/2} \left( \frac{1}{M} \right)^{d/8} \to \frac{1}{N_c} \left( \frac{1}{P^+} \right)^{(d-12)/8} m^{(d-24)/8},$$

where the $d$ independent factor is due to wave function normalization and the $d$ dependence comes from the overlap of initial and final wave functions. There are three distinct cases. For $d < 24$ the amplitude blows up in the continuum limit $m \to 0$. This is the sub-critical string. By taking a double scaling limit, $N_c \to \infty$, as $m \to 0$ a finite limit can be obtained, but Lorentz invariance is lost. The case $d = 24$ is the critical string and the vertex is finite and consistent with Lorentz invariance $(1/P^{+3/2})$. Finally, for $d > 24$ the vertex vanishes and the theory is trivial.

3.6 Why Bit Dynamics Must Sometimes Imply Gravity

If bit dynamics is to induce gravity, it must induce a Poincaré invariant higher dimensional theory which includes a massless spin 2 particle (the graviton) in its spectrum as interpreted in the effectively higher dimension theory. This is
a tall order. However, as argued by Mueller,[12] polymers of string bits generically scatter in a certain kinematic region in a way precisely consistent with this happening. Actually his argument was applied to light cone string, but it applies equally to our polymers. Consider forward scattering of a polymer of bit number $M_1$ off a polymer of bit number $M_2$. Both $M_1$ and $M_2$ are assumed large (of $O(1/m)$), but $M_2 >> M_1$. Then the energy of 2 is much less than that of 1 (remember $H \sim 1/mM$), which is of $O(1)$. The total energy of the system is of $O(1)$, so the interaction time of the polymers will be of order $1/E = O(1)$ The small polymer will strike the large polymer at any of $M_2$ locations. But because of the finite velocity of sound on the large polymer, a small polymer must be emitted from a location not very distant from the initial impact, because the emission must occur within a time (either before or after the impact) of order $O(1)$. As $M_2 \to \infty$ only a relatively tiny part of the large polymer is disturbed, so that the probability of scattering for each impact must approach a constant, and since there are $M_2$ possible impact locations, we conclude that

$$\text{Cross section } \sim \text{ const } \times M_2.$$  

What does this have to do with gravity? Not much, unless one knows that the physics predicted by the bit model turns out to be Poincaré invariant. Not all string bit models have this property—a simple counterexample being the sub-critical bosonic model. But if Poincaré invariance does hold with the identification $P^+ = mM$, then, for the kinematics considered above the Mandelstam invariant

$$s = 2(P_1 + P_2)^+(P_1 + P_2)^- - (P_1 + P_2)^2 \approx 2mM_2E_1$$

is large and the momentum transfer invariant $t = 0$ because forward scattering is being considered. This is just the kinematics of Regge behavior in which the cross section should have the behavior

$$\sigma \sim Ks^{\alpha(t)-1},$$

where $\alpha(t)$ is the highest lying Regge trajectory with vacuum quantum numbers. Comparison to the linear growth following from Mueller’s argument then implies
that
$$\alpha(0) = 2, \quad \text{if scattering is Poincaré invariant},$$

Since the vacuum Regge trajectory has even signature, 2 is a right signature point, so a massless spin 2 particle must lie on this trajectory. This particle must also couple at low momentum transfer, so that Weinberg’s soft graviton theorems will require couplings consistent with general covariance. Thus Mueller’s argument together with Poincaré invariance would require that gravitational phenomena be induced.

To understand what happens when Poincaré invariance is not obtained, consider the sub-critical bosonic model. In that case the 4 polymer cross section has the scaling behavior
$$\sigma \sim \left(\frac{1}{M_2}\right)^{(d-12)/12} M,$$
where $M$ is an integral over moduli space of an integrand that continues smoothly from the direct to crossed channel singularities. Thus it is the large $M_2$ behavior of $M$ that is controlled by crossed channel Regge trajectories. Then Mueller’s argument predicts
$$\alpha(0) = \frac{d}{12}, \quad \text{for subcritical bosonic model}.$$

This trajectory intercept implies that the spin 2 particle lying on the trajectory is massive in accord with expectations for the sub-critical case. Of course, for $d = 24$ the prediction reduces to the Poincaré invariant result.

4 Size of Polymer Ground State

In the previous section we enumerated several important insights string bit models provide into string dynamics. All of those results have appeared in the literature, some of them are many years old. In this section, I would like to briefly describe studies by Bergman and me on how string interactions might affect the size of the polymer ground state. The importance of this issue has been stressed by L. Susskind.

It has been known for a long time that vibrational fluctuations cause the size of
the polymer ground state to grow logarithmically with the bit number:

\[ R_0^2 \sim \frac{1}{T_0} \ln M. \]

Although one might be disturbed that there is any growth at all, Susskind has pointed out that the above growth is in fact too slow to expect a perturbative description of the ground state to have any validity at all. This is because the bit number density will grow linearly (up to powers of \( \ln M \)). Thus even though the string coupling might be quite small, at large \( M \) the density is so high that the interaction energy is a factor \( g^2 M \) times the unperturbed energy. Thus for \( M > 1/g^2 \), interactions will dominate.

In our bit models the \( g \sim 1/N_c \), so to examine the effects of interactions, we must look at finite \( N_c \) effects. There are many complications that arise. These include interaction between non nearest neighbor bits on a polymer chain as well as the possibility of rearrangements in the bond pattern. These two effects arise together and it is not really possible to separate them. Nonetheless, we thought it would be interesting to first consider the effects of non-nearest neighbor interactions, with the cyclic ordering of the bits assumed fixed. Because the bit Hamiltonian is positive definite, it can easily be seen that residual non-nearest neighbor interactions should be repulsive. Thus we expected them to increase the size of the ground state with an accompanying decrease in bit density.

We studied a model in which the nearest neighbor attractive interactions were supplemented by a short range repulsion between all pairs of bits. For details, I refer the reader to the poster session presented by Bergman [13]. I present here only a summary of our results. We introduced measures of size \( R_k^2 \), the mean squared distance between bits separated by \( k \) stems along the chain. We also introduced an overall size measure \( R^2 = (1/M) \sum_k R_k^2 \). We used did a variational calculation with a trial wave function given by the ground state of a harmonic system characterized by an arbitrary set of normal mode frequencies \( \omega_n \) which were our variational parameters. The main results are:

- \( R_1^2 \) stays finite at large \( M \). This means that the extra repulsions are not stressing
the bonds excessively.

- $R^2$ grows **quadratically** with $M$. On the one hand this means that the bit density is getting smaller, not blowing up. On the other hand, it is discouraging, since it is easy to show from dipole sum rules that $R^2 < 1/mE_{\text{GAP}}$. Thus quadratic growth of $R^2$ implies that $E_{\text{GAP}}$ the energy gap between ground and first excited states is closing faster than $1/M^2$, whereas a Poincaré invariant result would entail a gap of order $1/M$. In fact,

- $E_{\text{GAP}} \sim 1/gM^2$.

Although these results are discouraging, it must be remembered that we are not really analyzing the string bit model itself but rather a rough imitation of true bit dynamics. In particular bond rearrangements have been disallowed, and that could be quite important. We have also not attempted to study a supersymmetric model, and that is probably even more important. Thus it is premature to draw a negative conclusion about string bit ideas.

## 5 Conclusion

I hope to have convinced you that string bit models give important physical insight into the nature of string:

- **Stability**: the importance of statistics waves on polymers of superbits.
- **The induction of gravity in a unitary theory.**
- **Dimensional Enhancement**: The promotion of Galilei($d, 1$) to Poincaré($d+1, 1$).

In addition these models provide a framework for addressing a variety of questions unanswerable in string perturbation theory.

As should be clear, there are many open problems and issues that must be addressed. Probably most important is the construction of fully interacting superstring bit models in higher than 1 space dimension in which the maximal Galilei superalgebra closes. The question of whether interactions cause the size of the polymer
ground state to grow linearly with bit number and not ruin the “nice” properties of the string ground state is still open. An improved numerical study of these issues seems very feasible.

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