Bulk viscosity and r-modes of neutron stars

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Abstract. The bulk viscosity due to the non-leptonic process involving hyperons in $K^-$ condensed matter is discussed here. We find that the bulk viscosity is modified in a superconducting phase. Further, we demonstrate how the exotic bulk viscosity coefficient influences $r$-modes of neutron stars which might be sources of detectable gravitational waves.

1. Introduction

Neutron stars have different classes of quasi normal modes (QNMs) \[ \Pi \] depending on restoring forces acting on a perturbed fluid element. Here we are interested in Coriolis restored fluid modes known as the inertial $r$-modes. The $r$-modes become unstable due to gravitational radiation. It might be responsible for regulating the spin of newly born neutron stars as well as old, accreting neutron stars. It was shown that the leptonic and non-leptonic bulk viscosities might effectively damp the instability in different temperature regimes. The bulk viscosity coefficient due to non-leptonic weak processes in neutron star matter involving hyperons \[ n + p \leftrightarrow p + \Lambda \] in $K^-$ condensed matter and its role on damping the $r$-mode instability. In section 2, we describe the composition and equation of state (EoS) of neutron star matter. Results of hyperon bulk viscosity coefficients are explained in section 3. Section 4 gives the summary.

2. Composition and Equation of State

Here we describe the composition and EoS of neutron star matter under going a first order phase transition from hadronic to $K^-$ condensed matter. The constituents of charge neutral and $\beta$-equilibrated matter in both phases are neutrons ($n$), protons ($p$), $\Lambda$ hyperons, electrons, muons and also $K^-$ mesons in the condensed phase. The baryon-baryon interaction is mediated by the exchange of $\sigma$, $\omega$, $\rho$, $f_0(975)$ (denoted hereafter...
Figure 1. Bulk viscosity coefficient as a function of temperature with and without antikaon condensate for fixed values of baryon density ($n_b$) and angular velocity ($\omega$).

as $\sigma^*$) and $\phi(1020)$ mesons and described by the following Lagrangian density in Ref. [15, 16].

Similarly we adopt the Lagrangian density for (anti)kaons in the minimal coupling scheme as given by [17, 18, 19, 20, 21]. The mixed phase is determined by the Gibbs phase rules and global charge neutrality and baryon number conservation [22].

Nucleon-meson coupling constants which are determined by reproducing nuclear matter saturation properties such as binding energy $E/B = -16.3$ MeV, baryon density $n_0 = 0.153$ fm$^{-3}$, asymmetry energy coefficient $a_{asy} = 32.5$ MeV, incompressibility $K = 300$ MeV and effective nucleon mass $m_N^*/m_N = 0.70$, are taken from Ref. [23]. Further kaon-meson and hyperon-meson coupling constants are obtained from Ref. [11, 15]. In this calculation, we adopt the value of antikaon optical potential depth at normal nuclear matter density as $U_{\bar{K}}(n_0) = -160$ MeV.

When there is no first order phase transition from hadronic to $K^-$ condensed matter, $\Lambda$ hyperons appear at density $2.25n_0$. On the other hand, the early onset of $K^-$ condensate in a first order antikaon condensation, delays the appearance of $\Lambda$ hyperons. In our calculation, $K^-$ condensation begins at $2.23n_0$ and the mixed phase ends at $4.1n_0$ whereas $\Lambda$ hyperons appear at $2.51n_0$. Further we calculate energy density and pressure corresponding to above mentioned compositions.
3. Hyperon bulk viscosity coefficient and suppression of r-modes

The real part of bulk viscosity coefficient due to the non-leptonic process $n + p \rightleftharpoons p + \Lambda$ is given by \[4, 24\],

$$\zeta = \frac{P(\gamma_{\infty} - \gamma_0)\tau}{1 + (\omega\tau)^2}. \quad (1)$$

The difference of infinite and zero frequency adiabatic indices is,

$$\gamma_{\infty} - \gamma_0 = -\frac{n_b^2}{P} \frac{\partial P}{\partial n} \frac{d\bar{x}_n}{dn}, \quad (2)$$

where $\bar{x}_n = \frac{n_n}{n_b}$ is the neutron fraction in the equilibrium state. The relaxation time ($\tau$) for the non-leptonic process in the j-th (= hadronic / $K^{-}$ condensed) phase is given by \[4, 7, 11\],

$$\frac{1}{\tau} = \frac{(kT)^2}{192\pi^2 p_A} \frac{<|M_A|^2 >}{\delta \mu / \delta n_n}, \quad (3)$$

where $p_A$ is the Fermi momentum for $\Lambda$ hyperons and $<|M_A|^2>$ is the angle averaged matrix element squared in the corresponding phase.

Temperature dependence of hyperon bulk viscosities due to the non-leptonic process $n + p \rightleftharpoons p + \Lambda$ with and without $K^{-}$ condensate are shown at $n_b = 0.45 fm^{-3}$ and $\omega = 10^4 s^{-1}$ in Figure 1. The solid line represents the hyperon bulk viscosity in the absence of antikaon condensate. The dashed and dashed-dotted lines correspond to the hyperon bulk viscosities in the hadronic and antikaon condensed phases of the mixed phase in a first order hadronic to $K^{-}$ condensed phase transition \[11\]. The hyperon bulk viscosity in the absence of $K^{-}$ condensate is suppressed compared with hyperon bulk viscosities in the presence of antikaon condensate. Further we note that the hyperon bulk viscosity coefficient in the condensed phase is smaller than that of the hadronic phase above $3 \times 10^8 K$. This is also the temperature where the inversion of the temperature dependence of hyperon bulk viscosity in $K^{-}$ condensate happens. This feature is also found in the case without the condensate.

We investigate the r-modes of a neutron star having gravitational mass 1.6 $M_\odot$ corresponding to central baryon density 3.5$n_0$ and rotating at an angular velocity 2652 $s^{-1}$. We calculate damping time scales corresponding to hyperon bulk viscosity ($\tau_{BV}$), modified Urca bulk viscosity involving nucleons ($\tau_{MU}$), shear viscosity ($\tau_{SV}$) and the growth time scale of gravitational radiation ($\tau_G$) using the energy density and bulk viscosity profiles \[11\]. We compute the critical angular velocities ($\Omega_C$) as a function temperature solving $-\frac{1}{\tau_{GR}} + \frac{1}{\tau_{BV}} + \frac{1}{\tau_{MU}} + \frac{1}{\tau_{SV}} = 0$. Various damping time scales are exhibited as a function of critical angular velocity in Figure 2. It is noted that the r-mode instability of the neutron star is damped when the growth time scale due to gravitational radiation is comparable with damping time scales $\tau_{BV}$ and $\tau_{MU}$. We find that the hyperon bulk viscosity damps the instability at $T = 4 \times 10^9$ and below whereas it is the modified Urca bulk viscosity which effectively suppresses the r-mode instability above this temperature.
4. Summary

We have studied hyperon bulk viscosity in the presence of $K^-$ condensate. We find the inversion of the temperature dependence of hyperon bulk viscosity in our calculation. Though the hyperon bulk viscosity is suppressed in $K^-$ condensed matter than that in the hadronic phase, it still effectively damps the $r$-mode instability.

References

[1] Andersson N and Kokkotas K D 2001 *Int. J. Mod. Phys.* D 10 381
[2] Jones P B 2001 *Phys. Rev. Lett.* 86 1384
[3] Jones P B 2001 *Phys. Rev.* D 64 084003
[4] Lindblom L and Owen B J 2002 *Phys. Rev.* D 65 063006
[5] Haensel P, Levenfish K P and Yakovlev D G 2002 *Astron. Astrophys.* 381 1080
[6] van Dalen E N E and Dieperink A E L 2002 *Phys. Rev.* D 65, 063006
[7] Nayyar M and Owen B J 2006 *Phys. Rev.* D 73 084001
[8] Chatterjee D and Bandyopadhyay D 2006 *Phys. Rev.* D 74 023003
[9] Chatterjee D and Bandyopadhyay D 2007 *Astrophys. Space Sc.* 308 451
[10] Chatterjee D and Bandyopadhyay D 2007 *Phys. Rev.* D 75 123006
[11] Chatterjee D and Bandyopadhyay D 2008 *ApJ* 680 686
[12] Madsen J 1992 *Phys. Rev.* D 46 3290
[13] Madsen J 2000 *Phys. Rev. Lett.* 85 10
[14] Alford M G, Rajagopal K, Schaefer T and Schmitt A 2008 *Preprint* arXiv:0709.4635
[15] Schaffner J and Mishustin I N 1996 *Phys. Rev.* C 53 1416
[16] Boguta J and Bodmer A R 1977 *Nucl. Phys.* A 292 413
[17] Glendenning N K and Schaffner-Bielich J 1998 *Phys. Rev. Lett.* 81 4564
[18] Glendenning N K and Schaffner-Bielich J 1999 *Phys. Rev.* C 60 025803
[19] Pal S, Bandyopadhyay D and Greiner W 2000 *Nucl. Phys. A* 674 553
[20] Banik S and Bandyopadhyay D 2001 *Phys. Rev. C* 63 035802
[21] Banik S and Bandyopadhyay D 2001 *Phys. Rev. C* 64 055805
[22] Glendenning N K 1992 *Phys. Rev. D* 46 1274
[23] Glendenning N K and Moszkowski S A 1991 *Phys. Rev. Lett.* 67 2414
[24] Landau L D and Lifshitz E M 1999 Fluid Mechanics (Oxford:Butterworth-Heinemann)