Complementarity between Gauge-Boson Compositeness and Asymptotic Freedom — with scalar matter

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Abstract

We derive and solve the compositeness condition for the SU($N_c$) gauge boson coupling with $N_s$ scalar fields at the next-to-leading order in $1/N_s$ and the leading order in $\ln \Lambda^2$ ($\Lambda$ is the compositeness scale). It turns out that the argument of gauge-boson compositeness (with a large $\Lambda$) is successful only when $N_s/N_c > 22$, in which the asymptotic freedom fails, as is in the previously investigated case with fermionic matter.
The gauge theories have been widely applied not only in high energy physics, but also in various branches of physics, and the gauge bosons often appear as composites of matter fields \[1\] – \[7\]. For example, some people considered the composite models of weak bosons and other gauge bosons \[2\], some developed theory of induced gravity with composite graviton \[3\], some interpreted the vector mesons (which is known to be composite) as gauge bosons with hidden local symmetry \[4\], and others extensively studied dynamically induced ‘connections’ (i.e. gauge fields) in models of spacetime \[5, 6\], and in molecular, nuclear, and solid-state systems \[7\]. A gauge boson interacting with matter can be interpreted as composite under the compositeness condition \(Z_3 = 0\) \[8\], where \(Z_3\) is the wave-function renormalization constant of the gauge boson. The compositeness condition gives relations between the effective coupling constants and the compositeness scale \(\Lambda\). In our previous papers \[9\] – \[11\], we derived and solved the compositeness condition at the next-to-leading order in \(1/N\) in various models, and investigated its implications, where \(N\) is number of the matter field species. In particular in \[11\], considering compositeness of the SU\((N_c)\) gauge boson coupling with \(N_f\) fermion fields, we found a “complementarity” between the gauge-boson compositeness and asymptotic freedom \[12\] of the gauge theory. Namely, the gauge-boson can be considered as a composite due to \(Z_3 = 0\) only when \(N_f/N_c > 11/2\), in which the asymptotic freedom fails. Thus it is urgent for us to investigate if such a complementarity is accidental or really universal, by, for example, examining other cases of theories. In this paper, we perform a similar investigation for the SU\((N_c)\) gauge boson coupling with \(N_s\) scalar fields instead of the fermion fields. We find that the complementarity again holds, i.e. that the gauge-boson is considered as a composite only when \(N_s/N_c > 22\), in which the asymptotic freedom fails.

We consider the SU\((N_c)\) gauge theory for the gauge boson \(G^a_\mu (a = 1, \cdots, N_c^2 - 1)\) coupling with \(N_s\) complex scalar fields \(\phi_j (j = 1, \cdots, N_s)\), each of which belongs to the fundamental representation of SU\((N_c)\):

\[
\mathcal{L} = -\frac{1}{4} (G^a_\mu)^2 + \sum_j \left[ [D_\mu \phi_j]^2 - m^2 |\phi_j|^2 - \lambda (|\phi_j|^2)^2 \right] - \frac{1}{2\alpha} \left( \partial^\mu G^a_\mu \right)^2 + \partial^\mu \eta^a \left( \partial_\mu \eta^a - g f^{abc} \eta^b G^c_\mu \right) \tag{1}
\]

with \(G^a_\mu = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu\) and \(D_\mu \phi_j = \partial_\mu \phi_j - ig T^a G^a_\mu \phi_j\), where \(g\) and \(\lambda\) are the coupling constants, \(m\) is the mass of \(\phi_j\), \(f^{abc}\) is the structure constant of SU\((N_c)\), \(T^a\) is the representation matrix for the basis of the associated Lie algebra su\((N_c)\), \(\alpha\) is the gauge fixing...
parameter, and $\eta^a(a = 1, \cdots, N_s^2 - 1)$ is the Fadeev-Popov ghost. In order to absorb the ultraviolet divergences arising from quantum fluctuations, we renormalize the fields, mass, and coupling constants as

$$\mathcal{L} = -\frac{1}{4} Z_3 \left( G^a_{\mu\nu} \right)^2 + \sum_j \left[ Z_2 |D_\mu \phi_{rj}|^2 - Z_m m_r^2 |\phi_{rj}|^2 - Z_1 \lambda_r (|\phi_{rj}|^2)^2 \right]$$

$$- \frac{Z_3}{2Z_\alpha \alpha_r} \left( \partial^\mu G^a_{\mu\nu} \right)^2 + Z_\eta \partial^\mu \eta^a \left( \partial_\mu \eta^a_r - \frac{Z_1}{Z_2} g_r f^{abc} \eta^b \eta^c \right)$$

with $G^a_{\mu\nu} = \partial_\mu G^a_{\nu\mu} - \partial_\nu G^a_{\mu\nu} + (Z_1/Z_2) g_r f^{abc} G^b_{\mu\nu} G^c_{\nu\mu}$ and $D_\mu \phi_{rj} = \partial_\mu \phi_{rj} - i(Z_1/Z_2) q_r T^a G^a_{\mu\nu} \phi_{rj}$, where the quantities with the index “r” are the renormalized ones, and $Z_1$, $Z_2$, $Z_3$, $Z_\eta$, $Z_m$, $Z_\lambda$, and $Z_\alpha$ are the renormalization constants. Since the degree of freedom of the $Z_\alpha$ can be absorbed by that of the non-physical parameter $\alpha_r$, we simply fix it as $Z_\alpha = Z_3$.

If we impose the compositeness condition $Z_3 = 0$, the kinetic term of $G^a_{\mu\nu}$ disappear, and becomes a non-dynamical auxiliary field, whose Euler equation reduces to a constraint. The constraint, however, causes a self-interaction of $\phi_r$, which gives rise to a composite gauge field. The kinetic term is reproduced through the quantum effects of the matter fields. This can be treated in the most rigorous way by following the usual renormalization procedure with (2), and by imposing the compositeness condition $Z_3 = 0$ after that. As was argued in the case of the model with fermionic matters [11], the compositeness condition itself spoils the ordinary perturbation expansion in the coupling constant $g_r$, and the appropriate expansion parameter is the inverse of the number of the matter-field species, $1/N_s$. Here we calculate $Z_3$ based on the Lagrangian (2) in the leading and the next-to-leading orders in $1/N_s$, solve the condition $Z_3 = 0$ for the coupling constant $g_r$, and consider its implications with a particular attention on the complementarity between gauge-boson compositeness and asymptotic freedom.

In $1/N_s$ expansion, $\lambda$ should be $\leq O(1/N_s)$, because otherwise diagrams with the more loops become the larger. We assign $\lambda \sim O(1/N_s)$. We first choose the renormalization constant $Z_m$ so as to cancel out the constant (momentum-independent) contributions from the scalar self-mass parts order by order. Then the renormalization constant $Z_3$ should be chosen so as to cancel out all the divergences in the diagram A in Fig. 1 at the leading order in $1/N_s$ and in the diagrams B – N at the next-to-leading order. There is no mixing between the gauge boson line with one-scalar-loops inserted (Fig. 2a) and the scalar-loop chain due to $\lambda_4 |\phi_{rj}|^4$ interaction (Fig. 2b), because the sub-diagram in Fig. 3a vanishes. In calculation, we adopt the minimal subtraction prescription in the dimensional regularization. To avoid
the absurdity of vanishing coupling constants, we should keep \( \epsilon = (4 - d)/2 \) at a very small but non-vanishing value, where \( d \) is the analytically continued number of the spacetime dimensions. We retain only the part leading in \( \epsilon \), which amounts to retaining the terms of \( O(\epsilon^0) \) because the compositeness condition implies \( g^2 = O(\epsilon) \) and each diagram has at most the order of magnitude of \( g^{2\eta}/\epsilon^n = O(\epsilon^0) \).

Cancellations between the sub-diagrams in Fig. 3b extremely facilitate the calculations. Because the one-scalar-loop (denoted by \( \Pi_0^{\mu\nu} \)) inserted into a gauge-boson propagator (with momentum \( q \)) (Fig. 2a) is divergenceless (i.e. \( q_{\mu}\Pi_0^{\mu\nu} = 0 \)), it suppresses the \( \alpha_r \)-dependent part of the diagram. After lengthy calculations similar to those indicated in Ref. [11], we obtain the contributions from the A – H to \( Z_3 \) as are shown in Table 1, where we also cited the corresponding results for the model with fermionic matters in the previous paper [11]. The diagrams I and J in Fig. 1 contribute no leading divergences, because the scalar-loop chain behaves like \( (-q^2)^{-l\epsilon} \), \( (q \) is the total momentum transfer through the chain, and \( l \) is the number of loops in the chain.) and consequently the integration over \( q \) give rise to an extra factor of \( \epsilon \) in comparison with the leading contributions in \( \epsilon \). The diagram K is not leading in \( \epsilon \) because the sub-diagram Fig. 3c: converges, and the diagrams L – N cancel out because the sub-diagrams in Fig. 3d cancel.

Next, we renormalize the sub-diagram divergences by subtracting the divergent counter parts of (i) each scalar loop inserted into the gauge-boson lines in D – H, (ii) the scalar self-energy part in D, (iii) the scalar-scalar-gauge-boson vertex parts in \( D' \), E, G, and H, and (iv) the three-gauge-boson vertex part in G and H. The contributions from the counter parts cancel out for the diagrams with odd loops, and amount to minus twice the original terms for those with even loops. Collecting all the contributions in Table 1 and the counter terms, we finally obtain the compact expression

\[
Z_3 = 1 - \frac{1}{6} N_s g_t^2 I + \frac{11}{3} N_c g_t^2 I - \frac{\alpha_r}{2} N_c g_t^2 I (1 - \frac{1}{6} N_s g_t^2 I) + \frac{3}{2} N_c \left( \frac{6}{N_s} - g_t^2 I \right) \ln(1 - \frac{1}{6} N_s g_t^2 I) + O\left( \frac{1}{N_s^3} \right),
\]

(3)

where \( I = 1/16 \pi^2 \epsilon \). It is remarkable that this is much similar in form to the corresponding result for the previous model with fermionic matters ([11]), in spite that the contributions from the corresponding diagram differ. In fact (3) is what is obtained by replacing \( N_f \) by \( N_s/4 \) in the result Eq. (12) in [11]. We note that \( Z_3 \) does not depend on the scalar self-coupling constant \( \lambda_r \).
The compositeness condition \( Z_3 = 0 \) with expression \( (3) \) can be solved for \( g_t^2 \) by iterating the leading-order solution into itself. The solution is rather simple:

\[
g_t^2 = \frac{6}{N_s \alpha_r} \left[ 1 + \frac{22 N_c}{N_s} + O\left(\frac{1}{N_s}ight) \right].
\] (4)

The logarithmic term in \( (3) \) is suppressed in the solution \( (4) \) by the factor which vanishes in iteration of the leading solution. It is interesting that the solution does not depend on the gauge parameter \( \alpha_r \), in spite of the fact that \( Z_3 \) does. The \( \alpha_r \)-dependent term in \( (3) \) is also suppressed in the solution in the same way that the logarithmic term is. The solution of the compositeness condition should be gauge-independent because it is a relation among physically observable quantities.

Because the above argument relies on \( 1/N_s \) expansion including iteration, the absolute value of the next-to-leading contribution should not exceed that of the leading one. If we apply it to \( (4) \), we obtain

\[
N_s > 22 N_c.
\] (5)

The allowed region of \( N_s/N_c \) by \( (5) \) is complementary to that for asymptotic freedom in the gauge theory \([12]\), just like in the case of the previous model with fermionic matters \([11]\). When the gauge theory is asymptotically free, the next-to-leading contributions to the compositeness condition are so large that the gauge bosons cannot be composites of the above type. On the contrary, when the theory is asymptotically non-free, the next-to-leading order contributions are suppressed, and the gauge bosons can be interpreted as composite.

Though the complementarity in the scalar matter system is very parallel to that in the fermionic system, they are much different in several points. Because the scalar mass is not protected by any symmetry unlike fermions’, quadratic divergences arise in the scalar self-mass. Though they are not explicit in the dimensional regularization, they should be interpreted to exist from the physical points of view. Then they should be renormalized by unnatural fine tuning. Another characteristic of the scalar matter system is the existence of the self coupling constant \( \lambda \). At low energies in comparison with the cutoff scale, \( \lambda \) does not appear in the compositeness condition, as is mentioned above. It could, however, affect our result in various way. For example, it runs with the energy scale, and may blow up at some scale, invalidating the approximation at the order in \( 1/N_s \). On the other hand, the gauge coupling constant \( g \) remains fixed even for large scale \( \mu \) (but below the cutoff \( \Lambda \sim m_t e^{1/\epsilon} \)),
because the renormalization group beta function
\[ \beta(g_r) \equiv \mu \frac{\partial g_r}{\partial \mu} = -\epsilon g_r + \frac{1}{6} N_s g_r^3 I - \frac{11}{3} N_c g_r^3 I. \] (6)
vanishes due to solution (4) of the compositeness condition within the present approximation. In general, however, \( g \) would also blow up together with \( \lambda \) at the order where \( Z_3 \) depends on \( \lambda \).

It is straightforward to extend this result to the case where both the fermionic and scalar matters exist. We have only to replace \( N_s \) in (3) and (4) by \( N_s + 4N_f \), and consequently the complementality still exists because the same replacement rule holds also for the asymptotic freedom. It is tempting to extend it to supersymmetric models. For this case, there are a few problems to be overcome. First we should take into account the effects of the gaugeno, the superpartner of the gauge boson. It gives rise to further quantum corrections through intermediate virtual states, and at the same time it should be a composite since it is a partner of the composite gauge boson. Furthermore it has an additional interaction through gaugeno-scalar-fermion vertex, which would give rise to complexities of diagrams contributing to \( Z_3 \), while asymptotic freedom seems to be free of the effects of these additional interactions. These problems are now under investigation.

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Table 1. Contributions to $Z_3$ from the diagrams A – H

The diagrams A – H are defined in Fig. 1 in Ref. [11] for the model with spinorial matters and in Fig. 1 in this paper for the model with scalar matters. $I = 1/16\pi^2\epsilon$.

| diagram | with spinorial matter | with scalar matter |
|---------|-----------------------|--------------------|
| A       | $-\frac{2}{3} N_f g_t^2 I$ | $-\frac{1}{6} N_s g_t^2 I$ |
| B, C    | $(\frac{13}{6} - \frac{\alpha_t}{2}) N_c g_t^2 I$ | $(\frac{13}{6} - \frac{\alpha_t}{2}) N_c g_t^2 I$ |
| D, D', E | $\frac{1}{3} \alpha_t N_c N_f g_t^4 I^2$ | $\frac{3}{2} N_c \sum_{l=1}^{\infty} \left( -\frac{1}{6} \right)^l \frac{N_s^l (g_t^2 I)^{l+1}}{l(l+1)} +\frac{1}{12} \alpha_t N_c N_s g_t^4 I^2$ |
| F, G, H | $\frac{3}{2} N_c \sum_{l=1}^{\infty} \left( -\frac{2}{3} \right)^l \frac{N_s^l (g_t^2 I)^{l+1}}{l(l+1)}$ | $-\frac{2}{3} \alpha_t N_c N_f g_t^4 I^2$ |
FIG. 1: The gauge-boson self-energy parts at the leading order (A) and at the next-to-leading order (B–N) in $1/N_s$. The dashed, wavy, and dotted lines indicate the elementary scalar, gauge-boson, and Fadeev-Popov ghost propagators, respectively. The wavy lines with a blob and the chains of small circles are defined in Fig. 2.

\begin{align*}
a & = \sum_{l' = 0}^{\infty} \underbrace{\text{scalar-loops}}_{l'} \\
b & = \sum_{l' = 0}^{\infty} \underbrace{\text{scalar-loops}}_{l'}
\end{align*}

FIG. 2: The wavy line with a blob stands for the gauge boson propagator with a number of scalar-loops inserted. The chain of small circles stands for the chain diagram with scalar-loops.
FIG. 3: The diagram a vanishes, the diagrams in b cancel each other, the diagram c does not diverge, and the diagrams in d cancel each other.