Eigenmode Analysis of Radial Velocities

Y. Hoffman

Racah Inst. of Physics, Hebrew University, Jerusalem 91904, Israel

Abstract.
Radial velocity surveys are examined in terms of eigenmode analysis within the framework of CDM-like family of models. Rich surveys such as MARK III and SFI, which consist of more than $10^3$ radial velocities, are found to have a few tens of modes that are not noise dominated. Poor surveys, which have only a few tens of radial velocities, are noise dominated across the eigenmode spectrum. In particular, the bulk velocity of such surveys has been found to be dominated by the more noisy modes. The MARK III and SFI are well fitted by a tilted flat CDM model found by a maximum likelihood analysis and a $\chi^2$ statistics. However, a mode-by-mode inspection shows that a substantial fraction of the modes lie outside the 90% confidence level. This implies that although globally the CDM-like family of models seems to be consistent with radial velocity surveys, in detail it does not. This might indicate a need for a revised power spectrum or for some non-trivial biasing scheme.

1. Introduction

The statistical analysis of surveys of radial velocities plays a major role in the study of the large scale structure. Broadly speaking the analysis focuses on the estimation of the cosmological parameters and the reconstruction of the local cosmography. Radial velocities surveys are dominated by incomplete and anisotropic sky coverage, inhomogeneous, sparse sampling and distance measurements errors that are often larger than the individual velocities. Such surveys do not easily yield themselves to a statistical analysis and provide a real challenge for a proper analysis. The history of the field is therefore rich with controversies about the interpretation and cosmological implications of different surveys and seemingly conflicting results.

A prime goal of a statistical analysis is to provide a tool for confronting theoretical models with the data. This calls for a formalism for presenting models and data in the same language, a problem closely related to the problem of the functional representation. From the theoretical point of view, the choice of the representation is dictated by the symmetries of the theory. The cosmological principle makes the Fourier plane waves and the spherical Bessel/harmonics the natural choice. However typical astronomical observations break these symmetries, as the data is neither homogeneous nor isotropic. A better representation should reflect both the basic underlying theory and the particularities of the data. Here we follow the standard eigenmode analysis, also known as principal
components analysis (PCA) and the Karhunen-Loeve transform. This, or its slightly modified version of signal-to-noise eigenmodes, was suggested before as a method of analyzing redshift surveys (Vogeley and Szalay 1996 and references therein). Previous applications focused mostly on parameters estimation. Here we extend the method and use it as a general tool for understanding the nature of the data, its noise structure and information content. Dealing with velocity surveys the determination of bulk velocities of a given survey and its relation to the underlying velocity field is revisited. The PCA is used here to address the problem of the power spectrum determination.

2. Eigenmode Analysis: Radial Velocities and Bulk Velocities

Consider a data base of radial velocities \( \{u_i\}_{i=1,...,N} \), where

\[
u_i = \mathbf{v}(r_i) \cdot \hat{r}_i + \epsilon_i,
\]

\( \mathbf{v} \) is the three dimensional velocity, \( r_i \) is the position of the \( i \)-th data point and \( \epsilon_i \) is the statistical error associated with the \( i \)-th radial velocity. The assumption made here is of a cosmological model that well describes the data, that systematic errors have been properly dealt with and that the statistical errors are well understood. The data auto-covariance matrix is then written as:

\[
R_{ij} \equiv \langle u_i u_j \rangle = \hat{r}_j \langle \mathbf{v}(r_i) \mathbf{v}(r_j) \rangle \hat{r}_j + \sigma^2_{ij}.
\]

(Here \( \langle \ldots \rangle \) denotes an ensemble average.) The last term is the error covariance matrix. The velocity covariance tensor that enters this equation was derived by Górski (1988, see also Zaroubi, Hoffman and Dekel 1999) and it depends on the power spectrum and the cosmological parameters.

The eigenmodes of the data covariance matrix provides a natural representation of the data:

\[
R\eta^{(i)} = \lambda_i \eta^{(i)}
\]

The set of \( N \) eigenmodes \( \{\eta^{(i)}\} \) constitutes an orthonormal basis and the eigenvalues \( \lambda_i \) are arranged in decreasing order. A new representation of the data is given by:

\[
\tilde{a}_i = \eta_j^{(i)} u_j
\]

This provides a statistical orthogonal representation, namely:

\[
\langle \tilde{a}_i \tilde{a}_j \rangle = \lambda_i \delta_{ij}
\]

The normalized transformed variables are defined here by:

\[
a_i = \frac{\tilde{a}_i}{\sqrt{\lambda_i}}
\]

Eq. \( \mathbb{3} \) is written now as:

\[
\langle a_i a_j \rangle = \delta_{ij}
\]
Note that as the modes are statistically independent one can measure the $\chi^2$ of a given mode, independently of all other modes:

$$\chi_i^2 = a_i^2$$  \hspace{1cm} (8)

For normally distributed errors and a Gaussian random velocity field the $a_i$’s are normally distributed with zero mean and a variance of unity.

Velocity surveys are often analyzed in terms of their bulk flows, namely fitting the velocity field by a single constant velocity vector, ignoring any possible correlations of the underlying field. There are a variety of ways of defining the bulk velocity of a given survey and here we adopt the Kaiser (1988) algorithm which evaluates an error weighted bulk velocity. Thus, the full complexity of the underlying field of its N degrees of freedom is compressed into three parameters only. It is often argued that this data compression enables the extraction of more statistically robust quantities from the data, thus providing better constraints on the models. The bulk velocity properties of a survey is studied here within the PCA formalism.

The bulk velocity ($B$) of a survey is defined by means of a linear operator ($L$), $B = Lu$ (see Kaiser 1988 for the formal derivation). $B$ is expanded here by

$$B = \sum_i a_i B^{(i)} = \sum_i a_i \sqrt{\lambda_i} L \eta^{(i)}$$,  \hspace{1cm} (9)

where $B^{(i)}$ is the bulk velocity associated with the i-th mode. The bulk velocity covariance matrix is:

$$\langle B_\alpha B_\beta \rangle = \sum_i B^{(i)}_\alpha B^{(i)}_\beta$$  \hspace{1cm} (10)

In the case of an anisotropic sampling the bulk velocity covariance matrix is anisotropic as well. In the limit of a perfect survey (isotropic, no errors, dense, homogeneous) the eigenvectors are the spherical harmonics and Bessel functions. In such a case and assuming the data to lie on a thin shell, one expects:

$$B^{(i)}_\alpha = 0 \text{ for } i \neq 2, 3, 4$$  \hspace{1cm} (11)

### 3. Observations

The problem to be addressed here is the quality and expected significance of a survey given an assumed model, i.e. power spectrum, of the underlying velocity field. It follows that here one is more interested in the sampling, sky coverage and the errors then in the actual numerical value of the data points. Four data sets are studied here: The MARK III catalog (Willick et al. 1995), SFI (da Costa et al. 1996), LP10K (Willick 1999) and the nearby Type Ia supernova (hereafter SN; Riess 1999). The first two data sets consist of more than 1000 radial velocities, and are considered as rich catalogs, where the other two have velocities of only 15 Abell clusters (LP10K) and 44 Type Ia supernova (SN) and are considered here as poor catalogs. The analysis has been applied to a wide range of CDM-like models but only two models are explicitly presented here. One is a flat tilted CDM model ($n = 0.8, h = 0.75, \Omega_0 = 1$), where $n$ is the power index, $h$ is Hubble’s constant in units of 100$\text{Mpc}^{-1}\text{km/s}$ and $\Omega_0$ is the
density parameter), and the flat $\Lambda$-CDM model with $n = 1, \ h = 1, \ \Omega_0 = 0.4$. Both models are COBE normalized. The tilted model is the most probable CDM-like model for the MARK III data (Zaroubi et al. 1997), and is very close to the model favored by the SFI (Freudling et al. 1999). The other model is the currently most popular model obeying the age and geometrical constraints. The results obtained for the two models are basically very similar. The strategy followed here is to compute the eigenmodes and eigenvalues of a given survey and model with and without the noise. The comparison of these reveals how many independent modes are signal or noise dominated. It can also help assessing the degree to which the bulk velocity of the sample reflects the underlying velocity field or the observational errors.

For all data sets the noise-free eigenmode spectrum follows an approximate power law behavior over most of the range of modes. The addition of noise breaks this power law decline, and causes a flattening of the spectrum. The transition from one regime to the other marks the transition from the signal to noise dominated regimes. There is a striking difference between the rich surveys (MARK III and SFI, Fig. 1) and the poor surveys (SN and LP10K, Fig. 2). The formers exhibit a clear break, with some 10 modes or so that are virtually unaffected by the noise and a few tens of modes that are not noise dominated. In the poor samples, on the other hand, all modes are noise dominated! This happens for a wide spread of acceptable CDM-like models, both with COBE and clusters normalization.

Next, the bulk velocities of the spectrum of eigenvalues, $B^{(i)}$, is calculated. For an ideal survey only the first few modes are expected to be significant and the rest of the modes should have very small bulk velocities. MARK III and SFI indeed show such a behavior, namely the $B^{(i)}$ of the first few modes lie significantly above the noise level (Fig. 3). For the poor samples, SN and
LP10K, an opposite trend is found as $B_i$ does not decline, or even grows, with the mode number, namely the more dominant by the noise a mode is the higher is its $B_i$ (Fig. 4). It follows that the sample bulk velocity of the rich surveys indeed reflects the underlying velocity field (convolved with the sample window function). In the case of the poor samples the bulk velocity is dominated by the noise. It should be stated that the strong conclusions expressed here are valid only within the framework of CDM-like cosmogonies.

4. Power Spectrum

The optimal way of estimating the values of the cosmological parameters from a given survey is by performing a maximum likelihood analysis of the data given a range of models (Zaroubi et al. 1995). The maximum likelihood analysis does not provide an absolute measure of the quality of the parameter estimation, but rather finds the most probable model given the data and the assumed parameter space. A measure of the goodness of fit is provided by the $\chi^2$/d.o.f. However, it is often the case that the best model of a given parameter space and a given data set fits poorly on some scales and better on some others, small vs large scales say. This can result in a ‘conspiracy’, yielding an adequate $\chi^2$ even for the ‘wrong’ model. The PCA which projects the data into statistically independent normal variables enables the analysis of the data on a mode by mode basis and check the goodness of fit across the spectrum of the modes. One recalls here that the $\chi^2$ of a given mode is simply $a_i^2$ and the cumulative $\chi^2$ is

$$\chi^2_M = \frac{\sum_{i=1}^{M} a_i^2}{M}.$$

This combined PCA and $\chi^2$ analysis is applied here to the SFI and MARK III catalogs. The smaller data sets (SN and LP10K) are not discriminative.
Figure 3. The bulk velocities spectrum is plotted against the mode number for the MARK III (left) and SFI (right) data. The crosses correspond to the spectrum of the signal covariance matrix and the squares to the full (signal+noise) covariance matrix. This is calculated for the tilted-CDM model.

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enough to enable such a study.) Fig. 5 shows the cumulative $\chi^2_M$ of the MARK III and SFI surveys (assuming the tilted-CDM model). On top of these plots the lower and upper 90% confidence levels are plotted. For the the MARK III the total $\chi^2/d.o.f = 1.02$ is well within the 90% limit and therefore seems to be very consistent with the data. However, Fig. 5 shows that over most of the mode number range the $\chi^2_M$ lies outside the 90% confidence band. Actually from approximately the 100th mode to the last one there is a monotonic increase of $\chi^2_M$. A similar trend is also exhibited by the SFI data. To check the constraining power of the PCA/$\chi^2$ test it has been applied to the mock MARK III catalog of Kolatt et al. (1996). The cumulative $\chi^2_M$ is found to be fully consistent with the assumed model (figures are not shown here). Thus, a systematic inconsistency of the best CDM-like model with the data is found here that possibly suggests a fundamental problem to the CDM paradigm. A more detailed analysis is to be presented elsewhere.

5. Discussion

The analysis presented here consists of two parts. First, the constraining power of radial velocities surveys has been examined within the CDM-like family of models. Using PCA the structure of the expected data has been considered, rather then the actual numerical value of the data. The analysis reveals that rich surveys such as MARK III and SFI, have a few tens of modes that are not noise dominated, and hence are expected to reflect the underlying velocity field. Poor samples such as LP10K, SN and most probably all other surveys that consist of a few tens of objects are noise dominated. Not even a single eigenmode is signal dominated for such surveys, and the bulk velocity is dominated by the more noisy, and less significant, modes. This does not imply that such surveys are of no use in cosmology, but that they should be analyzed with great care. Direct reconstruction methods might be completely noise dominated and might
be very misleading. Indirect methods such as Wiener filtering and maximum entropy should be useful in analyzing such data. A note of cautious is due here. The statements made here are valid only within the framework of the standard cosmogony of CDM-like family of models.

Having convinced ourselves that surveys such as MARK III and SFI are powerful enough to constrain the CDM-like models, the consistency of these surveys with the models has been examined in detail. A mode-by-mode inspection finds significant discrepancies with the spectral behavior predicted by the ‘best’ model found by the maximum likelihood analysis and a global $\chi^2$ analysis. It seems that the overall agreement is obtained by some ‘conspiracy’, where the combination of the independent modes yields a reasonable $\chi^2$. This implies a gross disagreement of the most favorable cosmological model with the velocity data, or the need to invoke some non-trivial biasing.

PCA can also play a very important role in designing and planning new surveys. PCA is based on analyzing the data covariance matrix, which expresses the statistical properties of the data rather than its actual numerical values. It follows that it can be applied before a survey is done, and therefore can be used to design it. By studying the spectrum and structure of the eigenmodes of a survey of given geometry and depth and expected errors the constraining power of a survey can be properly evaluated in its planning phase.

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(1)

\( v \) is the three dimensional velocity, \( r_i \) is the position of the \( i \)-th data point and \( \epsilon_i \) is the statistical error associated with the \( i \)-th radial velocity. The assumption made here is of a cosmological model that well describes the data, that systematic errors have been properly dealt with and that the statistical errors are well understood. The data auto-covariance matrix is then written as:

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(2)

(Here \( \langle \cdots \rangle \) denotes an ensemble average.) The last term is the error covariance matrix. The velocity covariance tensor that enters this equation was derived by Górski (1988, see also Zaroubi, Hoffman and Dekel 1999) and it depends on the power spectrum and the cosmological parameters.

The eigenmodes of the data covariance matrix provides a natural representation of the data:

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R_{\eta^{(i)}} = \lambda_i \eta^{(i)}
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(3)

The set of \( N \) eigenmodes \( \{\eta^{(i)}\} \) constitutes an orthonormal basis and the eigenvalues \( \lambda_i \) are arranged in decreasing order. A new representation of the data is given by:

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This provides a statistical orthogonal representation, namely:

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(5)

The normalized transformed variables are defined here by:

\[
a_i = \frac{\tilde{a}_i}{\sqrt{\lambda_i}}
\]

(6)

Eq. 5 is written now as:

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Note that as the modes are statistically independent one can measure the $\chi^2$ of a given mode, independently of all other modes:

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3. Observations

The problem to be addressed here is the quality and expected significance of a survey given an assumed model, i.e. power spectrum, of the underlying velocity field. It follows that here one is more interested in the sampling, sky coverage and the errors then in the actual numerical value of the data points. Four data sets are studied here: The MARK III catalog (Willick et al. 1995), SFI (da Costa et al. 1996), LP10K (Willick 1999) and the nearby Type Ia supernova (hereafter SN; Riess 1999). The first two data sets consist of more than 1000 radial velocities, and are considered as rich catalogs, where the other two have velocities of only 15 Abell clusters (LP10K) and 44 Type Ia supernova (SN) and are considered here as poor catalogs. The analysis has been applied to a wide range of CDM-like models but only two models are explicitly presented here. One is a flat tilted CDM model ($n = 0.8$, $h = 0.75$, $\Omega_0 = 1$, where $n$ is the power index, $h$ is Hubble's constant in units of $100 H_0^{-1}\text{km s}^{-1}$ and $\Omega_0$ is the
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