A fractional difference returns for stylized fact studies

Rosmanjawati Abdul Rahman¹,a,* and Jibrin Sanusi Alhaji¹,2,b

¹School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Pulau Pinang, Malaysia
²Department of Statistics, Kano University of Science and Technology, Wudil, P.M.B. 3244-Kano State, Nigeria
*Corresponding author
E-mail: arosmanjawati@usm.my, bsanusijibrin46@gmail.com

Abstract. Stylized facts are properties observed and estimated after obtaining the first difference of natural logarithms of returns series, $r_t$. The fat tails, volatility clustering, persistence, leverage effect and half-life of a volatility are some of the stylized facts that have been studied in economics, finance and recently health data. This research considers the situations when the financial data or any other series are long memory or when they exhibit Interminable Long Memory (ILM) and volatility. In view of this, the research's aim is to modify the procedure for obtaining the conventional returns of series so therefore a new approach, the Fractional Difference Returns (FDR_t) is introduced. Then, both the FDR_t and the traditional or regular returns are computed and modelled by using the Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH) model and further, are compared based on the models' log likelihood and stylized fact characteristics. Information criteria, Log likelihood and diagnostic tests are the indicators in choosing the best approach between the FDR_t and the regular returns, $r_t$. As applied to four Asian emerging market currencies such as Malaysian Ringgit, Indonesia Rupiah, Hong Kong Dollar and Singaporean Dollar, the findings reveal that volatility analysis using FDR_t produced a better estimator compared to the $r_t$ specifically if the series exhibited ILM.

1. Introduction

In volatility analysis, returns and residuals from the auxiliary regression are used for volatility modelling. The returns are estimated by the first difference of natural logarithms of a series (see [1]). Meanwhile, the stochastic volatility models such as the Generalized Autoregressive Conditional Heteroscedastic (GARCH), Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) and Threshold Generalized Autoregressive Conditional Heteroscedastic (TGARCH) models are used to handle volatile series as shown in [2], [3], [4], [5], [6], [8], [9], [10], [11], [12] and [13]. However, if the series have deterministic trend and the Autocorrelation Functions (ACF) exhibit a slow decay, the long-range dependence volatility models such as the Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH) model should be considered. As shown in [4] and [14], the APARCH model is used to handle Long Memory (LM), volatility clustering and leverage or asymmetric effect. According to [15], absolute returns displayed a slower decaying ACF than square returns and they are used to estimate power models such as the APARCH model.

Statistical models were developed and modified to handle identified real-life problems and to avoid replicating past results. In addition, as the existing models require improvement to handle a shift in a process, therefore, the procedure of data transformation also needs some modification. The residuals of mean models such as the Autoregressive Integrated Moving Average (ARIMA), and the
Autoregressive Fractional Integrated Moving Average (ARFIMA) models are regularly affected or characterized by serial correlation problems, heteroscedasticity and non-normality. These problems usually occur due to large noise, trend, persistence and volatile nature of the data under consideration. The residuals analysis only expose few pattern in the original series that is not fully captured by the chosen and estimated models specifically when a process is dominated by severe LM and acute volatility. To effectively account for the volatility of stock markets activities, the LM modelling, the volatility analysis as well as residuals analysis were adequately employed in numerous studies (see [16],[17] and [18]). However, the fractional returns for an Interminable LM (ILM) and volatile process when the fractional differencing operator is in the interval 1<d<2, has not been introduced. For this reasons, in this work, we modify the procedure for estimating the returns of a series and introduce a Fractional Difference Returns (FDRt) that is applicable when a process exhibits a high persistence or ILM and volatility. Consequently, both the FDRt and the traditional or regular returns are computed and the APARCH model is estimated. Finally, the parameter estimates based on the proposed FDRt and rt are compared.

2 Material and Methodology
The derivation of the FDRt and discussion of the model are outlined in this section.

2.1. The dataset
The data for this study are obtained from the Morgan Stanley Capital International (MSCI) data stream and are 5218 daily (5 days) Singaporean Dollar (SGD), Hong Kong Dollar (HKD), Malaysian Ringgit (MYR) and Indonesian Rupiah (IDR) to one United State Dollar (USD) exchange rate between the 20th November 1997 and 20th November 2017.

2.2. The Returns
In [19], relationship between volatility and returns was examined by using the Autoregressive Conditional Heteroscedastic (ARCH) models. Let $Y_t$ denotes the closing value of the currency exchange rate on day $t$, then the daily returns $r_t$ is given by

$$ r_t = \ln \left( \frac{Y_t}{Y_{t-1}} \right) = \ln Y_t - \ln Y_{t-1} $$

In this case, it can also be called the continuously compounded exchange value per day that returns $Y_t$ on an exchange rate of $Y_{t-1}$ for a single day.

2.2.1 Fractional Difference Returns (FDRt)
If $\{Y_t\}, t=1,\ldots,T$ is an interminable or fractional unit root integral process with memory parameter in the range, 1<d<2, hence, the fractional difference returns is obtained based on the following steps:

2.2.2 Steps for estimating FDRt
1). If $\{Y_t\}, t=1,\ldots,T$ is a nonstationary process with time varying mean and variance, then examine the autocorrelations for a hyperbolic decay.
2). Test for a long memory and estimate $d$ such that 1<d<2 where $d=d^*+1, 0<d^*<1$.
3). Estimate the FDRt in two stages:
   (i) Let the first returns be
   $$ r_t = \ln Y_t - \ln Y_{t-1} $$

In this case, it can also be called the continuously compounded exchange value per day that returns $Y_t$ on an exchange rate of $Y_{t-1}$ for a single day.
Following the standard transformation practice in time series analysis and for a time varying mean, variance and nonstationary data \(Y_t, \ldots, Y_{t-q}\), the returns, \(r_t\) also represent first differencing.

(ii) Next, consider \(d^*\) and obtain the second returns, \(r_{t-1}^*\), where

\[
r_{t-1}^* = d^* (\ln Y_{t-1} - \ln Y_{t-2})
\]  

(3)

Since returns are calculated by the difference of natural logarithms of a series, then the FDR\(_t\) is derived as follows:

\[
FDR_t = r_t - r_{t-1}
\]  

(4)

Similarly, the FDR\(_t\) is obtained as follows:

\[
FDR_t = \ln Y_t - \ln Y_{t-1} - d^* (\ln Y_{t-1} - \ln Y_{t-2})
\]  

(5)

where \(d^*\) is such that \(0 < d^* < 1\).

A direct use of the first difference of natural logarithms series may cause two problems such as under and over-differencing specifically if the data generating process displays ILM. This refers to the situation when the integrating order of a series is greater than one and less than two and also volatile. The introduction of the FDR\(_t\) is expected to solve these two problems. Next, the FDR\(_t\) filters are used in estimating and capturing the stylized fact of volatility.

### 2.3 APARCH Model

APARCH model was introduced in [14]. It is designed to capture the volatility clustering, LM and leverage effects. Generally, APARCH model is written as

\[
x_t = \phi y_t + \varepsilon_t, \quad t = 1, \ldots, T,
\]  

(6)

\[
\delta
\]  

(7)

where

\[
\varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1)
\]  

(8)

Noted that although \(z_t\) is assumed to be Gaussian but it does not mean that returns \(r_t\) and FDR\(_t\) are Gaussian too. The \(c, \alpha, \beta, \gamma, \text{ and } \delta\) are parameters needed to be estimated with \(c > 0, \alpha_i \geq 0, i=1,\ldots,m, \beta_j \geq 0, j=1,\ldots,n, \text{ when } \alpha_i=0, i=1,\ldots,m, \beta_j=0, j=1,\ldots,n, \text{ then } \sigma^2_t = c\). Since variance is non-negative, therefore, \(c > 0\). The leverage effect, \(\gamma\) indicates how increasing asset prices are accompanied by declining volatility and vice-versa. The LM is captured by \(\delta\). LM occurs in volatility when the effect of volatility shocks decay slowly.

This study uses the gretl version 1.9.4 [21] and Ox Professional version 7.0 [22] for analysing the data.
3 Analysis and Results

Figure 1. The plot of daily Exchange rate for SGD, HKD, MYR and IDR to one USD (1997-2017).

Figure 1 shows the time series plot of the SGD, HKD, MYR and IDR to one USD data. All The graphs exhibit nonlinear trends and overall, the series is not stationary.

Figure 2. Correlogram for daily Exchange rate for SGD, HKD, MYR and IDR to one USD.

Figure 2 is the correlograms for the SGD, HKD, MYR and IDR to one USD and all shows a slow decay in the autocorrelation which indicates the long memory processes.
Figure 3. The plot of daily log-returns and FDR_t for both Malaysian ringgit and Indonesian rupiah

The daily log-returns henceforth called the traditional log-returns and the FDR_t for both Malaysian ringgit and Indonesian rupiah are shown in Figure 3. The two types of returns indicate periods of high volatility in 1998 and this is an evidence of volatility clustering ([23],[4] and [24]) and both returns are stable around mean between year 2000 and beyond. In this case, volatility clustering are caused by continuous external shocks or fluctuations rates of Ringgit and Rupiah to dollar. In a similar manner, the spike or outlier observed in both series is a period where the US dollar oppressing the two currencies. However, the dollar oppression was succeeded by period of relative calm as seen graphically from the year 2000 in both currency exchange rates. The graph also indicate a marginal improvement when the introduced FDR_t was used and later the numerical evidence will be given to support this.

Figure 4. Correlogram for daily absolute returns and absolute FDR_t for Malaysian ringgit and Indonesian rupiah
The autocorrelation function of absolute returns and absolute FDR\textsubscript{t} display a contrast behaviour compared to the returns and FDR\textsubscript{t}. As shown in Figure 4, autocorrelation function for ringgit and rupiah remains high, positive over years and decays gradually to zero which is a clear occurrence of volatility clustering. If returns are leptokurtic and having long memory, the use of autocorrelation function and LM test to determine the occurrence of LM is likely to fail. This is because the slow decay may be due to structural break and inconsistent autocorrelation estimate which are more serious for autocorrelation of square returns [25]. In the case of our present work, the observed slow decays in sample autocorrelations of absolute returns are natural and not man-made and similar to [12].

**Table 1.** Summary Statistics and Degree of Fractional Differencing Estimator.

| Statistics | \( S_t \) | \( H_t \) | \( M_t \) | \( I_t \) |
|------------|----------|----------|----------|----------|
| Mean       | 1.515    | 7.774    | 3.675    | 10010    |
| Variance   | 0.037    | 0.001    | 0.21     | 3299825  |
| Skewness   | 0.015    | 0.321    | 2.59     | 0.086    |
| Jarque-Bera test | 460.313 | 431.172  | 2517.15  | 419.289  |
| GPH        | 1.00996  | 1.00072  | 1.00837  | 1.01227  |
| ARFIMA(0,d,0) | 0.9959 | 0.99973  | 1.01894  | 1.06054  |

Let \( S_t \), \( H_t \), \( M_t \) and \( I_t \) be the level series of SGD, HKD, MYR and IDR respectively. The summary statistics and degree of fractional differencing estimator for the \( S_t \), \( H_t \), \( M_t \) and \( I_t \) are displayed in Table 1. The average exchange rate of the four series over the past twenty years was \$1.515, HK\$7.774, RM\$3.675 and Rp10010 respectively. All the exchange rate series are positively skewed while the Jarque-Bera test indicates that series are not normal. The two LM estimators, Geweke Porter Hudak (GPH) and ARFIMA(0,d,0) produce inconsistent results, where the GPH indicates that memory values are greater than unity whereas, the ARFIMA (0,d,0) estimator indicates the memory values are less than one for \( S_t \) and \( H_t \) exchange rates. The two LM estimators; the GPH and ARFIMA(0,d,0), indicate an average memory values of 1.002935, 1.00023, 1.01366 and 1.03641 for \( S_t \), \( H_t \), \( M_t \) and \( I_t \) respectively.

**Table 2.** The First Difference Exchange Rate and Degree of Fractional Differencing Estimators.

| LM Estimators | \( d_{\Delta S_t} \), \( \{d_{\Delta S_t} + 1\} \) | \( d_{\Delta H_t} \), \( \{d_{\Delta H_t} + 1\} \) | \( d_{\Delta M_t} \), \( \{d_{\Delta M_t} + 1\} \) | \( d_{\Delta I_t} \), \( \{d_{\Delta I_t} + 1\} \) |
|--------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| GPH          | -0.01388, \{0.986\}            | -0.03854, \{0.961\}            | 0.01927, \{1.019\}            | 0.04208, \{1.042\}            |
| ARFIMA(0,d,0) | -0.02534, \{0.975\}          | -0.03192, \{0.968\}          | 0.01858, \{1.019\}          | 0.06274, \{1.063\}          |

Values in the curly brackets are the estimated degree of fractional differencing based on \( d+1 \).

The GPH and ARFIMA(0,d,0) values of the first differenced exchange rate, the \( \Delta S_t \), \( \Delta H_t \), \( \Delta M_t \) and \( \Delta I_t \), each representing by \( d_{\Delta S_t} \), \( d_{\Delta H_t} \), \( d_{\Delta M_t} \) and \( d_{\Delta I_t} \), respectively are displayed in Table 2. The average GPH and ARFIMA(0,d,0) of each \( S_t \), \( H_t \), \( M_t \) and \( I_t \) based on \( d+1 \) are 0.981, 0.965, 1.019 and 1.053 respectively.

**Table 3.** Summary Statistics for Returns, \( r_t \) and Modified Returns, FDR\textsubscript{t}

| Statistics | SGDR\textsubscript{t} | SGDFDR\textsubscript{t} | HKDR\textsubscript{t} | HKDFDR\textsubscript{t} | MYRR\textsubscript{t} | MYRFDR\textsubscript{t} | INRR\textsubscript{t} | INRFDR\textsubscript{t} |
|------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| S.D        | 0.0038                 | 0.0075                 | 0.0003                 | 0.0283                 | 0.0078                 | 0.0078                 | 0.0140                 | 0.0139                 |
| C.V        | 118.67                 | 107.32                 | 149.78                 | 52.323                 | 254.11                 | 292.56                 | 54.924                 | 86.455                 |

S.D=standard deviation, C.V= coefficient of variation.

The minimum standard deviation 0.0038 for the Singapore exchange rate returns and 0.0003 for the Hong Kong returns confirm the superiority of regular return over both the Singapore and Hong Kong
FDR_t as displayed in Table 3. On the other hand, when the degree of fractional differencing is greater than one, the FDR_t has shown to be better due to the minimum variability as indicated in the Malaysian and Indonesia exchange rate. Eventually, we conclude that it is better to use the traditional returns to transform the Singapore and Hong Kong series as the degree of fractional differencing for both currencies are less than one and conversely, to use the FDR_t to transform the Malaysia and Indonesia exchange rate since the degree of fractional differencing is greater than one.

Table 4. The Summary Statistics for each r_t and FDR_t

| Return Type   | Mean   | S.D    | Skewness | Ex. Kurtosis | C.V (Risk) % |
|---------------|--------|--------|----------|--------------|--------------|
| MYRR_t        | 0.0000 | 0.0078 | 13.5010  | 951.9100     | 25410.000    |
| INRR_t        | 0.0003 | 0.0140 | 2.1673   | 81.6880      | 5492.400     |
| MYRFDR_t      | 0.0000 | 0.0078 | 13.5100  | 953.4400     | 29256.000    |
| INRFDR_t      | 0.0002 | 0.0139 | 1.6650   | 81.7400      | 8645.500     |
| AbsMYRR_t     | 0.0024 | 0.0074 | 27.2670  | 1128.0000    | 308.020      |
| AbsINRR_t     | 0.0054 | 0.0129 | 8.3851   | 98.6570      | 240.410      |
| AbsMYRFDR_t   | 0.0024 | 0.0074 | 27.3370  | 1132.5000    | 306.010      |
| AbsINRFDR_t   | 0.0054 | 0.0128 | 8.3988   | 99.0600      | 238.520      |

C.V=coefficient of variation.

The mean absolute returns values (in bold), of r_t and FDR_t in average are better with minimum coefficient of variation (risk-return trade off) compared to the non-absolute returns as displayed in Table 4, which contradict the financial market theory. In addition, absolute returns of FDR_t indicate minimum risk reduction in Malaysian ringgit exchange rate trading compares to the r_t. However, the FDR_t indicates that investment in Indonesian economy has less risk due to a smaller coefficient of variation which is 238.52% compared to Malaysia and this is also supported by the high volatility as indicated by excess kurtosis in the Malaysian exchange rate market for the period under study. Therefore, the merits of FDR_t highlighted here are enough to consider the FDR_t for the transformation of Malaysia and Indonesian exchange rate.

The aim of this study is to introduce a new approach for estimating returns, the FDR_t. Therefore, next, the APARCH model was used for modelling LM-volatility series, the regular returns and FDR to study the Malaysian and Indonesian exchange rate when the degree of fractional differencing is greater than one as observed in both series.

Table 5. The APARCH Model for r_t, Abs r_t, and FDR_t of Malaysia

| Model          | Returns Type | Distribution For Returns | Log likelihood |
|----------------|--------------|--------------------------|----------------|
| APARCH(1,1)    | MRt          | Skew-Student             | 28038.094      |
| APARCH(1,1)    | MFDRt        | Skew-Student             | 26580.197      |
| APARCH(1,1)    | AbsMFDRt     | Student                  | 22169.890      |
| APARCH(2,1)    | MRt          | Student                  | 26571.521      |
| APARCH(2,1)    | MFDRt        | Skew-Student             | 25875.183      |
| APARCH(2,1)    | AbsMFDRt     | Skew-Student             | 26676.286      |
| APARCH(1,2)    | MRt          | Gaussian                 | 21390.261      |
| APARCH(1,2)    | MFDRt        | Student                  | 29154.545*     |
| APARCH(1,2)    | AbsMFDRt     | Skew-Student             | 27726.009      |
### Table 6. The APARCH Model for $r_t$, Abs $r_t$ and FDR$_t$ of Indonesia

| Models           | Returns Type | Distribution for returns | Log likelihood |
|------------------|--------------|--------------------------|----------------|
| APARCH(1,1)      | IR$_t$       | Skew-Student             | 20376.316      |
| APARCH(1,1)      | IFDR$_t$     | Student                  | 20395.892      |
| APARCH(1,1)      | AbsIFDR$_t$  | Gaussian                 | 20093.376      |
| APARCH(2,1)      | IR$_t$       | Skew-Student             | 20714.387      |
| APARCH(2,1)      | IFDR$_t$     | Skew-Student             | 20710.268      |
| APARCH(1,2)      | IR$_t$       | Skew-Student             | 20705.988      |
| APARCH(1,2)      | IFDR$_t$     | Skew-Student             | 20701.733      |
| APARCH(1,2)      | AbsIFDR$_t$  | Student                  | 21306.758*     |

Comparing all the models in Table 5 and 6, for $r_t$, FDR$_t$ and AbsFDR$_t$ we observe that models APARCH(1,2)* of Malaysian Ringgit and APARCH(1,2)* of Indonesia Rupiah indicate a better fit because of the maximum log-likelihood values are 29154.545 and 21306758 respectively. Next, the estimation and residuals analysis of both models are considered.

### Table 7. APARCH(1,2) Model and Residuals Analysis of the FDR$_t$ for MYR Based on Student Dist.

| Parameters | Coefficient | P-value |
|------------|-------------|---------|
| Cst(M)     | 0.0000      | 0.0045  |
| Cst(V)     | 0.0005      | 0.0000  |
| ARCH(Alpha1)| 0.1312     | 0.0004  |
| ARCH(Alpha2)| 0.0109     | 0.7507  |
| GARCH(Beta1)| 0.8707     | 0.0000  |
| APARCH(Gamma1)| 0.1090     | 0.2450  |
| APARCH(Gamma2)| -0.0683    | 0.5747  |
| APARCH(Delta)| **1.9387**| **0.0000**|
| Student(DF) | 6.0172      | 0.0000  |
| Q-Stat.(50) Std. Res. | 0.0292 | 1.0000 |
| Q-Stat.(50) Sqr. Std. Res. | 0.0268 | 1.0000 |
| Constraint | 1.0102      | >1      |
| log likelihood | **29154.5450** |        |
| Akaike      | -11.1733    |         |

The standardized and square standardized residuals results as shown in Table 7, imply that the APARCH (1,2) model have uncorrelated and homoscedastic standardized errors and this could be attributed to the role of FDR$_t$ in eliminating the series large noise signals.
Table 8. APARCH(1,2) Model and Residuals Analysis for Abs FDR\(_t\) of IDR Based on Student Dist.

| Parameters              | Coefficient | P-value |
|-------------------------|-------------|---------|
| Cst(M)                  | 0.0006      | 0.0000  |
| Cst(V)                  | 0.0010      | 0.0000  |
| ARCH(Alpha1)            | 0.1512      | 0.8181  |
| ARCH(Alpha2)            | 0.0615      | 0.7936  |
| GARCH(Beta1)            | 0.8441      | 0.0000  |
| APARCH(Gamma1)          | 0.3497      | 0.9026  |
| APARCH(Gamma2)          | -0.1073     | 0.9759  |
| APARCH(Delta)           | 1.8696      | 0.0000  |
| Student(DF)             | 5.0539      | 0.2542  |
| Q-Stat.(5) Std. Res.    | 163.6700    | 0.0000  |
| Q-Stat.(5) Sqr. Std. Res.| 17.3476    | 0.9999  |
| Constraint              | 1.0577      | >1      |
| log likelihood          | 21306.7580  |         |
| Akaike                  | -8.1534     |         |

The LM estimates as shown in Table 7 and Table 8, are 1.9387 and 1.8696 respectively and these values plays the role in a Box-Cox transformation of \(\sigma\) for each MYR and IDR respectively. Also, the estimates values (1.9387 and 1.8696) are greater than one but less than 2 and these indicate the APARCH model is appropriate instead of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. Similar results can be found in ([26],[27],[14],[15] and [4]). In addition, the findings confirm that asset returns are dependence across time and a stylized fact of financial returns which is called ‘volatility clustering’ exist. Volatility clustering are adequately captured in GARCH model and showed by the autocorrelations of conditional standard deviation, \(\sigma\) and as shown by the constraint, \(\alpha + \beta\) ([12] and [24]). In a similar way as observed in our results, both the leverage effect and LM are greater than zero and the values are 1.0102 (from Table 7) and 1.0577 (in Table 8) for each MYR and IDR respectively. These show that both MYR and IDR series are ILM and integrated process that decays very slowly as lag increases.

4. Conclusion

This study highlights the merit of new FDR\(_t\) over the ordinary returns for stylized fact of financial asset prices such as stock price, market index, exchange rate, and crude oil prices. To show that the introduced FDR\(_t\) is useful specifically for series that are deterministic in trend, ILM and volatile, we examine the parameters of APARCH model using the ordinary returns, FDR\(_t\) and the absolute FDR\(_t\) based on the assumption that each follows Gaussian, Student and skewed-student distributions. Unless stated, GARCH models are estimated using Berndt Hall Hall Hausman’s (BHHH) optimisation [28]. In contrast, we use software G@RCH 7.0 and Broyden Fletcher Goldfarb Shanno (BFGS)-Bounds (for sample size greater than 1000) estimation method algorithms.

The findings show that FDR\(_t\) is better in a number of ways. First, the FDR\(_t\) has minimum variability when the degree of fractional differencing is greater than one as indicated in the Malaysian and Indonesia exchange rate. Consequently, we conclude that it is convenient to use the traditional returns to transform the Singapore and Hong Kong series whereby the FDR\(_t\) was used for the transformation of Malaysia and Indonesian exchange rate.

Second, after absolute returns transformation, FDR\(_t\) indicates minimum risk reduction in Malaysian ringgit exchange rate trading compared to the \(r_t\). In addition, by using the FDR\(_t\), results indicate that investment in Indonesian economy has minimum risk as shown by coefficient of variation value of 238.52% compared to Malaysia and this is also supported by the high volatility indicated by excess kurtosis in the Malaysian exchange rate market for the period under study. Third, the serial correlation
analysis carried out indicates that APARCH model is better fit to the Malaysian Ringgit which is
due to the strength of FDR in eliminating huge noise signal as often observed in financial series.
Fourth, the FDR indicates that the kurtosis tend to increase or decrease if the data are over-differenced
or under-differenced respectively. As shown in this study when ordinary returns of financial asset are
estimated by using the natural logarithms they are similar to when the first differenced series are used
when actually they are supposed to be integrated between 0 and 1 or between 1 and 2. The evidence
are from differencing by using d and d+1. Fifth, comparisons between the rt, FDRt and AbsFDRt
suggest that instead of modelling the rt of a LM or ILM process, it is better in this case to model FDRt,
or AbsFDRt instead.

Also, both the FDRt and absolute FDRt reveals that financial returns particularly exchange rate are
leptokurtosis or having heavy tail and in the case of this work, returns are student-t distributed as
shown in evidence of skewness and excess kurtosis. Sixth, the results of Malaysia ringgit and
Indonesia rupiah exchange rates confirm that the degree of self-similarity or long memory between
FDRt and absolute FDRt respectively are greater than ordinary and square returns. Evidence from the
results is that FDRt indicates LM value as 1.9387, which is slightly higher than 1.8696, produced by
the absolute FDRt. This suggests that the latter captured less dependence in ordinary returns compared
to the FDRt. Also, the LM values obtained from the FDRt(s) contradicts the theory of Efficient Market
Hypothesis (EMH) that stock prices are adjusting quickly to new information. However this findings
are similar to the findings of [4] and also consistent with [24] and [12].

Finally, the FDRt is better estimator to the rt since it shows that Malaysian Ringgit and Indonesian
Rupiah are having higher volatility clustering, high or bad persistence process that have high
probability of producing large half-life of volatility and all these together expose the degree of
explosion of both currencies to dollar rate. Eventually, our work observes that volatility clustering is a
non-parametric phenomena which means that it can only be observed or ascertained base on the
magnitude of volatility or variability.

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