Non-CKM induced flavor violation in “minimal” SUSY SU(5) models

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Patterns of flavor violation induced by neutrino Yukawa couplings are discussed in realistic “minimal” SUSY SU(5) models, obtained by adding nonrenormalizable operators to the minimal one, in order to fix the fermion spectrum and suppress proton decay. Results are presented for the three possible implementations of the seesaw mechanisms, i.e. of Type I, II and III.

I. INTRODUCTION AND MOTIVATIONS

Supersymmetry (SUSY) is still one of the most interesting possibilities to solve the hierarchy problem of the standard model (SM) of particle physics. A solution to this problem without excessive tuning requires that the massive parameters breaking SUSY softly are around the TeV scale, which, for simplicity, is hereafter identified with the electroweak scale $M_{\text{weak}}$.

Irrespectively of the extensions needed to solve the hierarchy problem, the leptonic sector requires also an extension of the originally proposed SM structure of only three left-handed SU(2) doublets, in order to accommodate neutrino masses. One way to proceed is to introduce SM singlets, or right-handed neutrinos (RHNs), which can couple to the lepton doublets with Yukawa couplings of $O(1)$ if their Majorana masses are superheavy. This is the conventional and well-known seesaw mechanism, which enjoys immense popularity because of its elegance, but which is difficult to test experimentally. It is therefore very important to search for signals that can give information on the existence of the heavy particles realizing this mechanism. An obvious magnifying glass for them could be precisely their large Yukawa couplings to the left-handed leptons, $Y_{\nu}$, and the large leptonic mixing angles in the MNS matrix. Indeed, these couplings can affect sizably the renormalization group (RG) flow of the soft SUSY-breaking parameters for the sleptons $\tilde{m}_{\text{soft}}$ from the cutoff scale, at which the breaking of SUSY is mediated to the visible sector, $M_{\text{cut}}$, down to the seesaw scale $M_{\text{seesaw}}$. They lead to non-vanishing off-diagonal elements of the charged-slepton mass matrix at $M_{\text{weak}}$, or lepton-flavor violations (LFVs) in the left-left sector of this matrix, $\tilde{m}^{\nu}_{\text{L-L}}$. The existence of intrinsic flavor violations in the slepton mass parameters at $M_{\text{cut}}$, however, could completely obscure the effects of the RHN interactions through RG equations (RGEs). Thus, we restrict ourselves to considering models with flavor-blind SUSY breaking and mediation of this breaking.

If in addition, we embed these SUSY models in a grand unified theory (GUT), the RHNs interact with these large Yukawa couplings also with the right-handed down quarks, which are the SU(5) partners of the doublet leptons. Hence, as pointed out by Moroi 2, these interactions can affect also the massive soft parameters of the down-squark sector, generating quark-flavor violations (QFVs) in the scalar sector different from those induced by the quark Yukawa couplings. In particular, in the superCKM basis for quark superfields, the scalar QFVs due to the RHNs are in the right-right sector of the down-squark mass matrix, $m_{\text{RR}}^2$, whereas those induced by the quark Yukawa couplings in non-GUT setups are in the left-left one $\tilde{m}_{\text{LL}}$ (GUT phases also appear when identifying the SM fields among the components of the SU(5) multiplets. Here, we neglect them altogether, postponing the discussion of their effect to a later occasion.) Thus, it has been argued that, in SUSY SU(5) models with RHNs and flavor-blind soft massive parameters at $M_{\text{cut}}$, scalar LFVs and QFVs at $M_{\text{weak}}$ are related to each other in a simple way.

The minimal model, however, is not realistic: it predicts a too rapid proton decay and the wrong relation between the down-quark mass matrix and the charged-lepton’s one. New physics beyond that of the minimal SUSY SU(5) model is needed to cure these problems and it is easy to imagine that such additional degrees of freedom can modify even drastically the simple relations between LFVs and QFVs of Ref. 2, and of many successive works. We refer to these relations as Moroi’s predictions. As is well known, one way to fix the incorrect fermion spectrum consists in the introduction of nonrenormalizable operators (NROs), suppressed by $1/M_{\text{cut}}^4$. The effects on flavor violation of only one such NRO of dimension-five (sufficient for the purpose) were studied in Ref. 3. They amount to introducing some arbitrariness in the choice of the flavor rotations of the SM fields when they are embedded in the SU(5) multiplets. This is expressed by the appearance of two additional unitary matrices (other than the RGE-evolved CKM and MNS ones), with arbitrary mixings among the first two generations, but with smaller ones among the third and the second/first generations. In the parameter space of mixings/phases opened up by the introduction of this NRO, there is however still a region in which these uni-

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tary matrices of additional mixings reduce to the unit matrix. In this region, the pattern of intergenerational sfermion mixings remains unchanged with respect to that obtained without NROs, i.e. Moroi’s predictions for flavor transitions, then, can be kept as viable. The authors of Ref. 3, however, did not discuss the problem of a too-large decay rate of the proton, induced by the exchange of colored Higgs fields. One way to suppress it, compatible with their analysis, is to assume that there exist other NROs, also suppressed by $1/M_{\text{cut}}$, that are baryon-number violating and that cancel (up to experimentally tolerable remnants) the colored-Higgs-fields induced operators responsible for proton decay. These, indeed, are dimensionally suppressed by the inverse mass of the colored Higgs fields, supposed to be larger than $1/M_{\text{cut}}$, but are also further suppressed by coefficients depending on small Yukawas and small CKM mixing angles. Hence, the cancellation is expected to be possible, and the model of Ref. 3 can be made realistic with some tuning (this is in addition to the intrinsic tuning required in this model for the doublet-triplet mass splitting). It remains, however, to be checked whether the parameter space of additional mixings/ phases relevant for flavor transitions remains unchanged for all values of $\tan \beta$, once this cancellation is enforced.

As outlined in Ref. 3, a technically different way to suppress proton decay becomes possible, if the number of NROs employed to fix the fermion spectrum is enlarged. It was shown by the authors of Ref. 3 that even only the addition of four NROs of dimension five is sufficient to introduce enough SU(5)-breaking effects to disentangle the Yukawa couplings contributing to the coefficient of the effective operators responsible for proton decay from the couplings giving rise to fermion masses and mixings. At the expenses of some tuning, then, it is possible to make these effects large enough to reduce the rate for proton decay below experimental limits, even for colored Higgs fields with mass of $O(M_{\text{GUT}})$, where $M_{\text{GUT}}$ is the so-called GUT scale. An enlargement of the number of NROs allows even more freedom to achieve this suppression.

Motivated by these considerations, we try to go one step further and study the relations between LFVs and QFVs in realistic “minimal” SUSY SU(5) models, with up to an infinite number of NROs added to the minimal SU(5) structure. These models share with the truly minimal one the fact that the Higgs sector solely responsible for the breaking of the SU(5) and the SM symmetries is given by the three Higgs multiplets $5_H$, $\overline{5}_H$, and $24_H$, with superpotential:

$$W_H = 5_H (M_5 + \lambda_5 24_H) \overline{5}_H + \frac{1}{2} M_{24} 24_H^4 + \frac{1}{6} \lambda_{24} 24_H^3.$$  

We remind here that $5_H$ and $\overline{5}_H$ contain the two weak Higgs doublets $H_u$ and $H_d$ of the minimal supersymmetric SM, and two color triplets, $H_U^c$ and $H_D^c$, i.e. $5_H = \{H_U^c, H_u\}$ and $\overline{5}_H = \{H_D^c, H_d\}$. The $24_H$ has among its components $G_H$, $W_H$ and $B_{ij}$, which are adjoint fields of SU(3), SU(2) and U(1), respectively. It contains also the vector-like pair $X_H$ and $\overline{X}_H$, with $X_H$ ($\overline{X}_H$) a triplet (antitriplet) of SU(3) and a doublet of SU(2). The SM quark and lepton fields $Q$, $U^c$, $D^c$, $L$, and $E^c$ are collected in the two matter multiplets $10_M = \{Q, U^c, E^c\}$ and $5_M = \{D^c, L\}$, with one replica of them for each generation, interacting according to

$$W_M = \sqrt{2} 5_M Y^{510} \overline{5}_H - \frac{1}{2} 10_M Y^{1010} \overline{5}_H. \quad (2)$$

Apart from the obvious extensions needed to accommodate neutrino masses, these models differ from the truly minimal one for the addition of NROs. We treat them in as much generality as it is possible, for example by including practically all classes of those needed for the fermion spectrum, of all dimensions, since we find that the problem is actually technically manageable. This, however, does not exclude that some of these coefficients are accidentally vanishing. In this sense, if enough NROs explicitly violating baryon number are introduced to suppress proton decay in the way outlined above, also the model of Ref. 3, with only one NRO used to fix the fermion spectrum, becomes part of this class of models. We refrain from studying here the flavor predictions for this modification of the model of Ref. 3, but we restrict ourselves to models in which the suppression of the proton-decay rate is achieved with a procedure of the type outlined in Ref. 3. Interestingly, this procedure is predictive. Since it involves a specific flavor ansatz for the Yukawa couplings mediating the proton decay rate, it fixes some of the additional mixings obtained in Ref. 3: it leaves Moroi’s predictions for flavor transitions between sfermions in the $\bar{5}_M$ representation of SU(5) unchanged, while induces modifications for those in the $10_M$ representations. As for flavor transitions in the $\bar{5}_M$ sector, we try to investigate what other type of ultraviolet physics may affect them. One obvious way to do that is to implement possible different types of the seesaw mechanism. We review them in Sec. 11. Another way is to disentangle the cutoff scale from the reduced Planck mass, $M_P$, by taking it as an adjustable scale varying from $M_P$ and $M_{\text{GUT}}$. Values of $M_{\text{cut}}$ below $M_P$ are for example typical of models with gauge mediation of SUSY breaking; they can occur also when the “minimal” models are embedded in higher-dimensional setups 8. We show some results in Sec. 11 after having specified the value of parameters used in this analysis.

![FIG. 1: The seesaw mechanism.](image-url)
II. SEESAW MECHANISM

The seesaw mechanism is a mechanism to generate the effective dimension-five operator for neutrino masses, $LH_uLH_u$, by integrating out heavy degrees of freedom at the scale $M_{\text{seesaw}}$. It is depicted schematically in Fig. 1. In this figure, a solid (broken) line indicates a fermion (boson) or, in a supersymmetric context, a superfield with an odd (even) $R$-parity. At the tree level, there are only two diagrams that can give rise to the effective seesaw mechanism, one mediated by a solid line and one by a broken line. At first glance, it might seem that the inner line, representing the mediator $M$, can be a singlet or triplet of SU(2) in both cases. In reality, the possibility of the singlet scalar is forbidden by the multiplication rule of SU(2); $2 \times 2 = 1_A + 3_S$, where the indices $A$ and $S$ indicate an anti-symmetric and symmetric product, respectively. Thus, there are only three types of seesaw mechanism, distinguished by the nature of the mediator, which can be an SU(2)

| singlet fermion | Type I |
|-----------------|--------|
| triplet scalar  | Type II|
| triplet fermion | Type III|

i.e. the RHNs $N^c$, a triplet Higgs $T$ and what we call matter triplets $W_M$, respectively.

Their interactions with the SU(2) lepton doublets are

$$N^cY^{1}_{1}LH_u, \quad \frac{1}{2}LY_{1}^{11}TL, \quad \sqrt{2}H_uW_MY^{11}_{\nu}L.$$ (3)

Integrating out the mediators and replacing $H_u$ by its vev $v_u$, we obtain the effective neutrino mass matrices:

$$m_\nu = \begin{cases} 
(Y^{11}_{\nu})^T \frac{1}{M^{11}_{M}} \left( Y^{11}_{\nu} \right) v_u^2 \\
Y^{11}_{\nu} \frac{\lambda_U}{M^{11}_{M}} v_u^2
\end{cases}$$ (4)

in the three cases. Here $M^{11}_{M}$ are mass matrices whereas $M^{11}_{M}$ is a number, and $\lambda_U$ is the coupling of $H_uTH_u$. In Type II, because the mediator has no flavor, the high-energy input in the neutrino mass matrix is just a number, i.e. the ratio $\lambda_U/M^{11}_{M}$, and the flavor structure of $Y^{11}_{\nu}$ is the same as that of the neutrino mass:

$$Y^{11}_{\nu} = \frac{1}{v_u} V_{MNS}^* m_\nu V_{MNS}^T \frac{M^{11}_{M}}{\lambda_U},$$ (5)

where $m_\nu$ is the diagonal form of $m_\nu$, and $V_{MNS}$ is the MNS matrix including here two Majorana phases. This is in great contrast with the Type I and III, in which the mediators carry flavor indices. In these cases, the flavor structure of $Y^{111}_{\nu}$ is different from that of $m_\nu$, and there is a large number of high-energy parameters contributing to the neutrino mass matrix, which can be expressed in terms of the three eigenvalues of $M^{111}_{M}$, $(M^{111}_{M})_{ii}$, and an arbitrary complex orthogonal matrix $R$ [4]:

$$\left( Y^{111}_{\nu} \right)^T = \frac{1}{v_u} V_{MNS}^* m_\nu V_{MNS}^T \sqrt{M^{111}_{M}} R.$$ (6)

TABLE I: The SU(5) Yukawa interactions of the seesaw mediators, together with their SM decompositions, and the expected patterns of flavor violations are listed.

| Type I | Type II | Type III |
|--------|---------|----------|
| $N^c$  | $15_H$  | $24_M$   |
| interaction | $N^c5_{M5H}$ | $5_{M15_{H5M}}$ | $5_{H24_{M5M}}$ |
| only LFV | $N^cLH_u$ | $LTL$ | $H_uW_M L, H_uB_M L$ |
| LFV & QFV | $D^\nu LQ_{15}$ | $D^\nu SD^\nu$ | $H_uX_M D^\nu$ |
| only QFV | $N^cD^\nu H_U^5$ | $D^\nu SD^\nu$ | $H_uX_M D^\nu$ |
| LFV/QFV | $>1$ | $\sim 1$ | $\sim 1$ |

Notice also that in these two cases, $m_\nu$ is quadratic in $Y^{111}_{\nu}$, whereas in the Type II seesaw it is linear in $Y^{111}_{\nu}$.

When embedded in an SU(5) GUT, the multiplets containing these mediators are matter singlets, $N^c$, in the case of the Type I seesaw, a Higgs field in a 15plet, $15_H$, in Type II, and finally in Type III, adjoint matter fields, $24_M$. The Yukawa interactions in Eq. (3) become now

$$-N^cY^{111}_{I}N^{5}_{M5H}, \quad \frac{1}{\sqrt{2}}Y^{111}_{II}15_{H5M}, \quad 5_{H24_{M5M}}Y^{1111}_{II5M},$$ (7)

which contain many more SM interactions than those listed in Eq. [3]. ($Y^{1111}_{I}$ and $Y^{1111}_{II}$ differ by phase factors, as discussed in Ref. [5].) As anticipated in the introduction, then, the large off-diagonal entries in $Y_\nu$ can affect not only the leptonic sector, but also the hadronic one. Indeed, the SM decomposition of the interactions in Eq. (7) is given in Table I. The SM interactions are accommodated in different lines depending on whether they give rise to off-diagonal terms in the left-right sector of the charged-slepton mass matrix, or in both. The fields $Q_{15}$ and $S$ in the column “Type II” and $B_M, X_M, X_M$ and $G_M$ in the column “Type III” are the SU(5) partners of the Higgs triplets $T$ and of the triplet fermion $W_M$, respectively. It should be noticed here that the colored Higgs field $H_U^5$ decouples at $M_{GUT}$, which is at least two orders of magnitude larger than $M_{\text{seesaw}}$, where $N^c, 15_H$, and $24_M$ are integrated out. Therefore, below $M_{GUT}$, only the interactions without $H_U^5$ remain active. Thus, in the Type I seesaw, LFVs in the scalar sector are in general larger than QFVs, as the interaction $N^cD^\nu H_U^5$ decouples earlier than $N^cLH_u$. In contrast, in the Type II seesaw, LFVs and QFVs are of the same order up to sub-leading SU(5)-breaking effects in the RG flows below $M_{GUT}$. This is simply due to the fact that the full SU(5) interaction remains active down to $M_{\text{seesaw}}$. As for the Type III, because two of the interactions inducing LFVs and one of those inducing QFVs survive between $M_{GUT}$ and $M_{\text{seesaw}}$, the relations between LFVs and QFVs depend on group-theoretical factors. An explicit calculation...
shows that their magnitudes are of the same order. The situation is summarized in the last line of Table I.

III. ANALYSIS AND SUMMARY

We summarize the choice of parameters made for our analysis. The cutoff scale $M_{\text{cut}}$ is varied from $M_{\text{GUT}}$ to $M_P = 2.4 \times 10^{16}$GeV. Of the four parameters in Eq. (1), two are needed to fix $M_{\text{GUT}}$ and the mass of the colored Higgs fields $H^U_D$ and $H^D_D$. We take both these parameters to be $2 \times 10^{16}$GeV. This choice is consistent with the unification of gauge couplings and with the bounds coming from the proton-decay rate $\beta$. One remaining parameter of the four in Eq. (1) is needed to finetune the electroweak scale: the fourth is free. We choose this to be $\lambda_{34}$. Throughout our analysis we take this to be of $O(1)$. In particular, in the plots that we show here, it is fixed to be 1/2. As for the parameters of the Type II seesaw, we set $M_U = 1/2$, and $M^H = M_{\text{seesaw}} = 10^{14}$GeV. For the Type I and III, we take the $R = 1$, and similarly $M^H = M_{\text{seesaw}}$, with the same value of $M_{\text{seesaw}}$ used for the Type II. In the light-neutrino sector, we adopt the normal hierarchy of masses. The mixing angle $\theta_{13}$ and all three phases of $V_{\text{MNS}}$ are set to zero. As for the soft SUSY-breaking parameters, we go beyond flavor blindness and assume universality at $M_{\text{cut}}$, as usually done in these analyses. We fix the gaugino mass, $M_{1/2}$, the common scalar mass, $\tilde{m}_0$, and the common proportionality constant in the trilinear couplings, $A_0$, to be 1 TeV. Finally we take $\tan \beta = 10$.

We are now in a position to show some results. We solve the RGEs from $M_{\text{cut}}$ to $M_{\text{weak}}$, reported in Ref. [5], for the entries (2,3) in the mass matrices $\tilde{m}^2_{e RR}$ and $\tilde{m}^2_{e LL}$, for the three possible implementation of the seesaw mechanism, and for different values of $M_{\text{cut}}$. We plot in Fig. 2 the absolute value of the ratio of these entries as a function of $M_{\text{cut}}$. The three different lines of dots correspond to the three different types of seesaw mechanism. As foreseen in Sec. II the mixing (2,3) induced in $\tilde{m}^2_{e LL}$ is larger than that in $\tilde{m}^2_{e RR}$ induced by the same neutrino Yukawa coupling in the seesaw of Type I. See lower line of black dots in this figure. As also expected, the down-squark mixing decreases when $M_{\text{cut}}$ approaches $M_{\text{GUT}}$ as the interval in which this mixing is induced becomes shorter. The two upper lines of red and green dots show the results obtained for the seesaw mechanisms of Type II and III, in agreement with the expectations discussed in Sec. II. The results shown in this figure remain pretty much unchanged for different choices of the GUT parameters, soft SUSY-breaking parameters, and type of neutrino mass-hierarchy chosen. They are obtained using a flavor ansatz as in Ref. [5], to suppress proton decay, having used an unlimited number of NROs to fix the fermion spectrum [7]. As explained in the introduction, they are consistent with the predictions by Moroi for the seesaw of Type I, with $M_{\text{cut}} = M_P$. We note, however, that the analysis of Ref. [3] would give results for the ratio of the (2,3) elements of $\tilde{m}^2_{e RR}$ and $\tilde{m}^2_{e LL}$ in general plagued by the uncertainty of additional mixings/phases (uncertainty possibly reduced when suppressing proton decay in the way outlined in the introduction). In summary, we conclude this section, with the observation that flavor transitions, do depend, in general, on the detailed implementations of NROs used to cure the problem of the minimal SUSY SU(5) model.

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[1] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986).
[2] T. Moroi, JHEP 0003, 019 (2000) and Phys. Lett. B 493, 368 (2000).
[3] S. Baek, T. Goto, Y. Okada and K. i. Okumura, Phys. Rev. D 64, 095001 (2001).
[4] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).
[5] D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B 661, 62 (2003).
[6] N. Haba and T. Ota, arXiv:hep-ph/0608244
[7] F. Borzumati, S. Mishima and T. Yamashita, to appear.
[8] H. Itoh, N. Okada and T. Yamashita, Phys. Rev. D 74, 055005 (2006).
[9] J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001).