On the Secrecy Outage Capacity of Physical Layer Security in Large-Scale MIMO Relaying Systems with Imperfect CSI

Xiaoming Chen†,‡, Lei Lei‡, Huazi Zhang* and Chau Yuen*
† College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, China.
‡ National Mobile Communications Research Laboratory, Southeast University, China.
* Department of ECE, North Carolina State University, USA.
Email: {chenxiaoming,leilei}@nuaa.edu.cn, hzhang17@ncsu.edu, yuenchau@sutd.edu.sg

Abstract—In this paper, we study the problem of physical layer security in a large-scale multiple-input multiple-output (LS-MIMO) relaying system. The advantage of LS-MIMO relaying systems is exploited to enhance both wireless security and spectral efficiency. In particular, the challenging issue incurred by short interception distance is well addressed. Under very practical assumptions, i.e., no eavesdropper’s channel state information (CSI) and imperfect legitimate channel CSI, this paper gives a thorough investigation of the impact of imperfect CSI in two classic relaying systems, i.e., amplify-and-forward (AF) and decode-and-forward (DF) systems, and obtain explicit expressions of secrecy outage capacities for both cases. Finally, our theoretical claims are validated by the numerical results.

I. INTRODUCTION

The open nature of wireless channel gives rise to many wireless security problems. Traditionally, wireless security is guaranteed through high layer encryption. With the development of interception technology, encryption becomes more complex, leading to high computation burden. Thanks to the physical layer security measures enlightened by information theory, we may exploit wireless channel characteristics to guarantee secure communications even without encryption.

The performance of physical layer security is measured by secrecy rate, namely the capacity difference between the legitimate channel from the information source to the destination and the eavesdropper channel from the information source to the eavesdropper [2] [3]. As expected, the introduction of relay into physical layer security can improve the legitimate channel capacity through cooperative diversity, and thus enhances transmission security [4]. Some feasible relaying schemes and their performances were discussed in [5]. Relay positions were then optimized from the perspective of minimizing the interception probability in [6]. Further, best relay selection was proposed to suppress the interception probability by exploiting selective gain in multiple relay systems [7]. Additionally, multiple relay cooperative beamforming combined with jamming was also adopted to maximize the secrecy rate [8]. In fact, if the relay is equipped with multiple antennas, more effective relaying strategy can be used to optimize the secrecy rate [9]. Linear precoding schemes were investigated in a MIMO relay network assuming global CSI [10]. Nevertheless, due to the fact that the eavesdroppers are usually well hidden, it is difficult for the relay to obtain the knowledge of the eavesdropper channel. Therefore, it is difficult to realize absolutely secure communications over the fading channel. Under this condition, the conception of secrecy outage capacity was adopted to guarantee a secure communication with a high probability [11] [12].

A challenging problem rises when the eavesdropper is close to the information transmitter. Even with a multi-antenna relay, the secrecy outage capacity is too small to fulfill the requirement of quality of service (QoS), due to the relatively high quality of intercepted signal. Recently, it is found that large-scale MIMO (LS-MIMO) systems can significantly improve the transmission performance by utilizing its enormous array gain [13] [14]. Inspired by this, we propose the use of an LS-MIMO relay to enhance wireless security. Note that a critical issue of the LS-MIMO system is that channel state information (CSI) may be imperfect due to duplex delay in time division duplex (TDD) systems, resulting in inevitable performance loss. In this paper, we focus on the performance analysis in terms of secrecy outage capacity in an LS-MIMO relay system with imperfect CSI under the amplify-to-forward (AF) and the decode-to-forward (DF) relaying strategies, respectively. The contributions of this paper are two-fold:

1) To the best of our knowledge, we are the first to introduce LS-MIMO into a relay system, and solve the challenging problem of short-distance interception.
2) We derive explicit expressions of secrecy outage capacity for AF and DF relaying systems, and then provide some guidelines for performance optimization.

This work was supported by the National Natural Science Foundation of China (No. 61301102, 6110195), the Natural Science Foundation of Jiangsu Province (No. BK20130820), the open research fund of National Mobile Communications Research Laboratory, Southeast University (No. 2012D16), the Doctoral Fund of Ministry of Education of China (No. 20123218120022) and the iTrust.
The rest of this paper is organized as follows. We first give an overview of the LS-MIMO relaying system employing physical layer security in Section II, and then derive the secrecy outage capacities based on AF and DF relaying strategies with imperfect CSI in Section III. In Section IV, we present some numerical results to validate the effectiveness of the proposed scheme. Finally, we conclude the whole paper in Section V.

II. SYSTEM MODEL

We consider a time division duplex (TDD) LS-MIMO relaying system, including one source, one destination equipped with a single antenna each and one relay deploying \( N_R \) antennas in presence of a passive single antenna eavesdropper, as shown in Fig. 1. It is worth pointing out that \( N_R \) is usually quite large in such an LS-MIMO relaying system, i.e. \( N_R = 100 \) or greater. The system works in the half-duplex mode, so the information transmission from the source to the destination via the aid of the relay requires two time slots. Specifically, in the first time slot, the source transmits the signal to the relay, and then the relay forwards the post-processing signal to the destination during the second time slot. Note that the direct link from the source to the destination is unavailable due to long distance. Meanwhile, the eavesdropper monitors the transmission from the relay to the destination, and tries to intercept the signal. Following [7], we also assume that the eavesdropper is out of the coverage area of the source, since it thought the information comes from the relay.

We use \( \sqrt{\alpha_{i,j}} h_{i,j} \) to denote the channel from \( i \) to \( j \), where \( i \in \{ S, R \}, j \in \{ R, D, E \} \) and \( S, R, D, E \) represent the source, the relay, the destination and the eavesdropper, respectively. \( \alpha_{i,j} \) is the distance-dependent path loss and \( h_{i,j} \) is the channel small scale fading. In this paper, we model \( h_{i,j} \) as Gaussian distribution with zero mean and unit variance. \( \alpha_{i,j} \) remains constant during a relatively long period and \( h_{i,j} \) fades independently slot by slot. Thus, the receive signals at the relay in the first time slot can be expressed as

\[
y_R = \sqrt{P_S \alpha_{S,R} h_{S,R}^H s + n_R},
\]

where \( s \) is the normalized Gaussian distributed transmit signal, \( P_S \) is the transmit power at the source, \( n_R \) is the additive Gaussian white noise with zero mean and unit variance at the relay. Through post-processing to \( y_R \) according to the CSI \( h_{S,R} \) and \( h_{R,D} \), the relay forwards a normalized signal \( r \) to the destination with power \( P_R \), then the received signals at the destination and the eavesdropper are given by

\[
y_D = \sqrt{P_R \alpha_{R,D} h_{R,D}^H r + n_D},
\]

and

\[
y_E = \sqrt{P_R \alpha_{R,E} h_{R,E}^H r + n_E},
\]

respectively, where \( n_D \) and \( n_E \) are the additive Gaussian white noises with zero mean and unit variance at the destination and the eavesdropper.

Assuming the legitimate channel and the eavesdropper channel capacities are \( C_D \) and \( C_E \), from the perspective of information theory, the secrecy capacity is given by \( C_{SEC} = [C_D - C_E]^+ \), where \([x]^+ = \max(x, 0)\) [2]. Since there is no knowledge of the eavesdropper channel at the source and relay, it is impossible to provide a steady secrecy capacity. In this paper, we take the secrecy outage capacity \( C_{SOC} \) as the performance metric, which is defined as the maximum rate under the condition that the outage probability that the transmission rate surpasses the secrecy capacity is equal to a given value \( \varepsilon \), namely

\[
P_r(C_{SOC} > C_D - C_E) = \varepsilon.
\]

III. SECRECY OUTAGE CAPACITY ANALYSIS

In this section, we concentrate on the analysis of secrecy outage capacity of physical layer security in an LS-MIMO relaying system. Considering that AF and DF are two commonly used relaying strategies, we study the two cases in sequence.

A. Amplify-and-Forward (AF) Case

In this case, the relay forwards the signal \( r^{AF} \) via multiplying the received signal \( y_R \) by a \( N_R \times N_R \) processing matrix \( F \), namely

\[
r^{AF} = F y_R.
\]

We assume the relay has full CSI \( h_{S,R} \) by channel estimation, and gets partial CSI \( h_{R,D} \) by channel reciprocity in TDD systems. Due to duplex delay between uplink and downlink, there is a certain degree of mismatch between the estimated CSI \( h_{R,D} \) and the real CSI \( h_{R,D} \), whose relation can be expressed as [15]

\[
h_{R,D} = \sqrt{\rho} h_{R,D} + \sqrt{1 - \rho} e,
\]

where \( e \) is the error noise vector with independent and identically distributed (i.i.d.) zero mean and unit variance complex Gaussian entries. \( \rho \), scaling from 0 to 1, is the correlation coefficient between \( h_{R,D} \) and \( h_{R,D} \). A large \( \rho \) means better CSI accuracy. If \( \rho = 1 \), the relay has full CSI \( h_{R,D} \). Additionally, due to the hidden property of the eavesdropper, the CSI \( h_{R,E} \) is unavailable. Therefore, \( F \) is designed only based on \( h_{S,R} \) and \( h_{R,D} \), but is independent of
we have the following theorem:

Based on the receive SNR in (9), we have the following system. Similar to (9), the SNR at the eavesdropper is given estimated as

Thus, the received signal at the destination and the corresponding signal-to-noise ratio (SNR) can be expressed as

\[
y^A_F = \frac{\sqrt{P_S P_{R \alpha_{S,R}, R,D} \| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2}}{\| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2 + 1} \| \hat{h}_{S,R} \|^2
\]

and

\[
\gamma^A_F = \frac{P_S P_{R \alpha_{S,R}, R,D} \| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2}{\| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2 + 1} \frac{P_{R \alpha_{S,R}, R,D} \| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2}{\| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2 + 1} + 1
\]

where \( a = P_S P_{R \alpha_{S,R}, R,D} \), \( b = P_{R \alpha_{S,R}, R,D} \) and \( c = P_{S \alpha_{S,R,R}, R,D} \).

Based on the receive SNR in (9), we have the following theorem:

**Theorem 1:** The legitimate channel capacity in an LS-MIMO relaying system in presence of imperfect CSI can be approximated as

\[
C^F_{S,R} = W \log_2 \left( 1 + P_{R \alpha_{S,R}, R,D} \| \hat{h}_{R,D} \|^2 / \| \hat{h}_{R,D} \|^2 \right)
\]

where \( W \) is the spectral bandwidth.

**Proof:** Please refer to Appendix I.

It is found that the legitimate channel capacity is a constant due to channel hardening in such an LS-MIMO relaying system. Similar to (9), the SNR at the eavesdropper is given by

\[
\gamma^E_F = \frac{d \| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2}{e \| \hat{h}_{S,R} \|^2 + \| \hat{h}_{R,D} \|^2 \| \hat{h}_{S,R} \|^2 + 1},
\]

where \( d = P_S P_{R \alpha_{S,R}, R,E} \) and \( e = P_{R \alpha_{S,R}, R,E} \). Hence, according to the definition of secrecy outage capacity in (4), we have the following theorem:

**Theorem 2:** Given the outage probability bound by \( \varepsilon \), the secrecy outage capacity based on the AF relaying strategy is

\[
C^F_{SOC} = C^F_D - W \log_2 \left( 1 + \frac{\ln \varepsilon}{1 - e^{-CN_{E,R}}} \right)
\]

where \( W \) is the spectral bandwidth.

**Proof:** Please refer to Appendix II.

Based on Theorem 2, we may obtain the interception probability \( P^F_0 \), namely the probability that the legitimate channel capacity is less than the eavesdropper channel capacity. By letting \( C^F_{SOC} = 0 \) in (20), we have

\[
P^F_0 = 1 - P \left( 2^{C^F_{SOC}/W} - 1 \right)
\]

B. Decode-and-Foward (DF) Case

Different from the AF relaying strategy, the DF decodes the receive signal at the relay, and then forwards the original signal to the destination, so as to avoid noise amplification. Also by using the MRC technique at the relay, the channel capacity from the source to the relay can be expressed as

\[
C_{S,R}^D = W \log_2 \left( 1 + P_{S \alpha_{S,R}, R,D} \| \hat{h}_{S,R} \|^2 / \| \hat{h}_{S,R} \|^2 \right)
\]

where (12) holds true based on channel hardening when \( N_R \to \infty \). Then, the relay performs MRT based on the estimated CSI \( \hat{h}_{R,D} \). The channel capacity from the relay to the destination is given by

\[
C_{R,D}^D = W \log_2 \left( 1 + P_{R \alpha_{S,R}, R,D} \| \hat{h}_{R,D} \|^2 / \| \hat{h}_{R,D} \|^2 \right)
\]

where (13) is obtained similarly to Theorem 1. Thus, the legitimate channel capacity under the DF relaying strategy can be expressed as

\[
C^D_{D} = \min(\gamma^D_F, \gamma^D_E)
\]

where \( W \) is the spectral bandwidth.

It is found that \( C^D_{D} \) is also a constant due to channel hardening in an LS-MIMO system. Meanwhile, the eavesdropper intercepts the signal from the relay, the corresponding channel capacity can be computed as

\[
C_{E}^D = W \log_2 \left( 1 + P_{R \alpha_{S,R}, E} \| \hat{h}_{R,E} \|^2 / \| \hat{h}_{R,E} \|^2 \right)
\]

For the secrecy outage capacity in an LS-MIMO DF relaying system, we have the following theorem:

**Theorem 3:** Given the outage probability bound by \( \varepsilon \), the secrecy outage capacity based on the DF relaying strategy is

\[
C_{SOC}^D = C^D_D - W \log_2 \left( 1 + P_{R \alpha_{S,R}, E} \ln \varepsilon \right)
\]

**Proof:** Please refer to Appendix III.

Similarly, we can also obtain the interception probability in this case. Let \( C_{SOC}^D = 0 \) in (25), we have

\[
P^D_0 = P \left( \frac{2^{C^D_{SOC}/W} - 1}{P_{R \alpha_{S,R}, E}} \right)
\]
IV. NUMERICAL RESULTS

To examine the accuracy of the derived theoretical expressions of the secrecy outage capacity for the LS-MIMO AF and DF relaying systems, we present several numerical results in the following scenarios: we set $N_R = 100$, $W = 10$ KHz and $\rho = 0.9$. The relay is in the middle of a line between the source and the destination. We normalize the path loss as $\alpha_{S,R} = \alpha_{R,D} = 1$ and use $\alpha_{R,E}$ to denote the relative path loss. For example, if $\alpha_{R,E} > 1$, then the eavesdropper is closer to the relay than the destination. In addition, we use $\text{SNR}_S = 10\log_{10}P_S$ and $\text{SNR}_R = 10\log_{10}P_R$ to represent the transmit signal-to-noise ratio (SNR) in dB at the source and the relay, respectively.

Firstly, we test the accuracy of the theoretical expression in AF relaying mode with $\text{SNR}_s = \text{SNR}_R = 20$ dB. As seen in Fig.2, the theoretical results are well consistent with the simulations in the whole $\alpha_{R,E}$ region with different outage probability requirements, which proves the high accuracy of the derived performance expressions. Given the outage probability bound by $\epsilon$, as $\alpha_{R,E}$ increases, the secrecy outage capacity decreases gradually, this is because the interception ability of the eavesdropper enhances due to the short interception distance. In addition, given $\alpha_{R,E}$, the secrecy outage capacity improves with the increase of $\epsilon$, since the outage probability is an increasing function of the secrecy outage capacity.

Secondly, we show the impact of $\text{SNR}_R$ on the secrecy outage capacity in AF relaying mode with $\epsilon = 0.01$, $\alpha_{R,E} = 1$, and $\text{SNR}_S = 20$ dB. As seen in Fig.3 the secrecy outage capacity is not an increasing function of $\text{SNR}_R$, since both the legitimate and eavesdropper channel capacities improve as $\text{SNR}_R$. Thus, it makes sense to find the optimal $\text{SNR}_R$ to maximize the secrecy outage capacity in LS-MIMO relaying systems.

Thirdly, we investigate the impact of $\rho$ on the secrecy outage capacity in AF relaying mode with $\text{SNR}_s = \text{SNR}_R = 20$ dB and $\epsilon = 0.01$. The CSI mismatch will result in the performance loss. However, in LS-MIMO relay systems, when the number of antennas is quite large, the impact of CSI mismatch can be weakened due to high spatial resolution. As shown in Fig.4, the performance loss by reducing $\rho$ from 1 to 0.8 is slight. In other words, the AF relaying scheme is insensitive to the CSI accuracy.

Then, we test the accuracy of the derived theoretical expressions based on DF relaying mode with $\text{SNR}_s = \text{SNR}_R = 20$ dB. As seen in Fig.5 the theoretical results coincide with the simulations nicely. Similar to the AF relaying mode, the secrecy outage capacity decreases as $\alpha_{R,E}$ increases and $\epsilon$ reduces. Note that, compared to the secrecy outage capacity of AF relaying mode in Fig.2, the secrecy outage capacity of DF relaying mode is better under the same conditions, since the DF mode avoids amplifying the noise at the relay.

Next, we investigate the impact of $\text{SNR}_R$ of the secrecy
outage capacity in DF relaying mode with $\varepsilon = 0.01$, $\alpha_{R,E} = 1$, and $\text{SNR}_S = 20\text{dB}$. From Fig. 5 as $\text{SNR}_R$ increases, unexpectedly the secrecy outage capacity decreases. This is because the legitimate channel capacity is independent of $\text{SNR}_R$ when $\text{SNR}_R$ is large, but the eavesdropper channel capacity is an increasing function of $\text{SNR}_R$. Hence, it is also necessary to choose an optimal $\text{SNR}_R$ to optimize the performance.

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.

APPENDIX A

PROOF OF THEOREM 1

Based on the SNR $\gamma_{AF}^{R,D}$ at the destination, the legitimated channel capacity can be expressed as

$$C_{AF}^{R,D} = W \log_2 \left( 1 + a \rho \frac{\|h_{R,D} \|^2}{\|h_{S,R} \|^2} \right)$$

$$(17)$$

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.

APPENDIX A

PROOF OF THEOREM 1

Based on the SNR $\gamma_{AF}^{R,D}$ at the destination, the legitimated channel capacity can be expressed as

$$C_{AF}^{R,D} = W \log_2 \left( 1 + a \rho \frac{\|h_{R,D} \|^2}{\|h_{S,R} \|^2} \right)$$

$$(17)$$

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.

APPENDIX A

PROOF OF THEOREM 1

Based on the SNR $\gamma_{AF}^{R,D}$ at the destination, the legitimated channel capacity can be expressed as

$$C_{AF}^{R,D} = W \log_2 \left( 1 + a \rho \frac{\|h_{R,D} \|^2}{\|h_{S,R} \|^2} \right)$$

$$(17)$$

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.

APPENDIX A

PROOF OF THEOREM 1

Based on the SNR $\gamma_{AF}^{R,D}$ at the destination, the legitimated channel capacity can be expressed as

$$C_{AF}^{R,D} = W \log_2 \left( 1 + a \rho \frac{\|h_{R,D} \|^2}{\|h_{S,R} \|^2} \right)$$

$$(17)$$

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.

APPENDIX A

PROOF OF THEOREM 1

Based on the SNR $\gamma_{AF}^{R,D}$ at the destination, the legitimated channel capacity can be expressed as

$$C_{AF}^{R,D} = W \log_2 \left( 1 + a \rho \frac{\|h_{R,D} \|^2}{\|h_{S,R} \|^2} \right)$$

$$(17)$$

Finally, we show the impact of $\rho$ on the interception probability based on DF relaying mode with $\text{SNR}_S = 20\text{dB}$ and $\varepsilon = 0.01$. As seen in Fig. 7 although the interception probability increases as $\rho$ decreases, the performance gap is nearly negligible, so the DF relaying scheme has high robustness.

V. CONCLUSION

A major contribution of this paper is the introduction of the LS-MIMO relaying technique into physical layer security to significantly enhance wireless security, especially when the interception distance is short. This paper focuses on the analysis of secrecy outage capacity under the AF and DF relaying strategies, and derives the closed-form expressions in terms of the transmit SNR and the channel condition. Altogether, these results provide some important guidelines for performance optimization of physical layer security in LS-MIMO relaying systems.
where \( R(x) \) denotes the real part of \( x \). \( \mathbf{h}_{R,D} \) is replaced by \( \sqrt{\mathbf{h}_{R,D} + \sqrt{1-p}} \) in [17]. [18] follows from the fact that \( \rho \| \mathbf{h}_{R,D} \|^2 \) scales with the order \( \Theta(\rho N_R) \) as \( N_R \to \infty \) while \( 2\sqrt{(1-\rho)R(\mathbf{h}_{R,D}) + (1-\rho)\| \mathbf{h}_{R,D} \|^2 / \| \mathbf{h}_{R,D} \|^2} \) scales as the order \( O(1) \), which can be negligible. [19] holds true because of \( \lim_{N_R \to \infty} \| \mathbf{h}_{R,D} \|^2 = 1 \) and \( \lim_{N_R \to \infty} \| \mathbf{h}_{R,D} \|^2 = 1 \), namely channel hardening [16]. Therefore, we get the Theorem 1.

**APPENDIX B**

**Proof of Theorem 2**

According to (4), given \( \varepsilon \), we have

\[
\varepsilon = P_r \left( C_{SOC}^{AF} > C_{DF}^{AF} - W \log_2(1 + \gamma_E^{AF}) \right) \\
= P_r \left( \gamma_E^{AF} > 2(C_{DF}^{AF} - C_{SOC}^{AF})/W - 1 \right) \\
= 1 - F \left( 2(C_{DF}^{AF} - C_{SOC}^{AF})/W - 1 \right), \tag{20}
\]

where \( F(x) \) is the cumulative distribution function (cdf) of \( \gamma_E^{AF} \). In order to derive the secrecy outage capacity, the key is to get the cdf of \( \gamma_E^{AF} \). Examining (10), due to channel hardening, we have

\[
\gamma_E^{AF} = \frac{dN_R \| \mathbf{h}_{R,E}^H \mathbf{h}_{R,D} \|^2}{\Theta R,E H(x) + cN_R + 1}. \tag{21}
\]

Since \( \| \mathbf{h}_{R,E} \| / \| \mathbf{h}_{R,D} \| \) is an isotropic unit vector and independent of \( \mathbf{h}_{R,E} \), \( \| \mathbf{h}_{R,E}^H \mathbf{h}_{R,D} / \| \mathbf{h}_{R,D} \|^2 \) is \( \chi^2 \) distributed with 2 degrees of freedom. Let \( y \sim \chi^2_2 \), we can derive the cdf of \( \gamma_E^{AF} \) as

\[
F(x) = P_r \left( \frac{dN_R y}{\Theta R,E H(x) + cN_R + 1} \leq x \right). \tag{22}
\]

If \( x < dN_R / \Theta \), then we have

\[
F(x) = P_r \left( y \leq \frac{(cN_R + 1)x}{dN_R - \Theta x} \right) \\
= 1 - \exp \left( \frac{(cN_R + 1)x}{dN_R - \Theta x} \right). \tag{23}
\]

Since \( x \geq N_R / \Theta \) is impossible when \( x = 2(C_{DF}^{AF} - C_{SOC}^{AF})/W - 1 \), we have

\[
\varepsilon = \exp \left( \frac{(cN_R + 1)\left(2(C_{DF}^{AF} - C_{SOC}^{AF})/W - 1\right)}{dN_R - \Theta \left(2(C_{DF}^{AF} - C_{SOC}^{AF})/W - 1\right)} \right). \tag{24}
\]

Hence, we get the Theorem 2.

**APPENDIX C**

**Proof of Theorem 3**

Similarly, according to the definition of secrecy outage capacity in (4), we have

\[
\varepsilon = P_r \left( C_{SOC}^{DF} > C_{DF}^{DF} - C_{E}^{DF} \right) \\
= P_r \left( \frac{\| \mathbf{h}_{R,E}^H \mathbf{h}_{R,D} \|^2}{\| \mathbf{h}_{R,D} \|^2} > \frac{2(C_{DF}^{DF} - C_{SOC}^{DF})/W - 1}{P_R,\alpha_{R,E}} \right) \\
= \exp \left( -\frac{2(C_{DF}^{DF} - C_{SOC}^{DF})/W - 1}{P_R,\alpha_{R,E}} \right), \tag{25}
\]

where (25) follows the fact that \( \| \mathbf{h}_{R,E}^H \mathbf{h}_{R,D} / \| \mathbf{h}_{R,D} \|^2 \) is \( \chi^2 \) distributed with 2 degrees of freedom as analyzed earlier. Based on (25), it is easy to get the Theorem 3.

**References**

[1] A. D. Wyner, “The wire-tap channel,” Bell Syst. Tech. J., vol. 54, pp. 1355-1387, Oct. 1975.

[2] P. K. Gopala, L. Lai, and H. El. Gamal, “On the secrecy capacity of fading channels,” IEEE Trans. Inf. Theory, vol. 54, no. 10, pp. 4687-4698, Oct. 2008.

[3] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.

[4] P. K. Gopala, L. Lai, and H. El. Gamal, “On the secrecy capacity of fading channels,” IEEE Trans. Inf. Theory, vol. 54, no. 10, pp. 4687-4698, Oct. 2008.

[5] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.

[6] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.

[7] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.

[8] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.

[9] X. Chen, and R. Yin, “Performance analysis for physical layer security in multi-antenna downlink networks with limited CSI feedback,” IEEE Wireless Commun. Lett., vol. 2, no. 5, pp. 503-506, Oct. 2013.