The Dynamical Environment of Dawn at Vesta

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Abstract

Dawn is the first NASA mission to operate in the vicinity of the two most massive asteroids in the main belt, Ceres and Vesta. This double-rendezvous mission is enabled by the use of low-thrust solar electric propulsion. Dawn will arrive at Vesta in 2011 and will operate in its vicinity for approximately one year. Vesta’s mass and non-spherical shape, coupled with its rotational period, presents very interesting challenges to a spacecraft that depends principally upon low-thrust propulsion for trajectory-changing maneuvers. The details of Vesta’s high-order gravitational terms will not be determined until after Dawn’s arrival at Vesta, but it is clear that their effect on Dawn operations creates the most complex operational environment for a NASA mission to date. Gravitational perturbations give rise to oscillations in Dawn’s orbital radius, and it is found that trapping of the spacecraft is possible near the 1:1 resonance between Dawn’s orbital period and Vesta’s rotational period, located approximately between 520 and 580 km orbital radius. This resonant trapping can be escaped by thrusting at the appropriate orbital phase. Having passed through the 1:1 resonance, gravitational perturbations ultimately limit the minimum radius for low-altitude operations to about 400 km, in order to safely prevent surface impact. The lowest practical orbit is desirable in order to maximize signal-to-noise and spatial resolution of the Gamma-Ray and Neutron Detector and to provide the highest spatial resolution observations by Dawn’s Framing Camera and Visible InfraRed mapping spectrometer. Dawn dynamical behavior is modeled in the context of a wide range of Vesta gravity models. Many of these models are distinguishable during Dawn’s High Altitude Mapping Orbit and the remainder are resolved during Dawn’s Low Altitude Mapping Orbit, providing insight into Vesta’s interior structure. Ultimately, the dynamics of Dawn at Vesta identifies issues to be explored in the planning of future EP missions operating in close proximity to larger asteroids.

Keywords: Vesta, Discovery Program, Electric Propulsion, Gravitational Perturbations, Spacecraft Operations
1. Introduction

The Dawn Discovery mission was successfully launched on September 27, 2007, and is the first NASA science mission making use of solar electric propulsion (EP), enabled by the earlier Deep Space 1 technology demonstration mission (Lehman, 1999). As a consequence of the efficiency of this low-thrust, low-acceleration system, Dawn is able to rendezvous with the two most massive objects in the asteroid belt, Vesta then Ceres. These targets were selected in order to study the earliest stages of planetary evolution for an object that formed dry (Vesta) and another that formed with substantial amounts of water (Ceres) (Russell et al., 2004).

Dawn’s first target, Vesta, has a semi-major axis of 2.36 AU and is located in the inner main asteroid belt. It is unique in having a basaltic crust that has survived over the age of the solar system, providing important constraints on models of the collisional evolution of the asteroid belt (e.g., Davis et al., 1985). It has been spectroscopically linked to HED meteorites on the Earth (McCord et al., 1970), which represent approximately 6% of all meteorite falls today (McSween, 1999). It has been inferred from those meteorites that Vesta is a differentiated object with an iron-rich core (Newsom, 1984; Ghosh and McSween, 1996; Righter and Drake, 1996). Hubble observations of Vesta revealed an object with an equatorial radius around 289 km and polar radius of 229 km, but with a substantial impact crater covering much of its southern hemisphere and distorting its shape (Zellner et al., 1997; Thomas et al., 1997a,b). This giant impact likely gave rise to the Vesta collisional family, which spans the inner main belt from the $\nu_6$ secular resonance with Saturn on its inner edge to the 3:1 mean motion resonance with Jupiter, separating it from the outer main belt. Some Vesta material entering these resonances would have their orbits pumped into Mars- and eventually Earth-crossing orbits, resulting in the HED meteorites recovered on the Earth (e.g., Migliorini et al., 1997). Dawn may provide connections between specific areas of Vesta’s surface and HED (and possibly other) meteorites.

Dawn executed a Mars gravity assist on February 17, 2009, to align its orbital inclination with that of Vesta. Dawn arrives at Vesta in July 2011 when it will enter an initial orbit having a radius of 2700 km (Survey orbit), from which it will obtain a preliminary shape model using the Framing Camera (FC) and spectrally map the entire illuminated surface using the Visible InfraRed mapping spectrometer (VIR) (Russell et al., 2006, 2007b). Assuming the rotational pole of Thomas et al. (1997a), Vesta’s obliquity is 27.2°. Dawn arrives at Vesta at the time of maximum illumination of the southern hemisphere and its large crater. After completing the Survey orbit phase, Dawn uses its EP thrusters to
descend to a High Altitude Mapping Orbit (HAMO) at approximately 950 km radius from which it will use the FC to map Vesta’s surface and determine its global shape and local topography using stereophotoclinometric techniques (e.g., Gaskell et al., 2008). Dawn will then descend to its Low Altitude Mapping Orbit (LAMO) of around 460 km radius from which it will map Vesta’s elemental composition using the Gamma-Ray Neutron Detector (GRaND). The Survey, HAMO, and LAMO phases are nominally 7, 27, and 90 days in duration (Russell et al., 2007a), but this is subject to further planning for the 8 month stay at Vesta, which may be extended to a year. Because Dawn uses its EP thrusters for orbit transfers, transitions between these different phases are expected to take around a month (Russell et al., 2007b).

To maximize the science return from the mission, we are interested in determining the lowest orbital radius from which Dawn can safely execute LAMO. The closer to the surface we can make observations, the better the spatial resolution of FC and VIR observations. However, a lower LAMO most benefits observations by GRaND. The spatial resolution of the GRaND instrument is approximately 1.5 times the altitude (Prettyman et al., 2003). At nominal 460 km orbital radius for LAMO, this altitude will vary between 175 km near the equator and 231 km near the pole. By decreasing LAMO to 400 km, the number of GRaND resolved elements on Vesta increases by more than 50%, improving our ability to identify geochemical units and relate them to HED meteorites (Prettyman et al., 2010). Going lower also improves GRaND signal-to-noise and may enable an accurate determination of Mg and Si, which are important discriminators among the various rock types expected on Vesta (Prettyman, priv. comm., and Prettyman et al. 2010). However, lower orbits and corresponding decreased orbital periods increases the need for desaturation of the spacecraft angular momentum, increasing the operational burden, which is not explored here.

A Vesta gravity model is also greatly improved with reduced altitude, allowing for better detection of mascons and determination of Vesta’s higher order gravitational terms. At HAMO, the gravity field is determined to at least degree 4 and at LAMO this is expected to improve to at least degree 10 (Russell et al., 2007b). Depending on the accuracy of Doppler and Doppler-rate data, simulations later in this manuscript show that these numbers can be significantly improved, reaching degree 10 at HAMO and degree 20 at LAMO.

While there are science benefits from a minimum radius LAMO, Vesta’s large mass and deviation from sphericity raises the question of how its gravity field will constrain the lowest orbital radius at which Dawn can safely operate. In addition, the long transfer times between Survey, HAMO and LAMO mean that Dawn will be slowly transiting commensurabilities between its orbital period and Vesta’s rotational period, where perturbations on Dawn’s orbit may be significant.

With this work we explore the dynamics of the Dawn spacecraft in a polar
orbit within 1000 km from Vesta. The gravitational potential of Vesta is determined assuming diverse and extreme scenarios for its interior structure, to ensure the dynamical environment is sufficiently explored and that the results are representative of what the mission is likely to experience once there. The orbital maneuver from HAMO to LAMO using EP to slowly spiral in is also simulated, to assess the effect of mean motion resonances.

2. Modeling the Gravitational Field of Vesta

In general, the gravitational potential of a body with arbitrary shape and mass distribution can be described using the spherical harmonics series (Kaula, 1966)

\[
U(r, \theta, \phi) = \frac{GM}{r} \left[ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left( \frac{r_0}{r} \right)^l P_{lm}(\cos \theta) \left( C_{lm} \cos m\phi + S_{lm} \sin m\phi \right) \right]
\] (1)

where \(G\) is the universal gravitational constant, \(M\) is the total mass of the body, \(\{r, \theta, \phi\}\) are the body-fixed barycentric spherical coordinates of the point where the potential \(U\) is computed, \(r_0\) is an arbitrary reference radius usually corresponding to the radius of the Brillouin sphere, the smallest sphere enclosing the body, \(P_{lm}(\cos \theta)\) is the associate Legendre function. Using this series, the Stokes coefficients \(\{C_{lm}, S_{lm}\}\) uniquely characterize the potential of the body. In cases where the mass density distribution \(\varrho(r, \theta, \phi)\) of a body is known, then the coefficients \(\{C_{lm}, S_{lm}\}\) of the series in Eq. (1) can be determined by integrating over the volume \(V\) of the body (Kaula, 1966)

\[
C_{lm} = \frac{1}{M} \int_V \varrho(r, \theta, \phi) \left( \frac{r}{r_0} \right)^l P_{lm}(\cos \theta) \cos m\phi \ dV
\] (2)

\[
S_{lm} = \frac{1}{M} \int_V \varrho(r, \theta, \phi) \left( \frac{r}{r_0} \right)^l P_{lm}(\cos \theta) \sin m\phi \ dV
\] (3)

The integration over the volume of the body is typically performed numerically, i.e. using Monte Carlo integration techniques (i.e., Press et al., 1992). In the Monte Carlo integration, each integral \(I = \int_V f dV\) is approximated by \(I \simeq V \langle f \rangle\), where \(\langle f \rangle\) is the mean value of the integrand inside the volume. Both \(V\) and \(\langle f \rangle\) are computed by randomly sampling points inside the body’s volume. The nominal error on the result of a Monte Carlo integration is \(\sigma_I = V \sigma_f / \sqrt{N}\), where \(\sigma_f\) is the variance of the integrand, and \(N\) is the number of sample points used.

The Stokes coefficients can be more readily compared and used in numerical work when their magnitude is normalized (i.e., Kaula, 1966), and in this
manuscript we use the following normalization rule:

\[
\{C_{lm}, S_{lm}\} = \sqrt{\frac{(l - m)!}{(l + m)!}} \frac{(2 - \delta_{0m})}{(2l + 1)} \{C_{lm}, S_{lm}\}
\]

This normalization produced results in agreement the Stokes coefficients of the asteroid 433 Eros produced by the NEAR mission, see §3.

In order to use this formalism to model Vesta’s gravitational potential, and thus the dynamics of a spacecraft in its proximity, we need a model for its shape and a model for its mass density distribution.

Analysis of Hubble Space Telescope (HST) observations of Vesta have yielded accurate determination of its size, shape and rotational state (Thomas et al., 1997a). The overall shape of Vesta can be fit by a triaxial ellipsoid of radii 289, 280, 229, ±5 km (Thomas et al., 1997a). The shape is not perfectly ellipsoidal, with departures of 15-20 km from the smooth ellipse, and a large indentation with depth of 20-30 km and diameter of about 200 km in the southern hemisphere (Thomas et al., 1997a). The rotation period used for Vesta is of 5.3421288 hours (Thomas et al., 1997a), and for the purpose of this work, we assumed Vesta to be a primary axis rotator. The 3D shape model obtained by Thomas et al. (1997a) was used as reference for the study presented in this manuscript.

Vesta is most likely a differentiated body. Mineralogical and isotopic data of HED meteorites suggests that heating, melting, formation of a metal core, a mantle, and a basaltic crust took place in the first few million years of solar system history (e.g., Keil, 2002, and references therein). Thermal modeling by Ghosh and McSween (1998) suggests that heating by $^{26}$Al would keep the mantle of Vesta hot for $\sim$ 100 My. It is possible that the mantle experienced a substantial if not complete melting that resulted in the formation of a metal core (Keil, 2002). Results by Lee and Halliday (1997) on the excess $^{182}$W measured on eucrites samples suggests that accretion, differentiation, and core formation on Vesta took place in the first 5–15 My. Ruzicka et al. (1997) estimated the radius of the core using mass balance from the density of Vesta and a variable fraction of silicates, with their best estimate of a core radius smaller than 130 km, an olivine-rich mantle with thickness $\sim$ 65–220 km, and a crust with thickness $\sim$ 40–85 km. The placement and integrity of that core is in question given the large impact event evidenced by the hemispheric southern crater. This impact may have fragmented the core. It may have caused it to be effectively displaced from the center of figure. This motivates us to consider several scenarios for Vesta’s interior mass distribution for modeling its gravitational potential.

In this work we choose several scenarios for the mass density distribution of Vesta:

- $U$ – uniform mass density;
- **C0** – a core centered on the origin of the tri-axial ellipsoid fit of the northern hemisphere;
- **CZ** – a core centered in the same center of mass of the U scenario;
- **CX50** – a core offset by 50 km along the x axis;
- **C0F20** – a core centered on the origin, plus 8 fragments with the same density as the core, each of 20 km radius, equally spaced along the equator, just below the surface;

Our selection was dictated by the desire to explore a wide range of conditions, from the very likely to the very extreme, in order to adequately characterize the dynamics of Dawn at Vesta. The different internal mass distributions are illustrated in Figure 1.

Along with these scenarios, all using the known shape model, we have included the EU scenario, where the shape is that of the smooth and symmetric triaxial ellipsoid, best fit of the northern hemisphere, with uniform mass density.

In modeling the core or core fragments, we assume a density of 7.90 g/cm$^3$, consistent with that of iron meteorites (Consolmagno and Britt, 1998). Core size is constrained by conserving the total mass of Vesta and by fixing the density of mantle material to be 3.12 g/cm$^3$, consistent with the grain densities of HED meteorites (ibid.). Vesta is massive enough for its gravity to compress interior material to the point where it is one of only a few asteroids thought to have no macro-porosity and little micro-porosity (Britt et al., 2002).

For each scenario, we have derived an expansion of the potential, using Eq. (2) and (3) with $10^8$ sampling points, sufficient to reach an integration error smaller than $10^{-6}$, and the results are displayed in Table 1. The root mean square (RMS) of all the normalized coefficients with a given degree $l$ is given by the formula $(\sum_{m=0}^{l} (\bar{C}_{lm}^2 + \bar{S}_{lm}^2)/(2l + 1))^{1/2}$ and provides a measure of the magnitude of the degree $l$ of the gravitational expansion. If we interpret the expansion as a spectrum, then the characteristic wavelength of each degree is $\lambda_l = 2\pi R/l$, with $R$ equatorial radius of the body. In Figure 2 we show the RMS of the coefficients for Vesta up to degree 20. These coefficients were obtained using the shape of Vesta (Thomas et al., 1997a) under the scenarios considered.

Finally, Vesta’s mass is derived from its influence on the motions of Mars and other asteroids, with a recent value of $1.35 \times 10^{-10} M_\odot$ proposed by Pitjeva and Standish (2009) and used throughout this work.

### 3. Simulating Dawn Motion Around Vesta

Given a gravitational potential and a rotational state for Vesta, we simulate Dawn’s orbital motion about Vesta under a wide range of initial conditions, primarily sampling the initial radius and orbital phase. We choose to run most of
our simulations at the 8th degree when studying the orbital motions of Dawn. This choice is supported by our tests of the orbital evolution of Dawn at LAMO in all the scenarios considered, where we have compared the dynamics of Dawn at degree 8 and 20. In our tests all the main features of the orbital evolution of Dawn were captured already at degree 8, and the difference in the orbital elements between the two cases were always of the order of 1% over a period of 100 days at LAMO.

The numerical simulations are performed using a code specifically designed for this task. This code is written in the C++ programming language, and makes very heavy use of a C++ framework designed specifically for celestial mechanics computations called ORSA\(^1\). The ORSA framework has been designed and continuously developed by Tricarico over the past decade, and has been used in several studies by Tricarico and collaborators. Particularly relevant for this study is that the ORSA framework implements the interaction between extended bodies with arbitrary shape and mass distribution, using a formalism that is specifically designed to avoid transcendental functions, that are computationally expensive (Tricarico, 2008). The numerical integrator used, based on the 15th order RADAU algorithm (Everhart, 1985), is a variable step scheme that was originally developed for the integration of cometary dynamics, and that provides excellent accuracy for a wide range of dynamical problems. This algorithm was modified to account for the effects of the primary’s potential on the spacecraft using the formalism in Tricarico (2008). This enhanced code was then validated for the dynamics in proximity of a rotating elongated primary, using the reconstructed orbital data of the NEAR spacecraft orbiting the asteroid 433 Eros, available in the Planetary Data System (PDS) (Miller et al., 2002; Konopliv et al., 2002). As showed in Figure 3, after 10 days the total offset between the reconstructed trajectory (PDS archived data) and trajectory integrated with our code is still smaller than 50 m. This also required the application of a simple model (Scheeres, 1999) of radiation pressure acting on the NEAR solar panels, using a mass to projected area ratio of 34.5 kg/m\(^2\). The 10 day period was the longest period free of trajectory changing maneuvers at this close distance from the asteroid, beginning on 7/14/2000 03:00 UTC, and corresponding to a 35 × 37 km orbit. As a comparison, the unmodeled stochastic accelerations present in the data, with a magnitude of 7.5 m/day\(^2\) and correlation time of 1 day (Miller et al., 2002), would easily account for an offset in excess of 100 m over the same period, so the numerical integration is well within the uncertainty of the reconstructed trajectory of the spacecraft.

For Dawn, the effects of solar radiation pressure can be estimated using the formalism in Scheeres (1999). The projected area of Dawn consists of ten 1.6×2.2

\(^1\)ORSA is open-source, available at http://orsa.sf.net.
m² solar panels composing the array, and a body of about $2 \times 2$ m², for a total of 39.2 m². The total mass of the spacecraft at Vesta will be of approximately 960 kg (Rayman et al. (2006) and Rayman, priv. comm.), so the mass to projected area ratio used here is $B \simeq 25$ kg/m².

4. Assessing the Vesta Dynamical Environment

The dynamics of a spacecraft orbiting within $\sim 1000$ km from Vesta is constantly perturbed from a purely Keplerian orbit, in particular when crossing spin-orbit resonances. When the orbital period of the spacecraft is close to a commensurability with the rotational period of Vesta, the effects of gravitational perturbations due to the high order terms of Vesta’s gravitational potential can be amplified, and this leads to large perturbations in the semi-major axis and eccentricity of the orbit of the spacecraft. This effect can be measured by monitoring the distance between the spacecraft and Vesta (center-to-center) over a period (50 days) longer than the time over which the perturbation acts (typically shorter than 10 days). Over this monitored period, orbits in a spin-orbit resonance have a distance range (farthest minus closest) significantly larger than orbits not affected by a spin-orbit resonance.

We have sampled all the circular polar orbits with an initial radius between 370 and 1000 km at 5 km increments. The dynamics of the spacecraft is numerically integrated over a period of 50 days, subject to gravitational perturbations from the Sun, the planets, Vesta, and to solar radiation pressure. We find that the two most important spin-orbit resonances are the 1:1 and the 2:3, located at about 550 km and 720 km, respectively, see Figure 4. The 1:1 resonance can affect orbits with an initial radius between about 500 and 590 km. Similarly, the 2:3 resonance operates between 680 and 740 km. The width of the resonances is not fully represented in Figure 4 because only one initial condition per initial radius is sampled for the data in that figure. Perturbations to the inclination of the polar orbits were also observed, with oscillations of up to 4° total for resonant orbits, while outside the resonances inclinations remained within one degree from polar for all distances larger than about 400 km.

Let us reiterate the fact that using an 8th degree gravitational potential is adequate for this study. As discussed in §3 the changes to the orbital elements at LAMO over 100 days due to degrees between 9 and 20 are of the order of 1%, so the dynamics of Dawn is completely captured in 8th degree simulations. Also, the fastest resonances detected are the 3:2 and 4:3, and the main resonant terms for these resonances are respectively degree 3 and 4, so including all degrees up to 8 includes the first and second harmonic terms for the two fastest resonances.

A graphical representation of the radial range variability is provided by Figure 5, where we show in polar coordinates the cloud of Dawn’s dynamical evolu-
tion at HAMO, 2:3, 1:1, and LAMO over a period of 50 days. An interesting result is that the naturally perturbed trajectories of Dawn are not north-south symmetric, as they tend to get closer to Vesta at the north pole and farther away at the south pole. When this effect is added to Vesta’s already asymmetric shape, it is clear that the altitude of the passages over the south pole will be even larger than what the shape alone imposes to a spacecraft on a circular polar orbit.

We have also investigated to what extent the different interior structures considered in this manuscript affect the dynamics of Dawn, using a similar approach of sampling circular polar orbits while monitoring radial range, and the results are displayed in Figure 6. We find it remarkable that over the range of reasonable diverse scenarios considered in this study, the radial range pattern is mostly unaffected by the details of the interior structure. An obvious exception is the EU scenario, where the shape used is that of a smooth and symmetric triaxial ellipsoid and not the shape model based on Thomas et al. (1997a) used in all the other scenarios. This leads us to conclude that Dawn’s dynamics, along with the multipole expansion of Vesta’s potential, depend primarily on Vesta’s shape and only secondarily on the details of the interior structure over the range considered, given the density constraints on mantle and core material provided by HED meteorites and the measure of Vesta’s mass. This is an advantage for the Dawn mission, because a detailed shape model will be constructed for Vesta from Survey data which, in combination with gravity data obtained at HAMO, will help navigate the spacecraft through the 2:3 and 1:1 resonances when descending from HAMO to LAMO.

One more question is to what extent the details of the initial conditions, such as initial orbital phase of Dawn and rotational phase of Vesta, affect the dynamical evolution of Dawn. We have performed once more the orbital sampling, but now for each initial radius we start 12 simulation at 30 degrees interval along the orbit, using the U scenario, and the results are displayed in Figure 7. We find that Dawn’s dynamics depends strongly on the details of its initial orbital phase relative to Vesta’s rotational phase. In Figure 7 we have plotted the same data first as a function of the initial radius, and then as a function of the mean radius, that here plays the role of an effective semi-major axis, i.e. the semi-major axis averaged over the simulated period of 50 days. The former makes it easier to spot the initial value of the radius at which a resonance can affect an orbit, while the latter clearly marks the exact position of the resonances. The 1:1 and 2:3 spin-orbit resonances have very different effects. At the 1:1 the largest magnitude of the radial range is reached for orbits starting at the edges of the resonance, with a minimum at its center located at about 550 km. At the 2:3 the magnitude of the radial range is mostly a function of the mean radius: orbits at the center of the resonance (~720 km) experience the largest perturbations, reaching a radial range of 150-200 km and quickly decreasing moving away from it. Out of spin-
orbit resonances, the radial range still depends strongly on the initial conditions, varying by up to a factor of 2. While this sensitivity to the initial conditions was analyzed in detail using the U scenario, we have also studied specific cases using the other scenarios considered in the manuscript, confirming these results.

The effects of the 1:1 resonance are displayed in more detail in Figure 8. Depending on its initial radius, the spacecraft will librate with different amplitudes and periods about the shortest equatorial radius of Vesta.

In Figure 9 we show the details of the orbital elements of Dawn at LAMO. In this case we included all the terms up to degree 20 of the gravitational potential expansion, to verify that no runaway effects are present even at this close distance, over the expected period of performance of about 100 days.

5. Operational Issues

In order to better understand the effects of commensurabilities on Dawn’s operation, we simulated its slow descent from HAMO using 20 mN of thrust (at the low end of Dawn’s thrust range, Rayman et al. 2007), which achieves transfer to nominal LAMO in about 30 days. An initial circular orbit and continuous thrusting are assumed, with the direction of thrust opposite to that of the velocity relative to Vesta. The results of the simulations are shown in Figure 10.

What is immediately obvious is the strong sensitivity to the starting position around the HAMO orbit for a given Vesta sub-longitude. Different initial Dawn orbital phases for a given Vesta rotational phase mean that the azimuthal asymmetries in Vesta’s gravitational field will be experienced at different phases and will have different effects. Away from commensurabilities the effects are somewhat randomized in the aggregate. However, near the 1:1 commensurability Dawn tends to pass over the same Vesta surface location twice for each orbit so the effects are reinforcing.

The most interesting effect of Vesta’s gravity field on Dawn is in the vicinity of the 1:1 resonance. There are circumstances under which the spacecraft can be trapped in the resonance even while continuing to thrust. This is also very sensitive to the initial orbital/rotational phase. Out of twelve simulations, only one evidences trapping (Figure 10) while most other phases exhibit an apparent temporary trapping. The trapping seems to occur when the Dawn spacecraft attempts to cross the 1:1 resonance near the longitude where Vesta’s equatorial radius is shortest, as demonstrated when showing how the 1:1 resonance operates in Figure 8. While trapped, Dawn’s orbit librates about this point. Trapping is not limited to scenarios using 20 mN thrust, as we have also observed that higher thrusting of 50 mN can exhibit a similar behavior.

Trapping can be escaped, but not by thrusting alone. Simulations in which the thrust is increased from 20 to 35 mN show that the libration phase of such
increased thrust is key (Figure 11). In the meantime, the orbital motion within the resonance is well-behaved and does not appear to introduce a risk of spacecraft loss.

We also note how in more than half of the spiraling cases of Figure 10 the amplitude of the radial oscillations increases significantly when crossing the 2:3 resonance between 700 and 750 km, pumping up the eccentricity of the orbit. We anticipate that it might be necessary for Dawn to take this effect into account and try to reduce the eccentricity of the orbit before reaching and crossing the 1:1 resonance when migrating from HAMO to LAMO.

6. Probing the Interior Structure of Vesta

The interior structure of Vesta is manifested by the higher order terms of its gravitational potential. The determination of these terms will depend upon the reconstruction of the spacecraft position and orbital motion from coherent X-band Doppler tracking (Asmar et al., 2009) and spacecraft-based observations of Vesta against background stars.

By simulating the dynamics of the spacecraft at different altitudes we can also determine the sensitivity to coefficients of a given degree of the gravitational expansion at HAMO or LAMO, as a function of the time free of trajectory maneuvers spent at each altitude. As we show in Figure 12, the perturbations due to terms of higher degree in the gravitational expansion take a longer and longer time to produce measurable effects. What we label as drift in the figure is the offset between the predicted and observed position of the spacecraft, where the prediction was based on a model that included only terms up to the degree marked on each curve. At HAMO, the effects due to the 8th degree become measurable after about 10 days, those due to the 10th degree after about 20 days. The effects of the 12th and 14th degree remain comparable over the 100 days period simulated, and this reflects a condition of measurability in presence of strong correlation, so the terms will not be determined uniquely at HAMO. At LAMO, because of the reduced altitude, the sensitivity to higher degrees is evident, and the effects due to the 14th and 16th degree are measurable within days, while the effects of the 18th and 20th degree remain correlated over about 60 days. This is in agreement with recent results by Asmar et al. (2009), who find that the 20th degree of the gravity field will be determined by Dawn. Here we assume that the sensitivity of the deep space network is of the order of meters or tens of meters, as is the case in similar conditions, when tracking spacecraft at Moon or Mars (e.g., Konopliv et al., 1998, 2001, 2006).

At this point it is an interesting exercise to determine the phase of the mission during which we can distinguish among all the scenarios considered. The RMS of the difference between normalized coefficients of different scenarios listed in
Table 1 provides a measure of how much the expansion of any two scenarios differs. We find that the difference between U scenario and any of the C scenarios is of the order of $10^{-3}$ and is the largest observed. The smallest difference observed is between C0 and CZ, equal to $7 \times 10^{-5}$. Finally, the difference between all the remaining pairs of scenarios is of the order of $4 \times 10^{-4}$. Using Figure 2 we can estimate the characteristic magnitude of the RMS of the coefficients (or their difference) of a given degree, and then using Figure 12 we can assign a timescale to each degree. What we obtain is that RMS coefficients of magnitude $10^{-3}$ correspond to the magnitude of terms of degree 4; coefficients of magnitude $4 \times 10^{-4}$ correspond to the magnitude of terms of degree 5 to 6; coefficients of magnitude $7 \times 10^{-5}$ correspond to the magnitude of terms of degree 8 to 10. Degrees below the 8th are not displayed in Figure 12, but it is clear that data necessary to reach sensitivity below that necessary to determine the gravitational terms of degree 8 will be acquired within the first few days of HAMO. This is the time over which will be possible to distinguish between all the scenarios except C0 and CZ. To distinguish between the latter two, data for up to 20 days at HAMO will be necessary. So going back to the original question we can say that if Vesta’s interior was described exactly by one of the scenarios considered, Dawn would be able to indicate which one this is within the first 20 days of HAMO.

7. Conclusions

Vesta presents novel and interesting operational challenges to the Dawn mission. Because of the low-thrust electric propulsion system, Dawn will pass through Vesta spin/Dawn orbit commensurabilities very slowly as it moves from its High Altitude Mapping Orbit to Low Altitude Mapping Orbit, maximizing the effects of the perturbation. In addition to the rapid oscillations in Dawn’s orbital radius as a consequence of Vesta’s complex gravity field, there is the potential that Dawn could be trapped near the 1:1 resonance, as it slowly decreases its radius through 550 km. Dawn can escape trapping by increasing thrust at the appropriate orbit libration phase. This is not an issue that is expected to recur at Ceres, which is observed to have a simple oblate spheroid shape. Once through this resonance, Dawn can continue to decrease its orbital radius to around 400 km (60 km less than the current LAMO) before the effect of perturbations begin to progressively increase orbital radial oscillations until impact with the surface becomes a hazard as Dawn reaches an average orbital radius of around 370 km.

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References

Asmar, S., Konopliv, A., Bills, B., Park, R., Raymond, C., Russell, C., Smith, D., Zuber, M. 2009. Simulations of Dawn Gravity for Vesta and Ceres And Implications For Interior Structure. AAS/Division of Planetary Sciences Meeting Abstracts 41, #50.01.

Britt, D. T., Consolmagno, G. J. 2004. Meteorite Porosities and Densities: A Review of Trends in the Data. Lunar and Planetary Institute Science Conference Abstracts 35, 2108.

Britt, D. T., Yeomans, D., Housen, K., Consolmagno, G. 2002. Asteroid Density, Porosity, and Structure. Asteroids III 485-500.

Consolmagno, G. J., Britt, D. T. 1998. The density and porosity of meteorites from the Vatican collection. Meteoritics and Planetary Science 33, 1231-1241.

Davis, D. R., Chapman, C. R., Weidenschilling, S. J., Greenberg, R. 1985. Collisional history of asteroids: Evidence from Vesta and the Hirayama families. Icarus 62, 30-53.

Everhart, E. 1985. An efficient integrator that uses Gauss-Radau spacings. Dynamics of Comets: Their Origin and Evolution, Proceedings of IAU Colloq. 83, held in Rome, Italy, June 11-15, 1984.Edited by Andrea Carusi and Giovanni B. Valsecchi. Dordrecht: Reidel, Astrophysics and Space Science Library. Volume 115, 1985., p.185 185.

Gaskell, R. W., and 15 colleagues 2008. Characterizing and navigating small bodies with imaging data. Meteoritics and Planetary Science 43, 1049-1061.

Ghosh, A., McSween, H. Y., Jr. 1996. An Extended Thermal History (100 MA Long) for Asteroid 4 Vesta Based on Radionuclide and Collisional Heating. Lunar and Planetary Institute Science Conference Abstracts 27, 407.

Ghosh, A., McSween, H. Y. 1998. A Thermal Model for the Differentiation of Asteroid 4 Vesta, Based on Radiogenic Heating. Icarus 134, 187-206.

Henderson, E. P., Perry, S. H. 1954. A discussion of the densities of iron meteorites. Geochimica et Cosmochimica Acta 6. 221-240.

Jacoby, W., Smilde, P. L. 2009. Gravity Interpretation. Gravity Interpretation by Wolfgang Jacoby and Peter L. Smilde. Berlin: Springer, 2009.

Kaula, W. M. 1966. Theory of satellite geodesy. Applications of satellites to geodesy. Waltham, Mass.: Blaisdell, 1966.

Keil, K. 2002. Geological History of Asteroid 4 Vesta: The "Smallest Terrestrial Planet". Asteroids III 573-584.

Konopliv, A. S., Binder, A. B., Hood, L. L., Kucinskas, A. B., Sjogren, W. L., Williams, J. G. 1998. Improved Gravity Field of the Moon from Lunar Prospector. Science 281, 1476.

Konopliv, A. S., Asmar, S. W., Carranza, E., Sjogren, W. L., Yuan, D. N. 2001. Recent Gravity Models as a Result of the Lunar Prospector Mission. Icarus 150, 1-18.

Konopliv, A. S., Miller, J. K., Owen, W. M., Yeomans, D. K., Giorgini, J. D., Garmier, R., Barriot, J.-P. 2002. A Global Solution for the Gravity Field, Rotation, Landmarks, and Ephemeris of Eros. Icarus 160, 289-299.

Konopliv, A. S., Yoder, C. F., Standish, E. M., Yuan, D.-N., Sjogren, W. L. 2006. A global solution for the Mars static and seasonal gravity, Mars orientation, Phobos and Deimos masses, and Mars ephemeris. Icarus 182, 23-50.

Lee, D.-C.; Halliday, A. N. 1997. Core formation on Mars and differentiated asteroids. Nature 388, 854-857.

Lehman, D. H. 1999. Deep Space 1 mission overview.. Bulletin of the American Astronomical Society 31, 1127.
McCord, T. B., Adams, J. B., Johnson, T. V. 1970. Asteroid Vesta: Spectral Reflectivity and Compositional Implications. Science 168, 1445-1447.

McSween, H. Y., Jr. 1999. Meteorites. The New Solar System 351.

Migliorini, F., Morbidelli, A., Zappala, V., Gladman, B. J., Bailey, M. E., Cellino, A. 1997. Vesta fragments from v6 and 3:1 resonances: Implications for V-type NEAs and HED meteorites. Meteoritics and Planetary Science 32, 903-916.

Miller, J. K., and 10 colleagues 2002. Determination of Shape, Gravity, and Rotational State of Asteroid 433 Eros. Icarus 155, 3-17.

Newsom, H. E. 1984. Molybdenum in Eucrites, New Evidence for a Metal Core in Vesta (eucrite Parent Body). Lunar and Planetary Institute Science Conference Abstracts 15, 603-604.

Pitjeva, E. V., Standish, E. M. 2009. Proposals for the masses of the three largest asteroids, the Moon-Earth mass ratio and the Astronomical Unit. Celestial Mechanics and Dynamical Astronomy 103, 365-372.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., Flannery, B. P. 1992. Numerical recipes in C. The art of scientific computing. Cambridge: University Press.

Prettyman, T. H., and 13 colleagues 2003. Gamma-ray and neutron spectrometer for the dawn mission to 1 ceres and 4 vesta. IEEE Transactions on Nuclear Science 50, 1190-1197.

Prettyman, T. H., McSween, H. Y., Raymond, C. A., Feldman, W. C., Li, J.-Y., McFadden, L. A., Russell, C. T., Tricarico, P. 2010. Dawn’s GRaND to Reveal the Complex Geochemistry of Vesta. Lunar and Planetary Institute Science Conference Abstracts 41, 2299.

Rayman, M. D., Fraschetti, T. C., Raymond, C. A., Russell, C. T. 2006. Dawn: A mission in development for exploration of main belt asteroids Vesta and Ceres. Acta Astronautica 58, 605-616.

Rayman, M. D., Fraschetti, T. C., Raymond, C. A., & Russell, C. T. 2007. Coupling of system resource margins through the use of electric propulsion: Implications in preparing for the Dawn mission to Ceres and Vesta. Acta Astronautica, 60, 930-938.

Righter, K., Drake, M. J. 1996. Core Formation in Earth’s Moon, Mars, and Vesta. Icarus 124, 513-529.

Russell, C. T., and 20 colleagues 2004. Dawn: A journey in space and time. Planetary and Space Science 52, 465-489.

Russell, C. T., and 20 colleagues 2006. Dawn Discovery mission to Vesta and Ceres: Present status. Advances in Space Research 38, 2043-2048.

Russell, C. T., and 26 colleagues 2007a. Exploring the asteroid belt with ion propulsion: Dawn mission history, status and plans. Advances in Space Research 40, 193-201.

Russell, C. T., and 15 colleagues 2007b. Dawn Mission to Vesta and Ceres. Earth Moon and Planets 101, 65-91.

Ruzicka, A., Snyder, G. A., Taylor, L. A. 1997. Vesta as the HED Parent Body: Implications for the Size of a Core and for Large-Scale Differentiation. Meteoritics and Planetary Science 32, 825-840.

Scheeres, D. J. 1999. Satellite Dynamics about small bodies: Averaged Solar Radiation Pressure Effects. Journal of the Astronautical Sciences 47, 25-46.

Thomas, P. C., Binzel, R. P., Gaffey, M. J., Zellner, B. H., Storrs, A. D., Wells, E. 1997a. Vesta: Spin Pole, Size, and Shape from HST Images. Icarus 128, 88-94.

Thomas, P. C., Binzel, R. P., Gaffey, M. J., Storrs, A. D., Wells, E. N., Zellner, B. H. 1997b. Impact excavation on asteroid 4 Vesta: Hubble Space Telescope results. Science 277, 1492-1495.

Tricarico, P. 2008. Figure figure interaction between bodies having arbitrary shapes and mass distributions: a power series expansion approach. Celestial Mechanics and Dynamical Astronomy 100, 319-330.

Zellner, B. H., Albrecht, R., Binzel, R. P., Gaffey, M. J., Thomas, P. C., Storrs, A. D., Wells, E. N. 1997. Hubble Space Telescope Images of Asteroid 4 Vesta in 1994. Icarus 128, 83-87.
|    | U   | C0   | CZ   | CX50  | C0F20 | EU   |
|----|-----|------|------|-------|-------|------|
| $\rho_m$ | 3.411 | 3.120 | 3.120 | 3.120 | 3.120 | 3.411 |
| $\rho_c$ | — | 7.900 | 7.900 | 7.900 | 7.900 | — |
| $R_c$ | — | 104.589 | 104.589 | 104.589 | 102.601 | — |
| $x_c$ | — | +0.000 | +0.107 | +50.000 | +0.000 | — |
| $y_c$ | — | +0.000 | −1.145 | +0.000 | +0.000 | — |
| $z_c$ | — | +0.000 | +8.503 | +0.000 | +0.000 | — |
| $x_0$ | +0.107 | +0.097 | +0.107 | +4.359 | +0.100 | +0.000 |
| $y_0$ | −1.145 | −1.049 | −1.145 | −1.050 | −1.057 | +0.000 |
| $z_0$ | +8.503 | +7.781 | +8.503 | +7.777 | +7.793 | +0.000 |
| $J_2$ | +0.068726 | +0.062803 | +0.062864 | +0.063890 | +0.064671 | +0.053887 |
| $J_3$ | −0.006257 | −0.005294 | −0.005751 | −0.005586 | −0.005396 | +0.000000 |
| $J_4$ | −0.009600 | −0.008835 | −0.008781 | −0.008847 | −0.009927 | −0.006243 |
| $C_{20}$ | −0.030735 | −0.028086 | −0.028114 | −0.028572 | −0.028922 | −0.024099 |
| $C_{21}$ | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 |
| $S_{21}$ | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 |
| $C_{22}$ | +0.004771 | +0.004363 | +0.004363 | +0.005003 | +0.004390 | +0.003687 |
| $S_{22}$ | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 | +0.000000 |
| $C_{30}$ | +0.002376 | +0.002001 | +0.002174 | +0.002036 | +0.002040 | +0.000000 |
| $C_{31}$ | −0.000739 | −0.000671 | −0.000677 | −0.000643 | −0.000672 | −0.000000 |
| $S_{31}$ | +0.000169 | +0.000168 | +0.000154 | −0.000105 | +0.000174 | +0.000000 |
| $C_{32}$ | −0.000926 | −0.000828 | −0.000848 | −0.000871 | −0.000829 | +0.000000 |
| $S_{32}$ | +0.000174 | +0.000159 | +0.000159 | +0.000088 | +0.000159 | +0.000000 |
| $C_{33}$ | +0.000184 | +0.000167 | +0.000168 | −0.000002 | +0.000167 | +0.000000 |
| $S_{33}$ | +0.000521 | +0.000474 | +0.000476 | +0.000452 | +0.000488 | +0.000000 |
| $C_{40}$ | +0.003200 | +0.002945 | +0.002927 | +0.002949 | +0.003309 | +0.002081 |
| $C_{41}$ | +0.000674 | +0.000610 | +0.000616 | +0.000487 | +0.000614 | +0.000000 |
| $S_{41}$ | −0.000142 | −0.000131 | −0.000130 | −0.000553 | −0.000112 | +0.000000 |
| $C_{42}$ | −0.000141 | −0.000135 | −0.000129 | −0.000172 | −0.000156 | −0.000376 |
| $S_{42}$ | +0.000293 | +0.000270 | +0.000268 | +0.000258 | +0.000273 | +0.000000 |
| $C_{43}$ | −0.000521 | −0.000475 | −0.000476 | −0.000433 | −0.000475 | +0.000000 |
| $S_{43}$ | −0.000045 | −0.000038 | −0.000041 | −0.000130 | −0.000040 | +0.000000 |
| $C_{44}$ | +0.000151 | +0.000139 | +0.000138 | +0.000078 | +0.000138 | +0.000040 |
| $S_{44}$ | +0.000261 | +0.000238 | +0.000238 | +0.000230 | +0.000253 | +0.000000 |

Table 1: Coefficients of one of the scenarios. The top section lists the input parameters: density of the mantle $\rho_m$, density $\rho_c$ and radius $R_c$ of the core, coordinates $\{x_c, y_c, z_c\}$ of the center of the core. Then follows the resulting center of mass position $\{x_0, y_0, z_0\}$, the zonal harmonics coefficients $\{J_l\}$, and the normalized tesseral harmonics coefficients $\{C_{lm}, S_{lm}\}$, expressed at a reference radius $R_0 = 300$ km. Only the terms up to the 4th degree are listed, but for most numerical simulations all terms up to the 8th degree were included.
Figure 1: Section of Vesta, shape model data by Thomas et al. (1997a). It fits within the envelope a triaxial ellipsoid with axes 577, 563, and 478 km (dashed-dashed-dotted line), but deviates from that shape as a consequence of the nearly hemispheric impact crater covering the southern pole. The characteristics of the four scenarios with a core are displayed. The C0F20 scenario is characterized by a core at the origin, plus eight spherical fragments with radius 20 km equally spaced along the equator with the same density as the core. North is up and scale units is in km.
Figure 2: RMS of the Stokes coefficients of the gravitational expansion as a function of the degree. The EU scenario is drastically different from the others (the power at odd degrees is zero), and the CF20 scenario presents larger power terms at even degrees starting at degree 8, corresponding to a wavelength of $(\pi/4)R$ that is equal to the spacing of the fragments on the equatorial plane.
Figure 3: Offset between the integrated NEAR trajectory using the ORSA code and the nominal trajectory as retrieved from the SPICE kernels on PDS, decomposed on its three spherical components: tangent is parallel to the relative velocity vector, normal is orthogonal to the orbit plane.
Figure 4: Distance range as a function of the initial radius of a circular orbit, computed over a period of 50 days. The central mark in each bar represents the median of the range. The rotation period used for Vesta is of 5.3421288 hours (Thomas et al., 1997a). Five spin-orbit resonances have been identified and marked in the plot. The 1:1 resonance affects the largest interval in initial radius, but the strongest perturbations come from the 2:3 resonance. The leftmost data point, with initial radius of 370 km, reaches a lowest distance just below 300 km, only a few km away from Vesta’s surface. The orbital radius of HAMO and LAMO is also marked.
Figure 5: Altitude variability for HAMO and LAMO configurations, and for the spacecraft in 1:1 and 2:3 spin-orbit resonance. Each cloud samples Dawn’s trajectory over a period 50 days. The north-south asymmetry in Vesta’s shape forces an opposite asymmetry on Dawn’s orbit, which tends to get closer to the north pole than to the south. North is up and scale units is in km.
Figure 6: Radial range for all the different scenarios considered. With the exception of the EU scenario, all scenarios show a similar radial range independently of the internal mass distribution assumed. Only one initial condition per initial radius was considered for each scenario. For an analysis of the dependence on the initial conditions, see Figure 7.

Figure 7: Exploring the dependence to the initial conditions, we have uniformly sampled circular orbits between 370 and 1000 km at 30° intervals along the orbit. Each line corresponds to a constant initial orbiting phase. Results obtained using the U scenario, and confirmed for specific cases using the other scenarios considered in the manuscript.
Figure 8: Dynamics at the 1:1 resonance. Top, left: ground track of the spacecraft starting at 555 km, in the middle of the resonance, where the radial range is the minimum; right: orbit radius as a function of time for the 555 km simulation. Bottom, left: ground track when starting at 585 km, at the external limit of the resonance, where the radial range is the largest; right: orbit radius for the 585 km simulation. Zero longitude is at the longest equatorial radius of Vesta, so the 1:1 dynamics is characterized by a libration about the shortest equatorial radius of longitude 90° or 270°.
Figure 9: Orbital evolution of Dawn at LAMO, using the \( a \) scenario and a gravitational potential including all terms up to degree 20, over a period of 100 days.
Figure 10: Dynamics of the spacecraft spiraling in from a circular orbit of 1000 km radius. The thrust is of 20 mN in all simulations, while the initial position is shifted by 30° along the polar orbit. Only in one simulation out of twelve the spacecraft gets trapped in the 1:1 spin-orbit resonance between 500 and 600 km. All the other simulations reach the target 400 km orbit.
Figure 11: Escaping the 1:1. While Dawn is spiraling in thrusting at 20 mN it is captured into the 1:1 resonance (dotted line). A first tentative to escape (dashed line) consists in increasing thrusting to 35 mN at time 30.0 days, but this does not succeed, as the spacecraft is still trapped, but with a different phase. A second tentative (continuous line), thrusting to 35 mN at time 32.0 days and thus at a different phase, successfully moves the spacecraft out of the resonance, towards LAMO.
Figure 12: Spacecraft drift due to the terms of a given degree in the expansion of the gravitational potential. At HAMO (top) for degrees \{8, 10, 12, 14\}, and at LAMO (bottom) for degrees \{14, 16, 18, 20\}. The highest degree used in each curve is marked on the curve. In order to effectively detect a drift between modeled and observed spacecraft dynamics, the monitored period must not include any trajectory maneuver or thrusting.