Gauge invariant effective action
for high energy processes in QCD

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Abstract
The Born amplitudes for a quasi-multi-Regge kinematics of produced gluons are constructed in accordance with the Feynman rules including apart from usual Yang-Mills vertices also an infinite number of induced vertices. The new vertices describe the interaction of physical gluons produced in direct channels with the reggeized gluons propagating in crossing channels. The nonlinear gauge invariant effective action reproducing these Feynman rules is constructed with the use of the Wilson contour integrals. After integrating over the physical degrees of freedom the reggeon action is derived.

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1 Introduction

The modern theoretical description of the deep inelastic $ep$ scattering at small Bjorken variable $x = Q^2/(2mv)$ is based on the GLAP [1] and BFKL [2] evolution equations. The GLAP equation allows one to predict the $Q^2$ dependence of the parton distributions $n_i(x)$ if they are measured at a sufficiently large value of $Q = Q_0$. In turn, the BFKL equation determines their $x$ dependence in the small $x$ range.

The most important distribution at small $x$ is the inclusive gluon density $g(x,k_\perp)$ which depends apart from the longitudinal Sudakov component $x$ of the gluon momentum $k$ also on its transverse projection $k_\perp$ in the infinite momentum frame of the proton. $g(x,k_\perp)$ is proportional to the total gluon-proton scattering cross-section in the Regge regime $s = -k_\perp^2/x \gg -k_\perp^2$. In this region the most probable process is the multi-gluon production.

In each order of perturbation theory for gluon-gluon collisions at high c.m. energies $\sqrt{s}$ ($s = 2p_Ap_B$) the main contribution to the total cross-section $\sigma_{tot}$ results from the multi-Regge kinematics for final state gluon momenta $k_0 = p_{A'}$, $k_1, ..., k_n, k_{n+1} = p_{B'}$:

$$s \gg s_i = 2k_{i-1}k_i \Rightarrow t_i = q_i^2 = (p_A - \sum_{r=0}^{i-1} k_r)^2, \quad \prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} k_i^2, \quad k_\perp^2 = -k^2. \quad (1)$$

In the leading logarithmic approximation (LLA) the $n$-gluon production amplitude in this kinematics has the multi-Regge form [2]:

$$A_{2+n}^{LLA} = A_{2+n}^{tree} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}. \quad (2)$$

Here $s_i^{\omega(t_i)}$ are the Regge-factors appearing due to the known fact [2], that gluons in each crossing channel $t_i$ are reggeized if one takes into account the radiative corrections to the Born production amplitude $A_{2+n}^{tree}$. The gluon Regge trajectory in LLA is $j = 1 + \omega(t)$, where

$$\omega(t) = -\frac{g^2 N_c}{16\pi^3} \int d^2k \frac{q^2}{k^2(q-k)^2}, \quad t = -q^2. \quad (3)$$

The infrared divergencies in the Regge factors cancel in the total cross section with the contribution of the real gluons. The production amplitude in the tree approximation has the following factorised form

$$A_{2+n}^{tree} = 2sgT_{A_{\perp} A}^{\epsilon_1} \Gamma_{A'r} \frac{1}{t_1} g T_{e_2 e_2}^{d_1} \Gamma_{2i}^{r_1} \frac{1}{t_2} g T_{e_3 e_3}^{d_2} ... g T_{e_{n+1} e_{n+1}}^{d_n} \Gamma_{n+1 r}^{n} \frac{1}{t_{n+1}} g T_{B'B'}^{e_{n+1} e_{n+1}} \Gamma_{B'B}. \quad (4)$$

Here $A, B$ and $A', B', d_r$ ($r = 1, 2...n$) are colour indices for initial and final gluons correspondingly. $T_{\epsilon_{r'}}^r = -if_{abc}$ are generators of the gauge group $SU(N_c)$ in the self-conjugated representation, $g$ is the Yang-Mills coupling constant,

$$\Gamma_{A'r} = \delta_{\lambda A', \lambda A}, \quad \Gamma_{r+1,r}^{r} = C_\mu(q_{r+1}, q_r)e_\mu (k_r) \quad (5)$$
are the reggeon-particle-particle (RPP) and reggeon-reggeon-particle (RRP) vertices correspondingly; the quantity \( \lambda_r = \pm 1 \) is the helicity of the gluon \( r \) in the c.m. system. In LLA the \( s \)-channel helicities of colliding particles are conserved. Note that in higher orders of perturbation theory for the RPP vertex this is not the case [3]. The effective RRP vertex \( C(q_2, q_1) \) can be written as [2]

\[
C(q_2, q_1) = -q_1 - q_2 + p_A \left( \frac{q_1^2}{k_1 p_A} + 2 \frac{k_1 p_B}{p_A p_B} \right) - p_B \left( \frac{q_2^2}{k_1 p_B} + 2 \frac{k_1 p_A}{p_A p_B} \right).
\]

It has the important property corresponding to the current conservation

\[
(k_1)_\mu C_\mu(q_2, q_1) = 0,
\]

which gives us a possibility to chose an arbitrary gauge for each of the produced gluons. For example, in the left (l) light cone gauges, where \( p_A e^l(k) = 0 \) and \( k e^l(k) = 0 \), one can use the following parametrisation of the polarization vector \( e^l(k) \)

\[
e^l = e^l_\perp - \frac{k_\perp e^l_\perp}{k p_A} p_A
\]

in terms of the two dimensional vector \( e^l_\perp \). In this gauge the RRP vertex takes an especially simple form, if we introduce the complex components \( e = e_x + i e_y, e^* = e_x - i e_y \) and \( k = k_x + i k_y, k^* = k_x - i k_y \) for transverse vectors \( e^l_\perp, k_\perp \) [4]

\[
\Gamma^1_{2,1} = C e^* + C^* e, \quad C = \frac{q^*_1 q_2}{k^*_1}.
\]

This representation was used in [4] to construct an effective scalar field theory for the multi-Regge processes. The Lagrangian of this theory includes apart from the effective RRP vertex \( C \) also the RPP vertex \( \Gamma_{1,2} \) and the helicity conservation is reformulated here as the charge conservation. The effective action was derived recently from the Yang-Mills theory by integrating over the fields which correspond to the highly virtual particles produced in the multi-Regge kinematics in the direct channels \( s_i \) [5]. As a result of this integration the induced terms appear in the Lagrangian.

The action for the effective field theory [4] describing multi-Regge processes can be written in a form which is invariant under the abelian gauge transformations \( \delta V_\mu^a = i \partial_\mu \chi^a \) for the fields \( V^a_\mu \) describing the physical gluons provided that the fields \( A^a_\pm \) corresponding to the reggeized gluons are gauge invariant (\( \delta A^a_\pm = 0 \):

\[
S_{mR} = \int d^4 x \left\{ \frac{1}{4} (F^a_{\mu\nu})^2 + \frac{1}{2} (\partial_\perp A^a_\perp)(\partial_\perp A^a_\perp) +
\right.
\]

\[
+ \frac{1}{2} g \left\{ - A^a_\perp (F^-_\perp T^a i \partial_\perp^{-1} F^-_\perp) - A^a_\perp (F^+_\perp T^a i \partial_\perp^{-1} F^+_\perp) + (\partial_\perp^{-1} F^a_{\perp\sigma}) (A_- T^a i \partial_\sigma A_+) +
\right.
\]

\[
+ (\partial_\perp^{-1} F^a_{\perp\sigma}) (A_+ T^a i \partial_\sigma A_-) + \left( \frac{1}{i \partial_+} + \frac{1}{i \partial_-} F^a_{\perp\sigma} \right) (\partial_\sigma A_+) T^a_\perp (\partial_\perp A_+) + i F^a_{\perp\perp} (A_- T^a A_+) \right\},
\]

\[
(9)
\]
where \( F_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu} \) and \( N_{\pm} = N_0 \pm N_3 \) are the light-cone components of the vectors \( V_{\mu} \) in the c.m. system \((p_A^+ = p_B^+ = \sqrt{s})\). In the interaction part of action (9) two first terms correspond to the gluon scattering vertices and four last terms lead to the production vertex (6). In these contributions one can use the equations \( \partial_{\pm} A_{\mp} = 0 \) valid in the Regge kinematics. The scalar field version \([4]\) of the effective theory for the multi-Regge processes can be obtained from action (9) if one will use the light cone gauge (8) to eliminate from \( V \) non-physical degrees of freedom. To avoid the double counting of contributions of the Feynman diagrams one should remember, that the induced terms in the effective vertices appear as a result of the integration over momenta of highly virtual particles in the direct channels \([5]\). Below we shall construct a more general effective action invariant under nonabelian gauge transformations.

The total cross-section calculated in LLA using the above expressions for production amplitudes grows very rapidly as \( s^\omega \) (\( \omega = (g^2 N_c/\pi^2) \ln 2 \)) and violates the Froissart bound \( \sigma_{\text{tot}} < c \ln^2 s \) \([2]\). One of the possible ways of the unitarisation of the LLA results is to use the above effective field theory \([4,5]\).

Another more simple (but not so general) method is related to the solution of the BKP equation \([6]\) for multi-gluon compound states. This equation has a number of remarkable properties, including conformal invariance \([7]\), holomorphic separability \([8]\), and the existence of nontrivial integrals of motion \([9]\). The Hamiltonian for the corresponding Schrödinger equation coincides with the Hamiltonian for a completely integrable Heisenberg model with spins belonging to an infinite dimensional representation of the noncompact Möbius group \([10]\). It means, that the Bethe ansatz can be used for finding eigen values and eigen functions of this Schrödinger equation. Faddeev and Korchemsky have shown \([10]\), that for this purpose one should find the solution of the Baxter equation for an integer function \( Q(\lambda) \).

For two gluons the solution is known \([10]\). In the case \( n > 2 \) one can present \( Q(\lambda) \) as a linear combination of these solutions, which leads to a recurrence equation for its coefficients \( d_k \). For example, for \( n = 3 \) this equation coincides with the fundamental relation for the orthogonal polynomials \( d_k(A) \) of the discrete variable \( A \) being the eigen value of the quantum integral of motion obtained in ref.\([9]\):

\[
A d_k(A) = \frac{k(k + 1)(k - m + 1)(k + m)}{2(2k + 1)}(d_{k+1}(A) + d_{k-1}(A)).
\]

Here \( d_0 = 0, d_1 = 1 \) and the quantization condition for \( A \) at integer values of the conformal weight \( m \) is \( d_{m-1}(A) = 0 \) (for \( A \neq 0 \)).

The orthogonality and completeness conditions for these polynomials can be constructed:

\[
\sum_{A \neq k} \frac{d_k(A) d_{k_1}(A)}{d_{m-2}(A) d'_{m-1}(A)} = \delta_{kk_1} \frac{k(k + 1)(k - m + 1)(k + m)}{2(2k + 1)},
\]

\[
\sum_{k=1}^{m-2} \frac{2(2k + 1) d_k(A) d_k(A_1)}{k(k + 1)(k - m + 1)(k + m)} = \delta_{AA_1} d_{m-2}(A) d'_{m-1}(A), \ A \neq 0.
\]

Nevertheless, their theory is not developed enough to find analytically the wave function and the intercept of the perturbative Odderon.
All these results are based on calculations of effective Reggeon vertices and the gluon Regge trajectory in the first nontrivial order of perturbation theory. Up to now we do not know the region of applicability of LLA including the low boundary for the initial energy and the momentum scale for the QCD coupling constant. Note, however, that recently the Born production amplitudes for a quasi-multi-Regge kinematics of final particles were calculated [11] and one-loop corrections to the reggeon vertices were found [3], which will allow one to find next to leading corrections to the BFKL equation.

In this paper we want to generalize the effective field theory approach of ref. [4] to processes for which the final state particles are separated in several groups consisting of an arbitrary number of gluons with a fixed invariant mass; each group is produced with respect to others in the multi-Regge kinematics. These conditions are more general than the requirements for the quasi-multi-Regge kinematics of Ref. [11] where only one additional group consisting of two gluons was considered. We show that the effective action for such a generalised quasi-multi-Regge process can be written in a gauge invariant form in terms of the Wilson contour integrals. (The Wilson contour integrals were used by Ya. Balitsky in his talk at the Fermi Lab Small-x Meeting (September 1994) to justify the application of the operator product expansion to the Regge processes in QCD.)

2 Quasi-elastic processes

The gluon-gluon elastic scattering amplitude in the Born approximation can be written as follows (see [2], cf.(4)):

\[
A_2^{\text{tree}} = \frac{1}{2} g T_{A'A}^c \Gamma^{\nu\nu+}(p_{A'},p_A) e_\nu^* e_\nu \frac{1}{t} g T_{B'B}^c \Gamma^{\mu\mu-}(p_{B'},p_B) e_\mu^* e_\mu ,
\]

where the tensor \( \Gamma^{\nu\nu}(p_{A'},p_A) \) contains apart from the light-cone projection of the Yang-Mills (YM) vertex

\[
\gamma^{\nu\nu}(p_{A'},p_A) = \delta^{\nu\nu}(p_{A'} + p_A)^{\sigma} + \delta^{\nu\sigma} (-2p_A + p_{A'})^{\nu'} + \delta^{\nu\sigma} (-2p_A + p_{A'})^{\nu'}
\]

an additional induced term (cf.[4,5]):

\[
\Gamma^{\nu\nu+}(p_{A'},p_A) = \gamma^{\nu\nu+}(p_{A'},p_A) - t n^{+\nu} \frac{1}{p_A^+} n^{+\nu}.
\]

Here the light-cone vectors \( n^+, n^- \) and the light-cone components \( k^+, k^- \) for particle momenta are defined in the c.m. system \((p_A^+ = p_B^+ = 2E, s = 4E^2, p_A^- = p_B^- = 0)\) by the equations

\[
n^+ = \frac{p_B^+}{E}, n^- = \frac{p_A^-}{E}, k^+ = kn^+, k^- = kn^-, \]

where \( n^+ n^- = n^+ n_+ = 2 \) and \( \partial_+ x^+ = 2 \). These definitions differ slightly from notations used in [4,5]. Using the Ward identity for the YM vertex

\[
(p_{A'})_{\nu'} \gamma^{\nu\nu}(p_{A'},p_A) = (t - p_{A'}^2)\delta^{\nu\nu} + p_{A'}^{\nu} q^{\sigma} + p_{A'}^{\sigma} p_A^{\nu}
\]
we obtain the corresponding identity for the effective vertex \( \Gamma^\nu\nu^+ \)

\[
(p_{A'})_\nu \Gamma^\nu\nu^+(p_{A'}, p_A) = -p_A^2 (n^\nu) + p_A^\nu(p_A)^\nu, \tag{15}
\]

where we neglected the small terms of the order of \((p_A q)/(p_B p_A)\). If the gluon \( A \) is on mass shell \((p_A^2 = 0)\) and its polarization vector \( e(p_A) \) satisfies the Lorentz condition \( p_A e = 0 \), we conclude from (15) that the effective vertex is gauge invariant \( (p_{A'})_\nu \Gamma^\nu\nu^+ = 0 \). This means that one can use an arbitrary gauge for the polarization vectors \( e \).

In the light cone gauge \( p_B e(p_A) = p_B e(p_{A'}) = 0 \) we obtain

\[
e_{\nu}(p_A') e_{\nu}(p_A) \Gamma^\nu\nu^+ = -4E(e_{\nu}'e_A) = -4E\delta_{\lambda\lambda',\lambda\lambda} \cdot \tag{16}
\]

Therefore taking into account relations (5) one can verify that (10) coincides with (4) for the elastic case \( n = 0 \).

Let us consider now the quasi-elastic process in which the final state contains apart from the particle \( B' \) with momentum \( p_{B'} \approx p_B \) also several gluons with a fixed invariant mass in the fragmentation region of the initial gluon \( A \). It is convenient to denote the colour indices of the produced gluons by \( a_1, a_2, ... a_n \) leaving the index \( a_0 \) for the particle \( A \). Further, the momenta of the produced gluons and of the particle \( A \) are denoted by \( k_1, k_2, ... k_n \) and \( -k_0 \) correspondingly. \( q = -\sum_{i=0}^{n} k_i \) is the momentum transfer. Omitting the polarization vectors \( e_{\nu}(k_i) \) for the gluons \( i = 0, 1, ... n \) we can write the production amplitude related with the single gluon exchange in the tensor representation

\[
A_{a_0a_1...a_nB'B} = -\phi_{a_0a_1...a_n}^+ \frac{1}{(2\pi)^4 g p_B^\nu B'B^\nu B} \delta_{\lambda\lambda',\lambda\lambda} \cdot \tag{17}
\]

Here the form-factor \( \phi \) depends on the invariants constructed from the momenta \( k_0, ... k_n \).

For the simplest case of one gluon production \( \phi \) was calculated in the Born approximation in [11]. We present this result in the form (cf. Appendix)

\[
\phi_{a_0a_1a_2c}^\nu\nu^+ = g^2 \left\{ \Gamma_{a_0a_1a_2c}^\nu\nu^+ - T_{a_1a_0} T_{a_2a} \frac{\gamma^{\nu\mu\sigma}(k_1, -k_0) \Gamma^\nu\mu\sigma(k_2, k_2 + q)}{(k_0 + k_1)^2} \right. - \\
- T_{a_2a_0} T_{a_1a} \frac{\gamma^{\nu\mu\sigma}(k_2, -k_0) \Gamma^\nu\mu\sigma(k_1, k_1 + q)}{(k_0 + k_2)^2} - \\
- T_{a_2a_0} T_{a_1a} \frac{\gamma^{\nu\mu\sigma}(k_2, -k_1) \Gamma^\nu\mu\sigma(k_0, k_0 + q)}{(k_1 + k_2)^2} \right\}. \tag{18}
\]

The last three terms in the brackets correspond to the Feynman diagram contributions constructed from the gluon propagator combining the usual Yang-Mills vertex (11) and the effective RPP vertex (12). The first term can be written as

\[
\Gamma_{a_0a_1a_2c}^\nu\nu^+ = \Lambda_{a_0a_1a_2c}^\nu\nu^+ + \Delta_{a_0a_1a_2c}^\nu\nu^+, \tag{19}
\]
where $\gamma$ is the light-cone projection of the usual quadri-linear Yang-Mills vertex

$$\gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = T^a_{a_1a_0} T^c_{a_2a}(\delta^{\nu_0 \nu_2} \delta^{\nu_1+} - \delta^{\nu_1+} \delta^{\nu_0 \nu_2} +$$

$$+ T^a_{a_2a_0} T^c_{a_1a}(\delta^{\nu_2\nu_1} \delta^{\nu_0+} - \delta^{\nu_2+} \delta^{\nu_0 \nu_1}) + T^a_{a_2a_1} T^c_{a_0a}(\delta^{\nu_2\nu_0} \delta^{\nu_1+} - \delta^{\nu_2+} \delta^{\nu_1\nu_0})$$

(20)

and $\Delta$ is a new induced vertex

$$\Delta^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c}(k_0^+, k_1^+, k_2^+) = -t (n^+)\nu_0(n^+)\nu_1(n^+)\nu_2\frac{T^a_{a_2a_0} T^c_{a_1a}}{k_1^+ k_0^+} + \frac{T^a_{a_2a_1} T^c_{a_0a}}{k_2^+ k_0^+}.$$ 

(21)

Note that due to the Jacobi identity

$$T^a_{a_0a} T^c_{a_1a} - T^a_{a_0a} T^c_{a_2a} = T^a_{a_1a} T^c_{a_0a},$$

(22)

and the momentum conservation law

$$k_0^+ + k_1^+ + k_2^+ = 0$$

(23)

which is valid in the quasi-elastic kinematics at large $s$ the tensor $\Delta$ is Bose-symmetric with respect to the simultaneous transmutation of momenta, colour and Lorentz indices of the gluons 0, 1, 2.

Using the Ward identities (14) and (15) for vertices $\gamma$ and $\Gamma$ one can verify that for $n = 2$ the amplitude (17) is gauge invariant

$$(k_i)_{\nu_i} A^{\nu_0\nu_1\nu_2} = 0 , i = 0, 1, 2.$$ 

(24)

Let us include the colour matrices in the trilinear YM vertices (see (11), (12)):

$$\gamma^{\nu_0\nu_1\sigma}_{a_0a_1c}(k_1, -k_0) = T^c_{a_1a_0} \gamma^{\nu_1\nu_0\sigma}(k_1, -k_0) , \Gamma^{\nu_0\nu_1+}_{a_0a_1c}(k_1, -k_0) = T^c_{a_1a_0} \Gamma^{\nu_1\nu_0+}(k_1, -k_0).$$

(25)

Then we obtain similarly to (19)

$$\Gamma^{\nu_0\nu_1+}_{a_0a_1c} = \gamma^{\nu_0\nu_1+}_{a_0a_1c} + \Delta^{\nu_0\nu_1+}_{a_0a_1c},$$

(26)

where, according to (12), we have

$$\Delta^{\nu_0\nu_1+}_{a_0a_1c}(k_0^+, k_1^+) = -t T^c_{a_1a_0} (n^+)\nu_1 \frac{1}{k_1^+} (n^+)\nu_0 , k_0^+ + k_1^+ = 0.$$ 

(27)

In the general case $n > 2$ for the gauge invariance of $\phi$ (17) one should take into account apart from the usual YM vertices $\gamma_3, \gamma_4$ and corresponding effective vertices (26,19) also the effective vertices $\Gamma$ coinciding with the induced vertices $\Delta$ for an arbitrary number $r > 3$ of external legs

$$\Gamma^{\nu_0\nu_1...\nu_{r-1}+}_{a_0a_1...a_{r-1}c}(k_0^+, k_1^+, ...k_{r-1}^+) = \Delta^{\nu_0\nu_1...\nu_{r-1}+}_{a_0a_1...a_{r-1}c}(k_0^+, k_1^+, ...k_{r-1}^+) .$$ 

(28)

Let us consider the contribution of all Feynman diagrams constructed from such induced vertex combined by a gluon propagator with the usual YM vertex $\gamma$ (25) describing the
interaction with an external gluon having the momentum $k_r$ (cf. (18)). Multiplying the corresponding expression for $\phi$ in eq. (17) by $k_r^\nu$ and using the Ward identity (14) for $\gamma$ we obtain the non-vanishing sum of terms:

$$- \sum_{i=0}^{r-1} T_{a_i a_1}^a \Delta_{a_0 a_1 ... a_{i+1} a_i}^{\nu_0 ... \nu_{r-1}+1}(k_0^+, ..., k_{i-1}^+, k_i^+, k_{r+1}^+, ..., k_{r-1}^+).$$

(29)

To compensate it by higher order contributions the infinite set of the quantities $\Delta$ should satisfy the recurrence relation:

$$\Delta_{a_0 a_1 ... a_r}^{\nu_0 ... \nu_r+1}(k_0^+, k_1^+, ..., k_r^+) =$$

$$\frac{(n^+)^{\nu_r}}{k_r^+} \sum_{i=0}^{r-1} T_{a_i a_1}^a \Delta_{a_0 a_1 ... a_{i+1} a_i}^{\nu_0 ... \nu_{r-1}+1}(k_0^+, ..., k_{i-1}^+, k_i^+, k_{r+1}^+, ..., k_{r-1}^+).$$

(30)

These induced vertices are invariant under arbitrary transmutations of indices $i$:

$$\Delta_{a_0 a_1 ... a_r}^{\nu_0 ... \nu_r+1}(k_0^+, k_1^+, ..., k_r^+) = \Delta_{a_0 a_1 ... a_r}^{\nu_0 ... \nu_r+1}(k_0^+, k_1^+, ..., k_r^+)$$

(31)

due to the Jacobi identity (22) for colour group generators $T$ and the energy-momentum conservation

$$\sum_{i=0}^{r} k_i^+ = 0.$$  

(32)

By constructing $\phi$ in (17) according to the Feynman rules including apart from the usual Yang-Mills vertices $\gamma$ also the effective vertices $\Gamma$ we obtain for it a gauge invariant expression.

### 3 Multi-gluon production in the central region

In the previous section we considered a generalization of the effective vertices $\Gamma^c_{A' A} = T^c_{A' A} \Gamma_{A' A}$ in eq. (4) to the case of the quasi-elastic processes. Here we want to generalize the effective vertices $\Gamma^d_{c_2 c_1} = T^d_{c_2 c_1} \Gamma_{c_2 c_1}$ for the multi-gluon production. For simplicity we discuss the following kinematics of the final state particles: the gluons $A'$ and $B'$ move almost along the momenta of the initial particles $A$ and $B$ and there is a group of produced gluons with a fixed invariant mass in the central region $y \sim 0$ of the rapidity $y = \frac{1}{2} \ln(k^+ / k^-)$. The momentum transfers $q_1 = p_A - p_{A'}$ and $q_2 = p_{B'} - p_B$ in this regime have (with a good accuracy) the Sudakov decomposition

$$q_1 = q_{1\perp} + \beta p_A, \quad q_2 = q_{2\perp} - \alpha p_B$$

(33)

where $\beta$ and $\alpha$ are the Sudakov parameters of the total momentum $k = \sum_{i=1}^{n} k_i$ of the produced gluons:

$$k = k_{\perp} + \beta p_A + \alpha p_B, \quad \kappa = k^2 = s\alpha\beta + (q_1 - q_2)_{\perp}$$

(34)
and $\sqrt{\kappa}$ is their invariant mass which is assumed to be fixed at high energies: $\kappa \ll s$.

In this kinematical region the production amplitude has the factorized form (cf. (4) and (17))

$$A_{d_1 d_2 \ldots d_n Ab^c b^d}^{\nu_1 \nu_2 \ldots \nu_n+} = -g P^+_A T_{A'}^{c_1} \Gamma_{AA'}^{\nu_1 \nu_2 \ldots \nu_n+} \frac{1}{t_1} \psi_{d_1 d_2 \ldots d_n a c_{2c_1}}^{\nu_1 \nu_2 \ldots \nu_n+} \frac{1}{t_2} g P^-_{B'} T_{B'B}^{c_2} \Gamma_{B'B}^{c_2}. \quad (35)$$

For the case of one gluon emission we have from eq.(6)

$$\psi_{d_1 c_{2c_1}}^{\nu_1+} = g \Gamma_{d_1 c_{2c_1}}^{\nu_1+}, \quad (36)$$

where $\Gamma$ is the sum of two terms

$$\Gamma_{d_1 c_{2c_1}}^{\nu_1+} = \gamma_{d_1 c_{2c_1}}^{\nu_1+} + \Delta_{d_1 c_{2c_1}}^{\nu_1+}. \quad (37)$$

The first term is the contribution from the tri-linear Yang-Mills vertex (see (11))

$$\gamma_{d_1 c_{2c_1}}^{\nu_1+} = T_{c_{2c_1}}^{d_1} \gamma^{\nu_1+}, \quad \gamma = 2(q_2 + q_1) - 2k_1^{+}n^- + 2k_1^{-}n^+. \quad (38)$$

The second term is the induced one

$$\Delta_{d_1 c_{2c_1}}^{\nu_1+} = T_{c_{2c_1}}^{d_1} \Delta^{\nu_1+}, \quad \Delta = -2t_1 \frac{n^-}{k_1^{+}} + 2t_2 \frac{n^+}{k_1^{-}}. \quad (39)$$

Due to the relation (see (6))

$$\gamma + \Delta = -2C, \quad (40)$$

expression (35) coincides with (4) for the particular case $n = 1$.

Let us consider now the production of two gluons in the central region. The amplitude of this process in the Born approximation was calculated in ref. [11]. The result can be written in the form of representation (35) where the tensor $\psi$ is (see Appendix):

$$\psi_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+} = g^2 \{ \Gamma_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+} - \frac{T^{d_1 d_2 1} \Gamma_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+}(k_2, -k_1)}{(k_1 + k_2)^2} \} -$$

$$- \frac{\Gamma_{d_1 d_2 c_{1c_2}}^{\nu_1 \nu_2+}(k_1, k_1 - q_1)}{(q_1 - k_1)^2} - \frac{\Gamma_{d_2 d_3 c_{2c_1}}^{\nu_1 \nu_2+}(k_2, k_2 + q_2)}{(k_2 + q_2)^2} - \frac{\Gamma_{d_2 d_3 c_{2c_1}}^{\nu_1 \nu_2+}(k_2, k_2 - q_1)}{(q_2 - k_1)^2} \} \quad (41)$$

The second term in the brackets describes the production of a pair of gluons through the decay of the virtual gluon in the direct channel. This contribution is a product of the effective vertex $\Gamma$, the usual YM vertex $\gamma$ and the gluon propagator. Analogously, the third and fourth contributions are products of two effective vertices (25) having the light cone components $\pm$ and the gluon propagator in the crossing channels.

The first term in the brackets is not expressed in terms of the effective vertices which appear in LLA. It can be presented as the sum of two terms

$$\Gamma_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+} = \gamma_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+} + \Delta_{d_1 d_2 c_{2c_1}}^{\nu_1 \nu_2+}. \quad (42)$$
where the contribution $\gamma$ is the light cone component of the quadri-linear Yang-Mills vertex (cf. (20))

$$\gamma_{d_1d_2c_1}^{\nu_1\nu_2} = T_{d_1c_1}^d T_{d_2}^{c_2} (\delta^{\nu_1\nu_2} \delta^{+-} - \delta^{\nu_1+\nu_2-}) +$$

$$+ T_{d_2c_1}^d T_{d_1}^{c_2} (\delta^{\nu_1+\nu_2-} - \delta^{\nu_2+\nu_1-}) + T_{d_2d_1}^d T_{c_1d}^{c_2} (\delta^{\nu_2-\nu_1+} - \delta^{\nu_2+\nu_1-})$$

(43)

and $\Delta$ is the new induced vertex (cf. (21))

$$\Delta_{d_1d_2c_1}^{\nu_1\nu_2} = -2 t_2 (n^+)^{\nu_1} (n^-)^{\nu_2} \left\{ \frac{T_{d_1c_1}^d T_{d_2}^{c_2}}{k_1^+ k_2^+} + \frac{T_{d_2d_1}^d T_{c_1d}^{c_2}}{(-k_1^- - k_2^+) k_2^+} \right\} -$$

$$- 2 t_1 (n^-)^{\nu_1} (n^-)^{\nu_2} \left\{ \frac{T_{d_1c_1}^d T_{d_2}^{c_2}}{k_2^- k_1^-} + \frac{T_{d_2d_1}^d T_{c_1d}^{c_2}}{(-k_1^- - k_2^+) k_1^-} \right\}. \quad (44)$$

One can verify that $\Delta$ is Bose invariant and symmetric with respect to the simultaneous transmutation of $n^-, t_1, d_1$ and $n^+, t_2, d_2$.

For the gauge invariance of the production amplitude in the case $n > 2$ one should introduce the induced vertices with an arbitrary number of external legs which are expressed in terms of the light cone projections of the vertices (28)

$$\Gamma^{\nu_1...\nu_n}_{d_1...d_n c_1} = \Delta^{\nu_1...\nu_n}_{c_1 d_1...d_n} (k_0^+ k_1^+ ... k_n^+) + \Delta^{\nu_1...\nu_n}_{c_2 d_1...d_n} (k_0^- k_1^- ... k_n^-), \quad (45)$$

where $k_0^+, k_0^-$ are determined by the momentum conservation

$$\sum_{i=0}^n k_i^+ = \sum_{i=0}^n k_i^- = 0. \quad (46)$$

In the next sections we consider the gauge-invariant functional formulation of the effective field theory for a general kinematics of the final state particles in high energy processes.

4 Field description of particles and reggeons in the Yang-Mills theory

Due to the gluon reggeization it is natural to expect that QCD at high energies can be reformulated as an interaction theory for physical particles (quarks and gluons) and reggeized gluons. Furthermore, after integration over physical degrees of freedom one should develop the Reggeon field theory analogous to the Pomeron calculus which was invented by V. Gribov many years ago [12]. In comparison with the previous publications [4,5] devoted to the multi-Regge processes in this paper we attempt to construct the Reggeon calculus for the gluodynamics in a form being similar to one of ref. [12] starting from the action for physical gluons interacting with the reggeons having fixed $j = 1$. The
gluon Regge trajectory \( j = j(t) \) appears as an one-loop correction to the effective action for the bare reggeons.

It is convenient for us to use the matrix representation of the Yang-Mills field \( v^a_\mu \).

\[
v_\mu = t^a v^a_\mu,
\]

(47)

where the anti-hermitial matrices \( t^a \) satisfy the commutation relations:

\[
[t^a, t^b] = f^{abc} t^c.
\]

(48)

Here \( f^{abc} \) are the structure constants and \( T^a = i t^a \) are the generators of the colour group \( SU(N_c) \) in the fundamental representation.

The virtual gluons in crossing channels, which lead to the Coulomb-like interaction between the particles with a big difference in their rapidities \( y = \frac{1}{2} \ln(k^+/k^-) \) and which are reggeized after taking into account radiative corrections, are described by the fields related with the light-cone components of the vector-potential \( v_\mu \). Indeed, only the longitudinal part \( \sim \delta_{\| \mu \nu} \) of the gluon propagator gives a big contribution proportional to \( s \). In accordance with ref. [4] we denote these reggeon fields by \( A_{\pm} \). \( A_{\pm} \) belong to the adjoint representation of the group \( SU(N_c) \) but they are considered to be invariant under the gauge transformations for which the local parameters \( \chi(x) \) decrease at large \( x \).

The particles which are produced in direct channels can be arranged in the groups consisting of gluons within some rapidity intervals \((y - \eta/2, y + \eta/2)\), where the auxiliary parameter \( \eta \) is chosen to be numerically big but significantly smaller than the relative rapidity of colliding particles \( Y = \ln s \)

\[
1 \ll \eta \ll Y.
\]

(49)

For the interactions among particles inside of each group the introduced parameter \( \eta \) is an analog of the ultraviolet cut-off in the relative longitudinal momenta. For the interactions between the neighbouring groups \( \eta \) plays the role of an infrared cut-off. The \( \eta \)-dependence should disappear in the final result analogously to the case of the normalization point dependence in hard processes. In the leading logarithmic approximation all transverse momenta \( k_\perp \) of gluons in the Feynman diagrams are of the same order of value as transverse momenta \( p_\perp \) of partons inside of colliding hadrons [2]. This means that the c.m. pair energy \( \sqrt{s_i} \) of the neighbouring gluons in the multi-Regge kinematics is significantly bigger than \( p_\perp \) and the effective parameter of the perturbation theory is \( g^2 \ln(s/p_\perp^2) \). Beyond LLA one should introduce the above parameter \( \eta \) in the analogous inequality for the pair energies of the neighbouring clusters of particles

\[
\ln \frac{s_i}{p_\perp^2} > \eta.
\]

We include this condition in the bare propagator of the reggeon fields \( A_{\pm} \) following the Gribov approach [12] for the pomeron case

\[
<A_{\pm}^\nu(z^\perp, \rho) A_{\pm}^{\mu'}(0, 0) > \sim \theta(y' - y - \eta) \delta^2(z) \ln |\rho|.
\]

(50)

Here \( z^\perp \) and \( \rho \) are correspondingly the light-cone and transverse components of the gluon coordinate \( x_\mu \). We took into account also that in the quasi-multi-Regge kinematics the
Reggeon momenta are transverse vectors \( q^2 = q_\perp^2 \) and in the effective vertices one can consider \( A_+ (A_-) \) as the fields independent of \( z_- (z_+) \) (cf. [4]):

\[
\partial_- A_+ = \partial_+ A_- = 0 . \tag{51}
\]

The vector-potential \( v \) can be presented in a symbolic form as the following sum of its components \( V^y \) and \( A^y_\pm \) describing correspondingly the gluons in the direct and crossing channels with the rapidities \( y \) inside of the interval \( \eta \):

\[
v = \sum_y (V^y + A^y) . \tag{52}
\]

Such representation is natural for the case, when \( V \) and \( A_\pm \) have the same transformation properties. However, according to our agreement \( V \) and \( A_\pm \) are transformed differently under the gauge group. Using the freedom of choosing an arbitrary parametrization of \( v \) in terms of \( V \) and \( A_\pm \) we shall modify later expansion (52) to satisfy the requirement of the gauge invariance (see eq. (75)).

Because the interaction is local in the rapidity except of the long-range correlation between \( A_+ \) and \( A_- \) in (50), we shall omit the index \( y \) for all fields further on. The usual Yang-Mills action for the gluons inside of each rapidity interval \( (y - \eta, y + \eta) \) after using decomposition (52) takes the form:

\[
S_{YM} = -\int d^4x \text{tr} \left\{ \frac{1}{2} G^2_{\mu\nu} - [D_\mu, G_{\mu\nu}] A_+ - [D_\nu, G_{\nu\mu}] A_- + [D_\mu, A_+] [D_\mu, A_-] - \frac{1}{2} [D_-, A_-] [D_+, A_+] + \frac{g}{2} G_{\mu\nu} [A_-, A_+] - \frac{1}{4} [D_+, A_-]^2 - \frac{1}{4} [D_-, A_+]^2 \right. \\
- \frac{1}{2} [D_-, A_-] [D_+, A_+] + \frac{g}{2} G_{\mu\nu} [A_-, A_+] - \frac{1}{4} [D_+, A_-]^2 - \frac{1}{4} [D_-, A_+]^2 \\
+ \frac{g}{2} [D_+, A_-] [A_-, A_+] + \frac{g}{2} [D_- A_+] [A_+, A_-] - \frac{g^2}{4} [A_+, A_-]^2 \right\} , \tag{53}
\]

where \( D_\mu \) and \( G_{\mu\nu} \) are respectively the covariant derivative and field strength tensor of the Yang-Mills field

\[
D_\mu = \partial_\mu + g V_\mu , \quad G_{\mu\nu} = \frac{1}{g} [D_\mu, D_\nu] = \partial_\mu V_\nu - \partial_\nu V_\mu + g [V_\mu, V_\nu] . \tag{54}
\]

The infinitesimal gauge transformations of \( V \) and \( G \) are

\[
\delta V_\mu = [D_\mu, \chi] , \quad \delta G_{\mu\nu} = g [G_{\mu\nu}, \chi] , \tag{55}
\]

where \( \chi \) is a small local parameter. Action (53) describes various interactions of produced particles and reggeons. But we know from the previous sections that one has to add to this action extra terms to reproduce the induced vertices (28) and (45). They correspond to the coherent emission of gluons belonging to a given rapidity interval by neighbouring groups of particles and are needed in particular to provide the gauge invariance property of \( A_\pm \) in accordance with the modified version (75) of representation (52). We construct these induced terms in the next sections.
5 Effective action for quasi-elastic processes

To begin with, let us consider in the effective action the linear terms which describe quasi-elastic processes:

\[ S_1 = -\int d^4x \, tr \left[ j_- A_+ + j_+ A_- \right]. \]  \tag{56}

Here each coefficient \( j_\pm \) is the sum of a modified Yang-Mills current \( j_{\pm}^{\text{YM}} \) and an induced contribution \( j_{\pm}^{\text{ind}} \):

\[ j_\pm = j_{\pm}^{\text{YM}} + j_{\pm}^{\text{ind}}. \]  \tag{57}

The Yang-Mills current \( j_{\pm}^{\text{YM}} \) appearing in action (53) is covariant under the gauge transformations (25)

\[ j_{\pm}^{\text{YM}} = -\left[ D_\mu, G_{\mu\pm} \right], \quad \delta j_{\pm}^{\text{YM}} = g\left[ j_{\pm}^{\text{YM}}, \chi \right]. \]  \tag{58}

It vanishes classically due to the equations of motion \( \delta G_{\mu\nu}^2 = 0 \) being valid in the tree approximation. The modified current \( j_{\pm}^{\text{YM}} \) is invariant under gauge transformations (see (61)) and vanishes also on the mass shell.

The induced currents \( j_{\pm}^{\text{ind}} \) depend only on \( V_- \) and \( V_+ \), respectively. Let us consider for example \( j_{\pm}^{\text{ind}} \) which describes the gluon production along \( p_A \). Using the recurrence relations (30) being equivalent to the requirement of gauge invariance of the current \( j_{\pm}^{\text{ind}} \), one can construct \( j_{\pm}^{\text{ind}} \) as a series in the QCD coupling constant

\[ j_{\pm}^{\text{ind}}(V_-) = \partial_{\perp\sigma}^2 \{ V_- - gV_- \frac{1}{\partial_-} V_- + g^2 V_- \frac{1}{\partial_-} V_- \frac{1}{\partial_-} V_- - ... \} = \partial_{\perp\sigma}^2 \partial_- \frac{1}{D_-} V_-, \]  \tag{59}

where the transverse Laplacian \( \partial_{\perp\sigma}^2 \) corresponds to the common factor \( q^2 = -t \) in the induced vertices (21, 27) for the quasi-elastic processes. The infinitesimal gauge transformation of \( j_{\pm}^{\text{ind}} \) is given by

\[ \delta j_{\pm}^{\text{ind}} = -\partial_{\perp\sigma}^2 \{ \partial_- \frac{1}{D_-} \chi \partial_- - \partial_- \chi \frac{1}{D_-} \partial_- \} = \partial_{\perp\sigma}^2 \partial_- \left[ \chi, \frac{1}{D_-} \right] \partial_- \]  \tag{60}

Using the smallness of \( \partial_{\perp} A_{\perp} \) (see (51)), we obtain from (60) in accordance with recurrence relations (30), that effectively \( \delta j_{\pm}^{\text{ind}} = 0 \) up to the terms giving a vanishing contribution in expression (56) after integrating by parts for \( \chi \) which decreases as \( x^\pm \to \infty \). Therefore for gauge invariance of \( S_1 \) (56) one should consider \( A_{\pm} \) as gauge invariant fields. In accordance with the invariant properties of \( j_{\pm}^{\text{ind}} \) one has to modify the Yang-Mills current (58) as follows

\[ j_{\pm}^{\text{YM}} = U^{-1}(V_\pm) j_{\pm}^{\text{YM}} U(V_\pm), \quad U(V_\pm) = 1 - g \frac{1}{\partial_\pm} V_\pm + g^2 \left( \frac{1}{\partial_\pm} V_\pm \right)^2 - ... = \frac{1}{D_\pm} \partial_\pm. \]  \tag{61}

Here \( U(V_-) \) is the matrix transformed according to the fundamental representation of the gauge group \( \delta U = -g\chi U \), which provides \( \delta j_{\pm}^{\text{YM}} = 0 \). Such a modification is possible,
because both $j^{YM}$ and $j^{mYM}$ are vanishing due to the equations of motion and lead therefore to the same production amplitude in the tree approximation as discussed in section 2. In particular this yields the cancellation of the poles $1/\partial^k_\pm$ in $j^{mYM}_\pm$ which otherwise would contradict the requirement of absence of simultaneous singularities of the production amplitudes in the overlapping channels $s_i$ and $t$. Note that this requirement is fulfilled for $j^{ind}$ (59) due to the common factor $\partial^2_\perp \sigma$ which cancels the pole $1/t$.

Thus, under the gauge transformation (55) with $\chi$ decreasing as $x_\pm \to \infty$ the total current (57) is invariant

$$\delta j_\pm = 0.$$ 

up to total derivatives $\partial_\pm$ giving due to (51) a negligible contribution to (56) for $\chi$ decreasing at $x_\pm \to \infty$. After integrating by parts in (56) the terms $\sim g^0$ in eqs. (59) and (61) cancel and the perturbative expansion for the total current $j$ starts with the contribution quadratic in $V$ and corresponding to effective vertex (12)

$$j_- = g\{[V_\mu, \partial_- V_\mu] - [\partial_\mu V_\mu, V_-] - 2[V_\mu, \partial_\mu V_-] - \partial_\mu^2 V_- \frac{1}{\partial_-} V_- - [j^{YM}_-, \frac{1}{\partial_-} V_-]\} +$$

$$+ g^2 \{[V_\mu, [V_-, V_\mu]] + \partial_\mu^2 V_- \frac{1}{\partial_-} V_- \frac{1}{\partial_-} V_- + [j^{YM}_-, \frac{1}{\partial_-} V_-] \frac{1}{\partial_-} V_- \} + O(g^3), \quad (62)$$

where

$$j^{YM}_- = \partial_\mu \partial_- V_\mu - \partial_\mu^2 V_- + g \{[V_\mu, \partial_- V_\mu] - [\partial_\mu V_\mu, V_-] - 2[V_\mu, \partial_\mu V_-] - g^2 [V_\mu, [V_\mu, V_-]] \} \quad (63)$$

vanishes due to the equations of motion valid for quasi-elastic amplitudes in the tree approximation. Note that the quantity $A^\perp_\pm$ can be considered as a classical component of the Yang-Mills field $v$. We shall discuss this possibility later.

The singular coefficients $1/\partial^k_\pm$ in expansions (59) and (61) are integral operators with unspecified boundary conditions. A similar problem occurs in the Feynman diagram approach to gauge theories if one uses the light cone gauge $V_- = 0$ because the gluon propagator in this gauge is proportional to the tensor

$$\Delta^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{\partial_-} (n^\mu \partial^\nu + n^\nu \partial^\mu), \quad (64)$$

containing the pole at $\partial_- = 0$. For this pole one can use the Mandelstam-Leibbrandt prescription [13]

$$\frac{1}{\partial_-} \to \frac{\partial_+}{\partial_+ \partial_- - i\epsilon}. \quad (65)$$

In our case due to the infrared cut-off (50) in the quasi-multi-Regge kinematics these singularities are absent in the integration region. The appearence of the poles in $\partial_-$ is related with the fact, that effective action (56) is nonlocal because it is expressed in terms of the Wilson contour integrals which depend generally on integration paths.
To clarify the last assertion we shall derive the first term of action (56) using another method. To begin with, note that in the right light cone gauge $V^r = 0$ where gluon propagators are proportional to tensor (64) all induced interactions are not essential and we can calculate the quasi-elastic amplitudes using the first two terms of the usual Yang-Mills action (53). Furthermore, the vector-potential $V^r$ in this gauge can be expressed in terms of the vector-potential $V$ in an arbitrary gauge by the following gauge transformation:

$$V^r_\mu = U^{-1}(V_-) \left(V_\mu + \frac{1}{g} \partial_\mu \right) U(V_-), \quad U(V_-) = P \exp \left(-\frac{g}{2} \int_{-\infty}^{x^-} dx^- V_-ight),$$

where the operator $P$ implies the ordering of colour matrices in the Wilson contour integral $U(V_-)$ in accordance with the increasing of the field arguments $x^-$. One can write down the operator representation for $U(V_-)$ and $U^{-1}(V_-)$:

$$U(V_-) = \frac{1}{1 + g \partial^{-1} V_-}, \quad U^{-1}(V_-) = 1 + g \frac{1}{\partial_-} V_-, \quad (67)$$

where it is implied, that $U$ and $U^{-1}$ act on the unit constant matrix from the left and right hand sides correspondingly.

The total current $j_-$ in the right gauge contains only the term bilinear in fields $V^r$:

$$j_- = j_{YM}^-(V^r) - \partial_\sigma \partial_- V^r_\sigma = g \left[V^r_\sigma, \partial_- V^r_\sigma \right], \quad (68)$$

where $j_{YM}^-$ is given in eq.(58).

Inserting expression (66) for $V^r_\sigma$ in the representation (68) and using the gauge covariance of $j_{YM}^-$ (58), we obtain the total current in the gauge invariant form:

$$j_- = U^{-1}(V_-) j_{YM}^- U(V_-) - \partial_\sigma \partial_- V^r_\sigma. \quad (69)$$

This result can be transformed to (57). Indeed, the first term is $j_{YM}^{µσ}(61)$ and the second term can be reduced to $j_{ind}^-$ (59) using the smallness of the factor $\partial_-$ after integration by parts in eq.(56) (see (51)) if it is not compensated by the singularity $1/\partial_-$ in expression (66) for $V^r_\sigma$:

$$- \partial_\sigma \partial_- V^r_\sigma \rightarrow -\partial_\perp V^r_\perp \frac{1}{g} \partial_\perp U(V_-) = j_{ind}^-(V_-). \quad (70)$$

If the equations of motion for $V$ are fulfilled the total current $j_\pm$ in (69) coincides with $j_{ind}^\pm$ (70):

$$j_\pm \rightarrow -\partial_\pm \partial_\perp \frac{1}{g} U(V_\pm). \quad (71)$$

We can use this expression for $j_\pm$ to construct the scattering amplitude in the approximation of the quasi-elastic unitarity for the kinematics where in the intermediate states of the $s$ and $u$ channel there are two gluon clusters with fixed invariant masses moving in the opposite directions along the momenta $p_A$ and $p_B$. To begin with, let us neglect all terms depending on $A_\pm$ in Lagrangian (53) except of $tr(\partial_\perp A_\perp)/\partial_\perp A_-$ and the linear
terms in (56) with substitution (71). After performing the gaussian functional integration over $A_\pm$ leading to the gluon propagators $1/\partial^2_{\perp}\sigma$ in the crossing channel, we obtain the following effective action for this double quasi-elastic process:

$$S_{\text{doubt}} = -\int d^4x \, tr \left( \frac{1}{2} G^2_{\mu\nu} + \Delta S \right), \quad \Delta S = -\frac{1}{g^2} \int d^4x \, tr \left( \partial_{-\partial_{\perp}} U(V_-) \right) \left( \partial_{+\partial_{\perp}} U(V_+) \right),$$

(72)

where $U(V_\pm)$ are the Wilson exponents (66). The integrals over $x^\pm$ in the second term can be calculated and we obtain for this contribution the action of a two-dimensional $\sigma$-model

$$\Delta S = -\frac{2}{g^2} \int d^2x_\perp \, tr \left( \partial_{\perp\sigma} T(V_-) \right) \left( \partial_{\perp\sigma} T(V_+) \right),$$

(73)

where

$$T(V_\pm) = P \exp(-\frac{g}{2} \int_{-\infty}^{\infty} dx V_\pm). \quad (74)$$

This $\sigma$ model was earlier derived by E. Verlinde and H. Verlinde [14] using other arguments. Note, that in our approach the Yang-Mills term $-\int d^4x \, tr \left( \frac{1}{2} G^2_{\mu\nu} \right)$ responsible for the gluon interactions inside of each of two produced clusters is essential. Indeed, only due to the Euler-Lagrange equations for the Yang-Mills action one can substitute $j$ by $j^{\text{ind}}$ in (57). Further, even after solving the above $\sigma$ model we should take into account non-eikonal corrections from the Yang-Mills interactions in the intermediate states to obtain the $S$-matrix with the full quasi-elastic unitarity. For example, in QED the contribution analogous to action (73) is trivial:

$$\Delta S = \frac{1}{2} \int d^4x \, (\partial_{\perp\sigma} V_-) (\partial_{\perp\sigma} V_+)$$

and corresponds to the generalized eikonal approximation for scattering amplitudes. Nevertheless, it is known, that in higher orders of the perturbation theory one should take into account apart from the elastic eikonal contribution also the screening effects appearing due to the $e^+e^-$ pair production. Action (72) can describe quasi-elastic processes, but even in LLA one should consider also the gluon emission in multi-Regge kinematics (1) leading to the gluon reggeization which can not be obtained in a simple way in the framework of the above $\sigma$ model. In the next sections we construct the other terms of the action which are responsible for more general quasi-multi-Regge processes.

6 Effective action for gluon production in the central rapidity region

As it was stressed above, there is an ambiguity in expansion (52) of the total Yang-Mills field $v$ in its components $V^y$ and $A^y$ which describe particles in direct channels and reggeized gluons in crossing channels, respectively. Using in fact this ambiguity we substituted the Yang-Mills current (58) by the modified current (61). This modified current $j^{\text{mYM}}$ appears in the action as a coefficient in front of the linear term in the
expansion over $A_{\pm}$ if we chose instead of (52) the different decompositions for each Lorentz projection of $v_\mu$

$$v_{\perp \mu} = \sum_y V_{\perp \mu} , \quad v_{\pm} = \sum_y \{ V_{\pm \mu} + U(V_{\mp}) A_{\pm} U^{-1}(V_{\mp}) \} , \quad (75)$$

where $U(A_{\pm})$ are determined in eqs (61, 66).

In particular, using representation (75) and results (37), (42) and (45) from our discussion of the gluon production in the central rapidity region we can write the corresponding action bilinear in fields $A_{\pm}$ in the following form (cf. (56) and (57))

$$S_2 = -\int d^4x \ ( L_2^{mY} + L_2^{m\text{ind}} ) . \quad (76)$$

The Lagrangians $L_2^{mY}$ and $L_2^{m\text{ind}}$ are certain modifications of the Yang-Mills contribution (see (53))

$$L_2^{mY} = tr\{ [ D_\mu, A_+ ] [ D_\mu, A_- ] - \frac{1}{2} [ D_-, A_- ] [ D_+, A_+ ] + \frac{g}{2} G_{+-} [ A_-, A_+ ] \} \quad (77)$$

and of the induced contribution (cf.(45)) related to the induced currents $j_{\pm}^{\text{ind}}$ (59) as

$$L_2^{\text{ind}} = tr A_+ \frac{\partial}{\partial V_-} tr \{ j_{-}^{\text{ind}}(V_-) - \partial^2_{\perp \sigma} V_- \} A_+ +$$

$$+ \quad tr A_+ \frac{\partial}{\partial V_+} tr \{ j_{+}^{\text{ind}}(V_+) - \partial^2_{\perp \sigma} V_+ \} A_- , \quad (78)$$

where it is implied, that after differentiating $j_-$ and $j_+$ over $V_-$ and $V_+$, respectively, the fields $A_-$ and $A_+$ substitute the fields $V_-$ and $V_+$ at the corresponding empty positions. In (78) we subtracted the kinetic terms because they appeared already in $L_2^{mY}$ as $tr(\partial_{\perp \mu} A_+)(\partial_{\perp \mu} A_-)$. The modification of contributions (77, 78) is needed to provide the gauge invariance of action (76) in accordance with our prescription for the fields $A_{\pm}$ as invariants of gauge transformations (55) with $\chi$ decreasing as $x^\pm \to \infty$ . In the case of the YM term (77) this invariance leads to the necessity of the redefinition of the reggeon fields $A_{\pm}$ in accordance with new decomposition (75) of fields $v$

$$A_{\pm} \to U(V_{\mp}) A_{\pm} U^{-1}(V_{\mp}) . \quad (79)$$

This procedure gives the following result for the modified Yang-Mills contribution $L_2^{mY}$ after using the covariance properties of $D_\mu$ and eq.(51) for the derivatives $\partial_{\pm} A_{\pm}$ :

$$L_2^{mY} = tr \{ [ \tilde{D}_{\pm \mu}, A_+ ] W [ \tilde{D}_{\pm \mu}, A_- ] \frac{1}{W} +$$

$$+ g tr \{ (\partial_+ \tilde{V}_+^{(-)}) A_- \frac{1}{W} A_+ W + (\partial_- \tilde{V}_+^{(-)}) A_+ W A_- \frac{1}{W} \} . \quad (80)$$
Here the gauge invariant matrix $W$ is given by
\[
W = U^{-1}(V_-) U(V_+) = (1 \pm g \partial_\mu V_\pm) U(V_+) U^{-1}(V_-) (1 \pm g \partial_\mu V_\pm)^{-1}
\] (81)
and the quantities $\tilde{D}^{(\pm)}, \tilde{V}^{(\pm)}$ are determined as
\[
\tilde{D}_\mu^{(\pm)} = U^{-1}(V_\pm) D_\mu U(V_\pm), \quad \tilde{V}_\mu^{(\pm)} = U^{-1}(V_\pm) V_\mu U(V_\pm) .
\] (82)

Now let us consider $L_{2}^{\text{ind}}$ (78). Its modification which is compatible with the gauge invariant properties of $S_2$ is given by
\[
L_{2}^{m\text{ind}} = \text{tr} U(V_+) A_- U^{-1}(V_+) \frac{\partial}{\partial V_-} \text{tr} j_-^{\text{ind}} A_+ + \text{tr} U(V_-) A_+ U^{-1}(V_-) \frac{\partial}{\partial V_+} \text{tr} j_+^{\text{ind}} (V_+) A_- - \text{tr} \{ A_- \partial^2_{\parallel} A_+ + A_+ \partial^2_{\parallel} A_- \}. \] (83)

This representation is again in accordance with our redefinition (79) of the fields $A_\pm$ in action (53) including now apart from the Yang Mills contributions also the induced terms. $L_{2}^{m\text{ind}}$ (83) is constructed in such a way that it can be obtained from the linear terms in (56) if we substitute the fields $V_\pm$ in eq. (59) for $j_\pm^{\text{ind}}$ by the total field $v_\pm$ and expand the result up to the linear term in $A_\pm$ after using the new decomposition (75) of fields $v$

\[
j_\pm^{\text{ind}} (V_\pm) \rightarrow j_\pm^{\text{ind}} (v_\pm) - j_\pm^{\text{ind}} (A_\pm) = j_\pm^{\text{ind}} (V_\pm) + U(V_\mp) A_\pm U^{-1}(V_\mp) \frac{\partial}{\partial V_\mp} j_\pm^{\text{ind}} (V_\pm) + ... . \] (84)

The perturbative expansion of the Lagrangian in action (76) into a series over $V_{\mu}$ is given by
\[
L_{2}^{mYM} + L_{2}^{m\text{ind}} = \text{tr} \left[ (\partial_{\parallel} A_+) (\partial_{\parallel} A_-) + \frac{g}{2} \{ [V_\mu, A_+] [V_\mu, A_-] + [V_\mu, A_+] [A_+, \partial_{\parallel} A_-] + \frac{1}{2} (\partial_+ V_+ - \partial_- V_-) [A_+, A_-] - \frac{1}{2} \partial_\mu [A_+, A_-] \} \right] +
\[
+ g \{ -V_\mu [\partial_\mu A_+ , A_-] + V_\mu [A_+, \partial_\mu A_-] + \frac{1}{2} (\partial_- V_+ - \partial_+ V_-) [A_+, A_-] - \frac{1}{2} \partial_\mu [A_+, A_-] \} +
\[
+ g^2 \{ [V_\mu, A_+] [V_\mu, A_-] + \frac{1}{2} [V_\mu, A_+] [V_\mu, A_-] - \frac{1}{2} [V_\mu, V_\mu] [A_+, A_-] +
\[
+ (\partial_{\parallel} A_+) \left( A_- \frac{1}{\partial_-} V_- \frac{1}{\partial_-} V_- + V_- \frac{1}{\partial_-} A_+ \frac{1}{\partial_-} V_- + V_- \frac{1}{\partial_-} V_- \frac{1}{\partial_-} A_- \right) +
\]
but instead of its stationarity in expression (87) is similar to the first Legendre transformation from the Yang-Mills action, \(0\) (see (51)) which are important for the gauge invariance of action (87). Note, that and to an additional term \(A\) including also the fields \(\dot{A}_\perp\) without other essential modifications of the theory. The more complicated effective action including also the fields \(a_\perp\) describing the reggeized quarks can be constructed for the backward scattering processes with the fermionic exchange in the crossing channel (cf.[5]). Note that for the case of the electroweak theory one should add to action (87)

\[
+ (\partial^2_{\mu} A_-) \left( A_+ \frac{1}{\partial_+} V_+ \frac{1}{\partial_+} A_+ + V_+ \frac{1}{\partial_+} A_+ V_+ \frac{1}{\partial_+} A_+ + \frac{1}{\partial_+} V_+ \frac{1}{\partial_+} A_+ \right) + K_2 \right) + O(g^3) \]. (85)

The total contribution of the sum \(K = \sum_{r=1}^{\infty} g^r K_r\) appearing in (85) and of the terms in (62) which contain \(j_{Y M}^\pm\) can be written as a perturbative expansion of the following expression:

\[
\Delta L_2 = L_2^{YM}(V + U A U^{-1}) - L_2^{YM}(V + A) +
\]

\[
\text{tr} \left\{ A_- [j^\text{ind}_+(V + U A U^{-1}) - j^\text{ind}_+(V + A)] + A_+ [j^\text{ind}_-(V + U A U^{-1}) - j^\text{ind}_-(V + A)] \right\}. \quad (86)
\]

Here we have used short-hand notations for two different representations (52) and (75) of the total field \(v\). Expression (86) is zero due to the equations of motion for the field \(V\) because it is a difference of the same effective Lagrangian for two various parametrizations of \(v\). In the next section we shall discuss the corresponding effective action \(S_{\text{eff}}(v)\) for quasi-multi-Regge processes in QCD for an arbitrary parametrization.

7 Reggeon calculus in QCD

From the results of the previous sections we conclude that the gauge-invariant action for the gluon-reggeon interactions which are local in each rapidity interval \((y - \eta, y + \eta)\) can be written in terms of the Yang-Mills field \(v\) and the reggeon fields \(A_\pm\) as follows

\[
S_{\text{eff}}(v, A_\pm) = -\int d^4x \ \text{tr} \left\{ \frac{1}{2} G_{\mu
u}^2(v) + [A_-(v_-) - A_-] j_{\text{reg}}^\pm + [A_+(v_+) - A_+] j_{\text{reg}}^\pm \right\}. \quad (87)
\]

Here \(A_\pm(v_\pm) = (1/\partial^2_{\pm\sigma}) j_{\text{ind}}^\pm(v_\pm)\) are composite fields (see (70)):

\[
A_\pm(v_\pm) = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - ... = -\frac{1}{g} \partial_\pm U(v_\pm)
\]

and \(j_{\text{reg}}^\pm = j_{\text{reg}}^\pm(A_\pm)\) are reggeon currents satisfying the kinematical constraints \(\partial_\pm j_{\text{reg}}^\pm = 0\) (see (51)) which are important for the gauge invariance of action (87). Note, that expression (87) is similar to the first Legendre transformation from the Yang-Mills action, but instead of its stationarity in \(j_{\text{reg}}^\pm\) one should use the on-shell condition for \(A_\pm\):

\[
j_{\text{reg}}^\pm = \partial^2_{\pm\sigma} A_\pm
\]

to express \(j_{\text{reg}}^\pm\) through \(A_\pm\). It is related with the fact, that action (87) describes only interactions in the given rapidity interval and the interactions between the particles with different rapidities will be taken into account in the framework of the reggeon field theory.

One can add to action (87) also the quark contribution \(\int d^4x \bar{\psi}(i\bar{D} - m)\psi\). It lead to an additional term \(g\bar{\psi}\gamma_\sigma\psi\) in the Yang-Mills current appearing in classical equations (88) without other essential modifications of the theory. The more complicated effective action including also the fields \(a_\perp\) describing the reggeized quarks can be constructed for the backward scattering processes with the fermionic exchange in the crossing channel (cf.[5]). Note that for the case of the electroweak theory one should add to action (87)
the terms which are responsible for interactions of quarks, leptons and the Higgs particle (cf. [2]).

Now we want to consider the problem of constructing the Reggeon calculus in QCD starting from the effective action (87). Because $A_\pm(v_\pm)$ has a linear term in $v_\pm$ the classical extremum of $S_{\text{eff}}$ (87) is situated at non-vanishing values of $v = \tilde{v}$ satisfying the gauge-invariant Euler-Lagrange equations

$$j_{\pm}^{YM}(v) = 0, \quad j_{\pm}^{YM}(v) = -\frac{\partial}{\partial v_\pm} tr A_\pm(v_\pm) j_{\pm}^{reg} = -U(v_\pm) (\partial_{\perp\sigma} A_\pm) U^T(v_\pm). \quad (88)$$

Here the matrix $U(v_\pm)$ is determined in eqs.(66, 67) and the transposed matrix $U^T(v_\pm)$ is

$$U^T(v_\pm) = \frac{1}{1 + g v_\pm \partial_{\perp}^{-1}}$$

which is supposed to be multiplied by a constant unit matrix from the left hand side.

Due to the invariance properties of $A_\pm$ under the solution $\tilde{v}$ of eqs.(88) is degenerate. We use here the Feynman gauge in which the gluon propagator of the field $v$ is $\delta^{\mu\nu}/k^2$.

Taking into account that $\partial^+ A_\pm$ and $\partial^- A_\pm$ are negligible in the multi-Regge kinematics (see (51)) one can construct the following perturbative solution of (88)

$$\tilde{v}_+ = A_+ + g \partial_\perp^{-2} \{ [(\partial_{\perp\sigma}^2 A_+), (\partial^{-1}_- A_-)] - \frac{1}{2}[A_-, \partial_+ A_+] \} + O(g^2),$$

$$\tilde{v}_- = A_- + g \partial_\perp^{-2} \{ [(\partial_{\perp\sigma}^2 A_-), (\partial_+^{-1} A_+)] - \frac{1}{2}[A_+, \partial_- A_-] \} + O(g^2),$$

$$\tilde{v}_{\perp\sigma} = \frac{1}{2} g \partial_\perp^{-2} \{ [A_+, \partial_{\perp\sigma} A_-] + [A_-, \partial_{\perp\sigma} A_+] \} + O(g^2). \quad (89)$$

The effective action calculated at the classical solution $v = \tilde{v}$

$$\tilde{S}_{\text{eff}}(A_\pm) = -\int d^4 x \ tr \{ \frac{1}{2}(\partial_{\perp\sigma} A_-)(\partial_{\perp\sigma} A_+) -$$

$$- \frac{1}{2} g \left( (\partial_{\perp\sigma}^2 A_-) [(\partial^{-1}_+ A_+), A_+] + (\partial_{\perp\sigma}^2 A_+) [(\partial^{-1}_- A_-), A_-] \right) + O(g^2) \} \quad (90)$$

describes generally all possible self-interactions of the Reggeon fields $A_\pm$ in the tree approximation. In particular, the tri-linear term is responsible for the transition of the reggeon corresponding to the field $A_\pm$ into the state constructed from two reggeons described by fields $A_{\mp}$. This transition is suppressed for the case of the elastic amplitude according to the Gribov signature conservation rule, because the signature of the reggeized gluon is negative. Nevertheless, it was argued [15,16], that the inclusion of this triple reggeon vertex simplifies the reggeon calculus and clarifies the mechanism of the gluon reggeization. Note that this vertex is proportional to $q^2$, and contains the singularity $\partial^{-1}_{\perp\sigma}$ corresponding to the contribution of diagrams in which there are highly virtual particles in the direct channels [5]. The quadri-linear term contains the transition of one reggeon into the state.
constructed from three reggeons [16]. This transition is not suppressed by the Gribov signature conservation rule. There is also a contribution which describes the scattering of two reggeons and gives in particular the integral kernel for the BFKL equation [2]. The six-linear term leads to to the pomeron self-interactions which are responsible for screening corrections [16]. All these terms can be obtained using the perturbative solution (89) of classical equations for the effective action (87). We hope to return to these problems in future publications.

To calculate higher loop corrections to the Reggeon Lagrangian one should write the field $v$ as a sum of the classical field $\bar{v}$ and the field $\epsilon$ describing quantum fluctuations near the classical field (cf.(52, 75)):

$$ v = \bar{v} + \epsilon, $$

(91)

expand the action in $\epsilon$ (cf(53)):

$$ \Delta S = S_{eff} - \bar{S}_{eff} = - \int d^4x \text{tr} \left\{ [D_\mu, \epsilon_\nu]^2 - [D_\mu, \epsilon_\nu][D_\nu, \epsilon_\mu] + g G_{\mu\nu} [\epsilon_\mu, \epsilon_\nu] + \frac{1}{2}(\epsilon_- - \frac{\partial}{\partial v_-})(\epsilon_+ - \frac{\partial}{\partial v_+}) j^{-\text{ind}}(v_-) A_+ + \frac{1}{2}(\epsilon_+ - \frac{\partial}{\partial v_+})(\epsilon_- + \frac{\partial}{\partial v_-}) j^{\text{ind}}(v_+) A_- + O(\epsilon^3) \right\}. $$

(92)

and calculate the functional integral over the quantum fluctuations $\epsilon$. Using the gaussian approximation one can find in particular the one-loop correction to the BFKL kernel in an independent way in comparison with the dispersion method of refs.[3],[11]. The possible advantage of this approach is a better infrared convergency of intermediate expressions.

To calculate the two-loop correction to the gluon Regge trajectory one should expand $S_{eff}$ up to $\epsilon^4$ taking into account the terms bilinear in $A_\pm$. These important problems will be discussed elsewhere. Here we use expressions (89) to find $\Delta S$ (92) only up to quadratic terms in $\epsilon$ and bilinear in $A_\pm$:

$$ \Delta S = - \int d^4x \text{tr} \left\{ (\partial_\mu \epsilon_\nu)^2 - (\partial_\mu \epsilon_\nu)(\partial_\nu \epsilon_\mu) + g \{ 2(\partial_\mu \epsilon_\nu)[\bar{v}_\mu, \epsilon_\nu] - 2(\partial_\nu \epsilon_\mu)[\bar{v}_\mu, \epsilon_\nu] + 2(\partial_\nu \bar{v}_\mu)[\epsilon_\nu, \epsilon_\mu] - (\partial_{\pm\sigma} A_+) \epsilon_- \frac{1}{\partial_-} \epsilon_- - (\partial_{\pm\sigma} A_-) \epsilon_+ \frac{1}{\partial_+} \epsilon_+ \} + \right. $$

$$ + \left. g^2 \{ [A_+, \epsilon_\nu] [A_-, \epsilon_\nu] - \frac{1}{2} [A_+, \epsilon_\nu] [A_-, \epsilon_-] - \frac{1}{4} [A_+, \epsilon_-]^2 - \frac{1}{4} [A_-, \epsilon_+]^2 - \right. $$

$$ - \frac{1}{2} [A_+, A_-] [\epsilon_+, \epsilon_-] + (\partial_{\pm\sigma} A_+) (\epsilon_- \frac{1}{\partial_-} \epsilon_- \frac{1}{\partial_-} A_- + \epsilon_- \frac{1}{\partial_-} A_- \frac{1}{\partial_-} \epsilon_- + A_- \frac{1}{\partial_-} \epsilon_- \frac{1}{\partial_-} \epsilon_- ) + $$

$$ + (\partial_{\pm\sigma} A_-) (\epsilon_+ \frac{1}{\partial_+} \epsilon_+ \frac{1}{\partial_+} A_+ + \epsilon_+ \frac{1}{\partial_+} A_+ \frac{1}{\partial_+} \epsilon_+ + A_+ \frac{1}{\partial_+} \epsilon_+ \frac{1}{\partial_+} \epsilon_+ ) \right\}, $$

(93)
where one should substitute $\tilde{v}$ by expressions (89). The terms bilinear simultaneously in $A_\pm$ and in $\epsilon$ can be used for finding next to leading corrections to the BFKL pomeron. We consider here only the contributions which are linear in $A_\pm$ and contain the singularities $\partial^{-1}_\pm$. Expanding the integrand $\exp(-iS_{eff})$ in the functional integral over $\epsilon$ up to the higher order terms containing linearly both $A_+$ and $A_-$ and substituting the products of $\epsilon_+$ and $\epsilon_-$ by the free propagators in the Feynman gauge

$$
<\epsilon_-(x)\epsilon_+(0)> = -2i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \exp(ikx), \quad (94)
$$

we obtain the corresponding one-loop contribution to the effective action in the form (cf.(51))

$$
S_{1eff}^1 = -\int d^4x \int d^2x_\perp \int dy \text{tr} (\frac{1}{2}(\partial_{\perp\sigma}A_+^\nu(x_-,x_\perp)) (\partial_{\perp\sigma}A_-^\nu(x_+,x_\perp)))
$$

$$
g^2 (2\pi)^3 N_c \{ \frac{\theta(|x_\perp - x'_\perp| - \delta)}{|x_\perp - x'_\perp|^2} - 2\pi \delta^2(x_\perp - x'_\perp) \ln \frac{1}{\delta\lambda} \}, \quad (95)
$$

where $\delta \to 0$ and $\lambda$ is proportional to a fictitious gluon mass, introduced to remove the infrared divergency. The fields $A_\pm^\nu$ are determined in section 4 (see (52)). The appearance of the integral over the rapidity $y = \frac{1}{2} \ln(k^+/k^-)$ is related with the fact, that due to the singularities $\partial^{-1}_\pm$ corresponding to the propagators of intermediate gluons in the direct channel we calculate really the contribution of the box diagram which contains $\ln s$. The Fourier transform of the expression in the last line of (95) gives just the gluon Regge trajectory (3)

$$
\omega(t) = \frac{g^2}{(2\pi)^3} N_c \int d^2x_\perp \exp(iqx_\perp) \{ \frac{\theta(|x_\perp| - \delta)}{|x_\perp|^2} - 2\pi \delta^2(x_\perp) \ln \frac{1}{\delta\lambda} \}. \quad (96)
$$

In accordance with our agreement (50) concerning the regularized propagator of the reggeon fields $A_\pm$ we modify their kinetic term in expression (90)

$$
S_{kin}^{1}(A_\pm) = -\int dy \int d^4x \text{tr} \frac{1}{2} (\partial_{\perp\sigma}A_+^\nu) \frac{\partial}{\partial y} (\partial_{\perp\sigma}A_-^{\nu+\eta}). \quad (97)
$$

It leads to an additional factor $\theta(y' - y - \eta)$ in the free correlation function of the fields $A_+^\nu$ and $A_-^\nu$. The dependence on the auxiliary parameter $\eta$ should disappear in the final result, as it was argued in section 4. With taking into account the one loop correction $S_{1eff}^1$ the renormalized correlation function corresponding to the sum of expressions (95) and (97) contains the Regge factor $\exp(\omega(q^2) (y'_- - y))$ in the momentum representation:

$$
\int d^2 x_\perp \exp(i x_\perp q) < A_+^{\nu'}(x_\perp)A_-^{\nu}(0) >_{\text{ren}} \sim \theta(y'_- - y - \eta) \exp(\omega(q^2) (y'_- - y)) . \quad (98)
$$

It is well known [2], that the infrared divergency in the gluon Regge trajectory (96) at $\lambda \to 0$ is cancelled in the BFKL kernel with the contribution corresponding to the real
gluon emission. This contribution appears in the classical reggeon action (90) as a quadri-linear term in $A_{\pm}$. We shall discuss the mechanism of this cancellation in the framework of this functional approach somewhere else.

Note, that if one wants to construct not only the reggeon vertices, but to compute also the production amplitudes in the quasi-multi-Regge kinematics in the tree approximation (see sections 5 and 6), the effective field theory describing all possible interactions of the reggeon fields $A_{\pm}$ and the particle fields $V_{\mu}$ can be derived from the action (87) using the known functional methods for calculating the $S$-matrix elements [18]. For this purpose one should find the solution of classical equations (88) with a fixed asymptotic behaviour $V_{\mu}$ for $v_{\mu}$ at $t \to \pm \infty$ and put this solution in expression (87), obtaining the generating functional for the $S$-matrix elements. Furthermore, the reggeon-reggeon-particle vertex in one loop approximation [3] can be calculated by finding the contribution from the quantum fluctuations near the classical solutions.

8 Conclusion

In this paper we constructed the gauge-invariant effective action (87) describing the interaction of bare reggeized gluons and physical gluons within the rapidity interval $\eta$. By eliminating the Yang-Mills field $v_{\mu}$ with the use of equations of motion (88) the reggeon action (90) is derived in the tree approximation of perturbation theory. One can calculate one loop corrections to this action, which leads in particular to the gluon reggeization (98). One loop corrections to the BFKL kernel are urgently needed for the consistent theoretical description of the small-x structure functions measured at HERA.

The possibility to represent the initial Yang-Mills theory in the form of a reggeon field model with the effective vertices calculated perturbatively is important, because the $s$-channel unitarity of the $S$-matrix for the theory with action (87) is transformed into various relations among the reggeon vertices. Therefore the multi-reggeon dynamics in the crossing channel turns out to be in the agreement with the unitarity constraints in the direct channels. The reggeon calculus can be presented as a field theory in the 3-dimensional space [12] where the two-dimensional coordinates $\rho = x_{\pm}$ describe the impact parameters of the reggeons and the time coordinate $y$ is their rapidity. The Schrödinger equation for the wave function $\Psi_{\omega}^{n}(\rho_{1}, \rho_{2}, ... \rho_{n})$ of the colourless glueball with a complex spin $j = 1 + \omega$ includes generally the components with an arbitrary number $n$ of the reggeized gluons and the transitions between these states can be obtained from action (87). However, in the generalised leading logarithmic approximation the number of the reggeised gluons is conserved. The BKP equations [6], obtained in this approximation, have a number of remarkable properties in the multi-colour QCD [7-10]. One can believe that at least some of these properties will survive in the above reggeon field theory.

The method of constructing the effective action, describing the quasi-multi-Regge processes in the Yang-Mills theory can be generalised to the case of quantum gravity, where the tri-linear effective vertices for the reggeon-particles interactions are known [18]. For the multi-Regge kinematics the action was given earlier [4]. It was used for finding the scattering amplitudes at super-Planck energies [19]. Recently this action was derived from the Hilbert-Einstein action by integrating over the heavy modes [20]. The action
responsible for the quasi-multi-Regge processes in gravity can be written in the form similar to eq.(87):

\[ S = I_G + I_{ind} = -\frac{1}{16\pi G} \int d^4 x \sqrt{g(x)} \left[ R - \frac{\sqrt{8\pi G}}{4} (R_{++}^{ind} A_{--} + R_{--}^{ind} A_{++}) \right] \quad (99) \]

where \( I_G \) is the Hilbert-Einstein action, the fields \( A^{++} \) and \( A^{--} \) describe the regeized gravitons in the \( t \)-channel. The quantities \( R_{++}^{ind} \) and \( R_{--}^{ind} \) are the light-cone components of the Ricci tensor \( R_{\mu\nu} \) calculated in the linear approximation over the graviton fields \( h^\rho_\sigma = g^\rho_\sigma - \delta^\rho_\sigma \) in the left (\( h^{-\sigma} = 0 \)) and right (\( h^{+\sigma} = 0 \)) gauges correspondingly. The discussion of this action will be given somewhere else.

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Appendix

Below we write down the results of ref. [11] for production amplitudes of quasi-multi-Regge processes in the Born approximation using slightly different notations and calculate corresponding matrix elements in the light-cone gauge. To begin with, let us consider the quasi-elastic process of one gluon emission in the fragmentation region of the colliding gluon A. The momenta of the final state gluons are $k_1, k_2$ and $p_B$, the momenta of two initial gluons are $-k_0$ and $p \equiv p_B$. The corresponding Lorentz and colour indices are $a_1, a_2, a_0, B', B$ and $\nu_1, \nu_2, \nu_0, \beta', \beta$. We introduce the invariants:

$$t = (k_0 + k_1 + k_2)^2 \equiv q^2, \quad s_{12} = (k_1 + k_2)^2, \quad t_1 = (k_0 + k_1)^2, \quad t_2 = (k_0 + k_2)^2$$

and the Sudakov components of momenta $k_1, k_2$:

$$\beta_{1,2} = \frac{(k_{1,2} \cdot p)}{(p_A \cdot p)}, \quad \beta_0 = -1.$$

Then the production amplitude given by eq. (22) from the second paper in ref.[11] can be written in the form of eq. (17) with

$$\phi^{\nu_0 \nu_1 \nu_2}_{a_0 a_1 a_2 c} = 8t \sqrt{s} \left[ T^{d}_{a_0 a_0} T^{\nu_0 \nu_1 \nu_2}_{a_2 d} a_{\nu_0 \nu_1 \nu_2} + ([a_1, \nu_1, k_1] \leftrightarrow [a_2, \nu_2, k_2]) \right],$$

where according to eq. (23) from ref.[11] the tensor $a^{\nu_0 \nu_1 \nu_2}$ is given by

$$a^{\nu_0 \nu_1 \nu_2} =$$

$$= \frac{p^\nu_0}{s} \left( \frac{p_1^\nu_1 p_2^\nu_2}{\beta_2 s^2} - \frac{p_1^\nu_1 k_{2}^\nu_2 - k_{2}^\nu_1 p_{2}^\nu_2}{s_{12} s} - \frac{k_{0}^\nu_1 p_{2}^\nu_2}{\beta_2 s \ t_1} + \frac{k_{0}^\nu_1 k_{1}^\nu_2}{t} \left( \frac{1}{t_1} + \frac{1}{s_{12}} \right) - \frac{k_{2}^\nu_1 k_{0}^\nu_2}{s_{12} \ t} + \frac{k_{0}^\nu_1 k_{0}^\nu_2}{t \ t_1} \right) +$$

$$+ p_1^\nu_0 \left( \frac{p_1^\nu_1 p_{2}^\nu_2}{\beta_2 s^2 t_1} + \frac{(k_{2}^\nu_1 p_{2}^\nu_2 - p_1^\nu_1 k_{1}^\nu_2)}{s \ t} \left( \frac{1}{t_1} + \frac{1}{s_{12}} \right) - \frac{p_1^\nu_1 k_{0}^\nu_2}{s_{12} \ t} \right) -$$

$$- p_2^\nu_0 \left( \frac{p_1^\nu_1 k_{2}^\nu_2 - k_{2}^\nu_1 p_{2}^\nu_2}{s_{12} s \ t} + \frac{k_{0}^\nu_1 p_{2}^\nu_2}{s \ t_1 t} \right) -$$

$$- \frac{\delta^{\nu_0 \nu_2}}{2} \left( \frac{(t_2 \beta_2 - t_2) p_{1}^\nu_0}{s_{12} s \ t} + \beta_2 \left( \frac{1}{t_1} + \frac{1}{s_{12}} \right) k_{1}^\nu_0 - \beta_1 k_{2}^\nu_0 \right) -$$

$$- \frac{\delta^{\nu_0 \nu_1}}{2} \left( \frac{(t \beta_2 - t_1) p_{2}^\nu_2}{s_{12} s \ t} - \beta_1 t_1 \ t \ t_{0}^\nu_2 - \left( \frac{1}{t_1} + \frac{1}{s_{12}} \right) k_{1}^\nu_2 \right) -$$

$$- \frac{\delta^{\nu_0 \nu_2}}{2} \left( - \frac{p_1^\nu_1}{s \ t} - \beta_{2} \ k_{0}^\nu_1 \ t_{0}^\nu_2 \right).$$
One can verify that this expression for $\psi^{\nu_1 \nu_2 \nu_{2+}}_{\alpha_0 \alpha_1 \alpha_2 c}$ coincides with (18) up to the terms which are proportional to $k_{i\perp}^\nu$ and give a vanishing contribution for the polarization vectors $e(k_i)$ satisfying the Lorentz condition $k_{i\perp}^\nu e^{\nu}(k_i) = 0$. They are gauge invariant and in the right light-cone gauge with $e(k_i) = e_\perp(k_i) - p_B (e_\perp k_i)/(p_B k_i)$ the matrix element of $a$ can be written as follows

$$e^{\nu_0}(k_0) e^{\nu_1}(k_1) e^{\nu_2}(k_2) a^{\nu_0 \nu_1 \nu_2} = \epsilon_\perp^{\nu_0} \epsilon_\perp^{\nu_1} \epsilon_\perp^{\nu_2} m^{\nu_0 \nu_1 \nu_2},$$

where the tensor $m$ has only transversal components and reads

$$m^{\nu_1 \nu_2 \nu_3} = \frac{\delta^{\nu_1 \nu_3}}{2} \left( - \frac{\beta_2}{t} \left( \frac{1}{t_1} + \frac{1}{s_{12}} \right) k_{1\perp}^{\nu_0} + \frac{\beta_1}{s_{12} t} k_{2\perp}^{\nu_0} \right) +$$

$$+ \frac{\delta^{\nu_2 \nu_3}}{2} \left( \frac{\beta_2}{t_1} k_{2\perp}^{\nu_1} + \frac{1}{t_1} \left( \frac{1}{s_{12}} + \frac{1}{s_{21}} \right) (k_{1\perp}^{\nu_1} - \frac{\beta_1}{\beta_2} k_{1\perp}^{\nu_0}) \right) +$$

$$+ \frac{\delta^{\nu_0 \nu_3}}{2} \left( \frac{\beta_2}{t_1} k_{2\perp}^{\nu_0} - \frac{1}{s_{12}} (k_{2\perp}^{\nu_0} - \frac{\beta_1}{\beta_2} k_{1\perp}^{\nu_0}) \right).$$

Here we took into account that $k_{0\perp} = 0$. This expression can be used for finding the one-loop correction to the residue of the BFKL pomeron.

Let us consider now the quasi-multi-Regge process of the double gluon emission in the central rapidity interval at high energy gluon collisions. The production amplitude of this process in the Born approximation was calculated in the second paper of ref. [11] and according to eqs (60),(61) and (62) from this paper it can be written in the form of above eq. (35) with

$$\psi^{\nu_1 \nu_2 \nu_{2+}}_{d_1 d_2 d_3 c_1} = 2 g^2 \{ T_{d_1 d_2} T_{d_3 c_1} A^{\nu_1 \nu_2} + ([d_1, \nu_1, k_1] \leftrightarrow [d_2, \nu_2, k_2]) \},$$

where

$$A^{\nu_1 \nu_2} = - \frac{a_1^{\nu_1} a_2^{\nu_2}}{t} + \frac{b_1^{\nu_1} b_2^{\nu_2}}{s} \left( 1 + \frac{s_{\alpha_1 \beta_2}}{t} \right) + \frac{c_1^{\nu_1} c_2^{\nu_2}}{s} \left( \frac{t_2}{s_{\beta_2 (\beta_1 + \beta_2)} t} - \frac{s_{\alpha_1 \alpha_2}}{t} \right) +$$

$$+ \frac{c_1^{\nu_1} b_2^{\nu_2}}{s} \left( \frac{t_1}{s_{\alpha_1 (\alpha_1 + \alpha_2)}} - \frac{s_{\beta_2 \beta_1} t}{t} \right) - \frac{c_1^{\nu_1} c_2^{\nu_2}}{s} \left( 1 + \frac{\kappa}{t} - \frac{s_{\alpha_2 \beta_1}}{t} \right) -$$

$$- 2 (\delta^{\nu_1 \nu_2} - \frac{2 k_{2\perp}^{\nu_1}}{\kappa}) \left( 1 + \frac{t_1}{\kappa} + \frac{s_{\alpha_1 \beta_2}}{t} + \frac{s_{\alpha_1 \beta_2} - s_{\alpha_2 \beta_1}}{\kappa} - \frac{t_1 \alpha_2}{\kappa (\alpha_1 + \alpha_2)} - \frac{t_2 \beta_1}{\kappa (\beta_1 + \beta_2)} \right).$$

Here we introduced the notations

$$a_1 = 2 \left( \alpha_1 p_B + q_1 - (\beta_1 + \frac{t_1}{s_{\alpha_1}}) p_A + \frac{t}{\kappa} k_2 \right),$$

$$25$$
\[ a_2 = 2 \left( \beta_2 p_A - q_2 - (\alpha_2 + \frac{t_2}{s \beta_2})p_B + \frac{t}{\kappa}k_1 \right), \]

\[ b_1 = 2(p_B - \frac{\beta_1 s}{\kappa}k_2), \quad b_2 = 2(p_A - \frac{\alpha_2 s}{\kappa}k_1), \]

\[ c_1 = 2(p_A - \frac{\alpha_1 s}{\kappa}k_2), \quad c_2 = 2(p_B - \frac{\beta_2 s}{\kappa}k_1), \]

\[ \kappa = (k_1 + k_2)^2, \quad t = (q_1 - k_1)^2, \quad k_i = \beta_i p_A + \alpha_i p_B + k_{i1} \]

and \( q_1 = p_A - p_{A'} \), \( q_2 = p_B - p_B' \).

Note, that due to the gauge invariance \( k_1^{\nu_1} A_{\nu_1 \nu_2} = 0 \) of the amplitude \( A_{\nu_1 \nu_2} \) the contribution of the Faddeev-Popov ghosts to the production cross-section is zero [11]. One can verify that the above expression for \( \psi \) verifies the Ward identity

\[ \gamma \Gamma_k e_\sigma (k_i) = 0 \] for the polarization vectors \( \sigma \) of the produced gluons. It is a consequence of the fact, that up to the same vanishing terms the tensor \( A_{\nu_1 \nu_2} \) can be written as the sum of contributions of the Feynman diagrams with effective vertices given in sections 2 and 3:

\[ A_{\nu_1 \nu_2} = -\frac{1}{2} \Gamma^{\nu_1 \sigma -}(k_1, k_1 - q_1) \Gamma^{\nu_2 \sigma +}(k_2, k_2 + q_2) \frac{1}{(q_1 - k_1)^2} - \frac{1}{2} \Gamma^{\nu_2 \nu_1 \sigma}(k_2, -k_1) \Gamma^{\sigma + -}(q_2, q_1) \]

\[ + \delta^{\nu_1 + \delta^{\nu_2 -} - \delta^{\nu_1 \nu_2} - \frac{1}{2} \delta^{\nu_2 +} + 4 t_1 \frac{p_A}{s^2 \alpha_1 (\alpha_1 + \alpha_2)} + 4 t_2 \frac{p_B}{s^2 \beta_2 (\beta_1 + \beta_2)}, \]

where

\[ \Gamma^{\nu_1 \sigma -}(k_1, k_1 - q_1) = \gamma^{\nu_1 \sigma -}(k_1, k_1 - q_1) - t_1 n^{-\nu_1} \frac{1}{k_1^+} n^{-\sigma}, \]

\[ \Gamma^{\sigma + -}(q_2, q_1) = \gamma^{\sigma + -}(q_2, q_1) - 2 t_1 \frac{n^-}{k_1^- + k_2^-} + 2 t_2 \frac{n^+}{k_1^+ + k_2^+} \]

and \( \gamma^{\nu \nu \sigma}(p_{A' A}, p_A) \) is the usual Yang-Mills vertex (11).

Because \( A_{\nu_1 \nu_2} \) satisfies the Ward identity

\[ k_1^\sigma A_{\sigma \nu_2} = k_2^{(\nu_2} \left( \frac{1}{2} \frac{k_1^- k_2^+}{(q_1 - k_1)^2} - \frac{1}{2} \frac{k_1^- k_2^+}{(k_1 + k_2)^2} + 1 \right) \]

the contribution of the Faddeev-Popov ghosts to the cross-section is fixed. Instead one can calculate the matrix element of \( A_{\nu_1 \nu_2} \) between polarization vectors \( e(k_i) \) with definite
helicities and sum its square over all possible helicity states. It is convenient to use for $e(k_1)$ and $e(k_2)$ the left and right gauges correspondingly

$$e'(k_1) = e'_{\perp} - \frac{e'_{\perp} k_1}{p_A k_1} p_A, \quad e''(k_2) = e'_{\perp} - \frac{e'_{\perp} k_2}{p_B k_2} p_B.$$ 

In these gauges the matrix element of the tensor $A^{\nu_1 \nu_2}$ can be written as follows

$$e'_{\nu_1}(k_1) e''_{\nu_2}(k_2) A^{\nu_1 \nu_2} = e'_{\perp}(k_1) e''_{\perp}(k_2) a^{\nu_1 \nu_2}.$$ 

Here the tensor $a^{\nu_1 \nu_2}$ has only transverse components and equals

$$a^{\nu_1 \nu_2}(q_1, q_2; k_1, k_2) = 4\left\{ \frac{q_{\perp 1} q_{\perp 2}}{t} - \frac{q_{\perp 1}}{\kappa} (k_{\perp 1} - \frac{\beta_1}{\kappa} k_{\perp 2}) + \frac{q_{\perp 2}}{\kappa} (k_{\perp 2} - \frac{\alpha_2}{\kappa} k_{\perp 1}) + \frac{k_{\perp 1} k_{\perp 2}}{\kappa} s_\lambda \left( \frac{1}{s_1} - \frac{1}{\kappa^2} \right) - \frac{2\delta_{\perp 1} \delta_{\perp 2}}{\kappa} \right\} -$$

$$2\delta_{\perp 1} \left( t + \frac{s_\beta_2 \alpha_1 - s_\alpha_2 \beta_1}{\kappa} - \frac{q_{\perp 2}}{\kappa} \frac{\alpha_2}{\kappa} + \frac{q_{\perp 1}}{\kappa} \frac{\beta_1}{\kappa} + \frac{k_{\perp 1} k_{\perp 2}}{\kappa^2} \right),$$

where $q = q_1 - k_1$, $t = q^2$. After summing over the polarizations of the intermediate gluons with the use of the relations

$$\sum e'_{\rho}(k) e''_{\sigma}(k) = -\delta^{\perp}_{\rho \sigma}, \quad \Omega_{\rho \sigma}(k) = \sum e'_{\rho}(k) e''_{\sigma}(k) = -\delta_{\rho \sigma} + 2 \frac{k_{\perp \rho} k_{\perp \sigma}}{k_{\perp}^2},$$

one can obtain one loop contribution to the BFKL kernel from the quasi-multi-Regge kinematics of intermediate gluons (see [22]). It is proportional to the integral over the produced gluon momenta from the quantity:

$$\sum A A = g_N a^{\nu_1 \nu_2}(q_1, q_2; k_1, k_2) a^{\nu_1 \nu_2}(q_1', q_2'; k_1, k_2) +$$

$$+ h_N \Omega_{\rho \sigma}(k_1) \Omega_{\rho' \sigma'}(k_2) a^{\rho \sigma}(q_1, q_2; k_1, k_2) a^{\rho' \sigma'}(q_1', q_2'; k_2, k_1) + (k_1 \leftrightarrow k_2).$$

Here $q_i' = q_i - Q$ and $Q$ denotes the total momentum transfer and $g_N$ and $h_N$ are the known colour factors

$$g_N = tr (T^{i_1})^2 (T^{i_2})^2, \quad h_N = tr (T^{i_1} T^{i_2})^2.$$ 

Matrices $T^i$ are the colour group generators. Note that the above expression for $\sum A A$ is much more complicated, than the result guessed in ref. [17]. It can not be written as a product of two factors depending on longitudinal and transverse momenta correspondingly. The integration over the squared mass $\kappa$ of the produced gluons contains the ultraviolet logarithmic divergency. According to our discussion in section 4 we should introduce an intermediate parameter $\eta$ which allows us to define clusters of particles in the final state. The invariant mass of the particles inside each cluster is restricted from
above by this parameter. Therefore, after integration over $\kappa$ the result will contain the
term linear in $\eta$ which cancels the analogous infrared divergency at a small relative ra-
pidity for two neighbouring gluons produced in the multi-Regge kinematics. The finite
one-loop contribution to the BFKL kernel is obtained by the subsequent integration of
the constant term $\sim \eta^0$ over transverse momenta $k_{1\perp}$ for fixed $q_{1\perp}, q_{2\perp}$. This constant
term can not be presented as a sum of contributions for contracted Feynman diagrams
in the transverse subspace contrary to the assumption made in ref. [17]. Note that in
the dispersive approach developed in ref. [11] one should take into account the next to
leading corrections to the production amplitude in the multi-Regge kinematics which also
can not be expressed in terms of the contracted diagrams.

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