Determination of Fracture Mechanics Parameters Using Simulation Based on Finite Element Analysis

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Abstract. Failures of assemblies and parts lead sometimes to cracks which can have disastrous results, even if the stress level in a structure considered „perfect” can indicate a satisfactory design. The study of crack initiation and propagation is a complex subject, which implies different notions from Physics, Chemistry and Mechanics. Understanding the main issues regarding the propagation of cracks – is treated in the Fracture Mechanics engineering field. For example, engineers desire to understand the conditions in which a pre-existing crack continues to propagate. Next, we will use Finite Element Analysis to determine the stress at the tip of a pre-existing crack, also other fracture mechanics related characteristics. Thus, we will use the Fracture Mechanics module for the Algor finite element analysis software, version 22.1.

1. Definition of the Stress Intensity Factor

These guidelines, written in the style of a submission to ACME 2016 Conference, show the best layout for your paper. In order to define the stress intensity factor, it is needed to consider a plate of unity thickness, made of an ideally-elastic material in which a thru crack of semi-length a exists, figure 1a in polar coordinates and figure 1b in cartesian coordinates. Plate dimensions are much larger than the crack size. The plate is subjected to a uniaxial stress state, their distribution being as such that at a certain distance from crack; the main principal stresses are perpendicular amongst them, also parallel to the major axis of the ellipse.

![Stress field in the vicinity of the crack](image)

Figure 1. Stress field in the vicinity of the crack.
In an infinitely thin plate we have a state of planar stress – the stress normal to the plate being zero. For a thick plate, a planar strain state is obtained – the deformation thru the thickness of the plate being zero. The crack is considered to be planar, with sharp tips. Considering an area element around the M point at r distance from the crack tip and at \( \theta \) angle related to its plane, Figure 1b. Stress state in the immediate vicinity of the crack is given by the relations:

\[
\begin{align*}
\sigma_x &= \frac{\sigma_0 \sqrt{2\pi a}}{2\sqrt{\pi r}} \cos \theta \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
\sigma_y &= \frac{\sigma_0 \sqrt{2\pi a}}{2\sqrt{\pi r}} \cos \theta \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
\tau_{xy} &= \frac{\sigma_0 \sqrt{2\pi a}}{2\sqrt{\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}
\end{align*}
\]

with \( \sigma_z = 0 \) for planar stress state or \( \sigma_z = \nu(\sigma_x + \sigma_y) \) for planar strains.

Stresses \( \sigma_x, \sigma_y \), and \( \sigma_z \) are proportional with the applied stress \( \sigma \), it varies with the square root of the crack size and they tend to \( \infty \) when \( r \) tends to zero. From the above relations, we observe that the stresses at the crack tip are calculated as product between the geometric factor \( \frac{1}{\sqrt{2\pi r}} \cdot f(\theta) \), which depends on the position of the area elements in which the stresses are calculated, and the factor \( \sigma \cdot \sqrt{2\pi a} \). This last factor, which represents a measure of the stress increase due to the crack’s existence, in relation to the residual stress in the plate when the crack is absent, was named stress intensity factor and was noted as \( K (K_i = \sigma \sqrt{2\pi a}) \). The I index is used to signal that we are in the mode I of fracture (opening). The stress intensity factor is measured in \( \text{MPa}\sqrt{\text{m}} \). The stress intensity factor depends on the fracture mode, on the geometry of the specimen and of the defect, on its orientation in relation with the applied stresses, all these elements being comprised into the \( Y \) coefficient from the general expression:

\[ K = Y \sigma \sqrt{2\pi a} \]  

Stress intensity factor is determined theoretically based on the elasticity theory, the most usual methods being: the method of conformal transform; the method of integral transform; alternating method; asymptotic approximations methods; finite element method.

For a plate with a centered crack of width \( W >> 4a \), subjected to uniaxial extension, the stress intensity factor \( K_i \) is calculated using the relation (2) where:

\[ Y = 1.77 \left[ 1 - 0.1 \left( \frac{2a}{W} \right)^2 \right] \]  

2. Introduction to the finite element analysis for Fracture Mechanics

The investigation of crack propagation using Algor software uses a post-processing function, meaning that, firstly the stress state is being analyzed, then the crack propagation is deduced based on the results from the table Results environment. The main steps taken in a crack propagation analysis are:

1. The model is being meshed, including the crack or defect. A crack being an separation between elements (a gap in the mesh). The crack tip should be a sharp angle (point). Check Figure 2;
2. It is mandatory to check the stress state in the elastic or elastic-plastic domain;
3. The crack or cracks parameters are defined in Results environment using a 3D view;
4. The crack propagation is analyzed in Results environment;
5. The obtained results are being examined.
The parameters pertaining to each crack are inserted in Results environment with the help of "Fracture Analysis".

Crack propagation direction is obtained under the assumption that the crack will propagate on the direction which satisfies at the crack tip, the condition: \( \frac{\partial \sigma}{\partial \theta} = 0 \). In crack analysis, especially in the numerical calculation, the crack propagation direction is calculated with:

\[
0 = \cos \left( \frac{\sqrt{3K_{II}^{2} + 8K_{I}^{2}K_{II}^{2}}}{3K_{I}^{2} + 9K_{II}^{2}} \right)
\]

where \( \theta < 0 \) if the factor \( K_{II} > 0 \) or \( \theta > 0 \) if the factor \( K_{II} < 0 \), where \( K_{I} \) and \( K_{II} \) are the stress intensity factors for mode I and II of crack formation. The propagation angle \( \theta \) respects the crack plane.

Hayashi and Nemat-Nasser (1981) give a series of directions which depend on the \( c_{ij} \) coefficients. At the crack tip, the stress intensity factors for different directions have the following expressions:

\[
K_{I}^k = c_{11}K_{I} + c_{12}K_{II}
\]

\[
K_{II}^k = c_{21}K_{I} + c_{22}K_{II}
\]

We also have at disposal the relation \( G = \frac{\left( k_{I}^k \right)^{\frac{3}{2}} + \left( k_{II}^k \right)^{\frac{3}{2}}}{E} \). The criterion for maximum released energy postulates that the crack propagates in the direction of maximizing \( G \).

Calculations regarding crack propagation are applicable only for elastic models under a quasi-static load. If the model is created for different cases, plasticity and dynamic analyses, the results will diverge from reality, in accordance to the amount of existing non-linearity and dynamics. Crack propagation analysis can be exerted on certain load cases or time steps, instead of being analyzed in its entirety.

3. Performing the crack analysis and viewing the results
In order to analyze all defined cracks, the "Fracture Analysis" module is being accessed. After the crack analysis is finished, the following results are obtained:

- "Fracture: J Integral": strain energy release rate for materials with non-linear elastic behavior. The J-Integral concept implies that, the calculation is adequate only for monotonic loading of elastic-plastic materials;
"Fracture: Stress Intensity Factor: "K_I", "K_{II}" and "K_{III}"": the stress intensity factors for the three modes of crack loading are obtained. Comparing the stress intensity factor with the critical value if stress intensity, fracture toughness (resistance to fracture) K_{IC}, will establish if the crack will propagate or not.

"Fracture: Crack Growth Direction": the direction of crack propagation is being calculated as a unity vector.

All results are displayed as vectors on the nodes along the crack front.

Variation of stress intensity factors and J integral in relation with the crack propagation in mode I of loading

In the finite element analysis of a cracked compact specimen, the initial crack is of 13 mm then the front of the crack is sequentially analyzed. Following are the 13 mm length crack and cracks of total lengths: 15.25 mm, 17.55 mm, 19.86 mm, 22.17 mm and 22.5 mm. All results were displayed as vectors on the nodes along the crack front. The arrow direction represents the crack propagation direction. To establish the stress state in the immediate vicinity of the crack for the mixed mode of loading, it was performed a finite element analysis in ALGOR software, the elastic module.

As seen in Figure 3, the maximum stress intensity factor is found on the node at the center of the crack front and has the value of K_{I} = 2324.43 N∙mm^{3/2} (=73.49 MPa∙m^{1/2}).

![Figure 3](image)

In Figure 4 the method of determining J integral is presented (for a crack length of 22.5 mm) as well as the stress distribution along zz direction and the displacements for a crack length of 15.5 mm. From Figure 4b it is concluded that the stress is maximal at the crack tip, which further on favors its propagation.

![Figure 4](image)
Using the data obtained from the finite element analysis, plots containing the variation of stress intensity factor $K_I$ were drawn, as well the J integral, for all 11 nodes found at the front of the crack, for all 6 crack lengths shown before. It is observed that the stress intensity factor increases as long as the crack propagates. From Figure 5 we observe that, for the same crack length, the stress intensity factor value remains almost constant in the nodes at the front of the crack (on 5 of these nodes).

![Figure 5](image)

**Figure 5.** Mode I – a) Variation of $K_I$ factor along crack front in relation to the crack length; b) Variation of J integral along crack front in relation to the crack length.

Regarding the J integral, the following is concluded: for small values of the propagated crack, a certain constancy of the J integral values along the crack front is observed; increasing the crack length, big dispersions of J integral appear in the points from the crack front.

Variation of stress intensity factor $K_I$, in relation to the length of the propagated crack, is presented in Figure 6a. It can be seen that, as long as the crack advances, the stress intensity factor increases accordingly. In Figure 6b it is shown the variation of maximum stress along Z direction in relation to the crack length for mode I of loading.

![Figure 6](image)

**Figure 6.** a) $K_I$ variation in relation to crack length; b) Maximum stress variation along Z direction, in relation to crack length – mode I.

4. Conclusions

Finite element analysis applied for cracked specimens offers important data in studying the initiation and the unstable propagation of the crack. As such, the stress intensity factor is compared to the fracture toughness. The moment when the allowable fracture toughness is exceeded, the crack propagates in an unstable manner. Full fracture is achieved after a certain amount of time that depends also on the crack propagation rate which needs to be found empirically. From Figure 6 it is observed that, for the same external loading value, as long as the crack advances, the stress along zz direction and the stress intensity factor increase. For example, the stress intensity factor for a crack longer than 22 mm is approx. 3500 N·mm$^{3/2}$ or 110.67 MPa·m$^{1/2}$. In these conditions, only a material which
exhibits fracture toughness greater than 110 MPa·m$^{1/2}$ will induce a stable propagation of the crack. Thus, the crack will propagate in an unstable manner, leading to fracture in the end. If the material chosen in finite element analysis does not meet the above condition, a new material will be imposed and a new analysis will be run until, for a certain crack length, the stress intensity factor becomes much smaller than the fracture toughness.

5. References
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