Cosmography by GRBs

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ABSTRACT

Aims. Relations connecting GRB quantities can be used to constrain cosmographic parameters of the Hubble law at medium-high redshifts.

Methods. We consider a sample of 64 GRBs to construct the luminosity distance - redshift relation and derive the values of the parameters \( q_0 \), \( j_0 \) and \( s_0 \).

Results. The results, calibrated by SNeIa data, agree with the \( \Lambda \)CDM model.

Key words. Gamma rays : bursts - Cosmology : cosmological parameters - Cosmology : distance scale

1. Introduction

A class of very accurate standard candles, the Supernovae Ia (SNeIa), has been highly developed in the last two decades of the twentieth century (Branch & Tammann 1992). However these objects are hardly detectable at redshifts higher than \( z = 1.7 \) hence the study of more distant regions of the Universe leads to the issue to implement more powerful standard candles. The problem becomes particularly crucial at intermediate redshift \( z = 6 - 7 \) where, up to now, not well defined distance indicators are available.

In the last years, several efforts have been made in order to implement Gamma Ray Burst (GRBs), the most powerful explosions in the Universe, as standard candles, and several interesting results have been recently achieved in literature (see, for example, Amati et al 2008, Basile and Perivolaropoulos 2008). Considering the standard model of such objects, the GRB phenomenon should originate from the black hole formation and reach huge amounts of energy (up to \( 10^{51} \) erg). These events are observed at considerable distances and there are several efforts to frame them into the standard of cosmological distance ladder.

In literature, several more detailed models give account for the GRB formation, see for example Ruffini et al 2008, Meszaros 2004, but, up to now, none of them is intrinsically capable of connecting all the observable quantities: for this reason GRBs cannot be used as standard candles, indeed. Despite of this shortcoming, there are some observational correlations among the photometric and the spectral properties of GRBs. These features allow to use GRBs as distance indicators (Schaefer 2007), also if they cannot be fully "enrolled" in the class of standard candles. In particular, it is possible to connect the peak energy of a GRB, \( E_p \), with the peak luminosity \( L \):

\[
\log L = a + b \log \frac{E_p(1 + z)}{300 \text{ keV}},
\]

where \( a \) and \( b \) are two calibration constants.

Another interesting relation connect the peak energy \( E_p \) with the isotropic energy released in the burst, \( E_{iso} \) and with the rest frame break - time of the afterglow optical light curve, \( t_b \):

\[
\log E_{iso} = a + b_1 \log \frac{E_p(1 + z)}{300 \text{ keV}} + b_2 \log \frac{t_b}{(1 + z)1 \text{ day}}
\]

where, also in this case, \( a \) and the \( b_i \), with \( b = 1, 2 \), are constants to calibrate.

These two relations show a discrepancy between data and theoretical curve which are less than the other relations available in literature, so we will consider them in this work. In (Schaefer 2007), the discrepancy between data and theoretical curves are shown for these relations. It is important to notice that the calibration is necessary in order to avoid the circularity problem: all the relations need to be calibrated for every set of cosmological parameters. Indeed all GRB distances, obtained only in a photometric way, are dependent on the cosmological parameters since there is no low-redshift set of GRBs to perform a cosmology-independent calibration.

In order to bypass such a circularity problem, we consider a cosmology-independent calibration using the SNeIa data, (Liang et al 2008); in fact, as we said, the SNeIa are very accurate standard candles, but their range is limited up to \( z \approx 1.7 \); hence, assuming that relations (1) and (2) works at any \( z \), and that at the same redshift GRBs and SNeIa have the same luminosity distance, it becomes possible to calibrate GRBs relations at low redshifts.

The next step is to use the calibrated GRB relations at every \( z \) in order to obtain two cosmology-independent relations. In this letter, we consider the results already obtained by Liang et al., (Liang et al 2008), which give reliable estimate of the parameters \( a \), \( b_i \). In Table 1 the calibration parameters are shown.

For the \( E_{iso} - E_p - t_b \) relation, the \( b \)-values in the first line is \( b_1 \) and in the second line is \( b_2 \).

The aim of this work is to achieve the cosmographic parameters (Weinberg 1972) using the above GRB relations. The only assumption that we make is that the Universe is described
by a Friedmann-Robertson-Walker geometry and the scale factor of the universe $a(t)$ can be expanded in a Taylor series (Sec.2). In Sec.3, after considering a sample of 64 GRBs, we derive, by a best-fit analysis, the cosmographic parameters discussed in the previous section. Conclusions are drawn in Sec.4.

### 2. Cosmography

The calibration which we want to achieve should be cosmologically model-independent. Hence, applying the above relations to a GRB sample in a given $z$-range, we want to derive the related cosmography. In particular, we want to obtain deceleration, jerk and snap parameters (Visser 2004) and compare them with the current values deduced by other methods and observations (see, for example, (Basilakos & Perivolaropoulos 2008; Capozziello et al 2008) and references therein).

Being only related to the derivatives of the scale factors, the cosmographic parameters allow to fit the data versus the distance-redshift relation without any a priori assumption on the underlying cosmological model.

In order to build a distance-redshift diagram, one has to calculate the luminosity distance for every GRB in a given sample. For the relation (1), the luminosity distance is:

$$d_l = \left( \frac{L}{4\pi S_{\text{bolo}}'} \right)^\frac{1}{2},$$

where $S'_{\text{bolo}} = S_{\text{bolo}}/(1+z)$ is the bolometric fluence of gamma rays in the burst, corrected with respect to the rest frame, while, for the relation (11), we have:

$$d_l = \left( \frac{L}{4\pi P_{\text{bolo}}} \right)^\frac{1}{2},$$

where $P_{\text{bolo}}$ is the bolometric peak flux. The luminosity distance can be connected to the Hubble series. Expanding the Hubble law up to the fourth order in redshift and considering the related luminosity distance, we get (Visser 2004):

$$d_H(z) = \frac{c}{H_0}z \left[ 1 + \frac{1}{2} (1 - q_0) z - \frac{1}{6} (1 - q_0 - 3q_0^2 + j_0 + \frac{k d^2_H}{a_0^2}) z^2 + \frac{1}{24} \left[ -2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0(1 + 2q_0) + s_0 + 2k d^4_H (1 + 3q_0) \right] a_0^2 \zeta^2 + \mathcal{O}(z^4) \right],$$

where $d_H = c/H_0$ is the Hubble radius and where the cosmographic parameters are defined as:

$$H(t) = \frac{1}{a} \frac{da}{dt};$$

$$q(t) = -\frac{1}{a} \frac{d^2a}{dt^2} / \left( \frac{1}{a} \frac{da}{dt} \right)^2;$$

$$j(t) = +\frac{1}{a} \frac{d^3a}{dt^3} / \left( \frac{1}{a} \frac{da}{dt} \right)^3;$$

$$s(t) = +\frac{1}{a} \frac{d^4a}{dt^4} / \left( \frac{1}{a} \frac{da}{dt} \right)^4.$$

They are usually referred to as the Hubble, deceleration, jerk and snap parameters, respectively. Their present values, which we denote with a subscript 0, may be used to characterize the evolutionary status of the Universe. For instance, $q_0 < 0$ denotes an accelerated expansion, while $j_0$ allows to discriminate among different accelerating models; a positive value of $j_0$ indicates that, in the past, the acceleration reversed its sign. Note that, in this work, according to the WMAP observations, we can consider the value of the Hubble constant $H_0 = 70 \pm 2$ km/sec/Mpc (Komatsu et al 2008).

The cosmographic parameters can be expressed in terms of the dark energy density and the equation of state (EoS). Following the prescriptions of the Dark Energy Task Force, (Albrecht et al 2006), we use the Chevallier-Polarski-Linder parameterization (CPL) for the EoS setting (Chevallier et al 2001), so that, in a flat universe filled with dust matter and dark energy, the dimensionless Hubble parameter $E(z) = H/H_0$ reads:

$$E^2(z) = \Omega_M (1 + z)^3 + \Omega_{\Lambda} (1 + z)^{3(1+w_0+w_a)} e^{-\frac{3w_a}{2}},$$

with $\Omega_{\Lambda} = 1 - \Omega_M$ because of the flatness assumption, and $w_0$ and $w_a$ the CPL parameterization. Such a relation can be used to evaluate the cosmographic parameters, obtaining, for a $\Lambda$CDM-universe:

$$q_0 = -1 + \frac{3}{2} \Omega_M;$$

$$j_0 = 1;$$

$$s_0 = 1 - \frac{9}{2}\Omega_M,$$

which are the quantities which we are going to fit using a given GRB sample.

### 3. GRB data fitting

Let us take into account the 69 GRBs reported in Schaefer (Schaefer 2007). We choose the 64 objects satisfying the relations considered above. The luminosity distance for each of the relations is given by Eqs. (3) and (4) and then we obtain a data distribution in the luminosity distance-redshift diagram $d_l - z$.

The errors on the data are only of photometric nature and, in a first analysis, we can exclude errors on the redshift.

Another version of the Hubble series can be used in order to improve the data fit. If we consider the equation for the distance modulus:

$$\mu = 25 + \frac{5}{2} (10) \ln \left[ \frac{d_H}{(1 Mpc)} \right] + 25,$$

and substitute the equation for $d_l$, we obtain a logarithmic version of the Hubble series:

$$\ln \left[ \frac{d_H}{(z Mpc)} \right] = \ln \left( \frac{d_H}{(1 Mpc)} \right) - \frac{1}{2} (1 - q_0) z$$

$$+ \frac{1}{24} \left[ -3 + 10q_0 + 9q_0^2 - 4j_0 + 1 + \frac{k d_H^2}{a_0^2} \right] z^2$$

$$+ \frac{1}{24} \left[ 4j_0 (j_0 + 1 + \frac{k d_H^2}{a_0^2}) + 5 - 9q_0 - 16q_0^2 - 10q_0^3 \right] z^3$$

$$+ j_0 (7 + 4q_0) + s_0 z^4 + \mathcal{O}(z^5).$$

### Table 1. Parameter values obtained by Liang et al 2008

| Relation       | a       | b        |
|----------------|---------|----------|
| $L - E_p$      | $52.26 \pm 0.09$ | $1.69 \pm 0.11$ |
| $E_{iso} - E_p - I_b$ | $52.83 \pm 0.10$ | $2.28 \pm 0.30$ |
| $d_l$          | $-1.07 \pm 0.21$ |          |
This logarithmic version shows the advantage that there is no need to transform the uncertainties on the distance modulus. With these considerations in mind, we perform a polynomial least-squares fit of the data considering Taylor series polynomials both in distance and in logarithmic distance. We stop at order \( n = 3 \) both for the polynomial fit and for the logarithmic fit. In the latter case, we obtain an estimate of the snap parameter. Note that we are using least squares since, in absence of any better data-fitting procedure, this is the standard procedure assuming Gaussian distributed uncertainties.

The truncated polynomial used in the fits is of the form

\[
d(z) = \sum_{i=1}^{3} a_i z^i,
\]

and

\[
\ln[d(z)/(zMpc)] = \sum_{i=1}^{3} b_i z^i
\]

for the logarithmic fit. In the latter case, the Hubble constant enters as the \( i = 1 \) component of the fit. In this work, we use \( H_0 \) as a constraint (a priori).

The fits can be used to estimate the deceleration and the jerk parameters. The logarithmic fit is better to estimate the snap parameter through the values of the coefficients \( a_i \) and \( b_i \) and their statistical uncertainties. Note that the statistical uncertainties on \( q_0 \) are linearly related to the statistical uncertainties on the parameter \( b_1 \), while the statistical uncertainties in \( j_0 \), and in \( s_0 \), depend non-linearly on \( q_0 \) and its statistical uncertainty. It is worth noticing the combination \( j_0 + k d_0^2/a_0^2 \), which is a well-known degeneracy in Eq. (15) (Weinberg 1972). It means that we cannot determine \( j_0 \) and \( \Omega = 1 + k d_0^2/a_0^2 \) separately, but we need an independent determination of \( \Omega \) in order to estimate the value of the jerk parameter.

The results of the fits are presented in Table 2. The weighted fits include the error bars on \( d_i \).

As said above, only statistical uncertainties have been considered and any other kinds of errors (systematics due to cosmological inference, modelling and also “historical” biases (Visser 2007b)) have been neglected. Note that if we do not assign \( H_0 \) as a constraint, the analysis gives \( H_0 = 56 \text{ km/s/Mpc} \).

![Fig. 1. Luminosity distance - redshift diagram. The circles are the GRBs. The solid line is the result of the fit](image1)

![Fig. 2. Logarithmic version of the luminosity distance versus redshift.](image2)

**Table 2. Results of the fits.** All the logarithmic fits are weighted.

| Fit          | \( q_0 \)         | \( j_0 + \Omega \) | \( s_0 \)         |
|--------------|-------------------|--------------------|-------------------|
| \( d(z) \)   | \(-0.48 \pm 0.37 \)| \(0.37 \pm 0.80 \)  |                   |
| \( d(z) \) weighted | \(-0.30 \pm 0.43 \) | \(1.06 \pm 2.78 \) |                   |
| \( \ln[d(z)] \) | \(-0.45 \pm 0.31 \) | \(-0.105 \pm 1.09 \) | \(1.35 \pm 14.21 \) |
| \( \ln[d(z)/d_0], H_0 = 65 \) | \(-0.58 \pm 0.30 \) | \(-0.02 \pm 1.05 \) | \(2.48 \pm 10.38 \) |
| \( \ln[d(z)/d_0], H_0 = 70 \) | \(-0.76 \pm 0.30 \) | \(0.24 \pm 1.05 \) | \(5.38 \pm 23.71 \) |

**Table 3. Goodness of the fits with the R-square.** Note the values \( \ll 1 \) for the logarithmic fits which are due to the discrepancy of the data.

| Fit          | R-square |
|--------------|----------|
| \( d(z) \)   | 0.5609   |
| \( d(z) \) weighted | 0.7617   |
| \( \ln[d(z)/d_0] \) | 0.07101 |
| \( \ln[d(z)/d_0], H_0 = 65 \) | 0.08335 |
| \( \ln[d(z)/d_0], H_0 = 70 \) | 0.07101 |

this means that the data sample needs to be improved with more GRBs to give more reliable results.

A further step is to test the goodness of fit statistics using the MATLAB package. In particular we have used the R-square method: A value closer to 1 indicates a better fit. In Table 3 the results of R-square are shown. In Figs. 3-4 the plots of the residuals of the fits are shown. For the logarithmic fit, the bad value of the R-square is due the logarithm of the Hubble series which spreads a lot the data on the \( \ln(d_i) \)-axis.

In summary, the results are in agreement with the \( \Lambda \)CDM model giving a universe which accelerates in the present epoch and underwent a decelerated phase in the past. The signature of this past phase is related to the sign change of the parameter \( q_0 \) and the positive value of the jerk parameter, unless a positive value of the spatial curvature constant \( k \) is considered. However this occurrence is excluded by the last observational results which are confirming a spatially flat universe (Komatsu et al 2008).
4. Discussion and Conclusions

Starting from some relations connecting the observable quantities of GRBs, we have used a sample of 64 GRBs in order to derive the luminosity distance - redshift diagram of the Hubble law. The relations have been conveniently calibrated by SNeIa in order to make them independent of any cosmological models. We have taken into account the Hubble law, in the Taylor series form, assuming the luminosity distance $d_l$ as a redshift function whose coefficients are combinations of the cosmographic parameters $H_0, q_0, j_0$ and $s_0$. The aim has been to evaluate such parameters starting from the GRB data. A direct analysis of the fits leads to the conclusion that, in the error range, the SNeIa results are confirmed also at higher redshifts (Visser 2007b). Besides, such results agree with the $\Lambda$CDM model according to Eqs. (11),(12),(13). In particular, the value of the parameter $q_0$ which we found is in agreement with the observed $\Omega_M$ (see Table 4).

However, the sample which we used is quite poor at high redshifts. In particular, at $z > 6$, we have only 2 GRBs showing a huge discrepancy with respect to the $d_l$ plot (see Fig. 1). These two data can substantially bias the fit results so we need some richer sample at medium-high redshifts in order to constrain better the results.

As final remark, considering these preliminary results, it seems that cosmography by GRBs could be a useful tool to constrain self-consistent cosmological models also if, up to now, GRBs are not standard candles in the proper sense.

### Table 4. Density parameters

| Fit                  | $\Omega_M$ | $\Omega_\Lambda$ |
|----------------------|------------|-------------------|
| $d_l(z)$             | 0.35 ± 0.36| 0.65 ± 0.73       |
| $d_l(z)$ weighted    | 0.46 ± 0.43| 0.54 ± 2.82       |
| $\ln[d_l/z Mpc]$, $H_0 = 65$ | 0.37 ± 0.31| 0.63 ± 1.13       |
| $\ln[d_l/z Mpc]$, $H_0 = 70$ | 0.28 ± 0.30| 0.72 ± 1.09       |

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