Comparison of Two Coronal Magnetic Field Models to Reconstruct a Sigmoidal Solar Active Region with Coronal Loops

Aiying Duan1,2,3, Chaowei Jiang4, Qiang Hu3,5, Huai Zhang1, G. Allen Gary3, S. T. Wu3, and Jinbin Cao6
1 Key Laboratory of Computational Geodynamics, University of Chinese Academy of Sciences, Beijing 100049, China; duanaiying@ucas.ac.cn, hzhang@ucas.ac.cn
2 SIGMA Weather Group, State Key Laboratory for Space Weather, National Space Science Center, Chinese Academy of Sciences, Beijing 100190, China
3 Center for Space Plasma and Aeronomic Research, The University of Alabama in Huntsville, Huntsville, AL 35899, USA
4 Institute of Space Science and Applied Technology, Harbin Institute of Technology, Shenzhen, 518055, China; chaowei@hit.edu.cn
5 Department of Space Science, The University of Alabama in Huntsville, Huntsville, AL 35899, USA
6 School of Space and Environment, Beihang University, Beijing 100191, China

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Abstract

Magnetic field extrapolation is an important tool to study the three-dimensional (3D) solar coronal magnetic field, which is difficult to directly measure. Various analytic models and numerical codes exist, but their results often drastically differ. Thus, a critical comparison of the modeled magnetic field lines with the observed coronal loops is strongly required to establish the credibility of the model. Here we compare two different non-potential extrapolation codes, a nonlinear force-free field code (CESE–MHD–NLFFF) and a non-force-free field (NFFF) code, in modeling a solar active region (AR) that has a sigmoidal configuration just before a major flare erupted from the region. A 2D coronal-loop tracing and fitting method is employed to study the 3D misalignment angles between the extrapolated magnetic field lines and the EUV loops as imaged by SDO/AIA. It is found that the CESE–MHD–NLFFF code with preprocessed magnetogram performs the best, outputting a field that matches the coronal loops in the AR core imaged in AIA 94 Å with a misalignment angle of ~10°. This suggests that the CESE–MHD–NLFFF code, even without using the information of the coronal loops in constraining the magnetic field, performs as good as some coronal-loop forward-fitting models. For the loops as imaged by AIA 171 Å in the outskirts of the AR, all the codes including the potential field give comparable results of the mean misalignment angle (~30°). Thus, further improvement of the codes is needed for a better reconstruction of the long loops enveloping the core region.

Key words: magnetic fields – magnetohydrodynamics (MHD) – methods: numerical – Sun: corona – Sun: flares

1. Introduction

Solar explosive events (e.g., flares and coronal mass ejections) observed within the corona are attributed to energy release in the coronal magnetic field. Hence, understanding the dynamics and physics of these eruptions is centered on understanding the structure and evolution of the magnetic field. The coronal field can evolve from reconnection and photospheric variations. However, direct measurement of the coronal magnetic field from emissions of extremely tenuous plasma by the corona through spectropolarimetric methods still proves to be difficult (Lin et al. 2004; Lin 2016). Currently, the routine measurement of the solar magnetic field that we can rely on is restricted to only a single layer of the solar surface, i.e., the photosphere. Most recently, chromospheric polarimetry has been beginning to show promising results; however, the height of formation is problematic (Quintero et al. 2017).

Due to the lack of measurement data, the three-dimensional magnetic field in the solar corona is usually “extrapolated” or “reconstructed” in numerical ways from the photosphere surface data based on particular assumptions or models. Such techniques of modeling the coronal magnetic field have been developed (e.g., see review papers of Sakurai 1989; McClymont et al. 1997; Solanki et al. 2006; Wiegelmann 2008; Wiegelmann & Sakurai 2012; Régnier 2013). The three magnetic field models often used include the potential field, linear force-free field (LFFF), and nonlinear force-free field (NLFFF). All these models are derived from the basic assumption that the Lorentz force in the corona vanishes in the case of extremely low plasma β (β is the ratio of the plasma pressure to the magnetic pressure) and quasi-static equilibrium of the coronal field. Consequently, the electric current \( J = \nabla \times B \) must be parallel to the magnetic field, i.e., \( J = \alpha B \), where \( \alpha \) is called the force-free parameter. In the potential field model, \( \alpha = 0 \); in the LFFF, \( \alpha \) is a constant; and in the NLFFF, \( \alpha \) is variable in space. The earliest models were based on potential field (Altschuler & Newkirk 1969; Sakurai 1982) and LFFF (Seehafer 1978), which are extrapolated from only the data of the line-of-sight (LoS) component of the photospheric field since the transverse components were not measured. Now, both the LoS and transverse components of the photospheric magnetic field can be measured (e.g., Hoeksema et al. 2014) and information on the electric current passing through the photosphere can be derived. Nonlinear force-free field (NLFFF) reconstructions, which are based on vector magnetograms, are more robust than those earlier models, and employ several numerical schemes (Sakurai 1981; Yang et al. 1986; Wu et al. 1990; Amari et al. 1997; Yan & Sakurai 2000; He & Wang 2006; Wheatland 2006; Wiegelmann & Neukirch 2006; Valori et al. 2010; Jiang & Feng 2012a). Existing NLFFF codes include the optimization method (Wheatland et al. 2000; Wiegelmann 2004; Wiegelmann & Neukirch 2006; Wiegelmann 2008), the magneto-frictional method (Valori et al. 2007, 2010; Guo et al. 2016), the Grad-Rubin method (Amari et al. 2006; Wheatland 2007; Amari et al. 2014), and the MHD-relaxation code based on the conservation-element/solution-element space-time scheme (CESE–MHD–NLFFF: Jiang et al. 2011; Jiang & Feng 2012a, 2013). NLFFF models are widely used in the solar physics community for exploring the 3D coronal structure prior to and post solar eruptions and for understanding the influence of the
magnetic topology on solar eruptions (e.g., Guo et al. 2008; Sun et al. 2012; Jiang et al. 2013; Cheng et al. 2014; Liu et al. 2014; Xue et al. 2016).

An alternative to NLFFF extrapolation is an approach based on a magnetic field model of the corona derived from the variational principle of the minimum energy dissipation rate (Hu & Dasgupta 2008; Hu et al. 2008, 2010). The formula governing the coronal magnetic field is more complex, and the solution, expressed as a superposition of three LFFFs (one being potential), is in general not force free. Such a non-force-free field (NFFF) extrapolation was presented in Hu & Dasgupta (2008) and was shown to be applicable to the NLFFF configuration given by the Low & Lou (1990) analytic solutions. It was also tested on numerical MHD simulation data for an active region (Hu et al. 2008). Later, it was further developed for practical applications to photospheric vector magnetograms obtained within an active region (Hu et al. 2010). The implementation of the algorithm is relatively simple and the full code written in Interactive Data Language (IDL) can handle the magnetograms of 1024 × 1024 pixels on a desktop PC. However, extensive testing and application of the NFFF algorithm have yet to be performed.

A critical assessment of the reliability of coronal magnetic field extrapolation modeling is to examine the goodness of matching of the geometry of the simulated magnetic field lines with that of the observed EUV coronal loops. This is because the plasma emission in the corona reflects the geometry of the invisible magnetic field, as in most parts of the corona the plasma is “frozen” with the magnetic fields. While earlier studies compared theoretical models with observed images in a rather qualitative way, recent studies go into quantitative measurements of the agreement between the modeled field lines and observed coronal-loop geometries. In an assessment of a variety of popular NLFFF codes for modeling AR 10953, DeRosa et al. (2009) compared the model field lines to 3D trajectories of coronal loops, which were observed stereoscopically by the twin Solar Terrestrial Relations Observatory (STEREO) spacecraft (Aschwanden et al. 2008). It was found that the misalignment angle between the field lines and the loops amounts to rather large values of 24°–44°, and unexpectedly, none of the examined NLFFF models improved significantly upon the value found for the potential field model. Another method employs a slight variation by measuring the distance between a loop and a projected field line (Chae & Moon 2005; Lim et al. 2007; Malanushenko et al. 2011). The same idea of a “distance” has also been used in 3D, with STEREO data (Wiegelmann & Neukirch 2002). Without tracing the trajectories of coronal loops, Wiegelmann et al. (2012) compared the model field lines with SDO/AIA images by calculating the sum of the gradient of the AIA image intensity along each projected field line, and the result is assumed to reach its minimum if all the field lines are co-aligned with the corresponding loops. The LFFF parameter α, derived from a visual fit to the loops, has been compared to that derived from a vector magnetogram (Burnett et al. 2004), providing an additional goodness metric.

Recently, Gary et al. (2014b) developed a new method of deriving the 3D structure of observed 2D coronal loops independent of heliostereoscopy, and suggested that it can determine the matching of the extrapolated magnetic fields with coronal loops. In that method, an automated loop recognition scheme (OCCULT-2; Aschwanden et al. 2013) is first used to extract 2D loop structures from EUV images and then the extracted loops are fitted with 2D cubic Bézier splines that are based on four control points (Gary et al. 2014a). The 2D splines are further extended to 3D with the heights of all four control points set as free parameters. The heights are determined by minimizing the misalignment angles of the 3D splines with the field lines of a given model of the coronal magnetic field, and the resulting 3D splines are regarded as the trajectories of the corresponding coronal loops in 3D. So, the image of these loops represents a 2D projection of these 3D structures. Naturally, different magnetic field models will result in different sets of misalignment angles, and a perfect matching model can presumably yield a misalignment angle of zero. Thus, this method provides a powerful tool for comparing different magnetic field models of the corona by using EUV images obtained from one single viewpoint.

Other coronal magnetic field models with forward-fitting of coronal loops has been developed to better reproduce the geometry of the loops (Malanushenko et al. 2012, 2014; Aschwanden 2013a, 2013b). For instance, unlike the traditional NLFFF codes that extrapolate the coronal field from the vector field at the photosphere, the coronal-loop forward-fitting quasi-NLFFF models of Aschwanden (2013a) use only the LoS magnetic field component, and the non-potentiality of the field (characterized by the force-free parameter α) is determined by the minimization of the misalignment of the field lines with the traced loops from the observed images. Aschwanden (2013b) shows that their code can be applied to either 3D loops determined stereoscopically or simply to 2D from LoS observations. In the latter case, the 3D height of the loops is used as a free parameter and a 2D misalignment angle is minimized. The forward-fitting method strongly depends on the recognition of coronal loops from the image, which has been automated by Aschwanden et al. (2013). We should note that even in such forward-fitting of the coronal loops, the misalignment angles of the field line with the loops is significant. For example, the misalignment angle that resulted in Aschwanden (2013b) is about 20° in the forward-fitting of TRACE loops.

In this paper, we will evaluate our extrapolation results from the NLFFF (CESE–MHD–NLFFF) and the NFFF codes by introducing a critical comparison of the modeled magnetic field lines with the coronal-loop geometry. In particular, we will employ Gary et al.’s (2014b) method to compute the misalignment angles of the model field lines and identified loops. The target region used in this examination is AR 12158 prior to its major eruption on 2014 September 10, which exhibits a well-shaped sigmoid that indicates a non-potential field. Employing the SDO AIA 94 Å images, we find that one NLFFF solution best fits (misalignment angle of ∼10°) the AR core loops, but not as well the loops that correspond to the large overlying field lines (misalignment angle of ∼30°) near the side boundaries. Our results are similar to those of DeRosa et al. (2009). In addition to the comparison of field lines with coronal loops, we also evaluate quantitatively the extrapolated fields by other means including the magnetic energy, helicity contents, magnetic twist, and squashing degree (Demoulin et al. 1996; Titov et al. 2002). The paper is organized as follows. We first briefly describe the CESE–MHD–NLFFF code and the NFFF code in Sections 2 and 3, respectively. Then, in Section 4, we
give a short review of the coronal-loop tracing and fitting method developed by Gary et al. (2014b). The results of assessing the extrapolated coronal magnetic fields for AR 12158 are given in Section 5, and finally discussions appear in Section 6.

2. The CESE–MHD–NLFFF Code

The CESE–MHD–NLFFF code (Jiang & Feng 2013) belongs to the class of MHD-relaxation methods. It solves a set of modified zero-\(\beta\) MHD equations with frictional force using an advanced conservation-element/solution-element (CESE) spacetime scheme on a non-uniform grid with parallel computing (Jiang et al. 2010). The modified MHD equations are written as

\[
\frac{\partial \rho}{\partial t} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nu \rho \mathbf{w}, \quad \rho = | \mathbf{B} |^2,
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

where \(\nu\) is the frictional coefficient. The initial condition for the computation sequence is a potential field extrapolated from the vertical component of the vector magnetogram. To drive the evolution of the field, we change the horizontal magnetic components at the bottom boundary gradually until they match the vector magnetogram, after which the system will relax to a new equilibrium. Details of this code can be found in Jiang & Feng (2012a) and Jiang et al. (2012). It is well tested by different benchmarks, including Low & Lou’s (1990) analytic force-free solutions and Titov & Démoulin’s (1999) magnetic flux rope model, and was recently applied to the \(SDO/\)HMI vector magnetograms (Jiang & Feng 2013; Jiang et al. 2014). Here we use the code in exactly the same way as Jiang & Feng (2013), without any parameter optimization for the present modeling.

Unlike the coronal field, the photospheric field is not necessarily force free because of much higher plasma \(\beta\), thus, it is usually required to remove the Lorentz force in the vector magnetogram for a boundary field consistent with the force-free assumption of NLFFF extrapolations (Wiegelmann & Neukirch 2006). Meanwhile, smoothing of original magnetogram is needed to reduce the data noise and smooth the very small-scale structures that cannot be properly resolved by the discretized grid. Such process of removing the force and smoothing is called preprocessing, and here we use the preprocessing code developed by Jiang & Feng (2014). Different from other preprocessing codes (Wiegelmann & Neukirch 2006; Fuhrmann et al. 2007, 2011), this code is unique in that it splits the vector magnetogram into a potential field part and a non-potential field part and handles the two parts separately. The potential part, as it is already force free, only needs smoothing, which is simply performed by taking the data sliced at a plane one pixel (of an HMI magnetogram) above the photosphere from the 3D extrapolated potential field. Then, the non-potential part is modified and smoothed by an optimization method to fulfill the constraints of total magnetic force-freeness and torque-freeness. One advantage of using such a splitting is that the preprocessing of the non-potential field part can be guided by the extents of force-freeness and smoothness of the smoothed potential field part. This is because in practical computation, particular attention needs to be paid to what extent the force needs to be removed and the smoothing can be performed. In practical computation based on numerical discretization, an accurate satisfaction of force-free constraints is apparently not necessary. Also, the extent of the smoothing for the data needs to be carefully determined, if we want to mimic the expansion of the magnetic field from the photosphere to some specific heights. Over-smoothing of the data may smear the basic structures while insufficient smoothing cannot filter the small-scale noise sufficiently. A careful choice of the weighting factors \(\mu\) is required to deal with these problems. In Jiang & Feng (2014), the values of force-freeness and smoothness calculated from the preprocessed potential field part are used as a reference, and the target magnetogram is required to have the same level of force-freeness and smoothness as its potential part in numerical precision. It is found that these requirements can restrict well the free parameters, i.e., the weighting factors \(\mu\) in the optimization function.

3. The Non-force-free Extrapolation Code

The non-force-free field is governed by the following equation (Hu & Dasgupta 2008; Hu et al. 2010):\n
\[
\nabla \times \nabla \times \nabla \times \mathbf{B} + a \nabla \times \nabla \times \mathbf{B} + b \nabla \times \mathbf{B} = 0.
\]

One solution is written \(\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3\), where each subfield \(\mathbf{B}_i\) satisfies the standard LFFF equation with distinct parameters \(\alpha_i\), \(i = 1, 2, 3\), such that \(a = -(\alpha_1 + \alpha_3)\) and \(b = \alpha_2\). To obtain a solution of this form, taking advantage of the relatively simple solutions to LFFFs, it turns out that \(\alpha_2 = 0\), and \(\alpha_1 \neq \alpha_3 \neq 0\). Therefore, one of the subfields, \(\mathbf{B}_2\), becomes the potential field solution for the boundary conditions.

For completeness, we give below a brief description of the algorithm for deriving an NNFFF solution to Equation (2) via the superposition of three LFFFs of distinct \(\alpha\) parameters. It can be shown that the result is the following equation,

\[
\begin{bmatrix}
\mathbf{B}_1 \\
\mathbf{B}_2 \\
\mathbf{B}_3
\end{bmatrix} = \mathcal{V}^{-1} \begin{bmatrix} 
\mathbf{B} \\
\nabla \times \mathbf{B} \\
\nabla \times \nabla \times \mathbf{B}
\end{bmatrix}.
\]

Here, the matrix \(\mathcal{V}\) is composed of the elements \(c_{ij}^{-1}\), \(i, j = 1, 2, 3\), which is a Vandermonde matrix and is guaranteed invertible as long as the \(\alpha\) are distinct (Hu & Dasgupta 2008). Therefore, each LFFF subfield can be solved for the known \(\alpha\) parameter and the given normal boundary condition by using a standard LFFF solver (Alissandrakis 1981). Ideally, the bottom boundary condition, as given by the right-hand side of Equation (3), has to be derived by utilizing two or more than two layers of vector magnetograms since the vertical gradient as well as the transverse gradients of the magnetic field has to be calculated. If multiple layer vector magnetograms are available, then the right-hand side of Equation (3) can provide the boundary conditions (vertical components) for each subfield, given known \(\alpha\) parameters. So, the optimal pair of \((\alpha_1, \alpha_3)\) parameters, while keeping \(\alpha_2 = 0\), is determined by a
of a Bézier curve, the boundary conditions for each subfield cannot be provided by the third-order system of Equation (3). An algorithm was devised by Hu et al. (2010) to work with the single layer vector magnetogram by adding an additional round of iterations over successive corrections to the potential subfield $B_2$. Starting with an initial guess, e.g., the simplest being $B_2 = 0$, the system of Equation (3) is reduced to second order, which allows for the determination of the boundary conditions for $B_1$ and $B_3$, and thus the normal trial-and-error process as described above. If the resulting minimum $E_n$ value is not satisfactory, then a corrector potential field to $B_2$ is derived from the difference transverse field, i.e., $B_1 - b_1$, and added to the previous $B_2$, in anticipation of an improved match between the transverse fields, as measured by $E_n$. The positive effect of such successive corrections is generally demonstrated by a monotonic decrease in $E_n$ from an initial value around 1.0 to less than 0.3, after 10,000 steps (Hu et al. 2010).

The algorithm relies on the implementation of fast calculations of the LFFFs including the potential field, such as the classic algorithm of Alessandrakis via Fast Fourier Transform (FFT) for an active region in a Cartesian box with periodic boundary conditions. It is highly desirable to extend the scheme to the whole solar sphere by taking advantage of the recently developed fast algorithm of Jiang & Feng (2012b) for global LFFF (including potential) extrapolation based on FFT as well. This extension also overcomes the limitation of flux imbalance intrinsic to the LFFF within a finite AR. This would contribute to the errors in our current NFFF extrapolation of AR magnetic field, which is based on LFFF solutions.

### 4. Coronal-Loop Tracing and Fitting

Here, we briefly review the procedures for tracing and fitting the coronal loops developed by Gary et al. (2014b). First, an automated loop recognition scheme (the code OCCULT-2) is employed to trace the loop structures from the EUV images taken by SDO/AIA in 171 Å. The OCCULT-2 code was developed by Aschwanden et al. (2013) and is included in SolarSoftWare (SSW), named “looptracing_auto4.pro,” and here we use the version dated 2015 December 8. The code provides a group of input parameters to control the tracing, which are given here respectively following Aschwanden et al. (2014): the low-pass-filter constant $\eta_{\text{min}} = 5$, the minimum loop curvature radius $r_{\text{min}} = 30$, the minimum loop length $l_{\text{min}} = 20$, $\eta_{\text{loop}} = 50$, and the base level factor $q_{\text{base}} = 5$. The code outputs a number of 2D curves corresponding to the 2D loop identified from the EUV images. Then, each of the traced 2D loops is fitted by a 2D cubic Bézier spline, which is defined by four control points as (Gary et al. 2014a, 2014b)

$$R(u) = [(1 - u)^3x_1 + 3u(1 - u)^2x_2 + 3u^2(1 - u)x_3 + u^3x_4]$$

$$= [(1 - u)^3y_1 + 3u(1 - u)^2y_2 + 3u^2(1 - u)y_3 + u^3y_4],$$

$$= [(1 - u)^3z_1 + 3u(1 - u)^2z_2 + 3u^2(1 - u)z_3 + u^3z_4],$$

where $u \in [0, 1]$ is a parameter along the curve, $(x_j, y_j, z_j)$ with $j = 1, 2, 3, 4$ are the coordinates of the four control points, and here $z_1 = 0$ and so $z = 0$ as for a 2D curve. The fitting is realized by minimizing the rms distances of 10 equally spaced points along the loop curve and the Bézier spline. It was proven that generally there is no need to use a higher-order curve than the cubic Bézier curve (of third order), which can sufficiently fit the EUV loops (Gary et al. 2014a, 2014b). After this, the 2D Bézier splines are extended to 3D by determining the four nonzero $z_j$ coordinates for which the curve (Equation (5)) has a minimal misalignment angle with the field vector along it. In particular, at a given point $R$ of a Bézier curve, the misalignment angle $\mu(R)$ between the direction of the 3D spline (or the 3D coronal loop) $L(R)$ and the magnetic field model $B(R)$ at the same point is defined as

$$\mu(R) = \arccos \left[ \frac{B(R) \cdot L(R)}{|B(R)||L(R)|} \right].$$

Thus, for a single loop, a mean misalignment angle can be defined by

$$\xi = \frac{1}{\Gamma} \sum_{k=1}^{\Gamma} \mu(R_k),$$

where $k$ is the node index of the curve, and here we use an evenly spaced 100 points ($\Gamma = 100$) along the loop length. As $\xi$ is a function of the four altitudes $z_j$, the best-fit altitudes $z_j^*$ are determined by minimizing $\xi$. After applying the minimization program to all of the loops, statistical assessments of the misalignment of the model field lines and loops can then be performed.

### 5. Results

AR 12158 on 2014 September 10 is selected as our target of study because of its clear sigmoidal configuration imaged by SDO/AIA in 171 Å channel as well as its near disk-center location. Figure 1 shows that AR 12158 was passing the central meridian on 2014 September 10, and moreover, the region appears to be isolated from neighboring ARs, although there are relatively long coronal loops reaching far from the core region. An eruptive flare of X1.6 class occurred in the core of this region, which began at 17:21 UT, reached its peak at 17:45 UT, and ended at 18:20 UT. This sigmoid and its flare have been studied by several authors. From observations of SDO/AIA and the Interface Region Imaging Spectrograph (IRIS), Cheng et al. (2015) suggested that prior to the major eruption, a magnetic flux rope is under formation by tether-cutting reconnection (Moore et al. 2001) between two groups of sheared arcades driven by the shearing and converging flows in the photosphere near the polarity inversion line. Li & Zhang (2015) reported slipping reconnection of flaring loop during the
flare process, which was also carefully studied by Dudík et al. (2016) in the context of the 3D standard flare model (Aulanier et al. 2012; Janvier et al. 2013). NLFFF extrapolation of the coronal magnetic field about 2 hr before the flare has been carried out by Zhao et al. (2016) using a Grad-Rubin code implemented in spherical coordinates (Gilchrist & Wheatland 2014). By computing a map of the magnetic field squashing factor (Titov & Démoulin 1999), Zhao et al. (2016) suggested that the locations of QSLs agree to some extent with the observed flare ribbons, which are predicted by the 3D standard flare model. They found a strongly twisted and complex magnetic flux rope in the extrapolation, the shape of which, however, is not seen in AIA images.

Here we perform extrapolations of the coronal magnetic field immediately before the X-class flare onset. Specifically, we use the HMI vector magnetogram at 17:00 UT on September 10, and the extrapolations are to be compared with the AIA observations at 17:10 UT. The reason for selecting AIA observations at 17:10 UT is that the sigmoid-shaped emission appears to be most distinctive in the 94 Å channel when we inspect the AIA images before the flare beginning time of 17:21 UT (see Figure 2). As can be seen in Figure 2, the loop-tracing code OCCULT-2 identifies a bundle of loops forming an inverse-S shape, which looks most apparent at the selected time. The Space weather HMI Active Region Patches (SHARP) vector magnetogram data product “hmi.sharp_cea_720s” (Bobra et al. 2014; Hoeksema et al. 2014) is used for our extrapolation. It includes vector magnetograms projected and remapped onto the cylindrical equal area (CEA; Snyder 1987) Cartesian coordinate system centered on the tracked AR, which is well-suited for the Cartesian version of our code. To compare the coronal loops with the modeled field, we realign the AIA images using the same CEA remapping to ensure that the AIA images are co-aligned with the magnetogram. We note that this is reasonable only when the target region is located near the disk center and should be small enough such that the solar radial direction is nearly co-aligned with the line of sight and the 2D loops in the AIA image correspond to the projection of 3D loops along the radial direction. Otherwise, comparison of the magnetic field lines with traced loops from EUV images should be performed directly in spherical geometry (Aschwanden et al. 2014) using a spherical version of Gary et al.’s (2014b)
loop-fitting code, and thus extrapolation in spherical coordinates is required (e.g., Jiang et al. 2012; Gilchrist & Wheatland 2014; Tadesse et al. 2014).

For the given SHARP vector magnetogram at the selected time, we apply three magnetic field models with different numerical method and different boundary conditions. The first one (NLFFF1) is CESE–MHD–NLFFF modeling using the original vector magnetogram. The second one (NLFFF2) is CESE–MHD–NLFFF with the preprocessed magnetogram. The third model is NFFF extrapolation, which also uses the vector magnetogram without preprocessing as in NLFFF1. All the extrapolations use the same resolution of 1 arcsec, and the results are compared within the same effective volume of $282 \times 266 \times 200$ arcsec$^3$.

In the following sections, we assess and compare the magnetic field models in different ways including the magnitude of the Lorentz force and the quality of divergence-freeness, the magnetic configuration and topology features by computing the magnetic field lines and squashing degree, misalignment of the magnetic field lines with the traced loops, and finally the magnetic energy and helicity content of the fields.

**5.1. Lorentz Force and Magnetic Field Divergence**

To check the quality of force-freeness and divergence-freeness of the reconstructed fields, two metrics are routinely used (e.g., Schrijver et al. 2006; DeRosa et al. 2009; Jiang & Feng 2013): the mean sine of the angle between the current $\mathbf{J}$ and $\mathbf{B}$ weighted by $\mathbf{J}$, named CWsin and defined by

$$\text{CWsin} \equiv \frac{\int_V J \mathbf{dV}}{\int_V J \mathbf{dV}}; \quad \sigma = \frac{\mathbf{J} \times \mathbf{B}}{JB},$$

where $B = |B|$, $J = |J|$, and $V$ is the computational volume; and a normalized divergence error measured by

$$\langle |f| \rangle = \frac{1}{V} \int_V \frac{\nabla \cdot \mathbf{B}}{6B/\Delta x} dV.$$

These two metrics are equal to zero for an exact or a perfect force-free field; hence, the smaller the metrics are, the better the extrapolation is for a force-free field. However, for a field with very small current, the CWsin does not necessarily give a reliable or meaningful value because of random numerical errors. As an example, the value of CWsin could be close to 1 for a potential field solution computed using the Green’s function method or other numerical realization. The reason is that the numerical finite difference, used for computing the current $\mathbf{J} = \nabla \times \mathbf{B}$ from $\mathbf{B}$, gives small but finite currents, whose directions are randomly from 0° to 180°, and thus the angle between $\mathbf{J}$ and $\mathbf{B}$ should have an average value of $\sim 90°$.

Consequently, the distribution of CWsin for an NLFFF is “contaminated” by this if a substantial portion of the volume is current-free. This issue has been previously noted (Jiang & Feng 2012a; Malanushenko et al. 2014). Small-scale structures (mainly in the weak field regions) in the solar magnetograms, as they are not sufficiently resolved, might also increase the
value of CWsin. This is because in these regions, although the magnetic field strength is small, the currents derived by numerical difference might not be small and their directions are often random. It seems to explain why many NLFFF extrapolations from real magnetograms usually give CWsin values, for example, \( \sim 0.30 \) (DeRosa et al. 2009), which is much larger than the results of benchmark tests with an “idealized magnetogram” (which are \( \sim 0.1 \) or even smaller; see Jiang & Feng 2012a). As demonstrated by Jiang & Feng (2013), the CWsin value decreases significantly in the regions of the AR core or with strong currents where the influence of random errors is suppressed.

The metrics for measuring the force-freeness and divergence-freeness can be defined in another way by analyzing the residual force in extrapolations (Jiang & Feng 2012a; Malanushenko et al. 2014). We note that the residual force actually consists of two parts, the Lorentz force and a force induced by a nonzero divergence of the field. This is because a nonzero \( \nabla \cdot B \) can be assumed as being a magnetic monopole, and analogous to a charge in electric field, it introduces a force \( F = BV\cdot B \) parallel to the field line (Dellar 2001). Of course, this force is unphysical and only results from numerical error. To define a reference value for these forces, we decompose the Lorentz force, \( (\nabla \times B) \times B \), into two components,

\[
(\nabla \times B) \times B = (\nabla \cdot B) B - \nabla(B^2/2),
\]

where the two terms on the right-hand side are called, respectively, the magnetic tension force and the magnetic pressure force. These two components should be balanced in a force-free field, but each is in general nonzero except in a uniform magnetic field. Thus, a metric of Lorentz force-freeness can be defined by the average ratio of the Lorentz force to the sum of the magnitudes of the two component forces, namely,

\[
E_{\nabla \times B} = \frac{1}{V} \int_V \frac{|B \times (\nabla \times B)|}{|B \cdot (\nabla \times B)| + |\nabla(B^2/2)|} dV.
\]

This is identical to Equation (10) of Malanushenko et al. (2014) and is similar to Equation (26) of Jiang & Feng (2012a), which only used the magnitude of magnetic pressure force as denominator. Similarly, we measure the magnitude of the other force \( B \nabla \cdot B \) by

\[
E_{\nabla \cdot B} = \frac{1}{V} \int_V \frac{|B \cdot (\nabla \cdot B)|}{|B \cdot (\nabla \cdot B)| + |\nabla(B^2/2)|} dV.
\]

These two metrics, \( E_{\nabla \times B} \) and \( E_{\nabla \cdot B} \), are meaningful in particular if the extrapolation field is used as the initial condition for the MHD simulations (e.g., Jiang et al. 2013; Kliem et al. 2013; Amari et al. 2014; Inoue et al. 2014), because they directly reflect the influence of the residual field on the numerical MHD system. Thus, we recommend checking these two metrics before using NLFFF (or any other) solutions to initialize MHD code, and examine carefully the related influence. For a typical plasma \( \beta \sim 0.01 \) in the lower corona (Gary 2001), the residual force that can be balanced by the gas pressure should be accordingly \( \sim 0.01 \) of the magnetic pressure force, and thus the two metrics should be as small as \( \sim 0.01 \) if the residual force can be considered negligible.

The results for the aforementioned metrics are given in Table 1 for all three extrapolation models as well as a potential field model that matches the magnetogram. From the results, we find that all of the metrics decrease from NLFFF models 1 to 2, showing that the NLFFF extrapolations are improved by the preprocessing of the vector magnetogram, and their values are close to the results in our previous work for the extrapolation of AR 11158 and AR 11283 (Jiang & Feng 2013), suggesting that the performance of the CESE–MHD–NLFFF code is not sensitive to different ARs. The divergence metric \( \langle |\nabla B| \rangle \) is on the order of \( 10^{-2} \), which is consistent with previous results (e.g., Metcalf et al. 2008; Valori et al. 2013; DeRosa et al. 2015). The value is smaller by one order for the NFFF as it is a superposition of three constant-alpha fields, and it is close to the value for the potential field model. Comparing \( E_{\nabla \times B} \) and \( E_{\nabla \cdot B} \) shows that the force induced by the divergence is significantly smaller than the Lorentz force. We note that the residual force as measured by these two metrics is larger than what can be ignored if the extrapolated field is input to an MHD model as a force-free state. On the other hand, the Lorentz force as measured by both CWsin and \( E_{\nabla \times B} \) is much larger in the NFFF extrapolation than in the NLFFFs, as it should be, although the numerical and measurement uncertainties would still contribute to this value as described earlier. The divergence-freeness condition is fulfilled much better in the NFFF solution, which is close to the values of that for the potential field model. This is because the NFFF solution is the sum of three LFFFs for which the divergence is guaranteed to be as small as the numerical scheme would allow. Finally, we note that CWsin for the potential field is indeed close to 1, which fails to indicate the force-freeness of a current-free field, as we have discussed, while \( E_{\nabla \times B} \) gives a reasonable value of \( \sim 10^{-2} \), which is as small as the value of \( E_{\nabla \cdot B} \).

### 5.2. Magnetic Field Lines, Current, and Topology

In Figure 3, we compare the magnetic lines, electric current distributions, and magnetic topology for three magnetic fields. The columns from left to right are respectively for NLFFF1, NLFFF2, and NFFF. The magnetic field lines are sampled and shown in the first row of the figure. Each field line color-coded by the mean value of the electric current density \( J = |\nabla \times B| \) in G arcsec\(^{-1}\) along the line. The second row of the figure shows a vertical integration of the current density \( \int J \, dz \) with units of Gauss). Overall, an inverse-S shape of strong current can be seen, especially in the NLFFFs, which is very roughly consistent with the AIA 94 Å image (Figure 1(c)). The strongest currents are associated with the magnetic field lines in the core region, which corresponds to the brightest loops in the AIA 94 Å image. The magnetic field lines of all the models here are mostly sheared arcades rather than fully S shapes, and show no presence of the strongly twisted
magnetic flux rope that is found in the extrapolation by Zhao et al. (2016). Our results seem to agree with the observations of AIA, from which the loop tracing code identifies approximately formed S-shaped loops. From a visual inspection of the magnetic field lines of the models, it appears that in the core region, the NLFFFs possess stronger shear than the NFFFs; in contrast, in the enveloping field region, the latter has slightly stronger sheared field lines than the former ones. Consistently, the current distribution of the NFFF is more diffused than that of the NLFFF models.

To further compare the magnetic topology, we compute the magnetic squashing degree at the bottom surface. The squashing degree, or $Q$ factor, is a quantity measuring the shape distortions of the elemental flux tubes based on the field-line mapping (Demoulin et al. 1996; Titov et al. 2002). Specifically, starting at footpoint $(x, y)$, the other footpoint of a closed magnetic field line is denoted by $(X(x, y), Y(x, y))$ for a given coronal field model. Then, $Q(x, y)$ is given by

$$Q = \frac{a^2 + b^2 + c^2 + d^2}{|ad - bc|},$$

where

$$a = \frac{\partial X}{\partial x}, \quad b = \frac{\partial X}{\partial y}, \quad c = \frac{\partial Y}{\partial x}, \quad d = \frac{\partial Y}{\partial y}. \quad (14)$$

Thus, a small value of $Q$ indicates that an infinitesimal circle at one footpoint is mapped to a circle at the other footpoint, while a very high value (e.g., $>100$) indicates extreme distortion of the circle, which indicates a steep gradient of the field-line mapping that occurs in magnetic quasi-separatrix layers (QSLs). Thus, the $Q$ factor is useful for searching for important topological structures like separatrices and QSLs. They prove to be relevant to the studies of reconnection sites in the corona and thus physics of flares. To compute $Q$, we use the approach recently proposed by Pariat & Démoulin (2012), which is computationally efficient and can be used to compute $Q$ inside the 3D domain. The third row of Figure 3 shows the maps of the $Q$ factor. Comparing these three models, we find limited difference between them, and there appears to be no well-shaped QSL defining the boundary of the shear core and the enveloping flux.

**Figure 3.** Comparison of the three extrapolation models (columns from left to right correspond to NLFFF1, NLFFF2, and NFFF, respectively) by magnetic field lines, electric current distributions, magnetic squashing degree, and twist degree (rows from top to bottom). In the plot of the magnetic field lines, each field line is color-coded by the mean value of the electric current density ($J = \nabla \times B$ with units of G arcsec$^{-1}$) along the line; the backgrounds show the map of $B_z$. The current distribution is shown by the vertical integration of the current density ($\int J dz$ in units of Gauss). The distributions of the magnetic squashing degree $\log(Q)$ and the twist degree are computed at a horizontal slice one pixel above the bottom surface of the extrapolation box. Their field of view is denoted by the white box in the top panels.
In the last row of Figure 3, we show the map of the twist \( T_n \) of the magnetic field lines (e.g., Inoue et al. 2011; Liu et al. 2016), which is defined by

\[
T_n = \frac{1}{4\pi} \int_L \frac{(\nabla \times B) \cdot B}{B^2} dl,
\]

where the integral is taken along each closed field line. The magnetic twist \( T_n \) measures the number of turns two infinitesimally close field lines make around each other (Berger & Prior 2006). Both signs of the twist are seen in the whole region, and the majority of the core field has a negative value of twist, i.e., a left-handed twisting, and with a relatively small value \( T_n \lesssim 1 \). This is consistent with the absence of a strongly twisted flux rope in the models. Comparison of the two NLFFF models shows that the preprocessing also results in a clear increase of the magnetic twist. The NFFF model gives a much more weakly twisted field than the NLFFF models, and the distribution of twist degree appears much more even. A further study of the magnetic topology and distribution of twist degree and their relation with the eruption is left to a future paper.

5.3. Misalignment of the Magnetic Field Lines and AIA Loops

Now we report the results of loop fitting to the magnetic field models as described in Section 4. Here we apply the procedures of Gary et al. (2014b) to two AIA channels, i.e., 94 Å and 171 Å, which represent two separate features of the AR field. The 94 Å channel shows loops at the AR core that are presumably associated with a significantly non-potential magnetic field, and they are usually compact and short, while the 171 Å loops correspond to the enveloping field that is often close to a potential state. As described in Section 4, the OCCULT-2 traced loops from the AIA images are first fitted by 2D four-point Bézier splines. The results are shown in Figure 4. As can be seen, although the four-point Bézier fitting is previously only applied to AIA 171 Å loops, here it also works pretty well for the AIA 94 Å loops, some of which exhibit slightly S shapes. Then, using the method described in Section 4, the Bézier splines (or loops) are extended to 3D by minimizing the misalignment angle between the loops and the 3D magnetic field. Figures 5 and 6 give the results of such a minimization of the misalignment angle for each loop, and the different panels in the figures are for different extrapolation models of the magnetic field. For convenience of comparison, the loops are shown by the thick curves, color-coded by the value of the corresponding misalignment angle with the magnetic field, and for each loop, a magnetic field line is traced passing through the midpoint of the loop, which is shown by the thin curves. The median and mean values of all of the misalignment angles for each magnetic field model are also shown in the figures.

We find that model NLFFF2 gives the best results in fitting to the AIA 94 Å loops as it has the smallest values for both the median and mean of the misalignment angles. For the AIA 171 Å loops, NLFFF2 also gives the smallest median value, while the smallest average value is given by the potential field model. Comparing the results for the two AIA channels, we notice that the fitting of the modeled field lines to loops is much better for the 94 Å than for the 171 Å. In particular in the best model (NLFFF2), the misalignment angles for the 94 Å loops, which have a median value of \( \sim 10^\circ \) and a mean value of \( \sim 15^\circ \), are less than half of those for the 171 Å loops (\( \sim 30^\circ \)). For the 94 Å loops, all the non-potential models give a significantly smaller value of misalignment than the potential field model, which confirms that these loops are associated with a magnetic field of significant non-potentiality. Improvement from the preprocessing of the magnetogram for NLFFF extrapolation is also confirmed by the decrease of the misalignment angles.

It is worth noting that our best results for the AIA 94 Å loops are on the order of the results using the coronal-loop forward-fitting NLFFF code shown in Aschwanden (2013b), who found that the 3D misalignment angle amounts to an average value of \( 19^\circ \pm 3^\circ \) for their studied ARs. However, for the 171 Å loops, our extrapolation models perform, at best, only close to the potential field model (with a misalignment of \( \sim 30^\circ \)). Such a value is comparable to the misalignment angles (\( 24^\circ - 44^\circ \)) resulting from a comparison of various NLFFF models for AR 10953 by DeRosa et al. (2009), who also found that those NLFFF models perform no better than a potential model.
NLFFF1 without preprocessing gives an even larger misalignment angle than the potential model, and it is slightly improved from the preprocessing. The different values for 94 Å and 171 Å show that the CESE–MHD–NLFFF code can reconstruct the AR core field (which is significantly non-potential) much more reliably than the envelope field (which is presumably close to potential). The large misalignment of the extrapolated field with the 171 Å loops is because the field lines corresponding to the long loops do not relax sufficiently during the MHD-relaxation process. Since in the code the relaxation speed is uniform and the relaxation time is thus proportional to the field-line length, the long field lines need more time to relax than the short ones. Moreover, the long field lines often extend close to or reach the side and top boundaries of the computational volume and are subject to boundary effects, as all these boundaries are fixed to the initial potential field (however, note that the computational volume is larger than the field of view of the magnetogram). Another factor that might be partially responsible for this effect is the departure from the Cartesian coordinates assumption farther away from the core. Thus, a further optimization of the relaxation process with the realization of spherical geometry is needed for a better reconstruction of the enveloping field.

5.4. Magnetic Energy and Helicity

In Table 2, we list the magnetic energy contents and relative helicity for the extrapolation solutions. Both the total magnetic energy $E_{\text{tot}}$ and the corresponding reference potential field energy $E_{\text{pot}}$ are computed, from which the free magnetic energy $E_{\text{free}} = E_{\text{tot}} - E_{\text{pot}}$ can be derived. The reference potential fields are computed numerically by solving the Laplace equation for the potential with Neumann boundary conditions based on the normal component of $B$ on all six boundaries of the analysis volume. As a consequence of each solution field having its own boundary value, there are separate reference potential fields for each of the solution fields, and thus the potential field energy contents are different among the models.
The relative magnetic helicity $H$ is defined following Berger & Field (1984):

$$H = \int_V (A + A_{\text{pot}})(B - B_{\text{pot}})dV,$$

(16)

where $A$ is the vector potential of the magnetic field, i.e., $B = \nabla \times A$, and $B_{\text{pot}}$ is the reference potential field and also $B_{\text{pot}} = \nabla \times A_{\text{pot}}$. The calculation of $H$ is performed using a rapid method developed by Valori et al. (2012). For reference, we also list values of the total unsigned magnetic flux $\Phi$ in the table.

Clearly, the free energy increases from NLFFF1 to NLFFF2. Such an increase of non-potentiality of the solutions is due to the improvement of the NLFFF extrapolations. Here, the free energy for NLFFF1 is even unphysically negative. We note that the energy of the extrapolated field being lower than the potential field energy has been reported previously, for instance, some solutions given in Table 3 of Metcalf et al. (2008) and Table 1 of Schrijver et al. (2008). Valori et al. (2013) shows that such a problematic extrapolation actually reflects the violation of Thomson’s theorem (that the total energy of a system can be expressed as the sum of its potential energy and its free energy), due to the finite divergence of the extrapolation field rather than the nonzero Lorentz force. The finite divergence is unavoidably induced in the course of seeking a force-free field with an inconsistent boundary condition (i.e., the unpreprocessed photospheric field). Since the MHD-relaxation code attempts to construct a force-free solution by reducing the Lorentz forces in the computing volume, while the boundary condition is incompatible with the force-free equation, the reduction of the Lorentz forces is at the expense of the solenoidal condition. In such case, the more inconsistent the boundary is, the higher the divergence of the solution will be, and Thomson’s theorem will be more severely violated, which could result in a total energy lower than the potential energy. The inconsistency of the photospheric field can be partly reduced by preprocessing. After preprocessing, the code outputs free energy on the order of $10^{32}$ erg, which could energize a major flare. Interestingly, the NFFF model outputs a significantly larger free energy content, which is about three times that from NLFFF2. This might be due to the fact that NFFF is a combination of LFFFs, whose current is distributed more evenly and result in a field much more non-potential than the NLFFFs.

All of the models have negative helicity, which complies with the hemispheric chirality rule (Seehafer 1990; Pevtsov et al. 1995), i.e., active regions in the southern and northern
hemispheres tend to have negative and positive helicities, respectively. A negative helicity indicates a left-handed twisting of the magnetic field lines, and can be easily confirmed by inspecting the magnetic field lines and their directions (see Figure 3) and also in the twist degree map (Figure 3), which shows that the majority of the values are negative. Consistent with the increase of free energy from NLFFF1, NLFFF2, to NFFF, the relative helicity contents also increase.

### 6. Discussion

Due to the absence of direct measurements, to determine the coronal magnetic field, one has to extrapolate the photospheric field. Critically assessing and comparing the results for different extrapolation codes is an important task to develop a reliable coronal magnetic field model consistent with observations and physics. In this paper, we have made a comprehensive assessment and comparison of an NLFFF code (CESE–MHD–NLFFF) and an NFFF code. For this, we extrapolated the coronal magnetic field for a sigmoidal AR near its central meridian passage immediately before a major flare. Vector magnetograms of the HMI SHARP data set are used, and three field models are calculated including the CESE–MHD–NLFFF extrapolations using the magnetogram with and without preprocessing and the NFFF extrapolation. The extrapolation solutions are evaluated and compared in different ways including the residual Lorentz force and divergence, the magnetic topology, and the energy/helicity contents. In particular, we use a method recently proposed by Gary et al. (2014b) to compute the misalignment angle of the model field lines with the 3D traces of coronal loops identified from two AIA images (wavelengths of 94 Å and 171 Å, respectively), which previously have not been tested for these codes.

The extrapolations show that the magnetic field lines in the AR’s core is consistent with sheared arches forming a sigmoidal shape in agreement with the observed AIA 94 Å loops. The twists of the field lines are mostly below one full turn, and there also seems to be no QSL that marks the clear presence of a magnetic flux rope. It is found that the best extrapolated field matches the AIA 94 Å loops with misalignment angles of mean value ∼15° and median value 9°, which are much smaller than that for the 171 Å loops (mean value of ∼32° and median value of ∼19°). Interestingly, the misalignment angles for AIA 94 Å loops are even comparable with those by coronal-loop forward-fitting method (Aschwanden 2013b), suggesting that the CESE–MHD–NLFFF can reproduce the magnetic configuration at the core region of an AR. On the other hand, the 171 Å loops are not well reproduced, which means that an improvement of the code is necessary for further relaxation of the long field lines. By comparing the NLFFF extrapolations made using both preprocessed and unpreprocessed input data, we confirm that preprocessing of the vector magnetogram considerably improves the extrapolation result. For best results, the residual Lorentz force is as large as 15% in normalized units (see above) while the residual divergence is one order of magnitude smaller. When comparing the NLFFFs and the NFFF results, it is found that the currents of the field by the NFFF is distributed more evenly than in the NLFFFs for this particular AR. In the future we plan to extend the comparison study with coronal loops to a number of AR samples, as well as including the magnetic field results from a data-driven MHD model (Jiang et al. 2016). Experiments using NFFF solutions as initial conditions to MHD simulations are also underway.

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Table 2
Magnetic Flux, Energy Contents, and Relative Helicity of the Extrapolation Models

| Model  | Φ  | E\(_{\text{rot}}\) | E\(_{\text{pot}}\) | E\(_{\text{rot}}\) / E\(_{\text{pot}}\) | H | H / Φ\(^2\) |
|--------|----|----------------|----------------|-----------------|---|---------|
| NLFFF1 | 3.41 | 11.5 | 11.7 | −0.18 | −1.5% | −1.60 | −1.38 × 10\(^{-2}\) |
| NLFFF2 | 3.18 | 11.1 | 10.0 | 1.10 | 10.9% | −2.03 | −2.00 × 10\(^{-2}\) |
| NFFF   | 3.41 | 14.8 | 11.8 | 3.02 | 25.6% | −2.36 | −2.04 × 10\(^{-2}\) |

**Note.** Units are, respectively, 10\(^{22}\) Mx for magnetic flux, 10\(^{32}\) erg for magnetic energy, and 10\(^{43}\) Mx\(^2\) for magnetic helicity.
