Implementation and validation of a three degrees of freedom steering-system model in a full vehicle model

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ABSTRACT
This paper describes the coupling between a three degrees of freedom steering-system model and a multi-body truck model. The steering-system model includes the king-pin geometry to provide the correct feedback torque from the road to the steering-system. The steering-system model is combined with a validated tractor semi-trailer model. An instrumented tractor semi-trailer has been tested on a proving ground and the steering-wheel torque, pitman-arm angle, king-pin angles and drag-link force have been measured during steady-state cornering, a step steer input and a sinusoidal steering input. It is shown that the steering-system model is able to accurately predict the steering-wheel torque for all tests and the vehicle model is accurate for vehicle motions up to a frequency where the lateral acceleration gain is minimum. Even though the vehicle response is not accurate above this frequency, the steering-wheel torque is still represented accurately.

1. Introduction

The number of road vehicles worldwide is growing every year. In 2010 the 1 billion units mark of vehicles in operation worldwide was exceeded. A total of 35 million heavy-duty trucks were registered in Europe in 2013. The growth in the number of trucks is estimated to be 17 % by 2030 compared to 2013. This growth is expected to be even stronger in Asia and the Middle East [1]. Trucks and in particular tractor-semitrailers are overrepresented in accident statistics. This can be attributed to unique risks which can lead to blind-spot crashes [2,3], loss-of-control accidents and road departures on longer trips due to fatigue and highway hypnosis [4]. The complex dynamics and growing length proves to be another risk factor [5].

Anti-lock braking system, electronic stability control, forward collision warning and automated emergency braking have become mandatory on new trucks to improve safety. Non-mandatory systems, but available for new trucks, include: adaptive cruise control, intelligent speed adaptation, blind spot information and curve-speed warning. In the future it is expected that fully automated trucks are driving on highways, where the truck operator...
is no longer a driver but a supervisor. To implement features such as automated steering, an active steering system is required which is controlled by electronics. To design and implement such a system, more knowledge of a conventional truck steering system is required. This knowledge can be used, for example, to determine the application of an electric actuator to make the steering-system active.

A challenging design aspect of a truck steering system is the on centre feel. Commercial vehicles in general make use of a hydraulic assist system, which amplifies the torque supplied by the driver using a so called boost-curve, as shown in Figure 1(a). The application of hydraulic assistance based on the steering-wheel torque input can result in an indirect steering feel, especially around the centre-position due to the low gradient of the boost-curve at this position [7]. Steering systems in truck handling simulation models, as found in literature, generally don't include a detailed steering-system model. In many cases they do not incorporate steering compliance and/or power-steering [8] and if they do, inertias of system components are often not considered [9].

Literature on cars shows that modelling of the hydraulic part of the steering-system is essential to predict the steering-wheel torque and vehicle response [7,10]. In [11,12] the steering-system is modelled using three degrees of freedom (DOF) with a stiffness between the steering wheel and the assistance motor and a stiffness between the steering rack and the assistance motor. Reference [13] uses a Coulomb friction model and describes the steering wheel torque during cornering sufficiently accurate, This is also shown by Pfeffer et al. [7], especially around the centre position. In [10] a concept is developed with an additional motor on the steering-wheel side to enable active features like a parking pilot, lane keeping assist, emergency lane assist, active yaw control and torque reference control. This paper also shows that the motor can be used for friction compensation within the steering system. In [14] a two DOF steering-system model in combination with speed dependent but further static nonlinear boost-curves is presented to design a lateral vehicle controller. This paper provides verification results of the linearised relation between the steering-wheel angle and steering-wheel torque, most results are presented in the frequency domain. This is one of the few works where the steering-system model of a truck is discussed. In order to develop an active steering system, which can support the driver or even steer autonomous, the modelling, parametrisation and verification of

![Figure 1](https://example.com/figure1.png)

**Figure 1.** An example of a typical steering-system and a boost-curve. (a) Example of a boost-curve and (b) typical steering system [6].
a truck steering-system to accurately describe the steering-wheel torque felt by the driver is researched in [15,16]. The model is validated for the full range of supply pressures in steady-state and up to frequencies above the maximum driver input frequency. In these papers it is concluded that a three DOF steering-system model with dry friction and a hydraulic model is required.

In this paper, this three DOF steering-system model is extended with two additional DOF to include the front wheels and the king-pin geometry is added. The steering-system and front-axle model are then implemented in a validated 44 DOF multi-body full vehicle model described in references [17,18]. The complete model is validated using experiments obtained with an instrumented tractor semi-trailer. An overview of these steps is given in Figure 2. This figure also reflects the goals of the paper:

1. Implement the king-pin geometry and tie-rod to provide the correct feed-back torque from the road to the steering-system and to connect the left and right wheel.
2. Connect the three DOF steering-system model to the left front wheel of the truck.
3. Integrate this front-axle model including steering-system model in the multi-body truck model.
4. Validate the full vehicle model by tests obtained with an instrumented tractor semi-trailer.

To the best of the authors knowledge a model of a truck steering-system with this level of detail and accuracy has not been described in literature yet. Most steering-system models consist of a single degree of freedom for the steering-wheel and a separate degree of freedom for the front wheels, the model presented in this paper has three DOF. This is achieved by introducing separate DOF for the steering-wheel, the input of the steering-house and the output of the steering-house. By separating the left wheel from the output of the steering-house and the left wheel from the right wheel, two additional DOF are introduced which adds up to five DOF. The advantage of this way of modelling is that friction can be implemented at all relevant places in correspondence with the physical components such as bearings and seals. Where in literature a lookup-table with a time-delay is often used to describe the power-steering assistance characteristics, this model uses a hydraulic model based on the physical components and the lag in the system is a result of the flexibility of the supply the hose and oil compressibility.

This paper is organised as follows: Section 2 describes the steering-system model. The coupling of the steering-system to the vehicle model is discussed in Section 3 and the instrumented test vehicle is discussed in Section 4. In Section 5 the measurements are compared with the simulation results. A discussion on the differences observed is presented in Section 6 and finally the conclusions and recommendations are given in Section 7.

![Figure 2. Integration of the steering-system model from [15,16] in the full vehicle model from [17,18].](image)
2. Steering-system model

A typical truck steering-system is shown in Figure 1(b). The driver actuates the steering-wheel, which is connected to the steering-column. Two universal joints are present to facilitate the height adjustment and the cabin motion. The second universal joint is connected to the input side of the steering-house, where the input torque is amplified by a hydraulic system. A simplified approach to model this hydraulic system is shown in Figure 1(a) where a boost-curve is shown. This boost-curve defines the resulting output torque of the steering-house as a function of input torque. The output of the steering-house is connected to the pitman-arm which moves the drag-link. The drag-link motion is converted to a rotation of the left wheel by means of a wheel-lever. The left wheel is connected to the right wheel via a tie-rod. Figure 3(a) shows the internals of a typical steering-house. The input torque $T_{th}$ is applied to the torsion-bar. This torsion-bar is connected to the spindle which converts the rotation into a translation of the piston. The piston has teeth on the bottom to actuate the sector-shaft which is the output of the steering-house. Upon deflection of the torsion-bar, the valve system gets actuated as shown in Figure 3(b). The inner part moves with respect to the outer part and the oil flow to one chamber gets restricted and the flow-path to the other chamber is opened. This results in a pressure difference across the piston. This pressure difference can only be influenced by torques on the output side of the steering-house resulting from the tyre feedback or by torques on the steering-wheel side resulting from the driver, there are no other inputs available.

In Figure 4 the steering-system model from [15,16] is shown. The driver interacts with the steering-wheel, having an inertia $J_{sw}$ by means of a prescribed angle $\delta_{sw}$ or by exerting a torque $T_{sw}$. Due to eccentricity of the steering-wheel centre of gravity (COG), an eccentricity torque $T_{ecc,sw}$ is present. Since the steering-wheel is supported in bearings that exhibit friction, $T_{fric,sw}$ is included here. The kinematics of the two universal joints are taken into account and the combined (variable) ratio of the universal joints is referred

![Figure 3](image.png)

**Figure 3.** Inside of a typical steering-house and a schematic overview of the valves. (a) Inside of a typical steering house and (b) valves and the flows through them.
Figure 4. Three DOF steering-system model with dry friction and a hydraulic model from [15,16].

to as $i_{uj}$. The compliance between the input of the steering-house and the steering-wheel is lumped into one equivalent stiffness $k_{sc}$. The input of the steering-house is also supported in bearings with friction $T_{fric, hua}$, where hua stands for Hydraulically Un-Actuated. The stiffness of the torsion-bar and spindle, respectively, $k_{tb}$ and $k_{sp}$ are modelled in series and a damping element, $d_{hua}$, is added to include relative damping between the input and output of the steering-house. A fixed reduction $i_{sh}$ is used to incorporate the reduction ratio between the spindle and the sector-shaft. The inertia of all moving parts between the spindle and the pitman-arm are lumped into one equivalent inertia $J_{sh, out}$. The output of the steering-house has angle $\delta_{pa}$. The stiffness of the pitman-arm, drag-link and wheel-lever is taken into account by the equivalent stiffness $k_{ha}$. This is the interface where the steering-system model is coupled to the multi-body model, which includes the kinematics of the pitman-arm and drag-link. The pitman-arm angle $\delta'_{pa}$ represents the resulting angle from the kinematics of the pitman-arm, drag-link and wheel-lever as shown in Figure 5.
The torque $T_{pa}$ results from the steering-system model and is an input to the multi-body model.

The equations of motion of the steering-system model are given by

$$ J_{sw} \ddot{\delta}_{sw} = T_{sw} - T_{fric,sw} - (\delta_{sw} i_{uj} - \delta_{sh,in}) k_{sc} i_{uj} - T_{ecc,sw}, \quad (1) $$

$$ J_{sh,in} \ddot{\delta}_{sh,in} = (\delta_{sw} i_{uj} - \delta_{sh,in}) k_{sc} - (\delta_{sh,in} - \delta_{pa,i_{sh}}) k_{hua} - (\delta_{sh,in} - \dot{\delta}_{pa,i_{sh}}) d_{hua} - T_{fric,hua}, \quad (2) $$

$$ J_{sh,out} \ddot{\delta}_{pa} = (\delta_{sh,in} - \delta_{pa,i_{sh}}) k_{hua} + (\dot{\delta}_{sh,in} - \dot{\delta}_{pa,i_{sh}}) d_{hua} + T_{ps} - (\delta_{pa} - \delta_{pa,i_{sh}}) k_{ha} - T_{fric,ha}, \quad (3) $$

where torque acting on the torsion-bar is calculated by:

$$ T_{tb} = \min \left( |\delta_{sh,in} - \delta_{sh,out,i_{sh}}| k_{hua}, k_{tb} \Delta_{tb} \right) \text{sign}(\delta_{sh,in} - \delta_{sh,out,i_{sh}}). \quad (4) $$

The stiffness $k_{hua}$ is calculated from the series connection of the stiffnesses $k_{tb}$ and $k_{sp}$ as:

$$ k_{hua} = \left( k_{tb}^{-1} + k_{sp}^{-1} \right)^{-1}. \quad (5) $$

The hydraulic model consists out of a symmetric Wheatstone bridge, where the hydraulic resistances $R_1$ and $R_2$ are determined by the torsion-bar. The flow resistances $R_1$ and $R_2$ are linked to the area opening of the orifice $A_1$ and $A_2$ via:

$$ A_i = \left( R_i C_d \sqrt{\frac{2}{\rho}} \right)^{-1} \quad \text{with } i \in \{1, 2\}, \quad (6) $$

where $C_d$ is the discharge coefficient and $\rho$ is the density of the hydraulic oil. The relation between the orifice area $A_i$ and the torsion-bar torque $T_{tb}$ is explained in depth in [16] and illustrated in Figure 6. A linear relation between $A_i$ and $T_{tb}$ is observed for small torsion-bar torques and for larger torsion-bar torques this relation becomes nonlinear.

The hydraulic system is fed by a fixed displacement pump where the pump flow $Q_s$ is coupled to the engine speed. The supply hose is assumed to be flexible by modelling a
capacity $C_{\text{hose}}$. Changing the resistances $R_1$ and $R_2$ results in changing supply pressure $P_s$ and changing chamber pressures $P_A$ and $P_B$. The hydraulic fluid in these chambers with volumes $V_A$ and $V_B$ is assumed to be compressible with bulk modulus $\beta$ since the volume is relatively big compared to the volume in the valve system. The return line of the Wheatstone bridge is connected to the reservoir which is connected to the pump again.

The pressure derivatives of the hydraulic model are given by:

$$\frac{dP_s}{dt} = \frac{Q_s - Q_{\text{bridge}}}{C_{\text{hose}}} = \frac{Q_s - \text{sign}(P_s)\sqrt{|P_s|}}{R_{\text{bridge}}},$$

$$(7)$$

$$\frac{dP_A}{dt} = \frac{\beta}{V_A} \left( \frac{\text{sign}(P_s - P_a)\sqrt{|P_s - P_a|}}{R_1} - \frac{\text{sign}(P_a)\sqrt{|P_a|}}{R_2} - A\dot{x}_p \right),$$

$$(8)$$

$$\frac{dP_B}{dt} = \frac{\beta}{V_B} \left( \frac{\text{sign}(P_s - P_b)\sqrt{|P_s - P_b|}}{R_2} - \frac{\text{sign}(P_b)\sqrt{|P_b|}}{R_1} + A\dot{x}_p \right),$$

$$(9)$$

where the bridge resistance is defined as:

$$R_{\text{bridge}} = \frac{1}{2} \sqrt{R_1^2 + R_2^2}$$

$$(10)$$

and the piston surface and position are indicated by $A$ and $x_p$, respectively. The volume in the chambers is calculated as:

$$V_A = V_{A0} + A x_p \quad \text{and} \quad V_B = V_{B0} - A x_p$$

$$(11)$$

where $V_{A0}$ and $V_{B0}$ are the initial volumes of the chambers. The pressure difference across the piston results in an assistance force, which is converted into an assistance torque $T_{ps}$ by using the sector shaft radius $R_{ss}$:

$$T_{ps} = (P_A - P_B) A R_{ss}.$$

$$(12)$$

The angle $\delta_{pa}$ is used to determine the position of the piston by:

$$x_p = \delta_{pa} R_{ss}.$$
3. Vehicle model

The vehicle model is a 44 DOF tractor semi-trailer as shown in Figure 7 and is modelled using MATLAB SimMechanics [17,18]. The tractor consists of separate front and rear chassis parts, which are coupled by a torsional spring to model the chassis flexibility. The cabin is modelled as a separate mass which is supported by the front part of the chassis. The engine and gearbox are modelled as a separate mass and include the stiffness and damping of the engine mounts. The rear axle is modelled as a rigid axle, which can translate along the vertical axis and can rotate around the longitudinal axis with respect to the rear chassis and is connected with springs and dampers to the chassis. The tyre–road interface is described by the TNO Delft-tyre model. The trailer consists of three separate bodies connected via torsional springs. Three independent trailer axles are used, the suspension is modelled in the same way as the tractor rear axle. The load is divided in three parts and attached to the chassis. The trailer is connected to the tractor by means of a fifth wheel with a yaw and pitch degree of freedom.

The front-axle is shown in Figure 8. This front-axle model consists out of two wheels with the king-pin geometry. The left and right wheel are connected via a tierod with stiffness $k_{\text{tierod}}$. The left wheel is steered by means of the wheel lever, drag-link and pitman-arm which are modelled as rigid bodies connected via two ball joints. Dry friction $T_{\text{fric,kp}}$ is present on both king-pin axis. This front-axle model adds two mechanical DOF to the steering-system model. The pitman-arm is connected to the steering-house and has angle $\delta'_{\text{pa}}$ and is actuated with a torque $T_{\text{pa}}$ from the steering-system model.

4. Validation by means of experiments

In addition to the steering-system validation presented in [16], the steering-system model is also validated in the context of a complete vehicle. An instrumented tractor semi-trailer is used and different tests are performed on a proving ground. In this section the instrumentation of the vehicle and the performed tests will be described.

*Figure 7.* 44 DOF vehicle model (engine not shown in this figure) [17,18].
Figure 8. Front-axle model. The front-axle model is modelled in MATLAB SimMechanics (multi-body model) and the steering-system model in MATLAB Simulink (block-diagram). The interface between the two models takes place at the pitman-arm.

Figure 9. Truck sensors, the following nomenclature is used: C: Cabin, F: Front, R: Rear, FL: Front Left, FR: Front Right, RL: Rear Left and RR: Rear Right.

4.1. Instrumentation of the test vehicle

The test vehicle is a laden tractor semi-trailer with a total weight of 40 tonnes. All instrumentation is present on the tractor, so the trailer is not instrumented. The measurement devices are shown in Figure 9. The front and the rear of the tractor chassis are instrumented with a gyroscope and an accelerometer which measure the yaw-rate and the lateral acceleration, respectively. The cabin is equipped with a 3-axis accelerometer and a 3-axis gyroscope close to the driver position. A side-slip angle sensor is used to determine the longitudinal and lateral velocities $V_x$ and $V_y$ and the vehicle sideslip angle $\beta_v$. String potentiometers are used to measure the relative vertical displacement between the axle and the chassis $\Delta_z$. The wheel-speeds are measured as well and are indicated with $\omega$.

The instrumentation of the steering-system is shown in Figure 10. The steering-wheel torque, $T_{sw}$ and angle $\delta_{sw}$ are measured. The pitman-arm angle $\delta_{pa}$ is measured. A strain gauge is used to measure the drag-link force $F_{dl}$ and the left and right king-pin angles are measured, $\delta_{kp,L}$ and $\delta_{kp,R}$, respectively.
4.2. Modelling of the sensors in the simulation model

To make a fair comparison between the vehicle model and the real vehicle the sensors are placed at the same location in the SimMechanics model for gyroscopes, accelerometers, the side-slip angle sensor and the displacement sensors. Since this paper focuses on the prediction of the steering-wheel torque, it is key to model the steering-wheel torque sensor correctly. As shown in Figure 10 the steering-wheel torque is measured at the steering-column. This is done by means of a strain gauge. By rewriting Equation (1), the torque $T_{sw}$ at the interaction point of the driver and the steering-wheel can be expressed as:

$$T_{sw} = T_{fric,sw} + (\delta_{sw}t_{uj} - \delta_{sh,in})k_{sc}i_{uj} + T_{ecc,sw} - J_{sw}\ddot{\delta}_{sw}. \quad (14)$$

However, due to the mounting of the sensor, not all the components will be registered. Since the driver is holding the steering-wheel, the acceleration of the steering-wheel is supported by the driver, as well as the eccentricity torque. The steering-wheel bearings are below the torque sensor and friction torque caused by these bearings is included in the measurement. When the driver holds the steering-wheel, the measured steering-wheel torque $T_{sw,meas}$ is given by:

$$T_{sw,meas} = T_{fric,sw} + (\delta_{sw}t_{uj} - \delta_{sh,in})k_{sc}i_{uj}. \quad (15)$$

4.3. Post-processing of the sensor data

To compare the measured signals with other literature or simpler vehicle models such as a single-track model, the measured signals are converted to the COG of the tractor. To estimate the yaw-rate at the COG the average of the two sensors is used:

$$r_{z,COG} = \frac{1}{2} (r_{zF} + r_{zR}). \quad (16)$$

To calculate the roll angle of the front and rear axle, the following equation is used:

$$\theta_{axle,i} = \arctan \left( \frac{\Delta z_{iL} - \Delta z_{iR}}{y_{sensor,i}} \right). \quad (17)$$
where $\theta_{axle,i}$ where $i \in \{F, R\}$ and $y_{\text{sensor}}$ is the lateral distance between the sensors. It is assumed that the roll angle between the front and rear axle varies in a linear way to determine the roll angle at the lateral accelerometer locations. The longitudinal distance between the front and rear roll sensors is $L$ and the distance from the front axle to the sensor is $X_{\text{sensor},i}$ which is positive when the sensor is mounted in front of the front-axle:

$$\theta_{ayi} = \theta_{axle,F} + X_{\text{sensor},i} \frac{\theta_{axle,F} - \theta_{axle,R}}{L}. \quad (18)$$

The measured lateral acceleration $a_{yi,\text{meas}}$ is compensated for by subtracting the contribution due to gravity:

$$a_{yi,\text{comp}} = \frac{a_{yi,\text{meas}} - \sin(\theta_{ayi})g}{\cos(\theta_{ayi})}. \quad (19)$$

To find the lateral acceleration in the COG both signals are converted to the COG by using the yaw-acceleration, $\dot{r}_z,\text{COG}$ and the average of both lateral accelerometers is used:

$$a_{y,\text{COG}} = \frac{1}{2} \left( a_{y,F,\text{comp}} - X_{\text{sensor,\text{COG},F}} \dot{r}_z,\text{COG} + a_{y,R,\text{comp}} + X_{\text{sensor,\text{COG},R}} \dot{r}_z,\text{COG} \right), \quad (20)$$

where $X_{\text{sensor,\text{COG},i}}$ is the longitudinal distance between the COG and the sensor, which is always positive. To determine the longitudinal and lateral velocity at the COG, the yaw-rate $\dot{r}_z,\text{COG}$ and the longitudinal and lateral distance of the sensor to the COG, $x_\beta$ and $y_\beta$, respectively, as shown in Figure 11(b), are used:

$$V_{x,\text{COG}} = V_{x,\text{sens}} - y_\beta \dot{r}_z,\text{COG}, \quad (21)$$

$$V_{y,\text{COG}} = V_{y,\text{sens}} + x_\beta \dot{r}_z,\text{COG}, \quad (22)$$

$$\beta_{\text{COG}} = \arctan \left( \frac{-V_{y,\text{COG}}}{V_{x,\text{COG}}} \right). \quad (23)$$

An average front wheel steering angle $\delta_{kp}$ is defined as the average of the left and the right king-pin angle.

$$\delta_{kp} = \frac{1}{2} \left( \delta_{kp,L} + \delta_{kp,R} \right). \quad (24)$$

This angle can be used to assess the vehicle response as a function of steering-wheel or front-wheel angle.

Figure 11. Compensation of the accelerometer for roll and conversion of the side-slip angle sensor velocities to the COG. (a) Roll compensation and (b) side-slip angle location compensation.
5. Test results

Three different tests will be discussed; the first test is a steady-state cornering test where the vehicle speed is increased gradually, while a constant cornering radius is maintained. The second test is a step-steer test where the vehicle drives at high speed and a step input is applied to the steering-wheel by the driver. The third test is a frequency response test where the driver applies a sinusoidal steering-wheel input.

To compare the model with the measurements the sensors are mounted at the same location in the SimMechanics model and the same post-processing is applied to have a fair comparison as described in the previous section. To assess the model performance an objective measure is used. The Pearson product-moment correlation coefficient $R$, indicates how well the measurements are replicated by the model, based on the proportion of total variation of outcomes explained by the model. This is a number between $-1$ and $1$. A value of $0$ indicates no correlation between the measurement and the prediction and a value of $1$ indicates a perfect correlation. A value of $-1$ indicates a perfect negative correlation, anti-phase) Suppose $\hat{g}$ is the measurement and $g$ is the prediction, $\text{cov}$ stands for covariance and $\text{std}$ for the standard deviation, then the following definition is used [19]:

$$ R = \frac{\text{cov}(\hat{g}, g)}{\text{std}(\hat{g})\text{std}(g)} $$

(25)

$R$ is blind to offsets by definition, therefore an additional measure is used. The average, indicated by mean, is used to calculate the offset between the two signals. The offset is divided by the range of the measured absolute signal to make it unitless. This makes it easier to compare different signals, but can lead to large values for signals that are not well excited.

$$ \text{offset} = \frac{\text{mean}(\hat{g}) - \text{mean}(g)}{|\text{max}(\hat{g}) - \text{min}(\hat{g})|} \cdot 100\%.$$  

(26)

For confidentiality reasons all graphs in this paper have been scaled and are made dimensionless. Four different linear scaling factors have been applied, one for all the angles and distances, one for all the forces and torques, one for the vehicle speed and one for the frequency axis. An example of scaling for an angle $\delta$ where the subscripts $\text{shown}$ and $\text{real}$ stand for the signals shown in this paper and the real unscaled signals, respectively, is given by:

$$ \delta_{\text{shown}} = \delta_{\text{real}}K_{\text{angle}}, $$

(27)

where $K_{\text{angle}}$ is the scaling factor for angles and has unit rad$^{-1}$.

5.1. Steady-state cornering

During the steady-state cornering test a circle is driven with a constant radius and an almost constant forward velocity. Due to practical reasons it is not possible to drive the complete speed range in one experiment and therefore different experiments have been combined. For the dimensionless velocity above 0.9, the step-steer experiment has been used. A selection of the data is taken from the step-steer experiment to ensure that the vehicle is in steady-state. All experiments have been done for both a left and right-hand turn.
Figure 12. Steady-state gains. From top to bottom: steady-state lateral acceleration gain, steady-state yaw-velocity gain and steady-state side-slip angle gain (scaled and made unit-less for confidentiality, velocity 1 corresponds with the peak in the yaw-velocity gain).

This experiment is utilised to scale the cornering stiffnesses of the tyres. An increase of 3% on both the front and trailer tyres cornering stiffness with respect to [17,18] is required. Figure 12 shows the measured and simulated steady-state gains. This figure shows a good resemblance between the measured and the simulated steady-state lateral acceleration gain and steady-state yaw velocity gain. The steady-state side-slip angle gain deviates from the measurements at high velocities.

5.2. Step-steer

This test is performed while driving at a constant forward velocity and applying a step input to the steering-wheel. This test is performed by a driver and therefore the steering-wheel rotation will not be an exact step. The top part of Figure 13 shows the steering-system during this manoeuvre. The velocity is kept constant during this manoeuvre by the cruise-control of the vehicle. This cruise-control is modelled by a PI-controller to keep the velocity constant during the simulation. At $t = 1.25$ [s] the driver rotates the steering-wheel. The steering-wheel angle is used as an input to the vehicle model. The steady-state part of this test can be used to tune the stiffness $k_{ha}$ by using the difference between the pitman-arm angle and the left king-pin angle. The value of the stiffness $k_{tierod}$ is tuned by using the difference in king-pin angles left and right under load.

For the pitman-arm angle $\delta_{pa}$ a small difference is seen at the time where the peak value is reached, but in steady-state the angle is accurately predicted by the model. The
Figure 13. Steering-system and chassis and cabin signals during a J-turn at motor-way speed. (a) Steering-system signals ($R$ and offset values are for the model with dry friction) and (b) chassis and cabin signals.
two king-pin angles $\delta_{kp,L}$ and $\delta_{kp,R}$ are nearly the same during straight-line driving, but diverge in steady-state after the step is applied. From this figure it becomes apparent that it is necessary to include a stiffness to model the tie-rod as the king-pin angles start to diverge under load. The drag-link force is noisier than the angles in the measurement which can be explained by the road roughness. During straight-line driving and in steady-state cornering the prediction is accurate. The steering-wheel torque $T_{sw,meas}$ already shows a non-zero value during straight-line driving. At the time of the step application the steering-wheel torque spikes and then settles to a noisy steady-state value. The prediction during straight-line driving moves around $T_{sw,meas} = 0$ since the road in the simulation is flat. The noise present in the simulation originates from the friction on the steering-wheel and the steering-house. The peak value is mainly caused by the friction in the hydraulic cylinder and the steady-state value is a result of forces and moments originating from the tyres in combination with the power-steering system.

To underline the importance of the modelling of dry friction in a steering-system, an additional simulation is included in Figure 13 where all dry friction components are set to zero, i.e. $T_{fric,sw} = T_{fric,hua} = T_{fric,kp} = 0$. A viscous damping coefficient is necessary for the stability of the system. The steering-wheel torque graph shows significant differences between the two models, the small steering-wheel movements do not result in a change in steering-wheel torque and the peak in torque at $t = 1.25\,[s]$ is not high enough. This results in an $R$ value of 0.872, significantly worse compared to the model with dry friction. The pitman-arm angle and drag-link force graphs also contain both models but for these signals the difference is insignificant.

Figure 13 shows the lateral acceleration, yaw-rate and side-slip angle of the COG of the vehicle. The lateral acceleration is predicted accurately, also at the time of the step application. The yaw-rate shows a good resemblance at the time of the step application, but the prediction and the measurement diverge before getting closer again for steady-state conditions. The tractor side-slip angle measurement is noisy and drawing a conclusion based on this figure is difficult since a general trend is missing. The right-hand side of the figure shows the signals measured in the cabin. The lateral acceleration in the cabin is also a noisy signal but the dynamics seem to be captured well by the model. The yaw-rate in the cabin shows similar differences compared to the yaw-rate measured at the COG, during settling a difference is observed. The roll-rate of the cabin is also an important quantity in how a driver experiences a vehicle. The amplitude and phase correspond well with the measurement.

A complete overview of the step steer experiments is presented in the appendix in Table A1.

5.3. Sinusoidal input

In the sinusoidal input test the forward velocity is again constant, but now the steering-angle is varied with a fixed frequency and amplitude. Due to practical reasons 22 separate tests have been performed for different frequencies, ranging from 0.1 to 2.7 normalised frequency units (NFU). The frequency is normalised by the frequency at which the lowest point in the lateral acceleration gain occurs. Again the cruise-control is used to keep the velocity constant and this test is performed by a driver, not a
steering robot, thus the sinusoidal steering-wheel input is an approximation of a true sine wave.

Figure 14 shows an example of a test where the steering-wheel is excited with a frequency of 0.4 NFU. This is a frequency, which is important from a vehicle dynamics point of view as, for example, rearward amplification tends to be largest at this frequency [20]. Figure 14(a) shows the steering-system-related signals. Both king-pin angles and the pitman-arm angle show a good agreement with the measurement. The drag-link force shows a good overall agreement, but the negative peaks are slightly under-predicted. The steering-wheel torque clearly shows the presence of friction since there are sudden jumps when the direction of movement is changed and also shows a curved shape which is caused by the boost-curve. The positive peaks are slightly under predicted and in some cases a difference is seen after reversal of the steering-wheel angle. Figure 14(b) shows the chassis and cabin-related signals where on the left side the chassis states are seen. The lateral acceleration and yaw-rate are predicted with a high accuracy ($R > 0.97$ and offset $< 4\%$). The side-slip angle measurement shows a lot of noise, but the general trend is consistent with the simulation model. The cabin lateral acceleration and yaw-rate are very similar to the chassis lateral acceleration and yaw-rate and the prediction is also accurate. Considering the roll-rate of the cabin some differences exist, especially at the start of the experiment and around the peaks.

The steering-system model without dry friction is also evaluated. The steering-wheel torque graph shows significant differences between the two models, the model without friction misses the jumps at reversal of the steering-wheel angle direction and the overall steering-wheel torque amplitude is too low. This results in an $R$-value of 0.748, again much worse than the model with dry friction.

Figure 15 shows the transfer function estimations from the steering-wheel angle to the lateral acceleration, yaw-rate and side-slip angle of the COG. The lateral acceleration shows a strong dip at 1 NFU and almost recovers to the steady-state value for higher frequencies. The model predicts the location of the peak close to the measurement, but does not fully recover to the steady-state value. The yaw-rate shows a similar dip at the same location and the simulation model here also does not fully recover to the measured value. The side-slip angle coherence drops drastically after 0.5 NFU in the measurement. Steady-state a difference is observed which was also seen in the steady-state measurement. For higher frequencies the prediction appears to be correct, but it does not contain the additional dynamics seen just after 1 Hz. Since the coherence is very low there this part is discarded as unreliable.

A complete overview of the $R$ values and offsets of the sinusoidal steering input experiment is presented in the appendix in Figure A1.

6. Discussion

The validation of the steering-system model shows that the model performs well for steady-state conditions. The vehicle response as well as the states in the steering-system can be predicted accurately with this model.

During the J-turn manoeuvre there is a constant offset in the steering-wheel torque. This is odd since the measured yaw-rate and lateral acceleration are close to zero during this manoeuvre. An explanation for this can be that the road is slightly banked for
Figure 14. Steering-system and chassis and cabin signals during a 0.4 Hz sinusoidal steering-wheel input at motor-way speed (scaled and made unit-less due to confidentiality). (a) Steering-system signals ($R$ and offset values are for the model with dry friction) and (b) chassis and cabin signals.
Figure 15. Transfer functions from the steering-wheel angle $\delta_{sw}$ to the lateral acceleration, yaw-rate and side-slip angle at the cog of the vehicle. The frequency axis is normalised on the lowest point in the lateral acceleration gain due to confidentiality.

drainage purposes, a-symmetry of the vehicle or improper zeroing of the sensors. The spike in steering-wheel torque when the step input is applied is mainly caused by dry friction as shown by an example of a model without dry friction. The steady-state value of the steering-wheel torque during cornering is caused by the tyre feed-back in combination with the power-steering system and the friction on the output side of the steering-house. The small variations on the steering-wheel torque are again caused by dry friction. The king-pin angles and pitman-arm angle show that it is necessary to implement a stiffness between the steering-house and the left wheel and a stiffness between the left and the right wheel. The drag-link force shows that the tyres and geometry of the upright have been modelled correctly since the measured and predicted force shows a good similarity. The lateral acceleration and yaw-rate of both the chassis and the cabin are accurately predicted. The side-slip angle measurement is noisy and therefore it is hard to draw
any conclusions based on this. The roll-rate of the cabin appears to be accurately repre-
sented which indicates an appropriate roll stiffness, height of the COG and inertia of the
cabin.

To show repeatability of the experiments, a table is with the quality assessment of more
step-steer experiments is presented in the appendix in Table A1. This table shows that the
steering-wheel torque averaged over four experiments has an $R$ value of 0.96 with an off-
set of 9.3%. All chassis states also have high $R$ values except for the sideslip angle $\beta_{\text{COG}}$,
the time signals show that this sensor is noisy and therefore no conclusion can be drawn
regarding this signal. The overview of the model quality of the sinusoidal experiments in
Figure A1 shows that the steering-wheel torque is predicted with an acceptable $R$ value
($> 0.95$) for the full frequency range and the offset is constant up till 1 NFU and decreases
afterward. This offset appears to be similar in magnitude in comparison to the offset seen
in the J-turn experiment. The angles in the steering-system are predicted with a high accu-
racy ($R > 0.98$) where no dependency on the frequency is observed. The same holds for
the drag-link force. The chassis and cabin quality assessment shows that the model is able
to predict the vehicle response accurately up till 1 NFU ($R > 0.97$), after this the model
starts to deviate from the measurement. Since the steering-system-related signals remain
correct, we assume that this deviation originates either in the tyres or the chassis model.
Since the chassis model is relative simple, we suspect that a more advanced chassis model
might resolve this issue.

7. Conclusions and recommendations

7.1. Conclusions

- Simulations with a 44 DOF multi-body tractor semi-trailer model in combination with
  a three DOF steering-system model and measurements with an instrumented tractor
  semi-trailer show that the complete model is an accurate representation of reality.
- Steady-state manoeuvres show that the previously developed 44 DOF multi-body trac-
tor semi-trailer model only requires an update of the dimensions and masses to match
the current test vehicle and a change in cornering stiffness of less than 5%.
- J-turn experiments show that the presence of a tie-rod stiffness is necessary since Ack-
erman steering by itself cannot fully explain the measured difference in the left and right
king-pin angles.
- Sinusoidal steering experiments show that the steering-system model is accurate for the
  complete measured range (0.1–3 NFU). The vehicle response deviates above 1 NFU.
- The sinusoidal steering experiments also confirm that the modelling of dry friction in
  steering-system in commercial vehicles is essential to predict the steering-wheel torque.
  When the steering direction changes, a sudden jump is observed in steering-wheel
torque.

7.2. Recommendations

- During the sinusoidal steering experiment a discrepancy between the measured and
  simulated vehicle response is observed above 1 NFU. At the same time the steering-
  system response is still valid above this frequency. Simulations with a more elaborate
chassis and suspension model such as a finite element model might provide more insight in this matter.

- An additional smaller discrepancy between the measured and simulated king-pin angles is observed which appears to be non-symmetrical. This could be friction related but might as well be geometry related. Additional tests with an instrumented tie-rod and measurement wheels to determine the exact wheel kinematics and forces may help to understand this better.

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Appendix 1. Complete quality assessment of step steer experiments

This appendix shows an overview of the quality assessment of additional step steer tests. The signals are compared by using the $R$-value and the offset and averaged over the total of four tests.

Table A1. Model quality of the steering-system states and chassis and cabin states (rounded to two significant digits).

| Signal | Exp. no. | $R$-values | Offset value [%] |
|--------|----------|------------|-----------------|
|        | 1 2 3 4  | Mean       | 1 2 3 4 Mean    |
| $\delta_{sw}$ | 1 1 1 1 | 1 1 0 0 | 0 0 0 0 0 0 0 0 |
| $\delta_{pa}$ | 1 1 1 1 | 1 1 | -2.6 2.6 -3.7 -4.4 2.6 |
| $\delta_{kp,L}$ | 1 1 1 1 | 1 1 | -1.5 -2.3 -2.3 -2.6 -2.1 2.6 |
| $\delta_{kp,R}$ | 1 1 1 1 | 1 1 | -2.1 1 -2.3 -0.52 -0.2 0.52 |
| $T_{sw,meas}$ | 0.94 0.96 0.97 0.98 | 0.96 | 7.7 8.5 12 9.3 9.3 |
| $F_{draglink}$ | 0.99 1 1 1 | 1 1 | 5.9 3.1 2.1 -2.1 2.1 |
| $\alpha_y,COG$ | 0.99 0.99 0.99 0.98 | 0.99 | 0.02 1.6 2.1 2.3 1.5 |
| $f_z,COG$ | 0.99 1 1 0.99 | 1 1 | -2.8 2.5 0.1 1.4 0.28 |
| $\rho_{COG}$ | 0.5 0.72 0.62 0.78 | 0.66 | -10 2.9 -11 -12 7.4 |
| $\alpha_y,C$ | 0.98 0.9 0.99 0.98 | 0.99 | 3.8 4 5.1 4.6 4.4 |
| $f_z,C$ | 0.99 0.99 1 0.99 | 0.99 | -2.6 1 0.01 1 0.14 |
| $f_x,C$ | 0.95 0.9 0.95 0.92 | 0.93 | -0.98 0.37 -1.4 -0.6 -0.64 |

Appendix 2. Complete quality assessment of sinusoidal steering experiments

This appendix shows an overview of the quality assessment of additional sinusoidal steering input tests. The signals are compared by using the $R$-value and the offset. To gain more insight in the model accuracy as a function of frequency, the $R$-values and offsets are plotted as a function of frequency.
Figure A1. Model quality of the steering-system states and chassis and cabin states as a function of frequency for a sinusoidal steering-wheel input. $f$ is the dominant frequency of the input signal at the steering-wheel. (a) Steering-system and (b) chassis and cabin.