Alpha Weakly Semi Closed Sets in Topological Spaces

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Abstract – N. Levine introduced the concept of generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets and they characterized that sets. In this paper, we introduce a new class of closed sets which is called Alpha weakly semi closed sets in topological spaces and we study the relationships of this set with some other generalized closed sets. Also we study some of its basic properties.

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I. INTRODUCTION

In 1970, Levine [1] introduced generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets. In 2000, M. Sheik John [2] introduced and investigated w-closed sets in topology. In 2017, Veeresha A Sajjanar [3] introduced weakly semi closed sets and investigated some of their properties. In this paper, Section I contains the concept of Alpha Weakly semi-closed (briefly αws-closed) set is introduced and their properties are investigated. Section II contains the Certain preliminary concepts, Section III contains the concept of αws-closed set is studied and a diagram also included which states the relationships among the generalized closed sets in topological spaces and Section IV contains the conclusions and Section V contains the references.

II. PRELIMINARIES

Throughout this paper X and Y represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, clA and intA denote the closure of A and the interior of A respectively. X – A denotes the complement of A in X. We recall the following definitions.

Definition 2.1: A subset A of a space X is called

(i) pre-open [4] if A ⊆ int clA and pre-closed if cl intA ⊆ A.
(ii) semi-open [5] if A ⊆ cl intA and semi-closed if int clA ⊆ A.
(iii) semi-pre-open [6] if A ⊆ cl int clA and semi-pre-closed if int cl intA ⊆ A.
(iv) a-open [7] if A ⊆ int cl intA and a-closed if cl int clA ⊆ A.
(v) regular open [8] if A = int clA and regular closed if cl intA = A.
(vi) b-open [9] if A ⊆ cl intA ∪ int clA and b-closed if cl intA ∩ int clA.
(vii) π-open [10] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The α-closure (resp. semi-closure, resp. semi-pre-closure, resp. pre-closure, resp. b-closure) of a subset A of X is the intersection of all α-closed (resp. semi-closed, resp. semi-pre-closed, resp. pre-closed, resp. b-closed) sets containing A and is denoted by aclA (resp. sclA, resp. spclA, resp. pclA, resp. bclA).

Definition 2.2: A subset A of a space X is called
Lemma 2.4: [34] In an extremally disconnected space X,

(i) \( pclA = spclA \).
(ii) \( aclA = sclA \).

Lemma 2.5: [34] In an extremally disconnected sub maximal space X,
\( clA = aclA = sclA = pclA = spclA \).
Lemma 2.6: [6] For any subset $A$ of $X$, the following results hold:

(i) $scl A = A \cup int cl A$.
(ii) $pcl A = A \cup cl int A$.
(iii) $spel A = A \cup int cl int A$.
(iv) $acl A = A \cup cl int cl A$.

III. ALPHA WEAKLY SEMI CLOSED SETS

In this section, we introduce a new type of closed sets namely $\alpha WS$-closed sets in topological spaces and study some of their properties.

Definition 3.1: A subset $A$ of a space $X$ is called Alpha Weakly Semi closed (briefly $\alpha WS$-closed) if $acl A \subseteq U$ whenever $A \subseteq U$ and $U$ is $ws$-open.

Proposition 3.2:

(i) Every closed set is $\alpha WS$-closed.
(ii) Every $\alpha$-closed set is $\alpha WS$-closed.
(iii) Every $\pi$-closed set is $\alpha WS$-closed.
(iv) Every regular closed set is $\alpha WS$-closed.
(v) Every $(gsp)^*$-closed set is $\alpha WS$-closed.

Proof:

(i) Let $A$ be a closed set in $X$. Let $A \subseteq U$ and $U$ is $ws$-open. Since $A$ is closed, $cl A = A$.
   But $acl A \subseteq cl A$. Therefore $acl A \subseteq U$. Hence $A$ is $\alpha WS$-closed in $X$.
(ii) Let $A$ be a $\alpha$-closed set in $X$. Let $A \subseteq U$ and $U$ is $ws$-open. Since $A$ is $\alpha$-closed, $acl A = A$.
   Therefore $acl A \subseteq U$. Hence $A$ is $\alpha WS$-closed in $X$.
(iii) Let $A$ be a $\pi$-closed subset of $X$. Since every $\pi$-closed set is closed [19] and by (i), we have $A$ is $\alpha WS$-closed.
(iv) Let $A$ be a regular-closed subset of $X$. Since every regular-closed set is closed [8] and
   By (i), we have $A$ is $\alpha WS$-closed.
(v) Let $A$ be a $(gsp)^*$-closed set in $X$. Let $A \subseteq U$ and $U$ is $ws$-open. Since every $ws$-open set is
   $gsp$-open and $A$ is $(gsp)^*$-closed, $cl A \subseteq A$. But $acl A \subseteq cl A$. Therefore $acl A \subseteq U$. Hence $A$
   is $\alpha WS$-closed in $X$.

The reverse implications are not true as shown in Examples 3.3, 3.4, 3.5 and 3.6

Example 3.3: Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. Then
   $\{b\}$ is $\alpha WS$-closed but not regular closed.

Example 3.4: Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then
   $\{b\}$ is $\alpha WS$-closed but not $\alpha$-closed.
   $\{b\}$ is $\alpha WS$-closed but not $(gsp)^*$-closed.

Example 5: Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. Then $\{c\}$ is $\alpha WS$-closed but not $\pi$-closed.

Example 3.6: Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, X\}$. Then
   $\{a, c, d\}$ is $\alpha WS$-closed but not $\alpha$-closed.

Proposition 3.7:

(i) Every $\alpha WS$-closed set is $ag$-closed.
(ii) Every $\alpha WS$-closed set is $gpr$-closed.
(iii) Every $\alpha WS$-closed set is $gb$-closed.
(iv) Every $\alpha WS$-closed set is $rgb$-closed.
(v) Every αws-closed set is gp-closed.
(vi) Every αws-closed set is gs-closed.
(vii) Every αws-closed set is agr-closed.
(viii) Every αws-closed set is gab-closed.
(ix) Every αws-closed set is sg-closed.
(x) Every αws-closed set is sgb-closed.

Proof:
(i) Let A be a αws-closed subset of a space X. Let A ⊆ U and U is open. Since every open set is
ws-open in X and A is αws-closed, aclA ⊆ U. Hence A is ag-closed.
(ii) Let A be a αws-closed set in X. Let A ⊆ U and U is regular open. Since every regular open
set is ws-open in X and since A is αws-closed, aclA ⊆ U. But pclA ⊆ aclA. Therefore
pclA ⊆ U. Hence A is gpr-closed.
(iii) Let A be a αws-closed set in X. Let A ⊆ U and U is open. Since every open set is ws-open
in X & Since A is αws-closed, aclA ⊆ U. But bclA ⊆ aclA. Therefore bclA ⊆ U. Hence A
is gb-closed in X.
(iv) Let A be a αws-closed set in X. Let A ⊆ U and U is regular open. Since every regular open
set is ws-open in X and since A is αws-closed, aclA ⊆ U. But bclA ⊆ aclA. Therefore bclA ⊆ U. Hence A
is rgb -closed.
(v) Let A be a αws-closed set in X. Let A ⊆ U and U is open. Since every open set is ws-open
and since A is αws-closed, aclA ⊆ U. But pclA ⊆ aclA. Therefore pclA ⊆ U. Hence A is
gp-closed.
(vi) Let A be a αws-closed set. Let A ⊆ U and U is open. Since every open set is ws-open and
since A is αws-closed, aclA ⊆ U. But aclA ⊆ aclA. Therefore pclA ⊆ aclA. Hence A is
gs-closed.
(vii) Let A be a αws-closed set. Let A ⊆ U and U is regular open. Since every regular open set
is ws-open and since A is αws-closed, aclA ⊆ U. Hence A is agr-closed.
(viii) Let A be a αws-closed set. Let A ⊆ U and U is α-open. Since every α-open set
ws-open and since A is αws-closed, aclA ⊆ U. But bclA ⊆ aclA. Therefore bclA ⊆ U.
Hence A is gab-closed.
(ix) Let A be a αws-closed set. Let A ⊆ U and U is semi-open. Since every semi-open set is
ws-open and since A is αws-closed, aclA ⊆ U. But aclA ⊆ aclA. Therefore aclA ⊆ U.
Hence A is sg-closed.
(x) Let A be a αws-closed set. Let A ⊆ U and U is semi-open. Since every semi-open set
ws-open and since A is αws-closed, aclA ⊆ U. But bclA ⊆ aclA. Therefore bclA ⊆ U.
Hence A is sgb-closed.

The reverse implications are not true as shown in Example 3.8, 3.9 And 3.10

Example 3.8: Let X = \{a, b, c, \} with topology \( \tau = \{\phi, \{a\}, \{a, b\}, X\} \). Then
\( \{a\} \) is ag-closed but not αws-closed.
\( \{a\} \) is gpr-closed but not αws-closed.
\( \{a, c\} \) is gb-closed but not αws-closed.
\( \{a\} \) is rgb-closed but not αws-closed.
\( \{a, c\} \) is gp-closed but not αws-closed.
\( \{a, c\} \) is gs-closed but not αws-closed.
\( \{a\} \) is agr-closed but not αws-closed.

Example 3.9: Let X = \{a, b, c, d\} with topology \( \tau = \{\phi, \{a, b\}, \{a, b, c\}, X\} \). Then
\( \{a\} \) is gab-closed but not αws-closed.

Example 3.10: Let X = \{a, b, c, d\} with topology \( \tau = \{\phi, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\} \). Then
\( \{a\} \) is sg-closed but not αws-closed.
\{a\} is sgb-closed but not αws-closed.

The concept “αws-closed” is independent from the concepts “g-closed”, “gr-closed”, “g*-closed”, “rg-closed”, “g\#p\#-closed”, “*g-closed”, “πg-closed” as seen in the following Examples 3.11 & 3.12

**Example 3.11:** Let $X = \{a, b, c\}$ with topology $τ = \{ϕ, \{a\}, \{a, b\}, X\}$.
- \{b\} is αws-closed but not g-closed and \{a, c\} is g-closed but not αws-closed.
- \{b\} is αws-closed but not gr-closed and \{a, c\} is gr-closed but not αws-closed.
- \{b\} is αws-closed but not g\#p\#-closed and \{a, c\} is g\#p\#-closed but not αws-closed.

**Example 3.12:** Let $X = \{a, b, c, d\}$ with topology $τ = \{ϕ, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$.
- \{c\} is αws-closed but not rg-closed and \{a, b\} is rg-closed but not αws-closed.
- \{c\} is αws-closed but not *g-closed and \{a, b, d\} is *g-closed but not αws-closed.
- \{c\} is αws-closed but not πg-closed and \{b, d\} is πg-closed but not αws-closed.

Thus the above discussions lead to the following diagram. In this diagram, “A → B” means A implies B but not conversely and “A ↔ B” means A and B are independent of each other.

**Figure 1**

**Theorem: 3.13**

The union of two αws-closed subsets of $X$ is αws-closed set.

**Proof:**

Let $A$ and $B$ be any two αws-closed sets in $X$. Let $A \subseteq U$ & $U$ is ws-open, $B \subseteq U$ & $U$ is ws-open. Then $acl(A) \subseteq U$ and $acl(B) \subseteq U$. Since $A \subseteq U$ and $B \subseteq U$, then $A \cup B \subseteq U$

$⇒ acl(A) \cup acl(B) \subseteq U$. We know that $acl(A \cup B) = acl(A) \cup acl(B)$ [35].
Hence \( acl (A \cup B) = acl(A) \cup acl (B) \subseteq U \). Hence \( acl (A \cup B) \subseteq U \). Therefore \( A \cup B \) is \( \text{aws-closed} \) in \( X \).

**Theorem: 3.14**

If a subset \( A \) of \( X \) is \( \text{aws-closed} \) in \( X \), then \( acl A - A \) does not contain any non-empty \( \text{ws-closed} \) set in \( X \).

**Proof:**

Let \( A \) be a \( \text{aws-closed} \) set in \( X \) and \( F \) be a \( \text{ws-closed} \) subset of \( acl A - A \).

Then \( F \subseteq acl A \cap (X - A) \implies F \subseteq acl A \& F \subseteq X - A \implies A \subseteq X - F \)

Since \( A \) is \( \text{aws-closed} \) set and \( X - F \) is \( \text{ws-open} \), then \( acl A \subseteq X - F \) (ie) then \( F \subseteq X - acl A \)

We have \( F \subseteq acl A \). Therefore, \( F \subseteq (X - acl A) \cap acl A = \phi \). Thus \( F \subseteq \phi \).

Hence \( acl A - A \) does not contain any non-empty \( \text{ws-closed} \) set in \( X \).

**Theorem: 3.15**

If a subset \( A \) is \( \text{aws-closed} \) set in \( X \) and, \( A \subseteq B \subseteq acl A \), then \( B \) is also \( \text{aws-closed} \) set.

**Proof:**

Let \( A \) be a \( \text{aws-closed} \) set in \( X \) such that \( A \subseteq B \subseteq acl (A) \). To prove \( B \) is also \( \text{aws-closed} \) set in \( X \). It is enough to prove \( acl (B) \subseteq U \). Let \( U \) be a \( \text{ws-open} \) set in \( X \) such that \( B \subseteq U \).

Since \( A \subseteq B, A \subseteq U \). Also since \( A \) is \( \text{aws-closed} \), \( acl (A) \subseteq U \). Now, \( B \subseteq acl (A) \)

\( \implies acl (B) \subseteq acl [ acl (A)] = acl A \subseteq U [36] \). (ie) \( acl (B) \subseteq U \). Therefore, \( B \) is \( \text{aws-closed} \) set in \( X \).

**Theorem: 3.16**

For every point \( x \) in a space \( X \), \( X - \{ x \} \) is \( \text{aws-closed} \) or \( \text{ws-open} \).

**Proof: Case (i)**

suppose \( X - \{ x \} \) is not \( \text{ws-open} \). Then \( X \) is the only \( \text{ws-open} \) set containing \( X - \{ x \} \)

Then using Definition 3.1 \( acl (X - \{ x \}) \subseteq X \). Hence \( X - \{ x \} \) is \( \text{aws-closed} \).

**Case (ii)**

Suppose \( X - \{ x \} \) is not \( \text{aws-closed} \). Then there exists a \( \text{ws-open} \) set \( U \) containing \( X - \{ x \} \) such that \( acl (X - \{ x \}) \not\subseteq U \).

Therefore \( acl (X - \{ x \}) \) is either \( X - \{ x \} \) or \( X \). Therefore Take

\( acl (X - \{ x \}) = X - \{ x \} \), then \( X - \{ x \} \) is \( \alpha \)-closed. By Proposition 3.2 (i) every \( \alpha \)-closed set is \( \text{aws-closed} \), \( X - \{ x \} \) is \( \text{aws-closed} \). This is contradiction to our assumption. Therefore

\( acl (X - \{ x \}) = X \). To prove \( X - \{ x \} \) is \( \text{ws-open} \). Suppose \( X - \{ x \} \) is not \( \text{ws-open} \). By case (i)

\( X - \{ x \} \) is \( \text{ws-open} \). Which is contradiction to our assumption. Therefore \( X - \{ x \} \) is \( \text{ws-open} \).

**Theorem: 3.17**

Let \( X \) and \( Y \) are topological spaces and \( A \subseteq Y \subseteq X \). Suppose that \( A \) is \( \text{aws-closed} \) set in \( X \) then \( A \) is \( \text{aws-closed} \) relative to \( Y \).

**Proof:**

Given \( A \subseteq Y \subseteq X \) and \( A \) is \( \text{aws-closed} \) in \( X \). To prove that \( A \) is \( \text{aws-closed} \) relative to \( Y \).
Let $A \subseteq Y \cap U$, where $U$ is ws-open in $X$. Since $A$ is $\alpha$ws-closed, then $\alpha cl A \subseteq U$. This implies $Y \cap \alpha cl A \subseteq Y \cap U$, where $Y \cap \alpha cl A$ is the $\alpha$-closure of $A$ in $Y$ and $Y \cap U$ is ws-open in $Y$. Therefore $\alpha cl A \subseteq Y \cap U$ in $Y$. Hence, $A$ is $\alpha$ws-closed set relative to $Y$.

**Theorem: 3.18**

Let $A$ be $\alpha$ws-closed in $X$. Then $A$ is $\alpha$-closed iff $\alpha cl A - A$ is ws-closed.

**Proof:**

Suppose $A$ is a $\alpha$-closed set. Then $\alpha cl A = A \Rightarrow \alpha cl A - A = \emptyset$ which is ws-closed.

Conversely, suppose $\alpha cl A - A$ is ws-closed. Since $A$ is $\alpha$ws-closed, then by Theorem 3.14, $\alpha cl A - A = \emptyset$, (ie) $\alpha cl A = A$. Hence $A$ is $\alpha$-closed.

**Theorem: 3.19**

Suppose $A$ is ws-open and $A$ is $\alpha$ws-closed. Then $A$ is $\alpha$-closed.

**Proof:**

Given that $A$ is ws-open and $A$ is $\alpha$ws-closed. Then $A \subseteq A \Rightarrow \alpha cl A \subseteq A\subseteq A$

Hence $A$ is $\alpha$-closed.

**Theorem: 3.20**

In a topological space if $X \alpha O(X) = \{X, \emptyset\}$ then every subset of $X$ is a $\alpha$ws-closed set.

**Proof:**

Given that $X$ is a topological space and $X \alpha O(X) = \{X, \emptyset\}$. Let $A$ be a subset of $X$. Suppose $A = \emptyset$, then by Theorem 3.4, $\emptyset$ is $\alpha$ws-closed set. Suppose $A \neq \emptyset$, then $X$ is only $\alpha$-open set containing $A$. Therefore $\alpha cl A \subseteq X$. Hence $A$ is $\alpha$ws-closed set in $X$.

**Theorem: 3.21**

If $A$ is regular open and $\alpha gr$-closed set then $A$ is $\alpha$ws-closed set in $X$.

**Proof:**

Suppose $A$ is a regular open set and $\alpha gr$-closed. Let $U$ be any ws-open set in $X$ $\exists: A \subseteq U$.

Since $A$ is regular open and $\alpha gr$-closed set in $X$, by Definition $\alpha cl A \subseteq A$, then $\alpha cl A \subseteq A \subseteq U$. Hence $A$ is $\alpha$ws-closed.

**Definition: 3.22**

The intersection of all ws-open subsets of $X$ containing $A$ is called the ws-kernel of $A$ and is denoted by $ws$-ker $(A)$.

**Theorem: 3.23**

If $A$ is a subset of $X$ is $\alpha$ws-closed iff $\alpha cl A \subseteq ws$-ker $(A)$.

**Proof:**

Suppose $A$ is $\alpha$ws closed. Then $\alpha cl A \subseteq U$ whenever $A \subseteq U \& U$ is $ws$-open.

To prove $\alpha cl(A) \subseteq ws$-Ker $(A)$, Take $x \in \alpha cl(A)$. To prove $x \in ws$-ker $(A)$

Suppose $x \notin ws$-ker$(A)$ then there exist a ws-open set $U$ containing $A$ such that $x \notin U$. Since $A$ is $\alpha$ws-closed, then $\alpha cl A \subseteq U \Rightarrow x \notin \alpha cl(A)$, Which is a contradiction to our assumption. Therefore $\alpha cl A \subseteq ws$-ker $(A)$. Conversely, Suppose $\alpha cl A \subseteq ws$-ker $(A)$. To prove $A$ is
aws-closed. If $U$ is any ws-open set containing $A$, then $ws-ker A \subseteq U \Rightarrow acl A \subseteq U$. Hence $A$ is $aws$-closed in $X$.

**Note: 3.24 [37]**

Let $x$ be a point of $X$. Then $\{x\}$ is either nowhere dense or pre-open.

**Remark: 3.25 [37]**

By the above note we take the following decomposition of a given topology $X$, namely

$$X = X_1 \cup X_2$$

Where $X_1 = \{x \in X; \{x\} \text{ is nowhere dense}\}$

$$X_2 = \{x \in X; \{x\} \text{ is pre-open}\}$$

This is called Jankovic-Reilly Decomposition.

**Theorem: 3.26**

For any subset $A$ of $X$, $X_2 \cap acl A \subseteq ws-ker (A)$

**Proof:**

To prove $X_2 \cap acl (A) \subseteq ws-ker(A)$. Consider $x \in X_2 \cap acl (A)$. To prove $x \in ws-ker(A)$

Suppose $x \not\in ws-ker (A)$, then there is a $ws$-open set $U$ containing $A$ such that $x \not\in U$.

If $F = X - U$, then $F$ is $ws$-closed. Now, $x \in acl (A) \Rightarrow acl (\{x\}) \subseteq acl (acl (A)) \subseteq acl (A)$

Since $a cl (\{x\}) \subseteq acl (A)$, we get $int (acl (\{x\})) \subseteq int (acl (A)) \subseteq A \cap int (acl (A))$

Therefore $int (acl (\{x\})) \subseteq A \cap int (cl (A))$. Now, take $x \in X_2$. We have $x \not\in X_1$ and so $int (cl (\{x\})) \neq \emptyset$. Let $y \in int (cl (\{x\}))$. Consider a point $y \in A \cap int (cl (\{x\}))$

$\Rightarrow y \in A \cap cl (\{x\}) \Rightarrow y \in A \cap F$ which is a contradiction to $x \not\in ws-ker (A)$ [38]. Therefore $x \in ws-ker (A)$. Hence $X_2 \cap acl (A) \subseteq ws-ker (A)$.

**Theorem 3.27:**

A subset $A$ of $X$ is $aws$-closed iff $X_1 \cap acl (A) \subseteq A$

**Proof:**

Consider $A$ is $aws$-closed. To prove $X_1 \cap acl (A) \subseteq A$. Let $x \in X_1 \cap acl (A)$, Then $x \in X_1$ and $x \in acl (A)$. Since $x \in X_1$, $int (cl (\{x\})) = \emptyset$. Hence $\{x\}$ is semi-closed. Every semi closed set is $ws$-closed in $X$ [15]. $\{x\}$ is $ws$-closed. If $x \not\in A$, then $U = X - \{x\}$ is $ws$-open set containing $A$ and so $acl A \subseteq U$. Since $x \in acl (A)$, $x \not\in U$ which is a contradiction to $x \not\in U$.

Hence $X_1 \cap acl (A) \subseteq A$. Conversely, let $X_1 \cap acl (A) \subseteq A$. To prove $A$ is $aws$-closed

Since $X_1 \cap acl (A) \subseteq A$, $X_1 \cap acl (A) \subseteq ws-ker (A)$.

Now, $acl (A) = X \cap acl (A) = (X_1 \cup X_2) \cap acl (A) = (X_1 \cap acl (A)) \cup (X_2 \cap acl (A))$

$acl (A) \subseteq ws-ker (A)$. Then by Theorem 3.23 $A$ is $aws$-closed.

**Theorem: 3.28**

Arbitrary intersection of $aws$-closed set is $aws$-closed.

**Proof:**

Let $\{A_i\}$ be the collection $aws$-closed sets of $X$. Let $A = \cap A_i$. Since $A \subseteq A_i$ for each $i$,

then $acl (A) \subseteq acl (A_i) \Rightarrow X_i \cap acl (A) \subseteq X_i \cap acl (A_i)$ for each $i$. 

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Since each $A_i$ is $\alpha$ws-closed, then by theorem 3.28, $X_1 \cap acl(A_i) \subseteq A_i$ for each $i$.
Thus $X_1 \cap acl(A) \subseteq X_1 \cap acl(A_i) \subseteq A_i \subseteq A$ for each $i$. By Theorem 3.27, $A$ is $\alpha$ws-closed.

**Theorem: 3.29**

In a door space $X$, every $\alpha$ws-closed set is $\alpha$-closed.

**Proof:**

Let $A$ be a $\alpha$ws-closed set in $X$. Since $X$ is a door space, by Definition 2.3(iii), $A$ is either open or closed. If $A$ is closed, then $A$ is $\alpha$-closed. If $A$ is open, then $A$ is $ws$-open.
Since $A$ is $\alpha$ws-closed & $A$ is $ws$-open, by Theorem 3.19, $A$ is $\alpha$-closed.

**Theorem: 3.30**

In an extremally disconnected space $X$, every $\alpha$ws-closed set is $gs$-closed.

**Proof:**

Let $X$ be an extremally disconnected space and $A$ be a $\alpha$ws-closed subset of $X$.
Let $A \subseteq U$ & $U$ be open. Since every open set is $ws$-open in $X$ and since $A$ is $\alpha$ws-closed, $aclA \subseteq U$. Since $X$ is extremally disconnected space, by Lemma2.4(ii), $sclA = aclA \subseteq U$
$\Rightarrow sclA \subseteq U$. Hence $A$ is $gs$-closed.

**Theorem: 3.31**

In an extremally disconnected sub maximal space $X$, every $\alpha$ws-closed set is $w$-closed.

**Proof:**

Let $X$ be an extremally disconnected space and $A$ be a $\alpha$ws-closed subset of $X$.
Let $A \subseteq U$ & $U$ is semi-open. Since every semi open set is $ws$-open & since $A$ is $\alpha$ws-closed, $aclA \subseteq U$. Since $X$ is extremally disconnected submaximal space, by Lemma 2.5, $clA \subseteq U$.
Hence $A$ is $w$-closed.

**Theorem: 3.32**

In a $T_b$ space $X$, Every $gs$-closed set is $\alpha$ws-closed.

**Proof:**

Let $A$ be a $gs$-closed set. Since $X$ is $T_b$ space, by Definition 2.3 (i), $A$ is closed. By Preposition 3.2(i) $A$ is $\alpha$ws-closed.

**Theorem: 3.33**

In an $\alpha$-space $X$, every $\alpha$-closed set is $ws$-closed.

**Proof:** Let $A$ be a $\alpha$-closed set in $X$. Since $X$ is an $\alpha$-space, by Definition 2.3(ii), $A$ is closed. By Preposition 3.2(ii), $A$ is $\alpha$ws-closed.

**IV. CONCLUSION**

In this paper, we have focused on Alpha Weakly Semi closed sets in topological spaces and found some important properties. In future this concept can be extended to bitopological and ideal topological spaces.

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