The relevance of virtual electroweak effects in the overall 
$t$-channel single top production at LHC

M. Beccaria$^{a,b}$, G. Macorini$^{c,d}$ F.M. Renard$^e$ and C. Verzegnassi$^{c,d}$

$^a$Dipartimento di Fisica, Università di Lecce
Via Arnesano, 73100 Lecce, Italy.
$^b$INFN, Sezione di Lecce
$^c$Dipartimento di Fisica Teorica,
Università di Trieste,
Strada Costiera 14, Miramare (Trieste)
$^d$INFN, Sezione di Trieste
$^e$Laboratoire de Physique Théorique et Astroparticules,
UMR 5207
Université Montpellier II,
F-34095 Montpellier Cedex 5.

Abstract

We compute the complete one loop electroweak effects in the MSSM for the eight processes of single top (and single antitop) production in the $t$-channel at hadron colliders, generalizing a previous analysis performed for the dominant $dt$ final state. The results are quite similar for all processes, showing an impressively large Standard Model effect and a generally modest genuine SUSY contribution in the mSUGRA scenario. The one loop effect on the total rate is shown and the possibility of measuring it at LHC is discussed.

PACS numbers: 12.15.-y, 12.15.Lk, 13.75.Cs, 14.80.Ly
I. INTRODUCTION

The relevance of a precise measurement of single top production at hadron colliders was already stressed in several papers in the recent years [1]. A well known peculiarity of the process is actually the fact that it offers the unique possibility of a direct measurement of the \( t \bar{b}W \) \( (V_{tb}) \) coupling, thus allowing severe tests of the conventionally assumed properties of the CKM matrix in the Minimal Standard Model, and for a very accurate review of the topics we defer to a very recent article [2].

For the specific purpose of a "precise" determination of \( V_{tb} \), two independent requests must be met. The first one is that of a correspondingly "precise" experimental measurement of the process. For the \( t \)-channel case on which we shall concentrate in this paper a very recent CMS study [3] concludes that, with 10 fb\(^{-1}\) of integrated luminosity, one could be able to reduce the (mostly systematic) experimental uncertainty of the cross section below the ten percent level (worse uncertainties are expected for the two other processes, the \( s \)-channel and the associated \( Wt \) production, whose cross section is definitely smaller than that of the \( t \)-channel). The second request is that of a similarly accurate theoretical prediction of the observables of the process. In this respect, one must make the precise statement that, in order to cope with the goal of measuring \( V_{tb} \) at the few (five) percent level, a complete NLO calculation is requested. In the SM this has been done for the QCD component of the \( t \)-channel, resulting in a relatively small (few percent) effect [4]. The electroweak effects have been computed very recently at the complete one loop level within the MSSM for the dominant \( dt \) component of the process (to be defined in Section 2) [5]. The main conclusion of that analysis was that, within the SM framework, the electroweak effect at one loop is sizable in the cross section, of the ten percent size or even more. This effect is larger than that of the QCD component, and would be essential for a genuine precision test of electroweak physics to be performed at LHC. As stressed in [5], the genuine SUSY effect would be modest (at the relative two-three percent level) for a class of considered benchmark points in the mSUGRA scenario, although possible special effects (typically of threshold kind) might be visible for particular light sparticles (e.e. stop)/ scenarios.

As we anticipated, from a numerical point of view the \( t \)-channel cross section is by far the largest in the single top production processes. In [5], we only considered one of the eight possible final states of the process (conventionally, four single top and four single antitop
cases, as shown in Section 2) i.e. the $dt$ state that gives more than fifty percent of the overall cross section. The aim of this paper is that of generalizing the analysis of [5] to the remaining seven final states. This will be done in full analogy with [5], computing the complete one loop electroweak effect in the MSSM. In this way, a full description of the overall single top production process in the dominant $t$-channel will be available for the electroweak sector, and concrete conclusions will be available concerning the possibility of performing meaningful tests of the structure of the CKM matrix, or of possible extensions of the Standard Model, at LHC.

Technically speaking, this paper is organized as follows. In Section 2 the definition of the eight considered processes will be given, and the main tasks that were fulfilled to perform a complete one loop electroweak calculation will be indicated. Since the problems to be solved were practically identical with those already met in [5], our description will be whenever possible quick and essential. In Section 3 we shall define the considered observables and show the results of our calculation, particular emphasis being given to the value of the total $t$-channel cross section. Short comments, future plans and conclusions will be given in the final Section 4.

II. THE $t$-CHANNEL PROCESSES AT ONE ELECTROWEAK LOOP

The complete description of the $t$-channel involves at partonic level four sub-processes for single $t$ production: $ub \rightarrow td$, $\bar{d}b \rightarrow t\bar{u}$, $cb \rightarrow ts$, $\bar{s}b \rightarrow t\bar{c}$, and the related four for the single $\bar{t}$ production.

The starting point is the cross section for the $ub \rightarrow td$ process with the complete set of one loop electroweak corrections (in the MSSM and SM) as computed in [5], where the reader can find all the relevant formulae and the technical details.

From the results of [5], the one loop cross sections for all the eight partonic processes can be obtained in a straightforward way, by using a "crossing" prescription, and replacing the masses.

At Born level $ub \rightarrow td$ is described by a single diagram describing $W$ propagation in the $t$-channel and shown in the left part of Fig. 1. We define the Mandelstam variables for $ub \rightarrow td$: 

\[ s = (p_b + p_u)^2 \]
\[ t = (p_b - p_t)^2 = m_b^2 + m_t^2 - 2E_bE_t + 2pp' \cos \theta \]
\[ u = (p_b - p_d)^2 = m_b^2 + m_d^2 - 2E_bE_d - 2pp' \cos \theta \]  

(1)

where

\[ E_u = \frac{(s + m_u^2 - m_b^2)}{(2\sqrt{s})} \]
\[ E_b = \frac{(s + m_b^2 - m_u^2)}{(2\sqrt{s})} \]
\[ E_t = \frac{(s + m_t^2 - m_d^2)}{(2\sqrt{s})} \]
\[ E_d = \frac{(s + m_d^2 - m_t^2)}{(2\sqrt{s})} \]

\[ p^2 = \frac{1}{4s}(s - (m_u + m_b)^2)(s - (m_u - m_b)^2) \]
\[ (p')^2 = \frac{1}{4s}(s - (m_t + m_d)^2)(s - (m_t - m_d)^2) \]  

(2)

and also introduce \( q' = p_b - p_t = p_d - p_u \), so \( t = q'^2 \). The Born amplitude is

\[ A^{Born} = \frac{e^2}{2s_W^2(t - M_W^2)}[\bar{u}(t)\gamma^\mu P_L u(b)][\bar{u}(d)\gamma_\mu P_L u(u)] \]  

(3)

The "crossed" t-channel process \( \bar{d}b \to \bar{u}t \) is described by starting from \( ub \to dt \) and only exchanging the \( d \) and \( u \) lines (leaving the \( b \) and \( t \) lines unchanged), as shown in the right part of Fig. (1). The incoming \( u \) becomes an outgoing \( \bar{u} \), the outgoing \( d \) becomes an incoming \( \bar{d} \). The scattering angle \( \theta \) is still the angle between \( p_t \) and \( p_b \) (and now also between \( p_u \) and \( p_d \)). In terms of Mandelstam variables this is equivalent to exchange \( t \to t, s \to u, u \to s \).

The "crossing" rule can be applied to obtain the unpolarized parton cross section of \( \bar{d}b \to \bar{u}t \):

\[ \frac{d\sigma}{d\cos \theta}(\bar{d}b \to \bar{u}t) = \frac{p'_{cross}}{128\pi s p_{cross}} \sum_{\text{spins}} |F(\bar{d}b \to \bar{u}t)|^2 \]
\[ = \frac{p'_{cross}}{128\pi s p_{cross}} \sum_{\text{spans}} |F(ub \to dt)|^2 (s \to u, u \to s) \]  

(4)

where \( p'_{cross} \) and \( p_{cross} \) are obtained from the above \( p', p \) by \( m_u \leftrightarrow m_d \).

For the \( \bar{t} \) production the cross sections can be calculated using the identities
\[
\frac{d\sigma}{d\cos\theta}(ub \rightarrow td) = \frac{d\sigma}{d\cos\theta}(\bar{u}\bar{b} \rightarrow \bar{d}t)
\]

\[
\frac{d\sigma}{d\cos\theta}(\bar{d}b \rightarrow \bar{t}u) = \frac{d\sigma}{d\cos\theta}(d\bar{b} \rightarrow \bar{u}t) (5)
\]

and finally the processes involving the second generation \(c, s\) and \(\bar{s}, \bar{c}\) quarks can be computed from the previous, simply replacing the masses of the external particles (and some masses in the loop corrections).

### III. OBSERVABLE QUANTITIES AND THEORETICAL PREDICTIONS

The analysis of this paper will be concentrated on the investigation of the virtual electroweak effects on unpolarized cross sections. The final top polarization can in principle be measured \[6\] probably not in the first LHC running period, and we shall devote a forthcoming paper to the study of its main features. Therefore we shall start with the calculation of the one-loop effects on the inclusive differential cross sections of the various processes, generally defined as:

\[
\frac{d\sigma}{ds}(PP \rightarrow t(\bar{t})q' + X) = \frac{1}{S} \sum_q \int_{\cos\theta_{\min}}^{\cos\theta_{\max}} d\cos\theta \left[ L_{q\bar{b}}(\bar{t}, \cos\theta) \frac{d\sigma_{q\bar{b}}(\bar{b}) \rightarrow t(\bar{t})q'}{d\cos\theta} (s) \right] (6)
\]

where the initial quark \(q\) and the relative final \(q'\) are \(q = u, \bar{d}, c, \bar{s}, q' = d, \bar{u}, s, \bar{c}\), for \(t\) production, and \(q = \bar{u}, d, \bar{c}, s, q' = \bar{d}, u, \bar{s}, c\), for \(\bar{t}\) production. \(\tau = \frac{S}{s}\), and \(L_{q\bar{b}}\) is the parton process luminosity,

\[
L_{q\bar{b}}(\bar{t}, \cos\theta) = \int_{\bar{y}_{\min}(tq')}^{\bar{y}_{\max}(tq')} d\bar{y} \left[ b(x)q(\frac{\tau}{x}) + q(x)b(\frac{\tau}{x}) \right] (7)
\]

where \(S\) is the total pp c.m. energy, and \(\bar{t}(x)\) the distributions of the parton \(i\) inside the proton with a momentum fraction, \(x = \frac{\sqrt{S} \ e^{\bar{y}}}{S}\), related to the rapidity \(\bar{y}\) of the \(tq'\) system \[7\]. The parton distribution functions are the latest LO MRST (Martin, Roberts, Stirling, Thorne) set available on \[8\]. The limits of integrations for \(\bar{y}\) depends on the cuts. We have chosen a maximal rapidity \(Y = 2\) and a minimum \(p_T\) which we shall specify later.

Note that we are at this stage considering as kinematical observable the initial partons c.m. energy \(\sqrt{s}\), and not the realistic final state invariant mass \(M_{tq'}\).
To relate these two quantities is relatively straightforward, and we expect from a previous analysis done for the top-antitop final state \[17\] that the difference between them is relatively small. More specifically, we shall concentrate the discussion of this paper on the properties of integrated cross sections, the integration being performed from threshold to a final realistic energy, so that at this stage the distinction between c.m. energy and invariant mass loses relevance although it would be important at a stage where the invariant mass distribution were actually measured, which again we believe will start after the first LHC period. Our starting quantities will therefore be, following the previous discussion, the inclusive differential cross sections of the eight processes, defined in Eq. (6). We have computed them in the MSSM model with mSUGRA symmetry breaking scheme, using a C++ numerical code called LEONE available upon request. The one loop amplitudes evaluated inside LEONE are obtained by crossing symmetry and simple modifications of the ones computed in \[5\]. As a consequence, they are automatically UV finite and reproduce the known asymptotic logarithmic Sudakov expansion, as we checked in full details in \[5\]. Infrared finiteness is less trivial, but it has also been checked in the present calculation. To this aim, we have introduced as usual a regulating photon mass $\lambda$ checking that observables are finite as $\lambda \to 0$.

We have analyzed several SUSY benchmark points and, anticipating a general result, we have systematically found a modest genuinely supersymmetric (i.e. beyond the pure Standard Model) effect. For this reason we shall only show the results for that benchmark point where the small effect is maximum, corresponding to the ATLAS DC2 SU6 point \[9\]. In all the remaining considered points the effect is slightly smaller. In practice, it remains constantly of the few percent relative size. In Fig. (2) we have shown the various distributions for the eight different processes. One sees that the dominant cross sections correspond to the final $(t, d)$ and $(\bar{t}, u)$ pairs, as expected from the corresponding larger initial states parton distribution functions. One also notices a generally smooth shape in energy for all the computed distributions. In the next Figures (3,4) we have shown the total distributions for final top (3) and final antitop (4), obtained summing in each Figure the corresponding four terms of Fig. (2), and separating the Born from the one loop term, the latter having been computed both for the SM and for the MSSM. As anticipated, one sees that the difference between the one-loop SM and MSSM is rather small. To better appreciate it, we have also shown the relative one-loop electroweak effects. One notices that, while the relative genuine
SUSY effect (i.e. the difference between SM and MSSM) remains systematically of the few percent size (in particular, two percent at the representative c.m. energy of 1 TeV), the relative SM effect is impressively large. Around the 1 TeV energy, chosen for pure indicative reasons, it reaches the 30% size. More relevant for our next analysis, when the energy is decreased toward threshold, it remains always negative and sizable, reaching the 10% value for low energy values. These features are valid for both final top and final antitop processes, and we shall return to this point in the final discussion.

It is likely that in the first period of LHC running a more experimentally meaningful observable is, rather than the energy distribution, the integrated cross section. In this spirit, we have shown in Figures (5, 6) the result of the energy integration of the previous distributions, performing the integration from threshold to a variable final energy, to be fixed by experimental arguments. More precisely, Fig. (5) shows the integrated cross section for final top, while in Fig. (6) we have shown the sum of the two integrated cross sections for final top and final antitop, which might be simpler to analyze in the first LHC period. The main (and partially unexpected) result of our analysis is, in our opinion, the fact that the electroweak effect at one loop is large. This is true for the SM and also, although not as a consequence of sizable genuine SUSY effects, for the MSSM in the mSUGRA scheme that we have chosen. One sees that an integration up to the (reasonable) 1 TeV limit exhibits a SM effect of $\approx 12\%$, slightly reduced in the MSSM. It should be stressed that the available calculations of the corresponding one-loop QCD effects \[10,11,12\] produce definitely smaller results, of the few percent size. This fact will be commented in the final discussion. At the expected LHC accuracy, the electroweak effect that we have computed might and should be experimentally detectable (a more accurate discussion will be the aim of a forthcoming dedicated paper \[13\], and would allow LHC to perform a rather remarkable precision test of electroweak physics.

IV. CONCLUSIONS

As a result of our analysis, we have found that in the process of single top and single antitop production at LHC a one-loop electroweak effect of relatively large size on the distributions and on the integrated cross sections is present in the MSSM, mostly caused by the SM component. There are two facts that, we believe, deserve a final comment. The
first is the fact that, to our knowledge, the presence of a one loop electroweak effect that is larger than the corresponding QCD one in a LHC process only appears for the process of single top production in the t channel. For this fact we have our personal view, that we present here. The starting point is that the possibility of finding at one loop an electroweak effect larger than the corresponding QCD one was already noticed in a pioneer paper investigating electron-positron scattering at asymptotic LC energies. The conclusion was that, at very large energies, the electroweak effect in the hadronic cross section became larger than the QCD one. This was a consequence of the assumed asymptotic Sudakov electroweak expansion and of the presence in it of large double logarithms. This effect was, though, asymptotic. For smaller energies, it was usually masked by next non leading non logarithmic terms. Quite generally, the LHC processes that we have examined can be decomposed in a sum of different helicity amplitudes. In this set, a subset of them obeys an asymptotic Sudakov expansion when the energy is much larger than the one loop involved masses, but another subset is ”reluctant” to behave in this way, typically for the SM when a top quark with relatively large mass is involved. At asymptotic energies one finds in fact in the total energy distribution a Sudakov expansion, but at low energies this is overwhelmed by the different values of the ”reluctant” amplitudes. The case of the single top production in the \( t \) channel is, in this respect, so peculiar because in this process only one helicity amplitude appears. This obeys a Sudakov logarithmic expansion at large energies. When one decreases the energy, since there do not exist other ”reluctant” amplitudes, it is not surprising that the smooth logarithmic energy behavior, in the absence of energy peaks or thresholds, survives until threshold remaining not too below the large asymptotic Sudakov logarithmic limit, which in this case is larger than the QCD one. We cannot claim that this is a proof of what we observed, but it seems to us, at least, conceivable. The second point is the conclusion that the genuine SUSY effect is small. We insist on the fact that this conclusion was obtained in the considered mSUGRA symmetry breaking scheme. We cannot exclude that for different symmetry breaking mechanisms the genuine effect is more sizable. This question remains open and, in our opinion, it would deserve a special dedicated rigorous analysis.

[1] T. M. P. Tait, Phys. Rev. D 61, 034001 (2000) \texttt{arXiv:hep-ph/9909352}.
[2] J. Alwall et al., “Is $V_{tb} = 1?$,” arXiv:hep-ph/0607115.

[3] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky and W. K. Tung, “New generation of parton distributions with uncertainties from global QCD JHEP 0207, 012 (2002) arXiv:hep-ph/0201195.

[4] T. Stelzer, Z. Sullivan and S. Willenbrock, Phys. Rev. D 56, 5919 (1997) arXiv:hep-ph/9705398.

[5] M. Beccaria, G. Macorini, F. M. Renard and C. Verzegnassi, Phys. Rev. D 74, 013008 (2006) arXiv:hep-ph/0605108.

[6] W. Bernreuther, M. Fuecker and Z. G. Si, Spin effects in hadronic top quark pair production, Prepared for TOP 2006: International Workshop on Top Quark Physics, Coimbra, Portugal, 12-15 Jan 2006, PoS TOP2006, 018 (2006).

[7] see e.g. R.K. Ellis, W.J. Stirling and B.R. Webber, ”QCD and Colliders Physics”, Cambridge University Press, eds. T.Ericson and P.V. Landshoff (1996).

[8] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, “MRST partons and uncertainties”, contribution to XI International Workshop on Deep Inelastic Scattering, St. Petersburg, 23-27 April 2003, hep-ph/0307262. General information and updated numerical routines for parton distribution functions can be found on the www site http://durpdg.dur.ac.uk/hepdata/.

[9] ATLAS Data Challenge 2 DC2 points; http://paige.home.cern.ch/paige/fullsusy/romeindex.html

[10] B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan and S. Weinzierl, Phys. Rev. D66 (2002) 054024 arXiv:hep-ph/0207055. Z. Sullivan, Phys. Rev. D70 (2004) 114012 arXiv:hep-ph/0408049. Z. Sullivan, Phys. Rev. D72 (2005) 094034 arXiv:hep-ph/0510224. S. Zhu, Phys. Lett. B524 (2002) 283–288 [Erratum: ibid B537 (2002) 351].

[11] J. Campbell, R. K. Ellis and F. Tramontano, Phys. Rev. D70 (2004) 094012 arXiv:hep-ph/0408158. Q.-H. Cao and C. P. Yuan, Phys. Rev. D71 (2005) 054022 arXiv:hep-ph/0408180. Q.-H. Cao, R. Schwienhorst and C. P. Yuan, Phys. Rev. D71 (2005) 054023 arXiv:hep-ph/0409040. Q.-H. Cao, R. Schwienhorst, J. A. Benitez, R. Brock and C. P. Yuan, Phys. Rev. D72 (2005) 094027 arXiv:hep-ph/0504230. J. Campbell and F. Tramontano, Nucl. Phys. B726 (2005) 109–130 arXiv:hep-ph/0506289.

[12] S. Frixione, E. Laenen, P. Motylinski and B. R. Webber, JHEP 0603, 092 (2006) arXiv:hep-ph/0512250. S. Frixione, AIP Conf. Proc. 792, 685 (2005).
FIG. 1: Born direct and crossed processes for single top production in the $t$ channel with first generation light quark current.

[13] M. Beccaria, M. Cobal, G. Montagna, F. M. Renard, C. Verzegnassi, in preparation.
[14] P. Ciafaloni and D. Comelli, Phys. Lett. B 446, 278 (1999) [arXiv:hep-ph/9809321].
[15] M. Beccaria, G. Macorini, F. M. Renard and C. Verzegnassi, Phys. Rev. D 73, 093001 (2006) [arXiv:hep-ph/0601175].
[16] M. Beccaria, F. M. Renard and C. Verzegnassi, Phys. Rev. D 71, 033005 (2005) [arXiv:hep-ph/0410089].
[17] M. Beccaria, S. Bentvelsen, M. Cobal, F. M. Renard and C. Verzegnassi, Phys. Rev. D 71, 073003 (2005) [arXiv:hep-ph/0412249].
FIG. 2: Differential distribution $d\sigma/dE_{\text{CM}}$ for the 4+4 partonic processes of single $t$ or $\bar{t}$ quark production.
FIG. 3: Differential distribution $d\sigma/dE_{CM}$ for the total rate of production of single $t$ quark at Born level and with full electroweak radiative corrections in the SM and in the MSSM. The right panel shown the percentual radiative effect in the SM and MSSM.
FIG. 4: Differential distribution $d\sigma/dE_{CM}$ for the total rate of production of single $\bar{t}$ antiquark at Born level and with full electroweak radiative corrections in the SM and in the MSSM. The right panel shown the percentual radiative effect in the SM and MSSM.
FIG. 5: Integrated cross section from threshold up to the energy $E_{CM}$ for the total rate of production of single $t$ quark at Born level and with full electroweak radiative corrections in the SM and in the MSSM. The right panel shows the percentual radiative effect in the SM and MSSM.
FIG. 6: Integrated cross section from threshold up to the energy $E_{CM}$ for the total rate of production of single $t$ quark or $\bar{t}$ antiquark at Born level and with full electroweak radiative corrections in the SM and in the MSSM. The right panel shown the percentual radiative effect in the SM and MSSM.