Central Limit Theorem in Lebesgue-Riesz spaces
for weakly dependent random sequences

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Abstract

We deduce sufficient conditions for the Central Limit Theorem (CLT) in the Lebesgue-Riesz space $L(p)$ defined on some measure space for the sequence of centered random variables satisfying the strong mixing (Rosenblatt) condition.

We investigate the asymptotical as well as non-asymptotical approach.

Key words and phrases

Random variables and vectors (r.v.), distributions, sigma algebras (fields), probability, estimations, metrisable spaces, separability, Utev’s constant, compactness, normed sums, Central Limit Theorem (CLT), Lebesgue-Riesz space, (strong) mixing conditions and mixing (Rosenblatt) coefficient, weak convergence of distributions.

1 Statement of problem, definitions, previous results.

Let $(T = \{t\}, B, \mu)$ be measurable space with sigma-finite separable non-zero measure $\mu$. The separability implies that the sigma-field $B$ is separable metric space relative the distance function
\[ d(A_1, A_2) := \mu(A_1 \setminus A_2) + \mu(A_2 \setminus A_1). \]

Recall that the classical Lebesgue-Riesz space \( L(p) = L(p, T, \mu) = L(p, T) \) consists on all the measurable numerical valued functions \( f : T \to \mathbb{R} \) having a finite norm

\[
\|f\|_{p, T} = \|f\|_{L(p, T)} \overset{\text{def}}{=} \left[ \int_T |f(t)|^p \mu(dt) \right]^{1/p}, \quad 1 \leq p < \infty.
\]

It is a complete separable rearrangement invariant (r.i.) Banach functional space.

Let also \( (\Omega = \{\omega\}, F, P) \) be non-trivial probability space. The Lebesgue-Riesz norm of arbitrary numerical valued random variable (r.v.), measurable function \( \zeta : \Omega \to \mathbb{R} \) may be denoted alike

\[
\|\zeta\|_{q, \Omega} = \|\zeta\|_{L(q, \Omega)} = \left[ \int_{\Omega} |\zeta(\omega)|^q P(d\omega) \right]^{1/q} = [E|\zeta|^q]^{1/q}, \quad 1 \leq q < \infty.
\]

Let also \( \xi_i(t) = \xi_i(t, \omega), \quad t \in T, \quad \omega \in \Omega, \quad i = 1, 2, \ldots, n, \ldots \) be a sequence of complete (total) measurable numerical valued centered \( \mathbb{E} \xi_i(t) = 0, \quad t \in T \) separable random fields (r.f.) or equally random processes (r.p.), defined in addition on our probability space \( (\Omega = \{\omega\}, B, P) : \)

\[ \xi_i : T \otimes \Omega \to \mathbb{R}. \]

Define

\[ S_n(t) \overset{\text{def}}{=} n^{-1/2} \sum_{i=1}^{n} \xi_i(t), \quad (1) \]

\[ R_n(t_1, t_2) := \text{Cov}(S_n(t_1), S_n(t_2)) = n^{-1} \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{E} \xi_i(t_1) \xi_j(t_2), \]

\[ R(t_1, t_2) = R_c(t_1, t_2) := \lim_{n \to \infty} R_n(t_1, t_2), \]

if there exists (and is finite) for all the values \( (t_1, t_2), \quad t_1, t_2 \in T. \)

Define the following r.f. \( S(t) = S(t, \omega) = S_c(t, \omega) \) as a Gaussian distributed separable centered r.f. with covariation function

\[ \mathbb{E} S(t_1) S(t_2) = \text{Cov}(S(t_1), S(t_2)) = R(t_1, t_2) = R_c(t_1, t_2). \quad (2) \]

**Definition 1.1.** We will say as ordinary that the sequence of the r.f. \( \{\xi_i(t)\} \) satisfies the Central Limit Theorem (CLT) in the space \( L(p, T) = L(p, T, \mu) \), iff all the r.f. \( \{\xi_i(t)\} \) are (bi - ) measurable, belongs to the space \( L(p, T) \) with probability one:
\[ P \left( \| \xi_i(\cdot) \|_{L(p, T)} < \infty \right) = 1 \]

as well as the limited Gaussian r.f. \( S(\cdot) \), and the sequence of normed r.f. \( S_n(\cdot) \) converges in distribution in this space \( L(p, T) \) as \( n \to \infty \) to the r.f. \( S(\cdot) \) for every bounded continuous numerical valued functional \( G : L(p, T) \to \mathbb{R} \)

\[ \lim_{n \to \infty} E G(S_n) = E G(S). \]

See the classical work of Yu.V.Prokhorov [37].

As a consequence:

\[ \lim_{n \to \infty} P \left( \| S_n(\cdot) \|_{L(p, T)} > u \right) = P \left( \| S(\cdot) \|_{L(p, T)} > u \right), \quad u > 0. \quad (3) \]

The asymptotic as well as non-asymptotic behavior for the right-hand side of the relation (3) as \( u \to \infty \) is investigated in particular in [8], [36].

The CLT in different Banach spaces, or more generally, in linear topological ones, is devoted the extensive literature, see e.g. [1], [2], [7], [10], [11], [12], [17], [22], [23], [24], [25], [26], [27], [28], [37], [38], [41], [44]. As a rule, in these works was investigated the case when the r.f. \( \{ \xi_i(t) \} \) are common independent; the case when these fields are weakly dependent makes up the content of the articles [11].

The case of CLT in Lebesgue-Riesz spaces for weakly dependent variables under super strong mixing condition is considered in [12], [26].

The applications of the CLT in linear spaces in the statistics are described for instance, in [28], [37], [38]; in the theory of Monte-Carlo method-in [13], [14].

We intend to generalize in this report the foregoing results [12], [26] relative the CLT in the space \( L(2, T) \) into more general case of the space \( L(p, T) \) when the source random sequence \( \{ \xi_i(t) \} \) forms relative the index \( i \) a sequence satisfying aside from certain moment assumptions the strong mixing condition.

Recall some used definitions. Let \( B_1, B_2 \) be two sigma - subfields of source field \( B \). The so-called mixing coefficient or equally Rosenblatt coefficient \( \alpha(B_1, B_2) \) between \( B_1 \) and \( B_2 \) is defined as ordinary

\[ \alpha(B_1, B_2) \overset{\text{def}}{=} \sup_{A_1 \in B_1, \ A_2 \in B_2} |P(A_1 \cap A_2) - P(A_1)P(A_2)|, \]

see [40].

Further, define a following family of sigma-algebras (fields)

\[ M^b_a \overset{\text{def}}{=} \sigma \{ \xi_i(t) \}, \quad a \leq i \leq b, \quad t \in T, \]

\[ \alpha(i) \overset{\text{def}}{=} \sup_{n} \max_{k \in [1, n]} \alpha(M^k_1, M^k_{i+k}). \quad (4) \]
By definition, the sequence of r.f. \( \{ \xi_i(t) \} \), \( i = 1, 2, \ldots \) satisfies a strong mixing condition, or equally \( \alpha \) mixing condition, iff \( \lim_{i \to \infty} \alpha(i) = 0 \).

Many examples of random processes and sequences obeys the strong mixing condition may be found in the articles [6], [9].

There are many works devoted to the one-dimensional CLT for such a sequences, including the invariance principle, see e.g. [15], [16], [18], [35], [39], [40], [42], [43] etc.

The non-asymptotical estimation for sums of weakly dependent r.v. is represented in particular in [11], [12], [42]. We will apply the estimate belonging to S.A.Utev [43]. Let \( \{ X_i \}, i = 1, 2, \ldots, n \) be a sequence of centered one-dimensional r.v. satisfying the mixing condition with correspondent coefficient \( \alpha(i) \). Then when \( s = 2, 4, 6, \ldots \) and for every real value \( v \geq s \)

\[
\mathbf{E} \left| \sum_{i=1}^{n} X_i \right|^{s} \leq a_s \left( \sum_{r=0}^{n-1} \alpha^{1-s/v}(r) (r+1)^{s/2-1} \right) \left( \sum_{i=1}^{n} \mathbf{E}^{2/v} |X_i|^v \right)^{s/2},
\]

where

\[
a_s = 12 \left( 1 + 2s/3 \right) (s - 1) 3^s \frac{[s!]^2}{[(s/2)!]^2}.
\]

It is no hard to evaluate

\[
a_s^{1/s} \leq K_U \cdot s, \quad K_U := 2^{-5/12} \cdot 3^{1/2} \cdot e^{2/e-23/24} \approx 4.760327\ldots;
\]

Let us denote

\[
Z(\alpha)(s, v) := \left\{ a_s \left( \sum_{r=0}^{\infty} \alpha^{1-s/v}(r) (r+1)^{s/2-1} \right) \right\}^{1/s},
\]

then if for some value \( v > s \Rightarrow Z(\alpha)(s, v) < \infty \) and \( X_i \in L(v, \Omega) \), we have under our conditions

\[
\left\| \sum_{i=1}^{n} X_i \right\|_{s, \Omega} \leq Z(\alpha)(s, v) \cdot \sqrt{\sum_{i=1}^{n} \left\| X_i \right\|_{v}^2}.
\]

Define the variable

\[
Y = Y(\{ X_i \})(v) := \sup_i \left\| X_i \right\|_{v, \Omega},
\]

then

\[
\sup_n \left\| \sum_{i=1}^{n} X_i \right\|_{s, \Omega} \leq Z(\alpha)(s, v) Y(\{ X_i \})(v),
\]

if of course the right-hand side is finite at last for one value \( v; v > s \).
If in particular all the centered variables \( \{X_i\} \) are in additional identical distributed, in particular, forms the strictly stationary centered sequence, we conclude denoting \( X = X_1 \):

\[
\sup_n \left\| n^{-1/2} \sum_{i=1}^n X_i \right\|_{s, \Omega} \leq Z[\alpha](s, v) \cdot \left\| X \right\|_{v, \Omega}, \quad v > s. \tag{11}
\]

Of course,

\[
\sup_n \left\| n^{-1/2} \sum_{i=1}^n X_i \right\|_{s, \Omega} \leq \inf_{v>s} \left\{ Z[\alpha](s, v) \cdot \left\| X \right\|_{v, \Omega} \right\}, \quad v > s. \tag{12}
\]

## 2 Moment estimates for the Lebesgue-Riesz norm of the sums of weakly dependent random fields.

Let as before \( \xi = \{\xi_i(t)\}, \quad t \in T, \quad i = 1, 2, \ldots \) be a sequence of centered random fields, \( t \in T \), satisfying the strong mixing condition relative the index \( i \) with correspondent Rosenblatt coefficient (more exactly, the sequence of coefficients) \( \alpha = \{\alpha(i)\} \).

We get using the estimate (10)

\[
\sup_n \left\| S_n(t) \right\|_{s, \Omega} \leq Z[\alpha](s, v) \sup_i \left\| \xi_i(t) \right\|_{v, \Omega}, \quad t \in T,
\]

or equally

\[
\sup_n \mathbb{E} \left| S_n(t) \right|^s \leq Z^s[\alpha](s, v) \left[ \sup_i \left\| \xi_i(t) \right\|_{v, \Omega} \right]^s = Z^s[\alpha](s, v) \left[ \sup_i \mathbb{E}^{1/v} |\xi_i(t)|^v \right]^s, \quad 1 \leq s < v.
\]

**Proposition 2.1.**

We deduce further after integration over the set \( T \) relative the measure \( \mu \) by virtue of theorem Fubini-Tonelli and Lyapunov-Hölder’s inequality

\[
\sup_n \mathbb{E} \left\| S_n \right\|_{s, T}^s \leq Z^s[\alpha](s, v) \int_T \left\{ \sup_i \left\{ \mathbb{E} |\xi_i(t)|^v \right\} \mu(dt) \right\}^{s/v}, \quad 1 \leq s < v. \tag{13}
\]

or equally

\[
\sup_n \mathbb{E} \left\| S_n \right\|_{s, T}^s \leq Z^s[\alpha](s, v) \sup_i \mathbb{E}^{s/v} |\xi_i(t)|^v_{\mu, T}, \quad 1 \leq s < v. \tag{14}
\]
Of course, the last estimate is reasonable if for instance the right-hand of (14) is finite.

3 Main result: Central Limit Theorem for the sums of weakly dependent random fields in the Lebesgue-Riesz space.

**Theorem 3.1.** Suppose that there exists a value $v$, $v > s$, for which $Z_\alpha(s, v) < \infty$ and $\sup_i \{ \int_T E|\xi_i(t)|^v \mu(dt) \} < \infty$. Suppose also that the sequence of r.v. $\{\xi_i(\cdot)\}$ converges weakly (in distribution) as $i \to \infty$ in the space $L(s, \mu, T)$, for instance, if all the r.v. $\{\xi_i(\cdot)\}$ are identical distributed.

Then this sequence $\{\xi_i(t)\}$ satisfies CLT in the Lebesgue-Riesz space $L(s, \mu, T)$.

**Proof.** The convergence as $n \to \infty$ of the characteristic functionals

$$\phi_n(x) := E \exp \left( i \int_T \xi(t) x(t) \mu(dt) \right)$$

for arbitrary fixed non-random element of conjugate space $x(\cdot) \in L((s-1), \mu, T)$ to the suitable for the limiting Gaussian ones follows immediately from the one-dimensional limit theorems for strong random variables (Cramer’s method), see e.g. [35], [39], [40]. The mentioned above one-dimensional r.v. haves the form

$$\eta_i[x] = \int_T \xi_i(t) x(t) \mu(dt).$$

It remains to ground the weak compactness of distributions of the r.v. $S_n(\cdot)$ in the space $L(s, T)$. As long as the Banach space $L(v, T, \mu)$ is separable, and since the function $y \to |y|^v$ satisfies the $\Delta_2$ condition, there exists a compact linear operator $U : L(v, T) \to L(v, T)$ such that the r.v. $U^{-1}\xi_i(\cdot) \in L(v, T)$ and moreover

$$\sup \{ \int_T E|U^{-1}\xi_i(t)|^v \mu(dt) \} < \infty,$$

see [33] and also [4], [31], [32]. One can apply Proposition 2.1 for the sequence of r.f. $U^{-1}\xi_i(t)$:

$$\sup_n E\|U^{-1} S_n\|_{s, T}^s \leq W_\alpha(s, v),$$

(15)

where
\[ W[\alpha](s, v) \overset{\text{def}}{=} Z^s[\alpha](s, v) \sup_i \left\{ \int_T E|U^{-1} \xi_i(t)|^v \mu(dt) \right\} < \infty, \ 1 \leq s < v. \quad (16) \]

We conclude on the basis of Tchebychev’s inequality for any value \( \epsilon \in (0, 1) \)
\[
\sup_n P \left( \||U^{-1}S_n||L(s, T) > Y \right) \leq \frac{W}{Y^s} \leq \epsilon
\]
for sufficiently greatest positive value \( Y \).

Therefore, the sequence of distributions of the r.v. \( S_n(\cdot) \) is weak compact in the space \( L(s, T) \).
Q.E.D.

**Remark 3.1.** We have investigated above only the case when the number \( s \) is even: \( s = 2, 4, 6, \ldots \). Let now the number \( s \) be arbitrary greatest than 2: \( s > 2 \). Suppose in addition that the set \( T \) has a finite measure relative the measure \( \mu : \mu(T) = 1 \). Define the number \( \bar{s} \) as a minimal even number greatest than \( s \):
\[
\bar{s} \overset{\text{def}}{=} \min\{ 2k, \ k = 1, 2, \ldots ; 2k \geq s \}.
\]

As long as by virtue of Lyapunov’s inequality \( \||f||L(s, T, \mu) \leq ||f|| L(\bar{s}, T, \mu) \), we conclude that if the conditions of Theorem 3.1 are satisfied for the value \( \bar{s} \) instead \( s \), the CLT in the space \( L(s, T, \mu) \) holds true.

## 4 The case of superstrong mixing condition.

Recall that the so-called superstrong mixing coefficient between two sigma-algebras \( B_1, B_2 \subset B \) is defined by the formula
\[
\beta(B_1, B_2) \overset{\text{def}}{=} \sup_{P(A_1 \in B_1), P(A_2 \in B_2) > 0} \left| \frac{P(A_1 \cap A_2) - P(A_1)P(A_2)}{P(A_1)P(A_2)} \right|. \quad (18)
\]

Define a following family of sigma-algebras (fields)
\[
M^n_a \overset{\text{def}}{=} \sigma\{ \xi_i(t) \}, \ a \leq i \leq b, \ t \in T,
\]
and consequently
\[
\beta(i) \overset{\text{def}}{=} \sup_n \max_{k \in [1,n]} \beta(M^n_k, M^n_{k+i}). \quad (19)
\]

By definition, the sequence of r.f. \( \{\xi_i(t)\}, \ i = 1, 2, \ldots \) satisfies a super strong mixing condition, or equally \( \beta \) mixing condition, iff \( \lim_{i \to \infty} \alpha(i) = 0 \). This notion belongs to Nachapetyan B.S. [29]; see also [3].
B.S. Nachapetyan in [29] proved that for the superstrong centered random sequence \( \{X_i\}, i = 1, 2, \ldots \)

\[
\sup_n \|n^{-1/2} \sum_{i=1}^n X_i\|_{L(s, \Omega)} \leq K_N[\beta](s) \sup_i \|X_i\|_{L(s, \Omega)}, \ s \in [2, \infty),
\]

where

\[
K_N[\beta](s) := 2s \left[ \sum_{k=1}^\infty \beta(k) (k+1)^{(s-2)/2} \right]^{1/s}.
\]

**Theorem 4.1.** Suppose that the source sequence of centered random fields \( \{\xi_i(t)\} \) satisfies the superstrong mixing condition relative the index \( i \),

\[
\sup_i \|\xi_i(t)\|_{L(s, \Omega)} \in L(s, T, \mu), \ s \geq 2,
\]

and

\[
K_N[\beta](s) < \infty.
\]

Suppose also that the sequence of r.v. \( \{\xi_i(\cdot)\} \) converges weakly (in distribution) as \( i \to \infty \) in the space \( L(s, \mu, T) \), for instance, if all the r.v. \( \{\xi_i(\cdot)\} \) are identical distributed.

Then this sequence \( \{\xi_i(t)\} \) satisfies CLT in the Lebesgue-Riesz space \( L(s, \mu, T) \).

**Proof** is much easier than one in theorem 3.1 and may be omitted.

5 Concluding remarks.

A. It is interest by our opinion to extend obtained in this report results into the the so-called anisotropic (Mixed) Lebesgue-Riesz spaces \( L(p_1, p_2, \ldots, p_d; T_1, T_2, \ldots, T_d) \), as well as to extend ones into the Sobolev’s spaces.

The independent case, i.e.when \( \alpha(r) = 0, \ r = 1, 2, \ldots \) is investigated in [34].

B. Perhaps, more general results in this direction may be obtained by means of the so-called method of majorizing measures, see e.g. [25], [41].

C. It is interest also by our opinion to deduce the non-asymptotical exponential decreasing as \( y \to \infty \) estimations for the tail of distribution for the normed sums

\[
Q(y) := \sup_n P(||S_n(\cdot)||_{L(s, T, \mu)} > y), \ y \geq 1.
\]
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