Particle acceleration and radiation friction effects in the filamentation instability of pair plasmas

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ABSTRACT

The evolution of the filamentation instability produced by two counter-streaming pair plasmas is studied with particle-in-cell (PIC) simulations in both one (1D) and two (2D) spatial dimensions. Radiation friction effects on particles are taken into account. After an exponential growth of both the magnetic field and the current density, a nonlinear quasi-stationary phase sets up characterized by filaments of opposite currents. During the nonlinear stage, a strong broadening of the particle energy spectrum occurs accompanied by the formation of a peak at twice their initial energy. A simple theory of the peak formation is presented. The presence of radiative losses does not change the dynamics of the instability but affects the structure of the particle spectra.

Key words: pair plasmas – filamentation instability – radiation friction.

1 INTRODUCTION

From the 1970s on, the long-standing problem of high-energy cosmic ray origin has involved beam-plasma instabilities in order to explain some aspects of the acceleration mechanism (see Blandford & Ostriker (1978); Bell (1978a,b) or Blasi (2013) for a more recent review). In particular the excitation of unstable modes by the accelerated particles propagating into the interstellar medium has been studied as a possible mechanism to generate strong magnetic turbulence predicted by the non-linear diffusive shock acceleration theory (see the reviews by Malkov & Drury (2001) and Blandford & Eichler (1987)). The study of a model problem characterized by two colliding electron-positron plasma clouds at relativistic energies can be relevant to various astrophysical scenarios including the fireball model of Gamma Ray Bursts (Piran 2005), pulsar wind outflows in Pulsar Wind Nebulae (Blasi & Amato 2011), and relativistic jets from Active Galactic Nuclei (Begelman et al. 1984). In this paper we examine the unstable modes generated by two counter-streaming neutral beams of pair plasmas in the ultra-relativistic regime. In particular, we address kinetic effects, such as particle acceleration, taking place during the nonlinear phase of the instability, and we take radiation friction (RF) effects into account. It is worth noticing that there is a current interest in kinetic simulations of pair plasmas with RF included in astrophysics, e.g. for the study of anomalous particle acceleration leading to flaring in the Crab Nebula (Jaroschek & Hoshino 2009; Cerutti et al. 2013). The problem of RF inclusion in the kinetic modeling of a relativistic plasma in high electromagnetic fields is also crucial in the context of ultraintense laser interaction with matter and plasma (Di Piazza et al. 2012 and references therein). It is therefore of interest to revisit classic plasma instabilities in highly relativistic regimes possibly dominated by radiation.

The system composed by two charge-neutral, counter-streaming pair plasmas is subject to a host of instabilities which depend on the orientation of the wavevector with respect to the direction of the beams (for a general review see Bret et al. (2010)). The unstable spectrum includes two limiting cases: the two stream instability, corresponding to an electrostatic mode with flow-aligned wavevector, and the filamentation instability (FI), corresponding to an electromagnetic mode with wavevector perpendicular to the beam direction. From analytical calculations based on first-order perturbation theory (Kazimura et al. (1998); Bret et al. (2004)), the growth rate $\Gamma$ in the linear phase for the two-stream instability $\Gamma \propto \gamma_0^{-3/2}$, while for the FI $\Gamma \propto \gamma_0^{-1/2}$, where $\gamma_0 = (1 + (p_0/m_e c)^2)^{1/2}$ is the initial beam Lorentz factor (with $p_0$ the initial drift momentum). This means that the instability which dominates the dynamics of the system in the ultra-relativistic regime is the FI, to which we restrict ourselves in this paper.

We use an electromagnetic, fully relativistic particle-in-cell
(PIC) code to perform simulations in one (1D) and two (2D) spatial dimensions. In both cases, the transition from the linear to the nonlinear phase is characterized by the coalescence of the current filaments. Subsequently, magnetic field and current filaments reach a quasi-stationary regime, with typical scales of several skin depths. Most particles are magnetically confined inside the current filaments. A group of particles are accelerated at twice their initial momentum, forming a peak in the energy spectrum of particles. A simple model for the formation of the spectral peak is presented.

In the simulations performed including the RF we observe a “cooling” effect in the high energy tail of the distribution function of each species without a substantial modification in the dynamics of instability and in the temporal evolution of fields.

2 SIMULATION SET-UP

We performed numerical simulations with an electromagnetic fully-relativistic PIC code in 1D and 2D. The initial configuration consists of two neutral beams of electron-positron pairs which propagate in opposite directions (corresponding to $p_e$ in the momentum space) and fill the entire simulation box. The system is symmetric, with the populations of the two beams having the same initial density, i.e. $n_{e1}^{(0)} = n_{e2}^{(0)} = n_{p1}^{(0)} = n_{p2}^{(0)} = 1/4n_T$, where $n_T$ is the total density, and the same momentum absolute value, i.e. $p_{e1}^{(0)} = p_{e2}^{(0)} = p_{p1}^{(0)} = p_{p2}^{(0)} = p_0$, and consistently the initial values of charge and current densities and of the electric and magnetic fields are zero. A very small temperature is introduced to seed the instability.

We performed simulations with different Lorentz factors $\gamma_0$ from 1 to $10^3$. Here we describe the case with $p_0/m_e c = 200$ as it is representative of the most relevant effects observed. For different values of $p_0$, there are no qualitative changes in the dynamics of the instability, the most important difference being the growth rate of the modes which scales as $\gamma_0^{-1/2}$ (see e.g. Kazimura et al. (1998), Bret et al. (2004)).

In the 1D case, the simulation box is aligned along the x-direction and it is divided into 15000 grid cells of equal length $\Delta x = 0.01 \lambda_p$ with $\lambda_p = c/\omega_p$ the skin depth and $\omega_p = (4\pi^2 n_T m_e)^{1/2}$. Each of the four plasma species is represented by $3 \times 10^6$ computational particles (200 particles per cell). The total simulation time is $t_{sim} = 1000 T_p$, where $T_p = 2\pi/\omega_p$, and the temporal resolution is $\Delta t = 0.01 T_p$.

In the 2D case, the simulation box is divided into $2000 \times 2000$ square cells. The grid dimensions are $L_{x,y} = 100 \lambda_p \times 100 \lambda_p$, and $\Delta x = \Delta y = 0.05 \lambda_p$. Each of the four species is represented by $2 \times 10^6$ computational particles. The total simulation time is $t_{sim} = 200 T_p$ with $\Delta t = 0.0325 T_p$. In both 1D and 2D cases we used periodic boundary conditions.

The inclusion of RF in the code is based on the Landau-Lifshitz approach (Landau & Lifshitz 1975). The numerical implementation in the PIC code is based on the algorithm presented in Tamburini et al. (2010). The Compton drag force is neglected.

In the case without RF inclusion, the equations of the PIC code are in dimensionless form with the density normalized to $n_T$, time to $1/\omega_p$, space to $c/\omega_p$, and fields to $m_e c \omega_p / e$. Thus, all the results of a simulation can be scaled with respect to a definite value for the density. In order to include RF it is necessary to set a dimensional value for the plasma density. We have performed simulations with RF included for density values up to $n_p = 10^{19} \text{cm}^{-3}$. For such values, it can be safely assumed that the plasma is optically thin to the high-frequency radiation (having a typical energy $\gtrsim 1$ MeV) which mostly contribute to radiation losses. In addition, the mean free path for Compton scattering of photons is $\gtrsim 1 \text{m}$ (estimated from the Klein-Nishina formula), typically much larger than the scale length on which the instability sets up (of the order of $l_p = c/\omega_p \sim \mu\text{m}$).

3 FLUID EQUATIONS

In order to facilitate the understanding of the simulation results of sections 4 and 5, it is worth to introduce fluid equations for the system, with emphasis on their symmetry properties. For simplicity we restrict ourselves to the 1D case.

Describing the colliding pair plasmas as a four fluids plasma (corresponding to electron and positrons with positive initial momentum, and electron and positrons with negative initial momentum) the system of equations has 16 variables: 4 number densities, 4 × 3 momenta (we assume cold fluids which means no pressure terms). We use the following notation: electron and positron variables with $p_0 > 0$ are indicated respectively with $f_\pm$ and $f_\pm^\prime$ (where $f$ is a generic fluid variable) while electrons and positrons with $p_0 < 0$ respectively with $f_\mp$ and $f_\mp^\prime$. For the EM field components, we take only transverse fields into account assuming that no charge separation occurs, thus $E_z = 0$ (this assumption follows from the difficulty of creating a charge separation in pair plasmas and it is confirmed in our simulations). The relevant field components $E_x$ and $B_y$ can be related to a vector potential $A_x$, as $E_x = -\partial_t A_y / c$ and $B_y = -\partial_z A_x$.

The fluid equations can be thus written as

$$\partial_t n_+^{\pm} + \partial_x (n_+^{\pm} u_+^{\pm}) = 0, \quad (1)$$

$$D_{t,\pm}^{\pm} \left( \gamma_e u_+^{\pm} \right) = \pm (e/m_e) n_+^{\pm} \partial_t A_x, \quad (2)$$

$$D_{t,\pm}^{\pm} \left( \gamma_e u_+^{\pm} \right) = \pm (e/m_e) D_{t,\pm}^{\pm} A_z, \quad (3)$$

where $D_{t,\pm}^{\pm} \equiv (\partial_t + u_+^{\pm} \partial_x)$. The vector potential satisfies the wave equation

$$\partial^2 + c^{-2} \partial_z^2 A_z = -4\pi J_z / c, \quad (4)$$

where

$$J_z = + e \left( n_+^{\pm} u_{z,\pm}^{\pm} + n_-^{\pm} u_{z,\mp}^{\pm} \right)$$

$$- e \left( n_-^{\pm} u_{z,\pm}^{\pm} + n_+^{\pm} u_{z,\mp}^{\pm} \right). \quad (5)$$

The system is symmetric under the transformation that reverses at the same time the charge and the direction of propagation of the populations, which simplifies the description of the dynamics. It is also possible to reduce the initial set of equations to a system involving only two populations and three pairs of dynamical variables, as done in Kazimura et al. (1998). The two populations are the sources of the positive ($J_+^\prime$) and negative ($J_-^\prime$) density current and are identified with + and - symbols. The two-fluid variables are defined as...
follows: \( n^+ = n^+_{e+}, n^- = n^+_{e-}, u^+ = u^+_{e+}, u^- = u^+_{e-}, u^+ = u^+_{x}, u^- = u^+_{x}, J^+ = J^+_{x}, J^- = J^+_{x} \) and \( J^+ = \varepsilon (n^+_{e+} u^+_{e+} - n^-_{e-} u^-_{e-}) \) and \( J^- = \varepsilon (n^+_{e-} u^+_{e-} - n^-_{e+} u^-_{e+}) \). The two-fluid system of equations which is obtained from Eqs. (6) is

\[
\partial_t n^\pm + \partial_x (n^\pm u^\pm_x) = 0, \quad (6)
\]

\[
\partial_t (n^\pm u^\pm_x) + \partial_x [n^\pm (u^\pm_x)^2] = \frac{1}{m_e c} (J^\pm_x \partial_x A_x), \quad (7)
\]

\[
\partial_t (\gamma J^\pm_x) + \partial_x (\gamma u^\pm_x J^\pm_x) = -\frac{e^2}{m_e c} n^\pm (\partial_t + u^\pm_x \partial_x) A_x. \quad (8)
\]

## 4 ONE-DIMENSIONAL RESULTS

We first study the FI in 1D and we compare the simulation results with the analytical model. Indeed, in 1D configuration, symmetry properties and conservation laws allow to obtain some analytical estimates. The 1D study also makes it possible to check in detail the importance of kinetic effects in the FI, especially related to generation of high energy particles, and to test the implementation and the effects of the RF force. We have also performed several tests changing the number of particles per cell and the spatial resolution in order to check the sensitivity of the results. These tests guided the choice of the numerical parameters of more expensive multidimensional runs.

At the beginning of the simulation, microfilaments of opposite current start to develop from the thermal noise of the plasma. Each of these fluctuations produces a magnetic field perturbation, thanks to the tendency of a micro-filament to shrink and to the repelling interaction between two opposite fluctuations. This magnetic perturbation acts on the fluctuations enhancing the current inhomogeneities. This positive feedback produces an exponential growth of all the quantities, characterizing the linear phase of the instability. The structure of the current density \( J_x \) as a function of \((x, t)\) is shown in Fig. 1 (a). The development of the instability can be divided into three phases: a linear phase for \( t < 50 T_p \), a transition phase for \( 50 T_p < t < 200 T_p \) and a nonlinear quasi-stationary phase for \( t > 200 T_p \).

In the linear phase, modes with a defined wavevector grow exponentially, as we verified by calculating the spatial Fourier transform of \( B_y \), i.e. \( \mathcal{F}[B_y] = B_y(k_x, t) \). The numerically obtained growth rate for every mode \( k_x \) agrees well with analytic calculations (Kazimura et al. 1998).

In Fig. 1 (b) a zoom on the structure of \( J_x \) during the linear and the transition phase is shown. During the exponential growth of the perturbations, \( J_x \) assumes a filamentary structure with a very small scale length (\( \ll \lambda_y \)). At \( t \approx 50 T_p \) separate filaments of opposite current, having a typical scale close to the electron skin depth, become distinguishable and start to merge. This coalescence characterizes the transition of the instability from the linear to the nonlinear quasi-stationary regime. For \( t > 200 T_p \), the merging phase of the filaments finishes and the size of each filament is constant, so that the configuration can be described as stationary except for some “vibration” which is observable in Fig. 1 (a).

To understand in more detail the nonlinear regime we consider the spatial profile of \( J_z \), \( B_y \) and \( n \) (total number density), at \( t = 700 T_p \), reported in Fig. 2. A filament with positive or negative current is identified by two consecutive maxima or minima, respectively. Within each positive filament, the current density assumes its maximum value near the edges. Moving towards the inner region of the filament \( J_z \) decreases assuming a local minimum at the center, whereas the total number density becomes flat-top. The same happens for negative current filaments. This feature corresponds to an anti-correlation between particle density and velocity, which will be further discussed below by looking at phase space distributions. An oscillatory pattern characterizes also the profile of the magnetic field \( B_y \), which has null points at the center of each filament, as it is shown in Fig. 2.

The spatial structures of \( J_x \), \( B_y \) and \( n \) indicate an accumu-
magnetic field is not sufficient to strongly deflect their trajectories, particles can cross field lines, because the amplitude of the energy density is of the order of the initial magnetic energy is of the form of the position with initial positive momentum at 

\[ \text{energy density:} \quad \epsilon \propto \frac{1}{x} \quad \text{for} \quad x \gg \lambda_p \]

The asymptotic state of the system may thus be described as a state in which the particles cluster into the minima of the effective potential.

### 4.1 Energy spectra and particle acceleration

Fig. 4(a) shows the kinetic energy spectrum of the \( f^+ \) population for different times. As expected from symmetry of the system, the spectrum is essentially identical for the other three populations. At \( t = 0 \) the spectrum is a delta-like function centered at the initial value of energy \( K/m_e c^2 = \gamma_0 - 1 \). As time goes on, the spectrum broadens and most of the particles lose part of their initial kinetic energy. This dynamics is also evident in the \( dN/dp_z \) spectra reported in Fig. 4(b) and Fig. 4(c).

A noticeable feature of the energy spectrum is the presence of a peak at twice the initial kinetic energy, see Fig. 4(a). Correspondingly, we observe a sharp cutoff at twice the initial drift momentum in the spectrum of \( p_z \), in Fig. 4(b) and Fig. 4(c).

For each of the two populations \( (f^+ \) and \( f^-) \) in the two-fluid description of the system, the high-energy particles having \( p_z = \pm 2p_0 \) are localized outside the filaments where most of the particles belonging to the other population are localized, as shown in Fig. 3(a) and Fig. 3(b). This feature gives a hint on how to explain the mechanism of acceleration which doubles the initial value of \( p_z \). In a given position \( x \) where the field \( E_z = E_z(x, t) \) acts on a species in such a way to reduce its initial momentum \( p_0 \), it necessarily acts on the counter-streaming species increasing its initial momentum.
Due to the symmetry of the system, the vector potential \(a(x)\) assumes the same values for its maxima and minima as a function of \(x\), so there will be another point \(x_0\) such that \(a(x_0) = p_0\). Substituting this value of \(a(x_0)\) in the eq. (10), we obtain

\[ |p_x^+| = 2p_0. \]  

(15)

This procedure can be inverted, starting from \(x\) and then going to \(x_0\), obtaining \(C_- = 1 + 4p_0^2\). Thus, we have demonstrated that in a one-dimensional geometry the energy of the particles reaches a maximum

\[ E_{\text{max}} = m_e c^2 (1 + 4 (p_0/m_e c)^2)^{1/2}. \]  

(16)

The formation of a peak in the energy spectrum (Fig. 4 (a)) around \(E_{\text{max}}\) and the very sharp cutoff in the \(dN/dp_z\) spectrum (Fig. 4 (b) and (c)) around \(\pm 2p_0\) are due to the formation of a caustic analogous to that at the base of the sharpness and intensity of a rainbow: the internally reflected rays cross and cluster to form a caustic sheet (see [Born et al. 1999] chap. 3, pag. 127), where classically the intensity is infinite. We can apply this principle to explain the peaks in the spectrum. These particles find themselves in positions where \(|a_z|\) is maximum. Because of the conservation of the canonical momentum, in these positions we have \(dp_z/dx = \pm da_z/dx = 0\). Thus, these particles all gain the same momentum to first order in their distance from the maximum of \(|a_z|\).

### 5 TWO-DIMENSIONAL RESULTS

We now describe the results of 2D simulations, in which the simulation plane \((x, y)\) is perpendicular to the direction of the beams.

For these simulations we have used PICCANTE [Sgattoni et al. 2014], an open-source, massively parallel particle in cell code.

Fig. 5 shows the 2D distributions of current density ((a) panel), magnetic field energy density ((b) panel) and number density of one species ((c) panel) at two different times \(t = 75 T_p\) (left panels) and \(t = 200 T_p\) (right panels). As a general trend, we observe the formation of isolated structures with a scale length of several electron skin depths at the earliest time \((t = 75 T_p)\). Later, these structures eventually merge forming a structure (“island”) of size close to the numerical box (see plots at \(t = 200 T_p\)) in the saturation regime.

The component of the current density that is relevant in our 2D configuration is \(J_z\) (as in 1D case) and is shown in Fig. 5 (a). The island is bounded by the null line of \(J_z\). The distribution of \(J_z\) inside the island at \(t = 200 T_p\) is indeed similar to that observed in 1D: the current peaks near the boundary of the island (which corresponds to the “horned” 1D profiles in Fig. 4) and has much weaker values well inside the island; locally, small scale filaments where the current changes sign are also observed. Fig. 5 (b) shows the energy density of the magnetic field, \(u_B = (B_x^2 + B_y^2)/8\pi\). The magnetic field is strongly localized along the island boundary, i.e. along the null line of \(J_z\). Fig. 5 (c) shows the density of a single population \(n_{-\gamma}\). The particles cluster in regions where the current density \(J_z\) has negative polarities (see Fig. 5 (a)), as in the 1D case.
The island profiles assumed by \( J_z, u_B \) and \( n_\perp \) during the non-linear phase are similar to those observed as asymptotic numerical solutions of 2D Navier-Stokes and magnetohydrodynamic equations (Hossain et al. 1983).

The distribution of \( p_z \) is shown in Fig. 6 (c) for \( t = 100 T_p \) and in Fig. 6 (d) for \( t = 200 T_p \). As in the 1D case, a peak at the upper cut-off \( p_z \approx 2 p_0 = 400 m_e c \) forms (see Fig. 6 (d)), while the spectral peak in the energy distribution (Fig. 6 (a)) disappears, probably because the energy in the tail of the distribution is smoothed out over an additional degree of freedom.

### 6 EFFECTS OF RADIATION FRICTION

The friction effect of the RF force physically corresponds to the incoherent emission of high-frequency radiation by ultra-relativistic electrons and positrons, seen (Di Piazza et al. 2012). Most of the emitted radiation has a frequency high enough to escape the plasma. From a numerical point of view, it is impossible to perform simulations with a spatial resolution high enough to resolve such a small wavelength radiation. Thus, it is assumed that such radiation escapes from the system without re-interacting with other electrons or positrons (as discussed in Section 2), and the RF acts as a loss term. To evaluate the amount of energy lost due to RF it is thus necessary to compare simulations with and without the inclusion of the RF force.

We checked that, for a fixed initial energy of the beams, the RF effects increase with the initial density \( n_T \). This can be explained by the scaling of the field amplitudes as \( \omega_p \sim n_T^{1/2} \). We show results obtained for the highest value considered \( n_T = 3 \times 10^{21} \text{cm}^{-3} \). RF effects have been found to be quite weak for densities \( < 10^{20} \text{cm}^{-3} \). Figure 7 reports the time evolution of the total energy (particle energy plus field energy) with respect to the initial kinetic energy of the beams, comparing the non-RF with the RF simulation, for both the 1D and the 2D case. We observe an energy

**Figure 5.** 2D simulation: contours of (a) current density \( J_z \), (b) magnetic field energy density \((B_x^2 + B_y^2)/8\pi\), and (c) density of electron population \( n_\perp \) all as a function of \((x,y)\) and for times \( t = 75 T_p \) (left) corresponding to the end of the linear phase and \( t = 200 T_p \) (right) corresponding to the saturation regime.

**Figure 6.** 2D simulation: Plots of the particle energy spectrum at two different time-steps: (a) \( t = 100 T_p \) and (b) \( t = 200 T_p \). The non-radiative (noRF) case is indicated with a red line while the radiative (RF) case (presence of RF force in the equation of motion) is indicated with a blue line. Plots of particle distribution in \( p_z \) at the same two different time-steps: (c) \( t = 100 T_p \) and (d) \( t = 200 T_p \). We have used the same choices of colors as before.

**Figure 7.** Percentage of the total energy in the system (normalized to the initial energy) as a function of time for a 1D simulation (red line) and for a 2D simulation (green line). At the end of the run, the energy loss due to radiation emission is about 50% in the 1D case, while it is about 30% in the 2D case.
decrease that reaches about 50% of the initial kinetic energy for the 1D case and 30% for the 2D case at the end of the run. The difference suggests a significant role of the field topology in determining the RF losses.

However, such major loss of energy due to the RF effect does not change strongly the dynamics characterizing the FI. The system organizes itself in filamentary structures for the current density which have almost the same size and features of the filaments obtained in the non-RF simulation. This behavior can be simply understood by noticing that the RF plays little role before the saturation phase. Moreover, in the ultra-relativistic case the dominant term of RF force (see Landau & Lifshitz (1975)) is $\sim \gamma^2$. Thus the RF contribution is strongly increased by the acceleration of some particles to higher energy, which is maximized at the instability saturation stage. Indeed, RF effects are more evident in the particle spectra. Figure 8 shows the kinetic energy distribution of particles at three different times. We observe that the distribution has a peak, i.e. a “bump on tail” as in the case without RF. However, with RF the energy corresponding to the peak decreases with time, and the peak smooths out. This means that the RF force is much stronger for particles belonging to the high-energy tail of the energy distribution, and its effect is thus to “cool down” such high-energy tail. In particular, RF effects play a much stronger role after the initial development of the FI because of the generation of both strong magnetic fields (which lead to synchrotron emission) and the acceleration of particles to high energy.

Similar features are observed in the spectra from the 2D simulations (Fig. 6, red lines). In the kinetic energy spectrum reported in Fig. 6 (a) and in Fig. 6 (b), differences due to the RF inclusion start to be appreciable at $t = 100 T_p$, becoming evident near the end of the run at $t = 250 T_p$. The peak near the cut-off value in the $dN/dp_z$ spectrum is present both without and with RF included, but in the latter case it corresponds to a lower value of $p_z$ and the corresponding number of particles is also smaller.

7 CONCLUSIONS

In this work we have studied the evolution of the filamentation instability produced by two counter-streaming pair plasmas using PIC simulations in both 1D and 2D spatial dimensions, with and without radiation friction effects. The instability development leads to the acceleration of a group of particles to high energy, forming a spectral peak in correspondence to twice the initial kinetic energy and a sharp cutoff at twice the initial drift momentum in the momentum distribution. A simple model has been outlined to account for the acceleration of such particles during the instability development, using conservation principles and a single particle approach. Radiation friction effects have been found to be significant only for relatively high density ($\sim 10^{20} \text{cm}^{-3}$) and to affect strongly the particle spectra, cooling down the distribution functions, while the instability development is weakly affected.

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