Transport phenomena in the urban street canyon

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1 Abstract

A generic proecological traffic control model for the urban street canyon is proposed by de-
vlopment of advanced continuum field hydrodynamical control model of the street canyon.
The model of optimal control of street canyon dynamics is also investigated. The mathemat-
ical physics' approach (Eulerian approach) to vehicular movement, to pollutants' emission,
and to pollutants' dynamics is used. The rigorous mathematical model is presented, us-
ring hydrodynamical theory for both air constituents and vehicles, including many types of
vehicles and many types of pollutants emitted from vehicles. The six proposed optimal mon-
ocriterial control problems consist of minimization of functionals of the total travel time, of
global emissions of pollutants, and of global concentrations of pollutants, both in the studied
street canyon, and in its two nearest neighbour substitute canyons, respectively. The six op-
timization problems are solved numerically. Generic traffic control issues are inferred. The
discretization method, comparison with experiment, mathematical issues, and programming
issues are discussed.

2 Description of the model.

In the present article we develop a continuum field model of the street canyon. In the
next article we will deal with numerical examples [1]. The vehicular flow in the canyon
is multilane bidirectional one-level rectilinear, and it is considered with two coordinated
signalized junctions [2, 3, 4]. The vehicles belong to different vehicular classes: passenger
cars, and trucks. Emissions from the vehicles are based on technical measurements and
many types of pollutants are considered (carbon monoxide CO, hydrocarbons HC, nitrogen
oxides NOx). The vehicular dynamics is based on a hydrodynamical approach [5]. The
governing equations are the continuity equation for the number of vehicles, and Greenshields’
equilibrium speed-density u-k model [6].
The model of dynamics of pollutants is also hydrodynamical. The model consists of a set of mutually interconnected nonlinear, three-dimensional, time-dependent, partial differential equations with nonzero right-hand sides (sources), and of boundary and of initial problem. The pollutants, oxygen, and the remaining gaseous constituents of air, are treated as mixture of noninteracting, Newtonian, viscous fluid (perfect or ideal gases). The complete model incorporates as variables the following fields: density of the mixture, mass concentrations of constituents of the mixture, velocity of mixture, temperature of mixture, pressure of mixture, intrinsic (internal) energy of mixture, densities of vehicles, and velocities of vehicles. The model is based on the assumption of local laws of balance (conservation) of: mass of the mixture, masses of its constituents, momentum and energy of the mixture, the numbers of the vehicles, as well as of the state equations (Clapeyron’s law and Greenshields’ model). The equations of dynamics are solved by the finite difference scheme.

The six separate monocriterial optimization problems are formulated by defining the functionals of total travel time, of global emissions of pollutants, and of global concentrations of pollutants, both in the studied street canyon and in its two nearest neighbour substitute canyons. The vector of control is a five-tuple composed of two cycle times, two green times, and one offset time between the traffic lights. The optimal control problem consists of minimization of the six functionals over the admissible control domain.

3 Equations of dynamics.

Under the above model specifications, the complete set of equations of dynamics of the model is formulated as follows (we follow the general idea presented in [7, 8]):

E1. Balance of momentum of mixture - Navier Stokes equation.

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \mathbf{S} \mathbf{v} = -\nabla p + \eta \Delta \mathbf{v} + (\xi + \frac{\eta}{3}) \nabla (\text{div} \mathbf{v}) + \mathbf{F},
\]

(1)

where \( \eta \) is the first viscosity coefficient \(( \eta = 18.1 \cdot 10^{-6} \text{[kg m$^{-1}$s]} )\) for air at temperature \( T = 293.16 \text{[K]} \), \( \xi \) is the second viscosity coefficient \(( \xi = 15.6 \cdot 10^{-6} \text{[^{kg m$^{-1}$s]}]} \) for air at temperature \( T = 293.16\text{[K]} \)), \( \mathbf{F} = \rho \mathbf{g} \) is the gravitational body force density, \( \mathbf{g} \) is the gravitational acceleration of Earth \(( \mathbf{g} = (0, 0, -9.81) \text{[m/s$^{2}$]} )\), \( \nabla \mathbf{v} \) is gradient of the vector (so it is a tensor of rank 2). We assume that the gaseous mixture is a compressible and viscous fluid.

E2. Balance of mass of mixture - Equation of continuity.

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = \mathbf{S}.
\]

(2)

We have assumed the source \( \mathbf{D}_{1} \).

E3. Balances of masses of constituents of mixture - Diffusion equations.

E3a.

\[
\rho \left( \frac{\partial c_{i}}{\partial t} + \mathbf{v} \cdot \nabla c_{i} \right) = S_{i}^{E} - c_{i} \mathbf{S} + \]

(3)
\[
+ \sum_{m=1}^{N-1} \{(D_{im} - D_{in}) \cdot \text{div}[\rho \nabla (c_m + \frac{k_{T,m} T}{T} \nabla T)]\}, \quad i = 1, \ldots, N_E.
\]

E3b.

\[
\rho \left( \frac{\partial c_i}{\partial t} + v \circ \nabla c_i \right) = -c_i S + \sum_{m=1}^{N-1} \{(D_{im} - D_{in}) \cdot \text{div}[\rho \nabla (c_m + \frac{k_{T,m} T}{T} \nabla T)]\}, \quad i = (N_E + 1), \ldots, N,
\]

where \( D_{im} = D_{mi} \) is the mutual diffusivity coefficient from the \( i \)-th constituent to \( m \)-th one, and \( D_{ii} \) is the autodiffusivity coefficient of the \( i \)-th constituent, and \( k_{T,m} \) is the thermodiffusion ratio of the \( m \)-th constituent. The diffusivity coefficients and thermodiffusion ratios are constant and known. In E3a we have assumed the sources \( D_1 - D_2 \). In E3b only the source \( D_1 \) is taken into account. Since the mixture is in motion, we cannot neglect the convection term: \( v \circ \nabla c_i \). We assume that the barodiffusion and gravitodiffusion coefficients are equal to zero.

E4. Balance of energy of mixture.

\[
\rho \left( \frac{\partial \epsilon}{\partial t} + v \circ \nabla \epsilon \right) = -\left( \frac{1}{2} v^2 + \epsilon \right) S + T : \nabla v + \text{div}(\mathbf{q}) + \sigma,
\]

where \( \epsilon \) is the mass density of intrinsic (internal) energy of the air mixture, \( T \) is the stress tensor, symbol : denotes the contraction operation, \( \mathbf{q} \) is the vector of flux of heat. We assume that [2]:

\[
\epsilon = \sum_{i=1}^{N} \epsilon_i,
\]

\[
\epsilon_i = \frac{1}{m_i} \left\{ c_i k_B T \exp\left(-\frac{m_i|\mathbf{g}|z}{k_B T}\right) \cdot \left[\left(\frac{-z}{c}\right) \cdot \left(1 - \exp\left(-\frac{m_i|\mathbf{g}|c}{k_B T}\right)\right) - \exp\left(-\frac{m_i|\mathbf{g}|c}{k_B T}\right)\right]\right\} + \tilde{\mu}_i c_i,
\]

\[
\tilde{\mu}_i = \frac{\mu_i}{m_i},
\]

\[
\mu_i = k_B T \cdot \left\{ \ln[(c_i p)(k_B T)^{-\frac{c_i}{k_B T}}] + \frac{m_i}{m_{\text{air}}} \left(\frac{2\pi h^2}{m_i}\right)^{\frac{3}{2}}\right\} + m_i |\mathbf{g}| z,
\]

\[
T_{mk} = -p \delta_{mk} + \eta \cdot \left[\left(\frac{\partial v_m}{\partial x_k} + \frac{\partial v_k}{\partial x_m} - \frac{2}{3} \delta_{mk} \text{div}(v)\right) \cdot \frac{\partial v_k}{\partial x_m}\right] + \xi \cdot \left[(\delta_{mk} \text{div}(v))^2\right], \quad m, k = 1, \ldots, 3,
\]

\[
T : \nabla v = \sum_{m=1}^{3} \sum_{k=1}^{3} T_{mk} \frac{\partial v_m}{\partial x_k},
\]

\[
\mathbf{q} = \sum_{i=1}^{N} \left\{ \left[\frac{\beta_i T}{\alpha_{ii}} + \tilde{\mu}_i \mathbf{j}_i\right] + \left[(-\kappa) \nabla T]\right\},
\]
\[ \mathbf{j}_i = -\rho D_{ii} \left( c_i + \frac{k_{T,i}}{T} \nabla T \right), \quad (13) \]

\[ \alpha_{ii} = \left[ \frac{\rho D_{ii}}{[\left( \frac{\partial \mu_i}{\partial c_i} \right)_{c_n=1,\ldots,N,\neq n,T,p}]_{n} = 1,\ldots,N,i \neq n,T,p} \right], \quad (14) \]

\[ \beta_i = [\rho D_{ii}] \cdot \left\{ \frac{k_{T,i}}{T} - \frac{\left[ \frac{\partial \mu_i}{\partial T} \right]_{c_n=1,\ldots,N,p}}{[\left( \frac{\partial \mu_i}{\partial c_i} \right)_{c_n=1,\ldots,N,\neq n,T,p}]_{n} = 1,\ldots,N,i \neq n,T,p} \right\}, \quad (15) \]

where \( \epsilon_i \) is the mass density of intrinsic (internal) energy of the \( i \)th constituent of the air mixture, \( m_i \) the molecular mass of the \( i \)th constituent, \( k_B = 1.3807 \cdot 10^{-23} \, [\text{J/kg}] \) is Boltzmann’s constant, \( \mu_i \) is the complete partial chemical potential of the \( i \)th constituent of the air mixture (it is complete since it is composed of chemical potential without external force field and of external potential), \( m_{\text{air}} = 28.966 \, [\text{u}] \) is the molecular mass of air (1[u] = 1.66054 \cdot 10^{-27} \, [\text{kg}]) , \( \delta_{mk} \) is Kronecker’s delta, \( c_{p,i} \) is the specific heat at constant pressure of the \( i \)th constituent of the air mixture, and \( \kappa \) is the coefficient of thermal conductivity of air. These magnitudes are derived from Grand Canonical ensemble with external gravitational Newtonian field.

**E5. Equation of state of the mixture - Constitutive equation - Clapeyron’s equation.**

\[ \frac{p}{\rho} = \frac{R}{m_{\text{air}}} \cdot T \quad (16) \]

is Clapeyron’s equation of state for a gaseous mixture, where \( R = 8.3145 \, \frac{\text{J}}{\text{mol} \cdot \text{K}} \) is the gas constant.

\[ p_i = c_i \cdot \frac{m_{\text{air}}}{m_i} \cdot p \quad (17) \]

are partial pressures of constituents according to Dalton’s law.

**E6. Balances of numbers of vehicles - Equations of continuity of vehicles.**

\[ \frac{\partial k_{l,vt}^s}{\partial t} + \text{div}(k_{l,vt}^s w_{l,vt}^s) = 0. \quad (18) \]

**E7. Equations of state of vehicles - Greenshields model.**

\[ w_{l,vt}^s(x,t) = (w_{l,vt,f}^s \cdot (1 - \frac{k_{l,vt}^s(x,t)}{k_{l,vt,jam}^s}), 0, 0). \quad (19) \]

The Greenshields equilibrium speed-density u-k model is assumed [6]. The values of maximum free flow speed \( w_{l,vt,f}^s \), and of jam vehicular densities \( k_{l,vt,jam}^s \), are given in Tables 3 and 8 of [1].

**E8. Technical parameters.**

The dependence of emissivity on the density and velocity of vehicles is assumed in the form:

**E8a.**
\[ \varepsilon_{l,ct,vt}^{s}(x,t) = k_{l,vt}^{s}(x,t) \cdot \left( \frac{|w_{l,vt}^{s}(x,t)| - \bar{w}_{vt,i}^{s}}{\bar{w}_{vt,i}^{s+1} - \bar{w}_{vt,i}} \right) \cdot (\bar{c}_{ct,vt,i} + \bar{c}_{ct,vt,i}) \],

(20)

where \( \bar{w}_{ct,vt,i} \) are experimental velocities, \( |w_{l,vt}^{s}(x,t)| \in (\bar{w}_{ct,vt,i}^{s}, \bar{w}_{ct,vt,i}^{s+1}) \), \( \bar{e}_{ct,vt,i} \) are experimental emissions of the \( ct \)th exhaust gas from single vehicle of \( vt \)th type at velocities \( \bar{w}_{ct,vt,i} \), respectively, measured in \([ \text{kg veh} \cdot \text{s}] \), \( N_{EM} \) is the number of experimental measurements. Similarly, the dependence of the change of the linear density of energy on the density and velocity of vehicles is taken in the form:

E8b.

\[ \sigma_{l,vt}^{s}(x,t) = q_{vt} \cdot k_{l,vt}^{s}(x,t) \cdot \left( \frac{|w_{l,vt}^{s}(x,t)| - \bar{w}_{vt,i}^{s}}{\bar{w}_{vt,i}^{s+1} - \bar{w}_{vt,i}} \right) \cdot (\sigma_{vt,i}^{s+1} - \sigma_{vt,i}) + \sigma_{vt,i}, \]

(21)

where \( \sigma_{vt,i} \) are experimental values of consumption of gasoline/diesel for a single vehicle of \( vt \)th type at velocities \( \bar{w}_{vt,i} \), respectively, measured in \([ \text{kg veh} \cdot \text{s}] \), \( q_{vt} \) is the emitted combustion energy per unit mass of gasoline/diesel \([ \text{J kg}^{-1}] \).

4 Optimization problems.

Our control task is the minimization of the measures of the total travel time (TTT) [5], emissions (E), and concentrations (C) of exhaust gases in the street canyon, therefore the appropriate optimization problems may be formulated as follows [2]:

F0. Vector of control.

\[ \mathbf{u} = (g_1, C_1, g_2, C_2, F) \in U_{adm}, \]

(1)

where \( \mathbf{u} \) is vector of boundary control, \( g_m \) are green times, \( C_m \) are cycle times, \( F \) is offset time, and \( U_{adm} \) is a set of admissible control variables (compare A8, A9, B5, B5S, B5SS).

We define six functionals F1-F6 of the total travel time, emissions, and concentrations of pollutants in single canyon, and in canyon with the nearest neighbour substitute canyons, respectively.

F1. Total travel time for a single canyon.

\[ J_{TTT}^{s} = \frac{2}{s=1} \sum_{s=1}^{2} \sum_{l=1}^{n_{s}} \sum_{vt=1}^{V_T} \int_{a}^{T_S} k_{l,vt}^{s}(x,t) \, dx \, dt. \]

(2)

F2. Global emission for a single canyon.

\[ J_{E}^{s} = \frac{2}{s=1} \sum_{s=1}^{2} \sum_{CT} \sum_{l=1}^{n_{s}} \sum_{vt=1}^{V_T} \int_{a}^{T_S} e_{l,ct,vt}^{s}(x,t) \, dx \, dt. \]

(3)
F3. Global pollutants concentration for a single canyon.

\[ J_C(u) = \rho_{STP} \cdot \sum_{i=1}^{N_E-1} \int_0^a \int_0^b \int_0^{T_S} c_i(x, y, z, t) \, dx \, dy \, dz \, dt. \] (4)

F4. Total travel time for the canyon in street subnetwork.

\[ J_{TTT,ext}(u) = J_{TTT}(u) + \]
\[ + a \cdot \sum_{s=1}^{2} \alpha_{TTT,ext}^s \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} k_{l,vt,jam}^s \cdot (C_s - g_s). \] (5)

F5. Global emission for the canyon in street subnetwork.

\[ J_{E,ext}(u) = J_E(u) + \]
\[ + a \cdot \sum_{s=1}^{2} \alpha_{E,ext}^s \sum_{l=1}^{n_s} \sum_{ct=1}^{CT} \sum_{vt=1}^{VT} e_{l,ct,vt,jam}^s \cdot (C_s - g_s). \] (6)

F6. Global pollutants concentration for the canyon in street subnetwork.

\[ J_{C,ext}(u) = J_C(u) + \]
\[ + \rho_{STP} \cdot a \cdot b \cdot c \cdot \sum_{i=1}^{N_E-1} c_{i,STP} \alpha_{C,ext}^s \cdot (C_s - g_s). \] (7)

The integrands \( k_{l,vt,jam}^s, e_{l,ct,vt,jam}^s, c_i \) in functionals F1-F6 depend on the control vector \( u \) through the boundary conditions B0-B8, through the equations of dynamics E1-E8, as well as, through the sources D0-D2. The value of the vector of control \( u \) directly affects the boundary conditions B5, B5S, B5SS, and then the boundary conditions B6-B8 for vehicular densities, velocities, and emissivities. It also affects the sources D0-D2. Next, it propagates to the equations of dynamics E1-E8 and then it influences the values of functionals F1-F6. We only deal with six monocriterial optimization problems O1-O6, and not with one multicriterial problem. We put the scaling parameters equal to unity: \( \alpha_{TTT,ext} = \alpha_{E,ext} = \alpha_{C,ext}^s = 1.0 \), in functionals F4-F6. \( \rho_{STP} \) is the density of air at standard temperature and pressure STP, \( c_{i,STP} \) is concentration of the \( i \)th constituent of air at standard temperature and pressure. \( J_{TTT} \) and \( J_{TTT,ext} \) are measured in \([veh \cdot s]\), \( J_E \) and \( J_{E,ext} \) are measured in \([kg]\), and \( J_C \) and \( J_{C,ext} \) are measured in \([kg \cdot s]\), respectively.

Now we formulate six separate monocriterial optimization problems O1-O6 that consist in minimization of functionals F1-F6 with respect to control vector \( F0 \) over admissible domain, while the equations of dynamics E1-E8 are fulfilled.

O1. Minimization of total travel time for a single canyon.

\[ J_{TTT}^* = J_{TTT}(u_{TTT}^*) = \min\{u \in U_{adm}: J_{TTT}(u)\}; \] (8)

O2. Minimization of global emission for a single canyon.

\[ J_E^* = J_E(u_E^*) = \min\{u \in U_{adm}: J_E(u)\}; \] (9)
O3. Minimization of global pollutants concentration for a single canyon.

\[ J_C^* = J_C(u_C^*) = \min\{u \in U^\text{adm} : J_C(u)\}; \quad (10) \]

O4. Minimization of total travel time for a canyon in street subnetwork.

\[ J_{\text{TTT,ext}}^* = J_{\text{TTT,ext}}(u_{\text{TTT,ext}}^*) = \min\{u \in U^\text{adm} : J_{\text{TTT,ext}}(u)\}; \quad (11) \]

O5. Minimization of global emission for a canyon in street subnetwork.

\[ J_{\text{E,ext}}^* = J_{\text{E,ext}}(u_{\text{E,ext}}^*) = \min\{u \in U^\text{adm} : J_{\text{E,ext}}(u)\}; \quad (12) \]

O6. Minimization of global pollutants concentration for a canyon in street subnetwork.

\[ J_{C,\text{ext}}^* = J_{C,\text{ext}}(u_{C,\text{ext}}^*) = \min\{u \in U^\text{adm} : J_{C,\text{ext}}(u)\}, \quad (13) \]

where \( J_{\text{TTT}}, J_{\text{E}}, J_C^*, J_{\text{TTT,ext}}, J_{\text{E,ext}}, J_{\text{E,ext}}, J_{C,\text{ext}}, J_{C,\text{ext}} \) are the minimal values of the functionals \( F_1-F_6 \), and \( u_{\text{TTT}}^*, u_{\text{E}}^*, u_C^*, u_{\text{TTT,ext}}^*, u_{\text{E,ext}}^*, u_{C,\text{ext}}^* \) are control vectors at which the functionals reach the minima, respectively.

5 Computer simulations.

From thousands of performed optimizations O1-O6 we selected some representable optimizations. The C language programme of more than 14000 lines written by the author was being run on workstations of Hewlett-Packard, Sun, Silicon Graphics, under UNIX operating system. It was being run also on Pentium personal computers under Linux operating system. We used C language UNIX/Linux compilers cc. The time of simulations varied from a couple of hours to several days or even weeks, depending on the discretization parameters. The discretization steps were tested in order to obtain finite solutions to the optimization problems in reasonable time of simulation. They were also studied from the point of view of sensitiveness of optima appearing on them.

6 Mathematical questions.

The problem of uniqueness of solutions of optimization problems O1-O6 is complex. If there are no vehicles allowed in the canyon, then all the six functionals O1-O6 are constant functions of parameters, the optima exist, and the number of optimal solutions is continuum. The values of functionals are then zero for total travel time and emissions, and are positive for concentrations. These results are analytical. For other cases there are no such analytical results for both the uniqueness and existence known to the author. The numerical solutions of optimization problems are approximately global within the error induced by discretization of the physical domain \( \Sigma \) and of the domain of control parameters \( U^\text{adm} \). The method of optimization was a full search.
7 Conclusions.

The proecological traffic control idea and advanced model of the street canyon have been developed. It has been found that the proposed model represents the main features of complex air pollution phenomena.

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