Solidarity, synchronization and collective action

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Abstract

For people to act collectively in actual situations—in contrast to public goods experiments—goal ambiguity, diversity of interests, and uncertain costs and benefits stand in their way. Under such conditions, people seem to have few reasons to cooperate, yet the Arab revolutions, as conspicuous examples, show that collective action can take place despite the odds. I use the Kuramoto model to analyze how people in a cohesive network topology can synchronize their salient traits (emotions, interests or other), and show that synchronization happens in a phase transition, when group solidarity passes a critical threshold. This model can yield more precise predictions of outbursts of collective action under adverse conditions, and casts a new light on different measures of social cohesion.

A remarkable feature of humans is that they can act collectively, even though individuals are tempted to defect and reap the benefits of others’ efforts [19]. This achievement is even more remarkable if one realizes that in contrast to public goods experiments of collective action, where the collective good—money—is clearly defined at the outset, public goods in actual situations, from revolutions to reorganizations, are often ambiguous. Yet more challenging, participants differ in their interests in, and shares of, those public goods. Under conditions of ambiguity, uncertainty and diversity, most people seem to have few reasons to cooperate. Nevertheless we can witness that in numerous cases, people somehow manage to get to action in the face of all odds.

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In this note I ask the question if under adverse conditions, when neither cost-benefit ratios nor reputations can win people over to contribute to collective goods, dilemma’s of cooperation can be solved through “interaction rituals” wherein individuals’ traits (emotions, interests or other) are synchronized. To this end I use Yoshiki Kuramoto’s model of synchronization [25], to be discussed first. Because synchronization is highly sensitive to network topology, I compare two concepts of social cohesion, $k$-core and $k$-connectivity, for their synchronization potential. Results are somewhat ambiguous and require laboratory experiments on human subjects.

Solidarity and synchronization

According to one of the founders of sociology, Emile Durkheim, groups of people are bond together by social cohesion [9]. With modern network theory in mind, one could say that cohesion consists in part of social ties that people have with others, and for another part of solidarity that people have with their group as a whole. Importantly, people are not identical, and $i \in \{1, 2, \ldots, N\}$ group members each have their own character $\omega_i$, shaped prior to current interactions on a given network. Their solidarity $\lambda$ can be enhanced through intensified interaction, such as collective singing or noise making (see documentaries on protests such as Tahrir square), religious practices [10], initiation boot camps in organizations [13], military training or other interaction rituals with a shared focus [6]. In some of these interaction rituals, people perform synchronized body movements [12], as to induce their emotional synchronization. Future research should detail out the relation between interaction rituals and solidarity; here I assume a monotonic relationship, exogenous to the model.

Although for the model (Eq.1) it makes no difference whether an interaction ritual increases solidarity, the strength of network ties, or both, I assume for simplicity that only solidarity varies; all ties between individuals are symmetric, $W_{ij} = W_{ji} = 1$ (and absent ties $W_{ij} = 0$), such that connected $i$ and $j$ mutually influence each other; and, the network does not change over the period of observation. For the moment I also assume that everybody has the same solidarity value (coupling strength), which will be loosened to individualized $\lambda_i$ in the next section. An individual’s current trait—emotion,

\footnote{A generalization of the Kuramoto model [2] allows for the network and its synchronization to co-evolve, such that the ties and their strengths $W_{ij}(t)$ change according to homophily and homeostasis. For homophily, see the main text below; homeostasis means individuals’ limited capacity to maintain ties, such that stronger ties with some people imply weaker or no ties with others, empirically found by [16] and others.}
goal, preference or other—\(\theta_i(t)\) depends on both her character \(\omega_i\) and on the ties \(W_{ij}\) she has with other people having their own characters and traits.

The Kuramoto model now enables to instantiate these sociological notions and to analyze the when and how of synchronization of individuals’ \(\theta_i(t)\) values. A group is perfectly synchronized when all pairwise differences \((\dot{\theta}_i(t) - \dot{\theta}_j(t))\) \(\to 0\) and stay stably synchronized over time. Dropping time indices for clarity, individual \(i\)’s trait evolves as follows,

\[
\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} W_{ij} \sin(\theta_j - \theta_i).
\]

(1)

Synchronization of the group can be expressed in terms of a complex order parameter, \(re^{i\psi(t)}\), where \(0 \leq r \leq 1\) measures the similarity, or coherence, of individuals’ traits and \(\psi(t)\) is the average trait \([25]\). Perfect synchronization \((r = 1)\) can only be achieved for identical characters and identical initial values, which are too unrealistic to be interesting.

In Kuramoto’s initial model, everybody was connected to everybody else. When solidarity increases such that it surpasses a critical value \(\lambda_c\), there is a phase transition from an incoherent state toward stable, although not perfect, synchronization of a majority of the nodes. This phase transition has also been found for many sparse networks, although not all networks are synchronizable \([7]\).

According to Randall Collins \([6]\), interaction rituals entail a “collective effervescence,” which he describes as “a mutually focused emotion and attention producing a momentarily shared reality.” Participants lose their sense of individuality and feel united. They also seem to get rosy expectations of the benefits and costs of collective action—else many wars would not have occurred. They reach a synchronized emotional state, and the model explains how and when this happens. Moreover, psychological experiments suggest that synchronized people are more likely to cooperate \([28]\). It goes without saying that at some point after a collective action, solidarity (or tie strength) will decrease, even though intensely experienced interaction rituals can have lasting effects.

Because synchronization makes people feel similar, it might also explain homophily at future encounters, i.e. a higher chance of tie formation or strengthening between people who perceive each other as similar \([15]\), consistent with evolutionary game theorists’ claim that homophily can be explained by clustering \([19]\). Furthermore, the phase transition predicts an outburst of collective action, which we often see in street protests, strikes and collective violence. This explanation is more subtle than the often used “critical mass” argument \([21]\), which depends on the number of participants and their
crossover points (analogous to $\lambda_i$) but addresses neither the network wherein people are embedded nor the trait changes ($\dot{\theta}_i$) caused by their interactions.

**Social cohesion**

Earlier studies [1] showed that a group’s synchronizability critically depends on its network topology. For social networks a relevant question is: do they synchronize better if they feature higher social cohesion? For social cohesion, there are two salient conceptions, both resulting in a Russian doll model of a given network, with more cohesive centers embedded in less cohesive peripheries.

The first conception of cohesion is the widely-known $k$-core, denoting that someone at the $k$-core of a network has at least $k$ contacts who in turn also have at least $k$ contacts within the same subgraph [22]; obviously, a network can have multiple $k$-cores that are mutually disconnected. One of the reasons for this notion’s popularity is that it makes possible to analyze $k$-cores’ resilience against random node removal, i.e. the critical (percolation) thresholds when these cores fall apart [8]. A drawback, however, is that a $k$-core can have topological bottlenecks, e.g. in the 3-core at the bottom of Fig.1 all information transmission between the two subgroups depends on a single node in between. In other words, $k$-cores are sensitive to targeted attacks on central nodes indicated by high betweenness centrality. The topology at the top of Fig.1 is also a 3-core—without bottleneck—showing that the two topologies are indistinguishable from a $k$-core point of view.

An alternative concept that does notice bottlenecks is $k$-connectivity [17], where all pairs of nodes at a given $k$-level are connected by minimally $k$ node-independent paths. A $k$-connective (sub)graph is also a $k$-core but not the other way around. Menger’s theorem can be used to prove that having $k$ independent paths between any pair of nodes is equivalent to saying that minimally $k$ nodes have to be removed to make the network fall apart [27].

The idea that $k$ people hold a group together provides a natural intuition of social cohesion. Multiple connections are advantageous because they can reduce noise and increase credibility of information, which is important for cooperation. For people to decide whether to cooperate with others, they need information about them—often through gossip—that is more reliable when received through multiple independent channels. Experiments have shown that when people receive multiple inconsistent gossips about a person, they tend to follow the majority [24]. Multi-connectivity also makes possible to meta-gossip about the quality of information sources, and to pin down and subsequently ignore noisy sources (personal observation by the author).
These arguments suggest that $k$-connectivity is better than $k$-coreness for gossip-based reputations [4], but would it also be better for synchronization?

To compare the two contenders, I use the following testbed, see Fig.1. The wheel at the top is 3-connected and the bow tie below is 1-connected. These two topologies can be well-compared because they have the same size (7); density (0.57); average path distance (1.43); degree distribution; degree centralization (0.6); and, both are 3-cores.

For the analysis, numerical solutions for the differential equations (Eq.1) are found by the deSolve package of R [23]. Individuals’ character values are drawn from a normal distribution with mean 2 and standard deviation 1. Initial values of $\theta_i$ are drawn from a uniform distribution ranging from 0 to $2\pi$. For Fig.1, solidarity values are drawn from the empirical distribution of Europe, i.e. the mean (3.68) and its standard deviation (.935) pertain to solidarity with sick and disabled people measured in a survey. Although its unit of measurement is not meaningful, it gives a realistic impression of the

\footnote{Thank you to Ferry Koster for providing me these data from EVS 3 (1999-2001).}
Figure 2: Synchronization of the wheel (almost straight red line), bow tie (black), and average synchronization of bow tie’s left and right hand subgraphs (blue).

As it turns out (Fig. 2), the 3-connected wheel topology synchronizes at a stable value, whereas the bow tie is unstable but has higher $r$ on average. Bow tie’s subgraphs $\{Z_1, Z_2, Z_6, Z_7\}$ and $\{Z_3, Z_4, Z_5, Z_7\}$ have yet higher $r$ (their average indicated by the blue line) but are equally unstable as the bow tie itself. If solidarity is increased, even up to two orders of magnitude, this result stays qualitatively the same. When solidarity is decreased, now setting its standard deviation to zero for ease of comparison, it turns out that the phase transition of the wheel is around $1.2 \leq \lambda_c \leq 1.7$. The exact value of $\lambda_c$ depends on the character values, drawn randomly from their distribution.
Discussion and conclusion

When diverse people want to achieve a collective goal that is ambiguous, and costs and benefits are uncertain at the outset, interaction rituals can synchronize people’s intentions and emotions such that collective action becomes possible [6]. Game theorists might learn from interaction rituals that when individuals lose their sense of individuality in a synchronized group, perceptions of costs and benefits may become much more optimistic than in daily life—and in standard game theory—which shows up in overconfidence at the onsets of protests, wars and revolutions. Whereas sociological theory is rather loose in explaining how this might happen, Kuramoto’s model helps to explicate how interaction rituals, solidarity and network cohesion lead to outbursts of collective action. Yet the possibility of synchronization critically depends on network topology. To investigate topology’s role, I compared two concepts of social cohesion on two networks that are identical for a series of socially relevant properties, such as degree distribution, average path distance and size. In one network (wheel), synchronization is stable whereas in the other network (bow tie) synchronization is higher but unstable. How human subjects react to these conditions psychologically is currently unclear, and requires laboratory experiments. I conjecture that for mutual trust, a stable value of synchronization is better than a higher but unstable value.

Also the functional relationship of interaction rituals and solidarity merits further study. For the latter, it seems that the speed and size of information cascades play a role [18], although neither of them is sufficient on its own to increase solidarity; shared focus and “emotional energy” are necessary, too, and oftentimes a physical co-presence as well [6]. Rare but strongly emotional rituals will undoubtedly have a stronger effect on solidarity than frequently occurring low arousal rituals such as prayers [3]. Groups also compete with each other, certainly on an evolutionary time scale [20], and some interaction rituals are targeted towards competing groups [5]. Ritualized animosity towards out-groups, or even innocent scapegoats, can also increase in-group solidarity. Individuals then have different levels of solidarity with different (sub)groups, $\lambda_s$, where $s$ is an index of these groups to be found by community detection [11].

Synchronization of actual groups has to be traded off with other characteristics. Groups have to adapt to a changing environment, at least sometimes, which has to be explored by individuals who are asynchronous with the majority. This is the well-known trade-off between exploitation and exploration, here at the group level [14]. For a group to change, subsequently, there is a trade-off between speed, through centralized command but at low synchronization, and accuracy, through decentralized negotiations wherein
everybody’s expertise is exploited and goals can be synchronized. How these and other trade-offs play out in practice is to be further investigated.

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