Photon-photon scattering in a 3-3-1 model

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Abstract. – We analyze the effects of a doubly charged vector bilepton as well as exotic quarks with charge $5/3\, e$ and $-4/3\, e$ on light by light scattering. We consider mass values in the range 0.3–1 TeV, which would be reached at the planned future linear colliders. It is found that such exotic particles, especially the doubly charged vector bilepton, give raise to remarkable deviations from the standard model cross section. The virtual effects arising from these particles would provide an indirect test to a particular model which is based on the $SU(3)\_c \times SU(3)\_L \times U(1)\_X$ gauge symmetry, known as 3-3-1 model, where such particles are a natural prediction.

Introduction. – In the past, light by light scattering was the subject of theoretical interest in an early attempt of applying perturbation theory beyond the tree level \cite{1}. The cross section has been studied to a great extent \cite{2}, but owing to technical troubles its experimental scrutiny has deserved little attention. Nevertheless, it is likely that $ee^\pm$ linear colliders (LC) would be built in the future \cite{3}. This class of colliders would also operate in the $\gamma\gamma$ and $\gamma e$ modes, opening up the possibility of performing a series of interesting experiments which are inaccessible at lepton and hadron colliders. The prospect of the construction of LC has renewed the interest in light by light scattering: it was found recently that this reaction might be an efficient mode to search for virtual effects of particles lying beyond the standard model (SM) \cite{4}. Following this approach, the virtual effects produced by particles which arise in supersymmetry (SUSY), left-right models (LRM), and supersymmetric left-right models (SUSYLR) have been examined in \cite{4,5}. In this letter we will consider the inclusion of a doubly charged gauge boson as well as exotic quarks with charge $5/3\, e$ and $-4/3\, e$. Although the analysis is applicable to any model predicting such exotic particles, the main motivation lies in an extension of the SM based on the $SU(3)\_c \times SU(3)\_L \times U(1)\_X$ gauge symmetry, known as 3-3-1 model, where these particles appear naturally.

A 3–3–1 model. – The idea of extending the standard model (SM) by embedding the electroweak gauge group into $SU(3)\_c \times U(1)\_X$ is previous to the appearance of the 3-3-1 model. One of the motivations of this model is the necessity of having a chiral theory of bilepton gauge bosons \cite{6}. These exotic particles arise also from an SU(15) grand unified theory with proton-stability, which has however the drawback of requiring artificial mirror fermions in order to cancel anomalies \cite{7}. On the contrary, the 3-3-1 model is appealing for suggesting a possible

\footnote{Non-gauge vector bileptons may appear in composite and technicolor theories.}
path to the solution of the flavor problem: anomaly cancellation is achieved only if all the fermion families are added up, instead of the usual cancellation occurring when it is summed over each fermion family separately. As a consequence, the number of fermion families must be a multiple of 3, the number of quark colors. In addition to this peculiarity, this model is interesting because it predicts striking effects which might be observed at energies to be reached by the next generation of colliders [8]. In the \textbf{3-3-1} model, leptons have the following representation under the SU(3)$_c \times$ SU(3)$_L \times$ U(1)$_X$ gauge group

\[
\begin{pmatrix}
e \\
v_e \\
e^c
\end{pmatrix}, \begin{pmatrix}\mu \\
v_\mu \\
\mu^c
\end{pmatrix}, \begin{pmatrix}\tau \\
v_\tau \\
\tau^c
\end{pmatrix} : (1,3^*,0).
\]

(1)

In the quark sector, the first two quark families are represented alike

\[
\begin{pmatrix}
u^\alpha \\
d^\alpha \\
D^\alpha
\end{pmatrix}, \begin{pmatrix}e^\alpha \\
s^\alpha \\
S^\alpha
\end{pmatrix} : (3,3,-1/3),
\]

(2)

whereas the third quark family, which is treated differently, is represented by

\[
\begin{pmatrix}
\nu_\alpha \\
e_\alpha \\
\tau_\alpha
\end{pmatrix}, \begin{pmatrix}u_\alpha \\
c_\alpha \\
D_\alpha
\end{pmatrix}, \begin{pmatrix}d_\alpha \\
s_\alpha \\
S_\alpha
\end{pmatrix} : (3,1,\pm 1/3),
\]

(3)

with \(\alpha = 1,2,3\) the quark color number. The charge operator is defined by \(Q/e = \lambda^3/2 + \sqrt{3} \lambda^8/2 + X\), where \(\lambda^3, \lambda^8\) are the usual Gell-Mann matrices. In this way, the new quarks \(D, S, T\) have charge \(-4/3\ e, -4/3\ e, 5/3\ e\), respectively. In the gauge sector there are five extra vector bosons besides those of the SM: a pair of singly charged gauge bosons \(U^\pm\), a pair of doubly charged ones \(U^{\pm\pm}\), and a neutral one \(Z'\). In the minimal version of the \textbf{3-3-1} model, three Higgs triplets \(\eta, \rho, \chi\) as well as one Higgs sextet \(\xi\) are required in order to accomplish symmetry breaking. The exotic quarks and the five new gauge bosons acquire mass when the triplet \(\eta\) breaks SU(3)$_c \times$ SU(3)$_L \times$ U(1)$_X$ down to the electroweak gauge group. The last stage in symmetry breaking occurs when the extra Higgs multiplets \(\rho, \chi\) and \(\xi\) break the electroweak gauge group down to U$_e$.

Numerical results and discussion. – It is evident that the most distinctive signal of the \textbf{3-3-1} model might be given by the doubly charged bilepton and the exotic quarks. Studies derived from low energy data show that the mass of the doubly charged bosons and that of the exotic quarks might lie below the TeV scale. In particular, from constraints on the electroweak parameters it was found that 230 GeV \(\leq M_{U^{\pm \pm}} \leq 800\) GeV [8], whereas LEP searches for SUSY particles give the bound \(M_{D,S,T} \geq 250\) GeV [9]. Recently, the more stringent bound \(M_{U^{\pm \pm}} \geq 850\) GeV was derived from muonium-antimuonium conversion, which would rule out the minimal \textbf{3-3-1} model but not those versions with an extended Higgs sector [10]. In fact, it has been argued that less stringent bounds are not excluded since in obtaining such constraint it was assumed that the couplings of bileptons to leptons are given by the identity matrix, which is a very restrictive condition indeed [11]. In this letter we will consider the range 0.3–1 TeV for the mass of both the doubly charged bileptons and the exotic quarks, because it is this range which would be of interest at a future LC.
The amplitude of light by light scattering receives contributions from the Feynman diagrams shown in Fig. 1. In addition to the SM contribution, any extended model produce virtual effects on this process through loops carrying charged particles. The new contributions depend exclusively on the mass and electric charge of the non standard particles, a feature which allows us to examine in a model independent way the virtual effects arising from a certain class of charged particles, namely scalars, vectors or fermions. Those new physics effects would manifest as deviations from the SM cross section, for this reason light by light scattering turns out to be useful: the invariant amplitude contributed by the new particle is proportional to the electric charge factor \( Q^2 \), which becomes \( Q^3 \) when the amplitude is squared to calculate the cross section. As a result, the total cross section is significantly enhanced in models including particles with a charge whose absolute value is greater than unity, in terms of the positron charge. Moreover, the enhancement factor \( Q^3 \) is very useful to distinguish clearly between the contributions coming from particles with the same spin and mass but a different electric charge. For instance, the contribution from a doubly charged particle is a factor of 2 larger than that from a singly charged particle of the same class.

The cross section for \( \gamma \gamma \rightarrow \gamma \gamma \) scattering can be obtained by means of the helicity amplitudes for loops with scalar bosons, fermions or gauge bosons, whose expressions in terms of scalar functions have been obtained previously \([4]\). We have used the FF numerical routines to evaluate the required scalar functions \([4]\). In calculating the cross section, the integration over the scattering angle \( \theta \) has been constrained to lie in the range \( 30^\circ \leq \theta \leq 150^\circ \), in order to avoid collinear (soft) photons escaping from the detector. In Figs. 2–5 we have plotted the unpolarized cross sections which are obtained when each new particle predicted by the \( 3\times3\times1 \) model is included together with the SM contribution. In all these graphs the resulting cross section deviations from the SM one at the threshold \( \sqrt{s} \geq 2 M_{\text{New}} \), with \( M_{\text{New}} \) the mass of the new particle. This fact can be understood from the fact that the additional contribution arises mainly from the interference between the SM amplitude \( A_{\text{SM}} \) and that of the new particle \( A_{\text{New}} \). Therefore, the new cross section will deviate from the SM one by the term \( 2 \text{Re}(A_{\text{SM}}A_{\text{New}}^*) \), which is approximately the same as \( 2 \text{Im}(A_{\text{SM}})\text{Im}(A_{\text{New}}) \) since at high energies the SM amplitude is almost purely imaginary. Another interesting fact which is evident in the plots is that the most distinctive effects come from the doubly charged vector bilepton, which dominates even to the exotic quarks contribution, although in the latter case the amplitude gets also enhanced by an additional factor corresponding to the quark color number. On the other hand, the less spectacular effects arise from a singly charged bilepton gauge boson, which together with a doubly charged scalar bilepton have been previously studied in the context of LRM and SUSYLR \([12]\).

The viability of using light by light scattering to search for SUSY particles has been examined with detail \([12]\). It was shown that under the conditions expected at the planned LC, the unpolarized cross section would be sensitive enough to allow the detection of contributions coming from charginos and sleptons. It is clear that, due to the enhancement produced by their charge, the detection of the exotic particles predicted by the \( 3\times3\times1 \) model is more promising than that of SUSY particles. In this respect, in searching for exotic particles at the present and future colliders, direct production represents also a viable mode. However, any theoretical analysis done in this direction has the disadvantage of requiring many assumptions about unknown model-dependent parameters. For instance, if direct production of bileptons is analyzed, one must consider parameters such as their couplings to leptons, the mass of the extra neutral boson \( M_{Z'} \), and the \( Z-Z' \) mixing angle \([13]\). This unwanted situation also arises in considering indirect search modes such as Møller and Bhabha scattering, which at tree level receive the contribution of a doubly charged bilepton through the \( u \) channel. Furthermore, an experimental study of direct production must consider also the respective backgrounds. In contrast, the cross section of light by light scattering involves just one free parameter, which is

\[ Q \]
the mass of the exotic particle. Although the cross sections are smaller than the ones predicted in other reactions, the facilities which would be offered at a LC open up the possibility of using light by light scattering as an alternative method to search for non-standard particles such as bileptons and exotic quarks.

Finally, by way of illustration, we show through Figs. 6–8 the total $\gamma\gamma \to \gamma\gamma$ unpolarized cross section for some scenarios which may arise in the 3-3-1 model. We have assumed the simple case in which the three exotic quarks have a mass $M_Q$ and both vector bileptons have a mass $M_U$. Such scenarios are very simplistic, but we only want to show how spectacular the enhancement produced by the full particle content of the 3-3-1 model. As shown in Fig. 3, the most notable scenario is the one with a relatively light doubly charged bilepton with a mass of a few hundreds of GeVs. In such a case the cross section may be one order of magnitude larger than the SM one. Although the doubly charged bilepton might produce spectacular effects by itself in this process and in any other ones, its detection would not be a sufficient evidence to support the 3-3-1 model, and further probes would be needed. In this respect, a reaction as light by light scattering might be an useful mode to test indirectly some details of the model. As the cross section is enhanced considerably by both bileptons and exotic quarks, the experimental study of light by light scattering would produce indirect evidence of the existence of these particles. In addition, since all the new contributions add up coherently, this mode would be suitable for counting the number of new particles, what would be helpful to identify a particular model. However, it is important to note that the scope of using light by light scattering to search for new particles is limited by the fact that any virtual effect appear at the threshold which corresponds to twice the mass value of the new particle.

Summary. – In closing, we stress that light by light scattering offers an interesting mode to search indirectly for exotic particles, especially the ones predicted by the 3-3-1 model, at a future LC. In particular, doubly charged vector bileptons give rise to spectacular new physics effects. If LC became a reality, light by light scattering would be useful to elucidate what is the adequate gauge group to extend the SM.

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Fig. 1 – Generic Feynman diagrams for $\gamma\gamma \to \gamma\gamma$ scattering in the nonlinear $R_\xi$ gauge. Charged particles of spin 0 and 1 contribute through all these diagrams, whereas fermions participate just via the box diagram.

Fig. 2 – Unpolarized cross section for $\gamma\gamma \to \gamma\gamma$ scattering when it is added to the SM a doubly charged vector bilepton $U^{++}$.

Fig. 3 – The same as in Fig. 2 for the case of a singly charged vector bilepton $U^+$. 

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Fig. 4 – The same as in Fig. 2 for the case of an exotic quark $S$ with charge $5/3 \, e$.

Fig. 5 – The same as in Fig. 2 for the case of an exotic quark $D$ with charge $-4/3 \, e$.

Fig. 6 – Unpolarized cross section for $\gamma\gamma \to \gamma\gamma$ scattering in the 3-3-1 model. We have assumed a simplistic scenario where the mass of the three quarks is $M_Q$. Both vector bileptons are given a mass of $M_U = 300 \, \text{GeV}$. 
Fig. 7 – The same as in Fig. 6 when the vector bilepton mass is $M_U = 600$ GeV.

Fig. 8 – The same as in Fig. 6 when the vector bilepton mass is $M_U = 900$ GeV.