World-Volume Locally Supersymmetric Born-Infeld Actions

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Abstract

We derive manifestly locally supersymmetric extensions of the Born-Infeld action with $p = 2$. The construction is based on a first order bosonic action for $Dp$-branes with a generalized Weyl invariance.
1 Introduction

In string theory, the Born-Infeld action [1] arises as (part of) the low energy effective action of open superstring theory [2, 3, 4, 5], and as (the bosonic part of) the effective actions for $Dp$-branes in type IIA and IIB theories [6]. Various supersymmetrizations of the BI action have been discussed. Global supersymmetrization of the ambient spacetime variables is discussed in [9, 10, 11, 12, 13]. This gives the analogue of the Green-Schwarz string action and thus also includes local $\kappa$-symmetry. Global world volume supersymmetrization is discussed in component form for $p = 3$ in [7, 8, 29]. For certain $p$’s global superspace formulation of world volume supersymmetry also exist [14, 15, 16, 17, 28].

In this paper we try to combine the advantages of having a geometrical first order formulation involving an auxiliary metric with the advantage of having supersymmetry manifest. This requires a world-volume superspace supergravity extension of the Born-Infeld action. As the form of superspace supergravity is highly dimension dependent we do not expect to find a general prescription. Below we restrict ourselves to $p = 2$. This case is also of interest since $D = 3$ supergravity is non-dynamical and one might hope to retain some simplicity even in the presence of the supergravity action.

We base our construction on a bosonic first order action for arbitrary $Dp$-branes. This action has a generalized type of Weyl invariance which reduces the number of degrees of freedom in the auxiliary second rank tensor field. It would have been nice to be able to extend this to a super-Weyl invariance for the locally supersymmetric theory, and indeed we find a super-Weyl invariant candidate action. Unfortunately it leads to unwanted bosonic terms, however.

The plan of the paper is as follows: In Sec.2 we review some background mainly concerning the spinning membrane. Sec.3 introduces the new bosonic Born-Infeld action and Sec.4 presents $D = 3$ superspace along with our superspace actions. In Sec.5 we give the reduction of the superfields to components preparing for Sec.6 where the component form of the actions are given. Sec.7, finally, contains our conclusions.

2 Background

In [18] the following general Weyl invariant action for $p$-branes was presented:

$$I_W = \int d^{p+1}\xi \sqrt{-g} (g^{mn} \gamma_{mn})^{p+1},$$

(1)
where \( X^\Lambda, \ \Lambda = 0, \ldots, D - 1 \) are coordinates in the \( D \)-dimensional (target) space-time, \( \xi^m, \ m = 0, \ldots, p \) coordinatize the \( p \)-brane world volume, \( \gamma_{mn} = \partial_m X^\Lambda \partial_n X^\Omega G_{\Lambda \Omega} \) is the world-volume metric induced from the space-time metric \( G_{\Lambda \Omega} \) and \( g \) is the determinant of the auxiliary metric \( g_{mn} \).

For \( p = 1 \) the action (1) agrees with the usual string action with an auxiliary metric \( g_{mn} \) \[19, 20\]. Coupling it to 2D supergravity in superspace leads to the spinning string:

\[
I = \int d^2 \xi d^2 \theta E^{-1} \nabla^\alpha X \nabla_\alpha X,
\]

where all fields are superfields, \( E \) is the (super-)determinant of the supervielbein, \( \nabla_\alpha \) are the spinorial covariant derivatives and we have suppressed the ambient space-time indices on \( X \).

For \( p = 2 \) the action (1) agrees with the standard cosmological term action \[21\]

\[
I = \int d^3 \xi \sqrt{-g} (g^{mn} \gamma_{mn} - 1).
\]

After eliminating \( g_{mn} \) they both give the Nambu-Goto type action representing the volume of the world-volume. However, whereas (3) is impossible to couple to 3D-supergravity \[22\], this is not so for the \( p = 2 \) version of the action (1). In \[23\] it was shown that there exist (at least) two different supersymmetrizations of (1) when \( p = 2 \). They read:

\[
I_1 = \int d^3 \xi d^2 \theta E^{-1} (\nabla^\alpha X \nabla_\alpha X)(\nabla^{\beta\gamma} X \nabla_{\beta\gamma} X)^{1/2},
\]

and

\[
I_2 = \int d^3 \xi d^2 \theta E^{-1} (i \nabla^\alpha X \nabla_\alpha X)(i \nabla^{\beta\gamma} X \nabla_{\beta\gamma} X)(\nabla^{\alpha\beta} X \nabla_{\alpha\beta} X)^{-1/2},
\]

where again all fields are superfields and we have introduced 3D-superspace supergravity, to be described in more detail below. In none of the actions (4) and (5) does the bosonic Weyl invariance extend to super-Weyl invariance. However, in \[23\] it was found that the combination \( I_1 - \frac{2}{3} I_2 \) is in fact super-Weyl invariant.

### 3 Born-Infeld actions

The Born-Infeld action

\[
I_{BI}^{p+1} = T_p \int d^{p+1} \xi \sqrt{- \det(\gamma_{mn} + F_{mn})}
\]

is a direct generalization of the Nambu-Goto type action for \( p \)-branes by inclusion of the two-form field \( F_{mn} \). In the context of \( D \)-branes the brane tension \( T_p \) is related to the fundamental string tension \((\alpha')^{-1}\) and the string coupling constant \( g_s \), and

\[
F_{mn} \equiv B_{mn} + 2\pi \alpha' F_{mn}.
\]

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with \( F = dA \) the \( U(1) \) field strength for the world-volume field \( A_m \) and \( B_{mn} \) the pull-back to the world-volume of the Kalb-Ramond antisymmetric tensor field. (There is also a multiplicative exponent of a dilaton field in the action which will play no role in our considerations. We set it to 1 w.l.o.g.). In analogy to the \( p \)-brane case there is a first order action of the cosmological term type for the action (6), \([18]\). It is given by (c.f. (3)),

\[
I^{BI}_p = T_p \int d^{p+1} \xi \sqrt{-s} \left[ s^{mn}(\gamma_{mn} + \mathcal{F}_{mn}) + (p - 1) \right],
\]

where \( s^{mn}(\xi) \) is a general tensor (it has no symmetry) and \( s \equiv \det s_{mn} \). This form of the action has been used in \([24]\) as the starting point for a discussion of the strong coupling limit of \( D \)-branes. The discussion was extended to include spin in that limit in \([25]\). However, the same objections to a direct local supersymmetrization that were raised in \([22]\) apply to the action (8) for \( p = 2 \), since then (8) becomes (3) for \( F = 0 = s^{[mn]} \). We thus have to look for an action analogous to (1) to use as a starting point. For \( p = 3 \) this was given in \([18]\) and for general \( p \) it is

\[
I^{BI}_p = T_p \int d^{p+1} \xi \sqrt{-s} \left[ s^{mn}(\gamma_{mn} + \mathcal{F}_{mn}) \right]^{p+1}. \tag{9}
\]

It is easy to see that eliminating \( s_{mn} \) the action (8) is recovered\(^1\). Furthermore we note that the action (3) has a (generalized) Weyl invariance and that it reduces to the action (1) in the limit \( F = 0 = s^{[mn]} \).

We now specialize to \( p = 2 \). Since our goal is a locally supersymmetric superspace action of the (4) or (5) type, we must reformulate (9) in terms of vielbeins. As a first step in that direction we separate the symmetric and antisymmetric part in \( s_{mn} \) according to

\[
s_{mn} = g_{mn} + \epsilon_{mnp} h_p / \sqrt{-g}. \tag{10}
\]

It follows that \( \sqrt{-s} = \sqrt{-g} / \sqrt{1 - h^2} \), and the action (9) becomes, (properly normalized for \( p = 2 \) and with \( T_p \) set to one),

\[
\frac{1}{3\sqrt{3}} \int d^3 \xi \sqrt{\frac{-g}{1 - h^2}} \left[ g^{mn} \gamma_{mn} + \epsilon_{mnp} h_p \mathcal{F}_{mn} / \sqrt{-g} \right]^{3/2}, \tag{11}
\]

where \( h^2 \equiv g^{mn} h_m h_n \). Introducing vielbeins \( e_a \equiv e_a^m \partial_m \) and their inverses \( e^a \equiv e^a_m d\xi^m \) according to

\[
g_{mn} = e_a^m e_b^n \eta_{ab}, \quad e_a^m e^a_n = \delta_m^n, \quad e^{-1} = \det(e^a_m), \tag{12}
\]

where \( a = 0, 1, 2 \) are tangent space indices and \( \eta_{ab} \) is the Minkowski metric, we may rewrite the action (11) as follows:

\[
\frac{1}{3\sqrt{3}} \int d^3 \xi \sqrt{\frac{-1}{1 - h^2}} \left[ e_a X e^a X + e_a^m e_b^n \epsilon^{abc} \mathcal{F}_{mn} \right]^{3/2}. \tag{13}
\]

\(^1\)We are being careless with normalisation. For exact equivalence we should specify the \( p \)-dependent numerical factors in front of the actions.
The independent fields are now $X^A, A_m, e^m_a$ and $h_a$. As a check, we have verified that (3) is indeed equivalent to the 3D Born-Infeld action (3). The Weyl invariance is now of the usual type, i.e., a rescaling of $e^m_a$ with $h_a$ inert. This form is suitable for supersymmetrization.

4 Superspace

We will use the 3D superspace conventions of [20]. The supergravity algebra is given by

\begin{align*}
\{\nabla_\alpha, \nabla_\beta\} &= 2i\nabla_{\alpha\beta}, \\
[\nabla_\alpha, \nabla_{\beta\gamma}] &= C_{\alpha(\beta}\gamma\rangle, \\
U_\alpha &= -iR\nabla_\alpha + i\frac{2}{3}(\nabla_\beta R)M_\alpha^\beta + iG_\alpha^\gamma M^\gamma_\beta, \\
\end{align*}

(14)

where $M$ are Lorentz generators and $R$ and $G_{\alpha\beta\gamma}$ are the basic superfields that solve the Bianchi identities ($G_{\alpha\beta\gamma}$ is completely symmetric in all three spinor indices.). In this notation a vector index is represented by a symmetrized pair of spinor indices. The covariant derivatives have the usual structure

\begin{align*}
\nabla_A = E_M^A D_M + \Phi_A \cdot M
\end{align*}

with $E_M^A$ being the supervielbein, $D_M$ the flat superspace covariant derivatives and $\Phi_A$ the connection. The matter superfields that we will consider are $X_{\Lambda}(\xi, \theta)$ whose $\theta$-independent part is the space-time coordinate and $H_\alpha$ which is a spinor superfield whose covariant derivative $H_\alpha = \frac{1}{2}\nabla_{(\alpha} H_{\beta)}$ has the bosonic vector $h_a$ as lowest component. Finally, the dual of the Maxwell field-strength\(^2\) is the lowest component of $F_{\alpha\beta} = \frac{1}{2}\nabla_{(\alpha} W_{\beta)}$, with $W_{\alpha}$ the electromagnetic spinor potential.

With the above ingredients we immediately write down two possible generalizations of (4) and (5):

\begin{align*}
I_1 &= \int d^3\xi d^2\theta E^{-1}(1 - H^2)^{-1/2}(\nabla^\alpha X \nabla_\alpha X - H^\alpha W_\alpha)S^{1/2}, \\
I_2 &= \int d^3\xi d^2\theta E^{-1}(1 - H^2)^{-1/2}\varphi^\alpha \varphi_\alpha S^{-1/2}, \\
\end{align*}

(15) and (16)

where

\begin{align*}
S &\equiv (\nabla^\beta_\gamma X \nabla_{\beta\gamma} X + H^\alpha_{\beta\gamma} F_{\alpha\beta\gamma}), \\
\varphi_\gamma &\equiv (i\nabla^\alpha X \nabla_{\alpha\gamma} X + H^\alpha_\gamma W_\alpha), \\
H^2 &\equiv H^\alpha_{\beta\gamma} H_{\alpha\beta}. \\
\end{align*}

(17)

These actions reduce to (4) and (5) when $H_\alpha = 0$, and in the next section we show that the bosonic parts of both (15) and (16) are equivalent to the action (13). Before going to components let us look at (possible) super-Weyl invariance [27]. Under an infinitesimal super Weyl transformation with parameter $\Lambda$ the superfields transform as follows:

\begin{align*}
\delta E_\alpha &= \Lambda E_\alpha, \quad \delta E_{\alpha\beta} = 2\Lambda E_{\alpha\beta} - i(E_{(\alpha}\Lambda) E_{\beta)} \Rightarrow \delta E^{-1} = -4\Lambda E^{-1},
\end{align*}

\(^2\)From now on we put the background $B$-field to zero.
\[ \delta H_\alpha = -\Lambda H_\alpha, \quad \delta H_{\alpha\beta} = 0, \]
\[ \delta W_\alpha = 3\Lambda W_\alpha, \quad \delta F_{\alpha\beta} = 4\Lambda F_{\alpha\beta} + 2(E_{(\alpha\Lambda})W_{\beta}), \] (18)

where we have used that [26]

\[ W_\alpha \equiv \nabla^\beta \nabla_\alpha \Gamma_\beta + 2R \Gamma_\alpha, \] (19)

and that

\[ \delta \Gamma_\alpha = \Lambda \Gamma_\alpha, \quad \delta R = 2\Lambda \rho - 2\nabla^2 \Lambda, \quad \delta \Phi_{\alpha\beta\gamma} = -(E_{(\gamma\Lambda)}C_{\beta\alpha} + \Lambda \Phi_{\alpha\beta\gamma}). \] (20)

The transformation of \( H_\alpha \) was determined from the requirement that \( \delta h_\alpha = 0 \), in agreement with the discussion in Sec.3.

The difficulty in finding an invariant action is entirely due to the inhomogeneous parts of the transformations in (18). We find the following super-Weyl invariant combination of the action in (16) and a modification of the action in (15):

\[ I_W = \bar{I}_1 - \frac{2}{3} I_2, \] (21)

where \( \bar{I}_1 \) indicates that we have made the replacement \( (\nabla^\alpha X \nabla_\alpha X - H^\alpha W_\alpha) \to (\nabla^\alpha X \nabla_\alpha X) \) in the Lagrangian. As will be seen in Sec.6, the action \( I_W \) in (21) has a bosonic part that differs from the action in (13), so it is not a supersymmetrization of that. It does however reduce to the super-Weyl invariant combination of (4) and (5) when \( H_\alpha = 0 \).

## 5 Components

There is a systematic procedure for deriving the components, the component action and the component local supersymmetry transformations of a theory in superspace. It is described in [26] for \( D = 4 \) supergravity actions. For the case at hand we use the definitions and results in [23] to which we add those pertaining to the \( H_\alpha \) and \( W_\alpha \) fields. We define

\[ X^\Lambda | \equiv A^\Lambda, \quad \nabla_\alpha X^\Lambda | \equiv \chi^\Lambda_\alpha, \quad \nabla^2 X^\Lambda | \equiv \frac{1}{2} \nabla^\alpha \nabla_\alpha X^\Lambda | \equiv T^\Lambda, \]

\[ \nabla_\alpha | \equiv \partial_\alpha, \quad \nabla_{\alpha\beta} | \equiv D_{\alpha\beta} + iS M_{\alpha\beta} + \Psi_{\alpha\beta} \gamma \nabla |, \]

\[ R | \equiv S, \] (22)

where | denotes “the \( \theta \) independent part of”. Then the superspace-component relations involving the matter superfield become (suppressing the space-time indices)

\[ \nabla_{\alpha\beta} X | = D_{\alpha\beta} A + \Psi_{\alpha\beta} \gamma \chi_\gamma \equiv \hat{D}_{\alpha\beta} A, \]
\[
\n\n\n\]

where

\[
G_{\alpha \beta \gamma} \equiv G_{\alpha \beta | \gamma} = \frac{1}{6} \left( D_{\delta (\alpha} \Psi_{\beta \gamma)}^{\delta} - t_{\delta (\alpha}^{\beta} \epsilon_{\beta \gamma}^{\delta} \Psi_{|\lambda|\lambda|\gamma} + \frac{i}{2} S \Psi_{(\alpha \beta \gamma)} \right),
\]

\[
\eta_{\alpha} \equiv \nabla_{\alpha} R_{\gamma} = - \frac{1}{2} \left( D_{\delta (\alpha} \Psi_{\beta \gamma)}^{\delta} - t_{\delta (\alpha}^{\beta} \epsilon_{\beta \gamma}^{\delta} \Psi_{|\lambda|\lambda|\gamma} + i S \Psi_{\alpha \beta \gamma} \right),
\]

\[
t_{\alpha \gamma \delta}^{\epsilon \lambda} = - \frac{1}{2} i \left( C_{\alpha \gamma} \Psi_{\sigma (\beta}^{\sigma} \epsilon_{\beta \gamma}^{\sigma} \Psi_{|\lambda|\lambda|\gamma} + C_{\beta \gamma} \Psi_{\sigma (\alpha}^{\lambda} \sigma (\beta}^{\sigma} \Psi_{|\lambda|\lambda|\gamma} \right).
\]

Our vielbein components are defined by \( \nabla_{\alpha} | \) and \( \nabla_{\alpha \beta} | \) in \([22]\) to be\(^3\)

\[
E_{\alpha}^{\mu} = \delta_{\alpha}^{\mu}, \quad E_{\alpha}^{\mu \nu} = 0, \quad E_{\alpha \beta}^{\mu} = \Psi_{\alpha \beta}^{\mu}, \quad E_{\alpha \beta}^{\mu \nu} = e_{\alpha \beta}^{\mu \nu} = e_{\alpha \beta}^{m}
\]

The components of the prepotential are

\[
W_{\alpha} | \equiv \lambda_{\alpha},
\]

\[
F_{\alpha \beta} | = \frac{1}{2} \nabla_{(\alpha} W_{\beta)} = f_{\alpha \beta},
\]

\[
\nabla_{\alpha} F_{\beta | \gamma} = \sigma_{\alpha \beta \gamma} + SC_{\alpha \beta \gamma}^{\lambda} - SC_{\alpha \beta \gamma}^{\lambda},
\]

\[
\sigma_{\alpha \beta \gamma} \equiv i \left[ D_{\alpha (\beta \gamma) \lambda} - D_{\beta \gamma \lambda} \cdot f_{\delta \alpha \lambda} \cdot \Psi_{\beta \gamma}^{\delta} + f_{\delta \beta \gamma} \cdot \Psi_{\alpha \lambda}^{\delta} \right],
\]

\[
\nabla^{2} F_{\beta | \gamma} = \sigma_{\beta \gamma} - \frac{i}{2} S \Psi_{(\beta \gamma)}^{\rho \lambda} \sigma_{\beta \gamma} = \sigma_{\beta \gamma} + \frac{i}{2} S \Psi_{(\beta \gamma)}^{\rho \lambda} \sigma_{\beta \gamma} + \frac{1}{2} D_{\rho (\beta \gamma)} \cdot f_{\gamma}^{\rho} + \frac{1}{2} \Psi_{\rho (\beta \gamma)}^{\lambda} \sigma_{\lambda}^{\rho},
\]

We define the components of the spinor superfield as follows

\[
H_{\alpha} | \equiv h_{\alpha},
\]

\[
\nabla_{\alpha} H_{\beta} \equiv (H_{\alpha \beta} + C_{\alpha \beta} B) | \equiv h_{\alpha \beta} + C_{\alpha \beta} b,
\]

\[
\nabla^{2} H_{\alpha} \equiv \mu_{\alpha} + \frac{1}{2} S h_{\alpha},
\]

---

\(^{3}\)As seen from the component of the spinorial derivative, we use a Wess-Zumino gauge.
\[ \nabla_\alpha H_{\beta\gamma} \equiv \phi_{\alpha\beta\gamma} = \frac{i}{2} D_{\alpha(\beta h_{\gamma})} - \frac{1}{2} C_{\alpha(\beta\mu\gamma)} + \frac{i}{2} \Psi_{\alpha(\beta\rho h_{\gamma})\rho} + \frac{i}{2} \Psi_{\alpha(\beta\gamma)} b, \]

\[ \nabla^2 H_{\alpha\beta} \equiv \phi_{\alpha\beta} = \frac{i}{2} D_{\gamma(\alpha h_{\beta})} - i D_{\alpha\beta} b + \frac{1}{3} \eta(\alpha h_{\beta}) - 2 g_{\alpha\beta\gamma} h_{\gamma} + \frac{i}{2} \Psi_{\gamma(\alpha\rho h_{\beta})\rho} + \frac{1}{2} \Psi_{\gamma(\alpha\rho h_{\beta})\rho} b - i S \Psi_{\alpha\beta h_{\rho}}, \]

\[ \nabla_\gamma B \equiv b_{\gamma} = \frac{1}{2} \mu_\gamma + \frac{i}{2} D_{\gamma(\alpha h_{\beta})} + S h_{\gamma} + \frac{i}{2} \Psi_{\gamma(\alpha h_{\beta})\rho} b + \frac{i}{2} \Psi_{\gamma(\alpha h_{\beta})} b, \]

\[ \nabla^2 B = \frac{i}{2} D_{\alpha\beta h_{\alpha\beta}} + \frac{i}{2} \Psi_{\alpha\beta h_{\gamma}} b - 2 S b - \eta_{\beta} h_{\beta}. \] (27)

Note that the explicit dependence on \( S \) in \( \nabla^2 H_{\alpha\beta} \) is canceled by the S-terms in \( G_{\alpha\beta\gamma} \) and \( \eta_{\beta} \). \( H_{\alpha\beta} \) is thus independent of \( S \), in agreement with the fact that it does not transform under the Super-Weyl transformations (18).

The component local supersymmetry transformations that leave the component action invariant are

\[ \delta A^\Lambda = -e^\alpha \chi_\alpha^\Lambda, \]
\[ \delta \chi_\alpha^\Lambda = e_\alpha T^\Lambda - i e^\beta \nabla_{\beta\alpha} A^\Lambda, \]
\[ \delta T^\Lambda = i e_\alpha \nabla_{\alpha\beta} A^\Lambda + \frac{1}{2} S e_\alpha A^\Lambda, \]
\[ \delta e_{\alpha\beta}^{\mu\nu} = -2 i e^\gamma \Psi_{\alpha\beta} e_{\gamma} e_{\gamma}^{\mu\nu}, \]
\[ \delta \Psi_{\alpha\beta} = D_{\alpha\beta} e^\rho - 2 i e^\mu \Psi_{\alpha\beta} e_{\mu\rho} + \frac{i}{2} S e_{(\alpha} \delta_{\beta)}, \]
\[ \delta S = -e^\alpha \eta_\alpha, \]
\[ \delta h_\alpha = -e_\alpha b - e^\beta h_{\beta\alpha}, \]
\[ \delta h_{\alpha\beta} = -e^\gamma \phi_{\gamma\alpha\beta}, \]
\[ \delta b = -e^\alpha b_\alpha, \]
\[ \delta \lambda_\alpha = -e^\beta f_{\beta\alpha}, \]
\[ \delta \mu_\alpha = e^\beta \left[ i D_{\beta\gamma} h_{\alpha}^\gamma + \frac{3}{2} S h_{\alpha\beta} - D_{\alpha\beta} b + i \Psi_{\beta\gamma} \phi_{\sigma h_{\gamma}} + i \Psi_{\alpha\beta} h_{\gamma} \right] \]
\[ + \frac{1}{4} e_\alpha (\eta^2 h_{\gamma} - 2 S b), \]
\[ \delta f_{\alpha\beta} = -e^\gamma \phi_{\gamma\alpha\beta} - S e_{(\alpha} \lambda_{\beta)}. \] (28)

6 Component actions

Having defined the components and derived the relations in (22) - (27), we are in a position to find the component actions from (13) and (16). We use the 3D density formula derived
in (\ref{eq:23}):

\[
\int d^3\xi d^2\theta E^{-1}\mathcal{L} = \int d^3\xi e^{-1} \left[ (\nabla^2 + i\Psi^\alpha_{\beta} \nabla_\alpha + 2S + \Psi_{\alpha(\beta} \Psi_{\gamma)^\alpha} ) \mathcal{L} \right].
\] (29)

We first want to establish the equivalence of the bosonic parts of (15) and (16) to (13). The bosonic contents of the actions simply follow from taking the bosonic part of the first term on the right hand side of (29)\footnote{Since the pure bose part of \(\mathcal{L}\) is zero, the \(S\)-term will not contribute.}. We thus find for the action in (15)

\[
I_1 \rightarrow \int d^3\xi \frac{e^{-1}}{\sqrt{1-h^2}} \left[ \Omega - 2T^2 \right] \Omega^{1/2},
\] (30)

where \(\Omega\) is the bosonic part of \(S\), defined in (17), i.e., it is the factor which is raised to 3/2 in the action (13). Obviously, integrating out the auxiliary field \(T\), we recover the action (13).

The action in (16) is a little more challenging. It gives, (keeping the spinor notation),

\[
I_2 \rightarrow -\int d^3\xi \frac{e^{-1}}{\sqrt{1-h^2}} \left[ \frac{1}{2}\Omega^2 + \left( -iT \cdot D_{\alpha\beta} A + \frac{1}{2}h^\sigma\_{(\alpha f_{\beta)\sigma}} \right) \left( -iT \cdot D^{\alpha\beta} A + \frac{1}{2}h^\gamma(\alpha f_{\beta)\gamma} \right) \right] \Omega^{-1/2},
\] (31)

where dot denotes contraction over the ambient spacetime indices. It is a remarkable fact, manifest in (31), that integrating out the auxiliary field \(T\), we again recover the action (13).

From the expressions (30) and (31) we may also read off the \(T\) equations that result from the super-Weyl invariant action \(\tilde{I}_1 - \frac{2}{3}I_2\), they are

\[
\Omega T^\Lambda + \frac{i}{3}D^{\alpha\beta} A^\Lambda h^\gamma_{\alpha f_{\beta\gamma}} + \frac{1}{3}D^{\alpha\beta} A^\Lambda (T \cdot D_{\alpha\beta} A) = 0,
\] (32)

which we only need to solve for \(T \cdot D_{\alpha\beta} A\), where

\[
(T \cdot D_{\alpha\beta} A)\mathcal{M}^\alpha_{\gamma\delta} = \frac{-i}{3\Omega} (D_{\gamma\delta} A \cdot D^{\alpha\beta} A)h^\gamma_{\alpha f_{\beta\gamma}},
\] (33)

and

\[
\mathcal{M}^\alpha_{\gamma\delta} = \frac{1}{2}\delta^\alpha_{(\gamma \delta)} + \frac{1}{3\Omega} (D_{\gamma\delta} A \cdot D^{\alpha\beta} A).
\] (34)

We note that for \(W_\alpha = 0\) the fact that \(\mathcal{M}\) is non-degenerate ensures that \(T^\Lambda = 0\) and thus that the correct bosonic action results in that case. For \(W_\alpha \neq 0\) the equation (33) is still solvable, the solution being expressed in the inverse \(\mathcal{M}^{-1}\) of the three by three matrix. It is clear, though, that the resulting action will be rather complicated and contain unwanted bosonic terms. For this reason we have not calculated it explicitly.
The component actions are

\[
I_1 = \int d^3\xi \frac{e^{-1}}{\sqrt{1-h^2}} \left[ \Omega_4^2 \right. \\
+ \Omega_4^2 \left\{ \left( 2i\chi^\alpha \cdot \hat{\nabla}_{\beta} \chi^\beta + \frac{1}{2} \sigma^\gamma_{\alpha} h^\alpha - 2T^2 - \mu^\alpha \lambda_\alpha + P\Psi_{\alpha(\beta} \Psi_{\gamma)\alpha\gamma} \\
+ i\Psi_{\beta} (2i\zeta_\alpha - 2T \cdot \chi_\alpha - h_{\alpha\gamma} \lambda^\gamma - b\lambda_\alpha + h^\gamma f_{\alpha\gamma}) + SP \right) \\
+ \Omega_4^{-\frac{3}{2}} \left\{ \left( 2i\zeta_\gamma + 2T \cdot \chi_\gamma + h_{\gamma\delta} \lambda^\delta + b\lambda_\gamma - h^\delta f_{\gamma\delta} \right) \xi^\gamma + \\
\frac{1}{4} P \left[ 4(\hat{\nabla}_{\delta} T \cdot \hat{\nabla}_{\delta} A - \frac{1}{2} \hat{\nabla}_{\delta} \chi_\gamma \cdot \hat{\nabla}_{\delta} \chi^\gamma - i\chi^\sigma \cdot \hat{\nabla}_{\delta} \chi^\alpha A_{\delta\sigma}) \\
+ \frac{8i}{3} \hat{\nabla}_{\delta} A \cdot \chi(\eta_\delta) + 2\phi_{\delta\epsilon} f^\epsilon - 2\phi_{\gamma\delta} \sigma^\gamma_{\delta\epsilon} + 2h_{\delta\sigma} \sigma^\delta_{\epsilon} - 2S\Psi_{\delta\epsilon} \rho_\chi \cdot \hat{\nabla}_{\delta} A \\
\right] + 4iS \chi^\beta \cdot \hat{\nabla}_{\beta} \chi^\gamma + 2iSh_{\delta\epsilon} (\Psi_{\rho\epsilon} \lambda^\rho - \Psi_{\rho\epsilon} \lambda^\rho) + 4S \phi_{\gamma\beta} \lambda^\beta \\
- 3S^2 \chi^2 + 2i\Psi\phi_{\beta} \beta \xi^\gamma \right\} \right] \\
- \frac{1}{8} \Omega_4^{-\frac{3}{2}} P\xi^2 \\
- \frac{1}{1-h^2} \left\{ \Omega_4^2 \left[ \left( \frac{1}{2} P[\phi_{\delta\epsilon} \phi_{\gamma\delta} - 2h_{\delta\epsilon} \phi_{\gamma\delta} - 2i\Psi_{\delta\epsilon} \phi_{\delta\rho} h_{\rho\sigma}] \\
+ \rho_{\gamma\beta} h_{\alpha\delta} (2i\zeta_\gamma + 2T \cdot \chi_\gamma + h_{\gamma\delta} \lambda^\delta + b\lambda_\gamma - h^\delta f_{\gamma\delta} \right) \\
+ \frac{1}{2} \Omega_4^{-\frac{3}{2}} P\phi_{\gamma\beta} h_{\alpha\delta} \phi_{\beta\sigma\rho} h_{\rho^\sigma} \right\} \\
+ \frac{3}{2(1-h^2)} \Omega_4^{-\frac{3}{2}} P\phi_{\gamma\beta} h_{\alpha\delta} \phi_{\beta\sigma\rho} h_{\rho^\sigma} \right]\right]
\]

and

\[
I_2 = \int d^3\xi \frac{e^{-1}}{\sqrt{1-h^2}} \left[ -\frac{1}{2} \Omega_4^2 \\
+ \Omega_4^2 \left\{ \left[ iT^\gamma + \frac{3}{2} S \chi^2 - \phi_{\gamma\alpha} \lambda_\alpha \right] + \Psi_\gamma \beta \xi^\gamma \right\} \\
+ \Omega_4^{-\frac{3}{2}} \left\{ \left[ -2\hat{\nabla}_{\alpha} \chi^\gamma \cdot \hat{\nabla}_{\alpha} A + 2\hat{\nabla}_{\alpha} A \cdot \hat{\nabla}_{\alpha\beta} \chi_\gamma + 3ST \cdot \chi_{\beta} + 2i\chi^\alpha \cdot \hat{\nabla}_{\alpha} T \\
- 2iT \cdot \hat{\nabla}_{\alpha} \chi^\alpha - 2\chi^2 \eta_\beta - \frac{1}{2} iS \chi^2 \Psi_{\beta\alpha} \\
+ 2\phi_{\gamma\alpha} \lambda_\alpha - 2\phi_{\beta\alpha} \gamma_{\alpha\gamma} + h_{\beta\gamma} \sigma^\gamma_{\alpha\gamma} - 3s_{\beta} h_{\alpha\gamma} \xi^\beta \\
- (iT^\beta \gamma - \frac{3}{4} S C_{\gamma\alpha} \chi^2 - \phi_{\alpha\gamma} \lambda_\alpha \right) \left( iT_{\gamma\beta} + \frac{3}{4} S C_{\gamma\alpha} \chi^2 - \phi_{\alpha\gamma} \lambda_\alpha \right) \\
+ 2\Psi_{\alpha\beta} \left( -iT_{\gamma\alpha} \gamma - \frac{3}{4} S C_{\gamma\alpha} \chi^2 + \phi_{\alpha\gamma} \lambda_\alpha \right) \right\} \right] \\
- \frac{1}{2} \xi^\gamma \left\{ \left[ \left( iT^\gamma + \frac{3}{4} S C_{\gamma\alpha} \chi^2 - \phi_{\gamma\alpha} \lambda_\alpha \right) i\xi^\alpha \xi^\gamma \\
+ \frac{1}{4} \xi^2 \left\{ 4 \left[ \hat{\nabla}_{\delta} T \cdot \hat{\nabla}_{\delta} A - \frac{1}{2} \hat{\nabla}_{\delta} \chi_\gamma \cdot \hat{\nabla}_{\delta} \chi^\gamma - i\chi^\sigma \cdot \hat{\nabla}_{\delta} \chi^\alpha A_{\delta\sigma} \right] \\
+ \frac{8i}{3} \hat{\nabla}_{\delta} A \cdot \chi(\eta_\delta) + 2\phi_{\delta\epsilon} f^\epsilon - 2\phi_{\gamma\delta} \sigma^\gamma_{\delta\epsilon} + 2h_{\delta\sigma} \sigma^\delta_{\epsilon} - 2S\Psi_{\delta\epsilon} \rho_\chi \cdot \hat{\nabla}_{\delta} A \\
+ 4iS \chi^\beta \cdot \hat{\nabla}_{\beta} \chi^\gamma + 2iSh_{\delta\epsilon} (\Psi_{\rho\epsilon} \lambda^\rho - \Psi_{\rho\epsilon} \lambda^\rho) + 4S \phi_{\gamma\beta} \lambda^\beta \\
- 3S^2 \chi^2 + 2i\Psi\phi_{\beta} \beta \xi^\gamma \right\} \right\} \right]\right]
\]

\[
10
\]
\[
-\frac{1}{(1-h^2)}\{i\Omega^{\frac{1}{2}}\phi_{\alpha\lambda\sigma}h^{\lambda\sigma}\zeta^\alpha
+\Omega^{-\frac{1}{2}}\{\frac{1}{2}\zeta^2(\phi_{\gamma\delta\epsilon}\phi^{\gamma\delta\epsilon} - 2h_{\delta\epsilon}\phi^{\delta\epsilon})
-2\phi_{\gamma\lambda\sigma}h^{\lambda\sigma}(-iT^\gamma - \frac{3}{4}SC^\gamma_\alpha\lambda^2 + \phi^{\gamma}_{\alpha}\delta_{\lambda\beta})i\zeta^\alpha
+i\Psi_{\alpha\beta}\zeta^2\phi_{\alpha\sigma\beta}h^{\sigma\rho}\}
+\Omega^{-\frac{1}{2}}\frac{1}{2}\zeta^2\phi_{\gamma\sigma\rho}h^{\sigma\rho}\zeta^\gamma
\frac{3}{2(1-h^2)^2}\Omega^{-\frac{1}{2}}\zeta^2\phi_{\delta\epsilon}\delta_{\delta\epsilon}\phi_{\gamma\sigma\rho}h^{\sigma\rho}\}
\]  
(36)

where we have introduced the definitions

\[
\Omega \equiv \tilde{\nabla}_{\alpha\beta}A\cdot\tilde{\nabla}^{\alpha\beta}A + h_{\alpha\beta}f^{\alpha\beta}
\]
\[
P \equiv \chi^2 + h_{\alpha}\lambda^\alpha
\]
\[
\zeta_\alpha \equiv \chi^\alpha\cdot\tilde{\nabla}_{\alpha\beta}A + ih_{\alpha\beta}\lambda^\beta
\]
\[
\xi_\gamma \equiv 2\tilde{\nabla}^{\alpha\beta}A\cdot\tilde{\nabla}_{\alpha\beta}\chi_\gamma + \phi_{\gamma\alpha\beta}f^{\alpha\beta} + \sigma_{\gamma\alpha\beta}h^{\alpha\beta} - 2iS\xi_\gamma
\]
\[
T_{\alpha\beta} \equiv \mathcal{T}\cdot\tilde{\nabla}_{\alpha\beta}A + \chi^\alpha\cdot\tilde{\nabla}_{\beta\gamma}\chi_\alpha + \frac{i}{2}h_{(\alpha}^{\gamma}f_{\beta)\gamma}
\]
\[
\chi^2 \equiv \chi^\alpha\cdot\chi_\alpha
\]
\[
\zeta^2 \equiv \zeta^\alpha\zeta_\alpha
\]
\[
\xi^2 \equiv \xi^\alpha\xi_\alpha
\]
\[
T^2 \equiv \mathcal{T}\cdot\mathcal{T}.
\]
(37)

The component version of the super-Weyl invariant combination (24) can be reassembled from (35) and (36) (omitting the terms from $H^\alpha W_\alpha$). The super-Weyl invariance should manifest itself in an independence of $S$. We have only checked the quadratic $S$-terms which indeed cancel.

\section{7 Discussions and conclusions}

We have studied a first order “Weyl invariant” bosonic action for $Dp$-branes for the special case of $p = 2$ and shown how it can be coupled to 3D supergravity in (at least) two different ways. We have further derived the component content of the model, both for the actions and the local supersymmetry transformations. Our results generalize those for the spinning membrane, except when it comes to super-Weyl invariance. The super-Weyl invariant action we find does indeed reduce to that of the spinning membrane when we turn off the worldvolume Maxwell fields, but it does not have the correct bosonic limit when they are non-zero. Although we have not been able to find one, we see no reason why another super-Weyl invariant action with the correct limit should not exist. To construct it one would have to have some additional guideline, however.
An interesting question is of course whether the present construction generalizes to any other \( p \). Many of the features we encountered in the above constructions are three dimensional. E.g., the fact that we were able to reduce \( s_{ab} \rightarrow e_a, h_a \) is based on the 3D equivalence of a vector and an antisymmetric tensor. For higher \( p \) we typically have \( h_{ab} \) instead of \( h_a \) and we thus expect the potential to have spin 3/2 instead of spin 1/2. The form of the superspace action will have to change because of the growth of the superspace measure. There is also the question of which multiplets to use. All in all, it looks as if each case has to be considered separately.

One of the perhaps unwanted features of the bosonic starting point is the non-linearity in (3). Usually, going from a Nambu-Goto type action to an action with an auxiliary metric leads to a quadratic behaviour in the path integral (after gauge fixing). This is not the case here, although we replace a square root of a determinant by powers of \( \partial X \). Further linearization may formally be achieved by introducing additional auxiliary fields and lagrange multipliers, but does not really seem very useful.

As far as we know, locally supersymmetric extensions of the Born-Infeld action have not been discussed previously. However, several globally (world volume) supersymmetric models in various dimensional superspaces exist. E.g., [7, 8, 14, 15] (\( p = 3, N = 1 \)) and [15, 16, 17] (\( p = 5, N = 1 \) and \( p = 3, N = 2 \)). It would be interesting to compare the present models with the globally supersymmetric D2-brane, which is discussed in [28].

As mentioned in the introduction, the motivation for studying the globally supersymmetric extensions of the \( BI \) action is usually either taken to be its appearance in the effective action of the \( 10D \) open superstring or its role in the \( D \)-brane effective action. In both these cases it is also interesting to investigate the possible existence of locally supersymmetric extensions, and the present results is a first contribution to this.

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