Similarity Transformation of Discrete Part Production Lines

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Abstract. In order to improve the computational efficiency of models of production lines with unreliable machines, we propose the similarity transformation method by adjusting the design parameters, such as times between machine failures, times to repair and buffer capacities, in an equal proportion. It is found that the system throughput remains almost unchanged under the similarity transformation. The typical manufacturing system, serial production line, is taken as an example to verify the proposed method. Theoretical analysis and numerical experiments have been performed to confirm the feasibility and effectiveness of the method. The results show that the throughput error is no more than 5% and the calculation time is much less than that of the original model. Therefore, it is helpful to apply the similarity transformation method to large production line models to reduce computational complexity.

1. Introduction

The manufacturing industry is the pillar industry of the national economy, and production lines are widely used in manufacturing industry, especially in large volume manufacturing. They are usually composed of a certain number of machines and buffers connected to each other. In general, most of the machines in production lines are unreliable and subject to random failures of machines and man-made operational error. The behaviors of production lines are very complicated and thus the analysis of production lines has become a growing concern of researchers.

In the literature, a large amount of research works have devoted to the modeling, performance analysis and optimization of production lines (see monographs [1]-[2] and review [3]-[5] for a comprehensive review). For the evaluation methods, existing works could be divided into simulation method and analytical method. Complex systems could be modelled by simulation method yet wasting time. The analytical method consists of exact method and approximate method. The former is mainly applied to two-machine system, and Li et.al. [6] summarized and compared the productivity analysis methods of some typical two-machine systems. Recently, some scholars contributed to the approximate method for complex two-machine systems which mainly included the decomposition method [7] and aggregation methods [8]. Afterwards, some research literatures have improved these approximate methods [9]-[11]. Most of the existing works focus on the performance evaluation, there are few studies on system-theoretic properties.

For example, Eginhard et.al [12] expounded the reversibility of production line under certain conditions and confirmed this view by rigorous mathematical analysis. David et.al [13] proposed the improvability of the production line which the system performance could be improved by reallocating the resources. PYUNG-HOIKOO et.al [14] proposed a new objective function of batch sorting based on
the queuing network model, alleviating the bottleneck of production line system through line search algorithm. Y. Narahari [15] et al. analysed the transient performance of discrete manufacturing system and took Markov model as an example, which owns important research value for studying the performance and sensitivity of production line system with limited resources. Li et al [16] studied the conditions for the establishment of monotonicity and reversibility for an abstract analysis model.

In this paper, we propose a new property, i.e., the similarity property, of discrete part production lines with unreliable machines. We introduce the concept of similarity transformation which indicates that the design parameters of a production line, such as times between machine failures, times to repair and buffer capacities, are adjusted in an equal proportion. It is found that the system throughput remains almost unchanged under the similarity transformation. Numerical experiments show that this method can be applied to long production lines to reduce the computational complexity of performance evaluation.

The rest of the paper is structured as follows: the system model of discrete part of serial line is established in Section 2. The similarity property is illustrated in Section 3 through theoretical analysis. Numerical experiments are conducted in Section 4. And in Section 5, conclusions are discussed.

2. System Model

The typical form of production lines, i.e., serial line, is taken as an example to illustrate the similarity transformation method. Fig.1 shows the structure of a serial production line, where circles represent the machines $M_i, i = 1, 2, \ldots, k$, and squares represent the buffer $b_i, i = 1, 2, \ldots, k - 1$. The materials flow into the system from $M_1$, passing through each machine on the production line in turn, and finally flows out from $M_k$. When the upstream machine $M_{i-1}$ of the machine $M_i$ fails, the amount of materials in the buffer $b_{i-1}$ between $M_i$ and $M_{i-1}$ decreases. When the number of materials in buffer $b_{i-1}$ is empty, the machine $M_i$ is said to be starved. On the contrary, when the downstream adjacent machine $M_{i+1}$ fails, the number of materials in the buffer $b_{i+1}$ between $M_i$ and $M_{i+1}$ increases, and when buffer $b_{i+1}$ is empty, the machine $M_i$ is said to be blocked.

![Serial line](image)

Due to the complexity of real production systems, the following assumptions are introduced:

1. Each machine may fail and it can be in either up state or down state. When it fails, the time waiting for maintenance can be neglected;

2. There is no starvation phenomenon in the first machine $M_1$ and no blocking phenomenon in the last machine $M_k$;

3. The machine up times (i.e., times between failures) and down times (i.e., times to repair) obey exponential distributions. The average up time (i.e., mean time between failures, MTBF) and down time (i.e., mean time to repair, MTTR) of machine $M_i$ are denoted as $t_{up}^i$ and $t_{down}^i$. $t_{up}^i$ is used to represent the machine up rate, then $1/t_{up}^i$ represents the average working time between the two adjacent faults; if $t_{down}^i$ is used to represent the machine failure rate, then $1/t_{down}^i$ represents the average repair time between the two adjacent faults.

4. Each machine is independent of each other. The cycle time of each machine is $c_i$, and the processing rate is $\mu_i = 1/c_i$;

5. Machine failures do not occur very frequently. More precisely, the frequency of machine failures is much less than the frequency of production, or equivalently, the MTBFs of machines are much larger than the machine cycle times.

6. Each buffer $b_i$ is characterized by its capacity $N$. 


The model described by the above assumptions can be formally denoted as a tuple \( (U, t_{up}, t_{down}, N) \), where \( U = [\mu_1, \mu_2, \ldots, \mu_i, \ldots, \mu_k] \) is the processing rate vector, \( t_{up} = [t_{up}^1, t_{up}^2, \ldots, t_{up}^k] \) and \( t_{down} = [t_{down}^1, t_{down}^2, \ldots, t_{down}^k] \) stand for times between failure and times to repair of each machine respectively, and \( N = [b_1, b_2, \ldots, b_{k-1}] \) represents for the buffer capacities.

3. Similarity Transformation and Properties

Our study is inspired by the similarity concept in geometry. It is well-known that two triangles are said to be similar if the corresponding sides are proportional. In a similar manner, we define the similarity concept of production systems. Consider two serial production lines with the same number of machines and buffers. They are said to be similar if the MTBFs and MTTRs of the corresponding machines and the capacities of the corresponding buffers are proportional, and the processing rates of the corresponding machines are the same. Formally, two production lines \( L = (U, t_{up}, t_{down}, N) \) and \( L' = (U', t_{up}', t_{down}', N') \) are similar if

\[
U = U', \ t_{up} = \alpha t_{up}', \ t_{down} = \alpha t_{down}', \ N = \alpha N' \tag{1}
\]

To derive the property of similarity transformation, we investigate how the system throughput is affected by changing time units and material units. Consider a production line \( L_1 = (U, t_{up}, t_{down}, N) \). If we change the time unit by a factor \( \alpha (\neq 0) \), then we obtain a different line \( L_2 = (\alpha U, \frac{1}{\alpha} t_{up}, \frac{1}{\alpha} t_{down}, N) \). Clearly, the two models represent the same physical system with different time scales. Therefore, the throughput of \( L_2 \) should be \( \alpha \) times that of \( L_1 \). Formally, we have the following property:

Property 1:

\[
TP_{L_2}(\alpha U, \frac{1}{\alpha} t_{up}, \frac{1}{\alpha} t_{down}, N) = \alpha TP_{L_1}(U, t_{up}, t_{down}, N) \tag{2}
\]

In our model, the material is discrete and thus we cannot change the material unit directly. However, due to Assumption 5 in Section 2, the behavior of the discrete model can be approximated by a continuous model. Intuitively, when the speed of discrete part flow is high, it can be viewed as a flow of continuous fluid. See review [17] and monograph [18] for detailed explanations. For continuous materials, we can change the unit of material as in the case of changing time unit. Accordingly, we have:

Property 2:

\[
TP_{L_2}(\alpha U, T_F, T_R, \alpha N) = \alpha TP_{L_1}(U, T_F, T_R, N) \tag{3}
\]

Combining the above two properties, we obtain an important property:

Property 3 (Similarity Property): For two similar systems: \( L_1 = (U, T_F, T_R, N) \) and \( L_2 = (\alpha U, \alpha T_F, \alpha T_R, \alpha N) \), where \( \alpha > 0 \), then the following equation holds:

\[
TP_{L_2}(\alpha U, \alpha T_F, \alpha T_R, \alpha N) = TP_{L_1}(U, T_F, T_R, N) \tag{4}
\]

It should be pointed out that the above similarity property holds exactly for continuous material model. Since the continuous model can be used as an approximate model for discrete part production lines, the property should hold approximately in discrete part production lines.

4. Numerical Experiments

In Section 3, the similarity property is justified through theoretical analysis. In this section, we further verify the property by numerical experiments and demonstrate it can be used to accelerate the speed of simulation-based performance evaluation of production lines. Two sets of experiments have been conducted. While the first set deal with serial lines with various number of machines, the second set handle lines with various machine processing rates.

4.1. Serial lines with various number of machines

We have conducted comparative experiments on production lines with \( k = 2, 4, 8, 16 \) machines, respectively. For each value of \( k \), we first evaluate the throughput of a production line where all
machines have the same parameters $t_{up} = 10$, $t_{down} = 10$, $U = 1$ and $N = 2$. Then, we change the factor $\alpha$ of similarity transformation at different levels, from 2, 4, 8, 10, 100 to 200. Accordingly, we get 6 production lines which are similar to the original line. The throughput of each line is evaluated through a discrete event simulation procedure. In simulation, the warm-up time is set to $100 \times \alpha$ and the simulation time is 10 times that of the warm-up time. The reason is that, for large values of $\alpha$, machine failures and repairs are small probabilistic events, and thus it is necessary to set the simulation time to a relatively large value. The line parameters and the obtained throughput (TP) of each line are shown in Table 1.

| $\alpha$ | 1   | 2   | 4   | 8   | 10  | 100 | 200 |
|----------|-----|-----|-----|-----|-----|-----|-----|
| $t_{up}$ | 10  | 20  | 40  | 80  | 100 | 1000| 2000|
| $t_{down}$ | 10  | 20  | 40  | 80  | 100 | 1000| 2000|
| $U$      | 1   | 1   | 1   | 1   | 1   | 1   | 1   |
| $N$      | 2   | 4   | 8   | 16  | 20  | 200 | 400 |
| TP       | 0.348604 | 0.352324 | 0.352978 | 0.358545 | 0.358784 | 0.365261 | 0.358302 |
|          | 0.241554 | 0.244388 | 0.245413 | 0.242343 | 0.247414 | 0.244450 | 0.245322 |
|          | 0.182436 | 0.185754 | 0.186842 | 0.188842 | 0.188625 | 0.184939 | 0.185066 |
|          | 0.154941 | 0.157499 | 0.159297 | 0.159171 | 0.161819 | 0.163087 | 0.162063 |

From Table 1, it can be seen that, for all values of $\alpha$, the system throughput fluctuates around a value. For each value of $k$, we consider the production line with $\alpha=1$ as the baseline. Then the error of the similar transformation method can be defined as the difference between the throughput of the baseline and that of other lines. In all tested cases, the percentage error is less than 3%, which are acceptable compared to the accuracy of the model. Note that for production lines with larger values of $\alpha$, the simulation time is also longer. Therefore, we can make a conclusion that the similarity transformation method is effective to reduce the computational complexity of simulation-based performance evaluation.

4.2. Serial lines with various machine processing rates

In this section, we carried out several experiments to illustrate how the error of the similar transformation method is affected by machine processing rates. We consider the production line with $k = 4$ and $\alpha=1$ in Table 1 as the baseline. Then, the machine processing rate is changed and is chosen from the set $\{2, 4, 6, \ldots, 20\}$. The results are shown in Fig.2, where the horizontal axis represents the machine processing rates and the vertical axis represents the error in units of $10^{-5}$. The figure shows that, with the increase of machine processing rate, the error decreases sharply at first and then remains at a steady level, especially when the machine processing rate is large. This phenomenon can be attributed to the fact that the larger the machine processing rate is, the more the discrete part model is similar to the continuous fluid model. Therefore, the similarity property should hold with a better accuracy.

Fig.2 Serial lines with various number of machine processing rates
5. Conclusion
In this paper, we have proposed the similarity transformation method of production lines with unreliable machines. Through theoretical analysis and numerical experiments, it is justified that two similar production lines have the same throughput. This result implies that a production line with large buffer capacities can be approximately reduced to a line with small buffer capacities. This finding can be used to improve the computational efficiency of discrete event similar models.

In the future, we will further study the similarity transformation method for manufacturing systems with complex structures, such as split/merge systems and assembly/disassembly systems. It is also possible to apply the proposed method to deal with some optimization problems, for instance, buffer allocation problems.

Acknowledgments
This study was conducted as a part of a project supported by National Nature Science Foundation of China. (Grant No.52075539)

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