The spin of the mesons and baryons

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It is shown that the spin of the $\pi^0$, $\eta$, $\Lambda$, $\Sigma^{\pm,0}$, $\Xi^{-,0}$, $\Lambda_+^-$, $\Sigma_c^0$, $\Xi_c^0$, and $\Omega^0_c$ mesons and baryons can be explained by the sum of the angular momentum vectors and the spin vectors of the electromagnetic waves which are in these particles according to the standing wave model. The spin of the $\pi^\pm$, $K^{\pm,0}$, $D^{\pm,0}$, and $D_S^{\pm}$ mesons and of the neutron is the sum of the angular momentum vectors of the oscillations and of the spin vectors of the neutrinos and the electric charges which are in the cubic lattice of these particles. Spin 1/2 is the consequence of the superposition of two perpendicular standing waves of equal frequencies and amplitudes shifted in phase by $\pi/2$. The spin of the antiparticles of the mesons and baryons is the same as the spin of the ordinary particles.

Introduction

The spin or the intrinsic angular momentum is, after the mass, the second most important property of the elementary particles. As is well-known the spin of the electron was discovered by Uhlenbeck and Goudsmit [1] more than 75 years ago. Later on it was established that the baryons have spin as well, but not the mesons. An explanation of the spin of the particles hassofar not been found. Biedenharn and Louck [2] state on p. 5 that, contrary to widespread perceptions, “internal angular momentum is a classical, non-relativistic concept” and that “spin 1/2 internal angular momentum is also a classical nonrelativistic field-theoretic concept”. The concept that spin 1/2 is a relativistic effect originates from Dirac’s equation. However, since electrons with practically zero velocity still have spin it is hard to see how spin 1/2 can be a relativistic effect. It is also difficult to see how the spin of a particle can be explained without consideration of the structure of the particle. Actually it appears to be crucial for the validity of a model of the elementary particles that the model can also explain the spin of the particles without additional
assumptions. The standard model of the particles cannot explain the spin because the spin is imposed on the quarks. For current efforts to understand the spin of the nucleon see Jaffe [3] and of the spin structure of the Λ baryon see Göckeler et al. [4].

We will look at the spin as a property of the standing waves in the cubic lattice of the particles which we have described in [5]. We cannot repeat the points made there. May it suffice to mention that we showed that the spectrum of the so-called stable mesons and baryons consist of a gamma-branch with the π⁰, η, Λ, Σ⁰, Ξ⁰, Ω⁻, Λ⁺, Σ⁺, Ξ⁺, Ω⁰, Λ⁺, Σ⁺, Ξ⁺, Ω⁰, c, Σ⁰, c, Ξ⁰, c, Ω⁰, c, particles and a neutrino branch with the π⁺, K⁺, n, D⁺, S particles, as follows clearly from the decays of the particles. The masses of the particles of the γ-branch are integer multiples of the mass of the π⁰ meson or are proportional to an integer quantum number n, on the average within 0.66%. The masses of the particles of the ν-branch are integer multiples of the mass of the π⁺ mesons times a factor 0.85 ± 0.02. Nambu [6] has first suggested that the masses of the elementary particles might show some regularity “if the masses were measured in a unit of the order of the π-meson mass”. Barut [7] has shown that the masses of the e, μ, τ leptons can be described with very good accuracy by the fourth power of a principal integer quantum number n. This rule has been extended by Gsponer and Hurni [8] to the masses of the quarks.

We have also shown that the masses of the particles of the γ-branch can be explained by standing electromagnetic waves in a cubic cavity. The standing waves come with a continuum of frequencies which must be in the particles after a high energy collision or according to Fourier analysis. The particles of the ν-branch consist of cubic neutrino lattices. The energy in the ν-branch particles is the energy of the lattice oscillations plus the energy in the rest masses of the neutrinos.

1 The spin of the particles of the γ-branch

We will now show that the spin of the particles can be explained by the sum of the angular momentum vectors and the spin vectors of the waves and neutrinos and electric charges in the mesons and baryons. It is striking that the particles which, according to the standing wave model, consist of a single oscillation mode do not have spin, as the π⁰, π⁺ and η mesons do, see Tables 1 and 2. The modes in these tables are improved versions of the modes in Tables 1 and 4 in [5], in which the consequences of electrical charge
Table 1: The spin and oscillation modes of the $\gamma$-branch particles. The \cdot marks coupled oscillations.

| particle | mode | spin |
|----------|------|------|
| $\pi^0$  | (1.1)$\pi^0$ | 0    |
| $\eta$   | (2.2)$\pi^0$ | 0    |
| $\Lambda$| 2·(2.2)$\pi^0$ | 1/2  |
| $\Sigma^0$| 2·(2.2)$\pi^0 + \pi^0$ | 1/2  |
| $\Sigma^\pm$ | 2·(2.2)$\pi^0 + \pi^\pm$ | 1/2  |
| $\Xi^0$  | 2·(2.2)$\pi^0 + 2\pi^0$ | 1/2  |
| $\Xi^-$  | 2·(2.2)$\pi^0 + \pi^0 + \pi^-$ | 1/2  |
| $\Omega^-$| 3·(2.2)$\pi^0 + \pi^-$ | 3/2  |
| $\Lambda_c^+$ | 2·(2.2)$\pi^0 + 2K^0 + \pi^+$ | 1/2  |
| $\Sigma_c^0$ | $\Lambda_c^+ + \pi^-$ | 1/2  |
| $\Xi_c^0$  | 2·(3.3)$\pi^0$ | 1/2  |
| $\Omega_c^0$ | 2·(3.3)$\pi^0 + 2\pi^0$ | 1/2  |

on the structure of the particles were not considered. It is also striking that particles whose mass is approximately twice the mass of a smaller particle have spin 1/2 as is the case with the $\Lambda$ baryon, $m(\Lambda) \approx 2m(\eta)$, and with the nucleon $m(n) \approx 2m(K^\pm) \approx 2m(K^0)$. The $\Xi_c^0$ baryon which is a doublet of one mode has also spin 1/2. Composite particles which consist of a doublet of one mode plus one or two other single modes have spin 1/2, as the $\Sigma_c^0$, $\Xi_c^0$ and $\Lambda_c^+$, $\Sigma_c^0$, $\Omega_c^0$ baryons do. The only particle which seems to be a triplet of a single mode, the $\Omega^-$ baryon, has spin 3/2. It appears that the relation between the spin and the oscillation modes of the particles is straightforward.

In the standing wave model the $\pi^0$ and $\eta$ mesons consist of $N = 2.85 \times 10^9$ standing electromagnetic waves, each with its own frequency. The oscillations in the photon lattice are longitudinal. All standing longitudinal waves of frequency $\nu_i$ in the $\pi^0$ and $\eta$ mesons do not have angular momentum or $\sum_i j(\nu_i) = 0$, with the index running from 1 to $N$. Longitudinal lattice oscillations cannot cause an intrinsic angular momentum because for longitudinal oscillations $\vec{r} \times \vec{p} = 0$. 


Table 2: The spin and oscillation modes of the $\nu$-branch particles. $(2.2)\pi_n^\pm = 340$ MeV is the second mode of the neutrino lattice oscillations plus the rest masses of the neutrinos in the lattice, as discussed in [5], section 6.

| particle | mode | spin |
|----------|------|------|
| $\pi^\pm$ | $(1.1)\pi^\pm$ | 0 |
| $K^\pm$ | $(2.2)\pi_n^\pm + \pi^0$ | 0 |
| $K^0$ | $(2.2)\pi_n^\pm + \pi^\mp$ | 0 |
| $n$ | $2(2.2)\pi_n^\pm + 2\pi^\mp$ | 1/2 |
| $D^\pm$ | $2[ (2.2)\pi_n^\pm + \pi^\mp ] + (2.2)\pi_n^\pm + \pi^0 + (2.2)\pi_n^\mp + \pi^\pm$ | 0 |
| $D^0$ | $2[ (2.2)\pi_n^\pm + \pi^\mp ] + (2.2)\pi_n^\pm + (2.2)\pi_n^\mp + 2\pi^0$ | 0 |
| $D_S^\pm$ | $4(2.2)\pi_n^\pm + 3\pi^\mp + 2\pi^0$ | 0 |

Each of the standing electromagnetic waves in the $\pi^0$ and $\eta$ mesons may, on the other hand, have spin $s = 1$ of its own, because circularly polarized electromagnetic waves have an angular momentum as was first suggested by Poynting [9] and verified by, among others, Allen [10]. The creation of the $\pi^0$ meson in the reaction $\gamma + p \rightarrow \pi^0 + p$ and conservation of angular momentum dictate that the sum of the angular momentum vectors of the $N$ electromagnetic waves in the $\pi^0$ meson is zero, $\sum_i j(s_i) = 0$. Either the sum of the spin vectors of the electromagnetic waves in the $\pi^0$ meson is zero, or each electromagnetic wave in the $\pi^0$ meson has zero spin. That would mean that they are linearly polarized. Linearly polarized electromagnetic waves are not expected to have angular momentum. That this is actually so was proven by Allen [10] who specifically states that “a wave linearly polarized to the [receiving] dipole” and “a wave linearly polarized perpendicular to the dipole” does “in neither instance ... exert a torque on the dipole”, which means that these waves do not have angular momentum, whereas “the dipole will experience a torque” when the “wave is circularly polarized” and consequently the wave has angular momentum. Since the standing longitudinal photon oscillations in the $\pi^0$ and $\eta$ mesons do not have angular momentum and since the sum of the spin vectors $s_i$ of the electromagnetic waves must
be zero, the intrinsic angular momentum of the $\pi^0$ and $\eta$ mesons is zero, or
\begin{equation}
\sum_i (j(\nu_i) + j(s_i)) = 0 \quad (1 \leq i \leq N).
\end{equation}

In the standing wave model the $\pi^0$ and $\eta$ mesons do not have intrinsic angular momentum or spin, as it must be.

We now consider particles such as the $\Lambda$ baryon which consist of superpositions of two perpendicular standing waves of equal frequencies and amplitudes shifted in phase by $\pi/2$ at each of the $N$ points of the lattice [5]. The oscillations in the particles are then coupled what we have marked in Tables 1,2 by the $\cdot$ sign. The particles then contain $N$ circular waves, each with its own frequency and each having an angular momentum of $\hbar/2$ as we will see.

The superposition of two perpendicular linearly polarized traveling waves of equal amplitudes and frequencies shifted in phase by $\pi/2$ leads to a circular wave with the constant angular momentum $j = \hbar$. The total energy of such a wave is the sum of the potential and the kinetic energy. If the motion is circular the kinetic energy is always equal to the potential energy. From
\begin{equation}
E_{pot} + E_{kin} = E_{tot} = h\omega,
\end{equation}
follows
\begin{equation}
E_{tot} = 2E_{kin} = 2 \frac{\Theta \omega^2}{2} = h\omega,
\end{equation}
with the moment of inertia $\Theta$. It follows that the angular momentum $j$ is
\begin{equation}
j = \Theta \omega = \hbar.
\end{equation}
This applies for one single circular oscillation and corresponds to spin $s = 1$, or to a circularly polarized electromagnetic wave.

We now add to one monochromatic standing oscillation with frequency $\omega$ a perpendicular second standing oscillation with the same frequency shifted in phase by $\pi/2$, having the same amplitude, as we have done before in [11]. In other words we consider the oscillations
\begin{equation}
x(t) = \exp[i \omega t] + \exp[-i(\omega t + \pi)],
\end{equation}
\begin{equation}
y(t) = \exp[i(\omega t + \pi/2)] + \exp[-i(\omega t + 3\pi/2)].
\end{equation}
If we replace $i$ in Eqs. (5,6) by $-i$ we have a circular wave turning in opposite direction. The energy of the superposition of the two waves is the sum of the energies of both individual waves, so according to Eq.(2) we have

$$4E_{\text{kin}} = 4\Theta \omega^2 / 2 = E_{\text{tot}} = \hbar \omega, \quad (7)$$

from which follows that the standing circular wave has an angular momentum

$$j = \Theta \omega = \hbar/2. \quad (8)$$

The superposition of two perpendicular monochromatic standing waves of equal amplitudes and frequencies shifted in phase by $\pi/2$ satisfies the necessary condition for spin $s = 1/2$ that the angular momentum is $j = \hbar/2$.

The standing wave model of the mesons and baryons treats the $\Lambda$ baryon, which has spin $s = 1/2$ and a mass $m(\Lambda) = 1.0190 \cdot 2m(\eta)$, as the superposition of two particles of the same type with $N$ standing electromagnetic waves. The waves are circular because they are the superposition of two standing waves with the same frequency and amplitude shifted in phase by $\pi/2$. The angular momentum vectors around a center axis of all circular waves in the lattice cancel, except for the wave at the center of the crystal, because for each oscillation with frequency $\omega$ there is at its mirror position a wave with the frequency $-\omega$, which consequently has a negative angular momentum since $j = mr^2\omega$. Oscillations with negative frequencies are permitted solutions in cubic isotropic lattices.

The frequency distribution of the axial longitudinal oscillations of a cubic lattice has to be corrected for the limitation of the group velocity to the value of the velocity of light. The oscillation frequencies are then given, according to [5], by

$$\nu = \nu_0 \phi = c \phi / 2\pi a, \quad (9)$$

with $\phi = 2\pi a / \lambda$, the lattice constant $a = 10^{-16}$ cm, the wavelength $\lambda$, and the velocity of light $c$. Since $\phi$ goes from $-\pi$ to $\pi$ there is for each positive frequency a negative frequency of the same absolute value at $-\phi$. Consequently the angular momentum vectors of all circular waves in the lattice cancel, but for the wave at the center of the lattice, whose frequency seems to be zero according to Eq.(9). This is a misleading result of the assumption of an infinite extension of the crystal introduced by the periodic boundary conditions used in the classic theory of lattice oscillations. The frequency at $\phi = 0$ is $\nu(0) = c / 2d$, where $d$ is the distance between two opposite sides of
the crystal, \( d \approx 10^3 a \). As for all circular oscillations in the lattice the wave at the center is also the superposition of two perpendicular standing waves and has an angular momentum of \( \hbar/2 \) according to Eq.(8). This is the only one of the waves whose angular momentum is not canceled. The net angular momentum of the \( N \) lattice oscillations which are the superpositions of two perpendicular standing waves is therefore \( \hbar/2 \). Since the circular standing waves in the \( \Lambda \) baryon lattice are the only possible contribution to an angular momentum the intrinsic angular momentum of the \( \Lambda \) baryon is \( \hbar/2 \) or

\[
j(\Lambda) = \sum_i j(\nu_i) = \hbar/2.
\] (10)

We have thus explained that the \( \Lambda \) and likewise the \( \Xi^0 \) baryon satisfy the necessary condition that \( j = \hbar/2 \) for \( s = 1/2 \). The intrinsic angular momentum of the \( \Lambda \) baryon is the consequence of the superposition of two perpendicular standing waves of the same frequency shifted in phase by \( \pi/2 \).

The other particles of the \( \gamma \)-branch, the \( \Sigma^0, \Xi^0, \Lambda^+, \Sigma^+_c \) and \( \Omega^0 \) baryons are composites of a baryon with spin 1/2 plus one or two \( \pi \) mesons which do not have spin. Consequently the spin of these particles is 1/2. The spin of all particles of the \( \gamma \)-branch, exempting the spin of the \( \Omega^- \) baryon, has thus been explained.

2 The spin of the particles of the \( \nu \)-branch

The characteristic particles of the neutrino-branch are the \( \pi^\pm \) mesons which have zero spin. At first glance it seems to be odd that the \( \pi^\pm \) mesons do not have spin, because it seems that the \( \pi^\pm \) mesons should have spin 1/2 from the spin of the charges \( e^\pm \) in \( \pi^\pm \). However that is not the case. The solution of this puzzle is in the composition of the \( \pi^\pm \) mesons which are, according to the standing wave model, made of a lattice of neutrinos and antineutrinos (Fig. 1), each having spin 1/2, whereas the \( \pi^0 \) meson is made of standing electromagnetic waves which are linearly polarized and do not have spin.

In a cubic lattice of \( N = 2.85 \cdot 10^9 \) neutrinos and antineutrinos the spin of nearly all neutrinos must cancel because conservation of angular momentum during the creation of the particle requires that the total angular momentum around a central axis is only a small integer or half-integer number. In fact the spin vectors of all but the neutrino in the center of the lattice cancel. In order for this to be so the direction of the spin of any particular neutrino in
the lattice has to be opposite to the direction of the spin of the antineutrino at its mirror position. Neutrinos and antineutrinos of the same type differ in the lattice only by the direction of their spin. As can be seen on Fig. 1 each neutrino has at its mirror position an antineutrino. The only angular momentum remaining from the spin of the neutrinos of the lattice is the angular momentum of the neutrino at the center of the lattice. Consequently the electrically neutral neutrino lattice consisting of \( N \) neutrinos, each with a spin \( n_i \), has an intrinsic angular momentum \( j = \sum_i j(n_i) = \hbar/2 \).

The standing longitudinal oscillations of the neutrinos in the lattice of the \( \pi^\pm \) mesons do not cause an angular momentum, \( \sum_i j(\nu_i) = 0 \). But electrons or positrons added to the neutral neutrino lattice have spin 1/2. If the spin of the electron or positron added to the neutrino lattice is opposite to the spin of the neutrino in the center of the lattice then the net spin of the \( \pi^+ \) or \( \pi^- \) mesons is zero, or

\[
j(\pi^\pm) = \sum_i j(n_i) + j(e^\pm) = 0 \quad (1 \leq i \leq N).
\]

It is important for the understanding of the structure of the \( \pi^\pm \) mesons to realize that \( s(\pi^\pm) = 0 \) can only be explained if the \( \pi^\pm \) mesons consist of a neutrino lattice to which an electron or positron is added whose spin is opposite to the net spin of the neutrino lattice. Spin 1/2 of the electric charges can only be canceled by something that has also spin 1/2, and the only conventional choice for that is the neutrino.

The spin of the \( K^\pm \) mesons is zero. With the spin of the \( K^\pm \) mesons we encounter the same oddity we have just observed with the spin of the \( \pi^\pm \)
mesons, namely we have a particle which carries an electrical charge with
spin 1/2, and nevertheless the particle does not have spin. The explanation
of $s(K^\pm) = 0$ follows the same lines as the explanation of the spin of the $\pi^\pm$
mesons. In the standing wave model the $K^\pm$ mesons are described by the
state $(2.2)\pi^\pm_n + \pi^0$, that means by the $(2.2)$ mode of the charged neutrino
lattice oscillations plus a $\pi^0$ meson. The $(2.2)\pi^\pm_n$ state of the neutrino lattice
is the sum of the energy of the $(2.2)$ oscillation of the lattice plus the sum of
the energies of the rest masses of the neutrinos and contains 340 Mev. The
$(2.2)$ mode of the longitudinal oscillations of a neutral neutrino lattice does
not have a net intrinsic angular momentum. But the spin of the neutrinos
contributes an angular momentum $\hbar/2$, which originates from the neutrino
in the center of the lattice, just as it is with the neutrino lattice in the $\pi^\pm$
mesons, so $\sum_i j(n_i) = \hbar/2$. Adding an electric charge with a spin opposite
to the net intrinsic angular momentum of the neutrino lattice creates the
charged $(2.2)\pi^\pm_n$ mode which has zero spin,

$$j((2.2)\pi^\pm_n) = \sum_i j(n_i) + j(e^\pm) = 0.$$  \hspace{1cm} (12)

As discussed in [5] it is necessary to add a $\pi^0$ meson to the $(2.2)\pi^\pm_n$
mode of the $\pi^\pm$ meson in order to obtain the correct mass and the correct decays
of the $K^\pm$ mesons. Since the $\pi^0$ does not have spin the addition of the $\pi^0$
meson does not add to the intrinsic angular momentum of the $K^\pm$ mesons.
So, according to Eq.(12), $s(K^\pm) = 0$ in agreement with the facts.

The explanation of $s = 0$ of the $K^0$ mesons described by the state $(2.2)\pi^\pm_n$
+ $\pi^\mp$ follows similar lines. The longitudinal oscillations of the $(2.2)\pi^\pm_n$
mode as well as of the basic $\pi^\mp$ mode do not create an angular momentum, $\sum_i j(\nu_i) = 0$. The first higher mode of the $\pi^\pm$ mesons, the $(2.2)\pi^\pm_n$ state, and the basic
$\pi^\mp$ mode each have $N$ neutrinos, so the number of neutrinos in the sum of
both states, the $K^0$ meson, is $2N$. Since the size of the lattice of the $K^\pm$ mesons
and the $K^0$ mesons is the same it follows that two neutrinos are at each lattice
point of the $K^0$ meson. We assume that Pauli’s exclusion principle applies
for neutrinos as well. Consequently each neutrino at each lattice point must
share its location with an antineutrino. That means that the contribution of
the spin of all neutrinos to the intrinsic angular momentum of the $K^0$ meson
is zero or $\sum_i j(2n_i) = 0$. The sum of the spin vectors of the two opposite
charges in the $K^0$ meson, i.e. in $(2.2)\pi^\pm + \pi^\mp$, is also zero. Since neither
the neutrino oscillations nor the spin of the neutrinos nor the electric charges
contribute an angular momentum or since

$$j(K^0) = \sum_i (j(\nu_i) + j(2n)) + j(e^+ + e^-) = 0,$$  \hspace{1cm} (13)

the intrinsic angular momentum of the $K^0$ meson is zero, or $s(K^0) = 0$, as it must be. In simple terms, since the structure of $K^0$ is $(2.2)\pi^\pm_n + \pi^\pm$, the spin of $K^0$ is the sum of the spin of $(2.2)\pi^\pm_n$ and $\pi^\mp$ in $K^0$, both of which do not have spin. It does not seem possible to arrive at $s = 0$ for the $K^0$ meson if the particle does not contain the $2N$ neutrinos required by the $(2.2)\pi^\pm_n + \pi^\mp$ state which we have suggested in section 6 of [5].

In the standing wave model the neutron, which has spin $1/2$ and a mass $\approx 2m(K^\pm)$ or $2m(K^0)$, is either the superposition of a $K^+$ and a $K^-$ meson or of two $K^0$ mesons. In the case of the neutron one must wonder how it comes about that a particle which seems to be the superposition of two particles without spin ends up with spin $1/2$. The intrinsic angular momentum of the sum of $K^+$ and $K^-$ comes in part from the superposition of two perpendicular standing longitudinal neutrino lattice oscillations. Similar to the case of the $\Lambda$ baryon the sum of the angular momentum vectors of the circular oscillations of all neutrinos in the lattice reduces to the angular momentum of the circular wave at the center of the lattice. Following Eq.(8) the angular momentum of the circular wave at the center or the net angular momentum of the entire neutrino lattice oscillations is $\sum_i j(\nu) = h/2$. The intrinsic angular momentum of the neutron consists consequently in part of the angular momentum of the circular neutrino lattice oscillations whose net sum is $h/2$.

The superposition of two $K^\pm$ mesons also means that the lattice contains $2N$ neutrinos because $K^+$ as well as $K^-$ each contains $N$ neutrinos. Assuming that Pauli’s exclusion principle applies, each neutrino in the lattice must share its place with a neutrino of opposite spin. That means that the spin vectors of all neutrinos of the lattice cancel, $\sum_i j(2n_i) = 0$. Neither the neutrinos nor the spin of the charges of the sum of the $K^+$ and $K^-$ mesons contribute to the net intrinsic angular momentum. But, and that is crucial, the superposition of the oscillations in the two $\pi^0$ mesons which are part of the $(2.2)\pi^\pm_n + \pi^0$ structure of $K^\pm$ adds an angular momentum $\sum_i j(2\pi^0) = h/2$ to the net angular momentum of the sum of $K^+ + K^-$, because the electromagnetic waves in the $\pi^0$ mesons are superposed at right angles and shifted in phase by $\pi/2$ just as the neutrino lattice oscillations. Consequently the sum of the intrinsic angular momentum vectors of the superposition of a
K\(^+\) and a K\(^-\) meson is
\[
j(K^+ + K^-) = \sum_i (j(\nu_i) + j(2n_i)) + j(e^+ + e^-) + j(2\pi^0) = 0 \text{ or } \hbar, \tag{14}
\]
which is incompatible with the experimental facts. We conclude that the neutron cannot be the superposition of a K\(^+\) and a K\(^-\) meson.

On the other hand the neutron can well be the superposition of two K\(^0\) mesons or of a K\(^0\) and a \(\bar{K}^0\) meson. A significant change in the lattice occurs when two K\(^0\) mesons are superposed. Since each K\(^0\) meson contains 2N neutrinos, as we discussed before in context with the spin of K\(^0\), the number of neutrinos in two superposed K\(^0\) lattices is 4N. Since the size of the lattice of the proton as well of the neutron is the same as the size of K\(^0\), (the measured \(r_p\) is within the experimental error the same as \(r_\pi\)), each lattice point now contains four neutrinos, a muon neutrino and an anti-muon neutrino as well as an electron neutrino and an anti-electron neutrino. The quartet of neutrinos oscillates just like individual neutrinos do because we have found in [5] that the ratios of the sum of the oscillation frequencies are independent of the mass as well as of the interaction constant between the lattice points. In the neutrino quartets the spin of the neutrinos cancels, \(\sum_i j(4n_i) = 0\). The superposition of the neutrino oscillations, that means of circular oscillations of frequency \(\nu_i\), contribute the angular momentum of the center circular wave, so \(\sum_i j(\nu_i) = \hbar/2\). The spin and charge of the four electrical charges hidden in the two K\(^0\) mesons cancel. There is no \(\pi^0\) component in K\(^0\) and consequently no contribution to the intrinsic angular momentum. It follows that the intrinsic angular momentum of a neutron created by the superposition of two K\(^0\) mesons comes from the circular neutrino lattice oscillations only and is
\[
j(n) = \sum_i (j(\nu_i) + j(4n_i)) + j(4\epsilon^\pm) = \hbar/2, \tag{15}
\]
as it must be. In simple terms, the spin of the neutron originates from the superposition of two perpendicular standing neutrino lattice oscillation with the same frequencies shifted in phase by \(\pi/2\), which produces the angular momentum \(\hbar/2\).

The spin of the proton is 1/2 and is unambiguosly defined by the decay of the neutron n \(\rightarrow p + e^- + \bar{\nu}_e\). The remaining particles of the neutrino branch, the D\(^{\pm,0}\) and D\(^{\pm}_S\) mesons both having zero spin, are superpositions of a proton and a neutron of opposite spin, or of their antiparticles, or of two neutrons of opposite spin in D\(^0\). The spin of D\(^\pm\) and D\(^0\) does therefore
not pose a new problem. We note in passing that in \(D^0\) a \(\pi^0\) meson replaces a \(\pi^\pm\) meson in \(D^\pm\), and consequently the mass difference \(m(D^\pm) - m(D^0) = 4.78\) MeV is very similar to \(m(\pi^\pm) - m(\pi^0) = 4.593\) MeV, comparable to the case \(m(K^0) - m(K^\pm)\) in [5]. Replacing a \(\pi^\pm\) meson by a \(\pi^0\) meson does not change the spin. In \(D^+_S\) only a \(\pi^0\) meson is added to \(D^\pm\) and therefore its spin is the same as that of \(D^\pm\).

The \(\Sigma^+\), the \(\Sigma^-\) and the \(\Xi^-\) baryons are afflicted by the same problem we have encountered with the \(\pi^\pm\) and \(K^\pm\) mesons, namely their spin does not seem to add up. When electric charge, either an electron or positron with spin 1/2, is added to the \(\Sigma^0\) or \(\Xi^0\) baryons, both of which have spin 1/2, the resulting \(\Sigma^\pm\) and \(\Xi^-\) baryons should not have the same spin \(s = 1/2\) as the \(\Sigma^0\) and \(\Xi^0\) have. This puzzle can be solved when the charge added to the \(\Sigma^0\) or \(\Xi^0\) is not added as an electron or positron but rather as \(\pi^+\) or \(\pi^-\) mesons which do not have spin. Considering the structure of the \(\Sigma^0\) baryon we have found in [5], which is \(2 \cdot (2.2) \pi^0 + \pi^0\), we can replace the single \(\pi^0\) meson in \(\Sigma^0\) with a \(\pi^+\) or \(\pi^-\) meson without changing the mass of \(\Sigma^0\) significantly and without changing the spin. The same applies correspondingly to \(\Xi^0\). The presence of either \(\pi^+\) or \(\pi^-\) in \(\Sigma^\pm\) becomes quite clear from the decay of these particles. \(\Sigma^+\) decays into \(p + \pi^0\) (51.57%) and \(n + \pi^+\) (48.31%), both make up 99.88% of the \(\Sigma^+\) decays. \(\Sigma^-\) decays into \(n + \pi^-\) (99.848%), whereas \(\Sigma^0\) decays into \(\Lambda + \gamma\) (100%). \(\Xi^-\) decays into \(\Lambda + \pi^-\) (99.887%) whereas \(\Xi^0\) decays into \(\Lambda + \pi^0\) (99.54%). In particular in the latter case it seems to be obvious that in the \(\Xi^-\) baryon a \(\pi^-\) meson replaces a \(\pi^0\) meson in \(\Xi^0\). If this is the case, then there is no problem with the spin of \(\Xi^-\) being the same as the spin of \(\Xi^0\). The same applies to the spin of the \(\Sigma^\pm\) baryons.

An explanation of the spin of the mesons and baryons can only be valid if the same explanation also applies to the antiparticles of these particles whose spin is the same as that of the ordinary particles. In the standing wave model the antiparticles of the \(\gamma\)-branch consist of the same photons as in the ordinary \(\gamma\)-branch particles, other than that the sign of the frequency of each of the standing electromagnetic waves is reversed. If a particle, such as the \(\Lambda\) baryon, consists of superpositions of standing perpendicular waves shifted in phase by \(\pi/2\), all waves in the cubic lattice of the particle are circular, and the angular momentum vectors of all waves in the lattice cancel, but for the wave at the center, regardless whether the frequencies are positive or negative. The remaining wave at the center has an angular momentum of \(\hbar/2\), regardless whether the frequencies are positive or negative. Reversing the sign of the frequency means only a phase shift of \(\pi\). Consequently the
intrinsic angular momentum of the antiparticles of the $\gamma$-branch particles have the same spin $s = 1/2$ as the ordinary particles. Of the particles of the $\nu$-branch only the neutron has spin $s = 1/2$. As shown above the spin of the neutron is a consequence of circular neutrino lattice oscillations. If we replace the positive frequencies of the neutrino lattice oscillations in the neutron with negative oscillations in the antineutrino then only the phase of the oscillations is shifted by $\pi$, but the net angular momentum of all circular oscillations is preserved. Consequently the spin of the antineutron is the same as the spin of the neutron, as it must be.

**Conclusions**

The intrinsic angular momentum of the $\gamma$-branch of the so-called stable elementary particles can be explained with the sum of the angular momentum vectors of the electromagnetic waves or photon lattice oscillations, plus the sum of the spin vectors of the electromagnetic waves, plus the sum of the spin vectors of the electric charges if any are in the particles of the $\gamma$-branch. Correspondingly, the intrinsic angular momentum of the particles of the neutrino branch is the sum of the angular momentum vectors of the neutrino lattice oscillations, plus the sum of the spin vectors of the neutrinos, plus the sum of the spin vectors of the electric charges which the particles carry.

The most simple particles, the $\pi^0$ and $\eta$ mesons, do not have spin because neither the longitudinal oscillations of the photon lattice nor the linearly polarized electromagnetic waves can contribute to an intrinsic angular momentum. The $\pi^\pm$ and $K^\pm$ mesons are quite different because the electric charges in either particle bring with them spin 1/2. Since $s(\pi^\pm) = 0$ and the spin of $e^\pm$ in $\pi^\pm$ and $K^\pm$ must be canceled by something that has likewise spin 1/2. We have shown that the sum of the spin vectors of the very many neutrinos which in our model are in $\pi^\pm$ and $K^\pm$ reduces to the spin of the neutrino at the center of the lattice. If the spin of the electric charge and of the neutrino at the center of the lattice are of opposite direction then $s(\pi^\pm)$ and $s(K^\pm) = 0$, as it must be. The spin of $K^0$ is likewise zero. There can be no contribution to the spin of $K^0$ from the two opposite charges which are in $K^0$ according to our model. There is also no contribution from the spin of the neutrinos because at each lattice point the spin of each neutrino is canceled by the spin of its antineutrino. Neither the longitudinal lattice oscillations,
nor the spin vectors of the neutrinos, nor the two electrical charges contribute
to an angular momentum of $K^0$, so $s(K^0) = 0$.

The spin $s = 1/2$ of the $\Lambda$ baryon and the neutron, which have a mass
$m(\Lambda) \approx 2m(\eta)$ and $m(n) \approx 2m(K^0)$, can be explained with the sum of the
angular momentum vectors of the superpositions of two perpendicular standing
waves of the same frequencies shifted in phase by $\pi/2$, as they are in $\Lambda$ and $n$ in the standing wave model. In either case an angular momentum $\hbar/2$
remains at the center of the lattice from the multitude of standing circular
waves in these particles. There can be no other contributions to the intrinsic angular momentum of these particles and hence $s(\Lambda)$ and $s(n) = 1/2$.
The other baryons are composites of $\Lambda$ and their spin does not pose a new problem.

Our explanation of the intrinsic angular momentum confirms the validity
of the structure of the particles which we considered. A convincing example
of this correlation is offered by the explanation of the puzzling absence of
spin in the $\pi^{\pm}$ mesons in spite of the electric charge these particles carry.
The standing wave model assumes that the $\pi^{\pm}$ mesons have a cubic neutrino
lattice. The net spin of the neutrinos in the lattice cancels the spin of the
electric charges in $\pi^{\pm}$. The spin, the mass and the decays of the $\pi^{\pm}$ mesons
require a neutrino lattice for the $\pi^{\pm}$ mesons. Spin $1/2$ of the $\Lambda$ baryon,
and consequently of the other baryons of the $\gamma$-branch, and spin $1/2$ of the
neutron originates from the superposition of two perpendicular standing oscillations in the particles, as the standing wave model postulates; and as is
indicated by the mass ratios $m(\Lambda)/m(\eta) \approx 2$ and $m(n)/m(K^0) \approx 2$. The spin
of the stable mesons and baryons can be explained with the standing wave
model without an additional assumption.

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