Abstract

In this work, we study the shadow of Kerr black hole surrounded by an axisymmetric plasma, whose density takes an Gaussian distribution in the angular direction. Along the radial direction, we consider two models: in model A the density of the plasma decays in a power law; in model B the density obeys a logarithmic Gaussian distribution. Using the numerical backward ray-tracing method, we find that the size of the shadow is sensitive to the inclination angle of the observer due to the angular distribution of the density of the plasma. In particular, we pay special attention to the model B and investigate the influence of the radial position of maximum density, the decay rate of the density towards the event horizon and the opening angle of the plasma on the shape and size of the Kerr black hole shadow. The effects of the plasmas studied in this work can be qualitatively explained by taking the plasmas as convex lenses with the refractive index being less than 1.

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1 Introduction

Black hole is one of the most fascinating objects in general relativity. The direct evidences for the existence of black holes have been accumulated in the past few years. In particular, the Event Horizon Telescope (EHT) Collaborate released the images of M87* three years ago [1] and the image of Sgr A* very recently [2]. As we know, black holes are black because they swallow any matter falling into the horizons and not even light can escape, so that we cannot actually see a black hole in any direct way. Thus, the images of black holes indicate that the supermassive black holes in the centers of galaxies are not isolated, and they are actually surrounded by plasma, corona and magnetic field configuration. Among these surroundings, the plasma is the most important one, and plays an indispensable role in forming the pictures of black holes.

On the one hand, the accretion disk of plasma can be used as a light source to illuminate the black hole. On the other hand, plasma as a dispersive medium would affect the path of the light traveling through it. In this present work, we would like to focus on the latter. Along this line, the null equation of motion including the influence of a non-magnetized pressureless plasma in the Schwarzschild spacetime was studied in [3–7]. The gravitational lensing and light deflection by other static black holes and rotational black holes in the same class of plasma medium can be found in [8–18]. The black hole shadow in the presence of a plasma are investigated in [19–31]. In these works, either the electron frequency of a non-magnetic cold plasma are assumed to take a special form to make the Hamilton-Jacobi equation of photons separable for rotating black hole spacetimes, or only weak gravitational lensings are considered, such that analytical methods can be employed. However, in order to gain more understandings of the influence of plasma on black hole shadow, more realistic models should be studied. In [32], the authors considered the models in which the radial density of the plasma takes a power-law or an exponential form, and calculated the Kerr black hole shadow using the numerical backward ray-tracing method. In their study, the angular distribution of the plasma was assumed to be still uniform, that is, the plasma is spherically symmetric.

For a realistic model, the plasma cannot be spherically symmetric around a spinning black hole. And for the radial density, it is expected to be mostly not monotonically decaying, instead it reaches maximum at some point and decay toward the event horizon and the infinity. It was supposed that the rate of decay toward the horizon is faster than that toward the infinity[33, 34]. In this work, we consider two models of plasma, in both of which the angular density takes an Gaussian distribution, and the radial density takes the power-law form in one model and a logarithmic Gaussian form in the other. In particular, we discuss the effect of plasma on the black hole shadow by showing that the plasma has a convex lens effect for the light rays.

The remaining parts of this paper are organized as follows. In section 2, we introduce the background spacetime and the plasma models to set up our problems. In section 3, we show our numerical method, present and discuss the results. The main conclusions are summarized in section 4. In this work, we have set the fundamental constants $c$, $G$, the vacuum permittivity $\varepsilon_0$ and the mass of the black hole $M$ to unity, and we will work in the convention $(-, +, +, +)$. 

2
2 Kerr black hole and plasma models

In this section, we would like to set up our problem. The background of interest is a Kerr spacetime, whose metric takes a form

\[
    ds^2 = - \left(1 - \frac{2r}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{1}{\Sigma} \left[(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta\right] \sin^2 \theta d\phi^2 - \frac{4ar}{\Sigma} \sin^2 \theta dt d\phi
\]  

(2.1)

in the Boyer-Lindquist coordinates, where

\[
    \Delta = r^2 - 2r + a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta.
\]  

(2.2)

Note that, we have set \(M = 1\) for simplicity and without loss of generality. The black hole horizon is located at \(r_h = 1 + \sqrt{1 - a^2}\) which is the larger root of the equation \(\Delta = 0\). We assume that outside the black hole there exist some refracting medium which have no backreaction to the background. In the presence of the refracting medium, the Hamiltonian of photons is given by[3]

\[
    H = \frac{1}{2} g_{\mu\nu} p^\mu p^\nu - (n^2 - 1)(p_\mu V^\mu)^2,
\]  

(2.3)

where, \(n\) is the refractive index and \(V^\mu\) is defined to denote the 4-velocity of the refracting medium. Considering the refracting medium is composed of a non-magnetized pressureless plasma, we have

\[
    n^2 = 1 - \frac{\omega^2}{\omega_p^2}.
\]  

(2.4)

Here \(\omega = -p_\mu V^\mu\) is the frequency of photon and \(\omega_p\) is the plasma electron frequency satisfying

\[
    \omega_p^2 = \frac{4\pi e^2}{m_e} N,
\]  

(2.5)

where \(e\) and \(m_e\) are the charge and mass of the electron, respectively, and \(N\) is the number density of the plasma which is generally a function of the spacetime coordinates \(x^\mu\). In addition, considering the Kerr spacetime is stationary and axisymmetric, it is plausible to assume the number density \(N\) is independent of the coordinates \(t\) and \(\phi\), that is, the plasma is distributed radially and angularly. In this work, in the angular direction, we take normal distribution for simplicity. As for the radial direction, we would like to consider two models. One is that the plasma decays as a power law along the radial direction and the other is set to obey logarithmic normal distribution. Precisely, for the first model A, the electron frequency of the plasma takes the form

\[
    \omega_{pA}^2(r, \theta) = \frac{k_A}{r^2} e^{-\frac{(r-h)^2}{2\xi^2}}, \quad r > r_h,
\]  

(2.6)

and correspondingly, we have the number density

\[
    N_A(r, \theta) = N_{\text{max}} \frac{r^2}{r_h^2} e^{-\frac{(r-h)^2}{2\xi^2}}, \quad r > r_h,
\]  

(2.7)
where $k_A$ is a certain constant number to characterize the number density, $N_{\text{max}} = \frac{k_A m_e}{4\pi e^2 r_h^2}$ is a rescaled parameter of $k_A$, being equal to the number density near the black hole horizon in this case, and $\xi_\theta$ characterizes the angular distribution. According to the normal distribution, we can see that the plasma is concentrated in the region $[\pi/2 - 2\xi_\theta, \pi/2 + 2\xi_\theta]$, and thus $4\xi_\theta$ can be seen as the opening angle of the plasma accretion when $\xi_\theta$ is small, as shown in Fig. 1. When $\xi_\theta$ is very large, angular normal distribution is approximately uniform, and our model reduces to one of the models studied in [32]. This allows us to check our calculations with the one in [32] by taking the large $\xi_\theta$ limit.

Figure 1: $\alpha$ is the opening angle of plasma. Roughly speaking, we have $\alpha \simeq 2\xi_\theta$ when $\xi_\theta$ is less than $\pi/4 \approx 0.785$, while strictly speaking, $\alpha \neq 2\xi_\theta$, where $\xi_\theta$ is defined as the shape parameter to characterize the dispersion degree of the normal distribution.

Figure 2: Density plots of the plasma for model A with $N_{\text{max}} = 1$ and $\xi_\theta = \pi/9$. The left plot shows the density contour map of a two dimensional slice $\phi = \text{constant}$. The horizontal and vertical coordinates are $\rho = r \sin \theta$ and $z = r \cos \theta$, respectively. The right one shows the density map of the plasma in 3D Cartesian coordinates: $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$ where $\theta$ and $\phi$ are polar and azimuthal angles in the Boyer-Lindquist coordinate, respectively.
In Fig. 2, we show the density plots of the plasma for model A with $N_{\text{max}} = 1$ and $\xi_\theta = \pi/9$. It is not hard to see that in this model, the plasma is mainly centered in the vicinity of the Kerr black hole horizon. Hence, although this model is able to capture some characteristics of the plasma around a black hole, such as the refractive index, rapid decay at large scales, etc., there are a few disadvantages that cannot be ignored. One of the most obvious shortcomings is that the density should not increase monotonically as the plasma gets closer to the event horizon. It is believed that the density reaches a maximum at a certain radius $r_m$ and falls off to the event horizon and the infinity, and the rate of decay towards the event horizon might be faster \cite{33, 34}. To capture this important feature of the plasma, we introduce the second model, which will be referred to as model B and has lognormal distribution along the radial direction. More precisely, we have

$$\omega_{pB}^2(r, \theta) = k_B e^{-\frac{(\log \frac{r}{r_m})^2}{2\sigma^2}} e^{-\frac{(\theta - \pi/2)^2}{4\xi^2}}, \quad r > r_h, \quad (2.8)$$

and the corresponding number density of the plasma

$$N_B(r, \theta) = N_{\text{max}} e^{-\frac{(\log \frac{r}{r_m})^2}{2\sigma^2}} e^{-\frac{(\theta - \pi/2)^2}{4\xi^2}}, \quad r > r_h, \quad (2.9)$$

where $r_m$ is the position of the maximum density, $N_{\text{max}} = N_B(r_m, \pi/2)$ and $\sigma$ is one of the parameters of lognormal distribution. We want to stress that our lognormal distribution is slightly different from the most common form, the reason is that we have taken into account of the normalization. In Fig. 3, we show the density plots of the plasma for model B with $N_{\text{max}} = 1$, $\sigma = 0.8$, $r_m = 3$ and $\xi_\theta = \pi/9$. From this figure, we can see that the position with the maximum density is no longer at the event horizon and the density declines faster towards the event horizon than to the infinity. These results are in line with our expectations for the plasma.

In addition, for model B, the plasma can form the so-called accretion disk. In our work, we assume the accretion disk is composed of these plasma, of which the number density $N_B$ is above $0.1 N_{\text{max}}$. Then, we would like to introduce a parameter $d$ to manifest the width of the plasma accretion

$$d = r_2 - r_1, \quad (2.10)$$

where $r_{1,2}$ are the cutoff positions of the plasma with

$$r_{1,2} = r_m e^{\pm \sqrt{2 \log 10} \sigma}, \quad (2.11)$$

being the roots of the equation $e^{-\frac{(\log \frac{r}{r_m})^2}{2\sigma^2}} = 0.1$. We give an example shown in Fig. 4, where we choose $r_m = 3$ and $d = 8$ with $r_1 = 1$ and $r_2 = 9$.

Now we have completed the introduction of the spacetime background and the plasma models. After plunging Eq. (2.4) into Eq. (2.3), we obtain the Hamiltonian

$$H(x^\mu, p_\mu) = \frac{1}{2} g_{\mu\nu} p^\mu p^\nu + \omega_p^2, \quad (2.12)$$

where $\omega_p^2$ are given by the Eq. (2.6) for model A and Eq. (2.8) for model B. From this Hamiltonian we can study the motions of the photons in the backgrounds.
Figure 3: Density plots of the plasma for model B. The coordinates are the same with those in Fig. N_{max} = 1, \sigma = 0.8, r_m = 3 and \xi_\theta = \pi/9.

Figure 4: A profile of a lognormal distribution with \( r_m = 3 \) and \( \sigma = 0.512 \). The width of the accretion is \( d = 8 \).

3 Black hole shadows

In this section, we are going to study the shadows of the Kerr black holes surrounded by a plasma with the distribution models mentioned in sec. 2. Considering the Eqs. (2.6), (2.8) and (2.12), we can see that the Hamiltonian including \( \omega_{pA}^2 \) or \( \omega_{pB}^2 \) is no longer separable, and the
standard analytical method to calculate black hole shadows can not apply here. Therefore, we would like to apply the numerical backward ray-tracing method proposed in [35, 36] for our study. As our interest is focused on the influence of plasma in this work, we fix $a = 0.998$ in the following discussion.

It is very convenient to do the numerical geodesic evolution by using the first-order differential equations. In the Hamiltonian canonical formalism, the geodesic equations read

$$\dot{p}_\mu = -\frac{\partial H}{\partial x^\mu}, \quad \dot{x}^\mu = \frac{\partial H}{\partial p_\mu}$$

(3.1)

where the dot denotes the derivative with respect to the affine parameter $\tau$. Recall that $\omega_p^2$ is independent on $t$ and $\phi$, we have two conserved quantities

$$E = -p_t, \quad L = p_\phi,$$

(3.2)

along the null geodesics. In the local rest frame of the observer at the position $(t_o, r_0, \theta_o, \phi_o)$, one can use celestial coordinates to denote the 3-momentum vector of $p^\mu$, and find the timelike component after considering the Hamiltonian $H = 0$. In addition, on the screen of the observer, one can also set up standard Cartesian coordinates and then build a map from the celestial coordinates and the Cartesian coordinates. Here, we also employ the stereographic projection, which is often called fisheye camera model. The details of this model can be found in [35]. Hereto, we have known the values of $(x^\mu, p_\mu)$ for given Cartesian coordinates on the screen of the observer, which can be set as the initial values of the Eq. (3.1), then we can determine the trajectories of the photons and identify whether the rays fall into the black hole or reach the infinity. To fix the black hole shadow, our strategy is to place a spherical source illuminating the system, that is, the black hole and the observer are both inside the spherical source. As a result, the pixels on the screen of the observer will be coloured if the corresponding rays can reach the source, otherwise, photons fall into the black hole and the pixels are dark. The boundary between the dark and coloured region is the shadow curve we want.

In order to quantitatively express the deformation of the shadow of a vacuum Kerr black hole and the variation of the shape of the shadow of a Kerr black hole surrounded with the plasma in different distributions, we introduce the parameter $\bar{R}$ to reflect the feature of the black hole shadow size. In Fig. 5 we show the basic coordinate parameters on the screen of the observer. The origin of the coordinates is the stereographic projection of the observer on the screen. The parameters $x_{\text{min}}$ and $x_{\text{max}}$ are the minimum and maximum of the shadow curve on the horizontal axis, respectively. Similarly, we have $y_{\text{min}}$ and $y_{\text{max}}$ along the vertical direction. It is worth noting that $y_{\text{min}} = -y_{\text{max}}$ due to the $\mathbb{Z}_2$ symmetry of the spacetime. Then we can define the centre of the shadow to be $(x_c, y_c)$ with $x_c = (x_{\text{max}} + x_{\text{min}})/2$ and $y_c = 0$. It is also convenient to introduce polar coordinates $(R, \psi)$ on the screen and the origin is placed at the centre $(x_c, y_c)$ such that $R = \sqrt{(x - x_c)^2 + y_c^2}$. Then we define the parameter $\bar{R}$ as

$$\bar{R} = \int_0^{2\pi} \frac{R(\psi)}{2\pi} d\psi.$$ 

(3.3)
Figure 5: A diagram of the Kerr black hole shadow curve, deformed by the plasma around the black hole.

It represents the average radius of the shadow curve, characterizing the size of the black hole shadow.

3.1 Model A

Firstly, let us show the results for model A. In Fig. 6, we set \( k_A = 16 \) and vary \( \xi_\theta \) with \( \theta_o = \pi/2 \), which corresponds to the observer being located on the equatorial plane. For (f) in Fig. 6, we set \( \xi_\theta = 10 \gg \pi/4 \) so that the distribution of plasma is approximately uniform in the angular direction and thus can be taken as a spherically symmetric plasma. Comparing our result (f) with the last plot of Fig. 7 in [32], we find that they agree with each other very well. In addition, it is not hard to see in Fig. 6 that the shadow curves in the first five plots do not show significant change with \( \xi_\theta < \pi/4 \) while the left side of the shadow curve in (f) gets sharp visibly. However, from the first five plots of Fig. 6, we can see that the images are distinctly different.

In Fig. 7, we show the results for model A at \( \theta_o = 17^\circ \) with \( k_A = 16 \). Similarly, we can see that the shapes of shadow curves change little as the angle \( \xi_\theta \) increases. In Fig. 8, we show the variations of \( \bar{R}/R_{Kerr} \) with respect to the opening angle \( \xi_\theta \) for \( \theta_o = \pi/2 \) and \( \theta_o = 17^\circ \) respectively. Obviously, we can see that when the observer is located on the equator the value of \( \bar{R}/R_{Kerr} \) is much smaller than that when the inclination angle \( \theta_o = 17^\circ \) for the same \( \xi_\theta \), even though \( \bar{R}/R_{Kerr} \) changes little with the increase of \( \xi_\theta \). The main reason is that the distribution density of the plasma along the angular direction is the largest at \( \theta_o/2 \), and the photons must travel through the thickest plasma. Meanwhile, roughly speaking the size of shadow can be regarded as the complement of...
Figure 6: The images of the Kerr black hole surrounded by a plasma of model A. The inclination angle of the observer is fixed at $\theta_o = \pi/2$, the spin is fixed at $a = 0.998$ and $k_A = 16$.

photon-escaping cone centred at the observer. Thus, the region of the plasma whose polar angle is near the value of the inclination angle $\theta_o$ is crucial to the formation of a black hole shadow in the sight of the observers, and the plasma with a higher density would play a greater influence on the trajectories of photons.

Next, we fix $\xi_\theta = 0.36$ and vary $k_A$ to see the influence of $k_A$ on the black hole shadow. The results can be found in Fig. 9, where we place the observer on the equatorial plane. One can see the size of the black hole shadow becomes smaller as $K_A$ goes up, and even the black hole shadow would disappear when $k_A$ is big enough. Our results are consistent with those in [32].

3.2 Model B

Now, we turn to the shadow of the Kerr black hole surrounded by the plasma of model B which is our main interest in this work. Note that physically the variations of $k_{A,B}$ have no essential difference between the model A and B, so we would like only focus on the influence of the radial position of maximum density $r_m$, the rate of the decay from the maximum towards both sides, and the opening angle $\xi_\theta$ of the plasma on the shape and size of the Kerr black hole. Recall that we have introduced the cutoff positions of the plasma as $r_1$ and $r_2$ around Eq. (2.10). Considering $r_2$ would be determined once $r_1$ is known for a given logarithmic Gaussian distribution, in order to
Figure 7: The images of the Kerr black hole surrounded by a plasma of model A. The inclination angle of the observer is fixed at $\theta_o = 17^\circ$ and the spin is fixed at $a = 0.998$.

Figure 8: The variation of $\bar{R}/\bar{R}_{Kerr}$ with respect to the opening angle $\xi_\theta$. Left: $\theta_o = \pi/2$; Right: $\theta_o = 17^\circ$.

describe the rate of the decay from the maximum, we let

$$ r_1 = \frac{r_m - r_h}{10}, \quad i = 1, 2, \ldots, 10, $$

which indicates a smaller $i$ corresponds to a faster decay rate.
At first, we consider the change of the shadow curves as the maximum density position moves and the decay rate of the density gets slower. As mentioned at the beginning of this section, we consider the near-extreme Kerr black hole and fix $a = 0.998$ to simplify the discussion. As a result, the radius of the innermost stable circular orbit on the equatorial plane is very near $r_h \simeq 1$. In this case, we let $r_m$ start at 2. In Fig. 10, we show the images of the Kerr black hole surrounded by a plasma of model B with various $r_m$’s and $i$’s while the other parameters being fixed. For the first three columns, the results of gravitational lensing are included while only the shadow region is presented for the last two columns to highlight the shape of the shadow curve. From Fig. 10, we can read a few remarkable features on the change of the black hole shadow.

1. The size of the shadow becomes small as $i$ goes small for each row. This means that the ability of the plasma that diverges the backward rays becomes strong as the decay rate from the maximum increases.

2. The differences of the shape and size of the Kerr black hole shadow are noticeable as the increase of $r_m$ qualitatively from each column of Fig. 10. More precisely, when $r_m$ gets larger, the size of shadow curve becomes smaller, while the change in the shape is too complicated to read a simple rule. At different density decay rates, namely for different values of $i$, the characteristics of the shape change are different. In Fig. 11, we show the the variation of...
Figure 10: The images of the Kerr black hole surrounded by a plasma of model B for various $i$ and $r_m$. The inclination angle of the observer is fixed at $\theta_o = \pi/2$, the spin is fixed at $a = 0.998$, $\xi_\theta = 0.36$ and $k_B = 0.9$. 
Figure 11: The variation of $\bar{R}/\bar{R}_{Kerr}$ with respect to $i$ and $r_m$.

Figure 12: A diagram of the light travelling on the slice of $\phi = \text{constant}$. For the sake of illustration qualitatively, we use circles instead of the actual isodensity lines of the plasma in Fig. 3. Considering the plasma density is larger as the the radius of the circle decreases and the refractive index of the plasma is less than 1, the light diverges after multiple refractions.

$\bar{R}/\bar{R}_{Kerr}$ with respect to $i$ and $r_m$ to quantitatively describe the change of the black hole shadow size.

3. In particular, for $i = 8$ and $i = 10$, with the increasing of $r_m$, there may first appear a pair of cusps in the shadow curves, which could further change into a couple of tails and disappear at last, as shown in the last two columns of Fig. 10.
In order to illustrate the effect of plasma on light rays, we project the wave vector of the light ray to the $\phi = \text{constant}$ plane. The remaining component is perpendicular to the $\phi = \text{constant}$ plane and lies on the $z = \text{constant}$ plane. Noting that the contour map of the plasma density is the contour map of the refractive index of plasma, thus it can be regarded as an interface. On the $z = \text{constant}$ plane, the contour maps of the plasma density are circles and the component of the wave vector of the light ray is tangent to the corresponding circle. As a result, there is no deflection.
Figure 15: The images of the Kerr black hole surrounded by a plasma of model B. The inclination angle of the observer is fixed at $\theta_0 = 17^\circ$ and the spin is fixed at $a = 0.998$ and $k_B = 0.8$.

Effect for the component of the wave vector.

The situation is different on the two dimensional slice $\phi = \text{constant}$. As shown in Fig. 3, the contour maps of the plasma density are convex curves. Considering that the refractive index of plasma is less than 1, the closer to the center, the greater the density, and the smaller is the refractive index. In any interface, the refraction angle should be greater than the incident angle. Therefore, such a plasma distribution will cause the light to diverge, as shown in Fig. 12. In our model, we keep the maximum refractive index constant by fixing $k_B$, so a narrower plasma disk corresponds to a larger change in the refractive index, which leads to a stronger light deflection.

Roughly speaking, we could take the plasma as a convex lens in the propagation of light rays. As in Fig. 13, there are three blue lines with arrows connecting to the observer, each line representing a light ray in a black hole spacetime without plasma, but with different terminal points. Among them, the top one can reach the infinity, the bottom one falls into the black hole and the middle one ends very close to the spherical photon orbit that determines a critical angle related to the black hole shadow. In the presence of a plasma of model B, the light rays would diverge after passing through the plasma, as shown in the red lines in Fig. 12. For example, the middle line can escape to the infinity. As a result, the size of black hole shadow would shrink. Moreover, since the increase of $i$ leads to the broadening of the plasma disk $d$, the thinner a plasma disk is, the smaller the size of the black hole shadow is.

Next, we study the influence of the opening angle $\xi_\theta$ of the plasma on the black hole shadow
for model B. The results for $\theta_0 = \pi/2$ and $\theta_0 = 17^\circ$ are shown in Fig. 14 and Fig. 15, respectively. Interestingly, compared with the results shown in Fig. 9 and Fig. 7 for the model A, we find that the opening angle $\xi_0$ has a greater influence on the shape of the black hole shadow for model B as shown in Fig. 14 and Fig. 15. In particular, for $\theta_0 = \pi/2$, the shape changes greatly with the increasing of the opening angle due to the large density distribution near $r_m$ for model B, while for model A, roughly speaking, the plasma density is almost concentrated near the event horizon, as seen in Fig. 2 and Fig. 3.

4 Summary

In this work, we studied the effect of plasma on the black hole shadow. We considered two models of axisymmetric plasma surrounding the Kerr black hole. Both models have a Gaussian distribution in the angular direction. Along the radial direction, the model A was assigned a distribution that the plasma density decays as power law, while for model B, the plasma density obeyed a lognormal distribution. These plasma distributions lead to a nonseparable Hamilton-Jacobi equation, so that we employed the numerical backward ray-tracing method to investigate the effects of the plasma on the Kerr black hole shadow. We have shown some interesting and important features in the black shadow in the presence of plasma. We discussed the changes in the shape and size of the black hole shadow with different opening angles of the plasma for both models, and different radial positions of the maximum density of plasma for model B, as well as different decay rate from the maximum density toward the event horizon.

We also observed that each contour map of the plasma density is a convex curve so that it can be seen as an interface with equal refractive index. The photon rays get diverged after passing through the plasma, considering the refractive index of plasma is less than 1, and the closer to the center, the smaller is the refractive index on the $\phi = \text{constant}$ plane. Then, we qualitatively explained our results and concluded the existence of plasma would shrink the size of black hole shadow based on the convex lens effect of the plasma.

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