I. INTRODUCTION

Knowledge of the neutrino-nucleus interaction and the electron-nucleus interaction is essential for current neutrino experiments with accelerators \cite{1, 2}. The emission of one nucleon is the most important contribution to the inclusive cross section in the quasi-elastic (QE) region, centered around \( \omega = |Q^2|/2m_N^* \), where \( \omega \) is the energy transfer, \( Q^2 = \omega^2 - q^2 < 0 \), and \( q \) is the momentum transfer to a nucleon with relativistic effective mass \( m_N^* \) \cite{8, 11}. Recent theoretical work has shown the importance of two-particle (2p2h) excitation in the QE cross section (about 20\% of the total cross section) \cite{12, 23}.

The emission of two particles requires interaction mechanisms with a pair of nucleons. Regardless of the final-state interactions, this can be achieved with meson-exchange currents (MEC) \cite{24, 25} and with short-range correlations (SRC) models \cite{26, 29, 34, 35}. Alternatively SRC have also been introduced as a two-body correlation operator \cite{12, 30–33}. In ref. \cite{13} the two-body one-pion exchange operator is modified to include SRC via the Landau Migdal parameters. One effect of SRC is to provide high momentum components to nucleons due to the nuclear force at short distances \cite{28, 29, 34, 55}. This is being exploited to extract the number of SRC pairs from semi-inclusive and inclusive \((e, e'N)\) reactions \cite{36, 38}.

In this work we present a method to compute the contribution of the correlations in inclusive electron scattering from the phenomenological scaling function. The QE scaling function is obtained from the \((e, e')\) data by dividing by a single nucleon cross section \cite{34, 11}. The starting point is the superscaling analysis with relativistic effective mass (SuSAM\*) that is based in the relativistic mean field model (RMF) of nuclear matter \cite{8, 9}. The SuSAM* scaling function describes most of the QE events but it contains residual effects due to SRC and MEC. In Ref. \cite{22} the MEC 2p2h contribution was subtracted from the \((e, e')\) data, using a RMF calculation, and a new scaling function was obtained without contamination from the MEC 2p2h channel. But this function still contains contributions from other non-QE mechanisms, in particular from 2p2h excitations produced by SRC.

To extract the 2p2h contribution of the correlations we assume that the tail of the scaling function, \( f^*(\psi^*) \), for high values of the scaling variable, \( \psi^* \) —or equivalently for high values of the energy transfer— is due mainly to SRC. In fact SRC produce high momentum components in the nuclear wave function while the excitation of nucleons with high momentum implies high values of the scaling variable, \( \psi^* > 1 \), which gives a contribution to the tail of \( f^*(\psi^*) \). In this work we assume that the SRC 2p2h response functions can be parametrized with semiepipirical formula similar to that of the MEC responses \cite{23}. By this assumption the SRC 2p2h response will be proportional to the 2p2h phase space and to the single-nucleon response. The effect of the SRC will be encoded in coefficients fitted to reproduce the tail of the scaling function. Our approach is alternative and complementary to current search on SRC pairs from inclusive electron scattering data in the zone of low energy and high transferred momentum \( q > 1.5 \text{ GeV/c} \) \cite{37}.

In this work we focus on intermediate momentum transfer \( q \leq 1 \text{ GeV/c} \). This region is of interest for neutrino experiments where we are going to apply our results. Our assumption in this study is that the emission of two particles is the result of energy-momentum transfer to a pair of correlated nucleons, whose wave function contains high-momentum components. In the independent pair approximation, the high-momentum component is thought to originate from the interaction between nucleon pairs in the nuclear medium, which can be mathematically expressed as the solution to the Bethe-
Goldstone (BG) equation. We will demonstrate that the product of the OB current with the high-momentum wave function of a nucleon pair leads to an effective two-body correlation current. The semi-empirical formula is obtained under the approximation of factorizing the one-body (OB) current in the 2p2h cross section multiplied by a correlation coefficient that accounts for the average high-momentum distribution of the nucleon pairs. After adjusting the correlation coefficients to the tail of the scaling function, we will see that the resulting 2p2h cross section due to SRCs in the QE region is large, of the same order as MEC.

In the SuSAM* approach the superscaling analysis of electron scattering data is used to predict QE neutrino-nucleus cross sections \[42\]. The neutrino (antineutrino) cross section was extended to the 2p2h sector with a relativistic MEC operator in the RMF model of nuclear matter \[23\], where the semiempirical formula for two-nucleon emission responses was fitted to the exact results for momenta in the range \(q = 200\) and 2000 MeV/c. The semiempirical formula allows to compute accurately the 2p2h MEC responses using an analytical formula, thus reducing the calculation time. In the same way in this work we extend the semiempirical formula for 2p2h correlations to neutrino scattering using the same correlations coefficients obtained from electron scattering, and using the single-nucleon weak responses.

The scheme of the paper is as follows. In Sect. II we present the superscaling formalism. In sect. III we relate the correlation current with the high-momentum component of the wave function of a nucleon pair. In sect. IV we introduce the semiempirical formula for the SRC response functions in the 2p2h channel. In sect. V we present the results. Finally, in Sect. VI we draw our conclusions.

II. FORMALISM

We follow the formalism of Ref. \[2\]. We start with the inclusive electron scattering cross section in plane-wave Born approximation with one photon-exchange

\[
d\sigma = \sigma_{Mott}(v_L R_L(q, \omega) + v_T R_T(q, \omega)),
\]

where the incident electron has energy \(\epsilon\), the scattering angle is \(\theta\), the final energy is \(\epsilon'\), and \(\Omega\) is the solid angle for the electron detection. The energy transfer is \(\omega = \epsilon - \epsilon'\), the momentum transfer is \(q\). In Eq. \(1\) \(\sigma_{Mott}\) is the Mott cross section, \(v_L\) and \(v_T\) are kinematic factors coming from the leptonic tensor

\[
v_L = \frac{Q^4}{q^4}, \quad v_T = \tan \frac{\theta}{2} \frac{Q^2}{2q^2}.
\]

Finally, the longitudinal and transverse response functions, only depend on \((q, \omega)\) and are the following components of the hadronic tensor

\[
R_L(q, \omega) = W^{00}, \quad R_T(q, \omega) = W^{11} + W^{22},
\]

where \(W^{\mu\nu}\) is the nuclear hadronic tensor, see Ref. \[2\] for details.

In refs. \[22, 23\] we developed a model of the reaction where the responses are the sum of QE plus 2p2h MEC contribution

\[
R_K(q, \omega) = R_K^{QE}(q, \omega) + R_K^{MEC}(q, \omega) + \cdots,
\]

for \(K = L, T\), where contributions of pion emission, and beyond are not included in our model. The MEC responses were studied in detail in \[23\], where a semiempirical formula was derived from a microscopical relativistic current in the RMF model of nuclear matter. We focus in this work on the QE responses that we describe using the SuSAM* approach \[22, 41\]. This model is an extension of the RMF model of nuclear matter \[8\], where the initial and final nucleons are interacting with the nuclear mean field and acquire an effective mass \(m_N^*\). The 1p1h response functions are

\[
R_K^{QE}(q, \omega) = \frac{V}{(2\pi)^3} \int dq \langle \frac{(m_N^*)^2}{\epsilon' - \epsilon} \delta(E' - E - \omega) \times \theta(p' - k_F) (= h) 2U_K, (5)
\]

The initial nucleon has momentum \(h\) below the Fermi momentum, \(h < k_F\), and on-shell energy \(E = \sqrt{\hbar^2 + m_N^{*2}}\). The final nucleon has momentum \(q' = h + q\), and the final on-shell energy is \(E' = \sqrt{\hbar^2 + m_N^{*2}}\). Pauli blocking implies \(p' > k_F\). The \(U_K\) are the single-nucleon responses for the 1p1h excitation

\[
U_L = w^{00}, \quad U_T = w^{11} + w^{22},
\]

corresponding to the single-nucleon hadronic tensor

\[
w^{\mu\nu} = \frac{1}{2} \sum_{ss'} j^{\mu}_{OB}(p', h)_{s's} j^{\nu}_{OB}(p', h)_{s's},
\]

and \(j^{\mu}_{OB}\) is the electromagnetic current matrix element

\[
j^{\mu}_{OB}(p', h)_{s's} = \epsilon(p')_{s's} \left[ F_1 \gamma^\mu + i \frac{F_2}{2m_N} \sigma^{\mu\nu} q^\nu \right] u(h)_{s'},
\]

where \(F_1\) and \(F_2\) are the Dirac and Pauli form factors. To compute the integral of Eq. \(5\), the usual procedure is to change from variable \(\theta_h\) to \(E'\). The integral over \(E'\) is made using the Dirac delta, this fixes the value of the angle between \(h\) and \(q\), \(\cos \theta_h = (2E\epsilon + Q^2)/(2hq)\), and the integration over the azimuthal angle \(\phi\) gives \(2\pi\) by symmetry of the responses when \(q\) is on the -axis \[2\]. We are left with an integral over the initial nucleon energy

\[
R^{QE}(q, \omega) = \frac{V}{(2\pi)^3} \frac{2\pi m_N^3}{q} \int_{\epsilon_0}^{\infty} d\epsilon \, n(\epsilon) 2U_K(\epsilon, q, \omega),
\]

where \(\epsilon = E/m_N^*\) is the initial nucleon energy in units of \(m_N^*\), and \(\epsilon_F = E_F/m_N^*\) the (relativistic) Fermi energy in the same units. Moreover we have introduced the energy distribution of the Fermi gas \(n(\epsilon) = \theta(\epsilon_F - \epsilon)\). The
lower limit, $\epsilon_0$ of the integral in Eq. (9) corresponds to the minimum energy for a initial nucleon that absorbs energy $\omega$ and momentum $q$. It can be written as (see Appendix C of ref. [2])

$$
\epsilon_0 = \text{Max} \left\{ \sqrt{1 + \frac{1}{\tau} - \lambda, \epsilon_F - 2\lambda} \right\},
$$

(10)

where we have introduced the dimensionless variables

$$
\lambda = \frac{\omega}{2m_N^*}, \quad \kappa = \frac{q}{2m_N^*}, \quad \tau = \kappa^2 - \lambda^2.
$$

(11)

Now we define a mean value of the single-nucleon responses [13] by averaging with the energy distribution $n(\epsilon)$

$$
\bar{U}_K(q, \omega) = \frac{\int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon)U_K(\epsilon, q, \omega)}{\int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon)}.
$$

(12)

Using these averaged single-nucleon responses we can rewrite Eq. (9) in the form

$$
R_{QE}^K(q, \omega) = \frac{V}{(2\pi)^3} \frac{2\pi m_N^*}{q} 2\bar{U}_K \int_{\epsilon_0}^{\infty} d\epsilon n(\epsilon).
$$

(13)

The superscaling function is defined as

$$
\frac{4}{\lambda} \xi_F f^*(\psi^*) = \int_{\epsilon_0}^{\infty} n(\epsilon)d\epsilon,
$$

(14)

where $\xi_F = \epsilon_F - 1$ is the kinetic Fermi energy in units of $m_N^*$. Note that this integral only depends on the variable $\epsilon_0$, which in turn depends on $(q, \omega)$. The definition (14) is, except for a factor, similar to that of the $y$-scaling function $f(y)$ [44, 45], where the scaling variable $y$ was the minimum moment of the initial nucleon. In this paper we use the $\psi^*$-scaling variable. The minimum energy of the nucleon, $\epsilon_0$, is transformed by a change of variable into the scaling variable, $\psi^*$, defined as

$$
\psi^* = \sqrt{\frac{\epsilon_0 - 1}{\epsilon_F - 1}} \text{sgn}(\lambda - \tau).
$$

(15)

In RFG and in nuclear matter with RMF $1 \leq \epsilon_0 \leq \epsilon_F$ and consequently the RFG superscaling function is zero outside this interval, that corresponds to $1 < |\psi^*|$ for all nuclei. However, in a real nucleus the momentum is not limited by $k_F$, since nucleons can have higher momentum, especially correlated nucleons can greatly exceed the Fermi momentum. This has the effect that the phenomenological superscaling function is not zero for $|\psi^*| > 1$. This high-momentum region of the phenomenological scaling function will allow us extracting information about the correlations.

Using $V/(2\pi)^3 = N/(2\pi k_F^3)$ we can write

$$
R_{QE}^K = \frac{\xi_F}{m_N^* \eta_F K} (Z\bar{U}_K^i + N\bar{U}_K^o) f^*(\psi^*),
$$

(16)

where we have added the contribution of $Z$ protons and $N$ neutrons to the response functions, and $\eta_F = k_F/m_N^*$. The SuSAM* approach, extends the formula (16) using a phenomenological scaling function, obtained from experimental data of $(\epsilon, \epsilon')$. In the SuSAM* of ref. [22] the 2p2h contribution of MEC was first subtracted from the inclusive cross section and then divided by the contribution of the single nucleon.

$$
\bar{f}_{QE}^i = \frac{d\sigma}{d\Omega_{d\epsilon_e}} - \frac{d\sigma}{d\Omega_{d\epsilon_e}}_{\text{MEC}},
$$

(17)

where

$$
\eta_F = \frac{Z}{m_N^* \eta_F K} (Z\bar{U}_K^i + N\bar{U}_K^o).
$$

(18)

Second a selection of the QE data points was done by noting that approximately half of the data collapse into a point cloud around the RFG scaling function. This point cloud constitutes the data that can be considered approximately QE and we reject the rest, which contribute to inelastic processes. Examples of the selected QE data are shown in Figs. 1 and 2.

### III. 2P2H RESPONSE FUNCTIONS WITH CORRELATED PAIRS

Before proposing our semiempirical parameterization of the 2p2h response due to correlations, in this section, we study in some detail the expected structure of 2p-2h response functions in a simple model. Our theoretical motivation is based on the RFG and the independent pair approximation or Bethe-Goldstone equation for the wave function of a pair of correlated nucleons in presence of the nuclear medium. Our aim is not to develop a detailed and exhaustive microscopic model, but rather to provide some basic outlines in a schematic model, as a starting point for proposing a formula that accounts for the fundamental ingredients that determine the nuclear response.

For convenience we use here square brackets in the notation of normalized states within the volume box $V$, indicating that the states are normalized to unity within the box

$$
| [p] \rangle \equiv \frac{\epsilon^{|p|} p_r}{\sqrt{V}} | \frac{1}{2} s \rangle \otimes | \frac{1}{2} t \rangle,
$$

(19)

while the absence of brackets indicates that the states are normalized over the entire space to a Dirac’s delta of momentum, i.e.

$$
| p \rangle \equiv \frac{\epsilon^{p_r} p_r}{(2\pi)^{3/2}} | \frac{1}{2} s \rangle \otimes | \frac{1}{2} t \rangle.
$$

(20)

The previous states carry implicit spin and isospin indices, which we do not write for convenience and to keep the equations short in order to enhance clarity.
In the absence of correlations, the only mechanism of interaction that can excite a 2p2h state is a two-body current, particularly meson exchange currents, whose matrix elements in the RFG can be written as:

$$\langle [p'_1, p'_2] | J_\mu(q, \omega) | [h_1, h_2] \rangle = \frac{(2\pi)^3}{V^2}$$ \hspace{1cm} (21)

$$\delta(p'_1 + p'_2 - q - h_1 - h_2) \delta^\mu(p'_1, p'_2, h_1, h_2),$$

where \( p'_1 > k_F \) and \( h_i < k_F \). The two-body current function \( J_\mu(p'_1, p'_2, h_1, h_2) \) also depends implicitly on spin-isospin indices.

The corresponding inclusive hadronic tensor in the 2p2h channel can be written as

$$W_{2p2h}^{\mu\nu}(q, \omega) = \frac{V}{(2\pi)^3} \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \left( \frac{m_N^*}{2} \right)^4$$

$$\times \phi_1^{\mu\nu}(p'_1, p'_2, h_1, h_2)$$

$$\times \theta(p'_1 - k_F^N) \theta(k_F^N - h_1) \theta(p'_2 - k_F^N) \theta(k_F^N - h_2)$$

$$\times \delta(E'_1 + E'_2 - E_1 - E_2 - \omega)$$

$$\times \delta(p'_1 + p'_2 - q - h_1 - h_2),$$ \hspace{1cm} (22)

where \( E_i, E'_j \) are the on-shell energies of nucleons with momenta \( h_i, p'_i \), and with relativistic effective mass \( m_N^* \).

The function \( \phi_1^{\mu\nu}(p'_1, p'_2, h_1, h_2) \) is the tensor for a single 2p2h transition, summed up over spin and isospin,

$$\phi_1^{\mu\nu}(p'_1, p'_2, h_1, h_2) =$$

$$= \frac{1}{4} \sum_{s_1,s'_1 s_2,s'_2 t_1,t_1' t_2,t_2'} j^{\nu}(1', 2', 1, 2) \delta(s_1 s'_1 s_2 s'_2) j^{\mu}(1', 2', 1, 2).$$ \hspace{1cm} (23)

where the two-body current function is antisymmetrized

$$j^{\nu}(1', 2', 1, 2) = j^{\nu}(1', 2', 1, 2) - j^{\nu}(1', 2, 2, 1).$$ \hspace{1cm} (24)

The factor \( 1/4 \) in Eq. (23) accounts for the anti-symmetry of the two-body wave function with respect to exchange of momentum, spin and isospin quantum numbers, to avoid double counting of the final 2p2h states.

Let’s now consider the element of the one-body current \( J_\mu(q, \omega) = J_\mu^{OB} = \sum_i J_i^\mu \), where the \( J_i^\mu \) acts on the \( i \)-th particle. We have

$$\langle [p'_1, p'_2] | J_\mu^{OB}(q) | [h_1, h_2] \rangle = \langle p'_1 p'_2 | J_i^\mu | h_1 h_2 \rangle = 0$$ \hspace{1cm} (25)

Indeed, a one-body current cannot produce 2p2h excitations due to the orthogonality of plane waves \( \langle p'_i | h_j \rangle = 0 \).

The situation changes when we consider that the two states \( h_1 \) are interacting in the medium, with a short-range NN potential \( V_{NN} \) that produces scattering to unoccupied states. If we turn-on the NN interaction the wave function \( |h_1 h_2\rangle \) of the pair is modified, but due to Pauli blocking it can only acquire momentum components above the Fermi momentum. In the independent pair approximation the wave function \( |h_1 h_2\rangle \) is replaced by the correlated wave function \( |\Phi_{h_1 h_2}\rangle \)

$$|h_1, h_2\rangle \rightarrow |\Phi_{h_1 h_2}\rangle = |h_1, h_2\rangle + |\Delta \Phi_{h_1 h_2}\rangle$$ \hspace{1cm} (26)

where \( |\Delta \Phi_{h_1 h_2}\rangle \) only has high momentum components. In the independent pair approximation the wave function is obtained from the solution of the BG equation for the correlated wave function \( |\Phi_{h_1 h_2}\rangle \). Since the NN interaction conserves the total momentum, the correlated part of the wave function in momentum space verifies

$$\langle p_1 p_2 | \Delta \Phi_{h_1 h_2} \rangle = \delta(p_1 + p_2 - h_1 - h_2) \Delta \varphi_{h_1 h_2}(p)$$ \hspace{1cm} (27)

where \( p = \frac{1}{2} (p_1 - p_2) \) is the relative momentum of the nucleon pair, and \( \Delta \varphi_{h_1 h_2}(p) \) is the relative wave function of \( \Phi_{h_1 h_2}(p) \). From the BG equation, it is given by

$$\Delta \varphi_{h_1 h_2}(p) = \frac{\theta(p_1 - k_F)\theta(p_2 - k_F)}{h^2 - p^2} \langle \mu | V_{NN} | \varphi_{h_1 h_2} \rangle$$ \hspace{1cm} (28)

where \( \varphi_{h_1 h_2}(p) \) is the relative wave function of \( \Phi_{h_1 h_2}(p) \), \( \mu = \frac{m_N}{2} \) is the reduced mass of the two-nucleon system and \( h = (h_1 - h_2)/2 \) the initial relative momentum, while \( p_1 = (h_1 + h_2)/2 + p \) and \( p_2 = (h_1 + h_2)/2 - p \). The Pauli blocking functions in Eq. (28) ensure that the wave function has high-momentum components. As we can see, it is not possible to remove the dependence on the total momentum \( h_1 + h_2 \) appearing in the Pauli-blocking step functions. Therefore a dependence of short-range correlations on the CM momentum of the pair appears.

Now, by applying a one-body current to a system of two correlated nucleons, it is possible to generate a two-particle state that exists above the Fermi level. The 2p2h matrix element of the one-body current is computed in Appendix A. It can be written similarly to Eq. (21)

$$\langle [p'_1, p'_2] | J_\mu^{OB}(q) | [\Phi_{h_1 h_2}] \rangle = \frac{(2\pi)^3}{V^2}$$

$$\delta(p'_1 + p'_2 - q - h_1 - h_2) j_{cor}^\mu(p'_1, p'_2, h_1, h_2).$$ \hspace{1cm} (29)

It is noteworthy that the matrix element describing the effect of the one-body current on the wave function of two correlated nucleons is formally similar to the matrix element of a two-body correlation current, which is represented by the function \( j_{cor}^\mu \)

$$j_{cor}^\mu(p'_1, p'_2, h_1, h_2) =$$

$$= (2\pi)^3 j_{OB}^\mu(p'_1, p'_2 - q) \Delta \varphi_{h_1 h_2} \left( p'_1 - \frac{q}{2} \right)$$

$$+ (2\pi)^3 j_{OB}^\mu(p'_2, p'_2 - q) \Delta \varphi_{h_1 h_2} \left( p'_2 + \frac{q}{2} \right)$$ \hspace{1cm} (30)

where \( p' = (p'_1 - p'_2)/2 \) is the relative momentum of the final particles.

The correlation current is the sum of the products of the OB current multiplied by the high-momentum wave function of the initial correlated pair. This is illustrated in the diagrams of Fig. 1. The shaded rectangle represents the correlations resulting in the production of high-momentum components of the correlated pair (\( h_1, h_2 \)). One of the high-momentum nucleons absorbs a photon with momentum \( q \), while the other nucleon is emitted through its interaction with the first nucleon. In Eq.
the products are, in fact, a multiplication of spin matrices, although we have left this out of equation (30) for clarity. The complete formula dependent on spin is provided in the appendix A, Eq. (A3). Note also that the current depends on isospin. Hence, when the initial state consists of a proton-neutron pair, the current acting on the first particle must correspond to the proton current, while the current acting on the second particle should correspond to the neutron current.

By inserting the correlation current (30) in Eq. (23) the hadronic tensor for a 2p2h transition is obtained. The isospin sums in Eq. (23) can be written as sums over pp, pn, and nn correlated pairs. Therefore

$$w^{\mu\nu}(p'_1, p'_2, h_1, h_2) = w^{\mu\nu}_{pp} + w^{\mu\nu}_{pn} + w^{\mu\nu}_{nn}. \quad (31)$$

Writing explicitly the isospin indices, $N, N' = p, n$, the diagonal components of the hadronic tensor $w^{\mu\nu}_{NN'}$ are the following

$$w^{\mu\nu}_{NN'}(p'_1, p'_2, h_1, h_2) =$$

$$(2\pi)^6 \left[ j_{NN'}^P(p'_1, p'_1 - q)\Delta_{\chi_{h_1h_2}}(p'_1 - \frac{q}{2}) \right]^2$$

$$+ (2\pi)^6 \left[ j_{NN'}^N(p'_2, p'_2 - q)\Delta_{\chi_{h_1h_2}}(p'_2 + \frac{q}{2}) \right]^2$$

$$+ 2(2\pi)^6 \text{Re} \left\{ j_{NN'}^s(p'_1, p'_1 - q)\Delta_{\chi_{h_1h_2}}(p'_1 - \frac{q}{2}) \right\} \times j_{NN'}^P(p'_2, p'_2 - q)\Delta_{\chi_{h_1h_2}}(p'_2 + \frac{q}{2}) \right\} \quad (32)$$

In the independent pair approximation the 2p2h hadronic tensor $W^{\mu\nu}_{NN'}(q, \omega)$ due to SRC, is defined as the integral of the correlated-pair tensor $w^{\mu\nu}_{NN'}(p'_1, p'_2, h_1, h_2)$ over the momentum space of 2p2h excitations, using Eq. (22). The computation of this tensor falls outside the scope of the present work. The proposed approach for such a calculation would require solving the Bethe-Goldstone equation for every nucleon pair $h_1, h_2$ while considering the center of mass of the two particles $h_1 + h_2 \neq 0$, and performing a seven-dimensional integration. In reference 47, the BG equation was solved for the particular case of back-to-back nucleons, $h_1 + h_2 = 0$, moving along the z-axis in a multipole expansion with a potential $V_{NN'}$ fitted to NN scattering data 48. Although the computation seems feasible, incorporating the center of mass for particles moving in any direction requires careful consideration of various technical details. As we move forward with the precise microscopic calculation, we introduce an alternative phenomenological approach in the upcoming section. This approach relies on the theory proposed in this section, with a few simplifications, to provide a rough estimation of the expected outcome for the SRC responses.

IV. SEMIEMPIRICAL 2P2H-SRC RESPONSE FUNCTIONS

The 2p2h response functions for two-body operators (MEC) was explored in Ref. 23. Our analysis revealed that the responses of a pair can be factorized out of the integral (22) with reasonable approximation. By doing so, we derived a semi-empirical expression for the MEC responses, with the coefficients to be determined by later fitting. In this work we assume that a similar factorization occurs in the case of correlated responses. Thus the SRC responses must be proportional to the phase-space for the emission of a nucleon pair leaving the nucleus. In the framework of the RMF of nuclear matter the 2p2h phase-space function is given by the integral

$$F_{NN'}(q, \omega) = \int d^3 p'_1 d^3 p'_2 d^3 h_1 d^3 h_2 \frac{(m^*_N)^4}{E_1 E_2 E'_1 E'_2} \times \theta(p'_1 - k_{F}^N)\theta(k_{F}^N - h_1)\theta(p'_2 - k_{F}^N)\theta(k_{F}^N - h_2)$$

$$\times \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \times \delta(p'_1 + p'_2 - q - h_1 - h_2), \quad (33)$$

where $N, N'$ can be proton or neutron, depending on the initial correlated state pp, pn or nn and $k_{F}^N$ is the Fermi momentum of the nucleon of $N$ kind. The phase space function is roughly proportional to the number of 2p2h excitations allowed by the kinematic for momentum and energy transfer $(q, \omega)$.

The concept of factorization in the 2p2h channel bears some resemblance to that employed in the 1p1h channel, where the response was written as a single-nucleon averaged response multiplied by the superscaling function. In the 2p2h case, we will also define the correlated-pair tensor averaged over the momentum space of 2p2h excitations, by dividing the hadronic tensor over the phase space function as follows

$$\langle w^{\mu\nu}_{NN'}(q, \omega) \rangle = \frac{W^{\mu\nu}_{NN'}(q, \omega)}{F_{NN'}(q, \omega)} \quad (34)$$

This leads to an exact factorization of the correlated hadronic tensor as the averaged tensor of a pair of nucle-
ons multiplied by the phase space.

\[ W^\mu_{NN'}(q, \omega) = \frac{V}{(2\pi)^3} F_{NN'}(q, \omega) \overline{\varphi}^\mu_{NN'}(q, \omega) \]  \hfill (35)

The averaged tensor of a pair accounts for the correlations in this formula, while the phase space does not contain any information about them. The phase space is exclusively related to the kinematics of independent particles without correlations in the Fermi gas.

The average of the tensor \( w^\mu_{NN'} \) in equation (32) is typically complicated. Two approximations lead to the simplification of this term. We will make the assumption that the average of the product can be written as the product of averages, and neglect on the average the interference term between the two particles, given by the last term in Eq. (32). In this way we obtain for the diagonal components

\[ \overline{\varphi}^\mu_{NN'}(q, \omega) \simeq (2\pi)^6 \left( |j^\mu_N| + |j^\mu_{N'}| \right)^2 \left| \Delta \varphi^N_{h_1h_2} \right|^2, \]  \hfill (36)

where we use the notation

\[ |j^\mu_N| \equiv |j^\mu_N(P'_1, P'_2 - q)|^2 = |j^\mu_N(P'_2, P'_2 - q)|^2 \]  \hfill (37)

\[ \left| \Delta \varphi^N_{h_1h_2} \right|^2 = \left| \Delta \varphi^N_{h_1h_2} \left( p' + \frac{q}{2} \right) \right|^2 = \left| \Delta \varphi^N_{h_1h_2} \left( p' - \frac{q}{2} \right) \right|^2. \]  \hfill (38)

By focusing on the minimal ingredients required to construct a semi-empirical formula for correlated nuclear responses, from Eqs. (35,36) we arrive at a simplified perspective that includes the product of three factors: the 2p-2h phase-space integral, the averaged response of a single nucleon, and the average high-momentum distribution of a nucleon pair. Thus we propose the following ansatz for the semi-empirical formula of 2p2h correlated responses in electron scattering.

\[ R^K_{SRC} = \frac{V}{(2\pi)^3} \frac{1}{m_N^2 m_N^2} \frac{2}{A(1 - 1)} \left[ \frac{Z(Z - 1)}{2} F_{pp}(q, \omega) c^K_{pp}(q)(2\overline{U}_K) \right. \]

\[ + N F_{pn}(q, \omega) c^K_{pn}(q)(\overline{U}_K + \overline{U}_{pp}) \]

\[ + N(N - 1) F_{nn}(q, \omega) c^K_{nn}(q)(2\overline{U}_K) \]  \hfill (39)

The mass of the pion \((m_\pi)^4\) in the denominator has been introduced for convenience by analogy to MEC responses \hfill (23) \hfill and the mass of the nucleon \(m_N^2\) in the denominator is set so that the correlation coefficients \(c^K_{NN'}(q)\) are dimensionless.

In Eq. (39), we have separated the contributions of the pairs, pp, pn and nn. Each of them is proportional to the sum of the corresponding single-nucleon responses, \(\overline{U}_K\) and/or \(\overline{U}_{pp}\). This is because the photon can be absorbed by both nucleons of the pair. Also, each contribution has been multiplied by the number of pairs pp, pn or nn and divided by the total number of pairs, \(A(A - 1)/2\), to take into account asymmetric matter \(Z \neq N\). This approach follows the lines of the model of ref. 85.

Since the coefficients \(c^K_{K}(q)\) are related to the high momentum components of a pair of nucleons (\(h_1, h_2\), they should strongly depend on isospin, given that the high momentum components of proton-neutron dominate over proton-proton and neutron-neutron. Here we will assume that they are proportional \(c^K_{pp}(q) = c^K_{nn}(q) = \alpha c^K_{nn}(q)\), where \(\alpha\) is a small constant. Experiments on \(^{12}\)C have reported a number of np pairs 18 times larger than their pp counterparts \hfill (49, 50) \hfill . Then a reasonable value is \(\alpha = 1/18\).

Is also reasonable to assume that the \(c^K_{NN'}\) coefficients are the same for both the longitudinal and transverse response, \(c^K_{L} = c^K_{T} = c^K_{NN'}\). This assumption greatly simplifies the semiempirical formula because it only depends on two parameters \(\epsilon^m\) and \(\alpha\). It is important to note that this assumption may not hold true in all cases, and care should be taken when interpreting results obtained using this simplification.

Based on the equation (36), the correlation coefficient \(\epsilon^m(q)\) in the independent pair approximation is related to the average momentum distribution of a proton-neutron pair in the 2p2h excitation.

\[ \frac{\epsilon^m(q)}{m_N^2 m_N^2} \simeq (2\pi)^6 \left( \sum_{s_1s_2s_1'2} |\Delta \varphi^m_{h_1h_2}(p' + q/2)|^2 \right). \]  \hfill (40)

In the case of symmetric nuclei, \(N = Z\), Eq. (39) reduces to

\[ R^K_{SRC} = \frac{V}{(2\pi)^3} F(q, \omega) \frac{Z + \alpha(Z - 1)}{2Z - 1} \frac{\epsilon^m(q)}{m_N^2 m_N^2} \left( \overline{U}_L + \overline{U}_T \right). \]  \hfill (41)

From the response functions we can write the semiempirical formula for the inclusive cross section in the 2p2h channel due to short-range-correlations

\[ \frac{d\sigma}{dQ'd\epsilon'}_{SRC} = \frac{g_{\text{int}} V}{(2\pi)^3} \frac{N^2 m_N^2 m_N^2}{2Z - 1} \left[ v_L(\overline{U}_L + \overline{U}_L') + v_T(\overline{U}_T + \overline{U}_T') \right]. \]  \hfill (42)

In the next section we use the \(^{12}\)C(\(e, e'\)) cross section data to fit the correlation coefficient \(\epsilon^m(q)\) for each \(q\).

In the case of CC neutrino scattering the semiempirical formula extends naturally by replacing the electromagnetic single nucleon responses with the corresponding to the \(n(\nu_{\mu}, \mu)p\) or \(p(\nu_{\mu}, \mu^n)n\), assuming the same relation between the correlation coefficients of the \(nn\) and (\(pp\) in the case of anti-neutrino) pairs to the \(np\) pairs in the
initial state $c^{nn} = c^{pp} = \alpha c^{nn}$

$$\left( \frac{d\sigma}{d\Omega d\epsilon'} \right)^\nu_{SRC} = \frac{\sigma_0 V}{2(2\pi)^3} \frac{1}{m_N^2 m_\pi^2} \frac{Z + \alpha(Z - 1)}{2Z - 1} \frac{1}{[\mathcal{V}_{CC} + 2 \mathcal{V}_{CL} \mathcal{U}_{CL} + \mathcal{V}_{LL} \mathcal{U}_{LL} + V_T \mathcal{U}_T \pm 2V_T \mathcal{U}_T]}, \quad (43)$$

where $\sigma_0$ is

$$\sigma_0 = \frac{G^2 c^2 \theta_c k'_F}{4\pi^2} \epsilon \left[ (\epsilon + \epsilon')^2 - q^2 \right], \quad (44)$$

with $G$ the Fermi constant and $\theta_c$ the Cabibbo angle. The lepton coefficients for neutrino scattering $V_K$ are given in Ref. [2]. The average single nucleon responses $\mathcal{U}_K$ are given in Appendix B. Here we use a new version of these responses using the definition (12) with the distribution $n(\epsilon)$ obtained from the phenomenological scaling function. This differs from the traditional definition [42] only for $\psi > 1$, where the contribution of the nucleons with momentum greater than $k_F$ is correctly taken into account, while the traditional definition is an extrapolation of the RFG where the nucleons are limited by $k_F$ [13].

Finally, in the semi-empirical formula will make use of the following approximation for the phase-space function $\psi$.

$$F_{NN}(q, \omega) = \left( 4\pi k'_F k_F^* \right)^3 m_N^2 \frac{1}{18} \sqrt{1 - \frac{4m_N^2}{(2m_N^2 + \omega)^2 - q^2}}. \quad (45)$$

Eq. (45) makes use of the frozen nucleon approximation to compute the integral [83]. The exact phase space was studied in depth in ref [54] as a function of $(q, \omega)$ for the kinematics of interest for neutrino experiments, $q \sim 1$ GeV, around the quasielastic peak. For these kinematics it was seen that the frozen approximation gives results very close to the exact value. The frozen approximation was also found to be quite accurate in the 2p2h MEC responses [55], and in the semi-empirical formula of the MEC responses [23].

V. RESULTS

In this section we fit the correlation coefficients of the 2p2h semiempirical formula and present predictions for the cross section of electron and neutrino scattering. We will compare with other calculations in nuclear models that introduce the SRC explicitly and we will see that the semiempirical formula is capable of reproducing the same results reasonably well taking into account the crude assumptions on which our phenomenological model is based.

For each value of $q$, the coefficient $c^{nn}(q)$ is fitted from the phenomenological scaling function under the hypothesis that the high energy tail ($\psi^* > 1$) is produced by the excitation of high-momentum nucleon pairs. In Fig. 2 we show the experimental data of $f_{QE}^*$ for $^{12}$C and for different $q$-values. These data have been obtained from the inclusive cross section by subtracting the 2p2h MEC contribution and dividing by the single nucleon cross section, Eq. (13). The Fermi momentum is $k_F = 225$ MeV/c and
TABLE I: Coefficients of the semiempirical formula for different values of the parameter $\alpha$. The estimated error of $c^\alpha(q)$ is 38%.

\[
\begin{array}{ccccccccccc}
q \ [\text{GeV/c}] & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
c^\alpha(\alpha = 0) & 866 & 2780 & 1588 & 1098 & 733 & 580 & 488 & 367 & 312 & 275 \\
c^\alpha(\alpha = \frac{1}{12}) & 7110 & 2628 & 1489 & 964 & 701 & 526 & 438 & 350 & 298 & 263 \\
\end{array}
\]

$M^* = m_N^*/m_N = 0.8$ \cite{22}. The errors of these parameters were estimated in ref \cite{41} in a $\chi^2$ fit. They are $\Delta E_F = 8$ MeV/c and $\Delta M^* = 0.044$. From the figure we see that the scaling is only approximate and the data are concentrated in a narrow band. The band is shown in Fig. 3 for $q \leq 1000$ MeV/c. The scaling model is based on two assumptions: first the single-nucleon factorization, which can be done assuming that the final state interactions (FSI) are small in the quasrelastic region. The second assumes that the scaling function depends solely on the scaling variable, which is a consequence of approximating the nuclear system by an infinite Fermi gas. The small scaling violation can be attributed to nuclear effects that breaks this approximation such as FSI, finite size and off-shell effects. Still the band of Fig. 3 is well defined following the shape of an asymmetric bell, with a evident tail on the right. The phenomenological scaling function of the SuSAM* model is obtained by fitting a sum of Gaussians to this band:

\[
f^*(\psi^*) = a_3e^{-(\psi^* - a_1)^2/(2a_2^2)} + b_3e^{-(\psi^* - b_1)^2/(2b_2^2)}.
\]

Note that there are no QE data above $\psi^* = 1.5$. Then it is not possible to know from the data how the tail would extend above this value. This is because the region $\psi^* > 1.5$ requires high values of the energy far from the QE region. For low $q$ there are not enough cross section data in this region, and for higher momentum transfer, $q > 500$ MeV/c we enter the inelastic zone with pion emission and the data no longer scale.

If we assume that the tail of the scaling function is produced by the emission of two correlated nucleons, then the semiempirical formula provides a theoretical way to extrapolate the model for high $\psi^*$ (of for high $\omega$). The tail hypothesis is based on previous calculations \cite{20,53,54,57} where the effect of the correlations in the 2p2h inclusive responses is a increasing function with the energy transfer contributing to produce a tail of the cross section at high energy. All these calculations are based on very different models with different kinds of correlations and provide numerical results that can differ in some cases even by an order of magnitude, but they are qualitatively similar to the 2p2h phase space, as in the present semiempirical approach.

In Fig. 2 we show the SRC contribution to the scaling function using the semiempirical formula, for different values of $q \leq 2000$ MeV/c. In all panels the SuSAM* scaling function, $f^*(\psi^*)$, is also shown. The correlated 2p2h results have been obtained by dividing the semiempirical formula, Eq. \cite{42} by the averaged cross section of the single nucleon as in Eq. \cite{137}

\[
I_{SRC}^* = \frac{(d\sigma_{SRC})_{SRC}}{\sigma_M(\nu_{LT} + \nu_{TT})}.
\]

Dividing Eq. \cite{12} over \cite{13} this gives

\[
f^{*}_{SRC}(q, \omega) = \frac{VF(q, \omega) m_N^* \eta_{p,n}^2 \kappa Z + \alpha(Z - 1) c^\alpha(q)}{\left(2\pi\right)^9} \frac{Z \xi_F}{2Z - 1} \frac{m_N^* m_e^*}{m_{2N}}
\]

Note first that $I_{SRC}^*(q, \omega)$ do not scales, i.e., it depends on $\psi$ but also on $q$. Second the dependency on energy of $I_{SRC}^*(q, \omega)$ comes solely from phase-space. This is because the single nucleon cross section has canceled with the denominator in Eq. \cite{47}. Then the effect of the correlations is a function increasing with energy as the phase space times a coefficient that is $q$-dependent. The coefficient $c^\alpha(q)$ is chosen to make $I_{SRC}^*$ to coincide with the tail of the scaling function for $\psi^* = 1.5$. They are given in Table 1 for $q = 100$ up to 1000 MeV/c. In the table we...
provide two sets of coefficients corresponding to the parameter $\alpha = 1/18$ and also $\alpha = 0$ (neglecting the pp and nn contribution). It is found that the coefficient $c^{\mu}(q)$ decreases with $q$. Roughly it behaves as $c^{\mu}(q) \sim 1/q$.

Since the experimental data of the scaling function are distributed in a thick band, as seen in Fig. 2, the coefficients $c^{\mu}(q)$ have an uncertainty that can be estimated by fitting the lower or upper part of the band for $\psi^* = 1.5$. These errors in $c^{\mu}(q)$ is approximately 38%. This error is large because it corresponds to the uncertainty of the scaling function in the tail zone for $\psi^* = 1.5$. This implies that the effect of the correlations...
on the 2p2h response in this approach presents the same uncertainty.

Our fit has been made for \( q \leq 1000 \text{ MeV}/c \) that is the region of interest for electron and neutrino scattering. For higher \( q \) the right side of the QE peak is missing from the data due to pion emission. In Fig. 3 we show the contribution \( f_{\text{SRC}}^{\ast} \) calculated with Eq. (18) for the the kinematics of all the QE experimental data with \( q < 1000 \text{ MeV}/c \). We see that these points generate a band that can explain the tail of the scaling function. To compute \( f_{\text{SRC}}^{\ast} \) for arbitrary \( q \)-values we have interpolated the coefficients \( c_{\text{pn}}^{\ast}(q) \) using the formula

\[
\sigma_{\text{pn}}^{\ast}(q) = a_0 \sqrt{\frac{m_N^2 + q m_N}{q}} \tag{49}
\]

that fits well the \( q \) dependence of \( c_{\text{pn}}^{\ast}(q) \) with the parameter \( a_0 = 197 \pm 74 \), see Fig. 4.

It can be observed from Eq. (28) that the high-momentum function \( \Delta \varphi_{h_1 h_2}(p) \) has a denominator of \( h^2 - p^2 \), causing the average distribution \( \langle \Delta \varphi_{h_1 h_2}(p + q/2) \rangle \) to rapidly diminish as \( q \) increases. This characteristic could be the reason for the \( q \)-dependence of the coefficients, although the precise relation is concealed by the averaging and not straightforward to establish.

Once the coefficients of the semiempirical formula have been fitted, we propose to carry out a new scaling analysis without the contribution of the emission of two particles. To do this we subtract the \( f_{\text{SRC}}^{\ast} \) contribution from the \( f_{\text{QE}}^{\ast} \) data. Since the MEC contribution had already been subtracted, the new data no longer contain 2p2h contribution and can thus be considered purely 1p1h data. The result of this new scaling analysis is shown in Fig. 5.

As we see, after the subtraction a new band of points is obtained that is symmetrical, since the emission of two nucleons has been eliminated. A new phenomenological scaling function can now be fitted with a Gaussian

\[
f_{\text{1p1h}}^{\ast}(\psi^\ast) = b e^{-(\psi^\ast)^2/a^2}. \tag{50}
\]

The fitted coefficients are \( a = 0.744 \pm 0.082 \) and \( b = 0.682 \pm 0.102 \). The errors in the parameters have been estimated by fitting Gaussians to the upper and lower part of the band.

With the present approach we can compute the inclusive effective section of electrons and neutrinos by adding separately the contributions of 1p1h, 2p2h-SRC and 2p2h-MEC contributions.

\[
\frac{d\sigma}{d\psi' d\epsilon'} = \left( \frac{d\sigma}{d\psi' d\epsilon'} \right)_{\text{1p1h}} + \left( \frac{d\sigma}{d\psi' d\epsilon'} \right)_{\text{SRC}} + \left( \frac{d\sigma}{d\psi' d\epsilon'} \right)_{\text{MEC}}, \tag{51}
\]

where the 1p1h cross section is computed with the scaling function \( \psi^\ast \), the 2p2h-SRC with the semiempirical formula fitted above, and the 2p2h-MEC with the semiempirical formula fitted in Eq. (22).

Results are shown if Fig. 6 for electrons and Fig. 7 for neutrinos. In Fig. 6 we show the inclusive cross section of \( ^{12}\text{C} \) for several kinematics as a function of \( \omega \). We show the separate contributions of the three terms of Eq. (51) and the total contribution of the model. The pion emission is not included.

The \( \omega \)-dependence of the 2p2h-SRC is similar to the 2p2h-MEC contribution and extends from the QE peak to the dip region and also to the \( \Delta \) region. The order of magnitude is similar to the MEC, except at the \( \Delta \)-peak where the MEC are larger. For incident electron energy \( \epsilon = 680 \text{ MeV} \) we compare with the SRC model of ref. \[13\] whose results are quite similar to ours. The present results are also similar to the SRC calculation of \[56\] for electron and neutrino scattering.

\[
\sigma_{\text{pn}}^{\ast}(q) = a_0 \sqrt{\frac{m_N^2 + q m_N}{q}} \tag{49}
\]

A similar behavior is seen in Fig. 7 where we show the \( \langle \nu\mu, \mu \rangle \) cross section from \(^{16}\text{O} \) and \(^{12}\text{C} \) for fixed neutrino energy and for two scattering angles as a function of \( \omega \). A comparison with the SRC results of \[13\] is also shown for \(^{16}\text{O} \) and \( \theta_\mu = 60^\circ \).

The comparison with the SRC models of \[20\] and \[13\] is shown in more detail in Fig. 8. Ghent model disagrees at high energy transfer where the response is above ours, Valencia model result is quite similar to ours even at high energy, except for fine details. One should keep in mind that the models of refs. \[20\] and \[13\] are non-relativistic. Taking into account the crude approximations that we have made to obtain the semiempirical formula, the agreement with the results of other more complex models is remarkable.

We must emphasize that the results in Fig. 8 are at least indicative that the semi-empirical formula is not completely off track, and that the tail of the scaling function may, at least in part, contain the average ef-
effect of correlations. The Ghent model calculates correlated responses using a cluster expansion in which nuclear states contain realistic operatorial correlation functions for different spin-isospin channels. The Valencia model introduces short-range correlations through the effective Landau-Migdal interaction. In this sense, the two models are completely different and our simultaneous agreement with both of them is unlikely to be mere coincidence. Indeed, the semiempirical calculation presented in this work is based on a direct fit to the tail of the scaling function to obtain the coefficients $c^{\text{PN}}(q)$. On the other hand, the theoretical models of Ghent and Valencia in Fig. 8 use more advanced approaches. Although these theoretical approaches are more sophisticated, the fact that the results obtained with the presented semiempirical model resemble the results of the theoretical models is an indication that the semiempirical approach can be a useful tool for understanding the structure of nucleon-nucleon correlations.

VI. CONCLUSIONS

We have introduced a semiempirical model for the calculation of SRC-driven two-nucleon emission cross section that can serve for the analysis of electron and neutrino inclusive reactions. Our model calculates these cross sections based on a factorized formula that is proportional to the 2p2h phase space and to an averaged single-nucleon cross section. The effect of the correlations is coded with a coefficient dependent on $q$. The correlation coefficient of the semiempirical formula is obtained by fitting the high-energy tail of the scaling function, which has been obtained from the experimental data of $(e,e')$. The model uses relativistic kinematics in the phase space and in the single nucleon responses, is analytical and it is valid for $q$ up to 1000 MeV/c although the formula can be extrapolated above this value. The effect of the correlations is of the same order of magnitude as the MEC and is comparable to other theoretical calculations of SRC in electron and neutrino scattering. We have seen that in the independent pair approximation, the coefficient $c^{\text{PN}}(q)$ would be associated with a mean value of the high-momentum distribution of proton-neutron pairs.

To further strengthen our empirical results, our plan is to perform a microscopic calculation of the 2p-2h response associated with the correlation current within the independent pair approximation. This would allow us to exactly calculate the value of the coefficient $c^{\text{PN}}$ in a model that includes correlations with a realistic nucleon-nucleon potential. Additionally, this would enable us to calculate the responses in the different pn, pp, and nn channels with the Bethe-Goldstone equation to verify the dominance of pn correlations. An appropriate model for this calculation would be the one in ref. [9], extended to solve the Bethe-Goldstone equation including the center of mass of all nucleon pairs. Work along these lines is in progress.

VII. ACKNOWLEDGMENTS

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Appendix A: Matrix element of the OB current with correlated nucleons

Here we derive the matrix element of the OB current between a correlated pair and a two-particle state above the Fermi level, Eqs. (20, 21). It is obtained as the sum of the OB current acting on each of the particles.

$$\langle [p'_1 p'_2] | J_{OB}^{\mu} (q) | [\Phi_{h_1 h_2}] \rangle = \langle [p'_1] [p'_2] | J_1^{\mu} + J_2^{\mu} | [\Delta \Phi_{h_1 h_2}] \rangle$$

(21)

Let’s calculate the matrix element of $J_1^{\mu}$ writing explicitly the spin indices. We assume that the two-hole uncorrelated state is $| h_1 s_1 h_2 s_2 \rangle$. The corresponding correlated wave function is denoted by

$$| \Phi_{h_1 h_2}^{s_1 s_2} \rangle = | h_1 s_1 h_2 s_2 \rangle + | \Delta \Phi_{h_1 h_2}^{s_1 s_2} \rangle$$

(22)

where $| \Delta \Phi_{h_1 h_2}^{s_1 s_2} \rangle$ carries the high momentum components. First we introduce a complete set of momentum states

$$\langle [p'_1 s'_1 p'_2 s'_2] | J_1^{\mu} | [\Delta \Phi_{h_1 h_2}^{s_1 s_2}] \rangle = \int d^3p_1 d^3p_2 \sum_{\sigma_1 \sigma_2} \langle [p'_1 s'_1 p'_2 s'_2] | J_1^{\mu} | p_1 \sigma_1 p_2 \sigma_2 \rangle \langle p_1 \sigma_1 p_2 \sigma_2 | [\Delta \Phi_{h_1 h_2}^{s_1 s_2}] \rangle$$

(23)

The first matrix element inside the integral is

$$\langle [p'_1 s'_1 p'_2 s'_2] | J_1^{\mu} | p_1 \sigma_1 p_2 \sigma_2 \rangle = \langle [p'_1 s'_1] | J_1^{\mu} | p_1 \sigma_1 \rangle \langle p'_2 s_2 | p_2 \sigma_2 \rangle$$

$$= \frac{(2\pi)^3}{V} \delta(p'_1 - p_1 - q) J_{OB}^{\mu} (p_1, p'_1 - q) s'_1 \sigma_1 \delta(p'_2 - p_2) \delta s'_2 \sigma_2$$

(24)
where $j_{OB}$ is the OB current function given in Eq. (8) in the electromagnetic case, and we have exchanged the bracket of $p'_2$ for $p_1$. The matrix element of the high-momentum wave function is

$$
\langle p_1 \sigma_1 p_2 \sigma_2 | [\Delta \phi_{{n_1 n_2}}] \rangle = \frac{(2\pi)^3}{V} \langle p_1 \sigma_1 p_2 \sigma_2 | \Delta \phi_{{n_1 n_2}} \rangle
$$

$$
= \frac{(2\pi)^3}{V} \delta(p'_1 + p'_2 - q - h_1 - h_2) \Delta \phi_{{n_1 n_2}}(p_{\sigma_1 \sigma_2})
$$

Inserting Eqs. (A4, A5) in (A3) and integrating using the Dirac deltas we obtain

$$
\langle [p_1 p_2] | J_1^\mu | [p_1 p_2] \rangle = \frac{(2\pi)^6}{V^2} \delta(p'_1 + p'_2 - q - h_1 - h_2) \sum_{\sigma_1} j_{OB}^\mu(p'_1, p'_2 - q) s'_1 \sigma_1 \Delta \phi_{{n_1 n_2}} \frac{(p'_1 - p'_2 - q)}{2} \sigma_1 s'_2.
$$

Proceeding analogously to calculate the matrix element of $J_2$, we obtain the equation

$$
\langle [p_1 p_2] | J_2^\mu | [p_1 p_2] \rangle = \frac{(2\pi)^6}{V^2} \delta(p'_1 + p'_2 - q - h_1 - h_2) \sum_{\sigma_2} j_{OB}^\mu(p'_2, p'_1 - q) s'_2 \sigma_2 \Delta \phi_{{n_1 n_2}} \frac{(p'_1 - p'_2 + q)}{2} s'_1 \sigma_1.
$$

By analogy with Equation (21), the 2p2h correlation current function turns out to be

$$
j_{cor}(p'_1, p'_2, h_1, h_2) = (2\pi)^3 \sum_{\sigma} j_{OB}^\mu(p'_1, p'_2 - q) s'_1 \sigma \Delta \phi_{{n_1 n_2}} \frac{(p'_1 - p'_2 - q)}{2} s'_2 \sigma + (2\pi)^3 \sum_{\sigma} j_{OB}^\mu(p'_2, p'_1 - q) s'_2 \sigma \Delta \phi_{{n_1 n_2}} \frac{(p'_1 - p'_2 + q)}{2} s'_1 \sigma.
$$

### Appendix B: Averaged single-nucleon responses in the SuSAM* model

The single-nucleon responses can be extended beyond values $|\psi^*| > 1$ using the energy distribution function (B3) obtained from the superscaling function by differentiating the two sides of the equation (14) with respect to $e_0$

$$n(e_0) = -\frac{2}{3} \frac{1}{\psi} \frac{d F^*(\psi^*)}{d \psi^*}.
$$

We use the non-correlated superscaling function $f^*(\psi^*) = b \exp(-\psi^*/a^2)$ to compute de energy distribution and the averaged single-nucleon responses, Eq. (13), giving $n(e_0) = -\frac{1}{3} f^*(\psi^*)$. The single-nucleon responses $U_K(\epsilon, q, \omega)$ for neutrinos (antineutrinos) are given in the Appendix C of Ref. [2]. The averaged single-nucleon responses after integration over $\epsilon$ are given by

$$
U_{CC} = \frac{1}{2} \left(\frac{2}{\kappa^2} - 1\right) w_2 \lambda^2 + 2 \lambda w_2 \left[1 + \frac{\xi_F}{\kappa^2} (\psi^* + a^2)\right]
$$

$$
+ w_2 \left[1 + 2 \xi_F (\psi^* + a^2) + \frac{\xi^2}{\kappa^2} (\psi^* + a^2 - 2 \psi^* + 2 a^4)\right]
$$

$$
+ 4 \lambda^2 w_4,
$$

(B2)

$$
U_{CL} = \frac{\lambda}{\kappa} \left\{\frac{1}{2} \left(\frac{1}{\kappa^2} - 1\right) - \lambda^2 - 2 \lambda w_2 \left[1 + \frac{\xi_F}{\kappa^2} (\psi^* + a^2)\right]
$$

$$
- w_2 \left[1 + 2 \xi_F (\psi^* + a^2) + \frac{\xi^2}{\kappa^2} (\psi^* + a^2 - 2 \psi^* + 2 a^4)\right]
$$

$$
- 4 \lambda w_4, \frac{1}{4}\right\}.
$$

(B3)

The single-nucleon structure functions, $w_1, w_2, w_3$ and $w_4$ are given by

$$
w_1 = \tau (2 G_M^Y)^2 + (1 + \tau) G_A^Y
$$

$$
w_2 = \frac{\xi_F}{\kappa^2} \left[1 + \frac{\xi_F}{\kappa^2} (\psi^* + a^2)\right]
$$

$$
w_3 = G_A^M G_M^Y,
$$

$$
w_4 = \frac{(G_A - \tau G_P^Y)^2}{4\tau},
$$

(B4)

(B5)

(B6)

(B7)

(B8)

(B9)

(B10)

where $G_M^Y$ and $G_A^Y$ are the electric and magnetic isovector form factors $G_M^Y = (G_E^Y - G_N^Y)/2$ modified in the medium with the relativistic effective mass [59]. For
the axial form factor, $G_A$, we use dipole parametrization with axial mass $M_A = 1.032$ GeV. Finally the pseudoscalar form factor $G_P^s = 4m_NN^*G_A/(m^2 - Q^2)$.

For electron scattering the equations are similar for $\mathcal{T}_E = \mathcal{T}_{CC}$ and $\mathcal{T}_T$, without the axial contribution, $w_3 = w_4 = 0$. For protons and neutrons the corresponding structure functions are

$$w_{1e}^e = \tau(G_M^*)^2,$$

$$w_{2e}^e = \frac{(G_E^*)^2 + \tau(G_M^*)^2}{1 + \tau}.$$

with the electric and magnetic form factors

$$G_E^* = F_1 - \tau \frac{m_N}{m_N} F_2, \quad G_M^* = F_1 + \frac{m_N}{m_N} F_2.$$

For the $F_i$ form factors of the nucleon, we use the Galster parameterization. 

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