Variations of constants as a test of gravity, cosmology and unified models. (Review)

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Gravitation as a fundamental interaction that governs all phenomena at large and very small scales, but still not well understood at a quantum level, is a cardinal missing link in unification of all physical interactions. Discovery of the present acceleration of the Universe, the dark matter and dark energy problems are also a great challenge to modern physics, which may lead to a new revolution in it. Integrable multidimensional models of gravitation and cosmology make up one of the proper approaches to studying basic issues and strong field objects, the early and present Universe and black hole (BH) physics in particular. Our main results within this approach are described for both cosmology and BH physics. Problems of the absolute $G$ measurements and its possible time and range variations are reflections of the unification problem.

The choice, nature, classification and precision of determination of the fundamental physical constants as well as their role in a transition, expected in 2011, to new definitions of the main SI units, supposed to be based on fundamental physical constants and stable quantum phenomena, are described. The problem of temporal variations of constants is also discussed, temporal and spatial variations of $G$ in particular. A need for further absolute measurements of $G$, its possible range and time variations is pointed out. The multipurpose space project SEE is briefly described, aimed at measuring $G$ and its stability in space and time, with precision 3-4 orders better than at present. It may answer many important questions posed by gravitation, cosmology and unified theories. A project of a laboratory experiment for testing possible deviations from the Newton law is also presented.

1. Introduction

Studies in the previous century in the field of gravitation were mainly devoted to theoretical studies and experimental verification of general relativity (GR) and alternative theories of gravity with a strong stress laid on relations between macro- and micro-world phenomena or, in other words, between classical gravitation and quantum physics. Very intensive investigations in these fields were done in Russia by M.A. Markov, K.P. Staniukovich, Ya.B. Zeldovich, A.D. Sakharov and their colleagues starting from mid-60s. As a motivation there were: the existence of singularities in cosmology and black hole physics, the role of gravity at large and very small (Planckian) scales, attempts to create a quantum theory of gravity as for other physical fields, problem of possible variations of fundamental physical constants etc. A lot of work has been done in such areas as [4]:

- exact solutions with different fields as sources in GR,
- particle-like solutions with a gravitational field,
- quantum field theory in a classical gravitational background,
- quantum cosmology with fields (e.g., scalar), with the cosmological constant etc.,
- self-consistent treatment of quantum effects in cosmology,
- development of alternative theories of gravity: scalar-tensor, gauge, with torsion, bimetric etc.,

- possible variations of fundamental physical constants \[ G \text{, } \delta G \text{, } \Delta G \].

As our main results of this period, one can mention [4] the first quantum cosmological model with a cosmological constant (creation from nothing) (1972); the first classical cosmological models for a conformal scalar field (1968) and quantum cosmological models with minimal and conformal scalar fields (1971), the first nonsingular cosmological model with spontaneous symmetry breaking of a nonlinear conformal scalar field (1978-79), exact solutions for nonlinear electrodynamics, including the Born-Infeld one, the first exact solution for dilaton-type interaction with an electromagnetic field in GR, the first nonsingular particlelike purely field solution with gravity (1979). One should also mention the conclusion that only $G$ may vary with respect to atomic system of measurements, or in the Jordan-Brans-Dicke frame (1978).

Since all attempts to quantize general relativity in a usual manner failed and it was proved that it is nonrenormalizable, it become clear that a promising trend goes along the lines of unification of all physical interactions which has started in the 70s. About this time, experimental studies of gravity in strong fields and gravitational waves started, giving a powerful speed-up in theoretical studies of such objects as pulsars, black holes, QSO’s, active galactic nuclei, the early Universe etc., which continue now.

But nowadays, when we think of the most important lines of future developments in physics, we can foresee that gravity will be essential not only by itself, but as
a missing cardinal link of some theory unifying all existing physical interactions: gravitational, weak, strong and electromagnetic ones. Even in experimental activities, some crucial next generation gravitational experiments verifying predictions of unified schemes will be important. Among them are: MICROSCOPE and STEP — testing the cornerstone Equivalence Principle, SEE — testing the inverse square law (or new non-Newtonian interactions), testing possible time variations of the Newtonian constant $G$, measurement of the absolute value of $G$ with unprecedented accuracy \[10, 11\]. All these experiments became tests of not only Gravity itself, but unified models of physical interactions as well. Of course, the gravitational-wave problem, verification of torsional, rotational (GPB), 2nd order and strong field effects remain important as well.

We can also predict that the studies of gravity itself and within the unified models will give, in the next century and millennium, even more applications to our everyday life, as the electromagnetic theory gave us in the 20th century after very abstract fundamental investigations of Faraday, Maxwell, Poинcaré, Einstein and others, who had never dreamed of such enormous applications of their works.

Another very important feature which may be envisaged is an increasing role of fundamental physics studies, gravitation, cosmology and astrophysics in particular, in space experiments \[12\]. Unique micro-gravity environments and the modern technology outbreak give nearly a perfect place for gravitational experiments which suffer a lot on Earth from its relatively strong gravitational field and gravitational fields of nearby objects due to the fact that there is no way of screening gravity.

In the development of relativistic gravitation and dynamical cosmology after A. Einstein and A. Friedmann, we may notice three distinct stages: first, studies of models with matter sources in the form of a perfect fluid, as was originally done by Einstein and Friedmann. Second, studies of models with sources as different physical fields, starting from the electromagnetic and scalar ones, in both classical and quantum setting (see \[4\]). And third, which is really topical now, application of ideas and results of unified models for treating fundamental problems of cosmology and black hole physics, especially in high energy regimes and for explanation of the greatest challenge to modern physics, explaining the present acceleration of the Universe, the so-called dark energy problem. Multidimensional gravitational models play an essential role in the latter approach.

The necessity of studying multidimensional models of gravitation and cosmology \[112\] is motivated by several reasons. First, the main trend of modern physics is unification of all known fundamental physical interactions: electromagnetic, weak, strong and gravitational ones. In the recent decades, there has been a significant progress in unifying weak and electromagnetic interactions and some more modest achievements in GUT, supersymmetric, string and superstring theories.

Now, theories with membranes, $p$-branes and more vague M-theory are being created and studied. Having no definite successful theory of unification now, it is desirable to study the common features of these theories and their applications to solving basic problems of modern gravity and cosmology. Moreover, if we really believe in unified theories, the early stages of the Universe evolution and black hole physics, as unique superhigh energy regions, and possibly even the low energy stage, when we observe the present acceleration, are the most proper and natural arena for them.

Second, multidimensional gravitational models, as well as scalar-tensor theories of gravity, are theoretical frameworks for describing possible temporal and range variations of fundamental physical constants \[4, 5, 6, 7\]. These ideas have originated from the earlier papers by E. Milne (1935) and P. Dirac (1937) on relationships between the phenomena of micro- and macro-worlds, and up till now they are under thorough study both theoretically and experimentally. The possible discovery of the fine structure constant variations is now under critical further investigation.

Lastly, applying multidimensional gravitational models to basic problems of modern cosmology and black hole physics, we hope to find answers to such long-standing problems as singular or nonsingular initial states, creation of the Universe, creation of matter and its entropy, the cosmological constant and coincidence problem, origin of inflation and specific scalar fields which may be necessary for its realization, the isotropization and graceful exit problems, stability and nature of fundamental constants \[5, 12\] \[13\], the possible number of extra dimensions, their stable compactification, new revolutionary data on present acceleration of the Universe, dark matter and dark energy etc.

Bearing in mind that multidimensional gravitational models are certain generalizations of GR which is tested reliably for weak fields up to 0.0001 and partly in strong fields (binary pulsars), it is quite natural to inquire about their possible observational or experimental windows. From what we already know, among these windows are:

- possible deviations from the Newton and Coulomb laws, or new interactions,
- possible variations of the effective gravitational constant with a time rate smaller than the Hubble one,
- possible existence of monopole modes in gravitational waves,
- different behaviour of strong field objects, such as multidimensional black holes, wormholes and $p$-branes,
- standard cosmological tests,
- possible nonconservation of energy in strong field objects and accelerators if the brane-world ideas
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about gravity in the bulk turn out to be true etc.

Since modern cosmology has already become a unique laboratory for testing the standard unified models of physical interactions at energies far beyond the level of existing and future manmade accelerators and other installations on Earth, there exists a possibility of using cosmological and astrophysical data for discriminating between future unified schemes. Data on possible time variations or deviations from the Newton law as a new important test should also contribute to the unified theory choice.

As there exist no accepted unified models, in our approach [121415] we have adopted simple (but general from the viewpoint of the number of dimensions) models, based on multidimensional Einstein equations with or without sources of different nature:

- the cosmological constant,
- perfect and viscous fluids,
- scalar and electromagnetic fields,
- their possible interactions,
- dilaton and moduli fields with or without potentials,
- fields of antisymmetric forms (related to p-branes)

The main objective of our programme was and is to obtain exact self-consistent solutions (integrable models) for these systems and then to analyze them in cosmological, spherically and axially symmetric cases. In our view, this is a natural and most reliable way of studying highly nonlinear systems. It is done mainly within Riemannian geometry. Some simple models in integrable Weyl geometry and with torsion were studied as well. In many cases, we tried to single out models which do not contradict the available experimental or observational data on variations of $G$. In some cases we have used the methods worked out for arbitrary dimensions in studying 4D models as well.

As our model [122], we use $n$ Einstein spaces of constant curvature with sources as $(m+1)$-component perfect fluid (or form fields), cosmological or spherically symmetric metrics, in manifolds obtained as a direct product of factor spaces of arbitrary dimensions. Then, in the harmonic time gauge, we show that the Einstein multidimensional equations are equivalent to Lagrange equations with a nondiagonal (in general) minisuperspace metric and some exponential potential. After possible diagonalization of this metric, we perform reduction to sigma-model and Toda-like systems and further to Liouville, Abel, generalized Emden-Fowler equations etc. and try to find exact solutions. We suppose that the behaviour of extra spaces is the following: they can be constant or dynamically compactified (e.g., toroidally), or large, but with barriers, walls etc.

So, we have been realizing the programme in arbitrary dimensions (from 1988) [1231415].

In cosmology:
we have obtained exact general solutions of multidimensional Einstein equations with sources:

- $\Lambda$, $\Lambda +$ scalar field (e.g. nonsingular, dynamically compactified, inflationary);
- perfect fluid (PF), PF + scalar field (e.g. nonsingular, inflationary solutions);
- viscous fluid (e.g. nonsingular, generation of mass and entropy; quintessence and coincidence in a 2-component model);
- stochastic behaviour near the singularity, billiards in Lobachevsky space, that $D = 11$ is critical and $\varphi$ destroys billiards (1994).

For all the above cases, Ricci-flat solutions were obtained for any $n$, and, in addition, solutions with curvature in one factor space; with curvatures in 2 factor spaces, solutions are known only for total $D = 10, 11$;

- fields: scalars, dilatons, forms of arbitrary rank (1998) — inflationary solutions, $\Lambda$ generation by forms (p-branes) \[71\];
- the first billiards for dilaton-forms (p-branes) interaction (1999);
- quantum systems (solutions of the WDW equation \[10\]) for all the above cases where classical solutions were obtained;
- dilatonic fields with potentials, billiard behaviour for them as well.

For many of these integrable models, we have calculated the time variation of the effective gravitational constant and compared it with the present experimental bounds, which allowed choosing particular models or singling out some classes of solutions.

Solutions depending on $r$ in any dimensions:

- generalized Schwarzschild and Tangherlini (BHs are singled out), solutions with a minimally coupled scalar field $\varphi$ (no BHs);
- generalized Reissner-Nordström (BHs are also singled out), the same plus $\varphi$ (no BHs);
- multitemporal solutions;
- for dilaton-like interaction of $\varphi$ and electromagnetic fields (BHs exist only in a special case);
- stability studies for the above solutions (only BH ones are stable);
- the same for dilaton-forms interaction (p-branes); stability was found only in some cases, e.g., for a single form.

The PPN parameters for most of the models were calculated.

Theory of experiments:
Space and laboratory experiments aimed at testing a possible violation of the Newton law and raising the precision of the absolute value of the Newton constant $G$ determination were suggested and worked out.
2. Multidimensional Models

The history of the multidimensional approach begins with the well-known papers of T.K. Kaluza and O. Klein on 5-dimensional theories, which aroused an interest in studies of multidimensional gravity. These ideas were continued by P. Jordan who suggested to consider the more general case \( g_{55} \neq \text{const} \) leading to a theory with an additional scalar field. They were, in some sense, a source of inspiration for C. Brans and R.H. Dicke in their well-known work on a scalar-tensor gravitational theory. After their work, a lot of investigations have been performed with material or fundamental scalar fields, both conformal and non-conformal (see details in [4]).

A revival of ideas of many dimensions started in the 70s and is continuing now, entirely due to the development of unified theories. In the 70s, an interest in multidimensional gravitational models was mainly stimulated by (i) the ideas of gauge theories leading to a non-Abelian generalization of the Kaluza-Klein approach and (ii) by supergravitational theories. In the 80s, the supergravitational theories were “replaced” by superstring models. Now, it is heated by expectations of unified theories. In the 70s, an inter-

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a low-energy limit of superstring models \[28,29\]. For \(D = 11\) supergravity \[28\] (considered now as a low-energy limit of a conjectured \(M\)-theory \[30\]), we have a metric and a 4-form in the bosonic sector. For \(D = 10\), one may consider type I supergravity with a metric, a scalar field and a 3-form; type IIA supergravity, with bosonic fields of type I supergravity, called the Neveu-Schwarz-Neveu-Schwarz (NS-NS) sector, and additionally a 2-form and a 4-form Ramond-Ramond (R-R) sector; type IIB supergravity with bosonic fields of type I supergravity (the NS-NS sector) and additionally a 1-form, a 3-form and a (self-dual) 5-form (the R-R sector). It is now believed that all five string theories (I, IIA, IIB and two heterotic ones with gauge groups \(\text{Spin}(32)/\mathbb{Z}_2\) as well as 11-dimensional supergravity \[28\]) are limiting cases of M-theory. All these theories are conjectured to be related by a set of duality transformations: \(S\), \(T\) (and more general \(U\)) dualities.

It was proposed earlier that the IIB string may have originate in a 12D theory, known as F-theory (Vafa). A low energy effective (bosonic) Lagrangian for F-theory was also suggested. The field content of this 12-dimensional field model is the following: a metric, a scalar field (with a negative kinetic term), a 4-form and a 5-form. In our work \[31\], a chain of so-called \(Bp\)-models in dimensions \(D = 11, 12, \ldots\) was suggested. The \(B_D\)-model contains \(l = D - 11\) scalar fields with negative kinetic terms (i.e. the so-called “phantom” fields) coupled to \((l + 1)\) different forms of ranks \(4, \ldots, 4 + l\). These models were constructed using \(p\)-brane intersection rules that will be discussed below.

For \(D = 11\) (\(l = 0\)), the \(B_D\)-model coincides with the truncated bosonic sector of \(D = 11\) supergravity. For \(D = 12\) (\(l = 1\)), it coincides with the truncated \(D = 12\) model. We have conjectured in \[31\] that these \(B_D\)-models for \(D > 12\) may correspond to low-energy limits of some unknown \(F_D\)-theories (analogues of \(M\)- and \(F\)-theories).

### Description of the models.

In our review \[15\], certain classes of \(p\)-brane solutions to field equations corresponding to the Lagrangian (4), obtained by us earlier, were presented.

These solutions have block-diagonal metrics defined on a \(D\)-dimensional product manifold, i.e.,

\[
g = e^{2\gamma}g^0 + \sum_{i=1}^{n} e^{2\varphi^i}g^i, \quad M_0 \times M_1 \times \ldots \times M_n, \quad (5)
\]

where \(g^0\) is a metric on \(M_0\) (our space) and \(g^i\) are fixed Ricci-flat (or Einstein) metrics on \(M_i\) (internal spaces, \(i > 0\)). The moduli \(\gamma, \varphi^i\) and the scalar fields \(\varphi^a\) are functions on \(M_0\), and fields of forms are also governed by several scalar functions on \(M_0\). Any \(F^a\) is supposed to be a sum of monomials, corresponding to electric or magnetic \(p\)-branes (\(p\)-dimensional analogues of membranes), i.e., the so-called composite \(p\)-brane ansatz is considered \[32,33\]. (In the non-composite case we have no more than one monomial for each \(F^a\).) \(p = 0\) corresponds to a particle, \(p = 1\) to a string, \(p = 2\) to a membrane etc. The \(p\)-brane world volume (world line for \(p = 0\), world surface for \(p = 1\) etc.) is isomorphic to some product of internal manifolds: \(M_{j} = M_{i_1} \times \ldots \times M_{i_k}\) where \(1 \leq i_1 < \ldots < i_k \leq n\) and has the dimension \(p + 1 = d_i + \ldots + d_k = d(I)\), where \(I = \{i_1, \ldots, i_k\}\) is a multi-index describing the location of the \(p\)-brane, and \(d_i = \dim M_i\). Any \(p\)-brane is described by the triplet (\(p\)-brane index) \(s = (a, v, l)\), where \(a\) is the colour index and labelling the form \(F^a\), \(v = (\text{electric}), m(\text{magnetic})\) is the multi-index defined above. For the electric and magnetic branes corresponding to a form \(F^a\), the world volume dimensions are \(d(I) = n_a - 1\) and \(d(I) = D - n_a - 1\), respectively. A sum of these dimensions is \(D - 2\). For \(D = 11\) supergravity we get \(d(I) = 3\) and \(d(I) = 6\), corresponding to an electric \(M2\)-brane and a magnetic \(M5\)-brane.

### Sigma-model representation.

In our paper \[34\], the model under consideration was reduced to a gravitating self-interacting sigma-model with certain constraints. The sigma-model representation for the non-composite electric case was obtained earlier in \[32,33\] (for the electric composite case see also \[35\]).

The \(\sigma\)-model Lagrangian, obtained from (4), has the form \[34\]

\[
\mathcal{L}_\sigma = R(g^0) - \hat{G}_{AB} g^{0\mu\nu} \partial_\mu \sigma^A \partial_\nu \sigma^B - \sum_s \varepsilon_s \exp(-2U^s) g^{0\mu\nu} \partial_\mu \Phi^s \partial_\nu \Phi^s - 2V, \quad (6)
\]

where \((\sigma^A) = (\phi^i, \varphi^a)\), \(V\) is a potential, \((\hat{G}_{AB})\) are components of the (truncated) target space metric, \(\varepsilon_s = \pm1\),

\[
U^s = U^A_{\alpha} \sigma^A = \sum_{a \in I_s} d_i \phi^i - \chi_s \lambda_{a, \alpha} \varphi^a
\]

are linear functions, \(\Phi^s\) are scalar functions on \(M_0\) (corresponding to the forms), and \(s = (a_s, v_s, I_s)\). The parameter \(\chi_s = +1\) for the electric brane \((v_s = e)\) and \(\chi_s = -1\) for the magnetic one \((v_s = m)\).

The pure gravitational sector of the sigma model was considered earlier in our paper \[13\]. For \(p\)-brane applications, \(g^0\) is Euclidean, \((\hat{G}_{AB})\) is positive-definite (for \(d_0 > 2\)), and \(\varepsilon_s = -1\) if pseudo-Euclidean (electric and magnetic) \(p\)-branes in a pseudo-Euclidean space-time are considered. The sigma model (6) may also be considered for the pseudo-Euclidean metric \(g^0\) of signature \((-+,+,+,+)\) (e.g. in studies of gravitational waves). In this case, for a positive-definite matrix \((\hat{G}_{AB})\) and \(\varepsilon_s = 1\), we get non-negative kinetic energy terms.

### The brane \(U\)-vectors.

The co-vectors \(U^a\) play a key role in studying the integrability of the field equations \[34,40\] and possible existence of stochastic behaviour near the singularity, see our paper \[30\]. An
important mathematical characteristic here is the matrix of scalar products \((U^s, U^{s'}) = \hat{G}^{AB}U^s_AU^{s'}_B\), where \((\hat{G}^{AB}) = (\hat{G}_{AB})^{-1}\). The scalar products for the co-vectors \(U^s\) were calculated in [34] (for the electric case see [32, 33, 35]):

\[
(U^s, U^{s'}) = d(I_s \cap I_{s'}) + \frac{d(I_s)d(I_{s'})}{2D} + \chi_\alpha\chi_\alpha'\lambda_{\alpha\alpha'}\lambda_{\alpha\alpha'} dh^{\alpha\beta},
\]

where \((h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}; s = (a_s, v_s, I_s), s' = (a_{s'}, v_{s'}, I_{s'}). They depend on brane intersections (the first term), the brane world-volume dimensions and the total dimension \(D\) (the second term), the scalar products of dilatonic coupling vectors and the electromagnetic types of branes (the third term). As will be shown below, the so-called “intersections rules” (i.e., relations for \(d(I_s \cap I_{s'})\)) are determined by the scalar products of \(U^s\)-vectors.

**Cosmological and spherically symmetric solutions**

A family of general cosmological-type \(p\)-brane solutions with \(n\) Ricci-flat internal spaces was considered in our paper [12], where a generalization to the case of \(n - 1\) Ricci-flat spaces and one Einstein space of non-zero curvature (say, \(M_1\)) was also obtained. These solutions are defined up to solutions to Toda-type equations and may be obtained using the Lagrange dynamics following from our sigma-model approach [31]. The solutions from [12] contain a subclass of spherically symmetric solutions (for \(M_1 = S^d\)). Special solutions with orthogonal and block-orthogonal [39] sets of \(U\)-vectors were considered previously in our works [31] and [13] [32], respectively. (For the non-composite case, see [37] [38] and references therein.)

**Toda solutions.** In [31], a reduction of \(p\)-brane cosmological-type solutions to Toda-like systems was first performed. General classes of \(p\)-brane (and spherically symmetric) solutions related to Euclidean Toda lattices associated with Lie algebras (mainly \(A_m\), \(C_m\) ones) were obtained by us in [11] [13] [35] [47] [48].

A class of space-like brane (\(S\)-brane) solutions (related to Toda-type systems) with a product of Ricci-flat internal spaces and \(S\)-brane solutions with special orthogonal intersection rules were considered in [58] [59], and solutions with accelerated expansion (e.g. with a power-law and exponential behaviour of scale factors) were singled out.

**Black-brane solutions.** In our papers [17] [48], a family of spherically symmetric solutions was investigated, and a subclass of black-hole configurations related to Toda-type equations under certain asymptotical conditions was singled out. These black-hole solutions are governed by the functions \(H_s(z) > 0\) defined on the interval \((0, (2\mu)^{-1})\), where \(\mu > 0\) is the extremality parameter, and obey the set of differential equations (equivalent to Toda-type ones)

\[
\frac{d}{dz} \left( \frac{1 - 2\mu z}{H_s} \frac{d}{dz} H_s \right) = B_s \prod_{s'} H_{\alpha\alpha'}^{-A_{ss'}},
\]

with the following boundary conditions:

(i) \(H_s((2\mu)^{-1} - 0) = H_{s0} \in (0, +\infty)\); (ii) \(H_s(+0) = 1\), \(s \in S\). Here \(B_s \neq 0\) and \((A_{ss'})\) is a quasi-Cartan matrix.

In Refs. [10] [17] [48], the following hypothesis was put forward: the functions \(H_s\) are polynomials when the intersection rules

\[
d(I_s \cap I_{s'}) = \frac{d(I_s)d(I_{s'})}{D - 2} + \frac{1}{2}(U^s, U^{s'})A_{ss'}, \quad s \neq s'.
\]

correspond to semi-simple Lie algebras, i.e., when \((A_{ss'})\) is a Cartan matrix. Here,

\[
(A_{ss'}) \equiv \left( \frac{2(U^s, U^{s'})}{(U^s, U^{s'})} \right),
\]

\(s, s' \in S\), is a quasi-Cartan matrix.

This hypothesis was verified for Lie algebras: \(A_m\), \(C_{m+1}\), \(m = 1, 2, \ldots\), in [17] [48]. It was also confirmed by special black-hole “block orthogonal” solutions considered earlier in [14] [43].

In our papers [10] [17] [48], explicit formulæ for the solution corresponding to the algebra \(A_2\) are presented. These formulæ are illustrated by two examples of \(A_2\)-dyon solutions: a dyon in \(D = 11\) supergravity (with \(M_2\)- and \(M_5\)-branes intersecting at a point) and a Kaluza-Klein dyon. Extremal configurations (e.g., with a multi-black-hole extension) were also obtained.

We note that special black-hole solutions with orthogonal \(U\)-vectors were considered in [30] (for the non-composite case) and [31]. These solutions have analogues in models with a multi-component perfect fluid [51] [54] [55].

The black-brane solution, corresponding to the Lie algebras \(C_2\) and \(A_3\) were obtained in [52].

In [36], some propositions related to i) interconnection between the Hawking temperature and the singularity behaviour, and ii) multi-temporal configurations were proved.

It should be noted that the polynomial structure was also found for the so-called flux-brane solutions which occur as generalizations of the well-known Melvin solution.

**Cosmological models in diverse dimensions.** Scalar fields play an essential role in modern cosmology though the problem of their origin still exists. They are attributed to inflation models of the early Universe and to models describing the present stage of the accelerated
expansion. There is no unique candidate potential of the minimally coupled scalar field. Typically a potential is taken as a sum of exponentials. Such potentials appear quite generically in a large class of theories: Kaluza-Klein models, supergravity and string/M-theories.

A single exponential potential was extensively studied in the Friedmann-Robertson-Walker (FRW) 4D model containing both a minimally coupled scalar field and a perfect fluid with a linear barotropic equation of state. The attention was mainly focused on the qualitative behavior of solutions, stability of exceptional solutions to curvature and shear perturbations and their possible applications within the known cosmological scenarios such as inflation and scaling ("tracking"). In particular, it was found by phase plane analysis that for "flat" positive potentials there exists a unique late-time attractor in the form of a scalar-dominated solution. It is stable within homogeneous and isotropic models with non-zero spatial curvature with respect to spatial curvature perturbations and provides power-law inflation. For "intermediate" positive potentials, a unique late-time attractor is the scaling solution where the scalar field "mimics" a perfect fluid, adopting its equation of state. The energy density of the scalar field scales with that of the perfect fluid. Using our methods for multidimensional cosmology, the problem of integrability by quadratures of the model in 4 dimensions was also studied. Four classes of general solutions, when the parameter characterizing the steepness of the potential and the barotropic parameter obey some relations, were found [58]. For the case of multiple exponential potential of the scalar field plus dust, an integrable model in 4D was obtained in [67].

As to scalar fields with a multiple exponential potential in any dimensions, it is not yet well studied, although a wide class of exact solutions were obtained in our papers [59, 60]. In our recent work [67], the behavior of this system near the singularity was studied using a billiard approach suggested earlier in our papers [56, 50]. A number of S-brane solutions were found in [68, 59]. See details for 2-component D-dimensional integrable models in Refs. [70, 44, 65].

Quite a different model with a dilaton, branes and a cosmological constant, with static internal spaces, was investigated in [71], where possible generation of the effective cosmological constant by branes was demonstrated. A model with variable equations of state was found in [68], with acceleration in our space and compactification of internal spaces.

**Cosmological models with time variations of $G$.**

As was mentioned before, cosmological models in scalar-tensor and multidimensional theories are frameworks for describing possible time variations of fundamental physical constants due to scalar fields present explicitly in STT and present initially or/and generated by extra dimensions in multidimensional approach. In [74], we have obtained solutions for a system of conformal scalar and gravitational fields in 4D and calculated the present possible relative variation of $G$ at the level of less than $10^{-12}$/year. Later, in the framework of a multidimensional model with a perfect fluid and 2 factor spaces (our 3D space could be open, closed or flat) and an internal 6D Ricci-flat space, we obtained the same limit for such variation of $G$ [9].

We have also estimated the possible variations of the gravitational constant $G$ in the framework of a generalized (Bergmann-Wagoner-Nordtvedt) scalar-tensor theory of gravity on the basis of the field equations, without using their special solutions. Specific estimates were essentially related to the values of other cosmological parameters (the Hubble and acceleration parameters, the dark matter density etc.), but the values of $G/G$ compatible with modern observations $10^{-12}$/year [90] were not exceeded.

In [59], we continued the studies of models in arbitrary dimensions and obtained relations for $G$ in a multidimensional model with Ricci-flat internal space and a multi-component perfect fluid. A two-component example (dust + 5-brane) was considered. It was shown that $G/G$ is less than $10^{-12}$/year. Expressions for $G$ were also considered in a multidimensional model with an Einstein internal space and a multicomponent perfect fluid [92]. In the case of two factor spaces with non-zero curvatures without matter, a mechanism for prediction of small $G$ was suggested. The result was compared with exact (1+3+6)-dimensional solutions which we obtained earlier.

A multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor-spaces $M_i$ in the presence of a one-component anisotropic fluid was considered in [119]. The pressures in all spaces were supposed to be proportional to the density: $p_i = w_i \rho_i$, $i = 0, ..., n$. Solutions with an accelerated power-law expansion of our 3-space $M_0$ and small enough variation of the gravitational constant $G$ were found. These solutions exist for two branches of the parameter $w_0$. The first branch describes superstiff matter with $w_0 > 1$, the second one may contain phantom matter with $w_0 < -1$, e.g., when $G$ grows with time, so this branch can describe not the present epoch but rather earlier stages.

Similar exact solutions, but nonsingular and with an exponential behaviour of the scale factors, were considered in [117] for the same multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor spaces $M_i$ in the presence of a one-component perfect fluid. Solutions with accelerated exponential expansion of our 3-space $M_0$ and small variation of $G$ were also found.

Exact S-brane solutions with two electric branes and two phantom scalar fields on the manifold

$$M = \mathbb{R}_+ \times \mathbb{R} \times M_2 \times M_3 \times M_4 \times M_5.$$  

were obtained and studied in [118]. We obtained asymptotic accelerated expansion of our 3D factor space and
variations of $G$ obeying the present experimental constraints, $\dot{G}/G \lesssim 10^{-12}$/year.

A D-dimensional cosmological model with several scalar fields and an antisymmetric $(p+2)$-form was also considered \[119\]. For dimensions $D = 4m + 1 = 5,9,13,...$ and $p = 2m - 1 = 1,3,5,...$, we obtained a family of new cosmological type solutions with a 4m-dimensional oriented Ricci-flat submanifold $N$ of Euclidean signature. These solutions are characterized by a self-dual or anti-self-dual parallel charge density form $Q$ of rank $2m$ defined on $N$. The (sub)manifold $N$ may be chosen to be either Kähler or hyper-Kähler, or an 8-dimensional manifold of $Spin(7)$ holonomy. A generalization of these solutions to a chain of extra (marginal) Ricci-flat factor spaces was also presented. Solutions with accelerated expansion of extra factor spaces were singled out. Certain examples of new solutions for IIA supergravity and for a chain of $BD$-models in dimensions $D = 14, 15,...$ were considered.

Spherically symmetric solutions, black holes and PPN parameters. In \[34\], it was shown that, after dimensional reduction on the manifold $M_0 \times M_1 \times \ldots \times M_n$ and when the composite $p$-brane ansatz is considered, the problem is reduced to the gravitating self-interacting $\sigma$ model with certain constraints. For electric $p$-branes, see also \[33, 35\] (in \[35\], the composite electric case was considered). This representation may be considered as a powerful tool for obtaining different solutions with intersecting $p$-branes. In \[34, 40\], Majumdar-Papapetrou type solutions were obtained (for the non-composite electric case see \[35\] and for the composite electric case see \[35\]). These solutions correspond to Ricci-flat ($M_i, g^i$), $i = 1, \ldots, n$ and were generalized to the case of Einstein internal spaces \[34\]. These solutions take place when certain orthogonality relations (on the couplings parameters, brane dimensions and the total dimension) are imposed. In this situation, a class of cosmological and spherically symmetric solutions was obtained \[31\]. Solutions with a horizon were considered in detail in \[36, 31\].

It should be noted that multidimensional and multitemporal generalizations of the Schwarzschild and Tangherlini solutions were considered in \[23, 72\], where the generalized Newton formulae in the multitemporal case were obtained.

We have also calculated the Post-Newtonian Parameters $\beta$ and $\gamma$ (the Eddington parameters) for general spherically symmetric solutions and black holes in particular \[14\]. These parameters, depending on $p$-brane charges, their world-volume dimensions, dilaton couplings and the total number of dimensions may be useful for physical applications.

Some specific models in classical and quantum multidimensional cases with $p$-branes were analyzed in \[31, 37\]. Exact solutions for a system of scalar fields and fields of forms with dilatonic type interactions for generalized intersection rules were studied in \[47\], where the PPN parameters were also calculated. Other problems connected with observations were studied in \[60, 63\], and general properties of BHs and wormholes in a brane world in \[62, 61\].

Stability analysis for solutions with $p$-branes was carried out in \[49, 85\]. It was shown there that, for some simple $p$-brane systems, multidimensional black branes are stable under monopole perturbations while other (non-BH) spherically symmetric solutions turned out to be unstable.

Below we mainly dwell upon some problems of fundamental physical constants, the gravitational constant in particular, upon the SEE and laboratory projects for measuring $G$ and its possible variations and briefly on some theoretical models with variations of the effective gravitational constant.

3. Fundamental physical constants

1. In any physical theory we meet constants which characterize the stability properties of different types of matter: objects, processes, classes of processes and so on. Some of them cannot be calculated via other physical constants. These constants are important because they arise independently in different situations and have the same value, at any rate within accuracies we have gained nowadays. That is why such constants are called the fundamental physical constants (FPCs) \[41, 12\]. It is impossible to define strictly this notion. It is because the constants, mainly dimensional ones, are present in certain physical theories. In the process of scientific progress, some theories are replaced by more general ones with their own constants, and there arise relations between old and new constants. So, we cannot speak of an absolute choice of FPC, but rather only of a choice corresponding to the present state of the physical sciences.

Really, before creation of the electro-weak interaction theory and some Grand Unification Models, the following choice of FPCs was considered:

$$c, h, \alpha, G_F, g_s, m_p \text{ (or } m_e), G, H, \rho, \Lambda, k, I, \ldots$$

where $\alpha, G_F, g_s$ and $G$ are constants of the electromagnetic, weak, strong and gravitational interactions, $H, \rho$ and $\Lambda$ are cosmological parameters (the Hubble constant, the mean density of the Universe and the cosmological constant), $k$ and $I$ are the Boltzmann constant and the mechanical equivalent of heat which play the role of conversion factors between temperature on the one hand, energy and mechanical units on the other. After adoption (in 1983) of a new definition of the meter ($\lambda = ct$ or $\ell = ct$), this role is also partly played by the speed of light $c$. It is now also a conversion factor between units of time (frequency) and length, it is defined with absolute accuracy, i.e., zero uncertainty (with the new suggested definitions of basic units of the International System of Units (SI), such a role may also
be played by $\hbar$ and $N_A$, where $N_A$ is the Avogadro number [99].

Now, when the theory of electro-weak interactions has a firm experimental basis and we have some good models of strong interactions, a more preferable choice is as follows:

$$h, (c), e, m_e, \theta_w, G_F, \theta_c, \Lambda_{\text{QCD}}, G, H, \rho, \Lambda, k, I$$

(9)

and maybe the three Kobayashi-Maskawa angles $\theta_2$, $\theta_3$ and $\delta$. Here $\theta_w$ is the Weinberg angle, $\theta_c$ is the Cabbibo angle and $\Lambda_{\text{QCD}}$ is the cut-off parameter of quantum chromodynamics. Of course, if a theory of the four interactions known now will be created (M-theory or some other), then we will probably have another choice. As we see, the macroscopic constants remain the same, though in some unified models, i.e. in multidimensional ones, they may be related in some manner (see below). From the point of view of these unified models, the above-mentioned ones are low-energy constants.

All these constants are known with different accuracies. The most precisely defined constant was $c$ and remains to be the speed of light: its uncertainty was $10^{-10}$ while now it is defined with an absolute accuracy. The atomic constants $c$, $h$, $m$ and others are determined with errors $10^{-6} \div 10^{-8}$, $G$ up to $10^{-4}$ or even worse, $\theta_w$ up to $10^{-3}$; the accuracy of $H$ is about a few per cent. Other cosmological parameters (FPCs): mean density estimations vary within 2 per cent; for $\Lambda$ we now have data that its corresponding energy density exceeds the matter density (0.7 vs. 0.3 of the total Universe mass). Here are some recent estimates from observational cosmology:

- Total mean density : $0.98 < \Omega_{\text{tot}} < 1.08$.
- Today’s Hubble parameter: $H_0 = 0.72 \pm 0.07$.
- Dark energy density parameter: $\Omega_{DE} = 0.7$.
- For dark matter: $\Omega_{DM} = 0.26$.
- For baryonic matter: $\Omega_B = 0.04$.
- For radiation: $\Omega_R = 5 \times 10^{-5}$.
- Power spectrum index: $n = 0.970 \pm 0.023$.
- Equation of state coefficient: $w = p/\rho < -0.78$.

As to the nature of the FPCs, we can mention several approaches. One of the first hypotheses belongs to J.A. Wheeler: in each cycle of the Universe evolution, the FPCs arise anew along with physical laws which govern this evolution. Thus the nature of the FPC and physical laws are connected with the origin and evolution of our Universe.

A less global approach to the nature of the dimensional constants suggests that they are needed to make physical relations dimensionless or they are measures of asymptotic states. Really, the speed of light appears in relativistic theories in factors like $v/c$, at the same time velocities of usual bodies are smaller than $c$, so it also plays the role of an asymptotic limit. The same sense have some other FPCs: $\hbar$ is the minimal quantum of action, $e$ is the minimal observable charge (if we do not take into account quarks which are not observable in free state) etc.

Finally, FPCs or their combinations may be considered as natural scales determining the basic units. While earlier the basic units were chosen more or less arbitrarily, i.e., the second, metre and kilogram, now the first two are based on stable (quantum) phenomena. Their stability is believed to be ensured by the physical laws which include FPCs. There appeared similar suggestions for a new reproducible realization of a kg, to fix the values of $N_A$ or other constants, e.g. $h$ [99].

Another interesting problem, which is under discussion, is why the FPCs have values in a very narrow range necessary for supporting life (stability of atoms, stellar lifetime etc.). There exist several possible explanations. First, that it is a good luck, no matter how improbable is the set of FPCs. Second, that life may exist in other forms and with another FPC set, of which we do not know. Third, that all possibilities for FPC sets exist in some universe. And the last but not least: that there is some cosmic fine tuning of FPCs, some unknown physical processes bringing them to their present values in a long-time evolution, cycles etc.

An exact knowledge of FPCs and precision measurements are necessary for testing the main physical theories, extension of our knowledge of nature and, in a long run, for practical applications of fundamental theories. Within this, certain theoretical problems arise:

1) development of models for confrontation of theory with experiment in critical situations (i.e., for verification of GR, QED, QCD, GUT or other unified models);

2) setting limits on spatial and temporal variations of the FPCs. It is becoming especially important now, with the idea to introduce new basic units of the International System of Units (SI), based completely on fundamental physical constants.

Of course, raising the precision of their absolute values is always a permanent task.

As to a classification of the FPCs, we can set them into four groups according to their generality:

1) Universal constants, such as $h$ which divides all phenomena into quantum and non-quantum ones (micro- and macro-worlds) and to a certain extent $c$, which divides all motions into relativistic and non-relativistic ones;

2) coupling constants like $\alpha$, $\theta_w$, $\Lambda_{\text{QCD}}$ and $G$;

3) constants of elementary constituencies of matter like $m_e$, $m_w$, $m_x$, etc., and

4) transformation multipliers such as $k$, $I$ and partly $c$ (conversion from the second to the metre). Soon
Of course, this division into classes is not absolute. Many constants move from one class to another. For example, $e$ was a charge of a particular object, the electron, class 3, then it became a characteristic of class 2 (electromagnetic interaction, $\alpha = e^2/(\hbar c)$ in combination with $\hbar$ and $c$); the speed of light $c$ has been in nearly all classes: from 3 it moved into 1, then also into 4. Some of the constants ceased to be fundamental (i.e. densities, magnetic moments, etc.) as they are calculated via other FPCs.

As to the number of FPCs, there are two opposite tendencies: the number of “old” FPCs is usually diminishing when a new, more general theory is created, but at the same time new fields of science arise, new processes are discovered in which new constants appear. So, in the long run, we may come to some minimal choice which is characterized by one or several FPCs, maybe connected with the so-called Planck parameters — combinations of $c$, $\hbar$ and $G$ (the natural, or Planck system of units [12][13]):

\[
L = (\hbar G/c^3)^{1/2} \sim 10^{-33} \text{ cm},
\]

\[
m_L = (\hbar c/2G)^{1/2} \sim 10^{-5} \text{ g},
\]

\[
\tau_L = L/c \sim 10^{-43} \text{ s}. \quad (10)
\]

The role of these parameters is important since $m_L$ characterizes the energy of unification of the four known fundamental interactions: strong, weak, electromagnetic and gravitational ones, and $L$ is a scale where the classical notions of space-time lose their meaning. There are other ideas about the final number of FPC (2, 1, or none, L. Okun’ et al.) Of course, everything will depend on a future unified theory.

2. The problem of the gravitational constant $G$ measurement and its stability is part of a rapidly developing field, called gravitational-relativistic metrology (GRM). It has appeared due to the growth of measurement technology precision, spread of measurements over large scales and a tendency to unification of fundamental physical interactions, where the main problems arise and are concentrated on the gravitational interaction. This was first formulated in [7].

The main subjects of GRM are:

- general-relativistic models on different astronomical scales: Earth, the Solar system, galaxies, cluster of galaxies, cosmology;
- time transfer, VLBI, space dynamics, relativistic astrometry etc. (pioneering works were done in Russia by Arifov and Kadyev and Brumberg in the 60s);
- development of generalized gravitational theories and unified models for testing their effects in experiments;
- measurement of fundamental physical constants, $G$ in particular, and their stability in space and time; MICROSCOPE, STEP, SEE...
- fundamental cosmological parameters as fundamental constants: cosmological models studies (quint-essence, k-essence, phantom, multidimensional models), measurements and observations; projects PLANCK, ...
- gravitational waves (detectors, sources...); LIGO, VIRGO, TAMÁ, LISA, RADIOASTRON,...
- basic standards (clocks) and other modern precision devices (atomic and neutron interferometry, atomic force spectroscopy etc.) in fundamental gravitational experiments, especially in space for testing GR and other theories: rotational, torsional and second-order effects (need an uncertainty of $10^{-6}$ or better), e.g. LARGEOS, Gravity Probe B, ASTROD, LATOR etc.

We are now at the level of $2.3 \times 10^{-5}$ in measuring the PPN-parameter $\gamma$ and $5 \times 10^{-4}$ for $\beta$; the Brans-Dicke parameter is $\omega > 40000$.

The proposed future missions aimed at improving the accuracy of $\gamma$ are:

1. GP-B (geodetic precession) — $10^{-5}$.
2. Bepi-Colombo (retardation) — $10^{-6}$.
3. GAIA (deflection) — $(10^{-5} - 10^{-7})$.
4. ASTROD I (W.-T. Ni) (retardation) — $10^{-7}$.
5. LATOR (Turyshev et al.) — $10^{-8}$.
6. ASTROD (W.-T. Ni) — $10^{-9}$.

There are three problems related to $G$, whose origin is mainly related to unified models predictions:

1) absolute $G$ measurements,
2) possible time variations of $G$,
3) possible scale variations of $G$ — non-Newtonian, or new interactions.

**Absolute measurements of $G$.** There are many laboratory determinations of $G$ with errors of the order $10^{-3}$, and only 4 have been on the level of $10^{-4}$ in the 80s. They are given in Table 1 (in $10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$).

| Experiment                | Value               | Error     |
|---------------------------|---------------------|-----------|
| 1. Facy and Pontikis, France | 1972 6.6714 ± 0.0006 |           |
| 2. Sagitov et al., Russia  | 1979 6.6745 ± 0.0008 |           |
| 3. Luther and Fowler, USA  | 1982 6.6726 ± 0.0005 |           |
| 4. Karagioz, Russia       | 1988 6.6731 ± 0.0004 |           |

From this table it is evident that the first three experiments contradict each other (the results do not overlap within their uncertainties). And only the fourth experiment is in accord with the third one.

The official CODATA value of 1986 was

\[
G = (6.67259 \pm 0.00085) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad (11)
\]
based on Luther and Towler’s determination. But after precise measurements of G by different groups, the situation became more vague.

As one may see from the Cavendish conference data of 1998 [88], the results of 7 groups could agree with each other only on the level of $10^{-3}$. So, CODATA adopted in 1999

$$G = (6.673 \pm 0.001) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \quad (12)$$

The most recent and precise G measurements do not agree, and some of them differ from the CODATA value of 1986. They are given in Table 2 (in $10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$).

But, from 2004 CODATA gives:

$$G = 6.6742(10) \cdot 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}. \quad (13)$$

So we see that we are not too far (a little more than one order) from Cavendish who obtained a value of G at the level of $10^{-2}$ two centuries ago. The situation with the measurement of the absolute value of G is really different from that with atomic constants values and their uncertainties. This means that either the limit of terrestrial accuracies of determining G has been reached or we have some new physics entering the measurement procedure [81]. The former means that maybe we should turn to space experiments to measure G [12][11], while the latter means that a more thorough study of theories generalizing Einstein’s GR or unified theories is necessary.

There also exist some satellite determinations of G (namely, $G \cdot M_{\text{Earth}}$) at the level of $10^{-9}$ (so, could we know G much better, our knowledge of masses of the Earth and other planets and consequently their models would be much better). There are also several less precise geophysical determinations in mines at the level of $10^{-3}$, but they do not improve the situation.

A precise knowledge of G is necessary, above all, since it is a FPC; next, for evaluation of mass of the Earth, planets, their mean density and, finally, for construction of Earth models; for transition from mechanical to electromagnetic units and vice versa; for evaluation of other constants through relations between them obtained by unified theories; for finding new possible types of interactions and geophysical effects; for some practical applications like increasing of modern gravimeter precision, since they demand calibration by a gravitational field of a standard body depending on G; the high accuracy of their calibration ($10^{-5} - 10^{-6}$) requires the same accuracy of G.

3. The knowledge of constants values has not only a fundamental but also a metrological meaning. The modern system of standards is mainly based on stable physical phenomena. So, the stability of constants plays a crucial role. And it may be even more important if new definitions of the units via fixed fundamental constants in the International System of Units (SI) will be introduced in 2011, as is suggested now. Let us give some short historical references to the evolution of systems of units.

Before the official adoption of the Metric Convention (1875) in 1799, two platinum basic standards (metre and kilogram) were introduced and placed in the Archive of the French Republic, which started the Metric System.

Gauss in 1832 created the first coherent system of units introducing the second and measured magnetic fields in terms of mechanical units: mm, gram and second.

In 1874, the British Association for Development of Science introduced the coherent system of units CGS based on Maxwell’s and Thomson’s suggestions of 1860: centimetre — gram — second.

In 1880, the British Association and International Congress of Electricians adopted practical units — ohm, volt and ampere.

In 1889, the First General Committee on Weights and Measures (BIPM) adopted the MKS System (metre, kilogram, second).

In 1939, the MKSA system (MKS + ampere) was suggested (based on Georgi’s suggestion of 1901) and officially adopted by BIPM in 1946.

In 1954. *kelvin and candela* were introduced by BIPM, and the whole system of 6 basic units was called SI (International System of Units) in 1960.

In 1971, the *mole* was added to SI, so up till now SI has 7 basic units: metre, kilogram, second, ampere, kelvin, candela and mole.

A new stage of SI evolution started in October 2005 when the International Committee for Weights and Measures (CIPM) adopted a recommendation on preparative steps towards redefining the *kilogram, ampere, kelvin and mole*, so that these units be linked to exactly known values of fundamental constants. Mills et al. [99] proposed that these four base units should be given new definitions linking them to exactly defined values of the Planck constant h, elementary charge e, the Boltzmann constant k and the Avogadro constant $N_A$, respectively. This would mean that six of the seven base units of the SI would be defined in terms of true invariants of nature. Not only would these four fundamental constants have exactly defined values, but also the uncertainties of many of the other fundamental constants of physics would be either eliminated or appreciably reduced. They suggested wordings for the four new definitions and presented a novel way to define the entire SI units explicitly using such definitions without making any distinction between base units and derived units:

The metre, unit of length, is such that the speed of light in vacuum c is exactly

299 792 458 metres per second.

Such a definition is simple, concise and makes clear the fundamental constant to which the unit is linked and the exact value of that constant.
If this general form were chosen, it would be appropriate to choose definitions of the same form for all seven base units.

Thus, for the second, candela, kilogram, ampere, kelvin and mole we would have:

The second, unit of time, is such that the ground state hyperfine splitting transition frequency of the caesium 133 atom is exactly

9 192 631 770 hertz.

The candela, unit of luminous intensity in a given direction, is such that the spectral luminous efficacy of monochromatic radiation of frequency $540 \times 1012$ hertz is exactly

683 lumens per watt.

The kilogram, unit of mass, is such that the Planck constant is exactly

$6.6260703 \times 10^{-34}$ joule second.

The ampere, unit of electric current, is such that the elementary charge is exactly

$1.60217665 \times 10^{-19}$ coulomb.

The kelvin, unit of thermodynamic temperature, is such that the Boltzmann constant is exactly

$1.3806505 \times 10^{-23}$ joule per kelvin.

The mole, unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, is such that the Avogadro constant is exactly

$6.022145 \times 10^{23}$ per mole.

Of course, there still remain a lot of problems to be solved before final introduction of these new definitions (raising the precision of absolute values of some constants, choosing the variant of kg realization, testing the self-consistency of definitions, possible stability of constants, readiness of the world community to accept these changes (science, industry, trade, educational level etc.).

4. Time Variations of $G$. As all physical laws were established and tested during the last 2-3 centuries in experiments on Earth and in the near space, i.e., at rather short space and time intervals as compared with the radius and age of the Universe, the possibility of slow variations of constants (i.e., with the rate of the evolution of the Universe or slower) cannot be excluded a priori. So, the assumption of absolute stability of constants is an extrapolation, and each time we must test it.

The problem of FPC variations arose with the attempts to explain the relations between micro- and macro-world phenomena. Dirac was the first to introduce (in 1937) the so-called “Large Number Hypothesis” which relates some known very big (or very small) numbers with the dimensionless age of the Universe $T \sim 10^{105}$ (age of the Universe in seconds $10^{17}$, divided by the characteristic elementary particle time $10^{-23}$ seconds). He suggested that the ratio of the gravitational to strong interaction strengths, $Gm_p^2/(\hbar c) \sim 10^{-40}$, is inversely proportional to the age of the Universe: $Gm_p^2/(\hbar c) \sim T^{-1}$. Then, as the age varies, some constants or their combinations must vary as well. The atomic constants seemed to Dirac to be more stable, so he chose variation of $G$ as $T^{-1}$.

After the original Dirac hypothesis, some new ones appeared (Gamow, Teller, Landau, Terazawa, Stanikovich etc., see [4,12]), as well as some generalized theories of gravitation admitting variations of an effective gravitational coupling. We can single out three stages in the development of this field:

1. Studies of theories and hypotheses with FPC variations, their predictions and confrontation with experiments (1937-1977).

2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analyses of experimental and observational data within these theories [73][4] (1977-present).

3. Analyses of FPC variations within unified models [5][1] (present).

In the development of the first stage from the analysis of the whole set of the then available astronomical, astrophysical, geophysical and laboratory data, a conclusion was made [73][74] that variations of atomic constants were excluded at the level of $10^{-15}$ per year, but variations of the effective gravitational constant in the atomic system of units do not contradict the experimental data on the level of $10^{-12} \div 10^{-13}$ year$^{-1}$. Moreover, in [73][74][75], a conception was worked out that variations of constants are not absolute but depend on the system of measurements (choice of standards, units and devices using this or that fundamental-
tual interaction). Each fundamental interaction, through dynamics described by the corresponding theory, defines the system of units and the corresponding system of basic standards, e.g., atomic and gravitational (ephemeris) seconds.

Earlier reviews of some hypotheses on variations of FPCs and experimental tests can be found in [3][5][7].

5. There are different astronomical, geophysical and laboratory data on possible FPC variations [12].

**Astrophysical data.** Here follow some recent ones.

Comparing the data from absorption lines of atomic and molecular transition spectra in high-redshift QSO’s, Varshalovich and Potekhin (Russia) [82] obtained for $z = 2.8 - 3.1$:

$$|\dot{\alpha}/\alpha| \leq 1.6 \cdot 10^{-14} \text{ year}^{-1},$$

and Drinkwater et al. [83]:

$$|\dot{\alpha}/\alpha| \leq 10^{-15} \text{ year}^{-1} \text{ for } z = 0.25$$

and

$$|\dot{\alpha}/\alpha| \leq 5 \cdot 10^{-16} \text{ year}^{-1} \text{ for } z = 0.68$$

for a model with zero deceleration parameter and $H = 75 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$.

A less precise conclusion was made on the basis of **geophysical data.** Indeed, according to F. Dyson (1972), $\beta$-decay of $Re_{187}$ to $Os_{187}$ give:

$$|\dot{\alpha}/\alpha| \leq 5 \cdot 10^{-15} \text{ year}^{-1}. \quad (17)$$

We must point out that all astronomical and geophysical estimations are strongly model-dependent. So, of course, it is always desirable to have laboratory tests of FPC variations.

a) Such a test was first done by a Russian group in the Committee for Standards (Kolosnitsyn, 1975). Comparing rates of two different types of clocks, one based on a Cs standard and another on a beam molecular generator, they found that

$$|\dot{\alpha}/\alpha| \leq 10^{-10} \text{ year}^{-1}. \quad (18)$$

b) More recent data were obtained by J. Prestage et al. [84] by comparing mercury and $H$-maser clocks. Their result is

$$|\dot{\alpha}/\alpha| \leq 3.7 \cdot 10^{-14} \text{ year}^{-1}. \quad (19)$$

All these limits were placed on the fine structure constant variations. From the analysis of decay rates of $K_{40}$ and $Re_{187}$, a limit on possible variations of the weak interaction constant was obtained (see ab approach for variations of $\beta$, e.g., in [76])

$$|\dot{\beta}/\beta| \leq 10^{-10} \text{ year}^{-1}. \quad (20)$$

But the most stringent data on variations of strong ($G_s$), electromagnetic and weak ($G_W$) interaction constants were obtained by A. Schlyachter (USSR) in 1976 from an analysis of the ancient natural nuclear reactor data in Gabon, Oklo, because the event took place $2 \cdot 10^{10}$ years ago. They are as follows:

$$|G_s/G_s| < 5 \cdot 10^{-18} \text{ year}^{-1},$$

$$|\dot{\alpha}/\alpha| < 10^{-17} \text{ year}^{-1},$$

$$|G_F/G_F| < 2 \cdot 10^{-12} \text{ year}^{-1}. \quad (21)$$

Some studies of the strong interaction constant and its dependance on transferred momenta may be found in [115]. A recent review on variations of $\alpha$ see in [97].

There have appeared some data on a possible variation of $\alpha$ on the level of $10^{-16}$ at some $z$ [88]. Other groups do not support these results. There also appeared data on possible variation of $m_e/m_p$ (Varshalovich et al.) The problem may be that even if they are correct, all these results are mean values of variations at some epoch of the evolution of the Universe (a certain $z$ interval). In essence, variations may be different at different epochs (if they exist at all), and at the next stage observational data should be analyzed with the account of evolution of the corresponding (“true”?) cosmological models.

6. Now we are still having no unified theory of all four interactions. So it is possible to construct systems of measurements based on any of these four interactions. But in practice it is now done on the basis of the mostly worked out theory, electrodynamics (more precisely, on QED). Of course, it may also be done on the basis of the gravitational interaction (as was partly the case earlier). Then, different units of basic physical quantities arise, based on dynamics of a particular interaction, i.e., the atomic (electromagnetic) second, defined via frequency of atomic transitions, or the gravitational second defined by the mean Earth motion around the Sun (the ephemeris time).

It does not follow from anything that these two seconds are always synchronized in time and space. So, in principle, they may evolve relative to each other, for example, at the rate of the evolution of the Universe or at some slower rate.

That is why, in general, variations of the gravitational constant are possible in the atomic system of units ($c$, $\hbar$, $m$ are constant, Jordan frame) and masses of all particles — in the gravitational system of units ($G$, $\hbar$, $c$ are constant by definition, Einstein frame). In practice, we can test only the first variant since the modern basic standards are defined in the atomic system of measurements. Possible variations of the FPCs must be tested experimentally, but for this purpose it is necessary to have the corresponding theories admitting such variations and their certain effects.

Mathematically, these systems of measurement may be realized as conformally related metric forms. Arbi-
tary conformal transformations give us a transition to an arbitrary system of units.

We know that the scalar-tensor and multidimensional theories are the corresponding frameworks for these variations. So, one of the ways to describe variable gravitational coupling is the introduction of a scalar field as an additional variable of the gravitational interaction. It may be done by different means (e.g. Jordan, Brans-Dicke, Canuto and others). We have suggested a variant of gravitational theory with a conformal scalar field (Higgs-type field \[77, 4\]), where Einstein’s GR may be considered as a result of spontaneous breaking of the conformal symmetry (Domokos, 1976) \[4\]. In our variant, a spontaneous breaking of the global gauge invariance leads to a nonsingular cosmology \[78\]. Besides, we may get variations of the effective gravitational constant in the atomic system of units when \(m, c, h\) are constant and variations of all masses in the gravitational system of units \((G, c, h\) are constant). It is done on the basis of approximate \[79\] and exact cosmological solutions with local inhomogeneity \[80\].

The effective gravitational constant is calculated using the equations of motion. Post-Newtonian expansion is also used in order to confront the theory with existing experimental data. Among the post-Newtonian parameters, the parameter \(f\) describing variations of \(G\) is included. It is defined as

\[
\frac{1}{GM} \frac{d(GM)}{dt} = fH. \tag{22}
\]

According to Hellings’ data \[81\] from the Viking mission,

\[
\dot{\gamma} - 1 = (-1.2 \pm 1.6) \cdot 10^3, \quad f = (4 \pm 8) \cdot 10^{-2}. \tag{23}
\]

In the theory with a conformal Higgs field \[79,80\], we have obtained the following relation between \(f\) and \(\dot{\gamma}\):

\[
f = 4(\dot{\gamma} - 1). \tag{24}
\]

Using Hellings’ data for \(\dot{\gamma}\), we can calculate \(f\) in our variant and compare it with \(f\) from \[81\]. Then we get \(f = (-9, 6 \pm 12, 8) \cdot 10^{-3}\), which agrees with (24) within its accuracy.

We have used here only Hellings’ data on variations of \(G\). Other theoretical calculations in different models give the following predictions: less than \(10^{-12}\) per year in multidimensional models \[89,91,92\], less than \(10^{-14}\) per year \[90\].

But the situation with the experiment and observations is not so simple. Along with \[81\], there are some other data \[4,5,12\], but the most precise are:

1. Hellings’ result

\[
|\dot{G}/G| < (2 \pm 4) \cdot 10^{-12} \text{ year}^{-1}. \tag{25}
\]

2. A result from nucleosynthesis (Acceta et al., 1992):

\[
|\dot{G}/G| < (\pm 0.9) \cdot 10^{-12} \text{ year}^{-1}. \tag{26}
\]

3. E.V. Pitjeva’s result (Russia) \[93\], based on satellites and planets motion:

\[
|\dot{G}/G| < (0 \pm 2) \cdot 10^{-12} \text{ year}^{-1}. \tag{27}
\]

4. Some new results from pulsars and Big Bang nucleosynthesis (BBN) at the level of \(10^{-12}\) per year.

There are also BBN data of Copi et al., 2003:

\[
-3 \cdot 10^{-13} < \dot{G}/G < 4 \cdot 10^{-13}. \tag{28}
\]

As to other experimental or observational data, the results are of different quality. The most reliable ones are based on lunar laser ranging (LLR) (Muller et al, 1993, Williams et al, 1996, Nordtvedt, 2003). They are not better than \(10^{-12}\) per year. Here, once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions like Earth-SEE satellite \[10,11,12,13\] or missions to other planets and lunar laser ranging will be a decisive step in solving the problem of temporal variations of \(G\) and determining the fates of different theories which predict them, since the greater is the time interval between successive measurements and, of course, the more precise they are, the more stringent results will be obtained.

As we saw, different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac type, where \(G\) is replaced with \(G(t)\).

2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where \(G\) depends on the scalar field \(\sigma(t)\).

3. Gravitational theories with a conformal scalar field arising in different approaches \[72,74,77,80\].

4. Multidimensional unified theories in which there are dilaton fields and effective scalar fields appearing in our 4-dimensional spacetime from additional dimensions \[57,11\]. They also may help in solving the problem of a variable cosmological constant changing from very large Planckian to present (0.7 of total energy) values.

As was shown in \[5,57,11\], temporal variations of FPCs are connected with each other in multidimensional models of unification of interactions. So, experimental tests on \(\dot\alpha/\alpha\) may at the same time be used for estimation of \(\dot{G}/G\) and vice versa. Moreover, variations of \(G\) are also related to the cosmological parameters \(\rho\), \(\Omega\) and \(q\), which gives opportunities of raising the precision of their determination.

Since FPC variations are closely connected with the behaviour of internal scale factors, it is a direct probe of properties of extra dimensions and the corresponding unified theories \[89,11\]. From this point of view, it is an additional test of not only gravity and cosmology, but of unified theories of physical interactions as well.
7. Non-Newtonian interactions, or range variations of $G$. Nearly all modified theories of gravity and unified theories also predict some deviations from the Newton law (inverse square law, ISL) or composition-dependent violations of the Equivalence Principle (EP) due to appearance of new possible massive particles (partners) [5]. Experimental data exclude the existence of these particles on a very good level in nearly all ranges except less than the millimetre and metres and hundreds of metres ranges. Our recent analysis of experimental bounds and new limits on possible ISL violation using a new method and modern precession data from satellites, planets, binary pulsar and LLR data were obtained in [95].

In the Einstein theory $G$ is a true constant. But, if we think that $G$ may vary with time, then, from a relativistic point of view, it may vary with distance as well. In GR massless gravitons are mediators of the gravitational interaction, they obey second-order differential equations and interact with matter with a constant strength $G$. If any of these requirements is violated, we come in general to deviations from the Newton law with range (or to a generalization of GR).

In [6] we have analyzed several classes of such theories:

1. Theories with massive gravitons like bimetric ones or theories with a $A$-term.
2. Theories with an effective gravitational constant like the general scalar-tensor ones.
3. Theories with torsion.
4. Theories with higher derivatives (4th-order equations etc.), where massive modes appear, leading to short-range additional forces.
5. More elaborated theories with other mediators in addition to gravitons (partners), like supergravity, superstrings, M-theory etc.
6. Theories with nonlinearities induced by any known physical interactions (Born-Infeld etc.)
7. Phenomenological models where a detailed mechanism of deviation is not known (fifth or other force).
8. Modifications of the Newton law at large ranges (MOND etc.), small acceleration at $a > a_0$ (Pioneer anomaly, etc.)

In all these theories, some effective or real masses appear leading to Yukawa-type (or power-law) deviations from the Newton law, characterized by the strength $\alpha$ and the range $\lambda$.

There exist some model-dependent estimates of these forces. The most well-known one belongs to Scherk (1979) from supergravity where the graviton is accompanied by a spin-1 partner (graviphoton) leading to an additional repulsion. Other models were suggested by Moody and Wilczek (1984) — introduction of a pseudoscalar particle leading to an additional attraction between macro-bodies with the range $2 \cdot 10^{-4} \text{ cm} < \lambda < 20 \text{ cm}$ and strength $\alpha$ from 1 to $10^{-10}$ in this range. Another supersymmetric model was elaborated by Fayet (1986, 1990), where a spin-1 partner of a massive graviton gives an additional repulsion in the range of the order $10^3 \text{ km}$ and $\alpha$ of the order $10^{-13}$.

A scalar field to adjust $\alpha$ was introduced by S. Weinberg in 1989, with a mass smaller than $10^{-5}\text{eV}/c^2$, or a range greater than 0.1 mm. One more variant was suggested by Peccei, Sola and Wetterich (1987), leading to additional attraction with a range smaller than 10 km.

Some $p$-brane models (ADD, brane worlds) also predict non-Newtonian additional interactions of both Yukawa or power-law types, in particular, in the sub-millimetre range, which is intensively discussed nowadays [13,96]. On PPN parameters for multidimensional models with $p$-branes see above, Sec. 2.

The Pioneer anomaly. A more serious evidence on a possible violation of Newton’s law has come to us from space, namely, from data processing on the motion of the spacecrafts Pioneer 10 and 11, referring to length ranges of the order of or exceeding the size of the Solar system. The discovered anomalous (additional) acceleration is [101]

$$(8.60 \pm 1.34) \times 10^{-8} \text{ cm/s}^2,$$

it acts on the spacecrafts and is directed towards the Sun. This acceleration is not explained by any known effects, bodies or influences related to the design of the spacecrafts themselves (leakage etc.), as was confirmed by independent calculations.

Many different approaches have been analyzed both in the framework of standard theories and invoking new physics (Chongming Xu’s talk at ICGA-7, Taiwan, November 2005), but none of them now seems to be sufficiently convincing and generally accepted. There are the following approaches using standard physics:

- an unknown mass distribution in the Solar system (Kuiper’s belt), interplanetary or interstellar dust, local effects due to the Universe expansion [100];
- employing the Schwarzschild solution with an expanding boundary [101,102] etc.

Among the approaches using new physics one can mention:

- a variable cosmological constant [103];
- a variable gravitational constant [106];
- a new PPN theory connecting local scales with the cosmological expansion [104];
- the five-dimensional Kaluza-Klein (KK) theory with a time-variable fifth dimension and varying fundamental physical constants [105];
- Moffat’s [107] non-symmetric gravitational theory;
- Milgrom’s [108,109] modified Newtonian dynamics (MOND);
- special scalar-tensor theories of gravity [110];
This Pioneer anomaly has caused new proposals of space missions with more precise experiments and a wide spectrum of research at the Solar system length range and beyond:

- Cosmic Vision 2015-2025, suggested by the European Space Agency, and
- Pioneer Anomaly Explorer, suggested by NASA [13].

So, we hope they can contribute a lot to our knowledge of gravity and unified models.

8. Space project SEE and laboratory projects.
We have seen that there are three problems connected with $G$. There is a promising new multi-purpose space experiment SEE (Satellite Energy Exchange) [10,11] which addresses all these problems and may be more effective in solving them than other laboratory or space experiments.

This experiment is based on a limited 3-body problem of celestial mechanics: small and large masses in a drag-free satellite and the Earth. Unique horse-shoe orlemma of celestial mechanics: small and large masses in a space missions with more precise experiments and a wide spectrum of research at the Solar system length range and beyond:

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This experiment is based on a limited 3-body problem of celestial mechanics: small and large masses in a drag-free satellite and the Earth. Unique horse-shoe orbits, which are effectively one-dimensional, are used in it.

The aims of the SEE-project are to measure the Inverse Square Law (ISL) and the Equivalence Principle (EP) at ranges of metres and the Earth radius, $\dot{G}$ and the absolute value of $G$ with unprecedented accuracies.

We have studied many aspects of the SEE project [11,12]:

1. A wide range of trajectories with the aim of finding optimal ones:
   - circular in a spherical field;
   - the same plus the Earth’s quadrupole modes;
   - elliptic with eccentricity less than 0.05.

2. Estimations of other celestial bodies’ influence.

3. Estimation of the sensitivity of trajectories to changes in $G$ and the Yukawa-type interaction strength parameter $\alpha$.

4. Modelling the measurement procedures for $G$ and $\dot{G}$ by different methods for different ranges and for different satellite altitudes:
   - optimal, 1500 km,
   - ISS free flying platform, 500 km and
   - a high orbit, 3000 km.

5. Estimations of some sources of error were made for:
   - radial oscillations of the shepherd’s surface;
   - longitudinal oscillations of the capsule;
   - transversal oscillations of the capsule;
   - shepherd’s non-sphericity;
   - limits on the quadrupole moment of the shepherd;
   - limits on admissible charges and time scales of charging by high energy particles etc.

6. Error budgets for $G$, $\dot{G}$ and $G(r)$ were calculated.

On the basis of all these studies, the general conclusion was that realization of the SEE project may improve our knowledge of $G$, $\dot{G}$ and $G(r)$ by 3-4 orders of magnitude.

A laboratory experiment was also suggested in our paper [114] to test the possible range variations of $G$. It is an experiment for possible detection of new forces, or test of the inverse square law, parameterized by a Yukawa-type potential with strength $\alpha$ and range $\lambda$. The installation comprises a ball with a spherical cavity whose centre is shifted with respect to the ball centre. The ball is placed on a turn-table being subject to uniform rotation. A torsion balance as a sensitive element is placed inside the cavity. A uniform gravitational field created inside the ball does not affect the balance, but any non-gravitational forces create a torque which acts periodically during the rotation of the ball. The spectrum of harmonics was calculated. It is shown that preferable to use is the first harmonic in the measurements. Sensitivity of the method was evaluated, which is limited by uncertainties due to manufacturing of the elements and temperature fluctuations of the sensitive element. It was shown that the sensitivity of the method suggested may be at the level of $10^{-10}$ in $\alpha$ in the range of $\lambda$ from 0.1 to $10^{7}$ m in the space of the Yukawa parameters ($\alpha, \lambda$).

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