Study of the electrodes length influence on the trajectories of water droplets dispersed in oil and affected by non-uniform electric field

S N Kharlamov[^1][^2], V V Zaikovskii[^1][^3] and I V Shenderova[^1][^4]

1 Institute of Natural Resources, National Research Tomsk Polytechnic University, 30 Lenin Ave., Tomsk, 634050, Russia

E-mail: ^2^ kharsn@mail.ru, ^3^ alkast@tpu.ru, ^4^ inna-shenderova@yandex.ru

Abstract. The paper presents the results of numerical modelling of the processes accompanying movement of drop viscous media (water) in oil under the influence of exterior forces of the electric and dynamic nature. Systematic calculations of influence on the electric field heterogeneity drops, created by a symmetric and asymmetrical configuration of electrodes are carried out both in inter electrode and behind electrode areas taking into account a complex operation of dielectrophoresis forces, buoyancies and drag, as well as the variability of electrode sizes. The analysis of drop movement trajectories shows that the asymmetrical configuration of electrodes can be applied for an electro-coalescence intensification of water-in-oil emulsion. Correctness of calculations of the mathematical model and numerical methods are confirmed by good results if compared with the available data of the other authors.

1. Introduction

Nowadays there are two basic physical principles, which are applied during electroseparation of water-in-oil emulsions: electrophoresis (the motion of charged particles along the lines of the electric field) and dielectrophoresis (motion of polarized substances toward the greatest increase of electric field).

The analysis of literature has shown [2-4] that in recent years the attention of researchers focuses on the issues of understanding the nature of the influence of "small-scale" dielectrophoresis effect. It is known that this physical mechanism has a number of disadvantages as it functions only at small distances between polarized droplets [5, 6]; in the certain range of parameters of the electric field it may be a reason for secondary droplets of dispersed phase (which are much smaller size) origination [7]; there is a possibility for a short circuit between non-insulated electrodes [8].

In an inhomogeneous electric field, water droplets will tend to move in the direction of the greatest increase of the electric field [6, 10]. Thus, an additional bonding mechanism emerges. Non-uniform electric field can be generated due to the asymmetric, confuser, diffuser or cylindrical configuration of the electrodes [9].

Considering the abovementioned, the aim of this paper is: 1) to perform the development and verification of a mathematical model and numerical algorithm of calculation of dispersed conductive phase movement processes in water-oil emulsions under electric field action; 2) to establish the efficient algorithm application in systematic researches of an intensification problems of electro coalescence process at asymmetrical configurations of electrodes, and in the forecast of the potential
changes, electric field intensity and drop movement trajectories of water in emulsions; 3) explanation and an assessment of influence character of dielectrophoresis forces on drop movement trajectory.

2. Numerical simulation of the electric field
Currently the different approaches to the calculation of the non-uniform electric field characteristics and its effect on polarized dielectrics are described in detail in the literature. In particular, there is an approach in which the potential of the electric field is modeled by implicit analytic function obtained by the method of conformal mappings [11]. Another approach, which allows obtaining analytical solution on the more general geometry of the electrodes with mixed boundary conditions, is the method of Wiener – Hopf [12]. However, its usage is presented in the literature only by cases of electric field calculation in the interelectrode space.

A much more common method of obtaining values of the electric field potential on arbitrary geometry of the electrodes and arbitrary distance from them - is the numerical solution of the Laplace equation [13, 14].

In the assumption of free charges absence in a simulated two-dimensional region, Laplace equation can be written in the form [15]:

\[ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \]  

(1)

This equation must be supplement by boundary conditions of several types. For the boundaries of the computational domain, which should be at a sufficient distance from the plates of the capacitor, the potential is zero. For left plate (index "lp") and right plate (index, "rp") of the condenser the potential should be known and specified by initial conditions of the calculation.

Relation to the stationary electric field, based on the determination of this value, could be formulated for the two-dimensional case, as a vector [15]:

\[ \mathbf{E} = \left\{ \frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y} \right\} \]  

(2)

Numerical integration of equation (1) is performed with the involvement of implicit finite-difference schemes and approximation of the derivative with the second order accuracy [O(Δx^2), O(Δy^2)] regarding the steps for axial and longitudinal variables on a "cross" grid-type pattern.

Verification of the numerical algorithm was implemented on a two-dimensional flat vertical condenser, in the ranges of define parameters, that are presented in table 1.

| Parameter, designation, measurement unit | Value |
|------------------------------------------|-------|
| Length of the left capacitor plate, L_{lp}, 10^{-2} m | 1 ÷ 10 |
| Length of the right capacitor plate, L_{rp}, 10^{-2} m | 10 ÷ 1 |
| Voltage on the left capacitor plate, U_{lp}, V | 0 ÷ 5000 |
| Voltage on the right capacitor plate, U_{rp}, В | 0 |
| Distance between capacitor plates, d, 10^{-2} m | 1 ÷ 0,05 |
| Discretization step along the longitudinal coordinate, Δx, 10^{-3} m | 1 ÷ 0,05 |
| Discretization step along the transverse coordinate, Δy, 10^{-3} m | 1 ÷ 0,05 |

Individual results of the calculation are shown on figures 1-2, which illustrate combined field potential \( \varphi \) and the electric field strength \( \mathbf{E} \) in the study area at fixed \( \Delta x = \Delta y = 10^{-4} \) m, \( U_{lp} = 5000 \) V, \( U_{rp} = 0 \) V, \( d = 0.5 \cdot 10^{-2} \) m. Variety of the scalar field potential \( \varphi(x, y) \) is illustrated by the background color of figures 1-2, the vector electric field \( \mathbf{E}(x, y) \) is indicated by arrows.
Figure 1. The change in the potential $\phi(x,y)$ and electric field $E(x,y)$ at $L_{lp} = 5 \cdot 10^{-2}$ m, $L_{rp} = 1 \cdot 10^{-2}$ m

Figure 2. The change in the potential $\phi(x,y)$ and electric field $E(x,y)$ at $L_{lp} = 1 \cdot 10^{-2}$ m, $L_{rp} = 5 \cdot 10^{-2}$ m

To validate and assess the accuracy of calculations graphical comparison of power distribution dielectrophoresis (8) obtained using the described algorithm and obtained by other researchers (for example, [13]) was carried out. The comparison of results is shown on figure 3. Graphical field mapping shows that the features associated with the predominant direction of the field vectors to the edges of the capacitor plates are forecasted quite accurately.

Figure 3. Graphical comparison of the dielectrophoretic force spatial distribution at $U_{lp} = 0$ V, $U_{rp} = 70$ V, $d = 200 \cdot 10^{-6}$ m, $L_{mm} = 1000 \cdot 10^{-6}$ m, $L_{nn} = 200 \cdot 10^{-6}$ m.

The results of the present calculation

Results of Zhenqian Chen, Xiaozhong Shen [13]
3. Numerical modeling of water droplets motion

For modeling the motion of single spherical incompressible water droplets in oil under the influence of an inhomogeneous electric field, it is necessary to determine the forces which influence such a drop. They are: gravity force, the force of Archimedes, drag force, the dielectrophoretic force.

The effect of gravity and Archimedes force is modeled by the ratio for buoyancy force $F_{buoyancy}$ [16]:

$$ F_{buoyancy} = F_{gravity} + F_{Archimedes} = - (\rho_{drop} - \rho_{oil}) \cdot g \cdot V_{drop} $$

(3)

The effect of drag force $F_{drag}$ for liquid particles, moving with a slight acceleration (taking into account the internal fluid circulation inside the droplet), is modeled by Hadamard – Rybczynski ratio [16]:

$$ F_{drag} = -3\pi \cdot \frac{\lambda + \frac{2}{3}}{\lambda + 1} \cdot \mu_{oil} \cdot d_{drop} \cdot \nu_{drop}, \text{ where } \lambda = \frac{\mu_{water}}{\mu_{oil}} $$

(4)

The effect of dielectrophoretic force $F_{DEP}$ on conducting drop of water, that dissolved in a non-conductive oil environment and being under the influence of an inhomogeneous electric field is modeled by the ratio [11]:

$$ F_{DEP} = 2\pi \cdot \varepsilon_{oil} \cdot \frac{d^3}{8} \cdot \frac{\varepsilon_{water} - \varepsilon_{oil}}{\varepsilon_{water} + 2\varepsilon_{oil}} \cdot \nabla E^2 $$

(5)

Thus, the equations of motion in projections on coordinate axes for a water drop could be written as follows:

$$ m_{drop} \cdot a_y = F_{buoyancy} + F^y_{drag} + F^y_{DEP}, \quad a_y = \frac{d^2 y}{dt^2} $$

$$ m_{drop} \cdot a_x = F^x_{drag} + F^x_{DEP}, \quad a_x = \frac{d^2 x}{dt^2} $$

(6) (7)

4. The discussion of research results

Application of numerical integration methods, in the computer algebra system “Mathematica”, the trajectory of single water droplets were simulated in the computational domain $H = 0.125 \times 0.125$ m with initial coordinates: $y_0 = 0.125$ m (upper edge of computational domain), $x_0 \in [0.015;0.045]$ m with a step. The range of varying physical parameters of the simulated environment is presented in table 2.

Table 2. The values of physical quantities in the simulated water drop movement.

| Parameter, designation, measurement unit | Value |
|-------------------------------|-------|
| Length of the left capacitor plate, $L_{lp}$, $10^{-2}$ m | 2 ÷ 5 |
| Length of the right capacitor plate, $L_{rp}$, $10^{-2}$ m | 5 ÷ 2 |
| Voltage on the left capacitor plate, $U_{lp}$, V | 5000 |
| Voltage on the right capacitor plate, $U_{rp}$, V | 0 |
| Distance between capacitor plates, d, $10^{-2}$ m | 1 |
| Discretization step along the longitudinal coordinate, $\Delta x$, $10^{-3}$ m | 0.05 |
| Discretization step along the transverse coordinate, $\Delta y$, $10^{-3}$ m | 0.05 |
| Droplet diameter, $d_{drop}$, $10^{-6}$ m | 50 |
| Density of water, $\rho_{water}$, kg/m$^3$ | 1000 |
| Density of oil, $\rho_{oil}$, kg/m$^3$ | 850 |
| Dynamic viscosity of water, $\mu_{water}$, $10^{-3}$ Pa·sec | 1.002 |
| Dynamic viscosity of oil, $\mu_{oil}$, $10^{-3}$ Pa·sec | 7 |
| Dielectric permittivity of water, $\varepsilon_{water}$, $10^{-12}$ F/m | 81 |
| Dielectric permittivity of oil, $\varepsilon_{oil}$, $10^{-12}$ F/m | 2.5 |
| Step of varying the initial droplet position, $x_0$, $10^{-4}$ m | 2.483 |
It should be noted that the above model doesn't consider the coalescence of water droplets – the trajectory of each droplet is considered separately, as if it was moving in the absence of other drops. The trajectories of water droplets with different initial position $x_0$ are placed on a single figure in order to provide a more effective visualization of the non-uniform electric field effect.

To study the dependence of the trajectory of water droplets from the ratio of plates dimensions and, consequently, influence of inhomogeneity of the electric field generated by the asymmetric configuration of the plates, various calculations were carried out with changing of the parameters $L_{lp}$, $L_{rp}$ in the range $(2 \div 5) \times 10^{-2}$ m. Some results of numerical simulation are presented in figures 4-5.

In particular, these results allow us to conclude that the configuration of the electrodes had a major influence on water droplets trajectories.

In the case of asymmetrical flat capacitor and the prevalence of the length of a charged plate over the length of the grounded plate (figure 4), there was a proportional deviation of one half of the drops in the direction of a charged plate, the other half – towards the grounded plate. This is a consequence of large-scale inhomogeneity of the electric field in the interelectrode space. In this case, due to the large number of horizontal deviations, the length of the path of water droplets in this field increases. The number of drops that slipped between zones of influence, with this configuration of electrodes, visually is about 3%, and the zone of influence of a grounded electrode in the “right” outer space is slightly less than the zone of influence of a charged electrode in the “left” outer space.

![Figure 4](image-url)

**Figure 4.** Water droplets trajectories in oil with the asymmetric configuration of the electrodes, $L_{lm} = 5 \cdot 10^{-2}$ m, $L_{nm} = 2 \cdot 10^{-2}$ m, $U_{lm} = 5000$ V, $U_{nm} = 0$ V.

![Figure 5](image-url)

**Figure 5.** Water droplets trajectories in oil with the asymmetric configuration of the electrodes, $L_{lm} = 2 \cdot 10^{-2}$ m, $L_{nm} = 5 \cdot 10^{-2}$ m, $U_{lm} = 5000$ V, $U_{nm} = 0$ V.
In case when the length of the grounded plate is more than the length of a charged plate (figure 5), almost all particles in the interelectrode space are attracted to a charged plate. However, about 15% of the particles do not reach the boundary of the zone of attraction, which is much greater than for the case when the charged plate length is more than the length of the grounded one. The distance between the "slipped out" drops increases if compared to the original, which creates an even greater negative effect.

The overall effect after treating the plane capacitor by asymmetrical vertical electric field, in both cases, is expressed in reduction in the number of drops in the interelectrode space and some distance behind the electrodes, due to their attraction to the edges of the plates.

5. Conclusion
The paper presents the results of mathematical modeling and numerical calculation of the movement process of water droplets in oil, as well as the efficiency of the algorithm in the range of investigated parameters. Verification of the model and algorithm is performed by comparison with the known results of other authors and with high degree of agreement in the prediction of the effects of power dielectrophoresis on the motion of the dispersed phase. For the single incompressible spherical water drops in oil, the calculation of the trajectories of water droplets in an inhomogeneous electric field, created by different configurations of electrodes, was performed. Detailed calculations have shown, that the capacitors configuration with prevalence of a charged plate length over the grounded plate length provides the least amount of "slipped" drops, greater path length of drops in the interelectrode space and a greater area of outer space impact on the drops trajectories.

References
[1] Levechenko D N, Bergshtein N V, Hudyakova A D and Nikolaeva N M 1967 Emulsii nefti s vodoi i metody ih razrusheniya (Moskow: Khimiya)
[2] Eow J S, Ghadiri M, Sharif A O and Williams T J 2001 Chemical Engineering Journal 84 173
[3] Mohammadi M, Shahhosseini S and Bayat M 2014 Chemical Engineering Research and Design 92 2694
[4] Eow J S and Ghadiri M 2002 Chemical Engineering Journal 85 357
[5] Panchenkov G M and Tzabek L K 1969 Povedenie emulsii vo vneshem elektricheskom pole (Moskow: Khimiya)
[6] Vinogradov V M and Vinokurov V A 2007 Obrazovanie, svoistva I metody razrusheniyaneftyanih emulsii (Moskow: RGU nefti i gaza imeni I.M. Gubkina)
[7] Mousavi S H, Ghadiri M and Buckley M 2014 Chemical Engineering Science 120 130
[8] Mhatre S, Vivacqua V, Ghadiri M, Abdullah A M, Al-Marri M J, Hassanpour A, Hewakandamby B, Azzopardi B and Kermani B 2015 Chemical Engineering Research and Design 96 177
[9] Kharlamov S N and Zaykovskiy V V 2013 Proc. 8th Int. Forum on Strategic Technology IFOST (Ulaanbaatar, Mongolia, 28 June–1 July 2013) 1 784
[10] Jones T B 1995 Electromechanics of particles (New York: Cambridge University Press)
[11] Khainovskii V I, Kozirev A E and Nikitin P V 2013 Vestnik APK Stavropoliya 4 99
[12] Kua C H, Lam Y C, Yang C, Youcef-Toumi K and Rodriguez I 2008 Journal of Electrostatics 66 514
[13] Chen Z and Shen X 2010 Building and Environment 45 968
[14] Tarantzev K V, Krasnaya E G and Korosteleva A V 2011 Izvestia Penzenskogo gosudarstvenogo pedagogicheskogo universiteta 26 654
[15] Nagel J R 2014 Antennas and Propagation Magazine 56 209
[16] Melheim J A and Chiesa M 2004 Proc.5th Int. Conf. on MultiphaseFlow (Yokohama, Japan, 30 May–4 June 2004) 1 126