Fluxes in Heterotic and Type II String Compactifications

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ABSTRACT

In this paper we consider heterotic compactifications on $K3 \times T^2$ as well as type II compactifications on $K3$-fibred Calabi-Yau spaces with certain fluxes for the gauge and RR field strengths $F$ and $H$ turned on. By providing an identification of corresponding fluxes we show that the well-known $\mathcal{N} = 2$ heterotic/type II string-string duality still holds for a subset of all possible fluxes, namely those which arise from six-dimensional gauge fields with internal magnetic flux on the common two-sphere $\mathbb{P}^1_0$, which is the base space of the type II $K3$-fibration. On the other hand, $F$- and $H$-fluxes without $\mathbb{P}^1_0$-support, such as heterotic $F$-fluxes on the torus $T^2$ or type II $H$-fluxes on cycles of the $K3$-fibre cannot be matched in any simple way, which is a challenge for heterotic/type II string-string duality. Our analysis is based on the comparison of terms in the effective low-energy heterotic and type II actions which are induced by the fluxes, such as the Green-Schwarz couplings related to flux-induced $U(1)$ anomalies, the effective superpotential and the Fayet-Iliopoulos scalar potential.

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1 Introduction

String compactifications with background fluxes constitute an interesting class of string vacua. In type IIA/B compactifications on Calabi-Yau three-folds background fluxes are provided by vacuum expectation values of internal NS and R H-fields [1, 2, 3, 4, 5, 6, 7]. In the local case a dual description of closed type II strings on Calabi-Yau spaces with H-fluxes was proposed in terms of open topological strings, corresponding to a duality of superstrings on non-compact Calabi-Yau spaces with H-fluxes and certain large N gauge systems with $\mathcal{N} = 1$ supersymmetry in four dimensions [8]. In $\mathcal{N} = 2$ heterotic string compactifications on $K3 \times \mathbb{T}^2$ vacua with background fluxes can be obtained by turning on internal magnetic fields (magnetic F-fluxes) on certain internal two-dimensional subspaces [9, 10].

In general, the heterotic F-fluxes or, respectively, the type II H-fluxes cause several interesting effects after compactification to four dimensions: they can introduce warped space-times [19, 20, 21], they lift the vacuum degeneracy in the moduli space, they induce a spontaneous breaking of $\mathcal{N} = 2$ supersymmetry, where only at special points in the moduli space supersymmetry may be unbroken; they can imply spontaneous breaking of $U(1)$ gauge symmetries, they may create tachyonic scalar fields, which destabilize the vacuum, and they generate chiral fermion spectra. All these effects are in principle of vital phenomenological interest.

Based on the $\mathcal{N} = 2$ heterotic/type II string duality, which was established in the absence of background fluxes [22], we will discuss in how far this duality still holds when heterotic F-fluxes resp. type II H-fluxes are turned on. In particular we will show how at least part of these fluxes will be mapped onto each other via the heterotic/type II string duality. However other heterotic F-fluxes or type H-fluxes find no obvious dual interpretation and are therefore in apparent conflict with the heterotic/type II string-string duality. Our discussion will be mainly based on the comparison of several terms in the four-dimensional effective action:

- In type IIA on a Calabi-Yau space $M$ the internal $H_R^{(n)}$-fluxes generate a moduli dependent superpotential [3]

$$W = \sum_{n=0}^{3} \int_M H_R^{(2n)} \wedge J^{3-n} = \epsilon_1 \ X^I(z) - m^I \ F_I(z), \quad (1.1)$$

Type I compactifications with internal magnetic fluxes on tori have also been constructed [11, 12, 13, 14, 15, 16, 17, 18].

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where $J$ is the Kähler class and $(X^I, F_I)$ ($I = 0, \ldots, N_V$) is the symplectic vector of $\mathcal{N} = 2$ special geometry which depends on the scalar fields $z^A$ ($A = 1, \ldots, N_V$) in the vector multiplets. This superpotential generically breaks supersymmetry and the masses of the gravitini are of the order $M_3^2/2 \simeq |W|^2$. In addition the ground state of the corresponding potential determines in general the values of the complex scalar fields $z^A$. The $e_I$ and $m_I$ are the quantized values of the H-fluxes on the 0,2,4 and 6-cycles of $M$, namely $e_0 = \int H^{(6)}_R$, $e_A = \int H^{(4)}_R$, $m^A = \int H^{(2)}_R$, $m^0 = \int H^{(0)}_R$. We will show that those type IIA H-fluxes, which have support on a certain two-sphere $\mathbb{P}^1$, correspond in the heterotic string to gauge F-fluxes on the ‘same’ $\mathbb{P}^1$. These are essentially $e_0$ and all $e_A$, except the one without $\mathbb{P}^1$ support. On the other hand, IIA H-fluxes which no support on $\mathbb{P}^1$, namely all except one $m^A$ as well as $m^0$, have no immediate heterotic interpretation.

- In the heterotic string internal F-fluxes on some two-cycle $C_i$ induce four-dimensional Green-Schwarz couplings via the ten-dimensional Chern-Simons term $B_2 \wedge \text{Tr}(F^4)$ as well as through the Chern-Simons interactions in $H \wedge \ast H$. They involve the $U(1)$ vector fields $A^I$ and the internal B-field $B = a_{C_i} J_{C_i} + \ldots$, where $C_i$ will be either a certain 2-sphere $\mathbb{P}^1$ or a certain 2-torus $T^2$. The four-dimensional couplings proportional to internal fluxes $e^i_I$ of $F^I = dA^I$ on $C_i$ are of the form $(\ast_4 da_{C_i} =: dB_{C_i})$

$$\mathcal{L}_{GS} = e^i_I B_{C_i} \wedge F^I. \quad (1.2)$$

These couplings are responsible for the longitudinal component of a $U(1)$ vector boson which becomes massive due to the fluxes $[23, 24]$. In addition the Green-Schwarz term can cancel possible $U(1)$ triangle anomalies due to massless chiral fermions (see below). For the case of $\mathbb{P}^1$-fluxes the same Green-Schwarz action is obtained from the ten-dimensional type IIA action turning on H-fluxes with $\mathbb{P}^1$ support. In fact, the Green-Schwarz action gives a very direct mean to map the heterotic gauge F-fluxes to the type H-fluxes and vice versa.

- Along with this Green-Schwarz coupling there will be in general a mass term of the $U(1)$ gauge bosons,

$$M^2_{A^I} \sim e^2_I, \quad (1.3)$$

which signals a spontaneous gauge symmetry breaking due to the fluxes $e_I$. In the heterotic string this term comes from the ten-dimensional kinetic term for the CS-improved $H$. However in the type II compactifications the same term is not present in the tree-level effective action, but comes as a one-loop effect.
The two-dimensional index theorem for the Dirac operator relates the net number of chiral fermions to the fluxes $e_I$. These fermions are in general charged under the corresponding $U(1)$ gauge group, and hence a $U(1)$ triangle anomaly may be implied. It will be canceled by the Green-Schwarz term described above, together with a second coupling of the form

$$\mathcal{L} = c_I a c_i F^i \wedge F^i,$$

(1.4)

where the $c_I$ denote the strength of this interaction. Then the chiral anomaly is given as the product $e^I_I c_I$. For non-zero fluxes $e_I$ and non-vanishing Green-Schwarz term the chiral anomaly is nevertheless absent, provided the coefficients $c_I$ are zero.

In other words, the mass generation for the $U(1)$ gauge boson by the Green-Schwarz term is not necessarily linked to a non-trivial $U(1)$ anomaly. Precisely this, non-vanishing Green-Schwarz couplings without anomaly, will happen for the $\mathbb{P}^1_b$-fluxes.

On the contrary, chiral anomalies are in general present for heterotic $\mathbb{T}^2$-fluxes, which has also been confirmed in a large class of type I compactifications with $\mathbb{T}^2$-fluxes (see also the discussion about $U(1)$ anomalies in [18]).

In general, a Fayet-Iliopoulos (FI) D-term scalar potential will be generated [25], depending on the fluxes $e_I$, which contains tachyonic mass terms for charged scalar fields.

In our paper we will consider on the heterotic side two equivalent string compactifications to four dimensions, which proceed however via an inequivalent intermediate six-dimensional compactification. In compactification scheme A the $E_8 \times E_8$ heterotic string is compactified from ten to six dimensions on a $\mathbb{T}^2_f \times \mathbb{T}^2_c$. The six-dimensional theory is then fibred over a base $\mathbb{P}^1_b$, where the first torus $\mathbb{T}^2_f$ varies over the $\mathbb{P}^1_b$ to build the K3 and the second one is constant. This will provide 20 perturbative Abelian gauge multiplets $A_\mu$ already in six dimensions from which one can build the magnetic F-fluxes $\int_{\mathbb{P}^1_b} dA_\mu$ on $\mathbb{P}^1_b$.

On the other hand, in the heterotic string one can also have fluxes on the $\mathbb{T}^2_c$; then one needs to have a gauge field in six dimensions from a compactification of the heterotic string on K3, i.e. in compactification scheme B one compactifies from ten to six dimensions on K3 and then from six to four dimensions on a $\mathbb{T}^2_c$. In order to have Abelian gauge fields in six dimensions, the moduli of the gauge bundle on K3 must be tuned to specific values.

On the dual type IIA side we will deal with a Calabi-Yau space $M$ which is a K3-fibration [26, 27] over the same base $\mathbb{P}^1_b$. Using the adiabatic principle we will be able to completely
map the IIA RR H-fluxes with support on $\mathbb{P}^1_b$ to the magnetic F-fluxes on the heterotic side, again on the ‘same’ $\mathbb{P}^1_b$. Note that it was argued already in [1] that the IIA 6-flux on $M$, corresponding to $e_0$, is mapped under the six-dimensional string-string duality to a magnetic field on the heterotic torus. We will find however that the type IIA H-fluxes investigated in [3] and [7] are mapped to heterotic fluxes on $\mathbb{P}^1_b$, cf. [10]. However the heterotic F-fluxes on the $\mathbb{T}^2_c$ are of a different nature and cannot be simply transferred to the type IIA side. In the other direction the type IIA H-fluxes on $K3_{IIA}$ have no simple heterotic interpretation. These H-fluxes lead to a massive IIA supergravity theory in six dimensions, for which the heterotic/type IIA duality apparently does not work [28]. Finally, in type IIB compactifications there are also fluxes from the NS 3-form field $H^{(3)}_{NS}$. These H-fluxes are already difficult to understand in type IIA [8]. Also in the heterotic models these fluxes have no obvious interpretation.

The paper is organized as follows. In section 2.1 we review the $\mathcal{N} = 2$ theory in four dimensions, which emerges from from the heterotic compactification on $K3 \times \mathbb{T}^2_c$. In section 2.2 we collect those facts about the six-dimensional duality that are relevant to the precise $\mathbb{P}^1_b$ flux mapping, which is presented in section 3. In particular the identification of the fluxes can be done by comparing the Chern-Simons terms, as discussed in 3.1. We discuss in some detail the four-dimensional terms induced in the superpotential from the presence of non-trivial heterotic F-fluxes 3.2. The heterotic $\mathbb{T}^2_c$ fluxes are discussed in section 4. In section 5 we discuss the possible emergence of tachyons due to fluxes. The paper ends with some conclusions.

2 Type II/heterotic string duality without fluxes

2.1 Heterotic strings on $K3 \times \mathbb{T}^2_c$ and type IIA on $M$

Let us recall the spectrum of the $E_8 \times E_8$ heterotic string compactified on $K3 \times \mathbb{T}^2_c$ (for a review on $\mathcal{N} = 2$ string compactifications see [23, 30, 31, 32]). It consists out of the $\mathcal{N} = 2$ gravity multiplet, containing the graviphoton field $\gamma$, $N_V$ vector multiplets $z^A$ and $N_H$ hyper multiplets $q_i$. The following three $U(1)$ vector fields do not depend on the specific gauge bundle of $K3$ and are therefore universal. The heterotic dilaton is the scalar component in a vector multiplet $S$. Further the complexified Kähler class and the complex structure of $\mathbb{T}^2_c$ are scalar components in the vector multiplets $T$ and $U$.

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In [8] it was argued that the type IIB $H^{(3)}_{NS}$-fluxes might correspond to NS 4-form fluxes in the dual type IIA models.
In addition there are in general 20 hyper multiplets, namely the moduli of $K3$, one of these contains the volume of the base $\mathbb{P}_1^1$ and will be denoted $\text{vol}(\mathbb{P}_1^1)_{het}$. In fact, one can consider situations, i.e. gauge bundles, where on the vector side only the fields $\gamma$, $S$, $T$ and $U$ are present ($N_V = 3$). The corresponding classical (i.e. $S \to \infty$) $\mathcal{N} = 2$ prepotential $\mathcal{F}$ then takes the form:

$$
\mathcal{F} = -i(X^0)^2 S T U,
$$

where $S = -iX^1/X^0$, $T = -iX^2/X^0$ and $U = -iX^3/X^0$ and $X^0$ corresponds to the graviphoton. This prepotential determines the gauge couplings of the $U(1)^4$ gauge group via these general formulas:

$$
\mathcal{L}^{\text{gauge}} = -\frac{i}{8} \left( N_{IJ} F^{IJ}_{\mu\nu} F^{\mu\nu J} - \tilde{N}_{IJ} F^{-IJ}_{\mu\nu} F^{-\mu\nu J} \right),
$$

where $F^{IJ}_{\mu\nu}$ ($I = 1, \ldots, N_V$) denote the self-dual and anti-self-dual electric field-strength components and

$$
N_{IJ} = \tilde{\mathcal{F}}_{IJ} + 2i \frac{\text{Im}(\mathcal{F}_{IK})\text{Im}(\mathcal{F}_{IL})X^K X^L}{(\mathcal{F}_{KL})X^K X^L}. 
$$

Hence $N_{IJ}$ is the field-dependent tensor that comprises the inverse gauge couplings $g_{IJ}^{-1} = \frac{i}{4} (\tilde{N}_{IJ} - \tilde{\mathcal{N}}_{IJ})$ and the generalized $\theta$ angles $\theta_{IJ} = 2\pi^2 (N_{IJ} + \tilde{\mathcal{N}}_{IJ})$. We can also define magnetic field strength tensors $G^{+\mu\nu}_{IJ}$ as

$$
G^{+\mu\nu}_{IJ} = N_{IJ} F^{+\mu\nu}_{IJ}, \quad G^{-\mu\nu}_{IJ} = \tilde{N}_{IJ} F^{-\mu\nu}_{IJ}.
$$

The duality group acts on the field strength vectors $(F^{+\mu\nu}_{IJ}, G^{+\mu\nu}_{IJ})$ as $Sp(2N_V + 2, \mathbb{Z})$ transformation

$$
\tilde{F}^{+\mu\nu}_{IJ} = U^I_{J} F^{+\mu\nu}_{IJ} + Z^{IJ} G^{+\mu\nu}_{IJ}, \\
\tilde{G}^{+\mu\nu}_{IJ} = W_{IJ} F^{+\mu\nu}_{IJ} + V_{IJ} G^{+\mu\nu}_{IJ}
$$

and similarly on the period vector $(X^I, \mathcal{F}_I)$

$$
\hat{X}^I = U^I_{J} X^I + Z^{IJ} \mathcal{F}_J, \\
\hat{\mathcal{F}}_I = W_{IJ} X^J + V_{IJ} \mathcal{F}_J.
$$

This induces the following transformations on the gauge coupling constants

$$
\tilde{N}_{IJ} = (V_I^K \mathcal{N}_{KL} + W_{IL})[(U + Z\mathcal{N})^{-1}]_J^L.
$$

From the heterotic string theory we know that all the physical low-energy couplings become weak in the large-dilaton limit, which suggests that the strongly-coupled $F^{+S}_{\mu\nu}$
field strength in the dilaton $\mathcal{N} = 2$ superfield should be replaced with its dual (which is weakly coupled in the large-dilaton limit). In $\mathcal{N} = 2$ terms, this is achieved by the symplectic transformation $\hat{X}^1 = \mathcal{F}_1$, $\hat{\mathcal{F}}_1 = -X^1$. The gauge couplings in the new basis is

$$\hat{\mathcal{N}}_{IJ} = -2i\hat{S}_{IJ} + 2i(S + \bar{S})\eta_{IK}\eta_{JL}(\hat{z}^K\hat{\bar{z}}^L + \bar{z}^K\hat{z}^L).$$

Note that now all $\text{Im} \hat{\mathcal{N}}_{IJ}$ are proportional to $S + \bar{S}$ and hence all the gauge couplings become weak in the large-dilaton limit. Among the duality transformations (2.11) acting on $(\hat{X}^I, \hat{\mathcal{F}}_I)$ we distinguish the perturbative T-dualities, which do not mix $\hat{X}^I$ with $\hat{\mathcal{F}}_I$, in contrast to the non-perturbative electro-magnetic S-duality transformations, which do mix $\hat{X}^I$ with $\hat{\mathcal{F}}_I$ (see [33, 34, 35] for more details).

On the dual type IIA side the massless spectrum is determined by the cohomology of the three-fold $\mathcal{M}$, namely by the two Hodge numbers $h^{(1,1)}$ and $h^{(2,1)}$. Specifically, the number of vector multiplets is $N_V = h^{(1,1)}$. Their $h^{(1,1)}$ complex scalar moduli in the NS-NS sector correspond to the deformations of the Kähler form $J$ of $\mathcal{M}$ plus the internal $B_{MN}$ fields; the $h^{(1,1)}$ $U(1)$ R-R vectors originate from the ten-dimensional 3-form gauge potential $A_{MNP}$ with two indices in the internal space. Second, there are $N_H = h^{(2,1)} + 1$ massless $\mathcal{N} = 2$ hyper multiplets. $h^{(2,1)}$ of them correspond to the complex structure deformations of $\mathcal{M}$, where the two additional R-R scalar degrees of freedom, needed to fill an $\mathcal{N} = 2$ hyper multiplet, come from $A_{MNP}$ with all indices in the internal direction. The additional hyper multiplet contains together with the NS-NS axion field $a$ the four-dimensional dilaton $e^{-2\phi_{\text{IIA}}}$ plus two more R-R scalar fields.

As stated already we assume that $\mathcal{M}$ is $K3_{\text{IIA}}$ fibered as well as elliptically fibered by the elliptic fibre $f$ (which is also the elliptic fibre of the elliptically fibered $K3_{\text{IIA}}$). In accordance with our earlier assumption that on the heterotic side there are just the vector multiplets $S, T, U$ and $\gamma$ present, we consider $\mathcal{M}$ to be one of the three $(3,243)$ CY’s over the Hirzebruch surfaces $F_0, F_1, F_2$ related to the heterotic instanton numbers $(12,12), (11,13), (10,14)$ [36]. The first of these is elliptically fibred over $\mathbb{P}^1_b \times \mathbb{P}^1_f$ and has the further heterotic/heterotic duality related to the exchange of the two $\mathbb{P}^1$’s, the second one was studied in connection with the five-brane transition and its geometric counterpart of blowing down a del Pezzo surface lying in the elliptic fibration over $\mathbb{P}^1_b$, and finally the last one is the Calabi-Yau hypersurface of degree 24 in the weighted projective space $\mathbb{P}_{1,1,2,8,12}$. Let us denote the base and fibre $\mathbb{P}^1$ of the base surface $F_n$ respectively by $\mathbb{P}^1_b$ and $\mathbb{P}^1_f$. Having Hodge numbers $h^{(1,1)} = 3$ there are just three Kähler moduli, corresponding to the volumes of $\mathbb{P}^1_b, \mathbb{P}^1_f$ and the elliptic fibre $\mathbb{T}^2_f$, respectively. Via the heterotic/type IIA string duality these fields can be mapped to the heterotic fields.
$S$, $T$, and $U$ as follows:

\[ S \sim \text{vol}(\mathbb{P}^1_{I1A}), \quad T \sim \text{vol}(\mathbb{T}^2_{IIA}), \quad U \sim \text{vol}(\mathbb{P}^1_{I1A}). \tag{2.13} \]

Note that this is the correct identification for the model with instanton number (12,12) which corresponds to the Hirzebruch base $F_0$. For $F_1$ and $F_2$ the identification of $\text{vol}(\mathbb{P}^1_{I1A})$ is slightly different, e.g. for $F_2$ one has $U - T \sim \text{vol}(\mathbb{P}^1_{I1A})$. On the other hand, the type II dilaton, which is inside the universal hyper multiplet, is mapped to the volume of the heterotic $\mathbb{P}^1_{I}$:

\[ e^{-2\phi_{IIA}} \sim \text{vol}(\mathbb{P}^1_{I1})_\text{het}. \tag{2.14} \]

### 2.2 The six-dimensional duality

Following [37] and [38] we compare the actions in six dimensions for the heterotic string on $T^4$ and the type IIA string on $K3$. Consider first the heterotic string on $T^4 = T^2_f \times T^2_c$. Then the vector fields $\gamma$, $S$, $T$ and $U$ are already present in six dimensions. They correspond to the left- and right-moving gauge group $U(1)^2_L \times U(1)^2_R$ which is associated to the torus $T^2_c$. In addition there are 4 $U(1)$ vectors from $T^2_f$ and 16 from the gauge group in ten dimensions. Including the Chern-Simons terms in the field strength of the heterotic $B$ field,

\[ H_{\mu \nu \rho} = (\partial_{\mu} B_{\nu \rho} + 2 A^I_{\mu} L_{IJ} F^J_{\nu \rho}) + \text{cyclic}, \tag{2.15} \]

one finds for the heterotic string action

\[ S_{\text{het}} \sim \int d^6x \sqrt{-g_{\text{het}}} e^{-\phi_{\text{het}}} \left( R_{\text{het}} + (\partial_{\mu} \phi_{\text{het}})^2 - \frac{1}{2 \cdot 3!} H^2_{\text{het}} \right. \]

\[ \left. + \frac{1}{8} \text{Tr}(\partial_{\mu} M_{\text{het}} L \partial_{\mu} M_{\text{het}} L) - F^I_{\text{het}} (L M_{\text{het}} L)_{IJ} F^J_{\text{het}} \right) \tag{2.16} \]

where $F^I_{\text{het}} (I = 1, \ldots, 24)$ are the 24 abelian gauge field strengths and $M_{\text{het}}$ is the (symmetric $24 \times 24$ matrix valued) scalar field representing an element of the $O(4,20)$ coset with $M_{\text{het}} L M^T_{\text{het}} = L$ for the intersection form

\[ L = \begin{pmatrix} I_4 \\ -I_{20} \end{pmatrix}. \tag{2.17} \]

Note that the $T, U$ fields are not in a diagonal basis but in one with the $H$ intersection form consisting of a hyperbolic plane. Upon further compactification on $\mathbb{P}^1_{I}$ the $U(1)$
gauge fields $A_\mu^g$, $A_\mu^S$, $A_\mu^T$ and $A_\mu^U$ will always be present in four dimensions. Then the matrix $(LM_{het}L)_{IJ}$, which determines the four-dimensional gauge couplings, is given by

$$N_{IJ} = \text{vol}(\mathbb{P}_b^{1/2}(LM_{het}L)_{IJ})$$

(2.18)

where the indices run over the 4-dimensional gauge fields (2.16).

On the other hand one has in the dual type IIA on $K3$

$$S_{IIA} \sim \int d^6 x \sqrt{-g_{IIA}} e^{-\phi_{IIA}} \left( R_{IIA} + (\partial_\mu \phi_{IIA})^2 - \frac{1}{2 \cdot 3!} H_{IIA}^2 \right)$$

$$+ \frac{1}{8} \text{Tr}(\partial_\mu M_{IIA} L \partial_\mu M_{IIA} L) - F_{IIA}^I (LM_{IIA} L)_{IJ} F^J_{IIA} - \frac{1}{4} B \wedge F_{IIA} \wedge L F_{IIA}^T$$

(2.19)

with $M_{IIA}$ the symmetric moduli matrix with $M_{IIA} L M_{IIA}^T = L^{-1}$ for the lattice intersection form

$$L = \begin{pmatrix} -H & 0 \\ 0 & d_{ij} \end{pmatrix}$$

(2.20)

Here $d_{ij} = \int_{K3} \omega^i \wedge \omega^j$ $(i, j = 1, \ldots, 22)$ denotes the intersection matrix on the middle cohomology (the $\omega^i$ an integral basis of harmonic two-forms).

Naively the field strength vector is given as ($C_i$ is a basis of two cycles)

$$F^I_{IIA} = \begin{pmatrix} F_2 \\ J_2 \\ K^i_2 \end{pmatrix} = \begin{pmatrix} H_R^{(2)} \\ \int_{K3} H_R^{(6)} \\ \int_{C_i} H_R^{(4)} \end{pmatrix}$$

(2.21)

Here $F_2 = H_R^{(2)} = dA_1$ is the Abelian field strength of the Ramond-Ramond 1-form $A_1$, $H_R^{(6)}$ is the dual Abelian field strength of the Ramond-Ramond 3-form $C^{(3)}$ with $H_R^{(4)} = dC_3$ ($C^{(3)}$ is dual to a vector in six dimensions) and the 22 field strengths $K^i_2 := dC^i_1$ correspond to $dC^{(3)}$ where two internal indices are on $K3$. However, to obtain closed gauge invariant field strengths one has to implement the following axionic shifts [38]

$$\hat{J}_2 = J_2 - J_2^i a^i d_{ij} + \frac{1}{2} F_2 a^i a^j d_{ij}$$

$$\hat{K}^i_2 = J_2^i - F_2 a^i = K^i_2 - d(A_1 a^i).$$

(2.22)

We denote by $a^i$, $C^i_1$ and $J_2^i = K^i_2 + A_1 a^i$ the parts associated with $\omega^i$ in the mode decomposition for $B_2$, $C_3$ and $J_4 = H_R^{(4)} + A_1 \wedge H_{NS}^{(3)}$ respectively. Regarding $F_2, \hat{J}_2, J_2$ as
candidate abelian field strengths note that neither $J^i_2$ nor $\tilde{J}_2 := \int_{K^3} * J_4$ are closed: instead one has $d J^i_2 = F_2 da^i$ and $d \tilde{J}_2 = J^i_2 a^i d_{ij}$ (the latter from the field equation $d * J_4 = H_3 \wedge J_4$ for $J_4$ coming from the CS-term).

Hence the IIA field strength vector is given by the following object:

$$F^I_{IIA} = \begin{pmatrix} F_2 \\ \hat{J}_2 \\ \tilde{K}^i_2 \end{pmatrix} = \begin{pmatrix} \int_{K^3} H_R^{(6)} + \int_{K^3} (A_1 \wedge H^{(3)}_{NS}) - J^i_2 a^i d_{ij} + \frac{1}{2} F_2 a^i a^j d_{ij} \\ \int_{C_i} H_R^{(4)} - d(a_1 a^i) \end{pmatrix} . \quad \text{(2.23)}$$

Note that with regard to the $\mathbb{P}^1_b$ fluxes we can use the naive field strength vector (2.21).

Now we can identify the type IIA $U(1)$ gauge fields with their dual heterotic counterparts. We will first focus on the $\gamma, S, T, U$ part of the gauge group with corresponding intersection form $H \oplus H$. The relevant part of the $K3_{IIA}$ integral cohomology lattice consists in one hyperbolic plane $H$ from $H^{1,1}(K3_{IIA}) = H \oplus H \oplus E_8 \oplus E_8$ related to the section (base) $\sigma$ (which is $\mathbb{P}^1_f$) and fibre $f$ of the $K3_{IIA}$ which we assume to be elliptically fibered and another hyperbolic plane from $H^0(K3_{IIA}) \oplus H^4(K3_{IIA})$. For the elliptic Calabi-Yau with Hirzebruch base $F_0$ the intersection matrix of $\sigma$ and $f$ is indeed in the hyperbolic form. (For the Calabi-Yau hypersurface of degree 24 in $\mathbb{P}_{1,1,2,8,12}$, which is an elliptic fibration over $F_2$, the corresponding intersection matrix $H$ is only reached for the basis consisting of $\sigma + f$ and $f$). This structure $H \oplus H$ of signature $(2, 2)$ is related to the coset $O(2,2)/2 \times O(2,2)$ and is realized heterotically in the sector $U(1)_I^2 \times U(1)_R^2$ which is associated to the $\mathbb{T}_c^2$ sector of the heterotic compactification space.

Now utilizing the six-dimensional string/string duality one has the following associations of the $\gamma, S, T, U$ sector (for the Calabi-Yau with base $F_0$)

$$F^\gamma_{het} \leftrightarrow F_2, \quad F^S_{het} \leftrightarrow \hat{J}_2, \quad F^T_{het} \leftrightarrow \tilde{K}^f_2, \quad F^U_{het} \leftrightarrow \tilde{K}^\sigma_2 . \quad \text{(2.24)}$$

Note that this identification already holds in six dimensions and of course persists upon further compactification on $\mathbb{P}^1_b$.

In four dimensions, one gets analogous assertions for the electro-magnetic dual gauge field strengths $G^\gamma_{het}, G^S_{het}, G^T_{het}, G^U_{het}$: they correspond to the (Hodge) dual forms on the (Poincare) dual cycles, i.e. in terms of the naive field strengths

$$G^\gamma_{het} \leftrightarrow \int_M H_R^{(8)}, \quad G^S_{het} \leftrightarrow \int_{\mathbb{P}^1_b} H_R^{(4)}, \quad \text{(2.25)}$$

$$G^T_{het} \leftrightarrow \int_{\mathbb{P}^1_b \times \sigma} H_R^{(6)}, \quad G^U_{het} \leftrightarrow \int_{\mathbb{P}^1_b \times f} H_R^{(6)}. \quad \text{(2.26)}$$
In $\mathcal{N} = 2$ supergravity, these electric/magnetic duality transformations, $A \leftrightarrow \tilde{A}$, are given in terms of the symplectic transformations $X^I \leftrightarrow \mathcal{F}_I$, as discussed before.

3 Fluxes on $\mathbb{P}^1$

After the discussion of the well established type II/heterotic string duality without fluxes let us now investigate whether this duality survives turning on fluxes on the type II and on the heterotic side. We start considering fluxes on the 2-sphere $\mathbb{P}^1_b$. Recall that we have identified the relevant six-dimensional gauge fields. Let us now map the fluxes. For that purpose we compactify further down to four dimensions on $\mathbb{P}^1_b$, the base over which the six-dimensional space is fibred. This gives $K3_{\text{het}} \times \mathbb{T}_c^2$ on the heterotic side or the Calabi-Yau space $M$ on the type IIA side, respectively. The internal fluxes $e_I$ will then be simply given as the internal F-fields integrated over the base $\mathbb{P}^1_b$:

$$e_I = \int_{\mathbb{P}^1_b} F_I.$$  \hspace{1cm} (3.27)

Here the $F_I$ correspond to six-dimensional field strengths, where the index $I$ ranges over those fields which survive the compactification to four dimensions. In addition to the $e_I$ we also like to introduce the ‘magnetic’ fluxes $m^I$. Together with the $e_I$ they build, in analogy to the magnetic/electric field strength tensors $(G_I, F^I)$, a symplectic vector $(e_I, m^I)$. On the type IIA side the $m^I$ will correspond to certain forms integrated over certain cycles inside the Calabi-Yau, namely those which do not contain $\mathbb{P}^1_b$, i.e. those inside $K3_{\text{IIA}}$, which do not have an interpretation as fluxes of any six-dimensional gauge fields. Accordingly, we will see that also on the heterotic side the $m^I$ are not just fluxes of such six-dimensional gauge field strengths.

As stated in the introduction, we will be interested in two types of terms in the Lagrangians: those giving the scalar potential $V$ of the superpotential $W$ induced by the fluxes (these are given by the kinetic terms shown below), and those giving the relation to induced Green-Schwarz couplings; these are essentially given by Chern-Simons terms.

3.1 Green-Schwarz terms, flux mapping and chiral anomalies

We will now compare the four-dimensional couplings of the (reduced) six-dimensional gauge fields with the derivative of the type IIA (model-independent) axion $a_{\text{IIA}}$ respectively the heterotic axion modulus $a_{\mathbb{P}^1_b}$ related to $\mathbb{P}^1_b$. Note that just as one has in string/string duality the well known association between $1/g_{\text{het}}^2$ and $\text{vol}(\mathbb{P}^1_{\text{IIA}})$, one
has equally the association of $1/g^2_{IIA}$ with $\text{vol}(\mathbb{P}^1_b \text{het})$ (see eqs. (2.13) and (2.14)) which leads to the corresponding association of the universal IIA B-field $B_{IIA}$ and the internal heterotic axion $a_{\mathbb{P}^1_b}$:

$$a_{IIA} \leftrightarrow a_{\mathbb{P}^1_b}. \quad (3.28)$$

The coupling constants are then proportional to H-fluxes of type IIA RR field strength or to F-fluxes of heterotic $U(1)$ gauge fields on the common base $\mathbb{P}^1_b$. By identifying the dual gauge fields on both sides we can thus map the fluxes.

Consider the CS-improved six-dimensional kinetic terms of the heterotic NS 2-form $B$. One has with $H_{het} = dB - \omega_{CS}^Y - \ldots$ that $H^2_{het}$ contains the term $dB \cdot (A^I \wedge dA^J L_{IJ})$ for any six-dimensional gauge field $A^I$. With the axion field in $B = a_{\mathbb{P}^1_b} \cdot J_{\mathbb{P}^1_b} + \ldots$ this leads to the four-dimensional heterotic Green-Schwarz couplings ($e_I = \int_{\mathbb{P}^1_b} dA_I$)

$$\mathcal{L}_{GS}^{het} = e_\gamma B_{\mathbb{P}^1_b} \wedge F^\gamma_{het} + e_s B_{\mathbb{P}^1_b} \wedge F^s_{het} + e_T B_{\mathbb{P}^1_b} \wedge F^U_{het} + e_U B_{\mathbb{P}^1_b} \wedge F^T_{het} \quad (3.29)$$
as $(T, U)$ are non-diagonal w.r.t. $L_{IJ}$.

On the other hand one has on the type IIA side the actual kinetic term $(H^4_R - 2dB \wedge A^{(1)})^2$ which contains the term $H^6_R \wedge B \wedge H^2_R$ furthermore one has the topological term $H^4_R \wedge H^4_R \wedge B$. The first term leads to

$$B_{IIA} \wedge \hat{J}_2 \int_{\mathbb{P}^1_b} H^2_R + B_{IIA} \wedge F_2 \int_M H^6_R, \quad (3.30)$$

whereas the second term provides

$$B_{IIA} \wedge \hat{K}_2 d_{ij} \int_{\mathbb{P}^1_b \times C_i} H^4_R. \quad (3.31)$$

For the $S-T-U$ model this leads to the following four-dimensional type IIA Green-Schwarz couplings with precoefficients involving 2-, 4-, 6-flux

$$\mathcal{L}_{IIA}^{GS} = B_{IIA} \wedge \hat{J}_2 \int_{\mathbb{P}^1_b} H^2_R + B_{IIA} \wedge \hat{K}_2 \int_{\mathbb{P}^1_b \times f} H^4_R$$

$$+ B_{IIA} \wedge \hat{K}_2 \int_{\mathbb{P}^1_b \times \sigma} H^4_R + B_{IIA} \wedge F_2 \int_M H^6_R. \quad (3.32)$$

Now we compare the heterotic, eq.(3.29), and the type IIA, eq.(3.32), Green-Schwarz coupling, and from matching these two effective actions we obtain the following mapping of the fluxes (for the IIA model with Hirzebruch base $F_0$):

$$e_\gamma \leftrightarrow \int_M H^6_R, \quad e_s \leftrightarrow \int_{\mathbb{P}^1_b} H^2_R,$$

$$e_T \leftrightarrow \int_{\mathbb{P}^1_b \times f} H^4_R, \quad e_U \leftrightarrow \int_{\mathbb{P}^1_b \times \sigma} H^4_R, \quad (3.33)$$
of course in consistency with the $\mathbb{P}^1_b$-integrated version of (2.24).

Note that the flux $e_S$ corresponds on the type IIA side to the vacuum expectation value of the 2-form $H^{(2)}$ over $\mathbb{P}^1_b$. This is in contrast to the fluxes $e_T$ and $e_U$ (and for the further gauge fields - see section (2.4)) which are determined by the vevs of $H^{(4)}$. However this observation is just a reflection of the fact that the weakly coupled heterotic S-gauge field corresponds to the period $\mathcal{F}_1$, as explained before.

Now let us come to the dual magnetic fluxes $m^I$. They correspond in the type IIA theory to the (Hodge) dual forms on the (Poincare) dual cycles. Hence we obtain the following identifications

\[
\begin{align*}
    m^U &\leftrightarrow \int \! H^{(2)}_R, \\
    m^S &\leftrightarrow \int \! H^{(4)}_R, \\
    m^T &\leftrightarrow \int \! H^{(2)}_R, \\
    m^\gamma &\leftrightarrow \int \! H^{(0)}_R .
\end{align*}
\]  

(3.34)

Starting from the ten-dimensional type IIA action with terms $H^{(6)} \wedge B \wedge H^{(2)}_R$ and $H^{(4)}_R \wedge H^{(4)}_R \wedge B$, the dual fluxes generate the following ‘dual’ Green-Schwarz terms in four dimensions:

\[
L^{\text{dual}}_{\text{GS}} = m^S B_{IIA} \wedge G_S + m^U B_{IIA} \wedge G_T + m^T B_{IIA} \wedge G_U .
\]

(3.35)

Note that the still missing GS-coupling

\[
L^{\text{dual}}_{\text{GS}} = m^\gamma B_{IIA} \wedge G_\gamma ,
\]

(3.36)

cannot be derived from the two ten-dimensional terms written above, but requires in ten dimensions a coupling of the form $H^{(8)} \wedge B$. This term is not present in the effective action of the ‘normal’ type IIA superstring, but it precisely arises in the massive IIA theory which was discussed in [39]. Specifically, in the massive IIA theory there is a cosmological constant term, $\Lambda \sim (m^\gamma)^2$, and the 2-form field strength has to be modified in the following way:

\[
H^{(2)}_R = dA_1 + 2m^\gamma B_{IIA} .
\]

(3.37)

Then the term in eq. (3.36) arises in the kinetic term $(H^{(2)}_R)^2$.

Note that there is no integration on the base $\mathbb{P}^1_b$ in any of these integrals (3.34) involving the fluxes $m^I$. Therefore the fluxes $m^I$ already exist in six dimensions as integral, topological numbers on the type IIA side. In fact, decompactifying $\mathbb{P}^1_b$ in type IIA, the fluxes $m^I$ are already present in six dimensional IIA compactification on $K3$. All these fluxes
arise in massive type IIA supergravity on $K3$ due to duality transformations from $m^\gamma$, as it was recently discussed in [28].

However on the heterotic side the interpretation of the magnetic fluxes $m^I$ is not obvious, since the corresponding integrals cannot be defined (but see 3.38 below) Therefore the heterotic/type IIA string-string duality breaks down in the presence of the magnetic fluxes $m^I$. In fact, the problematical point to match these two occurrences of Poincare-duality is the following: the electro-magnetic dual field strengths $G_\gamma$, $G_S$, $G_T$, $G_U$ are defined entirely in four dimensions; this is in manifest contrast to the original gauge fields which are survivors from a dimensional reduction. If one wants to contemplate on $\mathbb{P}_b^1$-fluxes of the electro-magnetic duals they should exist already in six dimensions. In conclusion there are no fluxes of the electro-magnetic duals of the four-dimensional gauge fields. In other words, the four IIA fluxes listed in eq.(3.34) with no support on the $\mathbb{P}_b^1$ cannot be interpreted as heterotic F-fluxes of six-dimensional gauge fields on $\mathbb{P}_b^1$.

Let us briefly discuss the action of the duality symmetries, being certain symplectic transformations, on the fluxes $e_I$ and $m^I$. First, the perturbative, heterotic T-duality transformations act within the fluxes $e_I$ and $m^I$ but do not mix them into each other. For example, the transformation $T \to 1/T$ has the following action on the fluxes: $e_S \leftrightarrow e_U$, $e_\gamma \leftrightarrow e_T$, $m^S \leftrightarrow m^U$, $m^\gamma \leftrightarrow m^T$ as inspection of the scalar potential (3.43) shows. On the other hand, non-perturbative S-duality transformation exchange some of the fluxes $e_I$ with some $m^I$. Therefore, in the heterotic string, the absence of the fluxes $m^I$ breaks these S-duality transformations. In the type IIA string, non-perturbative duality transformations which exchange $e_I$ with $m^I$ are in principle possible and depend on the details of the geometry. For example, as discussed in [20, 40], the $S - T - U$ models discussed here possess a non-perturbative exchange symmetry $S \leftrightarrow T$. In the model with Hirzebruch base $F_0$, this symmetry has a very simple geometric interpretation, namely it just reflects the freedom of exchanging the two $\mathbb{P}_b^1$'s of $F_0$, i.e. $\mathbb{P}_b^1 \leftrightarrow \sigma$. Using the fluxes in eqs.(3.33) and (3.34) the $S \leftrightarrow T$ symmetry has the following natural action on the fluxes:

$$e_T \leftrightarrow m^S, \quad e_S \leftrightarrow m^T, \quad (3.38)$$

and all other fluxes are unchanged. One gets of course similar mappings if one assumes a full triality among the moduli $S, T$ and $U$ (see e.g. [41, 42]).

Now, at the end of this chapter let us discuss the question of anomalies in relation to the $\mathbb{P}_b^1$-fluxes. The net number of chiral fermions, the index of the Dirac operator, which is
proportional to the triangle $U(1)^3$-anomaly is given by

$$n_+ - n_- = \int_{\mathbb{T}_c^2 \times K_{3\text{het}}} \text{ch}(F) \wedge \hat{A}(R), \quad (3.39)$$

where $\hat{A}(R)$ and $\text{ch}(F)$ are the $\hat{A}$ genus of the tangent bundle and the Chern character of the gauge bundle. The index is zero due to the simple fact that on the ‘constant’ $\mathbb{T}_c^2 F$ and $R$ both vanish. This reflects the statement that we can figure the heterotic compactification as a trivial dimensional reduction on a torus after compactifying to six dimensions on $K_{3\text{het}}$, which does not produce any chiral fermions in four dimensions. This is still completely consistent with the appearance of Green-Schwarz couplings. A contribution to the triangle anomaly would require the coupling $a_{\mathbb{P}_b^{1}} F \wedge F$. In general, this term can be obtained from the ten-dimensional GS-term $B \wedge F^4$ via dimensional reduction. However for the case of $\mathbb{P}_b^{1}$-fluxes we obtain

$$\mathcal{L} = a_{\mathbb{P}_b^{1}} F^I \wedge F^I \int_{\mathbb{T}_c^2 \times \mathbb{T}_c^2} F \wedge F, \quad (3.40)$$

which vanishes since $F$ has only support on $\mathbb{P}_b^{1}$.

The gauge symmetry not being anomalous, the GS-term $B_{\mathbb{P}_b^{1}} \wedge F$ nevertheless generates a longitudinal component for the $U(1)$ gauge fields. Since there is only one B-field, which couples to the gauge fields via the GS-term, namely the internal B-field $B_{\mathbb{P}_b^{1}}$ in the heterotic string or the universal B-field $B_{IIA}$ in the type IIA string, only one linear combination of $U(1)$ gauge fields will become massive, where the exact linear combinations depends on the fluxes turned on. In the heterotic string this mass term comes from the ten-dimensional kinetic term of the CS-improved NSNS 3-form field strength, which exists at string tree-level. The generated mass for the $A^I_{\mu}$ is proportional to $e_I^2 / \text{vol}(\mathbb{P}_b^{1})^2$. However using the heterotic/type IIA duality relation $(2.14)$, it follows that these gauge boson mass terms arise only at the one-loop level in the type IIA compactifications.

### 3.2 The superpotential couplings

Turning on non-trivial fluxes induces a potential for the vector multiplet scalar fields $z^A = X^A / X^0$. Quite generically, this scalar potential is lifting the vacuum degeneracy in the vector multiplet moduli space and is also breaking space-time supersymmetry. In the heterotic case the scalar potential originates from the gauge kinetic term eq. $(2.16)$ in six dimensions, which is determined by the scalar field matrix $(LM_{het}L)_{IJ}$. Specifically, the four-dimensional scalar potential $V$ is obtained by replacing in eq. $(2.16)$ the gauge field strength tensors $F^{het}$ by their corresponding internal electric fluxes $e_I$. Using eqs. $(2.8)$,
(2.18) and (3.27) this leads to following expression written in terms of four-dimensional variables:

\[ V = \frac{1}{\text{vol}(\mathbb{P}_1^1)_{\text{het}}} e_I (\text{Im} \hat{N})^{IJ} e_J, \tag{3.41} \]

where the coupling constants \( \hat{N}_{IJ} \) are computed using the period vector \( (\hat{X}^I, \hat{F}_J) \) which was obtained from the exchange \( \hat{X}^I = \mathcal{F}_I, \hat{F}_J = -X^J \). Using the well-known supergravity relation between \( W \) and \( V \), this scalar potential corresponds to the following superpotential

\[ W = e_D X^0 - e_S \frac{\partial \mathcal{F}}{\partial X^1} + e_i X^i, \tag{3.42} \]

where \( S = -iX^1/X^0 \) and the fields \( \phi^i = -iX^i/X^0 \) are the remaining moduli. In heterotic perturbation theory, the prepotential \( \mathcal{F} \) depends only linearly on \( S \), and hence \( W \) does not depend on \( S \); therefore the \( S \)-field is not fixed by the minimization condition \( W = 0 \) and \( d_i W = 0 \) in perturbation theory. Only non-perturbatively, after instanton corrections in \( e^{-S} \), \( \mathcal{F} \) and hence also \( W \) will depend on \( S \), which will then be fixed by the minimization conditions. For the case with only four vector fields \( A_\gamma, A_S, A_T \) and \( A_U \) with corresponding prepotential eq.(2.12), the couplings \( \hat{N}^{IJ} \) are easily computed using eq.(2.12), and we get the following heterotic scalar potential (for real values of the moduli):

\[ V = \frac{1}{\text{vol}(\mathbb{P}_1^1)_{\text{het}}} \left( (e_I)^2 + (e_S)^2 T^2 U^2 + (e_T)^2 T^2 + (e_U)^2 U^2 \right), \tag{3.43} \]

which corresponds to the following superpotential cf. [7]:

\[ W = e_D X^0 + e_S T U + i e_T T + i e_U U. \tag{3.44} \]

On the type IIA side the flux induced scalar potential originates from the kinetic terms \( (H^{(n)}_R)^2 \) in ten dimensions. Alternatively we can also use the derivation [3] of the scalar potential in the type IIB mirror compactification. There the scalar potential comes from the term \( H_R^{(3)} \wedge *H_R^{(3)} \) in the ten-dimensional effective action. After dimensional reduction the following expression is obtained [3]:

\[ V = -(2\text{Im} \tau)^{-1} \left( m^I (\text{Im} \mathcal{N})_{IJ} m^J + (e_I + m^J \hat{N}_{IJ}) (\text{Im} \mathcal{N})^{IK} (e_K + m^J \hat{N}_{JK}) \right). \tag{3.45} \]

The \( e_I \)-part of eq.(3.44). In type IIA, \( V \) can be derived from the following superpotential

\[ W = \sum_{n=0}^{3} \int_W H_R^{(2n)} \wedge J^{3-n} = e_I X^I - m^I \mathcal{F}_I. \tag{3.46} \]
With the six-dimensional mode decompositions $H^{(4)}_R = H^i_2 \wedge \omega_i$ (apart from contributions with a 0- respectively 4-form on $K3_{IIA}$) and $J = X^i \cdot \omega_i$ (where $X^i = a^i + i \text{vol}(C^i_2)_{IIA}$) on $K3_{IIA}$ one then obtains that

$$\int_M H^{(4)}_R \wedge J = X^i d_{ij} \int_{P^1_b} H^i_2 = X^j d_{ij} \int_{P^1_b \times C^i_2} H^{(4)}_R = e_i X^j d_{ij}$$

(3.47)

and furthermore

$$e_0 = \int_M H^{(6)}_R = e_\gamma .$$

(3.48)

$H^{(2)}_R \wedge \frac{1}{2} J^2$ contains the contribution

$$m^1 \int_{K3_{IIA}} \frac{1}{2} J^2 = \int_{P^1_b} H^{(2)}_R \int_{K3_{IIA}} \frac{1}{2} J^2 = e_S \frac{\partial F}{\partial X^1} .$$

(3.49)

So in total one gets the following superpotential

$$W = e_i X^j d_{ij} + e_0 X^0 - m^1 \frac{\partial F}{\partial X^1}$$

$$= ie_T T + ie_U U + e_\gamma + e_S TU ,$$

(3.50)

where we have used the prepotential $F$ in eq.(2.3) and $X^0 = 1$ in the second line of this equation. The agreement of this equation with heterotic superpotential eq.(3.44) shows how the superpotential can be directly transported from type IIA to the heterotic side, whereas above we compared these terms by explicitly evaluating the associated scalar potentials on both sides independently.

### 3.3 Further gauge fields

It is straightforward to extend these associations to the case that one has heterotically not only the $T$ and $U$ fields from the $g$ and $B$ sector but also further gauge fields, called $V$. So if one of the 16 $U(1)$’s survives the fibration down to 4 dimensions one can consider its associated flux. Such a model will be dual to type IIA on a Calabi-Yau three-fold with a larger $h^{(1,1)}$ reflecting the enhanced number of vector multiplets \[36, 43\]. Its $K3_{IIA}$ fibre has a correspondingly higher Picard number which indicates the existence of at least one further (beyond the $\sigma$ and $f$) algebraic 2-cycle $C^2_V$ whose cohomology class in the $K3_{IIA}$ is an integral (1,1) class and which exists generically in all $K3_{IIA}$ fibers. This gives a new 2-cycle for the Calabi-Yau and a new 4-cycle $P^1_c \times C^2_V$ from the adiabatic extension over the base $P^1_b$. Of course, the Wilson lines then correspond to the Cartan sub lattice of the $E_8 \oplus E_8$ part of the $K3_{IIA}$ middle cohomology.
The heterotic gauge field (existing already in six dimensions) with field strength $F_{het}^V$ maps to the the RR 3-form $C^{(3)}$ reduced on the new 2-cycle $C_{het}^2$, i.e.

$$F_{het}^V \leftrightarrow F_{IIA}^V = \int_{C_{het}^2} H_R^{(4)} ,$$

(3.51)

whereas the electro-magnetic dual field strengths $G_{het}^V$ (existing only in four dimensions) corresponds again to the dual 5-form $C^{(5)}$ on the Poincare dual 4-cycle $C_4$, i.e.

$$G_{het}^V \leftrightarrow G_{IIA}^V = \int_{C_4} H_R^{(6)} .$$

(3.52)

Given these identifications of the heterotic versus type IIA $U(1)$ field strengths one has immediately the following associations of fluxes on $\mathbb{P}_b^1$

$$e_V = \int_{\mathbb{P}_b^1} F_{het}^V \leftrightarrow \int_{\mathbb{P}_b^1 \times C_{het}^2} H_R^{(4)} ,$$

$$m^V \leftrightarrow \int_{(\mathbb{P}_b^1 \times C_{het}^2)^\perp} H_R^{(2)} .$$

(3.53)

4 $\mathbb{T}_2$-Fluxes

We now turn to heterotic fluxes on the constant torus $\mathbb{T}_2$. Therefore, we figure the compactification space $K3 \times \mathbb{T}_2$ to be build up in the other order B: One decompactifies first to six dimensions on $K3$ and then further to four dimensions on $\mathbb{T}_2$. Any abelian gauge field $A$ that survives the compactification on K3 may also be endowed with a flux $f$ on $\mathbb{T}_2$,

$$f = \int_{\mathbb{T}_2} F \in \mathbb{Z} .$$

(4.54)

There is no straightforward way to interpret these fluxes on the dual type IIA side, but we would like give some indications.

First recall that for the heterotic $\mathbb{P}_b^1$ fluxes in eq.(3.27) the vacuum expectation value of the internal magnetic field is inversely proportional to the $\mathbb{P}_b^1$-volume,

$$F_{\mathbb{P}_b^1} \sim \frac{1}{\text{vol}(\mathbb{P}_b^1)_{het}} .$$

(4.55)

On the type IIA side vol$(\mathbb{P}_b^1)_{het}^{-1}$ corresponds to the square of the type II coupling constant (see eq.(2.14)), such that

$$F_{\mathbb{P}_b^1} \sim g_{IIA}^2 .$$

(4.56)
Using the same arguments the heterotic $T^2_c$ fluxes scale like

$$F_{T^2_c} \sim \frac{1}{\text{vol}(T^2_c)_{het}}, \quad (4.57)$$

and keeping the overall volume of $K3 \times T^2_c$ as well as the volume of $T^2_f$ fixed we arrive at

$$F_{T^2_c} \sim \text{vol}(\mathbb{P}^1_{b})_{het} \sim 1/g_{IIA}^2. \quad (4.58)$$

So in the dual type II description these fluxes are large for weak coupling, i.e. they are of non-perturbative nature. A similar feature is encountered in type IIB Calabi-Yau compactifications in the presence of NS-fluxes, where the flux vectors are $e = e_R + \tau e_{NS}$, $m = m_R + \tau m_{NS}$, which originate from vacuum expectation values of $H^{(3)} = \tau H_{NS}^{(3)} + H_R^{(3)}$.

As opposed to the earlier case of the $\mathbb{P}^1_b$ fluxes we now expect to obtain a spectrum of chiral fermions in four dimensions together with an anomalous contribution to the chiral gauge anomaly. The index theorem for the Dirac operator relates the net number of chiral fermions to the magnetic flux on the torus

$$n_+ - n_- = \int_{K3} \hat{A}(R) \int_{T^2_c} F \sim f \int \text{tr} R^2. \quad (4.59)$$

But now we also face the presence of appropriate Green-Schwarz couplings to cancel the anomaly. The kinetic term of the NSNS 3-form $H_{het}$ decomposes into

$$H_{het} \wedge \ast H_{het} = 2dB \wedge \ast (AdA) + AdA \wedge \ast (AdA) + \cdots \quad (4.60)$$

The second term is the term that gives a positive mass to the particular gauge boson via $f^2 A_{\mu}A^{\mu}$. Note that this mass square is proportional to the square of the flux. The first term is a contribution to the Green-Schwarz coupling $B \wedge F$, which simply is of the following form in four dimensions:

$$\mathcal{L}_{GS} = f B_{T^2_c} \wedge F. \quad (4.61)$$

Via the index theorem, the coupling constant is proportional to the net number of chiral fermions, thus appropriate to cancel the anomaly induced by the triangle diagrams with fermion loops. To complete the Green-Schwarz couplings needed for the anomalous tree diagram the ten-dimensional term $B \wedge F^4$ leads in four dimensions to a coupling

$$\mathcal{L} = a_{T^2_c} F \wedge F \int_{K3} F \wedge F, \quad (4.62)$$

\footnote{In \cite{8} it was argued that the $H^{(3)}_{NS}$ fluxes correspond on the type IIA side to fluxes of an NS 4-form field strength.}
which is non-vanishing and independent of the flux. Using the Bianchi identity
\[ \int dH_{\text{het}} = \int (\text{tr} F^2 - \text{tr} R^2) = 0 \] (4.63)
the total GS coupling is proportional to the index and the two terms precisely surface to cancel the non-vanishing $U(1)$ anomaly.

5 Tachyons due to fluxes

It is well known that it can lead to unstable vacua when Yang-Mills gauge theories are compactified to lower dimensions with magnetic fluxes on the internal space \([44]\). The internal components of charged gauge bosons become scalars whose masses are modified due to their coupling to the internal magnetic field and thus may become negative eventually. Then the vacuum shifts to a new groundstate with a condensate of these tachyons. The analysis can be performed on purely field theoretical grounds by considering six-dimensional Yang-Mills theory compactified to four dimensions. It equally applies to both, the type IIA and heterotic string models.

First let us discuss tachyons due to $P^1$-fluxes. Later we will consider this mechanism in the more familiar case of a toroidal compactification of gauge fields with magnetic flux. There we will also explicitly show that the tachyon potential that derives from the gauge kinetic term $\text{tr} F^2$ is stabilized by a quartic term, the entire potential taking the form of a D-term. But, at first sight surprising, tachyons can also appear for a compactification on a $P^1_b$. One might expect that the absence of harmonic 1-forms, which implies the absence of massless scalars in the dimensional reduction of a gauge field on $P^1_b$, also excludes their appearance when magnetic flux is present on the sphere. However, the analysis of the spectrum of the Laplacian on a sphere acting on internal components of charged vector fields shows that it indeed does have negative eigenvalues as well \([45, 46]\). Formally, one needs to consider sections in the tangent bundle twisted by the flux, which may be trivial even if the tangent bundle itself is not. The flux being a discrete parameter the existence of tachyonic modes is not in contradiction with being massive Kaluza-Klein excitations in its absence. An important feature is that the modification of their masses is essentially given by $\delta M^2 \sim sqF$, $s$ denoting the internal spin, $q$ the charge and $F$ the flux, a linear dependence on the flux $F$. Hence, the spacing of mass levels of KK states and the shift induced by the flux is comparable, both being proportional to $1/\text{vol}(P^1_b)$. While this explains how the mass shift can let the massive KK modes jump to negative eigenvalues, it also raises the general question if the approach based on the effective action of fields massless in the absence of flux is appropriate at all. The two scales being equal one would
have to include all higher KK modes from the beginning, because all of them could come down to zero mass when coupling to appropriate internal fluxes. Thus, the starting point of our analysis appears to be ruined. This problem does not occur as long as we consider only abelian gauge symmetries without charged gauge bosons.

Another more exotic situation where charged scalar fields have negative masses can be met at very special points of the moduli space. When the compactification space degenerates and certain cycles shrink to zero size, black holes may condense and need to be included in the effective action even at small coupling. Let us look at the well known example of the conifold singularity of type IIB, which occurs at the co-dimension one locus, where in the Calabi-Yau space a cycle \(A_1\) with the topology of \(S^3\) vanishes, while the remaining cycles stay finite. More precisely the Calabi-Yau space \(M\) exhibits a nodal singularity, i.e. it is described locally by the eq. \(\sum_{i=1}^{4} \epsilon_i = \mu\). For \(\mu \to 0\) the real part of this local equation describes the vanishing \(S^3\). In the vicinity of a conifold point, \(X^1 = \int_{A_1} \Omega \to 0\), an additional hypermultiplet, the ground state of a singly wrapped 3–brane around the \(A_1\), with mass proportional to \(|X^1|\) becomes light \([47]\). It is charged with respect to the \(U(1)^{N_V}\) gauge symmetry of the vector multiplets. It corresponds to a magnetic monopole or dyon in the effective gauge theory.

Consider the case where the flux \(e_1\) which is aligned to the vanishing cycle of the conifold is turned on. In type IIB the corresponding superpotential is

\[
W = e_1 \mu + \mu \phi \tilde{\phi},
\]

where we have set

\[
\mu = \frac{X^1}{X^0}.
\]

The supersymmetric, stationary points of the corresponding scalar potential are at \(W = 0\) and \(dW = 0\), which leads to \(\mu = 0\) and in addition to the condensation of the hyper multiplets, \(\phi \tilde{\phi} = -e_1\), as discussed in \([3]\). In fact the supergravity scalar potential in the vicinity of the conifold point \([1, 7]\),

\[
V(\mu) = |W_\mu|^2 e^K K^{-1}_{\mu\bar{\mu}} = -\frac{e_1^2}{\log |\mu|^2},
\]

is stable in the \(\mu\)-direction and has a supersymmetry preserving minimum at \(\mu = 0\) with \(v = 0\). On the other hand due to the non-vanishing flux \(e_1\), two of the real scalars of \(\phi, \tilde{\phi}\) will become real massive scalars whereas the remaining two scalars will become tachyons. Explicitly, besides eq.(5.66) the scalar potential will also contain the term

\[
V(\phi, \tilde{\phi}) = e_1 (\phi \tilde{\phi} + \text{c.c.}) = 2e_1 (\phi_1 \tilde{\phi}_1 - \phi_2 \tilde{\phi}_2),
\]

\(20\)
where $\phi = \phi_1 + i\phi_2$ and $\tilde{\phi} = \tilde{\phi}_1 + i\tilde{\phi}_2$. So the fields $\phi_2$ and $\tilde{\phi}_2$ are tachyons with negative mass square $M^2 = -2e_1$.

Actually, the conifold and other known examples are no good examples for $\mathbb{P}^1$-fluxes in the context of the dual heterotic/type IIA models discussed above. In IIA language the entire CY shrinks to zero. This however corresponds to a magnetic flux, dual to the flux of $A_\gamma$, i.e. not a $\mathbb{P}^1$-flux.

For the toroidal case one can demonstrate the appearance of tachyonic charged scalars more explicitly by the dimensional reduction of the gauge kinetic term with background flux. We therefore assume that a whole non-abelian $SU(2)_V$ remains unbroken in six dimensions. Then the six-dimensional kinetic term

$$L^V_{YM} = \frac{1}{g_{YM}^2} \text{tr} F \wedge * F, \quad F^a = dA^a + f^{abc} A^b \wedge A^c$$

(5.68)

decomposes according to

$$F^a \wedge * F^b = dA^a \wedge * dA^b + 2f^{bcd}dA^a \wedge *(A^c \wedge A^d)$$

$$+ f^{acd} f^{bef} A^c \wedge A^d \wedge *(A^e \wedge A^f).$$

(5.69)

With the abelian flux $f$ for $A_0$ as in eq.(4.54) the second term is a mass term for the internal components of the six dimensional charged vector fields $A^a_{5,6}, a \neq 0$. Changing to complex coordinates

$$A^a_{\pm} = \frac{1}{\sqrt{2}} (A^a_5 \pm iA^a_6)$$

(5.70)

on the torus and to the Cartan basis in group space, the mass matrix becomes diagonal

$$2f^{0bc} A^b_5 A^c_6 = -\frac{1}{2} f^{0bc} (|A^b_+ + iA^c_+|^2 - |A^b_- + iA^c_-|^2)$$

(5.71)

such that we get two real massive scalars and two tachyons. The quartic term in (5.69) stabilizes the tachyons, it just completes the square to get a D-term potential

$$\text{tr} (F_{56})^2 = \left(f - \frac{1}{2} f^{0bc} (|A^b_+ + iA^c_+|^2 - |A^b_- + iA^c_-|^2)\right)^2.$$  

(5.72)

This is an agreement with the potential derived in [10], [11], which was obtained by computing string corrections to a quantum mechanical mass formula

$$M^2 = (2n + 1)|qf| - 2sqf,$$

(5.73)

where $n$ is the angular momentum quantum number, the Landau level, and $s$ the internal spin. A characteristic property of the mass square shift described by this formula is the
linear dependence on the flux \( f \), which appears as a generic property of D-term induced masses. The mass spectrum thus obtained satisfies the condition

\[
\text{Str} M^2 = 0
\]

for spontaneous supersymmetry breaking. For \( s = \pm 1 \) the above mass formula produces one positive and one negative mass-square scalar field. Concerning the spectrum of massless fermions the mass shift affects one chirality with \( s = \pm 1/2 \) to become massive, the opposite chirality to stay massless, such that the resulting four-dimensional spectrum will be chiral. While the expression (5.73) has been deduced from more heuristic arguments, it can be reproduced for a compactification of the heterotic string on \( T^6 \) with a background \( U(1) \) flux from the exact CFT treatment of \[9\]. It appears as the leading term of the exact mass formula for small values of the flux.

6 Conclusion

In this paper we have seen that in general F- and H-fluxes in heterotic and type II compactifications apparently break the string-string duality symmetry between heterotic/type II pairs which were dual to each other before turning on these fluxes. However for a subset of fluxes the string-string duality still holds, namely for those F- and H-fluxes which have support on the two-sphere \( \mathbb{P}^1 \) which is common to both string compactifications. One should ask what is the reason for the breakdown of the string-string duality symmetry by the other fluxes, or whether one can reconcile the string-string duality symmetry by turning on new fluxes which are so far not yet investigated. But unfortunately we cannot find any real trace for such a possibility in our discussion.

Another interesting question is the vacuum structure of heterotic and type II compactifications in the presence of fluxes. As discussed in \[1, 2, 3, 6, 7\] in the context of the effective supergravity action vacua with completely unbroken \( \mathcal{N} = 2 \) supersymmetry are possible at certain degeneration points in the moduli space (conifold points, large volume limit, etc); otherwise supersymmetry will be completely broken, and it seems to be no room for partial \( \mathcal{N} = 2 \rightarrow \mathcal{N} = 1 \) supersymmetry breaking in the flux induced supergravity action. However this situation changes in the rigid field theory limit where certain so far dynamical fields are frozen. This field theory limit can be conveniently described by replacing the compact type II Calabi-Yau spaces by their non-compact counterparts which can be then utilized to compute the flux induced field theory \( \mathcal{N} = 1 \) superpotentials for the corresponding Yang-Mills gauge theories without \[3\] and also with matter fields \[48\]. A nice way to understand the related large N duality between D-branes and H-fluxes, which
is based on topological transitions in the non-compact type II Calabi-Yau spaces, was provided in terms of geometric transitions in M-theory on non-compact spaces with $G_2$-holonomy \cite{9}. In the light of this result it would be also interesting to see whether also type II compactifications on compact Calabi-Yau spaces with H-fluxes, or their heterotic counterparts, can be lifted to some geometric M-theory compactifications. In case one would succeed to have partial supersymmetry breaking from $\mathcal{N} = 2 \to \mathcal{N} = 1$ even in the compact case, M-theory should be compactified on a compact seven-dimensional space with $G_2$-holonomy. However for the generic case of complete supersymmetry breaking by the fluxes, the situation looks much more complicated.

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