Multi-Level Design for Multiple-Symbol Non-Coherent Unitary Constellations for Massive SIMO Systems

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Dedicated to Prof. Ha H. Nguyen, who passed away before the submission of this letter.

Abstract—This letter investigates non-coherent detection of single-input multiple-output (SIMO) systems over block Rayleigh fading channels. Using the Kullback-Leibler divergence as the design criterion, we formulate a multi-symbol constellation optimization problem, which turns out to have high computational complexity to construct and detect. We exploit the structure of the formulated problem and decouple it into a unitary constellation design and a multi-level design. The proposed multi-level design has low complexity in both construction and detection. Simulation results show that our multi-level design has better performance than traditional pilot-based schemes and other existing low-complexity multi-level designs.

Index Terms—Constellation design, Kullback-Leibler (KL) divergence, non-coherent detection, ultra-reliable low-latency communications (URLLC).

I. INTRODUCTION

ULTRA-RELIABLE low-latency communication (URLLC) has been introduced as one of the pillars of the fifth generation (5G) and beyond cellular networks. Short packet communications (SPCs) with packets length of tens of bytes have been widely adopted to achieve low latency, while massive multiple-antenna systems with coherent detection have been well-studied as a diversity resource to achieve the stringent BLER requirement. However, to enable coherent detection, the transmitter must acquire accurate channel state information by sending a long sequence of training symbols, which decreases the spectral efficiency (SE) of SPCs. Thus, it is important to investigate the non-coherent detection of SPCs.

The design of non-coherent constellations with encoding and decoding of one symbol at a time has been well-studied in massive single-input multiple-output (SIMO) systems [1], [2]. However, the scheme suffers from loss of the SE since only the amplitude of the signal carries information. That said, to improve the SE and error performance, we aim to design non-coherent constellations that carry information in both amplitude and phase and are encoded and decoded over several symbols duration.

For SPCs, traditional non-coherent communications based on differential encoding and decoding, e.g., [3], may suffer from SE loss due to the insertion of pilots every coherence time, especially in the case of transmitting a few symbols. That said, there is a need to have other design objectives for non-coherent communications for SPCs. The Kullback-Leibler (KL) divergence is recognized as an effective design criterion for non-coherent communications [4]. In particular, the authors in [4] proved that the pairwise error probability (PEP) performance achieved by the maximum likelihood (ML) detector of any two signals is bounded by the KL divergence. This means that by maximizing the KL divergence between any two signals, one also maximizes the symbol error rate (SER) performance of the constellation. By utilizing the KL divergence as a constellation design criterion, the authors in [4] divided the constellation into different energy levels, where each level contains constellation points with the same energy. The optimization of the energy levels is called multi-level design while designing the constellation points within the same energy level is called unitary constellation design.

While researchers had focused on designing unitary constellations, e.g., see [5], [6], [7] and the references therein, the multi-level design over several symbols has attracted little attention [4], [8], [9]. For instance, the authors in [4] found the optimal multi-level design using an exhaustive search, and the ML detection of the optimal multi-level design was also obtained through an exhaustive search which limits its application to only a small number of symbols. In [8], [9], the authors provided a low-complexity multi-level design over only two symbols. However, both the designs in [8], [9] applies only to a specific unitary constellation and cannot be generalized to other structures or to other numbers of symbols.

To the best of our knowledge, a low-complexity multi-level design over any number of symbols for non-coherent detection of SPCs has not been considered in the literature. That said, we design a multi-level scheme based on the KL divergence to improve the SER performance of existing non-coherent unitary constellations designs. The proposed multi-level constellation has low complexity in both its construction and detection to meet the latency requirement of URLLC systems. Simulation results show that our multi-level design significantly improves the SER performance of existing unitary constellations and outperforms other low-complexity non-coherent schemes.

Notation: Matrices, column vectors, and scalar variables are denoted by uppercase bold letters (e.g., $\mathbf{A}$), lowercase bold letters (e.g., $\mathbf{a}$) and lowercase letters (e.g., $a$), respectively. We use $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ to denote the conjugate, transpose, and conjugate transpose, respectively. We use $|\cdot|$, $\|\cdot\|$, $\det(\cdot)$, and $\text{tr}(\cdot)$ to denote absolute value, Euclidean norm, determinant, and trace operations, respectively. We use $\mathbb{R}^{m \times n}$ and $\mathbb{C}^{m \times n}$ to indicate the set of real and complex matrices with dimension $m \times n$. 

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Consider the uplink transmission of a SIMO system in which a single-antenna transmitter communicates with an $M$-antenna base station. Let $\mathbf{h} \sim \mathcal{C}\mathcal{N}(0, \mathbf{I}_M)$ be the vector of independent Rayleigh fading channel coefficients between the transmitter and the receiver, and it is unknown to both of them. We assume a block fading channel where the channel coefficients stay constant over a block of consecutive $K$ symbols and then change to an independent realization in the coming block. Please note that each block of $K$ different transmit symbols $\{s_0, s_1, \ldots, s_{K-1}\}$ represents one constellation point $s = [s_0, s_1, \ldots, s_{K-1}]^T \in \mathbb{C}^{K \times 1}$ in our proposed multi-level constellation. The received signal $\mathbf{Y} \in \mathbb{C}^{M \times K}$ at the receiver can be formulated as:

$$\mathbf{Y} = \mathbf{h} \mathbf{s}^T + \mathbf{N},$$

where $\mathbf{N} \in \mathbb{C}^{M \times K}$ is the noise matrix, whose elements are independent zero-mean circular Gaussian random variables with variance $\sigma^2$. The transmitted sequence is assumed to satisfy the power constraint $\mathbb{E}([|\mathbf{Y}|^2]) = 1$. Thus, the average SNR at each receiving antenna is given as $SNR = 1/(K\sigma^2)$.

Given the transmit signal $\mathbf{s}$, the probability density function of the received signal $\mathbf{Y}$ can be written as follows [4, 9]:

$$f(\mathbf{Y}|\mathbf{s}) = \frac{\exp \left(-\frac{\text{tr}(\mathbf{Y}^H \mathbf{Y})}{\sigma^2} + \frac{\text{tr}(\mathbf{Y}^H \mathbf{Y} \mathbf{s}^H \mathbf{s})}{\sigma^2(\sigma^2 + ||\mathbf{s}||^2)} \right)}{\pi^{K M} \left(\sigma^2 + ||\mathbf{s}||^2\right)^{2K-2} \left(2\pi\right)^{M}}.$$  (2)

Given a known constellation of $\mathbf{s}$, i.e., $\Omega_s$, the ML detector to detect the transmit signal $\mathbf{s}$ is [9]:

$$\mathbf{s}_{ML} = \arg \max_{\mathbf{s} \in \Omega_s} \left(\frac{\text{tr}(\mathbf{Y}^H \mathbf{Y} \mathbf{s}^H \mathbf{s})}{\sigma^2(\sigma^2 + ||\mathbf{s}||^2)} - M \ln(\sigma^2 + ||\mathbf{s}||^2)\right).$$  (3)

Please note that the KL distance depends on the channel fading model. For Rayleigh fading channels, the KL distance between two transmit signals $\mathbf{s}_i$ and $\mathbf{s}_k$ is defined as the average KL divergence of their two conditional probability density functions per antenna, which is given by [4, eq. (38)]:

$$D_{KL}(f(\mathbf{Y}|\mathbf{s}_i), f(\mathbf{Y}|\mathbf{s}_k)) = \frac{1}{M} \mathbb{E}_{f(\mathbf{Y}|\mathbf{s}_i)} \left(\ln \left(\frac{f(\mathbf{Y}|\mathbf{s}_i)}{f(\mathbf{Y}|\mathbf{s}_k)}\right)\right),$$

$$= \frac{||\mathbf{s}_k||^2||\mathbf{s}_i||^2 - ||\mathbf{s}_i^T \mathbf{s}_k||^2}{\sigma^2(\sigma^2 + ||\mathbf{s}_k||^2)} + \frac{\sigma^2 + ||\mathbf{s}_i||^2}{\sigma^2 + ||\mathbf{s}_k||^2} - \ln \left(\frac{\sigma^2 + ||\mathbf{s}_i||^2}{\sigma^2 + ||\mathbf{s}_k||^2}\right) - 1.$$  (4)

III. KL-BASED MULTI-LEVEL CONSTITUTION DESIGN

As pointed out in [4], the KL distance between any two transmit sequences $\mathbf{s}_i$ and $\mathbf{s}_k$ determines the pairwise symbol error probability between them. For this reason, the pair with the smallest KL distance will produce the highest error probability and contribute the most to the error rate of the whole constellation set. That said, we aim to design a constellation set $\Omega_s$ that minimizes the maximum KL distance, which can be formally expressed as:

$$\max_{\Omega_s} \min_{\mathbf{s}_i, \mathbf{s}_k \in \Omega_s, \mathbf{s}_i \neq \mathbf{s}_k} D_{KL}(\mathbf{s}_i, \mathbf{s}_k).$$  (5)

Since any sequence $\mathbf{s}$ is a $(K \times 1)$-dimensional complex vector, it can be characterized by its magnitude $\alpha \in \mathbb{R}$ and its direction vector $\mathbf{v} \in \mathbb{C}^{K \times 1}$, which are defined as follows:

$$\alpha = ||\mathbf{s}||, \quad \mathbf{v} = \frac{\mathbf{s}}{||\mathbf{s}||}.$$  (6)

Mathematically, $\mathbf{v}$ is a unitary vector obtained by normalizing the original vector $\mathbf{s}$ such that any two sequences $\mathbf{s}_i$ and $\mathbf{s}_k$ with the same direction will have the same unitary vector $\mathbf{v}$.

The maximum likelihood detector in (3) can be rewritten in terms of $\alpha$ and $\mathbf{v}$ as follows:

$$\{\alpha_{ML}, \mathbf{v}_{ML}\} = \arg \max_{(\alpha, \mathbf{v})} \left(\frac{\alpha^2 \text{tr}(\mathbf{Y}^H \mathbf{Y} \mathbf{v} \mathbf{v}^H)}{\sigma^2(\sigma^2 + \alpha^2)} - M \ln(\sigma^2 + \alpha^2)\right).$$  (7)

Similarly, the KL divergence in (4) is also rewritten in terms of $\alpha$ and $\mathbf{v}$ as follow:

$$D_{KL}(\alpha_k, \mathbf{v}_k, \alpha_i, \mathbf{v}_i) = D_1(\alpha_k, \mathbf{v}_k, \alpha_i, \mathbf{v}_i) + D_2(\alpha_k, \alpha_i)$$

$$= \frac{\alpha_k^2}{\sigma^2(\sigma^2 + \alpha_k^2)} \left(1 - \frac{||\mathbf{v}_k||^2}{2}\right) + \frac{\left(\frac{\sigma^2}{\sigma^2 + \alpha_k^2} - \ln \left(\frac{\sigma^2 + \alpha_k^2}{\sigma^2 + \alpha_i^2}\right) - 1\right)}{2}. \quad (8)$$

As can be seen from (8), the KL distance between two constellation points $\mathbf{s}_i$ and $\mathbf{s}_k$ consists of two terms $D_1$ and $D_2$. The first term $D_1$ is due to the difference of direction vectors $\mathbf{v}_i$ and $\mathbf{v}_k$, and it is scaled by the energy levels $\alpha_k$ and $\alpha_i$ of the two points. On the other hand, the second term $D_2$ is mainly due to the difference between the energy levels of the two points.

Equation (8) indicates that designing the optimal $\Omega_s$ based on the KL distance requires a joint optimization over both $\alpha$ and $\mathbf{v}$, which is an extremely complex process. One can observe that the KL distance in (8) reduces to $D_2$, i.e., $D_1 = 0$, when the two constellation points have the same direction $\mathbf{v}$ but different energies; and reduces to $D_1$, i.e., $D_2 = 0$, when the two constellation points have the same energy levels. That said, to strike a balance between the SER performance and complexity, we split the constellation set $\Omega_s$ into different subsets $\mathcal{W}_{n_0}, n = 0, \ldots, N-1$, where each subset $\mathcal{W}_{n_0}$ comprises of $\mathcal{W}_{n_0}$ points with the same energy level $\alpha_{n_0}$. By doing so, we transform the constellation design problem in (8) into optimizing the KL distance between constellation points inside a given subset $\mathcal{W}_{n_0}$ (which only contains $\mathcal{W}_{n_0}$) and optimizing the KL distance between different subsets $\mathcal{W}_{n_1}, \mathcal{W}_{n_2}$. Please note that both $D_{	ext{intra}}(\mathcal{W}_{n_0})$ and $D_{\text{inter}}(\mathcal{W}_{n_0}, \mathcal{W}_{n_2})$ will be formally defined in Section IV.

For $b_k$ bits allocated to this constellation set $\Omega_s$, the number of points of all subsets $\mathcal{W}_{n_0}, n = 0, \ldots, N-1$, must be summed up to $2^b$, i.e., $\sum_{n=0}^{N-1} \mathcal{W}_{n_0} = 2^b$. To find the optimal combination of $\{w_0, w_1, \ldots, w_{N-1}\}$, an exhaustive search algorithm can
be used [4]; however, its computational complexity prohibits its practical implementation, especially for a higher number of bits per constellation. To overcome such a high computational complexity, we propose a constellation structure with subset $\mathcal{W}_n$, $n = 0, 1, \ldots, N - 1$, defined as

$$\mathcal{W}_n = \{\alpha_n v_0, \alpha_n v_1, \ldots, \alpha_n v_{2^l_0 - 1}\}, \quad n = 0, 1, \ldots, N - 1,$$

where $N = 2^{l_0}$. We define the set $\Omega_{\alpha} = \{\alpha_0, \alpha_1, \ldots, \alpha_{2^{l_0} - 1}\}$ as the set of $\alpha$ that satisfies $\alpha_0 < \alpha_1 < \cdots < \alpha_{2^{l_0} - 1}$, and we define the set $\Omega_{\alpha_v} = \{v_0, \ldots, v_{2^l - 1}\}$ as the set containing all possible unitary vectors $v$, with $l_0$ and $l_0$ be the number of bits allocated to $\Omega_{\alpha}$ and $\Omega_{\alpha_v}$, respectively.

One can observe that the set $\Omega_{\alpha_v}$ can be seen as the Cartesian product of $\Omega_{\alpha}$ and $\Omega_{\alpha_v}$, and such an observation serves two purposes: the low-complexity construction and detection of our proposed multi-level constellation design as discussed below.

1) Low-Complexity Construction of the Multi-Level Constellation: The number of possible choices for $\mathcal{W}_n$ for our proposed structure in (9) is only $l_0 + 1$. In contrast, the method in [4] requires an exhaustive search with exponential complexity. Furthermore, [4] requires the construction of unitary constellations $\Omega_{\alpha_v}$ of any sizes that are not necessarily a power of two. Instead, our method only requires unitary constellations whose cardinality range is the power of two, which are readily available in the literature.

2) Low-Complexity ML Detection of the Multi-Level Constellation: Instead of searching over all possible $s = \alpha v$, as shown in (7), we only need to search over $\alpha$ and $v$ separately, which is represented as follows:

$$v_{\text{ML}} = \arg\max_{v \in \Omega_{\alpha_v}} \text{tr} \left( Y^H Y v^* v^T \right).$$

$$\alpha_{ML} = \arg\max_{\alpha \in \Omega_{\alpha}} \frac{\alpha^2 \text{tr} \left( Y^H Y v_{\text{ML}}^\alpha v_{\text{ML}}^T \right)}{\sigma^2 (\sigma^2 + \alpha^2)} - M \ln(\sigma^2 + \alpha^2).$$

By doing this, the ML decoding complexity of the whole constellation can be reduced from $O(2^{l_0} + l_v)$ to $O(2^{l_0}) + O(2^{l_v})$, where $O(2^{l_0})$ and $O(2^{l_v})$ are the complexities of detecting $\alpha$ and $v$, respectively. The decoding complexities can be further reduced if we embed special unitary constellations with low decoding complexity.

IV. Optimization of Multi-Level Constellations Under Fixed Bit Allocation

In this section, we optimize the multi-level constellation given fixed $l_0$ and $l_v$. Given the proposed structure in (9), $D_{\text{intra}}(\mathcal{W}_n)$ and $D_{\text{inter}}(\mathcal{W}_{n_1}, \mathcal{W}_{n_2})$ can be written as follows:

$$D_{\text{intra}}(\mathcal{W}_n) = \frac{\alpha_n^4}{\sigma^2 (\sigma^2 + \alpha_n^2)} \min_{v_i \neq v_i} (1 - |v_i^T v_i^*|^2),$$

$$D_{\text{inter}}(\mathcal{W}_{n_1}, \mathcal{W}_{n_2}) = \frac{\sigma^2 + \alpha_{n_1}^2}{\sigma^2 + \alpha_{n_2}^2} - \ln \left( \frac{\sigma^2 + \alpha_{n_1}^2}{\sigma^2 + \alpha_{n_2}^2} \right) - 1.$$  

For a given $l_0$ and $l_v$, our problem in (5) can be reformulated as follows:

$$\{\Omega_0, \Omega_v\} = \arg\max_{\Omega_0, \Omega_v} \left( \min_{\mathcal{W}_n \subseteq \Omega_0} D_{\text{intra}}(\mathcal{W}_n), \min_{\mathcal{W}_{n_1}, \mathcal{W}_{n_2} \subseteq \Omega_v} D_{\text{inter}}(\mathcal{W}_{n_1}, \mathcal{W}_{n_2}) \right),$$  

s.t. $\|v_i\| = 1, \forall v_i \in \Omega_{v}, \text{card}(\{\Omega_v\}) = 2^{l_v}. \quad (14a)$

$$\frac{1}{\Omega_{\alpha}} \sum_{\alpha \in \Omega_{\alpha}} \alpha_n^2 = 1, \text{card}(\{\Omega_{\alpha}\}) = 2^{l_0}. \quad (14b)$$

Since $\alpha_n^4 / (\sigma^2 (\sigma^2 + \alpha_n^2))$ is an increasing function of $\alpha_n$ for $\alpha_n \geq 0$, $D_{\text{intra}}(\mathcal{W}_n)$ is minimized when $\alpha_n$ is minimized, i.e., $\alpha_n = \alpha_0$. Since $1/x - \ln(1/x) - 1 - x - \ln(x) - 1$ for $x > 1$, $D_{\text{inter}}(\mathcal{W}_{n_1}, \mathcal{W}_{n_2}) < D_{\text{inter}}(\mathcal{W}_{n_2}, \mathcal{W}_{n_1})$ for $n_1 < n_2$. Thus, the minimum inter distance must be in the cases of $D_{\text{inter}}(\mathcal{W}_{n_1}, \mathcal{W}_{n_2})$ where $n_1 < n_2$. Also, since $D_{\text{inter}}$ increases when the energy difference between two subsets increases, the minimum inter distance must be the cases of two consecutive subsets. As a result, we simplify our problem in (14) as follows:

$$\{\Omega_0, \Omega_v\} = \arg\max_{\Omega_0, \Omega_v} \left( \min_{\mathcal{W}_n \subseteq \Omega_0} (D_{\text{intra}}(\mathcal{W}_0)), \min_{\mathcal{W}_{n+1} \subseteq \Omega_v} (D_{\text{intra}}(\mathcal{W}_{n+1})) \right),$$  

s.t. (14b) and (14c).

We can see that the distance $D_{\text{intra}}(\mathcal{W}_0)$ contains the variable $v$ in the form of $\min_{v_i \neq v_i} (1 - |v_i^T v_i^*|^2)$, and this optimization problem of $v$ is not affected by $\alpha$. Thus, we can decouple our problem into two separate optimization problems, i.e., unitary set optimization and multi-level optimization. The unitary set optimization is given by:

$$\Omega_v = \arg\max_{\Omega_v} \left\{ \min_{v_i \neq v_i} (1 - |v_i^T v_i^*|^2) \right\},$$

s.t. (14b),

which is a classic problem called sphere packing on Grassmannian manifolds and has been well-studied in the literature [5], [6], [7]. Thus, we can use any available unitary set $\Omega_v$ from the literature and focus on multi-level optimization. Given $\Omega_v$, let $T_v = \min_{v_i \neq v_i} (1 - |v_i^T v_i^*|^2)$ be the minimum distance of the unitary constellation $\Omega_v$. The multi-level optimization problem for a given $T_v$ can be written as follows:

$$\Omega_{\alpha} = \arg\max_{\Omega_{\alpha}} \left\{ \min_{n_0, \ldots, n_{N-2}} \left( \frac{\alpha_0^4 T_v}{\sigma^2 (\sigma^2 + \alpha_0^2)} - \frac{1}{r_n} - \ln \left( \frac{1}{r_n} \right) - 1 \right) \right\}, \quad (17a)$$

s.t. (14c).

with $r_n = (\sigma^2 + \alpha_{n+1}^2)/(\sigma^2 + \alpha_n^2)$. The optimal $\{\alpha_0, \bar{r}_1, \ldots, \bar{r}_{N-2}\}$ must satisfy the conditions:

$$\alpha_0^4 T_v \sigma^2 (\sigma^2 + \alpha_0^2) = \frac{1}{\bar{r}_0} - \ln \left( \frac{1}{\bar{r}_0} \right) - 1.$$  

The proof is presented in the Appendix. Now we proceed to construct the optimal set $\Omega_{\alpha} = \{\alpha_0, \ldots, \alpha_{N-1}\}$ given the
two conditions in (18). From (18a), it is easy to prove that 
\[ \sigma^2 + \alpha_i^2 = (\sigma^2 + \alpha_0^2) (\tilde{r}_0)^i. \] 
The solution of (18b) and (19) can be obtained by conventional methods such as bisection or Newton method. Finally, after obtaining all the multi-level constellations for all possible bit allocation, i.e., \((\ell_\alpha, l_v) \in \{(0, l_\alpha), (1, l_\alpha - 1), \ldots, (l_\alpha, 0)\}\), we choose the bit allocation \(l_\alpha\) and \(l_v\) and its corresponding constellation with the highest KL distance.

V. Simulation Result

In this section, we evaluate the SER performance of our proposed multi-level constellation and compare it to other constellations from the literature. Since already-optimized unitary constellations are prerequisites for our method, we evaluate our proposed multi-level constellations with two types of unitary constellations: the general unitary constellations obtained by numerically solving the optimization problem in (16) and the cube-split constellations in [7]. While the general unitary constellations have the best SER performance among all unitary constellations, the cube-split constellations have low decoding complexity while still attaining acceptable performance compared to other non-coherent schemes.

A. Optimal Bit Allocation to \(\Omega_\alpha\)

First, we plot the number of bits allocated to \(\Omega_\alpha\), i.e., \(l_\alpha\). Since our scheme is SNR-adaptive, given a collection of unitary constellations \(\Omega_i\), \(l_\alpha\) and \(l_v\) will only depend on the SNR level. In general, if \(l_\alpha\) is greater than zero, it means our multi-level design achieved higher KL distance than unitary constellations. In contrast, if \(l_\alpha\) is equal to zero, our multi-level design simply converges to unitary constellations, and no performance gain is achieved from our multi-level design. Figure 1 shows that \(l_\alpha\) tends to decrease when the SNR increases and converges to 1-level design (unitary constellations) in the high SNR regime. This is in line with the statement of optimality of unitary constellations in the high SNR regime in [4]. It can also be seen from Fig. 1 that for higher \(K\), our multi-level design still has advantages over unitary constellations, though less frequently than the case of \(K = 2\).

B. SER Performance Evaluation

In this sub-section, we evaluate the SER performance gain achieved by our multi-level design (combined with unitary constellations) when compared to only unitary constellations without multi-level design. We also use two non-coherent schemes as baselines for comparisons: non-coherent detection of pilot-based QAM, which includes a pilot symbol and a sequence of QAM symbols, and the multi-level constellations from [8] and [9]. Figure 2 shows that multi-level design improves the SER performance in both general unitary constellations and cube-split constellations for both low and high SNR regimes. By embedding our multi-level design into cube-split constellations, the cube-split design achieves a significant improvement in SER performance and outperforms the conventional pilot-based QAM scheme. Additionally, the proposed multi-level design improves the performance of general unitary constellations and also outperforms both of the reference schemes. Please note that the multi-level design will converge to the 1-level design when SNR further increases, as discussed in Fig. 1.

In Fig. 3, we evaluate the SER performance versus different numbers of antennas \(M\), i.e., \(\{8, 16, 32, 64, 128, 128, 256\}\), given two fixed SNR levels: 3 dB and 8 dB. In general, the SER performance of all the schemes decreases exponentially, which is in line with the fact that for sufficiently large \(M\), the PEP of the ML detector (3) decreases exponentially as \(M\) increases [4]. In the case of SNR = 3 dB, both the multi-level design of the general unitary constellations and the cube-split constellations outperform that of 1-level design when \(M\) is large enough. However, in the case of SNR = 8 dB, there...
is no performance gain in the multi-level design of the general constellations compared to the 1-level design because the multi-level design of the general constellations converges to the 1-level design in that SNR level. Also, for large values of $M$, the multi-level design of the general constellations outperforms that of [9] and pilot-based QAM. In that scenario, the multi-level design of the cube-split constellations has similar or slightly lower performance than [9] and better performance than pilot-based QAM.

Since our method can be applied to any number of time slots $K$, we also evaluate the SER performance among different values of $K$ given the same SE of 4 bits/s/Hz. In the case of $K = 1$, energy encoding and detection in [2], which is also a specific case of our multi-level design for $K = 1$, will be used as a reference. Firstly, Fig. 4 shows that our multi-level design also improves the performance of the cube-split constellation for $K = 4$ and general unitary constellation for $K = 3$. In general, since the overall performance of non-coherent constellations improves when $K$ increases, it is reasonable to apply our multi-level design to enhance the performance of unitary constellations in a higher number of time slots. In contrast, the multi-level designs in [8], [9] (which is only designed for $K = 2$) and [4] (which has high complexity and therefore cannot be applied for large $K$) cannot achieve performance gain by increasing the number of time slots.

VI. CONCLUSION

In this letter, we proposed a low-complexity design for multi-level constellations based on KL divergence. Our multi-level design is shown to improve the SER performance of non-coherent schemes based on unitary constellations as well as outperforms other non-coherent schemes while having low complexity in construction and detection.

APPENDIX

We prove the set $\Omega_{\hat{\alpha}}$ is the optimal solution of (18) by contradiction. In this case, the objective value corresponding to $\Omega_{\hat{\alpha}}$ is 

$$\frac{\alpha_0^4}{\sigma^2(\alpha_0^2 + \alpha_0^2)} > \frac{\alpha_0^4}{\sigma^2(\alpha_0^2 + \alpha_0^2)}$$

(20)

$$\frac{1}{\bar{r}_n} - \ln\left(\frac{1}{\bar{r}_n}\right) - 1 > \frac{1}{\bar{r}_n} - \ln\left(\frac{1}{\bar{r}_n}\right) - 1,$$

(21)

Since $\frac{\sigma^2}{\alpha_0^2}$ is an increasing function of $\alpha_0 > 0$ and $1/r - \ln(1/r) - 1$ is an increasing function of $r > 1$, (20) and (21) are equivalent to $\alpha_0 > \alpha_0$ and $\bar{r}_n > \bar{r}_n$, respectively. Thus,

$$\sigma^2 + \alpha_0^2 = (\sigma^2 + \alpha_0^2) \prod_{i=0}^{n-1} \tilde{r}_i > (\sigma^2 + \alpha_0^2) \prod_{i=0}^{n-1} \tilde{r}_i$$

(22)

$$\Omega_{\hat{\alpha}}$$ clearly does not satisfy the power constraints because:

$$\frac{1}{2\kappa_0} \sum_{i=0}^{2\kappa_0-1} \alpha_i^2 > \frac{1}{2\kappa_0} \sum_{i=0}^{2\kappa_0-1} \alpha_i^2 = 1,$$

(23)

which concludes the proof.

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