Holographic Cosmic Quintessence
on
Dilatonic Brane World

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Abstract

Recently quintessence is proposed to explain the observation data of supernova indicating a time-varying cosmological constant and accelerating universe. Inspired by this and its mysterious origin, we look for the possibility of quintessence as the holographic dark matters dominated in the late time in the brane world scenarios. We consider both the cases of static and moving brane in a dilaton gravity background. For the static brane we use the Hamilton-Jacobi method motivated by holographic renormalization group to study the intrinsic FRW cosmology on the brane and find out the constraint on the bulk potential for the quintessence. This constraint requires a negative slowly varying bulk potential which implies an anti-de Sitter-like bulk geometry and could be possibly realized from the higher dimensional supergravities or string theory. We find the similar constraint for the moving brane cases and that the quintessence on it has the effect as a mildly time-varying Newton constant.
1 Introduction

Cosmic quintessence \([2]\) is proposed recently to be an alternative to understand the astronomical data of supernova \([1]\) (see also \([3]\) for a summary of the data constraints,) indicating an everlasting accelerating universe, which can also be interpreted as the effect of a small slowly time-varying cosmological constant \([2]\). Recently there are discussions on the difficulty about the string theory in a spacetime with future cosmological horizon \([4, 5]\) which exists both for de Sitter space and the universe dominated by cosmic quintessence at late time, and hinders the notion of S-matrices in the perturbative string or quantum field theory. However, from the cosmological point of view, the quintessence proposal has milder cosmological constant problem without severe fine tuning since it does not require a zero cosmological constant at all time, instead it models a slowly decaying cosmological constant and its effect at the present moment will fit the observational data by properly tuning its energy density.

Let us start with the usual (3+1)-dim. Friedman-Robertson-Walker (FRW) metric

\[
d s^2_{FRW} = -d\tau^2 + a^2(\tau)d\Sigma^2_k , \tag{1.1}
\]

and the Einstein equations are reduced to the FRW equations

\[
H^2 = \frac{-k}{a^2} + \sum_i \frac{\rho_i}{3} \left(\frac{a_0}{a}\right)^{\kappa_i} , \tag{1.2}
\]

\[
6\frac{\ddot{a}}{a} = \sum_i (2 - \kappa_i) \rho_i \left(\frac{a_0}{a}\right)^{\kappa_i} . \tag{1.3}
\]

Note that the second FRW equation is independent of \(k\) where \(k = -1, 0, 1\) corresponds to open, flat and closed universe with the corresponding hypersphere \(\Sigma_k\); \(a(\tau)\) is the cosmological scale factor with \(a_0\) parameterizing the initial size of the universe; \(H = \dot{a}/a\) is the Hubble parameter with \(\dot{a} \equiv \frac{da}{d\tau}\). In the above we have assumed the universe contains only the perfect fluids of pressure \(p_i\) and the energy density \(\rho_i = \rho_{0i}(a_0/a)^{\kappa_i}\), and obeying the equation of state \(p_i = \omega_i \rho_i\) with

\[
\kappa_i = 3(1 + \omega_i) . \tag{1.4}
\]

Usually one could have different types of matters present in the universe, for example, a CFT matter or photon has \(\omega = 1/3\) (or \(\kappa = 4\)), and a cosmological constant corresponds to \(\omega = -1\) (or \(\kappa = 0\)), and the usual matters have \(\omega = 0\) (\(\kappa = 3\)). In the late time or for the large scale factor, the dynamics of the FRW cosmology is dominated by the component with small \(\kappa > 0\).
In this paper, the dominated matter in the late time is assumed to be the quintessence in accord with the observation data in [1] giving the following constraint on $\omega$

$$-1 < \omega_{\text{obs}} < \frac{-2}{3} \quad \text{or} \quad 0 < \kappa < 1,$$

(1.5)

which indicates the universe is accelerating at the present stage according to the 2nd FRW equations for both flat, open and closed universes. In general an accelerating universe requires

$$-1 < \omega < \frac{-1}{3} \quad \text{or} \quad 0 < \kappa < 2,$$

(1.6)

which is driven by some matters coined as quintessence dominated at the late time of cosmic evolution [2]. As emphasized in [3], an observer in an accelerating universe will see a future horizon.

In this paper we will investigate the possibility of having the quintessence as the holographic imprint of the brane-world scenarios. Since the quintessence can be in general modelled as a slowly-rolling scalar which does not exist in the usual Randall-Sundrum(RS) type brane-world scenarios [9] with a brane embedded in the pure AdS space. However, in the dilatonic gravity it is natural to have a Liouville type potential, that is, in the exponential form. As one follows the AdS/CFT correspondence [8], the bulk gravity will induce holographic matters with similar on-shell Liouville type potential on the brane, which can be interpreted as the holographic dark energy from the point of view of brane cosmology; and for some specific dilaton gravity one may obtain the holographic cosmic quintessence.

Our answer to this investigation is positive. In the next section 2 and 3 we will review the general holographic principle in dilaton gravity which is intimately related to the holographic renormalization group [15] and induces intrinsic geometry on the brane. Based upon this intrinsic brane gravity we will discuss the possibility of having the holographic cosmic quintessence on the static brane. In section 4 and 5 we will turn to the more general bulk dilatonic background and discuss the holographic quintessence on a moving brane based upon the FRW cosmology derived from the extrinsic geometry of the brane, the Israel junction condition. It turns out that the quintessence on the moving brane has the similar effect as a mildly time-varying Newton constant. In the last section we conclude with some remarks and discussions.

2 Dilatonic Brane Gravity from Holography

The Randall-Sundrum model [9] of branes embedded in a bulk (anti-de Sitter) spacetime can be thought as a variant of the AdS/CFT correspondence [8], that is, the dual CFT is coupled
to the brane gravity as the induced holographic dark matters \[10, 12, 13\]. After integrating out the hidden CFT matters we will obtain an effective gravity theory in which we can add the local matters like the Standard model on the curved brane. This consideration of holography can be generalized to the bulk dilatonic gravity with the action

\[
S = \int_M d^{n+2}x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right), \tag{2.1}
\]

where \(V(\phi)\) is the dilaton potential which usually have its origin in higher dimensional supergravity or superstring theory. After integrating out the dual holographic field theory on the brane, one will obtain an effective dilaton gravity theory on the brane with the following action

\[
S_\Sigma = \int_\Sigma d^{n+1}x \sqrt{-\gamma} \left( Z(\phi) \mathcal{R} - \frac{1}{2} M(\phi) \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right) + \Gamma[\gamma, \phi] , \tag{2.2}
\]

where \(\mathcal{R}\) is the scalar curvature with respect to the boundary metric \(\gamma_{\mu\nu}\). The UV part \(\Gamma\) contains the higher-derivative and non-local terms and thus can be neglected in the low energy regime.

The boundary functionals \(Z(\phi), M(\phi)\) and \(U(\phi)\) on the brane worldvolume \(\Sigma\) are not arbitrary but are determined or induced by the bulk gravity theory and the geometry, and can be viewed as the holographic dark energy on the brane (or domain wall). In the bulk geometry of the Randall-Sundrum type with the metric

\[
g_{MN} dx^M dx^N = dr^2 + ds^2_{brane} = dr^2 + \Lambda^2(r) \bar{\gamma}_{\mu\nu}(x^\mu) dx^\mu dx^\nu , \tag{2.3}
\]

where \(\bar{\gamma}_{\mu\nu}\) depends only on the brane coordinates not on \(r\), and the on-shell bulk and boundary potentials are related by the following "superpotential-like" relation \[10, 11\]

\[
\frac{1}{2} \left( \frac{\partial U}{\partial \phi} \right)^2 - \frac{2}{3} U^2 = V(\phi) . \tag{2.4}
\]

A more general and systematic discussions based upon the Hamilton-Jacobi formalism about the holographic relations was given in \[13\] (see also \[17\] for the recent attempt in the cosmology context). This formalism is a generalization of AdS/CFT correspondence, and the dilaton is interpretated as the effective coupling of the holographic renormalization group flow in the boundary theory with its beta function defined by

\[
\beta = \Lambda \frac{\partial \phi}{\Lambda} , \tag{2.5}
\]

\(^1\)The trace of the variation of \(\Gamma[\gamma, \phi]\) with respect to \(\gamma_{\mu\nu}\) is the conformal anomaly which is argued in \[14, 12\] to drive the inflation of the brane universe, and is thus relevant in the early universe in accord with its UV nature.
where $\Lambda$ is the overall scale factor in (2.3) and is interpreted as the holographic energy scale in the dual theory.

On the other hand, one can determine the intrinsic brane gravity described by $S_\Sigma$ from the Hamilton-Jacobi equation given in [13] which is

$$\mathcal{H} = \frac{1}{\sqrt{\det \gamma}} \frac{\delta S_\Sigma}{\delta \gamma} \Pi = \frac{1}{\sqrt{\det \gamma}} \frac{\delta S_\Sigma}{\delta \phi} = 0,$$

where $\mathcal{H}$ is the Hamilton density constructed from the action (2.1) in the standard ADM formalism. The $\pi_{\mu\nu}$ and $\Pi$ are the canonical conjugate momenta with respect to $\gamma_{\mu\nu}$ and $\phi$. The boundary action $S_\Sigma$ is then identified as the Hamilton-Jacobi functional as indicated above. From the Hamilton-Jacobi equation, we obtain the following independent holographic relations [15, 16]

$$-\frac{4}{3} U Z + 2U' Z' = 1,$$

$$\beta = 12 \frac{Z'}{M},$$

$$-4M - 24Z'' + \beta M' = \frac{6}{U},$$

where $'$ means the derivative with respect to $\phi$. Moreover, the beta function can be determined from inverting the defining equations of the canonical momenta, and it turns out to be

$$\beta = -\frac{6U'}{U}.$$

From the set of the holographic relations we can determined the brane gravity theory with a given boundary dilaton potential. It is easy to see that there is no nontrivial solution for constant $M$ and $Z$, so it implies that the brane gravity usually has a nontrivial dilaton potential. Since we are interested in the possibility of the holographic quintessence which can be modelled with a power-law or exponential form dilaton potential, we try the ansatz for such kind of the potential form. We find that there is no simple solution for the power-law potential, while we have the solution for the Liouville type as the following

$$U = e^{b(\phi - \phi_0)},$$

$$M = 2Z = \left( \frac{-6}{4 + 6b^2} \right) \frac{1}{U},$$

\[2\]Our convention taken here is different from the one in [13, 14] with an additional factor $1/2$ in front of the bulk scalar curvature term so that the coefficients in the following holographic relations change accordingly.
for arbitrary constants $b$ and $\phi_0$. Moreover, the bulk and boundary potential is related by (2.4) as

$$V = \left( \frac{3b^2 - 4}{6} \right) U^2.$$  

(2.13)

Note that for $3b^2 = 4$, $V = 0$ which corresponds to the self-tuning brane model of [26]. In the next section we will study the FRW cosmology of the brane gravity $S_\Sigma$ to check the possibility of the holographic cosmic quintessence for the static brane.

### 3 Quintessence on the Holographic Brane

Before going to the FRW cosmology of the brane gravity described by $S_\Sigma$ we first recall some facts about the FRW cosmology of the quintessence modelled by the following (3+1)-dim. scalar Lagrangian [6]

$$L = \sqrt{\gamma} \left( \frac{1}{2} (\partial \Phi)^2 + W(\Phi) \right),$$

(3.1)

With respect to the FRW metric (1.1), the energy density $\rho$ and the pressure $p$ take the form

$$\rho = \frac{1}{2} \dot{\Phi}^2 + W(\Phi), \quad p = \frac{1}{2} \dot{\Phi}^2 - W(\Phi),$$

(3.2)

and the equation of state is

$$\omega \equiv \frac{p}{\rho} = \frac{\dot{\Phi}^2 - 2W(\Phi)}{\dot{\Phi}^2 + 2W(\Phi)}.$$  

(3.3)

Note that when $\dot{\Phi}^2 < W(\Phi)$ in the late time, then $\omega < -1/3$ (or $\kappa < 2$) giving the quintessence. In the case of $k = 0$, it has been shown in [6] that the only stable attractive fixed point of the solution space for $\kappa = 3(1 + \omega) < 2$ is that $\Phi$ scales as

$$a \frac{\partial \Phi}{\partial a} = \sqrt{\kappa},$$

(3.4)

and the scalar energy density red-shifts as

$$\rho = \rho_0 \left( \frac{a_0}{a} \right)^\kappa.$$  

(3.5)

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3 In fact, the general solutions for $Z$ and $M$ are

$$Z = c_0 e^{\frac{2\phi}{3b^2}} - \frac{3}{4 + 6b^2} e^{-b\phi},$$

$$M = \frac{4c_0}{3b^2} e^{\frac{2\phi}{3b^2}} - \frac{6}{4 + 6b^2} e^{-b(\phi - \phi_0)},$$

where $c_0$ is an integration constant. Since the $c_0$ terms have the opposite running behavior from the other terms and will destroy the UV/IR relation in the holographic RG scheme, thus we should put it to zero for consistency.
as usual. Note that the scalar-field equation of motion implies the conservation of the scalar-field stress tensor. At this fixed point it is easy to see that

$$W(\Phi) = W_0 e^{-\sqrt{\kappa}\Phi} .$$

(3.6)

The above indicates that a scalar field with Liouville type potential could be the quintessence within the usual Einstein gravity. However, the brane gravity in the last section is the gravity not in the Einstein frame but in the stringy frame with nontrivial $Z(\phi)$ and $M(\phi)$ in the action $S_\Sigma$. In order to study the usual FRW cosmology, we need to change to the Einstein frame by the following Weyl transformation

$$\hat{\gamma}_{\mu\nu} = Z(\phi) \gamma_{\mu\nu} ,$$

(3.7)

and with the $Z$, $M$ and $U$ of (2.12) and (2.11) the new action in the Einstein frame is

$$S_E = \int d^{3+1}x \sqrt{-\hat{\gamma}} [R - \frac{1}{2} (\hat{\partial} \Phi)^2 - (\frac{4 + 6b^2}{3})^2 e^{\frac{3b}{\sqrt{2+3b^2}}(\Phi-\Phi_0)}] ,$$

(3.8)

where we have defined

$$\Phi = \sqrt{2 + 3b^2} \phi .$$

(3.9)

so that the scalar kinetic term is in the canonical form. Note that the resulting potential in the Einstein frame is positive definite so that the positive energy density of the holographic matters is ensured.

In the Einstein frame the brane gravity has the scalar-field Lagrangian $L$ of (3.1) with a specific potential $W(\Phi)$ by identifying

$$\sqrt{\kappa} = -\frac{3b}{\sqrt{2 + 3b^2}} ,$$

(3.10)

which then requires $b < 0$ and the quintessence condition $0 < \kappa < 2$ requires

$$0 < b^2 < \frac{4}{3} .$$

(3.11)

4There is an issue about the equivalence of the different conformal frames as discussed in [28, 30, 31], we adopt the point of view that the standard FRW cosmology should be considered in the Einstein frame, we also find that the boundary gravity in the stringy frame (2.2) will lead to negative energy density for the FRW cosmology which agrees with the observation in [28].

5In this section we adopt the convention in [1] such that there is no 1/2 factor in front of the Einstein-Hilbert term in (3.8).

6The condition $b < 0$ is consistent with the usual runaway behavior of the string dilaton so that the string coupling $g_s = e^{-b\phi} \to 0$ as $\phi \to \infty$. 

6
Note that the marginal point for quintessence $b^2 = 4/3$ is the self-tuning brane condition for zero cosmological constant\(^7\). From the holographic relation (2.13), we conclude that a bulk potential $V(\phi) = V_0 e^{2b\phi} < 0$ with $0 < b^2 < 4/3$ will induce holographic quintessence on the brane cosmology. We want to emphasize the fact $V_0 < 0$ for $0 < b^2 < 4/3$ because this leads to an anti-de Sitter-like bulk geometry which is far more easy to be realized in the supergravity than the de-Sitter-like one [11]. It is interesting to look for the origin of $V(\phi)$ in the higher dimensional supergravities or string theory.

There are further complications by adding the standard matter fields besides the above Liouville scalar to cook up the more realistic cosmological model, the related issues and results have been discussed in [4, 7] and can be directly carried to the holographic brane scenarios discussed here.

4 FRW Cosmology on the Moving Brane

In the last section we have considered the brane world scenarios as the generalization of AdS/CFT correspondence, by which the intrinsic geometry on the static brane is induced by the dual QFT on the brane. Although it is very intriguing to have covariant intrinsic gravity on the brane, it is difficult to generalize such program to a moving brane since the energy scale on the brane keeps changing all the time, which looks bizarre from the holographic RG point of view. Instead an extrinsic approach is more appropriate in which the brane cosmology is derived from the extrinsic geometry of the brane by using the Israel patching condition [18]. However, from the general holographic principle point of view, we should expect the equivalence of the intrinsic and extrinsic approaches, which remains to be explored. Moreover, the FRW cosmology for a moving brane is also known as the mirage cosmology from the point of view of the effective theory on the brane [24], that is, the holographic image of the bulk gravity serves as the dark energy which drives the evolution of the brane universe.

To derive the brane FRW equations for a brane moving along the radial direction, we can add the following Gibbons-Hawking boundary action to (2.1)

$$S_{GH} = - \int_\Sigma d^{n+1}x \sqrt{-\gamma} K,$$

(4.1)

\(^7\)For $b^2 = 4/3$ the potential in the Einstein frame is nonzero, this may indicate the inequivalence between the intrinsic holographic gravity and the extrinsic approach based upon some generalization of the method developed by [27, 28]; however, it needs further investigations to clarify this point.
where $K$ is the extrinsic curvature. The brane FRW equations can be obtained from the following Israel junction condition with brane situated at $r = R$

$$\Delta K_{\mu\nu}(r = R_+) - K_{\mu\nu}(r = R_-) = -(T_{\mu\nu} - \frac{1}{n}T\gamma_{\mu\nu}) ,$$

(4.2)

which specifies the patching condition of the bulk geometry on the two sides of the brane with the boundary energy-momentum tensor $T_{\mu\nu} \equiv \frac{2}{\sqrt{-\gamma}}\delta S_\Sigma \delta \gamma_{\mu\nu}$. As remarked at beginning of this section, only the effective tension term of the brane, $U(\phi)$ of $S_\Sigma$ is kept since we are taking an extrinsic approach, of which there is no essential information about the intrinsic geometry on the brane. Moreover, by assuming the bulk spacetime is symmetric under the reflection with respect to the brane, (4.3) becomes

$$K_{ij} = -\frac{1}{2n} \gamma_{ij} U(\phi).$$

(4.3)

Rewriting the junction condition (4.3) into the FRW equations on the brane has been done in [10, 23, 22] for the AdS gravity of which $U(\phi)$ is a constant, especially a moving brane embedded in a Anti-de Sitter-Schwarzschild space results in a brane FRW cosmology with the CFT matters [10, 22]. Here we will generalize it to the dilatonic gravity with nontrivial $U(\phi)$ for the following bulk geometry

$$g_{MN}dz^M dz^N = -f(r)dt^2 + g(r)dr^2 + h(r)\delta_{ij}dx^i dx^j \equiv g(r)dr^2 + \gamma_{\mu\nu} dx^\mu dx^\nu ,$$

(4.4)

with $i,j$ run from 1 to $n$.

For the simplicity, we now restrict to the $n = 3$ case, and consider a moving brane with the trajectory described by $R(\tau)$ where $\tau$ is the proper time defined by

$$-d\tau^2 = -f(R)dt^2 + g(R)dR^2 ,$$

(4.5)

then the brane metric takes the FRW form of (1.1) with $a^2(\tau) = h(R(\tau))$. Once the trajectory of the moving brane is assumed with respect to the proper time, we can calculate the extrinsic curvature $K_{\mu\nu} \equiv \frac{1}{2}n^M \partial_M \gamma_{\mu\nu}$ with the unit normal $n^M$ defined with respect to the velocity vector $u^M = (\dot{t}, \dot{R}, 0, 0, 0)$, where dot denotes the derivative with respect to $\tau$. The junction condition (4.3) then turns out to have the form of FRW equation for the cosmology on the moving brane as

$$\frac{\dot{a}}{a}^2 = -\frac{1}{4g} \left( \frac{h'}{h} \right)^2 + \left( \frac{U}{6} \right)^2 .$$

(4.6)

To get familiar with the above FRW equation, we consider the simplest case for the bulk to be the pure black hole background without dilaton, we have $f(r) = g^{-1}(r) = k - V_0 r^2 / 3 - M/r$
with $k = -1, 0, 1$, $h(r) = r^2$ and $U = U_0, V = V_0$ with $U_0, V_0$ constants, then the FRW equation (4.6) becomes
\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{-k}{a^2} + \frac{M}{a^4} + \frac{V_0}{3} + \left(\frac{U_0}{6}\right)^2.
\] (4.7)

It is clear that this corresponds to the (3+1)-dim. radiation dominated FRW cosmology with a cosmological constant proportional to $\frac{\dot{a}}{a} + \left(\frac{U_0}{6}\right)^2$ which can be fine tuned to zero only for bulk anti-de Sitter space with $V_0 < 0$. This kind of fine-tuning is the brane version of cosmological constant problem.

Another interesting example is the dilatonic background for the self-tuning (static) brane proposed in [25, 26] to solve the brane version of the cosmological constant problem, in which $V_0 = 0$ but $U = U_0 e^{-\sqrt{\frac{3}{4}} \phi}$, and the metric is\(^8\)
\[
ds^2 = dr^2 + e^{2A(r)} dx^2_{brane},
\] (4.8)
where $A(r)$ is related to the dilaton profile which is
\[
\phi(r) = 2\sqrt{3} A(r) = \sqrt{\frac{3}{4}} \log \left| \frac{4r}{3} + c_1 \right| + c_0,
\] (4.9)
with $c_{0,1}$ the integration constants which can be exploited to tune the brane cosmological constant [25, 26]. It is easy to see that the bulk geometry has a naked singularity at finite $r$ although it has been argued that this kind of singularity are harmless according to the criterions given in [32, 33].

Instead of considering the static brane, we now embed a moving brane in such a background, the brane cosmology is then described by the following FRW equations
\[
\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{-e^{\frac{4A}{3}}}{9} + \left(\frac{U_0}{6}\right)^2\right) \frac{1}{a^8}.
\] (4.10)

Note that the scale behavior of the holographic matter on the RHS of the above FRW equation is neither of CFT nor of the quintessence. In some sense the above FRW cosmology has zero cosmological constant since the holographic matter is highly damped for the large scale factor and then leave nothing on the RHS of the FRW equation. This fact coincides with the claim of self-tuning cosmological constant for the static brane case [25, 26].

Moreover, if the holographic matter in (4.10) has positive energy by proper choice of the parameters $C_0$ and $U_0$, then the brane is inflating and moving away from the naked

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\(^8\)Since we have a different normalization of the scalar kinetic term from the one in [25, 26], the corresponding factors in the $A$ and $\phi$ change accordingly.
singularity which thus will not cause any physical problem in the history of the evolution of the brane universe. In this sense, the naked singularity is harmless. On the other hand, if the holographic matter has negative energy, then the brane world is moving toward the singularity and will finally crash on it even our universe is flat. The analogue of a naked singularity in the brane world scenarios and the big bang (crunch) singularity has been exploited in [33] to give some criterion for the allowing naked singularities which appear generically in many bulk geometry, and this kind of criterion is argued to be equivalent to the one in [32].

Despite that the self-tuning brane model provide a possible solution to the cosmological constant problem, from the point of view of the FRW cosmology, the holographic matters induced on the brane do not yield the quintessence. In the next section we will see that some more general dilatonic background does induce holographic quintessence.

5 Holographic Quintessence on the Moving Brane

There are three types of dilatonic backgrounds [21] (see also [19].) which solve the bulk equations of motion of (2.1) with the bulk and boundary dilaton potentials taken the following form

\[ V = V_0 e^{\beta \phi}, \quad U = U_0 e^{\alpha \phi}. \] (5.1)

The cosmological behaviors on the brane in these backgrounds have been discussed in details in [21], in the following we recapitulate their results but focus on the possible existence of the quintessence.

The 1st type background is just the one given in the last section with \( \alpha = \beta = 0 \) which gives the radiation dominated FRW cosmology. We then look into the 2nd type dilatonic background with \( \alpha = \beta/2, k = 0 \) and

\[ f(r) = g^{-1}(r) = (1 + \delta^2)^2 r^{2(1+\delta^2)} (-2M r^{-\frac{4-\delta^2}{1+\delta^2}} - \frac{2V_0 e^{2\delta \phi_0}}{3(4 - \delta^2)}), \] (5.2)

\[ h(r) = r^{\frac{2}{1+\delta^2}}, \quad \phi(r) = \sqrt{3}(\phi_0 - \frac{\delta}{1 + \delta^2} \log r), \] (5.3)

where \( M \) and \( \phi_0 \) are integration constants and \( \delta = \frac{\sqrt{3} \beta}{2} = \sqrt{3} \alpha \).

The FRW equation (4.6) of the brane in this background becomes

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{2M}{a^{4+\delta^2}} + \frac{\dot{V}_0}{a^{2\delta^2}}. \] (5.4)
where $\dot{V}_0 \equiv \left(\frac{2V_0}{3(4-\delta^2)} + \frac{U_0^2}{36}\right)e^{2\delta\phi_0}$. In the late time the second term dominates and is quintessential if

$$\delta^2 < 1 . \quad (5.5)$$

To ensure the positive energy condition for the all time, it requires $M, \dot{V}_0 > 0$ which corresponds to a bulk dilatonic black hole background for $V_0 < 0$ [19, 21]. Moreover, the factor $e^{2\delta\phi_0}$ implies no need for fine-tuning for the energy density of the quintessence.

Similarly, the brane FRW cosmology in the 3rd type dilatonic background is

$$\left(\frac{\dot{a}}{a}\right)^2 = 2\delta^4 M \left(\frac{a_0}{a}\right)^{4+1\over 2} + \frac{2\delta^4 V_0 e^{2\delta\phi_0}}{3(1+2\delta^2)} \left(\frac{a_0}{a}\right)^2 + \left(\frac{U_0 e^{\phi_0}}{6}\right)^2 \left(\frac{a_0}{a}\right)^{3\over 2} . \quad (5.6)$$

and the background geometry is given by

$$f(r) = g^{-1}(r) = (1 + \delta^2)^2 r_0^{4+1\over 2} (-2Mr^{1+2\delta^2} - \frac{2V_0 e^{2\delta\phi_0}}{3(1+2\delta^2)}) , \quad (5.7)$$

$$h(r) = a_0^2 r_0^{2\delta^2} , \quad \phi(r) = \sqrt{3}(\phi_0 - \frac{\delta}{1+\delta^2} \log r) , \quad (5.8)$$

and $\delta = \sqrt{3\beta} \equiv 1 \over \sqrt{3\alpha}$. $a_0 = \left|V_0 e^{2\delta\phi_0}(1-\delta^2)^{-1/2}\right|^{-1/2}$.

It is then easy to see that in the late time the quintessence requires

$$\delta^2 > 1 . \quad (5.9)$$

For the bulk geometry to be a black hole type, it requires $V_0 < 0, M > 0$ [19, 21], then the positive energy condition may be violated.

To explore a little bit on the nature of the above quintessence, we can add the local matter on the brane. As usual, we focus on the perfect fluid matter with energy-momentum tensor

$$T^{(m)}_{\mu\nu} = \rho^{(m)} u_\mu u_\nu + p^{(m)} \left(\gamma_{\mu\nu} + u_\mu u_\nu\right) \quad (5.10)$$

where $\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$. Then the FRW equation from the Israel junction condition changes to

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{4g} \left(\frac{h'}{h}\right)^2 + \left(\frac{\rho^{(m)} + U}{6}\right)^2 . \quad (5.11)$$

For the 1st type bulk background $U$ is constant, so we have the well-known $(\rho^{(m)})^2$ term different from the conventional FRW cosmology [27]. On the other hand, for the 2nd and 3rd type quintessential brane, $U$ is not a constant and will dress its scale dependence on the
\( \rho^{(m)} \) and \((\rho^{(m)})^2\). If we restore the Newton constant dependence in the FRW equation, that is,

\[
H^2 = \frac{8\pi G_N}{3} \rho ,
\]

(5.12)

we can absorb the dressing scale dependence of the \( \rho^{(m)} \) term into the Newton constant \( G_N \) so that the matter density \( \rho^{(m)} \) scales as required by the energy conservation and equation of state. This implies that the quintessence on the moving brane has the nature of a mildly time-varying Newton constant since the dressing scale dependence due to the quintessence is very slowly varying. Similar conclusion about the time-varying Newton constant for a moving brane of zero cosmological constant is also discussed in \[20\]. We also expect the similar situation for the static brane case. Moreover, these dressing will red-shift the matter energy.

6 Conclusion

Dilaton definitely has its deep origin in string theory and supergravity. In this paper we show that some dilaton potentials will induce the holographic quintessences on the brane, which provides a natural candidate to conform to the cosmological observational data in the brane world scenarios. By the way, it is interesting to compare our result with the efforts in getting the quintessences from some dilaton potentials in supersymmetric theory of supergravity origin. In \[3\] it has been shown that the quintessential behavior is incompatible with the positive energy condition in the case of the dilaton potential in the supersymmetric theory. However, as shown in this paper, the holographic principle shows that the boundary potential is related to the bulk potential by some inverse super-potential like relation, and there is no inconsistency between quintessence and positive energy condition.

We also see that the intrinsic brane gravity based upon the Hamilton-Jacobi formalism of holographic RG approach provide a useful tool in studying the brane cosmology. But it is speculative to claim the equivalence between this intrinsic approach and the extrinsic ones \[27\] in generalizing to the dilaton gravity \[29\] \[28\], which requires more investigations. It deserves more study on the phenomenological aspects of the above quintessential brane world scenarios. Especially the quintessence on the moving brane has the effect as the time-varying Newton constant which may lead to some novel effect beyond the standard FRW cosmology. Moreover, it is also interesting to look into the interplay between the sectors of holographic and normal matter to find a way out of the eternal acceleration of the quintessential universe in the brane world scenarios.
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