Sub-barrier fusion hindrance and absence of neutron transfer channels

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Received: 02 November 2022 / Accepted: 14 March 2023 / Published online: 6 April 2023

Abstract: The sub-barrier fusion hindrance has been observed in the domain of very low energies of astrophysical relevance. The measured fusion cross sections can be well estimated by using formula obtained by folding a Gaussian function for barrier height distribution with the expression for classical fusion cross section for fixed barrier. The parameters of the formula vary smoothly implying its usage in estimating the excitation function. In the present work, the effects of deformation and transfer on fusion reactions have been studied and the absence of neutron transfer channels has been correlated to the hindrance in heavy-ion fusion reactions.

Keywords: Fusion reactions; Excitation function; Barrier distribution; Nucleosynthesis

1. Introduction

The sub-barrier fusion hindrance has been observed in the domain of very low energies of astrophysical relevance. The energy generation by heavy-ion fusion reactions can be affected to a large extent due to fusion hindrance. In the region of extreme sub-barrier energies, even in the lighter systems such as the carbon and oxygen burning stages in heavy stars [1–3] including their evolution and elemental abundances are also influenced. This phenomenon was analyzed previously [4] using an elegant formula derived by folding a Gaussian function representing the fusion barrier height distribution with the expression for classical fusion cross section for fixed barrier. The variation of fusion cross section as a function of energy described well the existing data on sub-barrier heavy-ion fusion cross sections of interacting nuclei from ¹⁶O + ¹⁸O to ¹²C + ¹⁹⁸Pt, all of which were measured down to < 10 μb. In the framework of any theory of nuclear reactions describing fusion reactions which involves coupling to several collective states [5–12], this coexistence of different barriers is naturally accounted.

In the present work, the extent of coexistence of different barriers is analyzed in terms of transfer and deformation effects. The one neutron and two neutron transfer Q-values from projectile to target nuclei and from target to projectile nuclei have been calculated. Both the negative and positive fusion Q-value cases have been studied. The absence (presence) of neutron transfer channels has been correlated to the hindrance (enhancement) in heavy-ion fusion reactions.

2. Theoretical formalism

In order to replicate the dependence of nuclear cross sections of fusion reaction on the collisional kinetic energy, specifically measured at low energies near fusion threshold, the assumption of a fusion barrier height distribution becomes necessary to simulate the effects resulting from the coupling to other channels. In the coupled-channel calculations it is naturally achieved which involve, in both colliding nuclei, the coupling to collective states down to the lowest level. The nuclear structure effects influencing the distribution of potential energy barriers have been considered negligible and hence ignored in this work. A Gaussian form \( D(h) \) simulating the shape of the diffused barrier has been conceptualized [13] for the fusion barrier height distribution. Therefore, the distribution of barriers is provided by \( D(h) = \frac{1}{\sqrt{2\pi} \sigma_h} \exp \left( -\frac{(h-h_0)^2}{2\sigma_h^2} \right) \) where for each individual reaction, the parameters \( h_0 \) (mean barrier height) and \( \sigma_h \) (width of barrier distribution) are to be determined exclusively. A mathematical formula for the cross section

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can be derived for surmounting the barrier arising due to interacting nuclei by folding the Gaussian distribution for fusion barriers with classical nuclear fusion reaction cross-sectional expression

\[ \sigma_f(h) = \pi R_h^2 \left[ 1 - \frac{h^2}{E} \right] \quad \forall \ h \leq E, \quad \text{and} \quad 0 \ \forall \ h \geq E \] 

where \( R_h \) marks, approximately, the position of the barrier (effective radius corresponding to the relative distance), which results in the following expression [13, 14]

\[
\sigma_c(E) = \int_{E_0}^{E} \sigma_f(h)D(h)\,dh \\
= \int_{E_0}^{h_0} \sigma_f(h)D(h)\,dh + \int_{h_0}^{E} \sigma_f(h)D(h)\,dh \\
= \pi R_h^2 \frac{\sigma_h}{E\sqrt{2\pi}} \left[ \xi^2 \text{erf}(\xi) + \text{erf}(\xi_0) \right] + e^{-\xi^2} - e^{-\xi_0^2} 
\]

where \( E_0 = 0 \) for positive \( Q \) value reactions and \( E_0 = Q \) for negative \( Q \) value reactions, \( Q \) value being the sum of the rest masses of fusing nuclei minus rest mass of the resultant fused nucleus, \( \xi = \frac{E-h}{\sigma_{\theta}\sqrt{2}} \), \( \xi_0 = \frac{E-h}{\sigma_{\theta}\sqrt{2}} \) and \( \text{erf}(\xi) \) is the Gaussian error integral for argument \( \xi \). The three parameters \( R_h, h_0 \) and \( \sigma_h \) have to be determined by a least square fitting of Eq. (1) to the measured excitation functions for fusion reactions. While deriving the formula of Eq.(1), the quantal effects of barrier penetration have not been taken explicitly into account. The structure of a given excitation function for fusion reaction is, however, influenced by the sub-barrier tunneling which has been included effectively through the parameter \( \sigma_h \) which describes the width of barrier distribution.

3. Results and discussion

3.1. The fusion excitation functions

The excitation functions for nuclear reactions of heavy-ion fusion for lighter systems of astrophysical interest at sub-barrier energies have been analyzed theoretically. This has been facilitated through Eq.(1) which is obtained by folding an elegant diffused barrier distribution of Gaussian shape together with the classical mathematical fusion cross-sectional expression for single fixed barrier. Identical combinations of projectile-target system involved in heavy ion sub-barrier fusion reactions have been chosen which have been recently [15–19] studied. By using the method of least-square fitting, the values of three parameters \( h_0 \) (mean barrier height), \( \sigma_h \) (width) and \( R_h \) (effective radius) have been extracted. The values of these are tabulated in Table 1 arranging it in the ascending order of projectile masses.

The task of estimating cross section \( \sigma_c(E) \) for a particular reaction rests upon predicting the parameter values for \( h_0 \) (mean barrier height), \( \sigma_h \) (width) and \( R_h \) (effective radius) reliably. Since \( h_0 \) is necessarily the mean barrier height, it should be a function of \( z = Z_1Z_2/(A_1^{1/3} + A_2^{1/3}) \) (Coulomb parameter) in neighborhood of the fusion barrier. The third entity, the effective barrier radius \( R_h \), unquestionably should depend upon \( r_0(A_1^{1/3} + A_2^{1/3}) = r_0A_{12} \) where \( r_0 \) is the nuclear radius parameter. The extrapolation of the tendencies of \( \sigma_h \) is extra difficult, which basically arise due to nuclear deformation, vibrations and quantum mechanical barrier tunneling probability. It may be observed from Table 1 that the mean barrier height \( h_0 \) increases with the Coulomb parameter \( z \) for all cases while the effective radius \( R_h \) also increases with \( A_{12} \) except for \(^{16}\text{O} + ^{18}\text{O} \) and \(^{28}\text{Si} + ^{64}\text{Ni} \) and interestingly, as may be seen from Figs.-1–7 of Ref.[4], for \(^{16}\text{O} + ^{18}\text{O} \) case, the theoretical calculations are slightly off at lower energies as well.

The results for the colliding \(^{12}\text{C} + ^{24}\text{Mg} \), \(^{12}\text{C} + ^{30}\text{Si} \), \(^{12}\text{C} + ^{198}\text{Pt} \), \(^{16}\text{O} + ^{18}\text{O} \), \(^{28}\text{Si} + ^{64}\text{Ni} \), \(^{58}\text{Ni} + ^{58}\text{Ni} \) and \(^{64}\text{Ni} + ^{64}\text{Ni} \) systems are illustrated in Figs.-1–7 of Ref.[4]. It may be easily identified from the plots that accurately measured excitation functions for fusion reactions yield a systematized information on the cardinal attributes of the nucleus-nucleus interaction potential, viz., \( h_0 \) (mean barrier height) and \( \sigma_h \) (width) of its distribution for collisions between two nuclei. In Table 1, the magnitudes of the rms errors associated with the parameters \( \sigma_h, h_0, R_h \) and \( r_0 \) imply that except for the case of \(^{58}\text{Ni} + ^{58}\text{Ni} \), these values are quite small. Thus it may be concluded that all capture events lead to complete fusion except at lower energy end of \(^{58}\text{Ni} + ^{58}\text{Ni} \) reaction. It is interesting to mention here that in a semi-microscopic calculation based on realistic (DDM3Y) nucleon-nucleon potential [20], the values of \( h_0 \) and \( R_h \) obtained were 100.5 MeV and 10.39 fm, respectively, which are strikingly close to the values listed in Table 1. In Fig.-6 of Ref.[4], the theoretical curve slightly overestimates the measured data at energies below 95 MeV, but as \( z \) is particularly high in this case, it necessitates further experimental investigation in this region before concluding incomplete fusion. The capture or fusion cross sections for planning experiments can be also guessed using Eq.(1) along with the theoretically extracted values of \( h_0 \) and \( \sigma_h \) parameters.

Figures 1–7 of Ref.[4] show that the theoretical estimates by the diffused barrier formula resulted in good fits to the measured experimental data. This observation obviously infers that almost all the events leading to capture proceed to fusion for the chosen set of nuclei resulting in capture cross sections being essentially identical to fusion cross sections [21]. It may further imply that the for
the barrier distribution, the choice of Gaussian form describes quite well the nuclear cross sections for fusion reactions at energies below the barrier. This fact justifies the model ‘beyond single barrier’ which arises out of vibration and deformation of nuclei and more importantly tunneling. Whereas theoretically ‘barrier distribution’ is a valid concept under a few approximations, the fact that fits to the experimentally measured data are good implies certainly that in fusion reactions involving heavy-ions, it remains a meaningful concept at least for lighter systems of astrophysical interest.

It is pertinent to mention here that although the single-Gaussian parametrization for barrier distribution is reasonably successful in providing a good description of fusion process in general, neither the formula derived for fusion cross section nor the method using the barrier distribution can be put to use for all fusing systems. One can visualize from Eq.(1) that the excitation function is all the time a monotonically rising function of energy. This puts a limitation on Eqs.(1, 2) which cannot be used for describing fusion reactions at higher energies when incomplete fusion as well as deep-inelastic scattering can cause a lowering of the fusion cross section. Similar limitation arises for lighter systems (e.g., $^{12}\text{C} + ^{13}\text{C}, ^{12}\text{C} + ^{16}\text{O}, ^{16}\text{O} + ^{16}\text{O}$, etc.) as well when excitation functions possess oscillations and resonance structures. The possibility of better agreement to data may further be explored by opting for a more intricate formula for the barrier distribution, which, nonetheless, will bring in more additional adjustable parameters than just three used in the present work. Such refinements for the barrier distribution used in Eq. (1) may include distributions having different widths on lower or higher energy sides, certain moderation of the exponent in Gaussian distribution form or multi-component distributions.

### Sub-barrier fusion hindrance and absence of neutron

The anticipation of the swing of $\sigma_b$ is easier said than done. The reason lies in the fact that $\sigma_b$ crops up mainly due to nuclear vibrations, deformation and quantum mechanical barrier tunneling probability. A nucleus (tagged $i$) with a static deformation of magnitude $\beta_{2}(i)$ can have all possible orientations which lead to a standard deviation (SD) of $\Delta R_i$ in the effective radius $R_i$ given by

$$\Delta R_i = \frac{\beta_{2}(i)R_i}{\sqrt{4\pi}}$$

where except for the quadrupole, all other higher multipoles have been neglected. Thus, for a fixed distance between the centers of mass of two nuclei, the distribution of the sub-sequential surface-surface distance results in the SD $\sigma_i$ for the barrier height distribution given by

$$\sigma_i = \left[ \frac{\partial V}{\partial r} \right]_{r=R_i} \Delta R_i$$

$$= Z_1Z_2e^2 \frac{\beta_{2}(i)}{R_b} \frac{R_i}{\sqrt{4\pi R_b}} \left[ 1 - \frac{3R_i}{5R_b} \right]$$

Therefore, $\sigma_b$ can be given by

$$\sigma_b = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_0^2}$$

where $R_b = R_1 + R_2 = r_0[A_1^{1/3} + A_2^{1/3}],$ $\Delta R_1$ and $\Delta R_2$ are the SDs of the radius vectors specifying the surfaces of the target as well as the projectile nuclei having mean radii $R_1$ and $R_2$ and quadrupole deformation parameters $\beta_{2}(1)$ and $\beta_{2}(2)$, respectively. The quantity $\sigma_0$ in the above Eq. (4) is an adjustable parameter which, at least approximately, takes into account the nuclear vibrations and quantum mechanical barrier tunneling probability. Manifestly, for semi-magic as well as magic nuclei, $\sigma_1 = \sigma_2 = 0$ and then $\sigma_b = \sigma_0$.

#### Table 1

| Reaction [Ref.] | $\text{Z}$ | $A_{12}$ | $\sigma_b$ [MeV] | $h_0$ [MeV] | $R_b$ [fm] | $r_0 = \frac{h_0}{h_b}$ [fm] |
|----------------|-------------|-----------|------------------|-----------|----------|------------------|
| $^{12}\text{C} + ^{24}\text{Mg}$ [15] | 13.916 | 5.174 | 0.815±0.028 | 11.483±0.049 | 6.646±0.110 | 1.285±0.021 |
| $^{12}\text{C} + ^{30}\text{Si}$ [16] | 15.565 | 5.397 | 1.090±0.060 | 13.540±0.144 | 8.300±0.213 | 1.538±0.039 |
| $^{12}\text{C} + ^{108}\text{Pt}$ [17] | 57.650 | 8.118 | 1.749±0.038 | 55.140±0.095 | 11.179±0.113 | 1.377±0.014 |
| $^{16}\text{O} + ^{16}\text{O}$ [15] | 12.450 | 5.141 | 0.859±0.020 | 9.797±0.053 | 7.743±0.183 | 1.506±0.036 |
| $^{28}\text{Si} + ^{64}\text{Ni}$ [18] | 55.709 | 7.037 | 1.402±0.072 | 50.403±0.109 | 7.182±0.091 | 1.021±0.013 |
| $^{58}\text{Ni} + ^{58}\text{Ni}$ [19] | 101.269 | 7.742 | 2.275±1.022 | 98.278±1.799 | 8.550±1.494 | 1.104±0.193 |
| $^{64}\text{Ni} + ^{64}\text{Ni}$ [19] | 98.000 | 8.000 | 1.466±0.091 | 92.646±0.138 | 8.862±0.093 | 1.108±0.012 |
The fusion $Q$ values, being the sum of the rest masses of fusing nuclei minus rest mass of the resultant fused nucleus, have been calculated using atomic mass excesses [23]. In Table 2, the fusion $Q$ values, the effective radius parameter $r_0$ and the quantities $\sigma_1$, $\sigma_2$ and $\sigma_0$, obtained from the analyses of the measured fusion excitation functions, have been listed. For obtaining the values of $\sigma_1$ and $\sigma_2$, the theoretical values of static deformation $\beta_2(i)$ [24] from recent tabulation [25] have been used while the measured nuclear deformations are available in Ref.[26].

### 3.3. Fusion hindrance and neutron transfer $Q$-values

The one neutron and two neutron transfer $Q$-values from projectile to target nuclei $Q_{1n}$ and $Q_{2n}$, respectively, and those from target to projectile nuclei (except for $^{28}$Si+$^{64}$Ni) are all negative. The reactions studied in the present work, the transfer effects are, therefore, absent leading to fusion hindrance. Thence the width $\sigma_h$ of the distribution function is not enhanced due to any mixture of transfer and deformation effects. It is worthwhile to mention here that for reaction $^{32}$S+$^{110}$Pd for which neutron transfer $Q$-values are positive (and hence a mixture of transfer and deformation effects) leading to large $\sigma_h = 3.10$ MeV which is significantly greater than $\sigma_h = 1.92$ MeV for the reaction $^{36}$S+$^{110}$Pd for which neutron transfer $Q$-values are negative (and only pure deformation effect due to the deformed Pd) [27]. In general, for positive fusion $Q$-values, the fusion cross section should be more. But for the fusion reactions $^{12}$C+$^{34}$Mg, $^{12}$C+$^{30}$Si and $^{16}$O+$^{18}$O with positive $Q$-values, fusion hindrance remains owing to particularly small $\sigma_0$ values as is evident from Eqs.(3, 4). Such an effect is due to the saturation properties of nuclear matter, which hinders density built up and prevents substantial overlap of light nuclei participating in reactions causing hindrance in quantum tunneling. This leads to rapid decrease in fusion cross section characterizing a major impact on the estimations of thermonuclear reaction rates which play a very significant role in stellar evolution studies.

### 4. Conclusions

In the region of sub-barrier energies, the fusion reaction cross sections spanning a broad energy range can be estimated using the diffused-barrier formula [13, 27]. The values of the essential parameters $h_0$ (mean barrier height), $\sigma_h$ (width) and $R_h$ (effective radius) can be determined by least-square fit of the measured data. In order to have a systematic knowledge on the essential characteristics of the interacting potential (viz., $h_0$, $\sigma_h$, $R_h$), a set of accurately measured excitation functions of fusion reactions has been studied for two colliding nuclei. In the fusion reactions studied here, the absence of neutron transfer channels has been correlated to the hindrance in heavy-ion fusion reactions. Thus, widths of the barrier distribution are not enhanced due to any mixture of transfer and deformation effects. Even for positive fusion $Q$-values, the fusion hindrance remains because of small $\sigma_h$.

It is possible to examine the prospect of higher compliance to the observed data by selecting a more complicated barrier distribution formula [28], which, however, will introduce extra adjustable parameters than just three utilized in the present work. Such improvements in barrier distribution can be realized through distributions having distinctive widths on the lower or higher energy sides, multi-component distributions or a modification of the exponent appearing in distribution represented by a Gaussian form.

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**Table 2** The list of extracted values of effective radius parameter $r_0$ and the quantities $\sigma_1$, $\sigma_2$ and $\sigma_0$, obtained from the analysis of the measured fusion excitation functions along with the deformations used in this work are presented. The table is arranged in order of increasing projectile mass along with the fusion $Q$ values. The one neutron and two neutron transfer $Q$-values from projectile to target nuclei $Q_{1n}$ and $Q_{2n}$, respectively, are also listed while neutron transfers from target to projectile nuclei are provided within parentheses.

| Reaction (1) + (2) | $Q$ (fusion) [MeV] | $Q_{1n}$ [MeV] | $Q_{2n}$ [MeV] | $r_0$ [fm] | $\beta_2(1)$ Ref.[25] | $\beta_2(2)$ Ref.[25] | $\sigma_1$ [MeV] | $\sigma_2$ [MeV] | $\sigma_0$ [MeV] |
|-------------------|-------------------|----------------|----------------|------------|---------------------|---------------------|----------------|----------------|----------------|
| $^{12}$C+$^{34}$Mg | 16.298            | -11.391 (-3.866) | -13.418 (-0.739) | 1.285      | 0.000               | 0.393               | 0.000          | 0.6417         | 0.5025         |
| $^{12}$C+$^{30}$Si | 14.114            | -12.134 (-5.663) | -16.051 (-5.960) | 1.538      | 0.000               | -0.236              | 0.000          | 0.3656         | 1.0268         |
| $^{12}$C+$^{108}$P | -13.955           | -13.166 (-2.611) | -19.004 (-0.281) | 1.377      | 0.000               | -0.115              | 0.000          | 0.7992         | 1.5557         |
| $^{16}$O+$^{18}$O | 24.413            | -11.709 (-3.910) | -17.323 (0.0)    | 1.506      | -0.010              | 0.010               | 0.0116         | 0.0119         | 0.8588         |
| $^{28}$Si+$^{60}$Ni | -1.787            | -11.082 (-1.185) | -15.441 (2.587)  | 1.021      | -0.363              | -0.094              | 2.5738         | 0.7806         | 0.0000         |
| $^{58}$Ni+$^{64}$Ni | -65.855           | -3.218 (-3.218)  | -2.079 (-2.079)  | 1.104      | 0.000               | 0.000               | 0.0000         | 0.0000         | 2.2750         |
| $^{64}$Ni+$^{64}$Ni | -48.797           | -3.560 (-3.560)  | -1.446 (-1.446)  | 1.108      | -0.094              | -0.094              | 1.1823         | 1.1823         | 0.0000         |
Acknowledgements One of the authors (DNB) acknowledges support from Science and Engineering Research Board, Department of Science and Technology, Government of India, through Grant No. CRG/2021/007333.

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