**$D^0$ Mixing**

B. Golob  
*University of Ljubljana and Jozef Stefan Institute, Ljubljana, Slovenia*

An overview of selected experimental results in the field of $D^0$-$\bar{D}^0$ oscillations is presented. The average results for the mixing parameters, $x = (0.89 \pm 0.27)\%$ and $y = (0.75 \pm 0.18)\%$, exclude the no-mixing hypothesis at the level of 6.7 standard deviations. No sign of CP violation in the $D^0$ system is observed. The measurements impose constraints on the parameter space of many New Physics models.

1. Introduction

Last year, 31 years after the discovery of $D^0$ mesons, the first evidence of a mixing phenomena in the system of neutral charm mesons has been obtained [1,2]. Following this breakthrough were several additional measurements enabling - through an averaging procedure - a quite precise determination of the parameters governing the mixing. The results presented in the paper follow from the data collected by the two B-factories experiments, Belle and BaBar, from the charm-factory experiment Cleo-c, as well as from the proton collider experiment CDF. At the B-factories the cross-section for neutral charm meson production with a transverse momentum larger than 5.5 GeV/c yields a starting data sample of $10^8$ pairs produced in a coheren $C = -1$ state. And while at the Tevatron the experimental environment for the presented measurements is more difficult, the cross-section for neutral charm meson production with a transverse momentum of an initially produced $D^0$ meson is achieved by reconstruction of decays $D^{*-} \rightarrow D^0 \pi^+_s$ or $D^{*-} \rightarrow \bar{D}^0 \pi^-_s$. The charge of the characteristic low momentum pion $\pi_s$ determines the tag. The energy released in the $D^*$ decay,

$$q = M(D^*) - M(D^0) - m_\pi^0$$

has a narrow peak for the signal events and thus helps in rejecting the combinatorial background. Here, $M(X)$ is used to denote the invariant mass of the $X$ decay products, and $m_X$ stands for the nominal mass of $X$. $D^0$ mesons produced in $B$ decays have different decay time distribution and kinematic properties than the mesons produced in fragmentation. In order to obtain a sample of neutral mesons with uniform properties one selects $D^*$ mesons with momentum above the kinematic limit for $B$ meson decays (B-factories) or uses the impact parameter distribution to isolate primary charm mesons (CDF).

2. Measurements

Several methods and selection criteria are common to the presented measurements. Tagging of the flavour of an initially produced $D^0$ meson is achieved by reconstruction of decays $D^{*-} \rightarrow D^0 \pi^+_s$ or $D^{*-} \rightarrow \bar{D}^0 \pi^-_s$. The charge of the characteristic low momentum pion $\pi_s$ determines the tag. The energy released in the $D^*$ decay,

$$q = M(D^*) - M(D^0) - m_\pi^0$$

has a narrow peak for the signal events and thus helps in rejecting the combinatorial background. Here, $M(X)$ is used to denote the invariant mass of the $X$ decay products, and $m_X$ stands for the nominal mass of $X$. $D^0$ mesons produced in $B$ decays have different decay time distribution and kinematic properties than the mesons produced in fragmentation. In order to obtain a sample of neutral mesons with uniform properties one selects $D^*$ mesons with momentum above the kinematic limit for $B$ meson decays (B-factories) or uses the impact parameter distribution to isolate primary charm mesons (CDF).

2.1. Decays to CP eigenstates

In the limit of negligible CPV the mass eigenstates $D_{1,2}$ are also CP eigenstates. In decays $D^0 \rightarrow f_{CP}$ only the mass eigenstate component of $D^0$ with the CP eigenvalue equal to the one of $f_{CP}$ contributes. By measuring the lifetime of $D^0$ in decays to $f_{CP}$ one thus determines the corresponding $1/\Gamma_1$ or $1/\Gamma_2$. On the other hand, flavour specific final states like $K^-\pi^+$...
have a mixed CP symmetry. The measured value of the effective lifetime in these decays corresponds to a mixture of $1/T_1$ and $1/T_2$. The relation between the two lifetimes can be written as

$$\tau(f_{CP}) = \frac{\tau(D^0)}{1 + \eta_f y_{CP}} ,$$

where $\tau(f_{CP})$ and $\tau(D^0)$ are the lifetimes measured in $D^0 \to f_{CP}$ and $D^0 \to K^-\pi^+$, respectively. $\eta_f = \pm 1$ denotes the CP eigenvalue of $f_{CP}$. The relative difference of the lifetimes is described by the parameter $y_{CP}$.

CP-even final states $f_{CP} = K^+K^-, \pi^+\pi^-$ were used to measure $y_{CP}$ [6]. Expressed in terms of the mixing parameters, $y_{CP}$ reads

$$y_{CP} = y \cos \phi - \frac{1}{2} A_M \sin \phi ,$$

with $A_M$ and $\phi$ describing the CPV in mixing and interference between mixing and decays, respectively. In case of no CPV ($A_M, \phi = 0$) and $y_{CP} = y$.

Simultaneous fits to decay time distributions of selected $D^0 \to K^+K^-, K^-\pi^+$ and $\pi^+\pi^-$ candidates were performed with $y_{CP}$ as a common free parameter. The fit is presented in Fig. 1(a)-(c). The agreement of the fit function with the data is excellent, $\chi^2/n.d.f = 312/289$. The final value obtained is

$$y_{CP} = (1.31 \pm 0.32 \pm 0.25)\% .$$

The largest systematic uncertainties arise from the assumed resolution function (common offset in individual decay modes), possible deviations of acceptance dependence on decay time from a constant (estimated by a fit to the generated t distribution of reconstructed MC events) and variation of selection criteria (effect estimated using high statistics MC samples).

The resulting $y_{CP}$ is more than 3 standard deviations above zero and hence represents clear evidence of $D^0 - \overline{D^0}$ mixing, regardless of possible CPV. The difference of lifetimes is made visually observable by plotting the ratio of decay time distributions for decays to $f_{CP}$ and $K^-\pi^+$ in Fig. 1(d).

Recently the BaBar collaboration performed a similar measurement [7], with results for individual lifetimes shown in Fig. 2. The obtained value of the mixing parameter is

$$y_{CP} = (1.24 \pm 0.39 \pm 0.13)\% .$$

![Figure 1](image1.png)

Figure 1: (a)-(c): Result of the simultaneous fit to decay time distributions in $D^0$ decays to $KK$, $K\pi$ and $\pi\pi$ final states. The hatched areas represent the contribution of backgrounds. (d): Ratio of $D^0 \to f_{CP}$ and $D^0 \to K^-\pi^+$ decay time distributions. The slope visualizes the difference of effective lifetimes.

![Figure 2](image2.png)

Figure 2: Lifetimes of $D^0$ mesons measured in decays to $K\pi$, $KK$ and $\pi\pi$ (separately for $D^0$ and $\overline{D^0}$ for the latter two modes) [8]. The average value of the lifetime as measured in decays to CP eigenstates is lower than the one measured in flavour specific final state.

The two measurements make $y_{CP}$ the most precisely measured individual mixing parameter in the $D^0$ system.

Final states of definite CP allow also a search for possible CPV. Two methods, decay time dependent and time integrated, have been exploited. For the former, the lifetime in $D^0 \to f_{CP}$ is measured separately for $D^0$ and $\overline{D^0}$ tagged events. The asymmetry is [6]

$$A_T = \frac{\tau(D^0 \to f_{CP}) - \tau(D^0 \to f_{CP})}{\tau(D^0 \to f_{CP}) + \tau(D^0 \to f_{CP})} = \frac{1}{2} A_M y \cos \phi - x \sin \phi .$$

The values of $A_T$ measured by Belle [1] and BaBar [7]
are
\[ A_T = (0.01 \pm 0.30 \pm 0.15)\% \]
\[ A_F = (0.26 \pm 0.36 \pm 0.08)\% \] respectively, and show no sign of CPV at the level of around 0.3%.

With the time integrated method one measures the asymmetry
\[ A_{CP} = \frac{\Gamma(D^0 \rightarrow f_{CP}) - \Gamma(\bar{D}^0 \rightarrow f_{CP})}{\Gamma(D^0 \rightarrow f_{CP}) + \Gamma(\bar{D}^0 \rightarrow f_{CP})} = a^f_{dec} + a_{mix} + a_{int} \] \[ (9) \]

\( A_{CP} \) receives contribution from all three types of CPV, direct, CPV in mixing and in the interference between decays with and without the mixing. The latter two are independent of the final state. Experimenal the measured asymmetry must be corrected for possible charge asymmetries in the detection of the slow pion as well as the forward-backward asymmetry (\( A_{FB} \)) in the production of fermion pairs in \( e^+e^- \) collisions. The method of determination of \( \pi_0 \) correction factors was developed in \[ 8 \] using the untagged \( D^0 \rightarrow K^-\pi^+ \) decays. The forward-backward asymmetry is separated on the basis of its symmetry properties as a function of the \( D \) meson polar angle in the center-of-mass system. Figure 3 shows the measured \( A_{CP} \) ((a),(b)) and \( A_{FB} \) ((c),(d)) as a function of the \( D \) meson polar angle \[ 8 \]. Averaging over the polar angle yields the value
\[ A_{KK}^{CP} = (0.00 \pm 0.34 \pm 0.13)\% \] \[ (10) \]

Measurement of the Belle collaboration \[ 9 \]
\[ A_{KK}^{CP} = (-0.43 \pm 0.30 \pm 0.11)\% \] \[ (11) \]
is also consistent with no CPV.

2.2. Wrong-sign decays to hadronic final states

Decays of \( D \) mesons to two-body hadronic final states accessible to both, \( D^0 \) and \( \bar{D}^0 \), have traditionally been used to search for the mixing. Final state \( K^+\pi^- \) can be reached through a doubly Cabibbo suppressed (DCS) \( D^0 \) decay as well as through the \( D^0 \rightarrow \bar{D}^0 \) mixing followed by a Cabibbo favoured (CF) decay. The time evolution for these decays has three terms:
\[ |\langle K^+\pi^- | D^0(t) \rangle|^2 \propto [R_D + \sqrt{R_D}y't + (x'^2 + y'^2)4t^2]e^{-t}. \] \[ (12) \]
The first one is due to DCS decays, the third one due to the mixing, and the middle term represents the interference of the two contributions. \( R_D \) is the Cabibbo suppression factor relative to CF decays. \( x' \) and \( y' \) are the mixing parameters, rotated by a strong phase difference between DCS and CF decays, \( x' = x\cos \delta + y\sin \delta \) and \( y' = y\cos \delta - x\sin \delta \). The dimensionless time \( t \) is measured in units of \( \tau(D^0) \).

The BaBar collaboration obtained the first evidence for \( D^0 \) mixing by performing the decay time study of wrong-charge decays \( D^{+} \rightarrow D^0\pi^+ \), \( D^0 \rightarrow K^+\pi^- \) to separate the DCS and the mixing contribution. The parameters obtained from the fit are presented in terms of likelihood contours in Fig. 4 (top). The central value lies slightly in the non-physical region (\( x'^2 < 0 \)) and the no-mixing point (\( x'^2 = 0, y' = 0 \)) is excluded at the level corresponding to 3.9 standard deviations. Recently CDF collaboration obtained the result of similar significance \[ 10 \] shown in Fig. 4 (bottom left). Result form the Belle collaboration \[ 11 \] takes into account the presence of a physical boundary and is presented in Fig. 4 (bottom right) as a 95% C.L. contour calculated using the Feldman-Cousins method.

2.3. Time dependent Dalitz analyses

Several intermediate resonances can contribute to a hadronic multi-body final state. In a specific decay channel \( D^0 \rightarrow K_S\pi^+\pi^- \), recently analyzed by Belle \[ 12 \], contributions from CF decays (e.g. \( D^0 \rightarrow K^+\pi^- \)), DCS decays (e.g. \( D^0 \rightarrow K^+\pi^- \)) and decays to \( CP \) eigenstates (e.g. \( D^0 \rightarrow \rho^0K_S \)) are present. Individual contributions can be identified by analyzing the Dalitz distribution of the decay. Moreover, for a self-conjugated final state these different types of decays interfere and it is possible to determine their relative phases (unlike in the case of \( D^0 \rightarrow K^+\pi^- \) decays). Since these types of intermediate states also exhibit a specific time evolution one can determine directly the mixing parameters \( x \) and \( y \) by studying the time evolution of the Dalitz distribution.
The signal p.d.f. for a simultaneous fit to the Dalitz and decay-time distribution is

\[ \mathcal{M}(m_2^2, m_2^2, t) = \langle K_S^+ \pi^- | D^0(t) \rangle = \frac{1}{2} \mathcal{A}(m_2^2, m_2^2) [e^{-i\lambda_1 t} + e^{-i\lambda_2 t}] + \frac{1}{2} \mathcal{A}(m_2^2, m_2^2) [e^{-i\lambda_1 t} - e^{-i\lambda_2 t}] . \]  

The matrix element is composed of an instantaneous amplitude for $D^0$ decay, $\mathcal{A}(m_2^2, m_2^2)$, and an amplitude for the $\bar{D}^0$ decay, $\mathcal{A}(m_2^2, m_2^2)$, arising due to a possibility of mixing. They both depend on the Dalitz variables $m_2^2 = M^2(K_S^0 \pi^-)$ and $m_2^2 = M^2(K_S^+ \pi^-)$. The dependence on the mixing parameters is hidden in $\lambda_{1,2} = m_{1,2} - i\Gamma_{1,2}/2$. If CPV is neglected the amplitude for $D^0$ tagged decays is $\mathcal{M}(m_2^2, m_2^2, t) = \mathcal{M}(m_2^2, m_2^2, t)$. Amplitudes for $D$ decays are parametrized in the isobar model as a sum of 18 Breit-Wigner resonances and a constant non-resonant term. The result of the fit in terms of mixing parameters is presented in Fig. 5.

Numerically, the fit which allows for the CPV results in

\[
\begin{align*}
x &\approx (0.80 \pm 0.29 \pm 0.13\% \\ y &\approx (0.33 \pm 0.24 \pm 0.16\% \\ |q/p| &\approx 0.86 \pm 0.30 \pm 0.10\% .
\end{align*}
\]

The measurement represents the most accurate determination of $x$. The CP violating parameters $|q/p|$ and $\phi = arg(q/p)$ are consistent with no CP violation.

BaBar has performed the time dependent Dalitz analysis of $D^0 \rightarrow K^+ \pi^- \pi^0$ decays. The final state is flavor specific and hence the wrong-sign decays again receive contribution from mixing and DCS process. The combined Dalitz-decay-time signal distribution has the form

\[
\begin{align*}
|\langle K^+ \pi^- \pi^- | D^0(t) \rangle|^2 &\propto |A_f|^2 + |\bar{A}_f||A_f|(y'' \cos \delta_f - x'' \sin \delta_f)t + |\bar{A}_f|^2 \frac{y''^2 + y'^2}{4} t^2 .
\end{align*}
\]

$A_f$ (depending on the Dalitz variables $M^2(K^+ \pi^-)$ and $M^2(K^+ \pi^0)$) is the amplitude for $D^0 \rightarrow K^+ \pi^- \pi^0$ decays determined from the fit to the Dalitz distribution of wrong-sign decays. The amplitude for $D^0$ decays, $\bar{A}_f$, is fixed to the values obtained in the fit to the time integrated Dalitz distribution of right-sign $D^0 \rightarrow K^+ \pi^- \pi^0$ decays. The relative phase $\delta_f$ also depends on $M^2(K^+ \pi^-)$, $M^2(K^+ \pi^0)$, and is determined from the fit to the wrong- and right-sign Dalitz distributions. Parameters $x''$ and $y''$ are, similar as in the case of $D^0 \rightarrow K^+ \pi^- \pi^0$ decays, a rotated mixing parameters $x$ and $y$, now by an unknown strong phase shift $\delta_{K^+ \pi^0}$ between two points in phase spaces of DCS and CF decays to $K^- \pi^+ \pi^0$.

A fit to the time evolution of the wrong-sign Dalitz distribution results in

\[
\begin{align*}
x'' &\approx (2.61 \pm 0.67\% \\ y'' &\approx (-0.06 \pm 0.14\% .
\end{align*}
\]
2.4. \( \psi(3770) \rightarrow D^0 \bar{D}^0 \)

Pairs of neutral \( D \) mesons are produced at the threshold in a coherent \( C = -1 \) state. With the Cleo-c detector one exploits this quantum coherence to determine several parameters related to the mixing. The effective branching ratios for \( \psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow f_1 f_2 \) are modified with respect to the values measured from uncorrelated \( D^0 \) meson decays. The branching fractions can be obtained by integration of the time dependence for \( \psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow f_1 f_2 \) [14],

\[
\frac{d\Gamma(\psi(3770) \rightarrow f_1 f_2)}{d\Delta t} \propto (|a_+|^2 + |a_-|^2) \cosh y \Delta t + 2|a_+|^2 |a_-|^2 \cos x \Delta t - 2R(a_+^* a_-) \sinh y \Delta t + 23(a_+^* a_-) \sin x \Delta t ,
\]

where \( \Delta t \) is the time interval between two \( D \) mesons decays, and amplitudes \( a_+ \) and \( a_- \) are in the limit of no \( CPV \) defined as \( a_+ \equiv A_{f_1} A_{f_2} - A_{f_1} A_{f_2} \), \( a_- \equiv A_{f_1} A_{f_2} - A_{f_1} A_{f_2} \). The branching fraction for decays to the final state composed of a CP-even state (\( S_+ \)) and semileptonic state (\( e^- X \)), for example, is found to be \( Br(\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow S_+ e^- X) \approx Br(D^0 \rightarrow S_+) Br(D^0 \rightarrow e^- X)(1 - y) \). In a similar manner other double-tagged (both \( f_1 \) and \( f_2 \) reconstructed) branching fractions depend on the parameters \( y \), \( R_M = (x^2 + y^2)^2/2 \) and \( \sqrt{R_D} \cos \delta [13] \). Single-tagged (only a single final state reconstructed) branching fraction \( Br(D^0 \rightarrow S_+) \) and \( Br(D^0 \rightarrow e^- X) \) in the above example) remain unchanged compared to the uncorrelated measurements.

The measurements consist of determination of a set of double- and single-tagged branching ratios. Examples of reconstructed double-tagged signal yields are shown in Fig. 6 Parameters mentioned above are obtained from a simultaneous fit to the measured single- and double-tagged branching ratios (including additional world-average information on individual branching fractions). The result is [15]

\[
y = (-5.2 \pm 6.0 \pm 1.7)\% 
\]

\[
R_M = (2.0 \pm 1.2 \pm 1.2) \times 10^{-3} 
\]

\[
\sqrt{R_D} \cos \delta = 0.089 \pm 0.036 \pm 0.009 .
\]

This is the only direct measurement of the strong phase difference \( \delta \) between DCS and CF decays \( D^0 \rightarrow K^\pm \pi^\mp \).

2.5. Average of results

Measurements presented in the previous sections as well as others constrain the possible values of the \( D^0 \) mixing and \( CPV \) parameters in a way specific to the decay mode under consideration. A schematic view of constraints on \( x \) and \( y \) posed by some of the measurements is presented in Fig. 7 Heavy flavour averaging group [16] performs a \( \chi^2 \) fit to the \( D^0 \) mixing and \( CPV \) related observables. Correlations among the measured quantities are provided by the experiments and included in the fit. The average values from the fit in which \( CPV \) is allowed for are presented in Tab. 1

![Figure 7: Illustration of constraints in the (x, y) plane imposed by measurements in individual decay modes. R_M corresponds to measurements of semileptonic D^0 decays.](image)

| Parameter | Value |
|-----------|-------|
| \( x \) [%] | \( 0.89 \pm 0.26 \) |
| \( y \) [%] | \( 0.75 \pm 0.17 \) |
| \( \delta \) [°] | \( 21.9 \pm 11.3 \) |
| \( R_D \) [%] | \( 0.3348 \pm 0.0086 \) |
| \( A_D \) [%] | \( -2.0 \pm 2.4 \) |
| \(| q/p | [°] | \( 0.87 \pm 0.18 \) |
| \( \phi \) [°] | \( -9.1 \pm 7.8 \) |
| \( \delta_{K\pi\pi} \) [°] | \( 33.0 \pm 25.9 \) |

The no-mixing scenario \( (x, y) = (0, 0) \) is excluded by the world average of results at the level corresponding to almost 7 standard deviations. Since positive
values of $x$ and $y$ are preferred the almost $CP$ even state of neutral $D$ mesons seems to be shorter-lived and heavier.

3. Prospects and summary

A rich harvest of results in the field of $D^0$ mixing, arising from a range of various experiments, only in the last two years established the oscillations of this neutral mesons with a significance of around seven standard deviations. The measured mixing parameters $x$ and $y$ of $O(10^{-2})$ are at the upper edge of the values that can be accommodated within the SM. The results impose severe constraints on the parameter space of a wide range of New Physics models \cite{17}. To illustrate this, Fig. 8 shows the constraints on the squark mass - coupling constants plane arising from the established value of $x$ in the supersymmetric R-parity violating models.

![Figure 8: Constraints in squark mass - R-parity violating couplings plane arising from the established value of $x$. The hatched area represents the excluded region from $x < 1.2 \%$. The lines represent excluded regions for $x$ below 1.5\% - 0.3\% \cite{17}.](image)

Several measurements focus on the search for $CP$ violation in the $D^0$ system where a positive signal at the current level of sensitivity would represent a clear indication of New Physics processes. At the moment there is no such hint, and $CPV$ is in several processes found to be below the 0.3\% level.

In the near future the major experimental task in the field is to measure $x$ with a higher accuracy and to further limit the range of the $CP$ violation in various decay modes. Recent experimental results might trigger further theoretical efforts in the SM predictions for the mixing parameters, although at the moment more precise calculations appear to be difficult.

A more or less educated guess tells that with a modest integrated luminosity of $5\ ab^{-1}$ collected by the future Super-B factory it would be possible to measure $x$ and $y$ with a precision of 0.1\% - 0.15\%. Furthermore, the $CPV$ parameters $|q/p|$ and $\phi$ could be measured to around $\pm 0.1$, enabling tests of the $CP$ asymmetries at around $10^{-3}$ level.

References

[1] M. Starić et al. (Belle Coll.), Phys. Rev. Lett. \textbf{98}, 211803 (2007).
[2] B. Aubert et al. (BaBar Coll.), Phys. Rev. Lett. \textbf{98}, 211802 (2007).
[3] G. Burdman, I. Shipsey, Ann. Rev. Nucl. Sci. \textbf{53}, 431 (2003).
[4] A.F. Falk et al., Phys. Rev. D\textbf{69}, 114021 (2004).
[5] I.I. Bigi, N. Uraltsev, Nucl. Phys. B\textbf{592}, 92 (2001).
[6] S. Bergmann et al., Phys. Lett. B \textbf{486}, 418 (2000).
[7] B. Aubert et al. (BaBar Coll.), Phys. Rev. D\textbf{78}, 011105 (2008).
[8] B. Aubert et al. (BaBar Coll.), Phys. Rev. Lett. \textbf{100}, 061803 (2008).
[9] M. Starić et al. (Belle Coll.), arXiv:0807.0148 subm. to Phys. Lett. B.
[10] T. Aaltonen et al. (CDF Coll.), Phys. Rev. Lett. \textbf{100}, 121802 (2008).
[11] L.M. Zhang et al. (Belle Coll.), Phys. Rev. Lett. \textbf{96}, 151801 (2006).
[12] L.M. Zhang et al. (Belle Coll.), Phys. Rev. Lett. \textbf{99}, 131803 (2007).
[13] B. Aubert et al. (BaBar Coll.), arXiv:0807.3544 subm. to Phys. Rev. Lett.
[14] CP violation in meson decays, in C. Amsler et al., Phys. Lett. B \textbf{677}, 1 (2008).
[15] D.M. Asner et al. (Cleo Coll.), Phys. Rev. D\textbf{78}, 012001 (2008).
[16] E. Barberio et al. (HFAG), arXiv:0808.1204 and updates at \url{http://www.slac.stanford.edu/xorg/hfag/}.
[17] E. Golowich et al., Phys. Rev. D\textbf{76}, 095009 (2007).