Energy-Aware Aggregation of Dynamic Temporal Workload in Data Centers
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Abstract—Data center providers seek to minimize their total cost of ownership (TCO), while power consumption has become a social concern. We present formulations to minimize server energy consumption and server cost under three different data center server setups (homogeneous, heterogeneous, and hybrid hetero-homogeneous clusters) with dynamic temporal workload. Our studies show that the homogeneous model significantly differs from the heterogeneous model in computational time (by an order of magnitude). To be able to compute optimal configurations in near real-time for large scale data centers, we propose two modes, aggregation by maximum and aggregation by mean. In addition, we propose two aggregation methods, static (periodic) aggregation and dynamic (aperiodic) aggregation. We found that in the aggregation by maximum mode, the dynamic aggregation resulted in cost savings of up to approximately 18% over the static aggregation. In the aggregation by mean mode, the dynamic aggregation by mean could save up to approximately 50% workload rearrangement compared to the static aggregation by mean mode. Overall, our methodology helps to understand the trade-off in energy-aware aggregation.

Index Terms—Data Center; Energy-Aware; Server Cost Optimization; Workload Aggregation

I. INTRODUCTION

Cloud computing has many advantages such as flexibility, manageability, and scalability \cite{2}. Data centers serving cloud computing face a number of challenges. A key aspect is to minimize the total cost of ownership (TCO), while meeting customers’ workload needs. Thus, from the perspective of data center operators’ TCO, which comprises both energy consumption and infrastructure costs, it is important to minimize both together. It has been reported that the power consumption in cloud data centers has increased 400\% over the past decade \cite{16}. Data centers consumed 61 billion kilowatt-hours of power in 2006, according to a report of the U.S. Environmental Protection Agency (EPA) in 2007 \cite{11}. That is, 1.5 percent of all power consumed in the United States—at a cost of 4.5 billion dollars. Data center energy costs are approaching overall hardware costs \cite{3} and even worse, continue to increase at a rate that is faster than any others \cite{6}. The energy consumption is comprised of multiple elements, such as servers, cooling, and power distribution loss. We focus on the costs of servers in this paper, i.e., the energy consumption to run servers and the amortized capital expenditures (CAPEX) of servers.

Data center workload traces reveal that the workload is highly dynamic \cite{4,11,24}. Due to the variety of Internet services forming workloads for different data center operators, the workloads can be significantly different from one data center operator to another. Ideally, we want to predict the workload by stochastic distributions or by certain deterministic patterns by history information. However the data center work load is highly dynamic. Namely, it is difficult to predict for long term. To predict the workload accurately, the workload should be predicted every once a while. Data center operators must provision sufficient resources to satisfy the workload as per the service level agreements (SLA) with their customers. If such workloads have temporal peaks, it can result in over-provisioning for off-peak workloads. It has been found that there is significant power consumption when the CPU is idle, that is, at base power \cite{24}. It is measured that idle servers consume more than 66\% of the peak power \cite{11,15}. On the the other hand, for certain type of workloads, the deadline to complete the workload may be somewhat relaxed, which allows for evenly distributing the workload over a shorter window of time without any additional penalty. This situation leads to consideration for the aggregation by mean mode rather than the aggregation by maximum mode.

The above suggests that an important strategy for data center operators is to consolidate jobs (which comprise workloads) into a minimum number of servers and switch idle servers off. However, switching servers on and off impacts the wear-and-tear cost and consolidation cost. It has been observed that the hard disk is the most vulnerable part in a data center infrastructure; the majority (78\%) of hardware failure/replacement is due to hard disks \cite{31}. To represent the wear-and-tear costs due to switching on and off, we amortize the CAPEX of servers by dividing the average price of servers by the average number of switching on/off cycles of the hard disk. On the other hand, the source server needs to run for additional amount of time to keep the states of running application when doing consolidation. This then leads to consuming additional energy. Therefore, we consider two cost components in this paper: the energy consumption cost and the switching on/off cost. The presence of relationships between adjacent time slots (when workloads are estimated and reviewed) requires a global optimization framework over a temporal window spanning several hours, which may form a planning window or a forecast window. In addition, the optimal solution should be solved quickly enough, especially for large scale data centers with short-term predictability. Otherwise, when the solution is computed, the time window has past. We found that if all time slots are considered over a planning window, the computational time to obtain the

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solution is high. Thus, we propose to aggregate time slots of workload demand to reduce computational complexity, using two different strategies: static (periodic) and dynamic (aperiodic).

Another aspect to consider about a data center is machine variations. It is rare that all machines at a data center are of the same type (“homogeneous”). Often, a data center is heterogeneous with machines of different types. More realistically, a data center has heterogeneity but with clusters of homogeneous machines, where machines in a cluster get replaced about the same time from another cluster. Thus, we wish to understand how the problem at hand is impacted when data center machine configurations are of three possible types: homogenous, all heterogeneous, or a homogeneous-heterogeneous mix. While a data center with all homogeneous or all heterogeneous machines is unlikely in reality, we use it for the purpose of benchmarking and to show how we develop the model for the more realistic mixed homogeneous-heterogeneous data center.

To summarize, our major contributions are:

1) To present integer linear programming (ILP) formulations to determine the optimal number of running servers over a temporal window where load adjacencies are taken into account. In particular, these formulations are presented for three different data center configurations: homogeneous, heterogeneous, and mixed homogeneous-heterogeneous.
2) To propose two workload aggregation methods, static (periodic) and dynamic (aperiodic), to reduce the computational time to determine the optimum. In addition, two workload aggregation modes (aggregation by maximum and aggregation by mean) are introduced to address differing workload deadlines and service level agreements. To combat the pitfalls of static aggregations for both modes, we further enhance how dynamic aggregations are considered.
3) To consider workloads for a number of different distributions and conduct a comprehensive study to understand the impact on the optimal solution as well as on the aggregation schemes, in order to present the trade-off between energy-aware aggregation and the impact on the overall cost.
4) To present a sensitivity analysis on the optimal solution by varying weights of the power cost and the switching on-off cost components.

The remainder of this paper is organized as follows. We present optimization formulations in Section III. Two aggregation modes, aggregation by maximum and aggregation by mean, and two aggregation methods, static aggregation and dynamic aggregation, are presented in Section IV. Our numeric studies are discussed in Section V. Section II summarizes the related work. Finally, Section VI concludes the paper.
Because certain workload analysis suggests a daily cycle, a sinusoidal workload pattern is additionally considered in this paper. Furthermore, the aggregation by mean method is also proposed since some jobs can tolerate a certain amount of delay. Lastly, we consider sensitivity analysis on the optimal solution by varying weights of two cost components. It is also worth noting that the numeric study in this paper covered as many as 5,000 servers, while most existing works consider a small number of servers (such as up to 100 servers).

Finally, multi-time period problems have been studied over three decades in several related areas such as transportation research, inventory management, and telecommunication network capacity design [13, 17, 20, 25, 29, 32]. While in many of these problems, the dependency arises in the form of constraints due to the remaining capacity of one period being used in a subsequent period, the dependency in the data center resource management problem discussed here is primarily in the form of the switching on/off cost.

III. OPTIMIZATION PROBLEM FORMULATION

Consider a data center with \( I \) servers. Let \( \mathcal{I} \) denote the set of servers, while the cardinality of this set is denoted by \( I \), i.e., \( \#(\mathcal{I}) = I \). Consider a temporal window or forecasting window of \( \Upsilon \) hours. In practice, we expect \( \Upsilon \) to be six to eight hours, for which the workload can be reasonably forecasted according to historical observations. There are many techniques for forecasting, e.g., [18, 30]. However, load prediction or forecasting techniques are outside the scope of this paper. The duration of \( \Upsilon \) hours for the forecasting window is divided into \( T \) equal time slots or periods\(^1\) and the duration of a time slot, also referred to as slot size, is \( \tau = \Upsilon / 60 \) minutes. We assume that the workload on CPU needs is forecasted at the beginning of the entire planning window. Servers are reconfigurable at the beginning of each time slot, which is labeled as review points. The capacity of server \( i \) is denoted by \( v_i \). We want to determine when and how many servers to reconfigure at review points, so that the total cost of energy consumption and amortized server CAPEX is minimized over the entire planning window.

A. Heterogeneous Server Model

Consider first that all servers are heterogeneous; this model is denoted by Model-Het. The power consumption per time unit of running server \( i \in \mathcal{I} \) is denoted by \( c_i^p \). Binary decision variables \( x_{it} \) denote 1 if server \( i \) is turned on at review point for time slot \( t \). The switching costs of turning server \( i \) on and off are denoted by \( c_{i}^{s+} \) and \( c_{i}^{s-} \), respectively. \( c_{i}^{s+} \) is composed by wear-and-tear cost due to turning the server on \( (c_{i}^{w+}) \), power consumption to turn the server off \( (c_{i}^{p-}) \), and power consumption to run additional time to sustain the states of running applications due to consolidation in the server\( (c_{i}^{n-}) \). Thus we have

\[
c_{i}^{s+} = c_{i}^{w+} + c_{i}^{p-} + c_{i}^{n-}. \tag{2}
\]

\( x_{it} = 1, 0 \) represents whether the states of server \( i \) is “on” and “off” at time slot \( t \), respectively. Let “1” represent state change while “0” stands for no change between adjacent time slots. Then the state change from “on” to “off” can be represented by \( x_{it} \cdot (x_{it} - x_{i(t-1)}) \). Similarly, the state change from off to on can be represented by \( x_{i(t-1)} \cdot (x_{i(t-1)} - x_{it}) \). Our objective is to minimize the energy cost as well as the cost of switching servers on/off over the planning horizon. Therefore, the objective function is given by

\[
F = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left( c_{i}^{p} \cdot x_{it} + c_{i}^{s+} \cdot x_{it} \cdot (x_{it} - x_{i(t-1)}) + c_{i}^{s-} \cdot x_{i(t-1)} \cdot (x_{i(t-1)} - x_{it}) \right) \tag{3}
\]

Next we consider constraints. The main set of constraints in this problem is that the load must be satisfied in every time slot \( t \). Let \( d_t \) be the workload demand at time slot \( t \). Thus we require

\[
\sum_{i \in \mathcal{I}} v_i \cdot x_{it} \geq d_t, \quad t = 1, 2, \ldots, T \tag{4}
\]

Note that the objective function \((3)\) is a quadratic function over binary variables. We can transform this special form of a quadratic function into a linear function by introducing additional variables and constraints without resorting to any approximation. We introduce two binary variables \( x_{i+}^{t}(t) \) and \( x_{i-}^{t}(t) \) to represent switching on and off at the review point of time slot \( t \). Specifically, \( x_{i+}^{t}(t) = 1 \) represents that server \( i \) is turned on at the review point of time slot \( t \). Conversely, \( x_{i+}^{t}(t) = 1 \) means server \( i \) is turned off at the review point for time slot \( t \). \( x_{i+}^{t}(t) = 0 \) indicates that the state of server \( i \) does not change from time slot \( t-1 \) to \( t \). Thus, we have

\[
x_{it} - x_{i(t-1)} - x_{i+}^{t} - x_{i-}^{t} = 0, \quad i = 1, 2, \ldots, I; \quad t = 1, 2, \ldots, T \tag{5}
\]

Because \( x_{i+}^{t}(t) \) and \( x_{i-}^{t}(t) \) cannot both be 1 at any time slot \( t \), we add the following inequalities to enforce this requirement:

\[
x_{i+}^{t} + x_{i-}^{t} \leq 1, \quad i = 1, 2, \ldots, I; \quad t = 1, 2, \ldots, T \tag{6}
\]

With the aid of \( x_{i+}^{t}(t) \) and \( x_{i-}^{t}(t) \), we now transform the original quadratic objective function \((3)\) to the following linear function:

\[
F = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left( c_{i}^{p} \cdot x_{it} + c_{i}^{s+} \cdot x_{it} + c_{i}^{s-} \cdot x_{it} \right). \tag{7}
\]

In order to separate the energy cost \((F^{p})\) and wear-and-tear cost \((F^{w})\), we plug \((1)\) and \((2)\) into \((7)\). We have

\[
F^{p} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} (c_{i}^{p} \cdot x_{it} + c_{i}^{s+} \cdot x_{it} + (c_{i}^{p-} + c_{i}^{n-}) \cdot x_{it}), \tag{8}
\]

\[
F^{w} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} (c_{i}^{w+} \cdot x_{it} + c_{i}^{s-} \cdot x_{it}), \tag{9}
\]

\[
F = F^{p} + F^{w}. \tag{10}
\]

\(^1\)The terms time slot and period are used interchangeably in this paper.
We assume that all servers are set to the off status at time slot 0 (the beginning of the planning window); thus, we have the initial set of conditions as follows:

\[ x_{i0} = 0, \ i = 1, \cdots, I. \]  
(11)

To summarize, in Model-Het, the objective is to minimize (7), which is subject to (4), (5), and (6), where the initial conditions are given by (11) and all variables are binary variables.

B. Homogeneous Server Model

In the case of the homogeneous server configuration, all servers are considered to be identical. This model is denoted by Model-Hom. Although the heterogeneous model introduced in the previous section is generic to be applied to this model, we present a different formulation where the number of variables and constraints are reduced significantly. Let \( c^p \) be the power consumption of running a server in a time slot. Let \( y_t \) denote the number of running homogeneous servers at time slot \( t \). Therefore, we can reduce the \( I \) binary variables in Model-Het to a single integer variable for each time slot in Model-Hom. The power consumption for each server per time slot is denoted by \( c^p \). The cost of switching one server on and off is denoted by \( c^{s+} \) and \( c^{s-} \), respectively. \( c^{s+} \) is composed by wear-and-tear cost due to turning one server on (\( c^{w+} \)), power consumption to turn one server on (\( c^{p+} \)). Thus we have

\[ c^{s+} = c^{w+} + c^{p+}. \]  
(12)

And \( c^{s-} \) is composed by wear-and-tear cost due to turning one server off (\( c^{w-} \)), power consumption to turn one server off (\( c^{p-} \)) and power consumption to run additional time to sustain the states of running applications due to consolidation in one server (\( c^{c-} \)). Thus we have

\[ c^{s-} = c^{w-} + c^{p-} + c^{c-}. \]  
(13)

Similar to Model-Het, the first constraint is the workload requirement using the new variables \( y_t \):

\[ v_t \cdot y_t \geq d_t, \ t = 1, \cdots, T. \]  
(14)

Let \( y^+_t \) denote the number of servers that is switched on at the review point of time slot \( t \). Then \( y^+_t \) should take the maximum between 0 and \( y_t - y_{t-1} \). That is,

\[ y^+_t = \max\{0, y_t - y_{t-1}\}, \ t = 1, \cdots, T. \]  
(15)

Let \( y^-_t \) be the number of servers that are switched off at the review point of time slot \( t \). Similar to (15), we have

\[ y^-_t = \max\{0, y_{t-1} - y_t\}, \ t = 1, \cdots, T. \]  
(16)

Note that (15) and (16) are not directly usable constraints. Because we are considering a minimization problem with the cost coefficient being non-negative, we can substitute (15) by following two linear inequalities:

\[ y^+_t \geq y_t - y_{t-1}, \ t = 1, \cdots, T. \]  
(17)

Likewise, we can substitute (16) by following two linear inequalities:

\[ y^-_t \geq y_{t-1} - y_t, \ t = 1, \cdots, T. \]  
(19)

\[ y^-_t \geq 0, \ t = 1, \cdots, T. \]  
(20)

The final set of constraints is on the number of running servers should not be larger than total number of servers

\[ y_t \leq I, \ t = 1, \cdots, T. \]  
(21)

Because it is a minimization problem with the cost coefficient being non-negative, this constraint can be omitted when solving the optimization problem. Since all servers are off at the beginning of the planning window (i.e., at the review point for the beginning time slot 0), we have the initial condition

\[ y_0 = 0. \]  
(22)

The objective is to minimize the total energy and switching on/off cost, which is given by

\[ F = \sum_{t \in T} (c^p \cdot y_t + c^{s+} \cdot y^+_t + c^{s-} \cdot y^-_t). \]  
(23)

In order to separate the energy cost (\( F^p \)) and wear-and-tear cost (\( F^w \)), we plug (12) and (13) into (23). We have

\[ F^p = \sum_{t \in T} (c^p \cdot y_t + c^{s+} \cdot y^+_t + (c^{c-} + c^{w-}) \cdot y^-_t), \]  
(24)

\[ F^w = \sum_{t \in T} (c^{w+} \cdot y^+_t + c^{w-} \cdot y^-_t), \]  
(25)

\[ F = F^p + F^w. \]  
(26)

To summarize, in the Model-Hom, we minimize (23), which is subject to (14), (17), (18), (19), and (20) and the initialization condition is given by (22). This is an integer linear programming (ILP) problem.

C. Heterogeneous Homogeneous-Server-Cluster Model

In a data center, it is more realistic that there are different clusters of servers and each cluster has a certain number of homogeneous servers while servers may be different from one cluster to another, rather then all being homogeneous or all being heterogeneous. Thus, this is a hybrid of the aforementioned configurations. This model can also be used when homogeneous servers are required to be partitioned into multiple clusters for ease of management. We denote this model by Model-HH. Denote the set of clusters by \( J \) and its cardinality, \( J = \#(J) \), where \( 1 \leq J \leq I \), represents the number of clusters. The two models presented so far correspond to the two extremes in this model: when \( J = 1 \), it is Model-Hom; when \( J = I \), it becomes Model-Het.

Let the energy consumption of running a server in cluster \( j \) at a time slot be \( z_{jt} \). Denote the number of servers in cluster \( j \) by \( I_j \), where \( \sum_j I_j = I \). We denote the set of the number of running servers for cluster \( j \) by \( N_j \), which can be \( 0, 1, \cdots, I_j \). Let \( z_{jt} \) be the number of running servers in cluster \( j \) at time
slot $t$. The costs of switching a server in cluster $j$ on and off are represented by $c_j^+$ and $c_j^-$, respectively. $c_j^+$ is composed by wear-and-tear cost due to turning the server in cluster $j$ on ($e_j^{on}$), power consumption to turn the server in cluster $j$ on ($e_j^{on}$). Thus we have

$$c_j^+ = e_j^{on} + e_j^{on}.$$  \hspace{1cm} (27)

And $c_j^-$ is composed by wear-and-tear cost due to turning the server in cluster $j$ off ($e_j^{off}$), power consumption to turn the server in cluster $j$ off ($e_j^{off}$) and power consumption to run additional time to sustain the states of running applications due to consolidation in the server of cluster $j$ ($e_j^{off}$). Thus we have

$$c_j^- = e_j^{off} + e_j^{off} + e_j^{off}.$$  \hspace{1cm} (28)

We next introduce constraints in this problem. First, the workload requirements need to be satisfied all the time:

$$\sum_{j \in J} z_{jt} \cdot z_{jt} \geq d_t, \ for \ t = 1, \cdots, T.$$  \hspace{1cm} (29)

Secondly, the number of running servers cannot be larger than the total number of servers in that cluster:

$$z_{jt} \leq I_j, \ for \ j = 1, \cdots, J; \ t = 1, \cdots, T.$$  \hspace{1cm} (30)

The third set of constraints is similar to (19) and (20) in Model-Hom. But they need to be applied to each of the $J$ clusters. Let $z_{jt}^j$ be the number of servers turned on in cluster $j$ at the review point of time slot $t$ while $z_{jt}^j$ is the number of servers turned off in cluster $j$ at the review point of time slot $t$. We use the same technique as in Model-Hom to transform the constraints to linear constraints

$$z_{jt}^j \geq 0, \ for \ j = 1, \cdots, J; \ t = 1, \cdots, T.$$  \hspace{1cm} (31)

$$z_{jt}^j \geq z_{jt} - z_{jt}(t-1), \ for \ j = 1, \cdots, J; \ t = 1, \cdots, T.$$  \hspace{1cm} (32)

$$z_{jt}^j \geq 0, \ for \ j = 1, \cdots, J; \ t = 1, \cdots, T.$$  \hspace{1cm} (33)

And the objective function is given by

$$F = \sum_{t \in T} \sum_{j \in J} (c_j^+ \cdot z_{jt} + c_j^+ \cdot z_{jt} + c_j^- \cdot z_{jt}^-).$$  \hspace{1cm} (35)

Similar to previous two models, we separate the energy cost ($F^p$) and wear-and-tear cost($F^w$) and we have

$$F^p = \sum_{t \in T} \sum_{j \in J} (c_j^+ \cdot z_{jt} + c_j^+ \cdot z_{jt} + (p_j - c_j^-) \cdot z_{jt}^-),$$  \hspace{1cm} (36)

$$F^w = \sum_{t \in T} \sum_{j \in J} (c_j^+ \cdot z_{jt} + c_j^+ \cdot z_{jt} + c_j^- \cdot z_{jt}^-).$$  \hspace{1cm} (37)

$$F = F^p + F^w.$$  \hspace{1cm} (38)

Because we assume that at the beginning of the planning period, all servers are off, we have

$$z_{j0} = 0, \ for \ j = 1, \cdots, J.$$  \hspace{1cm} (39)

IV. AGGRGATING DEMAND TO REDUCE COMPUTATIONAL TIME

The number of variables for the heterogeneous case, the homogenous case, the heterogeneous homogeneous-server-cluster case, presented in Section IV are $3 \times I \times T$, $3 \times T$, and $3 \times J \times T$, respectively. Since the computational time grows with the number of variables for these models (to be discussed further in Section V.B), the time complexity for the heterogeneous and heterogeneous homogeneous-server-cluster cases differs significantly for large scale data centers (large $I$ and $J$, respectively) compared to the homogenous case.

To reduce the computational time, we need to either reduce the number of variables in the original problem or develop a heuristic to approximate the original problem. In this paper, we propose an aggregation heuristic strategy in which a certain number of contiguous time slots are combined into a single aggregated slot to reduce the total number of time slots for the workload, by considering load affinity. The value of the workload demand on an aggregated time slot is decided by the workload of the original time slots and the service-level agreement (SLA).

A. Aggregation by Maximum

If an SLA stringently requires that the workload should be satisfied all the time, then the workload of an aggregated time slot takes the maximum of the demand of the original time slots. For example, we want to aggregate contiguous slots $[d_k, d_{k+1}, \cdots, d_{k+\ell-1}]$ into an aggregated time slot; then the new demand $d_k$ over the $\ell$ times of the original slot size is given by

$$d_{k}^{max} = \max\{d_k, d_{k+1}, \cdots, d_{k+\ell-1}\}.$$  \hspace{1cm} (40)

This method introduces an artificial increase in the demand, which in turn, causes extra consumption of power energy. On the other hand, aggregation smooths out the irregularity of workload, which affects the switching cost. This raises the issue of trading off the computing time of running the model at the expense of extra cost on energy consumption.

1) Static Aggregation by Maximum: In the static approach (Fig. 1(a)), we aggregate every $M$ contiguous time slots into one, i.e., the aggregation window is periodic. We have $\hat{T} = T/M$, where $\hat{T}$ denotes the reduced number of time slots. The slot size of the aggregated workload (except the last slot) is $M$ times the slot size of the original demand. The slot size of the last slot is $T-M \times (T-1)$. The aggregated workload $[t/M]$ is given by

$$d_{[t/M]}^{max} = \max\{d_{[t/M]-1}, d_{[t/M]-1}, \cdots, d_{[t/M]-1}\}.$$  \hspace{1cm} (41)

2) Dynamic Aggregation by Maximum: Our dynamic aggregation approach (Fig. 1(b)) improves on the static (periodic) approach, with a goal to improve the overall cost. Instead of aggregating the workload in fixed numbers of original time slots statically, the adaptive aggregation method
aggregates an arbitrary number of time slots (aperiodic) as long as the number of aggregated time slots is $T$ such that the sum of the difference between the aggregated workload and the original workload is minimized. That is, we seek to minimize

$$
\tilde{T} \sum_{k=1}^{\tilde{T}} \sum_{\ell=1}^{T_k} (d_{k,\ell}^{\text{max}} - d_k), \quad \text{where } d_{k,\ell}^{\text{max}} = \max\{d_{k1}, \ldots, d_{kT_k}\},
$$

which is subject to

$$
\sum_{k=1}^{\tilde{T}} T_k = T. \tag{42}
$$

To this end, we need to choose $\tilde{T} - 1$ review points out of $T - 1$, requiring $(\tilde{T} - 1)$ operations. This extra computational time complexity is contradictory to the purpose of doing aggregation. Thus, we propose local smooth heuristics to implement this idea. We aggregate the time slots that are locally “smoother” together. In order to do this, we first define the smooth index of workloads. The smooth index is the absolute value of the difference between the workload demand of adjacent time slots (adjacent workloads for short). Therefore, we obtain $T - 1$ smooth indices. Then we pick the smallest non-zero smooth index and compare two adjacent workloads associated with this smooth index. The smaller workloads are aggregated into a maximum workload over these slots; the slot size of the aggregated workload is the sum of two slot sizes of adjacent slots and the new workload for the aggregated slot takes the maximum of these demands. The smooth index is updated for the new aggregated demand series. We repeat this procedure until the target number of slots is reached. We call this procedure the local smooth algorithm (see Algorithm 1). Denote “InMin” to be the procedure to find the index of minimum value in a vector, “Max” be the procedure to find the maximum value while “Mean” be the procedure taking the average, which is used in the aggregation by mean mode. The value of the workload in time slot $i$ is denoted by $d[i]$.

It is possible to make further improvements to Algorithm 1. For this, denote the smooth index vector by $si$. Scalar $ap$ records the aggregation point. Vector $ss$, which is initialized to $T$ ones, records the size of each time slot. The improved local smooth algorithm is presented in Algorithm 2.

**Algorithm 1 Local Smooth Algorithm**

1: $\tilde{T} \leftarrow T$ \hspace{1cm} \text{Initialization}
2: \hspace{1cm} while $j > \tilde{T}$ do
3: \hspace{2cm} for $i := 2 \to j$ do
4: \hspace{3cm} $si[i - 1] \leftarrow |d[i] - d[i - 1]|$ Compute smooth index
5: \hspace{2cm} end for
6: \hspace{1cm} $ap \leftarrow \text{InMin}(si)$ \hspace{0.5cm} Find aggregation point
7: \hspace{1cm} $d[ap] \leftarrow \text{Max/Mean}(d[ap], d[ap + 1])$ \hspace{0.5cm} Aggregate
8: \hspace{1cm} $d[ap + 1] \leftarrow \text{Max/Mean}(d[ap], d[ap + 1])$ \hspace{0.5cm} Adjust the index of slots behind
9: \hspace{1cm} $ss[ap] \leftarrow ss[ap] + ss[ap + 1] \hspace{0.5cm} \text{Compute the size of aggregated slots}$
10: \hspace{1cm} if $ap \neq j - 1$ then
11: \hspace{2cm} for $k := ap + 1 \to j - 1$ do
12: \hspace{3cm} $d[k] = d[k + 1]$ \hspace{1cm} Adjust the corresponding slot size
13: \hspace{2cm} end if
14: \hspace{1cm} end while
15: \hspace{1cm} $j \leftarrow j - 1$ Decrease the number of slots by 1
16: \hspace{1cm} return $d$, $ss$

It is easy to see that Algorithm 1 has complexity $O(T^2)$, while Algorithm 2 has linear complexity $O(T)$, which is achieved by recomputing two smooth indexes adjacent to the selected aggregation point only. Note that the computational time for the dynamic aggregation has no significant difference with its static aggregation counterpart since they have the same number of variables and constraints. To differentiate between the proposed dynamic aggregation and the implemented dynamic aggregation, we call the proposed dynamic aggregation as the strict dynamic aggregation.

**B. Aggregation by Mean**

The workload demand of the aggregated time slot can also take a certain percentile of the workload demand of the original slots. Note that Aggregation by maximum uses the workload to be the 100th percentile of the workload of the original slots. For many long-lived jobs that do not need to be executed in real-time, such as data warehousing or scientific computing, the workload can be arranged over time as long as the average workload over an acceptable time window is completed. Thus, we also introduce the mode of aggregation by mean: the workload demand of the aggregated time slot takes the mean of the workload demand of the original slots. Consider aggregating loads for contiguous slots $[d_k, d_{k+1}, \ldots, d_{k+\ell-1}]$ into one time slot; then, the new demand $\tilde{d}_k$ with $\ell$ times of the original slot size is given by

$$
\tilde{d}_k = \text{mean}\{d_k, d_{k+1}, \ldots, d_{k+\ell-1}\}. \tag{44}
$$
Algorithm 2 Improved Local Smooth Algorithm

1: \( j \leftarrow T \) \hspace{2cm} \text{Initialization}
2: \textbf{for} \( i := 2 \rightarrow j \) \textbf{do}
3: \( si[i - 1] \leftarrow (d[i] - d[i - 1]) \) \hspace{1cm} \text{Compute smooth index}
4: \textbf{end for}
5: \textbf{while} \( j > \hat{T} \) \textbf{do}
6: \( ap \leftarrow \text{InMin}([si]) \) \hspace{1cm} \text{Find aggregation point}
7: \( d[ap] \leftarrow \text{Max/Min}(d[ap], d[ap + 1]) \)
8: \( d[ap + 1] \leftarrow \text{Max/Min}(d[ap], d[ap + 1]) \) \hspace{0.5cm} \text{Aggregate}
9: \( si[ap - 1] \leftarrow d[ap] - d[ap - 1] \) \hspace{1cm} \text{Update smooth index}
10: \( si[ap] \leftarrow d[ap + 2] - d[ap + 1] \) \hspace{1cm} \text{Update smooth index}
11: \( ss[ap] \leftarrow ss[ap] + ss[ap + 1] \) \hspace{1cm} \text{Compute the size of aggregated slots}
12: \textbf{if} \( ap \neq j - 1 \) \textbf{then}
13: \( k := ap + 1 \rightarrow j - 1 \) \textbf{do}
14: \( d[k] \leftarrow d[k + 1] \) \hspace{1cm} \text{Adjust the index of slots behind the aggregation point}
15: \( ss[k] \leftarrow ss[k + 1] \) \hspace{1cm} \text{Adjust the corresponding slot size}
16: \textbf{end if}
17: \textbf{if} \( k < j - 1 \) \textbf{then}
18: \( si[k] \leftarrow si[k + 1] \) \hspace{1cm} \text{Adjust the smooth index}
19: \textbf{end if}
20: \textbf{end if}
21: \( j \leftarrow j - 1 \) \hspace{1cm} \text{Decrease the number of slots by 1}
22: \textbf{end while}
23: \textbf{return} \( d, ss \)

Compared to the aggregation by maximum mode, the aggregation by mean mode does not introduce the artificial increase of the workload demand and smoothes out the irregularity of the workload.

1) Static Aggregation by Mean: As before, we aggregate every \( M \) contiguous time slots into 1. The only difference is that the aggregated workload takes the average of the original workload:

\[
\tilde{d}_{\lfloor t/M \rfloor} = \text{mean}\{d_{\lfloor t/M \rfloor - 1} \cdot M + 1, \cdots, d_{\lfloor t/M \rfloor - 1} \cdot M + M \} \tag{45}
\]

The application of this method is based on the ability to rearrange user requests within a certain time window, which is a subset of the planning window, for some applications that do not require real-time execution. In other words, the requested load can be either executed in advance or delayed in the data center. An example of static aggregation by average is shown in Fig. 2.

2) Dynamic Aggregation by Mean: Similar to the previous instance, but this time to avoid delay or advance workload as much as possible, we also propose the counterpart of the dynamic aggregation by max. The objective function is to minimize

\[
\hat{t} \sum_{k=1}^{T} \sum_{t=1}^{T_k} |d_{kt} - \bar{d}_k|, \text{ where } \bar{d}_k = \text{mean}\{d_{k1}, \cdots, d_{kT_k}\} \tag{46}
\]

which is subject to (43).

Fig. 2(b) illustrates the dynamic aggregation by average. It is worth noting that although (42) and (46) look similar, the objectives of dynamic aggregation by maximum and dynamic aggregation by mean are different. Aggregation by maximum aims to reduce cost due to energy waste while aggregation by mean targets to reduce the movement of the workload. Thus, compared to the static aggregation, the aggregation by maximum results in less energy cost while in aggregation by mean, the energy cost is always the same. However, the proposed dynamic aggregation method has no constraints on the number of original slots combined to create an aggregated slot. In practical applications, the workload can only be executed in advance or delayed up to a certain time. Let \( S \) be the maximum number of continuous time slots that can be aggregated to meet the delayed requirement. Consequently, this problem is also subject to

\[
\max\{s_1, \cdots, s_{\hat{T}}\} \leq S. \tag{47}
\]

The exact solution based on improved local smooth algorithms requires \( n! \) time. We propose an approximation scheme with low complexity that relaxes the target number of aggregated workload slots to guarantee that the movement of the workload is less than a certain threshold. The modified algorithm is shown in Algorithm 3. To illustrate, Fig. 2(c) presents an example of dynamic aggregation constrained by \( S = 8 \). A problem with this implementation is that it may not have a feasible solution. To address this issue, we swap lines 22 and 23 in Algorithm 3 to relax the target number of time slots to guarantee that there is a solution.

V. RESULTS AND DISCUSSIONS

Before we discuss our results, we summarize a few key points of the approaches presented so far. The aggregation methods are proposed to reduce the computation time of the workload planning problem for large scale data centers. In practice, if the workload cannot be rearranged over time,
aggregation by maximum should be used; otherwise, aggregation by mean can be adopted. The price of aggregation by maximum is over-provisioning which causes extra energy consumption. The price of aggregation by mean is the workload rearrangement. Dynamic aggregation is proposed to alleviate over-provisioning and workload rearrangement in aggregation by maximum and by mean, respectively. In this section, we quantitively study the pros and cons of proposed aggregation methods.

A. Experiment Setup

In our study, the server’s CPU frequency set and power consumptions are adopted from [12], except that we use the maximum frequency only. For ease of comparison, the capacity of each server is normalized to 1. Greenberg et al. [19] use $.07 per kilowatt-hours (kWh) as the utility price. We assume the same utility price. The server energy consumption in a time slot is the product of the power consumption, the utility price, and the slot size. We use server CPU frequency to be 2.6GHz, power consumption to be 100 watts with a power cost of $0.07 per kWh.

Google reported [5] that the hard disk is the most vulnerable part in a server and the personnel cost for each repair is $100 and the replacement cost is 10% of the server cost ($2,000). We assume the lifetime for a disk to be 60,000 switching-on-and-off cycles. Using this, we arrive at 0.5 cents for the wear-and-tear cost per switching-on-off cycle. Out of 0.5 cents, wear-and-tear cost due to switching on is usually higher than that due to switching off; thus, we split 0.5 cents to 0.3 cents and 0.2 cents for wear-and-tear cost due to switching on (\(c_{i}^{w} / c_{i}^{w+} / c_{i}^{w+}\)) and wear-and-tear cost due to switching off (\(c_{i}^{w-} / c_{i}^{w-} / c_{i}^{w-}\)). Since turning on draws much more power than turning off in most cases we assume that the power consumption for switching on (\(c_{i}^{w+} / c_{i}^{w+} / c_{i}^{w+}\)) and switching off (\(c_{i}^{w-} / c_{i}^{w-} / c_{i}^{w-}\)) is 0.02 cents and 0.005 cents, respectively. This analysis of the turning on/off cost is similar to that in [12] except that we differentiate the cost of switching on and off. The cost of switching on (\(c_{i}^{w+} / c_{i}^{w+} / c_{i}^{w+}\)) is 0.32 cents. When consolidation is performed, the source server needs to run for an additional amount of time to sustain the state of running applications. We assume the average extra time to be 77 seconds. We arrive at 0.015 cents per server switching off (\(c_{i}^{w-} / c_{i}^{w-} / c_{i}^{w-}\)) as the cost of consolidation. Because the consolidation cost is not considered in our previous work [28], the cost of switching off (\(c_{i}^{w-} / c_{i}^{w-} / c_{i}^{w-}\)) is 0.22 cents in lieu of 0.205 cents in [28].

We next consider the scenario that the cost components may change due to the fluctuation of utility price and technology advancements. We define a cost model to consider this factor for cost sensitivity analysis. We weight the utility price by \(\beta\) and the wear-and-tear cost by \(1 - \beta\). We define the cost of running a 100-watt server for 5 minutes as \(\beta \cdot 0.14 \cdot (100/1000) \cdot 5/60\). The wear-and-tear cost due to switching on is defined by \((1 - \beta) \cdot 0.6\). The wear-and-tear cost due to switching off is defined by \((1 - \beta) \cdot 0.4\). Therefore, the power consumption of switching a server on and off is given by \(\beta \cdot 0.04\) and \(\beta \cdot 0.01\), respectively. The power consumption to do consolidation when switching off a server is given by \(\beta \cdot 0.03\). Thus, we have the cost of \(\beta \cdot 0.04\) for running a 100 watt server for 5 minutes, the cost of 0.6 - 0.56\(\beta\) for switching a server on and 0.4 - 0.36\(\beta\) for switching a server off. \(\beta = 0.5\) is used for the cost model in all other cases in our study.

We assume that the utilization of the data center is 20%. Therefore, the average workload to the cloud is \(I \times 0.2\). We also assume that the workload can be forecasted and profiled every 5 minutes. Due to the diurnal behavior associated with human beings’ working cycles, we chose the 8 hour work time as the planning horizon where the dynamically changing workload from one time slot to another is generated for our study. The sinusoidal function and three different random distributions with the same average are used to generate temporally dynamic workload profiles. The three random distributions are Erlang-2 (smooth), exponential, and two-state hyper-exponential (bursty). Each random distribution is generated 101 times using 101 independent random streams. Note that for the workload generated by these distributions that are over the maximum capacity, i.e., \(I\), we truncate the

---

**Algorithm 3 Improved Local Smooth Algorithm with Constraints on Advance and Delay**

1. \(j \gets T\) \hspace{1cm} \text{Initialization}
2. for \(i := 2 \rightarrow j\) do
3. \(s_i[i - 1] \gets (d[i] - d[i - 1])\) \hspace{1cm} \text{Compute smooth index}
4. end for
5. while \(j > T\) do
6. if \(ss[ap] + ss[ap + 1] \leq S\) then
7. \(ap \gets \text{InMin}(si)\) \hspace{1cm} \text{Find aggregation point}
8. \(d[ap] \gets \text{Mean}(d[ap], d[ap + 1])\) \hspace{1cm} \text{Aggregate}
9. \(d[ap + 1] \gets \text{Mean}(d[ap], d[ap + 1])\) \hspace{1cm} \text{Aggregate}
10. \(s_i[ap - 1] \gets d[ap] - d[ap - 1]\) \hspace{1cm} \text{Update smooth index}
11. \(s_i[ap] \gets d[ap + 2] - d[ap + 1]\) \hspace{1cm} \text{Update smooth index}
12. \(ss[ap] \gets ss[ap] + ss[ap + 1]\) \hspace{1cm} \text{Compute the size of aggregated slots}
13. if \(ap \neq j - 1\) then
14. for \(k := ap + 1 \rightarrow j - 1\) do
15. \(d[k] \gets d[k + 1]\) \hspace{1cm} \text{Adjust the index of slots behind the aggregation point}
16. \(ss[k] \gets ss[k + 1]\) \hspace{1cm} \text{Adjust the corresponding slot size}
17. if \(k < j - 1\) then
18. \(s_i[k] \gets s_i[k + 1]\) \hspace{1cm} \text{Adjust the smooth index}
19. end if
20. end for
21. \(j \gets j - 1\) \hspace{1cm} \text{Decrease the number of slots by 1}
22. end if
23. end if
24. end while
25. return \(d, ss\)
maximum workload to \( I \) to make the problem feasible. We also wish to study the workload that can be represented by certain deterministic cyclic functions. Assume that a day’s workload can be represented by a full cycle of the sinusoidal function and the 8 hour workload window is in the range of 0 degrees and 120 degrees. We first generate the value given by the plain sinusoidal function in the range of 0 degree and 120 degree:

\[
d_t = \sin(t \times 2 \times \pi/360), \forall t = 1, \ldots, 96. \quad (48)
\]

Then, the 8 hour workload load demand with an average of 0.2 \( \times I \) is given by:

\[
d_t = (d_t - \sum_{t \in T} d_t/96 + 1) \times 0.2 \times I, \forall t = 1, \ldots, 96. \quad (49)
\]

Compared to three workloads generated by random distributions, since this workload is deterministic, we call it the deterministic sinusoidal workload.

We ran the optimization model using CPLEX through Matlab on an Intel(R) Core(TM)2 Duo CPU U9400 1.40GHz with 4GB memory.

### B. Computational Time of Different Models and Different Number of Workload Slots

We first study how the number of servers and the number of time slots of the workload affects computational time. We fix the number of time slots in the workload as 96 and vary the number of servers from 10 to 100 with an incremental step of 10. The optimal cost and computational time is shown in Fig. 3(a) and (b), respectively. The optimal cost and computational time are both linear with respect to the number of servers. Then we fix the number of servers at 100 and vary the number of time slots in the workload from 10 to 100 in an incremental steps of 10 slots. The optimal cost and computation time is shown in Fig. 3(a) and (b), respectively. The optimal cost and computational time are both linear with respect to the number of servers. Then we fix the number of servers at 100 and vary the number of time slots in the workload from 10 to 100 in an incremental steps of 10 slots. The optimal cost and computation time is shown in Fig. 3(a) and (b), respectively. Note that Fig. 3(b) is on a log scale on the y-axis. In Fig. 3(a), the optimal cost is linear with respect to the number of slots. From Fig. 3(b), we can see that the computational time is nearly exponentially increasing with respect to the number of slots. The computational time increases linearly with respect to the increase in the number of servers because increasing the number of servers does not increase the number of constraints.

Next, to quantify the difference in computation time among three proposed models, we run the same problem in three models. The data center consists of 100 identical servers (i.e., \( I = 100 \)). The workload with 96 time slots is generated according to the four aforementioned distributions. This problem fits into Model-Hom. To run Model-Het, we assume that the servers are different (although they are not) so that we can make comparisons. By dividing the servers into 10 and 20 clusters, we consider three instances of Model-HH; these three instances are denoted by HH-10 and HH-20, respectively. Fig. 5 shows that the homogenous case has the least computational time while the heterogeneous case has the most computational time consistently for all four workloads. The computational time in heterogeneous case is ten times more than that for the homogeneous case. It also shows that a lesser number of clusters results in less computational time for Model-HH.

For the same workload distribution for a specific generated seed, we obtain the same optimum; since multiple seeds are generated for each distribution, we also present the confidence interval on the optimal solution for each distribution, which is summarized in Table I. Because the sinusoidal workload generated is deterministic, there is no confidence interval for the solutions of this workload. In Table I, Fixed Configuration means the solution obtained from statically keeping all servers running all the time; Local Optimum is the solution when the switching on and off cost is not considered; Global Optimal represents the optimal solution obtained from our method. For the cost of Local Optimum, we show its Switch Cost component. Note that Switch Cost here includes wear-and-tear cost and the energy cost of performing consolidation and switching on and off. Due to the significance of Switch Cost, the result from Local Optimum is worse than that from Fixed Configuration for Exponential and Hyper-2. Switch Cost is correlated to the regularity of workload shape. By applying Global Optimum, the cost savings over Fixed Configuration and Local Optimum are significant. Compared to the Static Configuration, we achieve approximately 78%, 51%, 40%, and 27% savings for the sinusoidal workload, Erlang-2, Exponential and Hyper-2, respectively. Compared to the Local Optimum, we achieve approximately 46%, 46%, and 41% for Erlang-2, Exponential and Hyper-2, respectively. Note that the local optimum is equal to the global optimum in the sinusoidal workload; this is because of the structure of the sinusoidal workload. To the best of our knowledge, [27] is the first effort to decompose the workload into certain substructures and compute the optimal solution by the substructures. A more theoretical study is presented in [22].

### C. Insights on Aggregation

We now present three insights that we have learned from aggregation.
Insight-1: We define the degree of aggregation as the ratio of the number of original demands and aggregated demands. Denoting the degree of aggregation by $\alpha$, we have $\alpha = \lceil T/T' \rceil$. Aggregation by maximum always causes over-provisioning. The energy required to keep more than the necessary servers running is wasteful due to over-provisioning. On the other hand, the aggregation by maximum smoothes out the regularity of the workload, i.e., the fluctuation of the workload is alleviated, which lowers the switching requirements. Moreover, the higher the degree of aggregation, the more smooth the workload becomes. Strict dynamic aggregation is better than static aggregation only in terms of avoiding as much over-provisioning as possible. As to the second component of the cost, i.e., the switching cost, it is difficult to conclude whether dynamic aggregation is better than static aggregation or not. Consequently, when considering the total of two cost components, it is possible that the total cost of static aggregation is even less than dynamic aggregation. This happens in the case that static aggregation gains more switching cost savings than dynamic aggregation and this difference is larger than the gain of over-provisioning energy consumption of dynamic aggregation over static aggregation. For the same reason, the optimum of the high degree of aggregation may be better than that of the low degree of aggregation.

Insight-2: The local smooth implementation may not be better than static aggregation in terms of avoiding as much over-provisioning as possible since the implemented algorithm is an approximation based on local information. However, the local smooth implementation always favors a smoothed workload, and thus, tends to reduce switching cost.

Insight-3: In aggregation by mean, the static aggregation and dynamic aggregation end up with the same level of average offered capacity since “mean” is used. That is, aggregation does not cause over-provisioning and thus, the energy consumption of static and dynamic aggregation by mean is equivalent. Thus, whether the aggregation method costs less is solely decided by the switch cost, which is impacted by the fluctuation of the workload during the planning window.

### D. Aggregation by Maximum

We now study the pros and cons of aggregating the workload by maximum. For this study, we consider 5,000 identical servers in a data center and the servers are clustered into fifty 100-homogenous-server groups for the purpose of management. Thus, this falls into Model-HH. We run the optimization for this system with different degrees of aggregation i.e., $\alpha = 1, 2, 3, 6, 8, 12$. Note that $\alpha = 1$ means that there is no aggregation.

We first consider static aggregation. As shown in Fig. 5, the cost at optimum increases while the computational time decreases when the degree of aggregation increases. The gradient of the computational time with regard to the degree of aggregation decreases as the degree of aggregation goes up in all four cases. This means that the degree of aggregation in a smaller range (e.g., 1-3 in this experiment) has a more significant effect than that in a larger range (e.g., 6-12 in this experiment). Compared to the computational time for $\alpha = 12$, the computational time for $\alpha = 1$ increases more than 100 times for all four workload cases. This also confirms that the computational time pattern increases exponentially with respect to the number of time slots. On the other hand, the gradient of the optimum, with regard to the degree of aggregation, does not change much in the entire observed range in all workloads. Compared to the optimal cost for $\alpha = 1$, the cost at optimality for $\alpha = 12$ only increases by approximately 5%, 16%, 19%, 24% for Sinusoidal, Erlang-2, exponential and hyper-exponential-2 cases, respectively. It is observed that a small degree of aggregation reduces the computational time noticeably without significantly increasing the overall cost. Fig. 6 presents the energy cost in the the static aggregation by max case. The the energy cost increases monotonically with respect the degree of aggregation and is the dominant components in the cost structure.

Next we consider dynamic aggregation for the same set of workload distributions and the degree of aggregation. The results are shown in Fig. 6. The pattern of change and the order of magnitude of the computational time are similar to those in the case of static aggregation since the number of variables
and the number of constraints in dynamic aggregation are both the same as their counterparts in the static aggregation case. The optimum of static aggregation consistently increases when the degree of aggregation increases. However, this is not the case for some workloads in dynamic aggregation. It does not always follow this pattern in three random workload cases. It is because our local smooth implementation of dynamic aggregation relies on local information, which may not be able to achieve the global optimum of minimizing energy cost but favors reducing switching costs as we have mentioned in Insight 1. It is noteworthy that (a) the violation of the pattern is relatively minor; (b) there is no statistical difference due to the confidence interval overlap in most pattern violation events. More importantly, as we can see in Fig. 10, the dynamic aggregation outperforms static aggregation in all cases except for $\alpha = 48$ in the sinusoidal workload, $\alpha = 2, 3, 4$ in Hyper-exponential-2 (there is no statistical difference due to the confidence interval overlap). Fig. 7 presents the energy cost in the the dynamic aggregation by max case. Because the energy cost is the dominant cost components, the total cost in Fig. 10 shows similar pattern as the energy cost. The energy consumption as a result of over-provisioning due to aggregation by maximum is presented in Fig. 11. For randomly distributed workload cases, the implemented dynamic aggregation reduces over provisioning when the degree of aggregation is small ($\leq 4$ for Erlang-2 and Exponential, $\leq 6$ for Hyper-exponential). This also confirms that our implemented dynamic aggregation helps reduce workload fluctuation. As we have shown earlier, the computational time is not related to how to aggregate. Therefore, our implementation of dynamic aggregation shows that it is a good choice over static aggregation in most cases. As we can see from Fig. 10 (b), (c), and (d), the discrepancy between static and dynamic aggregation forms an elliptical shape: it starts from 0, i.e., when no aggregation needed to be performed for $\alpha = 1$. Then the discrepancy increases, then decreases and converges back to 0 when $\alpha = 96$, i.e., when the number of time slots is 1. Thus, the aggregated workload demand becomes the largest workload demand of all original time slots.

E. Aggregation by Mean

Fig. 12 shows the optimal costs and computational time when we use the static aggregation by mean approach. In all three randomly distributed workloads, the optimal costs and computational time are both monotonically decreasing with respect to the degree of aggregation as we explained in Insight 3. The gain for both optimum and time complexity comes with the price of reallocating the workload demand. The optimal costs and computational time when applying dynamic aggregation by mean are shown in Fig. 13.

Now consider comparing dynamic aggregation with static aggregation. For computational time, there is no difference. On
the other hand, when it comes to optimal costs, conclusions are vastly different. As shown in Fig. 14, static aggregation outperforms dynamic aggregation in all three non-deterministic workload cases when aggregation by mean is considered while dynamic aggregation outperforms static aggregation for sinusoid workload. Recall Insight 3: the target of dynamic aggregation in the aggregation by mean is to reduce reallocation of the workload. Therefore, we also present comparative results of the amount of workload rearrangements in both static and dynamic aggregation by mean in Fig. 15. For all three randomly distributed workload cases, the amount of workload
rearrangements of dynamic aggregation is smaller than those of static aggregation while that of dynamic aggregation is bigger than that of static aggregation in the sinusoidal workload. Thus local smooth heuristics is a good approximation for non-deterministic workloads. This suggests that the implemented dynamic aggregation is suitable to be used non-deterministic workloads. As the degree of aggregation changes from 1 to 96, the gap starts from 0, then it increases to the largest value, then converges back to 0.

F. Impacts of Varying Weights of Cost Components

In addition, we want to understand the impact of cost components change. This study also uses the 5,000 server scenario that was used for the aggregation study discussed earlier. Note that the three models yield the same optimum but require different computational times. In other words, there is no difference for these three models when only optimum is considered. We vary the weight ($\beta$) of utility price from 0 to 1. That is, we consider the range from when the utility price is negligible to when the wear-and-tear cost is negligible.

Fig. 16 presents the optimum obtained under aggregation by maximum. As we can see in Fig. 16(a),(e) for sinusoidal workload, the optimum cost is approximately linear with respect to $\beta$ for all degrees of aggregation and for both static aggregation and dynamic aggregation. Because the switching cost in the sinusoidal workload is very small compared to the energy cost. This is not the case for the other three non-deterministic workload cases since their wear-and-tear cost is comparable to the energy cost. The exception is when $\alpha = 96$, the optimum is also linear with respect to $\beta$ since there is no switching cost in these cases. As shown in Fig. 16(b),(c),(d),(f),(g),(h), plots of the optimum with respect to $\beta$ are concave. For $\alpha < 96$, the optimum increases first then decreases with respect to the increase of $\beta$. It shows the dominant component to change the optimum shifted from running energy cost (increasing) to switching cost (decreasing). The degree of concavity is negatively proportional to the degree of aggregation ($\alpha$). The value of $\beta$ for optimum “turning around” increases as the degree of aggregation increases because the degree of aggregation goes higher, the workload becomes smoother and causes the switching cost to be smaller. This important observation suggests that if the weight of switching cost is high, we can use the high degree of aggregation to save computational time without compromising the optimum. The gaps among different degrees of aggregation increase as $\beta$ increases.

In aggregation by mean, we see a similar pattern regarding the concavity (the optimum with respect to beta) and the degree of concavity with respect to the degree of aggregation. Fig. 17 presents the same set of plots for aggregation by mean. The optimum for static aggregation decreases as the degree of aggregation increases because the mean energy does not change while workload is smoothed out. The optimum for dynamic aggregation does not have such pattern, since the objective is not to minimize the cost.

VI. CONCLUSION AND FUTURE WORKS

In this paper, we first presented three formulations for different data center environments: homogeneous data center, heterogeneous data center, and heterogeneous homogenous-server-cluster data center. The computational time to obtain the optimum varies significantly in these three cases. In order to achieve on-line (or close to on-line) computation for large scale data centers, we proposed to aggregate the workload to fewer time slots. Depending on the requirements
of applications and SLA allowance, there are two types of aggregation modes. Aggregation by maximum guarantees that the workload demand of every time slot is satisfied while aggregation by mean needs to delay or advance the workload demand. On the other hand, aggregation by maximum causes over-provisioning while aggregation by mean does not.

For each of aggregation modes, we propose two aggregation methods: static and dynamic aggregation. Aggregating a fixed number of time slots into one is called static aggregation while aggregating with a certain objective is named dynamic aggregation. In the aggregation by maximum mode, the objective of dynamic aggregation is to minimize the over-provisioned capacity. In the aggregation by mean mode, the objective of dynamic aggregation is to minimize the delay and advance workload demands. An approximation implementation of dynamic aggregation is introduced to alleviate the computational overhead of implementing the exact algorithm.

Our numerical results show that aggregation is an efficient method to reduce the computational time. Choosing the appropriate degree of aggregation is a tradeoff between the cost and the computational time. We observed that the dynamic aggregating method in both modes can achieve significant gain compared to the static aggregation approach in terms of their individual objective function. The sensitivity study on varying the cost component weights shows that the appropriate degree of aggregation also depends on the weights. While our study is based on artificially generated workloads using a number of random distributions as well as a deterministic shape, the proposed methods are general to be applicable to realistic workloads.

For future work, we plan to consider decomposing and using the decomposed substructures of a workload to determine optimal solutions. Another important direction we plan to pursue is to explore the solutions for unpredictable and partially predictable workloads. The partial predictability refers that (a) we can only accurately predict for a certain length of time, but not for the entire time horizon; (b) The predicted workload demand is not accurate (for example, in a certain range); (c) combination of points (a) and (b). Results from these directions will be reported elsewhere.

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