Measurements of Discharge through a Pump-Turbine in Both Flow Directions Using Volumetric Gauging and Pressure-Time Methods

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Abstract: This article presents the original procedures for measuring the flow rate using the pressure-time and the volumetric gauging method in the case of performance tests of a reversible hydraulic machine in either turbine or pump modes of operation. Achieving the lowest possible measurement uncertainty was one of the basic conditions during implemented machine tests. It was met using appropriate measuring procedures and high-class measuring equipment. Estimation of the uncertainty for both methods was made on the basis of an analysis consistent with current requirements in this respect. The pressure-time method was supplemented by the computational fluid dynamics (CFD) analysis that allowed reducing the impact of the pipeline complex irregular geometry on the uncertainty of flow measurement. Appropriate modifications of the calculation procedure enabled accurate measurements of flow during the pump mode of operation of the tested machine as well. The volumetric gauging method, thanks to a special procedure used for accurate measurement of the water level in the upper reservoir of the power plant, allowed measuring the discharge through the tested reversible machine with very low uncertainty. The obtained results allowed for a detailed comparison and mutual verification of the methods used to measure the discharge of the tested reversible machine in both modes of its operation. The most possible causes of obtained results are discussed and summarized in the paper. The need for further research was pointed out to explain the differences obtained and their influence on the accuracy of discharge measurement using the pressure-time method in pump operation mode.

Keywords: reversible hydraulic machines; penstocks; pressure pipelines; performance tests; flow rate measurements; volumetric gauging method; pressure-time method; water-hammer

1. Introduction

Fluid flow rate measurements are one of the most complex measurements that are carried out in engineering practice. These measurements, due to the need to maintain a very narrow uncertainty band, usually require the use of sophisticated, precise measuring equipment and the use of appropriate rigorous measurement procedures [1–3].

Liquid flow rate measurements in closed conduits or open channels of small size, for instance up to 1–2 m of diameter, are usually carried out using standard measuring devices such as measuring orifice plates, nozzles, Venturi tubes, measuring weirs, electromagnetic and ultrasound flow meters, calibrated bends, and others. Such devices are usually installed in properly prepared measuring sections of conduits or channels and provide a relatively easy and fairly accurate method of measuring the flow rate.

The situation is definitely more complicated when the liquid flow rate is to be measured in large-size conduits with a diameter of several meters or more. Measurements of the flow rate in this
type of structure, usually used in hydropower, are very difficult and expensive, especially when it is necessary to ensure the lowest possible uncertainty of measurement results.

According to international standards [4–6], a few primary methods for flow rate measurement can be used in hydropower plants:

- The velocity-area method—utilizing the distribution of local liquid velocities, measured using propeller current meters (especially in cases of large conduit diameters) or Pitot tubes (for smaller diameters and flow of liquids free of sediments). The volumetric flow rate is determined by integrating the velocity distribution over the entire area of the measuring cross-section.
- The pressure-time method (often called the Gibson method [7,8])—consisting of measuring the time course of changes in the pressure difference between two cross-sections of a closed conduit while stopping the liquid stream by means of a shut-off device. The volumetric flow rate of the liquid at the initial conditions, prior to the stoppage of the flow, is determined by appropriate integration of the change in pressure difference measured during the stoppage of the flow.
- The tracer method—consisting of measurements of the passing time, or concentration, of the radioactive or non-radioactive marker (e.g., salt) between two cross-sections of a conduit. The method requires long conduits and suitable conditions for good mixing of the marker.
- The volumetric gauging method—consisting of determining the variation of the water volume stored in the headwater or tailwater reservoir on the basis of the variation of the water level in this reservoir over time.
- The acoustic method—based on vector summation of the sound wave propagation speed and the average liquid flow velocity—it uses a difference in frequencies or passing times of the emitted and received acoustic signal.

It can be concluded that the first four methods on the above list belong to the group of traditional methods, while the acoustic method is relatively new and has been recently the object of numerous research activities oriented on its improvement and validation [9,10]. This method has not yet reached proper acceptance among the specialists. Standard [4] suggests conditional use of this method, i.e., in case of mutual agreement between interested parties. Its basic advantage is that it can be used for continuous flow rate measurement and monitoring. Such a feature is impossible or extremely hard to achieve using other primary methods of measuring absolute flow rate.

The volumetric gauging method and tracer method are those which are less frequently used in hydropower engineering. The first method is characterized by a very limited application, mainly to hydropower plants with artificial reservoirs, especially in pumped-storage plants. The second one requires very long measuring segments of flow conduits and special conditions facilitating the mixing process of the injected markers (e.g., the use of turbulizers).

The velocity-area method and the pressure-time method are primary methods that are the most commonly used for measuring the flow rate in the pipelines of hydraulic turbines [3,11–13]. It is also worth noting that the velocity-area method using propeller current meters, very popular in the past, nowadays is being replaced by the pressure-time method in hydropower plants equipped with pipelines longer than 10–20 m. One of the main reasons for this is the much lower cost of preparing and performing flow measurements using the pressure-time method and the use of computer techniques in recent years, which facilitate measurements and give the possibility for getting higher accuracy of results obtained with this method.

For low and very low head power plants, particularly with short intakes of hydraulic turbines, (with no penstocks) the situation is different. Up to now, generally only the velocity-area methods, especially current meter method, are basically available in such kind of plants. Flow rate measurements with this method are still quite expensive and alternatives are being sought. One such alternative is the acoustic scintillation technique, under development [14,15].

Relative (index) methods are also used to measure the flow rate in hydropower plants. For example, the Winter–Kennedy method and the methods utilizing non-standardized pressure difference devices,
non-standardized overflows (weirs), some simple variants of the acoustic method or local velocity measurement, which can be used for determining the relative value of the flow rate, or even the physical value, provided that calibration has been done on site by comparing with the results of measurements using the primary method [16–18].

As is the case concerning every measurement technique, obtaining the appropriate measurement precision is of the utmost importance. This is absolutely necessary wherever there are low uncertainty requirements, e.g., in the case of performance tests of hydraulic machines. The measurement conditions occurring in the flow systems of these machines require experience and knowledge about the flow phenomena prevailing in these systems, and also force the search for additional, unconventional techniques to ensure sufficiently low measuring uncertainty.

The bases of the analysis presented in the paper are measurement examples of flow rate through a high-head reversible hydraulic machine. Measurements were conducted using the volumetric gauging method and the pressure-time method, recommended (as mentioned earlier) by international standards [4–6] as the primary methods for discharge measurements used for performance tests (warranty, acceptance) of hydraulic turbines, pump turbines, and storage pumps. However, there are some restrictions on applicability as in the case of the pressure-time method, but work is continuously ongoing to expand and update these standards (A. Adamkowski, one of the authors of this work is a member of the PTC 18 Committee that is currently developing a revision to the ASME Performance Test Code PTC 18-2011 “Hydraulic Turbines and Pump Turbines”).

The simultaneous application of the pressure-time method and the volumetric gauging method to measure discharge through the tested hydrounit with reversible Francis turbine opened the possibility of their peer verification, which was the main goal of the work.

As part of this task, the suitability of the pressure-time method for measuring flow rate in the pump mode of operation was tested. The use of this method in such conditions is not recommended by standards [4–6], therefore the obtained results are of particular importance for the development of this method.

The tests were performed ensuring a low level of measurement uncertainty. It required a number of procedures, some of which are innovative solutions, such as:

- Applying a special procedure for measuring of water level changes in the upper reservoir using the volumetric method.
- Taking into account the complex geometry of measuring section of the pipeline and its impact on flow phenomena using techniques based on computational fluid dynamics (CFD) and applying these results in the pressure-time method.

Moreover, in order to reliably estimate the measurement uncertainty of the applied methods, a procedure that takes into account general requirements concerning uncertainty assessment gathered in [19] has been proposed. This task is an attempt to systematize the problem of estimating measurement errors with the use of the analyzed methods.

Comparison concerning results obtained using chosen flow measurement methods, which is an example quite rarely seen in the literature concerning this subject, provides a unique source of knowledge about the features of the methods and the possibilities of their practical use.

2. Materials and Methods

2.1. The Research Object

Both discussed methods for discharge measurement—pressure-time and volumetric gauging method—were used for performance tests of a reversible hydrounit in a Polish pumped-storage power plant (PSPP). The considered plant is equipped with four similar reversible hydraulic machines (pump-turbines) working under the head of approximately 440 m and generating/consuming power over 120 MW.
The artificial head water reservoir is connected to pump-turbines using two underground penstocks, branching close to the inlets of the pump-turbines, prior to the shut-off ball-valves. The pump-turbines are connected via the tailrace tunnel with the surge tank to the tail water tank. A schematic diagram of the PSPP flow system with its main dimensions is shown in Figure 1.

Figure 1. Flow system of the pump-turbine.

2.2. The Volumetric Gauging Method

Determining discharge using the volumetric gauging method consists in measuring the volume of water $\Delta V$ flowing through the tested hydraulic machine during time $\Delta t$. The discharge is determined using the following formula:

$$ Q_V = \frac{\Delta V}{\Delta t} = \frac{V(z(t_f)) - V(z(t_0))}{t_f - t_0} \quad (1) $$

where $\Delta V [m^3]$ stands for measured increase or decrease in volume of water in the head water reservoir, $\Delta t = t_f - t_0 [s]$—the time interval in which the increase/decrease in water volume occurred, and $z$—level of water in the head reservoir.

When using the volumetric gauging method, there are several issues that can significantly affect the accuracy of the measured flow rate [11,18]. The main task is to determine the relationship between the volume and the water level of the reservoir $V(z)$. This relationship should be determined on the basis of precise reservoir geometry measurements (particularly useful for artificial reservoirs) or accurate bathymetric scanning. The issue of determining the reservoir volume also involves measuring the water level in this reservoir.

In common situations, transmitters designed to control this level usually included in the power plant equipment are not suitable for use in the volumetric gauging method as they have a wide measuring range and low accuracy class. In order to achieve low uncertainty of measurements, the change in the water level in the reservoir should be determined using special methods. The schematic diagram of the proposed method is shown in Figure 2. Its most important element is measuring the increase in water level $\Delta z$ in the power plant reservoir by means of a precise transducer measuring the
pressure difference in the reservoir and a constant pressure level set using small auxiliary tank, hung at the appropriate height. The configuration of such an installation should ensure the possibility of carrying out an approximately one-hour measurement at a fixed operating point of the tested hydrounit.

Figure 2. The water level change measurement technique used in the volumetric gauging method.

The proposed method allows for the significant reduction of the measurement uncertainty giving an additional possibility for taking into account the unfavorable phenomenon of water surface waving occurring during the tests. This phenomenon can affect the results of the measured flow rate in the most significant way. Traditional ways for measuring the water level used in the volumetric gauging method cannot ensure required accuracy of discharge measurements. Using a measuring system with appropriate characteristics and applying linear regression for the results of measuring the level of water in the reservoir leads to eliminate the effect of water waving on measurement results (Figure 3). It’s worth pointing out that it is very important to base the regression line on the boundaries selected at the extreme points of the peaks or valleys of the differential pressure signal. This is a prerequisite for obtaining the correct final flow measurement results.

Figure 3. The volumetric gauging method—basic rules of flow rate determination.
Owing to the solutions applied, a very narrow uncertainty range was possible to achieve and the results of its estimation are presented in the next chapter of the paper. The uncertainties (standard and expended) were estimated according to the procedure described in Appendix B that was developed basing on the general recommendations presented in [19].

2.3. The Pressure-Time Method

2.3.1. Basic Information

The pressure-time method is based on the relationship between flow rate at steady state conditions and pressure-time change occurring in the pipeline during cutting off the flow [7,8]. The value of $Q_0$ indicating the discharge at initial liquid flow conditions is calculated using the definite integral over a time interval in which the flow varies from initial conditions to conditions after the flow is completely shut off [4,6,11]:

$$Q_0 = \frac{1}{\rho F} \int_{t_0}^{t_f} \left( \Delta p(t) + \Delta p_d(t) + \Delta P_r(t) \right) dt + Q_f$$  \hspace{1cm} (2)

where:

- $\rho$ is the density of a liquid,
- $t_0$ and $t_f$ are the initial and final time-limits of integration, respectively,
- $Q_f$ is the discharge under final steady-state conditions (after complete closing of the shut-off device) due to the leakage through the closed shut-off device,
- $\Delta p$ is the difference in pressures measured between the pipeline measuring cross-sections $B-B$ and $A-A$, which geometrical centers are at level $z_A$ and $z_B$, respectively (Figure 4):

$$\Delta p = p_B \rho + \rho g z_B - p_A - \rho g z_A$$  \hspace{1cm} (3)

$\Delta p_d$ is the difference in dynamic pressures between the pipeline measuring cross-sections with area of each section equal $A_A$ and $A_B$:

$$\Delta p_d = \alpha_2 \frac{\rho Q^2}{2A_B} - \alpha_1 \frac{\rho Q^2}{2A_A}$$  \hspace{1cm} (4)

where:

- $\alpha_1, \alpha_2$ are the kinetic energy correction factors for $A-A$ and $B-B$ sections (the value of the kinetic energy correction factor for fully developed turbulent flow in the pipeline, dependent on $Re$ number is within the limits from 1.03 to 1.11 [20,21]);
- $\Delta P_r$ is the pressure loss caused by hydraulic resistance in pipeline between the measurement cross-sections—quantity calculated as proportional to the square of flow rate (accounting for its direction):

$$\Delta P_r = C_r|Q|^2$$  \hspace{1cm} (5)
One of the most important parameters in Formula (2) is the \( F \) factor. Its value depends on the geometry of the pipeline flow system between the pressure measurement cross-sections. The following formula can be used to calculate the \( F \) factor in case of the pipeline segment with length \( L \) and \( j \) sub-segments with different sizes:

\[
F = \int_{0}^{L} \frac{dx}{A(x)} = \sum_{j=1}^{J} \frac{\Delta x_j}{A_j}, \quad \text{with} \quad \sum_{j=1}^{J} \Delta x_j = L
\]  

(6)

where \( \Delta x_j \) and \( A_j \) indicate the length and internal cross-sectional area of the \( j \)-th sub-segment, respectively. As shown in Equation (2), the pressure loss, \( \Delta P_r \), representing hydraulic resistance and the dynamic pressure difference, \( \Delta P_d \), should be separated from the pressure difference measured between the pipeline measurement cross-sections, \( \Delta P \). In total, the integral expression of Equation (2) defines the pressure difference resulting from the inertia force of the mass of liquid retained in the pipeline measuring section (segment). The values of \( \Delta P_r \) and \( \Delta P_d \) can be calculated with good accuracy using their dependence on the square of the flow rate in the forms written in Equation (4) and (5).

Measurements made using the pressure-time method, as was the case concerning the volumetric gauging method, were carried out for both flow directions through a reversible machine equipped with Francis type runner. Measuring flow rate in the pump direction requires appropriate modifications of the pressure-time method to the calculation procedures described in the standards, which were postulated by the authors in earlier publications [11,22,23] and which resulted with formula in Equation (5) (introducing term \( Q|Q| \) instead of \( Q^2 \)).

A comprehensive discussion of some problems related to the computational procedures in the pressure-time method is provided in standards [4,6] as well as in monograph [11]. A description of some important problems related to the use of the pressure-time method for measuring flow rate in hydropower plants can also be found in publications [22–29]. Calculation of friction losses according to the quasi-stationary hypothesis is consistent with the conclusions presented in [30]. It was proved that the modelling of unsteady friction losses has little effect on the course of water hammer in its initial time-phase that is taken into account in the pressure-time method. Nevertheless, it should be emphasized that including the transient nature of friction losses into the calculation method, under certain circumstances, may improve predictions of the pressure-time method as described in [27–29].

Several variants of the pressure-time method are used in practice. They differ mainly in methods of measuring the pressure differences between pipeline measurement cross-sections. In the considered case, the pressure-time method was used in the variant based on measuring the pressure changes at the cross-section of the pump-turbine spiral case outlet/inlet and relating these changes to the pressure.
exerted by the water column from the head water reservoir. This variant requires the determination of the geometric factor $F$ accounting the entire penstock of the tested machine, starting from the inlet section and ending with the outlet/inlet cross-section of the spiral case.

The recommendations of the standards [4–6] allow the use of the $F$ factor for straight-axis measuring pipelines of variable diameter (according to the Formula (6), taking into account their geometry). However, in the case of more complex changes in the geometry occurring in the measuring section of the pipeline (changes in the shape of the flow section, changes in the direction of the pipeline axis or branches), there is a need to take into account the influence of these changes on the flow conditions.

Irregular parts (components) of the penstock cause flow disturbances in the form of non-uniform water velocity distribution. This should be taken into account in order to ensure better accuracy of discharge measurement. In the considered case, except for the straight pipe sections with constant internal diameters, the penstock has three elbows (two vertical and one horizontal), a number of short conical sections connecting pipes of different diameters, and two short branches, where one branch remained closed during the tests. In addition, the square cross-section as well as transition section from square to the circular cross-section in the highest part of the penstock had to be taken into consideration. In the previously published work [24], authors presented the procedure, based on CFD, used for correction of $F$-factor calculated in case of penstocks with elbows. The assumption of equal kinetic energy resulting from the simulated and the uniform water flow velocity distributions in the same flow parts of the penstock was the main, except mass conservation law, theoretical basis for this procedure. In this work, using CFD, an extended procedure was developed and applied to correct the value of the $F$-factor for the above-mentioned irregular components of the penstock under consideration. The procedure is presented in detail in Appendix A. The selected results of CFD calculations and the $F$-factor correction for the studied case are presented later in this paper.

2.3.2. CFD Based Correction of Penstock Geometrical Factor

The NUMECA/Hexpress commercial software [31] was used for generating the computational grid representing the penstock geometry (Figure 5). The unstructured grids consisted of hexahedral elements.

![Figure 5. Geometry of hydraulic system (calculation domain): head water reservoir (hydraulic diameter of virtual half-cylindrical inlet 30 m) → square pipeline (4.3 × 4.3 m) → cylindrical pipe (4.3 m) → conical pipe (4.3/3.9 m) → cylindrical pipe (3.9 m) → conical pipe (3.9/3.6 m) → cylindrical pipe (3.6 m) → conical pipe (3.6/3.2 m) → cylindrical pipe (3.2 m) → pipe branch for two pump-turbines (2.276 m) → conical pipe (2.276/1.654 m) → outlet cylindrical pipe (1.654 m).](image)

For flow calculations, ANSYS/Fluent commercial software was used [32]. The flow was simulated by solving the steady-state Reynolds Average Navier-Stokes (RANS) equations with the $k-\omega$ SST turbulence model. Many studies demonstrate the great usefulness of this turbulence model in the calculation of industrial flow systems [33,34]. It’s commonly known that the $k-\omega$ SST model integrates advantages of both $k$- turbulence model and standard $k-\omega$ turbulence model [35].
The second-order upwind discretization was used with the SIMPLE scheme of pressure-velocity coupling. Non-dimensional distance from wall $y^+$ was assumed to be in range 1 to 5 according to the used turbulence model. Initialization of calculation was done from all zones limiting the computational domain. The calculations were conducted until all of the residuals (continuity residual, velocity components, turbulent kinetic energy, and specific rate of dissipation) reached values less than 0.001. The parameters for a closure of turbulence model were hydraulic diameter and turbulence intensity. First of them was calculated using formula: $D_h = 4A/P$ [m], in which $A$ is the area and $P$ is the perimeter (hydraulic diameter was 1.654 m at inlet/outlet of lower part the penstock and 30 m at inlet/outlet of upper part of the penstock). The second parameter was calculated using the formula [32]: $I = 0.16 \frac{Re}{(Re)^{-1/8}}$ in which $Re$ is Reynolds number at inlet or outlet cross-section. At the outlet of the measuring section, constant static pressure was assumed for all calculation cases. The free surface of the reservoir was assumed as a no-slip boundary condition.

The CFD calculations were conducted for four discharge values (20, 25, 30, and 35 m$^3$/s) in the turbine operation modes and for two discharge values (26 and 28 m$^3$/s) in the pump operation modes. The sample of calculation results in the form of water velocity distributions in cross-sections for three chosen flow parts of the penstock were presented in Figures 6–8 for both flow directions, for analyzed discharge of 35 m$^3$/s in turbine regime, and 28 m$^3$/s in pump regime.

**Figure 6.** The water velocity contours in the penstock inlet part with first elbow for discharge of $Q = 35$ m$^3$/s in turbine regime (left view) and for discharge of $Q = 28$ m$^3$/s in pump regime (right view).

**Figure 7.** The water velocity contours in the penstock part containing the cone pipe for discharge of $Q = 35$ m$^3$/s in turbine regime (left view) and for discharge of $Q = 28$ m$^3$/s in pump regime (right view).
These values were used as correction quantities of the geometrical factor $\Delta F$. However, it presents different level for both turbine and pump operational modes: the largest irregular flow occurs in the penstock branch and despite the fact that it only affects the velocity distribution locally, the propagation of these irregularities in the direction of the water flow is clearly more visible than in the opposite direction. The intensity of the flow disturbance decreases rapidly with distance. On the other hand, the smallest flow irregularities in the penstock are induced by the existing short tapered pipe sections.

The velocity distributions in the elbows also differ depending on the direction of flow, which is quite obvious—the elbows induce disturbances in the flow pattern, which propagate to the next penstock components with decreasing intensity. For example, for the turbine operation mode, the flow achieving the elbow #2 is almost uniform because of the long straight section of pipe before this elbow (looking in turbine flow direction), while in pump operation mode, a similar effect takes place in elbow #1.

The CFD simulation results received for the analyzed penstock flow parts (Figures 6–8) can be characterized as follows:

- The water velocity distributions inside the area of the penstock inflow/outflow (in the cross-sections near the head water reservoir) are different for the turbine and pump operation modes.
- The largest irregular flow occurs in the penstock branch and despite the fact that it only affects the velocity distribution locally, the propagation of these irregularities in the direction of the water flow is clearly more visible than in the opposite direction. The intensity of the flow disturbance decreases rapidly with distance. On the other hand, the smallest flow irregularities in the penstock are induced by the existing short tapered pipe sections.
- The velocity distributions in the elbows also differ depending on the direction of flow, which is quite obvious—the elbows induce disturbances in the flow pattern, which propagate to the next penstock components with decreasing intensity. For example, for the turbine operation mode, the flow achieving the elbow #2 is almost uniform because of the long straight section of pipe before this elbow (looking in turbine flow direction), while in pump operation mode, a similar effect takes place in elbow #1.

The CFD results taking account flow irregularities induced in the penstock were used to calculate the equivalent factor $F_e$ according to the original procedure presented in Appendix A.

The deviation factor, $\Delta f$, representing a relative difference between the equivalent penstock factor, $F_e$, (obtained using CFD calculations) and the penstock geometrical factor, $F$, was included in discharge determination according to the pressure-time method. This factor is calculated as follows:

$$\Delta f = \frac{F_e - F}{F}$$  \hspace{1cm} (7)

The values of quantity, $\Delta f$, determined for chosen discharge values for both flow directions are presented in Table 1. It can be stated that $\Delta f$ is kept almost constant for both flow directions separately. However, it presents different level for both turbine and pump operational modes: the average value of $\Delta f$ is about +0.13% and about +0.77% for turbine and pump modes of operation, respectively. These values were used as correction quantities of the geometrical factor $F$ calculated based only on the geometry of the entire penstock.
2.3.3. Flow Rate Measurement, Uncertainty

The values of flow rate (discharge) were calculated based on the difference of pressures measured between the inlet/outlet cross-section of the tested pump-turbine (cross-section (B-B)) and the reservoir (cross-section (A-A)) and accounting for $F_r$ factor obtained using CFD. Calculations were carried out using the computer program GIB-ADAM that has been tested and successfully verified on many occasions related to the implementation of laboratory tests as well as e.g., efficiency tests in hydropower plants [11]. Examples of the results measured or calculated for both modes of operation of the pump-turbine under investigation are shown in Figure 9. Measurements begin ca. 30–40 s before shut-off device start closing and end about 30–60 s after its complete closure or after extinction of the free pressure oscillation remaining in the flow system after the flow cut off. The time of closing the wicket gates of the tested machine was about 25 s and 20 s during turbine and pumping mode of operation, respectively. These time intervals were (8–10) times longer than the pipeline pressure wave period of about 2.5 s. Closing of the wicket gates was carried out in two stages in both modes—the faster stage followed by slower one. The reason for this common method of closing the wicket gates is to maintain the safety of the hydraulic system by preventing excessive pressure oscillations caused by too rapid shut-off of the flow, especially in the final phase of wicket gates closing.

The analysis of the influence of the above-mentioned and other parameters on the uncertainty of the results of the flow rate measurement with the applied method is presented in Appendix C.

| Machine Operation Mode | Discharge, $Q_0$ ($m^3/s$) | Relative Difference of $F$-Factor, $\Delta f$ (%) |
|------------------------|----------------------------|-----------------------------------------------|
|                        | 20                         | 0.15                                          |
|                        | 25                         | 0.14                                          |
| Turbine operation mode | 30                         | 0.13                                          |
|                        | 35                         | 0.11                                          |
|                        | 26                         | 0.77                                          |
|                        | 28                         | 0.77                                          |

Figure 9. Examples of measured values of wicket gates opening and pressure difference and discharge through the machine calculated using the pressure time method. Left view: turbine operation mode, right view: pump operation mode.
3. Results and Discussion

The volumetric gauging method of flow measurement, due to the high requirements that must be met, is difficult to apply when testing real objects. For this reason, the examples of its practical application are quite rarely published. More valuable are the results presented in this paper, which were obtained for a pumped-storage power plant equipped with an artificial head water reservoir with known geometric characteristics. This made it possible to use the volumetric gauging method to measure the flow rate through the tested reversible hydrounit. The required narrow uncertainty band was obtained by supplementing the method with a special solution for accurate measurement of the water level change in the reservoir that also allowed including the impact of waves, as well as the amount of rainfall and leaks during measurements. It should be emphasized that measuring the upper water level in a standard way usually cannot ensure sufficient accuracy of the volumetric gauging method used for measuring flow in hydroelectric power plants.

The application of the pressure-time method to measure the flow rate in a real flow system with complex geometry additionally requires the use of an innovative calculation methodology to determine the $F$-factor—one of the critical parameters for maintaining a sufficiently narrow measurement uncertainty band. Owing to this factor, the geometrical characteristics of a pipeline measuring segment and impact of its flow elements on flow irregularities are taken into account. Disregarding changes in flow velocity profiles resulting from the variable shapes of pipeline elements leads to an increase in the inaccuracy of measurement using the pressure-time method, which cannot be corrected only by improving the modeling of friction losses in these elements, as discussed in [27–29] or by improving the computational model [22,28]. In addition, increasing the accuracy of estimation of the leakage rate through closed-flow shut-off devices is not enough [26]. In order to take into account changes in liquid velocity profiles in pipeline bends, the authors proposed a special calculation procedure (described in [24]) using CFD analysis for correction of the $F$ factor. Verification of this procedure based on the analyzed examples confirmed that its application significantly increases the measurement accuracy of the pressure-time method. In this paper, the procedure based on CFD has been extended and used for piping systems with complex geometry (including curves, branches, conical elements, and inlets with changes in the shape of the flow section). In contrast to such a solution, the standard application of the pressure-time method does not provide the required uncertainty of flow rate measurement results. This innovative procedure provides the basis for using the pressure-time method in case of geometrically complex pipelines, and not only in turbine mode of operation, but also in the pump flow conditions of the tested reversible machine.

The uncertainties (standard and expanded) of the flow rate measurement results using both methods under consideration were as follows:

- **Volumetric gauging method**: standard and extended uncertainties were not greater than $+/-0.38\%$ and $+/-0.76\%$, respectively, for all measured flow rates—Appendix B;
- **Pressure-time method**: standard and extended uncertainties were not greater than $+/-1.0\%$ and $+/-1.1\%$, respectively, for all measured flow rates—Appendix C.

3.1. Turbine Operation Mode

Because it was not possible to measure water discharge through the tested machine using both methods (volumetric gauging and pressure-time methods) simultaneously, the comparison of the results measured for the turbine operation mode was performed using the Winter–Kennedy method. According to this method, the measurement of discharge is based on the relationship between the discharge, $Q$, and the difference of pressures, $\Delta p_{wk}$, between the outer and the inner side of a spiral case of the machine under test:

$$Q = k \Delta p_{wk}^n$$  \hspace{1cm} (8)

where $k$ and $n$ are constant coefficients experimentally determined during the calibration process. A value of the exponent, $n$, was assumed from the theory as equal to 0.5. Such assumption insignificantly
influenced the measuring results as was proven in [17] and it is negligible for purposes of comparison presented in this paper. For the tested machine, the values of \( k \) coefficient were determined independently on the basis of discharge measurement conducted using the volumetric gauging and the pressure-time methods—in Figure 10. The difference between \( k \) coefficient values obtained using these two different methods is very small, only about 0.2%. It should be emphasized that for the penstock geometric factor, \( F \), used in the pressure-time method without the \( \Delta f \) correction, the difference in the value of the \( k \) coefficient is slightly larger and amounts to approximately 0.33%. Although in the case under consideration the difference is not large, taking into account the various pipeline geometries that encounter in practice, it is recommended to support the pressure-time method by means of CFD analysis in the case of measuring sections of pipelines with irregular elements causing disturbances in the flow.

![Calibration of Winter-Kennedy installation](image)

**Figure 10.** Turbine operation mode of the tested hydrounit: Comparison of the volumetric measurement method and the pressure-time method based on the results of calibration of the technical installation of the Winter–Kennedy method, with which the tested pump-turbine was equipped.

### 3.2. Pumping Operation Mode

The use of the Winter–Kennedy method for measuring flow rate in the pump mode of operation of hydraulic machines is not recommended by the standards [4–6]. This made it impossible to compare the pressure-time and volumetric gauging methods in a manner analogous to that used for turbine mode of operation, i.e., based on the results of simultaneous flow measurements. The comparison of results obtained using the analyzed methods was made by referring them to the head of the plant—Figure 11. The analysis also covered the impact of the penstock geometry irregularities on the results obtained using the pressure-time method. The differences between the discharge results obtained from the volumetric gauging method and pressure-time method were from −0.16% to +0.58% for lower (426 m) and higher head (439 m), respectively. Without correction of \( F \) geometrical factors, the differences were much greater—their values were +0.6% and +1.35%, correspondingly.

The comparison shows that the differences between the results obtained using the analyzed methods are much larger for pump mode of operation than for turbine mode of operation. At this stage of research, the causes of such observations cannot be clearly explained. Measurement of the hydraulic machine discharge using the pressure-time method is much more difficult to perform in pump operation than in turbine operation. This fact may suggest the reasons for this comparison results. This may also be the main reason why current standards do not recommend using this method in pump mode of operation of tested machines. However, it should be emphasized that the differences obtained in the analyzed case are still within the range of the measurement uncertainty characterizing the compared methods.
In addition, it should be emphasized that between the pump and turbine modes, in addition to obvious differences, there are those that can significantly affect the results obtained using the pressure-time method:

- Shut-off during pumping is characterized by much more irregular pressure changes than when cutting off flow during turbine mode of operation. This is related to the fact that during turbine operation, the flow was cut off while maintaining the generator connected to the network, while in pump operation, complete flow cut-off with the motor connected to the network was unacceptable.
- At the final stage of closing the machine’s wicket gates, during pump operation, there is a short change in the direction of fluid flow—from the pump to the turbine direction;
- Due to the direction of flow, it should be noted that the pump operation mode, in contrast to the turbine operation mode, induces pressure pulsations with a much higher level, which propagate along the pipeline and have a direct impact on the measured pressure difference.
- The flow in the pump direction takes place along the expanding flow elements of the pipeline (diffusers), which is the reason for greater hydraulic losses (pressure losses occur due to local losses caused by greater turbulence in the boundary layers) and as a result requires greater correction of the \( F \) geometrical factor compared to turbine flow (and flow through the confusors).

Precise identification of how these differences may affect the final accuracy of flow measurement results obtained with the pressure-time method in the pump mode of operation of the hydraulic machines requires thorough professional testing and analysis. Currently, there is insufficient data on this topic, which hinders the extension of the applicability of this method and can also lead to excessive simplifications resulting in increased measurement uncertainty.

![Figure 11. Pump operation mode of the tested hydrounit: Comparison of the discharges measured by the volumetric gauging method and the pressure-time method.](image)

4. Conclusions

The paper presents experiences concerning the use of the volumetric gauging method and the pressure-time method for measuring the water discharge through a reversible hydraulic machine at a pumped storage power plant. Research using these methods concerned both turbine and pump mode of operation of the tested machine. As part of the research, new original procedures have been used aimed at significant reduction of the measurement uncertainty.

In the case analyzed in the article by appropriate treatment consisting of the use of high-quality transducers, with the use of appropriate measurement techniques and procedures supporting the measurements of the flow rate with the use of both methods, a satisfactorily low measurement uncertainty was achieved.
The use of a high-class transducer measuring the pressure difference between the upper reservoir and the auxiliary tank in the volumetric gauging method, as well as the original method of analysis of the measured pressure difference, allowed to increase the accuracy of measuring the change in water volume over time significantly, and also allowed us to take into account water waving, which, when ignored, can meaningly distort measurements.

The pressure-time method, which required taking into account the complex geometry of the pipeline connected to the tested hydrounit, was supported by CFD analysis of flow in the area of geometric irregularities (inlet, diameter changes, elbows, changes in cross-sectional shape). The original procedure using the results of this analysis provided the information necessary to introduce appropriate adjustments (correction) to the geometric factor $F$, which in turn, contributed to a significant reduction in the flow rate measurement uncertainty.

In contrast to the very good compliance of the results of discharge measurements obtained with the analyzed methods for the turbine operation of the tested machine (differences in the range of $\pm 0.2\%$), in the case of pump operation, larger differences between the results were observed; however, they were still in the uncertainty band for measuring each of these methods independently (differences from $-0.16\%$ to $+0.58\%$). At this stage, it is difficult to clearly explain these observations. The authors point out the differences in the course of flow phenomena during shut-off in turbine and pumping operation carried out as part of tests executed using the pressure-time method. There is a need for further research to explain the reasons of the obtained differences and their influence on the accuracy of discharge measurement using the pressure-time method in pump operation mode.

It is worth emphasizing the positive effect achieved by using the CFD procedure to support the pressure-time method. A measure of this effect is the reduction of the differences between the measurement results obtained using the volumetric gauging method and the pressure-time method. In the turbine operation mode, the CFD-based correction of the $F$ factor resulted in a 1.5-fold increase in the convergence of the compared results. In the case of pumping mode of operation, the convergence has improved several times (more than 2- to almost 5-fold, depending on the point of operation). This result proves the correctness of the assumptions made when using the CFD procedure and using its results for the pressure-time method of measuring the flow.

Particularly noteworthy are the results obtained for the pumping mode of operation, for which the use of the pressure-time method is not recommended by the standards. The comparison and consistency of these results with the results obtained with the volumetric gauging method confirmed the correctness of the assumptions underlying the proposed and applied modifications to the calculation procedure of the pressure-time method. This includes also the correct consideration of the temporary change in the flow direction occurring during its cutting off in the pumping mode of operation. Such experience from using this method in practice can help working out the relevant changes in the standards leading to the recommendation of the pressure-time method also for the pumping mode of reversible hydraulic machine operation.

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**Nomenclature**

$A$ area; $[m^2]$,

$D$ internal diameter of a pipeline; $[m]$,

$e_k$ kinetic energy per unit of mass (specific kinetic energy); $[J/kg]$,

$F$ geometrical factor of a pipeline; $[m^{-1}]$,

$L, l$ pipeline length; $[m]$,
\( m \) mass flow rate; [kg/s],
\( p \) pressure; [Pa],
\( \dot{Q} \) volumetric flow rate; [m\(^3\)/s],
\( t \) time; [s],
\( u \) absolute standard uncertainty,
\( V \) water volume [m\(^3\)] or water flow velocity; [m/s],
\( x \) distance along pipeline axis; [m],
\( x, y, z \) coordinates; [m],
\( Y \) turbine guide vane opening; [%],
\( \Delta f \) relative deviation factor of F factor; [%],
\( \rho \) liquid density; [kg/m\(^3\)],
\( \delta \) relative standard uncertainty [%].

**Indexes**

d dynamic pressure value,
e equivalent value,
f final value,
l total number of numerical cross-sections in a considered pipeline; [-],
I total number of sub-segments with different dimensions (geometry) in a considered pipeline; [-],
m average (mean) value,
r hydraulic resistance
0 initial value.

**Appendix A. Procedure for Calculating Equivalent Geometrical F Factor in The Pressure-Time Method for Pipelines with Irregular Shape Sections of the on the Basis of CFD Analysis**

The determination of the geometrical \( F \)-factor from Equation (3) is fully acceptable for straight measuring sections of pipelines where there are no flow irregularities. This equation does not take into account changes in the flow velocity profiles in irregularly shaped pipeline elements, such as elbows, bifurcations, cones, pipe inlets, etc. Therefore, the authors of this paper recommend a special calculation procedure to consider the effect of these irregular shaped flow elements on the pressure-time measurement results.

The procedure shown below is an extension of the procedure for the curved pipe sections published in [24].

**Step 1:** Determination of the geometry of the considered pipeline flow system, discharge \( Q_j \), etc., and the computational control flow space—Figure A1.

**Step 2:** Division of the computational control flow space into \( I \) numerical elements using cross-sections normal to the axis of the considered \( i \)-th (\( i = 1, 2, ..., I \)) pipe elements.

![Figure A1. A pipe elbow with marked computational space.](image-url)
Step 3: Simulation of velocity field $V(x,y,z)$ in the flow elements of the considered pipeline within the frame of the computational control space using CFD computer software.

Step 4: Computation of mean flow velocity, $V_{ai}$, for each $i$-th numerical cross-section from the previously derived CFD results (step 3), and assumption of equal kinetic energy resulting from the simulated and the uniform flow velocity distributions:

$$e_{kCFDi} = e_{kai}; \quad \rho = \text{const}$$  \hspace{1cm} (A1)

$$e_{kCFDi} = \frac{1}{m} \int \frac{1}{2} \rho V_i^2 \rho dA = \frac{\rho}{2m} \int V_i^3 dA_i; \quad \dot{m} = \rho V_{ai} A_i$$  \hspace{1cm} (A2)

$$e_{kai} = \frac{1}{2} V_{ai}^2 = \frac{1}{2m} \rho A_i V_{ai}^3$$  \hspace{1cm} (A3)

$$V_{ai} = \left[ \int A_i \left( \frac{V^3 dA}{A_i} \right) \right]^{1/3}$$  \hspace{1cm} (A4)

where $V_{i}$ is the flow velocity axial component—the component perpendicular to the $i$-th numerical cross-section.

Step 5: Computation of the equivalent cross-sectional area, $A_{ei}$, for each numerical cross-section ($i = 1, 2, ..., I$) using the continuity equation $Q_j = V_{ai} A_{ei} = \text{const}$:

$$A_{ei} = \frac{Q_j}{V_{ai}}$$  \hspace{1cm} (A5)

Step 6: Computation of coordinates of flow velocity centers in each $i$-th numerical cross-section, $i = 1, 2, ..., I$:

$$x_{Ci} = \frac{\int A_i xV(x,y,z) dA}{V_{ai} A_{ei}}; \quad y_{Ci} = \frac{\int A_i yV(x,y,z) dA}{V_{ai} A_{ei}}; \quad z_{Ci} = \frac{\int A_i zV(x,y,z) dA}{V_{ai} A_{ei}}$$  \hspace{1cm} (A6)

Step 7: For the considered flow rate $Q_j$ through the analyzed pipe element, computing the equivalent factor $F_{eq_j}$ from the formula:

$$F_{eq_j} = \frac{\sum_{i=1}^{I-1} \frac{l_{i-i+1}}{0.5(A_{ei} + A_{ei+1})}}{m}$$  \hspace{1cm} (A7)

where $l_{i-i+1}$ denotes the distance between the resultant velocity centers for computational sections $i$ and $i + 1$, $A_{ei}$ and $A_{ei+1}$—equivalent areas of computational cross-sections $i$ and $I + 1$, respectively.

The above computation should be performed for several discharge values ($Q_j, j = 1, 2, ..., m$) from the whole scope of its variation ($Q_{\text{min}} < Q_j \leq Q_{\text{max}}$). The average value of equivalent factor, $F_{eq}$, can be calculated from the relationship:

$$F_{eq} = \frac{1}{m} \sum_{j=1}^{m} F_{eq_j}$$  \hspace{1cm} (A8)

In the above procedure, it was assumed that the changes in velocity profiles are the same under steady and transient flow conditions. This assumption is correct for cases where the flow shut devices are not closed very quickly when using the pressure-time method. Practically, such cases occur in all hydraulic machines, due to the need to protect their flow systems against the destructive effects of the water hammer phenomenon.
Taking the equivalent value of $F_e$ instead of the value $F$ calculated directly from the geometry of pipeline sections it is possible to increase the pressure-time method accuracy in cases when pipelines have irregular flow elements.

**Appendix B. Analysis of the Uncertainty of Measuring the Flow Rate by the Volumetric Gauging Method**

The estimation of uncertainty of measuring the flow rate by the volumetric gauging method takes into account the following factors influencing the measured flow rate, both of a systematic and random nature:

1. Accuracy of geodetic measurements of the geometry of the head water reservoir of the power plant in order to determine the volume of water contained in it as a function of the water level
2. Accuracy class of the differential transducer used
3. The accuracy of the measurement data acquisition system used
4. Sampling frequencies of the differential transducer and accuracy of measuring the time interval in which the measurement took place
5. Selection of the time interval from $t_0$ to $t_f$ used to calculate the change in the volume of water in the reservoir taking into account waves on water surface

The uncertainty of measurement of the water level change resulting from rainfall while it was occurring was disregarded as irrelevant. It was also assumed that uncertainties resulting from water evaporation and leaks through the concrete embankments of the reservoir and steel pipelines connected to it are negligible.

The uncertainty of determining gravitational acceleration and water density in the studied conditions was neglected as practically irrelevant in measuring the change in water level with a differential transducer, and, as follows from further considerations, very small uncertainties of time registration and water level changes related to the resolution of the applied data acquisition system were not taken into account.

The relative accuracy of determining the volume of the reservoir was determined at $\delta_\Delta V = 0.4\%$, which resulted from the available documentation of the geodetic measurements of the reservoir, made more than 30 years ago after the completion of its construction. According to the principles, the relative standard uncertainty type B associated with it was determined as:

$$\delta_B(\Delta V) = \frac{\delta_\Delta V}{\sqrt{3}} = \sim 0.23\%$$  \hspace{1cm} (A9)

The pressure difference transducer with the measuring range set at range $\Delta z_{\text{range}} = 5$ m of water column and accuracy class $K_z = 0.075\%$ was used to measure the water level change in the reservoir $\Delta z$. The standard uncertainty of type B concerning measurement of this quantity was calculated from the formula:

$$u_B(\Delta z) = \frac{K_z \cdot \Delta z_{\text{range}}}{\sqrt{3}} = \sim 0.0022 \text{ m w.c.}$$  \hspace{1cm} (A10)

Due to the fact that flow rate values were measured for the water level in the reservoir changing by at least 1 m, the relative standard uncertainty type B resulting from the measurement of this changes was not worse than:

$$\delta_B(\Delta z) \leq 0.22\%$$  \hspace{1cm} (A11)

For registering $\Delta z$, a computer data acquisition system with a measurement card of an absolute accuracy of $\Delta DAQ = 0.55$ mV was used. In order to determine the measurement uncertainty of the water level resulting from using such a measurement card, the scaling of the water level transducer should be taken into account (in the considered case the full measuring range of the transducer corresponded to
the voltage change $U\Delta z_{\text{range}} = 3.5 \text{ V}$). The standard uncertainty of water level measurement resulting from that can be determined using formula:

$$u_B(\Delta z_{\text{DAQ}}) = \frac{1}{\sqrt{3}} \frac{\Delta z_{\text{range}}}{U_{\Delta z_{\text{range}}}} \approx \frac{1}{\sqrt{3}} \frac{0.00055-5}{3.5} \approx 4.5 \times 10^{-4} \text{ m}$$ (A12)

After referring this uncertainty to the maintained minimum change of the water level in the reservoir (1 m of water), the relative standard uncertainty was not worse than:

$$\delta_B(r_{\Delta z}) \approx 0.05\%$$ (A13)

Type B standard uncertainty regarding the measurement of the time range from $t_0$ to $t_f$ and resulting from the accuracy and time resolution of a digital recorder (computerized data acquisition system) can be determined from the formula:

$$u_B(\Delta t) = \frac{\Delta t_{\text{DAQ}}(t_f - t_0)}{\sqrt{3}} \approx 0.1 \text{ s}$$ (A14)

where $\Delta t_{\text{DAQ}} = 50 \times 10^{-6}$ is the time accuracy of the measuring card used in the data acquisition system, including its resolution.

Given the measurement time of each flow rate value that was not less than 1 h, the relative standard uncertainty of type B achieves negligible small value $\delta_B(\Delta t) \approx 0\%$.

The last of the above factors had random character and the standard uncertainty of type A that results was determined by statistical means. The recorded measurement signal of the water level change in the reservoir was characterized not only by changes resulting from waves on water surface, but also by random changes. The uncertainty arising from such nature of water changes was taken into account when calculating the $Q_V$ value as described below. The calculations were started with the selection of the first time limits $t_0$ and $t_f$ corresponding to the intersection of the trend line with the recorded signal $\Delta z(t)$—Figure 3. Then, the $t_0$ limit was shifted to the left to the next intersection of the trend line and the next $Q_{V_i}$ value was calculated while maintaining the $t_f$ limit. Then, the next $Q_{V_i}$ calculations were made by shifting the $t_0$ limit to the right from the original value to the intersection of the trend line with the signal $\Delta z(t)$. Similar calculations were carried out for the $t_f$ time limit shifted in a similar way. The obtained $Q_{V_i}$ calculation results were then subjected to statistical analysis, i.e., the average $Q_{V_m}$ value and standard uncertainty type A were calculated from the formula:

$$Q_{V_m} = \frac{1}{n} \sum_{i=1}^{n} Q_{V_i}$$ (A15)

$$u_A(Q_V) = k \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (Q_{V_i} - Q_{V_m})^2}$$ (A16)

where $k$ is the extension coefficients calculated for the Student’s $t$-distribution at a confidence level of 68.2% and the number of degrees of freedom $(n-1)$, $n$—the number of $Q_{V_i}$ values calculated.

The $Q_{V_m}$ value was treated as the flow rate value measured by the method discussed. The uncertainty calculated according to the above procedure took different values depending on the measured case, but in none of the examined cases in relation to the measured flow rate was not greater than:

$$\delta_A(Q_V) = 0.2\%$$ (A17)

Finally, using the law of uncertainty propagation, the total relative standard uncertainty was determined from the formula:
\[ \delta_c(Q_V) = \sqrt{\delta_A^2(Q_V) + \delta_B^2(\Delta V) + \delta_B^2(\Delta z) + \delta_B^2(\Delta r) + \delta_B^2(\Delta t)} \quad (A18) \]

This value of this uncertainty is as follows:

\[ \delta_c(Q_V) = \pm 0.38\%. \quad (A19) \]

It should be emphasized that the above-estimated standard uncertainty relates to a confidence level of about 68% and by using a coverage factor of \( k = 2 \), we obtain expanded uncertainty for measuring the flow rate by volumetric gauging method with a confidence level of about 95% of:

\[ \delta(Q_V) = k \cdot \delta_c(Q_V) = \pm 0.76\%. \quad (A20) \]

A summary of the estimated uncertainty of measuring \( Q \) by the volumetric method is presented in Table A1.

**Table A1.** The results of calculations of uncertainty of the flow rate measurement results obtained using the volumetric gauging method.

| Name                                                      | Designation | Value  | Unit |
|-----------------------------------------------------------|-------------|--------|------|
| relative uncertainty in determining the reservoir volume | \( \delta(\Delta V) \) | 0.4000 | %    |
| relative standard uncertainty in determining the reservoir volume | \( \delta_B(\Delta V) \) | 0.2309 | %    |
| standard uncertainty of water level measurement            | \( \mu_B(\Delta z) \) | 0.0022 | m    |
| relative standard uncertainty of measurement of water level related to a change in level of 1 m | \( \delta_B(\Delta z) \) | 0.2165 | %    |
| standard uncertainty of water level measurement resulting from the measurement card used | \( \mu_B(\Delta z_{DAQ}) \) | 0.0005 | m    |
| relative standard uncertainty of water level measurement resulting from the measurement card used | \( \delta_B(\Delta z_{DAQ}) \) | 0.0454 | %    |
| standard uncertainty of time interval measurement          | \( \mu_B(\Delta t) \) | 0.1000 | s    |
| relative standard uncertainty of a time interval measurement | \( \delta_B(\Delta t) \) | 0.0028 | %    |
| relative standard uncertainty due to the nature of the changes in the measured change in water level | \( \delta_A(Q_V) \) | 0.2000 | %    |
| total standard uncertainty of flow rate measurement        | \( \delta_t(Q_V) \) | 0.3772 | %    |
| expanded uncertainty of flow rate measurement \((k = 2)\)  | \( \delta(Q_V)_{k=2} \) | 0.7544 | %    |

**Appendix C.** Uncertainty Analysis of Flow Rate Measurements by Means of the Pressure-Time Method

Standards [4,6] specify the requirements that must be met so that the uncertainty of the flow rate measurement obtained using the pressure-time method is in the range of \( \pm 1.5\% \) (2.3%) according to [4] and \( \pm 1.0\% \) according to [6]. However, a way to calculate this uncertainty is not provided. The algorithm for estimating this uncertainty was the subject of only few available papers [36,37] but the presented algorithms do not comply with the applicable principles of expressing measurement uncertainty, presented in [19].

Below is a method for estimating the uncertainty of flow rate measurement under the considered conditions. The method is currently used by the authors of this contribution and complies with the recommendations presented in [19]. To present it, a simplified formulation of Equation (2) is introduced in the following form:

\[ Q_0 = \frac{1}{\rho_F} (\Delta p_m + \Delta p_{dm} + P_{rm}) (t_f - t_0) + Q_f \quad (A21) \]
where Δp_m, Δp_d_m, and P_r are the values of Δp, Δp_d, and P_r, respectively, after averaging over the time interval from t_0 to t_f.

Treating all the constituent quantities (components) in the above dependence as uncorrelated with each other, the value of the relative standard total uncertainty \( \delta_r(Q_0) \) can be calculated from the formula resulting from the law of uncertainty propagation:

\[
\delta_r(Q_0) = \sqrt{\delta^2(p) + \delta^2(F) + \delta^2(\Delta p_m) + \delta^2(\Delta p_d_m) + \delta^2(P_r) + \delta^2(t_f - t_0) + \delta^2(Q_f)} \tag{A22}
\]

The largest uncertainty component is related to the measurement and recording of the pressure difference. In the measurement procedure used by the authors of this work, the initially recorded pressure difference signal Δp(t_i) is numerically corrected taking into account characteristic of signal between limits t_f and t_g as well as the flow rate at final conditions (Q_f) and the C_r coefficient of frictional resistance characterizing the pipeline between measuring cross-sections. All measurement results of differential pressure values Δp(t_i) are corrected according to the formula:

\[
\Delta p(t_i)\text{correction} = \Delta p(t_i) - \left( \frac{1}{N_f} \sum_{t_f}^{t_g} \Delta p(t_i) - C_r Q_f |Q_f| - \Delta p_d \right) \tag{A23}
\]

where the second component on the right is the average value calculated from the recorded signal Δp(t_i) in the time interval (t_f,t_g), i.e., in the phase of suppression of free pressure oscillations after the flow is cut off, N_f is the number of recorded values of Δp(t_i) in the time interval (t_f, t_g), and Δp_d means the difference of dynamic pressures in the final steady state conditions (the method of calculating the difference of dynamic pressures is analogous to the calculation of the average difference of dynamic pressures—see the further part of the Appendix).

The C_r factor is determined from the formula (A23) on the basis of the measured pressure difference Δp_correction = P_r0 + Δp_d0 caused by friction losses in the pipeline measuring section and dynamic pressure difference in the initial steady flow conditions, i.e., immediately before the closing of the flow shutoff device. Thus, the value of P_r0 is calculated as the average of the measured pressure difference (after correction) in the time interval (t_00, t_0):

\[
P_r0 = \Delta p_0 - \Delta p_d0 = \frac{1}{N_0} \sum_{t_00}^{t_0} \Delta p(t_i)_{\text{correction}} - \Delta p_d0 \tag{A24}
\]

where N_0 is the number of recorded values of Δp(t_i) in the time interval from t_i = t_00 to t_i = t_0, and Δp_d0 means the difference of dynamic pressures in the initial steady.

The method of correction according to formula (A23) allows us to get rid of the most important part of uncertainty arising from the exact determination of the “zero” pressure differential transducer. The residual uncertainty associated with it is estimated when analyzing the effect of t_f limit on the uncertainty value. It should be emphasized that the correction applied takes place in the iterative process of calculating the Q_0 value.

Therefore, the mean pressure difference Δp_m is calculated from the measured and corrected values of Δp(t_i)_{correction} using the formula:

\[
\Delta p_m = \frac{1}{N} \sum_{t_0}^{t_f} \Delta p(t_i)_{\text{correction}} \tag{A25}
\]

where N is the number of recorded values of Δp(t_i) in the time interval from t_i = t_0 to t_i = t_f.

The absolute standard uncertainty of type B measurement of pressure difference Δp, resulting from the classes of transducers used, was determined as follows:
\[ u_{kB}(\Delta p_m) = \frac{K_{Ap} \cdot \Delta p_{range}}{\sqrt{3}} \quad (A26) \]

After considering the pressure difference transducer class \( K_{Ap} = 0.075\% \) and its range \( \Delta p_{range} = \pm 500 \text{ kPa} \) (1 MPa), this uncertainty was: \( u_{kB}(\Delta p_m) = 0.43 \text{ kPa} \).

To record \( \Delta p_m \), a computer data acquisition system with a measurement card with an absolute accuracy of 0.55 mV was used. In order to determine the measurement uncertainty of the water level resulting from the used measurement card, the scaling of the level transducer should be taken into account (in the case under consideration the full width of the transducer measuring range corresponded to a 3.5 V voltage change). The resulting standard uncertainty of level measurement can be determined by the formula:

\[ u_B(\Delta p_m) = \frac{\DeltaDAQ \cdot \Delta p_{range}}{\sqrt{3} \cdot U_{\Delta p_{range}}} \cong \frac{0.00055-1000}{\sqrt{3} \cdot 3.5} \cong 0.09 \text{ kPa} \quad (A27) \]

In connection with the above, the total standard uncertainty \( u(\Delta p_m) \), calculated from the formula:

\[ u(\Delta p_m) = \sqrt{u_{kB}^2(\Delta p_m) + u_B^2(\Delta p_m)} \quad (A28) \]

was not worse than:

\[ u(\Delta p_m) = 0.44 \text{ kPa} \]

After referring these uncertainty values to the average pressure difference increases caused by the inertia forces after flow cut-off during the measurement, i.e.,

\[ \Delta p_{m-inertia} = (\Delta p_m + \Delta p_{dm} + P_{rm}) \quad (A29) \]

the relative standard uncertainty \( \delta (\Delta p_m) \) is determined, which, together with other uncertainty components, has been presented in the uncertainty balance table Table A2. This uncertainty is approximately 0.36\% and 0.43\% for turbine and pump mode of operation, respectively.

In addition to the \( P_r0 \) value resulting from the measurement and calculations, the values of friction pressure drop \( P_r \) during flow cut off are calculated according to the relationship (A24) in the time interval \((t_0, t_f)\). For this range, the average pressure drop \( P_{rm} \) can be calculated from the formula:

\[ P_{rm} = \frac{C_f}{N} \sum_{t_0}^{t_f} Q(t_i) |Q(t_i)| - C_f Q_f |Q_f| - \Delta p_{df} \]

where \( N \) is the number of calculated \( Q(t_i) \) values in the range \((t_0, t_f)\). The values of the second and third components to the right of the above dependence are negligibly small, so it can be neglected when estimating their uncertainty.

The standard uncertainty type B resulting from the calculation of the \( P_{rm} \) value was estimated from the formula:

\[ u_B(P_{rm}) = u(P_{rm}) = \frac{\delta_{P_{rm}} P_{rm}}{\sqrt{3}} \quad (A30) \]

where \( \delta_{P_{rm}} \) is the average, relative difference in friction losses calculated using the quasi-stationary model (friction coefficient depending on the \( Re \) number) and the stationary model (constant friction coefficient)—the \( \delta_{P_{rm}} \) value was adopted according to approximately parabolic dependence of this difference on the flow rate proposed in monograph [11]: \( \delta_{P_{rm}} = \delta_{P_{max}} / 3 \approx 0.025 / 3 = 0.0083 \). It is worth emphasizing here that for calculating the flow rate, \( \delta_{P_{rm}} \) value was not used to correct friction loss calculations, i.e., the calculations were carried out assuming a constant \( C_f \) factor, not dependent on \( Re \).

The effect of other factors on uncertainty \( u(P_{rm}) \), e.g., unsteadiness of flow, was omitted as irrelevant from the practical point of view. References [38,39] indicate that dissipation of mechanical energy during flow deceleration (taking place when the pressure-time method is applied) is only
slightly less than that obtained from the quasi-steady hypothesis. It is the opposite to accelerating flow where energy dissipation is much larger than according the quasi-steady modeling. Some unsteady friction loss models in the closed conduits use these features [40]. These models have been confirmed experimentally—there is a high conformity between experimental and numerical results of the water hammer course [30]. With reference to the pressure-time method, the above assessment is confirmed by [27–29].

Finally, after the referring the \( u(P_m) \) to the value of \( \Delta p_{\text{m-inertia}} \), the relative standard uncertainty associated with the calculation of \( P_r \), for the highest value of flow rate measured is presented in the uncertainty balance Table A2.

The uncertainty of calculating the dynamic pressure difference between the pipeline measuring cross-sections, \( u(\Delta p_{dm}) \) was estimated as follows. The average dynamic pressure difference, \( \Delta p_{dm} \), in the time interval \( (t_0, t_f) \) was calculated from the formula:

\[
\Delta p_{dm} = \frac{1}{2} \left( \frac{\alpha_B \rho}{A_B^2} - \frac{\alpha_A \rho}{A_A^2} \right) \frac{1}{N} \sum_{t_0}^{t_f} [Q(t_i)]^2
\]

(A32)

in which \( N \) denotes the number of calculated \( Q(t_i) \) values in the interval \( (t_0, t_f) \), and \( A_A \) with \( A_B \) are the cross-sectional areas of the upper and lower pipeline measuring cross-sections, and \( \alpha_A \) and \( \alpha_B \)—Coriolis coefficients.

In the considered case, it was assumed that \( A_A = \infty \) and the effect of calculating \( \Delta p_m \) on the uncertainty of flow measurement results from changes in the Coriolis coefficient in the lower measuring cross-section of the pipeline. In calculations \( \alpha_B = 1.05 \) was taken as the average value of the Coriolis coefficient for fully developed turbulent flow in the pipeline determined within the limits from 1.04 to 1.06 [23]. On this basis, the standard uncertainty type B resulting from the calculation of the \( \Delta p_{dm} \) value was calculated using the following formula:

\[
u_B(\Delta p_{dm}) = \frac{0.01 \Delta p_{dm}}{\sqrt{3}}
\]

(A33)

For the cases with the highest values of measured flow rates, the values of standard uncertainty \( u(\Delta p_{dm}) \) determined in this way was 0.21 kPa and 0.16 kPa for turbine and pump mode of operation, respectively.

Table A2 of the uncertainty balance lists the relative standard uncertainty associated with the calculation of \( \Delta p_{dm} \), after relating \( u(\Delta p_{dm}) \) to the value of \( \Delta p_{\text{m-inertia}} \) for the highest values of measured flow rates in the turbine and pump mode of operation of the tested machine.

The time accuracy of the computer acquisition system measuring the pressure difference signal \( p(t_i) \) was omitted as having no impact on the standard uncertainty type B regarding the measurement of the time interval from \( t_0 \) to \( t_f \). It can be calculated using the following formula:

\[
u_B(\Delta t) = u_B(t_f - t_0) = \frac{\Delta t_{DAQ}(t_f - t_0)}{\sqrt{3}}
\]

(A34)

where \( \Delta t_{DAQ} = 50 \times 10^{-6} \) is the time accuracy of measurement card used in the data acquisition system including its resolution.

The value of \( u_B(t) \) is about 0.0007 s and 0.0005 s for turbine and pump mode of operation, respectively.

For the flow rate measurements, the time interval \( (t_0, t_f) \) during turbine mode of operation was not longer than \( T = \sim 25 \) s, and during pump mode of operation \( T = \sim 20 \) s, using a sampling frequency of 200 Hz. Table A2 of the uncertainty balance lists the relative standard uncertainty associated with the measurement of the time interval \( (t_0, t_f) \).
The method of determining the \( t_f \) time limit, i.e., the upper limit of integration in the pressure-time method, was presented in [21]—an earlier author’s publication. This method significantly influences the uncertainty of measuring \( Q_0 \) in cases where the free pressure oscillations after the closing of the shut-off device have relatively high amplitudes compared to the average \( \Delta p_m \) values. The value of \( t_f \) should be selected at the top of the peak or the bottom of the valley of free oscillations of pressure differences, with its exact determination taking place in the calculation program. It is recommended to choose the limit \( t_f \) from the first clear peak or valley of free oscillations in order to minimize the impact of these oscillations on the measurement result \( Q_0 \). Pulsations superimposed on these oscillations, which are random in nature, have been included in the estimation of uncertainty type A. For this reason, a series of calculations of \( Q_{bi} \), values for several values of time \( t_{fi} \), selected in close proximity of the original value of \( t_f \) selected in accordance with the above principle, was carried out in the range covering only one valley and one peak visible in the measured quick-change pressure difference signal (pressure difference pulsation). It should be emphasized that it is not advisable to significantly shift the \( t_f \) value from the tops of peaks and bottom of valleys of free differential pressure oscillations. The obtained \( Q_{0i} \) calculation results were subjected to statistical analysis, i.e., the average \( Q_{0m} \) value and standard uncertainty were calculated using the formula:

\[
u_{t_fA}(Q_0) = k \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (Q_{0i} - Q_{0m})^2}
\]

(A35)

where \( k \) is the extension coefficients calculated from the Student’s \( t \)-distribution for the confidence level \( p = 68.2\% \) and the number of degrees of freedom \( (n - 1) \), \( n \)—number of calculated \( Q_{0i} \) values.

After relating the \( u_{t_fA}(Q_0) \) values determined in the above described manner to the measured flow rate \( Q_0 \), the relative standard uncertainties \( \delta_{t_fA} \) did not exceed \( \delta_{t_fA}(Q_0) = 0.08\% \) for turbine mode of operation and \( \delta_{t_fA}(Q_0) = 0.1\% \) for pump mode of operation.

The uncertainty \( \delta(p) \) results from the variability of water density with pressure change and from the accuracy of its determination for given temperature and average absolute pressure in the pipeline occurring during tests. This uncertainty is very small; therefore, it was omitted in calculating the uncertainty of flow rate measurement.

The standard uncertainty \( \delta(F) \) for determining the geometric factor \( F \) results from the accuracy of measuring the length of individual pipeline segments \( (L_i) \) and the area of their internal cross-sections \( (A_i) \) and from the accuracy of the correction of the \( F \) factor using CFD calculations. The uncertainty of determining the \( F \) factor based on the available post-completion documentation of the pipeline, positively verified by direct measurement of \( L_i \) and \( A_i \), was not worse than:

\[
\delta(F_{geom}) = 0.15\%
\]

(A36)

Due to the fact that the uncertainty of the \( F \) factor correction introduced reaches about 0.75\%, the uncertainty of this correction based on CFD calculations is of the same order assuming even 20\% accuracy of CFD calculations, and as a result we get standard uncertainty:

\[
\delta(F) = \sqrt{\delta^2(F_{geom}) + \delta^2(F_{CFD})} \approx 0.21\%
\]

(A37)

The flow rate under final conditions, being that the leakage through the closed wicket gates of the pump-turbine, \( Q_f \), was measured in a separate way. For this purpose, under closed wicket gate conditions, pressure changes in the pipeline were recorded when closing the shut-off valve characterizing with very high tightness. On this basis, also using the pressure-time method, \( Q_f \) values were determined. For turbine mode of operation, it was equal \( Q_f \approx 0.14 \text{ m}^3/\text{s} \), while for pump mode of operation \( Q_f \approx 0.18 \text{ m}^3/\text{s} \), which is about 0.7\% in relation to the minimum flow rate values for turbine
and about 0.65% for pump flow direction. No detailed analysis of the $Q_f$ uncertainty was carried out, but it was assumed with a large excess that it is not worse than 10%, which gives uncertainty:

- $\delta(Q_f) = \sim 0.07\%$ for turbine mode of operation,
- $\delta(Q_f) = \sim 0.065\%$ for pump mode of operation.

The uncertainty resulting from the iterative algorithm for calculating the flow rate is $\delta(Q_{\text{iter}}) = 0.1\%$. This is due to the condition used for ending the calculations, which assumes that the calculations are finished when two subsequent values $Q_{\text{iter}-1}$ and $Q_{\text{iter}}$ do not differ by more than 0.1%.

The balance of the estimated uncertainty of $Q$ measurement using the pressure-time method is presented in Table A2.

**Table A2.** Summary results of calculations of uncertainty of flow rate results measured using the pressure-time method.

| Name                                                   | Symbol                        | Value   | Unit    |
|--------------------------------------------------------|-------------------------------|---------|---------|
| standard uncertainty of pressure measurement resulting from the applied differential pressure transducer | $u_{kg}(\Delta p_m)$         | 0.4330  | kPa     |
| standard uncertainty of pressure measurement resulting from the measurement card used                   | $u_{rg}(\Delta p_m)$         | 0.0907  | kPa     |
| total standard uncertainty of pressure measurement                                                 | $u(\Delta p_m)$              | 0.4424  | kPa     |
| relative standard uncertainty of pressure measurement related to the average differential pressure increase | $\delta(\Delta p_m)$         | 0.3600  | %       |
| standard uncertainty of calculating friction losses                                                  | $u_B(P_{rm})$                | 0.0555  | kPa     |
| relative standard uncertainty of calculating friction losses                                           | $\delta_B(P_{rm})$           | 0.0584  | %       |
| standard uncertainty of calculating the dynamic pressure difference                                   | $u_B(\Delta p_{dm})$         | 0.2100  | kPa     |
| relative standard uncertainty of calculating the dynamic pressure difference                          | $\delta(\Delta p_{dm})$      | 0.2211  | %       |
| standard uncertainty of time interval measurement                                                     | $u_B(\Delta t)$              | 0.0007  | s       |
| relative standard uncertainty of time measurement                                                     | $\delta_B(\Delta t)$         | 0.0029  | %       |
| standard uncertainty resulting from setting the upper limit of integration                             | $u_{QA}(Q_0)$                | 0.0270  | m³/s    |
| relative uncertainty resulting from setting the upper limit of integration                             | $\delta_{QA}(Q_0)$           | 0.0800  | %       |
| standard uncertainty of determining the geometrical factor                                             | $\delta(F_{\text{geom}})$    | 0.1500  | %       |
| standard uncertainty of CFD calculations                                                               | $\delta(F_{\text{CFD}})$     | 0.1500  | %       |
| total standard uncertainty of determining the geometric factor                                          | $\delta(F)$                  | 0.2100  | %       |
| uncertainty of determining the flow rate at final conditions                                            | $\delta(Q_f)$                | 0.0700  | %       |
| uncertainty resulting from iterative calculation of the flow rate                                      | $\delta(Q_{\text{iter}})$    | 0.1000  | %       |
| relative total standard uncertainty                                                                   | $\delta_c(Q_0)$              | 0.4973  | %       |
| relative expanded uncertainty ($k=2$)                                                                  | $\delta(Q_0)_{k=2}$          | 0.9946  | %       |

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