The time dependences of the main properties of the vortex loops just after reconnection in superfluid helium

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Abstract. We investigate a single vortex reconnection event in superfluid helium at zero temperature, when mutual friction is absent, and at 1.9 K temperature, when mutual friction essentially changes the evolution of lines. The vortex filament method with full Biot-Savart equation is used for numerical investigation of the time dependences of the main properties of the vortex loops just after reconnection. In this paper, we present the obtained dependencies of the average curvature of the loop, Lamb momentum, angular momentum, and energy from time. The effect of mutual friction force and initial conditions on these properties is studied.

1. Introduction
Quantum turbulence has a significant effect on the properties of superfluid helium. When using helium in technical devices as a coolant, it is necessary to know its properties in order to prevent all possible accidents. Many factors influence the dynamics of the vortex loops. Reconnection processes change the topology of the vortex structure, so they play an important role in their dynamics. A change in the dynamics of quantized vortex filaments, in turn, leads to a change in the properties of superfluid helium. During the event of reconnection of the vortex loops, it is possible that the small vortex ring may split off [1], [2]. Any reconnection leads to a change in the topology of vortex lines. This change in turn leads to a noticeable perturbation of the vortex lines (the Kelvin waves). That is the mean curvature of filaments due to this should increase. In work [3] using visualization of the movement of particles of submicron size, which were dispersed in superfluid helium, the Kelvin waves which are excited during reconnection were directly observed. When the scale of the small rings becomes of the order of atomic distances, the vortex motion is degenerated into thermal excitations. The perturbations along the vortex lines lead to additional damping of the vortex energy due to the action of the mutual friction force. At a temperature near to absolute zero, when there is no friction force, the decay of a vortex tangle is observed in a series of experiments, see for example [4], [5]. To date, the main mechanisms responsible for the decay of quantum turbulence at zero temperature are considered to be the processes of reconnection and the nonlinear cascade of Kelvin waves. At the moment of reconnection, a part of the vortex energy is transformed into the energy of phonons. As far as we are aware, there are as yet no experimental observations of dependences of the average curvature of the loop, Lamb momentum, angular momentum, and energy on time. Therefore, at present, the evolution of these parameters should be obtained using computer simulation. This is the main purpose of this article.
2. Equations of motion and numerical simulation

In the framework of a 'vortex filament method', the velocity of a point belonging to the vortex line is determined by the following equation:

\[
V_L = \frac{ds}{dt} = V_s + V_{\text{BSE}} + \alpha s' \times (V_{ns} - V_{\text{BSE}}) - \alpha' s' \times [s' \times (V_{ns} - V_{\text{BSE}})] + V_{sc}.
\]

This velocity is a sum of 4 terms: i) the macroscopic velocity, ii) the velocity induced by the configuration of vortex lines, iii) the boundary term, which is chosen in such a way that this velocity satisfies the boundary conditions, and iv) the friction terms. Here \(s(\xi)\) is the radius-vector of the vortex line points, \(\xi\) is a label parameter, in this case the arc length, \(V_{ns} = V_n - V_s\) is the relative velocity between normal fluid and superfluid component, \(\alpha, \alpha'\) are the temperature-dependent friction coefficients. There are various approximations for calculation of the velocity, induced by the configuration of vortex lines: such as the local-induced approximation or the full Biot-Savart law. In this work we used full Biot-Savart law:

\[
V_{\text{BSE}}(s_i) = \beta s' \times s^* + \{s_i - s_j\} \times ds_j .
\]

\(\prime\) denotes derivatives with respect to the arc length, \(\beta = (\kappa/4\pi) \ln(2s_i/s_j/e^{\frac{1}{2\alpha}a_0})\), \(a_0\) is the radius of vortex line core, \(\kappa\) is the quantum of circulation, \(s_{\pm}\) are the lengths of two line vortex elements connected to \(s_i\). In this paper it is assumed that the fluid is at rest. Vortex lines can merge and break up into smaller loops, i.e., to reconnect. The processes of reconnection lead to changes in the topology of the vortex structure. Different researchers use different criteria to trigger reconnections in their models [6]. In this paper we used “geometric-energetic” criterion. We used the 4th order Runge – Kutta scheme to integrate the equation of motion over time. Details of the implementation of the numerical scheme can be seen in [6]. The simulations were carried out for zero temperature and \(T = 1.9\) K. The parameters \(\alpha = 0.21\) and \(\alpha' = 0.009\) . The initial space resolution is \(\Delta \xi = 10^{-3}\) cm. The elements of the vortex loop may increase or decrease while moving. To maintain the accuracy of the calculations, the length of these elements was controlled so that their values changed in the following range \(\Delta \xi/1.8 < s_{\pm} < \Delta \xi \cdot 1.8\).

3. Integrals for calculating the physical parameters

When studying the dynamics of vortex loops, the curvature of the lines is one of the important global properties. First of all, it is directly included in the definition of the velocity of the vortex segments; it determines the smoothness of the vortex loop. Knowing the change in curvature over time, it is possible to determine whether it is being smoothed or is being kinked by abrupt changes in the value of curvature; it is possible to determine the change in its change to smoothness, etc. Quantity of the curvature is determined as:

\[
s^* = \frac{1}{l} \int s^* d\xi.
\]

Here \(l\) is the vortex length. Hereinafter, integration is carried out along the vortex loops. In order to determine the causes leading to the dissipation of vortex energy, it is important to know the change in momentum, the angular momentum. The following expression defines the total impulse of superfluid motion associated with the presence of vortex lines in the fluid:

\[
p = \rho \oint V_s dV.
\]

In this expression, \(V_s\) is the velocity induced by the vorticities, integration is performed over the entire velocity field caused by the vortices. This integral diverges far from and near the vortex line. It
is known that in this case the vortex pulse is determined, i.e. Lamb momentum [7], [8]. In the case of quantized vortex lines, the vortex pulse is determined by the following equation:

\[ p = \frac{1}{2} \rho \kappa \int s \times s' d\xi. \]

Here \( \rho \) is the density of superfluid component of liquid helium. To calculate the angular momentum \( L \) and energy \( E \), the following expressions were used:

\[ L = \frac{1}{3} \rho \kappa \int s \times [s \times s'] d\xi, \]

\[ E = \rho \kappa \int V_{\text{ext}} \times [s \times s'] d\xi. \]

Initially, two identical smooth vortex rings were created. The initial radius of the rings \( R_0 \) was \( 10^{-3} \) cm. For rings originally located in the same plane, the distance between the centers was equal to \( 3R_0 \). For rings originally located in perpendicular planes, the distance between the centers was equal to \( R_0 \). Further, the calculation continued until the implementation of reconnection. This paper considers the dynamics immediately after reconnection, i.e. the zero moment of time corresponds to the moment of the reconnection.

4. Results and discussion

Consider the change in curvature over time. The obtained results are presented in Figure 1. Hereinafter, all the investigated quantities are normalized to the corresponding values at the time of the reconnection.

![Figure 1. Time dependence of vortex curvature for different temperatures and initial positions of vortices planes. The values of parameters are presented in legend of figure.](image)

Just after reconnection of two initial loops, a large vortex loop is formed, as a rule. Sometimes several small loops may split off. Usually one or two loops split off. Small loops start moving quickly, leaving a large loop. As it moves, the small vortex rings quickly decrease in size. When their sizes become of the order of interatomic distances, their vortex energy turns into thermal perturbations. In our calculations, this ring consists of three vortex points, therefore, of three segments. The total length of these segments is very small compared to the length of the formed larger loop. The energy of vortex line per unit length is determined as: \( \varepsilon_v = \left( \rho \kappa^2 / 4\pi \right) \ln \left( \langle R \rangle / a_0 \right) \), where \( \langle R \rangle \) is radius of ring. Since the energy of the vortex is proportional to its length, the energy that is carried away is very small compared with the energy of the large loop. Therefore, without losing generality, the properties of a single large vortex loop will be studied.
Just after reconnection, the velocity of the elements of the filament closest to the intersection point increases sharply with a change in their direction. A substantially deformed section is formed on the new vortex loop. These segments of the loop begin to move away from each other. Since the motion of the vortex points is related to each other according to the equation of Bio-Savart, the velocity and its direction of other loop elements change. This interaction leads to a disturbance along the vortex line, to the so-called Kelvin waves. Further, the disturbance begins to propagate along the vortex loop. In figure 1, where the dependence of the mean curvature on time is shown, we see first a sharp increase in the curve at zero temperature, which corresponds to the above loop dynamics. At a temperature of 1.9 K, the increase in curvature is about 20 percent. This is an insignificant increase compared to an increase at zero temperature, where the average curvature increases by almost 7.5 times. This suggests that the frictional force significantly smoothes out disturbances before and after reconnection. It should also be noted that at zero temperature, the curves reach certain stationary values, near which oscillations occur, regardless of the initial conditions. This means that the vortex line is curved, Kelvin waves run along it. The question arises: will their non-linear interaction lead to energy dissipation? It should be noted that a significant change in curvature occurs during first 50 microseconds after reconnection. During this time, the nearest points of the reconnected loop diverge by a distance of the order of the radius of formed vortex loop, i.e. relaxation of the initial perturbation occurs. For details, see the figures of article [1].

Next, we consider the time dependencies of the Lamb vortex momentum, angular momentum and energy, see Figures 2–4.

An interesting question is whether the process of reconnection leads to a transition from one type of movement to another, namely, does this lead to a partial loss of translational motion of the loop and an increase in its twisting or vice versa? As it can be seen from the figures (see figure 2, 3.), the Lamb momentum and angular momentum at zero temperature are maintained regardless of the initial conditions. That is, we get a definite answer that there is no transition. Let’s move on to consideration the question of energy dissipation in the nonlinear interaction of Kelvin waves. Thus, despite the observation of Kelvin waves, as evidenced by a significant change in the curvature of the vortex configuration (Figure 1), a cascade of Kelvin waves responsible for the disintegration of vortices at zero temperature is not observed. This may be due to its implementation on scales much smaller than those considered in this work. Note, the angular momentum for the initial location of the loops in one plane is zero and does not change with time.
Figure 3. Time dependence of vortex angular momentum for different temperatures and initial positions of vortices planes. The values of parameters are presented on legend of figure.

Figure 4. Time dependence of vortex energy for different temperatures and initial positions of vortices planes. The values of parameters are presented on legend of figure.

At a nonzero temperature, a decrease in the values of the momentum, angular momentum and energy is observed. The frictional force of the vortex filament with normal component of superfluid helium is responsible for these changes, since Kelvin wave cascade is not observed, see the reasoning above. After 50-100 microseconds after the moment of reconnection, the rate of change in the momentum and energy for different initial data becomes almost the same (see Figures 2, 4). At the same time, the rate of curvature change for different initial conditions also becomes almost the same, Figure 1. In other words, by this time the initial perturbation relaxed and the dissipation entered the stationary regime. The differences in the indicated dependences for different initial conditions at the first stage (from 0 to 50–100 μs) of evolution are related to the difference between the initial configurations and associated with their perturbations.

As a result, it can be argued that the nature of perturbations arising during the reconnection is determined by the initial conditions and their further dynamics is determined by the helium temperature.
Conclusion
In this work, in the framework of the vortex filament method with the complete Biot-Savart equation, the effect of reconnections on the main properties of vortex loops in superfluid helium (mean curvature of vortex loop, Lamb momentum, angular momentum and energy) is numerically studied. At the time of reconnection of two vortex loops, one larger loop is formed. A change in the topology of the vortices leads to the appearance of Kelvin waves, while the average curvature of the loop formed during reconnection increases. At temperatures close to absolute zero (the force of mutual friction is negligible), the average curvature initially increases significantly, and then does not change over time (see Figure 1). In this situation (at these temperatures), one can expect the observation of energy dissipation due to energy transfer from large scales to small scales (owing to nonlinear coupling between Kelvin waves of different scales). However, the studies showed that the energy, Lamb momentum and angular momentum do not change with time (see Figures 2-4), i.e. a cascade of Kelvin waves is not observed. Apparently, this is happening on a smaller scale. A similar result was also obtained in [9].

At a temperature of 1.9 K, the arising Kelvin waves lead to an additional increase in energy dissipation, a decrease in the average curvature of the loops, Lamb momentum and angular momentum due to the increased mutual friction force (see Figures 1-4).

As a result, it can be argued that the nature of perturbations arising during the reconnection is determined by the initial conditions and their further dynamics is determined by the helium temperature. Since to date there are no experimental data on the properties of vortex loops under the study, the results are important for studies and understanding the dynamics of vortex structures in superfluid helium.

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