Quantum control in foundational experiments

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We describe a new class of experiments designed to probe the foundations of quantum mechanics. Using quantum controlling devices, we show how to attain a freedom in temporal ordering of the control and detection of various phenomena. We consider wave-particle duality in the context of quantum-controlled and the entanglement-assisted delayed-choice experiments. In the absence of superluminal communication, realism (defined as a property of photons being either particles or waves, but not both) is incompatible with determinism in a broad class of hidden-variable theories. We further show that, contrary to the standard Bohr complementarity, the complementary properties can be measured in a single experiment, provided that a component of the apparatus is a quantum object in a state of superposition. We discuss several recent experiments and indicate future applications.

I. INTRODUCTION

The Bohr-Einstein discussions on the nature of quantum theory [1,2] were responsible for the appearance of the first modern gedanken experiments. These thought experiments became the weapons of choice in the struggle of our classical intuition with quantum mechanics. In the last decades they developed into common lab procedures. Former paradoxes of quantum foundations are now resources of quantum information science and technology [3]. This new technological ability allows refining of now classic experiments [4,5], as well as probing other aspects of quantum foundations.

Wave-particle duality, superposition and entanglement are just some of the quantum concepts that run afoul of our classical expectations. Hidden-variable (HV) theories were proposed to remove or explain these non-classical features. Moreover, an additional set of rules (measurement description) draws quantum possibilities into an irreversible classical record [4,5]. This happens despite measuring devices being built from quantum constituents.

In the von Neumann’s discussion of measurement [8], a quantum system is used to observe the preceding one, until the chain of systems is cut by a classical observer (or a device). Keeping one link in this chain entails quantum controlling devices to perform the switching between different classical set-ups. The first example of a quantum control is a radioactive atom in the Schrödinger’s cat gedankenexperiment [9]. By correlating decayed and un-decayed states of an atom with dead and alive states of a cat, it demonstrated non-classical properties of entanglement. A modern example is a superposition of motional states of a mirror [10], which spurred a lot of work on quantum control in nano- and mesoscopic systems [11].

Familiar concepts — “particle” or “wave” — represent only one aspect of quantum objects. Although we observe single-photon interference (a definite wave-like behaviour), the pattern is produced click-by-click, in a discrete, particle-like manner [6,14,15]. Hence we adopt, as an operational definition, being a ‘wave/particle’ to stand for ability/ inability to produce interference [12,13]. As illustrated in Fig. 1(a), these properties are observed using two mutually exclusive set-ups of the Mach-Zender interferometer (MZI).

Bohr’s complementarity principle [17] ascribes a fundamental significance to this situation. “The information provided by different experimental procedures that in principle cannot . . . be performed simultaneously, cannot be represented by any mathematically allowed quantum state of the system” [18].

The dual nature of quantum systems enables the use of optical analogies [19], and motives the hydrodynamic models [20]. It also prompts the following inquiry: can we think about, say, photons as sometimes waves and sometimes particle, or must they be treated as being both? The complementarity principle makes conceivable a “conspiracy theory” of light [16]. The photon could...
“know” before entering the apparatus whether the latter has been set up in the wave configuration with BS$_2$ in place or the particle one (BS$_2$ removed) and adjust its behaviour accordingly.

![Fig. 1](image.png)

**FIG. 1:** Schematics of the delayed-choice experiments (adapted from [12])
(a) Mach-Zender interferometer. A quantum random number generator (QRNG) determines whether BS$_2$ is inserted (the output is 1) or not (the output is 0).
(b) The equivalent quantum network. An ancilla (red line), initially prepared in the state $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ then measured, acts as QRNG.
(c) Delayed-choice with a quantum beamsplitter. A quantum device plays the role of the QRNG controlling the Hadamard gate; this makes possible to delay the measurement revealing its output after the application of the $H$ gate.
(d) Biasing the QRNG by preparing the ancilla in an arbitrary state $|\psi\rangle = \cos\alpha|0\rangle + \sin\alpha|1\rangle$ is crucial in interpreting the experimental results as supporting wave-particle duality.

Wheeler’s delayed-choice experiment [33, 21, 22] is designed to eliminate this possibility. As shown on Fig. 1(a) one randomly chooses whether or not to insert the second beamsplitter only when the photon is already inside the interferometer and before it reaches BS$_2$.

The rationale behind the delayed-choice is to avoid a possible causal link between the experimental setup and photon’s behaviour: the photon should not “know” beforehand how to behave. The choice of inserting or removing BS$_2$ is controlled by a random number generator. The most spectacular to date implementation of WDC was done by the group of Aspect and Grangier [14]. We discuss the interpretation of the experiment and the key components of its design in Sec. [V].

### III. QUANTUM CONTROL

A quantum circuit model [31, 13] enables us to analyze the gedanken experiment at a higher level of abstraction and to understand the information flow between different subsystems. The delayed-choice experiment is equivalent to the quantum network in Fig. 1(b), where Hadamard gates $H$ play the role of beamsplitters; we call the top (black) line the photon and the bottom (red) line the ancilla. The quantum random number generator is modelled by an ancilla prepared in the equal-superposition state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, then measured; the result of this measurement (0 or 1) controls if BS$_2$ is inserted or not. The classical control after the measurement of the ancilla in Fig. 1(b) is equivalent to a quantum control before the measurement of the ancilla, Fig. 1(c). This seemingly innocuous transformation radically changes the setup and has two profound implications. First, since now we have a quantum beamsplitter in superposition of being present or absent, the interferometer is in a superposition of being closed or open.

Second, a quantum control allows us to reverse the temporal order of the measurements. We can now detect the photon before the ancilla, i.e., before finding out the interferometer is open or closed. This implies that we can choose if the photon behaves as a particle or as a wave after it has been already detected. A quantum control thus allows us to explore a regime outside the classical realm: in any classically-controlled experiment the choice of inserting or not the second beamsplitter has to be made before the photon is detected. Since the photon and the ancilla interact at the $C(H)$ gate, the ancilla is always prepared before the photon reaches BS$_2$.

In Fig. 1(d), the photon-ancilla system starts in the state $|\psi\rangle = \cos\alpha|0\rangle + \sin\alpha|1\rangle$ and the final state is

$$|\psi'\rangle = \cos\alpha|p\rangle|0\rangle + \sin\alpha|w\rangle|1\rangle,$$

where the wavefunctions $|p\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle)$ and $|w\rangle = e^{i\varphi/2}(\cos\frac{\varphi}{2}|0\rangle - i\sin\frac{\varphi}{2}|1\rangle)$ describe particle and wave behaviour, respectively. The two states are in general not orthogonal $\langle p|w\rangle = \frac{1}{\sqrt{2}}\cos\varphi$. Eq. (1) implies that if the ancilla is measured to be $|0\rangle (|1\rangle)$, the interferometer is open (closed) and the photon behaves like a particle (wave).

The interference pattern measured by the photon detector $D$ is $I_1(\varphi) = \text{tr}(p_1|1\rangle\langle 1|)$, with $p_1 = \text{tr}_2[|\psi\rangle\langle\psi|] = \frac{1}{2}(|p\rangle\langle p| + |w\rangle\langle w|)$ the reduced density matrix of the photon. The visibility of the interference pattern is $V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$, where the min/max values are calculated with respect to $\varphi$. If the interferometer is closed, the photon shows a wavelike behaviour with $I_w(\varphi) = \sin^2\frac{\varphi}{2}$ and visibility $V = 1$. For an open interferometer the photon behaves like a particle and $I_p(\varphi) = \frac{1}{2}$, resulting in $V = 0$. For the entangled state (1) the result is

$$I_1(\varphi, \alpha) = I_p(\varphi)\cos^2\alpha + I_w(\varphi)\sin^2\alpha.$$

Without correlating the photon data with the ancilla we observe an interference pattern with reduced visibility $V = \sin^2\alpha$: the photon has a mixed behaviour between a particle and a wave. On the other hand, if we do correlate the photon with the ancilla we observe either a perfect wave-like behaviour (ancilla $|1\rangle$) or a particle-like one (ancilla $|0\rangle$). By varying $\alpha$ we have the ability to...
modify continuously the interference pattern, morphing from wave to particle patterns (Fig. 4).

Before discussing the interpretation of this experiment in Sec. [X] we make two observations. Since the decision about type of the test is made only after the photon’s detection, the idea that it can be either a particle or a wave seems even more dubious. However, unlike its classically-controlled counterpart, no spacelike separation between the ancilla (taking on the role of a QRNG) and the photon is possible. We will discuss the consequences of this failure in Sec. [X].

One way to ensure the quantumness of the controlling device is to use the entangled ancilla [25], Fig. 2(b). Replacing the ancilla by one qubit of a maximally entangled pair, and introducing the bias \( \alpha \) into the second half of the pair before it is measured, allows for the ancilla qubit (loosely speaking) to “not have a state” before the interaction.

![FIG. 2: Entanglement-controlled QDC](image)

(a) We bias the quantum random number generator (QRNG) by preparing the ancilla in an arbitrary state \( \cos \alpha |0\> + \sin \alpha |1\> \)
(b) A pair of maximally entangled qubits replaces a single ancilla. The first qubit serves as a control for the Hadamard gate, while the bias \( \alpha \) is introduced to the second.

This set-up leads to even stronger refutation of HV, and to a surprising conclusion that wave-particle realism and determinism, both individually tenable, are mutually incompatible [26].

![FIG. 3: Quantum-controlled CHSH experiment](image)

A quantum control can be used in other experiments as well. For example, a modification of the CHSH experiment [1] to include quantum control is rather simple. The gate \( X \) creates a desired pair, prepared in a maximally entangled (or perhaps some other) state. On both Alice’s and Bob’s side the measurement direction \( (A \text{ or } A', B \text{ or } B') \) is chosen not by a random number generator, as in [5], but by a quantum-controlled gate. Similar to the delayed-choice experiment, the entangled photons are measured before the choices of the directions are made.

On the one hand such a design makes it impossible to talk about the measurement settings determining HV. On the other hand, the entire system including two qubits that Alice and Bob measure and the two ancillas, may be treated as a single entity thus possibly allowing for a consistent HV theory [7, 28].

IV. HIDDEN-VARIABLE MODELS

How do we know that, e.g., the delayed-choice experiment rules out the wave-particle dichotomy? Hidden variables help to obtain an answer. To this end we introduce a binary hidden variable \( \lambda = p, w \) that represents randomly created photons that are “really” particles or waves.

Hidden-variable theories typically incorporate three requirements that supplement the standard quantum mechanics [29]. In addition to matching the quantum predictions,

\[
q(a, b, \ldots |A, B, \ldots) = \sum_\lambda p(a, b, \ldots |A, B, \ldots, \lambda)p(\lambda |A, B, \ldots),
\]

where \( A, B, \ldots \) are measurement set-ups and \( a, b, \ldots \) the respective measurement results, a number of additional assumptions of various strength are made. Following [29] we present the relevant definitions.

(i) The strong determinism hypothesis asserts that for every measurement \( A \) that is described by a HV theory and for every value of \( \lambda \), there is an outcome \( a \) such that

\[
p(a |A, \lambda) = 1,
\]

and similarly for the measurements \( B, C, \ldots \)  

**Weak determinism** asserts that once the full set of measurements and the HV values are fixed, the totality of outcomes is determined. Since there is no measurement with the outcomes “particle” and “wave”, determinism is not imposed on the outcomes of \( D_a \). However, it plays key assumption in the entanglement-assisted delayed-choice experiment [26].

(ii) A HV model satisfies parameter independence if, for all \( a, A, B, \ldots, \lambda \),

\[
p(a |A, B, C, \ldots, \lambda) = p(a |A, \lambda)
\]

and similarly for \( b, A, B, C, \ldots \). This is a requirement that the outcome of any measurement depends only on the HV and the set-up of this measurement, and not
on any other measurements. Strong determinism implies parameter independence. In our analysis this hypothesis is not directly applicable since only one measurement setup is considered.

(iii) A HV model satisfies \( \lambda \)-independence if for all \( A, A', B, B', \ldots \)

\[
p(\lambda|A, B, \ldots) = p(\lambda|A', B', \ldots),
\]

where \( A \) and \( A' \) are two different set-ups of the same measurement. This asserts that the process determining the value of the hidden variable is independent of which measurements are chosen. A spacelike separation in Bell-type or delayed-choice experiments, together with an assumption of absence of the superluminal propagation, are the rationale for considering this property enforced.

In this context locality is a derived concept. Several combinations of different versions of the three basic properties lead to it \[29\].

We consider the requirements of “being a wave” and “being a particle” as “real objective properties”. This is a specific example of realism in a HV theory, that is (R) an imposition of some constraints on the probabilities \( p(a, \ldots|A, \ldots, \lambda) \) that represent classical expectations of systems’s behaviour.

As a result two of the conditional distributions \( p(a|b, \lambda) \) are constrained by the expectation of how particles (waves) behave in open (closed) interferometers. Consistent with our previous definition, a particle in an open interferometer (waves) behave in open (closed) interferometers. Confirming with our previous definition, a particle in an open interferometer \( (b = 0) \) and a wave in a closed MZI \( (b = 1) \) satisfy

\[
p(a|b = 0, \lambda = p) = \left( \frac{1}{2}, \frac{1}{2} \right),
\]

\[
p(a|b = 1, \lambda = w) = \left( \cos^2 \frac{\alpha}{2}, \sin^2 \frac{\alpha}{2} \right),
\]

respectively. Note that it is a weaker requirement than determinism, where the knowledge of HV fully determines the outcomes.

V. ANALYSIS OF THE DELAYED-CHOICE EXPERIMENTS

Consider now the experiments on Fig. 1(d) (or Fig. 1(a) with the biased QRNG). In dealing with HV theories we assume the standard conditions for probability distributions; for all variables \( i, j \) we have: (i) \( p(i) = \sum p(i, j) \) (marginals) and (ii) \( p(i, j) = p(i|j)p(j) = p(j|i)p(i) \) (conditionals). A hidden-variable theory should be adequate, i.e predict the correct quantum probabilities,

\[
q(a, b) \equiv p(a, b) = \sum_\lambda p(a, b|\lambda)p(\lambda).
\]

Without loss of generality \[12\] we represent this as

\[
p(a, b) = \sum_\lambda p(a|b, \lambda)p(b|\lambda)p(\lambda).
\]

The behaviour of a wave \( (\lambda = w) \) in an open \( (b = 0) \) and of a particle \( (\lambda = p) \) in a closed \( (b = 1) \) interferometer are unconstrained. We denote these two unknown distributions by \( x \) and \( y \), respectively

\[
p(a|b = 0, \lambda = w) = (x, 1 - x), \quad p(a|b = 1, \lambda = p) = (y, 1 - y).
\]

The probability distribution of the ancilla/QRNG \( p(b) \) is

\[
p(b) = \left( \cos^2 \alpha, \sin^2 \alpha \right).
\]

We assume that the source randomly emits particle- or wave-like photons with probability \( p(p) = f \) and \( p(w) = 1 - f \). The remaining two variables are the conditional probability distributions of the ancilla \( b \) and the hidden variable \( \lambda \):

\[
p(b|\lambda = p) = (z, 1 - z), \quad p(b|\lambda = w) = (v, 1 - v),
\]

satisfying the consistency condition \( p(b) = \sum_\lambda p(b|\lambda)p(\lambda) \). Writing explicitly the adequacy conditions Eq. \[9\] and manipulating the resulting equations we obtain:

\[
v(1 - f)(x - \frac{1}{2}) = 0,
\]

\[
f(1 - z)(y - \cos^2 \frac{\alpha}{2}) = 0,
\]

\[
zf + v(1 - f) - \cos^2 \alpha = 0.
\]

Five of the non-trivial solutions of this system essentially restore wave-particle duality, making behaviour of the photon independent of \( \lambda \). The last solution is:

\[
v = 0, \quad z = 1, \quad f = \cos^2 \alpha
\]

with \( x, y \) undetermined. In other words, the source randomly emits particles and waves with a distribution \( p(\lambda) = (\cos^2 \alpha, \sin^2 \alpha) \) identical to the probability distribution \( p(b) \) of the ancilla, being thus perfectly correlated with MZI being open or closed. In a classically-controlled delayed-choice experiment the spacelike separation (and the subuminal propagation of signals) enforce \( p(b, \lambda) = p(b)p(\lambda) \), i.e., \( v = z \) and the last solution is impossible. In this case the conclusion is either a wave-particle duality or deeper conspiratorial correlations (e.g. between \( \lambda \) and the settings of QRNG).

In the quantum delayed-choice experiment if the ancilla \( B \) is considered as part of the measuring device then a formal conclusion is that realism and \( \lambda \)-independence are incompatible. On the one hand, it can be argued that our result is stronger than the one obtained with a classically-controlled device, since the required correlation is not with the experimental settings (the beam splitter is present or absent), but with the set-up of the random number generator driving it. On the other hand, having a quantum ancilla allows for its hidden variables \( \Lambda \) that may somehow compromise the conclusion. Our analysis \[20\] demonstrates that this is not the case and provides experimental signatures for the refutation of possible HV theories.
VI. INTERPRETATION, EXPERIMENTS AND FUTURE WORK

The use of quantum control necessitates a reassessment of Bohr’s complementarity \[30\]. Partial information about complementary quantities can be obtained in a single experiments \[15, 34\]. Contrary to Bohr’s opinion, we do not have to change the experimental setup in order to measure complementary properties \[12\] — we can measure both properties in a single experiment, provided that a component of the apparatus is a quantum object in a superposition state. The behaviour is post-selected by the experimenter after the photon has been detected, by correlating the data with the appropriate value of the ancilla.

A quantum control makes impossible the spacelike separation between the device settings and the system. A spacelike separation can be reintroduced by having an additional classical device or creating sub-systems at mutually spacelike events \[26\]. Quantum delayed-choice experiment can force the proponents of the wave-particle realism to accept the same level of conspiracy as the classically-controlled experiment with the spacelike separation does. Adding the spacelike separation between the photon and the ancilla leads to new results \[26\]. However, it is still not clear to what extent its introduction is necessary in other experiments.

Not having to deal with the spacelike separation allows realization of the delayed-choice experiment with NMR \[31\] or on a chip \[32\].

One of the consequences of the quantum control is the morphing between wave and particle statistics, see Eq. (2). In the experiment of Ref. \[33\] the states \(|0\rangle\) and \(|1\rangle\) were encoded in the horizontal and vertical polarization of a photon. The observed wave-particle morphing of a single photon statistics is in an excellent agreement with the theoretic prediction (Fig. 4). In addition, it was clearly demonstrated that due to the interference between the “wave” and “particle” states superposition of \(|p\rangle\) and \(|w\rangle\) states is markedly different from their classical mixture.

Quantum control showed its usefulness in the delayed-choice experiments. As it should be clear from our presentation, it can be used in any experiment where several alternative set-ups are employed. We expect to see both the conceptual surprises and practical benefits stemming from its use.

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[1] Wheeler, J. A., Zurek, W. H., eds.: Quantum Theory and Measurement, Princeton University Press, Princeton (1984).

[2] Schlipp, P. A., ed.: Albert Einstein, Philosopher-Scientist, The Library of Living Philosophers (1949); MJF Books, New York, (1970).

[3] Nielsen, M. A., and Chuang, I. L.: Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, (2000); Bruß, D., Leuchs, G.: Lectures on Quantum Information Wiley-VCH, Weinheim (2007).

[4] Greenberger, D., Hentschel, K., Weinert, F., eds.: Compendium of Quantum Physics, Springer, Berlin (2009).

[5] Aspect, A., Dalibard, J., Roger, G.: Phys. Rev. Lett. 49, 1804 (1982); Reid, M. D., et al., Rev. Mod. Phys. 81, 1727 (2009).

[6] Grangier, P., Roger, G., Aspect, A.: Europhys. Lett. 1, 173 (1986).

[7] Peres, A.: Quantum Theory: Concepts and Methods, Kluwer, Dordrecht (1995).

[8] von Neumann, J.: Mathematische Grundlagen der Quantenmechanik, Springer, Berlin (1932); transl. by Beyer, E. T.: Mathematical Foundations of Quantum Mechanics, Princeton University Press, Princeton (1955), Ch. VI.

[9] Schrödinger, E.: Naturwiss. 23, 807, 823 (1935); transl. in [1], p. 152–167.

[10] Marshall, W., Simon, C., Penrose, R., Bouwmeester, D.: Phys. Rev. Lett. 91, 130401 (2003).

[11] Hornberger K., et al.: Rev. Mod. Phys. 84, 157 (2012); Poot, M., van der Zant, H. S. J.: Phys. Rep. 511, 273 (2012).

[12] Ionicioiu, R., Terno, D. R.: Phys. Rev. Lett 107, 230406 (2011).

[13] Barenco, A., Deutsch, D., Ekert, A., Jozsa, R.: Phys. Rev. Lett. 74, 4083 (1995).

[14] Jacques, v, et al.: Science 315, 966 (2007).

[15] Scully, M. O., Englert, B. G., Walter, H.: Nature 351, 111 (1991).

[16] Greenstein, G., Zajonc, A. G.: The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, Jones and Bartlett, Boston (1997), Ch. 2.

[17] Bohr, N.: in [1], pp. 9–49.

[18] Stapp, H.: in [2], pp. 111–113.

[19] Longhi, S.: Laser & Photon. Rev. 3, 243 (2009).

[20] Edlén, A., et al.: J. Fluid Mech. 674, 433 (2011).

[21] Wheeler, J. A.: in Mathematical Foundations of Quantum Mechanics, Marlow, A. R., ed.: Academic Press, New York (1978), pp. 9–48.

[22] Leggett, A. J.: in [1], pp. 161–166.

[23] von Weizsäcker, C. F. F.: Z. Phys. 118, 489 (1941).

[24] Bohr, N.: in [2], p. 230.

[25] Kaiser, F. et al.: Science 338, 637 (2012).

[26] Ionicioiu, R., Jennewein, T., Mann, R. B., Terno, D. R.: arXiv:1211.0979.

[27] Clauser, F., Horne, M. A., Shimony, A., Holt, R. A.: Phys. Rev. Lett. 23, 880 (1969).

[28] Bell, J. S.: Rev. Mod. Phys. 38, 447 (1966).

[29] A. Brandenburger and N. Yanofsky, J. Phys. A 41, 425302 (2008).

[30] Ananthaswamy, A.: New Scientist, 217 (2898), 37 (2013).

[31] Roy, S., Shukla, A., Mahesh, T. S.: Phys. Rev. A 85, 022109 (2012); Auccaise, R. et al.: Phys. Rev. A 85, 032121 (2012).

[32] Peruzzo, A., et al.: Science 338, 634 (2012).

[33] Tang, J.-S., et al.: Nature Phot. 6, 600 (2012).

[34] Wootters, W. K., Zurek, W. H.: Phys. Rev. D 19, 473 (1979).

[35] It was first discussed by von Weizsäcker [23] and briefly mentioned by Bohr in his review of the Einstein-Bohr discussions [24].