We prove that the correlations present in a multiparticle quantum state have an operational quantum character as soon as the state does not simply encode a multiparticle classical probability distribution, i.e. does not describe the joint state of many classical registers. Even unentangled states may exhibit such quantumness, that is pointed out by the new task of local broadcasting, i.e. of locally sharing pre-established correlations: this task is feasible if and only if correlations are classical and derive a no-local-broadcasting theorem for quantum correlations. Thus, local broadcasting is able to point out the quantumness of correlations, as standard broadcasting points out the quantum character of single system states. Further, we argue that our theorem implies the standard no-broadcasting theorem for single systems, and that our operative approach leads in a natural way to the definition of measures for quantumness of correlations.

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The characterization of correlations present in a quantum state has recently drawn much attention [1, 2, 3, 4, 5, 6]. In particular, efforts have been made to analyze whether and how correlations can be understood, quantified and classified as either classical or quantum. Even if such classical/quantum distinction may not be possible in clear-cut terms, understanding to some extent the quantumness of correlations is not only relevant from a fundamental point of view, but also in order to make more clear the origin of the quantum advantage [5, 6], with respect to the classical scenario. Therefore, while entanglement [8] may be considered the essential feature of quantum mechanics, it is relevant to study how and in what sense even correlations present in unentangled states may exhibit a certain quantum character.

In this Letter we provide an operational characterization of those multipartite states whose correlations may be considered as completely classical, hence, by contrast, also of quantumness. W e do this in two ways. First, we consider the process of extracting classical correlations (correlations that can be transferred to classical registers) from quantum states, and we prove that this classical-classical distribution is able to point out the quantumness of correlations, as standard broadcasting points out the quantum character of single system states. Further, we argue that our theorem implies the standard no-broadcasting theorem for single systems, and that our operative approach leads in a natural way to the definition of measures for quantumness of correlations.

of its most natural multipartite versions. For the sake of clarity, we derive them in the bipartite case.

We start by recalling [2, 3, 12] several definitions that make clear what we mean when we discuss classicality and quantumness, both of bipartite states and of correlations.

Definition 1. A bipartite state $\rho$ is: (i) separable [20] if it can be written as $\sum_k p_k \sigma_k^A \otimes \sigma_k^B$, where $p_k$ is a probability distribution and each $\sigma_k^A$ is a quantum state, and entangled if non-separable; (ii) classical-quantum (CQ) if it can be written as $\sum_i p_i |i\rangle \langle i| \otimes \sigma_i^B$, where $\{|i\rangle\}$ is an orthonormal set, $\{p_i\}$ is a probability distribution and $\sigma_i^B$ are quantum states; (iii) classical-classical (CC), or (strictly) classically correlated [3, 4], if there are two orthonormal sets $\{|i\rangle\}$ and $\{|j\rangle\}$ such that $\rho = \sum_{ij} p_{ij} |i\rangle \langle i| \otimes |j\rangle \langle j|$, with $p_{ij}$ a joint probability distribution for the indexes $(i,j)$.

One could consider a CC state to correspond simply to the embedding into the quantum formalism of a classical joint probability distribution. It is possible to go from a bipartite quantum state to a CC state and further to a CC state by local measurements.

Definition 2. A (quantum-to-classical) measurement map [21] $\mathcal{M}$ acts as $\mathcal{M}[X] = \sum_i \text{Tr}(M_i X) |i\rangle \langle i|$, where $\{M_i\}$ is a POVM, i.e. $0 \leq M_i \leq 1$ and $\sum_i M_i = 1$, and $\{|i\rangle\}$ is a set of orthonormal states.

A measurement map performs a POVM measurement and writes the result in a classical register (i.e., that can be perfectly read and copied), thus any POVM corresponds to a measurement map. Hence, to any bipartite state $\rho$ and any POVM $\{M_i\}$ (on $A$, in this case) we can associate a CQ state $\rho^{\text{CQ}}(\{M_i\}) = (\mathcal{M}_A \otimes \text{id}_B) |\psi\rangle = \sum_i p_i |i\rangle \langle i| \otimes \sigma_i^B$, where $\mathcal{M}_A$ is the measurement map associated to the POVM, so that $p_i = \text{Tr}(M_i^A \rho)$ and $\sigma_i^B = \text{Tr}_A(M_i^A \rho)/p_i$. Similarly, given POVMs $\{M_i\}$ and
{N_j} on A and on B respectively, we can associate to ρ the CC state ρ^{CC}(\{M_l\},\{N_j\}) = (\mathcal{M}_A \otimes \mathcal{N}_B)[\rho] = \sum_{i,j} p_{ij} |i\rangle \langle i| \otimes |j\rangle \langle j|, with \mathcal{M}_A, \mathcal{N}_B the two local measurement maps associated to the two POVMs, and p_{ij} = \text{Tr}(M^A_i \otimes N^B_j \rho). Notice that that in this case one may always think that the passage from the initial state ρ to the CC state ρ^{CC}(\{M_l\},\{N_j\}) happens in two separate (and commuting) steps corresponding to the two local POVMs. Both from an axiomatic and an operative point of view, we are led to look at Mutual Information (MI) as a measure of total correlations. 

**Definition 3.** (Quantum) mutual information I(ρ_{AB}) of a bipartite quantum state ρ_{AB} is given by I(ρ_{AB}) = S(A) + S(B) − S(AB), where S(X) = −\text{Tr}(ρ_X \log ρ_X) is the von Neumann entropy of ρ_X.

Quantum Mutual Information (QMI) is the generalization to the quantum scenario of the classical MI for a joint probability distribution \{p^A_i\}: I(\{p^A_i\}) = H(p^A_i) + H((p^A_i)) − H(p^A_i), with p^A_i = \sum_j p_{ij} the marginal distribution for A (similarly for B), and H(\{q_k\}) = −\sum_k q_k \log q_k is the Shannon entropy of the classical distribution \{q_k\}. QMI can be written as the relative entropy between the total bipartite state and the tensor product of its reductions, i.e. I(ρ_{AB}) = S(ρ_{AB}||ρ_A \otimes ρ_B), with ρ_X = Tr_Y(ρ_{XY}). Thus, QMI is positive, and vanishes only for factorized states. Most importantly, it cannot increase under local channels Λ_A \otimes Λ_B, i.e. I(ρ_{AB}) ≥ I((Λ_A \otimes Λ_B)[ρ_{AB}]) [2].

From an operative point of view, QMI provides the classical capacity of a noisy quantum channel when entanglement is a free unlimited resource [13]. Moreover, for a given state ρ^{AB}, I(ρ^{AB}) gives the smallest rate of classical randomness necessary and sufficient to erase all correlations between A and B in the asymptotic setting [2].

We will consider two other measures of correlations.

**Definition 4.** Given a bipartite state ρ_{AB} we define: the CQ mutual information as I_{CQ}(ρ_{AB}) = \max_{\{M_l\}} I(ρ^{CQ}(\{M_l\})); the CC mutual information as I_{CC}(ρ_{AB}) = \max_{\{M_l\},\{N_j\}} I(ρ^{CC}(\{M_l\},\{N_j\})).

The maxima are taken with respect to (local) measurement maps. Notice that both CQ mutual information and CC mutual information correspond to the QMI of the state after a local processing, more precisely after the application of a measurement map. I_{CC} corresponds exactly to the classical MI of the joint classical distribution p_{ij} = \text{Tr}(M_i \otimes N_j \rho). I_{CQ} was considered – though not in terms of MI – in [2] as a measure of classical correlations, but one may argue that in principle there is still a certain degree of quantumness in the CQ state entering in the corresponding definition. I_{CC} was first defined in [15] and provides the maximum amount of the correlations that are present in the state and that can be considered classical, in the sense that can be revealed by means of local measurements, and in this way from the quantum to the classical domain (i.e. recorded in classical registers). We have already seen that MI does not increase under local operations. In [2] this was proved also for I_{CQ}, and the same holds for I_{CC}, as local operations on both sides can be absorbed in the measurements. Moreover, I, I_{CQ}, I_{CC} are related by local operations themselves and each of them vanish only for uncorrelated state [2, 14]. We collect this results in the following

**Observation 1.** Mutual information functions I, I_{CQ}, I_{CC}: (i) are non-increasing under local operations; (ii) satisfy I ≥ I_{CQ} ≥ I_{CC} ≥ 0; (iii) vanish if and only if the state is factorized.

We will prove, with the help of simple lemmas, that all quantum states, that are not CC from the beginning, contain correlations that are not classical, in the sense made precise by Theorem 1.

**Lemma 1.** Given a CQ state ρ = \sum_i p_i |i\rangle \langle i| \otimes σ^B_i, we have I(ρ) = I_{CQ}(ρ) = χ(|p_i, σ_i\rangle), with the Holevo quantity χ(|p_i, σ_i\rangle) = S(\sum_i p_i σ_i) − \sum_i p_i S(σ_i). Moreover, we have I(ρ) = I_{CC}(ρ) if and only if the states σ^B_i commute and ρ is CC.

**Proof.** In order to prove I(ρ) = I_{CQ}(ρ), consider the measurement on A corresponding to a complete measurement on the basis comprising the orthogonal states \{|i\rangle\}. I(ρ) = χ(|p_i, σ_i\rangle) is checked straightforwardly. Thus, I_{CC}(ρ) is the classical MI between two parties, where party A sends a state σ_i labeled by i with probability p_i, and B proceeds to a generalized measurement that gives outputs j with conditional probabilities p(j|i) [2]. It is known [17] that χ is an upper bound to the classical MI of \{p_{ij} = p_i p(j|i)\}, that is saturated if and only if the states σ_i commute, i.e. can be diagonalized in the same basis.

**Lemma 2.** If I((Λ_A \otimes Λ_B)[ρ]) = I(ρ), there exist Λ^*_A and Λ^*_B such that (Λ^*_A \otimes Λ^*_B) ∘ (Λ_A \otimes Λ_B)[ρ] = ρ.

**Proof.** A theorem [10] by Petz states that, given two states ρ, σ and a channel Λ[Y] = \sum_i K_i Y K_i^T, then S(ρ||σ) = S(Λ[ρ]||Λ[σ]) if and only if there exists a channel Λ^* such that Λ^*[ρ] = ρ and Λ^*[σ] = σ. Moreover, the action of Λ^* on Λ[σ] can be given the explicit expression Λ^*[X] = σ^T Λ^* [Λ[σ]]−1/2 X (Λ[σ])−1/2 σ^1/2, where Λ^*[Y] = \sum_i K^*_i Y K_i. With this result, if furthermore σ = σ_A \otimes σ_B and Λ = Λ_A \otimes Λ_B, one easily checks that Λ^* = Λ^*_A \otimes Λ^*_B.

We are now ready to state our main result.

**Theorem 1.** We have I_{CC}(ρ) = I(ρ) if and only if ρ is classical-classical.
Proof. If the state is CC, it is immediate to check that $I_{CC} = I$. On the other hand, let us assume $I(r) = I_{CC} = I(r_{CC}) = I(r_{CC}^{p}) = I(r_{CC}^{p} (\{ M_{i} \}, \{ N_{j} \}))$, with $r_{CC}^{p} (\{ M_{i} \}, \{ N_{j} \}) = \sum_{ij} p_{ij} | i \rangle \langle i | \otimes | j \rangle \langle j |$ for some optimal $\{ M_{i} \}, \{ N_{j} \}$. Thanks to Lemma 2 we have that there exist maps $M^{*}$ and $N^{*}$ which invert the measurement maps, i.e. such that $\rho = (M^{*} \otimes N^{*}) [r_{CC}^{p}] = \sum_{ij} p_{ij} M^{*} (| i \rangle \langle i |) \otimes N^{*} (| j \rangle \langle j |)$. Let us consider $\rho_{CC} = (M^{*} \otimes N^{*}) [r_{CC}^{p}] = \sum_{ij} p_{ij} M^{*} (| i \rangle \langle i |) \otimes N^{*} (| j \rangle \langle j |)$, where $p_{ij} = \sum_{k \in k_{i}^{j}} C$ and $\sigma_{j}^{A} = \sum_{k} \rho_{k}^{A} | \phi_{k} \rangle \langle \phi_{k} | \otimes \tau_{k}$. This is a QC state such that $I(\rho_{CC}) = I_{CC} (\rho_{CC}) = I_{CC} (\rho)$. Therefore, all $\sigma_{j}^{A} = \sum_{k} q_{k}^{(j)} \rho_{k}^{A} | \phi_{k} \rangle \langle \phi_{k} | \otimes \tau_{k}$ are diagonal in the same basis $\{| \phi_{k} \rangle \}$ by Lemma 1. The original state can now be written as $\rho = \sum_{ij} p_{ij}^{B} \sigma_{j}^{A} \otimes N^{*} (| j \rangle \langle j |) = \sum_{i} \tau_{i}^{A} \rho_{i}^{A} | \phi_{i} \rangle \langle \phi_{i} | \otimes \tau_{i}^{B}$, where $\tau_{i}^{A} = \sum_{j} \rho_{j}^{B} \rho_{i}^{A} | \phi_{j} \rangle \langle \phi_{j} |$ and $\tau_{i}^{B} = \sum_{j} \rho_{j}^{B} \rho_{i}^{A} \tau_{j}^{A}$ $\tau_{j}^{A}$ are diagonal in the same basis $\{| \phi_{j} \rangle \}$. We thus have that $\rho$ is a QC state with $I = I_{CC}$, therefore it is CC, again by Lemma 1.

We depict here another operational way to characterize CC states which regards local broadcastability. We first recall the standard broadcasting condition [10].

**Definition 5.** Given a state $\rho$ we say that $\rho_{XY}$ is a broadcast state for $\rho$ if $\rho_{XY}$ satisfies $\rho_{X} = \rho_{Y} = \rho$.

We now specialize to the bipartite scenario $\rho = \rho_{AB}$. In this case, one can consider two cuts: one between the copies, and one between the parties. The latter defines locality. Thus, the copies are labeled by $X = AB$ and $Y = A'B'$, while the two parties are $(A, A')$ and $(B, B')$.

**Definition 6.** We say that the state $\rho = \rho_{AB}$ is locally broadcastable (LB) if there exist local maps $\Theta_{A} : A \rightarrow A', \Theta_{B} : B \rightarrow B'$ such that $\sigma_{AA',BB'} = \Theta_{A} \otimes \Theta_{B} (\rho_{AB})$ is a broadcast state for $\rho$.

No entangled state is LB, as no entangled state can be broadcast even by LOCC (see Proposition 1 in [18]). On the contrary, every CC state is LB by cloning locally its biorthonormal eigenbasis. We provide now a necessary and sufficient condition for local broadcastability in terms of QMI.

**Theorem 2.** A state $\rho_{AB}$ is LB if and only if there exist a broadcast state $\sigma_{AA',BB'}$ for $\rho_{AB}$ such that $I_{CC} (\rho_{AB}) = I_{CC} (\sigma_{AA',BB'})$. Moreover, any broadcast state $\sigma_{AA',BB'}$ satisfying the latter condition can be obtained from $\rho$ by means of local maps.

Proof. If $\rho = \rho_{AB}$ is LB then there exist a broadcast state $\sigma = \sigma_{AA',BB'} = (\Theta_{A} \otimes \Theta_{B}) (\rho_{AB})$. Since $\sigma$ is obtained from $\rho = \rho_{AB}$ by local operations, we have that $I(\sigma) \leq I(\rho)$, because local operations can not increase MI. Moreover, since $\sigma$ is a broadcast state, $\rho$ can be obtained by local operations from it, more precisely by local tracing. Indeed, $\rho = (T_{A} \otimes T_{B}) [\sigma]$, so that it must be $I(\sigma) \geq I(\rho)$. Therefore $I(\rho_{AB}) = I(\sigma_{AA',BB'})$. On the other hand, let us now suppose there exist a broadcast state $\sigma$ for $\rho$ such that $I(\rho_{AB}) = I(\sigma_{AA',BB'})$. We want to see it can be obtained by local broadcasting. Indeed, by taking $\Lambda_{AA'} = T_{A}$ and $\Lambda_{BB'} = T_{B}$, we have $I(\sigma) = I(\rho) = I((\Lambda_{AA'} \otimes \Lambda_{BB'}) [\sigma])$. By applying Lemma 2 we see there are local maps $\Theta_{A} = \Lambda_{AA'}$ and $\Theta_{B} = \Lambda_{BB'}$ that locally broadcast $\rho$ into $\sigma$.

From Theorem 2 we see that local broadcastability can be assessed by checking the existence of broadcast states with the same MI as the starting state.

We state now our second main result.

**Theorem 3.** Classical-classical states are the only states that can be locally broadcasted.

Proof. Given a LB state $\rho_{AB}$, consider any broadcast state $\sigma_{AA',BB'}$ satisfying $I(\rho) = I(\sigma)$, and let measuring maps $M$ and $N$ be optimal for the sake of $I_{CC}(\rho)$. Applying $M$ and $N$ on subsystems $A$ and $B$ of $\sigma$, we obtain: $\sigma = (M_{A} \otimes N_{B}) [\sigma] = \sum_{ij} p_{ij} | i\rangle\langle i |_{A} | j \rangle\langle j |_{B}$, where $p_{ij} = Tr(M_{A}^{i} \otimes N_{B}^{j} | \sigma \rangle \langle \sigma |)$. This is the optimal classical probability distribution for $\rho = (M_{A}^{i} \otimes N_{B}^{j}) | \sigma \rangle \langle \sigma |$. Hence, $Tr_{AB} (| \sigma \rangle \langle \sigma |) = Tr_{A} (| \sigma \rangle \langle \sigma |) = Tr_{B} (| \sigma \rangle \langle \sigma |)$. For the same reason, $Tr_{AB} (| \sigma \rangle \langle \sigma |) = Tr_{AB} (| \rho \rangle \langle \rho |)$. Thus, $I(\hat{\sigma}) = I(\rho)$, and at the same time

$$I_{CC}(\rho) = I(\{ p_{ij} \}) + \sum_{i} p_{i}^{A} S(\rho_{i}^{A}) + \sum_{j} p_{j}^{B} S(\rho_{j}^{B})$$

$$= \sum_{i} p_{ij} S(\rho_{ij}^{A,B'})$$

where $p_{i}^{A} = \sum_{j} p_{ij}, \rho_{i}^{A} = \sum_{j} p_{ij} \rho_{ij}^{A}$ (similarly for $p_{j}^{B}$ and $\rho_{j}^{B}$). The inequality comes from the concavity of entropy: $\sum_{i} p_{i}^{A} S(\rho_{i}^{A}) \geq \sum_{ij} p_{ij} S(\rho_{ij}^{A})$ (similarly for $B$), and we have used the fact that $I(\{ p_{ij} \}) = I_{CC}(\rho)$. Consider now any other measuring maps $M$ and $N$, and let them act on the (still quantum) systems $A'$ and $B'$ of $\hat{\sigma}$, getting a state $\sigma_{CC}$. This corresponds simply to transforming each $\rho_{ij}^{A,B'}$ into some CC state $\rho_{ij}^{CC} = T_{A'} (\{ M_{i} \}, \{ N_{j} \})$. Thus, we have $I(\sigma_{CC}) = I(\sigma_{CC}) + I_{CC}(\rho) + \sum_{ij} p_{ij} I((\rho_{ij}^{CC})_{A'B'} (\{ M_{i} \}, \{ N_{j} \}))$, for arbitrary $\{ M_{i} \}, \{ N_{j} \}$, because the measurement maps $M_{A} \otimes M_{A'}$ and $N_{B} \otimes N_{B'}$ may not be the optimal ones to get $I_{CC}(\sigma)$. By the assumptions and by Theorem 2 $\sigma$ may be obtained from $\rho$ via local broadcasting, and by Observation 3 it must be $I_{CC}(\sigma) \leq I_{CC}(\rho)$. Therefore, we have $I_{CC}(\sigma) = I_{CC}(\rho)$. This means that $I(\{ p_{ij}^{CC} \}_{A'B'} (\{ M_{i} \}, \{ N_{j} \}))$ must be zero for any non vanishing $p_{ij}$. Choosing $M, N$ repeatedly to be optimal for every $\rho_{ij}^{A,B'}$, one concludes that it must be $I_{CC}(\rho_{ij}^{A,B'}) = 0$ for every $i, j$ such that $p_{ij} > 0$, so that, according to Observation 3 it must be $\rho_{ij}^{A,B'} = \rho_{ij}^{A'} \otimes \rho_{ij}^{B'}$. Moreover to have
equality in [1] it must be that $\rho_{A}^{\mu} = \rho_{A}^{i}$ and $\rho_{B}^{\mu} = \rho_{B}^{i}$, because of the strong concavity of entropy. Thus, we have found that actually $\tilde{\sigma}$ is a classical-classical state, $\tilde{\sigma} = \sum_{i} p_{ij} |i_{A} \rangle \langle i_{A}| \otimes \rho_{A}^{i} \otimes (|j_{B} \rangle \langle j_{B}| \otimes \rho_{B}^{j})$, so that $I(\rho) = I(\sigma) = I_{CC}(\tilde{\sigma}) = I_{CC}(\sigma) = I_{CC}(\rho)$, because of Observation 1. Therefore, according to Theorem 1, $\rho$ is also classical-classical.

One immediately realizes that the essential assumptions used to prove that $\rho_{AB}$ is CC are: (i) $\sigma_{A'A'B'}$ is obtained from $\rho$ by local maps; (ii) $I(\sigma_{AB}) = I(\sigma_{A'B'}) = I(\rho_{AB})$. Indeed, thanks to Lemma 2 these conditions mean that $\rho_{AB}, \sigma_{A'A'B'}, \sigma_{AB}, \sigma_{A'B'}$ are all connected by local maps. Thus, with slight changes in the proof of Theorem 4 one can obtain the following stronger result.

**Theorem 4.** Given a state $\rho_{AB}$, there exists a state $\sigma_{A'A'B'}$ with $I(\sigma_{AB}) = I(\sigma_{A'B'}) = I(\rho_{AB})$, that can be obtained from $\rho_{AB}$ by means of local operations, if and only if $\rho_{AB}$ is classical-classical.

The just stated result represents a no-broadcasting theorem, more precisely, a no-local-broadcasting theorem, for correlations as measured by a single number, local mutual information quantities. All Theorems remain valid, as Observation 1 and Lemma 2 are immediately extended, while Lemma 1 generalizes to the case of a state that is classical with respect to all the parties but one.

In conclusion, we characterized operationally the set of classical-classical states, i.e. states that correspond essentially to the description of correlated classical registers. We showed that they are the only states for which correlations, as measured by mutual information, can be totally transferred from the quantum to the classical world. Furthermore, they are the only states that can be locally broadcast. A even stronger result was derived in terms of mutual information alone, without imposing the broadcast condition for states: correlations, as quantified by such a scalar quantity, can be locally broadcast only for classical-classical states. Thus, our results show that also separable non-CC states exhibit a certain degree of quantumness, and also lead to some natural ways to quantify the degree of non-classicality. E.g., one may consider the gap $\Delta_{CC}(\rho) = I(\rho) - I_{CC}(\rho)$, or, similarly to what done in [1], the minimal difference $\Delta_{\delta}(\rho_{AB}) = \min_{\sigma_{A'A'B'}} I(\sigma_{A'A'B'}) - I(\rho_{AB})$, between the mutual information of a two-copy broadcast state $\sigma_{A'A'B'}$ and the mutual information of the state $\rho_{AB}$ itself. Theorems 1 and 2 and 3 respectively, make sure that such quantities are strictly positive for all non-classical-classical states, and in particular entangled states. Actually, the gap $\Delta_{CC}$ resembles the discord introduced in [1]: the latter corresponds to the gap $I - I_{CQ}$, where $C$ means that the measuring map which gives rise to $I_{CQ}$ is chosen among complete projective measurements rather than POVMs, as in the case of $I_{CQ}$. A further analysis of the role of entanglement in the quantumness of correlations, as well as how our approach may lead to a non-trivial quantification of entanglement will appear somewhere else.

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[1] H. Ollivier and W.H. Zurek, Phys. Rev. Lett. 88, 17901 (2001)
[2] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. 34, 6899 (2001).
[3] J. Oppenheim, M. Horodecki, P. Horodecki, and R.
Horodecki, Phys. Rev. Lett. 89, 180402 (2002); M. Horodecki et al., Phys. Rev. A 71, 062307 (2005).

[4] B. Groisman, S. Popescu, A. Winter, Phys. Rev. A 72, 032317 (2005).

[5] B. Groisman, D. Kenigsberg and T. Mor, \texttt{arXiv:quant-ph/0703103}

[6] R. Jozsa and N. Linden, Proc. R. Soc. A 459, 2011 (2003)

[7] M. A. Nielsen and I. L. Chuang, “Quantum Computation and Quantum Information”, Cambridge University Press, Cambridge (2000)

[8] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, \texttt{arXiv:quant-ph/0702225}

[9] This task is related to, but different from, the standard broadcasting task \cite{Barnum}. For recent developments concerning the latter, see \cite{Barnum2}. H. Barnum, C.M. Caves, C.A. Fuchs, R. Jozsa, and B. Schumacher, Phys. Rev. Lett. 76, 2818 (1996).

[11] H. Barnum, J. Barrett, M. Leifer, and A. Wilce, \texttt{arXiv:0707.0620}

[12] R.F. Werner, Phys. Rev. A 40, 4277 (1989).

[13] C. H. Bennett et al., Phys. Rev. Lett. 83, 3081 (1999).

[14] D. P. DiVincenzo et al., Phys. Rev. Lett. 92, 067902 (2004).

[15] B. Terhal et al., J. Math. Phys. 43, 4286 (2002).

[16] D. Petz, Commun. Math. Phys. 105, 123 (1986).

[17] A. S. H. (Kholevo), Probl. Peredachi Inf. 9, 3 (1973).

[18] D. Yang et al., Phys. Rev. Lett. 95, 190501 (2005).

[19] M. Horodecki et al., Int. J. Quant. Inf. 4, 105 (2006).

[20] In the seminal paper by Werner \cite{Werner} the term \textit{classically correlated} is used, but for the sake of clarity we prefer here to say “separable”.

[21] All maps will be understood as channels, i.e. trace preserving and completely positive maps.