Quantum Cosmology

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Abstract

We comment on two issues in quantum cosmology, in the context of the Wheeler–De Witt equation and wave function of the Universe: (i) arrow of time and interpretation of the wave function in the classically allowed regions; (ii) stability of an approximation of the Born–Oppenheimer type in classically forbidden regions of the scale factor.

1 Introduction

Most of the evolution of the Universe is likely to have proceeded classically, in the sense that the dominant phenomenon is the classical expansion while quantum fluctuations are small and can be treated as perturbations. Prior to this stage, however, genuinely quantum phenomena almost certainly took place. Although it is not clear whether they have left any observable footprints, it is of interest to try to understand them, as this may shed light on such issues as initial conditions for classical cosmology (are inflationary initial data natural? how long was the inflationary epoch? is open Universe consistent with inflation?), properties of space-time near the cosmological singularity, origin of coupling constants, etc.

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To describe the Universe at its quantum phase, one ultimately has to deal with full quantum gravity theory, well beyond the Einstein gravity. It is, however, legitimate to take more modest attitude and consider quantum phenomena below the Planck (or string) energy scale. Then the quantized Einstein gravity (plus quantized matter fields) provides an effective “low energy” description which must be tractable, at least in principle, within quantum field theory framework. Processes that should be possible to consider in this way are not necessarily perturbative, as the example of tunneling in quantum mechanics and field theory shows.

Surprisingly, not so many phenomena are well understood even within this modest approach. Perhaps the most clear process is the decay of a metastable vacuum [1]. It has been clarified recently [2] that at least in the range of parameters where the treatment of space-time in terms of background de Sitter metrics is reliable, the Coleman–De Luccia instanton indeed describes the false vacuum decay, provided the quantum fluctuations above the classical false vacuum are in de Sitter-invariant (conformal) vacuum state. This is in full accord with the results of Refs. [1, 3]. It is likely that this conclusion holds also when the quantum properties of metrics are taken into account. Furthermore, the Hawking–Moss instanton [4] can be interpreted [2] as a limiting case of constrained instantons that describe the false vacuum decay in an appropriate region of parameter space, again in agreement with previous analyses [5, 6]. Hence, there emerges a coherent picture of the false vacuum decay with gravity effects included.

One may try to apply laws of quantum mechanics to the Universe as a whole, and consider the wave function of the Universe. Although research in this direction began more than 30 years ago [7], the situation here is still intriguing and controversial. The main purpose of this contribution is to make a few comments on this subject. Namely, we will discuss which analogies to ordinary quantum mechanics are likely to work in quantum cosmology, and which are rather misleading. We begin with quantum mechanics, and only then turn to the wave function of the Universe.
2 Wave function in quantum mechanics

To set the stage, let us consider a quantum mechanical system with two dynamical coordinates, \( x \) and \( y \). Let the Hamiltonian be

\[
\hat{H} = \hat{H}_0 + \hat{H}_y
\]

where

\[
\hat{H}_0 = \frac{1}{2} \hat{p}_x^2 + V_0(x)
\]
\[
\hat{H}_y = \frac{1}{2} \hat{p}_y^2 + \frac{1}{2} \omega^2(x)y^2 + \frac{1}{4} \lambda(x)y^4 + \ldots
\]

Let us assume that the potential \( V_0(x) \) is such that the motion along the coordinate \( x \) is semiclassical, while the dynamics along the coordinate \( y \) can be treated in perturbation theory about the semiclassical motion along \( x \). This approach is close in spirit to the Born–Oppenheimer approximation. We will consider solutions to the stationary Schrödinger equation with fixed energy \( E \).

Let us first discuss the dynamics in the classically allowed region of \( x \), where \( E > V_0(x) \). In this region, there are two sets of solutions with the semiclassical parts of the wave functions equal to

\[
\Psi \propto e^{+iS(x)}
\]
\[
\Psi \propto e^{-iS(x)}
\]

where

\[
S(x) = \int^x dx' \sqrt{2(E - V_0(x'))}
\]

These two sets of solutions correspond to motion right and left, respectively. Note that this interpretation is based on the fact that there exists *extrinsic time* \( t \) inherent in the problem: the complete, time-dependent wave functions are \( \exp(-iEt+iS(x)) \) and \( \exp(-iEt-iS(x)) \); the wave packets constructed out of the wave functions of these two types indeed move right and left, respectively, as \( t \) increases.
Let us now consider the dynamics along the coordinate $y$, still using the time-independent Schrödinger equation in the allowed region of $x$. This is done for, say, right-moving system by writing, instead of eq.(1),
\[
\Psi(x, y) = \frac{1}{\sqrt{p_x(x)}} \tilde{\Psi}(x, y)e^{iS(x)}
\]
where $p_x = \partial S/\partial x$. To the first order in $\hbar$ one obtains that the time-independent Schrödinger equation reduces to
\[
i\frac{\partial \tilde{\Psi}}{\partial x} \frac{\partial S}{\partial x} = \hat{H}_y \tilde{\Psi}
\]
This can be cast into the form of time-dependent Schrödinger equation by changing variables from $x$ to $\tau$ related by $x = x_c(\tau)$, where $x_c(\tau)$ is the solution of the classical equation of motion for $x$ in “time” $\tau$, which has energy $E$ and obeys
\[
\frac{\partial S}{\partial x}(x = x_c) = \frac{\partial x_c}{\partial \tau}
\]
After this change of variables, $\tilde{\Psi}$ becomes a function of $y$ and $\tau$ and obeys the following equation,
\[
i\frac{\partial \tilde{\Psi}(y; \tau)}{\partial \tau} = \hat{H}_y(\hat{y}, \hat{p}_y; \tau)\tilde{\Psi}(y; \tau)
\]
where the explicit dependence of $\hat{H}_y$ on $\tau$ comes from $x_c(\tau)$. We see that there have emerged intrinsic time $\tau$ which parameterizes the classical trajectory $x_c(\tau)$ and also the $y$-dependent part of the wave function. We note again that in quantum mechanics, the arrow of intrinsic time, which is set by the sign convention in eq.(3), is determined by the arrow of extrinsic time $t$.

Note also that one is free to choose any representation for operators $\hat{y}$ and $\hat{p}_y$ and write, instead of eq.(4),
\[
i\frac{\partial |\tilde{\Psi}\rangle}{\partial \tau} = \hat{H}_y(\tau)|\tilde{\Psi}\rangle
\]
To solve this equation, one may find convenient to switch to the Heisenberg representation, as usual.
We now turn to the discussion of the region of $x$ where the classical motion is forbidden and the system has to tunnel. To simplify formulas, we set $E = 0$ in what follows. If the system tunnels from left to right, the dominant semiclassical wave function is

$$\Psi \propto e^{-S(x)}$$

where $S(x) = \int^x dx' \sqrt{2V_0(x')} \, \, \, \text{and obeys the following equation,}$

$$-\frac{1}{2} \left( \frac{\partial S}{\partial x} \right)^2 + V_0(x) = 0$$

This equation may be formally considered as the classical Hamilton–Jacobi equation in Euclidean (“imaginary”) time. The zero energy classical trajectory $x_c(\tau)$ in Euclidean time $\tau$ obeys

$$\frac{d^2 x_c}{d\tau^2} = + \frac{\partial V_0}{\partial x}(x = x_c)$$

and hence

$$\frac{dx_c}{d\tau} = \frac{\partial S}{\partial x}(x = x_c)$$

Then $S(x)$ can be calculated as the value of the Euclidean action along this trajectory.

To find the equation governing the dynamics along $y$-direction in the classically forbidden region of $x$, we again write

$$\Psi(x, y) = \frac{1}{\sqrt{p_x}} \tilde{\Psi}(x, y)e^{-S(x)}$$

and obtain, changing variables from $x$ to $\tau$, $x = x_c(\tau)$, that $\tilde{\Psi}$ obeys the time-dependent Schrödinger equation, now in Euclidean time,

$$\frac{\partial \tilde{\Psi}(y; \tau)}{\partial \tau} = -\hat{H}_y(\hat{\hat{y}}, \hat{\hat{p}_y}; \tau)\tilde{\Psi}(y; \tau)$$

(6)

The minus sign on the right hand side of this equation is crucial for the stability of the approximation we use. Indeed, the system described by eq.(6) tends to de-excite, rather than excite, as “time” $\tau$ increases, so that the part $\tilde{\Psi}$ of the wave function remains always subdominant as compared to the leading semiclassical exponential. The physics behind this property is quite clear: we consider tunneling at fixed energy, so the de-excitation of
fluctuations along $y$ means the transfer of energy to the tunneling subsystem, which makes tunneling (exponentially) more probable. Inversely, if fluctuations along $y$ get excited, the kinetic energy along $x$ decreases, and tunneling gets suppressed stronger.

3 Wave function of the Universe

To discuss specific aspects of quantum cosmology, let us consider the closed Friedmann–Robertson–Walker Universe with the scale factor $a$. Let us introduce the cosmological constant $\Lambda$, minimal scalar field $\phi(x)$ with a scalar potential $V(\phi)$ and also massless conformal scalar field. We are going to treat the dynamics of the scale factor in a semiclassical manner; in this respect $a$ is analogous to the variable $x$ of the previous section. The minimal scalar field (as well as gravitons) will be considered within perturbation theory, so each of the modes $\phi_k$ will be analogous to the variable $y$ of the previous section.

The basic equation in quantum cosmology is the Wheeler–De Witt equation, which in our case reads

$$\left[ -\frac{1}{2}\hat{p}_a^2 - \frac{1}{2}a^2 + \Lambda a^4 + \hat{H}_\phi \right] \Psi = -\epsilon \Psi \quad (7)$$

where we have set $3M_{Pl}^2/16\pi = 1$ and ignored the operator ordering problems which are irrelevant for our discussion. Here

$$\hat{H}_\phi = \int \frac{d^3x}{2\pi^2} \left[ \frac{1}{2a^2} \hat{p}_\phi^2 + \frac{a^2}{2} (\partial_t \hat{\phi})^2 + a^4 V(\hat{\phi}) \right]$$

is the term due to the minimal scalar field; at the classical level $\hat{H}_\phi$ is the energy of matter defined with respect to conformal time. The non-negative constant $\epsilon$ on the right hand side of eq.(7) is the contribution of the conformal scalar field; the only purpose of introducing the latter field is to allow for non-zero $\epsilon$. We do not consider gravitons in what follows, as they are similar to the quanta of the minimal scalar field $\phi$.

In the spirit of the Born–Oppenheimer approximation, let us first neglect the conformal energy of the field $\phi$, i.e., omit the term $\hat{H}_\phi$ in eq.(7). Then the Wheeler–De Witt equation
takes the form of the time-independent Schrödinger equation in quantum mechanics of one generalized coordinate $a$ with energy $\epsilon$ and potential

$$U(a) = \frac{1}{2}a^2 - \Lambda a^4$$

At $16\Lambda^2\epsilon < 1$, there are two classically allowed regions: at small $a$ ($0 < a^2 < [1 - \sqrt{1 - 16\Lambda^2\epsilon}] / 4\Lambda$) and at large $a$ ($\infty > a^2 > [1 + \sqrt{1 - 16\Lambda^2\epsilon}] / 4\Lambda$). At the classical level, the former region corresponds to an expanding and recollapsing Friedmann-like closed Universe, while the latter corresponds to the de Sitter-like behavior. As $\epsilon \to 0$, the first classically allowed region disappears, while the second becomes exactly de Sitter.

In between these two regions, classical evolution is impossible (if one neglects $\hat{H}_\phi$), and one has to consider classically forbidden “motion”. Let us discuss classically allowed and classically forbidden regions separately.

### 3.1 Classically allowed region: issue of arrow of time

To be specific, let us consider classically allowed de Sitter-like region where the scale factor $a$ is large. In the leading order, there are again two types of semiclassical wave functions,

$$\Psi \propto e^{-iS(a)}$$  \hspace{1cm} (8)

and

$$\Psi \propto e^{+iS(a)}$$  \hspace{1cm} (9)

where

$$S(a) = \int^a da' \sqrt{2(\epsilon - U(a'))}$$

Classically, the momentum is related to the derivative of the conformal factor with respect to conformal time,

$$\frac{da}{d\eta} = -p_a$$
For the two semiclassical wave functions one has

\[ \hat{p}_a \Psi = \left( \mp \frac{\partial S}{\partial a} \right) \Psi \]

where upper and lower signs refer to eq.(8) and eq.(9), respectively. Hence, one is tempted to interpret the wave functions (8) and (9) as describing expanding and contracting Universes, respectively. Indeed, the Hartle–Hawking wave function \[8\] that in the allowed region is a superposition

\[ \Psi_{HH} \propto e^{-iS(a)} + e^{+iS(a)} \]

is often interpreted as describing a collapsing and re-expanding de Sitter-like Universe. Similar interpretation is often given to the Linde wave function \[9\]. On the other hand, the tunneling wave functions \[10, 11, 12\] which contain one wave in the allowed region,

\[ \Psi_{tun} \propto e^{-iS(a)} \]

are often assumed to be the only ones that correspond to an expanding, but not contracting, Universe; this is, at least partially, the basis for the tunneling interpretation.

An important difference with conventional quantum mechanics is, however, the absence of extrinsic time in quantum cosmology. Hence, the arrow of intrinsic time has yet to be determined. In other words, there is no \textit{a priori} reason to interpret the wave functions (8) and (9) as describing expanding and contracting Universes, respectively. The sign of the semiclassical exponent does not by itself determine the arrow of time.

Were the scale factor the only dynamical variable, it would be impossible to decide whether, say, the wave function (8) corresponds to expanding or contracting Universe. If the matter fields (and/or gravitons) are included, this should be possible. Before discussing this point, let us derive the equation for the wave function describing matter \[13, 14, 15\], again in the spirit of the Born–Oppenheimer approximation.

Let us extend the wave functions (8) and (9) to contain the dependence on the matter variables,

\[ |\Psi(a)\rangle = \frac{1}{\sqrt{\hat{p}_a}} e^{\mp iS(a)} |\tilde{\Psi}(a)\rangle \]

(11)
where at given \( a \) both \( |\Psi(a)\rangle \) and \( |\tilde{\Psi}(a)\rangle \) belong to the Hilbert space in which \( \hat{\phi}(x) \) and \( \hat{p}_\phi(x) \) act. As an example, one may (but does not have to) choose the generalized coordinate representation; then \( |\Psi(a)\rangle \) becomes a function \( \Psi(\{\phi_k\}; a) \) of the Fourier components of \( \phi \). In the first order in \( \hbar \) one obtains from eq.(7)

\[
\pm i \sqrt{\epsilon - U(a)} \frac{\partial |\tilde{\Psi}(a)\rangle}{\partial a} = \hat{H}_\phi |\tilde{\Psi}(a)\rangle
\]

(12)

in complete analogy to eq.(2).

The arrow of time is determined now by where (at what \( a \)) and which initial conditions are imposed on \( |\tilde{\Psi}(a)\rangle \). As an example, let us assume that the initial conditions for the evolution in real intrinsic time are imposed at small \( a \) (at the turning point \( a^2 = [1 + \sqrt{1 - 16\Lambda^2\epsilon}]/4\Lambda \)), and that at that point \( |\tilde{\Psi}\rangle \) describes smooth distribution of the scalar field. This type of initial data are characteristic, in particular, to the Hartle–Hawking no-boundary wave function. As \( a \) increases, the system will become more and more disordered, independently of the sign in eq.(12). With thermodynamical arrow of time, both wave functions (11) will describe expanding Universe.

If, with these initial conditions, one changes variables from \( a \) to \( \eta \) using

\[
\frac{\partial a}{\partial \eta} = \sqrt{\epsilon - U(a)}
\]

then \( \eta \) increases with \( a \), so that \( \eta \) is the conformal intrinsic time, independently of the choice of sign in eq.(11). In the case of positive sign, eq.(12) becomes the conventional Schrödinger equation for quantized matter in the expanding Universe,

\[
i \frac{\partial |\tilde{\Psi}\rangle}{\partial \eta} = \hat{H}_\phi(\eta) |\tilde{\Psi}\rangle
\]

(13)

where the matter Hamiltonian depends on \( \eta \) through \( a(\eta) \). On the other hand, in the case of negative sign one obtains “wrong sign” Schrödinger equation,

\[
i \frac{\partial |\tilde{\Psi}\rangle}{\partial \eta} = -\hat{H}_\phi(\eta) |\tilde{\Psi}\rangle
\]

This little problem is easily cured by considering, instead of \( |\tilde{\Psi}\rangle \), its \( T \)-conjugate, \( |\tilde{\Psi}^{(T)}\rangle \); if the generalized coordinate representation is chosen for \( |\tilde{\Psi}\rangle \), then \( T \)-conjugation is merely
complex conjugation, $\tilde{\Psi}^{(T)}(\phi_k; \eta) = \tilde{\Psi}^*(\phi_k; \eta)$. The $T$-conjugate wave function obeys conventional Schrödinger equation, but with $CP$-transformed Hamiltonian. Hence, the interpretation of both wave functions (11) as describing the expanding Universe is self-consistent; the only peculiarity is that the wave function $e^{+iS}|\tilde{\Psi}\rangle$ corresponds to the Universe in which matter is $CP$-conjugate. In particular, we argue that both components of the Hartle–Hawking wave function (10) correspond to expanding Universes.

In more generic cases (in particular, when the matter degrees of freedom cannot be treated perturbatively, see, e.g., Refs. [16, 17] and references therein), the situation may be much more complicated. Still, the arrow of time is generally expected to be one of the key issues in the interpretation of the wave function of the Universe.

### 3.2 Classically forbidden region: issue of stability of the Born-Oppenheimer approximation

We now consider the region of the scale factor that is classically forbidden in the absence of $\hat{H}_\phi$, i.e., $a_1 < a < a_2$, where

$$a_{1,2}^2 = \frac{1 \mp \sqrt{1 - 16\Lambda^2\epsilon}}{4\Lambda}$$

If $\hat{H}_\phi$ is switched off, there are two semiclassical solutions to the Wheeler–De Witt equation,

$$\Psi \propto e^{-S(a)} \quad (14)$$

and

$$\Psi \propto e^{+S(a)} \quad (15)$$

where

$$S(a) = \int_{a_1}^{a} da' \sqrt{2(U(a') - \epsilon)}$$

is defined in such a way that it always increases at large $a$. The wave function (14) decays as $a$ increases, so it may be interpreted as describing tunneling from classically allowed
Friedmann region to de Sitter-like one. It is convenient to introduce Euclidean conformal time parameter $\tau$ and consider Euclidean trajectory $a_c(\tau)$ obeying
\[
\frac{da_c}{d\tau} = \frac{\partial S}{\partial a}(a = a_c)
\]
At $\epsilon = 0$ the Euclidean four-geometry corresponding to this trajectory is a four-sphere, the standard de Sitter instanton.

Let us now turn on the scalar field Hamiltonian $\hat{H}_\phi$, and try to apply the procedure of the Born–Oppenheimer type. We write, instead of eq.(14), for the wave function decaying at large $a$,
\[
|\Psi(a)\rangle = \frac{1}{\sqrt{P_a}} e^{-S(a)} |\bar{\Psi}(a)\rangle
\]
and obtain in the first order in $\hbar$ that $|\bar{\Psi}(\tau)\rangle$ obeys the “wrong sign” Euclidean Schrödinger equation \[12\]
\[
\frac{\partial |\bar{\Psi}(\tau)\rangle}{\partial \tau} = +\hat{H}_\phi(\tau) |\bar{\Psi}(\tau)\rangle
\] (16)
where the change of variables from $a$ to $\tau$ with $a = a_c(\tau)$ has been performed. The sign on the right hand side of eq.(16) is opposite to that appearing in the usual quantum mechanics, eq.(6), and is directly related to the sign of $\hat{p}_a^2$-term in the Wheeler–De Witt equation (7).

The “wrong” sign in eq.(16) implies that the approximation we use is in fact unstable, if generic “initial” conditions are imposed at small $a$, say, at $a = a_1$. Note that imposing initial conditions in this way is natural if one interprets the wave function decaying at large $a$ as describing tunneling from small to large $a$. The formal reason for the instability of the approximation is that the degrees of freedom of the scalar field get excited as $a$ increases in the forbidden region. The rate at which this excitation occurs is generically high \[12\], and the approximation breaks down well before $a$ gets close to the second turning point $a_2$.

In the path integral framework, breaking of the Born–Oppenheimer-type approximation for the wave function decaying at large $a$ is also manifest \[18\]. This wave function corresponds to the Euclidean path integral with “wrong” sign of the action,
\[
\int Dg \, D\phi \, e^{+S[g,\phi]}
\]
The instanton action then gives the factor $e^{-S_{\text{inst}}}$, but the integral over $\phi$ (and gravitons) diverges.

The physics behind this instability is that tunneling of a Universe filled with matter is exponentially more probable as compared to empty Universe. Hence, the matter degrees of freedom tend to get excited in the forbidden region, thus making tunneling easier. Note that this property is peculiar to quantum cosmology: in quantum mechanics the situation is opposite, as we discussed in the previous section.

There are exceptional cases in which matter degrees of freedom do not get excited in the forbidden region, e.g., because of symmetry. In our model this would be the case if $\epsilon = 0$ and the scalar field $\phi$ was in the de Sitter-invariant state, cf. Ref. [19]. Such cases do not seem generic, however.

Breaking of the Born–Oppenheimer approximation does not necessarily mean that tunneling-like transitions from small $a$ to large $a$ with generic state of matter at small $a$ do not make sense. Rather, it is the semiclassical expansion that does not work in this case, so the state of the Universe after the transition may be quite unusual. Presently, neither the properties of this state, nor the properties of the wave function in the forbidden region are understood (except for special cases mentioned above).

The situation is different for the wave functions increasing towards large $a$, eq. (15). In that case the matter wave function obeys the usual Euclidean Schrödinger equation, $\partial |\tilde{\Psi}(\tau)\rangle / \partial \tau = -\hat{H}_\phi(\tau) |\tilde{\Psi}(\tau)\rangle$, where $\tau$ is still assumed to increase with $a$. Hence, it is possible to impose fairly general initial conditions at small $a$, and the approximation will not break down. In particular, the Hartle–Hawking wave function is a legitimate approximate solution to the Wheeler–De Witt equation in the forbidden region. This is in accord with the path-integral treatment: the increasing wave function (15) corresponds to the standard sign of the Euclidean action in the path integral.

The non-semiclassical behavior of the tunneling wave functions, signalled by the instability of the Born–Oppenheimer-type approximation, is a special, and potentially interesting,
feature of quantum cosmology. It is a challenging technical problem to develop techniques adequate to this situation. It is not excluded also that the properties of the tunneling wave functions are rich and complex, and that understanding them may shed light on the beginning of our Universe.

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