On the metrizability of the affine connectivity space and the unified theory of fundamental interactions

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Abstract. We propose the background connectivity interpret as the empirical connectivity, received as the particular case of the transition operator, which had been by induced by the collision integral. Approximating the transition operator by the differential operator we shall demand minimum variance of the given operator from the transition operator for the definition of the metric tensor. As a result we receive with the help of the variation formalism the differential equation of the regression, solving of which can find the metric tensor. By the help of the given tensor we receive the Christoffel symbols. Using the metric tensor and the Christoffel symbols can construct both the Einstein gravitation theory and the Chernikov gravitation theory with the two connectivity’s. Constructing the empirical connectivity theory we shall use the generalized functions, that implies the change of the space-time topology.

1. Introduction
We use the postulate, that in the real world it can determine only the most plausible physical laws. We shall not confuse the real world with the mathematical one, in which will be used sets of continuum cardinality. As a result we state the moderate problem: to construct the most plausible physical theory, in which the divergences will be lacking. Naturally, that in this case it needs to use the principle of least action and the theory of generalized functions. Besides, we shall use the privilege principle: the description must be the most simple.

2. “Ghosts”
For the construction of physical theories we propose to use Fadeev-Popov “ghosts” (unobservable zero-spin fermions) introduced in the quantum chromodynamics for the renormalizability [1]. We shall rely on the Landau theory of the Fermi liquid in which it is offered to replace the consideration of strong interacting original particles on the consideration of the almost ideal gas of repeat quasi-particles. We shall consider “ghosts” as primary particles of the Universe Fermi liquid and all Universe observable particles as repeat quasi-particles of the given Fermi liquid. If observable particles are absent in the Universe, then we shall consider that the Fermi liquid entropy is minimal, so as to use the second principle of thermodynamics. The given symmetrical state of the Universe matter is unlikely can consider a stable state if only on account of fluctuations. So in the first phase as a result of fluctuations must appeared particles in the excited state (quarks and antiquarks, having the additional degrees of freedom), among “ghosts”, existing in the ground state. As a result of an increase of freedom degrees (the
appearance of quarks and antiquarks) the Universe temperature must be decreased so, that must goes the spontaneous breaking of symmetry with the freezing-out of freedom degrees for quarks and antiquarks (the appearance of neutrinos and antineutrinos). At present quarks and antiquarks attend only in hadrons and black holes (inside quark bags). As is known, the whole of observable fundamental elementary particles have the spin (we can model them as vortices in the ghost Fermi liquid) and that the spin takes only the integer value or the semi-integral value of $h/(2\pi)$ corroborates the “ghosts” existence in the indirect fashion [2]. As a result the manifold must not be considered the connected space (closed trajectories of observable particles may not be pulling together in a point). We connect the cutoff parameter of quantum chromodynamics with the Fermi energy of University background fermions. We obtain the natural physical interpretation of the cutoff parameter and also we obtain the physical justification of the lattice model.

In our opinion further it must take place Cooper pairing of noncharged leptons with a formation of a Bose liquid, which must cause to the density reduction of primary leptons and to the appearance of photons, fundamental massive vector bosons, charged leptons and hadrons [2]. In the Universe standard model the given process, having an explosive behavior (Big Bang) was accompanied an immense energy emission [2]. As a result of a Bose condensation it was showed the asymmetry of noncharged leptons relative to a polarization. Sterile neutrinos and sterile antineutrinos (nonobservable particles in processes of beta decay) turned out to be bigger than observable particles with the “regular” polarization. With this moment we can consider that the Universe matter is at the modern stage and “ghosts” will play the distinct role only in a description of hadrons. The main role goes to neutrinos and antineutrinos. We shall consider these particles as background particles. The physical properties of sterile neutrinos and sterile antineutrinos must define geometrical properties of the space-time just therefore. Taking into consideration the Ginsburg-Landau theory of superconductority, the scalar field in the standard model of electroweak interactions ought to interpret as the order parameter of neutrino Cooper pairs. Masses of massive vector bosons must be an outcome of their interaction with neutrino Cooper pairs, at the same time as photons (which’s do not interact with this pairs) will stay massless particles.

3. **Neutrinos**

Because we consider the space-time as the not connected space all parameters, characterizing of fundamental elementary particles are quantized [2]. Considering “ghosts” as original particles of an Universe matter, which’s are characterized by the SU(3) symmetry, the transition to leptons (neutrinos and antineutrinos), which’s are characterized by the SU(2) symmetry, may be regard as the phase transition and may be interpreted as the process, which is connected with the spontaneous breaking of symmetry (we have the freezing-out of freedom degrees).

Because we assume that the energy $E$ of any fundamental interactions must depend on a number of all particles and quasi-particles participating in the interactions, then the it dependence on space coordinates is defined by means of an average number of bosons, which’s exchanged two particles. As a result the energy $E$ (the collision integral) has the form ($n$ is a number of bosons):

$$E = -2 \int \left\{ \sum_{(\sigma_1, \sigma_2)} \left[ \frac{\epsilon (\sigma_1, \sigma_2) \epsilon_1 \epsilon_2 \sum_{n=0}^{N} ne^{-2n\sigma_1}e^{-2n\sigma_2}e^{-2n\sigma_1}/T}{T \sum_{n=0}^{N} e^{-2n\sigma_1}/T} \right] \right\} dV_1 dV_2$$

where $\epsilon (\sigma_1, \sigma_2) = \pm 1$, $\epsilon_1$ is the energy density of first body particles; $\epsilon_2$ is the energy density of second body particles; $\epsilon$ is the energy density of dark matter particles, $T$ is theirs temperature;
\( \sigma_1 \) and \( \sigma_2 \) are cross-sections with the emission or the absorption of boson (we use the energy system of units: \( h/(2\pi) = c = 1 \), where \( h \) is the Planck constant and \( c \) is the velocity of light). By this the charge form of the fundamental particle is defined by the help of the given boson form and as well as by the help of the rate of its emission. The rate of its emission defines the “coat” quantity of virtual particles, surrounding the charge, which can become a unobservable one at large distances. Because of it the magnetic charge (the monopole) is the unobservable charge [3].

The appearance of photons (playing the role of standards in the relativity theory) caused to the domination of electromagnetic interactions in experimental data and to the division of the all Universe matter into two subsystems (slow and rapid). The matter of the slow subsystem (a thermostat) does not participate in electromagnetic interactions and play the catalyst role of stochastic processes, which it may observes by the matter of the rapid subsystem. As a result the particles of the rapid subsystem play the role of Brownian particles. They allow studying the properties of slow subsystem particles.

We extend Dirac hypothesis on the presence of sea electrons in the Universe for the provision of the radiation stability of electrons with the positive energy relative to the photon emission. That is to say it is hypothesized, that observable fermions are stable, because they are found in the excited state and all levels of particles in the ground (degenerate) level are engaged. By this the particle charge is defined by the ratio of the frequency of emitted particular virtual bosons to the frequency of the collision with Universe background particles. We shall follow the example of Pauli, who preferred to work with closed systems. Conservation laws take place in that kind of systems (in contrast to open systems). He suggested to introduce an unobservable (as then it seemed to him) particle, called as a neutrino subsequently (Fermi), for a description of a beta decay. Precisely neutrinos constitute a particle background, being a catalyst of stochastic processes and characterizing by two parameters (the density \( \rho \) and the temperature \( T \)). In our opinion the Universe neutrino background exists in two degenerate states: Fermi liquid (so named “the dark energy”), Bose liquid (so named “the dark matter”). The density of its Fermi liquid state is defined by the Fermi energy \( E_F \), which must exceed its temperature \( T \) to a great extent. As a result the statistical physics acquires a high weight for us and the Boltzmann equation will have the fundamental importance.

4. The transition operator

We propose in the gravitation theory the background connectivity (which for the first time had been introduced by A.N. Chernikov [4]) interpret as the empirical connectivity, received as the particular case of the transition operator, which had been by induced by the collision integral. Approximating the transition operator by the differential operator and demanding minimum variance of the given operator from the transition operator (employing the method of least squares for the definition of the metric tensor). As a result we receive with the help of the variation formalism the differential equation of the regression, solving of which can find the metric tensor. By the help of the given tensor we receive the Christoffel symbols, which are interpreting as the components of the theoretical connectivity. Using the metric tensor and the Christoffel symbols can construct both the Einstein gravitation theory and the Chernikov gravitation theory with the two connectivity’s and the one metric tensor [4]. Constructing the empirical connectivity in our gravitation theory we shall use the generalized functions. It implies the change of the space-time topology.

We study the system, for the description of which will used function \( \Phi, \Psi, X, ... \in L \) where \( L \) is the function complex linear space with semi-scalar product. The complex-valued function \( \langle \Psi, \Phi \rangle \) been semi-scalar product must come up to the conditions:

1) \( \langle \Psi, \Phi \rangle = \langle \Phi, \Psi \rangle^* \), 2) \( \langle \lambda \Psi + \nu X, \Phi \rangle = \lambda \langle \Psi, \Phi \rangle + \nu \langle X, \Phi \rangle \), 3) \( \langle \Psi, \Psi \rangle \geq 0 \),
where * is the symbol of the complex conjugation, λ and ν are complex numbers.

In the scattering theory the initial state of particles is described by the help of the vector |Ψ_in⟩, referring to the infinity remote past and the final state is described by the help of the vector |Ψ_out⟩, referring to the infinity remote future. In the both states we neglect the interaction between particles and define S-matrix by the relationship:

\[ |Ψ_{out}⟩ = S |Ψ_{in}⟩, \]

that is to say the process of the collision is studied as the “black box”, which described by the quantity S, transforming in-state in out-state of the system. Because only transitions between various states represent the interest, then from the quantity S is subtracted the unit operator I. As a result we define the transition operator:

\[ T = S - I \]

or

\[ T |Ψ_{in}⟩ = |Ψ_{out}⟩ - |Ψ_{in}⟩. \]

We shall rely on the approach developed in works of A.M. Perelomov [5], which introduce the definition of generalized coherent states. Given states arise by the action of the representation operator of a transformation group on a fixed vector in the space of this representation. Further, we introduce the density matrix, considering that its off-diagonal elements characterize the coherence of system states. If the parameter space is not compact, then for sufficiently large changes of parameters we must take account the finiteness of the information propagation velocity. By this the state coherence must be masked. It causes to the chaotization and the disturbance of quantum patterns on the macroscopic level. It causes introduce the change in the Perelomov definition, regarding that it takes place only in the neighborhood of the group unit. As a result we must generalize the given definition to the local groups and the local loops. Thus we shall understand the states, arising by the infinitesimal operator action of a local loop transformation representation on a fixed vector in the space of this representation as generalized coherent states.

We introduce the equivalence relation for the set \{Ψ(x)\} by infinitesimal transformations:

\[ Ψ → Ψ + δΨ = Ψ + δT(Ψ), \]

where \( δT = δx^iT_i \) is the transition infinitesimal operator \((i, j, k, ..., = 1, 2, ..., n)\). In this case there is no the sense to separate functions, describing in-states and out-states, which’s can prove to be coherent functions in the causally link region. The introduction of a macroscopic observer causes to seek the representation of transition operators by differential operators. As a result we must use the differentiable manifold \( M_n \), points \( x \) of which will have coordinates \( x^i \).

As is known, according to the Feynman’s hypothesis the probability amplitude of the system transition from the state \( Ψ(x) \) in the state \( Ψ'(x') \) equal to the following integral

\[ K(Ψ, Ψ') = \int_{Ω(Ψ, Ψ')} exp(iA)DΨ \]

\[ = \lim_{N→∞} I_N \int dΨ_1...∫ dΨ_k...∫ dΨ_{N-1} exp \left( i \sum_{k=1}^{N-1} A(Ψ(x_k)) ΔV_k \right) \]

\( (i^2 = -1 \) the constant \( I_N \) is chosen so that the limit was existed). Therefore the functions \( Ψ(x) \), received from the requirement of the minimality of the action A, are also the maximum
likelihood ones only. In this approach the Lagrangian $\Lambda$ plays the more fundamental role than differential equations which are received from it.

For this across a point $x \in \mathcal{M}$ we run smooth curves, by a help of which’s we define a set vector fields $\{\delta\xi(x)\}$, and then we define deviations of fields $g(x)$ in the form:

$$\delta_0 \Psi = \delta X(\Psi) = \delta T(\Psi) - \delta \xi(\Psi). \quad (7)$$

We shall require that these deviations were minimal ones even if in “the mean”. Let the square of the semi-norm $|X(\Psi)|$ has the form as the following integral

$$A = \int_{\Omega_n} \kappa \bar{X}(\Psi) \rho X(\Psi) d^nV \quad (8)$$

(A is an action, $\Lambda$ is a Lagrangian, $\kappa$ is a constant). Here and further $\rho = \rho(x)$ is the density matrix ($\text{tr} \rho = 1$, $\rho^* = \rho$ the top index “+” is the symbol of the Hermitian conjugation) and the bar means the Dirac conjugation, which is the superposition of Hermitian conjugation and the spatial inversion of the space-time $\mathcal{M}_4$. The given integral is the analog of the field $\Psi(x)$ variance in the region $\Omega_n$ under consideration. As is known, we can obtain equations of fields $\Psi(x)$ which’s are differential regression equations, requiring the action minimum by the variation on fields $\Psi(x)$, corresponding physical states. Solutions $\Psi(x)$ (and even one solution) of equations, which are being produced by the requirement of the minimality of the integral (8) can be used for the construction of the all set of functions $\{\Psi(x)\}$ (generated by the transition operator).

We consider the variation of the action $A$ with respect to unrestricted infinitesimal substitutions:

$$x \to x, \quad \Gamma_k \to \Gamma_k + \delta_0 \Gamma_k, \quad \partial_i \Gamma_k \to \partial_i \Gamma_k + \delta_0 \partial_i \Gamma_k, \quad (9)$$

where

$$\delta_0 \partial_i \Gamma_k = \partial_i \delta_0 \Gamma_k, \quad (10)$$

by this increments of field functions $\delta_0 \Gamma_k(x)$ must are reduced to zero at a border $\Omega_{n-1}$ a region $\Omega_n$.

As a result the variation of the action writes down in the form:

$$\delta_0 A = \int_{\Omega_n} \left( \frac{\partial \Lambda}{\partial \Gamma_k} \delta_0 \Gamma_k + \frac{\partial \Lambda}{\partial (\partial_i \Gamma_k)} \partial_i \delta_0 \Gamma_k \right) d^nV$$

$$= \int_{\Omega_n} \left( \frac{\partial \Lambda}{\partial \Gamma_k} - \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i \Gamma_k)} \right) \right) \delta_0 \Gamma_k d^nV + \int_{\Omega_n} \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i \Gamma_k)} \delta_0 \Gamma_k \right) d^nV. \quad (11)$$

We transform the last integral in the formula (11) in the integral on a surface after the Green theorem, taking its equal to zero. Thus for unrestricted increments of field functions $\delta_0 \Gamma_k(x)$ in the region $\Omega_n$ the action variation $\delta_0 A$ can make vanish only by the condition:

$$\frac{\partial \Lambda}{\partial \Gamma_k} - \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i \Gamma_k)} \right) = 0. \quad (12)$$

The theoretical connectivity $\Gamma_k$ is the solution of the given differential equation, which $\Gamma_k$ defines the theoretical space of the affine connectivity.
5. Riemannian space

Because of the formula (4) the transition operator $T_i$ is an empirical one, a particular case of which is an affine connectivity $\Gamma_i$. The given connectivity defines the space of the affine connectivity. The given space is not a quite comfortable one for the description of physical models. The Riemannian space is a more comfortable one for which we must define a metrical tensor $g(x)$, using the Green theorem.

For this across a point $x \in M_n$ we run smooth curves, by a help of which’s we define a set vector fields $\{\delta \xi(x)\}$, and then we define deviations of fields $g(x)$ in the form:

$$\delta_o \Psi = \delta X(g) = \delta \Gamma(g) - \delta \xi(g).$$  

(13)

We shall require that these deviations were minimal ones even if in “the mean”. Let the square of the semi-norm $|X(Y)|$ has the form as the following integral

$$A = \int_{\Omega_n} \kappa X(g) \rho X(g) d_n V.$$  

(14)

We consider the variation of the action $A$ with respect to unrestricted infinitesimal substitutions:

$$x \to x, \quad g \to g + \delta_o g, \quad \partial_i g \to \partial_i g + \delta_o \partial_i g,$$

(15)

where

$$\delta_o \partial_i g = \partial_i \delta_o g,$$

(16)

by this increments of field functions $\delta_o g(x)$ must are reduced to zero at a border $\Omega_{n-1}$ a region $\Omega_n$.

As a result the variation of the action writes down in the form:

$$\delta_o A = \int_{\Omega_n} \left( \frac{\partial \Lambda}{\partial g} \delta_o g + \frac{\partial \Lambda}{\partial (\partial_i g)} \delta_o \partial_i g \right) d_n V =$$

$$= \int_{\Omega_n} \left( \frac{\partial \Lambda}{\partial g} - \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i g)} \right) \right) \delta_o g d_n V + \int_{\Omega_n} \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i g)} \delta_o g \right) d_n V$$

(17)

We transform the last integral in the formula (17) in the integral on a surface after the Green theorem, taking its equal to zero. Thus for unrestricted increments of field functions $\delta_o g(x)$ in the region $\Omega_n$ the action variation $\delta_o A$ can make vanish only by the condition:

$$\frac{\partial \Lambda}{\partial g} - \partial_i \left( \frac{\partial \Lambda}{\partial (\partial_i g)} \right) = 0.$$  

(18)

The metric tensor $g_{ij}$ is the solution of the given differential equation. The given metric tensor defines the theoretical connectivity of the Riemannian space $\Gamma_{ij}^{\alpha}$ (Christoffel symbols). As is known, Christoffel symbols are defined in the form:

$$\Gamma_{ij}^{\alpha} = g^{\alpha k} \frac{1}{2} (\partial_i g_{jk} + \partial_j g_{ik} - \partial_k g_{ij}).$$  

(19)

We consider the formula

$$T_{ij}^{k} = \Gamma_{ij}^{k} - \Gamma_{ij}^{\alpha k},$$  

(20)

where $T_{ij}^{k}$ are the tensor components of the affine deformation and must describes gravitational waves. The given tensor defines the representation of the translation group or of the translation...
loop. The structure of this object is not a trivial one for the describing of black holes and hadrons. Tensor of the affine deformation defines the gravitational field, quanta of which are gravitons. Naturally, that the structure of the affine deformation tensor is defined by the empirical connectivity. The empirical connectivity of the most of the general form must describe the matter of the all University (in the form of coherent mixtures [5]), but by the description of the local region we can use it in the simplified form (in the form of the incoherent mixture). In hadrons and black holes we have the coherent mixture of quarks which is characterized by the group of $SU(3) \times U(1)$. In atomic nuclei and neutron stars we have the coherent mixture of hadrons which is characterized by the group of $SU(3) \times SU(2) \times U(1)$. The dark matter of external this objects is the coherent mixture of neutrinos which is characterized by the group of $SU(2) \times U(1)$. Because the dark matter exists in the ultra cold state ($E_F \gg T$), then we can consider that the visible matter exists in the incoherent state.

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