Bistable dissipative soliton in cubic-quintic nonlinear medium with multiphoton absorption and gain dispersion

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We studied the propagation of optical pulse in a dissipative cubic-quintic nonlinear medium with multiphoton absorption and gain dispersion. Variational method in conjunction with Rayleigh dissipative function is used to analytically solve the governing complex cubic-quintic Ginzburg–Landau equation. Three cases of multiphoton absorption are studied: (i) two-photon absorption (TPA), (ii) three-photon absorption (3PA) and (iii) both TPA and 3PA. For all three cases, the pulse intensity decays and width broadens. The greater the gain dispersion, the wider the pulse. Increasing quintic nonlinearity marginally affects pulse amplitude but significantly reduces pulse broadening. TPA and 3PA, which are functions of positive imaginary parts of, respectively, third- and fifth-order susceptibility, individually cause pulse decay and pulse broadening. Thus, their combination only expedites pulse degradation. Conversely, negative imaginary part of fifth-order susceptibility may lead to an effect that impedes pulse degradation. Matching numerical results are obtained that validates the entire analytical outcome. A suitable gain can arrest both pulse broadening and decay. Such dissipative solitonic pulses, bistable owing to quintic nonlinearity, are found for all cases.

Keywords: two-photon absorption; three-photon absorption; cubic-quintic nonlinearity; gain dispersion; complex Ginzburg–Landau equation; bistable dissipative soliton

1. Introduction

Pulse dynamics through dispersive nonlinear media has substantial importance in optical communication system and in all-optical device fabrication.[1] The volume and quality of theoretical research on nonlinear pulse dynamics in optical fiber have attained a high level during last few decades.[2] Generally, in fiber materials, the third-order susceptibility $\chi^{(3)}$ gives rise to the most prominent nonlinear effects. Its real part results in focusing cubic/Kerr nonlinearity through cubic/Kerr nonlinear coefficient $n_2 = 3 \text{Re} (\chi^{(3)}) / 8 n_0$, where $n_0$ is the linear refractive index. Kerr nonlinearity in turn can suppress the group velocity-induced pulse broadening. A perfect counter balancing of the group velocity dispersion and the nonlinearity-induced self-phase modulation leads to the temporal soliton.[3,4] Imaginary part of $\chi^{(3)}$ gives rise to two-photon absorption (TPA) in the material through the TPA coefficient $\alpha_2 = 3 \omega \text{Im} (\chi^{(3)}) / 2 n_0^2 c^2 \epsilon_0$, where $\omega$ is the frequency and $\epsilon_0$ is the vacuum permittivity.[5] TPA technology has applications in microscopy, imaging, lithography, and even in data storage. TPA can be used to eliminate most of the background linked with the image. It can also cause

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damage of the optical materials. Since it constitutes a nonlinear loss mechanism, it can limit the efficiency of optical switches. In case of fiber optics, reshaping and broadening of primary soliton take place under the effect of TPA. In the presence of TPA, an input pulse can be simultaneously amplified and compressed even though the compression factor is small. The amplified pulse splits into several chirped solitons, and their number depends on the gain and length of the fiber amplifier.[6] Effect of TPA has been investigated in breaking up of higher order solitons into two or more primary solitons.[7] However, in a fiber optic system, it introduces significant loss.[8] Thus, TPA is considered to have a detrimental effect on the pulse propagation in optical fiber. In some applications, e.g. switching, high nonlinearity is required to reduce the operating power. With the advent of such highly nonlinear fiber material, fifth-order nonlinear phenomena become relevant. In some fiber material, the fifth-order susceptibility $\chi(5)$ is traceable even at moderate laser intensity; please see [9] and the references therein. For highly intense ultra-short pulses, the fifth-order effects are considerable. The quintic nonlinear coefficient arises due to the real part of $\chi(5)$ through the relation $n_4 = 5Re(\chi^{(3)})/16n_0$. Although, it is usually very tiny in comparison to the cubic one, quintic nonlinearity can significantly modify the pulse dynamics. A combination of self-focusing Kerr nonlinearity and self-defocusing quintic nonlinearity promises intriguing dynamics of the optical signal.[10] This higher order nonlinearity helps to achieve stability of the pulse.[11] Besides, a positive-valued imaginary part of $\chi(5)$ gives rise to three-photon absorption (3PA) with the coefficient $\chi_3 = 5\text{Im}(\chi^{(3)})/2n_0^2c^3\varepsilon_0^3$. For example, in As$_2$S$_3$-based glass, 3PA coefficient has been found to be $2.0 \times 10^{-27}$ m$^3$ W$^{-2}$ at a wavelength of 1.55 µm.[12] For fibers made from such glass 3PA significantly modifies the soliton condition, propagation dynamics, and the factor of merit for all-optical switching. The influence of 3PA is notable for operating wavelengths corresponding to photon energies below half of the electronic band gap.[13] Since 3PA is proportional to the cube of the field intensity, it leads to a higher degree of spatial confinement. At communication wavelength 3PA has potential applications in wavelength shifting, pulse reshaping, and stabilization in short pulse fiber communication system.[14,15] Dissipative pulse propagation has been studied under the influence of TPA and gain dispersion.[16] But 3PA is not given proper attention in the context of fiber optic communication. Moreover, the situation with a negative-valued imaginary part of $\chi(5)$ has never been discussed in this context. Such negative imaginary part of $\chi(5)$ may give rise to an effect opposite to 3PA, which can be considered as three-photon emission (3PE) and provides a gain to the system. It is interesting to note the interplay between TPA-induced loss and 3PE-induced gain on pulse propagation. Also, the effect of quintic nonlinearity has been rarely studied in optical fiber.[17]

Thus, we investigate dissipative pulse propagation in a cubic-quintic nonlinear medium under the influence of TPA and 3PA. Three different cases of multiphoton absorption have been considered: (i) TPA, (ii) 3PA, and (iii) both TPA and 3PA. We add up gain dispersion in our model since it is prominent for an ultra-short pulse propagating through a gain medium. For femto-second and few pico-second pulses, the power spectrum is so wide that the effect of finite gain bandwidth becomes inevitable.[18] In the wide spectrum, gain varies with different spectral component, i.e. dispersion in gain occurs. This gain dispersion couples with the group velocity dispersion, thus significantly modifies the pulse dynamics and energy profile. It can be further noted that a suitable gain can help in minimization of the system loss, thus gives rise to dissipative soliton. We demonstrated the generation of such dissipative soliton for all three cases.
of multiphoton absorption. The solitons are found to be bistable, which is due to the coexistence of third- and fifth-order effects in the system.

For the mathematical modelling of the system, a complex cubic-quintic Ginzburg–Landau equation (CGLE5) is considered. The presence of loss/gain terms makes the system dissipative as well as nonintegrable one. With a proper choice of the parameters (basically vanishing the dissipative terms) one can transform the CGLE5 to a nonlinear Schrödinger equation (NLSE). Several analytical methods have been either originated or adopted to handle such NLSE and its modified version in order to achieve conservative soliton solution. For example, inverse scattering method [19], AKNS method [20,21], Bäcklund transformation technique [22], Hirota bilinear method [23,24], Lux pair method.[25] The early stage of theoretical investigation mostly used the aforesaid methods which are intended for achieving exact soliton solution. The lower order soliton solution is not only exact but also simple in nature. These methods also success-fully used to derive higher order soliton solutions, but mostly in form of complicated functions. Generally, the exact analytical methods have been found to be more success-ful for integrable systems. Now, with the advancement of theoretical research much more complex equations (e.g. some modified NLSE and CGLE5) are being proposed to capture the real effects of a system. Although otherwise strong, the exact methods either fail or become too tedious to solve such ‘realistic’ equations, which are mostly nonintegrable. In this context, a number of approximate analytical methods have been proposed. Variational method and moment method are of this category. For the current investigation we adopted variational approach in conjugation with Rayleigh’s dissipation function.[26] This is basically a combination of Lagrangian-based variational method [27] and perturbation method that has been successfully used to determine the pulse dynamics in a dissipative system.[28] Variation method leads to a solution which is very close to the exact solution. Moreover, it yields the evolution equation of the beam/pulse parameters that provides more insight into the dynamics of the system. The advantage of variational method is best felt for a system with no exact solution.

The layout of the study is as follows. Section 2 contains the mathematical model and analytical method for obtaining pulse evolution equations. Dissipative pulse propagation is discussed in Section 3, wherein Subsection 3.1 illustrates the analytical results. To validate the analytical results, a split-step Fourier method-based numerical scheme is employed in Subsection 3.2. Section 4 shows generation of dissipative soliton both analytically and numerically. The findings have been summarized, and potential applications are presented in the concluding Section 5.

2. Mathematical formulation

To describe the propagation of ultra-short pulse through a lossy dispersive fiber with cubic-quintic nonlinearity, gain dispersion, TPA and 3PA, we consider the following CGLE5:

\[
\begin{align*}
    i \frac{\partial E}{\partial z} + \frac{1}{2} \frac{\partial^2 E}{\partial t^2} + |E|^2 E + \gamma |E|^4 E = 
    \frac{i}{2} g_0 E + \frac{id}{2} \frac{\partial^2 E}{\partial t^2} - 
    \frac{i}{2} x E - i K |E|^2 E - iv |E|^4 E,
\end{align*}
\]

where \( E \) is the normalized field distribution, \( z \) is the normalized distance of propagation, \( t \) is retarded time. The first term in the left-hand side of Equation (1) represents the evolution of the pulse envelope with propagation. The second term is the GVD term. The third and fourth terms arise due to cubic and quintic nonlinearities,
respectively. The CGLE5 has been so normalized that coefficient of cubic nonlinearity becomes unity. \( \gamma \) is quintic nonlinearity coefficient normalized with respect to the cubic one. If the pulse width is larger than the intraband relaxation time, the gain spectrum \( g(\omega) \) can be expanded in Taylor series about the carrier frequency \( \omega_0 \). We consider up to second-order derivative terms in the expansion and an operating frequency equal to the carrier frequency. This leads to the first two terms in right-hand side of Equation (1). The first term in the right-hand side can be interpreted as the gain saturation term, which is meaningful for a pulse of energy comparable to the saturation energy of the amplifier. The second term is the second-order gain dispersion term that comes into play when the pulse spectral width and the gain bandwidth are comparable. \( g_0 \) and \( d \) symbolize the coefficients of gain saturation and gain dispersion, respectively. The third term accounts for the system loss, which is inherent in the medium. Here, \( \alpha \) stands for the dimensionless wave guide loss coefficient. The fourth and fifth terms are due to the TPA and 3PA, respectively. \( K \) denotes the TPA coefficient, while \( \nu \) represents that for 3PA. Since we choose a self-focusing cubic and defocusing quintic nonlinearity, the sign of the third term in left-hand side of Equation (1) is positive, while \( \gamma \) is negative. Sign of both \( K \) and \( \nu \) is positive as they correspond to TPA and 3PA-induced losses. At this point, it is worthy to mention that both cubic and quintic nonlinearity can be observed at a given wavelength, but dominancy of TPA and 3PA are observed at different wavelengths. Therefore, three possibilities may arise. Firstly, at a shorter wavelength TPA will be prominent, 3PA will not occur. Secondly, at a longer wavelength 3PA will be dominant and TPA will vanish. Thirdly, at an intermediate wavelength both TPA and 3PA can be observed simultaneously. For general mathematical development (and also for the third possibility), we consider both of them and finally discuss the first two cases by setting either \( K = 0 \) or \( \nu = 0 \).

Equation (1) is not completely integrable by the inverse scattering method. Such systems are generally solved following numerical methods. However, analytical approach, even approximate one, is very important as it gives very useful understanding of the system. In this context, Kankratovitch method, moment method, and variational method in conjugation with Rayleigh’s dissipative function (RDF) [29] can be named. Particularly, Variational method has been widely used to describe the nonlinear pulse propagation in optical fibers, even in such dissipative environment. The evolution of pulse parameters obtained using this method is in close proximity with those observed by other methods, including direct numerical simulations. The benefit of this approach is that it gives us explicit evolution for individual system parameters. It can be noted that the left-hand side of Equation (1) is conservative part, while the right-hand side of the equation is dissipative part. The analytical method involves Lagrangian formulation for the conservative part and RDF generation for the dissipative part. The Lagrangian corresponding to the conservative part of Equation (1) is given by,

\[
L = \frac{i}{2} \left( E E_z^* - E^* E_z \right) + \frac{1}{2} |E|^{2} - \frac{1}{2} |E|^{4} - \frac{\gamma}{3} |E|^{6},
\]

(2)

whereas the Rayleigh’s dissipation function for the dissipative part can be constructed as follows:

\[
R = \frac{id}{2} \left( E_{z'} E_z - E_z E_{z'}^* \right) + iK |E|^2 \left( E E_z^* - E^* E_z \right) + iv |E|^4 \left( E E_z^* - E^* E_z \right) - \frac{i}{2} \left( g_0 - \alpha \right) \left( E E_z^* - E^* E_z \right).
\]

(3)
We consider a sech ansatz of the following form:

$$E(z,t) = A(z) \sec \left( \frac{t}{W(z)} \right) \exp(i\phi(z)), \quad (4)$$

where $A(z)$, $W(z)$, and $\phi(z)$ represent the complex amplitude, temporal width, and phase of the pulse, respectively. The reduced Lagrangian and reduced RDF can be found using the $L_g = \int_{-\infty}^{\infty} L dt$, and $R_g = \int_{-\infty}^{\infty} R dt$, in conjugation with the ansatz (i.e. Equation (4)) and take the form:

$$L_g = i(\mathcal{A}_z^* - A^*A_z) W(z) + 2W(z)|A(z)|^2 \frac{\partial \phi(z)}{\partial z} + \frac{1}{3} |A(z)|^2 W(z) - 2 |A(z)|^4 W(z)$$

$$- \frac{16}{45} \gamma |A(z)|^6 W(z), \quad (5)$$

$$R_g = \frac{id}{3W(z)} \left[ (\mathcal{A}_z^* - A^*A_z) - 2i|A(z)|^2 \frac{\partial \phi(z)}{\partial z} \right]$$

$$+ \frac{4i}{3} KW(z)|A(z)|^2 (\mathcal{A}_z^* - A^*A_z) + \frac{8}{3} K |A(z)|^4 W(z) \frac{\partial \phi(z)}{\partial z}$$

$$+ \frac{32}{15} v |A(z)|^6 W(z) \frac{\partial \phi(z)}{\partial z} + \frac{16i}{35} v |A(z)|^4 W(z) (\mathcal{A}_z^* - A^*A_z)$$

$$- (g_o - \alpha) W(z) \left[ 2|A(z)|^2 \frac{\partial \phi(z)}{\partial z} + i(\mathcal{A}_z^* - A^*A_z) \right] \quad (6)$$

Now, four equations of motion can be obtained employing the Euler-Lagrangian equation,

$$\frac{d}{dz} \left( \frac{\partial L_g}{\partial \dot{q}_j} \right) - \frac{\partial L_g}{\partial q_j} + \frac{\partial R_g}{\partial \dot{q}_j} = 0, \quad (7)$$

where $\dot{q}_j = \frac{\partial q_j}{\partial z}$, and $q_j$ is the generalized coordinate, namely $A(z)$, $A^*(z)$, $W(z)$ and $\phi(z)$. The four equations are as follows:

$$-i \frac{d}{dz} (W(z)A^*(z)) = iW(z)A_z^*(z) + 2W(z)A^*(z) \frac{\partial \phi(z)}{\partial z} + \frac{1}{3} A^*(z) A(z) W(z)$$

$$+ \frac{idA^*(z)}{3W(z)} + \frac{4i}{3} KW(z)|A(z)|^2 A^*(z) - \frac{16}{15} \gamma |A(z)|^4 A^*(z) W(z)$$

$$+ \frac{16}{15} ivW(z)|A(z)|^4 A^*(z) - i(g_o - \alpha) W(z) A^*(z) \quad (8)$$

$$i \frac{d}{dz} (W(z)A(z)) = -iW(z)A_z(z) + 2W(z)A(z) \frac{\partial \phi(z)}{\partial z} + \frac{1}{3} A(z) A(z) W(z)$$

$$- \frac{idA(z)}{3W(z)} - \frac{4i}{3} KW(z)|A(z)|^2 - \frac{16}{15} \gamma |A(z)|^4 A(z) W(z)$$

$$- \frac{16}{15} ivW(z)|A(z)|^4 A(z) + i(g_o - \alpha) W(z) A(z) \quad (9)$$
\[ i(AA^*_z - A^*A_z) = -2|A(z)|^2 \frac{\partial \phi(z)}{\partial z} + \frac{1}{3} \frac{|A(z)|^2}{W^2(z)} + \frac{2}{3} |A(z)|^4 + \frac{16}{45} |A(z)|^6 \]  

(10)

and

\[ 2W(z) \left[ A(z) \frac{\partial A^*(z)}{\partial z} + \frac{\partial A(z)}{\partial z} A^*(z) \right] + 2|A(z)|^2 \frac{\partial W}{\partial z} + \frac{2d|A(z)|^2}{3W(z)} + \frac{8K}{3} |A(z)|^4 W(z) + \frac{32v}{15} |A(z)|^6 W(z) - 2(g_o - \alpha)|A(z)|^2 W(z) = 0. \]  

(11)

Equation (8) × A(z) − Equation (9) × A^*(z) gives,

\[ \frac{d}{dz} \left[ 2|A(z)|^2 W(z) \right] = -\frac{2d|A(z)|^2}{3W(z)} - \frac{8}{3} KW(z)|A(z)|^4 - \frac{32v}{15} W(z)|A(z)|^6 + 2(g_o - \alpha) W(z)|A(z)|^2, \]  

(12)

The integrated intensity of the pulse is given by \( \int_{-\infty}^{\infty} |E(z, t)|^2 = 2|A(z)|^2 W(z). \) Therefore, Equation (12) describes the energy dissipation with propagation. In absence of TPA, 3PA, and gain dispersion the right-hand side of Equation (12) vanishes. This leads to the constant intensity of the pulse be fitted for a conservative system.

Again Equation (8) × A(z) + Equation (9) × A^*(z) gives the following equation:

\[ i(AA^*_z - A^*A_z) = -2|A(z)|^2 \frac{\partial \phi(z)}{\partial z} - \frac{1}{3} \frac{|A(z)|^2}{W^2(z)} + \frac{4}{3} |A(z)|^4 + \frac{16}{15} |A(z)|^6. \]  

(13)

Comparing Equations (10) and (13), we get:

\[ |A(z)|^2 W^2(z) + \frac{16\gamma|A(z)|^4 W^2(z)}{15} = 1 \]  

(14)

The entity given by Equation (14) remains conserved throughout the propagation distance. For cubic case such condition was referred as ‘fundamental soliton condition’ (see [16]). It may be noted that although the ‘fundamental soliton condition’ remains conserved, it doesn’t guarantee zero broadening and decay of the pulse due to the dissipative effects. However, for a conservative system, the aforesaid condition can give rise to a shape preserving soliton. In a dissipative system one needs to accomplish the loss-gain balance to achieve a self-similar solitonic pulse.

The above equation gives rise to the evolution equations of amplitude \( A(z) \) and pulse width \( W(z) \) as follows:

\[ \frac{dW(z)}{dz} = \left[ \frac{-d}{3aW} \left[ -1 + \sqrt{M} \right] - \frac{2KW}{3a^2} \left[ -1 + \sqrt{M} \right]^2 \right] \]  

\[ - \frac{4vW}{15a^3} \left[ -1 + \sqrt{M} \right]^3 + (g_o - \alpha) W \frac{1}{a} \left[ -1 + \sqrt{M} \right], \]  

(15)
where $M = 1 + 4a/W^2$ and $a = 16\gamma/15$.

The integrated intensity can be obtained by

$$P_z(\tau) = \int_{-\infty}^{\infty} |u(z, t)|^2 dt.$$  

(17)

The coupled Equations (15) and (16) are the key equations for investigating the dissipative pulse propagation.

The model can be simplified if the gain saturation can perfectly counter balance the system loss, i.e. $g_0 = \alpha$. For this case, the pulse width and amplitude vary as follows:

$$\frac{dW(z)}{dz} = \frac{A(1 + aA^2)}{W(1 + 2aA^2)} \left[ -\frac{4W}{15a^2} \left[ -1 + \sqrt{M} \right] \right] \left( \frac{a}{W^2 \sqrt{M}} + \frac{1}{a} \left[ -1 + \sqrt{M} \right] \right),$$  

(18)

$$\frac{dA(z)}{dz} = \frac{A(1 + aA^2)}{W(1 + 2aA^2)} \left[ -\frac{4W}{15a^2} \left[ -1 + \sqrt{M} \right] \right] \left( \frac{a}{W^2 \sqrt{M}} + \frac{1}{a} \left[ -1 + \sqrt{M} \right] \right).$$  

(19)

Thus, we investigate the system under two conditions:

Case I: When gain saturation $g_0$ is equal to system loss $\alpha (g_0 = \alpha)$.

Case II: When gain saturation $g_0$ is greater than system loss $\alpha (g_0 > \alpha)$.

3. Dissipative pulse propagation: ($g_0 = \alpha$)

In this section, we discuss the pulse dynamics for a situation when the gain saturation is equal to the system loss, i.e. $g_0 = \alpha$. In spite of this gain–loss balance, the system will still remain dissipative because of the detrimental influence of gain dispersion and multiphoton absorption on pulse dynamics. Besides simplification, the condition $g_0 = \alpha$ will increase the ‘visibility’ of the gain dispersion, TPA, and 3PA.

3.1. Analytical results

Equations (18) and (19) are solved to find the dissipative pulse dynamics. Depending on the wavelength of the pulse we consider three different situations: a shorter wavelength that corresponds to TPA, a longer wavelength that corresponds to 3PA, and an intermediate wavelength, which shows both TPA and 3PA. A very recent work reported (see [30]) deals with such cases for wavelengths ranging from 1.15 to 1.55 µm in hydrogenated amorphous silicon. Such a material is being used in fabricating optical fiber core due to the high nonlinearity, low loss, and capacity of high power transmission at
telecommunication wavelength. At lower wavelengths, namely at 1.15 and 1.25 µm, TPA dominates over 3PA. At a larger wavelength, i.e. at 1.5 µm 3PA is dominating. At the intermediate wavelength 1.3 µm, the coexistence of TPA and 3PA is observed. This situation is indeed interesting as in this state the dominance of TPA or 3PA is intensity dependent. Since TPA and 3PA are proportional to the square and cube of the field intensity, respectively, 2PA dominates nonlinear absorption at low intensity, while 3PA dominates at high intensity. Other aspects such as gain dispersion and cubic-quintic nonlinearity are present for all three cases of multiphoton absorption. First, we consider a shorter wavelength where TPA is dominant and 3PA is too small to consider. Setting $\nu = 0$ in Equations (18) and (19), the effect of 3PA can be discarded. The variation of peak intensity (i.e. $|A|^2$) with propagation distance is plotted in left-hand-side column of Figure 1, taking quintic nonlinearity (Figure 1(a)), gain dispersion (Figure 1(d)), and TPA (Figure 1(g)) as parameters. Corresponding variation of pulse width with normalized propagation distance is demonstrated in the middle column of Figure 1, i.e. in Figure 1(b), (e) and (h), respectively. Plots in the right-hand-side columns (i.e.

![Figure 1](image_url)

Figure 1. Pulse degradation with normalized propagation distance in presence of TPA. (a) Decay of peak intensity, (b) pulse broadening and (c) integrated pulse intensity decay for different $\gamma$. $d=0.05$ and $K=0.01$. (d) Decay of peak intensity, (e) pulse broadening and (f) decay of integrated pulse intensity for different $d$ with $\gamma = -0.1$ and $K=0.01$. (g) Decay of peak intensity, (h) pulse broadening and (i) integrated pulse intensity decay for different $K$ with $\gamma = -0.1$ and $d=0.05$. For 1(a)–(i), $\nu = 0$. 


Figure 1(c), (f) and (i) portray the corresponding variation of integrated pulse intensity (P). Due to the detrimental effect of TPA and gain dispersion, a common trend of pulse amplitude decay (Figure 1(a), (d) and (g)) and pulse width broadening (Figure 1(b), (e) and (h)) is observed with propagation along the fiber length. Similar decay-trend is observed in the integrated pulse intensity too. Further it is noticed in Figure 1(a) that the variation in quintic nonlinearity marginally modifies the pulse decay behavior. But visible pulse width contraction is noticed with increasing quintic nonlinearity (Figure 1(b)). The integrated pulse intensity decreases with increasing strength of quintic nonlinearity.

The influence of the gain dispersion can be understood by noting Figure 1(d), (e) and (f), wherein the gain dispersion considerably degrades the pulse quality during propagation. With increasing gain dispersion the decay rate of both peak intensity (Figure 1(d)) and integrated pulse intensity (Figure 1(f)) increases. Side by side, pulse broadening becomes faster as displayed in Figure 1(e). TPA is expectedly found to be detrimental to the pulse quality. The rate of pulse decay (Figure 1(g) and (i)) and pulse

![Graphs showing the evolution of pulse parameters](image)

Figure 2. Evolution of pulse parameters with normalized distance of propagation in presence of 3PA. (a) Decay of peak intensity, (b) pulse broadening and (c) integrated pulse intensity decay for different $\gamma$ with $d = 0.05$ and $\nu = 0.01$. (d) Decay of peak intensity, (e) pulse broadening and (f) integrated pulse intensity decay for different $d$ and $\gamma = -0.1$ and $\nu = 0.01$. (g) Decay of peak intensity, (h) pulse broadening and (i) integrated pulse intensity decay for different $\nu$ with $\gamma = -0.1$ and $d = 0.05$. For 2(a)–(i), $K = 0$. 

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broadening (Figure 1(h)) become faster with stronger TPA. We now investigate the pulse propagation at a wavelength where 3PA is dominant and TPA is negligible. Like the case of TPA, in this case also the peak intensity and integrated pulse intensity decay and pulse broadens with propagation as depicted in Figure 2. The effects of quintic nonlinearity (Figure 2(a)–(c)) and gain dispersion (Figure 2(d)–(f)) are similar to the TPA case. A careful comparison of these figures with their TPA counterpart in Figure 1(a)–(f) reveals that the pulse decay and broadening rate are significantly slower in 3PA case. Thus, shifting from TPA to 3PA wavelength would be beneficial in terms of maintaining pulse quality during propagation. Figure 2(g), (h) and (i), respectively, display the variation of peak intensity, integrated pulse intensity, and pulse width with propagation distance, taking $v$ as a parameter. Like the case of TPA, increasing strength of 3PA ($v$) leads to pulse decay and broadening but at a much smaller rate. A third situation may arise when in both TPA and 3PA are sizable at an intermediate wavelength. The combined influence of TPA, 3PA, and gain dispersion ultimately leads to the decay

![Diagram](image_url)

**Figure 3.** Pulse degradation with normalized distance of propagation under combined influence of TPA and 3PA. (a) Decay of peak intensity, (b) pulse broadening and (c) decay of integrated pulse intensity for different $\gamma$ with $d = 0.05$, $K = 0.01$ and $v = 0.01$. (d) Decay of peak intensity, (e) pulse broadening and (f) integrated pulse intensity decay for different $d$ and $\gamma = -0.1$, $K = 0.01$ and $v = 0.01$. (g) Peak intensity decay, (h) pulse broadening and (i) decay of integrated pulse intensity for different $v$ with $\gamma = -0.1$, $d = 0.05$ and $K = 0.01$. 
Figure 4. Variation of normalized (a) peak intensity, (b) pulse width and (c) integrated pulse intensity along the fiber in presence of both TPA and 3PE for different values of 3PE coefficient with $d = 0.05$ and $K = 0.01$ and $\gamma = -0.1$. The corresponding insets show the blown-up views.

Figure 5. Pulse intensity profile under the effect of TPA and gain dispersion with propagation distance for (a) $\gamma = -0.001$, (b) $\gamma = -0.05$, (c) $\gamma = -0.1$ respectively. Insets on left hand side show the variation of peak intensity and those on right hand side show the variation of normalized pulse width along the fiber. Here $K = 0.01$, $d = 0.05$. 
as well as broadening of the pulse. The variation of peak intensity, pulse width, and integrated pulse intensity with propagation distance is plotted in Figure 3(a), (b) and (c), respectively, taking \( \gamma \) as a parameter. Corresponding sets of plots are presented in Figure 3(d)–(f) for different \( d \) and in Figure 3(g)–(i) for different \( \nu \). By comparing Figure 3(a)–(i) with their corresponding plots in Figures 1(a)–(i) and 2(a)–(i), it can be noted that the major share of the detrimental effect on peak intensity, width, and integrated intensity of the pulse goes to TPA. The monotonous decay or broadening profiles turn interesting with a choice of negative value of the imaginary part of \( \chi(5) \) and hence a negative \( \nu \). This leads to an effect opposite to 3PA and thus can be considered as three-photon emission. Since 3PA introduces loss in the system, 3PE should provide gain. Once again, we plot the variation of peak intensity, pulse width, and integrated pulse intensity with propagation distance under the influence of gain dispersion, TPA but for different strength of 3PE. It can be noted that increasing strength of 3PE reduces, although slightly, the rate of both pulse decay (Figure 4(a) and (c) and the corresponding insets) as well as pulse broadening (Figure 4(b)). Thus, 3PE will be favorable for arresting pulse decay and broadening and hence for the generation solitonic pulse. At this point, it is better to mention that the reports on 3PE are rather rare.

Figure 6. 3D plot showing the effect of 3PA and gain dispersion on pulse propagation along the fiber for (a) \( \gamma = -0.001 \), (b) \( \gamma = -0.05 \), (c) \( \gamma = -0.1 \) respectively. Left hand side insets show the variation of peak intensity and right hand side insets depict the variation of normalized pulse width with propagation. \( \nu = 0.01 \) and \( d = 0.05 \).
However, polarization-correlated 3PE from a positively charged triexciton in a self-assembled GaAs quantum dot has been reported \cite{32}. More reports of multiphoton emission in semiconductor devices are coming out recently. The first experimental observations of two-photon emission (TPE) from semiconductors has been reported in \cite{33}, wherein spontaneous and singly stimulated TPE were demonstrated in optically pumped GaAs and in current-driven GaInP/AlGaInP quantum well. However, the experimental realization of 3PE in a semiconductor is yet to come. In this report, we intend to provide the mathematical modelling for a fiber link with 3PE and to highlight its advantage for compensating loss in the process of obtaining dissipative solitons.

### 3.2. Numerical results

The results obtained analytically so far need to be validated by directly solving the governing CGLE. Figure 5 displays the pulse propagation under the influence of gain dispersion and TPA for different strength of quintic nonlinearity. Figure 5(a)
corresponds to $\gamma = -0.001$, while Figure 5(b) and (c) are plotted for $\gamma = -0.05$ and $\gamma = -0.1$, respectively. Corresponding peak intensity and pulse width are depicted in the left- and right-hand-side insets, respectively. The decay of peak intensity and broadening of pulse width are almost in accordance with the analytical result. Only an additional oscillation in peak intensity and width is present. This seems to be originated due to the counteracting effects of focusing cubic and defocusing quintic nonlinearity. However, because of the approximate nature of the variational method such oscillations are too weak to observe in analytical results. The numerically obtained pulse evolution profile thus qualitatively matches with the analytical one. Pulse propagation is also numerically obtained by taking $d$ and $K$ as parameters. The nature of pulse evolution for those cases too (not shown) qualitatively matches with that found analytically. We then plot the pulse propagation in 3PA wavelength in presence of gain dispersion for different strength of quintic nonlinearity. Figure 6 shows the pulse propagation profile and corresponding variation in peak intensity and pulse width for (a) $\gamma = -0.001$, (b) $\gamma = -0.05$, and (c) $\gamma = -0.1$. The decay and broadening trend is found to be similar to that of the analytical case presented in Figure 2. Similar study under combined influence of TPA and 3PA is presented in Figure 7 for varying strength of $\gamma$. This case too resembles with the analytical one depicted in Figure 3. The pulse propagation behavior

Figure 8. 3D pulse evolution in presence of both TPA and 3PE along the fiber length for (a) $\gamma = -0.001$, (b) $\gamma = -0.05$, (c) $\gamma = -0.1$ respectively. Left hand side insets show the variation of peak intensity and right hand side insets show the variation of normalized pulse width with propagation along the fiber. Here, $K = 0.01$, $d = 0.05$ but $\nu = -0.01$. 
has been extensively studied with different values of $d$ and $K$ for both 3PA and combined case of TPA and 3PA. For all cases, the numerical results are found to be identical to the corresponding analytical case, but not shown just to make the study less crowded with figures. Furthermore, we numerically verified the case of Figure 4 with 3PE and got matching gain behavior of 3PE. This is presented in Figure 8.

4. Solitonic pulse propagation: ($g_0 > \alpha$)

The investigation in the previous section shows pulse decay and broadening for all three cases of multiphoton absorption. The dissipative nature can be arrested by applying right amount of gain in the system. This can be done by a suitable choice of $\Delta g(= g_0 - \alpha)$, where $g_0 > \alpha$. If $g_0$ is not sufficiently larger than $\alpha$, an additional gain might be introduced in the system. An effective and suitable gain is must, and we refer that by $\Delta g$. In this section, we search for the desired gain and demonstrate the evolution of solitonic pulse. Since these solitons originate in dissipative media they can be considered as dissipative soliton. Such dissipative solitons might be bistable due to the quintic term in the fundamental soliton condition given by Equation (14). Figure 9(a) plots the bistability curves for $\gamma = -0.1$ and $\gamma = -0.05$. All the points on the curves satisfy soliton condition. Figure 9(d) shows the analytically obtained soliton condition with one such point, while Figure 9(b) and (c) show the peak intensity and pulse width variation. We found from Figure 9(a) that solitons of same width but of two different amplitudes can arise and lead to bistability. One such bistable soliton corresponding to

Figure 9. (a) Bistability curve corresponding to Equation (14). Solid line is for $\gamma = -0.1$ and dashed line for $\gamma = -0.05$. (b) and (c) show the peak intensity and width respectively whereas (d) depicts the soliton condition, which is conserved.
Figure 9(a) is shown in Figure 10(a)–(c) and (e)–(g) analytically. Corresponding numerically obtained bistable soliton is presented in Figure 10(d) and (h) and their insets. We now find the dissipative soliton for all three cases of multiphoton absorption with gain dispersion and cubic-quintic nonlinearity. For TPA case, a normalized $\Delta g$ of $0.805 \times 10^{-6}$ leads to constant peak intensity, width and integrated intensity as described in Figure 11(a), (b) and (c), respectively, using the analytical method. Their corresponding numerical curves are presented in Figure 11(d), (e) and (f), respectively. Other than the slight fluctuation due to quintic terms, the numerical results also show self-similar propagation of the pulse. This shows the generation of dissipative soliton. Similar evolution of dissipative soliton is shown for 3PA in Figure 12 with a normalized $\Delta g = 0.500 \times 10^{-6}$. When it comes to the combined case of TPA and 3PA normalized $\Delta g = 0.810 \times 10^{-6}$ leads to the formation of dissipative soliton. The analytical as well as numerical results are presented in Figure 13.
Figure 11. Evolution of dissipative soliton under effect of TPA. The parameters taken are $\gamma = -0.001$, $d = 0.05$ and $K = 0.01$.

Figure 12. Evolution of dissipative soliton in presence of 3PA. Here, $\gamma = -0.001$, $d = 0.05$ and $\nu = 0.01$. 
5. Conclusion

We studied the combined dissipative effect of three cases of multiphoton absorption in conjugation with gain dispersion on the pulse propagating in a medium having cubic-quintic nonlinearity. The results are obtained both analytically and numerically. When compared, they come out to be in close proximity with each other. The presence of quintic nonlinearity in the system helps in self-trapping of the pulse. It is found that shifting from TPA to 3PA wavelength is more beneficial as the later has a less detrimental effect on pulse dynamics. A material with negative-valued imaginary part $\chi(5)$ is found to be promising for arresting pulse degradation. Dissipative solitons are obtained by introducing different gains in TPA, 3PA, and combined case of TPA and 3PA. The dissipative solitons thus obtained are bistable in nature. The results presented in this study have potential applications in fiber optic communication system and devices. 3PA occurs at longer wavelength and therefore introduces smaller scattering loss. Since, at longer 3PA wavelength, the penetration capability into biological cells is greater and damaging effect to the tissues is lesser, the current investigation has potential application in biology and medical science, e.g. bio-imaging and light-activated therapy.

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References
[1] Agrawal GP. Nonlinear fiber optics. 3rd ed. San Diego (CA): Academic Press; 2001.
[2] Kivsher YS, Agarwal GP. Optical solitons: from fibers to photonic crystals. San Diego (CA): Academic Press; 2003.
[3] Hasegawa A, Tappert F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion. Appl. Phys. Lett. 1973;23:142–145.
[4] Hasegawa A, Tappert F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion. Appl. Phys. Lett. 1973;23:171–172.
[5] Mizrahi V, DeLong KW, Stegeman GI, et al. Two-photon absorption as a limitation to all-optical switching. Opt. Lett. 1989;14:1140–1142.
[6] Agrawal GP. Effect of two-photon absorption on the amplification of ultrashort optical pulses. Phys. Rev. E. 1993;48:2316–2318.
[7] Silberberg Y. Solitons and two-photon absorption. Opt. Lett. 1990;15:1005–1007.
[8] Aitchison JS, Oliver MK, Kapon E, et al. Role of two photon absorption in ultrafast semiconductor optical switching devices. Appl. Phys. Lett. 1990;56:1305–1307.
[9] Jana S, Konar S. Stable and quasistable spatio-temporal solitons in cubic quintic nonlinear medium. J. Nonlinear Opt. Phys. Mater. 2004;13:25–36.
[10] Konar S, Jana S, Mishra M. Induced focusing and all optical switching in cubic quintic nonlinear medium. Opt. Commun. 2005;255:114–129.
[11] Firth WJ, Paulau PV. Soliton lasers stabilized by coupling to a resonant linear system. Eur. Phys. J. D. 2010;59:13–21.
[12] Bindra KS, Bookey HT, Kar AK, et al. Nonlinear optical properties of chalcogenide glasses: observation of multiphoton absorption. Appl. Phys. Lett. 2001;79:1939–1941.
[13] Slusher RE, Lenz G, Hodelin J, et al. Large Raman gain and nonlinear phase shifts in high-purity As₂Se₃ chalcogenide fibers. J. Opt. Soc. Am. B. 2004;21:1146–1154.
[14] Hé GS, Markowicz PP, Lin TC, et al. Observation of stimulated emission by direct three-photon excitation. Nature. 2002;415:767–770.
[15] Cronstrand P, Luo Y, Norman P, et al. Ab initio calculations of three-photon absorption. Chem. Phys. Lett. 2003;375:233–239.
[16] Roy S, Bhadra S. Study of nonlinear dissipative pulse propagation under the combined effect of two-photon absorption and gain dispersion: a variational approach involving Rayleigh’s dissipation function. Phys. D. 2007;232:103–107.
[17] Xian-Qiong Z, Xiao-Xia Z, Ke C, et al. Ultrashort pulse breaking in optical fiber with third-order dispersion and quintic nonlinearity. Chinese Phys. B. 2014;23:064207-1–064207-7.
[18] Agrawal GP. Effect of gain dispersion on ultrashort pulse amplification in semiconductor laser amplifiers. IEEE J. Quantum Electron. 1991;27:1843–1849.
[19] Zakharov VE, Sabat AB. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. Sov. Phys. JETP. 1972;34:62–69.
[20] Ablowitz MJ, Kaup DJ, Newell AC, et al. Nonlinear-evolution equations of physical significance. Phys. Rev. Lett. 1973;31:125–127.
[21] Ablowitz MJ, Kaup DJ, Newell AC, et al. The inverse scattering transform Fourier analysis for nonlinear problems. Stud. Appl. Math. 1974;53:249–315.
[22] Rogers C, Schief WK. Bäcklund and Darboux transformations. Cambridge: Cambridge University Press; 2002.

[23] Hirota R. Exact envelope soliton solutions of a nonlinear wave equation. J. Math. Phys. 1973;14:805–809.

[24] Hirota R. Exact N-soliton solutions of the wave equation of long waves in shallow water and in nonlinear lattices. J. Math. Phys. 1973;14:810–814.

[25] Lakshmanan M, Rajaseekar S. Nonlinear dynamics: integrability, chaos and patterns. New Delhi: Springer Science & Business Media; 2003.

[26] Roy S, Bhadra SK. Solving soliton perturbation problems by introducing Rayleigh’s dissipation function. J. Lightwave Technol. 2008;26:2301–2322.

[27] Anderson D. Variational approach to nonlinear pulse propagation in optical fibers. Phys. Rev. A. 1983;27:3135–3145.

[28] Hasegawa A. Soliton-based ultra-high speed optical communications. Pramana. 2001;57:1097–1127.

[29] Lemos NA. Remark on Rayleigh’s dissipation function. Am. J. Phys. 1991;59:660–661.

[30] Gai X, Choi D, Luther-Davies B. Negligible nonlinear absorption in hydrogenated amorphous silicon at 1.5 μm for ultra-fast nonlinear signal processing. Opt. Express. 2014;22:9948–9958.

[31] Mehta P, Healy N, Baril NF, et al. Nonlinear transmission properties of hydrogenated amorphous silicon core optical fibers. Opt. Express. 2010;18:16826–16831.

[32] Arashida Y, Ogawa Y, Minami F. Polarization-correlated three-photon emission from charged triexcitons in a single quantum dot. Phys. Rev. B. 2012;85:235318-1–235318-5.

[33] Hayat A, Ginzburg P, Orenstein M. Observation of two-photon emission from semiconductors. Nat. Photonics. 2008;2:238–241.