SOLAR WIND TURBULENT SPECTRUM AT PLASMA KINETIC SCALES

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ABSTRACT

The description of the turbulent spectrum of magnetic fluctuations in the solar wind in the kinetic range of scales is not yet completely established. Here, we perform a statistical study of 100 spectra measured by the STAFF instrument on the Cluster mission, which allows us to resolve turbulent fluctuations from ion scales down to a fraction of electron scales, i.e., from ~10^2 km to ~300 m. We show that for k⊥ρe ∈ [0.03, 3] (which corresponds approximately to the frequency in the spacecraft frame f ∈ [3, 300] Hz), all the observed spectra can be described by a general law E(k⊥) ∝ k⊥−8/3 exp(−k⊥ρe), where k⊥ is the wavevector component normal to the background magnetic field and ρe the electron Larmor radius. This exponential tail found in the solar wind seems compatible with the Landau damping of magnetic fluctuations onto electrons.

Key words: plasmas – solar wind – turbulence

Online-only material: color figures

1. INTRODUCTION

In neutral, homogeneous, and isotropic fluids, the turbulent fluctuations are unpredictable, but their statistics are predictable and universal (Frisch 1995); the turbulent spectra follow the power law ∼k−5/3 for any local conditions (k being the wavenumber). This empirical result was explained by Kolmogorov (Kolmogorov 1941) assuming self-similarity of the turbulent fluctuations between the energy injection scale and the dissipation one ℓd.

In the magnetized solar wind, collisions are very rare (the mean free path is of the order of 1 AU); the dissipation process at work and the dissipation length are not known precisely. Moreover, in a magnetized plasma, it is difficult to imagine self-similarity over all scales where turbulent fluctuations are observed, since there exist several spatial and temporal characteristic scales, such as the ion Larmor radius ρi = √2kTiv/ni/(2πfci), the ion inertia length λi = c/ωpi, the corresponding electron scales ρe, λe, and the ion and electron cyclotron frequencies fci, fec. At these scales, the dominant physical processes change, which affects the scaling of the energy transfer time and furthermore the energy transfer rate, leading to spectral shape changes.

The first clear spectral change appears at ion scales. At 1 AU, the ion scales are nearly equal, λi ≃ ρi ≃ V/2πfci, so it is difficult to determine which of these scales is responsible for the ion break. Independent measurements at different distances from the Sun, between 0.3 AU and 0.9 AU (Bourouaine et al. 2012), and a statistical study at 1 AU (Leamon et al. 2000) indicate that the spectral break is related to the ion inertia length λi. Nearly incompressible magnetic fluctuations cascading from the inertial range may undergo kinetic effects in the vicinity of the ion scales. At these scales, ion temperature anisotropy instabilities occur (Gary et al. 2001) and can remove or inject energy in the turbulent cascade. However, for most of the solar wind observations, the plasma is stable (Mattei et al. 2007, 2011; Bale et al. 2009). The energy re-distribution among the fluid and kinetic degrees of freedom in the vicinity of ion scales is still a matter of debate and is probably at the origin of the spectral variations observed between 0.3 and 3 Hz in the satellite frame: the spectral index here varies between −4 and −2 (Leamon et al. 1998; Smith et al. 2006; Sahraoui et al. 2010). This spectral range is usually attributed to the ion dissipation range (Leamon et al. 1998, 1999, 2000; Smith et al. 2012) or to another fluid cascade, which may continue down to electron scales (Biskamp et al. 1996; Stawicki et al. 2001; Li et al. 2001; Galtier & Bhattacharjee 2003; Galtier & Buchlin 2007).

Between ion and electron scales, the fluctuations of the electron fluid form a small-scale inertial range (Alexandrova et al. 2007, 2008), or, following the nomenclature of Smith et al. (2012), an electron inertial range. Here, indeed, a reproducible spectrum ∝ k⊥−8/3 is observed (Alexandrova et al. 2009; Chen et al. 2010; Sahraoui et al. 2010). Approaching electron scales, one may expect to observe an electron dissipation range, as was suggested by Alexandrova (2008). At such small scales, there are only a few observations (Alexandrova et al. 2009; Sahraoui et al. 2010) and the descriptions are different. Larger statistical studies are needed to establish more firmly the properties of turbulent spectra at electron scales.

In this paper, we present a large statistical study of magnetic spectra starting at ion scales and going beyond electron spatial scales. We use data from the STAFF instrument (Cornilleau-Wehrlin et al. 1997) on the Cluster mission (Escoubet et al. 1997), which is able to measure such a range of scales. In a previous study, Alexandrova et al. (2009) described the electron inertial and the electron dissipation ranges separately: a power law ∼k−8/3 for the inertial range and a curved spectrum ∝ exp(−√k/k0) for the dissipation range. This model is rather complicated and has a large number of free parameters. In the present study, we propose a single algebraic description for both ranges, namely, an exponential with a power-law pre-factor: E(k⊥) = Ak−8/3 exp(−k⊥ℓd). We find that this model describes well the totality of the observed spectra at scales smaller than λi and ρi and that its cutoff scale ℓd correlates with ρi. The power-law exponent α is found to be close to −8/3. This model (henceforth called “the exp model”) has only one free parameter, the amplitude of the spectrum.
Previous authors (Sahraoui et al. 2010) have used a double power-law model with a break to fit the observations in the electron inertial and dissipation ranges. We have applied this model as well to our data, and we find that the first power-law exponent is consistent with the previous studies (Alexandrova et al. 2009; Chen et al. 2010) while the second exponent varies a lot. Despite the fact that the double power-law model has more free parameters than the exponential model used here, we find that it describes only 30% of the observed spectra and that the associated break scale does not present any clear correlation with an electron characteristic scale.

2. OBSERVATIONS

For our statistical study, we select homogeneous intervals of 10 minutes (long enough to study kinetic scales) within the five years interval (2001–2005) of Cluster. We eliminate time intervals during which Cluster is magnetically connected to the bow shock by using electrostatic wave spectrograms, which show clearly waves typical of the electron foreshock (Etcheto & Fauchoux 1984; Lacombe et al. 1985), and by using the shock model described by Filbert & Kellogg (1979). For small angles $\Theta_{BV}$ between the interplanetary magnetic field $\mathbf{B}$ and the solar wind velocity $\mathbf{V}$, Cluster is connected to the shock. Thus, our data set only contains intervals for which the angle $\Theta_{BV} > 60^\circ$. If the turbulent fluctuations have a phase speed $V_\phi \ll V$, Cluster detects by Doppler shift the fluctuations with $k|V$. As $\mathbf{B}$ and $\mathbf{V}$ are quasi-perpendicular, Cluster measures fluctuations with $\mathbf{k} \perp \mathbf{B}$. We apply the Taylor hypothesis to get the wavenumber from the frequency, $k_\perp = 2\pi f/V$. However, about ~10% of the pre-selected intervals show the presence of right-hand polarized whistlers in quasi-parallel propagation. For these waves the Taylor hypothesis is not applicable because $V_\phi > V$. We discard these intervals in the present study. This data selection process gives us 100 intervals. Within this statistical sample, the plasma conditions vary as usually in the solar wind in fast and slow streams at 1 AU (see the legend of Figure 1).

Figure 1 shows the total power spectral density (PSD) of magnetic fluctuations, for 27 intervals randomly chosen among 100, as a function of frequency in the spacecraft frame $P(f)$, as measured by STAFF with the Search Coil sensors (SC) at $f \in [0.5, 9]$ Hz and with the Spectrum Analyser (SA) at $f \geq 8$ Hz. The spectra are analyzed only for the frequencies where the signal-to-noise ratio ($S/N$) is larger than 3. The spectral parts below this threshold are not shown to avoid any erroneous interpretation. As one can see from Figure 1, this instrumental noise limit allows us to data up to 30–400 Hz, depending on the turbulence intensity (i.e., for the most intense spectrum, we have valid observations up to 400 Hz). The analyzed range of frequencies corresponds to $f \in [3, f_c]$. An example of a raw spectrum without the correction can be found in Figure 2.

A poor calibration of the first three frequencies of SA (at 8, 11, and 14 Hz; Y. de Conchy & N. Cornilleau 2011, private communication) was corrected by an interpolation of these points between the highest SC frequency and the 4th point of the SA spectra. The linear interpolation between $\log_{10} P(f)$ and $\log_{10} f$ is possible as far as the spectra follow a power law at these frequencies. An example of a raw spectrum without the correction can be found in Figure 2.

3. ALGEBRAIC DESCRIPTION OF TURBULENT SPECTRA AT SCALES SMALLER THAN $\rho_i$ AND $\lambda_i$

3.1. Exponential Model

Here we propose a model to describe the whole turbulent spectrum at scales smaller than $\rho_i$ and $\lambda_i$ and down to a fraction of the electron scales with the smaller possible number of parameters, namely, an exponential with a characteristic scale $\ell_d$ and with a power-law pre-factor

$$E(k_\perp) = Ak_\perp^{-\alpha}e^{-k_\perp \ell_d}.$$  

This $exp$ model has three free parameters: the amplitude $A$, the spectral index $\alpha$, and the cutoff or “dissipation” scale $\ell_d$.

We start by fitting the model (1) to the 100 observed spectra (with $\log_{10} S/N > 3$, as explained in Section 2) for $k_\perp$ corresponding to $f > 3$ Hz (see vertical dotted line in Figure 1), assuming that the three parameters have independent variations.

Figure 2 gives the fit with the most intense spectrum of Figure 1 as a function of the wavenumber $P(k_\perp) = P(f) V/2\pi$, which is determined using the Taylor hypothesis and the energy conservation law $\int P(k_\perp) dk_\perp = \int P(f) df$. Green crosses show the Morlet wavelet spectrum (Torrence & Compo 1998)
of STAFF-SC measurements. Red stars display the STAFF-SA data for the same time period. (In this plot we keep the STAFF-SC measurements. Red stars display the STAFF-SA data for the same time period. (In this plot we keep the

Figure 3. Results of the fitting with the $\exp$ model for the 100 observed spectra: (a) histograms on the spectral index $\alpha$; (b) cutoff scale $\ell_d$ as a function of $\alpha$; (c) $\ell_d$ as a function of the electron Larmor radius $\rho_e$; and (d) $\ell_d$ as a function of the electron inertial length $\lambda_e$.

of STAFF-SC measurements. Red stars display the STAFF-SA data for the same time period. (In this plot we keep the three first poorly calibrated data points; one can see them around $k = 0.1$ km$^{-1}$ and compare with the result of the interpolation in Figure 1.) The error bars are estimated from the variance over 10 minutes of the PSD at each frequency (Alexandrova et al. 2010). This spectrum is valid up to $\approx 400$ Hz, which gives us the maximum wavenumber $k \sim 4$ km$^{-1}$ (while $1/\rho_e \approx 1$ km$^{-1}$). This is the smallest scale ever measured with a good sensitivity at 1 AU in the solar wind. The $\exp$ model (1) fitting is shown by the black solid line. The parameters of the fit in this case are $\alpha = 2.70 \pm 0.15$ and $\ell_d = 0.90 \pm 0.25$ km, while $\rho_e = 0.95 \pm 0.05$ km.

Figure 3 summarizes the results of the fitting for the 100 spectra. Panel (a) shows the histogram of the spectral index, $\alpha = 2.63 \pm 0.15$, the error being the standard deviation of the mean. Note that $(\alpha) \approx 8/3$. It appears that the variations of $\alpha$ and $\ell_d$ are not independent since the dispersion in $\alpha$ is due to the variations of the cutoff scale $\ell_d$ as observed in Figure 3(b). A linear fit gives $\ell_d(\text{km}) = 12.9 - 4.4\alpha$, and so

$$\alpha = 2.9 - \ell_d/4.4, \quad (2)$$

e.g., if $\ell_d$ was small, $\alpha$ would be approximatively equal to 2.9, a value close to the one found by Alexandrova et al. (2009).

On the other hand, the variations of $\ell_d$ are related to the variations of the electron Larmor radius, $\ell_d \sim 1.35\rho_e$, as shown in Figure 3(c), with a relatively high correlation coefficient of 0.70. Figure 3(d) shows a positive but much weaker correlation of 0.34 between the dissipation scale $\ell_d$ and the electron inertial length $\lambda_e$.

The results presented in Figure 3 suggest that within the framework of the exponential model there is only one free parameter, the amplitude of the turbulent spectra, $A$, and the observed spectra can be described approximately by

$$E(k_{\perp}) \simeq A k_{\perp}^{-8/3} \exp(-k_{\perp}\rho_e). \quad (3)$$

We verify this point in Figure 4, where we superpose the 100 spectra analyzed here with the 7 spectra covering fluid and kinetic scales from Alexandrova et al. (2009). The spectra are shifted vertically by a parameter $E_0$ (a relative spectral level), in the same way as in Figure 2 of Alexandrova et al. (2009). The superposition of the 107 spectra is nice, which indicates the generality of the turbulent spectrum $E(k_{\perp})$ in the solar wind: it follows $\propto k_{\perp}^{-5/3}$ at MHD scales and $\propto k_{\perp}^{-8/3}\exp(-k_{\perp}\rho_e)$ at scales smaller than the ion kinetic scales $\lambda_i$ and $\rho_i$ (i.e., $k\rho_i > 0.03$). The bottom panel shows the 100 spectra $E(k_{\perp}\rho_i)$ compensated by a function $F = (k_{\perp}\rho_i)^{8/3}\exp(k_{\perp}\rho_i)$ for $k\rho_i > 0.03$: the resulting spectra are flat, indicating that the $\exp$ model with one free parameter describes well all the turbulent spectra in the solar wind at these scales and is valid for nearly two decades in wave numbers. Note that a damping length $\ell_d$ variation of more than 20% with respect to the mean $\rho_e$ values results in a strong departure of the compensated spectra.

The amplitude $A$ of the turbulent spectrum (related to the parameter $E_0$) is found to be correlated with the ion thermal pressures $nkT_i$, as within the MHD range of turbulence (Grappin et al. 1990), and with the ion temperature anisotropy $T_{\perp}/T_{\parallel}$ (not shown). Other plasma parameters seem to be less important, but still it is impossible to exclude completely the influence of the magnetic and kinetic energies in the solar wind (paper in preparation).

### 3.2. Break Model

Is there another simple model which represents well the observations with a small number of free parameters? Let us compare the turbulent spectra within the electron inertial and...
dissipation ranges (i.e., scales smaller than $\rho_i$ and $\lambda_i$) with the double power law or break model

$$E(k) = A_1 k^{-\alpha_1} (1 - H(k - k_b)) + A_2 k^{-\alpha_2} H(k - k_b),$$

where $H(x)$ is the Heaviside function, $k_b = 1/\rho_i$, $A_{1,2}$ are the amplitudes of the two power-law functions, and $\alpha_{1,2}$ are the spectral indices.

The values of the spectral indices are correlated to the position of the break scale $\ell_b$, which is determined from the fitting with 30 spectra, i.e., (a) histograms on the spectral indices, (b) break scale $\ell_b$ as a function of $\alpha_1$ (open circles) and $\alpha_2$ (filled circles), (c) $\ell_b$ as a function of the electron Larmor radius $\rho_e$, and (d) $\ell_b$ as a function of the electron inertial length $\lambda_e$.

Figure 5. Same format as Figure 3, but for the parameters of the break model determined from the fitting of the 30 spectra, i.e., (a) histograms on the spectral indices, (b) break scale $\ell_b$ as a function of $\alpha_1$ (open circles) and $\alpha_2$ (filled circles), (c) $\ell_b$ as a function of the electron Larmor radius $\rho_e$, and (d) $\ell_b$ as a function of the electron inertial length $\lambda_e$.

4. DISCUSSION AND CONCLUSION

The exp model $E(k) = A k^{-8/3} \exp(-k^2/\rho_i)$ proposed in this study provides a single algebraic description of the solar wind spectrum at scales smaller than the ion characteristic scales, $\lambda_i$ and $\rho_i$, and going beyond the electron scales (i.e., within the electron inertial and dissipation ranges). This model describes well the totality of the observed spectra and has only one free parameter—the amplitude $A$ of the spectrum. The amplitude seems to be a function of the ion thermal pressure and the ion temperature anisotropy in the solar wind. However, it is difficult to exclude the role of the magnetic and kinetic energies: more work is needed to determine the exact relationship between the amplitude of the turbulent spectrum and the energy budget in the solar wind.
The spectral index close to $-8/3$ observed in the solar wind at scales smaller than ion characteristic scales is in agreement with quasi-bidimensional strong Electron MHD turbulence ($k_\parallel \gg k_\perp$) when parallel cascade is weak (Galtier et al. 2005). Recently, the same spectral index was found as well in strong kinetic Alfvén turbulence (Boldyrev & Perez 2012).

In usual fluid turbulence, the far dissipation range is described by $E(k) \sim k^3 \exp(-ck\ell_d)$ (with $c \simeq 7$; Chen et al. 1993). This is due to the resistive damping rate $\propto k^2$ valid in a collisional fluid, which gives an exponential spectral tail. In the collisionless plasma of the solar wind there is no resistive damping, and thus this coincidence deserves an explanation.

Howes et al. (2011) consider a model (“weakened cascade model”) which includes the nonlinear transfer of energy from large to small scales in Fourier space (see Equation (23) in Howes et al. 2011) and the damping of kinetic Alfvén waves (KAWs). The spectral laws are respectively $E_k \propto k_\perp^{-5/3}$ at large scales and $E_k \propto k_\perp^{-7/3}$ between ion and electron scales. When taking into account the damping term, Howes et al. (2011) obtain numerically the same spectral laws, with a final curved tail at scales smaller than electron scales. Superficially, this spectrum thus resembles the analytic form which we have found to be valid to describe the solar wind turbulence.

We last remark that the damping term in the model of Howes et al. (2011) is obtained by linearizing the Vlasov–Maxwell equations in the gyrokinetic limit ($k_\parallel \ll k_\perp$, with frequencies $f \ll f_{ci}$). For $k_\perp \rho_i \gg 1$ it has the form $\gamma \simeq k_\perp V_A(k_\perp \rho_i)^2 \propto k_\parallel k_\perp^2$ (see Equation (63) in Howes et al. 2006). Taking into account the assumption of critical balance $\tau = \tau_A$ (i.e., $k_\perp u = k_\parallel V_A$; Goldreich & Sridhar 1995), with $\tau = 1/k_\perp u$ being the nonlinear time, $u$ the velocity fluctuation, $\tau_A$ the Alfvén time, and $V_A$ the Alfvén speed, and the spectral index $-7/3$ (i.e., $u \sim k_\perp^{2/3}$), one gets $k_\parallel \propto k_\perp^{1/3}$. Therefore, the damping term takes the form $\gamma \propto k_\perp^{4+1/3}$. The exponent of the damping rate is thus very close to the $k^2$ scaling of the Laplacian viscous term, which is known to lead in hydrodynamical turbulence to an exponential tail in the dissipation range.

This model does not take into account the cyclotron damping. So, while the proposed phenomenology may explain the exponential tail of the $k_\perp$-spectrum studied here, it cannot describe more isotropic wavevectors, which might be present as well in the solar wind. It is possible that the dissipation mechanism could also be due to electron–cyclotron absorption of oblique short-wavelength whistler waves, or even of lower-hybrid waves. More observations under different field-to-flow angles $\Theta_{BV}$ are needed within the electron inertial and dissipation ranges to address this point.

To build a realistic model of the dissipation in the solar wind, we need to resolve still an open question on the nature of the turbulent fluctuations. Some authors argue that the electron inertial range is a whistler mode turbulence (Saito et al. 2010; Narita & Gary 2010), while others suggest KAW turbulence (Bale et al. 2005; Schekochihin et al. 2009; Sahraoui et al. 2010; Salem et al. 2012), or a combination of both types of linear waves (Podesta & Gary 2011). The model of Howes et al. (2011) described above is based on KAW turbulence as well. However, it is still not clear whether we can describe turbulence in the solar wind as a mixture of linear waves (weak turbulence) which will dissipate homogeneously in space (or in the plane perpendicular to $\mathbf{B}$), or if it is a strong turbulence with dissipation restricted to intermittent coherent structures. What is the topology of these structures—current sheets, shocks, or coherent vortices?

In the present study, we have limited ourselves to observations of the spectral shape in the electron inertial and dissipation ranges. Our results give observational constraints for future theoretical models.

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### APPENDIX A

**PLASMA PARAMETER VARIATIONS DURING THE SPECTRA INTEGRATION TIME**

One could argue that the exponential bending found in the $k$-spectra of the magnetic fluctuations is due to variations of the solar wind speed $V$ or of the gyroradius $\rho_e$ during the 10 minutes of each considered interval. Indeed, each $k$-spectrum is obtained with an average $P(f)$ over 150 frequency spectra, itself shifted in the $k$-domain with the average $V$. The standard deviation $dV$ over 10 minutes is very small. Figure 7(a) displays the histogram of the ratio $dV/V$: 91% of the 100 considered intervals have $dV/V < 0.02$. Thus, the shift of $P(f)$ in the $k$-domain with the average $V$ cannot change the spectral bending. Similarly, the standard deviation $d(\rho_e)$ over 10 minutes is small. Figure 7(b) displays the histogram of the ratio $d(\rho_e)/\rho_e$: 96% of the 100 considered intervals have $d(\rho_e)/\rho_e < 0.1$. Thus, the use of the average $\rho_e$ for an interval, in place of the exact $\rho_e$ for each of the 100 considered intervals, is justified in what follows.
the 150 $k$-spectra, cannot produce the observed bending which covers a wide range of $k$, and cannot smooth a possible spectral break.

APPENDIX B
LOG-SPACED FREQUENCIES OF THE CLUSTER/STAFF-SA INSTRUMENT

Another argument against the observed spectral bending could be that it is an artifact due to the logarithmic frequency binning of the SA on Cluster.

Indeed, the center frequency of the output channels of SA are distributed logarithmically (between 8.8 Hz and 3.56 kHz), and each channel has a bandpass proportional to its center frequency, $2df = 26f/100$. As the onboard waveforms used by SA are lost, we have no way to check whether a different frequency binning would give different frequency spectra. We only make a crude comparison between an analytical frequency spectrum $P_A(f)$ and the same spectrum integrated by a trapeze method in the logarithmic frequency bands of SA, $P_T(f)$. Note that the trapeze integration has to be made on the logarithms of $P_A(f)$ because the gain of the STAFF-SA receivers is proportional to the logarithm of the power. Figure 8 gives the results of this comparison for a spectral break model (with two spectral indices $\alpha_1 = 2.8$, $\alpha_2 = 4$; left panel) and for an exponential model (right panel) $P_A(f) = f^{-2.7} \exp(-f/f_0)$, with $f_0 \simeq 115$ Hz. The solid lines give $P_A(f)$, and the crosses give $P_T(f)$ in the 18 lowest frequencies of SA. It is clear that the trapeze integration in logarithmic channels does not change the shape of the spectra. The ratio $R = P_T/P_A$ is very close to unity: 1.02–1.03. There is thus a slight systematic overestimation of the spectrum by the trapeze integration in logarithmic bands. This overestimation (2%–3%) cannot increase the downward bending of the spectrum. We conclude that the logarithmic frequency binning of the Spectral Analyser cannot smooth a possible spectral break, and cannot produce an artificial downward bending of the spectra.

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Figure 8. Comparison between an analytic power spectrum $P_A(f)$ (solid line) and the power spectrum $P_T(f)$ (crosses) calculated by a trapeze integration of $P_A(f)$ with the logarithmic frequency binning of the Spectral Analyser ($2df = 26\%$), for the two models discussed in the paper. The difference between $P_T(f)$ and $P_A(f)$ is less than 3%.