Minimum Disturbance Attitude Control Algorithm for DFFSR

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Abstract. Aiming at the attitude disturbance problem of Dual-arm Free Flying Space Robot (DFFSR), firstly, the kinematics equation of DFFSR has been established by using the geometric relationship of each link of DFFSR, the characteristic equation of manipulator, and the linear momentum and angular momentum conservation law of the system in microgravity environment; Secondly, based on the established Generalized Jacobian Matrix (GJM) describing the relationship between the end-effector motion velocity of DFFSR manipulator and the motion velocity of each joint, the relationship between the motion velocity of each joint of DFFSR manipulator and the angular velocity of satellite base has been derived, and the attitude interference model of DFFSR has been established; Finally, the Minimum Attitude Disturbance Map (MADM) algorithm for calculating the DFFSR manipulator has been established, and the attitude control algorithm based on MADM has been proposed. The attitude control method can ensure that the attitude of the satellite base of DFFSR remains unchanged during the movement of the manipulator.

Keywords. DFFSR, MADM, Attitude Control, Kinematics, Attitude Disturbance.

1. Introduction

In the space microgravity environment, the movement of the manipulator mounted on the Dual-arm Free Flying Space Robot (DFFSR) satellite base will produce reaction force and torque on the robot satellite base, which will change the position and attitude of the robot satellite base [1-2]. To complete complex space operations, DFFSR must ensure that the communication system carried in the robot satellite base works normally, so it is necessary to ensure that the antenna carried on the satellite base points to a certain position, such as the communication station on the ground or the communication satellite in orbit [3-4]. Therefore, attitude control is an important research field of DFFSR. Attitude control mainly studies how to ensure that the attitude of the satellite base of DFFSR remains unchanged during the movement of the manipulator, or change the attitude of the satellite base and manipulator according to the specified way to make them reach the preset angle [5-6].

At present, the research on attitude control can be divided into three categories: one is to use the reaction wheel and reaction jet device carried in the robot satellite base to ensure that the attitude or position of the satellite base remains unchanged; Secondly, a manipulator with special geometry is designed to balance the reaction force and torque generated by the manipulator motion on the satellite base by using symmetry, so that the attitude of the robot satellite base remains unchanged; Thirdly, by controlling the trajectory of the manipulator accurately, the attitude of the satellite base of the robot remains unchanged or rotates according to the specified requirements [7-8].

Based on the research of DFFSR kinematics and DFFSR attitude disturbance modelling [5, 9-10], an attitude control algorithm based on the Minimum Attitude Disturbance Map (MADM) has been
proposed. By combining the method of DFFSR dual-arm coordination, the attitude control can be carried out quickly and accurately.

2. Kinematics Equation of DFFSR

For a DFFSR system, the centroid of the system can be expressed as (when $c=r'$, it represents the velocity vector of the end effector of the right arm; when $c=l'$, it represents the velocity vector of the end effector of the left arm):

$$m_ir_i + \sum_{c=1}^{r} \left( \sum_{i=1}^{n} m_i^c \dot{r}_i^c \right) = r_G \left[ m_i + \sum_{c=1}^{r} \left( \sum_{i=1}^{n} m_i^c \right) \right]$$  \hspace{1cm} (1)

where, $r_G \in \mathbb{R}^3$ is the centroid position vector of the system in the inertial coordinate system, $m_i$ is the mass of robot satellite base, $r_i$ is the position vector of satellite base centroid in an inertial coordinate system.

Because the system in a microgravity environment is not affected by external force and moment, the DFFSR system satisfies the conservation law of linear momentum and angular momentum:

$$m^c_i \dot{r}^c_i + \sum_{c=1}^{r} \left( \sum_{i=1}^{n} m^c_i \dot{r}^c_i \right) = 0$$  \hspace{1cm} (2)

$$I_s^c \dot{\omega}_c + m_s^c \dot{r}^c_s \times \dot{r}^c_s + \sum_{c=1}^{r} \left( \sum_{i=1}^{n} (I_i^c \dot{\omega}_c^i + m_i^c \dot{r}_i^c \times \dot{r}_i^c) \right) = 0$$  \hspace{1cm} (3)

The geometric relationship of each connecting link in the DFFSR system can be obtained:

$$I^c_r - I^c_{(i-1)} = I^c_a + (i-1)b^c_{(i-1)}$$  \hspace{1cm} (4)

The characteristic equation of the left (right) arm of the DFFSR system is:

$$I^c_p = I^c_r + b^c_i + \sum_{i=1}^{n} I^c_i, c \in \{l, r\}$$  \hspace{1cm} (5)

The linear velocity of the DFFSR left (right) arm end effector $\dot{p}^c$ is:

$$\dot{p}^c = \dot{r}^c_i + \omega_c \times (p^c - r_i) + \sum_{i=1}^{n} u_i^c \times (p^c - d_c^i) \dot{\theta}_c^i, c \in \{l, r\}$$  \hspace{1cm} (6)

where, $\dot{r}_i$ is the linear velocity of the robot satellite base, $\omega_c$ is the angular velocity of the robot satellite base, $p^c$ is the position vector of the robot left (right) arm end effector, $u_i^c$ is the unit vector of the rotation axis of the ith joint of the left (right) arm, $d_i^c$ is the position vector of the ith joint of the left (right) arm, $\dot{\theta}_c^i$ is the angular velocity of the ith joint of the left (right) arm.

The linear velocity $\dot{p}^c$ of DFFSR left (right) arm end effector united is derived:

$$\dot{P} = \begin{bmatrix} \dot{p}^c_r \\ \dot{p}^c_l \end{bmatrix} = J_s \dot{\theta}_s + J_m \dot{\theta}_m$$  \hspace{1cm} (7)
By substituting the geometric relationship of each link and the characteristic equation of the left (right) arm of the DFFSR system into the equation of the law of conservation of angular momentum is derived:

\[ I_s \ddot{\theta}_s + I_M \ddot{\theta}_M = 0 \]  \hspace{1cm} (8)

Substituting equation (8) into (7), the following equation can be derived:

\[ \dot{P} = (J_M - J_s J_s^{-1} J_M) \dot{\theta}_M = J^* \dot{\theta}_M \]  \hspace{1cm} (9)

where, \( J^* \in R^{4\times n} \) is a generalized Jacobian matrix of DFFSR, which describes the relationship between the end velocity of manipulator and the angular velocity of each joint. \( \dot{\theta}_M \in R^{6\times 1} \) is the joint angular velocity matrix. If \( (J^*)^+ \) is the generalized inverse of \( J^* \), then equation (9) can be written as:

\[ \dot{\theta}_M = (J^*)^+ \dot{P} \]  \hspace{1cm} (10)

Equation (10) decomposes the end motion velocity into angular velocity of each joint, according to \( \dot{\theta}_M \) and \( \dot{P} \) corresponding relationship between them can be applied to the control algorithm of ground robot.

3. DFFSR Attitude Disturbance Model

In this paper, the generalized Jacobian matrix(GJM) describing the relationship between the end motion velocity of DFFSR manipulator and the motion velocity of each joint has been derived. On this basis, the relationship between the motion velocity of each joint of DFFSR manipulator and the angular velocity of satellite base has been derived, and then the attitude interference model of DFFSR has been established.

Angular velocity of DFFSR left (right) arm end effector \( \omega^c \) is:

\[ \omega^c = \omega_s + \sum_{i=1}^{n} \mu_i \dot{\omega}_i, c \in \{l, r\} \]  \hspace{1cm} (11)

General expression of velocity vector of DFFSR left (right) arm end effector is:

\[ \nu^c = \left[ (\dot{p}^c)^T, (\omega^c)^T \right]^T, c \in \{l, r\} \]  \hspace{1cm} (12)

when \( c = 'r' \), it represents the velocity vector of the end effector of the right arm, and when \( c = 'l' \), it represents the velocity vector of the end effector of the left arm, then \( \nu^c \) can be written as follows:

\[ \nu^c = J^c_i \left[ \frac{\nu_s}{\omega_s} \right] + J^c_M \dot{\theta}_M, c \in \{l, r\} \]  \hspace{1cm} (13)

where, \( J^c_i \in R^{6\times 6} \) is the Jacobian matrix related to the robot satellite base, \( J^c_M \in R^{6\times (m+n)} \) is the Jacobian matrix related to the manipulator, \( \dot{\theta}_M \) is the angular velocity of each joint of the robot, \( \dot{\theta}_M = [\dot{\theta}_1^c, \dot{\theta}_2^c, \dot{\theta}_3^c, \ldots, \dot{\theta}_n^c]^T \). In equation (13),

\[ J^c_s = \begin{bmatrix} E_{3\times 3} & -\hat{P}_{rs}^c \\ 0_{3\times 3} & E_{3\times 3} \end{bmatrix}, c \in \{l, r\} \]  \hspace{1cm} (14)
\[ p_{rs}^c = p^r - r_s, c \in \{l, r\} \]  

when \( c = r' \):

\[
J^c_M = \begin{bmatrix}
    u_1' \times (p^r - d_1^l) & \cdots & u_m' \times (p^r - d_m^l) \\
    u_1' & \cdots & u_m'
\end{bmatrix}
\]

when \( c = r' \):

\[
J^l_M = \begin{bmatrix}
    0_{3 \times n} & u_1' \times (p^l - d_1^l) & \cdots & u_n' \times (p^l - d_n^l) \\
    0_{3 \times n} & u_1' & \cdots & u_n'
\end{bmatrix}
\]

where, \( E_{3 \times 3} \) is identity matrix, \( 0_{sz \times z} \) is zero matrix.

Total Linear momentum \( \Omega \) and total angular momentum \( \Theta \) can be respectively expressed as:

\[
\Omega = m_{r_t} \dot{r}_t + \sum_{c=1}^{r} \left( \sum_{i=1}^{n_c} m_i^c \dot{r}_i^c \right)
\]

\[
\Theta = I_r \omega_r + m_{r_t} r_s \times \dot{r}_t + \sum_{c=1}^{r} \left[ \sum_{i=1}^{n_c} \left( I_i^c \omega_i^c + m_i^c \dot{r}_i^c \times \dot{r}_i^c \right) \right]
\]

Write equations (18) and (19) as the function of \( \dot{\nu}_s, \dot{\omega}_s \) and \( \dot{\theta}_M \), then there is:

\[
\begin{bmatrix}
    \dot{\nu}_s \\
    \dot{\omega}_s \\
    \dot{\theta}_M
\end{bmatrix} = \begin{bmatrix}
    ME_{3 \times 3} & -M\dot{r}_{gs} \\
    M\dot{r}_g & I_r
\end{bmatrix} \begin{bmatrix}
    \nu_s \\
    \omega_s
\end{bmatrix} + \begin{bmatrix}
    J_M
\end{bmatrix} \dot{\theta}_M
\]

where, \( M \) is the total mass of DFFSR system:

\[
M = m_t + \sum_{c=1}^{r} \left( \sum_{i=1}^{n_c} m_i^c \right)
\]

\[
r_{gs} = r_g - r_s
\]

where, \( r_g \) is the position vector of the center of mass of DFFSR system, \( r_i^c \) is the position vector of the center of mass of the left (right) arm connecting link.

\[
I_i = I_s + \sum_{c=1}^{r} \left[ \sum_{i=1}^{n_c} \left( I_i^c - m_i^c \dot{r}_i^c \dot{r}_i^c \right) \right]
\]

where, \( r_{is} \)

\[
r_{is} = r_i - r_s
\]

\[
J_M = \sum_{c=1}^{r} \left( \sum_{i=1}^{n_c} m_i^c J_M^c \right)
\]
\[ I_M = \sum_{c=1}^{r} \left[ \sum_{i=1}^{n_c} \left( I^c_{i} \tilde{r}_{c,i}^c + m_i \tilde{r}_{c,i}^c \right) \right] \]  

(26)

Assuming that there is no external force or moment acting on the system and the initial total momentum of the **DFFSR** system is zero, then there is:

\[ \nu_s = \left( -\tilde{r}_s J_s^{-1} I_M - J_M / M \right) \hat{\theta}_M = J_v \hat{\theta}_M \]  

(27)

\[ \omega_s = -I_s^{-1} I_M \hat{\theta}_M = J_\omega \hat{\theta}_M \]  

(28)

where, \( I_s \in R^{3\times3} \) is the inertial matrix of **DFFSR** satellite base, \( I_M \in R^{3\times3} \) is the inertial matrix of manipulator.

\[ J_v = \left( -\tilde{r}_s J_s^{-1} I_M - J_M / M \right) \]  

(29)

\[ J_\omega = -I_s^{-1} I_M \]  

(30)

Replace equations (27) and (28) into equation (13) and eliminate \( \nu_s \) and \( \omega_s \):

\[ \nu^c = \left[ \left( J_v + \tilde{p}_v J_\omega \right) + J^c_M \right] \hat{\theta}_M = J^c \hat{\theta}_M, \quad c \in \{l, r\} \]  

(31)

\[ J^c = \left[ \left( J_v + \tilde{p}_v J_\omega \right) + J^c_M \right], \quad c \in \{l, r\} \]  

(32)

where, the relationship between the movement speed of the manipulator and the attitude angular velocity of the satellite base \( J^c \) is described, that is, the interference of the manipulator motion on the attitude of the robot satellite base is described.

4. Minimum Attitude Disturbance Control Algorithm

4.1. Minimum Attitude Disturbance Map

Singular value decomposition of \( J^c \) in equation (32):

\[ J^c = UDV^T, \quad c \in \{l, r\} \]  

(33)

where, \( U \) and \( V \) are orthogonal matrices and \( D \) is a diagonal matrix. After singular value decomposition of \( J^c \), the directions of maximum disturbance and minimum disturbance caused by manipulator motion on satellite base can be obtained respectively. If the end of the robot is controlled to move along the minimum attitude interference curve, the motion of the robot can produce the minimum interference to the attitude of the satellite base.

The algorithm for calculating left (right) arm Minimum Attitude Disturbance Map(\textbf{MADM}) of **DFFSR** is as follows:

Step 1: input the mass \( m \) of each connecting link of **DFFSR** \( m_i^c \), length \( (a^c_i, b^c_i, l^c_i) \) and moment of inertia \( I^c_i \) and other parameters.

Step 2: calculate the workspace of **DFFSR** and map the target to the joint space.
Step 3: divide each joint angle of DFFSR linearly between the maximum and minimum, and then cycle through each joint angle $\theta_d = [\theta_1', \ldots, \theta_m', \theta_1', \ldots, \theta_n']^T$, do the following calculation:

Step 4: according to equations (16) and (17), calculate the left (right) arm $J^c_J$ value.

Step 5: according to equation (15), calculate the left (right) arm $p^c_s = p^c - r_s$ value.

Step 6: according to equation (14), calculate the left (right) arm $J^c_1$ value.

Step 7: according to equation (29), calculate the left (right) arm $J_v = (-\hat{r}_g I_s^{-1} M_M - J_M^c I_M)$ value.

Step 8: according to equation (30), calculate the left (right) arm $J_\omega = -I_s^{-1} M_M$ value.

Step 9: according to equation (32), calculate current attitude $J^c$ value of DFFSR.

Step 10: singular value decomposition of $J^c$ to get the minimum attitude interference direction of DFFSR.

4.2. Attitude Control Algorithm Based on MADM

An attitude control algorithm based on minimum attitude disturbance map (MADM) has been provided. The attitude control algorithm based on MADM has been described as follows:

Step 1: input the initial state of DFFSR and target. Let the satellite base position of DFFSR is $p_g(0)$, the attitude angle of the satellite base is $\theta_d(0)$, the angle vector of each joint of manipulator is $\theta_d(0)$ . The target location is $p_g$ , the target attitude is $\theta_b$ .

Step 2: read in the left (right) arm MADM.

Step 3: according to equation (10), map the initial attitude of DFFSR to joint space, and set this point as $N_b$ . And put it into $T_o$ table. According to equation (10), the target node is mapped to the joint space, and this point is set as $N_g$ . Set $T_e$ table is empty. Set DIR = 0.

Step 4: loop. If $T_o$ table is empty, it will fail to exit. Go to step 6. Otherwise, select point with the minimum f value in $T_o$ table is called $N_{best}$ . If $N_{best}$ is the target node, then the capture is successful, and then go to step 6. Otherwise, $N_{best}$ put into $T_e$ table, and the following calculation is performed:

1. Calculate attitude angles of front node of $N_b$ named $N_{pre}$ and post node named $N_{next}$ . If $\|N_{pre}, N_g\| > \|N_b, N_g\|$, then DIR=1; Otherwise, if $\|N_{next}, N_g\| > \|N_b, N_g\|$, then DIR=-1

2. If $N_p$ and $N_b$ is on the same attitude interference curve, go to step 5.

3. Generate successor node $N_{su}$ of $N_{best}$ if $N_{su}$ in $T_o$ table or if it appears in $T_e$ table, the point will be abandoned, otherwise it will be put into $T_o$ table, calculate $f(N_{su}) = g(N_{su}) + h(N_{su})$ .

4. The velocity of right (left) arm coordinated motion of DFFSR $\dot{\theta}_M$ is calculated.

Step 5: If $N_b = N_g$ , then the capture is successful, go to step 6. Otherwise,

1. If DIR=1, then $N_b = N_{pre}$ ; if DIR=-1, then $N_b = N_{next}$.

2. The velocity of right (left) arm coordinated motion of DFFSR $\dot{\theta}_M$ is calculated.

Step 6: end of algorithm.

By using the MADM based attitude control algorithm proposed in this paper and adopting an effective search strategy, DFFSR can capture the target quickly and accurately, and at the same time ensure the minimum attitude interference caused by the movement of the manipulator to the satellite base.
5. Conclusion
Based on the kinematic model of DFFSR and the attitude disturbance model of DFFSR, the algorithm of calculating the left (right) arm MADM of DFFSR has been established, and then the attitude control algorithm based on MADM has been proposed. By using this attitude control algorithm, DFFSR can capture the target quickly and accurately, and at the same time, it can ensure the minimum attitude disturbance caused by the movement of the manipulator to the satellite base.

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