Neutrino Bremsstrahlung in Neutron Matter from Effective Nuclear Interactions

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We revisit the emissivity from neutrino pair bremsstrahlung in neutron-neutron scattering, \(nn \to nn \nu \bar{\nu}\), which was calculated from the one-pion exchange potential including correlation effects by Friman and Maxwell. Starting from the free-space low-momentum nucleon-nucleon interaction \(V_{\text{low } k}\), we include tensor, spin-orbit and second-order medium-induced non-central contributions to the scattering amplitude in neutron matter. We find that the screening of the nucleon-nucleon interaction reduces the emissivity from neutrino bremsstrahlung for densities below nuclear matter density. We discuss the implications of medium modifications for the cooling of neutron stars via neutrino emission, taking into account recent results for the polarization effects on neutron superfluidity.

\[\text{26.60.+c, 21.30.Fe, 95.30.Cq, 97.60.Jd}\]

I. INTRODUCTION

The observation of neutron star properties and their evolution provides a challenging astrophysical setting for the study of dense nuclear matter \([1,2]\). On the theoretical side, the main objectives are a more comprehensive understanding of the equation of state and of microscopic nuclear properties, such as pairing and transport phenomena, where progress is tied to improving many-body calculations and techniques for densities ranging from sub-nuclear to a few times saturation density. Accordingly, and spurred by the study of rare isotopes, future theoretical research is also directed towards a systematic study of neutron-rich matter.

A unique method to probe the internal composition of a neutron star is by tracing its temperature evolution with cooling simulations, see e.g., [3–6]. Neutron stars are born in supernova explosions and the interior temperatures initially exceed \(T \sim 10^{11}\) K. As the neutron star cools, neutrinos begin to free stream and essentially leave the star without further interaction. As a consequence, after about 30 s, the long-term cooling of neutron stars is controlled by neutrino emission. This stage lasts up to about \(10^5\) years of age, when cooling by emission of photons becomes more effective. Importantly, the neutron stars remain luminous enough during the cooling, so that the surface temperature can be extracted from space telescope data and the theoretical cooling curves can be confronted with observations.

For proton fractions \(n_p/(n_n + n_p) < 1/9\), direct beta decay does not proceed in neutron star matter due to the imbalance of the neutron and proton Fermi momenta. Therefore, and in the absence of nucleon superfluidity, the dominant neutrino emission comes from \(\nu \bar{\nu}\) bremsstrahlung in nucleon-nucleon collisions and the so-called modified Urca process. The latter corresponds to in-medium beta decay, where the momentum difference between the decaying neutron and the final proton is absorbed by scattering off a second nucleon. For low temperatures \(T \ll E_{F_{n,p}}\) (where \(E_F\) denotes the Fermi energy and we use \(k_B = 1\)), the temperature dependence of these different emission channels is easily power-counted via the degeneracy of the fermions involved in the emission. It follows that the emissivity \(\varepsilon\), which is the total neutrino energy emitted per unit volume and unit time, scales as \(\varepsilon \sim T^8\).

However, the density dependence and strength of the emissivity requires, as input, accurate calculations of the nucleon-nucleon scattering amplitude in the many-body medium. Specifically, the bremsstrahlung processes involve the scattering amplitude for nucleons on the Fermi surface, whereas the modified Urca process involves also off-shell scattering, when e.g., the scattered neutron beta decays and thus its momentum is approximately given by the proton Fermi momentum. A further theoretical challenge lies in the fact that the dominant contribution to neutrino emissivities comes (for bremsstrahlung in \(nn\) collisions exclusively) from the non-central parts of the nucleon-nucleon amplitude, in particular from the tensor force \([7]\). In addition to neutrino emissivities, the general importance of non-central interactions for neutron star properties has been revived recently, where the effects on the magnetic susceptibility \([8]\) as well as on P-wave pairing in neutron star cores \([9]\) have been demonstrated to be crucial.

For the nucleon scattering amplitude, the benchmark calculation of Friman and Maxwell takes into account the long-range one-pion exchange tensor force explicitly and estimates the effects of short-range correlations by cutting off the interaction at short distances and including the short-range rho-exchange tensor force \([7]\). In this work, we start from the free-space low-momentum nucleon-nucleon interaction \(V_{\text{low } k}\) \([10,11]\). The construction of \(V_{\text{low } k}\) is motivated by the differences of the realistic nucleon-nucleon potential models at short distances. The model dependence at short distances \(r < d \approx 0.5\) fm originates from the fact that the interaction cannot be resolved from scattering experiments probing low momenta \(p < \Lambda \approx 2.0\) fm\(^{-1}\) (where \(\Lambda = 1/d\)). Note that the realistic potential models are fitted to phase
shifts below $E_{\text{lab}} \approx 350$ MeV, corresponding to $\Lambda \approx 2.1$ fm$^{-1}$. A systematic method to remove the model dependence is provided by the renormalization group (RG), where the high momentum modes with $p \geq \Lambda$ are integrated out to construct the physically equivalent effective theory. The renormalization group in this context is used as a tool to guarantee that the phase shifts are preserved by the low-momentum interaction under the renormalization. Since $V_{\text{low } k}$ does not have momentum components larger than the cutoff $\Lambda$, it does not have a strongly repulsive core. As a consequence, one does not have to compute a Brueckner $G$ matrix from $V_{\text{low } k}$ in many-body applications. In the sense of the RG, short-range correlation effects are implicitly included in $V_{\text{low } k}$ \cite{12}.

In addition to the phenomenological short-range correlation effects discussed by Friman and Maxwell, the scattering amplitude is screened by the particle-hole polarization of the many-body medium. For the in-medium tensor force, particle-hole screening effects are very important. This follows from a general spin-recoupling argument due to the interference of the central spin-spin part and the tensor part of the nucleon-nucleon interaction, leading to a substantial decrease of the latter \cite{9,12}. Moreover, the presence of the Fermi sea defines a preferred frame, which leads to novel non-central parts in the effective interaction and the scattering amplitude in the many-body medium \cite{9,13}. In pure neutron matter, the scattering amplitude on the Fermi surface has been computed to second-order in $V_{\text{low } k}$, with particular attention to the spin-dependence and non-central interactions, where it was found that the particle-hole screening leads to a substantial decrease of the tensor force and a significant long-wavelength center-of-mass tensor force induced by the medium \cite{9}. As our understanding of the renormalization of the nucleon-nucleon interaction in dense matter improves, it is thus important to include the in-medium modifications of the nuclear force as well as the novel non-central contributions in the calculation of the neutrino emissivities. Furthermore, with recent results for the neutron superfluid S- and P-wave pairing gaps including polarization effects obtained in \cite{9,14}, a consistent calculation of the emissivity is needed.

The neutrino emissivities also receive contributions from the spin-orbit force, which were included in recent calculations of Hanhart et al. \cite{15} and van Dalen et al. \cite{16} starting from the free-space scattering amplitude. As for the tensor force, particle-hole polarization effects reduce the spin-orbit interaction in neutron matter \cite{9}. As a consequence, the $^{3}$P$_{2}$ superfluid pairing gaps in neutron star cores are strongly suppressed to below few keV at second order in $V_{\text{low } k}$ for the pairing interaction \cite{9}. Therefore, one expects that the transition to the P-wave superfluid phase of neutrons is only reached in the very late stages of neutron star cooling. The consistency with data in present cooling simulations of Yakovlev et al. also requires low critical temperatures of the $^{3}$P$_{2}$ superfluid, $T_{c} < 2 \cdot 10^{8}$ K \cite{6}. This corresponds to an angle-averaged gap (as calculated in \cite{9}) in the $m, j = 0$ state of $\Delta < 30$ keV. We note that on the level of the free-space nucleon-nucleon interaction considerably larger P-wave pairing gaps of $\Delta \approx 0.35$ MeV are predicted at nuclear matter density \cite{17}.

If we thus consider neutron star matter above nuclear matter density, the $^{3}$S$_{0}$ superfluidity of neutrons ceases to exist due to the repulsion in the nuclear force and one expects that, at the relevant core temperatures, the neutrons are in the normal phase and the protons are superconducting. In this case, the modified Urca process is strongly suppressed due to proton superfluidity, and the dominant neutrino emission process will be neutrino bremsstrahlung in $nn$ collisions. This is the motivation to focus on the in-medium modification of the bremsstrahlung rate in this work. The second motivation comes from the fact that even at lower densities, where both neutrons and protons pair in the $^{1}$S$_{0}$ channel, the neutrino pair emissivity is more effective compared to the modified Urca process \cite{18}, although this conclusion depends on the values of the pairing gaps and neutrino emission mainly proceeds through the so-called Cooper Pair-Breaking and Formation (PBF) process in the regime $0.2 T_{c} \lesssim T < T_{c}$ \cite{19}.

In this work, we use as input the second-order results for the neutron-neutron scattering amplitude on the Fermi surface computed in \cite{9}. We start by giving the general spin-dependence of the scattering amplitude on the Fermi surface, with a short discussion of the novel contributions in Section II. The non-central scattering amplitudes are included in the calculation of the emissivity from neutrino bremsstrahlung in Section III. This Section follows closely the derivation of Friman and Maxwell \cite{7}. The results for the emissivity over a range of densities is given at the end of Section III. Finally, we conclude with a discussion of superfluidity in neutron stars, which takes into account recent results for the pairing gaps \cite{9,14}. We present some general arguments for the constraints on the density dependence of the gaps and compare the bremsstrahlung rate to the PBF process for various temperatures. Revised estimates of the $np$ and $pp$ bremsstrahlung and modified Urca rates will be reported in a subsequent publication, as they require the nucleon-nucleon scattering amplitude off the Fermi surface as well as an extension to asymmetric matter.

II. SPIN-DEPENDENCE OF THE SCATTERING AMPLITUDE IN NEUTRON MATTER

The scattering amplitude for neutrons on the Fermi surface contains the free-space central (scalar and spin-spin) and non-central (spin-orbit and tensor) parts. In addition, the many-body medium can induce effective interactions, which depend on the two-body center of mass momentum $P = p_{1} + p_{2} = p_{3} + p_{4}$ and one has in general \cite{9}.

\[ P = p_{1} + p_{2} = p_{3} + p_{4} \]
The matrix element

$$S_{\alpha_1, \alpha_2}(q, q') i(i_1 + i_2) \cdot \hat{q} \times \hat{q}'$$

when the emission occurs in the final state or vice versa. The function

$$f(E_f - E_i) \delta^3(P_f - P_i)$$

with unit vectors. Finally, the tensor operators given in Eq. (2) are linearly dependent, with

$$\epsilon_{nn} = N_{\nu} \int \left( \prod_{i=1}^{4} \frac{d^3p_i}{(2\pi)^3} \right) \frac{d^3Q_1}{2\omega_1(2\pi)^3} \frac{d^3Q_2}{2\omega_2(2\pi)^3} (2\pi)^4 \delta(E_f - E_i) \delta^3(P_f - P_i) \frac{1}{s} \left( \sum_{\text{spin}} |\mathcal{M}_{nn}|^2 \right) \omega_\nu F(E_{p_i})$$

where $p_i$ denote the momenta of the incoming and outgoing neutrons and $Q_{1,2} = (\omega_{1,2}, Q_{1,2})$ label the neutron energies and momenta. The delta functions account for energy and momentum conservation, and $\omega_\nu = \omega_1 + \omega_2$ is the total neutrino energy. $N_{\nu}$ denotes the number of neutrino species and $s = 2$ is a symmetry factor for the initial neutrons, when the emission occurs in the final state or vice versa. The function $F(E_{p_i}) = f(E_{p_i}) f(E_{P_i}) (1 - f(E_{P_i})) (1 - f(E_{P_i}))$ is the product of Fermi-Dirac distribution functions $f(E) = (\exp(E/T) + 1)^{-1}$, with neutron energies $E_{p_i}$. The matrix element $\mathcal{M}_{nn}$ includes the nucleon-nucleon scattering part and the coupling to the emitted neutrino pair. For the bremsstrahlung process the corresponding Feynman diagrams at tree-level and with second-order contributions are shown in Fig. 1. The second-order particle-hole intermediate states include all possible excitations for interacting

FIG. 1. Feynman diagrams contributing to the neutrino emissivity from bremsstrahlung to second-order in $V_{\text{low } k}$. The $V_{\text{low } k}$ vertex includes both the direct and the exchange term and the dashed line corresponds to the neutral current. As in [7], the emissivity includes permutations of the neutral current attached to all external lines, whereas the coupling to the internal nucleon lines is suppressed, because it does not lead to the small energy denominator in the additional nucleon propagator as in Eq. (5).
particles on the Fermi surface. In the particle-particle channel, the cutoff in $V_{\text{low-k}}$ provides a regulator, and we evaluate the phase space (including hole-hole states) exactly without angle-averaging approximation. We note that the logarithmic contribution to the quasiparticle interaction (due to the BCS singularity) is integrable and leads to finite Fermi liquid parameters [20], and thus finite emissivities.

Neutrino pair emission from a neutron line is given by the neutral current $V - A$ weak interaction

$$E^\nu_{\text{neutral}} = \frac{G_F}{2\sqrt{2}} \chi_1 \left( \delta_{\mu 0} - g_A \delta_{\mu i} \sigma_i \right) \chi_2 l_\mu,$$

(4)

with Fermi coupling constant $G_F = 1.166 \cdot 10^{-5}$ GeV$^{-2}$, the neutrino current $l_\mu = \bar{u}(Q_1) \gamma_\mu (1 - \gamma_5) u(Q_2)$ and the weak axial-vector coupling constant $g_A = 1.26$. The non-relativistic nucleon spinors are denoted by $\chi$ and $u(Q_{1,2})$ are relativistic spinors for the neutrinos, which are taken to be massless. As in [7], we use a non-relativistic approximation for all nucleon propagators, where the lowest term in an expansion in inverse powers of the nucleon mass is retained. In addition, one neglects the neutrino pair energy $\omega_\nu$ compared to the Fermi energy, since the emitted neutrinos are thermal. For the nucleon propagator $G$, this approximation yields

$$i G(p \pm Q_\nu, E_p \pm \omega_\nu) = \pm i \omega_\nu^{-1},$$

(5)

where the positive sign holds if the weak current is attached to an outgoing nucleon, negative otherwise. It follows that, in the non-relativistic approximation for the nucleon propagators, the vectorial part of the neutral current does not contribute [7].

The matrix element $M_{nn}$ includes the strong interaction part with spin-dependence given by Eq. (2). The spin sum over the squared matrix element is carried out independently for the pieces coming from the nucleon-nucleon exchange channels, where only the non-central parts in the amplitude are found to contribute. After contraction with the lepton trace given by

$$\text{Tr}(l_i l_j^\dagger) = 8 \left( Q_{1,j} + Q_{2,j} Q_{1,j} - g_{ij} Q_1 \cdot Q_2 + i \epsilon_{i\alpha j\beta} Q_1^\alpha Q_2^\beta \right),$$

(6)

one finds

$$\sum_{\text{spin}} |M_{nn}|^2 = 64 \frac{g_A^2}{G_F^2} \frac{\omega_1 \omega_2}{\omega_p} \frac{\pi^4}{m_n^2 k_{F_n}^2} A_{nn}^2(q, q')$$

$$= 64 \frac{g_A^2}{G_F^2} \frac{\omega_1 \omega_2}{\omega_p} \frac{\pi^4}{m_n^2 k_{F_n}^2} \left( \tilde{A}_{\text{tensor}}^2(q, q') + \tilde{A}_{\text{exch. tensor}}^2(q, q') - \tilde{A}_{\text{exch. tensor}}(q, q') \tilde{A}_{\text{exch. tensor}}(q, q') \right)$$

$$+ A_{\text{spin-orbit}}^2(q, q') + A_{\text{diff. vector}}^2(q, q') + 3 A_{\text{cross vector}}^2(q, q'),$$

(7)

where we have dropped terms that vanish upon angular integrations over $Q_{1,2}$ in the emissivity, Eq. (3), when the neutrino momenta are neglected in the momentum-conserving delta function.

For the evaluation of the phase space, one can easily perform the integrals over the neutrino momenta with $|Q_{1,2}| = \omega_{1,2}$ by inserting

$$1 = \int d\omega_\nu \delta(\omega_\nu - \omega_1 - \omega_2),$$

(8)

as the energy-conserving delta function depends only on the total neutrino energy. Moreover, one can decouple the angular parts in the neutron phase space and trade the radial momentum for energy integrals, by restricting the interacting neutrons to the Fermi surface, since they are strongly degenerate for typical neutron star temperatures. Corrections to this approximation scale as $T/E_{F_n}$. For this purpose, one replaces

$$d^3 p_i \rightarrow d^3 p_i \frac{m_n^2}{k_{F_n}^2} \delta(p_i - k_{F_n}) \int dE_{p_i}.$$  

(9)

Finally, we introduce the integration over momentum transfers through respective delta functions in the direct and the exchange channels,

$$1 = \int d^3 q \delta^3(q - p_1 + p_3) = \int d^3 q' \delta^3(q' - p_1 + p_3).$$

(10)
After carrying out the angular integrations, we find the general expression for the emissivity

\[
\varepsilon_{nn} = \frac{64 g_A^2 G_F^2 m_n^2}{15 \cdot 2^9 \pi^6} N_\nu I_{\nu \bar{\nu}} \langle A_{nn}^2 \rangle = 0.781 T_9^8 N_\nu \left( \frac{m_n^*}{m_n} \right)^2 \left( \frac{1.7 \text{ fm}^{-1}}{k_{F_n}} \right) \langle A_{nn}^2 \rangle \text{ erg cm}^{-3} \text{ s}^{-1},
\]

where \( I_{\nu \bar{\nu}} \) is the result of the integrals over the total neutrino and neutron energies convoluted with the thermal distribution factors \( F(E_{\nu \bar{\nu}}) \) which is identical to the expression in [7], and the square of the scattering amplitude averaged over the Fermi surface \( \langle A_{nn}^2 \rangle \) is given by

\[
\langle A_{nn}^2 \rangle = \int_0^{2k_{F_n}} \frac{dq}{k_{F_n}} \int_0^{2k_{F_n}} \frac{dq'}{k_{F_n}} \Theta(4k_{F_n}^2 - q^2 - q'^2) A_{nn}^2(q, q') = \int_0^{\pi/2} \frac{dq}{k_{F_n}} \int_0^{\pi/2} \frac{dq'}{k_{F_n}} \sin^2 \phi A_{nn}^2(q, \sqrt{4k_{F_n}^2 - q^2 \sin^2 \phi}).
\]

The remaining two-dimensional integral is calculated numerically and our results for \( N_\nu = 3 \) neutrino flavors are shown in Fig. 2 for densities ranging from \( k_{F_n} = 1.0 - 2.0 \text{ fm}^{-1} \). All results given in Fig. 2 include the effective mass obtained from the lowest order \( V_{\text{low } k} \). The effective mass varies from \( m_n^*/m_n = 0.95 \) at \( k_{F_n} = 1.0 \text{ fm}^{-1} \) to \( m_n^*/m_n = 0.78 \) at \( k_{F_n} = 2.0 \text{ fm}^{-1} \), and is in this range well approximated by a linear curve versus Fermi momentum. We note that one expects an increase of the effective mass in the induced interaction (which is compensated by the quasiparticle strength \( z_{k_F} \)), as can be seen from the results of the full RG calculation for neutron matter [14]. Furthermore, our results do not include a renormalization of \( g_A \) in the medium to \( g_A \approx 1.0 \).

![FIG. 2. The neutrino emissivity from bremsstrahlung in neutron-neutron scattering \( \varepsilon_{nn} \) versus Fermi momentum \( k_{F_n} \) in neutron matter. All curves include the lowest-order effective mass obtained from \( V_{\text{low } k} \), see [9]. The curve labeled V denotes the lowest-order emissivity obtained from the free-space low-momentum interaction. In comparison, we give the results obtained by Friman and Maxwell (curve F) using the uncorrelated one-pion exchange (OPE) tensor force without the exchange terms [7]. (In [7], the inclusion of exchange terms, the \( \rho \) tensor force, and short-range correlation effects yields a multiplicative suppression factor of \( \approx 0.62 \) at nuclear matter density or \( k_{F_n} = 1.7 \text{ fm}^{-1} \).) The thin curves correspond to the \( V_{\text{low } k} \) result with, respectively, second-order renormalization of the tensor, spin-orbit or spin non-conserving forces in the medium included. The curve labeled T is the full second-order result, where both particle-hole channels and the particle-particle channel are taken into account.

First, we compare the lowest-order \( V_{\text{low } k} \) results to the calculation of Friman and Maxwell from uncorrelated one-pion exchange (OPE), see Eq. (52) in [7]. We remark that exchange terms, the inclusion of the \( \rho \) tensor force and correlation effects lead to a multiplicative suppression factor in the calculation of Friman and Maxwell of \( \approx 0.62 \) at nuclear matter density. We find that \( V_{\text{low } k} \) gives similar rates, but without the need to estimate the correlation distance, which is experimentally unconstrained in neutron matter.

Recently, Hanhart et al. [15] and van Dalen et al. [16] also computed the neutrino pair emissivity from bremsstrahlung employing Low’s theorem for soft emission, with the free-space on-shell \( T \) matrix as input. These results provide a model-independent low-density limit on the emissivity from neutrino bremsstrahlung. In both works,
the emissivity was found to be reduced by a factor $\approx 1/4$ compared to the OPE result of Friman and Maxwell at saturation density. However, the applicability to relevant neutron star densities is limited when one starts from the free-space scattering amplitude. This is due to the fact that at second-order in the $T$ matrix the spin-spin and tensor part of the amplitude mix due to screening in the particle-hole channel. If one denotes the tensor part of the free-space amplitude by $T_{\text{tensor}}$, then the second-order contribution will be proportional to $T_{\text{tensor}} k_F a_8$ at low momenta, where $a_8$ is the S-wave scattering length coming from the spin-spin part of the $T$ matrix. Even at low-densities, e.g., $k_{F_n} = 1/2 \text{fm}^{-1}$ (i.e., $\rho = 1/40 \rho_0$), this would be a very large correction, which is not accounted for in [15,16].

Beyond the lowest-order result, we find that the renormalization of the non-central parts of the nucleon-nucleon interaction in the medium considerably reduces the emissivity, especially at sub-nuclear densities, as shown in Fig. 2. We emphasize that a second-order calculation cannot give final results, but it provides a range for the effects due to particle-hole screening. From the denominators of the induced interaction [22], one expects higher orders to somewhat decrease the second-order results. However, this argument is not straightforward for non-central interactions and a definitive conclusion requires explicit calculations of higher-orders with full non-central spin-dependence. Referring to a previous calculation of the induced quasiparticle interaction (i.e., for $q = 0$) including tensor forces [23], it was also found that the tensor force is reduced in the medium. For the cooling of neutron stars, as well as spin-isospin response in supernovae, the modification of transport properties in the medium is very important. Our work shows that particle-hole effects must be taken into account in a realistic calculation of the emissivities.

As discussed in the Introduction, an accurate estimate of the bremsstrahlung rate is important particularly at high densities, since the competing processes except for possible emission by PBF are expected to be strongly suppressed due to proton superfluidity. Therefore, we will compare the strength of the open PBF channel to the emission from a non-superfluid core of neutrons in the next Section.

IV. SUPERFLUIDITY AND COMPARISON WITH PBF PROCESSES

The temperatures in the interior of cooling neutron stars can be well below the critical temperature for neutron or proton superfluidity. This leads to a strong reduction of the neutrino emissivities since the fraction of particles that are unpaired scales exponentially with the temperature as $\exp(-2\Delta/T)$, where $\Delta$ denotes the zero temperature gap. Thus, the modification of the emissivities relies on an accurate calculation of the proton and neutron pairing gaps in neutron star matter, which must include the particle-hole polarization in the medium.

Before discussing recent results for the neutron pairing gaps in pure neutron matter [9,14], we proceed with some remarks on the superfluid properties and rather general in-medium modification arguments. The strongest attraction in the nuclear force is in the S-wave for laboratory energies below $E_{\text{lab}} \lesssim 250 - 260 \text{MeV}$, i.e., for back-to-back scattering of particles with momenta $k = k_{F_n} \lesssim 1.7 - 1.8 \text{fm}^{-1}$. Thus, for densities below nuclear matter density $\rho \lesssim \rho_0$, one expects that both neutrons and protons form a superfluid in the isotriplet $^1S_0$ channel (Note that for typical proton fractions of $n_p/(n_n + n_p) \approx 0.05$, one has for the proton Fermi momentum $k_{F_p} \approx 1/3 k_{F_n}$). At higher densities, one concludes from the free-space scattering phase shifts that the neutrons are expected to pair in the $^3P_2$ state [21]. Eventually, the ground state of matter at high densities has to be determined in a model of dense matter, as realistic interactions are constrained to relative momenta $k \lesssim 2.1 \text{fm}^{-1}$.

It is well known that polarization effects on the nucleon-nucleon interaction, which are necessary in order to satisfy the Pauli principle in microscopic calculations [22], lead to a strong reduction of the superfluid gaps in neutron stars [9,14,24–26]. The effect of the induced interaction on pairing can be understood by considering the second-order contributions. Higher-order terms modify the strength of the second-order result, but usually do not alter whether the induced interaction is attractive or repulsive in the particular channel.

For S-wave pairing of neutrons, the dominant part in the quasiparticle interaction is the central spin-spin delta function $G_0 \approx 0.6 - 0.8$ [14]. Neglecting the smaller contributions, one has for the induced pairing interaction in the $S = 0$ state,

$$A_{\text{central}}^{\text{ind}}(\cos \theta_q) = 3 G_0^2 (U(q/k_{F_n}) + U(q'/k_{F_n})), \quad (13)$$

where direct and exchange particle-hole channels are accounted for, $U(q/k_{F_n})$ denotes the (positive) static Lindhard function and for back-to-back scattering $q = k_{F_n} \sqrt{2 - 2 \cos \theta_q}$ ($q' = k_{F_n} \sqrt{2 + 2 \cos \theta_q}$) with scattering angle $\theta_q$. The projection of the sum of Lindhard functions in Eq. (13) on S-wave yields $\langle U(q/k_{F_n}) + U(q'/k_{F_n})\rangle_{=0} = 2(1+2 \log 2)/3 \approx 1.59$ (see also [27]). As a result, spin fluctuations will reduce the pairing interaction and consequently the superfluid gap will close at somewhat lower densities than one would expect from the free-space $^1S_0$ phase shifts. In the RG calculation of the effective interaction and the scattering amplitude on the Fermi surface [14] (for a discussion of the RG approach see also [28]), it is found that the maximum $^1S_0$ pairing gap is reduced to 0.8 MeV at $k_{F_n} \approx 0.8 \text{fm}^{-1}$.
and that the gap disappears at $k_{Fn} \approx 1.5\,\text{fm}^{-1}$ or $\rho \approx 2/3\,\rho_0$. This corresponds to a maximum critical temperature of $T_c = 5.3 \cdot 10^9\,\text{K}$.

The study of polarization effects on P-wave pairing at higher densities is more involved, since spin-orbit and tensor forces are crucial for the reproduction of the P-wave phase shifts in vacuum and consequently one has to address the renormalization of non-central interactions in the medium. In particular, the medium-induced spin-orbit force leads to a strong suppression of the $^3\text{P}_2$ gaps, which is e.g., due to the interference of the central spin-spin and the spin-orbit force at second order [9]. A similar argument as for S-wave pairing reproduces the effect of induced pairing interactions qualitatively and one has

$$A_{\text{ind}}(\cos \theta_q) = -\frac{1}{2} G_0 \langle V_{SO} \rangle (U(q/k_{Fn}) + U(q'/k_{Fn})) \frac{qq'}{k_{Fn}^2},$$

(14)

where the dimensionless $\langle V_{SO} \rangle < 0$ denotes an averaged spin-orbit matrix element. The induced contributions due to the mixing of spin-orbit and tensor forces are also repulsive, with a similar but more complicated momentum dependence. For the induced $^3\text{P}_2$ pairing matrix element, the largest contribution comes from the same $l = 0$ projection of the sum of the Lindhard functions, where the additional momentum-dependent factors $qq'$ are absorbed in the spin-orbit operator $L \cdot S \sim i(\sigma_1 + \sigma_2) \cdot q \times q'$ (note the unit vectors in Eq. (2)). In [9] it was found that second-order polarization effects lead to a strong depletion of the $^3\text{P}_2$ pairing gap from $\Delta \approx \frac{3}{c} \text{MeV}$ (obtained from the free-space $V_{\text{bare}}(k)$) to superfluid gaps on the level of few keV at nuclear matter density. This corresponds to a ratio of 0.45 of the second-order to lowest-order pairing interaction, and therefore it is expected that the suppression of the gap at second order is robust with pairing gaps below $\Delta \lesssim 1 - 10\,\text{keV}$. The latter value corresponds to a critical temperature of $T_c = 10^{6.8 - 7.8}\,\text{K}$. With surface temperatures from observational data $T_s \gtrsim 10^{5.6}\,\text{K}$, i.e., core temperatures $T \gtrsim 10^{7.4}\,\text{K}$ (see e.g., Table 2 in [29]), it is thus expected that the $^3\text{P}_2$ superfluid phase is only reached at late cooling stages. In fact, Yakovlev et al. have checked that for critical temperatures $T_c < 2 \cdot 10^8\,\text{K}$ the $^3\text{P}_2$ phase has no impact for the cooling of middle-aged neutron stars [6]. We therefore proceed and compare the bremsstrahlung emissivity in normal matter to the emission of neutrinos from the $^1\text{S}_0$ superfluid condensate only.

![FIG. 3. Comparison of the emissivity from neutrino pair bremsstrahlung in non-superfluid neutron matter (total in Fig. 2) and the $^1\text{S}_0$ PBF process as a function of Fermi momentum and for various temperatures. Results are shown using the $^1\text{S}_0$ superfluid gaps obtained in the RG approach with adaptive $z_{\kappa\rho}$ factor [9].](image)

Although superfluidity strongly suppresses the standard neutrino emission channels, there is a powerful mechanism for neutrino emission in superfluid matter due to the PBF process [19]. This is a significant source for neutrinos from the density range, where the temperature lies between $0.2\,T_c(k_{Fp}) \lesssim T < T_c(k_{Fn})$. The emissivity from the $^1\text{S}_0$ neutron pair-breaking and formation process is given by

$$\varepsilon_{\text{PBF}} = 1.170 \cdot 10^{21} \, N_\nu \, \frac{m_n^2}{m_n \cdot m_n} \, k_{Fn} \, T_\tau^2 \, F(\tau) \, \text{erg cm}^{-3} \text{s}^{-1},$$

(15)
where the function $F(\tau)$ depends on the critical temperature $\tau = T/T_c(k_{F_n})$ at given neutron Fermi momentum (for S-wave pairing $T_c(k_{F_n}) = 0.57\Delta(k_{F_n})$). Details and a parametrization of $F(\tau)$ which we employ can be found in [18]. In Fig. 3 we compare our results for the bremsstrahlung emissivity in the normal state to the $^{1}\!S_0$ PBF process. While it demonstrates that the PBF process is extremely effective at higher temperatures $T \gtrsim 10^{8.5}\text{ K}$, at lower temperatures the density range for emission via PBF is rather narrow and we expect the volume-integrated bremsstrahlung emission to dominate. Fig. 3 also nicely illustrates how the cooling of neutron stars is able to probe the internal structure, with the enhanced emissivity from the PBF process serving as a clear signal of the superfluid phase.

We finally note that at higher neutron densities the proton PBF process is also in effect. However, at lower temperatures the cooling through neutrino bremsstrahlung in normal-state $nn$ collisions will be larger for the same reason as above, and in addition due to the smaller number of protons.

V. SUMMARY

We have computed the emissivity from neutrino pair bremsstrahlung in nucleon-nucleon collisions in pure neutron matter, within an effective theory of quasiparticle interactions in the vicinity of the Fermi surface. The effective scattering amplitude is calculated from the model-independent low-momentum nucleon-nucleon interaction $V_{\text{low}k}$ to second order, keeping the full non-central spin dependence [9]. We find that inclusion of medium modifications, in particular the renormalization of the tensor force, reduces the emissivity compared to the tree-level $V_{\text{low}k}$ result by a multiplicative factor 0.64 at nuclear matter density (or a factor 0.5 relative to the direct one-pion exchange estimate). At sub-nuclear densities, the reduction is 0.2 at a fifth of nuclear matter density. Furthermore, we find that the effect of spin non-conserving parts in the scattering amplitude is rather small. While the temperature dependence of the emissivity is naturally very important for cooling simulations, the density dependence needs to be considered as well, since the luminosity of the neutron star involves an integral over the volume of the star.

When polarization effects are included in the $^3P_2$ neutron pairing interaction, a considerable reduction of the superfluid gaps was found [9]. Therefore, the neutrino bremsstrahlung process could be more important for the cooling of neutron stars than believed, since superfluidity of protons suppresses Urca processes as well as bremsstrahlung in $np$ and $pp$ scattering, whereas neutrons remain in the non-superfluid phase at higher densities at typical core temperatures in the neutron cooling stage. Even at lower densities, where neutrons form a $^1S_0$ superfluid, the lore is that neutrino bremsstrahlung from neutrons dominates the modified Urca process in the presence of superfluidity [18], although the PBF channel is considerably more effective. We also note that the bremsstrahlung process, in contrast to the Urca channels, produces $\nu_\mu$ and $\nu_\tau$ neutrinos. In order to compare our results for the bremsstrahlung rates to the emission through the PBF process, we have shown results for the PBF process for realistic pairing gaps obtained in the RG approach [14]. This nicely demonstrates that the PBF channel is very effective and can act as a powerful signal of superfluidity for temperatures comparable to the maximal critical temperature, but is restricted only to a narrow density range for lower temperatures. In the latter regime, one thus expects the integrated emissivity from bremsstrahlung to dominate.

Our results provide neutrino bremsstrahlung rates derived from successfully used effective nuclear interactions. The emissivities can be used as microscopic input for neutron star cooling simulations, in conjunction with the superfluid gaps calculated in [9,14]. A RG calculation of effective interactions in asymmetric matter, which will address both the effects of induced interactions on proton pairing in neutron star matter as well as higher-order contributions to non-central interactions, is in preparation [30]. A self-consistent treatment of the tensor force is necessitated by a substantial renormalization at second order. Such studies will further constrain neutrino emissivities microscopically in a consistent framework. Our results can then be incorporated in cooling simulations of neutron stars to possibly constrain the structure of neutron stars and their densest interiors.

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