Particle Aspects of Cosmology
and
Baryogenesis*

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December 3, 2021

Abstract

An introduction to particle aspects of cosmology with particular reference to primordial nucleosynthesis, dark matter and baryogenesis is provided. In particular, various scenarios—Grand Unified Theory baryogenesis, electroweak baryogenesis and baryogenesis through leptogenesis are reviewed.

*Lectures delivered at Department of Physics, KFUPM, Dhahran, Saudi Arabia (April 2002 and September 2002).
1 Introduction

I will first discuss Nucleosynthesis and show how it leads to two big problems in cosmology: Dark Matter and Baryogenesis. Before I discuss Primordial Nucleo-synthesis, I give you some background:

\[
\text{Cosmology – Physics of Early Universe} \quad \downarrow \\
\quad \text{High Temperature} \quad \uparrow \\
\quad \text{High Energy} \quad \uparrow \\
\text{Particle Physics – Physics at Short Distances}
\]

2 Thermal Equilibrium

Consider an arbitrary volume \( V \) in thermal equilibrium with a heat bath at temperature \( T \). The particle density \( n_i \) (i, particle index) at temperature \( T \) is given by

\[
n_i = \frac{N_i}{V} = \frac{g_i}{2\pi^2} \left( \frac{k_B T}{\hbar c} \right)^3 \int_0^\infty \left[ \exp \left( \frac{E}{k_B T} \right) \pm 1 \right]^{-1} z^2 dz. \tag{1}
\]

The energy density is given by

\[
\rho_i \, c^2 = \frac{g_i}{2\pi^2} \left( \frac{k_B T}{\hbar c} \right)^3 \left( k_B \right) \int_0^\infty \left[ \exp \left( \frac{E}{k_B T} \right) \pm 1 \right]^{-1} \left( \frac{E}{k_B T} \right) z^2 dz, \tag{2}
\]

where

\[
z = \frac{q c}{k_B T}, \quad E = \left[ (q c)^2 + (m_i c^2)^2 \right]^{1/2} \tag{3}
\]

and \( g_i \) are the number of spin states, \( q \) is the momentum of the particle and \( m_i \) is its mass. The + sign is for the fermions (\( F \)) and – sign is for the bosons (\( B \)). In particular for \( i = \text{photon}, m = 0, \, g = 2 \). In writing Eqs. (1) and (2),
we have put the chemical potential \( \mu_i = 0 \). For photon \( \mu = 0 \). Since particles and antiparticles are in equilibrium with photons \( \mu_i = -\mu_i \). If there is no asymmetry between the number of particles and antiparticles, \( \mu_i = \mu_i = 0 \). If the difference between the number of particles and antiparticles is small compared with the number of photons,\[ \left| \frac{\mu_i}{k_B T} \right| \ll 1 \quad (4) \]
and the chemical potential can be neglected. For the photon gas, we get from Eqs. (1) and (2) \([T_0 = 2.725^0 \text{ K}, \text{ subscript 0 denotes the present value of the temperature of cosmic background (CMB) radiation}]\)
\[
\begin{align*}
n_\gamma &= 2 \frac{\zeta(3)}{\pi^2} \left( \frac{k_B T}{\hbar c} \right)^3 = 2 \frac{1.2}{\pi^2} \left( \frac{1}{\hbar c} \right)^3 (k_B T)^3 \quad (5) \\
n_{\gamma_0} &= \frac{2.4}{\pi^2} \left( \frac{1}{\hbar c} \right)^3 (k_B T_0)^3 = 410.50 \text{ cm}^{-3} \quad (6) \\
n_\gamma (T) &= 410.50 \left( \frac{T}{2.725} \right)^3 \text{ cm}^{-3}, \\
\rho_\gamma c^2 &= 6 \frac{\zeta(4)}{\pi^2} \left( \frac{1}{\hbar c} \right)^3 (k_B T)^4 \\
&= \frac{\pi^2}{15} \left( \frac{1}{\hbar c} \right)^3 (k_B T)^4 \approx 2.7 n_\gamma (k_B T) \quad (8) \\
\rho_{\gamma_0} &= 2.60 \times 10^{-10} \text{ GeV cm}^{-3} \quad (9)
\end{align*}
\]

The zeta functions are defined as follows
\[
\begin{align*}
\int_0^\infty \frac{z^2 dz}{e^z - 1} &= \Gamma (3) \zeta (3) \\
\int_0^\infty \frac{z^2 dz}{e^z + 1} &= (1 - 2^{-2}) \frac{3}{4} \Gamma (3) \zeta (3) \\
\int_0^\infty \frac{z^3 dz}{e^z - 1} &= \Gamma (4) \zeta (4) \\
\int_0^\infty \frac{z^3 dz}{e^z + 1} &= (1 - 2^{-3}) \frac{7}{8} \Gamma (4) \zeta (4)
\end{align*}
\]

For a gas of extreme relativistic particles (ER), \( k_B T \gg m_i c^2, qc \gg m_i c^2 \), we thus get
\[
\begin{align*}
n_B &= \left( \frac{g_B}{2} \right) n_\gamma, \\
\rho_B &= \left( \frac{g_B}{2} \right) \rho_\gamma \quad (10a)
\end{align*}
\]

\[ 3]
\[ n_F = \frac{3}{4} \left( \frac{g_F}{2} \right) n_\gamma, \quad \rho_F = \frac{7}{8} \left( \frac{g_F}{2} \right) \rho_\gamma. \]  

(10b)

The entropy \( S \) for the photon gas is given by

\[ S = \frac{R^3}{T} 4 \frac{1}{3} \rho_\gamma (T). \]  

(11)

For any relativistic gas

\[ S = \frac{R^3}{T} 4 \frac{1}{3} \rho (T). \]  

(12)

Thus for a gas consisting of extreme relativistic particles (bosons and fermions): \((\hbar = c = 1)\)

\[ n(T) = \frac{1}{2} g'(T) n_\gamma(T) \]

\[ = \frac{1.2}{\pi^2} g'(T) \left( k_B T \right)^3 \]  

(13)

\[ \rho (T) = \frac{1}{2} g_*(T) \rho_\gamma(T) \]

\[ = \frac{\pi^2}{30} g_*(T) \left( k_B T \right)^4 \]  

(14)

\[ S = \frac{R^3}{T} 2 \frac{1}{3} g_*(T) \rho_\gamma(T), \]  

(15)

where

\[ g'(T) = \sum_B g_B + \frac{3}{4} \sum_F g_F \]  

(16a)

\[ g_*(T) = \sum_B g_B + \frac{7}{8} \sum_F g_F \]  

(16b)

are called the “effective” degrees of freedom. We note that entropy per unit volume is given by

\[ \frac{1}{k_B} S = \frac{s}{k_B} = \frac{2\pi^2}{45} g_*(T) \left( k_B T \right)^3. \]  

(17)
For non-relativistic gas $k_B T \ll m_i c^2$, we use the Boltzmann distribution

$$n_i = \frac{g_i}{2\pi^2} \left( \frac{k_B T}{\hbar c} \right)^3 \int_0^\infty \exp \left( -\frac{E}{k_B T} \right) z^2 \, dz$$  \hspace{1cm} (18)

$$E \approx m_i c^2 \left[ 1 + \frac{1}{2} \frac{q^2 c^2}{(m_i c^2)^2} \right].$$  \hspace{1cm} (19)

This gives

$$n_i = \left[ \frac{g_i}{(2\pi)^{3/2}} \right] \left( \frac{k_B T}{\hbar c} \right)^3 \left[ \left( \frac{m_i c^2}{k_B T} \right)^{3/2} e^{-m_i c^2/k_B T} \right]$$  \hspace{1cm} (20)

$$\rho_i = n_i m_i.$$  \hspace{1cm} (21)

Expansion rate is given by the Hubble Parameter

$$H = \frac{\dot{R}}{R}$$

where $[R(t)$ is a scale factor for distances in co-moving coordinates and describes the expansion of the universe$]$

$$\dot{R}^2 = \frac{8\pi G_N \rho}{3} R^2 - k c^2 + \frac{\Lambda c^2 R^2}{3}$$  \hspace{1cm} (22)

The second term on the right hand side is due to curvature of space while the third term containing the cosmological constant $\Lambda$, which being very small ($|\Lambda| < 3 \times 10^{-52} \text{ m}^{-2}$) is usually neglected. There is now evidence that $k = 0$ (in any case for early universe $\rho \sim 1/R^4$ and as such the second and third terms on r.h.s. of Eq. (22) can be neglected compared to the first term). Thus

$$H = \frac{\dot{R}}{R} \simeq \left( \frac{8\pi G_N \rho}{3} \right)^{1/2}$$  \hspace{1cm} (23)

For relativistic particles as already seen

$$\rho(T) = \frac{\pi^2}{30} g_*(T) (k_B T)^4$$

and

$$g_*(T) = \sum_{\text{Bosons}} g_B + \frac{7}{8} \sum_{\text{F}} g_F$$
denote the effective degrees of freedom. Thus

\[ H = \sqrt{\frac{4\pi^3}{45}} \frac{[g^*(T)]^{1/2} (k_B T)^2}{\hbar M_{pl}} \]

\[ = 1.66 [g^*(T)]^{1/2} \frac{(k_B T)^2}{\hbar M_{pl}} \]

\[ = 0.21 [g^*(T)]^{1/2} \left( \frac{k_B T}{\text{MeV}} \right)^2 \text{s}^{-1} \quad (24) \]

where \( \hbar M_{pl} = G_N^{-1/2} \) is the Planck mass: \( M_{pl} \simeq 10^{19} \text{ GeV} \).

### 2.1 Freeze Out

At high temperatures \( (k_B T \gg m) \), thermodynamic equilibrium is maintained through the processes of decays, inverse decays and scatterings. As the universe cools and expands, the reaction rates will fail to keep up with the expansion rate and there will come a time when equilibrium will no longer be maintained. At various stages then, depending on masses and interaction strengths, different particles will decouple with a “freeze out” surviving abundance. We now determine conditions under which the statistical equilibrium is established.

From dimensional analysis, the reaction rate for a typical process can be written as follows. For the decay of a \( X \)-particle, the decay rate is given by

\[ \gamma_X = g_d \alpha_X m_X \frac{m_X}{[(k_B T)^2 + m_X^2]^{1/2}}, \quad (25) \]

where \( m_X \) is the mass of the \( X \)-particle, \( \alpha_X = \frac{f_X^2}{4\pi} \) is the measure of coupling strength of \( X \)-particle to the decay products, and \( g_d \) are number of spin states for the decay channels. Note that

\[ \gamma_X \approx \begin{cases} g_d \alpha_X m_X & k_B T \ll m_X, \\ g_d \alpha_X \frac{m^2}{k_B T} & k_B T \gg m_X. \end{cases} \quad (26) \]

The reaction rate for the collision processes is given by

\[ \gamma_c = \langle \sigma v \rangle \left[ \text{number of target particles per unit volume which is proportional to } (k_B T)^3 \right]. \]

6
Thus

\[ \gamma_c \simeq \frac{g_d \alpha_X^2 (k_B T)^5}{\left[ (k_B T)^2 + m_X^2 \right]^2} \]  

(27)

\[ \gamma \geq H \quad \text{Equilibrium} \]  

(28)

\[ \gamma \sim H \quad \text{Freeze Out} \]  

(29)

\[ \gamma \ll H \quad \text{Out of Equilibrium} \]  

(30)

3 Primordial Nucleosynthesis

After the discovery of cosmic back ground radiation (CMB), the first success of big bang cosmology was the correct prediction of primordial abundance of He (\( \sim 24\% \)). This was cooked by nuclear reactions when the universe was seconds old (\( T \sim 10^{10} \text{ K} = 1 \text{ MeV} \))

At temperatures \( \geq 1 \text{ MeV} \), the weak reactions such as

\[ \bar{\nu}_e + p \leftrightarrow e^+ + n \]

\[ e^- + p \leftrightarrow \nu_e + n \]  

(31)

are still fast compared with the expansion rate of the universe to maintain thermodynamic equilibrium between \( p \) and \( n \). The abundance ratio at equilibrium is given by

\[ \frac{n}{p} \sim e^{-\Delta m/(k_B T)}, \quad k_B T > k_B T_D \sim 1 \text{ MeV}, \ t = 1 \text{ sec}. \]  

(32)

Using \( \Delta m = (m_n - m_p) = 1.3 \text{ MeV} \) and \( k_B T = k_B T_D = 1 \text{ MeV} \), we find \( n/p = 0.27 \). The rates for the above reactions are given by weak interactions except that we have to take into account Pauli’s exclusion principle. Then

\[ \gamma^{(n \rightarrow p)} = \frac{1}{\pi^2 \hbar^4} \frac{G_F^2}{\pi} A \int E_e^2 p^2 d\nu \left[ 1 + e^{E_e/k_B T} \right]^{-1} \left[ 1 - \left[ 1 + e^{E_e/kT} \right]^{-1} \right] \]  

(33)

where

\[ A = g_V^2 + 3g_A^2 = g_V^2 \left( 1 + 3g_A^2/g_V^2 \right), \quad g_A/g_V \simeq 1.26, \ g_V \simeq 0.9750, \]  

(34)

g_V and \( g_A \) are vector and axial vector coupling constants of the nucleon. The second factor in the integral is due to Pauli Principle which suppresses...
the rate by a factor equal to fraction of all states that are unfilled. For 
\( k_B T \gg Q = (m_n - m_p) \) we have

\[
E_e \simeq E_\nu \simeq p_\nu c = qc = (k_B T) z.
\]

Thus we obtain

\[
\gamma^{(n \to p)} \simeq \gamma^{(p \to n)} \approx \frac{1}{\pi^2} \frac{G_F^2}{\pi} A \left( \frac{k_B T}{\hbar c} \right)^3 (k_B T)^2 \int_0^\infty z^4 dz \left[ 1 + e^z \right]^{-1} \left[ 1 + e^{-z} \right]^{-1}
\]

The integral can be evaluated by differentiating by parts

\[
\frac{7}{8} \Gamma (4) \zeta (4) = \int_0^\infty \frac{z^3}{1 + e^z} \left[ 1 + e^z \right]^{-1} - \int_0^\infty \frac{z^4}{4} (-1) e^z (1 + e^z)^{-2}
\]

\[
= \frac{1}{4} \int_0^\infty z^4 [1 + e^z]^{-1} \left[ 1 + e^{-z} \right]^{-1}
\]

Thus we get

\[
\gamma = \frac{1}{\pi^2} (k_B T) \frac{G_F^2}{\pi} A \cdot \frac{7}{2} 6 \zeta (4) = \frac{7\pi}{30} G_F^2 g_\nu^2 (1 + g_A^2) (k_B T)^5 = 0.8 \left( \frac{k_B T}{\text{MeV}} \right)^5 \text{s}^{-1}
\]

The decoupling temperature is given by

\[
\gamma = H
\]

\[
= 0.21 g^{*1/2} \left( \frac{k_B T}{\text{MeV}} \right)^2 \text{s}^{-1}
\]

where

\[
g^* = 2 \left[ 1 + \frac{7}{8} \cdot 2 + \frac{7}{8} \cdot 1 \cdot N_\nu \right]
\]

\[
= \frac{22 + 7N_\nu}{4}
\]

This gives the decoupling temperature

\[
0.8 \left( \frac{k_B T_D}{\text{MeV}} \right)^5 = (0.21) \left( \frac{22 + 7N_\nu}{\Delta} \right)^{1/2} \left( \frac{k_B T_D}{\text{MeV}} \right)^2
\]

\[
k_B T_D \text{MeV} = \left[ \frac{0.21}{0.8} \left( \frac{22 + 7N_\nu}{0.2} \right)^{1/2} \right]^{1/3}
\]

\[
\simeq 1
\]

(37)
if $N_\nu = 3$.

As the temperature cools past the decoupling temperature $k_B T_D \approx 1$ MeV, it is no longer possible to maintain the thermal equilibrium. The ratio $n/p$ thereafter is frozen out and is approximately constant (it decreases slowly due to weak decay of neutron). The freeze out $n/p$ ratio is given by

$$X_n^* = \frac{n}{p} \approx e^{\frac{-Q}{k_B T_D}} \approx 0.16,$$

$$X_n(t) = X_n^* e^{-t/\tau_n}$$

where $\tau_n$ is neutron life time and we have used the $Q$–value $Q = (m_n - m_p) + m_e = 1.8$ MeV for the reactions (31). Helium nucleosynthesis occurs at $T < T_S$ because of deuteron bottle neck. For $T > T_S$, the deuteron formed is knocked out by photo dissociation

$$\gamma + D \rightarrow p + n,$$

since the binding energy $\Delta B$ for the deuteron is only 2.2 MeV. The formation of deuteron actually starts after $k_B T_S \approx 0.1$ MeV; $T_S$ is called nucleosynthesis temperature. The estimate that $k_B T_S \approx 0.1$ MeV can be obtained as follows

$$[\eta = \frac{n_B}{n_{\gamma 0}}]:$$

$$\frac{n_{\gamma \text{diss}}}{n_B} \sim \frac{1}{\eta} e^{-\frac{\Delta B}{k_B T_S}} \leq 1. \quad (39)$$

Thus

$$-\frac{\Delta B}{k_B T_S} \approx \ln \eta. \quad (40)$$

Using $\Delta B \approx 2.2$ MeV, and $\eta \approx 10^{-10}$, we find $k_B T_S \approx 0.1$ MeV.

For $T > T_S$, photodissociation is so rapid that deuteron abundance is negligibly small and this provides a bottleneck to further nucleosynthesis. The deuteron “bottleneck” thus delay nucleosynthesis till $k_B T \leq 0.1$ MeV. But once the bottleneck is passed, nucleosynthesis proceeds rapidly and essentially all neutrons are incorporated into $^4He$:

$$n + p \rightarrow D + \gamma$$

$$D + D \rightarrow ^3H + p, ^3He + n$$

$$^3H + D \rightarrow ^4He + n$$

$$^3H + ^4He \rightarrow ^7Li$$
It is clear from the above reactions that $^4He$ abundance is given by
\begin{equation}
Y = \frac{2 (n / p)}{1 + n / p} = \frac{0.32}{1.16} = 0.276.
\end{equation}

(41a)

The ratio $Y$ changes from $T_D$ to $T_S$ due to the neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$. During this time $n/p$ changes from 0.16 to 0.14. Thus at $T = T_S$,
\begin{equation}
Y = \frac{0.28}{1.14} = 0.246.
\end{equation}

(41b)

It is clear from Eqs. (37) and (40) that
\begin{align*}
N_\nu & \uparrow \implies T_D \uparrow \implies X_n^* \uparrow \implies X_n (\tau_N) \uparrow \implies Y \uparrow \\
\eta & \downarrow \implies T_S \downarrow \implies X_n \downarrow \implies Y \downarrow
\end{align*}

We take $N_\nu = 3$ as given by LEP data and use $Y$ to constraint $\eta$. It turns out that a small amount of $D$ remained unburned. The amount of unburned $D$ is very sensitive to $\eta$:
\begin{equation}
\eta \uparrow \quad T_S \uparrow, \quad Y \uparrow \implies \text{less unburned } D
\end{equation}

Now $D/H$ ratio in primeval samples of the universe has been measured. The UV light (neutral $H$ and $D$ are seen by their UV absorption) came from distant quasars and absorbers were pregalactic gas clouds. The abundance was found to be:
\begin{equation}
D/H = (3.0 \pm 0.1) \times 10^{-5},
\end{equation}

pinning
\begin{equation}
\eta = (6 \pm 3) \times 10^{-10} \quad (42)
\end{equation}

We now want to express it in terms of baryon density as a fraction of the critical density $\rho_c$
\begin{align*}
\rho_B &= m_N n_B, \quad \eta = \frac{n_{B_0}}{n_{\gamma_0}}, \quad n_{\gamma_0} = 410.50 \text{cm}^{-3} \\
\Omega_B &= \frac{\rho_B}{\rho_c} \\
\rho_c &= \frac{3 H_0^2}{8 \pi G_N} = 1.054 \times 10^{-5} h_0^2 \text{GeVcm}^{-3}
\end{align*}
\( H_0 \) (the present value of Hubble parameter)

\[
\begin{align*}
H_0 &= 100h_0 \text{km s}^{-1} \text{Mpc}^{-1} \\
&= h_0 \left(1 \times 10^9 \text{yr} \right)^{-1} \\
h_0 &= 0.65 \pm 0.05
\end{align*}
\]

Combining these relations

\[
\begin{align*}
\Omega_B h_0^2 &= 366 \times 10^5 \eta = (0.019 \pm 0.001) \\
\Omega_B &= 0.045 \pm 0.01
\end{align*}
\]

(43)

This is confirmed by an independent determination of \( \Omega_B \) involving measurements of microwave background (CMB) anisotropy, where the underlying physics is very different, gravitational rather than nuclear. This gives

\[
\Omega_B = 0.042 \pm 0.008
\]

(44)

in very good agreement with the value inferred from Big-Bang Nucleosynthesis (BBN).

4 Dark Matter

At a large scale, measurements of velocity flows of galaxies give

\[
\Omega_m = \frac{\rho_m}{\rho_c} = 0.35 \pm 0.007
\]

(45)

Such a matter density is much larger than the visible matter density. This implies that most of the mass in the universe is dark; it does not emit or absorb any of the electromagnetic ratio. The value of \( \Omega_B \) given in Eq. (43) is far below the amount of dark matter needed to hold structures in the universe together. The situation is summarized in the pyramid shown in Fig. 1.

The detection of the CMB signature of acoustic oscillations in measurements of CMB anisotropy also implies

\[
\begin{align*}
\Omega_T &= 1.03 \pm 0.06 \\
\Omega_k &= 0
\end{align*}
\]
In the inflationary scenario of the universe, $\Omega$ is driven to unity, in agreement with the above observed value of $\Omega$. Now from the relation (22)

$$H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi}{3} G_N \rho - \frac{k c^2}{R^2} + \frac{1}{3} \Lambda c^2$$

we obtain

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda$$

(46)

where we have expressed the cosmological constant in terms of vacuum energy $\rho_\Lambda$:

$$\Lambda c^2 = 4\pi G_N \rho_\Lambda$$

(47)

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$$

Further $\Omega_k = 0$ and $\Omega_m = 0.35$ implies that

$$\Omega_\Lambda = 1 - \Omega_m = 0.65$$

(48)

In analogy with Dark matter, it is called the Dark energy. Finally

$$\rho_\Lambda \simeq \frac{2}{3} \rho_c \simeq 3 \times 10^3 \text{eVcm}^{-3}$$

(49)

giving

$$\Lambda = 4 \times 10^{-58} \text{cm}^{-3}$$

(50)

The nature of both the Dark Matter and the Dark energy is a mystery.

### 4.1 Matter-antimatter asymmetry

We also note that $\eta$ has another role. It appears that the Universe is matter-antimatter asymmetric. For example anti-proton $\bar{p}$ to proton $p$ ratio in cosmic rays is $[\frac{\bar{p}}{p} \sim 10^{-4}]$. In general $p$ and $\bar{p}$ may annihilate if they are brought together. In a typical reaction

$$p + \bar{p} \rightarrow \gamma + \gamma.$$
The question we wish to answer is, how does the interchange affect the nuclear density \( n_B \) or \( n_{\bar{B}} \). We start with \( n_B = n_{\bar{B}} \). At \( T \leq 1 \text{GeV} \), the equilibrium abundance of nucleons and antinucleons is, using Eqs. (20) and (5)

\[
\eta = \frac{n_B}{n} = \frac{n_{\bar{B}}}{n} = \frac{g_i}{2.44} \left( \frac{2\pi}{k_B T} \right)^{1/2} \left( \frac{m_N}{k_B T} \right)^{3/2} e^{-m_N/k_B T} \tag{51}
\]

The freeze out temperature \( T \) is given by

\[
\gamma_{\text{ann}} \simeq H = 1.66 \left( \frac{g^*}{2} \right)^{1/2} \left( k_B T^* \right)^2 \frac{1}{M_{\text{pl}}} \tag{52}
\]

where

\[
\gamma_{\text{ann}} = n_B \langle \sigma v \rangle
\]

\( \sigma \) is the nucleon-antinucleon annihilation crosssection which we may take as \( \frac{1}{m^2} \) with \( v \simeq c = 1 \). Thus using Eq. (20) for \( n_B \)

\[
g_i \left( \frac{m_N k_B T^*}{2\pi} \right)^{3/2} e^{-m_N/k_B T^*} \frac{1}{m^2} = 1.66 \left( \frac{g^*}{2} \right)^{1/2} \left( k_B T^* \right)^2 \frac{1}{M_{\text{pl}}} \tag{53}
\]

For nucleons \((p,n)\) and antinucleons, \( g_i = 8 \), and

\[
g^* = g_\gamma + \frac{7}{8} g_F = 2 + \frac{7}{8} = 9
\]

Putting \( x^* = \frac{m_N}{k_B T^*} \), we have

\[
8 \frac{1}{(2\pi)^{3/2}} m_N x^{1/2} \frac{1}{m^2} e^{-x^*} = 1.66 (9)^{1/2} \frac{1}{M_{\text{pl}}} \tag{54}
\]

\[
x^* e^{-x^*} = \frac{8}{3 (1.66) (2\pi)^{3/2}} \frac{1}{m^2} m_N \simeq 5 \times 10^{19} \tag{55}
\]

Hence \( x^* = 47 \). Thus \( T^* = 20 \text{ MeV} \).

With \( T = T^* = 20 \text{ MeV}, g_i = 8 \), Eq (51) gives

\[
\eta = \frac{n_B}{n} = \frac{n_{\bar{B}}}{n} = 2 \times 10^{-18} \tag{56}
\]
This contradicts $\eta = (6 \pm 3) \times 10^{-10}$, which then reflect some primordial baryon asymmetry in the universe. To summarize

$$n_B \to n_B - n_{\bar{B}}$$
$$\eta = \frac{n_B - n_{\bar{B}}}{n_{\gamma}} \simeq (6 \pm 3) \times 10^{-10} \quad (57)$$

5 Baryogensis

5.1 Sakharov’s Conditions

Towards finding a solution of the second big problem of Cosmology, namely, that of baryogensis ($\eta \simeq 3 \times 10^{-10}$), Sakharov’s three conditions, which we enumerate below, must be satisfied: Assuming that the universe started with a complete matter-antimatter symmetry in a standard big bang picture, one can obtain matter-antimatter asymmetry in the universe provided that the following three conditions are satisfied

(i) Underlying theory must possess

$$\Delta B \neq 0$$

where $B$ is the baryon number.

(ii) Charge Conjugation, and $CP$ symmetry must be violated; otherwise

$$n_B \rightarrow_{C,CP} n_{\bar{B}}$$

So even if $B$ is violated, one can never establish baryon-antibaryon asymmetry unless $C$ and $CP$ are violated.

(iii) Departure from thermal equilibrium of X-particles mediating $\Delta B \neq 0$ processes is necessary. This is because if all processes, including those which violate baryon number, are in thermal equilibrium, the baryon asymmetry vanishes. This is a direct consequence of the $CPT$ invariance.

Proof:

The density matrix at time $t$ is

$$\rho (t) = e^{-\beta(t)H(t)} \quad (58)$$
where $\beta = \frac{1}{k_B T}$. Equilibrium average of $B$ is

$$\langle B \rangle_T = Tr \left( e^{-\beta H} \hat{B} \right) = Tr \left( \theta^{-1} e^{-\beta H} \hat{B} \right) = Tr \left( \theta e^{-\beta H} \hat{B} \theta^{-1} \right) = Tr \left( \theta e^{-\beta H} \theta^{-1} \hat{B} \theta^{-1} \right) = Tr \left( e^{-\beta H} (-\hat{B}) \right) = -Tr \left( e^{-\beta H} \hat{B} \right) = -\langle B \rangle_T \quad (59)$$

where we have used the fact that $H$ commutes with $CPT \equiv \theta$. Thus $\langle B \rangle_T = 0$.

Finally to establish asymmetry dynamically, $B$ violating processes must be out of equilibrium in the Universe. This can be seen as follows:

$$\frac{d\Delta n_B}{dt} = -\gamma_B e^{-\left(\frac{m-\mu}{k_B T}\right)} - \gamma_B e^{-\left(\frac{\bar{m}-\bar{\mu}}{k_B T}\right)} \quad (60)$$

where $\gamma_B$ denotes the rate for $\hat{B}$ and $\mu$ is the chemical potential: $\bar{\mu} = -\mu$. Since $m = \bar{m}$ by $CPT$ theorem, $e^{-\frac{m-\mu}{k_B T}}$ is not relevant and we omit it. Then for $k_B T \gg \mu$,

$$\frac{d\Delta n_B}{dt} = -\frac{2\mu}{k_B T} \gamma_B \quad (61)$$

On the other hand

$$\Delta n_B = \frac{2\zeta(3)}{\pi^2} g' (k_B T)^3 \left[ e^{\frac{\mu}{k_B T}} - e^{-\frac{\mu}{k_B T}} \right] \approx \frac{2}{\pi^2} g' (k_B T)^3 \frac{2\mu}{k_B T} \quad (62)$$

Thus eliminating $\frac{2\mu}{k_B T}$,

$$\frac{d\Delta n_B}{dt} = -\frac{\pi^2}{2} \frac{\gamma_B}{g'(k_B T)^3} \Delta n_B = -\frac{\pi^2}{2} \Gamma_B \Delta n_B \quad (63)$$

15
where $\Gamma_B = \frac{\gamma_B}{g'(k_B T)} = \frac{\gamma_B}{n_B}$ gives the rate for $\bar{B}$. The solution of above equation gives
\[
\Delta n_B = (\Delta n_B)_{\text{initial}} e^{-\frac{\pi^2}{4} \Gamma_B t}.
\] (64)

What we learn from this result is that if $B$-violating processes are ever in equilibrium, then these processes actually washes out any initial condition for $\Gamma_B t \geq 1$.

After establishing the above preliminaries we shall consider baryogenesis at three levels: (i) Grand Unification (GUT) (ii) Electroweak (iii) Baryogenesis induced by Leptogenesis.

### 5.2 Baryogenesis at GUT Level

In a typical GUT, quarks and leptons are assigned in one representation so that the Sakharov condition (i) is naturally satisfied. Taking the example of the simplest GUT, $SU(5)$, the fermions are assigned to the irreducible representations $\bar{5}_f$ and $10_f$

\[
\bar{5}_f = [d_L^c, l_L], \quad 10_f = \{d_L, u_c^L, e_c^L\}
\]

Coupling with gauge bosons is
\[
\frac{G}{\sqrt{2}} 24_V \left[ (\bar{5}_f)^\dagger (\bar{5}_f) + (10_f)^\dagger (10_f) \right]
\]

There are 24 gauge bosons [$W^\pm, Z, \gamma, 8$ gluons and 12 lepto-quarks $X$, $\bar{X}$ belonging to the adjoint representation $24_V$].

The condition (iii) of Sakharov is supplied by the expansion of the Universe. As already mentioned the condition (i) is naturally satisfied in a GUT, e.g. by lepto-quarks $X$, $\bar{X}$ predicted by GUTs.

At $T = T_D$ (the decoupling temperature i.e. the temperature at which $X$–particles go out of equilibrium), the number density of $X$–particles is given by [cf. Eq. (13)]:
\[
n_{XD} = \frac{2.4 g_X}{\pi^2} \frac{\pi}{2} (k_B T_D)^3
\] (65)

where $g_X$ is the total number of $X$ (and $\bar{X}$) spin states. Now the entropy density at $T_D$ is given by [cf. Eq. (17)]
\[
s = \frac{S}{R^3} = \frac{4}{3} \rho (T) = k_B \frac{\pi^2}{15} (\frac{4}{3}) (k_B T_D)^3 g_*/2,
\] (66)
where \( g_* \) is the effective number of degrees of freedom. The number of baryons at \( T_D \) are given by
\[
 n_B = n_{XD} \Delta B. \tag{67}
\]
where \( \Delta B \) denotes baryon asymmetry in \( X \)-decays. Thus from Eqs. (65) and (66)
\[
 k_B \left( \frac{n_B}{s} \right)_D = (2.4) \frac{45}{4\pi^4} \left( \frac{g_X}{g_*} \right) \Delta B = 0.28 \left( \frac{g_X}{g_*} \right) \Delta B. \tag{68}
\]
Now \( g_* \) is over 100 in a typical GUT. [In SU(5): \( \gamma, W^\pm, Z^0, 8G's, 34 \) Higgs, 6 quarks, 3 leptons, 3 neutrinos, 12 \( X \)'s. Thus \( g_* = (24 \times 2) + 34 + \frac{5}{2}(18 \times 4 + 3 \times 4 + 3 \times 2) = 160.8 \). We, therefore, expect \( g_X/g_* \approx 10^{-2} \) to \( 10^{-1} \). Thus we have
\[
 k_B \left( \frac{n_B}{s} \right)_D \approx 0.28 \times \left( 10^{-2} - 10^{-1} \right) \Delta B_X \approx 3 \times \left( 10^{-3} - 10^{-2} \right) \Delta B. \tag{69}
\]
But \( (n_B/s)_D = (n_B/s)_0 \), where 0 denotes the present time. Thus
\[
 k_B \left( \frac{n_B}{s} \right)_0 \approx 3 \times \left( 10^{-3} - 10^{-2} \right) \Delta B. \tag{70}
\]
\[
 \left( \frac{s}{k_B} \right)_0 = \left( \frac{s}{k_B} \right)_{\gamma_0} + \left( \frac{s}{k_B} \right)_{\nu_0} = \frac{2\pi^4}{45 \times 1.2} \left[ \frac{1}{2} g_\gamma n_{\gamma_0} + \frac{1}{2} \times \frac{4}{3} \sum g_{\nu_i} n_{\nu_0} \right] = 3.6 \left[ 1 + \frac{41}{11} \right] n_{\gamma_0} \approx 7 n_{\gamma_0}. \tag{71}
\]
Now [cf. Eqs. (17) and (5)] where we have used that \( n_{\nu_0} = (3/11) n_{\gamma_0} \) and \( \sum_i g_{\nu_i} = 3(7/8)1.2 \). Hence from Eq. (70), we get
\[
 \left( \frac{n_B}{n_\gamma} \right)_0 \approx 21 \times \left( 10^{-3} \text{ to } 10^{-2} \right) \Delta B \approx 2 \times \left( 10^{-2} \text{ to } 10^{-1} \right) \Delta B.
\]
\[
 \approx A(\Delta B), \tag{72}
\]
where $A \sim 10^{-1} - 10^{-2}$. Now we can write

$$
\Delta B = \sum_f B_f \frac{\Gamma (X \to f) - \Gamma (\bar{X} \to \bar{f})}{\Gamma_{tot} (X)} ,
$$

(73)

$$
f \equiv \{ql, \bar{q}\bar{q}\} , \quad B_f = \frac{1}{3}, \frac{2}{3}.
$$

$\Delta B$ vanishes if CP and C are conserved. The $X-$particles can generate $\Delta B$, by the processes of the following type [$r$ is the branching ratio]

$$
\begin{align*}
X & \rightarrow ql : r \quad B_1 = 1/3 \\
X & \rightarrow \bar{q}\bar{q} : 1 - r \quad B_2 = -2/3 \\
\bar{X} & \rightarrow q\bar{l} : \bar{r} \quad \bar{B}_1 = -1/3 \\
\bar{X} & \rightarrow \bar{q}\bar{q} : 1 - \bar{r} \quad B_2 = 2/3.
\end{align*}
$$

The mean baryon number per decay

$$
B_X = r B_1 + (1 - r) \bar{B}_2 \\
B_{\bar{X}} = \bar{r} \bar{B}_1 + (1 - \bar{r}) B_2.
$$

(74)

Thus

$$
\Delta B = \frac{1}{2} \left[ r B_1 + (1 - r) \bar{B}_2 + \bar{r} \bar{B}_1 + (1 - \bar{r}) B_2 \right]
$$

$$
= \frac{1}{2} \left[ r \left( B_1 - \bar{B}_2 \right) + \bar{r} \left( \bar{B}_1 - B_2 \right) + \left( \bar{B}_2 + B_2 \right) \right]
$$

$$
= \frac{1}{2} (r - \bar{r}).
$$

(75)

From Eqs. (75) and (72), we see that we can explain the baryon number generation if $r \neq \bar{r}$ i.e. $X-$interactions violate $C$ and CP. Also we require $\Delta B \sim 10^{-8}$ in order to explain the present baryon number $\eta = n_B/n_\gamma \approx 10^{-10}$.

Let us now obtain an estimate for $T_D$. If $k_B T_D > m_X$, the thermal equilibrium can be maintained by inverse decays. Thus the condition for departure from equilibrium is [cf. Eqs. (24) and (26) $k_B T_D \simeq m_X$]:

$$
\frac{1}{3} \alpha_X g_d \left( k_B T_D \right) \approx 1.66 \ g^2 \frac{(k_B T_D)^2}{M_{pl}}.
$$

(76)
The factor 1/3 is due to spin average \([X \text{ is a vector particle}]\). Now using \(g_d \approx 12 \times 2 = 24\) and \(g_s \approx 160\), we get

\[
k_B T_D \approx \alpha_X \times (4.0) \times 10^{18} \text{GeV}.
\] (77)

Using \(\alpha_X \approx 1/40\) \([\text{SU(5) value}]\), we get

\[
k_B T_D \approx 10^{17} \text{GeV}.
\] (78)

Thus if \(X\)–bosons are vector bosons, \(k_B T_D > \text{mass of vector bosons of SU(5)}\) \([\approx 10^{15} - 10^{16} \text{GeV}]\) and therefore vector bosons of SU(5) cannot give rise to baryon asymmetry.

### 5.3 Can Higgs particles give rise to the baryon asymmetry?

Whereas the gauge sector of the SU(5) structure is uniquely determined by the gauge group, in the Higgs sector the results depend on the choice of representation. Now the Higgs fields that couple to fermions belong to

\(5_H, 10_H, 15_H, 45_H\) and \(50_H\).

Let us first consider the minimal \(5_H\) which contains Higgs doublet of the SM:

- **Color singlet**
  
  \((1, 2, 1/2),\)

  the latter two quantum numbers refer to

  \(SU(2) \times U(1)\).

- **B violating color triplet**
  
  \(H_3 \ (3, 1, -1/3)\).

At tree level, the Higgs coupling with fermions are shown in Fig. 2. In any vertex of \(SU(5)\), \(B - L\) is preserved.

\(h_U, h_D\) are complex Yukawa coupling matrices. CP–violation arises from complex phases of the Yukawa couplings, which can not be absorbed by field redefinitions. At tree level, these phases do not give any contribution to baryon asymmetry since \(\text{Im} \ Tr \left[h_D^\dagger h_D\right] = 0\). One has to go to loop level,
where denoting by $\chi$, a member of $H_3$, where $\phi$ is some exchanged state, another Higgs or a gauge boson. Thus $r$ is given by Fig. 3.

$$r \sim \left| \gamma_0 + \gamma_1 I \left( M_\chi^2 - i\epsilon \right) \right|^2$$  \hspace{1cm} (79)

where $\gamma_0$ and $\gamma_1$ are complex and $I$ has an analytical structure. Then

$$\frac{1}{2} (r - \bar{r}) \sim \frac{1}{2} \left\{ \left| \gamma_0 + \gamma_1 I \left( M_\chi^2 - i\epsilon \right) \right|^2 - \left| \gamma_0^* + \gamma_1^* I \left( M_\chi^2 - i\epsilon \right) \right|^2 \right\}$$

$$\sim 2 \text{Im} (\gamma_0 \gamma_1^*) \text{Im} \left( I \left( M_\chi^2 - i\epsilon \right) \right)$$  \hspace{1cm} (80)

where the first part is the $CP$ violating and the second part is determined by the rescattering dynamics. Thus

$$\eta = A \Delta B_\chi = A \frac{1}{2} (r - \bar{r}) \approx A \text{Im} (\gamma_0 \gamma_1^*) \text{Im} I$$  \hspace{1cm} (81)

where $A$ comes from out of the equilibrium condition as seen previously; $\text{Im} (\gamma_0 \gamma_1^*)$ gives $CP$ and $C$ violation while $\text{Im} I$ comes from GUT’s dynamics. There is no firm pediction for $\eta$.

In particular, take $\phi$ in the above figure as $5_H$ as shown in Fig. 4. We can choose $f$ to be real and $h$ has 3 phases. Even so we can not generate in the lowest non-trival order CP violation. This is because $\gamma_0 \sim f$ real and $\gamma_1 \sim fh\bar{h}$, which has no $CP$ phase. Also gauge exchange $[\phi \equiv G^\mu]$ give no phase. One can eventually generate $\eta$ to higher order loop graphs. But then $\eta \sim 10^{-10}$, which is too small.

It is possible to obtain $\eta \sim 10^{-10}$ by either (i) adding Higgs in the 45 representation or (ii) by using more elaborate GUT’s e.g. $SO(10)$. In $SO(10)$ there exists a fermion that is singlet under SM, carries $L = -1$ and is identified with $\nu_R$. $CP$ violation may be provided by the complex Yukawa couplings between the right-handed and the left-handed neutrinos and scalar Higgs. The right-handed neutrinos acquire a Majorana mass $M_N = O(B - L)$ i.e. at the scale where $U(1)_{B-L}$ is broken, and its out of equilibrium decays may generate a non-vanishing $(B - L)$ asymmetry. We shall come back to the role of right-handed neutrino in generating $\eta$ when we consider Leptogenesis.
6 Electroweak Baryogenesis

In the SM, both baryon number \( B \) and lepton number \( L \), symmetries hold at the classical level

\[
\mathcal{L}_{\text{SM}} \to B,L \mathcal{L}_{\text{SM}}.
\]

However, because of the chiral nature of electroweak theory, at quantum level both \( J_B^\mu \) and \( J_L^\mu \) are not conserved, so called electroweak anamoly:

\[
\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = N_g \left( \frac{\alpha_2}{\pi} W^\mu_{a\nu} \tilde{W}^a_{\mu\nu} - \frac{\alpha'}{8\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} \right) \quad (82)
\]

where \( N_g \) is the number of generations, \( \alpha_2 = g^2/4\pi \), \( \alpha' = g^\prime_2/4\pi \) are couplings corresponding to \( SU_L(2) \times U(1); \tilde{W}_{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} W_{\alpha\beta} \). Then

\[
\partial_\mu (J_B^\mu - J_L^\mu) = \partial_\mu (J_B^\mu - J_L^\mu) = 0
\]

\[
\partial_\mu (J_B^\mu + J_L^\mu) \neq 0 \quad (83)
\]

Further

\[
\Delta B = B(+\infty) - B(-\infty) = \int_{-\infty}^{\infty} dt \partial_0 \int d^3x J_B^0 (\vec{x},t)
\]

\[
= \int_{-\infty}^{\infty} dt \int d^3x \partial_\mu J_B^\mu (\vec{x},t) \quad (84)
\]

since by Gauss’s theorem we can convert \( \int d^3x \partial_\mu J_B^\mu (\vec{x},t) \) into a surface integral which we can put equal to zero. Similarly for \( \Delta L \). Note that \( \Delta (B - L) = 0 \); the electroweak anamoly preserves \( B - L \). But

\[
\Delta (B + L) \neq 0
\]

\[
= 2N_g \nu \quad (85)
\]

where

\[
\nu = \frac{\alpha_2}{8\pi} \int d^4x W^\mu_{a\nu} \tilde{W}^a_{\mu\nu}
\]

Now in some theories, one can find classical solutions of the Euclidean field equations. These solutions, called instantons, are localized in Euclidean time
as well as in space. Let us consider a pure Yang-Mills field: Time $t$ in Minkowsky space must be replaced by $it$ in Euclidean space. Then the Euclidean Action

$$S_E = \int d^4x E(x_E) = -iS$$

$$\mathcal{L}_E(x_E) = \frac{1}{g^2} F^a_{\mu\nu} F^{a}_{\mu\nu}$$

(86)

There is no distinction between covariant and contravariant indices in Euclidean space. Here

$$A^a_\mu \rightarrow \frac{i}{g} A^a_\mu$$

$$F^a_{\mu\nu} \rightarrow \frac{i}{g} F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + [A^a_\mu, A^a_\nu]$$

$$A_\mu = \sum_a T^a A^a_\mu$$

$$F_{\mu\nu} = \sum_a T^a F^a_{\mu\nu}$$

It can be shown that

$$S_E[A] \geq \frac{1}{2} \int d^4x T r[F_{\mu\nu}\tilde{F}_{\mu\nu}]$$

$$= 8\pi^2 g |\nu[A]|,$$

(87)

the lower bound is obtained when

$$\tilde{F}_{\mu\nu} = \pm F_{\mu\nu}.$$}

It is necessary to find an interpretation for instanton solution in Euclidean four-space (imaginary $t$) within Minkowsky $3+1$ space-time (time real) in order to understand their physical significance. It has been shown that instantons become ‘tunneling events’ between two different Minkowski events. The gauge vacuum is given by

$$|\theta\rangle = \sum_i e^{-in\theta} |n\rangle.$$  

(88)
It has been shown by t’Hooft that the transition probability between two closest vacuua is given by
\[ \sim e^{-8\pi^2/g^2} \quad \text{for } \nu = 1. \quad (89) \]

This is the zero temperature solution:
\[ S_E \sim \frac{8\pi^2}{g^2} \quad (90) \]

For our problem \( \frac{g^2}{4\pi} = \alpha_2 \). Thus \((B + L)\) violating amplitude is
\[ A \sim e^{(-2\pi/\alpha_2)\nu} \quad (91) \]

which is extremely small \( \sim 10^{-90} \). Thus the probability of B-violating processes is highly suppressed at zero temperature.

The situation is different at high temperature which is relevant at early Universe. Here one goes from one vacuum to another by thermal fluctuations rather than tunneling [see Fig. 5].

The transition probability (per unit time per unit volume) is given by
\[ P \sim e^{-V_0(T)/T} \quad (92) \]

where \( V_0(T) \) is the height of the barrier. If the system is able to perform a transition from one vacuum to the closest one, then \( \Delta (B + L) = 2N_g = 6 \).

Each transition creates nine left-handed quarks as only these are coupled to \( W \) bosons (3 color states for each generation) and left-handed leptons (one per generation). However, adjacent vacua of the EW theory are separated by a ridge of configurations with energies larger than that of the vacua. The lowest energy point on this ridge is a saddle point solution to the equations of motion, and is referred to as the Sphaleron.

The thermal rate of B-violation in the broken phase is proportional to [in units \( k_B = 1 \)]
\[ \exp(-S_3/T) \]

where \( S_3 \) is the three dimensional action computed along the sphaleron configuration:
\[ S_3 \equiv E_{sp}(T) \equiv C \left( \frac{m_H}{m_W} \right) \frac{\pi m_W(T)}{\alpha_2} \quad (93) \]
\( C \left( \frac{m_H}{m_W} \right) \) is a function of \( \lambda, \frac{m_H^2}{m_W} \sim \lambda \):

\[
E_{sp} \simeq 7 - 14 \text{ TeV as } \lambda \text{ increases from 0 to } \infty.
\]

Then, the transition probability per unit time per unit volume is

\[
P_{sp}(T) = \mu \left[ \frac{m_W}{\alpha_2 T} \right]^3 m_W^4 \exp \left[ -\frac{E_{sp}(T)}{T} \right]
\]

where \( \mu \) is a dimensionless constant and the Boltzmann suppression appears large.

However, it is to be expected that, when EW symmetry becomes restored at temperature of around 100 GeV [\( m_W(T) \to 0 \), Electroweak phase transition] there will no longer be an exponential suppression. Now the only important scale in the symmetric phase is \( \alpha_2 T \) so that dimensional ground, we expect

\[
P_{sp}(T) = \mathcal{K} (\alpha_2 T)^4
\]

where numerical estimates yielded \( \mathcal{K} \sim 0.1 - 1 \).

Lattice simulation indicates

\[
P_{sp} \sim 30 \alpha_2^5 T^4
\]

not very different from above as \( \alpha_2 \sim \frac{1}{30} \).

Now \( B \) and \( L \) processes are in thermal equilibrium for

\[
\Gamma_{sp} = \frac{P_{sp}}{T^3} > H \simeq 1.66 g^*^{1/2} T^2 \quad \text{M}_{pl}
\]

\[
\mathcal{K} \alpha_2^4 T > 1.66 g^*^{1/2} T^2 \quad \text{M}_{pl}
\]

\[
\alpha_2 = \frac{\alpha_e}{\sin^2 \theta_W} = \frac{4}{137} \simeq 0.029
\]

\[
\alpha_2^4 \simeq 10^{-6}
\]

\[
T < \left( \frac{\mathcal{K}}{1.66 g^*^{1/2}} \right) \alpha_2^4 \text{M}_{pl} \approx 10^{12} \text{ GeV}
\]

Thus \( B \) and \( L \) processes are in thermal equilibrium for the temperature in the range

\[
T_{EW} \simeq 100 \text{ GeV} < T < T_{sp} \simeq 10^{12} \text{ GeV}
\]
This implies any $\Delta n_{B+L}$ established above
\[ T_{\text{max}} \left( \sim \alpha_2^4 M_{\text{pl}} \right) \]
e.g., GUT, will get washed out down to $T_{\text{EW}}$.

Given the above result, to generate $\eta$ at the EW phase transition: First the baryon number is violated, as we have seen above. Second $CP$ is violated, in the standard model. The third condition of Sakharov can be satisfied if the EW transition is of first order since then the coexistence of broken and unbroken phases at the phase transition is a departure from equilibrium. However one can not get the first order transition unless Higgs is light, $m_H < 60$ GeV, which is ruled out by LEP, $m_H \geq 114$ GeV. Another problem is the size of CP-violation:
\[ \eta \simeq \alpha_2^4 \epsilon_{CP} \simeq 10^{-6} \epsilon_{CP} \quad (101) \]
where
\[ \epsilon_{CP} \sim \lambda^6 \sin \delta \quad (102) \]
due to several GIM suppression factors in CKM, $\lambda \simeq 0.22$ so that
\[ \lambda^6 \simeq 5.5 \times 10^{-6}. \quad (103) \]
This gives
\[ \eta \sim 5.5 \times 10^{-12} \delta \sim 10^{-18} \]

The other possibility is the leptogenesis, where one tries to generate $L \neq 0$ but no $B$ from neutrino physics well before the electroweak transition, and $L$ gets partially converted into $B$ due to electroweak anamoly. This is discussed in the next section.

7 Baryogenesis via Leptogenesis

As already seen sphaleron transitions lead to
\[ \Delta (B - L) = 0 \]
\[ \Delta (B + L) = 2N_g = 6 \]
the baryon asymmetry can be generated by the lepton asymmetry.
Further $B + L$ asymmetry generated before EW transition i.e. at $T > T_{EW}$, will be washed out. However, since only left handed fields couple to sphalerons, a non zero value of $B + L$ can persist in the high temperature symmetric phase if there exist a non vanishing $B - L$ asymmetry [see below]. As already seen

$$n_i - \bar{n}_i = \frac{2}{\pi^2} g' T^3 \left( \frac{2\mu_i}{T} \right)$$

This also implies

$$n_B = B \left( \frac{4}{\pi^2} g' T^2 \right)$$
$$n_L = L \left( \frac{4}{\pi^2} g' T^2 \right)$$

(104)

where $B$ and $L$ are baryon and lepton asymmetry respectively.

Note that in SM

$$q_{Li} = \left( \begin{array}{c} u_{Li} \\ d_{Li} \end{array} \right) \quad B = \frac{1}{3}, \quad L = 0$$
$$u_{Ri}, d_{Ri}$$

$$\ell_{Li} = \left( \begin{array}{c} \nu_{Li} \\ e_{Li} \end{array} \right) \quad B = 0, \quad L = 1$$
$$\nu_{Ri}, e_{Ri}$$

Thus in Eq. (104)

$$B = 3 \times \frac{1}{3} \sum_i \left( 2\mu_{qi} + 2\mu_{ui} + 2\mu_{di} \right)$$
$$L = \sum_i \left( 2\mu_{li} + 2\mu_{ei} \right)$$

(105)

In high temperature plasma quarks, leptons and Higgs interact via Yukawa and gauge couplings and in addition, via the non perturbative sphaleron processes. In thermal equilibrium all these processes yield constraints between various chemical potentials. The effective interaction

$$O_{B+L} = \Pi_i (q_{Li}q_{Li}q_{Li}\ell_{Li})$$

yields

$$\sum_i \left( 3\mu_{qi} + \mu_{li} \right) = 0$$

(106)
Another constraint is provided by vanishing of total charge of plasma

\[ \sum_i \left[ +3 \left( -\frac{2}{3} \right) \mu_{d_i} + (-1) 2 \mu_{l_i} + (-2) \mu_{e_i} + \frac{1}{N} (1) \mu_\phi \right] = 0 \]

where we have used

\[ Y_q = \frac{1}{3}, \quad Y_u = \frac{4}{3}, \quad Y_d = -\frac{2}{3}, \quad Y_l = -1, \quad Y_{e^-} = -2, \quad Y_\phi = 1 \]

The above equation can be written as

\[ \sum_i \left( \mu_{qi} + 2 \mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{2}{N} \mu_\phi \right) = 0 \] (107)

Further invariance of Yukawa couplings \( q_{Li} \phi d_{Ri}, \) etc give

\[
\begin{align*}
\mu_{qi} - \mu_\phi - \mu_{dj} &= 0 \\
\mu_{qi} - \mu_\phi - \mu_{uj} &= 0 \\
\mu_{li} - \mu_\phi - \mu_{ej} &= 0
\end{align*}
\] (108)

When all Yukawa interactions are in equilibrium, these interactions establish equilibrium in different generations

\[
\mu_{li} = \mu_l, \quad \mu_{qi} = \mu_q \text{ etc.}
\]

Thus we obtain from Eqs (106) and (107)

\[
\begin{align*}
\mu_q &= -\frac{1}{3} \mu_l \\
\mu_q + 2 \mu_u - \mu_d - \mu_e + \frac{2}{N} \mu_\phi &= 0 \\
giving \quad -\frac{4}{3} \mu_l + 2 \mu_u - \mu_d - \mu_e + \frac{2}{N} \mu_\phi = 0
\end{align*}
\] (109)

Further Eqs. (108) imply

\[
\begin{align*}
-\frac{1}{3} \mu_l - \mu_\phi - \mu_d &= 0 \\
-\frac{1}{3} \mu_l - \mu_\phi - \mu_u &= 0 \\
\mu_l - \mu_\phi - \mu_e &= 0
\end{align*}
\] (110)
Using the above equations, we can write (109) as

\[-\frac{4}{3} \mu_l + 2 \left( -\frac{1}{3} \mu_l + \mu_\phi \right) - \left( -\frac{1}{3} \mu_l - \mu_\phi \right) - (-\mu_l - \mu_\phi) + \frac{2}{N} \mu_\phi = 0\]

Thus finally we can express \( \mu_q, \mu_u, \mu_d, \mu_e, \) and \( \mu_\phi \) in terms of \( \mu_l \).

\[
\begin{align*}
\mu_\phi &= \frac{8}{3} N \frac{1}{4N + 2} \mu_l = \frac{4N}{6N + 3} \mu_l \\
\mu_d &= -\frac{1}{3} \mu_l - \mu_\phi \\
&= -\frac{1}{3} \mu_l - \frac{4N}{6N + 3} \mu_l \\
&= \frac{6N + 1}{6N + 3} \mu_l \\
\mu_u &= -\frac{1}{3} \mu_l + \mu_\phi \\
&= -\frac{1}{3} \mu_l + \frac{4N}{6N + 3} \mu_l \\
&= \frac{2N - 1}{6N + 3} \mu_l \\
\mu_e &= \mu_l - \mu_\phi \\
&= \mu_l - \frac{4N}{6N + 3} \mu_l \\
&= \frac{2N + 3}{6N + 3} \mu_l \\
\end{align*}
\]

Hence from Eqs. (105)

\[
\begin{align*}
B &= N \left\{ -\frac{2}{3} \mu_l + \frac{2N - 1}{6N + 3} \mu_l - \frac{6N + 1}{6N + 3} \mu_l \right\} \\
&= \left[ -4N - 2 + 2N - 1 - 6N - 1 \right] \frac{\mu_l}{6N + 3} \\
&= -\frac{N}{3} \left( 8N + 4 \right) \frac{\mu_l}{2N + 1} \\
&= -\frac{4N}{3} \mu_l \\
L &= N \left( 2\mu_l + \frac{2N + 3}{6N + 3} \mu_l \right)
\end{align*}
\]
\[
\begin{align*}
B - L &= \frac{14N^2 + 9N}{6N + 3} \mu \tau \\
B &= \frac{8N^2 + 4N + 14N^2 + 9N}{6N + 3} \mu \tau \\
\frac{B}{B - L} &= \frac{22N^2 + 13N}{6N + 3} \mu \tau \\
\end{align*}
\]
(113)

These relations hold for \( T \gg v \). In general \( B/B - L \) is a function of \( v/T \).

For SM, \( N_g = 3, n_H = 1 \) so that \( a = 28/79 \).

Thus finally we obtain

\[
Y_B(\equiv \frac{n_B - n_{\bar{B}}}{s}) = aY_{B-L} = \frac{a}{a-1}Y_L 
\]
(116)

Note that by using Eq. (71),

\[
Y_B = \eta \left( \frac{\eta_s}{s} \right) \approx \frac{1}{7} \eta \\
\frac{1}{7} (6 \pm 3) \times 10^{-10} 
\]

In SM as well as in SU(5), \( B - L \) is conserved and no asymmetry in \( B - L \) can be generated. However, adding a right handed Majorana neutrino to the SM breaks \( B - L \), and the primordial lepton asymmetry may be generated by the out of equilibrium decay of heavy right handed Majorana neutrino \( N_R \).

The simple extension of SM can be embedded in GUTs with gauge group containing \( SO(10) \). Majorana neutrinos can also lead to See-saw mechanism, explaining the smallness of light neutrino \( \nu \) masses.

The relevant couplings are

\[
\mathcal{L} = \bar{\ell}_L \phi \nu N_R + \frac{1}{2} \bar{N}_R^c M N_R + h.c. 
\]
(117)
where $\phi$ is the usual Higgs doublet under $SU(2)_L$ while the second term gives Majorana mass for the right handed neutrino $N$. The vacuum expectation value of the Higgs field $\langle \phi \rangle$ generates neutrino Dirac masses

$$m_D = h_\nu \langle \phi \rangle$$  \hspace{1cm} (118)

The Lagrangian (117) also generates an effective $\Delta L = 2$ Lagrangian

$$\mathcal{L}_{\Delta L=2} = \frac{G}{M} (\ell_L^T i \sigma_2 \phi) C^{-1} (\phi^T i \sigma_2 \ell_L)$$  \hspace{1cm} (119)

This generates a Majorana mass for light neutrinos

$$m_\nu = \frac{G}{M} \langle \phi \rangle_0^2$$  \hspace{1cm} (120)

Further

$$m_N = M \gg m_D$$  \hspace{1cm} (121)

If the $\Delta L = 2$ interactions are in equilibrium, but the right handed electrons are not, then $\mu_l - \mu_\phi - \mu_e = 0$ is replaced by [c.f. Eq. (117)]

$$\mu_l + \mu_\phi = 0 \Rightarrow \mu_l = -\mu_\phi$$  \hspace{1cm} (122)

Thus using all the other previous equations

$$\mu_q = \frac{1}{3} \mu_l$$
$$\mu_d = \frac{2}{3} \mu_l$$
$$\mu_u = \frac{4}{3} \mu_l$$
$$\mu_\phi = -\mu_l$$

$$\mu_q + 2\mu_u - \mu_d - \mu_l + \frac{2}{N} \mu_\phi = 0$$  \hspace{1cm} (123)
The equations give

\[
\begin{align*}
\mu_l &= -\frac{3N}{14N + 6} \mu_e \\
\mu_d &= -\frac{2N}{14N + 6} \mu_l \\
\mu_u &= \frac{4N}{14N + 6} \\
\mu_q &= \frac{N}{14N + 6},
\end{align*}
\]

so that from Eq. (105)

\[
\begin{align*}
B &= N \left[ \frac{2N}{14N + 6} + \frac{4N}{14N + 6} - \frac{2N}{14N + 6} \right] \mu_e \\
&= \frac{4N^2 \mu_e}{14N + 6} \\
L &= N \left[ -\frac{6N}{14N + 6} + 1 \right] \mu_e \\
&= \frac{8N^2 + 6N}{14N + 6} \mu_e \\
B - L &= \frac{-4N^2 - 6N}{14N + 6} \\
\frac{B}{B - L} &= \frac{4N^2}{-4N^2 - 6N} = \frac{-2N}{2N + 3} = a; \quad a - 1 = \frac{-4N - 3}{2N + 3} \\
\frac{B}{L} &= \frac{2N}{4N + 3} = \frac{a}{a - 1}
\end{align*}
\]

(125)

The above relations hold if the corresponding interactions are in thermal equilibrium i.e. in the range

\[T_{ew} \sim 100 \text{ GeV} < T_{sph} \sim 10^{12} \text{ GeV}\]

which is of interest for baryogenesis; this is the case for all gauge interactions. This is not always true for Yukawa interactions. The rate of a scattering process between left and right handed fermions, Higgs bosons and W⁻-bosons

\[\psi_L \phi \longrightarrow \psi_R W\]
is [c.f. Eq. (27), $k_B = 1$]

$$\gamma \sim \alpha_2 h^2 \frac{(k_B T)^5}{(k_B T)^4} = \alpha_2 h^2 T$$  \hspace{1cm} (126)

The equilibrium condition is satisfied for

$$\gamma > H = 1.66 g_*^{1/2} \frac{T^2}{M_{Pl}}$$

$$T < \alpha_2 h^2 M_{Pl} \frac{1}{1.66 g_*^{1/2}} \sim h^2 \frac{3}{1.66 g_*^{1/2}} 10^{17}$$

$$T < h^2 10^{16} \text{ GeV}$$ \hspace{1cm} (127)

where we have used

$$g^* = 4 \times 2 + 1 + \frac{7}{8} (9 \times 4 + 2 \times 4 + 3 \times 2) \simeq 50$$

Now $[\langle \phi \rangle = v/\sqrt{2} = 175 \text{ GeV}]$

$$h_e = \frac{m_e}{\langle \phi \rangle} = \frac{5 \times 10^{-1} \text{ MeV}}{(250) \times 10^3 \text{ MeV}} = 3 \times 10^{-6}$$

$$T_e \sim 10^{-11} \times 10^{16} \text{ GeV} \simeq 10^5 \text{ GeV}$$

$$T_u \sim 10^7 \text{ GeV}$$

$$\vdots$$

$$T_s = 10^7 \left( \frac{m_s}{m_u} \right)^2 \sim (0.9) 10^{10} \simeq 10^{10} \text{ GeV}$$ \hspace{1cm} (128)

At temperature $T \simeq 10^{10} \text{ GeV}$, which is characteristics of leptogenesis, $e_R$, $\mu_R$, $d_R$, $s_R$ and $u_R$ are out of equilibrium. When the Majorana right-handed $\nu$'s (these existed in early universe) decay into leptons and Higgs scalar,

$$N_R \rightarrow \bar{\phi} + \ell$$
$$N_R \rightarrow \phi + \bar{\ell}$$

they violate lepton number. The interference between the tree-level amplitude and the absorption part of the one-loop vertex leads to lepton asymmetry (and baryon asymmetry $\eta_B = \frac{\alpha}{\alpha-1} \eta_L$) of the right order of magnitude
to explain the observed $\eta_B$. It has been observed that CP violation may be considerably enhanced if two heavy right handed $\nu$'s are nearly degenerate in mass.

As already seen $\Delta L = 2$ interaction of the form

$$\frac{G}{M} (\ell_L \ell_L \phi \phi) = \frac{m_\nu}{\langle \phi \rangle^2} \ell_L \ell_L \phi \phi$$  \hspace{1cm} (129)

is generated through the exchange of $N_R$ [Fig. 7].

These processes will take place with the rate

$$\gamma_{\Delta L=2} (T) = \frac{T^3}{\pi^3 \langle \phi \rangle^4} \sum_{i=e,\mu,\tau} m_{\nu_i}^2$$

The requirement for the harmless lepton number violation

$$\gamma_L < H$$

give

$$\frac{1}{\pi^3} \frac{T^3}{\langle \phi \rangle^4} \sum_{i=e,\mu,\tau} m_{\nu_i}^2 < 1.66g^{*1/2} \frac{T^2}{M_{Pl}}$$

$$\sum_{i=e,\mu,\tau} m_{\nu_i}^2 < 1.66g^{*1/2} \pi^3 \langle \phi \rangle^4 \frac{1}{M_{Pl} T}$$

$$= 1.66g^{*1/2} \pi^3 \frac{10^9 \text{ GeV}^4}{10^{19} \text{ GeV} T}$$

$$\sum_{i=e,\mu,\tau} m_{\nu_i}^2 = < 1.66g^{*1/2} \times 31 \left( \frac{10^8 \text{ GeV}}{T} \right) \text{ eV}^2$$

$$= g^{*1/2} \times 0.56 \left( \frac{10^{10} \text{ GeV}}{T} \right) \text{ eV}^2$$

$$= 4 \left( \frac{10^{10} \text{ GeV}}{T} \right) \text{ eV}^2$$

$$\sum_{i=e,\mu,\tau} m_{\nu_i}^2 \leq \left[ 2 \text{ eV} \left( \frac{T_X}{10^{10} \text{ GeV}} \right)^{-1/2} \right]^2$$  \hspace{1cm} (130)

where

$$T_X \equiv \text{Min} \left( T_{B-L}, 10^{12} \text{ GeV} \right)$$  \hspace{1cm} (131)
$T_{B-L}$ is the temperature at which $B - L$ number production takes place. Now $10^{12}$ is the temperature at which sphaleron transitions enter in equilibrium. Thus

$$
\sum_{i=e,\mu,\tau} m^2_{\nu_i} \leq \left[ 0.4 \text{ eV} \left( \frac{T_X}{T_{\text{SPH}}} \right)^{-1/2} \right]^2
$$

We can reverse the argument and for $T_{B-L} \simeq 10^{16}$ GeV as in $SO(10)$, Eq. (130) implies

$$
m_\nu \leq 2 \text{ eV} \left( 10^6 \right)^{-1/2} = 2 \times 10^{-3} \text{ eV}
$$

which is of interest in neutrino oscillations.

8 Thermal Leptogenesis

One starts from a thermal distribution of heavy Majorana neutrinos which have $CP$ violating decay modes into standard leptons: Natural candidates are $\nu_{Ri}, i = 1, 2, 3$; one in each of the three lepton families, while the Lagrangian of electroweak interactions keep invariance under the $SU(2)_L \times U(1)_Y$ gauge transformations.

In this case Yukawa interactions are described by

$$
\mathcal{L}_Y = -\bar{\ell}_L^i \phi h_{Lij} e_{Rj} + \bar{\ell}_L^i \tilde{\phi} h_{Lij}^* \nu_{Rj} - \frac{1}{2} \bar{\nu}_{R}^c M \nu_{R} + h.c.
$$

the lepton number violation is induced by the third term. $M$ is a Majorana mass matrix while $h_L$ are the Yukawa couplings. After spontaneous symmetry breaking the vacuum expectation value of the Higgs field $\langle \phi \rangle = v \simeq 175$ GeV generates the Dirac mass term $(m_D)_{ij} = h_{ij} v$, assumed to be small compared to $M$. Light neutrino mass matrix $M_\nu$ arises from the diagonalizing the $6 \times 6$ neutrino mass matrix

$$
M_\nu = \begin{pmatrix}
0 & m^T_D \\
m_D & M
\end{pmatrix}
$$

and takes the seesaw form

$$
m_\nu = -m^T_D M^{-1} m_D
$$
This also yields light and heavy neutrino mass eigenstates
\[
\begin{align*}
\nu & \simeq V^T_{\nu} \nu_L + \nu^c_L V^*_{\nu} \\
N & \simeq \nu_R + \nu^c_R \\
m_{N_i} & = M_i
\end{align*}
\]
where \(V_{\nu}\) is the neutrino mixing matrix. We shall restrict our discussion to the case of hierarchical Majorana neutrino masses, \(M_1 \ll M_2, M_3\) so that if the interactions of \(N_1 = N\) are in thermal equilibrium when \(N_2\) and \(N_3\) decay, the asymmetry produced by \(N_2\) and \(N_3\) can be erased before \(N_1\) decays. The asymmetry is then generated by the out of equilibrium \(CP\) violating decays of \(N \to \ell H\) versus \(N \to \bar{\ell} H\) at a temperature \(T \sim M \equiv M_1 \ll M_2, M_3\).

The crucial ingredients in leptogenesis scenario is \(CP\) asymmetry generated through the interference between tree level and one-loop Majorana neutrino decay diagrams. In the simplest extension of SM, these are shown below in Fig. 8.

Then the \(CP\) asymmetry is caused by interference between the above diagrams:

\[
\begin{align*}
\epsilon_1 & = \frac{\Gamma (N_1 \to \ell_i H) - \Gamma (N_1 \to \bar{\ell}_i H^*)}{\Gamma (N_1 \to \ell_i H) + \Gamma (N_1 \to \bar{\ell}_i H^*)} \\
& = \frac{1}{8\pi} \left| h_{1\ell_i} \right|^2 \sum_{\ell = 2,3} \text{Im} \left[ h_{1\ell_1} h_{1\ell_2}^* h_{1\ell_3}^* \right] \left[ f \left( \frac{M_2^2}{M_1^2} \right) + g \left( \frac{M_2^2}{M_1^2} \right) \right] \\
& = -\frac{3}{16\pi} \left| h_{1i} \right|^2 \text{Im} \left[ h_{1i} h_{1k}^* h_{2k}^* \right]
\end{align*}
\]

Thus

\[
\begin{align*}
\epsilon_1 & = -\frac{3}{10\pi} \left[ I^{1\ell_i}_{1\ell_i} \frac{M_1}{M_2} + I^{1\ell_i}_{1\ell_i} \frac{M_1}{M_3} \right]
\end{align*}
\]
The lepton asymmetry \( Y_L \) is related to the \( CP \) asymmetry through the relation
\[
Y_L = \frac{n_L - \bar{n}_L}{s} = \mathcal{K} \frac{\epsilon_1}{g^*}
\]
(143)
where \( g^* \) is the effective number of relativistic degrees of freedom contributing to the entropy and \( \mathcal{K} \) is the so called dilution factor which accounts for the wash out processes [inverse decay and lepton number violating scattering; such processes can create a thermal population of heavy neutrinos of high temperature \( T > M \)] and it can be obtained through solving the Boltzmann equations. In the SM, \( g^* = 12 \times 2 + \frac{7}{8} (18 \times 4 + 3 \times 4 + 3 \times 2) = 103.75 \).

The produced lepton asymmetry through \( Y_L \) is converted into a baryon asymmetry as already seen [\( a = -\frac{2}{3} \)]
\[
Y_B = \frac{a}{a - 1} Y_L \approx 0.4 Y_L
\]
(144)

Now
\[
(m_D)_{ij} = h_{ij} v \quad \quad \quad (m_D m_D^\dagger)_{11} = (m_D)_{1i} (m_D^\dagger)_{i1} = (h_{1i} h_{1i}^*) v^2 = |h_{1i}|^2 v^2 \quad \quad \quad (m_D m_D^\dagger)_{12} = (m_D)_{1i} (m_D^\dagger)_{i2} = (h_{1i} h_{2i}^*) v^2 \equiv (h_{1k} h_{2k}^*) v^2
\]
(145)

Thus from Eq. (142)
\[
I_{12}^{ik} = \frac{1}{v^2 (m_D m_D^\dagger)_{11}} \operatorname{Im} \left[ \left( (m_D m_D^\dagger)^2 \right) M_1 \frac{M_1}{M_2} + \operatorname{Im} \left[ \left( (m_D m_D^\dagger)^2 \right) M_1 \frac{M_3}{M_2} \right] \right]
\]

For illustrative purposes we consider two right handed neutrinos \( N_{1,2} \) and take
\[
m_D = \begin{pmatrix} a & a' & 0 \\ 0 & b & b' \end{pmatrix}
\]
(146)
Then the seesaw form (136) becomes

\[ m_\nu = \begin{pmatrix} a & 0 \\ a' & b' \end{pmatrix} \begin{pmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{pmatrix} \begin{pmatrix} a & a' \\ 0 & b \end{pmatrix} = \begin{pmatrix} \frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\ \frac{a'a}{M_1} & \frac{a'^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\ 0 & \frac{bb'}{M_2} & \frac{b'^2}{M_2} \end{pmatrix} \] (147)

We have to diagonalize it in order to go to mass basis:

\[
\begin{vmatrix}
\frac{a^2}{M_1} - \lambda & \frac{aa'}{M_1} & 0 \\
\frac{a'a}{M_1} & \frac{a'^2}{M_1} + \frac{b^2}{M_2} - \lambda & \frac{bb'}{M_2} \\
0 & \frac{bb'}{M_2} & \frac{b'^2}{M_2} - \lambda
\end{vmatrix} = 0
\] (148)

Under the assumption, \( \frac{b^2}{M_2}, \frac{b'^2}{M_2} \ll \frac{a^2}{M_1} \), this gives the mass eigenvalues 0, \( \frac{a^2}{M_1}, \frac{a^2 + a'^2}{M_1} \). We may identify

\[
\Delta m^2_{\text{atm}} = m^2_3 - m^2_1 = \left( \frac{a^2 + a'^2}{M_1} \right)^2
\] (149)

\[
\Delta m^2_s = m^2_2 - m^2_1 = \left( \frac{b^2}{M_2} \right)^2
\] (150)

so that

\[
\frac{a^2 + a'^2}{M_1} \simeq \left( \Delta m^2_{\text{atm}} \right)^{1/2} \simeq \left( 3 \times 10^{-3} \text{ eV}^2 \right)^{1/2} \\
\simeq 5 \times 10^{-2} \text{ eV}
\] (151)

\[
\frac{b^2}{M_2} \simeq \left( \Delta m^2_s \right)^{1/2} = \left( 5 \times 10^{-2} \text{ eV}^2 \right)^{1/2} \\
\simeq 7 \times 10^{-3} \text{ eV}
\] (152)

Now

\[
(m_D m_D^\dagger)_{11} = |a|^2 + |a'|^2 \\
(m_D m_D^\dagger)_{12} = a'b^* 
\] (153)
Therefore [c.f. Eq. (141)]
\[
\epsilon_1 = -\frac{3}{16\pi v^2 |a|^2 + |a'|^2} \left\{ \text{Im} \left[ (a'b^*)^2 \right] \frac{M_1}{M_2} \right\}
\]
(154)

Take now \( a' = Yae^{i\delta} \) and \( b' = b \) as real. Then
\[
\epsilon_1 = -\frac{3}{16\pi v^2} \frac{Y^2}{1 + Y^2} \frac{b^2}{M_2} \sin 2\delta \\
\approx -\frac{3}{16\pi} \frac{M_1}{v^2} \frac{Y^2}{1 + Y^2} 7 \times 10^{-3} \text{eV} \sin 2\delta \\
\approx -1.36 \times 10^{-17} \frac{M_1}{\text{GeV}} \frac{Y^2}{1 + Y^2} \sin 2\delta
\]
(155)

Taking \( Y \approx 1, \sin 2\delta \approx -1, M_1 \approx 10^{10} \text{GeV} \)
\[
\epsilon_1 = 0.68 \times 10^{-7} = 6.8 \times 10^{-8}
\]
(156)

Thus from equations (143) and (144)
\[
Y_B = \mathcal{K}(2.7) \times 10^{-10}
\]
(157)

This gives the right order magnitude as typical numbers one expects for \( \mathcal{K} \approx 10^{-1} \) to \( 10^{-2} \).

Acknowledgements: The author would like to acknowledge the support of King Fahd University of Petroleum and Minerals for this work.

9 Figure Captions

Figure 1: Cosmic density pyramid

Figure 2: Tree level Higgs couplings with fermions in SU(5)

Figure 3: Loop level Higgs and gauge particles contribution to baryon asymmetry

Figure 4: Loop level contribution of \(^5\)H Higgs boson to baryon asymmetry

Figure 5: Transition from one vacuum to another by thermal fluctuations

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Figure 6: Feynmann diagram for scattering process between left and right handed fermions, Higgs bosons and W boson

Figure 7: See-saw mechanism for Majorana light neutrino masses

Figure 8: Tree and one-loop level Majorana (heavy) neutrino decay diagrams
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Figure 1
