Pair plasma cushions in the hole-boring scenario

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Received 18 March 2013, in final form 16 July 2013
Published 7 August 2013
Online at stacks.iop.org/PPCF/55/095016

Abstract

Pulses from a 10 PW laser are predicted to produce large numbers of gamma-rays and electron–positron pairs on hitting a solid target. However, a pair plasma, if it accumulates in front of the target, may partially shield it from the pulse. Using stationary, one-dimensional solutions of the two-fluid (electron–positron) and Maxwell equations, including a classical radiation reaction term, we examine this effect in the hole-boring scenario. We find the collective effects of a pair plasma ‘cushion’ substantially reduce the reflectivity, converting the absorbed flux into high-energy gamma-rays. There is also a modest increase in the laser intensity needed to achieve threshold for a non-linear pair cascade.

(Some figures may appear in colour only in the online journal)

1. Introduction

Particle-in-cell (PIC) simulations of ultra-intense laser pulses interacting with plasma now probe the regime in which non-linear QED processes are important [1–3], and predict the production of large numbers of gamma-rays and electron–positron pairs when the laser interacts with either an over-dense or an under-dense plasma. Next generation lasers (10 PW) will be able to test these predictions. However, it is still not clear whether or not the fully non-linear pair cascade predicted by Bell and Kirk [4] and Fedotov et al [5] will be achieved. For counter-propagating pulses, the threshold is expected to lie below a single pulse intensity of $10^{24}$ W cm$^{-2}$. But simulations of interactions with over-dense plasmas [2], which are the more straightforward experimental set-up, have identified several effects that might raise the required threshold intensity.

One effect with similar consequences that has so far not been analyzed is the screening by a cloud or ‘cushion’ of pair plasma in the laser pulse just ahead of the target. As this cushion approaches the critical density, collective effects in the pair plasma can be expected to slow down the laser pulse and reflect or absorb it. Such cushions are observed in PIC simulations of linearly polarized laser pulses interacting with dense, solid targets (see, for example, figure 1 in [2]), but their dynamics are complex. In this paper we do not attempt an analysis of simulation results. Instead, we try to gain a qualitative understanding of pair cushions by investigating stationary solutions in a highly simplified situation. Although they might be difficult to realize in practice, these solutions provide an easily quantifiable framework in which to interpret PIC simulations and discuss experimental set-ups.

We consider the interaction of a circularly polarized laser beam with a solid target in the hole-boring scenario [6–8], as sketched in figure 1. A pair plasma cushion, located in the region excavated by the beam, is described by a one-dimensional, cold, two-fluid model that includes a classical radiation reaction term. Stationary solutions are found and matched to the boundary conditions of the incoming laser beam on one side, and the standing wave in the vacuum gap, on the other. Section 2 describes the hole-boring scenario and, in particular, the relativistic dynamics of the hole-boring front; section 3 presents the two-fluid equations and the method of their solution in the rest frame of the hole-boring front; section 4 analyzes the properties of these solutions and section 5 discusses a practical application. We conclude with a discussion of the physical significance of the solutions.

2. The hole-boring scenario

2.1. Dynamics of the hole-boring front

The one-dimensional model of hole-boring by a circularly polarized plane wave is based on the properties of the ‘hole-boring front’ [6–8] that divides the vacuum waves on
one side from the high density plasma (originally solid) on the other. The front reflects the ions and electrons that stream into the solid, by means of a charge-separated region that supports a strong, longitudinal electric field. At the same time, the front perfectly reflects the circularly polarized laser that is incident on its outer surface. A closer look reveals that electrons are absent in the charge-separated region, which is bounded on the target side by an ‘electron sheath’, a thin enhancement in the electron density at which the laser is reflected. The target ions stream in through the electron sheath, are brought to rest by the longitudinal electrostatic field, and subsequently accelerated back through the electron sheath [7]. The key parameter in this scenario is the ratio of the incident laser intensity $I^*$, assumed be a circularly polarized, monochromatic plane wave of (angular) frequency $\omega_0$ propagating in the positive $x$ direction, and the solid target density $\rho$ (both measured in the laboratory frame), which are combined into the dimensionless quantity

$$X = I^*/(\rho c^3)$$  \hspace{1cm} (1)

For a laser pulse of intensity $I^* < 10^{24}$ W cm$^{-2}$ impacting a metal target, $X \ll 1$. The thickness of the charge-separated region is then approximately $\sqrt{X \epsilon/3\omega_0}$ [7], where $\omega_0$ is the ion plasma frequency in the fully ionized target. This length is small compared to the laser wavelength, which is the characteristic dimension of the pair cushion. Therefore, we will treat the hole-boring front as a singular surface current—a perfect mirror, at which the electric field vanishes.

The speed of advance of the front into the solid, as well as the energy in the laboratory frame of the reflected ions, is given by equating the pressure exerted by the laser with that exerted by the in-streaming and reflected ions, neglecting the electron inertia. Allowing for a reflected wave of intensity $I^*$, the energy–momentum tensor $T^{\mu\nu}$ of the radiation field has the following non-vanishing elements

$$T^{00} = T^{11} = (I^* + I^-) / c = I^*/(1 + R) / c$$
$$T^{01} = T^{10} = (I^* - I^-) / c = I^*/(1 - R) / c,$$  \hspace{1cm} (2)

where the reflectivity is $R = I^-/I^*$. The front is assumed to move into the solid (which is at rest in the laboratory frame) at constant speed $c\beta_f$, and the elements $T^{\mu\nu}$ of the energy–momentum tensor of the laser in the rest frame of the front (the ‘HB-frame’) follow from Lorentz boosting (2):

$$T^{01} = T^{10} = \Gamma_f^2 \left[ (1 - \beta_f)^2 - (1 + \beta_f)^2 R \right] I^*/c$$  \hspace{1cm} (3)
$$T^{00} = T^{11} = \Gamma_f^2 \left[ (1 - \beta_f)^2 + (1 + \beta_f)^2 R \right] I^*/c,$$  \hspace{1cm} (4)

where $\Gamma_f = (1 - \beta_f^2)^{-1/2}$. Thus, the reflectivity in the rest frame of the hole-boring front is

$$R' = \left( \frac{1 + \beta_f}{1 - \beta_f} \right)^2 R,$$  \hspace{1cm} (5)
$$D = D^{-R} R,$$  \hspace{1cm} (6)

where the Doppler factor

$$D = \left( \frac{1 - \beta_f}{1 + \beta_f} \right)^{1/2}$$  \hspace{1cm} (7)

is the ratio of the laser frequency $\omega$ in the HB-frame to its value $\omega_{lab}$ in the laboratory frame.

On the other hand, the elements of the energy–momentum tensor on the target side of the hole-boring front are found by assuming perfect reflection of the ions. In the HB-frame, therefore, two mono-energetic beams of velocity $\pm c\beta_f$ and proper density $\rho$ exist immediately behind the front, so that

$$T^{00} = 2\rho c^2 \Gamma_f^2$$
$$T^{01} = 2\rho c^2 \beta_f^2 \Gamma_f^2$$
$$T^{10} = 0.$$  \hspace{1cm} (8)

The requirement that $T^{01}$ be continuous across the front determines the advance speed, provided $R'$ is known:

$$\beta_f = \sqrt{\xi} / \left( 1 + \sqrt{\xi} \right),$$  \hspace{1cm} (10)

where the reflection-modified $X$ parameter is defined as

$$\xi = X(1 + R')/2.$$  \hspace{1cm} (11)

The Doppler factor and (dimensionless) $x$-component of the four-velocity of the front, $u_x = \beta_f \Gamma_f$, are also functions of $\xi$ alone:

$$D = \left( 1 + 2\sqrt{\xi} \right)^{-1/2}$$  \hspace{1cm} (12)
$$u_x = D \sqrt{\xi},$$  \hspace{1cm} (13)
and the reflectivity in the laboratory frame is

$$R = \left( 1 + 2\sqrt{\xi} \right)^{-2} R'.$$  \hspace{1cm} (14)

In the standard hole-boring scenario, perfect reflection is assumed in the rest frame of the hole-boring front: $R' = 1$, $\xi = X$, in which case these expressions agree with those given by [6]. In this case, the electric and magnetic fields, as seen in the HB-frame, form a standing wave. They are everywhere...
parallel, and lie in a plane that contains the $x$-axis and rotates about it. The field magnitudes are constant in time at each position, but at a given instant vary sinusoidally in $x$ with equal amplitudes

$$E_{\text{ampl}} = 2D\sqrt{4\pi \Gamma^2/c}$$

and a phase difference of $\pi/2$. As we show below, a pair cushion greatly reduces $R$, leading to a smaller speed of advance of the hole-boring front, and a reduced electric field amplitude.

3. Two-fluid model of the pair cushion

3.1. Governing equations

In the presence of a pair plasma, the incident and reflected waves in the excavated channel are strongly coupled, and the relevant solutions are not vacuum waves, but non-linear, transverse electromagnetic modes of superluminal phase speed. We use a cold, two-fluid (electron and positron) description to analyze these waves. The continuous charge constituents. The Cartesian four-velocity components and force, thus taking into account the discrete nature of the fluid speed. We use a cold, two-fluid (electron and positron)

In order to find non-linear solutions that are homogeneous in the $y$–$z$ plane, we make a number of simplifications: firstly, in the transverse electromagnetic waves of interest here, electrons and positrons have the same density and oppositely directed transverse momenta: $n^- = n^+ = n$ and $u_{1-} = -u_{1+} = u_\perp$. It follows that $E_z = 0$. Secondly, we look for solutions in which the fluids do not stream along $x$ in the frame in which the hole-boring front is stationary: $u_x = 0$. In the following, the $\pm$ notation is dropped and the equations presented apply to the electric fluid in this frame.

The classical radiation reaction force in the Lorentz–Abraham–Dirac formulation is

$$g^\mu = \frac{2e^2}{3mc^3} \left( \frac{d^2u^\mu}{dt^2} - u^\mu \left| \frac{du^\nu}{dt} \right|^2 \right),$$

and it is clear that the spatial components lie in the $y$–$z$ plane when $u_x = 0$. This property is shared by the Landau–Lifshitz formulation of radiation reaction, in which the derivatives in (17) are replaced using the Lorentz equation of motion (see [9] for a review). Thus, the $x$-component of the fluid equation of motion is unaffected by radiation reaction:

$$\left( \gamma \frac{\partial}{\partial t} + cu_x \frac{\partial}{\partial x} \right) u_x = -\frac{e}{mc} \text{Im} (u_\perp B^*).$$

where $\gamma = u^0$. Solutions with $u_x = 0$ for all $x$ and $t$, therefore, require the transverse velocity and magnetic field vectors to be parallel:

$$\text{Im} (u_\perp B^*) = 0.$$  (19)

On the other hand, the (complex) equation of motion in the transverse plane contains a term due to radiation reaction:

$$\gamma \frac{\partial u_\perp}{\partial t} = -\frac{e}{mc^2} \gamma E + g_\perp,$$  (20)

where $g_\perp$ is the spatial part of $g^\mu$ in rotating coordinates, and we have set $u_x = 0$. For these transverse fields ($E_z = 0$, $B_z = 0$), the set of governing equations is completed by the Faraday and Ampère equations:

$$\frac{\partial E}{\partial x} - i \frac{\partial B}{c \partial t} = 0$$

and the equation of continuity:

$$\frac{\partial}{\partial t} (\gamma \rho) = 0.$$  (23)

3.2. Method of solution

We seek solutions that are separable in $x$ and $t$ in the HB-frame. In particular, for a monochromatic wave of angular frequency, $\omega$, the quantities $E$, $B$ and $u_\perp$ are proportional to $e^{i\omega t}$, whereas $n$ and $|u_\perp|$ are constant in time. Since force balance along $x$ (19) requires the fluid velocity to be parallel to the magnetic field, the complex variables $E$, $B$ and $u_\perp$ can be replaced by three real, positive, dimensionless amplitudes $a$, $b$ and $u$, together with two phases, $\phi$ and $\delta$, all of which are functions of $x$ only:

$$E = \left( \frac{mc \omega}{e} \right) a e^{i\phi + i\omega t}$$

$$B = \left( \frac{mc \omega}{e} \right) i b e^{i\phi + i\omega t}$$

$$u_\perp = i u e^{i\phi + i\omega t}.$$  (26)

Substituting these into (20), the transverse equations of motion become

$$u = a \cos \delta$$

$$\delta = \arctan (\epsilon u^3),$$  (28)

where

$$\epsilon = \frac{2}{3} \frac{mc^2}{\omega^3}$$

$$= 1.18 \times 10^{-8} \lambda_{\text{m}},$$  (30)

with $\lambda_{\text{m}}$ the laser (vacuum) wavelength in the laboratory frame in micrometers. The Lorentz–Abraham–Dirac form (17) of the radiation reaction term was used in deriving (28), and only the leading contribution in an expansion in $1/\gamma$ was retained:

$$g_\perp \approx -\left( \frac{2e^2 \omega}{3mc^3} \right) \omega \gamma^4 u_\perp.$$  (31)

The Landau–Lifshitz form yields exactly the same result in this limit.
The pair fluids do not contribute to the (1,1) and (0,1) components of the energy–momentum tensor. The latter is, therefore, just the Poynting flux:

\[ T^{01} = \left( \frac{m^2 c^2 \omega^2}{8\pi e^2} \right) P \]

\[ P = 2ab \cos \delta, \]  
and the former is the energy-density of the fields:

\[ T^{11} = \left( \frac{m^2 c^2 \omega^2}{8\pi e^2} \right) U \]

\[ U = a^2 + b^2 \]

The Faraday and Ampère equations take the form:

\[ \frac{da}{dx} = b \sin \delta \quad \frac{db}{dx} = -a \sin \delta \]

and can be used to evaluate the divergence (in this case, derivative with respect to \(x\)) of \( T^{01} \) and \( T^{11} \):

\[ \frac{dP}{dx} = -2 \left( \frac{n}{n_{cr}} \right) a^2 \sin \delta \cos \delta \]

\[ \frac{dU}{dx} = 0, \]

where \( n_{cr} = \frac{m \omega^2}{8\pi e^2} \) is the critical proper density.

The solution of the system (27), (28), (34) and (35) can be reduced to a quadrature:

\[ x = \int \frac{d\delta}{\sin \delta} \left\{ \frac{r_c \cos^2 \delta}{\sin \delta} \right\}^{2/3} - 1 \]

where

\[ r_c = \epsilon U^{1/2} = \text{constant.} \]

Having found \( \delta(x) \), \( a \) and \( u \) follow from (27) and (28). The constant pressure condition, (37), determines the magnetic field \( b \) in terms of the constant \( a_0 = \sqrt{a^2 + b^2} = \sqrt{U} \) and the density follows from energy conservation, (36):

\[ \frac{n}{n_{cr}} = \frac{2a^2 + 3a^2 \sin^2 \delta - a_0^2}{a^2 (4 - 3 \cos^2 \delta)}. \]

Finally, the phase \( \phi \) is evaluated from

\[ \phi = -\int \frac{\tan(\delta)}{a} \]

This solution depends on only one parameter, \( r_c \), which determines the importance of radiation reaction, and is equivalent to \( R_c \) in the notation of de Piazza et al [9]. The ‘classical radiation-dominated regime’ corresponds to \( r_c > 1 \).

### 3.3. Boundary conditions

At the downstream boundary, \( x = x_2 \), (see figure 1) the fields of the pair cushion must match those of either the hole-boring front or a vacuum gap. Continuity of the energy–momentum tensor components \( T^{01} \) requires zero Poynting flux, because the hole-boring front is assumed perfectly reflecting in the H-frame.

Using the notation \( a_{1,2} = a_{1,2} \), etc., this implies either \( b_2 = 0 \) or \( a_2 = a_0 = 0 \). The latter possibility is, however, unphysical, since it implies \( da/dx > 0 \) for \( x \to x_2 \), leading to a negative value of \( a \) just upstream of this boundary. Thus, \( b_2 = 0 \), and \( a_2 = a_0 \), and \( \delta_2 \) follows from (27) and (28).

This means that the edge of the pair cushion cannot be located at the mirror, where continuity of the tangential component of \( E \) requires it to vanish. Because the finite-density cushion cannot carry a singular current sheet at the surface \( x = x_2 \), the transverse component of \( B \) must also be continuous across it. Therefore, this point lies at a node of the magnetic field not only of the plasma wave, but also of the standing wave that occupies the vacuum region \( x > x_2 \). The cushion must, therefore, be separated from the mirror by a vacuum gap of thickness \( j + 1/2) \pi c/\omega \), where \( j = 1, 2, \ldots \). According to (40), the pair density reaches the critical value at the edge of this gap: \( n_2 = n_{cr} \).

At the upstream boundary, \( x = x_1 \), the fields in the cushion must match the vacuum fields of the incident and reflected laser beams. The location of this point is fixed by the number of pairs contained per unit area of the cushion, which rises with increasing cushion thickness from zero when \( x_1 = x_2 \) to a maximum value determined by the point at which the proper pair density vanishes. The electric field amplitude \( a = a_{\min} \) at which this occurs can be found from (27), (28) and (40):

\[ \epsilon^2 (5a_{\min}^2 - a_0^2)^4 + 54a_{\min}^2 - 27a_0^2 = 0. \]

The corresponding position sets the maximum thickness, \( \Delta x \) (in units of \( c/\omega \)), of the pair front compatible with a physically acceptable solution. In general, a quadrature is needed to find this quantity. However, for \( r_c \ll 1 \), one finds \( \Delta x = r_c/2 \) and \( a_1 \approx \alpha_0/\sqrt{2} \). This leads to the approximate expression:

\[ \Delta x = \frac{1}{r_c} \int_{1/\sqrt{2}}^1 \frac{dy}{y^2 + \frac{1}{1 - y^2}} \approx 0.1478/r_c. \]

Figure 2 compares this result to the numerically evaluated quadrature.

### 4. Results

The spatial dependence of the electromagnetic fields is shown in figure 3, for \( r_c = 0.03 \) at time \( t = 0 \) (the fields are proportional to \( e^{i\omega t} \)). In this example, the pair front has been chosen to have its maximum thickness, i.e. the density vanishes at the upstream edge. It then rises monotonically, reaching the critical value at the downstream edge of the front, where \( x = 0 \). For \( r_c \ll 1 \), the thickness of the front is large compared to the
Figure 2. Top: the phase-shift of the magnetic field caused by radiation reaction at the upstream (δ1) and downstream (δ2) edges of the pair cushion (the angle between the electric and magnetic field vectors is π/2 + δ). Bottom: the maximum thickness Δx = x2 - xnew of the cushion (in units of c/ω) compared to the approximate expression (43), both as functions of the radiation reaction parameter rc defined in (39).

Figure 3. Top: the spatial profile of the electromagnetic fields, as seen in the rest frame of the hole-boring front at t = 0, normalized to the value aωm/e. Bottom: the proper density, normalized to the critical density aωm/e (8πe2). The edges of the pair front are indicated by vertical lines at x1 = -38.3 and x2 = 0. Note that the fields are continuous, but the current-density discontinuity at x = 0, imposes a discontinuity on the x derivative of B (but not of E). In this figure, the radiation reaction parameter is rc = 0.03.

Figure 4. The field and pair density profiles in the presence of a pair cushion. (shaded red) for rc = 0.2. The target is shaded blue. The boundaries of the pair cushion are at x1 = -5.6 and x2 = 0.

Vacuum waves propagate both upstream and downstream of the pair front. Upstream (the region x < 38.3 in figure 3), the amplitude of the backward propagating wave is given by the reflectivity of the overall system consisting of pair front plus hole-boring front. For a pair front of maximum thickness, the edge of the pair cushion at r = 0, the density, however, is discontinuous (note that u0 = 0), although only at the downstream edge for a front of maximum permitted thickness. A discontinuity in the density implies a discontinuity also in the current density. As a result, the x-derivative of the magnetic field has a discontinuity, but that of the electric field does not.

At both the upstream and downstream edges, the electromagnetic fields are continuous, as are the fluxes of energy, T'01, and x-momentum, T'11. The density, however, is discontinuous (note that u0 = 0), although only at the downstream edge for a front of maximum permitted thickness. A discontinuity in the density implies a discontinuity also in the current density. As a result, the x-derivative of the magnetic field has a discontinuity, but that of the electric field does not.

5. Applications

The hole-boring front itself, however, can be located at any of the nodes of the electric field in this wave. Choosing the minimum vacuum gap size, the fields in the pair cushion and vacuum gap are illustrated in figure 4, for rc = 0.2.
that of the standard scenario without a pair front, \( R' = 1, \)
\( R = 1/\left(1 + 2\sqrt{X}\right)^2. \) However, the minimum value of \( R' \),
which is attained for the maximum number of pairs consistent
with a stationary solution, depends on the radiation reaction
parameter \( r_c \), which, in turn, depends not only on the hole-boring
parameter \( X \), but also on the laser wavelength and the number of pairs contained in the cushion.

As an example, we consider the effect of a pair cushion
when an intense laser pulse of wavelength \( \lambda_{em} = 1 \) impacts
an aluminum target (\( \rho = 2.7 \times 10^3 \text{ kg} \text{ m}^{-3}, Z = 13 \)).
This target is over-dense, since the ratio of the electron density
to the critical density is 698. Therefore, the hole-boring scenario can be expected to apply provided
the laser is not intense enough to render it relativistically
under-dense, which implies the restriction \( I_{24} < 1.33 \rho_{\text{em}}^{2/3} \),
assuming a circularly polarized pulse. For laser intensities
in this range, the stationary solution with the largest number
of pairs is found by iteratively solving (10) and (11) together
with (44).

The top panel of figure 5 shows the reflectivities in the
laboratory and hole-boring frames assuming a pair front of
maximum extent is present in the channel. At low intensities,
the laser is almost completely absorbed by the pair cushion.
In the approximation used to describe radiation reaction, (31),
the energy absorbed is converted entirely into transversely
directed, synchrotron-like radiation. Thus, in the HB-frame
the efficiency of conversion of laser light into high-energy
photons is \( 1 - R' \), i.e. close to 100% at low intensities and
87% at an intensity of \( 10^{23} \text{ W cm}^{-2} \).

The middle panel of figure 5 shows the kinetic energy in
the laboratory frame of an aluminum ion reflected back into
the target off the advancing hole-boring front: \( E_{\text{ion}} = 2Mc^2\beta^2T_f^2. \) In the case shown here, the speed of
advancement of the hole-boring front \( c\beta \), remains non-relativistic,
even for the highest intensity plotted. It is assumed here that,
for a given laser intensity, the pair cushion attains its maximum
possible thickness, which is plotted in units of \( c/\omega \) as a function
of laser intensity in this panel. Also shown is the parameter \( r_c \)
which characterizes the importance of the classical radiation
reaction force. Above \( I \approx 3 \times 10^{23} \text{ W cm}^{-2} \), this parameter
exceeds unity, i.e. the energy radiated by a single electron or
positron in one laser period is greater than the particle energy.
Radiation reaction is responsible for the change in slope at
roughly this intensity of the curves depicting \( R' \) and \( R \) in the
upper panel. When quantum effects are small (see lower panel)
the synchrotron-like radiation emitted at the downstream edge
of the cushion peaks at an energy
\[ h\nu \approx 0.7m^2c^2/r_c \left( \frac{c}{\omega} \right) \cos^3 \delta, \]
where \( \alpha \) is the fine-structure constant.

The lower panel of figure 5 plots the the QED parameter \( \eta \),
the ratio of the electric field seen in the electron or positron
rest frame to the critical field \( E_c = m^2c^3/e\hbar \) at the edge
of the pair front. As \( \eta \) approaches unity, quantum effects such
as electron recoil on emitting a photon and pair production
begin to become important. Three curves are shown. At low
intensities, where the number of pairs that can be contained
in a stationary cushion is relatively large, the solid red line
(depicting \( \eta \) in the presence of a maximal pair cushion) lies well
below the dashed red line (depicting \( \eta \) in the absence of a pair
cushion.) At intensities close to \( 10^{24} \text{ W cm}^{-2} \), the reduction
is less marked, being roughly 15%. Both of these curves lie
below that predicted for counter-propagating vacuum waves
of intensity \( I^+ \) [4], shown as a blue dashed line. This is
because of the recoil of the target, which effectively reduces
the pressure and frequency of the incident laser pulse [2].
Assuming \( a_0 \gg 1 \), we find
\[ \eta_{\text{pairs}} = \left( \frac{h\omega}{mc^2} \right) a_0^2 \cos^2 \delta. \]
At low intensity, \( \eta_{\text{pairs}} \propto a_0^2 \propto \sqrt{I^+} \), but the factor \( \cos^2 \delta \),
 arising from the phase shift between the force on the charged
fluids and the electric field which is brought about by radiation
reaction, causes \( \eta \) to rise less rapidly with laser intensity when
\( I^+ > 3 \times 10^{23} \text{ W cm}^{-2} \) [4].

6. Discussion

The solutions presented above fulfill the fully non-linear
coupled fluid and Maxwell equations including radiation
reaction. Several studies have treated classical radiation reaction in the contexts of the hole-boring scenario and of counter-propagating laser beams [11–13], but these neglect the influence of the radiating particles on the laser fields. Classical radiation reaction terms have been incorporated in PIC simulation codes, which, in principle, treat the fields self-consistently [8, 14, 15]. However, the solutions we find are analytical, in the sense that they can be reduced to the quadratures (38) and (41). This makes them a useful and flexible tool for the interpretation of both simulations and experiments.

The unique aspect of these solutions is the inclusion of classical radiation reaction. This force, acting on a single charged (relativistic) particle moving in the coherent laser field, is approximately anti-parallel to the particle speed in the laboratory frame. The energy dissipated is carried off primarily as short-wavelength photons. We assume the fluids in our treatment consist of electrons and positrons that radiate independently of each other, which is reasonable provided the wavelength of the radiated photons in the particle rest frame is small compared to the inter-particle spacing in that frame. For photons of frequency \(a_0^2 \omega_{\ln} \approx \left(\frac{2\pi}{\lambda}\right)^7\), the characteristic energy of the synchrotron-like spectrum and for a pair plasma at the critical density, this ratio can be estimated as

\[
e = \frac{c^4}{(4\pi^2 e^2/m^2 a_0^2)} = (\frac{a_0^2 c}{\lambda})^{-1/3} = 6 \times 10^{-4} D^{-1/3} (1 + R)^{-1} \lambda^{-5/3} (I^*/10^{24} \text{ W cm}^{-2})^{-1},
\]

(48)

so that our approach is justified for optical lasers with \(I^* > 10^{21} \text{ W cm}^{-2}\).

In this regime, classical radiation reaction introduces an effective friction term into the equations of motion for the fluids. For the one-dimensional problem considered here, this force lies in the \(y-z\) plane and has no component in the direction of laser propagation. As a result, the effective friction term into the equations of motion for the pair cushion remains close to the peak of the synchrotron-like spectrum and for a rather long incident pulse. Also, it is not clear that a significant number of pairs will be available to form a cushion in a realistic experimental configuration, particularly at low laser intensity. At high intensity, the underlying hole-boring scenario itself is in doubt. Piston oscillations can be become pronounced, and hot electrons may leak from the target into the vacuum gap [6–8]. Eventually, even solid targets become relativistically under-dense and are unable to reflect the incident pulse. In figure 5 we have implicitly adopted the conventionally estimated threshold for this effect, although possible departures from it have been discussed in the literature [18–20].

7. Conclusions

We present solutions to the coupled set of Maxwell’s equations and those for two cold, charged, relativistic fluids (pair plasma), including classical radiation reaction. The pair plasma reaches critical density and is bounded by regions containing vacuum fields: an incident laser and its reflected beam on one side, and a standing vacuum wave separating the pair plasma or ‘cushion’ from an over-dense target on the other. The solutions have two main properties, both of which are shown in figure 5: first, the pair cushion forms an efficient device for converting the energy flux in the laser into high energy photons, as is evident from the substantial reduction in the reflectivity. Second, the laser intensity at which quantum effects become important is increased somewhat, as can be seen from the difference between \(\eta_{\text{pairs}}\) and 1.

However, it is not possible, using our calculations, to make a reliable estimate of the threshold for the onset of a non-linear pair cascade, since important effects such as ‘straggling’ [21] are not considered.

Acknowledgments

ARB and CPR thank the UK Engineering and Physical Sciences Research Council for support under grant EP/G055165/1.
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