Screening of a Moving Parton in the Quark-Gluon Plasma

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The screening potential of a parton moving through a quark-gluon plasma is calculated using the semi-classical transport theory. An anisotropic potential showing a minimum in the direction of the parton velocity is found. As consequences possible new bound states in the quark-gluon plasma and $J/\psi$ dissociation are discussed.

Screening of charges in a plasma is one of the most important collective effects in plasma physics. In the classical limit in an isotropic and homogeneous plasma the screening potential of a point-like test charge $Q$ at rest can be derived from the linearized Poisson equation, resulting in Debye screening. In this way the Coulomb potential of a charge in the plasma is modified into a Debye-Hückel or Yukawa potential [1]

$$\phi(r) = \frac{Q}{r} \exp(-m_D r)$$  \hspace{1cm} (1)

with the Debye mass (inverse screening length) $m_D$ ($\hbar = c = k_B = 1$).

A special kind of plasma is the so-called quark-gluon plasma (QGP), where the electric charges in a plasma are replaced by the color charges of quarks and gluons, mediating the strong interactions between them. Such a state of matter is expected to exist at extreme temperatures, above 150 MeV, or densities, above about 10 times nuclear density. These conditions could be fulfilled in the early Universe for the first few microseconds or in the interior of neutron stars. In accelerator experiments high-energy nucleus-nucleus collisions are used to search for the QGP. In these collisions a hot and dense fireball is created which might consist of a QGP in an early stage (less than about 10 fm/c) [2]. Since the masses of the lightest quarks and of the actually massless gluons are much less than the temperature of the system, the QGP is an ultrarelativistic plasma. To achieve a theoretical understanding of the QGP, methods from quantum field theory (QCD) at finite temperature are adopted [3]. Perturbative QCD should work at high temperatures far above the phase transition where the interaction between the quarks and gluons becomes weak due to a specific property of QCD called asymptotic freedom. An important quantity which can be derived in this way is the polarization tensor describing the behavior of interacting gluons in the QGP. From the polarization tensor important properties of the QGP, such as the dispersion relation and damping of the plasma modes or the Debye screening of color charges in the QGP can be derived [4].

In the QGP the Debye mass of a chromoelectric charge follows from the static limit of the longitudinal polarization tensor in the high-temperature limit [4],

$$\Pi_{00}(\omega = 0, k) = -m_D^2 = -g^2 T^2 \left( 1 + \frac{n_f}{6} \right),$$  \hspace{1cm} (2)

where $g$ is the strong coupling constant and $n_f$ the number of light quark flavors in the QGP with $m_q \ll T$.

The high-temperature limit of the polarization tensor corresponds to the classical approximation. For instance, it is closely related to the dielectric function following from the semi-classical Vlasov equation describing a collisionless plasma. E.g., the longitudinal dielectric function following from the Vlasov equation is given by [5–7]

$$\epsilon_l(\omega, k) = 1 - \frac{\Pi_{00}(\omega, k)}{k^2} = 1 + \frac{m_D^2}{k^2} \left( 1 - \frac{\omega}{2k} \ln \frac{\omega + k}{\omega - k} \right).$$  \hspace{1cm} (3)

(The only non-classical inputs here are Fermi and Bose distributions instead of the Boltzmann distribution.) Quantum effects in the Debye mass have been considered using the hard-thermal-loop resummation scheme [8], dimensional reduction [9], and QCD lattice simulations [10]. The Debye mass and polarization tensor have also been computed in the case of an anisotropic QGP [11–13].

The modification of the confinement potential below the critical temperature into a Yukawa potential above the critical temperature might have important consequences for the discovery of the QGP in relativistic heavy-ion collisions. Bound states of heavy quarks, in particular the $J/\psi$ meson, which are produced in the initial hard scattering processes of the collision, will be dissociated in the QGP due to screening of the quark potential and break-up by energetic gluons [14]. Hence the suppression of $J/\psi$ mesons have been proposed as one of the most promising signatures for the QGP formation [15]. Indeed, a suppression of $J/\psi$ mesons has been observed experimentally [16] and interpreted as a strong indication for the QGP formation in relativistic heavy-ion collisions [17].

In most calculations of the screening potential in the QGP so far, it was assumed that the test charge is at rest. However, quarks and gluons coming from initial hard processes receive a transverse momentum which causes them to propagate through the QGP [18]. In addition, hydrodynamical models predict a radial outward flow in the fireball [19]. Hence, it is of great interest to estimate the
screening potential of a parton moving relatively to the QGP. Chu and Matsui [20] have used the Vlasov equation to investigate dynamic Debye screening for a heavy quark-antiquark pair traversing a quark-gluon plasma. They found that the screening potential becomes strongly anisotropic.

In the case of a non-relativistic plasma the screening potential of a moving charge $Q$ with velocity $v$ follows from the linearized Vlasov and Poisson equations as 

$$
\phi(\vec{r}, t; \vec{v}) = \frac{Q}{2\pi^2} \int d^4k \exp \left[ -i \frac{\vec{k} \cdot (\vec{r} - \vec{v}t)}{k^2} \right] \frac{2}{k^2} \text{Re}[\epsilon_i(\omega = k \cdot \vec{v}, k)].
$$

(4)

It is easy to show that this expression reduces to the Yukawa potential in the case of small velocities, $v = |\vec{v}| \ll v_{th}$, where $v_{th}$ is the thermal velocity of the plasma particles. In the opposite case, $v \gg v_{th}$, the Coulomb potential is recovered since a screening charge cloud cannot be formed for fast particles.

The above equation (4) also holds in the case of a relativistic plasma. We only have to use the relativistic expression (3) for the longitudinal dielectric function. For small velocities, $v \to 0$, i.e., $\omega \ll k$, we obtain

$$
\epsilon_i(\omega \ll k) = 1 + \frac{m_D^2}{k^2},
$$

(5)

from which again the (shifted) Yukawa potential results [22]

$$
\phi(\vec{r}, t; \vec{v}) = \frac{Q}{|\vec{r} - \vec{v}t|} \exp(-m_D|\vec{r} - \vec{v}t|).
$$

(6)

It should be noted that the opposite limit $v \gg v_{th}$, leading to a Coulomb potential in the non-relativistic case, cannot be realized in an ultrarelativistic plasma because the thermal velocity of the plasma particles is given by the speed of light, $v_{th} = c = 1$.

In the general case, for parton velocities $v$ between 0 and 1, we have to solve (4) together with (3) numerically. Since the potential is not isotropic anymore due to the velocity vector $\vec{v}$, we will restrict ourselves only to two cases, $\vec{r}$ parallel to $\vec{v}$ and $\vec{r}$ perpendicular to $\vec{v}$, i.e., for illustration we consider the screening potential only in the direction of the moving parton or perpendicular to it.

In Fig.1 the screening potential $\phi/Q$ in $\vec{v}$-direction is shown as a function of $r' = r - vt$, where $r = |\vec{r}|$, between 0 and 6 fm for various velocities. For illustration we have chosen a strong fine structure constant $\alpha_s = g^2/(4\pi) = 0.3$, a temperature $T = 0.25$ GeV, and the number of quark flavors $n_f = 2$. The shifted potentials depend only on $v$ and not on $t$ as it should be the case in a homogeneous and isotropic plasma. For $r' < 1$ fm one observes that the fall-off of the potential is stronger than for a parton at rest. The reason for this behavior is the fact that there is a stronger screening in the direction of the moving parton due to an enhancement of the particle density in the rest frame of the moving parton.

In addition, a minimum in the screening potential at $r' > 1$ fm shows up. For example, for $v = 0.8$ this minimum is at about 1.5 fm with a depth of about 8 MeV. The occurrence of a minimum in the potential in the direction of the velocity is also observed in so-called complex plasmas. Complex plasmas are classical, low-temperature plasmas containing particles with a diameter of a few microns [23]. These particles are charged in the plasma by collecting electrons on their surface. In the presence of an ion flow the positively charged ions are deflected by the microparticles, leading to an anisotropic distortion of the Debye sphere by an enhancement of the ion density in front of the microparticle. This positive charge cloud leads to an attraction between microparticles in the direction of the ion flow and the formation of string like structures [24], observed in experiments.

A minimum in the screening potential is also known from non-relativistic, complex plasmas, where an attractive potential even between equal charges can be found. If the finite extension of the charges is considered [25]. A similar screening potential was found for a color charge at rest in Ref. [26], where a polarization tensor beyond the high-temperature limit was used. However, this approach has its limitation as a gauge dependent and incomplete approximation for the polarization tensor was used [27]. Obviously, a minimum in the interparticle potential in a relativistic or non-relativistic plasma is a general feature if one goes beyond the Debye-Hückel approximation by either taking quantum effects, finite velocities, or finite sizes of the particles into account.

Note that Chu and Matsui [20] did not report the existence of a minimum in the potential of a quark traversing the QGP. However, in their Fig.1(d) a negative value of the potential of a fast quark ($v = 0.9$) in the direction of the quark velocity is shown. Since the potential has to tend to zero for large distances, this implies a minimum in the screening potential. This minimum was not found because the potential was plotted only for the limited range $0 < r < 1/m_d \approx 0.35$ fm for our choice of the parameters.

The minimum could give rise to bound states, e.g., of diquarks, if thermal fluctuations do not destroy them. The two-body potential, associated with the dipole fields created by two test charges $Q_1$ and $Q_2$ at $\vec{r}_1$ and $\vec{r}_2$ with velocities $\vec{v}_1$ and $\vec{v}_2$, can be written as

$$
\Phi(\vec{r}_1 - \vec{r}_2, \vec{v}_1 - \vec{v}_2, t) = \frac{Q_1Q_2}{4\pi^2} \int d^4k \left\{ \frac{\exp \left[ i \vec{k} \cdot ((\vec{r}_1 - \vec{r}_2) - (\vec{v}_1 - \vec{v}_2)t) \right]}{k^2 \text{Re}[\epsilon_i(\omega = k \cdot \vec{v}_1, k)]} \right\}.
$$

(7)

For comoving quarks, $\vec{v}_1 = \vec{v}_2$, this two-body potential reduces to the one-body potential (4), showing the attrac-
tion between the quarks which could give rise to a bound state. Colored bound states, e.g., diquarks, of partons at rest have also been claimed by analyzing lattice data [28]. For a quark-antiquark system, where \( Q_1Q_2 < 0 \), the two-body potential is inverted, showing a maximum. This may lead to short living mesonic resonances and an enhancement of the attraction between quarks and anti-quarks of mesonic states moving through the QGP.

The details of the potential, e.g., the depth of the minimum, depend on the choice of the parameters, such as the coupling constant. For a value \( \alpha_s = 0.3 \), as it is typical for the temperature reachable in heavy-ion collisions, the semi-classical approach corresponding to the weak coupling limit might not be reliable. Quantum effects and collisions between plasma particles are important at those temperatures and will change the dielectric functions and dispersion relations. Within the transport theoretical approach collisions can be considered, for example by using the relaxation time approximation [29].

Non-abelian effects (beyond color factors, e.g., in the Debye mass) will be important at realistic temperatures. Unfortunately they cannot be treated by the methods used here and are therefore beyond the scope of this work. However, as we discussed above, also in a complex plasma which is a strongly coupled plasma as it is also the case for the QGP close to the critical temperature, there is an attraction between the microparticles in the presence of an ion flow. This appears to be a general feature of weakly as well as strongly coupled plasmas. Therefore we do not expect a qualitative change of the screening potential due to non-perturbative and non-abelian effects.

The results for \( \vec{r} \) perpendicular to \( \vec{v} \) are shown in Fig.2, where the potential is shown as a function of \( |\vec{r} - \vec{vt}| = \sqrt{r^2 + v^2t^2} \) between 0.1 and 1 fm. Here we consider only the case \( t = 0 \) since at \( t > 0 \) and \( v > 0 \) there is no singularity in the potential due to \( \sqrt{r^2 + v^2t^2} > 0 \) for all \( r \). Hence the potential is cut-off artificially at small distances if plotted as a function of \( \sqrt{r^2 + v^2t^2} \). In contrast to the parallel case (Fig.1) the fall-off of the potential at larger values of \( v \) is less steep, i.e. the screening is reduced as it is expected since the formation of the screening cloud is suppressed at large velocities. Also no minimum in the potential is found.

Summarizing, we have calculated the screening potential of a color charge moving through the QGP from semi-classical transport theory corresponding to the high-temperature limit. As in Ref. [20] we obtained a strongly anisotropic screening potential. The screening is reduced in the direction perpendicular of the moving parton but increased in the direction of the moving parton, which may lead to a modification of the \( J/\psi \) suppression. In addition, we found a new feature of the screening potential of a fast parton in a QGP: a minimum in the potential shows up which could give rise to bound states of, for example, diquarks if not destroyed by thermal fluctuations. For a quark-antiquark pair this minimum turns into a maximum which could cause short living mesonic resonances. Combining the effect of reduced screening
in perpendicular direction and the presence of a maximum in parallel direction we expect a stronger binding of moving $J/\psi$ mesons than of $J/\psi$ mesons at rest with respect to the QGP. The consequences, e.g., for the $J/\psi$ yield should be investigated in more detail using hydrodynamical models or event generators for the space-time evolution of the fireball. Finally, let us note that our results also applies to other ultrarelativistic plasmas such as an electron-positron plasma in Supernova explosions. In this case one simply has to replace the Debye mass $m_D$ by $eT$, where $e$ is the electron charge.

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