Hyper neu anom al y in Be isotope s and the neutron spatial d istribution; a three-cluster m odel for $^9$Be.

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The study of the hyper neu (hfs) anom al y in neutron rich nuclei can give a very speci c and unique way to study the neutron d istribution and the clustering structure. We study the sensitivity of the hfs anom al y to the clustering e cts in the $^9$Be isotope using two di erent nuclear wave functions obtained in the three-cluster $^9( + + n)$ model. The results are compared to those obtained for $^{11}$Be in a two-body core + neutron model to examine whether the hfs anom al y is sensitive to a halo structure in $^{11}$Be.

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I. INTRODUCTION

The study of the hyper neu (hfs) anom al y in neutron rich nuclei, in particular, those with loosely bound neutrons can give a very speci c and unique way to measure the neutron d istribution. In a previous paper [1], we have obtained the values of the hfs anom al y calculated in a two-body core + neutron model for the $^{9,11}$Be isotopes. The hfs anom al y is de ned as the sum of the hfs anom al y related to the Bohr-W einkopf e cts $\delta_W$ and of the B reit-Rosenthal-C rawford-Shaw low (BRCS) correction $\delta_r$, $\delta = \delta_W + \delta_r$.

It was found in Ref. [1] that in Be isotopes, the value $\delta_W$ is very sensitive to the weights of the partial states in the ground state wave function and might vary with 50% depending on those weights. In [1], we found a d ifference of about 25% for the total hfs anom al y value in the $^{9,11}$Be isotopes.

In the present paper, we calculate the hfs anom al y in a three-cluster model of $^9$Be and study the sensitivity of the hfs anom al y to the clustering e cts. The calculations are performed with two di erent three-body wave functions $\delta$ [2] and [3]. These wave functions do er from each other by the choice of the cluster-cluster interaction potential used in their calculations. They are characterized by the same partial states but contributing with di erent weights. They both reproduce the $^{9}$Be ground state and the three-cluster $(+ + n)$ dissociation energy (1.573 MeV). We compare the results obtained with these two wave functions, and also compare the present results with the core + neutron model calculations [1,3].

The accuracy of the description of the $^9$Be ground state wave function can be explored by calculating the magnetic dipole and electric quadrupole mo ments which are determined by the weights and quantum num bers of the states. The $^9$Be magnetic dipole mo ment is independent of the radial behavior of the wave function but this is not the case for the electric quadrupole mo ment. Calculation of both the magnetic dipole and electric quadrupole mo ments thus provides a precise test of the accuracy of all aspects of the ground state wave function (for the experimental data on the magnetic dipole mo ment in the Be isotopes, we refer to Refs. [1,3]; no experimental data exists on the hyper neu anom al y for these nuclei). We also compare the results for $^9$Be to that for $^{11}$Be, to investigate the sensitivity of the hfs anom al y to di use (halo) neutron structures.

II. THREE-BODY WAVE FUNCTION

The wave function $^3b_{\Lambda \Sigma M}$ of the fragment relative mo tion in the three-cluster model $(+ + n)$ of $^9$Be is described with the Jacobi coordinates in the method of hyperspherical harmonics $^3l_{\Lambda \Sigma}$. It is written as

$$^3b_{\Lambda \Sigma M}(\ell; 2) = \sum_{K l_{\Lambda \Sigma}} z_{\Lambda \Sigma M}^{l_{\Lambda \Sigma}} \chi_{\Lambda \Sigma M}^{l_{\Lambda \Sigma}};$$

where $z_{\Lambda \Sigma M}^{l_{\Lambda \Sigma}}$ is the "active part" of the three-body wave function carrying the total orbital angular mo ment $L$ with the projection $M_L$, $\sum_{\Lambda \Sigma}$ is the total spin function of the whole system with the total spin $S$ and projection $M_S$ (here restricted to the neutron spin function). The wave function $z_{\Lambda \Sigma M}^{l_{\Lambda \Sigma}}(\ell; 2)$ depends on the relative coordinates (hyperradius and relative angles) and the neutron spin

$$z_{\Lambda \Sigma M}^{l_{\Lambda \Sigma}}(\ell; 2) = \sum_{K l_{\Lambda \Sigma}} z_{K l_{\Lambda \Sigma}}^{l_{\Lambda \Sigma}}(\ell; 2) = \sum_{K l_{\Lambda \Sigma}} z_{K l_{\Lambda \Sigma}}^{l_{\Lambda \Sigma}}(\ell; 2);$$

where $\ell_1$ and $\ell_2$ are the angular mo ments of the relative mo tion corresponding to the 1 and 2 coordinates. The sum $\sum_{L} = l_1 + l_2$ gives the total angular mo ment $\hbar L$ and $K$ is the hyperradius. The hyperradius $\hbar L = \frac{\ell_1^2 + \ell_2^2}{2}$ is a collective rotationally and permutationally invariant variable, 1 and 2 are the
translationaly invariant norm alized sets of Jacobi coordinates. We choose the Jacobi coordinates as follows:

\[ x = (A_{23})^{1/2} r_{23}; \]
\[ y = (A_{12})^{1/2} r_{12}; \]

where \( r_{23} \) is the relative coordinate of fragments 2 and 3, and \( r_{12} \) is the coordinate of fragment 1 relative to the center of mass of the fragments 2 and 3. \( A_{23} \) is the reduced mass number for the pair \((2,3)\), similarly, \( A_{12} \) is the reduced mass of the fragment 1 with respect to the mass of the subsystem \((1,2)\).

The hyperphysical polar angles are \( s = f; \phi; \gamma \) where \( f \) is defined by the relations

\[ 1 \quad x = \sin f; \quad (2) \]
\[ 2 \quad y = \cos f. \]

The choice of three different systems of Jacobi coordinates leads to three-body wave functions with different phase factors; the three different Jacobi coordinate systems being connected together by defined rotations. For \(^{9}\)Be, the \( I \)-basis correspond to choose the neutron as fragment 1, the two \(-\)particles as fragments 2 and 3. The \( Y \)-basis associates the fragment 1 with one of the \(-\)particles.

The values of the hypermomentum are \( K = \frac{1}{2} + \frac{3}{2} + 2n \) (\( n = 1; 2; \ldots \)). The hyperphysical Hamiltonians have the form

\[ \sqrt{K} L M \chi (\phi) = \sqrt{K} L \chi (\phi) \chi_{LM}; \]

where \( \sqrt{K} L (\phi) = N_{K}^{L} \chi (\phi) \chi_{LM}; \chi_{LM} \) is the coefficient of normalization

\[ N_{K}^{L} = \frac{2(2n + 2)!}{(n + 1 + 3)! (n + 1/2 + 1)!} \]

and the normalization condition for the function \( \sqrt{K} L (\phi) \) is

\[ \int_{0}^{2\pi} \chi_{LM}^{2} d\phi = \frac{1}{2} \pi. \]

The charge density distribution of the three-body nucleus entering the calculation of the electronic wave functions can be obtained as

\[ \chi(r) = \frac{3}{J M} Z_{i} \chi_{i}^{2} (\chi_{i} \times \chi_{y}) \chi_{i}^{2} (\chi_{i} \times \chi_{y}) \]

where \( \chi_{i}^{2} \) is a normalization factor. In the case of \(^{9}\)Be, the \(-\)particle density distribution is approximated by a sum of Gaussians, with the parameters taken from \([11]\) and giving a charge radius equal to 1.676 fm.

In the proceeding calculations two different evaluations of \( \chi_{i}^{2} \) \([\text{\[3,5\]}\] as noted in Section 4 are used.

### III. MAGNETIC HYPERFINE STRUCTURE

Here, we briefly mention the main points of the HFS anomaly calculations. For one details we refer to \([1] \) and references therein.

The magnetic hyperfine interaction Hamiltonian is defined by

\[ H = \frac{1}{2} J (\chi) A \chi \frac{d}{d\chi}; \]

where \( J \) is the nuclear current density and \( A \) is the vector potential created by the atomic electrons.

The hyperfine interaction couples the electronic angular momentum \( J \) and the nuclear one \( I \) to a hyperfine moment \( F = J + I \). The magnetic hyperfine splitting energy \( \Delta W \) for a state \( |J, F, M_{F}, M_{I} \rangle \) is defined as the matrix element of the Hamiltonian \( H \).

The functions \( J, G \) entering the matrix elements are the radial parts of the large and small components of the Dirac electron wave function, with the quantum number \( n = (J + \frac{1}{2}) \) for \( J = \frac{1}{2} \) and the orbital angular momentum \( l \). The calculations are performed with a realistic electronic wave function (see \([2]\)).

The magnetic dipole contribution to the hyperfine splitting \( \Delta W \) has the form

\[ \Delta W_{(J, F)FF} = \frac{1}{2} \left[ F_{1} (F + 1) I_{1} (I + 1) J_{1} (J + 1) a_{1}; \right. \]

where \( a_{1} \) is defined as

\[ a_{1} = \frac{2e}{1J (J + 1)} \left[ \sum_{i=1}^{X} \chi_{i} \chi_{i}^{2} (\chi_{i} + \frac{1}{2}) \chi_{i} \right] \]

with the \( Z \) components of the magnetic dipole moment \( M_{i} \) \((r_{i})\) related to the angular moment \( l_{i} \) and \( s_{i} \) of each nucleon; the summation runs over all the nucleons.

For an extended nuclear charge \( Z \) we have

\[ M_{i} = \sum_{i=1}^{Z} \left[ \frac{1}{2} \chi_{i} \chi_{i}^{2} (\chi_{i} + \frac{1}{2}) \chi_{i} \chi_{i}^{2} (\chi_{i} + \frac{1}{2}) \chi_{i} \right] \]

with \( D_{i} = \frac{1}{2} \chi_{i} \chi_{i}^{2} (\chi_{i}) \) and \( C_{i}^{\pm} = \frac{1}{2} \chi_{i} \chi_{i}^{2} (\chi_{i}) \).

The quantity \( a_{1} \) can be expressed through the HFS constant for a point nucleus \( a_{1}^{(0)} \) as

\[ a_{1} = \frac{2e}{1J (J + 1)} \left[ \sum_{i=1}^{Z} \chi_{i} \chi_{i}^{2} (\chi_{i} + \frac{1}{2}) \chi_{i} \right] \]

with

\[ a_{1}^{(0)} = \frac{2e}{1J (J + 1)} \left[ \sum_{i=1}^{Z} \chi_{i} \chi_{i}^{2} (\chi_{i} + \frac{1}{2}) \chi_{i} \right]. \]
Here, $\langle II \mid j^a_{i1}(g^{i1}_l s_1 + g^{i1}_l l)j II \rangle$ denotes the magnetic dipole moment of the point nucleus in nuclear magneton units $N$. The functions $F^a_0$, $G^b_0$ are the radial parts of the Dirac electronic wave function in the point nucleus approximation.

The HFS anomaly in the Bohr-W eisskopf exact is

$$b_w = \sum_{i=1}^{2} \frac{2 x^3}{4} \left< II \mid j^a_{i1}(g^{i1}_l s_1 + g^{i1}_l l)K^a(c_1)j II \right>$$

where $b = R^a_0 F^a_0 J^a_0 dr^a_0$, and

$$K^a(c_1) = \frac{E}{r} g^a(c_1) \quad (9)$$

$$K^b(c_1) = \frac{E}{r} g^b(c_1) \quad (10)$$

The index $j$ is related to the $A$ nucleons; $i(1,3)$ denotes one of the three clusters of $n_i$ nucleons. $g^{i1}_l$ and $g^{i1}_l$ are the gyromagnetic ratios of the $j$-th nucleon orbital and spin, respectively.

The hfs anomaly can be approximated by

$$b_w = \sum_{i=1}^{2} \frac{2 x^3}{4} \left< II \mid j^a_{i1}(g^{i1}_l s_1 + g^{i1}_l l)K^a(c_1)j II \right>$$

The index $i$ is related to the three-clusters of relative coordinate $r_1$ and angular momentum $l_1$ in respect to the center of mass system, and with the appropriate expressions $g^{i1}_l$, $g^{i1}_l$, and $D_1$, calculated for each cluster. The BCRS correction is defined as $= 1 \ h^2 (1 \ )$.

This term is defined by the nuclear charge distribution and information on the neutron distribution is contained solely in $b_w$.

IV. HFS ANOMALY AND NUCLEAR MOMENTS OF $^9$Be IN THE CLUSTER MODEL

To calculate the magnetic dipole moment and the hfs anomaly, we use three systems of coordinates related to each other by rotation: The T-basis with the neutron as cluster $i = 1$, and the Y (g) system $(g = 1$ or $2)$ with one of the particles as cluster $i = 2$ or $3$. The index $q = 1$ and $q = 2$ denote respectively the rotations 1(23) ! 2(31) and 1(23) ! 3(12) between the T and Y-basis.

The transformation of the hyperspherical harmonic function $E$ from the T-basis to the Y-basis is defined by

$$E_{KL} = \frac{1}{2} \ h^2 \left< \ y_{i}^q, \ y_{j}^q \right| \ y_{i}^q, \ y_{j}^q \rangle$$

where $\ h^2 \left< \ y_{i}^q, \ y_{j}^q \right| \ y_{i}^q, \ y_{j}^q \rangle$ are the Raynal-Reid coe cients.

The transformation of the spin part of the wave function is written as

$$s_1(s_2, s_3) \left| \ y_{i}^q \right| \ y_{i}^q \rangle = \left( S_1(s_2, s_3), S_2(s_2, s_3) \right)$$

with $s_0 = s_2(s_1, s_3)$, $s_0 = s_3(s_1, s_2)$, and $s_0 = s_1(s_1, s_2)$. The hfs anomaly can be approximated by

$$b = \sum_{i=1}^{2} \frac{2 x^3}{4} \left< II \mid j^a_{i1}(g^{i1}_l s_1 + g^{i1}_l l)K^a(c_1)j II \right>$$

where $s = s_2(s_1, s_3)$, $s = s_3(s_1, s_2)$, and $s = s_1(s_1, s_2)$.

To calculate the magnetic dipole moment, we consider the contribution of each fragment with respect to the rest system. In $^9$Be, the neutron spin and the -particle orbital moment contribute to the magnetic dipole moment as

$$\frac{1}{2} \ h^2 \left< \ y, \ y \right| \ y, \ y \rangle = \frac{1}{2} \ h^2 \left< \ y, \ y \right| \ y, \ y \rangle$$

and

$$\frac{1}{2} \ h^2 \left< \ y, \ y \right| \ y, \ y \rangle$$

are the expectation values of the spin and angular momentum projections of the fragments. $\ h^2$ is found in the T-basis and associated with the neutron spin; $\ h^2$ is found in the Y-basis obtained by the rotation $q = 1(2)$ for $i = 2(3)$ (see $E$ and $E$) and associated with the -particle orbital momentum. In Eq. $E$ is the weight of the partial state in the T-basis and for $\ h^2$ in the Y-basis. Here, we mean by channel the set of quantum number characterizing the partial waves contributing to the ground state wave function. In the case of $^9$Be represented by $\ h^2$, this set of quant $s = L, l, j, l$, and $s = L_1, l_1, j_1, l_1$ in the Y-basis. The nuclear wave function is summed over the hypermomentum $K$ which is not included explicitly as the matrix element does not depend on it.

In Eq. $E$, $n = \frac{1}{2} g^{(n)}_l$ denotes the magnetic dipole moment of the neutron and $g^{(n)}_l$ is measured in the T-basis with $l_1$ as angular momentum. Note that the magnetic dipole moment is obtained using the $g$ factors $g^{(n)}_l = \ 3260854 (90)$ and $g^{(n)}_l = \ 1$. The factor $\frac{1}{2}$ in $E$ is the center of mass factor $\frac{A}{A+1}$ in the Y-basis corresponding to the + (+ + n) system.
where the sign corresponds to \( I = \frac{n}{2} \), and where is replaced by the experimental value \((I = 1.1778(9)\) of the \(^7\)Be magnetic dipole moment. Here we denote

\[
K^a_k = \int_{A_A} (R) f^k A A (A\,A) R^2 dR
\]

where the ratio \( \frac{A_A}{A} \) takes into account the center of mass motion, \( A_i \) is the valence fragment mass (the neutron mass \( i = 1 \) in the \( T \)-basis, and the \( ^{2}\)He mass \( i = 2;3 \) in the \( N \)-basis).

So we can write

\[
X_{BW} \quad \frac{I}{2} \frac{I}{2}^k_{BW}
\]

where \( X_{BW} \) is obtained for each channel \( k > \).

V. RESULTS AND DISCUSSION

1. Three-body model of \(^7\)Be

In the calculations we use two ground state wave functions of the \(^7\)Be described as the three body \( ^2 + \pi n \) system. These wave functions are obtained with different and \( n \) interaction potentials.

The first wave function, \( \Psi_{1}(\text{BW}) \), is obtained with supersymmetric equivalent potential \( \Psi_2 \). With the \( \pi \) particle charge radius \( 1.676 \text{ fm} \) \( \Psi_{1}(\text{BW}) \) this wave function gives the value 2.564 \text{ fm} for the \(^7\)Be charge m.s. radius. This value agrees with the experimental ones, 2.519(12) and 2.50(9) \text{ fm} (see \( \text{[11]} \) and references therein). The \(^7\)Be magnetic dipole moment \( \mu_{^7\text{Be}} = 1.0531 \text{ mB} \) is less by 10% compared to the experimental values, 1.077342(3) \text{ mB} \( \text{[13]} \) and 1.0778(9) \text{ mB} \( \text{[7]} \). The calculated electric quadrupole moment is 53.39 \text{ mB} which is in good agreement with the experimental value 52.88(38) \text{ mB} \( \text{[12]} \).

The second wave function, \( \Psi_{2}(\text{BW}) \), is obtained with the Al-Bodmer potential \( \Psi_2 \) and the \( \pi \)-neutron interaction potential \( \Psi_3 \). The three-body interaction potentials are adjusted to the three-cluster dissociation energy \( 1.573 \text{ MeV} \). The value of the \(^7\)Be charge radius, 2.707 \text{ fm}, is overestimated compared to the experimental one. The \(^7\)Be magnetic dipole moment \( \mu_{^7\text{Be}} = 1.316 \text{ mB} \) is 10% larger than the experimental value. The electric quadrupole moment is \( Q = 65.42 \text{ mB} \).

The difference between these wave functions is in the radial dependence of the valence neutron wave function (obtained by integration over the \( x \) coordinate and the angles), the core charge radius and the weights of the partial states \( L_L, L_L, L_L \) for the ground state wave function (see Tables \( \text{[5]} \) and \( \text{[6]} \)). Tables \( \text{[5]} \) and \( \text{[6]} \) show the neutron contribution \( \Psi_{BW} \) to the \(^7\)Be anomaly (\( \text{[7]} \) and \( \text{[6]} \)) and the root mean square distance of the valence neutron \( r_s \) from the \(^7\)Be center of mass in each channel.

The contributions to the \(^7\)Be anomaly in the Bohr-Wicksskopfe ect, \( \Psi_{BW} \) and \( \Psi_{BW} \), and the BCRS correction calculated with \( \Psi_{1}(\text{BW}) \) and \( \Psi_{2}(\text{BW}) \) are listed in Table \( \text{[7]} \). The contribution of the \( \Psi_{BW} \) anomaly from the -
particle orbital motion, \( \frac{}{B_W} \) (second term in LW) is small compared to that from the neutron, \( \frac{\pi}{B_W} \); and its variation is small also (see Table III).

Table III: The \(^9\)Be charge m s radius, the neutron radial distance \( \pi \), the contributions to the hfs anomaly from the neutron spin (\( \frac{\pi}{B_W} \)) and from the p-particle orbital motion (\( \frac{\pi}{B_W} \)), the values of \( B_W \) and of calculated for the two wave functions \( W_F1 \) and \( W_F2 \) are compared to the (core+ neutron) results. \( W_F2 \) refers to another choice of the p-particle radius (1.46 fm).

| Value | core neutron | \( W_F1 \) | \( W_F2 \) | \( W_F2 \) |
|-------|--------------|------------|------------|------------|
| m s (fm) | 2.519 | 2.564 | 2.707 | 2.533 |
| \( \pi \) (fm) | 3.207 | 3.621 |
| \( B_W \) (\%) | -0.02281 | -0.03059 |
| \( B_W \) (\%) | 0.00085 | 0.00088 |
| \( B_W \) (\%) | -0.0236 | -0.02112 | -0.02882 | -0.03032 |
| \( B_W \) (\%) | -0.0451 | -0.04644 | -0.04926 | -0.04634 |
| \( B_W \) (\%) | -0.0687 | -0.06756 | -0.07809 | -0.07666 |

To explore the sensitivity of the results to the charge radius of the \(-\) clusters in \(^9\)Be, we vary the value of the \(-\)particle charge radius from 1.676 to 1.636 fm. The last value of the charge radius is obtained with regard to the negative contribution of the neutron charge distribution, \( (0.34)^2 \) fm \(^2 \) (see Fig. 16).

Correspondingly, the \(^9\)Be charge radius changes from 2.564 to 2.534 fm for \( W_F1 \) and from 2.707 to 2.678 fm for \( W_F2 \). Owing to the radial behavior of the electronic wave functions (see Fig. 9), entering the expressions \( B_W \), \( \pi \), and \( \frac{\pi}{B_W} \), with a smaller charge radius \( m_s \) and \( m_B \), we get a larger value of the hfs anomaly in the Bohr-Welsius effect. Thus, in our case, we get an increase in the hfs anomaly value \( B_W \) by 1.6% for \( W_F1 \) and 0.5% for \( W_F2 \).

The distance is smaller when calculated with \( W_F1 \), so correspondingly, the \(^9\)Be charge radius is smaller and the value of the BCFS correction is less (see Tabl III).

varies within \( 6 \% \) depending on the description of the nuclear wave function.

\( B_W \) and the total hfs anomaly \( \frac{}{B_W} \) which is the sum \( \frac{}{B_W} + \frac{}{B_W} + \), are mainly determined by the neutron distribution. There is no direct correspondence of the hfs anomaly value to the nuclear charge radius. For example, if we put the radius of the \(-\)particle at 1.46 fm, the \(^9\)Be charge radius obtained with \( W_F2 \) is 2.533 fm and the hfs anomaly \( = -0.07666 \) (see Tabl III \( W_F2 \)). These values are larger than those obtained with \( W_F1 \) with the charge radius 2.564 fm. Thus, even with smaller charge radius values one can get larger values of \( B_W \) and .

Therefore, we can conclude that the hfs anomaly is more sensitive to the neutron spatial distribution than to the charge distribution of the whole nucleus, and the value \( \pi \) is a crucial parameter for the hfs value in the Bohr-Welsius effect.

With the different nuclear wave functions, the \( \pi \) value varies of about 10% and the hfs anomaly \( B_W \) of about 30%. As the value of the hfs anomaly \( B_W \) is about 2 times smaller than the total hfs anomaly varies within 14% with the choice of the wave function.

We should also mention that the value of the hfs anomaly is very sensitive to the contribution of the different partial states in the \(^9\)Be ground state wave function. In particular, the hfs anomaly \( \pi \) in the channels \( \frac{}{B_W} = 112 \) and \( \frac{}{B_W} = 221 \) (that of 50% of the nuclear wave function) is too small as \( \frac{}{B_W} = 221 \) in the channel \( \frac{}{B_W} = 101 \). Thus, the relative weights of these states are of some importance. As found in [21] the magnetic dipole moment in the \( + \) + cluster model is also rather sensitive to these weights. In the case of \(^9\)Be, three channels mostly contribute to the magnetic dipole moment and the hfs anomaly, \( \frac{}{B_W} = 101 \), \( \frac{}{B_W} = 121 \), and \( \frac{}{B_W} = 221 \). The magnetic dipole moment in this case is [21]

\[ t \begin{array}{c} 1857 \end{array} + 191 \begin{array}{c} 112 \end{array} + 1914 \begin{array}{c} 221 \end{array} \]

According to [21] the experimental magnetic dipole moment value can be reproduced under the condition \( \frac{}{B_W} < 16 \% \). From the calculated value is underestimated.

Let us analyze the correlation between the weights and the values of the \(^9\)Be hfs anomaly, magnetic dipole and electric quadrupole moment, noting that the analysis is model dependent.

To estimate how the hfs anomaly \( \pi \) \( \frac{}{B_W} \) varies with the weights of the different states, we consider the contributions of these three channels only (so that \( \frac{}{B_W} + \frac{}{B_W} + \frac{}{B_W} \) = 1) and find the weights satisfying the experimental value of the \(^9\)Be magnetic dipole moment \( \frac{}{B_W} = 1917432(3) \). Under this condition, we get the hfs anomaly value plotted in Fig. 2 as a function of \( \frac{}{B_W} \) for the wave functions \( W_F1 \) and \( W_F2 \) (solid and dashed lines, respectively).

Similarly, one can express the electric quadrupole moment through the weights of these dominant states. For
quantify to test the nuclear wave function, allocate other parts.

FIG. 2: The variation of the Bohr-W eisskopf hfs anomaly value from the neutrons with the weight \(1_{01}\) for W F1 (solid line) and W F2 (dashed line) (particle radius: 1.676 fm). The dots on the lines are the values in agreement with the electric quadrupole moment (see text). The weight \(1_{221}\) is indicated for each point.

For W F1 we get

\[
\begin{align*}
Q(t) &= 3.1571_{101}^2 + 4.5531_{121}^2 + 4.5271_{221}^2 + 2!_{101}!_{121}!_{221}! 32.859 + 2!_{101}!_{221}!_{221}! 32.039 + 2!_{121}!_{221}! 7.314 + Q_{\text{res}},
\end{align*}
\]

where \(Q_{\text{res}} = 11.97\) m b.

For W F2 this relation is

\[
\begin{align*}
Q(t) &= 4.2041_{101}^2 + 6.635_{121}^2 + 6.662_{221}^2 + 2!_{101}!_{121}!_{221}! 41.908 + 2!_{101}!_{221}!_{221}! 41.610 + 2!_{121}!_{221}! 9.918 + Q_{\text{res}},
\end{align*}
\]

where \(Q_{\text{res}} = 13.11\) m b.

\(1_{res}\) and \(Q_{res}\) are the contributions of the neglected channels respectively to the weights, the magnetic dipole and electric quadrupole moments. The square dots on the lines in Fig. 2 point the hfs values obtained with the weights which also satisfy the experimental value of the \(^9\)Be electric quadrupole moment 52.88 (38) m b. For each wave function we get a few points — two for W F1 or even four for W F2. On this graph, we also report the corresponding values of the weight \(1_{221}! (1_{221} < 16)\). One can see the ranges of the hfs anomaly values obtained with W F1 and W F2, respectively. Therefore, an experimental estimation of the hfs anomaly in the Bohr-W eisskopf effect could give values for the weights of the partial states in the ground state wave function.

The graph shows that the hfs anomaly is a very critical quantity to test the nuclear wave function, all other parameters being equivalently well described, in particular, the electronic part.

2. Comparison with the core+ neutron model

In Ref. \([6]\), the hfs anomaly for \(^9\)Be was calculated within the core+neutron model of \(^9\)Be. The \(^9\)Be ground state wave function was given by the superposition of states

\[
\begin{align*}
^9\text{Be } 3\!\!^2 &= 1^+ \cdot \tilde{F}\text{e } 0^* n_{P_{3/2}} l_{3/2} = 24 \\
+ 1^+ \cdot \tilde{F}\text{e } 2^* n_{P_{3/2}} l_{3/2} = 26
\end{align*}
\]

corresponding to the \(^9\)Be core in the ground \(0^*\) and excited \(2^*\) states with the neutron separation energies 1.665 and 4.705 M eV, respectively.

With the weights \(1_{0^*} = 0.535\) and \(1_{2^*} = 0.465\) obtained with the spectroscopic factors from Ref. \([24]\) the magnetic dipole moment is \(\mu_{B\text{w}} = 1.0687\) and \(\mu_{B\text{w}}\), and have the values reported in Table III. The \(B_{\text{w}}\) is close to the values \(B_{\text{w}} = 0.0249\) \(3\) obtained with the weights \(1_{0^*} = 0.535\) and \(B_{\text{w}} = 0.0243\) from \(23\).

The \(B_{\text{w}}\) values obtained for the \(0^*\) state in the \(\text{core}+\text{neutron}\) model \(B_{\text{w}} = 0.0440\) and for the \(l^* = 0\) state of W F1 or W F2 in the \((+ + n)\) model \(B_{\text{w}} = 0.0332\) or \(0.0402\) are relatively close to each other. On the contrary, the \(B_{\text{w}}\) values for the different partial states \(l^* = 2\) in the three cluster model exceed by a few times (Table 1 and 2) the value obtained for the \(2^*\) state in the \(\text{core}+\text{neutron}\) model \(B_{\text{w}} = 0.063\).

The BRCS correction obtained with the two-body wave function is \(0.451\%), close to that obtained in the three-body calculations. Thus in the core+ neutron model we got \(0.0687\), to be compared with the values \(0.0756\) and \(0.0780\) obtained with the three clusters \(+ + n\) wave functions.

Thus the clustering effect, revealing itself in the set of states contributing to the ground state wave function, lead to a variation of the hfs value of less than 2% for W F1 and of about 14% for W F2. Com pared to the \(^{11}\)Be nucleus the difference in the value of the hfs anomaly in the Be isotopes is about 25%. This value gives us the accuracy of the measurement of the hfs anomaly needed to study clustering effects in light nuclei.

This result corroborates the conclusion in Ref. \([6]\), that the value of the hfs anomaly reflects the extended neutron distribution in \(^{11}\)Be and might indicate a neutron halo, but the difference for the different isotopes is not so pronounced as was found in Ref. \([6]\).

VI. CONCLUSION

In the present paper, we have calculated the hfs anomaly in the \(^9\)Be' ion with the nucleus described in a three-cluster model. The \(1s^2 2s\) electron wave functions are obtained taking into account the charge distribution of the clustered \((++n)\) nucleus and the shielding effect of two electrons in the \(1s^2\) configuration.
The result of the calculations strongly depends on the weights of the partial waves contributing to the ground state wave function. Together with the magnetic dipole and electric quadrupole moments the value of the hfs anomaly can be used to study the clustering effects in neutron rich light nuclei.

The total hfs anomaly is the sum of $a_1$ and $a_2$. The BRCS correction is only determined by the nuclear charge distribution and varies slightly from isotope to isotope. The value of the BRCS correction is comparable or larger than the value of $a_2$. The hfs anomaly in $^{11}$Be differs from that in $^9$Be by 25%. The clustering effect leads to variations of the hfs value within 15%. The calculated magnitude and differential change in the value of the hfs anomaly is indicative of the experimental precision that must be achieved to study the clustering effect and the neutron distribution in neutron rich light nuclei.

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