The MHD based numerical analysis of Rayleigh-Bénard convection flow of liquid metal in the smoothly constricted enclosure from the top using OpenFOAM

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Abstract. The influence of the existence of the Lorentz force in the Rayleigh- Bénard (RB) convection flow in the smoothly constricted cavity from the top is presented in this paper. The liquid metal with Prandtl number of \( Pr = 0.02 \) is used with the Rayleigh number of \( Ra = 10^5 \), and the magnetic field is imposed in terms of Hartmann number (\( Ha = 0-50 \). The vertically driven buoyancy force is kept constant for all simulation by maintaining the same \( Ra \). The present flow solver with magnetohydrodynamics (MHD) principle is developed in the open source CFD toolkit OpenFOAM. The Navier-Stokes equation is coupled with Maxwell’s equation of electrodynamics to cope up the MHD based flow physics in the cavity. The thermal energy equation with Boussinesq approximation is added in the solver to study natural convection flow in the presence of the magnetic field. The orientation of magnetic field has different nature and direction of induced Lorentz force in the cavity. The imposed magnetic field normal to the gravity has the tendency to suppress the convection roll formation. Conversely, it has been observed that the magnetic field imposed in the direction to parallel to gravity bifurcates the flow and assist in the formation of several convection rolls. The detail discussion of the variation of Lorentz force in the cavity and its effect on the streamlines, isotherms, and the average Nusselt number is reported.

Keywords: OpenFOAM, MHD, Rayleigh-Bénard convection, Boussinesq approximation.

1. Introduction
The flow of heat and mass transfer by buoyancy-induced force are relevant to several practical systems. In particular, the problem of Rayleigh-Bénard (RB) convective motion of flow in the confined geometries is encountered by many geophysical and engineering applications, such as differential heating and cooling in the lakes, estuaries or dam reservoir, solar energy collectors, nuclear reactors, cooling of electronic circuits, etc. [1-2]. The RB convection involves the vertical temperature gradient causes the thermal convection in the horizontal layer of fluid and convectional rolls starts to appear due to the buoyancy force in the enclosure beyond the critical Rayleigh-Bénard (\( Ra_c \approx 1708 \)). The heat dissipation or the convective rolls formation in the enclosure is found to be enhanced by increasing the Rayleigh number (\( Ra \)) [3]. De et al. [4] had performed the DNS of dynamics of plumes developed at various Rayleigh number (\( 7 \times 10^4 < Ra < 2 \times 10^6 \)) and strong correlations between dynamics of the horizontal velocity field and drift of the thermal plumes have been produced. Similarly, Aghighi et al. [5] had performed the finite element analysis of non-Newtonian fluid at different \( Ra (5 \times 10^3 < Ra < 10^5) \) and Prandtl number (\( Pr = 10, 100, 1000 \)) and the correlations for mean Nusselt number and maximum Bingham number (\( Bn_{max} \)) as functions of \( Ra, Pr, \)
and Bingham number \((Bn)\) is obtained. Thereafter, it has been observed that the heat transfer in the Casson model is lesser than Bingham and Herschell-Bulkley model. The heat transfer enhancement in the ordinary convection flow in the enclosure can be possible either by the increasing Rayleigh number or by the modification in the geometry, surfaces and Reynolds number for forced convection flow. Singh et al. [6] had performed the experimentation and computational analysis by fitting the rib turbulators in a criss-cross pattern for increasing the chaos in the flow which promotes the heat transfer of the system by the factor of \(2.5 - 3\). Similarly, Corcoine et al. [7] had performed the numerical analysis to \(RB\) convection flow with the aim of the enhancement of the heat transfer of the system by inserting the honeycomb in the cavity. The suspended honeycomb structure pushes the flow to get self-organized which evolve efficient heat transfer. Furthermore, the liquid metal flow and heat transfer in the enclosure are superiorly controlled or regulated with the use of magnetohydrodynamics (MHD) principle. The MHD is the interdisciplinary science which deals with the flow of electrically conducting fluid in the presence of the magnetic field [8]. The natural convection flow with different shapes of enclosure offers the dissimilar flow path and rate of heat transfer in the presence of the magnetic field. Sheremet et al. [9] numerically studied the natural convection flow with the wavy-walled cavity, where it has been observed that with the increase in the magnetic field double-core convective rolls formations take place for high value of Hartmann number \((Ha)\). The position and strength of magnetic field also affect significantly in enhancing the overall heat transfer of the system, Song and Tagawa [10] had developed FORTRAN code to perform the numerical analysis for the convection flow of gaseous oxygen in the square enclosure with different location of the magnetic field. The magnetic field placed near hot wall offers the maximum Nusselt number \((Nu_b = 152.7\%)\) over the hot surface. Yu et al [11] had studied the three-dimensional \(RB\) convection flow in the presence of magnetic field, where it has been observed that by the increase in the \(Ha\) range, the \(Nu\) of the system increases by the shift in the flow pattern of the fluid in the domain. Thereafter, it has been observed for convection flow of liquid metal in laminar zone that the flow pattern in the enclosure drastically changes by the application of magnetic field with different orientation, and the heat dissipation in domain shifted from dominating convection flow for \(Ha = 0\) to dominating diffusion heat transfer at higher \(Ha\) [12, 13].

According to the above literature studies, it has been found that the several \(RB\) convective flow analysis is performed on a two-dimensional or three-dimensional regular shape domain with or without application of the magnetic field. However, the effect of \(RB\) convectional flow of liquid metal with the smoothly constricted cavity from the top has got the least focus by the researchers. In this present study, the emphasized is given on the two-dimensional \(RB\) convection flow of liquid metal of \(Pr = 0.02\) in the gradually constricted cavity from the top at a fixed Rayleigh number of \(Ra = 10^5\). The numerical analysis is performed using an in-house developed MHD based flow solver in open source CFD tool OpenFOAM. The variation of streamlines, isotherms, and the average Nusselt number with the application of magnetic field is reported in detail.

2. Problem definition, grid independence and validation test

The two-dimensional rectangular enclosure with the gradual constriction is considered with an aspect ratio of \(4 (L/w = 4)\). Figure 1 (a and b) shows the schematic of the gradually constricted enclosure and mesh distribution in the domain respectively. The coordinates of the top wall are mentioned as \(f_t\) and the constraint ratio is given by \(C\), the function \(f_t\) and the constraint ratio \(C\) is written as follow:

\[
f_t = 1 - \frac{h}{2} [1 - \cos(2\pi x / L)]; \quad C = (1 - \frac{w - h}{h}) \times 100
\]

Here, \(w\) is side width of cavity has the value of 1 and \(L\) is the horizontal span length of enclosure has fixed value of 4. The computational analysis is performed for three different constraint ratio and the magnetic field is imposed on the system parallel to acceleration due to gravity \((g)\) is \(B_t\) and the normal to gravity is \(B_n\). the bottom surface of enclosure is heated and top surface is kept as cold and the temperature difference between bottom and top surface is maintained as 1K. The walls of the enclosure is maintained as electrically insulated.
The grid independence test is performed for three different mesh size at Hartmann number of $Ha = 50$ and $Ra = 10^5$, at fixed constraint ratio of $C = 50\%$. The probe location is set at the center of the cavity as shown in Figure 1 (a). The undulation in the temperature is observed for all three different grids (Grid 1 = 100 × 200, Grid 2 = 125 × 250, and Grid 3 = 150 × 300) as shown in Figure 2. The variation of temperature at the given probe location for all grids shows the uniform pattern with the constant amplitude of undulation. Hence, grid 2 is fine enough to capture flow physics and chosen for further simulation in this study. The performance of present solver is compared with results obtained in the reference [14]. The boundary condition and geometry detail is maintained same as available in the reference [14]. The normalized velocity is compared in the vertical y-direction at $x = 2$, the present results satisfy the pattern available in the reference [14].

3. Mathematical equations and numerical scheme

The governing mathematical equation is set for incompressible, viscous and electrically conducting fluid in the existence of Lorentz force. The Navier-Stokes (N-S) equation is coupled with Maxwell’s equation of electric potential to cope up the effect of induced electric potential as well as electric current. The Boussinesq approximation is added along with Lorentz force in the Navier-Stokes equation as source term. The energy equation is activated to get the temperature distribution in the enclosure. The complete sets of the equation used in the present is mentioned as follow:

Continuity equation:
\[ \nabla \cdot \vec{U} = 0 \]

N-S equation with Lorentz force and Boussinesq approximation:
\[ \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla) \vec{U} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{U} + \frac{j \times \vec{B}}{\rho} - \beta (T - T_{ref}) \vec{g} \]
Ohm’s law Current density for fluid:

\[ \vec{j} = \sigma (-\nabla \phi + \vec{U} \times \vec{B}) \]

Conservation of charge:

\[ \nabla \cdot \vec{j} = 0 \]

Thermal Energy equation:

\[ \frac{\partial T}{\partial t} + (\vec{U} \cdot \nabla)T = \nabla \cdot (\alpha \nabla T) \]

The Poisson’s equation for electric potential is attained by correlating the Ohm’s law of current density (Eq. 3) and conservation of charge (Eq. 4) is mentioned as follow:

\[ \nabla^2 \phi = \nabla \cdot (\vec{U} \times \vec{B}) \]

The three non-dimensional numbers are employed in this analysis to examine the fluid behaviour under the effect of magnetic field are as follows:

\[ H_a = \frac{B \omega}{\sqrt{\rho \nu}} \quad R_a = \frac{g \beta (T - T_{ref}) \omega^3}{\nu \alpha} = Gr \cdot Pr \quad \rho_{ref} = \frac{\rho}{\rho_{ref}} \]

Where the \( H_a \) is the Hartmann number, which equilibria the ratio of electromagnetic force to the viscous force. It is used to define the strength of the magnetic field. The \( Ra \) is the Rayleigh number, is the interaction of Grashof number (\( Gr \)) and Prandtl number (\( Pr \)). The Direct numerical simulation (DNS) is performed to solve the equations (Eqn. 1-6) by the in-house developed solver using finite volume approach (FVM). The PIMPLE algorithm is adopted in the solver to solve the complete set of the equation with the two outer correctors. The second-order central difference scheme is employed to discretize the convection and diffusion term. The time derivative term is discretized by first-order accurate Euler scheme. The relaxation factor for velocity and pressure is kept as 0.7 and 0.3 respectively. Table 1 shows the boundary condition and non-dimensional parameters employed in the present study to perform the computation.

### Table 1. Boundary conditions and non-dimensional parameters

| Sr. No | Parameters | Bottom          | Top            | Other sides          |
|--------|------------|-----------------|----------------|---------------------|
| 1      | \( U \)    | No slip         | No slip        | No slip             |
| 2      | \( p \)    | Fixed flux pressure | Fixed flux pressure | Fixed flux pressure |
| 3      | \( T \)    | 0.5             | -0.5           | Insulating          |
| 4      | \( \phi \) | Insulating      | Insulating     | Insulating          |
| 6      | \( Ha \)   | 0, 10, 25 and 50 |               |                     |
| 7      | \( Ra \)   | \( 10^5 \)      |               |                     |
| 8      | \( Pr \)   | 0.02            |               |                     |

### 4. Results and Discussions

In this section, we have discussed the alteration in the flow pattern and heat dissipation phenomena of liquid metal with different orientation of magnetic field at different Hartmann number (\( Ha \)) at fixed Rayleigh number (\( Ra \)). The constraint ratio of cavity also plays the significant role in the variation in the fluid flow and heat transfer characteristic in the enclosure.

The flow at \( Ha = 0 \), has the ordinary convection flow where the flow physics is solitarily transformed because of the presence of buoyancy force and with different geometrical section. Hence, in this study, the buoyancy force is kept constant for all simulations and the electromagnetic force (Lorentz force) is varied with increasing the intensity of magnetic field. Figure 3 (a-c) and Figure 4 (a-c) shows the streamlines variation at \( Ha = 0 \) at different \( C \) (0, 0.25, and 0.5). It has been observed from streamlines at \( Ha = 0 \) that the formation of convection rolls and its stability is solely concerned by the alteration in the geometrical section. The streamlines at \( Ha = 0 \) and \( C = 0 \) shows non-uniformity in the pattern. However, the streamlines for \( C = 0.25 \) and 0.5 has uniform pattern. This variation in streamlines
occurs due to constraint in geometrical section where fluid does not have enough cross-section ($C = 0.5$) to flow and streamlines get bifurcated into equal number of convection rolls with increase in the number of convectional rings is observed. The convectional rolls for $Ha = 0$ with increase in the constraint ratio ($C$) shows the increasing tendency of convectional rings with four primary rolls is observed at $C = 0$ and at $C = 0.5$ six primary convectional rings are observed in the cavity.

Figure 3. Streamlines variation at different $Ha$ and constraint ratio for $B_x$ magnetic field.

Figure 4. Streamlines variation at different $Ha$ and constraint ratio for $B_y$ magnetic field.
Figure 5. Isotherms variation at different $Ha$ and constraint ratio for $B_x$ magnetic field.

Figure 6. Isotherms variation at different $Ha$ and constraint ratio for $B_y$ magnetic field.

The increases in the convectional rolls in the cavity shows that the unsteadiness in the flow increases with increases in the constraint ratio, which is solely responsible for the increase in the heat dissipation even at same $Ra$. The buoyancy flow in the presence of the magnetic field produces the electric current which further intermingles with the magnetic field to produce Lorentz force. The orientation of Lorentz force is governed by the direction of the magnetic field applied to the system. The magnetic field applied in the direction parallel to gravity produces the Lorentz force in the direction normal to the gravity. Conversely, the magnetic field imposed normal to gravity has Lorenz force direction
parallel to the gravity. Therefore, it has been observed from Figure 3 (d-l) the streamlines for the increasing magnetic field and for same constraint ratio the number of convectional rings tends to decrease. This reduction in the convectional rolls occurs due to the presence of vertical opposing Lorentz force in the enclosure, the Lorentz force in the cavity has strong direct opposing nature to buoyancy force in the domain. Therefore for $B_x$ magnetic field with the higher intensity of $Ha$ ($Ha = 50$) the flow is highly suppressed and the convectional motion of fluid in the cavity is retarded. Therefore fluid is forced to flow in the entire cavity with lesser number of convectional rings, which further causes the decrease in the area occupancy of secondary flow (eddies).

However, for $B_y$ magnetic field, the induced Lorentz force is normal to the vertical buoyancy force hence the flow in the enclosure for $B_y$ magnetic field is stretched along the horizontal span length and the convectional rolls in the enclosure increase with the intensity of magnetic field as shown in Figure 4 (d-l). The convectional rolls in the cavity have even number of rolls with maximum twelve numbers of rings in the enclosure for $Ha = 50$ ($B_y$) is observed. This shows that the convectional motion in the domain for $B_y$ magnetic field has more influencing with respect to the magnetic field imposed in the $x$-direction ($B_x$).

Similarly, the isotherms variation in the enclosure for all the constraint ratio and magnetic field is observed as shown in Figure 5 and Figure 6. The formation of convectional rings has the directional influence of the formation of the thermal plumes. As the magnetic field imposed in the $x$-direction has the severe suppression effect on the flow the thermal plumes formation in the cavity also has severally affected by the presence of Lorentz force. It has been observed from Figure 5 (a-l) that the maximum four plumes are generated at $Ha = 0$ and $C = 0.5$. Thereafter, the increase in the strength of the magnetic field at $Ha = 50$ and $C = 0.5$, the thermal plumes have almost vanished and the heat dissipation occurs mostly because of diffusion. Conversely, it has been observed for $B_y$ magnetic field in Figure 6 (a-l) that the thermal plumes formation is increased with an increase in the magnetic field. The six number of plumes are observed at $Ha = 50$ and $C = 0.5$, whereas the three plumes are emerged at $Ha = 0$ and $C = 0$. Therefore, in addition to the magnetic field, the constraint in the geometry also plays the significant role in the enhancement of the flow distribution and the heat dissipation.

Figure 7 shows the average Nusselt number ($Nu_{avg}$) variation at different $Ha$ and constraint ratio ($C$) for $B_x$ and $B_y$ magnetic field. As we have seen that the flow features and thermal plumes generation are highly affected by the magnetic field and constraint ratio. Hence at $Ha = 0$, with an increase in the constraint ratio ($C$), the flow in the cavity becomes more unstable and thus the heat dissipation in the domain increases. Conversely, with increase in the magnetic field, as the flow in the cavity is suppressed consequently the heat dissipation also retarded and hence $Nu_{avg}$ tends to decrease at higher $Ha$. The suppression effect in the case of $B_y$ magnetic field is observed higher than the $B_x$ magnetic field, this is due to the presence of vertically opposing Lorentz force to buoyancy force as shown in Figure 7.

5. Conclusions

The present study deals with the influence of the magnetic field and the constraint ratio of the enclosure in the flow features and heat dissipation. The Rayleigh number for the present study is fixed at $Ra \approx 10^6$ at $Pr = 0.02$. The constraint ratio of the top surface is varied in between $C = 0, 0.25, \text{ and } 0.5$. It has been observed from results that the constraint ratio ($C$) has the favorable role in the increasing the heat dissipation of domain by increasing the unsteadiness in the flow at same $Ha$. The highest $Nu_{avg}$ is observed for the $C = 0.5$ at the same $Ha$. The constriction in the geometry causes the flow bifurcation hence the multiple thermal plumes emerged in the cavity which causes more heat dissipation. The magnetic field in the enclosure regulates the flow and produces the opposing Lorentz force. Hence, the flow in the cavity suppressed and heat dissipation in the cavity retards. The retardation effect is found to be more in the case of $B_x$ magnetic field than $B_y$. The production of vertically downward Lorentz force opposite to the buoyancy force is the sole reason for higher suppression rate.
Figure 7. The $N_u_{avg}$ at different $Ha$ and Constraint ratio ($C$), for $B_x$ (left), and $B_y$ (right)

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