Some properties of synchrotron radio and inverse-Compton gamma-ray images of supernova remnants

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ABSTRACT

The synchrotron radio maps of supernova remnants (SNRs) in a uniform interstellar medium and interstellar magnetic field (ISMF) are analysed, allowing for different ‘sensitivity’ of the injection efficiency to the obliquity of the shock. The very-high-energy γ-ray maps arising from inverse Compton processes are also synthesized. The properties of images in these different wavelength bands are compared, with particular emphasis on the location of the bright limbs in bilateral SNRs. Recent High-Energy Stereoscopic System (HESS) observations of SN 1006 show that the radio and inverse Compton γ-ray limbs coincide, and we found that this may happen if (i) injection is isotropic but the variation of the maximum energy of electrons is rather fast to compensate for differences in the magnetic field, or (ii) the obliquity dependence of injection (either quasi-parallel or quasi-perpendicular) and the electron maximum energy are strong enough to dominate the magnetic field variation. In the latter case, the obliquity dependences of the injection and the maximum energy should not be opposite. We argue that the position of the limbs alone, and even their coincidence in radio, X-rays and γ-rays, as discovered by HESS in SN 1006, cannot be conclusive as regards the dependence of the electron injection efficiency, the compression/amplification of the ISMF and the electron maximum energy on the obliquity angle.

Key words: acceleration of particles – radiation mechanisms: non-thermal – shock waves – cosmic rays – supernova remnants.

1 INTRODUCTION

The observation of supernova remnants (SNRs) in very-high-energy (VHE) γ-rays by the High-Energy Stereoscopic System (HESS) and Major Atmospheric Gamma-ray Imaging Cherenkov (MAGIC) experiments is an important step toward understanding the nature of Galactic cosmic rays and the kinematics of charged particles and magnetic fields in the vicinity of strong non-relativistic shocks. However, the spectral analysis of multiwavelength data allows both for leptonic and hadronic origin of VHE γ-ray emission (e.g. RX J1713.7−3946: Berezhko & Völk 2006; Aharonian et al. 2007). In this context, the broad-band fitting of the spectrum of the non-thermal emission from SNRs is one of the hot topics in present studies of SNRs. At the same time, another very important source of scientific information, the distribution of the surface brightness, is not in great demand: there are only a few discussions emphasizing that observed correlations of brightness in radio, X-rays and γ-rays may be considered to favour electrons as being responsible for VHE emission in RX J1713.7−3946, Vela Jr and some other SNRs (e.g. Aharonian et al. 2006; Plaga 2008). However, should the patterns of surface brightness in radio, X-rays and γ-rays really correlate if the VHE γ-ray radiation originates from electrons? What should be the limitations for theory when observed patterns are really quite similar, especially in symmetrical bilateral SNRs such as SN 1006 (HESS Source of the Month, 2008 August)?

Another key issue for particle kinetics is the three-dimensional morphology of bilateral SNRs in general and SN 1006 in particular. Is it polar-cap or barrel-like? The answer to this question is strongly related to the model of injection (quasi-parallel in the former case and isotropic or quasi-perpendicular in the latter case), therefore giving an important hint regarding acceleration theory. The properties of the brightness distribution may be the most conclusive issue on this subject (e.g. the criterion of Rothenflug et al. (2004) and the azimuthal profile comparison in Petruk et al. (2009)).

An experimental investigation of SNR images needs to be complemented by theoretical modelling of SNR maps in different energy domains. Radio and X-ray synchrotron images in a uniform...
interstellar medium (ISM) and uniform interstellar magnetic field (ISMF) are modelled by Reynolds (1998). The role of the gradient of ISM density and ISMF strength on the radio morphology of SNRs is studied by Orlando et al. (2007). These papers are based on classical magnetohydrodynamics (MHD) and assume unmodified shocks. Studies of non-thermal images of SNRs with non-linear acceleration theory are undergoing development (Lee, Kamae & Elliston 2008). The profiles of the synchrotron brightness in such SNRs are the subject of investigation in Ellison & Cassam-Chenaï (2005) and Cassam-Chenaï et al. (2005).

In the present paper, we present for the first time the inverse-Compton γ-ray images of SNRs in a uniform ISM and ISMF produced on the basis of the model of Reynolds (1998). In addition to this model, we allow for different ‘sensitivity’ of the injection efficiency to the shock obliquity, as is apparent in the numerical results of Ellison, Baring & Jones (1995). The synthesized maps are compared with radio ones. Some consequences for the origin of VHE emission of SNRs and the electron injection scenario are drawn.

2 MODEL

We consider a SNR in a uniform ISM and a uniform ISMF. At the shock, the energy spectrum of electrons is taken as \( N(E) = K E^{-\gamma} \exp(-E/E_{\text{max}}) \), where \( E_{\text{max}} \) is the maximum energy of electrons and \( \gamma = 2 \) is used throughout this paper. We follow Reynolds (1998) in calculation of the magnetic field and relativistic electrons (see also Petruk 2006; Petruk & Beshley 2008). The compression factor for the ISMF, \( \sigma \), increases from unity at a parallel shock to 4 at a perpendicular one. The fiducial energy at a parallel shock, which is responsible for the ‘sensitivity’ of relativistic electrons to radiative losses (Reynolds 1998) and which is used in inverse Compton (IC) images, is set to \( E_{\text{max}} \). The synchrotron losses are considered to be the dominant channel for radiative losses of relativistic electrons. We assume that \( K \) is constant in time; the eventual evolution of \( K \) affects the radial thickness of the rims and does not modify the main features of the surface brightness pattern (Reynolds 1998).

Electrons emitting IC photons have energies \( E \sim E_{\text{max}} \). Like \( K \), \( E_{\text{max}} \) is assumed to be constant in time. Its possible variation in time does not change the pattern of IC brightness, and leads to effects similar to those originating from the time dependence of \( K \). Namely, features in IC images have to be radially thicker if \( E_{\text{max}} \) decreases with time (i.e. increases with the shock velocity): since \( E_{\text{max}} \) was larger at previous times, there are more electrons in the SNR interior able to emit IC photons at the present time. If \( E_{\text{max}} \) increases with time (i.e. decreases with the shock velocity) then maxima in brightness are expected to be radially thinner.

Reynolds (1998) considered three models for injection: quasi-parallel, isotropic and quasi-perpendicular. The pattern of the radio surface brightness distribution in the case of quasi-perpendicular injection is quite similar to the isotropic injection case, though with different contrasts (Fullbright & Reynolds 1990; Orlando et al. 2007). The numerical calculations of Ellison et al. (1995) show that the obliquity dependence of the injection efficiency \( \zeta \) (fraction of accelerated electrons) may be either flatter or steeper than in the classic quasi-parallel case \( \zeta \propto \cos^2 \theta_o \), where \( \theta_o \) is the obliquity angle, the angle between the ISMF and the normal to the shock: see Fig. 1). In order to be more general than Reynolds (1998), we allow the injection efficiency to vary with obliquity angle with different

\begin{equation}
\zeta(\theta_o) = \zeta_1 \exp \left[-(\theta_o/\Theta K)^2\right],
\end{equation}

where \( \zeta_1 \) is the efficiency for a parallel shock. This expression approximately restores the results of Ellison et al. (1995) with \( \Theta K = (\pi/9)/(\pi/4) \). Classic quasi-parallel injection may be approximated with \( \Theta K = \pi/6 \). Isotropical injection assumes \( \Theta K = \infty \), but values \( \Theta K \geq 2\pi \) produce almost the same images as \( \Theta K = \infty \), because the range for obliquity angle is \( 0 \leq \theta_o \leq \pi/2 \).

We consider also quasi-perpendicular injection:

\begin{equation}
\zeta(\theta_o) = \zeta_1 \exp \left\{-[(\theta_o - \pi/2)/\Theta K]^2\right\},
\end{equation}

where \( \Theta K \) is a parameter and \( E_{\text{max}} \) the maximum energy at a parallel shock. This formula, with different values of \( \Theta K \), is able to approximately restore the different cases considered by Reynolds (1998).

The surface brightness is calculated by integrating emissivities along the line of sight within SNRs. The synchrotron emissivity at some radio frequency is \( q_{\text{syn}} \propto K B^2 \), where \( B \) is the strength of the magnetic field. The γ-ray emissivity of electrons due to inverse Compton processes is calculated as

\begin{equation}
q_{\zeta}(\epsilon) = \int_{0}^{\infty} N(E) p_{\zeta}(E, \epsilon) dE,
\end{equation}

where \( \epsilon \) is the photon energy. The spectral distribution \( p_{\zeta} \) of the radiation power of a ‘single’ electron in a blackbody photon field with temperature \( T \) is

\begin{equation}
p_{\zeta}(\gamma, \epsilon) = \frac{2e^2 \epsilon}{\pi \hbar^2 c^2} \gamma^{-2} I_{\zeta}(\eta_\gamma, \eta_\epsilon),
\end{equation}

where

\begin{equation}
\eta_\gamma = \frac{\gamma}{1 + \gamma}, \quad \eta_\epsilon = \frac{\epsilon}{1 + \epsilon}.
\end{equation}
where $\gamma$ is the Lorenz factor of the electron, $\epsilon_c = kT$,

$$\eta = \frac{\epsilon_c e}{(m_e c^2)^2}, \quad \eta_0 = \frac{\epsilon^2}{4 \gamma m_e c^2 (\gamma m_e c^2 - \epsilon)}.$$  

and $m_e, e, c, h$ and $k$ have their typical meaning. $I_c(\eta_1, \eta_0)$ may be approximated as (Petruk 2008)

$$I_c(\eta_1, \eta_0) \approx \frac{\pi^2}{6} \eta_1 \exp \left[ -\frac{5}{4} \left( \frac{\eta_0}{\eta_1} \right)^{1/2} \right] + 2 \eta_0 \exp \left[ -\frac{5}{7} \left( \frac{\eta_0}{\eta_1} \right) \right] \exp \left[ -\frac{2}{3} \eta_0 \right].$$

This approximation is quite accurate, and represents $I_c$ in any regime, from Thomson to extreme Klein–Nishina. The maximum of the spectral distribution $I_c(\epsilon)$ for electrons with energy $E$ is at (Petruk 2008)

$$\epsilon_{\text{max}}(E) \approx \frac{E \Gamma_c}{1 + \Gamma_c}, \quad \Gamma_c = \frac{4 \epsilon_c E}{(m_e c^2)^2}.$$  

(8)

All IC images in the present paper (except for Fig. 10) are calculated for an initial photon field with $T = 2.75 \text{ K}$ and $\gamma$-ray photon energy $\epsilon = 0.1 \epsilon_{\text{max}}(E_{\text{max}})$, which is, for example, $\epsilon = 0.3 \text{ TeV}$ for $E_{\text{max}} = 30 \text{ TeV}$.

3 RESULTS

3.1 Synchrotron radio images

We stress that all figures in the present paper have been computed using a complete MHD model.

Let us define the aspect angle $\phi_o$ as the angle between the interstellar magnetic field and the line of sight (Fig. 1). It is shown that the azimuthal variation of the radio surface brightness $S_\nu$ at a given radius of projection $r$, in a SNR that is not centrally brightened, is mostly determined by the variation of the magnetic field compression (and/or amplification) $\sigma_B$ and the electron injection efficiency $\zeta$ (Petruk et al. 2009):

$$S_\nu(\varphi) \propto \zeta(\Theta_{o,\text{eff}}(\varphi, \phi_o)) \sigma_B(\Theta_{o,\text{eff}}(\varphi, \phi_o))^{(r+1)/2}.$$  

(9)

where $\varphi$ is the azimuthal angle. The effective obliquity angle $\Theta_{o,\text{eff}}$ is related to $\varphi$ and $\phi_o$ as

$$\cos \Theta_{o,\text{eff}}(\varphi, \phi_o) = \cos \varphi \sin \phi_o;$$

(10)

and one should consider an injection model that strongly depends on the obliquity ($\Theta_{o,\text{eff}} \leq \pi/6$, Figs 2a and b). Instead, if the barrel is the preferred model (i.e. the ISMF is parallel to the symmetry axis between two limbs) then the injection efficiency should be almost.

![Figure 2](https://example.com/figure2.png)

Figure 2. Radio images of SNRs for an aspect angle $\phi_o = 90^\circ$ and different $\Theta_{o,\text{eff}}$: (a) $\pi/12$, (b) $\pi/6$, (c) $\pi/4$, (d) $\pi/2$, (e) $\pi$ and (f) $2\pi$. The ambient magnetic field is oriented along the horizontal axis. Hereafter, the increment in brightness is $\Delta S = 0.1S_{\text{max}}$. 

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independent of obliquity ($\Theta_k \geq \pi$). Figs 2c and f) or else prefer quasi-perpendicular shocks. 

Gaensler (1998) measured the angle $\psi$ between the symmetry axis in 17 'clearly' bilateral SNRs and the Galactic plane. Axes are more or less aligned with the Galactic plane in 12 SNRs ($\psi < 30^\circ$), 2 SNRs have $\psi \approx 45^\circ$ and 3 SNRs are almost perpendicular ($\psi > 60^\circ$). If we assume that the ISMF is parallel to the plane of Galaxy then most bilateral SNRs should be three-dimensional barrels, thus preferring isotropic (or quasiperpendicular) injection.

An interesting feature appears on images for $\Theta_k = (\pi/4)-(\pi/2)$ (Figs 2c and d). Namely, the SNR has 'quadrilateral' morphology. With an increase in obliquity, the injection efficiency decreases while the compression factor of the ISMF increases. The variation of injection $\zeta(\Theta_k)$ dominates $\sigma_{B}(\Theta_k)$ for $\Theta_k \leq \pi/6$. If $\Theta_k \geq \pi$ (i.e. injection is almost isotropic) then $\sigma_{B}(\Theta_k)$ plays the main role in the azimuthal variation of radio surface brightness. In the intermediate range of $\Theta_k$, the significance of the two types of variation is comparable, leading therefore to an azimuthal migration of the brightness maxima in the modelled images. There is no 'quadrilateral' SNR reported in the literature. If no such SNR exists at all, the range $\Theta_k \simeq (\pi/4)-(\pi/2)$ may be excluded. However, we stress that a complete statistical study of the morphology of radio SNRs would be needed to assess definitively the lack of quadrilateral SNRs.\(^1\)

The visual morphology of a SNR is different for different aspect angles. Fig. 4 shows SNR images for quasi-parallel injection with $\Theta_k = \pi/12$ (upper panel) and for isotropic injection ($\Theta_k = 2\pi$, lower panel). We expect that the ISMF may have a different orientation relative to the observer in various SNRs. If quasi-parallel injection is not a rare exception, then the pole-cap SNRs should be projected in a different way and we might expect to observe not only 'bipolar' SNRs (Figs 4c and d) but also SNRs with one or two radio eyes within the thermal X-ray rim (Figs 4a and b). Fullbright & Reynolds (1990) developed this thought statistically and showed that the quasi-parallel injection model was unlikely, but we would need a complete study to verify this statement.\(^2\) The statistical arguments of Fullbright & Reynolds (1990) may be affected by the fact that centrally bright radio SNRs (lines 1–2 in Fig. 3) are expected to be fainter than bilateral or circular SNRs with the same characteristics (lines 4–6 in Fig. 3): it could be that most of the centrally peaked SNRs are not observable.

3.2 IC $\gamma$-ray images

Let us consider first the case in which the maximum energy of electrons is constant over the SNR surface; this allows us to clearly see the role of injection efficiency and magnetic field variations.

Synthesized IC $\gamma$-ray images of SNRs are presented in Fig. 5 for different aspect angles. These images assume almost isotropic injection ($\Theta_k = 2\pi$) and should be compared with the radio maps in the lower panel of Fig. 4. The component of the ISMF that is perpendicular to the line of sight is along the horizontal axis in all images. An important difference is prominent in these two figures: namely, the two lobes develop with increasing $\psi$, in both radio and $\gamma$-rays, however their location with respect to the ISMF is opposite for the two cases. The line connecting the two maxima in the radio is perpendicular to the ISMF, while it is parallel to the ISMF in IC images (cf. Figs 5d and 4h).

The reason for this effect is the following. For assumed isotropic injection, the azimuthal variation of the radio brightness is determined only by the dependence of $\sigma_{B}$ on obliquity (the azimuth angle equals the obliquity angle for $\phi_o = \pi/2$). Electrons emitting VHE $\gamma$-rays have energies $E \sim E_{\text{max}}$ and experience substantial radiative losses (this effect is negligible for radio-emitting electrons). The magnetic field does not appear directly in the formulae for IC emission, but it affects the downstream distribution of relativistic electrons emitting IC $\gamma$-rays. The larger the post-shock magnetic field, the greater the radiative losses. The downstream distribution of IC-emitting electrons is therefore steeper where the magnetic field is stronger. This leads to lower IC brightness in SNR regions with a larger magnetic field (while the radio brightness increases there because of proportionality to $B^{3/2}$).

In the VHE $\gamma$-ray image of SN 1006 recently reported by the HESS collaboration (HESS Source of the Month, August 2008), the two maxima coincide in location, with limbs in radio and nonthermal X-rays. This fact, in view of the 'limb-inverse' property, could be considered as an argument against the leptonic origin of $\gamma$-ray emission in SN 1006 (if injection is isotropic). However, these IC images are obtained under the assumption that $E_{\text{max}}$ does not vary over the SNR surface. If $E_{\text{max}}$ is high enough in regions with large magnetic field (at a perpendicular shock), then the 'limb-inverse' effect may be less prominent or may even be unimportant (see below).

In the case in which injection strongly prefers parallel shocks (the limbs in SN 1006 are polar caps), the dependence $\zeta(\Theta_k)$ might dominate $\sigma_{B}(\Theta_k)$. The maxima of brightness in radio and IC $\gamma$-rays are therefore located in the same regions of SNR projection (Fig. 6, tobe compared with Figs 4a and d), in agreement with the Chandra and HESS observations of SN 1006.

The effect of intermediate values of $\Theta_k$ for injection that prefers a parallel shock, equation (1), on profiles of IC brightness is shown in Fig. 7. An increase of the sensitivity of injection to obliquity leads to radially thinner and more contrasting features.

If the injection prefers a perpendicular shock, equation (2), its increase in regions of larger magnetic field may compensate for the lack of $\gamma$-ray-emitting electrons. In that case, the position of the limbs coincides in radio and IC $\gamma$-rays if the dependence $\zeta(\Theta_k)$ is strong enough (Figs 8b and d). In the range of intermediate $\Theta_k$, the quadrilateral morphology also appears in models of IC $\gamma$-rays (Fig. 8c), as an intermediate morphology between those in Figs 5(d) and 8(d). (The contrast between maxima in the image of the quadrilateral SNR is so small that this feature will probably not be observable.)

Note that the quasi-perpendicular injection model leads to radio images similar to those in the isotropic injection case, cf. Figs 8(a)

\(^1\)G338.3–0.0 could be an example of a quadrilateral SNR.

\(^2\)G311.5–0.3 and G337.2–0.7 could be examples of SNRs with two radio 'eyes'.
Figure 4. Radio images of SNRs for different aspect angles $\phi_o$: $0^\circ$ (a, e), $30^\circ$ (b, f), $60^\circ$ (c, g) and $90^\circ$ (d, h). $\Theta_K = \pi/12$ (upper panel) or $2\pi$ (lower panel). The component of the ambient magnetic field that is perpendicular to the line of sight is oriented along the horizontal axis.

Figure 5. IC $\gamma$-ray images of SNRs. Isotropic injection, $E_{\text{max}}$ is constant over the SNR surface. Aspect angle $\phi_o$ is (a) $0^\circ$, (b) $30^\circ$, (c) $60^\circ$ or (d) $90^\circ$. The component of the ambient magnetic field that is perpendicular to the line of sight is oriented along the horizontal axis.

Figure 6. IC $\gamma$-ray images of SNRs. Quasi-parallel injection (equation 1) with $\Theta_K = \pi/4$, $E_{\text{max}}(\Theta_\phi) = \text{constant}$. Aspect angles $\phi_o$ are (a) $0^\circ$ and (b) $90^\circ$. In the latter case, the ISMF is along the horizontal axis.
Figure 7. Profiles of the IC surface brightness along the x-axis for aspect angles $\phi_o = 0^\circ$ (i.e. the radial profile of brightness is the same for any azimuth; to be compared with Fig. 3) and 90° (i.e. the ISMF is along the horizontal axis). The dependence of the injection is given by equation (1) with $\Theta_K$ (from bottom) = $\pi/12$, $\pi/6$, $\pi/4$, $\pi/2$, $\pi$, $2\pi$, $\infty$. $E_{\text{max}}$ is constant over the SNR surface.

and (b) and Fig. 2(f) (see also Orlando et al. 2007), because the magnetic field and injection efficiency both increase at perpendicular shocks, leading to greater synchrotron emission. In contrast, there is a lack of IC radiating electrons around perpendicular shocks, which may or may not (depending on $\Theta_K$ in equation 2) be compensated by injection. Thus IC images involving quasi-perpendicular injection may differ radically from those with isotropic injection, cf. Figs 8(d) and 5(d).

The obliquity variation of the electron maximum energy is an additional factor affecting the IC $\gamma$-ray brightness in SNRs. Actually, Rothenflug et al. (2004) have shown that the cut-off frequency increases in the radio limbs of SN 1006, which may (partially) be due to the larger $E_{\text{max}}$ there. Therefore $E_{\text{max}}$ is expected to be largest in this SNR at the perpendicular shock (at the equatorial belt) if the injection is isotropic or quasi-perpendicular, or at the parallel shock (at the polar caps) if the injection is quasi-parallel. In the latter case, the calculations of Reynolds (1998) suggest that the only possible model for $E_{\text{max}}$ in SN 1006 should be a loss-limited one in the Bohm limit.

The case of $E_{\text{max}}$ increasing with obliquity, equation (3), is shown in Fig. 9. The ‘limb-inverse’ property may not be important and the limbs may coincide in radio, X-rays and IC $\gamma$-rays even for isotropic injection if the maximum energy is large enough at the perpendicular shocks to provide energetic electrons despite the radiative losses (Fig. 9b, cf. Figs 4h and 5d). Note also that the limbs are thicker in this case, because of the more effective radiative losses at a perpendicular shock (owing to larger ISMF compression), compared with the limbs at a parallel shock.

The dependence of $E_{\text{max}}$ on $\Theta_m$ may also cause the splitting and rotation of IC limbs in the case of quasi-parallel injection (Fig. 9d, cf. Fig. 6b) or quasi-perpendicular injection. There is a possibility of quadrilateral SNRs appearing in $\gamma$-rays due to the interplay between the dependences $E_{\text{max}}(\Theta_m)$, $\zeta(\Theta_m)$ and $\sigma_B(\Theta_m)$ (Figs 9a and d).

All the above IC images are calculated for the photon energy $\epsilon = 0.1E_{\text{max}}(E_{\text{max}})$. The pattern of the $\gamma$-ray surface brightness remains almost the same with increasing photon energy, though the regions of maximum brightness become radially thinner and also the contrasts change (Fig. 10). This is because the electrons that contribute most of the emission at higher photon energies experience higher radiative losses, and therefore the downstream distribution of these electrons is steeper.

To this end, the main properties of the IC surface brightness may simply be derived from the approximate analytical formula for the azimuthal variation of IC surface brightness $S_{\varphi}(\varphi; \phi_o, \epsilon)$ of an
adiabatic SNR in a uniform ISM and uniform ISMF (Appendix):

\[ S_\nu(\varphi) \propto \zeta(\Theta_{\text{eff}}) \exp \left( -\frac{E_m \tilde{\vartheta}^{-1 - 5\eta}(\Theta_{\text{eff}}^2 E_m/2E_B)}{E_{\text{max}}^\eta F(\Theta_{\text{eff}})} \right), \]  

(11)

where \( E_m \propto \epsilon^{1/2} \) (equation A8), \( \tilde{\vartheta} = \vartheta/R \leq 1 \) and \( \vartheta \) is the distance from the centre of the SNR projection. This formula may not be used for a SNR that is centrally bright in \( \gamma \)-rays, and is valid for \( \vartheta/R \) larger than \( \approx 0.9 \).

4 CONCLUSIONS

In the present paper, we analyse the synchrotron radio and inverse-Compton \( \gamma \)-ray images of Sedov SNRs synthesized on the basis of the Reynolds (1998) model. Ellison et al. (1995) have shown that the dependence of the efficiency of injection \( \zeta \) on obliquity angle \( \Theta_\text{a} \) may differ from the commonly used expression in the quasi-parallel case. We therefore parametrize the dependence \( \zeta(\Theta_\text{a}) \) as given by equation (1). It is shown that the variation of the parameter \( \Theta_K \) provides a smooth transition from polar-cap (\( \Theta_K \leq \pi/6 \)) to barrel-like (\( \Theta_K \geq \pi \)) models of a SNR and that the assumed orientation of the ISMF should be related to a particular injection model. Some constraints on injection models, which follow from morphological considerations, are pointed out. The azimuthal variation of radio brightness is mostly due to variations of \( \zeta \) and \( \sigma_B \), in agreement with the approximate formula (9).

Theoretical \( \gamma \)-ray images of SNRs arising from the inverse Compton effect are reported for the first time. We analyse the properties of these images and compare them with the corresponding radio maps of SNRs. The azimuthal variation of IC brightness is mostly determined by variations of \( \zeta \), \( \sigma_B \) and \( E_{\text{max}} \), in agreement with the approximate formula (11) derived in the Appendix.

In the case in which \( E_{\text{max}} \) is constant over the SNR surface, we found an opposite behaviour of the azimuthal variation of surface brightness in radio and IC \( \gamma \)-rays, when the injection is isotropic and the aspect angle is larger than \( \approx 60^\circ \). Namely, the lines crossing the two limbs in the radio are perpendicular to the ISMF, while they are parallel in IC \( \gamma \)-rays. In particular, bright radio limbs correspond to dark IC areas, in disagreement with X-ray and HESS observations of SN 1006. This happens because the IC image is affected by large radiative losses of the emitting electrons behind a perpendicular shock, while the larger magnetic field increases the radio brightness.
there. Variation of $E_{\text{max}}$ over the SNR surface may (to some extent) hide this effect. The maximum energy should increase with obliquity in this case.

In the case of the polar-cap model of a SNR (quasi-parallel injection), the maxima in surface brightness are expected to coincide in radio and IC $\gamma$-rays (in agreement with the Hess observation of SN 1006), unless the increase of $E_{\text{max}}$ with obliquity is very strong, which is unlikely in the case of SN 1006 because the cut-off frequency is larger for limbs that are at the parallel shock in this injection model.

Limbs may also coincide in the case of quasi-perpendicular injection, if the lack of electrons (due to radiative losses) in regions of large magnetic field is compensated for by a strong enough increase in $\zeta$ and/or $E_{\text{max}}$ with $\Theta_0$.

Isotropic compression/amplification of the ISMF at the shock (i.e. independent of the shock obliquity), as might happen under highly effective acceleration, may also be responsible for limbs having the same position in the radio and in the IC $\gamma$-rays, for the quasi-parallel or quasi-perpendicular injection scenarios. In this case the dependence of $E_{\text{max}}(\Theta_0)$ must follow variation $\zeta(\Theta_0)$, namely it should be largest (smallest) at the parallel shock for quasi-parallel (quasi-perpendicular) injection, otherwise the morphology of SNRs in IC $\gamma$-rays may differ from the radio one.

We conclude that the location of the $\gamma$-ray limbs versus the radio and X-ray ones, recently discovered by Hess for SN 1006, cannot be conclusive as regards the actual dependence of the electron injection efficiency, the compression/amplification of the ISMF and the electron maximum energy on the obliquity angle for this SNR. Detailed features of the SNR maps in different wavebands should be considered for this purpose.

The interplay between dependences $\zeta(\Theta_0)$, $\sigma_B(\Theta_0)$ and $E_{\text{max}}(\Theta_0)$ may cause a quadrilateral morphology in SNR models, owing to splitting of the maxima in surface brightness. The absence of quadrilateral SNRs in IC $\gamma$-rays, if confirmed observationally, may result in limitations on $\Theta_0$ and $\Theta_0$.

The detailed characteristics of features of the IC image (e.g. the thickness of the rim) depend on the photon energy. They are radially thinner at larger photon energies, as expected.

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**REFERENCES**

Aharonian F. et al., 2006, A&A, 449, 223
Aharonian F. et al., 2007, H. A., 464, 235
Berezko E. G., Volkh H. J., 2006, A&A, 451, 981
Cassam-Chenaï G., Decourchelle A., Ballet J., Ellison D. C., 2005, A&A, 443, 955
Ellison D., Cassam-Chenaï G., 2005, ApJ, 632, 920
Ellison D. C., Baring M. G., Jones F. C., 1995, ApJ, 453, 873
Fulbright M. S., Reynolds S. P., 1990, ApJ, 357, 591
Gaensler B. M., 1998, ApJ, 493, 781
Kesteven M. J., Casswell J. L., 1987, A&A, 183, 118
Lee S.-H., Kamae T., Ellison D. C., 2008, ApJ, 686, 325
Orlando S., Bocchino F., Reale F., Peres G., Petruk O., 2007, A&A, 470, 927
Petruk O., 2006, A&A, 460, 375
Petruk O., 2008, A&A, in press (arXiv:0807.1690)
Petruk O., Beshley V., 2008, Kinematics and Physics of Celestial Bodies, 24, 159
Plaga R., 2008, New Aastron., 13, 73
Reynolds S. P., 1998, ApJ, 493, 375
Rothenflug R., Ballet J., Dubner G., Giacani E., Decourchelle A., Ferrando P., 2004, A&A, 425, 121
Schlickeiser R., 2002, Cosmic Ray Astrophysics. Springer, Berlin

**APPENDIX A: APPROXIMATE ANALYTICAL FORMULA FOR THE AZIMUTHAL VARIATION OF THE IC $\gamma$-RAY SURFACE BRIGHTNESS IN A SEDOV SNR**

An approximate formula for azimuthal variation of the IC $\gamma$-ray surface brightness allows one to avoid detailed numerical simulations and may be useful if approximate estimation for the variation is reasonable. It gives a deeper insight into the main factors determining the azimuthal behaviour of the IC surface brightness in SNRs.

Let the energy of relativistic electrons be $E$ in a given fluid element at present time; this energy was $E_{t_1}$ at the time at which this element was shocked. These two energies are related as

$$E = E_{t_1} \frac{E_{\text{rad}}}{E_{\text{ad}}} \frac{E_{\text{rad}}}{E_{\text{rad}}} $$

where $E_{\text{rad}}$ accounts for the adiabatic losses and $E_{\text{rad}}$ for the radiative losses. There are approximations valid close to the shock (Petruk & Beshley 2008):

$$E_{\text{rad}} \approx \bar{a}, \quad E_{\text{rad}} \approx 3 \sigma_B \frac{E}{2 E_{t_1}}. $$

where $\bar{a} = a/R$, $a$ is Lagrangian coordinate of the fluid element and $E_{t_1}$ is the fiducial energy for a parallel shock. The downstream evolution of $K$ in a Sedov SNR is

$$K \propto \zeta(\Theta_0) \bar{K} (\bar{a}). $$

With the approximations (A2), the distribution $N(E)$ may be written in the model of Reynolds (1998) as

$$N(E, \Theta_0) \propto \zeta(\Theta_0) \bar{K} (\bar{a}) E^{-4} \exp \left( - \frac{E_{t_1} \psi(E, \Theta_0)}{E_{max} \bar{F}(\Theta_0)} \right),$$

where

$$\psi(E, \Theta_0) = 1 + \frac{5 \sigma_B(\Theta_0)}{E_{t_1}} E$$

and the obliquity variation of the maximum energy of electrons is given by $E_{\text{max}} = E_{max} \bar{F}(\Theta_0)$.

Electrons with Lorentz factor $\gamma$ emit most of their IC radiation in photons with energy $\varepsilon_m$. Let us use the ‘delta-function approximation’ (Petruk 2008):

$$p_m(\gamma, \varepsilon) \approx p_m(\gamma) \delta(\varepsilon - \varepsilon_m), \quad p_m(\gamma) = \int_{0}^{\infty} p_m(\gamma, \varepsilon) \, d\varepsilon.$$
Substitution of equation (4) with equation (A6) yields

\[ q_{\text{IC}} = \frac{ec\sigma_T m_{e}^2 \varepsilon^{3/2}}{12\epsilon^{3/2}} N(E_{\text{in}}), \]  

(A7)

where

\[ E_{\text{in}} = \frac{m_{e}c^2\varepsilon^{1/2}}{2(kT)^{1/2}} \]  

(A8)

is the energy of electrons that give the maximum contribution to the IC emission for photons with energy \( \varepsilon \).

Let us consider the azimuthal profile of the IC \( \gamma \)-ray brightness \( S_{\gamma} \) at a given radius \( \varrho \) from the centre of the SNR projection.

The obliquity angle \( \Theta_o \) is different for each radial sector of a three-dimensional object. It is determined, for any position within a SNR, by the set \( (\varrho, \varphi, \phi_o) \). Integration along the line of sight gathers information from different radial sectors, with different obliquities. Let us determine the ‘effective’ obliquity angle by the relation

\[ \Theta_{o,\text{eff}}(\varrho, \phi_o) = \Theta_o(\varrho, 1, \phi_o), \]  

(A9)

Actually, \( \Theta_{o,\text{eff}} \) for a given azimuth equals the obliquity angle for a sector with the same azimuth lying in the plane of the sky (i.e. in the plane perpendicular to the line of sight and containing the centre of the SNR). \( \Theta_o \) varies around \( \Theta_{o,\text{eff}} \) during integration along the line of sight. The closer \( \varrho \) is to the edge of SNR projection, the smaller the range for variation of \( \Theta_o \) and the more accurate our approximation.

The surface brightness of SNR projection at a distance \( \varrho \) from the centre and at azimuth \( \varphi \) is

\[ S(\varrho, \varphi) = 2 \int_{\epsilon(\varrho)}^{1} q_{\text{IC}}(\varrho) \frac{\hat{F}_2 \hat{d} \hat{e}}{\sqrt{\hat{e}^2 - \varrho^2}}, \]  

(A10)

where \( \hat{F}_2 \) is the derivative of \( \hat{F}(\varrho) \) in respect to \( \varrho \). The azimuthal variation of the IC brightness for fixed \( \varrho \) is approximately

\[ S_{\varrho} \propto \varphi(\Theta_{o,\text{eff}}) \exp \left( -\frac{E_{\text{in}} \varrho^{-\varphi}(\varrho, \Theta_{o,\text{eff}})}{E_{\text{max},\text{eff}} F(\Theta_{o,\text{eff}})} \right) \times \int_{\epsilon(\varrho)}^{1} \frac{\hat{K} \hat{F}_2 \hat{d} \hat{e}}{\sqrt{\hat{e}^2 - \varrho^2}} \exp \left( -\frac{E_{\text{in}}(\varrho^{-\varphi} - \varrho^{-\varphi})}{E_{\text{max},\text{eff}} F(\Theta_{o,\text{eff}})} \right). \]  

(A11)

If \( \varrho \to 1 \) then \( \hat{a}(\varrho) \to 1 \). Thus, the exponent in the integral is roughly unity because \( \hat{a}(\varrho) \leq \hat{a}^{-1} \leq 1 \) and \( \hat{a}(\varrho) \leq \hat{\varrho} \leq 1 \). The integral in (A11) is therefore roughly the same for any azimuthal angle \( \varphi \). The azimuthal variation of the IC \( \gamma \)-ray brightness \( S_{\varrho}(\varphi, \phi_o, \varepsilon) \) is thus determined mostly just by

\[ S_{\varrho}(\varphi) \propto \varphi(\Theta_{o,\text{eff}}) \exp \left( -\frac{E_{\text{in}} \varrho^{-\varphi}(\varrho, \Theta_{o,\text{eff}})}{E_{\text{max},\text{eff}} F(\Theta_{o,\text{eff}})} \right) \]  

with \( E_{\text{in}} \) given by equation (A8), i.e. \( S_{\varrho} \) depends in this approximation on the temperature \( T \) of the seed blackbody photons and the energy \( \varepsilon \) of observed \( \gamma \) photons. The relation between the azimuthal angle \( \varphi \), the obliquity angle \( \Theta_{o,\text{eff}} \) and the aspect angle \( \phi_o \) is as simple as

\[ \cos \Theta_{o,\text{eff}}(\varrho, \phi_o) = \cos \varphi \sin \phi_o, \]  

(A13)

for the azimuth angle \( \varphi \) measured from the direction of the ISMF in the plane of the sky.

The approximation (A12) may be used for \( \varrho \) larger than \( \approx 0.9 \).

Like equation (9), equation (A12) does not give the correct profiles in the case of centrally bright SNRs, i.e. when \( \Theta_k \leq \pi/4 \) in equation (1) and \( \phi_o < 30^\circ \).

This paper has been typeset from a \TeX/XeLaTeX file prepared by the author.