Facts and Fictions about Anti de Sitter Spacetimes with Local Quantum Matter

Dedicated to the memory of Harry Lehmann

Bert Schroer
Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany
presently: CBPF, Rua Dr. Xavier Sigaud, 22290-180 Rio de Janeiro, Brazil
schroer@cbpf.br

November 1999

Abstract

It is natural to analyse the AdS$_{d+1}$-CQFT$_d$ correspondence in the context of the conformal-compactification and covering formalism. In this way one obtains additional insight about Rehren’s rigorous algebraic holography in connection with the degree of freedom issue which in turn allows to illustrate the subtle but important differences between the original string theory-based Maldacena conjecture and Rehren’s theorem in the setting of an intrinsic field-coordinatization-free formulation of algebraic QFT. I also discuss another more generic type of holography related to light fronts which seems to be closer to ’t Hooft’s original ideas on holography. This in turn is naturally connected with the generic concept of “Localization Entropy”, a quantum pre-form of Bekenstein’s classical black-hole surface entropy.

1 Historical background

There has been hardly any problem in particle physics which has attracted as much attention as the problem if and in what way quantum matter in the Anti de Sitter spacetime and the one dimension lower conformal field theories are related and whether this could possibly contain clues about the meaning of quantum gravity.

In more specific quantum physical terms the question is about a conjectured correspondence between two quantum field theories in different spacetime dimensions; the lower-dimensional conformal one being the “holographic image” or projection of the AdS theory. Conjectures, different from mathematical
proofs; allow of course almost always a certain margin in their precise mathematical formulation and in their physical interpretation. The field theoretic content of this conjecture has often been interpreted as a correspondence between two Lagrangian field theories (e.g. between a conformally invariant 4-dimensional SYM and a higher dimensional spin=2 gravitational-like theory). The exact theorem says that such a correspondence cannot exist; one side has to be non-Lagrangian. There is no exception to this proposition; not even the assumption of supersymmetry helps here. One of our goals is to spell this out in detail and to illustrate this interesting point with a simple model.

The community of string physicists has placed this correspondence problem in the center of their interest. Remembering the great conceptual and calculational achievements as e.g. the derivation of scattering theory and dispersion relations from field theory with which the name of Harry Lehmann (to whose memory this article is dedicated) is inexorably linked, I will limit myself to analyze the particle physics content of the so-called Anti deSitter conformal QFT-correspondence from the conservative point of view of a quantum field theorist who, although having no active ambitions outside QFT, still nourishes a certain curiosity about present activities in particle physics as e.g. string theory or noncommutative geometry. In the times of Harry Lehmann the acceptance of a theoretical proposal in particle physics was primarily coupled to its experimental verifiability and/or its conceptual standing within physics.

The AdS model of a curved spacetime has a long history as a theoretical laboratory of what can happen with particle physics in a universe which is the extreme opposite of globally hyperbolic in that it possesses a self-closing time, whereas the proper de Sitter spacetime was once considered among the more realistic models of the universe. The recent surge of interest about AdS came from string theory and is different in motivation and more related to the hope (or dream) to attribute a meaning to “Quantum Gravity” from a string theory viewpoint.

Fortunately for a curious outsider (otherwise I would have to quit right here), this motivation has no bearing on the conceptual and mathematical problems posed by the would be AdS-conformal QFT correspondence; the latter turned out to be one of those properties discovered in the setting of string theory which allow an interesting and rigorous formulation in QFT which confirms some, but not all the conjectured properties.

The rigorous treatment however requires a reformulation of (conformal) QFT within a more algebraic setting. The standard formalism based on pointlike “field coordinatizations” which underlies the Lagrangian (and the Wightman) formulations does not provide a natural setting for the study of isomorphisms between models in different spacetime dimensions, even though the underlying physical principles are the same. One would have to introduce many additional concepts and auxiliary tricks into the standard framework to the extend that the formulation appears contrived containing too many ad hoc prescriptions. The important aspects in this isomorphism are related to space and time-like (Einstein,Huygens) causality, localization of corresponding objects and problems of degree of freedom counting. All these issues are belonging to real-time physics
and in most cases their meaning in terms of Euclidean continuation (statistical mechanics) remains obscure; but this of course does not make them less physical.

This note is organized as follows. In the next section I elaborate the kinematical aspects of the AdS\textsubscript{d+1} -CQFT\textsubscript{d} situation as a collateral result of the old (1974/75) compactification formalism for the “conformalization” of the d-dimensional Minkowski spacetime. For this reason the seemingly more demanding problem of studying QFT directly in AdS within a curved spacetime formalism\footnote{This was also done in the 70ies by Fronsdal. There was a good reason why he missed the isomorphism to CQFT despite his musterful handling of (noncompact) group theory: it was the degree of freedom (multiplicity) problem which will be addressed later.} can be bypassed. The natural question whose answer would have led directly from CQFT\textsubscript{d} to AdS\textsubscript{d+1} in the particle physics setting (without string theory as a midwife) is: does there exist a quantum field theory which has the same SO(4,2) symmetry and just reprocesses the CQFT\textsubscript{d} matter content in such a way that the “conformal Hamiltonian” (the timelike rotational generator through compactified \(M\)) becomes the true hamiltonian? This theory indeed exists, it is an AdS theory with a specific local matter content computable from the CQFT matter content. The answer is unique, but as a result of the different dimensionality one cannot describe this one-to-one relation between spacetime indexed matter contents in terms of pointlike fields. This will be treated in section 3, where we will also compare the content of Rehren’s isomorphism \[\text{\cite{6}}\] with the Maldacena, Witten at al. \[\text{\cite{1,2,3}}\] conjectures and notice some subtle but potentially serious differences in case one interpretes the conjecture (as it was done in most of the subsequent literature) as a relation between two Lagrangian theories. Who is aware of the fact that subtle differences often have been the enigmatic motor of progress, will not dismiss such observations.

The last section presents some general results of AQFT on degrees-of-freedom-counting and holography. Closely connected is the idea of “chiral scanning” i.e. the encoding of the full content of a higher dimensional (massive) field QFT into a finite number of copies of one chiral theory in a carefully selected relative position within a common Hilbert space. In this case the prize one has to pay for this more generic holography (light-front holography) is that some of the geometrically acting spacetime symmetry transformations become “fuzzy” in the holographic projection and some of the geometrically acting symmetries on the holographic image are not represented by diffeomorphisms if pulled back into the original QFT.

## 2 Conformal Compactification and AdS

The simplest type of conformal QFT is obtained by realizing that zero mass Wigner representation of the Poincaré group with positive energy (and discrete helicity) and allow for a natural extension to the conformal symmetry group SO(4,2)/Z\textsubscript{2} without any enlargement of the Hilbert space. Besides scale transformations, this larger symmetry also incorporates the fractional transforma-
tions (proper conformal transformations)

\[ x' = \frac{x - bx^2}{1 - 2bx + b^2x^2} \quad (1) \]

It is often convenient to view this formula as the action of the translation group \( T(b) \) conjugated with a (hyperbolic) inversion \( I \)

\[ I : x \rightarrow \frac{-x}{x^2} \quad (2) \]

\[ x' = IT(b)Ix \quad (3) \]

\( I \) does not belong to the above conformal group, although it is unitarily represented (and hence a Wigner symmetry) in these special Wigner representations. For fixed \( x \) and small \( b \) the formula (1) is well defined, but globally it mixes finite spacetime points with infinity and hence requires a more precise definition in particular in view of the positivity energy-momentum spectral properties in its action on quantum fields. Hence as preparatory step for the adequate formulation of quantum field theory concepts, one has to achieve a geometric compactification. This starts most conveniently from a linear representation of the conformal group \( SO(d, 2) \) in \( d+2 \)-dimensional auxiliary space \( \mathbb{R}^{(d, 2)} \) (i.e. without field theoretic significance) with two negative (time-like) signatures

\[ G = \begin{pmatrix} g_{\mu\nu} & -1 \\ +1 \end{pmatrix} \quad (4) \]

and restricts this representation to the \((d+1)\)-dimensional forward light cone

\[ LC^{(d, 2)} = \{ \xi = (\xi, \xi_1, \xi_3); \xi^2 + \xi_d^2 - \xi_{d+1}^2 = 0 \} \quad (5) \]

where \( \xi^2 = \xi_0^2 - \xi'^2 \) denotes the d-dimensional Minkowski length square. The compactified Minkowski space \( \bar{M}_d \) is obtained by adopting a projective point of view (stereographic projection)

\[ \bar{M}_d = \left\{ x = \frac{\xi}{\xi_d + \xi_{d+1}}; \xi \in LC^{(d, 2)} \right\} \quad (6) \]

It is then easy to verify that the linear transformation, which keep the last two components invariant consist of the Lorentz group and those transformations which only transform the last two coordinates, yield the scaling formula

\[ \xi_d \pm \xi_{d+1} \rightarrow e^{\pm s}(\xi_d \pm \xi_{d+1}) \quad (7) \]

leading to \( x \rightarrow \lambda x, \lambda = e^s \). The remaining transformations, namely the translations and the fractional proper conformal transformations, are obtained by composing rotations in the \( \xi_i-\xi_d \) and boosts in the \( \xi_i-\xi_{d+1} \) planes.
A convenient description of Minkowski spacetime $M$ in terms of this $d + 2$ dimensional auxiliary formalism is obtained in terms of a “conformal time” $\tau$

$$M_d = (\sin \tau, e, \cos \tau), \ e \in S^{d-1}$$

(8)

$$t = \frac{\sin \tau}{\sqrt{e^d + \cos \tau}}, \ \vec{x} = \frac{\vec{e}}{\sqrt{e^d + \cos \tau}}$$

(9)

$$e^d + \cos \tau > 0, \ -\pi < \tau < +\pi$$

so that the Minkowski spacetime is a piece of the $d$-dimensional wall of a cylinder in $d+1$ dimensional spacetime which becomes tiled with the closure of infinitely many Minkowski worlds. If one cuts the wall on the backside appropriately, this carved out piece representing $d$-dimensional compactified Minkowski spacetime has the form of a $d$-dimensional double cone symmetrically around $\tau = 0, e = (0, e^d = 1)$ without its boundary. The above directional compactification leads to an identification of boundary points at “infinity” and give e.g. for $d=1+1$ the compactified manifold the topology of a torus. The points which have been added at the infinity to $M$ namely $\bar{M} \setminus M$ are best described in terms of the $d-1$ dimensional submanifold of points which are lightlike with respect to the past infinity apex at $m_{-\infty} = (0, 0, 0, 1, \tau = -\pi)$. The cylinder walls form the universal covering $\tilde{M_d} = S^{d-1} \times \mathbb{R}$ which is “tiled” in both $\tau$-directions by infinitely many Minkowski spacetimes (“heavens and hells”) \([12]\). If the only interest would be the description of the compactification $\bar{M}$, then one may as well stay with the original $x$-coordinates and write the $d+2$ $\xi$-coordinates follow Dirac and Weyl as

$$\xi^\mu = x^\mu, \ \mu = 0, 1, 2, 3$$

(10)

$$\xi^4 = \frac{1}{2}(1 + x^2)$$

$$\xi^5 = \frac{1}{2}(1 - x^2)$$

i.e. $(\xi - \xi')^2 = (x - x')^2$

Since $\xi$ is only defined up to a scale factor, we conclude that lightlike differences retain an objective meaning in $\bar{M}$ even though the space- and time-like separation does loose its meaning. An example of a physical theory on $\bar{M}$ are free photons. The impossibility of a distinction between space- and time-like finds its mathematical formulation in the Huygens principle which says that the lightlike separation is the only one where the physical fields do not commute and hence where an interaction can happen. In the terminology of local quantum physics this means that the commutant of an observable algebra localized in a double cone consists apparently of a (Einstein causal) connected spacelike- as well as two disconnected (Huygens causality) timelike- pieces. But taking

\(2\)The graphical representations are apart from the compactification (which involves identifications between past and future points at time/light-infinity) the famous Penrose pictures of $M$.\]
the compactification into consideration one realizes that all three parts are connected and the space/time-like distinction is meaningless on $\bar{M}$. In terms of Wightman correlation functions this is equivalent to the rationality of the analytically continued Wightman functions of observable fields which includes an analytic extension into timelike Jost points \[37,18\]. Therefore in order to make contact with particle physics aspects, the use of either the covering $\tilde{M}$ or of more general fields (see next section) on $\bar{M}$ is very important since only in this way one can implement the pivotal property of causality together with the associated localization concepts. As first observed by I. Segal \[11\] and later elaborated and brought into the by now standard form in field theory by Lüscher and Mack \[12\], a global form of causality can be based on the sign of the invariant

$$\left(\xi(e, \tau) - \xi(e', \tau')\right)^2 \geq 0,$$

hence

$$|\tau - \tau'| \geq 2 \left| Arcsin \left(\frac{e - e'}{4}\right)\right|^\frac{1}{2} = |Arccos (e \cdot e')|$$

where the $<$ inequality characterizes global spacelike distances and $>$ corresponds to positive and negative global timelike separations. Whereas the globally spacelike region of a point is compact, the timelike region is not. The concept of global causality solves the so called Einstein causality paradox of CQFT \[13\]. In the next section we will meet a global decomposition method which also avoids this paradox without the necessity of using covering space.

The central theme, namely the connection with QFT on AdS enters this section naturally if one asks the question whether one can instead of the surface of the forward light cone alternatively use a mass hyperboloid $H_{d+1}$ inside the forward light cone of the same ambient $d+2$ dimensional space

$$H_{d+1} = \{ \eta; \eta^2 = 1 \}$$

\[\eta^0 = \sqrt{1 + r^2 \sin \tau}\]

$$\eta^i = re^i, \; i = 1,...d$$

$$\eta^{d+1} = \sqrt{1 + r^2 \cos \tau}$$

This space which because of its formal relation to the analogous deSitter spacetime (which is defined by the spacelike hyperboloid) is called “Anti deSitter” spacetime is noncompact. It is obvious from its construction that its asymptotic part is the same as $\bar{M}_d$. It was conjectured by Maldacena and others \[1,2,3\] that there is also a correspondence between quantum field theories. This conjecture implies the tacit assumption (not explicitly stated in these papers) that an $AdS_{d+1}$ QFT which coalesces asymptotically\[ with an CQFT_d theory has a unique extension into the AdS bulk. Since there can be no mapping between

\[3\] Using the previous cylindric representation of the conformal covering, the covering of AdS corresponds to the full cylinder of which its mantel is the conformal covering.
pointlike fields on spacetimes of different dimensions the question of the origin of this unique extension is non-trivial. The conjecture came from some speculations concerning possible relations of string theory with some supersymmetric gauge theories (SYM) i.e. from ideas far removed from the present particle physics setting which will not be explained here.

In the 70s, at the time of the conformal compactifications, free fields on $AdS_4$ were studied from a particle physics viewpoint by Fronsdal \[4\]. The correspondence to $CQFT_3$ was overlooked; probably because of the fact that despite the obvious group theoretical connection through the common $SO(3, 2)$, the multiplicities of the discrete AdS free Hamiltonian turned out too big for matching those of the rotational conformal Hamiltonian; a fact which will find its explanation in the next section.

Although the two spacetime cannot be mapped into each other, their shared spacetime symmetry group $SO(4, 2)$ suggests that there is at least a correspondence between certain subsets which may be obtained from projecting down wedge regions from the ambient space onto the two spacetime manifolds. Wedges have a natural relation to $SO(4, 2)$; they may all be generated from standard wedge in the ambient auxiliary space $W_{st} = \{ \xi^1 > |\xi^0| \}$. The fixed point group of this transitive action on wedges consists of a boost and transversal translations and rotations\[4\]. The projected wedges $pW$ on AdS are by definition again wedges in $AdS/CQFT$ and the $SO(d, 2)$ symmetry group has the same transitive action i.e the system of wedges is described by $SO(d, 2)$ modulo the fixed point subgroup. This geometric situation clearly suggests that one should consider algebras associated with these wedges instead of looking for a relation between pointlike fields. On the conformal side this includes all double cone algebras of arbitrary small size since the noncompact wedge regions are conformally equivalent to compact double cones regions. The logic of algebraic QFT requires to continue this algebraic correspondence to all intersections obtained from wedges. In this way one expects to arrive at an isomorphism which carries the full content of both theories and which includes the asymptotic relation (on the conformal surface of the aforementioned cylinder) in terms of field coordinatizations used by Maldacena et al. In order to obtain a rigorous proof, one must check some consistency conditions in the conversion of maps between spacetime regions and algebras indexed by those regions. This was achieved by Rehren \[6\] and will be briefly comment on his theorem (including its relation to the original conjecture) in the next section.

According to our previous remarks, interacting conformal local fields live on the covering space $\tilde{M}$. Fortunately the geometric isomorphism between wedge regions can be lifted to an $\tilde{AdS}_{d+1} \sim \tilde{M}$ correspondence. The conformal decomposition theory of the next section avoids the use of the rather complicated coverings by using an operator analog of fibre bundles on $\tilde{M}$.

\[\text{If one adds the two longitudinal lightlike translations which in one direction cause a compression into the wedge, one obtains a 8-dimensional Galilei group.}\]
3 The conformal Hamiltonian as the true Hamiltonian

There is another less geometric, but more particle physics type of argument, which leads to the AdS-CQFT correspondence.

For this one should recall that in $SO(d, 2)$ there are besides the usual translations with infinity as a fixed point also “conformal translations” which act on the compactified $\bar{M}$ without fixed points as some kind of “timelike rotations”. They are the analogs of the light-like chiral rotation $R^{(\pm)}$ ($P^{(\pm)}_0$ in standard Virasoro algebra notation) and their connection with the light ray translation $P^{(\pm)}$ with which they share the positivity of their spectrum is

$$
R^{(\pm)} = P^{(\pm)} + K^{(\pm)}
$$

$$
K^{(\pm)} = I^{(\pm)} P^{(\pm)} I^{(\pm)}
$$

where $I^\pm$ is the representer of the chiral conformal reflection $x \rightarrow -\frac{1}{2} x$ (in linear lightray coordinates $x$) and $K$ is the generator of the fractional special conformal transformation (1). For free zero mass fields the discrete $R$-spectrum can be understood in terms of that of a Hamiltonian for a massless model in a spatial box. This is however not possible for the $R$-spectrum of chiral theories with anomalous scale dimension (the R-spectrum is known to be identical to the that of scale dimensions). In that case the only theory for which the spectrum is that of its Hamiltonian is the QFT on $AdS_2$. So if one wants to read the $SL(2, \mathbb{Z})$ modular characters of chiral conformal field theory in the spirit of a Hamiltonian Gibbs formula one should use the AdS side. An analogous statement holds in higher dimensions where the $\bar{M}$ rotation is described in terms of a Lorentz vector $R_\mu$

$$
R_\mu = P_\mu + IP_\mu I
$$

where the inversion $I$ was defined at the beginning of the previous section. It leads to a family of operators with discrete spectrum of $e \cdot R$ which are dependent on a timelike vector $e_\mu$. Again the operator $R_0$ is the true Hamiltonian of only one theory with the same symmetry group and the same system of algebras (but with a different spacetime indexing): the associated $d+1$ dimensional $AdS$ theory.

Now it is time to quote (adapted to our purpose) Rehren’s theorem and comment on it.

**Theorem 1** The geometric bijection between projected wedges $pW$ on $AdS_{d+1}$ and the conformal double cones in $\bar{M}_d$ which constitute the asymptotic infinity of $pW$ (as described in the previous section) extends to an isomorphism of the corresponding algebras. Both theories share the same Hilbert space and the same family of operator algebras but their spacetime organization and with it their physical interpretation changes.
For the proof we refer to Rehren [6].

Some comments are in order. There is no additional restrictive assumptions (supersymmetry, vanishing $\beta$-functions) on either side. If the algebras of the AdS theory are generated by pointlike fields then the associated conformal algebra cannot be generated by a field which has an energy-momentum tensor or obeys a causal equation of motion. This is one of Rehren’s conclusions and it is very instructive to illustrate this with an example.

Consider a free scalar AdS field [7]. A simple calculation which will not be repeated here reveals that it corresponds to a conformal generalized free field with homogeneous Kallen-Lehmann spectral function. Generalized free fields always have been physically suspect and if there spectral functions increases in the manner as the homogeneous degree demands in this case, one can even prove that primitive causality [17] is violated since the algebra on a piece of time-slice (represented as a chain of small double cones which approximate the compact slice from the inside) is not equal to its causal completion (causal shadow) algebra. As one moves up in time inside the causal shadow from the time-slice more and more degrees of freedom coming from the inner parts of the bulk enter the causal shadow which were not in the time-slice. Rehren’s graphical representation [33] of the CQFT world on the wall of a full AdS cylinder makes this undesired sidewise propagation geometrically visible. This free field situation is generic in the sense that pointlike AdS fields always carry too many degrees of freedom [8] which leads to a violation of causal propagation in the aforementioned sense [11]. Such theories have to be abandoned for general physical reasons (not just because they do not fit into a Lagrangian picture which automatically implies causal propagation). Therefore the nice idea [34] to circumvent the scarcity in constructing Lagrangian conformal models (the $\beta$-function restrictions) by starting instead with AdS Lagrangians does not work, since the resulting conformal theories all share the above defect.

In passing we mention that the brane idea shares the same causality conflict with pointlike field. Whereas from a mathematical viewpoint a manifold of interest may in certain cases be considered a brane of a larger dimensional space, the assignment of a physical reality to the ambient spacetime generates causality problems of the above kind for restrictions to the brane in case of pointlike field theories in the larger ambient spacetime. Only if the ambient degrees of freedom are carefully tuned to the brane can such causality violations be avoided. Note that it is always the causal shadow property which may get lost in such constructions and not the Einstein causality. This is not visible if one restricts ones attention to (semi)classical solutions concentrated on a brane and or to euclidean formulations. Whereas the principles of AQFT confirm in a very precise way that there exists an isomorphism it is very interesting that there is a clash with certain concepts which have been used in string theory for the last two decades. This clash extends beyond the above remarks on the

---

Contrary to a widespread belief, the number of degrees of freedom of causally propagating AdS theories is always larger than that of causally propagating conformal theories so that the isomorphism cannot be one among causally propagating theories. If the AdS theory is pointlike and causally propagating, the associated conformal theory has no causal propagation.
AdS-CQFT and the brane concept and casts doubt on the consisteny of such quasiclassical pictures as the Kaluza-Klein dimensional reduction.

As a matter of fact not even the quasiclassical Klein-Kaluza reduction idea has been shown to be consistent with causal QFT. For this one would have to demonstrate that the idea works on the ready made QFT and not just on the objects involved in the formal quantization approach which is used in the tentative construction of a QFT. As far as the strict conceptual requirement of causality and Haag duality in AQFT are concerned, the K-K mechanism, to the extend that it is not just a mathematical trick (but an asymptotic property of a genuine inclusion of two local quantum physics worlds) has at best remained an enigmatic speculative idea (and at worst a tautology caused by not doing what one actually is claiming to do).

The above degree of freedom discussion creates the suspicion that “good” causal conformal theories may have too few degrees of freedom in order to yield AdS pointlike fields as the other side of the coin of the above observation that pointlike AdS fields create causally bad conformal theories. This is indeed the case and can be seen by starting from the Wigner zero mass representation space of the Poincaré group

\[ H_{Wig} = \left\{ \psi(p) \mid \left| \int |\psi(p)|^2 \frac{d^{d-1}p}{2|\vec{p}|} \right| < \infty \right\} \]

(14)

which without extension provides an irreducible representation space of \( SO(d, 2) \). The subspace of modular wedge-localized Wigner wave functions consists of boundary values of wave functions \( \psi \) which are analytic in the rapidity strip \( 0 < \text{Im}\theta < \pi \) where for the standard wedge

\[ p_x = \sqrt{p_y^2 + p_z^2 \sinh \theta}, \quad p_0 = \sqrt{p_y^2 + p_z^2 \cosh \theta} \]

(15)

This wave function space is in common regardless whether we are talking about the standard wedge on \( M_d \) or the corresponding \( AdS_{d+1} \) wedge. Whereas this space in the \( M_d \) interpretation is easily rewritten in terms of covariant x-space wave functions with the expected support properties, an analog \( \eta \)-spacetime covariantization for AdS does not work. The best one can do is to introduce (Olsen-Nielsen) string-like wave functions in \( \eta \)-spacetime which do not depend on \( w \) and which behave under \( SO(d, 2) \) transformations as objects which depend on an additional direction (the string direction). So instead of pointlike fields one obtains strand-like objects (with weaker covariance properties) which emanate from the points on the asymptotic boundary and extend through the bulk and which are linear in the same momentum space creation/annihilation operators as those which appear in the free conformal fields. This is the way in which the AdS formulation maintains the conformal degrees of freedom and the primitive causality.

At this point an ardent string theorist might say: didn’t I tell you that starting from a conformal field theory you should expect to encounter AdS strings! However the strand-like objects of the Rehren theorem [6] are perfectly within
causal localizable AQFT, and hence they are not objects of string theory proper. The main characteristics of the strings of string theory namely the enhancement of the degrees of freedom due to the internal excitation structure is missing in the case of our strands. In order to avoid misunderstandings we emphasize again that the degree of freedom issue is not related to Einstein causality which remains valid irrespective of whether the local algebras are generated by point-like fields or not, but rather to the causal propagation property which requires Einstein causality as a prerequisite, but is not guarantied by the latter.

The Maldacena et al conjecture \[1\] is that some high dimensional true (i.e. not the above kinematical strands) string theories in some effective and not precisely specified sense is equivalent with conformally invariant supersymmetric Yang-Mills (SYM) theories. To the extend in which the argument supporting this conjecture uses a correspondence between pointlike (Lagrangian) AdS\(_5\) QFT and a conformal SYM it is contradicted by the above theorem.

Antinomies and contradictions about important topics in earlier times were often the source of progress and removal of prejudices and one would hope that they continue to receive their due attention. The issue is somewhat delicate as a result that the euclidean functional integral formalism in which the original conjectures were presented is at most an heuristic starting point since Feynman-Kac representations in strictly renormalizable QFTs are simply not valid for the physical (renormalized) results. For this reason the chosen method poses an obstacle against converting the conjecture into a proof. This conceptual flaw of the functional action approach is one of the raisons d’etre of algebraic QFT which succeeds to balance the starting calculational definitions with the properties of the constructed models. One could of course try to argue exclusively in terms of differential geometric concepts by abstacting from the action formulation purely geometric definitions of what constitutes SYMs and the associated string theories. But in doing this one will lose the relation to local quantum physics and the obtaines theorem may be void of an particle physics content.

If one admits that, as argued above, the Lagrangian perturbation method applied to the AdS side of the correspondence cannot be used for the construction of additional conformal QFTs, the question arises whether there are other construction methods. A closer investigation reveals that the spectrum of anomalous dimensions of interacting conformal field theories is determined in terms of a timelike braid group structure. A convenient way of presenting this structure is to work with nonlocal component fields \[14\] which result from the decomposition of the charge carrying globally local fields \(F\) on \(\tilde{M}\) under the reduction of the center of the conformal covering group \((S(D,2))\)

\[F(x) = \sum_{\alpha,\beta} F_{\alpha,\beta}(x), \quad F_{\alpha,\beta}(x) \equiv P_\alpha F(x) P_\beta\]

\[Z = \sum_\alpha e^{2\pi i \theta_\alpha} P_\alpha\]

in terms of projectors \(P_\alpha\) which appear in the spectral decomposition of the generator \(Z\) of the center \((\tilde{S(D,2)}) = \{Z^n, n \in \mathbb{Z}\} \). In a way the existence of this
decomposition facilitates the use of the standard parametrization of Minkowski space augmented by the quasiperiodic central transformation

\[ ZF_{\alpha,\beta}(x)Z^* = e^{2\pi i(\theta_\alpha - \theta_\beta)}F_{\alpha,\beta}(x) \]  
(17)

and hence one may to a large part avoid the use of the complicated covering parametrization and its \( SO(D,2) \) transformations which the unprojected fields \( F \) would require. For the latter fields on \( \bar{M} \) the notation would be insufficient; one also has to give an equivalence class of path (the number \( n \) of the heaven/hell one is in) with respect to our copy of \( M \) embedded in \( \bar{M} \). The projected fields on the other hand are analogous to sections in a trivialized vectorbundle. With the help of conformal 3-point functions one shows that the \( \theta \)-phases are related to the anomalous dimensions. The component fields \( F_{\alpha,\beta}(x) \) are the suitable objects for the formulation of the timelike braid group commutation relations which take the form of an exchange algebra

\[ F_{\alpha,\beta}(x)G_{\beta,\gamma}(y) = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'} G_{\alpha,\beta'}(y)F_{\beta',\gamma}(x), \quad x > y \]  
(18)

where the R-matrices are determined from admissible braid group representations. For more on the timelike braid group structure in higher dimensional conformal QFT we refer to [18].

Since the AdS-CQFT isomorphism implies a radical reprocessing on the physical side, it would be interesting to perform the timelike commutation relation analysis directly within the AdS setting. This has not been done yet.

### 4 Generalized Holography in Local Quantum Physics

The message we can learn from the AdS-conformal correspondence is two-fold. On the one hand there is the recognition that there are situations where it is necessary to avoid the use of “field coordinates” in favor of directly working with local algebras. In most concrete situations there were always convenient field coordinatizations available in terms of which the calculations simplified. For the AdS-conformal correspondence is however a new type of problem for which the best way is to stay intrinsic, i.e. to use the net of algebras.

The second message is that there may exist a holographic relation between QFT’s and their lower dimensional boundaries. We have argued that the degrees of freedom of \( AdS_{d+1} \) are the same as in the corresponding \( CQFT_d \) on the boundary even though the Hamiltonians and the associated thermal aspects are different. This is the only known case of a bijection of nets of algebras

\[ ^6 \text{Contrary to a widespread belief, the number of degrees of freedom of causally propagating pointlike AdS}_{d+1} \text{ theories is always larger than that of a causally propagating conformal theories CQFT}_d \text{ so that the isomorphism cannot be one among causally propagating pointlike theories i.e. if the AdS theory is pointlike and causally propagating, the associated conformal theory has no causal propagation and hence has to be discarded as unphysical.} \]
associated with spacetimes of different dimensions but with the same maximal spacetime diffeomorphisms group.

Another more frequent kind of holography occurs for spacetimes with a causal horizon. In that case certain spacetime diffeomorphisms of the original spacetime act in a “fuzzy” nongeometric manner, thus accounting for the fact that the diffeomorphism group of the horizon is smaller. Let us consider a simple example: the holographic image of a two-dimensional massive theory in the vacuum representation restricted to the standard wedge i.e. a Rindler-Unruh situation. We want to restrict the restrict the $d=1+1$ wedge algebra $A(W)$ to its upper half-line horizon $\mathbb{R}_+$. In a massive theory we expect that both operator algebras are globally identical

$$A(W) = A(\mathbb{R}_+)$$  \hspace{1cm} (19)

although their local net structure is quite different. Classically this corresponds to the fact that characteristic data on either of the two horizons determine uniquely the function in the wedge.\footnote{Since our approach tries to relate the holographic aspects via modular localization ideas to the old principles of particle physics, we do not have to invoke a new “holographic principle”.} It is very important to control the data on the entire upper horizon $\mathbb{R}_+$; in contradistinction to a spacelike interval compact intervals on $\mathbb{R}_+$ do not cast two-dimensional causal shadows. The physical reason is of course that each point in a small neighborhood below that interval is in the backward influence cone of some points on $\mathbb{R}_+$ which are far removed to the right outside that interval. Only if we take all of $\mathbb{R}_+$, we will have $W$ as its two-dimensional causal shadow.

In the general approach to QFT the von Neumann algebra of a compact spacetime region is, according to the causal shadow property of AQFT (which is a local version of the time-slice property mentioned in the previous section), identical to the algebra of its causally completed region. Each field theory with a causal propagation (in particular Lagrangian field theory) fulfills this requirement. If one takes a sequence of spacelike intervals which approximate a lightlike interval, the causal shadow region becomes gradually smaller and approaches an interval on the light ray in the limit. The only way to counteract this shrinking is to extend the spacelike interval gradually to the right in such a way that the larger lower causal shadow part becomes the full wedge in the limit.

The correctness of this intuitive idea which suggests the correctness of (19) can be checked against other rigorous results. One rigorous result from Wigner representation theory (which therefore is limited to free field theories) together with the application of the Weyl- or CAR- (for halfinteger spin) functor is the statement that the cyclicity spaces for an interval $I$ on $\mathbb{R}_+$ agree with the total space $\mathbb{H}$

$$\mathcal{A}(I)\Omega = \mathcal{A}(\mathbb{R}_+)\Omega = \mathcal{A}(W)\Omega = H$$  \hspace{1cm} (20)

\footnote{This is true in any dimension. The only exception is $d=1+1$, mass=0 in which case both horizons are needed to specify the two chiral components of conformal theories.}
i.e. the validity of the Reeh-Schlieder theorem on the light ray subalgebra. In fact this holds for all positive energy representations including zero mass, except zero mass in d=1+1 in which case the decomposition in two chiral factors prevents its validity. Therefore one only needs to proof the spatial statement $\mathcal{A}(\mathbb{R}+)\Omega = \mathcal{A}(W)\Omega = H$ in order to derive (19). But this spatial completeness follows from the causal shadow property for spacelike half-lines $L$ starting at the origin since the space $\mathcal{A}(L)\Omega = H$ and this completeness property cannot get lost in the light ray limit $L \to \mathbb{R}+$. The step from spaces to (19) is done with the help of Takesaki’s theorem (mentioned later).

We still have to rigorously define the holographic algebra $\mathcal{A}(\mathbb{R}+)$ (which turns out to be chiral conformal) and its net structure $\mathcal{A}(I)$ from $\mathcal{A}(W)$. This is done by the modular inclusion technique which is one of AQFT most recent mathematical achievements [20][19].

The modular way of associating a chiral conformal theory with e.g. a $d=1+1$ massive theory is the following. Start from the right wedge algebra $\mathcal{A}(W)$ with apex at the origin and let an upper lightlike translation $a^+$ (which fulfills the energy positivity!) act on $\mathcal{A}(W)$ and produce an inclusion (all the algebras are von Neumann algebras)

$$\mathcal{A}(W_{a^+}) \subset \mathcal{A}(W)$$

(21)

This inclusion is halfsided “modular”, i.e. the modular group $\Delta^t$ of $(\mathcal{A}(W), \Omega)$ which is the Lorentz boost acts on $\mathcal{A}(W_{a^+})$ for $t < 0$ as a “compression”

$$Ad\Delta^t \mathcal{A}(W_{a^+}) \subset \mathcal{A}(W_{a^+}), \ t < 0$$

(22)

The assumed nontriviality of the net i.e. the intersections of wedge algebras entails that the relative commutant (primes on algebras denote their commutant in $B(H)$)

$$\mathcal{A}(W_{a^+})' \cap \mathcal{A}(W)$$

(23)

is also nontrivial. Such inclusions are called “standard”. It is known that standard modular inclusions correspond to chiral conformal theories, i.e. the classification problem for the latter is identical to the classification of all standard modular inclusions. In the case at hand the emergence of the chiral theory is intuitively clear since the only “living space” in agreement with Einstein causality (within the closure of $W$ and spacelike with respect to the open $W_{a^+}$) which one can attribute to the relative commutant is the lightray interval of length $a_+$ starting at the origin. From the abstract modular inclusion setting the Hilbert space which the relative commutant generates from the vacuum

---

For presentations of the Tomita-Takesaki modular theory which are close to the present concepts and notations see [10]. A more extensive presentation which pays due attention to the importance of modular theory for the new conceptual setting of QFT is that by Borchers [36].

The nontriviality of the intersections is in some sense the algebraic counterpart of the renormalizable short distance behaviour in a quantization approach which is believed to be required by the mathematical existence of the Lagrangian theory.
could be a subspace \( H_+ \subset H, \ H_+ = PH \) of the original one, but the already mentioned causal shadow property assures that \( H_+ = H \), i.e. \( P = 1 \). With the help of the L-boost (=modular group \( \Delta^i \) of \((A(W), \Omega)\)) one then defines a net on the halfline \( \mathbb{R}_+ \) and a global algebra

\[
A(\mathbb{R}_+) = \text{alg} \left\{ \bigcup_{t<0} \text{Ad} \Delta^i (A(W_a)^t \cap A(W)) \right\} \subset A(W)
\]

The modular group \( \Delta^i \) of the original algebra leaves this lightray algebra invariant and hence we are in the situation of the Takesaki theorem [19] which states that a subalgebra together with the vacuum which is left invariant by the modular group of the larger algebra, has modular objects which are restrictions of those of \((A(W), \Omega)\) and the algebras coalesce iff \( A(\mathbb{R}_+) \Omega = H \). The identity (19) means that the original modular inclusion was standard and hence the theory on the light ray is conformal.

The identity of this conformal theory with the massive wedge algebra also shows that identification of chiral conformal theories with zero mass is a prejudice. Whether chiral theories are describing massless or massive situations depends on the identification of the mass operator. In the present case there exists another second lightlike translation along the lower horizon and the mass operator is given by the product \( P_+ P_- \). Even though the spectrum of each \( P_\pm \) is gapless, as required by conformal invariance, there is a mass gap in the physical mass operator. Since the lightray algebra is identical to the wedge algebra, the lower \( a_- \) lightray translation also acts on it; but not as a diffeomorphism but rather in a fuzzy [21] i.e. totally nonlocal way relative to the local Moebius group action coming from the geometry of the upper lightray. So in the lightray representation of the wedge algebra only the Lorentz boost (which becomes a scale transformation on \( \mathbb{R}_+ \)) and the upper lightray translation are shared as local diffeomorphism operations in both representations. The lower lightray translation is nonlocal on \( A(\mathbb{R}_+) \) and the Moebius rotation (after compactification of \( \mathbb{R}_+ \)) is newly created and acts only partially geometrically on \( W \).

It is one of the characteristic features of this generalized holography that in addition to the local and nonlocal encoding of diffeomorphisms of the original theory into its lower dimensional holographic image, there are also partially geometric symmetries as the Moebius rotations transferred back from the image into the original theory. The degrees of freedom of the chiral conformal \( A(\mathbb{R}_+) \) are in some intuitive sense “more” than in a standard chiral conformal theory associated with a chiral energy momentum tensor, because such a standard model would algebraically be too small in order to carry an additional fuzzy \( a_- \) lightray translation.

Still another related idea about relations of QFTs on different but this time equal dimensional spacetimes which uses modular techniques in an essential way has recently appeared under the name “Transplantation of Local Nets” [38].

The present modular inclusion approach to “lightray physics”, including the localization and degrees of freedom aspect is another illustration of the conceptual power of the field coordinate free approach and the modular inclusion.
method. In the standard setting there are several fake as well as genuine (requiring a change of field coordinates) problems with light cone restrictions and quantizations. Standard approaches are usually entirely formal; they tend to overlook localization problems whose understanding is vital for the physical interpretation of the formalism and furthermore often use field coordinatizations which become singular on the lightray.

These problems continue in higher dimensions where the wedge horizons are lightfronts. A typical case which requires new concepts is $d=1+2$. In that case the modular method, applied to one wedge, only transfers a small fraction of the geometric structure of the original theory into a chiral conformal theory obtained by modular inclusion, which localization-wise should really be associated with the upper light front horizon of the wedge. The lightfront quantization (or “infinite momentum frame” method) with respect to one lightfront only cannot account for the full locality informations. Since its transversal localization remains completely unresolved, the so obtained theory only contains the longitudinal localization data of a chiral conformal net.

Let me explain the way to get a transversal resolution. In that case one tilts the wedge by a $L$-boost which leaves the upper defining light ray for the initial wedge invariant \[ [9] \[10]. One then convinces oneself that this newly positioned second wedge has a modular associated chiral conformal theory which, though being unitarily equivalent to the first one, carries the missing information (which is needed for the reconstruction of the original $d=1+2$ theory) in form of its relative position in the common Hilbert space $H$. The tilted wedge together with the original one can be used to give a net structure to the original wedge in the transversal direction. Again the holographic projection of the original net into the horizon has besides geometric actions also fuzzy and partially local actions.

But instead using the transversal resolution of the 2-dim. horizon for a constructive approach based on the modular inclusion and intersection method, it would be somewhat more natural to describe the original theory in terms of the two chiral theories which the modular inclusion associates with the original wedge and the tilted wedge. In the general $d$-dimensional case one would encode the original theory in terms of $d-1$ copies of one and the same chiral theory in different positions within one Hilbert space. The name “chiral scanning” would hence be more appropriate than holography for such a procedure.

Adding nice names to structural relations is of course by itself not very constructive. The hope is that by more profound future studies one may develop criteria which allow a more universal intrinsic algebraic characterization of those relative positions and chiral theories which allow to construct a $d$-dimensional QFT. Chiral theories are the simplest and best understood QFTs and the study of $d-1$ copies of them seems to be simpler than to confront higher dimensional field theories directly.

In fact 't Hooft’s original holography \[23\] proposal and Susskind’s \[22\] subsequent work appear much more related to the light front encoding and/or the related scanning than to the AdS holography with its high geometric symmetry restriction. The present use of modular inclusions may be seen as an attempt
to find a firmer conceptual and mathematical basis for those ideas.

The importance of causal horizons in the above considerations suggests to look for a “localization entropy” of causally localized matter as a first step towards a quantum explanation of the universal Bekenstein area law in black hole physics. But there is a hurdle right at the start: unlike QM where a quantization box defines an inside/outside division of the Hilbert space and the quantum mechanical algebra (type $I_\infty$ von Neumann factor) through a tensor product factorization, the nature of the double cone algebras (the relativistic causally closed analogs of boxes) in QFT is totally different, since as hyperfinite type $III_1$ von Neumann factors they contain neither minimal projectors nor are there any pure states among its normal states [36]. This unusual state of affairs requires the introduction of the “split property” in order to construct the relativistic analogue of the QM box [16]. The physical mechanism behind this property is the strong vacuum fluctuations of partial charges at the surface of its localization volume $V$, one of the oldest and most characteristic phenomena which set apart QFT from QM.

Let us first try to understand this phenomenon in a mathematically refined formulation of its original discovery by Heisenberg. Using a smooth spacetime smearing function consisting of a spatial part $g_{R,\delta}(x)$ with thickness $\delta$ and localization radius $R$ multiplied by a compact support time-smearing $f$ in the definition of the partial charge

$$Q_R = \int j_0(x)f(x_0)g_{R,\delta}(x)d^nx$$

$$g_{R,\delta}(x) = \begin{cases} 
1, & |x| < R \\
0, & |x| > R + \delta 
\end{cases}$$

$$f(x_0) \geq 0, \quad \int f(x_0)d\,x_0 = 1$$

one finds that the square norm of the partial charge applied to the vacuum $\langle Q_RQ_R \rangle$ diverges with $\delta \to 0$ and increases for fixed $\delta$ in the limit $R \to \infty$ as $R^{d-2}$ where $d$ is the spacetime dimension [24].

But what, if any, could be the message of this area law with that of the would be localization entropy? We first have to understand the algebraic analogue of the surface vacuum fluctuation of the partial charges. This turns out to be the split property i.e. the necessity to work with fuzzy space time boxes in the form of double cones with a “collar” region of thickness $\delta$ separating the inside of the smaller box of radius $R$ from the outside of the bigger with radius $R+\delta$. In this split situation we do recover the quantum mechanical inside/outside tensor factorization which refers to a fuzzy box algebra $\mathcal{N}$ which extends beyond the smaller box into the collar without sharp geometric boundaries [16]. This sets the stage for defining von Neumann entropy which needs the type I tensor-factorization of boxes in QM.

There remains however another important difference to Schrödinger quantum mechanics in that the vacuum state remains entangled i.e. does not split
into an inside/outside part but rather remains a highly correlated state with the Hawking-Unruh temperature. This has paradigmatic consequences for the conceptual framework of the measurement process in local quantum physics [25]. It is also the origin of the localization entropy which we have been looking for. One can show that the vacuum state restricted to the fuzzy QFT box leads to a nontrivial entropy which diverges with \( \delta \to 0 \) and increases with the size \( R \) of the box in agreement with the above analogy which intuitively pictures the box entropy of the vacuum as being related to a partial “Hamiltonian charge” via a Gibbs formula in the above sense. As in that case one also would expect the validity of an entropical area law at least for large ratios of the diameter divided by the collar size and that the matter dependence would show up, if not in the coefficient of the area law itself, at least in its correction terms. The “Hamiltonian charge” which we intuitively relate with a Gibbs formula is not expected to be associated to a geometrical symmetry but rather to one of the infinitely many modular-generated fuzzy/hidden symmetries which any QFT possesses. In particular we find the use of the conformal rotational Hamiltonian which appeared in the recent literature [26] physically ad hoc, especially in view of the fact that Bekenstein’s area law does not require conformal invariance. Even if in very special conformal situations its spectrum happens to be similar to that of the logarithm of the modular operator of the splitting box algebra with respect to the vacuum and the resulting entropy complies with the Bekenstein area law, such an enigmatic observation will be helpful only if it leads to a general physical concept; by itself it cannot be a substitute for a deeper conceptual understanding. The Minkowski analog of black hole thermodynamics/statistical mechanics in our view is more the understanding of thermal aspects resulting from (modular) localization rather than the application of the heat-bath Gibbs formalism.

Various intuitively equivalent forms of localization entropy related to the split inclusion situation were introduced via the concept of relative entropy for a pair of states in the work of H. Narnhofer [27]. The most manageable version for the purpose of extracting a possible area law which refers directly to the states seems to be the relative entropy of the vacuum relative to the “split vacuum” on the restricted tensor product algebra \( \mathcal{A} \otimes \mathcal{B}' \), \( \mathcal{A} \subset \mathcal{N} \), \( \mathcal{B}' \subset \mathcal{N}' \) where \( \mathcal{A} \) is the smaller double cone algebra and \( \mathcal{B}' \) the commutant of the bigger one. There exists [28] a nice variational formula in terms of states only for such relative entropy of a von Neumann algebra \( \mathcal{M} \) between two states \( \omega_j, j = 1, 2 \)

\[
S(\omega_1 | \omega_2)_\mathcal{M} = - \langle \log \Delta_{\omega_1, \omega_2} \rangle_{\omega_2} \\
= \sup \int_0^1 \left[ \frac{\omega(1)}{1 + t} - \omega_1(y^*(t)y(t)) - \frac{1}{t} \omega_2(x^*(t)x(t)) \frac{dt}{t} \right] \\
x(t) = 1 - y(t), \ x(t) \in \mathcal{M}
\]

Here \( \Delta_{\omega_1, \omega_2} \) is the relative modular operator and for the case at hand we have to identify \( \omega_1 = \Omega, \omega_2 = \Omega \otimes \Omega \) (the split vacuum) and \( \mathcal{M} = \mathcal{A} \otimes \mathcal{B}' \). Using some previous nuclearity estimates of Buchholz and Wichmann [16], Narnhofer carried out a rough estimate for this entropy and found that it increases less
than the volume of the relativistic box of size $R$. In the present setting her result may be interpreted as a first indication in favor of a Bekenstein area law for localized quantum matter. I order to obtain more structural inside into this fundamental and universal phenomenon I started to investigate this problem in the mathematically more controllable situation of two double cones separated by a collar in conformal theories. By conformal invariance the large $R$ behaviour becomes coupled to the short distance behavior in the limit of vanishing collar size $\delta \to 0$. One expects to have an easier conceptual grasp on this ultraviolet behavior as a consequence of the fact that it reflects truely intrinsic properties of the local algebras and has nothing to do with short distance divergencies of particular field coordinates. Besides, conformal theories from an analytic viewpoint are the simplest theories after free fields. There are as yet no sufficiently concrete results worthwhile to be reported here.

To avoid misunderstanding, I am not saying that the area law for black holes is a simple consequence of the area law for localized quantum matter. It would be a pity if it would, because then not much would be revealed by black hole physics about the still extremely speculative issue of quantum gravity. Rather I believe that it is the seemingly very nontrivial conversion of localization entropy of local quantum physics into the more geometric Killing horizon entropy of black holes which will be the crucial step.

5 Epilogue

The present analysis of the AdS-CQFT correspondence has its roots in the LSZ setting of particle physics from which the conformally invariant QFT should result in the zero mass limit. The step from the traditional use of pointlike (Lagrangian) fields to operator algebras indexed by spacetime regions has been taken a long time ago with the intention to obtain a more profound understanding of the observed insensitivity of the S-matrix obtained as the asymptotic limit in the setting of the Lehmann Symmanzik-Zimmermann formalism to changes of field coordinates (“interpolating fields”) within the (Borchers) equivalence class. This led to a more intrinsic formulation of QFT called algebraic QFT (AQFT) which relegates the role of fields to a coordinatization of local algebras in terms of selection of particular generators. If one wants to use field-coordinatizations altogether, as was needed in Rehren’s proof of the AdS-CQFT isomorphism, it is appropriate to avoid the word “field” and talk about Local Quantum Physics. As the step from differential geometry with coordinates to the modern intrinsic coordinate-free formulation did not represent a change of the geometrical content, one does not change physical principles (but only some concepts for their implementation) by passing from QFT to LQP. Since certain problems, as e.g. the abominable short-distance problem in the pointlike formu-

\[\text{[1]}\text{The role of the double cone restricted vacuum in the black hole situation is played by the Hartle-Hawking state restricted to the outside of the black hole [29].} \]
lation (~coordinate singularities? of which field coordinate?) which always seemed to threaten the existence of Lagrangian QFT through its long journey through renormalization theory, become deemphasized in favor of apparently different aspects (ultraviolet divergencies → nontriviality of certain intersection of algebras) in the new formulation, this reprocessing of concepts represents a very healthy change.

The conjecture about the AdS-CQFT correspondence comes from string theory. Although string theory has been the dominant way of thinking in particle physics publications for at least two decades, its main achievements seems to be that (with some training and coaching) it allows theoretical physicists to make contributions to mathematics. Its historical origin in the dual S-matrix model of Veneziano was very close to the framework of LSZ scattering theory; in fact it started as a proposal for a nonperturbative crossing symmetric S-Matrix which fulfilled a very strong form (not suggested by QFT) of crossing called duality (saturation of crossing on the level of reggeized one-particle states). This forced the S-matrix to live in a high-dimensional spacetime of at least 10 dimensions (by invoking another invented structure: supersymmetry).

The next step in the LSZ logic would have been to ask for the understanding of this high dimensional QFT (i.e. the unique equivalence class of fields or the local net of algebras) which has this S-matrix as a bona fide physical S-matrix i.e. as a large time LSZ limit. Unfortunately this never happened; instead the off-shell transition was performed at a completely different purely technical place. It was based on the auxiliary observation that the particle towers which appeared in the lowest order (or lowest genus of Riemann surfaces which is the analogue of Schwinger’s auxiliary eigentime in QFT) can be reproduced by the mass spectrum of a string. In the original strong interaction representation of the model this tower was thought to lead to resonances (poles in the second Riemann sheet) resulting from higher order interactions destabilizing the higher-lying particles in the tower. It was this step (which occurred even before the decree of the use of string theory as a quantum theory of gravity) which is responsible for the lost (and never recovered since) relation to causality and localization which are the cornerstones of QFT. Whereas in earlier times quantum field theorist have thought (without success) about nonlocal alternatives in the form of an elementary length or a cutoff, recent developments in algebraic QFT have made abundantly clear that Einstein causality and its strengthened form Haag duality is inexorably linked with the mathematics of the Tomita-Takesaki modular theory. This is an extremely deep theory which is able to convert abstract domain properties of operators and subspaces obtained by applying algebras of local quantum physics to distinguished state vectors into concrete spacetime localization geometry (without the necessity to impose any additional structure from the mathematics of noncommutative geometry).

\[12\] There are also intrinsic ultraviolet aspects of the local algebras. For example if one uses the “split property” for the definition of the vacuum entropy of a local algebra with a “collar” for controlling the vacuum fluctuations near the causal horizon of the localization region, the entropy diverges with shrinking collar size in a way which is characteristic for the model but not for one of its field coordinates \[4].
All structural insight obtained up to now, the charge superselection structure, TCP, braid group statistics, V. Jones-as well as the new modular-inclusion theory mentioned in this paper, the universal nature of holography and the concept of localized entropy, all these properties depend on the causality aspects of QFT. So the reasons for giving them up must be very strong (theoretical or experimental) and amount to much more than the esthetics of differential geometric consistency observation. The biggest difference to the a more scholarly and less marketing Zeitgeist of previous times becomes visible if one looks at the terminology. Whereas e.g. the quasiclassical Bohr-Sommerfeld theory was presented in a way that left no doubt about its transitory character and the step towards quantum mechanics was the de-mystification of the quasiclassical antinomies and loose ends, string theorist often praise their product as a theory of everything and invite their fellow physicist to read the big latin letter M as “mystery” in a science whose main aim used to be de-mystification.

Having enjoyed the good fortune of proximity to Harry Lehmann to whose memory I have dedicated this paper, the present crisis often reminds me of good and healthy times in particle physics when he made his lasting contributions to particle physics. It would seem to me that in the present absence of profound experimental discoveries it would be more reasonable and safer to develop local quantum physics according to its very strong intrinsic logic and guidance of its underlying physical principles instead of taking off into the blue yonder under the maxim “everything goes”.

But apart from a few exceptions there is a lamentable dominance of ideas which despite their long age have not contributed anything tangible to particle physics. This danger emanating from this dominance which seriously threatens the chance of our most gifted and original young minds to contribute to the progress of particle physics (and which may even wipe out the very successful scholarly traditions in the exact sciences altogether) was certainly realized by the late Harry Lehmann who reacted to it with his characteristic mocking irony which his friends and collaborators will not forget easily, and which besides his scientific achievements probably explains Wolfgang Pauli’s sympathy and support extended to him.

There are indications that members of the older generation (who have been keeping silence in the face of the mathematical brilliance and exclusiveness behind some of the present dominant fashion in particle physics) are slowly becoming aware of the potential danger.

Note added: Although the majority interest has recently shifted away from the field theoretic AdS-CQFT problem, we find that it serves as an ideal illustration how the powerful concepts of AQFT can solve a problem which otherwise (despite a very large number of papers) would have remained unsolved.

Acknowledgement I am indebted to Gerhard Mack for valuable suggestions and encouragements.

13 Including the appearance of temporal plektonic structures in higher dimensional conformal field theories mentioned at the end of the third section.
References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2, (1998) 231
[2] S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys.Lett. B428 (1998) 105
[3] E. Witten, Adv. Theor. Math. Phys. 2, (1998) 253
[4] C. Fronsdal, Phys. Rev. D10, (1974) 589
[5] S. J. Avis, C. J. Isham and D. Storey, Phys. Rev. D18, (1978) 3565
[6] K.-H. Rehren, Ann, Henri Poincaré 1, (2000) 607
[7] M. Bertola, J. Bros, U. Moschella and R. Schaeffer, AdS/CFT correspondence for n-point functions, hep-th/9908144
[8] D. Buchholz, M. Florig and S. J. Summers, Hawking-Unruh temperature and Einstein Causality in anti-de Sitter space-time, hep-th/9905178
[9] B. Schroer, Ann. Phys. (N.Y.) 275, (1999) 190 and references therein
  B. Schroer, “Local Quantum Theory beyond Quantization”, in Quantum Theory and Symmetries, ed. H.-D. Doebner, V.K Dobrev, J.-D. Henning and W. Luecke, World Scientific (2000), hep-th/9912008
[10] B. Schroer, J. Math. Phys. 41, (2000) 3801
[11] I. E. Segal, Causality and Symmetry in Cosmology and the Conformal Group, Montreal 1976, Proceedings, Group Theoretical Methods In Physics, New York 1977, 433 and references therein to earlier work of the same author.
[12] M. Luescher and G. Mack, Commun. Math. Phys. 41, (1975) 203
[13] M. Hortacsu, B. Schroer and R. Seiler, Phys. Rev. D5, (1972) 2519
[14] B. Schroer and J. A. Swieca, Phys. Rev. D10, (1974) 480, B. Schroer, J. A. Swieca and A. H. Voelkel, Phys. Rev. D11, (1975) 11
[15] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. B247, (1984), 83
[16] R. Haag, Local Quantum Physics, Springer Verlag (1992)
[17] R. Haag and B. Schroer, J. Math. Phys. 3, (1962) 248
[18] B. Schroer, Anomalous Scale Dimensions from Timelike Braiding, hep-th/0005134
  B. Schroer, Space- and Time-Like Superselection Rules in Conformal Quantum Field Theory, hep-th/0010290
[19] D. Guido, R. Longo, J.E. Roberts and R. Verch, *Charged Sectors, Spin and Statistics in Quantum Field Theory on Curved Spacetimes*, math-ph/9906019
[20] B. Schroer and H.-W. Wiesbrock, RMP Vol 12 No 2 (Feb 2000) 301-326
[21] B. Schroer and H.-W. Wiesbrock, RMP Vol 12 No 1 (Jan 2000) 139
   B. Schroer and H.-W. Wiesbrock, *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models*, to appear in RMP Vol 12 No 3 (March 2000)
[22] L. Susskind, J. Math. Phys. 36, (1995) 6377
[23] G.´t Hooft, *Dimensional reduction in quantum gravity*, in Salam-Festschrift, A. Ali et al. eds., World Scientific 1993, page 284
[24] D. Buchholz, S. Doplicher, R. Longo and J. H. Roberts, Rev. Math. Phys. *Special Issue* (1992) 49
[25] Rob Clifton and Hans Halvorson, *Entanglement and open Systems in Algebraic Quantum Field Theory*, University of Pittsburgh preprint Jan.2000
[26] E. Verlinde, *On the Holographic Principle in a Radiation Dominated Universe*, hep-th/0008140.
[27] H. Narnhofer, in *The State of Matter*, ed. by M. Aizenman and H. Araki (World-Scientific, Singapore) 1994
[28] H. Kosaki, J. Operator Theory 16, (1986) 335
[29] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press (1994)
[30] B. Schroer, Phys. Lett. B 494, (2000) 124, hep-th/0005110
[31] I. T. Todorov, Two-dimensional conformal field theory and beyond. Lessons from a continuing fashion, math-phys/0011014
[32] R. Penrose, *how to compute-help-and hurt scientific research*, Convergence Winter 1999, page 30
[33] K.-H. Rehren, *Local Quantum Observables in the Anti de Sitter-Conformal QFT Correspondence*, hep-th/0003120
[34] J. Kupsch, W. Ruehl and B. C. Yunn, Ann. Phys. (N.Y.) 89, (1975) 141
[35] H.-W. Wiesbrock, Lett. Math. Phys. 31, (1994) 303, D. Guido, R. Longo and H.-W. Wiesbrock, Commun. Math. Phys. 192, (1998) 217
[36] H. J. Borchers, J. Math. Phys. 41, (2000) 3604
[37] Nicolay M. Nikolov and Ivan T. Todorov, *Rationality of conformally invariant local correlation functions on compactified Minkowski space*, hep-th/0009004

[38] D. Buchholz, J. Mund and S. J. Summers, *Transplantation of Local Nets and Geometric Modular Action on Robertson-Walker Space-Times*, hep-th/0011237

D. Buchholz, *Algebraic Quantum Field Theory, A Status Report*, hep-th/0011013