Scalar field reconstruction of power-law entropy-corrected holographic dark energy

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Abstract
A so-called ‘power-law entropy-corrected holographic dark energy’ (PLECHDE) was recently proposed to explain the dark energy (DE)-dominated universe. This model is based on the power-law corrections to black hole entropy that appear when dealing with the entanglement of quantum fields between the inside and the outside of the horizon. In this paper, we suggest a correspondence between the interacting PLECHDE and the tachyon, quintessence, $K$-essence and dilaton scalar field models of DE in a non-flat Friedmann–Robertson–Walker universe. Then, we reconstruct the potential terms accordingly, and present the dynamical equations that describe the evolution of the scalar field DE models.

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1. Introduction
One of the most dramatic fields of research in theoretical physics these days is the investigation of the cause of an unpredicted phase of accelerated expansion in our Universe. According to different cosmological observations, our Universe is undergoing a phase of accelerated expansion, the cause of which is still known [1–5]. A component that is responsible for this acceleration usually called dark energy (DE). Most of the observations confirm that DE constitutes more than 70% of the energy content of our Universe and the nature of such a component is still unknown [6]. The simplest candidate for DE is the cosmological constant, which leads to $w = -1$ [7]. However, it suffers the so-called fine-tuning and cosmic coincidence problems [2]. However, observations detect a deviation from the models with constant equation of state (EoS) parameter and show that time-varying DE gives a better fit than a cosmological constant. Therefore, the approach to the DE problem involving the cosmological constant is extended due to the inclusion of the dynamic cosmological constant term [8, 9]. In these models, the DE candidate has dynamical behavior and leads to a variable EoS parameter. The simplest candidate in the dynamic approach is the scalar field $\phi$. This approach for probing the nature of DE has been extensively studied in the literature. Some famous examples of these models are quintessence, tachyon, $K$-essence, dilaton field and so on (see [10–13] and references therein). For a recent review of DE models, see [14].

Among various attempts toward understanding the DE puzzle, the holographic dark energy (HDE) model attracted much interest recently. This model, which has been widely studied in the literature [15–26], motivated the holographic hypothesis [27] and has been tested and constrained by various astronomical observations [28, 29]. It is important to note that in the derivation of HDE density the black hole entropy $S$ plays a crucial role. Indeed, the definition and derivation of holographic energy density ($\rho_D = 3c^2M_p^2/L^2$) depend on the entropy–area relation $S \propto A \propto L^2$ of black holes, where $A$ represents the area of the horizon [15]. However, quantum corrections to the area law have been introduced in recent years, namely logarithmic and power-law corrections. Logarithmic corrections arise from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations [30],

$$S = \frac{A}{4G} + \gamma \ln \frac{A}{4G} + \delta,$$

where $\gamma$ and $\delta$ are dimensionless constants of the order of unity. The exact values of these constants have not yet
been determined and are still an open issue in quantum gravity. This logarithmic term also appears in a model of entropic cosmology that unifies the inflation and late time acceleration [31]. The logarithmic corrections to the entropy of black holes are associated not only with the loop quantum gravity but also with other sources, e.g. the generalized uncertainty principle (GUP) [32]. The HDE models with logarithmic correction have been explored in ample detail [33].

The second form of the correction to the area law, namely the power-law correction, appears when dealing with the entanglement of quantum fields in and out of the horizon. The entanglement entropy of the ground state obeys the Hawking area law. Only the excited state contributes to the correction, and more excitations produce more deviation from the area law [34] (also see [35] for a review of the origin of black hole entropy through entanglement). The power-law-corrected entropy is written as [36, 37]

\[ S = \frac{A}{4G} \left( 1 - K_\alpha A^{1-\alpha/2} \right), \]  

where \( \alpha \) is a dimensionless constant whose value is currently under debate, and

\[ K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{-\alpha}}. \]  

Here \( r_c \) is the crossover scale. The second term in equation (2) can be regarded as a power-law correction to the area law, resulting from entanglement, when the wavefunction of the field is chosen to be a superposition of the ground state and the excited state [36]. The entanglement entropy of the ground state obeys the Hawking area law. Only the excited state contributes to the correction, and more excitations produce more deviation from the area law [34]. This lends further credence to entanglement as a possible source of black hole entropy. The correction term is also more significant for higher excitations [36]. It is important to note that the correction term falls off rapidly with \( A \) and hence in the semi-classical limit (large \( A \)) the area law is recovered. So for large black holes the correction term falls off rapidly and the area law is recovered, whereas for the small black holes the correction is significant. This can be interpreted as follows: for a large area, i.e. at low energies, it is difficult to excite the modes and hence the ground state modes contribute to most of the entanglement entropy. However, for a small horizon area, a large number of field modes can be excited and contribute significantly to the correction, causing large deviation from the area law.

Motivated by the power-law-corrected entropy, a so-called ‘power-law entropy-corrected holographic dark energy’ (PLECHDE) was recently proposed by Sheykhi and Jamil [38]. The energy density of the PLECHDE is written as [38]

\[ \rho_D = 3c^2M_p^2L^{-2} - \beta M_p^2L^{-\alpha}. \]  

In the special case \( \beta = 0 \), the above equation yields the well-known holographic energy density. It was shown that this model is capable of providing an accelerated expansion. The advantages of the PLECHDE model compared to the usual HDE scenario is that in the presence of the power-law correction term, the identification of IR cutoff with the Hubble radius, \( L = H^{-1} \), can drive the accelerated expansion [38]. This is in contrast to the ordinary HDE where \( \omega_D = 0 \) if one chooses \( L = H^{-1} \) in the absence of interaction [18]. Due to the variety of models in the literature, to explain the DE problem, it seems essential to develop a correspondence between different approaches clarifying the theoretical status of the current models. To this end, in this paper we would like to implement the correspondence between the scalar field energy density and the PLECHDE model. This connection allows us to reconstruct the potential as well as the dynamics of the scalar fields that describe the acceleration of the universe expansion.

This paper is organized as follows. In the next section, we establish the correspondence between the PLECHDE and the tachyon field DE model. In section 3, we study the quintessence DE model based on the interacting PLECHDE. Section 4 contains the reconstruction procedure of the \( K \)-essence PLECHDE. In section 5, we consider the reconstructed model of the dilatonic DE based on the PLECHDE. We end the paper with some concluding remarks in section 6.

2. Tachyon reconstruction of PLECHDE

The tachyon model of DE originates from string theory and seems to have interesting cosmological consequences. One of the interesting properties of a rolling tachyon is its EoS parameter that changes from \(-1 \) to 0 [39]. This EoS parameter convinces people to consider the tachyon scalar field as a candidate for DE. It was demonstrated that DE driven by tachyon decays to cold dark matter in the late accelerated Universe and this phenomenon yields a solution to the cosmic coincidence problem [40]. Choosing different self-interacting potentials in the tachyon field model lead to different consequences for the resulting DE model. Due to this fact, we would like to reconstruct the tachyon equivalent of the PLECHDE to see which tachyon scalar field model can demonstrate quantum gravity effects. The connection between tachyon field and HDE [41], agegraphic dark energy (ADE) [42], ECHDE [43] and ECADE models [44] has already been established. The effective Lagrangian for the tachyon field is described by

\[ L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \]

where \( V(\phi) \) is the tachyon potential. The corresponding energy momentum tensor for the tachyon field can be written in a perfect fluid form

\[ T_{\mu\nu} = (\rho_\phi + p_\phi)u_\mu u_\nu - p_\phi g_{\mu\nu}, \]

where \( \rho_\phi \) and \( p_\phi \) are, respectively, the energy density and pressure of the tachyon, and the velocity \( u_\mu \) is

\[ u_\mu = \frac{\partial_\mu\phi}{\sqrt{\partial_\nu\phi\partial^\nu\phi}}. \]

We assume the background Friedmann–Robertson–Walker (FRW) metric that is described by the line element

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right), \]
where $a(t)$ is the scale factor and $k$ is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat and closed universes, respectively. The first Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_m + \rho_D),$$

(9)

where $\rho_m$ and $\rho_D$ are, respectively, the energy densities of pressureless matter and DE. The dimensionless density parameters are defined as usual

$$\Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_D = \frac{\rho_D}{\rho_c}, \quad \Omega_k = \frac{k}{a^2 H^2}, \quad$$

(10)

where the critical energy density is $\rho_c = 3H^2 M_p^2$. Thus, the first Friedmann equation can be rewritten as

$$1 + \Omega_k = \Omega_m + \Omega_D.$$  

(11)

The energy density and pressure of the tachyon field are given by

$$\rho_\phi = -T_0^0 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad$$

$$p_\phi = T_i^j = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \quad$$

(12)

(13)

The EoS parameter is

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1. \quad$$

(14)

Following [38], we rewrite the energy density of the PLECHDE in the following form:

$$\rho_D = \frac{3c^2 M_p^2}{L^2} \gamma_c, \quad$$

(15)

where

$$\gamma_c = \frac{1 - \frac{\beta}{3c^2 L a^{-2}}}{1 - \frac{\alpha}{3c^2 L a^{-2}}}. \quad$$

(16)

and $L$ is a length scale that provides an IR cut-off for the holographic model of DE. In the literature, a variety of IR cut-offs have been assumed. Li [16] discussed three choices for the length scale $L$, which is supposed to provide an IR cut-off. The first choice is the Hubble radius, $L = H^{-1}$ [18], which leads to an incorrect EoS, namely that for dust. The second option is the particle horizon radius. In this case, it is impossible to obtain an accelerated expansion. Only the third choice, the identification of $L$ with the radius of the future event horizon, gives the desired result, namely a sufficiently negative EoS to obtain an accelerated Universe. However, as soon as the interaction between DE and dark matter is taken into account, the identification of the IR cut-off with the Hubble (apparent horizon) radius can simultaneously drive accelerated expansion and solve the coincidence problem [19]. In recent years, some new IR cut-offs have also been proposed in the literature. In [20], an HDE model with the Ricci scalar as the IR cut-off was proposed, whereas in [21] the authors have added the square of the Hubble parameter and its time derivative to the definition of HDE. A linear combination of the particle horizon and the future event horizon was also proposed in [22]. In this paper, as the system’s IR cut-off, we take the radius of the event horizon measured on the sphere of the horizon defined as [23]

$$L = ar(t), \quad$$

(17)

where $r(t)$ can be obtained from

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^{\infty} \frac{dr}{a} = \frac{R_h}{a}. \quad$$

(18)

Solving the above equation for $r(t)$, we obtain

$$r(t) = \frac{1}{\sqrt{k}} \sin \left( \sqrt{k R_h} \right). \quad$$

(19)

Here $R_h$ is the radial size of the event horizon measured in the $r$-direction. Assuming an interaction between DE and dark matter, the conservation equations read

$$\dot{\rho}_m + 3H \rho_m = Q, \quad$$

(20)

$$\dot{\rho}_D + 3H \rho_D (1 + w_D) = -Q, \quad$$

(21)

where $Q = 3b^2 H (\rho_D + \rho_m)$ is an energy exchange term, $b^2$ is a coupling constant and $w_D$ is the EoS parameter of the DE component. Differentiating (15) with respect to cosmic time $t$ and using (11) and (17)-(19), we have

$$\dot{\rho}_D = H \rho_D \left( \frac{\alpha - 2 - \alpha}{\gamma_c} \right) \left[ 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right]. \quad$$

(22)

where $y = \sqrt{\frac{k R_h}{a}}$. Substituting the above relation into equation (21), we obtain [38]

$$w_D = -1 - \frac{1}{3} \left( \frac{\alpha - 2}{\gamma_c} \right) \left[ 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right] - \frac{b^2 (1 + \Omega_k)}{\Omega_D}. \quad$$

(23)

We can also obtain the equation of motion of DE. To this end, taking the time derivative of $\Omega_D \rho_D/(3M_p^2 H^2)$ yields

$$\dot{\Omega}_D = \Omega_D \left[ \frac{\dot{\rho}_D}{\rho_D} - 2 \frac{\dot{H}}{H} \right]. \quad$$

(24)

Using equation (21) and the time derivative of equation (9), as well as the above equation, one obtains

$$\dot{\Omega}_D = H \Omega_D \left[ 1 - \Omega_D \right] \left[ 3 \left( \frac{\alpha - 2}{\gamma_c} \right) \right. \times \left. \left[ 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right] - 3b^2 (1 + \Omega_k) + \Omega_k \right]. \quad$$

(25)

Taking into account that $\dot{\Omega}_D = \Omega_D' H$, the evolution equation of $\Omega_D$ can be written as

$$\Omega_D' = \Omega_D \left[ 1 - \Omega_D \right] \left[ 3 \left( \frac{\alpha - 2}{\gamma_c} \right) \right. \times \left. \left[ 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right] - 3b^2 (1 + \Omega_k) + \Omega_k \right]. \quad$$

(26)
To develop the correspondence between the PLECHDE and the tachyon field, we identify \( w_D = w_\phi \) and obtain
\[
\phi = \sqrt{1 + w_D} = \sqrt{\frac{1}{3} \left( \alpha - \frac{2}{\gamma_c} \right) \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} \right] - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D}}. \tag{27}
\]
Using \( \phi' = \phi' H \), we can write
\[
\phi' = \frac{1}{H} \times \sqrt{\frac{1}{3} \left( \alpha - \frac{2}{\gamma_c} \right) \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} \right] - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D}}, \tag{28}
\]
where the prime denotes derivative with respect to \( x = \ln a \). Integration yields
\[
\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{1}{aH} \times \sqrt{\frac{1}{3} \left( \alpha - \frac{2}{\gamma_c} \right) \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} \right] - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D}} \, da. \tag{29}
\]
where \( a_0 \) is the value of the scale factor at the present time \( t_0 \). Alternatively, we can rewrite equation (29) as
\[
\phi(t) - \phi(t_0) = \int_{t_0}^{t} \sqrt{\frac{1}{3} \left( \alpha - \frac{2}{\gamma_c} \right) \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} \right] - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D}} \, dt'. \tag{30}
\]
Also using relation (12), we have
\[
V(\phi) = 3M_p^2 H^2 \Omega_D \times \left[ 1 - \frac{1}{3} \left( \alpha - \frac{2}{\gamma_c} \right) \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} \right] - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D} \right]. \tag{31}
\]
Therefore, we have established an interacting power-law ECH tachyon DE model and reconstructed the potential and dynamics of the tachyon field. Such a tachyon scalar field model with potential (31) and dynamical equation (30) can play the role of DE like the PLECHDE.

3. Quintessence reconstruction of PLECHDE

We use the term ‘quintessence’ to denote a canonical scalar field \( \phi \) with a potential \( V(\phi) \) that can interact with all other components only through standard gravity. The quintessence model is therefore described by the Lagrangian
\[
\mathcal{L} = -\frac{1}{2} g^{\nu\mu} \partial_\mu \phi \partial_\nu \phi - V(\phi). \tag{32}
\]
The energy–momentum tensor of quintessence is
\[
T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right]. \tag{33}
\]
In the FRW framework, the energy density and pressure of the quintessence field can be written as
\[
\rho_\phi = -T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi), \tag{34}
\]
\[
p_\phi = T_{ij} = \frac{1}{2} \dot{\phi}^2 - V(\phi), \tag{35}
\]
where \( \rho_\phi \) and \( p_\phi \) denote the energy density and pressure, respectively. Using the above relations, it can be easily seen that the kinetic term and the scalar potential are
\[
\dot{\phi}^2 = (1 + w_D) \rho_\phi, \tag{36}
\]
\[
V(\phi) = \frac{1}{2} \rho_\phi. \tag{37}
\]
where \( w_D = p_\phi / \rho_\phi \). Identifying \( \rho_\phi = \rho_D = \frac{\gamma_c w_D^2}{4} \gamma_c \) and using (36), one obtains
\[
\dot{\phi}^2 = 3M_p^2 H^2 \Omega_D \left( \frac{1}{3} (\alpha - \frac{2}{\gamma_c}) \right) \times \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D} \right]. \tag{38}
\]
Using \( \dot{\phi} = \dot{\phi}' H \), we have
\[
\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{1}{aH} \times 3M_p^2 \Omega_D \left( \frac{1}{3} (\alpha - \frac{2}{\gamma_c}) \right) \times \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} - \frac{b^2(1 + \Omega_\Lambda)}{\Omega_D} \right] \, da. \tag{39}
\]
where \( a_0 \) denotes the present value of the scale factor. From equation (32), the scalar potential is obtained as
\[
V(\phi) = 3M_p^2 H^2 \Omega_D \left( 1 - \frac{1}{6} (\alpha - \frac{2}{\gamma_c}) \right) \times \left[ 1 - \frac{1}{c \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y} + \frac{b^2(1 + \Omega_\Lambda)}{2 \Omega_D} \right]. \tag{40}
\]
One can easily check that the usual holographic quintessence DE model can be retrieved in the limiting case \( \gamma_c = 1 (\alpha = \beta = 0) \).

4. K-essence reconstruction of PLECHDE

Quintessence relies on the potential energy of the scalar field which leads to the late time acceleration. It is possible to have a situation where the accelerated expansion arises out of modifications to the kinetic energy of the scalar field. K-essence is characterized by a scalar field with a non-canonical kinetic energy. The most general scalar-field action that is a function of \( \phi \) and \( X = -\dot{\phi}^2/2 \) is given by [45]
\[
S = \int d^4x \sqrt{-g} P(\phi, X). \tag{41}
\]
where the Lagrangian density \( P(\phi, X) \) corresponds to a pressure density. According to this Lagrangian, the energy density and pressure can be written as [11]
\[
\rho_\phi = f(\phi)(-X + 3X^2),
\]
\( f(\phi) = X + X^2). \tag{42}
\]
Therefore, the EoS parameter of the K-essence will be
\[
w_K = \frac{P_\phi}{\rho_\phi} = 1 - \frac{X}{1 - 3X}.
\]
To implement the correspondence between the K-essence and the PLECHDE, we set \( w_K = w_D \) and solve (44) for \( X \). We find that
\[
\begin{align*}
X &= \frac{1 - w_D}{1 - 3w_D} = \frac{2 - \frac{1}{3} \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] + \frac{\lambda^2 (1 + \Omega_0)}{40a} \} }{4 - \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] + \frac{3\lambda^2 (1 + \Omega_0)}{40a} } d a.
\end{align*}
\]
\[
\begin{align*}
\phi(a) &= \phi(a_0) + \int_{a_0}^{a} \frac{1}{H a} d a
\end{align*}
\]
\[
\times \left[ \frac{-4 + \frac{1}{3} \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] - \frac{\lambda^2 (1 + \Omega_0)}{40a} }{4 - \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] + \frac{3\lambda^2 (1 + \Omega_0)}{40a} } d a.
\end{align*}
\]
\[
\text{5. Dilaton field reconstruction of PLECHDE}
\]

It is quite possible that gravity is not given by the Einstein action, at least at sufficiently high energy scales. The most promising alternative seems to be that offered by string theory, where the gravity becomes scalar tensor in nature. In the low-energy limit of string theory, one recovers Einstein gravity along with a scalar dilaton field that is non-minimally coupled to gravity [46]. The dilaton field can be used for the explanation of the DE puzzle and avoids some quantum instabilities with respect to the phantom field models of DE [47]. The Lagrangian density of the dilatonic DE corresponds to the pressure density of the scalar field that has the following form [48]:
\[
P = -X + c e^{\lambda \phi} X^2,
\]
where \( c \) and \( \lambda \) are positive constants and \( X = \dot{\phi}^2/2 \). Such a pressure (Lagrangian) leads to the following energy density [48]:
\[
\rho = -X + 3c e^{\lambda \phi} X^2.
\]
The EoS parameter of the dilatonic DE can be written as
\[
w = \frac{P}{\rho} = \frac{1 - c e^{\lambda \phi} X}{1 - 3c e^{\lambda \phi} X}.
\]
Seeking a correspondence between the dilatonic DE and PLECHDE, we set \( w = w_D \), where \( w_D \) comes from the PLECHDE. Solving this equation for \( c X e^{\lambda \phi} \), we obtain
\[
c X e^{\lambda \phi} = \frac{w_D - 1}{3w_D - 1}.
\]

Taking into account that \( X = \dot{\phi}^2/2 \), we can rewrite equation (50) as
\[
\frac{c}{2} \left( \frac{d}{d\tau} e^{\lambda \phi} \right)^2 = \frac{w_D - 1}{3w_D - 1}.
\]

Using \( \frac{d}{d\tau} = H \frac{d}{d\ln a} \), we obtain
\[
\phi(a) = \frac{2}{\lambda} \ln \left[ e^{\lambda \phi(a)/2} + \frac{\lambda}{\sqrt{2c}} \int_{\ln a_0}^{\ln a} \frac{1}{H} \left( \frac{w_D - 1}{3w_D - 1} \right)^{1/2} d \ln a \right],
\]
which represents the evolution equation of \( \phi \). Using the expression for \( w_D \), we can further rewrite the above equation as
\[
\phi(a) = \frac{2}{\lambda} \ln \left[ e^{\lambda \phi(a)/2} + \frac{\lambda}{\sqrt{2c}} \int_{\ln a_0}^{\ln a} \frac{1}{H} \left( \frac{2 - \frac{1}{3} \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] - \frac{\lambda^2 (1 + \Omega_0)}{40a} }{4 - \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] + \frac{3\lambda^2 (1 + \Omega_0)}{40a} } d a \right]^{1/2}
\times \left( \frac{2 - \frac{1}{3} \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] - \frac{\lambda^2 (1 + \Omega_0)}{40a} }{4 - \left( \alpha - \frac{\mu - 2}{\gamma} \right) \left[ 1 - \frac{1}{\tau} \sqrt{\frac{\Omega_0}{\gamma}} \cos y \right] + \frac{3\lambda^2 (1 + \Omega_0)}{40a} } \right)^{1/2}.
\]

6. Concluding remarks

A possibility to explain the origin of the black hole entropy is the entanglement of quantum fields between the inside and outside of the horizon [36]. It was shown in [36] that the black hole entropy is proportional to the horizon area when the field is in its ground state, while a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. For large horizon areas, these corrections are relatively small and the area law is recovered. Taking into account the correction to area law the HDE density is modified as well. Based on this, a so-called PLECHDE was proposed recently [38] to explain the acceleration of the cosmic expansion. On the other hand, there are many different models of DE in the context of a scalar field such as tachyon, quintessence, K-essence and dilaton fields. One exciting question is whether a scalar field model can act as a DE model, while they have apparently distinct origin. In this paper, a connection between the scalar field models and the PLECHDE has been established. As a result and using proposal (14), we have reconstructed the potentials as well as the evolutionary forms of scalar fields. In section 2, we presented a tachyon field version of the PLECHDE. Then, we reconstructed the potential term and the dynamical equation governing the evolution of the tachyon field. In section 3, we established a correspondence between the PLECHDE and a quintessence model. The potential for the K-essence holographic correspondence was derived in section 4. We also considered the dilaton condensate without potential, and used the correspondence to find the evolutionary form of the scalar field. In the limiting case \( \gamma = (\alpha = 0 = \beta) \), where the power-law correction becomes trivial, all our results reduce to their corresponding expressions in standard HDE scalar field models.
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References

[1] Perlmutter S et al 1998 Nature 391 51
[2] Riess A G et al 1998 Astron. J. 116 1009
Riess A G et al 1999 Astron. J. 117 707
[3] Spergel D N et al (WMAP Collaboration) 2003 Astrophys. J. Suppl. 148 175
Spergel D N et al 2006 arXiv:astro-ph/0603449
[4] Teigmamp M et al (SDSS Collaboration) 2004 Phys. Rev. D 69 103501
[5] Abazajian K et al (SDSS Collaboration) 2004 Astron. J. 128 502
Abazajian K et al (SDSS Collaboration) 2004 Astron. J. 129 1755
[6] Choudhury T R and Padmanabhan T 2005 Astron. Astrophys. 429 807
[7] Sahni V and Starobinsky A 2000 Int. J. Mod. Phys. D 9 373
Peelbes P J and Katra B 2003 Rev. Mod. Phys. 75 559
[8] Ma Y Z and Zhang X 2008 Phys. Lett. B 661 239
Ma Y Z, Gong Y and Chen X 2009 Eur. Phys. J. C 60 303
Ma Y Z, Gong Y and Chen X 2010 Eur. Phys. J. C 69 509
[9] Shapiro I L and Sola J 2009 Phys. Lett. B 682 105
Cai R G, Hu B and Zhang Y 2009 Commun. Theor. Phys. 51 954
Fabris I C, Shapiro I L and Sola J 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)016
[10] Wetterich C 1988 Nucl. Phys. B 302 668
[11] Chiha T, Okabe T and Yamaguchi M 2000 Phys. Rev. D 62 023511
[12] Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 Phys. Rev. Lett. 85 4438
[13] Armendariz-Picon C, Mukhanov V and Steinhardt P J 2001 Phys. Rev. D 63 103510
[14] Copeland E J, Sami M and Tsujikawa S 2006 Int. J. Mod. Phys. D 15 1753
[15] Cohen A, Kaplan D and Nelson A 1999 Phys. Rev. Lett. 82 4971
[16] Li M 2004 Phys. Lett. B 603 1
[17] Huang Q G and Li M 2004 J. Cosmol. Astropart. Phys. JCAP08(2004)013
[18] Hsu S H 2004 Phys. Lett. B 594 13
[19] Pavon D and Zimdahl W 2005 Phys. Lett. B 628 206
Shydkhi A 2010 Class. Quantum. Grav. 27 025007
[20] Gao C et al 2009 Phys. Rev. D 79 043511
[21] Granda L N 2009 Phys. Lett. B 669 275
Granda L N and Oliveros A 2009 Phys. Lett. B 671 199
[22] Sadjadi H M and Hoonardooost M 2007 Phys. Lett. B 647 231
[23] Wang B, Gong Y and Abdalla E 2005 Phys. Lett. B 624 141
[24] Elizalde E, Nojiri S, Odintsov S D and Wang P 2005 Phys. Rev. D 71 103504
Gublerina B, Horvat R and Stefancic H 2005 J. Cosmol. Astropart. Phys. JCAP05(2005)001
Wang B, Abdalla E and Su R K 2005 Phys. Lett. B 611 21
Wei H 2009 Nucl. Phys. B 819 210
Karami K and Fehrni J 2010 Phys. Lett. B 684 61
[25] Nozari K and Rashidi N 2010 Int. J. Mod. Phys. D 19 219
[26] Wang B, Lin C Y and Abdalla E 2006 Phys. Lett. B 637 357
[27] Wang B, Lin C Y, Pavon D and Abdalla E 2008 Phys. Lett. B 662 1
[28] Sheykhi A 2009 Phys. Lett. B 681 205
Sheykhi A and Jamil M 2011 Phys. Lett. B 694 284
[29] t’Hooft G 1993 arXiv:gr-qc/9310026
Susskind L 1995 J. Math. Phys. 36 6377
[30] Zhang X and Wu F Q 2005 Phys. Rev. D 72 043524
Zhang X and Wu F Q 2007 Phys. Rev. D 76 023502
Huang Q G and Gong Y G 2004 J. Cosmol. Astropart. Phys. JCAP08(2004)006
Enqvist K, Hannestad S and Sloth M S 2005 J. Cosmol. Astropart. Phys. JCAP02(2005)004
[31] Feng B, Wang X and Zhang X 2005 Phys. Lett. B 607 35
Kao C, Lee W L and Lin F L 2005 Phys. Rev. D 71 123518
Shen J Y, Wang B, Abdalla E and Su R K 2005 Phys. Lett. B 609 200
[32] Meissner K A 2004 Class. Quantum Grav. 21 5245
Ghosh A and Mitra P 2004 Phys. Rev. D 71 027502
Chatterjee A and Majumdar P 2004 Phys. Rev. Lett. 92 141301
[33] Cai Y F, Liu J and Li H 2010 Phys. Lett. B 690 213
Medved A J M and Vagenas E C 2004 Phys. Rev. D 70 124021
[34] Wei H 2009 Commun. Theor. Phys. 52 743
Jamil M and Farooq M U 2010 J. Cosmol. Astropart. Phys. JCAP03(2010)001
Karami K et al 2011 Gen. Rel. Grav. 43 27
Sheykhi A et al 2010 arXiv:1005.4541
[35] Das S, Shankaranarayan S and Sur S 2010 arXiv:1002.1129
[36] Das S, Shankaranarayan S and Sur S 2008 arXiv:0806.0402
[37] Das S, Shankaranarayan S and Sur S 2008 Phys. Rev. D 77 064013
[38] Radicella N and Pavon D 2010 Phys. Lett. B 691 121
[39] Sheykhi A and Jamil M 2010 Gen. Rel. Grav. doi:10.1007/s10714-011-1190-x (arXiv:1011.0134)
[40] Gibbons G W 2002 Phys. Lett. B 537 1
[41] Srivastava S K 2004 arXiv:gr-qc/0409074
[42] Setare M R 2007 Phys. Lett. B 653 116
[43] Sheykhi A Phys. Lett. B 682 329
[44] Khodam-Mohammadi A and Malekzani M 2010 arXiv:1004.1720
[45] Jamil M and Sheykhi A 2011 Int. J. Theor. Phys. 50 625
Armendariz-Picon C, Damour T and Mukhanov V F 1999 Phys. Lett. B 458 209
Green M B, Schwarz J H and Witten E 1987 Superstring Theory (Cambridge: Cambridge University Press)
Carroll S M, Hoffman M and Trodden M 2003 Phys. Rev. D 68 023509
Piazza F and Tsujikawa S 2004 J. Cosmol. Astropart. Phys. JCAP07(2004)004