GLOBAL MONOPOLES IN DILATON GRAVITY

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Abstract

We analyse the gravitational field of a global monopole within the context of low energy string gravity, allowing for an arbitrary coupling of the monopole fields to the dilaton. Both massive and massless dilatons are considered. We find that, for a massless dilaton, the spacetime is generically singular, whereas when the dilaton is massive, the monopole generically induces a long range dilaton cloud. We compare and contrast these results with the literature.

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I. INTRODUCTION

Cosmologists have been attracted to topological defects as a possible source for the density perturbations which seeded galaxy formation \[1\]. Phase transitions in the early universe can give rise to various types of defect. Briefly, a defect is a discontinuity in the vacuum, and can be classified according to the topology of the vacuum manifold of the field theory model being used. Disconnected vacuum manifolds give domain walls, non-simply connected manifolds, strings, and vacuum manifolds with non-trivial \(\Pi_2\) and \(\Pi_3\) homotopy groups give monopoles and textures respectively. Strings and monopoles can be further subdivided into local and global defects depending on whether the symmetry broken is local or global. With the exception of the domain wall, which results from the breaking of a discrete symmetry and has no Goldstone boson, global defects are usually characterised by a power law fall-off in the energy density of the defect leading to divergent energies. Local defects on the other hand typically have no Goldstone bosons and are characterised by a well-defined core and finite energy per unit defect area. We therefore expect local and global defects to have significantly different behaviour, and nowhere is this more evident than in their coupling to gravity. Whilst local strings \[2\] and monopoles \[3\] produce only localised spacetime curvature, asymptoting locally flat and flat spacetimes respectively, global strings \[4,5\] and monopoles \[6,7\] have strong effects even at large distances. Indeed, the spacetime of both domain walls \[8\] and global strings \[5\] appears to be time dependent, with a de-Sitter expansion along the spatial extent of the defect. The spacetime of a global monopole is static, non-singular but not asymptotically flat; it asymptotes a locally flat spacetime, with a deficit solid angle of \(8\pi G\eta^2\) where \(\eta\) is the symmetry breaking scale \[6\], but this deficit angle can lead to potentially strong tidal forces \[3\].

Of course, this discussion has taken place within the context of general relativity, however, at sufficiently high energy scales it seems likely that gravity is not given by the Einstein action. The most promising alternative seems to be that given by string theory, where in the low energy limit gravity becomes scalar-tensor in nature \[10\]. Scalar-tensor gravity is not new, it was pioneered by Jordan, Brans and Dicke \[11\], who sought to incorporate Mach’s principle into gravity, and indeed the scalar-tensor part of the low energy superstring action is equivalent to Brans-Dicke theory for a particular value of the Brans-Dicke parameter: \(\omega = -1\). The implications of superstrings for cosmology is a subject of intense investigation, however, in this paper we are interested in the implications of superstring gravity for topological defects, in particular global monopoles, and how these effects are dependent on the mass of the dilaton.

Recently, Damour and Vilenkin \[12\] argued that a low mass superstring dilaton would be incompatible with a local string network formed at a GUT phase transition, however, by considering the fully coupled nonlinear field equations of a particular local string model with dilaton gravity, one can show that this conclusion is strongly dependent on the coupling of the defect to the dilaton \[13\]. Here we consider the gravi-dilaton field of the global monopole in superstring gravity. We consider a general form for the interaction with the dilaton, assuming, as in \[13\], that the monopole lagrangian couples to the dilaton via an arbitrary coupling \(e^{2\phi}\mathcal{L}\) in the string frame. We consider both massive and massless dilatons, which unsurprisingly turn out to be qualitatively rather different.

The layout of the paper is as follows. We first review the work of Barriola and Vilenkin,
deriving the Einstein metric of a global monopole. We then present the analysis for the
global monopole in superstring gravity, for both massless and massive dilatons. Finally, we
consider the implications of the dilaton for cosmological bounds on the monopole.

II. THE GLOBAL MONOPOLE

In this section we briefly review the work of Barriola and Vilenkin [6]. The simplest
model that gives global monopoles is described by the Lagrangian

\[ \mathcal{L}(\psi^i) = \frac{1}{2} \nabla_a \psi^i \nabla^a \psi^i - \frac{\lambda}{4} (\psi^i \psi^i - \eta^2)^2 \]

where \( \psi^i \) is a triplet of real scalar fields, \( i = 1, 2, 3 \). This model has a global O(3) symmetry,
which is spontaneously broken to a global U(1) symmetry by a choice of vacuum \( |\psi^i| = \eta \).
We look for a spherically symmetric, static configuration describing the global monopole at
rest. The field configuration describing a monopole may then be written as

\[ \psi^i = \eta f(r) \hat{x}^i \]

where \( \hat{x}^i \) is the unit radial vector in the internal space. The metric for the static monopole
is then written as

\[ ds^2 = B(r) dt^2 - A(r) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

In terms of the new variable \( f(r) \) the Lagrangian becomes

\[ \mathcal{L} = -\eta^2 \left( \frac{f'^2}{2A} + \frac{f^2}{r^2} + \frac{\lambda \eta^2}{4} (f^2 - 1)^2 \right) \]

and the field equation for \( f \) is

\[ \frac{1}{(AB)^{1/2} r^2} \left( \left( \frac{B}{A} \right)^{1/2} r^2 f' \right)' = \frac{2f}{r^2} + \lambda \eta^2 f (f^2 - 1) \]

Even in a flat space background \( (A = B = 1) \), this equation does not have an analytic
solution, however, it can be integrated numerically (see figure 1). Note that \( f(r) \sim 1 - 1/r^2 \)
as \( r \to \infty \).

The monopole couples to the metric via its energy momentum tensor

\[ G_{ab} = 8\pi G T_{ab} \]

Without loss of generality, we will assume that the scale of the coordinates in \( \mathbb{R}^4 \) is such
that the size of the monopole core is of order unity, i.e. we set \( \sqrt{\lambda} \eta = 1 \). We also rescale the
energy-momentum tensor of the monopole, \( \tilde{T}_{ab} = T_{ab}/\eta^2 \), which is given by

\[ \tilde{T}_t^t = \frac{f'^2}{2A} + \frac{f^2}{r^2} + \frac{1}{4} (f^2 - 1)^2 \]

\[ \tilde{T}_r^r = -\frac{f'^2}{2A} + \frac{f^2}{r^2} + \frac{1}{4} (f^2 - 1)^2 \]

\[ \tilde{T}_\theta^\theta = \frac{f'^2}{2A} + \frac{1}{4} (f^2 - 1)^2 \]
The $tt$ and $rr$ components of Einstein’s equation are

$$\frac{A'}{rA^2} + \frac{1}{r^2} \left( 1 - \frac{1}{A} \right) \epsilon \hat{T}^t_t = (8)$$

$$\frac{B'}{ABr} - \frac{1}{r^2} \left( 1 - \frac{1}{A} \right) = -\epsilon \hat{T}^r_r = (9)$$

where $\epsilon = 8\pi G\eta^2$ is the gravitational strength of the monopole.

Outside the monopole core, $f(r) \approx 1$ and the energy-momentum tensor can be approximated by

$$\hat{T}^t_t \approx \hat{T}^r_r \approx 1 \quad r^2, \quad \hat{T}^\theta_\theta \approx 0 \quad (10)$$

The general solution to Einstein’s equations is

$$B = A^{-1} = 1 - \epsilon - \frac{2GM}{r} \quad (11)$$

where $M$ is a constant of integration, the ADM mass of the monopole (see [14] for a rigorous definition of ADM mass in quasi-asymptotically flat spacetimes), which from (8) is

$$2GM = \epsilon \int_0^\infty \left[ r^2 \hat{T}^t_t - 1 \right] dr \quad (12)$$

We obtain $GM = -0.73\epsilon$ in a linearised approximation, in agreement with [7]. Ignoring the mass term, which is negligible on astrophysical scales, and rescaling the $r$ and $t$ variables we can write the monopole metric as

$$ds^2 = dt^2 - dr^2 - (1 - \epsilon) r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (13)$$

This metric describes a spacetime with deficit solid angle $\epsilon$. The spacetime is not asymptotically flat, but it is asymptotically locally flat. Note that the mass is negative, which at first seems suprising since the usual ADM mass for asymptotically flat spacetimes is necessarily positive [13]. However, as discussed by Nucamendi and Sudarsky [14], there is no such positivity requirement for quasi-asymptotically flat spacetimes, and indeed the negativity of the global monopole mass is in keeping with the gravitationally repulsive domain wall [8], which is also a global defect.

### III. GLOBAL MONOPOLES IN DILATON GRAVITY

We are interested in the behaviour of the global monopole metric when gravitational interactions take a form typical of low energy string theory. In its minimal form, string gravity replaces the gravitational constant, $G$, by a scalar field, the dilaton. To account for the unknown coupling of the dilaton to the monopole, as in [13] we choose the action

$$S = \int d^4x \sqrt{-g} \left[ e^{-2\phi} \left( -\hat{R} - 4(\nabla\phi)^2 - \hat{V}(\phi) \right) + e^{2\phi} \mathcal{L} \right] \quad (14)$$
where $\mathcal{L}$ is as in (1). The potential for the dilaton $\hat{V}(\phi)$ is for the moment assumed general. This action is written in terms of the string metric which appears in the string sigma model. To facilitate comparison with the previous section we instead choose to write the action in terms of the ‘Einstein’ metric

$$g_{ab} = e^{-2\phi} \hat{g}_{ab}$$

in which the gravitational part of the action appears in the normal Einstein form

$$S = \int d^4x \sqrt{-g} \left[ -R + 2(\nabla \phi)^2 - V(\phi) + e^{2(a+2)\phi} \mathcal{L}(\psi^i, e^{2\phi} g) \right]$$

where $V(\phi) = e^{2\phi} \hat{V}(\phi)$. The energy-momentum tensor is now

$$T_{ab} = 2 \frac{\delta \mathcal{L}(\psi^i, e^{2\phi} g)}{\delta g^{ab}} - g_{ab} \mathcal{L}(\psi^i, e^{2\phi} g) = e^{-2\phi} \nabla_a \psi^i \nabla_b \psi^i - g_{ab} \mathcal{L}$$

Einstein’s equation becomes

$$G_{ab} = \frac{1}{2} e^{2(a+2)\phi} T_{ab} + S_{ab}$$

where

$$S_{ab} = 2 \nabla_a \phi \nabla_b \phi + \frac{1}{2} g_{ab} V(\phi) - g_{ab} (\nabla \phi)^2$$

is the energy-momentum of the dilaton, which has as its equation of motion

$$-\square \phi = \frac{1}{4} \frac{\partial V}{\partial \phi} - \frac{a+2}{2} e^{2(a+2)\phi} \mathcal{L}(\psi^i, e^{2\phi} g) + \frac{1}{4} e^{2(a+1)\phi} g^{ab} \nabla_a \psi^i \nabla_b \psi^i$$

As before, we choose the general static, spherically symmetric metric (3) and write the field configuration (2) for the monopole. The Lagrangian now is

$$\mathcal{L} = -\eta^2 \left( e^{-2\phi} \left( \frac{f^2}{2A} + \frac{f^2}{r^2} \right) + \frac{\lambda \eta^2}{4} (f^2 - 1)^2 \right)$$

Again we take $\sqrt{\lambda} \eta = 1$ and the rescaled modified energy-momentum tensor is then

$$\hat{T}_t^t = e^{2(a+2)\phi} \left( e^{-2\phi} \left( \frac{f^2}{2A} + \frac{f^2}{r^2} \right) + \frac{1}{4} (f^2 - 1)^2 \right)$$

$$\hat{T}_r^r = e^{2(a+2)\phi} \left( e^{-2\phi} \left( -\frac{f^2}{2A} + \frac{f^2}{r^2} \right) + \frac{1}{4} (f^2 - 1)^2 \right)$$

$$\hat{T}_\theta^\theta = e^{2(a+2)\phi} \left( e^{-2\phi} \frac{f^2}{2A} + \frac{1}{4} (f^2 - 1)^2 \right)$$

The $tt$ and $rr$ components of Einstein’s equation are now
\[
\frac{A'}{rA^2} + \frac{1}{r^2} \left(1 - \frac{1}{A}\right) = \epsilon \hat{T}_t^t + \frac{1}{2} V(\phi) + \frac{\phi'^2}{A} \tag{23}
\]
\[
- \frac{B'}{ABr} + \frac{1}{r^2} \left(1 - \frac{1}{A}\right) = \epsilon \hat{T}_r^r + \frac{1}{2} V(\phi) - \frac{\phi'^2}{A} \tag{24}
\]
where \(\epsilon = \eta^2/2\). The dilaton equation is

\[
- \Box \phi = \frac{1}{4} \frac{\partial V}{\partial \phi} + \epsilon (a + 1) \hat{T}_t^t + \frac{\epsilon}{4} e^{2(a+2)\phi} (f^2 - 1)^2 \tag{25}
\]
and the equation of motion for \(f\) is

\[
\frac{1}{(AB)^{1/2}r^2} \left(\frac{B}{A}\right)^{1/2} e^{2(a+1)\phi} f' = \frac{2fe^{2(a+1)\phi}}{r^2} + e^{2(a+2)\phi} f (f^2 - 1) \tag{26}
\]
We now consider massless and massive dilatons in turn.

### A. Massless dilatonic gravity

For the massless dilaton \(V(\phi) = 0\). Hence the dilaton equation is

\[
- \Box \phi = \frac{1}{4} \frac{\partial V}{\partial \phi} + \epsilon (a + 1) \hat{T}_t^t + \frac{\epsilon}{4} e^{2(a+2)\phi} (f^2 - 1)^2 \tag{27}
\]

As a preliminary step, we consider linearizing the equations of motion, expanding the functions in powers of the small parameter \(\epsilon\)

\[
A = 1 + \epsilon A_1 + \ldots \\
B = 1 + \epsilon B_1 + \ldots \\
\phi = \phi_0 + \epsilon \phi_1 + \ldots \tag{28}
\]

To \(O(1)\), (23) gives

\[
\phi_0'^2 = 0 \tag{29}
\]

Hence \(\phi_0 = \text{const.}\) Then to \(O(\epsilon)\) we have

\[
\frac{A'}{r} + \frac{A_1}{r^2} = e^{2(a+1)\phi_0} \left(\frac{f'^2}{2} + \frac{f^2}{r^2}\right) + \frac{1}{4} e^{2(a+2)\phi_0} (f^2 - 1)^2 \\
- \frac{B'}{r} + \frac{A_1}{r^2} = e^{2(a+1)\phi_0} \left(-\frac{f'^2}{2} + \frac{f^2}{r^2}\right) + \frac{1}{4} e^{2(a+2)\phi_0} (f^2 - 1)^2 \tag{30}
\]
\[
\phi_0'' + \frac{2\phi_1'}{r} = (a + 1) e^{2(a+1)\phi_0} \left(\frac{f'^2}{2} + \frac{f^2}{r^2}\right) + \frac{a + 2}{4} e^{2(a+1)\phi_0} (f^2 - 1)^2
\]

For large \(r\), \(f \approx 1\) and
\[(A_1 r)' = e^{2(a+1)\phi_0}
B_1 = -A_1\]
\[\phi_1'' + \frac{2\phi_1'}{r} = \frac{(a + 1)e^{2(a+1)\phi_0}}{r^2}\] 
(31)

Then
\[A = 1 + \epsilon \left( e^{2(a+1)\phi_0} + \frac{a_1}{r} \right) + \ldots\]
\[B = 1 - \epsilon \left( e^{2(a+1)\phi_0} + \frac{a_1}{r} \right) + \ldots\]
\[\phi = \phi_0 + \epsilon \left( c_1 + \frac{c_2}{r} + (a + 1)e^{2(a+1)\phi_0} \ln r \right)\]
(32)

This agrees with the linearised result of Barros and Romero [16] (see also Banerjee et al. [17]) who studied global monopoles in Brans-Dicke gravity, however, as we will show, it is not enough to find a linearised solution, one must also consider self-consistency of the approximation one is using.

It appears from (32) that we may have an asymptotically locally flat spacetime, at least in the Einstein frame, however, note that \(\phi_1\) is divergent unless \(a = -1\). Hence, unless \(a = -1\), the linearized approximation ceases to be valid for \(r \simeq e^{1/\epsilon}\) and we must therefore consider the back reaction of the dilaton on the spacetime. In fact by studying the full field equations, we can show that if \(a \neq -1\) no such asymptotically locally flat spacetime exists for the global monopole in massless dilatonic gravity.

First note that from (23) and (24) we have
\[\left[ r(1 - A^{-1}) \right]' = \epsilon r^2 \left( e^{2(a+1)\phi} \left( \frac{f'^2}{2A} + \frac{f^2}{r^2} \right) + \frac{1}{4} e^{2(a+2)\phi} (f^2 - 1)^2 \right) + \frac{\phi'^2r^2}{A}\] 
(33)
\[\left[ \ln(AB) \right]' = \epsilon e^{2(a+2)\phi} rf'^2 + 2r\phi'^2\] 
(34)

For a locally asymptotically flat spacetime we want \(A, B \to \text{const}\) as \(r \to \infty\). Hence by integrating (34) between zero and infinity we see that the integrals
\[I_1 = \int_0^\infty e^{2(a+2)\phi} rf'^2 dr\]
\[I_2 = \int_0^\infty r\phi'^2 dr\] 
(35)

must be convergent. Now consider \(G_0^0 - G_r^r - 2G_\theta^\theta = 2R_0^0\):
\[\left( \frac{r^2(\sqrt{B})'}{\sqrt{A}} \right)' = -\frac{1}{4} \epsilon \sqrt{AB} r^2 e^{2(a+2)\phi} (f^2 - 1)^2\] 
(36)

Since \(A, B \to \text{const.}\), asymptotically this gives
\[\sqrt{AB} r^2 e^{2(a+2)\phi} (f^2 - 1)^2 = o(1)\] 
(37)
as \(r \to \infty\). Since \(A \to \text{const} \neq 1\) as \(r \to \infty\), (33) gives
\[ \epsilon e^{2(a+1)\phi} f^2 r^2 + \epsilon e^{2(a+1)\phi} f^2 + \frac{1}{4} \epsilon e^{2(a+2)\phi} r^2 (f^2 - 1)^2 + \frac{\phi^2 r^2}{A} \sim \kappa_1 \] (38)

at infinity, where \( \kappa_1 \) is a constant. But the convergence of the integrals \( I_i \) implies the integrands are \( o(1/r) \) as \( r \to \infty \). Together with (37) this means all but one term of the left hand side of (38) disappears at infinity. That is

\[ \epsilon e^{2(a+1)\phi} f^2 \sim \kappa_1 \] (39)

as \( r \to \infty \).

Now consider the dilaton equation (27). Near infinity we have

\[ \frac{1}{\sqrt{AB}} \left( \sqrt{\frac{B}{A}} r^2 \phi' \right)' \sim \epsilon (a + 1) e^{2(a+1)\phi} f^2 \sim \kappa_2 \] (40)

since \( a \neq -1 \). That is

\[ \phi' \sim \frac{A\kappa_2}{r} \] (41)

Hence

\[ r^2 \phi'^2 \sim A^2 \kappa_2^2 \] (42)

But from above, the convergence of \( I_2 \) implies \( r^2 \phi'^2 \to 0 \) as \( r \to \infty \). Hence we have a contradiction, and no non-singular, locally asymptotically flat spacetime exists for the monopole. What has happened is that the energy density depends on \( e^{2(a+1)\phi} \), which in turn is driven by the energy density, thus causing a disastrous feedback effect.

Now consider the case \( a = -1 \). The dilaton equation is now

\[ \left( \sqrt{\frac{B}{A}} r^2 \phi' \right)' = \frac{\epsilon}{4} r^2 \sqrt{AB} e^{2\phi} (f^2 - 1)^2 \] (43)

which, in the linearized approximation, can be integrated to give

\[ \phi_1 = -\frac{1}{4r} \int_0^r \xi^2 (f^2(\xi) - 1)^2 d\xi - \frac{1}{4} \int_r^\infty \xi (f^2(\xi) - 1)^2 d\xi \sim -\frac{\gamma_1}{r} \text{ as } r \to \infty \] (44)

where

\[ \gamma_1 = \frac{1}{4} \int_0^\infty \xi^2 (f^2(\xi) - 1)^2 d\xi \simeq 0.675 \] (45)

Thus \( \epsilon \phi_1 \) remains safely of order \( \epsilon \), and the linearized approximation is consistent, \( A \) and \( B \) taking the Barriola-Vilenkin (BV) form [1].

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B. Massive dilatonic gravity

Now consider the case of a massive dilaton. We use $V(\phi) = 2m^2\phi^2$ where the mass $m$ is measured in units of the Higgs mass. Obviously we do not expect this to be the exact form of the potential, however, for small perturbations of the dilaton away from its vacuum value, we might expect a quadratic form to be a good approximation. Naturally we will have to check the self consistency of such an approach. We expect $10^{-11} \leq m \leq 1$, representing a range for the unknown dilaton mass of 1 TeV - $10^{15}$ GeV. The dilaton equation is

$$\left(\frac{B'}{2AB} - \frac{A'}{2A^2} + \frac{2}{Ar}\right)\phi' + \frac{\phi''}{A} - m^2\phi =$$

$$\epsilon \left((a + 1)e^{2(a+1)\phi} \left(\frac{f'^2}{2A} + \frac{f^2}{r^2}\right) + \frac{a + 2}{4} e^{2(a+2)\phi}(f^2 - 1)^2\right)$$

(46)

Again we expand about flat space

$$A = 1 + \epsilon A_1 + \ldots$$

$$B = 1 + \epsilon B_1 + \ldots$$

(47)

To $O(\epsilon)$, (46) gives

$$\phi'' + \frac{2}{r}\phi' - m^2\phi = (a + 1) \left(\frac{f'^2}{2} + \frac{f^2}{r^2}\right) + \frac{a + 2}{4}(f^2 - 1)^2$$

(48)

which can be integrated to give

$$\phi = -\frac{1}{m} e^{-mr} \int_0^r \xi \sinh m\xi \left((a + 1) \left(\frac{f'^2}{2} + \frac{f^2}{r^2}\right) + \frac{a + 2}{4}(f^2 - 1)^2\right) d\xi$$

$$- \frac{1}{m^2} \sinh mr \int_1^\infty e^{-m\xi} \left((a + 1) \left(\frac{f'^2}{2} + \frac{f^2}{\xi^2}\right) + \frac{a + 2}{4}(f^2 - 1)^2\right) d\xi$$

(49)

Outside the core we find the leading order behaviour of $\phi_1$ to be

$$\phi_1 \simeq -\frac{(a + 1)}{m} \int_0^r \frac{e^{-mu}}{r^2 - u^2} du \simeq -\frac{(a + 1)}{m^2 r^2}$$

(50)

for $a \neq -1$, and $\phi_1 = O(1/(m^2 r^4))$ for $a = -1$. $A$ and $B$ will once again take their BV forms. Thus in contrast to the local cosmic string, for all values of $a$ there is a diffuse dilaton cloud.

We must now check that the dilaton remains small for all values of $m$ under consideration. This is not only to verify the consistency of our linearization prescription for solving the equations of motion, but also to justify taking a quadratic approximation to the dilaton potential. To see this, note that

$$\phi_1(0) = -\int_0^{\infty} \xi e^{-m\xi} \left((a + 1) \left(\frac{f'^2}{2} + \frac{f^2}{\xi^2}\right) + \frac{a + 2}{4}(f^2 - 1)^2\right) d\xi$$

$$= -\gamma_2 - (a + 1) \int_1^{\infty} \frac{e^{-m\xi}}{\xi} d\xi$$

(51)
where
\[ \gamma_2 = \int_0^\infty \xi e^{-m\xi} \left( (a + 1) \frac{f^2}{2} + \frac{a + 2}{4} (f^2 - 1)^2 \right) d\xi + (a + 1) \int_0^1 \frac{f^2 e^{-m\xi}}{\xi} d\xi \simeq 1 \] (52)
is approximately independent of \( m \), and the core of the monopole is taken to be 1 for convenience. But,
\[ \int_1^\infty \frac{e^{-m\xi}}{\xi} d\xi = \int_m^\infty \frac{e^{-u}}{u} du \sim -\ln m + \ldots \] (53)
hence \( \phi_1(0) = \mathcal{O}((a + 1) \ln m) \), (for \( a = -1 \), \( \phi_1(0) \simeq 0.35 \)) and we can therefore justify the approximation of taking a quadratic potential for \( \phi \) (as well as the linearized approximation) provided \( |\ln m| \ll \epsilon^{-1} \), which is easily satisfied by the parameter ranges under consideration. A plot of the dilaton field for various values of \( m \) and \( a \neq -1 \) is shown in figure 2. A plot of the dilaton field for \( a = -1 \) is shown in figure 3.

**IV. DISCUSSION**

We have derived the metric and dilaton field of a global monopole in low energy string gravity for an arbitrary coupling of the monopole lagrangian to the dilaton: \( e^{2a\phi} \mathcal{L} \). For the massless dilaton, this modification generically destroys the good global behaviour of the monopole, making it singular. This is because the dilaton multiplies the energy density of the monopole worsening the already strong gravitational effect. For \( a = -1 \), the metric is nonsingular and of the BV form in the Einstein frame, the dilaton taking the asymptotic form \( \phi \simeq -\epsilon \gamma_1/r \), where \( \gamma_1 \) is given by (45). In the string frame, the metric is given asymptotically by
\[ \hat{ds}^2 = \left( 1 - \frac{2(M + \epsilon \gamma_1)}{\hat{r}} \right) d\hat{r}^2 - \left( 1 - \frac{2(M + \epsilon \gamma_1)}{\hat{r}} \right)^{-1} d\hat{r}^2 - (1 - \epsilon) \hat{r}^2 d\Omega_\perp^2 \] (54)
with respect to suitably rescaled coordinates \( \hat{r}, \hat{t} \). Inputting our values for \( M \) and \( \gamma_1 \), we see that in the string frame the ADM mass of the global monopole is \( M \simeq -0.055\epsilon \) which is substantially smaller than the Einstein ADM mass, but still negative.

For the massive dilaton, to leading order in the Einstein frame the metric takes the BV form. The dilaton asymptotes \(-\epsilon/\sqrt{m^2 r^2}\) for \( a \neq -1 \), and \(-\epsilon/(m^2 r^4)\) for \( a = -1 \). This power law fall-off of a massive scalar field seems counterintuitive until one remembers that the dilaton is in fact part of the gravitational sector of the theory, and therefore couples to the energy momentum of the global monopole. The slow fall off of this energy momentum is what supports the rather diffuse dilaton cloud. We therefore have a rather different, nebulous, dilaton cloud surrounding the global monopole as opposed to the well defined

\(^1\)We find for \( a = 0 \), that \( \gamma_2 = 0.3 \) for \( m = 1 \), 0.9 for \( m = 0.1 \), and 1.1 for \( m \leq 0.01 \) to 2 significant figures.
cloud surrounding a local cosmic string [13]. To leading order in $1/r$, the asymptotic metric in both string and Einstein frames is the same, and is identical to the BV result.

Our results indicate that astrophysical bounds [9,18] on global monopoles obtained from their gravitational or metric field will be little altered by the dilaton, except if we approach close enough to feel its gravitational mass. On the cosmological scales therefore, the gravity of global monopoles is unchanged. One might wish to ask how the dilaton effects monopole bounds more directly, following the lines put forward by Damour and Vilenkin [12]. Here we note that for $a \neq -1$, the dilaton fall-off is as a power law in $mr$, therefore determined by a scale $m^{-1}$. We therefore expect the Damour-Vilenkin bound to hold, and such global monopoles will be inconsistent with a low (TeV) mass dilaton. For $a = -1$, the dilaton falls off as $(mr^2)^{-2}$, hence determined by $m^{-1/2}$ (or $m^{-1/2}(\sqrt{\eta})^{1/2}$ before rescaling). We should therefore substitute $m_{\phi}^{1/2} m_{\text{Higgs}}^{1/2}$ for $m_{\phi}$ in the Damour-Vilenkin bound. This has the effect of weakening the bound. For example, for a TeV mass dilaton, Damour and Vilenkin quote a bound on the symmetry breaking scale of $\eta \leq 10^{13}$GeV for the global monopole [12]. For $a = -1$, this is weakened to $\eta \leq 10^{14}$ GeV. Although this is obviously a weaker bound, it is in contrast to the local cosmic string where the bound would appear to be removed for $a = -1$.

In conclusion, the gravitational field of a global monopole in the presence of a massive dilaton cloud is broadly similar to that in Einstein gravity, however, the presence of a diffuse dilaton cloud leads to bounds on the energy scale of the global monopole due to its dilaton production [12]. In contrast, for a massless dilaton the spacetime is only regular if $a = -1$, and even then the ADM mass is significantly different from Einstein gravity.

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FIG. 1. The monopole field $f(r)$. 
FIG. 2. The dilaton field surrounding a global monopole. The factor of \((a + 1)\epsilon\) has been factored out of the dilaton. Note the contrast in the logarithmic dependence of the amplitude compared to the reciprocal dependence of the fall off on the mass of the dilaton.
FIG. 3. The dilaton field for $a = -1$. Note that while the horizontal scale is the same as for figure 2, the vertical scale is an order of magnitude less. The effect of changing $m$ is much less pronounced, and one sees the amplitude approaching a limit of approximately $-0.35$. 