Correlation among extinction efficiency and other parameters in an aggregate dust model

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Abstract We study the extinction properties of highly porous Ballistic Cluster-Cluster Aggregate dust aggregates in a wide range of complex refractive indices \((1.4 \leq n \leq 2.0, 0.001 \leq k \leq 1.0)\) and wavelengths \((0.11 \mu m \leq \lambda \leq 3.4 \mu m)\). An attempt has been made for the first time to investigate the correlation among extinction efficiency \((Q_{\text{ext}})\), composition of dust aggregates \((n, k)\), wavelength of radiation \((\lambda)\) and size parameter of the monomers \((x)\). If \(k\) is fixed at any value between 0.001 and 1.0, \(Q_{\text{ext}}\) increases with increase of \(n\) from 1.4 to 2.0. \(Q_{\text{ext}}\) and \(n\) are correlated via linear regression when the cluster size is small, whereas the correlation is quadratic at moderate and higher sizes of the cluster. This feature is observed at all wavelengths (ultraviolet to optical to infrared). We also find that the variation of \(Q_{\text{ext}}\) with \(n\) is very small when \(\lambda\) is high. When \(n\) is fixed at any value between 1.4 and 2.0, it is observed that \(Q_{\text{ext}}\) and \(k\) are correlated via a polynomial regression equation (of degree 1, 2, 3 or 4), where the degree of the equation depends on the cluster size, \(n\) and \(\lambda\). The correlation is linear for small size and quadratic/cubic/quartic for moderate and higher sizes. We have also found that \(Q_{\text{ext}}\) and \(x\) are correlated via a polynomial regression (of degree 3, 4 or 5) for all values of \(n\). The degree of regression is found to be \(n\) and \(k\)-dependent. The set of relations obtained from our work can be used to model interstellar extinction for dust aggregates in a wide range of wavelengths and complex refractive indices.

Key words: light scattering — ISM: dust, extinction

1 INTRODUCTION

The studies of cometary and interplanetary dust indicate that cosmic dust grains are likely to be fluffy, porous and composites of many small grains fused together, due to dust-gas interactions, grain-grain collisions and various other processes (Krueger & Kissel 1989; Greenberg & Hage 1990; Wolff et al. 1994). Porous, composite aggregates are often modeled as clusters of small spheres (known as “monomers”), agglomerated under various aggregation rules. Here grain aggregates are assumed to be fluffy sub-structured collections of very small particles loosely attached to one another. Each particle is assumed to consist of a single material, such as silicates or carbon, as formed in the various separate sources of cosmic dust. Extinction generally takes place whenever electromagnetic radiation propagates through a medium containing small particles. The spectral dependence of extinction, or extinction curve, is a function of the structure, composition and size distribution of the particles. The study of interstellar extinction provides us with useful information for understanding the properties of the dust.

It is now well accepted from observation and laboratory analysis of interplanetary dust particles that cosmic dust grains are fluffy aggregates or porous with irregular shapes (Brownlee 1985; Mathis & Whiffen 1989; Greenberg & Hage 1990). Using the Discrete Dipole Approximation (DDA) technique, several investigators studied the extinction properties of the composite grains (Wolff et al. 1994, 1998; Voshchinnikov et al. 2006; Vaidya & Gupta 1999; Vaidya et al. 2007; Vaidya & Gupta 2009). Iatì et al. (2004) studied opti-
cal properties of composite grains as grain aggregates of amorphous carbon and astronomical silicates, using the superposition transition matrix approach. Recently, Mazarbchubia & Das (2017) studied the light scattering properties of aggregate particles in a wide range of complex refractive indices and wavelengths to investigate the correlation among different parameters, e.g., the positive polarization maximum, the amplitude of negative polarization, geometric albedo, refractive indices and wavelength. The simulations were performed using the Superposition T-matrix code with Ballistic Cluster-Cluster Aggregate (BCCA) particles of 128 monomers and Ballistic Aggregate particles of 512 monomers.

The extinction efficiency of dust aggregates depends on aggregate size, composition and wavelength of incident radiation. The dependence of complex refractive index \((n, k)\) on \(Q_{\text{ext}}\) was studied by many groups in the past for spherical and irregular particles using different scattering theories (Mie theory, DDA approach, T-matrix theory, etc.). However, no correlation equations were reported earlier by any group. In this paper, we study the extinction properties of randomly oriented porous dust aggregates with a wide range of complex refractive indices and wavelengths of incident radiation. An attempt has been made for the first time to investigate the correlation among extinction efficiency \(Q_{\text{ext}}\), complex refractive indices \((m = n \pm ik)\), wavelength \((\lambda)\) and the size parameter of a monomer \((a)\).

2 NUMERICAL COMPUTATIONS

We have constructed the aggregates using the ballistic aggregation procedure (Meakin 1983, 1984) by applying two different models of cluster growth; first via single-particle aggregation and then through cluster-cluster aggregation. These aggregates were built by randomly hitting and sticking particles together. The first one is called Ballistic Particle-Cluster Aggregate (BPCA) when the method allows only single particles to join the cluster of particles. If the method allows clusters of particles to stick together, the aggregate is called a BCCA. Actually, BPCA clusters are more compact than BCCA clusters (Mukai et al. 1992). The porosity of BPCA and BCCA particles of 128 monomers has the values 0.90 and 0.94, respectively. The fractal dimensions of BPCA and BCCA particles are given by \(D \approx 3\) and \(\approx 2\), respectively (Meakin 1984). A systematic explanation with the dust aggregate model is already discussed in our previous work (Das et al. 2008). It can be noted that the structures of these aggregates are similar to those of Interplanetary Dust Particles collected in the stratosphere of Earth (Brownlee 1985). It is also well understood from laboratory diagnosis that particle coagulation in the solar nebula grows under the BCCA process (Wurm & Blum 1998).

The general extinction \(A_\lambda\) is given by (Spitzer 1978)

\[
A_\lambda = -2.5 \log \left( \frac{F(\lambda)}{F_0(\lambda)} \right) = 1.086 N_d Q_{\text{ext}} \sigma_d, \quad (1)
\]

where \(F(\lambda)\) and \(F_0(\lambda)\) are the observed and expected fluxes, \(N_d\) is the dust column density, \(Q_{\text{ext}}\) is the extinction efficiency factor determined from the Superposition T-matrix code and \(\sigma_d\) is the geometrical cross-section of a single particle.

The interstellar extinction curve (i.e., the variation of extinction with wavelength) is usually expressed by the ratio \(A_\lambda/E(B-V)\) versus \(1/A_\lambda\). The extinction curve covers the wavelength range 0.11 to 3.4 \(\mu\)m. The entire range consists of ultraviolet (UV), visible and infrared (IR) regions. The IR range corresponds to near IR, i.e. 0.750 to 2.5 \(\mu\)m, the visible range (0.38 to 0.76 \(\mu\)m) and the UV range (with the last part of violet in the visible spectrum to 0.11 \(\mu\)m).

The radius of an aggregate particle can be described by the radius of a sphere of equal volume given by \(a_v = a_m N^{1/3}\), where \(N\) is the number of monomers in the aggregate and \(a_m\) is the monomer’s radius of aggregates. We have found from a literature survey that most of the work related to interstellar extinction considered a normal size range of 0.001 to 0.250 \(\mu\)m, with a size distribution (mainly an MRN distribution) (Jones 1988; Whittet 2003; Vaidya et al. 2007; Das et al. 2010). They found an ‘optimum’ for the range of the cluster size generally used. The above size range of the monomer is more or less capable of evaluating an average observed interstellar extinction curve. If we consider \(N = 64\) and \(a_m\) in the range 0.001 to 0.065 \(\mu\)m (with a step size of 0.004 \(\mu\)m), then \(a_v\) will be 0.004 to 0.26 \(\mu\)m. This size range is almost comparable to the size range used by other investigators.

We use JaSTA-2 (second version of the Java Superposition T-matrix Application) (Halder & Das 2017), which is an upgraded version of JaSTA (Halder et al. 2014), to execute our computations. This routine is based on Mackowski & Mishchenko (1996)’s Superposition T-matrix code. All versions of JaSTA are freely available for download from
Table 1 Coefficients of Eq. (2) are shown for $\lambda = 0.11, 0.20, 0.30$ and 0.90 $\mu m$

| $\lambda$ | $k$ | $A_k$ | $B_k$ | $C_k$ |
|-----------|-----|-------|-------|-------|
| $0.11 \mu m$ | 0.001 | $-13.254$ | 46.876 | $-33.026$ |
|           | 0.01  | $-12.937$ | 45.862 | $-32.263$ |
|           | 0.1   | $-10.123$ | 36.537 | $-25.149$ |
|           | 0.5   | $-2.651$  | 10.490 | $-3.731$  |
|           | 1.0   | $-0.346$  | 1.555  | 4.661     |
| $0.20 \mu m$ | 0.001 | $-7.815$  | 36.995 | $-33.540$ |
|           | 0.01  | $-7.572$  | 35.969 | $-32.527$ |
|           | 0.1   | $-5.439$  | 26.899 | $-23.511$ |
|           | 0.5   | $-0.969$  | 6.779  | $-2.590$  |
|           | 1.0   | 0.010     | 1.141  | 4.359     |
| $0.30 \mu m$ | 0.001 | 1.651     | 2.290  | $-5.271$  |
|           | 0.01  | 1.647     | 2.191  | $-5.054$  |
|           | 0.1   | 1.589     | 1.335  | $-3.095$  |
|           | 0.5   | 3.527     | $-0.999$ | 3.939    |
|           | 1.0   | 0.681     | $-0.749$ | 5.243    |
| $0.90 \mu m$ | 0.001 | 0.217     | $-0.278$ | 0.032    |
|           | 0.01  | 0.217     | $-0.285$ | 0.068    |
|           | 0.1   | 0.213     | $-0.363$ | 0.429    |
|           | 0.5   | 0.230     | $-0.794$ | 2.046    |
|           | 1.0   | 0.328     | $-1.543$ | 4.092    |

Table 2 Coefficients of Eq. (2a)–(2c) at $\lambda = 0.11, 0.20, 0.30$ and 0.90 $\mu m$

| $\lambda$ | Coeff-1 | Coeff-2 | Coeff-3 | Coeff-4 |
|-----------|---------|---------|---------|---------|
| $0.11 \mu m$ | $A_k$  | 10.161  | $-31.889$ | 34.662  | $-13.280$ |
|           | $B_k$  | -26.328 | 94.623  | $-113.730$ | 46.990  |
|           | $C_k$  | 11.321  | -58.956 | 85.407  | $-33.111$ |
| $0.20 \mu m$ | $A_k$  | 15.474  | $-35.001$ | 27.379  | $-7.842$  |
|           | $B_k$  | -60.269 | 139.790 | $-115.490$ | 37.110  |
|           | $C_k$  | 55.593  | -131.620 | 114.040 | $-33.654$ |
| $0.30 \mu m$ | $A_k$  | 1.586   | $-2.127$  | $-0.431$  | 1.652    |
|           | $B_k$  | -8.473  | 16.673  | $-11.251$ | 2.302    |
|           | $C_k$  | 10.310  | -24.081 | 24.309  | $-5.295$  |
| $0.90 \mu m$ | $A_k$  | 0.001   | 0.171   | $-0.060$  | 0.217    |
|           | $B_k$  | -0.024  | $-0.430$ | $-0.812$  | $-0.277$ |
|           | $C_k$  | 0.002   | 0.053   | 4.009    | 0.028    |

http://ausastro.in/jasta.html. Computations with the T-matrix code are fast and this technique gives rigorous solutions for randomly oriented ensembles of spheres. It can be noted that the results obtained from the DDA approach and the T-matrix approach are almost same. Kimura (2001) showed the results with aggregates using the DDA and the T-matrix code, and found almost the same results with both codes. We perform the computations with a wide range of complex refractive indices ($n = 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0$, and $k = 0.001, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7$ and 1.0) and wavelengths (0.11, 0.12, 0.13, 0.16, 0.175, 0.185, 0.20, 0.207, 0.22, 0.23, 0.26, 0.30, 0.365, 0.40, 0.55, 0.6, 0.7, 0.8, 0.90 and 3.4 $\mu m$). In general, the range of $n$ and $k$ which is considered in our work almost covers the range of complex refractive indices of silicate and carbon at different wavelengths. The numerical computation in the present work has been executed with a BCCA cluster consisting of 64 monomers.

We present the results for $a_m = 0.001, 0.017, 0.041$ and $0.065 \mu m$ where $a_v$ is given by 0.004, 0.068, 0.16 and 0.26 $\mu m$. The monomer size parameter ($x = \ldots$)
2\pi a_m/\lambda) is taken in a range from 0.01 to 1.6. This study mainly concentrates on investigation of correlation among $Q_{\text{ext}}$, $(n, k)$, $\lambda$ and $x$.

3 RESULTS

3.1 Dependence on Monomer Size ($a_m$)

3.1.1 Correlation between $Q_{\text{ext}}$ and $n$

We perform the computations with a wide range of complex refractive indices (1.4 ≤ $n$ ≤ 2.0 and 0.001 ≤ $k$ ≤ 1.0) and wavelengths of incident radiation (0.11 ≤ $\lambda$ ≤ 3.4 μm). To study the dependence of $n$ and $k$ on the extinction efficiency ($Q_{\text{ext}}$), we can either plot $Q_{\text{ext}}$ versus $k$ by keeping $n$ fixed or plot $Q_{\text{ext}}$ versus $n$ by keeping $k$ fixed. We first show the results for moderate size of the cluster, i.e., at $a_v = 0.16$ μm. The results at the other three sizes are also presented thereafter.

We plot $Q_{\text{ext}}$ versus $n$ at $a_v = 0.16$ μm, for $k = 0.001, 0.01, 0.1, 0.5$ and 1.0 (although we have executed the code with all eight values of $k$ mentioned above), in a single frame, for $\lambda = 0.11, 0.20, 0.30$ and 0.90 μm, which is shown in Figure 1. It can be noted that the plots are shown for four wavelengths although the computations have been performed for all the wavelengths.

It can be observed from Figure 1 that if $k$ is fixed at any value between 0.001 and 1.0, $Q_{\text{ext}}$ increases with an increase of $n$ from 1.4 to 2.0 at all wavelengths. However, a small decrease in $Q_{\text{ext}}$ is also noticed at $\lambda = 0.11$ μm when $n > 1.7$ and $k$ is low. The variation of $Q_{\text{ext}}$ with $n$ is small when $k \geq 0.5$. The value of $Q_{\text{ext}}$ is small when $\lambda$ is large, i.e., $Q_{\text{ext}}$ decreases when size parameter of the monomer ($x = 2\pi a_m/\lambda$) decreases. We have also investigated that the variation of $Q_{\text{ext}}$ with $n$ is very small when $\lambda \geq 0.70$ μm.

We have found that $Q_{\text{ext}}$ and $n$ can be fitted by quadratic regression where the coefficient of determination $^1$ ($R^2$) for each equation is $\approx 0.99$. The best fit equation is given by

$$Q_{\text{ext}} = A_k n^2 + B_k n + C_k,$$

1.4 ≤ $n$ ≤ 2.0, 0.001 ≤ $k$ ≤ 1,

where $A_k$, $B_k$ and $C_k$ are $k$-dependent coefficients of Equation (2).

$^1$ The coefficient of determination is a key output of regression analysis which is interpreted as the proportion of the variance in the dependent variable that ranges from 0 to 1. A higher coefficient is an indicator of a better goodness of fit for the observations.
Fig. 2 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the real part of the refractive index ($n$) for $k = 0.001, 0.01, 0.1, 0.5$ and $1.0$ at $a_v = 0.004 \, \mu m$. The best fit curves correspond to linear regression for wavelengths (i) $0.11 \, \mu m$, (ii) $0.20 \, \mu m$, (iii) $0.30 \, \mu m$ and (iv) $0.90 \, \mu m$.

Fig. 3 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the real part of the refractive index ($n$) for $k = 0.001, 0.01, 0.1, 0.5$ and $1.0$ at $a_v = 0.068 \, \mu m$. The best fit curves correspond to quadratic regression for wavelengths (i) $0.11 \, \mu m$ and (ii) $0.60 \, \mu m$.

The coefficients obtained for different values of $k$ (only five values of $k$ are shown) are depicted in Table 1. If we plot coefficients $A_k$, $B_k$ and $C_k$ versus $k$ (where $k = 0.001, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7$ and $1.0$), we find that the best fit curves correspond to cubic regression, which have $R^2 \approx 0.99$. We do not show any figures in this case.

The coefficients are correlated with $k$ by the relations:

$$A_k = D_1 k^3 + D_2 k^2 + D_3 k + D_4 , \quad (2a)$$
$$B_k = E_1 k^3 + E_2 k^2 + E_3 k + E_4 , \quad (2b)$$
$$C_k = F_1 k^3 + F_2 k^2 + F_3 k + F_4 . \quad (2c)$$

All coefficients of Equation (2a)–(2c) are shown in Table 2. Thus knowing the coefficients, the extinction ef-
Correlation among Extinction Efficiency and Other Parameters

Fig. 4 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the real part of the refractive index ($n$) for $k = 0.001, 0.01, 0.1, 0.5$ and $1.0$ at $a_v = 0.26 \mu m$. The best fit curves correspond to cubic regression at (i) $\lambda = 0.11 \mu m$ and quadratic regression at (ii) $\lambda = 0.60 \mu m$.

Fig. 5 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the imaginary part of the refractive index ($k$) at $\lambda = 0.11, 0.16, 0.20$ and $0.26 \mu m$ at $a_v = 0.16 \mu m$. The best fit curves correspond to polynomial regression equation where the degree of equation depends on the value of $n$ and $\lambda$ (please see Sect. 3.1.2 for details).

Efficiency ($Q_{\text{ext}}$) can be calculated for any value of $n$ and $k$ from Equation (2).

In Figure 2, we report the results for $a_v = 0.004 \mu m$ at $\lambda = 0.11, 0.20, 0.30$ and $0.90 \mu m$. A strong linear correlation between $Q_{\text{ext}}$ and $n$ is seen at this size for all wavelengths from $0.11$ to $3.4 \mu m$.

In Figure 3, we plot $Q_{\text{ext}}$ versus $n$ for $a_v = 0.068 \mu m$ at $\lambda = 0.11 \mu m$ and $0.60 \mu m$. We have found that the nature is quadratic at all wavelengths from $0.11$ to $3.4 \mu m$. Finally, we show the results for $a_v = 0.26 \mu m$ at $\lambda = 0.11 \mu m$ and $0.60 \mu m$, in Figure 4. We have noticed that $Q_{\text{ext}}$ and $n$ are correlated via a cubic regression at $0.11 \mu m$, whereas the dependence is quadratic at other higher wavelengths. We do not show any equation or table in the above three cases. In summary, we can conclude that the correlation between $Q_{\text{ext}}$ and $n$ is linear.
when the cluster size is small whereas the correlation is quadratic at moderate and higher sizes of the cluster.

3.1.2 Correlation between $Q_{\text{ext}}$ and $k$

We now plot $Q_{\text{ext}}$ versus $k$ for $n = 1.4, 1.5, 1.6, 1.7, 1.8, 1.9$ and $2.0$, at $a_v = 0.16 \mu m$ where $\lambda$ is taken between $0.11 \mu m$ and $3.4 \mu m$. When $\lambda$ is between $0.11 \mu m$ and $0.26 \mu m$, we have found that $Q_{\text{ext}}$ and $k$ can be fitted via a polynomial regression equation where the degree of equation depends on the value of $n$ and $\lambda$.

In Figure 5 we show the plots for $\lambda = 0.11, 0.16, 0.20$ and $0.26 \mu m$. At $\lambda = 0.11 \mu m$, we find that $Q_{\text{ext}}$ and $k$ are correlated by (i) quartic regression when $n = 1.4$, (ii) cubic regression when $n = 1.5$ and (iii) quadratic regression when $n = 1.6, 1.7, 1.8, 1.9$ and $2.0$. Further, at $\lambda = 0.16 \mu m$, the correlation is cubic when $n = 1.5, 1.6, 1.7$ and $1.8$ and quadratic when $n = 1.4, 1.9$ and $2.0$. The correlation at $\lambda = 0.20 \mu m$ is cubic when $n = 1.6, 1.7$ and $1.8$ and quadratic when $n = 1.4, 1.5, 1.9$ and $2.0$. We also note that the correlation at $0.26 \mu m$ is quadratic when $n = 1.4, 1.5, 1.6, 1.7$ and $1.8$, and cubic when $n = 1.9$ and $2.0$. At low values of $n$, $Q_{\text{ext}}$ increases with an increase of $k$ whereas the trend is exactly opposite when $n$ is high, e.g., at $\lambda = 0.20 \mu m$, $Q_{\text{ext}}$ increases with $k$ when $n \leq 1.6$, but it decreases when $n \geq 1.7$. The vertical range of $Q_{\text{ext}}$ in the plot also decreases when $k$ increases. This range is maximum at $k = 0.001$ and minimum at $k = 1.0$.

In Figure 6, we show the plots for $0.30 \leq \lambda \leq 3.4 \mu m$ and we have found that the fit is quadratic for all values of $n$. An increase in $Q_{\text{ext}}$ with $k$ is noticed at almost all wavelengths. The vertical range of $Q_{\text{ext}}$ also decreases when $k$ increases. Further, the plot of $Q_{\text{ext}}$ with $k$ is the same at all values of $n$ when $\lambda$ is high (please see Fig. 6(iv)).

The best fit equation in the wavelength range $0.30 \mu m$ to $3.4 \mu m$ is given by

$$Q_{\text{ext}} = A_n k^2 + B_n k + C_n ,$$

$$1.4 \leq n \leq 2.0, \quad 0.001 \leq k \leq 1$$

where $A_n, B_n$ and $C_n$ are $n$-dependent coefficients of Equation (3).

The coefficients obtained for different values of $k$ are shown in Table 3. If we plot coefficients $A_n, B_n$ and $C_n$ versus $n$ (figures are not shown), we note that the best fit
curves correspond to quadratic regression, which have \( R^2 \approx 0.99 \).

Thus coefficients are given by

\[
A_n = D_1 k^2 + D_2 k + D_3, \quad (3a)
\]
\[
B_n = E_1 k^2 + E_2 k + E_3, \quad (3b)
\]
\[
C_n = F_1 k^2 + F_2 k + F_3, \quad (3c)
\]

All coefficients of Equation (3a)–(3c) are shown in Table 4. Thus, knowing the coefficients of Equation (3a–3c), the extinction efficiency \( Q_{\text{ext}} \) can also be estimated for any value of \( n \) and \( k \) from Equation (3).

In Figure 7, we plot \( Q_{\text{ext}} \) against \( k \) for \( a_v = 0.004 \) \( \mu m \) at \( \lambda = 0.11, 0.30, 0.60 \) and 0.90 \( \mu m \), although the computations have been performed for a wide range of wavelengths from 0.11 to 3.4 \( \mu m \). We observe a linear dependence at this size for all wavelengths. This linear nature becomes quadratic when the size of the cluster is \( a_v = 0.068 \) \( \mu m \).

Figure 8 shows the results for \( a_v = 0.068 \) \( \mu m \) at \( \lambda = 0.11 \) and 0.60 \( \mu m \). In Figure 9, the results obtained for \( a_v = 0.26 \) \( \mu m \) are plotted. In this case, the nature of the dependence looks similar to \( a_v = 0.16 \) \( \mu m \). \( Q_{\text{ext}} \) and \( k \) are correlated via polynomial regression equation (of degree 2, 3 or 4) where the degree of the equation depends on the real part of the refractive index \( (n) \) at \( \lambda = 0.11 \mu m \) (please see the caption of Fig. 9). The nature is quadratic when \( \lambda > 0.11 \mu m \). In summary, we can conclude that the relationship of \( Q_{\text{ext}} \) on \( k \) depends on the cluster size. The correlation is linear for small size and quadratic/cubic for moderate and higher sizes.

It is important to mention that the real part of the complex index of refraction \( (n) \) controls the effective phase speed of electromagnetic waves propagating through the medium, while the imaginary part \( k \) describes the rate of absorption of the wave. In any material, \( n \) and \( k \) are not free to vary independently of one another but rather are tightly coupled to one another via the so-called Kramer-Kronig relations. Therefore, the results presented above are quite expectable.

### 3.2 Dependence on Wavelength of Radiation \( (\lambda) \)

We now study the dependence of \( Q_{\text{ext}} \) on \( \lambda \) for a particular set of \((n, k)\) in the case of \( a_v = 0.16 \) \( \mu m \) only. We observe the following results:

(i) For \( n = 1.4, 1.5 \) and 1.6, \( Q_{\text{ext}} \) versus \( \lambda \) can be fitted via a quartic regression for \( k = 0.001, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7 \) and 1.0 (please see Fig. 10).

(ii) For \( n = 1.7, 1.8, 1.9 \) and 2.0, \( Q_{\text{ext}} \) versus \( \lambda \) can be fitted via a quartic regression in the wavelength range 0.11 – 0.40 \( \mu m \) [Fig. 11(i,iv) and Fig. 12(i,iv)] and a quadratic regression in the wavelength range 0.55 – 0.90 \( \mu m \) [Fig. 11(ii,vi) and Fig. 12(ii,vi)] for \( k = 0.001, 0.01, 0.05, 0.1 \) and 0.3. Further, \( Q_{\text{ext}} \) versus \( \lambda \) can be fitted via a quartic regression in the wavelength range 0.11 – 0.90 \( \mu m \) for higher values of \( k = 0.5, 0.7 \) and 1 [Fig. 11(iii,vi) and Fig. 12(iii,vi)]. We did not include the plots for \( \lambda = 3.4 \) \( \mu m \).

It can be noticed from Figures 10, 11 and 12 that \( Q_{\text{ext}} \) decreases with increase of \( \lambda \) when \( n \leq 1.6 \). When \( n \geq 1.7 \), \( Q_{\text{ext}} \) initially increases with increase of \( \lambda \) and reaches a maximum value, then it starts decreasing if \( \lambda \) is increased further. We also observe that \( Q_{\text{ext}} \) is maximum at \( k = 0.001 \) and minimum at \( k = 1.0 \) when \( \lambda = 0.11 \mu m \), but this trend changes at a critical value of wavelength \( (\lambda_c) \) where an exactly opposite nature is noticed. We see that \( \lambda_c \) is (i) 0.16 \( \mu m \) at \( n = 1.4 \), (ii) 0.185 \( \mu m \) at \( n = 1.5 \), (iii) 0.207 \( \mu m \) at \( n = 1.6 \), (iv) 0.22 \( \mu m \) at \( n = 1.7 \), (v) 0.26 \( \mu m \) at \( n = 1.8 \), (vi) 0.26 \( \mu m \) at \( n = 1.9 \) and (vii) 0.30 \( \mu m \) at \( n = 1.4 \). The value of \( Q_{\text{ext}} \) is maximum at \( k = 1.0 \) when \( \lambda > \lambda_c \). We do not show any equations or tables in this case.

### 3.3 Dependence on the Size Parameter of Monomer \( (x) \)

We now study the dependence of \( Q_{\text{ext}} \) on the size parameter of monomer \( (x = 2\pi a_m/\lambda) \) for \( n = 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 \) and 2.0, and \( k = 0.001, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7 \) and 1.0. A wide range of size parameter, \( 0.01 \leq x \leq 1.6 \) (where \( N = 64 \)), is considered to investigate the correlation between \( Q_{\text{ext}} \) and \( x \). The results are plotted in Figures 13, 14, 15 and 16 for \( k = 0.001, 0.01, 0.1 \) and 1.0, respectively. It can be seen from the figures that if \( x \) is fixed at any value between 0.01 and 1.6, \( Q_{\text{ext}} \) increases with an increase of \( n \). This increase is prominent when \( x > 0.2 \). The vertical range of \( Q_{\text{ext}} \) also increases with an increase of \( n \) from 1.4 to 2.0. This range is maximum (i) at \( x = 1.4 \) in the case of \( k = 0.001 \) (where \( Q_{\text{ext}} = [9.34, 3.45] \)), (ii) at \( x = 1.4 \) in the case of \( k = 0.01 \) (where \( Q_{\text{ext}} = [9.26, 3.50] \)), (iii) at \( x = 1.4 \) in the case of \( k = 0.1 \) (where \( Q_{\text{ext}} = [8.56, 3.90] \)), and (iv) at \( x = 1.12 \) in the case of \( k = 1.0 \) (where \( Q_{\text{ext}}(\text{max}) = [6.74, 5.88] \)). The slope of the \( Q_{\text{ext}} \) versus \( x \) curve increases with the increase of \( n \) from 1.4 to 2.0, which can be noted for all values of \( k \). It is also interest-
Table 3 Coefficients of Equation (3) at $\lambda = 0.30, 0.40, 0.60$ and $0.90 \mu m$

| $\lambda$ | $n$ | $A_n$  | $B_n$  | $C_n$  |
|-----------|-----|--------|--------|--------|
| 0.30 \(\mu\)m | 1.4 | -2.579 | 6.890  | 1.195  |
|           | 1.5 | -2.107 | 5.844  | 1.896  |
|           | 1.6 | -1.630 | 4.774  | 2.632  |
|           | 1.7 | -1.148 | 3.678  | 3.401  |
|           | 1.8 | -0.661 | 2.556  | 4.204  |
|           | 1.9 | -0.169 | 1.410  | 5.041  |
|           | 2.0 | 0.327  | 0.238  | 5.911  |
| 0.40 \(\mu\)m | 1.4 | -1.769 | 6.047  | 0.579  |
|           | 1.5 | -1.655 | 5.675  | 0.912  |
|           | 1.6 | -1.519 | 5.256  | 1.290  |
|           | 1.7 | -1.362 | 4.789  | 1.712  |
|           | 1.8 | -1.184 | 4.274  | 2.178  |
|           | 1.9 | -0.984 | 3.712  | 2.688  |
|           | 2.0 | -0.763 | 3.102  | 3.242  |
| 0.60 \(\mu\)m | 1.4 | -0.727 | 4.222  | 0.198  |
|           | 1.5 | -0.704 | 4.081  | 0.307  |
|           | 1.6 | -0.676 | 3.931  | 0.433  |
|           | 1.7 | -0.642 | 3.771  | 0.574  |
|           | 1.8 | -0.602 | 3.602  | 0.731  |
|           | 1.9 | -0.556 | 3.423  | 0.905  |
|           | 2.0 | -0.505 | 3.234  | 1.094  |
| 0.90 \(\mu\)m | 1.4 | -0.260 | 2.771  | 0.064  |
|           | 1.5 | -0.257 | 2.674  | 0.100  |
|           | 1.6 | -0.250 | 2.575  | 0.139  |
|           | 1.7 | -0.240 | 2.475  | 0.183  |
|           | 1.8 | -0.227 | 2.374  | 0.231  |
|           | 1.9 | -0.210 | 2.272  | 0.284  |
|           | 2.0 | -0.190 | 2.169  | 0.341  |

Table 4 Coefficients of Equation (3a–3c) at $\lambda = 0.30, 0.40, 0.60$ and $0.90 \mu m$

| $\lambda$ | Coeff-1 | Coeff-2 | Coeff-3 |
|-----------|---------|---------|---------|
| 0.30 \(\mu\)m | $A_n$ | 0.243  | 4.016  | -8.679 |
|            | $B_n$  | -1.269 | -6.771 | 18.857 |
|            | $C_n$  | 1.682  | 2.142  | -5.101 |
| 0.40 \(\mu\)m | $A_n$ | 1.069  | -1.956 | -1.126 |
|            | $B_n$  | -2.385 | 3.201  | 6.241  |
|            | $C_n$  | 2.202  | -3.048 | 0.530  |
| 0.60 \(\mu\)m | $A_n$ | 0.290  | -0.614 | -0.434 |
|            | $B_n$  | -0.481 | -0.010 | 5.179  |
|            | $C_n$  | 0.796  | -1.214 | 0.337  |
| 0.90 \(\mu\)m | $A_n$ | 0.171  | -0.466 | 0.056  |
|            | $B_n$  | -0.060 | -0.799 | 4.008  |
|            | $C_n$  | 0.217  | -0.278 | 0.028  |

To notice that the vertical range of $Q_{ext}$ at $x = 1.6$ decreases with the increase of $k$ and is lowest at $k = 1.0$. Further, the $Q_{ext}$ value does not depend much on $n$ for highly absorptive particles ($k = 1.0$) when $x < 0.5$. The variation is also small when $x > 0.5$.

We have found that $Q_{ext}$ and $x$ can be fitted by a cubic regression for all values of $n$ except 2.0, in case of $k = 0.001, 0.01$ and 0.1. The coefficient of determination ($R^2$) for each equation is $\approx 0.99$. The best fit equation is
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Table 5 All Coefficients of Eqs. (4), (5) and (6)

| $k$ | $n$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|-----|-----|------------|------------|------------|------------|
| 0.001 | 1.4 | -0.425 | 2.844 | -0.728 | 0.034 |
| 0.01 | 1.4 | -0.502 | 2.931 | -0.660 | 0.032 |
| 0.1  | 1.4 | -0.891 | 3.035 | 0.273 | 0.023 |
| 1.0  | 1.4 | -2.709 | 12.371 | -18.540 | 6.182 |
| 2.0  | 1.4 | -2.164 | -2.689 | 15.031 | -3.120 |
| 0.001 | 1.5 | -1.179 | 5.041 | -1.316 | 0.056 |
| 0.01 | 1.5 | -1.248 | 5.085 | -1.231 | 0.055 |
| 0.1  | 1.5 | -1.588 | 4.882 | -0.186 | 0.038 |
| 1.0  | 1.5 | -2.973 | 14.016 | -22.167 | 9.438 |
| 2.0  | 1.5 | -2.540 | 7.120 | -0.715 | 0.055 |
| 0.001 | 1.6 | -2.239 | 7.681 | -1.976 | 0.082 |
| 0.01 | 1.6 | -3.714 | 10.772 | -2.617 | 0.080 |
| 0.1  | 1.6 | -3.726 | 12.358 | -1.276 | 0.072 |
| 1.0  | 1.6 | -3.253 | 15.774 | -26.024 | 12.846 |
| 2.0  | 1.6 | -4.222 | 21.861 | -39.192 | 24.073 |
| 0.001 | 1.7 | -3.654 | 10.775 | -2.712 | 0.109 |
| 0.01 | 1.7 | -5.367 | 14.060 | -3.475 | 0.136 |
| 0.1  | 1.7 | -7.189 | 17.404 | -4.011 | 0.153 |
| 1.0  | 1.7 | -3.540 | 17.603 | -30.058 | 16.378 |
| 2.0  | 1.7 | -4.625 | 24.319 | -44.317 | 28.233 |
| 0.001 | 1.8 | -5.362 | 14.173 | -3.475 | 0.136 |
| 0.01 | 1.8 | -7.242 | 17.624 | -4.175 | 0.158 |
| 0.1  | 1.8 | -9.182 | 21.044 | -5.672 | 0.180 |
| 1.0  | 1.8 | -6.569 | 15.046 | -2.302 | 0.097 |
| 2.0  | 1.8 | -8.625 | 24.319 | -44.317 | 28.233 |

**Example:**

The correlation between $Q_{\text{ext}}$ and $x$ is found to be a quintic regression for all values of $n$ at $k = 1.0$. The best fit equation is given by

$$Q_{\text{ext}} = \gamma_1 x^5 + \gamma_2 x^4 + \gamma_3 x^3 + \gamma_4 x^2 + \gamma_5 x + \gamma_6,$$  \hspace{1cm} (6)

where $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$ and $\gamma_6$ are $n$-dependent constants of Equation (6). The constants are given in Table 5.

Equations (4), (5) and (6) are very useful in estimating $Q_{\text{ext}}$, where one can generate a large data set for $Q_{\text{ext}}$ for a selected set of $n$, $k$, $a_m$ and $\lambda$.

### 4 RESULTS FROM CORRELATION EQUATIONS

In the previous sections, we have obtained a set of correlation equations which can be used to calculate the extinction efficiency of dust aggregates with a wide range of size of aggregates and wavelength of radiation. We first calculate $Q_{\text{ext}}$ from relations (2) and (3) for BCCA particles with $N = 64$ and $a_m = 0.041 \mu m$, for selected values of $n$, $k$ and $\lambda$. The calculated values are compared with the computed values obtained using the Superposition T-matrix code. The results are shown in Tables 6 and 7. We also estimate $Q_{\text{ext}}$ using relations...
Table 6 $Q_{ext}$ for selected values of $n$ and $k$ from the relation (using Eq. (2)) and computations (from the simulations) where $a_m = 0.041 \, \mu m$ and $0.11 \leq \lambda \leq 3.4 \, \mu m$. The difference between correlation equation and computed value is $\text{Diff.} = |Q_{ext} \ (corr) - Q_{ext} \ (comp)|$.

| $\lambda$ | $n$ | $k$ | $Q_{ext} \ (corr)$ | $Q_{ext} \ (comp)$ | Diff. |
|-----------|-----|-----|-------------------|-------------------|-------|
| 0.11      | 1.5 | 0.001 | 7.487           | 7.555            | 0.068 |
|           | 1.7 | 0.01  | 8.316           | 8.353            | 0.037 |
|           | 1.9 | 0.1   | 7.729           | 7.683            | 0.046 |
| 0.20      | 1.5 | 0.001 | 4.368           | 4.305            | 0.063 |
|           | 1.7 | 0.01  | 6.738           | 6.771            | 0.033 |
|           | 1.9 | 0.1   | 7.963           | 7.975            | 0.012 |
| 0.30      | 1.5 | 0.001 | 1.881           | 1.842            | 0.039 |
|           | 1.7 | 0.01  | 3.431           | 3.441            | 0.010 |
|           | 1.9 | 0.1   | 5.179           | 5.205            | 0.026 |
| 0.40      | 1.5 | 0.001 | 0.906           | 0.895            | 0.011 |
|           | 1.7 | 0.01  | 1.753           | 1.749            | 0.004 |
|           | 1.9 | 0.1   | 3.057           | 3.070            | 0.013 |
| 0.55      | 1.5 | 0.001 | 0.390           | 0.388            | 0.002 |
|           | 1.7 | 0.01  | 0.768           | 0.768            | 0.000 |
|           | 1.9 | 0.1   | 1.513           | 1.513            | 0.000 |
| 0.70      | 1.5 | 0.001 | 0.205           | 0.203            | 0.002 |
|           | 1.7 | 0.01  | 0.406           | 0.406            | 0.000 |
|           | 1.9 | 0.1   | 0.879           | 0.880            | 0.001 |
| 0.90      | 1.5 | 0.001 | 0.103           | 0.102            | 0.001 |
|           | 1.7 | 0.01  | 0.208           | 0.208            | 0.000 |
|           | 1.9 | 0.1   | 0.508           | 0.509            | 0.001 |
| 3.4       | 1.5 | 0.001 | 0.002           | 0.002            | 0.001 |
|           | 1.7 | 0.01  | 0.008           | 0.009            | 0.001 |
|           | 1.9 | 0.1   | 0.056           | 0.056            | 0.001 |

Table 7 $Q_{ext}$ for selected values of $n$ and $k$ from the relation (using Eq. (3)) and computations (from the simulations) where $a_m = 0.041 \, \mu m$ and $0.30 \leq \lambda \leq 3.4 \, \mu m$. The difference between correlation equation and computed value is $\text{Diff.} = |Q_{ext} \ (corr) - Q_{ext} \ (comp)|$.

| $\lambda$ | $n$ | $k$ | $Q_{ext} \ (corr)$ | $Q_{ext} \ (comp)$ | Diff. |
|-----------|-----|-----|-------------------|-------------------|-------|
| 0.30      | 1.5 | 0.001 | 1.902           | 1.842            | 0.060 |
|           | 1.7 | 0.01  | 3.438           | 3.441            | 0.003 |
|           | 1.9 | 0.1   | 5.180           | 5.205            | 0.025 |
| 0.40      | 1.5 | 0.001 | 0.918           | 0.895            | 0.023 |
|           | 1.7 | 0.01  | 1.760           | 1.749            | 0.011 |
|           | 1.9 | 0.1   | 3.049           | 3.070            | 0.021 |
| 0.55      | 1.5 | 0.001 | 0.394           | 0.388            | 0.006 |
|           | 1.7 | 0.01  | 0.770           | 0.768            | 0.002 |
|           | 1.9 | 0.1   | 1.513           | 1.513            | 0.000 |
| 0.70      | 1.5 | 0.001 | 0.205           | 0.203            | 0.002 |
|           | 1.7 | 0.01  | 0.407           | 0.406            | 0.001 |
|           | 1.9 | 0.1   | 0.879           | 0.880            | 0.001 |
| 0.90      | 1.5 | 0.001 | 0.102           | 0.102            | 0.000 |
|           | 1.7 | 0.01  | 0.208           | 0.208            | 0.000 |
|           | 1.9 | 0.1   | 0.509           | 0.509            | 0.000 |
| 3.4       | 1.5 | 0.001 | 0.002           | 0.002            | 0.000 |
|           | 1.7 | 0.01  | 0.008           | 0.009            | 0.001 |
|           | 1.9 | 0.1   | 0.056           | 0.056            | 0.000 |
Fig. 7 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the imaginary part of the refractive index ($k$) for $n = 1.4, 1.5, 1.6, 1.7, 1.8, 1.9$ and $2.0$ at $a_v = 0.004 \mu m$. The best fit curves correspond to linear regression for wavelengths (i) $0.11 \mu m$, (ii) $0.30 \mu m$, (iii) $0.60 \mu m$ and (iv) $0.90 \mu m$.

Fig. 8 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the imaginary part of the refractive index ($k$) for $n = 1.4, 1.5, 1.6, 1.8$ and $2.0$ at $a_v = 0.068 \mu m$. The best fit curves correspond to quadratic regression for wavelengths (i) $0.11 \mu m$ and (ii) $0.60 \mu m$.

(4), (5) and (6) for selected values of $x$, $n$ and $k$, and the result is shown in Table 8. It can be seen that the values obtained from computation match well with the results obtained from correlation equations.

In general, to model the interstellar extinction, one needs to execute the light scattering code with different values of $a_m$ (or $a_v$) and wavelength ($\lambda$), which is very time consuming. Using a size distribution for aggregates, it is possible to obtain the average extinction curves for silicate and graphite (and/or amorphous carbon) particles. With a suitable mixing among them, the extinction curve versus different wavelengths can be generated, which can be fitted well with an observed extinction curve. Some good pieces of work on modeling were already done for aggregate particles. Some preliminary results on modeling interstellar extinction us-

\[ Q_{\text{ext}}(i) = \lambda = 0.11 \mu m, \quad a_v = 0.004 \mu m \]

\[ Q_{\text{ext}}(ii) = \lambda = 0.30 \mu m, \quad a_v = 0.004 \mu m \]

\[ Q_{\text{ext}}(iii) = \lambda = 0.60 \mu m, \quad a_v = 0.004 \mu m \]

\[ Q_{\text{ext}}(iv) = \lambda = 0.90 \mu m, \quad a_v = 0.004 \mu m \]
Extinction efficiency ($Q_{\text{ext}}$) is plotted against the imaginary part of the refractive index ($k$) for $n = 1.4, 1.5, 1.6, 1.8$ and $2.0$ at $a_v = 0.26 \mu m$. $Q_{\text{ext}}$ and $k$ are correlated via polynomial regression equations, where the degree of regression is found to be wavelength dependent: (i) at $\lambda = 0.11 \mu m$, the correlation is cubic for $n = 1.4, 1.5, 1.6, 1.7$ and $2.0$, and quartic at $n = 1.8$ and $1.9$, and (ii) at $\lambda = 0.60 \mu m$, the correlation is quadratic at all values of $n$.

Table 8 $Q_{\text{ext}}$ for selected values of $n$, $k$ and $x$ from the relation (using Eqs. (4), (5) and (6)) and computations (from the simulations). The difference between correlation equation and computed value is Diff. = $|Q_{\text{ext}} \text{(corr)} - Q_{\text{ext}} \text{(comp)}|$. 

| $x$ | $n$ | $k$ | $Q_{\text{ext}} \text{(corr)}$ | $Q_{\text{ext}} \text{(comp)}$ | Diff. |
|-----|-----|-----|------------------|------------------|------|
| 0.286 | 1.5 | 0.001 | 0.064 | 0.092 | 0.028 |
| 0.286 | 1.7 | 0.01 | 0.152 | 0.207 | 0.055 |
| 0.286 | 1.9 | 0.1 | 0.514 | 0.506 | 0.008 |
| 0.468 | 1.5 | 0.001 | 0.424 | 0.388 | 0.036 |
| 0.468 | 1.7 | 0.01 | 0.862 | 0.768 | 0.094 |
| 0.468 | 2.0 | 1.0 | 4.183 | 4.205 | 0.022 |
| 0.706 | 1.5 | 0.001 | 1.225 | 1.136 | 0.089 |
| 0.706 | 1.9 | 0.1 | 3.660 | 3.688 | 0.028 |
| 0.706 | 2.0 | 0.1 | 4.169 | 4.252 | 0.083 |
| 0.972 | 1.5 | 0.001 | 2.459 | 2.446 | 0.013 |
| 0.972 | 1.7 | 0.01 | 4.333 | 4.397 | 0.064 |
| 0.972 | 2.0 | 1.0 | 6.693 | 6.664 | 0.029 |
| 1.171 | 1.5 | 0.001 | 3.534 | 3.622 | 0.088 |
| 1.171 | 1.9 | 0.1 | 7.485 | 7.522 | 0.037 |
| 1.171 | 2.0 | 1.0 | 6.728 | 6.733 | 0.005 |
| 1.289 | 1.5 | 0.001 | 4.208 | 4.305 | 0.097 |
| 1.289 | 1.7 | 0.01 | 6.674 | 6.771 | 0.097 |
| 1.289 | 1.9 | 0.1 | 8.058 | 7.975 | 0.083 |
| 1.473 | 1.5 | 0.001 | 5.287 | 5.268 | 0.019 |
| 1.473 | 1.7 | 0.01 | 7.754 | 7.702 | 0.052 |
| 1.473 | 1.9 | 0.1 | 8.357 | 8.264 | 0.093 |

An aggregate dust model were already reported by Bhattacharjee et al. (2010). The present study shows that it is possible to study the extinction properties of interstellar dust aggregates for a given size of the particles and wavelength using relations (4), (5) and (6). The set of correlation equations can be used to estimate the general extinction $A_{\lambda}$ using Equation (1) for a given size distribution which will help to model the interstellar extinction curve. At this stage, we are not interested in modeling as this study is primarily intended to investigate the depen-
Fig. 10 Extinction efficiency ($Q_{\text{ext}}$) is plotted against wavelength ($\lambda$) for $k = 0.001, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7$ and $1.0$ at $a_v = 0.16$ $\mu$m when (i) $n = 1.4$, (ii) $n = 1.5$ and (iii) $n = 1.6$. The best fit curves correspond to quartic regression.

dency of extinction efficiency on size, wavelength and composition of particles. We show how this dependency can be framed with some correlation equations to study the extinction properties of interstellar dust.

Tamanai et al. (2006) experimentally investigated morphological effects on the extinction band in the IR region for amorphous silica (SiO$_2$) agglomerates. They also compared the measured band profiles with calculations for five cluster shapes applying Mie, T-matrix and DDA codes. Our correlations will also be helpful to study the experimental data, but it is also important to check the input parameters (size parameter, composition, etc.) of the experimental setup before using the correlation equations, because the relations are based on some selected set of parameters.

5 SUMMARY

- We have first studied the dependency of $Q_{\text{ext}}$ on size of the aggregates ($a_m$) to investigate the correlations between $Q_{\text{ext}}$ and complex refractive indices ($n, k$) at a particular size. Computations are performed at four different sizes ($a_v = 0.004, 0.068, 0.16$ and $0.26$ $\mu$m).

If $k$ is fixed at any value between $0.001$ and $1.0$, $Q_{\text{ext}}$ increases with increase of $n$ from $1.4$ to $2.0$. $Q_{\text{ext}}$ and $n$ are correlated via linear regression when the cluster size is small whereas the correlation is quadratic at moderate and higher sizes of the cluster. This feature is observed at all wavelengths (UV to optical to IR). We have also found that the variation of $Q_{\text{ext}}$ with $n$ is very small when $\lambda$ is high.

We have observed that $Q_{\text{ext}}$ and $k$ are correlated via polynomial regression equation (of degree 2, 3 or 4) where the degree of equation depends on the cluster size, real part of the refractive index of the particles ($n$) and wavelength ($\lambda$) of incident radiation. At $a_v = 0.16$ $\mu$m, $Q_{\text{ext}}$ and $k$ are found to be correlated with a polynomial regression equation (of degree 2 or 3) when $\lambda$ is between $0.11$ $\mu$m and $0.26$ $\mu$m. However, when $\lambda > 0.26$ $\mu$m, we have found that the correlation between them is quadratic for all values.
Fig. 11  Extinction efficiency ($Q_{\text{ext}}$) is plotted against wavelength of incident radiation ($\lambda$) at $a_v = 0.16 \mu$m. The left panel (top and bottom) shows the plot for the imaginary part of refractive indices ($k$) = 0.001, 0.01, 0.05, 0.1 and 0.3 in the wavelength range 0.11 µm to 0.40 µm, the middle panel (top and bottom) is for $k$ = 0.001, 0.01, 0.05, 0.1 and 0.3 in the wavelength range 0.55 to 0.90 µm and the right panel (top and bottom) is for $k$ = 0.5, 0.7 and 1 in the wavelength range 0.11 to 0.90 µm. The best fit corresponds to quartic regression (left panel), quadratic regression (middle panel) and quartic regression (right panel). The real part of the refractive index ($n$) is fixed at 1.7 and 1.8.

Fig. 12  Same as Fig. 11 but with $n = 1.9$ and 2.0.
Fig. 13 Extinction efficiency ($Q_{\text{ext}}$) is plotted against the size parameter of monomer ($x = 2\pi a_m/\lambda$, where $0.01 \leq x \leq 1.6$) for $N = 64$ and $k = 0.001$. The best fit curves represent a cubic regression for all values of $n$ except 2.0, where a quartic regression (degree 4) can be noticed. In all cases, the coefficient of determination ($R^2$) $\approx 0.99$.

Fig. 14 Same as Fig. 13 but with $k = 0.01$.

of $n$. The vertical range of $Q_{\text{ext}}$ in the plot also decreases when $k$ increases. This range is maximum at $k = 0.001$ and minimum at $k = 1.0$. If we include results for four different cluster sizes, we can summarize that the correlation of $Q_{\text{ext}}$ and $k$ depends mainly on cluster size. The correlation is linear for small size and quadratic/cubic/quartic for moderate and higher sizes.

- We study the dependence of $Q_{\text{ext}}$ on $\lambda$ for $a_v = 0.16 \mu$m. $Q_{\text{ext}}$ decreases with increase of $\lambda$ when $n \leq 1.6$. When $n \geq 1.7$, $Q_{\text{ext}}$ initially increases with increase of $\lambda$ and reaches a maximum value, then it starts decreasing if $\lambda$ is increased further.
For $n = 1.4, 1.5$ and $1.6$, $Q_{\text{ext}}$ versus $\lambda$ can be fitted via a quartic regression for all values of $k$. For other values of $n$, the correlation is polynomial regression where the degree of the equation depends on the value of $n$, $k$ and $\lambda$.

- We have found that $Q_{\text{ext}}$ and $x$ are correlated via a polynomial regression (of degree 3, 4 or 5) for all values of $n$. The degree of regression is found to be $n$ and $k$-dependent.

- The correlation equations can be used to model interstellar extinction for dust aggregates in a wide range of sizes of the aggregates, wavelengths and complex refractive indices.

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References

Bhattacharjee, C., Das, H., & Sen, A. 2010, Assam University Journal of Science and Technology, 6, 39
Brownlee, D. E. 1985, Annual Review of Earth and Planetary Sciences, 13, 147
Das, H. S., Das, S. R., Paul, T., Suklabaidya, A., & Sen, A. K. 2008, MNRAS, 389, 787
Das, H. K., Voshchinnikov, N. V., & Il’in, V. B. 2010, MNRAS, 404, 265
Greenberg, J. M., & Hage, J. I. 1990, ApJ, 361, 260
Halder, P., & Das, H. S. 2017, Comp. Phy. Comm. doi:10.1016/j.cpc.2017.08.020
Halder, P., Chakraborty, A., Deb Roy, P., & Das, H. S. 2014, Computer Physics Communications, 185, 2369
Iatì, M. A., Giusto, A., Saija, R., et al. 2004, ApJ, 615, 286
Jones, A. P. 1988, MNRAS, 234, 209
Kimura, H. 2001, J. Quant. Spec. Radiat. Transf., 70, 581
Krueger, F. R., & Kissel, J. 1989, Origins of Life and Evolution of the Biosphere, 19, 87
Mackowski, D. W., & Mishchenko, M. I. 1996, J. Quant. Spectrosc. Radiat. Transfer, 13, 2266
Mathis, J. S., & Whiffen, G. 1989, ApJ, 341, 808
Mazarbuiya, A. M., & Das, H. S. 2017, Ap&SS, 362, 161
Meakin, P. 1983, Journal of Colloid and Interface Science, 96, 415
Meakin, P. 1984, Phys. Rev. A, 29, 997
Mukai, T., Ishimoto, H., Kozasa, T., Blum, J., & Greenberg, J. M. 1992, A&A, 262, 315
Spitzer, L. 1978, Physical Processes in the Interstellar Medium, 333 (New York: Wiley-Interscience)
Tamanai, A., Mutschke, H., Blum, J., & Neuhäuser, R. 2006, J. Quant. Spec. Radiat. Transf., 100, 373
Vaidya, D. B., & Gupta, R. 1999, A&A, 348, 594
Vaidya, D. B., Gupta, R., & Snow, T. P. 2007, MNRAS, 379, 791
Vaidya, D. B., & Gupta, R. 2009, J. Quant. Spac. Radiat. Transf., 110, 1726
Voshchinnikov, N. V., Il’in, V. B., Henning, T., & Dubkova, D. N. 2006, A&A, 445, 167
Whittet, D. C. B., ed. 2003, in Dust in the Galactic Environment, 76 (2nd edn., Bristol: IOP Publishing Ltd.)
Wolff, M. J., Clayton, G. C., Martin, P. G., & Schulte-Ladbeck, R. E. 1994, ApJ, 423, 412
Wolff, M. J., Clayton, G. C., & Gibson, S. J. 1998, ApJ, 503, 815
Wurm, G., & Blum, J. 1998, Icarus, 132, 125