“Weak” Control for Human-in-the-loop Systems
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Abstract—In this letter, we propose a control framework for human-in-the-loop systems, in which many human decision makers are involved in the feedback loop composed of a plant and a controller. The novelty of the framework is that the decision makers are weakly controlled; in other words, they receive a set of admissible control actions from the controller and choose one of them in accordance with their private preferences. For example, the decision makers can decide their actions to minimize their own costs or by simply relying on their experience and intuition. A class of controllers which output set-valued signals is proposed, and it is shown that the overall control system is stable independently of the decisions made by the humans. Finally, a learning algorithm is applied to the controller that updates the controller parameters to reduce the achievable minimal costs for the decision makers. Effective use of the algorithm is demonstrated in a numerical experiment.

Index Terms—Human-in-the-loop system, stability, optimization, internal model control, robust control

I. INTRODUCTION

This letter is devoted to constructing a control framework for human-in-the-loop (HIL) systems, in which multiple decision makers are involved in the feedback loop composed of a plant and a controller. In the last five decades, the HIL concept has been realized and developed significantly in the literature. Most works focus on cooperative operation of the human and autonomous plants such as robots. There have been a variety of frameworks for the analysis and design of such human-robots interaction (see, e.g. the pioneering works and survey papers [1], [2], [3], [4], [5] and recent trials [6], [7], [8], [9], [10]).

Applications of HIL systems are now being proposed beyond such human-robots systems, where cooperation between human and robot is the key. Potential applications of HIL systems include for example, demand response in power grids involving humans decisions [11], air traffic management that must include human factors for pilots and control centers [12], incentive-based control of intelligent transportation systems relying on humans smart decisions [13], and so on. In such systems, the priorities of the humans in the loop may be unknown to and misaligned with those of the system designer. To realize such systems and to further broaden the applications, a broader control framework for HIL systems is necessary.

Some works have tried to construct more general control frameworks for HIL systems in e.g. [7], [8], [9], [14], [15], [16], [17]. In [16], [17], humans are modeled as uncertainties or constraints, and various methods of compensating their negative actions are proposed. In [7], [8], [9], [14], [15], humans are positively involved in the feedback loop of the controlled systems. In [7], [8], [9], humans are modeled as reference generators for autonomous controlled robots. This can be viewed as human decision-making being involved in the outer feedback loop of the overall control system. The cooperation of the human and inner controller is achieved by model predictive control (MPC) scheme or passivity-property. In the problem setting of [14], [15], humans are involved in the inner feedback loop. In particular, humans handle both actuation and measurement of the plant based on the request by the controller. Humans are characterized by the intermittency of their control actions or measurements and their spatial mobility. Then, an MPC-based method is proposed and applied to the practical control problem of an irrigation canal system.

In this letter, we propose a novel control framework for the HIL systems. In the framework, humans are interpreted as decision makers and are involved in the inner feedback loop of a plant and a controller. The humans handle the actuation to plant based on the request by the controller. We aim to realize “weak control” of the HIL system; the controller does not impose “too severe” requests for the decision makers that completely consume the degree of freedom (DOF) of their decisions. Instead, the controller provides a set of admissible control actions to enable the decision makers to pursue their own aims by utilizing the remaining DOF.

In the rest of the letter, first, the problem of the weak control for the HIL systems is formulated, in which the decision makers choose one control action $u$ from a given set of admissible actions $\mathcal{U}$ as illustrated in Fig. 1. Then, the solution is derived based on the idea of the internal model control (IMC, [18]). The resulting controller generates a set-valued signal, and it is shown that the overall control system is stable independently of the decisions. Finally, a learning algorithm is applied to the controller that updates the controller parameters in order to reduce the achievable cost for the decision makers. Effective use of the framework is demonstrated in a numerical experiment of an HIL control problem.

Notation: Let $v$ and $\mathcal{V}$ be a signal and set-valued signal, respectively. Then, their sum is defined as $v + \mathcal{V} := \{v + \bar{v} \mid \bar{v} \in \mathcal{V}\}$. The symbol $I$ represents the identity operator, i.e., for any signal $u$, $I_u = u$ holds. For a given set $\mathcal{V}$, the symbol $S(\mathcal{V})$ represents an element of $\mathcal{V}$, i.e., $S(\mathcal{V}) \in \mathcal{V}$ holds. For a given input-output system $\Sigma$, the symbol $||\Sigma||$ represents some performance criterion of interest.

II. HUMAN-IN-THE-LOOP CONTROL SYSTEMS

A. Problem Setting: Weak Control

In this section, we formulate and solve the problem of weak control for the HIL systems.
We consider here that multiple decision makers \( H_i \), \( i \in \{1, 2, \ldots, m\} \) are included in \( H \) and thus choose from among a set of possible actions. It should be noted that \( U \) is input-output stable for all decisions by \( H \).

We see that this \( \Sigma_{\text{HIL}} \) expands the signal \( v \) such that the sum of the elements is invariant.

The set-valued signals \( \mathcal{U} \) generated by \( \mathcal{E}_i \), \( i \in \{1, 2\} \) are illustrated in Fig. 3. Such generated \( \mathcal{U} \) must be a constraint for \( H \) of decision making.

Remark 1: Consider here that multiple decision makers \( H_i \), \( i \in \{1, 2, \ldots, m\} \) are included in \( H \) and they choose \( u_i \), \( i \in \{1, 2, \ldots, m\} \), respectively by pursuing their own aims. It should be noted that \( \mathcal{U} \) generated by \( \mathcal{E}_2 \) implicitly requires cooperation or negotiation between \( H_i \), \( i \in \{1, 2, \ldots, m\} \) for weakly controlled. This weak control framework allows us to express the case that decision makers can freely choose their own actions to some extent. Thus, they can pursue their own benefits or simply rely on their experience and intuition for their choices. This freedom can be a useful feature in many problems involving humans in smart infrastructures, where the priorities of the humans may be private information or misaligned with those of the system operator, yet the system operator should give the human users sufficient freedom to choose from among a set of possible actions.

The HIL control problem addressed in this letter is summarized in the following problem.

**Problem 1 (HIL control problem):** Find \( K \) and \( \mathcal{E} \) such that \( \Sigma_{\text{HIL}} \) is input-output stable for all decisions by \( H \).

Note again that any strategy or model of \( H \) is unavailable for the design of \( K \) and \( \mathcal{E} \) in the general problem setting. Only the rule \( \mathcal{E} \) is known and available to the designer.

**B. Signal Expander**

Examples of signal expanders \( \mathcal{E} \) are given in this subsection.

**Example 1:** An example of the expander is given by the following rectangular prism \( \mathcal{E}_1 \):

\[
\mathcal{E}_1(v) = \left\{ \text{diag}(\delta_1, \ldots, \delta_m) v \mid \delta_i \in [-\gamma_i, \gamma_i] \right\},
\]

where \( \gamma_i, i \in \{1, 2, \ldots, m\} \) are positive constants. Equivalently, this \( \mathcal{E}_1 \) is written as

\[
\mathcal{E}_1(v) = \left\{ \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix} \mid \xi_i \in \left[ -\gamma_i v_1, \gamma_i v_1 \right] \right\}.
\]

**Example 2:** The expander \( \mathcal{E}_1 \) is generalized to \( \mathcal{E}_2 \) with some coordinate transformation as:

\[
\mathcal{E}_2(v) = \left\{ E_L \text{diag}(\delta_1, \ldots, \delta_p) E_R^\top v \mid \delta_i \in [-\gamma_i, \gamma_i] \right\},
\]

where \( p \leq m \) is a natural number, \( E_L \in \mathbb{R}^{m \times p} \) and \( E_R \in \mathbb{R}^{m \times p} \) are matrices of full column ranks. By the introduction of \( E_L \) and \( E_R \), the signal \( v \) is expanded more flexibly than \( \mathcal{E}_1(v) \). Let us consider a simple example of \( \mathcal{E}_2 \). We define

\[
E_L = E_R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.
\]

Then, \( \mathcal{E}_2(v) \) is reduced to

\[
\mathcal{E}_2(v) = \left\{ \frac{1}{2} \begin{bmatrix} \varepsilon \\ -\varepsilon \end{bmatrix} \mid \varepsilon \in \left[ -\gamma_2|v_1 - v_2|, \gamma_2|v_1 - v_2| \right] \right\}.
\]

We see that this \( \mathcal{E}_2(v) \) expands the signal \( v \) such that the sum of the elements is invariant.

The main characteristics of the proposed HIL system are the existence of a set-valued signal in the feedback loop. Due to this set-valued signal \( \mathcal{U} \), we say that the HIL system is valid.
their decision-making, while $\mathcal{E}_1$ does not. The decision makers $\mathcal{H}_i$, $i \in \{1, 2, \ldots, m\}$ must cooperate each other to determine their actions $u_i$, $i \in \{1, 2, \ldots, m\}$ under the constraint $u \in \mathcal{U}$ for the case $\mathcal{E}_2$.

C. Weak Control: IMC-based Approach

In this subsection, we give a general solution to the HIL control problem, which is formulated in Problem \[1\].

First, the HIL control problem is reduced to a robust control problem \[19\] as follows. Noting that $\mathcal{U} = v + \mathcal{V}$, the behavior of $\mathcal{H}$ is equivalently expressed as

$$
\mathcal{H} : \begin{cases} d(t) = S(v(t)), \\ u(t) = v(t) + d(t).
\end{cases}
$$

This transformation is illustrated in Fig. 4. Letting $\Delta$ be

$$
\Delta : \begin{cases} d(t) = S(\mathcal{E}(v(t))) 
\end{cases}
$$

or more simply $\Delta : d(t) \in \mathcal{E}(v(t))$ as illustrated in Fig. 4(b), we reduce the overall control system $\Sigma_{\text{HIL}}$ to the system illustrated in Fig. 5. The system illustrated in Fig. 5 represents a control system addressed in a robust control problem with the time-varying uncertainty $\Delta$.

This transformation implies that the HIL control problem is essentially a robust control problem. Still, there are some practical differences between the problems considered in \[19\] and here. The HIL control positively utilizes the uncertainty for the signal expansion, which brings some benefit to $\mathcal{H}$. On the other hand, robust control focuses mainly on the negative effect of the uncertainty. In addition, the uncertainty in the HIL control is designable to achieve some aims, while that in the robust control is not. Details of design examples and applications are given in Section \[II-B\] and Section \[III\].

Next, we derive a design method for $K$ based on the system illustrated in Fig. 5. In particular, we propose a special controller-structure in $K$ to guarantee the stability of the overall control system $\Sigma_{\text{HIL}}$. Independently of the decisions made by $\mathcal{H}$.

To this end, the internal controller $K$ is given by

$$
v = K_e(e + P(u)),
$$

where $K_e$ is an operator. The controller structure in (3), which involves the plant model $P$, is based on the idea of the internal model control (IMC, \[13\]). By applying (3) to the system illustrated in Fig. 5, we obtain the following theorem.

\[\textänge{Theorem 1:} \text{Suppose that } K(e, u) \text{ is given by (3). Then, if } P \text{ and } K_e \text{ are } L_2\text{-stable, } \Sigma_{\text{HIL}} \text{ is } L_2\text{-stable for all decisions of } \mathcal{H}.\]

\[\text{Proof:} \text{ We recall that}
\]

$$
\begin{align*}
&y = P(u) + w, \\
&u = v + \Delta (v), \\
&v = K_e(r - y + P(u))
\end{align*}
$$

hold, where $\Delta$ is the operator that represents the input-output map (2). By summarizing the equations, we obtain the expression

$$
\Sigma_{\text{HIL}} : y = P(K_e(r - w) + \Delta (K_e(r - w))) + w. \quad (4)
$$

From the cascaded and parallel structure in (4), we see that the statement of the theorem holds.

As stated in the proof of the theorem, the implementation of the IMC-based controller (3) results in the cascaded and parallel structure in $\Sigma_{\text{HIL}}$. The structure contributes to the stability guarantee independently of the expander $\mathcal{E}$ and the decision in $\mathcal{H}$, which is described by $S$ in (2). In addition, the structure enables us to easily evaluate the performance of $\Sigma_{\text{HIL}}$ as follows. For simplicity, let us consider the linear regulation control problem; it is assumed that $r = 0$ and that $P$ and $K_e$ are linear. Then, the expression (4) is reduced to

$$
\Sigma_{\text{HIL}} : y = (I - P(I + \Delta)K_e)w. \quad (5)
$$

Supposing $\Delta = 0$, i.e., $v$ is not expanded in $\mathcal{E}$ or $u \equiv v$ is chosen in $\mathcal{H}$, we can evaluate the nominal performance $\|I - PK_e\|$ in some criterion such as the $L_2$ gain. We
emphasize that the performance \( ||I - P(I + \Delta)K_e|| \) is continuously and linearly deteriorated from the nominal one with the increase of \( ||\Delta|| \). This enables us to simply evaluate the bound of \( ||I - P(I + \Delta)K_e|| \). The continuity of the performance deterioration is called persistence and analyzed for general uncertain systems in [20].

Remark 2: A design strategy of \( K_e \) and \( \mathcal{E} \) is given in this remark. First, we design \( K_e \) such that the desired nominal performance is achieved; for example, minimize the performance as \( \min ||I - P K_e|| =: \rho \). Then, determine the degree of the expansion in \( \mathcal{E} \), which is characterized by e.g. \( \gamma_i \) of \( \mathcal{E}_i \), \( i \in \{1,2\} \). We design \( \mathcal{E} \) such that the performance deterioration is admissible for the designer, who is responsible for the overall control system; for example, for a given \( \Delta \rho > 0 \), find or maximize \( \gamma \) such that

\[
||I - P(I + \Delta)K_e|| \leq \rho + \Delta \rho \tag{6}
\]

holds for all decisions in \( \mathcal{H} \) satisfying \( [1] \).

III. LEARNING OF HUMAN PREFERENCES FOR UPDATING THE CONTROLLER

In the general problem formulated in Section III. no assumption is imposed on the decision maker \( \mathcal{H} \) except for the rule [1]. In this section, it is assumed that \( \mathcal{H} \) is rational and determines the control action \( u \) based on an optimization; a cost function is minimized under the constraint [1]. Then, we design and implement a mechanism of learning a part of the model in \( \mathcal{H} \) and of updating the expander \( \mathcal{E} \) online.

A. Problem Setting

The models of the plant \( P \) and controller \( K \) are specialized in the following discussion. For simplicity, we consider a linear regulation problem under the step disturbance; \( r = 0 \), \( w \) is the step signal, \( P \) and \( K \) are linear, and the overall control system is expressed by [5]. The following discussion can be extended to other practical cases, e.g. tracking control with \( r \neq 0 \), persistent disturbance to \( w \), nonlinear plant systems, and so on, with some modification. In addition, the structure of \( \mathcal{E} \) is fixed at \( \mathcal{E}_2 \), which is defined in Example [2]. In addition, \( \mathcal{E} \) has only one dimensional degree of freedom; letting \( E_L \) and \( E_R \) be vectors in \( \mathbb{R}^m \), \( \mathcal{E} \) is described by

\[
\mathcal{E}(v) = \mathcal{E}_2(v) = \left\{ \delta E_L E_R^T v \mid \delta \in [-\gamma, \gamma] \right\}, \tag{7}
\]

where \( \gamma \) is a positive constant. Note here that \( E_L \) represents the direction of the expansion, while \( \gamma |E_R^T v| \) represents the degree of the expansion.

We consider that the following optimization algorithm is implemented in \( \mathcal{H} \).

\[
\mathcal{H} : \begin{cases} 
\min & f(u) \\
\text{subject to} & u \in \mathcal{U}.
\end{cases} \tag{8}
\]

The global minimizer of the unconstrained optimization, simply \( \min f(u) \), is denoted by \( u^* \), while that of the constrained one, described by \( [3] \), is denoted by \( u^\dagger \). Trivially, \( f(u^*) \leq f(u^\dagger) \) holds. Note that the achievable minimum cost \( f(u^\dagger) \) depends on \( \mathcal{U} \) and therefore, it depends on the designed \( \mathcal{E} \). The aim of this section is to find \( \mathcal{E} \) that minimizes the achievable minimum cost \( f(u^\dagger) \) subject to some performance specification on \( \Sigma_{HIL} \).

To formulate the problem in a clearer manner, we define a specific set of expanders \( \mathcal{E} \), which is essentially the same as a set of triplets \( \{E_L, E_R, \gamma\} \), as follows. Let \( \rho \) be the nominal performance \( \rho := ||I - P K_e|| \).

**Notation 1**: For a given positive constant \( \Delta \rho \), the symbol \( \{\mathcal{E}\}_{\Delta \rho} \) represents the set of the expanders \( \mathcal{E} \) such that for any element in \( \{\mathcal{E}\}_{\Delta \rho} \), the inequality in \( [9] \) holds for all decisions by \( \mathcal{H} \), i.e., all realizations of \( \Delta \). In addition, \( \{\mathcal{U}(v)\}_{\Delta \rho} := \{v + \mathcal{E}(v) \mid \mathcal{E}(v) \in \{\mathcal{E}(v)\}_{\Delta \rho}\} \), which represents the set of all input candidates \( \mathcal{U} \) generated by \( \mathcal{E}(v) \in \{\mathcal{E}(v)\}_{\Delta \rho} \).

The problem addressed in the rest of this section is formulated as follows. follows.

**Problem 2**: For a given \( \Delta \rho \), find \( \mathcal{E} \in \{\mathcal{E}\}_{\Delta \rho} \) that minimizes \( f(u^\dagger) \) at the steady state.

In the next subsection, the solution method by updating \( \mathcal{E} \in \{\mathcal{E}\}_{\Delta \rho} \) is given.

B. Learning Algorithm for Updating Expander

The graphical interpretation of \( u^*, u^\dagger \), \( v \), \( \{\mathcal{U}(v)\}_{\Delta \rho} \), and \( f(u) \) is illustrated in Fig. [4]. We see that the generated \( \mathcal{U}(v) \in \{\mathcal{U}(v)\}_{\Delta \rho} \) illustrated in Fig. [4] (b) is more beneficial for \( \mathcal{H} \) than Fig. [4] (a); the achievable cost \( f(u^\dagger) \) is reduced by the update of \( \mathcal{E} \). We aim to find the best \( \mathcal{E} \in \{\mathcal{E}\}_{\Delta \rho} \) in this sense.

For updating \( \mathcal{E} \), we first estimate \( u^* \) by using some data set \( \{v_k, u^\dagger_k\} \), where \( k \) is the discrete time. Let \( E_{L0}, E_{L1}, \ldots, E_{Lk} \) be the sequence of the updated \( E_L \). We suppose that

\[
|u^\dagger_k - v_k| < \gamma |E_{Lk} E_R^T v_k| \tag{9}
\]

holds, which implies that \( u^\dagger_k \) is located on the interior of \( \mathcal{U}(v_k) \) as illustrated in Fig. [4] (a). Then, it follows that \( u^* \) is located on the hyperplane described by \( E_{Lk}^T (u^* - u^\dagger_k) = 0 \), which is
graphically shown in Fig. 6 (a). The set of the hyperplanes is expressed by the vector form

\[ [E_{L0}, E_{L1}, \cdots, E_{Lk}]^T u^* - [E_{L0}^T u_0, E_{L1}^T u_1, \cdots, E_{Lk}^T u_k]^T = 0. \]

If \( E_{\text{ex}} \) is of full row rank, we obtain the estimate of \( u^* \) as

\[ u^* = (E_{\text{ex}}^T E_{\text{ex}})^{-1} E_{\text{ex}}[E_{L0}^T u_0, E_{L1}^T u_1, \cdots, E_{Lk}^T u_k]^T. \]  

(10)

The estimate of \( u^* \) is utilized for updating \( E \). The algorithm for the update is briefly stated as follows.

**Algorithm: Updating Expander \( E \)**

1. **Initialization:** \( E_{Lk}, E_{Rk}, \gamma_k \) at \( k = 0 \)
2. repeat
3. get data \( \{v_k, u_k^T\} \) that satisfies (9)
4. if \( E_{\text{ex}} \) is of full row rank then
5. \( E_{Lk+1} \leftarrow v_k - u^*, \) where \( u^* \) is given by (10)
6. else
7. \( E_{Lk+1} \leftarrow E_{Lk} + \delta, \) where \( \delta \) is a small perturbation
8. end if
9. find \( E_{Rk+1}, \gamma_{k+1} \) maximizing \( \gamma_{k+1}|E_{Rk+1}^T v_k| \) subject to \( \gamma \in \{\gamma \}\}_{\Delta \rho}
10. return \( E_{Lk+1}, E_{Rk+1}, \gamma_{k+1} \)
11. \( k \leftarrow k + 1 \)
12. until \( E_{Lk}, E_{Rk}, \gamma_k \) converge

In the algorithm above, it is assumed that \( u_k^1 \) is available for updating \( E \). We justify the assumption as follows. We emphasize that the update can bring benefits only to the decision maker \( \mathcal{H} \), not to the system manager or controller designer who is responsible for the performance of \( \Sigma_{\text{HIL}} \). The benefits for \( \mathcal{H} \) can be incentive to disclose some information of \( \mathcal{H} \). It is thus natural to assume that the result of the decision, denoted by \( u^1 \), is disclosed and available for the update of \( E \).

**IV. NUMERICAL EXPERIMENT**

The plant \( P \), decision maker \( \mathcal{H} \), and controller \( K \) are given as follows. The plant \( P \) is the linear dynamical system described by

\[
\begin{align*}
\dot{x} &= \begin{bmatrix}
-1 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & -0.5
\end{bmatrix} x + \begin{bmatrix}
1 & 1
\end{bmatrix} w + \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} u, \\
y &= \begin{bmatrix}
1 & 1 & 1
\end{bmatrix} x.
\end{align*}
\]

The step disturbance is injected to \( w \) to drive the plant system. The transfer matrices from \( u \) and \( w \) to \( y \) are denoted by \( P_u(s) \) and \( P_w(s) \), \( s \in \mathbb{C} \), respectively. Then, the DC gains of \( P_u(s) \) and \( P_w(s) \) are given by

\[ P_u(0) = \begin{bmatrix} 1 & 0.5 & 2 \end{bmatrix}, \quad P_w(0) = 3.5, \]

respectively. In the decision maker \( \mathcal{H} \), the following optimization algorithm is implemented.

\[
\mathcal{H} : \begin{cases}
\min_{u} f(u) := 2u^T u - \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} u, \\
\text{subject to } u \in \mathcal{U}.
\end{cases}
\]

This optimization model is blind for the design of the controller. The controller \( K \) is described by

\[
K : \begin{cases}
v = K_e (r - y + P_u(u)), \\
U = v + E(v),
\end{cases}
\]

where \( K_e \) is a static system, i.e., it is simply a constant matrix, and \( P_u \) is the operator representation of \( P_u(s) \). In this section, we demonstrate the design procedure of \( K_e \) and \( E \).

The performance criterion for \( \Sigma_{\text{HIL}} \) is the DC gain, which represents the disturbance suppression performance \( |y(t)| \) as \( t \to \infty \) corresponding to the unit step disturbance \( w(t) \). The performance criterion for \( \mathcal{H} \) is the value of \( f(w^1(t)) \). We aim to minimize \( f(w^1(t)) \) as \( t \to \infty \) subject to the specification \( |y(t)| \leq 0.2 \) as \( t \to \infty \), denoted by \( |\Sigma_{\text{HIL}}|_{\text{dc}} \leq 0.2 \).

First, \( K_e \) is designed as

\[ K_e = \frac{1}{6} \begin{bmatrix} 2 & 4 & 1 \end{bmatrix}, \]

which achieves \( |y(t)| \to 0 \) as \( t \to \infty \) in the nominal situation; in other words, if the expander \( E \) is inactive, the step disturbance \( w(t) \) does not propagate to \( y(t) \) as \( t \to \infty \). We see this fact as follows. Note that \( (1 - P_u(s)K_e)P_w(s) \) represents the transfer function of \( \Sigma_{\text{HIL}} \) when \( E(v) = 0 \). The above \( K_e \) guarantees that \( (1 - P_u(0)K_e)P_w(0) = 0 \) holds.

Next, the structure of \( E \) is fixed as (7). The initial condition of \( E_L \) and \( E_R \) is given by

\[ E_{L0} := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T, \quad E_{R0} := \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \]

The value of \( \gamma \) is determined such that the DC gain specification \( |\Sigma_{\text{HIL}}|_{\text{dc}} \leq 0.2 \) holds. The specification is expressed as

\[ |(1 - P_u(0)(I_3 + \delta E_L E_R^T)K_e)P_w(0)| = \delta |P_u(0)E_L E_R^T K_e P_w(0)| \leq 0.2 \]

holds for all \( \delta \in [-\gamma, \gamma] \). By maximizing \( \gamma \) under the inequality, we obtain the initial value of \( \gamma \) as

\[ \gamma_0 = 0.2 / |P_u(0)E_{L0} E_{R0}^T K_e P_w(0)| = 0.0848. \]

Then, the updating algorithm proposed in Section III is applied to update \( E_L \) and \( \gamma \), while \( E_R \) is fixed at \( E_{R0} \).

The numerical experiments are performed for the following four cases: 1) no feedback controller is applied, 2) the controller \( K \) is applied without the expander \( E \), 3) the controller \( K \) is applied with fixed \( E \), i.e., \( E \) is composed of \( E_L = E_{L0} \), \( E_R = E_{R0} \), and \( \gamma = \gamma_0 \), and 4) the controller \( K \) is applied with updating \( E \). The experiment, the time step is fixed at 1 sec, and the continuous models in \( P \) and \( K \) are discretized. At each time step, the optimization problem in \( \mathcal{H} \) is solved, and the expander \( E \) is updated.

The trajectories \( y(t) \) for all cases are illustrated in Fig. 7. We see that the feedback control effectively suppresses the disturbance effects in \( y(t) \). The control in Case 2 results in the best performance, while the weak control in Cases 3 and 4 satisfies the specification, \( |y(t)| \leq 0.2 \) at a large \( t \).

The values of the cost \( f(w^1(t)) \) for all cases are illustrated in Fig. 8. We see that the costs achieved by the weak control in Cases 3 and 4 are smaller than that by the control in Case 2.
This demonstrates that the expander $E$ brings smaller costs for decision makers $H$. Furthermore, we compare Cases 3 and 4 to show the effectiveness of the updating algorithm. The weak control with updating $E$ in Case 4 contributes to reducing the cost compared with no updating case in Case 3 as illustrated in Fig. 8. It should be emphasized that Case 4 further reduces the cost while keeping the same DC gain performance in $\Sigma_{HIL}$ as illustrated in Fig. 7

V. CONCLUSION

In this letter, we proposed a framework for weak control for human-in-the-loop systems. In this framework, a signal expander is embedded in the controller and generates candidate control actions with some DOF. The DOF allows the human decision-makers to pursue their own aims, while guaranteeing the stability and the specified performance in the overall control system. A simple algorithm of updating the expander was also given, which was beneficial to human decision makers.

There are a variety of future works for the weak control; more sophisticated algorithms of updating the expander can be derived under practical problem setting, and the weak control can be applied to the demand response for power grids [11].

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