Eternal inflation and chaotic terminology

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Abstract

“Eternal inflation” is often confused with “chaotic inflation”. Moreover, the term “chaotic inflation” is being used in three different meanings (all of them unrelated to eternal inflation). I make some suggestions in an effort to untangle this terminological mess. I also give a brief review of the origins of eternal inflation.

I. INTRODUCTION

The idea of eternal inflation was introduced more than 20 years ago. For most of this time it remained an esoteric subject, pursued by a few enthusiasts and largely ignored by the rest of the physics community. But in the last few years a new “multiverse” cosmological paradigm has emerged with eternal inflation at its core. This led to proliferation of papers on the subject, both in technical journals and in the popular science literature. Unfortunately, many of these papers (see, e.g., [1–4]) confuse eternal inflation with another important idea, or rather two ideas, known under the name of “chaotic inflation”.

I am writing this brief note in an effort to disentangle these different and largely unrelated concepts. The next section reviews the origins of the key ideas of eternal inflation. (As almost everything in eternal inflation, this has also become a somewhat controversial subject.) Section 3 explains the different meanings of “chaotic inflation”. Section 4 reviews the cases where inflation is both “chaotic” and eternal. Finally, in Section 5 I propose some modest measures that might help to restore clarity. I also briefly delve in psychology and discuss possible causes for the mixup.

II. A BRIEF HISTORY OF ETERNAL INFLATION

The eternal character of inflation was encountered in Guth’s 1981 paper [5], where he introduced the idea of inflation. Guth realized that the horizon and flatness problems of standard cosmology can be solved by inflation – a period of accelerated expansion driven by false vacuum energy. He discovered, however, that inflation was not easy to end. Metastable false vacuum decays through bubble nucleation. Bubbles expand at speeds approaching the speed of light, but the false vacuum regions that separate them expand even faster. Thus, the bubbles never fill the entire space and inflation never ends. This is the graceful exit problem of the old inflationary scenario.
The problem was solved with the advent of “new” inflation [6,7]. This type of models assumes a scalar field – the inflaton – with a slowly varying potential. The field starts somewhere near the top of the potential and slowly rolls downhill. In the meantime, its potential energy drives the inflationary expansion of the universe.

The early versions of this scenario postulated a small barrier near the top of the potential – a leftover from the “old” model of inflation. With this additional feature, the model combines the characteristics of “old” and “new” inflation. It has a metastable false vacuum, which decays through bubble nucleation. The slow roll of the field then occurs inside the bubbles. Steinhardt [8] pointed out that the global structure of the universe in this type of model is the same as in “old” inflation: the bubbles are driven apart by the inflating false vacuum and never fill the space, so inflation never ends in the entire universe. A narrow barrier on top of a smooth energy hill is a rather unnatural feature, and once it was recognized that this feature is unnecessary, it was quickly abandoned [9].

Eternal inflation as a generic phenomenon, which is common to a very wide class of models, was first discussed in my paper [10] (see also [11]). The evolution of the inflaton field is influenced by quantum fluctuations, which can be pictured as a random walk of the field with a step $\Delta \phi \sim H/2\pi$ on a horizon scale ($l \sim H^{-1}$) per expansion time ($\Delta t \sim H^{-1}$). (Here, $H$ is the expansion rate of the universe.) In models of “new” inflation, quantum fluctuations represent a small perturbation to the classical motion of the field, except near the maximum of the potential, where the classical driving force is small and the quantum random walk dominates the dynamics. The key point is that when the field is hovering near the top of the potential, the corresponding regions exponentially multiply. In some of these regions the field “walks” to the steeper part of the slope and starts rolling down, but this “decay” of inflating regions is much slower than their “reproduction”. As a result, the total volume occupied by inflating regions grows (exponentially) with time.

Eternal inflation provides a natural arena for the “anthropic principle”. Different thermalized regions of the eternally inflating universe can be characterized by different values of the low-energy constants of Nature. The values we observe here can then be largely determined by anthropic selection. This was first emphasized by Linde (see [12] and references therein).

Mathematically, an eternally inflating universe can be described by a distribution $P(\phi, t) d\phi$, which gives the fraction of the comoving volume of the universe where the inflaton field $\phi$ is in the interval $d\phi$ at time $t$. This distribution satisfies a Fokker-Planck equation. Its approximate form for a flat potential was given in [10], and the general form was derived by Starobinsky [13]. This formalism was later extended from the comoving to the physical volume distribution [14]; for a review see [15].

The Fokker-Planck equation has also been extended to include the variable “constants” of Nature, and the resulting distributions have been used to assign probabilities to different values of the constants. The question of how observational predictions are to be extracted from this formalism is still being debated. The main difficulty is that the distributions are sensitive to how one defines the time variable $t$ [15–17], and no particular choice appears to be preferred. Possible ways to resolve this problem have been proposed in [18–20].

Although inflation is eternal to the future, inflationary spacetimes are necessarily past-incomplete (see [21] and references therein). Hence, inflation must have some sort of a beginning. (This issue is somewhat related to the topic of “eternal chaotic inflation”, to be
discussed below.)

III. THE THREE MEANINGS OF CHAOTIC INFLATION

The term “chaotic inflation” first appeared in Linde’s 1983 paper [22], where he proposed two different ideas. First, he realized that inflation is possible with potentials $V(\phi)$ which grow unboundedly with $\phi$ and do not have a maximum. The simplest example is a power-law potential, $V(\phi) \propto \phi^n$. Linde showed that in order to have inflation in this type of model, the field has to start its slow roll at a large value, $\phi > M_p$, where $M_p$ is the Planck mass.

As a motivation for the large initial value of the field, Linde suggested that the universe might have started in a chaotic initial state, with the inflaton field varying wildly from one place to another. Inflation will then occur in regions where the field happened to be large. It should be emphasized, however, that such a chaotic beginning is not mandatory for inflation with an unbounded potential. One alternative is provided by quantum cosmology: small closed universes can spontaneously nucleate with all possible values of the field. The corresponding probability distribution depends on the choice of the boundary conditions for the wave function of the universe (see [23] for a critical review), but regardless of the choice, there is always a nonzero probability for $\phi > M_p$.

The term “chaotic inflation” is now being used in reference to inflation with a chaotic beginning, with an unbounded potential, or with both of the above. This is, of course, confusing, and the confusion is exacerbated by the mixup with eternal inflation.

IV. ETERNAL CHAOTIC INFLATION

In 1986 Linde showed that inflation with an unbounded potential is also generically eternal [24]. This is somewhat unexpected, because, for a power-law potential the slope of the potential grows with $\phi$. However, the step of the random walk $\Delta \phi \sim H/2\pi$ is a growing function of $\phi$ as well, and Linde found that quantum fluctuations dominate over classical dynamics at sufficiently large $\phi$. Hence, inflation is eternal, by the same argument as before.

Eternal inflation with unbounded potentials has some qualitatively new features. The higher the field gets up the slope of the potential, the faster is the rate of inflationary “reproduction”, and the larger are the quantum jumps of the field. As a result, in some regions the field exhibits a runaway behavior and is driven all the way up to the Planck density.

This scenario is sometimes called “eternal chaotic inflation”. The term is rather ambiguous, because of the multiple meanings of “chaotic”.

V. A PROPOSAL

It would be interesting to figure out why eternal and chaotic inflations got so much confused in the first place. One obvious reason is that they are associated with the same

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1The term “eternal inflation” was introduced in this paper.
physicist who made important contributions to both subjects. Another reason, I think, is the stochastic character of eternal inflation. The inflating regions form a self-similar fractal [25,26], and computer simulations of eternal inflation [25,15,19] present a rather “chaotic” appearance. An even more haphazard picture, with episodes of inflation recurring within already thermalized regions, resulting in new islands of eternal inflation, was presented in [27].

History aside, chaotic usage of terms has reached epidemic proportions. I, therefore, humbly suggest that the reader considers taking the following preventive steps.

1. If you write about eternal inflation, refrain from calling it “chaotic”.

2. To differentiate between the different meanings of “chaotic inflation”, I suggest using one of the following: (i) “inflation with an unbounded potential” (or, if you like, “topless inflation”), (ii) “inflation with a chaotic beginning”, or, if necessary, (iii) “inflation with an unbounded potential and a chaotic beginning”.

3. If you want to refer to eternal inflation which is also chaotic, in any of the three meanings of the word, I suggest that you follow the same strategy as in 2 above and say, for example, “eternal inflation with an unbounded potential”, etc.

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