Chronology Protection in Two-Dimensional Dilaton Gravity

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ABSTRACT

The global structure of 1 + 1 dimensional compact Universe is studied in two-dimensional model of dilaton gravity. First we give a classical solution corresponding to the spacetime in which a closed time-like curve appears, and show the instability of this spacetime due to the existence of matters. We also observe quantum version of such a spacetime having closed timelike curves never reappear unless the parameters are fine-tuned.

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Callan, Giddings, Harvey and Strominger[1] (abbreviated as CGHS) provided a useful toy model of (1+1)-dimensional gravity. They attempted to analyze the back-reaction of the Hawking radiation[2] in the two-dimensional analogue of black hole geometry in a consistent way by the use of this model. In their first paper, CGHS developed their original scenario as follows: First introducing 1+1 dimensional dilaton gravity, CGHS found the solutions corresponding to black hole formation, and showed occurrence of the Hawking radiation in the spacetime background they obtained. Next taking account of quantum effect of quantized conformal matters as the Polyakov term[3], they analyzed the back-reaction of the Hawking effect in a leading semi-classical approximation consistently. However their first scenario on the quantum black hole remains to be elaborated. Up to the present time, many people have studied the behavior of quantum black hole in this model[4,5].

On the other hand, in general relativity, there are other interesting problems which must be resolved taking account of quantum effects. For example, since Morris, Thorne and Yurtsever pointed out the possibility of making a time-machine[6], quantum effect on the spacetime in which closed time-like curves appear (abbreviated as CTC-spacetime) has been discussed by several authors [7-11]. One of the most important problems in this subject is whether CTC-spacetime continues to exist or not if the quantum effect is taken into account. Hawking suggested the possibility that the laws of physics may prevent the appearance of closed time-like curves (so called ‘Chronology Protection Conjecture’). In four dimensions the analysis of this problem so far have been done only in fixed background spacetimes. So it will be interesting to treat such spacetimes including back-reaction in closed form even in lower dimensions.

In this paper we will investigate the quantum back-reaction problem on the stability of the CTC-spacetime in two-dimensional model of dilaton gravity. Following the CGHS scenario, we apply this model to a compact one dimensional universe. In the first half of this paper we give a classical solution corresponding to CTC-spacetime: an analogue of the Misner universe. Then we show the disappearance of this spacetime due to the existence of conformal matters even if the
parameters are fine-tuned. In the second half we investigate whether the extension to such a CTC-spacetime can be made or not if quantum back-reaction is taken into account.

We consider the 1 + 1 dimensional renormalizable theory of gravity coupled to a dilaton scalar field $\phi$ and $N$ massless conformal fields $f_i$. The classical action is

$$S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} [e^{-2\phi} (R + 4(\nabla \phi)^2 - 4\lambda^2) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2], \quad (1)$$

where $R$ is the scalar curvature, $\lambda^2$ is a cosmological constant. This model differs from the original C.G.H.S. model in the sign of the cosmological term.

The equations of motion derived from (1) are

$$0 = -4\partial_+ \partial_- \phi + 2\partial_+ \partial_- \rho + 4\partial_\rho \partial_- \phi - \lambda^2 e^{2\rho},$$

$$0 = \partial_+ \partial_- \phi - \partial_+ \partial_- \rho,$$

$$0 = \partial_+ \partial_- f_i,$$

in the conformal gauge: $g_{\mu \nu} dx^\mu dx^\nu = -e^{2\rho} dx^+ dx^-$, where $x^\pm = t \pm x$. In addition the following constraints have to be imposed:

$$4e^{-2\phi} (\partial_\pm^2 \phi - 2\partial_\pm \rho \partial_\pm \phi) = \sum_{i=1}^{N} \partial_\pm f_i \partial_\pm f_i. \quad (3)$$

In the following we adopt the periodic boundary condition that the spacetime point $(t, x)$ is identified with $(t, x + L)$ and the initial condition that the Universe starts from a static cylinder spacetime endowed with the usual Minkowski metric at the past infinity. Then the general form of the solutions is given by:

$$e^{-2\phi} = u_+ + u_- + e^{-2\lambda t},$$

$$e^{2\rho} = e^{-2\lambda t} e^{2\phi}, \quad (4)$$

where $u_+$ and $u_-$ are chiral periodic functions which satisfy the following equations.
from the constraints (3):

$$0 = \partial_{\pm}^2 u_{\pm} + \lambda \partial_{\pm} u_{\pm} + \frac{1}{2} \partial_{\pm} f \partial_{\pm} f. \quad (5)$$

If there is no matter field, general solutions satisfying the periodic boundary condition depend only on time:

$$e^{2\phi} = (M + e^{2\lambda t})^{-1},$$
$$e^{2\rho} = e^{-2\lambda t} e^{2\phi}, \quad (6)$$

where $M$ is an arbitrary constant and corresponds to an initial value for $\phi$ imposed at some past time. We classify the behavior of the solutions into three types with respect to the sign of $M$.

When $M$ equals to 0, the solution becomes an analogue of the Linear Dilaton Vacuum solution in C.G.H.S. model. The world is a static cylinder spacetime.

When $M$ is negative, we see from (6) that the observer meet some singularity in a finite proper time. From the expression of scalar curvature $R$:

$$R = -\frac{4\lambda^2 M}{M + e^{-2\lambda t}}; \quad (7)$$

we can see that this singularity is a true singularity. In fact this singularity is the same as the one in the 1 + 1 dimensional black-hole treated in CGHS.

On the other hand when $M$ is positive, the space collapses into zero volume in a finite proper time (as coordinate time $t$ goes to $+\infty$). But from (7) the scalar curvature still remains a finite value $-4\lambda^2$ at the point. Hence we expect that the spacetime can be extended. In fact, if one defines the coordinates:

$$\begin{cases}
\eta = -e^{-2\lambda t}, \\
\psi = t \pm x,
\end{cases} \quad (8)$$

the metric becomes

$$ds^2 = \frac{-1}{M - \eta}(\eta d\psi^2 + \frac{1}{\lambda}d\psi d\eta), \quad (9)$$

which is analytic in the extended manifold defined by $\psi$ and by $-\infty < \eta < M$. 

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The behavior of the extended manifold is shown in Fig.1. The region $\eta < 0$ is isometric with the previous manifold. The region $\eta > 0$ is extended part, where the closed timelike curves appear, because the roles of $t$ and $x$ are interchanged. It should be noted that the spacetime has a naked singularity on $\eta = M$ where the dilaton field becomes $+\infty$. The surface $\eta = 0$ is the boundary of the Cauchy Development; that is the Cauchy Horizon, where the dilaton remains finite. This extension is achieved by the same way as in the case of Misner space (we have two-dimensional version when $\lambda = 0$) and the Taub-NUT space [12]. Hawking used the Misner space to discuss the chronology protection conjecture [10]. In the rest of the present paper we investigate the conjecture in this model both at classical and the quantum levels.

Next we consider the Universe with classical massless scalar fields. The solutions of (3) satisfying the periodic boundary condition always exist for an arbitrary configuration of the scalar fields. If we expand the scalar fields in Fourier series:

\[
f(x^+, x^-) = f_+(x^+) + f_-(x^-),
\]

\[
f_\pm = \alpha_\pm + \sqrt{\frac{\varepsilon}{2}} x^\pm + \sum_{n=1}^{\infty} \left( a_n^\pm \sin \frac{2\pi n}{L} x^\pm + b_n^\pm \cos \frac{2\pi n}{L} x^\pm \right),
\]

where $\alpha_\pm$, $a_n^\pm$ and $b_n^\pm$ are the expansion coefficients. We obtain the solution as follows

\[
e^{-2\phi} = M - \left\{ \frac{\varepsilon}{2\lambda} + \frac{1}{2\lambda} \Sigma \right\} t + e^{-2\lambda t} + \left( \text{oscillation part} \right),
\]

\[
\Sigma = \sum_{(\cdot) = \pm} \sum_{n=1}^{\infty} \left( \frac{2\pi n}{L} \right)^2 \left[ (a_n^{(\cdot)})^2 + (b_n^{(\cdot)})^2 \right],
\]

where the fourth term on the right hand side of the first equation in (11) is the oscillation part, that is the sum of the trigonometric functions. From (11) it should be noted that any classical configuration of matter fields makes a finite contribution to the term proportional to time, which causes divergence of the scalar curvature. Therefore the Universe inevitably meets singularity at $t = \infty$ and cannot extend to the region with closed time-like curves.
From now on we study how the classical solution change if we include backreaction. In two dimensions, the quantum effect of massless matter fields is completely determined by conformal anomaly\cite{3}. The quantum effective action is sum of the classical action (1) and the Polyakov term induced by the $N$ matter fields:

$$S_{\text{quantum}} = -\frac{\kappa}{8\pi} \int d^2x \sqrt{-g(x)} \int d^2x' \sqrt{-g(x')} R(x, x') R(x'), \quad (12)$$

where $\kappa$ is $\frac{N}{12}$ and $G(x, x')$ is a Green’s function of the scalar fields. We assume that $\kappa$ is to be a large number and use the $1/N$-expansion. Further we add the following term introduced by Russo et al.\cite{13} to the above action:

$$S'_{\text{quantum}} = -\frac{\kappa}{8\pi} \int d^2x \sqrt{-g} 2\phi R. \quad (13)$$

Making the field redefinition:

$$\chi = \sqrt{\kappa} \left( \rho - \frac{1}{2} \phi + \frac{1}{\kappa} e^{-2\phi} \right),$$

$$\Omega = \sqrt{\kappa} \left( \frac{1}{2} \phi + \frac{1}{\kappa} e^{-2\phi} \right), \quad (14)$$

we see that the semi-classical equations of motion is simplified:

$$\partial_+ \partial_- \chi = -\frac{\chi^2}{\sqrt{\kappa}} e^{\frac{2}{\sqrt{\kappa}}(\chi - \omega)},$$

$$\partial_+ \partial_- (\chi - \omega) = 0, \quad (15)$$

and

$$\partial_\pm f \cdot \partial_\pm f - 2\kappa t_\pm (x_\pm) = -\partial_\pm \chi \partial_\pm \chi + \partial_\pm \Omega \partial_\pm \Omega + \sqrt{\kappa} \partial_\pm^2 \chi, \quad (16)$$

where $\cdot$ denotes the sum over $i$ and $t_\pm$ are arbitrary chiral functions to be determined by the boundary conditions. In this case the classical matter fields $f_i$ are
interpreted as macroscopic behavior of quantized fields. For an arbitrary macroscopic part of quantized matter fields as shown in (10), the solution of the equations (15) and (16) is given by

\[
\sqrt{\kappa} \chi = M - \frac{2\kappa}{\lambda} \left( \frac{\lambda^2}{4} - t_{vev} + \frac{\varepsilon + \Sigma}{4\kappa} \right) t + e^{-2\lambda t} + \text{(oscillation part)},
\]

\[
\sqrt{\kappa} \Omega = M + \frac{2\kappa}{\lambda} \left( \frac{\lambda^2}{4} + t_{vev} - \frac{\varepsilon + \Sigma}{4\kappa} \right) t + e^{-2\lambda t} + \text{(oscillation part)},
\]

where \( t_{vev} \equiv t_\pm \) is determined to be \( \pi^2 L^2 \) due to the Casimir effect, and \( \Sigma \) is the same as in (11).

From the equations (14) and (17), we can see qualitative behavior of \( \phi \) as a function of time \( t \), when the position \( x \) is fixed (Fig.2).

We examine whether quantum version of CTC-spacetime realizes or not. From (14) and (17), we obtain \( 2\rho = 2\phi - 2\lambda t \). To extend the spacetime to the region with CTC’s, \( 2\rho \) must become linear in \( t \) as \( t \to \infty \) in the conformal flat gauge, and also the coefficient of the linear term must be negative. By comparing \( \Omega - \phi \) and \( \Omega - t \) relations, we recognize that there are two distinct types of solutions. One is realized in the case (i): \( M > \sqrt{\kappa} \Omega_{cr} \) in the following parameters:

\[
a_n^{(\pm)} = b_n^{(\pm)} = 0 \quad \forall n, \quad \varepsilon = \kappa \lambda^2 + 4\kappa t_{vev},
\]

where \( \Omega_{cr} \) is the local minimum of \( \Omega \) at \( \phi = \phi_{cr} \) (Fig.2(a)), and the first condition in (18) restrict the universe to a homogeneous one. The other is realized in the case (ii): \( \Omega_{min} = \Omega_{cr} \) (\( \Omega_{min} \) is the local minimum of \( \Omega \) in \( \Omega - t \) relation (Fig.2(b))), \( a_n^{(\pm)} = b_n^{(\pm)} = 0 \ (\forall n) \) and some appropriate negative \( M \) is chosen. In the case (i), the value of dilaton at \( \eta = 0 \) can be adjusted to be so small that the semi-classical approximation is valid. On the other hand in the case (ii), the spacetime can be extended over \( \phi = \phi_{cr} \) to the strong coupling region smoothly and the appearance of CTC-spacetime may occur in the limit of \( t = \infty \) where \( \phi \) becomes infinite. However in the case (ii), the analysis with full quantization is needed. The statement that whether CTC’s appear or not may becomes meaningless.
Finally we conclude that the spacetime never been extended across $\eta = 0$ with any configuration of quantized matters except the fine-tuned example as in (18). Thus it can be said that the chronology protection holds in a *weaker* sense in the semi-classical level including back-reaction.

In order to determine whether the chronology protection holds or not in a *strong* sense, We must go on to extend the classical and semi-classical treatments of the compact universe in this paper to the full quantization of two-dimensional dilaton gravity (for example, see [14]). For the existence of any consistent solution in the extended region ($\eta > 0$) inevitably depends on the information from the naked singularity($\eta = M$), whose neighborhood is very strong coupling region. To proceed the analysis, the construction of the physical states having such classical and semi-classical behaviors will be intriguing especially.

Recently another interesting case has been reported in [15], in which two-dimensional analogue of inflation is treated. CGHS scenario has been shown to be extensively useful for the study of back-reaction problems in general relativity.

We are most grateful to A. Hosoya for a collaboration in an early stage of the work and reading the manuscript. One of the authors(T. M.) would like to acknowledge Y. Onozawa, M. Siino and K. Watanabe for enjoyable discussions.
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**Figure Captions**

Fig. 1 Misner-type universe;
\[ t = +\infty (\eta = 0) \] is a closed null geodesics. The region: \( \eta < 0 \) is globally hyperbolic spacetime, and the region: \( \eta > 0 \) have closed time-like curves.

Fig. 2 \( \Omega - \phi \) and \( \Omega - t \) relations;
(a) \( \Omega - \phi \) relation, (b) \( \Omega - t \) relation