Dressed Polyakov loop and phase diagram of hot quark matter under magnetic field

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We evaluate the dressed Polyakov loop for hot quark matter in strong magnetic field. To compute the finite temperature effective potential, we use the Polyakov extended Nambu-Jona Lasinio model with eight-quark interactions taken into account. The bare quark mass is adjusted in order to reproduce the physical value of the vacuum pion mass. Our results show that the dressed Polyakov loop is very sensitive to the strength of the magnetic field, and it is capable to capture both the deconfinement crossover and the chiral crossover. Besides, we compute self-consistently the phase diagram of the model. We find a tiny split of the two aforementioned crossovers as the strength of the magnetic field is increased. Concretely, for the largest value of magnetic field investigated here, $eB = 19m_q^2$, the split is of the order of 10%. A qualitative comparison with other effective models and recent Lattice results is also performed.

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I. INTRODUCTION

The nature of the Quantum Chromodynamics (QCD) vacuum is one of the most intriguing aspects of modern physics. Besides, it is very hard to get a full understanding of its properties, because its most important characteristics, namely chiral symmetry breaking and color confinement, have a non-perturbative origin, and the use of perturbative methods is useless. One of the best strategies to overcome this problem is offered by Lattice QCD simulations at zero chemical potential (see \textsuperscript{[1–4]} for several examples and see also references therein). At vanishing quark chemical potential, it is almost established that two crossovers take place at nearly the same temperature; one for quark deconfinement, and another one for the (approximate) restoration of chiral symmetry. It is still under debate whether two crossovers should occur at exactly the same temperature, see for example the report in Ref. \textsuperscript{[2]}.

An alternative approach to the physics of strong interactions, which is capable to capture some of the non-perturbative properties of the QCD vacuum, at the same time being easy to manage mathematically, is the Nambu-Jona Lasinio (NJL) model \textsuperscript{[5]}, see also Refs. \textsuperscript{[6]} for reviews. In this model, the QCD gluon-mediated interactions are replaced by effective interactions among quarks, which are built in order to respect the global symmetries of QCD. Since dynamical gluons are absent in this model, it is not a gauge theory. However, it shares the global symmetries of the QCD action; moreover, the parameters of the NJL model are fixed to reproduce some phenomenological quantity of the QCD vacuum: in its simplest version, the pion decay constant, the vacuum pion mass and the vacuum chiral condensate are reproduced. Therefore, it is reasonable that the main characteristics of its phase diagram represent, at least qualitatively, those of QCD.

Critically speaking, the worst aspect of the NJL model is that it lacks confinement: massive quark poles of the quark propagator are present at any temperature and/or chemical potential. It is well known that color confinement can be described in terms of the center symmetry of the color gauge group and of the Polyakov loop \textsuperscript{[7]}, which is an order parameter for the center symmetry. Motivated by this property, the Polyakov extended Nambu-Jona Lasinio model (P-NJL model) has been introduced \textsuperscript{[8, 9]}, in which the concept of statistical confinement replaces that of the true confinement of QCD, and an effective interaction among the chiral condensate and the Polyakov loop is achieved by a covariant coupling of quarks with a background temporal gluon field. In the literature, there are several studies about various aspects of the P-NJL model. Its phase structure with two flavors and symmetric quark matter has been investigated in Refs. \textsuperscript{[10–13]}; PNJL model with a Van der Monde term has been considered in \textsuperscript{[14]}; phase structure with 2+1 flavors has been studied in Refs. \textsuperscript{[15]}; possible realization of the quarkyonic phase \textsuperscript{[16]} has been discussed in Refs. \textsuperscript{[17, 18]}; mass dependence of the phase diagram, and a possible emergence of the quarkyonic phase, has been investigated in \textsuperscript{[18]}; phase diagram with imaginary chemical potential has been studied in \textsuperscript{[19]}; dual quark condensate has been computed in \textsuperscript{[20]}; neutral phases have been investigated in \textsuperscript{[21]}; phase diagram with asymmetric quark matter have been studied in \textsuperscript{[22]}; non-local extension has been introduced in \textsuperscript{[23]}; role of eight-quark interactions in the PNJL context has been elucidated in \textsuperscript{[24]}. The modification of the QCD vacuum, and of its thermal excitations as well, under the influence of external fields, is an attractive topic. Firstly, it is extremely interesting to understand how an external field can modify the...
main characteristics of confinement and spontaneous chiral symmetry breaking. Lattice studies on the response to external magnetic fields can be found in Refs. [26, 29, 30]. Previous studies of QCD in magnetic fields, and of QCD-like theories as well, can be found in Refs. [31, 32]. A self-consistent model calculations of magnetic catalysis and of deconfinement pseudo-critical temperature in magnetic field, has been performed firstly in Ref. [33] within the PNJL model, and then in Ref. [34] using the Polyakov extended quark-meson model. Effective models in chromo-magnetic fields have been considered in Refs. [35]. Besides, strong magnetic fields might be produced in non-central heavy ion collisions [36, 37]. In this case, it has been argued that the non-trivial topological structure of thermal QCD gives rise to Chiral Magnetic Effect (CME) [38, 39].

Moreover, we compute the dressed Polyakov loop, $\Sigma_1$, in a magnetic field. Along this line, we anticipate one of our results, namely that the dressed Polyakov loop, $\Sigma_1$, is capable to feel both the Polyakov loop and the chiral condensate crossovers, whatever the strength of the magnetic field is. This occurs despite the tiny split of the two crossovers, which we observe at sufficiently strong magnetic field strength. Therefore, in view of an effective theory for finite temperature QCD in terms of just one order parameter, our results are encouraging.

The plan of the paper is as follows. In Section II, we present the model we use. In Section III, we show and discuss our numerical results. Finally, in Section IV, we draw our conclusions.

II. DRESSED POLYAKOV LOOP IN THE EFFECTIVE MODEL

In this article, we model quark matter by the following Lagrangian density

$$\mathcal{L} = \bar{q} (i\gamma^\mu D_\mu - m_0) q + g_\sigma \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \tau q)^2 \right] + g_8 \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \tau q)^2 \right]^2,$$  \tag{1}

which corresponds to the NJL lagrangian with multi-quark interactions [15]. The covariant derivative embeds the quark coupling to the external magnetic field and to the background gluon field as well, as we will see explicitly below. In Eq. (1), $q$ represents a quark field in the fundamental representation of color and flavor (indices are suppressed for notational simplicity); $m_0$ is the bare quark mass, which is fixed to reproduce the pion mass in the vacuum, $m_\pi = 139$ MeV. Our interaction in Eq. (1) consists of a four-quark term, whose coupling $g_\sigma$ has inverse mass dimension two, and an eight-quark term, whose coupling constant $g_8$ has inverse mass dimension eight.

The evaluation of the bulk thermodynamic quantities requires us to compute the quantum effective action of the model. This cannot be done exactly. Hence, we rely ourselves to the one-loop approximation for the partition function, which amounts to take the classical contribution plus the fermion determinant. The one-loop thermodynamic potential of
quark matter in external fields has been discussed in \cite{35,37}, in the case of canonical antiperiodic boundary conditions; following \cite{21}, it is easy to generalize it to the more general case of twisted boundary conditions:

\[
\Omega = U(P,P,T) + \frac{\alpha^2}{g_\sigma} + \frac{3\sigma^4 g_\sigma}{g_\sigma^4} - \sum_{f=u,d} \frac{|q f e B|}{2\pi} \sum_k \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} g_\Lambda(p_z,k)\omega_k(p_z)
\]

\[
- T \sum_{f=u,d} \frac{|q f e B|}{2\pi} \sum_k \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} \log \left( 1 + 3 Pe^{-\beta \xi} - 3 Pe^{-2\beta \xi} + e^{-3\beta \xi} \right)
\]

The standard choice of the parameters reads \cite{10}:

\[
a_0 = 3.51, \quad a_1 = -2.47, \quad a_2 = 15.2, \quad b_3 = -1.75.
\]

The parameter \(T_0\) in Eq. \ref{eq:10} sets the deconfinement scale in the pure gauge theory, i.e. \(T_0 = 270\) MeV.

Following \cite{42}, in order to define the dressed Polyakov loop, we work in a finite Euclidean volume with temperature extension \(\beta = 1/T\). We take twisted fermion boundary conditions along the compact temporal direction,

\[
q(x,\beta) = e^{-i\varphi} q(x,0), \quad \varphi \in [0, 2\pi],
\]

while for spatial directions the usual periodic boundary condition is taken. The canonical antiperiodic boundary condition for the quantization of fermions at finite temperature, is obtained by taking \(\varphi = \pi\) in the previous equation. The dual quark condensate, \(\tilde{\Sigma}_n\), is defined as

\[
\tilde{\Sigma}_n(m,V) = \int_0^{2\pi} \frac{d\varphi e^{-i\varphi n}}{V} \langle \bar{q} q \rangle G ,
\]

with \(\varphi\) defined in Eq. \ref{eq:9}.

The vacuum part of the thermodynamic potential, \(\Omega(T = 0)\), is ultraviolet divergent. This divergence is transmitted to the self-consistent equations which determine the chiral condensate and the expectation value of the Polyakov loop. In this article, we use a smooth regularization procedure by introducing a form factor \(g_\Lambda(p)\) in the diverging zero-point energy. Our choice of \(g_\Lambda(p)\) is

\[
g_\Lambda(p) = \frac{\Lambda^{2N}}{\Lambda^{2N} + (p_k^2 + 2|q f e B|k)^N};
\]

we choose two values of \(N\), namely \(N = 5\) and \(N = 7\).

The potential term \(U[P,P,T]\) in Eq. \ref{eq:2} is built by hand in order to reproduce the pure gluonic lattice data \cite{10}. Among several different potential choices \cite{49} we adopt the following logarithmic form \cite{44,43},

\[
U[P,P,T] = T^4 \left\{ -\frac{a(T)}{2} \Phi \Phi + b(T) \ln \left[ 1 - 6PP + 4(PP^3 + P^3) - 3(PPP) \right] \right\},
\]

with three model parameters (one of four is constrained by the Stefan-Boltzmann limit),

\[
a(T) = a_0 + a_1 \left( \frac{T_0}{T} \right)^2 + a_2 \left( \frac{T_0}{T} \right)^\frac{5}{2},
\]

\[
b(T) = b_3 \left( \frac{T_0}{T} \right)^3.
\]
If we denote by $z$ an element of the center of the color gauge group, then it is easy to show that $\tilde{\Sigma}$ is a regulator specified in Eq. (5) with $N$ gauge group, then it is easy to show that $\tilde{\Sigma}$ is a regulator.

Then, in order to compute the dressed Polyakov loop using Eq. (10), the twisted boundary condition, Eq. (9), must be imposed only in $\phi$. Therefrom, we firstly compute the expectation value of the Polyakov loop and to the chiral condensate, taking the twisted boundary condition, Eq. (12), keeping the expectation value of the Polyakov loop fixed at its value at $\phi = \pi$ [21]. From the very definition of the Polyakov loop, it is only approximately relevant for the center symmetry, with the same transformation rule of the thin Polyakov loop. Since the center symmetry is spontaneously broken in the deconfinement phase and restored in the confinement phase [2] (in presence of dynamical quarks, it is only approximately restored), the dressed Polyakov loop can be regarded as an order parameter for the confinement-deconfinement transition as well.

For later convenience, we scale the definition of the dressed Polyakov loop in Eq. (10), and introduce

$$\Sigma_1 = -2\pi g_\sigma \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \langle qq \rangle G,$$

where $G(\phi)$ corresponds to the expectation value of the $G$ field computed keeping twisted boundary conditions for fermions.

### III. NUMERICAL RESULTS

In this Section, we show our results. The main goal to achieve numerically is the solution of the gap equations,

$$\frac{\partial \Omega}{\partial \sigma} = 0, \quad \frac{\partial \Omega}{\partial B} = 0.$$  \hspace{1cm} (12)

This is done by using a globally convergent algorithm with backtrack [54]. From the very definition of the dressed Polyakov loop in Eq. (10), the twisted boundary condition, Eq. (9), must be imposed only in $D_p$. Therefore, we firstly compute the expectation value of the Polyakov loop and to the chiral condensate, taking $\phi = \pi$. Then, in order to compute the dressed Polyakov loop using Eq. (11), we compute the $\phi$-dependent chiral condensate using the first of Eq. (12), keeping the expectation value of the Polyakov loop fixed at its value at $\phi = \pi$ [21].

In this study, we report results obtained using the UV-regulator specified in Eq. (5) with $N = 5$ and $N = 7$. As expected, there is no qualitative difference among the pictures that the two regularization schemes lead to. As a consequence, concrete results are shown only for the case $N = 5$; for what concerns the case $N = 7$, we collect the pseudo-critical temperatures in Table I. We have also checked that the results are qualitatively unchanged if we use a hard cutoff scheme instead of the smooth UV-regulator. The parameter set for both cases is specified in Table II. In the case $N = 5$, they are obtained by the requirements that the vacuum pion mass is $m_\pi = 139$ MeV, the pion decay constant $f_\pi = 92.4$ MeV and the vacuum chiral condensate $\langle \bar{u}u \rangle \approx -241$ MeV$^3$. In this case, the chiral and deconfinement pseudo-critical temperatures at zero magnetic field are $T_{c0} = T_{d0} = 175$ MeV. Similarly, for the case $N = 7$, the chiral and deconfinement pseudo-critical temperatures at zero magnetic field are $T_{c0} = 176$ MeV and $T_{d0} = 175$ MeV; respectively; the zero temperature chiral condensate at zero magnetic field strength is fixed to $\langle \bar{u}u \rangle \approx -246$ MeV$^3$.

We remark that the main effect of the eight-quark interaction in Eq. (11) is to lower the pseudo-critical temperature of the crossovers. This has been already discussed several times in the literature [13, 45], in the context of both the NJL and the PNJL models. Therefore, it is not necessary to discuss it further here, while at the same time we prefer to stress the results that have not been discussed yet.

In order to identify the pseudo-critical temperatures, we have defined the effective susceptibilities as

$$\chi_A = (m_\pi)^9 \left| \frac{dA}{dT} \right|, \quad A = \sigma, P, \Sigma_1.$$  \hspace{1cm} (13)

Strictly speaking, the quantities defined in the previous equation are not true susceptibilities. Nevertheless, they allow to represent faithfully the pseudo-critical region, that is, the range in temperature in which the various crossovers take place. Therefore, for our purposes it is enough to compute these quantities. In Equation (13), the appropriate power of $m_\pi$ is introduced just for a matter of convenience, in order to have a dimensionless quantity; therefore, $g = 0$ if $A = \sigma, \Sigma_1$, and $g = 1$ if $A = P$.

#### A. Condensates and dressed Polyakov loop

From now on, we fix $N = 5$ unless specified. The results for this case are collected in the form of three-dimensional plots in Fig. 1 (for the case $N = 7$ the plots do not differ qualitatively). In the left panel, we plot the chiral condensate $\langle \bar{u}u \rangle^{1/3}$, the expectation value of the Polyakov loop, and the dressed Polyakov loop $\Sigma_1$, as a function of temperature and magnetic field. In the right panel, we show the contour plots of the raw data of the effective susceptibilities. The lighter the color, the higher the susceptibility. In the contour plots, the vertical axes correspond to temperature (measured in MeV); the horizontal axes represent the magnetic field $eB/m_\pi^2$.

We slice the three dimensional plots in Fig. 1 at fixed value of the magnetic field strength, and show the results in Fig. 2 where we plot the chiral condensate $S = \langle \bar{u}u \rangle^{1/3}$ (upper panel), the Polyakov loop (middle panel) and $\Sigma_1$ (lower panel) as a function of temperature, for several values of the applied magnetic field strength,
FIG. 1. *Left panel.* Chiral condensate, Polyakov loop and dressed Polyakov loop as a function of temperature and magnetic field, for the case $N = 5$. *Right panel.* Contour plots of the raw data of the effective susceptibilities. The lighter the color, the higher the susceptibility. Vertical axes correspond to temperature (in MeV); horizontal axes represent magnetic field $eB/m^2$. For the dressed Polyakov loop susceptibility, the bifurcation of the critical region is evident.
FIG. 2. Left panel. Chiral condensate $S = |\langle \bar{u}u \rangle|^{1/3}$ (upper panel), Polyakov loop (middle panel) and $\Sigma_1$ (lower panel) as a function of temperature, for several values of the applied magnetic field strength, measured in units of $m^2$. In the figures, $N = 5$ corresponds to the order of the UV-regulator in Eq. (5). Right panel. Effective susceptibilities, defined in Eq. (13), as a function of temperature, for several values of $eB$. Conventions for lines are the same as in the left panel.

measured in units of $m^2$. In the right panel, we plot fits of the effective susceptibilities in the critical regions, as a function of temperature. The fits are obtained from the raw data, using Breit-Wigner-like fitting functions. The details of the fitting procedure are not relevant for the present discussion. For graphical reasons, in Fig. [1] we plot the chiral condensate with its sign; on the other hand, in Fig. [2] we take the absolute value of this quantity.

The qualitative behavior of the chiral condensate, and of the Polyakov loop as well, is similar to that found in a previous study within the PNJL model in the chiral limit [37]. Quantitatively, the main difference with the case of the chiral limit, is that in the latter the chiral restoration at large temperature is a true second order phase transition (in other model calculations it has been reported that the phase transition might become of the first order at very large magnetic field strengths [34]). On the other hand, in the case under investigation, chiral symmetry is always broken explicitly because of the bare quark masses; as a consequence, the second order phase transition is replaced by a smooth crossover.

Another interesting aspect, observed also in the chi-
r al limit \(52\), is that the Polyakov loop crossover temperature, is less sensitive to the strength of the magnetic field than the same quantity computed for the chiral condensate. It is useful, for illustration purpose, to quantify the net shift of the pseudo-critical temperatures, for the largest value of magnetic field we have studied, \(eB = 19m_{\pi}^2\). In this case, if we take \(N = 5\) (for \(N = 7\) the results are similar), then the two crossover occur simultaneously at \(eB = 0\), at the temperature \(T_\chi^0 = T_{\Pi}^0 = 175\) MeV; for \(eB = 19m_{\pi}^2\), we find \(T_\chi = 219\) MeV and \(T_{\Pi} = 190\) MeV. Therefore, the chiral crossover is shifted approximately by \(25.1\%\), to be compared with the more modest shift of the Polyakov loop crossover, which is \(\approx 8.6\%\).

The split of the two critical temperatures at a so large value of the magnetic field strength is only of 15\%; on the Lattice, no split is observed \(26\), and a modest increase of the critical temperature is measured. Therefore, we are in partial agreement with the Lattice results, in the sense that the raising of the critical lines is observed also in our model calculation; for what concerns the split of the two crossovers, we can take our \(O(10\%)\) split as a consequence of the crudeness of the model at hand. On the Lattice, the smaller pion mass used is of the order of 200 MeV \(26\). We have verified that our qualitative picture is unchanged if we increase artificially the vacuum pion mass up to this value. In passing, we notice that using a running coupling as in \(52\), but adding at the same time two further free parameters in the model, we expect a better agreement with the Lattice. The reason is that in \(52\), the coupling \(g_{\rho}\) is a function of the Polyakov loop, and it decreases as \(P\) is increased. As a consequence, near the Polyakov loop crossover temperature, the strength of the interaction is lowered, and a partial suppression of the chiral condensate is expected. Quantitatively, it is not clear a priori if the suppression is enough to rejoin the two crossovers; only a detailed numerical study can give the answer. We leave this important investigation to a future study.

The tiny decoupling of the two crossovers found within the PNJL model, both in the chiral limit \(52\) and in the case of physical pion mass considered here, is observed also within the Polyakov quark-meson model \(34\), when in the latter the zero point energy is considered (if the vacuum energy is subtracted, then the Polyakov loop and the chiral crossovers occur always simultaneously, but the pseudo-critical temperature is a decreasing function of \(eB\), which seems in disagreement with the recent Lattice results \(26\); see also \(51\) for a recent discussion of the role of the vacuum energy within the quark-meson model). Since the Polyakov loop is coupled to quarks in the same manner both in the PNJL and in the PQM model, the tiny split of the two crossovers as \(eB\) is increased does not appear as an artifact of the PNJL model; instead, it seems to be a consequence of the link among the chiral condensate and the Polyakov loop, which is common in the two kinds of models.

In the lower panels of Figures. 1 and 2, we plot the dressed Polyakov loop as a function of temperature, for several values of \(eB\). Our definition, Eq. (11), differs from the canonical one \(42\) for an overall factor, which gives mass dimension one to our \(\Sigma_1\). For small values of \(eB/m_{\pi}^2\), the behavior of \(\Sigma_1\) as temperature is increased, is qualitatively similar to that at \(eB = 0\), which has been discussed within effective models in \(21, 14\). In particular, the dressed Polyakov loop is very small for temperatures below the pseudo-critical temperature of the simultaneous crossover. Then, it experiences a crossover in correspondence of the simultaneous Polyakov loop and chiral condensate crossovers. It eventually saturates at very large temperature (for example, in \(21\) the saturation occurs at a temperature of the order of 0.4 GeV, in agreement with the results of \(14\)). However, we do not push up our numerical calculation to such high temperature, because we expect that the effective model in that case is well beyond its range of validity.

As we increase the value of \(eB\), as noticed previously, we observe a tiny splitting of the chiral and the Polyakov loop crossovers. Correspondingly, the qualitative behavior of the dressed Polyakov loop changes dramatically: the range of temperature in which the \(\Sigma_1\) crossover takes place is enlarged, if compared to the thin temperature interval in which the crossover takes place at the lowest value of \(eB\) (compare the solid and the dotted lines in Fig. 2, as well as the the lower panel of Fig. 1).

The effective susceptibility, \(d\Sigma_1/dT\), plotted in the lower right panel of Fig. 2, is qualitatively very interesting. We observe a double peak structure, which we interpret as the fact that the dressed Polyakov loop is capable to feel (and hence, describe) both the crossovers. If we were to interpret \(\Sigma_1\) as the order parameter for deconfinement, and the temperature with the largest susceptibility with the crossover pseudo-critical temperature, then we obtain almost simultaneous crossover even for very large magnetic field. If this were the case, then the Polyakov loop computed within the PNJL model, should be interpreted only as an indicator of statistical confinement, and the deconfinement would be described by \(\Sigma_1\). Of course, this picture would not contradict the well established picture at zero magnetic field \(9–11\). Indeed, in the case of small \(eB\), we find simultaneous crossover of chiral condensate, Polyakov loop and dressed Polyakov loop. In the latter case, it would be just a matter of taste which quantity one uses to identify the deconfinement crossover. Even if it is tempting to give this kind of interpretation, which would lead to simultaneous crossover also at finite \(eB\), it is very hard to accept it without more convincing microscopic arguments. Therefore, in the prosecution of this work, we prefer to associate the deconfinement crossover to that of the Polyakov loop. Nevertheless, the dressed Polyakov loop is a new quantity which is interesting to compute. In particular, the double peak structure in the \(\Sigma_1\) effective susceptibility, which is produced if the magnetic field is strong enough, offers the evidence that the dressed Polyakov loop is intimately related to both chiral condensate and (thin) Polyakov loop, and it is ca-
TABLE II. Coefficients of the fit function defined in Eq. (14).

| $\chi$ | $T_0$ (MeV) | $\varepsilon$ |
|--------|-------------|--------------|
| $a$    | $\alpha$    |              |
| $T_\chi$, $N = 5$ | $2.4 \times 10^{-1}$ | 1.85 | 175 | 0.21 |
| $T_r$, $N = 5$    | $2.1 \times 10^{-1}$ | 1.41 | 175 | 0.08 |
| $T_\chi$, $N = 7$ | $7.8 \times 10^{-1}$ | 1.29 | 176 | 0.19 |
| $T_r$, $N = 7$    | $3.9 \times 10^{-1}$ | 1.08 | 176 | 0.01 |

Table II. As an estimator of the goodness of the various fits, we report in Table II the percentage error defined as

$$\varepsilon = 100 \times \sum \left( \frac{f_A(x_i) - y_i}{y_i} \right)^2,$$

where $\sum$ denotes the numerical value of the fit function evaluated at the data $eB$.

The picture discussed in the previous Section is made clear by the phase diagrams in Fig. 1. We measure an increase of both deconfinement and chiral crossovers; the tiny split of the two critical temperatures is of the order of 10% for the largest value of the magnetic field strength $eB$. In the quark sector, we have used both a four-quark and an eight-quark interactions. Bare quark masses are fixed to reproduce the physical value of the vacuum pion mass. This model allows to treat self-consistently both chiral symmetry breaking and (effective, or statistical) confinement. We improve the previous work [23] in three ways: we set the vacuum pion mass to its physical value; we introduce eight-quark interaction; finally, we compute the dressed Polyakov loop. Our results on the dressed Polyakov loop, $\Sigma_1$, in magnetic field show that this quantity is capable to describe both Polyakov loop and chiral crossovers. This is resumed in the double peak structure of the effective susceptibility $d\Sigma_1/dT$, see Figs 4 and 5. Moreover, we find that $\Sigma_1$ is capable to feel both the Polyakov loop crossover and the chiral condensate crossover, and suggests itself as the the possibly unique order parameter of effective QCD.

The results on the pseudo-critical temperatures as a function of $eB$ are resumed in the phase diagrams in Fig. 3. These results were anticipated in a previous work [23] in which only the chiral limit was considered, and the eight quark interaction was neglected. Our results agree qualitatively with those of Ref. [33], in which a quark-meson model coupled to the Polyakov loop is considered.

As improvement of our results, it would be interesting to consider the effects of non-locality [24]. In that arguments, which are reproduced within the PNJL model as well, as the final results on critical temperatures show. In the case of the quark-meson model, however, the picture can change even qualitatively, depending on the fate of vacuum energy contribution. If they are not included, then a simultaneous first order transition is observed at every value of $eB$ (only if $eB$ is very small the transition is a smooth crossover), and the deconfinement temperature as a function of the magnetic field strength decreases. This picture confirms the scenario anticipated in a previous work [34]. In the case of the PNJL model, we cannot reproduce the latter scenario, because of a technical reason: indeed, in our case the vacuum contribution cannot be subtracted (as a matter of fact, we do not have a further effective potential term at zero temperature, which leads to spontaneous breaking of chiral symmetry when vacuum quark contributions are subtracted). Therefore, we can limit ourselves only to a comparison with the quark-meson model with vacuum contributions taken into account.

IV. CONCLUSIONS

We have computed, for the first time in the literature, the dressed Polyakov loop for hot quark matter in external magnetic field. To compute the finite temperature effective potential in magnetic field, we have used the Polyakov extended Nambu-Jona Lasinio model, with a logarithm effective action for the Polyakov loop. In the quark sector, we have used both a four-quark and an eight-quark interactions. Bare quark masses are fixed to reproduce the physical value of the vacuum pion mass.
case, however, the computation of the fermion spectrum in the magnetic field is not trivial because of the non-local structure of the action. Another interesting possibility is the use of Monte Carlo methods to compute the PNJL partition function in magnetic field, going beyond the saddle approximation. Encouraging results along this research line in the context of the PNJL model have been reported in [53]. Therefore, it might be interesting to extend the computation of [53] to the case of quarks in external magnetic field. Even more, we expect that the running coupling introduced by the Kyushu’s group [52] would help (at least partly) to get closer crossovers in magnetic field. A numerical investigation of this subject is left to a future study. Finally, the extension of our calculation to finite quark chemical potential, and to quark matter in external chromo-magnetic fields, the latter being motivated by Lattice results [23, 30], would deserve further attention.

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