Scalar charmonium and glueball mixing
in $e^+e^- \rightarrow J/\psi X$

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Abstract

We study the possibility of the scalar charmonium and glueball mixing in $e^+e^-$ annihilation at $\sqrt{s} = 10.6$ GeV. The effects can be used to explain the unexpected large cross section (12 $\pm$ 4 fb) and the anomalous angular distribution ($\alpha = -1.1^{+0.8}_{-0.6}$) of the exclusive $e^+e^- \rightarrow J/\psi \chi_{c0}$ process observed by Belle experiments at KEKB. We calculate the helicity amplitudes for the process $e^+e^- \rightarrow J/\psi H(0^{++})$ in NRQCD, where $H(0^{++})$ is the mixed state. We present a detailed analysis on the total cross section and various angular asymmetries which could be useful to reveal the existence of the scalar glueball state.

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1 Introduction

The charmonium production has long been considered as a good process for investigating both perturbative and nonperturbative properties of quantum chromodynamics (QCD), because of the relatively large difference between the scale at which the charm-quark pair is produced at the parton level and the scale at which it evolves into a quarkonium. In particular, comparing to hadron colliders, $e^+e^-$ colliders, provide a cleaner environment to study the charmonium productions and decays. However, some puzzles arise from the recent measurements on the prompt $J/\psi$ productions at BaBar and Belle [1, 2, 3]. For the inclusive $J/\psi$ productions, the cross section is much larger than the predictions of nonrelativistic quantum chromodynamics (NRQCD) [4]; there is also an over-abundance of the four-charm-quark processes including the exclusive $J/\psi$ and charmonium productions; there is no apparent signal in the hard $J/\psi$ spectrum which has been predicted by the $J/\psi gg$ production mode as well as the color-octet mechanism in NRQCD. To provide plausible solutions and explanations for these conflicts, theorists have studied the possibilities of the contribution from two-virtual-photon mediate processes [5], large higher-order QCD corrections [6, 7], collinear suppression at the end-point region of the $J/\psi$ momentum [8, 7], contribution from the $J/\psi$-glueball associated production [9] and contribution from a very light scalar boson [10].

Although all of the above corrections significantly enhance the cross sections for the prompt $J/\psi$ productions, they can be well distinguished by their momentum and angular distributions. The two-virtual-photon processes have a very hard spectrum for the $J/\psi$ momentum and an enhancement in the large $|\cos \theta|$ region ($\theta$ is the $J/\psi$ scattering angle in the center-of-mass frame) due to the $t$-channel electron-exchange contribution [5]. The collinear suppression softens the hard spectrum of the color-octet production [8] as well as that of the color-singlet $J/\psi gg$ process [7], while the high-order QCD corrections are generally expected not to change the angular distribution strongly and their effect could be represented by a large renormalization $K$ factor [7].

$J/\psi$-glueball associated production in $e^+e^-$ annihilation has been shown to be of a special interest recently [9]. As a basic prediction of QCD, glueball states have been studied in the QCD sum rules [11, 12], bag model [13, 14], nonrelativistic potential model [15], and lattice QCD [16]. In the search of the glueballs, radiative decays of heavy quarkonium, such as $J/\psi \rightarrow \gamma X$ and $\Upsilon \rightarrow \gamma X$, are regarded as favorable processes for glueball production and high statistics data have been studied by BES and CLEO Collaborations [17, 18]. The quarkonium-glueball associated productions at B factories should be another good place to discover glueballs because their amplitudes are essentially the same as those of the quarkonium decays. If glueball masses lie in the charmonium mass region, the glueball may be hidden in the charmonium resonance peak in the exclusive production, and this mechanism may contribute to the excess of exclusive charmonium-pair production events. In Ref. [9], $J/\psi$-glueball productions $e^+e^- \rightarrow J/\psi gg \rightarrow J/\psi G$ have been studied, and at the leading-twist level and the leading-order of the NRQCD velocity-power-counting rule, the production of the scalar glueball state $G_0(0^{++})$ is found to be dominant. As we shall see, the angular distribution of $e^+e^- \rightarrow J/\psi G_0$ is dramatically different from those of the other exclusive
$J/\psi$-charmonium productions, such as $e^+e^- \rightarrow J/\psi\eta_c$ and $e^+e^- \rightarrow J/\psi\chi_{c0}$. The angular distributions of all those processes with single virtual photon exchange can be parametrized as

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \alpha \cos^2\theta,$$

by using an asymmetry parameter $\alpha (-1 \leq \alpha \leq 1)$. For $e^+e^- \rightarrow J/\psi\eta_c$ and $e^+e^- \rightarrow J/\psi\chi_{c0}$, $\alpha = 1$ and 0.25 respectively [19], while for the $J/\psi-G_0$ production mode, $\alpha$ is estimated to be $-0.85$ in NRQCD [9]. We note here that preliminary result on the $J/\psi$ angular distribution in the $J/\psi-\chi_{c0}$ region is $\alpha = -1^{+0.8}_{-0.6}$, which is more consistent with $\alpha \sim -1$ rather than $\alpha \sim 0.25$ [3]. Conservatively, the present Belle data [3] suggest that $\alpha$ should be negative.

This unexpected angular distribution leads us to consider further the possibility of the scalar charmonium $\chi_{c0}$ and glueball $G_0$ mixing. In this Letter, we will focus on the mixing effects on various angular distributions which could be useful to reveal the existence of the scalar glueball state.

## 2 Formulae

Let us start with the angular momentum and parity considerations. The $J/\psi$-scalar($H$) associated production via a virtual photon in the $J^{PC}$ notation is $1^{-+} \rightarrow 1^{-+} + 0^{++}$. If we denote by $L$ the orbital angular momentum between $J/\psi$ and $H$, the angular momentum conservation tells $L = 0, 1, 2$. Conservation of parity gives $-1 = 1 \times (-1) \times (-1)^L$, and hence $L$ should be even. We are left with $L = 0$ or 2, that is, only $S$-wave and $D$-wave $J/\psi-H$ are allowed.

Now we present the helicity amplitudes for the $J/\psi-\chi_{c0}$ and $J/\psi-G_0$ productions separately in NRQCD. One of the crossed Feynman diagrams for $e^+e^- \rightarrow J/\psi\chi_{c0}$ is shown in Fig. 1(a), and that for $e^+e^- \rightarrow J/\psi G_0$ is shown in Fig. 1(b). We adopt the standard covariant projection formalism for the color singlet productions [20]. If a charmonium carries the momentum $P$, then the charm quark and anti-quark within the charmonium are on-shell and carry momenta $P/2 - p$ and $P/2 + p$, where $p$ is the relative momentum of the quark pair and satisfies $p \cdot P = 0$. The amplitudes for the charm-pair production are furthermore projected into a certain spin component by the projection operator

$$P_{S,S_z} = \sum_{s_1,s_2} \langle 1/2, s_1; 1/2, s_2|S, S_z\rangle.$$

After expanding the amplitudes in $p$ and convoluting them with the wave functions at the origin, one obtains the helicity amplitudes for $S$-wave and $P$-wave charmonia, such as $J/\psi$ and $\chi_{c0}$. For the glueball production as shown in Fig. 1(b), we follow the method of Ref. [9] and extract the leading-twist contribution by expanding the gluon momenta in the light-cone direction and requiring the gluon pair to be collinear. We define the light-cone direction as $n^\mu = (1, 0, 0, 1)$, while the $J/\psi$ momentum direction is along $\bar{n} = (1, 0, 0, -1)$. The formation of the scalar glueball $G_0$ from the gluon pair is described by a collinear wave
Figure 1: (a) One of the four Feynman diagrams for \( e^+ e^- \to J/\psi \chi_{c0} \); (b) One of the six Feynman diagrams for \( e^+ e^- \to J/\psi G_0 \).

function \( I_0 = \int_0^1 dx \phi_0(x) \), where \( \phi_0 \) is the distribution wave function for \( G_0 \) and \( x \) is the light-cone momentum fraction. We denote the \( G_0 \) momentum by \( k = (k^+ = k \cdot \bar{n}, k^- = k \cdot n, 0_\perp) \), and further represent the transverse momentum of a gluon inside \( G_0 \) as \( k_\perp \). By integrating over \( k_\perp \) and transforming from the momentum to the coordinate space, the distribution wave function \( \phi_0(x) \) is obtained as

\[
\phi_0(x) = \frac{F^0_{\alpha\beta}}{\sqrt{2(N_c^2 - 1)}} \int \frac{d^2 k_\perp dz^+ d^2 z_\perp}{(2\pi)^3 k^- x(1 - x)} e^{-i(2k^- z^+ - k_\perp \cdot z_\perp)} \langle G_0 | TG^{+\alpha}_a (0^-, z^+, z_\perp) G^{-\beta}_a (0) | 0 \rangle.
\] (3)

Here \( F^0_{\alpha\beta} = [-g_{\alpha\beta} + (n_\alpha \bar{n}_\beta + \bar{n}_\alpha n_\beta)/2]/\sqrt{2} \) is the scalar projector. \( G_a^- \) is the gluon field along the light-cone direction and \( a \) is the color index.

Under the above approximations, the helicity amplitudes for \( J/\psi - \chi_{c0} \) production are given by,

\[
M_\chi(\sigma, \lambda) = g_s^2 e^2 Q_\chi \frac{16}{\sqrt{3 s^4 m_c}} \left[ (9s^2 - 56m_c^2) j_\sigma \cdot \epsilon^*_\lambda + (-14s + 24m_c^2) j_\sigma \cdot P_1 \epsilon^*_\lambda \cdot P_2 \right] \Phi(0) \Phi'(0),
\] (4)

and those for \( J/\psi - G_0 \) production are

\[
M_g(\sigma, \lambda) = g_s^2 e^2 Q_c \frac{16 \sqrt{m_c}}{s} \left[ j_\sigma \cdot \epsilon^*_\lambda / s - j_\sigma \cdot n \epsilon^*_\lambda \cdot n / 2s \right] \Phi(0) I_0,
\] (5)

where \( j \) is the electron-positron current and \( \epsilon \) is the polarization vector of \( J/\psi \). \( \sigma \) and \( \lambda \) denote the helicities of the electron-positron current (along the electron beam direction) and \( J/\psi \), respectively. \( \Phi(0) \) and \( \Phi'(0) \) are the wave functions at the origin for \( J/\psi \) and \( \chi_{c0} \). In the center-of-mass frame, the amplitudes are reduced to

\[
M_{\chi,g}(\sigma = \pm, \lambda = 0) = g_s^2 e^2 Q_c a_{\chi,g} \sin \theta,
M_{\chi,g}(\sigma = \pm, \lambda = \pm) = g_s^2 e^2 Q_c b_{\chi,g} \frac{\sigma + \lambda \cos \theta}{\sqrt{2}},
\] (6)

where the coefficients \( a_{\chi,g} \) and \( b_{\chi,g} \) are for the longitudinally and transversely polarized \( J/\psi \) amplitudes respectively. We find

\[
a_{\chi} = C_F \frac{32}{\sqrt{3} s^2 r^2} \left( 4 + 10r^2 - 3r^4 \right) \Phi(0) \Phi'(0),
\]

\[b_{\chi} = C_F \frac{32}{\sqrt{3} s^2 r^2} \left( 4 + 10r^2 - 3r^4 \right) \Phi(0) \Phi'(0),\]
\[ b_\chi = C_F \frac{32}{\sqrt{3}} \frac{1}{s^2 r} (-18 + 7r^2) \Phi(0) \Phi'(0), \]  

(7)

for \( J/\psi - \chi_{c0} \) production, and

\[ a_g = \sqrt{C_F} \frac{32}{s \sqrt{m_c}} \frac{1}{4 - r^2} \Phi(0) I_0, \]
\[ b_g = -\sqrt{C_F} \frac{16}{s \sqrt{m_c}} \frac{r}{4 - r^2} \Phi(0) I_0, \]

(8)

for \( J/\psi - G_0 \) production. Here \( r = 4m_c/\sqrt{s} \) and \( C_F = (N_c^2 - 1)/(2N_c) = 4/3 \) is the color factor. These results are consistent with those in Refs. [9, 19, 21].

It is natural to expect the mixing between the pure states of \( \chi_{c0} \) and \( G_0 \) if they carry the same quantum numbers \( J^{PC} = 0^{++} \) and have small mass difference. The pure charmonium and glueball states, \( \chi_{c0} \) and \( G_0 \), and the mass eigenstates \( H_1, H_2 \) are then linked by

\[
\begin{pmatrix} \chi_{c0} \\ G_0 \end{pmatrix} = U \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, \quad U = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix},
\]

(9)

where \( \xi \) is the mixing angle. The mass-squared matrix for \( \chi_{c0} \) and \( G_0 \) has the following form and is diagonalized as

\[
U^\dagger \begin{pmatrix} (M_\chi)^2 & \Delta \\ \Delta & (M_g)^2 \end{pmatrix} U = \begin{pmatrix} (M_1)^2 & 0 \\ 0 & (M_2)^2 \end{pmatrix}.
\]

(10)

We assume that the observed \( \chi_{c0} \) state is \( H_1 \), and hence \( M_1 = M(\chi_{c0})_{\text{observed}} = 3.415 \) GeV [22]. Because \( H_1 \) properties observed in charmonium radiative decays are consistent with the pure \( \chi_{c0} \) assumption, we expect the mixing angle to be small \( \sin^2 \xi \leq 0.1 \). The mixing angle \( \xi \) is real if the transition matrix element \( \Delta \) is real. If \( M_\chi \geq M_g \), \( M_1 > M_2 \), while if \( M_\chi \leq M_g \), \( M_1 < M_2 \). The mixing angle can be non-negligible \( (\sin^2 \xi \sim 0.1) \) if \( |\Delta| \sim |M_\chi^2 - M_g^2| \).

The helicity amplitudes for \( J/\psi - H_1 \) and \( J/\psi - H_2 \) productions keep the forms in Eq. (6) and the coefficients \( a_{\chi,g} \) and \( b_{\chi,g} \) have to be replaced by \( a_{H_1,H_2} \) and \( b_{H_1,H_2} \) respectively. With Eq. (9), we have

\[
\begin{align*}
     a_{H_1} &= a_\chi \cos \xi - a_g \sin \xi, & a_{H_2} &= a_\chi \sin \xi + a_g \cos \xi, \\
     b_{H_1} &= b_\chi \cos \xi - b_g \sin \xi, & b_{H_2} &= b_\chi \sin \xi + b_g \cos \xi.
\end{align*}
\]

(11)

In the \( \sin \xi = 0 \) limit, \( H_1 \) is a pure \( \chi_{c0} \), while \( H_2 \) is a pure glueball.

## 3 Results

For \( J/\psi \) productions in \( e^+ e^- \) annihilation, the mixing effects can be measured via the angular distributions of \( J/\psi \) and its leptonic decays. According to the helicity amplitudes in Eq.
(6), the triple angular distribution for the process $e^+e^- \to J/\psi H \to l^+l^- H$ ($H$ denotes $H_1$ or $H_2$) is given by

$$\frac{d\sigma}{d\cos \theta d\cos \phi d\phi} = \frac{3g_4^2 e^4 Q_2^2 \sqrt{1-r^2} B}{2048\pi^2 s} \times \left[ |b_H|^2 (1 + \cos^2 \theta)(1 + \cos^2 \phi) + 2|a_H|^2 \sin^2 \theta \sin^2 \phi \right] + 2\text{Re}(a_H^{*}b_H) \sin 2\theta \sin 2\phi \cos \phi^* + |b_H|^2 \sin^2 \theta \sin^2 \phi \cos 2\phi^*, \quad (12)$$

where $\theta^*$ and $\phi^*$ are the polar and azimuthal angles of $l^-$ in the $J/\psi$ rest frame. The polar angle $\theta^*$ is measured from the $J/\psi$ momentum direction in the $e^+e^-$ collision rest frame, and $\phi^*$ is measured from the scattering plane. In Eq. (12), $B$ is the branching fraction $B(J/\psi \to l^+l^-) = 5.9\%$ for $l = e, \mu$.

With respect to the mass difference of the mass eigenstates $H_1$ and $H_2$, we meet with two different cases: (1) If the mass difference is large enough, only $H_1$ has been identified as the $\chi_{c0}$ resonance. (2) It is also possible that the mass difference is quite small and both $H_1$ and $H_2$ contribute to the observed resonance peak. Therefore we will consider these two cases separately.

In the first case, we rename $H_1$ as $H$. By integrating over $\theta^*$ and $\phi^*$ and comparing with Eq. (1), we obtain

$$\alpha = \frac{b_H^2 - a_H^2}{b_H^2 + a_H^2} = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}, \quad (13)$$

where $\sigma_T$ and $\sigma_L$ are the cross sections for the transversely and longitudinally polarized $J/\psi$ productions, and $\sigma = \sigma_T + \sigma_L$ is the total cross section. One can obtain the $\cos \theta^*$ distribution by integrating over $\theta$ and $\phi^*$ in Eq. (12), and obtain $d\sigma/d\cos \theta^* \sim 1 + \alpha^* \cos^2 \theta^*$. It is easy to find $\alpha^* = \alpha$ according to Eq. (12). The interference term $2\text{Re}(a_H^{*}b_H)$ can be measured through the combination of the $\theta$, $\theta^*$ and $\phi^*$ distributions. We introduce a new quantity $\alpha_{off}$ as the normalized interference term

$$\alpha_{off} = \frac{2\text{Re}(a_H^{*}b_H)}{b_H^2 + a_H^2}. \quad (14)$$

Both $\alpha$ and $\alpha_{off}$ are bounded in the interval $[-1, 1]$.

Now we present our numerical results based on the input parameter values $e^2/4\pi = 1/129.6$, $g_4^2/4\pi = 0.26$, $m_c = 1.5$ GeV, and $\sqrt{s} = 10.58$ GeV. The value of the $J/\psi$ wave function at the origin can be obtained from $\Gamma(J/\psi \to e^+e^-)$, $|\Phi(0)|^2 = 0.0336$ GeV$^3$ in the leading order of NRQCD [19, 6, 7].

The $\chi_{c0}$ wave function at the origin $\Phi'(0)$ is estimated from the observed width $\Gamma(\chi_{c0} \to \gamma\gamma)$ [19]. Since we assume that the observed state is a mixture $H$, the partial width is actually proportional to $|\cos \xi \Phi'(0)|^2$. Here we assume that only the charmonium content of $H$ contributes to the decay $\chi_{c0} \to \gamma\gamma$ at the leading order in $\alpha_s$. Therefore we obtain $|\cos \xi \Phi'(0)|^2 = 0.0117$ GeV$^5$. It should be noted that the partial widths and the observed radiative transition rates can be affected at several percent level for the mixing angle of the order of $\sin^2 \xi \sim 0.1$. 

5
The upper bound of the glueball wave function $|I_0|$ is obtained from the radiative $\Upsilon$ decay. The CUSB Collaboration reported a 90%-C.L upper limit for the branching ratio $Br(\Upsilon \to 3 \mathrm{GeV})$ which is about $0.01\%-0.15\%$ for the scalar boson $H$ mass between 2 and 8.5 GeV \cite{23}. The upper bound is obtained in terms of the CUSB mass resolution, 20 MeV. As discussed in Ref. \cite{9}, if the $H$ decay width $\Gamma_H$ is larger than the resolution, the upper bound should be less by a factor of $\Gamma_H/(20\text{MeV})$. In Ref. \cite{9}, the authors considered the pure glueball production process, $e^+e^- \to J/\psi G_0$, and they assumed the glueball decay width should be less than about 100 MeV, which is the full width at half maximum of the $\chi_{c0}$ peak in the Belle fit to the $J/\psi$ recoil mass distribution. They found the upper bound for the glueball wave function $|I_0|^2 < 5.8 \times 10^{-3} \text{GeV}^2$. Accordingly, the rate of the cross sections of the pure glueball and charmonium productions $\sigma_{J/\psi\chi_{c0}}/\sigma_{J/\psi H}$ which is proportional to $|I_0|^2/|\Phi'(0)|^2$, is less than 0.72. Similar upper bound applies in our case, except that the radiative $\Upsilon$ decay rate constrains $\sin \xi I_0$ instead of $|I_0|^2$, since $\Upsilon$ can decay to the glueball content of $H$ only, at the leading order in $\alpha_s$. In the $e^+e^- \to J/\psi H$ process, because the contributions from the charmonium and glueball contents are proportional to $\cos^2 \xi$ and $\sin^2 \xi$, respectively, the upper bound of Ref. \cite{9} leads to the bound $(\sin^2 \xi \sigma_{J/\psi\chi_{c0}})/(\cos^2 \xi \sigma_{J/\psi H}) < 0.72$. We therefore define the following quantity

$$\mathcal{R} = \text{Sign} \left[ \sin \xi \frac{I_0}{\Phi'(0)} \right] \times \frac{\sin^2 \xi \sigma_{J/\psi\chi_{c0}}}{\cos^2 \xi \sigma_{J/\psi H}} \tag{15}$$

to present our numerical results. Here $\text{Sign} \left[ \sin \xi \frac{I_0}{\Phi'(0)} \right]$ determines the sign of the interference terms. The $\xi$-dependence of our results enters only through the rate $\mathcal{R}$. For simplicity, we assume that the wave functions $\Phi'(0)$ and $I_0$ are real.

The magnitude of the parameter $\mathcal{R}$ can be as large as unity if $H_1$ is the only gluon-rich state which contribute to the radiative $\Upsilon$ decay. It is, however, unlikely that the $H_2$ mass lies outside of the reach of the CUSB search \cite{23}. If the $H_2$ mass is in the region 2-8.5 GeV, the radiative $\Upsilon$ decay rate should constrain $|\cos \xi I_0|^2$, and therefore gives an upper bound on $|\mathcal{R}|$ in Eq. (15) of the order of $\tan^2 \xi (\Gamma_{H_2}/100\text{MeV}) \times 0.72$. If $\sin^2 \xi < 0.1$ and if the $H_2$ width $\Gamma_{H_2}$ is below 500 MeV, the allowed range of $\mathcal{R}$ is reduced to $|\mathcal{R}| \leq 0.4$.

In Fig. 2, we show the cross section for $e^+e^- \to J/\psi H$ in the interval $-1 < \mathcal{R} < 1$. We find the cross section from the contribution of the $\chi_{c0}$ content, $\cos^2 \xi \sigma_{J/\psi\chi_{c0}}$, is 2.7 fb, much smaller than $12 \pm 4$ fb which we estimate from the preliminary Belle data \cite{24}. We note that the contribution from the interference term can significantly increase the cross section in the region $-1 < \mathcal{R} < 0$, and decrease the cross section for $0 < \mathcal{R} < 1$. For instance, at $\mathcal{R} = -0.1$, the total cross section is about 4.3 fb, while $\sin^2 \xi \sigma_{J/\psi\chi_{c0}} = |\mathcal{R}| \cos^2 \xi \sigma_{J/\psi\chi_{c0}} = 0.27$ fb. The interference term makes up the difference $4.3 - 2.7 - 0.27 = 1.3$ fb of the total cross section, which is about five times greater than the direct contribution from the glueball content. At $\mathcal{R} = -0.4$, the total cross section is about 6.5 fb among which 2.7 fb comes from the interference contribution. While at $\mathcal{R} = -1$, the total cross section can be as large as 9.6 fb (4.2 fb from the interference contribution), which is comparable with the central value of the experimental data.

In Fig. 3, we plot the angular asymmetry parameters $\alpha$ (solid line) and $\alpha_{\text{off}}$ (dashed line)
Figure 2: The cross section for $e^+e^- \rightarrow J/\psi H$ in the interval $-1 < \mathcal{R} < 1$ for the case that only one of the mass eigenstates lies in the ‘$\chi_{c0}$’ resonance.

Figure 3: The asymmetries $\alpha$ and $\alpha_{\text{off}}$ in the interval $-1 < \mathcal{R} < 1$ for the case that only one of the mass eigenstates lies in the ‘$\chi_{c0}$’ resonance.

which are defined in Eqs. (13) and (14) as functions of $\mathcal{R}$. $\alpha$ for the charmonium content is 0.25 at $\mathcal{R} = 0$, while for the glueball content is $-0.85$. The Belle fit finds $\alpha = -1.1^{+0.8}_{-0.6}$, consistent with $-1$ [3]. Although the error is still quite large, we may state that $\alpha < 0$ is favored by the Belle data [3]. In Fig. 3, $\alpha < 0$ corresponds to the region $\mathcal{R} < -0.045$ and $\mathcal{R} > 0.85$. At $\mathcal{R} = -0.4$ and $\mathcal{R} = -1$, $\alpha$ can be as small as $-0.30$ and $-0.43$, respectively. The region $\mathcal{R} > 0.85$ is not favored because of the suppressed cross section in Fig. 2. For the case of the asymmetry parameter $\alpha_{\text{off}}$, $\alpha_{\text{off}} = -0.97$ for the charmonium content ($\mathcal{R} = 0$), and $\alpha_{\text{off}} = -0.52$ for the glueball content ($\mathcal{R} \rightarrow \pm \infty$). According to Eqs. (13) and (14), $\alpha_{\text{off}} = \pm 1$ should correspond to $\alpha = 0$ which reflects the produced $J/\psi$ mesons are unpolarized ($\sigma_T = 2\sigma_L$). $\alpha_{\text{off}} = 0$ corresponds to $\alpha = 1$ or $-1$, that is, all $J/\psi$ are transversely or longitudinally polarized. These can be observed in Fig. 3 at $\mathcal{R} = -0.045$ ($\alpha = 0$ while $\alpha_{\text{off}} = -1$), $\mathcal{R} = 0.27$ ($\alpha = 1$ while $\alpha_{\text{off}} = 0$), and $\mathcal{R} = 0.85$ ($\alpha = 0$ while $\alpha_{\text{off}} = 1$). In the region $-1 < \mathcal{R} < -0.045$ favored by the Belle data, $\alpha_{\text{off}}$ varies slightly from $-1$ to $-0.90$. 

7
Figure 4: The asymmetries $\alpha$ and $\alpha_{\text{off}}$ in the interval $\bar{\mathcal{R}} = \sigma_{J/\psi G_0}/\sigma_{J/\psi \chi_{c0}} < 1$ for the case that both of the mass eigenstates $H_1$ and $H_2$ lie in the $`\chi_{c0}'$ resonance.

Now we turn to the second case in which both of the two mass eigenstates $H_1$ and $H_2$ are hidden in the $`\chi_{c0}'$ resonance peak of the $J/\psi$ recoil mass distribution in the Belle data [3]. In this case, one can obtain the total cross section and asymmetries by summing over the triple angular distributions in Eq. (12) for $H_1$ and $H_2$. By using the relation between the coefficients $a_{H_1 H_2}, b_{H_1 H_2}$ and $a_{\chi g}, b_{\chi g}$ in Eq. (11), we find that

$$
\begin{align*}
\frac{a_{H_1}^2 + a_{H_2}^2}{b_{H_1}^2 + b_{H_2}^2} &= \frac{a_{\chi}^2 + a_{g}^2}{b_{\chi}^2 + b_{g}^2}, \\
\frac{a_{H_1}^* b_{H_1} + a_{H_2}^* b_{H_2}}{b_{H_1}^2 + a_{H_2}^2} &= \frac{a_{\chi}^* b_{\chi} + a_{g}^* b_{g}}{b_{\chi}^2 + a_{g}^2}.
\end{align*}
$$

(16)

From the above relations, we can find that the sum of the contributions from the two mixed states $H_1$ and $H_2$ is essentially identical to the sum of the contributions from the pure $\chi_{c0}$ and $G_0$ states.

The sum of the cross sections for $J/\psi - \chi_{c0}$ and $J/\psi - G_0$ productions is simply to be $\sigma_{J/\psi \chi_{c0}} + \sigma_{J/\psi G_0} = (1 + \bar{\mathcal{R}})\sigma_{J/\psi \chi_{c0}}$, where

$$
\bar{\mathcal{R}} = \frac{\sigma_{J/\psi G_0}}{\sigma_{J/\psi \chi_{c0}}}.
$$

(17)

For the leading order estimate of $\sigma_{J/\psi \chi_{c0}} = 2.7$ fb, the sum of the cross sections can be as large as 5.4 fb for $\bar{\mathcal{R}} = 1$, which is to be compared with our estimate of $12 \pm 4$ fb [24].

The asymmetry $\alpha$ for this case is given by

$$
\alpha = \frac{b_{\chi}^2 - a_{\chi}^2 + b_{g}^2 - a_{g}^2}{b_{\chi}^2 + a_{\chi}^2 + b_{g}^2 + a_{g}^2},
$$

(18)

and $\alpha_{\text{off}}$ is expressed as

$$
\alpha_{\text{off}} = \frac{2\text{Re}(a_{\chi}^* b_{\chi} + a_{g}^* b_{g})}{b_{\chi}^2 + a_{\chi}^2 + b_{g}^2 + a_{g}^2}.
$$

(19)
where $a_{\chi,g}$ and $b_{\chi,g}$ follow from Eqs. (7) and (8). They are plotted against $R$ in Fig. 4. One can find $\alpha = 0.25$ and $\alpha_{\text{off}} = -0.97$ by setting $R = 0$ for the $J/\psi$-$\chi_{c0}$ production, and $\alpha = -0.85$ and $\alpha_{\text{off}} = -0.52$ for the $J/\psi$-$G_0$ production in the $R \to \pm \infty$ limit, which are the same as those for the previous case. $\alpha$ falls off and $\alpha_{\text{off}}$ goes up with increasing $R$. $R > 0.20$ gives $\alpha < 0$ and $\alpha_{\text{off}} > -0.87$. At $R = 1$, $\alpha = -0.41$ and $\alpha_{\text{off}} = -0.70$.

In this paper, we studied the disagreement between the experiment and the NRQCD prediction for the $J/\psi$-$\chi_{c0}$ associated production, in the production cross section and the $J/\psi$ angular distribution. We introduced a charmonium-glueball mixing mechanism to explain this discrepancy and propose to study various angular distributions to explore the possible mixing effects. Our results may facilitate the present and future experimental measurements to resolve possible glueball contents in the charmonium resonances.

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[24] Ref. [3] presents the accumulated event numbers for the $J/\psi$-$\eta_c$ and $J/\psi$-$\chi_{c0}$ productions are $175\pm23$ and $61\pm21$ respectively. With the published result for the $J/\psi$-$\eta_c$ cross section $\sigma(e^+e^- \rightarrow J/\psi \eta_c) \geq 33 \pm 11$ fb [2, 3], we estimate the $J/\psi$-$\chi_{c0}$ cross section to be about $12 \pm 4$ fb, provided that the detection efficiencies do not differ much between the two channels.