Weak annihilation rare radiative B-decays

Anastasiia Kozachuk\textsuperscript{a,b}, Nikolai Nikitin\textsuperscript{a,b,c}

\textsuperscript{a}M. V. Lomonosov Moscow State University, Physical Faculty, 119991, Moscow, Russia
\textsuperscript{b}D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University, 119991, Moscow, Russia
\textsuperscript{c}A. I. Alikhanov Institute for Theoretical and Experimental Physics, 117218 Moscow, Russia

Abstract

We give predictions for the branching ratios of several rare radiative B-decays which proceed only through weak annihilation mechanism. In addition to previous analysis we estimate the branching ratios of $B_s^0 \to D^{*0} \gamma$ and $B_c^+ \to \rho^+ \gamma$ decays and obtain $2.87 \times 10^{-9}$ and $2.59 \times 10^{-7}$ for them correspondingly. The accuracy of the predictions is at the level of 20%.

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1. Introduction

The investigation of rare $B$ decays forbidden at the tree level in the Standard Model provides the possibility to probe the electroweak sector at large mass scales. Interesting information about the structure of the theory is contained in the Wilson coefficients entering the effective Hamiltonian which take different values in different theories and have not been observed. So far, only upper limits on the branching ratios of these decays have been obtained: In 2004, the BaBar Collaboration provided the upper limit $B(B^0 \to J/\psi \gamma) < 1.6 \times 10^{-6}$ \cite{1}. Last year, the LHCb Collaboration reached the same sensitivity to the $B^0_s$-decay and set the limit on the $B^0_s$ decay: $B(B_s^0 \to J/\psi \gamma) < 1.7 \times 10^{-6}$ and $B(B_s^0 \to J/\psi \gamma) < 7.4 \times 10^{-6}$ at 90\% CL \cite{2}. Obviously, with the increasing statistics, the prospects to improve the limits on the branching ratios by one order of magnitude or eventually to observe these decays in the near future seem very favorable.

In this paper we consider more annihilation type decays in addition to our recent work \cite{3}. The paper is organized as follows: In Section 2 the effective weak Hamiltonian and the structure of the amplitude are recalled. In Section 3 we consider the photon emission from the $B$-loop and present the $B \to \gamma$ transition form factors within the relativistic dispersion approach based on constituent quark picture. Section 4 contains the analysis of the $V \to \gamma$ transition form factors. Finally, in Section 5 the numerical estimates are given.

2. The effective Hamiltonian, the amplitude, and the decay rate

We consider the weak annihilation radiative $B \to V \gamma$ transition, where $V$ is the vector meson. The corresponding amplitude is given by the matrix element of the effective Hamiltonian \cite{4}

\[ A(B \to V \gamma) = (\gamma(q_1) V(q_2) H_{\text{eff}}(p)), \]

where $p$ is the $B$ momentum, $q_2$ is the vector-meson momentum, and $q_1$ is the photon momentum, $p = q_1 + q_2$, $q_1^2 = 0$, $q_2^2 = M_V^2$, $p^2 = M_B^2$. The effective weak Hamiltonian of the transition has the form (we provide in this Section formulas for the effective Hamiltonian with the flavor structure $d \bar{c} u \bar{b}$, but all other decays of interest may be easily described by an obvious replacement of the quark flavors and the corresponding CKM factors $\xi_{\text{CKM}}$):

\[ H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \xi_{\text{CKM}} (C_1(\mu) O_1 + C_2(\mu) O_2), \]

$G_F$ is the Fermi constant, $\xi_{\text{CKM}} = V^{*}_{ud} V_{ub}$, $C_{1,2}(\mu)$ are the scale-dependent Wilson coefficients \cite{4}, and we only show the relevant four-quark operators

\[ O_1 = d_u \gamma_5(1 - \gamma_5)c_u \bar{u}_b \gamma_s(1 - \gamma_5)b_d, \]

\[ O_2 = d_u \gamma_5(1 - \gamma_5)c_d \bar{u}_b \gamma_s(1 - \gamma_5)b_u. \]
We use notations \( e = \sqrt{4\pi\alpha_{em}} \), \( \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \), \( \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2 \), \( \epsilon^{0123} = -1 \) and \( \text{Sp} \left( \gamma^0\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta \right) = 4i\epsilon^{\mu\nu\alpha\beta} \).

It is convenient to parameterize the amplitude in the following way

\[
A = \frac{eG_F}{\sqrt{2}} \left[ \epsilon_{\mu\nu} \langle \gamma^\mu q_1 - \gamma^\nu q_1 \rangle F_{PC} \right] + ie_v \epsilon^\mu \left( \langle \gamma^\mu q_1 - \gamma^\nu q_1 \rangle F_{PV} \right),
\]

where \( F_{PC} \) and \( F_{PV} \) are the parity-conserving and parity-violating invariant amplitudes, respectively. Here \( \epsilon_2(\epsilon_1) \) is the vector-meson (photon) polarization vector. We use the short-hand notation \( \epsilon_{abcd} = \epsilon_{\alpha\beta\gamma\delta}b^\alpha c^\beta d^\gamma \) for any 4-vectors \( a, b, c, d \).

For the decay rate one finds

\[
\Gamma(B \rightarrow V\gamma) = \frac{G_F^2}{16} \frac{\alpha_{em}}{M_B^3} \left( 1 - M_V^2/M_B^2 \right)^3 \times \left( |F_{PC}|^2 + |F_{PV}|^2 \right).
\]

Within the naive factorization approach the matrix element of the amplitude can be simplified and the parity-conserving and parity-violating parts of the amplitude can be represented as follows:

\[
F_{PC} = \xi_{\text{CKM}} a(\mu) \left[ F_{V} \frac{f_V}{M_B} f_V M_V + f_B H_P \right],
\]

\[
F_{PV} = \xi_{\text{CKM}} a(\mu) \left[ \frac{f_A}{M_B} f_V M_V + f_B H_S \right]
- 2Q_B f_B f_V M_V \left[ \frac{M_B^2}{M_B^2} - \frac{M_V^2}{M_V^2} \right].
\]

Summing up this Section, within the factorization approximation the weak annihilation amplitude may be expressed in terms of four form factors: \( F_A, F_V, H_P \) and \( H_S \). It should be emphasized that each of the form factors \( F_A, F_V, H_P \) and \( H_S \) actually depends on two variables: The B-meson transition form factors \( F_A, F_V \) depend on \( q_1^2 \) and \( q_2^2 \) and \( F_{A,V}(q_1^2, q_2^2) \) should be evaluated at \( q_1^2 = 0 \) and \( q_2^2 = M_V^2 \). The vector-meson transition form factors \( H_P, H_S \) and \( H_{P,S} \) depend on \( q_1^2 \) and \( p^2 \), and \( H_{P,S}(q_1^2, p^2) \) should be evaluated at \( q_1^2 = 0 \) and \( p^2 = M_B^2 \).

3. Photon emission from the B-meson loop and the form factors \( F_A \) and \( F_V \).

In this section we calculate the form factors \( F_{A,V} \) within the relativistic quark model, which is a dispersion approach based on constituent quark picture. This approach has been formulated in detail in [5] and applied to the weak decays of heavy mesons in [6].

The pseudoscalar meson in the initial state is described in the dispersion approach by the following vertex [7]: \( \tilde{q}_i(k_1) i\gamma_5 q(-k_2) G(s)/\sqrt{N_c} \), with \( G(s) = \phi_B(s)(s - M_P^2), s = (k_1 + k_2)^2, k_1^2 = m_1^2 \) and \( k_2^2 = m_2^2 \). We represent the pseudoscalar-meson wave function \( \phi_B(s) \) in the form \( \phi_B(s) = f(s)w(k^2), k^2 = \lambda(s, m_1^2, m_2^2)/4s \). For \( w(k^2) \) we use a simple Gaussian parametrization \( w(k^2) = A_\beta e^{-k^2/(2\beta^2)} \). For the concrete form of \( f(s) \) and the normalization condition which gives \( A_\beta \) see [7]. The parameter \( \beta \) is extracted from the condition

\[
f_p = \sqrt{N_c} \int_{(m_1 + m_2)^2}^\infty ds \phi_B(s) \rho_p(s),
\]

with

\[
\rho_p(s) = \left( \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{8\pi^2 s} \right) \frac{s - (m_1 + m_2)^2}{s}.
\]

Here \( \lambda(a, b, c) = (a - b - c)^2 - 4bc \) is the triangle function.

Recall that the form factors \( F_{A,V} \) describe the transition of the B-meson to the photon with the momentum \( q_1, q_2 = 0 \), induced by the axial-vector (vector) current with the momentum \( q_1, q_2 = M_V^2 \). The form factors \( F_{A,V} \) are given by the diagrams of Fig 1 and 2.

![Diagram 1](image1.png)  
*Figure 1: Diagrams for the form factor \( F_A \): a) \( F_A^{(b)} \), b) \( F_A^{(a)} \).*

![Diagram 2](image2.png)  
*Figure 2: Diagrams for the form factor \( F_V \): a) \( F_V^{(b)} \), b) \( F_V^{(a)} \).*

3.1. The form factor \( F_A \)

Let’s consider form factor \( F_A \) first. Fig 1 shows \( F_A^{(b)} \), the contribution to the form factor of the process when the \( b \) quark interacts with the photon; Fig 2 describes the contribution of the process when the quark utf- interacts while utf-d remains a spectator. It is convenient to change the direction of the quark line in the loop diagram of Fig 1. This is done by performing the charge
conjugation of the matrix element and leads to a sign change for the $\gamma_\gamma \gamma_5$ vertex. For the form factor we then obtain the following expression:

$$F_A = Q_b F_A^{(b)} - Q_u F_A^{(u)},$$  \hspace{1cm} (10)

where $F_A^{(b)} = F_A^{(b)}(m_b, m_u)$, $F_A^{(u)} = F_A^{(u)}(m_u, m_b)$, and

$$\frac{1}{M_B} F_A^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2} \int_{(m_1+m_2)^2}^{\infty} ds \phi_b(s) (s - M_V^2),$$

$$\rho_+(s, m_1, m_2) = \frac{m_1 - m_2}{s - M_V^2} \rho_+(s, m_1, m_2),$$

$$\rho_+(s, m_1, m_2) = \frac{s + m_1^2 - m_2^2}{2s} \rho_+(s, m_1, m_2) - m_2^2 \log \left( \frac{s + m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)} \right).$$  \hspace{1cm} (13)

Employing the fact that the wave function $\phi_b(s)$ is localized near the threshold in the region $s - m_b - m_u \leq \Lambda$, it is easy to show that in the limit $m_b \rightarrow \infty$ the photon emission from the light quark dominates over the emission from the heavy quark [8].

$$\frac{1}{M_B} F_A^{(b)} = \frac{f_b}{\Lambda m_b} + ..., \hspace{1cm} \frac{1}{M_B} F_A^{(u)} = \frac{f_b}{m_b^2} + ...$$  \hspace{1cm} (14)

3.2. The form factor $F_V$

The consideration of the form factor $F_V$ is very similar to the form factor $F_A$. $F_V$ is determined by the two diagrams shown in Fig[2a]. Fig[2b] gives $F_V^{(b)}$ the contribution of the process when the $b$ quark interacts with the photon; Fig[2c] describes the contribution of the process when the quark $u$ interacts.

It is again convenient to change the direction of the quark line in the loop diagram of Fig[2b] by performing the charge conjugation of the matrix element. For the vector current $\gamma_v$ in the vertex the sign does not change (in contrast to the $\gamma_\gamma \gamma_5$ case considered above). The calculation of $F_V$ within the dispersion approach gives the following result:

$$F_V = Q_b F_V^{(b)} + Q_u F_V^{(u)},$$  \hspace{1cm} (15)

where $F_V^{(b)} = F_V^{(1)}(m_b, m_u)$, $F_V^{(u)} = F_V^{(1)}(m_u, m_b)$, and

$$\frac{1}{M_B} F_V^{(1)}(m_1, m_2) = -\frac{\sqrt{N_c}}{4\pi^2} \int_{(m_1+m_2)^2}^{\infty} ds \phi_b(s) (s - M_V^2) \rho_+(s, m_1, m_2).$$  \hspace{1cm} (16)

The function $\rho_+(s, m_1, m_2)$ is given in [12]. In the heavy-quark limit $m_b \rightarrow \infty$ one finds

$$\frac{1}{M_B} F_V^{(b)} = -\frac{f_b}{\Lambda m_b} + ..., \hspace{1cm} \frac{1}{M_B} F_V^{(u)} = -\frac{f_b}{m_b^2} + ...$$  \hspace{1cm} (17)

The dominant contribution in the heavy quark limit again comes from the process when the light quark emits the photon. As seen from Eqs. (14) and (17), one finds $F_A = F_V$ in the heavy quark limit, in agreement with the large-energy effective theory [9].

4. Photon emission from the vector meson loop. The form factors $H_S$ and $H_P$.

We now calculate the form factors $H_{PS}$ using the relativistic quark model. The vector meson in the final state is described in this approach by the vertex $q_\gamma(-k_2)q_\gamma(k_1')$, $\Gamma_\gamma = \left( -\gamma_\mu + \frac{v_1 \cdot k_2}{\sqrt{m_1 + m_2}} \right) G(s) / \sqrt{N_c}$, with $G(s) = \phi_v(s)(s - M_2^2)$, $s = (k_1' + k_2)^2$, $k_1'^2 = m_1^2$ and $k_2^2 = m_2^2$. The vector meson wave function $\phi_v(s)$ has the same structure and normalizing conditions as that of the pseudoscalar meson described in Section[3]. All the details were given in [3]. For obtaining the parameter $\beta$ we use the following condition

$$f_V = \sqrt{N_c} \int_{(m_1+m_2)^2}^{\infty} ds \phi_v(s) \rho_V(s),$$  \hspace{1cm} (18)

with

$$\rho_V(s) = \frac{2\sqrt{s} + m_1 + m_2 A^{1/2}(s, m_1^2, m_2^2)}{3 \frac{8\pi^2 s}{8\pi^2 s}} \times \frac{s - (m_1 - m_2)^2}{s}.$$  \hspace{1cm} (19)

The form factors $H_S$ and $H_P$ are given by the diagrams of Fig[3] and [4].
4.1. The form factor $H_S$

Fig 3 shows $H_S^{(d)}$, the contribution to the form factor of the process when the $d$ quark interacts with the photon; Fig 4 describes the contribution of the process when the $u$ quark interacts while $d$ remains spectator. Changing the direction of the quark line in the loop diagram of Fig 3 leads to a sign change for the scalar factor of the process when the $d$ quark interacts with the photon; Fig 4b describes the contribution of the process when the $c$-quark interacts. We again change the direction of the quark line in the loop diagram of Fig 4b by performing the charge conjugation of the matrix element. For the pseudoscalar current ($m_c + m_d$)ψc in the vertex the sign does not change and we obtain:

$$H_p = Q_d H_p^{(d)} + Q_c H_p^{(c)},$$  \hspace{1cm} (23)

where $H_p^{(d)} = H_p^{(1)}(m_d, m_c)$, $H_p^{(c)} = H_p^{(1)}(m_c, m_d)$,

$$H_p^{(1)}(m_1, m_2) = \frac{\sqrt{N_c}}{4\pi^2} \int_{(m_1+m_2)^2}^{\infty} \frac{ds \phi(s)}{(s - p^2 - i0)} \rho_+(s, m_1, m_2) \rho_\perp(s, m_1, m_2),$$  \hspace{1cm} (24)

with $\rho_+(s, m_1, m_2)$ and $\rho_\perp(s, m_1, m_2)$ given in (12) and (13). For the behaviour of $H_p^{(d, c)}$ at large-mQ for the heavy-light vector meson Qq we get

$$H_p^{(d)} \rightarrow \frac{f_v}{\Lambda} \frac{m_Q^2}{m_Q^2 - p^2}, \quad H_p^{(c)} \rightarrow \frac{f_v m_Q}{m_Q^2 - p^2}.$$ \hspace{1cm} (25)

For the B-decays of interest, we need the value of the form factors $H_{pS}(p^2, q^2 = 0)$ at $p^2 = M_B^2$, which lies above the threshold $(m_1 + m_2)^2$. The spectral representations for $H_{pS}(p^2 = M_B^2)$ develop the imaginary parts which occur due to the quark-antiquark intermediate states in the $p^2$-channel. It should be emphasized that no anomalous cuts emerge if one consider the form factors in the double spectral representation at $q^2 \leq 0$. In all cases considered in this paper, the value of $p^2 = M_B^2$ lies far above the region of resonances which occur in the quark-antiquark channel. Far above the resonance region, local quark-hadron duality works well and the calculation of the imaginary part based on the quark diagrams is trustable. The imaginary part turns out to be orders of magnitude smaller than the real part of the form factor and for the practical purpose of the decay rate calculation may be safely neglected.

5. Numerical results

We have received the spectral representations of the form factors and now can obtain numerical estimations for the form factors and, finally, for the branching ratios.

5.1. Parameters of the model

We use here the following values of the constituent quark masses:

$$m_d = m_u = 0.23 \text{ GeV}, \quad m_s = 0.35 \text{ GeV},$$  \hspace{1cm} (26)

$$m_c = 1.45 \text{ GeV}, \quad m_b = 4.85 \text{ GeV}.$$
With the quark masses $m_q$ and the meson wave function parameters $\beta$ quoted in Table I, the decay constants from the dispersion approach reproduce the well-known decay constants of pseudoscalar and vector mesons also summarized in Table I.

Table 1: Meson masses from [16], leptonic decay constants, and the corresponding wave function parameters $\beta$ [13].

| Meson | Mass, $M$, GeV | Decay constant, $f$, MeV | $\beta$, GeV |
|-------|---------------|--------------------------|--------------|
| $B$   | 5.279         | 192 ± 8 [12]             | 0.565        |
| $B_s^0$ | 5.370         | 226 ± 15 [12]           | 0.62         |
| $B_c^0$ | 6.275         | 427 ± 6 [13]             | 0.94         |
| $D^*$  | 2.010         | 248 ± 2.5 [14]           | 0.48         |
| $D_s^*$ | 2.11          | 311 ± 9 [14]             | 0.54         |
| $J/\psi$ | 3.097         | 405 ± 7 [15, 16]         | 0.68         |
| $\rho$ | 0.775         | 209 ± 2 [16]             | 0.31         |

5.2. Predictions for the decay rates

Using the values given above we now obtain the estimations for the branching ratios. We present results for six weak annihilation rare radiative B-decays:

- $\mathcal{B}(B_s^0 \to J/\psi \gamma) = 1.2 \times 10^{-7} \left( a_2^2 \right)$, (27)
- $\mathcal{B}(B_d^0 \to J/\psi \gamma) = 5.5 \times 10^{-9} \left( a_2^2 \right)$, (28)
- $\mathcal{B}(B_d^0 \to D^{0} \gamma) = 5.2 \times 10^{-8} \left( a_2^2 \right)$, (29)
- $\mathcal{B}(B_s^0 \to D^{0} \gamma) = 2.9 \times 10^{-9} \left( a_2^2 \right)$, (30)
- $\mathcal{B}(B^+ \to D^+ \gamma) = 2.1 \times 10^{-7} \left( a_1^2 \right)$, (31)
- $\mathcal{B}(B_c^+ \to \rho^+ \gamma) = 2.6 \times 10^{-7} \left( a_1^2 \right)$, (32)

For the scale-dependent Wilson coefficients $C_i(\mu)$ and $a_{1,2}(\mu)$ at the renormalization scale $\mu \approx 5$ GeV we use the following values [3]: $C_1 = 1.1$, $C_2 = -0.241$, $a_1 = C_1 + C_2/N_c = 1.02$, and $a_2 = C_2 + C_1/N_c = 0.15$.

Conclusions

We obtained predictions for the branching ratios of weak annihilation rare radiative B-decays. The calculations were done in the naive factorization approximation, taking into account both the photon emission from the $B$-meson loop and the vector-meson loop ($V$-loop). The form factors containing non-perturbative QCD effects were calculated within the relativistic quark model. In comparison to previous analysis we obtained estimations for the decays $B^0_s \to D^{*0}\gamma$ and $B^+_c \to \rho^+\gamma$. The accuracy of the predictions is at the level of 20%.

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