Off equilibrium properties of vortex creep in superconductors

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We study a model for the dynamics of vortices in type II superconductors. In particular, we discuss glassy “off equilibrium” properties and “aging” in magnetic creep. At low temperatures a crossover point is found, \( T_g \), where relaxation times seem to diverge à la Vogel-Tammann-Fulcher. Magnetic creep changes by crossing \( T_g \); above \( T_g \) power law creep is found asymptotically followed by stretched exponential saturation; below \( T_g \) the creep is logarithmic and vortex motion strongly subdiffusive. In this region violation of time translation invariance is found along with important dynamical scaling properties.

Despite apparent structural differences, the dynamics of vortices in superconductors \cite{1} has many common features with glass formers such as supercooled liquids, spin glasses or polymers \cite{4}. In superconductors even simple quantities such as the sample magnetisation, \( M \), exhibit strong “glassy” behaviours and a variety of “history dependent” effects has been discussed \cite{5}. Common examples are hysteretic magnetisation loops which at low temperature exhibit “memory” effects: the value of \( M \) for a given applied field, \( H \), depends on the sweep rate of \( H \), i.e., on the history of the sample. \cite{6} “Aging” has been also recently observed in magnetic relaxation in type II superconductors \cite{6}. Interestingly, these phenomena are very similar to those found in other glassy systems such as supercooled liquids. For instance, after a quench, “cooling rate” dependences and “aging” effects are usually recorded in particle density measurements \cite{7}.

Those similarities are striking when one considers the differences which exist at a microscopic level between these systems. One is apparent: the interaction ranges of vortices in superconductors are typically much larger than those of molecules in glass formers. In this perspective our aim is to discuss the above connections and relate off equilibrium dynamics of vortex matter to the dynamics of glass former liquids, where relevant progresses have been recently accomplished \cite{8}. In all these systems, the glassy features are due to some general mechanisms and unrelated to specific material parameters. Thus, a statistical mechanics approach, disregarding sample details, seems appropriate to describe them. For instance, many of the features of glassy behaviours in fragile glasses, such as colloidal suspensions, are well described in the context of spin glass models \cite{8}.

Extensive work has been done on the equilibrium or close to equilibrium properties of the so-called vortex glass phase proposed by M.P.A. Fisher (see ref.s in \cite{9}). Here we discuss, in particular, the dynamics of the system when \textit{far from equilibrium}, a typical situation due to the enormous relaxation times found in the low temperatures region. We consider a simple tractable model for vortices \cite{9} which can be well understood in the context of standard statistical mechanics of disordered systems \cite{10}. Such a model was already shown to successfully depict a unified picture of creep and transport phenomena in vortex physics, ranging from magnetisations loops with “anomalous” second peak, logarithmic relaxation at vanishing \( T \), Bean profiles, to history dependent behaviours in vortex flow and I-V characteristics, to the reentrant nature of the equilibrium phase diagram \cite{10}.

As mentioned, here we study in details the system \textit{off-equilibrium} creep dynamics. The model gives the following scenario for vortex dynamics. At low temperatures a crossover point is found, \( T_g \), where relaxation times become longer than the typical observation scale \cite{11}. Actually, they seem to diverge at a lower temperature, \( T_c \), where an “ideal” thermodynamic glass transition can be located (in 2d \( T_c \) is numerically zero). Magnetic creep changes when \( T_g \) is crossed: above \( T_g \) power law creep is found asymptotically followed by stretched exponentials; below \( T_g \) the creep is logarithmic and shows strong form of “aging”, such as violation of time translation invariance with apparent dynamical scaling properties. Microscopically, \( T_g \) corresponds to a drastic change in vortex motion which becomes strongly subdiffusive. We also outlines the connections of vortex off equilibrium dynamics with other glassy systems.

We study a Restricted Occupancy Model (ROM) \cite{10}, a statistical mechanics model which in the limit of zero temperature and infinite upper critical field reduces to a cellular automaton introduced in \cite{10} to study vortex avalanches. The model is a coarse grained version \cite{10,7} of a system of straight vortex lines: a set of repulsive particles diffusing in a disordered pinning environment. The coarse graining length, \( l_0 \), is taken to be of the order of the natural screening length of the problem (which in our case is the London penetration length \( \lambda \)). Thus, the coarse graining technique \cite{12} reduces the original interaction potential to an effective short ranged one. The price to pay is a loss of information on scales smaller than \( l_0 \) and the necessity to allow multiple occupancy on the lattice sites of the coarse grained system.

The ROM model is defined by the Hamiltonian \cite{10}:

\[
\mathcal{H} = \frac{1}{2} \sum_{ij} n_i A_{ij} n_j - \frac{1}{2} \sum_i A_{ii} n_i - \kappa \sum_i A^0_i n_i,
\]

with the constraint \( 0 \leq n_i \leq N_{c2} \) on the integer occupancy variable \( n_i \) representing the number of vortices of the coarse grained cell corresponding to site \( i \) (bounded by the upper critical field).

The first term in \( \mathcal{H} \) represents the repulsion between particles \cite{10}. For the above considerations on vortex in-
teractions, in the present coarse grained representation (where \( I_0 \sim \lambda \)) a finite range potential must be considered. For simplicity we choose \( A_{ii} = A_0 \); \( A_{ij} = A_1 \) if \( i \) and \( j \) are nearest neighbours; \( A_{ij} = 0 \) for all others couples of sites. The second term in \( H \) concerns the particle self-interaction energy and the third one is a pinning potential, with a given distribution \( P(Ap) \), acting on a fraction \( p \) of lattice sites (below \( p = 1/2 \)). For simplicity we choose: \( P(Ap) = (1 - p)\delta(Ap) + p\delta(Ap - A_0^*) \). \( A_0 \) sets the energy scale and we choose \( A_0 = 1.0; A_0^* = 0.3; N_{ext} = 27; \) \( \kappa^* \equiv A_1/A_0 \in [0,0.3] \) (below we typically discuss data for \( \kappa^* = 0.28 \)). The relation of the parameters of the model to material parameters is shown elsewhere, here we only recall that \( \kappa^* \) is an increasing function of the Ginzburg-Landau \( \kappa \) and \( N_{ext} \propto H_{ext} \).

Through its surfaces, the system is in contact with an external reservoir of “particles”, which schematically corresponds to the applied field present in magnetic experiments on superconductors. During a ramp with \( \gamma \), we also record the magnetic correlation function \( \langle w \rangle \), which gives richer information.

The magnetisation in the system at time \( t \) is defined as: \( M(t) = N_{in}(t) - N_{ext}(t), \) where \( N_{in} = \sum n_i/L^2 \) is the overall density inside the system and \( N_{ext} \) is the applied field. As in usual zero field cooled experiments, we record the isothermal relaxation of \( M(t) \) after ramping at a given “sweep rate”, \( \gamma \), the field from zero up to a given working value \( N_{ext} \).

The presence, at low temperature, of sweep rate dependent hysteretic cycles, slowly relaxing magnetisation, and similar effects, indicate that our system, on the observed time scales, can be far from equilibrium. Off-equilibrium behaviour is appropriately described by two times correlation functions. Thus, along with \( M(t) \), we also record the magnetic correlation function \( (t > t_w) \):

\[
C(t, t_w) = \langle |M(t) - M(t_w)|^2 \rangle .
\]

Since the general dynamical features of \( C \) and \( M \) are very similar, for simplicity we mainly discuss \( C \) which gives richer information.

At not too low temperatures, for instance at \( T = 1.0 \), the system creep is characterised by finite relaxation times, but the dynamics is already highly non trivial. The two times correlator \( C(t, t_w) \) is plotted in Fig.1 after a ramp with \( \gamma = 10^{-3} \) at \( T = 1.0 \). At long times, \( C(t, t_w) \) is well fitted by the so called Kohlrausch-Williams-Watts (KWW) law (see Fig.1):

\[
C(t, t_w) \simeq C_\infty \left\{ 1 - e^{-(t-t_w)/\tau} \right\}^\alpha
\]

The KWW decay is observed in the asymptotic relaxation of superconductors as well as in glass formers (their so called \( \alpha \)-relaxation) above the glass transition. The time scale \( \tau \) and the Kohlrausch-exponent \( \beta \) depend on the temperature \( T \) (see Fig.2) and on the overall field \( N_{ext} \). The pre-asymptotic dynamics (i.e., \( t \ll \tau \)) is also interesting and characterised by various regimes. In particular, for not too short times, a power law is observed over several decades (see Fig.3):

\[
C(t, t_w) \simeq C_0 \left( \frac{t-t_w}{\tau} \right)^\alpha
\]

The exponent \( \alpha \) is almost independent of \( N_{ext} \), \( \alpha \simeq 1.7 \), except at very small or high fields. Notice that the \( \tau \) in eq.4 is the \textit{same} as in eq.3, but the exponents \( \alpha \) and \( \beta \) are numerically different (see Fig.3).

Interestingly, the above finding in the model of power laws followed by KWW relaxations is typically observed, for not too low \( T \), in superconductors. This behaviour is also typical of supercooled liquids, where the power law regime is called the \( \beta \)-relaxation.

At \( T = 1.0 \) or, generally, at not too low temperatures, no “aging” is seen: \( C(t, t_w) \) is a function of \( t - t_w \). This is clearly shown in Fig.1 where we plot \( C(t, t_w)/C_\infty \) for several different values of \( N_{ext} \) as a function of the scaling variable \( (t-t_w)/\tau \). All the data (for all \( t_w \) and \( N_{ext} \)) fall on the same master function (which is more general than the above KWW fit).

The scenario described for \( T = 1.0 \) is found in a broad region at low temperatures. However, around \( T = 0.5 \), \( \beta \) drastically decreases and, at the same time, a steep increase of \( \tau \) is found (see Fig.2). For instance, at \( N_{ext} = 10 \) for temperatures below \( T_g \simeq 0.25 \), the characteristic time gets larger than our recording window. Thus, below \( T_g(N_{ext}) \) the system definitely loses contact with equilibrium during our observation and, as shown in details below, typical glassy phenomena, such as “aging”, are observed. The crossover temperature \( T_g \) is itself a function of \( \gamma \). It has a physical meaning similar to the so called phenomenological definition of the glass transition point in supercooled liquids. Exploiting this analogy we will call this temperature the glass temperature despite the fact that in glassy systems it is loosely defined.

The presence of an underlying “ideal” glass transition point, \( T_c(N_{ext}) \), is a non trivial possibility which, in many cases (as supercooled liquids) is still under debate. \( T_c \) is often located by some fit of raw \( \tau \) data collected in the high \( T \) regime.

In our model, in the region where \( \tau \) has a steep increase a Vogel-Tamman-Fulcher law (VTF) fits the data (see inset of Fig.2):

\[
\tau = \tau_0 \exp \left( \frac{E_0}{T-T_c} \right)
\]

For example, at \( N_{ext} = 10 \), the characteristic time \( \tau_0 \) is very large, \( \tau_0 = 1.0 \times 10^3 \), and the characteristic activation energy, \( E_0 \), is ten times larger than \( T_c \): \( E_0 = 1.1 \) and \( T_c = 0.1 \), thus the above fit is consistent with an Arrhenius low
(i.e., with $T_c = 0$). The presence of a strong increase of $\tau$ close to a power law or a VTF law is again a mark of the apparent similitude with glassy features of supercooled liquids and glasses \[19\].

Since below $T_g$ relaxation times are huge, one might expect that the motion of the particles essentially stops, apart from their vibration inside cages of other vortices. Instead, as shown below, the off equilibrium dynamics has remarkably rich properties. In particular, the properties of the system slowly evolve with time. These phenomena are usually summarised by claiming that the system is “aging”. They occur in many different systems ranging from polymers, to supercooled liquids \[5\], spin glasses \[4\] or granular media \[17\], and their origin and apparent universality are still an open problem \[4,8,19\].

In the inset of Fig.4 we show that $C(t, t_w)$, at $T = 0.1$, exhibits strong “aging” \[20\]. Notice that $C$ depends on both times $t$ and $t_w$: the system evolution is slower the older is its “age” $t_w$ (here, as in eq. \[1\]), $t_w$ is the time elapsed from the sample preparation at the working field and temperature). Note the contrast with the case at $T = 1.0$ of Fig.4. Such a dynamical “stiffening” is typical of glass formers \[5\]. It is important to stress that the presence of slowly relaxing quantities, such as $M(t)$, must not lead to conclude that the system is close to equilibrium \[3\].

In the entire low $T$ region ($T < T_g$), after a short initial power law behaviour, $C(t, t_w)$ can be well fitted by a generalisation of a known interpolation formula, often experimentally used \[1\], which now depends on the waiting time, $t_w$:

$$C(t, t_w) \simeq C_\infty \left\{ 1 - \left[ 1 + \frac{\mu T}{U_c} \ln \left( \frac{t + t_0}{t_w + t_0} \right) \right]^{-1/\mu} \right\}$$  \hspace{1cm} (5)

We found that to take $\mu \simeq 1$ is consistent with our data. In the above fit $U_c/\mu T$ only depends on $N_{ext}$ and, interestingly, $t_0$ is approximately a linear function of $t_w$: $t_0 \propto t_w + t_0^g$, where $t_0^g$ is a constant. Very interesting is the presence of scaling properties of purely dynamical origin in the off-equilibrium relaxation. This is shown in Fig.3, where data for different fields, $N_{ext}$, and different waiting times, $t_w$, are rescaled on a master function. The above results imply that for times large enough (but smaller than the equilibration time), $C(t, t_w)$ is a universal function of the ratio $t/t_w$: $C(t, t_w) \sim S(t/t_w)$. Such a behaviour (called “simple aging” \[20\]) is in agreement with a general scenario of off-equilibrium dynamics (see Ref. \[10\]) and has strong analogies with other glassy systems \[3\]. Experimental data on $C(t, t_w)$ do not exist in vortex matter, but would be extremely important to shed light on the real nature of dynamical phenomena in superconductors.

Finally, we stress that in vortex physics transitions from low temperature logarithmic to higher temperature power law creep are usually experimentally found (see references in \[3\]) and give a way to approximately locate the position of $T_g$ in real samples.

In the above scenario, microscopic quantities concerning the internal rearrangement of the system, such as the vortex mean square displacement, $R^2(t)$, can be insightful. Since vortices can enter and exit the sample, $R^2(t)$ must be properly defined. To this aim, we also made a different kind of computer runs where after ramping the field from zero to $N_{ext}$ we close the system (i.e., remove the reservoir). Since now the number of particles is fixed \[18\], their positions, $\vec{r}_i(t)$, are well defined at each time step and we record: $R^2(t) = \frac{1}{N} \sum_i (\vec{r}_i(t) - \vec{r}_i(0))^2$, were $N$ is the total number of present particles.

$R^2(t)$ is plotted in Fig.4 for $N_{ext} = 10$. At high enough $T$, $R^2(t)$ is linear in $t$, but at lower temperatures it shows a pronounced bending. Finally, below $T_g$, the process at long times becomes subdiffusive:

$$R^2(t) \sim t^{\nu}$$  \hspace{1cm} (6)

with $\nu \ll 1$. From this point of view, $T_g$ is the location of a sort of structural arrest of the system, where particle displacement is dramatically suppressed. Interestingly, a very similar scenario has been recorded in real superconducting samples: for instance in Ref. \[20\] it was clearly shown that vortices are definitely mobile in the low temperature phase and only “freeze” below a characteristic field dependent temperature.

Summarising, the schematic coarse grained model we considered shows magnetic properties very close to those experimentally found in superconductors \[3\]. Here, in particular, we have focused on the off equilibrium creep dynamics and an interesting scenario emerges. At low temperatures the system relaxation time $\tau(N_{ext}, T)$ grows enormously with an “ideal” divergence à la VTF at some $T_c(N_{ext})$ (interestingly, molecular dynamics simulations of London-Langevin models seem to confirm these results \[6,21\]). In fact, below a crossover temperature, $T_g > T_c$, vortex displacement becomes sub-diffusive, signalling the presence of a structural arrest. It is impossible to equilibrate the system on the observation time scale. At low temperatures creep is logarithmic and becomes a power law above the crossover $T_g(N_{ext})$.

In typical experiments or computer simulations, magnetic properties are measured after ramping the external field at a given rate $\gamma$. Whenever $\gamma$ is much larger than the inverse of the characteristic relaxation time, $\tau(N_{ext}, T)$, the system is naturally driven off equilibrium, simply because it is unable to follow the drive, and “hysteresis”, “memory” effects, along with dependences on the sweep rate, occur. In the present framework, the strict correspondence between the dynamics of vortices and other glass formers can be also rationalised.

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FIG. 1. **Main frame** The two time magnetic correlation function $C(t, t_w)$ recorded at $T = 1.0$ for the shown values of the external field and waiting times $t_w$. All data are collapsed on the same curve when plotted as a function of $(t - t_w)/\tau$. Here $\tau(N_{ext}, T)$ is the characteristic creep time. No “aging” is present. The bold continuous line is a fit to the KWW function of the asymptotic region, and the dotted line is a power law fit. **Inset** The same data as above for $N_{ext} = 16$ as a function of $t - t_w$. 
FIG. 2. The parameters of the Kohlrausch-Williams-Watts (KWW) asymptotic relaxation of the magnetisation correlation, $C$, as a function of the temperature $T$, recorded at $N_{\text{ext}} = 10$. **Main frame** The equilibration time $\tau$ enormously grows by decreasing the temperature $T$. Below the crossover temperature $T_g \sim 0.25$, the system relaxation times are larger than the observation window. **Inset left** The KWW exponent $\beta$ as a function of $T$. **Inset right** Close to $T_g$, $\tau$ plotted as a function of $1/T$ approximately shows a Vogel-Tamman-Fulcher behaviour (see eq.(4)).

FIG. 3. **Inset** Logarithmic time relaxation at $T = 0.1$ (i.e., below $T_g$) of the two-times correlation function, $C(t, t_w)$, recorded at $N_{\text{ext}} = 16$ (a value close to the 2nd peak in magnetic loops). **Main Frame** Off equilibrium dynamical scaling. Superimposed on the same master function, $1 - (1 + x)^{-1/\mu}$ ($\mu^{-1} \sim 1$), are relaxation data of $C(t, t_w)$ recorded for $N_{\text{ext}} = 4, 10, 16$ for each of the shown $t_w$. The asymptotic scaling is $C(t, t_w) \sim S(t/t_w)$.

FIG. 4. The vortex mean square displacement $R^2(t)$ at $N_{\text{ext}} = 10$ for several temperatures. Below $T_g \sim 0.25$, $R^2(t)$ is strongly subdiffusive: $R^2(t) \sim t^\nu$ with $\nu < 1$. Straight lines are guides for the eye.