Research on derivation of calculation formula and application scope of simplified formula about camber of gantry crane cantilever based on coordinate transformation

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Abstract. For the gantry crane with cantilever, its cantilever camber value has a great influence on the operation and stop reliability of the lifting trolley, which is an important performance index of the bearing structure of the equipment. When the height of the outriggers on both sides of the gantry crane is inconsistent, the vertical height difference measured at the cantilever end is not the actual upwarping of the equipment, and the numerical change caused by the height difference of the outriggers needs to be corrected. In this paper, the calculation formula of the upwarping degree based on the transformation of coordinate system is proposed, which can accurately express the upwarping degree of the cantilever. Based on this, the simplified formula is derived and the scope of application is limited, which is suitable for the inspection and calculation in engineering practice, and the error is within the allowable range.

1. Introduction
As an important category of lifting machinery, gantry crane has irreplaceable advantages in lifting large goods. For some typical gantry cranes, such as general gantry crane and electric hoist gantry crane, they are widely used in the production and operation of enterprises and goods lifting process. For the gantry crane with cantilever, because of the large elastic deformation of the cantilever after loading, it must be prefabricated\textsuperscript{[1]}. The cantilever upwarping is mainly used to eliminate the downwarping caused by the self weight of the main beam, and to make the trolley run horizontally when working on the beam\textsuperscript{[2]}.

The relevant product standards stipulate that for the general gantry crane, electric hoist gantry crane and other cranes with cantilever, the cantilever should be upturned, and the actual upturned value of the effective cantilever measured after the static load test should not be less than. In terms of inspection rules, the inspection rules of crane supervision and inspection (gzjg [2002] No. 296) has clear provisions on the inspection content, requirements and methods of cantilever camber in TSG Q7016-2016 "Crane Installation, Modernization & General overhaul Supervision Inspection" has deleted the relevant inspection items, but the cantilever camber is still an important performance index to characterize the bearing structure of the equipment in the design and manufacture, installation and maintenance, inspection and evaluation of the lifting machinery.

There are many papers about the detection and calculation of cantilever camber in China, but they are similar. When there is height difference between the two legs of gantry crane, plane geometric methods such as similar triangle or trigonometric function method are used to approximately correct the numerical influence of camber caused by the height difference of the legs. Some scholars have earlier compared the difference of the actual camber of cantilever under different benchmarks, and simply
analyzed the error[3]. Some scholars try to use trigonometric function method to deduce the exact formula of the actual upwarping degree, but it uses the cantilever length to replace the inclined side of the similar triangle on the cantilever side, which has a certain deviation from the actual upwarping degree[4-5]. Other scholars calculate the cantilever upwarping degree based on the height difference angle of outrigger and the cantilever upwarping angle, but the formula needs to use the inverse trigonometric function, and the height measurement value has a great influence on the angle value obtained by the inverse trigonometric function, which easily leads to a large error between the calculated value and the actual value of the upwarping degree, so its application in engineering is limited[6].

In this paper, based on the transformation formula of two-dimensional rotating coordinate system, the accurate calculation formula of cantilever camber is derived for the first time, and the simplified formula suitable for daily engineering practice is obtained, and the scope of application is given.

2. Transformation formula of two dimensional rotating coordinate system

For the point A in the absolute coordinate system, its real position does not change with the transformation of the coordinate system. Let the coordinates of point A in the coordinate system XoY (x, y). The coordinate system X'oY' is obtained by rotating counterclockwise the coordinate system XoY around the point o at an angle of $\alpha$. Let the coordinate of point A in the coordinate system X'oY' (x', y').

![Schematic diagram of coordinates in different coordinate systems of point A](image)

Polar coordinates can be used to find the relationship with point A in coordinate system XoY and X'oY'. Let the distance from point A to point o be $\rho$ and the angle between line oA and oX axis is $\theta$. Then the polar coordinates of point A in the coordinate system XoY are:

$$
\begin{align*}
    x &= \rho \cos \theta \\
    y &= \rho \sin \theta
\end{align*}
$$

(1)

The coordinate system XoY rotates counterclockwise at an angle of $\alpha$ around point A. So the coordinate system X'oY' is obtained as follows:

$$
\begin{align*}
    x' &= \rho \cos(\theta - \alpha) = \rho(\cos \theta \cos \alpha + \sin \theta \sin \alpha) \\
    y' &= \rho \sin(\theta - \alpha) = \rho(\sin \theta \cos \alpha - \cos \theta \sin \alpha)
\end{align*}
$$

(2)

The results are as follows:

$$
\begin{align*}
    x' &= x \cos \alpha + y \sin \alpha \\
    y' &= y \cos \alpha - x \sin \alpha
\end{align*}
$$

(3)

The transformation formula of coordinate system is obtained

$$
\begin{pmatrix}
    x' \\
    y'
\end{pmatrix} =
\begin{pmatrix}
    \cos \alpha & \sin \alpha \\
    -\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
$$

(4)
Order in the formula \[ T = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \], it is the transformation matrix of coordinate system. Then the coordinate system transformation formula is also written as follows:

\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix}
\]  

(5)

3. Derivation of calculation formula of cantilever camber based on coordinate system transformation

For the gantry crane as shown in Figure 2, there is an arc-shaped arch in the middle of the main beam span, and an arc-shaped warping at the cantilever end.

Figure 2 general layout of gantry crane

In the process of calculating the camber of the right cantilever BC, the arc camber of the main beam AB and the arc camber of the right effective cantilever BC can be simplified as a straight line, as shown in Figure 3. It is known that the span of the main beam is s and the effective cantilever length on the right side is L. The height of outrigger and main beam joint a, B and cantilever effective arm length outer end C relative to the ground and vertical height ya, Yb, YC are measured. Usually, due to manufacturing and assembly errors, the height of A and B at both ends of the main beam is not equal, that is, Ya and Yb are not equal. At this time, the cantilever upwarping f is not equal to YC - Yb, but the distance from point C to line AB, that is, the length of line segment CD.

Fig. 3 Schematic diagram of cantilever camber calculation in rotating coordinate system

In the process of calculating the length of line segment CD, in order to simplify the calculation, we can take point A as the left origin and ground ox as the abscissa to establish the coordinate system XoY. When the coordinate system XoY rotates counterclockwise along point o at an angle of \( \alpha \) to make ox coincide with main beam AB, and get the coordinate system X'oY'. At this time, the distance of \( y_c' \) and \( y_b' \) is the cantilever upwarping degree f, that is:
\[ f = yc' - yb' \quad (6) \]

For the convenience of calculation and expression, the height difference of the two legs is reduced \( H = yb - ya \), vertical camber of cantilever \( h = yc - yb \). According to the formula (5), the coordinate system can be obtained after transformation \( yc' \) and \( yb' \) by substituting into formula (6), and we can get:

\[
\begin{align*}
    f &= yc' - yb' = (-xc \sin \alpha + yc \cos \alpha) - (-xb \sin \alpha + yb \cos \alpha) \\
    &= (yc - yb)\cos \alpha - (xc - xb)\sin \alpha = h \cos \alpha - H \sin \alpha
\end{align*}
\]

(7)

According to Fig. 3, it is obvious that:

\[
\begin{align*}
    S &= H \sin \alpha, \\
    S^2 &\approx L - h^2.
\end{align*}
\]

By substituting (7), the calculation formula of the cantilever camber of the gantry crane can be obtained

\[
    f = h\sqrt{S^2 - H^2} - H\sqrt{L^2 - h^2} \quad (8)
\]

Where:

- \( S, L \) — The span of main girder and effective arm length of cantilever;
- \( H, h \) — The height difference of the two legs and the vertical camber of the cantilever.

For formula (8), if \( H = 0 \), then \( f = h \). That is, when the two legs of the gantry crane are equal in height, the cantilever upwarping is equal to the vertical height difference between the outer end of the effective arm length \( C \) of the cantilever and the leg connection \( B \) of the main beam relative to the ground.

4. Derivation of simplified formula for calculation and correction of cantilever camber

For the calculation formula (8), if \( H \ll S, \) and \( h \ll L \), a simplified formula is obtained

\[
    f = hS - HL = h - \frac{L}{S} \times H \quad (9)
\]

Thus the practical formula for calculating and correcting the camber of gantry crane commonly used in engineering is obtained.

The geometric meaning of the simplified formula is to correct the value of the cantilever camber caused by the height difference of the outrigger by the similar triangle-principle. As shown in Figure 4, when the height difference of Outrigger \( H \) (i.e. segment BG) and the vertical upwarping of the cantilever \( h \) (i.e. segment CF) is small, the camber of cantilever BC is small \( CD \approx CE \), cantilever BC length \( L \approx BE \).
It is obvious from Fig. 4 that: $\angle BAG = \angle EBF = \alpha$, so $\Delta BAG \sim \Delta EBF$ According to the similar triangle principle, $BF = \frac{BE}{EF}$ Therefore $EF = \frac{BG \times BE}{AB} \approx \frac{L}{S} \times H$. Then we can get the following results:

$$CE = CF - EF \approx h - \frac{L}{S} \times H$$

It is a simplified formula for the calculation and correction of cantilever camber commonly used in daily inspection.

5. Study on error analysis and application scope of simplified formula for calculating and correcting cantilever camber

There are two parts of errors in the derivation of the simplified formula (9), that is, the error $\Delta 1$ caused by the approximate substitution of the actual camber CD with the vertical height difference CE, and the error $\Delta 2$ caused by the approximate substitution of the diagonal with the cantilever length L in the similar triangle.

5.1. The error $\Delta 1$ caused by the approximate substitution of vertical height difference CE for the actual camber CD

For error $\Delta 1$, it is obvious from Fig. 4 that: $\angle BAG = \angle DBF = \angle DCE = \alpha$ Therefore, the error is:

$$\Delta 1 = \frac{CE - CD}{CE} = \frac{CE - CE \cos \alpha}{CE} = 1 - \cos \alpha \quad (10)$$

At present, the relative error limit of crane basic parameter measurement is recommended to be 0.5% [7]. Namely: $\Delta 1 = 1 - \cos \alpha \leq 0.5\%$ And then the $\alpha \leq 5.73^\circ$ That is, when the height difference between the legs of the gantry crane leads to the deflection angle between the main beam and the ground $\alpha \leq 5.73^\circ$ Error, error $\Delta 1$ It can be ignored. Again by

$$\cos \alpha = \frac{\sqrt{S^2 - H^2}}{S} = \sqrt{1 - \left(\frac{H}{S}\right)^2}$$

It can be obtained $H \leq 0.09987S$ That is, when the height difference between the legs of the gantry crane $H$ When it is not more than 0.09987 times of the span of the main beam, the error is less than 0.09987 $\Delta 1$ It can be ignored.

Combined with the specific values, the national standards have provisions on the span and leg height difference of gantry crane, such as the span of general gantry crane is not less than 10m, the span of electric hoist gantry crane is not less than 7m, and the leg height difference is not more than 10mm [8-9]. On this basis, the error can be obtained

$$\Delta 1 = 1 - \cos \alpha = 1 - \sqrt{1 - \left(\frac{H}{S}\right)^2} \leq 1 - \sqrt{1 - \left(\frac{10}{7000}\right)^2} = 1.02 \times 10^{-6}$$

It is far less than the recommended error of 0.5%. Therefore, for the gantry crane designed and manufactured in accordance with the standard requirements, the error is less than 0.5%. So it can be ignored.

5.2. The error $\Delta 2$ caused by the approximate substitution of diagonal be with cantilever length L in similar triangles

According to figure 4, it is obvious that,

$$BE = \frac{BF}{\cos \alpha} = \frac{\sqrt{L^2 - h^2}}{\cos \alpha}$$

Therefore, it can be obtained that:
The span and cantilever length of gantry crane are specified in national standards. For example, the recommended effective cantilever length of general gantry crane is shown in Table 1; the effective cantilever length of electric hoist gantry crane is generally not greater than $S/3$.\cite{8-9}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Span $S$ & Effective cantilever length L1 or L2 \\
\hline
10~14 & 3.5 \\
18~26 & 3~6 \\
30~35 & 5~10 \\
40~60 & 6~15 \\
\hline
\end{tabular}
\caption{Value of effective cantilever length of general gantry crane}
\end{table}

Therefore, in general, it is advisable to $S/L \leq 3$. So the theoretical error can be obtained $\Delta 2$:

$$\Delta 2 = 1 - \frac{1}{1 - \left(\frac{H}{S}\right)^2} = 1 - \frac{1}{1 - \left(\frac{1 + \frac{H}{S}}{1 - \frac{H}{S}}\right) \times \left(\frac{1 + \frac{S}{L} \times \frac{h}{h}}{1 - \frac{S}{L} \times \frac{h}{h}}\right)} \leq 1 - \frac{1}{1 - \left(\frac{1 + \frac{3h}{S}}{1 - \frac{3h}{S}}\right) \times \left(\frac{1 + \frac{H}{S}}{1 - \frac{H}{S}}\right)}$$

It has been concluded that when $H \leq 0.09987S$, error $\Delta 1$ can be ignored and substituted into formula (12) to calculate according to the recommended accuracy of 0.5%, that is

$$\Delta 2 \leq 1 - \frac{1}{1 - \left(\frac{1 + \frac{3h}{S}}{1 - \frac{3h}{S}}\right) \times \left(\frac{1 + \frac{H}{S}}{1 - \frac{H}{S}}\right)} \leq 0.5\%$$

, and then the $h \leq 0.04696S$. That is, the vertical distance between the outer end of the effective arm length of one side cantilever and the connection of the nearest leg of the main beam $h$ When it is not more than 0.04696 times of the span of the main beam, the error is less than 0.04696 $\Delta 2$. It can be ignored.

To sum up, the simplified formula for calculating the camber is $f = h - \frac{L}{S} \times H$. The scope of application is $H \leq 0.09987S$ and $h \leq 0.04696S$.

It is worth noting that, as mentioned at the beginning of this paper, when some scholars use trigonometric function method to solve the correction formula of cantilever camber, they directly substitute the length of be with the length of cantilever $L$, and thus deduce the approximate calculation formula of approximate camber. This formula has errors with the actual camber derived from coordinate system transformation in this paper, but the error is negligible in daily inspection of influence.\cite{4-5}

It has also been mentioned at the beginning of this paper that some scholars based on the height difference angle of outrigger $\alpha$ and cantilevered up angle $\beta$ (i.e. $\angle CBF$), according to $\sin \alpha = \frac{H}{S}$ and $\sin \beta = \frac{h}{L}$, and substituting it into the formula $f = CD = L \times \cos(\alpha - \beta)$, so the accurate calculation formula of cantilever upwarping is obtained. But the formula needs to use the
inverse trigonometric function, and the measured values of H and h are important to the inverse trigonometric function \( \alpha \) and \( \beta \). The influence of the value f is great, which greatly affects the quality of the products \( \cos(\alpha - \beta) \). Therefore; the application of this method in engineering is limited.

6. Concluding remarks

The calculation of the cantilever camber of gantry crane is a content that must be mastered by the inspection personnel of lifting machinery, and it is also an item that is often examined in the certificate examination of inspection personnel over the years. It is the premise to correct the calculation deviation of the cantilever camber caused by the unequal height of the legs on both sides of the gantry crane. In this paper, based on the transformation formula of two-dimensional rotating \( \text{c coordinate system} \), the accurate calculation formula of cantilever camber is derived for the first time, and the simplified formula meeting the accuracy of engineering practice is obtained, and its application scope is given.

Acknowledgments

Supported by Quality and Technology Supervision of Zhejiang Province (No:20180357).

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