Quantization of gauged Floreanini-Jackiw chiral boson with Faddeevian anomaly

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Abstract

We consider the gauged model of Siegel type chiral boson with a Lorentz non-covariant mass-like term for the gauge fields which is found to be equivalent to the chiral Schwinger model with Faddeevian anomaly when it is described in terms of Floreanini-Jackiw type chiral boson. We carry out the quantization of gauge non-invariant version this model in both the Lagrangian and Hamiltonian formulation. The quantization of the gauge-invariant version of this model in the extended phase space also has been carried out in the Lagrangian formulation. The gauge-invariant version of this model in the extended phase space is found to map onto the physical phase space with the appropriate gauge fixing condition. BRST symmetry associated with this model has been studied with different gauge fixing terms. It has been shown that the same model shows off-shell as well as on-shell BRST invariance depending on the choice of gauge fixing term.

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I. INTRODUCTION

Chiral boson is relevant for the understanding of several field theoretical models and quantization of chiral boson is interesting in the regime of lower-dimensional field theory. It appeared initially in the study of heterotic string theory [1, 2]. The study of quantum Hall effect too got important input from chiral boson [3, 4]. We have found an interesting description of chiral boson in the pioneering work of Siegel [5]. An alternative illustration of chiral boson was offered by Srivastava in the article [6]. The Lagrangian of chiral boson was formulated with the second-order time derivative of the field in both these illustrations [5, 6], however, there is a foundational difference in the process of implementation of chiral constraint through the Lagrange multiplier. In the original version of Siegel, the chiral constrain was inserted in a quadratic form, on the contrary, in the description of Srivastava it was placed in a linear form. An innovative and sophisticated interpretation of chiral boson also came from the description of Floreanini and Jackiw [7].

An illuminating illustration towards quantization of free chiral boson was carried out in the article [8]. Extension of free chiral boson for the different purpose have been made in the articles [7, 14]. The study of free chiral boson is still acquiring a prominent position in the active field of research. A very recent development towards the BFV quantization of the free chiral boson along with the study of Hodge decomposition theorem in the context of conserved charges has been perused in the article [14]. In the article [13], and an application of augmented super field approach to derive the off-shell nilpotent and absolutely anti-commuting anti-BRST and anti-co-BRST symmetry transformations for the BRST invariant Lagrangian density of a free chiral boson has been executed in a significant manner. An equivalence between gauge invariant and gauge non-invariant solution of gauged chiral boson with Faddeevian anomaly with BRST quantization is made by us in [15].

Study of interacting chiral boson has also received a great deal of attention. A spontaneous generalization of a free chiral boson is to take into consideration its interaction with the gauge field, and this interacting field theoretical model is commonly known as gauged model of chiral boson. In the article [10] the interacting theory of chiral boson was initiated and described in detail by Bellucci, Golterman, and Petcher. The basic foundation of generalization towards taking into account the interaction with the gauge field however is laid in the Siegel’s construction of free chiral boson. So the theory of interacting chiral boson for the Faddeev-Jackiw (FJ) type description appears like a natural extension when the description of free FJ type chiral boson became accessible from the article [9]. An ingenious illumination was brought forward by Harada [17] in that context. The illuminating extension of Harada concerning interacting chiral boson based on FJ type kinetic term receives a great deal of attention [18-25], although this theory of interacting chiral boson was not derived from the iterating theory of chiral boson that developed in the article [16]. Actually Harada obtained it from Jackiw-Rajaraman (JR) version chiral Schwinger model [26] with

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the fruitful use of chiral the constraint in the phase space this theory. So there laid a missing link between these two types of interacting gauged chiral boson. An attempt towards the search for a link was therefore a natural pursuit which we have attempted to link up in our article [25]. In the article [24] we have shown that gauged model of chiral boson with FJ type chiral boson remains in disguise in the gauged model of chiral boson described in [10] with Siegel type chiral boson. However the gauged model of chiral boson associated with FJ type description of free chiral boson as extracted out by us from the description of the article, [10] is identical to the chiral Schwinger model with the standard Jackiw-Rajaraman type of anomaly when it described in terms of chiral boson [17].

If we look towards the background it reveals that chiral generation of Schwinger model [24] due to Hagen [27] had to suffer for a long period due to the non-unitarity problem. That problem was eradicated taking anomaly into consideration by Jackiw and Rajaraman [24]. However, this is not the only possibility to get out of this unitarity problem which we found in the article of Mitra [28]. In the article [28], he has ingeniously shown that the unitarity problem of chiral Schwinger model can be eradicated with a special type of anomaly termed as Faddeevan anomaly by Mitra since Gauss law commutator of the model with itself along with this anomaly remains non-vanishing. So the formulation of a gauged chiral boson with FJ type free chiral boson which would be consistent with chiral Schwinger model with the Faddeevan anomaly would certainly be of interest. A description of chiral Schwinger model with Faddeevan anomaly in terms of chiral boson is found in [20]. However, in this the article, it was described in an ad hoc manner: the direct link of it with its mother version [28] was not transparent. In [25], we have made an attempt to shown that this model also can be generated from the model described by Mitra in [28] imposing a chiral constraint in a similar way Harada [17] did it for the usual Chiral Schwinger mode. So a natural extension which would be of interest and instructive as well to investigate whether the root of chiral Schwinger model with the Faddeevan anomaly described in terms of a chiral boson is inlaid in the discrimination of gauged chiral boson in the pioneering work [10] which was done for Harada’s version in [24]. So, by all means, the investigation towards the exploration of that and the systematic study towards determination of spectrum in the Lagrangian formulation, its correlation with the hamiltonian formulation and the gauge and BRST invariance of this model as well is attempted in this article.

It would be beneficial from another point of view too because a careful look over the previous studies reveals that the gauged model of chiral boson has a crucial link with anomaly [17, 20, 24, 28, 37], because it has been found that gauged model of a chiral boson is crucially connected to chiral Schwinger model and that very chiral Schwinger model got secured from the non-unitarity problem when the anomaly was taken into consideration by Jackiw and Rajaraman [24]. In this respect, the recent chiral generation of The Thirring-Wess model is of worth mentioning [34, 35].

This article is organized as follows. Sec. II contains the formulation of gauged Floreanini-Jackiw type chiral boson that corresponds to the Faddeevan anomaly. Sec. III is devoted to the determination of theoretical spectra of this model in the Lagrangian formulation. In Sec. IV, a transparent description of the evaluation of the theoretical spectra of this model in the Hamiltonian formulation is presented. Sec. V holds a description of the determination of the theoretical spectrum of the gauge-invariant version of the theory in the Lagrangian formulation. In Sec. VI, an attempt is made to map the gauge symmetric version of the theory in the extended phase space onto the gauge non-invariant version of it in the usual Phase space. Sec. VII is devoted to the study of BRST symmetry of this model. And the final Sec. VIII contains a brief summary and discussion over the work.

II. FORMULATION OF GAUGED FLOREANINI-JACKIW TYPE CHIRAL BOSON THAT CORRESPONDS TO FADDEEVIAN ANOMALY

A gauged model of Siegel type chiral boson which resembles chiral Schwinger model with Jackiw-Rajaraman’s one parameter class of regularization was discussed in [23]. A natural extension that trails with is the gauge model of of Siegel type chiral boson which would be consistent with gauged Lagrangian of Floreanini-Jackiw type chiral boson with Faddeevan anomaly. To formulate that let us proceed with the following Lagrangian containing a Lorentz non-covariant mass like term for the gauge field.

\[ L = \int dx \left[ \frac{1}{2} (\phi'^2 - \phi^2) + e(\phi + \phi')(A_0 - A_1) + \frac{\Lambda}{2} ((\phi - \phi') + e(A_0 - A_1))^2 \right. \\
\left. + \frac{1}{2} (\dot{A} - A_0')^2 + \frac{1}{2} e^2 (A_0 + A_1)(A_0 - 3A_1) \right] \] (1)

This Lagrangian is constructed following the article [10]. Here \( \phi \) represents scalar field. The gauge field has two components \( A_0 \) and \( A_1 \) in \((1 + 1)\) dimensional space time. \( A \) is lagrange multiplier field. The coupling constant \( e \) has the dimension of mass. The model is holding a Lorentz non-covariant masslike term for the gauge field. It looks strange but what it renders is interesting and does not violate physical Lorentz invariance. It also establishes whether the root of chiral bosonized version of chiral Schwinger model with Faddeevan anomaly is in laid in the description
of gauged chiral boson or not. We are going to illustrate it in details in this section. We need to compute the canonical momenta corresponding to the fields $A_0$, $A_1$, $\phi$ and $\Lambda$.

\[
\frac{\partial L}{\partial \dot{A}_0} = \pi_0 \approx 0
\]

\[
\frac{\partial L}{\partial \dot{A}_1} = \pi_1 = (\dot{A}_1 - A'_0)
\]

\[
\frac{\partial L}{\partial \dot{\phi}} = \pi_\phi = (1 + \Lambda)\dot{\phi} - \Lambda\phi' + e(1 + \Lambda)(A_0 - A_1)
\]

\[
\frac{\partial L}{\partial \dot{\Lambda}} = \pi_\Lambda \approx 0
\]

A Legendre transformation $H = \pi_0 \dot{A}_0 + \pi_1 \dot{A}_1 + \pi_\phi \dot{\phi} + \pi_\Lambda \dot{\Lambda} - L$ with the use of the expression of momenta (2), (3), (4), (5) leads to the canonical Hamiltonian

\[
H_C = \int dx H_C = \int dx \left[ \frac{1}{2} \pi_1^2 + \pi_1 A'_0 + \pi_\phi \phi' - e(\pi_\phi + \phi')(A_0 - A_1)
\right.

\left. + 2e^2 A_1^2 + \frac{1}{2(1 + \Lambda)}(\pi_\phi - \phi')^2 \right]
\]

The equations (2) and (5) do not contain any time derivative so these are the primary constraint of the theory. The preservation of these constraints lead to new constraints. Repeating this preservation criteria of the usual constraint and the forth coming secondary constraints we find that the phase space of the system is embedded with the following six constraints.

\[
\Omega_1 = \pi_0 \approx 0
\]

\[
\Omega_2 = \pi_\Lambda \approx 0
\]

\[
\Omega_3 = \pi'_\phi + e(\pi_\phi + \phi') \approx 0
\]

\[
\Omega_4 = \pi_\phi - \phi' \approx 0
\]

\[
\Omega_5 = A_0 + A_1 \approx 0
\]

\[
\Omega_6 = \Lambda - f \approx 0
\]

Therefore, the generating functional of the theory can be written down as

\[
Z = \int |\det[\Omega_1, \Omega_m]|^{\frac{1}{2}} dA_1 d\pi_1 d\phi d\pi_0 dA_0 d\pi_0 e^{i \int d^2 x (\pi_1 \dot{A}_1 + \pi_\phi \dot{\phi} + \pi_\Lambda \dot{\Lambda} + \pi_0 \dot{A}_0 - H_C)}
\times \delta(\Omega_1)\delta(\Omega_2)\delta(\Omega_3)\delta(\Omega_4)\delta(\Omega_5)\delta(\Omega_6)
\]

The subscripts $l$ and $m$ runs from 1 to 6. After simplification by the use of gaussian integral we land on to

\[
Z = \int d\phi dA_1 dA_0 e^{\int d^2 x L_{CH}}
\]

where

\[
L_{CH} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2 A_1^2 + \frac{1}{2}(\dot{A}_1 - A'_0)^2
\]

So it is now evident that the Lagrangian is the appropriate gauged Lagrangian of Siegel type Chiral Boson that corresponds to the gauged chiral boson with Floreanini-Jackiw type chiral boson which can generated from chiral Schwinger model with Faddeevian anomaly introducing a chiral constraint in the phase space of the theory. We will now turn to find out the theoretical spectrum of this system in the Lagrangian formulation because what a model with Lorentz non-covariant structure offers would be of interest.
III. DETERMINATION OF THEORETICAL SPECTRUM IN THE LAGRANGIAN FORMULATION

The gauged Lagrangian density of chiral boson with a the Mitra type faddeevian anomaly anomaly is given by

$$\mathcal{L}_{CH} = \dot{\phi}^2 - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2A_1^2 + \frac{1}{2}(\dot{A}_1 - A_0')^2$$  \hspace{1cm} (16)

This Lagrangian is not gauge symmetric in its usual phase space and the structure of this Lagrangian is not Lorentz non-covariant. We will proceed to obtain the theoretical spectrum of the system described in a gauge non-invariant way in the Lagrangian formulation. Using Euler-Lagrangian equations we obtain the following three equations of motion corresponding to the field $\phi$, $A_1$ and $A_0$

$$\ddot{\phi} - \phi'' + e(A_0 - A_1) = 0, \hspace{1cm} (17)$$

$$-\ddot{A}_1 + \dot{A}_0' - 2e\phi' - 4e^2A_1 = 0, \hspace{1cm} (18)$$

$$-A_0'' + \dot{A}_1' + 2e\phi' = 0. \hspace{1cm} (19)$$

To solve these coupled differential equations (17), (18) and (19) we introduce the ansatz for the fields $A_\mu$ and $\phi$ as

$$A_\mu = \frac{1}{4e^2} \partial F, \hspace{1cm} (20)$$

$$\phi = \frac{1}{4e} F. \hspace{1cm} (21)$$

Equation (17) gives

$$\Box F = -\frac{1}{4e^2}(\dot{A}_1 - A_0') $$  \hspace{1cm} (22)

Plugging in the ansatz (20) and (21) in the equations (17), (18) and (19) and using (22) the following solution of $F$ is obtained:

$$(\Box + 4e^2)\Box F = 0, \hspace{1cm} (23)$$

with the restriction

$$A_0 + A_1 = 0. \hspace{1cm} (24)$$

Note that the equation (23) is Lorentz co-variant although the Lagrangian was not manifestly Lorentz covariant. The equation (23) indicates that the physical spectrum contains a massive boson only with mass $2\varepsilon$. At a first glance it seems that this restriction is put by hand on the dynamics of fields $A_0$ and $A_1$, but that is not the case and it will be transparent if we look carefully the Hamiltonian analysis of the model which has already been studied in [28]. In the following section however we will describe the Hamiltonian formulation in a coherent manner to ensure that the restriction is already there in the phase space of this constrained system and the spectrum obtained in the Lagrangian formulation agrees with the Spectrum in the Hamiltonian formulation. It will make this article self contained too.

IV. A TRANSPARENT DESCRIPTION OF THEORETICAL SPECTRUM IN THE HAMILTONIAN FORMULATION

To start with we should mention that main result as we are going to produce is already known from the articles [28, 29]. To make this article self contained and to make an easy comparison of the Lagrangian and Hamiltonian formulation in connection with the theoretical spectrum we are furnishing it in a desired cohered manner. To obtained the theoretical spectrum in the Hamiltonian formulation we calculate the momenta corresponding to the fields $A_0$, $A_1$ and $\phi$ from the Lagrangian density (16) using the standard definition of momenta.

$$\frac{\partial L_{CH}}{\partial \dot{A}_0} = \pi_0 \approx 0, \hspace{1cm} (25)$$
\[ \frac{\partial L_{CH}}{\partial A_1} = \pi_1 = (\dot{A}_1 - A'_0), \]  
\[ \frac{\partial L_{CH}}{\partial \phi} = \pi_\phi = \phi'. \]  

The equation (25) and (27) are the two primary constraint of the theory. The effective Hamiltonian (according to the terminology of Dirac) therefore is given by

\[ H_{EF} = \int dx \{ H_c + u \pi_0 + v(\pi_\phi - \phi') \} \]  

where the canonical Hamiltonian \( H_{CH} \) reads

\[ H_{CH} = \int dx \left[ \frac{1}{2} \pi_1^2 + \pi_1 A'_0 + \phi'^2 - 2e(A_0 - A_1)\phi' + 2e^2 A_1^2 \right] \]  

The Lagrange multiplier \( u \) and \( v \) are are found out in due course. The Gauss law constraints of this theory is found out to be

\[ G = \pi'_1 + 2e\phi' \approx 0. \]  

from the preservation of the constraint (25). The preservation of constraint \((\pi_\phi - \phi') \approx 0\) with respect to the Hamiltonian gives a new constraint

\[ (A_1 + A_0)' \approx 0. \]  

The Lagrangian multiplier \( u \) and \( v \) are given by

\[ u = -\pi_1 + A'_0, \]  
\[ v = \phi - e(A_0 - A_1). \]  

Imposing the constraints (25), (27) and (31) we obtain the reduced Hamiltonian:

\[ H_R = \int dx \left[ \frac{1}{2} \pi_1^2 + \pi_1 A'_0 + \frac{1}{4e^2} \phi'^2 + 4e^2 A_1^2 \right] \]  

The phase space of the system are endowed with following four constraints.

\[ \Omega_1 = \pi_0 \approx 0, \]  
\[ \Omega_2 = \pi_\phi - \phi' \approx 0, \]  
\[ \Omega_3 = \pi_1 + 2e\phi \approx 0, \]  
\[ \Omega_4 = A_1 + A_0 \approx 0. \]  

It is constrained theory so Poisson bracket become insufficient to get the correct equations of motion. The Dirac brackets [38] give the correct equations of motion which is defined by:

\[ [A(x), B(y)]^* = [A(x), B(y)] - \int [A(x), \Omega_i(\eta)] C_{ij}^{-1} [\Omega_i(\eta), B(x)] d\eta dz \]  

where \( C_{ij}^{-1} \) is obtained using the following relation

\[ \int C_{ij}^{-1} (x, z)[\omega_i(z), \omega_j(y)] dz = 1 \]
The Dirac brackets between the fields describing the reduced Hamiltonian are computed to be
\[
[A_1, \pi_1]^* = \delta(x - y),
\]
\[
[A_1, A_1]^* = -\frac{1}{2e^2} \delta'(x - y).
\]

The following two first order equations of motion are followed from the reduced Hamiltonian with the use of the Dirac brackets (41), and (42):
\[
\dot{\pi}_1 = \pi_1' - 4e^2 A_1,
\]
\[
\dot{A}_1 = \pi_1 - A_1'.
\]

These two equations lead to two second order differential equations
\[
[\Box + 4e^2] A_1 = 0.
\]
\[
[\Box + 4e^2] \pi_1 = 0.
\]

Equation (45) represents a massive boson with square of the mass $4e^2$ and equation (46) stands for the momentum of the massive field $A_1$. Let us now look back carefully to the spectrum obtained in Sec. II through the Lagrangian formulation. Note that the field $F$ can expressed in terms of the field $A_0$ and $A_1$ by the expression
\[
\pi_1 = -\epsilon_{\mu\nu} \partial_\nu A_\mu = \Box F \approx F
\]
So the field $F$ corresponds to the field $\pi$. In Sec. II we have imposed a condition $A_0 + A_1 = 0$ in an ad hoc manner. But the Hamiltonian formulation shows that the condition $A_0 + A_1 = 0$ is lying hidden within the system which manifests itself as a constraint in the phase space of the theory.

V. DETERMINATION OF THEORETICAL SPECTRUM OF THE GAUGE INVARIANT THEORY IN THE LAGRANGIAN FORMULATION

In this section we extend the phase space of the theory introducing some new fields following Stuckelberg formalism. The theory is made gauge invariant putting the Wess-Zumino term $L_{WZ}$ within the Lagrangian $L_{CH}$
\[
L_{GS} = L_{CH} + L_{WZ},
\]
where $L_{WZ}$ stands for Wess-Zumino term [39]:
\[
L_{WZ} = -\dot{\zeta} \zeta' - \frac{1}{2} \dot{\zeta} \zeta' - 2e(A_0 + A_1)\zeta'.
\]

It is straightforward to examine that the Lagrangian (48) is invariant under the transformation $A_\mu \rightarrow A_\mu + \frac{1}{2} \theta_\mu \lambda$, $\phi \rightarrow \phi + \lambda$ and $\zeta \rightarrow \zeta - \lambda$. In order to determine the physical specter we need to introduce the gauge fixing condition which is offered here by the following Lagrangian. So the Lagrangian with which we will proceed here is
\[
L_T = L_{CH} + L_{WZ} + L_{GF},
\]
where
\[
L_{GF} = B \partial_\mu A^\mu + \frac{1}{2} \alpha B^2
\]

The Euler-Lagrange equations of motion that follows from the Lagrangian (50) are the following.
\[
-\dot{A}_1 + \dot{A}_0' - 2e\phi' - 4e^2 A_1 - 2\zeta' = 0,
\]
\[
A_0'' + \dot{A}_1' + 2e\phi' + 2\zeta' = 0,
\]
\[ \partial_+ \phi' + e(A_0 - A_1) = 0, \quad (54) \]
\[ \partial_+ \zeta' - e(A_0 - A_1) = 0, \quad (55) \]
\[ \partial_\mu A^\mu + \alpha B = 0. \quad (56) \]

The ansatz for the fields which are found to be appropriate to solve the above set of coupled differential equations are

\[ A_\mu = \frac{1}{4e^2} \partial F + \partial_\mu B + \partial_\mu \chi, \quad (57) \]
\[ \zeta = -\frac{1}{4e} F - B - \chi, \quad (58) \]
\[ \phi = \frac{1}{4e} F - B - \chi. \quad (59) \]

Using the ansatz (57), (58) and (59) in the equations of motion (52), (53), (54), (55) and (56) we obtain the following three second order differential equations:

\[ (\Box + 4e^2) \Box F = 0, \quad (60) \]
\[ \Box B = 0, \quad (61) \]
\[ \Box \chi + \alpha B = 0, \quad (62) \]

where

\[ \pi_1 = \dot{A}_1 - A'_0 = \frac{\Box F}{4e^2} \quad (63) \]

The field \( F \approx \Box F \) is representing the massive field with mass \( 2e \). The corresponding equation we have obtained in both the Lagrangian and Hamiltonian formulation of the theory with its gauge non-invariant version in equation (23) and (45) or (46) respectively. The equation (61) appears because in the gauge fixed Lagrangian we have used an auxiliary field \( B \) and the field \( \chi \) represents the zero mass dipole field playing the role of gauge degrees of freedom that can be eliminated by operator gauge transformation. So the spectrum agrees with spectrum obtained in Sec. III.

VI. TO MAKE AN EQUIVALENCE BETWEEN THE GAUGE INVARIANT AND GAUGE NON-INVARIANT VERSION

To make an equivalence between the gauge invariant and the gauge non-invariant version of this model we proceed with the gauge symmetric Lagrangian. So we add up the Wess-Zumino term with the usual Lagrangian.

\[ L_{GS} = \int \! dx [\mathcal{L}_{CH} + \mathcal{L}_{WZ}] \quad (64) \]
\[ L_{GS} = \int \! dx [\dot{\phi} \phi' - \dot{\phi}' \phi + 2e(A_0 - A_1) - 2e^2 A_1^2 - \dot{\zeta}' - \zeta'' + 2e(A_0 + A_1) \zeta' + \frac{1}{2} (\dot{A}_1 - A_0')^2] \quad (65) \]

Equations of motion are

\[ \frac{\partial L_{GS}}{\partial \phi} = \pi_\phi = \phi', \quad (66) \]
\[
\frac{\partial L_{GS}}{\partial \zeta} = \pi = -\zeta',
\]
\[(67)\]

\[
\frac{\partial L_{GS}}{\partial A_0} = \pi_0 \approx 0,
\]
\[(68)\]

\[
\frac{\partial L_{GS}}{\partial A_1} = \pi_1 = (\dot{A}_1 - A'_0).
\]
\[(69)\]

By the use of equations (66), (67), (68) and (69) the canonical Hamiltonian that follows from the Lagrangian (65) is

\[
H_{CGS} = \int dx [\pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \pi_\phi \dot{\phi} + \pi_\zeta \dot{\zeta}] - L_{GS}
\]
\[(70)\]

Therefore, the effective Hamiltonian for this situation is

\[
H_{GSE} = \int dx \left[ \frac{1}{2} \pi_1^2 + \pi_1 A'_0 + \phi' - 2e(A_0 - A_1)\phi' + 2e^2 A_1^2 + \zeta'^2 - 2e(A_0 + A_1)\zeta' + u\pi_0 + v(\pi_\phi - \phi') + w(\pi_\zeta + \zeta') \right]
\]
\[(71)\]

The Gauss law constraint that comes out from the preservation condition of constraint (68) is

\[
G = [\pi_0, H] = \pi'_1 + 2e(\phi' + \zeta').
\]
\[(72)\]

The velocities \( u \) and \( v \) are found out as

\[
v = \phi' + e(A_0 - A_1),
\]
\[(73)\]

\[
w = -\zeta' + e(A_0 + A_1).
\]
\[(74)\]

Using the velocities and the successive use of the condition of preservation of the constraints the following set of second class constraints are found to be embedded within the phase space of the theory.

\[
C_1 = \pi_0 \approx 0
\]
\[(75)\]

\[
C_2 = \pi_\phi - \phi' \approx 0
\]
\[(76)\]

\[
C_3 = \pi_\zeta + \zeta' \approx 0
\]
\[(77)\]

\[
C_4 = \pi'_1 + 2e(\phi' + \zeta') \approx 0
\]
\[(78)\]

We are in a position to chose gauge fixing conditions those which are very crucial in this situation. The inappropriate use of gauge fixing leads to different effective theory which may mislead to reach to the goal. The appropriate gauge fixing which meet our need are the following.

\[
C_5 = \zeta' = 0,
\]
\[(79)\]

\[
C_6 = \pi_\zeta = e(A_0 + A_1).
\]
\[(80)\]

These inputs therefore enables us to write down the generating functional.

\[
Z = \int [det[C_i, C_m]]^\frac{1}{2} dA_1 d\pi_1 d\phi d\pi_0 dA_0 d\pi_0 d\zeta d\pi_\zeta e^{i\int d^2x (\pi_1 \dot{A}_1 + \pi_\phi \dot{\phi} + \pi_\zeta \dot{\zeta} - \pi_0 A'_0 - H_{CGS})} \times \delta(C_1)\delta(C_2)\delta(C_3)\delta(C_4)\delta(C_5)\delta(C_6).
\]
\[(81)\]
Here \( l \) and \( m \) runs from 1 to 4. Integrating out of the fields \( \zeta \) and \( \pi_\zeta \) we find that equation (81) reduces to

\[
Z = \int dA_1 d\pi_1 d\phi d\pi_0 dA_0 d\pi_0 e^{i \int d^2x (\pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 - H_{GSF})} \\
\times \delta(C_1) \delta(C_2) \delta(C_3) \delta(C_4). \tag{82}
\]

where

\[
H_{GSF} = \frac{1}{2} \pi_1^2 + \pi_1 A_0' + \phi' - 2e(A_0 - A_1)\phi' + 2e^2 A_1^2. \tag{83}
\]

Again integrating out of the momenta \( \pi_0, \pi_1 \) and \( \pi_\phi \) leads us to

\[
Z = \int d\phi dA_1 dA_0 e^{i \int d^2x L_{GSF}} \tag{84}
\]

where

\[
L_{GSF} = \dot{\phi} \phi - \phi'^2 + 2e\phi'(A_0 - A_1) - 2e^2 A_1^2 + \frac{1}{2}(\dot{A}_1 - A_0')^2. \tag{85}
\]

Note that the system now contains the usual four constraint \( C_1, C_2, C_3, C_4 \) and \( L_{GSF} \) is the identical to usual Lagrangian \( L_{CH} \) having the same Hamiltonian \( H_{GSR} = H_R \). So the gauge invariant Lagrangian maps on to gauge non-invariant Lagrangian of the usual phase space. It also ensures that the physical contents in both the version are identical.

**VII. STUDY OF BRST SYMMETRY OF THE MODEL**

Let us now turn towards the study of BRST symmetry of the effective action. It is an important symmetry which ensures the unitarity and renormalization of a theory [40–42]. In the articles [43–47] different method of construction of BRST invariant effective action of a given theory have been discussed and the applications of these formalism have been pursued in the articles [48–53]. Since BRST is a powerful tool to ensure the the unitarity and renormalization of a theory the study of BRST symmetry of any field theoretical model would be of interest. In this section we are intended to study the BRST and anti-BRST properties of this model both in off-shell and on-shell environment. With out going through the formal construction of BRST invariant effective action we discuss here the symmetry properties of this model with different gauge fixing term. To this end we consider gauge invariant version of the action with the Lorentz gauge fixing term

\[
S_{BRST} = \int d^2x [L + L_{WZ} + \partial_\mu A^\mu B + \frac{1}{2} \alpha B^2 + \partial_\mu \bar{C} \partial^\mu C]. \tag{86}
\]

It is straightforward to see that the Lagrangian is off-shell invariant under the BRST transformation

\[
\delta_B A_\mu = -\frac{1}{e} \lambda \partial_\mu C, \quad \delta_B \phi = \lambda C, \quad \delta_B \zeta = -\lambda C \tag{87}
\]

\[
\delta_B \bar{C} = \lambda B, \quad \delta_B C = 0, \quad \delta_B B = 0. \tag{88}
\]

The above Lagrangian is found to be off-shell invariant under the anti-BRST transformation

\[
\delta_{ab} A_\mu = -\frac{1}{e} \lambda \partial_\mu \bar{C}, \quad \delta_{ab} \phi = \lambda \bar{C}, \quad \delta_{ab} \zeta = -\lambda \bar{C} \tag{89}
\]

\[
\delta_{ab} C = \lambda B, \quad \delta_{ab} \bar{C} = 0, \quad \delta_{ab} B = 0. \tag{90}
\]

The gauge fixing condition can be chosen in different ways keeping the physical contents intact. We choose another important gauge fixing term which is known as ’t Hooft-Veltman gauge. The effective action with this gauge reads

\[
\tilde{S}_{BRST} = \int d^2x [L + L_{WZ} + B(\partial_\mu A^\mu + eA_\mu A^\mu) + \frac{1}{2} \alpha B^2 + \bar{C}(\Box + eA_\mu \partial^\mu)C]. \tag{91}
\]
This new effective action is found to be off-shell invariant with the following BRST transformation

\[ \delta_B A_\mu = - \frac{1}{e} \lambda \partial_\mu C, \quad \delta_B \phi = \lambda C, \quad \delta_B \zeta = - \lambda C, \]  
(92)

and the above effective action is off-shell invariant under the following anti-BRST transformation

\[ \delta_{ab} A_\mu = - \frac{1}{e} \lambda \partial_\mu \bar{C}, \quad \delta_{ab} \phi = \lambda \bar{C}, \quad \delta_{ab} \zeta = - \lambda \bar{C} \]  
(93)

\[ \delta_{ab} C = \lambda B, \quad \delta_{ab} \bar{C} = 0, \quad \delta_{ab} B = 0. \]  
(94)

Let us see whether the gauge fixing condition can be chosen in such a way when the effective action shows on-shell invariance under the BRST and anti-BRST transformation. To this end we consider the following effective action with a different gauge fixing term.

\[ \tilde{S}_{\text{BRST}} = \int d^2 x [L + L_{wz} + \frac{1}{2\alpha} (\partial \cdot A + \alpha e B)^2 + \partial_\mu \bar{C} \partial^\mu C + e^2 \bar{C} C]. \]  
(95)

Under the BRST transformation

\[ \delta_b A_\mu = - \frac{1}{e} \lambda \partial_\mu C, \quad \delta_b \phi = \lambda C, \quad \delta_b \zeta = - \lambda C \]  
(96)

\[ \delta_b C = 0, \quad \delta_b \bar{C} = \partial \cdot A + \alpha e B, \quad \delta_b (\partial \cdot A + \alpha e B) = (\Box + e^2) C, \]  
(97)

the effective theory is on-shell BRST invariant with the on-shell condition

\[ (\Box + e^2) C = 0 \]  
(98)

The effective theory is also found to be on-shell invariant under the anti-BRST Transformation

\[ \delta_{ab} A_\mu = - \frac{1}{e} \lambda \partial_\mu \bar{C}, \quad \delta_{ab} \phi = \lambda \bar{C}, \quad \delta_{ab} \zeta = - \lambda \bar{C} \]  
(99)

\[ \delta_{ab} \bar{C} = 0, \quad \delta_{ab} C = \partial \cdot A + \alpha e B, \quad \delta_{ab} (\partial \cdot A + \alpha e B) = (\Box + e^2) \bar{C} \]  
(100)

with the on-shell condition

\[ (\Box + e^2) \bar{C} = 0 \]  
(101)

This is in short BRST and anti-BSRT symmetric property of this theory in the off-shell and on-shell domain.

**VIII. SUMMARY AND DISCUSSION**

We have considered a gauged Lagrangian with a Siegel type chiral boson with the different masslike term for gauge fields. The masslike term which was chosen in [16] led to a gauged theory of Florenini-Jackiw type chiral boson which can be derived from Chiral Schwinger model with the Jackiw-Rajaraman type of electromagnetic anomaly [17]. An alternative masslike term is chosen here in order to derive the gauged model Florenini-Jackiw type chiral boson which gets generated from the chiral Schwinger model with Faddeevian type of anomaly [20, 53]. In the article [28] the author showed that the Chiral Schwinger model remains physically sensible in all respect with an independent type of masslike term where the nature of the anomaly belonged to the Faddeevian class.

This physical spectrum of the gauge non-invariant version of this model is found out both in the Lagrangian and Hamiltonian formulation. We should mention here that the determination of the spectrum in the Hamiltonian formulation was done in [28], but in the Lagrangian formulation, it was lacking. Here we have done it in a much transparent manner that enables us to correlate the spectrum of the theory in both the formulation. It was found that a condition \( A_0 + A_1 = 0 \) needed to be put in an ad-hoc manner to obtain the correct spectrum in the Lagrangian formulation. Though it seems to be unnatural a critical review shows that the condition \( A_0 + A_1 = 0 \) is basically a constraint of the theory which shows its mysterious appearance in the Hamiltonian formulation. In the article [23], the
similar type analysis was pursued for the bosonised version of chiral Schwinger model with the usual Jackiw-Rajaraman type of anomaly. In this work, we have extended it for the Chiral Schwinger model with Faddeevian anomaly proposed by Mitra in [28, 29]. These model are very much different so far constraint structure and the physical spectrum is concerned. So this work though looks similar to [23] as the computation is concerned it will shed light in the lower dimensional constrained field theoretical regime. For instance, the solvability of Lagrangian formulation here needs an extra condition which has appeared as a constraint in the Hamiltonian formulation. Without that condition, this model would not be solvable in the Lagrangian formulation.

The model is made gauge-invariant with the incorporation of Wess-Zumino field. The phase space determination of the model is then carried out in the Lagrangian formulation using Lorentz gauge condition since with this gauge the model does not lose its exactly solvable nature. The theoretical spectrum is found to be in exact agreement with its gauge non-invariant counterpart. The auxiliary fields found to remains allocated in the un-physical sector of the theory. An attempt is made to make an equivalence to the theory of the gauge-invariant and gauge non-invariant version using the ingenious formalism developed in [56]. It is found the role of gauge fixing is very crucial here.

BRST symmetry related to this model is also studied with different gauge fixing terms. A gauge fixing term has been used where the model shows on-shell gauge invariance.

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