Fayet–Iliopoulos Terms in Supergravity and Cosmology

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Abstract

We clarify the structure of $N = 1$ supergravity in 1+3 dimensions with constant Fayet–Iliopoulos (FI) terms. The FI terms $g\xi$ induce non-vanishing $R$-charges for the fermions and the superpotential. Therefore the $D$-term inflation model in supergravity with constant FI terms has to be revisited. We present all corrections of order $g\xi/M_P^2$ to the classical supergravity action required by local supersymmetry and provide a gauge-anomaly-free version of the model.

We also investigate the case of the so-called anomalous $U(1)$ when a chiral superfield is shifted under $U(1)$. In such a case, in the context of string theory, the FI terms originate from the derivative of the Kähler potential and they are inevitably field-dependent. This raises an issue of stabilization of the relevant field in applications to cosmology.

The recently suggested equivalence between the $D$-term strings and D-branes of type II theory shows that brane-anti-brane systems produce FI terms in the effective $4d$ theory, with the Ramond-Ramond axion shifting under the $U(1)$ symmetry. This connection gives the possibility to interpret many unknown properties of D–D systems in the more familiar language of $4d$ supergravity $D$-terms, and vice versa. For instance, the shift of the axion field in both cases restricts the possible forms of the moduli-stabilizing superpotential. We provide some additional consistency checks of the correspondence of $D$-term-strings to D-branes and show that instabilities of the two are closely related. Surviving cosmic D-strings of type II theory may be potentially observed in the form of $D$-term strings of $4d$ supergravity. We study such string solutions of supergravity with constant FI terms with one half supersymmetry unbroken and explain some of the puzzling properties of the zero modes around cosmic strings, such as the difference between the numbers of fermionic and bosonic modes.

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1 Introduction

Current cosmological observations suggest that it may be important to study the effective four-dimensional gravitational theory derivable from a fundamental theory, like M/string theory. If the effective theory has local supersymmetry, it is described by $d = 4$, $N = 1$ supergravity. For cosmological applications one is interested particularly in any possibility to find a de Sitter type configuration (dS) with broken supersymmetry to describe the currently accelerating universe as well as a slow-roll stage of the early universe inflation.

The potential in $N = 1$ supergravity\(^1\) is well known: there is an F-term potential, constructed in a standard way from the Kähler potential and superpotential, in some cases

\(^1\)We will limit ourselves to gravitational, vector and chiral multiplets.
there is also a non-trivial $D$-term potential, derivable in the standard fashion [1–3]. It is extremely important in the context of cosmological applications that the $D$-term potential is always positive, whereas the F-term in general has both positive and negative contributions.

Since the $D$-term potential is positive definite it may lead to de Sitter type solutions, particularly in presence of a constant FI term.

At present there is no known way to derive the effective $d = 4$, $N = 1$ supergravity with constant FI terms from M/string theory. Only field-dependent $D$-terms have been identified so far (one should keep in mind that FI terms studied in the context of open string theory are not immediately relevant for gravity and cosmology). There is no strict no-go theorem about the absence of constant FI terms in string theory, however, for all practical purposes, the situation is close to the existence of such theorem.\(^2\)

Since constant FI terms in effective supergravity in 1+3 dimensions lead to dS spaces, the possibility to get such terms from M/string theory may require new developments in the understanding of string theory. It is worth reminding here that the second string revolution has allowed to treat M-theory and 11-dimensional supergravity as leading to effective theories with 1+3 dimensional chiral fermions. Before the compactifications on orbifolds and orientifolds were studied, it was believed that it is impossible to get $d = 4$ chiral fermions from 11-dimensional supergravity. At present we may only hope that some new possibility will realize in M/string theory that will allow to derive constant FI terms and dS and near dS spaces.

The purpose and the results of this paper can be summarized as follows:

- Firstly, we will study the general case when local $N = 1$ supersymmetry in 1+3 dimensions admits constant FI terms and provide the supersymmetry rules in such theories.\(^3\) As an application of these rules we will revisit the $D$-term inflation model and correct the supergravity version of it to comply with the restrictions on the superpotential required when constant FI terms are present. We will also study $D$-term strings and their properties in supergravity with constant FI terms.

- Secondly, we will study the so-called anomaly generated FI terms originating from string theory and explain that a procedure of stabilization of certain moduli is required for these models to be used in the cosmological context in 1+3 dimensions.

- The recently suggested [4] equivalence between the $4d$ supergravity $D$-term strings and D-branes of type $II$ theory shows that brane-anti-brane systems in an effective $4d$ theory can be described as gauge theories with non-zero FI term. The axion shifting under the $U(1)$-symmetry is dual to the Ramond-Ramond form. This connection gives the possibility to establish a useful dictionary between the two descriptions and interpret many important properties of D–D systems in a simpler language of $4d$

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\(^2\)We are grateful to S. Kachru and J. Maldacena for numerous discussions of this issue.

\(^3\)In globally supersymmetric theories the constant FI terms can be added without any constraints on the theory. However, in the local case this is not true anymore.
supergravity gauge theories with non-zero $D$-terms, and vice versa. In particular, we can use our knowledge of the stability of supergravity vacua with non-zero $D$-terms for understanding the stability of the string vacua with brane-anti-brane systems and their various cosmological applications, such as $D$-brane inflation. For instance, the shift of the axion in both cases restricts the possible forms of the moduli-stabilizing superpotential.

We provide some additional consistency checks of the correspondence between $D$-term strings and D-branes and show that, not surprisingly, the instabilities of the two are closely related. Thus, not only the cosmic $D$-term-strings, formed after $D$-term inflation, do not cause any cosmological trouble, but in fact they may be potentially detected. Hence, the D-brane strings of type $II$ theory, could in principle be observed in the sky in the form of the supergravity $D$-term strings!

A standard expectation is that any Kähler potential and any holomorphic superpotential may define a version of $N = 1$ supergravity in 1+3 dimensions. We will clarify here the situation with constant FI terms, when this expectation is not valid and certain restrictions on the choice of the superpotential are required.

The very first version of supergravity with locally supersymmetric extension of the FI term of the Abelian vector multiplet was constructed in [5]. It has positive cosmological constant, $\Lambda > 0$. It was also shown there that local supersymmetry requires the axial gauging of gravitino and gaugino (local $R$-symmetry). This theory involves only the gravitational supermultiplet and the vector supermultiplet and has one-loop axial anomalies [6]. More general classes of models with constant FI terms and scalar supermultiplets were constructed in [7–10]. More recently there were few important developments in studies of some anomaly-free models with gauged $R$-symmetry and constant FI terms in supergravity [11, 12]. At that time the main focus of such investigations was towards particle physics with vanishing cosmological constant.

On the other hand, in the cosmology community, the role of $D$-terms has become extremely important as a possible origin of de Sitter configurations and inflation in supergravity [13,14]. It remains, however, not well known that the presence of constant FI terms poses specific restrictions on supergravity theories (see however [15,16]).

The existing versions of supergravities with FI terms are mostly incomplete for our purposes. The $D$-term inflation model has an important property that in the unstable de Sitter vacuum as well as in the absolute Minkowski vacuum the superpotential vanishes, $W_{\text{min}} = 0$. However, outside the minimum, the superpotential does not vanish, $W \neq 0$. Thus formulations of supergravity [1,9], where the Lagrangian depends not on two functions, the Kähler potential $K(z, z^*)$ and the superpotential $W(z)$, but only on one combination $G(z, z^*) = -K(z, z^*) - \ln |M_P^3 W|^2$ are not suitable\(^4\) since they are not well defined at $W = 0$.

\(^4\)In [9] there is a short “Note added” how to treat the case with vanishing superpotential.
The superspace approach with a non-singular dependence on the superpotential \( W \) presented in [2, 8] has all terms depending on constant FI. However, the holomorphic kinetic function \( f_{\alpha\beta}(z) \) for the vector multiplets is the simplest one, equal to 1. On the other hand, in [3] where there is an arbitrary scalar dependent \( f_{\alpha\beta}(z) \), the constant FI terms are not introduced. The significance of a generic, scalar dependent \( f_{\alpha\beta}(z) \) has to do with axial coupling \( aFF^* \) which sometimes plays an important role in the mechanism of anomaly cancellation.

In section 2 we give a summary of the ingredients of the construction of the supergravity action with superconformal symmetry. For our purpose it is most useful to study the formulation of supergravity with the superconformal origin which was recently constructed in [10]. It has all 3 generic functions, the Kähler potential \( K(z, z^*) \), the superpotential \( W(z) \), and the kinetic function \( f_{\alpha\beta}(z) \) for the vector multiplets and the theory is regular at \( W = 0 \). One furthermore has to define the symmetry transformations. This includes for any \( U(1) \) factor the possible occurrence of a FI constant \( \xi_{\alpha i} \).

In the superconformal approach, one constructs in a first step the action with full superconformal symmetry. It contains an extra chiral multiplet, which was often called ‘compensating multiplet’, but was baptized ‘conformon’ in [10] to reflect its significance. In the next step, the gauge symmetries that are not present in Poincaré supergravity, such as local dilations, local chiral \( U(1) \)-symmetry and local \( S \)-supersymmetry, are gauge fixed. This is discussed in the beginning of section 3.

The formulation of the theory in [10] has several advantages. For example, it simultaneously incorporates two different formulations of phenomenological supergravity depending on the gauge fixing of the local chiral \( U(1) \)-symmetry. The first formulation, in a Kähler-covariant gauge, which is more standard, corresponds to [1]. The other one, in a new gauge where the conformon is real, is closer to [2, 3], and has a non-singular dependence on the superpotential \( W \).

The new formulation [10] allowed to give a detailed explanation of the superconformal origin of FI-terms by including gauge transformations of the conformon field as first suggested in [9]. The conformon field \( Y \) is one of the extra superfields of the superconformal version of the theory, which gets fixed to remove the local dilatation and local chiral \( U(1) \)-symmetry. However, at the superconformal level before the gauge fixing such a field may participate in gauge transformations, which turn out to provide the FI terms:

\[
\delta_\alpha Y = i \frac{g\xi_\alpha}{3M_P^2} Y, \quad \delta_\alpha z_i = \eta_{\alpha i}(z), \quad (1.1)
\]

where \( g \) is the gauge coupling constant. When \( \xi_\alpha \) are some real constant terms in some of the

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\(^5\)In the context of superconformal \( SU(2,2|1) \) symmetry one often calls the superconformal chiral \( U(1) \) symmetry “\( R \)-symmetry”, since it rotates the supercharges (transforms the gravitinos). However, in the Poincaré supergravity, after the superconformal \( U(1) \) symmetry is fixed, there are sometimes other \( U(1) \) symmetries which are combinations of the superconformal \( U(1) \) symmetry and some additional \( U(1) \) gauge symmetries which were present in the superconformal theory. These \( U(1) \) symmetries one also calls \( R \)-symmetries. In what follows, we will use the term “local \( R \)-symmetries” in the context of Poincaré supergravity only.
U(1)’s, they turned out to be constant FI terms $\xi_\alpha$ in the related $U(1)$ in the supergravity theory. All corrections to the supergravity action in such a case can be deduced from the original superconformal action.

The scaling of fields that allows a suitable rigid limit is discussed in section 3.1. It allows us to present the action and transformation rules of supergravity with chiral and vector multiplets in a simple form in section 3.2. The different contributions to $R$-symmetry and $D$-terms and the implications for the superpotential are collected in section 3.3. There the difference between field-dependent and constant FI terms is clearly exhibited, but also it is shown how terms can be reinterpreted by performing Kähler transformations. The final part of section 3 shows how effective constant FI terms may result from field-dependent FI terms by replacing a chiral multiplet by its constant value without breaking local supersymmetry. This procedure is obvious in rigid supersymmetry, but cannot be done in supergravity in general. We treat in section 3.4 a case with a Kähler potential that splits in two parts.

Section 4 shows how the $D$-term inflation is modified by this connection to $R$-symmetry. We present here corrections to the action proportional to $g\xi/M_P^2$, which are required for consistency of the most general $N = 1$ supergravity with FI terms. We give an explicit example of such corrections for the supergravity theory describing $D$-term inflation [13, 14]. We also show that such corrections vanish in the limit of rigid supersymmetry. Such corrections to the $D$-term inflation model have not been exposed so far. Therefore, we will revisit this model and present a corrected form of it as an example of the general supergravity with constant FI terms.

The supergravity theory with constant FI terms has recently been shown to have $D$-term string solutions with unbroken supersymmetry in [4]. A short summary of the $D$-term string solution is presented in section 5.1. In section 5.2 we study these solutions and their zero modes in an extended model in which the $D$-term string is coupled to an arbitrary number of chiral superfields. We present the fermionic zero modes coming from these superfields and verify explicitly, as well as deduce from the superalgebra that the bosonic ones are absent, despite the unbroken supersymmetry.

Then, we turn to field-dependent FI terms in section 6. We revisit the issues of the $D$-term inflation model for the case of string theory inspired anomalous FI terms. We discuss the relation with the anomaly cancellation by the Green-Schwarz mechanism.

We show that the true derivation of such cosmological models from string theory requires to find a stabilization mechanism for the dilaton and/or volume moduli. We describe some preliminary efforts in this direction existing in the literature, which may eventually lead to a stringy version of $D$-term inflation. Furthermore we discuss the scales of $F$ and $D$-terms.

The cosmological implications of $D$-term strings for D-brane systems and D-brane inflation are discussed in section 7. Especially the stability is discussed first in the supergravity formulation (section 7.1). Then the relation of the instabilities of type $II$ D-strings and the $D$-term strings in supergravity is exhibited in section 7.2.

The connection of the supergravity $D$-term description to the D-brane-anti-brane con-
2 Short overview of the local superconformal action

We start in [10] with the $SU(2,2\mid1)$-invariant action for $n + 1$ chiral multiplets $X_I$ and $(X_I)^\ast \equiv X^I$ and some number of Yang–Mills vector multiplets $\lambda^a$ superconformally coupled to supergravity. The supergravity is represented by a Weyl multiplet consisting of a vielbein, a gravitino and a gauge field for $U(1)$ gauge symmetry, gauged $R$-symmetry. The action consists of three parts, each of them being separately conformally invariant.

$$
\mathcal{L} = [\mathcal{N}(X, X^*)]_D + [\mathcal{W}(X)]_F + \left[f_{\alpha\beta}(X)\bar{\lambda}_I^\alpha \lambda_I^\beta\right]_F. \tag{2.1}
$$

The superconformal action has a number of extra gauge symmetries, by comparison with ordinary supergravity. The function $\mathcal{N}(X, X^*)$ is an homogeneous function of degree one in $X$ and $X^*$. Upon gauge-fixing of extra gauge symmetries it will be related to the Kähler potential. The holomorphic function $\mathcal{W}(X)$ encodes the superpotential. The holomorphic function $f_{\alpha\beta}(X)$ encodes the kinetic terms for the vector multiplet fields.

The extra symmetries include local dilatation, local $R$-symmetry and $S$-supersymmetry. In particular, fixing local dilatation removes one of the chiral scalars so that in supergravity there are only $n$ of them. Fixing special supersymmetry removes an extra fermion field. One first performs a change of variables [10] of the $n + 1$ variables $X_I$ to $Y$, which will be the conformon scalar, and $n$ physical scalars $z_i$, which are hermitian coordinates for parametrizing the Kähler manifold in the Poincaré theory. One defines

$$
X_I = Y x_I(z_i). \tag{2.2}
$$

Here $z_i$, $(z_i)^\ast \equiv z^i$ are $n$ chiral superfields of ordinary supergravity and $Y$ is the so-called conformon superfield. The Kähler potential and metric in supergravity are related to the original superconformal structures as follows:

$$
\mathcal{K}(z, z^*) = -3 \ln \left[-\frac{1}{3}\mathcal{N}'/(YY^*)\right],
$$

$$
g_{ij} \equiv \partial^i \partial^j \mathcal{K} = -3(\partial^i X_I)(\partial^j X^J)\partial^I \partial^J \ln \mathcal{N}. \tag{2.3}
$$

In addition to local $SU(2,2\mid1)$ symmetry, the action may have some Yang-Mills gauge symmetries when there are Killing symmetries on the scalar manifold:

$$
\delta_\alpha X_I = k_{\alpha I}(X), \quad \delta_\alpha X^I = k^I_\alpha(X^*). \tag{2.4}
$$
Here the Killing vectors \( k_{aI}(X) \) are holomorphic functions and their commutators define the structure constants. There are also corresponding transformations for the vector fields \( W_{\mu}^{\alpha} \) and fermionic fields. These Yang-Mills gauge symmetries commute with local superconformal symmetries. The functions \( \mathcal{N}, \mathcal{W} \) should be invariant and \( f_{\alpha\beta} \) covariant under the Yang-Mills gauge symmetries, e.g.

\[
\mathcal{N}^{I} \partial_{aI} + \mathcal{N}_{aI} = \mathcal{W}^{I} \partial_{aI} = 0 .
\]

We use here and below the notation where \( \partial_{aI} = \partial X_{aI} \), and similar for \( \partial_{aI} \) and \( \mathcal{W}_{aI} \).

These symmetries of the superconformal action become a main focus of our attention in the present study of FI terms in supergravity. One finds that

\[
k_{aI} = Y \left[ r_{a}(z) x_{I}(z) + \eta_{aI}(z) \partial x_{I}(z) \right] ,
\]

where the Yang-Mills transformations of all chiral superfields in the superconformal action are

\[
\delta_{a} Y = Y r_{a}(z) , \quad \delta_{a} z_{i} = \eta_{ai}(z) ,
\]

where \( r_{a}(z) \) and \( \eta_{aI}(z) \) are \( n + 1 \) holomorphic functions for every symmetry.

The Yang–Mills gauge transformations of the scalars, which may also act on the conformon multiplet when \( r_{a}(z) \neq 0 \), are the Killing isometries that are gauged.

The invariance of \( \mathcal{N} \) as written in (2.5) leads to

\[
0 = \mathcal{N}^{I} \partial_{aI} + \mathcal{N}_{aI} = \mathcal{N} \left[ r_{a}(z) + r_{a}(z^{*}) - \frac{1}{3} \left( \eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) + \eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) \right) \right] .
\]

Thus \( r_{a}(z) \) describes the non-invariance of the Kähler potential:

\[
\delta_{a} \mathcal{K} = \eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) + \eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) = 3(r_{a}(z) + r_{a}(z^{*})) .
\]

Imaginary constants in \( r_{a} \) do not show up here. In the special case that the transformation of the conformon superfield \( Y \) is given by imaginary constants (this is consistent with the YM algebra for \( U(1) \) factors)

\[
r_{a} = i \frac{g_{\alpha} \xi_{a}}{3M_{P}^{2}} , \quad \partial_{I} \xi_{a} = 0 ,
\]

one finds that the Kähler potential is invariant,

\[
\delta_{a} \mathcal{K} = 0 .
\]

The vector multiplets have the auxiliary field \( D^{\alpha} \), whose value is given by

\[
D^{\alpha} = \left( \text{Re} f_{\beta\gamma} \right)^{-1} \mathcal{P}_{\beta} + \text{fermionic terms} ,
\]

\[
\mathcal{P}_{a}(z, z^{*}) = \frac{i}{2} M_{P}^{2} \left[ \left( \eta_{aI}(z) \partial^{I} \mathcal{K}(z, z^{*}) - \eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) - 3r_{a}(z) + 3r_{a}(z^{*}) \right) \right] = i M_{P}^{2} \left( -\eta_{aI} \partial^{I} \mathcal{K}(z, z^{*}) + 3r_{a}(z^{*}) \right) .
\]
These real functions $P_\alpha(z, z^*)$, called Killing potentials, encode the Yang-Mills transformations. Indeed, their derivatives determine the Killing vectors:

$$\partial_i P_\alpha(z, z^*) = i M_P^2 \eta_{aj} g^j_i.$$  (2.13)

In the case where $r_\alpha = i g \xi_\alpha / (3 M_P^2)$,

$$P_\alpha(z, z^*) = -i M_P^2 \eta_{i}^a \partial_i K(z, z^*) + g \xi_\alpha = i M_P^2 \eta_{ai} \partial^i K(z, z^*) + g \xi_\alpha.$$  (2.14)

The properties of $W$ under chiral and dilatational symmetries imply that it is of the form

$$W = Y^3 M_P^{-3} W(z).$$  (2.15)

Invariance of $W$ under YM transformations,

$$\delta_\alpha W = 0,$$  (2.16)

requires that

$$\eta_{ai} \partial^i W = -3 r_\alpha W.$$  (2.17)

Thus, one finds that the superpotential $W$ cannot be Yang-Mills invariant (which would correspond to $\eta_{ai} \partial^i W = 0$) in all models where the conformon multiplet transforms under gauge transformations. In particular, when $r_\alpha = i g \xi_\alpha / (3 M_P^2)$, which will be shown later to correspond to a constant FI term in supergravity $\xi_\alpha$, we find

$$\eta_{ai} \partial^i W = -i \frac{g \xi_\alpha}{M_P^2} W.$$  (2.18)

## 3 From superconformal theory to supergravity with FI

The gauge fixing of the local dilatational invariance introduces the mass scale $M_P$ which was absent in the superconformal theory.\footnote{We use $M_P \equiv M_{\text{Planck}} / \sqrt{8\pi} \sim 2 \times 10^{18}$ GeV.} It also fixes $|Y|$ in terms of a Kähler potential depending only on the physical scalars $z, z^*$,

$$YY^* \exp \left( -\frac{1}{3} K(z, z^*) \right) = M_P^2 = -\frac{1}{3} N.$$  (3.1)

The Kähler invariance has its origin in the non-uniqueness of the splitting (2.2). This creates an invariance under a redefinition

$$Y' = Y e^{\frac{1}{3} \Lambda_Y(z)}, \quad x'_I = x_I e^{-\frac{1}{3} \Lambda_Y(z)}$$  (3.2)

for an arbitrary holomorphic function $\Lambda_Y(z)$. This redefinition changes the Kähler potential to

$$K' = K + \Lambda_Y(z) + \Lambda_Y^*(z^*).$$  (3.3)
The Kähler transformations act therefore on the superpotential $W$ as

$$W' = W e^{-N_Y(z)},$$

leaving $W$ invariant. The $U(1)$ invariance can be fixed in case of an always non-vanishing gravitino mass by choosing

$$\text{Kähler symmetric } U(1) \text{- gauge : } W = W^*.$$  \hspace{1cm} (3.5)

This choice of gauge fixing of the chiral $U(1)$ leads to the action of phenomenological $N = 1$ supergravity as given in [1]. This gauge makes sense only for $W \neq 0$, as for $W = 0$ the condition is empty. An alternative gauge was suggested in [10] for the theories where $W = 0$ at some points in field space

$$\text{non-singular at } W = 0 \text{ } U(1) \text{- gauge : } Y = Y^*.$$  \hspace{1cm} (3.6)

In this gauge for the $U(1)$ symmetry, the theory is non-invariant under the Kähler transformations (3.2). This implies that the remaining invariance is a combination of chiral $U(1)$ and Kähler transformations. The action in this form will be closer to the action of the phenomenological N=1 supergravity as given in [2, 3], where it was derived by superspace methods.

We will now give the action, including the fermions. First we fix the notation. The gravitinos are usually written as Majorana spinors $\psi_\mu$, but sometimes it is convenient to split them into their complex chiral parts,

$$\psi_{\mu L} = \frac{1}{2}(1 + \gamma_5)\psi_\mu, \quad \psi_{\mu R} = \frac{1}{2}(1 - \gamma_5)\psi_\mu.$$  \hspace{1cm} (3.7)

The same notation applies to the gauginos $\lambda^a$. The spinors of the chiral multiplets are always denoted by their chiral parts. We use the position of the index $i$ to indicate the chirality, with

$$\chi_i = \frac{1}{2}(1 + \gamma_5)\chi_i, \quad \chi^i = \frac{1}{2}(1 - \gamma_5)\chi^i.$$  \hspace{1cm} (3.8)

The action can be written as

$$e^{-1} \mathcal{L} = -\frac{1}{2}M_P^2 \left[ R + \bar{\psi}_\mu \gamma^{\mu \rho \sigma} D_{\rho} \psi_\sigma \right] - g_2 \left[ M_P^2 (\partial_\mu z^i) (\partial_\nu z^j) + \bar{\chi}_i \mathcal{D}^i + \bar{\chi}^i \mathcal{D} \chi^i \right]$$

$$+ \frac{1}{4} \left( \text{Re } f_{\alpha \beta} \right) \left[ - \frac{1}{4} F_\mu^a F^\mu_{\nu \beta} - \frac{1}{2} \bar{\lambda}^\alpha \partial_\mu \lambda^\alpha \right] + \frac{i}{4} \left( \text{Im } f_{\alpha \beta} \right) \left[ F_\mu^a \tilde{F}^\mu_{\nu \beta} - \partial_\mu \left( \bar{\lambda}^\alpha \gamma_5 \gamma^\mu \lambda^\beta \right) \right]$$

$$- M_P^{-2} \mathcal{K} \left[ -3WW^* + (D^i W) g^{-1} i_j (D_j W^*) \right] - \frac{1}{2} (\text{Re } f)^{-1 \alpha \beta} \mathcal{P}_\alpha \mathcal{P}_\beta$$

$$+ \frac{1}{8} (\text{Re } f_{\alpha \beta}) \bar{\psi}_\mu \gamma^{\mu \rho \sigma} \left( \bar{F}_\rho^a + \tilde{F}_\rho^a \right) \gamma^\rho \lambda^\beta$$

$$+ \left\{ M_P g_{ij} \bar{\psi}_{\mu L} (\partial_\mu z^j) \gamma^\mu \chi_i + \bar{\psi}_R \cdot \gamma \left[ \frac{1}{2} \lambda_j^L \mathcal{P}_\alpha + \chi_i \mathcal{K}/2 M_P^{-1} D^i W \right] \right\}$$

$$+ \frac{1}{2} \bar{\psi}_\mu L \gamma^{\mu \rho \sigma} \psi_\rho - \frac{1}{4} M_P^{-1} f_{\alpha \beta} \bar{\chi}_i \gamma^\mu \tilde{F}_{\mu \nu}^\alpha \lambda_L$$

$$- e^{\mathcal{K}/2} M_P^{-2} (D^i D^j W) \bar{\chi}_i \chi_j + \frac{1}{2} (\text{Re } f)^{-1 \alpha \beta} \mathcal{P}_\alpha M_P^{-1} f_{\beta \gamma} \bar{\chi}_i \lambda^\gamma - 2 M_P g_2 \xi^i \bar{\chi}_i \lambda^j$$

$$+ \frac{1}{2} M_P^{-2} e^{\mathcal{K}/2} (D^j W) (g^{-1})_j f_{\alpha \beta \gamma} \bar{\lambda}_R \lambda^\gamma + \text{h.c.} \right\} + \text{4-fermion terms},$$  \hspace{1cm} (3.9)
where

\[ \hat{F}^\alpha_{\mu
u} = F^\alpha_{\mu
u} + \bar{\psi}^{[\mu} \gamma_{\nu]} \chi^\alpha, \quad F^\alpha_{\mu
u} = 2 \partial_{[\mu} W^\alpha_{\nu]} + W^\beta_{\mu} W^\gamma_{\nu} \tilde{f}^\alpha_{\beta\gamma}, \]

\[ \hat{F}^\alpha_{\mu\nu} = \frac{1}{2} e^{-1} \varepsilon^{\mu
u\rho\sigma} \hat{F}^\alpha_{\rho\sigma}, \]

\[ \hat{F}^\alpha_{\mu\nu} = \frac{1}{2} \left( \hat{F}^\alpha_{\mu
u} - \hat{F}^\alpha_{\nu\mu} \right) = F^\alpha_{\mu
u} - \frac{1}{4} \tilde{\psi}^{[\mu} \gamma_{\nu]} \gamma^\rho \lambda^\alpha_R + \frac{1}{4} \tilde{\psi}^{[\mu} \cdot \lambda^\rho \gamma_{\nu]} \lambda^\alpha_R; \]

\[ \mathcal{D}^i W = \partial^i W + \left( \partial^i \mathcal{K} \right) W, \]

\[ \mathcal{D}^i \mathcal{D}^j W = \partial^i \mathcal{D}^j W + \left( \partial^i \mathcal{K} \right) \mathcal{D}^j W - \Gamma^{ij}_{k} \mathcal{D}^k W, \quad \Gamma^{ij}_{k} = (g^{-1})^i_{\ell} \partial^j \hat{g}^\ell_{k}. \quad (3.10) \]

We have skipped the 4-fermion terms here, referring the reader to eq. (5.15) in [10]. These terms will not be affected by the presence of constant FI terms. Here \( g_{\ell}^j \) is the Kähler metric, see [2, 3]. The covariant derivative of \( z \) is

\[ \hat{\partial}_\mu z_i = \partial_\mu z_i - W^\alpha_{\mu} \eta_{i\alpha}. \quad (3.11) \]

The covariant derivatives on gaugino and gravitino are

\[ \mathcal{D}_\mu \lambda^\alpha = \left( \partial_\mu + \frac{1}{4} \omega^{ab}_\mu (e) \gamma_{ab} + \frac{1}{2} i A^B_{\mu} \gamma_5 \right) \lambda^\alpha - W^\alpha_{\mu} \lambda^\beta \tilde{f}^\alpha_{\beta\gamma}, \]

\[ \mathcal{D}_{[\mu} \psi_{\nu]} = \left( \partial_{[\mu} + \frac{1}{4} \omega^{ab}_{\mu} (e) \gamma_{ab} + \frac{1}{2} i A^B_{\mu} \gamma_5 \right) \psi_{\nu]} . \quad (3.12) \]

where the \( U(1) \) connection \( A^B_{\mu} \) in our gauge \( Y = Y^* \) is given by

\[ A^B_{\mu} = \frac{1}{2} \left[ (\partial_\mu \mathcal{K}) \partial_\mu z^i - (\partial^i \mathcal{K}) \partial_\mu z_i \right] + \frac{1}{M^2} W^\alpha_{\mu} \mathcal{P}_\alpha. \quad (3.13) \]

In the superconformal theory, this field was the gauge field of the \( U(1) \) \( R \)-symmetry of the superconformal algebra, but here it is simply an auxiliary field. The covariant derivative on the fermions of the chiral multiplets \( \chi_i \) is

\[ \mathcal{D}_\mu \chi_i = \left( \partial_\mu + \frac{1}{4} \omega^{ab}_\mu (e) \gamma_{ab} - \frac{1}{2} i A^B_{\mu} \gamma_5 \right) \chi_i + \Gamma^{jk}_{i} \chi_j \hat{\partial}_\mu z_k - W^\alpha_{\mu} \left( \partial^i \eta_{i\alpha} \right) \chi_j , \quad (3.14) \]

where the Kähler connection \( \mathcal{D}^{3,10} \) has been used.

The action \( \mathcal{S} \) is invariant under the local Poincaré group and \( Q \)-supersymmetry, which are standard local symmetries of supergravity. In addition, there are also some gauge symmetries with gauge fields \( W^\alpha_{\mu} \). We concentrate further on \( U(1) \) factors. Under these symmetries the gravitino, gaugino and chiral fermions are charged when the constant FI terms \( \xi_\alpha \) are present in \( \mathcal{P}_\alpha \).

Thus there are quite a few places where FI terms appear. They obviously appear in the potential and in covariant derivatives. Less obviously, they also appear via \( \eta_{i\alpha} \) since a particular combination of Killing vectors must satisfy the condition \( \mathcal{D}_\mu \eta_{i\alpha} = 0 \) that the superpotential \( W \) transforms under \( U(1) \)'s.

In the usual context of Kähler geometry, Killing potentials are determined up to constants (see e.g. [2], appendix D), since only the differential equation of the type shown in [2, 13] is
available. The superconformal approach and that of [2] have a different starting point. In [2] the arbitrary constants in the solution of the differential equation (2.13) are the cause of the FI terms. In [10], as first recognized in [9], the gauge transformations of the conformon multiplet, encoded in \( r_\alpha(z) \), are responsible for the FI terms. These gauge transformations then induce constants in \( P_\alpha \).

First observe that \( r_\alpha \neq 0 \) signals the mixture of chiral transformations and gauge transformations with index \( \alpha \). Indeed, after fixing the modulus of \( Y \) by some gauge choice, the remaining invariance is the linear combination of gauge transformations that leaves \( Y \) invariant, and this depends on \( r_\alpha \). Another way to see this is that the gravitino field couples to \( A^B_\mu \).

A short summary of the \( R \)-symmetry charges in supergravity with constant FI terms is the following. For gravitino and gaugino we have the axial coupling with some vector fields \( W^\alpha_\mu \), and the couplings are proportional to \( G_\alpha \equiv \xi_\alpha/(2M_P^2) \):

\[
D^\mu \lambda^\alpha = \left( \partial^\mu + igG_\beta W^\beta_\mu \gamma_5 + \cdots \right) \lambda^\alpha ,
\]

\[
D^\mu \psi_{ij} = \left( \partial^\mu + igG_\beta W^\beta_\mu \gamma_5 + \cdots \right) \psi_{ij} ,
\]

\[
D^\mu \chi_i = \left( \partial^\mu - igG_\beta W^\beta_\mu + \cdots \right) \chi_i - W^\alpha_\mu \eta_{\alpha j} k^j_i \chi_j - W^\alpha_\mu \left( \partial^j \eta_{\alpha j} \right) \chi_j ,
\]

(3.16)

For the scalars in the chiral multiplet and for the superpotential we have

\[
\dot{\partial} \mu z_i = \partial \mu z_i - W^\alpha_\mu \eta_{\alpha i} ,
\]

\[
\eta_{\alpha i} \partial W = -2igG_\alpha W .
\]

(3.17)

### 3.1 Supergravity and a rigid limit

The limit from a supergravity theory to a supersymmetry theory is not always obvious. In many versions of supergravity a notation \( M_P = 1 \) was used which does not make such limit easy. The purpose of this subsection is to show how to parametrize the scalar fields in a way that the rigid limit can be taken easily. For \( N = 1 \), the following procedure has been proposed in [10] for the simplest situation in which all scalar fields appear in the rigid limit.

A convenient choice is to expand around the point \( z = z^0 \) and take the special coordinates

\[
z_i = z^0_i + M_P^{-1} \phi_i ,
\]

(3.18)

with in (2.2)

\[
x_0 = 1 , \quad x_i = M_P^{-1} \phi_i
\]

(3.19)

and the Kähler potential in a form

\[
K = M_P^{-2} K(\phi, \phi^*, M_P^{-1}) ,
\]

(3.20)
where $K(\phi, \phi^*, M_P^{-1})$ is regular at $M_P^{-1} = 0$. Note that the Kähler metric is

$$g^i_j = \frac{\partial}{\partial z^i} \frac{\partial}{\partial z^j} K = \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi^j} K,$$

(3.21)
i.e., it does not change under the reparametrizations. Therefore the kinetic term for the scalars in (3.9) will lose its dependence on $M_P^{-1}$ by the reparametrization as chosen in (3.18) which was the motivation for the proportionality factor $M_P^{-1}$ in (3.18). It follows that the fields $\phi_i$ have again the same dimension as the conformal fields $X_I$. One finds that

$$Y = M_P \exp\left[K/(6 M_P^2)\right], \quad X_0 = Y = M_P + O(M_P^{-1}), \quad X_i = Y \phi_i = \phi_i \exp[K/(6 M_P^2)] = \phi_i + O(M_P^{-2}).$$

(3.22)

So the fields $\phi_i$ are in lowest order of $M_P^{-1}$ equal to the conformal fields that we started from. The same happens with the conformal fermions.

From now on, we will thus use the fields $\phi_i$ and complex conjugates $\phi^i$ rather than the $z_i$ and $\bar{z}^i$ to indicate the scalar fields. Therefore, derivatives $\partial^i$ will stand for derivatives w.r.t. $\phi_i$ rather than w.r.t. $z_i$. The difference is thus a factor $M_P$. E.g., from now on,

$$f_{i\beta} = \frac{\partial f_{\alpha \beta}}{\partial \phi^i} = M_P^{-1} \frac{\partial f_{\alpha \beta}}{\partial z_i}.$$ 

Thus to use the equations of the previous section we have to replace

$$\partial^i \rightarrow M_P \partial^i, \quad f_{i\beta} \rightarrow M_P f_{i\beta}, \quad D^i \rightarrow M_P D^i.$$ 

(3.23)

One checks also that

$$\eta^i = \delta_{\alpha i} \phi_i = M_P^{-1} \delta_{\alpha i}, \quad \frac{\partial}{\partial z_i} K = \frac{\partial}{\partial \phi^i} K.$$ 

(3.24)

One can also check that $P_\alpha$ in (2.12) has a finite rigid limit

$$P_\alpha = i(M_P \eta^i) \partial^i K - 3i(M_P^2 r_\alpha),$$ 

(3.25)

under the condition that

$$r_\alpha(z) = M_P^{-2} \tilde{r}_\alpha(\phi, M_P^{-1}),$$ 

(3.26)

where $\tilde{r}_\alpha(\phi, M_P^{-1})$ is regular at $M_P^{-1} = 0$.

In the limit of rigid symmetries when $M_P \rightarrow \infty$ and the FI terms $\xi_\alpha$ stay fixed, the axial coupling $G_\alpha = \xi_\alpha/(2 M_P^2)$ vanishes. The only $\xi_\alpha$-dependent term which survives this limit is the term in the potential $\frac{1}{2} g^2 \xi^2$. Thus the case of supersymmetric gauge theories, which allows any number of $U(1)$ groups with FI terms, is obtained in the proper limit from supergravity.

Consider a simple example with (3.19), a trivial Kähler metric and a single $U(1)$ group ($\alpha$ takes only one value, and we write therefore $\xi$ for $\xi_\alpha$), under which the scalars have a charge $q_i$:

$$K = \phi_i \phi^i, \quad f_{i\beta} = \delta_{i\beta}, \quad \delta_{\alpha i} \phi_i = ig q_i \phi_i, \quad \eta_{ai} = ig M_P^{-1} q_i \phi_i,$$

$$r_\alpha = \frac{1}{3} ig \xi M_P^2 = \frac{2}{3} ig G, \quad G = \frac{1}{2} \xi M_P^{-2}.$$ 

(3.27)
Then the superpotential $W$ has to be gauge-invariant, and with (2.15),

$$
\delta W = i \sum_{i=1}^{n} g q_i \phi_i \partial^i W(\phi) = -i \frac{g \xi}{M_P^2} W(\phi) = -2i g GW(\phi).
$$

(3.28)

If this is the case, the action is invariant and we find

$$
P = g \left( \xi - \sum_{i=1}^{n} q_i \phi_i \phi_i \right).
$$

(3.29)

The potential gets a contribution $\frac{1}{2} P^2$, which yields the Fayet–Iliopoulos cosmological constant $\frac{1}{2} g^2 \xi^2$.

An important constraint on the combination of charges which appears in a superpotential of the form $W = \lambda \phi_1 \cdots \phi_n$ is

$$
\sum_{i=1}^{n} q_i = -\frac{\xi}{M_P^2} = -2G.
$$

(3.30)

This property is in agreement with potentials in gauge theory being invariant under gauge transformations since in the limit $M_P \to \infty$ the sum of charges vanishes. However, in supergravity the sum of charges cannot vanish, which will lead to a particular correction in the supergravity potential for $D$-term inflation.

### 3.2 Simplified action

In this section we are trying to present the rules for the most general case of $N = 1$ supergravity with constant FI terms and make this section completely self-contained. For all practical purposes, if one is not interested in the reasons behind the rules, one should find here all information for the generic case.

The supergravity action is defined by $W(\phi)$, $K(\phi, \phi^*)$ and $f_{\alpha\beta}(\phi)$ as usual. Some of the $U(1)$ gauge groups may contain constant FI terms $\xi_\alpha$. In this section we are considering a simple case when in the superconformal theory $\tilde{r}_\alpha(\phi) = ig \xi_\alpha/3$. The bosonic part of the action is

$$
e^{-1} \mathcal{L}_{\text{bos}} = -\frac{1}{2} M_P^2 R - g_{ij} (\hat{\partial}_\mu \phi^i)(\hat{\partial}^\mu \phi^j) - V
\begin{align*}
&\quad -\frac{1}{4} (\text{Re} f_{\alpha\beta}) F^\alpha_{\mu\nu} F^{\mu\nu\beta} + \frac{1}{4} (\text{Im} f_{\alpha\beta}) F^\alpha_{\mu\nu} F^{\mu\nu\beta}.
\end{align*}
$$

(3.31)

The potential consists of an $F$-term and a $D$-term:

$$
\begin{align*}
V &= V_F + V_D, \\
V_F &= e^{(K/M_P^2)} \left[ (D^i W)(g^{-1})^j (D^j W^*) - 3 M_P^2 W W^* \right], \\
V_D &= \frac{1}{2} (\text{Re} f_{\alpha\beta}) D^\alpha D^\beta \bigg|_{\text{bos}} = \frac{1}{2} (\text{Re} f)^{-1} \alpha\beta \mathcal{P}_\alpha \mathcal{P}_\beta,
\end{align*}
$$

(3.32)
where

\[
\mathcal{D}^i W = \partial^i W + M_P^2 (\partial^i K) W ,
\]

\[
\mathcal{P}_\alpha (\phi, \phi^*, M_P^{-1}) = i \left[ M_P \eta_\alpha \partial^i K (\phi, \phi^*, M_P^{-1}) - 3 \tilde{r}_\alpha (\phi, M_P^{-1}) \right] = i \left[ - M_P \eta_\alpha^i \partial_i K (\phi, \phi^*, M_P^{-1}) + 3 \tilde{r}_\alpha^i (\phi^*, M_P^{-1}) \right],
\] (3.33)

where \( M_P \eta_\alpha = \delta_\alpha \phi \) and \( M_P \eta_\alpha^i = \delta_\alpha \phi^i \).

Under \( U(1) \) gauge transformations in the directions in which there are FI terms \( \xi_\alpha \), the superpotential must transform as

\[
\delta_\alpha W = - \frac{g_\xi_\alpha}{M_P^2} W (\phi). \] (3.34)

The part of the action quadratic in fermions reads

\[
e^{-1} \mathcal{L}_{\text{fer}} = - \frac{1}{2} M_P^2 \bar{\psi}_\mu \gamma^{\mu \nu} \mathcal{D}_\nu \psi_\sigma + \frac{1}{2} m \bar{\psi}_\mu R \gamma^{\mu \nu} \psi_\nu R + \frac{1}{2} m^* \bar{\psi}_\mu L \gamma^{\mu \nu} \psi_\nu L
\]

\[
- g j \left[ \bar{\chi}_j \mathcal{D}_j \chi^i + \bar{\chi}^i \mathcal{D}_j \chi_j \right] - m^i \bar{\chi}_i \chi_j - m_{ij} \bar{\chi}_i \chi_j + e^{-1} \mathcal{L}_{\text{mix}}
\]

\[
2 m_{ia} \bar{\chi}^i \lambda^a - 2 m_{a} \bar{\chi}_i \lambda^a - m_{R, \alpha \beta} \bar{\lambda}^a_R \lambda^\beta_R - m_{L, \alpha \beta} \bar{\lambda}^a_L \lambda^\beta_L
\]

\[
+ (\text{Re} f_{\alpha \beta}) \left[ - \frac{1}{2} \bar{\lambda}^a \mathcal{D} \lambda^a \right] + \frac{1}{4} (\text{Im} f_{\alpha \beta}) \left[ - \partial_\mu \left( \bar{\lambda}^a \gamma_5 \gamma^\mu \lambda^a \right) \right]
\]

\[
+ \frac{1}{4} \left\{ (\text{Re} f_{\alpha \beta}) \bar{\psi}_\mu \gamma^{\nu \rho} F_{\nu \rho} \gamma^\mu \lambda - \left[ f_{\alpha \beta} \bar{\chi}^i \gamma^{\mu \nu} F_{\mu \nu} \lambda L + \text{h.c.} \right] \right\},
\] (3.35)

where

\[
m = e^{K/2} W,
\] (3.36)

which is related to the (real) gravitino mass, \( m_{3/2} = |m| M_P^{-2} \). Also the following notation are used

\[
m^i \equiv \mathcal{D}^i m = e^{K/2} \mathcal{D}^i W = \partial^i m + \frac{1}{2} (\partial^i K) m , \quad \mathcal{D}_i m = \partial_i m - \frac{1}{2} (\partial_i K) m = 0 ,
\]

\[
m_i \equiv \mathcal{D}_i m^* = e^{K/2} \mathcal{D}_i W^* = \partial_i m^* + \frac{1}{2} (\partial_i K) m^*, \quad \mathcal{D}^i m^* = \partial^i m^* - \frac{1}{2} (\partial^i K) m^* = 0 ,
\] (3.37)

and

\[
m^{ij} = \mathcal{D}^i \mathcal{D}^j m = \left( \partial^i + \frac{1}{2} (\partial^i K) \right) m^j - \Gamma^{ij}_k m^k ,
\]

\[
m_{\alpha a} = - i \left[ \partial_\alpha \mathcal{P}_a - \frac{1}{4} (\text{Re} f) \eta^{ij} \mathcal{P}_\beta f_{\gamma a i} \right] ,
\]

\[
m_{R, \alpha \beta} = - \frac{1}{4} f_{\alpha \beta i} (g^{-1})^{ij}_m m^j.
\] (3.38)

\( \mathcal{L}_{\text{mix}} \) can be written in different ways:

\[
e^{-1} \mathcal{L}_{\text{mix}} = g j \bar{\psi}_\mu L (\partial^i \phi)^j \gamma^\mu \chi_i + \bar{\psi}_R \gamma^1 \psi_L + \text{h.c.},
\]

\[
= 2 g_j \bar{\psi}_\mu R \gamma^\mu \chi^i \partial_\nu \phi_i + \bar{\psi}_R \gamma^1 \psi_L + \text{h.c.},
\] (3.39)
where
\[
\begin{align*}
v_L &= v^1_L + v^2_L, \\
v^1_L &= \frac{1}{2} i \mathcal{P}_\alpha \Lambda_L^\alpha + m^i \chi_i, \\
v^2_L &= (\hat{\partial} \phi_i) \chi^j g^i_j.
\end{align*}
\]
(3.40)

The covariant derivatives on the scalar fields still contain gauge connection, while the one on the fermions \(\chi_i\) contain also Lorentz, gauge and Kähler connections:
\[
\mathcal{D}_\mu \chi_i = \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab}(e) \gamma_{ab} \right) \chi_i - W^\alpha_\mu \chi_j \partial^j \eta_{ai} - \frac{i}{2 M_P^2} W^\alpha_\mu \mathcal{P}_\alpha \chi_i
\]
\[
+ \frac{1}{4} \left[ (\partial_j \mathcal{K}) \partial_\mu \phi^j - (\partial^j \mathcal{K}) \partial_\mu \phi_j \right] \chi_i + \Gamma^j_k \chi^k_\mu \hat{\partial} \phi_j,
\]
(3.41)

where
\[
\partial^j \eta_{ai} = \frac{\partial}{\partial z_j} \delta^\alpha_{zi}, \quad \partial^j \phi_i = \frac{\partial}{\partial \phi_j} \delta_{a\phi_i}.
\]
(3.42)

The parts of the supersymmetry transformation laws of the fermions where they transform to bosons, and boson transformations linear in fermions, are:
\[
\delta e^\alpha_\mu = \frac{1}{2} \tilde{\epsilon} \gamma^\alpha_\mu \psi_i, \quad \delta \phi_i = \tilde{\epsilon}_L \chi_i, \quad \delta W^\alpha_\mu = -\frac{1}{2} \tilde{\epsilon} \gamma^\mu \lambda^\alpha,
\]
\[
\delta \psi_{\mu L} = \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab}(e) \gamma_{ab} + \frac{1}{2} i A^B_\mu \right) \epsilon_L + \frac{1}{2} M_P^2 m^i \epsilon_R,
\]
\[
\delta \chi_i = \frac{1}{2} \hat{\partial} \phi_i \epsilon_R - \frac{1}{2} (g^{-1})^i_j m_j \epsilon_L,
\]
\[
\delta \lambda^\alpha = \frac{1}{4} \gamma^{\mu\nu} F^\alpha_{\mu\nu} \epsilon + \frac{1}{2} \gamma_5 (\text{Re} f)^{-1} \alpha \beta \mathcal{P}_\beta \epsilon.
\]
(3.43)

Here
\[
A^B_\mu = \frac{1}{2 M_P^2} \left[ (\partial_i K) \partial_\mu \phi^i - (\partial^j K) \partial_\mu \phi_j \right] + \frac{1}{M_P^2} W^\alpha_\mu \mathcal{P}_\alpha.
\]
(3.44)

It is instructive to rewrite this expression for the composite gauge field \(A^B_\mu\) in a slightly different form, where we split the \(\mathcal{P}_\alpha\)-term in two parts, as shown in (3.33). The first part turns the derivatives of the fields \(\phi^i\) and \(\phi_i\) into covariant derivatives, the second part contains the FI terms so that in the simple case that \(\tilde{r}_\alpha(\phi) = i g \xi_\alpha / 3\) we find
\[
A^B_\mu = \frac{1}{2 M_P^2} \left[ (\partial_i K) \hat{\partial}_\mu \phi^i - (\partial^j K) \hat{\partial}_\mu \phi_j \right] + \frac{g}{M_P^2} W^\alpha_\mu \xi_\alpha,
\]
(3.45)

where \(\hat{\partial}_\mu \phi^i = \partial_\mu \phi^i - M P \eta_{ai} W^\alpha_\mu \).

### 3.3 Summary on R-symmetry, FI term and superpotential

In summary, there are 4 types of contributions to the local R-symmetry connection:

1. In the superconformal algebra there is a \(U(1)\) that acts as R-symmetry. Its gauge vector is an auxiliary field, i.e. it becomes a composite once the field equations are used. Even when there are no other gauge vectors present, this \(U(1)\) couples to the gravitino. This gives rise to the first terms in (3.13) or (3.44).
2. When we gauge further symmetries by vector multiplets, new terms contribute to the \( R \)-symmetry gauge field. These are the covariantizations of the terms in (1). They contribute to \( \mathcal{P}_\alpha \) as the first two terms of the second line in (2.12). These will be important for section 6. Their vacuum expectation value will be different from zero only if a field 'shifts' under the symmetry, i.e. the vacuum expectation value (vev) of the transformation law is non-zero.

3. If the symmetries gauged by vector multiplets do not leave the Kähler potential invariant, but give rise to a Kähler transformation as in (2.9) depending on the real part of a holomorphic function \( r_\alpha(z) \), then the latter contribute to the connection \( A^B_\mu \) via additions to \( \mathcal{P}_\alpha \). This is the part that does not appear in the ‘simplified’ case in section 3.2.

4. Finally, as the previous part defines \( r_\alpha(z) \) only up to an imaginary constant, one may add a constant, that has been called \( \xi_\alpha \), again contributing to \( \mathcal{P}_\alpha \), and hence to \( A^B_\mu \). This term is shown explicitly\(^8\) in (3.45). It plays a crucial role in establishing an unbroken supersymmetry of \( D \)-term strings, as shown in [4].

These four types of terms contribute to the \( R \)-symmetry \( U(1) \) connection, but as the first type of terms do not contribute to \( \mathcal{P}_\alpha \), only the terms of the types 2–4 contribute to the FI term, which is the vev of \( \mathcal{P}_\alpha^2 \).

Furthermore, the terms of type 2 do not contribute to \( r_\alpha(z) \), while the requirement on the superpotential, (2.17), only depends on \( r_\alpha(z) \). Thus, in case that we have only terms of type 2, the superpotential has to be invariant. A generalization is that both the Kähler potential and superpotential are not invariant, but transform under a simultaneous Kähler transformation. That is case 3.

If the Kähler potential is invariant (‘simplified case’), the requirement on the superpotential only depends on \( \xi_\alpha \) (terms of type 4) as shown in (3.28).

The final gauged \( U(1) \) symmetries are the linear combinations of the \( U(1) \) of the superconformal group and the \( U(1) \) of vector multiplets that preserve the gauge choice (3.6). Thus, if \( r_\alpha(z) = 0 \) the symmetry is preserved without corrections of the superconformal \( U(1) \). If, on the other hand, \( Y \) transforms under a \( U(1) \), i.e. \( r_\alpha(z) \neq 0 \), then the gauge symmetry in the Poincaré theory is a mixing of the gauge symmetry with the superconformal \( U(1) \) and hence the gravitino transforms under this symmetry. Thus, this mixing appears through terms of type 3 and 4.

Note also that the distinction between the different terms is dependent on the Kähler gauge choice. This is easily understood in the conformal framework. Remember that the significance of the value of \( r_\alpha \) is the transformation of the conformon field \( Y \), see (1.1). On the other hand, Kähler transformations are redefinitions of the conformon field by holomorphic functions of the scalars as shown in (3.2). Consider as an example a field \( \Phi \) with \( \delta^\alpha \Phi = 0 \). We

\(^8\)There is a typo in an analogous equation in [2] where the constant FI term is omitted in the gravitino supersymmetry transformations.
can then consider a Kähler transformation $Y' = Y \exp(c \Phi)$ for a constant $c$, which changes the Kähler potential to $K' = K + 3c(\Phi + \Phi^*)$. Then the new conformon transforms under the $U(1)$ as $\delta_\alpha Y' = icY'$, or $r_\alpha = ic$. The change in Kähler potential changes the contributions to $P_\alpha$ of type 2, but this is exactly compensated by the fact that the new $r_\alpha$ means an explicit FI term (type 4) with $g_{\xi}M_P^2 = 3c$. We thus see that the distinction between the terms depends on the Kähler gauge, but this gauge choice then has repercussions on the superpotential.

Briefly summarizing, there are two cases to be distinguished for the superpotential. First is the case with constant FI term, where the superpotential transforms as (2.17). The second case is when an effective FI term is produced as the vev of a field that shifts under $U(1)$. In this case, the superpotential is invariant (case 2 above), unless the symmetry only preserves the Kähler potential up to a Kähler transformation, in which case the superpotential should also have a homogeneous Kähler transformation (case 3). The fact that the shift of a field is promoted into an invariance group of the superpotential, implies important constraints on its form, as discussed in different parts of our paper. The question we wish to briefly address now is the interpolation between the two regimes. In a theory with several chiral fields, one would expect that if we “freeze” the vev of one of the shifting fields, the resulting structure of the remaining theory must be such as in the situation with a constant FI term. For instance, we should be able to define an effective superpotential which starts transforming non-trivially. In the following section, we shall discuss this connection in an explicit example.

### 3.4 Supersymmetrically removing chiral multiplets

Some chiral multiplets may be massive, and it can be interesting to consider the theory for fixed values of their complex scalars (and zero fermions).\(^9\)

We consider here the case that the Kähler potential splits between the heavy fields (denoted by $\rho$) and the light fields (denoted by $S$)

$$K(\rho, S, \rho^*, S^*) = K(\rho)(\rho, \rho^*) + K(S)(S, S^*) .$$

We may have several fields of each kind. We continue here with one light field $S$ and one heavy $\rho$, but the generalization is straightforward. The superpotential should satisfy (2.17), or explicitly, after the rescaling (3.23),

$$\delta_\alpha W = M_P \eta_{\alpha \rho} \partial^\rho W + M_P \eta_{\alpha S} \partial^S W = -3r_\alpha W .$$

The Killing potentials are

$$P_\alpha = iM_P \eta_{\alpha \rho} \partial^\rho K + iM_P \eta_{\alpha S} \partial^S K - 3iM_P^2 r_\alpha ,$$

\(^9\)Freezing the vev’s of the scalars that shift under the $U(1)$ symmetry, and integrating them out is a rather delicate thing. We are interested in the situation when the scalar vev that sets the FI term is stabilized in such a way that the effect of the FI term on the remaining fields is non-vanishing. Such a situation can be achieved only for certain ranges of parameters as we discuss in section 6.
Now we want to remove $S$ in a supersymmetric way. Considering (3.43), this implies that the stabilized value of $\rho$, which we indicate as $\rho_0$ should be such that
\[
m^\rho|_{\rho=\rho_0} = D^\rho W|_{\rho=\rho_0} = 0, \quad \rightarrow \quad \partial^\rho W\bigg|_{\rho=\rho_0} = -M_P^{-2}\partial^\rho K(\rho)\bigg|_{\rho=\rho_0}.
\]
Note that the right-hand side does not depend on $S$. This is the case for superpotentials of the form
\[
W(\rho, S) = w(\rho)w(S)(S).
\]
(If the condition (3.49) would be valid for all $\rho$, then the potential should be of this form). For these superpotentials, the invariance condition (3.47) is
\[
\frac{\delta_\alpha w(\rho)}{w(\rho)}(\rho) + \frac{\delta_\alpha w(S)}{w(S)}(S) = -3r_\alpha(\rho, S).
\]
(3.51)
The condition (3.49) implies that the terms with $\rho$-derivatives in both (3.47) and (3.48) can be omitted if we use instead of $r_\alpha$
\[
r'_\alpha(S) = r_\alpha + \frac{\delta_\alpha w(\rho)}{w(\rho)}(\rho_0) = r_\alpha - M_P^{-1}\eta_{\alpha\rho}\partial^\rho K(\rho)\bigg|_{\rho=\rho_0}.
\]
(3.52)
In the case of (3.50) we thus obtain
\[
r'_\alpha(S) = r_\alpha(\rho_0, S) + \frac{\delta_\alpha w(\rho)}{3w(\rho)}(\rho_0) = -\frac{\delta_\alpha w(S)}{3w(S)}(S).
\]
(3.53)
E.g. if the original Kähler potential and superpotential were invariant, $r_\alpha = 0$ in (3.51), but the two terms of that equation are opposite constants, then the final $r'_\alpha$ is a constant. In the case that the separate Kähler potentials are invariant, this is an imaginary constant, the FI-term. Thus here we see how the field-dependent first term of (3.48) gives rise to a constant FI term,
\[
g_\xi = -iM_P\frac{\delta_\alpha w(\rho)}{3w(\rho)}(\rho_0),
\]
(3.54)
in the effective model at constant value of $\rho$.

We obtain the same potential if we use the new superpotential
\[
W'(S) = We^{K(\rho)/(2M^2)}\bigg|_{\rho=\rho_0},
\]
(3.55)
which for (3.50) is proportional to $w(S)(S)$.

We may also compare with the situation in $N = 2$ supergravity. Then arbitrary FI terms are constant values of the triholomorphic moment maps. Such constant terms are only allowed when there are no physical hypermultiplets. In presence of hypermultiplets, the triholomorphic moment maps are functions of the hyperscalars. Fixing the hypermultiplets to a constant value is then consistent with a constant FI term for the effective special Kähler model of only vector multiplets.

\[^{10}\text{It is here that we use that the metric is diagonal between the left and right fields.}\]
4 D-term inflation revisited

We first discuss the standard D-term inflation model with constant FI terms taking into account the fact that the superpotential transforms as in (3.34). We thus consider the simplest model:

\[ K = |\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2, \]
\[ W = \lambda \phi_0 \phi_+ \phi_-, \]
\[ f_{\alpha\beta} = \delta_{\alpha\beta}, \]

(4.1)

where the fields \( \phi_0, \phi_+, \phi_- \) in the globally supersymmetric theory have charges \( (Q^+ = \pm 1, Q^0 = 0) \). To promote this model to local supersymmetry, i.e. to supergravity with constant FI term, we have to change the charge assignments for the chiral superfields, so that the superpotential transforms under local \( R \)-symmetry. We choose

\[ q_i = Q_i - \rho_i \frac{\xi}{M_P}, \quad \sum_{i=\pm0} \rho_i = 1. \]

(4.2)

The last equation follows from (3.30).

The complete scalar potential reads, following (3.32),

\[ V = V_F + V_D, \]
\[ V_F = e^{(|\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2)/M_P^2} \left\{ \lambda^2 |\phi_+|^2 |\phi_-|^2 \left( 1 + \frac{|\phi_0|^4}{M_P^4} \right) + \lambda^2 |\phi_0|^2 |\phi_-|^2 \left( 1 + \frac{|\phi_+|^4}{M_P^4} \right) \right. \]
\[ + \lambda^2 |\phi_0|^2 |\phi_+|^2 \left( 1 + \frac{|\phi_-|^4}{M_P^4} \right) + 6 \lambda^2 \frac{|\phi_0 \phi_+ \phi_-|^2}{M_P^4} \} \]
\[ V_D = \frac{g^2}{2} \left[ q_0 |\phi_0|^2 - \xi \right]^2 + \sum_{i=\pm} |\phi_i|^2 \left[ (\lambda^2 e^{i|\phi_0|^2/M_P^2} + g^2 q_i q_0) |\phi_0|^2 - g^2 \xi q_i \right] + O(\phi^4). \]

(4.3)

(4.4)

For fixed \( \phi_0 \), we have

\[ V = \frac{g^2}{2} \left[ q_0 |\phi_0|^2 - \xi \right]^2 + \sum_{i=\pm} |\phi_i|^2 \left[ (\lambda^2 e^{i|\phi_0|^2/M_P^2} + g^2 q_i q_0) |\phi_0|^2 - g^2 \xi q_i \right] + O(\phi^4). \]

(4.5)

Hence for \( |\phi_0|^2 > g^2 \xi q_\pm / (\lambda^2 e^{i|\phi_0|^2/M_P^2} + g^2 q_0 q_\pm) \), the minimum is found for \( \phi_\pm = 0 \), direction along which

\[ V(\phi_0, \phi_\pm = 0) = \frac{g^2}{2} \left[ q_0 |\phi_0|^2 - \xi \right]^2. \]

(4.6)

Since \( q_0 = -\rho_0 \xi / M_P^2 \), we conclude that for \( \rho_0 \neq 0 \), the mass of the neutral scalar is given by \( m^2_{\phi_0} \sim g^2 \xi^2 / M_P^4 \sim H^2 \). To recover the properties that motivated D-term inflation we must require \( \rho_0 = 0 \), i.e.

\[ q_0 = 0, \]

(4.7)

in which case the potential has a plateau at \( V_0 = g^2 \xi^2 / 2 \).
We now turn to the evaluation of the one-loop potential along this flat direction. The corresponding scalar and fermion masses are

\[
\begin{align*}
    m_{\phi \pm}^2 &= \lambda^2 e^{2|\phi_0|^2/M_p^2} |\phi_0|^2 - g^2 \xi q_\pm, \\
    m_{\chi \pm}^2 &= \lambda^2 |\phi_0|^2 e^{2|\phi_0|^2/M_p^2}.
\end{align*}
\]  

(4.8)

In fact, the quantities in (3.38) are all zero at \( \phi_\pm = q_0 = 0 \) except for

\[
m^{++} = m^{-+} = \lambda q_0 e^{2|\phi_0|^2/(2M_p^2)}. \tag{4.9}
\]

The contribution to the effective potential is proportional to \( \text{STr} M^4 \ln(M^2/\Lambda^2) \). We neglect the \( \xi q_\pm \) term in the ln factor, and the \( e^{2|\phi_0|^2/M_p^2} \) factor that went with \( \lambda^2 \). Hence

\[
\text{STr} M^4 \ln \frac{M^2}{\Lambda^2} = \left[ -2g^2 \xi^2 |\phi_0|^2 (q_+ + q_-) \right. + g^4 \xi^2 \left. (q_+^2 + q_-^2) \right] \left( \ln \frac{\lambda^2 |\phi_0|^2}{\Lambda^2} + \frac{|\phi_0|^2}{M_p^2} \right), \tag{4.10}
\]

where, following (4.12) and (4.7), \( q_+ + q_- = -\xi/M_p^2 \) and \( q_+^2 + q_-^2 \sim 2 \). Thus

\[
V(\phi_0, \phi_\pm = 0) = V_0 + \frac{1}{64\pi^2} \text{STr} M^4 \ln \frac{M^2}{\Lambda^2} = \frac{g^2}{2} \xi^2 \left[ 1 + \frac{1}{16\pi^2} \left( 1 + \frac{\lambda^2 |\phi_0|^2}{M_p^2} \right) \ln \frac{\lambda^2 |\phi_0|^2}{\Lambda^2} \right]. \tag{4.11}
\]

Next let us discuss the problem of anomalies. The charge of the fermion \( \chi_i \) is the strength of the coupling to \( W_\mu \) in the covariant derivative (5.16). This can be written as

\[
D_\mu \chi_i = \left( \partial_\mu + \frac{i}{2} \omega_\mu^{\ ab} \gamma_5 \right) \chi_i + \frac{1}{4} M_p^{-2} (\phi_j \partial_\mu \phi^j - \phi^i \partial_\mu \phi_j - i g \tilde{q}_\Lambda W_\mu \gamma_5) \chi_i. \tag{4.12}
\]

Hence the charge is

\[
\tilde{q}_i = q_i + G = q_i + \frac{1}{2} \frac{\xi}{M_p^2}. \tag{4.13}
\]

Similarly, (3.15) yields \( \tilde{q}_\lambda = -G \) for the gaugino and \( \tilde{q}_\psi = -G \) for the gravitino.

Then the \( U(1)^3 \) anomaly coefficient \( C \) reads

\[
C = \sum_{i=0, \pm} (q_i + G)^3 + (-G)^3 + 3(-G)^3, \tag{4.14}
\]

where the last two terms are respectively the gaugino and gravitino contributions. Using \( \rho_+ + \rho_- = 1 \), we have \( q_+ + G = 1 + (1 - 2\rho_+) G = 1 - (1 - 2\rho_-) G = -(q_- + G) \) and the anomaly coefficient reduces to the gravitino contribution: \( C = -3G^3 \).

Adding 3 neutral \( q = 0 \), hence \( \tilde{q} = G \) chiral multiplets can cancel these anomalies. These extra fields should not occur in the superpotential \( W \). This gives a generalization of the \( D \)-term inflation model taking into account the terms in supergravity that provide an exact local supersymmetry of the classical action.
In the theories where the gauge anomaly is not cancelled (anomalous U(1)) it is possible to introduce a coupling to the axion $a$ of the type $aFF^\ast$. The shift of the axion field under $U(1)$ may remove the anomaly. However, this requires to introduce a coupling of the form $\text{Re}f(z)F^2 + \text{Im}f(z)FF^\ast$ and stabilize the additional field $z$. We will discuss this case in section 6. But here, since we have a cancellation of all $FF^\ast$, the $D$-term inflation model with constant FI term, supplemented by 3 neutral chiral multiplets, is valid in the original version with constant kinetic function for the vector multiplet.

For the gravitational anomaly we find:

$$C_g = \sum_{i=0,\pm} (q_i + G) + (-G) - 21(-G) + 3G = 24G. \quad (4.15)$$

The 3 extra chiral multiplets introduced to cancel the gauge anomalies cannot cancel the gravitational ones. The relevant terms of the form $RR^\ast$ are higher-derivative terms which are not present in the classical supergravity action. One can think that the anomaly terms of this kind should be taken care in the context of other higher-derivative terms in the action and most likely in the context of the full M/string theory.

Our observation about neutral chiral multiplets cancelling the gauge anomaly of gaugino and gravitino can be applied to the Freedman model [5]. When it is supplemented by 4 neutral chiral multiplets, they cancel the $FF^\ast$ anomaly.

5 \quad D-term strings

In theories with FI $D$-term inflation, the $U(1)$ symmetry gets spontaneously broken. This breaking in general can result into the formation of the cosmic strings, as explained in [4], where these were referred to as $D$-term strings. It was shown that the $D$-term strings are (the only) BPS-saturated strings in $N = 1, 4d$ supergravity, and therefore are the natural candidates for the low energy description of $D$-brane strings. We shall postpone exploration of this connection to sections 7-8 and here we will study some unusual properties of these objects within $4d$ supergravity.

5.1 The string configuration

The string configuration can be obtained from one vector multiplet with minimal kinetic term, and one chiral multiplet, charged under the $U(1)$ of the former with charge $q = 1$ and with minimal Kähler potential $K = \phi \phi^\ast$. The solution that we consider is purely bosonic. The scalar of the chiral multiplet depends only on two coordinates of the $3 + 1$ dimensional space, which are parametrized by a distance $r$ from the string, and an azimuthal angle $\theta$, and has the form

$$\phi(r, \theta) = f(r)e^{in\theta}. \quad (5.1)$$
\( f(r) \) is a real function that outside the string core approaches the vacuum value \( f^2 = \xi \). The gauge potential takes the form

\[
gW_\mu \, dx^\mu = n\alpha(r) \, d\theta \quad \rightarrow \quad F = \frac{1}{2} F_{\mu\nu} \, dx^\mu \, dx^\nu = \frac{n\alpha'(r)}{g} \, dr \, d\theta = \frac{n\alpha'(r)}{gC(r)} e^1 e^2 ,
\]

where we already used vierbein forms

\[
e^1 = dr , \quad e^2 = C(r) d\theta .
\]

They live in a space which can be described by a metric

\[
ds^2 = -dt^2 + dz^2 + dr^2 + C^2(r) d\theta^2 ,
\]

which leads to the spin connections

\[
\omega^1_{12} = 0 , \quad \omega^2_{12} = -C'(r) .
\]

This defines a BPS configuration if the following differential equations are satisfied:

\[
C(r) f'(r) = |n| f(r) [1 - \alpha(r)] ,
\]

\[
\frac{\alpha'(r)}{gC(r)} = \frac{g}{|n|} \left[ \xi - f^2(r) \right] ,
\]

\[
1 - C'(r) = \pm A_\theta = \frac{|n|}{M_P^2} \left[ \xi \alpha(r) - \frac{C(r)}{2g^2} \right] ,
\]

where \( \pm = n/|n| \). Then there is a residual supersymmetry

\[
e = e^{\mp i\theta \gamma_5/2} \frac{1}{2} \left( 1 \pm i \gamma_5 \gamma_{12} \right) \epsilon_0
\]

where \( \epsilon_0 \) is a constant spinor, of which the previous factor, \( \Pi_{\pm} \) in the terminology of [A.6], selects 2 independent real components. In rigid supersymmetry this preservation of 1/2 of supersymmetry was found in [17, 18]. In supergravity the BPS condition on the gravitino is also satisfied due to a conspiracy of the spin connection (5.5) and the \( R \)-symmetry connection (3.45) similar to the mechanisms in 2 + 1 dimensions [19, 20].

We thus find that the \( D \)-term strings with elementary flux are BPS-saturated states, and preserve half of the supersymmetry.

### 5.2 Zero modes on D-strings

We have shown that D-strings are BPS saturated objects and preserve one half of the original supersymmetry. In view of this fact, the zero modes on the string exhibit a somewhat puzzling behaviour. To see this, let us complicate the model a bit in the following way.
shall couple the Higgs field $\phi$, that forms a D-string, to some number of chiral superfields $\Phi_i$ in the superpotential

$$W = \frac{a_i}{2} \phi \Phi_i^2,$$

(5.8)

where $a_i$ are coupling constants. We shall denote the fermionic components of these superfields by $\chi_i$, and the scalar component by $\Phi_i$. For simplicity, let us consider a single species of such fermions, the generalization to an arbitrary number of species being trivial. Also, the puzzle that we wish to discuss already appears in the limit of rigid supersymmetry, and switching on supergravity does not change it much. So for the beginning let us discuss the issue in this limit. In the case of rigid supersymmetry, the $U(1)$ is not an $R$-symmetry and the charges of the scalar $\Phi$ and fermion $\chi$ are equal. This charge (call it $q_\chi$) must be exactly $-\frac{1}{2}$ of the $\phi$ charge (which we normalize to one), and thus contributes to $P$ with the same sign as the FI term (taking $\xi > 0$). This implies that the expectation value of the scalar $\Phi$ is identically zero everywhere in the string background, and our $D$-string solution is unaffected. Indeed, the mass-square of the $\Phi$-scalar is

$$m^2_\Phi = |a\phi|^2 + \frac{1}{2} g^2 \left[-|\phi|^2 + \xi \right].$$

(5.9)

This quantity is positive definite everywhere (for $|a|^2 \geq \frac{1}{2} g^2$), including the string core, and hence the lowest energy configuration implies $\Phi = 0$. Hence, even in the presence of the $\Phi_i$-fields, the string background preserves half of the supersymmetry.

We shall now discuss the zero modes in the string background. Notice that the phase of the mass of the fermion $\chi$ changes by $2\pi$ around the strings and therefore according to standard index theorems [21,22] there must be a normalizable fermionic zero mode trapped in the core of the string. Let us find this mode explicitly. We put $\Phi = 0$ and take the configuration (5.1) with $n = 1$.

The Dirac equation for $\chi$ in the string background is

$$\mathcal{D} \chi_L = -af(r)e^{-i\phi} \chi_R.$$

(5.10)

After standard separation of variables by transverse and longitudinal functions,

$$\chi = \alpha(t,z) \tilde{\chi}(r,\theta),$$

(5.11)

we arrive to the following Dirac equation for the spinor $\tilde{\chi}$

$$\left[\gamma^1 \partial_r + \frac{\gamma^2}{C(r)} \left(\partial_\theta - \frac{1}{2} \gamma_{12} C'(r) - i \left( g q_\chi W_\theta + \frac{1}{2} A^B_\theta \right) \right) \right] \tilde{\chi}_L = -af(r)e^{-i\phi} \tilde{\chi}_R,$$

(5.12)

where the term $-\frac{1}{2} \gamma_{12} C'$ had to be added to the $\theta$-derivative as spin-connection term in the curved basis $(r, \theta)$. We then look for a solution\(^{11}\)

$$\tilde{\chi}(r,\theta) = e^{\gamma_1 \phi / 2} \chi_0(r),$$

(5.13)

\(^{11}\)The $\chi_0$ spinor is in fact the spinor in a flat basis.
such that (5.12) reduces to
\[
\gamma^1 \left[ \partial_r + \frac{1}{C(r)} \left( -\frac{1}{2} + \frac{1}{2} C' - i \gamma^{12} \left( gq_\chi W_\theta + \frac{1}{2} A^B_\theta \right) \right) \right] e^{\gamma^{12}/2} \chi_{0L}(r) \\
= -af(r) e^{-i\theta} e^{\gamma^{12}/2} \chi_{0R}(r).
\] (5.14)

Using projected spinors (see appendix A)
\[
\gamma_{12} \chi_L^\pm = \mp i \chi_L^\pm, \quad \gamma_{12} \chi_R^\pm = \pm i \chi_R^\pm,
\] (5.15)
and the third of (5.6) we find
\[
\gamma^1 \left[ \partial_r + \frac{1}{C(r)} \left( -\frac{1}{2} A^B_\theta - i \gamma^{12} \left( gq_\chi W_\theta + \frac{1}{2} A^B_\theta \right) \right) \right] \left( e^{-i\theta/2} \chi_{0L}(r) + e^{i\theta/2} \chi_{0L}(r) \right) \\
= -af(r) \left( e^{-i\theta/2} \chi_{0R}(r) + e^{-i\theta/2} \chi_{0R}(r) \right).
\] (5.16)

Therefore only the + modes exist, for which we get the relation
\[
\gamma^1 \left[ \partial_r - \frac{1}{C(r)} \left( gq_\chi W_\theta + A^B_\theta \right) \right] \chi_0^+(r) = -af(r) \chi_0^+(r).
\] (5.17)

The solution is thus given by two modes, decomposing the + mode using further projections \((1 \pm \gamma_1)\)
\[
\chi_0^+(r) = e^{\int_0^r \left[ (gq_\chi W_\theta + A^B_\theta)(r')/C(r') - af(r') \right] dr'} (1 + \gamma_1) \chi_0^+ + \\
+ e^{\int_0^r \left[ (gq_\chi W_\theta + A^B_\theta)(r')/C(r') + af(r') \right] dr'} (1 - \gamma_1) \chi_0^+.
\] (5.18)

The constants \((1 \pm \gamma_1)\) are two real modes of fermions.

Since \(W_\theta(r)\) goes to a constant, \(C(r)\) is linear in \(r\) in the rigid limit and \(f(r)\) approaches the constant \(\sqrt{\xi}\) at infinity, one of the zero modes is normalizable. Hence, for the string with unit winding number, there is one normalizable fermionic zero mode in the spectrum of string excitations. For \(n\) windings the number of zero mode solutions is \(n\) according to the index theorem. This result can be trivially generalized to an arbitrary number of \(\Phi_i\) superfields. In the background of the string with winding number \(n\), each \(\chi_i\) fermion deposits \(n\) normalizable zero modes. Hence, we can arbitrarily increase the number of the fermionic zero modes, either by increasing the winding number or the number of chiral fermions species coupled to the Higgs field.

Taking into the account the fact that the D-string leaves half of the supersymmetry unbroken, we might have expected to find an equal number of bosonic zero modes, coming from the \(\Phi_i\) fields. However, we find none! In the \(\Phi_i\) spectrum, the localized scalar excitations only exist if \(a \sim g\) or larger (whereas localized fermionic zero modes exist for any \(a \neq 0\)), and even in this case the masses of the lowest scalar excitations are \(\sim \xi\). Thus, the number of bosonic and fermionic zero modes is clearly unbalanced.
The resolution of the puzzle is that the unbroken supersymmetry acts on the fermionic zero modes trivially, and therefore there are no bosonic partners. In other words, whereas in the usual case we would obtain the bosonic partner by super-shifting the fermion, here the shift of the zero mode fermion vanishes

\[ \Phi_i(\epsilon) = \bar{\epsilon}_L \Delta \chi_i = 0, \quad (5.19) \]

(referring to the notations in appendix B). Indeed, the transformation parameters of unbroken supersymmetry and the zero modes are both + spinors under the projections \( (A.6) \), and the property \( (A.8) \) then kills the bosonic mode \( (5.19) \).

This can be understood from the algebra of the remaining supersymmetry. In fact, using the same type of projections, the non-zero part of the algebra is only non-zero between two supersymmetries of opposite type, see \( (A.10) \). Therefore the preserved supersymmetry is nilpotent. An equal number of bosonic and fermionic modes is only expected when the supersymmetry squares to an invertible operator (e.g. the translations).

We would like to note that in [19] (realizing an observation by Witten [23]) a 2 + 1-dimensional BPS vortex solution without Fermi-Bose degeneracy was found. Our effect, however, is of the different origin, since, unlike in [19], in our case the Fermi-Bose non-degeneracy persists even in the globally-supersymmetric limit.

6 Discussion of FI term from pseudo-anomalous \( U(1) \).

We now turn to the case of field-dependent FI terms. As is well-known, such field-dependent FI-terms emerge whenever there is a chiral superfield, which we denote \( \Phi \), shifting under \( U(1) \). In particular, this is mandatory whenever the \( U(1) \) symmetry exhibits a chiral anomaly that is cancelled by the Green-Schwarz mechanism [24]. In such a case, the imaginary part of the \( \Phi \)-scalar plays the role of an axion, and cancels the chiral anomaly by shifting under the \( U(1) \) symmetry. Such a \( U(1) \) is sometimes referred to as “anomalous” or more appropriately “pseudo-anomalous” since the total anomaly is of course zero. The FI term in such a case depends on the real part of \( \Phi \).

The supersymmetric set up for anomalous FI terms was developed in 1987 in [25] in the context of the heterotic string theory. It was used since then in the cosmology literature under the assumption that the dilaton field \( \Phi \) is somehow stabilized and therefore that the dilaton-dependent \( D \)-term produces a constant FI term. Now that we are trying to understand the \( D \)-term potentials in supergravity and string theory at the fundamental level, we have to revisit this approach.

For the purpose of illustration, we consider a “dilaton field” \( \Phi \), adding to the Kähler potential a part \( K_{(\Phi)}(\Phi + \bar{\Phi}) \). This field transforms under the \( U(1) \) as

\[ \delta \Phi = i\alpha, \quad \text{or} \quad \eta_\Phi = iM_-^{-1}. \quad (6.1) \]

The Kähler potential is invariant, implying that the real part of \( r \) is zero, and we do not add an imaginary constant to \( r \), which would imply the presence of the constant FI term.
However, (6.1) gives an extra contribution to $P$:

$$P(\Phi) = -K'(\Phi)(\Phi + \bar{\Phi}).$$ (6.2)

If this has a non-zero constant, it acts as a FI term.

Actually in [25] a “stringy” Kähler potential is considered, with appropriate vector kinetic term

$$K = -\ln(\Phi + \bar{\Phi}), \quad f = \Phi, \quad \Phi \equiv \phi^{-2} + i b.$$ (6.3)

The gauge coupling thus depends on the dilaton, $(\text{Re}f)^{-1} = \phi^2$. The theory also has an axion coupling proportional to $b FF^*$. With the shift transformation of the axion field under $U(1)$ as in (6.1), this term serves to remove the anomaly proportional to $FF^*$.

This gives

$$P(\Phi) = -K'(\Phi)(\Phi + \bar{\Phi}) = \frac{1}{\Phi + \bar{\Phi}} = \frac{1}{2} \phi^2.$$ (6.4)

The $D$-term potential is

$$\frac{1}{2}(\text{Re}f)^{-1}P^2 = \frac{1}{8} \phi^6.$$ (6.5)

Note that if this would be the only dependence on $\phi^2$, the potential would tend to make $\phi^2 \to 0$ and the FI term would disappear.

The stabilization of the dilaton was assumed in [25] and therefore it was considered that the dilaton-dependent $D$-term can be qualified as a constant FI term. Its value was computed in the context of the weakly-coupled heterotic string for a constant string coupling $g_s$ (constant dilaton, $g_s = \phi^2$) and found to be [26]:

$$\xi_{GS} = \frac{g_s^2 \text{Tr}Q_{192}}{192 \pi^2 M_P^2}.$$ (6.6)

The potential energy is given by $V_D = \frac{1}{2} g_s^2 \xi_{GS}^2 \sim g_s^6$. Clearly, without an assumption that $\phi$ is constant, this potential would behave, as we have just seen, as $\phi^6$ and tends to zero at small $\phi$.

Based on this specific analysis, two criticisms were made to the $D$-term inflation scenario. First (see for example [27]), the inflation scale provided by (6.6) is too close to the Planck scale to be consistent with the COBE normalization. Secondly, the necessary stabilization of the dilaton field requires some non-vanishing $F$-terms which, in the context of the simple model described by (6.3), drown any $D$-term [16]. We will discuss this latter question at the end of this section and first address the former criticism.

Quite generally, the role of the field $\Phi$ may be played by any modulus, whether it is the dilaton or any volume modulus. The situation with the stabilization of dilaton and volume in heterotic string theory is quite complicated. A few recent studies [28–30] have shown that stabilization of moduli in heterotic string theory requires significant deviations from the weakly coupled regime of the heterotic theory, such as strong coupling, presence of M5-branes etc. Therefore the actual numbers for FI terms, used in the cosmological context on the basis of weakly coupled heterotic string theory, have to be revised. One more
important phenomenological ingredient for the $D$-term inflation model was the choice of the gauge coupling: it was assumed to be of the same magnitude as the coupling of the standard model $U(1)$. One may, however, expect few more $U(1)$ in string theories, some of which may have different scales of gauge coupling and therefore give different values for the $D$-term inflation model.

In other string theories, the $D$-term may depend on some other moduli, like in type IIB theory where it is the volume of the compactified directions [31]. The gauge coupling for the vector field on the D7 brane has a dependence on the volume of the compactification modulus: $\text{Re} f \sim (\rho + \bar{\rho})$. The Kähler potential is $K = -3 \ln(\rho + \bar{\rho})$. As a result, the field-dependent $D$-term related to the non-self-dual fluxes on D7 brane is given by

$$\frac{1}{2}(\text{Re} f)^{-1}P^2 = \frac{C}{(\rho + \bar{\rho})^3}.$$  \tag{6.7}

In both cases the $D$-term potential has a runaway behaviour. Thus, using these potentials we cannot rely on any assumption of stabilization. To find models for cosmology from string theory, one has to consider the total potential where the dilaton and/or the volume have to be stabilized. Only when this is done, one will be able to keep the version of $D$-term inflation based on anomalous stringy $U(1)$ valid.

In short, the stringy FI term required for GS mechanism of anomaly cancellation, is a field-dependent $D$-term. Before one stabilizes it, it cannot be used in the cosmological context. As we already explained in the Introduction, it is not known how to derive constant FI terms from string theory, despite the fact that they are allowed under certain conditions in $N = 1, d = 4$ supergravity.

Let us finally discuss some generic issues that arise when one tries to stabilize FI terms in the field-dependent case.

A naive approach would be to assume that the vev of $\Phi$ is stabilized at some arbitrarily high scale $M_{\text{st}}$, by some superpotential, in such a way that below the scale $M_{\text{st}}$ we can integrate out $\Phi$ in order to be left at low energies with a supersymmetric theory with a constant FI term. This is, however, not possible as can be seen from the two following arguments.

First, if we manage to give a large supersymmetry-preserving mass to the real part of $\Phi$, then by supersymmetry this must also be the mass of its imaginary part. However, this imaginary part is the axion that shifts under $U(1)$, and thus gets a mass through the Higgs effect by becoming the longitudinal component of the $U(1)$-gauge field. Hence, if supersymmetry is preserved, $\Phi$ simply becomes a part of a massive vector supermultiplet with mass $M_{\text{st}}$. Hence, $\Phi$ cannot be integrated out in a supersymmetric way, unless we integrate out the whole massive vector supermultiplet. This fact immediately implies that if we integrate out $\Phi$ below scale $M_{\text{st}}$ we cannot be left with a non-zero FI term. Presence of a non-zero FI term in an effective supersymmetric theory at any scale requires an existence of a corresponding vector superfield, unless supersymmetry is broken.

Hence, $\Phi$ cannot be stabilized at scales bigger than the effective low energy value of the
FI term in a susy-preserving way, without jeopardizing the very existence of FI in the low-energy theory. Thus, in general, stabilization of \( \Phi \) requires some additional supersymmetry breaking, e.g. via some non-zero \( F \)-terms. We will see below that these \( F \)-terms can be parametrically (depending on the parameters in the Kähler potential) smaller than the FI-terms.

The fact that \( \Phi \) cannot be stabilized without the additional supersymmetry breaking is also clear from the following argument based on the form of the superpotential. Because \( \Phi \) shifts under the gauged \( U(1) \) symmetry, the superpotential must be invariant under this shift (up to an arbitrary Kähler transformation, and assuming now that there is no explicit FI term \( \xi \)). Hence it can only depend on \( \Phi \) through invariants of the form (or functions of them)

\[
W = e^{\Phi c} w_0
\]

where \( c \) is some constant, and \( w_0 \) is a holomorphic function of other chiral superfields such that it carries an overall charge \( Q = -c \) under the \( U(1) \) symmetry. \( w_0 \) may in particular be a composite operator, generated by some strong dynamics, such as e.g., gaugino condensation in a strongly coupled \( SU(N) \) group. In each particular case \( w_0 \) will be subject to the \( U(1) \)-selection rules resulting from charge assignment dictated by GS anomaly cancellation.

Thus, at the minimum of the \( \Phi \)-potential, the \( F \)-term of \( \Phi \) is generically non-zero. Indeed, it is impossible to satisfy the equations \( D_\Phi W = 0 \), at the true minimum of the potential, because the \( D \)-term has no minimum, only a runaway. So to stabilize \( \Phi \), the \( F \)-terms should become non-zero. On the other hand, the \( F \)-terms have to be smaller than the \( D \)-terms if we want to have the \( D \)-term inflation. This can result from a suitable Kähler function.\(^{12}\) This Kähler function can only stabilize \( \Phi \) if there are some non-zero \( F \)-terms. As said above, such terms are expected due to the form of the stabilizing superpotential, and due to the tendency of the FI term to push \( \Phi \) to the runaway branch. To have the \( D \)-term inflation we require that

\[
F \ll D,
\]

and also we need to satisfy the following inequality:

\[
H^2 = g^2 \xi^2 / M_p^2 \ll m_\Phi^2 \ll g \xi,
\]

where \( m_\Phi^2 \) is the stabilizing mass of \( \Phi \). The above implies that we need the \( \Phi \) mass to be bigger than the Hubble parameter during inflation, which is also of the same order as the contribution from the \( D \)-terms into the mass of \( \Phi \). In other words, we need \( F \) and \( D \)-terms to split their roles in the following way, that \( D \)-terms drive inflation, but \( F \)-terms stabilize \( \Phi \). This puts a requirement on the Kähler function.

If a stabilization of the dilaton and volume would be established in some models, then one would have a mechanism for deriving \( D \)-term inflation from string theory. Recently a proposal for stabilization of the dilaton and volume modulus was suggested in [32] in the context of the \( F \)-term potential. Some versions of brane inflation [33] were studied in [34]

\(^{12}\) Its origin is of course a separate issue, not to be discussed here.
where particular difficulties with realization of the brane inflation with dilaton and volume stabilization were pointed out. More recently, some proposals were made [35, 36] towards improvement of this situation due to a particular shift symmetry of the potential associated with the BPS states of branes. These proposals are supported by the supergravity analysis of type IIB compactifications in presence of fluxes and branes in [37, 38]. There is a hope that these efforts may lead to a satisfactory derivation of inflation from string theory.

7 Cosmological applications: Stability of the \textit{D-term} strings

We wish now to briefly discuss some aspects of the cosmological implications of the \textit{D-term} strings. It is sometimes assumed that \textit{D-term} strings are necessarily formed at the end of the \textit{D-term} inflation, and their tension may be in conflict with the observed spectrum of density perturbations. However, this expectation is too naive and in reality the issue is much more subtle. In brief, we will see that there is no reason to think that the \textit{D-term} cosmic strings cause any observable problems: in many models \textit{D-strings} may not be topologically stable.

More importantly perhaps, it has been conjectured recently [4] that there is a correspondence between the \textit{D-strings} of type \textit{II} string theory, and \textit{D-term} strings. This idea was further explored in [39]. According to this conjecture, BPS \textit{D_{1+q}} branes wrapped on a \textit{q}-cycle, are seen from the point of view of 4\textit{d} supergravity as \textit{D-term} strings. This conjecture has some immediate implications for the \textit{D-term} and \textit{D-brane} cosmology. For instance, since according to current understanding [40], \textit{D_{1+q}}-branes can be thought of as the tachyonic vortices formed in the annihilation of (\textit{D_{3+q}} - \overline{\textit{D_{3+q}}})-branes, it then immediately follows from the conjecture in [4] that the (\textit{D_{3+q}} - \overline{\textit{D_{3+q}}})-system corresponds to a non-zero \textit{D-term}.\textsuperscript{13} Moreover, if the compactification volume is somewhat larger than the string scale, the \textit{D-string} tension can be easily lowered in order to accommodate the current observational bounds. Hence, in type \textit{II} theories the cosmic \textit{D-strings} do not cause any cosmological problems and in fact may be potentially observable in the form of supergravity \textit{D-term} strings.

Recently, the suppression of the cosmological production of cosmic \textit{D-strings} in type \textit{II} theories was studied in [41]. Some of the interesting potential instabilities of these objects were pointed out in [42]. \textit{D-brane}–\textit{D-term-string} correspondence then allows to look for the counterparts of all these effects in the supergravity \textit{D-term} strings. A number of such connections were demonstrated in [4]. Below we wish to provide additional links. We shall first discuss instabilities of \textit{D-term} strings in 4\textit{d} supergravity, and then relate these instabilities to the ones of \textit{D-strings} in type \textit{II} theory discussed in [42].

\textsuperscript{13}We learned from J. Maldacena that M. Douglas also noticed that \textit{brane-anti-branes} are \textit{D-terms}.
7.1 D-term string stability in supergravity

We first discuss the case of supergravity. As shown in [4], in the simplest models, in which the constant FI $D$-term gets compensated by a single complex scalar at the end of the $D$-term inflation, there are topologically stable D-strings. These strings are topologically stable because of the non-trivial homotopy $\pi_1$ of the vacuum of the broken $U(1)$ symmetry. This manifold is $S_1$ and hence it contains uncontractible loops that can be labeled by an integer $n$. This fact guarantees the stability of the cylindrically symmetric Higgs configurations when the phase of the Higgs field winds by $2\pi n$ around some axis $\phi = \sqrt{\xi} e^{i\theta}$.

However, this is only true as long as $\phi$ is not transforming under any other non-abelian symmetry group. A priori, there is nothing that forbids such a transformation, and in fact in view of the conjectured D-brane connection [4], such a situation is very likely. So let us see what will happen with the topological stability of the $D$-term strings in the case where $\phi$ is in a representation of a larger symmetry group. For simplicity, let us assume that $\phi$ transforms as a doublet of some gauge $SU(2)$ symmetry. This symmetry combined together with our $D$-term $U(1)$ promotes the full gauge group into $SU(2) \times U(1)$. When $\phi$ condenses, compensating the $D$-term, the group is broken down to $U(1)'$, with the vacuum manifold now being $SU(2) \times U(1)/U(1)$. This manifold is a three sphere, and $\phi$ can take an expectation value at any point on it. However, unlike $S_1$, any closed loop on $S_3$ can be continuously contracted into a point. Hence strings are no more stable, and unwind without any topological obstruction. In such a scenario no cosmic strings would form in the phase transition after inflation.

Interestingly, to destabilize the $D$-term strings, the existence of a non-abelian gauge group is strictly speaking unnecessary. All we need is that there is at least one more complex field $\phi'$ that carries exactly the same charge under the $D$-term $U(1)$ as $\phi$ does, and that the value of this field is not fixed by the couplings in the superpotential. In such a case, the global structure of the vacuum manifold is again $S_3$, although the gauge structure is $S_1$. That is, there is an accidental global $U(2)$ symmetry. When the gauge group breaks down to nothing by the vev of $\phi$, the global group breaks down to $U(1)$. What matters for the topological stability is precisely the global structure of the vacuum. The $D$-term string now can unwind without any cost of potential energy. In other words, in this situation the $D$-term strings become semi-local strings [43]. We must stress however, that it may still cost a finite gradient energy to unwind such a string, and hence they may still play some interesting role in cosmology. This issue is beyond the scope of the present work.

7.2 D-string instability as D-term string instability.

We wish now to show that according to the conjecture of [4], the above discussed topological instability of the $D$-term strings can be viewed as the corresponding instabilities of D-brane strings of string theory [42]. It is known that $D_1$ strings become topologically unstable in

14Before we submitted this paper, [44] appeared which also uses this point.
the presence of D$_3$-branes. That is, if the D$_1$ is placed on top of a D$_3$ it can “dissolve” in the D$_3$ brane. If the D$_1$ is placed at some distance apart, there is a tunneling process by which it can break with two ends attached to the D$_3$ brane. These attached ends look like monopoles on the D$_3$-brane. To connect this instability with the one of the D-term strings, it is convenient to view it in the following way. Think of the D$_1$ − D$_3$ system as being formed from the system of two D$_3$ and one ¯D$_3$ branes, after one of the D$_3$-branes got annihilated by the ¯D$_3$. This annihilation proceeds via tachyon condensation, which cancels the energy of D$_3$ − ¯D$_3$ pair. This energy breaks all the supersymmetries, and according to [4] it is in form of the D-term. Consider first a situation when all the parent branes are on top of each other. The gauge symmetry of the original system is then U(2) × U(1), and the tachyon transforms as (2, −1) under it. This is identical to our example with SU(2) × U(1) symmetry , in which the role of the D-term-compensating field is assumed by the tachyon.$^{15}$ The instability of the string is clear in this language. The tachyonic vacuum is topologically trivial, and so is the vacuum with the canceled D-term. So D-term strings are topologically unstable and can unwind, spreading flux out. In the string language the same process is seen as dissolving D$_1$ string on the D$_3$-brane, with its flux spreading out.

Now let us displace one of the original D$_3$-branes in the perpendicular direction. This Higgses the original symmetry to U(1) × U(1) × U(1) and there are monopoles in this system. In the language of four-dimensional supergravity this is equivalent to giving a vev to the SU(2)-adjoint Higgs. After the tachyon condenses and compensates the D-term, the symmetry is broken to U(1) × U(1), and strings can be formed. However, these strings are not really stable but can break into monopole anti-monopole pairs.

We see that there is a complete correspondence with the D-term case.

8 FI D-terms from D-branes, and moduli stabilization

The connection suggested in [4] between the D-term strings and BPS D-brane strings (D$_{1+q}$-branes of type II string theory, wrapped on q-cycles) allowed us to view many properties of non-BPS brane-anti-brane systems from the point of view of FI D-term supersymmetry breaking. Let us briefly review some of our conclusions. Consider a pair of D$_{3+q}$ − ¯D$_{3+q}$ branes wrapped on a q-cycle of radius $R$. We shall assume that 6 − q remaining additional dimensions are also compactified on a cycle of radius $R_\perp$. The D$_{3+q}$ − ¯D$_{3+q}$ system breaks all the supersymmetries. The low-energy gauge group consists of two U(1)-symmetries. Their field strengths couple with opposite sign to the Ramond-Ramond (2+q)-form $C_{(2+q)}$. Hence, we can choose two orthogonal combinations of these U(1)-s according to their RR couplings. We shall be interested in the diagonal U(1) (from now on we shall call it simply U(1)) that has a non-zero RR charge. We shall denote the corresponding two-form field strength by $F_{(2)}$. Since the net RR charge of D$_{3+q}$ − ¯D$_{3+q}$ is zero, the system is unstable towards annihilation.

$^{15}$On a compact space, there should be something else that absorbs RR and gravitational fluxes of the remaining D$_3$-brane, in order to keep the tachyonic vacuum, without the D-term, flat. This does not affect our discussion.
Annihilation can be described as condensation of the open string tachyon $\phi$, which Higgses the diagonal $U(1)$-symmetry. The tachyonic vacuum is a closed string vacuum with no $D_{3+q}$ branes.

In our $D$-term description, the above stringy picture translates as follows. The supersymmetry breaking by the $D_{3+q} - \bar{D}_{3+q}$-system is seen in effective 4$d$, $N = 1$ supergravity as breaking by the FI $D$-term. In the limit $R_\perp \to \infty$, the world-volume 4$d$ supergravity approaches the rigid limit, and the FI term is related to the brane tension and the radius of a $q$-cycle in the following way

$$2(2\pi R)^q T_{3+q} = \frac{2R^q}{g_s (2\pi)^3 \alpha'^3} = \frac{g^2}{2} \xi^2,$$

where $g_s$ and $g$ are the string and the world-volume gauge couplings, respectively.

Notice that the above form is valid in the limit in which gravity is decoupled ($M_P = \infty$). In this limit, the FI term cannot drive any 4$d$ $D$-term inflation. After taking the finite compactification volume and bringing the 4$d$ action to the Einstein form by Weyl rescaling, the effective four-dimensional source will, of course, become dependent on the volume modulus, in accordance to [31], and in order to get inflation one has to address the issue of the volume stabilization. However, at the moment we shall work in the infinite-volume limit, in which the low-energy world-volume theory reduces to a gauge theory with the above FI term.

After annihilation of branes, this $D$-term is compensated by the vev of the tachyon $\phi$, which corresponds to a scalar component of a chiral superfield. The tachyonic vacuum is a supersymmetric vacuum with vanishing $D$-term. When branes are far apart, the mass$^2$ of the tachyon becomes positive, and the $D$-term is non-zero. As suggested in [4], the tachyon superpotential is

$$W = \phi_0 \phi \bar{\phi},$$

where $\bar{\phi}$ is the supersymmetric tachyonic partner, a chiral superfield with opposite $U(1)$-charge. $\phi_0$ is a $U(1)$-neutral superfield that corresponds to the inter-brane separation ($r$)

$$\phi_0 = M_s^2 r,$$

where $M_s$ is the string scale. Although there are other fields with masses comparable to tachyon, we have only focused our attention to the superfields whose chiral fermionic components become massless at zero brane separation. These are fermionic partners of $\phi_0$, $\Phi$, $\bar{\phi}$ as well as gaugino and 4$d$ gravitino. The latter is decoupled in the infinite compactification volume limit. There are also massless fermionic modes corresponding to the center of mass motion, which are not of our interest. All other fermions are massive. Note also that fermionic partners of $\phi$, $\bar{\phi}$ are massless only at the zero separation point ($\phi_0 = 0$).

Hence, we see that in the rigid limit, the low-energy dynamics of $(D_{3+q} - \bar{D}_{3+q})$-system is described by the globally supersymmetric limit of the $D$-term inflation model discussed in section 4. We now wish to consider the case of finite $M_P$ (that is finite $R_\perp$). A useful
guideline for understanding what may happen in the finite-volume case is the cancellation of the $U(1)$-gauge anomaly. First let us observe that in the rigid limit, there is no $U(1)$-anomaly, since the gaugino and the fermionic partner of $\phi_0$ are neutral, the 4d-gravitino is decoupled ($M_P = \infty$) and the anomalies of the chiral fermionic partners of $\phi$ and $\bar{\phi}$ exactly cancel. Now, for finite $M_P$, the situation may change and the charges of the chiral fermions may shift. One supergravity example of such a situation when going to finite $M_P$ re-arranges charges of the chiral fermions was discussed in section [4]. In that example, the charges of chiral fermions shift as described in (4.13), and the gaugino and gravitino acquire charges equal to $G$. However, with shifted fermionic charges the chiral gauge anomaly is non-vanishing anymore, and is equal to the gravitino contribution according to (4.14). Due to this fact, the analogous charge-shift in the present case may create a seeming puzzle. Starting from anomaly-free high-dimensional theory, we cannot create an anomaly by simply changing the compactification radius! So consistency of the compactification requires that if there is such an anomalous shift in fermionic charges, there must be an axion whose shift exactly cancels the anomaly by the GS mechanism. The axion that is charged under $U(1)$ indeed exist in the theory in form of the RR axion, as we shall now demonstrate.

The $U(1)$ field strength has the following world-volume coupling to the RR $(2 + q)$-form

$$g_s (2 \pi \alpha') T_{(3+q)} \int_{3+1+q} F_{(2)} \wedge C_{(2+q)}. \quad (8.4)$$

The 4d dual of the 4d-zero mode of $C_{(2+q)}$ is precisely the axion that shifts under $U(1)$. To see this, let us concentrate on the components of $C_{(2+q)}$ that take only two indices in the four non-compact dimensions, and the rest on a $q$-cycle. These components effectively define a two form, which we can call $C_{(2)}$. We shall only be interested in a 4d zero mode of this two-form (higher KK-modes are irrelevant for the anomaly cancellation). We can now go into the dual description of the $C_{(2)}$-form in terms of an axion

$$dC_{(2)} \rightarrow * da, \quad (8.5)$$

where star denotes a 4d Hodge-dual. Under this duality transformation we have to replace

$$(dC_{(2)})^2 \rightarrow (da - gQ_a W)^2, \quad (8.6)$$

Under the $U(1)$, the axion shifts as

$$a \rightarrow a + gQ_a \alpha(x). \quad (8.7)$$

$Q_a$ can be found from (8.4) by integrating over the extra coordinates and using the relation (8.1) between $\xi$ and the brane tension, as well as the relation $g^2 = 8 \pi g_s \alpha'^{q/2}/R^q$. The result\(^\dagger\) is $Q_a = \xi/M_P^2$. Hence, we see that in the 4d theory there is always the axion with right transformation properties, which could potentially cancel the anomaly appearing from the chiral fermion sector, provided there is a coupling

$$a F \wedge F. \quad (8.8)$$

\(^\dagger\)As shown in [4], this is also the right value that correctly reproduces the long range RR field of the $D$-term string formed by the tachyon.
Existence of such a coupling is in general compactification dependent and hence so must be the resulting fermionic charge assignment. In case that it is absent, the anomaly of the chiral fermions must be separately zero. When such coupling is present, the chiral fermion set must be anomalous, and exactly compensate the anomaly resulting from the shift of the axion via the GS mechanism. Hence, in such case the effective 4d description of D–D systems will be in terms of the “anomalous” $U(1)$ symmetry.

Let us finally tress that the existence of the axion that shifts under the $U(1)$ gauge symmetry has important consequences for the issue of moduli-stabilization.

In the 4d theory, the axion becomes a lowest component of a chiral supermultiplet $\Phi = \sigma + i a$, where the role of $\sigma$ is model dependent. For instance, in type II B theories, $\sigma$ can either be some combination of the dilaton and the volume modulus, or an NS-NS two form. The fact that the axion shifts under $U(1)$, implies that (in 4d Einstein frame) the FI term is $\Phi$-dependent. This of course raises the issue of the stabilization of the real part of $\Phi$. Stabilization could in principle happen via some non-perturbatively generated superpotential. However, because of the $U(1)$-symmetry, the superpotential must transform homogeneously under the shift of $\Phi$, that is, be invariant up to the Kähler transformation. Hence, up to an arbitrary Kähler transformation the $\Phi$-dependence of the superpotential can only be through the invariant(s) of the form

$$e^{iQ_\Phi}w_0,$$

where $c$ is a constant and $w_0$ is some holomorphic function of the chiral superfields (in our case $\phi_0, \phi, \bar{\phi}$) that carries the overall charge $Q = -c$ with respect to $U(1)$. We see that holomorphy and $U(1)$ invariance can potentially severely constrain the form of the stabilizing potential for $\Phi$.

9 Conclusion

In this paper we have clarified the status of constant FI terms $\xi$ in $N = 1, d = 4$ supergravity in general and in examples. Their presence shows up in covariant derivatives of all fermions and in the supersymmetry transformation laws, since the relevant local $U(1)$ symmetry is a gauged $R$-symmetry. These new couplings proportional to $g\xi/M_P^2$ lead to gauge and gravitational anomalies. Under certain conditions it is possible to cancel the gauge anomalies.

One of the important restrictions on supergravity with constant FI terms is the following: the superpotential $W$ has to transform under $U(1)$ gauge symmetry, $\delta W = -i\frac{g\xi}{M_P^2} W$, otherwise the constant FI term $\xi$ has to vanish. This requirement is consistent with the fact that in the gauge theory at $M_P \to \infty$ the potential is $U(1)$ invariant. However, we consider local supersymmetry of the classical supergravity action, in which terms of the order $g\xi/M_P^2$ are all taken into account.

\footnote{We thank Juan Maldacena for discussions on this and other issues.}
In the example of a $D$-term inflation [13], it is possible to generalize the original model with rigid supersymmetry to exact local supersymmetry. A gauge theory potential of the $D$-term model $W = \phi_0 \phi_+ \phi_-$ is neutral under $U(1)$ symmetry in gauge theory with constant FI terms. In this paper we have found how to promote this model to the supergravity level with constant FI terms: we required that the total charge of $\phi_+$ and $\phi_-$ fields does not vanish but is equal to $-\xi/M_P^2$. The gauge theory anomaly from gravitino, gaugino, original chiral multiplets $\phi_0$, $\phi_+$ and $\phi_-$ and additional 3 chiral multiplets can be cancelled.

In string theory there are no known examples of constant FI terms $\xi$. Only moduli-dependent $D$-terms are available [25,31]. In absence of constant FI terms, the rules of local supersymmetry require the superpotential to be invariant under the $U(1)$-gauged symmetry (for invariant Kähler potential). In such theories, where the cancellation of gauge anomaly is due to the Green-Schwarz mechanism with the shift of the axion and the coupling $aFF^*$, one has to stabilize the scalar partner of the axion to get the effective supergravity with constant FI terms. Some efforts in this direction have been made recently in [31,32,34–38]. It is possible that a stringy version of the D-brane inflation [33] with improvement with respect to volume and dilaton stabilization will be derived in the future and that the problems with inflation in string theory, pointed out in [34], will be resolved. This kind of string cosmology program requires a better understanding of the structure of 3+1 dimensional $N = 1$ supergravity with constant FI terms as well as the one with field-dependent $D$-terms. This paper has clarified the properties of such theories.

In this paper we have investigated another interesting role that FI terms can play in string theory. This role is based on the recently-suggested [4] equivalence between the 4d supergravity $D$-term strings and D-branes of type $II$ string theory. According to it, brane-anti-brane systems in an effective 4d theory can be viewed as gauge theories with non-zero FI term, in which the axion shifting under the $U(1)$-symmetry comes from the RR sector. The tachyonic instability of the brane-anti-brane system is seen as the instability triggered by the FI-term. Thus, many important properties of D–Đ systems can be understood in the light of 4d supergravity with FI $D$-terms. Certain aspects of stability of some string compactifications with branes and anti-branes can be understood in the language of supergravity vacua with non-zero $D$-terms. For instance, the shift of the axion under the gauge $U(1)$ symmetry gives selection rules for the possible invariants of the stabilizing superpotential.

On the cosmic string front, we have provided some additional consistency checks of the conjectured correspondence [4] between $D$-term-strings and D-branes, by mapping the instabilities of the two.

Finally, we have studied in non-minimal model the zero mode content on the BPS $D$-term cosmic string solutions of $N = 1$ supergravity with constant FI terms [4]. We have discovered a puzzling property that the numbers of bosonic and fermionic zero modes can be arbitrarily different. We have explained this puzzling behaviour by unusual properties of unbroken supersymmetry.
Acknowledgments

It is a pleasure to thank N. Arkani-Hamed, M. Douglas, D. Freedman, G. Gibbons, S. Kachru, A. Linde, D. Lyth, J. Maldacena, R. Myers, K. Stelle, P. West and B. de Wit for useful discussions. The work of R.K. was supported by NSF grant PHY-0244728. The research of G.D. is supported in part by a David and Lucile Packard Foundation Fellowship for Science and Engineering, and by the NSF grant PHY-0070787. The work of A.V.P. was partially supported by the European Community’s Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime and in part by the Federal Office for Scientific, Technical and Cultural Affairs, Belgium, through the Inter-university Attraction Pole P5/27. R. K. and P. B. are grateful to the organizers of Santa Barbara string cosmology workshop, and R.K., P.B. and A.V.P. to the organizers of the workshop in Bad Honnef on dark matter and dark energy, where part of this work was performed. P. B., G. D. and A.V.P. thank the Institute of theoretical physics of Stanford for the hospitality.
A Residual superalgebra of the $D$-term string

The commutator between two supersymmetry transformations is given by

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \frac{1}{2} \delta_{\text{CGCT}}(\xi^\mu(\epsilon_1, \epsilon_2)),$$

$$\xi^\mu(\epsilon_1, \epsilon_2) \equiv \bar{\epsilon}_2 \gamma^\mu \epsilon_1,$$

where the covariant general coordinate transformation (CGCT) is a combination of general coordinate transformations, Lorentz transformations and gauge transformations (at least the bosonic part):

$$\delta_{\text{CGCT}}(\xi^\mu) = \delta_{\text{GCT}}(\xi^\mu) - \delta_{\text{Lor}}(\xi^\mu \omega_{\mu}^{ab}) - \delta_{\text{G}}(\xi^\mu W_\mu).$$

We have e.g. on the scalars and the vectors

$$\delta_{\text{CGCT}}(\xi^\mu) \phi = \xi^\mu \hat{\partial}_\mu \phi, \quad \delta_{\text{CGCT}}(\xi^\mu) W_\mu = \xi^\nu F_{\nu \mu}.$$

Introducing notations

$$\delta(\epsilon) = \bar{\epsilon} Q = \epsilon^\alpha Q_\alpha, \quad \delta_{\text{GCT}}(\xi^\mu) = \xi^\mu P_\mu, \quad \delta_{\text{Lor}}(\Lambda_{ab}) = \frac{1}{2} \Lambda_{ab} L_{ab}, \quad \delta_{\text{G}}(\alpha) = \alpha T,$$

we can write the supersymmetry algebra as

$$\{Q_\alpha, Q_\beta\} = \frac{1}{2} (\gamma^\mu C^{-1})_{\alpha \beta} \left( P_\mu - \omega_{\mu}^{12} L_{12} - W_\mu T \right).$$

The split between the 4 initial supersymmetries that is relevant for the $D$-term strings occurs through the projectors

$$\Pi_\pm = \frac{1}{2} \left( 1 \pm i \gamma_5 \gamma^{12} \right).$$

These projectors satisfy

$$\gamma^1 \Pi_\pm = \Pi_\pm \gamma^1, \quad \gamma^2 \Pi_\pm = \Pi_\pm \gamma^2,$$

and Majorana conjugates exchange the projections, e.g.

$$\Pi_\pm \epsilon = \tau \Pi_\mp \epsilon.$$

This implies that $\xi^\mu(\epsilon_1, \epsilon_2)$, for $\mu = 1, 2$, is only nonzero for two spinors of different $\pm$-type. Using notations as $\epsilon_\pm = \Pi_\pm \epsilon$, we have

$$\xi^\mu(\epsilon_1, \epsilon_2) = \bar{\epsilon}_2 + \gamma^\mu \epsilon_1 - + \bar{\epsilon}_2 - \gamma^\mu \epsilon_1 + \quad \text{for } \mu = 1, 2.$$

In terms of supersymmetry generators, this implies that the only nonzero anticommutators are

$$\{Q_\alpha, Q_\beta\} = \frac{1}{2} (\Pi_+ \gamma^\mu C^{-1})_{\alpha \beta} \left( P_\mu - \omega_{\mu}^{12} L_{12} - W_\mu T \right).$$
B Relating bosonic and fermionic modes

In this appendix, we will repeat the argument why in supersymmetric theories one can construct a fermionic mode for every bosonic mode and vice versa. This leads to the theorem about the equality of the number of fermions and bosons in a supersymmetric theory. This exposé will clarify that the theorem depends crucially on the algebra, as the fact that the fermion mode constructed out of a bosonic mode and vice versa could be degenerate if the algebra is not suitable.

The construction starts from an expansion around a solution of the theory where the bosonic fields \( \phi^i \) have a value \( \phi^i_0 \) and the fermions \( \psi^a \) are zero \( \psi^a_0 = 0 \). It will use the concept of residual supersymmetry as exposed in [45], and the notations are taken from the final pages of that paper. We consider a supersymmetry transformation of the bosons and fermions, which, using the condensed notation of B. DeWitt [46], can be written as

\[
\delta(\epsilon)\phi^i = R^i_{\alpha}(\phi, \psi)\epsilon^\alpha, \quad \delta(\epsilon)\psi^a = R^a_{\alpha}(\phi, \psi)\epsilon^\alpha. \tag{B.1}
\]

The index \( \alpha \) represents only the residual supersymmetry at the solution of the theory, which implies \( R^a_{\alpha}(\phi_0, 0) = 0 \). We also have \( R^i_{\alpha}(\phi_0, 0) = 0 \), which is already obvious from the fact that this is a fermionic quantity and thus vanishes for zero fermions. We write the fields as

\[
\phi^i = \phi^i_0 + \Delta\phi^i, \quad \psi^a = \Delta\psi^a, \tag{B.2}
\]

and expand the action up to second order in the perturbations \( \Delta\phi^i \) and \( \Delta\psi^a \):

\[
S = S^0 + \frac{1}{2}\Delta\phi^i S^0_{ij}\Delta\phi^j + \frac{1}{2}\Delta\psi^a S^0_{ab}\Delta\psi^b + \mathcal{O}(\Delta^3),
\]

\[
S^0 \equiv S(\phi^i_0, 0), \quad S^0_{ij} \equiv \frac{\delta}{\delta\phi^i}\frac{\delta}{\delta\phi^j}S(\phi^i_0, 0), \quad S^0_{ab} \equiv \frac{\delta}{\delta\psi^a}\frac{\delta}{\delta\psi^b}S(\phi^i_0, 0). \tag{B.3}
\]

First order perturbations are absent because the zero state is a solution of the classical field equations, and mixed terms are absent due to their fermionic nature and \( \psi^a_0 = 0 \). As the supersymmetry transformations on the vacuum states are zero, supersymmetry acts only on the perturbations and in first order is

\[
\delta(\epsilon)\Delta\phi^i = R^i_{\alpha,a}(\phi_0, 0)\Delta\psi^a\epsilon^\alpha + \mathcal{O}(\Delta^2), \quad \delta(\epsilon)\Delta\psi^a = R^a_{\alpha,i}(\phi_0, 0)\Delta\phi^i\epsilon^\alpha + \mathcal{O}(\Delta^2), \tag{B.4}
\]

where the notations \( , i \) and \( , a \) denote derivatives of the transformation laws. Again, other terms are absent due to \( \psi^a_0 = 0 \). The statement that the action is supersymmetric is

\[
0 = \delta(\epsilon)S = \left[ \Delta\phi^i S^0_{ij} R^j_{\alpha,a}(\phi_0, 0)\Delta\psi^a + \Delta\psi^a S^0_{ab} R^b_{\alpha,i}(\phi_0, 0)\Delta\phi^i \right] \epsilon^\alpha + \mathcal{O}(\Delta^3). \tag{B.5}
\]

Consider now a solution of the fermionic field equations \( \Delta\psi^a S^0_{ab} = 0 \) and consider arbitrary \( \Delta\phi^i \). Then we find

\[
S^0_{ij} R^j_{\alpha,a}(\phi_0, 0)\Delta\psi^a = 0, \tag{B.6}
\]
which shows that

$$\Delta \psi^a \text{ solution of field eqs.} \rightarrow \phi^i(\epsilon) = R^i_{a,a}(\phi_0,0)\Delta \psi^a \epsilon^a \text{ solution of field eqs.} \quad (B.7)$$

for any $\epsilon$ that parametrizes a residual supersymmetry. Similarly, for any solution of the bosonic field equations one constructs a solution of the fermionic field equations as

$$\Delta \phi^i \text{ solution of field eqs.} \rightarrow \psi^a(\epsilon) = R^a_{i,i}(\phi_0,0)\Delta \phi^i \epsilon^a \text{ solution of field eqs.} \quad (B.8)$$

For counting the number of bosonic and fermionic modes, one may imagine that these mappings can be either degenerate or not. However, applying the map twice on a bosonic mode $\Delta \phi^i$ gives that there should be a bosonic solution of the field equations

$$\tilde{\Delta} \phi^i(\epsilon_1, \epsilon_2) = R^i_{a,a}(\phi_0,0)R^a_{\beta,j}(\phi_0,0)\Delta \phi^j \epsilon_2^\beta \epsilon_1^a. \quad (B.9)$$

The involved square of the transformation operators is what appears at first order in the anticommutator of the supersymmetry with parameter $\epsilon_1$ with the one with parameter $\epsilon_2$. Therefore, if this anticommutator is non-degenerate, then this gives an invertible mapping between bosonic states. This is only possible if the intermediate states are non-degenerate and thus there are at least as many fermionic modes as bosonic modes. Starting the argument with the fermions, we arrive at the statement that there are at least as many bosonic modes as fermionic modes. This gives the theorem that the number of bosonic modes and fermionic modes are equal.

This counting depends on the invertibility of the square of supersymmetry. The latter gives in the simple cases just the energy (or time derivative of the fields) such that the theorem applies. In some cases the algebra involves gauge transformations, and the theorem only applies to the gauge-invariant states. In our case the supersymmetries are nilpotent, and the theorem does not apply. In fact the construction (5.19) can be seen to give a vanishing bosonic mode.

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