Anisotropy of the superconducting state properties and phase diagram of MgB$_2$ by torque magnetometry on single crystals

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The angular and temperature dependence of the upper critical field $H_{c2}$ in MgB$_2$ was determined from torque magnetometry measurements on single crystals. The $H_{c2}$ anisotropy $\gamma_H$ was found to decrease with increasing temperature, in disagreement with the anisotropic Ginzburg-Landau theory, which predicts that the $\gamma_H$ is temperature independent. This behaviour can be explained by the two band nature of superconductivity in MgB$_2$. An analysis of measurements of the reversible torque in the mixed state yields a field dependent effective anisotropy $\gamma_{\text{eff}}$, which can be at least partially explained by different anisotropies of the penetration depth and the upper critical field. It is shown that a peak effect in fields of about 0.85 $H_{c2}$ is a manifestation of an order-disorder phase transition of vortex matter. The $H$-$T$ phase diagram of MgB$_2$ for $H \parallel c$ correlates with the intermediate strength of thermal fluctuations in MgB$_2$, as compared to those in high and low $T_c$ superconductors.

1. Introduction

Superconducting MgB$_2$ exhibits a number of rather peculiar properties, originating from the involvement of two sets of bands of different anisotropy and different coupling to the most relevant phonon mode \cite{1,2}. Among them are pronounced deviations of the upper critical field, $H_{c2}$, from predictions of the widely used anisotropic Ginzburg-Landau theory (AGLT).

Apart from two-band superconductivity, MgB$_2$ provides a link between low and high $T_c$ superconductors on a phenomenological level, particularly concerning vortex physics. In both high and low $T_c$ superconductors, for example, a phase transition of vortex matter out of a quasi-ordered “Bragg glass” have been identified, with rather different positions in the $H$-$T$ plane. Studying the intermediate MgB$_2$ may help establishing a “universal vortex matter phase diagram”.

Here, we present a torque magnetometry study of the anisotropic upper critical field, equilibrium magnetization, and the vortex matter phase diagram of single crystalline MgB$_2$ \cite{3}. We will show direct evidence of a temperature dependence of the $H_{c2}$ anisotropy, discuss strong indications of a difference between the anisotropies of the penetration depth and $H_{c2}$, and present the $H$-$T$ phase diagram for $H \parallel c$.

Single crystals were grown with a cubic anvil high pressure technique, described in this issue \cite{4}. Three crystals were used in this study, labeled A, B, and C. Sharp transitions to the superconducting state indicate a high quality of the crystals. An $M(T)$ curve of crystal B with $T_c = 38.2$ K can be found in Ref. \cite{5}.

The torque $\tau = \vec{m} \times \vec{B} \simeq \vec{m} \times \vec{H}$, where $\vec{m}$ is the magnetic moment of the sample, was recorded as a function of the angle $\theta$ between the applied field $\vec{H}$ and the $c-$axis of the crystal in various fixed fields \cite{6}. For measurements close to $T_c$, in fields up to 14 kOe, a non-commercial magnetometer with very high sensitivity was used \cite{7}. For part of these measurements, a vortex-shaking process was employed to speed up the relaxation of the vortex lattice \cite{8}. Crystal A was measured in this...
2. Upper critical field and its anisotropy

Early measurements on polycrystalline or thin film MgB$_2$ samples with various methods and single crystals by electrical transport yielded values of the anisotropy parameter of the upper critical field $\gamma_H = H_{c2}^\parallel / H_{c2}^\perp$ in a wide range of values of $1.1 \leq \gamma_H \leq 9$. More recently, several papers reported a temperature dependence of the $H_{c2}$ anisotropy, ranging between about 5–8 at 0 K and 2–3 close to $T_c$. In this section, we present direct evidence of a temperature dependence of the $H_{c2}$ anisotropy $\gamma_H$ and discuss details of its behaviour, comparing the torque data with numerical calculations.

Four angular torque dependences are shown in Fig. 1. Panels a) and b) correspond to measurements at 22 K. For fields nearly parallel to the $c$-axis, both curves are flat, apart from a small background visible in panel b). Only when $H$ is nearly parallel to the $ab$-plane there is an appreciable torque signal. The curve can be interpreted in a straight-forward way: for $H$ parallel to the $c$-axis the sample is in the normal state, while for $H$ parallel to the $ab$-plane it is in the superconducting state. The crossover angle $\theta_{c2}$ between the normal and the superconducting state is the angle for which the fixed applied field is the upper critical field. From the existence of both superconducting and normal angular regions follows immediately that $H_{c2}^\parallel (22 \text{ K}) < 19 \text{kOe}$ and $85 \text{kOe} < H_{c2}^\perp (22 \text{ K})$. In panel c), on the other hand, the crystal is seen to be in the superconducting state for all values of the angle $\theta$, and therefore $4 \text{kOe} < H_{c2}^\parallel (34 \text{ K})$. Finally, the data in panel d) show only a small background contribution – form and angular regime of the deviation from a straight line are incompatible with a superconducting signal. Therefore, the crystal is here in the normal state for any $\theta$, and we have $H_{c2}^\parallel (34 \text{ K}) < 14 \text{kOe}$.

From figure 1 we therefore have two limitations for the upper critical field anisotropy, hereafter called $\gamma_H$, without any detailed $H_{c2}$ criterion, and without any model fits:

$$\gamma_H (22 \text{ K}) > \frac{85}{19} \approx 4.47; \quad \gamma_H (34 \text{ K}) < \frac{14}{4} = 3.5. \quad (1)$$

These relations show that the upper critical field anisotropy $\gamma_H$ of MgB$_2$ cannot be temperature independent. As an immediate implication, the anisotropic Ginzburg-Landau theory (AGLT) in its standard form does not hold for MgB$_2$. The deviation is strong, within a change of temperature of about 0.3$T_c$, $\gamma_H$ changes, at least, by a fifth of its value.

Although it is clear that AGLT with its effective mass anisotropy model cannot describe the data measured at different temperatures consistently, the detailed analysis of the $\theta$ dependence of $H_{c2}$ we used is based on AGLT. We will show that as long as we stay at a fixed temperature, AGLT is able to describe $H_{c2}(\theta)$ remarkably well [20]. Although the location of $\theta_{c2}$, for example, under these conditions, as first sight, this clarity disappears, when examining the transition region in a scale necessary for the precise determination of $\theta_{c2}$ (see Fig. 1 in Ref. [4]). For an strict analysis, it is necessary to take into account that the transition at $H_{c2}$ is rounded off by fluctuations.

In sufficiently high fields, $H > H_{\text{LLL}}$, the so-called “lowest Landau level” (LLL) approximation was used successfully to describe the effects of fluctuations around $H_{c2}$ [3][22]. In the case of the cuprates, the value of $H_{\text{LLL}}$ and thus of the regime of applicability of the LLL approximation, is a controversial issue (see, e.g., Ref. [23] for a theoretical discussion of the limits of the LLL approximation). However, in the case of MgB$_2$, even using the theoretical criterion of Ref. [23], which led to the high estimation $H_{\text{LLL}} \approx 200 \text{kOe}$ in the case of YBa$_2$Cu$_3$O$_{7-\delta}$, we obtain an upper limit of $H_{\text{LLL}} \approx 5 \text{kOe}$. We therefore used a LLL scaling analysis for the determination of $\theta_{c2}(H)$ or $H_{c2}(\theta)$ [4].

From the resulting $H_{c2}(\theta)$ curve, the anisotropy parameter $\gamma_H$ is then extracted by an analysis
Figure 1. Torque $\tau$ vs. angle $\theta$ of MgB$_2$ single crystal B. The raw data have been antisymmetrized around 90 deg in order to subtract a symmetric background (which was of the order of 0.05−0.2 dyn cm, depending on $H$ and $T$). $\theta_{c2}$ indicates the angle for which the applied field is the upper critical field. The schematic drawing in a) shows the definition of the angle $\theta$. The inset in b) shows the data before antisymmetrizing.

with AGLT, which predicts the angular dependence of the upper critical field to be

$$H_{c2}(\theta) = H_{c2}^{\parallel c} \left( \cos^2 \theta + \frac{\sin^2 \theta}{\gamma_H^2} \right)^{-1/2}.$$  \hspace{1cm} \text{(2)}$$

We note that in the rescaling of the torque according to the LLL fluctuations theory, the target parameter $\gamma_H$ is used, which is obtained only later with Eq. (2). Therefore, scaling analysis and determination of $\gamma_H$ with Eq. (2) had to be performed iteratively in order to self consistently find $\gamma_H$. However, the $\theta_{c2}(H)$ and $H_{c2}(\theta)$ points obtained with the scaling analysis depend not very strongly on the value of $\gamma_H$ used in the scaling and the procedure converges rather fast.

Figure 2 shows the angular dependence of $H_{c2}$ of crystal B and C. The curves shown in the figure are fits of Eq. (2) to the data, showing that the angular dependence of $H_{c2}$ is well described by AGLT at both temperatures. On the other hand, the anisotropy parameter $\gamma_H$ needed to describe the data with Eq. (2) is temperature dependent, as is best seen in the inset. The irreversible properties of the two crystals (B and C) are different in a pronounced way (see Secs. 3 and 4), showing that they have a rather different defect structure. The good agreement both in value and angular dependence of $H_{c2}$ of crystals B and C that is observable in Fig. 2 indicates that such differences in the defect structure do not influence the upper critical field much,
at least in the region between 22 and 34 K, and therefore cannot influence our conclusion of a $T$ dependent $H_{c2}$ anisotropy.

Small, but systematic, deviations from the angular dependence of $H_{c2}$ according to Eq. (2) were observed only at temperatures close to $T_c$. It may indicate that we are approaching $H_{LL}$ in this region and the values of $\theta_{c2}(H)$ and $H_{c2}(\theta)$ obtained from the LLL scaling analysis (see above) start to deviate from the mean field values. The good general approximation of $H_{c2}(\theta)$ by Eq. (3) is in agreement with recent calculations [19]. However, the calculations predict small deviations at low temperatures [25], rather than close to $T_c$. Our experimental limitation of fields up to 90 kOe may prevent the observation of deviations from Eq. (3) at low temperatures.

The upper critical fields parallel and perpendicular to the layers obtained with the scaling analysis and Eq. (2) are shown in Fig. 3a). Results obtained for two crystals measured in two magnetometers are depicted as different symbols. The $T$-dependence of $H_{c2}$ is in agreement with (isotropic) calculations by Helfand et al. [20], with $H_{c2}^{ab}(0) \approx 31$ kOe. On the other hand, $H_{c2}^{ab}(T)$ exhibits a slight positive curvature near $T_c$. These features are common to highly anisotropic (layered) superconductors. Although MgB$_2$ as a whole is rather isotropic, superconductivity is dominant on the quasi-2D bands, which may well account for the different $T$ dependence of $H_{c2}^{ab}$ and $H_{c2}^{\parallel c}$. This may also be the origin of the positive curvature of $H_{c2}$ observed in other measurements of bulk, thin film and single crystal MgB$_2$ [3]. Due to the lack of low $T$ data
and the $\gamma_H(T)$ dependence, only an estimation $180 \text{kOe} \lesssim H_{c2}^{ab}(0) \lesssim 230 \text{kOe}$ can be given.

The anisotropy data [Fig. 3b)] show that $\gamma_H$ systematically decreases with increasing temperature, from $\gamma_H \simeq 6$ at 15 K to 2.8 at 35 K. From the experimental data shown in Fig. 3 we estimate $\gamma_H(T_c) = 2.3 - 2.7$, while at zero temperature, $\gamma_H$ may become as large as 8.

Comparing our data with the data reported by other authors \cite{10,11,12,13,14,15,16,17,18,19}, we note that electrical transport measurements \cite{14,15} yield too high values of $H_{c2}^{ab}$ \cite{11,12,13,14,15,16,17}. All bulk measurements (torque \cite{14}, magnetization \cite{12,13,16,18}, thermal conductivity \cite{11}, and specific heat \cite{13,17}) agree well on the $H_{c2}^{ab}(T)$ dependence and value. Concerning $H_{c2}^{ab}(T)$, and consequently $\gamma_H(T)$, however, reported values differ from each other. Exchanging the samples between different groups could help clarifying, whether the discrepancies of $H_{c2}^{ab}(T)$ values are mainly due to sample differences or due to differences in the experimental methods employed.

Very recently, $H_{c2}^{ab}(T)$, $H_{c2}^{bc}(T)$, and $\gamma_H(T)$, have been calculated for MgB$_2$ \cite{13}. The Fermi surface was modeled as consisting of two separate sheets, approximated as simple spheroids, but with average characteristics taken from first principles calculations. The result of these calculations are compared with our experimental data in Fig. 3. Very good agreement is seen for the upper critical field perpendicular to the layers (||c). Qualitatively, calculations and experiment also agree well for the upper critical field parallel to the layers (||ab). This shows that the essential source of the deviations of the upper critical field from AGLT predictions is captured with a simple effective two band model, while further details of the Fermi surface and superconducting gap are negligible. In Fig. 3 we see good quantitative agreement between experimental data and the theoretical curve between 20 and 25 K.

The deviations at lower $T$ may, on the one hand, be due to a decreased the accuracy of our analysis because the field limitation of 90 kOe restricts the angular range where $H_{c2}$ data could be obtained. This can lead to deviations larger than the estimated error bars, especially since the theoretical calculations indicate deviations of the $H_{c2}(\theta)$ dependence from the prediction of Eq. 2 at low $T$ \cite{25}. On the other hand, $H_{c2}$ at low temperatures depends on the shape of the Fermi surface in rather subtle manner, and the model Fermi surface used for the calculations \cite{13} may be too simple for a quantitatively correct description at low $T$. The deviations at higher $T$ may be due to the limitations of the LLL scaling approach in low fields, and or due to the influence of disorder, which is not accounted for in the calculations.

Close to $T_c$, non-locality is not important, and consequently, AGLT is expected to hold even if this is not the case at lower $T$. Despite of this, Fig. 3 clearly indicates that the variation of $\gamma_H$ with temperature is the strongest close to $T_c$. Therefore, in MgB$_2$, AGLT seems to have a very limited range of applicability indeed.

3. Reversible and irreversible torque below $H_{c2}$

An alternative method to obtain the anisotropy parameter $\gamma$ of a superconductor, used often and with success in the case of cuprates \cite{22,23}, consists of measuring the torque, as a function of angle, well below $H_{c2}$, and analyzing the data with a formula developed by Kogan \cite{29}:

$$\tau = -\frac{\Phi_0 HV}{64\pi^2\lambda_{ab}^2} \left(1 - \frac{1}{\gamma_{\text{eff}}^2} \right) \sin 2\theta \ln \left(\frac{\eta H_{c2}^{ab}}{\epsilon(\theta) H}\right), \quad (3)$$

where $\epsilon(\theta) = (\cos^2 \theta + \sin^2 \theta / \gamma_{\text{eff}}^2)^{1/2}$, $\gamma_{\text{eff}} = (m_0^*/m_{ab}^*)^{1/2}$ is the effective mass anisotropy, $\lambda_{ab}$ is the in-plane penetration depth, $V$ is the volume of the crystal, $\Phi_0$ is the flux quantum, and $\eta$ is a constant of the order of unity depending on the vortex lattice structure. Equation (3) is valid in the limits of fields $H_{c1} \ll H \ll H_{c2}$ and not too close to $T_c$. A further restriction is that Eq. (3) describes the reversible torque only. To obtain the true reversible torque, we employed a vortex-shaking process \cite{30}. In the investigated field and temperature region, the shaken torque was found to be well reversible.

In Fig. 4, normalized torque $\tau/\tau_{\text{max}}$ vs angle $\theta$ curves, measured in different fields at 34 K, are compared. Increasing $H$ from 2 [panel a)] to
1.) A few points are worth to be emphasized: temperatures from 1 to 10 kOe and from 27 to 36 K and of the upper critical field, \( \gamma_H \), differ in general. Calculations of \( \gamma_\lambda \) of MgB\(_2\) [32,1] indeed found values much lower than the upper critical field anisotropy values. There is also experimental support for a low \( \gamma_\lambda \) [2].

In Eq. (3), \( \gamma_{\text{eff}} \) appears twice, and in a first approximation [33], the appearance outside of the logarithm can be thought of as due to the \( \lambda \) anisotropy, while the appearance in the logarithm is linked to the \( H_{c2} \) anisotropy. A corresponding calculation with different (fixed) \( \gamma_\lambda \) and \( \gamma_H \) yields [34] a field dependent (common) effective anisotropy \( \gamma_{\text{eff}} \) similar to the experimental observations.

Field dependent point-contact spectroscopy [35] and specific heat [36] measurements indicate that the small gap disappears in fields of the order of 4–5 kOe, i.e., superconductivity in the (3D) \( \pi \) Fermi sheets is rapidly suppressed by even low fields, whereas it persists in the (2D) \( \sigma \) sheets up to much higher fields. This should result in an increase of the effective (bulk) anisotropy with increasing \( H \). Further studies are needed for a complete understanding of the detailed interplay of the effects described above.

In the “unshaked” torque data of crystals A and B, a pronounced peak in the irreversible torque for field alignments close to \( H||ab \), was observed (see, e.g., upper inset of Fig. 5 of Ref. [11]). It is tempting to ascribe this feature, also observed by other authors [17], to “intrinsic pinning”, in analogy to observations on strongly anisotropic cuprate superconductors. However, the observation of such “intrinsic pinning” in MgB\(_2\) is rather counter-intuitive, since the “intrinsic pinning” is mostly determined by the ratio of the \( c \)-axis coherence length to the separation of the superconducting layers, which is much larger in MgB\(_2\) than in the case of cuprates in the region where “intrinsic pinning” is commonly observed.

From the analysis of reversible torque data for crystal A measured in the range of fields and temperatures from 1 to 10 kOe and from 27 to 36 K [10], a few points are worth to be emphasized: 1.) \( \gamma_{\text{eff}} \) is field dependent, increasing nearly linearly from 2 in zero field to 3.7 in 10 kOe. 2.) No clear \( T \) dependence is visible between 27 and 36 K. 3.) The effective anisotropy \( \gamma_{\text{eff}} \), as obtained from the analysis with Eq. (3) is different from the \( H_{c2} \) anisotropy \( \gamma_H \).

Especially concerning point 3.), it is important to recognize that, also theoretically, the anisotropy \( \gamma_{\text{eff}} \) is not necessarily the same as the \( H_{c2} \) anisotropy \( \gamma_H \). When AGLT is not applicable, the anisotropies of the penetration depth, \( \gamma_\lambda \), and of the upper critical field, \( \gamma_H \), differ in general. Calculations of \( \gamma_\lambda \) of MgB\(_2\) [32,1] indeed found values much lower than the upper critical field anisotropy values. There is also experimental support for a low \( \gamma_\lambda \) [2].

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5 kOe [panel b)] leads to an unexpectedly large shift of the maximum torque towards 90 deg, which may indicate an increase of the anisotropy \( \gamma_{\text{eff}} \) with increasing \( H \). This is confirmed by the analysis of the data with Eq. (3). The best agreements of the equation with the data are obtained for \( \gamma_{\text{eff}} = 2.4 \) in 2 kOe and \( \gamma_{\text{eff}} = 3.0 \) in 5 kOe (full curves in Fig. 4). Although descriptions with \( \gamma_{\text{eff}} = 3.0 \) in 2 kOe or \( \gamma_{\text{eff}} = 2.4 \) in 5 kOe are also possible without obvious discrepancies to the data, the corresponding qualities of the fit as expressed by the parameter \( \chi^2 \) are worse by more than an order of magnitude in both cases.

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The presence of the PE in two crystals with such small number of stacking faults in the former one. The irreversibility field change is indicated by arrows. Inset: $\tau/H$ vs angle $\theta$ in 75 kOe at 15 K (crystal B) b) magnification of the peak effect region of crystal B. The irreversibility field $H_{\text{irr}}$ and the onset and maximum fields $H_{\text{on}}$ and $H_{\text{max}}$ of the PE for the $H$ increasing ($\uparrow$) and decreasing ($\downarrow$) branch are marked. Inset: Curve obtained under the same external conditions for crystal C.

by torque magnetometry (see, e.g., [28]). The apparent paradox is resolved by further measurements: torque measurements on crystal C with the same conditions show no sign of “intrinsic” pinning for $H\parallel ab$ [38, 34], indicating the extrinsic origin of the feature. The most likely cause of the peak in the irreversible torque for $H\parallel ab$ is a small amount of stacking faults. It may indicate the presence of some stacking faults in crystals A and B, while they would seem to be absent in crystal C [39].

4. Peak effect and vortex matter phase diagram

In low $T_c$ superconductors, such as NbSe$_2$, the order-disorder transition is signified experimentally by a peak effect (PE) in the critical current density [40]. We observed such a PE by torque measurements on MgB$_2$ single crystals B and C, both in $\tau(\theta)$ and $\tau(H)$ measurements, as can be seen in Fig. 6. In Sec. 3 we have noticed that crystals B and C behave quite differently for $H\parallel ab$, which may be due to the presence of a small number of stacking faults in the former one. The presence of the PE in two crystals with such pronounced differences strongly indicates that the PE, or rather it’s underlying mechanism, is an intrinsic feature of MgB$_2$. A study with a “minor hysteresis loop” technique on crystal B [4] revealed a history dependent critical current density in the PE region, compatible with and expected for the behaviour at the order-disorder transition.

Figure 6 shows the irreversible part $\Delta \tau(H) = \tau(H^\downarrow) - \tau(H^\uparrow)$ of the torque, scaled by $H \sin 2\theta$, vs field, at 18 K for various angles. The scaling was chosen to minimize the angle and field dependence intrinsic to the torque. Since the peak is not visible at all temperatures and angles as well as in Fig. 6, onsets and maxima were determined from irreversible torque curves as those shown in Fig. 6. $H_{\text{on}}$ was defined as the field, where the irreversible torque starts to deviate from a straight line behaviour, as indicated in the figure for the curve measured at 71.5 deg. $H_{\text{on}}$, defined in this way, is close to $H_{\text{on}}$ as indicated in Fig. 61b). However, we note that with the determination of onsets and maxima from the irreversible torque, the fine details of the differences in the field increasing and decreasing branch of the hysteresis loops...
The curve measured at $71^\circ$ for the determination of the irreversibility line. However, the smaller ratio of disorder present in this crystal than in crystal B. In crystal C, the location in higher fields of the peak effect are not affected by stacking faults, the extent of hysteresis may be. The difference of how pronounced the peaks of crystals B and C are [see Fig. 5b] supports such a scenario. The location in higher fields of the peak effect in crystal C indicates that there is less point-like disorder present in this crystal than in crystal B. However, the smaller ratio $\tau_{irr}(H_{max})/\tau_{irr}(H_{on})$ in crystal C, compared to crystal B, is difficult to explain with only one sort of disorder. Individual strong pinning, e.g., by sparse stacking faults, should be much more efficient in the disordered phase than in the Bragg glass with its nearly perfect ordered lattice [41].

If the peak height observed in crystal B is affected by stacking faults, a more pronounced PE close to $H||ab$ is natural, since the pinning efficiency of stacking faults, similar to twin boundary pinning, is strongly direction-dependent [42].

On the other hand, the peak height can be influenced by the natural angle dependence of the torque, despite the scaling made. This is because...
Figure 8. Vortex matter phase diagram of MgB$_2$ (crystal B) for $H \parallel c$. Data points shown are from the projection of the torque vs field data presented in Ref. [5] and the projection of additional $\tau(\theta)$ measurements in fixed $H$. The dashed line is a calculation [19] of the $H_{c2}$ dependence (cf. Fig. 3), the dotted lines are guides for the eye. The different phases of vortex matter are labeled (see text).

The angular dependence of the onsets and maxima of the PE tracks the one of $H_{c2}$, i.e., it follows Eq. (2). This indicates that the PE (or rather its underlying mechanism) is a feature for all directions of the applied field, and not just of the angular region where it is readily discernible. To directly check the situation for $H \parallel c$ and $H \parallel ab$, where torque measurements are not possible, SQUID and ac susceptibility measurements were performed.

In Fig. 8, we compare measurement curves obtained on crystal B at 20 K, using different experimental techniques. Torque measurements performed at an angle of 77.5° show [Fig. 8a)] a clearly discernible PE located in a field of about 60 kOe. Scaled with Eq. (2) to $H \parallel c$, this corresponds to 15 to 20 kOe. As can be seen in Fig. 8b), there is no sign of a PE observable in SQUID data in this field region. Generally, no sign of a peak effect was observed by SQUID magnetometry at any temperature, for both field directions. This is likely due to insufficient sensitivity of the SQUID. In ac susceptibility data [Fig. 8c)], on the other hand, a PE is visible for $H \parallel c$ in the appropriate field region. A report of the ac susceptibility results will be published elsewhere [17]. A PE in MgB$_2$ was also reported recently by other authors, in the case of $H \parallel c$ from transport data [13,17] and ac susceptibility with a local Hall probe [17,18], in the case of $H \parallel ab$ from transport data [17].

The phase diagram for $H \parallel c$ obtained from torque magnetometry, based on both $\tau(\theta)$ and $\tau(H)$ measurements and the angular scaling of Eq. (2) is presented in Fig. 8. The magnitude of the peaks is reduced quickly by increasing the temperature, and above 27 K, the PE is no longer discernible in the torque data. This is due to the decreased sensitivity of the torque magnetometer in lower fields and due to thermal smearing of the effective pinning potential. In a recent report of low frequency ac susceptibility measurements [17], the peak effect was observed for $H \parallel c$ at temperatures up to about 25 K, and interpreted in terms of the order-disorder transition as well. In Ref. [45], the transformation of the PE into a “step-like” ac susceptibility is reported for the temperature interval between 25 and 27.5 K, and interpreted as a signature of thermal melting. In our case, no step-like feature in the reversible torque was observed in the continuation of the PE. It should be emphasized, that thermal melting so far below $H_{c2}$ would be at odds [5] with theoretical expectations [10].

The equilibrium order-disorder transition, which corresponds to $H_{\text{max}}$, is located in fields of about 0.85 $H_{c2}$ in crystal B and in about 0.9 $H_{c2}$ in crystal C. The peak effect observed in other crystals by transport was reported to be located even closer to $H_{c2}$ [13,17]. These differences are natural for a disorder-induced phase transition in crystals with varying degrees of disorder. Form and location of the PE observed in MgB$_2$ resembles results obtained on NbSe$_2$ single crys-
tals with varying degrees of disorder but are rather different from the order-disorder transition in cuprate superconductors.

5. Conclusions

In summary, studying the anisotropic superconducting state properties of MgB$_2$ revealed a strong temperature dependence of the upper critical field anisotropy $\gamma_H$, and indicated a difference of the anisotropies of the penetration depth and the upper critical field. These findings, which imply a breakdown of the standard form of the widely used anisotropic Ginzburg-Landau theory in MgB$_2$, can be explained by superconductivity in this compound involving two band systems of different dimensionality, in accordance with microscopic studies.

A pronounced peak effect in the magnetic hysteresis is a signature of an “order-disorder” transition of vortex matter, similar to transitions in both high $T_c$ cuprate and low $T_c$ superconductors. Despite the intermediate importance of thermal fluctuations in MgB$_2$, the phase diagram resembles quite closely the one of the low $T_c$ superconductor NbSe$_2$. On the other hand, chances of a proper identification of mainly thermally induced melting at higher temperatures are better in MgB$_2$, due to the increased thermal fluctuations.

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