Multi-objective reasoning with constrained goal models

Chi Mai Nguyen 1 • Roberto Sebastiani 1 • Paolo Giorgini 1 • John Mylopoulos 1

Abstract Goal models have been widely used in computer science to represent software requirements, business objectives, and design qualities. Existing goal modelling techniques, however, have shown limitations of expressiveness and/or tractability in coping with complex real-world problems. In this work, we exploit advances in automated reasoning technologies, notably satisfiability and optimization modulo theories (SMT/OMT), and we propose and formalize: (1) an extended modelling language for goals, namely the constrained goal model (CGM), which makes explicit the notion of goal refinement and of domain assumption, allows for expressing preferences between goals and refinements and allows for associating numerical attributes to goals and refinements for defining constraints and optimization goals over multiple objective functions, refinements, and their numerical attributes; (2) a novel set of automated reasoning functionalities over CGMs, allowing for automatically generating suitable refinements of input CGMs, under user-specified assumptions and constraints, that also maximize preferences and optimize given objective functions. We have implemented these modelling and reasoning functionalities in a tool, named CGM-Tool, using the OMT solver OptiMathSAT as automated reasoning backend. Moreover, we have conducted an experimental evaluation on large CGMs to support the claim that our proposal scales well for goal models with 1000s of elements.

Keywords Requirements engineering • Goal models • SAT/SMT/OMT

1 Introduction

The concept of goal has long been used as useful abstraction in many areas of computer science, for example artificial intelligence planning [29], agent-based systems [34], and knowledge management [21]. More recently, software engineering has also been using goals to model requirements for software systems, business objectives for enterprises, and design qualities [2, 3, 11, 18, 42].

Goal-oriented requirements engineering approaches have gained popularity for a number of significant benefits in conceptualizing and analysing requirements [42]. Goal models provide a broader system engineering perspective compared to traditional requirements engineering methods, a precise criterion for completeness of the requirements analysis process, and rationale for requirements specification, as well as automated support for early requirements analysis. Moreover, goal models are useful in explaining requirements to stakeholders, and goal refinements offer an accessible level of abstraction for validating choices among alternative designs.

Current goal modelling and reasoning techniques, however, have limitations with respect to expressiveness and/or scalability. Among leading approaches for goal modelling, KAOS offers a very expressive modelling language but reasoning isn’t scalable (in fact, it is undecidable). i*, on the other hand, is missing constructs such as...
preferences, priorities and optimization goals. Although more recent proposals, such as Techno [22, 26], offer expressive extensions to goal models, they still lack some features of our proposal, notably optimization goals, and also lack scalable reasoning facilities.

As a result of these deficiencies, no goal modelling framework can express goals such as “Select which new requirements to implement for the next release, such as to optimize customer value while maintaining costs below some threshold” and be able to reason about it and generate a specification/solution for it. As another example, consider a situation where a goal model changes and a new specification/solution needs to be generated for the new goal model. In this case, the new specification may be required to fulfil the evolution goal “Minimize implementation effort” or “Maximize user familiarity by changing as little as possible the new functionality of the system relative to the old one”. (For the latter case, see also [30].) In both cases, we are dealing with requirements that are beyond the state of the art for goal modelling and reasoning. As we will discuss in Sect. 3, our proposal can accommodate such requirements with respect to both modelling and scalable reasoning.

We are interested in advancing the state of the art in goal models and reasoning by proposing a more expressive modelling language that encompasses many of the modelling constructs proposed in the literature, and at the same time offers sound, complete and tractable reasoning facilities. We are aiming for a goal modelling language in the spirit of Sebastiani et al. [36], rather than a social dependencies modelling language, such as i*. To accomplish this, we exploit advances in automated reasoning technologies, notably satisfiability modulo theories (SMT) [5] and optimization modulo theories (OMT) [38], to propose and formalize an extended notion of goal model, namely constrained goal model (CGM).

CGMs treat (AND/OR) refinements as first-class citizens allowing associated constraints, such as Boolean formulas or SMT/OMT formulas. For instance, when modelling a meeting scheduling system, we may want to express the fact that, to fulfil the nice-to-have requirement of keeping the scheduling fast enough (e.g. strictly <5 h), and we cannot afford both the time-consuming tasks of performing the schedule manually (3 h) and of calling the participant one by one by phone (2 h). CGMs provide user-friendly constructs by which we can encode constraints like this, either by adding Boolean formulas on the propositions which label such requirement and tasks, or by associating to those propositions numerical variables and by adding SMT formulas encoding mixed Boolean-arithmetical constraints on those variables and propositions. (See Sect. 3.)

To the best of our knowledge, this was not possible with previous goal modelling techniques, including that in [36]. At the same time, the CGM tool we developed can cope with goal models an order of magnitude beyond what has been reported in the literature in most cases. In some cases involving optimization goals, e.g. “minimize development costs for the next release of software product S”, the CGM tool performs more modestly, but can still handle models of size in the 100s of elements.

The main contributions of this work include:

I. An integration within one modelling framework of constructs that have been proposed in the literature in a piecemeal fashion, specifically,

(i) Allow for explicit labelling of goal refinements with Boolean propositions that can be interactively/automatically reasoned upon;

(ii) Provide an explicit representation of domain assumptions to represent preconditions to goals;

(iii) Allow for Boolean constraints over goals, domain assumptions, and refinements;

(iv) Provide a representation of preferences over goals and their refinements, by distinguishing between mandatory and nice-to-have requirements and by assigning preference weights (i.e. penalties/rewards) to goals and domain assumptions. Alternatively, preferences can be expressed explicitly by setting binary preference relations between pairs of goals and pairs of refinements;

(v) Assign numerical attributes (e.g. resources like cost, worktime, and room) to goals and/or refinements and define constraints and multiple objective functions over goals, refinements and their numerical attributes.

(vi) Define optimization goals over numerical attributes, such as cost or customer value;

II. Fully support automated reasoning over CGMs that is both sound and complete, i.e. returns only solutions that are consistent with CGM semantics, and all such solutions;

III. Establish that reasoning with CGM models is scalable with models including thousands of elements.

Taking advantage of CGMs’ formal semantics and the expressiveness and efficiency of current SMT and OMT solvers, we also provide a set of automated reasoning functionalities on CGMs. Specifically, on a given CGM, our approach allows for:
(a) the automatic check of the CGM’s realizability (i.e., check whether the goal model has any solution);
(b) the interactive/automatic search for realizations;
(c) the automatic search for the “best” realization in terms of penalties/rewards and/or of user-defined preferences;
(d) the automatic search for the realization(s) which optimize given objective functions.

Our approach is implemented as a tool (CGM-Tool), a stand-alone Java application based on the eclipse RCP engine. The tool offers functionalities to create CGM models as graphical diagrams and to explore alternatives scenarios running automated reasoning techniques. CGM-Tool uses the SMT/OMT solver OptiMathSAT [38–40], which is built on top of the SMT solver MathSAT5 [8], as automated reasoning backend.¹

The structure of the paper is as follows: Sect. 2 provides a succinct account of necessary background on goal modeling and on SMT/OMT; Sect. 3 introduces the notion of CGM through an example; Sect. 4 introduces the syntax and semantics of CGMs; Sect. 5 presents the set of automated reasoning functionalities for CGMs; Sect. 6 gives a quick overview of our tool based on the presented approach; Sect. 7 provides an experimental evaluation of the performances of our tool on large CGMs, showing that the approach scales well with respect to CGM size; Sect. 8 gives overview of related work, while in Sect. 9 we draw conclusions and present future research challenges.

2 Background

Our research baseline consists of our previous work on qualitative goal models and of Satisfiability and Optimization Modulo Theories (SMT and OMT, respectively). Our aim in this section is to introduce the necessary background notions on these topics, so that the reader can follow the narrative in subsequent sections. As prerequisite knowledge, we assume only that the reader is familiar with the syntax and semantics of standard Boolean logic and of linear arithmetic over the rationals.

2.1 Goal models

Qualitative goal models are introduced in [28], where the concept of goal is used to represent, respectively, a functional and non-functional requirement in terms of a proposition. A goal can be refined by means of AND/OR refinement relationships and qualitative evidence (strong and weak) for/against the fulfillment of a goal is provided by contribution links labelled +, −, etc. In [18], goal models are formalized by replacing each proposition g, standing for a goal, by four propositions (FSg, PSg, PDg, FDg) representing full (and partial) evidence for the satisfaction/denial of g. A traditional implication such as (p ∧ q) → r is then translated into a series of implications connecting these new symbols, including (FSp ∧ FSq) → FSr, (PSp ∧ PSq) → PSr, as well as FDp → FDr, FDq → FDr. The conflict between goals a and b is captured by axioms of the form FSp → FDr, and it is consistent to have both FSp and FDr evaluated to true at the same time. As a result, even though the semantics of a goal model is a classical propositional theory, inconsistency does not result in everything being true. In fact, a predicate g can be assigned a subset of truth values {FS, PS, FD, PD}.

Sebastiani et al. [36] extended the approach further by including axioms for avoiding conflicts of the form FSa ∧ FDA. The approach recognized the need to formalize goal models so as to automatically evaluate the satisfiability of goals. These goal models, however, do not incorporate the notion of conflict as inconsistency; they do not include concepts other than goals, cannot distinguish “nice-to-have” from mandatory requirements and have no notion of a robust solution, i.e., solution without “conflict”, where a goal cannot be (fully or partially) denied and (respectively, fully or partially) satisfied at the same time.

2.2 Satisfiability and optimization modulo theories

Satisfiability modulo theories (SMT) is the problem of deciding the satisfiability of a quantifier-free first-order formula Φ with respect to some decidable theory T (see [5, 35]). In this paper, we focus on the theory of linear arithmetic over the rationals, LRA: SMT(LRA) is the problem of checking the satisfiability of a formula Φ consisting in atomic propositions A1, A2,… and linear-arithmetic constraints over rational variables like “(2.1x1 − 3.4x2 + 3.2x3 ≤ 4.2)”, combined by means of Boolean operators ¬, ∧, ∨, →, ↔. (Notice that a Boolean formula is also a SMT(LRA) formula, but not vice versa.) An LRA-interpretation μ is a function which assigns truth values to Boolean atoms and rational values to numerical variables; μ satisfies Φ in LRA , written μ |= Φ”—aka, μ is a solution for Φ in LRA—if μ makes the formula Φ evaluate to true; Φ is LRA-satisfiable iff it has at least one LRA-interpretation μ s.t. μ |= Φ.

An optimization modulo theories over LRA (OMT(LRA)) problem {Φ, {obj1, ..., objk}} is the problem of finding solution(s) to an SMT(LRA) formula Φ which optimize the rational-valued objective functions

¹ The OMT solver OptiMathSAT can be used also as an SMT solver if no objective function is set: in such case, it works as a wrapper of MathSAT5.
3 Constrained goal models

The narrative of the next three sections is in line with the following schema.

In this section (Sect. 3), we introduce the notions of constrained goal model (CGM), and of realization of a CGM; we also present the automated reasoning functionalities of our CGM-Tool through a meeting scheduling example (Fig. 1), without getting into the formal details yet.

In Sect. 4, we present the abstract syntax and semantics of CGMs, defining formally the building blocks of a CGM and of its realizations, to which the reader has already been introduced informally in Sect. 3.

In Sect. 5, we describe how to support automated reasoning functionalities on CGMs by encoding them into SMT and OMT. We first show how to encode a CGM \( M \) into a SMT\((\mathcal{LRA})\) formula \( \Psi_M \), so that the search for an optimum realization of \( M \) reduces to an OMT\((\mathcal{LRA})\) problem over the formula \( \Psi_M \), which is then fed to an OMT solver. Then we present the reasoning functionalities over CGMs we have implemented on top of our OMT solver.

3.1 The CGM backbone: goals, refinements, and domain assumptions

We model the requirements for a meeting scheduling system, including the functional requirement ScheduleMeeting, as well as non-functional/quality requirements LowCost, FastSchedule, MinimalEffort,
and \textit{GoodQualitySchedule}. They are represented as root goals.

Notationally, round-corner rectangles (e.g. ScheduleMeeting) are root goals, representing stakeholder requirements; ovals (e.g. CollectTimetables) are intermediate goals; hexagons (e.g. CharacteriseMeeting) are tasks, i.e. non-root leaf goals; rectangles (e.g. ParticipantsUseSystemCalendar) are domain assumptions. We call elements both goals and domain assumptions. Labelled bullets at the merging point of the edges connecting a group of source elements to a target element are refinements [e.g. \( \text{GoodParticipation} \rightarrow \text{GoodQualitySchedule} \)], while the \( R \)'s denote their labels.

\textbf{Remark 1} Unlike previous goal modelling proposals, refinements are explicitly labelled, so that stakeholders can refer to them in relations, constraints and preferences. (This fact will be eventually discussed with more details.) The label of a refinement can be omitted when there is no need to refer to it explicitly.

Intuitively, requirements represent desired states of affairs we want the system-to-be to achieve (either mandatorily or preferably); they are progressively refined into intermediate goals, until the process produces actionable goals (tasks) that need no further decomposition and can be executed; domain assumptions are propositions about the domain that need to hold for a goal refinement to work. Refinements are used to represent alternatives of how to achieve a non-leaf element, i.e. a refinement of an element represents one of the alternative of sub-elements that are necessary to achieve it.

The principal aim of the CGM in Fig. 1 is to achieve the requirement \textit{ScheduleMeeting}, which is mandatory. (A requirement is set to be mandatory by means of user assertions.) \textit{ScheduleMeeting} has only one candidate refinement \( R_1 \), consisting in five sub-goals: Characterise Meeting, CollectTimetables, FindASuitableRoom, ChooseSchedule, and ManageMeeting. Since \( R_1 \) is the only refinement of the requirement, all these sub-goals must be satisfied in order to satisfy it. There may be more than one way to refine an element; e.g. CollectTimetables is further refined either by \( R_{10} \) into the single goal ByPerson or by \( R_2 \) into the single goal BySystem. Similarly, FindASuitableRoom and ChooseSchedule have three and two possible refinements, respectively. The sub-goals are further refined until they reach the level of domain assumptions and tasks.

The requirements that are not set to be mandatory are “nice-to-have” ones, like LowCost, MinimalEffort, FastSchedule, and GoodQualitySchedule (in blue in Fig. 1). They are requirements that we would like to fulfil with our solution, provided they do not conflict with other requirements.

\subsection{3.2 Boolean constraints: relation edges, Boolean formulas, and user assertions}

Importantly, in a CGM, elements and refinements are enriched by user-defined \textit{Boolean constraints}, which can be expressed either graphically as \textit{relation edges}, or textually as \textit{Boolean or SMT(\( \mathcal{L} \mathcal{R} \mathcal{A} \)) formulas}, or as \textit{user assertions}.

\subsection{3.3 Relation edges}

We have three kinds of relation edges. \textit{Contribution edges} \( “E_i \xrightarrow{\text{+}} E_j” \) between elements (in green in Fig. 1), like “ScheduleAutomatically \( \xrightarrow{\text{++}} \text{MinimalConflicts} \)”, mean that if the source element \( E_i \) is satisfied, then also the target element \( E_j \) must be satisfied (but not vice versa). \textit{Conflict edges} \( “E_i \xrightarrow{\text{--}} E_j” \) between elements (in red), like “ConfirmOccurrence \( \xrightarrow{\text{+}} \text{CancelMeeting} \)”, mean that \( E_i \) and \( E_j \) cannot be both satisfied. \textit{Refinement bindings} \( “R_i \xleftarrow{\text{}} R_j” \) between two refinements (in purple), like “\( R_2 \xleftarrow{\text{}} R_7” \), are used to state that, if the target elements \( E_i \) and \( E_j \) of the two refinements \( R_i \) and \( R_j \), respectively, are both satisfied, then \( E_i \) is refined by \( R_i \) if and only if \( E_j \) is refined by \( R_j \). Intuitively, this means that the two refinements are bound, as if they were two different instances of the same global choice.

For instance, in Fig. 1, the refinements \( R_2 \) and \( R_7 \) are bound because such binding reflects a global choice between a manual approach and an automated one.

\subsection{3.4 Boolean formulas}

It is possible to enrich CGMs with Boolean formulas, representing arbitrary constraints on elements and refinements. Such constraints can be either \textit{global} or \textit{local to elements or to refinements}, that is, each goal \( G \) can be tagged with a pair of \textit{prerequisite} formulas \( \{ \phi_G^+, \phi_G^- \} \) called \textit{positive and negative} prerequisite formulas, respectively—so that \( \phi_G^+ \) (resp. \( \phi_G^- \)) must be satisfied when \( G \) is satisfied (resp. denied). (The same holds for each requirement \( R \.).

For example, to require that, as a prerequisite for FastSchedule, ScheduleManually, and CallParticipants cannot be both satisfied, one can add a constraint to the positive prerequisite formula of FastSchedule:
\( \phi_{\text{FastSchedule}} \overset{\text{def}}{=} \ldots \land \neg (\text{ScheduleManually} \land \text{CallParticipants}), \)  
(1)

or, equivalently, add globally to the CGM the following Boolean formula:

\[ \text{FastSchedule} \implies \neg (\text{ScheduleManually} \land \text{CallParticipants}). \]  
(2)

Notice that there is no way we can express (1) or (2) with the relation edges above.

### 3.5 User assertions

With CGM-Tool, one can interactively mark [or unmark] requirements as satisfied (true), thus making them mandatory (if unmarked, they are nice-to-have ones). In our example, \( \text{ScheduleMeeting} \) is asserted as true to make it mandatory, which is equivalent to add globally to the CGM the unary Boolean constraint:

\[ (\text{ScheduleMeeting}). \]  
(3)

Similarly, one can interactively mark/unmark (effortful) tasks as denied (false). More generally, one can mark as satisfied or denied every goal or domain assumption. We call these marks user assertions, because they correspond to asserting that an element must be true, i.e. it is part of the solutions we are interested in, or false, i.e. we are interested in solutions that do not include it.

Notice that the process of marking/unmarking elements is conceived to be more interactive than that of adding/dropping relation edges or constraints.

### 3.6 Arithmetical constraints: numerical attributes and SMT(\(LRA\)) formulas

#### 3.6.1 Numerical attributes

In addition to Boolean constraints, it is also possible to use numerical variables to express different numerical attributes of elements (such as cost, worktime, space, and fuel) and to add arithmetical constraints in the form of SMT(\(LRA\)) formulas over such numerical variables.

For example, suppose we estimate that fulfilling \( \text{UsePartnerInstitutions} \) costs 80€, whereas fulfilling \( \text{UseHotelsAndConventionCenters} \) costs 200€. With CGM-Tool, one can express these facts straightforwardly by adding a global numerical variable \( \text{cost} \) to the model.

Then, for every element \( E \) in the CGM, CGM-Tool automatically generates a numerical variable \( \text{cost}_E \) representing the attribute \( \text{cost} \) of the element \( E \), and it adds the following default global constraint and prerequisite formulas:

\[
\left( \text{cost} = \sum_E \text{cost}_E \right),
\]
(4)

for every element \( E, \phi_E^+ \overset{\text{def}}{=} \ldots \land (\text{cost}_E = 0) \)  
(5)

\[
\phi_E^- \overset{\text{def}}{=} \ldots \land (\text{cost}_E = 0),
\]
(6)

that set the default value 0 for each \( \text{cost}_E \). (Notice that (4) is a default global constraint: the user is free to define his/her own objective functions.) Eventually, for the elements \( E \) of interest, one can set a new value for \( \text{cost}_E \) in case \( E \) is satisfied, e.g. \( \text{cost}_{\text{UsePartnerInstitutions}} := 80 \)€ and \( \text{cost}_{\text{UseHotelsAndConventionCenters}} := 200 \)€. When so, CGM-Tool automatically updates the values in the positive prerequisite formulas (5), e.g.

\[
\phi_{\text{UsePartnerInstitutions}}^+ \overset{\text{def}}{=} \ldots \land (\text{cost}_{\text{UsePartnerInstitutions}} = 80)
\]
(7)

\[
\phi_{\text{UseHotelsAndConventionCenters}}^- \overset{\text{def}}{=} \ldots \land (\text{cost}_{\text{UseHotelsAndConventionCenters}} = 200),
\]

whereas the corresponding constraint (6) is not changed. Similarly, one can set a new value for \( \text{cost}_E \) in case \( E \) is denied by updating the values in the negative prerequisite formulas (6).

**Remark 2** Notationally, we use variables and formulas indexed by the element they belong to (e.g. \( \text{cost}_{\text{UsePartnerInstitutions}} \) and \( \phi_{\text{UsePartnerInstitutions}}^+ \)) rather than attribute variables and formulas of the elements in an object-oriented notation (e.g. \( \text{UsePartnerInstitutions}.\text{cost} \) and \( \text{UsePartnerInstitutions}.\phi^+ \)) because they are more suitable to be used within the SMT(\(LRA\)) encodings (Sects. 4, 5).

#### 3.6.2 SMT(\(LRA\)) formulas

Suppose that, in order to achieve the nice-to-have requirement \( \text{LowCost} \), we need to have a total cost smaller than 100€. This can be expressed by adding to \( \text{LowCost} \) the prerequisite formula:

\[
\phi_{\text{LowCost}}^- = \ldots \land (\text{cost} < 100).
\]
(8)

Hence, e.g. due to (4)–(8), \( \text{LowCost} \) and \( \text{UseHotelsAndConventionCenters} \) cannot be both satisfied, matching the intuition that the latter is too expensive to comply to the nice-to-have \( \text{LowCost} \) requirement.

Similarly to \( \text{cost} \), one can introduce, for example, another global numerical attribute \( \text{workTime} \) to reason on working time and estimate, for example, that the total working time for \( \text{ScheduleManually}, \text{ScheduleAutomatically}, \text{EmailParticipants}, \text{CallParticipants}, \text{CollectFromSystemCalendar} \) are 3, 1,
1, 2, and 1 h(s), respectively, and state that the nice-to-have requirement FastSchedule must require a global time smaller than 5 h. As a result of this process, the system will produce the following constraints.

\[
\text{workTime} = \sum E \text{workTime}_E
\]

\[
\phi^\dagger_{\text{FastSchedule}} \triangleq \ldots \land \text{(workTime} < 5) \tag{10}
\]

\[
\phi^\dagger_{\text{ScheduleManually}} \triangleq \ldots \land \text{(workTime} \text{ScheduleManually} = 3) \tag{11}
\]

\[
\phi^\dagger_{\text{ScheduleAutomatically}} \triangleq \ldots \land \text{(workTime} \text{ScheduleAutomatically} = 1) \tag{12}
\]

\[
\phi^\dagger_{\text{EmailParticipants}} \triangleq \ldots \land \text{(workTime} \text{EmailParticipants} = 1) \tag{13}
\]

\[
\phi^\dagger_{\text{CallParticipants}} \triangleq \ldots \land \text{(workTime} \text{CallParticipants} = 2) \tag{14}
\]

\[
\phi^\dagger_{\text{CollectFromSystemCalendar}} \triangleq \ldots \land \text{(workTime} \text{CollectFromSystemCalendar} = 1),
\]

plus the corresponding negative prerequisite formula, which force the corresponding numerical attributes to be zero.

As with the previous case, e.g. the arithmetic constraints make the combination of ScheduleManually and CallParticipants incompatible with the nice-to-have requirement FastSchedule.

Notice that one can build combinations of numerical attributes. For example, if labour cost is 35€/h, then one can redefine cost as \(\text{cost} = \sum E \text{cost}_E + 35 \cdot \text{workTime}\), or introduce a new global variable totalCost as \(\text{totalCost} = \text{cost} + 35 \cdot \text{workTime}\).

**Remark 3** Although the nice-to-have requirements LowCost and FastSchedule look isolated in Fig. 1, they are implicitly linked to the rest of the CGM by means of arithmetic constraints on the numerical variables cost and workTime, respectively, which implicitly imply Boolean constraints like:

\[
\text{LowCost} \rightarrow \neg \text{UseHotelsAndConventionCenters} \tag{12}
\]

\[
\text{FastSchedule} \rightarrow \neg (\text{ScheduleManually} \land \text{CallParticipants}) \tag{13}
\]

\[
\text{FastSchedule} \rightarrow \neg \left(\text{EmailParticipants} \land \text{CollectFromSystemCalendar}\right) \tag{14}
\]

Nevertheless, there is no need for stakeholders to consider these implicit constraints, since they are automatically handled by the internal OMT(\(\mathcal{L}\mathcal{R}\mathcal{A}\)) reasoning capabilities of CGM-Tool.

### 3.7 Realizations of a CGM

We suppose now that ScheduleMeeting is marked satisfied by means of an user assertion (i.e. it is mandatory) and that no other element is marked. Then the CGM in Fig. 1 defines a set of possible realizations. The sub-graph which is highlighted in yellow describes one of them.

Intuitively, a realization of a CGM under given user assertions represents one of the alternative ways of refining the mandatory requirements (plus possibly some of the nice-to-have ones) in compliance with the user assertions and user-defined constraints. It is a sub-graph of the CGM including a set of satisfied elements and refinements: it includes all mandatory requirements and [resp. does not include] all elements satisfied [resp. denied] in the user assertions; for each non-leaf element included, at least one of its refinement is included; for each refinement included, all its target elements are included; finally, a realization complies with all relation edges and with all Boolean and SMT(\(\mathcal{L}\mathcal{R}\mathcal{A}\)) constraints. (Notationally, in Figs. 1, 2, and 3 a realization is highlighted in yellow, and the denied elements are visible but they are not highlighted.)

Apart from the mandatory requirement, the realization in Fig. 1 allows to achieve also the nice-to-have requirements LowCost, GoodQualitySchedule, but not FastSchedule and MinimalEffort; in order to do this, it requires accomplishing the tasks CharacteriseMeeting, CallParticipants, ListAvailableRooms, UseAvailableRoom, ScheduleManually, ConfirmOccurrence, GoodParticipation, MinimalConflicts, and it requires the domain assumption LocalRoomAvailable.

### 3.8 Setting preferences in a CGM

In general, a CGM under given user assertions has many possible realizations. To distinguish among them, stakeholders may want to express preferences on the requirements to achieve, on the tasks to accomplish, and on elements and refinements to choose. The CGM-Tool provides various methods to express preferences:

- attribute penalties and rewards for tasks and requirements;
- introduce numerical objectives to optimize;
- introduce binary preference relations between elements and between refinements.

These methods, which are described in what follows, can also be combined.

#### 3.8.1 Preferences via penalties/rewards

First, stakeholders can define two numerical attributes called Penalty and Reward, and then, stakeholders can
assign penalty values to tasks and reward values to (non-mandatory) requirements (the numbers “Penalty = …” and “Reward = …” in Fig. 1). This implies that requirements [resp. tasks] with higher rewards [resp. smaller penalties] are preferable. Next, stakeholders can define another numerical attribute Weight, that represents the total difference between the penalties and rewards. [This can be defined as a global constraint: \(\text{Weight} = \text{Penalty} - \text{Rewards}\).] When a model represents preferences, an OMT solver will look for a realization that minimizes its global weight. For instance, one minimum-weight realization of the example CGM, as shown in Fig. 2, achieves all the nice-to-have requirements except MinimalEffort, with a total weight of -65, which is the minimum which can be achieved with this CGM. Such realization requires accomplishing the tasks CharacteriseMeeting, EmailParticipants, UsePartnerInstitution, ScheduleManually, ConfirmOccurrence, GoodParticipation, and MinimalConflicts and requires no domain assumption. (This was found automatically by our CGM-Tool in 0.008 s on an Apple MacBook Air laptop.)

### 3.8.2 Preferences via multiple objectives

Stakeholders may define rational-valued objectives \(\text{obj}_1, \ldots, \text{obj}_k\) to optimize (i.e. maximize or minimize) as functions of Boolean and numerical variables—e.g. cost, workTime, totalCost can be suitable objectives—and ask the tool to automatically generate realization(s) which optimize one objective, or some combination of more objectives (like totalCost), or which optimizes lexicographically an ordered list of objectives \(\text{obj}_1, \text{obj}_2, \ldots\). (We recall that a solution optimizes lexicographically an ordered list of objectives \(\text{obj}_1, \text{obj}_2, \ldots\) if it makes \(\text{obj}_1\) optimum and, if more than one such solution exists, it makes also \(\text{obj}_2\) optimum, ..., etc.) Notice that lexicographic optimization allows for defining objective functions in a very fine-grained way and for preventing ties: if the stakeholder wants to prevent tie solutions on objective \(\text{obj}_1\), he/she can define one further preference criterion \(\text{obj}_2\) in case of tie on \(\text{obj}_1\), and so on.

Importantly, our CGM-Tool provides some pre-defined objectives of frequent usage. Weight (see last paragraph) is one of them. Other examples of pre-defined objectives...
stakeholders may want to minimize, either singularly or in combination with other objectives, are:

- **numUnsatRequirements**: the number of nice-to-have requirements which are not included in the realization;
- **numSatTasks**: the number of tasks which are included in the realization;
- **numUnsatPrefs**: the number of user-defined binary preference relations which are not fulfilled by the realization (see later).

For example, the previously mentioned optimum-weight realization of Fig. 2 is such that \( \text{Weight} = -65 \), \( \text{workTime} = 4 \) and \( \text{cost} = 80 \). Our CGM has many different minimum-weight realizations s.t. \( \text{Weight} = -65 \), with different values of \( \text{cost} \) and \( \text{workTime} \). Among them, it is possible to search, for example, for the realizations with minimum \( \text{workTime} \), and among these for those with minimum \( \text{cost} \), by setting lexicographic minimization with order \( \langle \text{Weight, workTime, cost} \rangle \). This results into one realization with \( \text{Weight} = -65 \), \( \text{workTime} = 2 \) and \( \text{cost} = 0 \) achieving all the nice-to-have requirements, as shown in Fig. 3, which requires accomplishing the tasks: CharacteriseMeeting, CollectFromSystemCalendar, GetRoomSuggestions, CancelLessImportantMeeting, and the denied elements are visible but are not highlighted (color figure online).

ScheduleAutomaticaly, ConfirmOccurrence, GoodParticipation, MinimalConflicts, CollectionEffort, MatchingEffort, and which requires the domain assumptions: ParticipantsUseSystemCalendar, LocalRoomAvailable. (This was found automatically by our CGM-Tool in 0.016 s on an Apple MacBook Air laptop.)

### 3.8.3 Preferences via binary preference relations

In general, stakeholders might not always be at ease in assigning numerical values to state their preferences, or in dealing with \( \text{SMT(} \mathcal{LRA} \text{)} \) terms, constraints, and objectives. Thus, as a more coarse-grained and user-friendly solution, it is also possible for stakeholders to express their preferences in a more direct way by stating explicitly a list of binary preference relations, denoted as \( \langle P_1 \geq P_2 \rangle \), between pairs of elements of the same kind (e.g. pair of requirements, of tasks, of domain assumptions) or pairs of refinements. \( \langle P_1 \geq P_2 \rangle \) means that one prefers to have \( P_1 \) satisfied than \( P_2 \) satisfied, that is, that he/she would rather avoid having \( P_1 \) denied and \( P_2 \) satisfied. In the latter case, we say that a preference is unsatisfied. Notice that \( P_1 \geq P_2 \) allows for having both \( P_1 \) and \( P_2 \) satisfied or both denied.
Remark 4 These are binary preferences, so that they say nothing on the fact that each \( P_i \) is singularly desirable or not, which in case must be stated separately (e.g. by penalties/rewards). Thus, the fact that a binary preference \( P_1 \geq P_2 \) allows for having both \( P_1 \) and \( P_2 \) denied should not be a surprise: if both \( \{ P_1 = \text{false}, P_2 = \text{true} \} \) and \( \{ P_1 = \text{false}, P_2 = \text{false} \} \) violated \( P_1 \geq P_2 \), then \( P_2 \) would play no role in the preference, so that it would reduce to the unary preference “I’d rather have \( P_1 \) than not have it.” A dual argument holds for the fact that \( P_1 \geq P_2 \) allows for having both \( P_1 \) and \( P_2 \) satisfied.

Also, this choice is a very general one, since it implements the case in which \( \langle P_1, P_2 \rangle \) are both desirable/rewarding (“I prefer winning the Turing Award than winning at the lottery.”) like the preference between two tasks, plus obviously the trivial case in which \( P_1 \) is desirable and \( P_2 \) is undesirable. If this choice is considered too general, then the stakeholder can add mutual-exclusion constraints, or combine it lexicographically with penalty/rewards, or directly use penalty/rewards instead.

With CGM-Tool, binary preference relations can be expressed either graphically, via a “prefer” arc “\( P_1 \text{prefer} P_2 \)” , or via and ad hoc menu window. Once a list of binary preference relations is set, the system can be asked to consider the number of unsatisfied preference relations as a pre-defined objective (namely \text{numUnsatPrefs}), and it searches for a realization which minimizes it. It is also possible to combine such objective lexicographically with the other objectives.

One typical usage we envision for binary preferences is between pairs of refinements of the same element—or equivalently, in case of single-source refinements, between their relative source elements. This allows for expressing stakeholders’ preferences between possible ways one intermediate element can be refined.

For example, suppose we want to minimize the total weight of our example goal model. As previously mentioned, there is more than one realization with minimum weight —65. Unlike the previous example, as a secondary choice we disregard \text{workTime} and \text{cost}; rather, we express also the following binary preferences:

\[
\text{BySystem} \succeq \text{ByPerson},
\text{UseLocalRoom} \succeq \text{UsePartnerInstitutions},
\text{UseLocalRoom} \succeq \text{UseHotelsAndConventionCenters}.
\tag{15}
\]

[Notice that the goal preferences in (15) are pairwise equivalent to the following refinement preferences:

\[
R_2 \succeq R_1, \quad R_3 \succeq R_1, \quad \text{and} \quad R_3 \succeq R_4 \tag{16}
\]

because the refinements in (16) are all single-source ones, whose sources are pairwise the goals in (15).]

Then we set \text{numUnsatPrefs} as secondary objective to minimize after \text{Weight}, that is, we set the lexicographic order \langle \text{Weight}, \text{numUnsatPrefs} \rangle. Then our tool returned the same realization of Fig. 3—that is, the same as with minimizing \text{workTime} and \text{cost} as secondary and tertiary choice—instead of that in Fig. 2. (This solution was found in 0.018 s on an Apple MacBook Air laptop.)

4 Abstract syntax and semantics

In this section, we describe formally the abstract syntax and semantics of CGMs.

4.1 Abstract syntax

We introduce first some general definitions. We call a goal graph \( D \) a directed acyclic graph (DAG) alternating element nodes and refinement nodes (collapsed into bullets), s.t.: (a) each element has from zero to many outgoing edges to distinct refinements and from zero to many incoming edges from distinct refinements; (b) each refinement node has exactly one outgoing edge to an element (target) and one or more incoming edges from distinct elements (sources).

We call a root element node any element node that has no outgoing refinement edges, a leaf element node any (non-root) element node that has no incoming refinement edges, and an internal element node any other element node. (Hereafter we will usually drop the word “node”, simply saying “refinement” for “refinement node”, “element” for “element node”, etc.)

Notice that, by construction, only elements can be roots and leaves of a goal graph. The sets of root, leaf and internal elements of a goal graph \( D \) are denoted as \text{Roots}(D), \text{Leaves}(D), \text{Internals}(D), \) respectively. Given a refinement \( R \) with outgoing edge to the element \( E \) and incoming edges from the element \( s E_1, \ldots, E_n \), we call \( E_1, \ldots, E_n \) the source elements of \( R \) and \( E \) the target element of \( R \), which are denoted by \text{Sources}(R) \) and \text{Target}(R), respectively. We say that \( R \) is a refinement of \( E \) and that \( R \) refines \( E \) into \( E_1, \ldots, E_n \), denoted \( “(E_1, \ldots, E_n) \overset{R}{\rightarrow} E” \). The set of refinements of an element \( E \) is denoted with \text{Refinements}(E).

Elements are goals or domain assumptions, subject to the following rules:

- a domain assumption cannot be a root element;
- if the target of a refinement \( R \) is a domain assumption, then it sources are only domain assumptions;
• if the target of a refinement $R$ is a goal, then at least one of its sources is a goal.

We call root goals and leaf goals requirements and tasks, respectively.

Notationally, we use the symbols $R$, $R_i$ for labelling refinements, $E$, $E_i$ for generic elements (without specifying if goals or domain assumptions), $G$, $G_i$ for goals, $A$, $A_i$ for domain assumptions. Graphically (see Fig. 1) we collapse refinements nodes into one bullet, so that we see a refinement as an aggregation of edges from a set of other goals. (See Table 1). Hence, in a goal graph we consider element nodes as the only nodes, and refinements as (grouped) edges, which are labelled by atomic propositions in $B$—which is augmented with an SMT($\mathcal{L}_{RA}$) formula $\Psi$ on the element and refinement labels in $\nu$ and on the numerical variables in $\mathcal{N}$. The SMT($\mathcal{L}_{RA}$) formula $\Psi$ is a conjunction of smaller formulas encoding relation edges, global and local Boolean/SMT($\mathcal{L}_{RA}$) constraints, user assertions, and the definition of numerical objectives, all of which will be described later in this section.

Intuitively, a CGM describes a (possibly complex) combination of alternative ways of realizing a set of requirements in terms of a set of tasks, under certain domain assumptions and constraints. A couple of remarks are in order.

**Definition 1** *(Constrained goal model)* A CGM is a tuple $\mathcal{M} \equiv \langle B, \mathcal{N}, D, \Psi \rangle$, s.t.

- $B \equiv G \cup R \cup A$ is a set of atomic propositions, where $G \equiv \{G_1, \ldots, G_N\}$, $R \equiv \{R_1, \ldots, R_K\}$, $A \equiv \{A_1, \ldots, A_M\}$ are, respectively, sets of goal, refinement and domain-assumption labels. We denote with $\mathcal{E}$ the set of element labels: $\mathcal{E} \equiv G \cup A$;
- $\mathcal{N}$ is a set of numerical variables in the rationals;
- $D$ is a goal graph, s.t. all its goal nodes are univocally labelled by a goal label in $G$, all its refinements are univocally labelled by a refinement label in $R$, and all its domain assumption are univocally labelled by a assumption label in $A$;
- $\Psi$ is a SMT($\mathcal{L}_{RA}$) formula on $B$ and $\mathcal{N}$.

A CGM is thus a “backbone” goal graph $D$—i.e. an and-or directed acyclic graph (DAG) of elements, as nodes, and refinements, as (grouped) edges, which are labelled by atomic propositions in $B$—which is augmented with an SMT($\mathcal{L}_{RA}$) formula $\Psi$ on the element and refinement labels in $\nu$ and on the numerical variables in $\mathcal{N}$. The SMT($\mathcal{L}_{RA}$) formula $\Psi$ is a conjunction of smaller formulas encoding relation edges, global and local Boolean/SMT($\mathcal{L}_{RA}$) constraints, user assertions, and the definition of numerical objectives, all of which will be described later in this section.

Thus, each non-leaf element $E$ is or-decomposed into the set of its incoming refinements $\{R_i\}_i$, and each refinement $R$ is and-decomposed into the set of its source elements $\{E_j\}_j$.

---

**Remark 5** The fact that the goal graph $D$ is an and–or graph can be deduced from the propositional encoding of Goal refinement and Closed World in Table 1: by combining the propositional encodings of goal refinement and Closed World in Table 1, we can infer the formulas\(^2\):\(^2\)

$$E \leftrightarrow \left( \bigvee_i R_i \right) \quad \text{and} \quad R \leftrightarrow \left( \bigwedge_j E_j \right). \quad (17)$$

---

\(^2\) We recall that in Boolean logic the formula $\bigwedge_i (R_i \rightarrow E)$, which comes from the goal refinement encoding in Table 1, is equivalent to $E \leftrightarrow (\bigvee_i R_i)$. The latter, combined with the encoding of Closed World $E \leftrightarrow (\bigvee_i R_i)$, gives the left formula in (17). The right formula in (17) is the other part of the goal refinement encoding in Table 1.
Fig. 4 Top and-decomposition and its translation into CGM format as a single multi-source refinement. Middle or-decomposition and its translation into CGM format as multiple single-source refinements. Bottom a simple piece of CGM (right) and its translation into standard and-or goal model format (left); it is necessary to introduce two auxiliary goals $G'$ and $G''$ to encode the refinements $R_1$ and $R_2$.

Remark 6 CGMs are more succinct in terms of number of goals than standard and-or goal models. On the one hand, a standard n-ary and-decomposition of a goal can be represented straightforwardly in a CGM by one refinement with n sources (Fig. 4, top), and an or-decomposition by n one-source refinements (Fig. 4, middle), so that no extra goals are added. On the other hand, in order to represent a piece of CGM with n non-unary refinements by standard goal models, we need introducing n new auxiliary intermediate goals to encode refinements, which CGMs encode natively (Fig. 4, bottom). We recall from Sect. 3 that refinements do not need to be explicitly labelled unless they need to be mentioned in other parts of the model.

Stakeholders might not be at ease in defining a possibly complex global SMT($\mathcal{LRA}$) formula $\Psi$ to encode constraints among elements and refinements, plus numerical variables. To this extent, as mentioned in Sect. 3, apart from the possibility of defining global formulas, CGMs provide constructs allowing the user to encode graphically and locally desired constraints of frequent usage: relation edges, prerequisite formulas $\{\phi_1^+, \phi_1^-\}$ and $\{\phi_2^+, \phi_2^-\}$, and user assertions. Each is automatically converted into a simple SMT($\mathcal{LRA}$) formula as follows and then conjoined to $\Psi$.

Element-contribution edges $E_1 \rightarrow E_2$, meaning that satisfying $E_1$ forces $E_2$ to be satisfied (but not vice versa). They are encoded into the formula $(E_1 \rightarrow E_2)$. (The edge $E_1 \rightarrow E_2$ can be used to denote the merging of the two contribution edges $E_1 \rightarrow E_2$ and $E_2 \rightarrow E_1$ into one.)

Element-conflict edges $E_1 \leftarrow E_2$, meaning that $E_1$ and $E_2$ cannot be both satisfied. They are encoded into the formula $\neg(E_1 \land E_2)$.

Refinement-binding edges $R_1 \leftarrow R_2$, meaning that, if both the target goals of $R_1$ and $R_2$ (namely $E_1$ and $E_2$, respectively) are satisfied, then $R_1$ refines $E_1$ if and only if $R_2$ refines $E_2$. They are encoded into the formula $(E_1 \land E_2) \rightarrow (R_1 \leftarrow R_2)$.

User assertions $E_i := \top$ and $E_i := \bot$, are encoded into the formulas $(E_i)$, $(\neg E_i)$, respectively.

Prerequisite formulas $\{\phi_1^+, \phi_1^-\}$ [resp. $\{\phi_2^+, \phi_2^-\}$] are encoded into the formulas $(G \rightarrow \phi_1^+)$ and $(\neg G \rightarrow \phi_1^-)$ [resp. $(R \rightarrow \phi_2^+)$ and $(\neg R \rightarrow \phi_2^-)$].

The following are instead encoded into SMT($\mathcal{LRA}$) “soft”3 constraints:

Preference edges $E_1 \triangleright E_2$ [resp. $R_1 \triangleright R_2$], and their equivalent binary preference relations $E_1 \triangleright E_2$ [resp. $R_1 \triangleright R_2$], are implemented into the soft constraint $\phi_{E_1 \triangleright E_2} \stackrel{\text{def}}{=} (E_1 \lor (\neg E_2))$ [resp. $\phi_{R_1 \triangleright R_2} \stackrel{\text{def}}{=} (R_1 \lor (\neg R_2))$]. (See also Remark 4 in Sect. 3.8.) Notice that $E_1$ and $E_2$ [resp. $R_1$ and $R_2$] must be of the same kind, i.e. they must be both tasks, or both requirements, or both refinements, or both intermediate goals, or both domain assumptions.

Unlike with other constraints, these soft constraints are not added directly to $\Psi$. Rather, the following SMT($\mathcal{LRA}$) constraint, which defines a numeric Pseudo-Boolean cost function, is added to $\Psi$:

3 In constraint programming and other related disciplines (e.g. MaxSAT, MaxSMT, OMT) constraints which must be satisfied are called “hard”, whereas constraints which are preferably satisfied but which can be safely violated, although paying some penalty, are called “soft.”
\[
\text{numUnsatPrefs} = \sum_{R \in P} \text{ite}(\phi_{E \in E}, 0, 1) + \sum_{(R) \in P} \text{ite}(\phi_{R \in R_E}, 0, 1),
\]

where \( P \) is the list of binary preference relations, and “\( \text{ite}(\phi, 0, 1) \)” denotes an if-then-else arithmetical term, which is evaluated to 0 if \( \phi \) is evaluated to true, to 1 otherwise. Hence, \( \text{numUnsatPrefs} \) counts the number of unsatisfied preferences, that is, the number of binary preferences \( P_1 \geq P_2 \), s.t. \( P_1 \) is false and \( P_2 \) is true.\(^4\)

Notice that, unlike refinements, relation edges, and preference edges are allowed to create loops, possibly involving refinements. In fact, refinements are acyclic because they represent the and-or decomposition DAG or the CGM requirements. Other arcs (and formulas) represent relations and constraints among elements, and as such they are free to form loops, even with refinements.

Finally, we provide the user of a list of syntactic-suggesting constructs, which allow for defining, both globally and locally, the most standard and intuitive constraints among assumption, goal, and refinement labels, with no need of defining the corresponding complicate or less-intuitive propositional formulas. (In what follows, \( P_1, \ldots, P_n \) denote atomic propositions in \( B \).

\begin{itemize}
  \item \( \text{Alt}(P_1, P_2) \) denotes the fact \( P_1 \) and \( P_2 \) are alternative, e.g. that one and only one of them is satisfied. This is encoded by the formula \( (P_1 \leftarrow \neg P_2) \).
  \item \( \text{Causes}(P_1, P_2) \) denotes the fact that satisfying \( P_1 \) causes \( P_2 \) to be satisfied. This is encoded by the formula \( (P_1 \rightarrow P_2) \).
  \item \( \text{Requires}(P_1, P_2) \) denotes the fact that satisfying \( P_1 \) requires \( P_2 \) to be satisfied. This is encoded by the formula \( (P_1 \rightarrow P_2) \).\(^5\)
  \item \( \text{AtMostOneOf}(\{P_1, \ldots, P_n\}) \) denotes the fact that at most one of \( \{P_1, \ldots, P_n\} \) must be satisfied. This is encoded by the formula \( (\bigwedge_{1 \leq i \leq n} \neg P_i \lor \neg P_j) \).
  \item \( \text{AtLeastOneOf}(\{P_1, \ldots, P_n\}) \) denotes the fact that at least one of \( \{P_1, \ldots, P_n\} \) must be satisfied. This is encoded by the formula \( (\bigvee_{1 \leq i \leq n} P_i) \).
\end{itemize}

\(^4\) In practice, the OMT solver OptiMathSAT \([39]\) provides more efficient ad hoc encodings for soft constraints like those in (18), which we have exploited in the implementation of CGM-Tool; we refer the reader to [39] for details.

\(^5\) Notice that the relation edge \( P_1 \rightarrow P_2 \), and the Boolean constraints \( \text{Causes}(P_1, P_2) \), \( \text{Requires}(P_1, P_2) \), and \( (P_1 \rightarrow P_2) \) are equivalent from the perspective of Boolean semantics. Nevertheless, stakeholders may use them in different contexts, e.g. “\( \text{Causes}(P_1, P_2) \)” is used when event \( P_1 \) occurs before \( P_2 \) and the former causes the latter, whereas “\( \text{Requires}(P_1, P_2) \)” is used when \( P_1 \) occurs after \( P_2 \) and the former requires the latter as a prerequisite.

OneOf(\( \{P_1, \ldots, P_n\} \)) denotes the fact that exactly one of \( \{P_1, \ldots, P_n\} \) must be satisfied. This is encoded by the conjunction of the previous two formulas.

### 4.2 Semantics

The semantics of CGMs is formally defined in terms of the semantics of simple Boolean expressions, as follows.

**Definition 2 (Realization of a CGM)** Let \( \mathcal{M} \triangleq (\mathcal{B}, \mathcal{N}, \mathcal{D}, \Psi) \) be a CGM. A realization of \( \mathcal{M} \) is a \( \mathcal{LRA} \)-interpretation \( \mu \) over \( \mathcal{B} \cup \mathcal{N} \) such that:

\begin{itemize}
  \item[(a)] \( \mu \models ((\bigwedge_{i=1}^n E_i) \leftrightarrow R) \land (R \rightarrow E) \) for each refinement \( (E_1, \ldots, E_n) \stackrel{R}{\rightarrow} E \);
  \item[(b)] \( \mu \models (E \rightarrow (\bigvee_{R \in \text{Ref}(E)} R_i)) \), for each non-leaf element \( E \);
  \item[(c)] \( \mu \models \Psi \).
\end{itemize}

We say that \( \mathcal{M} \) is realizable if it has at least one realization, unrealizable otherwise.

Alternatively and equivalently, (a) and (b) can be substituted by the conditions:

\begin{itemize}
  \item[(a')] \( \mu \models ((\bigwedge_{i=1}^n E_i) \leftrightarrow R) \) for each refinement \( (E_1, \ldots, E_n) \stackrel{R}{\rightarrow} E \);
  \item[(b')] \( \mu \models (E \rightarrow (\bigvee_{R \in \text{Ref}(E)} R_i)) \), for each non-leaf element \( E \),
\end{itemize}

which reveal the and-or structure of \( \mathcal{D} \). (Recall Remark 5 and Footnote 2.)

In a realization \( \mu \) for a CGM \( \mathcal{M} \triangleq (\mathcal{B}, \mathcal{N}, \mathcal{D}, \Psi) \), each element \( E \) or refinement \( R \) can be either satisfied or denied (i.e. their label can be assigned true or false, respectively, by \( \mu \)), and each numerical value is assigned a rational value. \( \mu \) is represented graphically as the sub-graph of \( \mathcal{D} \) which includes all the satisfied elements and refinements and does not include the denied elements and refinements. As an example, consider the realization highlighted in yellow in Fig. 1, where \( \text{cost}_{E} = 0 \) and \( \text{cost}_{E} = 0 \) for every element \( E \). From Definition 2, a realization \( \mu \) represents a sub-graph of the CGM, such that:

\begin{itemize}
  \item[(a)] A refinement \( R \) is part of \( \mu \) if and only if all its source elements \( E_i \) are also included. Moreover, if \( R \) is part of \( \mu \), then also its target element \( E \) is part of it. (See, for example, refinement \( R_1 \) for \text{ScheduleMeeting}, with all its source goals.)
  \item[(b)] If a non-leaf goal is in a realization sub-graph, then at least one of its refinements is included in the realization. (See, for example, refinement \( R_5 \) for \text{FindASuitableRoom}.)
\end{itemize}
A realization complies with all Boolean and SMT(ŁRA) constraints of the CGM, including relational edges, global and local formulas, user assertions, and the definitions of the numerical attributes and objectives. In particular:

\[ E_1 \rightarrow E_2; \] if \( E_1 \) is in \( \mu \), then \( E_2 \) is in \( \mu \). (See, for example, the contribution edge BySystem \( \rightarrow \) CollectionEffort.)

\[ E_1 \rightarrow E_2; \] \( E_1 \) and \( E_1 \) cannot be both part of \( \mu \). (See, for example, the conflict edge Byperson \( \rightarrow \) CollectionEffort.)

\( R_1 \rightarrow R_2; \) if both the target goals of \( R_1 \) and \( R_2 \) are part of the realization \( \mu \), then \( R_1 \) is in \( \mu \) if and only if \( R_2 \) is there. (See, for example, the binding \( R_{16} \rightarrow R_{17} \).)

User assertions If \( E_i \) is marked satisfied [resp. denied], then \( E_i \) is [resp. is not] a part of a realization \( \mu \). (See, for example, the requirement ScheduleMeeting, which is mandatory, i.e. it is marked satisfied.)

\( \phi_G; \) if \( G \) is a part of a realization \( \mu \), then \( \phi_G \) must be satisfied in \( \mu \). (For example, LowCost is part of \( \mu \), so that \( \phi_G \simeq \ldots \wedge (\text{cost} < 100) \) is satisfied, in compliance with the fact that \( \mu \) sets \( \text{cost} = 0 \).)

\( \phi_G; \) if \( G \) is not part of a realization \( \mu \), then \( \phi_G \) must be satisfied in \( \mu \). (E.g. UsePartnerInstitutions is not part of \( \mu \), so that \( \phi_{\text{UsePartnerInstitutions}} \simeq \ldots \wedge \phi_{\text{costUsePartnerInstitutions}} = 0 \) by (6)—is satisfied, in compliance with the fact that \( \mu \) sets \( \text{costE} = 0 \) for every \( E \).)

Global formulas and attribute definitions The realization complies with all global formulas and attribute definitions. (E.g. the global formula \( \text{cost} = \sum_{E \in E} \text{costE} \), which defines the attribute \text{cost}, is satisfied by \( \mu \) because \( \text{cost} = 0 \) and \( \text{costE} = 0 \) for every element \( E \).)

Remark 7 Importantly, in the definition of objectives only nonzero terms of the sums need to be considered. (E.g. the sum in \( \text{cost} = \sum_{E \in E} \text{costE} \) can be safely restricted to the elements UsePartnerInstitutions and UseHotelsAndConventionCenters.) This allows for reducing drastically the number of rational variables involved in the encoding. In the implementation of CGM-Tool, we have exploited this fact.

5 Automated reasoning with constrained goal models

In this section, we describe how to perform automated reasoning functionalities on CGMs by encoding them into SMT and OMT.

5.1 Encoding of constrained goal models

Definition 3 Let \( \mathcal{M} \subseteq (\mathcal{B}, \mathcal{N}, \mathcal{D}, \mathcal{W}) \) be a CGM. The SMT(ŁRA) encoding of \( \mathcal{M} \) is the formula

\[
\Psi_M \equiv \Psi_R \land \Psi_N \land \Psi_E,
\]

where

\[
\Psi_R \equiv \bigwedge_{(E_1, \ldots, E_n) \rightarrow E, R \in \mathcal{R}} \left( \bigwedge_{i=1}^n E_i \rightarrow R \right) \land (R \rightarrow E)
\]

(19)

\[
\Psi_E \equiv \bigwedge_{E \in \text{Roots}(\mathcal{D}) \cup \text{Internals}(\mathcal{D})} \left( E \rightarrow \bigvee_{R_i \in \text{Refinements}(E)} R_i \right)
\]

(20)

Roots(\mathcal{D}) and Internals(\mathcal{D}) being the root and internal elements of \( \mathcal{D} \), respectively. We call \( \Psi_M \) the SMT(ŁRA) Encoding of the CGM \( \mathcal{M} \).

Notice that the formulas \( \Psi_R \) and \( \Psi_E \) in (19) and (20) encode directly points (a) and (b) in Definition 2, for every element and refinement in the CGM. In short, the \( \Psi_R \land \Psi_E \) component of \( \Psi_M \) encodes the relation induced by the and/or goal graph \( \mathcal{D} \) in \( \mathcal{M} \). The component \( \Psi_R \) is the formula described in point (c) in Definition 2, which encodes all Boolean and SMT(ŁRA) constraints of the CGM, including relational edges, global and local formulas, user assertions, and the definitions of the numerical attributes and objectives.

Therefore, the following facts are straightforward consequences of Definitions 2 and 3 and of the definition and OMT(ŁRA).

Proposition 1 Let \( \mathcal{M} \subseteq (\mathcal{B}, \mathcal{N}, \mathcal{D}, \mathcal{W}) \) be a CGM; let \( \Psi_M \) its SMT(ŁRA) encoding as in Definition 3; let \( \mu \) a ŁRA-interpretation over \( \mathcal{B} \cup \mathcal{N} \). Then \( \mu \) is a realization of \( \mathcal{M} \) if and only if \( \mu \models \Psi_M \).

In short, Proposition 1 says that \( \mu \) is a realization for the CGM \( \mathcal{M} \) if and only if \( \mu \) is a model in SMT(ŁRA) for the formula \( \Psi_M \). Therefore, a realization \( \mu \) for \( \mathcal{M} \) can be
found by invoking a SMT(ŁRA) solver on the CGM encoding $\Psi_M$.

**Proposition 2** Let $M$ and $\Psi_M$ be as in Proposition 1, and let $\mu$ be a realization of $M$. Let $\{obj_1,\ldots, obj_k\}$ be numerical objectives occurring in $\Psi_M$. Then we have that:

(i) for every $i$ in $1,\ldots, k$, $\mu$ minimizes [resp. maximizes] $obj_i$ if and only if $\mu$ is a solution of the OMT(ŁRA) minimization [resp. maximization] problem $\langle \Psi_M, \{obj_i\} \rangle$;

(ii) $\mu$ lexicographically minimizes [resp. maximizes] $\langle obj_1,\ldots, obj_k \rangle$ if and only if $\mu$ is a solution of the OMT(ŁRA) lexicographic minimization [resp. maximization] problem $\langle \Psi_M, \{obj_1,\ldots, obj_k\} \rangle$.

In short, Proposition 1 says that $\mu$ is a realization for the CGM $M$ which optimizes lexicographically $\langle obj_1,\ldots, obj_k \rangle$ if and only if $\mu$ is a model in SMT(ŁRA) for the formula $\Psi_M$ which optimizes lexicographically $\langle obj_1,\ldots, obj_k \rangle$. Therefore, one such realization can be found by invoking a OMT(ŁRA) solver on $\Psi_M$ and $\langle obj_1,\ldots, obj_k \rangle$. Notice that we are always looking for one realization at a time. Multiple realizations require multiple calls to the OMT solver.

### 5.2 Automated reasoning on constrained goal models

Propositions 1 and 2 suggest that realizations of a CGM $M$ can be produced by applying SMT(ŁRA) solving to the encoding $\Psi_M$ and that optimal realizations can be produced by applying OMT(ŁRA) to $\Psi_M$ and a list of defined objectives $\{obj_1, \ldots, obj_k\}$. Notice that such list may include also the pre-defined objectives Weight, numUnsatRequirements, numSatTasks and numUnsatPrefs of Sect. 3 and (18) to be minimized.

This allowed us to implement straightforwardly the following reasoning functionalities on CGMs by interfacing with a SMT/OMT tool.

**Search/enumerate realizations** Stakeholders can automatically check the realizability of a CGM $M$ —or to enumerate one or more of its possible realizations— under a group of user assertions and of user-defined Boolean and SMT(ŁRA) constraints; the tool performs this task by invoking the SMT solver on the formula $\Psi_M$ of Definition 3.

**Search/enumerate minimum-penalty/maximum reward realizations** Stakeholders can assert the desired requirements and set penalties of tasks; then, the tool finds automatically realizations achieving the former while minimizing the latter, by invoking the OMT solver on $\Psi_M$ with the pre-defined Weight objective. The vice versa is obtained by negating undesired tasks and setting the rewards of nice-to-have requirements. Every intermediate situations can also be obtained.

**Search/enumerate optimal realizations w.r.t. pre-defined/user-defined objectives** Stakeholders can define their own objective functions $obj_1,\ldots, obj_k$ over goals, refinements and their numerical attributes; then, the tool finds automatically realizations optimizing them, either independently or lexicographically, by invoking the OMT solver on $\Psi_M$ and $obj_1,\ldots, obj_k$. User-defined objectives can also be combined with the pre-defined ones, like Weight, numUnsatRequirements, numSatTasks and numUnsatPrefs.

In particular, notice that numUnsatPrefs allows for addressing the fulfillment of the maximum number of binary preferences as the optimization of a pre-defined objective.

**Example 1** As a potentially frequent scenario, stakeholders may want to find a realization which minimizes, in order of preference, the number of unsatisfied non-mandatory requirements, the number of unsatisfied binary preferences, and the number of satisfied tasks. This can be achieved by setting the following ordered list of pre-defined objectives to minimize lexicographically:

$\langle$numUnsatRequirements,numUnsatPrefs,numSatTasks$\rangle$.

Notice that all the above actions can be performed interactively by marking an unmarking (nice-to-have) requirements, tasks and domain assumptions, each time searching for a suitable or optimal realization.

Importantly, when a CGM is found unrealizable under a group of user assertions and of user-defined Boolean and SMT(ŁRA) constraints, it highlights the sub-parts of the CGM and the subset of assertions causing the problem. This is implemented by asking the SMT/OMT solver to identify the unsatisfiable core of the input formula—i.e. the subset of sub-formulas which caused the inconsistency, see, for example, [9]—and mapping them back into the corresponding information.

### 6 Implementation

CGM-Tool provides support for modelling and reasoning on CGMs. Technically, CGM-Tool is a stand-alone application written in Java and its core is based on eclipse RCP engine. Under the hood, it encodes CGMs and invokes the OptiMathSAT SMT/OMT solver [39] to support

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6 http://optimathsat.disi.unitn.it.
reasoning on goal models. It is freely distributed as a compressed archive file for multiple platforms.\(^7\) CGM-Tool supports:

**Specification of projects** CGMs are created within the scope of project containers. A project contains a set of CGMs that can be used to generate reasoning sessions with OptiMathSAT (i.e., scenarios);

**Diagrammatic modelling** the tool enables the creation (drawing) of CGMs in terms of diagrams; furthermore, it enhances the modelling process by providing real-time check for refinement cycles and by reporting invalid refinement, contribution and binding links;

**Consistency/well-formedness check** CGM-Tool allows for the creation of diagrams conform with the semantics of the modelling language by providing the ability to run consistency analysis on the model;

**Automated reasoning** CGM-Tool provides the automated reasoning functionalities of Sect. 5.2 by encoding the model into an SMT formula. The results of OptiMathSAT are shown directly on the model as well as in a tabular form.

One essential feature of the tool is that expressive constructs (which may be more complex and difficult to use) are only available on demand: there are easy-to-use default settings for everything, so that the user can decide the level of expressiveness he/she feels at ease with.

CGM-Tool extends the STS-Tool [32] as an RCP application by using the major frameworks shown in Fig. 5: rich client platform (RCP), a platform for building rich client applications, made up of a collection of low-level frameworks such as OSGi, SWT, JFace, and Equinox, which provide us a workbench where to get things like menus, editors, and views; graphical editing framework (GEF), a framework used to create graphical editors for graphical modelling tools (e.g., tool palette and figures which can be used to graphically represent the underlying data model concepts); eclipse modelling framework (EMF), a modelling framework and a code generation facility for building tools and applications based on a structured data model.

With CGM-Tool, a CGM is built progressively as a sequence of scenarios, which are versions of the CGM to which the automated reasoning functionalities of the CGM-Tool can be applied. Figure 6 shows the graphical user interface (GUI) of the tool. Figures 7 and 8 show, respectively, how to define a numerical attribute of an element and how to set its value. Figure 9 shows how to set objective functions from the numerical attributes (e.g., set the priorities, choose the form of optimization (maximize/minimize), ...). Figure 10 shows how to define the global constraints in the model. Figures 11 and 12 show how to create and open a scenario. Figure 13 shows how the user assertions can be added by using the option “Force True” (element that must be included in the realization) and “Force False” (element that must not be included in the realization). Figure 14 shows how to automatically generate a realization for the current scenario by invoking the automated reasoning functionalities.

7 Scalability of the reasoning tool

We address the issue of the scalability of the automated reasoning functionalities of Sect. 5.2 w.r.t. the size of CGMs, by providing an empirical evaluation of the performance of CGM-Tool on increasingly large CGMs. (For the sake of readability, here we provide only a qualitative description, whereas the data and plots are reported in an “Appendix”.) As in Sect. 3, all experiments have been run on a MacBook Air laptop, Intel Core i5 1.8 GHz, 2 cores, 256 kb L2 Cache per Core, 3 MB L3 Cache, 4 GB RAM.

For the readers’ convenience, a compressed directory containing all the material to reproduce these experiments (models, tools, scripts, etc.) is available at http://www.cgm-tool.eu/experiment-version/.

We consider first the schedule-meeting CGM of Sect. 3 as a seed model. The model consists in 32 goals—among which there are one mandatory requirement, four nice-to-have requirements, and 18 tasks—plus 20 refinements and two domain assumptions, totalling 54 nodes. The CGM contains also three numerical objectives: cost, workTime, and Weight. The user-defined objectives cost and workTime involve, respectively, two and five tasks and no requirement, while the pre-defined attributes Weight involves 16 tasks plus all four non-mandatory requirements. This involves \(3 + 2 + 5 + 0 + 0 + 16 + 4 = 30\) rational variables (recall Remark 7). There are also three binary preference relations (15).

In the example reported in Sect. 3 with different configurations, the tool returned the optimal solutions in negligible time (all took <0.02 s). This is not surprising: as mentioned in Sect. 2.1, in previous empirical evaluation of OMT-encoded problems from formal verification, OptiMathSAT successfully handled optimization problems with up to 1000s Boolean/rational variables [38], so that handmade CGMs resulting into SMT formulas with few tens of Boolean and rational variables, like that in Sect. 3, are not a computational challenge.

In perspective, since CGM-Tool is supposed to be used to design CGMs representing possibly large projects, we wonder how its automated reasoning functionalities will scale on large models. To do this, we choose to build benchmark CGMs of increasing size, by combining

\(^7\) http://www.cgm-tool.eu/.
Fig. 5 CGM-Tool: component view

Fig. 6 CGM-Tool: graphical user interface as in the tool manual [27] (green notes are for description) (color figure online)

Fig. 7 CGM-Tool: how to define numerical attributes (instructions in red) (color figure online)
Fig. 8 CGM-Tool: how to define the value of the numerical attributes associated with elements (instructions in red) (color figure online)

1st: choose the Node Variable tab
2nd: click on an element

Then you will see (and be able to define) all the local numerical attributes of the chosen element

Choose Model Variable tab to be able to access to the global numerical attributes.

You can change the value of Priority of an attribute. The lower the value (> 1), the higher its optimization priority. 0 means that we do not care about optimization of this attribute.

Choose to either maximize the total value of the attribute or to minimize it.

Fig. 9 CGM-Tool: how to define objectives from numerical attributes (instructions in red) (color figure online)

Fig. 10 CGM-Tool: how to define global constraints (instructions in red) (color figure online)
different instances of the schedule-meeting CGM of Sect. 3 in various ways and testing them with different combination of objectives.

7.1 Experiment setup

In all our experiments, CGMs were produced as follows, according to three positive integer parameters $N$, $k$, and $p$, and some choices of objectives.

Given $N$ and $k$, we pick $N$ distinct instances of the schedule-meeting CGM of Sect. 3, each with a fresh set of Boolean labels and rational variables, we create an artificial root goal $G$ with only one refinement $R$ whose source goals are the $N$ mandatory requirements “ScheduleMeeting,” of each CGM instance. Hence, the resulting CGM has $54 \cdot N + 2$ nodes and $30 \cdot N$ rational variables (see Table 3). In another group of experiments (see Table 2), we dropped the non-mandatory requirements and their four direct sub-tasks, so that each instance contains 24 goals, two domain

**Fig. 11** CGM-Tool: how to create a scenario (instructions in red) (color figure online)

**Fig. 12** CGM-Tool: how to open the created scenario (instructions in red) (color figure online)

different instances of the schedule-meeting CGM of Sect. 3 in various ways and testing them with different combination of objectives.

**Fig. 13** CGM-Tool: how to add user’s assertions (instructions in red) (color figure online)
assumptions, and 18 refinements, and the resulting CGM has $44 \cdot N + 2$ nodes and $26 \cdot N$ rational variables.

Then we randomly add $(k - 1) \cdot N$ contribution relations $\rightarrow$ and $N$ conflict relations $\leftarrow$ between tasks belonging to different instances. When binary preference relations are involved, we also randomly add $p \cdot N$ binary preference relations, each involving two refinements of one same goal.

In each group of experiments, we fix the definition of the objectives and we set the value of $k$ (and $p$ when it applies)
and increase the values of $N$. For every choice of $N$, we automatically\footnote{To perform this test automatically, we developed an automated problem generator/manipulator which interfaces directly with the internal data structure representing the CGMs inside CGM-Tool.} generate 100 instances of random problems as in the above schema, which we feed to our tool, and collect the median CPU times over the solved instances—including both encoding and solving times—as well as the number of unrealizable instances as well as the number of instances which OptiMathSAT could not solve within a timeout of 1000 s.

Notice that, following some ideas from a different context \[20, 33\], the parameters $N$, $k$, and $p$ have been chosen so that to allow us to increase monotonically and tune some essential features of the CGMs under test, which may significantly influence the performances. For example,

- $N$ increases linearly the number of Boolean and rational variables,
- $k$ (and, to some extent, $p$) increases the connectivity of the graph and the ratio between unrealizable and realizable CGMs.
- Importantly, $k$ and $p$ also play an essential role in drastically reducing the symmetry of the resulting CGMs and insert some degree of randomness.

Another important parameter, which we borrowed from the schedule-meeting CGM, is the number of Boolean atoms per objective.

\textit{Remark 8} We are aware that the CGMs produced with this approach may not represent realistic problems. However, we stress the fact that here we focus only on providing a test on the \textit{scalability} of our automated reasoning functionalities.

### 7.2 Results

We run two groups of experiments in which we focus on optimizing, respectively:

- \textit{numerical attributes} like cost, worktime, penalty/rewards;
- \textit{discrete features} like the number of binary preferences, of want-to-have requirements and of tasks to accomplish.

In the first group of experiments, we consider the reduced version of the CGMs (i.e. without nice-to-have requirements) without random binary preference relations. We fix $k = 2, 4, 5, 8$. In each setting, we run experiments on three functionalities:

(a) plain realizability check (without objectives),
(b) single-objective optimization on cost, workTime, and Weight, respectively, 
(c) lexicographic optimization, respectively, on \( \langle \text{cost, workTime, Weight} \rangle \) and on \( \langle \text{Weight, workTime, cost} \rangle \).

Figure 15 shows the overall median CPU time over the solved instances of the first group of experiments, which are plotted against the total number of nodes of the CGM under test.\(^9\) (For more details about the experiment data and the median CPU time over the solved instances for each special case, please see Tables 4, 5, 6, and 7, Figs. 17, 18, 19, and 20 in “Appendix”.)

First, we notice that checking the realizability of the CGM, that is, finding one realization or verifying there is none, requires negligible time, even with huge CGMs (>8000 nodes, >5000 rational variables) and even when the CGM is not realizable. Second, the time taken to find optimal solutions on single objectives seems to depend more on the number of variables in the objective than on the actual size of the CGM: for cost (2 \( \cdot N \) variables) the solver can find optimum solutions very quickly even with huge CGMs (>8000 nodes, >5000 rational variables) while with Weight (16 \( \cdot N \) variables) it can handle problems of up to \( \approx 400 \) nodes and \( \approx 200 \) rational variables. Third, lexicographic optimization takes more time than single-objective optimization, but the time mostly depends on the first objective in the list.

In the second group of experiments, we consider the full version of the CGMs (with nice-to-have requirements) and introduce the random binary preference relations. We fix \( k = 2 \), and we run different experiments for \( p = 6 \), \( p = 8 \) and \( p = 12 \). In each setting, we run experiments on three functionalities:

(a) plain realizability check (without objectives),
(b) lexicographic optimization on \( \langle \text{numUnsatPrefs, numUnsatRequirements, numSatTasks} \rangle \) (PRT),
(c) lexicographic optimization on \( \langle \text{numUnsatRequirements, numUnsatPrefs, numSatTasks} \rangle \) (RPT).

Figure 16 shows the overall median CPU time over the solved instances of the second group of experiments. (For more details about the experiment data and the median CPU time over the solved instances for each special case, please see Tables 8, 9, and 10 Figs. 21, 22, and 23 in “Appendix”.)

First, checking realizability is accomplished in negligible time even with huge CGMs (>10,000 nodes, >6000 rational variables), as before. Second, we notice that optimal solutions, even with a three-level lexicographic combination of objectives, can be found with large CGMs (>1000 nodes, >600 rational variables).

On the negative side, for some problems, in particular large ones with objectives involving large amounts of elements, we notice that the search for the optimal realization could not be accomplished within the timeout.

To this extent, a few remarks are in order.

First, when interrupted by a timeout, OptiMathSAT can be instructed to return the current best solution. Since

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\(^9\) The choice of using the total number of nodes for the X axis in all our plots aims at providing an eye-catching indication of the actual size of the CGMs under test.
OptiMathSAT typically takes most of its time in fine-tuning the optimum and in checking there is no better one (see [38]), we envisage that good sub-optimal solutions can be found even when optimal ones are out of reach.

Second, our CGMs are very large in breadth and small in depth, with a dominating percentage of tasks over the total number of goals. We envisage that this may have made the number of variables in the sums defining Weight and numSatTasks unrealistically large wrt. the total size of the CGMs. This underscores the need for further experimentation to confirm the scalability of our proposal.

Third, in our experiments we did not consider user assertions which, if considered, would force deterministic assignments and hence reduce drastically the size of the OMT search space.

Fourth, OMT is a recent technology [37], which is progressing at a very high pace, so that it is reasonable to expect further performance improvements for the future versions of OMT tools. In particular, a recent enhancement for handling Pseudo-Boolean cost functions as in (18) has provided interesting preliminary results [41].

Overall, our evaluation showed that CGM-Tool always checks the realizability of huge CGMs in negligible time and finds optimal realizations on problems whose size ranges from few 100s to 1000s of nodes, mostly depending on the number of variables involved in the objective functions.

8 Related work

We next offer a quick overview of, and comparison with some the state-of-the-art goal-oriented modelling languages. Lapouchnian [24], Jureta et al. [23], and Borgida et al. [6] provide better and deeper comparisons on requirements modelling languages and the goal-oriented approach, including their advantages and limitations.

8.1 KAOS

KAOS [11] supports a rich ontology for requirements that goes well beyond goals, as well as a linear temporal logic (LTL)-grounded formal language for constraints. This language is coupled with a concrete methodology for capturing and analysing requirements problems. KAOS supports a number of analysis techniques, including obstacle, inconsistency, and probabilistic goal analysis. However, unlike our proposal, KAOS does not support nice-to-have requirements and preferences, nor does it exploit SAT/SMT solver technologies for scalability.

8.2 Sebastiani et al.

Sebastiani et al. [18, 36] propose a formal goal modelling language that supports scalable reasoning using SAT-solving techniques. Our proposal subsumes that work in many ways, including a more expressive language and much more advanced SMT/OMT-solving technology.

There is one construct of [18, 36] that was left out of the CGM language: \(+\) and \(-\) contributions from goals to goals. There are several reasons for this decision. In un-constrained) goal models, formalizing \((+,-)\) contributions require a four-value logic (fully/partially satisfied/denied). In principle, our CGM framework could be extended to such a logic, with the following drawbacks:

(a) The size of the Boolean search space would extend from \(2^N\) to \(4^N\). Given that reasoning functionality in
this paper is much more sophisticated and computationally more demanding than those in our earlier papers, this might drastically reduce the efficiency of the approach.

(b) Unlike standard two-value logic, which allows us to give a clear semantics of “realization”, without any vagueness, it is not obvious to us what a “realization” could be in such logic. (E.g. should realizations admit partially satisfied/denied tasks/requirements/assumptions? If yes, how should an user interpret a partially satisfied/denied requirement/task/assumption in a realization returned by the system? In which sense a realization involving partial values can be considered “optimal” or “optimum”?

There are other differences between the two proposals. In CGMs, we have made and/or-decompositions explicit by making refinement a first-class citizen that can be named and talked about (see Fig. 4 and Remark 5). Moreover, unlike with [18, 36], we have a backbone and/or DAG, where arbitrary constraints can be added. This DAG is such that a non-leaf goal is equivalent to the disjunction (“or”) of its refinements, and each refinement is equivalent to the conjunction (“and”) of its source goals. Relation edges, constraints, and assertions further constrain this structure.

8.3 \( i^* \) and Tropos

\( i^* \) [43] focuses on modelling actors for a requirements engineering problem (stakeholders, users, analysts, etc.), their goals and interdependencies. \( i^* \) provides two complementary views of requirements: the actor strategic dependency model (SD model) and the actor strategic rationale model (SR model). Typically, SD models are used to analyse alternative networks of delegations among actors for fulfilling stakeholder goals, while SR models are used to explore alternative ways of fulfilling a single actor’s goals. \( i^* \) is expressively lightweight, intended for early stages of requirements analysis, and did not support formal reasoning until recent thesis work by Horkoff [19]. Tropos [7] is a requirement-driven agent-oriented software development methodology founded on \( i^* \). Goal models can be formalized in Tropos by using Formal Tropos [17], an extension of \( i^* \) that supports LTL for formalizing constraints. The main deficiencies of this work relative to our proposal are that Formal Tropos is expressive but not scalable.

8.4 Techne and Liaskos

Techne [22] is a recent proposal for a family of goal modelling languages that supports nice-to-have goals and preferences, but it is strictly propositional and uses hand-crafted algorithms and therefore does not support optimization goals. Ernst et al. [15] constitutes a first attempt to reason with nice-to-have requirements (aka preferences). The scalability experiments conducted used the SAT solver of Sebastiani et al. [36] and added local search algorithms to deal with preferences. All experiments were conducted on a model with about 500 elements and the search algorithms returned maximal consistent solution but also near solutions. Ernst et al. [12] focuses on finding new solutions for a goal model that has changed (new goals were added/removed), such that the change minimizes development effort (EvoR1) or maximizes familiarity (EvoR2). Note that EvoR1, EvoR2 are evolution requirements. The paper uses a truth-maintenance system (TMS) and builds algorithms on top for finding solutions to EvoR1, EvoR2 that “repair” the previous solution and construct a new one. The search algorithms would need to be redone if we used different evolution algorithms, unlike the CGM tool where you can formally express EvoR1, EvoR2 or variants, and search is handle by the backend OMT/SMT solver. Ernst et al. [13, 14] continue the study of reasoning with Techne models and use SAT solvers and hand-crafted search algorithms to establish scalability for models size O(1K). Nevertheless, the resulting tools from this work still cannot handle quantitative optimization problems and other features of CGMs.

Liaskos [25, 26] has proposed extensions to qualitative goal models to support nice-to-have goals and preferences, as well as decision-theoretic concepts such as utility. This proposal is comparable to our proposal in this paper, but uses AI reasoners for reasoning (AI planners and GOLOG) and, consequently, does not scale very well relative to our proposal.

8.5 Feature models

Feature models [10] share many similarities with goal models: they are hierarchically structured, with AND/OR refinements, constraints, and attributes. However, each feature represents a bundle of functionality or quality and as such, feature models are models of software configurations, not requirements. Moreover, reasoning techniques for feature models are limited relative to their goal model cousins.

8.6 Search-based software engineering

Scalable reasoning for optimization problems has been studied by Harman et al. in the context of formalizing and solving the next release problem [44]: given a set of preferences with associated cost and customer value attributes, select a subset of preferences to be included in the next release that optimizes given attributes. That work uses genetic algorithms and other search techniques that may
return close-to-optimal solutions and use heuristics (meaning that reasoning is not complete).

9 Conclusions and future work

We have proposed an expressive goal-based modelling language for requirements that supports the representation of nice-to-have requirements, preferences, optimization requirements, constraints and more. Moreover, we have exploited automated reasoning solvers in order to develop a tool that supports sound and complete reasoning with respect to such goal models and scales well to goal models with thousands of elements. Our proposal advances the state of the art on goal modelling and reasoning with respect to both expressiveness and scalability of reasoning.

The contributions of this work are being exploited in several directions. Aydemir et al. [4] has proposed an expressive modelling framework for the next release problem that is founded on the same OMT/SMT solver technology as this work. Angelopoulos et al. [1] has offered a formalization of the next adaptation problem that chooses a next adaptation for an adaptive software system that minimizes the degree of failure over existing requirements. And Nguyen et al. [30] has exploited CGMs to capture evolution requirements, such as “System evolution shall minimize implementation costs” and showed how to conduct scalable reasoning over models that include such requirements.

As future work, we have planned to do an empirical validation of the CGM-Tool with modellers and domain experts. We are currently working in this direction within our research group with PhD students and postdocs who are expert in the modelling field. Next, we will extend the validation to industrial experts of different domains. We have also planned to do different case studies with real-life-complex-large-scale goal models of a specific domain, such as Air-Traffic Control Management, healthcare, and smart cities and smart environments.

Our proposal does not address another notorious scalability problem of goal models, namely scalability-of-use. Goal models have been shown empirically to become more difficult to conceptualize and comprehend as they grow in size [16], and therefore become unwieldy for use. As with other kinds of artifacts (e.g. programs and ontologies) where scalability-of-use is an issue, the solution lies in introducing modularization facilities that limit interactions between model elements and make the resulting models easier to understand and evolve. This is an important problem on our agenda for future research on goal models.

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Appendix: Data tables and plots

First group of experiments

See Tables 4, 5, 6, 7 and Figs. 17, 18, 19, 20.
Table 4  First group of experiments, $k = 2$: median time over solved instances

| Experiment | Number of instances | Number of replicas ($N$) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Optimization time (2$N$ terms) | % Timeout |
|------------|---------------------|--------------------------|-----------------------|-----------------------------|--------------|-------------|-------------------------------|--------------------------------|-----------|
| 1          | 100                 | 2                        | 90                    | 52                          | 1            | 0.00        | 0.00                          | 0.00                           | 0         |
| 2          | 100                 | 3                        | 134                   | 78                          | 1            | 0.00        | 0.00                          | 0.01                           | 0         |
| 3          | 100                 | 4                        | 178                   | 104                         | 3            | 0.00        | 0.00                          | 0.01                           | 0         |
| 4          | 100                 | 5                        | 222                   | 130                         | 3            | 0.00        | 0.00                          | 0.02                           | 0         |
| 5          | 100                 | 6                        | 266                   | 156                         | 2            | 0.00        | 0.00                          | 0.01                           | 0         |
| 6          | 100                 | 7                        | 310                   | 182                         | 5            | 0.00        | 0.00                          | 0.01                           | 0         |
| 7          | 100                 | 9                        | 398                   | 234                         | 7            | 0.00        | 0.00                          | 0.02                           | 0         |
| 8          | 100                 | 11                       | 486                   | 286                         | 4            | 0.00        | 0.00                          | 0.02                           | 0         |
| 9          | 100                 | 13                       | 574                   | 338                         | 7            | 0.00        | 0.00                          | 0.05                           | 0         |
| 10         | 100                 | 15                       | 662                   | 390                         | 13           | 0.00        | 0.00                          | 0.04                           | 0         |
| 11         | 100                 | 17                       | 750                   | 442                         | 15           | 0.00        | 0.00                          | 0.04                           | 0         |
| 12         | 100                 | 21                       | 926                   | 546                         | 14           | 0.00        | 0.00                          | 0.06                           | 0         |
| 13         | 100                 | 26                       | 1146                  | 676                         | 13           | 0.00        | 0.00                          | 0.07                           | 0         |
| 14         | 100                 | 31                       | 1366                  | 806                         | 14           | 0.00        | 0.00                          | 0.09                           | 0         |
| 15         | 100                 | 36                       | 1586                  | 936                         | 19           | 0.00        | 0.00                          | 0.11                           | 0         |
| 16         | 100                 | 41                       | 1806                  | 1066                        | 26           | 0.00        | 0.00                          | 0.13                           | 0         |
| 17         | 100                 | 46                       | 2026                  | 1196                        | 24           | 0.00        | 0.00                          | 0.18                           | 0         |
| 18         | 100                 | 51                       | 2246                  | 1326                        | 32           | 0.00        | 0.00                          | 0.20                           | 0         |
| 19         | 100                 | 101                      | 4446                  | 2626                        | 49           | 0.00        | 0.00                          | 0.49                           | 0         |
| 20         | 100                 | 151                      | 6646                  | 3926                        | 68           | 0.00        | 0.00                          | 0.77                           | 0         |
| 21         | 100                 | 201                      | 8846                  | 5226                        | 71           | 0.00        | 0.00                          | 0.93                           | 0         |

| Experiment | Optimum time (5$N$ terms) | Optimum weight (16$N$ terms) | Lexic. order cost time weight | Lexic. order weight time cost |
|------------|----------------------------|------------------------------|-------------------------------|------------------------------|
|            | Optimization time          | Optimization weight          | Optimization time             | Optimization weight          |
|            | % Timeout                  | % Timeout                    | % Timeout                     | % Timeout                    |
| 1          | 0.01                       | 0.02                         | 0.01                          | 0.02                         |
| 2          | 0.01                       | 0.03                         | 0.01                          | 0.25                         |
| 3          | 0.01                       | 0.41                         | 0.01                          | 1.53                         |
| 4          | 0.03                       | 5.51                         | 0.02                          | 7.48                         |
| 5          | 0.02                       | 24.74                        | 0.18                          | 29.74                        |
| 6          | 0.04                       | 533.90                       | 0.02                          | 329.34                       |
| 7          | 0.09                       | 185.23                       | 0.02                          | 494.33                       |
| 8          | 0.11                       | -                            | 7.29                          | -                            |
| 9          | 0.30                       | -                            | 17.98                         | -                            |
| 10         | 18.13                      | -                            | -                             | -                            |
| 11         | 3.11                       | -                            | -                             | -                            |
| 12         | 58.08                      | 11                           | -                             | -                            |
| 13         | 600.99                     | 78                           | -                             | -                            |
| 14         | -                         | -                            | -                             | -                            |
| 15         | -                         | -                            | -                             | -                            |
| 16         | -                         | -                            | -                             | -                            |
| 17         | -                         | -                            | -                             | -                            |
| 18         | -                         | -                            | -                             | -                            |
| 19         | -                         | -                            | -                             | -                            |
| 20         | -                         | -                            | -                             | -                            |
| 21         | -                         | -                            | -                             | -                            |
Table 5  First group of experiments, $k = 4$: median time over solved instances

| Experiment | Number of instances | Number of replicas ($N$) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Optimum cost ($2N$ terms) | Optimization time % | Timeout |
|------------|---------------------|--------------------------|-----------------------|----------------------------|---------------|-------------|------------------------------|----------------------------|--------------------|---------|
| 1          | 100                 | 2                        | 90                    | 52                         | 2             | 0.00        | 0.00                         | 0.00                       | 0                  |         |
| 2          | 100                 | 3                        | 134                   | 78                         | 1             | 0.00        | 0.00                         | 0.01                       | 0                  |         |
| 3          | 100                 | 4                        | 178                   | 104                        | 2             | 0.00        | 0.00                         | 0.01                       | 0                  |         |
| 4          | 100                 | 5                        | 222                   | 130                        | 4             | 0.00        | 0.00                         | 0.01                       | 0                  |         |
| 5          | 100                 | 6                        | 266                   | 156                        | 7             | 0.00        | 0.00                         | 0.01                       | 0                  |         |
| 6          | 100                 | 7                        | 310                   | 182                        | 7             | 0.00        | 0.00                         | 0.02                       | 0                  |         |
| 7          | 100                 | 9                        | 398                   | 234                        | 7             | 0.00        | 0.00                         | 0.02                       | 0                  |         |
| 8          | 100                 | 12                       | 486                   | 286                        | 10            | 0.00       | 0.00                         | 0.02                       | 0                  |         |
| 9          | 100                 | 13                       | 574                   | 338                        | 9             | 0.00       | 0.00                         | 0.03                       | 0                  |         |
| 10         | 100                 | 15                       | 662                   | 390                        | 11            | 0.00       | 0.00                         | 0.04                       | 0                  |         |
| 11         | 100                 | 17                       | 750                   | 442                        | 9             | 0.00       | 0.00                         | 0.04                       | 0                  |         |
| 12         | 100                 | 21                       | 926                   | 546                        | 15            | 0.00       | 0.00                         | 0.05                       | 0                  |         |
| 13         | 100                 | 26                       | 1146                  | 676                        | 22            | 0.00       | 0.00                         | 0.09                       | 0                  |         |
| 14         | 100                 | 31                       | 1366                  | 806                        | 25            | 0.00       | 0.00                         | 0.12                       | 0                  |         |
| 15         | 100                 | 36                       | 1586                  | 936                        | 27            | 0.00       | 0.00                         | 0.16                       | 0                  |         |
| 16         | 100                 | 41                       | 1806                  | 1066                       | 32            | 0.00       | 0.00                         | 0.14                       | 0                  |         |
| 17         | 100                 | 46                       | 2026                  | 1196                       | 36            | 0.00       | 0.00                         | 0.17                       | 0                  |         |
| 18         | 100                 | 51                       | 2246                  | 1326                       | 40            | 0.00       | 0.00                         | 0.20                       | 0                  |         |
| 19         | 100                 | 101                      | 4446                  | 2626                       | 55            | 0.00       | 0.00                         | 0.80                       | 0                  |         |
| 20         | 100                 | 151                      | 6646                  | 3926                       | 77            | 0.00       | 0.00                         | 0.72                       | 0                  |         |
| 21         | 100                 | 201                      | 8846                  | 5226                       | 85            | 0.00       | 0.00                         | 1.18                       | 0                  |         |

| Experiment | Optimum time ($5N$ terms) | Optimization time | Optimization weight ($16N$ terms) | Lexic. order cost time | Lexic. order weight time |
|------------|---------------------------|-------------------|-----------------------------------|------------------------|-------------------------|
| 1          | 0.01                      | 0                 | 0.02                              | 0.03                   | 0.04                    |
| 2          | 0.01                      | 0                 | 0.03                              | 0.04                   | 0.05                    |
| 3          | 0.02                      | 0                 | 0.08                              | 0.06                   | 0.13                    |
| 4          | 0.01                      | 0                 | 0.41                              | 0.08                   | 0.79                    |
| 5          | 0.02                      | 0                 | 1.09                              | 0.11                   | 2.82                    |
| 6          | 0.05                      | 0                 | 9.53                              | 0.13                   | 14.47                   |
| 7          | 0.06                      | 0                 | 13.54                             | 0.64                   | 447.13                  |
| 8          | 0.14                      | 0                 | –                                 | 1.53                   | 5                       |
| 9          | 0.43                      | 0                 | –                                 | 36.78                  | 23                      |
| 10         | 2.42                      | 0                 | –                                 | 368.94                 | 55                      |
| 11         | 28.45                     | 0                 | –                                 | –                      | –                       |
| 12         | 97.86                     | 2                 | –                                 | –                      | –                       |
| 13         | 537.73                    | 62                | –                                 | –                      | –                       |
| 14         | –                         | –                 | –                                 | –                      | –                       |
| 15         | –                         | –                 | –                                 | –                      | –                       |
| 16         | –                         | –                 | –                                 | –                      | –                       |
| 17         | –                         | –                 | –                                 | –                      | –                       |
| 18         | –                         | –                 | –                                 | –                      | –                       |
| 19         | –                         | –                 | –                                 | –                      | –                       |
| 20         | –                         | –                 | –                                 | –                      | –                       |
| 21         | –                         | –                 | –                                 | –                      | –                       |
Table 6 First group of experiments, $k = 5$: median time over solved instances

| Experiment | Number of instances | Number of replicas ($N$) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Optimum cost (2$N$ terms) | Optimization time | % Timeout |
|------------|---------------------|--------------------------|-----------------------|----------------------------|----------------|-------------|-----------------------------|-----------------------|----------------|----------|
| 1          | 100                 | 2                        | 90                    | 52                         | 4              | 0.00        | 0.00                        | 0.01                  |                | 0         |
| 2          | 100                 | 3                        | 134                   | 78                         | 3              | 0.00        | 0.00                        | 0.00                  |                | 0         |
| 3          | 100                 | 4                        | 178                   | 104                        | 6              | 0.00        | 0.00                        | 0.01                  |                | 0         |
| 4          | 100                 | 5                        | 222                   | 130                        | 6              | 0.00        | 0.00                        | 0.01                  |                | 0         |
| 5          | 100                 | 6                        | 266                   | 156                        | 7              | 0.00        | 0.00                        | 0.03                  |                | 0         |
| 6          | 100                 | 7                        | 310                   | 182                        | 5              | 0.00        | 0.00                        | 0.01                  |                | 0         |
| 7          | 100                 | 9                        | 398                   | 234                        | 7              | 0.00        | 0.00                        | 0.02                  |                | 0         |
| 8          | 100                 | 11                       | 486                   | 286                        | 9              | 0.00        | 0.00                        | 0.02                  |                | 0         |
| 9          | 100                 | 13                       | 574                   | 338                        | 17             | 0.00        | 0.00                        | 0.03                  |                | 0         |
| 10         | 100                 | 15                       | 662                   | 390                        | 14             | 0.00        | 0.00                        | 0.04                  |                | 0         |
| 11         | 100                 | 17                       | 750                   | 442                        | 24             | 0.00        | 0.00                        | 0.05                  |                | 0         |
| 12         | 100                 | 21                       | 926                   | 546                        | 24             | 0.00        | 0.00                        | 0.07                  |                | 0         |
| 13         | 100                 | 26                       | 1146                  | 676                        | 27             | 0.00        | 0.00                        | 0.12                  |                | 0         |
| 14         | 100                 | 31                       | 1366                  | 806                        | 32             | 0.00        | 0.00                        | 0.12                  |                | 0         |
| 15         | 100                 | 36                       | 1586                  | 936                        | 33             | 0.00        | 0.00                        | 0.12                  |                | 0         |
| 16         | 100                 | 41                       | 1806                  | 1066                       | 33             | 0.00        | 0.00                        | 0.16                  |                | 0         |
| 17         | 100                 | 46                       | 2026                  | 1196                       | 53             | 0.00        | 0.00                        | 0.16                  |                | 0         |
| 18         | 100                 | 51                       | 2246                  | 1326                       | 48             | 0.00        | 0.00                        | 0.23                  |                | 0         |
| 19         | 100                 | 101                      | 4446                  | 2626                       | 73             | 0.00        | 0.00                        | 0.51                  |                | 0         |
| 20         | 100                 | 151                      | 6646                  | 3926                       | 76             | 0.00        | 0.00                        | 3.33                  |                | 0         |
| 21         | 100                 | 201                      | 8846                  | 5226                       | 93             | 0.00        | 0.00                        | 1.49                  |                | 0         |

Experiment | Optimum time (5$N$ terms) | Optimization time | % Timeout | Optimum weight (16$N$ terms) | Optimization time | % Timeout | Lexic. order cost time weight | Optimization time | % Timeout | Lexic. order weight time cost | Optimization time | % Timeout |
|------------|---------------------------|-------------------|-----------|-----------------------------|-------------------|-----------|--------------------------------|-------------------|-----------|--------------------------------|-------------------|-----------|
| 1          | 0.01                      | 0                 | 0.02      | 0                           | 0                 | 0.03      | 0                              | 0.03              | 0         |                                | 0                 |           |
| 2          | 0.01                      | 0                 | 0.06      | 0                           | 0                 | 0.04      | 0                              | 0.08              | 0         |                                | 0                 |           |
| 3          | 0.01                      | 0                 | 0.07      | 0                           | 0                 | 0.04      | 0                              | 0.11              | 0         |                                | 0                 |           |
| 4          | 0.02                      | 0                 | 0.56      | 0                           | 0                 | 0.07      | 0                              | 0.67              | 0         |                                | 0                 |           |
| 5          | 0.03                      | 0                 | 1.51      | 0                           | 0                 | 0.14      | 0                              | 1.69              | 0         |                                | 0                 |           |
| 6          | 0.03                      | 0                 | 0.45      | 0                           | 0                 | 0.11      | 0                              | 0.69              | 0         |                                | 0                 |           |
| 7          | 0.27                      | 0                 | 284.79    | 31                          | 0                 | 0.71      | 0                              | 557.00            | 36        |                                | 0                 |           |
| 8          | 0.13                      | 0                 | 852.66    | 80                          | 0                 | 0.92      | 0                              | 705.92            | 85        |                                | 0                 |           |
| 9          | 0.17                      | 0                 | –         | –                           | 47.55             | 9         | –                              | –                 | –         |                                | –                 |           |
| 10         | 1.23                      | 0                 | –         | –                           | 111.58            | 28        | –                              | –                 | –         |                                | –                 |           |
| 11         | 11.87                     | 0                 | –         | –                           | 35.31             | 56        | –                              | –                 | –         |                                | –                 |           |
| 12         | 104.67                    | 0                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 13         | 455.20                    | 51                | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 14         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 15         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 16         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 17         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 18         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 19         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 20         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
| 21         | –                         | –                 | –         | –                           | –                 | –         | –                              | –                 | –         |                                | –                 |           |
Table 7: First group of experiments, \( k = 8 \): median time over solved instances

| Experiment | Number of instances | Number of replicas (\( N \)) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Optimum cost (\( 2N \) terms) |
|------------|---------------------|-----------------------------|-----------------------|----------------------------|---------------|--------------|-------------------------------|-----------------------------|
| 1          | 100                 | 2                           | 90                    | 52                         | 10            | 0.00         | 0.00                          | 0.01                        |
| 2          | 100                 | 3                           | 134                   | 78                         | 15            | 0.00         | 0.00                          | 0.01                        |
| 3          | 100                 | 4                           | 178                   | 104                        | 9             | 0.00         | 0.00                          | 0.01                        |
| 4          | 100                 | 5                           | 222                   | 130                        | 11            | 0.00         | 0.00                          | 0.01                        |
| 5          | 100                 | 6                           | 266                   | 156                        | 21            | 0.00         | 0.00                          | 0.01                        |
| 6          | 100                 | 7                           | 310                   | 182                        | 24            | 0.00         | 0.00                          | 0.01                        |
| 7          | 100                 | 9                           | 398                   | 234                        | 33            | 0.00         | 0.00                          | 0.01                        |
| 8          | 100                 | 11                          | 486                   | 286                        | 23            | 0.00         | 0.00                          | 0.03                        |
| 9          | 100                 | 13                          | 574                   | 338                        | 28            | 0.00         | 0.00                          | 0.03                        |
| 10         | 100                 | 15                          | 662                   | 390                        | 36            | 0.00         | 0.00                          | 0.04                        |
| 11         | 100                 | 17                          | 750                   | 442                        | 20            | 0.00         | 0.00                          | 0.05                        |
| 12         | 100                 | 21                          | 926                   | 546                        | 48            | 0.00         | 0.00                          | 0.05                        |
| 13         | 100                 | 26                          | 1146                  | 676                        | 43            | 0.00         | 0.00                          | 0.06                        |
| 14         | 100                 | 31                          | 1366                  | 806                        | 61            | 0.00         | 0.00                          | 0.10                        |
| 15         | 100                 | 36                          | 1586                  | 936                        | 67            | 0.00         | 0.00                          | 0.23                        |
| 16         | 100                 | 41                          | 1806                  | 1066                       | 71            | 0.00         | 0.00                          | 0.39                        |
| 17         | 100                 | 46                          | 2026                  | 1196                       | 77            | 0.00         | 0.00                          | 0.17                        |
| 18         | 100                 | 51                          | 2246                  | 1326                       | 75            | 0.00         | 0.00                          | 0.17                        |
| 19         | 100                 | 101                         | 4446                  | 2626                       | 98            | 0.00         | 0.00                          | 1.47                        |
| 20         | 100                 | 151                         | 6646                  | 3926                       | 97            | 0.00         | 0.00                          | 40.11                       |
| 21         | 100                 | 201                         | 8846                  | 5226                       | 100           | 0.00         | 0.00                          | –                          |

| Experiment | Optimum time (5\( N \) terms) | % Timeout | Optimum weight (16\( N \) terms) | % Timeout | Lexic. order cost time weight | % Timeout | Optimum order weight time cost | % Timeout |
|------------|--------------------------------|-----------|----------------------------------|-----------|-------------------------------|-----------|-------------------------------|-----------|
| 1          | 0.01                           | 0         | 0.03                             | 0         | 0.02                          | 0         | 0.04                          | 0         |
| 2          | 0.01                           | 0         | 0.03                             | 0         | 0.04                          | 0         | 0.06                          | 0         |
| 3          | 0.01                           | 0         | 0.14                             | 0         | 0.04                          | 0         | 0.19                          | 0         |
| 4          | 0.02                           | 0         | 0.07                             | 0         | 0.06                          | 0         | 0.09                          | 0         |
| 5          | 0.03                           | 0         | 1.85                             | 0         | 0.07                          | 0         | 2.25                          | 0         |
| 6          | 0.02                           | 0         | 14.71                            | 0         | 0.10                          | 0         | 14.11                         | 0         |
| 7          | 0.11                           | 0         | 17.37                            | 1         | 0.15                          | 0         | 25.14                         | 1         |
| 8          | 0.31                           | 0         | 79.55                            | 19        | 0.51                          | 0         | 253.57                        | 28        |
| 9          | 0.22                           | 0         | 131.37                           | 55        | 0.64                          | 0         | 240.96                        | 59        |
| 10         | 0.41                           | 0         | 6.89                             |           |                               |           |                               |           |
| 11         | 0.86                           | 0         |                                 |           |                               |           |                               |           |
| 12         | 149.86                         | 7         |                                 |           |                               |           |                               |           |
| 13         | 406.31                         | 23        |                                 |           |                               |           |                               |           |
| 14         |                                 |           |                                 |           |                               |           |                               |           |
| 15         |                                 |           |                                 |           |                               |           |                               |           |
| 16         |                                 |           |                                 |           |                               |           |                               |           |
| 17         |                                 |           |                                 |           |                               |           |                               |           |
| 18         |                                 |           |                                 |           |                               |           |                               |           |
| 19         |                                 |           |                                 |           |                               |           |                               |           |
| 20         |                                 |           |                                 |           |                               |           |                               |           |
| 21         |                                 |           |                                 |           |                               |           |                               |           |
Fig. 17 First group of experiments, $k = 2$, median run times over solved instances

Fig. 18 First group of experiments, $k = 4$, median run times over solved instances

Fig. 19 First group of experiments, $k = 5$, median run times over solved instances
Second group of experiments

See Tables 8, 9, 10 and Figs. 21, 22, 23.
Table 8  Second group of experiments, \( k = 2, p = 6 \): median time over solved instances

| Experiment | Number of instances | Number of replicas (N) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Lexic. order PRT | Lexic. order RPT |
|------------|---------------------|------------------------|-----------------------|-----------------------------|---------------|-------------|-----------------------------|-----------------|-----------------|
|            |                     |                        |                       |                             |               |             |                             | Optimization time | % Timeout | Optimization time | % Timeout |
| 1          | 100                 | 2                      | 110                   | 60                          | 1             | 0.00        | 0.00                        | 0.04            | 0               | 0.08            | 0           |
| 2          | 100                 | 3                      | 164                   | 90                          | 2             | 0.00        | 0.00                        | 0.07            | 0               | 0.08            | 0           |
| 3          | 100                 | 4                      | 218                   | 120                         | 1             | 0.00        | 0.00                        | 0.11            | 0               | 0.09            | 0           |
| 4          | 100                 | 5                      | 272                   | 150                         | 3             | 0.00        | 0.00                        | 0.12            | 0               | 0.15            | 0           |
| 5          | 100                 | 6                      | 326                   | 180                         | 2             | 0.00        | 0.00                        | 0.13            | 0               | 0.20            | 0           |
| 6          | 100                 | 7                      | 380                   | 210                         | 3             | 0.00        | 0.00                        | 0.21            | 0               | 0.26            | 0           |
| 7          | 100                 | 9                      | 488                   | 270                         | 2             | 0.00        | 0.00                        | 0.52            | 0               | 0.45            | 0           |
| 8          | 100                 | 11                     | 596                   | 330                         | 7             | 0.00        | 0.00                        | 0.90            | 0               | 0.50            | 0           |
| 9          | 100                 | 13                     | 704                   | 390                         | 8             | 0.00        | 0.00                        | 2.42            | 0               | 2.33            | 0           |
| 10         | 100                 | 15                     | 812                   | 450                         | 4             | 0.00        | 0.00                        | 39.45           | 0               | 1.55            | 0           |
| 11         | 100                 | 17                     | 920                   | 510                         | 6             | 0.00        | 0.00                        | 1.64            | 0               | 1.57            | 0           |
| 12         | 100                 | 21                     | 1136                  | 630                         | 7             | 0.00        | 0.00                        | 694.50          | 52              | 468.88          | 20          |
| 13         | 100                 | 26                     | 1406                  | 780                         | 6             | 0.00        | 0.00                        | –               | –               | –               | –           |
| 14         | 100                 | 31                     | 1676                  | 930                         | 14            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 15         | 100                 | 36                     | 1946                  | 1080                        | 15            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 16         | 100                 | 41                     | 2216                  | 1230                        | 19            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 17         | 100                 | 46                     | 2486                  | 1380                        | 16            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 18         | 100                 | 51                     | 2756                  | 1530                        | 27            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 19         | 100                 | 101                    | 5456                  | 3030                        | 33            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 20         | 100                 | 151                    | 8156                  | 4530                        | 46            | 0.00        | 0.00                        | –               | –               | –               | –           |
| 21         | 100                 | 201                    | 10,856                | 6030                        | 56            | 0.00        | 0.00                        | –               | –               | –               | –           |
Table 9  Second group of experiments, $k = 2$, $p = 8$: median time over solved instances

| Experiment | Number of instances | Number of replicas ($N$) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Lexic. order PRT Optimization time | % Timeout | Lexic. order RPT Optimization time | % Timeout |
|------------|---------------------|-------------------------|-----------------------|----------------------------|---------------|--------------|-------------------------------|-----------------------------------|-----------|-----------------------------------|-----------|
| 1          | 100                 | 2                       | 110                   | 60                         | 0             | 0.00         | 0.00                          | 0.06                              | 0         | 0.07                              | 0         |
| 2          | 100                 | 3                       | 164                   | 90                         | 1             | 0.00         | 0.00                          | 0.08                              | 0         | 0.08                              | 0         |
| 3          | 100                 | 4                       | 218                   | 120                        | 0             | 0.00         | 0.00                          | 0.18                              | 0         | 0.09                              | 0         |
| 4          | 100                 | 5                       | 272                   | 150                        | 2             | 0.00         | 0.00                          | 0.18                              | 0         | 0.14                              | 0         |
| 5          | 100                 | 6                       | 326                   | 180                        | 1             | 0.00         | 0.00                          | 0.36                              | 0         | 0.18                              | 0         |
| 6          | 100                 | 7                       | 380                   | 210                        | 2             | 0.00         | 0.00                          | 0.21                              | 0         | 0.20                              | 0         |
| 7          | 100                 | 9                       | 488                   | 270                        | 6             | 0.00         | 0.00                          | 0.28                              | 0         | 0.30                              | 0         |
| 8          | 100                 | 11                      | 596                   | 330                        | 4             | 0.00         | 0.00                          | 0.61                              | 0         | 0.47                              | 0         |
| 9          | 100                 | 13                      | 704                   | 390                        | 6             | 0.00         | 0.00                          | 0.73                              | 0         | 0.53                              | 0         |
| 10         | 100                 | 15                      | 812                   | 450                        | 12            | 0.00         | 0.00                          | 1.38                              | 0         | 0.69                              | 0         |
| 11         | 100                 | 17                      | 920                   | 510                        | 6             | 0.00         | 0.00                          | 1.81                              | 0         | 0.99                              | 0         |
| 12         | 100                 | 21                      | 1136                  | 630                        | 10            | 0.00         | 0.00                          | 7.00                              | 0         | 3.92                              | 0         |
| 13         | 100                 | 26                      | 1406                  | 780                        | 11            | 0.00         | 0.00                          | 330.39                            | 10        | 9.38                              | 1         |
| 14         | 100                 | 31                      | 1676                  | 930                        | 11            | 0.00         | 0.00                          | 327.86                            | 72        | 8.40                              | 10        |
| 15         | 100                 | 36                      | 1946                  | 1080                       | 14            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 16         | 100                 | 41                      | 2216                  | 1230                       | 13            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 17         | 100                 | 46                      | 2486                  | 1380                       | 14            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 18         | 100                 | 51                      | 2756                  | 1530                       | 20            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 19         | 100                 | 101                     | 5456                  | 3030                       | 33            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 20         | 100                 | 151                     | 8156                  | 4530                       | 40            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
| 21         | 100                 | 201                     | 10,856                | 6030                       | 59            | 0.00         | 0.00                          | -                                 | -         | -                                 | -         |
Table 10  Second group of experiments, $k = 2$, $p = 12$: median time over solved instances

| Experiment | Number of instances | Number of replicas ($N$) | Total number of nodes | Number of rational variables | % Unrealizable | Solving time | Time for proving unrealizable | Lexic. order PRT Optimization time % Timeout | Lexic. order RPT Optimization time % Timeout |
|------------|---------------------|--------------------------|-----------------------|-----------------------------|---------------|--------------|-----------------------------|---------------------------------|---------------------------------|
| 1          | 100                 | 2                        | 110                   | 60                          | 1             | 0.00         | 0.00                        | 0.06                            | 0                               |
| 2          | 100                 | 3                        | 164                   | 90                          | 0             | 0.00         | 0.00                        | 0.09                            | 0                               |
| 3          | 100                 | 4                        | 218                   | 120                         | 0             | 0.00         | 0.00                        | 0.13                            | 0                               |
| 4          | 100                 | 5                        | 272                   | 150                         | 0             | 0.00         | 0.00                        | 0.15                            | 0                               |
| 5          | 100                 | 6                        | 326                   | 180                         | 0             | 0.00         | 0.00                        | 0.25                            | 0                               |
| 6          | 100                 | 7                        | 380                   | 210                         | 0             | 0.00         | 0.00                        | 0.39                            | 0                               |
| 7          | 100                 | 9                        | 488                   | 270                         | 0             | 0.00         | 0.00                        | 0.43                            | 0                               |
| 8          | 100                 | 11                       | 596                   | 330                         | 0             | 0.00         | 0.00                        | 0.81                            | 0                               |
| 9          | 100                 | 13                       | 704                   | 390                         | 1             | 0.00         | 0.00                        | 1.15                            | 0                               |
| 10         | 100                 | 15                       | 812                   | 450                         | 1             | 0.00         | 0.00                        | 1.32                            | 0                               |
| 11         | 100                 | 17                       | 920                   | 510                         | 2             | 0.00         | 0.00                        | 14.66                           | 0                               |
| 12         | 100                 | 21                       | 1136                  | 630                         | 0             | 0.00         | 0.00                        | 602.22                          | 23                              |
| 13         | 100                 | 26                       | 1406                  | 780                         | 2             | 0.00         | 0.00                        | 911.26                          | 87                              |
| 14         | 100                 | 31                       | 1676                  | 930                         | 4             | 0.00         | 0.00                        | –                               | –                               |
| 15         | 100                 | 36                       | 1946                  | 1080                        | 0             | 0.00         | 0.00                        | –                               | –                               |
| 16         | 100                 | 41                       | 2216                  | 1230                        | 1             | 0.00         | 0.00                        | –                               | –                               |
| 17         | 100                 | 46                       | 2486                  | 1380                        | 2             | 0.00         | 0.00                        | –                               | –                               |
| 18         | 100                 | 51                       | 2756                  | 1530                        | 1             | 0.00         | 0.00                        | –                               | –                               |
| 19         | 100                 | 101                      | 5456                  | 3030                        | 5             | 0.00         | 0.00                        | –                               | –                               |
| 20         | 100                 | 151                      | 8156                  | 4530                        | 5             | 0.00         | 0.00                        | –                               | –                               |
| 21         | 100                 | 201                      | 10,856                | 6030                        | 10            | 0.00         | 0.00                        | –                               | –                               |
Fig. 21 Second group of experiments, $k = 2, k = 6$, median run times over solved instances

Fig. 22 Second group of experiments, $k = 2, k = 8$, median run times over solved instances
Fig. 23 Second group of experiments, $k = 2$, $k = 12$, median run times over solved instances

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