Neighborhoods of certain $p$–valent analytic functions defined by using generalized differential operators

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Abstract

In this paper, by use of the familiar concept of neighborhoods of $p$–valently analytic functions, the authors show several inclusion relations associated with the $(n, \tau)$–neighborhood of certain subclasses of analytic functions.

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1. Introduction and preliminaries

Let $A(p)$ denote the class of functions $f$ of the form:

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \quad (z \in \mathbb{U}, \ p, n \in \mathbb{N} = \{1, 2, 3, \ldots\}) \quad (1.1)$$

which are analytic and univalent in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. The function $f(z)$ in $A(p)$ is said to be $p$–valent starlike functions of order $\alpha$ and univalent convex of order $\alpha$ in $\mathbb{U}$ if it satisfies:

$$\text{Re} \left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha \quad (p \in \mathbb{N}, \ z \in \mathbb{U}, \ 0 \leq \alpha < p) \quad (1.2)$$

and

$$\text{Re} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \alpha \quad (p \in \mathbb{N}, \ z \in \mathbb{U}, \ 0 \leq \alpha < p) \quad (1.3)$$

respectively.

For the function $f \in A(p)$, we define the following new generalized differential operator as
follows:

\[
A_{\mu,\lambda,p}^0(\alpha, \beta, \delta)f(z) = f(z), \\
A_{\mu,\lambda,p}^1(\alpha, \beta, \delta)f(z) = \left( \frac{\mu + p (1 - \beta (\lambda - \alpha))}{\mu + \lambda} \right) f(z) + \left( \frac{\beta (\lambda - \alpha) + (1 - p) \delta}{\mu + \lambda} \right) zf'(z) \\
+ \frac{\delta}{\mu + \lambda} zf''(z),
\]

and for \( m = 1, 2, 3, \ldots \) generalises various operators as follows.

\[
A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)f(z) = A_{\mu,\lambda,p}^m(\alpha, \beta, \delta)(A_{\mu,\lambda,p}^{m-1}(\alpha, \beta, \delta)f(z)).
\]

(1.4)

\[
= z^p + \sum_{n=p+1}^{\infty} \left[ 1 + \left( \frac{1}{\mu + \lambda} (\lambda + n \delta) \right)^m \right] a_n z^n.
\]

(1.5)

for \( f \in A(p) \), \( \alpha, \beta, \mu, \lambda \geq 0 \), \( \beta, \lambda > 0 \), \( \lambda \neq \alpha \), \( p \in \mathbb{N} \), \( p \neq n \) and \( m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\} \). This generalises various operators as follows.

(i) \( A_{0,\lambda,1}^m(\alpha, \beta, \delta)f(z) = z + \sum_{n=2}^{\infty} \left[ 1 + \left( \frac{1}{\mu + \lambda} (\lambda + n \delta) \right)^m \right] a_n z^n \), the operator introduced and studied by Amourah and Darus (2016) [4].

(ii) \( A_{0,\lambda,1}^m(\alpha, \beta, 0)f(z) = z + \sum_{n=2}^{\infty} \left[ 1 + \left( \frac{1}{\mu + \lambda} (\lambda + n \delta) \right)^m \right] a_n z^n \), the operator introduced and studied by Darus and Ibrahim (2009) [12].

(iii) \( A_{0,\lambda,1}^m(0,1,0)f(z) = z + \sum_{n=2}^{\infty} \left[ 1 + \left( \frac{1}{\mu + \lambda} (\lambda + n \delta) \right)^m \right] a_n z^n \), the operator introduced and studied by Al-Oboudi (2004) [1].

(iv) \( A_{0,\lambda,1}^m(0,1,0)f(z) = z + \sum_{n=2}^{\infty} \left[ n^m \right] a_n z^n \), the operator introduced and studied by Sălăgean (1983) [13].

Further, let \( T^*(p) \) denoted the supclass of \( A(p) \) consisting functions of the form

\[
f(z) = z^p - \sum_{n=p+1}^{\infty} a_n z^n \quad (a_n \geq 0, \, n, p \in \mathbb{N}, \, z \in \mathbb{U}).
\]

(1.6)

Now, we define the \( (n, \tau) \)- neighborhood of the function \( f \in T^*(p) \) by

\[
N_{n,\delta}(f) = \left\{ g \in T^*(p) : g(z) = z^p - \sum_{n=p+1}^{\infty} |b_n| z^n \text{ and } \sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau \right\}.
\]

(1.7)

In particular, for the identity function

\[
h(z) = z^p
\]

we immediately have

\[
N_{n,\delta}(h) = \left\{ g \in T^*(p) : g(z) = z^p - \sum_{n=p+1}^{\infty} b_n z^n \text{ and } \sum_{n=p+1}^{\infty} n |b_n| \leq \tau \right\}.
\]

(1.9)

The concept of the neighborhood was first presented by Goodman [14], and then generalized by Ruscheweyh [15], and applied recently to Neighborhoods of certain \( p \)-valent analytic functions by Amourah.
and Darus [3], [9], [2]), and others including Al-Hawary et al. [10], [4], [6] and Anakira et al. [7], [11], [8].

We define and study some \((n, \tau)-\) neighborhood properties. The classes \(G^*(b, p)\) and \(G^*(b, p, \chi)\) are defined as follows:

**Definition 1.1.** A function \(f \in T^*\) is said to be in the class \(G^*(b, p)\) if and only if

\[
\left| \frac{1}{b} \left[ \frac{z(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)} - p \right] \right| < \zeta,
\]

for some \(\alpha \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}\) and \(m \in \mathbb{N}_0\) and for all \(z \in \mathbb{U}\).

**Definition 1.2.** A function \(f \in T^*(p)\) is said to be in the class \(G^*(b, p, \chi)\) if and only if

\[
\left| \frac{1}{b} \left[ (1 - \chi) \frac{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)}{zp} + \chi \frac{(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{pz^{p-1}} - 1 \right] \right| < \zeta,
\]

for some \(\alpha, \chi \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}\) and \(m \in \mathbb{N}_0\) and for all \(z \in \mathbb{U}\).

2. Neighborhoods for the classes \(G^*(b, p)\) and \(G^*(b, p, \chi)\)

In our investigation of the inclusion relations involving \(N_{n, \delta}(h)\), we shall require theorem 2.1 and theorem 2.2 below:

**Theorem 2.1.** Let the function \(f(z) \in T^*(p)\) be defined by (1.6). Then \(f\) is in the class \(G^*(b, p)\) if and only if

\[
\sum_{n=p+1}^{\infty} (n - p + \zeta |b|) \left[ 1 + \frac{(n - p) [(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right] a_n \leq \zeta |b|, \quad (2.1)
\]

where \(\alpha \geq 0, 0 < \zeta \leq 1, \beta, \lambda, \mu > 0, \lambda \neq \alpha, p \in \mathbb{N}, p \neq n, b \in \mathbb{C} - \{0\}\) and \(m \in \mathbb{N}_0\).

**Proof.** We first suppose that \(f \in G^*(b, p)\). Then by appealing to the condition (1.10), we readily obtain

\[
\text{Re} \left\{ \frac{z(A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z))'}{A_{\mu, \lambda, p}^m(\alpha, \beta, \delta)f(z)} - p \right\} > -\zeta |b|.
\]

or, equivalently,

\[
\text{Re} \left\{ \frac{- \sum_{n=p+1}^{\infty} (n - p) \left[ 1 + \frac{(n - p) [(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right] a_n z^{n-p}}{1 - \sum_{n=p+1}^{\infty} \left[ 1 + \frac{(n - p) [(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right] a_n z^{n-p}} \right\} > -\zeta |b|.
\]

We now choose values of \(z\) on the real axis and let \(z \to 1^+\) through real values. Then the inequality (2.2) immediately yields the desired condition (2.1).

Conversely, by applying the hypothesis (2.1) and letting \(|z| = 1\), we find from (1.10) that
Proof. We first suppose that immediately yields the desired condition (2.7).

Theorem 2.2. Let the function $f(z) \in T^*(p)$ be defined by (1.6). Then $f$ is in the class $G^*(b, p, \chi)$ if and only if

$$\sum_{n=p+1}^{\infty} \left( p + \chi(n - p) \right) \left[ 1 + \frac{(n - p) \mu}{\mu} \right] \leq \frac{p}{\mu + \lambda} \leq \frac{1}{\lambda - \alpha}.$$ (2.7)

Proof. We first suppose that $f \in G^*(b, p, \chi)$. Then by appealing to the condition (1.11), we readily obtain

$$\text{Re} \left\{ (1 - \chi) - \frac{A_m^{\mu}(\alpha, \beta, \delta) f(z)}{z^p} + \frac{A_m^{\mu}(\alpha, \beta, \delta) f(z)}{p z^{p-1}} - 1 \right\} > -\zeta |b|$$

or, equivalently

$$\text{Re} \left\{ - \sum_{n=p+1}^{\infty} \left( 1 - \chi + \frac{n}{p} \right) \left[ 1 + \frac{(n - p) \mu}{\mu + \lambda} \right] \leq \frac{p}{\mu + \lambda} \leq \frac{1}{\lambda - \alpha}.$$ (2.8)

We now choose values of $z$ on the real axis and let $z \to 1$ through real values. Then the inequality (2.8) immediately yields the desired condition (2.7).
Conversely, by applying the hypothesis (2.7) and letting \( z = 1 \), we find from (1.11) that
\[
\left| (1 - \chi) \Lambda_{\mu, \lambda, p}^m (\alpha, \beta, \delta) f(z) \right|_p + \chi \left| \frac{\Lambda_{\mu, \lambda, p}^m (\alpha, \beta, \delta) f(z)}{pz^{p-1}} \right|_p - 1
\]
\[
\leq \sum_{n=p+1}^\infty (1 - \chi + \frac{n}{p}) \left[ 1 + \frac{(n - p) \left[ (\lambda - \alpha) \beta + n \delta \right]}{\mu + \lambda} \right]^m a_n z^{n-p}
\]
\[
\leq \sum_{n=p+1}^\infty (1 - \chi + \frac{n}{p}) \left[ 1 + \frac{(n - p) \left[ (\lambda - \alpha) \beta + n \delta \right]}{\mu + \lambda} \right]^m a_n \leq \zeta |b|.
\]
Thus, we have
\[
\sum_{n=2}^\infty (p + \chi(n - p)) \left[ 1 + \frac{(n - p) \left[ (\lambda - \alpha) \beta + n \delta \right]}{\mu + \lambda} \right]^m a_n \leq p \zeta |b|.
\]
Hence, we have \( f \in G^*(b, p, \chi) \).

Our first inclusion relation involving \( N_{k, \delta}(h) \) is given by theorem 2.3 below.

\begin{align*}
\text{Theorem 2.3. If} \quad \tau &= \frac{\zeta |b| (1 + p)}{(1 + \zeta |b|) \left[ 1 + \frac{(n - p) \left[ (\lambda - \alpha) \beta + n \delta \right]}{\mu + \lambda} \right]^m} \quad (|b| < 1), \\
\text{then} \quad G^*(b, p) &\subseteq N_{n, \delta}(h). (2.10)
\end{align*}

\textbf{Proof.} Let \( f(z) \in G^*(b, p) \). Then, in view of the assertion (2.1) of theorem 2.1, we have
\[
(1 + \zeta |b|) \left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m \sum_{n=p+1}^\infty a_n \leq \sum_{n=p+1}^\infty (n - p + \zeta |b|) \left[ 1 + \frac{(n - p) \left[ (\lambda - \alpha) \beta + n \delta \right]}{\mu + \lambda} \right]^m a_n \leq \zeta |b|,
\]
which readily yeilds
\[
\sum_{n=p+1}^\infty a_n \leq \frac{\zeta |b|}{(1 + \zeta |b|) \left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m}. (2.11)
\]
Making use of (2.1) again, in conjunction with (2.11), we get
\[
\left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m \sum_{n=p+1}^\infty n a_n \leq \zeta |b| + \left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m (1 - \zeta |b|) \sum_{n=p+1}^\infty a_n
\]
\[
\leq \frac{\zeta |b| (1 + p)}{(1 + \zeta |b|) \left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m}.
\]
Hence
\[
\sum_{n=p+1}^\infty n a_n \leq \frac{\zeta |b| (1 + p)}{(1 + \zeta |b|) \left[ 1 + \frac{[\lambda - \alpha \beta + n \delta]}{\mu + \lambda} \right]^m} = \tau, \quad (|b| < 1)
\]
which, by means of the definition (1.9), establishes the inclusion (2.10) asserted by theorem 2.3.
Similarly, by applying theorem 2.2 instead of theorem 2.1, we now prove theorem 2.4 below.
Theorem 2.4. If
\[ \tau = \frac{p \zeta |b| (1 + p)}{(p + \chi) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m} \quad (|b| < p), \]
then
\[ G^*(b, p, \chi) \subset N_{n, \delta}(h). \quad (2.12) \]

Proof. Let \( f(z) \in G^*(b, p, \chi) \). Then, in view of the assertion (2.7) of theorem 2.2, we have
\[ \left( \frac{p + \chi}{p + \chi(n - p)} \right) \left[ 1 + \frac{(n - p) [(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m a_n \leq p \zeta |b|, \]
which readily yeilds
\[ \sum_{n=p+1}^{\infty} a_n \leq \frac{p \zeta |b|}{(p + \chi) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m}. \quad (2.13) \]
Making use of (2.7) again, in conjunction with (2.13), we get
\[ \chi \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m \sum_{n=p+1}^{\infty} n a_n \leq p \zeta |b| + p(\chi - 1) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m \sum_{n=p+1}^{\infty} a_n \leq \frac{p \zeta |b| (p + 1) \chi}{(p + \chi)}. \]
Hence
\[ \sum_{n=p+1}^{\infty} n a_n \leq \frac{p \zeta |b| (p + 1)}{(p + \chi) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m} = \tau, \quad (|b| < p) \]
which, by means of the definition (1.9), establishes the inclusion (2.12) asserted by theorem 2.4. \( \Box \)

3. Neighborhoods for the classes \( G^*_\tau(b, p) \) and \( G^*_\tau(b, p, \chi) \)

In this section, we determine the neighborhood for the each classes \( G^*_\tau(b, p) \) and \( G^*_\tau(b, p, \chi) \), which we define as follows. A function \( f(z) \in \mathcal{A}(p) \) is said to be in the class \( G^*_\tau(b, p) \) if there exists a function \( g(z) \in G^*_\tau(b, p) \) such that
\[ \left| \frac{f(z)}{g(z)} - 1 \right| < p - \varepsilon, (0 \leq \varepsilon < p). \quad (3.1) \]
Analogously, a function \( f(z) \in \mathcal{A}(p) \) is said to be in the class \( G^*_\tau(b, p, \chi) \) if there exists a function \( g(z) \in G^*_\tau(b, p, \chi) \) such that the inequality (3.1) holds true.

Theorem 3.1. If \( g(z) \in G^*_\tau(b, p) \) and
\[ \varepsilon = 1 - \frac{\tau (1 + \zeta |b|) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m}{(p + 1) (1 + \zeta |b|) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m - \zeta |b|}, \quad (3.2) \]
then
\[ N_{n, \delta}(g) \subset G^*_\tau(b, p). \]
Proof. Suppose that \( f(z) \in N_{n,\delta}(g) \). We find from (1.7) that
\[
\sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau, \tag{3.3}
\]
which readily implies that
\[
\sum_{n=p+1}^{\infty} |a_n - b_n| \leq \frac{\tau}{p+1}.
\]
Next, since \( g(z) \in G^*(b, p) \), we have [cf. equation 2.11]
\[
\sum_{n=p+1}^{\infty} b_n \leq \frac{\zeta |b|}{(1 + \zeta |b|) \left[ 1 + \frac{(\lambda - \alpha) \beta + n \delta}{\mu + \lambda} \right]^m},
\]
letting \( |z| \to 1 \), so we have
\[
\left| \frac{f(z)}{g(z)} - 1 \right| \leq \frac{\sum_{n=p+1}^{\infty} |a_n - b_n|}{1 - \sum_{n=p+1}^{\infty} |b_n|} \leq \frac{\tau}{(p+1)(1 + \zeta |b|) \left[ 1 + \frac{(\lambda - \alpha) \beta + n \delta}{\mu + \lambda} \right]^m - p \zeta |b|} = p - \epsilon
\]
provided that \( \alpha \) is given by (3.2). Thus, by the above definition, \( f(z) \in G^*_\epsilon(b, p) \) for \( \epsilon \) given by (3.2).

This evidently proves theorem 3.1. \( \blacksquare \)

Our proof of theorem 3.2 below is much akin to that of theorem 3.1.

Theorem 3.2. If \( g(z) \in G^*_\epsilon(b, p, \chi) \) and
\[
\epsilon = p - \frac{\tau (p + \chi) \left[ 1 + \frac{(\lambda - \alpha) \beta + n \delta}{\mu + \lambda} \right]^m}{(p+1)(p+\chi) \left[ 1 + \frac{(\lambda - \alpha) \beta + n \delta}{\mu + \lambda} \right]^m - p \zeta |b|}, \tag{3.5}
\]
then
\[
N_{n,\delta}(g) \subset G^*_\epsilon(b, p, \chi).
\]

Proof. Suppose that \( f(z) \in N_{n,\delta}(g) \). We find from (1.7) that
\[
\sum_{n=p+1}^{\infty} n |a_n - b_n| \leq \tau, \tag{3.6}
\]
which readily implies that
\[
\sum_{n=p+1}^{\infty} |a_n - b_n| \leq \frac{\tau}{p+1}.
\]
Next, since \( g(z) \in G^*_\epsilon(b, p, \chi) \), we have [cf. equation 2.13]
\[
\sum_{n=p+1}^{\infty} b_n \leq \frac{p \zeta |b|}{(p+\chi) \left[ 1 + \frac{(\lambda - \alpha) \beta + n \delta}{\mu + \lambda} \right]^m},
\]
letting $|z| \to 1$, so we have

$$\frac{|f(z) - 1|}{|g(z)|} \leq \frac{\sum_{n=p+1}^{\infty} |a_n - b_n|}{1 - \sum_{n=p+1}^{\infty} |b_n|} \leq \frac{\delta}{p+1} \frac{(p + \chi) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m}{(p + \chi) \left[ 1 + \frac{[(\lambda - \alpha)\beta + n\delta]}{\mu + \lambda} \right]^m - p\zeta |b|}$$

provided that $\varepsilon$ is given by (3.5). Thus, by the above definition, $f(z) \in G^*_\varepsilon(b,p,\chi)$ for $\varepsilon$ given by (3.5).

This evidently completes our proof of theorem 3.2.

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