From the spin-fermion model to anisotropic superconductivity

Lizardo H. C. M. Nunes\textsuperscript{a,}\textsuperscript{*}, Eduardo C. Marino\textsuperscript{a}

\textsuperscript{a}Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, 21941-972, Brazil

Abstract

We use the spin-fermion model to describe the CuO\textsubscript{2} planes of the high-$T_c$ superconductors. Using a large wavelength approach, we show that the ferromagnetic component of the Cu spin fluctuations couple to the oxygen holes producing a pairing interaction that leads to a superconducting gap whose symmetry is determined by the anisotropy of the Kondo interaction. We calculate $T_c$ as a function of the hole concentration in a mean-field approximation and our numerical results are in good agreement with the experiments.

Key words: cuprate superconductors

PACS: 74.25.Ha; 74.72.-h

The spin-fermion model is very useful in the description of a variety of strongly correlated electronic systems, ranging from heavy-fermions to manganites and high-$T_c$ superconductors. Our purpose in this note is to show that, integrating over the spin degrees of freedom, in the large wavelength limit of this model, we obtain an effective interaction for the electrons that produces a $d$-wave symmetric superconducting gap. We numerically solve the gap equation and display the superconducting transition temperature $T_c$ as a function of the occupation number. A crucial ingredient for this result, as we shall see, is the anisotropy of the Kondo interaction.

We start from the spin-fermion Hamiltonian \cite{1} envisaging the description of the CuO\textsubscript{2} planes of the cuprates,

\begin{equation}
H = -t_{pp} \sum_{\{ll\}, \sigma} c_{l,l',\sigma}^\dagger c_{l',\sigma} + J_B \sum_{\{ij\}} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\{i,j,j'\}} J_{ij,j'}^K \mathbf{S}_i \cdot \mathbf{s}_{j,j'},
\end{equation}

where $\mathbf{s}_{j,j'} = \sum_{\alpha, \beta} c_{j,\alpha}^\dagger c_{j',\beta}^\dagger \sigma_{\alpha, \beta}$ and $c_{j,\alpha}^\dagger$ creates a hole with spin $\sigma$ in the oxygen $p_{x,y}$-orbitals. The site indices $i$ and $j$ denote the Cu and O sites respectively and the last term in the above Eq. (1) is a non-local Kondo like interaction between the spin on the Cu site, which is represented by $\mathbf{S}$, and the holes on the four surrounding O sites.

We shall now use the long-wavelength continuum field theory associated to Eq. (1). We make $c_{j,\sigma} \rightarrow \psi_\sigma(x)$ and using spin coherent states, we have $\mathbf{S} \rightarrow SN(x)$, which are the continuum eigenvalues of the spin in these states.

The grand-partition function becomes

\begin{equation}
Z = \int D\psi D\psi^\dagger D\mathbf{L} D\mathbf{n} e^{S_B - \int_0^\beta d\tau \left( \int d^2 x \mathcal{L}_0 + H_K + H_H \right)},
\end{equation}

where $S_B$ is the Berry’s phase and $\mathbf{n}$ and $\mathbf{L}$ are, respectively, the antiferromagnetic and ferromagnetic fluctuations of $\mathbf{N}$.

\begin{equation}
\mathcal{L}_0 = \sum_{\sigma} \psi_\sigma^\dagger(x) \left( \partial_\tau + \frac{\nabla^2}{2m^*} - \mu \right) \psi_\sigma(x),
\end{equation}

is related to the hopping term of the Hamiltonian,

\begin{equation}
H_H = \frac{1}{2} \int d^2 x \left( \rho_\sigma |\nabla \mathbf{n}|^2 + \chi_\perp S^2|\mathbf{L}|^2 \right)
\end{equation}
is the well-known Heisenberg Hamiltonian in the continuum limit [2], and

\[ H_K = \frac{S}{\alpha^2} \int d^2 x d^2 y _1 d^2 y _2 J(\mathbf{x}; y _1; y _2) \left( \sum_{, } \psi _{\alpha, \beta}^\dagger(\mathbf{x}) (\mathbf{L} \cdot \sigma)_{\alpha, \beta} \psi _\beta(\mathbf{x}) \right), \tag{5} \]

is the Kondo part of the Hamiltonian, where we have neglected oscillating terms corresponding to the antiferromagnetic fluctuations. Moreover, the Kondo coupling in Eq. (5) is given by

\[ J_K(\mathbf{x}; y _1; y _2) = J_K \sum_{a_i, a_j} \eta _{a_i, a_j} \delta[y _1 - (\mathbf{x} + a_i)] \times \delta[y _2 - (\mathbf{x} + a_j)] , \tag{6} \]

where the coefficients \( \eta _{a_i, a_j} = \pm 1 \) reflect the anisotropy of the copper and oxygen wave functions, and the vector \( a_i \) runs over the four first-neighbors oxygen sites around a copper atom.

Integrating over \( \mathbf{L} \), we obtain

\[ s^2 = -\frac{3}{2} \sum_{\alpha, \beta} \sum_{a_i, a_j, b_i, b_j} \eta _{a_i, a_j} \eta _{b_i, b_j} \psi _\alpha^\dagger \psi _\beta \psi _\beta^\dagger \psi _\alpha + 4 \sum_{\sigma, a_i, a_j} \eta _{a_i, a_j} \psi _\sigma^\dagger (\mathbf{x} + a_i) \psi _\sigma (\mathbf{x} + a_j) \equiv \mathcal{L}_\text{eff}^1 , \tag{7} \]

in addition to the usual dynamical term for \( \mathbf{n} \) and a crossed term, which is eliminated by a canonical transformation [3].

We arrive at the effective Lagrangian

\[ \mathcal{L}_\text{eff} = \mathcal{L}_0 + \sum_\sigma \phi _\sigma \psi _\sigma^\dagger \psi _\sigma + \mathcal{L}_\text{eff}^1 + \mathcal{L}_\text{NLS} \tag{8} \]

where \( \phi _\sigma(\mathbf{z}) = \sum_{i=1,2} (\pm) z _i^\dagger \partial _i z _i + \nabla z _i \nabla z _i \) and \( \mathcal{L}_\text{NLS} \) is the well-known non-linear sigma model Lagrangian. Eq. (8) potentially describes both the magnetic and superconducting orderings.

Using the CP\(^1\) representation for the localized spin degrees of freedom and integrating over the CP\(^1\) \( z _i \)-fields in the large wavelength (small \( k \)) regime, assuming \( |z _i| \approx \text{constant} \), we get the final electronic Lagrangian

\[ \mathcal{L}_\psi = \mathcal{L}_0 + \mathcal{L}_\text{eff}^{(1)} + \mathcal{L}_\text{eff}^{(2)} \tag{9} \]

where

\[ \mathcal{L}_\text{eff}^{(2)} = \frac{\rho}{4} \sum_\sigma \psi _\sigma^\dagger \psi _-^\dagger \psi _- \psi _\sigma + \frac{\rho}{4} \sum_\sigma \psi _\sigma^\dagger \psi _\sigma . \tag{10} \]

Notice that the first term above is a BCS interaction, which produces s-wave isotropic superconductivity for a constant Heisenberg interaction \( J_H \).

Estimated values for the coupling constants in the case of the cuprates indicate that we may neglect the s-wave (BCS) term. Then, using a phenomenological choice for the coefficients \( \eta _{a_i, a_j} \), namely

\[ \eta _{a_x, a_x} = 1 \quad \eta _{-a_x, a_x} = -1 \]
\[ \eta _{a_y, a_y} = 1 \quad \eta _{-a_y, a_y} = 1 \]
\[ \eta _{a_x a_y, a_y a_x} = 1 \quad \eta _{-a_x a_y, a_y a_x} = 1 \]

we Fourier transform the final fermionic Hamiltonian, obtaining

\[ H_{\text{SC}} = \sum_\sigma \int d^2 k \left( \epsilon _k + \mu + \frac{\rho}{4} \right) \psi _\sigma^\dagger (\mathbf{k}) \psi _\sigma (\mathbf{k}) - \int d^2 k d^2 k' g_{k,k'} \psi _\sigma^\dagger (\mathbf{k}) \psi _\sigma (\mathbf{k}) \psi _\sigma^\dagger (\mathbf{k}') \psi _\sigma (\mathbf{k}') , \tag{12} \]

where \( g_{k,k'} = \frac{\rho}{4} \eta _k \eta _{k'} \), with \( \eta _k = \sin \left( k_x a \right) - \sin \left( k_y a \right) \), which produces a superconducting gap with line nodes, \( \Delta (\mathbf{k}) = \Delta_0 \eta _k \).

From Eq. (12), we calculate \( T_c \) as a function of the hole occupation in a mean-field approximation, as in the standard BCS approach, and the chemical potential is calculated self-consistently.

![Fig. 1. \( T_c \) as a function of the hole concentration for \( J_H = 0.13 \) eV, \( J_K = 0.2 \) eV and \( t_{pp} = 0.65 \) eV.](image-url)
Acknowledgement

This work has been supported in part by FAPERJ and CNPq.

References

[1] A.P. Kampf, Phys. Rep. 249 (1994) 219.
[2] A. M. Tsvelik, Quantum Field Theory in Condensed Matter Physics (Cambridge University Press, 1995).
[3] E. C. Marino, and M. B. Silva Neto, Phys. Rev. B. 66 (2002) 224512.