Nature of time and causality in Physics

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The conceptual definition and understanding of the nature of time, both qualitatively and quantitatively is of the utmost difficulty and importance, and plays a fundamental role in physics. Physical systems seem to evolve in paths of increasing entropy and of complexity, and thus, the arrow of time shall be explored in the context of thermodynamic irreversibility and quantum physics. In Newtonian physics, time flows at a constant rate, the same for all observers; however, it necessarily flows at different rates for different observers in special and general relativity. Special relativity provides important quantitative elucidations of the fundamental processes related to time dilation effects, and general relativity provides a deep analysis of effects of time flow, such as in the presence of gravitational fields. Through the special theory of relativity, time became intimately related with space, giving rise to the notion of spacetime, in which both parameters cannot be considered as separate entities. As time is incorporated into the proper structure of the fabric of spacetime, it is interesting to note that general relativity is contaminated with non-trivial geometries that generate closed timelike curves, and thus apparently violates causality. The notion of causality is fundamental in the construction of physical theories; therefore time travel and its associated paradoxes have to be treated with great caution. These issues are briefly analyzed in this review paper.

I. INTRODUCTION

Time is a mysterious ingredient of the Universe and stubbornly resists simple definition. St. Augustine, in his Confessions, reflecting on the nature of time, states: “What then is time? If no one asks me, I know: if I wish to explain it to one that asketh, I know not” (The Confessions, ch. XI, sec. 17). Perhaps the reason for being so illusive is that being a fundamental quantity, there is nothing more fundamental to be defined in terms of. Citing Alfred North Whitehead, the philosopher-mathematician: “It is impossible to mediate on time ... without an overwhelming emotion at the limitations of human intelligence.” However, intuitively, we do verify that the notion of time emerges through an intimate relationship to change, and subjectively may be considered as something that flows. This view can be traced back as far as Aristotle, a keen natural philosopher, who stated that “time is the measure of change.” Throughout history, one may find a wide variety of reflections and considerations on time, dating back to ancient religions. For instance, a linear notion of time may be encountered in the Hebrew and the Zoroastrian Iranian writings, and in the Judaico-Christian doctrine, based on the Bible and the unique character of historical events, which possesses a beginning, namely, the act of creation. In ancient Greece the image of Chronos, the Father Time, was conveyed, and Plato further assumed the notion of a circular time, where the latter had a beginning and looped back unto itself. This was probably inspired in the cyclic phenomena observed in Nature, namely the alternation of day and night, the repetition of the seasons, etc. Eastern religions also have a cyclic notion of time, consisting of a repetition of births and extinctions. However, it was only in the 17th century that the philosopher Francis Bacon clearly formulated the concept of linear time, and through the influence of Newton, Barrow, Leibniz, Locke and Kant amongst others, by the 19th century the idea of linear time dominated both in science and philosophy.

In a scientific context, it is perhaps fair to state that reflections on time culminated in Newton’s concept of absolute time, which assumed that time flowed at the same rate for all observers in the Universe. Quoting Newton from his Principia [1]: “Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration.” However, in 1905, Albert Einstein changed altogether our notion of time, through the formulation of the special theory of relativity and stating, in particular, that time flowed at different rates for different observers. Three years later, Hermann Minkowski formally united the parameters of time and space, giving rise to the notion of a fundamental four-dimensional entity, spacetime. Citing Minkowski: “Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

If we consider that time is empirically related to change, which is a variation or sequence of occurrences, then, intuitively, the latter provides us with a notion of something that flows, and thus the emergent character of time. In Relativity, the above empirical notion of a sequence of occurrences is substituted by a sequence of events. The concept of an event is an idealization of a point in space and an instant in time. It is interesting to note that the concept of an instant, associated to that of duration, or an interval of time, is also an extremely

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subtle issue, and deserves a brief analysis. One may consider a duration as an ordered set of instants, contrary to being a sum of instants. A duration is infinitely divisible into more durations, and not into an instant. Now the classical concept of time being a linear continuum implies that between any infinitesimally neighboring instants, an infinity of instants exist, and the flow of instants constituting a linear continuum of time is reminiscent of Zeno’s paradoxes. Perhaps the problem can be surpassed by quantizing time in units of the Planck time, $10^{-43}$ s, in an eventual theory of quantum gravity. Now, a sequence of events has a determined temporal order, which is experimentally verified as specific events—effects—are triggered off by others—causes—thus providing us with the notion of causality.

In the literature, one may find two exclusively mutual concepts of time, which can be characterized as the relational theories and the absolute theories of time. The latter imply that time exists independently of physical spacetime events, contrary to relational theories that defend that time is but a mere relationship of the causal ordering of events, i.e., time is an abstract concept, non-existent as a physical entity, but useful in describing processes. In particular, an example of an absolute theory of time can be traced back to early 17th century, with Isaac Barrow’s refutal of the Aristotelian notion that time is related to change, stating that time is an entity which exists independently of change or motion. This is reflected in his *Lectiones Geometricae*, in 1676, where he states: “Whether things run or stand still, whether we sleep or wake, time flows in its even tenor.” His student, Isaac Newton, extended this idea and compared time and space with an infinitely large vessel, containing events, and existing independently of the latter. This notion was, in turn, refuted by Gottfried Leibniz, who defended a relationship between time and an ordering of non-simultaneous events: “Time is the order of possibilities which cannot coexist and therefore must exist successfully.”

Now, for an emergent subjective notion of time to occur, it seems that a changing configuration of matter is necessary. For instance, in an empty universe, a hypothetical observer cannot measure time nor length, i.e., in a universe without processes one may argue that the observer cannot experience an emergent notion of time, or for that matter, of space. However, the absolute time theorists defend that the container spacetime, i.e., space and time, still exists. The fundamental question is whether time does exist independently, in the absence of change. Albert Einstein seems to provide the controversial answer: “Till now it was believed that time and space existed themselves, even if there was nothing – no Sun, no Earth, no stars – while now we know that time and space are not the vessels for the Universe, but could not exist at all if there were no contents, namely, no Sun, no Earth, and other celestial bodies” (New York Times, 3 December 1919). But, in another stage of his life contradicts himself, by stating: “The conceptions of time and space have been such that if everything in the Universe were taken away, if there were nothing left, there would still be left to man time and space” (New York Times, 4 April 1921). Note that the above ideas further complicate the issue of the flow of time. Some theorists deny the objective flow of time, but nevertheless admit the presence of change, and argue that the empirical notion of the flow is merely subjective. Their opponents defend an objective flow of time, where events change from being indeterminate in the future to being determinate in the present and past.

An interesting example of an absolute theory of time is the “Block Universe” description of spacetime as an unchanging four-dimensional block, where time is considered a dimension. In this representation, a preferred ‘now’ is non-existent and past and future times are equally present. All points in time are equally valid frames of reference, and whether a specific instant is in the future or past is frame dependent. However, despite the fact that each observer does indeed experience a subjective flow of time, special relativity denies the possibility of universal simultaneity (which shall be treated in more detail below), and thus the possibility of a universal now. Now, if future events already exist, why don’t we remember the future? We do remember the past, and this time asymmetry gives rise to a subjective arrow of time. It appears to the “Block Universe” representation that the notion of the flow of time is a subjective illusion. Note that the Block Universe point of view inflicts a great blow to the notion of “free will”, as it proposes that both past and future events are as immutably fixed, and consequently impossible to change. A Block Universe advocate may argue that free will is but mere determinism in disguise. We refer the reader to Ref. 2 for more details on the objections to the Block Universe viewpoint.

An important aspect of the nature of time is its arrow. The modern perspective in physics is that essentially “dynamical laws” govern the Universe, namely, given initial conditions of the physical state, the laws specify the evolution of a determined physical system with time. However, the dynamical equations of classical and quantum physics are symmetrical under a time reversal, i.e., mathematically, one might as well specify the final conditions and evolve the physical system back in time. But, several issues are raised by thermodynamics, general relativity and quantum mechanics on the theme of time asymmetry. In principle, the latter would enable an observer to empirically distinguish past from future. For instance, the Second Law of Thermodynamics, which states that in an isolated system the entropy (which is a measure of disorder) increases provides a thermodynamic arrow of time. One may assume that the Second Law of Thermodynamics and the thermodynamic arrow of time are a consequence of the initial conditions of the universe, which leads us to the cosmological arrow of time, that inexorably points in the direction of the universe’s expansion. In the context of quantum mechanics, a fundamental aspect of the theory is that of quantum uncertainty,
i.e., it is not possible to determine a unique outcome of quantum events. It is interesting to note that despite the fact that there is time-symmetry in the evolution of a quantum system, the reduction of the wave function is essentially time-asymmetric. These aspects are explored in more detail below.

As time is incorporated into the proper structure of the fabric of spacetime, it is interesting to note that general relativity is contaminated with non-trivial geometries that generate closed timelike curves, and apparently violates causality. A closed timelike curve allows time travel, in the sense that an observer who travels on a trajectory in spacetime along this curve, returns to an event that coincides with the departure. The arrow of time leads forward, as measured locally by the observer, but globally he/she may return to an event in the past. This fact apparently violates causality, opening Pandora’s box and producing time travel paradoxes [3], throwing a further veil over our understanding of the fundamental nature of time. The notion of causality is fundamental in the construction of physical theories; therefore time travel and its associated paradoxes have to be treated with great caution [4].

As this chapter is aimed for students and researchers in Psychology or Neuroscience, the mathematics is kept at a minimum. We refer the reader to the remaining chapters for the psychological aspects of time, and only the objective nature of time will be considered in this chapter, which is outlined in the following manner: In Section II, the relativistic aspects of time in special and general relativity will be considered in detail, where much emphasis will be attributed to spacetime diagrams. In Section III, time irreversibility and the arrow of time will be treated in the context of thermodynamics and quantum mechanics. In Section IV, closed timelike curves and causality violation will be analyzed, and in Section V, we conclude.

II. RELATIVISTIC TIME

The conceptual definition and understanding of time, both quantitatively and qualitatively is of the utmost difficulty and importance. Special relativity provides us with important quantitative elucidations of the fundamental processes related to time dilation effects. The general theory of relativity provides a deep analysis to effects of time flow in the presence of strong and weak gravitational fields. The general theory of relativity has been an extremely successful theory, with a well established experimental footing, at least for weak gravitational fields. Its predictions range from the existence of black holes, gravitational radiation to the cosmological models predicting a primordial beginning, namely the big-bang [5, 6].

A. Time in special relativity

To set the stage, perhaps it is important to emphasize that one of the greatest theoretical triumphs of the 19th century physics was James Clerk Maxwell’s formulation of electromagnetism, which, in particular, predicted that light waves are electromagnetic in nature. Now, as was believed, in Maxwell’s time, all wave phenomena required a medium to propagate, and the latter for light waves was denoted as the “luminiferous ether.” Thus, it was predicted that experiments would allow the absolute motion through the ether to be detected. However, the famous Michelson-Morley experiment, devised to measure the velocity of the Earth relative to the ether came up with a null result. To explain the latter, Lorentz deduced specific relationships, denoted as the Lorentz transformations, which are shown below. Einstein also later derived these transformations in formulating his special theory of relativity. Explicitly, using the Lorentz transformation, Lorentz and Fitzgerald explained the Michelson-Morley null result, by inferring the contraction of rigid bodies and the slowing down of clocks when moving through the ether. It is also worth mentioning that the Maxwell equations were not invariant under the Galilean transformations, i.e., they appeared to violate the Principle of Galilean Relativity, which essentially states that the dynamical laws of physics are the same when referred to any uniformly moving frame. At first, it was thought that Maxwell’s equations were incorrect, and were consequently modified to be invariant under Galilean transformations. But, this seemed to predict new electromagnetic phenomena, which could not be experimentally verified. However, applying the Lorentz transformations, it was found that the Maxwell equations remain invariant.

To make the above statements more precise, we shall briefly consider the Galilean transformations, and analyze the Lorentz transformations in more detail. Note that the first law of Newtonian physics essentially states that: “A body continues in its state of rest or of uniform motion, unless acted upon by an external force.” The frame of reference of a body at rest or in uniform motion, i.e., possessing a constant velocity, is denoted an inertial frame.

Consider now an inertial reference frame O’, with coordinates (t’, x’, y’, z’), moving along the x direction with uniform velocity, v, with respect to another inertial frame O, with coordinates (t, x, y, z). The Galilean transformation relates an event in an inertial frame O to another O’, and are given by the following relationships

\[ x’ = x - vt, \]
\[ y’ = y, \]
\[ z’ = z, \]
\[ t’ = t. \]

Note that the last equation is the mathematical assumption of absolute time in Newtonian physics.
In Euclidean space the distance between two arbitrary points, $A$ and $B$, with coordinates $(t_A, x_A, y_A, z_A)$ and $(t_B, x_B, y_B, z_B)$, respectively, is given by
\[(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2,\] (1)
where $\Delta x$, $\Delta y$, and $\Delta z$ are the Cartesian coordinate intervals between $A$ and $B$. One may infer some interesting properties from the above relationship. First, one verifies that $\Delta l = 0$ if and only if $\Delta x = \Delta y = \Delta z = 0$, which states that both points $A$ and $B$ coincide when the Euclidean distance between them is zero. Now, one may show that both the time difference $\Delta t = t_B - t_A$ and the relationship (1) are separately invariant under any Galilean transformation, which leads one to consider that time and space are separate entities in Newtonian physics.

However, in 1905, Einstein abandoned the postulate of absolute time and assumed the following two postulates: (i) the speed of light, $c$, is the same in all inertial frames; (ii) the principle of relativity, which states that the laws of physics take the same form in every inertial frame. Considering, once again, an inertial reference frame $O'$, with coordinates $(t', x', y', z')$, moving along the $x$ direction with uniform velocity relative to another inertial frame $O$, with coordinates $(t, x, y, z)$, and taking into account the above two postulates, Einstein deduced the Lorentz transformation, which are given by
\[
t' = \gamma(t - vx/c^2),
\]
\[
x' = \gamma(x - vt),
\]
\[
y' = y,
\]
\[
z' = z,
\] where $\gamma$ is defined as
\[
\gamma = (1 - v^2/c^2)^{-1/2}.\] (2)

One immediately verifies, from the first two equations, that the time and space coordinates are mixed by the Lorentz transformation, and hence, the viewpoint that the physical world is modelled by a four-dimensional spacetime continuum.

Considering two events, $A$ and $B$, respectively, with coordinates $(t_A, x_A, y_A, z_A)$ and $(t_B, x_B, y_B, z_B)$ in an inertial frame $O$, then the interval between the events is given by
\[
\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2,\] (3)
where $c$ is the speed of light, and $\Delta t$ is the time interval between the two events $A$ and $B$. Note that for this case if $\Delta s = 0$, one cannot conclude that $\Delta t = \Delta x = \Delta y = \Delta z = 0$, due to the minus sign associated with the temporal interval. One verifies that the expression (3) is invariant under a Lorentz transformation, and as advocated by Minkowski, space and time are united in a four-dimensional entity, denoted as spacetime. Thus, the interval (3) may be considered as an underlying geometrical property of the spacetime itself. The sign $\Delta s^2$ is also invariantly defined, so that
\[
\Delta s^2 < 0, \quad \text{timelike interval},
\]
\[
\Delta s^2 = 0, \quad \text{null interval},
\]
\[
\Delta s^2 > 0, \quad \text{spacelike interval}.
\]

It is useful to represent the nature of space and time using spacetime diagrams, as depicted in Fig. 1. The diagrams present a view of the entire spacetime, without a special status associated to the present time, as will be shown below. For simplicity, in the spacetime diagrams presented the $y$ and $z$ spatial dimensions have been suppressed. Observers moving with a relative velocity $v < c$ travel along timelike curves, for instance as depicted by the curve in Fig. 1 which is denoted by the worldline of the observer. Now, there is a unique time measured along a worldline, denoted as proper time. A photon travels along null curves, $ct = \pm x$, which are depicted by the dashed curves in Fig. 1 and constitute the light cone of $A$. Events $A$ and $B$ are separated by a timelike interval; events $A$ and $C$ by a null interval; and events $A$ and $D$ are separated by a spacelike interval. All events within the upper light cone of $A$ are in the future of $A$, and all events with the lower light cone constitute the past of $A$. Events outside the light cone, such as event $D$ only become visible to $A$ when it enters the light cone of $A$.

The interior of the future light cone of $A$ constitutes the region that can be influenced by $A$ with objects travelling with less than the speed of light. The boundary of the future light cone can only be influenced by signals with the speed of light from $A$. In counterpart, the past light cone constitutes the region in spacetime with events that may influence $A$. The exterior of the light cones, denoted the “elsewhere region of $A$”, constitutes the region of events which cannot influence nor be influenced by $A$.

To illustrate the latter feature, consider the following example, which is depicted in Fig. 2. Assume the existence of two stationary space-stations, $A$ and $B$, respectively. $A$ is observing $B$ using a powerful telescope, and at event $O$ observes event $E$, a threatening asteroid on collision course with $B$. Despite of sending a warning signal which arrives at $B$ at event $R$, it will be impossible to warn $B$ of the impending danger in time to avoid the collision. This is due to event $C$, collision $B$-asteroid, being outside the causal future of $A$, i.e., in the “elsewhere” region of $A$. This is depicted in Fig. 2.

The special theory of relativity challenges many of our intuitive beliefs about time. For instance, the theory is inconsistent with the common belief that the temporal order in which two events occur is independent of the observer’s reference frame. To illustrate this fact, consider an inertial frame $O'$, with coordinates $(t', x')$, moving along the $x$ direction with uniform velocity, $v$, with respect to another inertial frame $O$, with coordinates $(t, x)$. In Fig. 3 the dashed line parallel to the $x$-axis represents
events for a constant time \( t \), so that events \( A \) and \( B \) are simultaneous in the observer’s \( O \) reference frame. The dashed line parallel to the \( x' \)-axis depicts simultaneous events in observer’s \( O' \) reference frame so that despite the fact that events \( C \) and \( D \) are simultaneous in \( O' \), one verifies that \( C \) precedes \( D \) for the observer \( O \).

Consider the following example to further illustrate this point, which is depicted in Fig. 4. Consider two space-stations \( A \) and \( B \), separated by a distance \( D \). Assume now a stationary satellite \( O \) midway between the stations, and a second satellite \( O' \) moving towards station \( B \) with a velocity \( v \) with respect to \( O \). At the instant that both satellites are midway between the stations, these send out a simultaneous signal, \( A' \) and \( B' \), respectively, as measured by \( A \) at event \( C \). However, satellite \( O' \) will receive the signal from station \( B \), at event \( D \), before the signal from \( A \), event \( E \), as depicted in Fig. 4. Thus, whether a specific instant is in the future or past is frame dependent. Special relativity denies the possibility of universal simultaneity, and hence the possibility of a universal now.

This raises the problem of the synchronization of distant clocks in defining simultaneity in spacetime. A simple conceptual definition of a clock will be provided below. Consider two clocks at rest with respect with an observer, located at spacetime points \( A \) and \( B \), respectively. Suppose now that at time \( t_1 \), \( A \) sends out a signal to \( B \), which is reflected and returns to \( A \) at time \( t_2 \). Taking into account the constancy of the speed of light, \( A \) will conclude that the event reflection from \( B \), is simultaneous with the time \( T \) is his worldline, which is precisely half the interval of travel time, i.e.,

\[
T = \frac{1}{2} (t_1 + t_2).
\] (4)

This is a simple and practical way of determining simultaneity and of synchronizing clocks, which is depicted in Fig. 5.

Another feature of our intuitive beliefs challenged by the special theory of relativity is that related with time dilation effects. For instance, consider the following thought experiment suggested by Einstein. Suppose that an observer travels along a tram moving with a relativistic velocity, whilst observing a large clock through a powerful telescope. The observer sees the clock, as the emitted light catches up with the tram. Now, if the tram moves at a speed close to that of light, the light rays emitted from the clock take longer to catch up with the observer. It seems that time has slowed down as measured by the latter. If the tram now attains the speed of light, then the light reflected from the clock cannot catch up with the observer, and it seems that time would come to a standstill.
FIG. 3: Consider an inertial observer $O$, moving along the $x$-direction with uniform velocity, $v$, relative to another inertial observer $O'$. The dashed line parallel to the $x$-axis represent events for a constant time $t$, such that events $A$ and $B$ are simultaneous in the observer’s $O$ reference frame. The dashed line parallel to the $x'$-axis depicts simultaneous events in observer’s $O'$ reference frame. Note that $C$ and $D$ are simultaneous in $O'$, but $C$ precedes $D$ for observer $O$. Thus, special relativity denies the possibility of universal simultaneity, and consequently the possibility of a universal now.

One may infer time dilation from the constancy of the speed of light. First it is useful to provide a simple conceptual definition of a clock, namely, that of a ‘light clock’. The latter is constructed by two mirrors separated by a distance $d$, with a photon being continuously reflected in between. A ‘click’ of this idealized clock is constituted by the time interval, $2\Delta t'$, with which the photon traces the distance $2d$, as depicted in Fig. 3. Thus, one deduces the following expression $2\Delta t' = 2d/c$, which implies $\Delta t' = d/c$. Now, consider that this clock is at rest in an inertial frame $O'$ travelling with a relativistic velocity $v$ with respect to a frame $O$. From the special relativistic postulate that the speed of light is constant in all frames, the time interval traced out by the photon, as measured by the observer at rest $O$ is $2\Delta t$, and taking into account Pythagoras’ theorem, we have

$$c^2(\Delta t)^2 = v^2(\Delta t)^2 + d^2. \tag{5}$$

Using $\Delta t' = d/c$, one finally deduces the following relationship

$$\Delta t = \gamma \Delta t'. \tag{6}$$

As $\gamma > 1$, then $\Delta t > \Delta t'$, so that time as measured by the moving reference frame $O'$ slows down relatively to $O$.

The above relationship may also be deduced directly from the Lorentz transformations. Suppose that a clock sits at rest with respect to the inertial reference frame $O'$, in which two successive clicks, represented by two events $A$ and $B$ are separated by a time interval $\Delta t'$. To determine the time interval $\Delta t$ as measured by $O$, it is useful to consider the inverse Lorentz transformation, given by

$$t = \gamma (t' + vx'/c^2), \tag{7}$$

which provides

$$t_B - t_A = \gamma \left[ t'_B - t'_A + v(x'_B - x'_A)/c^2 \right], \tag{8}$$

where $t_A$ and $t_B$ are the two clicks measured in $O$. As the events are stationary relative to $O'$, we have $x'_B = x'_A$, so that one finally ends up with Eq. (6), taking into account $\Delta t = t_B - t_A$ and $\Delta t' = t'_B - t'_A$. We note that the fact that a moving clock slows down is completely reciprocal for any pair of inertial observers, and this is essentially explained as both disagree about simultaneity.

An interesting example of the time dilation effects is the so-called “twin paradox”, depicted in Fig. 4. Consider two identical twins, $A$ and $B$, respectively, where $A$ remains at rest, while $B$ travels away from $A$ at a relativistic velocity, close to the speed of light [2]. As a practical example, consider that $B$ initially recedes away.
from A at a speed of $v = 4c/5$ for 12 years, as measured by B’s clock, then returns at the same speed for 12 years. Thus, B measures a total journey time of 24 years. One may ask what the total travel time is, as measured by the twin A. To answer this, consider that the event K, relatively to the twin A, is where B begins his outward journey; L is the event when B turns around; and M the event when B arrives back at A. From Eq. 2, we verify that in both the outward and return journey, we have $\gamma = \left[ 1 - (4/5)^2 \right]^{-1/2} = 5/3$. Considering that the event N, relative to A, is simultaneous to L, then one verifies that $t_{KN} = \gamma t_{KL}' = 5/3 \times 12 = 20$ years, so that the total travel time as measured by A is 40 years.

However, time dilation effects are reciprocal between two inertial frames, and one may wonder how it is possible to reconcile the difference between both observers. It is important to emphasize that the difference between both observers, A and B, is that twin B is not an inertial observer, as his trajectory consists of two inertial segments joined by a period of acceleration. It is interesting to note that this feature has been observed experimentally, in particular, in the Hafele-Keating experiment [9], which we refer to below.

### B. Time in general relativity

The analysis outlined above has only taken into account flat spacetimes, contrary to Einstein’s general theory of relativity, in which gravitational fields are represented through the curvature of spacetime. In the discussion of special relativity, the analysis was restricted to inertial motion, and in general relativity the principle of relativity is extended to all observers, inertial or non-inertial. In general relativity it is assumed that the laws of physics are the same for all observers, no matter what their state of motion [8]. Now it is clear that a gravitational force measured by an observer essentially depends on his state of acceleration, which leads to the principle of equivalence, which states that “there is no way of distinguishing between effects on an observer of a uniform gravitational field and of constant acceleration.”

The general theory of relativity has been an extremely successful theory, with a well established experimental...
footing, at least for weak gravitational fields. Of particular interest in this work are the gravitational time dilation effects. For this, imagine the following idealized thought experiment, suggested by Einstein [10], which is depicted in Fig. 8. Consider a tower of height \( h \) hovering on the Earth’s surface, with a particle of rest mass \( m \) lying on top. The particle is then dropped from rest, falling freely with acceleration \( g \) and reaches the ground with a non-relativistic velocity \( v = (2gh)^{1/2} \). Thus, an observer on the ground measures its energy as

\[
E = mc^2 + \frac{1}{2}mv^2 = mc^2 + mgh. \tag{9}
\]

The idealized particle is then converted into a single photon \( \gamma_1 \) with identical energy \( E \), which returns to the top of the tower. Upon arrival it converts into a particle with energy \( E' = m'c^2 \). Note that to avoid perpetual motion \( m' > m \) is forbidden, so that we consider \( m = m' \), and the following relationship is obtained

\[
\frac{E'}{E} = \frac{mc^2}{mc^2 + mgh} \approx 1 - \frac{gh}{c^2}, \quad \tag{10}
\]

with \( gh/c^2 \ll 1 \). From the definitions \( E = h\nu \) and \( E' = h\nu' \), where \( \nu \) and \( \nu' \) are the frequencies of the photon at the bottom and top of the tower, so that from Eq. (10), one obtains

\[
\nu' = \nu \left( 1 - \frac{gh}{c^2} \right). \tag{11}
\]

This is depicted in Fig. 8.

Now, to obtain the important result that clocks run at different rates in a gravitational field, consider the following thought experiment. The observer at the bottom of the tower emits a light wave, directed to the top. The relationship of time between two crests is simply the inverse of the frequency, i.e., \( \Delta t' = 1/\nu' \), so that from Eq. (11), one obtains the approximations, considering \( gh/c^2 \ll 1 \),

\[
\Delta t' = \Delta t \left( 1 + \frac{gh}{c^2} \right). \tag{12}
\]

This provides the result that time flows at a faster rate on top of the tower than at the bottom. Note that this result has been obtained independently of the gravitational theory.

**C. Experimental tests**

A well-known experiment to test the time dilation effects in general relativity, in particular, that clocks should run at different rates at different places in a gravitational field, is the Pound-Rebka experiment [11], which confirmed the predictions of general relativity to a 10% precision level [12]. These results were later improved to a 1% precision level by Pound and Snider [13].

The Hafele-Keating experiment [9], realized in October 1971 was an interesting test of the theory of relativity. It essentially consisted of travelling four cesium-beam atomic clocks aboard commercial airliners, and flying twice around the world, first eastward, then westward. The results were then compared with the clocks of the United States Naval Observatory. To within experimental error, the results were consistent with the relativistic predictions.

A modern application of the special and general relativistic time dilation effects are the synchronization of atomic clocks on board the Global Positioning System (GPS) satellites. The GPS has become a widely used aid to navigation worldwide, enabling a GPS receiver to determine its location, speed and direction. Now, as verified above, general relativity predicts that the atomic clocks at GPS orbital altitudes will tick more rapidly, as they are in a weaker gravitational field than atomic clocks on Earth’s surface; whilst atomic clocks moving at GPS orbital speeds will tick more slowly than stationary ground clocks, as predicted by special relativity. When both effects are combined, the experimental data shows that the on-board atomic clock rates do indeed agree with ground clock rates to the predicted extent.
III. TIME IRREVERSIBILITY AND THE ARROW OF TIME

The modern perspective in physics is that the Universe is essentially governed by “dynamical laws”, i.e., they specify the evolution of a determined physical system with time, given the initial conditions of the physical state. One normally considers the evolution of physical systems into the future, which are governed by differential equations [14]. In this context, it is not common practice to evolve the physical systems into the past, despite the fact that the dynamical equations of classical and quantum physics are symmetrical under a time reversal. Mathematically, one might as well specify the final conditions and evolve the physical system back in time. One specifies data at some initial instant and these data evolve, through dynamical equations, to determine the physical state of the system in the future, or to the past, i.e., detailed predictability to the future and past is in principle possible. However, several issues are raised by thermodynamics, general relativity and quantum mechanics on time irreversibility and the arrow of time.

A. The arrow of time in thermodynamics

A classical example of an irreversible process is the dissipation of smoke from a lit cigarette. In principle, the evolution of the system and its outcome is possible if the microscopic dynamics of each individual particle is possible, but in practice one has little knowledge of the position and velocity of every particle in the system. The overall behavior of the system is well described in terms of appropriate averages of the physical parameters of the individual particles, such as the distribution of mass, momentum and of energy, etc. One may argue that the knowledge of these averaged parameters is sufficient to determine the dynamical behavior of the system and the respective final outcome. However, this is not always the case, as in the specific examples of ‘chaotic systems’.

Chaotic systems are classical systems, where a small change in the initial conditions modifies the behavior of the system exponentially, resulting in an unpredictability of the final outcome. This chaotic unpredictability is closely related to the Second Law of Thermodynamics, which states that the entropy of the system increases (or at least does not decrease) with time. The entropy is essentially a measure of the disorder or randomness in the system. For instance, note the increased randomness of the convoluted path of the diffusion of smoke from a lit cigarette. As a simple example, consider a body moving through the air. The body possesses kinetic energy, in an organized form, and as it slows down from the air resistance, the kinetic energy has been transferred to the random motion of the air particles and the individual particles of the body [14].

Consider the flow of heat from a hot body to a cooler body. The evolution of this system is deterministic in character and predicted by the Second Law of Thermodynamics. If one theoretically considers the time-reversed evolution of the system, then one would have the following scenario: Two bodies of the same temperature evolve to bodies of unequal temperature. It would even be a practical impossibility to know which body would be the hotter, and which the cooler. Note that this difficulty of dynamical retrodiction applies to most macroscopic systems, which possess a large number of constituent particles and behaving in accordance to the Second Law of Thermodynamics.

A particularly interesting example is that of friction [2]. Consider, for simplicity, a block of mass sliding along a plane, and being slowed down by a constant force of friction, consequently coming to rest at a determined instant. One may agree that providing the detailed microscopic properties, such as the distribution of heat on the plane, it may be possible to predict the initial conditions. However, using a macroscopic viewpoint, after the system has settled down, one cannot retrodict the initial conditions. One cannot even reconstruct the trajectory, and one could conjecture if the block came from the left or from the right.

In all of the examples outlined above one may argue that the micro-physics is completely deterministic, contrary to the outcome of macroscopic viewpoint [2]. This is essentially due to the fact that in the macroscopic viewpoint one does not have enough detail of the physical system’s micro-properties. One may provide a statistical prediction of the eventual outcome, but not a detailed and definite prediction. Note that the total energy of the system is conserved as dictated by the First Law of Thermodynamics. One may state that the disorder or randomness has increased. Thus, the increase of entropy generically provides a thermodynamic arrow of time. It is also possible to assume that the Second Law of Thermodynamics and the thermodynamic arrow of time are a consequence of the initial conditions of the universe, which leads us to the cosmological arrow of time, that inexorably points in the direction of the universe’s expansion.

B. The arrow of time in quantum mechanics

Quantum uncertainty is a fundamental aspect of quantum theory, i.e., it is not possible to determine a unique outcome of quantum events. Formally, consider the wave function \( \Psi(x) \) as a linear combination of eigenfunctions \( u_n(x) \), given by

\[
\Psi_1(x) = \sum_{i} a_i u_i(x) . \tag{13}
\]

Suppose now that a measurement takes place at \( t = \bar{t} \), thus reducing the wave function to

\[
\Psi_2(x) = a_n u_n(x) , \tag{14}
\]
for some specific value $i = n$. It is important to emphasize that the initial state $\Psi_0$ does not uniquely determine the final state $\Psi_f$. This is not due to lack of data, but is due to the nature of quantum physics. Furthermore, one cannot predict the final eigenstate $\Psi_f(x)$ from the initial state $\Psi_0$. One cannot also retrodict to the past at the quantum level, as once the wave function has collapsed to an eigenstate, one cannot know the initial state from the final state. Thus, despite the fact that there is time-symmetry in the evolution of a quantum system, the reduction of the wave function is essentially time-asymmetric.

It is illustrative to consider the following example [14]. Consider a photon source $S$ which emits individual photons. The latter are aimed at a beam-splitter $B$, which is simply a half-silvered mirror, and is placed at an angle of 45° to the beam. Thus, if a photon is reflected, it will be absorbed at the ceiling $C$; if it is transmitted, it will activate a detector $D$. Suppose that the probability of reflection and transmission is 50%, respectively. Now, suppose that a detection at $D$ is verified, which is equivalent to the reduction of the wave function corresponding to Eq. (14). Given this, one may ask what the initial probabilities are. For this, the relevant histories would be $SBD$ and $FBD$, where $F$ is a point on the floor. If a photon were emitted at $F$, it would be reflected at $B$ and be detected at $D$. Now, applying the quantum mechanical rules, one verifies that there is a 50% probability that the photon be detected at $D$, for the respective emissions at $S$ and at $F$. This is absurd, as there is a 0% probability that a photon would be emitted from $F$. From this simple example, one verifies time asymmetry related to the reduction of the wave function in quantum mechanics. This is depicted in Fig. 9.

![Diagram](image_url)

**FIG. 9:** Time asymmetry in the reduction of the wave function. See the text for details.

**IV. CLOSED TIMELIKE CURVES AND CAUSALITY VIOLATION**

As time is incorporated into the proper structure of the fabric of spacetime, it is interesting to note that general relativity is contaminated with non-trivial geometries which generate *closed timelike curves* [4]. A closed timelike curve (CTC) allows time travel, in the sense that an observer which travels on a trajectory in spacetime along this curve, returns to an event which coincides with the departure. The arrow of time leads forward, as measured locally by the observer, but globally he/she may return to an event in the past. This fact apparently violates causality, opening Pandora’s box and producing time travel paradoxes [3], throwing a veil over our understanding of the fundamental nature of time. The notion of causality is fundamental in the construction of physical theories, therefore time travel and its associated paradoxes have to be treated with great caution. The paradoxes fall into two broad groups, namely the *consistency paradoxes* and the *causal loops*.

The consistency paradoxes include the classical grandfather paradox. Imagine travelling into the past and meeting one’s grandfather. Nurturing homicidal tendencies, the time traveller murders his grandfather, imped ing the birth of his father, therefore making his own birth impossible. Another example is that of autoinfanticide, where the time traveller returns to the past, and kills himself as a baby. In fact, there are many versions of the grandfather paradox, limited only by one’s imagination. The consistency paradoxes occur whenever possibilities of changing events in the past arise.

The paradoxes associated with causal loops are related to self-existing objects or objects, trapped in spacetime. Imagine a researcher travelling forward in time and reading the details of the recently formulated, and anxiously anticipated, consistent theory of quantum gravity. Returning to his time, he explains the details to an ambitious younger colleague, who writes it up and the article is eventually published in the journal, where the first researcher read it after travelling into the future. The article on the theory of quantum gravity exists in the future because it was written in the past by the young researcher. The latter wrote it up, after receiving the details from his colleague, who in turn read the article in the future. Both parts considered by themselves are consistent, and the paradox appears when considered as a whole. One is liable to ask, what is the origin of the information, as it appears out of nowhere. The details for a complete and consistent theory of quantum gravity, which paradoxically were never created, nevertheless exist in spacetime. Note the absence of causality violations in these paradoxes.

A great variety of solutions to the Einstein field equations containing closed timelike curves exist, but two particularly notorious features seem to stand out [13]. Solutions with a tipping over of the light cones due to a rotation about a cylindrically symmetric axis; and solutions that violate the energy conditions of general relativity, which are fundamental in the singularity theorems and theorems of classical black hole thermodynamics [4].
A. Stationary, axisymmetric solutions

The tipping over of light cones seems to be a generic feature of some solutions with a rotating cylindrical symmetry, which is depicted in Fig. 10. The present work is far from making an exhaustive search of all the Einstein field equation solutions generating closed timelike curves with these features, but the best known spacetimes will be briefly mentioned. The earliest solution to the Einstein field equations containing closed timelike curves, is probably that of the van Stockum spacetime [4, 17]. It is a stationary, cylindrically symmetric solution describing a rapidly rotating infinite cylinder of dust, surrounded by vacuum. The centrifugal forces of the dust are balanced by the gravitational attraction. The light cones tip over close to the cylinder, due to the strong curvature of the spacetime, consequently inducing closed timelike curves. In 1949, Kurt Gödel discovered another exact solution to the Einstein field equations consisting of a uniformly rotating universe containing dust and a nonzero cosmological constant [17]. It is possible to show that moving away from the axis, the light cones open out and tilt in the angular direction, eventually generating closed timelike curves [5].

![Diagram of light cones tipping over](image)

**FIG. 10:** The tipping over of light cones, depicted in the figure is a generic feature of some solutions with a rotating cylindrical symmetry. The dashed curve represents a closed timelike curve.

An analogous solution to that of the van Stockum spacetime, although possessing a different asymptotic behavior, is that of an infinitely long straight string that lies and spins around the z-axis [4]. These latter solutions also induce closed timelike curves. An interesting variant of these rotating cosmic strings is an extremely elegant model of a time-machine, theoretically constructed by Gott [18]. It is an exact solution to the Einstein field equation for the general case of two moving straight cosmic strings that do not intersect. This solution produces closed timelike curves even though they do not violate the weak energy condition, which essentially prohibits the existence of negative energy densities, have no singularities and event horizons, and are not topologically multiply-connected as the wormhole solution, which will be considered below. The appearance of closed timelike curves relies solely on the gravitational lens effect and the relativity of simultaneity. However, it was shown that the Gott time machine is unphysical in nature, for such an acausal behavior cannot be realized by physical and timelike sources [14, 20].

B. Solutions violating the energy conditions

The traditional manner of solving the Einstein field equation consists in considering a plausible distribution of energy and matter, and then finding the geometrical structure. However, one can run the Einstein field equation in the reverse direction by imposing an exotic geometrical spacetime structure, and eventually determine the matter source for the respective geometry.

In this fashion, solutions violating the energy conditions have been obtained. One of the simplest energy conditions is the weak energy condition, which is essentially equivalent to the assumption that any timelike observer measures a local positive energy density. Although classical forms of matter obey these energy conditions, violations have been encountered in quantum field theory, the Casimir effect being a well-known example. Adopting the reverse philosophy, solutions such as traversable wormholes [4, 21, 22, 23], the warp drive [24, 25, 26], and the Krasnikov tube [27] have been obtained. These solutions violate the energy conditions and with simple manipulations generate closed timelike curves [28, 29, 30].

We shall briefly consider the specific case of traversable wormholes [21]. A wormhole is essentially constituted by two mouths, A and B, residing in different regions of spacetime [21], which in turn are connected by a hypothetical tunnel. One of the most fascinating aspects of wormholes is their apparent ease in generating closed timelike curves [28]. There are several ways to generate a time machine using multiple wormholes [4], but a manipulation of a single wormhole seems to be the simplest way [28]. The basic idea is to create a time shift between both mouths. This is done invoking the time dilation effects in special relativity or in general relativity, i.e., one may consider the analogue of the twin paradox, in which the mouths are moving one with respect to the other, or simply the case in which one of the mouths is placed in a strong gravitational field, so that time slows down in the respective mouth [4, 31].

To create a time shift using the twin paradox analogue, consider that the mouths of the wormhole may be moving one with respect to the other in external space, without significant changes of the internal geometry of the tunnel. For simplicity, consider that one of the mouths A is at rest in an inertial frame, whilst the other mouth B, initially at rest practically close by to A, starts to move out with a high velocity, then returns to...
its starting point. Due to the Lorentz time contraction, the time interval between these two events, $\Delta T_B$, measured by a clock comoving with $B$ can be made to be significantly shorter than the time interval between the same two events, $\Delta T_A$, as measured by a clock resting at $A$. Thus, the clock that has moved has been slowed by $\Delta T_A - \Delta T_B$ relative to the standard inertial clock. Suppose that the tunnel, between $A$ and $B$, remains practically unchanged, so that an observer comparing the time of the clocks through the handle will measure an identical time, as the mouths are at rest with respect to one another. However, by comparing the time of the clocks in external space, he will verify that their time shift is precisely $\Delta T_A - \Delta T_B$, as both mouths are in different reference frames, frames that moved with high velocities with respect to one another. Time is hooked up differently as measured through the interior or in the exterior of the wormhole. Now, consider an observer starting off from $A$ at an instant $T_0$ measured by the clock stationed at $A$. He makes his way to $B$ in external space and enters the tunnel from $B$. Consider, for simplicity, that the trip through the wormhole tunnel is instantaneous. He then exits from the wormhole mouth $A$ into external space at the instant $T_0 - (\Delta T_A - \Delta T_B)$ as measured by a clock positioned at $A$. His arrival at $A$ precedes his departure, and the wormhole has been converted into a time machine. See Figure [11].

### V. SUMMARY AND DISCUSSION

In this chapter, a brief review on the nature of time in Physics has been explored (see Ref. [32] for a recent review). In particular, it was noted that in Newtonian physics, time flows at a constant rate for all observers, providing the notion of absolute time, while in the special and general theories of relativity, time necessarily flows at different rates for different observers. It was shown that special relativity denies the possibility of universal simultaneity, and consequently the impossibility of a universal now. In this context, the Block Universe description emerges, where all times, past and future are equally present, and the notion of the flow of time is a subjective illusion. This leads one to the possibility of time being a dimension, contrary to a process. Indeed, intuitively, one verifies that the notion of time as something that flows arises due to an intimate relationship to change. Nevertheless, in relativity the concept of spacetime being an independent entity containing events predominates. Despite the fact of the great popularity of the Block Universe representation in the Physics (in particular the relativistic) community, this viewpoint often meets with resistance, and we refer the reader to an interesting paper by George F. Ellis [2].

Ellis argues that the Block Universe picture does not constitute a realistic model of the Universe for several reasons: It assumes simplified equations of state and thus does not apply to spacetimes including complex systems, e.g., biological systems; they don’t take into account several issues such as dissipative effects, feedback effects, and quantum uncertainty. Indeed, in our everyday experience, psychological time does contrast to the Block Universe picture, due to the subjective emergent notion of time as a process. Thus, adopting this viewpoint, it is plausible to consider that time is an abstract concept, non-existent as a physical entity, but useful in describing processes. Contrary to the Block Universe, representing spacetime as a fixed whole, Ellis argues in favor of an Evolving Block Universe model of spacetime [2], where “... time progresses, events happen, and history is shaped. Things could have been different, but second by second, one specific evolutionary history out of all the possibilities is chosen, takes place, and gets cast in stone” [2]. The Evolving Block Universe representation defends that spacetime is extended into the future as events occur along each worldline, which is determined by causal interactions.

A fundamental issue in the nature of time is its arrow. In modern physics, dynamical laws essentially govern the Universe, where one considers the evolution of physical systems into the future. Nevertheless, the dynamical equations of classical and quantum physics are symmetrical under a time reversal, and mathematically one may evolve the physical systems into the past. In principle, detailed predictability to the future and past is possible. However, several issues are raised by thermodynamics and quantum mechanics on time irreversibility and the arrow of time. In a thermodynamical context, one may argue that the micro-physics of a specific system is completely deterministic, contrary to the outcome of the macroscopic viewpoint [2]. This is essentially due to the fact that in the macroscopic viewpoint one does not have enough detail of the physical system’s micro-properties. One may provide a statistical prediction of the eventual outcome, but not a detailed and definite prediction. This fact is closely related to the Second Law of Thermodynamics. Thus, the increase of entropy generally provides a thermodynamic arrow of time. It is also possible to assume that the Second Law of Thermodynamics and the thermodynamic arrow of time are a consequence of the initial conditions of the universe, which leads us to the cosmological arrow of time, that inexorably points in the direction of the universe’s expansion. In a quantum mechanical context, it was also shown that despite the fact that there is a time-symmetry in the evolution of a quantum system, the reduction of the wave function is essentially time-asymmetric.

Relatively to causality violation, if one regards that general relativity is a valid theory, then it is plausible to at least include the possibility of time travel in the form of closed timelike curves. However, a typical reaction is to exclude time travel due to the associated paradoxes, although the latter do not prove that time travel is mathematically or physically impossible. The paradoxes do indeed indicate that local information in spacetimes containing closed timelike curves is restricted in unfamil-
FIG. 11: Depicted are two examples of wormhole spacetimes with closed timelike curves. The wormholes tunnels are arbitrarily short, and its two mouths move along two world tubes depicted as thick lines in the figure. Proper time $\tau$ and time $T$ at the wormhole throat is marked off, and note that identical values are the same event as seen through the wormhole handle. In Figure (a), mouth A remains at rest, while mouth B accelerates from A at a high velocity, then returns to its starting point at rest. A time shift is induced between both mouths, due to the time dilation effects of special relativity. The light cone-like hypersurface $H$ shown is a Cauchy horizon, beyond which predictability breaks down. Through every event to the future of $H$ there exist closed timelike curves, and on the other hand there are no closed timelike curves to the past of $H$. In Figure (b), a time shift between both mouths is induced by placing mouth B in strong gravitational field. See text for details.

The Principle of Self-Consistency stipulates that events on a closed timelike curve are self-consistent, i.e., events influence one another along the curve in a cyclic and self-consistent way. In the presence of closed timelike curves the distinction between past and future events are ambiguous, and the definitions considered in the causal structure of well-behaved spacetimes break down. What is important to note is that events in the future can influence, but cannot change, events in the past. According to this principle, the only solutions of the laws of physics that are allowed locally, and reinforced by the consistency constraints, are those which are globally self-consistent. Hawking’s Chronology Protection Conjecture is a more conservative way of dealing with the paradoxes. Hawking notes the strong experimental evidence in favour of the conjecture from the fact that “we have not been invaded by hordes of tourists from the future” [34]. An analysis reveals that the value of the renormalized expectation quantum stress-energy tensor diverges close to the formation of closed timelike curves, which destroys the wormhole’s internal structure before attaining the Planck scale. There is no convincing demonstration of the Chronology Protection Conjecture, but perhaps an eventual quantum gravity theory will provide us with the answers.

But, as stated by Thorne [35], it is by extending the theory to its extreme predictions that one can get important insights to its limitations, and probably ways to overcome them. Therefore, time travel in the form of closed timelike curves, is more than a justification for theoretical speculation, it is a conceptual tool and an epistemological instrument to probe the deepest levels of general relativity and extract clarifying views. Relative to the issue of time, one may consider that the underlying question is that of an ontological nature, and quoting Ellis [8]: “Does spacetime indeed exist as a real physical entity, or is it just a convenient way of describing relationships between physical objects, which in the end are all that really exist at a fundamental level?”
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