Statistical Hadronization of Supercooled Quark-Gluon Plasma

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The fast simultaneous hadronization and chemical freeze-out of supercooled quark-gluon plasma, created in relativistic heavy ion collisions, leads to the re-heating of the expanding matter and to the change in a collective flow profile. We use the assumption of statistical nature of the hadronization process, and study quantitatively the freeze out in the framework of hydrodynamical Bjorken model with different quark-gluon plasma equations of state.

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I. INTRODUCTION

The hydrodynamical models have been used extensively to study the evolution of the hot, strongly interacting matter created in relativistic heavy ion collisions. These models apply on the space-time region, where the initial, hard processes have reached the stage where the local thermal equilibrium can be assumed, and the strong interactions between constituent particles are very frequent.

On the other hand, the thermal statistical models applied to describe the final hadron abundances have been very successful for nucleus-nucleus (AA) [1, 2, 3, 4, 5], proton-nucleus (pA) [6] and even for elementary pp, p¯p and e+e− reactions. The latter ones, particularly, suggest the statistical nature of the hadronization process.

In this work, we study the fast hadronization and chemical freeze-out (CFO) of locally thermalized quark gluon plasma (QGP) described by hydrodynamical evolution. This process is idealized by sudden hadronization over a three dimensional hypersurface. The matter crossing the hypersurface is controlled by conservation of energy momentum- and relevant current conservation laws, and additionally, by assumption of apparent thermal and chemical equilibrium distribution of the resulting hadron gas (HG). By apparent equilibrium distribution we mean that although there is clearly no room for kinetic equilibration in elementary collisions – or in fast, simultaneous phase transition and CFO in nuclear collisions – the final hadron spectra are dictated by maximum entropy bound by conservation of energy and charge densities.

It is shown, that the first order quasistatic phase transition is far too slow [9, 10] to give the hadron chemical decoupling time of $t_{CFO} \lesssim 10 \text{ fm/c}$ determined experimentally, see [11] and references therein. Thus, in order to avoid the entropy decrease in the non-equilibrium hadronization, QGP must be allowed to supercool, i.e. to develop mechanical instability before the phase transition [11]. Although the simultaneous phase transition and CFO are assumed to be a non-equilibrium process, we can still exploit the apparent equilibrium parametrization – the consequence of statistical nature of hadronization.

We will show that the shock-like hadronization of supercooled QGP leads to the change in collective flow profile, and re-heating of the system. We study the system with various choices of the equation of state (EoS) on both sides of the FO hypersurface.

II. THE CHEMICAL FREEZE OUT PROCESS

The fast hadronization process, idealized to take place on the zero-volume hypersurface, leads to a discontinuity in energy-momentum- and charge conservation equations. In general, this leads not only to change in density quantities in LRF, but to the change in flow velocity profile as well [12, 13].

Denoting post freeze out quantities by subscript FO, the energy-momentum ($T$) conservation at the surface
element $d\sigma$ leads to a set of four equations

$$T^{\mu \nu}d\sigma = T^{\mu \nu}_{\text{FO}}d\sigma.$$  \hspace{1cm} (1)

In the isospin symmetric case, the electric charge conservation is trivial, and we are left with local baryon number and strangeness conservation equations:

$$N_B^\nu d\sigma = N_B^\nu_{\text{FO}}d\sigma$$  \hspace{1cm} (2)

$$N_S^\nu d\sigma = N_S^\nu_{\text{FO}}d\sigma.$$  \hspace{1cm} (3)

Here, we restrict our considerations to time-like FO hypersurfaces in order to avoid currents entering from post-FO to pre-FO side. The complications in the general case, where the space-like FO is included, are discussed extensively in [12, 13]. For the time-like hypersurface, there is always a proper Lorentz transformation for each point $x$ such that solving the equations (2, 3) can be carried out in a frame, where flow velocities are $u(x) = \gamma(1,0,0,v)$ and $u_{\text{FO}}(x) = \gamma_{\text{FO}}(1,0,0,v_{\text{FO}})$ on pre- and post-FO sides, respectively. After solving the FO equations, the actual flow velocity $u_{\text{FO}} = \gamma_{\text{FO}}(1,\vec{v}_{\text{FO}})$ is obtained by a simple spatial rotation. Provided with these tools, only two of the four equations (1) are independent.

Usually, the strangeness in hydrodynamical simulations is assumed to be homogeneously distributed, so the condition $n_S(x) = n_{S,\text{FO}}(x) = 0$ binds the parameters associated with strangeness conservation.

The process described is generally non-adiabatic, but the entropy constraint

$$S'^{\nu}d\sigma_{\nu} \leq S^{\nu}_{\text{FO}}d\sigma_{\nu}$$ \hspace{1cm} (4)

must be satisfied in every point on the FO hypersurface.

A. The Statistical Post Freeze Out Hadronization

Statistical models were applied for high energy collisions [14, 15, 16] from the beginning of this field. In the last decade a significant development of these models and the extension of the area of their applicability took place. The main reason for this is a surprising success of the statistical approach in reproducing new experimental data. The extension of the area of their applicability took place.

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This simple volume dependence is however not valid any more for a small system in which the mean particle multiplicity is low, like in $pp$ collisions. In this case the material conservation laws should be imposed exactly on each charge configuration of the system, i.e. canonical ensemble (CE) description should be used. This condition introduces a significant correlation between particles who carry conserved charges (see [18, 19], and references therein).

In $\text{PbPb}$ collisions at the SPS collider and $\text{AuAu}$ reactions at RHIC we produce large and hot systems, where the corrections are negligible and GC description is satisfactory for most of the produced particles. Nevertheless, the number of strange particles created is rather low, and the net strangeness is exactly zero. Therefore the CE description would be preferable for the strange particles (we are not going to include particles with charm or more heavy quarks). However, the basic assumption of the hydrodynamics is the local thermal and chemical equilibrium, whereas the canonical conservation laws can only be realized globally. This prevents us using CE in post FO, which is nothing but the extension of the hydrodynamical part with different EoS. This drawback is not of a serious concern, because the conditions reached in high energy heavy ion systems are expected to fulfill the requirements for GC description [19].

In the relativistic GC ensemble for ideal hadron gas (HG), the properties of the matter in unit volume are parametrized by temperature $T$ and fugacities $\lambda_i$ (or chemical potentials $\mu_i$) for conserved charges. For example, the pressure and energy density are $P = P(T, \lambda_B, \lambda_S)$ and $e = e(T, \lambda_B, \lambda_S)$ if we omit the fugacity $\lambda_Q$ associated with electric charge conservation.

In case of sudden freeze out there might not be sufficient time to achieve chemical equilibration of number of strange- and anti-strange quarks. Thus, an over- or underpopulation of strange hadron species may persist after hadronization. This can, however, be treated in the GC approach as we will see.

Now that the thermal ideal gas of hadrons provides us with tools to calculate post-FO quantities, we can collect the set of equations needed to describe the simultaneous FO and hadronization taking place at a time-like hypersurface. For the sake of simplicity, we choose a hypersurface of constant coordinate time, $d\sigma = (1,0,0,0)$, which is, of course, connected with any time-like hypersurface by a proper Lorentz transformation. Recalling the energy-momentum, baryon current and strangeness conservation equations yields

$$(e + P)\gamma^2 - P = (e_{\text{FO}} + P_{\text{FO}})\gamma_{\text{FO}}^2 - P_{\text{FO}} \hspace{1cm} (5)$$
\[(e + P)\nu \gamma^2 = (e_{\text{FO}} + P_{\text{FO}})\nu_{\text{FO}} \gamma^2_{\text{FO}} \quad (6)\]
\[n_B \gamma = n_{B,\text{FO}} \gamma_{\text{FO}} \quad (7)\]
\[n_S \gamma = n_{S,\text{FO}} \gamma_{\text{FO}} = 0. \quad (8)\]

Given the pre-FO quantities, the solution of this set of equations gives the post-FO parameters: \(v_{\text{FO}}, T_{\text{FO}}, \lambda_{B,\text{FO}}\) and \(\lambda_{S,\text{FO}}\), flow velocity of the fluid element, temperature and fugacities for conserved currents, respectively. These parameters describe completely the hadron spectra and other statistical quantities at the LRF of the fluid element.

In order to take into account the possible over- or underpopulation of strange- and anti-strange quarks in post-FO side, a parametrization of the conservation of the number of \(s, \bar{s}\) pairs must be introduced. In the GC formulation, a conservation law gives rise to a fugacity parameter. The one associated with the number of \(s, \bar{s}\) pairs, \(n_{s,\bar{s}}\), is usually called \(\gamma_{\text{S}}\), first introduced in Ref. [24]. The assumption of the survival of \(n_{s,\bar{s}}\) over the FO hypersurface yields yet another equation to be included in the set above:

\[n_{s,\bar{s}} \gamma = (n_{s,\bar{s}})_{\text{FO}} \gamma_{\text{FO}}. \quad (9)\]

### III. NUMERICAL STUDIES

For quantitative studies of the FO, we have chosen a framework of Bjorken model [21] for the expanding QGP. This allows us to cover many FO scenarios with relative ease. Within the Bjorken model, the evolution of matter in one spatial dimension \(z\) and charge densities \(n_i\) is governed by equations

\[
\frac{\partial e}{\partial \tau} + \frac{e + P}{\tau} = 0 \quad (10)
\]
\[
\frac{\partial P}{\partial y} = 0 \quad (11)
\]
\[
\frac{\partial n_i}{\partial \tau} + \frac{n_i}{\tau} = 0, \quad (12)
\]

where \(\tau = \sqrt{t^2 - z^2}\) is the proper time and \(y\) is the rapidity of a given fluid element. For vanishing net charges, thermodynamical quantities are constant along constant proper time curves on \((t, z)\) plane. Taking the FO to take place at constant coordinate time, the proper time of freezing out fluid element decreases with increasing spatial coordinate, so the temperature is an increasing function of \(z\). The constant coordinate time choice is made in order to have a clear picture of the FO process in different thermal circumstances. In addition to the equations above, one needs an equation to describe the thermodynamical structure of the evolving matter, the equation of state.

#### A. The Equation of State - Supercooled QGP

Three different equations of state for supercooled quark-gluon plasma are considered. The QGP is assumed to be an ideal gas of three flavors (u,d,s) of quarks and their antiquarks. We will model the thermodynamics of the QCD vacuum in terms of the MIT bag model [22] by introducing a phenomenological bag constant \(B\). In this model, confinement can be thought of as a rule which allows quarks to inhabit only regions of a "false vacuum", which has minimum energy density \(B\). This is due to the fact that quarks can not be found in physical vacuum but only inside hadrons that are small bags of the false vacuum containing triplets or pairs of quarks. In QGP, all the space is endowed with this energy density \(B\).

Light quarks \((u,\bar{u},d,\bar{d})\) can be considered as massless, but the mass of strange quark can not be neglected. For massless quarks and gluons, integral in partition function \(Z_{\text{QGP}}\) can be evaluated analytically, but the contribution of strangeness must be calculated numerically. It is convenient to split \(Z_{\text{QGP}}\) into four parts:

\[
\ln Z_{\text{QGP}} = \ln Z_f + \ln Z_b + \ln Z_s + \ln Z_{\text{vac}},
\]

where \(\ln Z_f, \ln Z_b\) and \(\ln Z_s\) are the contributions of massless quarks, gluons and massive strange quarks respectively. \(\ln Z_{\text{vac}}\) is the contribution of the bag constant added to the energy-momentum tensor. Evaluation of analytical parts of \(\ln Z_{\text{QGP}}\) leads to the expressions [23]

\[
\ln Z_f = V \left( \frac{7}{30} \pi^2 T^3 + \frac{\mu_q^2 T}{2 \pi^2} + \frac{1}{4 \pi^2} \pi^4 \right) \quad (13)
\]
\[
\ln Z_b = \frac{8}{45} \pi^2 T^3 \quad (14)
\]
\[
\ln Z_{\text{vac}} = - B V T, \quad (15)
\]

where \(\mu_q\) is a light quark chemical potential, \(\mu_q = \mu_B/3\). The total strangeness is zero during the whole collision evolution. If one assumes that the strangeness produced in a collision is spread homogeneously in QGP, then \(n_S\) must be zero in any given fluid element, so the contribution of \(s, \bar{s}\) quarks is

\[
\ln Z_s = \frac{6V}{\pi^2} \int_0^\infty dp p^2 \ln \left( 1 + e^{-\beta \sqrt{p^2 + m_s^2}} \right), \quad (16)
\]

where \(m_s\) is the mass of the strange quark, here chosen to vary between 150 and 250 MeV.

If one puts the bag constant to zero \((B=0)\), then \(\ln Z_{\text{vac}}\) term vanishes and we end up with the normal quantum distribution for the ideal gas. In this case, the explicit form of the three flavor QGP EoS is [23]

\[
P(T, \mu_i) = \frac{1}{3} e(T, \mu_i) = e_{SB}. \quad (17)
\]

In the case of finite bag constant, EoS reads:

\[
P(T, \mu_B, B) = \frac{1}{3} e_{SB} - B, \quad e(T, \mu_B) = e_{SB} + B. \quad (18)
\]
The third equation of state in addition to ones with finite and zero MIT bag constants considered here was suggested in the Ref. [24,25] (and first time applied for heavy ion collisions in [26]) for the case of baryon free QGP ($\mu_q = 0$) with two quark flavors:

$$P(T) = \frac{1}{3}e(T) - bT, \quad e(T) = e_{SB}. \quad (19)$$

We call this a spinodal EoS, inspired by the existence of a local minimum in pressure profile [19]. It is easy to check that $e$ and $P$ are connected by the thermodynamical equation

$$e = T \frac{dP}{dT} - P. \quad (20)$$

This EoS can be justified as resulting from the non-perturbative QCD effects: The lattice calculations (see for example [27]) show that close to the critical temperature for hadronic matter, QGP phase transition energy density has a sharp increase and soon saturates with an equilibrium Stefan-Boltzmann value $e(T) = e_{SB}(T)$, while pressure increases much more slowly and reaches Stefan-Boltzmann limit only at very high $T$. Such a pressure suppression is presented by eqs. (19).

The spinodal EoS considered here will be of form (19) but with three quark flavors and finite net baryon density:

$$P(T, \mu_i) = \frac{1}{3}e(T, \mu_i) - bT, \quad e(T, \mu_i) = e_{SB}. \quad (21)$$

Comparing these expressions with the bag model EoS one can see that for the pressure the bag constant, $B$, is replaced by $T$ and $\mu$ dependent term $bT$, while for energy density the analogy with bag model disappears due to the relation (20).

If the phase transition temperature $T_c$ is known, parameter $b$ can be fixed from the Gibbs condition $P_{HG}(T_c, \mu) = P_{QGP}(T_c, \mu)$. This same condition fixes the constant $B$ within the bag model.

### B. Freeze Out Illustrated

The post-FO matter is described by the quantum ideal gas, composed of hadrons up to a mass of 2.5 GeV, listed in the year 2000 issue of Review of Particle Physics [28]. In order to satisfy the conservation equation (1) for strange quark pairs, additional fugacity $\gamma_S$ is introduced for each strange particle $i$ as $\lambda_i \rightarrow \lambda_i \gamma_S[S_i]$. Additionally, the mesons carrying $s\bar{s}$ pairs must be taken into account. In this work, the number of meson $i$ is affected by the fugacity factor $\lambda_i = \gamma_S^{[S_i]}$, where $c_\phi$ is the relative $s\bar{s}$ content in the meson. We take $c_\phi = 0.5$ for $\eta$ mesons, and $c_s = 1$ for $\phi$, $f_0(980)$, $f_1(1510)$, $\phi(1680)$ and $\phi_3(1850)$.

In figure 1 we show the pressures of the HG and QGP as functions of temperature. The parameters appearing in different equations of state for QGP are fixed by setting the critical temperature to $T_c = 160$ MeV. The pressure minimum in the spinodal EoS is labeled by $T_m$. Of course, the critical temperature varies with baryon density (or $\mu_B$), but we find this variation to be negligible within reasonable range of FO density. The strangeness saturation parameter $\gamma_S$ is set to one, corresponding to the full strangeness equilibration. We always let the QGP side be in full strangeness equilibrium, so the Gibbs condition for the adiabatic, isothermic phase transition compels $\gamma_S = 1$. In the following, however, the phase transition is neither isothermic nor adiabatic, so the change in fugacities is allowed and unavoidable.

The figure 1 depicts the variation of some relevant variables along the coordinate time $t = 10$ fm/c in the Bjorken model. Going from the edge of the light cone ($z = 10$ fm) to the non-flowing center of the system ($z = 0$ fm), we describe the changes of quantities in proper time of the given fluid element from $\tau = 0$ to $\tau = 10$. This is, going from the origin to the right, we go from the later, cooler and more dilute stage of evolution to the earlier, hotter and more dense stage. We have chosen the initial values for energy- and baryon density $\epsilon_0 = 5$ GeV fm$^{-3}$ and $(n_B)_0 = 1$ fm$^{-3}$, respectively, at the time $\tau = 1$ fm/c. For the ideal gas QGP EoS, the higher $\epsilon_0 = 8$ GeV is used in order to keep the figure more descriptive. The model parameters $B$ in the MIT bag EoS and $b$ in the spinodal EoS are fixed to produce critical temperature 160 MeV, while for the ideal gas EoS there is no critical temperature. The arbitrary value $T_c = 160$ MeV is chosen to give similar scaling for temperatures in all cases. The ratio $R$ of entropy currents in HG and QGP stays well above one for bag and spinodal models all the way up to unrealistic FO conditions, so the substantial increase in entropy is expected in fast simultaneous FO and hadronization. For the reference, the $R$ for the ideal gas QGP is, as expected, far below...
FIG. 2: Various quantities on the FO surface. Panels from top to bottom: a) ideal gas EoS for QGP, b) MIT bag EoS, c) spinodal EoS. $R$ is the ratio of entropy currents in HG and QGP, $T_H$ and $T_Q$ are the temperatures on the HG and QGP side, and $T_c$ is the critical temperature. The model parameters in the bag- and spinodal EoS are fixed to produce critical temperature 160 MeV. For the ideal gas EoS, there is no real $T_c$, the chosen 160 MeV is arbitrary. The vertical line in the panel c marks the point, where the HG temperature crosses the critical value.

FIG. 3: Various quantities on the FO surface. Panels from top to bottom: a) ideal gas EoS for QGP, b) MIT bag EoS, c) spinodal EoS. $v_H$ and $v_Q$ are the HG and QGP flow velocities. $\gamma_{S_1}$ and $\gamma_{S_2}$ are the strangeness saturation fugacities with $s$ quark masses 150 MeV and 250 MeV, respectively. The model parameters in the bag- and spinodal EoS are fixed to produce critical temperature 160 MeV. For the ideal gas EoS, there is no real $T_c$, the chosen 160 MeV is arbitrary. The vertical line in the panel c marks the point, where the HG temperature crosses the critical value.
one. The crossing of HG temperature $T_H$ with $T_c$ marks the endpoint of the physically allowed FO. For the bag model, there is no such crossing, but the FO stays unphysical with all reasonable values of QGP temperature $T_Q$. We could choose lower initial value for the QGP energy density, but the the degree of supercooling, $T_Q/T_c$, would be of order 0.5 at the HG critical point. For the spinodal model QGP, the case is different. At the point $T_H/T_c = 1$, the degree of QGP supercooling is about 24%, and stays rather constant when going towards the cooler system.

In figure 3 we illustrate the changes in flow velocities and strangeness saturation parameter in the circumstances equal to ones in figure 2. For both cases, bag and spinodal QGP, the flow is decelerated at the FO to HG, indicated by the ratio of HG and QGP velocities, $v_H/v_Q$. At the point $T_H = 160$ MeV for the spinodal EoS, this ratio is 0.80, indicating the final HG flow velocity of 0.54c. The $\gamma_S$, resulting from the survival of $s\bar{s}$ pairs through the FO process, is calculated for two different values of $s$ quark mass $m_s$. $\gamma_S$ corresponds to $m_s = 150$ MeV and $\gamma_{S_2}$ is for $m_s = 250$ MeV. Comparing the figures, we find $\gamma_S$ to be very sensitive to the choice of QGP EoS and the $m_s$. It is worth noting, that fixing $\gamma_S$ to one or varying the $m_s$ gives only negligible change to other quantities in figures 2 and 3.

IV. CONCLUSIONS

We have found that the realistic and accurate study of the freeze-out process is important and cannot be neglected. Our results show that the FO process is very sensitive on the properties of the EoS. Since our main goal is to identify the EoS from the data this is an observation of basic importance.

The correct treatment shows that FO is not even possible from arbitrary kind of initial state, and the entropy constraint is a sensitive condition.

From the point of strangeness, we can also conclude that strangeness production is very sensitive to the correct FO treatment and to the pre FO EoS. Thus, strangeness data not only provide a signal of QGP formation, but with proper and realistic description of freeze out, strangeness provides the most sensitive signal indicating different properties of the pre FO EoS.

In conclusion, this study shows that collective, continuum reaction models, like fluid dynamical (FD) models (one fluid FD, multi-fluid FD, chiral FD), can and must be supplemented with realistic Freeze Out treatment to evaluate measurable data. These calculations indicate that this is now possible cell by cell in FD models, although it requires more than average computational capacity, and needs preferably high performance parallel computing.

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