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A Novel Behavior of Pump Power in the Instability Induced Supercontinuum Generation of Saturable Nonlinear Media

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Abstract. We investigate the modulational instability (MI) induced Supercontinuum generation (SCG) in exponential saturable nonlinearity. The pump power ($P$) is observed to behave in a unique way such that unlike the conventional Kerr case, the effective nonlinearity of saturable nonlinear system does not monotonously increases with an increase in power. The supercontinuum is observed at the shortest distance of propagation at power equal to the saturation power ($P_s$), whereas for all combinations of powers ($P < P_s$ or $P > P_s$) spectral broadening occurs at longer distance.

1. Introduction
One of the most advanced frontiers of nonlinear optics is the generation of intense ultrabroadband spectrum known to the scientific world as the supercontinuum generation (SCG). The potential applications of such as frequency metrology, optical coherence tomography, ultra-short pulse compression, sensor technique, fiber characterization etc., have made SCG as one of the dynamic and elegant nonlinear phenomena in nonlinear optics. Thus SCG in the modern days has been recognized as the “White-light laser” [1–3]. All the applications are powered by the exceptional properties such as high brightness, coherently pulsed nature, high degree of spatial coherence, broad bandwidth and the requirement of low pulse energies. The extensive interest stem not only on its application perspective but also on the scientific curiosity due to the rich physics. The invention of photonic crystal fiber (PCF) have revolutionized the concept of SCG, this is because of its exceptional featres such as the requirement of less input intensity, engineerable zero dispersion wavelength by tailoring the waveguide dispersion, endlessly single mode operation, and elevated nonlinearity. The first demonstration of SCG in PCF that span two octaves from UV to the infrared was reported by Ranka et al. This futuristic work sets the tone for new innovation in the context of SCG in PCF [4, 5].

The physics underlying SC generation is now generally well understood, the primary mechanism of generating broadband source is identified as soliton fission and modulational instability (MI) [1, 6]. Former is the well-known mechanism of generating high quality broadband spectrum driven by the soliton dynamics . Latter is the MI induced SCG (MI-SCG), where the initial dynamics are governed by the noise, which lead to the breakup of the pulses similar to the case of SF.
In recent times, there has been renowned interest in the concept of saturable nonlinearity (SNL), due to the inherent nature in some nonlinear materials. In reality, for any real glass material, there is an upper limit to the nonlinear refractive index that can be induced optically. Beyond a certain level of the pump power, higher-order nonlinear susceptibilities inevitably come into play and manifest themselves in qualitatively different ways depending on the type and nature of the dopants that are incorporated in the glass material. For instance, system such as semiconductor doped fiber [7, 8] and liquid core photonic crystal fiber [9, 10] the nonlinear response tends to saturate even at low power due to its enhanced nonlinear index. We consider such a system in the saturation window for our investigation and hence the inclusion of saturable nonlinearity is obvious to reveal the actual physics associated with such system. Nonetheless, high power operation not only saturates the nonlinear response of the medium but also elevates soliton order. This increase in soliton order favors the MI process over soliton fission, and thereby making MI as the prominent governing mechanism in the SCG in the SNL system. Lyra et al, reported the different functional form of the saturable nonlinearity, out of which exponential saturable nonlinearity (ESN) is observed to generate broadest spectrum at relatively shorter distance [11, 12]. In the similar lines to the earlier report [12], we describe in the manuscript about the unique role of power in the instability induced SCG.

2. Theoretical Model

The propagation of optical beam through a fiber with SNL is described by modified nonlinear Schrödinger equation (MNLSE). The MNLSE of the slowly varying envelope $U(z,t)$ along the $z$-axis in a retarded reference time centered at frequency $\omega_0$ is given by [7, 8, 12]

$$\frac{\partial U}{\partial z} + \sum_{n=2}^{4} \beta_n \frac{i^{n-1}}{n!} \frac{\partial^n U}{\partial t^n} = i \frac{\gamma |U|^2}{1 + \Gamma |U|^2} U; \quad (1)$$

where, $z$ and $t$ are the longitudinal coordinate and time in the moving reference frame, respectively. $\beta_n$ is the dispersion coefficient resulting from the Taylor expansion of the propagation constant around the center frequency $\omega_0$ and $\gamma = \frac{n_2 \omega_0}{A_{eff}}$ is the Kerr coefficient, where $A_{eff}$ is the effective core area, $n_2$ is the nonlinear index coefficient. $P_s$ is the power threshold corresponding to the saturation of the nonlinear response. $\Gamma$ is the saturation parameter whose expression is given by $\Gamma = 1/P_s$.

3. MI induced supercontinuum generation

The propagation of optical beam in ESN governed by Eq. (1) is numerically studied using split step Fourier method. The initial envelope of the soliton at $z = 0$ is given by $U(0,t) = \sqrt{P_0} \text{sech}(t)$ of width 10 ps at $\lambda_0 = 1.55\mu m$. The numerical simulation is carried out with a resolution of $2^{20}$ grid points. The system parameter are as follows: The dispersion coefficients takes $\beta_2 = -4.50 \times 10^{-2}$ ps$^2$/m, $\beta_3 = 6.710 \times 10^{-4}$ ps$^3$/m, $\beta_4 = -6.722 \times 10^{-7}$ ps$^4$/m and the nonlinear coefficient can be written as $\gamma = 2.18$ W$^{-1}$m$^{-1}$[9, 10]. The high power operation in the saturation window of the nonlinear response and the choice of longer pulse width in ps time scale favors the MI process over soliton fission and thereby confirms the MI as the dominant mechanism in SCG of SNL system.

Following our earlier report [12], the dispersion relation corresponding to the MI gain in ESN can be written as

$$K = \beta_3 \frac{\Omega^3}{6} \pm \left[ \left( \frac{\beta_2 \Omega^2}{2} + \beta_4 \frac{\Omega^4}{24} \right) \left( \frac{\beta_2 \Omega^2}{2} + \beta_4 \frac{\Omega^4}{24} + 2\gamma_1 P_0 \right) \right]^{1/2}, \quad \gamma_1 = \gamma e^{-\Gamma P_b} \quad (2)$$
In the case of anomalous dispersion regime, $\beta_2$ takes negative value and thereby serve the required phase matching for MI [6, 13]. Instability is found to be possible only within the threshold frequency known as the critical modulation frequency (CMF) and for frequencies $\Omega_{CMF} > \Omega$, $K$ becomes complex and the spectral sideband grows exponentially down the fiber. It is worth noting that when the saturation parameter $\Gamma = 0$, both CMF and $G_{max}$ revert back to the conventional Kerr type nonlinear response.

The MI/FWM process typically manifest as follows: (i) the primary stage is the emergence of spectral sidebands and the subsequent higher order sidebands (ii) the second stage is the evolution of ultra-broadband spectrum through the nonlinear phase matching. In time domain, the MI can be recognized as the growth of weak perturbation leading to the fast modulation of pulse envelope and results in the fine structure on the waveform.

4. Supercontinuum generation in saturable nonlinear media
This section features the insight analysis of the characteristics of ESN in the supercontinuum generation. To demonstrate the impact of pump power in the SCG, we have considered different input power at a constant saturation power. It is observed that the increase in input powers does not lead to monotonous enhancement of SCG. The enhancement we quote here is the ability of the system to achieve SCG at shorter distance of propagation as possible. The maximum enhancement of SCG is achieved only at operating the input power around the saturation threshold, for all other choice of input power the SCG inevitably gets suppressed. To illustrate, we discuss three combinations of powers as shown in the Fig. (1), one at the satur-
tion power $P = P_s$, others at below $P < P_s$ and above $P > P_s$ the saturation power. Without any loss of generality, we consider the saturation power as $P_s = 1500\text{W}$ and the operating input powers as $P(A \rightarrow 500, B \rightarrow 1500, C \rightarrow 3000)\text{W}$. Among the three cases, one can readily observe that at $P = P_s$, the sidebands evolve much earlier than the rest of the two cases. Precisely, the primary sideband appears at $z \approx 0.5 \text{ cm}$ and secondary sideband at $z \approx 1.6 \text{ cm}$, whereas on the other side, for all other representative power levels the sidebands manages to evolve only at higher distance of propagation. Similarly, the corresponding temporal evolution is given in the Fig. (2). The pulse breaking occurs at shortest possible distance corresponding to $P = P_s$ (i.e.) at $z \approx 1.9 \text{ cm}$ as shown in Fig. (2).

![Figure 2](image-url)

Figure 2: Temporal evolution of MI-SCG in SNL at $P (=500 \text{ W (a), 1000 W (b) and 3000 W (c) and } P_s = 1500 \text{ W for a propagation length } L = 5 \text{ cm}$.

To give an insight, we consider the unsaturated Kerr (USN) type nonlinear response case with the same parameters as in the case of SNL. The spectral and temporal evolutions corresponding to the power $P(A \rightarrow 500, B \rightarrow 1500, C \rightarrow 3000)\text{W}$ are given in Fig. (3) and Fig. (4). It is obvious from the figures that MI sidebands occur much earlier than the SNL counterpart. For instance, primary sideband occurs at $z \approx 0.3\text{cm}$ and secondary sideband occurs at $z \approx 0.8 \text{ cm}$ for an input power of $P = 1500\text{W}$ (Refer Table. 1). Also, one can straightforwardly notice that increase in power certainly increases the effective nonlinearity and thereby enhances the SCG by achieving at shorter distance of propagation. Thus, our numerical study strongly illustrates that the incorporation of the nonlinear saturation of the medium certainly suppresses the MI.

On the other hand, saturable nonlinear system behaves quite differently, than the conventional Kerr type nonlinearity. The difference in the distance for the evolution of sidebands in
SNL is attributed to the fact that the power is not directly proportional to the effective non-linearity ($\tilde{\gamma}$) as in the case of the conventional Kerr medium. It is observed that for power less than the saturation power, the $\tilde{\gamma}$ increases with an increase in power and for power greater than the saturation power, the $\tilde{\gamma}$ decreases with an increase in power. This change in the effective nonlinear coefficient changes the nonlinear length of the system. In principle, it is a proven fact that the MI occurs at the balance between the dispersion and the nonlinear length of the system. This change in the nonlinear length is in accordance with the required phase matching for the FWM process. For instance, the nonlinear length ($L_{nl} = \frac{1}{\tilde{\gamma} P}$) of the SNL at different power follows the order $L_{nl_B} < L_{nl_C} < L_{nl_A}$ and hence the distance of propagation required to achieve MI/FWM, or, in other words the ease of achieving PMC follows the order $PMC_B > PMC_C > PMC_A$.

Thus among the different combination of power, case B register the least value of $L_{nl}$ thus enables the pulse breaking at shorter distance. This is due to the fact that operating the system around the saturation power records the highest effective nonlinearity, thereby enables to achieve PMC at relatively shorter distance which results in the maximum enhancement of SCG and the associated nonlinear effects.

5. Results and Discussion

Thus, our numerical study strongly illustrates that the incorporation of the SNL of the medium certainly suppresses the MI and the associated dynamical behavior. Our numerical analysis
affirms the evolution of SCG from the MI, some of the valid reason to justify the same are as follows (i) no initial soliton contraction and propagation of higher order soliton as in SF, (ii) pulse breaking is purely due to the numerical noise included, (iv) extreme sensitivity to noise are some of the classical evidence of the MI induced SCG.

Figure 4: Temporal evolution of MI-SCG in USN at P (= 500 W (a), 1500 W (b) and 3000 W (c)) for a propagation length $L = 3$ cm.

To give a comprehensive idea of the impact of power in the SCG spectrum, we consider the evolution of SCG from MI for some representative cases of power at different propagation length. Fig. (5) infer that the pattern of evolution of the spectral sidebands in all cases remains the same. The saturation power is fixed as $P_s = 1500$ W and the input power corresponds to $P(A \rightarrow 500, B \rightarrow 1500, C \rightarrow 3000)$ W. It is evident from the figures that among the three representative powers, curve B corresponds to $P = P_s$ records the maximum gain (arb.units). Despite the high value of power curve C can only find less gain than curve B. Thus operating at saturation power enables one to achieve SCG at shorter distance than for all combinations of powers.

For numerical appreciation about the role of pump power in the SCG, we tabulated the distance corresponding to the pulse breaking and the evolution of sidebands in Table.1. For the better understanding Kerr type unsaturated nonlinear response is also considered alongside the saturable nonlinearity. One can straightforwardly infer from the table that for the case of Kerr response the distance corresponding to the evolution of sidebands and pulse breaking occurs at shorter distances for higher powers. Whereas in the SNL system both the pulse breaking and the spectral sideband evolution is no longer proportional to the power and register least at $P = P_s$. Interestingly one could also observe that power corresponding to the particular distance indeed
Figure 5: Sideband evolution of SNL for power $P(=500, 1500, 3000)\text{W}$ at a distance of 2 cm (a), 2.5 cm (b) and 3 cm (c).

occurs in pairs (i.e.) there exist two value of power at which the distance corresponding to the pulse breaking and spectral evolution register same value. This we reckon as the two state behavior in the context of MI, which can be readily observed in SCG, as well. This observation is a clear manifestation of evolution of SCG from MI.

6. CONCLUSION
In summary, we theoretically investigated SCG in the SNL system using MNLSE. We have reported that the behavior of pump power in the SNL system is indeed unique such that, the maximum enhancement of SCG (SCG at shortest distance) is observed only at $P = P_s$ and

| Power (W) | Primary Sideband(cm) | Secondary Sideband(cm) | Pulse breaking(cm) |
|----------|----------------------|------------------------|-------------------|
|          | USN | SNL | USN | SNL | USN | SNL |
| 500      | 0.9 | 1.7 | 1.7 | 3.2 | 2.3 | 4.2 |
| 1000     | 0.7 | 1.0 | 1.1 | 2.2 | 1.4 | 2.9 |
| **1500** | 0.3 | **0.5** | 0.8 | **1.5** | 0.9 | **1.9** |
| 3000     | 0.2 | 1.3 | 0.4 | 2.6 | 0.4 | 3.3 |
for all combinations of powers the SCG gets suppressed. To give a comprehensive idea about SCG in SNL system, Kerr type nonlinear response is also considered alongside the SNL, similar to the earlier published reports that the SNL inevitably suppresses the SCG. However, our present results deviate from the rest, such that we identified the active regime in SNL system, where maximum enhancement of the SCG is possible. Also, the two state behavior we reported enables one to manipulate the desired SCG characteristics in a relatively simple means by merely changing the input power. Thus in the report, we bring to the light on the unusual role of pump power in the SCG process in the SNL system. We understood that there is no experimental report till now in the context of SCG in SNL system, and we believe that our presented theoretically reported results will set benchmark for new experiments aiming to probe the various nonlinear effects in the SNL system.

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References
[1] Dudley J M, Genty G and Coen S 2006 Rev. Mod. Phys. 78 1135–1184
[2] Zheltikov A (2006) Physics-Uspekhi 49 605
[3] Alfano R R 2006 The Supercontinuum Laser Source (Berlin:Springer)
[4] RS Windeler, J and AJ Stentz (2000) Opt. Lett. Lett. 25 25
[5] Husakou A V and Herrmann J 2002 J. Opt. Soc. Am. B 19 2171–2182
[6] Agrawal G 2012 Nonlinear fiber optics (San Diego, 5th edition: Academic Press)
[7] Dinda P T and Porsezian K 2010 J. Opt. Soc. Am. B 27 1143–1152
[8] Hickmann J M, Cavalcanti S B, Borges N M, Gouveia E A and Gouveia-Neto A S 1993 Opt. Lett. 18 182–184
[9] Ganeev R, Ryasnyansky A, Baba M, Suzuki M, Ishizawa N, Turu M, Sakakibara S and Kuroda H 2004 Applied Physics B 78 433–438 ISSN 0946-2171
[10] Vieweg M, Gissibl T, Pricking S, Kuhlmeier B T, Wu D C, Eggleton B J and Giessen H 2010 Opt. Express 18 25232–25240
[11] Lyra M L and Gouveia-Neto A 1994 Opt. Communication 108 117–120
[12] K Nithyanandan Vasantha Jayakantha Raja R P K and Uthayakumar T 2013 Opt. Fib. Tech. 19 348
[13] Porsezian K, Nithyanandan K, Raja R V J and Shukla P K 2012 J. Opt. Soc. Am. B 29 2803–2813