Unsteady MHD nanofluid flow over a stretching sheet with chemical reaction

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Abstract. The unsteady magnetohydrodynamic (MHD) mass transfer flow of an incompressible, electrically conducting nanofluid past a porous stretching sheet has been considered. In this study we considered three different types of water based nanofluids viz., copper (Cu), aluminum oxide $\text{Al}_2\text{O}_3$, and titanium dioxide (TiO$_2$). By using appropriate similarity transformations, the governing partial differential equations of momentum and concentration are converted into a set of non-linear ordinary differential equations. The resulting equations are then solved numerically by using shooting method along with R-K fourth order scheme. The effects of diverse flow physical parameters on the flow fields and mass transfer are illustrated graphically. It is found that the influence of unsteady parameter on skin friction coefficient and Sherwood number is opposite.

1. Introduction
The term “nanofluid” refers homogenous mixture of base fluid and nanoparticles. These fluids are characterized by suspension of particle copper which the diameter is less than 100nm. In general, nanofluids possess to improve the thermo-physical properties such as melting diffusivity, consistency, melting potential and heat conduction characteristics as compared with the common base fluids like water, engine oils and lubricants, polymeric solution, bio-fluids and other common liquids. In fact, the main concept of using nanoparticles is to improve the melting potential of the based fluid. Many researchers [1-5] investigated the flow of a nanofluid over various flow fields due to its extensive use in engineering applications. Beg et al. [6] found the unsteady mixed convective nanofluid over a porous exponentially stretching sheet by using explicit finite difference scheme. Kumar Nandy et al. [7] examined the influence of Brownian motion and thermophoresis over unsteady permeable shrinking sheet. Freidoonimehr et al. [8] investigated numerically an unsteady magnetohydrodynamic laminar stream of a nanofluid past a perpendicular stretched layer with porous medium. Recently, Zin et al. [9] addressed the effect of thermal energy on unsteady magnetohydrodynamics flow over a permeable medium. Heat transfer analysis on Cu-water nanofluid stream past a stretching layer with slip is investigated by Pandey et al. [10]. An extensive variety of survey papers on nanofluids and different utilizations are make in [11-13].

In recent years, several researchers focused very much attention to analyze the unsteady flows over a stretching surface due to its uses in geophysics, paper production, polymer preparing of a biochemical mechanical plant that incorporate both polymer and metal sheets and so on. The chemical
reaction effects in boundary layer flows are highly significant for their various reasonable applications within the sight of homogeneous and heterogeneous mass exchange responses happens by diffusive operations which include the atomic dispersion of species. Harish Babu and Satya Narayana [14] investigated the joule heating on magnetohydrodynamic heat exchange of an incompressible fluid with heat source over stretching plate. Chamkha et al. [15] inspected the unsteady limit layer stream because of an extending surface installed in a permeable medium generation/absorption were acquired. Manjula et al. [16] addressed the Qualities of heat and mass exchange of an electrically directing and incompressible and viscous liquid through a permeable medium. Mishra et al. [17] proposed the soret effects on hydromagnetic micropolar stream along extending layer in the nearness of the chemical reaction. Satya Narayana et al. [18] analyzed the consequence of chemical response on magnetohydrodynamic heat and mass transfer of a non-Newtonian over extending layer. Daba and Devaraj [19] presented the chemically reacting varied convection flow through a permeable unsteady stretching surface with slip and thermal radiation. Recently, some of them studied MHD flows over a stretching sheet see ref. [20-22].

The objective of this model is to analyze the influence of three distinct kinds of nanoparticles (Cu, Al$_2$O$_3$, and TiO$_2$) over an extending layer in the presence of magnetic field and chemical response. By using similar transformations, the momentum and mass transfer equations, under boundary layer assumptions reduce to the set of nonlinear ODE. The resulting ODE’s are calculated statistically by using R-K rule along with shooting technique see ref. Matausek [23]. The results for emerging parameters are being illustrated through graphs and discussion. Physical behaviors of reduced skin-friction coefficient and Sherwood number are discussed through graphs.

2. Mathematical formulation

We are taken an unsteady magnetohydrodynamic laminar stream through an extending layer in a H$_2$O based incompressible nanofluid containing 3 unique sorts of nanoparticles, as appeared in Figure 1. Likewise, it is accepted that the base liquid and the nanoparticles are in current harmony and no slip condition exists between them. It is additionally gathered attractive Reynolds number is minor. In this way, it is possible to disregard the incited attractive field in contrast with the connected attractive field. Consider $C_w$ a chance to fixation at the plate and the focus from the layer is $C_{\infty}$ additionally, let the response of the species a chance to primary request homogeneous synthetic response of rate $k_1$ which changes with time. We expect that for time $t<0$, the liquid and mass streams are relentless. The insecure liquid and mass streams begin at $t=0$. The layer rises out of an opening at beginning ($x=0$, $y=0$) and moves with non-uniform $f(y)$ along the x-axis. The speed of the mass exchange opposite to the extending surface is $V_w(t)$.

The governing equations of such kind of flow see ref. [24] and mass transfer has usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hfill (1)

$$\rho_{nf} \left( \frac{\partial u}{\partial t} + u \left( \frac{\partial u}{\partial x} \right) + v \left( \frac{\partial u}{\partial y} \right) \right) = \mu_{nf} \left( \frac{\partial^2 u}{\partial y^2} \right) - \sigma B^2 u$$  \hfill (2)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_{\infty})$$  \hfill (3)

Here $u$&$v$ are the velocity components along with x and y-Coordinates, individually, $t$ indicates time, $\rho_{nf}$ & $\mu_{nf}$ are indicates density and the dynamic viscosity of the nanofluid, individually, $\mu_{nf}$ was suggested by Brinkman [25], $\sigma$ bethe electrical conductivity, magnetic field $B = B_0 (1 - e^{-t})$ imposed along the y-axis, the diffusion coefficient $D$ of the diffusing species in the liquid, the time-dependent
reaction rate \( k_i \) represent for destructive response whereas \( k_f \) indicate for constructive response, \( k_0 \) is a constant, the relaxation time of the period \( t_0 = \lambda \), \( \lambda_0 \) is a constant.

\[
\rho_{df} = (1 - \phi) \rho_f + \phi \rho_s; \quad \eta_{df} = \frac{\eta_f}{(1 - \phi)^{2/3}}
\]

Here, \( \phi \) is nanoparticle volume fraction parameter, \( f \) & \( s \) subscripts is refer to fluid and solid fraction properties. The properties of thermos-physical base fluid (H\( _2 \)O) and variety of nanoparticles are given in

The proper boundary conditions for the problem are given by

\[
u = U_w(x,t) = \frac{ax}{(1 - ct)}, \quad v = v_w(x,t) = \frac{v_0}{\sqrt{1 - ct}}, \quad C = C_w = C_\infty + \frac{ax}{(1 - ct)^2} \text{ at } y = 0
\]

\[u \to 0, \quad C \to C_\infty \text{ as } y \to \infty\]

where, \( a \) and \( b \) constants (where \( a > 0 \) and \( c \geq 0 \), with \( c t < 1 \)). These two constants have \( t \) measurement. Additionally, \( a \) is the underlying stretching rate and powerful stretching rate \( a/(1 - ct) \) which is creating with time. The accompanying dimensionless capacities \( f(\eta), \phi(\eta) \) are the comparability factors are utilized as:

\[
\eta = \left( \frac{a}{v_f(1 - ct)} \right)^{1/2}, \quad \psi(x,y) = \left( \frac{v_f^2 a}{1 - ct} \right)^{1/2} f(\eta) \cdot \phi(\eta) = \frac{C - C_w}{C_w - C_\infty}
\]

where, \( \psi(x,y) \) is the free stream function which fulfils the continuity Eq. (1) With

\[
u = \frac{\partial \psi}{\partial y} = \frac{ax}{1 - ct} f'(\eta) \quad \nu = -\frac{\partial \psi}{\partial x} = -\left( \frac{v_f^2 a}{1 - ct} \right)^{1/2} f(\eta)
\]

Exchange Eqs. (6) and (7) in Eqs. (2), (3) and (5), the below ordinary differential equations are getting:

\[
\frac{1}{(1 - \phi)^{1/2}} f''(\eta) - \left( 1 - \phi + \phi \left( \frac{\rho_f}{\rho_s} \right) \right) \left[ f''(\eta) - f(\eta) f''(\eta) \right] - Mf'(\eta) = 0
\]

\[
A \left( \frac{1}{2} \eta f''(\eta) + 2\phi(\eta) \right) + f'\phi(\eta) - f \phi'(\eta) = \frac{1}{Sc} \phi''(\eta) - \gamma \phi(\eta)
\]

The transformed boundary conditions become

\[f(\eta) = c a, \quad f'(\eta) = 1, \quad \phi(\eta) = 1 \text{ at } \eta = 0\]

\[f'(\eta) \to 0, \quad \phi(\eta) \to 0 \text{ as } \eta \to \infty\]

Here, separation as for \( \eta \) in prime demonstrated, \( A = \frac{\sigma}{\sigma_f} \) is the unsteadiness parameter, \( M = \sigma B_0^2/\alpha \rho_s \) is the magnetic parameter, Schmidt number is \( Sc = \nu/\nu_s \), \( f_0 < \nu_0/\nu_0 \) is the suction parameter, chemical reaction parameter is \( \gamma = k_0/\beta \). Here, \( \gamma > 0 \) speak to the destructive response, \( \gamma = 0 \) relates to no response and \( \gamma < 0 \) see for the generative response and furthermore Eqs. (7) and (8) alongside limit conditions (10) were understood numerically by applying shooting design. \( C_\infty \) and \( Sc \) are given by
\[ C_f = \frac{\tau_w}{\rho u_f^2}, \quad Sh = \frac{C u}{D(C_\infty - C_u)} \]  \hspace{1cm} (11)

Where, \( \tau_w = \mu_w \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_m = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} \)  \hspace{1cm} (12)

Implement the non-dimensional transformations to (11), we obtain
\[ C_f \text{Re}_{x/2}^{1/2} = \frac{1}{(1-\phi)} f''(0), \quad Sh \text{Re}_{x/2} = -\phi'(0) \]  \hspace{1cm} (13)

where local Reynolds number is \( \text{Re}_x = \frac{U x}{\nu} \).

3 Results and Discussion

The nonlinear ODE (8) & (9) subjected to the boundary conditions (10) are calculated numerically by testing shooting technique. Complete numerical calculations are solved for distinct values of magnetic field parameter \( M \), unsteadiness parameter \( A \), suction parameter \( f_w \), nanoparticle volume fraction \( \phi \) with three various kinds of nanofluids. The numerical results of \( f'(\eta) \) and \( \phi(\eta) \) distributions as well as \( f'(0) \) and \( C_f \text{Re}_{x/2}^{1/2} \) are presented in Figures 2-6.

![Figure 1. The geometry of physical model](image1)

![Figure 2(a). Portrait \( f'(\eta) \) for distant values of \( M \)](image2)

![Figure 2(b). Portrait on \( \phi(\eta) \) for distant values of \( M \)](image3)

The effect of \( M \) on \( f'(\eta) \) and \( \phi(\eta) \) are depicted in Figures 2(a) and 2(b) individually. It is observed that the \( f'(\eta) \) portrait dwindle with ascending of \( M \) for different types of nanofluids. Physically, Lorentz force which is a drag like power is made by depending upon infliction of the perpendicular \( M \) to the electrically conducting liquid. This force results in dwindle of \( \phi(\eta) \) of the stream through the extending layer. In addition, \( M \) shows same to on concentration distributions. Also, here seen that \( \phi(\eta) \) distribution and the velocity boundary layer thickness are larger in case of \( Cu \) than to \( Al_2O_3 \) nanofluids.
Figures 3(a) & 3(b) respectively present the consequence of $A$ on $f(\eta)$ & $\phi(\eta)$ portrait. It is recognized that, both $f(\eta)$ & $\phi(\eta)$ figures dwindle with raising of $A$. As the portrait $A$ ascending, less mass is exchanged from the liquid to the layer; hence, the focus dwindles. Since the liquid stream is caused just by the extending region and the territory surface fixation is bigger than free stream focus, the figure $\phi(\eta)$ dwindle with ascending $\eta$. Also the fluid $f(\eta)$ & $\phi(\eta)$ distributions are more when fluid is away from the wall.

Figures 4(a), 4(b) & 4(c) respectively depict $\phi(\eta)$ profile for distinct values of $\phi$, $Sc$ and $\gamma$. It is recognized that, $\phi(\eta)$ is dwindle with expanding of $\phi$, $Sc$ and $\gamma$. This is because of certainty that the solutal boundary layer thickness enhances with higher values of $\phi$. 

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**Figure 3(a).** Portrait $f(\eta)$ for distant values of $A$

**Figure 3(b).** Portrait $\phi(\eta)$ for distant values of $A$

**Figure 4(a).** Portrait $\phi(\eta)$ for distant values of $\phi$

**Figure 4(b).** Portrait $\phi(\eta)$ for distant values of $Sc$
Figure 4(c). Portrait of $\phi(\eta)$ for distant values of $\gamma$.

Figures 5(a) & 5(b) present the response of $f_w$ and $\phi$ against $A$ on $C_{Re}^{1/2}$ respectively. It is observed that $C_{Re}^{1/2}$ be dwindle of $f_w$ and $\phi$. It is realized that $C_{Re}^{1/2}$ is more in case of nanofluid ($\phi=0.1$) than to the viscous fluid ($\phi=0$). Moreover, $C_{Re}^{1/2}$ is bigger in $Cu-H_2O$ nanofluid related to the $Al_2O_3-H_2O$ nanofluid.

Figure 5(a). Variety of $C_{Re}^{1/2}$ against $A$ for distant values $f_w$.

Figure 5(b). Variety of the $C_{Re}^{1/2}$ against $A$ for distant values of $\phi$.

Figures 6(a) & 6(b) respectively show the reputation of $Sc$ and $A$ on Sherwood number for distinct values of $\gamma$. It is recognized that, the $\theta'(0)$ developed with increasing $\gamma$ as well as $Sc$ and $A$.

Figure 6(a). Variety of $\theta'(0)$ against $A$ for different values of $\gamma$.

Figure 6(b). Variety of $\theta'(0)$ against $Sc$ for distant values of $\gamma$. 
Table 1. Thermos physical goods of the base liquid and diverse nanoparticles. See ref. [26]

| Physical properties | Fluid phase (water) | Cu | Al₂O₃ | TiO₂ |
|---------------------|---------------------|----|-------|------|
| Cₚ(J/kg K)          | 4179                | 385| 765   | 686.2|
| P (kg/m³)           | 997.1               | 8933| 3970  | 4250 |
| k (W/m K)           | 0.613               | 401| 40    | 8.9538|

4 Conclusions
The mass move in insure magnetohydrodynamic stream of nanofluids past an extending layer with concoction response is explored numerically for various kinds of nanoparticles. From the present investigation we found that:

- The portrait \( f' \) distribution reduction with raising the field \( M \) and \( A \).
- The profile \( \phi' \) decrease for \( f_w \), \( Sc \) and \( A \) for three different kinds of nanofluids (like copper, titanium dioxide, aluminium oxide–H₂Onanofluids).
- By reducing the fields \( A \), \( \phi \), \( M \), and suction parameter the \( C_{i,Re}^{1/2} \) decrease in the presence nanofluids.
- The profile \( \phi(0) \) enhances by raising \( \gamma \) for both Cu-H₂O and TiO₂–H₂Onanofluids.

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