Multivariate Distribution in the Stock Markets of Brazil, Russia, India, and China

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Abstract
The purpose of this article is to analyze the dependence between Brazil, Russia, India, and China (BRIC) stock markets, adjusting the multivariate Normal Inverse Gaussian probability distribution (NIG) in 2010–2019 on data yields. Using the estimated parameters, a robust estimator of the correlation matrix is calculated, and evidence is found of the degree of integration in BRIC financial markets during the period 2000–2019. In addition, it is found that the Value at Risk presents a better performance when using the NIG distribution versus multivariate generalized autoregressive conditional heteroscedastic models.

Keywords
dependency, BRIC, multivariate normal inverse Gaussian distribution, stock returns

Introduction
The link among the securities markets of different countries is an important topic that requires analysis. The relevance of this issue is justified by the integration of the markets and the degrees of dependency of the economies, therefore, is important for decisions of investment and economic policy (Singh & Kaur, 2016).

Dependence between financial markets is beyond questioning because interrelation has been a natural process of so-called globalization, at least for the past three decades. Therefore, the transmission of volatilities and financial contagions is almost instantaneous nowadays (Mathur & Dasgupta, 2013).

This document investigates the degree of association that exists between the stock markets of Brazil, Russia, India, and China (BRIC) during the period 2000–2020. The analysis is carried out in three-time windows: 2000–2006, 2007–2009, and 2010–2020, since it is desired to study the degree of statistical dependence and integration before, during, and after the international financial crisis.

The analysis of these countries began in 2000 because around this year, some authors started to use more frequently the BRIC acronym (O’Neill, 2001) to group these emerging economies. Although the nations did not formally assume this term BRICS,—which included South Africa until the annual meeting of the group in 2011,—we will take only Brazil, Russian, India, and China, because of their importance and presence in the world economy (Mueller, 2011).

This document contributes by calculating the degree of association through the multivariate normal inverse Gaussian probability distribution (NIG). This distribution may have different specifications, but it consists of at least four parameters that allow capturing some of the most relevant stylized facts of financial returns, such as bias and high kurtosis (Hu, 2005). In addition, as part of the numerical estimation of the multivariate distribution, the correlations are included in the group of parameters, see Alayón (2015). As it is known, correlations measure the linear dependence between pairs of indexes of the BRIC.

In this way, correlation coefficients between daily returns are estimated with the magnitudes and signs depending on the behavior of the multivariate distribution. Therefore, the coefficients measure more accurately the relationship between returns, conditioned the NIG distribution adequately fits the set of available information. In this case, we used the McAssey (2013) test, which can verify if the multivariate probability distribution fits properly to a random vector.
Under the estimations made, we contribute to the understanding of the dependency patterns among the stock indexes of the BRICs, through the correlations matrix associated with the multivariate NIG distribution, which is different from the sample matrix. This information is undoubtedly useful for fund managers, traders, and economic authorities that make relevant investment decisions (Buraschi et al., 2010).

The article is organized as follows. The “Introduction” section presents a general overview. Section “Literature Review” shows the literature review, and Section “Estimation Procedures” develops the estimation procedure. Next, the “Estimations and Results” section presents the estimates and the results, and the “Conclusion” section shows conclusions, interpretation, limitations of the work, and future lines of research.

**Literature Review**

In portfolio diversification, one of the key elements is the correlation matrix among different assets in a way such that the composition of the portfolio can change dramatically depending on this matrix. Nowadays, in a world interconnected by the financial markets, long- and short-term investments, trade agreements, and many other factors, the linkages are stronger and diversification is more complicated. Moreover, it is necessary to study the dependence or co-movements in the periods before and after an economic or financial turmoil. Over the years, it has been important to measure the relationship, both in the short and long terms, among the stock markets of different countries. For this goal, various studies have used dynamic correlations, regression models, vector autoregression (VAR)/vector error correction (VEC) specifications, Granger causality and members of the autoregressive conditional heteroscedastic (ARCH)–generalized ARCH (GARCH) family to study the relationships in different periods for the BRIC group. For example, Singh and Kaur (2016) developed a VEC model to analyze the link in the BRIC group and the dynamics to the equilibrium. The authors demonstrated the existence of relationships among Brazil, Russia, and China for the periods 2007–2009 of the financial turmoil and afterward and there was a high Pearson correlation between India and Brazil (.923) and between India and China (.588). However, in the whole period from 2004 to 2013, there is no co-movement in the long term of the BRICS as a group, causing difficulties in the diversification of international portfolios.

Friedman and Shachmurove (1997) studied the behavior of the stock markets of the European Community through an autoregressive vector. They found that the stock markets of Great Britain, France, Germany, and the Netherlands are highly related. In contrast, the rest of the countries (smaller markets) present less interconnected and more independent stock markets. The context of this study is an opening period of more interrelated markets with better access to information.

Likewise, Wong et al. (2005) studied the stock markets of India, the United States, the United Kingdom, and Japan through Granger’s co-integration and causality techniques. The argument is that because of the opening of the financial markets in the world, there is a growing importance of the emerging markets in the international scenario. One of the more prominent emerging markets is that of India, and the study focused on its integration in the global markets. The authors found that both the U.S. and Japanese markets cause the Indian stock market, but not the other way around. On the contrary, Yang (2003) examined the links between the government bond markets of the United States, Japan, Germany, the United Kingdom, and Canada for the period 1986–2000 and found no evidence of a long-term relationship.

Frank and Hesse (2009) also used multivariate GARCH (dynamic conditional correlation [DCC]) models to analyze the joint movements during the financial turmoil of 2007 between liquidity and banking solvency measures (LIBOR-OIS and CDS spreads) in developed countries with bond spread, stock returns, and credit markets in emerging economies. The authors found evidence that the correlation between markets was higher in time of financial crisis.

Besides, Chittedi (2014) examined the integration of the stock markets among BRIC and the developed economies of the United States, the United Kingdom, and Japan. The period of analysis was from 1996 to 2011, and the study included the models of DCCs and asymmetric generalized dynamic correlations. The results of the authors confirm that the developed markets have a significant impact on BRIC, although there is an asymmetric relationship between these markets. The correlations among markets augmented during the periods of crisis and post-crisis.

In the study of Ji et al. (2018), they found that Brazil, Russia, and India show significant asymmetric effects in the uncertainty degree among the stock markets and the demand shocks in the international oil markets. It is discovered the existence of dynamic links between exchange rates and the return of the equities in BRICS through the analysis of wavelets. The findings establish that the relationships between exchange rates and returns of the equities are positive in the medium and long term, and it was demonstrated the existence of bidirectional Granger causality.

Visalakshmi and Lakshmi (2016) provided evidence that in the short term, domestic and global destabilizing factors caused different levels of vulnerability in equity markets of BRICs. Still, in the long term, the behavior is homogeneous and stable, with a diversification opportunity for the investors.

Under the fact that the emerging markets have a more profound influence in the international financial system and world economy, Yarovaya and Lau (2016) analyzed the movements of the different stock exchange of emerging markets, including the corresponding to the BRICS and Mexico, Indonesia, South Korea, and Turkey (MIST) economies.
They used cointegration and regime-switching methodologies to find that the conditional correlation among stock exchange markets shows a greater dependence when negative shocks affect the market; in some cases, the authors find contagion, and in other pairs of markets they find strong interconnectedness. One of the objectives was the analysis of the contagion and decoupling among BRICS and MIST since the position of UK investors. The asymmetry of the relations indicates the possibility of diversification and, therefore, is still an attractive market for investors among the emerging markets studied.

Chkili (2016) examined the dynamic relationships between gold and the BRICS stock markets. In the article, an asymmetric DCC model was developed using weekly data and estimating the time variable correlations for the different assets and checked the effectiveness of the gold as a hedging tool for the different equities markets. The empirical results revealed that the dynamic conditional correlations varied between positive and negative values along the period studied, and such correlations are low or negative during the main financial turbulences, which suggests that the gold could be a secure refugee against extreme movements of the markets.

Boubaker and Raza (2017) studied the contagion effects and the shocks between the oil prices and the stock markets of the BRICS using a multivariate VARMA-GARCH-DCC model and analysis of wavelets. Through these methodologies, they described that the Brent and BRICS stock markets are related.

Likewise, the authors evaluated the degree of diversification of the portfolio and the degree of effectiveness of the hedging strategy using gold and equities. Their findings suggest that the measure of return adjusted by risk is better if we add the gold to the portfolio of investments.

For the case of volatility, Bentes (2017) investigated the relationship between implicit volatility and the realized volatility through monthly data of the BRICS, and he evaluated the information contained in the implicit volatility to express the future realized volatility. The models used are the autoregressive distributed lag and the error correction to separate the dynamics of the variables in the short term and long term. The results pointed out that there exists evidence about the presence of effects in the short and long term only for India, and for the case of Russia, there are adjustments in the short term.

Kocaarslan et al. (2017) analyzed asymmetric DCCs and quantile regression to study the dependence structure in the nonlinear and asymmetric interactions of the BRICS group with the United States. The conclusions are that the impacts of the expectations of the volatility and the interdependence among markets are strongly explained through the perceptions of risk, both in the financial and nonfinancial markets.

In addition, Mensi et al. (2014) examined the structure of dependence between the stock exchange markets of the BRICS and different global factors. The authors used the quantile regression for the period between September 1997 and September 2013, and they found a significant asymmetric dependence between the BRICS markets and the global markets and commodities (S&P index, oil, gold, CBOE volatility index). Interestingly enough, there is no significant evidence that the uncertainty of the economic policy of the United States has an impact on the BRICS stock exchange markets.

In a different context, Stavroyiannis (2017) studied the significant fall of the oil prices in 2014, a fact that created a financial turbulence in the emerging markets of the oil exporters. Specifically, an analysis of the shock on the BRICS markets is developed with stochastic volatility and multivariate GARCH models. In both cases, there exists a mean reversion in the behavior of the BRICS markets prices together with an increase of the DCCs, which results in a higher association degree of the BRICS markets.

Similarly, Hammoudeh et al. (2013) described the interrelationship of the BRICS equity markets and their risk profiles according to the International Country Risk Guide. The results in this study showed that China is the only country whose market responded to all the country risk factors and therefore is an important finding about the importance of the Asiatic giant on the BRICS group.

Bouri et al. (2017) studied if the markets’ contemporaneous and lagged volatility of the basic/energy products can help to predict the volatility of the country risk of BRIC in different quantiles. To reach this goal, first defined the latent volatility using the specifications of the GARCH model and then used these specifications in a regression based on quantiles, augmented with a dichotomic variable which identifies the periods before and after the drop of the energy prices in the middle of 2014. The period studied used daily data in the years 2000–2016, which permits us to find that the relationship of volatility between products and CDS is not the same in different conditions of volatility. It is essential to point out that the volatility of the markets of raw materials/energy determines the sovereign risk in the quantiles of the medium and high volatility of the exportation of raw materials/energy (Brazil and Russia).

At the same time, in the case of the importers of raw material/energy, like China, the predictability is significant in the quantiles of high volatility. In every case, there is evidence of a differentiated impact of the volatility, given the interdependence of the BRICS.

Bouri et al. (2018) analyzed the dependence of the implicit volatility shocks of oil and sovereign risk of BRICS from July 2009 to March 2017. First, he studied the indirect effects on the value at risk using quantiles of regression and an impulse-response quantile. The findings are the presence of asymmetry in the transmission mechanisms of the shocks between the oil exporters (Russia and Brazil) and the oil importers (China and India). In the first case, the countries are more sensitive to positive oil impacts, and in the second case, the respective countries are more susceptible to
harmful effects. In addition, there is evidence that a low (high) volatility of the oil market predicts a sovereign risk low (high) in several quantiles and lags. In this analysis, there is no deeper discussion of the consequences of politics. Shahrokhi et al. (2017) studied the globalization process and the complex interaction of the economy with politics. As a consequence, there is an impact on the strength of BRICS agreement with an inhibition of the economic and social prosperity.

The author analyzed the current difficulties of the BRICS, the economic and social environment. As an expected result, the prosperity of the future of this group not only depends on themselves but also on the leading economies of the world. In a global world, there is a connection with many actors. However, it is worth mentioning that this statement does not consider the international financial crisis of 2007–2009. For this reason, Singh and Kaur (2016) analyzed through cointegration techniques the relationship among the stock markets of the BRIC in two periods: 2004–2013 and 2007–2013. In these two periods, there is no evidence of a long-term relationship, although co-movements are observed in the short term for Brazil, Russia, and China.

It is well known that the normal distribution is not adequate for the modeling of the returns of stocks and several alternatives have been proposed in the literature. Some characteristics of the observed data like heavy tails are studied, for example, in Eom et al. (2019). The conclusion in this study is that the heavy tails remain after controlling the effect of the 1997 crash in the Korean market and the cluster of volatility (with GARCH models). An application of the multivariate generalized hyperbolic distribution is found in Scoignia and Wilcox (2014) for seven stocks of the Johannesburg Stocks Exchange. Izzuan and Li (2014), downward and upward movements are studied resulting in two different extreme values distributions, adjusted for these movements of SSE composite index of China. The stocks are in the same sector and the authors encountered evidence in favor of the NIG distribution. Yan and Han (2019) studied the returns distributions of 10 indexes of emerging and developed markets together two individuals from United States and China. Using daily, weekly and monthly frequencies of data, they proposed a three-component mixture of normal distributions for the adjustment.

In a study taking into account the asymmetry of returns of 20 companies in Russian stock exchange (MOEX), Lakshina (2020) used the Taylor expansion of the exponential utility until the third degree. The NIG distribution is used for the residuals of a GO-GARCH model and concluded the superior performance of the three degrees over the two degree for the low levels of absolute risk aversion. The same conclusion is obtained for the forecasted portfolio returns in the period studied. As a recent contribution Choi and Yoon (2020), studied 12 alternatives modeling the distribution of the following indices: HSCEI, KOSPI 200, S&P 500, and EURO STOXX 50. In their risk analysis study, they found that the generalized hyperbolic distribution is the most suitable comparing indices and equity-linked securities.

In this article, we continue with this approach, and we analyze the interrelations among the stock markets of the BRICs for the period 2000–2019, considering three subperiods: 2000–2006, 2007–2009, and 2010–2019. These time windows have been used because it is desired to study the behavior of returns before the international financial crisis of 2007–2009 and to contrast the results with the previous and subsequent periods. The objective is to find evidence of the relationship among these markets, both in the time of crisis and in time of stability, but under a different tool from the traditional techniques of time series that have usually been used in literature.

Specifically, we want to study the correlations among the returns of the stock markets of the BRIC, but under the multivariate probability distribution that best fits the observed data. In this case, the multivariate NIG is proposed as a tool, since it has been observed that it captures the stylized facts of financial returns for emerging economies, Corlu et al. (2016) and Alayón (2015).

In this document, the adjustment of the NIG distribution in the three-time windows studied, the McAssey (2013) test, is applied. The test allows verifying whether a continuous multivariate probability distribution is suitable for a set of information described by a random vector.

The next section briefly describes the NIG distribution, as well as the McAssey hypothesis test (2013) that will be implemented on the daily performances of the BRIC in the time windows 2000–2006, 2007–2009, and 2010–2019.

Estimation Procedures

Throughout literature, multiple references to multivariate normal distribution can be found as an option to describe a random vector. However, it has been found that most of the time, the financial returns present several stylized facts that the normal distribution does not capture or describe adequately (Teräsvirta & Zhao, 2011). In this case, several alternative probability distributions have been proposed, such as Pareto, α-stable, t-Student generalized, generalized hyperbolic, among others (Santos et al., 2012).

Among these distributions, we test a member of the generalized hyperbolic family, called NIG, because it presents flexible statistical properties for modeling data sets (Hellmich & Kassberger, 2011).

In addition, since the aim is to estimate the correlation matrix between the returns, a multivariate GARCH model was estimated to contrast the results against the NIG probability distribution.

In the next two sections, the normal distribution, the NIG distribution, and GARCH models are briefly described. They will be compared with having a point of contrast on the daily returns of the BRICs. In each case, as mentioned above, the
McAssey hypothesis test (2013) will be performed to evaluate the goodness of fit of both density functions adjusted to the random vector associated with the BRIC.

**Multivariate Normal Distribution**

The multivariate normal distribution is a generalization of the univariate probability density function

\[ f(x) = \frac{1}{\sqrt{(2\pi)^p|\Sigma|}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right) \]

where \(\mu\) is the mean and \(\sigma^2\) the variance. So, in the case of \(p\) dimensions

\[ f(x) = \frac{1}{(2\pi)^{p/2} |\Sigma|^p} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right) \quad (1) \]

is the multivariate normal probability density function and is usually written as \(X \sim N(\mu, \Sigma)\), where \(X\) is an embodiment of the random vector \(X\) of size \(p \times 1\), \(\mu\) is the vector of expected values and \(\Sigma\) is the matrix of variance-covariance, that is \(E[X] = \mu\) and \(V[X] = \Sigma\) (Tong, 2012).

Note that this distribution is completely determined by the first and second moments of the random vector \(X\).

**Multivariate NIG**

A vector \(X\) of size \(p \times 1\) is said to follow a multivariate generalized hyperbolic distribution (Hu, 2005) if

\[ X = \mu + W\gamma + \sqrt{WAZ} \]

where

1) \(Z \sim N(0, I)\) is a multivariate normal of order \(k\),
2) \(A\) is a matrix of size \(p \times k\),
3) \(\mu, \gamma\) are vectors of size \(p \times 1\),
4) \(W \geq 0\) is a Generalized Inverse Gaussian random variable (GIG).

The probability density function of \(X\) is

\[ f(x) = \frac{1}{\left(\frac{\psi}{\chi}\right)^{\lambda/2} \left(\psi + \gamma^T\Sigma^{-1}\gamma\right)^{p/2}} \frac{\psi^{\lambda} K_{\lambda}\left(\frac{\sqrt{\psi} \chi}{\sqrt{\psi}}\right)}{(2\pi)^{p/2} |\Sigma|^p K_{\lambda}\left(\frac{\sqrt{\psi} \chi}{\sqrt{\psi}}\right)} K_{\lambda-p/2}\left(\frac{\sqrt{\psi} Q(x)}{\sqrt{\psi} + \gamma^T\Sigma^{-1}\gamma}\right) \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right) \quad (2) \]

where

1) \(Q(x) = (x - \mu)^T \Sigma^{-1} (x - \mu)\) is the distance of Mahalanobis (McAssey, 2013).
2) \(\lambda, \psi, \chi\) determine the shape of the distribution, both in the tails and in the central parts. In addition, in the case of the NIG we have \(\lambda = \frac{1}{2}\) (Paolella, 2007).
3) \(\mu\) is a location parameter and \(\Sigma = A^T A\) is the dispersion matrix, since we have \(E[X] = \mu + E[W]\gamma\) and \(V[X] = V[W]\gamma + E[W]\Sigma\) (Barndorff-Nielsen, 1997).
4) \(K_{\lambda}(x)\) is a third-order Bessel function (Hu, 2005), given by

\[ K_{\lambda}(x) = \frac{1}{2} \int_{0}^{\infty} e^{-\frac{1}{2} t^{\lambda}} dt \quad (3) \]

with \(\lambda \in \mathbb{R}\) and \(x > 0\).

5) The probability density function of \(W\) is

\[ f(w; \lambda, \gamma, \psi) = \frac{1}{k_{\lambda}(\gamma, \psi)} w^{\lambda-1} \exp\left(-\frac{1}{2}(\gamma w^{-1} + \psi w)\right) \quad (4) \]

With \(k_{\lambda}(\gamma, \psi) = 2\eta^2 K_{\lambda}(\alpha)\), \(\eta = \chi / \psi\) \(\gamma, \psi = \sqrt{\chi \psi}\). The parameters \(\lambda \in \mathbb{R}\), \(\gamma \geq 0\) and \(\psi \geq 0\) determine the location and dispersion of \(W\) (Protassov, 2004).

Usually to facilitate the numerical analysis and the calculation of the estimators by maximum likelihood (Lüthi, 2014) the variable change \(\alpha = \sqrt{\chi \psi}\) is defined, such that \(\psi = \alpha K_{\lambda+1}(\alpha)\) and \(\chi = \alpha^2 / K_{\lambda+1}(\alpha)\). This specification will be used in this study, so the parameters to be estimated are \(\alpha, \mu, \gamma\) and \(\Sigma\), since \(\lambda = -0.5\).

The McAssey (2013) test is performed to assess the goodness of fit between a data set and a multivariate continuous probability distribution. The null hypothesis is that the vector follows the proposed distribution. The algorithm follows the steps as presented:

1) We simulate a random sample of size \(N, \{\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_N\}\), using the maximum likelihood estimators that have been found.
2) Then the Mahalanobis distance is calculated for the simulated data, using the sample mean \(\hat{\mu}\) and the sample covariance matrix \(\hat{\Sigma}\):

\[ d_i = \left(\hat{u}_i - \hat{\mu}\right)^T \hat{\Sigma}^{-1} \left(\hat{u}_i - \hat{\mu}\right) \]

3) Then the cumulative empirical distribution over the interval \(\left(0, 2m \hat{d}_i \left\lfloor \hat{d}_i \right\rfloor\right)\) for the Mahalanobis distance is given by

\[ \hat{G}_N(t) = \frac{1}{N} \sum_{i=1}^{N} I(\hat{d}_i \leq t) \]

by means of the indicator function \(I(x)\).
4) A partition \( \{p_0, \ldots, p_T\} \) is built in the interval \([0,1] \) such that \( p_0 = 0, p_T = 1 \) allow to construct \( q_j = 0 \) and \( q_j = \min \{t \in \mathbb{R}; G_n(t) \geq p_j\} \) for \( j = 1, 2, \ldots, T \).

5) The test statistic is calculated

\[
A_T = \sum_{j=1}^{T} \left| \frac{E_j - O_j}{E_j} \right|
\]

With \( E_j = n(p_j - p_{j-1}) \) and \( O_j = \) equal to the number of observations in the interval \( \left[q_j-1, q_j\right] \).

The hypothesis is rejected if the \( p \)-value is less than some level of significance, where the \( p \)-value is calculated on the empirical distribution of the statistics \( A_T \) (McAssey, 2013).

The following presents the estimates made for the three probability distributions indicated, the goodness of fit test, and the corresponding interpretation.

**Multivariate GARCH Models**

Multivariate GARCH models allow to incorporate a flexible and dynamic structure. In this way, an autoregressive vector (VAR) can be estimated for the conditional mean and at the same time, an equation for the variance of each endogenous ARCH-GARCH type variable \( (p,q) \), where \( p \) is the number of lags of the random disturbance and \( q \) is the number of lags of the estimated conditional variance.

In general, the specification is \( y_t = Ax_t + u_t \) with \( u_t \sim B^{1/2}v_t \), where \( v_t \) is a random vector of size \( m \times 1 \) of dependent variables, \( A \) is an array of parameters \( m \times k \), \( x_t \) is a vector of size \( m \times 1 \) of explanatory variables, usually the lags of \( y_t \), \( B^{1/2} \) is the Cholesky factor of the conditional covariance matrix (Bollerslev et al., 1988) over time and \( v_t \) is a random vector, whose elements are white noise.

There are several usual parameterizations (Comte & Lieberman, 2003) for a multivariate GARCH, according to the assumed structure for the conditional covariance matrix. Some relevant models are constant conditional correlation GARCH (GARCH-CCC), GARCH-DCC, and asymmetric DCC GARCH (GARCH-ADCC).

The model with CCC assumes a matrix \( R \) of fixed correlations, invariant in time, such that:

\[
H_t = D_t^{1/2}RD_t^{1/2}
\]

where \( D_t \) is a diagonal matrix of conditional variances (Bollerslev et al., 1988) and the variance equation is given by:

\[
\begin{align*}
    h_{it} &= c_i + a_i u_{it}^2 + b_i h_{it-1} \\
    h_{jt} &= \sqrt{h_i h_j}
\end{align*}
\]

where \( r_{ij} \) is the constant unconditional correlation. On the contrary, GARCH-DCC model assumes that the correlation matrix changes at every instant of time under some dynamic specifications:

\[
R_t = \text{diag}(Q_t^{-1/2})Q_t\text{diag}(Q_t^{-1/2})
\]

where \( Q_t \) is the unconditional covariance of standardized residuals of the univariate GARCH models in each element of the vector of yields and the variance equation is

\[
\begin{align*}
    h_{it} &= c_i + a_i u_{it}^2 + b_i h_{jt-1} \\
    h_{jt} &= q_{jt}\sqrt{h_i h_j}
\end{align*}
\]

Similarly, the model GARCH-ADCC considers asymmetric in conditional correlations weighting by the magnitude and the negative sign of the standardized residuals such that

\[
Q_t = (\tilde{Q} - \alpha \tilde{Q} - \beta \tilde{Q} - \delta E[n_t, n_t']) + \alpha \tilde{e}_{t-1} \tilde{e}_{t-1} + \beta Q_{t-1} + \delta n_{t-1}n_{t-1}'
\]

These specifications are useful for interpreting the association, causality, and volatility between the elements of the random vector.

The following section presents the results of the previous procedures in the case of BRIC.

**Estimations and Results**

The estimations analyze the interrelation among stock indexes of the financial markets of Brazil (BOVESPA), Russia (IOMEX), India (SENSEX), and China (SSEC) in the period 2000–2019. Data set is obtained from the Bloomberg platform, where the natural logarithm has been taken, and the first difference represents the return daily.

Daily returns are taken from January 1, 2000 to December 31, 2019, separating them into three-time windows, which mark the before, during, and after the international financial crisis of 2007–2009. Besides, the year 2000 is taken as the cut-off point, because it is when the BRIC acronym arises informally.

Table 1 shows that the kurtosis is greater than three, the bias is different from zero and that the standard deviation is greater than the average of the returns, which suggests that the time series do not behave like the normal distribution, and that it is verified under the Jarque–Bera, Lilliefors, Cramer–von Mises, and Anderson–Darling tests. These finds are congruent with the usual stylized facts of financial returns (Cont, 2001).

In Annex 1, it can be seen the multivariate normal probability distribution does not fit the set of BRICS returns, since the null hypothesis is rejected. However, it can be stated that there is an underlying probability distribution with stable parameters since the population mean and variance of each time series are constant, as indicated by the unit root tests, where there is evidence of stationarity (see Annex 2).
Subsequently, Annexes 2 and 3 present the estimates of the parameters corresponding to the multivariate NIG distribution and multivariate GARCH models, together with the p-value of the McAssey (2013) test.

Annex 2 shows that there is evidence that the distribution NIG describes more robustly the daily returns in each of the three-time windows, although not necessarily with the same parameters. This can be said because the null hypothesis is not rejected under the McAssey (2013) test.

Table 2 shows the three correlation matrices that have been calculated under the multivariate NIG probability distribution for each of the time windows, and standard errors are in parenthesis. Specifically, according to Barndorff-Nielsen et al. (2012), the standard error of each element of the estimated parameter vector can be expressed as

\[ V[\Theta] = \frac{1}{m} (I^{-1})_{\Theta,\Theta} \]

where \( m \) is the sample size and \( I \) is the covariance matrix from

\[ S_i = \frac{\partial \ln f(x_i;\Theta)}{\Theta}. \]

In the matrices of Table 2, it can be seen that the magnitude of the correlation among the returns of the stock indices of BRIC...
has increased over time. In other words, despite the volatility, there is a higher degree of interdependence. The relationship among the returns of the BRIC is positive and significant, during the financial crisis, the association between India, Brazil, China, and Russia was temporarily higher.

This result agrees with the correlations estimated under the multivariate GARCH-ADCC model, which is the estimate of best fit to BRIC returns according to the Akaike and Schwarz criteria (see Annex 3).

Now, if the Markowitz portfolio is estimated using the correlation matrices obtained using the NIG probability distribution, the multivariate normal, and the GARCH-ADCC model, it can be seen the coefficient of variation is lower for the case of the NIG distribution (see Table 3). That is, a higher return is obtained with lower volatility in the case of the NIG.

Likewise, the GARCH model and the normal distribution underestimate the value at Risk and with a confidence level of 95%, the Kupiec test implies the better performance of the NIG and the GARCH-ADCC model.

Figure 1 presents the comparison between the curves (efficient frontier) for the normal distribution, NIG, and

| Statistic                  | Normal          | NIG             | GARCH-ADCC     |
|----------------------------|-----------------|-----------------|----------------|
| M return                   | 0.027%          | 0.027%          | 0.027%         |
| SD                         | 1.202%          | 1.147%          | 1.167%         |
| Coefficient of variation   | 44.211          | 42.221          | 43.239         |
| Value at risk              |                 |                 |                |
| 90%                       | −1.268%         | −1.297%         | −1.273%        |
| 95%                       | −1.873%         | −1.928%         | −1.867%        |
| 99%                       | −2.768%         | −3.587%         | −3.786%        |
| Test of Kupiec (p-value)   |                 |                 |                |
| 90%                       | 5.061 (0.024)   | 3.341 (0.068)   | 1.854 (0.169)  |
| 95%                       | 2.639 (0.102)   | 2.429 (0.119)   | 3.017 (0.071)  |
| 99%                       | 2.321 (0.093)   | 2.010 (0.156)   | 2.181 (0.147)  |

Source. Own elaboration with data from Bloomberg.

Note. ADCC = asymmetric dynamic conditional correlation; NIG = normal inverse Gaussian; GARCH = generalized autoregressive conditional heteroscedastic.
GARCH-ADCC. Graphically we can see that the estimation of the portfolios using the assumption of NIG distribution is superior considering the trade-off between risk and return.

**Conclusion**

In this article, we estimate the degree of association among the daily stock returns of the BRIC group in three-time windows: 2000–2006, 2007–2009, and 2010–2020. The correlation matrix for each sub-period is obtained by adjusting the multivariate probability distribution NIG, where evidence of a reasonable fit is found under the McAssey (2013) test.

In each of the time windows, the NIG distribution is adjusted appropriately, since the $p$-value of the test is greater than 10% and the null hypothesis is not rejected, although not necessarily under the same parameters. During the financial crisis, the parameter $\alpha$ is smaller, which can be explained by the fact that during this period the four stock indices follow a downward behavior, and it would seem there is less dispersion among them. The relationship among the returns of the stock indices of BRIC is positive and significant although higher during the crisis period.

Furthermore, to complement the analysis, Markowitz’s investment portfolio was estimated considering the correlation matrices of the GARCH-ADCC model, the NIG distribution, and the normal distribution. In this case, it was found that the portfolio associated with the NIG function presents a lower coefficient of variation; that is, it achieved a higher return with lower volatility with respect the other two cases. Likewise, the value at risk is higher, which suggests that the normal distribution and the GARCH-ADCC model underestimate the Value at Risk.

Finally, the interrelation between the stock markets of BRIC has strengthened, and the financial crisis of 2007–2009 contributed to this effect, in this regard, as the correlation is increasing and positive, the possibilities of diversified portfolios decrease.

**Annex 1**

**Multivariate Normal Distribution Test**

| Period      | Statistic’s McAssey | $p$-value |
|-------------|---------------------|-----------|
| 2000–2006   | 1465.68             | .0000     |
| 2007–2009   | 3064.63             | .0000     |
| 2010–2019   | 3313.18             | .0000     |

Source. Own elaboration with data from Bloomberg.

**Annex 2**

**Unit Root Test**

| Variable   | DFA       | PP        | KPSS     | ERS       |
|------------|-----------|-----------|----------|-----------|
| BOVESPA    | $-66.64^{***}$ | $-66.72^{***}$ | 0.11     | 0.19$^{***}$ |
| IOMEX      | $-66.19^{***}$ | $-66.15^{***}$ | 0.18     | 0.02$^{***}$ |
| SENSEX     | $-65.85^{***}$ | $-65.83^{***}$ | 0.17     | 0.03$^{***}$ |
| SSEC       | $-65.51^{***}$ | $-65.58^{***}$ | 0.08     | 0.06$^{***}$ |

Source. Own elaboration with data from Bloomberg. Note. *, **, ***: 10%, 5%, and 1% in level of significance, respectively. DFA = Dickey–Fuller Augmented; ERS = Elliott–Rothenberg–Stock; KPSS = Kwiatkowski–Phillips–Smichdt–Shin; PP = Phillips–Perron.

**Annex 3**

**Multivariate NIG**

| Parameters | BOVESPA            | IOMEX             | SENSEX            | SSEC              |
|------------|--------------------|-------------------|-------------------|-------------------|
| $\mu$      | 0.00017 (0.0010)   | 0.0041 (0.0025)   | 0.0012 (0.0007)   | $-0.0009$ (0.0005) |
| $\alpha$   | 1.3929 (0.7687)    |                   |                   |                   |
| $\gamma$   | $-0.0013$ (0.0007) | $-0.0029$ (0.0016) | $-0.0011$ (0.0006) | $0.0008$ (0.0004) |
| McAssey    | 3183.61            |                   |                   |                   |
| $p$-value  | .1273              |                   |                   |                   |

Source. Own elaboration with data from Bloomberg. NIG = normal inverse Gaussian. Note. Standard error between parentheses.
Table 7. Multivariate NIG Probability Distribution 2007–2009.

| Parameters | BOVESPA | IOMEX | SENSEX | SSEC |
|------------|---------|-------|--------|------|
| µ          | 0.0026(0.0013) | 0.0024(0.0015) | 0.0013(0.0007) | 0.0048(0.0027) |
|           | 0.000477(0.000272) | 0.000229(0.000119) | 0.000003(0.000002) | 0.000057(0.000033) |
|           | 0.000229(0.000119) | 0.000770(0.000454) | −0.000011(0.000006) | 0.000071(0.000036) |
| Δ         | 0.000003(0.000002) | −0.000011(0.000006) | 0.000124(0.000068) | 0.000003(0.000001) |
|           | 0.000057(0.000033) | 0.000071(0.000036) | 0.000003(0.000001) | 0.000499(0.000275) |
| γ         | −0.0019(0.0010) | −0.0021(0.0012) | −0.0005(0.0003) | −0.0038(0.0020) |
| α         | 0.6999(0.4029) | 1994.53 |
| p-value   | .1184 | |

Note. Standard error between parentheses. 
NIG = normal inverse Gaussian. 
Source. Own elaboration with data from Bloomberg.

Table 8. Multivariate NIG Probability Distribution 2010–2019.

| Parameters | BOVESPA | IOMEX | SENSEX | SSEC |
|------------|---------|-------|--------|------|
| µ          | 0.0003(0.0001) | 0.0010(0.0005) | 0.0002(0.0001) | 0.0012(0.0006) |
|           | 0.000244(0.000135) | 0.000072(0.000042) | −0.000002(0.000001) | 0.000024(0.000013) |
|           | 0.000072(0.000042) | 0.000181(0.000199) | 0.000004(0.000019) | 0.000030(0.000016) |
| Δ         | −0.000002(0.000001) | 0.000004(0.000019) | 0.000207(0.000119) | 0.000004(0.000022) |
|           | 0.000024(0.000013) | 0.000030(0.000016) | 0.000004(0.000022) | 0.000193(0.000002) |
| γ         | −0.0003(0.0002) | −0.0007(0.0004) | −0.0001(0.0001) | −0.0012(0.0007) |
| α         | 1.1388(0.6067) | 2694.05 |
| p-value   | .1018 | |

Note. Standard error between parentheses. 
NIG = normal inverse Gaussian. 
Source. Own elaboration with data from Bloomberg.

Annex 4

Multivariate GARCH Models in 2010–2019

Table 9. GARCH CCC, GARCH DCCC and GARCH ADCC.

| Parameters | GARCH CCC | GARCH DCC | GARCH ADCC |
|------------|-----------|-----------|------------|
| µ1         | 0.007*    | 0.006**   | 0.006**    |
| µ2         | 0.039*    | 0.038**   | 0.038**    |
| c11        | 0.003*    | 0.004**   | 0.004**    |
| c22        | 0.012*    | 0.013**   | 0.012**    |
| a11        | 0.294*    | 0.272**   | 0.271**    |
| a22        | 0.043*    | 0.044**   | 0.046**    |
| b11        | 0.495*    | 0.471**   | 0.469**    |
| b22        | 0.901*    | 0.954**   | 0.945**    |
| α          | 0.001**   | 0.034**   |            |
| β          | 0.804**   | 0.967**   |            |
| δ          | −0.048**  |           |            |

Source. Own elaboration with data from Bloomberg. 
Note. *, **, ***: 10%, 5%, and 1% in level of significance, respectively. 
ADCC = asymmetric dynamic conditional correlation; CCC = constant conditional correlations; DCC = dynamic conditional correlation; 
GARCH = generalized autoregressive conditional heteroscedastic.

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References

Alayón, J. L. (2015). Generalized hyperbolic distribution: An application in portfolio selection and quantification of market risk measures. Revista de Economía del Rosario, 18(2), 249–308.

Barndorff-Nielsen, O. E. (1997). Normal inverse Gaussian distributions and stochastic volatility modelling. Scandinavian Journal of Statistics, 24(1), 1–13.
Calibration of multivariate generalized hyperbolic
Hu, W. (2005).

Hellmich, M., & Kassberger, S. (2011). Efficient and robust port-

Hammoudeh, S., Sari, R., Uzunkaya, M., & Liu, T. (2013). The

Bouri, E., Jalkh, N., & Roubaud, D. (2017). Commodity volatility

Bouri, E., Shahzad, S. J. H., Raza, N., & Roubaud, D. (2018). Oil

Bouri, E., & Roubaud, D. (2017). Oil volatility and sovereign risk of BRICS. Energy Economics, 70, 258–269.

Burasi, A., Porchia, P., & Trojani, F. (2010). Correlation risk and optimal portfolio choice. The Journal of Finance, 65(1), 393–420.

Bollerslev, T., Engle, R. F., & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. Journal of Political Economy, 96(1), 116–131.

Chittedi, K. R. (2014). Global financial crisis and contagion: Evidence for the BRIC Economies. The Journal of Developing Areas, 48(4), 243–264.

Chkili, W. (2016). Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries. Research in International Business and Finance, 38, 22–34.

Choi, S. Y., & Yoon, J. H. (2020). Modeling and risk analysis using parametric distributions with an application in equity-linked securities. Mathematical Problems in Engineering, 2020, Article 9763065.

Comte, F., & Lieberman, O. (2003). Asymptotic theory for multivariate GARCH processes. Journal of Multivariate Analysis, 84(1), 61–84.

Cont, R. (2001). Empirical properties of asset return: Stylized facts and statistical issues. Quantitative Finance, 1(2), 223–236.

Corlu, C., Meterelli, M., & Tişcu, T. (2016). Empirical distributions of daily equity index return: A comparison. Expert Systems With Applications, An International Journal, 54(3), 170–192.

Eom, C., Kaizoji, T., & Scalas, E. (2019). Fat tails in financial return distributions revisited: Evidence from the Korean stock market. Physica A Volume, 256, 1–24.

Frank, N., & Hesse, H. (2009). Financial spillovers to emerging markets during the global financial crisis. Czech Journal of Economics and Finance, 59, 507–521.

Friedman, J., & Shachmurove, Y. (1997). Using vector autoregression models to analyze the behavior of the European community stock markets. CARESS Working Paper. 97–04. https://ideas.repec.org/p/cla/penntw/6c418113c19a91c029047e10212054f1.html

Hammoudeh, S., Sari, R., Uzunkaya, M., & Liu, T. (2013). The dynamics of BRICS’s country risk ratings and domestic stock markets, US stock market and oil price. Mathematics and Computers in Simulation, 94, 277–294.

Hellmich, M., & Kassberger, S. (2011). Efficient and robust portfolio optimization in the multivariate generalized hyperbolic framework. Quantitative Finance, 11(10), 1503–1516.

Hu, W. (2005). Calibration of multivariate generalized hyperbolic distributions using the EM algorithm, with applications in risk management, portfolio optimization and portfolio credit risk. https://www.researchgate.net/publication/254671602_Calibration_Of_Multivariate_Generalized_Hyperbolic_Distributions_Using_The_EM_Algorithm_With_Applications_In_Risk_Management_Portalio_Optimization_And_Portalio_Credit_Risk

Izzuin, S., & Li, S. (2014). Dependence structure between oil and other commodity futures in China based on extreme value theory and copulas. The World Economy.

Ji, Q., Liu, B. Y., Zhao, W. L., & Fan, Y. (2018). Modelling dynamic dependence and risk spillover between all oil price shocks and stock market returns in the BRICS. International Review of Financial Analysis, 13(1), 1–12.

Kocaarslan, B., Sari, R., Gormus, A., & Soytas, U. (2017). Dynamic correlations between BRIC and US stock markets: The asymmetric impact of volatility expectations in oil, gold and financial markets. Journal of Commodity Markets, 7, 41–56.

Lakshina, V. (2020). Do portfolio investors need to consider the asymmetry of returns on the Russian stock market? The Journal of Economic Asymmetries, 21, Article e00152.

Lüthi, D. (2014). Package ghyp. The Comprehensive R Archive Network, 1, 1–50.

Mathur, S., & Dasgupta, M. (2013). BRICS: Trade policies, institutions and areas of deepening cooperation. Swati Communications.

McAssey, M. P. (2013). An empirical goodness-of-fit test for multivariate distributions. Journal of Applied Statistics, 40(5), 1120–1131.

Mensi, W., Hammoudeh, S., Reboredo, J. C., & Nguyen, D. K. (2014). Do global factors impact BRICS stock markets? A quantile regression approach. Emerging Markets Review, 19, 1–17.

Mueller, M. (2011). New kids on the block: The rise of the BRIC and the reconfiguration of global economic ties. European Researcher, 12(15), 1615–1625.

O’Neill, J. (2001, October). Building better global economic BRICS (Goldman Sachs Global Economics Paper 66). https://www.goldmansachs.com/insights/archive/building-better.html

Paolella, M. S. (2007). Intermediate probability. John Wiley.

Protsassov, R. S. (2004). EM-based maximum likelihood parameter estimation for multivariate generalized hyperbolic distributions with fixed λ. Statistics and Computing, 14(1), 67–77.

Santos, A. A., Nogales, F. J., & Ruiz, E. (2012). Comparing univariate and multivariate models to forecast portfolio value-at-risk. Journal of Financial Econometrics, 11(2), 400–441.

Shahrokh, M., Cheng, H., Dandapani, K., Figueiredo, A., Parhizgar, A. M., & Shachmurove, Y. (2017). The evolution and future of the BRICS: Unbundling politics from economics. Global Finance Journal, 32, 1–15.

Singh, K., & Kaur, P. (2016). Do BRIC Countries’ equity markets co-move in long run? Theoretical Economics Letters, 6(1), 119–130.

Socgna, V., & Wilcox, D. (2014). A comparison of generalized hyperbolic distribution models for equity returns. Journal of Applied Mathematics, 2014, 1–15.

Stavroyiannis, S. (2017). Is the BRICS decoupling effect reversing? Evidence from dynamic models. International Journal of Economics and Business Research, 13(3), 303–315.
Teräsvirta, T., & Zhao, Z. (2011). Stylized facts of return series, robust estimates and three popular models of volatility. *Applied Financial Economics, 21*(1–2), 67–94.

Tong, Y. L. (2012). *The multivariate normal distribution*. Springer Science & Business Media.

Visalakshmi, S., & Lakshmi, P. (2016). BRICS market nexus for cross listed stocks: A VECX framework. *The Journal of Finance and Data Science, 2*(1), 76–88.

Wong, W. K., Agarwal, A., & Du, J. (2005). Financial integration for india stock market: A fractional cointegration approach. Department of Economics, National University of Singapore.

Yan, H., & Han, L. (2019). Empirical distributions of stock returns: Mixed normal or Kernel density? *Physica A: Statistical Mechanics and its Applications, 514*, 473–486.

Yang, M. (2003). European stock market integration: Does EMU matter? *Journal of Business Finance & Accounting, 30*(9), 1235–1276.

Yarovaya, L., & Lau, M. C. K. (2016). Stock market comovements around the Global Financial Crisis: Evidence from the UK, BRICS and MIST markets. *Research in International Business and Finance, 37*, 605–619.