New electro-acoustic waves in a degenerate quantum plasma system

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The existence of new electro-acoustic (EA) waves [named here ‘degenerate pressure driven EA’ (DPDEA) waves] propagating in a degenerate quantum plasma (DQP) system [containing non-inertial, cold, non-relativistically (NR)/ultra-relativistically (UR) degenerate electron species (DES), and inertial, cold, NR degenerate positive particle species (PPS)] is predicted for the first time. The DPDEA waves, in which the inertia is mainly provided by the mass density of the inertial PPS, and the restoring force is mainly provided by the non-inertial, NR/UR DES, are new since they are completely disappeared if the degenerate pressure of the plasma particle species is neglected. The dispersion relation (derived here for the first time) is applied in a white dwarf DQP system to show the dispersion properties of the new DPDEA waves.

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The degenerate quantum plasma (DQP) systems are significantly different from other plasma systems because of their extra-ordinarily high density and low temperatures. These DQP systems having different unique properties do not only exist in space environments \([1, 2]\), but also in laboratory devices \([3, 4]\). They mainly contain non-inertial, non-relativistically/relativistically degenerate electron and/or positron \([5]\) species \([1, 4]\), and inertial, non-relativistically, non-degenerate/degenerate \(1\text{H}\) or \(1\text{He}\) \([1] \) or \(12\text{C}\) or \(16\text{O}\) \([6]\) or and other heavy particle species \([7, 8]\). The degeneracy of the DQP particle species arises due to Heisenberg’s uncertainty principle, and that the uncertainty in momenta of extremely dense DQP plasma particle species are infinitely large. This introduces a pressure called ‘degenerate pressure’ which depends only on the DQP particle number density, but not on thermal temperature. The degenerate pressure \(\mathcal{P}\) exerted by any DQP particle species, and the corresponding degenerate energy \(\mathcal{E}\) are given by \([3, 11]\)

\[
\mathcal{P} = Kn^\gamma, \quad \mathcal{E} = Kn^{\gamma-1}, \quad (1)
\]

where \(n\) is the number density of any DQP particle species, and \(\gamma\) and \(K\) are defined as

\[
\gamma = \frac{5}{3}, \quad K = \frac{3}{5} \left(\frac{\pi}{3}\right)^{\frac{5}{2}} \frac{\pi^2 \hbar^2}{m} \simeq \frac{3}{5} \Lambda_c \hbar c, \quad (2)
\]

for non-relativistic limit \([3, 11]\), and

\[
\gamma = \frac{4}{3}, \quad K = \frac{3}{4} \left(\frac{\pi}{9}\right)^{\frac{2}{3}} \hbar c \simeq \frac{3}{4} \hbar c, \quad (3)
\]

for ultra-relativistic limit \([3, 11]\), where \(\Lambda_c = \pi \hbar / mc, \hbar\) is the reduced Planck constant, and \(m\) is the rest mass of any DQP particle species. We note that the subscript on the species-dependent parameters and variables will be used \(e\) or \(j\) depending on the DQP particle species \(e\) or \(j\). This degenerate pressure \(\mathcal{P}\) is balanced by either self-gravitational pressure or electrostatic pressure depending on the DQP system under consideration to bring the DQP system into an equilibrium state.

The DQP systems are significantly different first because they have unique properties (extra-ordinarily low density and temperature \([12]\), and next because they may introduce new kind of waves. This work is attempted to find the possibility for the existence of a new kind of waves [called here ‘degenerate pressure-driven electro-acoustic (DPDEA) waves’ in which the inertia is mainly provided by the mass density of the inertial positive particle (PPS) species, and the restoring force is mainly provided by the non-inertial, ultra-relativistically (UR)/non-relativistically (NR) degenerate electron species (DES)], and to identify their dispersion properties in such DQP systems.

We consider a DQP system containing non-inertial DES \([1, 3, 4]\) represented by the subscript \(e\), and inertial, degenerate PPS represented by the subscript \(j\), which can be \(1\text{H}\) or \(1\text{He}\) \([1, 2]\) or and \(12\text{C}\) or \(16\text{O}\) \([6]\) or/and other heavy particles \([3, 4, 6, 8]\). Thus, at equilibrium we have \(n_{e0} = Z_e n_{j0}\), where \(n_{e0}\) (\(n_{j0}\)) is the number density of the DES (PPS) \(e\) \((j)\), and \(Z_e\) is the charge state of the PPS \(j\). The dynamics of the new DPDEA waves, whose phase speed is much smaller than \(C_e\) \([6]\) \[where \(C_e = (\mathcal{E}_{e0}/m_e)^{1/2}\), \(\mathcal{E}_{e0}\) is the electron degenerate energy at equilibrium, and \(m_e\) is the rest mass of an electron\] is described by

\[
\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \quad (4)
\]

\[
\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = -\frac{Z_j e}{m_j} \frac{\partial \phi}{\partial x} + \frac{K_j}{m_j} \frac{\partial}{\partial x} (n_j^{\gamma-1}), \quad (5)
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e \left[ n_{e0} \left( 1 - \frac{e \phi}{\mathcal{E}_{e0}} \right)^{\frac{1}{\gamma-1}} - Z_j n_j \right], \quad (6)
\]

where \(n_j\) (\(u_j\)) is the number density (fluid speed) of the inertial, degenerate PPS \(j\); \(\phi\) is the electrostatic wave potential; \(m_j\) is the mass of the inertial PPS \(j\); \(x\) (\(t\)) is the space (time) variable; \(e\) is the charge of a proton. It may be noted here that the effect of the self-gravitational field is neglected since it is inherently small in comparison with the electric field in the DQP system (e. g. white dwarf DQP system) under consideration. It should be mentioned here that the first term inside the square bracket...
expressed as $j\omega_t$. It may be noted here that $\omega$ is the angular frequency (propagation constant) of the DPDEA waves. This assumption reduces (7) to

$$\omega^2 = \frac{(\gamma_e - 1)k^2C_q^2 + (\gamma_j - 1)k^2C_j^2}{1 + (\gamma_e - 1)k^2L_q^2},$$

where $C_q = (\varepsilon_{j0}/m_j)^{1/2}$ is the DPDEA wave speed, and $L_q = (\varepsilon_{j0}/4\pi n_j e^2)^{1/2}$ is the DPQ screening length. It may be noted here that $\omega_{pj} = C_q/L_q$. To examine the effect of degeneracy in PPS $j$ analytically, (10) can be expressed as

$$\omega = \frac{kC_q\sqrt{\gamma_e - 1}}{\sqrt{1 + (\gamma_e - 1)k^2L_q^2}}(1 + \beta_j)^{1/2},$$

where $\beta_j = (\gamma_j - 1)\varepsilon_{j0}/(\gamma_e - 1)Z_j\varepsilon_{j0}$ represents the effect of the degeneracy in PPS $j$. To visualize the basic characteristics of the new DPDEA waves, the values of $\varepsilon_{j0}$, $C_e$, $C_q$, $L_q$, $\varepsilon_{j0}/\varepsilon_{e0}$, $C_j/C_q$, and $\beta_j$ for the DES density as $n_{e0} = 10^{29}$ cm$^{-3}$, and for the PPS as $^{12}C$, are tabulated in Table I. The limit $\beta_j \ll 1$, which is a good approximation as obvious from Table I, leads (11) to

$$\omega = \frac{kC_q\sqrt{\gamma_e - 1}}{\sqrt{1 + (\gamma_e - 1)k^2L_q^2}}.$$

This is the dispersion relation for the DPDEA waves propagating in a DQP system (containing NR/UR DES, and non-degenerate inertial PPS), and it is valid for both short and long wavelength limits. The short-wavelength (compared to $L_q$) limit ($kL_q \gg 1$) reduces (11) to $\omega = \omega_{pj}$, which is the upper limit of $\omega$, and is independent of the ultra-relativistic effect of the DES. On the other hand, the long wavelength (compared to $L_q$) limit ($kL_q \ll 1$) reduces (11) to $\omega = (\gamma_e - 1)^{1/2}kC_q$. The latter can be expressed for NR and UR DES as

$$\frac{\omega_{nr}}{k} = \sqrt{\frac{2}{3}}\sqrt{\frac{P_{nr}^e}{\rho_{j0}}} = \sqrt{\frac{2\lambda_eehc}{5n_{j0}m_{j}}}$$

(13)

for NR DES, and

$$\frac{\omega_{ur}}{k} = \sqrt{\frac{1}{3}}\sqrt{\frac{P_{ur}^e}{\rho_{j0}}} = \sqrt{\frac{\lambda_eehc}{4n_{j0}m_{j}}}$$

(14)

for UR DES, where $P_{nr}^e$ ($P_{ur}^e$) is the degenerate pressure exerted by NR (UR) DES, $\rho_{j0} = m_j n_{j0}$ is the mass density of the PPS, and $\omega_{nr}$ ($\omega_{ur}$) is the angular frequency of the DPDEA waves for NR (UR) DES. The last two equations mean that i) in new DPDEA waves for the situation of NR (UR) DES, the inertia is provided by the mass density ($\rho_{j0}$) of the inertial PPS, and the restoring force is mainly provided by the degenerate pressure $P_{nr}^e$ ($P_{ur}^e$) exerted by the NR (UR) DES, and ii) $\omega_{ur}/\omega_{nr} \approx [5/(8\lambda_eehc)]^{1/2} \approx 1.05 \approx 1$. This means that for $kL_q \ll 1$, the UR DES does not have any effect on the new DPDEA waves.

We discussed the DPDEA waves for two extreme limits, viz. $kL_q \ll 1$ and $kL_q \gg 1$, in which the UR DES does not have any effect on them. Now to examine the dispersion properties of them in between these two limits, the dispersion relation (12) have been numerically solved. The numerical results are displayed in figure 1 which indicates that the nature of the dispersion curve

TABLE I: Typical values of the physical quantities, $\varepsilon_{j0}$, $C_e$, $C_q$, $L_q$, $\varepsilon_{j0}/\varepsilon_{e0}$, $C_j/C_q$, and $\beta_j$ for the DES density as $n_{e0} = 10^{29}$ cm$^{-3}$, and for the PPS as $^{12}C$.

| Physical quantity | NR DES          | UR DES          |
|-------------------|-----------------|-----------------|
| $\varepsilon_{j0}$ | $8.24 \times 10^{-3}$ | $1.46 \times 10^{-7}$ |
| $C_e$ (cm s$^{-1}$) | $9.50 \times 10^{9}$ | $1.27 \times 10^{10}$ |
| $C_q$ (cm s$^{-1}$) | $1.56 \times 10^{10}$ | $2.10 \times 10^{9}$ |
| $L_q$ (cm)         | $5.32 \times 10^{-10}$ | $7.10 \times 10^{-10}$ |
| $\varepsilon_{j0}/\varepsilon_{e0}$ | $1.37 \times 10^{-5}$ | $7.71 \times 10^{-6}$ |
| $C_j/C_q$          | $1.54 \times 10^{-3}$ | $1.13 \times 10^{-3}$ |
| $\beta_j$          | $2.30 \times 10^{-6}$ | $1.28 \times 10^{-6}$ |

FIG. 1: The dispersion curves of the DPDEA waves for NR (solid curve) and UR (dashed curve) DES, where $L_q$ is for the NR DES.
for the new DPDEA waves is similar to that for the well
known ion-acoustic (IA) waves, and that the UR DES has
insignificant effect on this new kind of DPDEA waves for
both short (compared to $L_q$) and long wavelength limits,
but it has significant effect in between these two limits.
The UR DES enhances the phase speed of the DPDEA
waves in between these two limits. It should be noted
here that the analytical analysis is completely agree with
the numerical analysis displayed in figure 1 for these two
limits. How the DPDEA waves are new, and different
from the IA waves can be pinpointed in Table II, where

TABLE II: The basic differences between the new DPDEA
waves and the well-known IA waves.

| Properties          | DPDEA Waves                  | IA Waves               |
|---------------------|-------------------------------|------------------------|
| Restoring force     | Degenerate pressure           | Thermal pressure       |
| Phase speed         | $\sqrt{\frac{E_{e0}}{m_j}}$  | $\sqrt{\frac{k_B T_e}{m_j}}$ |
| Existence at $T_e=0$| Possible                      | Not possible           |
| Length scale (cm)   | $\sqrt{\frac{E_{e0}}{4\pi n_e e^2}}$ | $\sqrt{\frac{k_B T_e}{4\pi n_e e^2}}$ |

$T_e$ is the electron temperature, and $k_B$ is the Boltzmann
constant. We note that $E_{e0}$, which is the equilibrium
electron degenerate energy defined by [1]-[3], is completely
independent of the thermal energy of any plasma species.
It is obvious from Table II that the DPDEA waves are new
not only from the view of restoring force, but also
from the view of their phase speed and length scale.

To conclude, the new results which have been found
from this investigation can be pinpointed as follows:

1. The existence of a new kind of electro-acoustic
waves (named here DPDEA waves) propagating in
an degenerate plasma system is predicted.

2. The DPDEA waves are new not only from the view
of the restoring force (which is essential for the exis-
tence of any kind of waves), but also from the view
of their phase speed and length scale.

3. The DPDEA waves completely disappear if the
degenerate pressure associated with any plasma
species is neglected.

4. The degenerate pressure associated with the iner-
tial PPS (providing here the inertia in the DPDEA
waves) is negligible compared to that associated
with the electron species (providing here the restor-
ing force in the DPDEA waves).

5. Unsatisfying the condition $kL_q \ll 1 \ll kL_q$, the
phase speed of the DPDEA waves for the situation
of UR DES is more than that in the situation of
NR DES. This is because that the energy of the
UR DES is more than that of the NR DES.

The physics of the DPDEA waves is that if any column
of any DQP medium is perturbed by any reason, i.e. the
column of the medium is compressed (expanded), the
degenerate (electrostatic) pressure tries to bring it back
to its equilibrium shape, but during this action, because
of Newton’s 1st law of motion and of inertial property
of the PPS, it is expanded (compressed) more than its
equilibrium shape, it is then compressed (expanded) by
the electrostatic (degenerate) pressure, but again during
this action, because of Newton’s 1st law of motion and of
inertial property of the PPS, it is expanded (compressed)
more than its equilibrium shape. This process will con-
tinue for infinite time to to produce and sustain the new
DPDEA waves in the DQP system under consideration.
Though the DPDEA wave dispersion relation (derived
for the first time) is applied in white dwarf DQP sys-
tem, it can be applied in any space and laboratory DQP
systems in understanding the basic features of the elec-
trostatic perturbation mode in such DQP systems.

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