High-frequency Quasi-periodic Light Variations from Arc-shaped Gas Clouds Falling onto a Black Hole

Kotaro Moriyama1, Shin Mineshige1, and Hiroyuki R. Takahashi2

1 Department of Astronomy, Kyoto University, Kitashirakawa, Oiwake-Chuo, Sakyo-ku, Kyoto 606-8502, Japan; moriyama@kusastro.kyoto-u.ac.jp
2 Center for Computational Astrophysics, National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan

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Abstract

We investigate the dynamical and radiative properties of arc-shaped gas clouds falling onto a stellar-mass black hole based on the three-dimensional general relativistic radiation-magnetohydrodynamics (3D-GRRMHD) simulation data. Assuming that the gas clouds radiate mainly due to the free–free emission and/or optically thick, inverse Compton scattering, we calculate how the emissivity distributions develop with time. We find that (1) gas clouds, each of which has a ring-like or arc shape, are intermittently formed, and that (2) they slowly fall onto the black hole, keeping nearly the Keplerian orbital velocity. These features support the dynamical properties of the gas clouds assumed in the spin measurement method proposed by Moriyama & Mineshige, but the radius of the inner edge of the accretion disk is larger than that of the marginally stable orbit (ISCO). Next, we examine how each gas cloud is observed by a distant observer by calculating the photon trajectories in the black hole spacetime. The luminosity of the accretion flow exhibits significant time variations on different timescales, reflecting the time evolution of the gas density distributions. The relatively slow variations on the timescales of 0.08–0.10 s is due to the formation and fall of gas clouds, while quasi-periodic flux peaks with short time intervals (0.01 s) are due to the quasi-periodic enhancement of light from the non-axisymmetric arc-shaped clouds through the beaming effect. This may account for the high-frequency quasi-periodic oscillations (HF QPOs) observed in black hole binaries. The observational implications and future issues are briefly discussed.

Key words: accretion, accretion disks – black hole physics – gravitation – magnetohydrodynamics (MHD) – radiative transfer – relativistic processes

1. Introduction

General relativity (GR) is the most widely accepted theory of gravitational fields, and its validity is tested by recent observations of gravitational waves from merging binary black hole systems (Abbott et al. 2016a, 2016b) and from a number of observations in the weak field regime (Will 2006). The observational determination of the black hole spacetime is one of the most important issues of the general relativity theory, since it can lead to the proof of the event horizon and the measurement of components of the metric tensor of the spacetime. It is known in general relativity that the black hole spacetime is completely determined by the only two parameters: a black hole mass, $M$, and spin parameter, $a$, where $a = J/M$ and $J$ is the angular momentum of the black hole (we hereafter take the speed of light, $c$, and the gravitational constant, $G$, to be unity, and neglect the electric charge of black holes, since they do not seem to be important in the astrophysical context). The black hole masses can be relatively easily measured by observing the motions of stars or gas, since the observed targets may not necessarily be close to the black hole (Shahbaz et al. 1999; Ghez et al. 2005; Orosz et al. 2011). However, the measurement of the spin is not easy, since its effect is only detected in the vicinity of the black hole, where full consideration of general relativistic effects is necessary (see Moriyama & Mineshige 2015, hereafter Paper I).

In paper I, we proposed a new method for spin measurement based on the non-periodic flux variation from an infalling gas blob, ring, or arc-shaped blob. There we postulated the following features of the gas cloud: (1) it has a ring or arc shape and is intermittently formed in the innermost region of an accretion disk; (2) it has nearly the Keplerian orbital velocity and slowly falls onto the black hole; and (3) the light variation from the gas cloud is significantly affected by the relativistic effects, such as the focusing effect around the photon circular orbit (see paper I, Section 3.1). By means of global magnetohydrodynamics (MHD) simulations of the gas flow in a pseudo Newtonian potential (Paczynski and Witt 1980), Machida & Matsumoto (2003) have proved the feature (1). They studied the time evolution of a torus and carefully analyzed the simulation results, finding that gas clouds with the spiral shape are intermittently formed and fall onto the black hole from the inner edge of the torus. It is, however, unclear whether the key features (1)–(3) can be reproduced in the general relativistic regime.

In order to examine the realistic behavior of the accretion flow near the black hole, it is essential to take into account a complex and sensitive interaction between general relativistic, radiation, and magnetic fields. This requirement is satisfied by the implementation of the 3D general relativistic radiation-magnetohydrodynamics (3D-GRRMHD) simulation (McKinney et al. 2014; Mishra et al. 2016; Takahashi et al. 2016; Sadowski et al. 2017). We use the latest simulation data of Takahashi et al. (2016) and examine whether the key features assumed in paper I are reproduced.

We analyze the time development of the accretion flow around a stellar-mass black hole ($M = 10 M_\odot$, where $M_\odot$ is the solar mass), and carefully examine how they are observed by a distant observer. Here, we consider the accretion flow from the relatively cold disc ($\geq 10^7$ K), which is truncated near the black hole. We calculate the light variation from the accretion flow using the general relativistic ray-tracing method.

Furthermore, the examination of the accretion flow in the vicinity of the black hole enables us to explore the origin of the
high-frequency quasi-periodic oscillations (HF QPOs; the frequency is \( f_{\text{HF}} = 100-450 \text{ Hz} \)), which have been detected in quite a few black hole low-mass X-ray binaries (Remillard & McClintock 2006). They are thought as the relativistic phenomenon occurring in the innermost part of the accretion disk, since \( f_{\text{HF}} \) is on the same order of the rotation period at the marginally stable orbit. The HF QPOs mainly occur in the very high state (the steep power-law state) with sub-Eddington luminosity, and their frequencies do not shift freely in response to luminosity changes (Remillard et al. 2002; Remillard et al. 2006). In order to explain the observed HF QPOs, a number of models have been proposed (Kato et al. 2001; Rezzolla et al. 2003; Remillard 2005; Kato et al. 2008; Kato et al. 2010), although there is no widely accepted one. By using the GR simulation data, we propose an extended physical model of a simple hotspot one (Schnittman & Bertschinger 2004; Beheshtipour et al. 2016) to explain the HF QPOs.

The plan of this paper is as follows. In Section 2, we describe our methods of calculating the observed flux variation from the simulated accretion flow. In Section 3, we will show the results and inspect the dynamical and radiative assumptions of our gas ring model for the spin measurement made in Paper I. Section 4 is devoted to discussions of the observational implications and future issues.

2. Model and Methods of Numerical Calculations

2.1. Overview of the GRRMHD Simulation

Takahashi et al. (2016) performed the GRRMHD simulation in the polar coordinates \((r_{\text{KS}}, \theta_{\text{KS}}, \phi_{\text{KS}})\) in Kerr–Schild spacetime with the black hole mass of \( M = 10 \, M_\odot \). The numbers of the numerical grid points are \((N_r, N_\theta, N_\phi)_{\text{KS}} = (264, 264, 64)\), and the computational domain consists of \( r_{\text{KS}} = [r_H, 250M] \), \( \theta_{\text{KS}} = [0, \pi] \), and \( \phi_{\text{KS}} = [0, 2\pi] \), where \( r_H = M + \sqrt{M^2 - a^2} \) is the radius of the event horizon. They start the simulation from the equilibrium torus given by Fishbone & Moncrief (1976).
are the gas density and electron temperature of the envelope, respectively. We note that the other elements remain constant. We postulate that photons do not undergo the electron scattering in $i < 60^\circ$, since $\tau_{es} < 1$. Here, the radius of the envelope is set to be $20M$, and $\rho_{\text{env}}$ and $T_{\text{env}}(=10^9 \, \text{K})$ are the gas density and electron temperature of the envelope, respectively.

$$V_0 = \sqrt{q - \cos^2 \theta (-a^2 + \lambda^2 / \sin^2 \theta)}.$$  \hfill (4)

Here, $\Delta = r^2 - 2Mr + a^2$, $u^\mu$ is the four-velocity of the gas element, and $\Lambda$ and $q$ are angular momenta of a ray with respect to the $\phi$ and $\theta$ directions per unit energy, respectively. Note that $\Lambda$ and $q$ satisfy

$$\Lambda = -x_{\text{obs}} \sin i,$$  \hfill (5)

$$q = y_{\text{obs}}^2 - a^2 \cos^2 i + \lambda^2 \cot^2 i,$$  \hfill (6)

along a ray that reaches a point $(x_{\text{obs}}, y_{\text{obs}})$ on the observer’s plane (Cunningham & Bardeen 1973). Here, $i$ is the inclination angle, and $x_{\text{obs}}$ and $y_{\text{obs}}$ are Cartesian coordinates on the observer’s plane. Further, the $x_{\text{obs}}$-axis is parallel to the equatorial plane of the black hole, and the $y_{\text{obs}}$-axis is perpendicular to the $x_{\text{obs}}$-axis (see Figure 1).

The actual calculation procedures are as follows.

1. Following Paper I (see Section 2.4), we calculate the total intensity of rays, $I = I(x_{\text{obs}}, y_{\text{obs}}, t)$, that reach each cell at the coordinates of $(x_{\text{obs}}, y_{\text{obs}})$ at the observer’s time of $t$ (see Figure 1). We note that Equations (7)–(9) in Paper I must be replaced by Equations (1)–(4).

2. We calculate the local flux reaching one cell at $t$, $df = I(x_{\text{obs}}, y_{\text{obs}}, t) \, ds_{\text{obs}} / 4\pi D^2$, where $ds_{\text{obs}}$ is the area of the cell, and the distance between the center of the observer’s plane and black hole, $D$, is set to be $1000M$.

3. In order to obtain the flux, $f(t)$, we integrate $df$ over the entire observer’s plane at each observer’s time, $t$,

$$f(t) = \frac{1}{4\pi D^2} \int I(x_{\text{obs}}, y_{\text{obs}}, t) \, ds_{\text{obs}}.$$  \hfill (7)

4. We define the normalized flux, $F(t) \equiv f(t) / f(t_{\text{max}})$, where $t_{\text{max}}$ denotes the time when $f(t)$ reaches its maximum.

In order to analyze the time variation of the gas accretion, we need to examine the emissivity distribution on the equatorial plane. We define the vertical average of the emissivity as

$$\overline{f}(x, y, t) = \frac{1}{4M} \int_{-2M}^{2M} f(t, x, y, z) \, dz,$$  \hfill (8)

where $x, y$, and $z$ are Cartesian coordinates transformed from the Boyer–Lindquist ones. Further, we neglect the radiation from the region of $|z/M| > 2$, since the emissivity in the region is $10^{-4}$ times smaller than the maximum emissivity at each time.

2.3. Free–Free Model: Contribution of Temperature Distribution

In the case of the simple model, we assume a constant and uniform electron temperature, $T_e = 10^7 \, \text{K}$. In order to investigate the impacts of this assumption on the results, we construct another model by considering the temporal and spatial variations of the temperature (free–free model). We assume a one-temperature plasma in which the electron temperature is equal to the ion temperature obtained by the GRRMHD simulation, and consider the free–free emission, i.e., the emissivity is proportional to $\rho^2 T_e^2 / 2$. We note that the other approximations and numerical procedures are the same with those of the simple model.

2.4. Scattering Model: Consideration of Inverse Compton Scattering

We finally construct a scattering model by considering the light variation under inverse Compton scattering and free–free absorption in the case of the non-rotating black hole. The electron-scattering optical depth, $\tau_{es}$, and effective optical depth, $\tau_{\text{eff}}$, are expressed as

$$\tau_{es} = \int_z^{z_{\text{max}}} \rho \kappa_{es} \, dz,$$  \hfill (9)

$$\tau_{\text{eff}} = \int_z^{z_{\text{max}}} \rho \sqrt{\kappa_{es} + \kappa_{\text{ff}}} \, dz,$$  \hfill (10)
where $\kappa_{\text{sc}} = 0.4 \text{ cm}^2 \text{ g}^{-1}$, $\kappa_{\text{fl}} = 6.4 \times 10^{22} \rho T_\text{e}^{-1.5} \text{ cm}^2 \text{ g}^{-1}$, and $z_{\text{max}} (= 20M)$ are set to be the maximum height of the radiation region. In order to construct the scattering model, we show in the left panel of Figure 2 the spatial distribution of the gas density averaged over $0 \leq \phi < 2\pi$ and $0.12 \leq t \leq 0.23$ s. There is the high-density gas cloud ($\rho = 10^{-3} - 10^{-5} \text{ g cm}^{-3}$) in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Time development of the spatial distribution of the vertically averaged emissivity, $f(x, y, t)$, near the non-rotating black hole. The color represents the logarithm of $f(x, y, t)$ normalized by the absolute maximum one, which is found at $(x, y) = (-5.9M, -7.1M)$ at the time of $t = 0.16$ s. The origin is set to be at the center of the black hole, the white circle indicates the event horizon, and the gas cloud is rotating in a counterclockwise direction. The observer’s time, $t(\text{s})$, is shown in the upper-left corner of each panel.}
\end{figure}
the region of \(|z| \lesssim 2M\), while the low-density gas envelope \((\rho \lesssim 10^{-5} \, \text{g cm}^{-3}\)) exists in the region of \(|z| \gtrsim 2M\). The solid curve in the left panel of Figure 2 plots the photosphere, where \(\tau_{\text{es}} = 1\), and roughly corresponds to \(i = 60^\circ\). In the region \(i \lesssim 60^\circ\), \(\tau_{\text{es}}\) is smaller than 1, since there is the high-temperature outflow and its gas density is much smaller than that of the gas envelope. Thus inverse Compton scattering mainly occurs at \(i \gtrsim 60^\circ\). From these properties of the accretion gas flow, we construct the following model of the gas clouds and envelope (see the right panel of Figure 2).

1. The electron temperature of the gas cloud region \((|z|/M \ll 2)\) is constant as \(T_{\text{cloud}} = 10^7 \, \text{K}\).
2. Gas clouds distribute to the equatorial plane and the initial intensity is given by \(I_0 \propto \ell\).
3. The soft photon is isotropically emitted from the gas cloud region with the energy, \(k_B T_{\text{cloud}}\), where \(k_B (=1.38 \times 10^{-16} \, \text{erg K}^{-1})\) is the Boltzmann’s constant.
4. In the region of \(|z| > 2M\) and \(i \gtrsim 60^\circ\), there is the static gas envelope with the gas density, \(\rho_{\text{env}} (=10^{-5} \, \text{g cm}^{-3})\), and electron temperature, \(T_{\text{env}} = 10^9 \, \text{K}\) (Sadowski \\& Narayan 2015).
5. In the region \(i < 60^\circ\), the radiation does not undergo the electron scattering, since \(\tau_{\text{es}} < 1\).

We note that the free–free absorption is neglected in the envelope region, since the free–free absorption \(\tau_{\text{ff}} \lesssim 10^{-7}\) is much smaller than 1, where we set \(z = -20M\) as the bottom height of the radiation region. We approximately calculate the observed intensity, \(I\), affected by inverse Compton scattering process (Rybicki \\& Lightman 1979; Kubota \\& Done 2004) as

\[
I = g^4 I_0 e^y,
\]  

where \(y = 4kT_{\text{env}} \max(\tau_{\text{scat}}, \tau_{\text{scat}}^2)/(m_e c^2)\) is the Compton \(y\) parameter, \(m_e\) is the electron mass, \(\tau_{\text{scat}}\) is the electron-scattering optical depth along the ray trajectory, and \(g\) is the energy-shift factor of the gas cloud (Equation (2)).

The procedure of the numerical calculation is as follows.

1. In the comoving frame of the gas cloud located at \((x, y)\) at the time of \(t_i\) \((0.12 \leq t_i \leq 0.23 \, \text{s})\), we calculate the trajectories of the rays using the symplectic method.
2. We calculate the electron-scattering optical depth, \(\tau_{\text{scat}}' = \int \rho v_{\text{es}} d\ell\), along a ray trajectory, where \(d\ell\) is the infinitesimal spatial interval of the ray in the Boyer–Lindquist coordinates. At the position where \(\tau_{\text{scat}}'\) reaches 1, we reselect the direction of the ray using an uniform random number, assuming isotropic emission in the comoving frame of the gas element. Then we take \(\tau_{\text{scat}}' = 0\), and calculate \(\tau_{\text{scat}}''\) along the new ray trajectory.
3. If the ray reaches the observer’s plane, we estimate the traveling time, \(t_i\), and other quantities, \(\max(\tau_{\text{scat}}, \tau_{\text{scat}}^2)\), \(g\), and \(I_0\), where \(\max(\tau_{\text{scat}}, \tau_{\text{scat}}^2)\) is given by the number of the scattering of the ray. We substitute them into Equation (11), and calculate the observed intensity emitted from the gas element located at \((x, y)\), \(d\ell(x, y, t)\), where \(t = t_i + t_i - D\), and \(D = 1000M\) is the distance between the center of the observer’s plane and the black hole.
4. We perform the upper procedure to all of the elements of gas clouds the whole time, and then obtain the total intensity, \(I_{\text{tot}}(t)\), by superposing each intensity profile, \(d\ell(x, y, t)\). By replacing the term of “\(I\)” in Equation (7) with “\(I_{\text{tot}}(t)\),” we obtain the observed energy flux, \(f(t)\).
5. Further, we calculate the Compton \(y\) parameter, and then we estimate the \(y\) parameter averaged over the all rays.
reaching the observer’s plane, \( \gamma_{av} \), where we use \( dI(x, y, t) \) as the weighted function.

### 3. Results

#### 3.1. Results of the Simple Model: Overall Behavior of the Accretion Flow (\( a/M = 0 \))

In order to understand the time development of the accretion flow around the non-rotating black hole, in Figure 3 we show the time evolution of the spatial distribution of the vertically averaged emissivity during \( t = 0.12-0.23 \) s, where \( t \) is the observer’s time and is shown in the upper-left corner of each panel. The color represents the logarithm of \( \tilde{f}(x, y, t) \) normalized by the absolute maximum one, \( \max [\tilde{f}(x, y, t)] \), which is found at \( (x, y) = (-5.9M, -7.1M) \) at the time of \( t = 0.16 \) s.

We confirm that arc-shaped gas clouds are intermittently formed, and fall onto the black hole in the following.

1. At \( t = 0.12 \) s, an arc-shaped gas cloud is formed at \( R/M = 14 \), and gradually falls to the black hole. At \( t = 0.13 \) s, the gas cloud grows and separates into two arc-shaped gas clouds \((C_1 \text{ and } C_{\text{transient}})\), and a low-emissivity gas cloud, \( C_2 \), is formed outside of \( C_{\text{transient}} \) (see Figure 4(a)).

2. During \( 0.13-0.16 \) s, \( C_{\text{transient}} \) is captured by \( C_1 \). In addition, low-emissivity gas clouds, \( C_{\text{low}} \), are formed around \( C_1 \) at \( t = 0.16 \) s (see Figure 4(b)) and are captured by \( C_1 \) and \( C_2 \) during \( 0.16-0.20 \) s.

3. At \( t = 0.19 \) s, the emissivity of \( C_1 \) decays, while that of \( C_2 \) increases. During \( t = 0.20-0.23 \) s, \( C_2 \) falls onto the black hole.

The density fluctuation of \( C_1 \) \((C_2)\) occurs during \( 0.12-0.21 \) \( (0.13-0.23 \) s, and so the lifetime is \( 0.09 \) \((0.10) \) s. During \( 0.13-0.18 \) \((0.20-0.23) \) s, the major radiation is emitted from \( C_1 \) \((C_2)\) which slowly falls onto the black hole.

We show in Figure 5(a) the light curve of the accretion flow for the case with \((a/M, i) = (0, 60^\circ)\). The flux variation can be divided into two stages.

1. After the formation of the first gas cloud at \( t = 0.12 \) s, the flux first increases until the flux maximum is at \( t = 0.147 \) s with the growth of \( C_1 \).

2. The flux then decays during \( 0.15-0.20 \) s, since \( C_1 \) falls onto the black hole and its emissivity decreases. The flux gradually decreases during \( 0.20 \leq t \leq 0.23 \) s, since \( C_2 \) falls onto the black hole.

In order to understand the dynamical properties of the arc-shaped gas clouds, we calculate how the radial coordinates and velocity components, \( v^\phi (=dx/dt) \), of the gas clouds vary with time. Here, we focus on the arc-shaped gas clouds, \( C_1 \) and \( C_2 \), and do not examine further properties of \( C_{\text{transient}} \) and \( C_{\text{low}} \) in this study, since their formations are transient. We define the following average of the physical quantity, \( G(=R, v^\phi) \), as

\[
\langle G \rangle = \int Gd^3x / \int d^3x,
\]

where we perform the volume integration over \( R_s - 2M \leq R \leq R_s + 2M, -2M \leq z \leq 2M, \) and \( 0 \leq \phi \leq 2\pi \). Further, \( R_s(t) \) is defined as the radial position where \( \tilde{p}(R, t) = \int \int \rho(R, \phi, z)Rd\phi dz / \int \int Rd\phi dz \) reaches its maximum at time, \( t \); that is, it represents the radial position of the arc-shaped gas cloud.

We show in Figures 5(b)-(d) the time developments of \( R \) and \( v^\phi \), where \( v_K[=M^{1/2}/(r^{3/2} + aM^{1/2})] \) is the Keplerian orbital velocity, and the solid (or the dashed) curve in each panel indicates the value of \( C_1 \) \((C_2)\).

The time variations of the radial positions of the gas clouds, \( \langle R \rangle \), indicate that the gas clouds slowly fall onto the black hole (Figure 5(b)). During \( 0.12-0.19 \) s, \( \langle R \rangle \) of \( C_1 \) gradually decreases, since \( C_1 \) slowly falls onto the black hole. During \( 0.19-0.23 \) s, \( C_2 \) grows and then falls onto the black hole, and so \( \langle R \rangle \) of \( C_2 \) decreases. From Figure 5(c), we also find that the radial velocities of gas clouds, \( \langle v^\phi \rangle \), are negative and small \((-0.01 < \langle v^\phi \rangle < 0.001)\); that is, each cloud gradually falls onto the black hole (see Figure 5(c)). Remarkably, Figure 5(d) proves that the azimuthal angular velocities are very close to \( v_K \) \((0.994 < \langle v^\phi/v_K \rangle < 1.005)\), while the polar angular velocities
are smaller than one percent of $v_K$ ($-0.006 < \langle v/v_K \rangle < 0.010$). Therefore each gas cloud has nearly the Keplerian orbital velocity, but slowly falls to the black hole with a small radial velocity. These features justify the basic assumptions made in Paper I.

### 3.2. Short-term Light Variation of the Arc-shaped Gas Cloud (a/M = 0)

In addition to the slow variations, the light curve has many peaks with short time intervals (see Figure 5(a)). In order to understand the physical origin producing such peaks, we focus on the typical flux peak (i) and valley (ii) in panel (a1) of Figure 6. We show the snapshots of the emissivity images at the time of the peak (i) and valley (ii) in Figures 6(a2) and (a3), respectively. The color represents the logarithm of the relative brightness of the gas cloud image, which is located at $(x_{\text{obs}}, y_{\text{obs}}, t) = (-11.3M, 0.3M)$ at the time of 0.147 s. Panels (b1)–(b3) are the same as panels (a1)–(a3), but for the low inclination angle case, $i = 20^\circ$, where the maximum local flux, $\max(df(x_{\text{obs}}, y_{\text{obs}}, t))$, is located at $(x_{\text{obs}}, y_{\text{obs}}) = (-11.3M, 0.3M)$ at the time of $t = 0.147$ s.

Panels (a2) and (a3) show that there are three bright regions due to the beaming effect from $C_1$, $C_2$, and $C_{\text{transient}}$. We note that the dominant radiation is emitted from the innermost region corresponding to $C_1$. During 0.13–0.19 (0.20–0.23) s, the major radiation is emitted from $C_1$ ($C_2$) (see Section 3.1), and so the averaged peak interval of $C_1$ ($C_2$) is $1.0 \times 10^{-2}$ ($6.6 \times 10^{-3}$) s.

We also examine the low-$i$ case (say, $i = 20^\circ$), and show the results in panels (b2) and (b3). We find similar tendencies but the variation amplitudes are smaller than in the case of $i = 60^\circ$. This can be understood, since it shows that the peak amplitudes are mainly determined by the beaming effect (see panels (a1) and (b1)). Panels (b2) and (b3) also show that the radiation from arc-shaped gas clouds is Doppler boosted (de-boosted).

### 3.3. Spiral Gas Clouds in the High-spin Case (a/M = 0.9375)

We show in Figure 7 the time evolution of the spatial distribution of the vertically averaged emissivity in the high-spin case (a/M = 0.9375). We also confirm that arc-shaped gas clouds are intermittently formed and fall onto the black hole in the following.

1. At $t = 0.11$ s, first arc-shaped gas cloud, $S_1$, is formed at $R/M = 14$, and gradually falls onto the black hole. At $t = 0.15$ s, other gas clouds ($S_{\text{transient}}$ and $S_2$) are formed outside $S_1$ (Figure 8(a)), and then $S_{\text{transient}}$ is captured by $S_1$ at 0.16 s (Figure 8(b)).
During 0.16–0.19 s, there are three arc-shaped gas clouds ($S_1, S_{low}$, and $S_2$, see Figure 8(b)), and $S_1$ is the brightest gas cloud.

The entire time the major radiation is emitted from $S_1$ and its emissivity decreases after 0.16 s. The density fluctuation of $S_1$ occurs during 0.11–0.19 s, and so its lifetime is 0.08 s.

Next, we show in Figure 9(a) the light curve of the accretion flow in the case of $i = 60^\circ$. The flux variation can be divided into two stages:

1. after the formation of $S_1$ at $t = 0.11$ s, the flux first increases until the flux maximum at $t = 0.154$ s with the growth of $S_1$;
2. the flux then decays during 0.154–0.19 s, since $S_1$ falls to the black hole and its emissivity decreases.

The time developments of the radial coordinate and $v^\phi$ of the arc-shaped gas cloud, $S_1$, have similar behaviors of those for the non-rotating black hole case (Figures 9(b)–(d)). Here, we analyze the dynamical property of $S_1$, and do not examine further properties of $S_2, S_{transient}$, and $S_{low}$, since their formations are transient, and the major radiation is emitted from $S_1$.

During $t = 0.11–0.19$ s, $\langle R \rangle$ of $S_1$ gradually decreases, and the radial velocity, $\langle v^r \rangle$, is negative and small ($-0.005 < \langle v^r \rangle < -0.001$), since it gradually falls to the black hole (see Figures 9(b) and (c)). We show in Figure 9(d) that the azimuthal angular velocity is very close to the Keplerian orbital velocity including the spin dependency, $0.997 < \langle v^\phi / v_K \rangle < 1.004$, while the polar angular velocity is smaller than one percent of the Keplerian one ($-0.005 < \langle v^\theta / v_K \rangle < 0.002$). Therefore, $S_1$ has nearly the Keplerian orbital velocity, but slowly falls to the black hole with a small radial velocity.

As is the case of the non-rotating black hole, the light curve has many peaks with short time intervals, and its origin is the beaming effect of $S_1$. The averaged peak interval is $9.7 \times 10^{-3}$ s.

**Figure 7.** The same as Figure 3, but for the high-spin case, $\alpha/M = 0.9375$. Here, $J(x, y, t)$ reaches its absolute maximum at $(x, y, z, t) = (5.5M, 9.9M, 0.16s)$.

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2. During 0.16–0.19 s, there are three arc-shaped gas clouds ($S_1, S_{low}$, and $S_2$, see Figure 8(b)), and $S_1$ is the brightest gas cloud.
In order to confirm this fact, in Figure 10 we show the light curve around the typical peak (i) and the snapshots at the time of the peak (i) and valley (ii), where $t = 60^\circ$ (panels (a1)–(a3)) or $20^\circ$ (panels (b1)–(b3)). By carrying out the same discussion for the non-rotating case (see Section 3.2), we conclude that the physical origin producing the peaks is the beaming effect of the non-axisymmetric gas cloud, and that the peak amplitudes are mainly determined by the beaming effect. The dominant radiation at the time of peak is emitted from $S_1$, and the other radiation from $S_{\text{transient}}$ and $S_{\text{low}}$ is smaller than that from $S_1$ (panel (a2) and (b2)).

3.4. Results of the Free–Free Model: Temperature Dependence of Gas Clouds

In the case of the simple model, we assume the constant electron temperature of $T_e = 10^7$ K. Next, we investigate how the dynamical and radiative properties of arc-shaped gas clouds depend on the temperature distribution using the free–free model (Section 2.3). We show in Figure 11 the spatial distribution of the emissivity and light curve of the free–free model (upper panels) and those of the simple one (lower panels), where we fix $a/M, i = (0, 0), 60^\circ$. We show in panel (a1) the emissivity distribution of the free–free model at the time of $t = 0.16$ s, and confirm that the shape of the arc-shaped gas cloud is similar to that of the simple one (this property is satisfied in the whole time, see, Figure 12). In addition, we compare panel (a1) with (b1) and find that the low-emissivity atmosphere around the arc-shaped gas cloud is thicker than that of the simple model, since the hot gas atmosphere exists around the gas cloud. Therefore, in the case of the free–free model, the whole variation of the emissivity of the gas clouds is gentler than that of the simple model. In order to show the contribution of the feature to the radiation property, we compare the light curve of the free–free model and that of the simple one (see panels (a2) and (b2)). At the time of $0.18 < t < 0.23$ s, the overall flux variation of the free–free model is slower than that of the simple one, and the amplitudes of the short-term light variation are larger. As a result, we confirm that the dynamical and radiative properties of the gas clouds are not significantly altered from those of the simple model.

3.5. Results of the Scattering Model: Contribution of Inverse Compton Scattering

We investigate the behavior of the short-timescale variation of the flux in the case of the scattering model. In Figure 13 we show the flux variation for the non-scattering model ($(\rho, y_{\text{en}}) = (0, 0)$, see panel (a)), the unsaturated-Compton scattering model ($(\rho, y_{\text{en}}) = (1 \times 10^{-7}, 0.3)$, see panel (b)), and the critical-saturated Compton scattering model ($(\rho, y_{\text{en}}) = (4.3 \times 10^{-7}, 6.0)$, see panel (c)). Here, $\chi_{\text{crit}}$ is the critical $\gamma$ parameter for the saturated Comptonization (see problem 7.1 in Rybicki & Lightman 1979):

$$\chi_{\text{crit}} = \ln\left(\frac{4T_{\text{env}}}{T_{\text{cloud}}} \right) = 6.0. \quad (13)$$

Actually, Equation (11) is invalid in such a critical case, but we can examine the qualitative feature of the light variation. We first confirm that the light curve of the non-scattering model has the same feature as that of the simple one (see panel (a) of Figure 5), and so we demonstrate the validity of the scattering calculation method. Next, we plot the light curve in the case of the unsaturated-Compton scattering model (panel (b)), and find that the relativistic light variation can be seen. The flux variation due to the beaming effect is gentler than the non-scattering model, since the electron scattering process spreads the peak width of the relativistic flux variation. Even in the case of the critical saturated Compton scattering ($\chi_{\text{en}} = 6.0$), the peak structures are marginally seen (see Figure 13(c)).

![Figure 8](image_url) Characteristic arc-shaped gas clouds for the high-spin case, $a/M = 0.9375$. The definition of the color is the same as that of Figure 7. Three arc-shaped gas clouds ($S_1$, $S_{\text{transient}}$, and $S_2$) are formed at $t = 0.15$ s. At $t = 0.16$ s, $S_{\text{transient}}$ has been captured by $S_1$, and low-emissivity gas clouds $S_{\text{low}}$ are formed.
conclusion, we find that the fluctuation of the relativistic radiation is not grossly altered, and may be observed even in the case that the inverse Compton scattering affects.

4. Discussion and Future Issues

4.1. Reproduction of the Ring-model Features

In this paper, we have confirmed that the accretion flow has the following features near the black hole by using the 3D-GRRMHD simulation data.

1. Arc-shaped gas clouds are intermittently formed near the inner edge of the accretion disk \( (R/M = 14 - 16; \text{Figures 3 and 7}) \).
2. The clouds have nearly the Keplerian orbital velocity and slowly fall onto the black hole \( \text{(Figures 5(b)-(d), and 9(b)-(d))} \).
3. The rotational velocities of the clouds do not depend on the spin value so much. This can be understood as follows. In the denominator of the rotational velocity \( \left[ \omega = \omega_K = \frac{M^{1/2}}{\sqrt{r^3/2 + aM^{1/2}}} \right] \), the factor depending on the spin, \( aM^{1/2} \), is much smaller than that of the other factor, \( r^{3/2} \), at \( r/M \approx 10 \).
4. The light curve has many peaks with short time intervals \((0.01 \text{ s})\) due to the beaming effect of the non-axisymmetric arc-shaped gas clouds and long time duration \((0.08-0.10 \text{ s})\) due to the density variation.

These features are consistent with the key assumptions of our ring model (Paper I). Therefore, these results provide support to the situation postulated in Paper I. We should note, however, that inverse Compton cooling is not considered in the simulation. Because of the lack of the cooling process, the gas temperature, \( T_{\text{gas}} \), is higher than \( 10^{10} \text{ K} \) at \( R/M < 8 \), and so the arc-shaped gas cloud may be dispersed at the region due to the pressure of the high-temperature atmosphere. In order to understand the accurate behavior of the arc-shaped gas cloud in \( R/M < 8 \), we need to incorporate the inverse Compton effect to the GRRMHD simulation.

In order to justify the black hole spin measurement proposed in Paper I, we need to separate the relativistic flux variation from the fluctuation due to the density variation. Therefore, we must examine whether the light variation due to relativistic effects is distinguished from that due to the density variation, by performing the simulation for a much longer time during which plenty of gas clouds are successively formed.

4.2. Observational Implications: Shot Analysis

In Paper I, we argue that the radiation from the gas ring may correspond to the X-ray shots that are flare-like light variations with sharp peaks. X-ray shots are flare-like light variations and observed during the low/hard state, whose spectra is characterized by a power-law profile (see, e.g., Done et al. 2007). Negoro et al. (1994) investigated the variabilities using the technique of superposed shots, adding plenty of shot profiles by aligning their peaks (called superposed shot analysis). It is known that the superposed shot profile of Cyg X-1 can well be fitted with the sum of two exponential functions: one with the time constant of \( \sim 0.01 \text{ s} \) and another of \( \sim 0.1 \text{ s} \) (see Paper I and references therein). By superposing light curves of the arc-shaped gas clouds (with different initial phases), we expect that the superposed light curve is equivalent to those of a gas ring (see Paper I, Section 5.4). In the next study, we must calculate the time development of the superposed light curve by performing the long-term simulation, and examine the relationship between the short-timescale variations \((0.01 \text{ s})\) due to relativistic effects and the observational smaller time constant \((\sim 0.01 \text{ s})\).

Here, we note that the mass accretion rate of the arc-shaped gas clouds, \( \langle \dot{M}_{\text{arc}} \rangle \), is consistent with the intensity for the X-ray shot \((\sim 10^{37} \text{ erg s}^{-1})\). We calculate \( \langle \dot{M}_{\text{arc}} \rangle = - \int 2\pi R z u d r d x / \int j d x \), and obtain \( \langle \dot{M}_{\text{arc}} \rangle = 10^{38-39} \text{ erg s}^{-1} \) for both of the spin cases. If we assume that the energy conversion efficiency is \( \eta = 10^{-2}-10^{-1} \), the luminosity is \( 10^{36-38} \text{ erg s}^{-1} \), which is on the same order of that of the X-ray shot.

4.3. Observational Implications: HF QPOs

So far, HF QPOs have been detected in seven sources, such as GRO J1655-40, XTE J1550-564, GRS 1915+105, H1743-322, 4U 1630-47, XTE J1859+226, and XTE J1650-500 (McClintock & Remillard 2006; Remillard & McClintock 2006). Three sources (XTE J1859+226, XTE J1650-500, and...
4U 1630-47 have single frequency (Cui et al. 2000; Homan et al. 2003; Remillard & McClintock 2006). The other four sources (GRO J1655-40, XTE J1550-564, H1743-322, and GRS 1915+105) display pairs of HF QPOs with frequencies in a 3:2 ratio (Strohmayer 2001; Remillard et al. 2002; Homan et al. 2005). Most often, these pairs of QPOs are not always detected.

The Galactic black hole binary system XTE J1550-564 is a typical source of the HF QPOs (with frequencies of 92, 184 and 276 Hz, see Remillard et al. 2002), where $M = 8.4-11.2 \, M_\odot$, $D = 5$ kpc, and the binary inclination angle is $70^\circ$ (Orosz et al. 2002). The outburst found in 1998 was used to examine the disk structure in the very high state when the HF QPOs were observed. Kubota & Done (2004) analyzed the X-ray spectra, and reported the following features.

1. The inner part of the optically thick disk does not reach the radius of the marginally stable orbit ($R/M \approx 10$).
2. The inner disk temperature, $T_{in}$, is relatively cold ($T_{in} \sim 10^7$ K).
3. The inner part of the disk is sandwiched by the hot gas regions.

Such features agree well with the simulation data, whereby simulation results indicate that the disk is truncated at $R/M = 20-30$, the disk temperature is $\gtrsim 10^7$ K, and the disk is sandwiched by the overheated regions (Takahashi et al. 2016).

From this consideration, we expect that the relativistic flux variation of the arc-shaped gas clouds is the origin of the HF QPOs. In the case of a non-rotating black hole, the averaged peak interval of the arc-shaped gas cloud, $\Delta t$, is $\Delta t = 1.0 \times 10^{-2} \, (6.6 \times 10^{-3}) \, s$ (where $1/\Delta t = 100 \, (159)$ Hz), and the corresponding radius is $R/M = 10 \, (7.4)$, where we assume that $\delta t$ is equal to the Keplerian orbital period. In the case of a high-spin black hole, the averaged peak interval of the gas cloud is $\Delta t = 9.7 \times 10^{-3} \, (1/\Delta t = 103$ Hz), and the corresponding radius is $R/M = 9.6$. These frequencies are on the same order of that of the HF QPOs ($100-450$ Hz).

In this study, we do not discuss the power spectra and the commensurability of the frequency of the HF QPOs, since long-term simulation data, with which statistical analysis can be employed, are unavailable at the present. In order to examine the origin of the pairs of HF QPOs, we need to analyze plenty of the formation of arc-shaped gas clouds.

4.4. Remaining Issues

Finally, we summarize future issues.

1. In this paper, we assumed that the electron temperature, $T_e$, is kept constant everywhere, or is equal to the ion temperature (one-temperature plasma). In order to accurately calculate the spectral variation and dynamics of arc-shaped gas clouds in the plunging region, it is necessary to evaluate the emissivity variation by carefully estimating the temporal and spatial fluctuation of $T_e$ due to inverse Compton cooling in the original GRRMHD simulation (Sadowski & Narayan 2015; Sadowski et al. 2015, 2017).
2. In this simulation, the formation of the arc-shaped gas cloud mainly occurred twice (once) for the case of

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**Figure 10.** The same as Figure 6, but for the high-spin case, $a/M = 0.9375$. Here, the maximum local flux is $\max(\delta f(x_{obs}, y_{obs}, t)) = \delta f(-13.0M, 0.5M, 0.141)$ in the case of $i = 60^\circ \, (20^\circ)$. 

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In order to examine the major properties of dynamics and radiation of the arc-shaped gas clouds, it is necessary to perform the simulation for a much longer time during which plenty of gas clouds are successively formed. By using the averaged features of the gas clouds, we need to modify our ring model to the more realistic one. With such longer simulation data, we accurately evaluate the profile of the power spectra, and may be able to detect the important features of HF QPOs, such as the pairs of QPOs with frequencies in a 3:2 ratio. We considered two spin cases ($a/M = 0$ and 0.9375), and reproduced the features of the ring model in Paper I. In order to examine the spin dependence of the light variation, we need to calculate various spin cases. Then, we must examine the spin dependence of the dynamical features of gas clouds, and test the method of the spin measurement proposed in Paper I.

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Figure 11. Comparison of the dynamical and radiative properties of the gas clouds of the free–free model (upper panels) with those of the simple one (lower panels) in the case of the non-rotating black hole. The left panels show the spatial distribution of the emissivity, where $t = 0.16$ s. The definition of the color of panels (a1) and (b1) are same as those of Figure 12 (Figure 3). The right panels plot the normalized light curve of each model, where $i = 60^\circ$.
Figure 12. The same as Figure 3, but for the case of the free-free model. Here, $\bar{J}(x, y, \tau)$ reaches its absolute maximum at $(x, y, z, \tau) = (-5.2M, -7.7M, 0.16\, s)$. 

**Figure 12.** The same as Figure 3, but for the case of the free-free model. Here, $\bar{J}(x, y, \tau)$ reaches its absolute maximum at $(x, y, z, \tau) = (-5.2M, -7.7M, 0.16\, s)$. 

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Appendix

In this paper, we transform the Kerr–Schild coordinates, \((r_{KS}, \theta_{KS}, \phi_{KS})\), and the four-velocity of the gas element, \(u^\mu_{KS}\), into the Boyer–Lindquist ones, \((t, r, \theta, \phi)\), and \(u^\nu\):

\[
t = t_{KS} - \int_{r_i}^{r} \frac{2Mr}{\Delta} \, dr, \tag{14}
\]
\[
r = r_{KS}, \quad \theta = \theta_{KS}, \tag{15}
\]
\[
\phi = \phi_{KS} - \int_{r_i}^{r} \frac{a}{\Delta} \, dr, \tag{16}
\]
\[
w^t = w^t_{KS} - \frac{2Mr}{\Delta} w^r_{KS}, \tag{17}
\]
\[
w^r = w^r_{KS}, \quad w^\theta = w^\theta_{KS}, \tag{18}
\]
\[
w^\phi = w^\phi_{KS} - \frac{a}{\Delta} w^r_{KS}. \tag{19}
\]

Here, \(r_i\) is numerical constant, and we choose \(r_i/M = 20\) in order to set \((t, \phi) = (r_{KS}, \phi_{KS})\) at the radius \(r/M = 20\), which is the inner edge of the initial torus.

ORCID iDs
Kotaro Moriyama @ https://orcid.org/0000-0003-1364-3761
Hiroyuki R. Takahashi @ https://orcid.org/0000-0003-0114-5378

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Figure 13. Normalized flux variation including the inverse Compton effect for the gas density of the envelope of \(\rho_{env} = 0 \text{ g cm}^{-3}\) (left), \(10^{-7} \text{ g cm}^{-3}\) (middle), and \(4.3 \times 10^{-7} \text{ g cm}^{-3}\) (right), respectively.