J/ψ production at the Tevatron and HERA: the effect of k_T smearing

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ABSTRACT

We study the effects of intrinsic transverse momentum smearing on J/ψ production both at the Tevatron and at HERA. For the case of large-\(p_T\) J/ψ production at the Tevatron, the effects due to \(k_T\) smearing are mild. On the other hand, inelastic J/ψ photoproduction at HERA is very sensitive to the \(k_T\) smearing and, in fact, with a reasonable value of \(\langle k_T \rangle\) it is possible to resolve the large-\(z\) discrepancy seen by comparing non-relativistic QCD (NRQCD) predictions with the HERA data. We conclude that, with the present kinematic cuts, photoproduction at HERA is not a good test of NRQCD.
Over the last few years, there has been a considerable advance in the understanding of quarkonium physics due to the development of the non-relativistic effective field theory of QCD, called non-relativistic QCD (NRQCD) \[1\]. The Lagrangian for this effective theory is obtained from the full QCD Lagrangian by neglecting all states with momenta larger than a cutoff of the order of the heavy quark mass, \( m \), and accounting for this exclusion by introducing new interactions in the effective Lagrangian, which are local since the excluded states are relativistic. Beyond the leading order in \( 1/m \) the effective theory is non-renormalisable. The scale \( m \) is an ultraviolet cut-off for the physics of the bound state; however the latter is more intimately tied to the scales \( mv \) and \( mv^2 \), where \( v \) is the relative velocity of the quarks in the bound state. The physical quarkonium state admits of a Fock expansion in \( v \), and it turns out that the \( Q\bar{Q} \) states appear in either colour-singlet or colour-octet configurations in this series. Of course the physical state must be a colour-singlet, so that a colour-octet \( Q\bar{Q} \) state is connected to the physical state by the emission of one or more soft gluons. In spite of the non-perturbative nature of the soft gluon emissions, the effective theory still gives useful information about the intermediate octet states. This is because the dominant transitions that occur from colour-octet to physical colour-singlet states are via \( E1 \) or \( M1 \) transitions with higher multipoles being suppressed by powers of \( v \). It then becomes possible to use the usual selection rules for these radiative transitions to keep account of the quantum numbers of the octet states, so that the production of a \( Q\bar{Q} \) pair in an octet state can be calculated and its transition to a physical singlet state can be specified by a non-perturbative matrix element. The cross-section for the production of a meson \( H \) then takes on the following factorised form:

\[
\sigma(H) = \sum_{n=\{\alpha, S, L, J\}} \frac{F_n}{m_{\alpha n}^{d_n - 4}} \langle \mathcal{O}_n^{2S+1L_J} \rangle,
\]

where the \( F_n \)'s are the short-distance coefficients and the \( \mathcal{O}_n \) are local 4-fermion operators, of naive dimension \( d_n \), describing the long-distance physics. The short-distance coefficients are associated with the production of a \( Q\bar{Q} \) pair with the colour and angular momentum quantum numbers indexed by \( n \). These involve momenta of the order of \( m \) or larger and can be calculated in a perturbation expansion in the QCD coupling \( \alpha_s(m) \). The \( Q\bar{Q} \) pair so produced has a separation of the order of \( 1/m \) which is pointlike on the scale of the quarkonium wavefunction, which is of order \( 1/(mv) \). The non-perturbative long-distance factor \( \langle \mathcal{O}_n^H \rangle \) is proportional to the probability for a pointlike \( Q\bar{Q} \) pair in the state \( n \) to form a bound state \( H \).

The existence of the colour-octet components of the quarkonium wave function is the new feature of the NRQCD approach. Before the development of NRQCD, the production and decay of quarkonia were treated within the framework of the colour-singlet model \[2, 3\]. In this model, it is assumed that the \( Q\bar{Q} \) pair is formed in the short-distance process in a colour-singlet state. The corrections from terms higher
order in $v$ were neglected. While this model gave a reasonable description of low-energy $J/\psi$ data, it was known that it was incomplete because of an inconsistency in the treatment of the $P$-state quarkonia. This was due to a non-factorising infra-red divergence, noted first in the application of the colour-singlet model to $\chi_c$ decays \cite{4}, and the proper resolution of this problem was obtained only by including the colour-octet components in the treatment of the $P$-states \cite{5}. The colour-octet components, however, had a more dramatic impact \cite{6} on the phenomenology of $P$-state charmonium production at large $p_T$ at the Tevatron $p\bar{p}$ collider \cite{7} where the colour-singlet model was seen to fail miserably. While the inclusion of the colour-octet components for the $P$-states was necessary from the requirement of theoretical consistency, there was no such problem with the $S$ states because the corresponding amplitude was finite and the colour-octet components were suppressed compared to the colour-singlet component by $O(v^4)$. But the data on direct $J/\psi$ and $\psi'$ production at the Tevatron \cite{7} seem to indicate an important contribution from the colour-octet components for the $S$-states as well \cite{8}.

While it is clear that the correct description of the Tevatron large-$p_T$ data requires that the colour-octet components of the quarkonium wave function have to be taken into account, the major problem is that the corresponding long-distance matrix elements are a priori unknown and can be obtained only by fitting to the Tevatron data \cite{9}. Clearly it is important to have other tests of NRQCD, so that the matrix elements which are obtained from the Tevatron data can be determined independently in other experiments.

One important cross-check is the inelastic photoproduction of $J/\psi$ at the HERA $ep$ collider \cite{10}. The inelasticity of the events is ensured by choosing $z \equiv p_p \cdot p_{J/\psi}/p_p \cdot p_\gamma$ to be sufficiently smaller than one. An upper limit of $z \sim 0.8$ would seem to be an appropriate choice \cite{11}. An additional cut needs to be imposed on the $p_T$ of the $J/\psi$ to ensure that the production process occurs at short distance. In the HERA experiments, the $z$ distributions have been studied using $p_T > 1$ GeV. The surprising feature of the comparisons \cite{12,13} of the NRQCD results with the data from HERA is that the colour-singlet model prediction is in agreement with the data while including the colour-octet component leads to violent disagreement with the data at large $z$. While the colour-singlet cross section dominates in most of the low-$z$ region, the colour-octet contribution increases steeply in the large-$z$ ($0.8 < z < 0.9$) region and this rise is not seen in the data. In these comparisons, the values of the non-perturbative matrix elements are taken to be those determined from a fit to the Tevatron large-$p_T$ data. Naively, one would think that this points to a failure of NRQCD. But this conclusion is premature and, in our opinion, incorrect. The reason is that while at the Tevatron the measured $p_T$ of the $J/\psi$ is greater than about 5 GeV, at HERA the $p_T$ can be as small as $O(1)$ GeV. At such small values of $p_T$ (and also for $z$ very close to unity), there could be significant perturbative and non-perturbative soft physics
effects. One way to parametrise these effects is to include the effects of the transverse momentum smearing of the partons inside the proton. This is the purpose of the present paper. We study the effects of the parton transverse momentum, $k_T$, on the $J/\psi$ distributions both at the Tevatron and at HERA. We demonstrate that the $z$ distribution measured at HERA is particularly sensitive to the effects of $k_T$ smearing, and argue that inelastic photoproduction at HERA, with the present kinematic cuts, is not a clean test of NRQCD. Other effects such as soft-gluon resummation \cite{14} and the breakdown of NRQCD factorisation near $z = 1$ \cite{15} have been discussed in the context of this discrepancy.

In our work, we first consider the effects of $k_T$ smearing on the extraction of the non-perturbative matrix elements from the large-$p_T$ direct $J/\psi$ production data from the CDF experiment at the Tevatron. Before we discuss the fits, we recall the inputs that go into the theoretical computations of Ref. \cite{9} where these fits are performed in the absence of smearing. The direct $J/\psi$ production (i.e. $S$-state production, with the $P$-wave contributions removed) cross section in the NRQCD approach receives contributions from 1) the colour-singlet $^3S_1^{[1]}$ channel, 2) the colour-octet $^3P_J^{[8]}$ connected to the physical $J/\psi$ state by an E1 transition, 3) the colour-octet $^1S_0^{[8]}$ channel which makes a M1 transition to the physical state, and 4) the colour-octet $^3S_1^{[8]}$ channel which connects to the $J/\psi$ state by a double E1 transition. All three colour-octet channels contribute at $O(v^7)$. The non-perturbative parameter for the colour-singlet channel (i.e. the radial wave-function at the origin) can be extracted from $J/\psi$ lepton decay or estimated from potential model calculations. Given this input, the three non-perturbative parameters $\langle O(^3P_J^{[8]}) \rangle$, $\langle O(^1S_0^{[8]}) \rangle$, $\langle O(^3S_1^{[8]}) \rangle$ (which we call matrix elements $M_1$, $M_2$ and $M_3$ respectively) have to be extracted by fitting to the CDF data. It turns out that for $p_T$ values greater than about 4 GeV, the $p_T$ dependence of the short-distance coefficients corresponding to the $^3P_J^{[8]}$ and the $^1S_0^{[8]}$ channels are identical. The $^3S_1^{[8]}$ channel on the other hand has a different $p_T$ distribution: the reason for this is that a fragmentation-type diagram where a single gluon attaches itself to the $c\bar{c}$ pair is present only for this channel and not for the $^3P_J^{[8]}$ and the $^1S_0^{[8]}$ channels. Due to the existence of this fragmentation-type diagram, the contribution from the $^3S_1^{[8]}$ channel dominates at large $p_T$ and so it is possible to use the experimental data points in this $p_T$ region to fit the value of the corresponding non-perturbative matrix element $M_3$. On the other hand, the contributions due to the $^3P_J^{[8]}$ and the $^1S_0^{[8]}$ channels are dominant at low $p_T$ and one can use data the points at the lower end of the measured $p_T$ spectrum to fit a linear combination of $M_1$ and $M_2$. The linear combination turns out to be $M_1/m_c^2 + M_2/3$. We use the phrase ‘fragmentation-type’ guardedly. The subprocess cross sections to which we refer here are obtained from fixed-order perturbation theory calculations. In a genuine fragmentation calculation, the effects of Altarelli-Parisi evolution of the fragmentation functions are taken into account, and these go beyond fixed-order perturbation theory. In comparing the $^3S_1^{[8]}$
Figure 1: The CDF data \cite{7} for $B d\sigma/dp_T$ (in nb/GeV) for $J/\psi$ production at 1.8 TeV with $-0.6 \leq \eta \leq 0.6$, compared to the model predictions with $\langle k_T \rangle = 0$, 0.7, 1.0 GeV, respectively.

prediction to the large-$p_T$ data, one must correct for the effect of this evolution, and the prescription for this correction (as given in Ref. \cite{9}) is to multiply the number obtained from the fixed order calculation with the correction factor $R$, where $R$ is the ratio of the cross sections (computed using the fragmentation approach) at the scales $p_T$ and $m_c$ i.e. with and without the $Q^2$ evolution of the fragmentation functions.

For the Tevatron, we perform similar fits to those described above except that we now include the effects of intrinsic transverse momentum smearing. The inclusion of $k_T$ smearing in large-$p_T$ reactions is done in the standard fashion, see for example Ref. \cite{16}. Each incoming parton is given an intrinsic transverse momentum, with a gaussian distribution whose width is an adjustable parameter. The gaussian shape is motivated by measurements of the transverse momentum distribution of $l^+l^-$, $\gamma\gamma$ and $\pi\pi$ pairs produced in hadron-hadron collisions. The cross section with the inclusion of
smearing is therefore given by

$$E \frac{d^3 \sigma}{dp^3}(p\bar{p} \to J/\psi X) = \sum_{a,b,c=q,g} \int d^2 k_{Ta} \int d^2 k_{Tb} \int dx_1 \int dx_2$$

$$f_{a/p}(x_1, \vec{k}_{Ta}, Q^2) f_{b/\bar{p}}(x_2, \vec{k}_{Tb}, Q^2) \frac{1}{\pi} \frac{d^3 \hat{\sigma}}{d^3 \hat{t}} (ab \to 2S + 1 L J c) \delta(\hat{s} + \hat{t} + \hat{u} - M^2),$$

(2)

where

$$f_{a/p}(x_1, \vec{k}_{Ta}, Q^2) = \frac{F_{a/p}(x_1, Q^2) D(\vec{k}_{Ta})}{x_1^2 + (4k_{Ta}^2/s)}$$

(3)

and similarly for $f_{b/\bar{p}}(x_2, \vec{k}_{Tb}, Q^2)$. $F_{a/p}$ is the usual parton distribution, and the function $D(\vec{k}_T)$ is the intrinsic transverse momentum distribution

$$D(\vec{k}_T) = \frac{b^2}{\pi} \exp(-b^2 k_T^2),$$

(4)

with

$$\langle k_T \rangle = \frac{\sqrt{\pi}}{2b},$$

(5)

and normalised such that

$$\int d^2 k_T D(\vec{k}_T) = 1.$$  

(6)

For the numerical phase-space integration, it is convenient to use the change of variables

$$k_{Ti} = \kappa \left( \ln \frac{1}{q_i} \right)^2, \quad (i = a, b)$$

(7)

so that

$$\int_0^\infty k_{Ti} dk_{Ti} \rightarrow \int_0^1 dq_i \frac{\kappa^2}{2q_i}.$$  

(8)

In terms of the variable $q_i$, the Gaussian distribution becomes

$$D(q_i) = \frac{b^2}{\pi} \left( \frac{q_i}{\kappa} \right)^2.$$  

(9)

By choosing $\kappa$ such that $b\kappa$ is slightly larger than unity, we obtain a relatively smooth integrand.

The results of the effects of including intrinsic $k_T$ for $J/\psi$ production at the Tevatron are shown in Fig. 1, where we consider the $J/\psi p_T$ distribution, $B d\sigma/dp_T$, with $B$ the branching ratio of the $J/\psi$ into $\mu^+\mu^-$. The cuts on the pseudorapidity of the $J/\psi$ are those used by the CDF experiment i.e. $-0.6 \leq \eta \leq 0.6$. In our computations we use MRS(D') parton densities [17] with factorisation scale $Q = M_T \equiv \sqrt{M_{\psi}^2 + p_T^2}$.  

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We present the results for three different values of \( \langle k_T \rangle \), viz. \( \langle k_T \rangle = 0, 0.7, 1.0 \) GeV. The choice \( \langle k_T \rangle = 0 \) corresponds to no smearing. For the colour-singlet parameter, \( \langle O^{(3S_1[1])} \rangle \), we use the value 0.81 GeV. This corresponds, of course, to the case where there is no smearing. For the case when smearing is included we need to know the effect of the smearing on the value of this parameter. For this purpose, we use the data on \( J/\psi \) production from the EMC collaboration \cite{18} in the limited \( z \) range \( 0.3 < z < 0.6 \). In this range, the colour-singlet contribution gives almost the entire cross-section \cite{13}. We can therefore use these data to extract the effect of the smearing on the singlet wave-function. For \( \langle k_T \rangle = 0.7, 1.0 \) GeV, we find the values to be 0.98 GeV and 1.41 GeV respectively. The values of the non-perturbative octet parameters extracted for the three choices of \( \langle k_T \rangle \) are tabulated in Table 1. We see that the effect of the \( k_T \) smearing on the parameters extracted from the large-\( p_T \) CDF data is very modest.

![Fig. 1](image)

**Fig. 1:** The fits to the data when \( k_T \) smearing is included are very good and comparable in quality to the case \( \langle k_T \rangle = 0 \).

| \( \langle k_T \rangle \) (GeV) | \( M_1/m_c^2 + M_2/3 \) (GeV) \(^3\) | \( M_3 \) (GeV) \(^3\) |
|-----------------|-----------------|-----------------|
| 0.0             | \((3.14 \pm 0.58) \times 10^{-2}\) | \((1.26 \pm 0.33) \times 10^{-2}\) |
| 0.7             | \((2.82 \pm 0.47) \times 10^{-2}\) | \((1.35 \pm 0.30) \times 10^{-2}\) |
| 1.0             | \((2.35 \pm 0.39) \times 10^{-2}\) | \((1.50 \pm 0.29) \times 10^{-2}\) |

Table 1: The values of the colour-octet non-perturbative matrix elements determined from the CDF data on \( J/\psi \) production, for three choices of intrinsic initial state transverse momentum.

Taking these fitted values of the parameters, we next consider inelastic \( J/\psi \) photoproduction at HERA. For photoproduction, the formalism for the inclusion of smearing effects is similar to that used for the \( p\bar{p} \) collisions described above, except that now we have only one hadron in the initial state. We take the same choice of parton distributions, scales etc. as used in the Tevatron fits. We compute the \( z \) distribution for a photon-proton centre-of-mass energy \( \sqrt{s_{\gamma p}} = 100 \) GeV, imposing a transverse momentum cut \( p_T > 1 \) GeV. Again we present results for \( \langle k_T \rangle = 0, 0.7, 1.0 \) GeV. For each choice of \( \langle k_T \rangle \), the values of the octet non-perturbative matrix elements are taken from Table 1, and the singlet matrix elements are taken to be the same as that used in the Tevatron fits.

With these inputs, we compute the \( z \) distribution for the three different values of \( \langle k_T \rangle \). The results are compared with HERA data \cite{10} in Fig. 2. In the absence of smearing, \( \langle k_T \rangle = 0 \), we see that the colour-octet component makes a large contribution at \( z \) close to 1 which is not supported by the data. However the introduction of \( k_T \) makes a substantial change to the octet contribution. Whereas the effect of \( k_T \)-smearing is very small for large-\( p_T \) production at the Tevatron, these effects are found to be very

\[\text{6}\]


important for $J/\psi$ production at HERA. In particular, smearing significantly reduces the size of the cross section (mainly due to the smearing function itself, but also due to the smaller octet matrix elements, see Table 1.) The $z$ distribution also becomes flatter, in better agreement with the HERA data. In fact, excluding the highest two data points at $z \approx 0.86$ we find that the $\chi^2$ is minimised for $\langle k_T \rangle \sim 0.7$ GeV and the resulting description of the data is very good. It is perhaps not surprising that the highest-$z$ data points are not exactly accounted for, since in this region other effects such as contamination from elastic scattering or breakdown of NRQCD factorisation may be important.

In conclusion, while a direct comparison of the predictions of NRQCD with the $z$-dependence of the inelastic photoproduction cross section for $J/\psi$ at HERA show a marked disagreement between the two, we argue that such a comparison is misleading.
The inelastic photoproduction process does not provide a clean test of NRQCD because in order to have a sizeable event rate, a very low $p_T$-cut ($\sim 1$ GeV) is necessary in the HERA experiments. At such low values of $p_T$ (and for values of $z$ reasonably close to one), we find that the effect of $k_T$-smearing is important and that, indeed, for $\langle k_T \rangle \sim 0.7$ GeV, the discrepancy between theory and experiment is no longer observed. On the other hand, the inclusion of $k_T$-smearing has a very modest effect on the large-$p_T$ $J/\psi$ data from the Tevatron. Better tests of NRQCD may be obtained by studying other observables at the Tevatron itself, such as the study of the polarisation of the produced $J/\psi$ \cite{19} or the production of other charmonium resonances \cite{20, 21} whose cross-section can be predicted in NRQCD.

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