Probing transverse coherence of x-ray beam with 2-D phase grating interferometer

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Abstract: Transverse coherence of the x-ray beam from a bending magnet source was studied along multiple directions using a 2-D π/2 phase grating by measuring interferogram visibilities at different distances behind the grating. These measurements suggest that the preferred measuring orientation of a 2-D checkerboard grating is along the diagonal directions of the square blocks, where the interferograms have higher visibility and are not sensitive to the deviation of the duty cycle of the grating period. These observations are verified by thorough wavefront propagation simulations. The accuracy of the measured coherence values was also validated by the simulation and analytical results obtained from the source parameters. In addition, capability of the technique in probing spatially resolved local transverse coherence is demonstrated.

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References and links

1. W. Yang, X. Huang, R. Harder, J. N. Clark, I. K. Robinson, and H. K. Mao, “Coherent diffraction imaging of nanoscale strain evolution in a single crystal under high pressure,” Nat. Commun. 4, 1680 (2013).
2. S. Marchesini, S. Bourret, A. E. Sakdinawat, M. J. Bogan, S. Bajt, A. Barty, H. N. Chapman, M. Frank, S. P. Hau-Riege, A. Szöke, C. Cui, D. A. Shapiro, M. R. Howells, J. C. H. Spence, J. W. Shaevitz, J. Y. Lee, J. Hajdu, and M. M. Seibert, “Massively parallel X-ray holography,” Nat. Photonics 2(9), 560–563 (2008).
3. A. Sakdinawat and D. Attwood, “Nanoscale X-ray imaging,” Nat. Photonics 4(12), 840–848 (2010).
4. F. Pfei ffer, M. Bech, O. Bunk, P. Kraft, E. F. Eikenberry, C. Brönnimann, C. Grünzweig, and C. David, “Hard-X-ray dark-field imaging using a grating interferometer,” Nat. Mater. 7(2), 134–137 (2008).
5. R. L. Sandberg, A. Paul, D. A. Raymondson, S. Härdich, D. M. Gaudiosi, J. Holtsnider, R. I. Tobey, O. Cohen, M. M. Murnane, H. C. Kapteyn, C. Song, J. Miao, Y. Liu, and F. Salmassi, “Lensless diffractive imaging using tabletop coherent high-harmonic soft-X-ray beams,” Phys. Rev. Lett. 99(9), 098103 (2007).
6. S. Schleede, F. G. Meinel, M. Bech, J. Herzen, K. Achterhold, G. Potdevin, A. Malecki, S. Adam-Neumair, S. F. Thieme, F. Bamberg, K. Nikolau, A. Bohla, A. O. Yildirim, R. Loewen, M. Gifford, R. Ruth, O. Eickelberg, M. Reiser, and F. Pfeiffer, “Emphysema diagnosis using X-ray dark-field imaging at a laser-driven compact synchrotron light source,” Proc. Natl. Acad. Sci. U.S.A. 109(44), 17880–17885 (2012).
7. L. W. Whitehead, G. J. Williams, H. M. Quiney, D. J. Vine, R. A. Dilanian, S. Flewett, K. A. Nugent, A. G. Peele, E. Bala ur, and I. McNulty, “Diffractive imaging using partially coherent x rays,” Phys. Rev. Lett. 103(24), 243902 (2009).
8. M. Born and E. Wolf, Principle of Optics, 7th ed. (Cambridge University, 1999).
9. B. J. Thompson and E. Wolf, “Two-beam interference with partially coherent light,” J. Opt. Soc. Am. 47(10), 895–902 (1957).
10. R. A. Bartels, A. Paul, H. Green, H. C. Kapteyn, M. M. Murnane, S. Backus, I. P. Cristov, Y. Liu, D. Attwood, and C. Jacobsen, “Generation of spatially coherent light at extreme ultraviolet wavelengths,” Science 297(5580), 376–378 (2002).
11. D. Paterson, B. E. Allman, P. J. McMahon, J. Lin, N. Moldovan, K. A. Nugent, I. McNulty, C. T. Chantler, C. C. Retsch, T. H. K. Irving, and D. C. Mancini, “Spatial coherence measurement of X-ray undulator radiation,” Opt. Commun. 195(1–4), 79–84 (2001).
12. W. Leitenberger, H. Wendrock, L. Bischoff, T. Panzer, U. Pietsch, J. Grenzer, and A. Pucher, “Double pinhole diffraction of white synchrotron radiation,” Physica B 336(1–2), 63–67 (2003).
1. Introduction

With the advent of brilliant and highly coherent x-ray sources like the third-generation synchrotron radiation facilities and x-ray free electron lasers (XFELs), the number of experiments using the coherence property of the source, such as coherent diffraction imaging (CDI), holography, and x-ray microscopy, has increased tremendously [1–3]. Other lab-based sources, such as standard x-ray tube sources, tabletop soft x-ray sources, and compact light sources, are being increasingly used for x-ray microscopy/imaging experiments [4–6]. Therefore, it is becoming important to characterize the beam coherence as well as the degradation of coherence and changes in the wavefront due to the optical elements along the beam path. With respect to matching the beam properties with the sample, it is essential to characterize the incident x-ray source in all transverse directions at the sample position [7].

The typical and most widely used method to demonstrate the transverse coherence effect is by generating an interference pattern using the Young’s double pinhole/slit arrangement [8]. This has been extensively used to characterize the coherence of the beam from optical light sources [9], XUV radiation [10], synchrotron sources [11, 12], and XFELs [13]. To obtain the full complex coherence function (CCF) of the beam requires a series of measurements with variable slit separations and positions, which is time consuming and therefore not practical. J. Lin et al. [14] introduced a different technique first used by K. A. Nugent et al. [15] for soft x-ray beams, where a uniformly redundant array (URA) was utilized as a phase-shifting mask to measure the coherence property of the undulator radiation. Though the technique can measure the full coherence of the x-ray beam with a single interferogram, it is not model-free and requires knowledge of the detailed structure of the URA as an input for data deconvolution. The spatial range of the coherence measurement is also limited for hard X-rays [16].

13. A. Singer, F. Sorgenfrei, A. P. Mancuso, N. Gerasimova, O. M. Yefanov, J. Gulden, T. Gorniak, T. Senkeibl, A. Saldinowat, Y. Liu, D. Atwood, S. Drzazybtski, D. D. Mai, R. Treusch, E. Weckert, T. Salditt, A. Rosenhahn, W. Wurth, and I. A. Vartanyants, “Spatial and temporal coherence properties of single free-electron laser pulses,” Opt. Express 20(16), 17480–17495 (2012).
14. J. A. Lin, D. Paterson, A. G. Peele, P. J. McMahon, C. T. Chanter, K. A. Nugent, B. Lai, N. Moldovan, Z. Cai, D. C. Mancini, and I. McNulty, “Measurement of the spatial coherence function of undulator radiation using a phase mask,” Phys. Rev. Lett. 90(7), 074801 (2003).
15. J. A. Nugent and J. E. Trebes, “Coherence measurement technique for short wavelength light sources,” Rev. Sci. Instrum. 63(4), 2146–2151 (1992).
16. F. Pfeiffer, O. Bunk, C. Schulze-Briese, A. Diaz, T. Weitkamp, C. David, J. F. van der Veen, I. Vartanyants, and I. K. Robinson, “Shearing interferometer for quantifying the coherence of hard X-ray beams,” Phys. Rev. Lett. 94(16), 164801 (2005).
17. P. Cloatens, J. P. Guigay, C. De Martino, J. Baruchel, and M. Schlenker, “Fractional Talbot imaging of phase gratings with hard x-rays,” Opt. Lett. 22(14), 1059–1061 (1997).
18. J. P. Guigay, S. Zabler, P. Cloatens, C. David, R. Mokso, and M. Schlenker, “The partial Talbot effect and its use in measuring the coherence of synchrotron X-rays,” J. Synchrotron Radiat. 11(6), 476–482 (2004).
19. A. Diaz, C. Mocuta, J. Stangl, M. Keplinger, T. Weitkamp, F. Pfeiffer, C. David, T. H. Metzger, and G. Bauer, “Coherence and wavefront characterization of Si-111 monochromators using double-grating interferometry,” J. Synchrotron Radiat. 17(3), 299–307 (2010).
20. R. Khuender, F. Masiello, P. van Vaerenbergh, and J. Härtwig, “Measurement of the spatial coherence of synchrotron beams using the Talbot effect,” Phys. Status Solidi A 206(8), 1842–1845 (2009).
21. O. Chubar, L. Berman, Y. S. Chu, A. Fluerasu, S. Hulbert, M. Idir, K. Kaznatcheev, D. Shapiro, Q. Shen, and J. Baltser, “Development of partially-coherent wavefront propagation simulation methods for 3rd and 4th generation synchrotron radiation sources,” Proc. SPIE 8141, 814107 (2011).
22. I. Zanette, C. David, S. Rutishauser, and T. Weitkamp, “2D grating simulation for X-ray phase-contrast and dark-field imaging with a Talbot interferometer,” in X-Ray Optics and Microanalysis, Proceedings of the 20th International Congress, CP1221, M. Denecke and C. Walker, eds. (American Institute of Physics, 2010), pp. 73–79.
23. H. H. Wen, E. E. Bennett, R. Kopace, A. F. Stein, and V. Pai, “Single-shot x-ray differential phase-contrast and diffraction imaging using two-dimensional transmission gratings,” Opt. Lett. 35(12), 1932–1934 (2010).
24. K. A. Nugent and J. E. Trebes, “Comparison of optical undulator and XUV source coherence,” Opt. Express 19(9), 8073–8078 (2011).
Different from the above-mentioned techniques, a periodic phase object can be used to generate fractional Talbot images, and the evaluation of these images as a function of defocusing distance provides information about the transverse coherence of the beam [17, 18]. Using two gratings and generating a Moiré pattern, F. Pfeiffer et al. [16] measured the complex coherence function (CCF) from decaying visibility of fractional Talbot distances. This technique is model-free and requires only a very simple experimental setup and easy data processing. However, for hard x-rays, fabrication of high aspect ratio amplitude gratings (either 1-D or 2-D) is very challenging. Using the same method, A. Diaz et al. [19] showed that the coherence-preserving capability of the pseudo-channel-cut monochromator is two times better than that of the double-crystal monochromator owing to the higher mechanical stability of the former.

To date, not many studies have focused on the simultaneous measurement of the transverse coherence along multiple directions. J. P. Guigay et al. [18] studied the CCF along both the vertical and horizontal directions using a 2-D binary square profile grating made of etched pattern on silicon. Based on the partial Talbot effect, it is shown that the variation of visibility with higher Fourier harmonics is fast, and therefore a reduced range of measuring distances is needed. No attempt was made to measure the data along transverse directions other than the horizontal and vertical directions. Moreover, they used a phase grating that produces a phase shift of 0.41 rad instead of $\pi$ or $\pi/2$ rad, which produces maximum visibility fringes. Recently, extending the work by P. Cloetens [17], R. Klünder et al. [20] implemented a better analysis method—using two Talbot images—that requires precise aligning of the two images.

The present work presents the simultaneous measurement of transverse coherence or CCF along four directions namely, 0° (horizontal), 90° (vertical), and 45° and 135° to the horizontal direction using a single 2-D $\pi/2$ phase grating with a checkerboard pattern of gold deposited on the Si$_3$N$_4$ membrane. Transverse coherence property of the x-ray beam passing through beamline optics including a Si (111) double-crystal monochromator (DCM), and three Be windows was studied. The Fourier transform (FT) of the measured interferograms contains harmonic peaks along many transverse directions, and each of these peaks is available to draw a map of the CCF along that direction. Since higher-order harmonic peaks are very weak in practice, only the first harmonic peaks along the four primary directions are analyzed. Images at different defocusing distances are recorded to extract the visibility and subsequently the CCF. This technique, like the Moiré technique [16], is model-free and also can be used to resolve the local variations in the coherence of the beam waveform over a small area. (Two region of interests (ROI’s) of area 25 × 20 µm$^2$ were shown in this work.) Recently it has been shown that the knowledge of the spatial coherence can be used as a priori information for the phase retrieval techniques (e.g., coherence diffraction imaging using ptychography) [7]. Here we emphasize that the present work deals with measurements of the transverse or spatial coherence of the beam due to the usage of the monochromatic beam. The technique proposed in this paper is suitable to provide the transverse coherence and phase profile of the beam waveform as input to these advanced coherence sensitive studies. Furthermore, if we compare a single 2-D phase grating interferometer system with a two-grating interferometer system, the later requires a 2-D phase grating as well as a 2-D amplitude grating and fabrication of these gratings for hard x-rays is challenging. On the other hand, the single 2-D phase grating interferometer is easier to set up and align while providing the same information on the phase profile of the beam waveform and the CCF map as the two-grating interferometer system does. However, for both of these techniques, the grating period must be optimized to ensure the fractional Talbot distances needed are within the manageable measuring range. We also modeled the measured coherence values by performing wavefront propagation simulation using the SRW package [21].

2. Theoretical background

The relation between the visibility of the interferograms and the measured CCF can be found in standard optics text books such as [8]. Here we repeat some of the basic equations of the
coherence theory to enhance the readability and to form a smooth transition to the equations derived for the particular case discussed in this paper. The correlation of field $E$ at two different points, $p_1$ and $p_2$, with a time lag of $\tau$, called the Mutual coherence, can be expressed as,

$$\Gamma_{12}(\tau) = \langle E(p_1, t)E^*(p_2, t+\tau) \rangle.$$  \hfill (1)

For an experiment with quasi-monochromatic source, average of the mutual coherence function ($\Gamma$) over a time interval larger than the typical fluctuations present in the source is called the Mutual intensity function ($J$) [16]. When the two points coincide ($p_1 = p_2$), a special case called self-coherence occurs and Eq. (1) reduces to,

$$J_{12} = \Gamma_{12}(0) = \langle E(p_1)E^*(p_2) \rangle \Rightarrow J_{11} = I_1, \quad J_{22} = I_2.$$

Practically this correlation effect can be observed by forming the interference of two light beams. The measured intensity at any point, due to the interference of two beams from an extended source, can be written as

$$I = \left[ \langle E(p_1) + E(p_2) \rangle \right]^2 \left[ \langle E^*(p_1) + E^*(p_2) \rangle \right] = I_1 + I_2 + 2\text{Re}\{J_{12}\}.$$  \hfill (2)

The normalized mutual intensity function $j_{12}$, which is called the complex coherence function, is written as (from Eq. (2))

$$j_{12} = \frac{J_{12}}{\sqrt{J_{11}J_{22}}} = \frac{J_{12}}{\sqrt{I_1I_2}}.$$  \hfill (3)

Combining Eq. (3) and Eq. (4)

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} |j_{12}| \cos(\varphi_{j_{12}}),$$  \hfill (4)

where $\varphi_{j_{12}} = \text{Arg}(j_{12})$. The third term on the right-hand side of Eq. (5) is responsible for the interference effect and $|j_{12}|$ takes the value 1 for complete coherence of the two beams and is 0 for complete incoherence. In the case of partial coherence, it takes values between 0 and 1.

Experimentally the coherence function cannot be obtained by only measuring the intensity at the observation plane. However, it can be obtained from the visibility measurements of interference fringes formed from two wavefronts. The visibility can be mathematically formulated as [8]

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},$$  \hfill (5)

where $I_{\text{max}}$ and $I_{\text{min}}$ are the intensity maxima and minima on the observation screen, respectively, and from Eq. (5),

$$I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1I_2} |j_{12}|, \quad I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1I_2} |j_{12}|.$$  \hfill (6)

Hence the visibility of the fringes is obtained by inserting Eq. (7) into Eq. (6) and rewriting as

$$|j_{12}| = \frac{V(I_1 + I_2)}{2\sqrt{I_1I_2}}.$$  \hfill (7)

For a phase grating, $I_1 = I_2$ and Eq. (8) reduces to

$$|j_{12}| = V.$$  \hfill (8)

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i.e., the visibility of fringes is equal to the degree of coherence of the beam at the grating position (Eq. (9)). A 2-D $\pi/2$ checkerboard phase grating illuminated with x-rays produces self-images at fractional Talbot distances ($d_n$) following the equation $d_n = n^*p^2/2\lambda$, where $n = 0.5, 1.5, 2.5\ldots$ [22]. For a fully coherent source, the visibility of the fringes should be the same at all fractional Talbot distances, though in practice the visibility decreases with defocusing distance due to the partial coherence of the beam. The envelope function, which describes all maximum visibility positions, is then the complex coherence function.

A Gaussian intensity distribution is a good approximation for a synchrotron radiation and can be written as

$$I_s(s_x, s_y) = I_0 \exp\left[-\frac{s_x^2}{2\sigma_x^2} - \frac{s_y^2}{2\sigma_y^2}\right],$$

where $s_x, s_y$ denote the coordinates in the source plane, and $\sigma_x$ and $\sigma_y$ are the source size along those directions, respectively. According to the propagation laws of mutual intensity function the CCF is also a Gaussian of the form

$$|j(x, y)| = j_0 \exp\left[-\frac{x^2}{2\xi_x^2} - \frac{y^2}{2\xi_y^2}\right],$$

$$\xi_x = \lambda D / 2\pi \sigma_x, \xi_y = \lambda D / 2\pi \sigma_y,$$

where $x$ and $y$ are the horizontal and vertical axes perpendicular to the direction of the beam propagation at the grating position, $\xi_x$ and $\xi_y$ are the coherence length in the $x$ and $y$ directions, respectively, and $D$ is the beam propagation distance from the source.

In this work, since the measurement of the visibility was performed by changing the grating-to-detector distance $d$, the co-ordinates in Eq. (11) has to be re-written in terms of $d$. For a $\pi/2$ phase-shift grating, the interference fringes are formed by the neighboring diffraction orders; therefore, the beam separation is related to the defocusing distance $d$ as

$$x = \frac{d\lambda}{p_0},$$

where $p_0$ is the period of the diffraction pattern along $x$ direction with the subscript $\theta = 0$ ($\theta$ is the angle between the direction of interest and the horizontal direction). Similarly we can write the coordinate transformations along other directions. Therefore, Eq. (11) can be written in one dimension as a function of $d$

$$|j'(d)| = j_0 \exp\left[-\frac{d^2}{2*(\sigma_d)^2}\right],$$

where $\sigma_d$ is the width of the Gaussian envelop function or the CCF, $\theta$ can take any values depending on the direction along which the width of the Gaussian envelope is measured. For the experimental setup of this work, $\theta$ equals 0, 45, 90, or 135. It is assumed that the beam intensity distribution is Gaussian along all four directions and Eq. (14) is valid along these directions. Following Eq. (13), the coherence length in each direction can then be obtained from

$$\xi_{\exp,\theta} = \frac{\lambda \sigma_{\theta}}{p_{\theta}},$$

with $p_{\theta}$ the period in the $\theta$ direction. Equation (15) indicates that to obtain the coherence length of the beam along a particular direction, we need to normalize the experimentally
obtained Gaussian width with the period of the interferogram along this direction. To validate the accuracy of the simple analytical model in Eq. (10), simulations with the wavefront propagation code SRW [21] were also performed with the same geometrical setup for a bending magnet source at the APS.

3. Experimental details

The experiment was performed at the 1-BM-B beamline of the Advanced Photon Source (APS). The schematic experimental setup is shown in Fig. 1. The monochromatic beam of 18 keV is provided by a Si (111) double-crystal monochromator (DCM). The 2-D checkerboard phase grating of $\pi/2$ phase shift and 4.8-$\mu$m period was placed about 34 m from the bending magnet source. A $1 \times 1 \text{ mm}^2$ slit was placed upstream of the DCM and 25 m from the source. Three beryllium windows are situated at 22 $\pm$ 0.05 m, 27 m, and 30.5 m from the source. The first upstream window is a double window, and each of these windows has a thickness of 250 $\mu$m. Hence beam travels through a Beryllium thickness of 1 mm. To record the interferograms directly without using the second analyzer grating, a high-resolution CCD detector was used. The detector system consisted of a CoolSnap HQ2 CCD with $1392 \times 1040$ imaging pixels of $6.45 \times 6.45 \text{ µm}^2$ pixel size. With a 10 $\times$ objective lens the effective pixel size is 0.64 µm. A LYSO (lutetium-yttrium oxyorthosilicate) scintillator with 150 µm thickness is used to convert x-rays into visible light.

Measurements were performed by acquiring the interferograms at multiple detector positions from close to the grating (43 mm) to the maximum distance physically possible (750 mm) at an interval of about 10 mm. Each interferogram contains the periodic interference pattern from the checkerboard phase grating, as shown in Fig. 2(a). A Fourier-transformed image yields the equivalent reciprocal or Fourier space image, which contains a set of peaks that are characteristic of a periodic pattern in real space. As shown in Fig. 2(b), the checkerboard structure contains periodicity not only along the horizontal and vertical directions but also in directions 45° and 135° to the horizontal. The amplitude of the zero-order peak ($A_0$) represents the average value of the amplitude of the image whereas the amplitude of the first-order peak ($A_1$) is the attenuated amplitude. Hence the visibility as represented by Eq. (6) can be written as $V = 2A_1/A_0$ [22]. In practice we obtained the visibility by first inverse Fourier transforming the 1st- and zeroth-order harmonic images and later taking the ratio of the average of the intensities. Due to the central symmetry of the Fourier transform figure, there are two first-order peaks in each direction. For simplicity, we used only one of the peaks for the analysis. A map of this visibility as a function of defocusing distance $d$ can be related to the spatial coherence of the beam as in Eqs. (9)–(15).

![Fig. 1. Schematic diagram of the experimental setup.](image-url)
Fig. 2. (a) Central portion of the measured interferogram at 83-mm distance. (b) Central portion of the Fourier transform of one of the interferograms showing the harmonic peaks along the 0°, 45°, 90°, and 135° directions. (c) SEM image of the 2-D checkerboard phase grating used for this experiment. The grating fabricated by electroplating Au onto Si3N4 membrane has a period of 4.8 µm and is optimized for a π/2 phase shift. (d) Orientation of the grating in the transverse plane perpendicular to the direction of the beam propagation. The angles θ are with respect to the horizontal direction.

4. 2-D grating fabrication

Figure 2(c) shows the scanning electron microscope image of the 2-D checkerboard phase grating fabricated in-house, in collaboration with the Center for Nanoscale Materials (CNM). The grating was fabricated by electroplating Au into polymer molds in a two-step fabrication process. The polymer mold, poly methyl methacrylate (PMMA), was patterned using a JEOL 9300FS 100-keV electron beam lithography tool. The bias-corrected pattern was written with 1100-µC/cm² exposure dose using 40 nA of current and a 10-nm shot size. The samples were developed for 40 sec in a 7:3 isopropanol:deionized water solution. The resulting PMMA mold was electroplated using a Technic 25 E Au-sulfate electroplating solution heated to 40°C. Current applied to the sample was set to achieve a current density of 2 mA/cm², which caused an estimated plating rate of 120 nm/min. After the grating was plated to the desired height, the PMMA mold was removed and the sample was ready for use.

5. Results

5.1 Results from a 2-D checkerboard phase-grating experiment

Figure 3 shows the evolution of the visibility along the 0°, 45°, 90°, and 135° directions with the defocusing distance. A sinusoidal oscillation of the visibility is due to the fractional Talbot effect imparted by a 2-D checkerboard phase grating that is optimized for π/2 phase shift for 18-keV x-rays. As expected, the visibility at the successive fractional Talbot distance decreases with defocusing distance due to the partial coherence of the bending magnet radiation source. The measurements were performed by placing the grating in the beam oriented 45°, as shown in Fig. 2(d). By rotating this way, the checkerboard structure forms continuous lines along the diagonal of the gold squares and produces higher visibility along the 0° and 90° directions. In the 45° and 135° directions, the position of the first-harmonic peak in the Fourier image corresponds to a period of 2.4 µm, which is the half-period of the
Fig. 3. Evolution of the experimental visibility ($\mathcal{V}$) as a function of the grating-to-detector distance ($d$) and the fitted Gaussian envelope function ($\mathcal{L}$) along four different directions: (a) 0°, (b) 90°, (c) 45°, and (d) 135° with $\sigma_\theta$ shown in the figure. The extracted coherence lengths are shown in Table 1. The solid lines and the schematic of the 2-D checkerboard grating are drawn as a visual aid.

grating. This periodicity is not easily identified from the physical shape of the grating, but corresponds to the unique 2-D interference pattern at these fractional Talbot distances.

The transverse coherence of the x-ray beam wavefront is related to the width of the envelope function, which can be modeled as Gaussian according to Eq. (12). Figures 3(a)–3(d) show fitted Gaussian envelope functions drawn through the maximum visibility points along four different directions. As expected, the fall-off of the envelope function is much slower along 90° due to the higher degree of coherence along the vertical direction. The 0° CCF is much steeper due to the larger horizontal source size and hence the lower coherence length. Along 45° and 135° it is in-between indicating an elliptical coherence area of the wavefront. The coherence values obtained from $\sigma_\theta$ are tabulated in section 5.3.

On closer observation of the visibility curves along 45° and 135° directions in Figs. 3(c) and 3(d), we found that the odd and even numbered sets of peaks have different heights, while the envelope function drawn on both of these sets produces a similar decay pattern. This was found to be due to the mismatch of the duty cycle of the grating as explained in section 5.2.

5.2 Simulation of coherence measurement using 2-D checkerboard phase grating

In order to fully understand the experimental observation and check the accuracy of the simple Gaussian model, wavefront propagation was carried out using SRW [21]. The electron source rms size obtained from the machine studies is $85 \times 37 \, \mu$m². The x-ray radiation is calculated at 18 keV from a 0.4-m-long bending magnet with a field of 0.6 T. The 2-D checkerboard phase grating was generated in the same orientation as in the experiments. Twenty half-periods were generated with fifty grid points within each half-period. The
interferograms at different distances $d$ were simulated and then analyzed with the same procedure as described in the previous sections.

Figure 4 shows the interference pattern at the first visibility maxima in the $0^\circ$ or $90^\circ$ ($d = 83.6$ mm) and the $45^\circ$ or $135^\circ$ ($d = 41.8$ mm) directions. In Fig. 4(a), a clear self-image of the checkerboard pattern was obtained. The high visibility in the $0^\circ$ and $90^\circ$ directions comes from the line-type structure with the period $p_0 = p_{90} = 3.4 \mu$m, while the visibility along the $45^\circ$ and $135^\circ$ directions is zero at this distance.

On the other hand, the visibility maxima in the $45^\circ$ and $135^\circ$ cases originates from the interference pattern shown in Fig. 4(b), which has a periodicity of $p_{45} = p_{135} = 2.4 \mu$m. The repeating patterns are not as strongly visible as in Fig. 4(a), which explains the lower visibilities observed in these two directions [cf. Figures 3(c) and 3(d)] in comparison with those in the other two directions [cf. Figures 3(a) and 3(b)]. This is a clear indication that the diagonal direction of the grating square blocks is the preferred direction for measurements.

The simulated visibility evolution as a function of $d$ is presented in Figs. 5(a)–5(d). The decay patterns are almost identical with those observed experimentally (cf. Figure 3). The relatively lower visibility in the experiment is mainly caused by the point spread function (PSF) of the detector system. The other contributions to the lower visibility are the limited detector resolution: There are only roughly 4 pixels within the half period of the checkerboard (2.4 $\mu$m) and to a small extent to the grating imperfections. Furthermore, the odd- and even-numbered sets of peaks in the experimental results have slightly different heights in the $45^\circ$ and $135^\circ$ directions [cf. Figures 3(c) and 3(d)], which is absent in the $0^\circ$ and $90^\circ$ directions. This was found to be due to a slight mismatch (the square gold pattern is a few tens of nanometers smaller than the non-gold position) of the duty cycle of the grating. Simulation of a checkerboard grating with the size of the gold blocks reduced by $1/50$ in comparison with the non-gold blocks confirms this effect, as shown in Fig. 6. The figure shows the simulated visibility along the $45^\circ$ (or $135^\circ$) direction for an ideal grating (black solid line) with $4.8 \mu$m period and a grating with the same period but with size of the gold blocks reduced by $1/50$ (red dashed line). The deviation of the peak height is obvious and is similar to the experimental observation in Figs. 3(c) and 3(d). On the other hand, simulations along the $0^\circ$ and $90^\circ$ directions show negligible difference for the two gratings. Since the curves are almost identical to those in Figs. 5(a) and 5(b) with differences much less than the line thickness, we do not show them again. These studies indicate that a slight deviation of the grating duty cycle affects the visibility only along the diagonal directions but not along the other two directions. The sensitivity of this measurement is very high thus can be used to characterize the duty cycle of the gratings to optimize the grating fabrication process.

![Fig. 4. Interference patterns simulated at (a) $d = 83.6$ mm and (b) $d = 41.8$ mm from the grating. The periodicities in different directions are also indicated.](image-url)
5.3 Discussion

Table 1 shows the parameters extracted from the measurement and calculated transverse coherence values. The $\sigma_\theta$ value was extracted from the width of the fitted Gaussian envelope function as shown in Fig. 3. The period of the diffraction pattern $p_\theta$ was obtained as the reciprocal of the first-harmonic peak position in the Fourier image along each direction. The
The coherence length $\xi_{\text{exp}}$ was calculated using Eq. (15) with parameters extracted from the measurement. For comparison, the coherence length was also calculated using Eq. (12) with the source size obtained from the machine studies. The slightly smaller coherence length value from the experiment suggests that the coherence of the beam is affected by the beamline optics. One should note that the source size and the coherence values are usually given only along $0^\circ$ and $90^\circ$, but with our interferometer the x-ray beam coherence along the $45^\circ$ and $135^\circ$ directions are also accessible.

The spatial coherence area of the wavefront is graphically represented in Fig. 7. For the present configuration of the beamline the degradation of the transverse beam coherence is very weak and the Gaussian beam approximation holds for any transverse plane along the downstream. It is known that the source profile from a bending magnet in a synchrotron source is an ellipse with the major axis in the horizontal direction. Therefore, the transverse coherence area of the wavefront downstream has been generally accepted as an ellipse with its major axis along the vertical direction because the coherence length is reciprocal to the source dimension. The measurement result along the $45^\circ$ and $135^\circ$ directions in the present study confirms this for the first time. Furthermore, when the x-ray beam propagates through optics, the wavefront may be altered differently along different directions. The proposed method allows measuring the coherence property along multiple directions simultaneously and quickly. It can be noted that by rotating the 2-D grating the coherence property along any direction can be obtained.

| Parameters | 0° | 45° | 90° | 135° |
|------------|----|-----|-----|------|
| Width of the Gaussian envelop function in Fig. 3 ($\sigma_\theta$) (mm) | 180 | 175 | 430 | 183 |
| Diffraction pattern period ($p$) (µm) | 3.4 | 2.4 | 3.4 | 2.4 |
| Experimental coherence length ($\xi_{\text{exp}}$) (µm) | 3.6 | 5.0 | 8.7 | 5.2 |
| Estimated coherence length calculated analytically with Eq. (12) ($\xi_{\text{ana}}$) (µm) | 4.3 | - | 10.0 | - |
| Simulated coherence length ($\xi_{\text{sim}}$) (µm) | 4.2 | 5.3 | 9.3 | 5.3 |

*Source size obtained from machine studies: $\sigma_x = 85$ µm, $\sigma_y = 37$ µm

Fig. 7. Graphical representation of the coherence area obtained from experiment, simulation, and analytical calculation.
One of the unique characteristics of this technique is its ability to measure the spatial coherence locally in a small region within the field of view due to the use of a high-resolution detector and small-period grating. Figure 8(a), shows two, $25 \times 20 \mu\text{m}^2$ area, adjacent to each other, selected for demonstrating this fact. Figures 8(b) and 8(c) illustrate, respectively, the visibility map along the $90^\circ$ direction for area 1 and area 2 marked in Fig. 8(a). The visibility curves in Figs. 8(b) and 8(c) are not as smooth as that in Fig. 3(b) due to the small area selected for the Fourier transform. The CCF for these two visibility maps gives $\sigma_\theta = 480 \text{ mm}$ for area 1 and $\sigma_\theta = 430 \text{ mm}$ for area 2, producing transverse coherence length of 9.7 $\mu\text{m}$ and 8.7 $\mu\text{m}$, respectively. For area 1, the increased value of the local coherence from that of the entire area (8.7 $\mu\text{m}$) may be attributed to the local transverse coherence property under the effect of upstream optics. However, the area 2 and most of the other regions have almost the same coherence length value as the entire area. This indicates that for the current configuration the beamline optics has minimal effect on the transverse coherence. For this study the upstream optics mainly consisted of three Be windows, with one double window and a DCM. It has been already verified that for a relatively well-polished Be window, degradation of the coherence is very weak [20]. It was also mentioned in [19] that the vibration in the DCM produces an effective source size larger than the actual source size and causes the degradation of the transverse coherence length. However, our measurement did not show any large change in the source size, indicating a relatively stable behavior of the DCM. In future, a systematic study of the local transverse coherence degradation by the beamline optics is planned.

Another reason this single-grating technique is superior to others is the possibility of reconstructing the wavefront from a single fractional Talbot image using the spatial harmonic analysis technique, which was initially developed by the visible light interferometry community and recently applied to x-ray phase-contrast imaging by H. Wen [23]. It has been already shown that the differential phase images generated along two orthogonal directions can be combined into a phase image [24]. Using the phase and the intensity profile, one can obtain the wavefront of the beam. The focus of the present work is to measure the beam coherence; therefore, the detailed analysis of the wavefront will be done separately.

The present study measured the coherence of a hard x-ray beam from a bending magnet source for which the degree of coherence was low compared to insertion device sources. This technique may be also applied to measure spatial coherence as well as the beam wavefronts from certain class of laboratory-based x-ray sources, for example, the newly developed ultra-bright x-ray sources [25].

![Fig. 8. Spatially resolved coherence length measurements within two small area ROI's of the field of view. (a) Measured interferogram at $d = 143$ mm with the ROI’s identified by the white rectangles. (b) and (c) Visibility measurements along the $90^\circ$ direction for the area 1 and area 2, respectively, as identified in Fig. 8(a). The coherence length values for the area 1 and area 2 are 9.7 $\mu\text{m}$ and 8.7 $\mu\text{m}$, respectively.](image-url)
6. Conclusions

The spatial coherence properties of a bending magnet beamline containing a Si (111) DCM and three Be windows were studied. The use of a single 2-D phase grating was demonstrated to measure coherence lengths along four different transverse directions simultaneously. The visibility measurements suggest that the preferred measuring orientation of a 2-D checkerboard grating is along the diagonal directions of the square blocks, where the interferograms have higher visibility and are not sensitive to the deviation of the duty cycle of the grating period. The measured transverse coherence values have been confirmed with the wavefront propagation simulation. The success of the simple analytical equation verified that the intensity distribution of the bending magnet source is close to Gaussian. This technique is model-free, that is, no previous assumption of the shape of the CCF is needed, and it is possible to resolve the spatial coherence locally within the field of view. The technique leads to the complete characterization of the transverse spatial coherence area of the wavefront, which later can be fed into the data processing of coherence-dependent techniques to overcome the partial coherence effects. Also, using this single-grating interferometer in combination with the spatial harmonic technique, it is possible to reconstruct the wavefront. This technique should find extensive applications in x-ray beam spatial coherence characterization in synchrotron sources.

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