

Effects of CDTT Model on the Dynamical Instability of Cylindrically Symmetric Collapsing Stars

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Abstract

We assume cylindrically symmetric stars which begin collapsing by dissipating energy in the form of heat flux. We wish to study the effects of Carroll-Duvvuri-Trodden-Turner (CDTT) model, $f(R) = R + \sigma L_{\text{ad}}^4$, on the range of dynamical instability. For this purpose, perturbation scheme is applied to all the metric functions, material functions and $f(R)$ model to obtain the full set of dynamical equation which control the evolution of the physical variables at the surface of a star. It is found that instability limit involves adiabatic index $\Gamma$ which depends on the density profile and immense terms of perturbed CDTT model. In addition, model is constrained by some requirement, e.g. positivity of physical quantities. We also reduce our results asymptotically as $\mu \to 0$, being the GR results in both the Newtonian and post Newtonian regimes.

1 Introduction

An increasing attention has been paid to the modification of Einstein-Hilbert (EH) action with the lagrangian density $\sqrt{-g}R$, where $g$ is the metric tensor

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and \( R \) is the curvature scalar. A simplest modification is to add terms which are proportional to \( \sqrt{-g} R^n \). It is known that for \( n > 1 \), terms help to understand the standard cosmology at early times showing de-Sitter behavior \[1\]. For \( n < 0 \), these corrections provide a possible gravitational alternative to dark energy by self-accelerating vacuum solutions and hence lead to the cosmic speed-up \[2\].

The most studied, simplest and definiteness \( f(R) \) models in the literature are CDTT model \( f(R) = R + \sigma^\mu R^\mu \), super gravity model \( f(R) = R + \alpha R^2 \) and their mixture generalized CDTT model \( f(R) = R + \sigma^\mu R^\mu + \alpha R^2 \), where \( \sigma = \pm 1 \), \( \alpha \) is a positive real number and \( \mu \) is a parameter with units of mass. Both the CDTT and generalized CDTT models allow de-sitter solution with \( R_0 = \sqrt{3} \mu^2 \), whenever \( \mu > 0 \). By choosing \( \mu \sim H_0 \), the present accelerating expansion can be achieved and even early time cosmological evolution can be recovered. Since laws of gravity gets modified on large distances in \( f(R) \) models, this allows several interesting observational signatures such as modification to the spectra of the galaxy clustering \[3, 4\], cosmic microwave background \[5, 6\] and weak lensing \[7, 8\].

In order to provide \( f(R) \) gravity as a consistent theory of gravity, \( f(R) \) models are severely constrained cosmologically and by gravitational physics \[9-11\]. The best known scale of gravitational physics is the solar system scale. To have stable stellar configuration, the function \( f(R) \) should satisfy the condition, \( d^2 f/dR^2 \geq 0 \) at each point inside the star \[12\]. If this condition is not fulfilled, the star would collapse to some new configuration due to small perturbation. Also, this theory should compared to parameterized post-Newtonian (PPN) approximation, where the non-trivial metric elements outside the stars are \( g_{00} = 1 - \frac{2m}{r} \) and \( g_{11} = \frac{1}{1-\gamma_{PPN} \frac{\gamma_{PPN}}{r}} \) with observational value \( |\gamma_{PPN} - 1| \ll 1 \). However, most of the \( f(R) \) models gives \( \gamma_{PPN} = \frac{1}{2} \), contradicting the observations.

Gravitation theory and relativistic astrophysics have gone through extensive developments to the discovery of stellar collapse. A star has a life cycle wherein it is born in gigantic clouds of dust and galactic material, then evolve and shine for millions of years and eventually enter the phase of dissolution and extinction. It happens when internal pressures subside by nuclear process, gravity takes over, and the star begins to contract and collapses onto itself. Due to the achievements of \( f(R) \) gravity in numerous areas of cosmology and astrophysics, it is quite natural to discuss dynamics of gravitational collapse in this theory. In recent years, we have explored some aspects of
dark energy and gravitational collapse in $f(R)$ theory [13]-[21].

The problem of dynamical instability is closely associated with formation and evolution of self-gravitating objects. The pioneer work in this direction was done by Chandrasekhar [22]. Afterwards, this issue has been investigated for adiabatic, non-adiabatic, anisotropic and shearing viscous fluids by Herrera et al. [23]-[27]. In general, dynamical instability discussed in terms of adiabatic index $\Gamma$. The expression for isotropic spheres may be given by $\Gamma \geq \frac{4}{3} + n \frac{M}{r}$, where $n$ is a number of order unity that depends on the structure of a star, $M$ and $R$ are the mass and radius of a star respectively. For example, for white dwarfs $n = 2.25$. Instability limit of collapsing stars also depends upon the physical quantities describing the properties of different fluids filling the stars. For example, the dissipating quantities increases the instability range at Newtonian corrections but makes the fluid less unstable at relativistic corrections [28]. Also, Chan et al. found interesting results by studying the effects of anisotropy, radiation and shearing viscosity at Newtonian (N) and post-Newtonian (PN) regimes [29]-[31]. Sharif and Azam [32, 33], have discussed the effects of electromagnetic field on the dynamical instability of spherical and cylindrical symmetric gravitational collapse. We have investigated the effects of a well known $f(R) = R + \alpha R^2$ model on the dynamical instability of expansion-free spherically symmetric gravitational collapse [34].

It is a fact that most of the collapse models analyzed so far are spherical due to their wide astrophysical significance. However, the study of non-spherical collapse remains a major uncharted territory, even many attempts have been made, e.g., [35, 36]. Specifically, final fate of collapse of a non-spherical cloud coming from numerical relativity and certain analytical solutions in cylindrical symmetry provide some new examples about gravitational collapse [37, 38]. Nevertheless, a comprehensive analytical treatment of collapse still needs to be done. In this connection, we would like to extend our work on dynamical instability to the cylindrically symmetric collapsing stars in $f(R)$ gravity.

The manuscript is laid out as follows. In Section 2, we provide a full description of the matter distribution, the line element, both inside and outside the fluid boundary, and the field equations. We also formulate the dynamical equations which governs the dynamics of gravitational collapse. In section 3, we consider CDTT model and discuss its feature on cosmology and gravitation. Also, we apply perturbation scheme to all the function and $f(R)$ model. Considering Newtonian and post-Newtonian limits instability range
in the form of Γ factor is obtained. In the last section 4, work is summarized followed by an Appendix.

2 Dynamical Equations

The modified form of EH action leading to \( f(R) \) gravity can be written as

\[
S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R),
\]

where \( \kappa \) is the coupling constant and \( f(R) \) is any arbitrary function of Ricci scalar. The field equations in metric approach are reached by varying above action with respect to \( g_{\alpha\beta} \) as follows

\[
F(R)R_{\alpha\beta} - \frac{1}{2} f(R)g_{\alpha\beta} - \nabla_\alpha \nabla_\beta F(R) + g_{\alpha\beta} \Box F(R) = \kappa T^m_{\alpha\beta}, \quad (\alpha, \beta = 0, 1, 2, 3),
\]

where \( F(R) \equiv \frac{df(R)}{dR} \) and \( \Box = \nabla_\mu \nabla^\mu \) representing the covariant derivative. The total field equations in metric approach are reached via variations of the EH action (2.1)

\[
G_{\alpha\beta} = \frac{\kappa}{F} [T^m_{\alpha\beta} + T^c_{\alpha\beta}],
\]

where

\[
T^c_{\alpha\beta} = \frac{1}{\kappa} \left[ \frac{f(R) - RF(R)}{2} g_{\alpha\beta} + \nabla_\alpha \nabla_\beta F(R) - g_{\alpha\beta} \Box F(R) \right].
\]

This way of writing field equations help to study the dark side of the universe in term of curvature terms provided that it does not satisfy the usual energy conditions. Consequently, this theory may be used to explain the expansion of the universe and other aspects of dark energy in the gravitational physics.

We consider a cylindrically symmetric collapsing star bounded by a hypersurface \( \Sigma \). The interior region inside the boundary can be represented by the following line element

\[
ds^2 = A^2(t, r) dt^2 - B^2(t, r) dr^2 - C^2(t, r) d\theta^2 - dz^2.
\]
In order to preserve cylindrical symmetry, following constraints may be imposed on the coordinates

\[-\infty \leq t \leq \infty, \quad 0 \leq r < \infty, \quad -\infty < z < \infty, \quad 0 \leq \theta \leq 2\pi. \quad (2.6)\]

The exterior region across the boundary of that star can be represented by a cylindrically symmetric manifold in the retarded time coordinate \(\nu\) as follows \([39]\)

\[ds^2_+ = -\frac{2M(\nu)}{r}d\nu^2 + 2drd\nu - r^2(d\theta^2 + \gamma^2dz^2), \quad (2.7)\]

where \(\gamma^2 = -\frac{\Lambda}{3}\), \(\Lambda\) being the cosmological constant while \(M(\nu)\) is the total mass inside the boundary surface. As gravitational collapse is a highly dissipative process, we assume that fluid filling the collapsing cylinder is dissipating energy in the form of heat flux \(q\) and can be represented by the following energy-momentum tensor \([28, 33]\)

\[T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta - pg_{\alpha\beta} + q_\alpha u_\beta + q_\beta u_\alpha, \quad (2.8)\]

where \(\rho\) stands for energy density, \(p\) for pressure and \(u_\alpha\) for four-velocity of the fluid. In co-moving coordinates, these quantities satisfy the relations

\[u^\alpha = A^{-1}\delta^\alpha_0, \quad u^\alpha u_\alpha = 1, \quad q^\alpha = qB^{-1}\delta^\alpha_1, \quad u^\alpha q_\alpha = 0. \quad (2.9)\]

The effects of dissipation describe a wide range of situations. For example, using quasi-static approximation, limiting cases of radiative transport have been studied in \([40]\). It is found that hydrostatic time scale is very small as compared to the stellar lifetimes for different phases of a star’s life. It is of the order of 27 minutes for the sun, 4.5 seconds for a white dwarf and \(10^{-4}\) seconds for a neutron star of one solar mass and 10 km radius \([41]\).

For interior metric, Eqs.\((2.3)\) yield the following set of field equations

\[\left(\frac{A}{B}\right)^2 \left(\frac{B'C'}{BC} - \frac{C''}{C}\right) = \frac{\kappa}{F} \left[\rho A^2 + \frac{A^2}{\kappa} \left\{\frac{f - RF}{2} + \frac{F''}{B^2} + \frac{F'}{B^2} \right\}\right],\]

\[-\frac{F}{A^2} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right], \quad (2.10)\]

\[\frac{\dot{C}'}{C'} + \frac{A'C'}{AC} + \frac{BC'}{BC} = \frac{\kappa}{F} \left[qAB + \frac{1}{\kappa} \left(\dot{F}' - \frac{A'}{A}F - \frac{\dot{B}}{B}F'\right)\right], \quad (2.11)\]
\[
\left( \frac{B}{A} \right)^2 \left( \frac{\dot{A} \dot{C} - \dot{C}}{AC} \right) + A' C' = \frac{\kappa}{F} \left[ p B^2 + \frac{B^2}{\kappa} \left\{ -\frac{f - RF}{2} + \ddot{F} \right\} \right],
\]

\[
+ \frac{\dot{F}}{A^2 C} + \frac{F'}{B^2} \left( \frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C} \right) \right],
\]

\[
\frac{1}{AB} \left( \frac{A''}{B} - \frac{\ddot{B}}{A} + \frac{\dot{A} \dot{B}}{A^2} - \frac{A'B'}{B^2} \right) = \frac{\kappa}{F} \left[ \rho - \frac{1}{\kappa} \left\{ -\frac{f - RF}{2} + \ddot{F} \right\} \right] - \frac{F''}{B^2} + \frac{\dot{F}}{A^2} - \frac{F'}{B^2} \left( \frac{A'}{A} + \frac{C'}{C} \right),
\]

\[
(2.12)
\]

\[
(2.13)
\]

\[
(2.14)
\]

Here dot and prime denote derivatives with respect to \( t \) and \( r \) respectively. Subtracting Eq. (2.13) from (2.12), we have

\[
\frac{1}{A^2} \left( \frac{\dot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{F}}{F} \right) \frac{\dot{C}}{C} + \frac{1}{B^2} \left( \frac{A'}{A} - \frac{B'}{B} - \frac{F'}{F} \right) C' + \frac{1}{C} \left( \frac{C''}{B^2} - \frac{\ddot{C}}{A^2} \right) = 0.
\]

\[
(2.15)
\]

The dynamical equations help to investigate the evolution of gravitational collapse with time and yield the variation of total energy inside a collapsing body with respect to time and adjacent surfaces. We have formulated these equations by using contracted Bianchi identities both for the usual matter and effective energy-momentum tensor carrying curvature terms. These are given by

\[
\left( \begin{array}{c}
\left( \frac{m}{(m)} \right) T^{\alpha \beta} + \left( \frac{c}{(c)} \right) T^{\alpha \beta}
\end{array} \right) \left. \right|_{\beta} u_{\alpha} = 0, \quad \left( \begin{array}{c}
\left( \frac{m}{(m)} \right) T^{\alpha \beta} + \left( \frac{c}{(c)} \right) T^{\alpha \beta}
\end{array} \right) \left. \right|_{\beta} \chi_{\alpha} = 0.
\]

\[
(2.16)
\]

The two independent components of Bianchi identities for the system under
consideration read

\[ \dot{\rho} - q' \frac{A}{B} + q \frac{A}{B} \left( \frac{2A'}{A} - \frac{C'}{C} \right) + (\rho + p) \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \phi_1(r, t) = 0, \]

(2.17)

\[ \rho' + \dot{q} A + \frac{B}{A} \left( \frac{2\dot{B}}{B} + \frac{\dot{C}}{C} \right) + (\rho + p) \frac{A'}{A} + \phi_2(r, t) = 0, \]

(2.18)

where \( \phi_1(r, t) \) and \( \phi_2(r, t) \) are given in Eqs. (5.1) and (5.2) respectively in Appendix.

3 CDTT Model and Perturbation Scheme

In this section, we consider the following form of CDTT model,

\[ f(R) = R + \sigma \frac{\mu^4}{R}, \]

(3.1)

where \( \sigma \) was taken equal to \(-1\) in the original paper [2]. It is claimed that with the parameter \( \mu \) chosen as the order of the inverse of the universe age, \( \mu^{-1} \sim 10^{18} \text{ sec} \sim (10^{33} \text{eV})^{-1} \), this model can be applied to cosmology and describes the present accelerating expansion of the universe. However, this additional term was not essential to describe earlier time inflation. Further, CDTT model has a homogeneous solution of de-Sitter metric with constant scalar curvature \( R = R_0 \equiv \sqrt{3} \mu^2 \). Thus, in order to have the desired physically plausible late time behavior, we have to set \( \sqrt{3} \mu^2 \sim H_0^2 \).

Henttunen, et al. [42] have analyzed the properties of polytropic stars by considering CDTT model with the conclusion that the density profiles in general resemble the Newtonian Lane-Emden solutions. They analyzed that interior solution for metric components and curvature of such stars are always singular, consequently, \( f(R) = R - \mu^4/R \) model is not experimentally suitable to describe the space-time around the sun. A possible way to avoid is to relax the requirement data set for the central boundary conditions, but a more plausible approach is to modify the functional form of \( f(R) \). As it is mentioned in the introduction that sign of \( f''(R) \) determine whether this modified theory approaches the GR limit or not at high curvature. For \( f''(R) > 0 \), the model behave very close to GR and is stable and vice versa. Due to this reason, the original CDTT model was severely criticized [43, 44].
In order to avoid instability, Hu and swaicky introduced CDTT model with $\sigma = 1$ [45]. Thus, throughout the paper, we shall use following form

$$f(R) = R + \frac{\mu^4}{R}.$$  \hfill (3.2)

For the present model, the stability condition is satisfied with extremely small and positive $f''(R)$ everywhere inside and near the polytrope. Also, it aims to explain the current accelerated expansion of the universe. At least for this model, Einstein evolution can be recovered for most of the cosmic history, especially standard epoch of matter domination can be obtained, providing a sufficiently long time to satisfy observations. It is mentioned here that the additional inverse term will not significantly alter standard evolution until today and that the solution lies well within present constraints from Big-Bang Nucleosynthesis [46].

It is noticeable that CDTT model adds a perturbative function in the form of inverse power to the EH action. Thus for such a modification, the Einstein cosmology may be recovered in the limit that the perturbation term disappear. This system has great similarity with perturbation theory [47]. First, we analyze our work on dynamical instability when $\mu \neq 0$, then we reduce our results asymptotically, $\mu \to 0$, being the Einstein solution.

In this paper, the perturbation scheme introduced is used to analyze the instability conditions of the dynamical equations. We assume that initially all the function whether material or metric have only radial dependence, i.e., lie in the static equilibrium. However, afterwards, all these quantities have time dependence in their perturbation. Taking $0 < \epsilon \ll 1$, quantities may be written in the following manner

\begin{align*}
A(t, r) & = A_0(r) + \epsilon T(t)a(r), \quad (3.3) \\
B(t, r) & = B_0(r) + \epsilon T(t)b(r), \quad (3.4) \\
C(t, r) & = rB(t, r)[1 + \epsilon T(t)e(r)], \quad (3.5) \\
\rho(t, r) & = \rho_0(r) + \epsilon \tilde{\rho}(t, r), \quad (3.6) \\
p(t, r) & = p_0(r) + \epsilon \tilde{p}(t, r), \quad (3.7) \\
m(t, r) & = m_0(r) + \epsilon \tilde{m}(t, r), \quad (3.8) \\
q(t, r) & = \epsilon \tilde{q}(t, r), \quad (3.9) \\
R(t, r) & = R_0(r) + \epsilon T(t)e(r), \quad (3.10) \\
f(R) & = [R_0(r) + 2\mu^4R_0^{-1}(r)] + \epsilon T(t)e(r)[1 - 2\mu^4R_0^{-2}(r)], \quad (3.11) \\
F(R) & = 1 - \mu^4R_0^{-2}(r) + 2\epsilon \mu^4R_0^{-4}(r)T(t)e(r). \quad (3.12)
\end{align*}
The static configuration of the field equations (2.10)-(2.12) and (2.15) is obtained as

\[
\frac{\kappa}{1 - \mu^4 R_0^2} \left[ \rho_0 + \frac{\mu^4 R_0^{-1}}{\kappa} \left\{ 1 + 2 R_0^{-2}(-3 R_0^{-1} R_0'' + R_0'') \right\} \right] \\
= \frac{1}{B_0^2} \left[ \left( \frac{B_0'}{B_0} \right)^2 - \frac{B_0'}{r B_0} - \frac{B_0''}{B_0} \right],
\]

(3.13)

\[
\frac{\kappa}{1 - \mu^4 R_0^2} \left[ p_{\nu 0} - \frac{\mu^4 R_0^{-1}}{\kappa} \left\{ 1 - \frac{2 R_0^{-2} R_0'}{B_0^2} \left( \frac{A_0'}{A_0} + \frac{2 B_0'}{B_0} + \frac{1}{r} \right) \right\} \right] \\
= \frac{1}{B_0^2} \frac{A_0'}{A_0} \left( \frac{B_0'}{B_0} + \frac{1}{r} \right),
\]

(3.14)

\[
\left( \frac{A_0'}{A_0} - \frac{B_0'}{B_0} - \frac{2 \mu^4 R_0^{-3} R_0'}{1 - \mu^4 R_0^{-2}} \right) \left( \frac{B_0'}{B_0} + \frac{1}{r} \right) + \frac{2 B_0'}{r B_0} + \frac{B_0''}{B_0} = 0.
\]

(3.15)

Applying static part of the perturbed system, the first dynamical equation (2.17) is identically satisfied while (2.18) yields

\[
p_r' + (\rho_0 + p_0) \frac{A_0'}{A_0} + \kappa \phi_{2s} = 0,
\]

(3.16)

where \( \phi_{2s} \) is the static part of equation \( \phi_2 \). The perturbed configurations of Eq.(2.17) leads to

\[
\dot{\bar{\rho}} - \bar{q} \frac{A_0'}{B_0} + \bar{q} \frac{A_0}{B_0} \left( \frac{2 A_0'}{A_0} - \frac{B_0'}{B_0} - \frac{1}{r} \right) + \left[ (\rho_0 + p_0) \left( \frac{2 b}{B_0} + \frac{c}{B_0} \right) + \kappa \phi_{1p} \right] \dot{T} = 0.
\]

(3.17)

Here \( \phi_{1p} \) denote the perturbed form of equation \( \phi_1 \) and is given in Appendix. Applying perturbation on Eq. (2.11) and eliminating \( \bar{q} \), we have

\[
\bar{q} = -\frac{1 - \mu^4 R_0^{-2}}{\kappa A_0 B_0} \left[ \frac{1}{B_0} \left\{ \frac{b + c}{r} + b' + c' + \frac{A_0'}{A_0} (b + c) \right\} \right. \\
+ \left. \frac{R_0^{-1}}{1 - \mu^4 R_0^{-2}} \left\{ 2 \mu^4 (e' + 4 e R_0^{-1} R_0' - e A_0') \right\} + \frac{b}{B_0} \right] \dot{T}.
\]

(3.18)

Differentiating above equation and substituting in the second and third term of Eq.(3.17) and then integrating resulting equation with respect to “t”, we
have

\[ \bar{\rho} = \left[ - (\rho_0 + p r_0) \left( \frac{2b}{B_0} + \frac{\bar{c}}{B_0} \right) + \phi_3(r) \right] T, \quad (3.19) \]

where \( \phi_3(r) \) is given in Appendix. Considering the second law of thermodynamics, we can express a relationship between \( \bar{\rho} \) and \( \bar{p}r \) as the ratio of specific heat by assuming an equation of state of Harrison-Wheeler type as follows \[48, 49\]

\[ \bar{p} = \Gamma \frac{p r_0}{\rho_0 + p r_0} \bar{\rho}. \quad (3.20) \]

Here \( \Gamma \) measures the variation of pressure for a given variation of density. We take it constant throughout the region that we want to study. Substituting value of \( \bar{\rho} \) from Eq. (3.19) in (3.20), we have

\[ \bar{p} = - \Gamma p_0 \left( \frac{2b}{B_0} + \frac{\bar{c}}{B_0} \right) T + \Gamma \frac{p_0}{\rho_0 + p_0} \phi_3 T, \quad (3.21) \]

The perturbed configurations of second Bianchi identity is

\[ \ddot{\bar{q}} + \bar{q} \frac{B_0}{A_0} + (\bar{\rho} + \bar{p}) \frac{A'_0}{A_0} + (\rho_0 + p_0) \left( \frac{a}{A_0} \right)' T + \frac{T}{\kappa} \phi_{2p} = 0, \quad (3.22) \]

where \( \phi_{2p} \) represents the perturbed part of \( \phi_2 \) and is provided in the Appendix. Substituting values of \( \bar{q}, \bar{\rho} \) and \( \bar{p} \) from Eqs. (3.18), (3.19) and (3.21) respectively in the above equation, we have

\[ \Gamma \left[ - p_0 \left( \frac{2b}{B_0} + \frac{\bar{c}}{B_0} \right) + \frac{p_0}{\rho_0 + p_0} \phi_3 \right] T + (\rho_0 + p_0) \left( \frac{a}{A_0} \right)' T 
- \frac{A'_0}{A_0} \left[ (\rho_0 + p_0 + \Gamma p_0) \left( \frac{2b}{B_0} + \frac{\bar{c}}{B_0} \right) - \left( \Gamma \frac{p_0}{\rho_0 + p_0} + 1 \right) \phi_3 \right] T 
+ \frac{T}{\kappa} \phi_{2p} - \frac{1 - \mu^4 R_0^{-2}}{\kappa A_0^2} \left[ \frac{1}{B_0} \left\{ b + \bar{c} + b' + \bar{c}' + \frac{A'_0}{A_0} (b + \bar{c}) \right\} 
+ \frac{R_0^{-4}}{1 - \mu^4 R_0^{-2}} \left\{ 2\mu^4 (e' + 4eR_0^{-1}R'_0 - e A'_0 \frac{A_0}{A_0}) \right\} \right] \dot{T} = 0. \quad (3.23) \]

It is mentioned here that above equation is the required evolution equation
for further analysis. The Ricci scalar curvature is given by
\[
R = -\frac{2}{B^2} \left( \frac{A''}{A} + \frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} \right)
- \frac{2}{A^2} \left( \frac{\dot{B}}{B} + \frac{\dot{A}'B}{AB} + \frac{\ddot{C}}{C} - \frac{\dot{A}'C}{AC} + \frac{\dot{B}'C}{BC} \right).
\] (3.24)

The static part of the above equation is obtained as follows
\[
R_0(r) = -\frac{2}{B_0^2} \left[ -\frac{A''_0}{A_0} - \frac{B''_0}{B_0} + \frac{2A'_0B'_0}{A_0B_0} + \frac{A''_0}{rA_0} - \frac{B'_0}{rB_0} + \frac{B^2_0}{B_0^2} \right].
\] (3.25)

The perturbed configuration of the Ricci scalar curvature with the use of Eq.(3.25) yields
\[
-\frac{1}{A_0B_0} \left( a'' - \frac{A''_0}{A_0} - \frac{2bA''_0}{B_0} \right) - \frac{2}{rB_0^3} \left( \frac{b'}{r} + \frac{c'B'}{r} - \frac{\ddot{c}B'_0}{r} - \frac{3bB'_0}{B_0} \right)
+ \frac{e}{2} + \frac{\ddot{c}}{r^3} - \frac{1}{A_0B_0^3} \left( a'B'_0 + \frac{bA'_0}{A_0} - \frac{aA'_0B'_0}{B_0} - \frac{3bA'_0B'_0}{B_0} \right)
+ \frac{2\ddot{c}}{rB_0^2} + \frac{1}{B_0^2r^2} \left( c' - \frac{b}{B_0} - \frac{\ddot{c}}{r} \right) - \frac{\ddot{T}}{T} \left( \frac{b}{rB_0} - \frac{\ddot{c}}{r} \right) \frac{1}{A_0^2} = 0.
\] (3.26)

This equation can also be written as
\[
\ddot{T}(t) - \phi_4(r)T(t) = 0,
\] (3.27)

where \(\phi_4(r)\) is given in the Appendix. In order to find the instability range, we assume that all the terms of equation \(\phi_4\) are such that it remains positive. Consequently, the solution of Eq.(3.27) is obtained as
\[
T(t) = -e^{\sqrt{\phi_4}t}.
\] (3.28)

In the following subsections, we use above value in the dynamical equation Eq.(3.23) to analyze the evolution of the physical variables at the surface of the star.

**Newtonian Limit**

In this regime, we assume that \(\rho_0 \gg p_0\) and \(A_0 = 1, B_0 = 1\) in order to fulfill the Newtonian limit. Thus substituting \(A'_0 = 0, B'_0 = 0\) in Eq.(3.23),
we obtain

\[- \frac{1 - \mu^4 R_0^{-2}}{\kappa} \left[ \frac{2b + \bar{c}}{r} + b' + \bar{c} + \frac{2 \mu^4 R_0^{-4} (e' - 4 e R_0^{-1} R_0')}{1 - \mu^4 R_0^{-2}} \right] \hat{T}
+ \left[ \Gamma \{ -p_0(2b + \bar{c}) \} + a' \rho_0 + \frac{\phi_{2p(N)}}{\kappa} \right] T = 0. \tag{3.29}\]

It is mentioned here that \(\phi_{2p(N)}\) represent terms belonging to the Newtonian regime of perturbed second Bianchi identity. Substituting value of \(T\) from Eq.(3.28), we have

\[
\Gamma < \frac{a' \rho_0 + \phi_5(r)}{p_0(2b + \bar{c})}', \tag{3.30}\]

where \(\phi_5(r)\) is given in Appendix. It is noticeable here that adiabatic index depends upon the energy density, pressure and curvature terms in the Newtonian regime. Thus the collapsing system would remains unstable until Eq.(3.30) satisfied.

**Asymptotic Behavior**

When \(\mu \to 0\), the expression for \(\Gamma\) becomes

\[
\Gamma < \frac{a' \rho_0 + \frac{\phi_5}{\kappa} \left( \frac{2b + \bar{c}}{r} + b' + \bar{c} \right)}{p_0(2b + \bar{c})}'. \tag{3.31}\]

This results represents the GR solution.

**Post Newtonian Limit**

Here, we include relativistic effects upt to order \(\frac{m_a}{r}\). In this case, we have

\[
A_0 = 1 - \frac{m_0}{r}, \quad B_0 = 1 + \frac{m_0}{r}, \tag{3.32}\]

\[
\Rightarrow \quad \frac{A'}{A_0} = \frac{m_0}{r(r - m_0)}, \quad \frac{B'}{B_0} = \frac{-m_0}{r(r + m_0)}. \tag{3.33}\]

Substituting above values in Eq.(3.23), we obtain

\[
\Gamma > \frac{\frac{m_0}{r}}{p_0(2b + \bar{c})(1 - \frac{m_0}{r}) - \frac{\rho_0 \phi_{3(pN)}}{\rho_0 + p_0}} \left[ p_0(2b + \bar{c})(1 - \frac{m_0}{r}) \right]' + \frac{\phi_6(r)}{p_0(2b + \bar{c})(1 - \frac{m_0}{r}) - \frac{\rho_0 \phi_{3(pN)}}{\rho_0 + p_0}}. \tag{3.34}\]
In the above expression \( \phi_{3(PN)} \) denote those terms of \( \phi_3 \) which belong to PN regime whereas \( \phi_6(r) \) is given in Appendix. Thus, system would be unstable in PN-approximation as long as the above inequality is satisfied. It can be observed that how relativistic and curvature terms are affecting the instability range of collapsing star. Further, in order to fulfill the dynamical instability condition, we need to keep all the terms positive. Hence, we assume that all the quantities involved in the above expression are positive. In addition, following constraints should be satisfied

\[
(r_0 + p_0)(2b + \bar{c}) > \phi_{3(PN)},
\]

and

\[
p_0(2b + \bar{c})(1 - \frac{m_0}{r}) > \frac{\rho_0 \phi_{3(PN)}}{\rho_0 + p_0},
\]

along with \( \frac{m_0}{r} < 1 \).

**Asymptotic Behavior**

As \( \mu \to 0 \), adiabatic index \( \Gamma \) has the same representation. However, in this case \( \phi_3 \) and \( \phi_6 \) respectively reduced to

\[
\phi_3(r) = \frac{1}{\kappa} \left[ \left( \frac{2b + \bar{c}}{r} + b' + \bar{c}' + \frac{bm_0}{r^2} \right) \frac{1}{r} - \frac{1}{\rho_0} \left( \frac{2b + \bar{c}}{r} + b' + \bar{c}' + \frac{bm_0}{r^2} \right) \right]
\]

\[
+ \frac{m_0}{r\kappa} \left[ \left( \frac{2b + \bar{c}}{r} + b' + \bar{c}' \right) \frac{1}{r} - \frac{2}{\rho_0} \left( \frac{2b + \bar{c}}{r} + b' + \bar{c}' \right) \right],
\]

\[
\phi_6(r) = \frac{\phi_4}{\kappa} \left[ \frac{3b + \bar{c}}{r} + \frac{3m_0}{r^2} + (b' + \bar{c}') \left( 1 + \frac{2m_0}{r} \right) \right].
\]

It is mentioned here that in both the N and PN regimes heat flux is not affecting limit of dynamical instability.

**4 Summary**

In this paper, we have assumed dissipative fluid configuration of a non-static cylindrically symmetric collapsing star and discussed the dynamical instability for the final stages of the star by a perturbative approach. As study of gravitational collapse could be used as a paradigm to understand stellar formation. We introduced this issue in modified \( f(R) \) theory of gravity.
We have considered CDTT model which leads standard cosmic history. As short-timescale instabilities are avoided with extremely small and positive $f''(R)$, so we have considered modified CDTT model with positive term $\frac{\mu^4}{R}$. It is worthwhile to mention here that this model satisfying the condition $f''(R) > 0$ have stable high curvature limits and a well behaved cosmological solution with proper era of matter domination.

To see the effects of CDTT model on the dynamical instability of fluid evolution during the collapsing process, we have applied a perturbation scheme on the field equation and dynamical equations. It is worth mentioned here that recently we have worked on spherically symmetric gravitational collapse evolving under the expansionfree condition \[34\]. In this study, the range of instability is independent of adiabatic index $\Gamma$ showing the consistency of obtained results with the expansionfree condition which requires that fluid would evolve without compressibility. However, no such condition is imposed in this work, hence results appeared in the form of adiabatic index.

Also, let us make the comparison of our results with Chandrasekhar's results on the limit of dynamical instability of spherically symmetric configuration of matter \[22\]. In this work, dynamical instability depends upon the numerical value of the adiabatic index. It is found that if $\Gamma > 4/3$, the pressure in a star is strong enough than the weight of the outer layers which make a star stable. Whereas, for $\Gamma < 4/3$, the weight increases very fast than the pressure and star collapses resulting a dynamical instability. As concerned to work done in this paper, results depends on the physical quantities, like energy density, pressure, curvature terms and mass of the cylinder. Thus, in Newtonian regime collapsing star yield the instability limit in the the form of Eq. (3.30). By reversing the sign stability of the system would be achieved. However, in PN regime instability range not only depends on Eq. (3.34) but also on certain constraints in Eq. (3.35) and (3.36).

Finally, it is mentioned here that our previous work had been done on super gravity model while present study made on CDTT model, hence the focus of future work is the combination of both the above models, i.e., generalized CDTT model in any of the symmetry.
5 Appendix

\[ \phi_1(r, t) = \frac{A^2}{\kappa} \left[ \frac{f - RF}{2A^2} + \frac{F''}{A^2B^2} - \frac{F}{A^4} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{F'}{A^2B^2C} \right] + \frac{\dot{A}}{\kappa A} \left[ \frac{f - RF}{2} + \frac{2F'}{B^2} \right] - \frac{\dot{F}}{A^2} \left( \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) \]

\[ + \frac{\dot{B}}{\kappa B} \left[ \frac{\ddot{F}}{A^2} + \frac{\dot{F}}{A^2} \left( \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) + \frac{F'}{B^2} \left( -\frac{2A'}{A} + \frac{C'}{C} \right) \right] \]

\[ + \frac{\dot{C}}{\kappa C} \left[ \frac{\ddot{F}}{A^2} - \frac{\dot{F}}{A^2} \left( \frac{2\dot{A}}{A} + \frac{\dot{C}}{C} \right) - \frac{F'}{B^2} \left( -\frac{2A'}{A} + \frac{C'}{C} \right) \right] + \frac{1}{\kappa B^2} \left( \dddot{F}' - \frac{A'}{A} \ddot{F} - \frac{\dot{B}}{B} F' \right) \right] + \frac{A^2}{\kappa} \left[ \frac{f - RF}{2B^2} + \frac{\ddot{F}}{A^2} + \frac{\dot{F}}{A^2} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{F'}{B^2} \left( \frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C} \right) \right], \quad (5.1) \]

\[ \phi_2(r, t) = -\frac{B^2}{\kappa} \left[ \frac{1}{A^2B^2} \left( \dddot{F}' - \frac{A'}{A} \ddot{F} - \frac{\dot{B}}{B} F' \right) \right] + \frac{A^2}{\kappa A} \left[ -\frac{\ddot{F}}{A^2} + \frac{F''}{A^2} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{2\dot{C}}{C} \right) - \frac{F'}{B^2} \left( \frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C} \right) \right] \]

\[ + \frac{B^2}{\kappa B} \left[ -\frac{f - RF}{2A^2} + \frac{\ddot{F}}{A^2} + \frac{\dot{F}}{A^2} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) - \frac{F'}{B^2} \left( \frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C} \right) \right] + \frac{C^2}{\kappa C} \left[ \frac{F''}{B^2} + \frac{\dot{F}}{A^2} \left( \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) + \frac{F'}{B^2} \left( \frac{2A'}{A} + \frac{B'}{B} + \frac{C'}{C} \right) \right] \]

\[ + \frac{1}{\kappa A^2} \left( \dddot{F}' - \frac{A'}{A} \ddot{F} - \frac{\dot{B}}{B} F' \right) \left( \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C} \right). \quad (5.2) \]

\[ \phi_{2s} = -\frac{2\mu^4 R_0^3 A'_0}{B'_0 A_0} \left[ 3R_0^{-1} R_0'^2 + R'_0 + R_0 \left( \frac{A'_0}{A_0} + \frac{B'_0}{B_0} + \frac{1}{r} \right) \right] \]
\[
\phi_3(r) = \frac{A_0}{\kappa B_0} \left[ \frac{1 - \mu^4 R_0^{-2}}{A_0 B_0^2} \left\{ \frac{b + \bar{c}}{r} + b' + \bar{c}' + \frac{A_0'}{A_0} (b + \bar{c}) \right\} \right. \\
- \left. \frac{2 A_0 \mu^4 R_0^{-4}}{B_0} \left\{ -e' + 4e R_0^{-1} R_0' + e \frac{A_0'}{A_0} \right\} + \frac{b}{A_0 B_0^2} \right], \\
- \frac{A_0}{\kappa B_0} \left[ \frac{1 - \mu^4 R_0^{-2}}{A_0 B_0} \left\{ \frac{b + \bar{c}}{r} + b' + \bar{c}' + \frac{A_0'}{A_0} (b + \bar{c}) \right\} \right. \\
- \left. \frac{2 A_0 \mu^4 R_0^{-4}}{B_0^2} \left\{ -e' + 4e R_0^{-1} R_0' + e \frac{A_0'}{A_0} \right\} + \frac{b}{A_0 B_0} \right] \\
\times \left( \frac{2 A_0'}{A_0} + \frac{B_0'}{B_0} - \frac{1}{r} \right) - \phi_{1p},
\]
\[\phi_{2p} = \frac{2\mu^4 R_0^{-3}}{B_0^2} \frac{\ddot{T}}{T} \left[ e'R_0^{-1} - 4eR_0^{-2}R'_0 + eR_0^{-1} \frac{A'_0}{A_0} - R'_0 \frac{b}{B_0} + \frac{eR_0^{-1}B'_0}{2B_0} \right]
- \left[ \frac{2\mu^4 R_0^{-3}R'_0}{B_0^2} \left\{ R'_0 \left( \frac{a'}{A_0} \right)' + \left( \frac{b}{b_0} \right)' \left( \frac{b + \bar{c}}{rB_0} \right) \left( 2 + r \frac{B'_0}{B_0} \right) + \frac{b'}{B_0} \right\} \right. + \left. \frac{\bar{c}}{B_0} + \frac{1}{R'_0} \left( 2R'_0 \frac{b}{B_0} - e' R_0^{-1} + 4 e R_0^{-2} R'_0 \right) \left( \frac{A'_0}{A_0} + \frac{2B'_0}{B_0} + \frac{1}{r} \right) \right] + \frac{A'_0}{A_0} \left[ \frac{2\mu^4 R_0^{-3}R'_0}{B_0^2} \left( -3R_0^{-1}R'_0 + R''_0 \frac{A'_0}{A_0} + \frac{2B'_0}{B_0} + \frac{1}{r} \right) \right] \times \left( \frac{a'}{A'_0} + \frac{a}{A_0} \right) - \frac{2\mu^4 R_0^{-3}}{B_0^2} \{ e'' + 4 e R_0^{-1} R'_0 - 4 R_0^{-1} (e' R'_0 - 5e R_0^{-2} R'_0) \}
- 5e R_0^{-1} R_0'' + 4 e R_0 R_0''} \right] + \frac{2\mu^4 R_0^{-3}}{B_0^2} \left( \frac{2R'_0}{B_0^2} + e' R_0^{-1} - 4e R_0^{-2} R'_0 \right) \times \left( \frac{A'_0}{A_0} + \frac{2B'_0}{B_0} + \frac{1}{r} \right) - \frac{2\mu^4 R_0^{-3}}{B_0^2} \left\{ \left( \frac{a'}{A_0} \right)' + \left( \frac{b}{B_0} \right)' \left( \frac{b + \bar{c}}{rB_0} \right) \left( 2 + r \frac{B'_0}{B_0} \right) + \left( \frac{b'}{B_0} \right)' \right\} \times \left( \frac{b}{B_0} + e' R_0^{-1} - 4e R_0^{-2} R'_0 \right) - c e R_0^{-1} - 2 c e R_0^{-3} R'_0 \times \left\{ \left( \frac{a}{A_0} \right)' + \left( \frac{b}{B_0} \right)' \left( \frac{b + \bar{c}}{rB_0} \right) \left( 2 + r \frac{B'_0}{B_0} \right) + \left( \frac{b'}{B_0} \right)' \right\} \right\} + \frac{1}{rB_0} \left[ \frac{2\mu^4 R_0^{-3}R'_0}{B_0^2} \left( -3R_0^{-1}R'_0 + R''_0 \frac{A'_0}{A_0} + \frac{2B'_0}{B_0} + \frac{1}{r} \right) \right] \times \left( \frac{b'}{B_0} + \bar{c} \right) (2 + r \frac{B'_0}{B_0}) + \frac{1}{rB_0} \left\{ 4 e R_0'' \right\} - R_0^{-1} R_0'' + 2 \mu^4 R_0^{-4} (e'' + 4 R_0^{-1} R_0' e'') - 8 \mu^4 R_0^{-5} (e' R'_0 - 5e R_0^{-1} R_0'') + 4 e R_0'' - 2 \mu^4 R_0^{-3} \left( \frac{2A'_0}{A_0} + \frac{2B'_0}{B_0} + \frac{1}{r} \right) \left( e' R_0^{-1} - 4e R_0^{-2} R'_0 - \frac{2R'_0}{B_0} \right) \times \left( \frac{b'}{B_0} + \bar{c} \right) (2 + r \frac{B'_0}{B_0}) + 2 \mu^4 R_0^{-3} R'_0 \left\{ \left( \frac{a}{A_0} \right)' + \left( \frac{b}{B_0} \right)' \left( \frac{b + \bar{c}}{rB_0} \right) \left( 2 + r \frac{B'_0}{B_0} \right) \right\} + \left( \frac{b'}{B_0} + \bar{c} \right) \right\} \right\} \right\} \right\} \right\}. \quad (5.6)\]
\[ \phi_4(r) = \frac{A_0^2B_0}{2(1 + b + \bar{c})} \left[ e - \frac{4b}{B_0} \left( -\frac{A_0''}{A_0} - \frac{B_0''}{B_0} - \frac{2A_0'B_0'}{AB_0} + \frac{A_0'}{rA_0} \right) \right. \\
+ \left. \frac{B_0' B_0''}{rB_0 + B_0''} \right] - \frac{2 B_0'}{B_0} \left\{ \frac{A_0''}{A_0} - \frac{a''}{a} \right\} + \frac{2B_0' + rB_0''}{rB_0} \left( b + \bar{c} \right) \\
- \frac{2b' + 2\bar{c}' + b''r + \bar{c}''r}{2B_0' + rB_0''} \left( B_0' \left( \frac{a}{A_0} \right) + \frac{A_0'}{A_0} \left( b \right) \right) + \frac{B_0 + rB_0'}{rB_0} \times \left\{ \left( \frac{a}{A_0} \right)' + \left( \frac{b}{B_0} \right)' \right\} + \left( b + \bar{c} \right) \left( \frac{A_0'}{A_0} + \frac{B_0'}{B_0} \right) \right] . \] (5.7)

\[ \phi_5(r) = \frac{\phi_4(1 - \mu^4R_0^{-2})}{\kappa} \left[ \frac{2b + \bar{c}}{r} + b' + \bar{c}' + \frac{2\mu^4R_0^{-4}(e' - 4eR_0^{-1}R_0')}{1 - \mu^4R_0^{-2}} \right] - \frac{\phi_{2p(N)}}{\kappa} . \] (5.8)

\[ \phi_6(r) = \frac{\phi_4(1 - \mu^4R_0^{-2})}{\kappa} \left[ \frac{2b + \bar{c}}{r} + b' + \bar{c}' + \frac{m_0(b + \bar{c})}{r^2} + \frac{b}{r} \left( 1 + \frac{m_0}{r} \right) \right] - \frac{2\mu^4R_0^{-4}}{1 - \mu^4R_0^{-2}} \left( -e' + 4eR_0^{-1}R_0' + e \frac{m_0}{r^2} \right) \right] - \frac{\phi_{2p(N)}}{\kappa} + \frac{\phi_4(1 - \mu^4R_0^{-2})}{\kappa} \times \frac{2m_0}{r} \left[ \frac{2b + \bar{c}}{r} + b' + \bar{c}' + \frac{2\mu^4R_0^{-4}}{1 - \mu^4R_0^{-2}}(e' + 4eR_0^{-1}R_0') \right] . \] (5.9)

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