Fully-heavy tetraquark spectra and production at hadron colliders

Ruilin Zhu\textsuperscript{1,2}\textsuperscript{*}

\textit{\textsuperscript{1}Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, China}
\textit{\textsuperscript{2}Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA}

(Dated: October 20, 2020)

Abstract

Motivated by the observation of exotic structure around 6900MeV in the $J/\psi$-pair mass spectrum using proton-proton collision data by the LHCb collaboration, we study the spectra of fully-heavy tetraquarks within Regge trajectory relation. The X(6900) may be explained as a radially excited state with quark content $cc\bar{c}\bar{c}$ and spin-parity $0^{++}$ or $2^{++}$ or an orbitally excited state with $1^{-+}$. New $cc\bar{c}\bar{c}$ structures around 5990MeV, 6450MeV, and 7280MeV are predicted. Other $bb\bar{b}\bar{b}$ and $bc\bar{b}\bar{c}$ structures which may be experimentally prominent are discussed. On the other hand, the fully-heavy S-wave tetraquark production at hadron colliders are investigated and their cross sections are obtained.

\textsuperscript{*}Electronic address: rlzhu@njnu.edu.cn
I. INTRODUCTION

Color confining is a long-standing open question since the invention of Quantum Chromodynamics (QCD). To understand its nature, one phenomenological way is to establish the hadron spectroscopy. In naive quark model, meson is composed of a quark-antiquark pair while baryon is composed of three quarks. However, one could also imagine in general that there would be the possibility of Glueball with parton content \((gg, ggg, \ldots)\) and multi-quark states with parton content \((qq\bar{q}, qqq\bar{q}, \ldots)\) since they do not violate the principle of QCD color confining. There is great progress in the search of multi-quark states. Many new exotic structures beyond the naive quark model have been observed since the discovery of \(X(3872)\) in 2003 [1, 2], for a recent review of these new structures, see Refs. [3–7].

Very recently, the LHCb collaboration have reported the discovery of one exotic structure around 6900 MeV in the invariant mass spectrum of \(J/\psi\) pairs at the Large Hadron Collider [8]. This is a possible candidate for a fully charm tetraquark and has the quark content \(cc\bar{c}\bar{c}\). Its mass and decay width are determined from two model scenarios

\[
m_{X(6900)} = 6905 \pm 11 \pm 7 \text{MeV},
\]

\[
\Gamma_{X(6900)} = 80 \pm 19 \pm 33 \text{MeV},
\]

for model-I with no interference with the non-resonance single parton scattering continuum;

\[
m_{X(6900)} = 6886 \pm 11 \pm 11 \text{MeV},
\]

\[
\Gamma_{X(6900)} = 168 \pm 33 \pm 69 \text{MeV},
\]

for model-II with interference with the non-resonance single parton scattering continuum.

Theoretically, many phenomenological models have investigated the fully heavy tetraquark states in Refs. [9–58]. In these studies, most of them verified the existence of stable states with four heavy quarks and predicted the ground states of fully heavy tetraquarks below \(X(6900)\). To explain \(X(6900)\), some of them assigned it as a radially or orbitally excited state, such as Refs. [49, 54]. Note that there is also a CMS report on possible structure in (18, 19) GeV region [59], hinting a potential fully bottom tetraquark. The discovery of a fully heavy tetraquark shall definitely deepen our knowledge of hadron structure.

For a \(cc\bar{c}\bar{c}\) system, the spin-parity \(J^{PC}\) can be \(0^{++}, 1^{−−}\) and \(2^{++}\) for S-wave states while \(0^{−−}, 1^{−+}, 1^{−−}, 2^{−−}\) and \(3^{−−}\) for P-wave states. Considering the \(X(6900)\) directly decays into a pair of \(J/\psi\), the allowed quantum numbers of spin-parity are \(0^{++}\) and \(2^{++}\) for S-wave states while \(0^{−−}, 1^{−+}\) and \(2^{−−}\) for P-wave states. In this paper, we will argue the \(X(6900)\) is more likely to be an excited state rather than a ground state of fully charm tetraquark within the Regge trajectory relation. Then we employ the Regge trajectories to study the spectra of fully heavy tetraquarks \(T_{cc\bar{c}\bar{c}}, T_{bb\bar{b}\bar{b}}\) and \(T_{bc\bar{b}\bar{c}}\). On the other hand, we will study the production mechanism of fully heavy S-wave tetraquarks at hadron colliders. The color-singlet and color-octet configurations will be discussed. We will employ the nonrelativistic QCD (NRQCD) [60] to predict the cross section of fully heavy tetraquarks \(T_{cc\bar{c}\bar{c}}, T_{bb\bar{b}\bar{b}}\) and \(T_{bc\bar{b}\bar{c}}\).

The rest of this paper is organized as follows. The the spectra of fully heavy tetraquarks \(T_{cc\bar{c}\bar{c}}, T_{bb\bar{b}\bar{b}}\) and \(T_{bc\bar{b}\bar{c}}\) are given in Sec. II. The total cross sections are presented in Sec. III. We also phenomenologically discuss how to hunt for the possible \(X(6900)\) partners. The differential cross section is investigated at low transverse momentum in Sec. IV. In the end we give a brief summary.
II. REGGE TRAJECTORIES FOR THE FULLY HEAVY TETRAQUARK SPECTRA

The $X(6900)$ is around 700MeV above the $J/\psi J/\psi$ threshold, thus it is not likely to be a ground state of fully charm tetraquark. Actually many literatures have predicted the ground state of fully charm tetraquark around 6GeV, see literatures such as [18, 21, 36, 49, 54] or Tab. VIII in Ref. [55]. In the following, we will use the Regge trajectories to study the excited fully heavy tetraquarks and attempt to understand the $X(6900)$ spectra.

In Regge theory, all hadrons (stable or unstable baryons and mesons) can be associated with Regge poles that move in the angular momentum plane as a function of hadron mass [61]. Later developments indicate this relation is approximately linear

$$ J = \alpha M^2 + \alpha_0, $$

where $J$ is the spin quantum number and $M$ is the hadron mass. $\alpha$ is the slope and $\alpha_0$ is the intercept, both of which are model parameters and different for different baryons and mesons. On the other hand, it is convenient to construct the hadron Regge trajectories in $(n_r, M^2)$ plane [62, 65]

$$ n_r = \beta M^2 + \beta_0, $$

where $n_r = n - 1$ with the radial quantum number $n$. The slope $\beta$ and the intercept $\beta_0$ are also the free parameters and dependent on certain hadron.

One interesting remark is that the slopes decrease when much heavier quark gets in the hadron. For the hadrons with identical constituent quark content, the slopes are at the same order. From the global fits of spectra of all known meson data and higher excited states from QCD-motivated relativistic quark potential model in Refs. [62, 64], $\alpha(q\bar{q}) \subset [0.828, 1.336] \sim \alpha(qs) \subset [0.780, 0.964] > \alpha(ss) \subset [0.684, 0.729] > \alpha(q\bar{c}) \subset [0.489, 0.557] \sim \alpha(sc) \subset [0.463, 0.497] > \alpha(q\bar{b}) \subset [0.243, 0.288] \sim \alpha(s\bar{b}) \subset [0.241, 0.290]$, and $\beta(q\bar{q}) \subset [0.679, 0.916] > \beta(s\bar{s}) \subset [0.559, 0.597] > \beta(q\bar{c}) \subset [0.339, 0.378] > \beta(s\bar{c}) \subset [0.309, 0.336] > \beta(q\bar{b}) \subset [0.172, 0.183] \sim \beta(s\bar{b}) \subset [0.169, 0.177]$. For charmonium, $B_c$ and bottomonium systems, the fitted slopes are $\alpha(c\bar{c}) \subset [0.414, 0.493] > \alpha(c\bar{b}) \subset [0.242, 0.298] \sim \alpha(b\bar{b}) \subset [0.184, 0.267]$ and $\beta(c\bar{c}) \subset [0.287, 0.325] > \beta(c\bar{b}) \subset [0.172, 0.190] \sim \beta(b\bar{b}) \subset [0.151, 0.178]$. Based on these fits, one could expect the slopes are approximately equal for hadrons with identical heavy quark content but with different spin-parity.

The other remark is that the parameters $\alpha_0$ and $\beta_0$ are not unpredictable. At least, its value can be well estimated by the ground states of hadron due to

$$ \alpha_0 = -\alpha M^2 (J = J_{\text{min}}) + J_{\text{min}}, \quad \beta_0 = -\beta M^2 (n = 1). $$

In the following, we will update the fitting of the slope and intercept for heavy quarkonium and $B_c$ meson systems. Compared to the fitting in Refs. [64], we increase the weight of experimental data of heavy quarkonium and $B_c$ meson spectra and decrease the weight of unobserved states such as discarding unobserved higher excited states ($J > 3; n_r > 3$) predicted from potential models. We adopt Chi-square fit and the Chi-square goodness of fit is defined as

$$ \chi^2 = \sum_{i=1}^{N} \left[ \frac{M_i(\alpha, \alpha_0; \beta, \beta_0) - M_i}{M_i} \right]^2. $$
Model I gives a narrow state while Model II gives a wide state. Considering the P-wave state with/without interference with the non-resonance single parton scattering continuum, where $X_{\text{In Fit-I}}$, allowed fits are.

\[
\alpha(\eta_c) = 0.35 \pm 0.04, \quad \alpha_0(\eta_c) = -3.17 \pm 0.43, \quad \chi^2 = 0.002, \quad (5)
\]
\[
\alpha(J/\psi) = 0.39 \pm 0.04, \quad \alpha_0(J/\psi) = -2.86 \pm 0.50, \quad \chi^2 = 0.001, \quad (6)
\]
\[
\beta(\eta_c) = 0.29 \pm 0.03, \quad \beta_0(\eta_c) = -2.67 \pm 0.38, \quad \chi^2 = 0.004, \quad (7)
\]
\[
\beta(J/\psi) = 0.31 \pm 0.02, \quad \beta_0(J/\psi) = -3.07 \pm 0.29, \quad \chi^2 = 0.002. \quad (8)
\]

Similarly, we can get the fit results for bottomonia and $B_c$ mesons.

\[
\alpha(B_c) = 0.20 \pm 0.02, \quad \alpha_0(B_c) = -7.79 \pm 0.89, \quad \chi^2 = 0.001, \quad (9)
\]
\[
\beta(B_c) = 0.15 \pm 0.02, \quad \beta_0(B_c) = -6.07 \pm 0.72, \quad \chi^2 = 0.001, \quad (10)
\]
\[
\alpha(\eta_b) = 0.13 \pm 0.01, \quad \alpha_0(\eta_b) = -11.43 \pm 1.27, \quad \chi^2 = 0.001, \quad (11)
\]
\[
\alpha(\Upsilon) = 0.14 \pm 0.01, \quad \alpha_0(\Upsilon) = -11.58 \pm 1.38, \quad \chi^2 = 0.001, \quad (12)
\]
\[
\beta(\eta_b) = 0.11 \pm 0.01, \quad \beta_0(\eta_b) = -9.75 \pm 0.95, \quad \chi^2 = 0.001, \quad (13)
\]
\[
\beta(\Upsilon) = 0.13 \pm 0.01, \quad \beta_0(\Upsilon) = -11.66 \pm 0.84, \quad \chi^2 = 0.005. \quad (14)
\]

Actually, we can estimate the intercepts using the Eq. (3). For example,

\[
\alpha_0(\eta_c) \approx -\alpha(\eta_c)(2m_c)^2 \approx -3.11, \quad \beta_0(\eta_c) \approx -\beta(\eta_c)(2m_c)^2 \approx -2.59, \quad (15)
\]
\[
\alpha_0(J/\psi) \approx -\alpha(J/\psi)(2m_c)^2 + 1 \approx -2.51, \quad \beta_0(J/\psi) \approx -\beta(J/\psi)(2m_c)^2 \approx -2.79, \quad (16)
\]

where the charm quark mass is adopted as $m_c = 1.5GeV$ [67]. These estimations are in agreement with the fit results. One can check the consistent estimations in bottomonia and $B_c$ mesons. In the following, we will apply these estimation and fit results in fully heavy tetraquarks.

\[
\alpha_0(T_{4Q}(0^{++})) \approx -\alpha(T_{4Q}(0^{++}))(4m_Q)^2, \quad \beta_0(T_{4Q}(0^{++})) \approx -\beta(T_{4Q}(0^{++}))(4m_Q)^2, \quad (17)
\]
\[
\alpha_0(T_{4Q}(2^{++})) \approx -\alpha(T_{4Q}(2^{++}))(4m_Q)^2 + 2, \quad \beta_0(T_{4Q}(2^{++})) \approx -\beta(T_{4Q}(2^{++}))(4m_Q)^2. \quad (18)
\]

If we assume that the $X(6900)$ is a fully charm tetraquark, then we can extract the slopes $\alpha(T_{4Q})$ and $\beta(T_{4Q})$. Considering the reasonable condition $\alpha(T_{4Q}), \beta(T_{4Q}) < \alpha(\bar{c}c), \beta(\bar{c}c)$, the allowed fits are

\[
\beta(T_{cc\bar{c}c}(0^{++})) \approx \beta(T_{cc\bar{c}c}(2^{++})) = 0.088 \pm 0.003, \quad \text{Fit – I} \quad (19)
\]
\[
\beta(T_{cc\bar{c}c}(0^{++})) \approx \beta(T_{cc\bar{c}c}(2^{++})) = 0.175 \pm 0.005, \quad \text{Fit – II} \quad (20)
\]
\[
\alpha(T_{cc\bar{c}c}(0^{++})) \approx 0.085 \pm 0.002, \quad \text{Fit – III} \quad (21)
\]

In Fit-I, $X(6900)$ is a $T_{cc\bar{c}c}(0^{++}(2S))$ or $T_{cc\bar{c}c}(2^{++}(2S))$ state; in Fit-II, $X(6900)$ is a $T_{cc\bar{c}c}(0^{++}(3S))$ or $T_{cc\bar{c}c}(2^{++}(3S))$ state; in Fit-III, $X(6900)$ is a $T_{cc\bar{c}c}(1^{-+}(1P))$ state.

From the LHCb data, the mass and the decay widths of the $X(6900)$ are different with/without interference with the non-resonance single parton scattering continuum, where Model I gives a narrow state while Model II gives a wide state. Considering the P-wave state...
decays into a pair of $J/\psi$ will be suppressed from a momentum ratio factor compared to the S-wave state decays, thus we may conjecture the $X(6900)$ is a $T_{cc\bar{c}\bar{c}}(1^{-+}(1P))$ state if its decay width is narrow, while the $X(6900)$ is a $T_{cc\bar{c}\bar{c}}(0^{++},2^{++})(2S,3S)$ state if its decay width is wide. We plot the $(n_r, M^2)$ plane Regge trajectories of $T_{cc\bar{c}\bar{c}}$ in Fig. 1. Similarly, one can easily get the $(J, M^2)$ plane Regge trajectories of $T_{cc\bar{c}\bar{c}}$.

To test of the validation of Fit-I, Fit-II, and Fit-III, one can look for the $X(6900)$ partners. For the states lower than $X(6900)$, both Fit-I and Fit-III predict $T_{cc\bar{c}\bar{c}}(0^{++},2^{++})(1S)$ state with mass $5.99 \pm 0.03 \text{GeV}$, while Fit-II predicts $T_{cc\bar{c}\bar{c}}(0^{++},2^{++})(1S)$ state with mass $5.99 \pm 0.03 \text{GeV}$ and $T_{cc\bar{c}\bar{c}}(0^{++},2^{++})(2S)$ with mass 6.45 $\pm 0.03 \text{GeV}$. For the states higher than $X(6900)$, both Fit-I and Fit-III predict a state with mass 7.66 $\pm 0.03 \text{GeV}$, while Fit-II predicts a state with mass 7.28 $\pm 0.03 \text{GeV}$. In the following, we list the Fit-II results

$$m(T_{cc\bar{c}\bar{c}}((0,2)^{++}(nS)) |_{\text{Fit-II}} = \begin{cases} 5.99 \pm 0.03 \text{GeV} , & n = 1, \\ 6.45 \pm 0.03 \text{GeV} , & n = 2, \\ 6.886 \pm 0.022 \text{GeV} , & n = 3, \\ 7.28 \pm 0.03 \text{GeV} , & n = 4. \end{cases}$$

(22)

To give a prediction for the spectra of $T_{bb\bar{b}\bar{b}}$ and $T_{bc\bar{c}\bar{c}}$, one could estimate the slopes as $\beta(T_{bc\bar{c}\bar{c}}) \sim \beta(B_c)/\sqrt{2}$ and $\beta(T_{bb\bar{b}\bar{b}}) \sim \beta(b\bar{b})/\sqrt{2}$. Then we will get the estimation as

$$m(T_{bc\bar{c}\bar{c}}((0,2)^{++}(nS)) = \begin{cases} 12.40 \pm 0.05 \text{GeV} , & n = 1, \\ 12.72 \pm 0.05 \text{GeV} , & n = 2, \\ 13.04 \pm 0.09 \text{GeV} , & n = 3. \end{cases}$$

(23)
FIG. 2: Regge trajectories for fully charm tetraquark $T_{bc\bar{b}\bar{c}}$ with the spin-parity $J^{PC} = 0^{++}, 2^{++}$. For $n \leq 3$, three states around $12.4\text{GeV}, 12.72\text{GeV}, 13.04\text{GeV}$ are predicted.

\[ m(T_{bc\bar{b}\bar{c}}((0,2)^{++}nS)) = \begin{cases} 18.80 \pm 0.02\text{GeV}, & n = 1, \\ 19.08 \pm 0.02\text{GeV}, & n = 2, \\ 19.37 \pm 0.04\text{GeV}, & n = 3. \end{cases} \quad (24) \]

FIG. 3: Regge trajectories for fully charm tetraquark $T_{bb\bar{b}\bar{b}}$ with the spin-parity $J^{PC} = 0^{++}, 2^{++}$. For $n \leq 3$, three states around $18.8\text{GeV}, 19.08\text{GeV}, 19.37\text{GeV}$ are predicted.

We plot the $(n_r, M^2)$ plane Regge trajectories of $T_{bc\bar{b}\bar{c}}$ in Fig. 2 and $T_{bb\bar{b}\bar{b}}$ in Fig. 3.
III. PRODUCTION AT HADRON COLLIDERS

The cross section of the fully heavy tetraquark at proton-proton collider shall be factorized as

\[
\sigma(p + p \rightarrow T_{4Q} + X) = \sum_{i,j=q,g} \int_0^1 dx_1 dx_2 f_{i/p}(x_1, \mu) f_{j/p}(x_2, \mu) \int_0^1 dz \hat{\sigma}^{(0)}_{ij} \times H_{ij}(z; \mu) \delta \left( z - \frac{m_{T_{4Q}}^2}{x_1 x_2 s} \right),
\]

(25)

where \( T_{4Q} \) denotes one of the fully heavy tetraquarks \( T_{cc\bar{c}\bar{c}}, T_{bb\bar{b}\bar{b}}, \) and \( T_{bc\bar{b}\bar{c}} \); \( \hat{\sigma}^{(0)}_{ij} \) is the LO cross section for the partonic subprocess \( i + j \rightarrow T_{4Q} + X \); \( H_{ij} \) is the hard kernel; \( x_i \) is the parton longitudinal momentum fraction and \( s \) is the centre-of-mass energy of incoming protons. To produce the fully heavy tetraquarks, two pair of heavy quarks should be created at first. Thus the two gluon fusion is the dominant production mechanism for the fully heavy tetraquarks. Up to NLO, the following processes should be considered

\[ p + p \rightarrow g + g \rightarrow T_{4Q}, \quad p + p \rightarrow g + g \rightarrow T_{4Q} + g. \]

The hard kernel \( H_{ij} \) can be expanded in powers of strong coupling constant

\[
H_{ij}(z; \mu) = \sum_n \left( \frac{\alpha_s}{2\pi} \right)^n H^{(n)}_{ij}(z; \mu),
\]

(26)

\[
H^{(0)}_{ij}(z; \mu) = \delta_{ig} \delta_{jg} \delta(1 - z).
\]

(27)

![Feynman diagrams](image)

FIG. 4: Typical Feynman diagrams for the production of fully heavy tetraquark \( T_{4Q} \).

The typical Feynman diagrams for \( g + g \rightarrow T_{4Q} \) are plotted in Fig. 4. The LO partonic cross section \( \hat{\sigma}^{(0)}_{gg} \) are related to the LO Feynman amplitude squared

\[
\hat{\sigma}^{(0)}_{gg} = \frac{\pi}{(D - 2)(N_c^2 - 1)^2} |M(g + g \rightarrow T_{4Q})|^2.
\]

(28)

In this paper, we only consider the S-wave tetraquark production. It is convenient to write...
the partonic amplitude into Lorentz invariant terms

\[ \mathcal{M}[g(\epsilon_1(p_1)) + g(\epsilon_2(p_2)) \rightarrow T_{4Q}(0^{++}, p_H)] = \left[ a g_{\mu\nu} + b \frac{p_H P_H}{p_H^2} \right] \epsilon^\mu_1 \epsilon^\nu_1, \]  
\[ \mathcal{M}[g(\epsilon_1(p_1)) + g(\epsilon_2(p_2)) \rightarrow T_{4Q}(2^{++}, \epsilon^*(p_H))] = \left[ c \epsilon^*_{\mu\nu} + d \frac{p_H P_H P_{1\beta} P_{1\gamma}}{p_H^2} + f \frac{g_{\mu\nu} P_{1\alpha} P_{1\beta}}{p_H^2} \right] \epsilon^\mu_1 \epsilon^\nu_1. \]

It is a hard task to calculate the four body production matrix elements for fully heavy tetraquarks. For an ab initio method, we will employ the NRQCD to simplify the LDMEs for fully heavy tetraquarks as the series of two-body LDMEs

\[ \langle 0 | \mathcal{O}^{T_{4Q}} | 0 \rangle = \sum_{i} c_{11} \langle 0 | \mathcal{O}^{Q\bar{Q}[2S+1 L_J]}^{[11]} | 0 \rangle \langle 0 | \mathcal{O}^{Q\bar{Q}}^{[2S+1 L_J]}^{[11]} | 0 \rangle \]
\[ + \sum_{j} c_{8j} \langle 0 | \mathcal{O}^{Q\bar{Q}}^{[2S+1 L_J]}^{[8]} | 0 \rangle \langle 0 | \mathcal{O}^{Q\bar{Q}}^{[2S+1 L_J]}^{[8]} | 0 \rangle, \]

where the number 1 and 8 denote the color singlet and octet. By Fierz transformation, the above decomposition can be performed in a diquark and anti-diquark configurations \(3 \otimes \bar{3}\) and \(6 \otimes 6\). One can see the decomposition of a diquark and anti-diquark configurations in Ref. [53].

Since the color-octet LDMEs of heavy quarkonium are small, we just consider the color-singlet contribution here. Since \(0^{++}\) can be produced by vector-vector and pseudoscalar-pseudoscalar configurations, while \(2^{++}\) can be produced by vector-vector coupling. Thus we denote the coefficient of vector-vector coupling to the tetraquark LDMEs is denoted as \(c_{11}\) and the coefficient of pseudoscalar-pseudoscalar coupling to the tetraquark LDMEs is denoted as \(c_{10}\). We leave a complete investigation of all other possible LDMEs contributions in future works. Then the LO partonic cross sections are

\[ \hat{\sigma}_{gg}(T_{bc\bar{c}})(0^{++}) = \frac{4\pi^5 \alpha_s^4}{27s^2 r^4 m_{T_{bc\bar{c}}}^2} c_{11} \left[ \langle 0 | \mathcal{O}^{bc}(3S_1^{[11]}) | 0 \rangle \right]^2, \]
\[ \hat{\sigma}_{gg}(T'_{bc\bar{c}})(0^{++}) = \frac{4\pi^5 \alpha_s^4}{81s^2 r^4 m_{T_{bc\bar{c}}}^3} c_{10} \left[ \langle 0 | \mathcal{O}^{bc}(1S_0^{[0]}) | 0 \rangle \right]^2, \]
\[ \hat{\sigma}_{gg}(T_{bc\bar{c}})(2^{++}) = \frac{64\pi^5 \alpha_s^4}{81s^2 r^4 m_{T_{bc\bar{c}}}^2} c_{11} \left[ \langle 0 | \mathcal{O}^{bc}(3S_1^{[11]}) | 0 \rangle \right]^2, \]

where \(r = m_c/m_b\) and \(s_J = 3\). One can easily get the LO partonic cross sections for fully charm or bottom tetraquarks. \(\hat{\sigma}_{gg}(T_{c\bar{c}c\bar{c}})(0^{++}, 2^{++})\) can be obtained by the replacement \(m_{T_{bc\bar{c}}} \rightarrow m_{T_{c\bar{c}c\bar{c}}}\), \(\langle 0 | \mathcal{O}^{bc}(3S_1^{[11]}) | 0 \rangle \rightarrow \langle 0 | \mathcal{O}^{c\bar{c}}(3S_1^{[11]}) | 0 \rangle\) (or \(\langle 0 | \mathcal{O}^{c\bar{c}}(1S_0^{[0]}) | 0 \rangle\)), and \(r \rightarrow 1\). \(\hat{\sigma}_{gg}(T_{bb\bar{b}b}(0^{++}, 2^{++})\) can be obtained by the replacement \(m_{T_{bc\bar{c}}} \rightarrow m_{T_{bb\bar{b}b}}\), \(\langle 0 | \mathcal{O}^{bc}(3S_1^{[11]}) | 0 \rangle \rightarrow \langle 0 | \mathcal{O}^{bb\bar{b}}(3S_1^{[11]}) | 0 \rangle\) (or \(\langle 0 | \mathcal{O}^{bb\bar{b}}(1S_0^{[0]}) | 0 \rangle\)), and \(r \rightarrow 1\).
Using the LDMEs of S-wave charmonium, bottomonium, and $B_c$ meson in Refs. [68, 71–73], the fully heavy tetraquark hadroproduction cross section can be obtained as

$$\sigma(X(6900), T_{cc\bar{c}c}(0^{++})) = c_{11} \begin{cases} (9.4, 18.2)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (21.4, 41.5)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (37.2, 72.3)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (35)$$

$$\sigma(X(6900), T_{cc\bar{c}c}'(0^{++})) = c_{10} \begin{cases} (3007, 5840)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (6845, 13293)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (12097, 23494)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (36)$$

$$\sigma(X(6900), T_{cc\bar{c}c}(2^{++})) = c_{11} \begin{cases} (2453, 4764)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (5584, 10845)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (9729, 18895)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (37)$$

$$\sigma(T_{bc\bar{b}c}(0^{++})) = c_{11} \begin{cases} (0.09, 0.16)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (0.25, 0.44)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (0.49, 0.86)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (38)$$

$$\sigma(T_{bc\bar{b}c}'(0^{++})) = c_{10} \begin{cases} (28.8, 50.9)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (79.3, 140.0)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (155.4, 274.5)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (39)$$

$$\sigma(T_{bc\bar{b}c}(2^{++})) = c_{11} \begin{cases} (23.5, 41.6)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (64.7, 114.2)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (126.8, 223.9)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (40)$$

$$\sigma(T_{bb\bar{b}c}(0^{++})) = c_{11} \begin{cases} (0.03, 0.05)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (0.09, 0.14)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (0.18, 0.30)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (41)$$

$$\sigma(T_{bb\bar{b}c}'(0^{++})) = c_{10} \begin{cases} (8.7, 14.7)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (27.3, 46.0)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (57.7, 97.3)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (42)$$

$$\sigma(T_{bb\bar{b}c}(2^{++})) = c_{11} \begin{cases} (7.1, 12.0)\text{nb} , & \sqrt{s} = 2.75\text{TeV} , \\ (22.2, 37.6)\text{nb} , & \sqrt{s} = 7\text{TeV} , \\ (79.3, 147.0)\text{nb} , & \sqrt{s} = 14\text{TeV} , \end{cases}$$  \hspace{1cm} (43)$$

where the scale is adopted at $(2m_{T_{4q}}, m_{T_{4q}})$.
From the above calculation, the cross section of a $2^{++}$ tetraquark from pseudoscalar-pseudoscalar configuration, both of which are greatly larger than the cross section of a $0^{++}$ tetraquark from vector-vector configuration, which is important to determine the nature of $X(6900)$ if the $X(6900)$ is a S-wave tetraquark. The cross section of tetraquark is scaled as $1/s$, which is another phenomenon to test the theoretical method. The cross section of $T_{bc\bar{b}}$ is one percent of that of $T_{cc\bar{c}}$, while the cross section of $T_{bb\bar{b}}$ is suppressed by a factor 1/300 compared to the cross section of $T_{cc\bar{c}}$

To hunt for the $T_{bc\bar{b}}$ states, one could study the process $p + p \rightarrow g + g \rightarrow T_{bc\bar{b}}$. For a $T_{bc\bar{b}}$ state around 12.4GeV, one could use the decay channel $T_{bc\bar{b}} \rightarrow \Upsilon + \ell^- + \ell^+$ or $T_{bc\bar{b}} \rightarrow J/\psi + \ell^- + \ell^+$. For a $T_{bc\bar{b}}$ state around 12.72GeV or 13.04GeV, one could use the decay channel $T_{bc\bar{b}} \rightarrow \Upsilon + J/\psi$.

To hunt for the $T_{bc\bar{b}}$ states, one could also study the process $p + p \rightarrow g + g \rightarrow T_{bb\bar{b}}$. For a $T_{bb\bar{b}}$ state around 18.8GeV, one could use the decay channel $T_{bb\bar{b}} \rightarrow \Upsilon + \ell^- + \ell^+$ or $T_{bb\bar{b}} \rightarrow \Upsilon + J/\psi$. For a $T_{bb\bar{b}}$ state around 19.08GeV or 19.37GeV, one could use the decay channel $T_{bb\bar{b}} \rightarrow \Upsilon + \Upsilon$.

IV. DIFFERENTIAL CROSS SECTION AT LOW TRANSVERSE MOMENTUM

Concerning about the differential cross section, the LO Feynman diagrams only give a delta function. We need to consider the process $g + g \rightarrow T_{4Q} + g$. But we can study its behaviour at low transverse momentum limit. In the low transverse momentum limit $p_\perp \ll m_{T_{4Q}}$, the differential cross section becomes

$$\frac{d\sigma}{dy dp_\perp^2} = \frac{\delta^{(4)}_{gg}}{2\pi^2} \int dx_1dx_2 f(x_1,\mu)f(x_2,\mu) \frac{1}{p_\perp^2} \left[ \frac{2(1 - \xi_1 + \xi_2^2)^2}{(1 - \xi_1)_+} \delta(1 - \xi_2) + \frac{2(1 - \xi_2 + \xi_1^2)^2}{(1 - \xi_2)_+} \delta(1 - \xi_1) + 2 \log \frac{m_{T_{4Q}}^2}{p_\perp^2} \delta(1 - \xi_2) \delta(1 - \xi_1) \right], \quad (44)$$

where $y$ is the rapidity; $p_\perp$ is the transverse momentum of tetraquark; $\xi_1 = m_{T_{4Q}} e^y/(x_1 \sqrt{s})$, and $\xi_2 = m_{T_{4Q}} e^y/(x_2 \sqrt{s})$. This formalism will break down when $p_\perp \rightarrow 0$. Thus we use the Collins-Soper-Sterman resummation formula [74] and the differential cross section can be rewritten as [75] [76]

$$\frac{d\sigma}{dy dp_\perp^2} \bigg|_{p_\perp \ll m_{T_{4Q}}} = \frac{1}{(2\pi)^2} \int d^2 p e^{i\mathbf{p}\cdot\mathbf{b}} e^{-S_{sud}(b, m_{T_{4Q}}, C_1, C_2)} W(b, m_{T_{4Q}}, \xi_1, \xi_2), \quad (45)$$

is where $S_{sud}(m_{T_{4Q}}, C_1, C_2)$ is the Sudakov factor

$$S_{sud}(b, m_{T_{4Q}}, C_1, C_2) = \int_{C_1^2/m_{T_{4Q}}^2}^{C_2^2/m_{T_{4Q}}^2} \frac{d\mu^2}{\mu^2} \left[ A \log \frac{C_2^2 m_{T_{4Q}}^2}{\mu^2} + B \right], \quad (46)$$

where both $A$ and $B$ can be expanded perturbatively as $A(B) = \sum_i \left( \frac{n_{gf}}{2\pi} \right)^i A^{(i)}(B^{(i)})$. For the lowest nontrivial order, $A^{(1)} = 2C_A$ and $B^{(1)} = -2b_0 = -(11C_A/3 - 2n_f/3)$. It is popular to choose $C_1 = 2e^{-\gamma_E}$ and $C_2 = 1$. 

10
$W(b, m_{T4Q}, \xi_1, \xi_2)$ can be written as

$$W(b, m_{T4Q}, \xi_1, \xi_2) = \hat{\sigma}^{(0)}_{gg} \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) C_{ga} \left( \frac{\xi'_1}{x_1}, b, C_1, C_2, \mu^2 \right) \times C_{gb} \left( \frac{\xi'_2}{x_2}, b, C_1, C_2, \mu^2 \right),$$

(47)

where $\xi'_1 = m_{T4Q} e^y / \sqrt{s}$, and $\xi'_2 = m_{T4Q} e^y / \sqrt{s}$. $C_{ij}$ rely on the fixed perturbative calculation and can be expanded as $C_{ij} = \sum_n \left( \frac{\alpha_s}{\pi} \right)^n C_{ij}^{(n)}$, and at leading order, $C_{gg}^{(0)}(x) = \delta(1-x)$ and $C_{gq}^{(0)}(x) = 0$. We will leave the higher-order QCD corrections in future studies.

For the $X(6900)$ production at proton-proton collision, we give a plot for its production at low transverse momentum in Fig. 5.

![Graph](image)

**FIG. 5**: The $X(6900)$ production with $\sqrt{s} = 14$ TeV and $2.5 < y < 5$ at the LHC. Pt=$p_\perp$ is the transverse momentum of the $X(6900)$.

V. SUMMARY

We have presented an analysis of the spectra of fully heavy tetraquarks within Regge trajectories and a calculation for the production of fully heavy tetraquarks at hadron colliders. The $X(6900)$ discovered by the LHCb collaboration could be explained as a radially excited S-wave fully heavy tetraquark or an orbitally excited P-wave tetraquark. The $X(6900)$ discovery indicates that the existence of fully heavy tetraquark partners, $T_{cc\bar{c}\bar{c}}$, $T_{bc\bar{b}c}$ and $T_{bb\bar{b}\bar{b}}$. The production of both $T_{bc\bar{b}c}$ and $T_{bb\bar{b}\bar{b}}$ have a suppression factor, however, these heavier states shall be tested within a larger data sample of proton-proton collision.

Acknowledgments

The author thanks the useful discussions with Peng Sun, Xiangpeng Wang and Kai Yi. This work is supported by NSFC under grant No. 11705092 and 12075124, and by Natural
Science Foundation of Jiangsu under Grant No. BK20171471 and Jiangsu Qinglan project.

[1] S. K. Choi et al. [Belle], Phys. Rev. Lett. 91, 262001 (2003) doi:10.1103/PhysRevLett.91.262001 [arXiv:hep-ex/0309032 [hep-ex]].
[2] D. Acosta et al. [CDF], Phys. Rev. Lett. 93, 072001 (2004) doi:10.1103/PhysRevLett.93.072001 [arXiv:hep-ex/0312021 [hep-ex]].
[3] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao and B. S. Zou, Rev. Mod. Phys. 90, no.1, 015004 (2018) doi:10.1103/RevModPhys.90.015004 [arXiv:1705.00141 [hep-ph]].
[4] S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. 90, no.1, 015003 (2018) doi:10.1103/RevModPhys.90.015003 [arXiv:1708.04012 [hep-ph]].
[5] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237-320 (2019) doi:10.1016/j.ppnp.2019.04.003 [arXiv:1903.11976 [hep-ph]].
[6] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao and B. S. Zou, Rev. Mod. Phys. 90, no.1, 1-154 (2018) doi:10.1103/RevModPhys.90.015004 [arXiv:1705.00141 [hep-ph]].
[7] G. Yang, J. Ping and J. Segovia, [arXiv:2009.00238 [hep-ph]].
[59] S. Durgut, Search for Exotic Mesons at CMS, APS April Meeting 2018, Ohio
[60] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125-1171 (1995) [erratum: Phys. Rev. D 55, 5853 (1997)] doi:10.1103/PhysRevD.55.5853 [arXiv:hep-ph/9407339 [hep-ph]]
[61] G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. 8, 41-44 (1962) doi:10.1103/PhysRevLett.8.41
[62] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 79, 114029 (2009) doi:10.1103/PhysRevD.79.114029 [arXiv:0903.5183 [hep-ph]].
[63] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 66, 197-206 (2010) doi:10.1140/epjc/s10052-010-1233-6 [arXiv:0910.5612 [hep-ph]].
[64] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 71, 1825 (2011) doi:10.1140/epjc/s10052-011-1825-9 [arXiv:1111.0454 [hep-ph]].
[65] X. G. He, W. Wang and R. Zhu, [arXiv:2008.07145 [hep-ph]].
[66] P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104
[67] R. Zhu, Nucl. Phys. B 931, 359-382 (2018) doi:10.1016/j.nuclphysb.2018.04.018 [arXiv:1710.07011 [hep-ph]].
[68] R. Zhu, Y. Ma, X. L. Han and Z. J. Xiao, Phys. Rev. D 95, no.9, 094012 (2017) doi:10.1103/PhysRevD.95.094012 [arXiv:1703.03875 [hep-ph]].
[69] C. F. Qiao, P. Sun, D. Yang and R. L. Zhu, Phys. Rev. D 89, no.3, 034008 (2014) doi:10.1103/PhysRevD.89.034008 [arXiv:1209.5859 [hep-ph]].
[70] C. F. Qiao and R. L. Zhu, Phys. Rev. D 87, no.1, 014009 (2013) doi:10.1103/PhysRevD.87.014009 [arXiv:1208.5916 [hep-ph]].
[71] R. Zhu, JHEP 09, 166 (2015) doi:10.1007/JHEP09(2015)166 [arXiv:1508.01445 [hep-ph]].
[72] R. Zhu, Phys. Rev. D 92, no.7, 074017 (2015) doi:10.1103/PhysRevD.92.074017 [arXiv:1507.02031 [hep-ph]].
[73] R. Zhu, Y. Ma, X. L. Han and Z. J. Xiao, Phys. Rev. D 98, no.11, 114035 (2018) doi:10.1103/PhysRevD.98.114035 [arXiv:1805.06588 [hep-ph]].
[74] J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250, 199-224 (1985) doi:10.1016/0550-3213(85)90479-1
[75] P. Sun, C. P. Yuan and F. Yuan, Phys. Rev. D 88, 054008 (2013) doi:10.1103/PhysRevD.88.054008 [arXiv:1210.3432 [hep-ph]].
[76] R. Zhu, P. Sun and F. Yuan, Phys. Lett. B 727, 474-479 (2013) doi:10.1016/j.physletb.2013.11.002 [arXiv:1309.0780 [hep-ph]].