Strong fluctuations in the vicinity of quantum phase transitions can induce exotic ground states and excitations [1], such as unconventional quantum critical scalings [1, 2], deconfined quantum critical point (QCP) [3, 4], and emergent enriched symmetries [5]. However, pursuing intrinsic features of these exotic states is a challenging quest and only few exactly solvable models provide significant insight. For example, an exotic spin liquid ground state can be characterized by the honeycomb Kitaev model [6], and stimulates serious hunting in materials [7]. Remarkably, an integrable model [8] emerges when the QCP of the paradigmatic 1D transverse-field Ising chain (TFIC) [1, 2] is perturbed by a longitudinal magnetic field. The excitations of this model are beautifully characterized by the interplay of eight particles governed by the $E_8$ exceptional Lie algebra. This $E_8$ picture is a compelling pattern of the general class of the universality of the 1D TFIC once perturbed by a longitudinal magnetic field, as shown originally by Zamolodchikov [8]. Therefore, finding and exploring the $E_8$ physics in condensed matter systems will be a significant milestone for realizing analytically predicted emergent exotic excitations and provides a manipulable platform to explore quantum magnetism.

Compelling though it may be, the manifestation of this exotic $E_8$ physics can only be established via a dynamics study. In experiments, it is very challenging to accurately determine the location of a 1D QCP and resolve all the eight massive states perturbed away from the 1D QCP. Despite these inherent difficulties, the observation of the lowest two $E_8$ states ($m_1$ and $m_2$) at the 1D ferromagnetic QCP of the quasi-1D magnet CoNb$_2$O$_6$ from inelastic neutron scattering (INS) measurements provided evidence of the quantum $E_8$ spectrum [9], and motivated further materials-based studies on this fascinating phenomenon [10]. Recently, the quasi-1D Heisenberg-Ising antiferromagnetic (AFM) materials, e.g., BaCo$_2$V$_2$O$_8$ (BCVO) and SrCo$_2$V$_2$O$_8$ (SCVO), has attracted numerous studies with rich quantum phases and excitations induced by transverse or longitudinal field [11–18]. It is desirable to explore whether this system can host a complete picture of $E_8$ physics. Along this line, excitations up to the fifth $E_8$ particle ($m_5$) has been resolved by a recent terahertz (THz) spectroscopy measurement on BCVO [19].
along the [010] direction. We use NMR to accurately locate the 1D QCP [$H_{1D}^{3D}$ in Fig. 1(b)], then perform the INS measurements to present the full $E_8$ spectrum for the first time. This result is highly consistent with both the numerical calculations with Eq. (1) for the BCVO material and the theoretical analysis of the essential integrable part of the model, providing an unambiguous evidence for the existence of the $E_8$ physics. Furthermore, our study also captures all the multi-particle modes in the studied energy window. This rare experimental realization of the $E_8$ physics and other coherent modes suggested by an integrable system provides a solid experimental test bed for exploring exotic feature of the dynamics and the excitations in quantum magnets.

To begin with, BCVO can be described by the Hamiltonian [13]

$$
H = J \sum_{n,i} [S^z_{n,i} S^z_{n+1,i} + \epsilon (S^x_{n,i} S^x_{n+1,i} + S^y_{n,i} S^y_{n+1,i})] + J' \sum_{n,i\neq j} S^z_{n,i} S^z_{n,j} - \mu_B \sum_{n,i} g\mathbf{H} \cdot \mathbf{S}_{n,i},
$$

(1)

which includes intra-chain coupling $J$, the anisotropic factor $\epsilon$, and the weak inter-chain coupling $J'$, with the spin-1/2 operator $S^\mu_{n,i}$ ($n$ and $i/j$ are chain and site labels respectively) and the Landé factor tensor $g$. Detailed parameter values are described in the Supplemental Material (SM) [21].

Without the $J'$ term, the system reduces to decoupled 1D AFM chains which accommodate a QCP [$H_{1D}^{1D}$ in Fig. 1(b)] of TFIC universality [11, 12, 31]. Although the existence of $J'$ hides the putative 1D QCP deeply inside the AFM ordered phase, the renaissance of strong 1D quantum fluctuations provides a finite temperature quantum critical region which can be detected outside the AFM phase through NMR experiments. On the other hand, at the hidden 1D QCP of a TFIC universality basis, the weak $J'$ interaction provides a longitudinal background perfectly satisfying all the prerequisites to exhibit exotic $E_8$ excitations [21], with the eight single-particle masses in unit of $m_1$ shown in Fig. 1(c). Note that due to the screw structure [Fig. 1(a)], characterized by the $g$-tensor $g$ [13], external field $\mathbf{H}$ along the $a$-axis induces an effective staggered in-plane field to lower both $H_{1D}^{1D}$ and $H_{1D}^{3D}$ significantly.

We first determined the location of the putative 1D QCP via carrying out $^{51}$V NMR measurements on BCVO with transverse field scanning from 3 T to 12 T. The spin-lattice relaxation rate $1/T_1$ with temperature at various transverse field values are shown in Fig. 2(a). Below 15 K, $1/T_1$ exhibits a strong field dependence consistent with the onset of 1D critical magnetic correlations. At low fields, the magnetic transitions are clearly evidenced by a peaked feature in the $1/T_1$, with strong low-energy spin fluctuations when magnetic ordering occurs. A sharp drop of $1/T_1$ is followed below Néel temperature $T_N$, dominated by the spin-wave excitations. The $T_N$ at each field is then determined from the drop and displayed in the phase diagram of Fig. 1(b), consistent with results from other measurement, e.g., magnetic susceptibility [13].

In the vicinity of the 1D QCP, the $1/T_1$ of the paramagnetic phase at high-temperature follows an analytical form of $1/T_1 \sim T^{-0.75}e^{-\Delta/k_B T}$, where $\Delta$ is the gap value [1, 32]. From 6 K to 12 K, $1/T_1$ follows the power-law form at 5 T and deviates from it at other fields, hence, a 1D QCP ($H_{1D}^{1D}$) is suggested at about 5 T [see Fig. 2(b) inset]. To better resolve the $H_{1D}^{1D}$, we fit the gap $\Delta$ and find it follows $\Delta(H) = 1.62 g_{xx}\mu_B (H - H_{1D}^{1D})$ with field, where $H_{1D}^{1D} = 4.7 \pm 0.3$ T (Fig. 2(b)). The linearly vanishing gap and the scaling exponent 0.75 near 5 T demonstrate a 1D QCP of TFIC universality in the relaxation rate [1]. As demonstrated in the inset of
FIG. 3: $E_8$ excitation spectrum near the 1D QCP. (a) (Zone center) INS Intensity at $Q = (002)$ in BCVO at the vicinity of the 1D QCP with $H = 4.7$ T at 0.4 K. Blue diamonds with error bars correspond to experimental data and black lines are the fit to the Gaussian functions. The red vertical lines at eight peaks correspond to the eight single $E_8$ particles. Other peaks come from multi-particle excitation and zone-folding effect identified by iTEBD method. The peak with mass $m_{11}...m_{16}$ labels multi-particle channel with particles masses $m_{11}...m_{16}$. The colored regions illustrate contributions from zone-folding effect with transfer momentum at $q=0$ for regions near the $F_1$ peak and at $q = \pi/2$ for regions near the $F_2$ peak. (b) The analytical dynamic structure factor $D^{xx}$ calculated from quantum $E_8$ integrable field theory [21]. Red curve stands for single-particle spectra, while black curve is obtained after including multi-particle contributions. In accord with the experiment, $m_1$ is set as 1.2 meV and the analytical data are broadened in a Lorentzian fashion with full-width at half-maximum fixed at $0.08 m_1$. (c) Neutron scattering intensity from iTEBD calculations at the zone center. The black and blue curves are results with and without zone-folding effect. DSF spectra for individual scattering channel: (d) single-, (e) two-, (f) three- and (g) four-particles contributions to the $D^{xx}$. $ijkl$ refer to excitations with combined mass modes of $m_i m_j m_k m_l$. Fig. 2(b), $1/T_1 \sim T^{-0.75}$ is at field near 5 T and and the spin dynamics at the 1D QCP is characterized.

Since the 1D QCP hides in the 3D ordering dome, the competition between this 1D quantum criticality and the ordered background results in interesting excitation. We then performed INS to measure the dynamical structure factor (DSF) of BCVO under the transverse magnetic field. Figure 3(a) shows many distinctive peaks in the constant-$Q$ (zero transfer momentum) spectra at zone center $Q = (002)$, with field near the identified (masked) 1D QCP. Assisted by two theoretical calculations: (i) analytical calculation from Eq. (2); (ii) infinite time-evolving block decimation (iTEBD) [33, 34] numerical calculation based on Eq. (1), these are found to match quantitatively a combination of three types of excitations, including single-$E_8$ states, multi-$E_8$ states, and zone-folding modes as labeled by $P_1$ to $P_{15}$ in the figure. Details are described below.

Starting from the Zeeman ladder of confinement bound states [9] at zero magnetic field, the analysis to separate the spin-flip and non-spin-flip contributions at finite magnetic field give a qualitative description of the low energy excitations [14]. However, a full picture of the full excitation feature especially at the high energy region is still missing. Fortunately, at $H \approx H_{1D}^\mathrm{C}$, the stable Néel ordering masks the desired QCP of TFIC on one hand, but provides a necessary effective perturbation field to bring in the $E_8$ physics on the other hand [21]. The slightly off-critical TFIC can be described by a central charge $c = 1/2$ conformal field theory (CFT) with the perturbation of its relevant magnetic field [8, 35, 36],

$$H_{E_8} = H_{1/2} + h \int \sigma(x) dx$$ (2)
where $H_{1/2}$ is the Hamiltonian of the $c = 1/2$ CFT which describes the $J, \mu_0$ term in Eq. (1) [11, 12], and the perturbation from the field $\alpha(x)$ with coupling $h$ corresponds to the effective field of the chain mean-field of the interchain term in Eq. (1) when the material is in the AFM phase, and is absent outside the ordering phase.

Following the form factor framework [35–39], we calculate the DSF, $D^{\alpha\alpha}(\omega, q = 0)$ ($\alpha = x, y, z$ along the crystalline [100], [010] and [001] direction, respectively), for the zero-momentum transfer. The analytical result for $D^{xx}$ is displayed in Fig. 3(b) for the total spectral weight and the $E_8$ particles for series of peaks are clarified in Fig. 3(d). Astonishingly, analytical calculations also identify multi-particle channels, which exhibit clearly distinguishable peaks for the spectrum continuum above the two-particle threshold, as shown in Fig. 3(e)-(g).

We then directly compare these excitations with the neutron dynamic spectra (Fig. 3(a)), and find the later matches excellently with the analytical prediction for the peak positions and the spectra weights (Fig. 3(b)).

First, all the eight single-$E_8$ particle energies, whose positions are marked by red vertical lines, are resolvable, and a common trend of reducing spectral weight with increasing energies. Note that the two lightest particles matching the golden mass ratio 1.618 as expected [8, 35, 36].

Second, various multi-$E_8$ particle excitations are also clearly resolved as pronounced spectral peaks, even in the high-energy region, leading to the accountability of these modes and full consistency with our theoretical results (Table S1 [21]). Due to the low dimensionality, each two-particle scattering channel contributes a non-trivial two-particle DSF continuum with a sharp edge at low-energy boundary and a peaked feature close to this boundary (Fig. 3(e)), leading to experimentally distinguishable peaks.

Finally, some additional features in the measured spectra are caused by microscopic details of the system, which can be fully taken care of by the iTEBD calculation. For example, due to the four-period screw structure of the BCVO, the DSF obtained from iTEBD (Fig. 3(c)) is able to distinguish a zone folding peak in between the $m_3$ and $m_4$ particle excitation, and another one coincident with multi-$E_8$-particle excitation in between the $m_7$ and $m_8$ particle excitation. As another example, the strong suppression of the $m_1$ observed in neutron data at $Q = (002)$ is also captured by the detailed iTEBD calculations (Fig. 3). As shown in section S3 of the SM [21], the suppression of the $E_8$ particle excitations is caused by the spin-flip term (the $c$ term) of Eq. 1, and this effect is the most significant for the $m_1$ peak.

To show that the realized $E_8$ physics in BCVO close to the QCP is not accidental, we further carry out neutron experiments with other two nearby magnetic fields $H = 4.5$ T and $4.8$ T at $Q = (002)$. The frequency of the spectral peaks for different fields are collected and displayed in Fig. 4, where an evolution of mass distribution continuously passes through the $E_8$ phase. Not surprisingly, the best agreement with the $E_8$ predictions is found at the field of $4.7$ T, at the putative 1D QCP predetermined by NMR. This field is lower than 5 T obtained by the Terahertz studies of the $E_8$ modes [19]. Details of dynamical spectra for those fields can be found in the SM [21].

The excellent agreement between the analytical dynamical $E_8$ spectrum and the neutron data at the putative 1D QCP in BCVO supports the unambiguous existence of complex $E_8$ physics. One striking feature revealed in our INS data is the robustness of the high-energy, single mass peaks, which do not decay into the multiple low-mass modes at the critical field. The underlying physics needs to be further addressed theoretically. Moreover, when the field deviates from the $E_8$ phase, the $E_8$ peaks gradually smear out (cf. Fig. S4 [21]). However, shift of spectra peaks and spectral weight transfer with clear DSF peaks are still resolved away from the critical field upon the decay of $E_8$ particles. This shows that quantum many-body ground state continuously deforms with the tuning of external parameters (field, pressure, etc.), meanwhile, the dynamic spectrum rearranges itself into new non-diffusive modes. Studying non-integrable effects is of great importance to give a comprehensive understanding on real materials since integrability is rare in reality. The BCVO now can serve as an ideal test bed: the corresponding 1D effective model turns into a perturbed quantum $E_8$ model, which is no longer integrable with heavy $E_8$ particles beginning to decay [39, 40].

In conclusion, the combined experiments on BCVO, detection of 1D QCP via NMR and the observation of dynamical spectrum through neutron scattering, together with their excellent agreements with the analytical results from the quantum $E_8$ integrable field theory, unambiguously realize the beautiful $E_8$ physics in this AFM material, leaving us with an exemplary realization of the $E_8$ spectrum. The precise iTEBD numerical simulation on the microscopic model fur-
ther resolves the details of the spectrum and bridges accurately the essential physics in the quantum integrable model and the realistic dynamics in the material. Our study sets a concrete ground to explore the physics of $E_0$ particles and excitations, as well as physics beyond the integrable model. A better control and identification of the excitations above the ground state, and the understanding of the robustness of the high-energy modes, are therefore crucial to arrive at a comprehensive understanding for the quantum many-body system and to study not only its equilibrium properties but also its non-equilibrium features (for instance, Ref. 41 and references therein).

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