Measuring the quantum states of a mesoscopic SQUID using a small Josephson junction

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(Dated: March 22, 2022)

We have experimentally studied the energy levels of a mesoscopic SQUID using inelastic Cooper-pair tunneling. The tunneling in a small Josephson junction depends strongly on its electromagnetic environment. We use this fact to do energy level spectroscopy of a SQUID-loop by coupling it to a small junction. Our samples with strong quasiparticle dissipation are well described by a model of a particle localized in one of the dips in a cosine-potential, while in the samples with weak dissipation we can see formation of energy bands.

The Josephson junction, despite its simple structure has proven to be surprisingly versatile and new applications are found in quantum computing and nanoelectronics \[1, 2\]. The devices are based on the quantum mechanical behavior of the superconducting phase variable \(\theta\), which has been previously studied with either rf-irradiation \[3\] or during rapid current ramping \[4, 5\]. We are using a different probe, namely, an additional mesoscopic Josephson junction. Our scheme is based on the theory of phase fluctuations \[6, 7\], according to which quasiparticle irradiation \[8\] or during rapid current ramping \[9, 10\]. We can see formation of energy bands.

In this Letter we present detailed spectroscopic investigations on small SQUID loops, which are driven from the nearly classical limit \((E_J/E_C \gg 1)\) deep into the quantum regime \((E_J/E_C \sim 1)\). Our results yield evidence for higher energy bands of the macroscopic phase variable in a regime \((E_J/E_C \gtrsim 1)\) where they have not been investigated before \[11\]. In addition, our experiment provides the first verification that multiphoton transitions between the levels of a quantum mechanical harmonic oscillator play a prominent role in electron tunneling in a mesoscopic tunnel junction.

As an energy detector in our measurement we use a voltage biased, superconducting tunnel junction which has a smaller size and critical current than the junction we want to study. For large (conventional) Josephson junctions the supercurrent is given by \(I = I_c \sin(\varphi)\), where \(I_c\) is the critical current, which is related to the Josephson coupling energy \(E_J = hI_c/(2e)\). The phase \(\varphi(t) = \int_{-\infty}^{t} \frac{2e}{h} V(t')dt'\) is defined as an integral of the voltage \(V\) across the tunnel barrier. For small junctions, where the charging energy \(E_C = e^2/(2C) \gg E_J\) Cooper pair tunneling is inelastic and given by

\[
I(V) = \frac{\pi e E_J^2}{\hbar} [P(2eV) - P(-2eV)],
\]

where \(P(E)\) is a function describing the probability of energy exchange between a tunnel junction and its electromagnetic environment and depends on the impedance seen by the junction \[12\].

At low temperatures, the junction environment, i.e. the heat bath, is in its ground state and \(P(E) \simeq 0\) for \(E < 0\). Thus, the latter term in Eq. (1) can be neglected and \(I(V)\) becomes directly proportional to \(P(2eV)\). The theory is valid for linear impedances constructed from lumped elements. Nevertheless, we argue that the idea of energy exchange can be generalized so that a discrete spectrum of energy levels in the environment will cause a set of discrete peaks in the IV-curve. Hence, the small detector junction can be used for spectroscopy.

A Josephson junction can be described by the Schrödinger equation \[13\]

\[
d^2\psi(\varphi) \over d(\varphi/2)^2 + \left( \frac{E}{E_C} + \frac{E_J}{E_C} \cos \varphi + \frac{I}{I_C} \varphi \right) \psi(\varphi) = 0,
\]

where \(I\) is the current flowing through the junction. The current in our measurements always satisfies \(I \ll I_c\), so the tilt in the potential is negligible and setting \(I = 0\) in Eq. (2) leads to the familiar Mathieu-equation. The single junction Hamiltonian can also be used to describe a SQUID-loop, where the loop size is so small that the geometric inductance can be neglected and the loop is perfectly symmetric. The only difference is that \(E_J\) then depends periodically on an externally applied magnetic flux \(\Phi\) according to \(E_J = 2E_J^{\text{single}} \cos(\pi \Phi/\Phi_0)^2\), where \(\Phi_0 = h/(2e)\) and \(E_J^{\text{single}}\) is the Josephson coupling energy for a single junction. For large \(E_J/E_C\), the particle is trapped in one of the potential wells. In this case, for currents \(I \ll I_c\), the Josephson junction can be described by an inductance \(L = \Phi_0/(2\pi I_c)\). Combined with the capacitance of the tunnel junction, the junction forms an
LC-oscillator with a characteristic resonance frequency of $\omega_p = 1/\sqrt{LC} = \sqrt{8E_JE_C}/\hbar$. Consequently, a Josephson junction behaves like a harmonic oscillator with a level spacing of $\omega_p$. When $E_J/E_C$ becomes smaller, the energy levels are not harmonic but they will depend on the shape of the cosine-potential.

Depending on the environmental resistance seen by the Josephson junction, i.e. in our case “the environment of the environment”, the junction can become completely delocalized and the whole periodicity of the cosine-potential has to be accounted for [11, 12]. The eigenstates are then given by Bloch-functions $\Psi_n(\varphi) = u_n(\varphi)e^{i\varphi q/(2\pi)}$, where $q$ is the quasi-charge, $n$ the band index and $u_n(\varphi)$ is a 2$\pi$-periodic function. This phase transition from the localized to delocalized state happens when $R > R_Q$, where $R_Q = \hbar/(4e^2)$, or 6.45 kΩ [13, 14]. In our measurement we need a clear voltage-bias and thus we have not fabricated any resistor close to the junction. The source of dissipation is, therefore, given by the quasiparticle resistance of the probe junction. This changes the periodicity of the wave-functions from $2\pi$ to $4\pi$ and each band is split into two [12].

We have carried out experiments with different circuit configurations; both 2- and 4-lead measurements including 1, 2 or 4 SQUID(s) coupled to a small detector junction. We will here describe measurements of two different samples: a 4-SQUID(s) sample, with four leads and a 1-SQUID sample with just two leads. A scanning electron micrograph of the 4-SQUID sample, together with a schematic drawing of it, is shown in Fig. 1. The SQUID configuration allows us to change the energy levels of the measured system and enables us to resolve the resonances due to the SQUID(s) from other resonances in the environment. The critical current, or equally, the value of $E_J$ could be tuned to less than 1% of the maximum, which shows that our SQUIDs were very homogeneous. The samples were made from aluminum with e-beam lithography and 2-angle evaporation in an UHV chamber.

![Fig. 1: SEM micrograph of a sample with 4 SQUIDs. The probe junction has an area of 100 x 100 nm² and the SQUID junctions 150 x 550 nm². In the samples covered in this paper, additional gates leads were available for the islands. Inset shows the schematic of the circuit in the 4-lead measurement.](image)

The four wire setup facilitates the determination of circuit parameters. The important parameters are $E_J$ and $E_C$, or rather their ratio. The Ambegaokar-Baratoff (A-B) formula, $E_J = \pi\Delta/(4e^2R_T)$, was used to find $E_J$ from the normal state resistances, $R_T$, while the capacitances where estimated from the junction areas (see Fig. 1) using a value of 45 fF/µm² [15]. The BCS-gap, $\Delta$, was about 215 µeV in our samples. The experimental parameters for the different circuits are summarized in Table 1.

The samples were mounted into an rf-tight copper enclosure and cooled down to 80 mK with a plastic dilution refrigerator. The measurement leads were filtered using 0.7 m long sections of Thermocoax. Mini-Circuits rf-filters with a cut-off frequency of 1.9 MHz were employed on the top of the cryostat, at room temperature.

Fig. 2 displays the measured IV-curve for zero magnetic flux, or maximum $E_J$, for the 4-SQUID sample together with an IV-curve simulated with $P(E)$-theory. The locations of the peaks were found to depend only on the magnetic flux, not at all on the gate voltages. To properly identify the energy levels of the SQUID we measure IV-curves for different magnetic fields. The peak positions as function of applied flux are shown in Fig. 3. The width of the resonance peaks (about 4 µeV) is smaller than $k_B T = 7\mu eV$. The width is therefore, either intrinsic or given by external noise. Our peak widths are thus comparable to or even smaller than what has been observed in similar spectroscopic studies [16].

The peak structure can be qualitatively explained with a 3-resonator model, where one resonator represents all the SQUIDs and the two other come from the rest of the measurement circuitry; bonding wires and pads. The parameters of the simulation and simulation circuitry are found in Fig. 2. The parameters for the SQUID were taken from independent measurements but the parameters of the two other resonator circuits were fitted to IV-curve. The resistances used in the simulation represent the broadening of peaks due to dissipation and noise.

The $P(E)$-function in Eq. (1) was calculated using the integral equation approach presented in Ref. [17]. The comparison of the IV-curve with the simulation clearly shows that multiphoton excitations are present in the experiment. However, the energy levels are not equally

| Sample                  | $R_T$ (kΩ) | $C$ (fF) | $E_J$ (µeV) | $E_C$ (µeV) | $E_J/E_C$ |
|-------------------------|------------|----------|-------------|-------------|-----------|
| 4-SQUID (detector)      | 166        | 0.5      | 3.6         | 160         | 0.023     |
| 4-SQUID (squid)         | 2.5        | 7.6      | 544 (272)   | 10.5        | 51.8      |
| 1-SQUID (detector)      | 70         | 0.8      | 8.5         | 100         | 0.08      |
| 1-SQUID (squid)         | 3.5        | 5.7      | 422 (188)   | 14          | 30.1      |

TABLE I: Parameters for the 4-squid and 1-squid samples. Energies are given in units of µeV. The Ambegaokar-Baratoff values for $E_J$ are given in parentheses.
In order to find a better quantitative agreement with the experimental peaks spaced as would be the case for a classical inductance.

In order to find a better quantitative agreement with the level spacing, the Schrödinger equation (2) was numerically solved under the assumption that the particle is localized in one of the wells. The experimental peaks together with the calculated transitions are shown in Fig. 9. The form of the cosine potential decreases the level spacing from the harmonic case. This deviation from the harmonic oscillator case is largest for the third transition as can be seen in Fig. 3 where also the transition from the first to fourth harmonic level is shown for comparison.

The parameter for $E_j$ used in the calculation is about twice the value given by the A-B relation (see Table 1). Typically, $E_j$ is expected to be renormalized downwards due to the low impedance environment, but in our case it is renormalized upwards. Similar disagreements between the A-B value and have been reported before [12, 16]. Our model does not, however, explain the double peak structure (see Fig. 3) found in all the IV-curves. This peak splitting is fairly constant over the whole measurement range but the position of the double peak is different for the three main transitions and, as can be seen in Fig. 2, the multi-photon excitations given by $P(E)$-theory are not (except for transition (1,1,0)) consistent with the observed peaks.

The 2-point measurements with both one and two SQUID(s) showed similar behavior as the 4 SQUID measurements. The number of SQUIDs in the sample did not seem to have any significant effect on the IV-curve. Rather, there are notable differences between the 2-lead and 4-lead samples. In the 1-SQUID sample with only two leads the current dropped very fast when tuning down $E_j$ (from 360 pA at $\Phi/\Phi_0 = 0$ to 40 pA at $\Phi/\Phi_0 = 0.4$). This behavior can be explained when considering that the current through the circuit is given by two rates: the excitation of oscillator modes in the SQUID and their subsequent relaxation, which depends on the environment seen by the SQUID. In the 2-lead circuits the current is limited by the down relaxation and the effect can be approximatively explained with the formula [15]

$$\Gamma_1 = 2 \sum_{l<n} \text{Re}\{Y(\text{E}_l/h)\}\{\text{E}_l/h\}R_0|\langle l | \varphi | n \rangle|^2, \quad (3)$$

where the admittance is given by $Y(\omega) = [1/(i\omega C_{Det}) + R_0]^{-1}$ and $C_{Det}$ is the capacitance of the detector junction (0.8 fF) in series with the resistance of the environment $R_0$ (100 $\Omega$).

The position of the clearest flux-dependent peaks for the 1-SQUID sample are shown in Fig. 11. In this case, better agreement with the measured resonances is found when considering the full periodicity of the cosine-potential in Eq. 2. However, the value of $E_j$ was taken to be twice the A-B value, as in the case with four SQUIDs. This apparent enhancement of $E_j$ is probably due to the charging energy, as discussed in Ref. [13]. The effect according to the theory is, however, smaller than what we observe.

Because the transitions are due to transfer of Cooper-pairs in the detector junction, the allowed first-order transitions can be found by calculating the matrix element $|\langle l | e^{-i\varphi} | n \rangle|$, between bands $l$ and $n$. In addition, due to van-Hove like singularities, the observed transitions are between band edges. The transition between
the first and fourth band is clearly visible as two distinct peaks. The lowest bands are so narrow that they only show up in the width of the resonance peaks. As the $E_J/E_C$ ratio is tuned down, the life-time of the states grows and this should lead to narrower peaks. But, instead we observe a broadening of the resonances, indicating a broadening of the bands as expected from theory.

The theory for Josephson junctions [12] tells that in order for band formation we need to suppress the ohmic or quasiparticle dissipation in the environment, which causes the phase to localize. In our system, this suppression is provided by the large quasiparticle resistance of the detector junction. Therefore, the wavefunctions are $4\pi$-periodic and each band from the $2\pi$-periodic case is split into two. The observed transitions are, however, the same as what would be expected for $2\pi$-period bands. Consequently, the true periodicity of the bands cannot be resolved in the experiment.

In summary, we have experimentally studied the quantum mechanical energy levels of the Josephson junction. Our results for samples with $E_J/E_C \gg 1$ can qualitatively be described by $P(E)$-theory, consistent with multi-photon excitations in the experiment. The non-linearity of the SQUID systems, prominent of $E_J \sim E_C$, can be taken into account by considering the exact form of the cosine-potential. Evidence of the existence of Bloch bands is observed in our 2-lead samples both in the form of van-Hoven like singularities between band edges and a broadening of resonance peaks. Our results show that a small superconducting junction can be employed as a detector for mesoscopic quantum circuits.

Fruitful discussion with D. Haviland, F. Hekking, F. Wilhelm, G. Schön, J. Siewert, E. Thuneberg, A. Zaikin and T. Heikkilä are gratefully acknowledged. This work was supported by the Academy of Finland and by the Large Scale Installation Program ULTI-3 of the European Union.

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