Comment on ”Experimental Demonstration of the Time Reversal Aharonov-Casher Effect”

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In a recent Letter [1], Bergsten et al. have studied the resistance oscillations with gate voltage \( V_g \) and magnetic field \( B \) in arrays of semiconductor rings, and interpreted the oscillatory \( B \)-dependence as Altshuler-Aronov-Spivak oscillations and oscillatory \( V_g \)-dependence as the time reversal Aharonov-Casher (AC) effect. This comment shows (i) that authors [1] incorrectly identified AAS effect as a source of resistance oscillations with \( B \), (ii) that spin relaxation in [1] is strong enough to destroy oscillatory effects of spin origin, e.g., AC effect, and (iii) the oscillations in [1] are caused by changes in carrier density and the Fermi energy by gate, and are unrelated to spin.

AAS effect is the \( h/2e \) oscillations of conductance with \( B \) in disordered diffusive rings. Oscillations occur because the interference of the two electron trajectories passing the whole ring clockwise and counterclockwise survives disorder averaging in conditions of diffusive regime \( l \ll L_\phi, L \), where \( l \) is the mean free path, \( L_\phi \) is the phase breaking length, and \( L \) is the circumference of the ring.

The mean free path in samples [1] is \( l \sim 1.5–2 \mu m \). From the ratio of \( h/2e \) and \( h/4e \) signal amplitudes [1], L\( _\phi \) is between 2.8 and 3.5 \( \mu m \). (Note that \( h/2e \) signal is due to interference of clockwise and counterclockwise paths, with magnitude defined by \( \exp(-2L/L_\phi) \), and \( h/4e \) oscillations are due to interference of paths going twice clockwise and twice counterclockwise, defined by \( \exp(-4L/L_\phi) \). The calculation of \( L_\phi \) in [1] misses a factor of two.). Thus, samples [1] are not in diffusive regime relevant to AAS oscillations, but are in the quasi-ballistic regime \( l \lesssim L \). Then \( h/2e \) oscillations are defined not only by interference of time-reversed paths, but also e.g., by the interference of the amplitude of propagation through the right arm clockwise and the amplitude of propagation via the three-segment path: the left arm, the right arm (counterclockwise) and again through the left arm. With all interference processes included, \( h/2e \) oscillations depend on the Fermi wave-vector and \( n_s \) [2, 3]. Averaging over few resistance curves does not eliminate contributions of of non time-reversed processes (certainly not beyond 0.3% of the overall signal for oscillations in [1]). Their importance is missed in [1] and is crucial.

(ii) Another mistake in [1] is the neglect of spin relaxation. For the spin-orbit constant \( \alpha = 5 \text{peV.m} \), the parameter \( \alpha ml \sim 2.5 \) (\( m \) is the effective mass), and spin simply flips due to a single scattering event. The spin-flip length \( L_S = l < L \). Thus, oscillations of spin origin are rather unlikely in [1]. The closest to [1] feasible setting requires ballistic regime \( l \gg L \) [3], which requires mobility an order of magnitude higher. Note that \( L_\phi > L_S \) and oscillations with \( B \) originating from charge coherence are plausible to observe.

(iii) The key to understanding the \( h/2e \) oscillations with \( V_g \) in [1] is its Fig. 4. It can be seen clearly that resistance oscillations are present only when \( n_s \) changes with \( V_g \), and are not present when \( n_s \) saturates. Therefore the reason for the observed oscillations is the variation of the \( n_s \). Oscillations of spin origin, particularly the AC effect, must persist when \( n_s \) is constant, while \( \alpha \) varies with \( V_g \). No such evidence is present in [1].

The origin of oscillations with \( n_s \) is the contribution to \( h/2e \) signal from interference of non-time reversed paths. These are independent of \( L_S \), and are governed by \( L_\phi > L_S \). That makes this effect dominant over any spin oscillations. With the account of the role of contacts connecting the ring and the leads [2, 3], in the absence of spin-orbit interactions and for strong coupling of leads and rings, the conductance of the single ring is

\[
G = \frac{2e^2}{h} \left[ 1 - \frac{1 - \cos (\pi \Phi/\Phi_0)}{1 - e^{i\pi k_F L} \cos^2 (\pi \Phi/\Phi_0)} \right] \tag{1}
\]

We note that disregard of transmission and reflection from contacts is yet another critical omission in [1], whose equation for conductance is incorrect in ballistic/quasi-ballistic regime. (It is also incorrect for AAS and AC effect in diffusive regime). The second harmonics in [1] depends on \( k_F \) and \( n_s \) in an oscillatory manner, leading to oscillations of conductance with \( V_g \). The system of the \( n \) interconnected rings can be described similarly to the setting in [4]. On Fig. 1, we show the dependence of the amplitude of the second harmonic on \( k_F \) for one and four rings. Conductance oscillates with electron density despite no spin effects are involved. To summarize, conclusions of [1] on the observation of the AC effect are unfounded.
FIG. 1: The amplitudes of the second harmonics in a single ring (solid curve) and four consequently connected rings (dashed curve). The spin-orbit interaction is absent.

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