The Gluon condensation in high energy cosmic rays

Wei Zhu\textsuperscript{a}, Jiangshan Lan\textsuperscript{b}, Jianhong Ruan\textsuperscript{a} and Fan Wang\textsuperscript{c}

\textsuperscript{a}Department of Physics, East China Normal University, Shanghai 200241, China
\textsuperscript{b}Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
\textsuperscript{c}Department of Physics, Nanjing University, Nanjing, Jiangsu 210093, China

Abstract

The gluon condensation (GC)-effects in high energy cosmic rays are investigated. After a brief review of the GC, two dozen examples including gamma-, electron-, positron-, proton- and antiproton-spectra in a broad GeV\textasciitilde TeV region are examined as the GC-evidence. We find that the GC in proton may break the power-law of the cosmic ray spectra if the energy of accelerated protons exceeds the GC-threshold. The nuclear dependence of the GC-effects are predicted based on the QCD evolution equation. The results are used to expose the GC-effects in neutron star and at the next high energy hadron colliders. Particularly, the galactic center GeV excess and the increase of the positron and antiproton fractions in cosmic rays are understood in Quantum Chromodynamics theory rather than in the special dark matter models.

\textbf{keywords}: Quantum Chromodynamics; Gluon condensation; Cosmic ray spectra

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1 Introduction

The majority of high energy particles in cosmic rays are protons. Proton-proton \((p-p)\) collisions are general events in the Universe. The gluons inside proton dominate the proton collisions at high energy and their distributions obey the evolution equations in Quantum Chromodynamics (QCD). QCD analysis shows that the evolution equations will become nonlinear due to the initial gluons correlations at high energy and it results in the chaotic solution beginning at a threshold energy \([1,2]\). Most surprisingly, the dramatic chaotic oscillations produce very strong shadowing and antishadowing effects, they converge gluons to a state with a critical momentum \([3]\). This is the gluon condensation (GC) in proton. For readers to understand the GC, we give a brief review about its history in Sec. 2.

The GC should induce significant effects in the proton processes, provided the GC-threshold \(E_{p-p}^{GC}\) enters the observable high energy region. Unfortunately, the value of \(E_{p-p}^{GC}\) can not been entirely determined in theory since it relates to the unknown input conditions. We have not directly observed the GC-effects until the 13 TeV \(p-p\) collisions at the Large Hadron Collider (LHC). Therefore, we turn to the collisions of cosmic rays. Protons accelerated in the Universe may exceed \(E_{p-p}^{GC}\) and cause the GC-effects in their collisions. The Auger collaboration indirectly used the cosmic ray data and found that the \(p-p\) cross section at \(\sqrt{s_{p-p}} \sim 100\ TeV\) is the normal value \(\sim 567\ mb\) without a big increment \([4]\). Thus, we suggest that the corresponding GC-threshold \(\sqrt{s_{p-p}^{GC}} > 100\ TeV\) in the center-of-momentum (CM) system.

Greisen-Zatsepin-Kuzmin (GZK) \([5]\) predicted a drastic reduction of the spectrum of cosmic proton rays near the energy \(E_{GZK} \sim 6 \times 10^{19}\ eV\), since the energy of the cosmic rays losses in the collisions with cosmic microwave background radiation during their long propagation. Let us give a simple estimation. The mean free path for photoproduction is calculated to be \(\lambda_{\gamma p} = 1/(N\sigma)\), where \(N\) is the number density of black-body photons
and \( \sigma(\gamma p \rightarrow \pi^0 p) \simeq 100 \mu b \) is the cross section at threshold. This leads to \( \lambda_{\gamma p} \simeq 10 \text{Mpc} \). The Markarian galaxies are the nearest possible ultra high energy cosmic rays (UHECR)-sources, which are residing at distances of approximately \( x \sim 100 \text{Mpc} \). The arrival probability of protons through these distances with energies exceeding \( 10^{20} \text{eV} \) is only \( \sim e^{-x/\lambda_{\gamma p}} = 10^{-4} - 10^{-5} \). However, the Auger data seem to diminish by steps only in one order of magnitude, but not by an abrupt descend as conceived above [6]. Many ideas and different models are proposed to understand the GZK puzzle even suspecting the Lorentz invariance and the Standard Model, however, the true answer of the GZK puzzle is still far from knowing [7]. We found that the sharp peak in the momentum distribution of gluons caused by the GC-effects implies a large enhancement of the cross section at \( p-p \) collisions. The sudden increase of \( p-p \) cross section by several orders of magnitude may compensate the GZK suppression. Using this result we expect that the GC-effects begin from the incident proton energy \( E^{GC}_{p-p} = E_{GZK} = 6 \times 10^{19} \text{eV} \), or \( \sqrt{s^{GC}_{p-p}} = 330 \text{TeV} \). We emphasize that this is only a possible choice and \( E^{GC}_{p-p} > E_{GZK} \) is also possible. However, it does not affect the following discussion, since there are only a few available data with large uncertainty at this energy range. Except \( p-p \) collisions, there are high energy proton collisions with dense matter in the Universe. The GC-threshold energy may decrease in high dense matter. Therefore, other values \( E^{GC}_{p-A} < E_{GZK} \) are possible. A detail discussion is given in Sec. 7.

The most stable secondary particles at \( p-p \) collisions are photon, electron-positron, proton-antiproton and neutrino-antineutrino. In this work we try to explore the possible GC-signature in the energy spectra of these cosmic rays. The power-law form of energy spectrum is a general rule of the cosmic ray spectra at high energy. It is described by a straight line of the energy spectrum with a fixed index in the double-logarithmic coordinator. This line may span over more than one order of magnitude. The small
change of the spectral index can be clearly visualized via multiplying the flux by some power of the energy. Usually the broken of power-law is thought to be related to the production mechanisms of cosmic rays. In Sec. 3 we indicate that the GC-effects in proton break up a single power line into two segments at a threshold energy $E_p^{GC}$. The reason is that the GC-effects increase suddenly the production of pions. This production mechanism has never been discussed before in the literature.

The energy spectra of cosmic rays have complex structure and may relate to different dynamics. If the GC-effects are true, they will be appeared in some spectra at individual energy band. Following the results of Sec.3, we calculate the gamma spectra of two supernova remnants (SNRs): Tycho [8] and Cas A [9] in Sec. 4. We assume that the protons accelerated in these SNRs may exceed $E_p^{GC}$, ($A$ indicates the nucleon number of a targeted nucleus). Thus, the GC-effects dominate their energy spectra if accelerated protons interact with the surrounding dense matter. We find that our predicted broken power-law is consistent with the observed data of the Very Energetic Radiation Imaging Telescope Array System (VERITAS) and the Fermi Large Area Telescope (Fermi-LAT). Following these examples, we calculate the gamma ray spectrum of Centaurus A (Cen A) [10] and compare the GC-effects with the special dark matter (DM)-model [11]. Note that we are talking about the DM models rather than a general DM theory in this work. Other examples of the TeV gamma ray of the Active Galactic Nuclei (AGN) [12] are presented. The energy spectrum of pulsars is interesting subject, where the GC-threshold becomes very low due to its extreme density. We present that the observed broken power-law of pulsar spectra with a suppressed factor at GeV band [13] is consistent with the GC-effects. The galactic center GeV-gamma excess [14, 15] is a hot topic in recent years. We find that the above excess can be understood by the GC-effects in the QCD theory rather than in the special dark matter models.
An excess of the cosmic ray positron spectra at $10 \text{ GeV} - 1 \text{ TeV}$ has been reported by the Alpha Magnetic Spectrometer (AMS02) [16], which was interpreted as the DM-signature. A similar excess structure in the electron spectrum at $300 \text{ GeV} - 700 \text{ GeV}$ was early reported by the Advanced Thin Ionization Calorimeter (ATIC) [17]. However, the later finding was not confirmed by more accurate observations of the Fermi-LAT [18], the High Energy Stereoscopic System (HESS) [19], the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) [20] and the VERITAS [21]. The new data show that a power broken in the electron energy spectrum may expand to a broader range rather than a narrow excess. We try to explain these results using the GC-effects in Sec. 5.

The study of antiproton in cosmic ray is an important topic in astrophysics and has received a large amount of attention in recent years. A strong motivation of this study is to find any exotic phenomena including the possible DM-information. For this sake, the spectral shape of cosmic ray antiprotons is accurately measured from $20 \text{ GeV}$ to $400 \text{ GeV}$ by the Payload for Antimatter Exploration and Light-Nuclei Astrophysics (PAMELA) [22], the AMS02 [23] and the balloon-borne Cosmic Ray Energetics And Mass (CREAM) [24]. The measured antiproton to proton ratio presents almost a constant. Antiparticles are created as secondaries in the ordinary hadronic collisions. The conventional mechanism of secondary production predicts that this ratio decreases with energy. It means that an extra enhancement of antiprotons in this energy range is necessary. Besides, a bump structure in the proton spectrum at $10^2 \text{ GeV} - 10^5 \text{ GeV}$ is reported by the CREAM experiment [25], which confirms previous measurements by the ATIC [26] and PAMELA [27] data. We will understand these proton and antiproton spectra using the GC-effects in Sec. 6.

Surprisingly, in the above mentioned examples, the GC-effects distribute over a wide range: from $10^{-3}$ to $10^{-12}$ for the GC-critical point $x_c$, or from $1 \text{ GeV}$ to $24 \text{ TeV}$ for
the GC-threshold $E_{\pi}^{GC}$. For deeply understanding this result, we study the nuclear dependence of the GC-effects based on the ZSR equation in Sec. 7. We give a reasonable dynamic explanation about it. An interesting result is that the GC-effects in neutron star are important in the gamma spectra of pulsars. Besides, we predict that the GC-effects can be observed at next planning $p - Pb$ or $Pb - Pb$ collisions in the LHC.

The discussions and a summary are presented in Sec. 8, where our assumptions about the GC-effects are further deliberated. We also discuss the contributions of the GC-effects to the gamma ray bursts (GRBs). Although the GRB-circumstance suppresses the gamma- and neutrino-rays at $GeV - TeV$ band, and we can not observe the GC-characteristic spectra, these results are just part of the GC-effects. Besides this, the GC-effects play an important role in the transition of a lot of supernova energy to the gamma- and neutrino-bursts.

The GC occurs on the elemental gluon-revel of protons, therefore, it may expose a serious secrets hidden behind the cosmic ray spectra: Why the gamma ray spectra present the broken power-law? Why these broken spectra of pulsars have a cutoff tail? Is there a connection between the electron-positron spectra and proton-antiproton spectra? Why the broken power-law caused by the GC-effects may span over $GeV \sim TeV$ energy region? Why we have not recorded the GC-effects at the LHC? Where we can directly observe them? What is the source of the galactic center $GeV$ excess? Particularly, why the GC-effects may reproduce the predictions of the DM models at the cosmic ray spectra? We will give our answers in the following sections. The GC in proton is an exciting theoretic result, since it changes our traditional understanding about the proton behavior at high energy and advances the astrophysics.
2 A brief review about the gluon condensation

The QCD evolution equations have different forms at different energy ranges. At high energy, or small Bjorken variable $x$, gluon distributions dominate the evolution processes (Fig. 1) according to the linear DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation [28] and BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation [29]. They both predict that the gluon density in proton grows with decreasing $x$ and it will cause the violation of unitarity for the scattering cross section. In consequence, a series of the nonlinear evolution equations, for example, the GLR-MQ-ZRS (Gribov-Levin-Ryskin, Mueller-Qiu, Zhu-Ruan-Shen) equation [30-32] and BK (Balitsky-Kovchegov) equation [33] were proposed, in which the corrections of gluon recombination are considered. An important result of the non-linearization is that the BK equation, or its generalization, the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) equation [34] predict the unintegrated gluon distribution $F(x, k) \rightarrow$ a constant when transverse momentum $k$ of gluon is smaller than a characteristic saturation momentum $Q_s(x)$ (Fig. 2 (a-c)). This saturation behavior implies a balance between gluon splitting and fusion, however it also has been understood as the color glass condensate (CGC), where “condensate” implies that the maximum occupation number of gluons is $\sim 1/\alpha_s > 1$, although it lacks a characteristic sharp peak in the momentum distribution.

Zhu, Shen and Ruan first pointed our that the BK equation in Fig. 3 is inconsistent with other three equations. Instead of the BK equation, a new nonlinear evolution equation (ZSR) based on Fig. 4 is derived in [1,2]. According to the standard quantum field theory, summing-up all possible amplitudes including so-called virtual processes are necessary for the regularization of the evolution equations. The derivation of the nonlinear evolution equations based on Fig. 4 includes the contributions from real 2-2, virtual 2-2, real 1-3 and virtual 1-3 amplitudes. A key point is how to calculate the above com-
\[
Q^2 \frac{\partial G(x, Q^2)}{\partial Q^2} = \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dx_1}{x_1} G(x_1, Q^2)
\]
\[
= \frac{\alpha_s N_c}{\pi} \int \frac{d^2 k_{ab}}{k_{ab}^2} \left[ F(x, k_{ab}) - \frac{1}{2} F(x, k_{ab}) \right]
\]

\[
\frac{\partial F(x, k_{ab})}{\partial x} = -x \frac{\partial F(x, k_{ab})}{\partial x}
\]

\[
= \frac{\alpha_s N_c}{\pi} \int \frac{d^2 k_{ab}}{k_{ab}^2} \left[ F(x, k_{ab}) - \frac{1}{2} F(x, k_{ab}) \right]
\]

\[
\frac{\partial G(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dx_1}{x_1} G(x_1, Q^2)
\]
\[
+ \frac{18 \alpha_s^2}{\pi R_0^2 Q^2 N_c^2} \int_x^{1/2} \frac{dx_1}{x_1} G^2(x_1, Q^2)
\]
\[
- \frac{36 \alpha_s^2}{\pi R_0^2 Q^2 N_c^2} \int_x^1 \frac{dx_1}{x_1} G^2(x_1, Q^2)
\]

\[
\frac{\partial N(k^2, x)}{\partial x} = -x \frac{\partial N(k^2, x)}{\partial x}
\]

\[
= \frac{\alpha_s N_c}{\pi} \int \frac{d^2 k_{ab}}{k_{ab}^2} \left[ \frac{N(k^2, x) - \frac{1}{2} N(k^2, x) + \frac{1}{2} N(k^2, x)}{\sqrt{k^2 + 4k^2}} \right]
\]

**DGLAP**

**BFKL**

**GLR-MQ-ZRS**

**BK**

Figure 1: A set of the QCD evolution equations for gluons at small $x$ and leading order approximation. Where the BK equation is taken in a full momentum space [36,37]. $G(x, Q^2)$ and $F(x, k)$ are integrated and unintegrated gluon distributions.

Figure 2: Schematic diagrams (a-c) for the saturation solutions of the BK/JIMWLK equation; (d-f) for the condensation solutions of the ZSR equation, which is evolved from a saturated GBW input.
Figure 3: The elemental amplitudes corresponding to Fig. 1.

Figure 4: A set of consistent elemental amplitudes if amplitude \((d)\) replaces \((d)\) in Fig. 3. Note that a complete evolution equation based on \((d)\) should contain complicated interference and virtual processes. These four evolution equations were unitary derived using the time ordered perturbative theory in Refs.[1,2,32].
plicated virtual diagrams? Fortunately, such a technique was established using the time
ordered perturbation theory (TOPT) in [31]. This TOPT-cutting rule was successfully
used to recover the momentum conservation in the GLR-MQ equation [32] and obtained
the support in serious of examples [35]. Using the TOPT cutting rule, the resulting
ZSR equation for the unintegrated gluon distribution $F(x, k^2)$ at the leading logarithmic
($LL(1/x)$) approximation is [2]

$$
-x \frac{\partial F(x, k^2)}{\partial x} = \frac{3\alpha_s k^2}{\pi} \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \left\{ \frac{F(x, k'^2) - F(x, k^2)}{|k'^2 - k^2|} + \frac{F(x, k^2)}{\sqrt{k^4 + 4k'^4}} \right\} \\
- \frac{81}{16} \frac{\alpha_s^2}{\pi R_N^2} \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \left\{ \frac{k'^2 F^2(x, k'^2) - k'^2 F^2(x, k^2)}{k'^2 |k'^2 - k^2|} + \frac{F^2(x, k^2)}{\sqrt{k^4 + 4k'^4}} \right\},
$$

(2.1)

where the first item on the right is the BFKL equation part and the second one is nonlinear
correction caused by the gluon fusions. The singular structure in the linear and nonlinear
evolution kernels both corresponds to the random evolution in the $k$-space, where $k^2 - k'^2$
may across over zero. This is a general property of the logarithmic ($1/x$) resummation, and
remember, this is also a key point of our following story. Note that the BK equation is a
special approximation of Eq. (2.1) [1,2,36], therefore, the saturation-, or the CGC-solution
of the BK equation is instable and they will continually evolve in the ZSR equation.

A major difference between the ZSR and BK equations is as follows. As we know that
the contributions of evolution along random chain of gluon-transverse momentum $k$
are important at small $x$. For example, it leads to a singular structure in the BFKL equation
at $k^2 = k'^2$. Comparing the nonlinear parts of the BK (in the full momentum space [37])-and
ZSR equations, one can find the BK equation lost this singularity. It means that the
resummation in the BK equation at small $x$ is incomplete.

It is interest that the nonlinear ZSR evolution equation results in the chaotic solution
if the Bjorken variable $x$ goes beyond a critical point $x_c$ (Fig. 5). This is a first chaotic example in the QCD evolution equations, although chaos is a popular natural phenomenon in nonlinear science. Most surprisingly, the dramatic chaotic oscillations produce the strong shadowing and antishadowing effects via the nonlinear terms of Eq. (2.1), they converge gluons to a state with a critical momentum. This is the gluon condensation (GC) in proton (Fig. 6 and Fig. 2(d)-(f)). We emphasize that the above either the chaotic solution or the convergence effect origin from the regularized nonlinear part in Eq. (2.1), and it is a general structure of the logarithmic ($1/x$) resummation even at higher ordered approximations (see Ref. [3] for details).

The cross section of inclusive particle production at high energy $p - p$ collisions is dominated by the product of the gluon distributions in incident and targeted protons. The rapidity- and transverse momentum-distributions at $p - p$ collisions with the GC-effects are plotted in Figs. 7 and 8. One can find that the large fluctuations arisen by the
Figure 6: The $x$- and $k_T^2$-dependence of $F(x, k_T^2)$ of the ZSR equation. The solution shows that the gluon distribution function $F(x, k_T^2)$ begins its smooth evolution under suppression of gluon recombination, but when $x$ approaches a small critical point $x_c$, $F(x, k_T^2)$ will oscillate aperiodically in a narrow $k_T^2$ range.

Figure 7: A schematic diagram for the inclusive gluon rapidity distribution at $p - p$ collisions (taking from Ref.[3]), where $\sqrt{s_{i+1}} > \sqrt{s_i}$. The results show the large fluctuations are arisen by the GC.
Figure 8: A schematic diagram for the $k_T$-distributions of gluon jet at $p - p$ collisions (taking from Ref.[3]).

GC-effects. These results lead to an extreme enhancement of the multiplicity of gluon jets and number of secondary particles at $p - p$ collisions. We will discuss these GC-effects at high energy cosmic rays in the following sections.
3 The characters of the gluon condensation in cosmic ray spectra

The secondary particles in cosmic rays may originate from the hadronic processes, for example, \( p + p(A) \rightarrow \pi^{\pm,0} + p + \bar{p} + \text{others} \). The GC increases suddenly the proton-proton or proton-nuclei cross sections. The shape of energy spectra of these secondary particles is an ideal subject to observe the GC-effects if the energy of incident proton is accelerated beyond the GC-threshold \( E_{p-p(A)} > E_{p-p(A)}^{GC} \).

The cross section of inclusive gluon mini-jet production at high energy \( p - p \) collisions [3,38] reads

\[
\frac{d\sigma_g}{dk^2 dy} = \frac{64N_c}{(N_c^2 - 1)k^2} \int dq \int_0^{2\pi} d\phi \phi_\Delta(\Omega) \frac{F(x_1, \frac{1}{4}(k + q)^2)F(x_2, \frac{1}{4}(k - q)^2)}{(k + q)^2(k - q)^2},
\]

where \( \Omega = \text{Max}\{k^2, (k + q)^2/4, (k - q)^2/4\} \); The longitudinal momentum fractions of interacting gluons are fixed by kinematics: \( x_{1,2} = k e^{\pm y}/\sqrt{s} \). The multiplicity of gluon mini-jet is

\[
N_g = \frac{1}{S} \int dy \int dk^2 \frac{d\sigma_g}{dk^2 dy},
\]

\( S \) is the inelastic cross section for the mini-jet.

We denote that \( N_\pi(E_{p-p}, E_\pi) \) and \( N_X(E_{p-p}, E_X) \) as pion- and \( (p + \bar{p}) \) pair-(denoted by X) numbers with energies \( E_\pi \) and \( E_X \) at \( p - p \) collisions; \( E_{p-p} \) is the energy of incident proton in the rest frame of targeted proton. In the normal case, these quantities are calculated using the multiplicity of gluon jet and the phenomenological fragmentation functions. Unfortunately, we lack the knowledge about the hadronization of gluons at the GC-environment. For illustrating the GC-effects in cosmic ray spectra quantitatively, we make the following deductions:

(i) The sharp peak in the gluon momentum distribution caused by the GC-effects leads
Figure 9: Schematic diagrams for the hadronization. (a)-(b): the normal hadronization without the GC-effects; (c)-(d) the saturated hadronization with the GC-effects.

to an extreme enhancement of the multiplicity of gluon jets and number of secondary particles at $p - p$ collisions, which can be larger than the normal value several orders of magnitude as we have examined in Ref. [3]. In general, the more the larger $N_g$, the more the secondary particles. However, energy conservation restricts the massive particle production. In fact, if $N_\pi + 2N_X$ is larger than a threshold value,

$$N_\pi m_\pi + 2N_X m_p > \sqrt{s_{p-p}}$$

(3.3)

violates energy conservation. Therefore, provided the multiplicity of the gluon jet much larger than the possible maximum number of massive particles due to the GC-effects, the production of massive particles will be saturated: all available kinetic energies of protons are almost used to create pions and $(p + \overline{p})$-pair in the CM system. We call it as the saturated hadronization (Fig. 9). The validity of this assumption will be checked by the
following experimental facts, but we use a simple estimation to explain it. Considering
$p-p$ collisions at $\sqrt{s_{p-p}} = 200 \text{ TeV}$ and all secondary particles are pions, one can find
that the above mentioned maximum number of pions are about $N_{\pi}^{\text{max}} \sim 10^6$. We take
the normalized kinetic energy distribution of pion without the GC-effects as

$$P(E) = \frac{1}{\sqrt{\pi m_{\pi} E}} e^{-E/(2E)}, \quad (3.4)$$

where the average kinetic energy $\bar{E} \simeq (\sqrt{s_{p-p}} - 2m_p - N_{\pi} m_{\pi})/N_{\pi}$. As an example,
assuming average transverse momentum of pion is about $\sim 5 \text{ GeV}$, the resulting $N_{\pi} \sim 10^3$.

Now we consider the GC-effects at the same collision energy $\sqrt{s_{p-p}} = 200 \text{ TeV}$. In this
case $N_{\pi} \to N_{\pi}^{\text{GC}} \sim 10^6$ has a big increment, which exceeds $N_{\pi}^{\text{max}}$ and the corresponding
distribution $P(E) \to \delta(E)$ at $N_{\pi} \to N_{\pi}^{\text{max}}$, i.e., the saturated hadronization.

According to the above discussions, we write directly the relativistic invariant and
energy conservation as

$$(2m_p^2 + 2E_{p-p} m_p)^{1/2} = 2m_p + N_{\pi} m_{\pi} + N_X (m_p + m_{\pi}), \quad (3.5)$$

$$E_{p-p} + m_p = [2m_p + N_{\pi} m_{\pi} + N_X (m_p + m_{\pi})] \gamma \quad (3.6)$$

at the saturated hadronization limit; $\gamma$ is the CM Lorentz factor. Note that although a
small part of particles may still take a large momentum tail, but it does not affect our
following discussions.

(ii) A lot of secondary particles with a certain energy accumulate in a narrow space
at per collision, they may transform each other in the formation time due to their wave-
functions overlap, for example, $p + \bar{p} \leftrightarrow X(1835)$ and $\pi^+ + \pi^- \leftrightarrow 2\pi^0$. X(1835) was
discovered by BES Collaboration [39] and is explained as a proton-antiproton bound
state [40]. However, the above balance between $p + \bar{p}$ and X(1835) (or $\pi^+$ and $\pi^0$) will be
broken since \( m_p + m_\pi > X(1835) \) (or \( m_{\pi^+} + m_{\pi^-} > 2m_{\pi^0} \)), and the kinetic energy of all secondary particles are almost zero in the CM system at the GC-circumstance. Besides, the lifetime of \( \pi^0 \) (10^{-16} s) is much shorter than the typical weak decay lifetimes of \( \pi^\pm \) (10^{-6} s – 10^{-8} s). Therefore Eqs. (3.5) and (3.6) becomes

\[
(2m_p^2 + 2E_{p-p}m_p)^{1/2} = 2m_p + N_\pi m_\pi + N_X m_X, \tag{3.7}
\]

\[
E_{p-p} + m_p = [2m_p + N_\pi m_\pi + N_X m_X] \gamma, \tag{3.8}
\]

where the contributions from \( \pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \) and \( \mu^\pm \to e^\pm + \nu_e(\bar{\nu}_e) + \nu_\mu(\bar{\nu}_\mu) \) to the spectra are suppressed. We will check this suggestion after its application in Sec. 8.

Using \( E_X/E_\pi = m_X/m_\pi = \zeta \) and \( N_X/N_\pi \sim \alpha_s^2 = \eta \) (\( \eta = 0.01 \) if \( Q^2 > 100 \text{ GeV}^2 \)), we rewrite Eqs. (3.7) and (3.8) as

\[
(2m_p^2 + 2E_{p-p}m_p)^{1/2} = 2m_p + N_\pi m^* \tag{3.9}
\]

\[
E_{p-p} + m_p = [2m_p + N_\pi m^*] \gamma, \tag{3.10}
\]

where \( m^* = (1 + \eta \zeta)m_\pi \). One can easily get the solutions \( N_\pi(E_{p-p}, E_\pi) \) (or \( N_X(E_{p-p}, E_X) \)) for \( p - p(A) \) collisions in GeV-unit

\[
\ln N_\pi = 2.19 + 0.5 \ln E_{p-p(A)}, \quad \ln N_\pi = 4.50 + \ln E_{\pi}, \tag{3.11}
\]

and

\[
\ln N_X = -2.42 + 0.5 \ln E_{p-p(A)}, \quad \ln N_X = -2.72 + \ln E_X. \tag{3.12}
\]
This extra power-law describes the GC-effects in cosmic ray spectra. We will show that Eq. (3.11) and (3.12) result in the broken power-law.

The relation between the GC-threshold $\sqrt{s_{p-p(A)}}$ and the GC-critical point $x_c$ is kinematically determined as follows [3]. The gluon condensation plays a role if the contributions of the gluon condensation peak local at $y_{max} = \ln(\sqrt{s_{p-p(A)}/k_c})$, or at

$$x_c = \frac{k_c}{\sqrt{s_{p-p(A)}}} e^{-y_{max}} \approx \frac{k_c^2}{s_{p-p(A)}}. \quad (3.13)$$

Combining $\sqrt{s} = \sqrt{2m_p E}$ and Eq. (3.11), we have

$$E_{\pi}^{GC} = \exp \left(0.5 \ln \frac{k_c^2}{2m_p x_c} - 2.3\right). \quad (3.14)$$

On the other hand, $p - p(A)$ cross section disappears if the contributions of the gluon density peak at $x_c$ move to the rapidity center $y = 0$ (see Fig. 7), i.e., at

$$x_c = \frac{k_c}{\sqrt{s_{G^{C,max}}}} e^{y=0}, \quad (3.15)$$

or at

$$E_{\pi}^{G^{C,max}} = \exp \left(0.5 \ln \frac{k_c^2}{2m_p x_c^2} - 2.3\right) = \frac{E_{\pi}^{GC}}{\sqrt{x_c}}$$

$$= e^{2.3} \sqrt{\frac{2m_p}{E_{\pi}^2}} \left(\frac{E_{\pi}^{G^{C}}}{}\right)^2 = 17\left(E_{\pi}^{G^{C}}\right)^2. \quad (3.16)$$

Therefore we predict that the GC-effects lead to a broken power-law at $E = E_{\pi}^{GC}$ with a cutoff tail.

Now we turn to the Universe. According to the Big Bang model, the light elements at the beginning of the Universe is created by the nuclear fusion reactions. However, the absence of stable nuclei with $A = 8$ basically stops the primordial big bang chain. Heavier nuclei are produced in stellar (up to Fe) or supernova (where the rich neutrons help to form
heavy nuclei with $A > 200$). Therefore, except $p - p$ collisions, high energy protons may collide with other nuclei even dense matter in the Universe. The GC-threshold decreases in high dense matter. Therefore, $E^{GC}_{p-A} < E^{GC}_{p-p}$ at $p - A$ collisions are certain, and the GC-effects may appear below the GZK scale.

Considering the collisions of an accelerated proton beam with a target, which is consisted by several elements. A leading contribution origins from the collisions of proton on the heaviest nucleus with an enough abundance. For illustrating this result we consider the following possible situations:

(a) Suppose the target $A$ is a heaviest nucleus and only $p - A$ collisions have the GC-effects. The GC-effects are absent at other $p - A'$ collisions since $E^{GC}_{p-A'} > E^{GC}_{p-p}$ at $A' < A$. In this case spectrum $\Phi(p - A')$ is negligible comparing with strong GC-effects in the spectrum $\Phi(p - A)$.

(b) Suppose $p - A$ and $p - A'$ collisions have the GC-effects, but $A' < A$. When $A' \sim A$, one can incorporate these two spectra to one spectrum. While $A' \ll A$, the spectrum $\Phi(p - A')$ origins from $p - A'$ collisions is much weaker than $\Phi(p - A)$ since the power-law $\Phi \sim E^{-\beta_p}_{p-A'}$ (note $E^{GC}_{p-A'} \gg E^{GC}_{p-p}$ since $A' \ll A$). Therefore, the contributions from the $p - A'$ collisions can either be incorporated into the suppressed factor (see $E^{-\beta_p}$ in Eq. (6.2)), or be neglected.

In consequence, for an individual GC-source in a certain energy band, we only consider the GC-effects originated from the collisions of proton on a heaviest element, if it has enough abundance.

The above discussion is focused on a single GC-source. The observed spectra in sky survey may origin from several GC-sources. These spectra either composite roughly a single broken power curve, or they appear precisely as the multi-broken power structure (see examples in Figs. 17 and 19). Particularly, the $p - p$ collision is a special GC-
source since the proton (hydrogen) is a popular matter in the Universe. Its characteristic spectrum is a broken power at $E \sim 24 \, TeV$ in the gamma-, electron-positron spectra, or at $E \sim 320 \, TeV$ in the proton-antiproton spectra. We will recognize them in following sections.
4 The GC effects in the gamma ray spectra

The SNRs are important gamma ray source. There are different shapes of gamma energy spectra, which corresponds to various production mechanisms. For example, gamma rays can be generated as bremsstrahlung radiation when electrons and positrons interact with ambient matter, or as a result of inverse Compton scattering of low energy photons. Hadronic interactions can also create gamma rays. Indeed, proton-proton (or proton-nuclei and nuclei-nuclei) collisions may create $\pi^0$ meson, which quickly decays into two gamma photons. The recent detection of the neutral pion-decay signature from two middle-aged SNRs: IC 443 and W44 has been demonstrated by the Fermi-LAT [41]. In normal case the gamma-ray spectrum \( \Phi_\gamma(E_\gamma) \) is symmetric about \( E_\gamma = m_{\pi^0}/2 = 67.5 \text{MeV} \). However, we will show that the GC-effects break the gamma power-law at GeV-TeV band.

Cosmic ray protons accelerated in some SNRs may reach the \( E_{p-p(A)}^{GC} \)-energy range and induce the GC-effects in their gamma spectra. We consider \( p + p(A) \rightarrow \pi^{\pm,0} + \text{others} \), \( \pi^- + \pi^+ \rightarrow 2\pi^0 \) and \( \pi^0 \rightarrow \gamma + \gamma \). The corresponding gamma flux is

\[
\Phi_\gamma(E_\gamma) = \Phi_0^\gamma(E_\gamma) + \Phi_{\gamma}^{GC}(E_\gamma),
\]

where \( \Phi_0^\gamma(E_\gamma) \) is the contributions without the GC effects. The measurements of gamma spectra in our examples are oriented, therefore \( \Phi_0^\gamma \ll \Phi_{\gamma}^{GC} \) and we take \( \Phi_0^\gamma \approx 0 \). We will check this approximation in the following Fig. 12. On the other hand,

\[
\Phi_{\gamma}^{GC}(E_\gamma) = C_{p-p(A)} \left( \frac{E_\gamma}{1 \text{ GeV}} \right)^{-\beta_{\gamma}} \int_{E_{\mu}^{min}}^{E_{\mu}^{max}} dE_{\pi} \left( \frac{E_{p-p(A)}}{E_{p-p(A)}^{GC}} \right)^{-\beta_{p}} N_{\pi}(E_{p-p(A)}, E_\pi) \frac{d\omega_{\pi-\gamma}(E_\pi, E_\gamma)}{dE_\gamma},
\]

where \( N_{\pi}(E_{p-p(A)}, E_\pi) \) is taking from Eq. (3.11); index \( -\beta_{\gamma} \) (or \( -\beta_{p} \)) relates to the propagating loss of gamma rays (or to the acceleration restriction of protons); a similar
factor \((E_\pi/1 \text{ GeV})^{-\beta_\pi}\) has been incorporated into \((E_{p-p(A)}/E_{p-p(A)}^{GC})^{-\beta_p}\) using Eq. (3.11); 

\(C_{p-p(A)}\) incorporates the kinematic factor with the flux dimension and the percentage of 

\(\pi^0 \rightarrow \gamma + \gamma\), therefore, the normalized spectrum for \(\pi^0 \rightarrow \gamma + \gamma\) is

\[
\frac{d\omega_{\pi-\gamma}(E_\pi, E_\gamma)}{dE_\gamma} = \frac{2}{\beta_\pi E_\pi} H[E_\gamma; \frac{1}{2} E_\pi (1 - \beta_\pi), \frac{1}{2} E_\pi (1 + \beta_\pi)], \tag{4.3}
\]

\(H(x; a, b) = 1\) if \(a \leq x \leq b\), and \(H(x; a, b) = 0\) otherwise. in consequence, Eq.(4.1) becomes

\[
\Phi_\gamma(E_\gamma) = C_{p-p(A)} \left(\frac{E_\gamma}{1 \text{ GeV}}\right)^{-\beta_\gamma} \int_{E_{p-p(A)}^{GC}}^{E_{p-p(A)}^{GC,\text{max}}} dE_\pi \left(\frac{E_{p-p(A)}}{E_{p-p(A)}^{GC}}\right)^{-\beta_p} N_\pi(E_{p-p(A)}, E_\pi) \frac{2}{\beta_\pi E_\pi}, \tag{4.4}
\]

where the low-limit of the integration takes \(E_\pi^{GC}\) (or \(E_\gamma\)) if \(E_\gamma \leq E_\pi^{GC}\) (or if \(E_\gamma > E_\pi^{GC}\)).

Note that the above processing will lead to a sharp change of power index at \(E_\pi = E_\pi^{GC}\).

Actually, the p-p cross section is not increased suddenly, therefore, we take the following way to smooth the resulting curves: the low-limit of the integration takes \(E_\pi^{GC}\) (or \(E_\gamma\)) if \(E_\gamma \leq E_\pi^{GC} \pm \varepsilon\) (or if \(E_\gamma > E_\pi^{GC} \pm \varepsilon\)), \(\varepsilon\) is an arbitrary small number. Without any complex calculations, one can find that the GC-effects break the power-law in \(\Phi_\gamma(E_\gamma)\) at \(E_\gamma = E_\pi^{GC}\).

Tycho and Cas A are two ideal SNRs since they located in a relatively clean environment and have been studied over a wide range of energies. Although the acceleration mechanism in SNRs is unclear, we assume that the protons accelerated by these two SNRs may reach \(E_{p-p(A)}^{GC}\)-energy range and they interact with the surrounding matter. Thus, the GC-effects appear in their energy spectra.

Figure 10 gives our predicted gamma spectrum and the comparison with Tycho’s spectrum [8]. The results clearly show the broken power-law at \(E_\pi^{GC} = 440 \text{ GeV}\) and \(24 \text{ TeV}\).
Recently the gamma spectra of Cas A are derived by the MAGIC Collaboration using accumulating data at $100 \text{ GeV} - 10 \text{ TeV}$ and combining 8 years of the Fermi-LAT data at $60 \text{ MeV} - 500 \text{ GeV}$ [9]. The result shows that the gamma-ray emission from $60 \text{ MeV}$ to $10 \text{ TeV}$ can be attributed to a broken power-law, but it cannot be reproduced using the leptonic model. Figure 11 shows our predicted gamma spectrum of Cas A and a comparison with observed data, where two broken points are $E = E_{\pi}^{\text{GC}} = 5.2 \text{ GeV}$ and $440 \text{ GeV}$. The former corresponds to $x_c = 1.3 \times 10^{-4}$. Such low GC-threshold implies a very big target. We suggest that it origins from $p-$neutron star collisions. We know very little knowledge about the structure of neutron star. We image that the neutron star may provide a large unfixed number $A^*$ although we don’t know the details of proton-neutron interactions. Thus, the neutron star in a pulsar provides the GC-source with very low threshold $E_{\pi}^{\text{GC}}$, it not only forms a broken power-law around this threshold, but also presents a cutoff according to Eq. (3.16). We introduce a fast suppressed factor $\exp\{1 - [E/(0.8E_{\pi}^{\text{GC,max}})]^2\}$ starting from $E = 0.8E_{\pi}^{\text{GC,max}}$. In this Cas A example, $E_{\pi}^{\text{GC,max}} \sim 460 \text{ GeV}$. Note that such cutoff does not appear at $E_{\pi}^{\text{GC}} = 440 \text{ GeV}$ since the value of $E_{\pi}^{\text{GC,max}} \sim 3 \times 10^6 \text{ GeV}$ (see Eq. (3.16) has beyond the measured range.

It is interesting that two “older” SNRs: IC 443 and W44 (3 kyr $\sim$ 30 kyr) present a completely different gamma ray spectra [41], which are symmetrical with respect to the photon energy $\sim m_\pi/2$ and they have smooth change of power indexes. Although the gamma-ray spectra of both (young and older) kinds of SNRs are explained by the decay of neutral pion, however, the distributions of $\pi^0$ in two cases are different. The energy of protons inside “order” SNRs may be lower than $E_{p-A}^{\text{GC}}$, and the remnants are diluted in their expansions. In this case, the pion spectrum at p-p collisions is estimated by using an experimental inclusive cross section at lower energy scale in a resonance model. On the other hand, the environment of young SNRs (Tycho and Cas A) are closer to that of
bursting supernova, where the protons are accelerated to beyond $E_{p-p(A)}^{GC}$. Therefore, the GC-effects appear in the gamma ray spectra of Tycho and Cas A.

Cen A is a closest known gamma ray emitting active galaxy. Its gamma ray spectrum [10] presents an excess structure and was understood as the possible DM-signature [11]. We present the GC-effects in this spectrum in Fig. 12, where the data are combined spectrum of the Fermi-LAT analysis [12] and the HESS spectrum [19]. This example also shows that the contributions of $\Phi_0^\gamma$ in Eq. (4.1) is negligible.

The $TeV$ gamma rays of the Active Galactic Nuclei (AGN) in extragalactic system are detected [12]. We present the comparisons of our predicted spectra with the Fermi-LAT data in Fig. 13. The broken power-law of the GC-effects still exists.

The galactic center GeV (gamma-ray) excess aroused great interest. Several groups reported the detection of excess emission at energies of a few GeV near the Milky Way center based on the data from the Fermi-LAT [15]. Gamma-ray emission in galactic center direction includes the products of interactions between cosmic rays with interstellar gas and radiation fields, as well as many individual sources such as pulsars, binary systems, supernova remnants, and even annihilation of weakly interacting massive particles (WIMPs) as a special DM model. We try to give an explanation using the GC-effects. We present the contributions of the GC-effects in $p-A^*$ collisions at $E \sim 2 GeV$ to the gamma ray spectrum of the Milky Way center in Fig. 14. The data are taken from Ref. [42]. The predictions of two DM models are compared. An obvious distinguish between two explanations is an excess or a broken line. We find that the data prefer to the GC-effects.

The gamma ray spectrum emitted from a galaxy M31 also appears a broken power-law [43]. We plot our prediction (solid line) in Fig. 15, where two broken powers at $E_\pi^{GC} = 1 GeV$ (with $x_c = 3.4 \times 10^{-3}$) and $E_\pi^{GC} = 440 GeV$ with $x_c = 1.8 \times 10^{-8}$ are
presented, respectively. The parameter $\beta_\gamma$ in Fig. 14 is larger than that in Fig. 15. We understand that the emitted gamma rays from the Milky Way center will interact with interstellar matter more than that from the center of M31, since the earth and the Milky Way center localize at the same disk plan.

We have shown that a broken power in the spectra of M31, Milky Way center and Cas A at 1, 3 and 5.2 GeV, they correspond to $x_c = 3.4 \times 10^{-3}, 3.8 \times 10^{-4}$ and $1.3 \times 10^{-4}$, respectively. According to the estimations in the following Sec. 7, these targets have big number $A'$ and they may explained the GC-effects in the neutron star. For conforming the above suggestion, we study the the gamma-ray spectra of pulsars. Pulsars are constructed by neutron stars. If our suggestion is correct, we shall observed the full GC-effects: a broken power-law with a cutoff tail in the pulsar gamma spectrum, since the value of $E_{\pi, max}^{GC}$ may reduce down to very low energy in neutron star. Using $E_{\pi}^{GC}$ as an input and Eqs. (3.14), (3.16), we plot our predicted pulsar gamma ray spectra in Fig. 16, where the contributions from $E_{\pi}^{GC} = 440$ GeV are added. Our predicted gamma spectra coincide with the data. We emphasize that the GC-threshold $E_{\pi}^{GC}$ and the range of GC-effects $E_{\pi, max}^{GC}$ seem two irrelevant quantities, however, we shed light on their relationship using a simple equation (3.16) and the predictions are supported by the data of pulsar spectra.
Figure 10: Predicted gamma ray spectra multiplied by $E^2$ and comparisons with the VERITAS- and Fermi-spectra for Tycho’s supernova remnant Ref.[8]. The indexes $\beta_\gamma = 1.8$, $\beta_p = 2.0$. The coefficients $C_{p-p} = 1.13 \times 10^{-8} \text{ eV cm}^{-2}\text{s}^{-1}$, $C_{p-A} = 1.5 \times 10^{-5} \text{ eV cm}^{-2}\text{s}^{-1}$. The broken lines are the p-A and p-p contributions, the solid line is the total contributions of the GC-effects.

Figure 11: Similar to Fig. 10 but for the Cas A supernova remnant [9]. The indexes $\beta_\gamma = 2.1$, $\beta_p = 2.0$. The coefficients $C_{p-p} = 1.7 \times 10^{-7} \text{ eV cm}^{-2}\text{s}^{-1}$, $C_{p-A} = 1.7 \times 10^{-4} \text{ eV cm}^{-2}\text{s}^{-1}$. Not that the first power-law breaks at $E = E_{\pi}^{GC} = 5.2 \text{ GeV}$ due to $p-A^*$ collisions and the corresponding cutoff begins at $0.8E_{\pi}^{GC,max} = 410 \text{ GeV}$. 
Figure 12: The gamma ray spectrum of Cen A [10] multiplied by $E^2$ consists with the GC-effects (solid line). The indexes $\beta_\gamma = 2.1$, $\beta_p = 2.0$. The coefficients $C_{p-p} = 1.7 \times 10^{-7} \text{ eV cm}^{-2} \text{s}^{-1}$, $C_{p-A} = 1.7 \times 10^{-4} \text{ eV cm}^{-2} \text{s}^{-1}$. Thick-dashed line is a prediction of the DM-model [11] and fine-line is the background contributions.

Figure 13: Some of TeV-gamma ray spectra of AGN derived from the Fermi-LAT [13] observations and the comparisons with our predicted GC-effects.
Figure 14: Contributions of the GC-effects in $p - A^*$ collisions at $E \sim 2 \text{ GeV}$ to the gamma ray spectrum of the Milky Way center. The data are taken from Ref.[42]. The predictions of two DM models are compared.

Figure 15: Predicted gamma ray spectrum of galaxy (M31) center. Data are taken from [43]. Two broken lines due to the GC-effects at $E = 1 \text{ GeV}$ and 440 GeV are used.
(PSR J0101-6422) GC effects

(PSR J0218+4232) GC effects

(PSR J0659+1414) GC effects
Figure 16: Predicted gamma ray spectra of pulsars. The GC-effects break the power-law at $E = E_{\pi}^{GC}$ and then the spectra are suppressed at $0.8E_{\pi}^{GC,max}$ due to $p - A^*$ collisions (see Table 6). The second broken power at $E_{\pi}^{GC} = 440$ GeV origins from $p - U$ collisions. The data are taken from Ref.[13]: (a) for pulsar J0101-6422; (b) for pulsar J0218+4232; (c) for pulsar J0659+1414; (d) for pulsar J1446.4701; (e) for pulsar J1459-6053; (f) for pulsar J1709-4429; (g) for pulsar J1713+0747; (h) for pulsar J1747.4036; (i) for pulsar J1954+2835.
5 The GC-effects in the electron-positron spectra

We discuss the GC effects in the electron-positron spectra via the creation and chain decay of \( \pi^0: p + p \to \pi^0 + \text{others}, \pi^0 \to \gamma + \gamma \to e^- + e^+ \). Note that \( p + p \to \pi^\pm + \text{others}, \pi^\pm \to \mu^\pm + \nu_\mu, \mu^\pm \to e^\pm + \nu_e + \overline{\nu}_e \) are suppressed due to \( \pi^- + \pi^+ \to 2\pi^0 \) (see Sec. 3).

It is different from the gamma spectra, electrons travel along random trajectories and all direction information lost before measured. We can not reconstruct the position of the electron source. Therefore, the isotropic measured electron and positron fluxes are

\[
\Phi_j(E_j) = \Phi^0_j(E_j) + \Phi_j^{\text{GC}}(E_j),
\]

where \( j = e^+ \) or \( e^- + e^+ \). Following Eq. (4.4), we have

\[
\Phi_j^{\text{GC}}(E_j) = C_j \left( \frac{E_j}{1 \text{ GeV}} \right)^{-\beta_j} \int_{E_j} dE_\gamma \left( \frac{E_\gamma}{1 \text{ GeV}} \right)^{-\beta_\gamma} \int_{E_{\gamma_{\min}}}^{E_{\gamma_{\max}}} dE_\pi \left( \frac{E_{p-p(\Lambda)}}{E_{p-p(\Lambda)}} \right)^{-\beta_p} N_\pi(E_{p-p(\Lambda)}, E_\pi) \frac{d\omega_{\gamma-e}(E_\pi, E_\gamma)}{dE_\gamma} \frac{d\omega_{e-e}(E_\gamma, E_e)}{dE_\gamma} \quad (5.1)
\]

\[
= C_j \left( \frac{E_j}{1 \text{ GeV}} \right)^{-\beta_j} \int_{E_j} dE_\gamma \left( \frac{E_\gamma}{1 \text{ GeV}} \right)^{-\beta_\gamma} \int_{E_{\gamma_{\min}}}^{E_{\gamma_{\max}}} dE_\pi \left( \frac{E_{p-p(\Lambda)}}{E_{p-p(\Lambda)}} \right)^{-\beta_p} N_\pi(E_{p-p(\Lambda)}, E_\pi) \frac{2}{E_\gamma E_\pi},
\]

where factor \( (E_\pi/1 \text{ GeV})^{-\beta_\pi} \) has been incorporated into \( (E_{p-p(\Lambda)}/E_{p-p(\Lambda)})^{-\beta_p} \) using Eq. (3.11). \( C_j \) incorporates the kinematic factor with the flux dimension and the percentage of \( \pi^0 \to 2\gamma \) and \( \gamma + \gamma \to e^- + e^+ \).

After taking average over possible directions, the energy of pair-produced electron-positron is uniformly distributed from zero to maximum value, i.e.,

\[
\frac{d\omega_{\gamma-e}(E_\gamma, E_e)}{dE_e} = \frac{1}{E_\gamma}.
\]

Figure 17a is a fitting data [19,44] of \( e^- + e^+ \) spectrum using \( E_\pi^{\text{GC}} = 880 \text{ GeV} \) for the first broken power, where \( \Phi^0_{e^- + e^+} \) is taken from the counting background events in the
Figure 17: (a) Predicted cosmic ray electron+positron spectrum multiplied by $E^{2.7}$ as a function of energy (solid line). Broken lines present the broken power-law of the GC-effects. $\Phi_{e^-+e^+}^0$ (dashed line) is taken from a count background events in the signal region in [20]. The free parameters see Table 1. The data are taken from Ref. [19,44]. (b) Same as (a) but using a fine analysis of spectrum, where the first broken point locks at $E_{e^-+e^+} = 440$ GeV.
Figure 18: Predicted cosmic ray positron spectrum multiplied by $E^3$ as a function of energy (solid line). Broken lines present the broken power-law of the GC-effects. The free parameters see Table 2. Thick- and fine-dashed lines are the prediction of a DM-model [11] and $\Phi_{e^+}$, using a conventional mechanism of secondary production [46]. The data are taken from Ref. [16,43,44].

The positron spectrum could show GC-signature clearer because a very low $e^+$-background. Our predicted positron flux is shown in Fig. 18, where $E_{\pi}^{GC} = 440 \, GeV$ for the first broken power is used and the data are taken from [16,43,45]. $\Phi_{e^+}$ is taken from a pair-production model [46]. The free parameters are listed in Table 2. As seen from Fig. 18, comparing with the DM-model, which presents a sharp drop off at high energies, the spectrum with the GC-effects exhibits a broad bump. Although they both agree with the data till 600 $GeV$, combining the electron plus positron spectrum (Fig. 17), the GC-effects seem

signal region [20]. The other free parameters are seen Table 1. We find that the broken power spectrum by the GC-effects is consist with the data. As we have pointed out in Sec. 3 that the broken curve in Fig. 17a may incorporated several GC-sources in the sky survey. A more fine analysis of the $e^- + e^+$ spectrum is presented in Fig. 17b, where the possible multi-GC sources are shown.
Why the $e^-$ (or $e^- + e^+$) spectrum presents more richer GC-sources than the $e^+$-spectrum? Antiparticles multiple-interact with ubiquitous particles and will be annihilated during their long propagation. Therefore, the most measured relativistic antiparticles originate from the neighboring strong source near the earth. While the electron spectrum is sky survey and it collects the contributions from all possible sources including extra-galaxies. Therefore, the particle spectrum contains more richer GC-sources.
6 The GC-effects in the proton-antiproton spectra

The measured proton (or antiproton) flux, which contains the contributions of the GC-effects and the background reads

\[ \Phi_j(E_j) = \Phi_0^j(E_j) + \Phi_{GC}^j(E_j), \]

where \( j = p \) or \( \bar{p} \); \( \Phi_0^j(E_j) \) is the background contributions without the GC effects, while the GC-effects contribute

\[ \Phi_{GC}^j(E_j) = C_j \left( \frac{E_j}{1 \text{ GeV}} \right)^{-\gamma_j} \int_{E_{X\text{min}}}^{E_{X\text{max}}} dE_X \left( \frac{E_{p-p(A)}}{E_{GC}^{p-p(A)}} \right)^{-\beta_p} N_X(E_{p-p(A)}, E_X) \frac{d\omega_{X-j}(E_X, E_j)}{dE_j}, \]

(6.2)

where \(-\gamma_j \) is the suppression index of secondary particle \( j \) during propagation; \(-\beta_p \) is the index of incident proton which carry information about their acceleration information; a factor \((E_X/1 \text{ GeV})^{-\beta_X}\) has been incorporated into \((E_{p-p(A)}/E_{GC}^{p-p(A)})^{-\beta_p}\) using Eq. (3.12). The relating percentages of every sub-processes are incorporated into \( C_j \). After taking average over possible directions, the energy distribution of observed proton (or antiproton) is equal probability, i.e.,

\[ \frac{d\omega_{X-j}(E_X, E_j)}{dE_j} = \frac{1}{E_X}. \]

(6.3)

The proton spectrum using Eq. (6.2) is plotted in Fig. 19a. A fine spectrum analysis is presented in Fig. 19b. Data are taken from [25-27]. \( \Phi_0^p = e^{9.76}(E/1 \text{ GeV})^{-2.80} \) (dashed line) is a tangent line at \( E_p = 100 \text{ GeV} \). The GC-effects predict the broken power-law at \( E_X^{GC} = 12 \text{ TeV} \) and \( 320 \text{ TeV} \), which correspond to \( E_{\pi}^{GC} = 880 \text{ GeV} \) and \( 24 \text{ TeV} \) in Fig. 17a via \( E_X^{GC} = E_{\pi}^{GC} m_\pi/m_\pi \). Note that \( \beta_p \) in Table 3 takes a different value from that
Figure 19: (a) Predicted cosmic ray proton spectrum multiplied by $E^{2.75}$ versus energy (GeV). The data are taken from Refs. [25-27]. Broken lines present the broken power-law of the GC-effects. The free parameters see Table 3; (b) Same as Fig. (a) but using a fine analysis of spectrum, where the first broken point locals at $E_p = 6 \, TeV$. 
Figure 20: Predicted cosmic ray antiproton spectrum multiplied by $E^{2.75}$ as a function of energy (GeV). The data are taken from Refs. [22-24]. Broken lines present a double broken power-law. The free parameters see Table 4.

| Spectrum | $E_{p-A}^{GC} = 8 \times 10^{16}eV$ | $E_{p-p}^{GC} = 6 \times 10^{19}eV$ |
|----------|---------------------------------|---------------------------------|
|          | $\beta_p$ $\beta_j$ $C_j$       | $\beta_p$ $\beta_j$ $C_j$       |
| $p$      | 1.5 1.85 3.3 $\times$ 10$^{-2}$ | 1.5 1.85 6 $\times$ 10$^{-5}$   |

| Spectrum | $E_{p-A}^{GC} = 2 \times 10^{16}eV$ | $E_{p-p}^{GC} = 6 \times 10^{19}eV$ |
|----------|---------------------------------|---------------------------------|
|          | $\beta_p$ $\beta_j$ $C_j$       | $\beta_p$ $\beta_j$ $C_j$       |
| $\bar{p}$ | 1.3 2.25 5 $\times$ 10$^{-4}$   | 1.3 2.25 8 $\times$ 10$^{-7}$   |
in Table 2, since they incorporating $\beta_X$ and $\beta_\pi$, respectively. A fine spectrum analysis is presented in Fig. 19b.

The antiproton spectrum and comparing with the data [22-24] are presented in Fig. 20, $\Phi_0^0 = e^{1.3}(E/1 \text{ GeV})^{-2.85}$ (dashed line) is a tangent line at $E_P = 20 \text{ GeV}$. Our predicted second bumped structure goes beyond the recent measurable range.
7 Nuclear dependence of the GC effects

In the above sections we find that the GC-parameters in cosmic ray spectra span over several orders of magnitude: $E_G^{GC}$ from 1 GeV to 24 TeV; $x_c$ from $10^{-3}$ to $10^{-12}$. In this section we try to use the ZSR equation to explain this result.

At first step, we consider $p - p$ collisions. The chaotic solutions of Eq. (2.1) exist around $Q_s \sim 1$ GeV, where the perturbative calculations are barely available. However, more lower $k$-range should be included in the calculations, for example, we have taken $k_0 = 0.1$ GeV. The region at $k^2 < 1$ GeV$^2$ is a complicate range, where coexisting perturbative and non-perturbative effects. Many works have discussed the low $Q^2$ transition region from the perturbative side [48]. They incorporate in an effective non-perturbative corrections into the evolution calculations. Considering the non-perturbative dynamics of QCD generate an effective gluon mass at very low $Q^2$ region, and its existence is strongly supported by the QCD lattice simulations [49]. This dynamical gluon mass is intrinsically related to an infrared finite strong coupling constant. According to this idea, we take a following restriction

$$\alpha_s \leq \alpha_{s,max}. \quad (7.1)$$

In Ref. [3] the Golec-Biernat and Wusthoff (GBW) saturation model [50] is taken as the input at $x_0 = 4 \times 10^{-5}$

$$F_{GBW}(x, k_T^2) = \frac{3\sigma_0}{4\pi^2\alpha_s} R_0^2(x) k_T^2 \exp(-R_0^2(x) k_T^2), \quad (7.2)$$

where $\sigma_0 = 29.12 \text{ mb}$, $\lambda = 0.277$, $R_0(x) = (x/x_0)^{\lambda/2}/Q_s$ and $Q_s = 1$ GeV; $F \equiv F/k_T^2$ and $\alpha_s = 0.2$ is fixed.

Figure 21 presents the relation of $x_c$ with the different parameter $\alpha_{s,max}$. We find that $x_c = 6 \times 10^{-12}$ corresponds to $\alpha_{s,max} = 0.110$ at $p - p$ collisions. Such low value of $\alpha_{s,max}$
implies that QCD coupling constant at very small $x$ has been frozen in the evolution. We emphasize that $\alpha_{s,max}$ in this work is an effective parameter, since it may incorporate either the unknown non-perturbative factor in the input and the possible higher order corrections to the evolution equation.

Now we consider the nuclear corrections at $p - A$ collisions:

(i) The nonlinear terms in Eq. (2.1) multiplies $A^{1/3}$ since the correlations among longitudinal nucleons in a nucleus with the nucleon number $A$ in a nucleus according to the European Muon Collaboration (EMC) effect [51];

(ii) The input point $x_0 \rightarrow A^{1/3} x_0$ as the same reason in (i);

(iii) The parameter $\alpha_{s,max}^A = \alpha_{s,max} A^\beta = 0.110 A^\beta$, $\beta$ is a free parameter.

Except neutron star, $E^{GC}_\pi = 440 \text{ GeV}$ is our used lowest GC-threshold originated in SNRs. On the other hand, we consider that the heaviest (with longer life) nucleus in SNRs is uranium ($A=238$). A very real possibility is that the GC-effects at $E^{GC}_\pi = 440 \text{ GeV}$ origins from $p - U$ collisions. Thus, we get $\beta = 0.027$ in (iii), it implies that the $A$-dependence of $\alpha_{s,max}^A$ is weak. Using this result we predict the GC-threshold for a series of typical nuclear targets in Table 5. We find the strong $A$—dependence of $x_c$ in Fig. 22.

What important particularly in Table 5 is that the GC-threshold at $p - Pb$ collisions is $E^{GC}_{p-Pb} = 3.4 \times 10^7 \text{ GeV}$ or $\sqrt{s^{GC}_{p-Pb}} = 8 \text{ TeV/nucleon}$. Note that the contributions of nuclei and proton in $p - A$ collisions dominate the rapidity distribution on the left and right side of rapidity space, the GC-threshold as a GC-signature should be $\sqrt{s^{GC}_{p-A}} \simeq \sqrt{s^{GC}_{A-A}}$. Although this value exceeds the existing energy range in $Pb - Pb$ collisions at the LHC ($\sqrt{s_{LHC}} \sim 5.02 \text{ TeV/nucleon}$), the GC effects will appear in $p - Pb$ or $Pb - Pb$ colliders at $\sqrt{s} \geq 8 \text{ TeV/nucleon}$, they maybe have recorded on tape at the LHC. Note that the above estimation is based on $E^{GC}_{p-p} = E_{GZK} = 6 \times 10^{19} \text{ eV}$. The GC-threshold becomes $\sqrt{s^{GC}_{p-Pb}} > 8 \text{ TeV}$ if $E^{GC}_{p-p} > E_{GZK}$. In this case, a more higher energy $p - Pb$ collider is
Figure 21: Relation of the GC-critical point $x_c$ and parameter $\alpha_{s,\text{max}}$ at $p - p$ collisions.

Figure 22: $A$—dependence of the GC-critical point $x_c$. 
necessary. Any way, because the enhancement of secondary particles due to the GC-effects begins at largest pseudo-rapidity \[^3\], the forward detectors setting at zero degrees with respect to the beam axis in these experiments should be particularly considered. This technique has been built in the LHC forward (LHCf) experimental plan [52].

We have the relation between the GC-critical point \(x_c\) and the target \(A\) based on the ZSR equation. Thus we can predict the GC-effects using a model about cosmic ray sources. As we have mentioned in Sec. 4 that the power-law of the gamma spectrum of pulsar is broken around \(E^{GC}_\pi \sim 1\) GeV, which corresponds to \(x^*_A \sim 3.4 \times 10^{-3}\). According to the ZSR equation, it origins from the collisions of proton on an extremely big "nucleus" with a large number \(A^*\) (see Fig. 22). In fact, using approximation

\[
x^*_c \simeq x^*_0 = x_0 A^{*1/3},
\]

since a big nonlinear effect at \(A^* \to \infty\), we obtain \(A^* \sim 10^6\). Such target is naturally suggested as the neutron star although we don’t know the mechanism of the proton-neutron star interactions. This result is consistent with the structure of pulsars. Furthermore, using Eqs. (3.14) and (3.16), we give \(E^{GC,max}_\pi\) and \(x_c\) corresponding to various observed \(E^{GC}_\pi\) in Table 6. The GC-effects will be suppressed starting from \(\sim 0.8E^{GC,max}_\pi\). It also coincides with the observed cutoff in the pulsar energy spectra.
Table 6  The GC-range at $p-A^*$ collisions (in GeV)

| Target                | $\chi_c$          | $E_{\pi}^{GC}$ (input) | $E_{\pi}^{GC,max}$ |
|-----------------------|-------------------|-------------------------|---------------------|
| Cas A                 | $1.27 \times 10^{-4}$ | 5.2                     | 460                |
| Galactic center       | $8.6 \times 10^{-4}$ | 2.0                     | 68                 |
| M31                   | $3.4 \times 10^{-3}$ | 1.0                     | 17                 |
| SR J0101-6422         | $1.7 \times 10^{-3}$ | 1.4                     | 34                 |
| PSR J0218+4232        | $3.4 \times 10^{-3}$ | 1.0                     | 17                 |
| PSR J0659+1414        | $3.8 \times 10^{-2}$ | 0.3                     | 1.5                |
| PSR J1446-4701        | $3.4 \times 10^{-3}$ | 1.0                     | 17                 |
| PSR J1459-6053        | $9.5 \times 10^{-3}$ | 0.6                     | 6                  |
| PSR J1709-4429        | $2.1 \times 10^{-4}$ | 4.0                     | 273                |
| PSR J1713+0747        | $7.0 \times 10^{-3}$ | 0.7                     | 8.4                |
| PSR J1747-4036        | $7.0 \times 10^{-3}$ | 0.7                     | 8.4                |
| PSR J1954+2835        | $3.8 \times 10^{-4}$ | 3.0                     | 154                |
8 Discussions and Summary

1. We discuss the assumption of suppressed $\pi^\pm$-decay in the GC-condition at Sec. 3. If this decay is allowable, similar to Eq. (4.2), the contributions of $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ and $\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \nu_\mu(\bar{\nu}_\mu)$ to electron flux are

$$\Phi_{eGC}^{eGC}(E_e) = C_e \left( \frac{E_e}{1 \text{ GeV}} \right)^{-\beta_e} \int_{E_p^{\text{max}}/E_e}^{E_p/1 \text{ GeV}} dE_\pi \left( \frac{E_{p-(A)}}{1 \text{ GeV}} \right)^{-\beta_p} N_{\pi^\pm}(E_{p-(A)}, E_\pi)$$

$$\frac{d\omega_{\pi-\mu}(E_\pi, E_\mu)}{dE_\mu} \frac{d\omega_{\mu-e}(E_\mu, E_e)}{dE_e},$$

(8.1)

where $N_{\pi^\pm}(E_{p-(A)}, E_\pi)$ is the number of $\pi^\pm$ with energy $E_\pi$; two factors $(E_\pi/1 \text{ GeV})^{-\beta_\pi}$ and $(E_\mu/1 \text{ GeV})^{-\beta_\mu}$ have been incorporated into $(E_{p-(A)}/E_{p-(A)}^{\text{GC}})^{-\beta_p}$; two normalized spectra are

$$\frac{d\omega_{\pi-\mu}(E_\pi, E_\mu)}{dE_\mu} = \delta(E_\mu - 0.8E_\pi),$$

(8.2)

and

$$\frac{d\omega_{\mu-e}(E_\mu, E_e)}{dE_e} = 4 \left( \frac{2E_e}{E_\mu} \right)^2 (1.5 - \frac{2E_e}{E_\mu}), \quad E_e \leq \frac{E_\mu}{2}.$$

(8.3)

The broken point $E_e^{\text{GC}}$ of the electron spectrum is fixed by the value of $E_{GZK}$ and $E_e = 0.4E_\pi$. Figure 23 is an example of $\Phi_e(E_e)$. One can find that the GC contributions steeply drop on the left of the peak. The different behaviors in Figs. 23 and 17 are arisen from the distributions Eqs. (8.3) and (4.3), respectively. The results in Fig. 24 are inconsistent with the data and show that the suppression of $\pi^\pm$-decay is necessary.

Another evidence is the neutrino spectra. If $\pi^\pm$- and $\mu^\pm$-decays exist in the GC-effects, they should enhance neutrino flux at $GeV - TeV$ band. However, we have not observed such neutrino spectra at $GeV - TeV$ band with the GC-character, although the ultra-
energy neutrino flux beyond 10 TeV have been detected. It confirms our assumption in Sec. 2: the GC-effects suppress the neutrino sources $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}), \mu^{\pm} \rightarrow e^{\pm} + \nu_{e} + \bar{\nu}_{e}$.

2. GRBs are extremely intense and fast shots of gamma radiation from astrophysical objects, both galactic and extragalactic. GRBs are one of the brightest events in Universe, briefly reaching luminosity comparable to the integrated luminosity of a few hundred thousand galaxies. The short variability timescale of a GRB and high total energy release imply an extremely high emissivity. Usually, the synchrotron emission and the inverse Compton process are used to explain the GRB phenomena. However, a serious problem with the synchrotron hypothesis concerns its efficiency [53]. Considering proton can obtain 1800 times as much kinetic energy of electron under the similar acceleration conditions, we see no reason to doubt the GC-effects through $p + p \rightarrow \pi^{0} \rightarrow \gamma + \gamma$ in GRBs. Besides, a strong proton beam randomly collide with the interstellar matter in sky, which may burst out gamma rays due to the GC effects and forms the magical fast GRBs. We noted that the gamma ray of GRBs distributes from Kev to several GeV, but lacks a strong
Figure 24: A map of the GC-effects, which presents the relationship among various physical quantities in this work.

GC-characteristic spectrum at $GeV - TeV$ band. The reason is that a lot of $TeV$ gamma photons caused by the GC-effects met surrounding dense soft photons inside an explosive supernova ejects and they are annihilated to electrons and positrons via $\gamma + \gamma \rightarrow e^- + e^+$ [54].

(iii) The GC opens a window to observe the new phenomena in astrophysics and high energy physics. However, this work still left some unresolved questions: what is the theory of the saturated hadronization? how can observe the GC-effects at next accelerators? how can predict the parameters that we have extracted from the experimental data? Of course, more precise and wide astrophysical data are necessary.

(iv) Because of our discussions are established on a common basis-the GC, we have the relationships among various GC-characteristic quantities as shown in Fig. 24. On the basis of these relationships, we find the general connections among different cosmic ray spectra through the GC-effects. The reads can take $E_{x}^{GC}$ or $A$ as an input, and obtain other relating parameters.

Let us give a summary. A new discovery in QCD is that the gluons in proton may converge to a state with a critical momentum at a high energy range. This gluon condensation (GC) increases suddenly the proton-proton or proton-nuclei cross sections. A natural suggestion is that many observed, but uncomprehended excesses in cosmic ray spectra ori-
gin from a common source—the GC. Using Eq. (2.1) combining with Eqs. (3.11), (3.12) and (3.16), we discuss quantitatively gamma ray, electron-positron and proton-antiproton spectra. We find that the GC-effects span over a broad energy range and they break the power-law of the cosmic ray spectra if the energy of accelerated protons exceeds the GC-threshold. We study the nuclear dependence of the GC effects. The results are used to expect the GC-effects at the next $p - Pb$ or $Pb - Pb$ collisions. In particular, we point out that the galactic center GeV excess and the increase of the positron and antiproton fractions in cosmic rays are really the GC-effects. The GC in proton is an exciting theoretic result, since it changes our traditional understanding proton behavior at high energy. It opens a window to expose the new phenomena about astrophysics and high energy physics.

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