Estimation of vulnerability of groundwater in a granitic aquifer to pollution using entropy theory

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Entropy theory was used to estimate the vulnerability of groundwater aquifer to pollution which could have a degree of uncertainty of liable different dynamic systems. Mainly three parameters such as precipitation, groundwater level, and total dissolved solids in groundwater in a granitic area from Peninsular India are considered and tested. Results show that interaction entropy is comparatively higher in high risk vulnerability zone, whereas it is lower to negligible in low risk vulnerability areas. Therefore, the risk areas of groundwater pollution could be demarcated by virtue of interaction entropy, which provided the same outcomes as achieved on the DRASTIC map. The significance of this work is in evaluating the degrees of aquifer vulnerability for groundwater pollution. An aquifer vulnerability map could be prepared for the whole country by selecting suitable sites for the development of industries.

Keywords: Entropy theory, DRASTIC map, estimation aquifer vulnerability, groundwater pollution, Peninsular India.

The concept of aquifer vulnerability because of groundwater pollution by exterior pollution loads on the surface was introduced by Margat\(^1\). Scientific workers then proposed several methods to evaluate aquifer vulnerability due to hydrogeological variations and impacts acting on it\(^2-14\). But assessment of aquifer vulnerability is costly and sometimes it takes more time on a large scale area\(^5\). Broadly the earlier proposed approaches are of two categories: (1) subjective rating method which is based exclusively on DRASTIC model\(^7\), and (2) process-based\(^15\) and statistical methods\(^16\).

In general, hydrogeological risk is assessed using DRASTIC model even though it has some limitations for the intrinsic properties in the data obtained from different sources\(^17\). For evaluating groundwater vulnerability, this model was utilized in many countries such as China, Palestine, Malaysia, Pakistan, Tunisia, etc.\(^3,18-21\). In India also, DRASTIC model was applied in Indo-Gangetic Plains, Nalgonda, Hazarika, Kanyakumari, Fatehgarh Sahib, Dindigul districts, etc.\(^22-27\).

The idea of entropy, which is a measure of the degree of uncertainty in a random process, was first hypothesized by Shannon\(^28\). Later it was used in diverse areas\(^29-33\).

Three distinctive dynamic factors like precipitation, water table and total dissolved solids (TDS) in groundwater were taken into account to assess the vulnerability of groundwater aquifer to pollution. The main aims of this article are (1) to discuss the applicability of entropy theory with the help of driving agents for aquifer vulnerability in a hard rock area from Peninsular India, and (2) to compare the aquifer risk assessed by the entropy method with that from the DRASTIC model and propose applicability of entropy theory for estimating the groundwater pollution.

A measure of the information and uncertainly associated with any random variable is called Entropy\(^28,29,31\). It is measured with the help of marginal entropy \(H(X)\), joint entropy \(T(X,Y)\), conditional entropy \(H(X|Y)\), and transinformation \(T(X,Y)\) for the two discrete random variables (i.e. \(X\) and \(Y\)) where \(X\) and \(Y\) are the independent variable and dependent variable respectively. These entropies are mathematically expressed as

Transformation

\[
T(X,Y) = - \sum_{i=1}^{n} p(X_i) \log p(X_i) - \sum_{j=1}^{m} p(Y_j) \log p(Y_j) \\
+ \sum_{i=1}^{n} \sum_{j=1}^{m} p(X_i,Y_j) \log p(X_i,Y_j),
\]

(1)

where \(X_i, i = 1, 2, \ldots n, Y_j, j = 1, 2, \ldots, m, p(X_i)\) and/or \(p(Y_j)\): discrete probability of occurrence for the \(X\) and \(Y\) variables, \(p(X_i, Y_j)\): Joint probability of \(X_i, Y_j\); and \(p(X_i/Y_j)\): probability of \(X_i\) conditional on \(Y_j\).

Further, the marginal and joint entropies are expressed as

Marginal entropy, \(H(X) = - \sum_{i=1}^{n} p(X_i) \log p(X_i)\),

Marginal entropy, \(H(Y) = - \sum_{j=1}^{m} p(Y_j) \log p(Y_j)\),

and

Join entropy, \(H(X,Y) = - \sum_{i=1}^{n} \sum_{j=1}^{m} p(X_i,Y_j) \log p(X_i,Y_j)\).

(2)

Multivariate joint entropy for more than two variables \((X_1, \ldots, X_n)\) with \(r_{i_1}, \ldots, r_{i_n}\) \((i_1 = 1,2, \ldots, N_1; i_2 = 1,2, \ldots, N_2; \ldots; i_n = 1,2, \ldots, N_n)\) being an \(n\)-dimensional probability distribution is written as\(^{14}\)

\[
H(X_{1,\ldots,n}) = - \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \ldots \sum_{i_n=1}^{N_n} p_{i_1,\ldots,i_n} \log p_{i_1,\ldots,i_n},
\]

(3)

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RESEARCH COMMUNICATIONS

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The Venn diagrams are shown for the bivariate (Figure 1 a) and multivariate (Figure 1 b) of all entropies along with total correlation and relationships among them.

If two random variables $U$ and $V$ constitute sources and the other $Y$ constitutes the effect, then $X$ of the two-dimensional case is substituted by $U$ and $V$. Equation (1) can be replaced by

$$ T(U, V; Y) = H(U, V) + H(Y) - H(U, V, Y), $$

(5)

where values of $U$: $k = 1, 2, 3, \ldots, K$; and values of $V$: $w = 1, 2, 3, \ldots, W$.

The $X$-subdivision is prepared such that the ranges of $U$ and $V$-values form together the $X$-values and is considered as an input event $i$. Then it could be substituted by the joint event $(k, w)$, which indicates as $n_{i} = n_{kw}$. For estimating $T(U, V; Y)$, a function of bivariate transmission between $U$ and $Y$, and $V$ and $Y$ is considered. The joint event $(k, w, j)$ has been structured into a 3D contingency table with the help of $UVY$ cells and $n_{kwj}$ entities.

In general, an independent random variable is the precipitation ($P$) in any watershed whereas the dependent random variables are the depth to groundwater table (DWT) and TDS of groundwater. They are symbolized as $U$, $V$ and $Y$ respectively. Therefore, marginal entropy, $H(U)$ is the potential information of the $P$ measurement, whereas $H(V)$ and $H(Y)$ are the potential information of the DWT and TDS measurements respectively. And the joint entropy, $H(U, V)$ is the information gained in the $P$ and DWT measurements. This entropy is measured as the first variable ($X$), whereas TDS value in individual well is measured as another dependent variable ($Y$). Then the transinformation, $T(U, V; Y)$ is estimated as the reduction

$$ C(X_1, \ldots, X_n) = \sum_{i=1}^{n} H(X_i) - H(X_1, \ldots, X_n). $$

(4)

Information redundancy within multiple variables is written with the total correlation as

$$ Figure 1. Venn diagram for bivariate and multivariate (a) $T(X, Y)$: Information common to $X$ and $Y$; $H(X|Y)$: information only in $X$; $H(Y|X)$: information only in $Y$; and $H(X, Y)$: total information only in $X$ and $Y$ together, and (b) Schematic diagram of three-dimensional transmitted information.

Figure 2. Study area with the experimental wells and hydrogeological risk zonation derived from the DRASTIC model.

in original uncertainty involved in the TDS measurements considering the knowledge of $P$ and DWT measurements$^{31,32,34}$. The contingency tables have been prepared and the marginal entropies of the $P$, DWT and TDS measurements are calculated$^{11}$. The interaction entropy, which is the measure of aquifer vulnerability for groundwater pollution, is also calculated with the help of eqs (1) and (5).

For a decade, monthly rainfall data was gathered from a weather station placed at Dindigul town, Tamil Nadu state in Peninsular India (as shown in Figure 2). This area is located between 77°53’08”–78°01’24”E long. and 10°13’44”–10°26’47”N lat.$^{36}$ spreading over 240 sq. km. Geologically, it is in a zone of Achaean granites and gneisses intruded by dolerite dykes at some places with low groundwater potential$^{37,38}$. The annual mean precipitation was 929 mm, but it was not uniform throughout the area because of the undulating topography and diverse meteorological conditions prevalent. However, the measured $P$ was considered as uniformly distributed in the area for the present analysis.

During the same period, monthly DWT was collected and the TDS of groundwater also gathered for both dry and wet periods in 4 monitoring wells, which are shown in Figure 2. The well inventory data are presented in Table 1. All monitoring wells except one, which was circular, were rectangular in shape. The $P$, DWT and TDS data were utilized to estimate the interaction entropies for assessing risk of groundwater-bearing aquifer to pollution. In addition, DRASTIC map was also collected as secondary data. This map indicates that the area has four vulnerable zones such as (1) high, (2) moderate, (3) low and (4) negligible. The calculated interaction was encountered in the well 83,515, whereas the minimum was in the well 83,029. These wells have fallen in high and negligible vulnerable zones respectively. The calculated transinformation between groundwater level and TDS were divided into eight class intervals individually. Their class intervals were 2.5 m, below ground level (bgl) and 500 mg/l respectively. In total, 21 joint events were obtained in a 3D ($5 \times 8 \times 8$) contingency table. Initially, a frequency contingency table between the precipitation and depth to groundwater level in individual wells was prepared. Then joint event ($k$, $w$ and $j$) was found as the occurrence of $k$ and $w$ because of the knowledge of $j$. Then the entries were designated as $n_{kjw}$, $n_{kj}$ and $n_{k}$. Further another contingency table between the groundwater table and precipitation corresponding to the TDS values at the same well was made separately considering $n_{kjw}$. It was done for the estimating transinformation and interaction entropies with the help of eqs (1) and (5). Similarly, the same entropies were calculated for the rest of the wells, which are presented in Table 2.

The estimated marginal entropy of precipitation was 1.801 bits and was considered to be uniform throughout the area. For groundwater level, it varied from 1.553 to 2.608 bits, whereas for TDS measurements it varied from 1.166 to 1.914 bits. In these cases, the comparatively high marginal entropies were detected in the high vulnerable zones. The estimated transinformation between precipitation and water level measurements ranged from 0.182 to 0.682 bits. Whereas this transinformation ranged from 0.342 to 0.642 bits between the disorder of rainfall and TDS concentration which had low TDS measurement at the wells when the amount of precipitation is known. Comparatively high uncertainty (ranging from 0.709 to 2.098 bits) of the dependence between groundwater level and TDS measurements was observed which implies that the calculated transinformation between groundwater level and hydrochemical processes is inversely related with respect to the vulnerable zones. But their impacts were nearly in a sequential order at the wells as evident from DRASTIC map. The estimated interaction entropies among the three variables ranged from 0.725 to 1.361 bits with an average of 1.058 bits. The maximum value of interaction was encountered in the well 83,515, whereas the minimum was in the well 83,029. These wells have fallen in high and negligible vulnerable zones respectively on the DRASTIC map (Figure 2).

Low uncertainties (Table 2) were observed in the wells 83,503 and 83,029, which are located in negligible vulnerable zones (Figure 2). The calculated interaction entropies $T(U, V; Y)$ were about 0.725 and 1.053 bits at wells 83,029 and 83,503 respectively. The difference was because of the interaction among the variables of $P$, DWT and TDS due to improper agricultural practices and developing urbanization in the area. When the remaining two wells were compared with the DRASTIC map, the transinformation values nicely matched with the vulnerability zones. Well 83,515 falls in high vulnerable zone and is highly polluted due to the untreated industrial disposal$^{42}$. The maximum uncertainty (around 1.361 bits) was estimated at this well (in Table 2). The DRASTIC
map was deduced based on the hydrogeological setting, whereas the entropy results included the impact of industries which had enhanced the entropies. Entropy model indicates that the interaction entropy was well-correlated with the vulnerability index, thereby, yielding a convincing aquifer vulnerability to the pollution.

The recommended size of a watershed is about 100–300 sq. km by Ground Water Resource Estimation Committee (GWREC, 1997) and at least three spatially well-distributed monitoring wells should be considered within that, or one monitoring well/100 sq. km either is more. Nearly 32,870 wells in an area of 3,287,240 sq. km are to be monitored in India to measure groundwater storage, and its sustainability for understanding the status of groundwater. Since 1969, CGWB (MoWR, RD&GR) started monitoring groundwater levels from observation wells located all over the country. There are about 15,653 wells for the measurement. Apart from these, the state government and other departments have their networks. Groundwater quality is monitored twice every year, one during the dry period and the other during the wet period of the year. The India Meteorological Department also measures rainfall data. Therefore, there is an opportunity to carry out aquifer vulnerability study for the entire country and prepare Aquifer Vulnerability (AVAIL) map adopting the entropy theory.

Entropy theory was applied to assess aquifer vulnerability to pollution in a hard rock aquifer from Peninsular India. It concludes that the interaction entropy among the precipitation (P), DWT and TDS of groundwater is employed to assess the aquifer vulnerability to pollution. The information contained in the P, DWT and TDS data is a measure of uncertainty related to aquifer vulnerability to groundwater pollution. The estimated interaction entropies are well-correlated with the aquifer vulnerability zones demarcated on the DRASTIC map, which are in the sequential order. The estimated minimum interaction entropy is about 0.725 bits in negligible vulnerable area whereas in the high vulnerable area it is about 1.361 bits. Therefore, the aquifer vulnerability to groundwater pollution could be deduced using the entropy theory apart from the DRASTIC model in any hydrogeological set-up. It will also support the preparation of AVAIL map of the whole country for sustainable management of groundwater resources.

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