Grip Analysis of Road Surface and Tire Footprint Using FEM

M Sabri*, S Abda
Computational & Experimental System Mechanics Research Centre,
Mechanical Engineering Department, Universitas Sumatera Utara, Medan, Indonesia

*E-mail: sabrimesin@gmail.com

Abstract. Road grip involve a touch between road pavement and the tire tread pattern. The load bearing surface, which depends on pavement roughness and local pressures in the contact patch. This research conducted to develop a Finite element model for simulating the experimentally testing of asphalt in Jl. AH Nasution Medan, North Sumatera Indonesia base on the value of grip coefficient from various tire loads and the various speed of the vehicle during contact to the road. A tire model and road pavement are developed for the analyses the geometry of tire footprint. The results showed that the greater the mass of car will increase grip coefficient. The coefficient of grip on the road surface contact trough the tire footprint strongly influence the kinetic coefficient of friction at certain speeds. Experimentally show that Concrete road grip coefficient of more than 34% compared to the asphalt road at the same IRI parameters (6–8). Kinetic friction coefficient more than 0.33 was obtained in a asphalt path at a speed of 30–40 Km/hour.

1. Introduction
The automobile tires base of Pneumatic work principle began to be manufactured since 1895. They have replaced the solid tires, which incriminate the mechanical principle of the vehicle and cause discomfort for passengers as the driving force and speed become larger. However, the refinement to be achieved is not only the greater Comfort, but the better grip of the pneumatic tire also needs to be corrected.

Part of the kinetic energy developed by a vehicle has to be absorbed by the suspension system, the brakes and the tires during cornering and braking. Where the car meets the road, there are only the vehicle’s tires to ensure the ultimate contact patch. The aim of this study is to identify the grip coefficient of tire-road contact using Finite Element Method. The mechanics of grip are to be explained by the astonishing visco-elastic properties of the tire’s rubber [2] which within the contact patch produce a host of physical phenomena that strive to counteract any untimely skidding over the road surface.

2. Tire/road interface model
The model requires essentially a contact numerical models (which includes a contact condition and a friction model) and a propagation model. This approach allows for the development of the individual models independently from the other two, without affecting the entire model structure.

The tire is modeled by a 2D flexible ring model, as described in [1]. A contact condition is used, which states that no part of the tire may penetrate the road surface (the condition is actually an inequality or constraint equation). The friction model is based on Coulomb’s friction. The calculated structural vibrations of the tire are used as input for a 3D Finite element code (FEM), to calculate the grip coefficient of tire-road interface.
A 2D flexible ring model is used to explain the suggested approach. The model is shown in figure (1). The ring is assumed to be a thin elastic body of thickness $h$ and radius $R$, the thickness being small compared to the radius and displacements of the ring. Stress stiffening due to the pressurized air in the tire is accounted for. The radial displacement, $x_r(\theta)$, is assumed to be constant over the thickness, whereas the tangential displacement $x_\theta(\theta)$ varies linearly over the thickness. The material is assumed to be isotropic and Hooke’s law is used to relate strains to stresses (Young’s modulus $E$). Rotary inertia is neglected. The ring is supported by an elastic foundation (a distributed radial and tangential stiffness ($k_r$ and $k_\theta$, respectively)), representing the side-wall stiffness.

![Ring model of tire. Degrees of freedom $x_r$ and $x_\theta$.](image)

The dynamic equations for the flexible tire are, see [1] and [2]:

$$
\rho h \ddot{x}_r + d \dot{x}_r + \frac{E}{R^4} \left( \frac{\partial^4 x_r}{\partial \theta^4} - \frac{\partial^2 x_\theta}{\partial \theta^2} \right) + \frac{p_a}{R} \left( x_r - \frac{\partial x_\theta}{\partial \theta} \right) + \frac{p_a}{R} \left( x_r - \frac{\partial^2 x_\theta}{\partial \theta^2} + 2 \frac{\partial x_\theta}{\partial \theta} \right) + k_r x_r = f_r \\
\rho h \ddot{x}_\theta + d \dot{x}_\theta + \frac{Eh}{R^2} \left( \frac{\partial^2 x_\theta}{\partial \theta^2} - \frac{\partial x_r}{\partial \theta} \right) + \frac{E}{R^4} \left( \frac{\partial^2 x_r}{\partial \theta^2} + \frac{\partial^2 x_\theta}{\partial \theta^2} \right) + \frac{p_a}{R} \left( x_\theta - \frac{\partial^2 x_r}{\partial \theta^2} + 2 \frac{\partial x_r}{\partial \theta} \right) + k_\theta x_\theta = f_\theta
$$

(1) (2)

Where $\rho$ is the density, $d$ the damping coefficient, $I = \frac{h^3}{12}$ denotes the area moment of inertia per unit width, $p_a$ is the air pressure and $f_r$ and $f_\theta$ are the distributed forces per unit width in the, respectively, radial and tangential direction. For convenience, the differential equations (1) and (2) are rewritten as:

$$
L_r(x_r, x_\theta) = f_r \\
L_\theta(x_r, x_\theta) = f_\theta
$$

(3) (4)

Essential in the development of a fast numerical solution procedure is that the tire model equations can be used to solve the unknown displacements $x_r, x_\theta$ if the forces $f_r, f_\theta$ are known, but also that they can be used to calculate the unknown forces $f_r, f_\theta$ if the displacements $x_r, x_\theta$ are given. This is the case for the position of points on the tire which are in the footprint, as these positions are equal to the position of the road surface (or, if friction is included, can be calculated from the position of the road and the friction model used).

For simplicity, we use a coordinate system which is fixed to the tire. This implies that, in the transient calculations, we let the road surface rotate around the tire. This is however not essential to the method.

3. Contact model

The tire will deform when in contact with the road surface. The footprint of the tire in obviously depends on the position of the road and also related to the friction forces between the tire and the road surface, as well as the tire model (note that the radial displacements $x_r$, Tangential displacement $x_\theta$ should also
satisfy the differential equations in the footprint!). The normal and friction force will cause the tire to adhere to the road surface and, as already indicated, the position of the tire in the footprint equals the position of the road surface. If the friction force exceeds a maximum friction force at some points in the footprint (as in Coulomb’s friction) these points will slip and their position, i.e. the displacements \( x_r, x_\theta \), is determined by the geometry of the road surface, the maximum friction force and the tire model.

The contact condition is simply a constraint equation, specifying that the tire cannot penetrate the road surface. For simplicity, we discuss a road surface with no texture, but we have extended the algorithm to various types of texture. For the tire model given above and a smooth road surface, this implies that the radial and tangential displacements of the tire should satisfy:

\[
f(x_r, x_\theta) = (R + x_r) \cos \theta - x_\theta \sin \theta + R - \delta \geq 0
\]  
(5)

where \( \delta \) denotes the deflection approach of the tire to the road surface, see figure (1). Thus, if \( f(x_r, x_\theta) < 0 \) the tire penetrates the road surface and the contact condition is violated. On the other hand, in the contact region or footprint the displacements of the tire satisfy the equality:

\[
f(x_r, x_\theta) = 0,
\]  
(6)

Which, as is discussed below, is one of the equations for the two unknown displacements \( x_r, x_\theta \) in the footprint.

The contact forces \( f_r, f_\theta \) and displacements \( x_r, x_\theta \) are, besides the contact condition, determined by the friction model. For given displacements \( x_r, x_\theta \) of any point of the tire, one can calculate the actual forces needed to keep that point at that position. Hence, one can also calculate the (normal) normal force \( f_n \) and shear stress \( \tau \). For a non-textured road surface, \( f_n \) and \( \tau \) follow from:

\[
f_n = f_r \cos \theta - f_\theta \sin \theta
\]  
(7)

\[
\tau = f_r \sin \theta + f_\theta \cos \theta
\]  
(8)

As the displacements \( x_r, x_\theta \) are known, we can use equations 3 and 4 to calculate, respectively, the forces \( f_r \) and \( f_\theta \). If the contact is assumed to be frictionless, i.e. if \( \tau = 0 \), this implies

\[
L_r(x_\theta, x_r) \sin \theta + L_\theta(x_\theta, x_r) \cos \theta = 0
\]  
(9)

and together with \( g(x_r, x_\theta) = 0 \), one can solve for \( x_r \) and \( x_\theta \). If friction is accounted for \( |\tau| \leq \mu |f_n| \) when there is no slip and \( x_r, x_\theta \) can, initially, be determined from the position of the road surface. If \( |\tau| > \mu |f_n| \) slip occurs and, besides the contact condition, \( \tau = \mu f_n \). Substitution of equations 7 and 8 yields:

\[
L_r(x_\theta, x_r) (\sin \theta \mp \mu \cos \theta) + L_\theta(x_\theta, x_r) (\cos \theta \pm \mu \sin \theta) = 0,
\]  
(10)

where the sign \((\pm)\) depends on the sign of \( \tau \).

4. Numerical model

The vibrations of the tire, calculated by the tire/contact model, are used as input for the 3D Finite element model. Hence, phenomena like the horn-effect are accounted for roughness absorption of the road surface can be included as well. As in the FEM code the source mesh is fixed, we need to transform the vibrations of the rotating tire to normal vibrations on a fixed source mesh. The average shape of the tire, as calculated by the tire/contact model, is used as the 3D source mesh, see figure 2.
This research is focused on grip coefficient analysis of road surface influenced by road surface roughness (IRI) and road surface deflection (dL). Fig. 3 show the deformation of the contact surface area between tire footprint and road should satisfy the differential equation (or element matrix equations). For all points in the footprint, the position is given and one can calculate the contact forces needed to put the tire at that position. We use this procedure to iterate the solution. The average value of IRI According to the experimental measurement in Abdul Haris Nasution Street Medan [3] are between 30 – 50 mm.

![Figure 2. Finite element source mesh](image)

**Figure 2.** Finite element source mesh

With respect to the current 3D tire model, we assume all points to vibrate along the entire width of the tire. The vibrations are subsequently interpolated to this fixed source mesh. Furthermore, the vibrations are show in fig. 4 in the finite element simulation.

![Figure 3. IRI along tire-road contact surfaces](image)

**Figure 3.** IRI along tire-road contact surfaces

![Figure 4. Deflection mod of road surface](image)

**Figure 4.** Deflection mod of road surface
Transforming the time signal to the Fourier domain seems trivial. However, for some point’s on the source mesh close to the contact region, the actual displacement can be such that they come in and out of contact (figure 5).

![Figure 5. Eigen Vector tire-road deflection contact](image)

Once these points are in contact, they should not be able to radiate the friction at all, as the position of that point in the contact is assumed to be fixed. Therefore, near the contact region, we keep track of the volume (between the tire and the road surface) in the very small region between the footprint and those points which are never in contact. The change of the volume in time is then converted to a harmonic velocity and this value is set to the velocity of an element radiating from the contact region into the ‘horn’. Note that a similar approach can be used to simulate air-pumping.

5. Grip on road surfaces
The Vehicles cannot be able to move, if there were no grip between tire and road. a moving vehicle has to deal with natural forces, such as the banking, the slope or the unevenness of the road, or rolling resistance, which are constantly trying to slow the vehicle down or push it off its path. As the only contact point between the vehicle and the road, the tire ensures two fundamental functions. It gives the vehicle its directional stability, which the driver needs to steer it. The tire acts as a transmission component for brake and drive torque.

The maximum values of longitudinal grip coefficient $\mu_{\text{max}}$ and transversal grip coefficient $\tau_{\text{max}}$ cannot therefore be reached simultaneously. The tire-road interface represents a global grip potential delineated approximately by a circle, and available to provide longitudinal or transversal grip or both. To obtain the results from the purpose of this study, initial research includes measurement of IRI, measurement of $dL$ and grip coefficient measurement ($F$, $F_f$, $\mu_k = f (\text{IRI})$, $\mu_k = f (dL)$).
Figure 6. Simulation $\mu_{kj}=f(\text{IRI})$ compare to experimental $\mu_{kj}=f(\text{IRI})$ Grip coefficient of tire-road contact.

Using the finite difference equations, the discrete analogue of the contact condition and the friction model calculated for each of the discrete points of the tire. Before proceeding with the next point, we check whether the contact condition is violated. If so, the position of that discrete point is fixed to the road surface (fig. 6). The forces required to keep that point at that position is calculated (the normal and shear forces $f_n$ and $\tau$). It is then checked whether $|\tau|$ is smaller than $\mu|f_n|$. If so, the forces are correct and the algorithm can proceed with the next point. If not, the position of that point of the tire is calculated from solving the displacements from the equations $g(x_r, x_\theta)=0$ and equation 10. For points already fixed to the road surface, one does need to check whether the normal force becomes negative.

6. Conclusion
The correlation between road surface roughness and the pattern of tire tread will generate the grip coefficient of tire-road contact. The Effect of $\mu_k=f(\text{IRI})$ for every increase speed, will decrease kinetic grip coefficient ($\mu_k$).

Reference

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