Turning up and down strong magnetic fields in relativistic nuclear collisions

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I show that the average transverse momentum, \( \langle p_t \rangle \), of the hadrons emitted in relativistic nuclear collisions can be used as a “knob” to control the strength of the magnetic field induced by the spectator protons over the region of nuclear overlap. I thus argue that any observable sensitive to this magnetic field is nontrivially correlated with \( \langle p_t \rangle \) at a given collision centrality.

Heavy atomic nuclei are smashed at relativistic energy at the BNL Relativistic Heavy Ion Collider (RHIC) and at the CERN Large Hadron Collider (LHC) to produce and characterize the quark-gluon plasma, the hot fluid-like state of strong-interaction matter. These processes involve the interaction of highly charged objects, i.e., ions with \( Z \approx 80 \), moving in opposite directions at nearly the speed of light, and are therefore associated with the emergence of magnetic fields of gigantic strength \([15] B \sim 10^{15} \text{T}, \) the strongest ever created in a laboratory. Experimental searches for signatures of the magnetic field in relativistic nuclear collisions are actively pursued at both RHIC \([6–13]\) and LHC \([14–18]\), and theoretical studies aimed at establishing a quantitative phenomenology of \( B \) field-related effects have recently appeared in the literature \([19–29]\). This growing effort is driven by the fact that a strong \( B \) field acting on the hot quark-gluon medium is believed to lead to the emergence of so-called chiral anomalous effects \([30–36]\), whose experimental observation would have far-reaching implications, bringing evidence in particular of local strong parity violation in high-energy nuclear experiments.

However, this is an outstanding challenge. The observable effects driven by the \( B \) field are typically of the same kind as the observable effects driven by the strong interaction governing the quark-gluon plasma \([37]\), and it is difficult to separate these two contributions in the data. In this paper, I introduce a new simple experimental method to attack this problem, which allows one to systematically enhance the strength of the \( B \) field sourced by the spectator protons.

The idea is to look at events that yield the same number of particles in the final state (i.e., fixed multiplicity), and then sort these events according the mean transverse momentum, \( \langle p_t \rangle \), of their final-state hadrons. In the hydrodynamic framework of high-energy nuclear collisions, the mean transverse momentum is a measure of the temperature (or energy) of the fluid from which the particles are emitted \([38–40]\). In one event, and assuming that the quark-gluon plasma is invariant under longitudinal boosts:

\[
\langle p_t \rangle = \frac{1}{N} \int_{p_t} \frac{dN}{dp_t} p_t,
\]

where \( N \) is the total number of particles (or multiplicity) detected in one event, and \( \frac{dN}{dp_t} \) is the spectrum of observed charged hadrons at a given rapidity. Collisions with fixed final-state multiplicity correspond to a good approximation to events where the entropy of the medium is fixed. This has a nice implication. If two events have the same entropy, but different volumes, then the event contained within a smaller volume corresponds to a medium with a larger temperature, and in turn a larger \( \langle p_t \rangle \). Therefore, at fixed multiplicity there exists a tight negative correlation between \( \langle p_t \rangle \) and the size of the system, a well-known feature of hydrodynamic simulations \([40–44]\).

Events with abnormally large values of \( \langle p_t \rangle \) correspond to events that are abnormally small in size, while events with low values of \( \langle p_t \rangle \) are larger than average in size. In this paper, I show that this decrease of the size corresponds in fact to an increase of the collision impact parameter, and consequently of the number of nucleons that do not participate in the collisions, i.e., the spectators. This effect implies that \( \langle p_t \rangle \) provides an experimental handle on the number of spectator nucleons at a given multiplicity, with nontrivial implications for the manifestation of the \( B \) field.

To show this, I perform simulations of the collision process using a phenomenological model. I use the TRENTo model of initial conditions \([45]\) tuned as in Ref. \([46]\) to simulate \( ^{197}\text{Au}+^{197}\text{Au} \) and \( ^{238}\text{U}+^{238}\text{U} \) collisions at RHIC, and as in Ref. \([47]\) to simulate \( ^{208}\text{Pb}+^{208}\text{Pb} \) collisions at LHC. This model provides a prescription for the entropy density, \( s(x, \tau_0) \), created in the interaction of two nuclei \( A \) and \( B: s(x, \tau_0) \propto \sqrt{T_A(x+b/2)T_B(x-b/2)}, \) where \( \tau_0 \) is the time at which the hydrodynamics description of the system becomes applicable, \( b \) is the impact parameter of the collision, and \( T_A(B) \) is a Lorentz-boosted density of nuclear matter. This model includes as well fluctuations in the deposited entropy density, produced by at the level of the participant nucleons. The TRENTo model does not allow to evaluate \( \langle p_t \rangle \), nevertheless, following recent studies \([38–39]\), an accurate approximation of the relative variation of this quantity can be achieved by exploiting the correspondence existing between \( \langle p_t \rangle \) and the thermodynamic properties of the system at \( \tau_0 \). I denote the initial total energy per rapidity and the initial total entropy per rapidity respectively by:

\[
E_0 = \tau_0 \int_x e(x, \tau_0), \quad S = \tau_0 \int_x s(x, \tau_0),
\]
FIG. 1. Top: Average impact parameter in \( {^{238}}\text{U}+{^{238}}\text{U} \) (diamonds), \( {^{197}}\text{Au}+{^{197}}\text{Au} \) (circles), and \( {^{208}}\text{Pb}+{^{208}}\text{Pb} \) (squares) collisions as a function of \( \langle p_t \rangle \). Bottom: Average number of spectator nucleons. Different panels correspond to different centrality classes.

where \( e(\mathbf{x}, \tau_0) \) is the energy density of the system, obtained for simplicity as \( e \propto s^{4/3} \). A good approximation of the relative variation of \( \langle p_t \rangle \) in a narrow multiplicity interval is then given by:

\[
\frac{\langle p_t \rangle - \langle \langle p_t \rangle \rangle}{\langle \langle p_t \rangle \rangle} = \kappa_0 \frac{E_0/S - \langle E_0/S \rangle}{\langle E_0/S \rangle}, \tag{3}
\]

where \( \kappa_0 \) is a constant which depends on the thermodynamic and viscous properties of the system \( [44, 48] \), and has to be chosen to reproduce the magnitude of the relative dynamical fluctuation of \( \langle p_t \rangle \) measured in experimen-tal data \( [49, 50] \). Doing so, one can study initial-state quantities as a function of the relative variation of the final-state \( \langle p_t \rangle \). I calculate observable at fixed centrality. Following the experimental procedure, where the centrality of a collision is defined by the amount of produced particles \( [51, 62] \), I define centrality classes from the amount of produced entropy, \( S \).

The upper panels of Fig. 1 show the collision impact parameter, \( b \), as a function of \( \langle p_t \rangle \), for three different centrality classes in \( {^{197}}\text{Au}+{^{197}}\text{Au} \), \( {^{238}}\text{U}+{^{238}}\text{U} \), and \( {^{208}}\text{Pb}+{^{208}}\text{Pb} \) collisions. I remark a positive correlation between \( b \) and \( \langle p_t \rangle \). The correlation is strong in particular in the ultracentral bin, 0.5-1.0%, where the impact parameter increases by a significant factor. The fact that the slope of the curves is weaker for \( {^{208}}\text{Pb}+{^{208}}\text{Pb} \) collisions is a subtle issue, due to the interplay between the value of \( \kappa_0 \) in Eq. (3), which is lower at RHIC energy than at LHC energy, and the fact that entropy fluctuations produced at the level of the participant nucleons are instead larger at RHIC energy than at LHC energy (see e.g. Ref. [63]), which naturally creates a wider range of impact parameters in a given centrality class.

I summarize these findings in Fig. 2, which illustrates my result for \( {^{208}}\text{Pb}+{^{208}}\text{Pb} \) collisions in the ultracentral bin, 0.5-1.0%. Collisions at low \( \langle p_t \rangle \) (left panel in the figure) correspond to events at smaller impact parameter, smaller number of spectator nucleons, and thus a smaller \( B \) field induced by the spectator protons (which essentially vanishes in ultracentral collision with \( N_s \sim 6 \)). Moving to high \( \langle p_t \rangle \) (right panel in the figure), the impact parameter increases, and this triggers an enhancement in the number of spectator nucleons, which does turn the \( B \) field up. The mean transverse momentum, hence, serves as a sort of knob to turn up and down the strong \( B \) field created in high-energy nuclear collisions at a given collision centrality. Note that, while the fine
Let me work out a specific example in the case of the observable used to infer signatures of the chiral magnetic effect (CME) in high-energy nuclear experiments. The CME is a manifestation of local strong parity violation which is expected to occur in relativistic nuclear collisions \cite{58, 59}. At the high temperatures achieved in the early stages of the quark-gluon plasma, one expects the emergence of local domains of chirally-imbalanced matter with a nonzero axial chemical potential, \( \mu_5 \). In presence of an external magnetic field, such as that produced by the spectator protons, an electric current is thus induced, \( \vec{J} \propto \mu_5 \vec{B} \). Positively- and negatively-charged particles get pushed (in opposite directions) along this current, i.e., along the direction of the \( B \) field \cite{60}. The CME is thus a dipole-like charge-dependent deformation of the system in momentum space. The signal of the CME is typically measured as a correlation between the direction of a charge-dependent dipolar flow, \( v_1 \), and the plane of elliptic flow, \( v_2 \) \cite{61}. This corresponds to the following 3-particle correlation:

\[
y^\pm = \left< \cos \left( \phi_1^\pm + \phi_2^\pm - 2 \phi_3 \right) \right> = (v_1^\pm)^2 v_2, \tag{6}
\]

where \( v_1^\pm \) is the charged-dependent dipolar flow, while \( v_2 = \cos(2(\phi_1 - \phi_2)) \) is the elliptic flow of all hadrons. The strength of the signal of the CME grows with the strength of the \( B \) field, and thus, according to Fig. \ref{fig:1}, it should increase with \( \langle p_t \rangle \) at a given collision centrality. The relevant measure of the correlation between the CME signal and \( \langle p_t \rangle \) is hence given by the following 3-particle correlator:

\[
\rho \left( \langle p_t \rangle, v_1^\pm \right) = \frac{\left< \delta(p_t) \cos \left( \phi_1^\pm - \phi_2^\pm \right) \right>}{\sqrt{\left< (\delta(p_t))^2 \right> (g^+/v_2)}}. \tag{7}
\]

This gives the statistical correlation between \( v_1^\pm \) and \( \langle p_t \rangle \). A baseline for this quantity in absence of CME could be estimated following the calculations of Ref. \cite{62}. In presence of CME, the correlator in Eq. \ref{eq:7} is positive, but its value depends on the specific system under consideration, due, e.g., to energy-dependent initial-state fluctuations, as found in Fig. \ref{fig:1} or to the deformation of the colliding species \cite{63, 64}. I recommend measurements of this quantity at both RHIC and LHC, as it provides a new sensitive probe of the CME that will nicely complement the ongoing investigations.

I reiterate that a correlation such as that given by Eq. \ref{eq:7} should be constructed for all observables that present a sensitivity to the magnetic field induced by the spectator protons.

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