Light as quantum back-action nullifying meter

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We propose a new method to overcome quantum back-action in a measurement process using oscillators. An optical oscillator is used as a meter to measure the parameters of another open oscillator. The optical oscillator is synthesized such that the optical restoring force counters any perturbations induced by the quantum back-action phenomena. As a result, it is shown that the quantum back-action in continuous measurement is suppressed in the low frequency regime i.e., for frequencies much smaller than the resonance frequency of the open oscillator. As the meter plays the role of measuring parameters as well as suppressing the quantum back-action, we call it as quantum back-action nullifying meter. As an application of this method, synthesis of quantum back-action nullifying optical oscillator for suppressing radiation pressure force noise in linear and non-linear optomechanics is described.

I. INTRODUCTION

Classical mechanics tells us that measurements can be infinitely accurate given the measurement scheme and the apparatus are perfect. On contrary to that, quantum mechanics imposes limits on the accuracy of our measurements via the Heisenberg uncertainty principle. The uncertainty principle [1, 2], which gives the fundamental quantum limit, is neither a consequence of the measuring device nor the measurement strategy. While the uncertainty principle is unbeatable, other quantum limits such as standard quantum limit are avoidable [3, 4]. Standard quantum limit is a consequence of quantum back action (QBA) [5, 9] which states that an accurate measurement at a particular time induces uncertainty in the same measurement performed at a later time. In a continuous measurement, QBA limits the accuracy of our measurements atleast to standard quantum limit. Recent experimental advances have shown that QBA limits the measurement accuracy in many physical systems [5, 6, 10–12]. Overcoming it requires performing a special kind of measurement known as quantum non-demolition measurement [3, 13]. However, a quantum non-demolition measurement is not possible in all the experiments as it requires the measured variable to commute with the Hamiltonian. Thus several alternative mechanisms [14–24] are being developed to overcome QBA. This article proposes a new method to suppress QBA by using a meter with intrinsic restoring force.

Quantum back-action in continuous measurement stems from the interplay between the canonically conjugate variables. Such an interplay can arise [25] from the natural evolution of the system or because of the interaction between system and meter. Even though the physics of QBA can be illustrated [26–27] by using isolated systems like a quantum free particle or a quantum simple harmonic oscillator, these systems are never isolated in experiments. A system on which a measurement is to be performed is generally coupled to (i) environment and (ii) meter. In my previous work [20], decoherence from the environment was used to suppress the QBA by erasing the memory of previous measurement from the system. In this article, we propose a new method to suppress QBA by using the restoring force of the meter to counter the perturbation induced by the QBA.

In a measurement process, generally, the experimenter has control over the meter and its coupling to the system on which the measurement is to be performed. Evasion of QBA can be achieved by choosing a meter with intrinsic restoring force, an oscillator for example, and then using the restoring force to counter any perturbation induced by the QBA phenomena. As the meter in this method serves the dual purpose of measuring, and nullifying the QBA, we call it as quantum back-action nullifying meter (QBNM).

The theory of QBNM is developed by modeling the system and the meter as oscillators. Even though the meter could be any measuring device with intrinsic restoring force, we restrict the meter to an optical oscillator as optics offer unique advantage in the experimental implementation of this technique. As many systems [25, 31] can be modeled with simple harmonic oscillators, the results presented in this article will be useful in many areas of physics.

Consider a simple harmonic oscillator (SHO) with momentum $\hat{p}_1$, position $\hat{z}_1$, mass $m_1$, and Eigen frequency $\omega_1$. The equations of motion are given as

$$\dot{\hat{p}}_1 = -m_1\omega_1^2\hat{z}_1, \quad m_1\ddot{\hat{z}}_1 = \hat{p}_1.$$  \hspace{1cm} (1)

Solving Eq. (1) gives the position of the simple harmonic oscillator at time $t$ as

$$\hat{z}_1 = \hat{z}_2 \cos(\omega_1 t) + \frac{\hat{p}_2}{(m_1\omega_1)} \sin(\omega_1 t),$$

where $\hat{z}_2$ is the initial position, $\hat{p}_2$
is the initial momentum, and \( t_e = t - t_2 \) with \( t_2 \) as initial time. The \( \dot{z}_s \) and \( \dot{p}_s \) leads to initial imprecision (IIP) and QBA, respectively. The competition between IIP and QBA limits the accuracy in continuous measurement of \( \dot{z}_1 \) at least to \( \sqrt{\hbar/m_j\omega_1} \), where \( \hbar \) is the reduced Planck constant, which is the standard quantum limit. The QBA in Eq. (1) arises from the canonically conjugate variables. We haven’t considered any specific measurement scheme or measuring device in Eq. (1), but it already implies standard quantum limit. The simple harmonic oscillator considered until now is an isolated case. In an experiment, the simple harmonic oscillator is coupled to the environment, and a measurement can be performed by coupling it to a meter. In order to account for every thing in a measurement process, we have to include both the reservoir and the meter.

For simplicity, the simple harmonic oscillator coupled to reservoir will be called as an open simple harmonic oscillator (OSHO). Suppose that we are interested in measuring \( \dot{z}_s \) of an OSHO whose momentum, mass, and Hamiltonian are represented by \( \dot{p}_s, m_s, \) and \( H_s \), respectively. Experimentally, \( \dot{z}_s \) is estimated by coupling the OSHO to a meter. We assume that the meter is an optical oscillator with Hamiltonian \( H_o \), effective momentum \( \dot{p}_o \), effective position \( \dot{z}_o \), and effective mass \( m_o \). The optical oscillator can be synthesized by driving an optical cavity with an external laser field. The optical field inside the cavity oscillates at the Eigen frequency \( \omega_o \) of the cavity and this creates an optical oscillator. As the optical oscillator is driven at frequency \( \omega_d \) of the driving laser, after making a unitary transformation with \( U = e^{i\omega_d H_o/\hbar \omega_o} \) the total Hamiltonian becomes \( H_s + \Delta H_o/\omega_o + H_{so} + H_r \), where \( \Delta = \omega_o - \omega_d \), \( H_{so} \) is the Hamiltonian for interaction between optical oscillator and OSHO, and \( H_r \) is the Hamiltonian of the reservoirs and their interaction with the OSHO and the optical oscillator. The explicit form of the \( H_{so} \) depends on the nature of coupling between the OSHO and the meter. We keep the \( H_{so} \) arbitrary. The equations of motion are given as

\[
\dot{z}_j = -i\hbar \frac{\partial}{\partial z_j} \left[ H_s + \Delta H_o/\omega_o + H_{ro} + \dot{z}_j \right] = \frac{i}{\hbar} [H_{so}, \dot{z}_j],
\]

\[
\dot{p}_j = \frac{i}{\hbar} \left[ H_s + \Delta H_o/\omega_o + H_{ro} + \dot{p}_j \right] = \frac{i}{\hbar} [H_{so}, \dot{p}_j],
\]

where \( j = s, o \). We define \( C_{\dot{p}_j} = i[H_{so}, \dot{p}_j]/\hbar \) and \( C_{\dot{z}_j} = i[H_{so}, \dot{z}_j]/\hbar \). The commutation relations \( [H_{so}, \dot{p}_j] \) and \( [H_{so}, \dot{z}_j] \) can not be determined without knowing the explicit form of the \( H_{so} \). However, we can guess that \( C_{\dot{z}_s} \) and \( C_{\dot{p}_o} \) are a function of \( \dot{z}_s, \dot{z}_o, \dot{p}_s, \) and \( \dot{p}_o \) as \( H_{so} \) is also a function of the same variables.

As we are interested in measuring \( \dot{z}_s \), without loss of generality, we assume that \( H_{so} \) is independent of \( \dot{p}_s \). These assumptions not only simplify Eq. (6) and Eq. (7) but also represent the realistic experimental scenario in optical metrology. For example: the position (or \( \dot{z}_s \)) coupling comes naturally as any OSHO coupled to optical oscillator experiences position dependent electromagnetic field because of the wave nature of light [32, 33]. Hence Eq. (2) and Eq. (3) can be rewritten as

\[
m_s(\ddot{z}_s + \gamma_s \dot{z}_s + \omega_s^2 z_s) = C_{\dot{p}_s} (\dot{z}_o, \dot{p}_o, \dot{z}_s) + \delta, \tag{4}
\]

\[
\ddot{p}_o + \gamma_o \dot{p}_o + m_o\omega_o \Delta \dot{z}_o = C_{\dot{p}_o} (\dot{z}_o, \dot{p}_o, \dot{z}_s) + \delta, \tag{5}
\]

where \( \gamma_j \) is the damping rate and \( j \) is the corresponding noise operator [34, 35]. We adopt the following notation throughout this letter: Any arbitrary quantum operator \( A \) is written [2] as a sum of its expectation term \( \bar{A} \) and its quantum fluctuation term \( \delta A \). Hence, by applying the Taylor expansion up to first order of quantum fluctuations, the dynamics of the quantum fluctuations in Eq. (4) and Eq. (5) are given as

\[
m_s(\dddot{z}_s + \gamma_s \ddot{z}_s + \omega_s^2 z_s) = \sum_{j=s,o} \frac{\partial C_{\dot{p}_j}}{\partial z_j} \left| \dot{z}_j = \dot{z}_j \right. \left. \dot{p}_j = \dot{p}_j \right. + \delta, \tag{6}
\]

\[
\dddot{p}_o + \gamma_o \ddot{p}_o + m_o\omega_o \Delta \ddot{z}_o = \sum_{j=s,o} \frac{\partial C_{\dot{p}_j}}{\partial p_j} \left| \dot{z}_j = \dot{z}_j \right. \left. \dot{p}_j = \dot{p}_j \right. + \delta, \tag{7}
\]

Note the partial derivatives in Eq. (6) and Eq. (7) are a function of expectation terms only (no quantum terms). The transient behavior of the expectation terms is damped out on a time scale \( t_e > 1/\gamma_j \) leading to a steady state (any fast oscillation at optical frequencies is eliminated by the rotating wave approximation). Hence all the partial derivatives in Eq. (6) and Eq. (7) become time independent because of decoherence. As all the partial derivatives in Eq. (6) and Eq. (7) are a function of expectation terms only, from now onward, for notation simplicity, we shall drop indicating that the partial derivatives are evaluated at expectation terms. As a result, by using the Fourier transform definition \( \hat{\delta}(A) = \int_{-\infty}^{\infty} \hat{A} e^{i\omega t} dt/\sqrt{2\pi} = \hat{A}(\omega) \) with \( \omega \) as Fourier frequency, Eq. (6) and Eq. (7) imply that

\[
\ddot{z}_s(\omega) = \frac{\partial C_{\dot{p}_s}}{\partial \dot{z}_s} \hat{D}_s(\omega) \hat{D}_s(\omega) + \frac{\partial C_{\ddot{p}_s}}{\partial \dot{p}_s} \hat{D}_s(\omega) \hat{D}_s(\omega), \tag{8}
\]

\[
\dddot{p}_o(\omega)(\gamma_o - i\omega - \frac{\partial C_{\ddot{p}_o}}{\partial \dot{p}_o} ) + m_o\Omega_o^2 \ddot{z}_s(\omega) = \ddot{\delta}_o(\omega) + \dddot{\delta}_s(\omega) \hat{D}_s(\omega), \tag{9}
\]

where \( \Omega_o^2 = \omega_o^2 - \omega_s^2 - i\gamma_s \omega \) with \( \Omega_s^2 = \omega_s^2 - \omega_s^2 - i\gamma_s \omega \). The second terms on the
left hand side (LHS) and the right hand side (RHS) of Eq. (9) establish the presence of QBA. However, these terms arise from $\hat{H}_o$ and $\hat{H}_{os}$. Thus, unlike the oscillator in Eq. (1), the QBA in Eq. (9) has contribution from the interaction between optical oscillator and OSHO as well. The optical oscillator can read the parameters of OSHO only if there is an interaction between them. This interaction generally involves exchange of forces which leads to perturbation of OSHO parameters. The quantum mechanical nature of the optical oscillator writes perturbations with quantum nature on to the OSHO and this manifests as QBA from the $\hat{H}_{os}$. We think the most popular example [34] for the QBA in optical metrology is the radiation pressure force noise in gravitational wave interferometer.

Coupled dynamics of optical oscillator and OSHO is given from Eq. (6) and Eq. (9) as

$$\delta p_o(\omega) = \frac{\langle \partial C_{p_o} \partial C_{p_o} \rangle - m_o \Omega_o^2 \delta z_o(\omega) + \partial C_{p_o} \partial \delta o(\omega) + \delta \omega(\omega)}{(\gamma_o - i\omega - \partial C_{p_o} \partial \delta o(\omega))^2}.$$  

(10)

On the RHS of Eq. (10), the first term in the numerator represents the QBA term. It reveals that QBA arises from previous measurement as well as from the interaction between the optical oscillator and the OSHO. The QBA in Eq. (10) can be completely nullified if

$$m_o \Omega_o^2 = \frac{\partial C_{p_o} \partial C_{p_o}}{\partial z_o} \sqrt{m_o^2 (\Omega_o^2 - \omega^2)^2 + m_o^2 \gamma_o^2 \omega^2}.$$  

(11)

where $\epsilon = \frac{\gamma_o \omega}{\Omega_o^2 - \omega^2}$. The LHS of Eq. (11) comes from optical spring constant as well as $\hat{H}_{os}$ while the RHS comes from $\hat{H}_{os}$. Satisfying Eq. (11) implies designing the measurement process such that the QBA perturbations from $\hat{H}_{os}$ are nullified by using the restoring force of the optical oscillator. The condition in Eq. (11) can not be satisfied perfectly as the LHS is real and the RHS is complex, which means complete elimination of QBA is not possible using this method. Nevertheless, Eq. (11) can lead to significant reduction in QBA if its real part is zero and its imaginary part is small (i.e., $\epsilon \ll 1$). For $\epsilon \ll 1$, the QBA in Eq. (10) can be suppressed if

$$m_o \Omega_o^2 = \frac{\partial C_{p_o} \partial C_{p_o}}{\partial z_o}.$$  

(12)

Equation (12) is nothing but the real part of Eq. (11) when $\epsilon \ll 1$. Using Eq. (12) and Eq. (10), we can write

$$S_{p_o p_o} \approx \frac{S_z z_o (\epsilon \frac{\partial C_{p_o} \partial C_{p_o}}{\partial z_o})^2 + S_{ss} (\frac{1}{\gamma_o} \frac{\partial C_{p_o} \partial C_{p_o}}{\partial z_s})^2 + S_{oo}}{(\gamma_o - i\omega - \frac{\partial C_{p_o} \partial \delta o(\omega)}{\partial p_o})^2}.$$  

(13)

where $\langle \delta B(\omega) \delta B(\omega') \rangle = B(\omega')$ with $B = p_o, s, o$. On the RHS of Eq. (13), in the numerator, the first, second, and third terms give noise spectral densities from QBA, OSHO reservoir and optical oscillator reservoir, respectively. As $\epsilon \rightarrow 0$, QBA noise in Eq. (13) goes to zero without affecting other terms. The condition $\epsilon \ll 1$ can be realized experimentally for $\Omega_o \gg \omega$ and $\Omega_o \gg \gamma_s$. As $S_{z z_o}, S_{p_o p_o}$, and all the partial derivatives in Eq. (13) are independent of $\omega$, the QBA noise in Eq. (13) goes to zero as $\epsilon \rightarrow 0$. Thus $\epsilon \ll 1$ is a necessary condition which limits QBA suppression using Eq. (12) only to the low frequency regime. We neglected the cross-correlation terms in Eq. (13) by setting $\langle \delta z_o \delta o(\omega) \rangle = 0$. Even if these cross-correlations are not neglected, noise contribution from them becomes zero upon frequency symmetrization. Despite of the complex nature of Eq. (11), Eq. (12) gives an experimentally feasible criteria to extract information from $\hat{H}_o$ without QBA in the low frequency regime by measuring $\hat{p}_o$ of the optical oscillator. The advantage of using an optical oscillator becomes obvious once we recognize that $\hat{p}_o$ is proportional to the optical oscillator’s phase quadrature which can be measured in a homodyne setup [33]. Using an optical oscillator as a meter not only allows us to measure $\hat{p}_o$ in a relatively simple way but also to coherently couple with variety of OSHOs [29,31,38] like a mechanical oscillator [39] or atoms in harmonic trap [40] etc. Hence the Eq. (10), Eq. (12) and Eq. (13) can be applied to many systems for QBA evasion in low frequency regime.

Before we go any further, it is useful to note that the condition in Eq. (11) is derived by assuming that $\hat{p}_o$ is not coupled to the OSHO. This is a reasonable but not a necessary assumption in the context of OSHO or quantum optical metrology. Thus it is natural to wonder about QBA evasion when $\hat{p}_o$ is coupled to the optical oscillator. Equation (6) and Eq. (7) is general enough to include both $\hat{z}_o$ and $\hat{p}_o$ coupling to the optical oscillator. Hence starting from Eq. (6), by following the same steps that led to Eq. (11), we obtain that the condition for QBA evasion as

$$m_o \Omega_o^2 = \frac{\frac{\partial C_{p_o} \partial C_{p_o}}{\partial z_o}}{\frac{1}{m_o^2} \frac{\partial C_{p_o} \partial p_o}{\partial p_o} + i \frac{\gamma_o \omega}{m_o^2 \omega^2}}.$$  

(14)

when $\hat{p}_o$, instead of $\hat{z}_o$, is coupled to the optical oscillator. Derivation Eq. (14) is given in the supplementary material. Again by noting that, in Eq. (14), the LHS is real and the RHS is complex. QBA evasion can be achieved
in the low frequency regime, i.e., $\omega \to 0$ when
\[ m_\omega \Omega^2 \left( \frac{1}{m_s} \frac{\partial C_{sz}}{\partial p_s} + \frac{\partial C_{\hat{p}_s}}{\partial \hat{p}_s} \right) = -\frac{\partial C_{\hat{p}_s}}{\partial \hat{z}_o} - \frac{\partial C_{\hat{p}_o}}{\partial \hat{p}_o}. \] (15)

Using Eq. (11) and Eq. (10), as a direct application of the theory developed so far, we will derive the conditions for achieving radiation pressure force noise suppression in optomechanics [39].

\[ \dot{\hat{z}} = \frac{\hat{p}_s - M_1 \hat{Z}_o}{M + \hat{Z}_o} \] (16)

where $\hat{p}_s$, $\hat{Z}_o$, $m$, and $\omega_m$ are the momentum, position, mass and eigen frequency of the mechanical oscillator, respectively. $\hat{P} = i\hbar \omega_0(\hat{a}^\dagger - \hat{a})/\sqrt{2}c$, $\hat{Z} = c(\hat{a}^\dagger + \hat{a})/\sqrt{2}\omega_a$ and $M = \hbar \omega_a/c^2$ with $\omega_a$, $\hat{a}$, $\hat{a}^\dagger$, as the Eigen frequency, annihilation operator and creation operator for the cavity mode, respectively. $\Delta_\omega$ is the detuning between drive and cavity Eigen frequency, $c$ is the speed of light in vacuum and $[\hat{Z}, \hat{P}] = i\hbar$. The first two terms on the RHS of Eq. (16) represent Hamiltonian of the mechanical oscillator, which plays the role of system operator, from which information is extracted by the cavity field which plays the role of optical oscillator. The third term on the RHS of Eq. (16) represents the Hamiltonian for the cavity field and its interaction with the mechanical oscillator. We didn’t include the reservoir terms in Eq. (16). By noting that $\hat{p}_s$, $\hat{Z}_o$, $\hat{P}$, and $\hat{Z}$ are similar to $\hat{p}_s$, $\hat{z}_o$, $\hat{p}_o$, and $\hat{Z}_o$, respectively, we can immediately apply Eq. (11) to derive the condition for achieving QBA evasion in optomechanics. By realizing that $C_{\hat{p}_s}$ is similar to $C_\hat{P} = M\omega_a^2 \hat{Z}/l$, and $C_{\hat{p}_o}$ is similar to $C_\hat{Z} = (\hat{P}^2/2M + M\omega_a^2 \hat{Z}^2/2)/l$, we can evaluate the condition in Eq. (17) for $\hat{H}_o$ as
\[ \frac{\hbar \omega_a^2 (\hat{a} + \hat{a}^\dagger)^2}{2(2lD(\omega))^2} = (\Delta_a - \omega_a \hat{z}_o)^2. \] (17)

Here $D(\omega) = m(\omega_m^2 - \omega^2 - i\gamma_m \omega)$ with $\gamma_m$ as the optomechanical oscillator damping, $\hat{a}^\dagger$ is the complex conjugate of $\hat{a}$, and $Z = (\hat{a} + \hat{a}^\dagger) \sqrt{\hbar/2M\omega_a}$. It can be directly verified whether Eq. (17) can suppress the optomechanical radiation pressure force noise by cross-checking with the dynamical equations of 1122 of $\hat{a}$. A discussion on more details about practical implementation of Eq. (11) on $\hat{H}_o$ will be interesting, but that is beyond the scope of this letter and the interested readers may find such details in [43]. Application of Eq. (11) to Eq. (16) is only one example. In fact, we can also apply Eq. (11) to non-linear optomechanics where $\hat{z}$ is kept to all orders. The Hamiltonian $\hat{H}_n$ for the non-linear optomechanics is given as
\[ \hat{H}_n = \frac{\hat{p}_s^2}{2m} + \frac{m\omega_m^2 \hat{z}_o^2}{2} + \left( \frac{\hat{P}^2}{2M} + \frac{M\omega_a^2 \hat{Z}_o^2}{2} \right) (1 + \hat{z}_o^2)^{-1}. \] (18)

Now by applying Eq. (11) to Eq. (18), one can derive the condition for avoiding QBA in non-linear optomechanics too.

In this paragraph we compare our techniques with other prominent methods like QND, squeezing, Coherent quantum noise cancellation (CQNC), quantum mechanics free subsystems (QMFS), and variational measurements. The sufficient condition for QND measurement is that the measured variable has to commute with the total Hamiltonian. While this can eliminate QBA completely, it is very rare to find systems for which the Hamiltonian commute with the measured variable. Sending squeezed vacuum through the empty port of the interferometer is another technique [45–47] to suppress the QBA, if the squeeze angle is appropriately optimized. The strength of squeeze parameter determines the effectiveness of this method. To our knowledge, highest squeezing achieved so far is 15 db [48]. Achieving high quality squeezing is the main challenge in this technique. The CQNC [49, 50] uses an auxiliary system which is coupled to the main system. The auxiliary system is to be synthesized [51–53] such that the QBA noise from the main system cancel with the noise from the auxiliary system. The success of CQNC depends on the finetuning of the auxiliary system. Another method to avoid QBA is through QMFS [54–55]. As the QBA arises because of the interplay between the canonically conjugate variables, an effective negative mass system is created to satisfy the relation $[\hat{x}_1 + \hat{x}_2, \hat{p}_1 - \hat{p}_2] = 0$. Where $\hat{x}_1$, $\hat{x}_2$ and $\hat{p}_1$, $\hat{p}_2$ are positions and momentum of two systems, respectively. The negative mass leads to minus sign before $\hat{p}_2$. In contrast to all these methods, a QBNM works by choosing a meter with intrinsic restoring force. The restoring force of the optical oscillator is used to reduce the randomness coming from the QBA. It is also possible to combine QBNM with other established techniques like squeezing etc to improve the overall effectiveness of both the methods.

Theory for quantum back-action nullifying meter is developed. By assuming the meter as an optical oscillator, a new method to achieve QBA evasion in the low frequency regime in quantum optical metrology is developed. Evasion of QBA is achieved by using the restoring force of the optical oscillator to counter the perturbation induced by FIG. 1. A generic Optomechanical cavity. The cavity field $\hat{a}$, when reflected from the optomechanical oscillator, displaces the position $\hat{z}$ of the optomechanical oscillator. The change in the length of the optical cavity modifies the optical response of the optomechanical cavity. $\gamma_m$ and $\varsigma$ are the decay rates of optomechanical mirror and optomechanical cavity, respectively.

Figure 1 represents a generic optomechanical system. The optomechanical Hamiltonian $\hat{H}_o$ is given [40] as
\[ \hat{H}_o = \frac{\hat{p}_s^2}{2m} + \hbar \omega_m \hat{z}_o^2 \] (16)
the QBA force. Application of QBNM for nullifying the quantum radiation pressure force noise in optomechanics is presented.

II. FUNDING

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