Group Leverage Centrality and its Applications in Brain Networks

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Abstract. The concept of leverage centrality is the relationship between the degree of a node relative to its neighbours and operates under the principle that a node in a network is central if its immediate neighbours rely on it for information. It was specifically formulated for brain networks. In this paper, we have built upon leverage centrality and introduced group leverage centrality, which is a measure of how important a subset of nodes is in the network. Then, the effects of meditation on different lobes of the brain were quantified using this new centrality measure.

1. Introduction

Graph theory is particularly useful in the study of the brain. The human brain can be modelled as a graph in which neurons are taken as nodes or vertices, and the physical links between the neurons are taken as edges. As the brain network is highly complex and vast, it can be clustered into working regions, or lobes, to effectively model it. These clusters could be considered as the regions of interest (Korhonen et al., 2017) [1]. It has been shown that meditation plays a vital role in improving brain performance (Fox et al., 2014) [2], and helps to improve grey matter concentration, focus, attention, and reduce work stress. Different lobes of the brain have different functions. With the electroencephalography results of the brain (before and after meditation) we can analyse the importance of the lobes, based on the electrical activity. To quantify which lobe is more active, we employ centrality measures. Centrality measures were introduced to study and identify the key nodes in the network. Over the years, many centrality measures have been developed and used in different real life applications (Das et al., 2018) [9]. In 2010, another centrality measure- leverage centrality- was introduced to study and identify the key nodes in the context of brain networks (Joyce et al., 2010) [4]. Since every lobe of the brain is a set of nodes, we have introduced a new centrality measure for a subset of nodes- group leverage centrality, an extension of leverage centrality.
2. Leverage Centrality

Leverage centrality considers the degree of a node relative to its neighbours and operates under the principle that a node in a network is central if its immediate neighbours rely on that node for information. Multiple examples and theorems have been studied and derived by Vargas et al. (2017) [3], which are given below.

2.1 Definition

Leverage centrality is a measure of the relationship between the degree of a given node \( v \), and the degree of each of its neighbours, averaged over all neighbours, and is defined as shown below:

\[
I(v) = \frac{1}{k_v} \sum_{j \in N_v} \left( \frac{k_v - k_j}{k_v + k_j} \right)
\]

(1)

Here, \( k_v \) stands for the degree of node \( v \) and \( N_v \) stands for the set of nodes adjacent to the node \( v \).

2.2 Example

![Image](2.4. Group Leverage Centrality)

2.3 Some Propositions of Leverage Centrality

- For any graph \( G \), \( \sum_{v \in G} I(v) \leq 0 \). The leverage centrality \( I(v) \) is zero in any regular graph. The sum of leverage centralities of all nodes in \( G \) will be less than 0 in any non-regular graph.

- The number of vertices with positive leverage centrality is \( n - 1 \).

- A vertex of lowest degree (highest degree) cannot have a positive (negative) leverage centrality. In any graph \( G \), all the vertices except one to have negative leverage centrality and one have positive leverage centrality. There are \( n - 1 \) vertices with negative leverage centrality in a star graph \( K_{1,n-1} \).

The concept of group centrality was introduced by Everett and Borgatti in 2010 [10]. Also, they also defined degree, closeness and betweenness centralities in terms of groups and classes as well as individuals.

2.4. Group Leverage Centrality

In literature, researchers have studied the Leverage Centrality and its different properties. In this work, we define a new metric of Leverage Centrality called the Group Leverage Centrality, which is leverage centrality not for a single node but for a group of nodes. We are also introducing two different ways in which Group Leverage Centrality could possibly be defined.
2.4.1 Total Group Leverage Centrality: Total Group Leverage Centrality (TG) is defined as the average of leverage centralities of all the nodes in a given set of nodes S, where S is the subset of V, the vertices of the graph in consideration.

\[ TG(S) = \frac{1}{|S|} \sum_{v \in S} l(v) \quad S \subseteq V \quad (2) \]

2.4.2 Complement Leverage Centrality: Complement Group Leverage Centrality (CG) is defined as the average of leverage centralities of all the nodes in a given set of nodes S, where S is the subset of V (the vertices of the graph in consideration) and only the vertices in the set \( V \setminus S \) are considered for computing the leverage centralities. Here, \( k_v \) and \( N_v \) represent the degree and set of neighbours of node v respectively.

\[ CG(S) = \frac{1}{|S|} \sum_{v \in S} \frac{1}{k_v} \sum_{i \in N_v \cap (S')} (k_i - k_v) \quad \text{where} \quad S' = V - S \quad (3) \]

3. Algorithm

3.1 Total Group Leverage Centrality

Total Group Leverage Centrality is found by finding the average of all the leverage centralities of the vertices (all neighbours) present in the specified subset.

**Input:** Graph \( G = (V, E) \), subset S containing n nodes

**Output:** \( TG(S) \)

\[
TSum \leftarrow 0 \\
\text{for node } v \text{ in } S \text{ do } \{ \\
\quad TgSum \leftarrow TgSum + l(v) \quad //l(v) \text{ gives the leverage centrality of node } v \\
\} \\
TG \leftarrow TgSum/n \\
\text{return } TG
\]

3.2 Complement Group Leverage Centrality

Complement Group Leverage Centrality is found by finding the average of all the leverage centralities of the vertices while considering only the neighbours outside the specified subset. Here, \( N(v) \) is the set of nodes which are neighbours to node v.

**Input:** Graph \( G = (V, E) \), subset S containing n nodes

**Output:** \( CG(S) \)

\[
CgSum \leftarrow 0 \\
\text{for node } v \text{ in } S \text{ do } \{ \\
\quad \text{complementNodeCentrality} \leftarrow 0 \\
\} 
\]
for node i in N(v) do {
    if \( i \in S' \) do {
        \text{complementNodeCentrality} \leftarrow \text{complementNodeCentrality} + \frac{k(v) - k(i)}{k(v) + k(i)}
    }
}
\text{CGSum} \leftarrow \text{CGSum} + \text{complementNodeCentrality}/\text{degree}[v]
CG \leftarrow \text{CGSum}/n
return CG

4. Group Leverage Centrality in Brain Networks

The concept of Group Leverage Centrality can be used to quantify the effect of meditation on different parts of the brain network and their relation to other parts.

4.1 EEG Data Acquisition

The tests were conducted in the Biomedical Engineering Department of Sri Sivasubramaniya Nadar College of Engineering, Kalavakkam. The modality of study used was electroencephalography (EEG). RMS EMG (SuperSpec) software was used to acquire the EEG. The 10-20 Electrode placement system was used to record the signals. The scalp electrodes were placed based on the different lobes in the brain, C3, C4, CZ, central electrodes; F3, F4, F7, F8, FZ, frontal lobe electrodes and FP1, FP2 were placed on the frontopolar area; P3, P4 and P, parietal lobe electrodes and O1, O2 and Oz were occipital lobe electrodes. For reference on the right and left earlobes, Electrodes A1 and A2 were placed. A ground electrode was placed on the mastoid process behind the ear. The odd numbers depict that they are located on the right side of the brain and even numbers depict that they are

![Figure 2: Flowchart for signal processing](image-url)
located on the left side of the brain. The electrodes used were silver/silver chloride (Ag/AgCl) electrodes placed on the scalp using a 10 20 EEG paste.

4.2 EEG Data Extraction

In this stage, the raw EEG was averaged by using a smoothing filter and separated into different frequency bands namely alpha, beta and theta (4 - 8 Hz). We removed the delta band. Their individual power spectral densities and their total and relative powers were calculated as shown in Fig. 2 (above) and the values were stored for later analysis.

4.3 Statistical Analysis

All the values that were calculated were tested for skewness and they were found not to be normally distributed. Hence the parameters that were measured were tested non-parametrically. The three independent samples are subjects classified as normal (DASS-normal). All the analysis was pre-meditation results. The task performances were tested. Using the Friedman test, the relative powers of all the four frequency bands were tested for significant differences. The significant level was taken as \( p = 0.05 \). A related sample test was done between task and rest for all 19 electrode locations to measure the individual significance between each of the variable pairs. The entire analysis was done in IBM SPSS Statistics, Version 24. For all the three groups, the mean relative alpha, beta, theta and delta component energies were found for all the 19 electrode locations during both rest and while performing the various linguistic and non-linguistic activities. The Friedman test was done for all the groups separately and a statistically significant difference was noted in the measured parameters (relative alpha, beta, theta and delta component energies).

4.4 Functional Brain Graph Modelling

The working procedure to get data from the EEG values and conversion to graph models has been studied and described in Graph theoretical analysis of structural and functional systems (Bullmore & Sporns, 2009) [5]. The functional brain was analysed by using an EEG graph. The correlation matrix was then derived from the time series data. This matrix can be converted to adjacency matrix - here we used a text file to store the edged directly without the need of adjacency matrix. Based on the given data, the graphs were modelled by fixing threshold values.

Following gives the brain graph model for alpha, beta and theta bands before and after meditation.

![Figure 3: Process of Converting EEG Data to Graph Model](image-url)
Figure 4: Alpha, Beta, and Theta Bands Before and After Meditation

Figure 5: Graphs Obtained After Modelling

After obtaining the graph, the complement group leverage centrality was computed for subsets based on the lobes for all the graphs in Fig. 5.

5. Results

The following table contains the computed values of the group leverage centrality. A detailed
description of the nodes and the subsets in the table is presented in the discussions section. A few readings in it are left blank because complement group leverage centrality cannot be defined for a subset which contains isolated nodes.

Table 1: Complement Group Leverage Centralities of the Lobes (Subsets) Before and After Meditation

| Subsets   | Alpha Before | Alpha After | Beta Before | Beta After | Theta Before | Theta After |
|-----------|--------------|-------------|-------------|------------|--------------|-------------|
| C3,C4,CZ  | 0.0132       | -0.0526     | -0.0076     | 0.0225     | 0.0457       | -0.2809     |
| F3,F4,F7,F8,FZ | 0.0146   | 0.0476      | -0.1368     | 0.0197     | 0.0152       | 0.0057      |
| O1,O2     | -            | -0.2777     | 0.0         | -          | -            | -0.6333     |
| FP1,FP2   | -            | -0.2777     | 0.0         | 0.0        | 0.0          | 0.0095      |
| P3,P4,PZ  | -0.0111      | 0.0643      | 0.0781      | -0.01      | -0.1253      | 0.0660      |
| T3,T4,T5,T6 | -0.1528      | -0.1093     | -0.0276     | -0.1908    | -0.2735      | -0.0927     |

6. Discussions

In this section, the group complement leverage centrality values before and after meditation graphs of alpha, beta and theta bands are compared. Alpha band is concerned about the relaxation of the body. Beta band involves high cortical activation of the brain areas and they are activated during stress and tension. Theta band is related to the body’s overall Deep relaxation. We have discussed the results of the impact of meditation with respect to the following groups/subsets of nodes.

1. **P3, P4, PZ**: These 3 nodes represent the central parietal lobe electrode locations. They are associated with cognitive processing and contribute to the activity of how things are perceived and segregated. From the results tabulated below, we can very well see that meditation has profound effects in the functioning of these three bands. The value of Alpha has increased to a positive CG; Beta band, which involves increased cortical activity has decreased to a negative value and Theta band which induces deeper relaxation to the body has also increased significantly to a positive value. Thus meditation has an enormous amount of positive effect on the perception capability of the brain and the connectivity between the different lobes.

2. **F3, F4, F7, F8, FZ**: The Dorsolateral Prefrontal cortex is related to short term memory that has been created based on the recently acquired information. It has effects on the logical performance, working memory, intentional and motivation focus. From the table, we can see that the Alpha band value has increased slightly, a dip in the value of Theta band and an increase in Beta. All these results essentially tell that meditation improves the connectivity and information encoding power of the brain.

3. **C3, C4, CZ**: These nodes refer to the central electrodes that control the sensory motor cortex, for example, these neurons are put to use and activated during hand movements. It is related to somatosensory information, thus the influence of meditation doesn't really matter.

4. **O1, O2**: Occipital lobes represent the primary visual areas of the brain. As far as Alpha and Beta
bands are concerned, the isolated nodes O1, O2 which did not communicate with the other lobes showed some range of connectivity (CG is still negative therefore they are not very influential) after the mediation period. In Beta band, after meditation the only connectivity between the 2 nodes has disappeared.

5. FP1, FP2: The Prefrontal cortex electrode locations (FP1,Fp2) are influential in the arousal, motivation and attention of the person. Before meditation, no active connections were noted. However after meditation the brain activation is visible in Alpha (from None to -0.2777) and in Theta band (to a positive value). No improvement during the Beta band.

6. T3, T4, T5, T6: The anterior temporal lobe electrode locations (T3,T4 limbic- amygdala) is associated with emotions while posterior temporal lobe electrode locations (T5,T6 hippocampus) are related to some memory related functions. From the table, we do not see any significant changes in the CG values after meditation in all the three bands.

7. Conclusion

The concept of Group Leverage Centrality to define the centrality of a group of nodes with respect to a brain network has been introduced. Furthermore, two different definitions for the same have been established of which the Complement Group Leverage Centrality has been used to find the effects of meditation on different lobes of the brain. We notice that meditation has profound effects on the human mind, and can cause increased cognitive processing and perception, decreased stress, and general well-being.

8. Acknowledgements

The authors also would like to thank SSN Management for their support and encouragement.

9. References

1. Korhonen,O., H. Saarimaki, E. Glerean, M. Sams and J. Saramaki. Consistency of regions of interest as nodes of fMRI functional brain networks, Network Neuroscience, 1:3, 254 - 274, 2017.

2. Fox, K.C., S. Nijeboer, ML. Dixon, JL. Floman, M. Ellamil, SP. Rumak, P. Sedlmeier, K. Christoff. Is meditation associated with altered brain structure? A systematic review and meta-analysis of morphometric neuroimaging in meditation practitioners, Neuroscience and Biobehavioral Reviews, 43,.48 -73, 2014.

3. Vargas, R., A. Waldron, A. Sharma, R. Flórez and D.A Narayan. A graph theoretic analysis of leverage centrality. AKCE Int J Graphs Comb. 14(3): 295- 306, 2017.

4. Joyce, K.E, P.J Laurienti, J.H. Burdette and S. Hayasaka. A new measure of centrality for brain networks, PLoS One, 5,1-13, 2010.,

5. Bullmore, E., O. Sporns. Complex brain networks: Graph theoretical analysis of structural and functional systems, Nature Reviews Neuroscience, 10, 186-198, 2009.

6. Garey, L. Cortex: statistics and geometry of neuronal connectivity, 2nd edition, Journal of Anatomy, 194, 153-157, 1999.

7. Mahesh, V., G. Bacak-Turan, G. Balaraman, R. Sundaeswaran and R. Sujatha. Group closeness centrality of graphs Springer- Conference Proceedings: Recent Advances in Computational and Engineering Mathematics, 2019.

8. Das, K., S. Samanta and M. Pal. Study on centrality measures in social networks: a survey. Soc Netw Anal Min. 8(1):13, 2018.
9. Lee, S.H., C.E. Kim, I.S. Lee, W.M Jung, H.G Kim, H. Jang, et al. Network analysis of acupuncture points used in the treatment of low back pain. Evid Based Complement Alternat Med 2013:402180.

10. M. G. Everett and S. P. Borgatti, The Centrality of Groups and Classes, Journal of Mathematical Sociology 1999, Vol. 23(3), pp. 181-201.