A new adaptive fuzzy hybrid unscented Kalman/H-infinity filter for state estimating dynamical systems

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Abstract
State estimation and dynamical model identification from observed data has been an attractive research area with a wide range of applications such as communication, navigation, radar target tracking, and system control. A method of Adaptive Fuzzy Unscented Kalman/H∞ Filter (AFUKH∞) to estimate non-linear systems is presented using a combination of the Unscented Kalman Filter (UKF) and Unscented H∞ Filter (UH∞F). The proposed filter does not need linearisation and is based on a combination of gain, a priori state estimation, and a priori measurement estimation in each time step. The performance of the filter is adaptively adjustable. Thus, its efficiency is better than the other two filters. Two fuzzy logic systems are proposed that determine the weight of the UKF and UH∞F filters at each step. These two fuzzy systems are designed to be independent of the dynamics of the system (problem). The proposed filter is referred to as a hybrid AFUKH∞-II. In the proposed method, the state of the feedback is used as input that improves the efficiency of the filter. The challenge of reentry vehicle tracking and the state estimation of a magnetic motor as two non-linear high-order problems are used as benchmarks, and the results are compared with the UKF, UH∞F, and AFUKH∞ filters. The experiments show that an estimation of the proposed hybrid filter (AFUKH∞-II) is improved against state-of-the-art filters. Also, estimation error and variance values of the proposed filter in the presence of Gaussian noise is decreased by 270% and 370%, respectively, compared with the AFUKH∞ filter.

1 | INTRODUCTION

The observation of any physical phenomenon is inherently uncertain. When a physical quantity is measured by a sensor, the observed value includes two types of errors: system and random. System error describes a situation in which the mean of a series of observations differs from the observed quantity in a specific manner and typically is resolved by appropriate sensor calibration. On the other hand, random errors are caused by factors that cannot easily be predicted and eliminated, such as vibrating sensors and oscillating electrical instruments. A probabilistic model is usually used to measure a physical state that has a probabilistic distribution. Therefore, in addition to reducing these errors, the development of models that can have appropriate resistance to their nature in terms of their possible Gaussian or non-Gaussian distribution is a goal of this study.

The model provided by Kalman is an effective tool for state estimation, but it is limited to linear systems. In this model, the observed value is a linear function of the system states. The system model is also presented linearly. In the Kalman model, it is assumed that the states and measured values are impregnated with Gaussian white noise with zero means. The existence of these assumptions is restrictive. For industrial problems, the exact model of the system and the statistical nature of process noise are not readily available because many practical systems are non-linear and the Kalman method cannot be applied. To estimate non-linear systems, some non-linear filters have been presented, such as Extended Kalman Filters, different kinds of unscented Kalman filters (UKFs), various sorts of filters based on Monte-Carlo methods such as particle filters, and all kinds of cubature Kalman filters (CKFs). Studies have shown that when the UKF filter is better
in one part of the estimate and the UH∞F filter in the other part, the dual filter can provide better results by providing an optimal estimate. We combine the UKF and Unscented H-infinity filter infinite filter (unscented H∞ filter [UH∞F]) to overcome this ignorance. This method provides an intermediate answer between the two filters in different problems and is more reliable than the two other filters. An analysis of variance also showed that the filter has high resistance to the statistical nature of noise and can be appropriately answered by changing its variable. The validity of the results was investigated using two non-linear problems of the free fall simulation and reverse pendulum in the field of mechanical engineering.

The performance analysis of the filters takes place in the field of minimum average error, and the development goal is usually to reduce the estimation error. Assessment of this type of performance requires a perception of the statistical nature of the error [1, 2]. Another estimation of the system efficiency can be to reduce the worst possible error, which requires no statistical recognition of the error. The result of this assessment was to present the H∞ filter in the 1980s, which, by considering non-linear issues, led to the presentation of various non-linear H∞ filters by various researchers. This type of uncertainty estimation is called robust estimation. However, robust estimators based on Kalman filter theory can be designed to meet these requirements, but they are partially non-generalized. The H∞ filter is specifically designed for robustness. Nevertheless, combining the two Kalman and H∞ filters can apply the properties of both of these filters: minimum average error and minimises the error increases in the estimation problem. Consequently, Khargonenkar was a researcher who presented the prototype of the hybrid Kalman filter using optimization [3]. Therefore, Simon et al. [4] presented two different methods to combine two filters, initially, using the steady gain linear combination of two filters and then extracting the gain of the hybrid filter from a Riccati equation [4]. Among other methods, optimization methods were used to combine the two objective functions of these filters [5–7]. Employing the linear combination of gains, the prior state estimation, and the magnitude of the two UKF and UH∞F filters, Tehrani et al. [8] presented a dual filter that exhibited higher robustness relative to the UKF and UH∞F filters in both two Gaussian and non-Gaussian noise states as well as optimization ability. Significant research was accomplished on the use of fuzzy logic systems in filtering issues. Major studies were conducted in the covariance setting of model error and measurement. Sasiadek et al. [9], presented a method based on variation in the covariance of model error and measurement. They developed the Takagi–Sugeno–Kang (TSK) fuzzy technique to adapt filter error covariance. Jin and Zhao [10, 11] corrected the covariance of the model error or measurement error using a fuzzy system. A difference was seen in the corrector function of covariance in linear or non-linear forms [12]. Tehrani et al. [13] presented a hybrid adaptive fuzzy unscented Kalman/H∞ filter (AFUKH∞) and demonstrated that the proposed filter provides the best estimation between UKF and UH∞F filters. The method in that study does not change the structure of these two filters and does not use state feedback.

Inspired by the approach of Tehrani et al. [13], our contribution proposes a hybrid filter using UKF and UH∞F based on the AFUKH∞ filter structure. We suggested the state feedback as input to improve the efficiency of the result. There are two major changes to AFUKH∞. First, the state recurrence relation between the two UKF and UH∞F filters has been disconnected. Second, the task of this recurrence relation is appointed to the dual output filter. In the last step, a fuzzy decision with proposed rules is applied to final estimation. Regardless of the approach of Tehrani et al. [8], our contribution does not employ state feedback in the state estimation. Also, the independence of the two UKF or UH∞F filters has been changed. Therefore, the input of the two filters in the state and value of the measurement is the same. Subsequently, this new state estimation filter is compared with three filters: UKF, UH∞F, and AFUKH∞. Adaptively fuzzy adjust the performance of the proposed filter, the efficiency of the new filter is better than both filters. The rest of the work is structured as follows: Section 2 describes the proposed AFUKH∞-II filter. In Section 3, we describe the fuzzy logic system. The experiments in a simulation format are in Section 4. Conclusions are given in Section 5.

2 | PROPOSED FILTER (AFUKH∞-II)

A combination of the UKF and UH∞F filters is presented, which adjusts its efficiency adaptively. Thus, its efficiency is better than that of the other two filters. This dual filter does not need to be linearised in the face of a non-linear problem and is based on a combination of the benefits of two filters, a priori state estimation of state and a priori measurement estimation at each time step. The combination of these two filters is based on an appropriate weight of the previous state estimate, a preliminary estimate of the size and appropriate interest of each filter. The mathematical equations of the two UKF and UH∞F filters are given in various references [13, 14]. Therefore, they are not mentioned again. In the proposed hybrid filter, the two non-linear filters, UKF and UH∞F, are combined. The basis of this combination is to allocate appropriate weights to the gain, previous state estimation, and magnitude. In a hybrid filter, two distinct weights are used: one for the linear combination of the gain and prior state estimation, and the second for the linear combination of the previous magnitude estimation. The value of these two numerical weights is between 0 and 1 and is extracted adaptively from two independent fuzzy logic engines. The combined relations of the AFUKH∞ and AFUKH∞-II filters are presented in Equations (1)–(4):

$$x_{k+1}^{\text{ Hybrid}} = x_k^{\text{ Hybrid}} + K_k^{\text{ Hybrid}} (y_k - y_k^{\text{ Hybrid}})$$  \hspace{1cm} (1)
Such that:

\[
K_k^{\text{Hybrid}} = d_s K_{UKF}^k + (1 - d_s) K_{H}^k \quad (2)
\]

\[
\hat{x}_k^{\text{Hybrid}} = d_s \hat{x}_{UKF}^k + (1 - d_s) \hat{x}_{H}^k \quad (3)
\]

\[
y_k^{\text{Hybrid}} = d_m y_{UKF}^k + (1 - d_m) y_{H}^k \quad (4)
\]

where, the superscript of each variable indicates its dependence on the filter, and variables with positive and negative superscripts are related to the subsequent and prior estimates of the state or magnitude, respectively. Accordingly, variable \( \hat{x}_k \) is the state estimation, \( y_k \) indicates the magnitude estimation, \( \bar{K} \) represents the gain, \( d_s \) is the combined weight of the gain and the previous state estimation and \( d_m \) is the combined weight of the magnitude previous estimation. Weights from 0 to 1 are divided into four equal intervals, and using experimental knowledge about the rules, a TSK fuzzy logic system is presented as:

**Rule (1)** If \( (e_{UKF} - e_{UH} F) < -2\sigma \), then \( d_s = 1 \)

**Rule (2)** If \( -2\sigma < (e_{UKF} - e_{UH} F) < 0 \) then \( d_s = 0.75 \)

**Rule (3)** If \( -0.2\sigma < (e_{UKF} - e_{UH} F) < +0.2\sigma \) then \( d_s = 0.5 \)

**Rule (4)** If \( 0 < (e_{UKF} - e_{UH} F) < +2\sigma \) then \( d_s = 0.25 \)

**Rule (5)** If \( +2\sigma < (e_{UKF} - e_{UH} F) \) then \( d_s = 0 \)

In rules (1)–(5), the eUKF equals the previous estimation error of the UKF. The eUH indicates the previous estimation error of the UH filter and \( \sigma \) is indicative of the STD of the previous state estimation of the two UKF and UH filters illustrated in Equation (5). Equally, the same rules are considered to estimate the magnitude in which the magnitude estimation error is deployed instead of the state estimation error:

\[
\sigma = \sqrt{\frac{1}{1 - n} \sum_{i=1}^{n} (\tilde{e}_i - \overline{e})}
\]

where \( \tilde{e}_i = e_{UKF} - e_{UH} F \) is the difference between the estimation error of the UKF and the UH filter in each iteration, \( \overline{e} \) is the mean and \( n \) represents the number of estimation errors before the current iteration, which is considered to be 10. In rules (1)–(5), when the errors are nearer each other, their average difference is close to 0 (the involved laws are laws 2, 3, and 4) and the weights of the UKF and the UH become approximately equal. By increasing the difference to the positive (negative), the error difference increases (decreases), and the weight of the UKF decreases (increases). The error difference becomes distant from 0. In this case, rules (4) and (5) will be employed. For high positive (negative) differences, the weight of the UKF filter, value \( d_s \) tends to 0 (1). The STD can be considered a scaling variable in which the bigness and smallness of the error difference is expressed by its coefficients. The membership functions of these five laws are depicted in Figure 4, in which the membership function of rule

### 3 | FUZZY LOGIC SYSTEM

Two integrated fuzzy systems are presented that dynamically derive weights \( d_s \) and \( d_m \) for Equations (2)–(4), such that an appropriate interaction for the gain, previous state estimation, and the magnitude between UKF and UH filters is achieved. The rules of the fuzzy system are regulated by the STD of the difference between the errors of the two UKF filters and UH filters in the previous state and magnitude estimation values. The STD of the difference between the nearer errors tends to 0, and as the difference increases, the STD variable also increases. According to this, if the two errors are close to each other, it indicates that there is a slight difference between the two UKF and UH filters. Therefore, a weight equal to 0.5 is sufficient for Equations (2) and (3). If the difference is significant, the weight should be inclined to the filter that provides a lesser error. Hence, the weights should tend to 1 or 0. Weights from 0 to 1 are divided into four equal intervals, and using experimental knowledge about the rules, a TSK fuzzy logic system is presented as:
is the right trapezoidal, the membership functions of rules (2), (3), and (4) are triangular and the membership function of law 5 is the left trapezoidal.

4 | SIMULATION

As a sequence, to assess the performance of the AFUKH∞-II filter and compare the results with the other three filters (UKF, UH∞F, and AFUKH∞), two non-linear simulation problems are considered.

FIGURE 1 Adaptive fuzzy unscented Kalman/H∞ filtering algorithm [13]. UH∞F, unscented H∞ filter; UKF, unscented Kalman filter

FIGURE 2 Adaptive fuzzy unscented Kalman/H∞ filtering algorithm (proposed method). UH∞F, unscented H∞ filter; UKF, unscented Kalman filter

FIGURE 3 Block diagram of using AFUKH∞-II filter in a system. AFUKH∞-II, adaptive fuzzy unscented Kalman/H∞-II

FIGURE 4 Membership functions of fuzzy system. UH∞F, unscented H∞ filter; UKF, unscented Kalman filter

4.1 | The problem of reentry vehicle tracking

This problem was employed by Julier et al. [15] to examine the efficiency of the UKF filter. It was also used by Sarkka et al. [16] to analyse the UKF continuous filter. The state-space is composed of object positions $x_1$ and $x_2$, and their peer velocities, $x_3$ and $x_4$. The aerodynamic property of the object is also marked with an $x_5$ state. Accordingly, the equations of motion of the object entering Earth can be expressed by Equations (6)–(15) as:

$$R(t) = \sqrt{x_1^2(t) + x_2^2(t)}$$  (6)

$$V(t) = \sqrt{x_3^2(t) + x_4^2(t)}$$  (7)

$$b(t) = b_0 \exp(x_5(t))$$  (8)

$$D(t) = b_0 \exp\left(\frac{R_0 - R}{H_0}\right) V(t)$$  (9)

$$G(t) = \frac{Gm_0}{R^3(t)}$$  (10)

$$\dot{x}_1 = x_3(t)$$  (11)
\[
\begin{align*}
\dot{x}_2 &= x_4(t) \quad (12) \\
\dot{x}_3 &= D(t)x_3(t) + G(t)x_1(t) + w_1 \quad (13) \\
\dot{x}_4 &= D(t)x_4(t) + G(t)x_2(t) + w_2 \quad (14) \\
\dot{x}_5 &= w_3 \quad (15)
\end{align*}
\]

where value \(D\) is the drag associated with force, \(G\) is the phrase related to gravity, \(R\) defines the distance from the centre of Earth and \(b\) is the ballistic coefficient. Other constants that have been taken from Tehrani et al. [13]:

\[
H_0 = 13.406\text{km} \quad R_0 = 6374\text{km}
\]

\[
Gm_0 = 3.986 \times 10^5\text{km}^3/\text{s}^2 \quad b_0 = -0.59783 1/(\text{km} \cdot \text{s})
\]

The observer models are similar to Equations (16) and (17), observed by a radar located in the position \((x_r, y_r) = (R_0, 0)\). In Equations (16) and (17), \(v_k\) is the measured noise in such a way that the STDs of \(v_t\) and \(v_2\) are 1 m and 17 milliradian, respectively. Also, the covariance of the process noise is \(Q = \text{diag}\{2.406, 2.406, 1\} \times 10^{-3}\). Measurement and discretisation of process equations happen every 0.1 s. The real initial conditions for the device to enter the Earth are \(X_{0, \text{true}} = [6500.4349, 14, -1.8093, 0.6932, 0]\), and \(P_{0, \text{true}} = \text{diag}\{10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 0\}\):

\[
\begin{align*}
\theta(t_k) &= \tan^{-1} \left( \frac{x_2(t_k) - y_k}{x_1(t_k) - x_k} \right) + v_2 \quad (17)
\end{align*}
\]

For filtering, the initial conditions are set to \(X_0 = [6500.4349, 14, -1.8093, 0]\), and \(P_0 = \text{diag}\{10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 1\}\). The simulation results for Gaussian noises are illustrated in Figure 5. This chart is based on the average mean error [13] in each stage of the filter’s implementation. Figure 5 demonstrates that the AFUKH\(\infty\) filter is more like a selective mechanism between the two UKF and UH\(\infty\)F filters, but the AFUKH\(\infty\)-II filter estimation indicates different and improved results.

The mean error and variance of the four filters in the presence of Gaussian noise are given in Table 1. According to experimental results, the error degree and the variance of the AFUKH\(\infty\)-II decreased relative to the other three filters. The error and variance of the AFUKH\(\infty\)-II improved 270% and 370%, respectively, compared with the AFUKH\(\infty\) filter, whereas the performance of the AFUKH\(\infty\) had only a relative improvement over the UKF filter.

### Figure 5
Average of the root mean square errors for the UKF, UH\(\infty\)F, AFUKH\(\infty\), and proposed filter (AFUKH\(\infty\)-II) with consideration of Gaussian noise for the reentry vehicle tracking problem. AFUKH\(\infty\)-II, adaptive fuzzy unscented Kalman/Hoe-II; UH\(\infty\)F, unscented Hoe filter; UKF, unscented Kalman filter.

### Table 1
Comparison of average values and state estimation error variances obtained from four UKF, UH\(\infty\)F, AFUKH\(\infty\), and AFUKH\(\infty\)-II filters with non-Gaussian noise for the tracking problem

| Method         | Average values | Variances |
|----------------|----------------|-----------|
| UH\(\infty\)F  | \(19.8 \times 10^{-3}\) | \(1.29 \times 10^{-4}\) |
| UKF            | \(11.7 \times 10^{-3}\) | \(3.82 \times 10^{-4}\) |
| AFUKH\(\infty\) | \(10.2 \times 10^{-3}\) | \(3.79 \times 10^{-4}\) |
| AFUKH\(\infty\) II (proposed) | \(3.02 \times 10^{-3}\) | \(8.02 \times 10^{-5}\) |

Abbreviations: AFUKH\(\infty\)-II, adaptive fuzzy unscented Kalman/Hoe-II; UH\(\infty\)F, unscented Hoe filter; UKF, unscented Kalman filter.

In the following discussion, conditions for non-Gaussian noise are investigated. In this case, a uniform noise is added to Equations (13) and (14) in intervals [0, 0.02] and [0, -0.02]. Figure 6 depicts the efficiency of the four filters based on the average mean error value [1, 2] at each stage of the filter implementation.

As expected, the AFUKH\(\infty\) filter has provided an estimation close to the UH\(\infty\)F. However, the AFUKH\(\infty\)-II provides a relatively better estimation compared with the other three filters. The numerical results of the mean and error estimation variance for 100 Monte-Carlo iterations are presented in Table 2. The performance of the AFUKH\(\infty\)-II filter has improved by about 150% compared with the AFUKH\(\infty\) filter in the average value of error and variance. The results of the simulations for the AFUKH\(\infty\) filter showed better performance relative to the other three filters.

### 4.2 Two-phase permanent magnet synchronous motor [1, 15]

To state the estimation of the two-phase motor, equations of this type of motor are:

\[
\frac{d}{dt}i_s = \frac{-R a}{L} i_s + \frac{\omega q}{L} \sin(\theta) + \frac{u_s + d_1}{L}
\]

(18)
FIGURE 6 Average of the root mean square errors for the UKF, UHooF, AFUKHoo, and AFUKHoo filters with non-Gaussian noises for the tracking problem. AFUKHoo-II, adaptive fuzzy unscented Kalman/Hoo-II; UHooF, unscented Hoo filter; UKF, unscented Kalman filter

TABLE 2 Comparison of average values and state estimation error variances obtained from four UKF, UHooF, AFUKHoo, and AFUKHoo-II (proposed) filters with non-Gaussian noise for the tracking problem

| Method          | Average values | Variances     |
|-----------------|----------------|---------------|
| UKF             | $5.49 \times 10^{-2}$ | $5.19 \times 10^{-3}$ |
| UHooF           | $3.46 \times 10^{-2}$ | $1.60 \times 10^{-3}$ |
| AFUKHoo         | $3.28 \times 10^{-2}$ | $1.52 \times 10^{-3}$ |
| AFUKHoo-II (proposed) | $1.28 \times 10^{-2}$ | $6.10 \times 10^{-4}$ |

Abbreviations: AFUKHoo-II, adaptive fuzzy unscented Kalman/Hoo-II; UHooF, unscented Hoo filter; UKF, unscented Kalman filter.

\[
\dot{I}_b = \frac{-R_a}{L} I_b - \frac{\omega \lambda}{L} \cos(\theta) + \frac{u_b + q_{2b}}{L} 
\]  

(19)

\[
\dot{\omega} = \frac{3\lambda}{2J} I_a \sin(\theta) + \frac{3\lambda}{2J} I_b \cos(\theta) - \frac{F_\omega}{J} + q_3 
\]  

(20)

\[
\dot{\theta} = \omega + q_4 
\]  

(21)

where the $I_a$ and $I_b$ flows are the currents in two windings, $\theta$ and $\omega$ are the angular positions and rotor speed, respectively. $R$ and $L$ are the resistance and inductance of the coil, respectively, and $F$ is the friction coefficient. The $u_a$ and $u_b$ are the applied voltages to the cross-section of two windings and $j$ is the inertial torque of the axis and the motor load. Afterwards, the state vector is defined by Equation (22) [17]:

\[
x = [I_a, I_b, \omega, \theta]^T
\]  

(22)

The constants of Equations (18)–(21), which were taken directly from [16], are also given as:

\[
R_a = 2\Omega \quad L = 0.003 \, \Omega \cdot s \quad \lambda = 0.01 \, V \cdot s
\]  

Also, $q_i$ components are the process noise caused by uncertainty in control inputs ($q_1$ and $q_2$), changes in acceleration owing to load torque ($q_3$), and momentary velocity changes ($q_4$). The $q_1$ and $q_2$ are independent white Gaussian noise with an STD of 0.001, and $q_3$ and $q_4$ are also independent white Gaussian noise with an STD of 0.02 and $q_4$ considered 0. The control inputs are the voltages of each coil, which are set in the form of Equations (23) and (24):

\[
u_a(t) = \sin(2\pi t)
\]  

(23)

\[
u_b(t) = \cos(2\pi t)
\]  

(24)

Besides, the measure currents of each coil, which are set in the form of Equations (25) and (26):

\[
y_a = I_a + \nu_a
\]  

(25)

\[
y_b = I_b + \nu_b
\]  

(26)

where $\nu_a$ and $\nu_b$ are the independent Gaussian white noise with zero mean and covariance $\text{diag}\{1,1\} \times 10^{-2}$. It is assumed that the vector measurement takes place every 0.01 s and the integral constant of the system simulation is 0.0005. The state estimation error of the Gaussian noise simulation for this model is for 200 iterations is depicted in Figure 7. Based on this figure, AFUKHoo-II (symbol □) provides a better estimate rather than two other filters and AFUKHoo (symbol ■) approximates the UKF (symbol ×).

Table 3 presents the average mean square error and error variance of four filters. According to the results, the efficiency
of the AFUKHoo-II filter is better in both error and variance sections at about 6% compared with AFUKHoo, whereas the efficiency of the AFUKHoo filter is improved by only 1.5% relative to UKF.

In the following discussion, to simulate non-Gaussian noise, uniform noise in interval [3, 0] is added to Equation (20) and more uniform noise in the range [0, 0.05]. These values affect acceleration and motor speed. The simulation results are presented in Figure 8. Performance of the AFUKHoo-II filter (symbol □) is better than for the other three filters. The UKF filter also provides the worst estimation in this situation. The results indicate that optimised filters based on Gaussian noise may show weaker performance under unknown conditions compared with robust filters such as the UHooF filter.

For a more detailed examination, the quantitative performance of the filters is presented in Table 4. The results indicate that the error of the AFUKHoo-II filter has improved about 2.5% relative to the AFUKHoo and its variance has decreased by about 16%.

## Conclusion

An adaptive filter (AFUKHoo-II) is proposed to estimate the states of non-linear problems. The proposed filter is based on the combination of two non-linear filters, UKF and UHooF. This hybrid filter is founded on three linear combinations of the previous state estimation, the magnitude, and the gain of the two filters. The advantage of the proposed filter is adaptive characteristics and its robustness to various noise types, even an unknown noise model. Using a fuzzy method, the proposed method does not need offline optimization. Of course, the AFUKHoo-II cannot guarantee the least error and best estimation because this depends on problem dynamics and the use of optimization algorithms. In estimation problem simulations, it was demonstrated that the use of state feedback and direct establishment of the filter's recurrence cycle compared with when this cycle did not have a direct form resulted in improved performance. For example, this proposed filter increased efficiency in the entered object tracking problem and reached 270% of Gaussian noise presence.

As future work, the UKF can be replaced with the CKF. The effect of coloured noise on the proposed filter may be evaluated. Also, the use of a fuzzy system to define the weights of filters based on the importance of a specific state can be studied.

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