Emergent Gravity from a Mass Deformation in Warped Spacetime

Tony Gherghetta\textsuperscript{a1}, Marco Peloso\textsuperscript{a2} and Erich Poppitz\textsuperscript{b3}

\textsuperscript{a}School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455, USA

\textsuperscript{b}Department of Physics
University of Toronto
Toronto, ON M5S 1A7, Canada

Abstract

We consider a deformation of five-dimensional warped gravity with bulk and boundary mass terms to quadratic order in the action. We show that massless zero modes occur for special choices of the masses. The tensor zero mode is a smooth deformation of the Randall-Sundrum graviton wavefunction and can be localized anywhere in the bulk. There is also a vector zero mode with similar localization properties, which is decoupled from conserved sources at tree level. Interestingly, there are no scalar modes, and the model is ghost-free at the linearized level. When the tensor zero mode is localized near the IR brane, the dual interpretation is a composite graviton describing an emergent (induced) theory of gravity at the IR scale. In this case Newton’s law of gravity changes to a new power law below the millimeter scale, with an exponent that can even be irrational.

\textsuperscript{1}E-mail: tgher@physics.umn.edu
\textsuperscript{2}E-mail: peloso@physics.umn.edu
\textsuperscript{3}E-mail: poppitz@physics.utoronto.ca
1 Introduction

It is a striking fact that four-dimensional (4D) gravity can be localized in AdS$_5$ by tuning the bulk and brane cosmological constants [1]. It is even more remarkable that this five-dimensional (5D) model has a dual 4D interpretation via the AdS/CFT correspondence [2]-[7]. The gauge-gravity duality has made the warped gravity setup an attractive setting for studying aspects of strongly coupled gauge dynamics, from both the formal and phenomenological perspective. A feature particularly useful for low-energy model building is that the nonfactorizable geometry localizes not only gravity, but also fields of different spin, such as scalars and fermions [8, 9].

An important difference of scalar and fermion localization from gravity is that fermion and scalar zero modes can be localized anywhere in the 5D bulk. This unrestricted localization is achieved by introducing bulk and boundary masses, with the degree of localization directly depending on the bulk mass parameter [9]. This has been particularly fruitful for model building, where models of UV/IR brane-localized fields, corresponding to hybrid theories of elementary and composite particles in the 4D dual, have opened new phenomenological possibilities for the supersymmetric standard model [10, 11] and the Higgs sector [12, 13]. In fact, this unrestricted localization can also occur for gauge fields. While it is well known that gauge bosons are not localized in the bulk [14, 15], it is possible to tune bulk and boundary mass terms so that a U(1) zero mode can be localized anywhere [16].

The graviton appears to be different, but it is natural to ask whether gravity can similarly be localized anywhere in the bulk. The AdS/CFT correspondence provides our primary motivation for studying the delocalization of the graviton in the Randall-Sundrum (RS) scenario. If the graviton zero mode could be localized on the IR brane, this would suggest that in the dual theory the graviton is a composite CFT state whereby dynamical gravity only emerges in the infrared. The dual UV theory would then be a pure gauge theory with no propagating massless spin-2 particle. It has been suggested that in an “emerging” (“fat”) gravity model, the graviton compositeness may alleviate the UV sensitivity of the cosmological constant [17]. One may worry that delocalization comes at the price of losing general covariance in the bulk, but as we will see, this does not preclude the existence of a massless tensor mode. Furthermore, the Weinberg-Witten theorem is avoided if gravity is induced at the quantum level [18]. Thus, there does not appear to be any obstacle to smoothly deforming the graviton wave function away from the UV brane, as can be done for other bulk fields, and one hopes that a similar mechanism can be implemented for gravity.

In this paper, we study a modification of the linear Einstein equations and show that, indeed, the graviton can be localized on the IR brane. Much like for bulk scalar and fermion fields, our modification requires the introduction of bulk and boundary mass terms for the metric perturbations. However adding a graviton mass is technically more involved compared to the case of lower spin fields. For instance,
already in 4D a Fierz–Pauli [19] mass term for the graviton leads to the well known vDVZ discontinuity at the linearized level [20] (while other choices lead to ghosts). This is due to the fact that the longitudinal polarization of a massive graviton remains coupled to matter in the limit of zero graviton mass. In the warped 5D case, the extra polarizations of the graviton can be systematically studied by exploiting the symmetries of the 4D Minkowski slices. This leads to the definition of tensor, vector, and scalar fluctuations with respect to the 4D Poincaré transformations. In this way, we show that by a special choice of the bulk and boundary mass terms there exist tensor and vector zero modes that can be localized anywhere in the bulk. A priori, one should also expect scalar modes but, remarkably, they are absent in our model. Thus, up to quadratic order in the action, we will see that our model is ghost-free. This is in contrast to many modifications of gravity where ghost perturbations typically arise in the scalar sector. In particular, a previous study of massive gravity in warped geometry, which only introduced boundary mass terms, concluded that ghosts were present in the spectrum [21].

Below, we present the structure and summarize the main results of this paper:

• We begin, in Section 2, with a study of the bulk equations of motion for the graviton perturbation with a bulk mass term added. The solutions of the linearized equations of motion for the metric perturbations lead to massless and massive tensor, vector, and scalar modes. These are subject to boundary conditions on the two branes, which we discuss next.

• Boundary mass terms are introduced and the junction conditions are derived, in Section 3, for all the metric perturbations. Some details are presented in the Appendices; in Appendix A we present the bulk gravity action to quadratic order. In particular, in Section 3 we also find that a massless tensor and vector mode is consistent with the boundary conditions and that the model is ghost-free. For any allowed value of the bulk mass parameter, there exist two possible choices of boundary terms admitting a zero mode (see Fig. 1). The quadratic action of the graviton zero mode for the two branches is given in Appendix B. At the massive level, we also obtain the Kaluza-Klein spectrum. Most importantly, we note that the scalar spectrum vanishes identically. Details of the quadratic action for scalar perturbations and the relevant boundary conditions are given in Appendix C.

Curiously, for zero bulk mass there are also two solutions admitting a zero mode. One of them is the original RS solution with zero boundary masses, while the other, requiring nonvanishing boundary mass terms, can be termed the specular RS solution: its zero mode wave function, $1/A^2(z)$ and peaked on the IR brane, is the inverse of the original RS solution’s wave function, $A^2(z)$; see Sections 3-5 and Fig.1 [specular: (adj.) of, relating to, or having the qualities of a mirror].
We continue, in Section 4, with a discussion of the properties of the massless modes. We show that the tensor mode can be localized anywhere in the bulk and gives rise to a finite 4D Planck mass. This then motivates us to consider localizing the graviton on the IR brane and to interpret the force it mediates as an emergent infrared phenomenon since there is no propagating massless spin 2 mode in the UV. In Sections 4.3, 4.4, and Appendix D, we study Newton’s law at short distances and show that it exhibits a new power law dependence, that can even be irrational. The crossover to a $V \propto 1/r^\beta$, $\beta > 1$ (and real) behavior is observable at distances shorter than the infrared scale—but much larger than the AdS curvature scale, unlike the RS model—which can be taken to be in the sub-millimeter range. Gravity at such distances is presently under experimental investigation [22]–[25].

In Section 5, we discuss the 4D holographic interpretation of our gravity solution. While it is necessarily speculative, even the remotest plausibility of a 4D dual picture inspires some confidence in the existence of a nonlinear extension of our linearized gravity analysis.

- In the first of the two branches—the one smoothly connected to the RS model—the dual involves a strongly coupled 4D CFT coupled to a source graviton (Section 5.1). The coupling is relevant precisely when the zero mode is localized near the IR brane. As a result of this coupling, the energy momentum tensor of the CFT should somehow acquire a large anomalous dimension. The conformal symmetry is broken at the infrared scale where a large Newton constant is induced by the CFT. The massless zero mode is thus a mixture of the source graviton and a CFT composite. The dominant component of the zero mode graviton, in the case of relevant coupling, comes from the broken CFT. We show how the change in Newton’s law can be explained in detail in the dual picture.

- The holographic dual of the other branch of the gravity solution, described in Section 5.2, where the graviton is always localized near the IR brane, is even more mysterious. Holography suggests that it involves a CFT coupled to a massive source graviton (with mass of order the AdS curvature scale). Thus the observed massless graviton mode is essentially “emergent” at low energies due to the strong infrared dynamics of the broken CFT.

We conclude, in Section 6, with a discussion of the effects of nonlinear terms to our analysis whose details are beyond the scope of the present paper.

Finally, let us comment on the relation of our model to the “fat” graviton idea of Sundrum [17], which was designed to turn off gravity at short distances (precisely at the scales close to the experimental limit [22]–[25]) and alleviate the cosmological constant problem. In the picture presented here—which is the first quantitative...
model of a “fat” graviton, where gravity in many cases is an “emergent” low-energy phenomenon—the gravitational interaction becomes stronger at short length scales. The stronger UV gravity may be a more general phenomenon, or be simply due to the particular setup: gravity in a slice of AdS$_5$, being dual to a large-$N$ CFT, has a tower of stable “mesons” (the graviton Kaluza-Klein modes) contributing to the gravitational potential at short length scales, and may just signify that the “fat” graviton, without the effects of the Kaulza-Klein modes, should be looked for elsewhere.

2 Warped gravity with a bulk mass

2.1 The Randall-Sundrum background solution

In the absence of mass terms, warped gravity in five dimensions is governed by the following 5D action:

$$S = \int d^5x \sqrt{-g} (M^3 R - 2\Lambda) - \sum_i \int d^4x \sqrt{-\gamma_i} (M^3 [K] + \lambda_i) ,$$  

where $M$ is the 5D Planck scale, $\Lambda$ is a bulk cosmological constant, and the sum is over two boundary three-branes with brane tensions $\lambda_i$. The quantity $[K]$ denotes the jump of the trace of the extrinsic curvature across the brane.

The solution to Einstein’s equations is a slice of AdS$_5$ where the fifth dimension is compactified on an orbifold $S^1/Z_2$ of radius $R$ with the Randall-Sundrum metric [1]:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 = A^2(z)(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) \equiv g^{(0)}_{AB} dx^A dx^B ,$$

where $0 \leq y \leq \pi R$ is the “fundamental domain” (where the bulk integral [1] is computed), $k$ is the AdS curvature scale, $\Lambda = -6k^2 M^3$, and the Minkowski metric $\eta_{\mu\nu}$ has signature $(-+++)$. The Latin indices ($A, B, \ldots$) label all the 5D coordinates, while Greek indices ($\mu, \nu, \ldots$) are restricted to the 4D coordinates. We will work with conformal coordinates defined by $z = (e^{ky} - 1)/k$ and $A(z) = (1 + kz)^{-1}$ is the warp factor. At the orbifold fixed points $z_0 = 0$ and $z_1 = (e^{\pi k R} - 1)/k$ there are two three-branes, the UV and the IR brane, respectively.

In the 5D bulk the action is invariant under general coordinate transformations. At linear order the infinitesimal coordinate transformations are:

$$x^M \rightarrow x^M + \xi^M(x) ,$$  

where it is convenient to split the transformation parameters into $\xi^M = (\xi^\mu + \partial^\mu \xi, \xi^5)$ with the condition $\partial^\mu \xi^\mu = 0$. In this way the coordinate transformations are separated into two scalar ($\xi, \xi^5$) and one vector ($\xi^\mu$) transformation parameters (with respect to the Poincaré symmetry of the 4D Minkowski background). In a covariant theory these can be used to eliminate five degrees of freedom (d.o.f).
The metric perturbations, \( h_{AB} \) around the background Randall-Sundrum metric \( g^{(0)}_{AB} \), correspond to fifteen degrees of freedom, and a useful way to parametrize them is:

\[
ds^2 = A^2(z) \left[ (1 + 2\phi)dz^2 + 2 (B_\mu + B_\mu)dz \, dx^\mu \right. \\
+ \left. \left( (1 + 2\psi)\eta_{\mu\nu} + 2E_{\mu\nu} + E_{(\mu,\nu)} + \tilde{h}_{\mu\nu} \right) dx^\mu dx^\nu \right] ,
\]

\[
ed \equiv (g^{(0)}_{AB} + h_{AB})dx^A \, dx^B ,
\]

where \( E_{(\mu,\nu)} \equiv \partial_\mu E_\nu + \partial_\nu E_\mu \). We see that the metric perturbations are divided into three sectors: scalar, vector, and tensor (with respect to the Poincaré symmetry of the 4D Minkowski background).

Specifically, the tensor mode \( \hat{h}_{\mu\nu} \), is taken to satisfy the transverse (\( \partial^\mu \hat{h}_{\mu\nu} = 0 \)) and traceless (\( \hat{h}^{\mu}_{\mu} = 0 \)) conditions. It is gauge invariant under the infinitesimal coordinate transformations \( (3) \), and being symmetric, it contains \( 10 - 5 = 5 \) d.o.f.

The vector modes \( B_\mu \), and \( E_\mu \) are both taken to be transverse (\( \partial^\mu B_\mu = \partial^\mu E_\mu = 0 \)), and consequently contain 6 d.o.f. Under the coordinate transformation \( (3) \) the two vector modes transform as:

\[
B_\mu \rightarrow B_\mu - \xi'_\mu ,
\]

\[
E_\mu \rightarrow E_\mu - \xi_\mu ,
\]

where prime (\( ' \)) denotes differentiation with respect to \( z \). Notice that there is a gauge invariant combination:

\[
\hat{B}_\mu \equiv B_\mu - E'_\mu .
\]

In a covariant theory, the orthogonal combination can be eliminated with \( \xi_\mu \) using the coordinate transformation \( (7) \), leaving only the three polarizations contained in \( \hat{B}_\mu \).

Lastly, the scalar modes \( \psi, \phi, B, \) and \( E, \) represent four real degrees of freedom. Under the remaining infinitesimal coordinate transformations they transform as:

\[
\psi \rightarrow \psi - \frac{A'}{A} \xi^5 ,
\]

\[
\phi \rightarrow \phi - \xi^{5'} - \frac{A'}{A} \xi^5 ,
\]

\[
B \rightarrow B - \xi' - \xi^5 ,
\]

\[
E \rightarrow E - \xi .
\]

There are now two gauge invariant combinations:

\[
\hat{\psi} \equiv \psi - \frac{A'}{A} (B - E') ,
\]

\[
\hat{\phi} \equiv \phi - \frac{A'}{A} (B - E') - (B - E')' .
\]
These two modes $\hat{\psi}$, and $\hat{\phi}$ represent two polarizations in the case of a covariant theory.

For later convenience, we also define the gauge invariant brane positions. In general, a boundary brane can be at the perturbed position $z_i + \zeta_i$, and, as we shall see, the perturbation $\zeta_i$ couples to the scalar perturbations of the metric. Under the infinitesimal change of coordinates, $\zeta_i \rightarrow \zeta_i + \xi^5$. Hence, we can form the invariant combination:

$$\hat{\zeta} \equiv \zeta + B - E' .$$  \hspace{1cm} (15)

In summary, the perturbations of the metric contain $5 + 6 + 4 = 15$ degrees of freedom in the three sectors, corresponding to the degrees of freedom of a symmetric $5 \times 5$ tensor. In the case of a covariant theory, such as the original Randall-Sundrum model, they are reduced to $5 + 3 + 2 = 10$ polarizations. Not all of them are necessarily dynamical, since (as we will see) some polarizations vanish due to constraint equations that result from the equations of motion (equivalently, they can be seen as Lagrange multipliers in the original action). This parametrization of the perturbations will be also useful when we add mass terms to the action.

### 2.2 Adding a bulk mass term

Let us now consider adding a mass term for the perturbations $h_{AB} \equiv g_{AB} - g_{AB}^{(0)}$, where $g_{AB}^{(0)}$ is the background Randall-Sundrum metric. The 5D bulk action becomes:

$$S = \int d^5x \sqrt{-g} \left[ M^3 R - 2\Lambda - M^3 k^2 g^{(0)MN} g^{(0)AB} (a h_M h_N + b h_M h_{AB}) \right] ,$$  \hspace{1cm} (16)

where $a$ and $b$ are real parameters. The previous classification of the perturbations $h_{AB}$ is useful since modes belonging to different representations are not coupled to each other at the linearized level. This means that we can write the Einstein equations in the three sectors independently. For the covariant case, it is easiest to compute the equations in the form $\delta G^M_N = 0$, where $\delta G$ is the linear perturbation of the Einstein tensor. This equation holds as long as there is only a cosmological constant in the bulk (adding dynamical fields in the bulk gives rise to a nonvanishing $\delta T^M_N$).

If we now include the bulk mass term, then the action (16) leads to the bulk Einstein equations:

$$\delta G^A_B + 2M^3 k^2 \left[ a h_B^A + bh^A_B \right] = 0 ,$$  \hspace{1cm} (17)

where $h_B^A = g^{(0)AC} h_{CB}$ and $h = h_A^A$. Let us now separately consider the nontrivial Einstein equations in each of the three sectors:
2.2.1 Tensor

The equation of motion for the tensor modes $\hat{h}_{\mu\nu}$ is the transverse–traceless part of the $\mu\nu$ component of (17), and is given by:

$$\Box \hat{h}_{\mu\nu} + \hat{h}''_{\mu\nu} + 3 \frac{A'}{A} \hat{h}'_{\mu\nu} - 4 a k^2 A^2 \hat{h}_{\mu\nu} = 0 ,$$

(18)

where $\Box \equiv -\partial^2_t + \partial^2_x$. Notice that this equation does not depend on the $b$ part of the mass term (16) since the tensor mode $\hat{h}_{\mu\nu}$ is traceless. The solution of the equation of motion is obtained by a separation of variables $\hat{h}_{\mu\nu}(x,z) = f(z) H_{\mu\nu}(x)$, where $\Box H_{\mu\nu}(x) = m^2 H_{\mu\nu}(x)$, with $m$ representing the mass of the four-dimensional Kaluza-Klein modes. The massless mode solution is:

$$\hat{h}^{(0)}_{\mu\nu}(x,z) = \left[ C_1 A(z)^{-2(1-\sqrt{1+a})} + C_2 A(z)^{-2(1+\sqrt{1+a})} \right] H^{(0)}_{\mu\nu}(x) ,$$

(19)

while the massive modes are:

$$\hat{h}^{(n)}_{\mu\nu}(x,z) = A^{-2}(z) \left[ C_1 J_2 \left( \frac{m_n}{k A(z)} \right) + C_2 Y_2 \left( \frac{m_n}{k A(z)} \right) \right] H^{(n)}_{\mu\nu}(x) ,$$

(20)

where $C_1, C_2$ are arbitrary constants. We will consider only values $a \geq -1$, which include the Randall-Sundrum case ($a = 0$), and, as we will see, provides a general and interesting phenomenology. In the limiting case ($a = -1$), the massless solutions are degenerate. Also note that in the limit $a \to 0$ these modes become:

$$\hat{h}^{RS,(0)}_{\mu\nu}(x,z) = \left[ C_1 + C_2 A(z)^{-4} \right] H^{(0)}_{\mu\nu}(x) ,$$

$$\hat{h}^{RS,(n)}_{\mu\nu}(x,z) = A^{-2}(z) \left[ C_1 J_2 \left( \frac{m_n}{k A(z)} \right) + C_2 Y_2 \left( \frac{m_n}{k A(z)} \right) \right] H^{(n)}_{\mu\nu}(x) ,$$

(21)

(22)

which, together with appropriate boundary conditions (see below) smoothly reproduce the Randall-Sundrum solution [1].

2.2.2 Vector

The 5$\mu$ and $\mu\nu$ components of (17) lead to the Einstein equations for the vector modes:

$$\Box \hat{B}_\mu - 4 a k^2 A^2 B_\mu = 0 ,$$

(23)

$$\hat{B}_\mu' + 3 \frac{A'}{A} \hat{B}_\mu + 4 a k^2 A^2 E_\mu = 0 .$$

(24)

These equations can be decoupled by eliminating $B_\mu$ using (8) to obtain equations which depend only on $\hat{B}_\mu$ and $E_\mu$. This gives rise to a second order equation solely in terms of $\hat{B}_\mu$:

$$\Box \hat{B}_\mu + \hat{B}''_\mu - k A \hat{B}_\mu' - (4 a + 3) k^2 A^2 \hat{B}_\mu = 0 ,$$

(25)
where (24) can be used to obtain $E_\mu$ from $\hat{B}_\mu$. As for the tensor mode we see that the vector modes do not depend on the $b$ part of the mass term (16) since the vector modes are transverse and do not contribute to the trace of the perturbation in the mass term. The equation (25) is again solved by separating the variables. When $a \neq 0$ the massless mode solutions are:

$$\hat{B}_\mu^{(0)}(x, z) = \left[ C_1 A(z)^{-1+2\sqrt{1+a}} + C_2 A(z)^{-1+2\sqrt{1+a}} \right] b_\mu^{(0)}(x), \quad (26)$$

$$E_\mu^{(0)}(x, z) = -\frac{1}{2a k} \left[ C_1 \left( 1 - \sqrt{1+a} \right) A(z)^{-2(1+\sqrt{1+a})} + C_2 \left( 1 + \sqrt{1+a} \right) A(z)^{-2(1-\sqrt{1+a})} \right] b_\mu^{(0)}(x), \quad (27)$$

while the massive mode solutions are given by:

$$\hat{B}_\mu^{(n)}(x, z) = A^{-1}(z) \left[ C_1 J_2 \left( \frac{m_n}{k A(z)} \right) + C_2 Y_2 \left( \frac{m_n}{k A(z)} \right) \right] b_\mu^{(n)}(x), \quad (28)$$

where $C_1, C_2$ are arbitrary constants and $E_\mu^{(n)}$ are obtained from (24).

We can compare this with the massless case. When $a = 0$ the equations of motion (23) and (24) only depend on $\hat{B}_\mu$. From (23) we see that the vector mode is always massless and since (24) is a first order differential equation there is only one solution given by:

$$\hat{B}_\mu^{RS}(x, z) = C_1 A^{-3}(z) b_\mu(x), \quad (29)$$

where $C_1$ is an arbitrary constant. We will see later that the boundary conditions will eliminate this mode, as is well known for the RS case.

### 2.2.3 Scalar

The scalar equations are obtained from the 55, 5$\mu$ and $\mu\nu$ components of (17). This leads to the following equations involving the scalar modes:

$$\Box \hat{\psi} + 4 \frac{A'}{A} \left( \psi' - \frac{A'}{A} \phi' \right) + \frac{4}{3} k^2 A^2 \left[ a \phi + b \left( 4 \psi + \phi + \Box E \right) \right] = 0, \quad (30)$$

$$\hat{\psi}' - \frac{A'}{A} \phi - \frac{2}{3} a k^2 A^2 B = 0, \quad (31)$$

$$\phi' + 2 \psi' - 4 a k^2 A^2 E = 0, \quad (32)$$

$$\frac{1}{3} \Box \left( \phi + 2 \psi \right) + \phi'' + 3 \frac{A'}{A} \psi' - \frac{A'}{A} \phi' = - \frac{A' \phi}{A^2} - \frac{A'^2 \phi}{A^2}$$

$$+ \frac{4}{3} k^2 A^2 \left[ a \psi + b \left( 4 \psi + \phi + \Box E \right) \right] = 0. \quad (33)$$

Notice that unlike the vector and tensor modes the coefficient $b$ now appears in the scalar equations. This is because the scalar modes do contribute to the trace of the metric perturbations.
When \(a = b = 0\), only the gauge invariant combinations of the metric perturbations appear in the Einstein equations. The system of equations is straightforward to solve and from (31) and (32) we obtain:

\[
\hat{\phi} = -2 \hat{\psi}, \quad \hat{\psi}(x, z) = C_1 A^{-2}(z) S(x), \tag{34}
\]

where \(C_1\) is an arbitrary constant. Hence, as in the vector sector, only one mode is present in the RS case. Using equation (30) this mode (the radion) is massless, \(\Box S(x) = 0\). The remaining equation (33) is degenerate and consistent with the solution (34).

The solution of the system of equations (30)-(33) for the scalar perturbations \(\psi, \phi, B\) and \(E\) when \(a \neq 0\) and \(b \neq 0\) is more involved. As first step we will eliminate \(\psi\) and \(\phi\) in terms of \(\hat{\psi}\) and \(\hat{\phi}\). Thus, using (13), (14), (31), and (32) we obtain:

\[
E = \frac{1}{4a k^2 A^2} \left( \hat{\phi} + 2 \hat{\psi} \right), \tag{35}
\]

\[
B = \frac{3}{2 a k^2 A^2} \left( \hat{\psi}' + k A \hat{\phi} \right), \tag{36}
\]

\[
\psi = \hat{\psi} - \frac{1}{4a k A} \left[ 4 \hat{\psi}' - \hat{\phi}' + 4 k A \left( \hat{\phi} - \hat{\psi} \right) \right], \tag{37}
\]

\[
\phi = \hat{\phi} + \frac{1}{4a k^2 A^2} \left( 4 \hat{\psi}'' - \hat{\phi}'' + 3 k A \hat{\phi}' \right). \tag{38}
\]

Using these equations the remaining two scalar equations, (30) and (33) can then be written in terms \(\hat{\psi}\) and \(\hat{\phi}\). This leads to two coupled second order differential equations which can be solved for \(\hat{\psi}\) and \(\hat{\phi}\). These equations are equivalent to a fourth order differential equation, which is difficult to solve in general. However, the coupled differential equations magically simplify for the “Fierz–Pauli” choice:\(^2\)

\[
b = -a, \tag{39}
\]

of the bulk mass parameters in (16), since the sum of the two coupled equations only involves the combination \(\hat{\psi} - \hat{\phi}\). The orthogonal combination \(\hat{\psi} + \hat{\phi}\) is then most easily obtained from (30). Therefore, defining the linear combinations:

\[
X \equiv \hat{\psi} - \hat{\phi}, \quad Y \equiv \hat{\psi} + \hat{\phi}, \tag{40}
\]

the remaining two scalar equations are:

\[
\Box X + X'' + k A X' - 4 (1 + a) k^2 A^2 X = 0, \tag{41}
\]

\[
Y = \frac{1}{(3 + 4a) k^2 A^2} \left[ 2 k A X' - (5 + 4a) k^2 A^2 X + \frac{1}{2} \Box X \right]. \tag{42}
\]

\(^2\)In fact the major motivation for this choice is provided by the presence of \((\Box E)^2\) in the quadratic action for the scalar perturbations. As in the 4D case [26], these terms cancel for the Fierz–Pauli choice; see Appendix C.
Hence, we have obtained a second order equation in terms of $X$ only. The remaining equation is an algebraic equation for $Y$, which is trivially solved in terms of $X$. It is straightforward to obtain the general solution of the remaining differential equation. Again separating the variables and writing $X(x,z) = f(z)S(x)$ we find that the massless modes solving these equations are:

$$
X^{(0)}(x,z) = \left[ C_1 A(z)^{-2\sqrt{1+a}} + C_2 A(z)^{2\sqrt{1+a}} \right] S^{(0)}(x) ,
$$

$$
Y^{(0)}(x,z) = -\frac{1}{4a+3} \left[ C_1 \left( 2\sqrt{1+a} - 1 \right)^2 A(z)^{-2\sqrt{1+a}} + C_2 \left( 2\sqrt{1+a} + 1 \right)^2 A(z)^{2\sqrt{1+a}} \right] S^{(0)}(x) ,
$$

where $C_1, C_2$ are arbitrary constants and the four dimensional mode obeys $\Box S^{(0)}(x) = 0$. If we now substitute these general solutions back into (35)-(38) we obtain:

$$
E^{(0)}(x,z) = -\frac{1}{2a \left( 3+4a \right) k^2} \left[ C_1 \left( 2a + 3(1-\sqrt{1+a}) \right) A(z)^{-2(1-\sqrt{1+a})} + C_2 \left( 2a + 3(1+\sqrt{1+a}) \right) A(z)^{-2(1+\sqrt{1+a})} \right] S^{(0)}(x) ,
$$

$$
B^{(0)}(x,z) = 0 ,
$$

$$
\psi^{(0)}(x,z) = 0 ,
$$

$$
\phi^{(0)}(x,z) = 0 .
$$

Remarkably, the bulk equations of motion have forced all the massless scalar perturbations to become zero, except for the $E^{(0)}$ mode! Furthermore, as shown in Appendix C this mode gives a vanishing contribution to the action for the perturbations, and therefore is not physical (at least, to quadratic order in the perturbations). Thus, the addition of the bulk mass term with the Fierz–Pauli choice (39) has completely eliminated the massless (radion) mode.

The massive mode solutions are:

$$
X^{(n)}(x,z) = \left[ C_1 J_2 \left( \frac{m_n}{kA(z)} \right) + C_2 Y_2 \left( \frac{m_n}{kA(z)} \right) \right] S^{(n)}(x) ,
$$

where $\Box S^{(n)}(x) = m^2 S^{(n)}(x)$ and $Y^{(n)}(x,z)$ is obtained from (42). As we shall see, also these modes disappear, once the boundary conditions are taken into account.

### 3 Brane localized mass terms

#### 3.1 Boundary conditions

In order to determine the mass spectrum from the general solutions we need to derive the boundary conditions satisfied by the bulk modes on the branes. We will...
also add boundary mass terms on the branes since this will be crucial for obtaining a deformation of the RS solution. Hence, consider the following brane action at the location $z_i$:

$$\Delta S_i = -k M^3 \int d^4x \sqrt{-\gamma_0} h_{\mu\nu} h_{\alpha\beta} \left( \alpha_i \gamma_{0\alpha}^{\mu} \gamma_{0\beta}^{\nu} + \beta_i \gamma_{0\alpha}^{\mu} \gamma_{0\beta}^{\nu} \right), \quad (50)$$

where $\gamma_{0\mu\nu} = A^2 \eta_{\mu\nu}$ is the background induced metric on the boundary, with $A$ evaluated at the (unperturbed) location of the brane, and $h_{\mu\nu}$ are the perturbations of the induced metric,

$$\gamma_{\mu\nu} = \gamma_{0\mu\nu} + h_{\mu\nu}. \quad (51)$$

The total induced metric $\gamma$ is evaluated at the perturbed brane position $z_i + \zeta_i (x^\mu)$. The brane displacements $\zeta_i$ constitute two additional scalar modes of the system which have support only on the two boundaries. It is worth noting that the action $\Delta S_i$ is not the unique boundary action which one could choose. This term itself is not covariant, and there are several inequivalent possibilities (as we discuss below in Section 3.4). Our choice was motivated by the fact that $\Delta S_i$ is rather simple and natural.

The boundary mass terms give a contribution to the energy-momentum tensor on the boundary:

$$\delta S_{\mu\nu} = -\frac{2}{\sqrt{-\gamma_0}} \frac{\delta \Delta S_i}{\delta h_{\mu\nu}} = -4k M^3 (\alpha h_{\mu\nu} + \beta h \gamma_{0\mu\nu}), \quad (52)$$

where to linear order in the perturbations:

$$h_{\mu\nu} = 2A^2(z_i) \left[ \left( \psi + \frac{A'(z_i)}{A(z_i)} \zeta_i \right) \eta_{\mu\nu} + E_{\mu\nu} + \frac{1}{2} \left( E_{\mu\nu} + \tilde{h}_{\mu\nu} \right) \right], \quad (53)$$

and $h = h^\mu_\mu = \gamma^\mu_0 \gamma^\mu_\mu$, with $\gamma^\mu_0$ the inverse background induced metric. This contribution must be added to the standard RS piece arising from the brane tensions, namely $S^{(0)}_{\mu\nu} = -\lambda_i \gamma_{\mu\nu}$, where $\lambda_i = \pm 6k M^3$ and $\pm$ refers to the UV/IR brane, respectively. Hence the total energy–momentum tensor is:

$$S_{\mu\nu} = S^{(0)}_{\mu\nu} + \delta S_{\mu\nu}. \quad (54)$$

This expression is related to the jump of the extrinsic curvature $K_{\mu\nu}$ across each brane. Note that the extrinsic curvature is computed from bulk quantities, and formally it is not affected by the bulk mass term in (16). The extrinsic curvature, up to first order in the perturbations, is given by:

$$K_{\mu\nu} = \nabla_\mu n_\nu = \partial_\mu n_\nu - \Gamma^5_{\mu\nu} n_5 = \partial_\mu n_\nu - \Gamma^5_{\mu\nu} A \left( 1 + \phi + \frac{A'}{A} \zeta_i \right). \quad (55)$$
Evaluating it from the bulk geometry gives:

$$\begin{align*}
K_{\mu\nu} &= A' \eta_{\mu\nu} + \delta K_{\mu\nu}, \\
\delta K_{\mu\nu} &= A \left\{ \left( \psi' + \frac{2A'}{A} \psi - \frac{A'}{A} \phi + \frac{A''}{A} \zeta_i \right) \eta_{\mu\nu} + \left( E' + \frac{2A'}{A} E - B - \zeta_i \right) \right\}_{,\mu\nu} \\
&+ \left( \frac{1}{2} E'_{(\mu} + \frac{A'}{A} E_{(\mu} - \frac{1}{2} B_{(\mu}} \right)_{,\nu} + \left( \frac{1}{2} \tilde{h}'_{\mu\nu} + \frac{A'}{A} \tilde{h}_{\mu\nu} \right) \right\}. \tag{56}
\end{align*}$$

The junction conditions are then:

$$M^2 \left[ \hat{K}_{\mu\nu} \right] = -S_{\mu\nu}, \tag{57}$$

where $\hat{K}_{\mu\nu} = K_{\mu\nu} - Kh_{\mu\nu}$ and $[\ldots]$ means the jump across the brane (with $Z_2$ symmetry imposed). Evaluating the junction conditions explicitly in the three sectors gives:

**Tensor:**

$$\hat{h}'_{\mu\nu} = \pm 4 \alpha_i k A \hat{h}_{\mu\nu}, \tag{58}$$

**Vector:**

$$B_\mu = \mp 4 \alpha_i k A E_\mu, \tag{59}$$

**Scalar:**

$$\psi' - \frac{A'}{A} \phi = \pm 4k A \left[ \alpha_i \left( \psi + \frac{A'}{A} \zeta_i \right) - \frac{\alpha_i + \beta_i}{3} \left( 4 \left( \psi + \frac{A'}{A} \zeta_i \right) + \Box E \right) \right], \tag{60}$$

$$E' - B - \zeta_i = \pm 4 \alpha_i k A E, \tag{61}$$

These expressions are to be evaluated at the brane locations $z_0$ (upper sign) and $z_1$ (lower sign). They are valid in general, both for the massless and massive modes, and we will use them to determine the mass spectrum.

In the scalar sector, there is an additional boundary condition, which is obtained directly from varying the quadratic action for the scalar perturbations with respect to the brane displacement $\zeta_i$ (see Appendix C for details). When the mass terms are absent, this equation is actually redundant with respect to the boundary conditions given above. This is not surprising since it is a consequence of general covariance of the model. In the present case, the inclusion of the mass terms removes this degeneracy, leading to the additional boundary condition. For $b = -a \neq 0$, and $\beta_i = -\alpha_i \neq 0$, the additional boundary condition can be rewritten as:

$$\text{Scalar} : \quad 4 \psi + \Box E = 0, \tag{62}$$

which, like the previous boundary conditions, only holds at the two boundaries.

### 3.2 Tensor modes

Applying the boundary condition (58) to the tensor mode general solution (19) gives:

$$\left( 1 - \sqrt{1 + a \mp 2\alpha_i} \right) C_1 A(z_i)^{2\sqrt{1 + a}} + \left( 1 + \sqrt{1 + a \mp 2\alpha_i} \right) C_2 A(z_i)^{-2\sqrt{1 + a}} = 0, \tag{63}$$

12
Figure 1: The range of bulk (\(a\)) and brane (\(\alpha\)) mass parameters that lead to a 4D massless graviton. There are two branches, \(\alpha \pm\), joined together at the limiting value \(a = -1\). The RS model (\(a = \alpha = 0\)) is a special case on the \(\alpha -\) branch. Its mirror image on the \(\alpha +\) branch is the “specular” RS solution (\(a = 0, \alpha = 1\)).

where this equation is evaluated at \(z_0\) (upper sign) or \(z_1\) (lower sign). For generic mass parameters \(a, \alpha_i\) the only solution is \(C_1 = C_2 = 0\), and there is no massless graviton. However, when:

\[
\alpha_0 = -\alpha_1 \equiv \alpha , \quad \alpha = \frac{1}{2} (1 \mp \sqrt{1 + a}) \equiv \alpha_{\mp} ,
\]

the coefficient \(C_1 (C_2)\) drops from the boundary condition, and \((63)\) simply gives \(C_2 = 0 (C_1 = 0)\). Hence, the massless tensor mode \((65)\) becomes:

\[
\hat{h}_{\mu\nu}(x, z) = N_T A(z) - 4\alpha H^{(0)}_{\mu\nu}(x) ,
\]

where \(N_T\) is the overall normalization constant which is determined from the quadratic action in the perturbations and cannot be determined by the boundary conditions.

This form of the solution is only meaningful when the bulk mass parameter \(a \geq -1\). This corresponds to \(\alpha = \alpha_+ (\alpha_-)\) for \(\alpha \geq 1/2 (\alpha \leq 1/2)\), so that the full range of the boundary mass parameter \(\alpha\) is covered by the two branches \(\alpha \pm\). This behavior is plotted in Figure 1. When \(a, \alpha_- \to 0\), the bulk and boundary mass terms become zero, and the \(\alpha_-\) mode reduces to the usual RS tensor mode that is constant in the \(z\) coordinate. Hence the \(\alpha_-\) mode is a smooth deformation of the RS tensor mode from \(\alpha = 0\) to \(-\infty < \alpha \leq 1/2\). On the other hand the \(\alpha_+\) mode can only exist when the boundary mass is nonzero and corresponds to the deformation of the RS tensor mode to values \(1/2 \leq \alpha < \infty\). Later, we will see that this corresponds to localizing modes continuously from the UV brane to the IR brane.

---

\[^3\text{Since we restrict ourselves to the Fierz–Pauli choice } \beta_i = -\alpha_i, \text{ we also set } \beta_0 = -\beta_1 \equiv \beta \text{ for the scalar modes.}\]
It is worth highlighting the presence of the massless zero mode for \( a = 0, \alpha_+ = 1 \), arising for a special value of the brane mass parameter, and without a mass term in the bulk. In this case, general covariance is broken only at the location of the two branes, while the 5D bulk is general coordinate invariant. As is clear in Fig. 1, this model is the “specular” version of the RS mode. Indeed, as we have seen, the bulk equation (18) is a second order differential equation, and in particular the value \( a = 0 \) leads to two linearly independent bulk solutions. The choice of \( \alpha_+ = 1 \) leads to the survival of the massless mode that is normally removed by the RS boundary conditions (and vice versa).

For the massive solutions (20) the boundary condition (58) for \( \alpha = \alpha_\pm \) can be written as

\[
\frac{J_{2\sqrt{1+a\pm1}}\left(\frac{m_n}{kA(z_0)}\right)}{Y_{2\sqrt{1+a\pm1}}\left(\frac{m_n}{kA(z_0)}\right)} = \frac{J_{2\sqrt{1+a\pm1}}\left(\frac{m_n}{kA(z_1)}\right)}{Y_{2\sqrt{1+a\pm1}}\left(\frac{m_n}{kA(z_1)}\right)}.
\]

Since \( A(z_0) = 1 \) and \( A(z_1) = e^{-\pi kR} \) we obtain in the limit \( ke^{-\pi kR} \ll m_n \ll k \) the Kaluza-Klein mass spectrum

\[
m_n \simeq \left(n + \sqrt{1 + a} - \frac{3}{4}\right) \pi ke^{-\pi kR}, \quad n = 1, 2, 3, \ldots.
\]

This approximation for the mass spectrum becomes increasingly better as \( n \) grows. When \( a = 0 \) we recover the RS Kaluza-Klein mass spectrum [1].

### 3.3 Vector modes

Since the massless tensor mode only exists for \( \alpha = \alpha_\pm \) we will also assume this value of \( \alpha \) for the vector mode. The boundary condition (59) for the massless vector mode (20) becomes:

\[
C_1(1 + \frac{4}{a} \alpha_\alpha) A(z_i)^{-(1+2\sqrt{1+a})} + C_2(1 + \frac{4}{a} \alpha_\alpha) A(z_i)^{-(1-2\sqrt{1+a})} = 0,
\]

where this condition is imposed at \( z_0 \) and \( z_1 \). When \( \alpha = \alpha_- \), the term proportional to \( C_2 \) in (68) vanishes (since \( \alpha_+ \alpha_- = -a/4 \)), which then simply enforces \( C_1 = 0 \). On the contrary, for \( \alpha = \alpha_+ \) the boundary condition (68) leads to \( C_2 = 0 \). Thus the massless vector mode solution becomes:

\[
\hat{B}_\mu^{(0)}(x, z) = N_V A(z)^{1-4\alpha} b_\mu^{(0)}(x), \quad (69)
\]

\[
E_\mu^{(0)}(x, z) = \frac{N_V}{a k} (1 - \alpha) A(z)^{-4\alpha} b_\mu^{(0)}(x), \quad (70)
\]

where \( N_V \) is a normalization constant and \( \alpha = \alpha_\pm \) for \( \alpha \geq 1/2 (\alpha \leq 1/2) \). Notice that when \( a, \alpha_- \to 0 \) the mode which gets killed, \( C_1 = 0 \), is precisely the solution which would have smoothly gone to the RS vector mode (29). Hence, for \( \alpha \leq 1/2 \)
(including, in particular, the RS limit $\alpha = 0$) this mode is always killed by the $Z_2$ symmetry. However, we see that for nonzero boundary masses $\alpha \neq 0$ there is a new massless mode present, given by (69), that is instead absent in the RS case. Just like the tensor mode, this mode can be localized anywhere in the bulk.

The boundary condition (69) for the massive solutions (28) leads to the same equation (66) as for the graviton tensor modes, and hence to an identical Kaluza-Klein mass spectrum (67). This is not too surprising since both modes originate from the five-dimensional graviton.

### 3.4 Scalar modes

Let us discuss the zero and massive scalar modes separately. For the zero modes, we have seen that the bulk equations enforce $\psi = \phi = B = 0$. The boundary condition (60) then leads to $\zeta_i = 0$ for both branes. This leaves only the scalar mode $E$ which can in principle be nonvanishing. However, an explicit calculation shows that if $E$ is the only mode present, it does not contribute to the quadratic action for the perturbations. Hence we conclude that, at least at the linearized level, there are no massless scalar modes in the theory.\(^4\)

For the massive modes, there are three nontrivial boundary conditions (60), (61), and (62). Since each equation holds on both branes, we have a system of six equations in four variables for each massive mode. The variables are the displacements of the two branes $\zeta_{0,1}$ and from the mode (49), the ratio $C_1/C_2$ and the mass $m_n$. Since this is an overdetermined system of equations we can only hope to have nontrivial massive scalar modes if some of these equations are degenerate.

To check if there is a degeneracy first note that, after some algebra, Eq. (62) forces the mode $X^{(n)}$ in (49) to vanish at both branes. The remaining equations can then be combined to eliminate $\zeta_i$, and together with (62), they also force $X'$ to vanish at the two branes. Thus there is no degeneracy and the requirements of:

\[
X^{(n)} = X^{(n)'} = 0 , \quad \text{at both boundaries} ,
\]

can only be satisfied for $C_1 = C_2 = 0$. This immediately implies that the massive scalar modes are also absent.

The absence of massive scalar modes is due to a “mismatch” between the bulk and boundary mass terms that we have introduced. Both terms can be considered as massive deformations of the original Randall-Sundrum proposal. We have previously shown that the deformation (52) of the boundary action “matches” with the deformation (16) of the bulk action to produce nontrivial tensor and vector modes. However, we now see that this is not the case for the scalar sector. It is possible that some inequivalent choice for the boundary actions can also accommodate nonvanishing scalar modes. As we already mentioned, the action (52) is not covariant, so other

\(^4\)We have verified that for the “specular” solution, $a = 0, \alpha = 1$, where the calculation is slightly different, there is also no scalar mode in the spectrum.
inequivalent actions can be considered. This is particularly relevant for the scalar modes, which have the additional ambiguity (with respect to the other sectors) of the brane positions. For instance, the perturbation of the induced metric in (52) is defined as:

$$h_{\mu\nu} = \gamma_{\mu\nu} - A^2(z_i) \eta_{\mu\nu},$$

(72)

where $\gamma_{\mu\nu}$ is the (total) induced metric, with scalar perturbations included. In this way, the background induced metric is evaluated at the unperturbed brane positions $z_{0,1}$. At linear order, this leads to the expression (53). Alternatively, one can choose to evaluate also the second term in (72) at the perturbed brane positions $z_i + \zeta_i$. This results in omitting the terms proportional to $\zeta_i$ in the expression (53). As a consequence, eq. (60) also appears without the two terms proportional to $\zeta_i$, while clearly the vector and tensor sectors are instead unaffected by this choice. We have verified that this choice also leads to vanishing scalar modes. Another possibility would be not to perturb the brane positions at all. This then leaves two fewer degrees of freedom, together with two fewer equations (i.e. Eq. (62) evaluated at the two branes). Again we have verified that in this case the remaining equations are not degenerate, so that no scalar modes are present.

In summary, all the inequivalent choices for the brane mass terms that we have considered lead to vanishing scalar modes. Although these choices appear as the most natural possibilities, clearly we have not exhausted all the possible choices. Hence, we cannot rule out the possibility that some other brane term can lead to nonvanishing modes also in the scalar sector.

### 4 Mode properties

#### 4.1 Localization of the zero modes

The existence of zero modes follows from the relation (64) between the bulk and boundary mass parameters. However, this relation only fixes one of the parameters, say $a$, so that there is still freedom to choose $\alpha$. Since the wavefunction depends on $\alpha$ we can arbitrarily localize the zero modes anywhere in the bulk. This is similar to previous studies [9] involving fields with spin < 2. Consider the effective four-dimensional action for the tensor modes. In terms of the RS variables $y$ we obtain:

$$S_{eff} = -\frac{1}{4} M^2 N^2 T \int d^4x \int_0^{R} dy \, e^{-2(1-4\alpha)ky} \partial_\rho H^{(0)}_{\mu\nu}(x) \partial_\rho H^{(0)}_{\mu\nu}(x) + \ldots ,$$

(73)

where we have used the solution (65) in the effective bulk action at quadratic order (see Appendix [B] note that indices in (73) and (75) are raised with $\eta_{\mu\nu}$). From (73) we see that the tensor zero mode wavefunction $f_T^{(0)}$ has a $y$ dependence:

$$f_T^{(0)}(y) \propto e^{-(1-4\alpha)ky}.$$  

(74)
When $\alpha = 0$ we obtain the RS tensor mode localized on the UV brane. However we now see that by varying $\alpha$ we can smoothly deform the tensor mode to be localized anywhere. For $\alpha < 0$ the mode becomes even more localized on the UV brane, compared to the original RS scenario. On the other hand, for $\alpha > 0$ the tensor mode is delocalized away from the UV brane towards the IR brane. The transition occurs for $\alpha = 1/4$ where the tensor mode is completely flat. As $\alpha$ becomes larger than 1/4 the tensor mode becomes more and more localized on the IR brane. Hence, we have a continuous deformation of the original RS tensor mode from being completely localized on the UV brane to being completely localized on the IR brane!

Similarly for the vector mode we obtain the effective action:

$$S_{\text{eff}} = -\frac{1}{4}M^3N^2 \int d^4x \int_0^{\pi R} dy \, e^{-4(1-2\alpha)ky} F^{(0)}_{\mu\nu}(x) F^{(0)}_{\mu\nu}(x) + \ldots ,$$

(75)

where we have used (69) in the quadratic bulk action and $F^{(0)}_{\mu\nu}(x) = \partial_{\mu} b^{(0)}_{\nu}(x) - \partial_{\nu} b^{(0)}_{\mu}(x)$. Therefore, the vector zero mode has the wavefunction dependence:

$$f^{(0)}_{V}(y) \propto e^{-2(1-2\alpha)ky} .$$

(76)

Again we see that the vector mode can be localized anywhere in the bulk. In particular when $\alpha = 1/2$ the vector mode is flat. Note that there is a difference between the tensor and vector mode wavefunction dependence so that these modes are never localized at the same place.

### 4.2 4D Planck mass

The 4D (reduced) Planck mass ($M_P$) is defined by assuming that there is matter on the branes described by an energy-momentum tensor $T^{(i)}_{\mu\nu}$:

$$S_{\text{matter}} = \int d^4x \, N_T \, A^{-4\alpha(z_i)} T^{(i)\mu\nu} H^{(0)}_{\mu\nu}(x) ,$$

$$\equiv \int d^4x \, \frac{1}{M_P} \, T^{(i)\mu\nu} \tilde{H}^{(0)}_{\mu\nu}(x) ,$$

(77)

where $\tilde{H}^{(0)}_{\mu\nu}(x)$ is the canonically normalized tensor zero mode. Thus, the four dimensional Planck mass is:

$$M^2_P = \frac{M^3}{k} \frac{A^{8\alpha(z_i)}}{(4\alpha - 1)} \left[ e^{2(4\alpha-1)\pi kR} - 1 \right] .$$

(78)

When $\alpha = 0$ this expression reduces to the RS result [1]:

$$M^2_P \simeq \frac{M^3}{k} ,$$

(79)
assuming $\pi k R \gg 1$. In particular notice that the RS result does not depend on where the matter is localized.

However, when $\alpha \neq 0$ the Planck mass depends on whether matter is localized on the UV brane or IR brane. For matter on the UV brane, $A(z_0) = 1$, and we obtain:

$$M_P^2 = \frac{M^3}{k(4\alpha - 1)} \left[ e^{2(4\alpha - 1)\pi k R} - 1 \right] \simeq \begin{cases} 
\frac{M^3}{k(4\alpha - 1)} e^{2(4\alpha - 1)\pi k R} & \alpha > \frac{1}{4}, \\
M^3 \pi k R & \alpha = \frac{1}{4}, \\
\frac{M^3}{k(1-4\alpha)} e^{-8\alpha \pi k R} & \alpha < \frac{1}{4}, 
\end{cases} \quad (80)
$$

where we have assumed $\pi k R \gg 1$ in the approximate expressions. These expressions are consistent with the behavior of the graviton wavefunction. In particular when $\alpha < 1/4$ the Planck mass is consistent with the fact that the graviton wavefunction is localized on the UV brane and there is no exponential expression. The $\alpha = 1/4$ result follows from the fact that in this limit the graviton wavefunction is flat and we recover the flat space result [27]. For $\alpha > 1/4$ the graviton is localized towards the IR brane and the coupling to matter in the UV brane is exponentially suppressed.

Instead when matter is localized on the IR brane, $A(z_1) = e^{-\pi k R}$, we obtain:

$$M_P^2 = \frac{M^3}{k(4\alpha - 1)} \left[ e^{-2\pi k R} - e^{-8\alpha \pi k R} \right] \simeq \begin{cases} 
\frac{M^3}{k(4\alpha - 1)} e^{-2\pi k R} & \alpha > \frac{1}{4}, \\
M^3 \pi R e^{-2\pi k R} & \alpha = \frac{1}{4}, \\
\frac{M^3}{k(1-4\alpha)} e^{-8\alpha \pi k R} & \alpha < \frac{1}{4}, 
\end{cases} \quad (81)
$$

again assuming $\pi k R \gg 1$ in the approximate expressions. These expressions are consistent with the localization properties of the graviton, except that now there is a nontrivial warp factor for the matter on the IR brane.

An interesting phenomenological scenario occurs when the graviton is localized on the IR brane and all the standard model matter is located on the UV brane. If we associate the IR brane with the millimeter scale $10^{-3}$ eV, then for $M \sim k \sim$ TeV, we obtain the usual Planck mass $M_P$ for $\alpha = 1/2$. The scale of the UV brane is the TeV scale. In the bulk the tensor mode has a profile $e^{ky}$, and it is therefore localized away from the UV brane. This explains the weakness of gravity with respect to the gauge interactions. One can also verify that the same exponential suppression also holds for the coupling of the massive tensor modes. The vector zero mode is instead flat in the bulk. However, the vector mode does not couple to brane sources with a conserved energy-momentum tensor. Hence, conserved matter on the brane interacts only with the graviton tensor mode.

We will see that the dual picture is particularly interesting since it indicates that the graviton is composite at the millimeter scale, and emerges from the dual CFT as a massless bound state. This emergent gravity picture is not in contradiction with the Weinberg-Witten theorem [18] because as in models of induced gravity, there is a source gravitational background which invalidates the assumptions of the theorem. The corresponding energy-momentum tensor is no longer conserved and can obtain
a (large) anomalous dimension. Since the UV scale is associated with the TeV scale the gauge hierarchy problem is trivially solved (as usual, additional dynamics may be necessary to generate and stabilize the IR scale).

### 4.3 Short range modifications of gravity

We are interested in the gravitational interaction between nonrelativistic matter sources on the UV brane. This interaction is due to the tensor perturbation of the geometry. It includes the contributions from both the zero mode (already studied in Section 4.2 above) and the massive Kaluza-Klein modes. As we have seen in (67), the lowest Kaluza-Klein mass is at the IR scale $m_1 \sim k A(z_1)$. This scale sets the distance above which gravity is standard. We are interested in the gravitational interactions at distances $r$ smaller than this, but still much greater than $1/k$ (so that they can be phenomenologically relevant):

$$\frac{1}{k} \ll r \ll \frac{1}{k A_1},$$

where we have denoted $A_1 \equiv A(z_1)$. The computation is given in Appendix D. In the following we present the final result and discuss it.

The contribution from the Kaluza-Klein modes presents two distinct behaviors depending on whether $\alpha$ is smaller or greater than $1/4$. For either region, we define a positive parameter $\xi \equiv |4\alpha - 1|$ (therefore, the two regions are joined at $\xi = 0$; notice however that we have chosen $\xi$ to be positive in both of them). The gravitational potential is found to be:

$$V(r) \simeq -\frac{\mu}{M_P^2 r} \left[ 1 + \frac{2 \Gamma(2\xi)}{\xi \Gamma^2(\xi)} \frac{1}{(2 k r)^{2\xi}} \right], \quad \xi = 1 - 4\alpha, \quad \alpha < 1/4,$$

$$V(r) \simeq -\frac{\mu}{M_P^2 r} \left\{ \begin{array}{ll} 1 + \frac{2 \Gamma(2\xi)}{\xi \Gamma^2(\xi)} \frac{1}{(2 k A_1 r)^{2\xi}} & , \quad r \gtrsim \frac{1}{k A_1} \\
\frac{2 \Gamma(2\xi)}{\xi \Gamma^2(\xi)} \frac{1}{(2 k A_1 r)^{2\xi}} & , \quad r \lesssim \frac{1}{k A_1} \end{array} \right\}; \quad \xi = 4\alpha - 1, \alpha > 1/4.$$

The $1/r$ term in these two expressions is the contribution from the zero mode, which reproduces the standard Newtonian gravity at large distances. Instead, the second term represents the interaction mediated by the Kaluza-Klein massive modes in each case.

Note that there is a striking phenomenological difference between the two cases, shown in (83) and (84), respectively. Indeed, in the first case $\alpha < 1/4$, the corrections to the Newtonian potential are relevant only at distances $r \lesssim k^{-1}$, that is only at the UV scale.\(^5\) However, in the second case of $\alpha > 1/4$, the gravitational potential

\(^5\)The RS model, characterized by $\alpha = 0$ belongs to this interval; $\alpha = 0$ corresponds to $\xi = 1$, so that we recover the $1/r^3$ correction computed in [28].
is strongly modified already at the much larger (and, possibly, phenomenologically relevant) IR scale, \( r \simeq (A_1 k)^{-1} \). Moreover, below this distance the \( 1/r \) term disappears and the potential is given solely by a power law \( 1/r^{2\xi+1} \). The cancellation of the \( 1/r \) term will be explained in the next section, but it is already clear that the corresponding force is stronger than the usual gravity. So, for \( \alpha > 1/4 \) standard gravity only emerges at infrared distance scales \( r \gtrsim (A_1 k)^{-1} \).

Actually, this behavior is due to the different localization of the zero mode in the two regimes.\(^6\) For \( \alpha < 1/4 \), the zero mode is localized towards the UV brane, and the relative contribution from the Kaluza-Klein modes can be neglected. On the contrary, for \( \alpha > 1/4 \) the zero mode is localized towards the IR brane and the relative contribution of the massive modes significantly increases becoming the dominant contribution for \( r \lesssim (A_1 k)^{-1} \)—the regime indicated in (82). We will see that for \( \alpha > 1/4 \) this behavior is consistent with the graviton being a composite at the IR scale.

Let us briefly examine the experimental consequences of our composite graviton model in the intermediate regime (82). Assuming that the IR (or compositeness) scale is related to the cosmological constant then the IR scale is \( \sim 10^{-3} \) eV. As we have seen above at energies smaller than this scale we have the usual Newton law. The striking experimental signal of our model would be that Newton’s law of gravity changes to a new power law \( r^{-1} \rightarrow r^{-2\xi-1} \) below \( \sim 0.1 \) mm, which could be fractional or even irrational!

### 4.4 The composite graviton and Green’s function analysis

The modifications of Newton’s law and the compositeness of the graviton can also be studied by computing the 5D bulk Green’s function associated with the tensor mode \( \hat{h}_{\mu\nu} \). The formalism for computing 5D propagators in a slice of AdS can be found in Appendix A of Ref. [10]. The Lorentz index structure of \( \hat{h}_{\mu\nu} \) can be neglected, and then the Green’s function of the tensor mode is identical to that of a bulk scalar mode with mass \( \hat{M}_5^2 = 4ak^2 \) and boundary condition \( r = 4\alpha \pm \) (using the notation of Ref. [10]). A straightforward substitution into the general expressions in Ref. [10] then gives rise to the Planck brane-Planck brane Green’s function:

\[
G(x, z_0; x', z_0) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} G_p(z_0, z_0),
\]

where:

\[
G_p(z_0, z_0) = \pm \frac{2\nu k}{p^2} - \frac{1}{p} \left[ \frac{I_{\nu}(pL)K_{\nu+1}(p/k) + K_{\nu}(pL)I_{\nu+1}(p/k)}{I_{\nu}(pL)K_{\nu}(p/k) - K_{\nu}(pL)I_{\nu}(p/k)} \right].
\]

\(^6\)We do not expect the potential to be discontinuous at \( \alpha = 1/4 \). However, the two expressions (83) and (84) become invalid for \( \xi \rightarrow 0 \), since the approximations adopted to derive them break down in this limit. We expect that in this region the exact potential quickly (but smoothly) interpolates between the values given on the two sides.
Here \( p \equiv \sqrt{p^2} \), \( \nu = \pm (4\alpha_{\pm} - 1) \equiv \nu_{\pm} \), \( L^{-1} = kA(z_1) = ke^{-\pi kR} \) is the IR scale, and \( I_{\nu}(z), K_{\nu}(z) \) are the modified Bessel functions. The Green’s function \([86]\) has been written as a sum of two terms \([29]\). The first term represents a part of the zero mode contribution, while the more complicated second term receives contributions from both the zero mode and the Kaluza-Klein tower of massive modes: the zeros of the denominator of the second term at \( p = im_n \) precisely reproduce the Kaluza-Klein mass spectrum \([67]\). Moreover, the contribution from the zero mode is clear from the fact that also this term scales as \( 1/p^2 \) for \( p \to 0 \).

The contribution of the massive states can be simplified by expanding the Green’s function in the 4D momentum \( p \). Since \( k \) is the AdS curvature scale we will assume \( p \ll k \). Consider first the case of \( \alpha < 1/4 \). This corresponds to the localization of the graviton on the UV brane. In the limit of either \( pL \gg 1 \) or \( pL \ll 1 \) the dominant contribution to the Green’s function is always:

\[
G_p(z_0, z_0) \simeq 2(4\alpha - 1) \frac{k}{p^2} + \cdots \simeq -\frac{2M^3}{M_P^2} \frac{1}{p^2} + \cdots ,
\]  

where in the second equality we have substituted for the Planck mass from \([80]\).

Thus, we see that at low energies \( pL \ll 1 \) only the zero mode contributes to the Green’s function and the massive modes are decoupled. Similarly, for \( pL \gg 1 \) the zero mode is again dominant. This behavior is consistent with the gravitational potential \([83]\), and the fact that the tensor zero mode is localized on the UV brane and remains pointlike for all energies \( p \ll k \).

Let us now consider the case where the graviton is localized on the IR brane. Assuming \( \alpha > 1/4 \), we find that in the limit \( pL \ll 1 \):

\[
G_p(z_0, z_0) \simeq -2\xi e^{-2\xi \pi kR} \frac{k}{p^2} + \cdots \simeq -\frac{2M^3}{M_P^2} \frac{1}{p^2} + \cdots ,
\]  

where \( \xi = 4\alpha - 1 \) and the Planck mass has been substituted from \([80]\). Again we see that the zero mode gives the dominant contribution at energies below the IR scale. There is now also an extra volume suppression that is absorbed into the gravitational coupling. On the other hand at energies above the IR scale, when \( pL \gg 1 \), we obtain:

\[
G_p(z_0, z_0) \simeq -\frac{1}{2(1-\xi)k} \left[ 1 - \frac{p^2}{4k^2(\xi - 1)(\xi - 2)} - \left( \frac{p}{2k} \right)^{2\xi - 2} \frac{\Gamma(2 - \xi)}{\Gamma(\xi)} + \cdots \right] ,
\]  

Remarkably, there is now no longer a dominant zero mode contribution since the massive Kaluza-Klein modes cancel the leading \( 1/p^2 \) contribution in \([80]\)! We will

\[\text{Remark}^{7}\text{This expansion assumes } \xi \neq 1/2 . \text{ For } \xi = 1/2 \text{, the analytic terms are not present in the series expansion of } \text{[80].} \]
see that this behavior is consistent with the fact that the graviton is composite at the IR scale $L^{-1}$. When $pL \ll 1$ the graviton is essentially pointlike and contributes in the usual way, but when $pL \gg 1$ the zero mode effectively disappears. Thus standard gravity only appears at energies below the compositeness scale $L^{-1}$ and is seen to emerge from the CFT, whereas above the compositeness scale (or $r \lesssim L$) the dominant CFT contribution changes Newton’s law to a different power law \(84\), which can be fractional and even irrational.

The cancellation of the leading $1/p^2$ term can also be seen in coordinate space for special values of $\alpha$ where the sum over Kaluza-Klein modes can be done analytically. Consider the case of $\nu_\perp = -1/2$ or $\alpha = 3/8$ (recall that $\alpha > 1/4$ corresponds to IR-brane localization). The Green’s function is:

$$G_p(z_0, z_0) = -\frac{1}{p} \coth(pz_1) = -\frac{1}{z_1} \left[ \frac{1}{p^2} + 2 \sum_{n=1}^{\infty} \frac{1}{p^2 + m_n^2} \right], \quad (90)$$

where $m_n = \pi n/z_1$ (note that this, in fact, is the Green’s function for an even field on a flat $S^1/Z_2$ orbifold of size $z_1$, evaluated at one of the fixed points). It is straightforward to check that the $1/p^2$ pole is cancelled for $pz_1 \gg 1$. Alternatively the gravitational potential can be calculated exactly using the expressions \(D.20\) and \(D.23\) in Appendix D. One obtains:

$$V(r) = -\frac{\mu}{M_P^2} \frac{2z_1}{\pi r} e^{-r/2z_1} + \ldots. \quad (91)$$

Note that the Fourier transform of $V$ is proportional to the Green’s function \(90\). For distances $r < z_1 \simeq L$ the series expansion gives:

$$V(r) \simeq -\frac{\mu}{M_P^2} \frac{2z_1}{\pi r^2} + \ldots. \quad (92)$$

This agrees with the gravitational potential \(84\) for $r < L = 1/(kA_1)$.

For $\alpha < 1/4$, one can also analytically verify that the $1/r$ term does not cancel when $\nu_\perp = 1/2$ (or $\alpha = 1/8$). This is obvious because in this case the exact Green’s function is:

$$G_p(z_0, z_0) = -\frac{1}{p^2} - \frac{1}{p} \coth(pz_1), \quad (93)$$

and the extra $-1/p^2$ term compared to \(90\), corresponding to the $1/r$ term in the potential, is no longer cancelled at high energies.

These two simple examples enlighten our gravitational potential calculation for $\alpha > 1/4$, and reveal that when the graviton is localized on the IR brane the usual Newton’s law of gravity only emerges as a low-energy phenomenon. Remarkably, as we will see in the next section, these properties can also be described purely in a 4D dual interpretation.
5 The holographic interpretation

Motivated by the string theory AdS/CFT correspondence, bulk theories in a slice of AdS$_5$ can be given a holographic interpretation as dual to a CFT (at large-$N$ and large 't Hooft coupling) with conformal invariance broken in the IR, coupled to gravity and possibly other fields [5,6,7]. As opposed to the string-theory AdS/CFT, the UV boundary value of bulk fields are not only sources of operators in the dual CFT but acquire their own dynamics due to the presence of a UV cutoff.

The nature of the holographic interpretation of bulk theories in a slice of AdS$_5$ is necessarily speculative; however, it passes some quantitative tests, despite the fact that one usually does not know what the dual CFT is, or even whether it actually exists! Nevertheless, in most cases one is able to give a reasonably convincing picture of the dual 4D strong dynamics, which may be useful as a guide to finding a theory with the desired features. With these caveats in mind, in this section we attempt a quantitative dual 4D description of the mass-deformed gravity in a slice of AdS$_5$ studied in the previous sections.

The study of the 4D dual begins by recalling that the UV-boundary values of the bulk fields are sources of operators in the dual CFT. The classical bulk action, evaluated on solutions of the bulk equations of motion with arbitrary values on the UV brane (and obeying the proper boundary conditions on the IR brane) is interpreted by the stringy AdS/CFT correspondence as being equal to the generating functional of connected Green’s functions of the 4D dual CFT. To continue, we begin with the solution of the bulk equation which obeys the boundary condition on the IR brane:

\[
\hat{H}(p,z) = \hat{H}(p) A^{-2}(z) \left( J_{\nu_+\mp 1}(iq) - Y_{\nu_+\mp 1}(iq) \frac{J_{\nu_+}(iq_1)}{Y_{\nu_+}(iq_1)} \right),
\]

(94)

where \( \nu_\pm = \pm (4\alpha_\pm - 1) \), \( q = \frac{p}{kA(z)} \); \( q_1 = \frac{p}{kA_1} \); \( q_0 = \frac{p}{kA_0} \); \( p^2 = -m^2 \) is the mass-shell condition, and \( \hat{H}(p,z) \) is the 4D Fourier transform of \( \hat{h}(x,z) \). Note that we will omit tensor indices, as we are only concerned here with the transverse-traceless components of the metric perturbation, which obey a scalar equation. The bulk action, which generates Green’s functions in the dual is:

\[
S_{\text{bulk}} = \frac{M^3}{4} \int \frac{d^4p}{(2\pi)^4} \left[ A^3 \hat{H}(p,z)(\hat{H}'(-p,z) - 4\alpha Ak\hat{H}(-p,z)) \right] \bigg|_{z=z_0},
\]

(95)
evaluated on the bulk solution (94). This can be rewritten as:

\[
S_{\text{bulk}} = \frac{M^3k}{4} \int \frac{d^4p}{(2\pi)^4} F(q_0,q_1) \hat{H}(p)\hat{H}(-p),
\]

(96)
where:

\[
F(q_0,q_1) = \mp iq_0 \left[ J_{\nu_+\mp 1}(iq_0) - Y_{\nu_+\mp 1}(iq_0) \right] \frac{J_{\nu_+}(iq_1)}{Y_{\nu_+}(iq_1)} \left[ J_{\nu_+}(iq_0) - Y_{\nu_+}(iq_0) \frac{J_{\nu_+}(iq_1)}{Y_{\nu_+}(iq_1)} \right].
\]

(97)
The dual theory two point function of the operator $\mathcal{O}$ sourced by the bulk field $\hat{H}$, $\langle \mathcal{O}\mathcal{O}(p) \rangle \equiv \int d^4x \ e^{-ipx} \langle T\mathcal{O}(x)\mathcal{O}(0) \rangle$, is contained—up to local counterterms which we will discuss later—in the second derivative, $\Sigma(p)$, of $S_{\text{bulk}}$ with respect to the boundary value of the metric perturbation $A_0^2 \hat{h}$. The correlator is, in various equivalent forms to be used later:

$$
\Sigma(p) = \int d^4x e^{-ipx} \frac{\delta^2 S_{\text{bulk}}}{\delta (A_0^2 \hat{h}(x, z_0)) \delta (A_0^2 \hat{h}(0, z_0))} \\
= \left( \frac{M}{k} \right)^3 \frac{k^4}{2} (\mp iq_0) \frac{J_{\nu_{\pm}}(iq_0)Y_{\nu_{\pm}}(iq_1) - Y_{\nu_{\pm}}(iq_0)J_{\nu_{\pm}}(iq_1)}{J_{\nu_{\pm}+1}(iq_0)Y_{\nu_{\pm}}(iq_1) - Y_{\nu_{\pm}+1}(iq_0)J_{\nu_{\pm}}(iq_1)} \\
= \left( \frac{M}{k} \right)^3 \frac{k^4}{2} q_0 (I_{\nu}(q_0)K_{\nu}(q_1) - I_{\nu}(q_1)K_{\nu}(q_0)) + I_{\nu}(q_1)K_{\nu+1}(q_0) \\
= \frac{M^3}{2A_0^4 G_p(z_0, z_0)},
$$

where $G_p$ is the boundary-to-boundary propagator of (86) divided by $A_0^3$ and with $k$ replaced by $A_0 k$.

The behavior of $\Sigma(p)$ can be studied for momenta $p$ such that $kA_0 \gg p \gg kA_1$, or equivalently, $q_0 \ll 1, q_1 \gg 1$. In this energy regime, the effects of the conformal symmetry breaking (i.e., the IR brane) are completely negligible. The leading nonanalytic piece in $\Sigma(p)$ is then interpreted, by “matching” to the string AdS/CFT correspondence in the $A_0 \to \infty$ limit, as due to the strong dynamics of the dual CFT above the scale of conformal symmetry breaking. On the other hand, the analytic pieces in the correlator, which, in string AdS/CFT, are subtracted away by adding appropriate counterterms, are now interpreted as kinetic (and higher-derivative terms) of the dynamical source field in the holographic dual.

5.1 $\alpha_-$ branch holography

We will first consider the $\alpha_-$ branch of our solution, which is the one continuously connected to the $\alpha = 0$ RS value (see Figure 1). For $\frac{1}{2} > \alpha_- > 0$, or $-1 < \nu_- < 1$, we find:

$$
\Sigma(p) \sim - \left( \frac{M}{k} \right)^3 \frac{k^4}{2} \left( \frac{q_0^2}{2\nu} + q_0^{2\nu+2} \frac{\Gamma(-\nu)}{2^{2\nu+1}\nu\Gamma(\nu)} + \ldots \right),
$$

while for all other values on the $\alpha_-$-branch ($\alpha_- < 0$ or $\nu = \nu_- > 1$):

$$
\Sigma(p) \sim - \left( \frac{M}{k} \right)^3 \frac{k^4}{2} \left[ \frac{q_0^2}{2\nu} \left( 1 + \ldots + cq_0^{2\nu} \right) + q_0^{2\nu+2} \frac{\Gamma(-\nu)}{2^{2\nu+1}\nu\Gamma(\nu)} + \ldots \right],
$$

---

8 Notice that the expression (99) does not contradict (86) since when $0 < \xi < 1$ (or $-1 < \nu_- < 0$) the nonanalytic term dominates in (86), so that the constant term disappears in the inverse. Also, for $\nu_- = -1/2$ there is no analytic term in the series expansion in this regime.
Figure 2: The scaling dimension $\Delta_O$ plotted as a function of the boundary mass parameter $\alpha$. The source coupling to the CFT is irrelevant for $\alpha < 1/4$ and $\alpha > 3/4$, marginal for $\alpha = 1/4$ and $\alpha = 3/4$ and relevant for $1/4 < \alpha < 3/4$. Note also that the RS value at $\alpha = 0$, and the specular RS value at $\alpha = 1$ both have $\Delta_O = 4$.

where $\lfloor 2\nu \rfloor$ denotes the largest integer smaller than $2\nu$. In each case (99), (100), we have included the leading analytic piece as well as all terms up to the leading nonanalytic piece, $q_0^{2\nu+2}$. The power of $q_0$ in the nonanalytic piece indicates that the scaling dimension $\Delta_O$ of the operator $O$—the energy momentum tensor of the dual theory—sourced by the metric perturbation $h$ is:

$$\Delta_O = 3 + \nu = 4 - 4\alpha_- , \quad (101)$$

on the $\alpha_-$ branch. The leading analytic piece in (99), (100) indicates that there is a kinetic term for the metric perturbation in the dual theory.

Thus, the holographic description of this branch is that of a metric fluctuation $\hat{h}_{\mu\nu}$ coupled to $T^{\mu\nu}_{CFT}$ of scaling dimension $4 - 4\alpha_-$. Note the unusual fact that the energy momentum tensor of the CFT has an anomalous dimension. This, however, is required if the 4D dual is to evade the Weinberg-Witten theorem—which assumes Poincare invariance, broken here by the presence of a nontrivial background metric, as in theories of induced gravity. Notice also, from (101), that the scaling dimension
of $T_{CFT}^{\mu\nu}$ can be as low as $2$, for $1/2 > \alpha_+ > 0$, as can be seen in Fig. 2. A scaling behavior of the CFT with such a large anomalous dimension should persist no matter how small the breaking of Poincare invariance (or, equivalently, the deviation of the metric background from Minkowskian)! Leaving aside the issue of existence CFTs with such behavior, we continue with our attempt at giving a (semi-)quantitative picture of the dual dynamics.

5.1.1 The dual theory and its dynamics

The Lagrangian of our dual theory is, then, at a UV scale $\sim k$, with a canonically normalized metric perturbation:

$$L_{UV} = \epsilon_{\nu} \frac{1}{4} h_{\mu\nu} \Box h^{\mu\nu} + \frac{\lambda_{UV}}{k} h_{\mu\nu} T_{CFT}^{\mu\nu} + \frac{\lambda_{UV}}{k} h_{\mu\nu} T_{\text{matter}}^{\mu\nu} + L_{CFT},$$

(102)

where $\epsilon_{\nu} = \text{sign } \nu$ and $\lambda_{UV} = |\nu|^{1/2} (M/k)^{-3/2}$. We have included the coupling to observable matter fields (UV-brane localized in the gravity dual).9

From eqn. (102), taking into account the anomalous scaling dimension of $T_{CFT}$ from (101), we conclude that the coupling of the metric perturbation to the CFT energy momentum tensor is relevant for $\alpha_+ > 1/4$, marginal if $\alpha_+ = 1/4$, and irrelevant for $\alpha_+ < 1/4$. Introducing a renormalization scale $\mu$, the dimensionless coupling is then $\lambda(\mu) \equiv (\mu/k)^{1-4\alpha_+} \lambda_{UV}$ and satisfies the RGE:

$$\mu \frac{d\lambda}{d\mu} = -(4\alpha_+ - 1)\lambda + \ldots$$

(103)

where the first term is a result of dimensional analysis and higher order terms due to the CFT’s interactions have been neglected.

If $\lambda$ is relevant, i.e. $\alpha_+ > 1/4$, the solution of the RGE:

$$\lambda_{IR} = \lambda_{UV} \left( \frac{k}{m_{IR}} \right)^{4\alpha_+ - 1},$$

(104)

indicates that the coupling of $h_{\mu\nu}$ to the CFT is enhanced in the IR. The conformal invariance is broken at the IR scale $m_{IR}$ and we expect that integrating out the CFT dynamics at the IR scale will induce a kinetic term for $h_{\mu\nu}$. From the CFT point of view the dynamics is both strong and unknown; however, it is clear (see, e.g., [30]) that producing a kinetic term requires two insertions of the dimensionless coupling $\lambda_{IR}$, as indicated in eqn. (106) below.

Clearly, in the weakly coupled gravity dual we can directly compute this contribution by calculating the two point function $\Sigma$ in the IR limit $p \ll k A_1$. The pure

---

9Note that in the case of irrelevant coupling, the sign of the kinetic term for $h_{\mu\nu}$ is the proper one, as $\epsilon_{\nu} = 1$ for $\alpha_+ < 1/4$; while it has the wrong sign in the case of relevant coupling; we will see below that the leading contribution to the $h_{\mu\nu}$ kinetic term of the right sign arises from IR physics not accounted for in (102).
IR contribution is obtained by subtracting the analytic piece of $\Sigma$ that arose in the limit $p \gg kA_1$, see (99). Thus, expanding (98) for small $q_0$ and $q_1$ and subtracting the analytic term already accounted for in (99) (and in the kinetic term in (102)) leads to the pure IR contribution to the correlator:

$$
\Sigma(p)_{IR} \simeq \left( \frac{M}{k} \right)^3 \frac{k^4}{2} \left( A_1^{2\nu} \frac{q_0^2}{2\nu} + \ldots \right),
$$

where $A_1 = m_{IR}/k$. Thus, the effective Lagrangian describing the long-wavelength fluctuations of $h_{\mu\nu}$ and its coupling to the observable matter sector is given by:

$$
\mathcal{L}_{IR} = \left[ \epsilon_{\nu} - \left( \frac{M}{k} \right)^3 \frac{\lambda_1^2}{\nu} \right] \frac{1}{4} h_{\mu\rho} \Box h^{\mu\rho} + \frac{\lambda_{UV}}{k} h_{\mu\rho} T_{\mu\rho}^{\text{matter}}.
$$

In the relevant case, where $-1 < \nu_- < 0$, our gravity dual calculation shows that the IR contribution to the kinetic term for $h_{\mu\rho}$ has the correct sign and dominates over the ghost-like $\epsilon_{\nu}$ contribution. Hence, using (104) and canonically normalising (106), the coupling of the observable matter to gravity at scales below $m_{IR}$ is:

$$
M_P = \sqrt{\frac{M^3}{k|\nu_-|}} \frac{\lambda_{IR}}{\lambda_{UV}} = \sqrt{\frac{M^3}{k|\nu_-|}} \left( \frac{k}{m_{IR}} \right)^{4\alpha_- - 1}.
$$

Upon identifying the ratio of UV to IR scales with the warp factor,

$$
\frac{k}{m_{IR}} = e^{\pi kR},
$$

Eq. (107) gives $M_P^2 = \frac{M^3}{|\nu_-|k} e^{2\pi kR(4\alpha_- - 1)}$, in agreement with the gravity dual result (80).

Consider next the irrelevant case with $\nu_+ > 0$. In this case the coupling of the metric perturbation to the CFT is irrelevant so that the induced contribution proportional to $\lambda_{IR}$ in (106) is negligible. Thus, in (106) the leading term proportional to $\epsilon_{\nu}$ dominates (and has the right sign!), leading to $M_P = k/\lambda_{UV}$, which precisely equals the last line in Eq. (80) as well as the RS result.

Finally, consider the case where the coupling of CFT to the background metric is marginal ($\nu_- = 0$). We have to take into account higher order terms in (103), which we can calculate using the weakly coupled gravity description. In the IR limit $p \ll kA_1$ we obtain:

$$
\Sigma(p)_{IR} = - \left( \frac{M}{k} \right)^3 \frac{k^4}{2} \left( q_0^2 \log \frac{A_0}{A_1} + \ldots \right),
$$

so that the IR Lagrangian becomes:

$$
\mathcal{L}_{IR} = \left( \frac{M}{k} \right)^3 \log \left( \frac{A_0}{A_1} \right)^2 \frac{1}{4} h_{\mu\rho} \Box h^{\mu\rho} + \frac{1}{k} h_{\mu\rho} T_{\mu\rho}^{\text{matter}},
$$

27
where $A_0/A_1 = k/m_{IR}$. Thus canonically normalising the kinetic term and using (108) leads to a Planck mass:

$$M_P^2 = k^2 \left( \frac{M}{k} \right)^3 2 \log \frac{A_0}{A_1} = M^3 2\pi R .$$

(111)

This again agrees with the corresponding result in (80).

Thus, we have a rather unusual "theory," particularly in the case of a relevant coupling of the CFT to gravity. We have two important hierarchical scales of nature, for definiteness take them to be meV ($m_{IR}$) and TeV ($k$), as in our discussion after Eq. (80). The infrared scale is presumably determined by some dynamical mechanism—unspecified both in the 5D gravity and the 4D CFT descriptions—from the UV scale. The TeV scale is the cutoff of the theory, where a more fundamental description takes over. Naturally, in the UV theory, a Newton constant $G_N$ of order TeV$^{-2}$ is expected (the first term in (102), possibly including additional bare contributions). Along the RG flow to lower and lower scales, the hidden CFT sector becomes stronger and remains so until, at the meV scale, conformal invariance is broken in the strongly coupled CFT. Thus, the meV scale "broken CFT" induces a Newton constant. It is strong enough that despite the fact that it operates at meV scales, the induced Planck scale is hierarchically larger: $M_P^2 \gg$ TeV$^2$! In other words, gravity is so weak because of the strength of the hidden CFT over a large interval of scales.

While this "scenario" sounds really unusual, and we are not aware of a CFT with the desired properties, it is not so difficult to come up with a weak coupling—in fact, free field theory—model of this phenomenon. Consider $N$ free fields (of whatever nature, so long as they induce the correct sign Newton constant), coupled only to gravity, with a characteristic mass scale $\sim$meV. Above the meV scale, this hidden theory is conformal, which protects the UV modes of these fields from generating a $G_N$. But conformal invariance is broken at the meV scale and so one expects a contribution to $M_P^2$ which will be of order $N \times$meV$^2$. Now if we take $N = M_P^2/\text{meV}^2$, then this clearly dominates the TeV$^2$ contribution of ordinary massive matter.

As far as the evolution of the universe, our picture would predict that the strength of gravity should change from TeV to $M_P$ during cosmological evolution; note that this does not affect BBN as the transition to "normal" strength gravity occurs before nucleosynthesis, when the Hubble size is of order meV$^{-1}$. There may, however, be relevant consequences for the physics of the earlier universe, as for instance inflation and baryogenesis.\textsuperscript{10}

\subsection*{5.1.2 The gravitational potential in the dual theory}

Let us now describe the leading correction to Newton’s law at intermediate distances, $r < 1/(kA_1)$, from the point of view of the dual interpretation. In the irrelevant case

\textsuperscript{10}It should be clear that we have nothing to say about the cosmological constant.
Figure 3: The Feynman diagrams in the 4D dual theory responsible for the gravitational potential corrections (which need to be summed up for the case of a relevant coupling). The source field \( h_{\mu\nu} \), indicated by a wavy line, interacts with the CFT contribution, indicated by the blob.

\( (\nu_- > 0) \), the coupling of matter to the CFT can be treated perturbatively and the leading correction arises from a single insertion of the CFT correlator and two insertions of the source field, as in the RS case (see Figure 3). The leading and first subleading contribution to the Newton potential is:

\[
V(r) = -\mu \frac{\lambda^2_{UV}}{k^2} \int \frac{d^3 p}{2\pi^2} e^{ipx} \left( \frac{1}{p^2} - \frac{\lambda^2_{UV} \langle \mathcal{O}\mathcal{O} \rangle(p)}{k^2 p^4} \right),
\]

where:

\[
\langle \mathcal{O}\mathcal{O} \rangle(p) = -\left( \frac{M}{k} \right)^3 k^4 \left( \frac{p}{2k} \right)^{2\nu+2} \frac{4\Gamma(-\nu)}{\nu \Gamma(\nu)},
\]

is the nonanalytic piece of \( \Sigma(p) \) of (99), interpreted as the CFT correlator at the relevant energy scale, and \( \lambda_{UV} \) is given after (102). A Fourier transform is performed by using the properly regulated and normalized 3d Fourier transform of \( p^\alpha \), which is (31):

\[
\int \frac{d^3 p}{2\pi^2} e^{ipx} p^\alpha = \left( \frac{2}{r} \right)^{3+\alpha} \frac{\Gamma(\frac{\alpha+3}{2})}{2\sqrt{\pi} \Gamma(-\frac{\alpha}{2})}.
\]

After using (31), and various gamma-function identities,\(^{11}\) we find precisely our result from the gravity calculation (83).

Consider next the correction, at \( r < 1/(kA_1) \), for the case when the interaction with the CFT is relevant (\( \nu_- < 0 \)). Then, we have to sum the chain of bubble graphs as indicated below (recall \( \epsilon_\nu = -1 \) now):

\[
V(r) = -\mu \frac{\lambda^2_{UV}}{k^2} \int \frac{d^3 p}{2\pi^2} e^{ipx} \left[ \epsilon_\nu - \frac{\lambda^2_{UV} \langle \mathcal{O}\mathcal{O} \rangle(p)}{k^2 p^2} + \epsilon_\nu \frac{\lambda^2_{UV}}{k^4} \left( \frac{\langle \mathcal{O}\mathcal{O} \rangle(p)}{p^2} \right)^2 - \cdots \right],
\]

\[
= \mu \frac{\lambda^2_{UV}}{k^2} \int \frac{d^3 p}{2\pi^2} e^{ipx} \frac{1}{p^2 - \frac{\lambda^2_{UV} \langle \mathcal{O}\mathcal{O} \rangle(p)}{k^2}} \approx -\mu \int \frac{d^3 p}{2\pi^2} e^{ipx} \frac{1}{\langle \mathcal{O}\mathcal{O} \rangle(p)},
\]

\( ^{11} \)A useful identity is: \( \Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma(x + \frac{1}{2}) \). Note also that a factor of \( 1/(4\pi) \) has been absorbed in the definition of \( V \).
where we notice that for the distance scales of interest the CFT correlator dominates over \( p^2 \) in the denominator, as appropriate for a relevant coupling. Finally, computing the Fourier transform as before, and using (80) we again recover precisely the leading term of the potential from Eq. (84) from the gravity side.

5.2 \( \alpha_+ \) branch holography

Let us now consider \( \nu = \nu_+ = 4\alpha_+ - 1 > 1 \). In this case the graviton is always localized on the IR brane. We find, for \( A_0 k \gg p \gg A_1 k \), taking the upper sign in (98):

\[
\Sigma(p) \simeq - \left( \frac{M}{k} \right)^3 k^4 \left[ (\nu - 1) + q_0^2 \frac{1}{4(\nu - 2)} + \ldots + cq_0^{2\nu - 2} \frac{\Gamma(2 - \nu)}{2^{2\nu - 2} \Gamma(\nu - 1)} \right],
\]

indicating that now that the scaling dimension of the operator \( \mathcal{O} \) is

\[
\Delta_\mathcal{O} = \nu + 1 = 4\alpha_+.
\]

Thus when \( \nu_+ > 2 \) the source coupling to the CFT is irrelevant, marginal for \( \nu_+ = 2 \), while for \( 1 \leq \nu_+ < 2 \) the coupling is relevant. This behavior is plotted in Fig. 2.

At low energies \( pL \ll 1 \), on the other hand, the series expansion of the correlator is given by:

\[
\Sigma(p)_{IR} \simeq - \left( \frac{M}{k} \right)^3 k^4 \left[ (\nu - 1) + q_0^2 \frac{1}{4(\nu - 2)} - 4\nu(\nu - 1)^2 \frac{A_1^{2\nu}}{A_0^{2\nu}} \frac{1}{q_0^2} + \ldots \right].
\]

To obtain (118), we first took the large-\( A_0 \) limit in (98), obtaining a power series in \( q_0 \) (of which we kept only the leading terms above), with \( q_1 \)-dependent coefficients, and subsequently expanded these coefficients for small \( q_1 \); a more formal way of obtaining this is by multiplying the entire correlator by \( A_0^{2\nu - 2} \) and taking the large \( A_0 \) limit (see footnote). Remarkably we see that the correlator now has a pole at \( p^2 = 0 \). Thus at low energies we can interpret the massless graviton to be predominantly a composite of the CFT. This is in contrast to the \( \alpha_- \) branch where no such pole exists. Similar pole structures have also been identified in Refs. [6, 11, 32].

\footnote{Note that (118) does not contradict (85), which asserts, instead, that \( \Sigma^{-1}(p) \sim G(p) \simeq 1/p^2 \) for \( p \ll L^{-1} \). In (118) the pole in \( \Sigma \) at \( p^2 = 0 \) appears only if the UV cutoff is taken larger than the AdS curvature scale; instead in (85), finite cutoff effects (i.e. mixing with the source) wash away the pole. Note that if we formally took \( A_0 \to \infty \) (the UV cutoff of the dual theory), after multiplication by \( A_0^{2\nu - 2} \) to isolate the leading nonanalytic piece in (116), the persistence of the pole in (118) would run afoul of the Weinberg-Witten theorem. Our dual field theory (119), however, is coupled to gravity perturbed by the graviton mass, hence the cutoff cannot be taken large (and the source decoupled), due to the strong coupling problem of massive gravity [26].}
In addition, we immediately see from (118) that the leading analytic piece is a constant, and corresponds to a mass term of order the curvature scale for the source field $h_{\mu\nu}$. The interpretation of this fact in the dual 4D theory is that the CFT generates a mass for the source so that it decouples at low energy and the propagating mode is predominantly the composite graviton (see also the calculation of the long-distance gravitational potential (121)).

The analytic terms of (118) can be used to obtain the long-distance Lagrangian:

$$L_{IR} = \frac{1}{4} h_{\mu\rho}(\Box - m_h^2) h^{\mu\rho} + \frac{\chi}{k} h_{\mu\rho} T_{CFT}^{\mu\rho} + \frac{X}{k} h_{\mu\rho} T_{\text{matter}}^{\mu\rho} + L_{CFT},$$

(119)

where $\chi = (\nu - 2)^{1/2}(M/k)^{-3/2}$ and $m_h^2 = 4(\nu - 1)(\nu - 2)k^2$. If we now write the small-momentum expansion of the correlator as:

$$\langle \mathcal{O}\mathcal{O} \rangle \simeq (Mk)^3 16\nu_+ (\nu_+ - 1)^2 A_1^{2\nu_+} \frac{1}{p^2},$$

(120)

where $A_0 = 1$, then the leading contribution to the gravitational potential at large distances is given by:

$$V(r) \simeq -\mu \frac{\chi^2}{k^2} \int \frac{d^3p}{2\pi^2} e^{ip\cdot x} \frac{\chi^2}{k^2} \langle \mathcal{O}\mathcal{O} \rangle(p) m_h^4,$$

$$= -\mu \frac{\chi^4 M^3}{m_h^4} 16\nu_+ (\nu_+ - 1)^2 A_1^{2\nu_+} \int \frac{d^3p}{2\pi^2} e^{ip\cdot x} \frac{1}{p^2},$$

$$= -\frac{\mu}{M_P^2 r},$$

(121)

where the source propagator has been approximated by $1/(p^2 + m_h^2) \simeq 1/m_h^2$ and the Planck mass is given by (using the values for $m_h$ and $\chi$ given after (119)):

$$M_P^2 = \left(\frac{M}{k}\right)^3 \frac{k^2}{\nu_+} A_1^{2\nu_+} = \frac{M^3}{k(4\alpha_+ - 1)} e^{(8\alpha_+ - 2)\pi kR}.$$

(122)

This agrees with the Planck mass formula (80) for $\alpha_+ > 1/2$. Note also that further insertions of $\langle \mathcal{O}\mathcal{O} \rangle$ in (121) are negligible at large distances.

In fact, the result for the leading long-distance contribution to the potential (121) indicates that at $r > L$, we can describe long-range physics as due to the exchange of a massless spin-2 field, $h_{\mu\nu}$—mostly a composite of the CFT—coupled directly to matter, thus forgoing the discussion of the massive source in (119). In this case the Lagrangian is given by

$$L_{IR} \simeq \frac{1}{4} \left(\frac{M}{k}\right)^3 \frac{k^2}{\nu} A_1^{-2\nu} \bar{h}_{\mu\rho} \Box h^{\mu\rho} + \bar{h}_{\mu\rho} T_{\text{matter}}^{\mu\rho},$$

(123)

31
where canonically normalising the kinetic term leads to the Planck mass \(80\). The IR lagrangian \(123\) can, equivalently, be obtained by directly considering the IR limit \(\langle q_0 \ll 1, q_1 \ll 1\rangle\) of the correlator \(\Sigma(p)\) of \(98\):

\[
\Sigma(p)_{IR} \simeq - \left( \frac{M}{k} \right)^3 \frac{k^4}{2} \left[ (A_1^{-2\nu_+} - 1) \frac{q_0^2}{2\nu_+} + \ldots \right],
\]

and interpreting the leading \(A_1^{-2\nu_+}\) term as the kinetic term of the interpolating long-distance field \(\bar{h}_{\mu\rho}\) in \(123\).

Continuing on to intermediate energies \(L^{-1} \ll p \ll k\), we see immediately from \(116\) that there is no longer any pole, since we are now above the compositeness scale. This is also consistent with the fact that for \(pL \gg 1\) there is no longer any \(1/p^2\) term in the Green’s function \(89\). The analytic terms are identical to those at low energies, so that the source remains massive. Hence, in this regime the dominant contribution to the gravitational potential arises from the CFT, which corresponds to the nonanalytic term in \(116\).

When \(\nu_+ > 2\) the source coupling to the CFT is irrelevant and the gravitational potential follows from the coupling to the CFT as depicted in Fig.3. From the analytic terms of \(116\) we obtain the UV Lagrangian:

\[
\mathcal{L}_{UV} = \frac{1}{4} h_{\mu\rho}(\Box - m_h^2)h^{\mu\rho} + \frac{\chi}{k} h_{\mu\rho}T_{CFT}^{\mu\rho} + \frac{\chi}{k} h_{\mu\rho}T_{matter}^{\mu\rho} + \mathcal{L}_{CFT},
\]

where \(\chi = (\nu_+ - 2)^{1/2}(M/k)^{-3/2}\) and \(m_h^2 = 4(\nu - 1)(\nu - 2)k^2\). Since there is no longer any massless pole, the leading contribution to the potential is given by:

\[
V(r) \simeq -\mu \frac{\chi^2}{k^2} \int \frac{d^3p}{(2\pi)^2} e^{ip \cdot x} \frac{\langle OO \rangle(p)}{m_h^4},
\]

\[
= \mu \frac{\chi^4}{m_h^4} \left( \frac{M}{k} \right)^3 \int \frac{d^3p}{(2\pi)^2} e^{ip \cdot x} \left( \frac{p}{2k} \right)^{2\nu - 2} \frac{4 \Gamma(2 - \nu)}{\Gamma(\nu - 1)},
\]

where \(\langle OO \rangle\) is the nonanalytic part of \(116\) given by:

\[
\langle OO \rangle = - \left( \frac{M}{k} \right)^3 k^4 \left( \frac{p}{2k} \right)^{2\nu - 2} \frac{4 \Gamma(2 - \nu)}{\Gamma(\nu - 1)}.
\]

Performing the Fourier transform leads precisely to the result derived purely on the gravity side \(81\).

When \(1 < \nu_+ < 2\) the source coupling to the CFT is relevant but the nonanalytic term is still subdominant compared to the leading mass term in \(116\). In this case no summation is needed beyond the leading CFT correction depicted in Fig.3 and so the contribution to the potential is identical to that obtained in \(126\). The corresponding Fourier transform then leads to the same expression \(81\).
6 Discussion and conclusion

We have seen that the graviton zero mode can be smoothly deformed away from the Planck brane. This deformation requires modifying the bulk covariant theory at quadratic order by introducing bulk and boundary mass terms. However, a massless mode only occurs for a special choice of the bulk and boundary masses. This is an additional tuning beyond the usual tuning of bulk and brane cosmological constants in the RS model. As in the RS case this tuning may be realized as the result of bulk supersymmetry \cite{9, 33}. At the linearized level there is a 4D general covariance, which is consistent with the fact that there is a massless tensor mode. In addition there is also a massless vector mode with a 4D U(1) gauge symmetry.

However, general relativity is an inherently nonlinear theory, and it is apparent that higher order nonlinear interactions will spoil this symmetry. Without further modification the gravity in our model is different from the full nonlinear Einstein theory. This situation may be remedied by modifying our model at the nonlinear level with the introduction of nonlinear terms in the bulk and brane, in order to at least preserve the 4D general covariance. Thus we expect our zero mode to remain massless in the nonlinear theory, although this analysis is beyond the scope of the present paper.

Nonetheless it is already interesting that a smooth deformation exists at quadratic order without the presence of ghosts. In particular the scalar sector is trivially zero because this is the only solution consistent with the bulk and boundary equations. Clearly this is due to the fact that we are working only to quadratic order, and the scalar modes can possibly appear at the nonlinear level. Scalar modes may also arise when matter is added on the brane. On the phenomenological side, they are certainly needed to reproduce the correct gravitational law if the stress–energy tensor of the matter fields is not traceless. A similar situation takes place in the usual RS case. In the compact version with a stabilized radion there are no massless scalar excitations. However, a massless scalar mode (most easily interpreted as the brane bending mode) arises when matter is present on the brane, and allows for the recovery of standard 4d gravity at large scales \cite{34}. A similar analysis should also be carried out in the set-up we have discussed here.

By the AdS/CFT correspondence there is an interesting 4D dual interpretation of our model, especially in the case when the graviton zero mode is localized on the IR brane. This is because zero modes localized on the IR brane correspond to CFT bound states and therefore the dual CFT interpretation would correspond to gravity emerging from the strongly coupled gauge theory. In this model of emergent gravity the UV theory is a gauge (string) theory at the TeV scale, and the graviton is a composite particle which can be associated with the millimeter scale. Thus gravity emerges as a low energy phenomenon in the IR. This is different from the conventional viewpoint that gravity is a fundamental degree of freedom in the UV theory, and our model is the first step in constructing and understanding this novel possibility.
Acknowledgments

We would like to thank Joel Giedt and Alex Pomarol for useful discussions. The work of T.G. and M.P. was supported in part by a Department of Energy grant DE-FG02-94ER40823 at the University of Minnesota. T.G. is also supported by a grant from the Office of the Dean of the Graduate School of the University of Minnesota, and an award from Research Corporation. E.P. acknowledges the support of the National Science and Engineering Research Council of Canada. T.G. also acknowledges the Aspen Center for Physics where part of this work was completed.

A Bulk gravity action to quadratic order

We will present the expansion of the bulk action (16) around the background RS solution, to quadratic order in the perturbation $h_{MN}$, where $g_{MN} = A^2 \eta_{MN} + h_{MN} = A^2 (\eta_{MN} + \tilde{h}_{MN})$, and $M, N = 0, \ldots, 3, 5$. The Lagrangian density to quadratic order in $\tilde{h}_{MN}$ is given by:

$$\mathcal{L}_5[A^2 \eta + h; M] = M^3 A^3 \left[ \tilde{h}^{MN} \partial_K \partial^K \tilde{h}_{MN} - 2 \tilde{h}^{MN} \partial_N \partial^K \tilde{h}_{MK} + \tilde{h}^{MN} \partial_M \partial_N \tilde{h} + \frac{3}{4} \partial^K \tilde{h}_{MN} \partial_K \tilde{h}^{MN} + \partial^K \tilde{h}_{MN} \partial_K \tilde{h}^{MN} \right]$$

where $\tilde{h} = \tilde{h}_{MN}^M$, prime (') denotes $\partial_5$, and indices are raised and lowered with $\eta_{MN}$. The last term in (A.1) is the contribution from the bulk mass term. The corresponding equation of motion arising from $\mathcal{L}_5$ is given by:

$$M^3 A^3 \left[ - \partial^2 (\tilde{h}_{MN} - \eta_{MN} \tilde{h}) - \eta_{MN} \partial_A \partial_B \tilde{h}^{AB} + \partial_M \partial^A \tilde{h}_{AN} + \partial_N \partial^A \tilde{h}_{AM} - \partial_M \partial_N \tilde{h} - 3 \frac{A'}{A} \left( \tilde{h}_{MN}^M - \eta_{MN} \tilde{h} + 2 \eta_{MN} \partial_A \tilde{h}^{A5} - \partial_M \tilde{h}_{N5} - \partial_N \tilde{h}_{M5} \right) - \frac{2 A^2}{A^2} \eta_{MN} \tilde{h}_{55} \right]$$

where $\partial^2 = \Box + \partial_5^2$. Note that the R-S solution requires that $\Lambda = -6k^2 M^3$, and the term involving the cosmological constant in the last line of (A.2) vanishes. If we
consider only the tensor fluctuations $\tilde{h}_{MN} = \hat{h}_{\mu\nu}$ as defined in (18). The remaining equations of motion for the vector (23), (24) and scalar modes (30)-(33) follow from the $\mu 5$ and 55 components of (A.2). A similar expansion to quadratic order in the metric perturbation, but without the bulk mass term, has also been performed in Ref. [21].

B Quadratic action of the graviton zero mode

We have seen that by appropriately tuning the bulk and brane mass parameters, there is a zero mode both in the tensor and vector sectors. The presence of these zero modes signal some symmetries of the starting action. In particular, the massless tensor mode signals a 4D general coordinate invariance, while the massless vector mode signals a 4D gauge invariance. Clearly, these symmetries only occur at the linearized level in the perturbations. A study beyond this order would require considering the inclusion of higher order terms in the original action, and it is beyond our current aims.

It is instructive to compute the quadratic action for the tensor zero mode, and to explicitly show how the mass term cancels. This calculation also reveals how to canonically normalize the graviton in the 4D theory, and therefore how to properly define the four dimensional Planck mass. Assuming the Fierz–Pauli choice $b = -a$, and $\beta_i = -\alpha_i$, with $-\alpha_1 = \alpha_0 \equiv \alpha$ for the bulk/brane mass terms, respectively, the total action up to second order in the tensor perturbations gives:

$$S_{\text{tensor}}^{(2)} = M^3 \left\{ \int d^5x \, A^3 \left[ \sqrt{-\hat{g}_4} \hat{R}_4 - \frac{1}{4} \hat{h}^{\mu\nu} \hat{h}_t^{\mu\nu} - 3 \frac{A'}{A} \hat{h}^{\mu\nu} \hat{h}_t^{\mu\nu} - \left( 3 \frac{A'^2}{A^2} + 3 k^2 A^2 + a k^2 A^2 \right) \hat{h}^{\mu\nu} \hat{h}_t^{\mu\nu} \right] \right\},$$

where $+/−$ refers to the UV/IR brane, respectively, and the $\hat{h}_{\mu\nu}$ spacetime indices are raised with the (inverse) Minkowski metric $\eta^{\mu\nu}$. The first term,

$$\sqrt{-\hat{g}_4} \hat{R}_4 = \frac{1}{4} \hat{h}^{\mu\nu} \hat{h}^{\rho\sigma} \hat{h}_t^{\rho\sigma},$$

has the tensorial structure of the quadratic 4D Einstein–Hilbert term for a transverse–traceless $\hat{h}_{\mu\nu}$, except that $\hat{h}_{\mu\nu}$ still depends on the fifth coordinate $z$. One can verify that the action (B.3) reproduces the tensor mode bulk and brane equations, (18) and (58), respectively.

To proceed further, we decompose $\hat{h}_{\mu\nu}$ into the eigenmodes (19) and (20), and integrate over the compact coordinate. Hermiticity of the action ensures that eigenmodes with different mass eigenvalues are decoupled, so that one is left with an
infinite sum over the decoupled actions $S^{(2)(n)}$, for each four dimensional mode $H^{(n)}_{\mu\nu}$. The last two lines of (B.3) combine to form the mass term for any given mode. In particular, let us consider the zero mode:

$$\hat{h}_{\mu\nu} = C_1 A(z)^{-2(1-\sqrt{1+a})} H^{(0)}_{\mu\nu}(x), \quad (B.5)$$

as given by (19), where we only include the part which is continuously connected to the RS graviton. In this case, the mass terms of (B.3) combine to give:

$$C_2^2 k M^3 \left\{ \int_{z_0}^{z_1} dz k \left[ -2 \sqrt{1+a} \left( 2 + \sqrt{1+a} \right) A(z)^{1+4\sqrt{1+a}} \right] 
+ \left( \frac{3}{2} - \alpha \right) \left[ A(z_0)^{4\sqrt{1+a}} - A(z_1)^{4\sqrt{1+a}} \right] \right\} \int d^4 x H^{\mu\nu(0)} H^{(0)}_{\mu\nu}, \quad (B.6)$$

Indeed we see that the bulk and brane contributions cancel when (64) is imposed for $\alpha = \alpha_+$, resulting in a massless tensor perturbation. Similarly, if we choose the $C_2$ part of (19) then we obtain a massless tensor perturbation for $\alpha = \alpha_+$. The quadratic 4D action for the zero mode then simply becomes:

$$S^{(2)(0)}_{\text{scalar}} = \frac{M^3 C_1^2}{2k \left( 4\alpha - 1 \right)} \left[ A(z_1)^{2(1-4\alpha)} - A(z_0)^{2(1-4\alpha)} \right] \int d^4 x \sqrt{-g_4} R_4, \quad (B.7)$$

where $g_4, R_4$ now refer to the standard 4D metric $g_{\mu\nu,4} = \eta_{\mu\nu} + H^{(0)}_{\mu\nu}(x)$. By adding a source term on the brane, it is straightforward to see that the coupling of $H^{(0)}_{\mu\nu}$ to brane fields is set by the 4D Planck mass $M_8$.

C Quadratic action of the scalar modes

The equations for the scalar perturbations can be directly obtained from the second order action in the scalar perturbation. The derivation of this action is quite involved (we extend the computation of Ref. [35], performed for the covariant case), but the result is relatively simple. The computation of the action supports the choice of the (generalized) Fierz–Pauli mass term for the perturbations. Indeed, the expansion of the mass terms gives the following higher derivative kinetic terms:

$$S^{(2)}_{\text{scalar}} \supset 4 k^2 \int d^5 x A^5(z) (a + b) (\Box E)^2 + \sum_i 4 k \int d^4 x A^4(z_i)(\alpha_i + \beta_i) (\Box E)^2_i, \quad (C.8)$$

where the sum over $i$ refers to the two boundary branes. As in the 4D case, the Fierz–Pauli choice $(a+b = \alpha_i + \beta_i = 0)$ eliminates these pathological higher derivative terms.
from the action \([26]\). Employing this choice, and relating the brane mass coefficients as \(\alpha_{0,1} = \pm \alpha\), the action for the scalar perturbations is:

\[
S_{\text{scalar}}^{(2)} = \int d^5x A^3 \times \left\{ -6\hat{\psi} \Box (\hat{\psi} + \hat{\phi}) + 12\hat{\psi}^2 + 24 \frac{A'}{A} \hat{\psi}' \left( 2\hat{\psi} - \hat{\phi} \right) + 12 \frac{A'^2}{A^2} \left( \hat{\phi}^2 + 8 \hat{\psi}^2 \right) \\
+ a k^2 A^2 \left[ 32 \phi \psi + 48 \psi^2 + 24 \psi \Box E + 8 \phi \Box E + 2 B \Box B \right] \right\},
\]

\[
\pm \int d^4x A^3 \left\{ 24 \frac{A'}{A} \hat{\psi}^2 + 6\hat{\psi} \Box \hat{\zeta} + 3 \frac{A'}{A} \hat{\zeta} \Box \hat{\zeta} \right. \\
+ \alpha k A \left[ 48 \left( \psi + \frac{A'}{A} \zeta \right)^2 + 24 \left( \psi + \frac{A'}{A} \zeta \right) \Box E \right] \right\}, \tag{C.9}
\]

where \(+/−\) refers to the UV/IR brane, respectively. Note that only gauge invariant combinations \(\hat{\psi}, \hat{\phi}, \) and \(\hat{\zeta}\) appear when there are no bulk or boundary masses. This is due to the fact that the usual action for gravity is general coordinate invariant. This symmetry has been made manifest in (C.9), by rewriting a total derivative in the bulk as a boundary term, which then produces a sum of two brane terms. For instance, this is the origin of an \(E''\) term which is not present in the original action, but which is needed to produce the \(\hat{\psi}^2\) term in the bulk. Clearly, this procedure does not change the action, but allows one to write it in a more compact and manifestly covariant form. The terms proportional to the bulk/brane masses are instead not general coordinate invariant, and for this reason they cannot be rewritten in terms of gauge invariant quantities only.

To obtain the equations of motion from (C.9), it is convenient to work with the original total derivative bulk terms, and compute the Euler-Lagrange equations by the usual procedure. In fact the simplest way to proceed is to rewrite (C.9) as a bulk action by promoting \(\zeta\) to a bulk field, which evaluates to \(\zeta_i\) at the two boundaries, and then vary this 5D action. In general when we vary terms containing \(f'\) (where \(f\) denotes any of the scalar perturbations), we produce terms proportional to \(\delta f'\). These terms are dealt with by integrating by parts the variation of the action:

\[
\int d^5x \delta f' \left[ \ldots \right] = \int d^5x \{ \delta f \left[ \ldots \right] \}' - \int d^5x \delta f \left[ \ldots \right]' \tag{C.10}
\]

The last term in (C.10) enters in the usual Euler–Lagrange equations, while the total derivative is usually assumed to vanish when evaluated at infinity. However, in the presence of branes, this term must also separately vanish on shell, for each brane. It is precisely this requirement that leads to the boundary conditions for the bulk fields.
Using this procedure, the bulk/brane equations of motion can be readily computed. There are four bulk equations:

\[ \Box \left[ \hat{\psi}' - \frac{A'}{A} \hat{\phi} - \frac{2}{3} a k^2 A^2 B \right] = 0 , \]

\[ \Box \left[ \hat{\psi}'' - \frac{A'}{A} \hat{\phi}' + 3 \frac{A'}{A} \hat{\psi}' - 4 \frac{A'^2}{A^2} \hat{\phi} - \frac{4}{3} a k^2 A^2 (3 \psi + \phi) \right] = 0 , \]

\[ \frac{A'}{A} \left( \hat{\psi}' - \frac{A'}{A} \hat{\phi} \right) - \frac{4}{3} a k^2 A^2 \psi + \Box \left[ \frac{1}{4} \hat{\psi} - \frac{a}{3} k^2 A^2 E \right] = 0 , \]

\[ \hat{\psi}'' - \frac{A'}{A} \hat{\phi}' + 3 \frac{A'}{A} \hat{\psi}' - 4 \frac{A'^2}{A^2} \hat{\phi} - \frac{4}{3} a k^2 A^2 (3 \psi + \phi) \]

\[ + \Box \left[ \frac{1}{2} \hat{\psi} + \frac{1}{4} \hat{\phi} - a k^2 A^2 E \right] = 0 , \]  

which are obtained by varying \( B, E, \phi, \) and \( \psi, \) respectively (since \( \zeta \) is defined only on the two boundaries).

For the boundary conditions one would naively expect five equations. However, one can verify that once the boundary term in (C.9) is rewritten as a bulk term, \( \phi' \) and \( B' \) do not appear in the total action. Hence, there are no boundary conditions arising from the variation of the action with respect to \( \phi \) and \( B. \) This leaves only three boundary conditions which follow from varying \( E, \psi, \) and \( \zeta \) in the action (C.9).

These are respectively given by the following equations evaluated on the two branes:

\[ \Box \left[ \psi' - \frac{A'}{A} \phi - 4 \alpha k A \left( \psi + \frac{A'}{A} \zeta \right) \right] = 0 , \]  

(C.12)

\[ \psi' - \frac{A'}{A} \phi - 4 \alpha k A \left( \psi + \frac{A'}{A} \zeta \right) + \frac{1}{4} \Box \left[ E' - B - \zeta - 4 \alpha k A E \right] = 0 , \]  

(C.13)

\[ 4 \alpha k A \left( \psi + \frac{A'}{A} \zeta \right) + \frac{\Box}{A} \left[ \alpha k A' E + \frac{1}{4} \left( \psi + \frac{A'}{A} \zeta \right) \right] = 0 . \]  

(C.14)

For the massless modes (i.e, imposing \( \Box \equiv \Box 0 \)) the solution of the above equations is \( \psi = \phi = \zeta = 0. \) The remaining combination \( B - E' \) is undetermined and one can verify that when the other modes are absent, this combination gives a vanishing contribution to the action (C.9). In the covariant case, we have seen that out of the original scalar modes \( \psi, \phi, E, B, \zeta \) only \( \hat{\psi}, \hat{\phi}, \hat{\zeta} \) appear in the quadratic action for the perturbations. This is a consequence of 5D general coordinate invariance, which guarantees that two modes are not present in the action at all orders. In the present case, higher order terms could make the remaining modes dynamical. However, to study in a consistent way the non–covariant theory beyond quadratic order in the action requires introducing higher-order terms in addition to the quadratic bulk/brane mass terms (16) and (50). This is beyond the scope of the present analysis.
For the massive modes, one can check that the bulk equations (C.11) are equivalent to the equations (30) - (33). Also, the boundary equations (C.12) and (C.13) are equivalent (for the massive modes) to the two junction conditions (60), and (61). However Eq. (C.14) is a third independent boundary condition, which as discussed in the main text, cannot be obtained through the usual junction/Israel condition procedure. Eliminating $\zeta$ in (C.14) through the other two boundary conditions gives:

$$4 \frac{A'}{A} \left( \hat{\psi}' - A' \hat{\phi} \right) + \Box \hat{\psi} = 0 \ .$$

(C.15)

Using the third equation of (C.11), this equation simplifies to: 13

$$a (4 \phi + \Box E) = 0 \ .$$

(C.16)

This boundary condition vanishes identically when $a = 0$. This shows that the “additional” boundary condition (C.14) is redundant in the standard case without bulk or boundary mass terms. Instead when $a \neq 0$ we obtain an independent boundary condition, which appears as Eq. (62) in Section 3.1.

### D Derivation of the gravitational potential

We will now derive the leading terms in the expressions (83)-(84). We start from the action for the Kaluza-Klein modes of the 5d tensor mode, coupled to a conserved matter source on the UV brane. The action can be written in the form:

$$S = \sum_{n} c_n \int d^4x \ H^{(n)}_{\mu\nu} \left( \Box - m_n^2 \right) H^{(n)}_{\mu\nu} + \frac{1}{2}Z_n(0) \int d^4x \ H^{(n)}_{\mu\nu} T^{\mu\nu} \ ,$$

(D.17)

where $T_{\mu\nu}$ denotes the energy-momentum tensor of the matter source, and the variables of the tensor mode wavefunctions are separated as:

$$\hat{h}_{\mu\nu}(x, z) = \sum_{n} H^{(n)}_{\mu\nu}(x) Z_n(z) \ .$$

(D.18)

The coefficients $c_n$ are given by:

$$c_n = \frac{M^3}{4} \int_0^{z_i} dz \ A^3 Z_n^2(z) \ .$$

(D.19)

Correspondingly, the gravitational potential generated by a static mass $\mu$ on the UV brane, measured at the distance $r$ from $\mu$, is given by:

$$V = -\frac{\mu}{8} \sum_{n} \frac{Z_n^2(0)}{c_n} \frac{e^{-m_n r}}{r} \ .$$

(D.20

---

13Note that there is no problem with combining bulk and boundary equations. Indeed, the value of any function $f(z_i)$ in the boundary equations is defined as lim $f(z)$ for $z \to z_i$ in the fundamental domain.
All the modes mediate a (gravitational) attraction. The sum (D.20) includes the long range contribution from the zero mode, giving rise to the standard Newtonian potential, characterized by the Planck mass, $M_P$ (80), while each Kaluza-Klein mode $n$ provides a Yukawa-type force contribution, which is relevant at $r \lesssim m_n^{-1}$. We are interested in computing the gravitational potential at the “intermediate” distances (82). For these distances, the largest contribution to the potential (D.20) is given by modes with mass $m$ satisfying:

$$A_1 k \ll m \ll k.$$  \hspace{1cm} (D.21)

We can use this condition in the expansion of the Bessel functions which characterize the bulk profile of the Kaluza-Klein modes (20). Moreover, the mass spectrum in this range is given in Eq. (67).

The “plus” and “minus” branches can be discussed simultaneously by introducing the parameters

$$\nu_{\pm} \equiv \pm (4\alpha_{\pm} - 1).$$

They satisfy

$$2\sqrt{1 + a} = \nu_{\pm} \pm 1,$$

so that $\nu_{\pm}$ ranges from $\pm 1$ to $+\infty$, as $a$ ranges from $-1$ to $+\infty$ (the RS point is at $\nu_- = 1$). We can set $C_1 = 1$ in the mode solutions (20), since the normalization cancels in (D.20) (also, for shorthand we write $\nu$ rather than $\nu_{\pm}$). We then have:

$$Z_n(z) = A^{-2} \left[ J_{\nu_{\pm}} \left( \frac{m_n}{kA} \right) + C_2 Y_{\nu_{\pm}} \left( \frac{m_n}{kA} \right) \right], \hspace{1cm} C_2 = - \frac{J_{\nu} \left( \frac{m_n}{kA_1} \right)}{Y_{\nu} \left( \frac{m_n}{kA_1} \right)}.$$  \hspace{1cm} (D.22)

which give the following exact expressions:

$$Z_n(0) = \mp \frac{2k}{m_n \pi Y_{\nu} \left( \frac{m_n}{kA} \right)} \hspace{0.5cm}, \hspace{0.5cm} c_n = \frac{M^3 k}{2 \pi^2 m_n^2} \left[ \frac{1}{Y_{\nu}^2 \left( \frac{m_n}{kA_1} \right)} - \frac{1}{Y_{\nu}^2 \left( \frac{m_n}{k} \right)} \right].$$  \hspace{1cm} (D.23)

Accordingly, we obtain:

$$\frac{Z_n^2(0)}{c_n} = \frac{8k}{M^3} \frac{Y_{\nu}^2 \left( \frac{m_n}{kA_1} \right) - Y_{\nu}^2 \left( \frac{m_n}{k} \right)}{Y_{\nu}^2 \left( \frac{m_n}{kA} \right)} \simeq \frac{8k}{M^3} \frac{Y_{\nu}^2 \left( \frac{m_n}{kA_1} \right)}{Y_{\nu}^2 \left( \frac{m_n}{k} \right)}.$$  \hspace{1cm} (D.24)

Inserting this ratio back into the potential (D.20), and taking the expression (80) for the 4D Planck mass on the UV brane, we obtain,

$$V \simeq - \frac{\mu}{M_P^2} \left[ 1 + \frac{k M_P^2}{M^3} \sum_{n>0} \frac{Y_{\nu}^2 \left( \frac{m_n}{kA_1} \right)}{Y_{\nu}^2 \left( \frac{m_n}{k} \right)} e^{-m_n r} \right].$$  \hspace{1cm} (D.25)
In the range (D.21), the sum becomes:

\[
\sum_{n>0} Y^2_\nu \left( \frac{m_n}{kA_1} \right) e^{-m_n r} \approx \frac{\pi A_1}{2^{2|\nu|-1} \Gamma(|\nu|)^2} \sum_{n>0}^{A_1-1} \left( \frac{m_n}{k} \right)^{2|\nu|-1} e^{-m_n r} 
\]

\[
\approx \frac{\pi A_1}{2^{2|\nu|-1} \Gamma(|\nu|)^2} \frac{\Gamma(2|\nu|)}{2^{2|\nu|-1} \Gamma(|\nu|)^2 (kr)^{2|\nu|}}, \quad (D.26)
\]

where we have first used the result (21), and then we have approximated the expression for the masses as \( m_n \approx \pi k A_1 n \) in the relevant regime. The potential (D.24) then rewrites:

\[
V \approx -\mu M_P^2 \frac{k}{M^3} \frac{1}{(kr)^{2|\nu|}} \frac{\Gamma(2|\nu|)}{2^{2|\nu|-1} \Gamma(|\nu|)^2} \left[ 1 + \frac{k M_P^2}{M^3} \frac{\Gamma(2|\nu|)}{2^{2|\nu|} \Gamma(|\nu|)^2} \right]. \quad (D.27)
\]

where we have reintroduced the suffix ± which characterizes the two branches.

The ratio \( k M_P^2 / M^3 \) controls the relative contribution of the zero mode and the Kaluza-Klein tower. For \( \alpha < 1/4 \), the zero mode is more localized towards the UV brane, and the massive Kaluza-Klein modes have a negligible effect in the regime (82). Substituting the value for \( M_P \) given in eq. (80), and identifying \( \nu_- = 1 - 4\alpha = \xi \), we obtain the result (83) of the main text. In the complementary interval \( \alpha > 1/4 \), the zero mode is localized towards the IR brane, and the relative contribution from the Kaluza-Klein modes significantly increases. This interval is covered both by \( \nu_- < 0 \), and \( \nu_+ \). We can describe it by a unique parameter \( \xi \equiv 4\alpha - 1 \), ranging from zero to infinity, which is identified with \( -\nu_- \) from 0 to 1, and with \( \nu_+ \) from 1 to \( \infty \). This leads to the expression (84) given in the main text.

References

[1] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].

[2] J. M. Maldacena, “The large-N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [arXiv:hep-th/9711200].

[3] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].
[4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].

[5] N. Arkani-Hamed, M. Porrati and L. Randall, “Holography and phenomenology,” JHEP 0108, 017 (2001) [arXiv:hep-th/0012148].

[6] R. Rattazzi and A. Zaffaroni, “Comments on the holographic picture of the RS model,” JHEP 0104, 021 (2001) [arXiv:hep-th/0012248].

[7] M. Perez-Victoria, “RS models and the regularized AdS/CFT correspondence,” JHEP 0105, 064 (2001) [arXiv:hep-th/0105048].

[8] Y. Grossman and M. Neubert, “Neutrino masses and mixings in non-factorizable geometry,” Phys. Lett. B 474, 361 (2000) [arXiv:hep-ph/9912408].

[9] T. Gherghetta and A. Pomarol, “Bulk fields and supersymmetry in a slice of AdS,” Nucl. Phys. B 586, 141 (2000) [arXiv:hep-ph/0003129].

[10] T. Gherghetta and A. Pomarol, “A warped supersymmetric standard model,” Nucl. Phys. B 602, 3 (2001) [arXiv:hep-ph/0012378].

[11] T. Gherghetta and A. Pomarol, “The standard model partly supersymmetric,” Phys. Rev. D 67, 085018 (2003) [arXiv:hep-ph/0302001].

[12] R. Contino, Y. Nomura and A. Pomarol, “Higgs as a holographic pseudo-Goldstone boson,” Nucl. Phys. B 671, 148 (2003) [arXiv:hep-ph/0306259].

[13] K. Agashe, R. Contino and A. Pomarol, “The minimal composite Higgs model,” [arXiv:hep-ph/0412089].

[14] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, “Bulk gauge fields in the RS model,” Phys. Lett. B 473, 43 (2000) [arXiv:hep-ph/9911262].

[15] A. Pomarol, “Gauge bosons in a five-dimensional theory with localized gravity,” Phys. Lett. B 486, 153 (2000) [arXiv:hep-ph/9911294].

[16] K. Ghoroku and A. Nakamura, “Massive vector trapping as a gauge boson on a brane,” Phys. Rev. D 65, 084017 (2002) [arXiv:hep-th/0106145].

[17] R. Sundrum, “Fat gravitons, the cosmological constant and sub-millimeter tests,” Phys. Rev. D 69, 044014 (2004) [arXiv:hep-th/0306106].

[18] S. Weinberg and E. Witten, “Limits on massless particles,” Phys. Lett. B 96, 59 (1980).
[19] M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” Proc. Roy. Soc. Lond. A 173, 211 (1939).

[20] H. van Dam and M. J. G. Veltman, Nucl. Phys. B 22, 397 (1970); V. I. Zakharov, JETP Lett. 12, 312 (1970).

[21] Z. Chacko, M. Graesser, C. Grojean and L. Pilo, “Massive gravity on a brane,” Phys. Rev. D 70, 084028 (2004) [arXiv:hep-th/0312117].

[22] J. Chiaverini, S. J. Smullin, A. A. Geraci, D. M. Weld and A. Kapitulnik, “New experimental constraints on non-Newtonian forces below 100-mu-m,” Phys. Rev. Lett. 90, 151101 (2003), [arXiv:hep-ph/0209325];

[23] J. C. Long, H. W. Chan, A. B. Churnside, E. A. Gulbis, M. C. M. Varney and J. C. Price, “Upper limits to submillimeter-range forces from extra space-time dimensions,” Nature 421, 922 (2003);

[24] E. G. Adelberger, B. R. Heckel and A. E. Nelson, “Tests of the gravitational inverse-square law,” Ann. Rev. Nucl. Part. Sci. 53, 77 (2003) [arXiv:hep-ph/0307284];

[25] C. D. Hoyle, D. J. Kapner, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt and H. E. Swanson, “Sub-millimeter tests of the gravitational inverse-square law,” Phys. Rev. D 70, 042004 (2004) [arXiv:hep-ph/0405262].

[26] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, “Effective field theory for massive gravitons and gravity in theory space,” Annals Phys. 305, 96 (2003) [arXiv:hep-th/0210184].

[27] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315].

[28] L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999) [arXiv:hep-th/9906064].

[29] S. B. Giddings, E. Katz and L. Randall, “Linearized gravity in brane backgrounds,” JHEP 0003, 023 (2000) [arXiv:hep-th/0002091].

[30] S. L. Adler, “Einstein gravity as a symmetry breaking effect in quantum field theory,” Rev. Mod. Phys. 54, 729 (1982) [Erratum-ibid. 55, 837 (1983)].

[31] I.M. Gelfand and G.E. Shilov, “Generalized functions and operations on them,” Dobrosvet, Moscow, 2000 (in Russian).

[32] R. Contino and A. Pomarol, JHEP 0411, 058 (2004) [arXiv:hep-th/0406257].
[33] R. Altendorfer, J. Bagger and D. Nemeschansky, “Supersymmetric RS scenario,” Phys. Rev. D 63, 125025 (2001) [arXiv:hep-th/0003117].

[34] T. Tanaka and X. Montes, “Gravity in the brane-world for two-branes model with stabilized modulus,” Nucl. Phys. B 582, 259 (2000) [arXiv:hep-th/0001092].

[35] L. Kofman, J. Martin and M. Peloso, “Exact identification of the radion and its coupling to the observable sector,” Phys. Rev. D 70, 085015 (2004) [arXiv:hep-ph/0401189].