Control of Dynamical Localization

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(Dated: November 3, 2018)

Control over the quantum dynamics of chaotic kicked rotor systems is demonstrated. Specifically, control over a number of quantum coherent phenomena is achieved by a simple modification of the kicking field. These include the enhancement of the dynamical localization length, the introduction of classical anomalous diffusion assisted control for systems far from the semiclassical regime, and the observation of a variety of strongly nonexponential lineshapes for dynamical localization. The results provide excellent examples of controlled quantum dynamics in a system that is classically chaotic and offer new opportunities to explore quantum fluctuations and correlations in quantum chaos.

PACS numbers: 05.45.Mt, 32.80.Qk, 05.60.-k

I. INTRODUCTION

The quantum kicked rotor (KR) and its classical analog, the standard map, have long served as a paradigm for quantum and classical chaos. Due to its atom optics realization, the KR has recently attracted renewed interest. The KR is also of considerable interest in a variety of other fields such as condensed matter physics, molecular physics, and quantum information science.

One well-known quantum effect in KR is “dynamical localization” (DL). That is, although a classical kicked rotor displays unrestricted diffusive energy increase due to classical chaos, only a finite number of rotational states are excited in the quantum dynamics, with the quantum excitation probability versus the rotational quantum number typically assuming a characteristic exponential lineshape. DL is a pure quantum coherence effect and is therefore very sensitive to decoherence. For example, previous studies have demonstrated that noise, nonperiodicity in the kicking sequences, and quantum measurements can destroy DL.

As a coherence effect, DL is also indicative of the possibility of controlling the KR dynamics via quantum effects. Indeed, we recently showed that the quantum phases describing the initial rotor quantum state can be manipulated to effectively control quantum fluctuations in quantum chaos and thus enhance or suppress quantum chaotic diffusion in KR. However, manipulating quantum phases in initial states cannot change the unitary evolution operator of the system. Hence, neither the average dynamical localization length nor the characteristic lineshape of dynamical localization can be altered in this way.

Motivated by interest in controlled classically chaotic quantum dynamics, and to gain more insights into the nature of DL, we consider controlling DL and the associated energy absorption via a modified kicked rotor (MKR) system, in which the phase of the kicking field, or the timing of the kicking sequences, is actively manipulated. In particular, we consider an MKR system in which the sign of the kicking potential is periodically changed, or alternatively time delayed, after a certain number of kicks. As shown below, such a slight modification of KR has profound effects on the dynamics: whereas periodically changing the sign of the kicking potential does not destroy DL, it dramatically changes the quantum diffusion dynamics of KR as well as the nature of DL, offering entirely new opportunities for controlling dynamics, as well as understanding quantum fluctuations and correlations in quantum chaos in periodically kicked systems. For example, we demonstrate that, with the sign of the kicking potential of KR periodically changed: (i) the dynamical localization length is significantly increased so that the energy absorption is strongly enhanced; (ii) classical anomalous diffusion (which can be slower than quantum anomalous diffusion under certain conditions) can enhance control even when the effective Planck constant is about an order of magnitude larger than the relevant classical phase space structures, and (iii) DL may display strong deviations from purely exponential lineshapes.

This paper is organized as follows: In Sec. II we introduce the modified kicked-rotor model. We then present results in Sec. III on the enhancement of dynamical localization length, with qualitative explanations based upon a known result from the Band Random Matrix theory. In Sec. IV we show control of DL in a different regime, where the dynamics can be tied to a new mechanism, i.e., the creation of new structures in classical phase space. Section V contains the results on coherent manipulations of the lineshapes for DL. We conclude the paper with a brief discussion in Sec. VI.

II. A MODIFIED KICKED-ROTOR MODEL

The KR Hamiltonian is given by

\[ H_{KR}(\hat{L}, \theta, t) = \hat{L}^2/2I + \lambda \cos(\theta) \sum_n \delta(t/T - n), \]

arXiv:quant-ph/0309215v1 29 Sep 2003
where \( \hat{L} \) is the angular momentum operator, \( \theta \) is the conjugate angle, \( I \) is the moment of inertia, \( \lambda \) is the strength of the kicking field, and \( T \) is the time interval between kicks. The basis states of their Hilbert spaces are given by \(|m\rangle\), with \( \hat{L}|m\rangle = m\hbar|m\rangle \). The quantum KR map operator for propagating from time \((N + 0^+)T\) to time \((N + 1 + 0^+)T\) is given by

\[
\hat{F}_{KR} = \exp \left[ i \frac{\tau}{2} \frac{\partial^2}{\partial \theta^2} \right] \exp[-ik \cos(\theta)],
\]

with dimensionless parameters \( k = \lambda T/\hbar \) and the effective Planck constant \( \tau = \hbar T/I \). For later use we also define the dimensionless scaled rotational energy as \( \tilde{E} \equiv \langle \hat{L}^2 \rangle /2\hbar^2 \), where \( \langle \cdot \rangle \) represents the average over the quantum ensemble. In the \(|m\rangle\) representation, \( \hat{F}_{KR} \) takes the following form [23]:

\[
\langle m_1|\hat{F}_{KR}|m_2 \rangle = \exp \left( i \frac{\tau}{2} m_1^2 \right) \tilde{J}_{m_1-m_2}(k),
\]

where \( \tilde{J}_{m_1-m_2}(k) \) is the Bessel function of the first kind of order \((m_1 - m_2)\).

The classical limit of the KR quantum map, i.e., the standard map, depends only on one parameter \( \kappa \equiv k\tau \) and is given by:

\[
\begin{align*}
\hat{L}_N &= \hat{L}_{N-1} + \kappa \sin(\theta_{N-1}); \\
\theta_N &= \theta_{N-1} + \hat{L}_N,
\end{align*}
\]

where \( \hat{L} \equiv L\tau/\hbar \) is the scaled c-number angular momentum and \( \langle \hat{L}_N, \theta_N \rangle \) represents the phase space location of a classical trajectory at \((N+1-0^+)T\). For later discussion we note that for particular values of \( \kappa \), the classical map Eq. (4) can generate accelerating trajectories whose momentum increases (or decreases) linearly with time (at least on the average). These trajectories are called \textit{transporting} trajectories [24]. To see this consider the initial conditions: \( \langle \hat{L} = 2p_1, \theta = \pm \pi/2 \rangle \) for \( \kappa = 2p_2 \), where \( l_1 \) and \( l_2 \) are integers. Clearly, these phase space points are shifted by a constant value \((\pm 2\pi l_2) \) in \( \hat{L} \) after each iteration, resulting in a quadratic increase of rotational energy. These transporting trajectories are rather stable insofar as they may persist for values of \( \kappa \) close to \( 2\pi l_2 \) (with their average momentum shift after each iteration oscillating around the constant value \( \pm 2\pi l_2 \)), thus giving rise to transporting regular islands [24], i.e., the accelerator modes in the KR case. If classical trajectories are launched from the accelerator modes, they simply jump to other similar islands located in adjacent phase space cells. For trajectories initially outside the accelerator modes, the “stickiness” of the boundary between the accelerator modes and the chaotic sea induces anomalous diffusion over the energy space, i.e., energy increases in a nonlinear fashion, but not quadratically. This is intrinsically different from the case of normal chaotic diffusion in which energy increases linearly with the number of kicks.

We introduce here a slightly modified kicked rotor (MKR) system whose Hamiltonian is given by:

\[
H^{MKR}(\hat{L},\theta,t) = \hat{L}^2/2I + \lambda \cos(\theta) \sum_n f_M(n)\delta(t/T - n),
\]

where \( f_M(n) \) is real, \( |f_M(n)| = 1 \), and \( f_M(n) \) changes sign after every \( M \) kicks. That is, the only difference between KR and MKR is that in the MKR the sign of the kicking potential is changed after every \( M \) kicks. The effect of changing the sign of the kicking potential can be further understood in terms of the time-evolving wave function, which can be expanded as a superposition of \(|m\rangle\) states: \( \sum_n C_m(\theta)|m\rangle \), with the expansion coefficients \( C_m \). Since \( \cos(\theta + \pi) = -\cos(\theta) \), and \( \sum_n C_m(\theta + \pi)|m\rangle = \sum_n (-1)^m C_m(\theta)|m\rangle \), changing the sign of the kicking potential is seen to be equivalent to adding a \( \pi \) phase difference between all neighboring basis states. Compare this now to the effect of a time delay \( t_d = 2\pi T/\tau \) between two neighboring kicks. Due to the free evolution of the rotor any two angular momentum eigenstates \(|m\rangle\) and \(|m+1\rangle\) will acquire in time \( t_d \) an additional relative quantum phase given by \( \exp[\mp \pi \tau d(m + 1)^2 - m^2]/(2T) \) = \( \exp(\pm m\pi) \). As such, the MKR can be also realized by introducing the time delay \( t_d \) after every \( M \) kicks.

For times \((N + 0^+)T\) to \((N + M + 0^+)T\), the MKR quantum propagator can be written as

\[
\hat{F}_{MKR} = \exp \left( i\pi \frac{\partial^2}{\partial \theta^2} \right) \hat{F}_{KR}^M \equiv \hat{D}\hat{F}_{KR}^M.
\]

where this equation defines \( \hat{D} \) as the free evolution operator over time \( t_d \). Here \( \hat{F}_{KR}^M \) denotes \( M \) applications of \( \hat{F}_{KR} \). From Eq. (5), one sees that the only difference in time propagation between KR and MKR for every \( M \) kicks is the \( \hat{D} \) operator, whose matrix elements \( \langle m_1|\hat{D}|m_2 \rangle \) are given by

\[
\langle m_1|\hat{D}|m_2 \rangle = (-1)^{m_1} \delta_{m_1,m_2}.
\]

Thus we have

\[
\langle m_1|\hat{F}_{MKR}|m_2 \rangle = (-1)^{m_1} \langle m_1|\hat{F}_{KR}^M|m_2 \rangle.
\]

Note also that the classical limit of the MKR quantum map [Eq. (6)] is given by

\[
\begin{align*}
\hat{L}_{N+1} &= \hat{L}_N + \kappa f_M(N)\sin(\theta_N), \\
\theta_N &= \theta_{N+1} + \hat{L}_{N+1}.
\end{align*}
\]

With respect to model systems in the literature, the MKR here can be regarded as a specific realization of the so-called generalized kicked-rotor model, which was first introduced in Ref. [25], in the context of quantum anti-resonance. However, it is very different from the amplitude-modulated kicked rotor systems [12, 26] previously studied because (i) the kicking field strength here remains constant, and therefore any interesting results arise from pure phase modulations; and (ii) as shown below, the dynamical localization is significantly altered, but not destroyed.
III. ENHANCED DYNAMICAL LOCALIZATION LENGTH

As an example, consider the case of $k = 4.0$, $\tau = 2.0$, and $M = 50$, whose classical limit for both the KR and MKR is fully chaotic and displays normal chaotic diffusion. We demonstrate below that the dynamical localization (DL) and the related energy absorption associated with $\hat{F}_{MKR}$ are dramatically enhanced over that associated with $F_{KR}^M$.

![Figure 1](image-url)

**FIG. 1**: Phase control of dynamical localization achieved by changing the sign of the kicking potential after every 50 kicks. $\kappa = 4.0$, $\tau = 2.0$, and the initial state is $|0\rangle$. (a) A comparison between KR (the narrow lineshape, solid line) and MKR (the broad lineshape, dashed line) in terms of the probability $P(m)$ of finding the system in the state $|m\rangle$ after $4 \times 10^5$ kicks. (b) The time dependence of the dimensionless scaled rotational energy $\hat{E}$ in each case of KR (solid line) and MKR (dashed line). Note that the solid line lies very closely to the $\hat{E} = 0$ axis.

Figure 1 displays the angular momentum distribution $P(m)$ after $4 \times 10^5$ kicks, starting with the initial state $|0\rangle$, for both KR and the MKR with $M = 50$. The exponential line shape of $P(m)$ shown in Fig. 1a indicates that DL occurs in both cases. By fitting $P(m)$ with exponentials $P(m) \sim \exp[-|m|/l_{KR}]$ and $P(m) \sim \exp[-|m|/l_{MKR}]$ for KR and MKR, respectively, one obtains that the dynamical localization length $l_{MKR} \sim 140.0$ is significantly larger than $l_{KR} \sim 7.0$. This clear difference in dynamical localization length is also reflected in the energy absorption shown in Fig. 1b. In particular, while the energy absorption of KR saturates after a few kicks, the MKR system continues to absorb energy in a more or less linear manner for as long as $10^4$ kicks. Enhancement of dynamical localization length and energy absorption is also observed for other values of $M$, and for a wide range of parameters $k$ and $\tau$.

As is well known, the DL of KR can be traced back to the localization properties of the eigenstates of the quantum map operator [Eq. (2)]. Note first that, due to the rapid decay of $J_{m, -m_2}(k)$ with increasing $|m_1 - m_2|$ and the pseudo-random nature of the function $\exp(\imath m_1 \tau / 2)$ in $m_1$, the quantum map operator $\hat{F}_{KR}$ in general assumes a band structure and behaves in a pseudo-random manner in the $|m\rangle$ representation. Hence, below we qualitatively consider the MKR results in terms of a well-known feature from Band-Random-Matrix theory [21, 22, 23], namely, that the larger the band width of the quantum map operator the larger the dynamical localization length.

Note first that the matrix $\langle m | \hat{F}_{KR} | m' \rangle$ is pseudo-random. However, we do not expect $\langle m | \hat{F}_{KR}^M | m' \rangle$ to be a pseudo-random matrix of the same type since multiplying $\hat{F}_{KR}$ by itself $M$ times is expected to establish correlations between the matrix elements $\langle m_1 | \hat{F}_{KR}^M | m_2 \rangle$. Nevertheless, we do assume that the matrix $\langle m | \hat{F}_{MKR}^M | m' \rangle$ is banded and pseudo-random since the eigenstates of $\hat{F}_{MKR}$ have no simple connection with those of $\hat{F}_{KR}$. Consider now an arbitrary matrix element $\langle m_1 | \hat{F}_{MKR}^M | m_2 \rangle$. Due to the quantum diffusive dynamics within the $M$ kicks, the $\langle m_1 | \hat{F}_{MKR}^M | m_2 \rangle$ with $|m_1 - m_2| >> 1$ should be much greater than $\langle m_1 | \hat{F}_{KR} | m_2 \rangle$ (nevertheless, both of them can be very small). According to Eq. (5), this implies that the matrix element $\langle m_1 | \hat{F}_{MKR}^M | m_2 \rangle$ is also much greater than $\langle m_1 | \hat{F}_{KR} | m_2 \rangle$. In this sense, we expect that the $\langle m | \hat{F}_{MKR}^M | m' \rangle$ matrix should display a wider band than does $\langle m | \hat{F}_{KR} | m' \rangle$.

The band width of the matrix $\langle m | \hat{F}_{KR} | m' \rangle$ can be defined by choosing a cut-off value for its matrix elements. One traditional choice is $|m_1 - m_2| \sim k$. In this case, $\langle m_1 | \hat{F}_{KR} | m_2 \rangle$ at the boundary of the band is on the order of $J_k(k)$, a number which is sufficiently small. However, this cut-off value, if applied to the matrix $\langle m | \hat{F}_{MKR} | m' \rangle$, would yield almost the same band width as $\hat{F}_{KR}$. Hence, quantitatively characterizing the band structure of the matrix $\langle m | \hat{F}_{MKR}^M | m' \rangle$ is subtle, since the very small matrix elements $\langle m_1 | \hat{F}_{MKR}^M | m_2 \rangle$ must play an important role in enhancing the dynamical localization length of MKR.

To gain more insight we computationally examined the $M$-dependence of the dynamical localization length of the MKR. Specifically, we numerically diagonalized $\hat{F}_{MKR}$, where each matrix is generated using $2^{14}$ basis states and is then truncated at dimension $d$ chosen below. We char-
FIG. 2: (a) The $M$-dependence of the dynamical localization length of MKR, characterized by the Shannon entropy $S_{MKR}$ averaged over all approximate eigenstates of the MKR propagator $\hat{F}_{MKR}$. (b) The $M$-dependence of the band width $b_{MKR}$ associated with the MKR propagator $\hat{F}_{MKR}$, defined by a cut-off value (as small as $10^{-20}$) for its matrix elements $\langle m_1|\hat{F}_{MKR}|m_2 \rangle$.

characterize the dynamical localization length by the Shannon entropy $S_{MKR}$ [27] (note that the Shannon entropy is simply proportional to the dynamical localization length [27]), even over all approximate eigenstates $|\phi_j\rangle$ of $F_{MKR}$, i.e.,

$$S_{MKR} = 2 \frac{\kappa}{\pi} \int \sum_{j=1}^{d} \exp \left[ - \frac{d/2}{\sum_{m=-d/2}^{d/2}} |\langle m|\phi_j\rangle|^2 \ln |\langle m|\phi_j\rangle|^2 \right] \ln (10)$$

where the constant $\alpha$ equals 0.96, and $d$ is chosen to be 2700. The results for the $M$-dependence of $S_{MKR}$, for $M = 2$ to $M = 400$ is shown in a log–log plot in Fig. 2a. $S_{MKR}$ is seen to behave initially as a smooth increasing function of $M$, and then to saturate at $M \sim 50$. To explain the results from the perspective of Band-Random-Matrix theory, we choose a cut-off value for matrix elements $\langle m_1|\hat{F}_{MKR}|m_2 \rangle$ so as to define the band width $b_{MKR}$. Interestingly, we find that this cut-off value must be extremely small (roughly speaking, at least as small as $10^{-10}$) in order that the $M$-dependence of $b_{MKR}$ resembles that of $S_{MKR}$. For example, Fig. 2b displays $b_{MKR}$ as a function of $M$ for a cut-off value of $10^{-20}$. The evident similarities between Fig. 2a and Fig. 2b, suggest (i) that we can indeed qualitatively explain the enhanced dynamical localization length in terms of the Band-Random-Matrix theory, and (ii) that even extremely small quantum fluctuations in the values of the matrix elements of the MKR quantum map operator affect its dynamical localization length.

In particular, comparing Fig. 2a with Fig. 2b, one sees that the saturation behavior of $S_{MKR}$ for large $M$ is consistent with the saturation behavior of $b_{MKR}$. The latter reflects the saturation of the matrix elements $\langle m_1|\hat{F}_{MKR}|m_2 \rangle$ and therefore the matrix elements $\langle m_1|\hat{F}_{MKR}^m|m_2 \rangle$. As such, we infer that the saturation behavior of $S_{MKR}$ is simply a result of DL in KR.

Detailed studies on numerous other cases with varying $k$ and $\tau$ show that the above result, i.e., $S_{MKR}$ first increases with $M$ and then saturates, is quite general, as long as the system has a completely chaotic classical limit and quantum correlations are insignificant. On the other hand, if $S_{MKR}$ behaves differently, then there are two possible origins: either the system is in the deep quantum regime or there are non-negligible regular islands in the classical phase space. For example, in the next section we show cases in which the energy absorption in the MKR with $M = 2$ is appreciably larger than that in the MKR with $M = 3$. In these cases one can obtain even more significant changes in DL.

IV. CLASSICAL ANOMALOUS DIFFUSION ASSISTED CONTROL

In the previous section we studied cases where both KR and MKR essentially have a fully chaotic classical limit. However, as shown below, the MKR can also display new non-chaotic classical phase space structures that are absent in KR. In particular, regular islands with very interesting transporting properties can be induced in MKR. In such cases one can achieve even more dramatic alteration of the DL than that shown above.

Consider first the classical MKR map Eq. [27] for $M = 2$. Interestingly, in this case there exists transporting trajectories that are different from those in KR. In particular, we have previously observed [20] that for $\kappa = (2l_x+1)\pi$, trajectories emanating from $L = (2l_y+1)\pi$, $\theta = \pm \pi/2$ will be shifted by a constant value $[\pm (2l_x+1)\pi] \in \hat{L}$ after each kick. This observation suggests that new transporting regular islands that differ from the accelerator modes in KR can be created by changing the sign of the kicking potential after every two kicks. This is confirmed in our extensive numerical studies, both here and in Ref. [20].

Note that in our previous work [21], we were most interested in the quantum-classical comparison in anomalous diffusion and considered relatively small effective Planck constants. In that case we found that quantum anomalous diffusion induced by the new transporting reg-
ular islands can be much faster than the underlying classical anomalous diffusion. Here, to make a closer connection to atom optics experiments we consider larger $\tau \sim 1.0$. In these cases the effective Planck constant is about an order of magnitude larger than the area of the phase space structures associated with classical anomalous diffusion. Intuitively, one would anticipate that such transporting regular islands are too small to be relevant to the quantum dynamics. Surprisingly, this intuition is incorrect, as shown below.

To be more specific, consider first the case of $\kappa = 5.0$. Figure 3 displays the classical phase space structures of both KR and the MKR with $M = 2$. While the regular islands seen in Fig. 3a (the KR case) are not transporting (that is, the momentum of the trajectories launched from these islands is bounded and oscillates periodically), a simple computation reveals that the small islands seen in Fig. 3b in the MKR case are transporting. That is, classical trajectories launched from the right (left) transporting regular island shown in Fig. 3b have their momentum shifted by $\pi (-\pi)$ on the average after each kick, indicating that these islands originate from the marginally stable point $L = (2l_1 + 1)\pi, \theta = \pm \pi/2$ with $\kappa = \pi$. Hence, in this case phase manipulation in going from KR to the MKR has both destroyed the nontransporting regular islands of KR and induced new transporting regular islands. Consider a second case with $\kappa = 10.0$. The corresponding classical phase space structures are shown in Fig. 4 (KR) and Fig. 4 (MKR) (Note that, to clearly display the transporting regular islands, only a part of one phase space cell is shown here). While there are hardly any regular islands seen in Fig. 4a, two small transporting regular islands are seen in Fig. 4b. The average momentum shift for each kick associated with these two islands is found to be $\pm 3\pi$, consistent with the fact that $\kappa = 10.0$ is close to $3\pi$.

Consider now the quantum dynamics of these systems. There have been only a few studies on the quantum dynamics of delta-kicked systems where classical chaos coexists with transporting regular islands. Of particular relevance is the previous result that the accelerator modes of KR enhance deviations from the normal DL behavior in KR [28, 29], even for systems far from the semiclassical limit. Since MKR displays new transporting islands, we therefore anticipate that DL may be strongly affected by modifying the Hamiltonian from the KR to MKR system. This is indeed seen below. The results are, however, counter-intuitive, since the classical transporting regular islands created by phase manipulation of the kicking field are found to have an area that is much smaller than the effective Planck constant.

For example, for each of KR and MKR, Figs. 5 and 6 display energy absorption for the cases of $\tau = 1.0$, $k = 5.0$ and $\tau = 1.0$, $k = 10.0$ (corresponding to $\kappa = 5$ and $\kappa = 10$), respectively. Also shown is the MKR case with $M = 3$, discussed below. It is seen that the energy absorption associated with the MKR with $M = 2$ (upper dashed line) is much larger than that of KR (solid line). Consider, for example, $E$ at a specific time $t = 1500T$ when the energy absorption of both KR and MKR has clearly shown signs of saturation (e.g., the average rotational energy may decrease with time due to statistical fluctuations). In the first case (Fig. 5), $E = 67.7$ for KR and $E = 1269.7$ for MKR. In the second case (Fig. 6), $E = 798.0$ for KR and $E = 10049.0$ for MKR. In both cases a control factor larger than an order of magnitude has been achieved in going from the KR Hamiltonian to the MKR case with $M = 2$.

This is not the case for $M = 3$, $\tau = 1.0$, $k = 5.0$ shown in Fig. 5. Here the energy absorption in the MKR with $M = 3$ is only slightly larger than in the KR and far less than in the MKR with $M = 2$. Similarly, for the case of $\tau = 1.0$, $k = 10.0$ shown in Fig. 6, although energy absorption in the MKR with $M = 3$ is much enhanced

![FIG. 3: Classical phase space structures of (a) the standard map and (b) the map of Eq. 8 with $M = 2$, in the case of $\kappa = 5.0$. All variables are in dimensionless units. Note that the small regular islands seen in panel (b) are transporting while those in panel (a) are not.](image-url)
FIG. 4: Same as in Fig. 3 except $\kappa = 10.0$. Note that the small regular islands seen in panel (b) are transporting.

FIG. 5: The time dependence of the dimensionless scaled rotational energy $\tilde{E}$ for KR (solid line), for the MKR with $M = 2$ (uppermost dashed line), and for the MKR with $M = 3$ (middle dashed curve), with $\tau = 1.0$, $k = 5.0$, and the initial state $|0\rangle$.

FIG. 6: Same as in Fig. 5 except $\tau = 1.0$, $k = 10.0$.

V. NONEXPONENTIAL LINESHAPES FOR DYNAMICAL LOCALIZATION

It was pointed out more than two decades ago [5] that the DL of KR can be mapped onto the problem of Anderson localization in disordered systems. In particular, an exactly soluble case of disorder in tight-binding models, i.e., the Lloyd model [30], suggests that dynamical localization should assume an exponential line shape, at least on the average. This has been confirmed by numerous computational studies on KR. For example, Fig. 5 clearly demonstrates, for both KR and MKR, that the distribution function $P(m)$ can be fit beautifully with an exponential function with a characteristic localization length.

However, the Hamiltonian nature of the KR and MKR implies that there always exist some subtle quantum phase correlations in the quantum dynamics. Hence, in addition to some universal properties of DL, the DL lineshape can display rich non-universal properties, e.g., the
system may display nonexponential dynamical localization. Nonexponential dynamical localization has been previously observed in KR but its origins are still poorly understood [28, 31].

Here, we demonstrate that the MKR with $M = 2$ can display strongly nonexponential line shapes for DL, rarely seen in KR. We focus on the MKR with $M = 2$ since the classical MKR with $M = 2$ has transporting regular islands that are absent in KR. Second, transporting regular islands may induce large fluctuations in DL [29]. However, we examine below cases with connections to anomalous diffusion as well as those without clear connections to anomalous diffusion. We have also studied other versions of MKR with $M \neq 2$, and have found that nonexponential lineshapes for DL in the latter case are much less common than in the $M = 2$ case.

The four cases of nonexponential lineshapes shown in Fig. 7 can be divided into two categories, based upon the properties of their underlying classical dynamics. The classical dynamics associated with the cases in Figs. 7a and 7b, shown in Figs. 3b and 3b, displays transporting regular islands. The presence of these classical structures implies an inhomogeneous classical phase space, and, as demonstrated in the previous section, may have a significant impact on the quantum dynamics even when their size is much smaller than the effective Planck constant. In this regard, the nonexponential lineshapes shown in Figs. 7a and 7b may not be totally surprising. However, for the other two MKR cases shown in Figs. 7c and 7d, we did not find any regular islands in their classical phase space even when examined on a very fine scale, suggesting that their classical dynamics is essentially fully chaotic. Thus, understanding the nonexponential lineshapes shown in Fig. 7 and Fig. 7 will be even more challenging.

The nonexponential lineshapes for DL arise from extremely small quantum fluctuations and residual quantum correlations in quantum chaos. To be able to resolve the nonexponential line shape for DL, $P(m)$ has to be known with high precision. For example, the two shoulders shown in Fig. 4 involve occupation probabilities $P(m)$ as small as $\sim 10^{-14}$. It is therefore not surprising that, while it is common to have strongly nonexponential lineshapes for DL in the MKR case, each individual lineshape is highly sensitive to the exact value of the effective Planck constant. For example, for the case shown in Fig. 5, increasing the value of $\tau$ from 2.0 to 2.0 + $10^{-5}$ can completely destroy the nonexponential lineshape! This drastic change in the lineshape for DL even occurs without causing an obvious difference in energy absorption behavior. Evidently then, both experimental observations and theoretical predictions of nonexponential dynamical localization are far from trivial and are in need of further study.

VI. DISCUSSION AND CONCLUSIONS

This paper has dealt with control of dynamical localization in kicked rotor systems. In all cases we manipulate the external kicking field to alter the properties of the rotor system, i.e. the distribution of population amongst rotor energy levels after saturation as well as the energy absorption. In particular, we have examined the effect of introducing a reversal of the kicking field after $M$ kicks.
which, within the framework of quantum mechanics, corresponds to introducing a phase shift amongst rotor energy levels.

Two parameter regimes have been examined, one which shows enhanced DL lengths with increasing \( M \), and the other which need not. This behavioral difference can be understood in terms of the character of the underlying classical phase space: the former systems are completely chaotic whereas the latter show a mixed phase space that includes transporting regular islands. Indeed, we have found that even if the transporting islands are tiny compared to the effective Planck constant, they still have a profound effect on the control of the DL. Further, a comparison of the traditional kicked rotor system to the modified kicked rotor systems shows that the latter is much more capable of displaying nonexponential dynamical localization. Thus, by modifying the kicking potential are able to control the dynamical localization in the kicked rotor.

The results of this paper are relevant to two fields of study: quantum control and kicked rotor dynamics. From the control perspective, modifying the kicking field changes the dynamics. However, this system does not obviously permit a picture in terms of interfering quantum pathways (the standard view of weak field coherent control) since (a) the kicking field is always on, and (b) there are a multitude of interfering transitions responsible for the observed behavior. Indeed, it is even difficult to isolate the interfering pathways that are responsible for dynamical localization in the simple kicked rotor, whose dynamics is easier than that of the MKR.

From the viewpoint of kicked rotor studies in the field of quantum chaos, this paper provides insights into the quantum dynamics in the case displaying classical anomalous diffusion. As one of the results of this study, we find that classical transporting regular islands can dramatically affect the quantum dynamics even when their size is much smaller than the effective Planck constant. Further, the MKR system proposed in this paper provides a new model for the study of quantum dynamics where the underlying classical chaos coexists with transporting regular islands. By choosing proper system parameters, we can create transporting regular islands whose size varies from being much larger to being much smaller than that of the accelerator modes of KR. Encouraged by this, we plan in the near future to further use the MKR to study quantum tunneling between the transporting regular island and the chaotic sea, and between transporting regular islands, and to study the phase space structure of quantum eigenstates.

In the fully chaotic case, our approach suggests the need for additional studies of dynamical localization from the Band-Random-Matrix theory perspective. For example, we have qualitatively explained the results in Sec. III in terms of well known features of Band-Random-Matrix theory. However, we found that the band width of the quantum map operator had to be defined using an extremely small cut-off value for the matrix elements, suggesting that a quantitative understanding of the MKR results such as Fig. 2 may require new models of Band-Random-Matrix ensembles.

The strongly nonexponential lineshapes for DL found in the MKR with \( M = 2 \) further demonstrate the need for more theoretical work on properties of DL. In particular, our results should motivate greater interest in characterizing and understanding nonexponential dynamical localization, with efforts directed towards explaining why nonexponential dynamical localization occurs for some system parameters and not for others. This is of importance in understanding the high sensitivity of nonexponential lineshapes for DL to the exact value of the effective Planck constant.

We have chosen the system parameters to be within the reach of current atom optics experiments on KR. Although experimental studies of nonexponential lineshapes for DL are difficult, we believe that it is straightforward to experimentally observe the results of Sec. III and Sec. IV. Apart from the atom optics realization of KR and MKR, it is also interesting to consider a molecular version of KR and MKR, i.e., diatomics periodically kicked in strong microwave fields. Preliminary computational studies confirm that directly observing quantum control of dynamical localization in molecular rotational motion is possible, e.g., in the case of quantum anomalous diffusion. Another promising experimental realization of KR and MKR requires kicked particles in a square-well potential. Along this direction an interesting model (which is very different from ours) has recently been proposed for the study of classical and quantum anomalous diffusion.

In summary, consideration of control in classically chaotic quantum systems is of general interest and importance to both fields of quantum chaos and quantum control. In this paper we have demonstrated, via a modified kicked rotor model, that dynamical localization, perhaps the best known phenomenon in quantum chaos, can be modified over a wide range. The results are of both experimental and theoretical interest.

This work was supported by the U.S. Office of Naval Research and the Natural Sciences and Engineering Research Council of Canada. HJW was partially supported by the Studien-stiftung des Deutschen Volkes and the Barth Fonds of ETH Zürich.

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