Anomaly-free ALP from non-Abelian flavor symmetry

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Abstract

Motivated by the Xenon1T excess in electron-recoil measurements, we investigate the prospects of probing axion-like particles (ALP) in lepton flavor violation experiments. In particular, we identify such ALP as a pseudo-Goldstone from the spontaneous breaking of the flavor symmetries that explain the mixing structure of the Standard Model leptons. We present the case of the flavor symmetries being a non-Abelian $U(2)$ and the ALP originating from its $U(1)$ subgroup, which is anomaly-free with the Standard Model group. We build two explicit realistic examples that reproduce leptonic masses and mixings and show that the ALP which is consistent with Xenon1T anomaly could be probed by the proposed LFV experiments.

1 Introduction

The structure of fermion masses and mixings remains one of the more mysterious and difficult questions still unanswered in high energy physics. The Standard Model (SM) accommodates the observed flavor structures through the Yukawa couplings of the fermions with the Higgs, but it cannot explain their intricate structures or even the existence of three families of fermions. The use of flavor symmetries, which generate the Yukawa couplings after spontaneous symmetry breaking, seems to be the most promising path to provide answers to these questions. In flavor models, we extend the SM with a flavor symmetry, under which the SM fermions are charged. The SM Yukawa couplings are forbidden by the symmetry, but higher dimensional operators with additional scalars, the flavons, and fermionic mediators are possible. After spontaneous symmetry breaking, the flavon vevs, normalized by the mediator masses, generate the dimensionless Yukawa couplings in a Froggatt-Nielsen mechanism [1]. In the literature, there is a large variety of models with different flavor symmetries [1–30], either continuous or discrete,
Abelian or non-Abelian, that try to explain the structure of Yukawas and neutrino masses. All these flavor symmetries can be gauged or global symmetries but both possibilities have different physical consequences. In particular, as it is well-known, when the flavor symmetry is global, its spontaneous breaking generates light (pseudo-)Goldstone bosons that can have important effects in the low-energy phenomenology.

Recently, the Xenon1T experiment reported an excess on electronic recoil events [31], indicating the possible existence of an axion-like particle (ALP) coupling to the electrons [32–34]. Then, it would be interesting to identify such ALP as the "flavored" pseudo-Goldstone boson accounting for the mixing structure of lepton sectors. However, to avoid the constraints from astrophysics observations, the global symmetry generating the ALP should be anomaly free with electromagnetic fields, resulting in additional requirements on the structure of this global symmetry. Actually such leptophilic ALP has attracted much attentions in these days [34–43].

In a recent paper [33], we studied an axion-like particle (ALP) with family-dependent charges as a possible source for the recent Xenon1T excess. In these models, we have a $U(1)_f$ global symmetry, part of a bigger flavor symmetry, spontaneously broken by the vev of a complex scalar field, $\phi$, whose angular component is identified with an ALP. The couplings of this $\phi$ field are flavor dependent (in the lepton sector) and thus the ALP will have lepton flavor-changing couplings. These couplings are basically determined by the mixing matrices diagonalizing the charged lepton Yukawas from the flavor basis with diagonal $U(1)_f$ charges. If this $U(1)_f$ symmetry is the only responsible for the observed hierarchy among the charged-lepton generations, the charged-lepton mixing matrices tend to be CKM-like. Then, ALP flavor changing effects are small and out of reach for the expected sensitivity of proposed experiments like Mu3e or MEGII-fwd.

However, from the observed PMNS matrix [44, 45], we know that leptonic mixings are large and, in fact, nearly maximal in some sectors. Thus, we can naturally expect large mixings both in the neutrino and charged-lepton mass matrices. Therefore, it is equally legitimate to explore the possibility that these large mixings come mainly from the charged-lepton or from the neutrino sector. In this paper, we follow the large charged-lepton mixing possibility and explore the associated phenomenology in the presence of a "flavored" pseudo-Goldstone boson. As pointed out in [33], in this case, we would have larger LFV effects that could be observable in the near-future proposed experiments. We will explore two different realistic flavor models with large charged-lepton mixings, able to reproduce lepton masses and mixings. Both examples are based in a non-Abelian $U(2)_f = SU(2)_f \times U(1)_f$ flavor symmetry, where the $U(1)_f$ is global and anomaly-free, with two different assignments for the $U(2)_f$ representations.

The paper is organized as follows. In the next section, we obtain the ALP couplings to SM leptons and show that they depend only on the charges and charged-lepton mixing matrices. Then we list the constraints and sensitivity from LFV experiments and Xenon1T observations. Section 3 analyzes the required ingredients to have sizeable LFV ALP couplings in flavor models. In section 4 we build two different realizations of $U(2)_f$, assigning to a doublet the first and second families in 4.1 and the second and third families in 4.2. Finally, in section 5 we present our conclusions.

2 ALP couplings and phenomenology

In this section, we present the ALP couplings with the SM fermions and the phenomenological constraints that must be taken into account for our analysis.

In general, if we have several flavon scalars charged under the $U(1)_f$ group, the pseudoscalar
parts of those fields, \( a_i \), mix to produce the physical ALP, \( a \), as
\[
a = \sum_i \frac{Q_i v_i a_i}{\sqrt{\sum_j Q_j^2 v_j^2}},
\]
where \( Q_i \) refers to the charge of the flavons under \( U(1)_f \) and \( v_i \) to the vevs.

Due to the Nambu-Goldstone nature of the ALP field, the interactions between the SM fermions and the ALP are derivative. In this case, they are flavor dependent if \( U(1)_f \) couplings are family-dependent, and, in the mass basis, are given by
\[
-\mathcal{L}_{ae} = i \frac{\partial_\mu a}{2 f_a} \bar{e}_i \gamma^\mu \left( V_{ij}^e + \gamma^5 A_{ij}^e \right) e_j,
\]
with \( f_a \approx O(v_i) \) the ALP decay constant and the axial and vector couplings defined as
\[
V_{ij}^e = \frac{1}{2} \left( U_R^{e\dagger} x_R U_R^e + U_L^{e\dagger} x_L U_L^e \right),
\]
\[
A_{ij}^e = \frac{1}{2} \left( U_R^{e\dagger} x_R U_R^e - U_L^{e\dagger} x_L U_L^e \right).
\]

Here the diagonal matrices \( x_L \) and \( x_R \) are determined by the \( U(1)_f \) charges of the corresponding left- (LH) and right-handed (RH) charged leptons:
\[
x_L = \text{Diag} (Q_1, Q_2, Q_3), \quad x_R = \text{Diag} (q_1, q_2, q_3),
\]
and \( U_R^e, U_L^e \) are the two rotations that diagonalize the lepton mass matrix
\[
U_L^{e\dagger} M_e U_R^e = \text{diag} (m_e, m_\mu, m_\tau).
\]

By using unitarity of \( U_L^e \) and \( U_R^e \), eqs. (3) and (4) can be written as:
\[
V_{ij}^e = \frac{1}{2} \left[ (q_3 + Q_3) \delta_{ij} + (q_1 - q_3) U_R^{e*} U_R^{e1i} U_R^{e1j} + (q_2 - q_3) U_R^{e*} U_R^{e2j} + (Q_1 - Q_3) U_R^{e*} U_R^{e1j} + (Q_2 - Q_3) U_R^{e*} U_R^{e2j} \right],
\]
\[
A_{ij}^e = \frac{1}{2} \left[ (q_3 - Q_3) \delta_{ij} + (q_1 - q_3) U_R^{e*} U_R^{e1i} U_R^{e1j} + (q_2 - q_3) U_R^{e*} U_R^{e2j} - (Q_1 - Q_3) U_R^{e*} U_R^{e1j} - (Q_2 - Q_3) U_R^{e*} U_R^{e2j} \right].
\]

ALP interactions, at scales much below the symmetry breaking scale, are given by these couplings, \( V^e \) and \( A^e \). From eqs. (7) and (8), it can be seen that, for non-equal charges \( O(1) \), the size of flavor-changing effects is basically determined by the LH and RH mixing, \( U_L^e \) and \( U_R^e \).

As discuss at refs.[35, 46], flavored ALPs can be searched through LFV processes such as \( \ell_j \to \ell_i a \), whose branching ratio is given by
\[
\text{BR} (\ell_j \to \ell_i a) = \frac{m_{\ell_i}^3}{16 \pi \Gamma (\ell_j)} \left| \frac{C_{ij}^e}{4 f_a} \right|^2 \left( 1 - \frac{m_{\ell_i}^2}{m_{\ell_j}^2} \right)^2,
\]
with \( \left| C_{ij}^e \right|^2 = \left| V_{ij}^e \right|^2 + \left| A_{ij}^e \right|^2 \). Limits from the LFV decays \( \ell_j \to \ell_i \gamma \) and \( \ell_j \to 3\ell_i \) can be imposed over the ALP process in such a way that, once the ALP couplings to leptons are known, they can be translated into bounds on \( f_a \). Present and expected sensitivities on the LFV transitions
 involving the ALP are collected at Table 1. The strongest restrictions come from the $\mu \rightarrow e a$ decay and imply,

$$f_a^{\text{Jodidio}} \geq (2.7 \times 10^{9}\text{ GeV}) |C_{21}^e|, \quad (10)$$

$$f_a^{\text{Mu3e}} \geq (1.6 \times 10^{10}\text{ GeV}) |C_{21}^e|. \quad (11)$$

In addition, there is an interesting proposal at PSI, the MEGII-fwd [35]. This proposal is based on a detector in the forward direction, where the SM background is suppressed, to collect energetic forward positrons. If we assume perfect $\mu^+$ polarization and with the detector in the forward direction, this decay is only sensitive to $|V_{21}^\nu + A_{21}^\nu|^2$ and not to $(V - A)$ couplings [35]. Therefore, the bound on $f_a$ would be,

$$f_a^{\text{MEGII-fwd}} \geq (1.2 \times 10^{10}\text{ GeV}) |V_{21}^\nu + A_{21}^\nu|, \quad (12)$$

where the most stringent constraint is obtained in the case of $V_{21}^\nu = A_{21}^\nu$. Moreover, if we require the ALP to explain the Xenon1T excess [31], it can only constitute the $\sim 7\%$ of the total dark matter abundance and its coupling to electrons must be$^6$ [32]:

$$A_{11}^e \simeq 10^{-13} \frac{f_a}{m_e}, \quad \text{for } m_a \in [2, 3] \text{ keV}. \quad (13)$$

Furthermore, for this range of masses, the ALP has to be anomaly-free, $\sum_i Q_i = \sum_i q_i = 0$, so that its interaction with photons is very suppressed and can evade present limits on X-ray emissions.

### 3 Building anomaly-free global flavour models

As seen in the previous section, ALP flavor-changing couplings in the lepton sector are strongly dependent on the unitary matrices diagonalizing the charged-lepton Yukawa matrix. Additionally, the mass of the ALP and its decay constant are fixed if we require it to explain the Xenon1T

$^6$In ref. [32], eq.(2) and discussion below, it is explained that the Xenon1T excess implies $f_a/q_e \simeq \sqrt{r} 10^{10}$ GeV, with $r$ the fraction of DM constituted by the ALP. Then, $g_{ae} \simeq 5 \times 10^{-14}/\sqrt{r}$. In our notation, $g_{ae}$ is the pseudoscalar coupling of the axion with the electron, $P_{11}^a/f_a$, with $P_{ij}^a = (m_i + m_j) A_{ij}^a$. Then, for $r = 0.07$, $A_{11}^e \simeq 2 \times 10^{-13}/(2 m_e) f_a = 10^{-13} f_a/m_e$. As our models fix $A_{11}^e$, a prediction for $f_a$ can be derived as $f_a = A_{11}^e m_e/10^{-13}$.

| Lepton decay | BR limit | Experiment |
|--------------|----------|------------|
| $\text{BR}(\mu \rightarrow e a)$ | $< 2.6 \cdot 10^{-6}$ | Jodidio et al. [47] |
| $\text{BR}(\tau \rightarrow e a)$ | $< 2.1 \cdot 10^{-5}$ | TWIST [48] |
| $\text{BR}(\tau \rightarrow e a \gamma)$ | $< 1.1 \cdot 10^{-9}$ | Crystal Box [49] |
| $\text{BR}(\tau \rightarrow e a)$ | $< 2.7 \cdot 10^{-3}$ | ARGUS [50] |
| $\text{BR}(\tau \rightarrow \mu a)$ | $< 4.5 \cdot 10^{-3}$ | ARGUS [50] |

Table 1: Limits over the axion decay constant from lepton decays. Belle-II limits are derived from the simulated result at Belle [53] by rescaling the luminosity [35].
excess. The absence of the electromagnetic anomaly to evade X-ray constraints puts an additional restriction on the fermionic charges. Following the strategy of our previous paper [33], we require anomaly cancellation with the SM particle content, without the addition of new fermions, and this leads us to consider flavor dependent charges.

In our scenario, the flavor symmetries describing fermion masses must include a global $U(1)$ group with non-universal charges and these charges must cancel the electromagnetic anomaly. Therefore, we have to assign different $U(1)$ charges to the three charged-leptons above the scale of flavor symmetry breaking. This implies that the three charged-leptons can not belong to a single triplet of flavor, which would have a single $U(1)$ charge, but there must be three different representations of dimension 1 or, at most, a doublet and a singlet in the left and right-handed sectors. Hence, we will consider a $U(1)_f$ and a $U(2)_f = SU(2)_f \times U(1)_f$ symmetries as examples of these two situations.

### 3.1 A $U(1)_f$ flavor symmetry

$U(1)$ flavor symmetries were used in the first attempts to explain the structure of fermion masses and mixings and they remain a viable option [1–3, 6, 7, 9, 11, 12, 18]. However, we must impose the additional requirement of anomaly cancellation with the leptonic charges, $-Q_1 - Q_2 = Q_3$ and $-q_1 - q_2 = q_3$.

If the Yukawa Lagrangian is

$$
\mathcal{L}_Y \supset Y^e_{ij} \tilde{e}_i L \tilde{H}_2 e_j + Y^\nu_{ij} \tilde{e}_i L H_2 \nu_j + M^\nu_{ij} \nu_i R \nu^c_j,
$$

we can always use the Higgs charge to make the $(3,3)$ element $O(1)$ or the required power, i.e.

$$
-Q_3 + q_3 + q_2 \geq 0,
$$

and we take the flavon charge $q = -1$. The required hierarchy imposes then, $(-Q_3) \leq (-Q_2) \leq (-Q_1)$ and $q_3 \leq q_2 \leq q_1$ with $(-Q_1), q_1 \geq 0$ and $(-Q_3), q_3 \leq 0$ (remember that we impose that the sum of charges must be zero, independently for left and right fields).

Now, we can consider, for simplicity, the $2–3$ sector, and we have,

$$
Y^e / y_r = \begin{pmatrix} 
\varepsilon Q_1 - Q_2 + q_1 - q_3 & \varepsilon Q_1 - Q_2 \\
\varepsilon q_2 - q_3 & 1
\end{pmatrix} = \begin{pmatrix} 
\varepsilon Q_1 - 2Q_2 + q_1 + 2q_2 & \varepsilon Q_1 - 2Q_2 \\
\varepsilon q_1 + 2q_2 & 1
\end{pmatrix},
$$

where, we have suppressed $O(1)$ coefficients in all the entries. Now, we must require that the second eigenvalue, the muon mass, is $O(\lambda_c^2)$. If we take $\varepsilon = \lambda_c$, we must have $-Q_1 - 2Q_2 + q_1 + 2q_2 = 2$, and the only possibilities would be: i) $-Q_1 - 2Q_2 = 1$ and $q_1 + 2q_2 = 1$, ii) $-Q_1 - 2Q_2 = 0$ and $q_1 + 2q_2 = 2$ and iii) $-Q_1 - 2Q_2 = 2$ and $q_1 + 2q_2 = 0$. From here, we can see that only the solution ii) would give large leptonic mixing in the left-handed charged lepton sector. However, in this case, we have $Q_1 = -2Q_2$ and $q_1 = 2 - 2q_2$. Given that $(-Q_1), q_1 \geq (-Q_2), q_2$, we have necessarily $(-Q_2), q_2 \leq 0$. Moreover, if we require $m_e / m_\mu \simeq \lambda_c^2 (m_e / m_\mu \simeq \lambda_c^2)$, the only possibilities would be $Q_2 = 0$ and $q_2 = 0$ ($Q_2 = 0$ and $q_2 = -1/3$).

Notice this structure is not changed if we add the first generation, as we have chosen the charges hierarchical and the first row or the first column can be at most of the same order as the elements in this submatrix. Therefore they can not change the order of magnitude of these elements. Furthermore, the charges of the first generation are already fixed by this submatrix and the anomaly cancellation condition. The full $3 \times 3$ matrix with $Q_2 = 0$ and $q_2 = -1/3$ is

$$
Y^e / y_r = \begin{pmatrix} 
\varepsilon^5 & \varepsilon^2 & 1 \\
\varepsilon^5 & \varepsilon^2 & 1 \\
\varepsilon^5 & \varepsilon^2 & 1
\end{pmatrix}.
$$

As we can see, this matrix gives rise to $O(1)$ LH mixings, but, in principle, small, CKM-like RH mixing. Nevertheless, the large LH mixings are due to $Q_1 = Q_2 = Q_3$ and therefore from
eqs. (7)–(8), left-handed mixings disappear in offdiagonal $V^e$ and $A^e$ ALP couplings. Thus, ALP couplings will depend only on the small RH mixings and therefore will be out of reach for the near future experiments. Unfortunately, the situation is the same in case iii) with $O(1)$ RH mixings, but equal RH charges and even case i), where we have medium, but still small, LH and RH mixings, can not reach the required size for the near future experiments.

**3.2 A $U(2)_f$ flavor symmetry**

Let us consider now the flavour group $U(2)_f = SU(2)_f \times U(1)_f$ [8, 10, 54–62]. Here we group the three generations in a doublet and a singlet of $SU(2)_f$ with different $U(1)_f$ charges, giving then rise to FC couplings. We consider two possibilities:

i) **12-doublet case** — LH and RH fermions of the third generation transform as singlets of $SU(2)_f$ whilst those of the first and second generation belong to doublets,

$$\ell_L = \begin{pmatrix} \bar{e}_L \\ \bar{\mu}_L \end{pmatrix}, \quad \bar{\nu}_L = \begin{pmatrix} \bar{\nu}_eL \\ \bar{\nu}_\mu L \end{pmatrix}, \quad \ell_R = \begin{pmatrix} e_R \\ \mu_R \end{pmatrix}. \quad (17)$$

ii) **23-doublet case** — LH and RH fermions of the first generation are singlets while those of the second and third generation transform as doublets,

$$\ell_L = \begin{pmatrix} \bar{\mu}_L \\ \bar{\tau}_L \end{pmatrix}, \quad \bar{\nu}_L = \begin{pmatrix} \bar{\nu}_\mu L \\ \bar{\nu}_\tau L \end{pmatrix}, \quad \ell_R = \begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}. \quad (18)$$

Considering $Q$ and $q$ the $U(1)_f$ charges of the LH and RH fields, the following charge matrices determine the ALP couplings to charged leptons in the 12– and 23–doublet case, respectively:

$$x_L^{(12)} = \text{Diag}(Q, Q, -2Q), \quad x_R^{(12)} = \text{Diag}(q, q, -2q). \quad (19)$$

$$x_L^{(23)} = \text{Diag}(-2Q, Q, Q), \quad x_R^{(23)} = \text{Diag}(-2q, q, q). \quad (20)$$

Notice that eqs. (7) and (8) can be simplified if the universal part is separated from the $x_L$ and $x_R$ charge matrices:

$$V^e_{ij} = \frac{1}{2} \left[ (q + Q) \delta_{ij} - 3q U^e_{R,si} U^e_{R,sj} - 3Q U^e_{L,si} U^e_{L,sj} \right], \quad (21)$$

$$A^e_{ij} = \frac{1}{2} \left[ (Q - q) \delta_{ij} - 3q U^e_{R,si} U^e_{R,sj} + 3Q U^e_{L,si} U^e_{L,sj} \right], \quad (22)$$

with $s = 3$ for the 12–doublet case and $s = 1$ for the 23–doublet case. From eq. (22), the Xenon1T requirement in eq. (13) can be recast as:

$$f^\text{X1T}_a = (2.5 \times 10^9 \text{ GeV}) \left| q - Q + 3Q \left| U^e_{L,si} \right|^2 - 3q \left| U^e_{R,si} \right|^2 \right|. \quad (23)$$

A general view can be obtained from eqs. (21)-(23) by setting $Q = -q = 1$, assuming $U^e_R \equiv U^\text{PMNS}_R$ and $U^e_L$ totally general. Figure 1 displays, in these conditions, the prediction of the ALP decay constant compatible with the Xenon1T result in the 12–doublet ($s = 3$, blue region) and 23–doublet case ($s = 1$, violet region). The dashed and continuous black ellipses correspond to the regions where the RH mixing is CKM- and PMNS-like, respectively. The different shape of the blue and violet regions can be understood from eqs. (21)-(23). For $Q = -q = 1$,

$$A^e_{11} = \left( -1 + 3/2 \left( \left| U^e_{L,31} \right|^2 + \left| U^e_{R,31} \right|^2 \right) \right), \quad (24)$$

$$C^e_{12} = \frac{3\sqrt{2}}{2} \left( \left| U^e_{R,31} U^e_{R,32} \right|^2 + \left| U^e_{L,31} U^e_{L,32} \right|^2 \right)^{1/2}, \quad (25)$$

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Figure 1: Values of the ALP decay constant compatible with the Xenon 1T result assuming a $U(2)_f$ flavor symmetry in the 12-doublet-case (blue region) and 23-doublet-case (violet region) and $Q = -q = 1$. The horizontal lines display the bounds in eqs.(10)-(11), where the continuous line is the current bound while the dashed lines indicate the limits expected from future experiments. The MEGII-fwd line has been drawn, as a reference, in the limit $V_{e2} = A_{e2}$. The full constraint must be obtained in each case from eq.(11). The LH-mixing is fixed to reproduce the PMNS at the 3σ level while the RH-mixing is completely general. The circles identify the regions where the RH-rotation is CKM-like (dashed circles) and PMNS-like (continuous circles). The stars show the predictions corresponding to the benchmark points in Table 3 and 6.

in the 12-doublet case or

$$A_{11}^c = \left(-1 + \frac{3}{2}\left(|U_{e,11}^c|^2 + |U_{R,11}^c|^2\right)\right),$$

$$C_{21}^c = \frac{3\sqrt{2}}{2} \left(|U_{e,11}^c U_{R,12}^c|^2 + |U_{e,11}^c U_{L,12}^c|^2\right)^{1/2},$$

in the 23-doublet case.

Given that $U_{e,1}^L$ is presumed to be PMNS-like, $|U_{e,31}^L| \sim 0.5$, $|U_{e,32}^L| \sim 0.6$, $|U_{e,11}^L| \sim 0.8$ and $|U_{e,12}^L| \sim 0.55$. If $U_{e,1}^R$ ranges from small to large values, it is understood that in the 23-doublet case $A_{11}^c$ can reach a factor of two larger than in the 12-doublet case. Similarly, as $f_a = 5 \times 10^9$ GeV,$A_{11}^c$, the slope of the region is given by $5 \times 10^9$ GeV/$|C_{12}^c|$ and in the 12-doublet case, for small $U_{e,1}^R$, the slope is proportional to $1/|C_{12}^e| \approx 1/0.3 \approx 3.3$ while, in the 23-doublet case, it is $1/|C_{12}^c| \approx 1/0.45 \approx 2.3$.

The horizontal lines display the bounds in eqs. (10)-(11), where the continuous line is the current bound while the dashed lines indicate the limits expected from future experiments. It is observed that for both scenarios most of the allowed region by present bounds is testable at future experiments. Only for the 12-doublet case, a small area above the expected Mu3e limit remains unconstrained.
4 Explicit $U(2)_f = SU(2)_f \times U(1)_f$ flavour models

Let us consider now explicit models with $U(2)_f$ flavor group. These models must reproduce lepton masses and mixings and simultaneously explain the Xenon1T excess. Additionally, we will investigate lepton flavor violation effects in these models and look for points that could be observed at the next generation of experiments.

As stated before, two possibilities for the leptonic sector are considered: one, where the first and second generation constitute a doublet and the third transforms as singlet, and another, where the role of the first and third generations are exchanged.

In this case, both Higgs doublets, which corresponds to a type-X 2HDM\(^7\), transform as singlets under $SU(2)_f$ but have different $U(1)_f$ charges. In either case three flavons are added: two transforming as doublets under $SU(2)_f$, $\Phi_1$ and $\Phi_2$, and one as a singlet, $\chi$. The flavor symmetry is spontaneously broken when the flavons get non-zero vevs in the directions:

$$\langle \Phi_1 \rangle \propto \Lambda \left(0 \varepsilon_1\right), \quad \langle \Phi_2 \rangle \propto \Lambda \left(\varepsilon_2 \varepsilon_3\right), \quad \langle \chi \rangle \propto \Lambda \varepsilon_\chi. \quad (28)$$

As we show below, the three vevs are of the same order, $\varepsilon_1 \sim \varepsilon_2 \sim \varepsilon_\chi$\(^8\). Here we note that all the examples below introduce three RH neutrinos that generate the neutrino masses through a type-I see-saw mechanism.

4.1 The 12-doublet case

| Fields   | $H_1$ | $H_2$ | $\tilde{\ell}_L, \tilde{\ell}_R$ | $\nu_e, \nu_\mu, \nu_\tau$ | $\tau_L, \tau_R$ | $\chi$ |
|----------|-------|-------|-------------------------------|--------------------------|-----------------|-------|
| SU(2)    | 1     | 1     | 2                             | 2                        | 1               |       |
| U(1)     | 0     | 2/3   | -1/3                          | -1/6                     | 2/3             | 1     |

Table 2: Particle content and charge assignment under $U(2)_f$ for the 12-doublet case.

The spectrum and quantum numbers for this version of the $U(2)_f$ model are detailed in table 2. From the symmetries, the most general set of non-renormalizable operators for charged leptons, at leading order in the flavon vevs, is given by the following couplings with $\Phi_2$:

$$-\mathcal{L}_c^{(12)} = \frac{c_5}{\Lambda^2} \left(\tilde{\ell}_L \tilde{\Phi}_2\right) \left(\ell_R \Phi_2\right) H_2 + \frac{c_6}{\Lambda^2} \left(\Phi_2 \tilde{\ell}_L\right) \chi \tau_R \tilde{H}_2 + \frac{c_5}{\Lambda^2} \tilde{\tau}_L \left(\Phi_2 \ell_R\right) \chi \tilde{H}_2 + \frac{c_5}{\Lambda^2} \tau_R \chi^2 \tilde{H}_2 + \text{h.c.,} \quad (29)$$

where $\tilde{\phi} = i \sigma_2 \phi^*$. The parenthesis in the operators make explicit the $SU(2)_f$ contractions of the fields. The first order corrections to the terms in eq.(29) come from the substitution $\tilde{\Phi}_2 \to \Phi_1 \chi$ as well as $\chi \to \tilde{\Phi}_1 \Phi_2$:

$$-\delta \mathcal{L}_c^{(12)} = \frac{c_5}{\Lambda^2} \left(\tilde{\ell}_L \Phi_1\right) \left(\tilde{\Phi}_2 \ell_R\right) \chi \tilde{H}_2 + \frac{c_6}{\Lambda^2} \left(\tilde{\ell}_L \tilde{\Phi}_2\right) (\Phi_1 \ell_R) \chi \tilde{H}_2 + \frac{c_5}{\Lambda^2} \tilde{\tau}_L (\ell_R \Phi_1) \chi^2 \tilde{H}_2 + \text{h.c.} \quad .$$

\(^7\)In type-X 2HDM, $H_1$ and $H_2$ couple only to quarks and leptons respectively due to an additional $Z_2$ symmetry. Our models are formulated in the large $\tan \beta = v_1/v_2$ regime where the leptophobic doublet can be identified with the SM Higgs in good approximation.

\(^8\)The alignment of these vevs, in particular of $\langle \Phi_2 \rangle$, is motivated by the presence of large leptonic mixing. This non-trivial structure can be obtained with an adequate scalar potential, (see for instance [17, 63–66]).
From eqs. (29)-(30), the resulting mass matrices are

\[
M^{(12)}_{\nu} = \frac{vH_2}{\sqrt{2}} \left( \begin{array}{ccc}
\varepsilon_1 & c_1^e & \sqrt{2} c_2^e \varepsilon_1 \\
c_1^e & \varepsilon_1 & \sqrt{2} c_2^e \varepsilon_1 \\
\sqrt{2} c_3^e \varepsilon_1 & \sqrt{2} c_3^e \varepsilon_1 & \varepsilon_1
\end{array} \right),
\]

(30)

\[
\delta M^{(12)}_{\nu} = \frac{vH_2}{\sqrt{2}} \varepsilon_1 \varepsilon_2 \left( \begin{array}{ccc}
c_6^\nu + c_9^\nu & \sqrt{2} c_7^\nu \varepsilon_1 & 0 \\
c_6^\nu & \varepsilon_1 & 0 \\
\sqrt{2} c_8^\nu \varepsilon_1 & 0 & \sqrt{2} c_9^\nu
\end{array} \right).
\]

(31)

In the neutral sector, the Yukawa and Majorana terms that contribute at leading order are\(^9\):

\[
-L^{(12)}_{\nu} = \frac{c_\nu^\nu}{\Lambda} (\Phi_2 \bar{\nu}_L) \nu_{\mu R} H_2 + \frac{c_\nu^\nu}{\Lambda^2} \left( \hat{\Phi}_1 \bar{\nu}_L \nu_{\mu R} H_2 + \frac{c_\nu^\nu}{\Lambda^2} \lambda^2 \bar{\nu}_L \nu_{\tau R} H_2 + \frac{c_\nu^\nu}{\Lambda^3} (\Phi_1 \bar{\nu}_L) \left( \tilde{\Phi}_1 \Phi_2 \right) \nu_{\mu R} H_2 + \frac{c_\nu^\nu}{\Lambda^3} (\tilde{\nu}_L \tilde{\Phi}_1) \chi^2 \nu_{\tau R} H_2 + \text{h.c.},
\]

(32)

\[
L^{(12)}_M = M_0 \left[ \frac{c_\nu^\nu}{2\Lambda} \chi^\dagger \tilde{\nu}_{\mu R} \nu_{\mu R} + \frac{c_\nu^\nu}{2\Lambda} \chi^\dagger \left( \tilde{\nu}_{\mu R} \nu_{\tau R} + \tilde{\nu}_{\tau R} \nu_{\mu R} \right) + \frac{c_\nu^{10}}{2\Lambda^2} \chi^2 \nu_{\mu R} \nu_{\mu R} + \frac{c_\nu^{11}}{2\Lambda^3} (\tilde{\Phi}_2 \Phi_1) \chi \nu_{\tau R} \nu_{\tau R} \right] + \text{h.c.},
\]

(33)

which produce the following Yukawa and Majorana matrices

\[
Y^{(12)}_{\nu} = \frac{\varepsilon_2}{\sqrt{2}} \left( \begin{array}{ccc}
0 & -c_1^\nu + c_6^\nu \varepsilon_1 & \left( c_2^\nu + c_9^\nu \varepsilon_1 \varepsilon_2 \right) \varepsilon_1 \\
0 & c_1^\nu + c_6^\nu \varepsilon_1 \varepsilon_2 & c_2^\nu \varepsilon_1 \\
0 & c_3^\nu \varepsilon_1 & \sqrt{2} c_8^\nu \varepsilon_1 \varepsilon_2
\end{array} \right),
\]

(34)

\[
M^{(12)}_{\nu} = M_0 \varepsilon_1 \left( \begin{array}{ccc}
c_6^\nu & 0 & 0 \\
c_1^\nu \varepsilon_1 & c_7^\nu & 0 \\
0 & c_8^\nu & \sqrt{2} c_9^\nu \varepsilon_1 \varepsilon_2
\end{array} \right).
\]

(35)

No coupling involving the RH electron neutrino is present in eqs. (32) and (33), except for the \(\tilde{\nu}_{\nu R} \nu_{\nu R}\) vertex. This is because the \(U(1)_Y\) charge of this field has been chosen so that it cannot be compensated by any combination of fields, unless it appears in pairs\(^10\). As a consequence, the zeros appearing in the Yukawa and Majorana matrices of eqs. (34) and (35) are exact at all orders and lead to one massless active neutrino. As in this formulation the first and second generation of LH neutrinos are in the same doublet, the natural scenario is having \(m_{\nu_3} = 0\), which implies an inverted hierarchy.

\(^9\)Equations (32) and (33) receive NLO corrections from the replacement of flavons discussed above eq. (30). However, these NLO operators contribute to exactly the same entries that the leading terms, so they can be safely reabsorbed in the leading \(O(1)\) coefficients by the redefinition \(c_i \to c_i - c_i \varepsilon_1 \varepsilon_2 / (\sqrt{2} \varepsilon_1)\).

\(^10\)Unlike \(\nu_{\mu R}\) and \(\nu_{\tau R}\), the \(\nu_{\nu R}\) charge is an odd multiple of \(q_\chi / 2\).
Regarding the ALP phenomenology, the benchmark point of the model produces a decay constant equal to

$$f_a = 1.3 \times 10^9 \text{ GeV}. \quad (39)$$

In figure 1, the position of our benchmark point in the (scaled) ALP parameter space is marked with a yellow diamond in the blue area\(^{11}\). As can be seen, an axial coupling to electrons of

\(^{11}\)Notice that the blue and purple regions are computed considering $Q = -q = 1$ while the charges of our model are $Q = -q = 1/3$
$|A_{11}| = 0.3$ is generated while its flavor-violating couplings in the 1-2 sector make feasible to prove its presence through the $\mu \rightarrow e\alpha$ decay at Mu3e. However, it is not detectable at MEGII-fwd which can only test up to $f_s/|C_{21}^s| = 8.9 \times 10^8$ GeV for couplings like the ones derived in this scenario ($A_{21} \approx -0.9V_{21}$).

4.2 The 23-doublet case

| Fields | $H_1$ | $H_2$ | $\nu_{eL}, \tilde{\nu}_{eL}$ | $\nu_R$ | $\nu_{\mu R}$ | $\nu_{\tau R}$ | $\tilde{\nu}_L, \tilde{\nu}_R$ | $\ell_R$ | $\Phi_1$ | $\Phi_2$ | $\chi$ |
|--------|-------|-------|-------------------|---------|------------|-------------|-------------------|-------|-------|-------|-------|
| SU(2)  | 1     | 1     | 1                 | 1       | 1          | 1            | 2                 | 2     | 2     | 2     | 1     |
| U(1)   | 0     | 2/3   | 2/3               | -1/12   | 1/2        | -2/3         | -1/3              | -1/3  | 5/6   | -2/3  | -1/6  |

Table 5: Particle content and charge assignment under $U(2)_f$ for the 23-doublet case.

This version of the $U(2)_f$ model is very similar to the previous 12-doublet case but with $\epsilon_L$, $\epsilon_R$ the singlets under $SU(2)_f$. The charges of $\Phi_1$ and $\chi$ under the $U(1)_f$ symmetry are also slightly modified, as can be seen in Table 5.

The leading operators in the charged sector are exactly those in eqs. (29) and (30) but making the replacements:

$$\tau_R \rightarrow \epsilon_R, \quad \tau_L \rightarrow \epsilon_L, \quad \chi \rightarrow \chi^2. \quad (40)$$

In view of this, the mass matrix for the charged leptons can then be precisely obtained as $M^{(23)}_e = P^T_{321} M^{(23)}_e \chi_1 \rightarrow \chi^2 \chi_2, P_{231}$, where $P_{231}$ is the 231 column permutation matrix of the identity. The resulting charged lepton mass matrix and its first order correction read as

$$M^{(23)}_e = \frac{\nu H_2 \epsilon_2^2}{\sqrt{2}} \left( \begin{array}{ccc} 2c_4^2 & 2c_2 \frac{\epsilon_1}{\epsilon_2} & \sqrt{2}c_2 \frac{\epsilon_1}{\epsilon_2} \\ \sqrt{2}c_2 \frac{\epsilon_1}{\epsilon_2} & c_1^2 & c_1^2 \\ \sqrt{2}c_2 \frac{\epsilon_1}{\epsilon_2} & c_1^2 & c_1^2 \end{array} \right), \quad (41)$$

$$\delta M^{(23)}_e = \frac{\epsilon_1 \epsilon_2 \epsilon_3}{2} \left( \begin{array}{ccc} 0 & \sqrt{2}c_2 \frac{\epsilon_1}{\epsilon_2} & \epsilon_1 \epsilon_2 \\ \sqrt{2}c_2 \frac{\epsilon_1}{\epsilon_2} & c_1^2 + c_4^2 + c_6^2 & c_1^2 + c_4^2 + c_6^2 \\ \epsilon_1 \epsilon_2 & c_1^2 + c_4^2 + c_6^2 & 0 \end{array} \right). \quad (42)$$

At variance with the 12-doublet case, in this model, given that the LH neutrinos of the second and third generations belong to the same doublet, a scenario with normal hierarchy for the neutrino mass spectrum, $m_{1\nu} \ll m_{2\nu} < m_{3\nu}$, is the natural prediction. With the charge assignment in Table 5, in the neutral sector the following leading order Majorana and Yukawa terms can be written in the Lagrangian

$$-\mathcal{L}^{(23)}_\nu = \frac{c_4}{\Lambda} (\tilde{\Phi}_1 \tilde{\nu}_L) \nu_{\mu R} H_2 + \frac{c_4^2}{\Lambda} (\tilde{\Phi}_2 \tilde{\nu}_L) \chi^2 \nu_{e R} H_2 + \frac{c_6^2}{\Lambda} (\tilde{\Phi}_1 \tilde{\nu}_L) \chi^2 \nu_{\mu R} H_2 + \frac{c_6^2}{\Lambda} (\tilde{\Phi}_1 \tilde{\nu}_L) \chi^2 \nu_{\tau R} H_2 + \text{h.c.}, \quad (43)$$

$$\mathcal{L}^{(23)}_M = M_0 \chi \left[ \frac{c_4}{\Lambda} (\tilde{\nu}_{e R} \nu_{e R} + \tilde{\nu}_{\mu R} \nu_{\mu R}) + \frac{c_4^2}{\Lambda} \tilde{\nu}_{e R} \nu_{\tau R} \right] + \text{h.c.} \quad (44)$$
\[ \chi^2_{3\sigma} \begin{array}{cccccccc} c_1 & c_2 & c_3^\prime & c_4 & c_5 & c_6 & c_7 & c_8 & c_9^\prime \\ 3.69 & 1.649 & -0.451 & -1.74761 & -0.728 & -1.883 & 2.152 & 0.863 & -2.343 & 2.914 \\ \end{array} \]

F.T. \[ \begin{array}{cccccccc} c_1^\prime & c_2^\prime & c_3^\prime & c_4^\prime & c_5^\prime & c_6^\prime & \varepsilon_1 & \varepsilon_2 & \varepsilon_\chi \\ 22 & -0.509 & 0.844 & -1.309 & -1.041 & 2.572 & 2.775 & 0.371 & 0.389 & 0.246 \\ \end{array} \]

| \( \theta_{12}^{\text{NH}}(^0) \) | \( \theta_{23}^{\text{NH}}(^0) \) | \( \theta_{13}^{\text{NH}}(^0) \) | \( m_\nu/m_\tau \) | \( m_\mu/m_\tau \) | \( \Delta m^2_{21}/\Delta m^2_{31} \) |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| b.f. \( \begin{array}{cccccccc} 33.56 & 48.0 & 8.55 & 2.88 \times 10^{-4} & 0.059 & 0.029 \\ \end{array} \) |
| exp. \( \begin{array}{cccccccc} 33.44 & 49.2 & 8.57 & 2.88 \times 10^{-4} & 0.059 & 0.029 \\ \end{array} \) |
| 3σ \( \begin{array}{cccccccc} 31.27 & -35.86 & 39.5 & 52.0 & 8.20 & -8.97 & 2.79 & -2.96 \times 10^{-4} & 0.0577 & 0.0612 & 0.027-0.033 \\ \end{array} \) |

Table 6: Values of the model parameters at the benchmark point with \( c_7^\prime = c_9^\prime = 1 \).

Table 7: Values of the measured observables from the NuFIT 5.1 (2021) global fit [45] and values obtained from the 23-model at the benchmark point. The absolute mass values can be obtained with \( v_{H_2} = v_H/26 \) and \( M_0 \equiv 3.17 \times 10^{10} \text{ GeV} \).

Similar to the previous case, we chose the \( U(1)_f \) charge of the RH tau neutrino to be a fraction of the smallest flavon charge: \( q_{\tau}\text{RH} = q_\chi/2 \), hence it is never compensated by the considered fields. Thus, by construction, in \( \mathcal{L}_\nu^{(23)} \) there are no couplings involving the RH tau neutrino. The resulting Yukawa and Majorana mass matrices display the following textures

\[
Y_{\nu}^{(23)} = \frac{\varepsilon_1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} c_2^\prime \varepsilon_1 & c_3^\prime & 0 \\
-c_2^\prime \varepsilon_1 & c_3 & c_4^\prime \\
c_5^\prime & -c_5 & c_6^\prime \\
\end{pmatrix}, \tag{45}
\]

\[
M_{\nu}^{(23)} = M_0 \varepsilon_\chi \begin{pmatrix}
0 & c_7^\prime & 0 \\
c_2^\prime & 0 & 0 \\
0 & 0 & c_8^\prime \\
\end{pmatrix}. \tag{46}
\]

Once more, the zeros in the third column of eq. (45) and the third column and row of eq. (46) are exact at all orders and lead to a simplified scenario with one massless neutrino. In particular, the simplification of \( m_{\nu_1} = 0 \), implies \( m_\nu = \sqrt{\Delta m^2_{21}} \) and \( m_\nu = \sqrt{\Delta m^2_{31}} \).

The effective neutrino mass matrix after the seesaw exhibits a the following structure:

\[
m_{\nu}^{(23)} = -\frac{v_{H_2}^2 \varepsilon_1 \varepsilon_2 \varepsilon_\chi}{2 M_0} \begin{pmatrix}
2 a_6 \varepsilon^4_\chi & a_5 \frac{\varepsilon_\chi}{\varepsilon_1} & a_4 \frac{\varepsilon_\chi}{\varepsilon_2} \\
-a_5 \frac{\varepsilon_\chi}{\varepsilon_1} & 2 a_2 \frac{\varepsilon_2 \varepsilon_\chi}{\varepsilon_1} & a_4 - a_3 \frac{\varepsilon_1}{\varepsilon_2} \\
a_4 \frac{\varepsilon_\chi}{\varepsilon_2} & a_1 - a_3 \frac{\varepsilon_1}{\varepsilon_2} & 2 a_1 \\
\end{pmatrix}, \tag{47}
\]

where the effective parameters are related with the parameters of the lagrangian as

\[
a_1 = c_1^\prime c_2^\prime, \quad a_2 = c_2^\prime c_5^\prime, \quad a_3 = c_1^\prime c_6^\prime, \quad a_4 = \sqrt{2} c_1^\prime c_4^\prime, \quad a_5 = \sqrt{2} c_4 c_5^\prime, \quad a_6 = \sqrt{2} c_3 c_4^\prime. \tag{48}
\]

In Table 6 and 7, the values of the coefficients and observables at the benchmark point are presented. From Table 7 we see that the PMNS is correctly reproduced even at the 1σ level, being the neutrino contribution 17% for the \( \theta_{23} \) angle, 50% for the \( \theta_{12} \) angle, and 80% for the \( \theta_{13} \) angle. From the Table 7, the values of the \( H_2 \) vev and the Majorana mass scale \( M_0 \)
can be determined by imposing to reproduce the absolute values of $m_\tau$ and $\Delta m^2_{31}$, then we have $v_{H_2} = v_H/26$ and $M_0 = 3.17 \times 10^{10}$ GeV. The prediction for the axial ALP coupling to electron is $|A^e_{11}| = 0.342$. Consequently, from eq.(13), the prediction for the ALP decay constant compatible with the Xenon 1T result reads

$$f_a^{\text{Xenon 1T}} = 1.75 \times 10^9 \text{ GeV},$$

(49)

From the requirement of large mixings to come from the charged lepton sector, we obtain large LFV axion couplings with $|A^e_{21}| \gg |V^e_{21}|$, in particular $|C^e_{21}| = 0.409$ and $|V^e_{21} + A^e_{21}| = 0.484$. With these values the bounds in eq.(10-12) translates into: $f_a^{\text{Jodidio}} \geq 1.10 \times 10^9$ GeV, $f_a^{\text{Mu3e}} \geq 6.5 \times 10^9$ GeV and $f_a^{\text{MEGII-fwd}} \geq 5.8 \times 10^9$ GeV. So, while the point evades the current limit imposed by the Jodidio et al. experiment, it lies within a region that will be fully tested by the future Mu3e and MEGII-fwd experiments, as it is displayed in Figure 1 (yellow diamond inside the violet region).

5 Conclusions

Motivated by the Xenon1T excess in electron-recoil measurements, we studied the possible lepton flavor violation effects from an ALP explaining this excess and originating from the spontaneous breaking of a flavor group. This flavor group is built to explain the masses and mixing patterns in the lepton sector.

A priori, we have two options for flavor symmetries with a non-anomalous $U(1)$ subgroup and flavor dependent charges, $U(1)$ or $U(2)$. We have seen that a plain $U(1)$ symmetry can not provide sizeable flavor-changing couplings reachable in near future experiments. Therefore, we have concentrated in the flavor group $U(2)_f = SU(2)_f \times U(1)_f$. We considered two scenarios, where the first-second or second-third generations of leptons are grouped in a doublet representation of $SU(2)_f$. In these models, we require the symmetry to reproduce correctly lepton masses and mixing with $O(1)$ coefficients and sizeable mixings in the charged lepton sector. We find that, in both scenarios, the ALP consistent with the Xenon1T anomaly could be probed by future LFV experiments. Interestingly, we find that for the 1-2-doublet case, an inverted hierarchy of neutrino masses is preferred while for the 2-3-doublet case, a normal hierarchy of the neutrino masses is more likely. Therefore, if LFV experiments find a positive signal from ALP interactions, future neutrino experiments could help to disentangle these two models.

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