FeynHiggs 2.7

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We present the Version 2.7 of FeynHiggs, a program for computing MSSM Higgs-boson masses and related observables, such as mixing angles, branching ratios, and couplings, including state-of-the-art higher-order contributions.

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1. Complex Parameters in the MSSM Higgs Sector

The Higgs sector of the Minimal Supersymmetric Standard Model with complex parameters (cMSSM) consists of two Higgs doublets

\[ H_1 = \left( v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i \chi_1) \right), \]
\[ H_2 = e^{i \xi} \left( v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i \chi_2) \right) \]

which form the following Higgs potential:

\[ V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - \frac{1}{2} \varepsilon_{\alpha \beta} H_1^\alpha H_2^\beta + \text{h.c.} + \frac{g_1^2 + g_2^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g_2^2}{2} |H_1 \bar{H}_2|^2. \]

The Higgs potential contains two complex phases \( \xi, \arg(m_1^2) \). The phase \( \arg(m_1^2) \) can be rotated away \([12]\) and, at tree level, \( \xi \) has to vanish in order to fulfill the minimum condition of the Higgs potential, so there is no CP-violation at tree level and the spectrum contains five states of definite CP-purity: \( h, H, A, H^\pm \).

CP-violating effects are induced by complex parameters that enter via loop corrections: the trilinear couplings \( A_{t,b,t} \), the Higgsino mass parameter \( \mu \), and the gaugino mass parameters \( M_{1,2,3} \). They yield \( \Sigma_{hA}, \Sigma_{HA} \neq 0 \) and induce mixing between \( h, H, \) and \( A \) \([3]\). The Higgs mass matrix has the form

\[ M^2 = \begin{pmatrix} q^2 - m_{h_{h}}^2 + \Sigma_{hh} & \Sigma_{hH} & \Sigma_{hA} \\ \Sigma_{hH} & q^2 - m_{H_{H}}^2 + \Sigma_{HH} & \Sigma_{HA} \\ \Sigma_{hA} & \Sigma_{HA} & q^2 - m_{A_{A}}^2 + \Sigma_{AA} \end{pmatrix} \]

where \( m_{h_{h}}, m_{H_{H}}, m_{A_{A}} \) denote the tree-level Higgs masses and the self-energies’ momentum dependence has only been suppressed for compact notation. It should be noted that in general \( M^2 \) is symmetric but not Hermitian. In the approximation of vanishing external momentum (\( q^2 = 0 \)), one can obtain the higher-order corrected mass eigenstates via a unitary transformation from the tree-level states:

\[ \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} h \\ H \\ A \end{pmatrix}. \]

2. Higgs-boson Self-Energy Corrections in FeynHiggs

2.1. Higgs-boson Masses

The following contributions to the mass matrix and the charged-Higgs-boson self-energy are taken into account:

\[ \begin{pmatrix} q^2 - m_{h_{h}}^2 + \Sigma_{hh} & \Sigma_{hH} & \Sigma_{hA} \\ \Sigma_{hH} & q^2 - m_{H_{H}}^2 + \Sigma_{HH} & \Sigma_{HA} \\ \Sigma_{hA} & \Sigma_{HA} & q^2 - m_{A_{A}}^2 + \Sigma_{AA} \end{pmatrix} \]

\[ \begin{pmatrix} \Sigma_{HH} & q^2 - m_{H_{H}}^2 + \Sigma_{HH} & \Sigma_{HA} \\ \Sigma_{hA} & \Sigma_{HA} & q^2 - m_{A_{A}}^2 + \Sigma_{AA} \end{pmatrix} \]

\[ \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} h \\ H \\ A \end{pmatrix}. \]
Leading $\mathcal{O}(\alpha_1 \alpha_s)$ cMSSM two-loop corr. [3].

2 Leading $\mathcal{O}(\alpha_1^2)$ and subleading $\mathcal{O}(\alpha_1 \alpha_s, \alpha_1 \alpha_b, \alpha_2^3)$ two-loop corrections evaluated in the real MSSM (rMSSM) [5,7]. For the treatment of phases see Sect. 2.2 below.

3 Full one-loop evaluation (all phases, $q^2$ dependence) [4] and leading non-minimal flavour-violating (NMFV) corrections [10].

FeynHiggs [11,12,13,14] performs a numerical search for the complex roots of det $M^2(q^2)$ which are denoted $M_{h_1,2,3}^2$. We decompose $M^2 = M^2 - iMT$, where $M$ is the mass of the particle and $\Gamma$ its width, and then define the loop-corrected masses according to $M_{h_1} \leq M_{h_2} \leq M_{h_3}$.

The Higgs masses are thus determined as the real parts of the complex poles of the propagator. Complex contributions to the Higgs mass matrix are denoted $\alpha = \text{only the cMSSM}$, $\alpha_s = \text{only the rMSSM}$, interpolated in the phases.

2.2. Two-loop Corrections in the cMSSM

The full phase dependence is taken into account in the complete one-loop [14] and $\mathcal{O}(\alpha_1 \alpha_s)$ two-loop [14] contributions to the Higgs self-energies. We used the approximation of vanishing external momenta and vanishing electroweak gauge couplings in the evaluation of all two-loop diagrams.

The $\text{tlCplxApprox}$ flag controls the treatment of phases in the part of the two-loop corrections known only in the rMSSM so far. The following values are possible:

- 0 = all corrections: $\mathcal{O}(\alpha_1 \alpha_s, \alpha_0 \alpha_s, \alpha_1^2, \alpha_1 \alpha_b, \alpha_2^3)$ in the rMSSM,
- 1 = only the cMSSM $\mathcal{O}(\alpha_1 \alpha_s)$ corrections,
- 2 = the cMSSM $\mathcal{O}(\alpha_1 \alpha_s)$ corrections combined with the remaining corrections in the rMSSM, truncated in the phases,
- 3–6 = the cMSSM $\mathcal{O}(\alpha_1 \alpha_s)$ corrections combined with the remaining corrections in the rMSSM, interpolated in the phases.
- New in 2.7 is the choice of interpolation variables: $A_t, A_b, M_3, \mu$ (3), $A_t, X_b, M_3, \mu$ (4), $X_t, A_b, M_3, \mu$ (5), $X_t, X_b, M_3, \mu$ (6).

FeynHiggs thus not only has the most precise evaluation of the Higgs masses in the cMSSM available to date (using the Feynmandiagrammatic approach), but can also deliver an estimate of the uncertainties due to the rMSSM-only parts, by comparing the output for different values of $\text{tlCplxApprox}$.

2.3. Improvements in the $\Delta_b$ Resummation

Corrections to the bottom-Yukawa coupling are potentially large and important phenomenologically, in particular for large values of $\tan \beta$.

A resummation of the terms $\mathcal{O}(\alpha_i^3 \tan^n \beta)$ and $\mathcal{O}(\alpha_i^2 \tan^n \beta)$ can be performed with the help of $\Delta_b$ [8,9].

Some care has to be taken when applying the resummed $\Delta_b$ corrections in FeynHiggs because the $\Delta_b$ terms inserted into the one-loop Higgs self-energies have to be consistent with the two-loop corrections selected. Whenever the corrections 2 in Eq. (2.1) are enabled ($\text{tlCplxApprox} \neq 1$), we also have to use the $\Delta_b$ of Ref. [6]. The determination of $\Delta_b$ was improved in this direction in Version 2.7, avoiding incomplete higher-order corrections at $\mathcal{O}(\alpha_0 \alpha_s)$ and beyond.

In all other instances (e.g. decays, production cross-sections), the $\Delta_b$ of [17] is used in FeynHiggs 2.7, computed self-consistently according to the procedure in [18].

3. Decay Rates and Production Cross-Sections

3.1. $h_i \rightarrow f_j \bar{f}_k$ at One-Loop Precision

The Higgs decays to fermions, $h_i \rightarrow f_j \bar{f}_k$ are now available at full one-loop precision [24]. The real gluon (photon) which cancels the IR pole is treated fully inclusively [25].

The routine $\text{FHCouplings}$ has an additional flag fast to switch off the computation of the off-diagonal decays ($j \neq k$) for efficiency. These decays hardly contribute to the total decay rate and can safely be neglected.

The phenomenologically important resummed $\Delta_b$ corrections are still taken into account in $h_i \rightarrow b \bar{b}$, with the corresponding one-loop contribution subtracted to prevent double counting.
3.2. Improvements in the $gg \to h$ Production Cross-Section

For the Standard Model estimate, we use the NNLL prediction of Ref. [19]. The MSSM production cross-section is estimated in the effective-coupling approximation [23] by multiplying pieces of the amplitude with the corresponding state-of-the-art amplitude correction factors as follows:

$$A_{\text{MSSM}} = c_t^{\text{NLO}} c_t^{\text{NNLO}} A_{\text{SM}} + c_b, r \text{ Re} A_b^{\text{MSSM,LO}} + c_f A_f^{\text{MSSM,LO}} + A_{\text{rest}},$$

$$A_{\text{SM}} = c_t^{\text{NLO}} A_{\text{SM}}^{\text{NLO}} + c_b, r \text{ Re} A_b^{\text{SM,LO}} + c_f A_f^{\text{SM,LO}} + A_{\text{rest}},$$

$$\sigma_{\text{MSSM}} = \frac{|A_{\text{MSSM}}|^2}{|A_{\text{SM}}|^2} \sigma_{\text{SM,NLO}}.$$  \hfill (6)

The factors $c_t^{\text{NLO}}, c_t^{\text{NNLO}}, c_b, r, c_b, i$ are taken from Ref. [21] and by construction reproduce the top and bottom loops at NLO including the interference term. $c_f$ is taken from Ref. [22]. The LO cross-sections are parameterized with $m_b(m_b)$.

The $\sigma_{\text{SM,NLO}}$ is also taken from Ref. [21] and contains only SM top and bottom loops.

4. Non-Minimal Flavour Violation

In the NMFV MSSM, the sfermion flavours are allowed to mix with each other, i.e. the mixing is $6 \times 6$ rather than $2 \times 2$:

| NMFV | MFV |
|------|-----|
| $\tilde{u}_i = R_{ij}^{u}$ | $\tilde{u}_i = U_{ij}^{u}$ |
| $\tilde{c}_i = R_{ij}^{u}$ | $\tilde{c}_i = U_{ij}^{u}$ |
| $\tilde{d}_i = R_{ij}^{u}$ | $\tilde{d}_i = U_{ij}^{u}$ |

and likewise for the sdown sector.

The mixing matrices $R^{u,d}$ diagonalize the mass matrices $(M^2)^{u,d} = (M_{\text{MFV}}^2)^{u,d} + \Delta^{u,d}$, where

$$M_{\text{MFV}}^2 = \begin{pmatrix} 0 & 0 & m_i X^i & 0 & 0 & 0 \\ 0 & 0 & m_j X^j & 0 & 0 & 0 \\ m_i X^i & m_j X^j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ m_k X_k & m_k X_k & 0 & 0 & 0 & 0 \end{pmatrix}.$$  \hfill (7)

The four sectors are clockwise labelled LL, LR, RR, RL with entries

$$M_{L,q}^2 = m_q^2 + \cos 2\beta (T_3^q - Q_q s_W^2)m_Z^2, \quad M_{R,q}^2 = m_q^2 + 2\beta Q_q s_W^2 m_Z^2, \quad X_q = A_q - \mu^* \tan 2\beta \tilde{c}_r .$$

Technical remark: FeynHiggs 2.7 keeps the MFV mixing matrices $U$ exactly ‘on top’ of the NMFV matrices $R$, such that the MFV entries are automatically updated whenever the NMFV entries change and vice versa.

The most immediately notable effect comes from the LR(RL) sector, as the $A_{ij}$ enter the couplings directly, e.g.

$$\sum_{g,g'} \left[ m_d R_{i,g}^{d} R_{j,g'}^{d} \delta_{gg'} \mu + A_{g}^{d*} \tan \beta \right] - m_d R_{i,g}^{d} R_{j,g'}^{d} \delta_{gg'} \mu^* + A_{g}^{d} \tan \beta \right]$$

This enters the Higgs masses through the $A$ self-energy and can lead to sizable effects.

The main constraints on the $\Delta^{u,d}$ come from low-energy observables. Currently included in FeynHiggs are $b \to s\gamma$ and $\Delta M_b$, both at one-loop level including NMFV effects, with more to follow.

5. Output of FeynHiggs 2.7

We give a short overview of the output routines of the FeynHiggs library.

FHiggsCorr – All Higgs-boson masses and mixings: $M_{h,1,2,3}, M_{H^\pm}, a_{\text{eff}}, \mu_{\text{Higgs, ZHiggs}}, \Delta M_b$.

FHUncertainties – Uncertainties of the masses and mixings.
FHCouplings – Couplings and Branching Ratios for the following decay channels:

\[
\begin{align*}
    h_{1,2,3} & \rightarrow f_i \bar{f}_j, \gamma \gamma, ZZ^{(*)}, WW^{(*)}, gg, \\
    h_i Z^+, h_i h_j, H^+ H^- & , \\
    \tilde{f}_i \tilde{f}_j, \tilde{\chi}_i^+ \tilde{\chi}_j^-, \tilde{\chi}_i^0 \tilde{\chi}_j^0 & , \\
    H^\pm & \rightarrow f_i^* \bar{f}_j^*, h_i W^{\pm*}, \tilde{f}_i \tilde{f}_j^*, \tilde{\chi}_i^\mp \tilde{\chi}_j^\pm, \\
    t & \rightarrow W^+ b, H^+ b,
\end{align*}
\]

plus the corresponding ones of an SM Higgs with mass \( M_h \): \( h_i^{SM,1,2,3} \rightarrow f_i \bar{f}_j, \gamma \gamma, ZZ^{(*)}, WW^{(*)}, gg \).

FHHiggsProd – Higgs production-channel cross-sections (SM total cross-sections multiplied with MSSM effective couplings, see Ref. [19,23,27])

- \( gg \rightarrow h_i \) – gluon fusion.
- \( WW \rightarrow h_i \), \( ZZ \rightarrow h_i \) – gauge-boson fusion.
- \( W \rightarrow Wh_i \), \( Z \rightarrow Zh_i \) – Higgs-strahlung.
- \( b \bar{b} \rightarrow b \bar{b}h_i \) – bottom Yukawa process.
- \( b \bar{b} \rightarrow b \bar{b}h_i \) – ditto, one \( b \) tagged.
- \( t \bar{t} \rightarrow t \bar{t}h_i \) – top Yukawa process.
- \( t \bar{t} \rightarrow t \bar{t}h_i \) – stop Yukawa process.

FHConstraints – Electroweak precision observables, see e.g. Ref. [26] for details:

- \( \Delta \rho \) at \( \mathcal{O}(\alpha, \alpha \alpha_s) \), including NMFV effects.
- \( M_W, \sin^2 \theta_{\text{eff}} \) via SM formula + \( \Delta \rho \).
- \( \text{BR}(b \rightarrow s \gamma) \) and \( \Delta M_s \) at one-loop level including NMFV effects [28].
- \( (g_\mu - 2)_{\text{SUSY}} \) including full one- and leading/subleading two-loop SUSY corrections.
- EDMs of electron (Th), neutron, Hg.

7. Usage

FeynHiggs has four modes of operation: Library mode, Command-line mode, Web mode, and Mathematica mode.

7.1. Library Mode

The FeynHiggs library libFH.a is a static Fortran 77 library. Its global symbols are prefixed with a unique identifier to minimize symbol collisions. The library contains only subroutines (no functions), so that no include files are needed (except for the couplings) and the invocation from C/C++ is hassle-free. Detailed debugging output can be turned on at run time. All routines are described in detail in the API guide and on man-pages.

7.2. Command-Line Mode

The user submits a text file, such as

\[
\begin{align*}
    MT & = 172.6 \\
    MSusy & = 500 \\
    MA0 & = 200 \\
    Abs(M_2) & = 200 \\
    Abs(MUE) & = 1000 \\
    TB & = 5 \\
    Abs(Xt) & = 1000 \\
    Abs(M_3) & = 800
\end{align*}
\]

to the FeynHiggs executable with a command like

\texttt{FeynHiggs file [flags]}

where the \texttt{flags} are optional. The output is a human-readable version of the results. The \texttt{table} utility converts the output to machine-readable format, for example

\texttt{FeynHiggs file [flags] | table TB Mh0 > out}

Loops over parameter values (parameter scans) are possible:

- \( MA0 \) 200 350 50, linear: 200, 250, 300, 350,
- \( TB \) 5 40 *2, logarithmic: 5, 10, 20, 40,
- \( TB \) 5 50 /4, number of steps: 5, 20, 35, 50.
7.2.1. Command-Line Mode Scripted

If a "-" is specified as file name in command-line mode, FeynHiggs reads the input from stdin. This allows to script FeynHiggs sessions, such that parameters, flags, and possibly environment variables are preserved in a compact shell script. For example:

```
#!/ /bin/sh
make || exit 1
FHDEBUG=2 ./build/FeynHiggs - ${1:-400302103} << END
MT 173.1
M SUSY 3000
M A 1000
Abs(M_2) 2500
Abs(M U NE) 2000
TB 5
Abs(Xt) 1000
Abs(M 3) 2000
END
make || exit 1
```

`make || exit 1` updates the FeynHiggs executable and terminates with an error if any problem occurred in the build.

The prefixed `FHDEBUG=2` sets an environment variable, here to increase the debugging level.

There are default flags, overridable through command-line argument 1 (`${1:-400302103}`).

The actual parameters are fed to FeynHiggs up to the `END` marker. The identifier `END` can be chosen freely but must match exactly the one at the bottom of the parameter list.

7.3. SUSY Les Houches Accord Format

The FeynHiggs executable can also process files in SUSY Les Houches Accord 2 (SLHA2) format \[29\]. It uses the SLHA Library \[30\]. Processing of SLHA2 files can also be done in Library Mode with the subroutine `FSetSLHA`.

FeynHiggs in fact tries to read each file in SLHA format first and falls back to its native format if that fails.

7.4. Web Mode

The FeynHiggs User Control Center (FHUCC) is online at feynhiggs.de/fhucc. It is a Web interface for the command-line frontend. The user gets the results together with the input file for the command-line frontend.

7.5. Mathematica Mode

A more powerful interactive environment is provided by the Mathematica interface of FeynHiggs. The MathLink executable `MFeynHiggs` must first be loaded with

```
Install["MFeynHiggs"]
```

and makes all FeynHiggs routines available as Mathematica functions. In combination with the arsenal of standard Mathematica functions such as `ContourPlot` and `Manipulate`, even sophisticated analyses can be carried out easily.

8. Summary: Main New Features

Version 2.7 of FeynHiggs includes the following new features:

- Inclusion of the full cMSSM two-loop $O(\alpha_t \alpha_s)$ corrections in highly optimized form. Extended interpolation options for rMSSM parts.
- Inclusion of the full one-loop corrections to the $h_i \rightarrow f_j \bar{f}_k$ decays.
- Inclusion of NMFV corrections in the Higgs self-energies, decays, and low-energy constraints.
- Improvements in the $\Delta_b$ resummation.
- Total Higgs production cross-sections in effective coupling approximation, with particular improvements in the $ggh$ channel.

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