On the possibility to measure the imaginary part of the spin-dependent amplitude of zero-angle coherent elastic scattering in a spin-filter experiment with an unpolarized proton beam interacting with a polarized deuterium target

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Abstract. As deuterons pass through an unpolarized target, there appear the phenomena of spin rotation (oscillation) and spin dichroism of deuterons; both phenomena are the manifestation of the deuteron birefringence effect. The magnitude of the effect is determined by the refractive index of deuterons in matter, which depends on the spin-dependent part $d_1$ of the amplitude of zero-angle coherent elastic scattering of the deuteron by the nucleus. Spin dichroism is determined by the imaginary part of this amplitude, and in view of the optical theorem, by the total scattering cross-section, which is different for different spin states of the deuteron. The phenomenon of spin dichroism of deuterons results in the fact that after passing through the target, the initially unpolarized deuteron beam acquires tensor polarization proportional to the difference in the total cross-sections of scattering of deuterons with different spin states by the nucleus. The stated difference was first measured for deuterons with energies up to 20 MeV, where the phenomenon of deuteron spin dichroism was experimentally revealed, and the deuteron tensor polarization was measured. The phenomenon of spin dichroism, i.e., the production of tensor polarization was also observed at high deuteron energy. In the present report we discuss the possibility that the imaginary part of the amplitude $d_1$ of deuteron scattering by the proton may be determined experimentally by measuring the rate of the decrease in the intensity of an unpolarized proton beam in a storage ring as the beam passes through a tensor-polarized deuterium target.

1. Introduction
Spin rotation and spin dichroism phenomena, also called birefringence effect, arise for deuterons and all other particles with spin $S \geq 1$ passing through unpolarized matter [1, 2]. The above phenomena are caused by the difference of the spin-dependent coherent forward scattering amplitude for deuterons with the spin projections $m = 0$ and $m = \pm 1$ ($m$ is the magnetic quantum number).

As a result, the refraction index and the absorption coefficient for a deuteron transmitted through matter depend on the deuteron spin state. Therefore, an unpolarized deuteron beam
passing through an unpolarized target acquires tensor polarization due to the deuteron spin dichroism, because the absorption coefficient depends on the spin state of the deuteron.

The first experimental study of the deuteron spin dichroism in carbon targets was carried out at the electrostatic HVEC tandem Van-de-Graaff accelerator with a deuteron energy up to 20 MeV at the Institut für Kernphysik of Universität zu Köln [3, 4]. In the experiment [3, 4, 5], it was found out that in the energy interval 5 – 20 MeV, the deuteron spin dichroism in a carbon target changes both magnitude and sign as function of the energy change in the target. In 2007, another experiment at the Nuclotron in JINR provided a measurement of spin dichroism for 5.5 GeV/c deuterons transmitted through carbon targets [6, 7].

2. Birefringence of deuteron in unpolarized matter

According to [1, 2], the index of refraction for a particle with spin can be written as follows:

\[ \hat{N} = 1 + \frac{2\pi \rho}{k^2} \hat{f}(0), \]  

(1)

where \( \rho \) is the density of matter (number of scatterers per cm\(^3\)), \( k \) is the deuteron wave number, \( \hat{f}(0) = \text{Tr} \hat{\rho}_f \hat{F}(0) \), where \( \hat{\rho}_f \) is the spin density matrix of the scatterers, and \( \hat{F}(0) \) is the operator of the forward scattering amplitude acting in the combined spin space of the particle and scatterer spin \( \vec{J} \).

Let us choose the direction of the particle wave vector \( \vec{k} \) as the axis of quantization \( z \). Then, for an unpolarized target the deuteron forward scattering amplitude \( \hat{f}(0) \) can be presented [1, 2] as

\[ \hat{f}(0) = d + d_1 \hat{S}_z^2, \]  

(2)

where \( \hat{S}_z \) is the operator of the spin projection on the \( z \) axis, \( d \) and \( d_1 \) are the spin-independent and spin-dependent parts of the forward scattering amplitude, respectively.

The refractive index for deuterons in unpolarized matter can be written as

\[ \hat{N} = 1 + \frac{2\pi \rho}{k^2} \left( d + d_1 \hat{S}_z^2 \right). \]  

(3)

According to equation (3), the index of refraction \( \hat{N} \) depends on the deuteron spin orientation relative to the deuteron momentum. The refractive index \( \hat{N}_m \) for a deuteron in the eigenstate of operator \( \hat{S}_z \) is

\[ \hat{N}_m = 1 + \frac{2\pi \rho}{k^2} f_m(0), \]  

(4)

where \( f_m(0) = d + d_1 m^2 \), and \( m \) denotes the magnetic quantum number of the deuteron. According to equation (4), the refractive indices for the deuteron in the spin states \( m = +1 \) and \( m = -1 \) are the very same, while those for \( m = \pm 1 \) and \( m = 0 \) are different (\( \Re \hat{N}(\pm 1) \neq \Re \hat{N}(0) \) and \( \Im \hat{N}(\pm 1) \neq \Im \hat{N}(0) \)). This can be explained by the non–spherical wave functions of the deuteron ground state. Therefore, the scattering cross-section \( \sigma_{\pm 1} \) for a deuteron with \( m = \pm 1 \) (deuteron spin parallel and antiparallel to its momentum \( \vec{k} \)) differs from the scattering cross-section \( \sigma_0 \) for a deuteron with \( m = 0 \), therefore \( \sigma_{\pm 1} \neq \sigma_0 \). As a result, according to the optical theorem, we find

\[ \Im f_{\pm 1}(0) = \frac{k}{4\pi} \sigma_{\pm 1} \neq \Im f_0(0) = \frac{k}{4\pi} \sigma_0, \]

\[ \Im d_1 = \Im (f_{\pm 1}(0) - f_0(0)). \]  

(5)

From equations (5) it follows that the deuteron spin dichroism appears even when a deuteron passes through an unpolarized target. Due to the different absorption, the initially unpolarized beam acquires polarization or, yet more precisely, alignment.
3. Deuteron spin dichroism in an unpolarized target

We now let a deuteron beam in state \( m = 1 \) pass through the target. The beam intensity changes according to \( I_1(z) = I_1^0 e^{-\sigma_1 \rho z} \), where \( I_1^0 \) is the beam intensity before entering the target. Similarly, for states \( m = -1 \) and \( m = 0 \), the intensity changes as \( I_{-1}(z) = I_{-1}^0 e^{-\sigma_{-1} \rho z} \) and \( I_0(z) = I_0^0 e^{-\sigma_0 \rho z} \), where \( I_{-1}^0 \) and \( I_0^0 \) denote the beam intensities before entering the target, respectively.

Let us consider the transmission of an unpolarized deuteron beam through an unpolarized target (see figure 1). The unpolarized deuteron beam can be described as a composition of three polarized beams with equal intensities \( I = I_1^0 + I_{-1}^0 + I_0^0 \). Usually, in an experiment \( \sigma_{\pm 0} \rho z \ll 1 \) and the change of intensity for each of its component can be expressed as \( I_{\pm 1}(z) = I_{\pm 1}^0 (1 - \sigma_{\pm 1} \rho z) \) and \( I_0(z) = I_0^0 (1 - \sigma_0 \rho z) \). Since \( \sigma_{-1} = \sigma_1 \), the deuteron spin dichroism can be characterized by the ratio \( D \) as follows,

\[
D(z) = \frac{I_{\pm 1}(z) - I_0(z)}{I_{\pm 1}(z) + I_0(z)} \approx \frac{1}{2} (\sigma_0 - \sigma_{\pm 1}) \rho z - \frac{2 \pi \rho z \Im(d_1)}{k}, \tag{6}
\]

where equations (4) and (5) are used. Equation (6) can then be rewritten as

\[
D(L) = -\frac{2\pi N_a L \Im(d_1)}{k M_r} = \frac{N_a L (\sigma_0 - \sigma_{\pm 1})}{2M_r}, \tag{7}
\]

where \( N_a \) is the Avogadro number, \( L \) is the target thickness in g/cm\(^2\), and \( M_r \) is the molar mass of the target matter.

According to [8], tensor polarization of the beam can be expressed as \( p_{zz} = \frac{I_{-1} + I_1 - 2I_0}{I_{-1} + I_1 + I_0} \). The tensor polarization of the initially unpolarized deuteron beam transmitted through the target of thickness \( L \) from deuteron spin dichroism reads as follows,

\[
p_{zz}(L) = \frac{I_{-1}(L) + I_1(L) - 2I_0(L)}{I_{-1}(L) + I_1(L) + I_0(L)} \approx \frac{2N_a L (\sigma_0 - \sigma_{\pm 1})}{3M_r} = \frac{8\pi N_a L \Im(d_1)}{3kM_r}. \tag{8}
\]

The relation between \( D \) and \( P_{zz} \) follows from equations (7) and (8),

\[
p_{zz} \approx \frac{4}{3D}. \tag{9}
\]

According to equation (8), the imaginary part of the spin-dependent forward scattering amplitude can be directly determined in a transmission experiment by measuring the tensor polarization of the deuteron beam.

![Figure 1](image_url)
4. Measuring the imaginary part of the spin-dependent amplitude of zero-angle coherent elastic scattering in a transmission experiment

There are several types of transmission experiments that can be carried out to perform deuteron spin dichroism measurements.

The first type is a transmission experiment of an extracted beam from an accelerator where the unpolarized deuterons pass through an unpolarized target (see figure 1). The experiments [3]-[7] were carried out this way. The main advantage of this method is its technical simplicity, because only an unpolarized deuteron beam, an unpolarized solid target, and a polarimeter are required. Since the magnitude of the effect is proportional to the target thickness, an increase in thickness increases the energy loss in the target. These losses limit the target thickness, especially at low beam energies, and are the reason that the measured spin-dependent amplitudes have to be averaged over the deuteron energy in the target.

Another method uses multiple transmissions of an unpolarized deuteron beam through an internal unpolarized gas target in a storage ring [9, 10]. In this case, the effect is accumulated, and is limited only by the lifetime of the stored beam. However, possible birefringence effects in electric and magnetic fields of the storage ring may contribute [11].

The third method is a spin-filter experiment. Since the total cross-sections $\sigma_0$ and $\sigma_{\pm 1}$ are invariant, it does not matter whether the deuteron is used as a target or beam probe. The main idea of the spin filtering method is that by passing an unpolarized beam (for example protons) in a storage ring for a sufficiently long time through the polarized target consisting of deuterons in state $m = -1$. As time $t$ passes, the intensity $I_{-1}(t)$ of the stored beam is measured, of course, the initial intensity $I_{-1}(0)$ of the beam is known as well. The measurement is repeated with new unpolarized beam and a polarized target consisting of deuterons in state $m = 1$, and the quantities $I_1(0)$ and $I_1(t)$ are determined. At last, a new unpolarized beam with known initial intensity $I_s(0)$ passes through the unpolarized deuterium target. The final intensity $I_s(t)$ of the beam is measured again. For all three measurements, the time of storing is the same. Since an unpolarized deuterium target contains a superposition of deuterons in states $m = -1$, $m = 1$ and $m = 0$ the final beam intensity in this measurement should be different from the average beam intensity in the two previous measurements, because of $\sigma_0 \neq \sigma_{\pm 1}$ (see figures 2 and 3). As a result, we obtain three independent beam intensity ratios,
\[
\frac{I_{-1}(t)}{I_{-1}(0)} = e^{-\sigma_{\pm 1} \rho_{-1} \nu t}, \quad I_1(t) = e^{-\sigma_{\pm 1} \rho_1 \nu t}, \quad I_s(t) = e^{-\sigma_{\pm 1} \rho_s \nu t} e^{\frac{i}{3} (\sigma_{\pm 1} - \sigma_0) \rho_s \nu t},
\]  
\tag{10}
\]

where \(\rho_{-1}, \rho_1, \rho_s\) are the density of the targets (number of scatterers per \(\text{cm}^3\)), and \(\nu\) is the revolution frequency. Let us now introduce new variables,

\[
R_{-1}(t) = \ln \left( \frac{I_{-1}(t)}{I_{-1}(0)} \right), \quad R_1(t) = \ln \left( \frac{I_1(t)}{I_1(0)} \right), \quad R_s(t) = \ln \left( \frac{I_s(t)}{I_s(0)} \right).
\]

Then we can obtain the following relations,

\[
\sigma_{\pm 1} - \sigma_0 = 3 \left( \frac{R_s(t)}{\rho_s \nu t} - \frac{1}{2} \left( \frac{R_{-1}(t)}{\rho_{-1} \nu t} + \frac{R_1(t)}{\rho_1 \nu t} \right) \right), \quad \delta d_1 = \frac{k}{2\pi} (\sigma_{\pm 1} - \sigma_0). \tag{12}
\]

This shows that the spin-dependent part of the total cross-section can be determined by measuring the rate of the intensity decrease of an unpolarized proton beam in a storage ring while the beam passes through polarized and unpolarized deuterium targets. For an estimation of the storage time we use in equation (10) the following parameters: \(\rho \sim 10^{15} \text{ deuterons/cm}^2\), \(\nu \sim 1.6 \text{ MHz}\), \(\sigma_{\pm 1} - \sigma_0 \sim 4.3 \text{ mb} \) \[12\], and \(\sigma_{\pm 1} \sim 70 \text{ mb} \) \[12\].

In order to obtain a 3\% difference in the proton beam intensity due to the spin-dependent part of the total cross-section (see equation (10)), a storage time of about 4 hours should be used. During that time period, the beam intensity decreases by about a factor of four.

5. Conclusion

It was shown that the imaginary part of the spin-dependent forward scattering amplitude of the deuteron can be measured directly due to the phenomenon of deuteron spin dichroism. Several types of experiments for the investigation of this phenomenon were considered briefly. One of them is based on the transmission (spin-filtering) of an initially unpolarized proton beam passing through polarized and/or unpolarized deuterium targets. It should be mentioned that the imaginary part of the amplitude \(d_1\) can be measured in such a spin-filtering experiment by changing the orientation of the tensor polarization of the deuterium target. According to our estimate, this type of experiment can be carried out at COSY and at GSI.

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