Urban Traffic Light Control Considering Capacity Difference between Public Bus and Private Vehicles

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This work was funded in part by The Science and Technology Development Fund (FDCT), Macau SAR (file no. 0017/2019/A1).

ABSTRACT Due to the rapid growth of transportation demands for economic development, the importance of effective urban traffic signal control can never be underestimated, and the severe space saturation greatly limit the operations of transportation. In this paper, public buses and other vehicles are treated as different traffic flows. We address the urban traffic signal control problem in a scheduling framework by considering that the capacity of a public bus is much greater than that of other vehicles. The dynamics of an urban traffic network controlled by traffic lights are described by a novel model, which is formed by inserting mixed logical constraints into a cell transmission flow dynamic model. It includes the nonlinear relationship among the current link volume from the upstream, the current remaining link capacity from the downstream and the state of the traffic light. With the objective of minimizing the total delay time in the traffic network for people in the private vehicles and buses, we obtain a mixed integer linear programming problem, which is converted by the traffic signal control problem. We further analyze the influence of the number of passengers on buses and the time interval. The efficiency of the proposed method is verified by comparing the models with and without considering the public transport regular. When the average number of passengers on a bus increases from 10 to 60, the degree of optimization ranges from 17% to 51% in the two-stage model and from 4% to 26% in the four-stage model. Finally, we consider the case without bus lanes and show the importance of dedicated bus lanes.

INDEX TERMS public transportation system, urban traffic signal scheduling, mixed integer linear programming.

I. INTRODUCTION

A. BACKGROUND

With the increasing demands for transportation due to fast economic development, the execution of effective urban traffic signal control becomes more and more important. However, the increasingly severe space saturation greatly restricts the performance of a transportation system. With the rapid increase in population and vehicles, traffic congestion in urban areas is getting worse and worse. The reduction of traffic congestion can improve traffic safety and efficiency, and reduce environmental pollution. Therefore, how to effectively arrange traffic lights is becoming more and more important to big cities. Reasonable traffic signal scheduling is one of the ways to reduce delay time for urban traffic network systems [1-5].

A set of road links forms an urban traffic network and the road links are connected with each other by intersections. A number of approaches and crossing areas form an intersection [6-7]. There is at least one lane in an approach. But at each approach, the queue is unique and independent. An intersection can be crossed safely by two compatible streams simultaneously and cannot be crossed by antagonistic streams. By traditional traffic signal control, one repetition of the basic series of stages at an intersection is called a signal cycle. Then, at each stage, a predefined compatible traffic streams cross the intersection simultaneously in the instruction of simultaneous traffic light signals. The cycle time is the duration of a cycle [8-9]. For safety reasons, in order to avoid interference between
antagonistic streams, among the consecutive stages, it is necessary to insert a constant lost (or inter-green) time with a few seconds. For each traffic light, the split is the ratio of the green light time and the red light time within one cycle, and the offset is the delay between the starting times of green periods of two neighboring traffic lights along the same traffic route [10-11].

In China, National Middle and Long-term Science and Technology Development Program proposes preferential policies for developing public transit. More and more cities endeavor to explore effective approaches to develop public transit, including some small-medium-scale cities. There is a sustained growth of traffic volume in big cities, while public transit in small-medium-scale cities develops at a quite slow rate [12-13].

B. LITERATURE REVIEW

For the traffic signal control, four basic strategies are considered, e.g., the combination of fixed time strategies or traffic responsive strategies, and isolated strategies or coordinated strategies. For isolated fixed-time strategies, they are only applicable to under-saturated traffic conditions. Stage-based strategies belong to this class, such as the work proposed in [8-9]. This kind of strategies determines the optimal splits and cycle time to minimize the total delay or maximize the intersection capacity utilization. Phase-based strategies belong to this class too, such as the work in [10]. This kind of strategies determines not only the optimal split and cycle time but also the optimal staging, which may be an important feature of complex intersections. For isolated traffic-responsive strategies, the work in [6] and [11] are the representative ones. These strategies use real-time measurements. Some more or less sophisticated vehicle-actuation logic is executed by the use of inductive loop detectors. In practice, the widely used strategies are the fixed time coordinated ones. Typical work in this direction includes the one in [7], which considers a two-way arterial with several signals (intersections) and specifies the corresponding offsets to maximize the number of vehicles that can travel within a given speed range without stopping at any signal (green light). The study in [14] utilizes platoon dispersion (i.e., a dynamic first-order time-delay system) to model flow progression along a link, and uses heuristic optimization algorithms such as hill-climbing to determine the splits, offsets, and cycle time and optimize the corresponding performance index such as the total number of vehicle stops.

In practice, the traffic situations are highly dynamic and the fixed time strategies become ineffective. Then, there has been a widespread concern to coordinated traffic-responsive strategies. For coordinated traffic-responsive strategies, the study in [15] is a representative work. The authors in [15] update the splits, offsets and cycle time based on actual traffic measurements, which can be considered as a traffic-responsive version. Model-based optimization methods are used in coordinated traffic-responsive strategies in [16-19]. They do not consider explicitly the splits, offsets, or cycles. Starting from the current time and the currently applied stage, they calculate the optimal values of the next few switching time in real-time according to the realistic traffic models. Generally, over a future time horizon, the models have 2-5 seconds as the sampling time. Control-theoretical-based approaches are also used in coordinated traffic-responsive strategies. For example, the work in [20-21] use the store-and-forward modeling for traffic networks with a linear quadratic regulator optimal control solution.

Based on Daganzo’s cell transmission models [22-23], the study in [24] addresses the urban traffic signal control in a scheduling framework with the objective of minimizing the total delay time in the traffic network. This model describes each outgoing flow rate in the way of a nonlinear mixed logical switching function, which is the novelty point. This function describes the density of the source link, the density and capacity of the destination link, and the response to the past situation of the traffic light from the driver’s potential psychology. The adoption of this dynamic flow model makes it applicable to situations including the under-saturated and over-saturated situations. In this model, the traditional concepts of cycles, offsets, and splits are not adopted. In this way, the urban traffic light control problem becomes a class of model-based optimization problems, where each traffic light is assigned with a green light period in a real-time way by the network controller. In the recent work for this problem, the work in [25] develops a harmony search with the ensemble of a local search operators for solving the urban traffic light control problem with the minimization of the total network-wise delay time as the objective. The study in [26] describes a heterogeneous traffic system with signalized and non-signalized intersections. The work in [27] addresses a bi-objective urban traffic light scheduling problem to minimize both the total network-wise delay time for the vehicles and total delay time of all pedestrians. The work in [28] presents a bi-objective urban traffic light scheduling problem to minimize the total network-wise delay time for the vehicles and optimize the control for drivers’ unhappy. The study in [29] addresses the traffic signal scheduling problem with a heterogeneous traffic network of signalized and non-signalized intersections in order to minimize the total network-wise delay time of all vehicles within a given finite-time window by using meta-heuristics.

The aforementioned studies do not consider the factors of the public transport regular. The authors in [30] evaluate the performance of bus routes within a public transportation system, concluding that the operation in the off-peak period is better than that in the peak period, and the average running time has the greatest impact on performance. For the buses’ running time, they are affected by traffic signal control, and reasonable traffic signal control can effectively
reduce running time. The facility-design-based measures provide priority to buses mainly through appropriate arrangements of the available infrastructure. The most common measure in this category is on the dedicated bus lanes [31-32]. As for priority strategies of public transport, it includes fixed time strategies such as the work in [33-34] and real-time strategies such as the work in [35-36]. The real-time strategies include rule-based strategies such as the work in [35] and optimization-based strategies such as the work in [36]. The main difficulty faced by the rule-based strategies comes from the requirements for responding to multiple requests, which can be solved by optimization-based strategies. Meanwhile, the computational effort may be significantly increased. The work in [36] develops a real-time, traffic-responsive signal control strategy in order to minimize the total person delay at traffic signals in the under-saturated traffic conditions.

The work in [35] considers the bus headway for the bus priority at traffic signals. The study in [36] considers different vehicle arrivals, including deterministic and stochastic, for reducing the total person delay at one intersection. They all consider public transport from a micro perspective. In our work, for a large traffic network, we analyze this problem from a macro perspective.

In the latest literature, the work in [37] presents a bi-objective algorithm to minimize both passenger delay and the deviation from bus schedule, where an example in Nanjing with two adjacent signalized intersections, a bus stop and five bus routes is tested. The authors in [38] maximize bus service reliability, including minimizing the bus schedule discrepancies and the total squared headway deviations by setting optimal signal timings. Finally, a problem with 5 intersections and 20 buses in London as the numerical experiments. The study in [39] describes a signal timing model for intersections with a center transit lane and bus stops considering private vehicles, buses, crossing pedestrians, and passengers, where an intersection in Shanghai as the example to verify the model. The work in [40] sets up a dynamic headway control to minimize the weight addition of deviation of bus headways and ratio of signal cycle length scaling by considering a high-frequency route with bus lane. The paper [41] develops a bi-level programming model to minimize average passenger delay at intersections and vehicle delay in lanes simultaneously. An example with one intersection is analyzed. The authors in [42] propose a traffic signal optimization model to minimize total travel delay by considering the interaction of traffic flows between the intersections and upstream curbside bus stops, in which one intersection is analyzed to verify the proposed model. The work in [43] proposes a mixed-integer nonlinear problem model to minimize the travel time of Bus Rapid Transit (BRT) considering on-time rate of the bus schedule and analyze the example of Fengxian-Pudong BRT line in Shanghai. The paper [44] presents an adaptive signal controller for managing traffic delays and urban bus service reliability with fully adaptable acyclic timing plans. One example in London with five intersections is analyzed.

C. Research Plan

In this paper, we describe the signal control problem based on the novel traffic flow model that is first applied in [24]. In [24], the authors compare the proposed scheduling strategy for fixed-time scheduling and the results show that the proposed scheduling strategy can significantly reduce delay time.

Since the public transport regular plays an important role in economic development and every one’s daily life, we formulate the problem of urban traffic light control considering the bus transportation system in order to reduce the total waiting time for the people in all vehicles including buses over a given finite horizon. Then, in each intersection, we adopt a real-time scheduling strategy to dynamically determine the RED traffic signal or Green traffic signal for each traffic light. Meanwhile, we also consider the situation without dedicated bus lanes. It is important to determine the traffic light signals of the intersections according to the real-time traffic situations. Hence, the urban traffic light control problem becomes a type of optimization-based problems, which belong to coordinated traffic-responsive strategies. We convert the traffic light control problem into a mixed integer linear program model. The techniques that we use are introduced in [45]. Compared with a mate-heuristic algorithm obtained by using evolutionary computation and reinforcement learning, this method can be solved by several standard optimization tools and get the global optimal solution. We solve this model by using standard optimization tool LINGO.

This study contributes to the performance of urban traffic light control in the following ways: (1) the factors of the public transport regular are taken into consideration and urban traffic signal control models based on Daganzo’s cell transmission model are formulated, achieving a better result, (2) by conversion, we get a mixed integer linear programming model, which is converted by the traffic signal control problem, and (3) we show the importance of dedicated bus lanes for the improvement of the urban transportation.

In this article, the research target is to optimize the urban traffic light control in considering the public transport regular. The rest of this paper is organized as follows. Section II develops the model for the traffic light scheduling problem considering the regular public transit system based on a cell transmission model. Meanwhile, the case without dedicated bus lanes is considered. Then, Section III describes how to convert the traffic light scheduling problem into a mixed integer linear programming problem. With the models, the results and analysis are given in Section IV. Finally, Section V concludes this paper.
II. PROBLEM FORMULATION

A set of intersections and links form a traffic network. In Fig. 1, we describe a simple unidirectional traffic network.

![Traffic Network Diagram]

**FIGURE 1. A simple unidirectional traffic network**

In this traffic network, there are two antagonistic traffic flows at each intersection. Based on the cell transmission model, a discrete time model is proposed. In this model, we consider the regular public transit system. There are some assumptions that are given as follows.

- The entrance and exit models for the traffic network are determined.
- The reason of the delay for each vehicle in the traffic network is the waiting time caused only by traffic signals. The vehicles will leave the network.
- The link turning ratios at each link in the traffic network are determined.

From Part A to Part D, we consider the situation with dedicated bus lanes. Then we have the following assumption.

- There is a dedicated bus lane with only buses on it.
- In Part E, we consider the situation without dedicated bus lanes.

In order to better illustrate the establishment of the model, we present the notation used in this model as follows.

- \( C_i(k) \): In the interval \( k \), the amount of vehicles except buses in the link \( i \).
- \( C_{ij}(k) \): In the interval \( k \), the amount of buses in the link \( i \).
- \( C_j \): In the link \( j \), the maximum capacity for vehicles except buses.
- \( C_{ij} \): In the link \( j \), the maximum capacity for buses.
- \( f_{0i}(k) \): In the interval \( k \), the amount of vehicles except buses exiting from links \( i \) to \( j \).
- \( f_{0ij}(k) \): In the interval \( k \), the amount of buses exiting from links \( i \) to \( j \).
- \( A \): The number of average people in vehicles except for those in buses.
- \( B \): The number of average people in buses.
- \( \Delta \): The sampling interval.
- \( M \): The amount of sampling intervals in one prediction horizon.
- \( L \): The set of all one-way links.
- \( J \): The set of all intersections.
- \( \omega \): The stage in an intersection. At each stage, the predefined compatible traffic streams cross the intersection simultaneously according to the simultaneous traffic light signals.
- \( \Omega_a \): The set of stages in intersection \( a \), \( a \in J \).
- \( F_a \subseteq L \times L \): In the intersection \( a \), the set of all streams such as \( (i, j) \in F_a \) means that there exists a traffic stream except buses from links \( i \) to \( j \) via the junction \( a \) and \( (ii, jj) \in F_a \) means that there exists a traffic stream of buses from links \( i \) to \( j \) via the junction \( a \).
- \( h_a \): \( \Omega \to 2^F \): The association of each stage to relevant compatible streams.
- \( \theta_a(k) \): In the interval \( k \), the traffic situation of stage \( \omega \).
- \( N \): The set of natural numbers.
- \( L_i \): The length of the link \( i \).
- \( v_i^* \): The maximum speed allowed on the link \( i \).
- \( d_{ij}(k) \): In the interval \( k \), the amount of entering vehicles except buses of the link \( j \).
- \( d_{ij}(k) \): In the interval \( k \), the amount of entering buses of the link \( j \).
- \( s_j(k) \): In the interval \( k \), the amount of exit vehicles except buses of the link \( j \).
- \( s_{ij}(k) \): In the interval \( k \), the amount of exit buses of the link \( j \).
- \( \lambda_{ij}(k) \): In the interval \( k \), the turning ratio of vehicles except buses in the link \( i \) towards the link \( j \).
- \( \lambda_{ij}(k) \): In the interval \( k \), the turning ratio of buses in the link \( i \) towards the link \( j \).
- \( l_{ij}^r \): The \( r \)-th speed profile of vehicles except buses in the link \( i \) towards the link \( j \).
- \( l_{ij}^r \): The \( r \)-th speed profile of buses in the link \( i \) towards the link \( j \).
- \( e \): A positive real number which is small enough such as the minimum positive precision for a computer.

In the proposed traffic flow model, we consider three types of constraints as follows. They are the stage status constraints, the link volume dynamic constraints and the flow dynamic constraints. Minimizing the delay time for the people in the vehicles at the traffic network is our goal. Next, we describe the three types of constraints and the objective function to be optimized.

A. STAGE STATUS CONSTRAINTS

At each sample interval \( k \), there exists only one action stage at intersection \( a \). The constraints are as follows. For vehicles except buses,

\[
(\forall \omega \in \Omega_a) \theta_a(k) = 0 \Rightarrow (\forall (i, j) \in h_a(\omega)) f_{0ij}(k) = 0 \quad (1a)
\]

For buses,

\[
(\forall \omega \in \Omega_a) \theta_a(k) = 0 \Rightarrow (\forall (ii, jj) \in h_a(\omega)) f_{0ij}(k) = 0 \quad (1b)
\]
For the traffic light,
\[ \sum_{\omega \in \Omega} \theta_\omega(k) = 1 \quad (1c) \]
\[ (\forall \omega \in \Omega, \forall k \in N) \theta_\omega(k) \in \{0,1\} \quad (1d) \]
where \( \theta_\omega(k) = 0 \) and \( \theta_\omega(k) = 1 \) represent that the RED traffic light and Green traffic light at stage \( \omega \) in the interval \( k \), respectively.

Constraints (1a) and (1b) mean that if the RED traffic light is on at stage \( \omega \) in the interval \( k \), then all the related flow rates must be zero. Constraints (1c) and (1d) mean that there is only one GREEN traffic stage at an intersection.

**B. LINK VOLUME DYNAMIC CONSTRAINTS**

According to conservation principle, each link \( j \in L \) for vehicles except buses, there are volume dynamics constraints (2a) - (2d) as follows.

\[ C_j(k+1) = C_j(k) + (d_j(k) - s_j(k))\Delta \quad (2a) \]
\[ (\forall k \in N) C_j(k) \in N \quad (2b) \]
\[ d_j(k) = \sum_{i \in S(j)} f_{ij}(k) \quad (2c) \]
\[ s_j(k) = \sum_{j \in S(i)} f_{ji}(k) \quad (2d) \]

Each link \( j \in L \) for buses, there are volume dynamics constraints (2e) - (2h) as follows.

\[ C_{jB}(k+1) = C_{jB}(k) + (d_{jB}(k) - s_{jB}(k))\Delta \quad (2e) \]
\[ (\forall k \in N) C_{jB}(k) \in N \quad (2f) \]
\[ d_{jB}(k) = \sum_{i \in S(jB)} f_{ijB}(k) \quad (2g) \]
\[ s_{jB}(k) = \sum_{j \in S(iB)} f_{jiB}(k) \quad (2h) \]

From Fig. 1, we can get that \( d_j(k) = f_{ijB}(k), s_j(k) = f_{ji}(k), d_{jB}(k) = f_{ij}(k) \) and \( s_{jB}(k) = f_{jiB}(k) \).

**C. FLOW DYNAMIC CONSTRAINTS**

According to the work in [24], for vehicles except buses, at each stage \( \omega \in \Omega_o \) and each stream \( (i,j) \in h_o(\omega) \), the exit flow \( f_{ij}(k) \) is determined by the following factors: (1) the current link volume from the upstream: \( C_i(k) \), (2) the current remaining link capacity from the downstream: \( C_j(k) \), and (3) the past \( r+1 \) time intervals of the traffic light: \( \theta_\omega(k-r), \ldots, \theta_\omega(k) \). Then, we get Constraint (3) represent it as follows.

\[ f_{ij}(k) = g_{ij}(C_i(k), C_j(k), \theta_\omega(k-r), \ldots, \theta_\omega(k)) \quad (3) \]

We can get that Constraint (3) is a nonlinear function. If the GREEN traffic signal stay for a long time, then the drivers intend to keep a high speed as long as the downstream link can receive the flow with sufficient capacity. Meanwhile, it embodies the delay caused by traffic state transition. We adopt the following method to define \( g_{ij}(\bullet) \). Assume that there are \( r+1 \) speed categories which are monotonically non-increasing. Let them as \( l_0^i \geq \cdots \geq l_r^i > 0 \), presenting the speed ranges from high to low. We determine the actual speed category \( l_i(k) \) in the following Constraints (4a) - (4e).

\[ l_i(k) = \sum_{p=0}^{r} \delta_{ij}^p(k) \cdot l_i^p \quad (4a) \]
\[ \sum_{p=0}^{r} \delta_{ij}^p(k) - \theta_\omega(k) = 0 \quad (4b) \]
\[ (\forall q : 0 \leq q \leq r-1)(1-\theta_\omega(k-q-1))\prod_{p=q}^{r} \theta_\omega(k-p) = 1 \quad (4c) \]
\[ \Leftrightarrow \delta_{ij}^{r-1}(k) = 1 \]
\[ \prod_{p=0}^{r} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{ij}^0(k) = 1 \quad (4d) \]
\[ (\forall p : 0 \leq p \leq r)\delta_{ij}^p(k) \in \{0,1\} \quad (4e) \]

Constraint (4b) indicates that if \( \theta_\omega(k) = 0 \), we can get that \( \sum_{p=0}^{r} \delta_{ij}^p(k) = 0 \). Then, according to Constraints (4e), \( \delta_{ij}^0(k) = 0 \), so \( l_i(k) = 0 \). That is to say, in the interval \( k \) at stage \( \omega \), when the traffic light is RED, the related speed must be zero. Similarly, if \( \theta_\omega(k) = 1 \), we can get that \( \sum_{p=0}^{r} \delta_{ij}^p(k) = 1 \), which means that there exists only one \( p \) such that \( \delta_{ij}^p(k) = 1 \). It means that, in the interval \( k \) at stage \( \omega \), when the traffic light is GREEN, the related speed category \( l_i(k) \) can select only one corresponding value. Constraints (4c) and (4d) mean that the number of continuously green light interval from past to now decide the actual speed category. When the number of continuously green intervals is larger, the speed category is higher. Especially, when there is a traffic state transition, from Constraint (4c), we can get that \( q = 0 \), then \( \delta_{ij}^0(k) = 1 \).

According to Constraint (4a), \( l_i(k) = l_i^r \). Since \( l_i^r \) is the lowest speed category, it chooses the lowest speed category when there is a traffic state transition.

In order to better illustrate the above constraints, we use an example to explain Constraints (4a) - (4e). In the speed category, we assume that there are six speed items, \( r = 5 \), and they are \( l_0^i, l_1^i, l_2^i, l_3^i, l_4^i, l_5^i \). At an intersection, if the continuously green light interval is 4, \( p = r-4+1 = 5-4+1 = 2 \). Then, the speed category \( \delta_{ij}^0(k) = 1 \), and the other speed category is 0, \( \delta_{ij}^0(k) = 0, \delta_{ij}^0(k) = 0, \delta_{ij}^0(k) = 0, \delta_{ij}^0(k) = 0 \) and \( \delta_{ij}^0(k) = 0 \). Thus, in this intersection, the speed category is selected as \( l_i(k) = l_i^2 \). Similarly, in another intersection, if the continuously green light interval is 1, we can obtain that \( p = r-1+1 = 5-1+1 = 5 \). Then the speed category \( \delta_{ij}^0(k) = 1 \), and the other speed category is 0. Then, in this intersection, the speed category is selected as \( l_i(k) = l_i^1 \). The speed category that is chosen is the lowest one. It means that at a traffic state transition, it chooses the lowest speed category, representing the delay caused by a traffic state transition. When the continuously green light interval is bigger, the selected speed category is higher. When the continuously green light number is equal to or greater than 6, according to Constraint (4d), we can get \( \delta_{ij}^0(k) = 1 \), so \( l_i(k) = l_i^6 \), for which the speed category is maintaining the highest one.

Similarly, for buses, the actual speed category \( l_{ij}(k) \) is given by the following Constraints (4f) - (4j).
\[ l_{ij}(k) = \sum_{p=0}^{q} \delta^p_{\text{ij}}(k) \rho^p_{\text{ij}} \]  \hspace{1cm} (4f)

\[ \sum_{p=0}^{q} \delta^p_{\text{ij}}(k) - \theta_{\text{ij}}(k) = 0 \]  \hspace{1cm} (4g)

\[(\forall q: 0 \leq q \leq r-1)(1-\theta_{\text{ij}}(k-q-1)) \prod_{p=0}^{q} \theta_{\text{ij}}(k-p) = 1 \]  \hspace{1cm} (4h)

\[ \Rightarrow \delta^q_{\text{ij}}(k) = 1 \]  \hspace{1cm} (4i)

\[ \prod_{p=0}^{q} \theta_{\text{ij}}(k-p) = 1 \Leftrightarrow \delta^0_{\text{ij}}(k) = 1 \]  \hspace{1cm} (4j)

After \( l_{ij}(k) \) and \( l_{ij}(k) \) are determined, the link flow rates \( f_i(k) \) and \( f_j(k) \) are given as follows.

\[ f_i(k) = \left[ \min \{ \lambda_{ij}(k)C_{ij}(k), l_{ij}(k) \delta_{ij}(k) \} \right] \]  \hspace{1cm} (5a)

\[ f_j(k) = \left[ \min \{ \lambda_{ij}(k)C_{ij}(k), l_{ij}(k) \delta_{ij}(k) \} \right] \]  \hspace{1cm} (5b)

\[ f_i(k) \Delta \in N \]  \hspace{1cm} (5c)

\[ f_j(k) \Delta \in N \]  \hspace{1cm} (5d)

\[ \sum_{\{i:j,i\in A,j\in j\}} f_j(k) = \Delta \]  \hspace{1cm} (5e)

\[ \sum_{\{i:j,i\in j\},j\in j} \lambda_{ij}(k) = 1 \]  \hspace{1cm} (5f)

where \( \lfloor \cdot \rfloor \) denotes the floor of the input argument, i.e., the largest integer smaller than the input argument, \( \lambda_{ij}(k) \) and \( \lambda_{ij}(k) \) are assumed to be known in advance.

From Constraints (5e) and (5f), each vehicle moves from the upstream link to the downstream link. In a one-time interval, \( f_i(k) \Delta \) is the amount of vehicles except buses moving from links \( i \) to \( j \) and \( f_j(k) \Delta \) is the amount of buses moving from links \( i \) to \( j \). According to Constraints (5a)–(5d), \( f_i(k) \Delta \) and \( f_j(k) \Delta \) are the largest integers that are not greater than the volume \( \lambda_{ij}(k)C_{ij}(k) \) and \( \lambda_{ij}(k)C_{ij}(k) \) from the upstream link \( i \), respectively. Meanwhile, \( f_i(k) \Delta \) and \( f_j(k) \Delta \) are the largest integers that are not greater than the remaining capacity \( \hat{C}_{ij}(k) - C_{ij}(k) \) and \( \hat{C}_{ij}(k) - C_{ij}(k) \) to the downstream link \( j \) weighted by the speed category \( l_i(k) \) and \( l_{ij}(k) \), respectively.

For the above constraints, Constraints (1a), (2a) - (2d), (4a) - (4e), (5a), (5c) and (5e) are for vehicles except buses; Constraints (1b), (2e) - (2h), (4f) - (4j), (5b), (5d) and (5f) are for buses; and Constraints (1c) and (1d) are for vehicles including buses.

D. OBJECTIVE FUNCTION

The total delay time within \( M \) time intervals for vehicles except buses in the traffic network can be calculated as follows.

\[ \sum_{i=1}^{M} \sum_{k=1}^{K} \Delta \left( C_i(k) - \frac{L}{V_i} \sum_{j \in L(i,j)} f_j(k) \right) \]  \hspace{1cm} (6c)

Similarly, the total delay time within \( M \) time intervals for the buses in the traffic network can be calculated as follows.

\[ \sum_{i=1}^{M} \sum_{k=1}^{K} \Delta \left( C_i(k) - \frac{L}{V_i} \sum_{j \in L(i,j)} f_j(k) \right) \]  \hspace{1cm} (6d)

In this work, we consider the bus transit system. Let the total delay within \( M \) time intervals in the traffic network for the people in the vehicles be the objective function. We use the following formula as the objective function:

\[ \min \left\{ \sum_{i=1}^{M} \sum_{k=1}^{K} A \left( C_i(k) - \frac{L}{V_i} \sum_{j \in L(i,j)} f_j(k) \right) \right\} \]  \hspace{1cm} (6e)

For the above objective function (6e), we can see that the function consists of two parts. The first part is the total delay time within \( M \) time intervals in the traffic network for the people in the private vehicles while the second part is the total delay time within \( M \) time intervals in the traffic network for the people in the buses.

E. THE MODEL OF THE CASE WITHOUT DEDICATED BUS LANES

We notice that there are some places without bus lanes. If there are none dedicated bus lanes, the model presented in Part A to Part D in this section should be improved. The buses and private vehicles use the same lanes and they are randomly distributed on every road. The buses are converted into private vehicles in a same stage. Then, some parameters are changed. In the previous model, we use the delay time for the people as the objective function. Then, in link \( i \), the modified maximum capacity \( \hat{C}_i(k) \) can be represented as follows.

\[ \hat{C}_i(k) = \frac{B}{A} C_i(k) \]  \hspace{1cm} (7a)

\[ (\forall k \in N) C_i(k) \in N \]  \hspace{1cm} (7b)

Since private vehicles and public buses share the same traffic lanes, in link \( i \), the total amount of vehicles including buses \( C_i(k) \) can be represented as follows.

\[ C_i(k) = C_i(k) + C_i(k) \]  \hspace{1cm} (7c)

In link \( i \), the modified maximum capacity \( \hat{C}_i(k) \) can be shown as follows.

\[ \hat{C}_i(k) = \hat{C}_i(k) + C_i(k) \]  \hspace{1cm} (7d)

Then, in the next time interval \( K+1 \), the modified maximum capacity \( \hat{C}_i(k+1) \) can be represented as formulas (7e) and (7f).

\[ \hat{C}_i(k+1) = \hat{C}_i(k) + d_i(k) \times \frac{C_i(k)}{C_i(k) + C_i(k)} - s_i(k) \times \frac{C_i(k)}{C_i(k) + C_i(k)} \]  \hspace{1cm} (7e)

\[ (\forall k \in N) C_i(k+1) \in N \]  \hspace{1cm} (7f)
In the next time interval $K+1$, the modified amount of buses $C_{ij}(k+1)$ can be shown as follows.

$C_{ij}(k+1) = C_{ij}(k) + d_{ij}(k) - s_{ij}(k) \cdot \frac{C_{ij}(k)}{C_{ij}(k) + C_{ij}(k)}$  

(7g)

$(\forall k \in \mathbb{N}) C_{ij}(k+1) \in \mathbb{N}$  

(7h)

Similarly, in the next time interval $K+1$, the amount of private vehicles $C_{ij}(k+1)$ can be represented as (7i) and (7j).

$C_{ij}(k+1) = C_{ij}(k) + d_{ij}(k) - s_{ij}(k) \cdot \frac{C_{ij}(k)}{C_{ij}(k) + C_{ij}(k)}$  

(7i)

$(\forall k \in \mathbb{N}) C_{ij}(k+1) \in \mathbb{N}$  

(7j)

For buses, the improved model ensures that the number of people passing an intersection per unit time is consistent with the proposed model. Then, the modified $r$-th speed profile of buses in the link $i$ towards the link $j$ is as follows.

$I_{ij}^r = I_{ij}^r \times \frac{B}{A}$  

(7k)

The modified $r$-th speed profile of vehicles including buses in the link $i$ towards the link $j$ is as follows.

$I_{ij}^r = C_{ij}(k) \times I_{ij}^r + C_{ij}(k) \times I_{ij}^r$  

(7l)

Finally, the improved model objective function can be shown as follows.

$$\text{min} \sum_{i,j \in A} \sum_{k=0}^{M} \left( C_{ij}(k) + \frac{B}{A} \frac{C_{ij}(k)}{C_{ij}(k) + C_{ij}(k)} - \sum_{p=0}^{M} \frac{1}{V_j} \sum_{r \in R_i} f_{ij}^r(k) \right) \Delta$$  

(8)

### III. CONVERSION OF MIXED LOGICAL CONSTRAINTS

In the description of Section II, in the situation of considering the dedicated bus lanes, there is a set of constraints including the stage status Constraints (1a) - (1d), link volume dynamics Constraints (2a) - (2h), the actual speed category Constraints (4a) - (4j), and link flow rates flow Constraints (5a) - (5f). Among these constraints, some of them are mixed logic constraints or non-linear constraints. In order to solve the problem, we convert them into mixed integer linear constraints in this section as follows.

We first convert the constraints for vehicles except buses. Constraint (1a) is converted into Constraint (9) as follows.

$(\forall \omega \in \Omega_a) \theta_a(k) = 0 \Rightarrow (\forall (i,j) \in h_a(\omega)) f_{ij}^r(k) = 0$  

(1a)

$(\forall \omega \in \Omega_a) (\forall (i,j) \in h_a(\omega)) f_{ij}^r(k) \leq M \theta_a(k)$  

(9)

where $M_1 = \max_{i,j \in \mathbb{C}_i}$, making $M_1$ big enough.

$M_1$ is the first value we introduce, which is as big as possible. It makes the right conversion from (1a) to (9).

**Proposition 1:** In the scheduling formulation of urban network traffic signal, Constraint (1a) is equivalent to Constraint (9). □

**Proof:** When $\theta_a(k) = 0$, according to Constraint (9), $f_{ij}^r(k) \leq 0$. With $f_{ij}^r(k) \geq 0$, we have $f_{ij}^r(k) = 0$. When $\theta_a(k) = 1$, $f_{ij}^r(k) \leq M_1$. According to Constraint (5a), it is trivially true. ■

Constraint (4c) is converted into Constraints (10a) - (10c) as follows.

$(\forall q : 0 \leq q \leq r-1) (1 - \theta_a(k-q-1)) \prod_{p=0}^{r} \theta_a(k-p) = 1$  

(4c)

$\Leftrightarrow \delta_{ij}^r(k) = 1$  

(10a)

$(\forall q : 0 \leq q \leq r-1) (\forall p : 0 \leq p \leq q) \theta_a(k-p) + \delta_{ij}^r(k) \leq 0$  

(10b)

$(\forall q : 0 \leq q \leq r-1) (1 - \theta_a(k-q-1)) + \sum_{p=0}^{r} \theta_a(k-p) - \delta_{ij}^r(k) \leq q + 1$  

(10c)

**Proposition 2:** In the scheduling formulation of urban network traffic signal, Constraint (4c) is equivalent to Constraint (10a) - (10c). □

**Proof:** When $(\forall q : 0 \leq q \leq r-1) (1 - \theta_a(k-q-1)) \prod_{p=0}^{r} \theta_a(k-p) = 1$, we can get $(\forall q : 0 \leq q \leq r-1) (\forall p : 0 \leq p \leq q) \theta_a(k-p) + \delta_{ij}^r(k) \leq 0$.

Then, Constraints (10a) - (10c) can be transformed as follows:

$(\forall q : 0 \leq q \leq r-1) \delta_{ij}^r(k-1) \leq 1$  

(10d)

$(\forall q : 0 \leq q \leq r-1) \delta_{ij}^r(k) \geq 1$  

(10e)

Then, we can get $(\forall q : 0 \leq q \leq r-1) \delta_{ij}^r(k-1) = 1$. Constraints (10a) - (10c) can be transformed as follows:

$(\forall q : 0 \leq q \leq r-1) \theta_a(k-q-1) \leq 0$  

(10f)

$(\forall q : 0 \leq q \leq r-1) (\forall p : 0 \leq p \leq q) \theta_a(k-p) + 1 \leq 0$  

(10g)

$(\forall q : 0 \leq q \leq r-1) (1 - \theta_a(k-q-1)) + \sum_{p=0}^{r} \theta_a(k-p) \leq q + 2$  

(10h)

It is obvious that Constraint (10h) is trivially true. From Constraints (10f) and (10g), we can get $(\forall q : 0 \leq q \leq r-1) \theta_a(k-q-1) = 0$ and $(\forall q : 0 \leq q \leq r-1) (\forall p : 0 \leq p \leq q) \theta_a(k-p) = 1$, leading to $(\forall q : 0 \leq q \leq r-1) \theta_a(k-q-1) = \cdots = \theta_a(k-q+1) = \theta_a(k-q) = 1$, $\theta_a(k-q-1) = 0$, which shows that $(\forall q : 0 \leq q \leq r-1) (1 - \theta_a(k-q-1)) \prod_{p=0}^{r} \theta_a(k-p) = 1$ holds. ■

Constraint (4d) is converted into Constraints (11a) - (11b) as follows.

$\prod_{p=0}^{r} \theta_a(k-p) = 1 \Leftrightarrow \delta_{ij}^r(k) = 1$  

(4d)

$(\forall p : 0 \leq p \leq r) \theta_a(k-p) + \delta_{ij}^r(k) \leq 0$  

(11a)

$\sum_{p=0}^{r} \theta_a(k-p) - \delta_{ij}^r(k) \leq r$  

(11b)

**Proposition 3:** In the scheduling formulation of urban network traffic signal, Constraint (4d) is equivalent to Constraints (11a) - (11b). □

**Proof:** When $\prod_{p=0}^{r} \theta_a(k-p) = 1$, we can get $(\forall p : 0 \leq p \leq r) \theta_a(k-p) = 1$. Then, Constraints (11a) and (11b) can be transformed as follows:
$\delta_y^b(k) \leq 1$  \hspace{1cm} (11c)  
$\delta_y^q(k) \geq 1$  \hspace{1cm} (11d)  

Then, we can get $\delta_y^q(k) = 1$.

When $\delta_y^q(k) = 1$, Constraints (11a) and (11b) can be transformed as follows:

$(\forall p: 0 \leq p \leq r)\theta_y(k-p) \geq 1$  \hspace{1cm} (11e)  

$\sum_{p=0}^r \theta_y(k-p) \leq r+1$  \hspace{1cm} (11f)

It is clear that Constraint (11f) is trivially true. From Constraint (11e), we can get $(\forall p: 0 \leq p \leq r)\theta_y(k-p) = 1$, which means that $\prod_{p=0}^r \theta_y(k-p) = 1$ holds.

Constraints (5a) and (5c) are converted into the mixed logical constraints (5c) and (12a) - (12c) as follows.

$f_y(k) \Delta = \left[ \min \{\lambda_y(k)C_y(k), l_y(k)(\hat{C}_y - C_y(k))\} \right]$  \hspace{1cm} (5a)  

$f_y(k) \Delta \in \mathbb{N}$  \hspace{1cm} (5c)  

$f_y(k) \Delta \leq \lambda_y(k)C_y(k)$  \hspace{1cm} (12a)  

$f_y(k) \Delta \leq l_y(k)(\hat{C}_y - C_y(k))$  \hspace{1cm} (12b)  

$[f_y(k) \Delta + 1 \geq \lambda_y(k)C_y(k) + \varepsilon]$  \hspace{1cm} (12c)  

$\cup [f_y(k) \Delta + 1 \geq l_y(k)(\hat{C}_y - C_y(k)) + \varepsilon]$  \hspace{1cm} (12d)

In this conversion and subsequent conversion, $\varepsilon$ refers to the positive real number which is small enough such as the minimum positive precision for a computer (shown in Section II). The functions of (12b) and (12c) are to satisfy the upper and lower bounds of integer requirements. Note that (12c) is a mixed logical constraint. In [24], the authors show that Constraint (12b) needs to be transformed into mixed logical Constraint (13a) as follows.

$f_y(k) \Delta \leq l_y(k)(\hat{C}_y - C_y(k))$  \hspace{1cm} (13a)  

$(\forall q: 0 \leq q \leq r)\delta_y^q(k) = 1 \Leftrightarrow f_y(k) \Delta \leq l_y^q(\hat{C}_y - C_y(k))$  \hspace{1cm} (13b)  

However, this is unnecessary since (12b) is already a linear constraint. Furthermore, we can show that (13a) results in a conflict as follows. According to the work in [45], The authors in [24] convert Constraint (13a) into mixed integer linear constraints (13b) - (13c) as follows.

$(\forall q: 0 \leq q \leq r)\delta_y^q(k) = 1 \Leftrightarrow f_y(k) \Delta \leq l_y^q(\hat{C}_y - C_y(k))$  \hspace{1cm} (13a)  

$(\forall q: 0 \leq q \leq r)f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)) \leq M_y(1-\delta_y^q(k)) \hspace{1cm} (13b)$  

$(\forall q: 0 \leq q \leq r)f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)) \geq \varepsilon + (m_y - \varepsilon)\delta_y^q(k) \hspace{1cm} (13c)$

where $M_y \geq \max_k (f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)))$, making $M_y$ big enough, and $m_y \geq \min_k (f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)))$, making $m_y$ small enough. We have the following result.

**Proposition 4:** In the scheduling formulation of urban network traffic signal, the replacement of Constraint (12b) with Constraints (13b) - (13c) would result in a conflict. \hspace{1cm} \Box  

**Proof:** When $\theta_y(k) = 0$, according to Constraints (1a) and (4b), we can get $f_y(k) = 0$, $\delta_y^q(k) = 0$. Then, Constraints (13b) - (13c) can be transformed as follows:

$(\forall q: 0 \leq q \leq r)f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)) \leq M_y$  \hspace{1cm} (13d)  

$(\forall q: 0 \leq q \leq r)f_y(k) \Delta - l_y^q(\hat{C}_y - C_y(k)) \geq \varepsilon$  \hspace{1cm} (13e)

It can be found that Constraint (13e) is a conflict since the left side is smaller than zero, while the right side is greater than zero.

The cause of this error is from Constraint (13a), it is wrong: 

$(\forall q: 0 \leq q \leq r)f_y(k) \Delta \leq l_y^q(\hat{C}_y - C_y(k)) \Rightarrow \delta_y^q(k) = 1$ holds. This situation is a conflict when $f_y(k) = 0$.

With (12c) being a mixed logical constraint, it needs to be converted into a linear constraint. In [24], the authors convert it into mixed integer linear Constraints (14a) - (14e) as follows.

$f_y(k) \Delta + 1 \geq \lambda_y(k)C_y(k) + \varepsilon$  \hspace{1cm} (12c)  

$\cup [f_y(k) \Delta + 1 \geq l_y(k)(\hat{C}_y - C_y(k)) + \varepsilon]$  \hspace{1cm} (12d)  

$m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14a)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14b)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14c)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14d)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14e)  

where $m_y \leq \min_k (f_y(k) + 1 - \lambda_y(k)C_y(k) - \varepsilon)$, making $m_y$ small enough, $m_y \leq \min_k (f_y(k) \Delta + 1 - l_y^q(\hat{C}_y - C_y(k)) - \varepsilon)$, making $m_y$ small enough, $M_y \geq \max_k (m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + l_y^q(\hat{C}_y - C_y(k)) + \varepsilon)$, making $M_y$ big enough, and $m_y \leq \min_k [(m_y(1 - \mu_y(k)) - f_y(k) \Delta - 1 + l_y^q(\hat{C}_y - C_y(k)) + \varepsilon)$, making $m_y$ small enough.

However, this is not correct since it results in a conflict as stated by the following proposition.

**Proposition 5:** In the scheduling formulation of urban network traffic signal, the replacement of Constraint (12c) with Constraints (14a) - (14e) results in a conflict. \hspace{1cm} \Box  

**Proof:** When $\theta_y(k) = 0$, according to Constraints (1a) and (4b), we can get $f_y(k) = 0$, $\delta_y^q(k) = 0$. Then, Constraints (14a) - (14e) can be transformed as follows:

$m_y(1 - \mu_y(k)) - 1 + \lambda_y(k)C_y(k) + \varepsilon \leq 0$  \hspace{1cm} (14f)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - 1 + l_y^q(\hat{C}_y - C_y(k)) + \varepsilon \leq M_y$  \hspace{1cm} (14g)  

$(\forall q: 0 \leq q \leq r)m_y(1 - \mu_y(k)) - 1 + l_y^q(\hat{C}_y - C_y(k)) + \varepsilon \leq M_y$  \hspace{1cm} (14h)  

When $\mu_y(k) = 1$, Constraint (14f) can be transformed as:

$\lambda_y(k)C_y(k) \leq 1 - \varepsilon$  \hspace{1cm} (14i)

We know that the value of $\lambda_y(k)C_y(k)$ is a non-zero integer. However, from (14i) we can get that $\lambda_y(k)C_y(k)$ must be zero, leading to a conflict. Then, according to Constraints (14d) and (14e), we should let $\mu_y(k) = 0$ and
\( \mu_2(k) = 1 \), then Constraint (14h) can be transformed as:

\[
(\forall q : 0 \leq q \leq r)\bigl(\hat{C}_j - C_j(k)\bigr) \geq 1 + \varepsilon \quad \text{(14j)}
\]

Then, when the vehicles on the road are saturated, this constraint creates contradictions, which means that the model has no solution.

From the above formulation, Constraints (12b) and (12c) are in the constraint set. It is sure that Constraint (5a) is satisfied. For Constraint (12b), there is no prerequisite, such as \( (\forall q : 0 \leq q \leq r)\delta^0_y(k) = 1 \). The reason is that even if \( (\forall q : 0 \leq q \leq r)\delta^0_y(k) = 0 \), Constraint (12b) is still satisfied. Hence, no need to convert Constraint (12b) and in fact it is a linear constraint. For Constraint (12c), there are prerequisites, such as \( (\forall q : 0 \leq q \leq r)\delta^0_y(k) = 1 \). The reason is that when \( (\forall q : 0 \leq q \leq r)\delta^0_y(k) = 0 \), Constraint (12c) is not satisfied. We let \( \theta_y(k) = 1 \) be the prerequisite, which is the same as \( (\forall q : 0 \leq q \leq r)\delta^0_y(k) = 1 \). Thus, Constraint (12c) can be converted into mixed logical Constraint (15a) as follows.

\[
\left[ f_y(k) + 1 \geq \lambda_y(k)C(k) + \varepsilon \right] \\
\text{where } \theta_y(k) = 1 \quad \text{(15a)}
\]

According to the work in [45], Constraint (15a) can be converted into mixed integer linear Constraints (15b) - (15e) as follows.

\[
\left[ f_y(k) + 1 \geq \lambda_y(k)C(k) + \varepsilon \right] \\
\iff \\
\left[ f_y(k) + 1 \geq \lambda_y(k)C(k) + \varepsilon \right] \\
\text{where } \theta_y(k) = 1 \Rightarrow \left[ f_y(k) + 1 \geq \lambda_y(k)C(k) + \varepsilon \right] \\
\text{where } \theta_y(k) = 1 \quad \text{(15a)}
\]

\[
M_1(1 - \theta_y(k)) - \lambda_y(k)C(k) + \varepsilon + f_y(k) + 1 \geq m_1(1 - \eta_y(k)) \quad \text{(15b)}
\]

\[
M_1(1 - \theta_y(k)) - l_y(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_y(k) + 1 \geq m_1(1 - \eta_y(k)) \quad \text{(15c)}
\]

\[
\eta_y(k), \eta_y(k) \in [0, 1] 
\]

where \( M_1 = \max_k (l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon + f_y(k) + 1) \), making \( M_1 \) big enough. \( M_2 \geq \max_k (l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon + f_y(k) + 1) \), making \( M_2 \) big enough, \( M_3 \leq \min_k \left( M_1(1 - \theta_y(k)) - \lambda_y(k)C(k) - \varepsilon + f_y(k) + 1 \right) \), making \( M_3 \) small enough, \( m_1 \) small enough, and \( m_2 \leq \min_k \left( M_1(1 - \theta_y(k)) - l_y(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_y(k) + 1 \right) \), making \( m_2 \) small enough.

In order to satisfy the equation \( \theta_y(k) = 1 \Rightarrow f_y(k) + 1 \geq \lambda_y(k)C(k) + \varepsilon \), the left part of (15b) introduces the sufficient big value \( M_2 \). In order to satisfy the equation \( \theta_y(k) = 1 \Rightarrow f_y(k) + 1 \geq l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon \), the left part of (15c) shows the sufficient big value \( M_3 \). At last, in order to satisfy the “or” relationship, we introduce the sufficient small value \( m_1 \) and \( m_2 \), and the binary valve \( \eta_y(k) \) and \( \eta_y(k) \), which are in the Constraints (15d) and (15e), and the right side of Constraints (15b) and (15c). Then, we have the following results.

**Proposition 6:** In the scheduling formulation of urban network traffic signal, Constraint (15a) is equivalent to Constraints (15b) - (15e).

**Proof:** When \( \theta_y(k) = 1 \), Constraints (15b) and (15c) can be transformed as:

\[
\lambda_y(k)(C(k) + \varepsilon + f_y(k) + 1) \geq m_1(1 - \eta_y(k)) \quad \text{(15f)}
\]

\[
l_y(k) + 1 \geq l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon + f_y(k) + 1 \geq m_1(1 - \eta_y(k)) \quad \text{(15g)}
\]

When \( \eta_y(k) = 1 \), \( f_y(k) + 1 \geq l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon + f_y(k) + 1 \geq m_1(1 - \eta_y(k)) \).

When \( \eta_y(k) = 0 \), according to the value of \( m_1 \), Constraint (15f) is trivially true. When \( \eta_y(k) = 1 \), \( f_y(k) + 1 \geq l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon + f_y(k) + 1 \geq m_1(1 - \eta_y(k)) \).

When \( \eta_y(k) = 0 \), according to the value of \( m_2 \), Constraint (15g) is trivially true. According to Constraints (15d) and (15e), there is at least one of \( \eta_y(k) \) and \( \eta_y(k) \) that can be chosen as 1.

That is to say, when \( \theta_y(k) = 1 \), we can get

\[
f_y(k) + 1 \geq l_y(k)(\hat{C}_j - C_j(k)) + \varepsilon \quad \text{(15h)}
\]

When \( \theta_y(k) = 0 \), \( f_y(k) = 0 \), Constraints (15b) and (15c) can be transformed as:

\[
M_2 \geq \lambda_y(k)(C(k) + \varepsilon + 1) \geq m_1(1 - \eta_y(k)) \quad \text{(15i)}
\]

\[
M_1 \geq l_y(k)(\hat{C}_j - C_j(k)) - \varepsilon + 1 \geq m_1(1 - \eta_y(k)) \quad \text{(15j)}
\]

When \( \eta_y(k) = 1 \), \( M_2 - \lambda_y(k)(C(k) + \varepsilon + 1) \geq m_1(1 - \eta_y(k)) \).

When \( \eta_y(k) = 0 \), \( m_1 \) is transformed into Constraints (15i) and (15j) as follows.

\[
(\forall \omega \in \Omega_s) \forall (i, j) \in h_\omega(\omega)\bar{f}_{ij}(k) \leq M_4 \theta_x(k) \quad \text{(16)}
\]

where \( M_4 > \max_{h \leq \hat{C}_j} \) makes \( M_4 \) big enough.

**Constraint (4h) is converted into Constraints (17a) - (17c) as follows.**

\[
(\forall q : 0 \leq q \leq r - 1) -(1 - \theta_x(k - q - 1)) + \delta_0^{-q}(k) \leq 0 \quad \text{(17a)}
\]

\[
(\forall q : 0 \leq q \leq r - 1)(\forall p : 0 \leq p < q) - \theta_x(k - p) + \delta_0^{-q}(k) \leq 0 \quad \text{(17b)}
\]

\[
(\forall q : 0 \leq q \leq r - 1)(1 - \theta_x(k - q - 1)) + \delta_0^{-q}(k) \leq q + 1 \quad \text{(17c)}
\]

**Constraint (4i) is converted into Constraints (18a) and (18b) as follows.**

\[
(\forall p : 0 \leq p \leq r) - \theta_x(k - p) + \delta_0^{-q}(k) \leq 0 \quad \text{(18a)}
\]

\[
\sum_{r = 0}^{r} \theta_x(k - r) - \delta_0^{-q}(k) \leq r \quad \text{(18b)}
\]

Constraints (5b) and (5d) are converted into Constraints (5d) and (19a) - (19c) as follows.

\[
f_{ij}(k) \Delta \in N \quad \text{(5d)}
\]
with bus lanes. The model includes the objective function (19a)-(19c), (19c) \rightarrow (20a) \rightarrow (1a)- (19c), (19c) \rightarrow (20a)

\[ \sum_{p=0}^{\gamma} \delta_{ij}^p \cdot \theta_{\alpha}(k) = 0 \]  
\[ (\forall q: 0 \leq q \leq r-1), (1 - \theta_{\alpha}(k-q-1) + \delta_{ij}^{r-q}(k) \leq 0 \]  
\[ (\forall q: 0 \leq q \leq r-1), (\forall p: 0 \leq p \leq q) - \theta_{\alpha}(k+p) + \delta_{ij}^{r-q}(k) \leq 0 \]  
\[ (\forall q: 0 \leq q \leq r-1), (1 - \theta_{\alpha}(k-q-1) + \delta_{ij}^{r-q}(k) \leq 0 \]  
\[ + \sum_{p=0}^{\gamma} \theta_{\alpha}(k+p) - \delta_{ij}^{r-q}(k) \leq q + 1 \]

\[ \sum_{p=0}^{\gamma} \theta_{\alpha}(k+p) - \delta_{ij}^{r-q}(k) \leq r \]

\[ (\forall p: 0 \leq p \leq r) \delta_{ij}^{r-q}(k) \in \{0, 1\} \]

\[ f_{ij}(k) \Delta + l_{ij}(k) (\hat{C}_{ij} - C_{ij}(k)) \]

\[ \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta + s_{ij} \Delta \]

subject to

\[ \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ \forall k \in N \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ \forall k \in N \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ (\forall \omega \in \Omega) \forall k \in N \theta_{\alpha}(k) \in \{0, 1\} \]

\[ (\forall \omega \in \Omega) \forall k \in N \theta_{\alpha}(k) \in \{0, 1\} \]

\[ (\forall k \in N) \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ (\forall k \in N) \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ (\forall k \in N) \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ (\forall k \in N) \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]

\[ (\forall k \in N) \forall (i, j) \in \sum_{k=1}^{K} \sum_{j=1}^{M} C_{ij}(k) - L_{ij} \sum_{j \in L \cup (j, i) \cup \cup j, F_{ij}} f_{ij}(k) \Delta \]
\( \eta_i(k), \eta_k(k) \in \{0,1\} \)  
\( f_{ij}(k) \Delta \in \mathbb{N} \)  
\[ \sum_{i \in L, j \in L} \lambda_{ij}(k) = 1 \]  
\[ \sum_{i \in L, j \in L} \lambda_{ij}(k) = 1 \]

For these constraints, we can get that the first two constraints are for vehicles including buses. The next 21 constraints are for vehicles except buses, while the last 21 constraints are for buses.

For the case without bus lane, the improved model is shown as follows:

\[ \min \sum_{i \in L} \sum_{j \in L} A \left( C_i(k) + \frac{B}{A} C_d(k) - \frac{L}{V_j} \sum_{i \in L, j \in L} \lambda_{ij}(k) f_{ij}(k) \right) \Delta \]  
\[ \text{(8)} \]

The constraints include (1c), (1d), (2c), (2d), (4e), (5c), (5e), (9), (10a) - (10c), (11a), (11b), (15d), (15e) and (7a) - (7l). In addition, the following constraints should be included. Since the total amount of vehicles including buses changes for the case without bus lane, the constraints (12a), (12b), (15b) and (15c) should be changed as follows, respectively.

\[ f_{ij}(k) \Delta \leq \lambda_{ij}(k) C_i(k) \]  
\[ f_{ij}(k) \Delta \leq l_{ij}(k) (C_j(k) - C_i(k)) \]  
\[ M_2 (1 - \theta_{ij}(k)) - \lambda_{ij}(k) C_j(k) + \varepsilon + f_{ij}(k) \Delta + 1 \geq m_i (1 - \eta_i(k)) \]  
\[ M_3 (1 - \theta_{ij}(k)) - l_{ij}(k) (C_j(k) - C_i(k)) + \varepsilon + f_{ij}(k) \Delta + 1 \geq m_i (1 - \eta_i(k)) \]  
\[ (21a) \]
\[ (21b) \]
\[ (21c) \]
\[ (21d) \]

### IV. NUMERICAL RESULTS AND ANALYSIS

In order to solve optimization problem formulated in the last section at time \( k \), some information is required. We need to know the past situation of traffic light, the current volume at each link and the incoming flow with the corresponding prediction. In this paper, we assume that the past situation of traffic light and the current volume can be measured by sensors. While the incoming flow with the corresponding prediction are provided by a traffic management system via another model. According to the objective function and the constraints, the solver gets the profile of optimal traffic signal control at time \( k \). After this step, the latest real-time data are provided by the traffic management system and sensors. Then, the solver gets the profile of optimal traffic signal control at time \( k + 1 \). This process is repeated. That is to say, we adopt a receding horizon manner to implement the scheduling. Thus, for the changing traffic conditions, we have the real-time response.

In this section, from Part A to Part D, we analyze the model with dedicated bus lanes. In Part E, we analyze the model without dedicated bus lanes. This section will be summarized in Part F.

#### A. THE CASE WITH TWO STAGES FOR AN INTERSECTION

This situation is the simplest one in the urban traffic network. It is the same as what we shown in Fig. 1. There are two stages in each intersection as shown in Fig. 2.

![FIGURE 2. The two stages of an intersection](image)

It can be seen from Fig. 2 that there are two stages and at each stage there is only one stream crossing the intersection. In this part, we consider a traffic network including different number of horizontal and vertical roads. Due to the consideration of the public transport regular, the size of the model is about twice that of the original model. There are two performance measurements. The first one is the computational time for different situations and the second one is the time delay reduction for the two models with and without considering the public transport and the maximum improvement degree for the model with public transport regular. We assume that there are \( n_h \) horizontal roads and \( n_v \) vertical roads, respectively. For the values of some parameters, we refer to the paper [24]. Then, some parameters are as follows. The incoming vehicle flow rate \( b_j(k) \) is given and \( L_j / V_j = 1 \) is used in this part. For each link \( j \), the maximal volume \( C_j \) and \( C_j \) are selected as 30 and 15, respectively. The scheduling horizon is \( H_p = M \Delta \). The sampling interval is 12 seconds. Two speed categories are chosen. We set the fast speed ratio and the low speed ratio as \( I_f = 0.8 \) and \( I_f = 0.4 \) for the vehicles except the buses. At the same time, \( I_{ig}^0 = 0.4 \) and \( I_{ig}^1 = 0.2 \) are set for the buses. The value of \( A \) is set to be 4. This is based on the calculation of private vehicles. In the front row, there are a driver and a passenger. Meanwhile, in the rear row, there are two passengers, i.e., four people in total. The value of \( B \) is set as 40. During May 2016, in a large city in China, the average number of passengers on the main road of buses was 39.93. Hence, we choose 40 in the model. The optimization problem is solved by LINGO on an ASUS Laptop Y4200F with an Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz and 1.80GHz CPU. We test cases of \( n_h = n_v = 2, 4, 6, 8, 10, 12, 24, 36, 48, \) and 60 seconds, respectively. The corresponding computation time for the two-stage model is summarized in Table I. The time values are given in seconds. When the computational time is very long (more than 120s), it is no sense to solve. Then we use a short solid line to represent them.
TABLE I

| Computation time for different scale in the case of 2 stages | (IN SECONDS) |
|-------------------------------------------------------------|---------------|
| $n_v = n_h$ | $H_p =$ | $H_p =$ | $H_p =$ | $H_p =$ | $H_p =$ |
| 12(s) | 24(s) | 36(s) | 48(s) | 60(s) |
|---|---|---|---|---|
| 2 | 0.05 | 0.11 | 0.41 | 1.03 | 2.22 |
| 4 | 0.06 | 0.51 | 5.98 | 75.22 | - |
| 6 | 0.07 | 4.05 | 91.64 | - | - |
| 8 | 0.09 | 9.13 | - | - | - |
| 10 | 0.13 | 11.96 | - | - | - |

From Table I, we can see that when $H_p = 12s$, each case can be solved in a short time. With the increasing of $H_p$, the computation time also increases rapidly. For a small-scale network, the increase in computation time is relatively slow. To ensure the real-time reaction on the diversification of traffic conditions, we should choose a suitable scheduling horizon according to the size of the network. For a large-scale problem, it can be divided into several small-scale problems. The solver calculates the optimal control signal based on the latest data repeatedly. That is to say, the scheduling is implemented in a receding horizon way.

Next, we compare the two models with and without considering the public transport regular. In some situations, these two models achieve the same result. Except for these situations, the model with public transport regular achieves a better result. Therefore, we consider the maximum optimization degree obtained by the model with public transport regular. At this time, the solutions obtained by the two models are different. We use three indicators for evaluation, the number of delay people, the total delay time in the road network and the average delay time per person. We set one-time interval to express it.

Table II shows the result of the two models of two stages. It can be seen that, for $n_v = n_h = 2$, the model with public transport regular can reduce the delay of 344 people at most by 12s, leading to 86 people on average at each intersection. Similar, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the delay of 1340, 2752, 4892, and 7876 people at most by 12s, respectively. If the result of original model and the model with public transport regular is different in corresponding intersections, the model with public transport regular can reduce the delay time per person about 4.15s at most in 12s.

In order to show the comparisons of the two models more intuitively, Fig. 3 shows the result for the average delay time per person in two stages model. It can be seen directly from Fig. 3 that the model with public transport regular is more optimized than the original model, and the maximum degree can reach about 40%.

B. THE CASE WITH FOUR STAGES FOR AN INTERSECTION

In four-stage situation, it is the normal one in the urban traffic network. There are four stages in each intersection as shown in Fig. 4.

It can be observed from Fig. 4 that, at each stage, there are two compatible streams crossing an intersection simultaneously. Compared with the two-stage system, there are two more stages in this system. The number of constraints for this model is about twice as that of a two-stage system. For each stage, the flow is twice as that of the original one. Overall, the size of the four-stage model is delay time per person, for $n_v = n_h = 2$, the model with public transport regular can reduce the average delay time of 4.25s at most in 12s. Similarly, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the average delay time of 4.35s, 4.03s, 4.03s, and 4.15s at most in 12s, respectively. If the result of original model and the model with public transport regular is different in corresponding intersections, the model with public transport regular can reduce the delay time per person about 4.15s at most in 12s.
about four times of that of the two-stage model. For the values of the parameters, we set them to be the same as the two-stage model. The corresponding computation time for the four-stage model are summarized in Table III. The times are also given in seconds. When the computational time is very long (more than 120s), it is no sense to solve. Then we use a short solid line to represent them.

![FIGURE 4. The four stages of an intersection](image)

| TABLE III | COMPUTATION TIME FOR DIFFERENT SCALE IN THE CASE OF 4 STAGES (IN SECONDS) |
|-----------|--------------------------------|
| $n_v = n_h$ | $H_p = 12(s)$ | $H_p = 24(s)$ | $H_p = 36(s)$ | $H_p = 48(s)$ | $H_p = 60(s)$ |
| 2 | 0.05 | 0.22 | 2.44 | 26.55 | 73.24 |
| 4 | 0.08 | 0.66 | - | - | - |
| 6 | 0.11 | 11.89 | - | - | - |
| 8 | 0.16 | 23.24 | - | - | - |
| 10 | 0.22 | - | - | - | - |

It can be observed from Table III that when $H_p = 12(s)$, each case can be solved in a short time, which is close to the two-stage model. With the increase of $H_p$, the computation time increases rapidly. The growth rate is significantly greater than that for the two-stage model. While for a small-scale network, there is relatively slow increase in computation time. In order to ensure the real-time response to the traffic conditions, choosing a suitable scheduling horizon according to the size of the network is necessary. Meanwhile, the big-scale problem can be divided into several small-scale problems. The solver gets the optimal control signal based on the latest data again and again. In other words, the performance of the scheduling problem is the way of receding horizon.

Next, we compare the two models with and without considering the public transport regular. We use the same approach as the two-stage model does with the same indicators for evaluation.

Table IV shows the result of the two models of four stages. It can be summarized that, for $n_v = n_h = 2$, the model with public transport regular can reduce the delay of 828 people at most by 12s, leading to 207 people on average at every intersection. Similarly, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the delay of 3364, 7524, 13300, and 20740 people at most by 12s, respectively, leading to 210, 209, 208, and 207 people on average at every intersection, respectively. We can get that if the result of the original model and the model with public transport regular is not the same in one intersection, the model with public transport regular can reduce the delay about 208 people at most in an intersection. For the indicators of the average delay time per person, when $n_v = n_h = 2$, the model with public transport regular can reduce the average delay time of 2.23s at most in 12s. Similarly, when $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the average delay time of 2.33s, 2.37s, 2.36s, and 2.33s at most in 12s, respectively. If the result of original model and the model with public transport regular is different in the corresponding intersection, the model with public transport regular can reduce the delay about 208 people at most in 12s.

For further comparison, Fig. 5 shows the result for the average delay time per person of the two models with four stages. It can be seen from the Fig. 5 that the model with public transport regular is better than the original one, and the maximum degree can be improved about 20%. Compared to the two-stage model, the optimization degree of the four-stage model is about half of that by the two-stage model. The reason is that the four-stage model passes a quarter of the total stage at each time, and the two-stage model passes one-half of the total stage. The degree of optimization is specific to the part of passing vehicles.

| TABLE IV | THE RESULTS OF THE TWO MODELS IN THE CASE OF FOUR STAGES |
|-----------|--------------------------------------------------|
| $n_v = n_h$ | The original model | The model with public transport regular |
| | The number of delay people (person) | The total delay time in the road network(s) | The average delay time per person (s/person) | The number of delay people (person) | The total delay time in the road network(s) | The average delay time per person (s/person) |
| 2 | 4192 | 50304 | 11.31 | 3364 | 40368 | 9.08 |
| 4 | 16308 | 195696 | 11.28 | 12944 | 155328 | 8.95 |
| 6 | 35632 | 427584 | 11.23 | 28108 | 337296 | 8.86 |
| 8 | 63056 | 756672 | 11.20 | 49756 | 597072 | 8.84 |
| 10 | 99988 | 119856 | 11.23 | 79248 | 950976 | 8.90 |
C. THE CASE OF DIFFERENT NUMBER OF PEOPLE ON BUSES

In practice, the number of passengers in buses changes dynamically over time, we further test the difference of the two models. To be more intuitive, we use the average delay time per person as an indicator for evaluation. The average number of people in a bus is set to 10, 20, 30, 40, 50, and 60, meaning that $B = 10, 20, 30, 40, 50,$ and 60, respectively. The other indicators as used in above discussion are kept. Fig. 6 and Fig. 7 show the results of two-stage model and four-stage model in the case of $n_v = n_h = 10$, respectively.

It can be seen that with the number of passengers increasing, the average delay time per person becomes less and less in the model with public transport regular, the corresponding value becomes larger and larger in the original model. In the case of $n_v = n_h = 10$, as the average number of passengers in a bus is increased from 10 to 60, the difference of these two models are 1.55s, 2.3s, 3.29s, 4.15s, 4.81s, and 5.15s, respectively in the two-stage model and 0.46s, 1.27s, 1.86s, 2.33s, 2.7s, and 3s, respectively in the four-stage model. With the number of people increasing from 10 to 60, the results of the model with public transport regular can improve those by the original model by 17%, 25%, 34%, 42%, 47%, and 51% respectively in the two-stage model. The corresponding improving values for the four-stage models are about 4%, 12%, 17%, 21%, 24%, and 26%, respectively. The reason is that the objective value is the number of people in all existing vehicles minus the number of people who can pass the intersection. As the number of average people in the bus increases, the number of people in all vehicles increases. Since the original model does not consider public transportation, the number of passing passengers is unchanged, resulting in the average delay time increasing with the increase of the average number of people in the bus. For the model with public transport regular, as the average number of people in the bus increases, the number of passing passengers increases at the same time, leading to the objective value maintained within a range. When the average number of people in the bus increases, the total number of people increases. As a result, the average delay time is reduced. We can get that with the increase of the number of people in the buses, the model with public transport regular has greater advantages.

D. THE CASE OF DIFFERENT SAMPLE INTERVAL

The choice of scheduling horizon affects the result. Also, the choice of the sample interval and the problem scale affects the choice of scheduling horizon. The choice of the sample interval has greater impact on the result. Here, we analyze the impact of different sampling intervals for these two models. Since the interval time is different, we use the new indicator, the percentage of average delay time per person. It is the ratio of average delay time per person and the sampling interval. The sample intervals are set to 3s, 6s, 9s, 12s, 15s and 18s. The other indicators are the same as that for the above discussion. Fig. 8 and Fig. 9 show the result of two-stage model and four-stage model in the case of $n_v = n_h = 10$, respectively.

It can be seen from Fig. 8 and Fig. 9 that when $\Delta = 3s$, two models get the same result. The reason is that there are no buses passing the intersection during this time. From $\Delta = 3s$ to $\Delta = 15s$, the percentage of average delay time per person drop rapidly for the model with public transport regular, while the percentage of average delay time per person drop slowly for the model without public transport regular. It means that there are buses passing the
intersection continuously. From $\Delta = 15s$ to $\Delta = 18s$, the percentage of average delay time per person drop slowly for the two models, which means that all buses of the corresponding stages pass through the intersection. With $\Delta$ increasing from 3s to 18s, the optimization degree of the model with public transport regular can improve those by the original model about 0%, 21%, 26%, 42%, 54%, and 54%, respectively, in the two-stage model. The corresponding values for the four-stage models are about 0%, 6%, 13%, 21%, 21%, and 21%, respectively. The reason is that the objective value is the number of people in all existing vehicles minus the number people passing the intersection. When there is none bus passing, the results of the two models are same. If there are some buses passing, as the number of buses passing increased, the number of passengers passing the intersection increases. In this situation, the degree of optimization for the model with public transport regular increases. In the end, when all the buses in the current stage passed, this two models returned to the same changing degree. In this example, the most reasonable sample interval is to ensure that all buses at the current stage pass.

![Figure 8](image1)

**FIGURE 8.** The results of the two models in different sampling intervals for the case of two stages in the scale of 10 $\times$ 10

![Figure 9](image2)

**FIGURE 9.** The results of the two models in different sampling intervals for the case of four stages in the scale of 10 $\times$ 10

### E. THE CASE WITHOUT DEDICATED BUS LANES

We then compare the model with and without dedicated bus lanes. To be more intuitive, the average delay time per person is used as an indicator for evaluation. The parameters are the same as previously used. Fig. 10 and Fig. 11 illustrate the results of two-stage model and four-stage model, respectively.

From the results, it is clear that the model with dedicated bus lanes performs better. When $n_v = n_b = 10$, the difference of these two models are 1.14s and 1.45s, respectively. Furthermore, the four-stage one shows a greater improvement with dedicated bus lanes. We can get that when the number of private vehicles increases, the buses have greater impact. The reason for this is the constraint (7i). When the proportion of private vehicles becomes larger, the speed profile will decrease, and the result of the model will also become larger. It is of great significance to set dedicated bus lanes, especially for places with a large number of vehicles.

![Figure 10](image3)

**FIGURE 10.** The results of the two models with and without dedicated bus lanes for the case of two stages

![Figure 11](image4)

**FIGURE 11.** The results of the two models with and without dedicated bus lanes for the case of four stages

### F. SUMMARY

From the perspective of solvability, as long as the number of vehicles meets the demand for capacity, the model is solvable. Hence, it is suitable for various saturation conditions. From the perspective of obtained result, as the average number of passengers in a bus from 10 to 60, the degree of optimization are from 17% to 51%
in the two-stage model, and 4% to 26% in the four-stage model respectively. Our model is better than the original model. Considering the problem of dedicated bus lanes, it is important to set up the dedicated bus lanes especially for places with a large number of vehicles.

V. CONCLUSION

In this paper, we propose a novel centralized scheduling formulation for minimizing the total delay time for all people in the buses and private vehicles by using urban traffic signal control. The model is based on a cell transmission model incorporated with novel link flow rate functions of the volume from the upstream, the available capacity from the downstream, and the past situation of traffic signals. We develop a mixed integer linear programming formulation, which is transformed by the traffic signal control problem. Meanwhile, we provide the proofs for the conversion. By experiments, we get the computation time for different cases including the different number of stages (two stages and four stages), the different number of intersections (2x2, 4x4, 6x6, 8x8 and 10x10 intersections), and different prediction horizon (12s, 24s, 36s, 48s and 60s). Furthermore, we analyze the impact of the number of people in buses and different sample intervals. In order to perform the efficient real-time scheduling, results indicate that we should choose a reasonable prediction horizon according to the size of the network (the total number of intersections) and the number of stages. Meanwhile, we should choose a reasonable sample interval base on the number of buses. The solver calculates the optimal control signal according to the latest data over and over again, which means that the execution of the scheduling process is a manner of receding horizon. We compare the model proposed in this study to the original one in [24] and show that the model proposed in this study achieves better results. When the average number of passengers in a bus from 10 to 60, the degree of optimization model are from 17% to 51% in the two stages model, and 4% to 26% in the four stages model, respectively. Finally, to analyze the scenarios without dedicated bus lanes, we modify the model to adapt to this situation for comparison. The comparisons and analysis show that it is important to set dedicated bus lanes, especially for the roads with a large number of vehicles.

There are some directions for the future work.

(1) In this paper, we assume that we know the network boundary flow rates and the turning ratios in advance. In fact, these values are stochastic in practice. It is meaningful to study the problem with this situation taken into account in our future work.

(2) This study only considers the effect of the people in vehicles, including the ones in buses. The effect of pedestrians may be considered at the same time. It is meaningful to integrate the effect of pedestrians on the proposed model, which can be seen as a bi-objective optimization problem.

(3) Each vehicle in the traffic network is delayed only by traffic signals is assumed. However, there may be some emergencies or traffic accidents in the traffic network. It is meaningful to study the traffic signal scheduling under these situations.

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI: 10.1109/ACCESS.2021.3119663, IEEE Access