Mirror Symmetry
of
K3 and Torus

Masaru Nagura\footnote{A Fellow of the Japan Society for the Promotion of Science for Japanese Junior Scientists} \footnote{E-mail address: nagura@tkyvax.phys.s.u-tokyo.ac.jp} and

Katsuyuki Sugiyama\footnote{E-mail address: ksugi@tkyvax.phys.s.u-tokyo.ac.jp}

Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo 113, Japan

ABSTRACT

We discuss a K3 and torus from view point of ”mirror symmetry”. We calculate the periods of the K3 surface and obtain the mirror map, the two-point correlation function, and the prepotential. Then we find there is no instanton correction on K3 (also torus), which is expected from view point of Algebraic geometry.
1 Introduction

It is a long time since the string theory attract the attention of physicist as a candidate of the most fundamental theory that explain all the physical phenomena in nature. In spite of the intense research activities, it remains an open problem to formulate these theories even now. One of the difficulties seems to stem from the treatment of the Calabi-yau spaces that should be compactified. In view of the particle physics, (co)homology classes of this Calabi-yau spaces correspond to zero mass fields in the low energy effective theory and this spaces play very important roles in researching the properties of the field contents, Yukawa couplings and so on [1, 2, 3]. In view of mathematics they correspond to the tangent spaces of the moduli space; the complex structure or the Kähler structure. It seemed hard to investigate these properties because of effects of quantum corrections [4, 5].

Recently great progresses have been made in the understanding of the moduli space in Calabi-Yau manifolds by the discovery of the new notion, called the "mirror symmetry" [10]. It became possible to extract the properties of the spaces. Especially some Yukawa couplings correspond to the complex moduli recieve no quntum correction either in loop or by instanton, owing to the nonrenormalization theorem. This suggests the new concept "quantized algebraic varieties" and attracts physical and mathematical attentions because of the close relations between string theories and algebraic geometries. For that reason, it is not too much to say that this shed new light on the algebraic geometry. When one discusses the mirror symmetries, the (complex) dimension of the Calabi-Yau manifolds is restricted to be three so far. Nevertheless generalized mirror manifolds with other dimensions are expected to be much richer in contents and to be fascinating objects [6].

The aim of this paper is to construct mirror manifolds paired with the (complex one dimensional) torus, the K3 surface and to investigate their properties as a first attempt to the generalization in other dimensions. Firstly we construct mirror manifolds by the method of the orbifoldization. That is, one operates a discrete symmetry which the original manifold has on it and removes singular points on this and obtains a mirror manifold. Primarily we study deformations of the Hodge structures which the original manifold or its mirror manifold have. The information on the Hodge structures is encoded in periods of these manifolds. Therefore one can extract properties of their moduli spaces associated with the deformations of the complex structures or Kähler structures by examining these periods concretely.

In the past, it was difficult to study the Yukawa couplings correspond to the Kähler
moduli spaces directly because of the quantum corrections on this spaces in contrast with
the complex moduli spaces which receive no quantum correction. In the case of the Calabi-
Yau manifolds, however, one can correlate the Kähler moduli space of the original manifold
with the complex moduli space of its mirror manifold. To be more precise, we map the
complex moduli space of the mirror manifold to the Kähler moduli space of the original
one and get the information on the Kähler moduli spaces of the original manifold exactly.
Especially we are interested in couplings of (co)homology elements. Using the mirror map,
we identify the Yukawa coupling of the complex moduli on the mirror manifold with that of
the Kähler moduli on the original manifold. And we find there is no instanton correction on
the K3 (and torus).

This paper is organized as follows. In section 2, we construct a mirror manifold paired
with a K3 surface, by orbifoldization. In section 3, we explain Picard-Fuchs equations that
periods of the algebraic varieties satisfy. We write down solutions of the Picard-Fuchs equa-
tion for the K3 case and construct a mirror map. In section 4, a coupling of (co)homology
elements is calculated. The result is suggestive of the nonexistence of the rational curve on
the K3 surface. In section 5, monodromy matrices associated with solutions of Picard-Fuchs
equation are obtained. Further a fundamental region of the monodromy group is written
down. Section 6 is devoted to conclusions and comments.

2 Construction of mirror manifold

In this section, we take a K3 surface as an original manifold $\mathcal{M}$ and construct its mirror
manifold $\mathcal{W}$ by orbifoldizing the original manifold $\mathcal{M}$, and resolving its singularities.

2.1 Orbifoldization

We take a K3 surface which is represented by a defining equation with order 4 in the
projective space $\mathbb{CP}^3$:

$$\mathcal{M} : p = X_1^4 + X_2^4 + X_3^4 + X_4^4 = 0$$
Hodge numbers of this variety $\mathcal{M}$, $h_{p,q} = \dim_{\mathbb{C}} \mathcal{H}_{p,q}(\mathcal{M}, \mathbb{Z})$ are known and illustrated in the following Hodge diamond [7]:

\[
\begin{array}{cccc}
1 & & & \\
& 0 & 0 & \\
1 & 20 & 1 & \\
& 0 & 0 & \\
1 & & & \\
\end{array}
\]

That is to say, non-zero numbers read,

$$h_{0,0} = h_{0,2} = h_{2,0} = h_{2,2} = 1 ,$$

$$h_{1,1} = 20 .$$

Nineteen cycles out of independent twenty cycles in the Dolbeault (co)homology $\mathcal{H}_{1,1}(\mathcal{M})$ can be expressed in monomials of $X_i$. The remaining one cycle is associated with the Kähler structure of $\mathcal{M}$ [8, 9]. Also the manifold $\mathcal{M}$ has discrete symmetry $G = \mathbb{Z}_4^2$,

$$(1,0,0,3) ,$$

$$(0,1,0,3) ,$$

$$(0,0,1,3) ,$$

where the symbol $(n_1,n_2,n_3,n_4)$ means that the manifold $\mathcal{M}$ is invariant under the transformation,

$$(X_1, X_2, X_3, X_4) \rightarrow (\alpha^{n_1}X_1, \alpha^{n_2}X_2, \alpha^{n_3}X_3, \alpha^{n_4}X_4) ,$$

$$(\alpha = e^{2\pi i} = i) .$$

Furthermore these three transformations are not independent because successive actions of these transformations result in $(1,1,1,1) = (0,0,0,0)$ in the $\mathbb{C}P^3$.

Following the recipe in the case of the quintic Calabi-Yau manifold [10], we construct a mirror manifold $\mathcal{W}$. Firstly we operate the original manifold with this symmetry $G$. There are $24 (= 4 \times 4C_2)$ singular points under this operation. So we have to remove these points from the manifold $\mathcal{M}$ and blow up each point. A manifold $\mathcal{W}$ obtained in this recipe is a mirror manifold of $\mathcal{M}$. Its Euler number is calculated, as follows;

$$\chi(\mathcal{W}) = \frac{24 - 24}{4^2} + \frac{4 \times 24}{4} = 24 .$$

In short,

$$\chi(\mathcal{W}) = \chi(\mathcal{M}) = 24 ,$$

where the symbol $(n_1,n_2,n_3,n_4)$ means that the manifold $\mathcal{M}$ is invariant under the transformation,
and the mirror manifold \( \mathcal{W} \) has the same Hodge diamond as \( \mathcal{M} \). This reflects a self-duality of the K3 surface. Note that there is one invariant monomial, \( X_1 X_2 X_3 X_4 \) in the (co)homology class \( \mathcal{H}_{1,1} \) under the action of \( G \). Since the \( \mathcal{W} \) is invariant under \( G \), one can identify the monomial \( X_1 X_2 X_3 X_4 \) as an element of \( \mathcal{H}_{1,1}(\mathcal{W}) \). We assume this monomial on the mirror manifold \( \mathcal{W} \) corresponds to the Kähler class of the original \( \mathcal{M} \) and we can deform the complex structure in the mirror \( \mathcal{W} \) with this monomial. That is to say, we consider one parameter family of a mirror manifold \( \mathcal{W}_\psi \),

\[
\begin{align*}
\mathcal{W}_\psi ; & \quad \{ p = 0 \}/G - 4\psi X_1 X_2 X_3 X_4 \quad (\psi \in \mathbb{C}) \\
& = \{ p_\psi = 0 \}/G , \\
p_\psi & := p - 4\psi X_1 X_2 X_3 X_4 \\
& = X_1^4 + X_2^4 + X_3^4 + X_4^4 - 4\psi X_1 X_2 X_3 X_4 .
\end{align*}
\]

(1)

The parameter \( \psi \) is a coordinate of the complex structure of the mirror manifold \( \mathcal{W} \). Using some map, called a mirror map, we can translate the information on the complex moduli in \( \mathcal{W} \) into that on the Kähler moduli on \( \mathcal{M} \). In later section 4, we explain these properties in more detail.

### 3 Picard-Fuchs equation and mirror map

In this section, we introduce periods of the mirror manifold and write down a differential equation, called a Picard-Fuchs equation satisfied by the periods \([11, 12]\). We can obtain solutions of this equation instead of carrying out the integral of the periods and define a mirror map of the K3 surface, which connects the complex structure of \( \mathcal{W} \) with the Kähler structure of \( \mathcal{M} \).

#### 3.1 Holomorphic form and periods

The structure of the moduli space of a Calabi-Yau manifold is described by giving the periods of its manifold \([11]\), which is expressed by a holomorphic form. Let us consider a \( d \)-dimensional hypersurface \( V \) defined by the following homogeneous polynomial \( W \) in \( \mathbb{C} \mathbb{P}^{d+1} \),

\[
W := X_1^{d+2} + X_2^{d+2} + \cdots + X_{d+1}^{d+2} + X_{d+2}^{d+2}
\]
It is known that $V$ defined by $W$ is a Calabi-Yau manifold. Then there is a holomorphic $d$-form $\Omega$ on $V$ defined globally and nowhere zero. $\Omega$ can be expressed by, \[ \Omega := \int_\gamma \frac{1}{W} \sigma, \]

\[ \sigma := \sum_{a=1}^{d+2} (-1)^a X_a dX_1 \wedge \cdots \wedge dX_a \wedge \cdots \wedge dX_{d+2}, \]

where $\gamma$ is a small one-dimensional curve winding around the $V$. The $(d+1)$-form $\sigma$ is invariant under discrete transformations $Z_{d+2}^{\sigma}$,

\[ \begin{aligned} (X_1, X_2, \ldots, X_{i-1}, X_i, X_{i+1}, \ldots, X_{d+1}, X_{d+2}) & \rightarrow (X_1, X_2, \ldots, X_{i-1}, \alpha_{d+2} X_i, X_{i+1}, \ldots, X_{d+1}, \alpha_{d+2} X_{d+2}), \\
\alpha_{d+2} & \equiv \exp \frac{2\pi i}{d+2}. \end{aligned} \]

The set of periods are defined to be the integral of $\Omega$ over homology cycles $\tilde{\Gamma}_a$ of $V$,

\[ \tilde{\omega}_a = \int_{\tilde{\Gamma}_a} \Omega, \]

\[ \tilde{\Gamma}_a \in \mathcal{H}_d(V). \]

It is known that if we consider the following series,

\[ W^{(N)} = \sum_{i=1}^{N} x_i^N - N\psi \prod_{i=1}^{N} x_i \quad (N \geq 3), \]

the differential equations satisfied by the periods correspond to $1, \prod_{i=1}^{N} x_i, (\prod_{i=1}^{N} x_i)^2, \ldots, (\prod_{i=1}^{N} x_i)^{N-2}$ can be written \[ \frac{d}{dz} - z \left( \frac{d}{dz} + \frac{1}{N} \right) \cdots \left( \frac{d}{dz} + \frac{N-1}{N} \right) \tilde{\omega}^{(N)} = 0, \]

where $\tilde{\omega}^{(N)} := -N\psi^{\tilde{\omega}^{(N)}}$ and $z := \psi^{-N}$. These equations are called Picard-Fuchs equations.

### 3.2 Application to Kummer surface

When we apply the formulae for $N = 4$, we can obtain the Picard-Fuchs equation for the mirror manifold of the K3 surface $W$ represented by the defining equation \[ \left( \frac{d}{dz} \right)^3 - z \left( \frac{d}{dz} + \frac{1}{4} \right) \left( \frac{d}{dz} + \frac{2}{4} \right) \left( \frac{d}{dz} + \frac{3}{4} \right) \tilde{\omega} = 0, \]

\[ z := \psi^{-4}, \]
We should note that this equation is the same as that of the original manifold \( M \). The periods of mirror are the same as the corresponding periods of the original up to multiplication factor \( 1/4^2 \). We think this fact is a direct result from mirror symmetry.

Firstly let us study the property of solutions of this differential equation around a point \( z = 0 \). This point \( z = 0 \) is a regular singular point \([13]\). In addition, all the coefficients of the terms \( (z \frac{d}{dz})^n \), \((n = 0, 1, 2)\) vanish at this point. So solutions of this equation have unipotent monodromy of index 3, i.e. maximally unipotent monodromy \([13]\). Independent solutions of equation (2) can be solved,

\[
W_1(\psi) = \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4 (4\psi)^{4n}} ,
\]

\[
W_2(\psi) = -4 \times W_1 \cdot \log(4\psi)
+ 4 \times \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4 (4\psi)^{4n}} \left[ \Psi(4n + 1) - \Psi(n + 1) \right] ,
\]

\[
W_3(\psi) = 4^2 \times W_1 \cdot \log(4\psi)^2
- 2 \times 4^2 \times \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4 (4\psi)^{4n}} \left[ \Psi(4n + 1) - \Psi(n + 1) \right] \cdot \log(4\psi)
+ 4^2 \times \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4 (4\psi)^{4n}} \left\{ \left[ \Psi(4n + 1) - \Psi(n + 1) \right]^2 + \Psi'(4n + 1) - \frac{1}{4} \Psi'(n + 1) \right\} .
\]

As another expression, we can take a basis of this equation \([12]\),

\[
\varpi_a := \frac{1}{z \cdot (z - 1)^{1/2}} \left( \frac{dz}{ds} \right) \cdot s^a \quad (a = 0, 1, 2) .
\]

In this formula, \( s \) is a function of \( z \) and is a solution of the schwarzian differential equation,

\[
\{ s, z \} = \frac{1}{2} \cdot \frac{1}{z^2} + \frac{3}{8} \cdot \frac{1}{(1 - z)^2} + \frac{13}{32} \cdot \frac{1}{z(1 - z)} .
\]

It can be represented by a triangle function \([16]\),

\[
s\left(0, \frac{1}{4}, \frac{1}{2}; z\right) .
\]

The \( s(z) \) is also rewritten as a ratio of two independent solutions \( y_I, y_{II} \) of Gauss’s hypergeometric differential equation \([16]\),

\[
z(1 - z) \frac{d^2 y}{dz^2} + \left( 1 - \frac{3}{2} z \right) \frac{dy}{dz} - \frac{3}{64} y = 0 ,
\]

\[
s = \frac{y_{II}}{y_I} .
\]
Obviously solutions of (8) satisfy a relation,

\[ \varpi_0 \varpi_2 - \varpi_1^2 = 0 \quad . \tag{10} \]

To be sure, there is arbitrariness in the choice of two independent solutions \( y_I, y_{II} \) resulting from the relation,

\[ \{ \frac{As + B}{Cs + D}, z \} = \{ s, z \} \quad , \tag{11} \]

with \( AD - BC \neq 0 \), \( (A, B, C, D \in \mathbb{C}) \).

Let us investigate this arbitrariness in the choice of \( s \). If one replaces \( s \) with \( \frac{As + B}{Cs + D} \), \((AD - BC \neq 0; A, B, C, D \in \mathbb{C})\), the solutions \( \varpi_a \) transform linearly,

\[ \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \end{pmatrix} \rightarrow \frac{1}{AD - BC} \begin{pmatrix} D^2 & 2CD & C^2 \\ BD & AD + BC & AC \\ B^2 & 2AB & A^2 \end{pmatrix} \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \end{pmatrix} \quad . \tag{12} \]

Even after this transformation, the relation (10) remains invariant. In order to research this transformation matrix in detail, let us introduce a new basis of the solution of (8),

\[ \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \end{pmatrix} \equiv \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \end{pmatrix} \quad . \tag{13} \]

Then the relation (10) is re-expressed,

\[ 0 = -\varpi_0 \varpi_2 + \varpi_1^2 = -\omega_0^2 + \omega_1^2 + \omega_2^2 \quad . \]

On this new basis, the transformation (12) can be written,

\[ \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \end{pmatrix} \rightarrow M \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \end{pmatrix} \quad , \tag{14} \]

\[ M = \frac{1}{AD - BC} \begin{pmatrix} \frac{1}{2}(A^2 + B^2 + C^2 + D^2) & AB + CD & \frac{1}{2}(-A^2 + B^2 - C^2 + D^2) \\ AC + BD & AD + BC & -AC + BD \\ \frac{1}{2}(-A^2 - B^2 + C^2 + D^2) & -AB + CD & \frac{1}{2}(A^2 - B^2 - C^2 + D^2) \end{pmatrix} \quad . \tag{15} \]

Because a bilinear form \((-\omega_0^2 + \omega_1^2 + \omega_2^2)\) is invariant under the transformation (13), the transformation matrix \( M \) is an element of the group \( SO(2,1) \).

Taking notice of the relation (10), we can decompose the holomorphic \((2,0)\)-form \( \Omega \) with a basis of homology cycles \( \alpha, \beta, \gamma \), which belong to \( \mathcal{H}_2(W) \), and the solutions (8) into

\[ \Omega = \varpi_0 \alpha + \varpi_1 \beta - \varpi_2 \gamma \quad , \tag{16} \]
such that these cycles satisfy relations,

\[ \int_W \alpha \wedge \gamma = \int_W \gamma \wedge \alpha = 1 , \]
\[ \int_W \beta \wedge \beta = 2 , \int_W \text{others} = 0 . \]  

Then the coupling of the holomorphic 2-form \( \Omega \) with itself vanishes obviously,

\[ \int_W \Omega \wedge \Omega = 0 . \]

We can easily return to the relation (11) from (15) (18). The existence of these homology cycles; \( \alpha, \beta \) and \( \gamma \) satisfying the above relations on \( K3 \) surface has been known in Algebraic geometry. Of course, the choice of the homology basis; \( \alpha, \beta \) and \( \gamma \) is not unique. If \( \alpha', \beta' \) and \( \gamma' \) consist another basis then they are related to \( \alpha, \beta \) and \( \gamma \) by a matrix \( N \);

\[
\begin{pmatrix}
\alpha' \\
\beta' \\
\gamma'
\end{pmatrix} = N \begin{pmatrix}
\alpha \\
\beta \\
\gamma
\end{pmatrix},
\]

where the matrix \( N \) takes value in integer and preserve a matrix \( \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} \) is invariant. In short, the following relation is satisfied.

\[
N \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} N^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \]  

3.3 Mirror map

Following the recipe of D.R.Morrison [13], we define the ”mirror map \( t(\psi) \)” by demanding a property \( t(e^{2\pi i z}) = t(z) + 1, \)

\[ t(z) := \frac{1}{2\pi i} \cdot \frac{W_2(z)}{W_1(z)} . \]

In the above formula, \( W_1(z), W_2(z) \) are solutions of the equation (2) and are given in (3) and (4). Remarkably, these solutions can also be written as the products of two solutions of a Gauss’s hypergeometric equation (8).

\[
W_1 = y_1 \cdot y_1 ,
\]
\[
W_2 = y_1 \cdot y_2 + \sqrt{2} \pi y_1 \cdot y_1 .
\]
The functions $y_1, y_2$ are defined,

$$y_1 := {}_2F_1\left(\frac{1}{8}, \frac{3}{8}; 1; z\right),$$

$$y_2 := {}_2F_1\left(\frac{1}{8}, \frac{3}{8}; 1; z\right) \log z + 2 \tilde{F}_1\left(\frac{1}{8}, \frac{3}{8}; 1; z\right),$$

$$(22)$$

$$2 \tilde{F}_1\left(\frac{1}{8}, \frac{3}{8}; 1; z\right) = \sum_{n=0}^\infty \frac{\Gamma(n+\frac{1}{8}) \Gamma(n+\frac{3}{8})}{\Gamma(\frac{3}{8}) \Gamma(n+\frac{5}{8}) (n!)^2} \times \left\{ \Psi\left(n+\frac{1}{8}\right) + \Psi\left(n+\frac{3}{8}\right) - 2 \Psi(n+1) \right\} z^n.$$  

Then the mirror map $t$ valid in the range $|\psi| > 1$ and $0 \leq \arg \psi < \pi$ can be expressed,

$$t = \frac{1}{2\pi i} \left( \frac{y_2}{y_1} + \sqrt{2\pi} \right) = \frac{1}{2\pi i} \left[ \sqrt{2\pi} - 4 \log \psi + \frac{2 \tilde{F}_1\left(\frac{1}{8}, \frac{3}{8}; 1; \psi^{-4}\right)}{2 \tilde{F}_1\left(\frac{1}{8}, \frac{3}{8}; 1; \psi^{-4}\right)} \right].$$

$$(24)$$

Also in the range $|\psi| \leq 1$ and $0 \leq \arg \psi < \pi/2$, this map $t$ can be written by the analytic continuation,

$$t = -\frac{1}{2} + \frac{i}{2} \cdot \frac{Z_1 - e^{\frac{3}{4}\pi i} (\tan \frac{\pi}{8})^{-1} \cdot Z_2}{Z_1 + e^{\frac{3}{4}\pi i} (\tan \frac{\pi}{8})^{-1} \cdot Z_2},$$

$$(25)$$

$$Z_1 := \left[ \frac{\Gamma\left(\frac{3}{8}\right)}{\Gamma\left(\frac{5}{8}\right)} \right]^2 \cdot {}_2F_1\left(\frac{1}{8}, \frac{1}{8}; \frac{3}{4}; \psi^4\right),$$

$$Z_2 := \left[ \frac{\Gamma\left(\frac{3}{8}\right)}{\Gamma\left(\frac{5}{8}\right)} \right]^2 \cdot \psi \cdot {}_2F_1\left(\frac{3}{8}, \frac{3}{8}; \frac{5}{4}; \psi^4\right).$$

Moreover the expression of $t$ reads for $|\psi - 1| \sim 0$,

$$t = \frac{i}{\sqrt{2}} \cdot \frac{U_1 + (\tan \frac{\pi}{8})^{-1} \cdot U_2}{U_1 - (\tan \frac{\pi}{8})^{-1} \cdot U_2},$$

$$(26)$$

$$U_1 := \left[ \frac{\Gamma\left(\frac{3}{8}\right)}{\Gamma\left(\frac{5}{8}\right)} \right]^2 \cdot {}_2F_1\left(\frac{1}{8}, \frac{1}{8}; \frac{1}{2}; 1 - \psi^4\right),$$

$$U_2 := \left[ \frac{\Gamma\left(\frac{3}{8}\right)}{\Gamma\left(\frac{5}{8}\right)} \right]^2 \cdot (\psi^4 - 1)^{1/2} \cdot {}_2F_1\left(\frac{5}{8}, \frac{5}{8}; \frac{3}{2}; 1 - \psi^4\right).$$

The map $t$ connects the complex structure of the mirror manifold $\mathcal{W}$ to the Kähler structure of the original manifold $\mathcal{M}$. The properties of $t$ concerning monodromies are postponed till section 5.
4 Yukawa coupling

In this section, we calculate a two-point correlation function of (co)homology classes associated with the Kähler structure of the original K3 surface or with the complex structure of its mirror surface. The notion of the Yukawa coupling is easily extended to d-dimensional case. This can be regarded as the d-point correlation function of (co)homology classes correspond to the complex or the Kähler structure.

4.1 Kähler potential and metric on the moduli space

A metric of the complex moduli space in the mirror $\mathcal{W}$ can be calculated from a Kähler potential $K$ \[3\]. The Kähler potential in two-dimension is the same form as in three-dimensional case;

\[
e^{-K} := \int_\mathcal{W} \Omega \wedge \bar{\Omega} = \varpi_0 \bar{\varpi}_2 + \varpi_2 \bar{\varpi}_0 - 2 \varpi_1 \bar{\varpi}_1 = \left| \frac{1}{z \cdot (z - 1)^{1/2}} \left( \frac{dz}{dt} \right)^2 \right| (t - \bar{t})^2 = \varpi_0 \bar{\varpi}_0 (t - \bar{t})^2,
\]
where the symbol "\(\bar{\cdot}\)" represents a complex conjugate. Also the metric of the complex moduli can be expressed,

\[
G_{\psi \bar{\psi}} = \frac{\partial^2 K}{\partial \psi \partial \bar{\psi}} = \frac{1}{2(\Im t)^2} \left| \frac{dt}{d\psi} \right|^2.
\] (27)

4.2 Yukawa coupling

Two-point coupling $\kappa_{\psi \psi}$ of the (co)homology class $\mathcal{H}^2(\mathcal{W})$ is defined as,

\[
\kappa_{\psi \psi} := - \int_\mathcal{W} \Omega \wedge \chi_\psi^\mu \wedge \chi_\psi^\nu \Omega_{\mu \nu},
\] (28)
where

\[
\chi_\psi^\mu := \frac{1}{2\|\Omega\|^2} \bar{\Omega}^{\mu \nu} \chi_\psi_{\rho \sigma} dx^\rho,
\] (29)
\[
\chi_\psi := \chi_\psi_{\rho \sigma} dx^\rho \wedge dx^\sigma \in \mathcal{H}^{1,1}(\mathcal{W}),
\] (30)
with the help of the Kodaira’s theorem: \( \frac{d\Omega}{d\psi} = \kappa_\psi \Omega + \chi_\psi \in \mathcal{H}^{2,0}(\mathcal{W}) \oplus \mathcal{H}^{\infty,\infty}(\mathcal{W}) \). The above formula is reduced to

\[
\kappa_{\psi\psi} := - \int_{\mathcal{W}} \Omega \wedge \frac{d^2\Omega}{d\psi^2} = 2\left\{ \left( \frac{d\varpi_1}{d\psi} \right)^2 - \left( \frac{d\varpi_0}{d\psi} \right) \cdot \left( \frac{d\varpi_2}{d\psi} \right) \right\} \\
= 2 \left[ \frac{-4\psi}{(1 - \psi^4)^{1/2}} \right]^2 \\
= \frac{32\psi^2}{1 - \psi^4} .
\]

Here we have to remember that the holomorphic \((2,0)\)-form takes value in the holomorphic line bundle. We have arbitrariness in the choice of \( \Omega \), i.e. a gauge symmetry in physical speaking,

\[
\Omega(\psi) \rightarrow f(\psi)\Omega(\psi) ,
\]

where \( f(\psi) \) is an arbitrary holomorphic function. Under this transformation, the Kähler potential \( K \) turns into \( K - \log f(\psi) - \log \bar{f}(\bar{\psi}) \). By the aid of the Kodaira’s theorem, the two point coupling \( \kappa_{\psi\psi} \) becomes \( (f(\psi))^2\kappa_{\psi\psi} \).

To relate with the Yukawa coupling of the Kähler structure, we convert the coordinate \( \psi \) to the one \( t \) given by the mirror map. We fix gauge arbitrariness by choosing the Kähler potential to be a simple form,

\[
K = -\log (t - \bar{t})^2 .
\]

In turns in order to get this formula, we have to perform a gauge transformation on the original \((2,0)\)-form \( \Omega \),

\[
\Omega \rightarrow \frac{1}{\varpi_0(\psi)} \Omega .
\]

After these transformation, the coupling \( \kappa_{\psi\psi} \) becomes \( \kappa_{\psi\psi} \left( \frac{d\psi}{dt} \right) \times \frac{1}{(\varpi_0)^2} \). The two point coupling \( \kappa_{tt} \) associated with the Kähler structure of the manifold \( \mathcal{M} \) becomes

\[
\kappa_{tt} = \kappa_{\psi\psi} \left( \frac{d\psi}{dt} \right)^2 \times \frac{1}{(\varpi_0)^2} \\
= 2 \left[ \frac{-4\psi}{(1 - \psi^4)^{1/2}} \left( \frac{dt}{d\psi} \right)^{-1} \right]^2 \times \frac{1}{(\varpi_0)^2} \\
= 2 (\varpi_0)^2 \times \frac{1}{(\varpi_0)^2} \\
= 2 .
\]
From [4, 5], it is shown that
\[ \kappa_{tt} = \int_{\mathcal{M}} e \wedge e + \text{(instanton corrections)}, \tag{33} \]
\[ e \in \mathcal{H}^{1,1}(\mathcal{M}). \tag{34} \]

Moreover the value of \( \int_{\mathcal{M}} e \wedge e \) is 2.

In contrast with the quintic case [10], the coupling in the Kähler moduli space contains no quantum (instanton) correction. Coefficients of correction terms, if exist, are interpreted as the number of rational curves embedded in this manifold [10, 17, 18]. Therefore our calculation shows that the K3 surface \( p_\psi = 0 \) contains no rational curve on it.

Let us consider this more Algebraic geometricaly. For simplicity we consider rational curves with degree one. A Riemann surface with genus 0 i.e. \( \mathbb{CP}^1 \) is parametrized by a projective coordinate \((u, v)\). If a rational curve on the K3 surface exists, there exists a set of complex numbers \((a_i, b_i)\) \((i = 1, 2, 3, 4)\) such that the point \((a_1 u + b_1 v, a_2 u + b_2 v, a_3 u + b_3 v, a_4 u + b_4 v)\) in \( \mathbb{CP}^3 \) is on the defining equation for arbitrary sets of \((u, v)\). In short, the relation should be satisfied,
\[ (a_1 u + b_1 v)^4 + (a_2 u + b_2 v)^4 + (a_3 u + b_3 v)^4 + (a_4 u + b_4 v)^4 = 4 \psi (a_1 u + b_1 v)(a_2 u + b_2 v)(a_3 u + b_3 v)(a_4 u + b_4 v), \]
for all values of \((u, v)\). In this case, there is five relations. But there exists one automorphism,
\[ (u, v) \rightarrow (\hat{A}u + \hat{B}v, \hat{C}u + \hat{D}v). \]
Collecting these, one can calculate the number of the freedom that the rational curve has,
\[ 8 - 5 - 4 = -1. \]
It is overdetermined. Therefore there exists no rational curve with degree one and we exhibit no quantum correction exists in the coupling of the K3 surface.

A prepotential \( F^{(t)} \) of \( \kappa_{tt} \) (32) can be defined in this gauge,
\[ F^{(t)} = t^2. \]

On the other hand in the old gauge, this prepotential is written by taking account of the fact that the prepotential is a homogeneous polynomial with degree 2,
\[ F^{(\varpi)} = (\varpi_0)^2 l^2 = \varpi_0 \varpi_2 = (\varpi_1)^2. \]
Also the differential operator $\partial_t$ acting on $\mathcal{F}^{(t)}$ is replaced by a covariant derivative for degree 2, which we introduce as follows;

$$D_t^{(2)} = \omega_0^2 \partial_t \omega_0^{-2} = \partial_t - 2 \frac{\partial_t \omega_0}{\omega_0} ,$$

such that

$$D_t D_t \mathcal{F}^{(\omega)} = 2(\omega_0)^2 ,$$

is the coupling obtained in the old gauge.

5 Monodromy

In this section, we study monodromy transformations and give monodromy matrices, fundamental regions.

5.1 Monodromy

The defining equation of one parameter family of K3 surface was represented in (9). If the conditions $\partial p_i / \partial X_i = 0$, $(i = 1, 2, 3, 4)$ are satisfied for some $\psi$, this variety is singular. These conditions are rewritten,

$$(X_1X_2X_3X_4)^3(\psi^4 - 1) = 0 ,$$

and this variety is singular at $\psi^4 = 1, \infty$, i.e. $\psi = \alpha^n, \infty$, $(\alpha = e^{\frac{2\pi}{4}} = i ; n = 0, 1, 2, 3)$. If one rotates the values of the parameter $\psi$ around the special value analytically in the complex $\psi$-plane, homology cycles $\gamma_i$ of the variety (9) turn into linear combinations of other homology cycles $\sum j c_{ij} \gamma_j$. As a consequence, the periods also change into linear combinations of others. The matrices associated with this linear transformation are called monodromy matrices and contain geometrical information on the variety.

5.2 Monodromy matrices

In order to obtain the monodromy matrices, let us consider two monodromy transformations $A, T$ [10, 19], which act on the mirror map $t$. The $A$ is a transformation of $t(\psi)$ associated with the rotation around the point $\psi = 0$ with an angle $\frac{\pi}{2}$ counterclockwise in the complex $\psi$-plane, i.e. $\psi \rightarrow e^{\frac{\pi}{4}i}\psi$. Also the $T$ is associated with the rotation around the point $\psi = 1$ with angle $2\pi$ counterclockwise, i.e. $\psi = 1 + \epsilon \rightarrow \psi = \psi$. 

\[13\]
$1 + e^{2\pi i} (\epsilon; \text{small complex number})$. As is well-known, one can express the monodromy transformation $T_m$ ($m = 0, 1, 2, 3, \infty$) [11][12], which are associated with the rotations around the points $\psi = \alpha^0, \alpha^1, \alpha^2, \alpha^3, \infty \ (\alpha = e^{2\pi i} i)$ with the angle $2\pi$ respectively,

$$T_m = \mathcal{A}^m \mathcal{T} A^{-m}, \quad (m = 0, 1, 2, 3), \quad (35)$$

$$T_\infty = (T_0 T_1 T_3)^{-1} = (\mathcal{T} \mathcal{A})^{-4}. \quad (36)$$

Also the monodromy transformation $\hat{T}_\infty$ associated with the rotation around $\psi = \infty$ with an angle $\frac{\pi}{2}$ is represented as,

$$\hat{T}_\infty = (\mathcal{T} \mathcal{A})^{-1}. \quad (37)$$

Firstly let us consider the actions of $\mathcal{A}$ on $Z_1, Z_2$ in equation (24),

$$\mathcal{A}Z_1(\psi) = Z_1(\psi), \quad (38)$$

$$\mathcal{A}Z_2(\psi) = \alpha Z_2(\psi).$$

We obtain the relation,

$$\mathcal{A}t(\psi) = \frac{\sqrt{2t(\psi)} + \frac{1}{\sqrt{2}}}{-\sqrt{2t(\psi)}} = -1 - \frac{1}{2t(\psi)}. \quad (39)$$

This transformation (37) is a composition of an inversion $-\frac{1}{2t(\psi)}$ and a translation $t(\psi) - 1$. We get monodromy matrices $\tilde{A}, \mathcal{A}$ associated with $\mathcal{A}$ on the bases $t'(w_0, w_1, w_2), t'(\omega_0, \omega_1, \omega_2)$, respectively,

$$\tilde{A} = \begin{pmatrix} 0 & 0 & 2 \\
 0 & -1 & -2 \\
 \frac{1}{2} & 2 & 2 \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} \frac{9}{4} & 1 & -\frac{7}{4} \\
 -2 & -1 & 2 \\
 -\frac{1}{4} & -1 & -\frac{1}{4} \end{pmatrix}. \quad (38)$$

Secondly the transformation $\mathcal{T}$ should be considered. Because of the actions of $\mathcal{T}$ on $U_1, U_2$ in the equation (28),

$$\mathcal{T}U_1(\psi) = U_1(\psi), \quad (40)$$

$$\mathcal{T}U_2(\psi) = -U_2(\psi), \quad (40)$$

the relation is obtained,

$$\mathcal{T}t(\psi) = -\frac{1}{2t(\psi)}. \quad (39)$$
This is an inversion of $t(\psi)$. Monodromy matrices $\tilde{T}$, $T$ on the bases $t'(\varpi_0, \varpi_1, \varpi_2)$, $t'(\omega_0, \omega_1, \omega_2)$ associated with $\mathcal{T}$ are given,

$$\tilde{T} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} \frac{5}{4} & 0 & -\frac{3}{4} \\ 0 & -1 & 0 \\ \frac{3}{4} & 0 & -\frac{5}{4} \end{pmatrix}.$$  \tag{40}

Collecting the above considerations, we list the results. Monodromy matrices on the basis $t'(\varpi_0, \varpi_1, \varpi_2)$ are expressed,

$$\tilde{A} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & -2 \\ \frac{1}{2} & 2 & 2 \end{pmatrix}, \quad \tilde{T} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix},$$  \tag{41}

$$\tilde{T}_1 = \begin{pmatrix} 2 & 12 & 18 \\ -1 & -5 & -6 \\ \frac{1}{2} & 2 & 2 \end{pmatrix}, \quad \tilde{T}_2 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & -4 \\ \frac{1}{2} & 4 & 8 \end{pmatrix},$$  \tag{42}

$$\tilde{T}_3 = \begin{pmatrix} 2 & 4 & 2 \\ -3 & -5 & -2 \\ \frac{9}{2} & 6 & 2 \end{pmatrix}, \quad \tilde{T}_\infty = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 8 & 1 \end{pmatrix},$$  \tag{43}

$$\widehat{T}_\infty = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}. \tag{44}
$$

Also monodromy matrices on the basis $t'(\omega_0, \omega_1, \omega_2)$ read,

$$A = \begin{pmatrix} \frac{9}{4} & 1 & -\frac{7}{4} \\ -2 & -1 & 2 \\ -\frac{1}{4} & -1 & \frac{1}{4} \end{pmatrix}, \quad T = \begin{pmatrix} \frac{5}{4} & 0 & -\frac{3}{4} \\ 0 & -1 & 0 \\ \frac{3}{4} & 0 & -\frac{5}{4} \end{pmatrix},$$  \tag{45}

$$T_1 = \begin{pmatrix} \frac{45}{4} & 7 & -\frac{35}{4} \\ -7 & -5 & \frac{5}{4} \\ \frac{35}{4} & 5 & -\frac{29}{4} \end{pmatrix}, \quad T_2 = \begin{pmatrix} \frac{77}{4} & 18 & -\frac{27}{4} \\ -18 & -17 & 6 \\ \frac{27}{4} & 6 & -\frac{13}{4} \end{pmatrix},$$  \tag{46}

$$T_3 = \begin{pmatrix} \frac{21}{4} & 5 & \frac{5}{4} \\ -5 & -5 & -1 \\ -\frac{5}{4} & -1 & -\frac{5}{4} \end{pmatrix}, \quad T_\infty = \begin{pmatrix} 9 & 4 & 8 \\ 4 & 1 & 4 \\ -8 & -4 & -7 \end{pmatrix},$$  \tag{47}

$$\widehat{T}_\infty = \begin{pmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 1 \\ -\frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix}. \tag{48}
$$

All these matrices belong to the group $SO(2,1)$. 
5.3 Fundamental region

Recall the result (27) in section 4, we obtain the metric of the Kähler moduli space,

\[ G_{\bar{t}t} = \frac{1}{2(3t)^2} . \]

A Ricci tensor and a scalar curvature are calculated,

\[ R_{\bar{t}t} = -\frac{1/2}{(3t)^2} , \]
\[ R = -4 . \]

It is sufficient to consider the upper half \( t \)-plane in considering the moduli space because the curvature is a negative constant number. The monodromy transformations are generated by two generators \( A, T \) or by other ones \( A, \tilde{T}_{\infty} \). These act on \( t, \tilde{T}_{\infty} \),

\[ \tilde{T}_{\infty} t(\psi) = t(\psi) + 1 , \]
\[ A t(\psi) = -1 - \frac{1}{2t(\psi)} . \]

The upper half \( t \)-plane is mapped to itself and is covered once and only once by combinations of these actions. A fundamental region is drawn by considering these expressions in Fig.1. The fixed point of \( A \) is \( t = -\frac{1}{2} + \frac{i}{2} \) and that of \( T \) is \( t = \frac{i}{\sqrt{2}} \). These correspond to the values of \( t \) at \( \psi = 0, \psi = 1 \) respectively. On the contrary, the infinity \( t = \infty \) is the fixed point of \( \tilde{T}_{\infty} \) and corresponds to the point \( \psi = \infty \). The \( t \) with large imaginary part in the fundamental region is mapped to some point with small imaginary part in some region by the action of \( A \). In short, large \( \Im t \) region is associated with small \( \Im t \) region by modular transformation. That is suggestive of some kind of duality.

Also one can take another function in order to study monodromy group \( SL(2, \mathbb{R}) \) by the standard method [20]. Firstly, let us introduce a function \( \gamma(\psi) \),

\[ \gamma(\psi) := \frac{i Z_1 - e^{\frac{3}{4} \pi i} Z_2}{Z_1 + e^{\frac{3}{4} \pi i} Z_2} , \]

\[ = \frac{i}{\pi} \left( \tan \frac{3\pi}{8} \right) \left\{ \log \psi^4 - \pi i \right. \]
\[ + \left[ \sum_{n=0}^{\infty} \frac{\Gamma \left( n + \frac{3}{8} \right) \Gamma \left( n + \frac{5}{8} \right)}{(n!)^2 \psi^{4n}} \left( 2\Psi(n+1) - \Psi \left( n + \frac{3}{8} \right) - \Psi \left( n + \frac{5}{8} \right) \right) \right] \]
\[ \times \left[ \sum_{n=0}^{\infty} \frac{\Gamma \left( n + \frac{3}{8} \right) \Gamma \left( n + \frac{5}{8} \right)}{(n!)^2 \psi^{4n}} \right]^{-1} \right\} , \]

\[ \left( 49 \right) \]
\[ Z_1^\gamma := \left[ \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{7}{4})} \right]^2 \cdot _2F_1\left( \frac{3}{8}, \frac{3}{8}, \frac{3}{4}; \psi^4 \right) , \]
\[ Z_2^\gamma := \left[ \frac{\Gamma(\frac{5}{8})}{\Gamma(\frac{5}{4})} \right]^2 \cdot _2F_1\left( \frac{5}{8}, \frac{5}{8}, \frac{5}{4}; \psi^4 \right) . \]

The \( \gamma \) is a map of the \( \psi^4 \)-plane to a pair of triangles in the upper half \( \gamma \)-plane, which together constitute a fundamental region of this transformation in Fig.2. On this \( \gamma \), the transformations \( A, T, \hat{T}_\infty \) are represented respectively,

\[ A^\gamma = \left( \begin{array}{cc} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{array} \right) , \quad T^\gamma = \left( \begin{array}{cc} -\frac{1}{\sqrt{2}} & 2 + \frac{3}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 2 + \frac{3}{\sqrt{2}} \end{array} \right) , \quad (51) \]
\[ \hat{T}_\infty^\gamma = \left( \begin{array}{cc} 1 \quad 2(\sqrt{2} + 1) \\ 0 \quad 1 \end{array} \right) . \quad (52) \]

Fixed points of these transformations are \( \gamma(0) = i \), \( \gamma(1) = \sqrt{2} + 1 \), and \( \gamma(\infty) = i\infty \) respectively.

6 Torus

We take a Torus which is represented by the following polynomial in \( \mathbb{CP}^2 \);

\[ \mathcal{M}; \ p = X_1^3 + X_2^3 + X_3^3 - 3\psi X_1 X_2 X_3 = 0 . \]

The Hodge diamond of torus is known,

\[
\begin{array}{ccc}
1 \\
1 & 1 \\
1 & & \\
\end{array}
\]

To obtain the mirror manifold, we divide \( \mathcal{M} \) by a discrete symmetry \( G = Z_3 \);

\[ (1, 0, 2), \quad (53) \]
\[ (0, 1, 2) , \quad (54) \]

where the symbol \( (n_1, n_2, n_3) \) means the transformation,

\[ (X_1, X_2, X_3) \to (\alpha^{n_1} X_1, \alpha^{n_2} X_2, \alpha^{n_3} X_3) , \quad (55) \]
\[ \alpha := e^{2\pi i/3} . \quad (56) \]
Let $\mathcal{W}$ be the mirror manifold. Euler number of $\mathcal{W}$ is:

$$\chi(\mathcal{W}) = \frac{0 - 0}{3} + 0 = 0.$$ 

Again torus is self-dual. We can calculate the periods on torus exactly with no help of Picard-Fuchs equation. Let $\alpha, \beta$ be the duals of $A$-cycle and $B$-cycle respectively. Then a holomorphic $(1,0)$-form $\Omega$ on the torus is decomposed with the basis $\alpha, \beta$ into

$$\Omega = \omega_0 \alpha - \omega_1 \beta,$$

where

$$\int_{\mathcal{W}} \alpha \wedge \beta = 1 \quad (57)$$

$$\int_{\mathcal{W}} \alpha \wedge \alpha = \int_{\mathcal{W}} \beta \wedge \beta = 0 \quad (58)$$

$$\alpha, \beta \in H_1(\mathcal{W}), \quad \Omega \in H^{1,0}(\mathcal{W}), \quad (59)$$

$$\omega_0 = \int_A \Omega = -\frac{1}{3\psi}(2\pi i)^3 \Gamma\left(\frac{1}{3}, \frac{2}{3}; 1; \psi^{-3}\right), \quad (60)$$

$$\omega_1 = \int_B \Omega = -\frac{1}{3\psi}(2\pi i)^3 \left[2\Gamma\left(\frac{1}{3}, \frac{2}{3}; 1; \psi^{-3}\right) \log \psi^{-3} + 2\Gamma\left(\frac{1}{3}, \frac{2}{3}; 1; \psi^{-3}\right)\right], \quad (61)$$

Also mirror map is defined,

$$t = \frac{\omega_1}{2\pi i \omega_0}. \quad (61)$$

It is a standard coordinate of the complex moduli of the torus. Corresponding to generators of the modular transformations, $t$ transforms as follows;

$$\begin{align*}
&\left\{ \begin{array}{c}
A \rightarrow A + B \\
B \rightarrow B
\end{array} \right\} \quad \left\{ \begin{array}{c}
A \rightarrow A \\
B \rightarrow B + A
\end{array} \right\} \\
&\quad t \rightarrow \frac{t}{1+t} \quad t \rightarrow t + 1
\end{align*}$$

These generate $SL(2; \mathbb{Z})$, which is widely known. The interesting object is one-point coupling;

$$\kappa_t = \int_{\mathcal{W}} e = \frac{1}{\omega_0^2} \times \int_{\mathcal{W}} \Omega \wedge \frac{d\Omega}{dt} = 1. \quad (62)$$

The factor in the right-hand side at the first line of the above equations $1/\omega_0^2$ is choosed when we fix the gauge, and it is known as the primitive factor in Algebraic geometry. We conclude there is no instanton correction in accordance with Algebraic geometry. Similarly, prepotential and covariant derivative can be constructed.
7 Conclusion

In this paper, we dealt with the K3 surface mainly. The coupling of (co)homology elements is calculated under some gauge choice. In this gauge, the Kähler potential becomes simple form in the variables $t, \bar{t}$. The two point coupling becomes constant number and no quantum correction exists. In the mathematical language, this fact about the coupling means that there is no rational curve on the K3 surface in contrast with the quintic hypersurface. The monodromy group acts on the mirror map $t$ as the $SL(2, \mathbb{R})$ or on the periods as the $SO(2, 1)$. There is no modification in this monodromy group owing to the quantum corrections. This property seems to stem from the fact that the mirror map in this case can be identified with a solution of the schwarzian equation (7) by using a $PSL(2)$ symmetry (11). In the quintic case, the periods cannot be expressed in the similar simple form and the monodromy group acting on the mirror map is not the $SL(2, \mathbb{R})$ because of the quantum corrections.

Acknowledgement

The authors greatly thank Prof. T. Eguchi for helpful discussions and kindful encouragement. They also thank M. Jinzenji for useful discussions.
References

[1] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.

[2] A. Strominger and E. Witten, Commun. Math. Phys. 101 (1985) 341.

[3] P. Candelas and X. de la Ossa, Nucl. Phys. B355 (1991) 415.

[4] M. Dine, N. Seiberg, X. Wen and E. Witten, Nucl. Phys. B278 (1986) 769; Nucl. Phys. B289 (1987) 319.

[5] J. Distler and B. Greene, Nucl. Phys. B309 (1988) 295.

[6] P. Aspinwall, B. Greene and D. Morrison, ”The Monomial-Divisor Mirror Map”, preprint IASSNS-HEP-93/43.
V. Batyrev and D. van Straten, ”Generalized Hypergeometric Functions and Rational Curves on Calabi-Yau Complete Intersections in Toric Varieties”, preprint

[7] P. Griffiths and J. Harris, Principles of Algebraic Geometry, (Wiley -Interscience, New York, 1978).

[8] S. Seiberg, Nucl. Phys. B303 (1988) 286.

[9] S. Cecotti, Int. J. Mod. Phys. A6 (1991) 1749.

[10] P. Candelas, X. de la Ossa, P. Green and L. Parkes, Phys. Lett. B258 (1991) 118; Nucl. Phys. B359 (1991) 21.

[11] P. Griffiths, Ann. Math. 90 (1969) 460, 469.

[12] W. Lerche, D. Smit and N. Warner, Nucl. Phys. B372 (1992) 87.

[13] M. Atiyah, R. Bott and L. Gårding, Acta Mathematica 131 (1973) 145.

[14] P. Candelas, Nucl. Phys. B298 (1988) 458.

[15] D. Morrison; Picard-Fuchs Equations and Mirror Maps For Hypersurfaces, in Essays on Mirror Manifolds, eds. S.-T. Yau, (Int. Press, Hong Kong, 1992).

[16] A. Erdélyi, F. Oberhettinger, W. Magnus and F. Tricomi, Higher transcendental functions, (McGraw-Hill, New York, 1953).
[17] P. Aspinwall, D. Morrison, Commun. Math. Phys. 151 (1993) 245.

[18] E. Witten, Mirror Manifolds and Topological Field Theory, in Essay on Mirror Manifolds, ed. S.-T. Yau, (Int. Press, Hong Kong, 1992), pp.120-180.

[19] J. Lehner, A short course in automorphic functions, (Holt, Rinehart and Winston, 1966).

[20] F. Klein and R. Fricke, Vorlesungen über die Theorie der elliptischen Modulfunktionen, Vol. I, (Teubner, 1890).