Production of Triply Charmed $\Omega_{ccc}$ Baryons in $e^+e^-$ Annihilation

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Abstract

The total and differential cross sections for the production of triply charmed $\Omega_{ccc}$ baryons in $e^+e^-$ annihilation are calculated at the Z-boson pole.

1. Introduction

Investigation into the properties of baryons containing two or three heavy $c$ and $b$ quarks, the features of their production at operating accelerators and those under construction, and their lifetimes and decay modes is topical in particle physics, but these issues have not yet received adequate study. All that is currently known in these realms from experiments amounts to the claim [1] that a doubly charmed baryon $\Xi_{cc}^+$ was observed in experiments with a beam of charged hyperons at FERMILAB. Theoretical investigations of baryons containing two heavy quarks are reviewed, for example, in [2]. Calculations available in the literature that deal with the cross sections for the production of baryons containing two heavy quarks treat primarily processes described in the fourth order of standard perturbation theory—that is, processes leading to the production of respective diquarks [3]. Only in [4] were sixth-order calculations performed, where the process $e^+e^- \rightarrow s\bar{s}c\bar{c}b\bar{b}$ was associated with the production of an $\Omega_{scb}$ baryon in $e^+e^-$ collisions. The production of baryons involving three heavy quarks has not yet been considered.

The present article reports on a continuation of the investigation begun in [4], providing a description of some features of the process involving the production of triply charmed baryons $\Omega_{ccc}$ in $e^+e^-$ annihilation. This case has nothing to do with the production $cc$

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diquarks, because they can transform, with a probability close to unity, only into $\Xi^{++}_{ccu}$ or $\Xi^{+}_{ccd}$ baryons, a negligible fraction of these diquarks going over to $\Omega_{ccc}$ baryons. As a matter of fact, calculations in the sixth order of perturbation theory for the elementary process $e^+e^- \rightarrow ccccc\bar{c}$ are the only possibility of theoretically studying triply charmed baryons. The main contribution to the amplitude of this process comes from 504 Feynman diagrams. In relation to the production of $\Omega_{scb}$ baryons, which was considered previously and where all components have different flavors, the calculations for $\Omega_{ccc}$ baryons are complicated by the need for taking into account the interference between identical particles.

In studying the production of $\Omega_{ccc}$ baryons in proton-proton collisions, it would be necessary to consider an order of magnitude greater number of Feynman diagrams corresponding to the subprocesses $q\bar{q} \rightarrow ccccc\bar{c}$ and $gg \rightarrow ccccc\bar{c}$. Moreover, the description of baryon production in hadron-hadron collisions would require a much greater effort in calculating the contributions to the amplitude of the production process from various parton color states than in the case of $e^+e^-$ annihilation.

In present study, the unification of three charmed quarks into an $\Omega_{ccc}$ baryon is described within the well-known nonrelativistic approximation [5]. Upon obtaining numerical results for the cross sections describing $\Omega_{ccc}$ baryon production, we analyze the possibility of constructing their approximate analytic description in terms of one known fragmentation function or another.

2. Amplitude of $\Omega_{ccc}$ production in $e^+e^-$ annihilation

We assume that the amplitude of the production of triply charmed baryons $\Omega_{ccc}$ in $e^+e^-$ annihilation corresponds to the elementary process

$$e^+(k_1) + e^-(k_2) \rightarrow c(p_1, \xi_1) + c(p_2, \xi_2) + c(p_3, \xi_3) + \bar{c}(p_4, \chi_1) + \bar{c}(p_5, \chi_2) + \bar{c}(p_6, \chi_3), \quad (1)$$

where $k_1$ and $k_2$ are the 4-momenta of colliding particles; $p_1, ..., p_6$ are the 4-momenta of product partons; and $\xi_i$ and $\chi_j \ (i, j = 1, 2, 3)$ are the color indices of quarks and antiquarks, respectively. As usual, we disregard the contribution of the electroweak interaction of quarks to the amplitude of process (1), since it is an order of magnitude less than the corresponding contribution of QCD interaction. Thereupon, all Feynman diagrams to be taken into account for process (1) reduce to the nine basic diagrams in Fig.1, which correspond to different positions of the quark-gluon vertices. Thirty-six nonequivalent dispositions of quark and antiquark lines characterized by specific 4-momenta, polarizations, and color indices are possible for each of the basic diagrams 1-7, and 18 nonequivalent dispositions of such lines
are possible for the basic diagrams 8 and 9. Since a collision between an electron and a positron leads to annihilation into either a photon or a $Z$ boson, the total number of relevant Feynman diagrams is 576.

First, we consider the color structure of the amplitude of $\Omega_{ccc}$ baryon production. Since an electron, a positron, and any baryon are singlets with respect to the $SU(3)_c$ color group, three product antiquarks $c$ must also form an $SU(3)_c$-singlet state. Therefore, the final state of process (1) must be fully antisymmetric in the color indices of the three charmed quarks bound into an $\Omega_{ccc}$ baryon and in the color indices of the three product charmed antiquarks. This requirement, together with the requirement of an appropriate normalization, is satisfied by introducing, in the amplitude of process (1), the product $(\varepsilon_{\xi_1\xi_2\xi_3}/\sqrt{6})(\varepsilon_{\chi_1\chi_2\chi_3}/\sqrt{6})$ of antisymmetric tensors and performing summation over the color indices of $\xi_i$ and $\chi_j$ ($i, j = 1, 2, 3$).

We set $T^a = \lambda^a/2$, where $\lambda^a$ ($a = 1, ..., 8$) are the Gell-Mann matrices, and denote by $N$ the total number of such permutations of different pairs of color indices of quarks and antiquarks that transform the sets $(\xi_1, \xi_2, \xi_3)$ and $(\chi_1, \chi_2, \chi_3)$ into the sets $(\xi_{i_1}, \xi_{i_2}, \xi_{i_3})$ and $(\chi_{i_1}, \chi_{i_2}, \chi_{i_3})$, respectively. The color factors associated with diagrams of the types 1-7 can then be found by means of direct analytic calculations. The result is

$$\sum_{a,b,\xi} \sum_{\xi_1,\xi_2,\xi_3} \sum_{\chi_1,\chi_2,\chi_3} 1/6 \varepsilon_{\xi_1\xi_2\xi_3} \varepsilon_{\chi_1\chi_2\chi_3} T^a_{\xi_1,\chi_1} T^b_{\xi_2,\chi_2} T^c_{\xi_3,\chi_3} = (-1)^N 4/9. \quad (2)$$

In the sum in expression (2), the index $\xi_1$, appears twice (as it must)–directly in the tensor $\varepsilon_{\xi_1\xi_2\xi_3}$ and indirectly as the substitute of one of the indices $\xi_{i_1}, \xi_{i_2}$ and $\xi_{i_3}$. The same is true for the other Greek indices in the above sum, with the exception of $\zeta$, and for the zeroth color factors corresponding to diagrams of types 8 and 9, for which we have

$$\sum_{a,b,c} \sum_{\xi_1,\xi_2,\xi_3} \sum_{\chi_1,\chi_2,\chi_3} \varepsilon_{\xi_1\xi_2\xi_3} \varepsilon_{\chi_1\chi_2\chi_3} f^{abc} T^a_{\xi_1,\chi_1} T^b_{\xi_2,\chi_2} T^c_{\xi_3,\chi_3} = 0, \quad (3)$$

where $f^{abc}$ are the structure constants of the $SU(3)$ group. The proof of the equality in (3) is given in [4]. This equality means that the total contribution to the amplitude of the process in (1) from the diagrams involving three-gluon vertices vanishes. Thus, the number of contributing diagrams reduces to 504.

Since the contribution to the amplitude of process (1) from the diagram that differs from a specific diagram by a permutation of $N$ fermion pairs involves the Feynman factor $(-1)^N$, it can be concluded, with allowance for (2), that all terms appearing in the amplitude of $\Omega_{ccc}$ baryon production have the same sign.

It should be noted that, in our calculations, we used an additional simplifying approximation, setting the $c$-quark mass to zero in all expressions entering into the numerators of
fermion propagators and in all traces. At the same time, we set $m_c = 1.5$ GeV and $p_i^2 = 2.25$ GeV$^2$ ($i = 1, \ldots, 6$) in all of the denominators of the propagators of virtual particles and in the expression for the final-state phase space (of course, the amplitude containing zero $c$-quark mass in the denominator would diverge). But if we used a nonzero $c$-quark mass everywhere in the amplitude and in the square of the relevant matrix element, the volume of information to be saved in the computer memory and the time required for the compilation of codes and for numerical calculations of the cross sections would grow enormously, which would render the problem in question unsolvable with our means.

In order to estimate the effect of the above approximation on the accuracy of the numerical results, we repeated the calculation of the cross section for $\Omega_{scb}$ baryon production in a similar approximation and compared the results obtained in this way with the results of the full calculation performed in [4]. It turned out that the cross sections obtained for $\Omega_{scb}$ baryon production within the “massive” and “massless” (for all quarks simultaneously) approximations differ only by 8%. It seems reasonable to expect an inaccuracy on the same order of magnitude for $\Omega_{exc}$ baryon production as well. Anyway, this inaccuracy does not exceed other theoretical uncertainties associated, for example, with the choice of the renormalization scale in the strong-interaction coupling constant or with the wave function for the triply charmed heavy baryon. Thus, this approximation appears to be numerically justified.

Taking into account the aforesaid, we can represent the matrix element for process (1) in the form

$$\mathcal{M} = \frac{g_s^4g^2}{9\cos^2\theta_W(s - M_Z^2 + iM_Z\Gamma_Z)}D^Z - \frac{4g_s^4e^2}{9s}D^\gamma,$$

where

$$D^Z = \sum_{i,j,k \in \{1,2,3\}} \sum_{i',j',k' \in \{1,2,3\}} \left\{[(p_j + p_i + p_{c'})^2 - m_c^2]^{-1}[(k_1 + k_2 - p_{k'})^2 - m_c^2]^{-1} \times \right.$$\n
$$\times (p_i + p_c)^{-2} (p_i + p_j + p_{c'})^{-2} \bar{u}(p_j)\gamma^\nu(\not{p_j} + \not{p_i} + \not{p_{c'}})\gamma_5v(-\not{p_{c'}}) \times$$ \n
$$\times \bar{u}(p_k)\gamma^\delta(\not{k_1} + \not{k_2} - \not{p_{k'}})\gamma_5(g_{V^c} - g_{A^c}\gamma_5)v(-\not{p_{k'}}) +$$ \n
$$+ [(p_{j'} + p_i + p_{c'})^2 - m_c^2]^{-1}[(k_1 + k_2 - p_{k'})^2 - m_c^2]^{-1} (p_i + p_{c'})^{-2} \times$$ \n
$$\times (p_i + p_j + p_{c'})^{-2} \bar{u}(p_j)\gamma_\beta(-\not{p_{j'}} - \not{p_i} + \not{p_{c'}})\gamma^\nu v(-\not{p_{j'}}) \times$$ \n
$$\times \bar{u}(p_k)\gamma_\epsilon(-\not{k_1} - \not{k_2} + \not{p_k})\gamma^\delta(g_{V}^c - g_{A^c}\gamma_5)v(-\not{p_{k'}}) +$$ \n
$$+ [(p_{j'} + p_i + p_{c'})^2 - m_c^2]^{-1}[(k_1 + k_2 - p_{k'})^2 - m_c^2]^{-1} (p_i + p_{c'})^{-2} \times$$ \n
$$\times (p_i + p_j + p_{c'})^{-2} \bar{u}(p_j)\gamma_\beta(-\not{p_{j'}} - \not{p_i} + \not{p_{c'}})\gamma^\nu v(-\not{p_{j'}}) \times$$ \n
$$\times \bar{u}(p_k)\gamma^\delta(\not{k_1} + \not{k_2} - \not{p_{k'}})\gamma_\epsilon(g_{V}^c - g_{A^c}\gamma_5)v(-\not{p_{k'}}) +$$ \n
$$+ [(p_{j'} + p_i + p_{c'})^2 - m_c^2]^{-1} [(k_1 + k_2 - p_{k'})^2 - m_c^2]^{-1} (p_i + p_{c'})^{-2} \times$$
\[(p_i + p_{i'})^{-2}\hat{u}(p_i)\gamma^\nu(\hat{p}_i + \hat{p}_{i'})\gamma^\delta v(-p_{i'}) \times \]
\[\times (p_i + p_{i'} + p_{i''} + p_{i'''})^{-2}\hat{u}(p_{i'})\gamma^\nu(\hat{p}_{i'} + \hat{p}_{i''} + \hat{p}_{i'''})\gamma^\delta v(-p_{i''}) \times \]
\[\times \hat{u}(p_{i''})\gamma^\nu(\hat{p}_{i''} + \hat{p}_{i'''} + \hat{p}_{i''''})\gamma^\delta v(-p_{i'''} \times \]
\[\times (p_{i''} + p_{i'''} + p_{i''''} + p_{i'''''})^{-2}\hat{u}(p_{i'''})\gamma^\nu(\hat{p}_{i'''} + \hat{p}_{i''''} + \hat{p}_{i'''''})\gamma^\delta v(-p_{i'''''}) \times \]
\[\times \gamma^\nu(\hat{k}_1 - \hat{k}_2 + \hat{p}_k)\gamma^\delta v(-p_k) + \]
\[\times [((p_{i''} + p_{i'''} + p_{i''''} + p_{i'''''})^2 - m^2_c)^{-1}\gamma^\nu(\hat{p}_{i'''} + \hat{p}_{i''''} + \hat{p}_{i'''''} + \hat{p}_{i''''''})\gamma^\delta v(-p_{i'''''}) \times \]
\[\times (p_{i'''} + p_{i''''} + p_{i'''''})^{-2}\hat{u}(p_{i''''})\gamma^\nu(\hat{p}_{i''''} + \hat{p}_{i'''''})\gamma^\delta v(-p_{i''''}) \times \]
\[\times \gamma^\nu(\hat{k}_1 - \hat{k}_2 + \hat{p}_k)\gamma^\delta v(-p_k) + \]
\[\times [((p_{i'''} + p_{i''''} + p_{i'''''})^2 - m^2_c)^{-1}\gamma^\nu(\hat{p}_{i''''} + \hat{p}_{i'''''})\gamma^\delta v(-p_{i''''}) \times \]
\[\times (p_{i''''} + p_{i'''''})^{-2}\hat{u}(p_{i'''''})\gamma^\nu(\hat{p}_{i'''''})\gamma^\delta v(-p_{i'''''}) \times \]
\[\times \gamma^\nu(\hat{k}_1 - \hat{k}_2 + \hat{p}_k)\gamma^\delta v(-p_k) \times \]
\[\times \gamma^\nu(\hat{k}_1 - \hat{k}_2 + \hat{p}_k)\] while the expression for \(D^\nu\) can be derived from \(D^Z\) by means of the substitutions \(g_V^c \to 1\), \(g_V^p \to 0\), \(g_V^c \to Q_c = 2/3\) and \(g_A^c \to 0\). Summation in (5) corresponds to 36 permutations of quark and antiquark lines in diagrams of types 1-7.

3. Method of orthogonal amplitudes

In order to derive the expression that is obtained for the square of the matrix element \(\langle M |^2 \rangle\) upon summation over the final-fermion polarizations and averaging over the polarizations of colliding particles, we use the method of orthogonal amplitudes and the REDUCE computer system for analytic calculations. The method of orthogonal amplitudes was proposed in [6] and was employed in calculations referring to \(\Omega_{c\bar{c}}\) baryon production in \(e^+e^-\) collisions [4].

A simple and mathematically rigorous validation of the method of orthogonal amplitudes is the following (to the best of our knowledge, it has not yet been given anywhere). Suppose that four-component spinors \(u(p)\) and \(u(p')\) describing particles of mass \(m\) and \(m'\), respectively, their 4-momenta being \(p\) and \(p'\) \((p^2 = m^2, p'^2 = m'^2)\), obey the Dirac equation. Of the four linear homogeneous equations for the components of the spinor \(u(p)\) \([u(p')]\), only two are independent; therefore, each of the four components under consideration can be represented as a linear combination of two arbitrary independent constants, denoted here
by \( X \) and \( Y \) (\( X' \) and \( Y' \)). Any quantity of the form \( \bar{u}(p')Ru(p) \),
where \( R \) is an operator specified in terms of the \( \gamma \)-matrices
and their contractions with some 4-vectors, can be represented as a linear combination
of four independent elements \( XX'^rs, YY'^rs, YX'^rs \), and \( YY'^rs \).
Therefore, quantities of the form \( \bar{u}(p')Ru(p) \) can be treated as vectors
of a linear four-dimensional space \( L \) spanned by the above elements.
Any four linearly independent quantities of the form \( w_n \equiv \bar{u}(p')O_nu(p) \),
where the operator \( O_n \) is either unity, \( \gamma_5, \hat{V}, \hat{V}'\gamma_5 \),
or \( (\hat{V}^n\hat{V}'^m - \hat{V}'^m\hat{V}^n)/2 \) (with \( V, V', V'' \), and \( V''' \) being arbitrary 4-vectors),
can be taken for basis vectors of the space \( L \). The scalar product \( (w_n, w_{n'}) \) of vectors \( w_n \) and \( w_{n'} \) belonging to
the linear space \( L \) is defined as the product \( \bar{w}_n w_{n'}^{*} \) summed
over the polarizations of fermions that are described by the spinors \( u(p) \) and \( u(p') \).

Here, we take, for basis vectors of the space \( L \), four quantities \( w_n \) specified by the
operators \( O_1 = 1, O_2 = \hat{K}, O_3 = \hat{Q}, \) and \( O_4 = \hat{K}\hat{Q} \) with the 4-vectors \( K \) and \( Q \) here being orthogonal
to the 4-momenta \( p \) and \( p' \) and to each other - that is, \( K_\mu p^\mu = 0, K_\nu p'^\nu = 0, Q_\mu p^\mu = 0, Q_\nu p'^\nu = 0, \) and \( K_\mu Q^\mu = 0 \). Otherwise, the 4-vectors \( K \) and \( Q \) are arbitrary. They
can be specified, for example, by the relations \( K^\mu = \varepsilon^{\mu\nu\rho\sigma} p_\nu p'_\rho a_\sigma \) and \( Q^\mu = \varepsilon^{\mu\nu\rho\sigma} p_\nu p'_\rho K_\sigma \),
where the 4-vector \( a_\sigma \) is entirely arbitrary. From the orthogonality of the 4-vectors \( \hat{K} \) and \( \hat{Q} \), it follows that \( \hat{K}\hat{Q} = (\hat{K}\hat{Q} - \hat{Q}\hat{K})/2 \). The four quantities \( w_n \) used are orthogonal to one another, \( (w_n, w_{n'}) = C_n\delta_{nn'} \), \( C_n \neq 0 \),
this proving their linear independence and justifying the name "orthogonal amplitudes." Thus, it was shown that any quantity of the form \( \bar{u}(p')Ru(p) \) can be represented as a linear combination of orthogonal amplitudes.

In order to solve the problem of calculating the square of the matrix element, we first introduce basic orthogonal amplitudes as

\[
\begin{align*}
w_{i1} &= \bar{u}(p_i)v(-p_{3+i}), & w_{i2} &= \bar{u}(p_i)\hat{K}_iv(-p_{3+i}), \\
w_{i3} &= \bar{u}(p_i)\hat{Q}_iv(-p_{3+i}), & w_{i4} &= \bar{u}(p_i)\hat{Q}_i\hat{K}_iv(-p_{3+i}), \\
w_{e1} &= \bar{v}(-k_1)u(k_2), & w_{e2} &= \bar{v}(-k_1)\hat{K}_eu(k_2), \\
w_{e3} &= \bar{v}(-k_1)\hat{Q}_eu(k_2), & w_{e4} &= \bar{v}(-k_1)\hat{Q}_i\hat{K}_eu(k_2),
\end{align*}
\]

where \( i = 1, 2, 3 \). We would like to note that the pair combinations of the spinors \( \bar{u}(p_i) \) and \( v(-p_j) \) can be chosen in six equivalent ways.

On the basis of the quantities in (6), we construct 256 orthogonal amplitudes as

\[
w_{nrst} = w_{1n}w_{2r}w_{3s}w_{4t},
\]

where \( n, r, s, t = 1, 2, 3, 4 \).
The expansion of the matrix element (4) in the amplitudes given by (7) has the form

\[ M = \sum_{n,r,s,t=1}^{4} c_{nrst} w_{nrst}. \]  

(8)

In order to derive the coefficients \( c_{nrst} \) in this expansion, we multiply both sides of (8) by the factor \( w_{nt'rs't'} \), take the sum of the result over the polarizations of all of the fermions, and make use of the orthogonality of different amplitudes. As a result, we arrive at

\[ c_{nrst} = \left\{ \sum_{\text{polar.}} M w_{nrst}^* \right\} / (w_{nrst}, w_{nrst}), \]  

(9)

where \( (w_{nrst}, w_{nrst}) \) is an analog of the scalar product defined above in the linear space \( L \) - that is, the sum of the squared modulus of the amplitude \( w_{nrst} \) over the polarization of all fermions. Since \( w_{nrst} \) involves arbitrariness associated with the choice of the 4-vectors \( K \) and \( Q \) in (6), there is also arbitrariness in the coefficients \( c_{nrst} \) in (9). The substitution of these coefficients into (8) leads to an identity whose left-hand side is determined unambiguously. Thus, summation on the right-hand side of (8) removes the above ambiguity.

Since electrons and positrons are treated as massless particles and since the charmed-quark mass is set to zero in the numerators of each term of the amplitude for process (1) and in respective traces, it is clear that 192 of the 256 coefficients in expansion (8) vanish, because they are linear combinations of the traces of an odd number of the Dirac \( \gamma \) matrices. We further list 64 orthogonal amplitudes in formula (7) that generate nonzero expansion coefficients: \( t = 2, 3 \) with either \( n, r, s = 2, 3 \), or one of the indices \( n, r, s \) is equal to 2 or 3, while the other two belong to the set \( \{1, 4\} \).

It can clearly be seen that the expression obtained for the square of the matrix element upon summation over the polarizations of final fermions and averaging over the polarizations of initial particles takes the form

\[ \overline{|M|^2} = \frac{1}{4} \sum_{n,r,s,t} |c_{nrst}|^2 \cdot (w_{nrst}, w_{nrst}). \]  

(10)

In actual calculations by the method of orthogonal amplitudes, we compose one REDUCE code for traces and tensor contractions that corresponds to 504 terms in any quantity \( M w_{nrst}^* \) and then, by means of any text editor (for example, ”joe”), perform obvious changes necessary for obtaining the REDUCE code for calculating all 64 nonzero quantities \( M w_{nrst}^* \).

4. Cross sections for \( \Omega_{ccc} \) baryon production at the \( Z \) pole in \( e^+e^- \) annihilation
In describing the $\Omega_{ccc}$ baryon as a bound state of three charmed quarks, we use the nonrelativistic approximation [5]. This means that we disregard the relative velocities of the $c$ quarks confined within the baryon - that is, in the laboratory frame, the velocities and momenta of all three $c$ quarks produced in process (1) are taken to be identical and equal to one-third of the momentum $p$ of the $\Omega_{ccc}$ baryon having the mass $M=3m_c$. With allowance for the unification of three charmed quarks into the baryon, the phase space of process (1) effectively becomes the 4-particle phase space of the process

$$e^+(k_1) + e^-(k_2) \rightarrow \Omega_{ccc}(p) + \bar{c}(p_4) + \bar{c}(p_5) + \bar{c}(p_6).$$

The differential cross section for process (11) takes the form

$$d\sigma = \frac{(2\pi)^4|\mathcal{M}|^2}{2s} \cdot \frac{|\psi(0)|^2}{M^2} \cdot \delta^4(k_1 + k_2 - p - p_4 - p_5 - p_6) \times$$

$$\times \frac{d^3p}{(2\pi)^32E} \cdot \frac{d^3p_4}{(2\pi)^32E_4} \cdot \frac{d^3p_5}{(2\pi)^32E_5} \cdot \frac{d^3p_6}{(2\pi)^32E_6},$$

where $\psi(0)$ is the value that the respective wave function takes in the case where all three $c$ quarks forming the $\Omega_{ccc}$ baryon are located at the same point, so that their relative coordinates are zero. The numerical value of $|\psi(0)|^2$ is taken to be identical to that in [7], where it was

$$|\psi(0)|^2 = 0.36 \cdot 10^{-3} \text{ GeV}^6.$$ (13)

In calculating the total and differential cross sections, we employed codes for numerical integration that are based on the Monte Carlo method and which are contained in the CompHEP package [8], which is broader. It appeared that the maximum computational errors in the differential cross sections came from the first iteration. Therefore, only the total cross section was calculated in the first iteration, while both the total cross section and the differential cross sections were determined in the next five iterations. Each iteration involved 200 000 Monte Carlo calls on the integrand. The errors in calculating the total cross section amounted to 1.0%, while the errors in calculating the differential cross sections were predominantly 2 to 3% (this is reflected below in the text and in the figures). As was indicated above, the error associated with the disregard of the charmed-quark mass in the numerators of the amplitude for process (1) and in the traces is a few percent. Moreover, we additionally tested the consistency of the cross-section values for two different admissible choices of the 4-vectors $K_\mu^i, Q_\mu^i, K^\mu_i,$ and $Q^\mu_i (i = 1, 2, 3)$ used to construct the quantities in (6), which specify the orthogonal amplitudes (7).
In addition to statistical errors, the calculations contain unavoidable theoretical uncertainties. First, there is the uncertainty associated with the running strong-interaction coupling constant as a function of the renormalization scale. Since all of the calculations were performed at an energy value that corresponds to the Z-boson pole ($\sqrt{s} = 91.2$ GeV), it is reasonable to specify the coupling-constant values as follows: $\alpha_s = \alpha_s(M_Z/2) = 0.134$ and $\alpha = \alpha(M_Z) = 1/128.0$; accordingly, $\sin^2 \theta_W = \sin^2 \theta_W(M_Z) = 0.2240$. However, it is not evident why it is $M_Z$, and not, for example, the invariant mass of some product quark pair or even the $\Omega_{ccc}$ baryon mass, that should be chosen for the characteristic scale of strong interaction. Since the cross section for process (11) is proportional to the fourth power of the strong-interaction coupling constant, this source of errors is the most important. Second, the accuracy of the potential model employed as a basis for calculating the baryon-wave-function value $\psi(0)$ is uncertain.

For the chosen set of model parameters, the total cross section for the process $\sigma_{\text{tot}}$ and the forward (backward) production asymmetry at the Z-boson pole are

$$\sigma_{\text{tot}} = (0.0404 \pm 0.0004) \text{ fb},$$

$$A_{FB} = (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B) = 0.101 \pm 0.005,$$

where $\sigma_F(\sigma_B)$ is the cross section for the production of an $\Omega_{ccc}$ baryon moving in the forward (backward) direction with respect to the electron-momentum direction. The cross-section value in (14) is close to that of the total cross section for $\Omega_{scb}$ baryon production in $e^+e^-$ collisions ($0.0534 \pm 0.0014$ fb) if the strange-quark mass is set to 300 MeV [4].

The differential cross sections with respect to the transverse momentum $p_T$ and the rapidity $Y$ of $\Omega_{ccc}$ baryons are presented in Fig. 2. The distribution $d\sigma/dY$ peaks at a small positive value of $Y$, while $d\sigma/dp_T$ has a maximum at $p_T \approx 12$ GeV. Note that the maximum of the differential cross section with respect to the transverse momentum of $\Omega_{ccc}$ baryons occurs at a $p_T$ value much lower than that for $\Omega_{scb}$ baryons produced under the same conditions, in which case $d\sigma/dp_T$ peaks within the $p_T$ interval 23-26 GeV.

It is desirable to associate our numerical results with some simple analytic form, which we will seek among well-known fragmentation functions [9-12]. It is clear that the production of a triply charmed baryon can hardly be interpreted as a fragmentation process, since each of the three $c$ quarks can be treated, on equal footing, as a fragmenting quark produced at the $\gamma/Z$ vertex and since the interference between identical quarks is likely to be significant in the process being considered. However, we accept not the physical concept of the fragmentation model but its mathematical form used in processing experimental data on $e^+e^-$ annihilation.
(see, for example, [13]); namely, we set
\[
\frac{d\sigma}{dz} = \sigma_{c\bar{c}} \cdot D_{c\rightarrow\Omega_{ccc}}(z),
\]
where \(\sigma_{cc}\) is the total cross section for the process \(e^+e^- \rightarrow c\bar{c}\) while \(D_{c\rightarrow\Omega_{ccc}}(z)\) is the respective fragmentation function. Instead of the variable \(z\), its approximate value \(x_p = p/p_{\text{max}}\) is used below.

Neglecting a small asymmetry in the angular distribution of \(\Omega_{ccc}\) baryons, we arrive at the following relation between the differential cross section with respect to the transverse momentum and the fragmentation function:
\[
\frac{d\sigma}{dp_T} = \frac{4\sigma_{c\bar{c}p_T}}{s} \int_{2p_T/\sqrt{s}}^1 \frac{D_{c\rightarrow\Omega_{ccc}}(z)dz}{z\sqrt{z^2 - 4p_T^2/s}}.
\]

We now compare our numerical results with those obtained according to expression (17) with various fragmentation functions. First, we consider the Peterson function [9]
\[
D(z) \sim \frac{1}{z} \left(1 - \frac{1}{z} - \frac{\varepsilon}{1 - z}\right)^{-2},
\]
which is often used in processing experimental data on charmed-hadron production in \(e^+e^-\) annihilation [14]. Also, this function provides a good approximation to numerical results on \(\Omega_{scb}\) baryon production in \(e^+e^-\) annihilation [4]. The best fit to our calculations (dash-dotted curve in Fig. 2) corresponds to \(\varepsilon = 0.92\). The agreement is clearly poor. The best fits with the Collins-Spiller fragmentation function [10]
\[
D(z) \sim \left(\frac{1 - z}{z} + \varepsilon \frac{2 - z}{z}\right) (1 + z^2) \left(1 - \frac{1}{z} - \frac{\varepsilon}{1 - z}\right)^{-2}
\]
at \(\varepsilon = 3.0\) and with the fragmentation function [11]
\[
D(z) \sim z^\alpha(1 - z),
\]
at \(\alpha = 0.8\) are also unsatisfactory. The results of the calculations according to (17) with the functions in (19) and (20) are displayed in Fig. 2 (dashed and dotted curves, respectively).

An acceptable analytic form for our numerical results is provided by the LUND fragmentation function [12]
\[
D(z) \sim \frac{1}{z}(1 - z)^a \exp \left(-\frac{c}{z}\right),
\]
at the parameters \(a = 2.4 \pm 0.2\) and \(c = 0.70 \pm 0.03\). The corresponding results calculated according to (17) are represented by the solid curve in Fig. 2.

It should be noted that the fragmentation functions (19)-(21), along with the functions in (18), were employed by the OPAL Collaboration [15] in processing experimental data on \(B\)-meson production.
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Fig. 1. Basic Feynman diagrams for the process $e^+ + e^- \rightarrow c + c + \bar{c} + \bar{c} + \bar{c}$. 
Fig. 2. Differential cross sections for $\Omega_{ccc}$ baryon production in $e^+e^-$ annihilation at the $Z$ pole with respect to the transverse momentum $p_T$ (left) and rapidity $Y$ (right). The results of Monte Carlo calculations and the errors in them are represented by crosses. The curves in Fig. 2 (left) correspond to expression (17) calculated with the fragmentation functions in the form (solid curve) (21) at $a = 2.4$ and $c = 0.70$, (dash-dotted curve) (18) at $\varepsilon = 0.92$, (dashed curve) (19) at $\varepsilon = 3.0$, and (dotted curve) (20) at $\varepsilon = 0.8$. 