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An extended picture fuzzy multicriteria group decision analysis with different weights: A case study of COVID-19 vaccine allocation

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ABSTRACT

The high contagion rates of COVID-19 and the limited amounts of vaccines forced public health authorities to develop vaccinations strategies for minimizing mortality, avoiding the collapse of health care infrastructure, and reducing their negative impacts to societies and economies. We propose a Multi Criteria Group Decision Making for prioritizing a set of COVID-19 vaccination alternatives, under a picture fuzzy environment, where the weights for Decisions Experts (DE) and criteria are unknown. A panel of six DEs assess six criteria for prioritizing four groups for vaccination. The weights for DE and criteria are handled in the form of fuzzy sets. Three types of weights are calculated: subjective, objective, and mixture weights.

According to our results, three out of the six criteria hold 60% of the strategic importance: 1) allocation and distribution, 2) COVID-19 strains and 3) capabilities and infrastructures. However, persons with comorbidities became the group with the highest priority, followed by essential workers, women, and adults older than 40 years. Governments, decision makers, and policy makers can find rigorous scientific evidence for articulating effective vaccinations campaigns from this work, and contribute to minimize undesired outputs, such as high mortality rates or collapse of hospitals.

1. Introduction

According to the Coronavirus Resource Centre at Johns Hopkins University, by the first half of February 2022, the SARS-CoV-2 virus (responsible of COVID-19 disease) caused approximately 5.7 million deaths worldwide [1]. It also yielded one of the most dramatic economic recessions of the last century, as the International Monetary Fund estimated a negative economic growth worldwide, equal to approximately −4.9 for 2020 [2]. While mitigation strategies such as social distancing, closures of non-essential activities, washing hands frequently, etc, have been effective for reducing the number of infected people, and therefore to avoid the collapse of healthcare systems, it is evident that massive vaccination campaigns are considered the most effective strategy to overcome this pandemic. By the end of 2021, there were around 300 vaccines under development across 80 countries, and six of these had received the Emergency Use Listing (EUL) certification [3]. Countries such as United States, Russia, China, and United Kingdom lead the race on designing and developing safe and effective vaccines [4].

Now that several vaccines have received the EUL certification and others are waiting, issues such as the limited availability of doses, how best to prioritize vaccines among different groups, and designing logistic plans for effective distribution are relevant topics. There are records of existing vaccination prioritization initiatives from previous pandemics as influenza HINI in 2009 [5], and Ebola in Africa in 2013 [6], which provides general guidelines for how groups should be prioritized. COVID-19 shows particularities that require closer attention. In contrast to other respiratory diseases, humans under 15 years of age have a lower susceptibility to infection, and lower odds of developing a severe status or die from infection. In most of the previous pandemics, the preferred strategy was to prioritize children (considering this group might be the most vulnerable) [7]. For the COVID-19 pandemic, this strategy would not be optimal considering that other groups like people older than 60 are at a higher risk. However, this last group also has lower mobility and social interaction than younger people do. Moreover, individuals working on activities considered essential for controlling the COVID-19 pandemic and keeping the society functioning are exposed to a higher risk, since they cannot fully implement social distance protocols. In this respect, there is active the debate whether essential workers should be prioritized over older adults or adults with comorbidities, given that the first group is at a higher risk of contagion. However, the latter groups are

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at a higher risk of developing severe conditions or death [8].

The main challenge for overcoming the COVID-19 pandemic consists of developing and allocating effective vaccines. To achieve this, new infrastructure such as manufacturing laboratories, specialized warehouses, transport equipment, and other related medical devices has to be deployed worldwide. An efficient and effective vaccination strategy should consider optimal values for the mentioned variables, which also are subject to restrictions such as vaccine price and convenience, mortality rate per country, distances to distributions centres, and virus strains. The World Health Organization (WHO) developed a framework, endorsed by the Strategic Advisory Group of Experts (SAGE), for equitably allocating COVID-19 vaccines around the world. The goal statement for this initiative is: "COVID-19 vaccines must be a global public good. The overarching goal is for COVID-19 vaccines to contribute significantly to the equitable protection and promotion of human well-being among all people of the world [9]."

The Centre for Disease Control and Prevention (CDC) is the main authority responsible for designing and deploying the strategy for optimal vaccine allocation in United States [10]. While this strategy was being designed, the points of view of several groups across the country, such as Johns Hopkins Coronavirus Resource Centre (JHCRC), National Association of Academies of Science (NAAS), The Association of Schools and Programs of Public Health (ASPPH), and Census Bureau, among others, were considered. Besides, mathematical models that predict how the pandemic is evolving under different scenarios (e.g., prioritizing essential workers over older adults and vice versa), are considered as supportive tools. On February 2021, the CDC published a vaccine allocation strategy that prioritized essential healthcare staff, followed by older adults and adults with comorbidities [11]. It must be noted that this national vaccination strategy serves as a general framework, and it is only a guideline for local governments. Each state or city is responsible for developing its own vaccination strategy, based on particularities. For instance, the Texas government stated that healthcare workers must be vaccinated first, followed by adults over 65, and adults with medical conditions [12]. The California government also prioritized healthcare workers, followed by Food-Agriculture and Education-childcare workers, and individuals at the highest risk for morbidity and mortality from COVID-19 were allocated the third place [13]. The Mexican government allocated healthcare workers as a priority group, followed by adults older than 60. Education workers were the third priority, followed by adults older than 50 [14]. The Saudi government provides free vaccination to citizens, residents, and individuals with medical conditions such as hypertension or hepatitis. For ensuring an ordered and secure vaccination process, three priority groups were formulated. The first comprises individuals at 65 years or older, workers performing essential activities, persons with obesity (BMI >40), immunosuppressed patients, and people with two or more chronic illness (e.g. hypertension or diabetes). The second priority group targets individuals over 50 years, healthcare workers, individual with a unique chronic illness, and persons with moderate obesity (BMI between 30 and 40). The last group comprises of all persons willing to get vaccinated, regardless of age, occupation, or any medical condition [15]. According to the above it is clear that different scenarios produced by geographical location, sociodemographic characteristics, or income level might lead to the prioritizing of different groups. Furthermore, COVID-19 vaccines should be supplied without delays or inventory shrinkages, assuring that everyone gets prompt access to them. Then governments and policy makers face an important decision-making problem under a sort of Multi-Criteria Decision-Making (MCDM) framework. In the context of vaccine allocation strategies, Decision Experts (DE) should decide which priority groups are set, while a cluster of criteria is already given. Along this process, DEs might deal with conflicting or missing information, and information might also be given qualitative and quantitative values, which creates additional complexity. Multi Criteria Group Decision Making (MCGDM) is a well-known framework that comprises of methods and techniques to assess, in a clear and transparent way, these multiple, conflicting, and unmeasurable criteria, which can be vague [16–19].

In this paper, a novel type of MCGDM is proposed to prioritize a set of COVID-19 vaccination alternatives, under a picture fuzzy environment, on which the weights for Decisions Experts (DE) and criteria are unknown. Under the Picture Fuzzy Environment (PFE), the Complex Proportional Assessment (COPRAS) method is introduced to investigate how vaccinations groups should be prioritized. The main objective for this paper is to prioritize a set of COVID-19 vaccination strategies, by proposing an extended picture fuzzy MCGDM algorithm based on the COPRAS method, on which weights for DEs and criteria are unknown. An original contribution of this work is the weighting method that is applied for assessing DEs and criteria. Thus, weights for groups of DEs and criteria are introduced under the picture fuzzy environment. Three types of weight are proposed: subjective, objective, and mixture weights. The form weights are obtained, in conjunction with practical guidelines from the policy maker’s perspective, are among the main contributions of this research. On [20] a decision support system for controlling and preventing COVID-19 contagious is proposed. Although the mathematical model is detailed explained by the authors, the work is not related with vaccination strategies. Singh & Haseeb [21] proposed a novel methodology based on correlations coefficients and picture fuzzy sets to select the most suitable mask for tackling COVID-19. Practical guidelines for policy makers are not provided by authors. On [22] the Inter-criteria Correlation and the Combined Compromise Solution are framed under a gray-based environment to allocate temporary COVID-19 hospitals. Authors provide managerial implications and practical guidelines which are complementary to this present work. Hezam et al. [23] proposed a robust methodology for allocating COVID vaccines, but it shows limitations from the managerial implications and practical perspectives. Due to space limitations, other works related to vaccination strategies and MCGDM fuzzy methods as [24–28] are not discussed. However from all literature available in the field, most of it showed the limitations that were previously mentioned. Either it is providing vast managerial implications or not suggesting practical guidelines from policy makers perspective.

The rest of the paper is structured as follows: A review of similar and related works available in the literature are provided in the next section. The proposed methodology, based on an extended COPRAS method under a PFs, by comprising weighting and complex proportional assessment methods, is presented in Section three. A case study for defining priority groups for allocating COVID-19 vaccines is provided in Section four. Section five is reserved for a discussion that contrast our results, with those from previous studies. Conclusions and future work are presented in the last section.

2. Literature review

A remarkable initiative, well-known in the literature as Fuzzy Sets (FSs), was introduced by Zadeh [29] as an approach for handling vagueness and incomplete MCGDM problems. FSs are capable of expressing DEs’ vague preferences through expressions such as ‘much more’, ‘slightly’, ‘much less’, etc. Later, Atanassov [30] introduced the concept of Intuitionistic Fuzzy Sets (IFSs), which is an extension of FSs that handles MCGDM problems from an intuitionistic perspective. Traditional FSs are based on membership degree ($\mu \in [0;1]$; favour), and non-membership degree ($\nu \in [0;1]$; against). IFSs introduce an additional element named a hesitancy degree ($\pi \in [0;1]$; abstain), on which $\mu - \pi - \nu$ is satisfied. For accurately describing vagueness on MCGDM problems, IFSs has shown limitations, since they are capable of describing fuzziness only in a dichotomous way (in favour vs against), whereas DEs are more comfortable if they are able to make assessments considering two or more possible alternatives. Picture Fuzzy Sets (PFs) emerged as an approach for overcoming those limitations, by introducing three possible memberships: in favour, against, and refusal [31]. PFs are an extension of IFSs, in which a new membership is incorporated
to yield a more precise problem description, through more accurate DE assessments. According to Cuong and Kreinovich [32], PFSSs are characterized by four elements: the positive (μ ∈ [0, 1], i.e. membership degree in IFs), neutral (δ ∈ [0, 1], i.e. hesitancy degree in IFs), negative (υ ∈ [0, 1], i.e. degree of non-membership in IFs) and refusal membership degrees (ξ ∈ [0, 1]); these elements satisfy 0 ≤ ξ + μ + δ + ι ≤ 1. Based on the above, the novel contribution of PFSSs is given by the introduction of refusal membership degree (ξ), which satisfies the x = 1 − μ − δ − ι. This last element enhances the consistency of the obtained results and avoids missing information along the decision making process [33–35].

Ashraf, Abdullah & Qadir [36] stated that PFs are capable of more accurately representing the way humans make decisions, in contrast to the earlier methods. There are several extended MCGDM methods available in the literature, in which PFSSs are incorporated. Gündoğdu et al. [37] proposed an extended Analytic Hierarchy Process (AHP) under the PFSSs. Meksavang et al. [38], introduced the ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method based on PFSSs. Tian et al. [39], presents another VIKOR extension under the PFSSs. Wang et al. and Sindhu et al. [18,19], utilized the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method based on PFSSs. Lu. et al. [40] proposed the extended Complex Proportional Assessment (COPRAS) under the PFSSs environment, leading to the PF-COPRAS. In the middle of COVID pandemic, Menekse and Camgoz Akdağ [41] proposed hybrid multi-criteria group decision-making based on the Analytic Hierarchy Process (AHP), the distance from average solution and spherical fuzzy sets for evaluating videoconferencing platforms. Their results suggest that Zoom was the most suitable platform for distance education during COVID pandemic. A MCGDM framework based on TOPSIS technique and COPRAS method under a fuzzy environment on [20] is proposed. Five criteria are prioritized for choosing the most suitable alternatives for mitigating and preventing COVID spreading. Not specifying or operationalizing the alternatives represents a relevant limitation on this work. Kumar et al. [42] proposed VICKOR-TOPSIS framework under a PFSS for improving voting systems during COVID pandemic. Although this methodology shows similarities with our proposal, it is not focusing on investigating COVID vaccination strategies. Most of the research available on the literature, framed on MCGDM fuzzy methods and focused on investigating covid vaccination strategies as [23–28] shows shortcomings as limited managerial implications, reduced guidelines to policy makers or marginal novelty on the mathematical methods proposed. From this perspective, our proposal makes an original contribution on this direction.

Zavadskas et al. [43] documented several advantages of the COPRAS algorithm, with respect to its predecessors. It is based on simpler calculations, which result in shorter processing times and more accurate results. Lu. et al. [40] state that one remarkable feature on COPRAS, that is not available in other MCGDM methods, is way their results are displayed. Results are more clearly and intuitively displayed, through the odds ratios that contrast maximum-minimum or positive-negative assessments. In recent literature, different extensions of COPRAS that investigate problems with different aims and contexts are available [44]. Hashemkhani Zolfani et al. [45] conducted a study aimed to evaluate hotel construction projects from an environmental perspective. The Step-Wise Assessment Ratio Analysis (SWARA) and COPRAS methods provided a unified framework for this study. While criteria weights were calculated through SWARA, the ranking of decision alternatives was conducted using COPRAS. Mishra et al. [46] presented a case study for assessing renewable energy production technologies, using the combining SWARA and COPRAS approaches. Criteria weights were obtained by the SWARA algorithm, and alternatives were assessed using COPRAS under fuzzy environment. The study provides evidence in favour of the cited approach, considering it yields more consistent results in comparison to those based on IFs. In Almulhim and Barahona [47], ELECTRE-I and VIKOR are integrated under an intuitionistic fuzzy environment. The extent to which judges’ assessments impact attributes at different periods is investigated in this work. To validate the suitability of this methodology, sustainability indicators for renewable energy systems were assessed at different periods of time. Ashraf, Abdullah and Almragbi [48] proposed a spherical intelligent fuzzy system for investigating the spreading and transmission of COVID-19. This approach integrates TOPSIS and COPRAS under a spherical fuzzy environment. Their results provide a mathematical model for selecting the best alternative for tackling COVID-19 transmission, given a path strategy. It is beyond the scope of this work to present a detailed explanation of COPRAS applications that are available in literature. Those interested in the COPRAS method are encouraged to read Rani et al. [2020], Rani et al. [2020]-A, Roozbahani et al. [2020] and Narayananmoorthy et al. [2021] [49–52]. A theoretical description of the mathematical formulations behind COPRAS algorithm is presented in the next section.

A common pattern in all research cited in this literature review is that they consider either objective or/and subjective weights for DE assessments. Similarly, most of MCGDM available in the literature, and based on the PFS environment, considers either only objective or/and subjective weights for DE assessments. From the above, it is clear that proposing PF-COPRAS, which considers objective and subjective weights for both — DEs assessments and criteria — represents a theoretical contribution in the field. Thus, we introduce a novel MCGDM methodology, based on PF-COPRAS that accurately handles objective and subjective weights for both DEs. A detailed explanation of the proposed methodology is provided in the next section. It is complemented with a case study for investigating priority groups in the COVID-19 vaccine allocations.

3. Materials and methods

This section is divided into three sub-sections. The first is the conceptual framework, which provides definitions for criteria and alternatives. Formal definitions for each element of our conceptual framework are also provided in this section. This is followed by a theoretical explanation of Picture Fuzzy Sets. Our proposed methodology, which integrates nine-steps for prioritizing groups when allocating COVID-19 vaccines, is the last subsection.

3.1. Conceptual framework

The conceptual framework illustrates how variables are related under this study. Namely, the relationships between criteria and alternatives are investigated. The framework is composed of six criteria (denoted by C) and four alternatives (denoted by A). While the alternatives were adapted from the work proposed by Hezam et al. [23], an exhaustive literature review was carried out for operationalizing the criteria. A summary of this review is shown in Table 1, and Fig. 1 is a graphical representation of our conceptual framework. With respect to alternatives, let $A = \{ A_i | i \in M \}$ be a set of alternatives, where $M = \{ 1, 2, \ldots, m \}$, represents each group that is being prioritized for COVID-19 vaccination. However, let $C = \{ C_j | j \in N \}$ be a set of evaluation criteria, where $N = \{ 1, 2, \ldots, n \}$ that represents evidence under which each DE is providing their assessments. Note that some criteria are conflicted or negatively correlated, and therefore investigating them can provide helpful scientific evidence for researchers, practitioners, or policy makers.

Two types of weights for DEs and criteria are defined for this methodology. At first, each criterion is related to specific weight (denoted as $w_i$); thus, $w = (w_1, w_2, \ldots, w_n)$ comprises of the vector of weights for the investigated criteria, conditioned to $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Besides, if $DE = \{ DE_k | k \in L \}$ is comprised of the group of DEs participating in the study, where $L = \{ 1, 2, \ldots, l \}$; then, DEs' weights are given by the vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_l)$, which satisfies $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{l} \alpha_i = 1$. Note that for the purposes of this research, both the vector weights for criteria and the DEs are unknown.

As it was previously mentioned, our proposed framework is
Table 1
Definitions of criteria.

| Criterion                              | Definition                                                                 | Related literature |
|----------------------------------------|---------------------------------------------------------------------------|--------------------|
| Vaccine convenience (C₁)               | It is given by indicators such as number of shots, effectiveness, cost per dose, storage and distribution issues, and documented side effects. | [53,54]            |
| Mortality rate (C₂)                    | There are two well-known approaches for calculating mortality rates. The first is the proportion of COVID-19 deaths, with respect to a given population. Mortality rate is also given by the proportion of COVID-19 deaths, out of the total number of infected persons. Typically, mortality rate is expressed per capita. | [55,56]            |
| Capabilities and infrastructure (C₃)   | This is defined as the capability of each country to provide health services to the larger population. The E-SPAR index is a quantitative indicator for measuring health care infrastructure. It is proposed by the World Health Organization and adapted for this research. | [57]               |
| Factors of health risks (C₄)           | Persons older than 65 years old are at a higher risk of presenting a serious medical condition due to COVID-19, or even die. Persons with comorbidities such as obesity, diabetes, hypertension, and chronic illnesses are at a higher risk of mortality. | [58]               |
| Allocation and distribution (C₅)       | As a result of the high rates of contagion, during early stages of COVID-19 emergence, the available doses tend to be insufficient. Thus, allocation refers to who should be vaccinated first. An exhaustive analysis should be conducted to define priority groups. However, distribution involves the required steps for the logistic process, which consists in moving doses from the manufacturing centre to the final user. | [59,60]            |
| COVID-19 strains (C₆)                  | It is defined as those mutations on SARS-CoV-2 virus structure or properties, that might lead to new variants. Some of these new variants can promptly disappear, while others can persist, and therefore, be more harmful. By the time this research was conducted, the emergence of several strains around the world was documented. | [61,62]            |

The formal definitions from the literature for criteria are summarized in Table 1, and complementary definitions for alternatives are given in Table 2.

3.2. Picture Fuzzy sets

This subsection introduces three basic definitions on which Picture Fuzzy Sets are founded. It is followed by a nine-step methodology, named Picture Fuzzy (PF) – Mixture Weighting and COPRAS. A case study for investigating priority groups in allocating COVID-19 vaccines is provided in the next section.

**Definition 1.** According to Cuong & Kreinovich [32], a Picture Fuzzy Set (PFS) denoted by A on the domain of X, is defined as follows:

\[ A = \{ x, \mu_A(x), \delta_A(x), \nu_A(x), \} \times \{ x \} \]  \hspace{1cm} (1)

where \( \mu_A(x) \in [0,1] \) refers to the degree of positive membership, \( \delta_A(x) \in [0,1] \) is called the degree of neutral membership, and \( \nu_A(x) \in [0,1] \) is related to the degree of negative membership. Note that for all elements of \( A = \{ x, \mu_A(x), \delta_A(x), \nu_A(x), \} \times \{ x \} \), the condition: \( 0 \leq \mu_A(x) + \delta_A(x) + \nu_A(x) \leq 1 \).

Table 2
Definitions of the alternatives.

| Criterion | Definition | Related literature |
|-----------|------------|--------------------|
| Age (A₁)  | It comprises of three groups of people: elderly people (65≥) and adults (40≥) without health chronical conditions. | [23] |
| Type of job (A₂) | It gathers persons working on activities considered as essential. Formal definitions for activities considered as essentials are given by governments, authorities, and international organizations. | [63,64] |
| Women (A₃) | It gathers pregnant, breastfeeding, and other health conditions women. | [23] |
| Comorbidities (A₄) | In the public health field, and taking a disease as the reference, comorbidity is related to more death rates, worse outcomes, and more expensive or complex medical treatments. According to the literature, the main comorbidities for COVID-19 are caused due to hypertension, cardiovascular disorders, chronic obstructive pulmonary disease, and chronic kidney disease, among others. | [65,66] |

Fig. 1. Conceptual framework with associations among criteria and alternatives.
Assess the relative alternatives weights through the PF-score degree

8. Calculate preferences degrees

9. Calculate utility degrees

Fig. 2. Flowchart for the proposed methodology.

### Definition 3
Let \( p = (\mu_p, \delta_p, \nu_p) \) and \( q = (\mu_q, \delta_q, \nu_q) \) be two PFNs; then, according to Cuong & Kreinovich [32], \( S(p) \) and \( S(q) \) denote the PF score degrees for \( p \) and \( q \), respectively. Besides, \( H(p) \) and \( H(q) \) represent the accuracy degrees for \( p \) and \( q \), respectively. While expression (6) shows how these scores are obtained, the accuracies are calculated based on formulation (7).

\[
S(p) = \mu_p - \nu_p ; \quad S(q) = \mu_q - \nu_q
\]

(6)

\[
H(p) = \mu_p + \delta_p + \nu_p ; \quad H(q) = \mu_q + \delta_q + \nu_q
\]

(7)

Note that for equations (6) and (7), the conditions \( S(p) \in [-1, 1] \) and \( H(p) \in [0, 1] \) are satisfied. The above also applies for PFN q. The literature proposes five general rules for any couple of picture fuzzy scores with the forms \( S(p) \) and \( S(q) \), as illustrated [32]:

1. If \( S(p) < S(q) \), then \( p < q \);
2. If \( S(p) > S(q) \), then \( p > q \);
3. If \( S(p) = S(q) \), \( h(p) < h(q) \), then \( p < q \);
4. If \( S(p) = S(q) \), \( h(p) > h(q) \), then \( p > q \);
5. If \( S(p) = S(q) \), \( h(p) = h(q) \), then \( p = q \);

### 3.3. Picture Fuzzy (PF) – mixture weighting and COPRAS

Picture Fuzzy (PF) – Mixture Weighting and COPRAS methodology, which incorporates both objective and subjective weights for DEs and criteria, is presented in this section. Works that combine COPRAS method with mixture weighting, under a MCGDM approach, were not found through the literature review. Therefore, this study makes an important contribution in that direction. As shown in Fig. 2, our methodology comprises of nine steps. It begins with defining the problem, goal, criteria, alternatives, and DEs group, and ends with the calculation of utility degrees. Note that this flowchart is unidirectional, and each step is carried out only once.

**Step 1.** Set overall goal, criteria and alternatives and group of DEs

The starting point is defining a research goal, which in this case consists of prioritizing a set of COVID-19 vaccination strategies, given a finite number of alternatives and criteria. Under the framework that is shown in Fig. 1, a group of DEs carry out the assessments of the alternatives, by considering the settled criteria.

**Step 2.** Build individual PF-evaluation matrices

Evaluation matrices are calculated in the second step. Here, each DE assesses alternatives based on the previously defined criteria, and using the five-level linguistic scale shown in Table 3. Note that each DE evaluates each alternative only once by considering all the settled criteria.

Regarding the formal notation for DEs assessments, let us denote \( D^{(k)} \) as the assessment provided by each individual DE. Besides, \( A \) denotes the PF - Evaluation matrix, which is composed by \( (d_{ij}^{(k)})_{m \times n} \) elements. The PF - Evaluation matrix is calculated based on the following expression.

\[
D^{(k)} = \left( d_{ij}^{(k)} \right)_{m \times n}
\]

According to expression (8) \( d_{ij}^{(k)} = (\mu_{ij}^{(k)}, \delta_{ij}^{(k)}, \nu_{ij}^{(k)}) \) is a picture fuzzy number, which refers to the assessment value provided by DE, \( k \)-ism DE to the alternative \( A_i \), given the criterion \( C_j \).

**Step 3.** Calculate the weights of DEs through PFs weighting method

Here, two types of weights are calculated. While the first is named objective, the second is identified as subjective. Objective weights are defined on the idea that they are not influenced by personal beliefs, feelings, interpretations, or individual judgments. However, subjective weights represent personal opinions or beliefs.

1. Objective weights \( \sigma_i^O = (\sigma_i^{O1}, \sigma_i^{O2}, ..., \sigma_i^{O9}) \) are directly calculated from rates that each DE provides to alternatives, by using the linguistic scale shown in Table 3 (by taking \( D^{(k)} \) value as input). Ju et al. [68] put forward a weighting method for calculating DEs’ objective weights, which is based on the entropy measure. Thus, the objective DEs’ weights, \( \sigma_i^O = (\sigma_i^{O1}, \sigma_i^{O2}, ..., \sigma_i^{O9}) \), are calculated with the following expression:

### Table 3
Linguistic terms and its respective PFNs for step 2. Adapted from Arya and Kumar [67].

| Linguistic Terms | PFNs          |
|-----------------|---------------|
| Extreme Priority (EP) | (0.65,0.10,0.20) |
| High Priority (HP) | (0.72,0.15,0.11) |
| Medium Priority (MP) | (0.60,0.33,0.05) |
| Simple Priority (SP) | (0.32,0.50,0.16) |
| Low Priority (LP) | (0.25,0.65,0.10) |
\[ \sigma_l^k = \frac{1 - \rho_k}{1 - \sum_{k=1}^{n} \rho_k}. \]  

(9)

According to (9), \( k = 1, \ldots, l \) satisfies \( \sum_{k=1}^{l} \rho_k = 1 \), and \( \sigma_l \) represents the Picture-Fuzzy Entropy value related to the judgment made by the \( k \)-ism DE (D(k)). The method proposed by Ju et al. [68] is incorporated here to compute the Picture-Fuzzy Entropy, as it is shown in the expression (10). Note that values for \( \mu, \delta, \) and \( \nu \) are obtained from definition 1, which was introduced in subsection 3.

\[ \rho_k = \sum_{i=1}^{n} \sum_{j=1}^{t} \left( \frac{1}{2} \left( |\mu_{ij}^{k} - \delta_{ij}^{k}| + |\mu_{ij}^{k} - \nu_{ij}^{k}| + |\delta_{ij}^{k} - \nu_{ij}^{k}| \right) \right) \]  

(10)

II) Subjective weights are computed based on the linguistic scale presented in Table 4. In contrast to objective weights, subjective weights are based on personal judgements or beliefs held by each DE. In this respect, each DE provides their personal judgments about each other, based on their expertise or personal background. This means obtaining additional evidence, on which each DE assess other members of the group of experts. The Picture-Fuzzy DE matrix, denoted as \( (\mathbf{r}_k)_{h=1}^{n} \), where \( \mathbf{r}_k = (\mu_{ij}, \delta_{ij}, \nu_{ij}) \) is the Picture-Fuzzy Number (PFN) that represents the importance evaluation of the k-ism DE from the q-ism DE, and \( k, q = 1, 2, \ldots \text{and} k \neq q \). Matrix R is represented based on the following expression:

\[
R = (\mathbf{r}_k)_{h=1}^{n} = \begin{pmatrix}
\mathbf{DE}_1 & \cdots & \mathbf{DE}_i & \cdots & \mathbf{DE}_l \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\mathbf{r}_1 & \cdots & \mathbf{r}_i & \cdots & \mathbf{r}_l
\end{pmatrix}
\]  

(11)

Then, the Picture-Fuzzy Weighted Interaction Geometric (PFWIG) operator [68] is used to aggregate values, as shown in the following expression.

\[
r_i = \left( \prod_{q<i} \left( 1 - \nu_{q,\mathbf{r}_k} - \delta_{q,\mathbf{r}_k} \right)^{\lambda} - \prod_{q<i} \left( 1 - \mu_{q,\mathbf{r}_k} - \delta_{q,\mathbf{r}_k} \right)^{\lambda} - \prod_{q<i} \left( 1 - \nu_{q,\mathbf{r}_k} - \delta_{q,\mathbf{r}_k} \right)^{\lambda} \right) \left( 1 - \nu_{i,\mathbf{r}_k} \right)^{\lambda} \\
- \prod_{q<i} \left( 1 - \delta_{q,\mathbf{r}_k} - \nu_{q,\mathbf{r}_k} \right)^{\lambda} - \prod_{q<i} \left( 1 - \delta_{q,\mathbf{r}_k} - \nu_{q,\mathbf{r}_k} \right)^{\lambda} \\
- \prod_{q<i} \left( 1 - \delta_{q,\mathbf{r}_k} - \nu_{q,\mathbf{r}_k} \right)^{\lambda} \right)
\]  

(12)

In (12), \( r_i = (\mu_{ij}, \delta_{ij}, \nu_{ij}) \) is a PFN that represents the importance group evaluation given by the k-ism DE, where \( k = 1, \ldots, l \) and \( \lambda = \frac{1}{1+1} \).

Finally, DEs’ subjective weights, denoted as \( \sigma_l^k = (\sigma_l^{k,1}, \sigma_l^{k,2}, \ldots, \sigma_l^{k,l}) \), are obtained. The method proposed by Svadlenka et al. [69] is incorporated as it is shown in the following expression:

\[
\sigma_l^{k} = \frac{\mu_{ij} + \delta_{ij} + \nu_{ij}}{1 + \mu_{ij} - \nu_{ij}}
\]  

(13)

On (13), \( k = 1, \ldots, l \) is the number of DEs and \( \sigma_l^k \) represents the subjective weight for the k-th DE in the group, conditioned to \( 0 \leq \sigma_l^k \leq 1 \) and \( \sum_{k=1}^{l} \sigma_l^k = 1 \).

III) Combining objective and subjective weights. Once two types of weights are obtained, the next step consists of aggregating them. The Mixture of Weights algorithm proposed by Svadlenka et al. [69], and denoted as \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_l) \) is calculated as illustrated in the following expression:

\[
\sigma_l = \gamma \sigma_l^o + (1 - \gamma) \sigma_l^s
\]  

(14)

In (14), the conditions \( 0 \leq \sigma_l \leq 1 \) and \( \sum_{k=1}^{l} \sigma_k = 1 \) are satisfied. \( \gamma \) represents the trade-off parameter also conditioned to \( 0 \leq \gamma \leq 1 \). For instance, weights are completely subjective when \( \gamma = 1 \). In contrast, weights are totally objective in the case that \( \gamma = 0 \). When the mixture of weights is present, the trade-off parameter is bounded in \( 0 < \gamma < 1 \).

Step 4 Build the group PF-evaluation matrix

Values for \( D = (d_{ij})_{m \times n} \) are calculated as illustrated in formulation (8). Note that D represents a group PF Evaluation matrix, where \( d_{ij} = (\mu_{ij}, \delta_{ij}, \nu_{ij}) \) takes the PFN form. Then, the PFWIG operator is incorporated for aggregating D(k) matrices by considering the DEs’ weights (\( \sigma_l \)), where \( k = 1, \ldots, l \) and \( l \) represents the number of DEs, as illustrated in the following expression:

\[
d_{ij} = \left( \prod_{k=1}^{l} \left( 1 - \nu_{ij}^{k} - \delta_{ij}^{k} \right)^{\sigma} - \prod_{k=1}^{l} \left( 1 - \mu_{ij}^{k} - \delta_{ij}^{k} - \nu_{ij}^{k} \right)^{\sigma} \right) \left( 1 - \nu_{ij}^{k} \right)^{\sigma} \\
- \prod_{k=1}^{l} \left( 1 - \delta_{ij}^{k} - \nu_{ij}^{k} \right)^{\sigma} - \prod_{k=1}^{l} \left( 1 - \delta_{ij}^{k} - \nu_{ij}^{k} \right)^{\sigma} \\
- \prod_{k=1}^{l} \left( 1 - \delta_{ij}^{k} - \nu_{ij}^{k} \right)^{\sigma} \right)
\]  

(15)

In (15), \( \sigma_l \) represents the weight value for the k-th DE, where \( i = 1, \ldots, m \) and \( j = 1, \ldots, n \) is the index for alternatives, \( i = 1, \ldots, m \) is reserved for criteria and \( k = 1, \ldots, l \) refers to the number of DEs participating in the study.

Step 5 Compute weights of criteria through PFSSs weighting method

Step five is reserved for calculating weights for criteria, a similar way to how it was calculated for DEs’ weights, by considering the following three-phase algorithm.

I) Objective weights for criteria, is denoted as \( w_j^o = (w_{j,1}^o, w_{j,2}^o, \ldots, w_{j,n}^o) \), and calculated based on the Picture-Fuzzy entropy measure proposed by Tian et al. [33] as illustrated in the following expression:

\[
w_j^o = \frac{1 - \tau_j}{\sum_{j=1}^{n} \tau_j}
\]  

(16)

Expression (16) is conditioned to \( j = 1, \ldots, n \), and \( \sum_{j=1}^{n} w_j^o = 1 \). \( \tau_j \) represents the PF entropy information for the criteria \( C_j \). It is obtained by the method proposed by Tian et al. [33], as shown below:

\[
\tau_j = \sum_{i=1}^{n} \left( 1 - \frac{1}{2} \left( \left| \mu_{ij} - \delta_{ij} \right| + \left| \mu_{ij} - \nu_{ij} \right| + \left| \delta_{ij} - \nu_{ij} \right| \right) \right)
\]  

(17)

II) Subjective weights for criteria are denoted as \( \sigma_j^s = (\mu_{j,1}, \delta_{j,1}, \nu_{j,1}) \), and represent the importance evaluation for the criterion \( C_j \), given by k-ism DE, for \( j = 1, \ldots, n \) and \( k = 1, \ldots, l \). Note that \( v_j^s \) is a PF number, and the linguistic scale introduced in Table 4 is used for its calculation. Thus, the Picture Fuzzy Criteria Weighted matrix \( V = (v_j^s)_{n \times 1} \) where \( v_j^s = (\mu_{j,1}, \delta_{j,1}, \nu_{j,1}) \) is calculated as follows:

\[
V = (v_j^s)_{n \times 1} = \begin{pmatrix}
\mathbf{C}_1 \\
\vdots \\
\mathbf{C}_n
\end{pmatrix}
\begin{pmatrix}
v_1^s & \cdots & v_n^s
\end{pmatrix}
\]  

(18)

The PFWIG proposed by Ju et al. [68], is applied to determine the
Group Picture Fuzzi Criteria Weight matrix GV = \( \{v_i\}_{i=1} \) by using the following equation:

\[
v_j = \left( \prod_{k=1}^{m} \left( 1 - \nu_{ik} - \theta_{ik} \right)^{\alpha_i} \right) - \left( \prod_{k=1}^{m} \left( 1 - \mu_{ik} - \theta_{ik} - \psi_{ik} \right)^{\alpha_i} \right),
\]

\[
\prod_{k=1}^{m} \left( 1 - \nu_{ik} \right)^{\alpha_i} - \left( \prod_{k=1}^{m} \left( 1 - \alpha_i \right) \right) - \left( \prod_{k=1}^{m} \left( 1 - \nu_{ik} \right)^{\alpha_i} \right)
\]

(19)

Note that (19) is indexed by criteria \( j = \ldots, i \), and DEs \( k = \ldots, 1 \).

Formulation (20) is applied for obtaining the subjective weights of criteria \( w^s_j = \left( \sigma_1, \sigma_2, \ldots, \sigma_n \right) \), as proposed by Svaldenka et al. [69]:

\[
w_j^s = \frac{\mu_{ij} + \delta_{ij} + \gamma_{ij}}{\sum_{i=1}^{n} \left( \mu_{ij} + \delta_{ij} + \gamma_{ij} \right)} \left( 1 + \mu_{ij} - \nu_{ij} \right)
\]

(20)

where \( j = \ldots, n \), also \( 0 \leq w_j^s \leq 1 \) and \( \sum_{j=1}^{n} w_j^s = 1 \) is satisfied.

III) The Mixture weights of the criteria, denoted as \( = (w_1, w_2, \ldots, w_n) \), are calculated as follows:

\[
w_j = w_j^s + (1 - \gamma)w_j^o
\]

(21)

where \( 0 \leq w_j \leq 1 \) satisfies \( \sum_{j=1}^{n} w_j = 1 \), \( \gamma \) is the trade-off parameter and \( 0 \leq \gamma \leq 1 \). In this form, the criteria weights are subjective when \( \gamma = 1 \). Otherwise, the criteria weights are objective when \( \gamma = 0 \). The criteria weights are mixed when \( 0 < \gamma < 1 \).

Step 6. Obtain criteria values for beneficial and non-beneficial types

At this point, each alternative \( A_i \) is expressed in a form where its summation maximizes the criteria \( a_i \). This maximizes the benefits, while also minimizing the criteria \( b_i \). The latter refers to a non-beneficial type, as is further illustrated. Let \( \Delta = \{1, \ldots, \beta \} \) be the set for maximizing (benefiting) the criteria \( a_i \), then the following expression is applied:

\[
a_i = \sum_{j=1}^{\beta} w_{ij} a_i, i = 1, 2, \ldots, m
\]

(22)

After applying expression (22), the criteria \( a_i \) is maximized (benefitted). Subsequently, the criterion \( b_i \) is minimized (non-beneficial). Let \( \nabla = \{g + 1, \ldots, m\} \) be a set for minimizing criterion \( b_i \), then expression (23) is applied, as illustrated below:

\[
b_i = \sum_{j=1}^{\eta} w_{ji} b_i, i = 1, 2, \ldots, m
\]

(23)

Note that \( w_i \) represents the importance degree of the \( j \)-ism criteria, as previously shown by expressions (22) and (23).

Step 7. Assess the relative alternatives weights through the PF- score degree

At this point the relative weight \( \theta_i \) for each alternative is calculated, as illustrated below [49,70]:

\[
\theta_i = \psi \cdot S \left( a_i \right) + S \left( b_i \right) \cdot \frac{\sum_{j=1}^{\beta} \left( \sum_{j=1}^{\eta} w_{ij} \right) a_i}{\sum_{j=1}^{\beta} \left( \sum_{j=1}^{\eta} w_{ij} \right)}, i = 1, 2, \ldots, m
\]

(24)

In expression (24), \( S \left( a_i \right) \) and \( S \left( b_i \right) \) represent the PF score degrees for \( a_i \) and \( b_i \), respectively. In addition, the parameter \( \psi \) denotes the strategic value of the DE in the closed interval \([0,1]\). For instance, if \( \psi < 0.5 \), the DE presents a pessimistic position; therefore, a high weight should be assigned to the non-beneficial criteria. However, if \( \psi > 0.5 \), the DE holds an optimistic position, and a high weight should be associated to the beneficial criteria \( a_i \). If \( \psi = 0.5 \), the DE presents neutral behaviour, and both beneficial and non-beneficial type criteria should have the same importance degree.

Step 8. Calculate preferences degrees

The preference degrees for alternatives \( A_j \) for \( j = 1, \ldots, m \) are calculated according to their relative importance. In this form, the alternative with the maximum relative, denoted as \( A^* \), is ranked in the first place, followed by the second highest value, and so on.

\[
A^* = \max \theta_i, i = 1, 2, \ldots, m
\]

(25)

Step 9. Calculate utility degrees

By contrasting each assessed alternative with the optimal one \( (A^*) \), the utility degree \( \eta_i \) is calculated. To display meaningful and intuitive results, a \([0–100]\) range is used. While the optimal alternative yields the highest score, the lowest is reserved for the worst. Expression (26) is used to calculate the utility degree \( \eta_i \).

\[
\eta_i = \frac{\theta_i}{\theta_{\text{max}}} \times 100, i = 1, 2, \ldots, m
\]

(26)

Note that \( \theta_i \) and \( \theta_{\text{max}} \) represent relative weights for alternatives \( A_i \) and \( A^* \) respectively. They were obtained through computing expression (25). Later, utility degrees are ranked in the decreasing order, with the highest degree is at the top, followed by the second highest, and so on.

4. Results (prioritization of the COVID-19 vaccination)

4.1. Define priority groups for allocating COVID-19 vaccines

In this subsection, the Picture Fuzzy (PF) – Mixture Weighting and COPRAS methodology is applied to real data, to yield empirical evidence related to COVID-19 vaccine allocation.

Step 1. After setting the main goal (see Fig. 1), an electronic questionnaire has been prepared according to the four alternatives — \( A_i (i = 1, \ldots, 4) \), and the six criteria — \( C_j (j = 1, \ldots, 6) \). The questionnaire was shared with a group of DEs. The group was composed of six DEs (k = 1, 2, \ldots, 6) details about DEs’ background and expertise are shown in Table 5.

This questionnaire allowed us to collect six matrices of order \( C_k \times A_i \), where \( k = 1, \ldots, 6 \) and \( i = 1, \ldots, 4 \). Besides, a unique linguistic term from Table 3 is allocated to each element \( (k,i) \) of these six matrices by the same number of DEs. The data under analysis comprises \( 6 \times 4 \times 6 = 144 \) elements and each one of them corresponds to a linguistic term, as it is shown on Table 6.

Step 2. The Picture Fuzzy (PF) Evaluation Matrices \( D^{(k)}(k = 1, 2, \ldots, 6) \) are built from the assessment provided by the group of DEs. The scale presented in Table 3 was used for this purpose. The resulting evaluation matrices are shown in Table 6. Subsequently, the PFNs are built by applying equation (8) in the PF-Evaluation matrices \( D^{(k)}(k = 1, 2, \ldots, 6) \).

Step 3. The Mixture of Weights of DEs are calculated as follows:

1) By using formulations (9) and (10), objective weights for DEs \( (\sigma_0^1, \sigma_0^2, \ldots, \sigma_0^n) \), are computed while the matrix \( D^{(k)} \) is taken as input.

Table 5

| Background and expertise for the DEs group |
|------------------------------------------|
| Decision Expert | (DE) | Background and expertise |
|-----------------|------|-------------------------|
| DE1 | Epidemiologist with 5–8 years of experience, working in a non-governmental organization |
| DE2 | Researcher with 1–4 years of experience, working in a government organization |
| DE3 | Doctor of Medicine with 1–4 years of experience, working in a government organization |
| DE4 | Professor with 17–20 years of experience, working in the education sector |
| DE5 | Doctor in Clinical Laboratory Testing with 9–12 years of experience, working in a government organization |
| DE6 | Researcher in Public Health and Epidemiology with 5–8 years of experience, working in a government organization |
Table 6

| Decision Expert (DE) | Alternative | C1 | C2 | C3 | C4 | C5 | C6 |
|----------------------|------------|----|----|----|----|----|----|
| DE1                  | A1         | HP | EP | SP | SP | LP |    |
|                      | A2         | HP | HP | HP | HP | MP |    |
|                      | A3         | SP | HP | SP | SP | SP |    |
|                      | A4         | MP | HP | HP | HP | MP | LP |
| DE2                  | A1         | LP | HP | LP | EP | MP | MP |
|                      | A2         | MP | EP | EP | SP | HP |    |
|                      | A3         | MP | SP | SP | EP | MP |    |
|                      | A4         | EP | EP | EP | EP | EP | MP |
| DE3                  | A1         | HP | ME | HP | LP | EP |    |
|                      | A2         | SP | EP | EP | SP | HP |    |
|                      | A3         | SP | HP | HP | EP | MP |    |
|                      | A4         | EP | HP | EP | EP | EP | MP |
| DE4                  | A1         | HP | HP | MP | SP | HP |    |
|                      | A2         | EP | EP | HP | HP | MP |    |
|                      | A3         | SP | SP | SP | SP | MP |    |
|                      | A4         | EP | EP | EP | EP | EP | MP |
| DE5                  | A1         | EI | EI | EI | EI | EI |    |
|                      | A2         | EI | EI | EI | EI | EI |    |
|                      | A3         | EI | EI | EI | EI | EI |    |
|                      | A4         | EI | EI | EI | EI | EI |    |
| DE6                  | A1         | SP | HP | MP | SP | HP |    |
|                      | A2         | MP | HP | MP | HP | MP |    |
|                      | A3         | MP | HP | SP | HP | MP |    |
|                      | A4         | HP | EP | HP | HP | HP | MP |

II) Based on the judgments that each DE provides to the other members of the group, and by using the scale presented in Table 4, subjective weights ($s_{1}^{DE}$, $s_{2}^{DE}$, ..., $s_{6}^{DE}$) are obtained and presented in Table 7.

III) Through expressions (11–13) these weights are converted into PFNs for deriving DEs’ objective and subjective weights.

IV) The Mixture of Weights of DEs ($w_{1}^{DE}$, $w_{2}^{DE}$, ..., $w_{6}^{DE}$) are calculated by employing equation (14). Note that, the value of a trade-off parameter equal to $\gamma = 0.5$ is assumed here. Objective, subjective, and mixture weights are presented in Table 8.

Step 4. Picture Fuzzy (PF) Linguistic terms, which are shown in Table 6, are transformed to PFNs, and these last are aggregated through the application of formulation (15). The group evaluation (aggregated) matrix, with the order $D = (d_{ij})_{3x6}$ where $d_{ij} = (P_{ij}, V_{ij}, E_{ij})$ and indices $i = 1, 2, 3$, and $j = 1, 2, 3, 4, 5, 6$, is provided in Table 9.

Step 5. The mixture of weights is computed by taking objective and subjective weights as inputs. Below is an explanation of how these calculations are carried out.

Table 7

| DE1 | DE2 | DE3 | DE4 | DE5 | DE6 |
|-----|-----|-----|-----|-----|-----|
| P   | EI  | VI  | EI  | VI  | EI  |
| VI  | EI  | VI  | EI  | EI  | EI  |
| EI  | I   | P   | P   | P   | P   |
| EI  | I   | I   | I   | I   | I   |
| EI  | I   | I   | I   | I   | I   |
| EI  | I   | EI  | EI  | EI  | EI  |
| EI  | P   | I   | LI  | VI  | VI  |

Table 8

| Objective, subjective, and mixture DEs weights. |
|------------------------------------------------|
| Decision Experts | DE1 | DE2 | DE3 | DE4 | DE5 | DE6 |
|------------------|-----|-----|-----|-----|-----|-----|
| Objective weights| 0.18| 0.16| 0.16| 0.21| 0.12| 0.17|
| Subjective weights| 0.18| 0.16| 0.18| 0.16| 0.17| 0.15|
| Mixture weights  | 0.18| 0.16| 0.17| 0.19| 0.14| 0.17|

Table 9

| The group PF-evaluation matrix. |
|---------------------------------|
| C1 | C2 | C3 | C4 | C5 | C6 |
|----|----|----|----|----|----|
| A1 | (0.54, 0.68, 0.17) | (0.50, 0.40, 0.09) | (0.69, 0.38, 0.48) | (0.55, 0.12) | (0.14) | (0.10) | (0.13) | (0.15) | (0.53) | (0.12) |
| A2 | (0.58, 0.68, 0.07) | (0.67, 0.16, 0.71) | (0.53, 0.32, 0.67) | (0.27, 0.12, 0.18) | (0.15) | (0.14) | (0.14) | (0.22) |
| A3 | (0.44, 0.62, 0.25) | (0.47, 0.40, 0.60) | (0.56, 0.58, 0.43, 0.13) | (0.13) | (0.22, 0.32, 0.11) | (0.29, 0.12) |
| A4 | (0.66, 0.68, 0.12) | (0.69, 0.69, 0.67, 0.16, 0.55) | (0.15, 0.18, 0.13, 0.16, 0.15) | (0.17, 0.16, 0.12) |

1) Objective weights ($w_{1}^{DE}, w_{2}^{DE}, ..., w_{6}^{DE}$) are calculated when the group PF-evaluation matrix is taken as input, and expressions (16) and (17) are applied.

II) Subjective weights ($w_{1}^{DE}, w_{2}^{DE}, ..., w_{6}^{DE}$) are obtained by taking the judgments that each DE provides to the given criteria. This is carried out by using the scale presented in Table 4. Linguistic terms evaluations provided by the group of DEs are shown in Table 10. Note that expressions (18–20) are used to derive these evaluations into PFNs, and therefore obtain subjective weights.

III) The Mixture of Weights of the criteria $w = (w_{1}, w_{2}, ..., w_{6})$ are calculated by applying equation (21). Here, the trade-off parameter is settled as being equal to $\gamma = 0.5$. The calculations obtained are presented in Table 11.

Step 6. Summarily for both: beneficial ($C_{1}, C_{3}, C_{5}$) and non-beneficial criteria ($C_{2}, C_{4}, C_{6}$) are computed by applying expressions (22) and (23), respectively.

Step 7. Relative weights for alternatives are obtained ($\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$) by taking the PF score degrees as input and applying expression (24). Table 12 shows these values.

Steps 8. & 9. Utility degrees ($\eta_{1}, \eta_{2}, \eta_{3}, \eta_{4}$) are calculated through equations (25) and (26). Table 12 presents the results obtained. Based on the above, the ranking order is $A_{4} > A_{2} > A_{3} > A_{1}$, when $\psi = 0.5$. Therefore, alternative $A_{4}$ is assigned as the highest priority.

4.2. Sensitivity analysis

A sensitivity analysis was conducted by changing values on the parameter $\psi$, which refers to the strategic importance assigned to each
Sensitivity analysis with respect to changes on the parameter \( \psi \) is the most robust, since it is the least sensitive to changes on the parameter \( \psi \), are presented in Table 13. A visualization with the obtained results, are provided in Fig. 3. While alternative \( A_1 \) is the most robust, since it is the least sensitive to changes on \( \psi \) parameter, \( A_1 \) was the most sensitive. It brings to our attention that the ranking alternatives remain the same for \( \psi \) values of \( \geq 0.4 \), that is \( A_4 > A_2 > A_3 > A_1 \). In this form, \( A_1 \) is at the top of the ranking order when \( \psi \geq 0.1 \). However, \( A_2 \) is at the bottom of the rank order when \( \psi \leq 0.2 \) (see Table 13). The above analysis implies that the strategic evaluations of DEs are pessimistic in nature, regarding the non-beneficial criteria type. Results throws partial evidence in favour of the robustness of this proposed methodology. The higher the parameter \( \psi \), the more robust the methodology. From the pessimistic perspective of DEs participating on this study, the more robust the model is, the lesser the utility is for alternatives \( A_1 \) (Age) and \( A_3 \) (Women). The foregoing is clearer when \( \psi \geq 0.4 \), and the ranking order of the alternatives remains the same.

5. Discussion and managerial implications

A novel MCGDM for prioritizing groups for COVID-19 vaccination is proposed in this study. It is based on an extended COPRAS method under a Picture Fuzzy Environment, which allows to split weights for DE and criteria, and therefore yield more consistent and reliable results. Three types of weights are calculated: subjective, objective, and mixture weights. Our approach takes judgments from six DEs as input. They assess the six criteria to rank four different groups for COVID-19 vaccination. Finally, a sensitivity analysis allowed us to investigate the extent to which the proposed methodology is robust to different values of the DEs’ strategic importance.

Considering an average of all three weights, the results of allocation and distribution (\( C_3 \)) has the biggest average weight. It includes all the steps required for logistic processes, to move vaccines from the manufacturing centre to the final user. Rastegar et al. [59] state that an effective distribution of vaccines, from laboratories to patients, requires the observation of demand rates, strategic inventory plans and designing suitable distribution schedules. Simulation-based approach for investigating the logistic performance of COVID-19 vaccine distributions is analysed in Sun, Andoh and Yu [71]. Improvements on routes optimization and dynamic distribution are proposed by their study. Their results suggest that factors such as fleet size, fleet composition, type of vehicle, and route should be holistically considered for achieving the optimal allocation and distribution. The second most important criterion was COVID-19 strains (\( C_4 \)). This is understood as mutations of the SARS-CoV-2 virus structure or properties that might lead to new disease variants. According to Vasireddy et al. [61] the emergence of new COVID-19 strains compels available vaccines to demonstrate its effectiveness. Mahase [72] states that some new strains (Omicron or Delta) are more than 50% more contagious than the first SARS-CoV-2 discovered in Wuhan. By the time this work was finished, there was limited evidence available in favour of the vaccines’ effectiveness against new strains. Capabilities and infrastructure (\( C_4 \)) resulted in the third most important weight and is defined as the network that gathers all hospitals, medical centres, and public health services in a country. Note that these three criteria together represent 60% of the total average weight importance.

Comorbidities (\( A_4 \)) was assigned as the alternative with the highest priority. In Duijzer et al. [5], and World Health Organization [6,60,65], these are defined as characteristics or conditions of an individuals’ health that are related to higher death rates, worse diagnostics, and more expensive or complex medical treatments. Bajgain et al. (2021) [66] conducted a literature review, in which hypertension, diabetes, and cardiovascular diseases were identified as the most common COVID-19 comorbidities. The results presented here are consistent with the World Health Organization (WHO) statements, in which the prioritization of high-risk groups and providing equal opportunities to less privileged groups is endorsed. Similarly, Hazem et al. [23] offer evidence in favour of prioritizing vaccines for critically ill individuals, the elderly, health care workers, and pregnant women. Type of job (\( A_2 \)) is second priority alternative. For this research, it indicates workers of activities considered as essential, for instance healthcare, energy and food industries, and police-security, among others. These findings are congruent with the available research in the discourse. A National Expert Group on Vaccines Administration (NEGVC) was created by the Indian government. It recommended the prioritizing of healthcare and frontline workers, and individuals above 45 years with comorbidities [73]. Women (\( A_3 \)) and Age (\( A_1 \)) were the alternatives with the lowest priority. While the first comprises pregnant, breastfeeding, and women with underlying health conditions, the second gathers adults older than 40 years old and elderly (65 years old or older). By showing supportive evidence that people with comorbidities (\( A_3 \)) and essential workers (\( A_1 \)) should be prioritized over Women (\( A_3 \)) and Age (\( A_1 \)), this study contributes to the discussion about essential workers and people with health conditions as priority groups. A sensitivity analysis showing that the more robust the model is, the less the utility for Women (\( A_3 \)) and Age (\( A_1 \)), thows complementary evidence this direction. The above allowed...
us to disclose the pessimistic perspective of DEs regarding less useful alternatives. These results are consistent with the other studies [23, 66, 74], and the policies stated by several governments and health organizations [9, 10].

The main contributions of this research are as follows: (1) proposing a consistent methodology for analysing data with a high degree of fuzziness. As far as we know, there is no study that combines the COPRAS method under Picture Fuzzy Environment (PFE), where the weights for Decisions Experts (DE) and criteria are unknown; (2) governments, decision makers and policy makers can find rigorous scientific evidence for articulating effective vaccinations campaigns that contribute to minimize undesired results (deaths or overrun hospitals) from this study; (3) From DEs’ perspective, this work is among the first to propose to split the weights into objective, subjective, and mixture weights. This leads to more transparent results and a clearer methodology.

However, this also has some limitations. Considering that DEs from only two countries were part of the panel, our findings are biased due to the cultures of their countries of origin. Therefore, these results should be applied cautiously. A wider study, including DEs from more countries or regions, is advised for future research. At the time of conducting this study, available research that considers minorities as ethnic, religious, or linguistic groups was limited. More research that prioritises minority groups as an alternative on the settled framework is also advised for future studies based on the proposed methodology.

6. Conclusions

This study proposes a novel MCGDM method based on COPRAS under PFE. It is demonstrated to be efficient for tackling ambiguity on DE’s opinions (hesitancy, in favour or against). The reliability and robusticity of this methodology was assessed by a sensitivity analysis. Prioritizing people affected by comorbidities for vaccinations reduces deaths during the pandemic, and complex, costly and delayed treatments. Besides, the prioritization of essential workers preserves the healthcare system reduces the negative effects to the economy and keeps some strategic activities available for the larger society. Governments, policy makers, and decisions makers can find empirical evidence that contributes to the discussion of how conflicted or negative correlated alternatives should be prioritized, by keeping a transparent methodology and a clear framework in this study.

The MCGDM based on COPRAS and PFE is a versatile methodology. It successfully deals with problems where high amounts of uncertainty are present, or the weights for criteria and DEs are unknown. This methodology is ideal for being replicated to investigate problems in fields such as demography, renewables energies, agriculture, mental health, or site allocation, among others.

Author statement

Dr. Tarifa Almulhim: Design methodology, data collection, data analysis, writing the main manuscript, editing, reviewing, and corresponding author.

Dr. Igor Barahona: Design methodology, data collection, data analysis, writing the main manuscript, editing, reviewing, and specialized software.

Declaration of competing interest

None.

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