Sensitivity analysis of seismic waveforms to upper-mantle discontinuities using the adjoint method

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SUMMARY
Using spectral-element simulations of wave propagation, we investigated the sensitivity of seismic waveforms, recorded on transverse components, to upper-mantle discontinuities in 1-D and 3-D background models. These sensitivity kernels, or Fréchet derivatives, illustrate the spatial sensitivity to model parameters, of which those for shear wave speed and the surface topography of internal boundaries are discussed in this paper. We focus on the boundaries at 400 and 670 km depth of the mantle transition zone. SS precursors have frequently been used to infer the topography of upper-mantle discontinuities. These seismic phases are underside reflections off these boundaries and are usually analysed in the distance range of 110°–160°. This distance range is chosen to minimize the interference from other waves. We show sensitivity kernels for consecutive time windows at three characteristic epicentral distances within the 110°–160° range. The sensitivity kernels are computed with the adjoint method using synthetic data. From our simulations we can draw three main conclusions: (i) The exact Fréchet derivatives show that in all time windows, and also in those centred on the SS precursors, there is interference from other waves. This explains the difficulty reported in the literature to correct for 3-D shear wave speed perturbations, even if the 3-D structure is perfectly known. (ii) All studies attempting to map the topography of the 400 and 670 km discontinuities to date assume that the traveltimes of SS precursors can be linearly decomposed into a 3-D elastic structure and a topography part. We recently showed that such a linear decomposition is not possible for SS precursors, and the sensitivity kernels presented in this paper explain why. (iii) In agreement with previous work, we show that other parts of the seismograms have greater sensitivity to upper-mantle discontinuities than SS precursors, especially multiply bouncing S waves exploiting the S-wave triplications due to the mantle transition zone. These phases can potentially improve the inference of global topographic variations of the upper-mantle discontinuities in the context of full waveform inversion in a joint inversion for (an)elastic parameters and topography.

Key words: Body waves; Computational seismology; Wave propagation.

1 INTRODUCTION
In the scientific community, the consensus is that upper-mantle discontinuities are most likely caused by phase changes of mantle minerals and are found around depths of 410 and 660 km (e.g. Revenaugh & Jordan 1991; Shearer 1991; Deuss 2009). We will refer to them as occurring at 400 and 670 km because in this paper we only present evidence from a synthetic study, which uses PREM (Dziewoński & Anderson 1981) as a background model. In PREM, the discontinuities are fixed at depths of 400 and 670 km, and this is the naming we adopt here. These discontinuities were first discovered using array studies (Niazi & Anderson 1965; Johnson 1967) in which the triplication of body waves caused by the upper-mantle discontinuities in the distance range 15°–30° was exploited. These triplications were further used in waveform modelling studies (e.g. Grand & Helmberger 1984; Tan & Helmberger 2007; Staelher et al. 2012) to more tightly constrain the nature of the transition zone discontinuities. However, to the best of our knowledge, they have not been exploited for the global mapping of discontinuity topography. The lateral resolution of these studies is usually low and confined to the corridor of the event connecting the array (Zhao & Helmberger 1993). Receiver functions have also been used to identify upper-mantle discontinuities (Vinnik 1977). This type of study samples the mantle structure directly beneath seismic stations...
and can thus result in high lateral resolution, provided the station
distribution is dense (e.g. Rondenay 2009). The downside of the
receiver function studies is that the access to the upper-mantle dis-
continuities is restricted beneath continents, where the stations are
usually located. Precursors to surface reflected phases (SS and to a
lesser extend PP) have proven to be much better at resolving the
nature of upper-mantle discontinuities globally. These precursors
follow a similar path to that of the SS wave, but instead of being
reflected at the free surface, they are reflected at the underside of the
discontinuities. The precursors arrive 2–4 min before the SS wave.
The reason for using them is that they are thought to be mainly sen-
titive to the structure beneath the bounce points (halfway between
source and receiver great circle path) and hence allow us to sample
oceanic as well as continental regions. For an overview the reader
is referred to Deuss (2009). In a different approach, Meier et al.
(2009) used surface wave overtones to infer the global structure of
the mantle transition zone.

It is important to understand the seismic structure around 400 km
and 670 km depth because it puts strong constraints on the tem-
perature and composition at the boundaries of the upper mantle
transition zone (e.g. Helffrich 1999; Shearer 2000; Schmerr &
Garnero 2006; Deuss 2009) and hence the geodynamic nature of the
mantle. Much effort has been dedicated to inferring the topographic
variations of the mantle transition zone discontinuities using observ-
ations of SS precursors together with ray theory (e.g. Shearer 1991,
1993, 2000; Shearer & Masters 1992; Flanagan & Shearer 1998,
1999; Gu & Dziewonski 1998; Gu & Dziewonski 2002; Gu
et al. 2003; Chambers et al. 2005; Schmerr & Garnero 2006; Houser
et al. 2008; Lawrence & Shearer 2008). Despite an intense research
effort, many discrepancies between the studies remain, which are
currently not completely understood. The problem is similar or even
worse for PP precursors, but we will not address the PP phases any
further here. The traveltimes of the SS precursors do not only depend
on the topography of the discontinuities, but also on the 3-D veloc-
ity structure of the mantle. A correction of the latter therefore needs
to be applied to the observed traveltimes, and this is a likely cause
of the discrepancies between the studies. Indeed, Bai et al. (2012)
demonstrated that even if the 3-D structure is perfectly known, ray theory
can only explain about 50 per cent of the traveltime anomalies pro-
duced by it. The complicated shape of the Fresnel zones associated
with minimax phases has been put forward to explain the problem
with traveltime corrections (Chaljub & Tarantola 1997; Zhao &
Chevrot 2003). Recently, Koroni & Trampert (2016) have identified
a more fundamental problem, that SS traveltimes cannot be lin-
early decomposed into a 3-D velocity structure and a discontinuity
topography part.

In this study, we analysed finite frequency kernels to understand
what causes the problems associated with inferring mantle transi-
tion zone structure from traveltimes of SS precursors. We calculated
exact seismograms in 1-D and 3-D background models and added
topography to the upper-mantle discontinuities. The study is en-
tirely synthetic. Analysing the evidence based on finite frequency
kernels, we found that within the precursor window there is always
interference of several seismic waves. This explains why a correct-
ion based on a single wave cannot properly account for the 3-D
seismic structure. We also found that the sensitivity of the travel-
times of SS precursors to the boundary denoting the discontinuity
changes significantly with the background model. We argue that this
is the reason why effects of 3-D velocity structure and topography
are not additive for a given traveltime anomaly. In agreement with
other studies, we identified areas in the seismogram, associated with
multiply bouncing S waves, that are far more sensitive to the upper-
mantle discontinuities than SS precursors, which to our knowledge
have never been used for a global mapping of the discontinuities.
In the following, the methods for computing the sensitivity ker-
nels will be described. We will show the resulting volumetric and
boundary sensitivity kernels. Lastly, we will discuss our results and
analyse their implications regarding the inference of topography of
upper-mantle discontinuities.

2 SYNTHETIC DATA AND CALCULATION OF THE SENSITIVITY KERNELS

To investigate the sensitivity of the waveforms and especially of the
SS precursors, we used part of the data generated in Koroni &
Trampert (2016) and a detailed description of it can be found in
their paragraph 2.1. We used both 1-D and 3-D elastic models for
the simulations of the synthetic data. The 1-D background model
is PREM (Dziewo´nski & Anderson 1981) and is used for the cal-
culations of most of the sensitivity kernels, the 3-D mantle model
is S20RTS (Ritsema et al. 1999) combined with the CRUST2.0
(Bassin et al. 2000) and the topography models are from Meier et al.
(2009) scaled to ±30 km. The models are used to study the effects of
3-D shear velocity and topographic perturbations on SS precursors
signals. We used 1-hr long transverse component seismograms and
PREM arrival times to select the time windows and to calculate the
Fr´echet derivatives (or sensitivity kernels). The earthquake source
is a pure strike-slip event of magnitude 7.9 at a depth of 20 km. The
source time function (STF) is set to a Gaussian of half-width equal
to 30 s and the resolution of the seismograms is such that they are
accurate down to a minimum period of 18 s. We use the STF to
smooth the kernels which otherwise would appear oscillatory and
it would be harder to identify the various phases. The STF is twice
as large as what would be expected for an event of magnitude 7.9
(Ekstr¨om et al. 2012) still our seismograms have dominant ampi-
tudes in the period range 30–150 s, the high frequency band being
close to the periods often used in SS precursor studies. For a single
source, at the equator at 300° longitude, we selected 3 receivers as
shown in Fig. 1 at epicentral distances of approximately 110°, 140°
and 160°. These three distances show the main characteristics of the
sensitivity kernels as illustrated in the next section. The simulations
were done with the spectral element package SPECFEM3D_GLOBE
(Komatitsch & Vilotte 1998; Komatitsch & Tromp 1999, 2002a,b).

The calculation of the Fr´echet derivatives is based on adjoint
methods. For a review of adjoint methods in geophysical problems,
the reader is referred to Tarantola (1984), Gauthier et al. (1986),
Talagrand & Courtier (1987) and Tarantola (1998). The mathemat-
ics of our implementation is developed in the papers of Tromp et al.
(2005); Liu & Tromp (2006, 2008) and coded up in the numerical
package SPECFEM3D_GLOBE, the same code we used for the cal-
culation of the synthetic data and sensitivity kernels. Prior to the
calculation of the sensitivity kernels, the synthetic data were pro-
cessed in the following way: we removed the mean, tapered and
filtered them using a band-pass with corners at 22 and 250 s. We
selected narrow time windows on the transverse component wave-
forms to investigate the sensitivity of seismic phases. The variable
time windows (see Tables 1–3) are 20 per cent cosine tapered cen-
tred around the arrival time we want to investigate.

The first step towards the computation of the Fr´echet deriva-
tives is to define an objective or misfit function. For simplicity
we only show sensitivity kernels for traveltime anomalies mea-
sured by cross-correlation (e.g. Luo & Schuster 1991; Marquering
boundary kernels in the 3-D model S20RTS plus CRUST2.0 (from now on denoted by S20RTS) to compare them to those in PREM. The time windows for each kernel are shown together with the corresponding transverse component seismogram. We use the naming convention of the TauP software (Crotwell et al. 1999) to identify phases seen on the sensitivity kernels. In what follows, the symbol ≤ denotes an underside reflection, while the symbol ≥ denotes a top-side reflection off the depth of the discontinuity.

3.1 110° distance

The Fréchet derivatives representing the sensitivity to relative volumetric changes in $V_{SH}$ are shown in Fig. 2 and the ray theoretical arrivals, calculated by TauP are shown in Table 1 per time window. Time window (a) corresponds to a kernel which shows a pure sensitivity to $S_{diff}$ in the shape of a banana-doughnut kernel. We see no clear sign of the core phases theoretically arriving in the same time window. Time window (b) is mainly sensitive to $S_{diff}$. Its sensitivity kernel shows higher order Fresnel zones of $S_{diff}$ sensitivity because the time window is in the coda of $S_{diff}$ and there are possible reverberations in $D′$. $D′$ is a second order discontinuity at 150 km above the core-mantle boundary in PREM, which has been used for the calculation of the sensitivity kernels. PREM also contains a first order discontinuity at a depth of 220 km. Therefore, top-side reflections of $S_{diff}$ off the 220 km discontinuity ($S Sylvia Sdiff$ and $S Sylvia Sdiff$) arrive in this time window, and as a consequence there is a hint of their sensitivity in the kernel. Time window (c) contains a 400 km top-side reflection of $S_{diff}$ and a double top-side reflection at the 220 km discontinuity, and hence the kernel is that of $S_{diff}$ with symmetric reflection legs. The reflections off the 220 and 400 km discontinuities interfere and are therefore only faintly visible. The underside reflection of the $SS$ wave at the 670 km discontinuity arrives in window (d). The arrival of the $SSS$ phase almost coincides with the top-side reflection of $S_{diff}$ at the same discontinuity. Therefore the kernel has a sensitivity to $S_{diff}$ besides the 670 km precursor of $SS$. The top-side reflection legs appear symmetric because $S Sylvia Sdiff$ arrives at the same time as $S Sylvia Sdiff$. There is also some interference with $S Sylvia Sdiff$ and $S Sylvia Sdiff$. Time window (e) corresponds to the arrival of the 400 km precursor to $SS$, but the sensitivity kernel also shows the higher order Fresnel zones of the $S_{diff}$ top-side reflection at the same discontinuity ($S Sylvia 400S Sdiff$). Note that due to the source time function of 60 s, 670-related waves can co-incide with the underside reflection of $S_{diff}$ at the same discontinuity. Therefore the kernel has a sensitivity to $S_{diff}$ besides the 670 km precursor of $SS$. The top-side reflection legs appear symmetric because $S Sylvia Sdiff$ arrives at the same time as $S Sylvia Sdiff$. There is also some interference with $S Sylvia Sdiff$ and $S Sylvia Sdiff$. Time window (e) corresponds to the arrival of the 400 km precursor to $SS$, but the sensitivity kernel also shows the higher order Fresnel zones of the $S_{diff}$ top-side reflection at the same discontinuity ($S Sylvia 400S Sdiff$). Note that due to the source time function of 60 s, 670-related waves can also be seen. It is interesting to note that no time window contains a pure precursory phase, but each time window contains multiple arrivals. If we looked at horizontal slices of the volumetric kernels at the depths of the discontinuities rather than vertical slices along the great circle, we would recognize the typical X-shaped kernels of the minimax phases $SS$ and its precursors.

Focusing on the boundary kernels computed for the time windows of the $SS$ precursors, we indeed see the typical X-shaped sensitivity kernels predicted for minimax phases (Neele et al. 1997; Dahlen 2005; Lawrence & Shearer 2008; Deuss 2009). These boundary sensitivity kernels are shown in Fig. 3. For the time window of the predicted $S Sylvia 670S$ (d), the sensitivity is almost

![Figure 1](https://academic.oup.com/gji/article-abstract/210/3/1965/3930845) The source (red star) and receiver (blue diamonds) configuration. The stations are at epicentral distances of 110.84°, 143.04° and 160.58°.
Figure 2. Top panel: the transverse component seismogram with the time windows of the successive kernels corresponds to a station at an epicentral distance of $110.84^\circ$. Bottom panel: the volumetric sensitivity kernels for relative shear wave speed variations in background model PREM. The source–receiver locations are shown by the grey spheres. The units are s km$^{-3}$.

3.2 $140^\circ$ distance

The sensitivity kernels for relative changes in $V_{SH}$ are shown in Fig. 4 and the ray theoretical arrivals, calculated by TauP, are shown.


Table 1. The time windows shown in Fig. 2 with respect to the centroid source time for the station at an epicentral distance of 110.84° and an event depth of 20 km. In parentheses including the ±30 s of the source time function. The names of the seismic phases and their traveltimes predicted by PREM within our time windows selections are shown as a guide to understand the volumetric kernels.

| Time windows and phase names | Traveltime (PREM) |
|-----------------------------|-------------------|
| (a) 1570–1650 s (1540–1680 s) | SKKS 1565.74 s, SKKKS 1572.49 s, Sdiff 1607.33 s |
| (b) 1650–1730 s (1620–1760 s) | S\(\vee\)220 sSdiff 1703.59 s |
| (c) 1730–1810 s (1700–1840 s) | S\(\vee\)220 sSdiff 1703.59 s, S\(\vee\)400 sSdiff 1774.59 s, S\(\vee\)220 sSdiff\(\wedge\)220 s 1799.85 s |
| (d) 1810–1880 s (1780–1910 s) | S\(\vee\)220 sSdiff\(\wedge\)220 s 1799.85 s, S\(\wedge\)670 s 1855.01 s, S\(\wedge\)670 sSdiff 1866.3 s |
| (e) 1880–1970 s (1850–2000 s) | S\(\wedge\)670 s 1855.01 s, S\(\wedge\)670 sSdiff 1866.3 s, S\(\wedge\)400 s 1930.14 s, S\(\wedge\)400 sSdiff\(\wedge\)400 s 1941.84 s |
| (f) 1970–2050 s (1940–2080 s) | S\(\wedge\)400 sSdiff\(\wedge\)400 s 1941.84 s, S\(\wedge\)220 s 1992.17 s, S\(\wedge\)400 sSdiff\(\wedge\)670 s 2033.56 s, ScS\(\wedge\)670 ScsS 2062.39 s, SS 2078.87 s |
| (g) 2050–2130 s (2020–2160 s) | S\(\wedge\)400 sSdiff\(\wedge\)670 s 2033.56 s, ScS\(\wedge\)670 ScsS 2062.39 s, SS 2078.87 s, S\(\wedge\)670 sS\(\wedge\)670 s 2125.06 s, ScS\(\wedge\)400 sScS 2156.67 s |

in Table 2. Time window (a) shows a strong sensitivity to S\(\text{diff}\) in the shape of a banana-doughnut kernel. Jumping to the S\(\wedge\)670 s window (b), we observe an underside reflection kernel due to S\(\wedge\)670 s and ScS\(\wedge\)670 ScSs. According to the TauP software, the double top-side reflected waves S\(\wedge\)400 sSdiff\(\wedge\)400 s and S\(\wedge\)220 sSdiff\(\wedge\)670 s arrive in this time window too. ScsS and various higher order Fresnel zones of Sdiff interfere, therefore the Sdiff part of the kernel mostly cancels. The top-side reflections off all the PREM discontinuities interfere but they remain clearly visible. This time window therefore has a complex sensitivity to all upper-mantle discontinuities. There is also sensitivity to a core phase, most likely SKKKS. Note that in a spherically symmetric model, although spheroidal and toroidal modes are separated, transverse components show both (e.g. Dahlen & Tromp 1998; Komatitsch & Tromp 2002a). In window (d), we see the S\(\wedge\)400 s and the ScS\(\wedge\)400 ScsSs sensitivity, but also a transition zone reverberation of S\(\wedge\)400 sSdiff\(\wedge\)670 s and its symmetric counterpart, again due to the finite source (see Table 2). We can further identify in this time window the underside reflection of the core phase SKS\(\wedge\)670 ScSs. This time window is therefore sensitive to both, the 400 km and the 670 km discontinuities. Time window (c) between the S\(\wedge\)670 s and S\(\wedge\)400 s arrivals shows a sensitivity corresponding to an underside reflection, with additional sensitivity covering most parts of the mantle. Arrivals in this time window have sometimes been attributed to a discontinuity around 520 km depth (e.g. Shearer 1990, 1996). PREM, however, has no discontinuity at this depth, despite the corresponding seismogram showing a faint arrival. Bock (1994) already reported such ‘fake’ arrivals due to a model with a simple gradient in the upper-mantle transition zone and the interaction of side-lobes during stacking. We see that this window is within the arrival of S\(\wedge\)400 sSdiff\(\wedge\)670 s and its symmetric counterpart and has a signature of arrivals in windows (b) and (d) due to the finite source in our simulations. Finally, time window (e) shows a strong SS wave sensitivity.

The boundary kernels for the precursor time windows (b) and (d) show the expected X-shaped sensitivity (Fig. 5). There is clear cross-sensitivity in the sense that the 670-precursor is sensitive to the 400-km discontinuity and vice versa. This contamination stems from the arrivals we identified in the volumetric kernels (see also Table 2).

3.3 160° distance

Fig. 6 shows the volumetric kernels for several time windows for a seismogram recorded at 160° and the corresponding TauP arrivals are given in Table 3. Time window (a) shows a strong S\(\text{diff}\) sensitivity...
and an effect of the SKKS core phase due to the 60-s duration of the source time function. Time window (b) is centred around the 670 km precursor and its sensitivity is seen in the kernel. However, there are also (partial) top-side reflections. The ray theoretical calculation identifies the likely candidate as $SV^\text{Sdiff}$ and its symmetric counterpart. However, because the 400 and 670 km top-side reflections interfere, they are not as strong in the kernel. The core phase is likely the 670 km underside reflection of SKS, namely the $SKS^\text{Sdiff}$, as theoretically predicted. Time window (d) is centred on the 400 km SS precursor and the kernel shows sensitivity to $S^\text{V}$ but also to the SKKS. The top-side reverberation seen in the kernel is stronger and is the $S^\text{Sdiff}$ due to our source time function of 60 s. Again, window (e) between the main upper-mantle SS precursors shows distinct discontinuity arrivals. These are the $ScS^\text{Sdiff}$ and $SV^\text{Sdiff}$ and their symmetric counterparts. Time window (e) contains the SS phase with a very faint signature of core phases.

The boundary kernels for this distance (Fig. 7) exhibit clean, wide X-shaped sensitivity to the topography. Although the volumetric shear wave speed kernels show cross-discontinuity sensitivity, the boundary kernels do not in this case.

4 DISCUSSION

4.1 Wave interference

From the observations made on the relative wave speed kernels in the previous section (Figs 2, 4 and 6), we consistently see that SS precursors do not arrive in isolation. In each time window, several waves interfere to produce a combined traveltime anomaly. We only showed a few examples, but this observation holds for all long-period SS precursors at other distance ranges. This poses a problem if there is a need to correct the traveltime anomalies for the 3-D velocity structure for inferring boundary topography. This point was already illustrated by Bai et al. (2012), who showed that a large amount of the delay times could not be explained by ray theoretical predictions of the arrival times of SS precursors, even in a perfectly known 3-D velocity model. The obvious reason for their observation is that the precursors account for only part of the signal in the precursor time windows. Since they did not account for the interference with the other arrivals, their predicted traveltime variations were off by 50 per cent from their measurements. Sometimes ray theory is blamed for the shortcomings in predicting delay times and banana-doughnut kernels are advocated (e.g. Zhao & Chevrot 2003; Bai et al. 2012), but theoretical sensitivity kernels for isolated seismic phases as those in Zhao & Chevrot (2003), for instance, will not solve the problem of interfering phases. Full sensitivity kernels, like the ones we showed above, are needed to account for the effect of many phases contributing to the delay times.

We would like to note that our long STF is not the reason for this observed interference, although, because of the extended STF, some extra phases might contribute. It is also worth noting that the kernels will not be substantially different because of the implemented STF, only smoothed. We checked that kernels for Green’s functions contained essentially the same information. A last point is that the kernel does not always show all the phases predicted by ray theory. This is mainly due to the focal mechanism, which has a profound effect on the kernel (e.g. Sieminski et al. 2009). It is important to realize that our kernels are exact for a given Earth model, focal mechanism and STF.

4.2 Boundary topography and/or 3-D velocity structure?

Ultimately, whether the multiple arrivals are a problem for the 3-D velocity corrections will depend on the relative importance of the contributions from the topographic and the 3-D velocity structure to the measured traveltime shift $\delta t$. To investigate this, we used the synthetic data calculated in Koroni & Trampert (2016) using the models PREM, PREM+TOPO, S20RTS and S20RTS+TOPO, where TOPO stands for the topography models of the 400 and 670 km discontinuities from Meier et al. (2009) scaled to $\pm 30$ km. We plotted histograms of the time residuals measured by cross-correlating seismograms in selected precursor time windows. More specifically, the time windows are centred around the $S^\text{V}$ arrivals calculated in PREM using TauP (Crotwell et al. 1999). The time windows have a width of 80 s and the measured delay time ($\delta t$) corresponds to the maximum of the cross-correlation. The effect of 3-D velocity structure is measured by cross-correlating the seismograms in S20RTS with those in PREM to give:

$$\delta t_{S20RTS-PREM} = t_{S20RTS} - t_{PREM}$$

where $t$ is the traveltime of the precursor in the given model shown by the superscript. To estimate the contribution from the topography model alone we define:

$$\delta t_{[\text{PREM+TOPO}]-PREM} = t_{[\text{PREM+TOPO}]} - t_{PREM}$$

for a 1-D velocity background model, and:

$$\delta t_{[\text{S20RTS+TOPO}]-S20RTS} = t_{[\text{S20RTS+TOPO}]} - t_{S20RTS}$$

for the 3-D velocity background model. The results are shown in Fig. 8 where histograms of $\delta t$ defined above are plotted for the two time windows corresponding to $S^\text{V}$ and $S^\text{Sdiff}$. For the
topography models used in our simulations, we observe that the 3-D velocity contributions are of the same order but slightly larger than those from the topography. Unless for the real earth the 3-D velocity perturbations are much smaller or the discontinuity topography variations much larger, it appears, given the multiple arrivals and their interference in each time window, that it will be difficult to make accurate corrections for the 3-D velocity structure in order to retrieve discontinuity topography.

Figure 3. Top panel: the transverse component seismogram calculated for background model PREM and the time windows around the predicted SS precursors arrivals. Bottom panel: boundary kernels for the time windows corresponding to the SS precursors. The sensitivity to the boundary topography is shown. The units for the boundary kernels are s km$^{-2}$. 
A more fundamental problem was recently identified by Koroni & Trampert (2016). They showed that the delay times of SS precursors caused by topographic variations and by 3-D velocity structure are not additive in the sense that:

$$t^{(S20RS+TOPO)} - t^{PREM} \neq t^{(S20RS+TOPO)} - t^{S20RTS}$$

- time delay due to topographic variation

$$+ t^{S20RTS} - t^{PREM}$$

- time delay due to 3-D velocity structure

(6)
or using expressions (3)–(5):

$$\delta t^{(S20RTS+\text{TOPO})-\text{PREM}} \neq \delta t^{(S20RTS+\text{TOPO})-S20RTS} + \delta t^{S20RTS-\text{PREM}}$$

To first order these delay times can be expressed using Fréchet derivatives of traveltimes in the appropriate background models (see eqs 1 and 2). If this linearizing assumption captures most of the physics, the following expressions should

Figure 5. Same as Fig. 3, but now for the receiver at an epicentral distance of 143.04°.
Figure 6. Same as Fig. 2, but now for the receiver at an epicentral distance of 160.58°.
Figure 7. Same as Fig. 3, but now for the receiver at an epicentral distance of 160.58°.
Figure 8. Histograms showing the distribution of traveltime delays caused by the relative perturbation in 3-D shear velocity (green bars) and those caused by topography in upper-mantle discontinuities in PREM (magenta line) and in S20RTS as background model (purple line). These time delays are measured by windowing around the PREM arrival of the $S^\wedge 400S$ (top panel) and $S^\wedge 670S$ (bottom panel) precursors.

hold:

$$
\delta t_{[S20RTS+TOPO] - PREM} = \int K_{r}^{PREM} d \ln V_{SH} dx^3 + \int K_{r}^{PREM} d \ln r dx^2
$$

$$
\delta t_{[S20RTS+TOPO] - S20RTS} = \int K_{r}^{S20RTS} d \ln r dx^2
$$

$$
\delta t_{S20RTS - PREM} = \int K_{r}^{PREM} d \ln V_{SH} dx^3
$$

where $\delta t$ is again the traveltime delay and the superscripts in the kernels ($K$) specify the background models in which they should be calculated. Inserting these expressions into eq. (7) we should find that

$$
\int K_{r}^{PREM} d \ln r dx^2 \neq \int K_{r}^{S20RTS} d \ln r dx^2.
$$

(9)

To check how far the inequality in expression (9) holds, we show in Fig. 9 the boundary kernels together with our sample seismogram at a distance of 143.04°, in a 1-D and 3-D background velocity model (PREM and S20RTS, respectively). The time windows used to compute the boundary kernels are selected based on the predicted arrival of the $S^\wedge S$ precursors in PREM. Although the arrival times of these seismic phases slightly differ in a 3-D model, the width of the time windows is sufficiently large to capture the energy of the $S^\wedge S$ precursors, in both the 1-D and 3-D background models (see Fig. 9 top panel). We observe that the boundary kernels indeed differ significantly. For the 400 km boundary, the kernels have the same magnitude but the characteristic $X$-shapes are slightly misplaced. Because these shapes are narrow and elongated, their integrated contribution to the delay time will be significantly different. For the 670 boundary, the kernels in this example have largely different amplitudes depending on the background model. These kernels demonstrate that the boundary sensitivity changes a lot with the background model. In particular, the kernel is almost 5 times larger in the 3-D background model. This can easily be understood as the waveform in the 3-D background model contains more energy than that in the 1-D background model (Fig. 9 top panel, window b). This means that the delay times caused by boundary topography and by 3-D velocity structure cannot be separated into two independent contributions. This effect was measured by Koroni & Trampert (2016) for all events and station configurations in their synthetic experiment. This non-linearity has nothing to do with imperfect shear velocity corrections, but with the strong change of boundary kernels with the background model. We speculate that this significant change in kernels is because of the $X$-shaped nature of the boundary kernel of the minimax phase. For minimum traveltime phases, the boundary sensitivity kernels are elliptic and there is therefore a higher chance of more overlap with changing background models. This is seen by Rutten (2017), who used synthetic seismograms in a model with CMB topography and measured traveltime differences due to CMB topography on seismic phases such as $PcP$, $ScS$, $ScP$, which are minimum traveltime phases. Following a similar process as Koroni & Trampert (2016), she found that these traveltimes were additive.
Figure 9. Top panel: the waveforms in the 1-D and 3-D background model are shown along with the selected time windows b and d, which correspond to the predicted arrivals of \( S^\wedge 400S \) and \( S^\wedge 670S \) in PREM. Boundary kernels for SS precursors in 1-D and 3-D background models: The \( S^\wedge 400S \) kernel (middle panel) and \( S^\wedge 670S \) kernel (bottom panel) are shown at different scales since we do not seek to compare them to each other, but rather to compare the 1-D to 3-D difference.
Note that most studies in fact use differential traveltimes with respect to \(SS\). Transforming the traveltime anomaly \(\delta t\) in eqs (7) and (8) into \(\Delta t\) by

\[
\Delta t = \delta t_{\text{precursor}} - \delta t_{SS}
\]

the kernels in eqs (8) and (9) then need to be replaced by:

\[
\Delta K = K_{\text{precursor}} - K_{SS}.
\]

All our arguments as well as the conclusions on additivity remain unchanged as there is no prospect of eq. (9) being violated.

By analysing our synthetic seismograms, we further noted that other parts show significantly more sensitivity to topography than the precursory wavefield. In particular, the multiples of the \(S\) wave, namely \(SS\), \(SSS\), \(SSSS\) and \(SSSSS\) express a high sensitivity to changes in the topographic structure at the 400 and 670 km boundaries. This is a consistent observation and to provide evidence, we plotted the difference of the seismograms calculated in PREM and in PREM+TOPO as a function of epicentral distance, which is shown in Fig. 10. The difference plots indicate that there are portions in the transverse component seismograms which are far more informative regarding the topographic structure of the 400 and 670 km discontinuities than the part corresponding to the \(SS\) precursors. These places of high sensitivity correspond to arrivals of multiples of \(S\) waves and are likely due to triplication in the traveltime curves caused by interaction with the upper-mantle discontinuities. The sensitivity of these phases to upper-mantle structure is well-known and has been investigated in waveform modelling studies (e.g. Grand & Helmberger 1984; Tan & Helmberger 2007; Staelhre et al. 2012). The multiples of the \(S\) wave are also taken into account in inversions for mantle structure using adjoint methods (e.g. Bozdag et al. 2016). However, this type of data has to date not been used for inferring the global topographic variations of upper-mantle discontinuities. One likely reason is the observed trade-off between shear wave speed and topographic structure (e.g. Chaljub & Tarantola 1997; Shearer 2000; Gu & Dziewonski 2002; Song et al. 2004) which may indeed hamper the inference of global topography maps of the ‘400’ and ‘670’ discontinuities. To deal with the problem of trade-off, some studies have performed joint inversions of shear wave speed and topographic structure (e.g. Gu et al. 2003; Houser et al. 2008) using mainly traveltime of \(SS\) precursors and ray theory. Based on the analyses above, we suggest that only a full waveform inversion for both shear wave speed and topographic structure using accurate forward models and exact Fréchet kernels will enable us to image the upper-mantle discontinuities. Such a full waveform imaging will clearly benefit from including multiply bouncing \(S\) waves.

5 CONCLUDING REMARKS

Taking into account the evidence seen on the volumetric and boundary sensitivity kernels, we can summarise the following points:

(i) Long-period \(SS\) precursors never arrive isolated in a long-period time window for the 110°–160° distance range investigated here. Therefore, it is impossible to make adequate 3-D velocity corrections of delay times of \(SS\) precursors, ray theory or other, unless one knows all the phases which arrive in the chosen time window.

(ii) The effect of the 3-D velocity structure and the topography on the delay times of \(SS\) precursors is likely to be of similar size and thus both effects are not easily distinguishable.
(iii) The X-shaped boundary kernels of SS precursors change significantly with the background model in which they are calculated. Therefore, the contributions of the 3-D velocity structure and the boundary topography are not additive in the delay time.

(iv) Other time windows than those of SS precursors exhibit much more sensitivity to the topography of upper-mantle discontinuities, especially those involving the triplications of multiples of the S wave. Given that these waves have not been explicitly used to map the ‘400’ and ‘670’ topography globally, we suggest that they should be used in a full waveform approach.

Because of the points made above, it seems impossible to infer boundary topography from delay times of SS precursors based on ray theoretical concepts. We suggest that a full waveform approach is essential to globally map upper-mantle discontinuities and that the 3-D mantle velocity and boundary structure of internal discontinuities should be inferred simultaneously.

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