The Phenomenology of Enhanced $b \rightarrow sg$

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Abstract

Potential hints for New Physics enhanced $b \rightarrow sg$ dipole operators are reviewed. Implications for inclusive kaon spectra, and corresponding search strategies are discussed. Remarkably, all $B$ meson rare decay constraints can be evaded. Large CP asymmetries are expected if new contributions to the dipole operators contain non-trivial weak phases. A critical comparison of $\eta'$ production in the Standard Model and in models with enhanced $b \rightarrow sg$ is presented. A Standard Model explanation of the large $B \rightarrow \eta' X_s$ rate measured by CLEO poses a challenge, evidently requiring novel non-perturbative order of magnitude enhancement of gluon anomaly mediated contributions. However, in the case of enhanced $b \rightarrow sg$ the existence of a cocktail solution is very likely.

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1 Introduction

Enhanced $b \rightarrow sg$ refers to the possibility that new physics enhances the $\Delta B = 1$ chromomagnetic dipole operators so that $\text{BR}(b \rightarrow sg) \sim 10\%$. This is in contrast to the Standard Model predictions of $\text{BR}(b \rightarrow sg) \sim .2\%$ for “on-shell” gluino, and $\text{BR}(B \rightarrow X_{\text{no charm}}) \sim 1 - 2\%$. We begin with an updated account of potential hints for enhanced $b \rightarrow sg$ from inclusive $B$ decays, then discuss implications of such a scenario for the inclusive $K$ momentum spectra and for exclusive rare $B$ decays. The possibility of large $CP$ violating asymmetries in rare decays is emphasized. The last section is devoted to a comparison of $B \rightarrow \eta'K$ and $B \rightarrow \eta'X$, in the Standard Model and in models with enhanced $b \rightarrow sg$, in light of the large rates for these processes reported by the CLEO Collaboration.

The theoretical motivation for enhanced $b \rightarrow sg$ follows from the observation that it is often associated with generation of particular combinations of quark masses or CKM mixing angles via new dynamics at TeV scales. The chirality flip inherent in new contributions to the dipole operators and quark mass matrices has a common origin, leading to direct correlations between the two. There are several known examples in which this connection can be realized without violating FCNC bounds: radiatively induced quark masses at one-loop via exchange of gluinos and squarks, or via exchange of new charge -1/3 vectorlike quarks and neutral scalars, and dynamically generated quark masses in technicolor models with techniscalars. Constraints from $\text{BR}(b \rightarrow s\gamma)$ rule out enhanced $b \rightarrow sg$ via one-loop diagrams containing a top quark and charged Higgs. The possibility of a large rate for $b \rightarrow sg$ in supersymmetric models was first noted in Ref. A detailed discussion of $b \rightarrow s\gamma$ in supersymmetric models of enhanced $b \rightarrow sg$ can be found in Ref. It is also worth mentioning that models of quark substructure with order TeV compositeness scales would be potential candidates since in this case gluon emission by an exchanged preon participating in quark mass generation might also lead to significant dipole operator contributions.

2 Hints for enhanced $b \rightarrow sg$

2.1 Charm counting

Some phenomenological consequences of enhanced $b \rightarrow sg$ for $B$ decays are a decrease in the average charm multiplicity and semileptonic branching ratio, and an increase in the kaon yields. Hints for all three are summarized below. More details are given in.

Updated inclusive $B$ to charmed hadron flavor blind branching ratios used
Table 1: Inclusive $B \rightarrow \text{charmed hadron}$, and $B \rightarrow K$ branching ratios [%]. $(c\bar{c})$ is any $c\bar{c}$ meson.

| Process          | Branching Ratio |
|------------------|-----------------|
| $\bar{B} \rightarrow D^0/D^+X$ | 62.1 ± 2.5      |
| $\bar{B} \rightarrow D^+/D^-X$  | 24.7 ± 2.0      |
| $\bar{B} \rightarrow D^+_{s}/D^0_{s}X$ | 9.9 ± 2.6      |
| $\bar{B} \rightarrow (c\bar{c})X_s$ | 2.7 ± .35      |
| $\bar{B} \rightarrow \Lambda^+_{s}/\Lambda^-_{s}X$ | 3.9 ± 2.0      |
| $\bar{B} \rightarrow \Xi^+_sX$ | 0.8 ± 0.5      |
| $\bar{B} \rightarrow \Xi^0_sX$ | 1.2 ± 0.9      |

to obtain the $B$ decay charm multiplicity at the $\Upsilon(4S)$ are given in Table 1. For the $D/D_s$ yields we have averaged the ARGUS, CLEO 1.5 and CLEO measurements. The $D^0$ and $D^+$ yields have been rescaled to the world averages determined in Ref.\[3\], BR($D^0 \rightarrow K^-\pi^+$) = 3.87 ± .09% and BR($D^+ \rightarrow K^-\pi^+\pi^0$) = 8.8 ± .6%, respectively. The $D_s$ yields correspond to the PDG average\[3\], BR($D_s \rightarrow \phi\pi$) = 3.6 ± .9%. The charmed baryon and charmonium yields are those recently used by the CLEO collaboration\[14\]. The resulting world-average $B$ decay charm multiplicity at the $\Upsilon(4S)$ is

$$n_c^{exp} = 108 \pm 4.8\%.$$ \hspace{1cm} (1)

Using only the recent CLEO $D/D_s$ yields\[14\] rather than the world averages gives $n_c = 112 \pm 5.3\%$. We will see that next-to-leading order Standard Model QCD predictions are somewhat larger.

The flavor specific charmed hadron branching ratios in Table 2 are obtained by combining the relative flavor specific yields with the corresponding flavor blind yields in Table 1. The charmonium yield is included to obtain the $\Upsilon(4S)$ world-average

$$\text{BR}^{exp}(\bar{B} \rightarrow X_{c\bar{c}}) = 19.4 \pm 3.5\%.$$ \hspace{1cm} (2)

Using only the recent CLEO $D/D_s$ flavor blind yields rather than the world averages gives $\text{BR}(\bar{B} \rightarrow X_{c\bar{c}}) = 21.2 \pm 3.6\%$. We'll see that this is consistent with next-to-leading order predictions. $\text{BR}(\bar{B} \rightarrow X_{c\bar{u}\bar{d}})$ can also be determined purely experimentally by combining flavor blind and flavor specific charmed hadron yields\[6\], giving

$$\text{BR}^{exp}(B \rightarrow X_{c\bar{u}\bar{d}}) = 45.7 \pm 6.6\%,$$ \hspace{1cm} (3)

\[a\] In the former we have added the recent CLEO measurement $\text{BR}(D^0 \rightarrow K^-\pi^+) = 3.81 \pm .22\%$, obtained from partial reconstruction of $\bar{B} \rightarrow D^{*+}X\ell^-\nu$.\[16\]
Table 2: Inclusive flavor tagged $B$ decay branching ratios [%]. $D$ denotes $D^0$ or $D^+$, and similarly for $\overline{D}$.

| $T$   | $\frac{B(B \to TX)}{B(\overline{B} \to T\bar{X})}$ | $B(\overline{B} \to T\bar{X})$ | $B(B \to TX)$ |
|-------|-------------------------------------------------|---------------------------------|----------------|
| $D$   | 0.100 ± 0.031                                     | 7.9 ± 2.2                       | 78.8 ± 3.7     |
| $D_s^-$ | 0.21 ± 0.10                                     | 1.7 ± 0.8                       | 8.2 ± 2.6      |
| $\Lambda_c^+$ | 0.19 ± 0.14                                 | 0.6 ± 0.5                       | 3.3 ± 1.7      |

also consistent with next-to-leading order predictions. The result using only the CLEO $D/D_s$ yields is essentially the same. Similarly, one finds

$$\text{BR}(\overline{B} \to X_{c\bar{u}d} \to DX/D_sX) = 41.0 \pm 6.2\%,$$

which is used to normalize estimates of kaon production from $s\bar{s}$ popping.

The charm multiplicity and $\text{BR}(\overline{B} \to X_{c\bar{c}s})$ can be used to bound $\text{BR}(\overline{B} \to X_{no\,charm})$ via the relation

$$n_c = 1 + \text{BR}(B \to X_{c\bar{c}s}) - \text{BR}(B \to X_{no\,charm}).$$

The above determinations of $n_c$ and $\text{BR}(B \to X_{c\bar{c}s})$ give

$$\text{BR}(B \to X_{no\,charm}) = 11.4 \pm 5.9\%.$$

Using only the recent CLEO $D/D_s$ yields gives 9.3 ± 6.4%. For comparison, a recent NLO analysis gives $\text{BR}(B \to X_{no\,charm}) = 1.5 \pm 0.8\%$ in the Standard Model. Bounds on $\text{BR}(B \to X_{sg})$ follow by subtracting $\approx 1\%$ to account for $b \to u$ transitions. This is a potential hint for enhanced $b \to sg$, although it is also consistent with no $b \to sg$ at the 2$\sigma$ level. In the error bars approximately ±3.4% is due to uncertainties in the $D$ decay branching fractions, i.e., $D^0 \to K^-\pi^+$, $D^+ \to K^+\pi^-\pi^+$, and $D_s \to \phi\pi$. The remainder will be considerably reduced at the $B$ factories, so that this method will provide an important measurement of the charmless branching ratio.

Finally, an upper bound on $\text{BR}(B \to X_{sg})$ can be obtained from the recent determination by CLEO of the ratio of ratios

$$R \equiv \frac{\Gamma(B \to T\bar{X})}{\Gamma(B \to all)} / \frac{\Gamma(B \to D\ell^+\nu)}{\Gamma(B \to X\ell^+\nu)} = 0.901 \pm 0.034 \pm 0.015,$$

using the relation

$$R = 1 + |V_{ub}/V_{cb}|^2 + (\text{BR}(B \to D_s^-\ell^+X_\nu) - \text{BR}(B \to D_s^-X_\nu))$$
\[ \text{BR}(B \to (c\bar{c})X) = \text{BR}(B \to \Lambda_c^+ X) = \text{BR}(B \to \Xi_c^{0+} X) = \text{BR}(B \to X_{sg}) \]  

Unlike in the previous method, this bound does not depend on \( \text{BR}(D^0 \to K\pi) \). Taking \(.008 \pm .003\) and \(-.01 \pm .005\) for the second and third terms, and using the most recent CLEO charmonium, \( \Xi_c^{0+} \), and flavor specific \( \Lambda_c^+ \) yields, which are given in Tables 1 and 2, we obtain \( \text{BR}(B \to X_{sg}) = 1.7 \pm 4.4\% \), or

\[ \text{BR}(B \to X_{sg}) < 9.0\% \quad @90\%\text{c.l.} \]  

In Ref. older values for the charmonium and charmed baryon yields were used, leading to an upper bound of 6.8\%. From the measured lower ratio in \( R \) a rather indirect determination of \( \text{BR}(D^0 \to K^-\pi^+) \) (they are inversely proportional) is possible, yielding \( 3.69 \pm 0.2\% \) which is lower than but consistent with the world average for direct measurements. Motivated by the reasonable prejudice that the latter gives a more reliable determination at this time, it is worth mentioning that a 1\( \sigma \) upwards shift in the lower ratio in \( R \) would simultaneously reproduce the central value for direct measurements of \( \text{BR}(D^0 \to K^-\pi^+) \) and increase the central value for \( \text{BR}(B \to X_{sg}) \) to 6\%. It will be very interesting to see what this method ultimately yields for \( \text{BR}(B \to X_{sg}) \) at the \( B \) factories.

### 2.2 Kaon counting

It is possible to check whether the potentially large charmless yield is due to \( b \to s \) transitions by comparing the measured flavor blind and flavor specific inclusive \( \overline{B} \to XK \) branching ratios with the kaon yields from intermediate charmed states (see Refs. for details). The latter are divided into two classes: kaon yields which are essentially determined by experiment and those which have to be estimated. For example, the largest known contributions are decays of intermediate \( D/D_s \), which have been obtained by combining inclusive \( \overline{B} \to DX/D_s X \) and PDG \( D/D_s \to KX \) branching ratios. Sizable 4.4\( \sigma \) and 5.6\( \sigma \) excesses remain in the total \( K^- \) and \( K^+/K^- \) yields, respectively, compared to known contributions.

The most important contribution to be estimated is \( s\bar{s} \) popping in \( \overline{B} \to X_{c\bar{c}d} \), leading to final states of the form \( DK\overline{K}X \) and \( D_s\overline{K}X \). The additional kaon yields per \( \overline{B} \to X_{c\bar{c}d} \to DX/D_s X \) decay have been estimated using a JETSET 7.4 string fragmentation model for \( B \to X_{c\bar{c}d} \). The total probability of \( s\bar{s} \) popping in such decays is found to be \( \approx 14 \pm 3\% \). Crude but generous

\( b \) \( s\bar{s} \) popping in other processes, e.g., \( \overline{B} \to \Lambda cX \) or \( \overline{B} \to X_{c\bar{s}d} \), can be safely neglected due to small rates for these processes or phase space suppression.
estimates for kaon production from \( \Lambda_c \), \( \Xi_c \) and charmonium decays make up the rest.

Including the above estimates gives [%]

\[
\begin{align*}
BR(\bar{B} \to K^- X) - BR(\bar{B} \to X_c \to K^- X) &= 16.8 \pm 5.6, 15.2 \pm 5.8 \\
BR(\bar{B} \to K^+ X) - BR(\bar{B} \to X_c \to K^+ X) &= 1.2 \pm 4.2, 0.8 \pm 4.2 \\
BR(\bar{B} \to K^+/K^- X) - BR(\bar{B} \to X_c \to K^+/K^- X) &= 17.8 \pm 5.3, 15.8 \pm 5.7 \\
BR(\bar{B} \to K^0/K^0 X) - BR(\bar{B} \to X_c \to K^0/K^0 X) &= 4.6 \pm 6.9, 2.9 \pm 7.2
\end{align*}
\]

The second set of numbers is obtained using only the recent CLEO \( B \to DX/D_s X \) branching ratios. We see that a 3\( \sigma \) \( K^- \) excess remains. The \( K^- \) excess is also reflected in the 3\( \sigma \) total charged kaon excess. The \( K^0/K^0 \) result is consistent with either no kaon excess, or sizable kaon excess.

The kaon excesses are consistent with expectations from enhanced \( b \to s g \).

Alternatively, if the excesses are due to underestimates of kaon yields then the most likely culprit is \( s \bar{s} \) popping. It is important to note that this can be determined at the \( B \) factories via measurements of \( BR(\bar{B} \to D K K X) \) and \( BR(\bar{B} \to D_s K X) \). A measurement of the additional kaon spectra in these decays is also extremely important since this is one of the largest uncertainties in determining the Standard Model inclusive kaon spectra. About 90\% of the uncertainty in the charged kaon excesses in Eq. 10 is due to measurements of \( BR(B \to K X) \), \( BR(B \to DX) \), and \( BR(D \to K X) \). Fortunately, the first two branching ratios will be measured much more precisely at the \( B \) factories.

2.3 \( n_c \) and \( BR(B \to X_{c\ell\nu}) \)

In Fig. 1 predictions of the Standard Model and models with enhanced \( b \to s g \) for \( n_c \), \( BR(\bar{B} \to X_{c\bar{s}s}) \), \( BR(\bar{B} \to X_{c\bar{u}d}) \), and \( BR(B \to X_{c\ell\nu}) \) are compared with their measured values at the \( \Upsilon(4S) \). For the measured semileptonic branching ratio we take:

\[
BR^{exp}(B \to X_{c\ell\nu}) = 10.23 \pm .39,
\]

which is the average of the nearly model-independent ARGUS and CLEO dilepton charge correlation measurements. The theoretical inputs include full on-shell scheme next-to-leading order QCD corrections and \( O(1/m_b^2) \) HQET corrections to the tree-level \( b \to c \) parton model decay widths. Next-to-leading order scheme-independent corrections to the \( \Gamma(b \to c\bar{s}s) \) \textit{penguin} contributions are also included. The remaining scheme-dependent corrections should be an order of magnitude smaller. The \( b \to u \) transitions have also been taken into account to NLO but the \( b \to s \) transitions have
been neglected. As in Ref. 26 the quark pole masses are varied in the range 4.6 \( < m_b < 5.0 \) and .25 \( < m_c/m_b < .33 \), and the renormalization scale is varied from \( m_b/4 < \mu < m_b \).

According to Figs. 1a,c,e, \( n_{exp} \) is lower than the Standard Model range for all \( \mu \) but, as also pointed out in Ref. 27, \( BR_{exp}(B \to X_{c\bar{s}s}) \) and \( BR_{exp}(B \to X_{c\bar{u}d}) \) are consistent with the Standard Model ranges. In particular, there is no indication that the discrepancy in \( n_c \) is due to poor theoretical control over hadronic decays beyond the sources of uncertainty already considered above, e.g., large deviations from local parton-hadron duality. \( BR_{exp}(B \to X_{c\ell\nu}) \) is consistent with the Standard Model range at low values of \( \mu \). However, we caution that whereas a low renormalization scale appears to be justified for the semileptonic decay widths 28 this may not be the case for the hadronic decay widths entering the branching ratio 11.

In Figs. 1b,d,f we take \( BR(X_{sg}) \approx 10\% \). \( n_c \) becomes consistent with experiment, \( BR_{exp}(B \to X_{c\bar{s}s}) \) and \( BR_{exp}(B \to X_{c\bar{u}d}) \) remain consistent with experiment, and \( BR_{exp}(B \to X_{c\ell\nu}) \) can now be reproduced at larger values of \( \mu \), i.e., without requiring large perturbative or non-perturbative QCD corrections.

### 3 K production from \( B \to X_{sg} \)

The main contribution of enhanced \( b \to sg \) to kaon production is fragmentation via soft \( q\bar{q} \) popping. In the \( b \) quark rest frame the gluon and \( s \) quark emerge back to back with energy \( m_b/2 \). In the string picture a string connects the \( s \) and spectator quarks and the gluon is a kink in the string which carries energy and momentum. The ensuing fragmentation is modeled using a JETSET 7.4 Monte Carlo with recent DELPHI tunings 29. The large energy released in \( b \to sg \) decays should lead to high multiplicity final states, or soft kaon momentum spectra. This expectation is confirmed by the Monte Carlo results, as we’ll see below. The number of kaons per \( \overline{B} \to X_{sg} \) decay produced in Monte-Carlo is .67 (\( K^- \)), .19 (\( K^+ \)), .62 (\( K^0 \)), and .15 (\( K^0 \)), so it is clear that enhanced \( b \to sg \) would significantly reduce the charged kaon excesses.

Hard \( q\bar{q} \) fragmentation of the gluon becomes important at large \( K \) momenta. In this case the decay \( b \to sg \to s\bar{q}q \) can be described by an effective four quark operator. The corresponding contribution to fast \( K \) production can be estimated using factorization, i.e., the meson is formed from the primary quarks in the decay. We return to the factorization model when discussing direct \( CP \) violation.

\(^c\) The transition from 5 to 4 flavors is taken into account for \( \mu < m_c \).
Figure 1: SM NLO predictions ($\alpha_s(M_Z) = .117$) for (a) $n_c$, (c) BR($B \to X_{cs\bar{s}}$), (e) BR($B \to X_{c\bar{s}d}$) vs. BR($B \to X_{c\ell\bar{\nu}_\ell}$). The impact of BR($b \to s g$) = 10% is shown in (b), (d), (f), respectively. Left (right) borders are for $\mu = m_b/4$ ($\mu = m_b$). Dashed lines are for $\mu = m_b/2$. Bottom (top) borders are for $m_b = 5.0$, $m_{c}/m_b = .33$ ($m_b = 4.6$, $m_{c}/m_b = .25$) in (a) - (d). This is reversed in (e), (f). The crosses are the experimentally determined ranges at the $\Upsilon(4S)$. 
Figure 2: \( \text{BR}(B \rightarrow K_sX) \) vs. \( p_{K_s} \) [GeV]. Branching ratios are for 0.1 GeV bins except CLEO upper limits. (a) ARGUS data (crosses), SLD/CLEO Monte Carlo (upper solid), Monte Carlo for \( \text{BR}(B \rightarrow X_{sg}) = 10\% \) (lower solid) and 15\% (dashed). (b) fast kaon spectra: CLEO 90\% CL UL's for 2.11 < \( p_{K_s} \) < 2.42, 2.42 < \( p_{K_s} \) < 2.84 (dot - dashed), SLD Monte Carlo (thick solid), Monte Carlo for \( \text{BR}(B \rightarrow X_{sg}) = 10\% \) (solid), 15\% (dashed).

In Fig. 2 inclusive \( K_s \) momentum spectra (in the \( \Upsilon(4S) \) rest frame) generated by the \( B \rightarrow X_{sg} \) and (SLD tuned) CLEO \( B \rightarrow X_c \) Monte Carlos are compared with the measured spectrum. In the \( b \rightarrow sg \) Monte Carlo the \( b \) and spectator quark momenta are modeled using the Gaussian distribution of Ref. 32, with \( p_F = 250 \) MeV. Parton showers are also included. For those momenta where most \( b \rightarrow sg \) kaons are produced the expected ratio of signal to Standard Model background is \( \approx 1 : 5 - 1 : 10 \). Clearly, resolving the presence of enhanced \( b \rightarrow sg \) at these momenta directly would be a very difficult task. A vertexing veto of charm would have to be extremely efficient to significantly enhance the \( b \rightarrow sg \) component. Perhaps the relative back-to-back geometry of signal events versus the more spherical geometry of background events could help discriminate between the two. A related question is to what extent do sphericity event shape cuts used to distinguish between continuum and \( \Upsilon(4S) \) decays, such as those commonly employed by CLEO, bias against \( b \rightarrow sg \) events.

A promising strategy is to search for kaons from enhanced \( b \rightarrow sg \) at higher momenta, e.g. \( p_{K_s} \gtrsim 1.8 \) GeV. A significant kaon signal at still higher momenta, e.g., above 2.1 GeV, where the background from intermediate charm states is highly suppressed would provide an unambiguous signal for charmless \( b \rightarrow s \) transitions. Unfortunately, because of very large theoretical uncertainties above 2.1 GeV it would be very difficult to determine whether such a
signal is due purely to Standard Model penguins, or intervention of enhanced $b \to sg$, unless it happened to be close to the current upper bound $34$. According to Fig. 2, the ratio of enhanced $b \to sg$ signal to Standard Model background for $p_K \gtrsim 1.8 \text{ GeV}$ is expected to be $\sim 1 : 1$. Although branching ratios are reduced to the $10^{-3}$ level, high statistics analysis will be possible at the $B$ factories. The background at large momenta can be determined experimentally with little theoretical input. For example, the dominant $B \to D \to K$ contributions can be obtained directly by folding measured $B \to DX$ and MARK III $D \to KX$ inclusive momentum spectra. In fact, fast kaons mainly originate from the lowest multiplicity $D$ decays, e.g., $D \to K\pi, K\rho, K^*\pi$, which are well measured, and the Cabbibo suppressed decays $B \to DK, D^*K$, which will be well measured in the future. Of course, the $D$ spectra will be determined to very high precision at the $B$ factories. Furthermore, the poorly known $s\bar{s}$ popping and $B \to D_s \to K$ contributions should be much softer and are therefore unlikely to contribute significantly. A vertexing veto of charm of modest efficiency, which should certainly be available at the $B$ factories, could significantly enhance the $b \to sg$ component above $1.8 \text{ GeV}$. It is worth mentioning that kaon momentum spectra obtained from a similar JETSET Monte Carlo for Standard Model $b \to s\bar{q}q$ decays are similar in shape (again the large energy release leads to high multiplicity decays or soft spectra). However, the parton level branching ratio is $\approx 1\%$ so the kaon yields are an order of magnitude smaller than in the case of enhanced $b \to sg$.

Fast kaon searches can be carried out at the $Z$ as well by studying high $p_T$ $K^\pm$ production. According to Fig. 3, the expected signal to background ratio for $p_T \geq 1.8 \text{ GeV}$ is again $\sim 1 : 1$. The SLD Collaboration has made a preliminary analysis using its ’93-’95 and ’96-’97 data samples. Unfortunately, the statistics are still too low to reach any definitive conclusions. Hopefully the situation will be further clarified at the 1998 summer conferences where a more extensive analysis including more recent data will be presented. DELPHI has also searched for an excess of charged kaons in the $p_T$ spectrum using their high statistics samples. However, to date DELPHI has pursued a different strategy, attempting to fit the entire measured spectrum with Monte Carlo $B \to X_{sg}$ and $B \to X_c$ components kept free. Unfortunately, this procedure suffers from too much model-dependence at the present time to reach definitive conclusions. As is clear from Fig. 3, the shapes of the two components are not expected to differ dramatically over a wide range of momenta, so they would have to be known fairly accurately in order to extract a $b \to sg$ contribution.

A significant excess relative to a well established $B \to X_c \to K$ background rate at large kaon momenta would provide model-independent evidence for new

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\[d\text{I thank Mark Convery for discussions of this point.}\]
physics. The model-dependence would then enter when attempting to extrapolate the excess to lower momenta via the $B \to X_{sg}$ Monte Carlo, in order to determine the total charmless rate. The Monte Carlo results for fast kaon production can only be regarded as order of magnitude estimates. However, we should mention that JETSET successfully describes the inclusive $K^\pm$ and $K^0$ momentum spectra measured in $Z$ decays\cite{bbo957,bbo960}, including the separately measured spectra for $Z$ decays to light ($u,d,s$) flavors\cite{bbo957,bbo960}. Unfortunately, useful data at $x_p > .8$ ($x_p \equiv p_K/p_{beam}$) is not currently available. JETSET also reproduces the measured portion of the $K^\pm$ spectrum in the $e^+e^-$ continuum near the $Y$ resonances\cite{bbo957}, however in this case there is no data available at $x_p > .5$. Precision high $x_p$ data will be very important for further tuning of Monte Carlo fragmentation parameters relevant to fast kaon production in $b \to s$ decays, and should be available from $B$ factory continuum studies. Spectra for kaons produced in the continuum at a charm factory, e.g., $\sqrt{s} \sim 4 \, GeV$, would of course be extremely useful. We also note that the $B \to X_{sg}$ Monte Carlo fast kaon yields obtained with default JETSET tunings are approximately 30% smaller than those obtained with the recent DELPHI tunings\cite{bbo957}, which gives an idea of the large systematic errors involved. Finally, fast kaon production is extremely sensitive to the modelling of Fermi motion. For example, setting the Fermi momentum parameter $p_F = 250 \, MeV$ in the Gaussian model of Ref.\cite{bbo957} reduces production of kaons with momenta above $1.8 \, GeV$ by about 50% compared to production without Fermi motion. More sophisticated treatments of Fermi motion are therefore required.

4 CP violation

Enhanced dipole operator coefficients can carry new CP violating weak phases. Furthermore, for BR($b \to sg) \sim 10\%$ the dipole amplitudes for rare hadronic decays are of same order as the Standard Model amplitudes\cite{bbo957}. Since the strong interaction phases associated with the two amplitudes must in general differ, their interference can lead to large direct CP asymmetries ($A^\text{dir}_CP$). We present factorization model results for $B^{\pm} \to \phi K^{\mp}, \phi X_s, B^{\pm} \to K^0\pi^{\pm}$ and $B^0 \to K^{\pm}\pi^{\mp}$. For the moment we will ignore soft final state interactions (FSI), which actually are expected to be important in $B$ decays\cite{bbo957,bbo960}. In the absence of FSI the Standard Model contributions for the first three modes above are dominated by the penguin $b \to s\bar{s}s$ and $b \to s\bar{d}d$ transitions, resulting in small CP violating asymmetries\cite{bbo957,bbo960} of order 1\%.

The relevant $\Delta B = 1$ effective weak Hamiltonian takes the form\cite{bbo957,bbo960,book}

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V^*_{us} \sum_{i=1}^{2} c_i Q^i_{us} - V_{tb} V^*_{ts} \left( \sum_{i=3}^{11} c_i Q_i + c'_1 Q'_{11} \right) \right]. \quad (12)
$$
Figure 3: $B(\bar{B} \to K^{\pm}X)$ vs. kaon $p_T$ [GeV]. Branching ratios are for 0.1 GeV bins. (a) SLD/CLEO Monte Carlo (upper solid), Monte Carlo for $B(\bar{B} \to X_{sg}) = 10\%$ (lower solid) and 15\% (dashed) with $p_T = 250$ MeV and showers included. (b) Fast kaon spectra: SLD Monte Carlo (thick solid), Monte Carlo for $B(\bar{B} \to X_{sg}) = 10\%$ (solid) and 15\% (dashed).

The flavor structures of the current-current, QCD penguin, and electroweak penguin operators are, respectively, $Q_{u1,2}$, $Q_{3,\ldots,6} \sim \bar{s}u\bar{d}b$, and $Q_{7,\ldots,10} \sim \bar{s}b\sum\bar{q}q'$, where the sum is over light quark flavors. The chromomagnetic dipole operators are given by

$$Q_{11} = \frac{g_s}{16\pi^2} m_b(\mu) \bar{s}\sigma_{\mu\nu} R^{\alpha} b G_\alpha^\mu \nu, \quad Q_{11}' = \frac{g_s}{16\pi^2} m_b(\mu) \bar{s}\sigma_{\mu\nu} L^{\alpha} b G_\alpha^\mu \nu. \quad (13)$$

They are included in the factorization model by allowing the gluon to go off shell and turn into a quark-antiquark pair. We parametrize the dipole operator Wilson coefficients as

$$c_{11} = -|c_{11}| e^{i\theta_{11}}, \quad c_{11}' = -|c_{11}'| e^{i\theta_{11}'}.$$  

(14)

In the standard model $c_{11}(m_b) \approx -0.15$, and $c_{11}'$ is a factor $m_s/m_b$ smaller, hence negligible. BR($b \to sg$) $\sim 10\%$ requires $|c_{11}|^2 + |c_{11}'|^2$ to be enhanced by nearly an order of magnitude. Strong phases originating at NLO from $c\bar{c}$ rescattering have been taken into account in the Standard Model penguin amplitudes using the NLO scheme-independent effective Wilson coefficient formalism of Refs.\textsuperscript{48,52}. For the numerical inputs we choose $\alpha_s(m_b) = 0.212$, $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, $m_s(m_b) = 0.1$ GeV, and $N_{c,ff} = 10$ for the effective number of colors parametrizing non-factorizable corrections (called $1/\xi$ in Ref.\textsuperscript{53}). We also take $\rho = 0.11$ and $\eta = 0.33$ for the Wolfenstein parameters, $\mu = m_b$ for the renormalization scale, and $q^2 \approx m_b^2/2$ for the
square of the virtual gluon momentum entering the penguin and dipole am-
plitudes. For the $B \to K, \pi$ form factors at zero momentum transfer we take $F_{1,0}^K(0) = F_{1,0}^\pi(0) = 0.33$ in the standard parametrization of Ref. [53]. A dipole behaviour is taken for the $q^2$ dependence of $F_1$ and a monopole behaviour is taken for $F_0$, according to the lattice results given in Ref. [54].

In Fig. 4 branching ratios (CP averaged) and direct CP asymmetries ob-
tained with enhanced $c_{11}$ and $c'_{11} = 0$ are compared to Standard Model predictions. Also included are CLEO measurements or upper bounds for the branching ratios [55−58]. In the case of $B \to \phi X_s$ the bound has been obtained for $p_\phi > 2.1$ GeV. The large dependence on $\theta_{11}$ confirms that there can be substantial destructive or constructive interference between the penguin and dipole amplitudes. Given the large uncertainties in the factorization model estimates, i.e., sensitivity to numerical inputs, non-factorizable contributions (parametrized by $N_c$), and absence of FSI, it is clear that the experimental branching ratio constraints can be satisfied for a significant range of $\theta_{11}$, even if $\text{BR}(b \to sg) \approx 15\%$. Although the charmless branching ratio with enhanced $b \to sg$ is an order of magnitude larger than in the Standard Model it is peaked at very low $q^2$, i.e., the “on-shell” gluon limit. At larger values of $q^2$ relevant to two-body or quasi two-body rare decays the enhanced $b \to sg$ amplitudes are reduced to the level of the Standard Model amplitudes, as is well illustrated by a parton level Dalitz plot analysis of $b \to s\bar{q}q$ decays [11]. We should mention that in the Standard Model the interference of the dipole and penguin amplitudes is destructive for the charmless $b \to s\bar{q}q$ transitions at the parton level, leading to a reduction in all corresponding factorization model decay rates by $\sim 10 − 20\%$. In general in models with enhanced $b \to sg$ there is no reason to expect $c_{11} \gg c'_{11}$ in the absence of extra flavor symmetries. As an illustrative example, in Fig. 5 we show the branching ratio dependences on $\theta_{11}$, $\theta'_{11}$ for $|c_{11}| = |c'_{11}|$ and $\text{BR}(b \to sg) \approx 10\%$. Thus, we see that the general case where both $Q_{11}$ and $Q'_{11}$ are enhanced is even less constrained.

It is clear from Fig. 4 that in the case of the semi-inclusive $\phi X_s$ and exclusive $\phi K^{\pm}, K^0\pi^{\pm}$ modes enhanced $b \to sg$ can lead to sizable direct CP asymmetries, which are easily more than an order of magnitude larger than the naive $O(1\%)$ asymmetries expected in the standard model when ignoring soft FSI. Large contributions to the $B \to \phi K_s$ time dependent CP asymmetry would also arise. However, as has been discussed recently [44−47], the impact of FSI on $A_{CP}^{\phi K_s}(B \to K \pi)$ is likely to be quite sizable. For example, in Ref. [11] a crude model based on Regge phenomenology was considered in which only two-body pseudoscalar intermediate states were taken into account.

\footnote{In Ref. [11] the CP asymmetries were incorrectly given with opposite sign due to a sign error in the $c\bar{c}$ rescattering strong phases.}
Figure 4: $10^5 \text{BR}(B \to \phi X_s, \phi K^\pm, K^0\pi^\pm, K^\pm\pi^\mp)$ (CP averaged) vs. $\theta_{11}$ and corresponding direct CP asymmetries vs. $\theta_{11}$ for BR($b \to s g$) $\approx$ 15% (dotted curves), 10% (solid curves), 5% (dashed curves). Horizontal solid lines are the corresponding Standard Model branching ratios and asymmetries. Horizontal dot-dashed lines are CLEO 90% c.l. branching ratio upper limits for $B \to \phi K^\pm, \phi X_s$ and measured 1$\sigma$ ranges for $B \to K^0\pi^\pm, K^\pm\pi^\mp$. 
Rescattering contributions of the doubly Cabbibo suppressed $b \to u\bar{s}s$ current-current operators to the $B^+ \to K^0\pi^+$ amplitude were found to be of order 10%. Furthermore, since almost all inelastic contributions were neglected the actual contributions could be significantly larger. It is therefore not inconceivable that in the Standard Model $A_{CP}^{dir}(B^+ \to K^0\pi^+)$ could be several tens of percent. It stands to reason that similar rescattering contributions to the $B^+ \to \phi K^+$ amplitude could also be of order 10%, again leading to sizable asymmetries.

Clearly, large direct CP violation in the exclusive modes $B^\pm \to \phi K^\pm, K^0\pi^\pm$ would not provide an unambiguous signal for New Physics. Fortunately, it is possible to use flavor $SU(3)$ tests to bound the magnitudes of FSI contributions and Standard Model asymmetries in these modes along the lines suggested in [63] for the $B \to \phi K_s$ time-dependent asymmetry. A bound which could turn out to be particularly useful since it only depends on the ratio

$$R_K = \frac{BR(B^+ \to K^+K^0) + BR(B^- \to K^-K^0)}{BR(B^+ \to K^0\pi^+)+BR(B^- \to K^0\pi^-)}$$

(15)

is

$$|A_{CP}^{dir}(B^\pm \to K^0\pi^\pm)| < 2\lambda\sqrt{R_K (1 + R_{SU(3)})} + O(\lambda^3, \lambda^2 R_{SU(3)}).$$

(16)

$\lambda \approx 0.22$ is the Wolfenstein parameter, and $R_{SU(3)}$ parametrizes $SU(3)$ violation and is typically of order 20% - 30%. The analogous bound for $A_{CP}^{dir}(B^\pm \to \phi K^\pm)$ follows by substituting for $R_K$ the ratio $[(\sqrt{BR(B^\pm \to K^{0(*)}K^\mp}) + \sqrt{BR(B^\pm \to \phi\pi^\mp)})^2/BR(B^\pm \to \phi K^\pm)]$. Direct CP asymmetries in excess of these bounds would provide a clear signal for New Physics. It should be noted that analogous tests for New Physics are possible in other modes, e.g., $B^\pm \to \phi K^{*\pm}, K^{0*}\pi^\pm$. Alternative strategies employing $B \to K_K$ decays to constrain FSI contributions to $B \to K\pi$ are discussed in Ref. [64].

5 $B^\pm \to \eta' K^\pm$ and $B \to \eta' X_s$

The CLEO collaboration has reported an exclusive rate

$$\mathcal{B}(B^\pm \to \eta'K^\pm) = (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5},$$

(17)

and a semi-inclusive rate

$$\mathcal{B}(B \to \eta'X_s) = (6.2 \pm 1.6 \pm 1.3^{+0.0}_{-1.5}) \times 10^{-4} \ (2.0 < p_{\eta'} < 2.7 \text{ GeV}).$$

(18)

The experimental cut on $p_{\eta'}$ is beyond the kinematic limit for most $b \to c$ decays and corresponds to a recoil mass $m_{X_s} < 2.5$ GeV in the laboratory frame.
Figure 5: $10^3 \text{BR}(B \rightarrow \phi X_s, \phi K^\pm, K^0\pi^\pm, K^\pm\pi^\mp)$ vs. $\theta_{11}$ (left axes) and $\theta'_{11}$, for $|c'_{11}| = |c_{11}|$ and $\text{BR}(b \rightarrow s\phi) \approx 10\%$. Also included are $10^3 \text{BR}(B^\pm \rightarrow \eta' K^\mp)$ and the quasi two-body contribution to $10^3 \text{BR}(B \rightarrow \eta' X_s)$ in the factorization model, as discussed in Section 5. All branching ratios are $CP$ conjugate averages.
There is no evidence for η′K* modes in the inclusive analysis, with most events lying at large recoil mass, \( m_{X_s} > 1.8 \) GeV. The surprisingly large branching ratios have prompted many phenomenological papers on η′ production, some of which are listed here. A consensus has emerged that the exclusive branching ratio measurement can be accounted for in the Standard Model in the factorization approach. However, the inclusive branching ratio is more problematic. We will discuss the impact of enhanced \( b \to s g \) on both rates in the factorization model. In the inclusive case we will also discuss the gluon anomaly mediated subprocess \( b \to s g^* \to η' s g \), first investigated in Ref. 74, and potential long-distance contributions. The results presented here are part of a collaboration with Alexey Petrov.

5.1 \( B^\pm \to η' K^\pm \): Factorization Model Contributions

We adopt the two-angle mixing formalism for η - η′ mixing:

\[
|η⟩ = \cos θ_8 |η_8⟩ - \sin θ_8 |η_0⟩, \quad |η'⟩ = \sin θ_8 |η_8⟩ + \cos θ_0 |η_0⟩.
\]

Our numerical inputs for the mixing angles and SU(3) octet and singlet axial vector current decay constants are the best-fit values found in Ref. 81:

\[
θ_8 = -22.2^\circ, \quad θ_0 = -9.1^\circ, \quad f_8 \frac{f_π}{f_π} = 1.28, \quad f_0 \frac{f_π}{f_π} = 1.20.
\]

For simplicity, we set \( ⟨η'|\bar{c}γ_\muγ_5c|0⟩ = 0 \) as it has become clearer that the intrinsic charm content of the η′ is not large enough to play a crucial role in η′ production. The anomaly is taken into account in the matrix element \( ⟨η'|\bar{s}γ_\muγ_5s|0⟩ \). SU(3) symmetry is imposed to relate the \( B \to η' \) form factors \( F_0', F_1' \) at zero momentum transfer to \( F_0^π(0) \), as in Ref. 54. For the \( q^2 \) dependences of \( F_0', F_1' \) we again use the monopole and dipole behaviours given in Ref. 54. Other inputs entering our factorization model calculations have been discussed in Section 4.

In Fig. 6, we compare BR(\( B^\pm \to η' K^\pm \) (CP averaged) and \( A_{CP}^{\text{IR}}(B^\pm \to η' K^\pm) \) obtained with BR(\( b \to s g \) \( \approx 10\% \) to the corresponding Standard Model values in the factorization model. The observed ±1σ range for the branching ratio is also included. Also see Fig. 5 for a plot of the branching ratio in the \( (θ_11, θ'_11) \) plane, given \( |c_{11}| = |c'_{11}| \). One feature that stands out immediately
is the sensitivity of the exclusive rate to the current mass \(m_s\), first noted in Ref. 59. This is because both \(\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle\) and \(\langle K^- | \bar{s} \gamma_5 u | 0 \rangle\) are inversely proportional to \(m_s\) via the equations of motion. Recent lattice and QCD sum rule studies give

\[
\begin{align*}
\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle & \approx 128 \pm 18 \text{ MeV} \\
\langle K^- | \bar{s} \gamma_5 u | 0 \rangle & \approx 100 \pm 21 \pm 10 \text{ MeV}
\end{align*}
\]

so that \(m_s(m_b) \sim 100 \pm 21 \pm 10 \text{ MeV}\) appears to be a reasonable choice. Given the large uncertainties inherent in the factorization approach it appears that the observed rate, particularly the lower half of the quoted range, can be accommodated in the Standard Model. For example, it has recently been shown that with a more general parametrization of non-factorizable corrections in which two different parameters \(N_{\text{eff}}^c(V-A)\) and \(N_{\text{eff}}^c(V+A)\) are assigned to matrix elements of four quark operators with \((V-A)(V-A)\) and \((V-A)(V+A)\) structure, respectively, it is possible to increase the Standard Model rate by more than 50% while satisfying constraints on other exclusive decays. However, it is also clear that enhanced \(b \to s \bar{q} q\) can significantly increase the rate.

\[A_{CP}^q\] can receive large contributions from enhanced \(b \to s \bar{q} q\) thus leading to asymmetries which are significantly larger than the naive Standard Model factorization estimates of a few percent 70, 85. However, one should not take Standard Model asymmetry estimates obtained in the factorization approach too seriously, particularly because of the unknown impact of soft FSI.

### 5.2 \(B \to \eta' X_s\): Factorization Model Contributions

Factorization model contributions to \(B \to \eta' X_s\) involve two types of amplitudes distinguished by their hadronization pattern, namely quasi two-body and quasi three-body decays. In the two-body decays an \(\eta'\) is formed from \(s \bar{s}, u \bar{u}\) or \(d \bar{d}\) pairs via the subprocess \(b \to s \bar{q} q\), which in the parton model corresponds to \(b \to \eta's\). The three-body decays involve hadronization of the spectator into the \(\eta'\), which in the parton model corresponds to \(B \to \eta' s \bar{q} (q = u, d)\). The two types of decay give rise to very different recoil spectra. In two-body decays \(E_{\eta'} \sim m_{b}/2\) and \(q^2 \sim m^2_{X_s}/2\), leading to a recoil spectrum peaked at low energies, e.g., \(m_{X_s} \sim 1.4 \text{ GeV}\), with a spread of a few hundred \(\text{MeV}\) when Fermi motion is taken into account. In three-body decays the recoil spectrum rises steadily with \(m_{X_s}\) and actually peaks beyond the signal region.

In Fig. 7 two-body and three-body branching ratios with \(\text{BR}(b \to s \bar{q} q) \approx 10\%\) are compared to the corresponding Standard Model branching ratios for the inputs discussed above. A plot of the two-body branching ratio in the

\[^b\text{A Dalitz plot analysis of quark level } b \to s \bar{q} q \text{ decays suggests that interference between the two-body and three-body amplitudes can be neglected. In any case this interference would have negligible impact at large } m_{X_s} \text{ where most of the signal is located.}\]

\[^i\text{The } \sim \alpha_s^2 c_1^2 \text{ contributions to } \Gamma(B \to \eta' s \bar{q}) \text{ and } \Gamma(b \to s \bar{q} q) \text{ diverge in the } q^2 \to 0 \text{ “on-shell” gluon limit. At the quark level this divergence must be canceled by the } O(\alpha_s)\]
Figure 6: \(10^3 \text{BR}(B^\pm \rightarrow \eta' K^\pm)\) (CP averaged) and \(A^{dir}_{CP}(B^\pm \rightarrow \eta' K^\pm)\) vs. \(\theta\) for \(\text{BR}(b \rightarrow s\gamma) \approx 10\%\) and \(m_s = 100\ MeV, 150\ MeV\). Solid curves are for \(c'_1 = 0\ (\theta = \theta_{11})\), long dashed curves are for \(|c_{11}| = |c'_{11}|\) and \(\theta'_{11} = 180^\circ\ (\theta = \theta_{11})\), short dashed curves are for \(c_{11} = 0\ (\theta = \theta'_{11})\).
Figure 7: $10^5 \text{BR}(b \rightarrow \eta's)$ and $10^5 \text{BR}(B \rightarrow \eta's \bar{q})$ (CP averaged) vs. $\theta$ for $\text{BR}(b \rightarrow s \bar{q}) \approx 10\%$ and $m_s = 100$ MeV. Solid curves are for $c'_{11} = 0$ ($\theta = \theta_{11}$), long dashed curves are for $|c_{11}| = |c'_{11}|$ and $\theta'_{11} = 180^\circ$ ($\theta = \theta_{11}$), short dashed curves are for $c_{11} = 0$ ($\theta = \theta'_{11}$). Horizontal lines are Standard Model branching ratios.

$(\theta_{11}, \theta'_{11})$ plane with $|c_{11}| = |c'_{11}|$ is also included in Fig. 5. For the three-body branching ratios we have for simplicity taken $p_{\eta'} > 2$ GeV in the $B$ rest frame, thus ignoring the small effect of the Lorentz boost to the laboratory frame. In the Standard Model we obtain $\text{BR}(b \rightarrow \eta's) \approx 1 \times 10^{-4}$ and $\text{BR}(B \rightarrow \eta's \bar{q}) \approx 1.4 \times 10^{-5}$. A similar result was obtained in Ref. [76]. In models with enhanced $b \rightarrow s \bar{q}$ both yields can be substantially increased. However, because the two-body contribution is generally larger than the three-body contribution by a factor 2 to 4 we conclude on the basis of the observed recoil spectrum that the bulk of the signal can not be accounted for in the factorization approach in models with enhanced $b \rightarrow s \bar{q}$. 

5.3 $B \rightarrow \eta'X_s$: Anomaly Mediated Processes

It has been suggested by Atwood and Soni (AS) that the large inclusive rate is connected to the QCD penguins via the gluon anomaly, leading to the subprocess $b \rightarrow s g^* \rightarrow \eta' s \bar{q}$. The effective $gg\eta'$ coupling was parametrized as

$$V_{\mu\nu}c_1^{\mu}\epsilon_2^{\nu} = H(q^2, k^2, q_{\eta'}^2)\epsilon_\alpha\beta_{\mu\nu}\epsilon^{\alpha\beta}k^\beta c_1^\mu c_2^\nu$$

(21)

correction to $\Gamma(b \rightarrow s \bar{q})$. The three-body $\eta'$ yield has been obtained with a cutoff $q^2 > 1$ GeV$^2$ imposed on the virtual gluon momentum. The dependence on cutoff is negligible in the Standard Model, and is sufficiently moderate near 1 GeV$^2$ in the case of enhanced $b \rightarrow s \bar{q}$, i.e., the branching ratio decreases by 25% as the cutoff is increased from 1 GeV$^2$ to 2 GeV$^2$. 

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where \( q^2 \equiv (p_b - p_s)^2 \). A constant form factor was assumed, i.e., \( H(q^2, k^2, q'^2_s) \simeq H(0, 0, m^2_{b'}) \equiv H_0 \), and \( H_0 \) was extracted directly from the decay rate for \( J/\psi \to \eta' \gamma \), yielding \( H_0 \approx 1.8 \text{ GeV}^{-1} \). With the central assumption of a constant form factor, AS found \( \text{BR}(b \to s g) \sim 8 \times 10^{-4} \) \( (p_{\eta'} > 2 \text{ GeV}) \), which would account for the observed signal. Moreover, the three-body decay leads to an \( m_{X_s} \) spectrum that is consistent with observation. However, it is clear that in order to obtain the total decay rate the differential distribution for this subprocess must be integrated over a wide range of \( q^2 \), spanning approximately 1 \( \text{GeV}^2 \) to \( m^2_b \). It is therefore of paramount importance to investigate the off-shell \( q^2 \) dependence of the form-factor.

Hou and Tseng\(^{75}\) (HT) argued that the factor \( \alpha_s \) implicit in \( H \) should be running and must therefore be evaluated at the scale of momentum transfer through the \( \eta' gg \) vertex. The mild logarithmic dependence of \( H \) on \( q^2 \) would lower AS’s result by roughly a factor of 3. HT therefore suggested that enhanced \( b \to sg \) might be required in order to account for the observed \( \eta' \) signal, noting that this could substantially increase the \( b \to s\eta'g \) rate. They have also made the interesting observation that this could lead to direct CP asymmetries for this subprocess which are as large as 10%. Still, given such mild \( q^2 \) dependence we note that a Standard Model cocktail solution could account for the observed signal, particularly when the large hadronic uncertainties involved are considered. The introduction of New Physics is therefore not warranted in this case. For example, the combination of a decreased \( b \to s\eta'g \) branching ratio of \( (2 - 3) \times 10^{-4} \), a factorization model branching ratio of \( 1 \times 10^{-4} \), and a branching ratio of \( 1.1 \times 10^{-4} \) (with experimental cut) for \( \eta' \) production via decay of intermediate charmonia\(^{74}\) would be consistent with the observed \( \eta' \) yield at the 1\( \sigma \) level. Furthermore, since \( b \to \eta' sg \) and intermediate charmonia decays lead to recoil spectra whose shapes are consistent with observation the total recoil spectrum could also be consistent.

In Ref.\(^{59}\) we suggested that the leading \( q^2 \) dependence of the \( gg\eta' \) form factor is much more severe than assumed in\(^{74, 75}\). First, we observe that the “direct” \( gg\eta' \) coupling is suppressed. Since the quantum numbers of the \( \eta' \) and gluon are \( 0^- \) and \( 1^- \), respectively, formation of an \( \eta' \) requires orbital angular momentum. This introduces a dependence on the transverse gluon momentum, so that asymptotically the coupling scales like \( 1/q^4 \). The leading order contributions in \( 1/q^2 \) are, in fact, due to hard amplitudes involving quark or gluon exchange which couple to the \( |\bar{q}q\rangle \) or \( |gg\rangle \) components of the \( \eta' \), respectively. To model the leading \( q^2 \) dependence in the region of interest we therefore consider a \( gg\eta' \) vertex in which a pseudoscalar current is coupled perturbatively to two gluons through quark loops.

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The calculation yields a form factor which can be parametrized as

$$H(q^2, 0, m_{\eta'}^2) = -\frac{H_0 m_{\eta'}^2}{q^2 - m_{\eta'}^2}. \quad (22)$$

In the general case of two virtual gluons with momenta $q$ and $k$ the denominator is replaced with $2q \cdot k$. Recently, it has been shown that quark condensate contributions do not modify the leading $q^2$ dependence, suggesting that it is not modified by non-perturbative QCD effects. The dependence of $H_0$ on $q^2$, which includes possible running of $\alpha_s$, must be subleading compared to the strong power dependence. However, it insures the absence of a pole at $q^2 = m_{\eta'}^2$. To first approximation it can be modeled by a constant $H_0$ which we identify with the value extracted from $J/\psi \to \eta' \gamma$. In the Standard Model this leads to $\text{BR}(b \to s g \eta') \sim 1.6 \times 10^{-5}$ including cuts (for $m_b = 5.0 \text{ GeV}$), which is more than an order of magnitude smaller than the observed $\eta'$ yield. On the other hand, for $\text{BR}(b \to sg) \approx 10\%$ we obtain $\text{BR}(b \to s g \eta') \sim 2 - 1.5 \times 10^{-4}$, with the largest values, as usual, corresponding to $\theta_{11} \sim 180^\circ$. The corresponding direct CP asymmetries are systematically smaller than those obtained by HT because of the shift in the decay distribution to lower $q^2$, thus leading to smaller strong phases in the chromoelectric form factor.

As a further indication that $H(q^2, 0, m_{\eta'}^2)$ has a strong power dependence we note that precisely the same $1/(q^2 - m_{\eta'}^2)$ behaviour is obtained via quark exchange in the non-relativistic quark model. Furthermore, the leading $q^2$ dependence associated with gluon exchange (or a gluon “loop”) will not be any less severe, as can be seen from the form of the gluon propagator. HT have suggested\textsuperscript{[9]} that intermediate gluonia arising via the gluon self coupling might postpone the onset of $q^2$ suppression by giving rise to a $1/(q^2 - m_G^2)$ dependence in the form factor, where $m_G$ is a glueball mass scale of $2 - 4 \text{ GeV}$. This possibility appears to us difficult to justify since it is more reasonable to expect the virtuality of the glueball, of order $m_{\eta'}^2$, rather than its mass to be the relevant scale in such a process.

Our result for the Standard Model $b \to s g \eta'$ branching ratio casts doubt on the gluon fusion mechanism proposed in Ref.\textsuperscript{[7]} as a possible explanation of the large exclusive $B \to \eta' K$ rate. In this process the gluon from the $b \to s g \eta'$ transition is highly virtual and is absorbed by the spectator. The authors of\textsuperscript{[10]} calculate the exclusive rate using a perturbative QCD method which we believe is unreliable in this case, given that a large contribution in their approach arises for $q^2 \lesssim 1 \text{ GeV}^2$. In fact, using essentially the same $g g \eta'$ form factor as in Eq.\textsuperscript{[12]} they obtain an exclusive rate which is at least as large

\textsuperscript{[9]}See note added in Ref.\textsuperscript{[7]}.\textsuperscript{[10]}

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as the corresponding inclusive $b \to s q \eta'$ rate, whereas one naturally expects it to be significantly smaller.

Thus far we have considered short-distance mechanisms for $\eta'$ production in $B \to X_{sg}$ decays. In particular, in both the factorization model and $b \to \eta' sg$ subprocesses, fast $\eta'$ production is associated with large gluon virtualities, i.e., $q^2 >> 1$ GeV$^2$. It is important to note that there could also be important long-distance quark and gluon fragmentation contributions. As in the case of kaon production, we obtain an order of magnitude estimate of soft $q \bar{q}$ popping contributions using the $b \to s q$ Monte Carlo. Taking BR($b \to s q$) $\approx$ 10%, JETSET 7.4 with DELPHI tunings gives BR($B \to \eta' X_s$) $\sim 1 \times 10^{-4}$ for $p_{\eta'} > 2$ GeV. As always in soft fragmentation processes, the large energy release leads predominantly to high multiplicity final states resulting in a soft $\eta'$ spectrum, or a recoil spectrum that would be consistent with observation.

In the Standard Model the $\eta'$ yield from quark fragmentation must be an order of magnitude smaller, i.e., BR($B \to \eta' X_s$) $\sim 10^{-5}$, since the underlying $b \to s q \bar{q}$ branching ratio is $\sim 1\%$. It is worth mentioning that JETSET 7.4 with similar tunings gives a reasonable description of both the $\eta'$ rate and momentum spectrum observed in Z decays\footnote{The ratio of the first two branching ratios is $\sim 0.6 \pm 0.3$.}. Again, we stress that the $B \to X_{sg}$ Monte Carlo only provides very rough order of magnitude estimates. For example, interference between contributions proceeding via the $u \bar{u}$, $d \bar{d}$, and $s \bar{s}$ components of the $\eta'$ can not be included in this approach.

Given the significant glue content of the $\eta'$ one might expect that soft “$gg$ popping” (long-distance gluon fragmentation) is as important as soft $q \bar{q}$ popping in chromomagnetic $B \to X_{sg}$ transitions. Because this is a long-distance process there would be no $q^2$ suppression, unlike in the case of the decays $b \to s q \eta'$. Again, the large energy release would lead predominantly to soft $\eta'$, or a hard recoil spectrum. Unfortunately, there is even less guidance from experiment in this case than in the case of quark fragmentation. The similarity of the $\eta_c \to gg$ mediated decay rates $\footnote{The ratio of the first two branching ratios is $\sim 0.6 \pm 0.3$.}$ BR($\eta_c \to \eta' \pi \pi$) $= 0.041 \pm 0.017$, BR($\eta_c \to \eta \pi \pi$) $= 0.049 \pm 0.018$, and BR($\eta_c \to K \bar{K} \pi$) $= 0.066 \pm 0.018$, suggests that the $\eta' \pi \pi$ mode, like the other two, proceeds via the intermediate transition $gg \to \bar{q} q' \bar{q} q'$. In particular, there is no evidence for $\eta'$ formation out of gluons in this decay. On the other hand, we know that soft fragmentation processes should lead mainly to high multiplicity final states for which no data is currently available. In any case, long-distance gluonic $\eta'$ yields from Standard Model $b \to s q \eta^{(*)}$ decays should be at least an order of magnitude smaller than those obtained with BR($b \to s q$) $\approx 10\%$, particularly since the Standard Model transition is dominated by the chromoelectric form factor, which unlike the chromomagnetic form factor is not peaked at low $q^2$.\footnote{The ratio of the first two branching ratios is $\sim 0.6 \pm 0.3$.}
In Ref. 77 an *ad hoc* effective Hamiltonian

\[ H_{\text{eff}} = a\alpha_s G_F \bar{s}_L b_R G_{\mu\nu} \tilde{G}^{\mu\nu} \]  

was proposed to model the long-distance gluon fragmentation contributions to \( \eta' \) production in the Standard Model. In this scenario the \( \eta' \) is formed out of \( G\tilde{G} \) and it is assumed that a large enough coefficient \( a \) is generated to account for the observed \( \eta' \) yield. In addition to leading to a quasi two-body recoil spectrum in conflict with experiment and with the expectation from soft fragmentation processes, this proposal suffers from another serious drawback. From the vacuum to \( \eta' \) matrix element of \( G\tilde{G} \), the authors of Ref. 95 conclude that the central value of the \( \eta' \) yield in Eq. 18 would correspond to \( a \approx 0.015 \text{ GeV}^{-1} \). It is then a simple matter to show that Eq. 23 would imply a fantastically large inclusive rate \( \text{BR}(b \rightarrow sg g) \approx 10^{-50\%} \), where the range is obtained by varying the scale at which \( \alpha_s \) is evaluated from \( m_b \) to \( m_{\eta'} \). The latter is the scale entering the \( G\tilde{G} \) matrix element, or the scale at which \( a \) is determined. In other words, a large \( \eta' \) yield would be associated with a non-perturbative enhancement of the \( b \rightarrow s \) glue rate in the Standard Model of more than an order of magnitude, which is not plausible. Conversely, it is unreasonable to expect a contribution to the \( B \rightarrow \eta'X_s \) branching ratio in this framework much larger than \( 10^{-5} \).

Table 3 summarizes the orders of magnitude of the various contributions to \( \text{BR}(B \rightarrow \eta'X_s) \) for \( 2 < p_{\eta'} < 2.7 \text{ GeV} \), in the Standard Model and in models with \( \text{BR}(b \rightarrow sg) \approx 10\% \). If long-distance gluon fragmentation is as important as quark fragmentation then there is an additional contribution of order \( 10^{-5} \) in the Standard Model and of order \( 10^{-4} \) for enhanced \( b \rightarrow sg \). The quasi two-body \( b \rightarrow \eta'X_s \) recoil spectrum is peaked at low recoil mass, e.g., \( m_{X_s} \approx 1.4 \text{ GeV} \). The shapes of the remaining recoil spectra are consistent with experiment. It is clear from the Table that the existence of a cocktail solution for the observed signal is very likely in models with enhanced \( b \rightarrow sg \). However, the situation is more problematic in the Standard Model. Apart from the \( \approx 10^{-4} \) branching ratio contribution due to decays of intermediate charmonia, all other contributions with consistent recoil spectra are of order \( 10^{-5} \). Should the measured branching ratio remain near the current value, our results suggest that a Standard Model explanation will require a novel non-perturbative mechanism leading to order of magnitude enhancement of anomaly mediated processes.
Table 3: Orders of magnitude for various contributions to $\text{BR}(B \to \eta' X_s)$ ($2.0 < p_{\eta'} < 2.7$ GeV) in the Standard Model and in models with $\text{BR}(b \to sg) \approx 10\%$. The entries, from top to bottom, correspond to decays of intermediate charmonia, the quasi two-body and quasi three-body factorization model contributions, the anomaly induced $b \to sg\eta'$ decay, and long-distance quark fragmentation, as explained in the text.

| Process | Standard Model | Enhanced $b \to sg$ |
|---------|----------------|---------------------|
| $B \to (\bar{c}c)X_s, (\bar{c}c) \to \eta' X_s$ | $\approx 1.1 \times 10^{-4}$ | $\approx 1.1 \times 10^{-4}$ |
| $b \to \eta's$ | $10^{-4}$ | $10^{-4}$ |
| $\bar{B} \to \eta's\bar{q}$ | $10^{-5}$ | $5 \times 10^{-5}$ |
| $b \to sgn'$ | $10^{-5}$ | $10^{-4}$ |
| LD quark fragmentation | $10^{-5}$ | $10^{-4}$ |

6 Conclusion

Before concluding I would like to mention three novel features of radiative $B$ decays which can arise in models with enhanced $b \to sg$:

- Large direct CP asymmetries in the inclusive decays $B \to X_s\gamma$ vs. $\bar{B} \to X_s\gamma$ of 10% - 50% are possible. In the Standard Model the asymmetry is only of order 1% due to a combination of CKM and GIM suppression, both of which can be lifted in New Physics scenarios with additional contributions to the dipole operators containing new weak phases. The asymmetries provide a unique probe of models with enhanced $b \to sg$ since they are particularly sensitive to interference of the next-to-leading order one-loop diagram containing a chromomagnetic dipole operator insertion (which generates a strong phase) and the tree-level diagram for the electromagnetic dipole operator.

- New chromomagnetic dipole operator mediated graphs in which the spectator quark radiates a photon can lead to large isospin violation in $B \to K^*\gamma$. In particular, rate asymmetries between the $K^{*-}\gamma$ and $K^{*0}\gamma$ final states could exceed 50%, compared to only a few % in the standard model.

- In models in which $b \to dg$ is also strongly enhanced, e.g., in association with generation of $V_{ub}$, $b \to d\gamma$ is necessarily enhanced so that $\text{BR}(B \to \rho\gamma, \omega\gamma)$ could be an order of magnitude larger than in the Standard Model.

To summarize, there are two potential hints for enhanced $b \to sg$ which are essentially experimental, a 2σ deficit in charm counting and a 3σ deficit in
kaon counting. Significantly improved precision in charm and kaon counting will require a reduction in uncertainties in the absolute $D/D_s$ branching scales. Improved precision in kaon counting will also require an experimental determination of the amount of $s\bar{s}$ popping in $B$ decays via future measurements of $\text{BR}(B \to DK\bar{K}X)$ and $\text{BR}(B \to D_s\bar{K}X)$. There are also well known hints from comparison of next-to-leading order Standard Model predictions for the charm multiplicity and semileptonic branching ratio with measurements at the $\Upsilon(4S)$. Unfortunately, improved theoretical precision poses a difficult challenge in the near future.

A promising direct search strategy is the search for high momentum kaon excesses in $B$ decays, e.g., $p_K \gtrsim 1.8$ GeV. A JETSET analysis indicates that for $\text{BR}(b \to sg) \sim 10\%$ the corresponding $B \to KX$ branching ratios are of order $10^{-3}$. The Standard Model background at these momenta is of same order and is dominated by kaon yields from intermediate $D^0/D^+$ decays, which can be determined experimentally. Remarkably, enhanced $b \to sg$ can evade all existing rare $B$ decay constraints. We have also seen that new weak phases in the chromomagnetic dipole operator coefficients can lead to large $CP$ violation in hadronic rare decays. For example, direct $CP$ asymmetries in $B^\pm \to \phi K^\pm$ and $B^\pm \to K^0\pi^\pm$ could be as large as $10\% - 50\%$. However, additional flavor $SU(3)$ tests will be required in order to distinguish such contributions from potentially large Standard Model asymmetries induced by soft final state rescattering. Furthermore, such large New Physics contributions are not restricted to models with enhanced $b \to sg$. In contrast, similarly large direct $CP$ asymmetries in radiative $B \to X_s\gamma$ decays would provide unambiguous evidence for strongly enhanced chromomagnetic dipole operators.

The large $B \to \eta^\prime K$ branching ratios measured by CLEO can be accounted for in the Standard Model in the factorization approach. Low values of $m_s$ and particular ranges of parameters for non-factorizable contributions are favored. Nevertheless, enhanced $b \to sg$ can lead to significant enhancement of the factorization model rates. A Standard Model explanation for the large inclusive $B \to \eta^\prime X_s$ rate measured by CLEO is more challenging. It seems that a novel non-perturbative mechanism is required which would enhance the gluon anomaly mediated contributions by an order of magnitude. In contrast, enhanced $b \to sg$ can increase the gluon anomaly, long-distance quark fragmentation, and quasi three-body factorization model contributions by an order of magnitude each so that the existence of a cocktail solution becomes very likely. The inclusive $\eta^\prime$ rate may therefore be providing us with another hint for enhanced $b \to sg$, or TeV scale flavor dynamics.
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