Technicolor and the First Muon Collider

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Technicolor and the First Muon Collider

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Abstract.

The motivations for studying dynamical scenarios of electroweak and flavor symmetry breaking are reviewed and the latest ideas, especially topcolor-assisted technicolor, are summarized. Technicolor’s observable low-energy signatures are discussed. The superb energy resolution of the First Muon Collider may make it possible to resolve the extraordinarily narrow technihadrons that occur in such models—$\pi^0_T$, $\rho^0_T$, $\omega_T$—and produce them at very large rates compared to other colliders.

I OVERVIEW OF TECHNICOLOR

Technicolor—the strong interaction of fermions and gauge bosons at the scale $\Lambda_{TC} \sim 1$ TeV—describes the breakdown of electroweak symmetry to electromagnetism without elementary scalar bosons [1]. In its simplest form, technicolor is a scaled-up version of QCD, with massless technifermions whose chiral symmetry is spontaneously broken at $\Lambda_{TC}$. If left and right-handed technifermions are assigned to weak $SU(2)$ doublets and singlets, respectively, then $M_W = \cos \theta_W M_Z = \frac{1}{2} g F_\pi$, where $F_\pi = 246$ GeV is the technipion decay constant, analogous to $f_\pi = 93$ MeV for the ordinary pion.

The principal signals in hadron and lepton collider experiments of “classical” technicolor were discussed long ago [2,3]. In the minimal technicolor model, with just one technifermion doublet, the only prominent collider signals are the enhancements in longitudinally-polarized weak boson production. These are the $s$-channel color-singlet technirho resonances near 1.5–2 TeV: $\rho^0_T \rightarrow W^+_L W^-_L$ and $\rho^\pm_T \rightarrow W^\pm_L Z^0_L$. The $O(\alpha^2)$ cross sections of these processes are quite small at such masses. This and the difficulty of reconstructing weak-boson pairs with reasonable efficiency make observing these enhancements a challenge.

1) Talk presented at the Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider

2) The only technipions in minimal technicolor are the massless Goldstone bosons that become, via the Higgs mechanism, the longitudinal components $W^\pm_L$ and $Z^0_L$ of the weak gauge bosons.
Nonminimal technicolor models are much more accessible because they have a rich spectrum of lower mass technirho vector mesons and technipion states into which they may decay. If there are \( N_D \) doublets of technifermions, all transforming according to the same complex representation of the technicolor gauge group, there will be \( 4N_D^2 - 1 \) technipions whose decay constant is

\[
F_T = \frac{F_\pi}{\sqrt{N_D}}.
\]

(1)

Three of these are the longitudinal weak bosons; the remaining \( 4N_D^2 - 4 \) await discovery.

In the standard model and its extensions, the masses of quarks and leptons are produced by their Yukawa couplings to the Higgs bosons—couplings of arbitrary magnitude and phase that are put in by hand. This option is not available in technicolor because there are no elementary scalars. Instead, quark and lepton chiral symmetries must be broken explicitly by gauge interactions alone. The most economical way to do this is to employ extended technicolor, a gauge group containing flavor, color and technicolor as subgroups [4–6]. Quarks, leptons and technifermions are unified into a few large representations of ETC. The ETC gauge symmetry is broken at high energy to technicolor \( \otimes \) color. Then quark and lepton hard masses arise from their coupling (with strength \( g_{ETC} \)) to technifermions via ETC gauge bosons of generic mass \( M_{ETC} \):

\[
m_q(M_{ETC}) \simeq m_\ell(M_{ETC}) \simeq \frac{g_{ETC}^2}{M_{ETC}^2} \langle \bar{T}T \rangle_{ETC},
\]

(2)

where \( \langle \bar{T}T \rangle_{ETC} \) and \( m_q, \ell(M_{ETC}) \) are the technifermion condensate and quark and lepton masses renormalized at the scale \( M_{ETC} \).

If technicolor is like QCD, with a running coupling \( \alpha_{TC} \) rapidly becoming small above \( \Lambda_{TC} \sim 1 \) TeV, then \( \langle \bar{T}T \rangle_{ETC} \simeq \langle \bar{T}T \rangle_{TC} \simeq \Lambda_{TC}^2 \). To obtain quark masses of a few GeV, \( M_{ETC}/g_{ETC} \lesssim 30 \) TeV is required. This is excluded: Extended technicolor boson exchanges also generate four-quark interactions which, typically, include \( |\Delta S| = 2 \) and \( |\Delta B| = 2 \) operators. For these not to be in conflict with \( K^0-\bar{K}^0 \) and \( B^0_d-\bar{B}^0_d \) mixing parameters, \( M_{ETC}/g_{ETC} \) must exceed several hundred TeV [5]. This implies quark and lepton masses no larger than a few MeV, and technipion masses no more than a few GeV—a phenomenological disaster.

Because of this conflict between constraints on flavor-changing neutral currents and the magnitude of ETC-generated quark, lepton and technipion masses, classical technicolor was superseded over a decade ago by “walking” technicolor [7]. Here, the strong technicolor coupling \( \alpha_{TC} \) runs very slowly—walks—for a large range of momenta, possibly all the way up to the ETC scale of several hundred TeV. The slowly-running coupling enhances \( \langle \bar{T}T \rangle_{ETC}/\langle \bar{T}T \rangle_{TC} \) by almost a

\[3]\)

The technipions of nonminimal technicolor include the longitudinal weak bosons as well as additional Goldstone bosons associated with spontaneous technifermion chiral symmetry breaking. The latter must and do acquire mass—from the extended technicolor interactions discussed below.
factor of $M_{ETC}/\Lambda_{TC}$. This, in turn, allows quark and lepton masses as large as a few GeV and $M_{\pi_T} \gtrsim 100$ GeV to be generated from ETC interactions at $M_{ETC} = O(100 \text{ TeV})$.

Walking technicolor requires a large number of technifermions in order that $\alpha_{TC}$ runs slowly. These fermions may belong to many copies of the fundamental representation of the technicolor gauge group, to a few higher dimensional representations, or to both.  

In many respects, walking technicolor models are very different from QCD with a few fundamental $SU(3)$ representations. One example of this is that integrals of weak-current spectral functions and their moments converge much more slowly than they do in QCD. Consequently, simple dominance of the spectral integrals by a few resonances cannot be correct. This and other calculational tools based on naive scaling from QCD and on large-$N_{TC}$ arguments are suspect [10]. Thus, it is not yet possible to predict with confidence the influence of technicolor degrees of freedom on precisely-measured electroweak quantities—the $S, T, U$ parameters to name the most discussed example [11].

The large mass of the top quark [12] motivated another major development in technicolor. Theorists have concluded that ETC models cannot explain the top quark’s large mass without running afoul of experimental constraints from the $\rho$ parameter and the $Z \to b\bar{b}$ decay rate [13]. This state of affairs has led to the proposal of “topcolor-assisted technicolor” (TC2) [14].

In TC2, as in top-condensate models of electroweak symmetry breaking [15], almost all of the top quark mass arises from a new strong “topcolor” interaction [16]. To maintain electroweak symmetry between (left-handed) top and bottom quarks and yet not generate $m_b \simeq m_t$, the topcolor gauge group under which $(t, b)$ transform is usually taken to be a strongly-coupled $SU(3) \otimes U(1)$. The $U(1)$ provides the difference that causes only top quarks to condense. Then, in order that topcolor interactions be natural—i.e., that their energy scale not be far above $m_t$—without introducing large weak isospin violation, it is necessary that electroweak symmetry breaking remain due mostly to technicolor interactions [14].

Early steps in the development of the TC2 scenario have been taken in two recent papers [17]. The breaking of topcolor $SU(3) \otimes U(1)$ near the electroweak scale gives rise to a massive color octet of $V_8$ colorons and a color-singlet $Z'$. The $SU(3)$ may be broken by some of the same technifermion condensates that break electroweak $SU(2) \otimes U(1)$, so that the colorons (which are expected to be broad) have mass near 500 GeV. However, in order that the strong topcolor $U(1)$ interaction not contaminate the ordinary $Z$ couplings to fermions, it and the weaker $U(1)$ acting on light fermions must be broken down to their diagonal subgroup, ordinary weak hypercharge, in the vicinity of 2 TeV. This suggests that the $Z'$ mass is in the range

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4) The last possibility inspired “multiscale technicolor” models containing both fundamental and higher representations, and having an unusual phenomenology [8]. In multiscale models, there typically are two widely separated scales of electroweak symmetry breaking, with the upper scale set by the weak decay constant, $F_\pi = 246$ GeV. Multiscale models in which the entire top quark mass is generated by ETC interactions are excluded by such processes as $b \to s\gamma$ [9].
1–3 TeV, out of reach of all but the highest energy colliders. As I discussed in my talk at the FMC workshop, the $Z'$ is so heavy that it may require a multi-TeV Big Muon Collider to find and study it. This subject deserves further study.

In TC2 models, ETC interactions are still needed to generate the light and bottom quark masses, contribute a few GeV to $m_t$,\(^5\) and give mass to the technipions. The scale of ETC interactions still must be hundreds of TeV to suppress flavor-changing neutral currents and, so, the technicolor coupling still must walk.

Thus, even though the phenomenology of TC2 is still in its infancy, it is expected to share general features with multiscale technicolor: many technifermion doublets bound into many technihadron states, some at relatively low masses, some carrying ordinary color and some not. The lightest technihadrons may have masses in the range 100–300 GeV and should be accessible at the Tevatron collider in Run III if not Run II. All of them are easily produced and detected at the LHC at moderate luminosities. If technihadrons exist, they will be discovered at hadron colliders before the First Muon Collider (FMC) is built. As we shall see, this is a good thing for the FMC: Several of the lightest technihadrons are very narrow and can be produced in the s-channel of $\mu^+\mu^-$ annihilations. In the narrow-band FMC, it would be exceedingly difficult to find them by a standard scan procedure without a good idea of where to look.

II TECHNICOLOR AT THE FMC

A Technihadron Decay Rates

I assume that the technicolor gauge group is $SU(N_{TC})$ and take $N_{TC} = 4$ in calculations. Its gauge coupling must walk and I assume this is achieved by a large number of isodoublets of technifermions transforming according to the fundamental representation of $SU(N_{TC})$. I consider the phenomenology of only the lightest color-singlet technihadrons and assume that the constraint from the $S$-parameter on their spectrum still allows the lightest ones to be considered in isolation for a limited range of $\sqrt{s}$, the $\mu^+\mu^-$ center-of-mass energy, about their masses. These technihadrons carry isospin $I = 1$ and 0 and consist of a single isotriplet and isosinglet of vectors, $\rho_T^0$, $\rho_T^\pm$ and $\omega_T$, and pseudoscalars $\pi_T^0$, $\pi_T^\pm$, and $\pi_T^{0'}$. The latter are in addition to the longitudinal weak bosons, $W_L^\pm$ and $Z_L^0$—those linear combinations of technipions that couple to the electroweak gauge currents. I adopt TC2 as a guide for guessing phenomenological generalities. In TC2 there is no need for large technifermion isospin splitting associated with the top-bottom mass difference. This implies that the lightest $\rho_T$ and $\omega_T$ are approximately degenerate. The lightest charged and neutral technipions also should have roughly the same mass, but there may be appreciable $\pi_T^0 - \pi_T^{0'}$ mixing. If that happens, the lightest technipions carry isospin $I = 1$ and 0 and consist of a single isotriplet and isosinglet of vectors, $\rho_T^0$, $\rho_T^\pm$ and $\omega_T$, and pseudoscalars $\pi_T^0$, $\pi_T^\pm$, and $\pi_T^{0'}$. The latter are in addition to the longitudinal weak bosons, $W_L^\pm$ and $Z_L^0$—those linear combinations of technipions that couple to the electroweak gauge currents. I adopt TC2 as a guide for guessing phenomenological generalities. In TC2 there is no need for large technifermion isospin splitting associated with the top-bottom mass difference. This implies that the lightest $\rho_T$ and $\omega_T$ are approximately degenerate. The lightest charged and neutral technipions also should have roughly the same mass, but there may be appreciable $\pi_T^0 - \pi_T^{0'}$ mixing. If that happens, the lightest.

\(^{5)}\) Massless Goldstone “top-pions” arise from top-quark condensation. This ETC contribution to $m_t$ is needed to give them a mass in the range of 150–250 GeV.
neutral technipions are really $\bar{U}U$ and $\bar{D}D$ bound states. Finally, for purposes of discussing signals at the FMC, we take the lightest technihadron masses to be

$$M_{\rho_T} \simeq M_{\omega_T} \sim 200 \text{ GeV}; \quad M_{\pi_T} \sim 100 \text{ GeV}.$$  \hfill (3)

The decays of technipions are induced mainly by ETC interactions which couple them to quarks and leptons. These couplings are Higgs-like, and so technipions are expected to decay into the heaviest fermion pairs allowed. Because only a few GeV of the top-quark’s mass is generated by ETC, there is no great preference for $\pi_T$ to decay to top quarks nor for top quarks to decay into them. Furthermore, the isosinglet component of neutral technipions may decay into a pair of gluons if its constituent technifermions are colored. Thus, the predominant decay modes of the light technipions are assumed to be

$$\pi_0^T \rightarrow \bar{b}b, \bar{c}c, \tau^+\tau^-$$

$$\pi_0'^T \rightarrow gg, \bar{b}b, \bar{c}c, \tau^+\tau^-$$

$$\pi_T^+ \rightarrow \bar{c}b, \bar{c}s, \tau^+\nu_\tau.$$  \hfill (4)

To estimate branching ratios we use the following decay rates (for later use in the technihadron production cross sections, we quote the energy-dependent width \[3,18\]):

$$\Gamma(\pi_T \rightarrow f\bar{f}) = \frac{1}{16\pi F_T^2} N_f p_f C_f^2 (m_f + m_f')^2$$

$$\Gamma(\pi_0'^T \rightarrow gg) = \frac{1}{128\pi^3 F_T^3} \alpha_S^2 C_{\pi_T} N_{TC}^2 s^3.$$  \hfill (5)

Here, $C_f$ is an ETC-model dependent factor of order one except that TC2 suggests $|C_t| \lesssim m_b/m_t$. $N_f$ is the number of colors of fermion $f$; $p_f$ is the fermion momentum; $\alpha_S$ is the QCD coupling evaluated at $M_{\pi_T}$; and $C_{\pi_T}$ is a Clebsch of order one. For $M_{\pi_T} = 110 \text{ GeV}$, $F_T = F_\pi/3 = 82 \text{ GeV}$, $m_b = 4.2 \text{ GeV}$, $N_{TC} = 4$, $\alpha_S = 0.1$, $C_b = 1$ for $\pi_0^T$ and $\pi_0'^T$, and $C_{\pi_T} = 4/3$: \[\text{6}\]

$$\Gamma(\pi_0^T \rightarrow \bar{b}b) = \Gamma(\pi_0'^T \rightarrow \bar{b}b) = 35 \text{ MeV}$$

$$\Gamma(\pi_0'^T \rightarrow gg) = 10 \text{ MeV}.$$

If technicolor were like QCD, we would expect the main decay modes of the lightest technivector mesons to be $\rho_T^0 \rightarrow \pi_T^0\pi_T^-$ and $\omega_T \rightarrow \pi_T^+\pi_T^-\pi_T^0$ with the technihadrons all composed of the same technifermions. However, the large ratio $\langle \bar{T}T \rangle_{ETC}/\langle \bar{T}T \rangle_{TC}$ occurring in walking technicolor significantly enhances technipion masses compared to technivector masses. Thus, $\rho_T \rightarrow \pi_T\pi_T$ decay channels may well be closed. If this happens, then $\rho_T^0$ decays to $W_L^+W_L^-$ or $W_L^+\pi_T^0$ and $\omega_T$ to $\gamma\pi_T^0$ or $Z\pi_T^0$. \[\text{8,19,20,6}\]

6) The amplitude is taken to be $\mathcal{M}(\pi_T \rightarrow \bar{f}(p_1)f(p_2)) = C_f (m_f + m_f')/F_T \bar{u}(p_2)\gamma_5v(p_1)$.  

We parameterize this for $\rho_T$ decays by adopting a simple model of two isotriplets of technipions which are mixtures of $W_L^\pm$, $Z_L^0$ and mass-eigenstate technipions $\pi_T^\pm$, $\pi_T^0$. The lighter isotriplet $\rho_T$ is assumed to decay dominantly into pairs of the mixed state of isotriplets $|\Pi_T\rangle = \sin \chi |W_L\rangle + \cos \chi |\pi_T\rangle$, where

$$\sin \chi = F_T/F_\pi.$$  \hfill (7)

Then, the energy-dependent decay rate for $\rho_T^0 \to \pi_A^+ \pi_B^-$ (where $\pi_{A,B}$ may be $W_L$, $Z_L$, or $\pi_T$) is given by

$$\Gamma(\rho_T^0 \to \pi_A^+ \pi_B^-) = \frac{2 \alpha_{\rho_T} C_{AB}^2 p_{AB}^3}{3}s,$$  \hfill (8)

where $p_{AB}$ is the technipion momentum and $\alpha_{\rho_T}$ is obtained by naive scaling from the QCD coupling for $\rho \to \pi \pi$:

$$\alpha_{\rho_T} = 2.91 \left( \frac{3}{N_{TC}} \right).$$  \hfill (9)

The parameter $C_{AB}^2$ is given by

$$C_{AB}^2 = \begin{cases} 
\sin^4 \chi & \text{for } W_L^+ W_L^- \\
2 \sin^2 \chi \cos^2 \chi & \text{for } W_L^+ \pi_T^- + W_L^- \pi_T^+ \\
\cos^4 \chi & \text{for } \pi_T^+ \pi_T^- 
\end{cases}$$  \hfill (10)

Note that the $\rho_T$ can be very narrow. For $\sqrt{s} = M_{\rho_T} = 210$ GeV, $M_{\pi_T} = 110$ GeV, and $\sin \chi = \frac{1}{3}$, we have $\sum_{AB} \Gamma(\rho_T^0 \to \pi_A^+ \pi_B^-) = 680$ MeV, 80% of which is $W_L^\pm \pi_T^\mp$.

We shall also need the decay rates of the $\rho_T$ to fermion-antifermion states. The energy-dependent widths are

$$\Gamma(\rho_T^0 \to \bar{f}_i f_i) = \frac{N_f \alpha^2}{3 \alpha_{\rho_T}} p_i (s + 2m_i^2) A_i^0(s).$$  \hfill (11)

Here, $\alpha$ is the fine-structure constant, $p_i$ is the momentum and $m_i$ the mass of fermion $f_i$, and the factors $A_i^0$ are given by

$$A_i^0(s) = |A_{iL}(s)|^2 + |A_{iR}(s)|^2,$$

$$A_{i\lambda}(s) = Q_i + \frac{2 \cos 2\theta_W}{\sin^2 2\theta_W} \zeta_{i\lambda} \left( \frac{s}{s - M_Z^2 + i\sqrt{s} \Gamma_Z} \right),$$

$$\zeta_{iL} = T_{3i} - Q_i \sin^2 \theta_W, \quad \zeta_{iR} = -Q_i \sin^2 \theta_W.$$  \hfill (12)

For $M_{\rho_T} = 210$ GeV and other parameters as above, the $\bar{f}f$ partial decay widths are:

$$\Gamma(\rho_T^0 \to \bar{u}_i u_i) = 5.8 \text{ MeV}, \quad \Gamma(\rho_T^0 \to \bar{d}_i d_i) = 4.1 \text{ MeV},$$
$$\Gamma(\rho_T^0 \to \bar{\nu}_i \nu_i) = 0.9 \text{ MeV}, \quad \Gamma(\rho_T^0 \to \ell_i^+ \ell_i^-) = 2.6 \text{ MeV}.$$  \hfill (13)
For the $\omega_T$, phase space considerations suggest we consider only its $\gamma\pi^0_T$ and fermionic decay modes. The energy dependent widths are:

$$\Gamma(\omega_T \to \gamma\pi^0_T) = \frac{\alpha p^3}{3M_T^2},$$

$$\Gamma(\omega_T \to \bar{f}_i f_i) = \frac{N_f \alpha^2 p_i (s + 2m_i^2)}{3\alpha p_T} B_i^0(s).$$ (14)

The mass parameter $M_T$ in the $\omega_T \to \gamma\pi^0_T$ rate is unknown a priori; naive scaling from the QCD decay, $\omega \to \gamma\pi^0$, suggests it is several 100 GeV. The factor $B_i^0$ is given by

$$B_i^0(s) = |B_{iL}(s)|^2 + |B_{iR}(s)|^2, \quad B_{i\lambda}(s) = \left[ Q_i - \frac{4\sin^2 \theta_W}{\sin^2 2\theta_W} \zeta_{\lambda \lambda} \left( \frac{s}{s - M_Z^2 + i\sqrt{s}\Gamma_Z} \right) \right] (Q_U + Q_D).$$ (15)

Here, $Q_U$ and $Q_D = Q_U - 1$ are the electric charges of the $\omega_T$'s constituent technifermions. For $M_{\omega_T} = 210$ GeV and $M_{\pi^0_T} = 110$ GeV, and choosing $M_T = 100$ GeV and $Q_U = Q_D + 1 = \frac{4}{3}$, the $\omega_T$ partial widths are:

$$\Gamma(\omega_T \to \gamma\pi^0_T) = 115 \text{ MeV}$$

$$\Gamma(\omega_T \to \bar{u}_i u_i) = 6.8 \text{ MeV}, \quad \Gamma(\omega_T \to \bar{d}_i d_i) = 2.6 \text{ MeV}$$

$$\Gamma(\omega_T \to \bar{\nu}_i \nu_i) = 1.7 \text{ MeV}, \quad \Gamma(\omega_T \to \ell_i^+ \ell_i^-) = 5.9 \text{ MeV}. \quad (16)$$

The beam momentum spread of the First Muon Collider has been quoted to be as narrow as $\sigma_p/p = 3 \times 10^{-5}$ at $\sqrt{s} = 100$ GeV and $10^{-3}$ at $\sqrt{s} = 200$ GeV. These correspond to beam energy spreads of $\sigma_E = 5$ MeV at 100 GeV and 300 MeV at 200 GeV. The resolution at 100 GeV is less than the expected $\pi^0_T$ widths. At 200 GeV it is sufficient to resolve the $\rho^0_T$, but not the $\omega_T$, for the parameters we used. It is very desirable, therefore, that the 200 GeV FMC’s energy spread be about factor of 10 smaller. Since each of these technihadrons can be produced as an $s$-channel resonance in $\mu^+\mu^-$ annihilation, it would then be possible to sit on the peak at $\sqrt{s} = M$. As we see next, the peak cross sections are enormous, 2–3 orders of magnitude larger than can be achieved at a hadron collider and even at a linear $e^+e^-$ collider because of the latter’s inherent beam energy spread.

### B Technihadron Production Rates

Like the standard Higgs boson, neutral technipions are expected to couple to $\mu^+\mu^-$ with a strength proportional to $m_\mu$. Compared to the Higgs, however, this coupling is enhanced by a factor of $F_\pi/F_T = 1/\sin \chi$. This makes the resolution of the FMC well-matched to the $\pi^0_T$ width. Thus, the FMC is a technipion factory, overwhelming the rate at any other collider. Once a neutral technipion has been
FIGURE 1. Theoretical (unsmearred) cross sections for $\mu^+\mu^- \rightarrow \pi^0 \rightarrow \bar{b}b$ (dashed), $gg$ (dot-dashed) and total (solid) for $M_{\pi^T} = 110$ GeV and other parameters defined in the text. The solid horizontal lines are the backgrounds from $\gamma, Z^0 \rightarrow \bar{b}b$ (lower) and $Z^0 \rightarrow \bar{q}q$ (upper). Note the energy scale.

found in $\rho_T$ or $\omega_T$ decays at a hadron collider, it should be relatively easy in the FMC to locate the precise position of the resonance and sit on it. The cross sections for $\bar{b}b$ and $gg$ production are isotropic; near the resonance, they are given by

$$
\frac{d\sigma(\mu^+\mu^- \rightarrow \pi^0 \text{ or } \pi^0' \rightarrow \bar{b}b)}{dz} = \frac{N_f}{2\pi} \left( \frac{C_\mu C_f m_\mu m_f}{F_T^2} \right)^2 \frac{s}{(s-M_{\pi^T}^2)^2 + s\Gamma_{\pi^T}^2},
$$

$$
\frac{d\sigma(\mu^+\mu^- \rightarrow \pi^0' \rightarrow gg)}{dz} = \frac{C_{\pi^T}}{32\pi^3} \left( \frac{C_\mu m_\mu \alpha_S N_{TC}}{F_T^2} \right)^2 \frac{s^2}{(s-M_{\pi^T}^2)^2 + s\Gamma_{\pi^T}^2}.
$$

(17)

Here, $z = \cos \theta$ where $\theta$ is the center-of-mass production angle.

The $\pi^0' \rightarrow gg$ production cross sections and the $Z^0$ backgrounds are shown in Fig. 1 for $M_{\pi^T} = 110$ GeV and other parameters as above ($C_\mu = C_f = 1$, $C_{\pi^T} = 4/3$, $F_T = 82$ GeV, $\alpha_S = 0.1$, $N_{TC} = 4$). The peak signal rates approach 1 nb. The $\bar{b}b$ dijet rates are much larger than the $Z^0 \rightarrow \bar{b}b$ backgrounds, while the $gg$ rate is comparable to $Z^0 \rightarrow \bar{q}q$. Details of these and the other calculations in this section, including the effects of the finite beam energy resolution, will appear in Ref. [21]. See Ref. [22] for another example of neutral scalars that may be produced in $\mu^+\mu^-$ annihilation.
FIGURE 2. Theoretical (unsmeread) cross sections for $\mu^+\mu^- \rightarrow \rho^0_T$, $\omega_T \rightarrow e^+e^-$ for input masses $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 212.5$ GeV and other parameters as defined in the text.

The cross sections for technipion production via the decay of technirho and techniomega $s$-channel resonances are calculated using vector meson ($\gamma$, $Z^0$) dominance [3,8,19,20]. They are given by:

\[
\frac{d\sigma(\mu^+\mu^- \rightarrow \rho^0_T \rightarrow \pi A\pi B)}{dz} = \frac{\pi\alpha^2 p^3_{AB} A_\mu^0(s) C^2_{AB} (1 - z^2)}{s^{\frac{3}{2}} (s - M_{\rho_T}^2)^2 + s \Gamma_{\rho_T}^2},
\]

\[
\frac{d\sigma(\mu^+\mu^- \rightarrow \omega_T \rightarrow \gamma\pi^0_T)}{dz} = \frac{\pi\alpha^3 s^2 p^3} {3\alpha_{\rho_T} M_{\omega_T}^2 (s - M_{\omega_T}^2)^2 + s \Gamma_{\omega_T}^2} B_\mu^0(s) (1 + z^2),
\]

where $A_\mu^0$ and $B_\mu^0$ were defined in Eqs. 12 and 15, respectively. For $M_{\rho_T} = M_{\omega_T} = 210$ GeV, $M_{\pi_T} = 110$ GeV, and other parameters as above, the total peak cross sections are [21]:

\[
\sum_{AB} \sigma(\mu^+\mu^- \rightarrow \rho^0_T \rightarrow \pi A\pi B) = 1.1 \text{ nb}
\]

\[
\sigma(\mu^+\mu^- \rightarrow \omega_T \rightarrow \gamma\pi^0_T) = 8.9 \text{ nb}.
\]

The technirho rate is 20% $W^+W^-$ and 80% $W^\pm\pi^\mp_T$.

Finally, it is reasonable to expect a small nonzero isospin splitting between $\rho^0_T$ and $\omega_T$. This would appear as a dramatic interference in the $\mu^+\mu^- \rightarrow \bar{f}f$ cross section provided the FMC energy resolution is good enough in the $\rho_T-\omega_T$ region. The cross section is most accurately calculated by using the full $\gamma-Z^0-\rho_T-\omega_T$ propagator...
matrix, $\Delta(s)$. With $M_V^2 = M_V^2 - i \sqrt{s} \Gamma_V(s)$ for $V = Z^0, \rho_T, \omega_T$, this matrix is the inverse of

$$
\Delta^{-1}(s) = \begin{pmatrix}
  s & 0 & -s f_{\gamma \rho_T} & -s f_{\gamma \omega_T} \\
  0 & s - M_Z^2 & -s f_{\gamma Z_T} & -s f_{Z \omega_T} \\
  -s f_{\gamma \rho_T} & -s f_{\gamma Z_T} & s - M_{\rho_T}^2 & 0 \\
  -s f_{\gamma \omega_T} & -s f_{Z \omega_T} & 0 & s - M_{\omega_T}^2
\end{pmatrix}.
$$

Here,

$$
f_{\gamma \rho_T} = \sqrt{\frac{\alpha}{\alpha_{\rho_T}}}, \quad f_{\gamma \omega_T} = \sqrt{\frac{\alpha}{\alpha_{\rho_T}}} (Q_U + Q_D),
$$

$$
f_{Z \rho_T} = \sqrt{\frac{\alpha}{\alpha_{\rho_T}}} \cos 2\theta_W, \quad f_{Z \omega_T} = -\sqrt{\frac{\alpha}{\alpha_{\rho_T}}} \sin \theta_W (Q_U + Q_D).
$$

Then, the cross section is given in terms of matrix elements of $\Delta$ by

$$
\frac{d\sigma(\mu^+ \mu^- \rightarrow \rho_T^0, \omega_T \rightarrow \bar{f}_i f_i)}{dz} = \frac{N_f \pi \alpha^2}{8s} \left\{ \left| D_{iLL} \right|^2 + \left| D_{iRR} \right|^2 \right\} (1 + z)^2
$$

$$
+ \left\{ \left| D_{iLR} \right|^2 + \left| D_{iRL} \right|^2 \right\} (1 - z)^2,
$$

where

$$
D_{i\lambda\lambda'}(s) = s \left[ Q_i Q_\mu \Delta_{\gamma\gamma}(s) + \frac{4}{\sin^2 2\theta_W} \zeta_{i\lambda} \zeta_{\mu\lambda'} \Delta_{ZZ}(s) + \frac{2}{\sin 2\theta_W} \left( \zeta_{i\lambda} Q_\mu \Delta_{Z\gamma}(s) + Q_i \zeta_{\mu\lambda} \Delta_{\gamma Z}(s) \right) \right].
$$

Figure 2 shows the theoretical $\rho_T^0 - \omega_T$ interference effect in $\mu^+ \mu^- \rightarrow e^+ e^-$ for input masses $M_{\rho_T} = 210$ GeV and $M_{\omega_T} = 212.5$ GeV and other parameters as above. The propagator $\Delta$ shifts the nominal positions of the resonance peaks by $\mathcal{O}(\alpha/\alpha_{\rho_T})$. The theoretical peak cross sections are 5.0 pb at 210.7 GeV and 320 pb at 214.0 GeV. This demonstrates the importance of precise resolution in the 200 GeV FMC.

### III CONCLUSIONS

Modern technicolor models predict narrow neutral technihadrons, $\pi_T, \rho_T$ and $\omega_T$. These states would appear as spectacular resonances in a $\mu^+ \mu^-$ collider with $\sqrt{s} = 100$–200 GeV and energy resolution $\sigma_E/E \ll 10^{-4}$. This is a very strong physics motivation for building the First Muon Collider.

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