Extensive Test of an SU(3)–based Partial Dynamical Symmetry

R.F. Casten
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, CT USA
E-mail: richard.casten@yale.edu

Abstract. The concept of symmetries pervades much of our understanding of nature. In nuclear structure, the IBA embodies a framework with three dynamical symmetries U(5), O(6) and SU(3). Of course, most nuclei break these symmetries. Leviatan has discussed a concept of Partial Dynamical Symmetry (PDS) in which the states of the ground and gamma bands, only, are exactly described by SU(3) while all others are not. With an $E2$ operator which is not a generator of SU(3), this PDS gives a parameter–free description of $\gamma$ to ground band relative $B(E2)$ values in $^{168}$Er that is virtually identical to the best collective model (IBA) calculations with 2-3 parameters. We have carried out the first extensive study of this PDS, in 47 rare earth nuclei. Overall, the PDS works very well, and the deviations from the data are usually understandable in terms of specific kinds of mixing.

The study of nuclear structure is typically approached in two complementary ways—in terms of the interactions and orbits of the individual nucleons and in terms of the symmetries, quantum numbers and structure of the many-body system as a whole. Very broadly, the second approach is a powerful tool for telling us what nuclei do, and given a structure for a nucleus, for generating a wide variety of detailed predictions, while the first can tell us why the nuclei do what they do. This work proceeds from the second perspective, focusing on the symmetries and partial symmetries of well-deformed collective even-even nuclei.

The concept of dynamical symmetries has long been a powerful technique to interpret nuclear structure. The IBA model [1] codifies this in terms of three symmetries, U(5)—a vibrator-like structure, SU(3)—an axial rotor-like structure, and O(6)—a non-axial type of rotor. We are concerned here about SU(3). A dynamical symmetry description is characterized by specific degeneracies, quantum numbers, and selection rules. While few nuclei actually manifest an unbroken or nearly unbroken symmetry, these symmetries act as benchmarks, much like the magic numbers do for spherical nuclei. Most well-deformed nuclei somewhat resemble SU(3) but exhibit strong deviations from it [2, 3, 4]. Such nuclei have often been described in terms of numerical calculations involving a specific IBA Hamiltonian that includes symmetry breaking terms [3, 4]. One of the best studied is $^{168}$Er [3, 5]. The IBA is able to account for the low lying intrinsic excitations as well as many absolute and relative $E2$ transition rates. Seemingly, this description identified $^{168}$Er as a well deformed nucleus with strong SU(3)-symmetry breaking.

However, a number of years ago Leviatan [6] proposed the idea of a Partial Dynamical Symmetry (PDS) in which certain aspects of the parent symmetry are exactly preserved while others are not. In the application to $^{168}$Er, the ground and intrinsic excitations are exactly described as pure SU(3) excitations while all other states involve significant SU(3) breaking and the mixing of SU(3) representations.
His description of $E2$ transition rates was remarkably successful. Using a generalized $E2$ operator the predictions for relative to ground $B(E2)$ values are parameter-free but yet reproduce the data as well as the best parameterized broken-SU(3) calculations. Indeed, the PDS results are almost identical to those of the broken-SU(3) numerical IBA calculations. Of course, they are similar as well to a standard SU(3) calculation with the same operator for the $\gamma$-ground band transitions.

This surprising result raises many questions. However, before the theoretical issues are investigated it is important to know if this success is unique to $^{168}$Er or if it characterizes other well-deformed nuclei. It is the purpose of this work to address this question. To this end, we have collected all relevant data on relative $\gamma$ to ground band $B(E2)$ values in the rare earth region. In all, we studied 47 nuclei. In some, the data is insufficient for a substantial test—often relevant $E2$-$M1$ mixing ratios, or weak transitions are unknown or poorly known. However, in about 22 nuclei there are sufficient data. We present examples of the results below.

First, a word about the PDS calculations. They were carried out with the Hamiltonian of Eq. 1 of Ref. [6]. Here the Hamiltonian has two parameters, $h_0$ and $h_2$. As long as the Hamiltonian is written in the form of Ref. [6] the solutions satisfy an SU(3) PDS with pure SU(3) structure solely for the $\gamma$ and ground representations. The calculations are parameter-free for relative $\gamma$ to ground transitions (intra-band $E2$ transitions do require one overall parameter for each nucleus so we ignore them for the present purposes) but they do depend weakly on boson (valence nucleon) number. The PDS calculations were done in each case for the appropriate number of valence nucleons.

Examples of the results are shown in Figs. 1-3. Figure 1 includes $^{168}$Er as well as several other well-deformed nuclei and show truly excellent agreement between the data and the PDS. The only notable discrepancies are for a few spin-increasing transitions. Figure 2 shows other nuclei in the same region with good agreement but slightly larger discrepancies—again primarily for the same transitions. Figure 3 shows two nuclei, in transitional regions, where there are huge disagreements. These latter are, of course, not surprising since an SU(3) PDS would not be expected to match the properties of transitional nuclei. However, it is nevertheless interesting to note that the discrepancies with the data, while much larger, are for exactly the same spin-increasing transitions (increasing relative to the normalized transition) as are the smaller discrepancies for well-deformed nuclei. Moreover, the discrepancies for spherically-deformed transitional nuclei at $N = 90$ are similar to those for nuclei in the prolate to soft transition seen in the Os nuclei. However, we note that, while $^{176,180,188}$Os all show large deviations, $^{184}$Os, with slightly larger $E_{1/2} = 3.20$, shows reasonable agreement, as seen in Fig. 2.

An interesting question is why the discrepancies are specifically and nearly always for the spin-increasing transitions, especially those such as $2g \rightarrow 4g$, $3g \rightarrow 4g$, $4g \rightarrow 6g$, and $5g \rightarrow 6g$. The reason is actually simple. Spin-increasing transitions always have small values in the case of a pure rotor with an uncoupled intrinsic $\gamma$ vibration. This is seen in the Alaga rules [7] and has a simple physical origin that is easily seen if one decomposes the wave function into rotational and intrinsic parts. Here we explain this for the decay of the $4^+_1$ level. The explanation is similar for higher spin band members. The $4^+_1$ level of the ground band has rotational angular momentum $R = 4$. The $4^+_1$ level has intrinsic angular momentum 2. It has rotational angular momentum components $R = 0, 2$ and 4 but $R = 0$ dominates. Therefore an $E2$ transition to the $4^+_1$ level of the ground band with $R = 4$ is highly hindered.

Actual deformed nuclei always show deviations from the Alaga rules, which are usually interpreted in terms of $\gamma$-ground bandmixing. Such bandmixing, depending on the relative signs of the direct and admixed components of the wave function can, in general, either increase or decrease the unperturbed $B(E2)$ value. However, for unperturbed transitions that are forbidden or very small, such as the $\gamma$-ground spin increasing transitions, the mixing can only increase the
Figure 1. Comparison of the data on $\gamma$ to ground band relative $B(E2)$ values in some well-deformed nuclei with the SU(3) PDS of Ref. [6]. For each initial level, the most spin-decreasing transition (the leftmost one) is normalized to 100. All data from NDS. The symbol $<$ next to a transition bar means that the height of the bar is an upper limit. Usually this is due to an unknown $M1$ component.

$B(E2)$ value since the sign of the mixed component gets squared. Therefore, one expects, and finds, that the spin increasing transitions are nearly always stronger than the Alaga rules. This is what one sees in the data and in comparison to the PDS as well. The extent of the deviations from the PDS seen in Figs. 1-3 measures approximately the amount of such bandmixing.

I am grateful to my collaborators R.B. Cakirli and A. Couture, and to A. Leviatan, P. Van Isacker, M. Macek, and N. Pietralla for discussions of the PDS and patient explanations of the physics behind it. Work supported by the US DOE under Grant No. DE-FG02-91ER-40609.

1. References
[1] Arima A and Iachello F 1974 Phys. Lett. 53 309–12
[2] Casten RF and Warner DD 1988 Rev. Mod. Phys. 60 389–469
[3] Warner DD and Casten RF 1982 Phys. Rev. Lett. 48 1385–1389
[4] McCutchan EA and Casten RF 2006 Phys. Rev. C 74 057302(1-4).
[5] Warner DD, Casten RF and Davidson WF 1980 Phys. Rev. Lett. 45 1761–5
[6] Leviatan A 1996 Phys. Rev. Lett. 77 818(1-4)
[7] Alaga G, Alder K, Bohr A and Mottelson BR 1955 Mat. Fys. Medd. Dan. Vid. Selsk. 29 No. 9
Figure 2. Similar to Fig. 1 for nuclei showing somewhat larger disagreements with the PDS.

Figure 3. Similar to Fig. 1 for transitional nuclei which, not surprisingly, show large disagreements with the PDS.