Experimental Control of Macroscopically Large, Schrödinger’s Cat like Quantum Coherent State of Bose-Einstein Condensate of Excitons as Qubits

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Abstract

We report experimental detection and control of Schrödinger’s Cat like macroscopically large, quantum coherent state of a two-component Bose-Einstein condensate of spatially indirect electron-hole pairs or excitons. This can provide access to millions of excitons as qubits to allow efficient, fault-tolerant quantum computation. In this work, we varied the number density of photo generated and bias driven excitons by changing the applied bias voltage and photo excitation intensity. Phase coherent periodic oscillations in photo generated capacitance as a function of the bias voltage and light intensity are measured respectively over a macroscopically large area. We then argue that presence of coherent resonant tunnelling in this well-dot heterostructure strongly restricts the available momentum space of the charge carriers within this quantum well. Consequently, the measured average electric polarization vector of the associated indirect excitons spontaneously increase below ~100 K as excitonic dipoles chooses a particular phase and collectively orients along the direction of applied bias. As a result, these excitons continuously undergo Bose-Einstein condensation below a transition temperature as the density threshold is approached at regular bias intervals. Moreover, periodic presence and absence of splitting of excitonic peaks in the optical spectra based on photocapacitance confirm such tunneling induced variations in quantum coupling of electrons between quantum well and quantum dots. Observation of negative ‘quantum capacitance’ due to screening of charge carriers by the quantum well indicate periodic variation of Coulomb correlations of interacting excitons with increasing bias as a precursor to condensation and vice versa. Generation of density dependent enhancement of quantum interference beats in photocapacitance oscillation with bias even under incoherent white light further confirm the presence of stable, long range spatial correlation among these indirect excitons as well as the existence excitonic matter waves. We also detected collective Rabi oscillations of these macroscopically large, ‘multipartite’, two-level, coupled and uncoupled quantum states of excitonic condensate which can be used as qubits. Moreover, some of these photo generated excitons undergoing condensation can be addressed independently of others using applied bias voltage over a localized region. Therefore, our study not only brings the physics and technology of Bose-Einstein condensation within the reaches of semiconductor chips, but also opens up experimental investigations of the fundamentals of quantum physics using similar techniques.
I. INTRODUCTION

Experimental detection of macroscopically large, quantum coherent state within semiconductor heterostructures will open up new paradigms for novel applications in quantum optoelectronics including quantum computation. Additionally, it will allow us to explore the fundamental physics of macroscopic quantum entanglements even in the steady state and to study these emergent quantum phenomena in condensed matter physics from a radically different perspective. Here we use excitons [1] which are quasiparticles of bound electron-hole pairs as the chosen platform to execute this. These excitons are composite bosons with integer spins. So a Bose-Einstein condensate (BEC) of excitons can provide a large number of identical, two-level quantum states as excitonic qubits. This is possible because we can use applied bias over a confined region to selectively modulate the macroscopic quantum state of some photo generated excitons independently of the rest. Usually, bottom-up approaches are used in quantum computation, where one assembles individual qubits into a quantum superposition state to fabricate N-qubit quantum registers in a step-by-step manner. However, in this work we explored a top-down approach where we start with macroscopically large, quantum coherent state having millions or more dipolar excitons undergoing BEC to build N-qubit quantum registers in the temperature range of 10-100 K. This was done using an opto-electrical method to detect the experimental signatures of spontaneous enhancement of electrical polarization, observation of quantum coherence as well as quantum Rabi oscillations using capacitance measured under photo excitation (or simply termed as photocapacitance hereafter). Interestingly, the observed capacitive detection and control of quantum coherence is already within the operational range of some cryogenically cooled, commercial supercomputers. Moreover, we argue that the availability of such a large number of excitonic qubits offer better computational capabilities and sufficient redundancy [2] during
quantum error corrections. Therefore, such device level quantum control of macroscopically large excitonic BEC state can be highly rewarding on many counts.

In this study, we use a single crystal sample of a double-barrier, resonant tunneling diode to investigate excitonic BEC. In general, coherent and incoherent tunneling of electrons and holes were thoroughly explored [3-5] in the past. However, here we demonstrate a unique feature of coherent resonant tunneling in a quantum coupled heterostructure having zero dimensional (0D) quantum dots (QDs) layer and two dimensional (2D) quantum well (QW). This structural uniqueness actually helped us to identify the presence of excitonic BEC coexisting with resonant tunneling from a different perspective in the first place. Multiple peaks [6] in photocapacitance as charging of QDs [7] and correlations of excitonic complexes [8] were studied earlier. Quantum oscillations of photocurrents in such 0D-2D heterostructures were also investigated by some of us in the past [9-11]. On the other hand, here we demonstrate why photocapacitance is unique in its ability to probe the Schrödinger’s Cat like macroscopically large, multi-partite quantum state of collective electric polarization of these indirect excitons undergoing BEC. Till now, excitonic BECs were mostly reported using planar 2D structures [12-18]. However, purely optical signatures of excitonic BEC are still being debated [19,20], specifically in the context of dark excitons [21,22] which are expected to form the usual BEC ground state in most semiconductors. In contrast to previous studies, we use spatially indirect excitons within a quantum coupled 0D-2D heterostructure to show the presence of excitonic BEC using photocapacitance from many different perspectives. First, we show an uncharacteristic enhancement of photocapacitance below ~100 K as regular bias intervals. Because photocapacitance is proportional to orientational average of electric polarization of excitonic dipoles, so it clearly indicates a continuous phase transition to enhanced spontaneous electrical polarization of these excitons as their density is changed with
applied bias voltage. Then we explain that the "co-existence" of resonant tunneling and excitonic optical spectra in this 0D-2D heterostructure directly indicate momentum space narrowing of these indirect excitons. This essentially leads to this enhancement of electric polarization of excitonic dipoles, thus indicating the presence of excitonic BEC at periodic bias intervals. Areal densities of these excitons as BEC order parameter are estimated and these are shown to match well with those calculated using our photocapacitance experiments. We then provide experimental evidences of long range order associated with such excitonic BEC by interfering spatially separated groups of excitons. Quantum interference beats of excitons measured by photocapacitance over a macroscopically large area even with incoherent white light reveal the stable, long range, spatial coherence of this excitonic matter wave state. Observation of negative quantum capacitance [23-25] points to positional correlations among the charge carriers within the 2D QW at first and then subsequently among the associated indirect excitons as well. Photo response of Rabi oscillations within a single QD as a function of light intensity were previously reported [26-30] in the literature. However, here we also establish collective quantum Rabi oscillations of macroscopically large, quantum coherent states of excitonic BEC involving millions of QDs as a function of photo excitation intensity within the time scales probed in our measurements. We also explain why quantum coherence of coupled and uncoupled state of this 0D-2D heterostructures coexists with tunneling in section XII. Finally, we propose how all these can be used as qubits using a semiconductor optoelectronics based quantum computing device.
II. PHASE-COHESMENT OSCILLATIONS OF PHOTOCAPACITANCE OVER A LARGE AREA

Schematic diagram of the sample is given in Fig. 1(a). Energy Band diagram under reverse bias after photo generation of electrons and holes is illustrated in Fig. 1(b). The triangular shaped 2D QW actually forms in GaAs when the photo excited charge carriers were driven in opposite directions under the applied bias and then accumulate near the AlAs barrier. A two-dimensional electron gas (2DEG) then form inside the GaAs TQW when the quasi Fermi level for accumulated electrons on the p-type side of AlAs barrier crosses the GaAs conduction band edge in an ideal zero Kelvin picture. The AlAs potential barrier physically separates the InAs QD layer from these charges accumulated in the 2DEG or TQW. Thus indirect excitons are formed, where electrons (within GaAs 2DEG) and holes (within InAs QD) in two different layers are spatially separated by AlAs along the growth direction (\( \hat{z} \)) of the sample. These spatially indirect excitons are also shown in Fig. 1(b). One expects larger photo generations in the top, illuminated p-type side of AlAs as compared to that in the lower n-type side. As a result, a sizably larger 2DEG can accumulate in the top side under reverse bias and we ignore relatively smaller accumulation of holes on the lower side of AlAs barrier. We measure photocapacitance \([31-33]\) at 10 kHz with \( \sim200 \) μm wide ring shaped electrical top-contact covering \( \sim10^{11}/\text{cm}^2 \) of InAs QDs. Detailed information of the sample structure and experimental methods can be found in the Appendix A. In Fig. 1(c), we show photocapacitance and photo conductance (photo-\(G/\omega\)) oscillations versus bias (\(V\)) measured at 10 K. In Fig. 1(d), we find that photo-\(G/\omega\) and \(dI_{ph}/dV\) oscillations coincide, where \(I_{ph}\) is the DC-photo current. So photo generation of currents (\(I_{ph}\)) anywhere in the sample structure also affect photo-\(G/\omega\). It was also known that photocapacitance can detect \([31,32]\) the average...
electric polarization of such spatially indirect excitonic dipoles. Incidentally, at this light intensity, maxima (minima) of photocapacitance coincide with maxima (minima) of tunneling conductance as photo-$\frac{G}{\omega}$ or $\frac{dI_{Ph}}{dV}$ at the inflection points of $I_{Ph}$. So the maxima of $I_{Ph}$ are not necessarily
maximizing the bias driven accumulation of excitonic dipoles around the 0D-2D heterojunction. However, later experiments also found the relative phase between photocapacitance and photo-\(G/\omega\) oscillation peaks to be dependent on photoexcitation intensity as well. This fact certainly rules out any calibration errors in extracting these oscillations of photocapacitance and photo-\(G/\omega\) from the measured impedance using the LCR meter. Presence of periodically negative regions of photo-\(G/\omega\) also reveal the absence of any simple-minded charging/discharging of QDs. In addition, any triangular shaped quantum well (TQW) as the 2D accumulation layer in p-GaAs with finite band bending need not have so many quantized levels (usually remains < 10) occupied by electrons for producing these many (~ 20 or more) [9] oscillation peaks and valleys through successive sequential tunneling. Most importantly, the periods of such oscillations are also shown to be changing with photo excitation intensities [9]. Although the spacing between successive maxima of the observed oscillations in Figs. 1(c) and 1(d) are nearly periodic, the energy levels of any TQW are certainly not equally spaced either. Hence, it is difficult to imagine a simplistic energy level matching and sequential tunneling process as the main causes of these oscillations. Moreover, it is also unlikely that electron Fermi levels will be situated at the top edge of the GaAs TQW barrier for this 2DEG, which can contribute to successive electron escape towards the p type side with increasing reverse bias. We also expect more electron and hole accumulations near AlAs barriers and subsequently a deeper TQWs with increasing biases. A deeper TQW and successive bound to unbound transition of quantized TQW levels with electron escape from the higher levels to top p-type side would have resulted in decreasing bias gaps of successive oscillation peaks with increasing biases, which are definitely not observed in Fig. 1. Therefore, any sequential, incoherent [9] tunneling processes as the driving mechanism for these oscillations through millions of InAs QDs are not sufficient explanations of these quantum oscillations. Only coherent tunneling can
allow the phase coherence to persists in these measurements made using such large area contacts covering so many QDs. Although, the number density of photo generated and bias driven excitons in this structure is strongly dependent on light intensity as well as on the applied bias. In section VI, we will present further experimental evidences that phase coherent oscillations of photocapacitance and photo-$G/\omega$ are not necessarily linked always to tunneling induced oscillations $I_p$ but connected to quantum Rabi oscillations at a macroscopic level as well. Moreover, in section XI we will quantitatively argue why quantum coherence and tunneling coexist to allow these experimental observations in the first place.

Normally, photo generated holes will drift and accumulate near the illuminated side of the AlAs barrier under forward bias. As a result, a similar band alignment with inversed tilt [31,32] is produced under forward bias due to significant accumulation of holes in the top, illuminated p-GaAs side of AlAs barrier. Similarly, a two-dimensional hole gas (2DHG) forms when the quasi Fermi level for holes accumulated on the p-type side of AlAs barrier crosses the respective GaAs valence band edge under forward bias. Tunneling rate of heavier holes are expected to be much less compared to the lighter electrons. Although photocapacitance oscillations are also less pronounced and more widely spaced in reverse bias [Fig. 1(e)] as compared to forward bias [Fig. 1(f)]. However, it was known that peak-to-valley ratio of Fabry-Perot oscillations of charge carriers during coherent resonant tunneling [34] in a double barrier structure can be larger if barrier transmission probabilities are smaller. This explains the observed enhancements of photocapacitance oscillations under forward bias as compared to reverse bias. As such, observations of these well-formed oscillations measured over such a large area evidently require the presence of long-range phase matching during coherent resonant tunneling through so many InAs QDs. This will be elaborated further in sections III-VI. Moreover, these oscillations survive
even when excited below 1.61 eV, but fully subside for photo excitations below GaAs bandgap at 10 K. It is also interesting to see that maxima and minima of photocapactance oscillations are shifting in both Figs. 1(e), 1(f) with photo excitation wavelengths as well.

III. SPONTANEOUS ENHANCEMENT OF ELECTRIC POLARIZATION AND WHY RESONANT TUNNELING IN 0D-2D HETEROSTRUCTURE INDICATES EXCITONIC BEC

Capacitive responses usually freeze as one lowers the temperature of any solid sample. However, we observed uncharacteristic enhancements of photocapitance and photo-G/ω oscillations for temperatures below ~100 K in Figs. 2(a) and 2(b), respectively. Although peak positions shift with changing temperatures but the period of these oscillations is independent of temperature. This suggests [9] that the oscillations arise from quantum effects. Surface charge density of indirect excitons (σ_{ph}) can be estimated [31] as \( σ_{ph}e = CV = \langle \vec{P} \rangle .2 \), where \( e \) is the electronic charge and \( C \) is photocapitance per unit area at each bias \( V \) and \( \langle \vec{P} \rangle \) is the orientational average of electric polarization vector of these indirect excitonic dipoles. We know that photogenerated charge carriers are bias driven towards the AlAs barrier to form indirect excitons. As a result, the number density of these indirect excitons also increase with increasing bias voltages. However, we observe that \( \langle \vec{P} \rangle \) or \( σ_{ph} \) not just simply increases below ~100 K with increasing accumulation of excitons, but it begin to oscillate with larger magnitudes with decreasing temperatures. We will investigate the cause of such phase coherent oscillations in later sections V-VII. Interestingly, there is a sudden, unusual increase of maximum oscillation magnitude of \( \langle \vec{P} \rangle \) or \( σ_{ph} \) from ~1.067×10^{11}/cm² around 70 K to ~1.099×10^{11}/cm² at 10.5 K as
FIG. 2. Sudden uncharacteristic enhancement of photocapacitance oscillations and spontaneous enhancement of electric polarization at certain bias values below ~100 K. (a) Photocapacitance and (b) Photo-G/ω oscillations at different temperatures. Here we used selective photoexcitation centered around 630 nm using a halogen lamp. (c) Variation of peak oscillation magnitudes at -0.4 V are shown as a function of temperature. Instead of usual dielectric freezing at lower temperatures, photocapacitance as orientational average of electric polarization of these 0D-2D excitonic dipoles suddenly increase below ~100 K, indicating a continuous phase transition. Percentage of excitons undergoing this phase transition increases with decreasing temperatures. (d) Summary of the above experimental observations using a schematic diagram of interacting dipoles of indirect excitons during BEC phase transition in this quantum coupled 0D-2D heterostructure as they try to collectively align towards the z direction. This is thereby leading to momentum space narrowing and consequent spontaneous enhancement of electric polarization \( \langle \vec{P} \rangle \) of indirect excitons whenever the resonant tunneling condition is fulfilled at periodic bias intervals. The red contours on the left represent the wave functions of individual indirect excitons before the onset of BEC below ~100 K. The large wave function on the right ‘roughly’ represents the macroscopic quantum state of excitonic BEC once it is formed.
shown in Fig. 2(c). This is understood as the bias (density) dependent onset of spontaneous electric polarization as excitonic dipoles collectively orient along the \( \hat{z} \) direction as we lower the temperature [as depicted in Fig. 2(d)]. Thus, the measured \( \langle \vec{P} \rangle \) as the orientational average of electric polarization vector of these indirect excitonic dipoles also increase in magnitude at specific bias values. Such increase in the oscillation magnitude of \( \langle \vec{P} \rangle \) below \( \sim 100 \) K indicates a continuous phase transition of the excitonic system whenever the excitonic dipoles choose a particle phase or angle and align collectively along one particular direction at a particular bias. In addition, the contrast of these phase coherent oscillations as \( \sim \frac{(C_{\text{Max}}-C_{\text{Min}})}{(C_{\text{Max}}+C_{\text{Min}})} \) acts like a measure of first order correlation which increases with decreasing temperature. We expect the cause of this increase in \( \langle \vec{P} \rangle \) to be excitonic BEC whenever the resonant tunneling conditions are fulfilled at periodic bias intervals as will be explained in detail in the next paragraph. This claim will be further substantiated with experimental observations of density dependent quantum interference of excitonic matter waves section V and with the observation of Rabi oscillations in subsequent section VI.

We assume that these well-formed, sharp oscillations of photo electrical responses with applied bias can’t be due to non-resonant tunneling having broad distribution of allowed energies as also discussed in the section II. However, resonant tunneling [35,36] of electrons between 2DEG and InAs QDs in reverse bias requires

\[
E_{QD}^e(V, I) = E_{2DEG}^e(V, I) + \left[ \frac{\hbar^2}{2m^*} (\vec{k}_x^2 + \vec{k}_y^2) + \Phi_{EX_{xy}}^e(V, I) \right] + (eV),
\]

where \( E_{QD}^e, E_{2DEG}^e \) are respective ground-state energies, \( \vec{k}_x, \vec{k}_y \) are in-plane momentum of 2DEG electrons, \( \Phi_{EX_{xy}}^e(V, I) \) is the net Coulomb interaction energy of these 0D-2D indirect excitons in the x-y plane and \( I \) is the light intensity used for photoexcitation. Therefore, resonant tunneling at \( V(z) = V_0, I = I_0 \) is possible, only if

\[
\left[ \frac{\hbar^2}{2m^*} (\vec{k}_x^2 + \vec{k}_y^2) + \Phi_{EX_{xy}}^e(V, I) \right]
\]

of all these
excitons remain identical. Moreover, for a direct photo excitation of indirect excitons at the minimum of Brillouin zone, excitonic momentum \( \vec{K}_{\text{Exciton}}(V_0) \) must also satisfy \( \vec{K}_{\text{Exciton}}(V_0) = \vec{k}_{e^{2\text{DEG}}}(V_0) + \vec{k}_{h^{\text{InAs}}}(V_0) \approx 0 \). So only those 2DEG electrons with an unique \( \vec{k}_{e^{2\text{DEG}}}(V_0) \) can form 0D-2D indirect excitons and simultaneously take part in coherent resonant tunneling with a constant phase \[ \theta = \vec{k}_{e^{2\text{DEG}}}(V_0) \cdot \vec{Z}_{\text{InAs}}, \]
assuming nearly monodisperse InAs QDs, specifically in the \( \vec{z} \) direction. Otherwise, any random phase differences during any sequential, incoherent tunneling through millions of InAs QDs could have averaged out these oscillations measured over an \( \sim 200 \mu \text{m} \) wide area. The sudden enhancement of \( \langle \vec{P} \rangle \) indicated in Fig. 2(c) also provides us the crucial evidence of momentum-space narrowing associated with bias dependent excitonic BEC. This happen when coherent resonant tunneling necessarily compels \( |\vec{k}_{e^{2\text{DEG}}}(V_0, I_0)| \rightarrow |\vec{k}_{h^{\text{InAs}}}(V_0, I_0)| \) and subsequently \( \langle \vec{P} \rangle \) orient towards the \( \vec{z} \) direction periodically with increasing voltage bias. Any slight variation of energies due to size distribution effects in QDs and in QW can be compensated by this local variations of \( \Phi_{\text{Ex}}^{xy}(V, I) \) in the x-y plane. As long as these small local variations doesn’t not affect the final BEC ground state having a long range order, we expect to see phase coherence among these excitons spread over this macroscopically large area. Further, experimental evidences of these cooperative effects will be discussed in sections V, VI and VII. This fine tuning of resonant tunneling and exciton BEC is achieved periodically as a function of increasing bias and \( \langle \vec{P} \rangle \) spontaneously increases till that resonant condition is reached and then decrease below the level of dark capacitance (Fig. 2(a)) as well. This is because the increase in bias not only increase the number density of excitons accumulated around the heterojunction, but also increase the excess accumulation of charge carriers until these excess charges tunnel out of
the heterojunction and the resonant tunneling conditions are satisfied again. Thereafter, the excitonic system recovers and again head towards the next maximum of $\langle \vec{P} \rangle$ with increasing bias.

It also should be noted that the spatial anchoring of these indirect excitons with InAs QDs are crucial to prevent excitonic Mott transitions. In a way, these QDs act as localized trapping potentials for excitons to sustain this BEC. Moreover, the estimated exciton densities ($\sim 10^{11}$/cm$^2$) as BEC order parameter match well with those calculated using our photocapacitance experiments. The thermal de-Broglie wavelength (around 100 nm at 10 K) of these indirect excitons easily exceeds the average $\sim 11 \pm 2$ nm spacing between the neighboring InAs QDs with surface density $\sim 10^{11}$/cm$^2$. Percentage of photo generated excitons undergoing excitonic BEC also increases with decreasing temperatures as shown in Fig. 2(c). We will now discuss additional details on why this excitonic BEC is observable at these parameter ranges along with the measured exciton densities in the next section.

IV. FEASIBILITY OF EXCITONIC BEC: ORDER PARAMETER AND EXPERIMENTAL ESTIMATES

An estimate of the critical surface density of 2D bosons as the order parameter [37] for BEC is given by $N^{2D}_{BEC} = 2 \left( \frac{1}{\lambda_{Th}} \right)^2 \ln \left( \frac{L}{\lambda_{Th}} \right)$, where $\lambda_{Th}$ is the thermal de-Broglie wavelength for indirect excitons in this case, $L$ is the lateral extent ($\sim 200$ µm) of the 2DEG being probed in x-y plane. Using $m^*$ for excitons in GaAs as 0.058$m_0$ where $m_0$ is the free electron mass, we find $\lambda_{Th} = \left( \frac{2\pi\hbar^2}{m^*k_BT} \right)^{1/2} \approx 100$ nm for $T = 10$ K, where $k_B$ is Boltzmann constant. Using this value, we obtained the threshold density for 2D excitons as $N^{2D}_{BEC} \sim 1.52 \times 10^{11}$/cm$^2$. Even this estimate is
This supports excitonic BEC around 100 K for indirect excitons with areal density $\sim 10^{11}/\text{cm}^2$ in this quasi 2D heterostructure of III-V semiconductors. Close to the measured dipolar density $\sigma_{ph}$ of 0D-2D indirect excitons at 10 K. Moreover, this $\lambda_{Th} \approx 100 \text{ nm}$ at 10 K is much bigger than the $\sim 11 \text{ nm}$ average in-plane separation between neighboring InAs QDs with surface density $\sim 10^{11}/\text{cm}^2$ (see Appendix A, Fig. 9). It is likely that actual BEC coherence length can be even larger and wave functions of these quantum coupled, 0D-2D indirect excitons in fact begin to overlap around 100 K. Interestingly, these quantum coherent oscillations slowly decohere and gradually vanish with enhanced charge accumulations under increasing DC bias voltages [Figs. 1(c)-1(f)] and also at higher photo excitation intensities [9]. These quantum oscillations certainly vanish with increasing temperatures above $\sim 100 \text{ K}$ [Fig. 2].

Critical density of BEC in this quasi-2D system can also be estimated using [38]

$$ n_{EX} \sim N_{BEC}^{2D} = -\left(\frac{1}{\lambda_{Th}^2(T_c^{BEC})}\right) \ln \left(1 - \exp\left(-\frac{|\varepsilon_0|}{k_BT_c^{BEC}}\right)\right). $$

In Fig. 3, we plot the critical temperature
for attaining excitonic BEC as a function $n_{EX}$, where $|\varepsilon_0| = 6.8$ meV is assumed to be the excitonic binding energy $[38, 39]$ in such III-V material systems. We also plot the following Berezinskii-Kosterlitz-Thouless (BKT) $[20, 40]$ transition temperature ($T_c^{BKT}$) in Fig. 3, where $a_0$ is the Bohr exciton radius. $a_0$ is assumed to be $\approx 10$ nm for $|\varepsilon_0| = 6.8$ meV and $k_B T_c^{BKT} = (\hbar^2/2m^*)4\pi n_{EX}^{\alpha_0^2}/ln(ln(1/n_{EX}a_0^2))$. We see that $T_c^{BKT} < T_c^{BEC}$, but both are still around 100 K or more for $n_{EX} \sim \sigma_{ph} \sim 10^{11}$/cm$^2$. This temperature of $\sim 100$ K is already well within the operational range of liquid nitrogen based cryogenics being used already for some commercial supercomputers.

Moreover, the areal density of these indirect excitons as BEC order parameter is also the limiting value of first order spatial correlation function $g_1(r)$ such that $[41, 42] n_{EX} \sim \sigma_{ph} \equiv \lim_{r \to \infty} [g_1(r)].$ We already estimated this as $n_{EX} \sim \sigma_{ph} \sim 10^{11}$/cm$^2$ even when $r \to 200 \mu m$. So, we expect the interacting Bose gas of dipolar, indirect excitons to exhibit quasi long-range order over such a macroscopically large area. Although we have not yet directly measured this $g_1(r)$, which is currently beyond the scope of this study. However, we will demonstrate that photocapacitance of two spatially separated ensembles of excitons generated even with white light can coherently interfere when light spots overlap. This will be shown in the next section V. Moreover, it is known that parallel configurations of excitonic dipoles usually support stable dipolar BECs $[43]$. Repulsive dipolar interactions between excitons also reduce $[44]$ long-range density fluctuations. Therefore, we predict that this BEC state of dipolar, indirect excitons can be further stabilized $[43]$ at even higher temperatures ($T_0$) with InAs layer having many more QDs and/or using materials having large excitonic binding energies ($E_b$) such that $E_b >> k_B T_0$. Moreover, it is also known $[45]$ that the critical temperature for any two-component excitonic BEC tends to be higher than that of any one-component excitonic system due the decrease in the net reduced mass over the one
component system. However, precise identification of BEC/BKT of these excitons and knowing
the exact fraction of dark [21,22] to bright excitons is currently beyond the scope of this study.

V. EVIDENCES OF LONG RANGE SPATIAL ORDER AND DENSITY DEPENDENT
CHANGE IN PHOTOCAPACITANCE

We tried to interfere two partially separated clusters of photo generated excitons as the
macroscopically large light spot is moved sideways across the ring shaped top contact [Fig. 4(a)].
Correspondingly, we observe gradual formation and decay of quantum interference ‘beats’ of
photocapacitance as function of applied bias in Fig. 4(b) as a result of phase coherent interference
between two macroscopically large groups excitons separately excited with different light spots.
These beat patterns are formed whenever spatially fragmented ensembles of excitons coherently
diffract across the opaque ring with a different in-plane Coulomb correlation energy \( \Phi_{xy}^{EX}(V, I) \)
and consequently oscillate with slightly different frequencies as a function of applied bias. This
results in somewhat different ‘stiffness’ (or the equivalent ‘spring constant’) for two different
groups of excitons undergoing interference. In Fig. 4(c), the spot size of laser beam was increased
slowly beyond the ring periphery to see similar formations of interference beats in
photocapacitance. Additional observation of interference beats even with incoherent white light
[Fig. 4(d)] prove that the phase ‘coherence’ witnessed by these photocapacitance oscillations is
not originating from the light source, but it is purely due to the formation of coherent, excitonic
matter wave states. In addition, the contrast as \( \sim \frac{(C_{\text{Max}}-C_{\text{Min}})}{(C_{\text{Max}}+C_{\text{Min}})} \) of these phase coherent interference
oscillations as a measure of first order correlation actually increases [Fig. 4(d)] with increasing
exciton density which also increases periodically with increasing light intensities. So this certainly
FIG. 4. Observation of quantum interference beats as evidence of long-range spatial coherence of excitonic BEC formed over a macroscopically large area at 10.5 K. (a) Schematic representation shows how the 633 nm, red laser spot is moving from left to right across the ring-shaped top electrical contact of diameter ~200 μm. Spatially fragmented light spots (red) create two separate groups of excitons which then interfere across the vicinity of gold ring (black) of width ~25 μm. (b) Formation and decay of quantum beats in photocapacitance are seen when spatially separated excitonic ensembles are interfered. (c) Spot size of the laser beam is gradually increased and shifted beyond the ring by moving a lens to see the formation of quantum interference beats in photocapacitance. (d) Observation of quantum interference beats even with incoherent white light rules out polarization interference from light and indicates the creation of phase coherent excitonic BECs with growing spot size at increasing intensities. (e) Most importantly, quantum interference beats with high contrast is also seen when two separate, uncorrelated light sources (633 nm and a white light) are used to photo excite the sample simultaneously.
indicates a density dependent phase transition to excitonic BEC state. Most interestingly, in Fig. 4(e), we see quantum interference beats with high contrast when a continuous wave (CW) laser beam of 633 nm and a purely white light source are used simultaneously. Intensities of these light sources were adjusted to get similar photocurrent values. Oscillatory changes in photocapacitance beats at simultaneous photo excitations are clearly much larger than the simple algebraic sum of photocapacitance produced by individual light beams once the dark background is subtracted. As such, interference of two independent, pulsed lasers can happen only within times scales smaller than their coherence times. So observation of interference beats from two different macroscopic populations of excitons simultaneously excited with 633 nm laser and incoherent white light again proves that the origin of phase ‘coherence’ is not coming from the light source, but it is purely due to the formation of phase coherent, matter wave states of excitons. We also understand that such phase correlation among two groups of excitons is a direct measure of the off-diagonal long range order (ODLRO) [46] of any quantum entangled state of BEC of excitons. Such interference, therefore, corresponds to linear superposition of quantum states of two excitonic BECs photo excited with unrelated light sources over two macroscopically large but overlapping regions. Further experimental evidences of such quantum superposition will be presented in the next section VI to substantiate this claim. Therefore, these quantum interference beats of photocapacitance provide direct experimental confirmations of not only long range, but also long lived quantum correlations between two separate groups of excitons along the x-y plane. However, such density dependent beat patterns are absent in photo-G/ω [see Appendix B, Fig. 10]. This is because longitudinal photo-G/ω tends to follow bulk dI_ph/dV sequentially as shown above in Fig. 1(d). It will be discussed again in the section VII. Moreover, it also highlights the importance of
photocapacitance in detecting such long-range, cooperative phenomena of excitons in the transverse x-y plain of this quantum coupled heterostructure.

VI. RABI OSCILLATIONS OF EXCITONIC BEC STATE INVOLVING MANY QUANTUM DOTS

Exciton density in our sample are changed with both increasing bias voltage as well as with increasing light intensity. Till now, we have reported oscillations of photocapacitance with respect to applied bias. We also argued that spontaneous and collective enhancement of electric polarization of excitons happen whenever the resonant tunneling conditions are fulfilled as a function of applied bias. In this section, we will now study the effect of light intensity variations on photocapacitance and photocurrent at a fixed applied bias. Any evidence of coherent oscillations of photocapacitance independent of resonant tunneling can further indicate that these are related to quantum Rabi oscillations of excitonic BEC state at a macroscopic level. As such, excitonic Rabi oscillation versus photoexcitation amplitude [~square root of light intensity] were mostly reported [26-30] using a single QD only. However, in Figs. 5(a) and 5(b), we show photocapacitance and photo-G/ω oscillations even as functions of (I)−0.5 (which is also proportional to time) involving ~millions of InAs QDs, where \( I = |\vec{E}|^2 \) and \( |\vec{E}| \) is electric field amplitude of ‘continuous’ photoexcitations with a 633 nm He-Ne laser. On the other hand, Fig. 5(c) shows that photocurrent increases monotonically, which hardly oscillates with increasing light intensity. This is because of the strong pinning of energy levels with applied bias voltage, which does not allow any changes in the condition for resonant tunneling. Therefore, such oscillations of photocapacitance and photo-G/ω as a function of increasing light intensity are certainly not due to
any sequential tunneling events as a result of energy level matching of InAs QDs and GaAs 2DEG when the quantized levels come into and out of resonance with each other.

It is known [47-49] that even with such ‘continuous’ excitations, two-level quantum systems can be driven into a stable, steady state, entanglement even in the presence of decoherence. As a result, oscillations of photocapacitance and photo-G/ω can be interpreted as collective Rabi oscillations of ‘macroscopically’ large, two-level, coupled and uncoupled BEC quantum states of 0D-2D indirect excitons as $\psi_{\text{Coupled}}^{EX}$ and $\psi_{\text{Uncoupled}}^{EX}$. Details of these quantum states will be described in the section IX. As such, the Rabi frequency is $\Omega_R = |\tilde{\mu}_{12}, \vec{E}/\hbar|$ and pulse-area is $\Theta = |\tilde{\mu}_{12}, \int_0^{\Delta \tau} \vec{E}(t) \left(\frac{\hbar}{\epsilon_{12}}\right)| = |\tilde{\mu}_{12}, \vec{F}|$, where $\tilde{\mu}_{12} \approx \tilde{\mu}_{12}(V, I)$ is the electric dipole matrix for excitonic
transition which is affected by both applied bias and light intensity, $\Delta \tau$ is the duration probed by our measurements. Usually Rabi oscillations are seen with pulsed lasers at times much shorter than dephasing time ($\tau_y$). Therefore, these experimental observations clearly illustrate negligible dephasing rate ($\propto \frac{1}{\tau_y}$) of such that $\Theta \gg \tau_y^{-1}$. In general, dephasing rates $\frac{1}{\tau_y}$ for solids are around $\sim 10^{12}$ Hz. Therefore, observed Rabi oscillations in the form of photocapacitance as a function of light intensity further validates the presence of excitonic BEC state having negligible dephasing rates even at this temperature and time scales probed in our measurements. Except for the temperature threshold, we have not seen any sharp onset of these oscillations at just only one particular exciton density. This is partly because of the involvement of resonant tunneling in this structure having a mixture of both excitons and bias driven accumulation of excess electrons/holes. This will be elaborated further in the next section. However, observation of phase coherent, density driven interferences [in both Figs. 4, 5] are definitely a signature of macroscopic phase coherence of excitonic matter waves probed with photocapacitance. This also supports the observation of spontaneous enhancement of oscillation magnitudes of electric polarization of the excitons below $\sim 100$ K indicating excitonic BEC in section II. We will also provide quantitative estimates on why quantum coherence sustains under resonant tunneling in this structure in section XI.
VII. NEGATIVE QUANTUM CAPACITANCE & IN-PLANE COULOMB CORRELATIONS

Oscillations of ‘negative’ quantum capacitance ($C_q$) [23-25] under reverse bias are shown in Fig. 6. Here we assume that the estimated quantum capacitance, due to the screening effect of the accumulated 2DEG, is in series with the dark capacitance to produce the measured photocapacitance. Holes confined within InAs QDs attract the electrons in 2DEG to create indirect excitons which are localized around these QDs. These dipolar excitons repel each other in the x-y plane as well. Formation of these indirect excitons yield negative $\left(\frac{d\mu}{dn}\right)$ and subsequently generate ‘negative’ quantum capacitance as $1/C_q \sim \left(\frac{d\mu}{dn}\right)$, where $n$ is the number of indirect excitons and $\mu$ is excitonic chemical potential such that $na_B^2 < 1$ as well as $nd^2 < 1$, where $n \sim 10^{11}/\text{cm}^2$ (estimated in section III), $a_B$ for electrons in GaAs as 11.6 nm and $d$ is the length of indirect excitonic dipoles ~5-65 nm as the width of the undoped region on p-type side. As a result, we can conclude that the ‘negative’ quantum capacitance produce ‘checkerboard’ like positional correlations between electrons of the 2DEG as a 2D Wigner lattice [13,23] formed over these InAs QDs. In consequence of this many-body cooperative effect in the x-y plane, these 2DEG electrons ‘collectively’ oscillate in the Coulomb potential valleys ($\varphi_{xy}^e(V)$) around each InAs QDs. Such Wigner crystallization actually serves as a precursor for the excitonic BEC and vice versa by spontaneously enhancing the collective electric polarization ($\langle \vec{P} \rangle$) of associated indirect excitons at periodic bias intervals. This aspect will now be elaborated in the next paragraph. In earlier works [24,25], Landau quantization under strong magnetic field lead to localization of electrons. Here, formation of excitons localize the 2DEG electrons around the InAs QDs.
As a whole, generation of these indirect excitons lower the overall potential energy of this excitonic system. So indirect excitons are formed at the minima of such Coulomb potential valleys ($\Phi^{EX}_{xy}$) localized near the InAs QDs. When more electrons accumulate in 2DEG with increasing reverse bias, then the in-plane coulomb correlation of these electrons get perturbed. Subsequently, a similar fate also awaits the associated 0D-2D indirect excitons as well. Therefore, increasing bias periodically drives the excitonic system away from the potential energy minima of in-plane interaction energy ($\Phi^{EX}_{xy}$) of these excitons as a consequence of repulsive Coulomb interactions among the 2DEG electrons. Thereafter excitonic system again comes back to the minima of $\Phi^{EX}_{xy}$ when these excess 2DEG electrons tunnel through the 0D-2D heterojunction. So this spatial organization due to Coulomb correlation of 2DEG electrons and subsequent formation of indirect excitons repeat periodically in sync with the coherent resonant tunneling. As a result, the above
mentioned self-organization of indirect excitons also periodically ‘crystallize’ in the x-y plane and subsequently melt away with increasing bias voltages.

We have already mentioned that holes accumulate inside a similar TQW formed near the illuminated, top p-type side of AlAs barrier under forward bias. Being heavier and less mobile, holes have more potential energy of Coulomb correlation \( (\varphi^h_{xy}(V)) \) than their kinetic energy. Thus holes form a much ‘stiffer’ lattice of indirect excitons in the x-y plane. It is similar to having a much larger equivalent spring constant for oscillations as compared with electrons under reverse bias. That explains a smaller negative quantum capacitance [inset of Fig. 6] for holes and why measured photocapacitance oscillations are more pronounced and more closely spaced under forward bias as compared to reverse bias (Figs. 1(e) and 1(f)). We have already explained in the above section II why long range cooperative effects of resonant tunneling between 0D and 2D reservoirs can dictate the BEC of these excitons which manifest itself in terms for density driven interference of matter waves of excitons over a macroscopically large area as well as the onset of spontaneous electric polarization below ~ 100 K. So, in principle, there could also be a formation of Wigner supersolid [52] as well. In practice, spatial arrangements of these self-assembled InAs QDs are not exactly periodic. However, precise experimental evidence of any translational symmetry breaking in the x-y plane can be somewhat intricate and currently beyond the scope of this study.
VIII. PERIODIC SPLITTING OF EXCITONIC PHOTOCAPACITANCE SPECTRA AND 0D-2D QUANTUM COUPLING

In Fig. 7, optical spectra measured with photocapacitance and photo-\(G/\omega\) are shown for two successive bias values of maximum (-0.40 V) and minimum (-0.53 V) of photocapacitance or \((\langle \hat{P} \rangle)\) oscillations. These bias values were taken from Fig. 1(c). It is important to note that, these spectra were measured using similar photoexcitation intensities used to observe oscillations described in Fig. 1. We observed two resonant excitonic transitions centered at \(\sim 1.53\) eV and at \(\sim 1.61\) eV in Fig. 7(a). However, these two sharp excitonic peaks are absent only from the photocapacitance spectra in Fig. 7(b). Instead, we see a somewhat broader peak in the middle. Interestingly, these excitonic peaks are barely seen at zero applied bias [Fig. 7(c)]. This clearly shows that these two spectral peaks are not predominantly originated from the optical generation of direct excitons within InAs QDs and also in GaAs layers. We also see similarly sharp splitting at all bias voltages of maximum photocapacitance or \((\langle \hat{P} \rangle)\) oscillations as marked in Fig. 1(c) [see Fig. 11 in Appendix B]. However, these excitonic peaks survive in the photo-\(G/\omega\) spectra at both biases. There we will also connect these periodic changes in the photocapacitance spectra of excitons with quantum coupling of 0D and 2D electrons during resonant tunneling under reverse bias and how all these can be modelled as quantum gate operations.

Because photocapacitance is always sensitive to average electric polarization [31] of these indirect excitonic dipoles, so it detects the collective modulation of electric polarization of these excitons as a result of periodic reorganization of the excitons with applied bias. As a result, we observe these photocapacitance oscillations and sharpening of excitonic spectra at specific bias
FIG. 7. Periodic presence and absence of sharp splitting of photocapacitance spectra at maximum and minimum of photo capacitance oscillation 10 K. (a) Photocapacitance and photo-G/ω spectra under -0.40 V bias corresponding to maximum of photocapacitance oscillation as shown in Fig. 1(c). We observe the presence of two sharp excitonic peaks around 1.53 eV and 1.61 eV in both spectra. (b) Photocapacitance and photo-G/ω spectra under -0.53 V bias corresponding to a minimum of photocapacitance oscillation as shown in Fig. 1(c). Unlike the photo-G/ω, those two excitonic peaks of Fig. 7(a) are strikingly missing only from the photocapacitance spectra in Fig. 7(b). Also see Fig. 11 in Appendix B. (c) Excitonic peaks are barely visible under zero bias at 10 K, which indicate the critical roles of the applied bias in generating these indirect excitons at these photo excitation intensities.

voltages whenever the resonant tunneling conditions are met and subsequently exciton BECs are formed as explained in sections II and III. On the other hand, whenever these indirect excitons either collectively assemble or disassemble in to a Bose condensate state around the QDs with changing biases, these also produce corresponding changes in photo current I_{Ph}. So, spectral signatures of optical generation of indirect excitons survive in the photo-G/ω (≈dI_{Ph}/dV) spectra at both biases in Fig. 7. We will again address this anomalous behavior of optical spectra measured with photocapacitance and photo-G/ω in the next section IX.
IX. TUNNELING INDUCED 0D-2D QUANTUM COUPLING MODELED AS QUANTUM GATE OPERATION OF EXCITONIC QUBITS

We will now explain how quantum tunneling in 0D and 2D heterostructure and associated mixing of macroscopically large, quantum states of excitonic BEC can be modeled as a Hadamard quantum Gate operation. Due to their smaller effective mass, electrons take part in quantum tunneling more efficiently than holes. As a result, we assume that the usual Hadamard Operator

$$\hat{H}(V, I) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

acts only on electron wave functions ($\phi_L^e$ and $\phi_R^e$) and not on hole wave functions ($\phi^h$) of these excitons, where subscripts $L \rightarrow$ InAs QD and $R \rightarrow$ GaAs 2DEG under reverse bias. If excitonic wave function is

$$\phi_{EX}(r_e, r_h) = \frac{1}{\sqrt{N}} \sum_{k_e, k_h} C(k_e, k_h) \phi_{k_e}(r_e) \phi_{k_h}(r_h),$$

then relevant excitons are: (a) direct excitons inside InAs QD as $\phi^{DX} = \phi_L^e \phi_R^h$ and (b) indirect excitons with holes inside InAs QD and electrons inside GaAs 2DEG as $\phi^{IX} = \phi_R^e \phi_L^h$. Here the summations over $k_e$ and $k_h$ in $\phi_{EX}(r_e, r_h)$ drop out during BEC as $|\vec{k}_{e2DEG}(V_0, I_0)| \equiv |\vec{k}_{nInAs}(V_0, I_0)| = k_0$ (constant). Ground state energy of 2DEG and consequently that of indirect excitons (IX) vary with $\phi(V)$. Following the classic example of a double well potential, capacitive coupling of 2DEG and the QDs at bias values of maximum photocapacitance oscillation can be written as

$$\hat{H}\psi_{Uncoupled}^{EX} = \hat{H} \begin{pmatrix} \phi_{DX}^{EX} \\ \phi_{IX}^{EX} \end{pmatrix} = \psi_{Coupled}^{EX},$$

where $\phi_{+}^{EX} = \phi_R^e \phi_L^h = \frac{1}{\sqrt{2}} [\phi_L^e + \phi_R^e] \phi_L^h$ and $\phi_{-}^{EX} = \phi_R^e \phi_L^h = \frac{1}{\sqrt{2}} [\phi_L^e - \phi_R^e] \phi_L^h$ are the entangled states of 0D-2D indirect excitons. These quantum coupled states have energies $E_+, E_-$. Experimentally, we identify this energy splitting as $|E_+ - E_-| \approx 100 \text{ meV}$. These observations are presented in Fig. 7(a) and also in Figs. 11(a), 11(b). There we only see these two sharply split excitonic peaks in the photocapacitance spectra measured at biases corresponding to maximum quantum coupling as measured with photocapacitance [these
voltages are marked in Fig. 1(c)]. It is also to be noted that the line widths of these two excitonic peaks are small compared to splitting of 100 meV as well. The $\hat{H}(V, I)$ acts back on two component BEC state of $\psi_{\text{Coupled}}^{\text{EX}}$ to yield $\psi_{\text{Uncoupled}}^{\text{EX}}$ at biases of minimum photocapacitance coupling and sharply split excitonic peaks subsequently vanishes from the photocapacitance spectrum [Fig. 7(b) and also in Appendix Figs. 11(c) and 11(d)] leaving aside a broad peak $\sim$1.55 eV. This broad composite peak originates because photo generations in both p and n type GaAs layers, GaAs 2DEG as well as in InAs QDs are contributing to the bias driven accumulation of these 0D-2D indirect excitons around the AlAs potential barrier. We had mentioned above in section VII that these bias driven reorganization of indirect excitons also produce corresponding changes in photo current $I_{\text{Ph}}$. As a result, spectral signatures of optical generation of indirect excitons remain intact in the photo-$G/\omega (-dI_{\text{Ph}}/dV)$ spectra at both biases [as shown in Figs. 7 and 11].

IX. WHY PHOTOCAPACITANCE CAN DETECT

A MACROSCOPIC QUANTUM COHERENT STATE?

It is not always advisable to visualize a physical situation in terms of classical picture of excitons as localized electron-hole pairs, as shown in Fig. 1(b), specifically when we are dealing with quantum phenomena like excitonic BEC, quantum coupling of QDs and 2DEG, coherent tunneling and quantum capacitance etc. Oscillatory changes in photocapacitance $(C)$ actually represent the variations of quantum mechanical average $(\langle \hat{P} \rangle)$ of the vector operator for electric polarization $\hat{P}$ of these dipolar excitons in the BEC quantum state. One can write this as $\langle \hat{P} \rangle = Tr[\hat{P} \rho(t)] = Tr[\hat{P} \exp(-i\hat{h}t/\hbar)\exp(i\vec{F} \cdot \vec{P})\rho(t_0)\exp(-i\vec{F} \cdot \vec{P})\exp(+i\hat{h}t/\hbar)]$, where $[53] \rho(t)$ is the density matrix of the BEC state and $\hat{h}$ is the interaction Hamiltonian under rotating-wave
approximation. If the dipole moment operator of j\textsuperscript{th} exciton in BEC quantum state is \( \hat{p}_j \sim (\hat{\mu}_{12})_j \) and \( \hat{P} = \sum_j \hat{p}_j \), then \( \exp(-i\vec{F}.\hat{P}) = \exp(-i\vec{F}.\hat{p}_1)\exp(-i\vec{F}.\hat{p}_2)\ldots\exp(-i\vec{F}.\hat{p}_n) \). So \( \langle \hat{P} \rangle \) is factorable in terms of ‘n’ multi-partite, quantum operators. Therefore, photocapacitance oscillations, in principle, detect the n-particle quantum correlations necessary [54,55] to establish the macroscopically large ‘Schrödinger’s Cat’ like quantum state of excitonic BEC. This is certainly evident from the density driven quantum interference of photocapacitance oscillations shown in Figs. 4(d) and 4(e) as well as the Rabi oscillations presented in Fig. 5.

XI. HOW QUANTUM COHERENCE COEXISTS WITH RESONANT TUNNELING

In practice, there are many physical situations in which macroscopic systems show quantum tunneling without quantum coherence. Incoherent tunneling of electrons under reverse bias [Fig. 1(b)] can disturb the excitonic system and destroy the excitonic BEC which is a required condition for resonant tunneling in such 0D-2D structure as explained in section III. However, it was known [54] that any observation of quantum coherence & tunneling between macroscopically different states can also indicate Schrödinger's Cat like quantum states under certain conditions. The main difficulties in achieving that comes from dissipative coupling with the environment which eventually lead to strong decoherence. However, if we are dealing with a quantum state with very small dissipation, possibly a BEC state then it can preserve that coherence. Therefore, in our case, we explore the possibilities of phase coherent oscillations of excitonic BEC state coexisting with coherent resonant tunneling in a double well potential (InAs QD and GaAs 2DEG). Under reverse bias, the electrons can move back and forth between the two wells with frequency
~2\Gamma. Equivalently, this leads to the splitting of the ground state as a doublet [54] with energy difference \( \Delta \approx 2h\Gamma \) as also mentioned in the last section X. In thermal equilibrium conditions, standard tunneling probability of the resonance oscillation is \( \sim \exp(-\gamma t) \). The coherence of tunneling process can be lost when the damping \( \gamma = \omega_0^{-1}\Gamma^2 \) multiplied by \( \frac{k_B T}{\hbar \Gamma} \) (if this is >> 1) becomes comparable to \( \frac{\Gamma}{2\pi} \) [54]. This is because, the time-scales for these two phenomena can be quite different. Experimental control of quantum coherence requires that the relative phase of the wave function should be preserved over times of order \( \Gamma^{-1} \). In principle, this \( \Gamma^{-1} \) may be very long for a BEC state. Any "observation" of the system at time intervals shorter than \( \Gamma^{-1} \) can destroy these quantum coherence effects and effectively localize the system on one side of the barrier in the double quantum well system. In other words, one can say that quantum coherence will be totally lost if the root mean-square fluctuations (at frequency \( \Gamma \)) of the difference in energy between the two originally degenerate states becomes comparable to the tunneling energy \( \hbar \Gamma \). Assuming, \( \Delta \sim 100 \text{ meV} \) from Fig. 7(a), \( \omega_0 \sim 1.5 \text{ eV}/h \), T=10 K we get \( \gamma \sim 10^{12} \text{ Hz} \) and \( \Gamma \sim 10^{14} \text{ Hz} \). We clearly see that \( k_B T \ll 2h\Gamma \) and \( \frac{k_B T}{\hbar \Gamma} \sim 1/100 \). As a result, \( \left( \frac{\gamma}{2\pi} \frac{k_B T}{\hbar \Gamma} \right) \sim 10^{10} \text{ Hz} \) and \( \frac{\Gamma}{2\pi} \sim \frac{10^{14}}{2\pi} \text{ Hz} \). As a result, \( \left( \frac{\gamma}{2\pi} \frac{k_B T}{\hbar \Gamma} \right) \sim 10^{10} \text{ Hz} \) is \( \ll \frac{\Gamma}{2\pi} \sim \frac{10^{14}}{2\pi} \text{ Hz} \). Therefore, these quantitative estimates clearly explain why we observe coherent resonant tunneling in the first place and why the macroscopic quantum coherence of exciton BEC state is sustained in this quantum coupled 0D-2D heterostructure even in presence of tunneling.
In summary, we observed the onset of phase coherent quantum oscillations of photocapacitance as a function of increasing number of photo generated and bias driven formation of excitons with increasing applied bias as well as with photoexcitation intensities below ~ 100 K. We understand these photocapacitance oscillations with respect to bias voltages in terms of modulations of capacitive coupling within the 0D-2D heterostructure as a result of cooperative effects of coulomb correlation and coherent resonant tunneling. We showed that coexistence of coherent resonant tunneling and formation of indirect excitons in a ‘capacitively’ coupled 0D-2D quantum structure strongly imposes the momentum space narrowing required for such oscillations with increasingly large electric polarization of excitonic dipole moments as a consequence of the underlying BEC of excitons at specific bias values. Onset of spontaneous increase in the collective electric polarization \( \langle \vec{P} \rangle \) of excitonic dipoles and density driven interference of excitonic matter waves over a macroscopically large area with a sharp transition temperature are demonstrated as direct evidences for the presence of exciton BEC in this mixed system of excitons with excess electrons under reverse bias. It must be noted that the excitonic system is driven towards and out of resonant tunneling and BEC state as more charge carriers accumulate near the heterojunction and subsequently tunnel out of it. This crucial structural aspect of realizing excitonic BEC coexisting with resonant tunneling which is tunable with applied bias was not reported earlier. BEC phase transition was also evident from the sudden increase in orientational average of electric polarization of these excitons below ~100 K. Observations of negative quantum capacitance and quantum interference beats of photocapacitance measured over a macroscopically large area revealed the presence of long range spatial coherence of this BEC state. We also see sharpening of excitonic photocapacitance spectra at bias voltages of photocapacitance oscillations.
corresponding to maximum $\langle P \rangle$ as shown in Figs. 7, 11. This clearly happen whenever excitonic BEC takes shape in the absence of excess charge carriers near the inflection points of $I_{ph}$. However, in our case, we also see splitting of photocapacitance spectra due to strong quantum coupling of electrons in the InAs quantum dots and the 2DEG under reverse bias. In fact, presence (absence) of splitting of excitonic peaks measured using photocapacitance spectroscopy at maxima (minima) of photocapacitance oscillation also proved that steady state, quantum entanglement between 0D-2D excitons are locally tunable like a Hadamard gate using applied bias voltage. Consequently, we also detected Rabi oscillations of multipartite, ‘Schrödinger’s Cat state’ [54] of two-level excitonic matter wave measured with photocapacitance even when photocurrent vary monotonically with increasing light intensity and does not oscillate at all. All these observations clearly demonstrate the presence of an underlying cooperative phenomena interconnecting coherent resonant tunneling in the 0D-2D heterostructure, long-range oscillation of Coulomb correlation of indirect excitons in the x-y plane and the Rabi oscillations of macroscopically large quantum states of this excitonic BEC. As such, BEC is usually explained by two-body interactions but we explained here why photocapacitance can measure the multi-partite nature of this BEC quantum state as well as probe the macroscopic phase coherence of a large number of excitonic ‘qubits’ as a function of both light intensity and applied bias. As explained in section XI, both coherent tunneling and quantum coherence over a spatially large area were necessarily required [54] to probe this macroscopic quantum state of excitonic BEC.

Therefore, in principle, a N-qubit register of two-component [56,57] excitonic condensate having steady state tunable quantum entanglement can be fabricated with multiple, overlapping light spots and applied biases and probed with localized sensing of photocapacitance as shown in Fig. 8. In addition, many of these qubits can be addressed collectively as well as independently of
FIG. 8. A schematic representation of excitonic BEC state as $\left(\psi^{EX}_{Coupled}\right)^n \otimes \left(\psi^{EX}_{Coupled}\right)^{n_2} \otimes \ldots \left(\psi^{EX}_{Coupled}\right)^{n_N}$ using a 0D-2D heterostructure within a light spot is shown in the top left corner. Another schematic of a N-qubit register having N numbers of electrical contact pads (blue) is depicted below. This can also be configured as $\left(\psi^{EX}_{Coupled}\right)^n \oplus \left(\psi^{EX}_{Uncoupled}\right)^{n_2} \oplus \ldots \left(\psi^{EX}_{Coupled}\right)^{n_N}$ etc. combinations when multiple light spots overlap as in Fig. 4(e) and the 0D-2D structure is maximally quantum coupled below some pads and not coupled in the other pads using locally different applied biases and light intensities. Pink colored spheres represent $n_i$ excitons within the 2D ‘checkerboard’ like potential below the $i^{th}$ contact pad. Shapes, sizes and overall design of the array of electrical contact pads and the distance ‘d’ between two neighboring pads can be customized using micro/nano lithography for different quantum applications.

the others using a combination of applied bias voltages and light over a limited area. So the abundant multiplicity (millions or more) of excitonic qubits in macroscopically large, coherent, quantum superposition(s) can allow experimentally tunable quantum entanglement. This can enhance computational capacity and can certainly permit adequate redundancy for efficient, fault-tolerant quantum operations. There were past studies on fault tolerant operations of bosonic qubits [58-60], however none actually used the concept of macroscopic quantum state of BEC explicitly
like we demonstrated here. Operational temperatures of these excitonic BEC can be raised further with more densely packed, ordered array of QDs in the x-y plane and/or using materials having larger excitonic binding energies. However, fabrications of single crystals of 0D-2D heterostructures using 2D materials (e.g. transition metal di-chalcogenides, oxides, perovskites etc.) having higher excitonic binding energies are still an open challenge for semiconductor optoelectronics. Additionally, most of the DiVincenzo’s criteria [61] for quantum computation are also fulfilled in this system in principle [see Fig. 8].

However, even now these 0D-2D heterostructures can be scaled up for mass production of miniaturized, portable quantum optoelectronics devices using the existing III-V and/or Nitride based semiconductor fabrication technologies. Such feasibilities of fabricating quantum devices, however, certainly difficult to implement for most other current approaches being developed for quantum computation outside the boundaries of a sophisticated research laboratory. It can bring in a paradigm shift in optoelectronics for faster and wider adaptation of quantum technologies in terms of miniaturization and portability.
APPENDIX A: SAMPLE AND EXPERIMENTAL METHODS.

The single crystal sample used in our measurements is exactly similar to one of those used by Vdovin et al. [9-11]. It has one layer of InAs quantum dots within GaAs/AlAs/InAs/AlAs/GaAs p-i-n diode structure. It was grown by Molecular Beam Epitaxy (MBE) system in University of Nottingham on a highly doped (100) n+ GaAs substrate. The following layers were grown subsequently on the substrate: a 1.0 µm heavily doped (4×10^{18}/cm^3) n+ GaAs layer was grown at temperature 550 °C on top of the substrate. This was followed by a 100 nm n-doped (2×10^{16}/cm^3) GaAs layer, and then 100 nm undoped spacer layer of GaAs, two intrinsic AlAs layers of thickness 5.1 nm each having a 1.8 monolayer of InAs quantum dots with areal density ~1×10^{11}/cm^2 in between these (See Fig. 9). Then a 60 nm undoped GaAs layer complete the intrinsic region. Finally, a 0.51 µm heavily doped (2×10^{18}/cm^3) p+ GaAs layer were grown. Net area of the ring-shaped gold contact pads with diameter around 200 µm is ~3×10^{-4} cm^2. This was deposited on a cylindrical mesa structure using photolithography. Width of such rings are around 25 µm.

For low temperature measurements, we placed our sample on a customized copper holder inside a closed-cycle cryostat CS-204S-DMX-20 from Advance Research Systems. Temperature of the cryostat was controlled with a Lakeshore (Model-340) temperature controller. The sample was illuminated from the top p-GaAs side using an Acton Research SP2555 monochromator having a 0.5-m focal length along with a 1000-W quartz-tungsten-halogen lamp from Newport as non-coherent light source or with a 633 nm and/or 488 nm laser as described. For photocapacitance measurements (C_{photo} = dQ/dV), we used Agilent’s E4980A LCR meter with small signal rms voltage of 30 mV with a frequency of 10 kHz. A simple series equivalent circuit of capacitance (C) and conductance (G) in parallel was used to extract these parameters. Spectral response of the lamp-monochromator combination is reasonably smooth and changes slowly and monotonically.
within the wavelength ranges used in our experiments. So Lambda square corrections [62] were not used while plotting these spectra.

FIG. 9. A bright field TEM image of a similar InAs QD layer without the top AlAs, GaAs layers present in the actual sample used in our study. (a) A larger scan is shown on the left with estimated plot of size distributions. (b) A magnified version having 128 nm x128 nm scan size is given on the right. Surface density of these InAs QDs were ~1.0 x 10^{11}/cm². Average spatial (x-y) extent of these QDs were around 11± 2 nm. The average z dimension of 1.6 nm was measured using AFM.
FIG. 10. Unlike photocapacitance [Fig. 4], we do not see any quantum interference beats in photo-G/ω. (a) Photo-G/ω does not show beat like features when the laser spot is moved as described in Fig. 4(a). (b) No beat formation in photo-G/ω as the laser spot was increased gradually beyond the periphery of the ring. (c) No beat formation even with white light. (d) Quantum interference beats were absent when two separate, uncorrelated, CW laser beams of 633 nm and 488 nm are used to photo excite the sample simultaneously. Therefore, unlike photocapacitance, photo-G/ω is not specific to the phase coherent, collective quantum properties of dipolar excitons at the 0D-2D heterojunction.
FIG. 11. Presence and absence in sharply split excitonic transitions in photocapacitance spectra at other bias voltages at 10 K. (a) and (b) Photocapacitance and photo-G/ω spectra under biases of successive maximum of photocapacitance oscillations from Fig. 1(c). Corresponding biases are -0.66 V, -0.92 V respectively as shown in Fig. 1(c). (c) and (d) Photocapacitance and photo-G/ω spectra under biases of successive minimum of photocapacitance oscillations from Fig. 1(c). Corresponding biases are -0.80 V, -1.07 V respectively as shown in Fig. 1(c). Therefore, unlike photocapacitance, photo-G/ω cannot sense the tunneling induced quantum coupling and reorganization of the 0D-2D indirect excitons.
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Data Availability. All data and materials used in the analyses will be available for purposes of reproducing and/or extending these analyses from the corresponding author upon reasonable requests.
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