Abstract

In Tendermint blockchains, the proof-of-stake mechanism and the underlying consensus algorithm entail a dynamic fault model that implies that the active validators (nodes that sign blocks) may change over time, and a quorum of these validators is assumed to be correct only for a limited period of time (called trusting period). The changes of the validator set are under control of the blockchain application, and are committed in every block. In order to check what is the state of the blockchain application at some height $h$, one needs to know the validator set at that height so that one can verify the corresponding digital signatures and hashes. A naïve way of determining the validator set for height $h$ requires one to: (i) download all blocks before $h$, (ii) verify blocks by checking digital signatures and hashes and (iii) execute the corresponding transactions so the changes in the validator sets are reproduced. This can potentially be very slow and computationally and data intensive.

In this paper we formalize the dynamic fault model imposed by Tendermint, and describe a light client protocol that allows to check the state of the blockchain application that, in realistic settings, reduces significantly the amount of data needed to be downloaded, and the number of required computationally expensive signature verification operations. In addition to mathematical proofs, we have formalized the light client protocol in TLA+, and checked safety and liveness with the APALACHE model checker.

1 Introduction

Tendermint is a leading state machine replication (SMR) engine [Buc16] that tolerates Byzantine faults. It supports arbitrary state machines written in any programming language by providing a flexible interface for application development. Its core consensus protocol [BKM18] is a variant of the algorithm for Byzantine faults with Authentication from [DLSS88], built on top of an efficient gossiping layer.

Tendermint is designed for public, open-membership networks, and a production deployment consists of a number of node types. The consensus-forming nodes, responsible for proposing blocks and voting on them, are called validators. Validators are not directly connected to one another, but rather through a gossip network consisting of non-validating nodes called full nodes, who execute the same consensus protocol and relay its messages but do not participate in block production or voting. Besides this restriction, a full node also maintains the copy of the blockchain, and executes the same set of protocol rules like validators. Since validator nodes are highly security-sensitive (as

*Supported by Interchain Foundation
Figure 1: Sequential Verification. If Block 1 is trusted to be from the blockchain, then NextV₁ defines the validators Val₂ that vote for Block 2. If a commit for Block 2 (recorded as the LastCommit in Block 3), contains votes from more than 2/3 of the voting power in NextV₁ = Val₂, then Block 2 can be trusted to be from the blockchain.

they need to manage private keys), their operators often run additional full nodes, termed sentry nodes, as gateways to the rest of the network. A validator node typically connects only to its sentry nodes, which then form connections to other full nodes and sentry nodes.

A final type of node is a light node, also called a light client. Light nodes do not participate formally in the network; they make requests for data from full nodes and verify, by checking hashes and signatures, that such data indeed came from the underlying blockchain. Light clients synchronize to the latest block of the blockchain by verifying validator signatures. Once synchronized, they can verify Merkle proofs about the state of the blockchain by using the Merkle root stored in the latest block. Effectively, light nodes perform read operations of the blockchain. However, since light nodes do not follow the full consensus protocol or execute transactions, they operate under a different security model than full nodes. A light client protocol is normally designed such that it does not have high computational and bandwidth requirements as it is supposed to be used also in constrained environments, for example in mobile devices.

In a traditional Byzantine-fault-tolerant SMR system, a read operation by a client is normally implemented by having the client send a request to all replicas and then waiting to receive the same response from at least \( f + 1 \) replicas, where \( f \) is the maximum number of faulty replicas. This approach is not applicable in the Tendermint model as (i) a client does not have direct access to validator nodes (as they are connected to the network only through sentry nodes that are not necessarily public) and (ii) validator set changes are dynamic so it is not obvious what is the current validator set. In Tendermint, no restrictions are placed on how the validator set may change from one block to the next; the intersection between adjacent validator sets may be empty. Changes to the validator set are determined entirely by the application state machine.

In this paper we describe a protocol that addresses the challenges (i) and (ii) for the design of a light client. As the light client cannot query a validator directly, it cannot query the required \( f + 1 \) validators needed for the traditional approach. The described shielding of validator nodes by sentries thus forces us to design a protocol that allows the light node to (a) obtain the required data from other nodes (different from validators), and (b) verify the received information by checking signatures (in contrast to checking \( f + 1 \) identical responses). In addition, in a context where the validator set is always changing, it is not clear to a light client which validators can currently be trusted, and who has the authority to sign a block as a validator. The essential operation of a light node thus becomes tracking the evolving validator set over time.

Tendermint is particularly popular for proof-of-stake blockchains. In such blockchains, becom-
ing a validator requires an economic commitment, often referred to as a bond (or stake), which guarantees the correct behaviour of the validator. Validators that behave incorrectly (i.e., by going offline, or signing conflicting blocks) lose some fraction of their bond. Thus, bonded validators have an incentive to behave correctly. For a validator to reclaim its stake, it must wait for a so-called unbonding period before their stake is returned. This unbonding period is sufficiently long to detect misbehavior and punish it. But this also means that participants only have the incentive to follow the protocol for some limited time. Once they can be sure that they will get their stake back, it is unsafe to rely on their cooperation. These Proof-of-Stake mechanics result in a Tendermint Security Model with time-dependent fault assumptions that depend on the blocks.

Tendermint is also a core component of the Cosmos Project [BK16], which consists of many independent proof-of-stake blockchains. At the heart of the Cosmos Project is the InterBlockchain Communication (IBC) protocol for reliable communication between independent blockchains; what TCP is for computers, IBC aims to be for blockchains. IBC is based on light client protocols, which enable one blockchain to perform read operations on another.

Contributions. The major contribution of this paper is to provide the formal underpinning for Tendermint light clients, which rests on three pillars

- A formalization of the Tendermint security model.
- A light client protocol based on the security model, and modeled in TLA+.
- Model Checking the correctness of the protocol against the security model with the Apalache model checker.

We start by formalizing the Tendermint Security model; we then discuss the design of the Light Client Verification that implements a fault-tolerant read based on the model. The formalization of the failure model, and the invariants introduced in this paper are based on the open source reference implementation [Cor20b] and the specifications of the data structures [Cor20a]. The challenge addressed here is that the light client might have a block of height \( h_1 \) and needs to read the block of height \( h_2 > h_1 \). Checking all block headers of heights from \( h_1 \) to \( h_2 \) might be too costly (e.g., in terms of energy for mobile devices). The described protocol tries to reduce the number of intermediate blocks that need to be checked, by exploiting the guarantees provided by the security model.

In the following we outline our approach and start by describing the Tendermint block structure and how nodes follow the state of the application in the standard way.

2 Overview

Tendermint Signature Scheme  Figure 1 shows an example of the first three blocks generated by a Tendermint blockchain. Block 1 is the result of an instance of Tendermint consensus. The instances are called heights. The validators who actively participated in this consensus instance and their IDs (public key) are stored in \( Val_1 \). In Tendermint, instead of having one vote each, validators may have different voting powers, which are also stored in \( Val_1 \). We say that in consensus a quorum is reached if validators that represent more than \( 2/3 \) of the voting power agree.

In addition to deciding upon data to be put into a block, these validators also decide on the nodes who are going to participate in the next height, and the IDs (public keys) of these nodes are stored in \( NextV_1 \). The same nodes are stored in \( Val_2 \), and they decide on Block 2. The proof that they indeed decided on Block 2 is given by a commit. A commit contains a set of signed messages that contain a hash called \( BlockID_2 \), and in the blockchain it is stored in Block 3. In order for a
commit to be valid, it must contain messages by a quorum of validators. In our example, we assume each validator has voting power 1, and \( \text{Votes}_3 \) contains signatures by more than \( \frac{2}{3} \) of the nodes in \( \text{Val}_2 \). Thus, if a node has obtained Block 1 from a trusted source, and it has downloaded Block 2 and Block 3, it can check that Block 2 is indeed from the blockchain, by checking the described relationships of hashes, quorums, and signatures.

An important feature of Tendermint is that the choice of \( \text{NextV} \) is application-specific and unrestricted; for any height \( i \), the sets \( \text{Val}_i \) and \( \text{NextV}_i \) need not intersect. Thus, if one needs to check some data, e.g., the existence of a transaction in a block of height \( \ell \), one a priori needs to download the blocks for all heights up to \( \ell + 1 \), and sequentially check all the blocks. This is computationally expensive due to checking hashes and signatures, and may hinder the access to blockchain, e.g., for mobile devices. That said, in many deployments of Tendermint, e.g., on the Cosmos Hub blockchain, we observe that large changes in the validator sets are rare. Thus the question is to design a protocol that allows to verify the block of height \( \ell \) without downloading and verifying all the blocks required by the sequential method. We call such a protocol a (skipping) light client. It implements a read operation of a block, by communicating with full nodes. As some full nodes may be faulty, this functionality must be implemented in a fault-tolerant way. To do so, we next formalize the fault assumption that is imposed by Tendermint.

**Tendermint Security Model** The staking and unbonding mechanism induces a security model: starting at the time a block gets generated (this time is stored in the block), more than two-thirds of the next validators of a new block are correct for the duration of the trusting period, a duration which is less than the unbonding period defined by the Proof-of-Stake mechanics. An example is sketched in Figure 2. Block 1 is created at time \( \text{Time}_1 \), and the full nodes \( p_1, p_2, p_3, \) and \( p_4 \) are decided to be the validators of the next block. More than two thirds, that is, at least three, are assumed to be correct during the time interval depicted by the dashed rectangle. Similarly, when Block 9 is generated, an additional constraint over \( p_3, p_4, p_5, \) and \( p_6 \) is added. This so-called security model can be seen as a Byzantine fault model with dynamic (or moving) faults.
The fault-tolerant read operation over a Tendermint blockchain needs to be designed for this security model. To do so, after formalizing the blockchain data structure in Section 3, we formally define this security model in Section 4.

It should be noted that Tendermint provides guarantees even outside this security model, where one third or more validators are faulty within a trusting period and may thus fork the blockchain. The security model introduced here enables clear separation of concerns between light client verification, which operates within the model, and fork detection, which operates outside it. We leave fork detection to future work.

**Skipping Verification under Tendermint Security Model** In Section 5 we formalize the distributed computing problem for which our light client is designed, and prove the central result that allows us to solve it in Section 6. The underlying intuition is as follows: Recall Figure 2. Assume a light client starts with Block 1 from the blockchain, and needs to verify Block 17 at time $t$. Consider the example in Figure 3. Here, $NextV_1 = \{p_1, p_2, p_3, p_4\}$, which by the Tendermint security model means that more than two thirds of these nodes are correct during the trusting period, which is depicted by the dashed rectangle in Figure 2. Let's assume the light client has downloaded blocks of height 17 and 18 with a validator set $Val_{17} = \{p_5, p_6, p_7, p_8\}$. The light client would like to verify Block 17 based on $LastCommit_{18}$, which would need to contain a quorum of $Val_{17}$. However, being aware only of the dashed rectangle, it is impossible to infer whether any of the nodes in $Val_{17}$ are correct. Thus Block 17 cannot be verified at this time.

The idea behind the light client is to try bisection, i.e., since the first check for Block 17 failed, the light client will download a header in the middle, i.e., Block 9 from some node in the Tendermint network. To verify Block 9, the light client also needs a commit for that block which is stored in the next block, so that it also downloads Block 10. Let's consider the node has downloaded Blocks 9 and 10 as shown in Figure 3. At time $t$, the node can still trust that less than one third of the nodes in $NextV_1$ is faulty. The set $Votes_{10}$ contains two of the nodes from $NextV_1$, namely, $p_3$ and $p_4$. By the security model, at least one of them is necessarily correct. As a result, at least one of the nodes who signed Block 9 is correct.

In Tendermint consensus, correct nodes only sign blocks that were properly generated; thus, Block 9 in the figure can be trusted. As the light client has now established that Block 9 is from the blockchain, $NextV_9$ from Figure 3 imposes a trust assumption that corresponds to the dotted rectangle in Figure 2. Thus, the light client may now try to verify Block 17 based on this new trust assumption.

As the Tendermint security model makes reference to real time, and the trusting period is a concrete time duration, we require that the light nodes local time is approximately synchronized to the time of the Tendermint blockchain. This is needed to check whether a block is within the trusting period. However, the trusting period is in the order of weeks, so that in typical scenarios a clock precision of several seconds is sufficient and easily achievable. Similarly, for liveness we require that downloading a header is faster than the duration of the trusting period (i.e., two weeks). While from a theoretical viewpoint, all this implies that we operate in a synchronous computation model, in practice, due to the order of time durations, this does not impose practical limitations.

In the implementation we do several performance improvements. Rather than downloading complete blocks, we download so-called lightblocks that just contain the required metadata to do the checks above. Such a lightblock for Block 9 contains the header of the Block 9 and also $LastCommit_{10}$, as this is the only information we need from Block 10. Thus, instead of downloading Blocks 9 and 10, the implementation downloads the lightblock 9, only.
Figure 3: Example for Skipping Verification. The light client has downloaded blocks of height 1, 9, 10, and has trust that Block 1 was generated by a Tendermint blockchain. Moreover assume that the trusting period of Block 1 has not expired, that is, the current time $t < \text{Time}_1 + TP$.

Consider the two checks (1) and (2): (1) $\text{Votes}_{10}$ contains one correct validator from $\text{NextV}_1$, that is, it contains validators that represent more than $\frac{1}{3}$ of the voting power in $\text{NextV}_1$. (2) The hash matches. By the behavior of a correct node executing Tendermint consensus, if (1) and (2) are satisfied, then $\text{Val}_9$ is indeed the validator set of height 9 on the blockchain, and by the Tendermint Security Model, the light client can trusts $\text{Commit}_{10}$, and thus Block 9 was generated by a Tendermint blockchain and can be trusted.

Model Checking and Implementation In Section 8 we discuss how we have formalized the blockchain and the light client protocol in TLA+. In addition to having a machine-readable protocol specification in TLA+ (that reads very similar to the mathematical description from Sections 3–7), we were able to produce non-trivial system executions with the symbolic model checker Apalache, and we checked the protocol for small instances of the blockchain. In Section 9 we discuss our implementation in Rust that is based on a modular architecture that simplifies testing.

3 Blockchain data structure

We give a formalization of the Tendermint block structure. We start with an abstract view, in particular with respect to the domains of data fields, to highlight concepts as independent of their implementation as possible. We will later refine towards the implemented data structures to address distributed aspects.

A set of transactions is stored in a data structure called block, which contains a field called header. In the implementation, hashes are used to reduce the amount of data that needs to be (re)transmitted and stored. Hashes are used within a block, where a header stores hashes of the data of the block. But hashes are also used to point to the previous block. The former usage is done just for performance, so that for our purposes we ignore these hashes, and we will assume that the blockchain is a list of headers, rather than a list of blocks. The hashes that point to previous blocks are needed to implement that chain, and we will treat them explicitly in this model.

Definition 1 (Header). A header contains the following fields, whose domain (except the height) we leave unspecified for now:

- **Height**: non-negative integer
• Time
• LastCommit
• LastBlockID
• Val
• NextV
• Data
• AppState
• LastResults

In the implementation, LastBlockID is also stored as part of LastCommit as indicated in Figure 1, and is a hash of the previous block. For our theoretical treatment it is more convenient to not treat it within the LastCommit. This redundancy is also subject to ongoing discussions in the Tendermint project [Iss20].

Tendermint consensus [BKM18] generates a sequence of such headers, that ensures the following invariants:

Definition 2 (Basic Invariants). A Tendermint blockchain is a list called chain of headers that, for all \( i < \text{len}(\text{chain}) - 1 \), satisfies:

1. \( \text{chain}[i].\text{Height} + 1 = \text{chain}[i + 1].\text{Height} \) (We do not write \( \text{chain}[i].\text{Height} = i \), to allow that a chain can be started at some arbitrary height, e.g., when there is social consensus to restart a chain from a given height/block.)

2. \( \text{chain}[i].\text{Time} < \text{chain}[i + 1].\text{Time} \)

3. \( \text{chain}[i + 1].\text{Val} = \text{chain}[i].\text{NextV} \)

Definition 2(3) captures the changing validator sets discussed in Section 2. In addition to these basic invariants there are invariants that are based on hashes and digital signatures. We start to introduce their semantics by defining some preliminary functions:

Definition 3 (Abstract auxiliary soundness functions). The system provides the following functions:

1. hash: We assume that every hash uniquely identifies the data it hashes

2. execute: used for state machine replication. The function maps Data (transactions) and an application state to a new state. It is a function (deterministic transitions).

3. PossibleCommit: There is a function PossibleCommit that maps a block (header) to the domain of LastCommit from Definition 1.

4. proof\( (b, \text{commit}) \): a predicate: true iff

   (a) \( b \) is in the chain, i.e., there is an \( i \) such that \( \text{chain}[i] = b \)

   (b) \( \text{commit} \) is in PossibleCommit(\( b \)).
In Tendermint consensus [BKM18], the validators sign a given block. A set of signatures by a quorum of validators for a block is called a commit. Some of the required semantics can be proven independently of these details. To capture these, we introduce Definition 3.

Because proof in Definition 3 refers to the chain, it depends on the execution, which results in a different quantifier order. For instance, we say "there exists a function hash such that for all runs", while we say "for each run there exists a function proof". The consequence is that in Definition 3 hash is a predetermined function (implemented), while proof will have to be computed at runtime as a function of the chain. The challenge in a distributed system is to locally compute proof without necessarily having complete knowledge of chain. In the context of the light client, we even want to infer knowledge about chain from the outcomes of the local computation of proof. We use digital signatures for that, and introduce them below when we introduce the distributed aspects.

**Definition 4** (Soundness predicates). Given two blocks $b$ and $b'$:

1. $\text{matchHash}(b, b')$ if $\text{hash}(b) = b'.\text{LastBlockID}$
2. $\text{matchProof}(b, b')$ if $\text{proof}(b, b'.\text{LastCommit})$

**Definition 5** (Security invariants). For all $i < \text{len}(\text{chain}) - 1$:

1. $\text{matchHash}(\text{chain}[i], \text{chain}[i + 1])$
2. $\text{matchProof}(\text{chain}[i], \text{chain}[i + 1])$
3. $\text{chain}[i + 1].\text{AppState} = \text{execute}(\text{chain}[i].\text{Data}, \text{chain}[i].\text{AppState})$

The function $\text{matchHash}$ formalizes the hash arrow in Figure 1. We will show in Section 6 how proof and thus $\text{matchProof}$ can be checked in a distributed system in the presence of Authenticated Byzantine faults, based on quorums in Votes.

## 4 Tendermint Security Model

In Section 1 we discussed that in Tendermint blockchains the proof-of-stake mechanism entails a time-dependent security model. We capture this by the following formalization, which states that once a new validator set ($\text{NextV}$) is chosen, we trust that it contains a correct quorum for some limited time, namely the trusting period. We start with preliminaries.

**Definition 6** (Validator Data Structures). Given a full node, a validator pair is a pair $(\text{peerID}, \text{vp})$, where peerID is the PeerID (public key) of a full node, and the voting power vp is an integer (representing the full node’s voting power in a certain consensus instance). A validator set is a set of validator pairs. For a validator set $V$, we write $\text{totalVP}(V)$ for the sum of the voting powers of its validator pairs.

**Definition 7** (Domain of Distributed Commit). A commit is a set of precommit messages sent and signed by validator nodes during the execution of Tendermint consensus [BKM18]. Each message contains the following fields:

1. **Type**: precommit
2. **Height**: positive integer
3. **Round** a positive integer
4. **BlockID** a hash value of a block

We assume the authenticated Byzantine fault model [DLS88] in which no node (faulty or correct) may break digital signatures, but otherwise, no additional assumption is made about the internal behavior of faulty nodes. That is, faulty nodes are only limited in that they cannot forge messages. This implies for Definition 7, e.g., that a faulty node \( p_f \) may sign a precommit message for a hash of a block that is not on the blockchain, but it may not generate a precommit message that appears to be signed by a correct node \( p_c \) (unless \( p_c \) actually signed that message before and \( p_f \) received it).

A Tendermint blockchain has the **trusting period** as a configuration parameter \( TP \). We define a predicate \( \text{correctUntil}(n, t) \), where \( n \) is a node and \( t \) is a time point. The predicate \( \text{correctUntil}(n, t) \) evaluates to true if and only if the node \( n \) follows all the protocols (at least) until time \( t \). (It is false if a node \( n \) deviates from the protocol once by time \( t \).)

**Definition 8** (Security Model). If a block \( h \) is in the chain, then there exists a subset \( C \) of \( h.NextV \), such that:

\[
\text{totalVP}(C) > \frac{2}{3}\text{totalVP}(h.NextV)
\]

and for every validator pair \((n, p) \in C\), it holds that

\[
\text{correctUntil}(n, h.Time + TP).
\]

The definition of correct refers to realtime, while it is used here with \( Time \) as stored in a block and the configuration parameter trusting Period \( TP \), which are "hardware times". To not clutter the presentation, we do not make a distinction here between real-time and hardware time, and we assume that the hardware clock is sufficiently synchronized to real time. Also, the trusting period \( TP \) is typically in the order of weeks, so that inaccuracies in time synchronization can be dealt with security margins.

**Definition 9** (Distributed Commit). For a block \( b \), each element \( PC \) of \( \text{PossibleCommit}(b) \) satisfies that

1. \( PC \) contains only votes by validators from \( b.Val \)
2. \( \text{totalVP}(PC) > \frac{2}{3}\text{totalVP}(b.Val) \)
3. and there is an \( r \) such that each vote \( v \) in \( PC \) satisfies:
   
   (a) \( v.Type = \text{precommit} \)
   (b) \( v.Height = b.Height \)
   (c) \( v.Round = r \)
   (d) \( v.BlockID = \text{hash}(b) \)

In a distributed commit necessarily all BlockIDs are equal, which can be checked locally.

We have now defined all the guarantees provided by a Tendermint blockchain that are necessary to formalize what it means to observe the state of the blockchain from the outside.

5 **The Light Client Verifier Problem**

In the most abstract viewpoint, the light client just implements a read of a header (block) of a given height from the blockchain. This header \( h \) needs to be generated by the Tendermint consensus. In particular, a header that was not generated by the blockchain should never be stored. Due
to the evolving validator sets, without constantly following the progress of the blockchain, in the presence of Byzantine faulty nodes, one cannot know a priori who are the relevant validators for a block that are allowed to sign a block: For instance, a set of Byzantine nodes, which never participated in Tendermint consensus may generate and sign a block that structurally is according to the definitions. Thus the Verifier has to locally check whether the nodes who sign a block can be trusted, more precisely, whether sufficiently many correct nodes have signed.

We will start with the sequential problem statement that considers the abstract case where the blockchain is just a data structure and there are no faults, and we will then introduce the distributed model we consider and the distributed problem statement.

**Definition 10 (Sequential Problem Statement).** The Verifier satisfies the following properties

**Safety.** The Verifier never stores a header which is not in the blockchain.

**Liveness.** The Verifier receives as input a height \( \text{targetHeight} \) (not greater than the current height of the blockchain), and eventually stores the header of height \( \text{targetHeight} \) of the blockchain.

**Distributed Problem Statement**  To address the sequential problem statement, we consider the following setup: The verifier communicates with a full node called primary. No assumption is made about the full node (it may be correct or faulty). Communication between the light client and a correct full node is reliable and bounded in time. Reliable communication means that messages are not lost, not duplicated, and eventually delivered. There is a (known) end-to-end delay \( \Delta \), such that if a message is sent at time \( t \) then it is received and processes by time \( t + \Delta \). This implies that we need a timeout of at least \( 2\Delta \) for a query/response communication (e.g., a remote procedure call) to ensure that the response of a correct peer arrives before the timeout expires.

As we do not assume that the primary is correct, no protocol can guarantee the combination of the sequential properties. Thus, in the (unreliable) distributed setting, we consider two kinds of termination, successful and failure, and we will specify below under what (favorable) conditions the verifier can terminate successfully, and satisfy the requirements of the sequential problem statement.

**Variables used by light client verification**  To formalize the problem, we need to define the state space of the protocol. We do so by defining problem variables: the local data structure \( \text{lightStore} \) contains lightblocks that contain a header. For each lightblock, we record its verification status, that is, whether it is verified. The local variable primary contains the PeerID of a full node. The container \( \text{lightStore} \) is initialized with a header \( \text{trustedHeader} \) that was correctly generated by Tendermint consensus. We use the convention that the status of \( \text{trustedHeader} \) is verified.

**Definition 11 (Distributed Problem Statement).** The light client satisfies the following properties

**Safety.** At all times, every verified header in \( \text{lightStore} \) was generated by an instance of Tendermint consensus.

**Liveness.** From time to time, a new instance of the verifier is called with a height \( \text{targetHeight} \). Each instance must eventually terminate. If the primary is correct, and \( \text{lightStore} \) always contains a verified header whose age is less than the trusting period, then the verifier adds a verified header \( \text{hd} \) with height \( \text{targetHeight} \) to \( \text{lightStore} \) and it terminates successfully.

These definitions imply that if the primary is faulty, a header may or may not be added to \( \text{lightStore} \). The definition allows that verified headers are added to \( \text{lightStore} \) whose height was not passed to the verifier (e.g., intermediate headers used in bisection; see Section 7). Note that for liveness, just initially having a \( \text{trustedHeader} \) within the trusting period is not sufficient. For
instance, if the trusting period expires before the first message round trip with the primary can be completed, the Tendermint security model does not provide any guarantees about correct and faulty nodes anymore. After giving the specification of the protocol in Section 7, we will discuss some liveness scenarios in Section 7.2.

Relation of the distributed to the sequential problem The specification in Definition 11 provides a partial solution to the sequential specification in Definition 10. The solution with respect to safety is complete, even if the primary is faulty. However, we can only guarantee liveness when the primary is correct and the verifier has a sufficiently recent trusted header. For these runs distributed liveness implies the sequential liveness. Ensuring complete liveness (or perhaps just almost sure termination) would require us to make additional assumptions about the total (expected) number of faulty nodes in the network. The security model imposes such assumptions on validator nodes only, which represent only a fraction of all the nodes in the Tendermint system (cf. Section 1). Adding incentives and punishment rules to nodes that communicate with light clients is subject of current discussion of the community, so that we cannot give reasonable additional assumptions in this paper. However, in practice, if a run of the verifier fails, the light client may pick a new primary and retry until it reaches a correct primary which then ensures liveness. In this regard, it is assumed that light clients have access to at least one correct full node.

6 Light Client Verification

The standard way of following the evolution of the blockchain is to download block after block and perform sequential verification as shown in Figure 1. Here we discuss a verification method that does not force a client to download the headers for all blocks of height up to targetHeight (Definition 11). The outline of the approach is given in Figure 3. The method consists in asserting that the commit for the header of height targetHeight contains the signature of at least one correct node. This can be checked by exploiting the security model of Definition 8. We have to consider the intersection of the set of validators in the commit and of correct nodes in a set NextV in a previously downloaded and trusted block. By Definition 8 more than 2/3 of the voting power in NextV is correct (for some time). Now, if the set of validators in a commit have more than 1/3 of the voting power in NextV, these two sets intersect, which implies that at least one validator is both (i) correct and (ii) signed the commit. The following proposition establishes the part (i) of this argument, and is a direct consequence of Definition 8.

Proposition 1. Given a (trusted) block tb of the blockchain, at a real-time t, a given set of full nodes N contains a correct node, if

1. \( t - TP < tb.\text{Time} < t \), and
2. the voting power in \( tb.\text{NextV} \) of nodes in N is more than \( 1/3 \) of totalVP(\( tb.\text{NextV} \))

We now need to make explicit a property of the commits that comes from the way commits are computed by Tendermint consensus. Analysis of the consensus algorithm in [BKM18] immediately shows that a correct validator node only sends \texttt{prevote} or \texttt{precommit} messages if LastBlockID of the new (to-be-decided) block is equal to the hash of the last block \( \ell \). This implies that at a time where due to Definition 8 more than two thirds of \( \ell.\text{NextV} \) are still correct, we can trust a commit that is consistent with \( \ell.\text{NextV} \). Due to this, and by the fact that in the authenticated Byzantine model signatures cannot be forged we obtain the following proposition.

Proposition 2. Let b be a block, and c a commit. If at real-time t
1. \( c \) contains at least one validator pair \((v, p)\) such that \( v \) is correct — that is, \( \text{correctUntil}(v, t) \) — , and

2. \( c \) is contained in \( \text{PossibleCommit}(b) \)

then the block \( b \) is on the blockchain.

The following central result is a direct consequence of Propositions 1 and 2

**Corollary 1.** Given a trusted block \( t_b \) and a block \( b \) with a commit \( c \), at real-time \( t \), if

1. \( t - TP < t_b.\text{Time} < t \), and

2. the voting power in \( t_b.\text{NextV} \) of nodes in \( c \) is more than \( 1/3 \) of \( \text{totalVP}(t_b.\text{NextV}) \), and

3. \( c \) is contained in \( \text{PossibleCommit}(b) \),

then the block \( b \) is on the blockchain.

As a result we need not resort to sequential verification, but can use the current time, and a block for which we have previously established trust to extend the trust to a new block. However, if the preconditions Corollary 1(1–3) are not satisfied, this does not imply that \( b \) is forged. It might be that between \( t_b \) and \( b \) the validator set has changed too much to ensure a sufficiently large intersection. The protocol in the following section will then download an intermediate header whose height lies between \( t_b.\text{Height} \) and \( b.\text{Height} \) and tries to get trust in the intermediate header and use this to eventually verify \( b \).

## 7 Protocol Description

The basic data structure of our verification protocol is the \( \text{lightStore} \) which is a container for the so-called lightblocks, which correspond to the headers from Definition 11. In the implementation, lightblocks contain actual validator sets, while headers in the Tendermint implementation only contain hashes of these sets:

**Definition 12 (Lightblock).** The core data structure of the protocol is the \( \text{LightBlock} \). It consists of the following fields:

- **Header**
- **Commit**
- **Validators**
- **NextValidators**
- **Provider**

The \( \text{lightStore} \) is a data structure that stores such lightblocks, together with their state. The states are from the set \{\( \text{StateUnverified}, \text{StateVerified}, \text{StateFailed} \)\}. The LightStore exposes the following functions to query stored lightblocks:

\( \text{Get(height Height)} \) (\( \text{LightBlock, bool} \)) returns a lightblock at a given height or false in the second argument if the LightStore does not contain the specified lightblock.

\( \text{LatestVerified()} \) \( \text{LightBlock} \) returns the highest verified lightblock.
func VerifyToTarget(primary PeerID, lightStore LightStore, targetHeight Height) (LightStore, Result) {
    nextHeight := targetHeight
    for lightStore.LatestVerified().header.height < targetHeight {
        // Get Light Block
        current, found := lightStore.Get(nextHeight)
        if !found {
            current = FetchLightBlock(primary, nextHeight)
            lightStore.Update(current, StateUnverified)
        }
        // Verify
        verdict = ValidAndVerified(lightStore.LatestVerified(), current)
        // Decide where to continue
        if verdict == OK {
            lightStore.Update(current, StateVerified)
        } else if verdict == CANNOT_VERIFY {
            // do nothing, the light block current passed validation,
            // but the validator set is too different to verify it.
            // We keep the state of current at StateUnverified. For a
            // later iteration, Schedule might decide to try
            // verification of that light block again.
        } else {
            // verdict is some error code
            lightStore.Update(current, StateFailed)
            return (lightStore, ResultFailure)
        }
        nextHeight = Schedule(lightStore, nextHeight, targetHeight)
    }
    return (lightStore, ResultSuccess)
}

Figure 4: Light Client Verification Main Function

Update(lightBlock LightBlock, v State) The
state of the lightblock is set to v.

Our light client protocol is depicted in Figure 4. It gets as input

- lightStore: a container that stores light blocks that have been downloaded and that passed
  verification. Initially it contains a lightblock with trustedHeader. As the function can be called
  multiple times, the lightStore may contain more lightblocks that have been downloaded and
  verified so far.

- primary: the address (peerID) of the full node that the verification queries for blocks

- targetHeight: the height of the needed header.

In this paper we consider the case where targetHeight is greater than (or equal to)
lightStore.LatestVerified().Header.Height as it is the most interesting. In the other case there
are two options, either there is a trusted lightblock (within the trusting period) with height less
than targetHeight, and we can use the same method as described here from that block, or we need
to download all headers between targetHeight and the height of a trusted lightblock in store and
just check hashes in LastBlockID in decreasing order of heights.

The function uses two auxiliary variables, namely nextHeight, which should be thought of as
the “height of the next header we need to download and verify”, and current, the header that is
currently under verification. nextHeight is initialized to targetHeight. Then the protocol enters a loop that consists of the following three stages:

**Get Lightblock** here a lightblock is assigned to current. If the required lightblock had been downloaded before then it is taken from the lightStore, otherwise FetchLightBlock is called to download a lightblock of a given height from the primary. This function is the only one that communicates with another node in the system.

**Verify** ValidAndVerified is local code that checks the lightblock. It encodes the checks of Corollary 1 or, if sequential lightblocks should be verified falls back to standard sequential verification (cf. Figure 1). If it can verify current it returns OK. If the precondition of Corollary 1 is violated (but otherwise current is well-formed) it returns CANNOT_VERIFY. Otherwise, that is, if the current has been proven to be corrupted, it returns an error code.

**Decide where to continue** Schedule decides which height to try to verify next. We keep this underspecified as different implementations (currently in Golang and Rust) may implement different optimizations here.

For Schedule, we provide the following necessary conditions on how the height may evolve: Schedule returns H s.t.

(S1) if lightStore.LatestVerified().Header.Height = nextHeight and lightStore.LatestVerified().Header.Height < targetHeight then nextHeight < H ≤ targetHeight

(S2) if lightStore.LatestVerified().Header.Height < nextHeight and lightStore.LatestVerified().Header.Height < targetHeight then lightStore.LatestVerified().Header.Height < H < nextHeight

(S3) if lightStore.LatestVerified().Header.Height = targetHeight then H = targetHeight

Case (S1) captures the case where the lightblock at height nextHeight has been verified, and we can choose a height closer to targetHeight. As Schedule gets the lightStore as parameter, the choice of the next height can depend on the lightStore, e.g., we can pick a height for which we have already downloaded a lightblock. Case (S3) is a special case when we have verified targetHeight. In Case (S2) the lightblock of nextHeight could not be verified, and we need to pick a smaller height.

**Invariant** The implementation enforces the invariant that it is always the case that

\[ \text{lightStore.LatestVerified().Header.Time} > \text{now} - \text{TP}. \]  

(1)

If the invariant is violated, the light client does not have a lightblock it can trust and it terminates with failure. A trusted lightblock must be obtained externally, its trust can only be based on social consensus.

7.1 Correctness

**Proposition 3.** The protocol satisfies safety.

**Proof.** It is sufficient to remark, that a lightblock is marked as verified in line 19 if ValidAndVerified returned OK, which is the case only if the preconditions of Corollary 1 are satisfied (in which case safety is ensured by the corollary) or if we fall back to sequential verification in which case Definition 9 is checked. \[ \Box \]
Proposition 4. The protocol satisfies liveness.

Proof. We proof by case distinction regarding the primary:

If the primary is correct then

- **FetchLightBlock** will always return a lightblock consistent with the blockchain
- **ValidAndVerified** will verify a correct lightblock once a sufficiently recent lower lightblock can be verified.
- If Invariant (1) holds, eventually every lightblock will be verified and core verification terminates successfully.
- As by Definition (1), if the primary is correct, for liveness we are restricted to the case when Invariant (1) holds, we concludes the proof.

If the primary is faulty then there are three cases:

- it either provides lightblocks in time that pass all the tests, and the function returns with the lightblock
- or it provides one lightblock that fails a test, and the function terminates with failure.
- or it is too slow in (or stops) providing lightblocks, such that eventually Invariant (1) is discovered to be violated, and the protocol terminates with failure.

This concludes the liveness argument.

7.2 Liveness Scenarios

The simplicity in the above liveness proofs is due to Invariant (1). The problem definition allows that a protocol does nothing: Once the invariant is violated we are allowed to terminate with a failure. Successful termination depends on the age of lightStore.LatestVerified() (for instance, initially on the age of trustedHeader) and the changes of the validator sets on the blockchain. We will now give some examples.

Let \( sh \) be lightStore.LatestVerified() when core verification is called (e.g., trustedHeader) and \( sTime \) be the time the verifier is invoked.

In order to ensure liveness, lightStore always needs to contain a verified (or initially trusted) lightblock whose time is within the trusting period. To ensure this, the verifier needs to add new lightblocks to lightStore and verify them, before all lightblocks in lightStore expire.

Many changes in validator set Let’s consider Scheduler implements bisection, that is, it halves the distance. Assume the case where the validator set changes completely in each block. Then the method in this specification needs to sequentially verify all lightblocks. That is, for \( W = \log_2(targetHeight - sh.Height) \), \( W \) lightblocks need to be downloaded and checked before the lightblock of height \( sh.Height + 1 \) is added to lightStore.

- Let \( Comp \) be the local computation time needed to check lightblocks and signatures for one lightblock.
- Then we need in the worst case \( Comp + 2\Delta \) to download and check one lightblock.
- Then the first time a verified lightblock could be added to lightStore is \( sTime + W(Comp + 2\Delta) \).
• However, it can only be added if we still have a lightblock in $\text{lightStore}$, which is not expired, that is only the case if

\begin{align*}
- \quad & sh.\text{Time} < sTime + W(\text{Comp} + 2\Delta) - TP, \\
- \quad & \text{that is, if core verification is started at } sTime < sh.\text{Time} + TP - W(\text{Comp} + 2\Delta)
\end{align*}

Starting from the above argument one may then do an inductive argument from this point on, depending on the implementation of $\text{Schedule}$. We may have to account for the lightblocks that are already downloaded, but they are checked against the new $\text{lightStore}.\text{LatestVerified}()$.

We observe that the worst case time it needs to verify the lightblock of height $\text{targetHeight}$ depends mainly on how frequent the validator set on the blockchain changes. That the verifier terminates successfully crucially depends on the check that the lightblocks in $\text{lightStore}$ do not expire in the time needed to download more lightblocks, which depends on the creation time of the lightblocks in $\text{lightStore}$. That is, termination of the verifier is highly depending on the data stored in the blockchain. The current light client verifier protocol exploits that in practice changes in the validator set are rare. For instance, consider the following scenario.

**No change in validator set**  Assume that on the blockchain the validator set of the block at height $\text{targetHeight}$ is equal to $\text{sh.NextV}$. Then there is one round trip in $\text{FetchLightBlock}$ to download the lightblock of height $\text{targetHeight}$, and $\text{Comp}$ to check it. As the validator sets are equal, $\text{ValidAndVerified}$ returns $\text{OK}$, if $\text{sh.Time} > \text{now} - TP$. That is, if $sTime < \text{sh.Header.Time} + TP - 2\Delta - \text{Comp}$, then the verifier terminates successfully.

### 8 Formalization in TLA$^+$

As part of our formalization efforts, we have specified the light client protocol in TLA$^+$ and checked its properties with the symbolic model checker Apalache [KKT19]. We found that TLA$^+$ allows us to express the protocol at the level that is quite close to the mathematical description, which we provide in Sections 3–7. In addition to having a machine-readable protocol specification, we were able to produce non-trivial system executions with the model checker as well as verify the protocol properties for small parameter values. The complete specification can be found in Appendix A. In this section, we highlight non-obvious decisions about our specification.

Our TLA$^+$ specification consists of several building blocks: the reference chain, the primary model, and the protocol specification. The reference chain is populated before the light client runs. Depending on whether the primary peer is correct or faulty, the communication with the primary is modelled as a non-deterministic action that either copies blocks from the reference chain, or it produces corrupted blocks.

Our specification has five parameters:

1. **CONSTANTS**
2. **AllNodes**, \* a set of potential validators
3. **ISPRIMARYCORRECT**, \* is primary correct (a Boolean)
4. **TRUSTINGPERIOD**, \* trusting period in discrete time units
5. **TRUSTHEIGHT**, \* the starting height of the client
6. **TARGETHEIGHT** \* the goal height of the client

By fixing the specification parameters, we can verify the protocol properties with the model checker and observe counterexamples to the false hypotheses.
8.1 Specifying the reference chain

The reference chain is simply a function from block heights to the lightblocks. Since the model checker supports only finite sets, we limit the domain of this function to the set $1..\text{TG}+1$. To this end, we first define the sets of block headers and lightblocks:

| Line | Description |
|------|-------------|
| 1    | $\text{BlockHeaders} \triangleq \{ \text{height: } 1..(\text{TG} + 1), \text{time: Int}, \text{VS: SUBSET AllNodes}, \text{NextVS: SUBSET AllNodes}, \text{lastCommit: SUBSET AllNodes} \} $ |
| 2    | $\text{LightBlocks} \triangleq \{ \text{header: BlockHeaders}, \text{Commits: SUBSET AllNodes} \} $ |

In TLA+, notation $[a : A, \ldots, z : Z]$ defines the set of records whose fields $a, \ldots, z$ are restricted to the sets $A, \ldots, Z$, respectively. Moreover, $\text{SUBSET } X$ defines the powerset of a set $X$.

Several comments about the block headers are in order. First, we model timestamps as integers, leaving the time resolution up to the user’s interpretation. Second, we do not explicitly model digital signatures and thus model the validator sets and commits as subsets of $\text{AllNodes}$. Third, we omit hashes and instead limit the power of faulty peers in the peer model. Although we could add hashes in the specification, we found that they do not improve the protocol understanding, as they are an implementation detail. Fourth, we restrict voting powers to $\{0, 1\}$; otherwise, we would have to use multisets instead of sets. (One can model a validator with a voting power of $k$ with $k$ validators with the voting power of 1.)

We define the predicate $\text{InitToHeight}$ that restricts the function $\text{blockchain}$ as per Definitions 2 and 8. This predicate also non-deterministically selects a set of faulty validators $\text{Faulty} \subseteq \text{AllNodes}$ and a value of the global clock $\text{now}$, which must be above the timestamp of the last block: $\text{now} \geq \text{blockchain}[1+\text{TG}].\text{time}$. Interestingly, we model a global clock as an integer variable, by following the Lamport’s approach [Lam05b].

8.2 Specifying the primary and light client

The light client maintains the following state variables:

| Line | Description |
|------|-------------|
| 1    | $\text{VARIABLES state, nextHeight, fetchedLightBlocks, lightBlockStatus, latestVerified} $ |

These variables are similar to those in Figure 4. The variable $\text{state}$ encodes the progress of the light client and ranges over $\{\text{"working"}, \text{"finishedSuccess"}, \text{"finishedFailure"}\}$.

The variable $\text{nextHeight}$ is as in Figure 4. The other three variables model the lightstore: The variable $\text{fetchedLightBlocks}$ is a function from a subset of heights to $\text{LightBlocks}$, which maintains the lightblocks received from the primary; $\text{lightBlockStatus}$ maps those heights to the states $\text{"StateVerified"}$, $\text{"StateUnverified"}$, and $\text{"StateFailed"}$. Finally, $\text{latestVerified}$ maintains a copy of the latest verified block.

We encode a system transition with the predicate $\text{Next}$ as follows:

| Line | Description |
|------|-------------|
| 1    | $\text{Next} \triangleq \text{state} \text{ = "working"} \land (\text{VerifyToTargetDone} \lor \text{VerifyToTargetLoop}) \land \exists t \in \text{Int}: t \geq \text{now} \land \text{now'} = t \land \text{UNCHANGED } (\text{blockchain, Faulty}) $ |

In $\text{Next}$, the light client either performs one iteration of the loop in Figure 4 (by performing action $\text{VerifyToTargetLoop}$), or terminates the loop (by performing action $\text{VerifyToTargetDone}$).
Table 1: Model checking experiments with Apalache. A configuration \( n/k/B \) represents \( n \) validator nodes and \( k \) blocks. The primary is correct when \( B = C \), and the primary is faulty when \( B = F \).

| Property                                      | 4/3/C       | 4/3/F       | 5/5/C       | 5/5/F       | 7/5/F       |
|-----------------------------------------------|-------------|-------------|-------------|-------------|-------------|
| PositiveBeforeTrustedHeaderExpires           | \( X = 1 \) 9s | \( X = 1 \) 9s | \( X = 1 \) 6s | \( X = 1 \) 6s | \( X = 1 \) 8s |
| Correctness                                   | \( \leq 4 \) 9s | \( \leq 4 \) 9s | \( \leq 11 \) 3m46s | \( \leq 11 \) 5m28s | \( \leq 11 \) 13m20s |
| Precision                                     | \( \leq 4 \) 8s | \( \leq 4 \) 8s | \( \leq 11 \) 2m38s | \( \leq 11 \) 2m51s | \( \leq 11 \) 4m35s |
| SuccessOnCorrPrimaryAndChainOfTrust           | \( \leq 4 \) 9s | \( \leq 4 \) 9s | \( \leq 11 \) 2m48s | \( \leq 11 \) 2m2s | \( \leq 11 \) 3m28s |
| NoFailedBlocksOnSuccess                       | \( \leq 4 \) 9s | \( \leq 4 \) 10s | \( \leq 11 \) 2m14s | \( \leq 11 \) 2m4s | \( \leq 11 \) 3m25s |
| StoredHeadersAreVerifiedOrNotTrusted          | \( \leq 4 \) 10s | \( \leq 4 \) 10s | \( \leq 11 \) 17s   | \( \leq 11 \) 16s   | \( \leq 11 \) 23s   |
| CorrectPrimaryAndTimeliness                   | \( \leq 4 \) 9s | \( \leq 4 \) 8s | \( \leq 11 \) 2m8s  | \( \leq 11 \) 2m4s  | \( \leq 11 \) 3m52s  |
| Complexity                                    | \( \leq 4 \) 9s | \( \leq 4 \) 9s | \( \leq 11 \) 2m8s  | \( \leq 11 \) 2m4s  | \( \leq 11 \) 3m52s  |

Simultaneously, the global clock now advances by a non-negative value. We omit the details of VerifyToTargetLoop and VerifyToTargetDone, as they closely follow the code in Figure 4.

The behavior of a primary node is captured by the operator \( \text{FetchLightBlockInto} \), which is shown below:

```plaintext
1 CopyLightBlockFromChain(block, height) ≜
2     LET refBlock ≜ blockchain[height] IN
3     LET lastCommit ≜ blockchain[height + 1].lastCommit IN
4     block = [header ↦ refBlock, Commits ↦ lastCommit]
5
6 IsLightBlockAllowedByDigitalSignatures (height, block) ≜
7     \( \lor \) either the block is produced by consensus
8     \( \lor \) (enforced by hashes), while commits are not restricted
9     \( \lor \) block.header = blockchain[height]
10    \( \lor \) or the block is signed only by the faulty validators
11    \( \lor \) block.Commits ⊆ Faulty ∧ block.header.height = height
12
13 FetchLightBlockInto (block, height) ≜
14     IF IS_PRIMARY_CORRECT
15        THEN CopyLightBlockFromChain (block, height)
16        ELSE IsLightBlockAllowedByDigitalSignatures (height, block)
```

In this code, a correct primary simply copies the block header and the respective commit from the reference chain. A faulty peer has more freedom, which is restricted with the predicate \( \text{IsLightBlockAllowedByDigitalSignatures} \). Like a correct primary, it can also produce a sound lightblock. Additionally, it may produce a sound block header, but an incorrect set of commits. Alternatively, if the block header is different from the block on the reference chain, then it may be signed only by the faulty validators.

### 8.3 Model Checking Experiments

Our main goal in the verification efforts is to prove safety and liveness of the protocol as per Definition 11. We have formalized the safety property of Definition 11 as a state invariant called Correctness. Assuming that the protocol terminates, the liveness property of Definition 11 can also be written as a state invariant, which describes the state upon termination (successful or not). We call this state invariant \( \text{SuccessOnCorrPrimaryAndChainOfTrust} \).
We perform bounded model checking with Apalache, which explores executions up to a given length. Although this activity can produce counterexamples, it does not guarantee absence of bugs. Interestingly, the light client always terminates in a fixed number of steps that depends on the difference between the target height and the trusted height. If we call this difference $\delta$, then the protocol should terminate in no more than $T(\delta) = \frac{\delta(\delta - 1)}{2}$ iterations. This is the worst-case bound for conditions (S1)-(S3), and a concrete implementation may schedule block queries more optimally, e.g., a worst-case linear bound or an expected sublinear bound.

To check the complexity bound, we have written a state invariant called Complexity, which tests that the protocol does not go over the worst-case bound. When we fix the specification parameters, Apalache finds a deadlock for the computations longer than $T(\delta)$. Together with Complexity, this gives us a termination argument (for fixed parameters). Hence, it suffices to check the above invariants.

To improve our understanding of the protocol, we have also specified a few additional properties. For instance, one could (wrongly) expect that the initially trusted block should still be within the trusting period, when the light client terminates. We formulate this property as a state invariant below:

1. PositiveBeforeTrustedHeaderExpires $\triangleq$
2. \text{LET trustedTime} $\triangleq$ blockchain[TRUSTED_HEIGHT].time \text{IN}
3. \text{state} = "finishedSuccess"
4. $\Rightarrow$ trustedTime $\geq$ now + TRUSTING_PERIOD

This invariant is violated, as the light client trusts the block at TARGET_HEIGHT upon successful termination, whereas the block at TRUSTED_HEIGHT may be not trusted anymore. The model checker produces a counterexample in one step.

Another false hypothesis is formulated in the invariant candidate StoredHeadersAreVerifiedOrNot-Trusted. It is a weaker version of SuccessOnCorrPrimaryAndChainOfTrust, as it is inspecting all blocks, not only the verified ones. Although this property holds for $\delta = 3$, it fails for $\delta = 5$. The model checker is showing us a counterexample, where the lightblock 5 is verified on the basis of block 3, while block 4 is kept unverified.

| StoredHeadersAreVerifiedOrNotTrusted $\triangleq$
| state = "finishedSuccess"
| $\Rightarrow$ $\forall lh, rh \in \text{DOMAIN fetchedLightBlocks}$:
| $\lor lh \geq rh$
| $\lor \exists mh \in \text{DOMAIN fetchedLightBlocks}$: $lh < mh \land mh < rh$
| $\lor$ \text{LET } l = fetchedLightBlocks[lh]
| r = fetchedLightBlocks[rh] \text{ IN}
| $\lor$ "OK" = ValidAndVerified(l, r)
| $\lor$ now $-$ l.header.time $> \text{TRUSTING_PERIOD}$

Table 1 summarizes the results of our experiments with Apalache. The experiments were run in an AWS instance equipped with 32GB RAM and a 4-core Intel® Xeon® CPU E5-2686 v4 @ 2.30GHz CPU. We write “$X_{=k}$” when a bug is reported at depth $k$, and “$\checkmark_{\leq k}$” when no bug is reported up to depth $k$. We ran the experiments for small parameter values such as 4 to 7 nodes and 3 to 5 blocks. For these parameters, the tool responds in a matter of minutes, which is fast enough for us to see interesting counterexamples. For larger parameter values, the model checking gets significantly slower. We believe that this is caused by powersets and cardinality tests.

We also tried to run the model checker TLC. However, we ran into two problems. First, TLC enumerates states, so it requires timestamps and the global clock to range over a finite domain. Second, even when we introduced a logical abstraction of time, TLC could not inspect all initial states, as it had to enumerate all combinations of multiple powersets. Although we believe that it
would be possible to use this model checker by introducing a more abstract version of the blockchain, we found that Apalache was sufficient for our purposes.

In conclusion, model checking has improved our understanding of the protocol. It also has confirmed our intuition by showing us counterexamples. In the future, we plan to construct an inductive invariant, to obtain a complete argument over block heights.

9 Implementation

We implemented the light client verification protocol in the Rust programming language.

Architecture The light client is architected for composability: It was expected that a light client running the verification algorithm would share the process space with other components running in separate threads. These separate components require a synchronous interface to fetch headers the light client verified.

To maintain simplicity of the core verification logic, the light client is implemented as a finite state machine operating on events fetched from an unbounded queue. Events processed by the light client can then be understood as atomic transformations, performed on state owned and encapsulated within the light client. Events sent to the light client can include a callback to facilitate synchronizing interactions between components. The queues in this case have the benefit of serializing all access to the light client core logic, eliminating the need for mutexes while guaranteeing memory safety.

Interactions with the light client are performed via a facade which acts as a thin interface exposing synchronous methods which serialize interactions with the light client runtime via the queue. The method set for this interface is abstracted in such a way as to allow mock replacements to be used during testing. This abstraction allows testing complex interactions between components.

Rust implementation Each aspect of the protocol is specified at the code level as an interface (called trait in Rust), hereafter called a component. Each component may (but need not) depend on other components. This allows us to unit-test each component independently by mocking out the others components it depends on. Moreover, this approach also enables us to implement deterministic and reproducible tests by mocking out components which perform intrinsically non-deterministic computations, such as performing network requests or fetching the current system time.

Figure 5 shows the definition of the Verifier trait, which consists of a single method taking in a untrusted block, a trusted one, and a set of options including the TP and the current time. The definition of a concrete verifier which depends on other components is provided in Figure 6.

The implementation spans around 2500 lines of code (not counting comments and whitespace), and is openly available [Bli20].

| pub trait Verifier { |
| fn verify(&self, untrusted: &LightBlock, trusted: &LightBlock, options: &Options) -> Verdict; |
|} |

Figure 5: Definition of the verifier trait
1 pub struct ProdVerifier {
2     predicates: Box<dyn VerificationPredicates>,
3     voting_power_calculator: Box<dyn VotingPowerCalculator>,
4     commit_validator: Box<dyn CommitValidator>,
5     header_hasher: Box<dyn HeaderHasher>
6 }

Figure 6: Definition of a concrete verifier

10 Related Work

Bitcoin introduced the notion of a light client protocol in the form of simplified payment verification (SPV) [Nak19]. In SPV, a client downloads complete chains of block headers in order to discover the longest chain, or more accurately, the chain with the greatest amount of computational work, which is deemed to be the canonical one. From there, it can verify proofs of transaction inclusion in any of the blocks. Notably, the protocol is linear in the number of blocks, which may be prohibitive for long chains. Sublinear variants of SPV have been proposed, including so-called Proofs-of-Proofs-of-Work [KLS16, KMZ17] and Flyclient [BKLZ19], which utilize probabilistic sampling to reduce the number of headers a light client must download. These solutions apply strictly to consensus protocols where agreement is determined by a heaviest-chain scoring metric, and are thus not relevant to BFT protocols like Tendermint, where chains are extended one block at a time by a quorum of validators.

Tendermint was the first system to lift traditional BFT consensus protocols [CL02, DLS88, Lam11, BSA14] into the blockchain domain [Kwo14]. In traditional BFT consensus, clients submit requests directly to validator nodes (known as replicas), and wait to receive a quorum of identical responses [CL02, BSA14]. That is, these systems expect clients to know the network addresses of validators and to maintain direct connections with them. And while some do not even support validator set changes (i.e., reconfiguration) [CL02], those that do expect clients to learn about the latest validator set from some unspecified directory service [BSA14]. In a public, open-membership, adversarial setting with arbitrary validator set changes and no trusted directory service, such an approach to servicing clients is wholly insufficient.

Most comparable blockchain systems avoid this problem by restricting validator set changes to happen in “epochs”, so that the set of validators is static for a period of time (an epoch) and can only change at the epoch boundary. In this setting, a lightclient could always skip from the first to last height in the epoch, and only needs to verify validator set changes at the epoch boundaries. Tendermint, however, does not restrict the changes of the validator sets; they can happen at every block.

Tendermint emerged in the context of Proof-of-Stake blockchains, where economic stake within the system, rather than resource consumption outside the system, is used to incentivize correct behaviour. Proof-of-Stake systems have long been known to suffer from the so-called nothing-at-stake attack, whereby past validators who have since exited their stake can forge arbitrary alternative histories [But14]. Such attacks are solved by subjective initialization, whereby a client subjectively decides which validators to initially trust, and by an unbonding period, during which validators can be punished for misbehaviour, and beyond which they can no longer be trusted by clients. While much has been written about this informally online [But14, But15, Unc17, PoS20], we are not aware of a formal treatment of the Proof-of-Stake light client problem.

From the viewpoint of more classic research literature, the light client problem is a modern
variant of performing a read operation from a replicated database [BHGS7], or reading a shared
state [ABD95], when some of the peers are faulty, or learning an accepted value in Paxos [Lam05a].
As this is an important problem, there is a vast literature on this subject, also with respect to diverse
consistency criteria, e.g., [GB17]. As Tendermint blockchains provide “immediate finality” — a.k.a.,
irrevocability in more classic consensus definitions [CBS09] —, for the light client we are interested
in strong consistency.

The first contribution of this paper is a formalization of the Tendermint Security model. It
shares the aspect of Authenticated Byzantine Faults with in the classic work in [DLSS88, CL02].
However, the staking mechanism requires us to formalize a notion close to Byzantine faults with
recovery which is less studied [CL02, BHNN00, ADGF+07]. A similar concept has also be considered
for communication faults [BCBG+07].

Other approaches to achieve a sublinear traversal of blockchains include the use of alternative
authenticated data structures or more advanced cryptography. For instance, the Merkle Mountain
Ranges [CW09, Hod12] used in Flyclient [BKLZ19] can be used in a BFT-based blockchain for
logarithmically verifying that a past block is an ancestor of a more recent trusted block. However,
as noted, they cannot be used for verifying that a future block is a child of a past trusted block
without a mechanism like that described in this paper. Skipchains [NKKJ+17] do allow clients
to skip from past to future blocks, though they require the retroactive addition of (aggregated or
collective) signatures to past blocks. Since past blocks cannot be directly modified, such protocols
should be considered as services layered on top of the underlying blockchain protocol. Finally,
recent advances in cryptography [Gro16, BSCTV17] enable blockchain designs where clients verify
succint proofs attesting to some set of state transitions. This can take the form of proofs that the
validator set changed in a particular way [GGJ+20], or, in a more extreme case, proofs attesting
to the correct execution of the entire blockchain protocol, which eliminate the need to traverse the
chain at all [MS18]. While exciting, such protocols tend to require more exotic assumptions beyond
the standard authenticated Byzantine fault model and are thus less proven in real-world systems.

While we have focused here on a sublinear light client protocol for verifying a Tendermint
blockchain under the Security Model we outlined, we have not addressed what guarantees remain
in the event the security model fails, i.e., when 1/3 or more of the voting power is faulty. While
such a scenario may cause our light client to accept a faulty chain (i.e., a fork), such faults may be
detected, so long as the client is connected to at least one honest full node — a standard assumption
among light client protocols. Furthermore, in future work, we intend to show that validators are
accountable, that is, detection of forks will result in the faulty validators being identified, and thus
punished accordingly. While such protocols may detect forks in the blockchain, other protocols
have focused on detecting invalid state transitions, where validators commit to a state that cannot
be derived from applying transactions in the blockchain to the previous state [ABSB18]. Such
protocols are complementary to ours; they use so-called fraud proofs and data-availability proofs
to allow light clients to detect invalid state transitions, even when a majority of validators are faulty.

We have used TLA+ [Lam02] for specification as it became a Lingua Franca for formal
specification of complex distributed systems [NRZ+15]. The APALACHE model checker [KKT19]
proved very effective to model check the protocol under different fault scenarios. Recently there
has been made significant progress in automated verification of fault-tolerant distributed algorithms
[LL+19, DHV+14, KLVW17, vGGKB+19, KQH18, BEJQ18, DDMW19]. While this work
typically focuses on consensus and Paxos-like algorithms, our work considers how to observe the
state of the result of consensus from the outside. In systems that solve consensus, the notions of
quorums or thresholds are crucial and at the core of verification approaches [DHV+14, KLVW17,
BLL+19]. In our system, these quorums appear in limited form, namely as data in the validator
sets and commits. For now, our current model checking results just consider small systems (up to
seven validators), but we are confident to be able to adapt the recent results in automated veri-
fication to our domain in order to be able to scale to realistic sizes, or even to the parameterized case [AK86, EN95, BJK+15], that is, for all numbers of validators.

Recently, Ognjanovic [Ogn20] used our TLA+ model as a base for implementing the light client verification protocol in Scala and verifying it with Stainless (https://stainless.epfl.ch).

11 Conclusions

Security. We have presented the first formalization of the Tendermint security model, which allows us to understand it as an Authenticated Byzantine model with dynamic Byzantine faults. We presented a light client verification protocol based on that model, and proved that it is always safe, and that it satisfies liveness if it communicates with a correct full node. It is clear that in principle faulty full nodes would benefit from lying to the light client, by trying to make the light client accept a block that deviates (e.g., contains additional transactions) from the one generated by Tendermint consensus. However, our safety properties guarantees that this cannot happen if the security model holds.

However, the question remains whether for liveness, full nodes would benefit from cooperating, i.e., from responding timely. This is indeed the case if we consider the broader context where the classic less than 1/3 model may be violated. In this case, the light client may help the correct full nodes to understand whether their header is a good one, or in other words, to detect forks on the chain. In parallel to the verification logic described in this paper, we also design a fork detector that probes multiple full nodes. In combination with the detector, the correct full nodes indeed have the incentive to respond, and we can base our liveness arguments on the assumption that correct full nodes reliably respond. The details of the fork detector is outside the scope of this paper.

Performance. It is obvious that in the case where validator set changes are rare (which is the case in the Cosmos Hub, the largest live network in the Cosmos ecosystem), skipping verification outperforms sequential verification: if the validator set does not change, then verifying a block on height 1000 based on height 100 needs one step with skipping verification and 900 steps with sequential verification; each step involving expensive operations as checking hashes and signatures. Still, there are several interesting performance measurement we are interested in so that we are currently setting up a framework for experimental performance evaluation.

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A APPENDIX: Complete TLA+ specifications

MODULE Blockchain_A1

This is a high-level specification of Tendermint blockchain that is designed specifically for the light client. Validators have the voting power of one. If you like to model various voting powers, introduce multiple copies of the same validator (do not forget to give them unique names though).

EXTENDS Integers, FiniteSets

Min(a, b) △= IF a < b THEN a ELSE b

CONSTANT

AllNodes,

a set of all nodes that can act as validators (correct and faulty)

ULTIMATE_HEIGHT,

a maximal height that can be ever reached (modelling artifact)

TRUSTING_PERIOD

the period within which the validators are trusted

Heights △= 1 .. ULTIMATE_HEIGHT

possible heights

A commit is just a set of nodes who have committed the block

Commits △= SUBSET AllNodes

The set of all block headers that can be on the blockchain. This is a simplified version of the Block data structure in the actual implementation.

BlockHeaders △= [

height : Heights,

the block height

time : Int,

the block timestamp in some integer units

lastCommit : Commits,

the nodes who have voted on the previous block, the set itself instead of a hash

in the implementation, only the hashes of V and NextV are stored in a block, as V and NextV are stored in the application state

VS : SUBSET AllNodes,

the validators of this bloc. We store the validators instead of the hash.

NextVS : SUBSET AllNodes

the validators of the next block. We store the next validators instead of the hash.
]

A signed header is just a header together with a set of commits

LightBlocks △= [header : BlockHeaders, Commits : Commits]

VARIABLES

now,

the current global time in integer units

blockchain,

A sequence of BlockHeaders, which gives us a bird view of the blockchain.

Faulty

A set of faulty nodes, which can act as validators. We assume that the set of faulty processes is non-decreasing.

If a process has recovered, it should connect using a different id.

all variables, to be used with UNCHANGED

vars △= (now, blockchain, Faulty)
The set of all correct nodes in a state

\[ \text{Corr} \triangleq \text{AllNodes} \setminus \text{Faulty} \]

APALACHE annotations

\[ a < : b \triangleq a \quad \text{type annotation} \]

\[ NT \triangleq \text{STRING} \]

\[ \text{NodeSet}(S) \triangleq S < : \{NT\} \]

\[ \text{EmptyNodeSet} \triangleq \text{NodeSet}\left(\{\}\right) \]

\[ BT \triangleq [\text{height} \mapsto \text{Int}, \text{time} \mapsto \text{Int}, \text{lastCommit} \mapsto \{NT\}, \text{VS} \mapsto \{NT\}, \text{NextVS} \mapsto \{NT\}] \]

\[ LBT \triangleq [\text{header} \mapsto BT, \text{Commits} \mapsto \{NT\}] \]

end of APALACHE annotations

the header is still within the trusting period

\[ \text{InTrustingPeriod}\left(\text{header}\right) \triangleq \text{now} \leq \text{header} \cdot \text{time} + \text{TRUSTING\_PERIOD} \]

Given a function \( pVotingPower \in D \to \text{Powers} \) for some \( D \subseteq \text{AllNodes} \) and \( pNodes \subseteq D \), test whether the set \( pNodes \subseteq \text{AllNodes} \) has more than 2/3 of voting power among the nodes in \( D \).

\[ \text{TwoThirds}\left(pVS, pNodes\right) \triangleq \]

\[ \text{LET } TP \triangleq \text{Cardinality}(pVS) \]

\[ \text{SP} \triangleq \text{Cardinality}(pVS \cap pNodes) \]

\[ \text{IN} \]

\[ 3 \cdot SP > 2 \cdot TP \quad \text{when thinking in real numbers, not integers: } SP > 2.0 / 3.0 \cdot TP \]

Given a set of \( \text{FaultyNodes} \), test whether the voting power of the correct nodes in \( D \) is more than 2/3 of the voting power of the faulty nodes in \( D \).

\[ \text{IsCorrectPower}\left(pFaultyNodes, pVS\right) \triangleq \]

\[ \text{LET } FN \triangleq pFaultyNodes \cap pVS \]

\[ CN \triangleq pVS \setminus pFaultyNodes \]

\[ CP \triangleq \text{Cardinality}(CN) \]

\[ FP \triangleq \text{Cardinality}(FN) \]

\[ \text{IN} \]

\[ CP > 2 \cdot FP \quad \text{Note: when } FP = 0, \text{ this implies } CP > 0. \]

This is what we believe is the assumption about failures in Tendermint

\[ \text{FaultAssumption}\left(pFaultyNodes, pNow, pBlockchain\right) \triangleq \]

\[ \forall h \in \text{Heights} : \]

\[ pBlockchain[h] \cdot \text{time} + \text{TRUSTING\_PERIOD} > pNow \Rightarrow \]

\[ \text{IsCorrectPower}\left(pFaultyNodes, pBlockchain[h]\right) \]

Can a block be produced by a correct peer, or an authenticated Byzantine peer

\[ \text{IsLightBlockAllowedByDigitalSignatures}\left(ht, block\right) \triangleq \]

\[ \forall \text{block.header} = \text{blockchain}[ht] \quad \text{signed by correct and faulty (maybe)} \]

\[ \forall \text{block.Commit} \subseteq \text{Faulty} \wedge \text{block.header.height} = ht \quad \text{signed only by faulty} \]

Initialize the \( \text{blockchain} \) to the ultimate height right in the initial states. We pick the faulty validators statically, but that should not affect the light client.

\[ \text{InitToHeight} \triangleq \]

---

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∧ Faulty ∈ subset AllNodes  some nodes may fail
pick the validator sets and last commits
∧ ∃ vs, lastCommit ∈ [Heights → subset AllNodes]
∧ timestamp ∈ [Heights → Int]
    now is at least as early as the timestamp in the last block
∧ ∃ tm ∈ Int : now = tm ∧ tm ≥ timestamp[ULTIMATE_HEIGHT]
    the genesis starts on day 1
∧ timestamp[1] = 1
∧ vs[1] = AllNodes
∧ lastCommit[1] = EmptyNodeSet
∧ ∀ h ∈ Heights \ {1}:
    ∧ lastCommit[h] ⊆ vs[h − 1]  the non-validators cannot commit
    ∧ TwoThirds(vs[h − 1], lastCommit[h])  the commit has > 2/3 of validator votes
    ∧ IsCorrectPower(Faulty, vs[h])  the correct validators have > 2/3 of power
    ∧ timestamp[h] > timestamp[h − 1]  the time grows monotonically
    ∧ timestamp[h] < timestamp[h − 1] + TRUSTING_PERIOD  but not too fast
    ∧ lastCommit[h] ⊆ vs[h]
    ∧ NextVS[h] = vs[h] or AllNodes
∧ blockchain = [h ∈ Heights ↦
    [height ↦ h,
    time ↦ timestamp[h],
    VS ↦ vs[h],
    NextVS ↦ if h < ULTIMATE_HEIGHT then vs[h + 1] else AllNodes,
    lastCommit ↦ lastCommit[h]]]

is the blockchain in the faulty zone where the Tendermint security model does not apply

InFaultyZone ≜ ¬FaultAssumption(Faulty, now, blockchain)

******************************************************************************* BLOCKCHAIN ACTIONS ****************************

Advance the clock by zero or more time units.

AdvanceTime ≜
∧ ∃ tm ∈ Int : tm ≥ now ∧ now' = tm
∧ UNCHANGED ⟨blockchain, Faulty⟩

One more process fails. As a result, the blockchain may move into the faulty zone. The light client is not using this action, as the faults are picked in the initial state. However, this action may be useful when reasoning about fork detection.

OneMoreFault ≜
∧ ∃ n ∈ AllNodes \ Faulty :
    ∧ Faulty' = Faulty ∪ {n}
    ∧ Faulty' ≠ AllNodes  at least process remains non-faulty
∧ UNCHANGED ⟨now, blockchain⟩

}
**MODULE Lightclient_A_1**

* A state-machine specification of the lite client, following the English spec:

* [https://github.com/informalsystems/tendermint-rs/blob/master/docs/spec/lightclient/verification.md](https://github.com/informalsystems/tendermint-rs/blob/master/docs/spec/lightclient/verification.md)

EXTENDS Integers, FiniteSets

**CONSTANTS**

- **TRUSTED_HEIGHT**, an index of the block header that the light client trusts by social consensus
- **TARGET_HEIGHT**, an index of the block header that the light client tries to verify
- **TRUSTING_PERIOD**, the period within which the validators are trusted
- **IS_PRIMARY_CORRECT**
  - is primary correct?

**VARIABLES**

- **state**, the current state of the light client
- **nextHeight**, the next height to explore by the light client
- **nprobes**, the lite client iteration, or the number of block tests
- **fetchedLightBlocks**, a function from heights to LightBlocks
- **lightBlockStatus**, a function from heights to block statuses
- **latestVerified**, the latest verified block

- **lcvars** ≝ (state, nextHeight, fetchedLightBlocks, lightBlockStatus, latestVerified)

****************** Blockchain instance **********************************

- **CONSTANTS**
  - **AllNodes**, a set of all nodes that can act as validators (correct and faulty)

- **VARIABLES**
  - **now**, blockchain, Faulty

- **bcvars** ≝ (now, blockchain, Faulty)

Create an instance of Blockchain. We could write **extends Blockchain**, but then all the constants and state variables would be hidden inside the Blockchain module.

**ULTIMATE_HEIGHT** ≝ **TARGET_HEIGHT** + 1

**BC** ≝ **instance Blockchain_A_1** with

- **now** ← **now**, **blockchain** ← **blockchain**, **Faulty** ← **Faulty**
the heights on which the light client is working

\[ \text{HEIGHTS} \equiv \text{TRUSTED\_HEIGHT . TARGET\_HEIGHT} \]

the control states of the lite client

\[ \text{States} \equiv \{ \text{“working”, “finishedSuccess”, “finishedFailure”} \} \]

* 
Check the precondition of \text{ValidAndVerified}.

[LCV-FUNC-VALID.1:: TLA-PRE.1]

\text{ValidAndVerifiedPre}(\text{trusted, untrusted}) \equiv \\
\text{LET} \ thdr \equiv \text{trusted.header} \\
\hspace{1em} \text{uhdr} \equiv \text{untrusted.header} \\
\text{IN} \\
\quad \land \ BC! \text{InTrustingPeriod}(\text{thdr}) \\
\quad \land \ thdr.\text{height} < \text{uhdr.height} \\
\hspace{1em} \text{the trusted block has been created earlier (no drift here)} \\
\quad \land \ thdr.\text{time} \leq \text{uhdr.time} \\
\quad \land \ \text{untrusted}.\text{Commits} \subseteq \text{uhdr}.\text{VS} \\
\quad \land \ \text{LET} \ TP \equiv \text{Cardinality(\text{uhdr}.\text{VS})} \\
\hspace{1em} SP \equiv \text{Cardinality(\text{untrusted}.\text{Commits})} \\
\text{IN} \\
\quad 3 \ast SP > 2 \ast TP \\
\quad \land \ thdr.\text{height} + 1 = \text{uhdr.height} \Rightarrow \text{thdr}.\text{NextVS} = \text{uhdr}.\text{VS} \\

As we do not have explicit hashes we ignore these three checks of the \text{English} spec:

1. “\text{trusted}.\text{Commit} is a commit is for the header \text{trusted}.\text{Header}, i.e. it contains the correct hash of the header”.
2. \text{untrusted}.\text{Validators} = \text{hash(\text{untrusted}.\text{Header}.\text{Validators})}
3. \text{untrusted}.\text{NextValidators} = \text{hash(\text{untrusted}.\text{Header}.\text{NextValidators})}

* 
* Check that the commits in an \text{untrusted} block form 1/3 of the next \text{validators} \\
* in a \text{trusted} header.

\text{SignedByOneThirdOfTrusted}(\text{trusted, untrusted}) \equiv \\
\text{LET} \ TP \equiv \text{Cardinality(\text{trusted}.\text{header}.\text{NextVS})} \\
\hspace{1em} SP \equiv \text{Cardinality(\text{untrusted}.\text{Commits} \cap \text{trusted}.\text{header}.\text{NextVS})} \\
\text{IN} \\
\quad 3 \ast SP > TP \\

* 
Check, whether an \text{untrusted} block is valid and verifiable \text{w.r.t.} a \text{trusted} header.

[LCV-FUNC-VALID.1:: TLA.1]

\text{ValidAndVerified}(\text{trusted, untrusted}) \equiv \\
\text{IF} \ \neg \text{ValidAndVerifiedPre}(\text{trusted, untrusted}) \\
\quad \text{THEN} \ “\text{FAILED\_VERIFICATION}” \\
\quad \text{ELSE IF} \ \neg \text{BC! \text{InTrustingPeriod}(\text{untrusted}.\text{header})} \\
\quad \text{THEN} \ “\text{FAILED\_TRUSTING\_PERIOD}” \\
\quad \text{ELSE IF} \ \text{untrusted}.\text{header}.\text{height} = \text{trusted}.\text{header}.\text{height} + 1
\[\forall SignedByOneThirdOfTrusted(trusted, untrusted)\]

\[\text{THEN "OK" }\]

\[\text{ELSE "CANNOT\_VERIFY"}\]

Initial states of the light client. Initially, only the trusted light block is present.

\[\text{LCInit } \triangleq \]
\[\land state = \text{"working"} \]
\[\land \text{nextHeight} = \text{TARGET\_HEIGHT}\]
\[\land nprobes = 0 \quad \text{no tests have been done so far}\]
\[\land \text{LET trustedBlock } \triangleq \text{blockchain}[\text{TRUSTED\_HEIGHT}]\]
\[\text{trustedLightBlock } \triangleq [\text{header } \mapsto \text{trustedBlock}, \text{Commits } \mapsto \text{AllNodes}]\]
\[\text{IN}\]
\[\land \text{fetchedLightBlocks is a function of one element, i.e., TRUSTED\_HEIGHT}\]
\[\land \text{fetchedLightBlocks } = [h \in \{\text{TRUSTED\_HEIGHT}\} \mapsto \text{trustedLightBlock}]\]
\[\land \text{lightBlockStatus is a function of one element, i.e., TRUSTED\_HEIGHT}\]
\[\land \text{lightBlockStatus } = [h \in \{\text{TRUSTED\_HEIGHT}\} \mapsto \text{"StateVerified"}]\]
\[\land \text{latestVerified } = \text{trustedLightBlock}\]

The block should contain a copy of the block from the reference chain, with a matching commit

\[\text{LCV\_FUNC\_FETCH.1:: TLA.1}\]
\[\text{FetchLightBlockInto} (\text{block}, \text{height}) \triangleq\]
\[\text{LET ref } \triangleq \text{blockchain[height]}\]
\[\text{lastCommit } \triangleq\]
\[\text{IF height } < \text{ULTIMATE\_HEIGHT}\]
\[\text{THEN blockchain[height + 1].lastCommit}\]
\[\text{ELSE blockchain[height].VS}\]
\[\text{IN}\]
\[\text{block } = [\text{header } \mapsto \text{ref}, \text{Commits } \mapsto \text{lastCommit}]\]

Either the primary is correct and the block comes from the reference chain, or the block is produced by a faulty primary.

\[\text{[LCV\_FUNC\_FETCH.1:: TLA.1]}\]
\[\text{FetchLightBlockInto} (\text{block}, \text{height}) \triangleq\]
\[\text{IF IS\_PRIMARY\_CORRECT}\]
\[\text{THEN CopyLightBlockFromChain} (\text{block}, \text{height})\]
\[\text{ELSE BC! IsLightBlockAllowedByDigitalSignatures} (\text{height}, \text{block})\]

Add a block into the light store

\[\text{[LCV\_FUNC\_UPDATE.1:: TLA.1]}\]
\[\text{LightStoreUpdateBlocks} (\text{lightBlocks}, \text{block}) \triangleq\]
\[\text{LET } \text{ht } \triangleq \text{block.header.height}\]
\[\text{IN}\]
\[\{h \in \text{domain lightBlocks } \cup \{\text{ht}\} \mapsto\]
\[\text{IF } h = \text{ht \ THEN block ELSE lightBlocks[h]}\]

Update the state of a light block
\textbf{[LCV-FUNC-UPDATE.1:: TLA.1]} \hfill LightStoreUpdateStates(\texttt{statuses, ht, blockState}) \triangleq \\
\{h \in \text{DOMAIN} \; \text{statuses} \cup \{\text{ht}\} \mapsto \\
\text{IF } h = \text{ht} \text{ THEN blockState ELSE statuses}[h]\}

Check, whether \texttt{newHeight} is a possible next height for the light client.

\textbf{[LCV-FUNC-SCHEDULE.1:: TLA.1]} \hfill CanScheduleTo(\texttt{newHeight, pLatestVerified, pNextHeight, pTargetHeight}) \triangleq \\
\text{LET } \text{ht} \triangleq pLatestVerified.header.height IN \\
\bigvee \quad \text{ht} = \text{pNextHeight} \\
\quad \wedge \text{ht} < \text{pTargetHeight} \\
\quad \wedge \text{pNextHeight} < \text{newHeight} \\
\quad \wedge \text{newHeight} \leq \text{pTargetHeight} \\
\bigvee \quad \text{ht} < \text{pNextHeight} \\
\quad \wedge \text{ht} < \text{pTargetHeight} \\
\quad \wedge \text{ ht} < \text{newHeight} \\
\quad \wedge \text{ newHeight} < \text{pNextHeight} \\
\bigvee \quad \text{ht} = \text{pTargetHeight} \\
\quad \wedge \text{ newHeight} = \text{pTargetHeight}

The loop of \texttt{VerifyToTarget}.

\textbf{[LCV-FUNC-MAIN.1:: TLA-LOOP.1]} \hfill VerifyToTargetLoop \triangleq \\
\text{the loop condition is true} \\
\wedge \text{latestVerified.header.height} < \text{TARGET\_HEIGHT} \\
\quad \text{pick a light block, which will be constrained later} \\
\wedge \exists \text{current} \in BC!LightBlocks : \\
\text{Get next LightBlock for verification} \\
\wedge \text{IF } \text{nextHeight} \in \text{DOMAIN} \; \text{fetchedLightBlocks} \\
\text{THEN} \\
\quad \text{copy the block from the light store} \\
\quad \wedge \text{current} = \text{fetchedLightBlocks}[\text{nextHeight}] \\
\quad \wedge \text{UNCHANGED} \; \text{fetchedLightBlocks} \\
\text{ELSE} \\
\quad \text{retrieve a light block and save it in the light store} \\
\quad \wedge \text{FetchLightBlockInto}(\text{current, nextHeight}) \\
\quad \wedge \text{fetchedLightBlocks}' = \text{LightStoreUpdateBlocks}(\text{fetchedLightBlocks, current}) \\
\text{Record that one more probe has been done (for complexity and model checking)} \\
\wedge \text{nprobes}' = \text{nprobes} + 1 \\
\text{Verify the current block} \\
\wedge \text{LET } \text{verdict} \triangleq \text{ValidAndVerified(\text{latestVerified, current})} IN \\
\text{Decide whether/how to continue} \\
\text{CASE } \text{verdict} = \text{"OK"} \rightarrow \\
\wedge \text{lightBlockStatus}' = \text{LightStoreUpdateStates(\text{lightBlockStatus, nextHeight, "StateVerified"})} \\
\wedge \text{latestVerified}' = \text{current} \\
\wedge \text{state}' = \\
\quad \text{IF } \text{latestVerified}'.header.height < \text{TARGET\_HEIGHT} \\
\quad \text{THEN } \text{"working"} \\
\text{ELSE } \text{"finishedSuccess"}
\( \exists \text{newHeight} \in \text{HEIGHTS} : \)
\( \land \text{CanScheduleTo(newHeight, current, nextHeight, TARGET\_HEIGHT)} \)
\( \land \text{nextHeight}' = \text{newHeight} \)

\( \square \text{verdict} = "\text{CANNOT\_VERIFY}" \rightarrow \)
do nothing: the light block current passed validation, but the validator set is too different to verify it. We keep the state of current at \( \text{StateUnverified} \). For a later iteration, Schedule might decide to try verification of that light block again.

\( \land \text{lightBlockStatus}' = \text{LightStoreUpdateStates(lightBlockStatus, nextHeight, "StateUnverified")} \)
\( \land \exists \text{newHeight} \in \text{HEIGHTS} : \)
\( \land \text{CanScheduleTo(newHeight, latestVerified, nextHeight, TARGET\_HEIGHT)} \)
\( \land \text{nextHeight}' = \text{newHeight} \)
\( \land \text{UNCHANGED (latestVerified, state)} \)

\( \square \text{OTHER} \rightarrow \)

\( \text{verdict is some error code} \)
\( \land \text{lightBlockStatus}' = \text{LightStoreUpdateStates(lightBlockStatus, nextHeight, "StateFailed" )} \)
\( \land \text{state}' = "\text{finishedFailure}" \)
\( \land \text{UNCHANGED (latestVerified, nextHeight)} \)

The terminating condition of \( \text{VerifyToTarget} \).

\[ \text{LCV\_FUNC\_MAIN.1:: TLA\_LOOPCOND.1} \]

\( \text{VerifyToTargetDone} \triangleq \)
\( \land \text{latestVerified.header.height} \geq \text{TARGET\_HEIGHT} \)
\( \land \text{state}' = "\text{finishedSuccess}" \)
\( \land \text{UNCHANGED (nextHeight, nprobes, fetchedLightBlocks, lightBlockStatus, latestVerified)} \)

*************** Lite client + Blockchain ******************

\( \text{Init} \triangleq \)

the blockchain is initialized immediately to the \( \text{ULTIMATE\_HEIGHT} \)
\( \land \text{BC!InitToHeight} \)

the light client starts
\( \land \text{LCInit} \)

The system step is very simple. The light client is either executing \( \text{VerifyToTarget} \), or it has terminated. (In the latter case, a model checker reports a deadlock.) Simultaneously, the global clock may advance.

\( \text{Next} \triangleq \)
\( \land \text{state} = "\text{working}" \)
\( \land \text{VerifyToTargetLoop} \lor \text{VerifyToTargetDone} \)
\( \land \text{BC!AdvanceTime} \)
the global clock is advanced by zero or more time units

*************** Types ******************

\( \text{TypeOK} \triangleq \)
\( \land \text{state} \in \text{States} \)
\( \land \text{nextHeight} \in \text{HEIGHTS} \)
\( \land \text{latestVerified} \in \text{BC!LightBlocks} \)
\( \land \exists \text{HS} \in \text{SUBSET HEIGHTS} : \)
\( \land \text{fetchedLightBlocks} \in [\text{HS} \to \text{BC!LightBlocks}] \)
\( \land \text{lightBlockStatus} \)
\( \in [\text{HS} \to \{"\text{StateVerified", "StateUnverified", "StateFailed" }\} ] \)
The properties to check
this invariant candidate is false

NeverFinish \(\triangleq\)
\[
\text{state} = \text{"working"}
\]
this invariant candidate is false

NeverFinishNegative \(\triangleq\)
\[
\text{state} \neq \text{"finishedFailure"}
\]
This invariant holds true, when the primary is correct.
This invariant candidate is false when the primary is faulty.

NeverFinishNegativeWhenTrusted \(\triangleq\)
\[
(\text{minTrustedHeight} \leq \text{TRUSTED\_HEIGHT})
\]
\[\text{BC!InTrustingPeriod}(\text{blockchain}[\text{TRUSTED\_HEIGHT}])\]
\[\Rightarrow \text{state} \neq \text{"finishedFailure"}\]
this invariant candidate is false

NeverFinishPositive \(\triangleq\)
\[
\text{state} \neq \text{"finishedSuccess"}
\]
* Correctness states that all the obtained headers are exactly like in the blockchain.
It is always the case that every verified header in LightStore was generated by an instance of Tendermint consensus.
[LCV-DIST-SAFE.1:: CORRECTNESS-INV.1]

CorrectnessInv \(\triangleq\)
\[
\forall h \in \text{DOMAIN } \text{fetchedLightBlocks}:
\text{lightBlockStatus}[h] = \text{"StateVerified"} \Rightarrow
\text{fetchedLightBlocks}[h].\text{header} = \text{blockchain}[h]
\]
* Check that the sequence of the headers in storedLightBlocks satisfies ValidAndVerified = \text{"OK"} pairwise. This property is easily violated, whenever a header cannot be trusted anymore.

StoredHeadersAreVerifiedInv \(\triangleq\)
\[
\text{state} = \text{"finishedSuccess"}
\]
\[\Rightarrow\]
\[
\forall lh, rh \in \text{DOMAIN } \text{fetchedLightBlocks}:
\forall lh \geq rh
\]
\[\text{either there is a header between them}\]
\[
\forall \exists mh \in \text{DOMAIN } \text{fetchedLightBlocks}:
lh < mh \land mh < rh
\]
\[\text{or we can verify the right one using the left one}\]
\[
\forall \text{"OK"} = \text{ValidAndVerified}(\text{fetchedLightBlocks}[lh], \text{fetchedLightBlocks}[rh])
\]

An improved version of StoredHeadersAreSound, assuming that a header may be not trusted.
This invariant candidate is also violated,
as there may be some unverified blocks left in the middle.

StoredHeadersAreVerifiedOrNotTrustedInv \(\triangleq\)
\[
\text{state} = \text{"finishedSuccess"}
\]
\[\Rightarrow\]
\[
\forall lh, rh \in \text{DOMAIN } \text{fetchedLightBlocks}:
\text{for every pair of different stored headers}
\]
\[ \forall lh \geq rh \]

\[ \exists mh \in \text{DOMAIN} \ \text{fetchedLightBlocks} : \\
\text{lh} < mh \land mh < rh \\
\text{either there is a header between them} \\
\text{or we can verify the right one using the left one} \]

\[ \forall \ "OK" = \text{ValidAndVerified} \left( \text{fetchedLightBlocks}[lh], \text{fetchedLightBlocks}[rh] \right) \]

\[ \text{or the left header is outside the trusting period, so no guarantees} \]

\[ \forall \sim \text{BC!InTrustingPeriod} \left( \text{fetchedLightBlocks}[lh].\text{header} \right) \]

\[ * \text{ An improved version of StoredHeadersAreSoundOrNotTrusted,} \]

\[ * \text{ checking the property only for the verified headers.} \]

\[ * \text{ This invariant holds true.} \]

\[ \text{ProofOfChainOfTrustInv} \triangleq \\
\text{state} = \text{"finishedSuccess"} \]

\[ \Rightarrow \\
\forall lh, rh \in \text{DOMAIN} \ \text{fetchedLightBlocks} : \\
\text{for every pair of stored headers that have been verified} \\
\forall lh \geq rh \\
\forall \text{lightBlockStatus}[lh] = \text{"StateUnverified"} \\
\forall \text{lightBlockStatus}[rh] = \text{"StateUnverified"} \\
\text{either there is a header between them} \\
\forall \exists mh \in \text{DOMAIN} \ \text{fetchedLightBlocks} : \\
\text{lh} < mh \land mh < rh \land \text{lightBlockStatus}[mh] = \text{"StateVerified"} \\
\text{or the left header is outside the trusting period, so no guarantees} \\
\forall \sim (\text{BC!InTrustingPeriod} \left( \text{fetchedLightBlocks}[lh].\text{header} \right)) \\
\text{or we can verify the right one using the left one} \]

\[ \forall \ "OK" = \text{ValidAndVerified} \left( \text{fetchedLightBlocks}[lh], \text{fetchedLightBlocks}[rh] \right) \]

\[ * \text{ When the light client terminates, there are no failed blocks. (Otherwise, someone lied to us.)} \]

\[ \text{NoFailedBlocksOnSuccessInv} \triangleq \\
\text{state} = \text{"finishedSuccess"} \Rightarrow \\
\forall h \in \text{DOMAIN} \ \text{fetchedLightBlocks} : \\
\text{lightBlockStatus}[h] \neq \text{"StateFailed"} \]

\[ \text{This property states that whenever the light client finishes with a positive outcome,} \\
\text{the trusted header is still within the trusting period.} \]

\[ \text{We expect this property to be violated. And Apalache shows us a counterexample.} \]

\[ \text{PositiveBeforeTrustedHeaderExpires} \triangleq \\
\left( \text{state} = \text{"finishedSuccess"} \right) \Rightarrow \text{BC!InTrustingPeriod} \left( \text{blockchain}[\text{TRUSTED HEIGHT}] \right) \]

\[ \text{If the primary is correct and the initial trusted block has not expired,} \\
\text{then whenever the algorithm terminates, it reports "success"} \]

\[ \text{CorrectPrimaryAndTimeliness} \triangleq \\
\left( \text{BC!InTrustingPeriod} \left( \text{blockchain}[\text{TRUSTED HEIGHT}] \right) \right) \\
\wedge \text{state} \neq \text{"working"} \wedge \text{IS PRIMARY CORRECT} \Rightarrow \\
\text{state} = \text{"finishedSuccess"} \]

\[ * \text{ If the primary is correct and there is a trusted block that has not expired, then whenever the algorithm terminates,} \\
\text{it reports "success".} \]
\[ \text{SuccessOnCorrectPrimaryAndChainOfTrust} \triangleq \\
(\exists h \in \text{domain fetchedLightBlocks} : \\
\quad \text{lightBlockStatus}[h] = \text{"StateVerified"} \land BC!InTrustingPeriod(\text{blockchain}[h]) \\
\land \text{state} \neq \text{"working"} \land \text{ISPRIMARYCORRECT}) \Rightarrow \\
\quad \text{state} = \text{"finishedSuccess"} \]

Lite Client Completeness: If header \( h \) was correctly generated by an instance of Tendermint consensus (and its age is less than the trusting period), then the lite client should eventually set \( \text{trust}(h) \) to true.

Note that Completeness assumes that the lite client communicates with a correct full node.

We decompose completeness into Termination (liveness) and Precision (safety).

Once again, Precision is an inverse version of the safety property in Completeness, as \( A \Rightarrow B \) is logically equivalent to \( \neg B \Rightarrow \neg A \).

\[ \text{PrecisionInv} \triangleq \\
(\text{state} = \text{"finishedFailure"}) \land \\
\quad \exists h \in \text{domain fetchedLightBlocks} : \\
\quad \text{let lightBlock} \triangleq \text{fetchedLightBlocks}[h] \text{IN} \\
\quad \quad \text{the full node lied to the lite client about the block header} \\
\quad \text{let lightBlock.header} \neq \text{blockchain}[h] \\
\quad \text{the full node lied to the lite client about the commits} \\
\quad \text{let lightBlock.Commits} \neq \text{lightBlock.header.VS} \\
\quad \text{the old invariant that was found to be buggy by TLC} \]

\[ \text{PrecisionBuggyInv} \triangleq \\
(\text{state} = \text{"finishedFailure"}) \land \\
\quad \exists h \in \text{domain fetchedLightBlocks} : \\
\quad \text{let lightBlock} \triangleq \text{fetchedLightBlocks}[h] \text{IN} \\
\quad \quad \text{the full node lied to the lite client about the block header} \\
\quad \text{lightBlock.header} \neq \text{blockchain}[h] \\
\quad \text{the worst complexity} \]

\[ \text{Complexity} \triangleq \\
\quad \text{let } N \triangleq \text{TARGETHEIGHT} - \text{TRUSTEDHEIGHT} + 1 \text{IN} \\
\quad \text{state} \neq \text{"working"} \Rightarrow \\
\quad (2 \star nprobes \leq N \star (N - 1)) \]

We omit termination, as the algorithm deadlocks in the end. So termination can be demonstrated by finding a deadlock. Of course, one has to analyze the deadlocked state and see that the algorithm has indeed terminated there.