Effect of system size on the traditional signatures of critical behavior in projectile multifragmentation

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Abstract. The effect of the system size on a number of traditionally accepted signatures of cluster approximation technique of critical behavior have been examined for projectile multifragmenting systems like Mg at 4.5 AGeV and Kr at 0.95 AGeV. The results obtained from analyzing our experimental data on the fluctuation of size of the largest fragments, reduced variance and the mean value of the second moments of charge distribution provide clear evidences of size effect in terms of the height and position of the peaks of the studied parameters.

1. Introduction
Nucleus-nucleus collisions at intermediate energies offer various possibilities to produce hot nuclei which may break up into smaller pieces having charges $Z_{pf} \geq 3$, resulting multifragmentation (MF). The measured fragment properties are expected to reveal information about a possible phase transition which was earlier theoretically predicted for nuclear matter [1-3]. The connection between MF as decay mechanism of excited nuclei and a possible liquid-gas phase transition taking place in nuclear matter was initially motivated by the strong resemblance between van-der-waals potential and nucleon-nucleon potential [4,5]. The presence of the power law in the mass yield distribution of the fragments along with the values of the exponents close to that of an ordinary fluid led Purdue [6] group to suggest that multifragmentation of nuclei might be analogous to a fluid undergoing a continuous phase transition from liquid to a gas.

Although a considerable amount of work in understanding the statistical aspects of multifragmentation have been carried out by several groups [7-19], the mechanism of nuclear multifragmentation is not yet completely understood. One of the most complicated questions related to the liquid-gas phase transition in nuclear matter which is yet to be answered is the order of the phase transition. EOS collaboration, from analyzing the results of interactions of 1 AGeV Au on C using cluster approximation technique, has suggested that, MF of Au can be understood as due to a continuous phase transition. Similar results have also been reported for 1 AGeV La on carbon [20-22].

A continuous phase transition in nuclear matter is generally characterized by the presence of some characteristic signatures. One of the most striking characteristics of the systems undergoing continuous phase transition that might have taken place in the final stage of fragmentation of heavy ion collisions is that certain experimentally observed quantities might undergo fluctuations that exist on all length scales in a small range of the control parameter. These observables

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may diverge or even tend to vanish near some critical value of the control parameter. However, due to finiteness in the size and the number of constituents of a nucleus, the experimental observables do not exhibit expected sharp singularities at critical point, it rather shows a rounded peak at \( m = m' \) (otherwise \( T = T' \)) where \( m' = m_c, m_c \) being the critical multiplicities for the infinite size system. The finite size effect thus lowers the critical point thereby resulting a possible change in the order of phase transition. A number of works \([16-27]\), both theoretical and experimental, have been carried out to realize the role of size of the fragmenting nuclei on the traditional signatures of critical behavior and hence on nuclear MF process.

In this work an attempt has been made, by studying Mg-Em at 4.5 AGeV and Kr-Em at 0.95 AGeV, to find how system size of fragmenting nuclei influence the various observables of critical behavior.

2. Results

The experimental data have been obtained by line scanning NIKFI BR-2 nuclear emulsion pel-lices that are irradiated parallel to their lengths by a 4.5 AGeV \(^{24}\)Mg beam from the JINR synchrophasotron at Dubna. All total 580 minimum biased Mg-Em and 542 \(^{84}\)Kr-Em interactions are considered for the present work.

Frequency distribution of various charged projectile fragments with \( Z_{pf} \geq 1 \) emitted from Mg-Em interactions are plotted in figure 1. Similar distribution fitted with a power law has also been reported in nuclear fragmentation of \(^{238}\)U at 0.96 AGeV, \(^{84}\)Kr at 1.25 AGeV, \(^{84}\)Kr at 0.95 AGeV and \(^{131}\)Xe at 1.22 AGeV in nuclear emulsion \([16,19,23,28-30]\).

For Mg, considering total number of system constituents as 12, the yields of the fragments charge distribution lying between 1-6 have been re-plotted in figure 2 in log-log scale and compared with the result of our earlier works on Kr-Em at 0.95 AGeV \([16]\). A straight line fit to the respective data points gives the values of the exponent \( \tau \), 2.54 ± 0.417 and 2.12 ± 0.15 respectively for Mg-Em and Kr-Em interactions. The slopes of the straight lines are estimated by least square approach and considering the error in the regression are normally distributed the cited values of the slopes lie within the 95% confidence level.
2.1. Size effect of various experimental observables
The total charged fragment multiplicity \( m \) as defined in reference [8-10, 16], is

\[
m = N_f + N_\alpha + N_{prot} \tag{1}
\]

where \( N_f, N_\alpha \) and \( N_{prot} \) denote the number of heavy PFs with charge \( Z_{pf} \geq 3 \), alpha particles with \( Z_{pf} = 2 \) and the number of emitted protons with \( Z_{pf} = 1 \) respectively. Here \( N_{prot} \) is determined using charge balance of the PFs, \( m \) is a parameter which is considered to be linearly related with the temperature \( T \) of the system. Thus, \( \varepsilon = m_c - m \) gives a measure of the distance of a given event from the critical point.

2.2. Fluctuations in \( Z_{\text{max}} \)
It is known that a system exhibits significant fluctuation in the neighborhood of the critical point in a small range of the control parameter and appears at increasingly large scale as \( \varepsilon \rightarrow 0 \). Elliot et al., while attempting to realize critical behavior of nuclear system in projectile multifragmentation process using the technique of cluster approximation, has pointed out that the most readily observed fluctuations in the cluster distribution are those in the size of the largest cluster. In figure 3, the standard deviation of \( Z_{\text{max}} \) normalized with respect to the charge of the projectile is shown as a function of multiplicity. With our experimental data, large fluctuations in the multiplicity range between 11-19 are readily seen from this plot with a peak at \( m = 17 \pm 1 \) for Kr. For Mg it is observed in the range of 5-9 with a peak at \( m = 6 \pm 1 \). Recently, Gulminelli et al [23] have pointed out that finiteness of the system under investigation smooth the fluctuation effect to such an extent that not only the transition point is loosely defined and shifted, but also the signal is qualitatively the same for a critical point, a first order phase transition or even a continuous change or cross over. Thus the heap like structures that has been observed in the figure 3 may not carry as much information as one would expect for cluster approximation of critical behavior analysis for an infinite system. Nevertheless, the distinct difference in the heights and positions of the two peaks in the figure are found to be in consistent with the source size effect as reported by other workers [18, 25].

![Figure 3. Standard deviation of normalized \( Z_{\text{max}} \) as a function of multiplicity \( m \) for experimental data.](image-url)
2.3. Charge moments and Conditional Moments

For a single event, Campi [8-10] has defined the kth moment of charge distribution as

\[ M_k = \sum_{Z_{pf}} n_{Z_{pf}}(\varepsilon)Z_{pf}^k \]  

(2)

and for large number of events, \( < M_k(\varepsilon) > \) in the small bins of multiplicity m as

\[ < M_k(\varepsilon) >= \frac{1}{N} \sum_i M_i^k(\varepsilon) = \frac{1}{N} \sum_i \left( \sum n_{Z_{pf}}(\varepsilon)Z_{pf}^k \right) \]  

(3)

Here \( n_{Z_{pf}} \) is the normalized charge distribution and is defined as \( n_{Z_{pf}} = N_{Z_{pf}}/Q_{PF} \), \( Q_{PF} \) is the sum of charges of all the projectile spectator protons, fast alpha particles and heavy projectile fragments with charges \( Z_{pf} \geq 3 \). \( N \) denotes the total number of events in a given small range of \( \varepsilon \), and \( M_i^k \) is the kth order charge distribution moment for ith event.

To examine the system size effect, we next consider another traditional signature of critical behavior, namely, \( \gamma_2 \) related to variance of charge, \( \sigma^2 \) is defined as [8-10]

\[ \gamma_2 = \frac{M_2 M_0}{M_1^2} = 1 + \frac{\sigma^2}{<Z>^2} \]  

(4)

Here \( M_0, M_1, M_2 \) are the zero, first and second order moments respectively. While \( M_0 \) and \( M_1 \) correspond to mean number and mean size of the clusters, \( M_2 \) is a quantity that is related to fluctuation in the size of the fragments. We have calculated \( \gamma_2 \) event by event as a function of total charged multiplicity m for Mg and plotted in figure 4. The result thus obtained is again compared with the result of our earlier works on Kr-Em interactions at 0.95 AGeV. The error bars show the standard deviations in \( < \gamma_2 > \). For Kr, as the peak in \( < \gamma_2 > \) is not well defined, the position and height of the peak value are therefore determined by polynomial fitting of the experimental data points. Errors in \( m_c \) values have been estimated by considering the errors in the coefficients of the polynomial as well as the error of our PF charge measurement which for the present investigation is considered to be \( \pm 1 \). From this figure, while for Kr, the height and

![Figure 4. Variation of \( < \gamma_2 > \) with total charged fragment multiplicity m for experimental data.](image-url)
confirms EOS finding that the position and height of the peak in $\gamma_2$ is essentially determined by the source size [25,26].

To have further insight into our sample of data, the mean values of 2nd moment of charge distribution, $< M_2 >$, is plotted against $m$ in figure 5 and compared with the Kr-Em results [16]. Possible effects of system size are seen again in the position and height of the peaks.

3. Summery
From the present investigation on Mg-Em interactions at 4.5 AGeV and a comparison of the results with our earlier works on Kr-Em interactions at 0.95 AGeV, it is seen that there exists clear evidences of system size dependence on the heights and positions of the peaks of traditional signatures of critical behaviour. Though no attempt has been made in this work to determine the order of phase transition, nevertheless the observations of heap like structure in the measured observables indicate the critical behavior of nuclear matter in the fragmentation of Mg projectile at this energy.

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4. References
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Figure 5. Average of second charge moments as a function of multiplicity m for $^{84}$Kr.
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