A double torsion pendulum with two cascade soft degrees of freedom

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Abstract. We report on a double torsion pendulum, where motion along two degrees of freedom (DoFs) is almost free. The Test Mass (TM) is enclosed in a replica of the LISA-Pathfinder electrostatic readout and actuation system. This apparatus is designed to perform extensive ground testing of undesired effects such as leakage of the readout noise from one DoF to another, or actuation cross talks with closed feedback loop. Such investigation is relevant to the noise budget of LISA and LISA-Pathfinder missions, as the TM will be sensitive to weak forces along all 6 degrees of freedom (DoFs). The instrument being developed in Firenze is capable of measuring the forces and stiffnesses acting simultaneously along the 2 soft DoFs. We have completed an upgrade of the apparatus to a definitive configuration and we report on both advances in the commissioning tests and on measurements of residual charge, with the first DoF released.

1. Introduction
One of the main requirements of the LISA mission is the drag-free operation mode, that means isolation of its test masses (TM) from all kind of spurious, i.e. non gravitational, disturbances.

The LISA requirement for the residual acceleration noise in the free-falling frame of the test mass is [1]:

$$S_n^{1/2} \leq 3 \cdot 10^{-15} \left[ 1 + \left( \frac{f}{0.003 \text{Hz}} \right)^2 \right] \frac{m}{s^2 \sqrt{\text{Hz}}}$$

in the frequency band $0.1 \text{mHz} \leq f \leq 0.1 \text{Hz}$.

Among the perturbing causes, special attention is to be given to the coupling with the electrostatic sensing and actuation system, that could introduce a coupling between test mass and spacecraft. The capacitive sensing and actuation device, called the Gravitational Reference System (GRS) [2], is used to provide the relative position input to the control loop that is closed on micro-thrusters, used to force the spacecraft to follow the test-mass. Extensive ground tests are required to study the residual weak forces that may couple the test mass to the GRS. The challenge for these tests is to cancel gravity, in order to simulate free fall conditions, along as many DoFs as possible. Up to now, excellent results in the measurement and reduction of residual acceleration have been obtained with torsion pendulums, that only allows to study one rotational single DoF [3], [4] or one translational single DoF [5]. The purpose of our experiment is to work with a double torsion pendulum that allows the test mass to be “almost free” on two DoFs. With our apparatus we aim to extend the above referenced work and to measure...
and analyze cross talks between two soft DoFs. A crucial goal of this experiment consists in evaluating the effects on one DoF, when feedback is applied to the second DoF. A similar experimental scheme is now pursued also by other groups [6], [7].

2. The PETER apparatus in Firenze

In the last few years we have built and begun to operate a double pendulum, nicknamed PETER (PEndulum Translational and Rotational). We use two Tungsten torsion fibers to suspend a hollow, cubic test mass (Figure 1): the lower, $\theta$ fiber allows the test mass to be quasi free in rotation around its symmetry vertical axis. This fiber hangs from the top of the arm of a crossbar that is, in turn, suspended to an upper, $\varphi$ fiber: torsion of this upper fiber allows quasi free motion of the test mass along an arc of a 30cm diameter circumference: for all practical purposes, this can be considered a translational motion. Dummy loads hang from the other three arms of the crossbar. The sensitivity goal for this apparatus, limited by the mechanical setup and the electronics noise is $\leq 10^{-13}$ ms$^{-2}$/Hz$^{1/2}$ around 1mHz (on each DoF), namely 1 order of magnitude worse than the LISA PF goal along the sensitivity axis. For further details we refer to [8].

![Figure 1. Scheme of the roto-translational pendulum.](image)

During the 2010 operation stop, we installed a new GRS (on loan from Trento University) with capacitive sensors in Mo and SHAPAL (Figure 2) and a new, purposely made, hollow Al TM with 46mm side (Figure 3). This set-up is a closer replica of the LISA-Pathfinder flight model geometry, and, very important for our set-up, leaves a 4mm gap between the sensors and the test mass, allowing larger amplitudes of initial oscillations without the TM hitting the GRS electrodes. At the same time we substituted our home-developed readout and sensing electronics with a replica of the flight model capacitive readout electronics produced by ETH Zurich. Each channel is read with a capacitive-inductive resonant bridge at 100kHz. The electronics also generates the waveform for the actuation. The apparatus was also equipped with an Optical Read Out (ORO) [9] that will provide an independent measurement of the test mass along the 2 soft DoFs. This is useful both for diagnostic purposes and because cross-correlation of two independent outputs can significantly reduce the readout noise.
2.1. Calibration

The first step in preparing the apparatus consists of fine tuning the input cables capacitance, so that all capacitors resonate at the resonance frequency of the read-out bridge. We then mounted the GRS on a traditional micropositioner and rigidly hanged the TM inside it. By moving the sensor assembly of a known amount of both translation and rotation, we obtained, for both channels, a calibration $51 \mu V/nm$ with a $V_{pump} = 10V_{pp}$. Given this calibration factor, we verified that the electronics noise is $\approx 1nm/Hz^{1/2}$, as specified by ETH Zurich.

3. Present sensitivity

We have so far operated the double pendulum with just the rotation degree of freedom. First, we implemented a feedback control [10], using the electrostatic actuators, to damp the rotational movement of the test mass. In the control loop we have also integrated the movement of the motorized stages, needed to center the test mass in the GRS. In Figure 5 we show the amplitude spectral density of the angular displacement. This can be converted in acceleration (amplitude) spectral density (Figure 6) by considering an arm length of 0.02m, i.e. the sensing electrodes separation, and assuming a TM of 2kg, i.e. the mass of the TM of LISA PF. It can be seen that, in the high frequency band, we are limited by the electronics noise while in the low frequencies we are a factor $\sim 50$ above the intrinsic limit given by thermal noise of the suspension fiber. We are presently investigating the origin of the excess noise.
4. DC bias measurement

Using our apparatus with only the rotational DoF free, we have performed preliminary measurements for the electrostatics characterization of the GRS. The procedure used here follows closely that outlined in [11]. We measured the electrostatic patch fields on the sensor surfaces that might arise due to contamination from the assembly and from material out-gassing over a long mission [12]. This spurious DC potentials couple to the test mass charge fluctuations and can be a relevant noise source on force for LISA and LISA PF:

\[
S_{F}^{1/2} = \frac{q}{C_{\text{tot}}} \left| \frac{\partial C_x}{\partial x} \right| S_{\Delta x}^{1/2} \approx 1.6 \text{fN/Hz}^{1/2} \left( \frac{q}{10^7 e} \right) \left( \frac{S_{\Delta x}^{1/2}}{100 \mu \text{V/Hz}^{1/2}} \right),
\]

where \( q \) is the accumulated test mass charge, \( C_{\text{tot}} \) is the total capacitance of the test mass to all sensor surfaces and \( C_x \) is the the capacitance of \( x \)-electrode to the test mass, Figure 8. \( \Delta x \) is a combination of the DC bias distribution inside to the sensor and can be represented as:

\[
\Delta x = \left| \frac{\partial C_x}{\partial x} \right|^{-1} \sum_i \frac{\partial C_i}{\partial x} V_i,
\]

where \( V_i \) are the spurious potentials of the surfaces inside to the sensor and \( C_i \) are the capacitance of each electrode to the test mass [11].

As our apparatus is sensitive to torque, we will consider the derivative with respect to the rotational angle \( \varphi \) rather than the displacement \( x \).

Here we want to show how it is possible to compensate this effect by applying DC voltages to the sensing electrodes, so that the resulting average DC potential is zero. In the electrostatic model considered here, we assume each electrode as having a single uniform potential, neglecting spatial variation [11]: in this case the instantaneous torque \( N_{\varphi} \) and the test mass potential \( V_M \) can be expressed as:

\[
N_{\varphi} = \frac{1}{2} \sum_i \frac{\partial C_i}{\partial \varphi} (V_i - V_M)^2,
\]

\[
V_M = \frac{q}{C_{\text{tot}}} + \frac{1}{C_{\text{tot}}} \sum_i C_i V_i.
\]
Stray DC biases, dielectric noise and test mass charge are denoted $\delta V_i$, $v_{ni}$ and $q$. In the main measurement described in the text, a modulated bias $V_\Delta$ is applied to the test mass via the $z$ electrodes. Actuation voltages $V_{ai}$ can be applied to compensate $\delta V_i$ [11].

The DC bias effect can be measured by applying a sinusoidal signal to $z$ electrodes, Figure 9: this simulates a modulated test mass charge. The torque produced at the voltage modulation frequency $\omega_z$ can be expressed as:

$$ (N_\varphi)_{\omega_z} = - \frac{4C_z}{C_{tot}} V_z \sin(\omega_z t) \left( \frac{\partial C_x}{\partial \varphi} \right) (\Delta \varphi + 4V_{comp}), $$

where $C_z$, $C_{tot}$ and $C_x$ are indicate in Figure 8 and $\Delta \varphi$ is the sum, with suitable signs, of the four sensing electrodes potential:

$$ \Delta \varphi = \delta V_{1B} + \delta V_{2B} - \delta V_{1A} - \delta V_{2A}. $$

We can now scan (Figure 10) the $V_{comp}$ amplitude of the demodulating signal at $\omega_z$ and, extracting the coherent torque, measure $\Delta \varphi$ and the best $V_{comp}$ that compensates it. Using

$$ \text{coeff} = - \frac{4C_z}{C_{tot}} V_z \sin(\omega_z t) \left( \frac{\partial C_x}{\partial \varphi} \right). $$
$V_z = 3V$ and $\omega_z = 3mHz$ for the modulation parameters we obtained a $\Delta \varphi \approx -14mV$ and consequently:

$$ (V_{comp})_{BEST} = -\frac{\Delta \varphi}{4} \approx 3.5mV. \quad (8) $$

With the measurement of the $V_{comp}$ needed to cancel the stray DC bias due to spatially varying DC surface potentials, we can also measure the residual charge on the TM. By applying a further modulation signal on the x-electrodes, as indicated in Figure 11, the torque produced at the voltage modulation frequency $\omega_{MOD}$ can be written as:

$$ (N_{\varphi})_{\omega_{MOD}} = -4V_{MOD}\sin(\omega_{MOD}t)\left( \frac{\partial C_x}{\partial \varphi} \right) \frac{q}{C_{tot}}. \quad (9) $$

Finally, using the modulation parameter $V_{MOD} = 3V$, $\omega_{MOD} = 3mHz$ and $V_{comp} = 3.5mV$, we obtained a residual test mass charge $q \approx 3.7 \cdot 10^7e$. This value is comparable to those derived in similar experiments, and shows that the test mass is properly isolated [13].

5. Next steps
We have satisfactorily tested the capabilities of our apparatus, when operated as a single DoF torsion pendulum. In the immediate future we shall release the upper fiber in order to enable the second, translational DoF and will implement a 2 DoF feedback control to keep the TM in a controlled motion. When that is achieved, we will have a test bench to simulate free fall in the laboratory along two degrees of freedom: this will enable us to carry on the desired measurements of cross talks between the 2 soft DoFs.

6. References
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