SPIN FILTERING IN STORAGE RINGS

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The spin filtering in storage rings is based on a multiple passage of a stored beam through a polarized internal gas target. Apart from the polarization by the spin-dependent transmission, a unique geometrical feature of interaction with the target in such a filtering process, pointed out by H.O. Meyer \(^1\), is a scattering of stored particles within the beam. A rotation of the spin in the scattering process affects the polarization buildup. We derive here a quantum-mechanical evolution equation for the spin-density matrix of a stored beam which incorporates the scattering within the beam. We show how the interplay of the transmission and scattering within the beam changes from polarized electrons to polarized protons in the atomic target. After discussions of the FILTEX results on the filtering of stored protons \(^2\), we comment on the strategy of spin filtering of antiprotons for the PAX experiment at GSI FAIR \(^3\).

1. Introduction

1.1. Future QCD spin physics needs polarized antiprotons: PAX proposal

The physics potential of experiments with high-energy stored polarized antiprotons is enormous. The list of fundamental issues includes the determination of transversity — the quark transverse polarization inside a transversely polarized proton — the last leading-twist missing piece of the QCD description of the partonic structure of the nucleon, which can only be investigated via double-polarized antiproton–proton Drell–Yan production. Without measurements of the transversity, the spin tomography of the proton would be ever incomplete. Other items of great importance for
the perturbative QCD description of the proton include the phase of the time-like form factors of the proton and hard proton–antiproton scattering. Such an ambitious physics program with polarized antiproton–polarized proton collider has been proposed recently by the PAX Collaboration \(^3\) for the new Facility for Antiproton and Ion Research (FAIR) at GSI in Darmstadt, Germany, aiming at luminosities of \(10^{31} \text{ cm}^{-2} \text{s}^{-1}\). An integral part of such a machine is a dedicated large–acceptance Antiproton Polarizer Ring (APR).

Here we recall, that for more than two decades, physicists have tried to produce beams of polarized antiprotons \(^4\), generally without success. Conventional methods like atomic beam sources (ABS), appropriate for the production of polarized protons and heavy ions cannot be applied, since antiprotons annihilate with matter. Polarized antiprotons have been produced from the decay in flight of \(\bar{\Lambda}\) hyperons at Fermilab. The intensities achieved with antiproton polarizations \(P > 0.35\) never exceeded \(1.5 \times 10^5 \text{ s}^{-1}\). Scattering of antiprotons off a liquid hydrogen target could yield polarizations of \(P \approx 0.2\), with beam intensities of up to \(2 \times 10^3 \text{ s}^{-1}\). Unfortunately, both approaches do not allow efficient accumulation of antiprotons in a storage ring, which is the only practical way to enhance the luminosity. Spin splitting using the Stern–Gerlach separation of the given magnetic subsates in a stored antiproton beam was proposed in 1985 \(^7\). Although the theoretical understanding has much improved since then \(^8\), spin splitting using a stored beam has yet to be observed experimentally.

1.2. FILTEX: proof of the spin-filtering principle

At the core of the PAX proposal is spin filtering of stored antiprotons by multiple passage through a Polarized Internal hydrogen gas Target (PIT) \(^3^,9\). In contrast to the aforementioned methods, convincing proof of the spin–filtering principle has been produced by the FILTEX experiment at TSR–ring in Heidelberg \(^2\). It is a unique method to achieve the required high current of polarized antiprotons.

In the FILTEX experiment at TSR \(^2\) the transverse polarization rate of \(dP_B/dt = 0.0124 \pm 0.0006\) (only the statistical error is shown) per hour has been reached for 23 MeV stored protons interacting with an internal polarized atomic hydrogen target of areal density \(6 \times 10^{13} \text{ atoms/cm}^2\). The principal limitation on the observed polarization buildup was a very small acceptance of the TSR–ring. Extrapolations of the FILTEX result, in conjunction with the then available theoretical re-interpretation \(^1^,10\) of the
FILTEX finding, suggested that in the custom-tailored large-acceptance Antiproton Polarizer Ring (APR) antiproton polarizations up to 35-40% are feasible.

1.3. Mechanisms of spin-filtering: transmission and scattering within the beam (pre-2005)

Everyone is familiar with the polarization of the light transmitted through the plate of an optically active medium, which is usually the regime of weak absorption and predominantly real light-atom scattering amplitude. In the realm of particle physics, the absorption becomes the dominant feature of interaction. The transmitted beam becomes polarized by the polarization-dependent absorption, which is the standard mechanism, for instance, in neutron optics. While the polarization of elastically scattered slow neutrons is a very important observable, the elastically scattered neutrons are never confused with the transmitted beam.

In his theoretical interpretation of the FILTEX result, H. O. Meyer made an important observation that the elastic scattering of stored particles within the beam is an intrinsic feature of the spin filtering in storage rings. First, one takes a particle from the stored beam. Second, this particle is either absorbed (annihilation for antiprotons, meson production for sufficiently high energy protons and antiprotons) or scatters elastically on the polarized atom in the PIT. Third, if the scattering angle is smaller than the acceptance angle $\theta_{\text{acc}}$ of the ring, the scattered particle ends up in the stored beam. Specifically, the polarization of the particle scattered within the beam would contribute to the polarization of a stored beam.

The FILTEX PIT used the hyperfine state of the hydrogen in which both the electron and proton were polarized. The familiar Breit Hamiltonian for the nonrelativistic $ep$ interaction includes the hyperfine and tensor spin-spin interactions. Meyer and Horowitz noticed that those spin-spin interactions give a sizeable cross section of the polarization transfer from polarized target electrons to scattered protons, which is comparable to that in the nuclear proton-proton scattering. (Incidentally, the transfer of the longitudinal polarization of accelerated electrons to scattered protons, suggested in 1957 by Akhiezer et al., is at the heart of the recent high precision measurements of the ratio of the charge and magnetic form factors of nucleons at Jlab and elsewhere, for the review see.) Furthermore, Meyer argued that the contribution from $pe$ scattering is crucial for the quantitative agreement between the theoretical expectation for the polar-
ization buildup of stored protons and the FILTEX result \(^1\), which prompted the idea to base the antiproton polarizer of the PAX on the spin filtering by polarized electrons in PIT \(^9\).

After the PAX proposal, the feasibility of the electron mechanism of spin filtering has become a major issue. Yu. Shatunov was, perhaps, the first to worry, and his discussions with A. Skrinsky prompted, eventually, A. Milstein and V. Strakhovenko of the Budker Institute to revisit the kinetics of spin filtering in storage rings \(^15\). Simultaneously and independently, similar conclusions on the self-cancellation of the polarized electron contribution to the spin filtering of (anti)protons were reached in J"ulich by the present authors within a very different approach.

After this somewhat lengthy and Introduction, justified by the novelty of the subject, we review the basics of the quantum mechanical theory of spin filtering with full allowance for scattering within the beam.

2. Spin filtering in storage rings: transmission, scattering, kinematics and all that

The sky is blue because what we see is exclusively the elastically scattered light. The setting sun is reddish because we see exclusively the transmitted light. The sun changes its color because the transmission changes the frequency (wavelength) spectrum of the unscattered light. In the typical optical experiments, one never mixes the transmitted and scattered light. An unique feature of storage rings, noticed by Meyer, is a mixing of the transmitted and scattered beams.

Some kinematical features of the proton-atom scattering are noteworthy. First, the Coulomb fields of the proton and atomic electron screen each other beyond the Bohr radius \(a_B\). To a good approximation, protons flying by an atom at impact parameters \(> a_B\) do not interact with an atom. The cancellation of the proton and electron Coulomb fields holds at scattering angles (all numerical estimates are for \(T_p = 23\) MeV)

\[
\theta \gtrsim \theta_{\text{min}} = \frac{\alpha_em_e}{\sqrt{2m_pT_p}} \approx 2 \cdot 10^{-2} \text{ mrad}, \quad (1)
\]

at higher scattering angles one can approximate proton-atom interaction by an incoherent sum of quasieelastic (E) scattering off protons and electrons,

\[
d\sigma_E = d\sigma_{pE} + d\sigma_{eE} \quad (2)
\]

As Horowitz and Meyer emphasized, atomic electron is too light a target
to deflect heavy protons, in $pe$ scattering

$$\theta \leq \theta_e = \frac{m_e}{m_p} \approx 5 \cdot 10^{-1} \text{ mrad.}$$

(3)

For 23 MeV protons in the TSR-ring, proton-proton elastic scattering is Coulomb interaction dominated for

$$\theta \lesssim \theta_{\text{Coulomb}} \approx \sqrt{\frac{2\pi \alpha_{\text{em}}}{m_p T_p}} \sigma^{\text{pp}}_{\text{tot,nucl}} \approx 100 \text{ mrad}$$

(4)

Finally, the FILTEX ring acceptance angle equals

$$\theta_{\text{acc}} = 4.4 \text{ mrad},$$

(5)

and we have a strong inequality

$$\theta_{\text{min}} \ll \theta_e \ll \theta_{\text{acc}} \ll \theta_{\text{Coulomb}}.$$  

(6)

The corollaries of this inequality are: (i) $pe$ scattering is entirely within the stored beam, (ii) beam losses by single scattering are dominated by the Coulomb $pp$ scattering.

At this point it is useful to recall the measurements of the $pp$ total cross section in the transmission experiments with the liquid hydrogen target. With the electromagnetic $pe$ interaction included, the proton-atom X-section is gigantic:

$$\hat{\sigma}^{pe}_{\text{tot}} = \hat{\sigma}^{e\ell}_{\text{min}} & \sim 4\pi \alpha_{\text{em}}^2 a_B^2 \approx 2 \cdot 10^4 \text{ Barn.}$$

(7)

How do we extract $\sigma^{pp}_{\text{tot,nucl}} \sim 40 \text{ mb}$ on top of such a background from $pe$ scattering? Very simple: in view of (3) and its relativistic generalization, elastic scattering off electrons is entirely within the beam and does not cause any attenuation!

**3. The in-medium evolution of the transmitted beam**

In fully quantum-mechanical approach, the beam of stored antiprotons must be described by the spin-density matrix

$$\hat{\rho}(p) = \frac{1}{2}[I_0(p) + \sigma s(p)],$$

(8)

where $I_0(p)$ is the density of particles with the transverse momentum $p$ and $s(p)$ is the corresponding spin density. As far as the pure transmission is concerned, it can be described by the polarization dependent refraction
index for the hadronic wave, given by the Fermi-Akhiezer-Pomeranchuk-Lax formula\textsuperscript{11}:

\[
\hat{n} = 1 + \frac{1}{2p}N\hat{F}(0).
\] (9)

The forward NN scattering amplitude \( \hat{F}(0) \) depends on the beam and target spins, and the polarized target acts as an optically active medium. It is convenient to use instead the Fermi Hamiltonian (with the distance \( z \) traversed in the medium playing the rôle of time)

\[
\hat{H} = \frac{1}{2}N\hat{F}(0) = \frac{1}{2}N[\hat{R}(0) + i\hat{\sigma}_{tot}],
\] (10)

where \( \hat{R}(0) \) is the real part of the forward scattering amplitude and \( N \) is the volume density of atoms in the target. The anti-hermitian part of the Fermi Hamiltonian, \( \propto \hat{\sigma}_{tot} \), describes the absorption (attenuation) in the medium.

In terms of this Hamiltonian, the quantum-mechanical evolution equation for the spin-density matrix of the transmitted beam reads

\[
\frac{d}{dz}\hat{\rho}(p) = i\left(\hat{H}\hat{\rho}(p) - \hat{\rho}(p)\hat{H}^\dagger\right)
= i\frac{1}{2}N\left[\hat{R}\hat{\rho}(p) - \hat{\rho}(p)\hat{R}\right] - \frac{1}{2}N\left(\hat{\sigma}_{tot}\hat{\rho}(p) + \hat{\rho}(p)\hat{\sigma}_{tot}\right)
\] (11)

In the specific case of spin-\( \frac{1}{2} \) protons interacting with the spin-\( \frac{1}{2} \) protons (and electrons) the total cross section and real part of the forward scattering amplitude are parameterized as

\[
\hat{\sigma}_{tot} = \sigma_0 + \sigma_1(\sigma \cdot Q) + \sigma_2(\sigma \cdot k)(Q \cdot k),
\]

spin-sensitive loss

\[
\hat{R} = R_0 + R_1(\sigma \cdot Q) + R_2(\sigma \cdot k)(Q \cdot k)
\] (12)

\( \sigma \)-Pseudomagnetic field

Then, upon some algebra, one finds the evolution equation for the beam polarization \( P = s/I_0 \)

\[
\frac{dP}{dz} = -N\sigma_1(Q - (P \cdot Q)P) - N\sigma_2(Qk)(k - (P \cdot k)P)
\] (Polarization buildup by spin-sensitive loss)

\[+ NR_1(P \times Q) + nR_2(Qk)(P \times k), \] (Spin precession in pseudomagnetic field)

(13)
where we indicated the rôle of the anti-hermitian – attenuation – and hermitian – pseudomagnetic field – parts of the Fermi Hamiltonian. It is absolutely important that the cross sections $\sigma_{0,1,2}$ in the evolution equation for the transmitted beam describe all-angle scattering, in the proton-atom case that corresponds to $\theta \geq \theta_{\text{min}}$.

Here we notice, that the precession effects are missed in the Milstein-Strakhovenko kinetic equation for the spin-state population numbers. The precession is the major observable in condensed matter studies with polarized neutrons. Kinetic equation holds only if the spin-density matrix is diagonal one. In the case of the spin filtering in storage rings with pure transverse or longitudinal (supplemented by the Siberian snake for the compensation of the spin rotation) polarizations of PIT, the kinetic equation can be recovered, though, from the evolution of the density matrix upon the averaging over the precession. Hereafter we focus on the transverse polarization studied in the FILTEX experiment.

For the sake of completeness, we cite the full system of coupled evolution equations for the spin density matrix

$$\frac{d}{dz} \left( \begin{array}{c} I_0 \\ s \end{array} \right) = -N \left( \begin{array}{cc} \sigma_0(> \theta_{\text{min}}) & Q \sigma_1(> \theta_{\text{min}}) \\ Q \sigma_1(> \theta_{\text{min}}) & \sigma_0(> \theta_{\text{acc}}) \end{array} \right) \cdot \left( \begin{array}{c} I_0 \\ s \end{array} \right), \quad (14)$$

In has the eigen-solutions $\propto \exp(-\lambda_{1,2} Nz)$ with the eigenvalues $\lambda_{1,2} = \sigma_0 \pm Q \sigma_1$. Eq. (13) reduces to the Meyer’s equation

$$\frac{dP}{dz} = -N \sigma_1 Q (1 - P^2). \quad (15)$$

The polarization buildup follows the law $P(z) = -\tanh(Q \sigma_1 N z)$.

4. Incorporation of the scattering within the beam into the evolution equation

For scattering angles of the interest, $\theta \gtrsim \theta_{\text{min}}$, the differential cross section of the quasielastic proton-atom scattering equals

$$\frac{d\hat{\sigma}_E}{d^2q} = \frac{1}{(4\pi)^2} \hat{F}(q) \hat{\rho} \hat{F}^\dagger(q) = \frac{1}{(4\pi)^2} \hat{F}_c(q) \hat{\rho} \hat{F}_c^\dagger(q) + \frac{1}{(4\pi)^2} \hat{F}_p(q) \hat{\rho} \hat{F}_p^\dagger(q) \quad (16)$$

The evolution equation for the spin-density matrix must be corrected for the lost-and-found protons, scattered quasielastically within the beam, $\theta \leq \theta_{\text{acc}}$. The formal derivation from the multiple-scattering theory, in which the unitarity, i.e. the particle loss and recovery balance, is satisfied rigorously, is too lengthy to be reproduced here. The result is fairly
transparent, though:

\[
\frac{d}{dz} \hat{\rho} = i[H, \hat{\rho}] = \frac{1}{2} N \left( \hat{R} \hat{\rho}(p) - \hat{\rho}(p) \hat{R} \right)
\]

\text{Pure precession & refraction}

\[- \frac{1}{2} N \left( \hat{\sigma}_{\text{tot}} \hat{\rho}(p) + \hat{\rho}(p) \hat{\sigma}_{\text{tot}} \right)
\]

\text{Evolution by loss}

\[+ \ N \int_{\Omega_{\text{acc}}} \frac{d^2 q}{(4\pi)^2} \hat{F}(q) \hat{\rho}(p-q) \hat{F}^\dagger(q) \]

\text{Lost & found: scattering within the beam}

Notice the convolution of the transverse momentum distribution in the beam with the differential cross section of quasielastic scattering. This broadening of the momentum distribution is compensated for by the focusing and the beam cooling in a storage ring.

5. Needle-sharp scattering off electrons does not polarize the beam

The relevant parts of the nonrelativistic Breit ep interaction, found in all QED textbooks, are

\[U(q) = \alpha_{\text{em}} \left\{ \frac{1}{q^2} + \mu_p \left( \sigma_p q \langle \sigma_e \rangle q - (\sigma_p \sigma_e) q^2 \right) \right\}, \tag{18} \]

and give the contribution to the total proton-atom X-section of the form (we suppress the condition \(\theta > \theta_{\text{min}}\))

\[\hat{\sigma}_{\text{tot}}^e = \sigma^e_0 + \sigma^e_1 (\sigma_p \cdot Q_e) + \sigma^e_2 (\sigma_p \cdot k)(Q_e \cdot k) \]

\text{Coulomb} \quad \text{Coulomb \times (Hyperfine + Tensor)}

\text{(19)}

with \(\sigma^e_2 = \sigma^e_1\).

The pure electron target contribution to the transmission losses equals

\[
\frac{1}{2} \frac{d}{dz} f_0(p)(1 + \sigma \cdot P(p)) = \]

\[- \frac{1}{2} N f_0(p) \left[ \sigma^e_0 + \sigma^e_1 P Q_e + \sigma^e_2 (P + \sigma^e_1 Q_e) \right] \]

\text{particle loss} \quad \text{spin loss}

\text{(20)}

Here \(\sigma^e_1 \approx -70 \text{ mb}\), which comes from the Coulomb-tensor and Coulomb-hyperfine interference \(10\), is fairly large on the hadronic cross section scale.
Now note, that pe scattering is needle-sharp, \( \theta \leq \theta_e \ll \theta_{acc} \), and the lost-and-found contribution from the scattering within the beam can be evaluated as (we consider the transverse polarization)

\[
N \int \frac{d^2q}{(4\pi)^2} \hat{F}_e(q) \hat{\rho}(p-q) \hat{F}^\dagger_e(q) = \frac{1}{2} NI_0(p) \int \frac{d^2q}{(4\pi)^2} \hat{F}_e(q) \hat{\rho}(p-q) \hat{F}^\dagger_e(q) = \frac{1}{2} NI_0(p) \left[ \sigma_0^E(\leq \theta_{acc}) + \sigma_1^E(\leq \theta_{acc})(P \cdot Q) \right]
\]

One readily observes the exact cancellation of the transmission, eq. (20), and scattering-within-the-beam, eq. (21), electron target contributions to the evolution equation (17). The situation is entirely reminiscent of the cancellation of the effect of atomic electrons in the transmission measurements of the proton-proton total cross section. One concludes that polarized atomic electrons will not polarize stored (anti)protons.

6. Scattering within the beam in spin filtering by nuclear interaction

The angular divergence of the beam at the target position is much smaller than the ring acceptance \( \theta_{acc} \). Consequently, the contribution from the elastic pp scattering within the beam can be approximated by

\[
\int d^2p \int_{\Omega_{acc}} \frac{d^2q}{(4\pi)^2} \hat{F}_p(q) \hat{\rho}(p-q) \hat{F}^\dagger_p(q) = \sigma^E(\leq \theta_{acc}) \cdot \int d^2p I_0(p)
\]
Now we decompose the pure transmission losses

\[
\frac{d}{dz} \hat{\rho} = -\frac{1}{2} N \left( \hat{\sigma}_{\text{tot}}(> \theta_{\text{acc}}) \hat{\rho}(p) + \hat{\rho}(p) \hat{\sigma}_{\text{tot}}(> \theta_{\text{acc}}) \right)
\]

Unrecoverable transmission loss

\[-\frac{1}{2} N I_0(p) \left[ \hat{\sigma}_0^{e1}(< \theta_{\text{acc}}) + \hat{\sigma}_1^{e1}(< \theta_{\text{acc}}) P Q \right]
\]

Potentially recoverable particle loss

\[+ \sigma \left( \hat{\sigma}_0^{e1}(< \theta_{\text{acc}}) P + \hat{\sigma}_1^{e1}(< \theta_{\text{acc}}) Q \right) \]

Potentially recoverable spin loss

(24)

into the unrecoverable losses from scattering beyond the acceptance angle and the potentially recoverable losses from the scattering within the acceptance angle. Upon the substitution of (23) and (24) into the evolution equation (17), one finds the operator of mismatch between the potentially recoverable losses and the scattering within the beam of the form

\[
\Delta \hat{\sigma} = \frac{1}{4} \left( \hat{\sigma}_0^{e1}(< \theta_{\text{acc}})(1 + \sigma P) + (1 + \sigma P) \hat{\sigma}_1^{e1}(< \theta_{\text{acc}}) \right) - \hat{\sigma}^{E}(\leq \theta_{\text{acc}})
\]

\[= \sigma \left( 2\Delta \sigma_0 P + \Delta \sigma_1 Q \right) \]

(25)

The lost-and-found corrected coupled evolution equations take the form

\[
\frac{d}{dz} \left( I_0 \right) = -N \left( \sigma_0(> \theta_{\text{acc}}) Q \sigma_1(> \theta_{\text{acc}}) + \Delta \sigma_1 + \Delta \sigma_0 + 2\Delta \sigma_0 \right) \left( I_0 \right),
\]

(26)

In the limit of vanishing mismatch, \( \Delta \sigma_{0,1} = 0 \), one would recover equations for pure transmission but with losses from scattering only beyond the acceptance angle. The corrections to the equation for the spin density do clearly originate from a difference between the spin of the particle taken away from the beam and the spin the same particle brings back into the beam after it was subjected to a small-angle elastic scattering. In terms of the standard observables as defined by Bystricky et al. (our \( \theta \) is the
scattering angle in the laboratory frame) \(16\)

\[
\sigma_1^{el}(> \theta_{acc}) = \frac{1}{2} \int_{\theta_{acc}} d\Omega \left( \frac{d\sigma}{d\Omega} \right) \left( A_{00mn} + A_{00ss} \right)
\]

\[
\Delta \sigma_0 = \frac{1}{2} \left[ \sigma_1^{el}(\leq \theta_{acc}) - \sigma_1^{E}(\leq \theta_{acc}) \right]
\]

\[
= \frac{1}{2} \int_{\theta_{min}}^{\theta_{acc}} d\Omega \frac{d\sigma}{d\Omega} \left( 1 - \frac{1}{2} D_{nn0n} - \frac{1}{2} D_{ss0s} \cos(\theta) - \frac{1}{2} D_{k's0s} \sin(\theta) \right)
\]

\[
\Delta \sigma_1 = \sigma_1^{el}(\leq \theta_{acc}) - \sigma_1^{E}(\leq \theta_{acc}) \frac{1}{2} = \int_{\theta_{min}}^{\theta_{acc}} d\Omega \frac{d\sigma}{d\Omega} \times \left( A_{00nm} + A_{00ss} - K_{nn0n} - K_{s'ss} \cos(\theta) - K_{k's0s} \sin(\theta) \right)
\]

The difference between the spin of the particle taken away from the beam and put back after the small-angle elastic scattering corresponds to the spin-flip scattering, as Milstein and Strakhovenko correctly emphasized \(15\). Here there is a complete agreement between the spin-density matrix and kinetic equation approaches.

7. Polarization buildup with the scattering within the beam

Coupled evolution equations with the scattering within the beam, eq. (26), have the solutions \(\propto \exp(-\lambda_{1,2}N_z)\) with the eigenvalues

\[
\lambda_{1,2} = \sigma_0 + \Delta \sigma_0 \pm Q \sigma_3
\]

\[
Q \sigma_3 = \sqrt{Q^2 \sigma_1 (\sigma_1 + \Delta \sigma_1) + \Delta \sigma_0^2},
\]

The polarization buildup follows the law (see also \(15\))

\[
I(z) = I(0) \exp[-(\sigma_0 + \Delta \sigma_0)N_z \cosh(Q \sigma_3 N_z)] \left\{ 1 + \frac{\Delta \sigma_0}{Q \sigma_3} \tanh(Q \sigma_3 N_z) \right\}, \quad \text{(28)}
\]

\[
P(z) = -\frac{Q (\sigma_1 + \Delta \sigma_1) \tanh(Q \sigma_3 N_z)}{Q \sigma_3 + \Delta \sigma_0 \tanh(Q \sigma_3 N_z)}, \quad \text{(29)}
\]

The effective small-time polarization cross section equals

\[
\sigma_P \approx -Q (\sigma_1 + \Delta \sigma_1). \quad \text{(30)}
\]

8. Numerical estimates and the FILTEX result

We recall first the works by Meyer and Horowitz \(^1\), Meyer \(^1\) initiated the whole issue of the scattering within the beam, correctly evaluated the principal double-spin dependent Coulomb-nuclear interference (CNI) effect,
but an oversight has crept in when putting together the transmission and scattering-within-the-beam effects, which we shall correct below.

The FILTEX polarization rate as published in 1993, can be re-interpreted as $\sigma_P = 63 \pm 3$ (stat.) mb. The expectation from filtering by a pure nuclear elastic scattering at all scattering angles, $\theta > 0$, based on the pre-93 SAID database, was

$$\sigma_1(\text{Nuclear}; \theta > 0) = 122 \text{ mb.} \tag{31}$$

The factor of two disagreement between $\sigma_P$ and $\sigma_1$ called for an explanation, and Meyer made two important observations: (i) one only needs to include the filtering by scattering beyond the acceptance angle, (ii) the Coulomb-nuclear interference angle $\theta_{Coulomb}$ is large, $\theta_{Coulomb} \gg \theta_{acc}$, and one needs to correct for the Coulomb-nuclear interference (CNI) effects. Based on the pre-93 SAID database, he evaluated the CNI corrected

$$\sigma_1(\text{CNI}; \theta > \theta_{acc}) = 83 \text{ mb.} \tag{32}$$

The effect of pure nuclear elastic $pp$ scattering within the acceptance angle would have been utterly negligible, this substantial departure from 122 mb of eq. (31) is entirely due to the interference of the Coulomb and double-spin dependent nuclear amplitudes - there is a close analogy to the similar interference in $pe$ scattering. As we shall argue below, for all the practical purposes Meyer’s eq. (32) is the final theoretical prediction for $\sigma_P$, but let the story unfold.

The estimate (32) was still about seven standard deviations from the above cited $\sigma_P$. Next Meyer noticed that protons scattered off electrons are polarized. They all go back into the beam. Based on the Horowitz-Meyer calculation of the polarization transfer from target electrons to scattered protons, that amounts to the correction to (32)

$$\delta \sigma_1^{ep} = -70 \text{ mb.} \tag{33}$$

Finally, adding the polarization brought into the beam by protons scattered elastically off protons within the acceptance angle,

$$\delta \sigma_1^{pp}(\text{CNI}; \theta_{min} < \theta < \theta_{acc}) = +52 \text{ mb,} \tag{34}$$

brings the theory to a perfect agreement with the experiment: $\sigma_1 = (83 - 70 + 52) \text{ mb} = 65 \text{ mb}$.

Unfortunately, this agreement with $\sigma_P$ must be regarded as an accidental one. In view of our discussion in Sec. 6, the starting point (32) corresponds to transmission effects already corrected for the scattering within
the beam. As such, it correctly omits the transmission effects from the scattering off electrons. Then, correcting for (33) and (34) amounts to the double counting of the scattering within the beam. These corrections would have been legitimate only if one would have started with the sum of $\sigma_1(> \theta_{\text{min}})$ for electron and proton targets rather than with (32).

In a more accurate treatment of the scattering within the beam, we encountered the mismatch X-sections $\Delta \sigma_{0,1}$. They correspond to spin effects at extremely small scattering angles $\theta_{\text{min}} < \theta < \theta_{\text{acc}} \ll \theta_{\text{Coulomb}}$. The elastic scattering that deep under the Coulomb peak can never be accessed in the direct scattering experiments, such observables can only be of the relevance to the storage rings. The existing SAID and Nijmegen databases have never been meant for the evaluation of the NN scattering amplitudes at so small angles. An important virtue of these databases is that they have a built-in procedure for the Coulomb-nuclear interference effects in all observables. If one would like to take an advantage of this feature, then one needs a careful extrapolation of these observables to the range of very extremely small angles of our interest. There are strong cancellations and it is prudent to extrapolate the whole integrands of $\Delta \sigma_{0,1}$ rather than the separate observables. Upon such an extrapolation, $\Delta \sigma_{0,1}$ are found to be negligible small, for the polarization cross section (30) of our interest we find $\Delta \sigma_1 \approx -6 \cdot 10^{-3} \text{mb}$.

Milstein and Strakhovenko took a very different path: they started with the nuclear scattering phases from the Nijmegen database, added in all the Coulomb corrections following the Nijmegen prescriptions, and evaluated directly all the CNI effects. The numerical results for $\Delta \sigma_1$ from the two different evaluations are for all the practical purposes identical. The technical reason for negligible $\Delta \sigma_1$ in contrast to a very large difference between (31) and (32) is a vanishing interference between the hadronic spin-flip and dominant Coulomb amplitudes.

The principal conclusion is that the polarization buildup of stored protons is, for all the practical purposes, controlled by the transmission effects, described by Meyer’s formula (32) for the CNI corrected nuclear proton-proton elastic scattering beyond the ring acceptance angle. Corrections to this formula for the spin-flip scattering prove to be negligible small. The polarization transfer from polarized electrons to scattered protons is a legitimate, and numerically substantial, effect, but it is exactly canceled by the electron contribution to the spin-dependent transmission effects.

The conversion of the FILTEX polarization rate, which by itself is the 20 standard deviation measurement, into the polarization cross section $\sigma_P$
depends on the target polarization and the PIT areal density. The recent reanalysis \(^{19}\) gave \(\sigma_P = 72.5 \pm 5.8\) mb, where both the statistical and systematical errors are included. The latest version of the SAID database, SAID-SP05 \(^{17}\), gives \(\sigma_1(CNI; \theta > \theta_{acc}) = 85.6\) mb, which is consistent with the FILTEX result within the quoted error bars. Following the direct evaluation of the CNI starting from the Nijmegen nuclear phase shifts, Milstein and Strakhovenko find for the same quantity 89 mb \(^{15}\).

9. Conclusions

We reported a quantum-mechanical evolution equation for the spin-density matrix of a stored beam interacting with the polarized internal target. The effects of the scattering within the beam are consistently included. An indispensable part of this description is a precession of the beam spin in the pseudomagnetic field of polarized atoms in PIT. In the specific application of our evolution equation to the spin filtering in the storage ring, the precession effects average out, and the spin-density matrix formalism and the kinetic equation formalism of Milstein and Strakhovenko become equivalent to each other.

Following Meyer, one must allow for the CNI contribution to the spin-dependent scattering within the beam, which has a very strong impact on the polarization cross section. There is a consensus between theorists from the Budker Institute and IKP, Jülich on the self-cancellation of the transmission and scattering-within-the-beam contributions from polarized electrons to the spin filtering of (anti)protons. Both groups agree that corrections from spin-flip scattering within the beam to eq. (32) for the polarization cross section are negligible small. There is only a slight disagreement between the reanalyzed FILTEX result, \(\sigma_P = 72.5 \pm 5.8\) mb \(^{19}\) and the theoretical expectations, \(\sigma_P \approx 86\) mb.

Regarding the future of the PAX suggestion \(^3\), the experimental basis for predicting the polarization buildup in a stored antiproton beam is practically non-existent. One must optimize the filtering process using the antiprotons available elsewhere (CERN, Fermilab). Several phenomenological models of antiproton-proton interaction have been developed to describe the experimental data from LEAR \(^{20,21,22,23,24,25}\). While the real part of the \(p\bar{p}\) potential can be obtained from the meson-exchange nucleon-nucleon potentials by the G-parity transformation and is under reasonable control, the fully field-theoretic derivation of the self-hermitian annihilation potential is as yet lacking. The double-spin \(p\bar{p}\) observables necessary to constrain
predictions for $\sigma_{1,2}$ are practically nonexistent (for the review see \textsuperscript{26}). Still, the expectations from the first generation models for double–spin dependence of $p\bar{p}$ interaction are encouraging, see Haidenbaur’s review at the Heimbach Workshop on Spin Filtering \textsuperscript{27}. With filtering for two lifetimes of the beam, they suggest that in a dedicated large–acceptance polarizer storage ring, antiproton beam polarizations in the range of 15–25\% seem achievable, see Contalbrigo’s talk at this Workshop \textsuperscript{28}.

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