Photon-neutrino scattering and the B-mode spectrum of CMB photons

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On the basis of the quantum Boltzmann equation governing the time-evolution of the density matrix of polarized CMB photons in the primordial scalar perturbations of metric, we calculate the B-mode spectrum of polarized CMB photons contributed from the scattering of CMB photons and CNB neutrinos (Cosmic Neutrino Background). We show that such contribution to the B-mode spectrum is negligible for small \( \ell \), however is significantly large for \( 50 < \ell < 200 \) by plotting our results together with the BICEP2 data. Our study and results imply that in order to theoretically better understand the origin of the observed B-mode spectrum of polarized CMB photons (\( r \)-parameter), it should be necessary to study the relevant and dominate processes in both tensor and scalar pertubations.

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I. INTRODUCTION

It is known that in the inflation cosmology, the power-law spectrum of either metric scalar perturbation \( P_S(k) = A_S(k/k_0)^{n_S-1} \) or tensor perturbation \( P_T(k) = A_T(k/k_0)^{n_T-1} \) has been produced in the inflationary era of the early universe \([1]\), where \( A_S \) and \( A_T \) are the amplitudes of scalar and tensor perturbations, and \( n_{S,T} \) are their spectral indexes. \( A_S \) and \( n_S \) have been determined through the measurements of microwave background temperature anisotropy \([2–4]\). The amplitude of metric tensor perturbation is characterized by the tensor-scalar ratio \( r = P_T/P_S \), relating to the B-mode spectrum of polarized CMB photons imprinted by the metric tensor perturbations of primordial gravitational waves. The BICEP2 collaboration recently reports \( r = 0.20^{+0.07}_{-0.05} \) \([5]\). If this report is verified, it is regarded as an important result that may reveal the existence of metric tensor perturbations in the inflationary era of the early universe.

However, there are alternative explanations of the BICEP2 data. Whether the BICEP2 data could be explained by the vector and tensor modes from primordial magnetic fields \([6]\). Some authors speculate that the BICEP2 observed B-mode polarization is the result of a primordial Faraday rotation of the E-mode polarization \([7, 8]\). In this article, using the result of photon polarization generated by the photon-neutrino scattering \([9]\), we investigate the possible contribution to the observed B-mode spectrum by considering the interaction between CMB photons and Cosmic Neutrino Background (CNB) in the background of scalar perturbations, without tensor perturbations. In order to quantitatively calculate such contribution in the scalar perturbation, we solve the quantum Boltzmann equation for the time-evolution of the matrix density (Stokes parameters) of polarized CMB photons which are involved in the Compton and photon-neutrino scatterings as the collision terms of the quantum Boltzmann equation. Our result is shown together with the BICEP2 data and its implication on the interpretation of the BICEP2 data is discussed.

II. THE PHOTON POLARIZATION FROM COMPTON AND PHOTON-NEUTRINO SCATTERINGS.

The linear and circular polarizations of an ensemble of photons can be described by the density operator

\[
\hat{\rho}_{ij} = \frac{1}{\text{tr} (\hat{\rho})} \int \frac{d^3k}{(2\pi)^3} \rho_{ij}(k) D_{ij}(k), \quad \hat{\rho}_{ij}(k) = \frac{1}{2} \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix},
\]
where $\rho_{ij}(k)$ represents the density matrix in terms of the Stokes parameters $I$, $Q$, $U$ and $V$ in the $2 \times 2$ polarization space $(i,j)$ of one photon of energy-momentum “$k$”.

The number operator $D_{ij}(k) = a_i^\dagger(k)a_j(k)$ and its expectation value

$$
\langle D_{ij}(k) \rangle \equiv \text{tr}[\hat{\rho}_{ij}D_{ij}(k)] = (2\pi)^3 \delta^3(0)(2k^0)\rho_{ij}(k).
$$

The time evolution of the number operator $D_{ij}(k)$ obeys the Heisenberg equation

$$
\frac{d}{dt}D_{ij}(k) = i[H_I, D_{ij}(k)],
$$

where $H_I$ is an interacting Hamiltonian. Using Eqs. (1), (2) and (3), one obtains the time evolution of $\rho_{ij}(k)$, quantum Boltzmann equation [10],

$$
(2\pi)^3 \delta^3(0)(2k^0)\frac{d\rho_{ij}(k)}{dt} = i\langle[H_I(t), D_{ij}(k)]\rangle - \frac{1}{2} \int dt\langle[H_I(t), [H_I(0), D_{ij}(k)]]\rangle,
$$

where $H_I(t)$ is the interacting Hamiltonian. On the right-handed side of Eq. (4), the first and second terms respectively represent the forward scattering and higher order collision terms.

There are a lot of papers which investigate the effects of the Compton scattering on the anisotropy and polarization of CMB (see for example [10–12]). In this article, we attempt to study the CMB photon polarization by considering the contribution of the photon-neutrino scattering to the polarization density matrix of photons obtained recently [9].

$$
2k^0\frac{d\rho_{ij}}{dt} = -\frac{\sqrt{2}}{6\pi} \alpha G_F \int dq \left[ \rho_{s'j}(k)\delta_{is} - \rho_{is}(k)\delta_{js'} \right] f_\nu(x, q)
\times \left( q^2 \epsilon_{s'} \cdot \epsilon_s + 2 q \cdot \epsilon_{s'} q \cdot \epsilon_s - \epsilon_{\mu\nu\rho\sigma} \epsilon_{s'}^\mu \epsilon_s^\nu k^\rho q^\sigma \right),
$$

where $dq = (2E_\nu)^{-1}d^3q/(2\pi)^3$ is the integration over the neutrino four-momentum ($q^0 = E_\nu \approx |q|$) with the distribution function $f_\nu(x, q)$, the polarization four-vectors $\epsilon_{i\mu}(k)$ and their indexes $i,j,s,s' = 1,2$, represent two transverse polarizations of the photon $k^0 = |k|$. $G_F$ and $\alpha$ are Fermi coupling constant and electromagnetic fine-structure constant.

Using the Stokes parameters in Eq. (1): the total intensity $I$, linear polarizations intensities $Q$ and $U$, as well as the $V$ indicating the difference between left- and right-circular polarizations intensities, we consider the both Compton and photon-neutrino scattering and write Eq. (4) as follows

$$
\frac{dI}{dt} = C_{e\gamma}^I
$$

$$
\frac{d}{dt}(Q \pm iU) = C_{e\gamma}^{\pm} \mp i\kappa_{\pm}(Q \pm iU) + O(V)
$$

$$
\frac{dV}{dt} = C_{e\gamma}^{V} + \kappa_Q Q + \kappa_U U,
$$

(6)
here \( C^i_{\gamma \gamma}, C^+_{\gamma \gamma} \) and \( C^V_{\gamma \gamma} \) respectively indicate the contributions from the Compton scattering to the time evaluation of \( I, Q \pm iU \) and \( V \) parameters, their expressions can be found from the literature for example \[10-12\]. Whereas the contributions from the photon-neutrino scattering \([5]\) are given by

\[
\dot{\kappa}_\pm = -\frac{\sqrt{2}}{6\pi k^0} \alpha G^F \int d\mathbf{q} \mathcal{F}_\nu(x, q) \times (\epsilon_{\mu\nu\rho\sigma} \epsilon_1^{\mu} k^{\nu} q^{\rho} q^{\sigma})
\]

\[
\dot{\kappa}_Q = -\frac{\sqrt{2}}{3\pi k^0} \alpha G^F n_\nu \langle v_\alpha q_\beta \rangle \epsilon_2^\alpha \epsilon_1^\beta
\]

\[
\dot{\kappa}_U = -\frac{\sqrt{2}}{6\pi k^0} \alpha G^F n_\nu \left( \langle v_\alpha q_\beta \rangle \epsilon_1^\alpha \epsilon_2^\beta - \langle v_\alpha q_\beta \rangle \epsilon_2^\alpha \epsilon_1^\beta \right),
\]

where we define the neutrino average velocity

\[
\langle v_\alpha \rangle = \frac{1}{n_\nu} \int \frac{d^3 q}{(2\pi)^3} \frac{q_\alpha}{q_0} f_\nu(x, q), \quad \langle v_\alpha q_\beta \rangle = \frac{1}{n_\nu} \int \frac{d^3 q}{(2\pi)^3} \frac{q_\alpha}{q_0} q_\beta f_\nu(x, q).
\]

where the neutrino number-density \( n_\nu(x) = \int d^3 q/(2\pi)^3 f_\nu(x, q) \) and energy-density \( \epsilon_\nu(x) = \int d^3 q/(2\pi)^3 q^0 f_\nu(x, q) \). In the second equation of Eq. \((4)\), we will neglect the small contribution \( O(V) \) from the circular polarization. In Eq. \((9)\), the first equation \( \dot{\kappa}_\pm \) yields

\[
\dot{\kappa}_\pm = \frac{\sqrt{2}}{6\pi k^0} \alpha G^F \int d\mathbf{q} \mathcal{F}_\nu(x, q) \times \left[ q^0 k \cdot (\epsilon_1 \times \epsilon_2) + k^0 q \cdot (\epsilon_1 \times \epsilon_2) \right]
\]

\[
= \frac{\sqrt{2}}{6\pi} \alpha G^F n_\nu \frac{2}{1 + \langle \mathbf{v} \rangle \cdot (\epsilon_1 \times \epsilon_2)} \approx \frac{\sqrt{2}}{6\pi} \alpha G^F n_\nu \frac{2}{2},
\]

where \( k \cdot (\epsilon_1 \times \epsilon_2) = |k| \). In this article, we apply these equations \((6-9)\) to the case of photons scattering with cosmic neutrino background (CNB), whose average velocities \((8)\) are small, will be discussed in the next section. As a result of the leading order approximation, the dominate contribution of photon-neutrino scattering to photon polarization comes from the first term of Eq. \((9)\).

**III. CNB NEUTRINO DISTRIBUTION FUNCTION AND AVERAGE VELOCITY**

We discuss the CNB neutrino distribution function \( f_\nu(x, q) \) and average velocity \( \langle \mathbf{v} \rangle \) by using the Boltzmann equation for massive neutrinos (see the Chapter 4 of Ref. \[13\]). Because the ensemble of massive neutrinos behaves as a hydrodynamic fluid described by the distribution function \( f_\nu(x, q) \), satisfying the following Boltzmann evolution equation derived from the conservations of neutrino number and energy-momentum,

\[
\frac{\partial f_\nu}{\partial t} + \frac{p_i}{a E_\nu} \frac{\partial f_\nu}{\partial x_i} - \frac{1}{a E_\nu} \left( \frac{\dot{a} p^2}{a E_\nu} + \frac{p^2}{E_\nu} \frac{\partial \Phi}{\partial t} + \frac{p_i}{a} \frac{\partial \Psi}{\partial x_i} \right) = 0,
\]

\((10)\)
where the longitudinal gauge \[14\] is adopted and the scalar modes of metric perturbations are characterized by two scalar potentials \( \phi \) and \( \varphi \) in the line element

\[
ds^2 = a^2(\tau)\{- (1 + 2\psi) d\tau^2 + (1 + 2\varphi) dx_i dx^i\}.
\]

We are in the comoving frame of expanding universe, where the homogeneous and isotropic density \( \bar{n}_\nu \propto a^{-3} \) and the neutrino average velocities \( \langle \bar{v}^i \rangle \) are zero. However, we observe the perturbation of the neutrino fluid density \( \delta n_\nu = (n_\nu - \bar{n}_\nu)/\bar{n}_\nu \) and the perturbation of the neutrino average velocities \( \delta v^i \equiv (\langle v^i \rangle - \langle \bar{v}^i \rangle) = (v^i) \). The perturbation of the neutrino average velocities \( \delta \langle v^i \rangle = \langle v^i \rangle \) is proportional to the perturbation of the neutrino fluid density \( \delta n_\nu \). This can be seen by the linear approximation of the Boltzmann equation \[10\]. Integrating the Boltzmann equation \[10\] over \( \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} \frac{p_i}{E_\nu} \) and \( \int \frac{d^3p}{(2\pi)^3} \frac{p_i}{E_\nu} \frac{p_j}{E_\nu} \), and considering the linear approximation, one obtains

\[
\frac{\partial \delta n_\nu}{\partial t} + \frac{1}{a} \frac{\partial \langle v^i \rangle}{\partial x^i} + 3 \frac{\partial \varphi}{\partial t} = 0 \quad (12)
\]

\[
\frac{\partial \langle v^i \rangle}{\partial t} + \frac{\dot{a}}{a} \langle v^i \rangle + \frac{1}{a} \frac{\partial \psi}{\partial x^i} = 0 \quad (13)
\]

\[
\frac{\partial \langle v^i v^j \rangle}{\partial t} + 2 \frac{\dot{a}}{a} \langle v^i v^j \rangle + \frac{1}{a} \left( \frac{\partial \psi}{\partial x_i} \langle v^j \rangle + \frac{\partial \psi}{\partial x_j} \langle v^i \rangle \right) = 0 \quad (14)
\]

where analogously to Eq. \[8\] the neutrino average velocities are

\[
\langle v^i \rangle = \frac{1}{n_\nu} \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{E_\nu} f_\nu, \quad \langle v^i v^j \rangle = \frac{1}{n_\nu} \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{E_\nu} \frac{p^j}{E_\nu} f_\nu, \quad (15)
\]

Rewrite Eqs. \[12\]-\[14\] in terms of the conformal time \( \eta \) and the Fourier components of variables (e.g. \( \bar{v}^i \) for \( \langle v^i \rangle \))

\[
\dot{\delta}_\nu + iK \bar{v} + 3\dot{\varphi} = 0 \quad (16)
\]

\[
\dot{\bar{v}} + \frac{\dot{a}}{a} \bar{v} + iK \bar{\psi} = 0 \quad (17)
\]

\[
\frac{\partial}{\partial \tau} \bar{v}^i \bar{v}^j + 2 \frac{\dot{a}}{a} \bar{v}^i \bar{v}^j + i(K^i \bar{v}^j + K^j \bar{v}^i) \bar{\psi} = 0, \quad (18)
\]

here it is assumed that the velocity is ir-rotational so \( v^i = \frac{K^i}{K} \bar{v} \). As shown in these equations, the neutrino average velocity is of the order of neutrino density perturbations, i.e., \( \bar{v} \sim \delta n_\nu/\bar{n}_\nu \sim \Delta T \sim 10^{-5} \). Therefore, in Eq. \[9\] the second term depending on the neutrino average velocity is negligible, compared with the first term depending on the neutrino density.

In addition, in the following calculations, we select the coordinate where the components of the
photon momentum \( \mathbf{k} \), polarization vectors \( \epsilon_1 \) and \( \epsilon_2 \) are

\[
\begin{align*}
    k_x &= \sin \theta \cos \phi, \quad \epsilon_{1x}(k) = \cos \theta \cos \phi, \quad \epsilon_{2x}(k) = \sin \phi, \\
    k_y &= \sin \theta \sin \phi, \quad \epsilon_{1y}(k) = \cos \theta \sin \phi, \quad \epsilon_{2y}(k) = \cos \phi, \\
    k_z &= \cos \theta, \quad \epsilon_{1z}(k) = -\sin \phi, \quad \epsilon_{2z}(k) = 0.
\end{align*}
\]

(19)

IV. TIME-EVOLUTION OF POLARIZED CMB PHOTONS

In this section, we focus on the linear polarization of CMB (E- and B- modes) due to the Compton and photon-neutrino (CNB) scattering in company with primordial scalar perturbations only. As usual, the CMB radiation transfer in the conformal time \( \eta \) is described by the multipole moments of temperature (I) and polarization (P)

\[
\Delta I, P(\eta, K, \mu) = \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^\ell \Delta I, P(\eta, K) P_\ell(\mu),
\]

where \( \mu = \mathbf{n} \cdot \mathbf{K} = \cos \theta \), the angle between the CMB photon direction \( \mathbf{n} = \mathbf{k}/|\mathbf{k}| \) and the wave-vectors \( \mathbf{K} \) of Fourier modes of scalar perturbations, and \( P_\ell(\mu) \) is the Legendre polynomial of rank \( \ell \).

We adopt the following Boltzmann equation obeyed by \( \Delta I, P(\eta, K, \mu) \), and expand the primordial scalar perturbations \( S \) of metric field in Fourier modes characterized by the wave-vector \( \mathbf{K} \). For a given Fourier mode, ones can select the coordinate system where \( \mathbf{K} \parallel \hat{z} \) and \( (\hat{e}_1, \hat{e}_2) = (\hat{e}_\theta, \hat{e}_\phi) \).

For each plane wave, the scattering can be described as the transport through a plane parallel medium \[15, 16\], and Boltzmann equations are

\[
\begin{align}
    \frac{d}{d\eta} \Delta I^{(S)} + i K \mu \Delta I^{(S)} + 4[\dot{\psi} - i K \mu \varphi] &= \hat{\tau} \left[ -\Delta I^{(S)} + \Delta I^{(0)} + i \mu v_b + \frac{1}{2} P_2(\mu) \Pi \right] \tag{20} \\
    \frac{d}{d\eta} \Delta P^{(S)} + i K \mu \Delta P^{(S)} &= \hat{\tau} \left[ -\Delta P^{(S)} - \frac{1}{2} [1 - P_2(\mu)] \Pi \right] + i a(\eta) \dot{\kappa}_\pm \Delta P^{(S)} \tag{21}
\end{align}
\]

where \( \hat{\tau} \equiv d\tau/d\eta \), the scaling factor \( a(\eta)|_{\eta_0} = 1 \) at the present time \( \eta_0 \), \( \Pi \equiv \Delta I^{2(S)} + \Delta P^{2(S)} + \Delta P^{0(S)} \) and the polarization anisotropy is defined by

\[
\Delta P^{(S)} = Q^{(S)} \pm i U^{(S)}. \tag{22}
\]

In the RHS of Eqs. (20) and (21), the scattering parts are determined by the Compton and photon-neutrino scattering terms in Eq. (6), in particular, the contribution of photon-neutrino (CNB) scattering to CMB polarization comes from the \( \dot{\kappa}_\pm \)-terms in Eq. (6). The temperature anisotropy \( \Delta T \) depends on the metric perturbations \( \varphi \) and \( \psi \) and baryon velocity term \( v_b \) in Eq. (20). Eq. (21) for the polarization anisotropy can be written as follows,

\[
\frac{d}{d\eta} \left[ \Delta P^{(S)} e^{i K \mu \eta} + \dot{i} \kappa(\eta, \mu) + \dot{\tau}(\eta) \right] = -e^{i K \mu \eta} \pm i \kappa(\eta) \left( \frac{1}{2} \hat{\tau} [1 - P_2(\mu)] \Pi \right), \tag{23}
\]
where

\[ \tilde{\kappa}(\eta, \mu) \equiv \int_0^\eta d\eta a(\eta) \kappa_\pm, \quad \tilde{\tau}(\eta) \equiv \int_0^\eta d\eta \dot{\tau}. \]

(24)

With the initial condition \( \Delta_P^{\pm}(0, K, \mu) = 0 \), the integration of Eq. (23) along the line of sight up to the present time \( \eta_0 \) yields

\[
\Delta_P^{\pm}(\eta_0, K, \mu) = \frac{3}{4} (1 - \mu^2) \int_0^{\eta_0} d\eta e^{ix\mu\pm i\kappa(\eta)\tau} \Pi(\eta, K) \]

(25)

where \( x = K(\eta_0 - \eta) \) and

\[
\kappa(\eta) = \int_\eta^{\eta_0} d\eta a(\eta) \kappa_\pm(\eta). \]

(26)

These are analogous to the optical depth \( \tau(\eta) \) with respect to the Compton scattering

\[
\dot{\tau} = an_e x_e \sigma_T, \quad \tau(\eta) = \int_\eta^{\eta_0} \dot{\tau}(\eta) d\eta, \]

(27)

where \( n_e \) is the electron density, \( x_e \) is the ionization fraction and \( \sigma_T \) is the Thomson cross-section.

V. THE B-MODE POWER SPECTRUM OF POLARIZED CMB PHOTONS

One can separate the CMB polarization \( \Delta_P^{\pm}(\eta_0, K, \mu) \) into the divergence-free part (B-mode \( \Delta_B^{(S)} \)) and curl-free part (E-mode \( \Delta_E^{(S)} \)) as following

\[
\Delta_E^{(S)}(\eta_0, K, \mu) \equiv -\frac{1}{2} \{ \delta^2 \Delta_P^{+(S)}(\eta_0, K, \mu) + \delta^2 \Delta_P^{-(S)}(\eta_0, K, \mu) \} \]

(28)

\[
\Delta_B^{(S)}(\eta_0, K, \mu) \equiv \frac{i}{2} \{ \delta^2 \Delta_P^{+(S)}(\eta_0, K, \mu) - \delta^2 \Delta_P^{-(S)}(\eta_0, K, \mu) \} \]

(29)

where \( \delta \) and \( \bar{\delta} \) are spin raising and lowering operators respectively, and one assumes scalar perturbations to be axially symmetric around \( K \) so that

\[
\bar{\delta}^2 \Delta_P^{\pm}(\eta_0, K, \mu) = \partial^2_{\mu}\{(1 - \mu^2) \Delta_P^{\pm}(\eta_0, K, \mu)\}, \]

(30)

where \( \partial_\mu = \partial/\partial \mu \). From Eqs. (25) and (30), we obtain the E- and B-modes

\[
\Delta_E^{(S)}(\eta_0, K, \mu) = \frac{3}{4} \int_0^{\eta_0} d\eta g(\eta) \Pi(\eta, K) \bar{\delta}^2 \{ (1 - \mu^2)^2 e^{ix\mu \cos \kappa(\eta)} \}, \]

(31)

\[
\Delta_B^{(S)}(\eta_0, K, \mu) = \frac{3}{4} \int_0^{\eta_0} d\eta g(\eta) \Pi(\eta, K) \bar{\delta}^2 \{ (1 - \mu^2)^2 e^{ix\mu \sin \kappa(\eta)} \}, \]

(32)

where \( g(\eta) = \dot{\tau} e^{-\tau} \). Eq. (32) shows that the photon-neutrino scattering \( (\kappa \neq 0) \) results in the nontrivial B-mode \( \Delta_B^{(S)} \) and the modifications of the E-mode \( \Delta_E^{(S)} \). This agrees that the Compton
scattering only can not generate B-mode without taking into account the tensor type of metric perturbations \[12, 18–20\].

Using Eq. (25), we can obtain the value of \(\Delta_E^{(S)}(\mathbf{n})\) at the present time \(\eta_0\) and in the direction \(\mathbf{n}\) by summing over all their Fourier modes \(K\), analogously to the normal approach \[11, 12, 17\],

\[
\Delta_{E,B}^{(S)}(\mathbf{n}) = \int d^3 K \xi(K)e^{\mp 2i\phi_{K,n}} \Delta_{E,B}^{(S)}(\eta_0, K, \mu),
\]

where \(\phi_{K,n}\) is the angle needed to rotate the \(K\) and \(\mathbf{n}\) dependent basis to a fixed frame in the sky. The random variable \(\xi(K)\) used to characterize the initial amplitude of the mode satisfies [see for example \[11, 12, 17\]]

\[
\langle \xi^*(K_1)\xi(K_2) \rangle = P_S(K)\delta(K_1 - K_2),
\]

where \(P_S(K)\) is the initial power spectrum of the scalar mode perturbation.

As a result, by integrating Eqs. (33) and (34) over the initial power spectrum of the metric perturbation, we obtain the power spectrum for \(E\)- and \(B\)- modes

\[
C^{\ell(S)}_{E,B} = \frac{1}{2\ell + 1} \frac{(\ell - 2)!}{(\ell + 2)!} \int d^3 K P_S(K) \left| \sum_m \int d\Omega Y^*_{lm} \Delta_{E,B}^{(S)}(\eta_0, K, \mu) \right|^2.
\]

Using identities \(\partial_\mu(1 - \mu^2)\mathbb{I}^{ix\mu} = (1 + \partial_\mu^2)x^2 \mathbb{I}^{ix\mu}\) and \(\int d\Omega Y^*_{lm} \mathbb{I}^{ix\mu} = (i)^\ell \sqrt{4\pi(2\ell + 1)} j_\ell(x) \delta_{m0}\), we obtain the polarized CMB power spectrum in multipole moments \(\ell\),

\[
C^{\ell(S)}_E = (4\pi)^2 \frac{2(\ell + 2)!}{(\ell - 2)!} \int d^3 K P_S(K) \left| \sum_m \int_0^{\eta_0} d\eta g(\eta) \Pi(\eta, K) \frac{j_\ell}{x^2} \cos(\kappa(\eta)) \right|^2,
\]

\[
C^{\ell(S)}_B = (4\pi)^2 \frac{2(\ell + 2)!}{(\ell - 2)!} \int d^3 K P_S(K) \left| \sum_m \int_0^{\eta_0} d\eta g(\eta) \Pi(\eta, K) \frac{j_\ell}{x^2} \sin(\kappa(\eta)) \right|^2,
\]

where \(j_\ell(x)\) is a spherical Bessel function of rank \(\ell\). In Fig. 1 we plot the numerical value \(C^{\ell(S)}_B\) of Eq. (37) together with the BICEP2 result. It is shown that the contribution of photon-neutrino (CNB) scattering to the B-mode is negligible for small \(\ell\), however is significantly large for \(50 < \ell < 200\).

To order to better understand our results \[36\] and \[37\], we approximately write \(C^{\ell(S)}_E\) and \(C^{\ell(S)}_B\) as follows

\[
C^{\ell(S)}_E \approx \bar{C}^{\ell(S)}_E (\cos^2 \kappa), \quad C^{\ell(S)}_B \approx \bar{C}^{\ell(S)}_E (\sin^2 \kappa)
\]

where

\[
\bar{C}^{\ell(S)}_E = (4\pi)^2 \frac{2(\ell + 2)!}{(\ell - 2)!} \int d^3 K P_S(K) \left| \sum_m \int_0^{\eta_0} d\eta g(\eta) \Pi(\eta, K) \frac{j_\ell}{x^2} \right|^2.
\]
FIG. 1. The solid line represents $\ell(\ell + 1)C_B^{(S)}/2\pi[\mu K^2]$ due to the primordial scalar perturbations and photon-neutrino (CNB) scattering. The experiment BICEP2 results (dots with their error bars) are plotted.

is the power spectrum of the E-mode polarization contributed from the Compton scattering in the case of scalar perturbation \cite{11}. In Eq. (38), the mean value $\bar{\kappa}$ of Eq. (26) is an average from the last scattering time (redshift $z_l \approx 10^3$) to the present time ($z_0 = 0$). Using the matter dominate Friedmann equation $H^2/H_0^2 = \Omega_M^0(1 + z)^3 + \Omega_\Lambda^0$, $H_0 \approx 74$ km/s/Mpc, $\Omega_M^0 \approx 0.27, \Omega_\Lambda^0 \approx 0.73$, and $ad\eta = -dz/H(1 + z)$, as well as the conservation of total neutrino number $n_\nu = n_\nu^0(1 + z)^3$, we obtain

$$\kappa(z) = \int_{\eta_0}^{\eta_0} a d\eta \bar{\kappa}_\pm = \frac{\sqrt{2}}{12\pi} G n_\nu^0 \int_z^{z_l} \frac{(1 + z')^2}{H(z')} dz'$$

where the present number-density of all flavor neutrinos and anti-neutrinos $n_\nu^0 = \sum(n_\nu^0 + \bar{n}_\nu^0) \approx 340$ cm$^{-3}$). Actually, the $\bar{\kappa}$ is the mean opacity of CMB photons against the photon-neutrino (CNB) scattering.

To end this section, we would like to point out that the B-mode power spectrum $C_B^{(S)}$ of Eq. (37) is attributed only to the scalar perturbations and photon-neutrino (CNB) scatterings. This result implies that the B-mode power spectrum could be contributed by other mechanisms with scalar perturbations, in addition to the contribution from the primordial tensor perturbations \cite{11}. Therefore, this is crucial how to interpret the measurement of $r$-parameter that is the ratio of the B-mode power spectrum and the E-mode power spectrum.
VI. SUMMARY.

Suppose that the total contribution to the polarized CMB B-mode comes from the primordial tensor perturbations \((T)\), one obtains the B-mode power spectrum \[ C_B^{(T)} = \frac{(4\pi)^2}{3} \int d^3 K \frac{P_T(K)}{K} \int_0^{\eta_0} d\eta g(\eta) S_T^T(\eta, K) \left( 2j_{1/2} + \frac{j}{x} \right)^2, \]

where \(P_T(K)\) is the initial power spectrum of primordial tensor perturbations. Based on this assumption, one can approximately obtain the \(r\)-parameter \(r = \frac{P_T}{P_S} \propto \frac{C_B^{(T)}}{C_E^{(S)}}\). Taking into account the contribution \([36, 38]\) of photon-neutrino (CNB) scattering and assuming the total observed B-mode power spectrum \(C_B^{(ob)}\) given by \(C_B^{(ob)} = C_B^{(T)} + C_B^{(S)}\), we have

\[ r = \frac{P_T}{P_S} \propto \left( C_B^{(ob)} - C_B^{(S)} \right) \propto \frac{C_B^{(T)}}{C_E^{(S)}} \approx \frac{C_B^{(T)}}{C_E^{(S)}} - \sin^2 \bar{\kappa}, \]

where \(\sin^2 \bar{\kappa} \sim \bar{\kappa}^2 \approx 0.025\). This implies that the measured \(r\)-parameter would not be completely originated from primordial tensor perturbations. In addition, there might be other contributions from either some astrophysical effects \([6, 8]\) or some microscopic effects, for example the CMB photon-photon scatterings \([21, 22]\). Therefore, it is important to study possibly significant contributions to the B-mode power spectrum of polarized CMB photons so that one can better understand the contribution of primordial tensor perturbations to the \(r\)-parameter experimentally measured, \(r = 0.2\) as reported by BICEP2.

In summary, we have studied the Quantum Boltzmann Equation governing the time-evolution of the density matrix (Stokes parameters) of polarized CMB photons by considering the both Compton and photon-neutrino (CNB) scattering in the background of primordial scalar perturbations. It is shown that in this case the B-mode spectrum of polarized CMB photons can also be generated without primordial tensor perturbations. We quantitatively calculate the generated B-mode spectrum which is related to the mean opacity \(\bar{\kappa}\) \([10]\) of CMB photons scattering with neutrinos (CNB). In the other hand, we compare our result with the B-mode spectrum generated by the Compton scattering in the background of primordial tensor perturbations, which seems to be dominated. We generally discuss the possible implication of our result on the interpretation of the BICEP2 measurement \(r = 0.2\) in terms of primordial tensor perturbations.

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