General expressions for downlink signal to interference and noise ratio in homogeneous and heterogeneous LTE-Advanced networks

Nora A. Ali, Hebat-Allah M. Mourad, Hany M. ElSayed, Magdy El-Soudani, Hassanein H. Amer, Ramez M. Daoud

Electronics and Communications Engineering Department, Cairo University, Giza, Egypt
Electronics and Communications Engineering Department, American University in Cairo, Cairo, Egypt
KAMA Trading, Engineering Office, Cairo, Egypt

The interference is the most important problem in LTE or LTE-Advanced networks. In this paper, the interference was investigated in terms of the downlink signal to interference and noise ratio (SINR). In order to compare the different frequency reuse methods that were developed to enhance the SINR, it would be helpful to have a generalized expression to study the
Accepted 6 September 2016
Available online 12 September 2016

Keywords:
LTE-Advanced
Signal to interference and noise ratio (SINR)
Fractional frequency reuse (FFR)
Soft frequency reuse (SFR)
Heterogeneous network

Introduction

Frequency reuse schemes are the most suited interference management techniques for the OFDMA based cellular networks wherein the cells are divided into separate regions with different frequencies [1,2]. The most famous technology using OFDMA is the Long Term Evolution (LTE). LTE was developed by the third Generation Partnership Project (3GPP) to complement the 3G technology with high data rate, low latency and high spectral efficiency. To further improve the network and set the requirements of the International Mobile Telecommunication-Union (IMT-U), 3GPP developed LTE-Advanced to be the 4G technology by using carrier aggregation, higher order MIMO and implementing low power nodes with the macrocells. However, using OFDMA results in a big problem which is the inter-cell interference (ICI) due to using the same frequency for all cells [1,2]. This results in performance degradation especially for the edge users. Fractional frequency reuse (FFR), soft frequency reuse (SFR) and the new hybrid frequency reuse (NHFR) are the most used interference management techniques that were generated to mitigate the ICI problem in LTE homogenous network [3,4].

In FFR, the whole system bandwidth is not used inside the cell, where the cell is divided into inner and outer regions; the inner regions use the same frequency (reuse factor = 1), but the outer regions use different frequencies (reuse factor > 1) as shown in Fig. 1 [5–7]. In SFR, the whole system bandwidth is used inside the cell. The cell is divided into inner and outer regions with different frequencies and different transmission powers (Fig. 2) using power control to mitigate the interference [5–7]. Signal to interference and noise ratio (SINR) is the most significant factor to measure the amount of ICI and to evaluate the performance of the proposed interference management technique.

In NHFR, the cell is not divided into inner and outer regions. But, the centre frequencies of the neighbouring base stations are changed to reduce the ICI as shown in Fig. 3 [3]. Changing these centre frequencies causes some overlapping performance of the different methods. Therefore, this paper introduces general expressions for the SINR in homogeneous and in heterogeneous networks. In homogeneous networks, the expression was applied for the most common types of frequency reuse techniques: soft frequency reuse (SFR) and fractional frequency reuse (FFR). The expression was examined by comparing it with previously developed ones in the literature and the comparison showed that the expression is valid for any type of frequency reuse scheme and any network topology. Furthermore, the expression was extended to include the heterogeneous network; the expression includes the problem of co-tier and cross-tier interference in heterogeneous networks (HetNet) and it was examined by the same method of the homogeneous one.

© 2016 Production and hosting by Elsevier B.V. on behalf of Cairo University. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
bands between the neighbouring cells to keep the total system bandwidth.

This paper considers homogeneous and heterogeneous networks. The homogeneous network is the network that consists of one type of base stations. For example in LTE or LTE-Advanced, the homogeneous network is the network that consists of macrocells only [1,6]. The heterogeneous network is defined according to the 3GPP standard for LTE advanced as the network of base stations of different transmission powers [8,9].

The objective of this paper was to derive general analytical expressions for SINR in homogeneous and in heterogeneous networks. The homogeneous SINR expression takes into account all assumptions with respect to the frequency reuse method, the network parameters, the network layout (number of tiers) and the user location (centre or edge). The derived expressions are applied to different scenarios with different assumptions [4–7]. Comparison with previous research works shows that the proposed expressions are more generic and all cases can be considered as special cases.

The heterogeneous SINR expression takes into account the two types of interference; the co-tier interference and the cross-tier interference [10–12]. The co-tier interference is the interference between the cells of the same type and the cross-tier interference is the interference between the cells of different types. Different scenarios were examined and the expression was applied for all of them.

The paper is organized as follows: the derived SINR expressions for the homogenous and heterogeneous networks are described in ‘Methodology’. The different scenarios where the derived expressions have been validated with the contribution and the justification are described in ‘Results and discussion’. Finally the paper is concluded in ‘Conclusion’.

Methodology

In this paper, the problem of interference is discussed in terms of finding general expressions of SINR for the different interference management techniques. Firstly, the expression for homogeneous network was derived to be used for the different interference management techniques such as FFR, SFR and also for the techniques that use different centre frequencies with overlapping frequency band such as NHFR. Secondly, the expression was extended to generate another expression for the interference management techniques in heterogeneous networks.

General analytical expression of SINR for homogenous network

As mentioned before, the interference in homogeneous networks comes from the neighbouring macrocells that use the same frequency. The SINR for a desired user $i$ depends on the total interference from all neighbouring macrocells ($\sum_m I_m$). This interference can be divided into different parts if the cell is divided into inner and outer regions or if the neighbouring cells use different centre frequencies. The general SINR expression for homogenous networks is as follows:

$$\text{SINR}_i = \frac{P_{r,i}}{\sum_{m\neq i} I_m + N_0} = \frac{P_{r,i}}{\delta (I_m + I_{\text{out}}) + \delta I + N_0}$$

where $P_{r,i}$ represents the received power of the desired user, $I_m$ represents the total interference from the neighbouring macrocell and $M$ is the number of interfering macrocells. $I_{\text{out}}$ represents the total interference coming from the inner region due to using the same centre frequency and the same frequency band. $I_{\text{out}}$ represents the total interference coming from the outer region due to using the same centre frequency and the same frequency band. $I$ represents a different quantity which is the total interference from the neighbouring macrocells that use different centre frequencies with overlapping frequency band between the desired cell and the neighbouring ones. $\delta$ is the overlapping parameter and $N_0$ represents the thermal noise power. Hence by identifying these terms, they are as follows:

$$P_{r,i} = P_t h_i G_d d_i^{-n}$$

$$I_m = \sum_{j \neq i} P_t h_j G_j d_j^{-n}$$

$$I_{\text{out}} = \sum_{j \neq i} P_t h_j G_j d_j^{-n}$$

$$I = \sum_{m=1}^M \gamma_m P_t h_m G_m d_m^{-n}$$

where all the above parameters are defined in Table 1 and according to the type of frequency reuse method and the location of the desired user (edge/centre), the SINR expression can take different forms as shown in Table 2.

In the NHFR method [3], the variable $\delta$ equals one and the variable $\gamma_m$ was used to compute the amount of interference due to this overlapping; it can take different values depending on the percentage of overlapping band from the total bandwidth. According to 3GPP TR 36.942 [13], the total transmitted power is equally distributed over the number of subcarriers. So, it can be assumed that, if the overlapping band between the desired and the neighbouring cell represents 50% of the total system bandwidth (the neighbouring cell uses 50% of subcarriers of the desired cell), the neighbouring cell interferes with 50% of its power ($\gamma_m = 50\%$) on the desired cell. Therefore, $\gamma_m$ equals 100% for the cells that use the same centre frequency and are not divided into inner and outer regions.

In some special cases, the noise has negligible effect on the performance and can be ignored and the interference becomes the dominant factor. In these cases, ICI can be measured as a function of the signal to interference ratio (SIR) as follows:

$$\text{SIR}_i = \frac{P_{r,i}}{\delta (I_{\text{inner}} + I_{\text{outer}}) + \delta I}$$

In other cases, the macrocell is divided into sectors to increase the spectral efficiency; in this case, a new summation is added to the SINR expression as follows:

$$\text{SINR}_i = \frac{P_{r,i}}{\sum_m \sum_{s=1}^S \gamma_m P_{s,i} + N_0}$$

where $S$ is the number of sectors inside the macrocell and $I_{m,s}$ is the interference from sector $s$ in macrocell $m$.  


As mentioned before, 3GPP developed LTE-Advanced which is the 4G technology to improve the network and set the requirements of the International Mobile Telecommunication-Union (IMT-U). The most significant improvement in the 4G technology is the ability of implementing heterogeneous networks to improve the spectral efficiency per unit area [9]. As mentioned above, the heterogeneous network is the network of base stations of different transmission powers. In other words, the heterogeneous network is the network that consists of macro-cells with low power nodes such as pico and/or femto cells as shown in Fig. 4 [10,11]. The main difference between the macro-cells and the low power nodes is the amount of transmission power of the base station (enodeB). Deploying these low power nodes with the macro-cells became more essential due to different reasons. Firstly, it can offload some traffic from the macro-cells and the neighbouring low power nodes. Therefore, the paper introduces a general SINR expression for the heterogeneous network including the co-tier and the cross-tier interference as follows:

\[
\text{SINR}_i = \frac{P_{t,i}G_i}{\sum_{m=1}^{M} P_{t,m}G_m + \mu \sum_{m=1}^{M} \sum_{l=1}^{P} \sum_{z=1}^{Z} P_{r,z,m}N_0 + \sum_{m=1}^{M} I_{in,m} + \sum_{m=1}^{M} I_{out,m}}
\]  

(8)

where

\[
P_{r,z,m} = P_{t,z}h_{z,m}G_zd_{z,m}^{-\eta}
\]

(9)

The expression in (8) is the generic expression for the heterogeneous network using the parameters in Table 3. The first term in the denominator is the same as the first term of the denominator of (1); it represents the co-tier interference or the ICI between the macro-cells. But, the second term belongs to heterogeneous networks only; it represents the interference due to the low power nodes (cross-tier interference). The above equation is the general one and it includes the homogeneous network by using the new parameter \(\mu\). It equals zero in case of homogeneous networks, in which there are no low power nodes. It equals one in case of heterogeneous network where the interference comes from the neighbouring macro-cells and the neighbouring low power nodes. Therefore,
the expression in (8) can be considered as a general one for both homogenous and heterogeneous networks and the homogenous expression in (1) is a special case.

According to the different assumptions, the network type and the type of interference, the SINR takes different forms and the expressions in Table 4 include these forms. The expressions in Table 4 include the co-tier and cross-tier interference because they contain the interference from the macrocells and from the low power nodes. For example, if the desired user is a macro user, the interference from the macrocells is co-tier interference and from the low power nodes is cross-tier interference. In contrast, if the user is a pico or femto user, the interference from macrocells is cross-tier interference and from the same pico or femtocells is co-tier interference. Also, the first expression in the table is the expression in case of the homogeneous network; this guarantees that the homogenous expression is a special case from the heterogeneous one.

### Results and discussion

The derived expressions of SINR for the homogenous and heterogeneous networks were examined to guarantee that they are valid for all interference mitigation methods. This validation was carried out by applying the expressions for the different methods in the literature.

#### Homogeneous SINR expression validation

The proposed generic expression in (1) was validated by comparing it with previously developed ones [4–7]. A network of two tiers (7 cells in the first tier and 12 cells in the second tier) was used as shown in Fig. 5 [4]. The transmitted power equals $P_{T,c}$ or $P_{T,f}$ depending on the region (inner or outer) and the antenna gain was ignored [4]. The paper used the FFR and SFR methods; therefore, the parameter $\delta$ equals zero. By keeping the notations of the proposed expression in (1), using the expression of FFR for an edge user in Table 2 and using the layout in Fig. 5, the SINR of an edge user is as follows:

$$\text{SINR}_{\text{FFR,edge}} = \frac{P_{T,c} h_{c,m} d_{c,m}^{\beta}}{\sum_{k=1}^{M_{\text{Im}}} P_{T,k} h_{k,m} d_{k,m}^{\beta} + N_0} = \frac{(\frac{P_{T,c}}{N_0})^{\beta}}{6 \left(\frac{n}{N_0}\right)^{\beta} + \frac{N_0}{\frac{N_0^2}{P_{T,c}^2}}}$$ (10)

where $\beta$ equals $P_{T,c}/P_{T,f}$.

The above equation is the same equation for an edge user using FFR and reuse factor 3 [4], where $h_3$ and $D_3$ denote the second tier, but with different notations. Also, by applying the same assumptions to the expression in (1) and using the expression of FFR for a centre user in Table 2 with $N_0$ equals 18 (6 in the first tier and 12 in the second tier as shown in Fig. 5), the SINR expression becomes the same expression for a centre user using FFR [4] as follows:

$$\text{SINR}_{\text{FFR,center}} = \frac{P_{T,c} h_{c,m} d_{c,m}^{\beta}}{\sum_{k=1}^{M_{\text{Im}}} P_{T,k} h_{k,m} d_{k,m}^{\beta} + N_0}$$

$$= \frac{(\frac{P_{T,c}}{N_0})^{\beta}}{6 \left(\frac{n}{N_0}\right)^{\beta} + 12 \left(\frac{n}{N_0}\right)^{\beta} + \frac{N_0}{\frac{N_0^2}{P_{T,c}^2}}}$$ (11)

The noise was ignored and the performance was measured as a function of SIR with the following assumptions [5,6]. Two tiers network (19 cells), the transmitted powers of the inner and outer regions were the same in case of FFR and different in case of SFR, the fading channels had unity gain and the antenna gain was ignored. These two papers used FFR and SFR; therefore, the parameter $\delta$ equals zero and the expression

---

**Table 3** Parameters of SINR expression in heterogeneous network.

| Parameter | Definition |
|-----------|------------|
| $L_p$     | Number of different types of low power nodes, it equals one in case of existing femto- or picocells only and equals two in case of existing femto and picocells together |
| $Z$       | Number of low power nodes |
| $P_{t,c}$ | Transmit power of low power node. It equals $P_{t,c}$ for picocell and $P_{t,f}$ for femtocell |
| $h_{z,m}$ | The fading channel gain between the desired macrocell and the low power node in another macrocell, $m$, it equals $h_{p,m}$ for picocell and $h_{f,m}$ for femtocell |
| $G_z$     | The antenna gain of low power node |
| $d_{p,m}$ | The distance between the desired cell and the low power node in macrocell, $m$ |

**Table 4** Different forms of SINR expression in heterogeneous network.

| The network type | SINR expression |
|------------------|-----------------|
| Homogenous       | $\frac{P_{T,c} h_{c,m} d_{c,m}^{\beta}}{\sum_{k=1}^{M_{\text{Im}}} P_{T,k} h_{k,m} d_{k,m}^{\beta} + N_0}$ |
| HetNet with pico and femto cells | $\frac{P_{T,c} h_{c,m} d_{c,m}^{\beta}}{\sum_{k=1}^{M_{\text{Im}}} P_{T,k} h_{k,m} d_{k,m}^{\beta} + \sum_{j=1}^{M_{\text{Im}}} P_{T,j} h_{j,m} d_{j,m}^{\beta} + N_0}$ |
| HetNet with pico and femto cells | $\frac{P_{T,c} h_{c,m} d_{c,m}^{\beta}}{\sum_{k=1}^{M_{\text{Im}}} P_{T,k} h_{k,m} d_{k,m}^{\beta} + \sum_{j=1}^{M_{\text{Im}}} P_{T,j} h_{j,m} d_{j,m}^{\beta} + \sum_{l=1}^{M_{\text{Im}}} P_{T,l} h_{l,m} d_{l,m}^{\beta} + N_0}$ |

**Fig. 5** Example of two tiers network [4].
in (1) with the special cases in Table 2 was used to calculate the SIR. By keeping these assumptions and using the notations in (1), the SIR of an edge user using SFR and FFR (reuse factor 3) is as follows:

$$\text{SIR}_{\text{SFR, edge, i}} = \frac{d_i^{-\alpha}}{\sum_{j \neq i} P_{r,j} d_j^{-\alpha} + \sum_{k=1}^{M} P_{r,k} d_k^{-\alpha}}$$

(12)

$$\text{SIR}_{\text{FFR, edge, i}} = \frac{d_i^{-\alpha}}{\sum_{j \neq i} d_j^{-\alpha}}$$

(13)

The above two equations are the same equations for an edge user using SFR and FFR [5,6], but with a different notation.

Also, the proposed expression was examined by comparing it with another expression, where the transmitted powers of the inner and outer regions were equal for FFR and different for SFR [7]. The antenna gain was ignored and the fading channel gain was unity. By keeping the notations in (1) and using the expression of SFR for edge user in Table 2, the SINR equation becomes the same equation for an edge user using SFR [7], but with different notations as follows:

$$\text{SINR}_{\text{SFR, edge, i}} = \frac{P_{r,i} d_i^{-\alpha}}{\sum_{j \neq i} P_{r,j} d_j^{-\alpha} + \sum_{k=1}^{M} P_{r,k} d_k^{-\alpha} + N_0}$$

(14)

Finally, a survey of the different interference avoidance techniques was introduced [14], and the research discussed the different frequency reuse methods: the conventional frequency reuse such as Reuse-1 and Reuse-3 and the fractional frequency reuse such as SFR and partial frequency reuse (PFR). In this paper, the proposed SINR expression was applied for all these methods in order to guarantee its validation for the various techniques. In Reuse-1, the total bandwidth is reused in all neighbouring cells in order to increase the system capacity and therefore, the interference on any user in any cell comes from all neighbouring cells [14]. By applying these assumptions to the general formula in (1), the SINR expression is as follows:

$$\text{SINR}_i = \frac{P_{r,i}}{\sum_{m=1}^{M} I_m + N_0}$$

(15)

In Reuse-3 method, the total bandwidth is divided into three parts as shown in Fig. 6 [14]. This prevents the interference among the cells in the same tier and the interference comes only from other different tiers. Therefore, the SINR expression for any user in the first tier using two tiers network as shown in Fig. 6 is as follows:

$$\text{SINR}_{i} = \frac{P_{r,j}}{\sum_{m=1}^{M} I_m + N_0}$$

(16)

In the SFR method [14], it is the same as the SFR that was previously discussed in this paper and is shown in Fig. 2. The SINR expression was applied for SFR and is shown in Table 2 for both centre and edge users. In the PFR method [14], it is the same as FFR that is shown in Fig. 1 and also the SINR expression was shown in Table 2 for both centre and edge users.

**Heterogeneous SINR expression validation**

The correctness of the SINR expression for the heterogeneous network was investigated by comparison with different developed ones in the literature [10–12]. The heterogeneous network with the following assumptions was considered [10]. Each macrocell was divided into three sectors and each sector was provided with a number of picocells; the antenna gain for all eNodeBs (eNBs) was ignored and also the distances. By keeping these assumptions and by substituting in the second expression in Table 4, the SINR of the macro user is as follows:

$$\text{SINR}_{\text{macro, i}} = \frac{P_{r,i}}{\sum_{m=1}^{M} I_m + \sum_{z=1}^{Z} P_{r,z} h_{z} d_{z}^{-\alpha} + N_0}$$

$$= \frac{P_{r,i}}{\sum_{m=1}^{M} P_{m} h_{m} + \sum_{z=1}^{Z} P_{r,z} h_{z} + N_0}$$

(17)

The above equation is the same equation for a macro user but with different notations [10]. Also by applying the same assumptions to get the SINR of a pico user, it was found that the resulting equation is the same equation for a pico user [10], but with different notations as follows:

$$\text{SINR}_{\text{pico, i}} = \frac{P_{r,i}}{\sum_{m=1}^{M} I_m + \sum_{z=1}^{Z} P_{r,z} h_{z} d_{z}^{-\alpha} + N_0}$$

$$= \frac{P_{r,i}}{\sum_{m=1}^{M} P_{m} h_{m} + \sum_{z=1}^{Z} P_{r,z} h_{z} + N_0}$$

(18)

A heterogeneous network of macro and femto cells was investigated with the following assumptions [11]. The system model consists of seven macrocells, each one is divided into three sectors and a number of femtocells are distributed randomly inside each cell. By applying these assumptions and substituting in the second expression in Table 4, the SINR of a macro and femto user in terms of received power is as follows:

$$\text{SINR}_{\text{macro, i}} = \frac{P_{r,i}}{\sum_{m=1}^{M} I_m + \sum_{z=1}^{Z} P_{r,z} + N_0}$$

$$= \frac{P_{r,i}}{\sum_{m=1}^{M} P_{m} h_{m} + \sum_{z=1}^{Z} P_{r,z} + N_0}$$

(19)

$$\text{SINR}_{\text{femto, i}} = \frac{P_{r,i}}{\sum_{m=1}^{M} I_m + \sum_{z=1}^{Z} P_{r,z} + N_0}$$

(20)

The above two equations are the same equations for macro and femto users [11], but with different notations.

Finally, one macrocell with N femtocells distributed randomly inside it to construct a heterogeneous network was used [12]. The antenna gain and the distances were ignored. By keeping these assumptions and substituting in the second expression in Table 4, the SINR expressions for macro and femto user are as follows:
The above two equations are the same equations for macro and femto users [12], but with different notations. All the previous results show that the proposed analytical SINR expressions that were derived in this paper for the homogenous and heterogeneous networks are general expressions. These expressions are valid for any network topology with any parameters and are valid for the different interference management techniques developed in the literature and for any user location. Also, the results show that the heterogeneous expression is the general one and all other cases, including the homogenous case, are special cases from it. This is because the expression contains all parameters that can be used in homogeneous or heterogeneous networks and it does not ignore any parameter whether it is effective or not. Furthermore, the expressions can be used to find generic expressions for the probability of coverage and capacity and to investigate the performance of different fading environments.

Conclusions

In this paper, generic analytical expressions for the downlink SINR in both homogeneous and heterogeneous networks were derived. In homogeneous networks, the expression was investigated for different scenarios including different frequency reuse techniques, different network parameters and different user locations (edge/centre). In heterogeneous network, the expression was investigated for different types of low power nodes such as pico and femto cells and it was derived for macro and low power node users. Validation of the expressions for both homogeneous and heterogeneous networks was examined by comparing with previously developed expressions and the comparison proved the correctness of both expressions. These expressions are very important and essential to study or to analyse any cellular network that uses OFDMA technique with any network parameters and any network layout.

Conflict of Interest

The authors have declared no conflict of interest.