Performance of Understanding Students’ Construction In The Naming Fraction of The Three Representation

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Abstract. The purpose of this study is to describe students' thinking processes in constructing the names of the fraction in circular, rectangular and number line representations by using APOS Theory (Action, Process, Object, and Scheme). For that purpose, the researcher used qualitative research approach. The subject of this research is the students of class XI in Malang. The process of collecting data begins with giving the questions to the subject to be completed. Then the students are interviewed to reinforce the answers that have given. From the performance of the subject and results of the interview, the data are analyzed using indicators of APOS Theory is action, process, object, and scheme. The results of this study indicate that for the thinking process students with circle representation S1 reached the stage of the scheme, while for S2 and S3 only reached the stage of the process. With the rectangular representation, S1 and S2 reach the stage of the scheme, but different with S3, S3 that reached the stage of the process. Moreover, the thinking process by using the number line representation S1, S2 and S3 can reach the stage of the scheme. To construct the name of the fraction with a circle and rectangular representation the subject is using the whole-part concept and area model of the fraction. As for the representation that uses the number line the subject using the concept of measurement and associates it with the length model in constructing the name of the fraction.

1. Introduction

Fractions are difficult learning throughout the country [1]. Generally, the student could see that the fractions require more understanding from the mistakes made. The knowledge of the fraction obtained from whole numbers [2]. As it knows that students' difficulties in learning fraction have many sources, one of them is often emphasized to see the fractions solely in part-whole concept [2]. Not only part-whole can be used to construct the concept of fractions. Fractions consist of several sub-contruct are interconnected that is whole-part, ratio, measurement, operator, and division [3]. Fraction is a pair of whole numbers that can represent the part-whole are writing by ab where b ≠ 0, a is called the numerator and b is the denominator [4]. Learning fractional concept begins in elementary school education. In the study related to the internal representation of elementary school students aged 8-11 years who have various mathematical abilities, this research is motivated by the circumstances when students in Midland, England have difficulty in constructing the concept of fractions [5]. When students are giving questions about the meaning of fractions, the students tend to answer something
very small, and circles divided into small, or a form with many lines. Moreover, also in junior high school, there is 13% faulty concepts and settlement related fractional operations, which guided discovery learning could minimize the difficulty.

In the school are various of characteristics, both of teachers and students. Each student has a unique personality in the learning process [6]. As well as in learning fractions that are naming fractions, it can start by using concrete objects then abstract objects as an understanding. Accordance with the Bruner Theory which states that learning begins from an inactive stage that is a learning stage of knowledge which the knowledge actively studied using concrete objects or using real situations [7]. Concrete objects or real situations used in naming fractions such as geometry images and the situation in the division of pizza. The use of such objects and situations can regard as a representation. Representation is an internal process that occurs in students' thinking about a problem, used as a tool to find solutions to the problem [8], [9]. The representation is dividing into two the are external and internal representations. The internal representation is the process of thinking related to the ideas or concept of the mathematics that allow the mind of someone to think on that basis [10], [11]. The process takes place in a person's mind so that the representation could not observe directly. While external representation is a manifestation of internal representation, where the result of the embodiment can express used diagrams, illustrations, spoken instructions and statement of written problem statement [11], [12].

In constructing knowledge related to fractional learning, [13] states that individuals construct knowledge can be explained by various theories, including APOS Theory (Action, Process, Object, Scheme). According to the theory of APOS, an action is an activity in the form of physical repetition or mental manipulation based on several algorithms explicitly. An action is a response from the external stimulus received by the subject. So the action is meant as a physical change of the object to obtain another object. When the action is repeated and done in a reflection, then the action is internalization into a process. Which process is an internal construction that carried out the same action without requiring external stimulus? Then the object can be constructed in two ways, the first when an individual reflects the action applied to a particular process and then conscious of the process as a totality, and could be done and construct actually as a transformation so that the individual reconstructs the process as a cognitive object. It can say that the process encapsulated into objects. Second, to construct a cognitive object, an individual reflects on a particular scheme and realizes that the scheme is totality and can act on it. In this case, it, that the individual schematizes the schema into an object. The schema for a particular piece of mathematics is a collection of individuals from actions, processes, objects, and other schemes that the linked in the framework of the individual's mind in the face of a mathematical problem [14].

When the students construct the name of fraction simultaneously, there is a process of thinking. Some researchers have reviewed the importance of thinking in mathematics learning. From the results of the research obtained the characteristics of critical thinking and the thinking of students in solving math problems [15], [16]. The thought is a mental activity performed by students in solving problems and could be seen behavior that appears to be the result of the completion of tasks [17]. Meanwhile, according [18] thinking is a higher cognitive activity and involves lower cognitive processes (e.g., lower perception, memory, and concepts). So thinking is a higher cognitive activity that involves lower cognitive processes as well as concepts to resolve problems that can see from visible behavior and outcomes from the performance of an anyone. In anyone thinking there is a stage or process, where the process of thinking is a process that begins with receiving data, processing and storing it in memory and recalling in memory when needed in subsequent processing [19]. While in the learning, the naming of fractions can use representations of circles, rectangles, and number line, which representation is used to petrify in constructing the name of the fraction itself. Therefore, the purpose of this research is to describe the thinking process of class XI students in constructing the name of the fraction.
2. Research Methods

This research uses qualitative descriptive research approach. This research intended to know genuinely and in detail about the problem or phenomenon researched. Qualitative data that have obtained will be analyzed and then described or elaborated to see the student's thinking process in constructing the name of the fraction. Subjects in this research are three students from class XI, where this subject has a concept or picture related to the naming of fractions that have studied at the previous level. The instrument in this research is a matter related to the fractional concept of naming fraction.

| Table 1. Research Instrument [1] |
|----------------------------------|
| **Questions**                     | **Answers** | **Reason** |
| ![Circle Representation](image)  | ![Rectangle Representation](image) | ![Number Line Representation](image) |

The procedure or method of data collection begins with the provision of research instruments in the form of questions given to 3 students of class XI within 10 minutes. The result of the student's work is used to conduct the interview. The interview used to reinforce or clarify the answer. Then from the test results and interviews, used to describe, and explain the process of thinking students in the contract the name of fractions.

3. Result and Discussion

3.1. Thinking Process with Circle Representation

In this type, the subject can identify the fractions. At the time the subject responds to the problem are given then S1, S2 and S3 have reached action. Where the subject understands the components of the problem, there are fractions and circle representation. Then experiencing the process of understanding formation by using the representation of the circle to solve the problem [19], [20]. In this case, S1 wrote \( \frac{2}{4} \) as the answer, The following is the performance of S1.

![Performance of S1 with Circle Representation](image)

In the formation of the understanding, S1 constructs the name of the fraction by stating that two parts are shaded from 4 parts. Which the process is said as the process of formation opinion as well as the subject has experienced a process, obtained from the performance of interviews researchers with S1. The following quote interviews researchers with S1.

P : “Why \( \frac{2}{4} \)? How did you get it?”

S1 :
While in the form of understanding S2 write down \( \frac{1}{2} \) as the answer. The following is the performance of S2.

![Figure 2. Performance of S2 with Circle Representation](image)

In the process of forming it, S2 replaces the answer of \( \frac{2}{2} \) is \( \frac{1}{2} \) and declare that there is one circle and two parts are crossed out. In this case, S2 formed his opinion related to the naming of fractions as well as has experienced a process. The process can be known during the interview with researchers. The following are quote interviews researchers with S2.

\[
P: \quad \text{"Where you get it?"}\]
\[
S2: \quad \text{"Because two parts are crossed out, and two does not (stay silent for a few minutes, then change the answer to \( \frac{1}{2} \))."} \]

Moreover, in the formation of understanding S3 write \( \frac{4}{2} \) as the answer. In the process of formation, S3 construct the name of the fraction by stating that the circle is broken down into four parts then shaded there is 2. Thus S3 has experienced a process informing of such opinion. The process could be known when researchers interviewed S3. The following are the quoted interview with the researcher with S3.

\[
P: \quad \text{"Where you get it?"}\]
\[
S3: \quad \text{"Course, the piece of circular, is 4 and shaded are 2."} \]

Can be known from the performance of students, S1, S2 and S3 using the concept of the part-whole to construct the name of the fraction. In this case, the subject has experienced as an object which can be seen from the characteristics and stated that at the object is a conceptual understanding [14]. Then S1 has experienced as scheme, cause in this phase S1 organizes stages of Action, Process, Object and other properties [14]. S1 associate it with the model of the fraction that is represented by the area model. Which, as [21] points out, is that fractions can be modeled using the region or area model, length model and also set model. While didn't organize APO stage and other properties, so S2 and S3 have not experienced scheme.

### 3.2. Thinking Process with Rectangular Representation

Similar to the thought process by circle representation, in this type the subject can identify the fractions. When the subject responds to the given problem, it can say that S1, S2, and S3 have reached action. Which subject understands the components of the problem are fractions and rectangular representation. Then the subject has experienced a sense of formation process using rectangular representation to solve the problem [19], [20]. In this case, S1 wrote the answer \( \frac{2}{4} \).

![Figure 3. Performance of S1 with Rectangular Representation](image)
In the formation of the understanding S1 constructs the name of the fraction by stating the whole of rectangles there are four parts, and two are shaded. Which this process is said to be the process of opinion formation and the subject has experienced a process, while the process is the procedural understanding [14]. While in the form of understanding S2 write down $\frac{2}{4}$ as the answer.

In the process of forming the definition, S2 constructs the name of the fraction by stating that there are two boxes crossed out and there are four boxes. In this case, S2 formed his opinion on the naming of the fractions as well as has experienced a process. The process can be more clearly know during the interview process with the researchers. The following quote interviews researchers with a researcher with S2.

P: “How to get $\frac{2}{4}$?”
S2: “There are four boxes, and that crossed out are 2.”

Just as with the circle representation, in the formation of S3 notes write down $\frac{4}{2}$ as the answer. In the process of formation, S3 construct the name of the fraction by stating that the rectangle is broken into four parts then the shaded there are 2. So S3 has experienced a process in this forming opinion. The process can be known when researchers interviewed S3. The following are the interview quote from the researcher with S3.

P: “$\frac{4}{2}$ again? What’s your reason?”
S3: “As in the circle representation, the fraction is 4, and the shaded are 2.”

Similarly, with circle representation, S1, S2 and S3 using the part-whole concept to construct the name of fraction represented by rectangles. In this case, the subject has experienced as an object which it can be seen from the characteristics, where the object is the conceptual understanding [13]. Moreover, then S1 and S2 organize stages of Action, Process, Object and other properties namely scheme [14]. S1 and S2 associate it with the model of fraction represented by area. Based on the report [21] stating that the area can be used to help the concept of the part-whole section. While S3 does not experience forming of the scheme, because it does not organize the APO (Action, Process, Object) and other properties.

3.3. The Thinking Process with Number Line Representation
In this type, the subject can identify the fractions represented by the number line. When the subject responds to the given problem, then it can be said that S1, S2, and S3 have reached action. Which subject understands the components of the problem that is fractions and the representation of the number line. Then the subject has experienced a sense of formation process using number line representation to solve the problem [19], [20]. In the formation of the understanding, S1 constructs the name of the fraction by stating that the distance between 0 and 1 to be divided by x, so it is the midline. To clarify the answer S1 researchers interviewed S1. The following quote interviews researchers with S1.

P: “Are you sure $\frac{1}{2}$? From where?”
S1: “From it, the distance of 0 to 1 is divided by x, that x is the midline.”

In the interview, S1 changed the midline statement with the midpoint. With amended that statement, it could be said that S1 has experienced a process of opinion formation, which is the procedural understanding [14]. While in the form of knowledge S2 write down $\frac{1}{2}$ as the answer. The following is the performance of S2.
In the process of forming the definition, S2 constructs the name of the fraction by stating that x in the middle of 0 and 1 and the number is $\frac{1}{2}$. In this case, S2 formed his opinion on the naming of fractions as well as has experienced a process. At the time of the formation of understanding, S3 write $\frac{1}{2}$ as an answer. In the process of formation, S3 constructs the name of the fraction by stating that x in the middle of 0 and 1. So that S3 has experienced a process of forming the opinion. The process can be known when researchers interviewed S3. The following are the quoted interview with the researcher with S3.

P : “Why $\frac{1}{2}$? what’s the reason?”
S3 : “Because it is in the middle 0 and 1.”

In the representation of the number lines, S1, S2, and S3 use the concept of measurement to construct the name of fraction represented by the number line. In this case, the subject experiences an object which can be seen from its characteristics. The object is a conceptual understanding [22]. Then S1, S2, and S3 organize stages of Action, Process, Object and other properties namely scheme [22]. The scheme can be seen that the subject relates it to the fractional model represented by the length. Which, as [21] points out, is that fractions can be modeled using area, length and also set model. So, in this case, S1, S2, and S3 experience as “scheme,” by organizing the APO (Action, Process, Object) with other properties.

4. Conclusion

Based on the theory of APOS that has described in the results of research and discussion, it can be concluding that the thinking process of class XI students in constructing the name of the fraction with representations reviewed using APOS Theory as follows. In the circle representation, the thinking process S1 reaches as forming scheme, whereas S2 and S3 only reach as the process. Which S1 constructs the fractional name using the part-whole concept and relates it to the fractional model of the area. In the rectangular representation the thinking process S1 and S2 reach forming the scheme, but unlike S3 which only achieves a process. Similarly, in the case of a circle, in this case, S1 and S2 construct the name of the fraction using the concept of part-whole and associate it with the fractional model of the area. In the number line representation the process of thinking S1, S2 and S3 reach forming the scheme. In the representation of the number line, the subject constructs the name of fraction using the concept of measurement and relates it to the length model of the fraction.

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