Description of transient processes in multiplying medium by the physical birth-and-death model

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Abstract. The work presents an approach to describe neutron and neutron-breeding medium interaction process based on the physical birth-and-death model. This model is an extended form of the basic birth-and-death model and operates with breeding medium parameters. Analytical solutions and physical interpretation of the processes within reactor can be obtained by such method. Recently, thermal reactor theory with an internal neutron source in the form of delayed neutrons on the basis of birth-and-death model within linear growth approximation was proposed. Particles radioactive decay properties and equation analysis for describing breeding medium parameters using point thermal reactor framework were also studied within the model using Poisson and binomial distribution formalism. Experimental data of three nuclear subcritical facilities were used in this work, KUCA (Japan), MASURCA (France) and GUINEVERE (Belgium). Reactivity and the average number of particles in the system were estimated in this work within physical model framework for three subcritical systems.

1. Introduction
Various methods are used to describe neutron and neutron-breeding medium interaction process in thermal nuclear reactor: point kinetics equations [1], statistical methods based on the branching processes theory [2, 3] and etc. Processes physical interpretation and analytical solutions for transient analysis of the reactor core can be obtained by such methods.

The birth-and-death model is a special case of the branching processes theory. It was obtained through solving direct Kolmogorov equations [4, 5]. The model is purely mathematical and uses the concept of the birth $\lambda$ and death $\mu$ intensities, but is not directly related to the breeding medium parameters. The thermal reactor theory with an internal neutron source in the form of precursor nuclei was proposed in early work on this topic. It was based on the birth-and-death model within linear growth approximation [6, 7]. The precursor radioactive decay properties [8] and the equation analysis for describing breeding medium parameters of the point thermal reactor example [9] were also studied as part of model.

The processes inside nuclear reactor core at constant power can be represented in the form of two cumulative processes: $a$) $1/\rho$ prompt neutrons are produced by one initial neutron when entering the subcritical breeding medium; $b$) as a result, the required number of precursor nuclei are formed before emission of another unique neutron. This neutron supports a nuclear reaction. Such approach is an augmented birth-and-death model and called the “physical birth-and-death model” (FMRG).
Current work is aimed at deriving expressions for the main breeding medium characteristics in the framework of the FMRG model. Partly, the reactivity $\rho$ and the average number of particles in the system $M(t)$ at the time $t$ are considered. These parameters were estimated for three subcritical nuclear facilities: KUCA (Japan) [10], MASURCA (France) [11] and GUINEVERE (Belgium) [12].

2. The physical birth-and-death model
The mathematical apparatus of the FMRG model is based on birth and death model formalism. The birth $\lambda$ and death $\mu$ intensities are expressed for breeding medium parameters in terms of physical properties of fission fuel: $\lambda_f$, $\lambda_c$ and $\nu$, forming the basis for the FMRG mathematical apparatus, where $\lambda_f$ is the neutron induced nuclear fission probability, $\lambda_c$ is neutron death probability (absorption by the medium without fission and leakage from the reactor core); $\nu$ is mathematical expectation of the secondary neutrons number in one fission act.

Connection of birth and death intensities with breeding medium parameters $\nu$, $\lambda_f$, $\lambda_c$ can be found by considering a homogeneous process ($\lambda(t) = \lambda = \text{const}$ and $\mu(t) = \mu = \text{const}$).

The effective neutron multiplication factor in the FMRG formalism is the mathematical expectation of single neutron direct descendants number $K = \lambda_{\nu} / (\lambda_f + \lambda_c)$ [2]. The average neutron lifetime is characterized by $\tau_n = 1/\mu$ in the birth-and-death model within the linear growth approximation. Then the average number of neutrons at the time is

$$M(t) = \exp(\lambda - \mu)t = \exp((\lambda / \mu - 1)\mu t)$$

where $\mu t$ parameter is the dimensionless time in units of one neutron generation lifetime. Then the theoretical effective neutron multiplication factor $K$ is the average number of neutrons formed in the system per unit time. From (1) expression for $K$ is derived as

$$K = \exp(\lambda / \mu - 1) \approx \lambda / \mu$$

Then, by definition, $\lambda / \mu - 1$ is close to 1, hence, the exponent will be limited by two terms in the expansion (2).

$$\lambda_{\nu} / (\lambda_f + \lambda_c) \approx \lambda / \mu$$

Therefore, $\mu = \lambda_c + \lambda_f$ and $\lambda = \nu \lambda_f$. The criticality factor in the expression for the average number of particles $M(t) = \exp(\alpha t) = \exp((\lambda - \mu)t)$ is

$$\alpha = \lambda - \mu = \nu \lambda_f - (\lambda_f + \lambda_c)$$

Reactivity is defined as

$$\rho = (K - 1) / K = [e^{(\lambda / \mu - 1)} - 1] / e^{(\lambda / \mu - 1)} = 1 - e^{-(\lambda / \mu - 1)} \approx \lambda / \mu - 1$$

The expression for reactivity through breeding medium parameters from (3) and (5) take the following form

$$\rho = \lambda / \mu - 1 = \lambda_f \nu / (\lambda_f + \lambda_c) - 1$$
If the time value $t$ in (1) corresponds to equation $\ln \left( \frac{M(t)}{M_0} \right) = -1$, the reactivity expression can be obtained

$$\rho \approx \frac{\lambda}{\mu} - 1 = -\tau_m / t_1$$  \hspace{1cm} (7)

The parameter $M_0$ is the number of particles in the system at the initial time, $t_1$ is the time period for which number of particles in the system decrease by a factor of $e^1$.

The equation for particles average number in the system $M(t)$ from (1) and (4) can be written as

$$M(t) = \exp \left\{ \left[ \nu \lambda_f / (\lambda_f + \lambda_c) - 1 \right] t (\lambda_f + \lambda_c) \right\}$$ \hspace{1cm} (8)

Formula (8) is an expression of the particles number in a point reactor in general form $M(t) = \exp (\rho t / \tau)$, where $\tau$ is prompt neutrons lifetime $\tau_m = 1/(\lambda_f + \lambda_c)$ and the reactivity is defined by (6). Expression (8) for $\rho$ allows to consider both prompt and delayed neutrons. They have the same source of origin: fuel nucleus decay by thermal neutrons. This means that delayed neutrons contribution is the addition of the effective delayed neutron fraction to the value of $\nu$. Expression for $M(t)$ is

$$M(t) = \exp \left\{ \left[ \lambda_f (\nu + \beta) / (\lambda_f + \lambda_c) - 1 \right] t / \tau_m \right\} \hspace{1cm} (9)$$

Combining (9) and (6) gives a complete equation for neutron average number in the system

$$M(t) = \exp \left\{ \left[ \rho + \lambda_f \beta / (\lambda_f + \lambda_c) \right] t / \tau_m \right\} \hspace{1cm} (10)$$

3. Kinetic parameters determination for subcritical nuclear systems

Experimental data analysis for three subcritical nuclear facilities, KUCA [10], MASURCA [11] and GUINEVERE [12], was performed based on (7) and (10). The reactivity value ($\rho_{th}$) was obtained using formula (7). The average number of particles in the system at the time (10) was estimated using experimental reactivity values ($\rho_{exp}$) and the neutron generation time ($\Lambda$).

The Kyoto University Critical Assembly (KUCA) was developed by Kyoto University in Japan. Experimental data with highly enriched uranium (93%) were used to check formulas (7) and (10) in this work (figure 1, a). The Fixed-Field Alternating Gradient (FFAG) accelerator was used as an external source of neutrons. Spallation neutrons were obtained by 100 MeV protons with a Pb-Bi target. The prompt neutrons generation time is $\Lambda = 30.5 \mu$s, the effective delayed neutrons fraction is $\beta_{eff} = 0.00804$ [10].

The MASURCA is a subcritical assembly at the research center in Cadarache in France. During MUSE-4 experimental program central zone of facility was filled with MOX (UO$_2$, PuO$_2$, enriched $\approx 25\%$) and Na rodlets. The GENEPi deuteron accelerator and a tritium target were used as an external neutron source (figure 2, a). The experimental effective neutron multiplication factor is $k_{eff} = 0.96$. The prompt neutrons generation time is $\Lambda = 0.58 \mu$s. The effective delayed neutrons fraction coefficient is $\beta_{eff} = 0.00335$ [11].

GUINEVERE (VENUS-F) is a zero power ADS system. It consists of a VENUS reactor (Mole, Belgium) and a GENEPi-3C deuteron accelerator (figure 3, a). Enriched metallic uranium of 30% ($^{235}$U) was used to obtain experimental data. A deuteron accelerator and a tritium target were used as an external source of neutrons. The prompt neutrons generation time is $\Lambda = 0.5 \mu$s, the effective fraction of delayed neutrons is $\beta_{eff} = 0.00722$ [12].
The average number of particles $M(t)$ in the system evaluating result is shown in Figures 1 (b), 2 (b) and 3 (b).

The reactivity values for three subcritical systems are reported in Table 1. The experimental reactivity values ($\rho_{exp}$) for the KUCA assembly were obtained by the area ratio method [10], MASURCA – by the source multiplication method (MSM) [11], GUINEVERE – by the pulsed neutron source (PNS) method [12].

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**Figure 1.** KUCA configuration (a), where Det1, Det2, Det3 are detectors at different zones of the nuclear system, and logarithm time spectra for two KUCA detectors (b) [10].

**Figure 2.** MASURCA configurations (a), where A, A’, B are detectors locations, and logarithm time spectra for B- detector of MASURCA (b) [11].
Figure 3. VENUS-F configuration (a), where D1...D10 are detectors, and logarithm time spectra for three VENUS-F detectors (b) [12].

Table 1. Theoretical values of $\rho$ compared to experimental values for two subcritical systems.

|       | $k_{eff}$ | $\tau_{m}$, $\mu$s | $t_1$, $\mu$s | $\rho_{exp}$ | $\rho_{th}$ | $\Delta \rho$ |
|-------|-----------|---------------------|--------------|--------------|-------------|-------------|
| KUCA  | Det1      | 0.974               | 29.7         | 1320         | -0.027      | -0.022      | 0.005       |
|       | Det2      | 0.968               | 29.5         | 1300         | -0.033      | -0.023      | 0.010       |
|       | Det3      | 0.972               | 29.6         | 1200         | -0.029      | -0.025      | 0.004       |
| MASURCA| ---       | 0.961               | 0.57         | 49           | -0.005      | -0.012      | 0.007       |
| GUINEVERE| D2       | 0.967               | 0.48         | 12           | -0.034      | -0.041      | 0.007       |
|       | D4        | 0.961               | 0.48         | 12           | -0.041      | -0.039      | 0.002       |
|       | D9        | 0.964               | 0.48         | 8            | -0.048      | -0.057      | 0.009       |

4. Conclusion
This work presents an analytical approach to the description of neutrons and multiplying medium interaction processes based on the physical birth-and-death model of particles. Expressions for reactivity ($\rho$) and the average number of particles at the time ($M(t)$) are given within physical model framework. These parameters were estimated for three nuclear subcritical installations: KUCA, MASURCA and GUINEVERE.

Reactivity assessment for three assemblies shows that the physical birth-and-death model makes it possible to correctly estimate (within 1% accuracy) the reactivity value. Comparison of particles average number calculated values with experimental data for three assemblies KUCA, MASURCA, and GUINEVERE confirms that the physical model can be used to estimate main neutron-physical characteristics of alike subcritical systems for the time range from 0 to 60 $\mu$s.

The physical birth-and-death model of particles requires further mathematical apparatus development for describing neutron and neutron-breeding medium interaction processes.

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