Supersymmetric calculations with component fields
in differential renormalization

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Two applications of the method of differential renormalization to supersymmetric
gauge theories are reviewed. The photon propagator in supersymmetric QED is
renormalized at one loop and the first supergravity contributions to the anomalous
magnetic moment of a charged lepton are obtained.

1 Introduction

Quantum Field Theories are in general plagued with infinities, which are usu-
ally regularized with some sort of cut-off. Then the renormalization program
allows to get rid of the singularities in a consistent way by a redefinition of the
parameters in the Lagrangian. In some methods, like BPHZ, renormalization is
carried out in one single step without intermediate regularization. Whatever
scheme is used, it is desirable that it maintains the symmetries of the theory
under study. Theories with a high degree of symmetry can be very demanding
in this respect. This is the case of gauge and supersymmetric theories. In a
scalar theory with very little symmetry, almost any regulator does the job.

Dimensional regularization is a remarkable method that explicitly pre-
serves gauge invariance. Because of its relative simplicity and the importance
of gauge theories in the description of Nature, this method has become stan-
dard in many computations in Quantum Field Theory. However, dimensional
regularization has difficulties in coping with supersymmetric theories: it can
break supersymmetry (SUSY). Essentially, the reason is that the equality of
Bose and Fermi degrees of freedom only holds for specific values of the space-

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time dimension, which is changed by this regularization. There exists a variant of the method, dimensional reduction \cite{footnote3}, in which the field components remain unchanged, while the loop integrals are performed in a $d$-dimensional space. This method preserves SUSY, at least at lower orders, but at higher ones the situation is more involved, especially in the case of broken SUSY (see Ref. \cite{footnote4} and references therein). Another approach is based on higher derivative regularization supplemented by Pauli-Villars \cite{footnote5}. What is clear in any case is the scarcity of simple methods suitable for supersymmetric theories.

The method of differential renormalization (DR) has appeared recently \cite{footnote6}. It is a method of renormalization without regulators or explicit counterterms which works in coordinate space. DR does not modify the space-time dimension, thus being a candidate for preserving SUSY. The method has been shown to be quite powerful in a three-loop computation for the massless $\lambda\phi^4$ theory. Other applications include lower dimensional theories, Chern-Simons, nonperturbative calculations, etc. \cite{footnote7}. Formal aspects like checks of unitarity \cite{footnote10}, its relation with dimensional regularization \cite{footnote8}, the inclusion of masses \cite{footnote9} and the consistency of the procedure to any order \cite{footnote11} have been also studied. Different versions of DR have been developed in Ref. \cite{footnote12}. The first application to supersymmetric theories was a calculation of the $\beta$-function to three loops in the Wess-Zumino model \cite{footnote13}. In Ref. \cite{footnote14} DR was applied to pure supersymmetric gauge theories, and the $\beta$-function was obtained to two loops and one loop, in the abelian and non-abelian case, respectively. These calculations were performed with superspace techniques. Very recently this method has also been employed in non-perturbative calculations in supersymmetric gauge theories \cite{footnote15}. In the superspace formalism, SUSY is manifestly preserved; however the situation is more involved in the physically interesting case of broken SUSY, where it is usually preferred to work with component fields (see, however, Ref. \cite{footnote16}). Here we review the two existing applications of DR to supersymmetric gauge theories in the component approach: the vacuum polarization in supersymmetric QED (SQED) \cite{footnote17} and the calculation of the anomalous magnetic moment of a charged lepton, $(g - 2)_l$, in unbroken supergravity \cite{footnote18}. These two examples provide non-trivial tests of the potential of DR to preserve (abelian) gauge invariance and SUSY. In standard DR, the fulfilment of the corresponding Ward identities is accomplished adjusting at the end the different scales that appear in the renormalization procedure. In Ref. \cite{footnote17} we described a procedure to constrain the scales appearing in DR in such a way that Ward identities are automatically satisfied. We have verified the consistency of this approach in different one-loop examples \cite{footnote17}. Here we shall use this constrained procedure of DR.

In what follows we briefly review the method of DR and its constrained
form. Then we present the two one-loop supersymmetric calculations with component fields. The first example, the renormalization of the photon propagator in SQED, illustrates the method. Afterwards the more involved case of the evaluation of \((g-2)\), in unbroken supergravity is revised, and the results are discussed. The last Section is devoted to conclusions.

2 Differential renormalization and its constrained version

In coordinate space the amplitudes are finite as long as point coordinates are kept apart; singularities only arise when points coincide. These singularities, when too severe, give rise to divergences in integrals on internal points or in Fourier transforms. In other words, the bare expressions are in general ill-defined distributions. The idea of differential renormalization is to rewrite these expressions as derivatives of less singular ones. The derivatives are understood in the sense of distribution theory, i.e., acting formally by parts. The amplitudes written in this way are identical to the bare ones for separate points but behave well enough at coincident points. One has then the renormalized amplitudes in coordinate space. In this process some dimensionful integration constants appear that will play the role of renormalization scales. An illustrative example is given in massless \(\lambda \phi^4\) theory, where the four point function at one loop has a factor \(1/x^4\) (\(x\) being the difference of any two external points). The renormalization is given by the substitution

\[
\frac{1}{x^4} \rightarrow \frac{1}{4} \frac{\log(M^2x^2)}{x^2}.
\]

The scale \(M\) is the renormalization scale alluded to above. Notice that the r.h.s. in Eq. (1) has a well defined Fourier transform. We will work in Euclidean space, where the handling of the expressions is simpler. Analogous equations can be obtained for other singularities (see Ref. [6] for details) and the program can be in general carried out for any theory at any number of loops [11].

In principle, the scales introduced for different diagrams are independent. These scales can be chosen in different ways (as long as the same scale appears in identical subgraphs), each corresponding to a choice of the renormalization scheme. The easiest one is to take all scales to be equal. This choice is appropriate for a scalar \(\lambda \phi^4\) theory, for instance. However, in theories with more symmetry much care must be taken when fixing the constants. In gauge theories for example, there are restrictions to be fulfilled which are dictated by the Ward identities. The usual way of proceeding is to renormalize the amplitudes and then relate the constants imposing the Ward identities. Although this is a possible procedure, one would prefer that gauge symmetry were automatically preserved, as occurs in dimensional regularization.
The constrained version of DR fixes the arbitrary constants from the beginning (see Ref. 17 for details). The idea is to restrict oneself to a set of consistent rules to manipulate the singular expressions. Besides the usual DR rules, differential equalities and formal integration by parts, two other conditions appear to be enough to conveniently fix the local terms at one loop. First, the factorization of delta-functions in the renormalization procedure, i.e.,

\[ F(x, x_1, \ldots, x_n) \delta(x - y)]^R = [F(x, x_1, \ldots, x_n)]^R \delta(x - y), \tag{2} \]

where \( F \) is an arbitrary function and \( R \) stands for “renormalized”. Second, one demands that the propagator equation \((\Box - m^2) \Delta_m(x) = -\delta(x)\) has a general validity when embedded in any amplitude. This is to say that

\[ F(x, x_1, \ldots, x_n)(\Box - m^2) \Delta_m(x) = F(x, x_1, \ldots, x_n)[-\delta(x)] \tag{3} \]

holds for \( F \) arbitrary, and not only for well-behaved enough functions. Above, \( \Delta_m(x) = \frac{1}{4\pi^2} m K_1(mx) \) is the Feynman propagator for a particle of mass \( m \), with \( K_1 \) a modified Bessel function. These rules allow to renormalize a set of basic functions which are used to expand the amplitudes. The basic functions for tadpole, bubble and triangular diagrams with massless propagators are denoted by

\[ A = \Delta(x) \delta(x), \tag{4} \]
\[ B[O] = \Delta(x) O^x \Delta(x), \tag{5} \]
\[ T[O] = \Delta(x) \Delta(y) O^x \Delta(x - y), \tag{6} \]

where \( \Delta(x) = \frac{1}{2\pi^2} \frac{1}{x^2} \) is the massless propagator and \( O^x \) is a differential operator. For example, using these rules one can easily obtain

\[ T^R[\Box] = -B^R[1](x) \delta(x - y) = \frac{1}{4} \frac{1}{(4\pi^2)^2} \log \frac{x^2 M^2}{x^2} \delta(x - y). \tag{7} \]

The massive counterparts are defined accordingly and will be denoted with a subindex \( m \). The DR substitutions for massive expressions can be obtained using recurrence relations among Bessel functions (see Refs. 9 and 17 for a more detailed discussion). The only one we shall need is

\[ \frac{m^2 K_1(mx)^2}{x^2} \rightarrow \frac{1}{2}(\Box - 4m^2) \frac{m K_0(mx)K_1(mx)}{x} + \pi^2 \log \frac{\tilde{M}^2}{m^2} \delta(x), \tag{8} \]

where \( \tilde{M} = \frac{2M}{\gamma_E} \), and \( \gamma_E = 1.781... \) is Euler’s constant. When one has to handle tensor structures, the general procedure involves decomposing the expressions.
into trace and traceless parts. Consistency with Eqs. (2, 3) may imply the appearance of local terms. For instance, in the renormalization of $T[\partial_\mu \partial_\nu]$ one has
\[ T^R[\partial_\mu \partial_\nu] = \frac{1}{4} \delta_{\mu\nu} T^R[\Box] + T[\partial_\mu \partial_\nu] - \frac{1}{4} \delta_{\mu\nu} \Box - \frac{1}{128\pi^2} \delta(x)\delta(y)\delta_{\mu\nu} . \] (9)

The last term is the local term alluded to, which is needed for the consistency of the equality:
\[ B^R[\partial_\mu](x)\delta(y) = -\partial_\mu^\nu \Box^\nu T[1] + \Box^\nu T[\partial_\nu] - 2\partial_\mu^\nu \partial_\sigma^\nu T[\partial_\sigma] - \partial_\mu^\nu T^R[\Box] + 2\partial_\mu^\nu T^R[\partial_\mu \partial_\sigma] + T^R[\partial_\mu \Box] . \] (10)

To complete the renormalization of $T[\partial_\mu \partial_\nu]$, one must note that only the first term in Eq. (9) is singular and requires renormalization (see Eq. (7)). Eq. (9) shows that in general
\[ \delta_{\mu\nu} T^R[\partial_\mu \partial_\nu] \neq [\delta_{\mu\nu} T[\partial_\mu \partial_\nu]]^R . \] (11)

3 Vacuum polarization in SQED

This is a simple example of a calculation in a supersymmetric abelian gauge theory using component fields. As a matter of fact we will only check the transversality of the photon self-energy and not any SUSY relation. The vacuum polarization in SQED has two contributions, one coming from the scalar loops (the corresponding diagrams are depicted in Fig. 1), and the other from QED. Before renormalizing, we will reduce their expressions to sums of basic functions and add the two parts. The Feynman rules in coordinate space can be found in Ref. 18.

The vacuum polarization in scalar QED reads
\[ \Pi^{(1)}_{\mu\nu}(x) = -e^2 \Delta_m(x) \partial_\mu \partial_\nu \Delta_m(x) , \] (12)
\[ \Pi^{(2)}_{\mu\nu}(x) = -2e^2 \delta_{\mu\nu} \Delta_m(x)\delta(x) , \] (13)

where $A \hat{\partial} B = A\partial B - B\partial A$. In terms of basic functions one has
\[ \Pi^{(1)}_{\mu\nu}(x) = -e^2 \{ 4B_m[\partial_\mu \partial_\nu] - \partial_\mu \partial_\nu B_m[1] \} , \] (14)
\[ \Pi^{(2)}_{\mu\nu}(x) = -2e^2 \delta_{\mu\nu} A_m . \] (15)

Now the propagator equation can be used to rewrite the tadpole contribution, Eq. (13), in terms of bubble functions,
\[ A_m = \Delta_m(x)\delta(x) \]
\[ \Pi_{\mu\nu}(x) = -\Delta_m(x)(\Box - m^2)\Delta_m(x) = -B_m[\Box] + m^2 B_m[1]. \quad (16) \]

On the other hand the QED contribution (diagram 1 in Fig. 1, but with a fermion running in the loop) can be written
\[ \Pi_{\mu\nu}(x) = 4e^2 \left( m^2 \delta_{\mu\nu} + \frac{1}{2} \delta_{\mu\nu} \Box - \partial_\mu \partial_\nu \right) B_m[1] + 2B_m[\partial_\mu \partial_\nu] - \delta_{\mu\nu} B_m[\Box]. \quad (17) \]

Then the supersymmetric QED vacuum polarization is the sum of the spinor diagram plus twice the scalar ones. One directly obtains a transverse result depending only on one basic function:
\[ \Pi_{\mu\nu}(x) = -2e^2 (\partial_\mu \partial_\nu - \delta_{\mu\nu} \Box) B_m[1]. \quad (18) \]

This equation exhibits the consistency of constrained DR and abelian gauge invariance in this simple one-loop supersymmetric calculation. The renormalization is completed substituting \( B_m[1] \) by its renormalized expression, given by Eq. (8):
\[ \Pi^R_{\mu\nu}(x) = -\frac{e^2}{(4\pi^2)^2} (\partial_\mu \partial_\nu - \delta_{\mu\nu} \Box) \left[ 4m^2 \frac{m K_0(mx) K_1(mx)}{x} + 2\pi^2 \log \frac{M^2}{m^2} \delta(x) \right]. \quad (19) \]
4 Supergravity contributions to $(g - 2)_l$

This calculation is more involved. In order to show how coordinate space techniques can be used to calculate a quantity typically defined for fixed momenta, we first work out the standard QED correction in coordinate space.

4.1 $(g - 2)_l$ in momentum and coordinate space

The anomalous magnetic moment of the electron, $(g - 2)_l$, is usually defined in momentum space as a static limit. For $p$ and $p'$ being the incoming momenta of the electrons and $q = p + p'$ being the outgoing momentum of the photon, the parity conserving vertex containing all radiative corrections can be expressed, for on-shell external electrons, in terms of two form factors:

$$\Lambda_\mu(p, p') = ie[(F_1(q^2) + F_2(q^2))\gamma_\mu + \frac{F_2(q^2)}{2m}(p_\mu - p'_\mu)].$$

Then the anomalous magnetic moment is defined as

$$\frac{g - 2}{2} = \lim_{q^2 \to 0} F_2(q^2).$$

The corresponding expression for the vertex in coordinate space is related to the momentum space one via

$$\Lambda_\mu(p, p') = \int d^4xd^4ye^{ip \cdot x}e^{ip' \cdot y}\Lambda_\mu(x, y).$$

The on-shell condition for the external electrons corresponds to imposing Dirac equation on their wave functions. This means that we must substitute terms of the form $\not\partial f(x, y)$ and $f(x, y)\not\partial$ in $\Lambda_\mu(x, y)$ by $-mf(x, y)$ and $mf(x, y)$, respectively. Similarly, the static limit corresponds in coordinate space to imposing $\Box A_\mu = 0$. Therefore, terms in $\Lambda_\mu(x, y)$ like $\Box f(x, y) = [-\partial_\mu^2 + \partial_\mu^\mu][-(\partial_\mu^\nu + \partial_\mu^\nu)]f(x, y)$ must be neglected. In practice, however, one cannot always extract all derivatives, and pieces containing internal derivatives remain.

Hence, Dirac and Maxwell equations cannot be directly imposed. The simplest solution is to perform at the end a Fourier transform to obtain $\Lambda_\mu(p, p')$ and to take the appropriate limits. This procedure may seem to end up in the usual momentum space one. The difference, however, is that renormalization is carried out before Fourier transforming, so the Fourier integrals are finite and can be computed in four dimensions without any regulator.

In QED, $(g - 2)_l$ vanishes at tree level (this is predicted by Dirac’s equation: $g = 2$ for a particle of spin 1/2), but it is well-known that a finite non-zero
$(g - 2)_l$ is generated at one loop. At this order the vertex correction, $V_\mu$, is given by the standard triangular diagram and reads

$$V_\mu(x, y) = (-ie)^3 \gamma_\alpha (\theta^\nu - m) \Delta_m(x) \gamma_\mu (\theta^\rho - m) \Delta_m(y) \Delta(x - y). \quad (23)$$

Using Leibnitz rule to rearrange derivatives, Eq. (23) can be expressed in terms of the triangular functions defined in Eq. (6), in this case with two massive propagators, i.e.,

$$T_m[O] = \Delta_m(x) \Delta_m(y) O^\mu \Delta(x - y). \quad (24)$$

Using Dirac's equation whenever possible the resulting expression is

$$V_\mu(x, y) = ie^3 \{ -4\gamma_\mu \partial^\rho \theta \theta^\rho T_m[\partial] + 4m T_m[\partial_\mu] - 2\gamma_\alpha (\partial^\rho - \partial^\rho') T_m[\partial_a] - 4\gamma_\alpha T_m[\partial_\mu \partial_a] \} \quad (25)$$

The terms proportional to $\gamma_\mu$ do not contribute to $(g - 2)_l$ and we ignore them in the following. Of the rest, only the last triangular function, $T_m[\partial_\mu \partial_a]$, is singular. As in Eq. (9) it is decomposed into a part proportional to $\delta_{\mu \nu}$ and a finite traceless part. Only this traceless part contributes to $(g - 2)_l$. Hence, $(g - 2)_l$ can be extracted from the finite expression,

$$V_\mu(x, y)^{g-2} = ie^3 \{ -4(\partial^\rho \theta - \partial^\rho') \theta T_m[\partial] + 4m T_m[\partial_\mu] - 4\gamma_\alpha T_m[\partial_\mu \partial_a - \frac{1}{4} \delta_{\mu \nu}] \} \quad (26)$$

which can be readily Fourier transformed (see Eq. (22)). The necessary integrals in the static limit,

$$p^2 = p'^2 = -m^2, \quad q^2 \to 0, \quad (27)$$

are

$$\hat{T}_m[\partial_\mu] = -\frac{i}{32\pi^2 m^2} (p_\mu - p_\mu'), \quad (28)$$

$$\hat{T}_m[\partial_\mu \partial_a - \frac{1}{4} \delta_{\mu \nu}] = -\frac{i}{32\pi^2 m^2} \left\{ -\frac{1}{6} (pa p_a' + p_a' p_a) + \frac{1}{3} (pa p_a + p_a' p_a') + \frac{1}{4} m^2 \delta_{\mu \nu} \right\} \quad (29)$$

Now the value of the anomalous magnetic moment can be read from the coefficient of $p_\mu - p_\mu'$ in $V_\mu(x, y)^{g-2}$ (see Eq. (26)):

$$\frac{g - 2}{2} = \frac{2m}{ie^3} \frac{16\pi^2 m}{4 - 4 \frac{1}{2} - 4 \frac{1}{4}} \frac{\alpha}{2\pi}. \quad (30)$$

This is the well-known Schwinger result.

Figure 2: Diagrams of order $e\kappa^2$ contributing to $(g - 2)_l$ in supergravity. A graviton is exchanged in diagrams D1-D5 and a gravitino in D6-D10.

4.2 $(g - 2)_l$ in unbroken supergravity

In a supersymmetric theory $(g - 2)_l$ vanishes because no such term appears in the Lagrangian of a chiral supermultiplet. Hence, as long as SUSY is preserved, all quantum corrections must cancel order by order. Therefore, the anomalous magnetic moment of the lepton besides being an observable, is also an ideal arena to check theoretical implications and to perform consistency tests of regularization methods. Ferrara and Remiddi also proved explicitly that in global SUSY the one-loop QED corrections, order $e^3$, do cancel. This is to say that the fermion contribution (Schwinger result) and the scalar one cancel each other. The latter results from twice the same triangular diagram but with the slepton and the photino replacing the lepton and the photon, respectively.

The one-loop gravitational corrections are of order $e\kappa^2 = 8\pi\kappa G_N$, resulting from a graviton or gravitino exchange (the corresponding diagrams are depicted in Fig. 2). Using dimensional regularization, Berends and Gasteans calculated the five diagrams where a graviton is exchanged. All five diagrams are infinite but their sum is finite. The finiteness of $(g - 2)_l$ in a non-renormalizable theory such as gravity seemed miraculous. Del Aguila et al. and Bellucci et al. checked that when gravitation is embedded in a supersymmetric theory (unbroken), the contributions from the graviton and the gravitino cancel, as required by SUSY. Bellucci et al. also traced back
to an effective chiral symmetry in the gravitino sector the finiteness of the gravitino contribution and then of the graviton contribution, if their sum has to vanish. Dimensional regularization does not yield a vanishing value for \((g-2)\). This is one example of a case where dimensional regularization breaks SUSY. A (one–loop) SUSY preserving method is required in order to obtain such a cancellation and this was shown to be the case in dimensional reduction. In Ref.~\cite{18} we calculated these contributions using DR (the Lagrangian, Feynman rules and other technical details can be found there). The use of the rule extending the validity of the propagator equation allowed us to relate diagrams with different topology before explicit renormalization. Then considering the sum of each diagram and its supersymmetric partner, we found that the contribution of each sum to \((g-2)\) depends only on one singular scalar basic function, \(T_m[\square]\), apart from finite terms. (This is analogous to what happens in the calculation of the vacuum polarization in SQED above.) The terms proportional to \(T_m[\square]\) cancel out in the complete sum and so do the finite terms. In this way SUSY is preserved, i.e., a vanishing value of \((g-2)\) is obtained. Terms proportional to \(q_\mu\) did not appear, either. Hence, on-shell gauge invariance, forbidding these terms, is also respected in this calculation. It is worth to note, however, that singular basic functions with other tensor structure do appear in individual graphs and in the total graviton (gravitino) contribution. Then, for these contributions which become physically meaningful if SUSY is broken, the tensor decomposition of these singular functions and the local terms they introduce (see for instance Eq. (9)) are relevant. In Ref.~\cite{18} we used the engineering trace-traceless decomposition neglecting local terms. We call this method ‘partially constrained’ DR. Therefore, the partial results may and do differ when using ‘partially constrained’ DR or constrained DR (for the latter includes the local terms), although the total sum is the same. In Table~\cite{18} we gather the results in dimensional regularization, dimensional reduction, ‘partially constrained’ DR and constrained DR. As can be observed, the last three methods (columns) preserve SUSY (add to zero), but ‘partially constrained’ DR gives a different value for the graviton (gravitino) contribution. The graviton (gravitino) contribution depends on the regularization method and is not well defined by itself. This is related to the fact that we are dealing with a non-renormalizable theory, and in the non-supersymmetric case no symmetry protects the value of \((g-2)\). The fact that dimensional reduction and constrained DR give the same result for this contribution seems to indicate that the requirement that the regularization/renormalization method be compatible with gauge invariance and SUSY greatly constrains the results at one loop.
Table 1: Contributions of the diagrams in Fig. 2 to \( (g-2) \), in units of \( \frac{G \pi \alpha}{2} \), obtained with dimensional regularization, dimensional reduction, ‘partially constrained’ DR and constrained DR.

| Diagram | Dimensional Regularization | Dimensional Reduction | ‘Partially Constrained’ DR | Constrained DR |
|---------|---------------------------|-----------------------|-----------------------------|----------------|
| D1      | \( \frac{1}{3} n-4 \)     | \( \frac{61}{36} \)  | \( \frac{1}{6} \log \left( \frac{M^2}{m^2} \right) - \frac{25}{18} \) | \( \frac{1}{6} \log \left( \frac{M^2}{m^2} \right) - \frac{25}{18} \) |
| D2+D3   | \( \frac{11}{3} \frac{1}{n-4} - \frac{37}{3} \) | \( \frac{11}{3} \frac{1}{n-4} - \frac{37}{3} \) | \( -\frac{11}{11} \log \left( \frac{M^2}{m^2} \right) - \frac{11}{11} \) | \( -\frac{11}{11} \log \left( \frac{M^2}{m^2} \right) - \frac{11}{11} \) |
| D4+D5   | \( -4 \frac{1}{n-4} + 7 \) | \( -4 \frac{1}{n-4} + 6 \) | \( 2 \log \left( \frac{M^2}{m^2} \right) + 1 \) | \( 2 \log \left( \frac{M^2}{m^2} \right) + 2 \) |
| Graviton (D1+D2+D3 +D4+D5) | 7/4 | 1/2 | -1 | 1/2 |
| D6      | \( \frac{8}{3} \frac{1}{n-4} - \frac{25}{18} \) | \( \frac{8}{3} \frac{1}{n-4} - \frac{27}{18} \) | \( -\frac{4}{3} \log \left( \frac{M^2}{m^2} \right) + \frac{19}{18} \) | \( -\frac{4}{3} \log \left( \frac{M^2}{m^2} \right) + \frac{19}{18} \) |
| D7+D8   | \( \frac{4}{3} \frac{1}{n-4} - \frac{13}{9} \) | \( \frac{4}{3} \frac{1}{n-4} - \frac{9}{9} \) | \( -\frac{4}{3} \log \left( \frac{M^2}{m^2} \right) + \frac{17}{18} \) | \( -\frac{4}{3} \log \left( \frac{M^2}{m^2} \right) + \frac{17}{18} \) |
| D9+D10  | \( -4 \frac{1}{n-4} + 4 \) | \( -4 \frac{1}{n-4} + 2 \) | \( 2 \log \left( \frac{M^2}{m^2} \right) - 1 \) | \( 2 \log \left( \frac{M^2}{m^2} \right) - 2 \) |
| Gravitino (D6+D7+D8 +D9+D10) | -1/2 | -1/2 | 1 | -1/2 |
| TOTAL (Graviton +Gravitino) | 5/4 | 0 | 0 | 0 |

5 Conclusions

DR is a method of renormalization recently proposed\(^3\), which works in coordinate space and does not introduce any intermediate regulator. It seems to have the potential of preserving gauge and chiral invariance. This procedure has been applied in several contexts and in particular to several supersymmetric calculations with component fields, which we have reviewed here. The method can be constrained to get rid of arbitrary constants, except for the
necessary renormalization group scale. Constrained DR is defined by a set of rules which completely fix the renormalization of singular expressions at least to one loop. In the two examples worked out, a transverse renormalized vacuum polarization in SQED and a vanishing \((g-2)_l\) in supergravity have been obtained. The use of the rule extending the validity of the propagator equation plays an essential role: it allows to relate the expressions appearing in different graphs and enforce the supersymmetric and gauge invariance constraints from the beginning. In both cases, SUSY cancellations make the result insensitive to the renormalization of basic functions with non-trivial tensor structure, and thus to the inclusion of local terms in the tensor decomposition. From only these two calculations, however, it cannot be said how general this effect is. At any rate, if SUSY is broken it is clear that the local terms become relevant. As shown in Ref. \[\text{17}\], the extensive use of the rules completely determines these terms, so constrained DR can in principle be used in the case of broken SUSY. Here we have only presented a simple (but non-trivial) consistency check of the constrained DR method for supersymmetric abelian gauge theories. A real test (or proof) should consider the Ward identities of both SUSY and gauge invariance, as well as higher orders.

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