Bridging the primordial $A = 8$ divide with Catalyzed Big Bang Nucleosynthesis

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Catalysis of nuclear reactions by metastable charged particles $X^-$ opens the possibility for primordial production of elements with $A > 7$. We calculate the abundance of $^9\text{Be}$, where synthesis is mediated by the formation of $(^6\text{Be}X^-)$ bound states, finding a dramatic enhancement over the standard BBN prediction: $^9\text{Be}/^3\text{H} \simeq 10^{-13} \times (X/10^{-3})$. Thus observations of $^9\text{Be}$ abundances at low metallicity offers a uniquely sensitive probe of many particle physics models that predict $X^-$, including variants of supersymmetric models. Comparing the catalytically-enhanced abundances of primordial $^6\text{Li}$ and $^9\text{Be}$, we find the relation $^9\text{Be}/^6\text{Li} = (2 - 5) \times 10^{-3}$ that holds over a wide range of $X^-$ abundances and lifetimes.

The first model of Big Bang Nucleosynthesis\(^1\) (BBN) made an ambitious attempt at explaining all elemental abundances as a result of successive primordial neutron capture. Two of the most important reasons why that theory did not work, and only very light nuclei can be generated in the Big, are the nuclear $A = 5$ and $A = 8$ divides, or the absence of stable nuclear isotopes with these mass numbers. After five decades of progress in nuclear physics, astrophysics and cosmology, we now have a very successful framework of Standard Big Bang Nucleosynthesis (SBBN) that makes predictions for elemental abundances of light elements, $\text{H}$, $\text{D}$, $\text{He}$ and $\text{Li}$, as functions of only one free parameter, the ratio of baryon to photon number densities $\eta_b$. With an additional CMB-derived\(^3\) input value of $\eta_b = 6 \times 10^{-10}$, the comparison of observed amounts of deuterium, helium and lithium serve as a stringent test of Big Bang cosmology and particle physics, and the importance of these tests is paramount for many theories seeking to extend the Standard Model\(^2\).

Last year it was realized that BBN is sensitive to a wider class of particle physics models than was previously thought through the phenomenon of catalyzed primordial nuclear fusion\(^4\), or CBBN. In particular it was shown that the presence of metastable heavy negatively charged particles, that unavoidably will form bound states with nuclei\(^4\),\(^5\),\(^6\),\(^7\),\(^8\),\(^9\), leads to the catalysis of $^6\text{Li}$ production via $(^4\text{He}X^-) + \text{D} \rightarrow ^6\text{Li} + X^-$\(^4\), with a rate exceeding the SBBN rate by many orders of magnitude. Moreover, the earlier presence of these particles at $T \sim 30$ keV leads to a moderate reduction of $^7\text{Li}$ abundance\(^5\), thus tantalizingly reproducing the observational pattern of $^6\text{Li}$ and $^7\text{Li}$\(^5\). Since then many groups have incorporated the catalyzed reactions into their calculations\(^9\), and the first dedicated three-body potential model calculation of the catalyzed rate was performed in\(^10\).

The catalysis of $^6\text{Li}$ can be viewed as an effective “bridging” of the $A = 5$ divide, and in this Letter we show that $A = 8$ is also bridged resulting in an enormous enhancement of the primordial $^9\text{Be}$ abundances over the SBBN value, $^9\text{Be}/^3\text{H} \lesssim 10^{-18}$\(^11\), thus effectively incorporating $^9\text{Be}$ into the BBN “family” of light nuclei. As was pointed out in the original papers\(^4\),\(^5\), the path to $A > 8$ nuclei is controlled by the $(^6\text{Be}X^-)$ bound state, to get to which $X^-$ has to go through a “double bottleneck” of successive $\alpha$-captures: $X^-\rightarrow(^4\text{He}X^-)\rightarrow(^8\text{Be}X^-)$. Once the $(^8\text{Be}X^-)$ bound state is formed, it participates in neutron capture, $\text{CBBN} : (^8\text{Be}X^-) + n \rightarrow ^9\text{Be} + X^-; \; Q_C = 0.26\text{MeV}$,\(^1\) that is catalyzed relative to neutron capture on $^8\text{Be}$, $\text{SBBN} : ^8\text{Be} + n \rightarrow ^9\text{Be} + \gamma; \; Q_S = 1.66\text{MeV}$.\(^2\)

The latter is, of course, never important for BBN because free $^8\text{Be}$ lives for under a femtosecond. According to the general scaling of quantum mechanics\(^4\), the cross section for\(^1\) is enhanced relative to\(^2\) by a large factor $\sim c^3/({\omega a})^3$, where $\omega = Q_S/c$ is the photon frequency in\(^2\), $a$ is the characteristic size of the $(^8\text{Be}X^-)$ system, and $c$ is the speed of light ($c = h = 1$ hereafter, and $\alpha = e^2$). In the rest of this Letter we analyze the rate for reaction\(^11\) in detail, calculate the rates for $(^8\text{Be}X^-)$ formation, and incorporating them in the reaction network, calculate the resulting $^6\text{Li}$, $^9\text{Be}$ synthesis at $T \simeq 8$ keV as a function of initial $X^-$ abundance per baryon $Y_X$, and lifetime $\tau_X$. In this work we use the following input values for the r.m.s. charge radii $r_N^c$ of $^4\text{He}$, $^8\text{Be}$, and $^9\text{Be}$ and calculate the binding energies of $(\text{N}X^-)$ in the limit of very heavy $X^-$ using a Gaussian charge distribution:

$$r_{^4\text{He}}^c = 1.67; \quad r_{^8\text{Be}}^c = 2.50; \quad r_{^9\text{Be}}^c = 2.50 \; \text{fm} \quad (3)$$

$$E_{^4\text{He}}^b = 347; \quad E_{^8\text{Be}}^b = 1408; \quad E_{^9\text{Be}}^b = 1477 \; \text{keV}. \quad (4)$$

While $(^4\text{He}X^-)$ binding energy has an uncertainty under a few keV, the (correlated) uncertainty in $E_{^8\text{Be}}^b$ and $E_{^9\text{Be}}^b$ can be as large as 50 keV, if e.g. the charge distribution is varied from Gaussian to square. More precise values of binding energies are possible through theoretical and experimental advances in determining the electromagnetic form factors of light nuclei\(^12\).

Catalyzed neutron capture. It is well known that the $^8\text{Be} + n$ system has a low-energy neutron $s$-wave resonance ($E_n \simeq 70\text{keV}$\(^13\)) that corresponds to a $1/2^+$
excited state of $^9\text{Be}$. Through the photodisintegration of $^9\text{Be}$, the energy and the width of this excited state are determined very accurately \cite{12},

$$E_{\frac{3}{2}^+} = 1735\pm 3\text{keV}; \quad \Gamma_n(E_n) \simeq 2\sqrt{\frac{192E_n}{\text{keV}}}; \quad \Gamma_\gamma = 0.57\text{eV}. \quad (5)$$

Notice the $\sqrt{E}$ behavior of the width for the near-threshold resonance \cite{14}. In Fig. 1 we show the isobar diagram for the $^8\text{Be} + n + X^-$ system, that holds the key for determining the rate of reaction \cite{11}. As is evident from this diagram, the neutron resonance occurs right at the threshold, $E_{\text{res}} \simeq 0\pm 30\text{keV}$, with the estimated error related to differences in charge distribution for $^9\text{Be}$ and $^8\text{Be}$. Fortunately, the neutron width in \cite{5} is larger than possible shifts of resonance energy, which makes this uncertainty tolerable for the calculation of the rate (even if the actual resonance is slightly below the threshold). Furthermore, there is no reason to assign a different width for the neutron resonance in the ($^9\text{Be}X^-)_p + n$ system relative to $^8\text{Be} + n$ and we adopt $\Gamma_n(E_n)$ from Eq. \cite{5}. An accurate calculation of the decay width for the ($^9\text{Be}_{\frac{3}{2}^+}X^- \rightarrow^9\text{Be}_{\frac{1}{2}^+}X^-$ process requires a dedicated nuclear many-body calculation that we will not pursue in this paper. Instead, we choose to re-process the existing experimental information on $\Gamma_\gamma$ by extracting the strength of the reduced matrix element for the nuclear $E1$ transition,

$$\langle 1/2^+ \parallel d/e \parallel 3/2^- \rangle|^2 = \frac{3\Gamma_\gamma}{2\alpha_0\omega^3} = (0.88 \text{fm})^2, \quad (6)$$

and connect it with an approximate expression for the ($^9\text{Be}_{\frac{3}{2}^+}X^-)_p$ p-wave decay width,

$$\Gamma_{\text{out}} \simeq \frac{\alpha^2}{2v} \times \langle 1/2^+ \parallel d/e \parallel 3/2^- \rangle|^2 \times I_r^2; \quad (7)$$

$$I_r = \int r^2 dr \times R_{10}(r) \frac{f(r)}{r^2} R_{p1}(r).$$

In Eqs. \cite{6} and \cite{7} $v$ is the velocity of the outgoing $^9\text{Be}$ in the catalyzed reaction, $I_r$ is the radial integral with dimension [distance]$^{-3/2}$. $R_{10}$ is the “atomic” 1s radial wave function of the decaying ($^9\text{Be}X^-)$ system, $R_{p1}$ is the final state $l = 1$ wave function with $p = \sqrt{2m_{^9\text{Be}}Q}$, normalized to the $p/2\pi$ scale \cite{14}, that in large $r$ limit becomes the wave function for the Coulomb problem, $f(r)$ represents a “form factor” that we have to assign to the interaction of a nuclear dipole operator with the electric field created by $X^-$, such that $f(r) \rightarrow 1$ if the orbit of $^9\text{Be}$ were far from $X^-$. For our calculation, we take $f(r) = 4\pi \int_0^\infty x^2 dx \rho_e(x)e, \text{so that for a constant charge density it scales as } f \sim r^3 \text{ inside the nuclear radius and } f = 1 \text{ outside.}$ Explicit numerical calculation gives

$$I_r \simeq (5.0 \text{ fm})^{-3/2}. \quad (8)$$

We note the resulting length scale in \cite{8} is six times larger than the effective size of the nuclear dipole in \cite{9} giving some \textit{a posteriori} justification to our procedure. Combining \cite{6}, \cite{7}, and \cite{8} we arrive at the following estimate for the decay width,

$$\Gamma_{\text{out}} = \frac{3\alpha}{4v} \frac{\Gamma_\gamma I_r^2}{\omega^3} \simeq 5 \text{ keV}, \quad (9)$$

which constitutes four orders of magnitude enhancement over $\Gamma_\gamma$. This estimate looks natural, perhaps on the lower side, given that $\Gamma_{\text{out}}$ contains no small parameters and the energy level spacing in this system is on the order of 1.5 MeV. Defining $\Gamma_{\text{tot}}(E_n) = \Gamma_{\text{out}} + \Gamma_n(E_n)$, we calculate the cross section of catalyzed photonless neutron capture \cite{11} using the Breit-Wigner formula,

$$\sigma_n(E_n) = \frac{g\pi}{k_0^2} \frac{\Gamma_n \Gamma_{\text{out}}}{(E - E_n)^2 + \Gamma_{\text{out}}^2/4}. \quad (10)$$

g is the spin factor, and $g = 1$ for \cite{11}. The result is a factor of $\sim 5$ lower than the unitarity limit at $E \sim 10\text{keV}$. Thus at temperatures $T/10^9\text{K} = T_9 = 0.1$ the reaction rate is calculated to be

$$N_A\langle \sigma_n v_n \rangle \simeq 2 \times 10^9 \text{ cm}^3\text{ mol}^{-1}\text{s}^{-1}, \quad (11)$$

which is large but not larger than \textit{e.g.} the rate of neutron capture on $^7\text{Be}$. Since the actual position of $E_n^{\text{res}}$ has an uncertainty of $\pm 30\text{keV}$ we do not have sufficient precision to obtain the variation of the rate with temperature. We did check, however, that variation in the resonant energy by $\sim 30\text{ keV}$ introduces only moderate shifts of a factor of $\sim 2$. Variations in $\Gamma_{\text{out}}$ directly affect the rate. A moderate increase of $\Gamma_{\text{out}}$ by a factor of 4 would increase the cross section to near the unitarity bound while a decrease of $\Gamma_{\text{out}}$ will obviously have an opposite effect. Fortunately, dedicated \textit{ab-initio} calculations of $\Gamma_n(E_n)$ and $\Gamma_{\text{out}}$ are feasible in state-of-the-art nuclear physics \cite{15}, with the clear potential of improving the accuracy of estimate \cite{11}.

\textbf{Formation of ($^8\text{Be}X^-$).} Before neutrons can undergo capture and form $^9\text{Be}$, first the bound states ($^4\text{He}X^-$)
The non-resonant part of the rate can be somewhat enhanced compared to a typical \((\alpha, \gamma)\) reaction due to the subthreshold \(2l\) resonances, but at \(T \sim 10\) keV it is totally negligible in comparison with \((12)\). At these temperatures and with this rate the capture of \(^4\)He on \((^4\)He\(^-\)) is about two orders of magnitude slower than the Hubble rate and rapidly dropping with \(T\). This is also important, as it shows that only a relatively small but non-negligible fraction of \((^4\)He\(^-\)) will be converted to \(^{8}\)Be\(^-\).

The rate depends very sensitively on the \(3p\) and \(3d\) energy levels, but subtleties of the charge distribution in \(^8\)Be make little difference for their energies. We find it remarkable that one can calculate the abundance of \(^{8}\)Be\(^-\) virtually free of major nuclear physics uncertainties. In fact, the main correction to \((12)\) comes from the \(m_{^8\text{Be}}/m_X\)-suppressed contribution to resonant energies that we ignore in this paper, but can account for very easily. For \(X^-\) as light as 100 GeV, these corrections amount to a 12 keV upward shift of \(E_{\text{res}}\), resulting in a factor of a few suppression in \(^{8}\)Be\(^-\) abundances, but quickly become negligible for heavier \(m_X\).

The molecular mechanism of forming \(^{8}\)Be\(^-\)) is completely different, as it is regulated by Coulomb unsuppressed processes. Due to the \(Y_X^2\) scaling, this mechanism is of secondary importance because \(Y_X\) is rather tightly constrained by \(^6\)Li overproduction \((11)\). We calculate the rate of molecular formation \((^4\)He\(^-\)) + \(^X\) \(\rightarrow\) \((^4\)He\(^-\))\(^-\)\(^X\)\(^+\) \(\rightarrow\) \((^4\)He\(^-\))\(^-\)\(^X\)\(^+\) + \(\gamma\), \(Q \approx 320\) keV, in the spirit of Kramers and ter Haar \((10)\), treating the “nuclear” motion of \(^X\) semiclassically, and “electron” motion of \(^4\)He quantum-mechanically, which is known to give a good approximation to a full quantum mechanical treatment. At temperatures \(T_9 < Q\) one finds

\[
\langle \sigma \nu \rangle_{\text{mol}} = 8\pi^{1/2}T_9^{-1/2}\int \sigma^2 dr \times \Gamma_{\gamma}(r)|V(r)|^{1/2},
\]

where \(r\) is the distance between two \(^X\)\(^-\) particles, \(V(r)\) is the potential energy of \((^4\)He\(^-\))\(^-\)\(^X\)\(^+\) interaction, and \(\Gamma_{\gamma}(r)\) is the probability per time for a quantum jump of \(^4\)He from the atomic to the molecular state with emission of a photon. Using the variationally determined molecular wave functions, we calculate \(\Gamma_{\gamma}(r), V(r)\) and find the following estimate for the rate of molecular formation

\[
N_A\langle \sigma \nu \rangle_{\text{mol}} \sim 40 \times T_9^{-1/2}\text{ cm}^3\text{mol}^{-1}\text{s}^{-1}.
\]

Note that the molecular rate is significantly smaller than the “atomic” \((^4\)He\(^-\)) recombination rate, \(8 \times 10^{9}T_9^{-1/2}\), but is not suppressed by \(m_{^4\text{He}}/m_X\), which is a direct consequence of the Coulomb attraction in the initial state. 

**Synthesis of \(^9\)Be:** Rates \((11)\), \((12)\), and \((14)\) enable us to calculate the \(^9\)Be freezeout abundance numerically. Before we do that, we would like to mention that in the hypothetical limit of both \(Y_X\) and \(\tau_X\) being large the production of \(^9\)Be will be neutron-supply-limited, and all neutrons produced in the DD and DT fusion below \(T_9 = 0.1\) may end up captured by \((^{8}\)Be\(^-\)) before they decay, leading to \(^9\)Be\((\text{large } Y_X, \tau_X) \approx O(10^{-9})\). Given that at lowest metallicities a \(10^{-14} - 10^{-13}\) range for \(^9\)Be is being probed \((12)\), this would constitute gross overproduction of \(^9\)Be, which is clearly excluded.

Figure 2 gives the result of our CBBN calculation at \(T_9 < 0.12\). Besides rates \((11)\), \((12)\), and \((14)\) calculated in this paper, we include the \(^6\)Li CBBN rate \((4)\) with the use of the \(S\)-factor properly calculated in \((10)\).

### Table I: Properties of \(^{8}\)Be\(^-\) Continuum

| \(nl\) | \(E_{\text{res}}(\text{keV})\) | \(E_{\text{res}}(\text{keV})\) | \(\Gamma_{\gamma}(\text{eV})\) |
|-------|------------------|------------------|------------------|
| 3s    | -265             | 173              | 0.1              |
| 3p    | -323             | 114              | 1.1              |
| 3d    | -351             | 88               | 1.0              |
| 2s    | -524             | -86              | 0.5              |
| 2p    | -706             | -267             | 4.5              |
| 1s    | -1408            | -                 | -                 |

Note that the molecular rate is significantly smaller than the “atomic” \((^4\)He\(^-\)) recombination rate, \(8 \times 10^{9}T_9^{-1/2}\), but is not suppressed by \(m_{^4\text{He}}/m_X\), which is a direct consequence of the Coulomb attraction in the initial state.
is exactly what is observed [17, 18], if we interpret the
situation (15) especially valuable. Close inspection of Fig. 2
predicts subsequent evolution after the Big Bang, making predic-
tions of metallicity. The lower plot represents
{\text{choices of } Y_X, \tau_X} input. The top figure represents very long life-
time \tau_X and abundance of \( Y_X = 5 \times 10^{-3} \). This option is excluded because \(^9\text{Be}\) is produced with \( O(10^{-11} - 10^{-10}) \) abundance. The increase in \(^{\text{Be}}X^-\) at low \( T \) is due to continued molecular formation. The lower plot represents \( \tau_X = 2000s \) and initial abundance of \( Y_X = 0.1 \), suggested by the solution to the \(^7\text{Li}\) problem [1]. For these parameters \(^6\text{Li}\) is \( 1.3 \times 10^{-11} \), and \(^{9}\text{Be}\) is \( 7 \times 10^{-14} \).

\[
^9\text{Be} \simeq 10^{-13} \times \left[ Y_X (t = 2 \times 10^6\text{sec}) / 10^{-5} \right]. \tag{15}
\]

Given that in some cases \(^9\text{Be}\) is detected below \( \times 10^{-13} \) [17], and typical abundances \( Y_X \geq O(10^{-3}) \) are expected from annihilation of \( X^-X^+ \) at the freezeout, this restricts the lifetime of \( X^- \) to a few thousand seconds, reinforcing the lithium bound [3]. It is also important that \(^9\text{Be}\) is far less fragile than \(^6\text{Li}\) and therefore is unlikely to experience a significant reduction of its abundance in subsequent evolution after the Big Bang, making prediction (15) especially valuable. Close inspection of Fig. 2 reveals a very similar behavior for \(^6\text{Li}\) and \(^9\text{Be}\). Dividing the two abundances, we eliminate the dependence on \( Y_X \), obtaining the following relation,

\[
^9\text{Be}_{\text{primordial}} / ^6\text{Li}_{\text{primordial}} \simeq (2 - 5) \times 10^{-3}, \tag{16}
\]

which is valid as long as \( \tau_X \geq 2000s \). Intriguingly, this is exactly what is observed [14, 18], if we interpret the

\(^6\text{Li}\) results as a “primordial plateau”, and take seriously [very tenuous] hints of elevated levels of \(^9\text{Be}\) at lowest metallicities. It is of course possible that all abnormalities observed in lithium and beryllium abundances will find astrophysical explanations having nothing to do with the Big Bang [14], and only more theoretical and observational work in this direction will clarify this issue.

To conclude, metastable negatively charged particles are predicted by many particle physics models, including some variants of supersymmetry. These particles, should they live in excess of 1000s, trigger CBBN and in this paper we show that sizable amounts of \(^9\text{Be}\) can be generated, and calculated with reasonable accuracy from first principles. Somewhat reduced \(^7\text{Li}\) abundance [1], strongly enhanced primordial values of \(^6\text{Li}\) and \(^9\text{Be}\), and \(^{9}\text{Be} / ^6\text{Li}, \text{few} \times 10^{-3}\) constitute a typical “footprint” of CBBN. In this light, further observational studies of \(^9\text{Be}\) and \(^6\text{Li}\) at low metallicities find an unexpected and very strong motivation from modern particle physics.

\[ \text{FIG. 2: } \log_{10} \text{ of the elemental abundances in CBBN for two choices of } \{ Y_X, \tau_X \} \text{ input. The top figure represents very long lifetime } \tau_X \text{ and abundance of } Y_X = 5 \times 10^{-3}. \text{ This option is excluded because } ^{9}\text{Be} \text{ is produced with } O(10^{-11} - 10^{-10}) \text{ abundance. The increase in } ^{\text{Be}}X^- \text{ at low } T \text{ is due to continued molecular formation. The lower plot represents } \tau_X = 2000s \text{ and initial abundance of } Y_X = 0.1, \text{ suggested by the solution to the } ^7\text{Li} \text{ problem. For these parameters } ^{6}\text{Li} = 1.3 \times 10^{-11}, \text{ and } ^{9}\text{Be} = 7 \times 10^{-14}. \]

\[ \text{The main result of this calculation is summarized as:} \]

\[ ^9\text{Be} \simeq 10^{-13} \times \left[ Y_X (t = 2 \times 10^6\text{sec}) / 10^{-5} \right]. \tag{15} \]

[1] R. A. Alpher, H. Bethe and G. Gamow, Phys. Rev. 73, 803 (1948).
[2] S. Sarkar, Rept. Prog. Phys. 59, 1493 (1996).
[3] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003).
[4] M. Pospelov, Phys. Rev. Lett. 98, 231301 (2007) [arXiv:hep-ph/0605215].
[5] A. De Rujula, S. L. Glashow and U. Sarid, Nucl. Phys. B 333, 173 (1990); S. Dimopoulos, et al., Phys. Rev. D 41, 2388 (1990).
[6] K. M. Belotsky, M. Y. Khlopov and K. I. Shibaev, arXiv:astro-ph/0604518; K. Kohri and F. Takayama, Phys. Rev. D 76, 063507 (2007); M. Kaplinghat and A. Rajaraman, Phys. Rev. D 74, 103004 (2006).
[7] C. Bird, K. Koopmans and M. Pospelov, arXiv:hep-ph/0703096.
[8] For latest account see e.g. P. Molaro, arXiv:0708.3922 [astro-ph].
[9] R. H. Cyburt et al., JCAP 0611, 014 (2006); M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 649, 436 (2007); K. Jedamzik, arXiv:0707.2070 [astro-ph], arXiv:0710.5153 [hep-ph]; M. Kusakabe et al., arXiv:0711.3854 [astro-ph], arXiv:0711.3858 [astro-ph].
[10] K. Hamaguchi et al., Phys. Lett. B 650, 268 (2007).
[11] D. Thomas et al., Astrophys. J. 406, 569 (1993).
[12] I. Sick, Prog. Part. Nucl. Phys. 47, 245 (2001).
[13] K. Sumiyoshi et al., Nucl. Phys. A709, 467 (2002).
[14] L. Landau and E. Lifshits, Quantum Mechanics: Non-Relativistic Theory, Third Edition, Pergamon Press.
[15] C. Forssen et al., Phys. Rev. C 71, 044312 (2005).
[16] H. A. Kramers and D. ter Haar, Bull. Astron. Inst. Neth. 10, 137 (1946).
[17] P. Primas et al., Astron. Astrophys. 364, L42 (2000) and references therein.
[18] M. Asplund et al., Astrophys. J. 644, 229 (2006).
[19] Vangioni-Flam et al., Astron. Astrophys., 337, 714 (1998); E. Rollinde, E. Vangioni-Flam and K. A. Olive, Astrophys. J. 627, 666 (2005).