Abstract

We calculate the rate of the decay \( \mu \to e\gamma \) and the electric dipole moment of the electron in the left-right supersymmetric model when the breaking of parity occurs at a considerably large scale. The low-energy flavor violation in the model originates either from the nonvanishing remnants of the left-right symmetry in the slepton mass matrix or from the direct flavor changing lepton-slepton-neutralino interaction. The result is found to be large for the masses of the supersymmetric particles not far from the electroweak scale and already accessible at the current experimental accuracy. It also provides nontrivial constraints of the lepton mixing in the model.
1 Introduction

The quest for a supersymmetric grand unified theory is plagued by lack of direct signals which would distinguish such a theory from supersymmetry in general. Supersymmetry, in particular the Minimal Supersymmetric Model (MSSM) \[1\], can be probed experimentally through the high energy production of superpartners. However, the MSSM, while filling in some of the theoretical gaps of the Standard Model, fails to explain other phenomena such as the weak mixing angle, the small mass (or masslessness) of the known neutrinos, the origin of CP violation, to mention a few. Extended gauge structures such as grand unified theories, introduced to provide an elegant framework for unification of forces \[2\], would connect the standard model with more fundamental structures such as superstrings, and also resolve the puzzles of the electroweak theory.

Phenomenologically, grand unified theories would predict either relationships between otherwise independent parameters of the standard model, or new interactions (i.e. interactions forbidden or highly suppressed in the standard model).

Among supersymmetric grand unified theories SO(10) \[3\] and SU(5) \[4\] have received significant attention. It is known that the rates of processes with flavor number violation, such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3\epsilon$ and $\mu - e$ conversion on nuclei are significantly enhanced in comparison with the pure MSSM case and are just one order of magnitude below the current experimental limit. In this article we shall study a model, the left-right symmetric extension of the MSSM, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ \[5\]. \[6\]. \[7\]. We show that this model shares some interesting features of the unified models providing the nonvanishing remnants of the left-right symmetry in the slepton mass matrix even if the right-handed scale is far from the electroweak one. Its attraction is twofold: first, the Left-Right Supersymmetric Model ($LRSUSY$) is an extension of the Minimal Supersymmetric Standard Model based on left-right symmetry, and second, it could be viewed as a low-energy realization of certain $SUSY - GUT$s, such as $SO(10)$. However, the left-right symmetry, being much less restrictive assumption than unification itself, does not relate the mixing angles in lepton and quark sectors. In particular, evaluating the $\mu \rightarrow e\gamma$ decay we show that this model can provide both smaller and larger rates than in unified theories depending mainly on the size of the mixing in the lepton sector.

Since the measurement of $\mu \rightarrow e\gamma$ has a very stringent bound $B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$
we are able to obtain values close to the experimental bound and restrict some of the parameters in the theory.

Our paper is organized as follows: in section 2 we give a brief description of the model, followed by the analysis of different flavor-violating mechanisms in section 3. The amplitude of the decay $\mu \to e\gamma$ is analyzed in section 4. Our conclusions are reached in section 5.

2 The Left-Right Supersymmetric Model

The $LRSUSY$ model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks). In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^+,-,W^0)_L$, $(W^+,-,W^0)_R$ and a singlet gauge boson $V$ corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bi-doublets, $\Phi_u(\frac{1}{2}, \frac{1}{2}, 0)$ and $\Phi_d(\frac{1}{2}, \frac{1}{2}, 0)$, which are required to give masses to both the up and down quarks. In addition, the spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by introducing the Higgs triplet fields $\Delta_L(1, 0, 2)$ and $\Delta_R(0, 1, 2)$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated for the right-handed neutrino and a small one for the left-handed neutrino. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1, 0, -2)$ and $\delta_R(0, 1, -2)$, with quantum number $B-L=-2$ to insure cancellation of the anomalies that would otherwise occur in the fermionic sector.

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire non-zero vacuum expectation values (VEV’s). These values are:

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ and } \langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\omega} \end{pmatrix}.$$  

$\langle \Phi \rangle$ causes the mixing of $W_L$ and $W_R$ bosons with $CP$-violating phase $\omega$. In order to simplify, we will take the VEV’s of the Higgs fields as: $\langle \Delta_L \rangle = 0$ and

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \langle \Phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \langle \Phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}.$$  

Choosing $v_L = \kappa' = 0$ satisfies the more loosely phenomenologically required hierarchy $v_R \gg max(\kappa, \kappa') \gg v_L$ and also the required cancellation of flavor-changing neutral
currents. The Higgs fields acquire non-zero VEV’s to break both parity and $SU(2)_R$. In the first stage of breaking the right-handed gauge bosons $W_R$ and $Z_R$ acquire masses proportional to $v_R$ and become much heavier than the usual (left-handed) neutral gauge bosons $W_L$ and $Z_L$, which pick up masses proportional to $\kappa_u$ and $\kappa_d$ at the second stage of breaking.

The supersymmetric sector of the model, while preserving left-right symmetry, has four singly-charged charginos (corresponding to $\tilde{\lambda}_L, \tilde{\lambda}_R, \tilde{\phi}_u$, and $\tilde{\phi}_d$), in addition to $\tilde{\Delta}_L, \tilde{\Delta}_R, \tilde{\delta}_L$, and $\tilde{\delta}_R$. The model also has eleven neutralinos, corresponding to $\tilde{\nu}_Z, \tilde{\nu}_{2L}, \tilde{\nu}_V, \tilde{\phi}^0_{1u}, \tilde{\phi}^0_{2u}, \tilde{\phi}^0_{1d}, \tilde{\phi}^0_{2d}, \tilde{\Delta}_L, \tilde{\Delta}_R, \tilde{\delta}_L$, and $\tilde{\delta}_R$.

In the scalar matter sector, the $LRSUSY$ contains two left-handed and two right-handed scalar fermions as partners of the ordinary leptons and quarks, which themselves come in left- and right-handed doublets. In general the left- and right-handed scalar leptons will mix together. Some of the effects of these mixings, such as enhancement of the anomalous magnetic moment of the muon, have been discussed elsewhere.

### 3 Sources of flavor violation in $LRSUSY$

The interaction of fermions with scalar (Higgs) fields has the following form:

$$\mathcal{L}_Y = Y_u Q_L \Phi_u Q_R + Y_d Q_L \Phi_d Q_R + Y_\nu L_L \Phi_u L_R + Y_e L_L \Phi_d L_R + H.c.;$$

$$\mathcal{L}_M = iF(L_L^T C^{-1} \tau_2 \Delta_L L_L + L_R^T C^{-1} \tau_2 \Delta_R L_R) + H.c.,$$

LR symmetry requires all $Y$-matrices to be hermitean in the generation space and $F$ matrix to be symmetric.

The off-diagonal entries in matrices $Y_\nu$, $Y_e$ and $F$ are responsible for the lepton flavor violation in the theory.

In what follows we consider the effects related to the neutrino Yukawa couplings $Y_\nu$. The seesaw mechanism allows to have large Yukawa couplings and at the same time escape the constraints coming from the neutrino mass data if the scale of the right-handed physics is higher than 100 TeV. Indeed, if $F v_R$ is large, the Yukawa couplings $Y_\nu$ could be very large and there is the possibility of having $Y_{\nu e}$ of the order 1. Therefore, the seesaw mechanism could provide large amount of flavor violation in the leptonic sector. In what follows we will concentrate on the specific $LRSUSY$-related mechanisms for the lepton flavor violation.
In complete analogy with the quark sector \cite{12} the flavor changing transitions between charged leptons are proportional to the Dirac Yukawa couplings of neutrinos:

\[ \mathcal{L}_{\text{FCNC}} = \bar{E}_L V^\dagger Y^\text{diag}_\nu E_R \phi_{2u}^0 + h.c., \]

where we use already the mass eigenstate representation. Here \( V \) is the Kobayashi-Maskawa matrix in the lepton sector. The neutral Higgs particles associated with \( \phi_{2u}^0 \) have to be very heavy in order to suppress the FCNC contribution to the neutral kaon mixing. Therefore, the direct contribution of this interaction to the lepton violating processes is negligibly small because corresponding one-loop diagram is suppressed as \( m_{\text{FCNC}}^2 \). However, in supersymmetric model we have to consider other possible interactions of the same origin involving higgsinos:

\[ \mathcal{L}'_{\text{FCNC}} = \bar{E}_R \phi_{2u}^0 V^\dagger Y^\text{diag}_\nu \bar{E}_L + \bar{E}_R^* V^\dagger Y^\text{diag}_\nu E_{2u} \bar{E}_L + h.c. \]

Here we do not have strict phenomenological restrictions on the mass of the corresponding higgsino and therefore the interaction (3) is very important for the muon conversion or decay.

If the FCNC higgsino happens to be heavy we have to consider different sources of flavor violation related with the scalar lepton mass matrix. Flavor violating terms in the slepton mass matrix arise as a result of the renormalization group evolution from the \( \Lambda_{\text{GUT}} \) scale and are caused by the admixture of the neutrino Yukawa couplings. Instead of calculating this matrix, which could be done only numerically, we adopt here the following anzatz for the charged slepton matrix with some elements of the left-right symmetry \cite{13}:

\[ \begin{pmatrix} \bar{E}_L^\dagger \\ \bar{E}_R^\dagger \end{pmatrix} \begin{pmatrix} M_L^2 + c_\nu \lambda_\nu^2 + c_e \lambda_e^2 & A_e \\ A_e^\dagger & M_R^2 + c'_\nu \lambda'_\nu^2 + c'_e \lambda'_e^2 \end{pmatrix} \begin{pmatrix} \bar{E}_L \\ \bar{E}_R \end{pmatrix}, \]

where \( A_e = A(M_e + a_e \lambda_e^2 M_e + a_\nu \lambda_\nu^2 M_e + a'_\nu M_e \lambda'_\nu^2) - M_e \mu \tan \beta \).

The coefficients \( c_\nu, c'_\nu, c_e, c'_e, a_e, a_\nu, a'_\nu \) appear either at the tree level or in the one-loop renormalization from \( \Lambda_{\text{GUT}} \). The requirement of the L-R symmetry is:

\[ M_L = M_R, \quad c_e = c'_e, \quad c_\nu = c'_\nu, \quad a_\nu = a'_\nu. \]

As a result, the mass matrix (4) differs from that of the MSSM where \( c'_\nu = 0 \) and \( a'_\nu = 0 \). The values of all these coefficients depend on many additional parameters and we simply assume here the following estimate:

\[ c_\nu \sim c'_\nu \sim m_{\text{susy}}^2 (16 \pi^2)^{-1} \ln(\Lambda_{\text{GUT}}^2 / M_{W_R}^2) \sim \mathcal{O}(m_{\text{susy}}^2). \]
When the left-right symmetry is broken, the relations (5) become approximate and we expect \( \frac{M^2_L - M^2_R}{M^2} \sim 10^{-2} - 10^{-1} \) [1, 3].

Next we consider the implications of these FCNC mechanisms in the LRSUSY in lepton-flavor violating decay \( \mu \to e\gamma \).

4 The Decay \( \mu \to e\gamma \)

The amplitude of the \( \mu \to e\gamma \) transition can be written in the form of the usual dipole-type interaction:

\[
M_{\mu \to e\gamma} = \frac{e}{2} d \tilde{\psi}_\mu \sigma^{\mu\nu} F_{\mu\nu} \psi_e
\]

It leads to the partial width

\[
\Gamma_{\mu \to e\gamma} = \frac{\alpha}{4} d^2 m^3_\mu
\]

Comparing it with the standard decay width, \( \Gamma_{\mu \to e\bar{\nu}\nu} = \frac{1}{192\pi^2} G_F^2 m^5_\mu \) and using the experimental constraint on the branching ratio, we get the following limit on \( d \) [14]:

\[
|d| < 3 \cdot 10^{-25} \text{ e cm}
\]

In what follows we calculate \( d \) due to different sources of flavor violation. We note that there are two general classes of contributions to \( d \). First, there are terms proportional to the mass of muon divided by the square of supersymmetric mass scale, \( m_\mu/M^2 \). The second class of contributions are terms proportional to the mass of the tau lepton, \( m_\tau/M^2 \).

We concentrate our analysis on the second class and calculate the corresponding \( d \). The enhancement factor, \( m_\tau/m_\mu \), associated with this subclass allows us to consider only the diagrams with the chirality flip on the slepton line and neglect at the moment all other contributions. In some sense, we assume from the very beginning that the mixing with the third generation is significant and of the same order as the mixing in the \( \mu - e \) sector.

To obtain the amplitude of \( \mu \to e\gamma \) derived from the flavor-violating mechanisms in the previous section we have to calculate the one-loop diagrams of Fig. 1.

In the neutral higgsinos exchange the leading contribution is due to the superpartner of the field \( \phi^0_{2u} \) corresponding to the FCNC Higgs [3]. It gives the amplitude:

\[
|d| = \frac{1}{16\pi^2} m_\tau \frac{(A - \mu \tan \beta) m_h}{M^4} |V_{33}|^2 |V_{32}V_{31}^*| Y^2_{\nu_\tau} F(m_h^2/M^2),
\]

(10)
where
\[ F(x) = \frac{1}{2(1-x)^4} \left[ 1 + 4x - 5x^2 + 4x \log x + 2x^2 \log x \right] \quad (11) \]

Let us suppose for a moment that at the GUT scale we have the unification of quarks and leptons in the form of a close correspondence between Yukawa couplings and mixing angles, \( V_{31} \sim V_{td}, V_{32} \sim V_{ts}, Y_{\nu_e} \sim Y_t. \) Then our result leads to the following numerical estimate, where we take the common mass for all supersymmetric parameters, \( M \sim m_h \sim (A-\mu \tan \beta), F \simeq 1/12: \]
\[ |d| \sim 24 \cdot 10^{-25} \left( \frac{100 \text{GeV}}{M} \right)^2 e \cdot \text{cm} \quad (12) \]

which is already larger than the current experimental limit. Alternatively, we can translate Eq. (10) into the following limit:
\[ |V_{33}|^2 |V_{32} V_{31}^*| Y_{\nu}^4 \left( \frac{100 \text{GeV}}{M} \right)^2 < 5 \cdot 10^{-5}. \quad (13) \]

Another group of diagrams proportional to \( m_\tau \) which we have to consider here is that of Fig. 2 where the fermion line in the loop corresponds to the propagation of the bino, the superpartner of the \( U(1) \) gauge boson. The flavor violation comes through mass insertions in the slepton line. These insertions are indicated as small bubbles. The superpartners of \( SU(2)_L \) or \( SU(2)_R \) are not interesting because they couple only to left- or right-handed fermions and cannot lead to \( m_\tau \)-proportional effects since these require a helicity flip. Using the estimate (6) and for \( m_h \sim M \), the result reads as follows:
\[ |d| \sim \frac{\alpha}{4\pi} \frac{1}{20} m_\tau \frac{A-\mu \tan \beta}{M^3} |V_{33}|^2 |V_{32} V_{31}^*| Y_{\nu}^4 \quad (14) \]

It gives a different dependence of Yukawas, \( Y_{\nu}^4 \). The constraints which could be obtained from (14) are one–two orders of magnitude weaker in comparison with (13), mainly due to the factor \( 4\pi \alpha \sim 0.1 \).

These two mechanisms exhaust all possible supersymmetric contributions to \( d \) proportional to the mass of the tau lepton. As to the diagrams with the charged higgsinos, they are irrelevant for our analysis since we know that large \( v_R \) leads to the decoupling of the right-handed sneutrinos through the corresponding couplings in the supersymmetric \( F \)-term.

Lastly, we obtain an estimate of the electric dipole moment of the electron in this model due to the complex phases in the lepton Yukawa couplings. The mechanism analogous to
leads to the following estimate:

$$d_{EDM} \sim J \frac{1}{16\pi^2} \frac{1}{20} m_\tau \frac{(A - \mu \tan \beta) M^2_L - M^2_R}{M^2} Y^3_\tau Y_\mu,$$

where $J = \text{Im}(V_{21} V^*_{31} V_{32} V^*_{22})$ is the CP-odd rephasing invariant of the leptonic KM matrix and $\frac{M^2_L - M^2_R}{M^2} \sim 10^{-2} - 10^{-1}$ is the relative difference between left- and right-handed slepton masses. The corresponding constraint on the CP-odd combination of mixing angles and Yukawa couplings reads as follows:

$$J Y^3_\tau Y_\mu \left(\frac{100\text{GeV}}{M}\right)^2 < 10^{-4} - 10^{-3}$$

which is definitely weaker than constraints coming from the muon decay if we believe that $Y_\mu \sim Y_\tau \sim 10^{-2}$.

## 5 Conclusions

We have evaluated the $\mu \to e\gamma$ decay rate and we have shown that the left-right supersymmetric model could provide large rates for the lepton flavor violating processes if the scale of the supersymmetric masses is not far from the left-handed electroweak scale. The most promising mechanism in this respect is related with the higgsino particle, the superpartner of $FCNC$ Higgs. The absence of direct phenomenological requirements for this fermion to be heavy makes the $\mu \to e\gamma$ decay large, even without any flavor-changing insertions into the slepton line. The specific numerical predictions are weakened, however, by the lack of knowledge of explicit values for the supersymmetric masses. In particular, the predictive power of the model cannot be improved unless one specifies a way to solve the $FCNC$ problem. If one believes that the masses of $FCNC$ higgsinos are related to the $v_R$ scale and thus very large, one has to use another source of flavor violation coming from the Yukawa dependence of slepton masses. In this case $LRSUSY$ is similar to MSSM at the usual electroweak scale and the right-handed scale plays the role of the intermediate scale considered in Ref. [15].

The $m_\tau$-proportionality of the amplitude for this process is very similar to the situation which occurs in unified theories [3, 4]. The similarity is not accidental. It is based on the presence of non-MSSM type slepton masses and mixings which survive in the effective low-energy theory even if the scale of the new physics, unification scale or $v_R$, is very high. In $LRSUSY$, however, the mixing angles and masses in the lepton and quark sectors are not...
related. The rate of flavor-changing decay $\mu \to e\gamma$ may be even bigger than in the unified theories. Therefore we conclude that at the current experimental accuracy the model already predicts nontrivial limits on mixing angles in the lepton sector.

In this letter we have concentrated only on $m_\tau$-proportional contributions to the amplitude of $\mu \to e\gamma$ decay. A complete analysis would include the calculation of subleading $m_\mu$-proportional terms which could be important because the number of diagrams involved is very large. One would have to consider also the effects of the neutralino mixing, the differences between masses of superparticles, etc; all these are model-dependent. We believe, however, that the large degree of arbitrariness in the choice of various parameters in the model cannot change the main features of the muon decay amplitude considered here.

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Figure captions

Fig. 1. One-loop contribution to the muon decay amplitude due to flavor-changing lepton-slepton-neutralino interaction. The cross on the tau slepton line indicates left-right mixing.

Fig. 2. One-loop contribution to the muon decay amplitude due to the flavor-changing insertions into the slepton line.