Light vector meson photoproduction at large $t$

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Abstract:

We have studied in perturbative QCD all independent helicity amplitudes describing the photoproduction of light vector mesons at large $t$. We found a new hard production mechanism which is related to the possibility for a real photon to fluctuate into a massless $q\bar{q}$ pair in a chiral-odd spin configuration. Each helicity amplitude is given as a sum of a usual chiral-even contribution (when the helicities of quark and antiquark are antiparallel) and this additional chiral-odd part (where the helicities of quark and antiquark are parallel). The chiral-odd contribution is large, it leads to a dominance of the non spin-flip amplitude in a very broad region of intermediately high $|t|$. All amplitudes are expressed in terms of short distance asymptotics of the light-cone wave functions of vector meson (photon). We demonstrate that for each helicity amplitude there exists a soft non-factorizable contribution. We give arguments that for dominant non spin-flip helicity amplitude the relative contribution of the soft nonfactorizable interactions is numerically not large.
1 Introduction

During the last few years a number of hard diffractive processes were suggested for probing the short-range structure of hadrons and the behaviour of perturbative QCD (pQCD) at high energies. The description of these processes is greatly simplified if a factorization theorem is valid which allows to write the amplitude as a convolution of the wave function of the produced meson, a hard interaction block and a block related to the density of partons in the target.

The key element of the factorization proof for exclusive meson production in DIS [1] is the selection of processes with longitudinally polarized incoming photons. This ensures that small $\sim 1/Q$ transverse interquark distances dominate in the $\gamma^*_L \rightarrow \text{meson}$ transition and that extra soft interactions between constituents moving in the same direction as the $\gamma^*_L$ and those moving in the same direction as the nucleon are suppressed by a factor $1/Q^2$. As a result the study of exclusive vector meson production can be used for probing “small dipole” - hadron interactions at high energies.

Another appealing alternative is to use hard diffractive processes with scattering of the colliding particles at large enough $t$ to reach the so called squeezed regime [2]. Here the key question is whether indeed the squeezing occurs, i.e. whether multiple interactions can be neglected. This is the case for $\gamma^*_L + p \rightarrow V + X$ [3]. However the rates in this case are low. The rates are much higher for the real photon case $\gamma + p \rightarrow V + X$.

The photoproduction of $J/\Psi$ meson at large $t$ was studied in the papers [5, 6]. The predictions of pQCD for photoproduction of longitudinally polarized light meson at large $t$ were derived in refs. [7, 8, 9]. Photoproduction of transversely polarized meson was discussed in a phenomenological model in [3].

Here we will derive pQCD predictions for all helicity amplitudes of this process $M_{\lambda_1, \lambda_2}$, where $\lambda_1 = \pm$ is the photon helicity and $\lambda_2 = \pm, 0$ is the vector meson helicity. We calculate the factorizing contribution of small quark-antiquark separation and indicate the non-factorizing contributions appearing in our calculations as singularities at the end point in momentum fraction. We shall show that the chiral-odd configuration (the helicities of the quark and the antiquark are parallel) in the quark loop of Fig.1 gives a very important contribution if $t$ is not asymptotically large. Within usual perturbation theory a photon can split only into a chiral-even (the helicities of the quark and the antiquark are antiparallel) massless quark pair. The violation of chiral symmetry, which is well known to be a soft QCD phenomenon, generates a nonperturbative chiral-odd component of the real photon wave function. The interaction of this additional chiral contribution can however be described in pQCD since high $t$ quark-dipole scattering chooses a $q\bar{q}$ configuration with small transverse interquark distances. As a result the chiral-odd contribution can be factorized into a convolution of two nonperturbative photon and vector meson light-cone wave functions with the hard scattering amplitude. The chiral-odd wave function of the real photon
is proportional to the quark condensate and its magnetic susceptability [15]. We shall show that the helicity amplitudes are very sensitive to the values of these fundamental parameters of the QCD vacuum. Let us stress that this new hard production mechanism for high $t$ photoproduction was not considered before.

We shall demonstrate that in the spin non-flip amplitude $M_{++}$ both the chiral-even and the chiral-odd mechanisms give contributions $\sim t^{-2}$ which are of the same sign. The contributions of these two mechanisms to the single spin-flip amplitude $M_{+0}$ are of opposite signs, they behave as $\sim t^{-3/2}$ for the chiral-even and $\sim t^{-5/2}$ for the chiral-odd case. For the double spin-flip amplitude $M_{+-}$ the contributions from both mechanisms are of opposite sign and $\sim t^{-2}$ for the chiral-even and $\sim t^{-3}$ for the chiral-odd case. Due to large numerical coefficient in front of the chiral-odd contributions, even in the case of $M_{+0}$ and $M_{+-}$ the chiral-odd mechanism is very important in a wide region of intermediate $t$ despite the fact that it is formally $1/t$ suppressed. This observation could explain why onset of the asymptotic regime, namely the dominance of the $M_{+0}$ helicity amplitude was not observed experimentally at large $t$.

The cross section for a reaction with rapidity gap $\eta_0$ (see the diagram of Fig.1) can be related to those for the photoproduction of $V$ off a quark and a gluon via the gluon and quark densities in a proton $G(x,t)$ and $q(x,t)$:  

\begin{center}
\includegraphics[width=\textwidth]{figure1.png}
\end{center}

Figure 1: The photoproduction of vector meson
\[
\frac{d^2 \sigma(\gamma p \rightarrow V X)}{dt dx} = \sum_f (q(x,t) + \bar{q}(x,t)) \frac{d\sigma(\gamma q \rightarrow Vq)}{dt} +
\]
\[
G(x,t) \frac{d\sigma(\gamma G \rightarrow VG)}{dt}; \quad x = \frac{4p^2}{s} \cosh^2 \frac{\eta_0}{2}. \quad (1.1)
\]

\(\eta_0\) is the difference in rapidity between the struck parton and produced meson. (We consider here the case of small angle scattering so that \(-t/xs \ll 1\).

The factorization formula (1.1) is valid if the typical transverse distances between quarks in the upper part of Fig. 1 are small. In this case the contribution of the additional soft interactions which are schematically depicted in Fig. 2 will be power suppressed and therefore the jet balancing the high transverse momentum of the meson would be produced close to the gap edge.

\[\gamma\]
\[\rightarrow\]
\[N\]
\[\rightarrow\]
\[X\]
\[V\]

Figure 2: The additional soft interaction

We shall show below for each helicity amplitude of the \(\gamma q \rightarrow Vq\) process that there exist soft non-factorizable contributions originating from the region of large transverse distances between quark and antiquark in Fig. 1. For such soft contributions the factorization (1.1) breaks down and the rapidity \(\eta\) of the leading jet for the system \(X\) can lie far away from the gap edge, \(\eta \gg \eta_0\).

We shall show that in the region of intermediately large \(t\) the relative contribution of these soft nonfactorizable interactions is not large numerically. Therefore in this \(t\) region eq. (1.1) should be a good approximation and we expect that the leading jet of \(X\) should be close to the gap edge. It would be very interesting to study the jet structure of the system \(X\) experimentally since this is a clean signature for the dominance of the hard factorizable production mechanism.
We shall discuss now the helicity amplitudes of the parton subprocess $\gamma q \rightarrow V q$ in detail. The two-dimensional polarizations vectors of a real photon and transversly polarized vector meson are denoted as

$$e^{(\pm)} = \mp \frac{1}{\sqrt{2}} (1, \pm i) .$$

(1.2)

There are three independent helicity amplitudes which we choose as

$$M_{++}(M_{--} = M_{++}) , \ M_{00}(M_{00} = -M_{00}) , \ M_{+-}(M_{--} = M_{+-}) .$$

Although in the following we calculate the helicity amplitudes only for the production of $\rho^0$ meson, the resulting formulas are valid - after a suitable change of the coupling constant - also for $\phi$ and $\omega$ meson production.

We shall use the results of [10], [11] for the to describe the $\rho$ and real photon $q\bar q$ light-cone wave functions at small interquark distances.

In the present letter we present only the main steps of our calculations and the main results. Technical details of the derivation as well as a comparison with data will be presented in a later publication.

We use the following definitions

$$\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 , \ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] , \ \varepsilon^{0123} = 1$$

(1.3)

The quark charge is denoted by $eQ_q$. The flavour structure of the $\rho^0$ vector meson can be described by the replacement $eQ_q \rightarrow e/\sqrt{2}$.

2 Factorization formulae for $\gamma q \rightarrow V q$ amplitude

The amplitude of the $\gamma q \rightarrow V q$ process can be written as a convolution of the hard scattering amplitude which describes the production of a quark pair $A_{\alpha\beta}(l, p - l)$ and the amplitude of the transition of this quark pair into a meson $\Psi_{\alpha\beta}(x, -x)$

$$M = \int A_{\alpha\beta}(l, p - l)e^{i(l\cdot l - p\cdot p)x}\Psi_{\alpha\beta}(x, -x)\frac{d^4l}{(2\pi)^4}\frac{1}{4^4}d^4x$$

(2.1)

$$\Psi_{\alpha\beta}(x, -x) = \langle \rho(p)|\bar{\Psi}_{\alpha}(x)|\Psi_{\beta}(-x)|0 \rangle ,$$

(2.2)

$p$ and $l$ are the momenta of the meson and the quark respectively, see Fig.1. The momentum of the target quark (gluon) is $p_2$. The factor $2^4$ is present in the above equation since the separation between quark and antiquark is $r = 2x$.

We introduce light cone variables

$$l_\pm = 1/\sqrt{2}(l_0 \pm l_3) , \ \ d^4l = dl_+dl_-d^2l , \ \xi = 2u - 1 , \ \ l_+ = up_+ .$$

(2.3)
The two-dimensional transverse vectors are denoted by bold face. We denote the longitudinal momentum fraction for the quark as \( u \), for the antiquark as \( (1 - u) = \bar{u} \).

The hard scattering amplitude \( A_{\alpha\beta} \) depends weakly on \( l_- \), neglecting this dependence we perform integration over \( l_- \) and \( x_+ \)

\[
M = 2 \int A_{\alpha\beta}(l, p - l) e^{-i(l, r)} d^2l d^2r \left( \frac{d}{2\pi} \right) \frac{1}{2} du \Psi_{\alpha\beta}(u, r) \quad (2.4)
\]

\[
\Psi_{\alpha\beta}(u, r) = \int \frac{d(p_+ x_+)}{(2\pi)} e^{i(p_+ x_+ - x_+)} \Psi_{\alpha\beta}(x, -x) \quad . (2.5)
\]

The \( x \) values contributing essentially to the integral are small \( \sim 1/q \), therefore \( x^2 = 2x_+ x_- - x^2 \to 0 \).

We use the usual Fierz transformation

\[
\delta_{\alpha\alpha'} \delta_{\beta\beta'} = \frac{1}{4} \Gamma^t_{\alpha\beta} \Gamma^t_{\alpha'\beta'} = \frac{1}{4} \Gamma^t_{\beta\alpha} \Gamma^t_{\beta'\alpha'} \quad (2.6)
\]

\[
\Gamma^t = \{1, \gamma^\mu, \gamma_5, \gamma^\mu \gamma_5, i\gamma^\mu\}, \quad \Gamma_t = \left(\Gamma^t\right)^{-1}, \quad (2.6)
\]

to disentangle the spinor indices of \( A_{\alpha\beta}(l, p - l) \) and \( \Psi_{\alpha\beta} \).

We use

\[
< \rho(p)|\bar{\Psi}(x)\Gamma(t)\Psi(-x)|0> = \left[ <0|\bar{\Psi}(-x)\Gamma(t)\Psi(x)|\rho> \right]^* \quad . (2.7)
\]

Expressions for the light-cone wave functions of vector mesons were derived in \[10\] and \[11\] and we shall adopt the notations used there. For instance (see eq.(2.8) of \[11\]),

\[
<0|\bar{u}(x)\gamma_\mu u(-x)|\rho, \lambda> = f_\rho m_\rho \left[ \frac{e^{(\lambda)x}}{px} p_\mu \int_0^1 du e^{i(\xi px)} \phi_{||}(u) \right.
\]

\[
\left. + \left( e^{(\lambda)x}_\mu - p_\mu \frac{e^{(\lambda)x}}{px} \right) \int_0^1 du e^{i(\xi px)} g_{\perp}^{(v)}(u) \right] \quad (2.8)
\]

Here we have neglected the terms proportional to \( m_\rho^3 \) which involve wave functions of twist-4. The polarization state of vector meson with definite helicity \( \lambda \) is described by \( e^{(\lambda)} \).

According to \[10\], \[11\] all wave functions like \( \phi_{||}(u), g_{\perp}^{(v)}(u) \) which parametrize the matrix elements \( <0|\bar{u}(x)\Gamma_t u(-x)|\rho> \) are normalized in the same way

\[
\int_0^1 \phi(u) du = 1 \quad . (2.9)
\]
3 Hard scattering amplitude.

The hard scattering amplitude in eq. (2.4) was calculated in [7]. The result is given as integral over the transverse gluon momentum $k$

$$A_{\alpha \beta} = is \int \frac{J_{\alpha \beta}^{\gamma \rightarrow q \bar{q}}(k, q)}{k^2 (k - q)^2} J_{qq}(-k, -q) d^2 k. \quad (3.1)$$

This impact factor representation is well known from the pioneering works [12]. $s$ is the total c.m.s energy squared of the photon-quark collision, $t = -q^2$. The impact–factors $J_{\gamma V}$ and $J_{qq}$ correspond to the upper and the lower blocks in Fig. 1. They are $s$–independent. For colorless exchange the impact–factors contain factors $\delta_{ab}$, where $a$ and $b$ are the color indices of the exchanged gluons.

Due to gauge invariance the impact–factors, which describe the coupling to colorless state vanish when the gluon momenta tend to zero:

$$J_{\gamma V}(k, q) \rightarrow 0 \quad \text{at} \quad \begin{cases} k & \rightarrow 0, \\ (p - k) & \rightarrow 0. \end{cases} \quad (3.2)$$

This property garanties the infrared safety of the integral (3.1). The quark and gluon impact factors are

$$J_{qq} = \alpha_s \frac{\delta_{ab}}{N}; \quad J_{gg} = -\alpha_s \delta_{ab} \frac{2N}{N^2 - 1}. \quad (3.3)$$

The helicity and color state of the quark or gluon target are conserved by these vertices. The relations (3.3) show that the cross section for photoproduction of vector mesons by gluons is about 5 times larger than by quarks:

$$d\sigma_{\gamma g \rightarrow V g} = \left(\frac{2N^2}{N^2 - 1}\right)^2 d\sigma_{\gamma q \rightarrow V q} = \frac{81}{16} d\sigma_{\gamma q \rightarrow V q}. \quad (3.4)$$

The impact factor describing the upper part of the diagram Fig. 1 is

$$J_{\gamma \rightarrow q \bar{q}} = eQ_q g^2 \frac{\delta_{ab}}{2N} \left(\left[ mR\hat{e} - 2u(P\hat{e}) - \hat{P}\hat{e} \right] \frac{\hat{p}_2}{s} \right)_{\alpha \beta}. \quad (3.5)$$

Here $R$ and the transverse vector $P = (0, P, 0)$ are:

$$P = \left[ \frac{q_1}{q_1^2 + m^2} + \frac{k - q_1}{(k - q_1)^2 + m^2} \right] - [q_1 \leftrightarrow q_2];$$

$$R = \left[ \frac{1}{q_1^2 + m^2} - \frac{1}{(k - q_1)^2 + m^2} \right] + [q_1 \leftrightarrow q_2]. \quad (3.6)$$

According to eq. (2.3) we have to switch to the mixed representation, i.e. the momentum representation with respect to gluon $t$–channel momenta and the
coordinate representation with respect to transverse distance between quark and antiquark \( r \).

To perform the corresponding Fourier transform we have to express the quark and antiquark momenta \( q_1, q_2 \) through the momentum of quark \( l \) which is transverse with respect to the meson momentum \( p \)

\[
q_1 = l + qu \quad q_2 = -l + q\bar{u}.
\]  

(3.7)

As result we obtain

\[
P(r) = \int \frac{d^2 l}{(2\pi)^2} e^{-iqr} P = \int \frac{d^2 l}{(2\pi)^2} \frac{1}{l^2 + m^2} e^{-iqr} f_{dipole}.
\]

(3.8)

\[
R(r) = \frac{1}{2\pi} K_0(rm) f_{dipole}.
\]

(3.9)

The dipole amplitude has now appeared. It is given by the formulae

\[
f_{dipole} = e^{iqr} \left( 1 - e^{-i\kappa r} \right) \left( 1 - e^{-i(q-k)r} \right)
\]

(3.10)

In the massless limit

\[
P(r) = -\frac{im}{2\pi} K_1(rm) \frac{r}{r} f_{dipole} |_{m \to 0} \quad \longrightarrow \quad -\frac{i}{2\pi} \frac{r}{r} f_{dipole}.
\]

(3.11)

The trace calculations for those hard scattering amplitudes with Fierz structures \((\gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu})\) which lead to the dominant \((\sim s)\) contribution is straightforward. Let us define

\[
\hat{Q} = (mR\hat{e} - 2u(P\hat{e}) - \hat{P}\hat{e}) \frac{p_2^2}{s},
\]

then the relevant traces which have to be calculated are

\[\begin{align*}
\frac{1}{4} Tr[\gamma_\mu \hat{Q}] &= (1 - 2u)(P\hat{e}) \frac{p_2^\mu}{s} \\
\frac{1}{4} Tr[\gamma_\mu\gamma_5 \hat{Q}] &= \frac{i}{s} \epsilon_{\mu\nu\sigma\tau} P^\nu e^\sigma p_2^\tau \\
\frac{1}{4} Tr[\sigma_{\mu\nu} \hat{Q}] &= \frac{im}{s} (p_2^\mu e_\nu - e_\mu p_2^\nu) R
\end{align*}\]

(3.12)

4 Meson wave functions

Now we transform \( \rho \) meson wave functions separately for longitudinal and transverse polarization into a form convenient for subsequent calculations.

a) \( \Gamma_t = \gamma_\mu \), longitudinal polarization
\[ < 0|\bar{u}(x)\gamma_\mu u(-x)|\rho(p, \lambda = 0) > = f_\rho p_\mu \int_0^1 du e^{i\xi(p+x^-)} \phi_\parallel(u) \quad (4.1) \]

The Fourier transform (FT) of this expression with respect to \((p_+ x^-)\), gives

\[ \frac{1}{2} p_\mu f_\rho \phi_\parallel(u) \quad (4.2) \]

b) \(\Gamma_t = \gamma_\mu\), transverse polarization

\[ < 0|\bar{u}(x)\gamma_\mu u(-x)|\rho(p, \lambda = T) > = -f_\rho m_\rho p_\mu \frac{(e^{(T)}x)}{(p_+ x^-)} \int_0^1 du e^{i\xi(p+x^-)} (\phi_\parallel(u) - g_\perp^{(v)}(u)) \quad (4.3) \]

\[ FT : \quad i f_\rho m_\rho p_\mu (e^{(T)}r) \int_0^1 du e^{i\xi(p+x^-)} \int_0^u dv (\phi_\parallel(v) - g_\perp^{(v)}(v)) \quad (4.4) \]

We consider only the contribution to the amplitude of the lowest Fock component of the meson wave function, i.e. the quark antiquark component, and we neglect quark masses. The twist-3 vector meson wave function \(g_\perp^{(v)}(u)\) is expressed through the twist-2 wave function \(\phi_\parallel(v)\) with the help of a relation (WW) derived in \([10]\) which is similar to the one derived by Wandzura and Wilczek for \(g_2\) structure function

\[ g_\perp^{(v)}(u) = g_\perp^{(v)WW}(u) = \frac{1}{2} \left[ \int_0^u \frac{dv}{\bar{v}} \phi_\parallel(v) + \int_0^u \frac{dv}{v} \phi_\parallel(v) \right] . \quad (4.5) \]

\[ FT : \quad i \frac{f_\rho}{2} m_\rho (e^{(T)}r) \left( \frac{\bar{u}}{2} \int_0^u \frac{dv}{\bar{v}} \phi_\parallel(v) - \frac{u}{2} \int_0^u \frac{dv}{v} \phi_\parallel(v) \right) . \quad (4.6) \]

c) \(\Gamma_t = \gamma_\mu \gamma_5\), longitudinal polarization: no contribution in our approximation

d) \(\Gamma_t = \gamma_\mu \gamma_5\), transverse polarization:

This matrix element is given by eq. (2.9) of \([11]\)

\[ < 0|\bar{u}(x)\gamma_\mu \gamma_5 u(-x)|\rho(p, \lambda) > = -\frac{1}{2} f_\rho m_\rho \varepsilon_{\mu\alpha\beta} \epsilon^{(\lambda)}_{\nu}p^\alpha x^\beta \int_0^1 du e^{i\xi(px)} g_\perp^{(a)}(u) . \quad (4.7) \]

The difference in sign between our definition for this matrix element and the corresponding one of \([11]\) is related to different sign conventions for \(\varepsilon_{\mu\alpha\beta}\).
\[ <0|\bar{u}(x)\gamma_\mu\gamma_5 u(-x)|\rho(p, \lambda = T) >= \]
\[ = -\frac{1}{2} f_\rho m_\rho \varepsilon_{\mu\nu\alpha\beta} e^{(T)\nu} p^\alpha x^\perp x^\beta \int_0^1 du e^{i\xi(p+u)} g_\perp^{(a)}(u) \quad (4.8) \]

\[ \text{FT}: \quad -\frac{1}{8} f_\rho m_\rho \varepsilon_{\mu\nu\alpha\beta} e^{(T)\nu} p^\alpha p^\beta g_\perp^{(a)}(u) \quad (4.9) \]

\[ g_\perp^{(a)}(u) = g_\perp^{(a)WW}(u) = 2\bar{u} \int_0^u \frac{dv}{v} \phi_{\perp}(v) + 2u \int_u^1 \frac{dv}{v} \phi_{\perp}(v) \quad (4.10) \]

e) \Gamma_\lambda = \sigma_{\mu\nu}, \text{longitudinal polarization:} \]
The parametrization of this matrix element is given by (2.16) of [11]

\[ <0|\bar{u}(x)\sigma_{\mu\nu} u(-x)|\rho(p, \lambda) >= i f_\rho^T \left[ \left( e^{(\lambda)}_{\mu} p_\nu - e^{(\lambda)}_{\nu} p_\mu \right) \int_0^1 du e^{i\xi(px)} \phi_{\perp}(u) \right. \]
\[ + (p_\mu x_\nu - p_\nu x_\mu) \frac{(e^{(\lambda)} x)}{(px)^2} m_\rho^2 \int_0^1 du e^{i\xi(px)} \left( h_{||}^{(t)}(u) - \frac{1}{2} \phi_{\perp}(u) - \frac{1}{2} h_3(u) \right) \]
\[ + \frac{1}{2} \left( e^{(\lambda)}_{\mu} x_\nu - e^{(\lambda)}_{\nu} x_\mu \right) \frac{m_\rho^2 (px)}{(px)^2} \int_0^1 du e^{i\xi(px)} \left( h_3(u) - \phi_{\perp}(u) \right) \left] \right. \quad (4.11) \]

\[ \text{FT}: \quad \frac{1}{2} f_\rho^T m_\rho \left( p_\mu r_\nu - p_\nu r_\mu \right) \int_0^1 dv \left( h_{||}^{(t)}(v) - \phi_{\perp}(v) \right) \quad (4.12) \]

\[ h_{||}^{(t)}(u) = h_{||}^{(t)WW}(u) = \xi \left( \int_0^u \frac{dv}{v} \phi_{\perp}(v) + \int_u^1 \frac{dv}{v} \phi_{\perp}(v) \right) \quad (4.13) \]

With the help of this relation we find the following formulae

\[ \text{FT}: \quad \frac{1}{2} f_\rho^T m_\rho \left( p_\mu r_\nu - p_\nu r_\mu \right) u\bar{u} \left( \int_0^1 \frac{dv}{v} \phi_{\perp}(v) - \int_0^1 \frac{dv}{v} \phi_{\perp}(v) \right) \quad (4.14) \]

f) \Gamma_\tau = \sigma_{\mu\nu}, \text{transverse polarization:} \]
We have to consider two cases: the case which gives the chiral-odd contribution to the amplitude without spin-flip and the case with double spin-flip. In both cases the spins of the quarks in the loop are parallel. But in the double spin-flip case a spin-flip of orbital angular momentum by two units occurs.
The wave function of a transversely polarized meson which contributes to the amplitude without spin-flip reads

\[ i \frac{g}{2} f_{\rho}^{T} \left( e_{\mu}^{(T)} p_{\nu} - e_{\nu}^{(T)} p_{\mu} \right) \phi_{\perp}(u). \] (4.15)

For double spin-flip it is given by the second term in the square bracket of eq. (4.11)

\[ \frac{g}{2} f_{\rho}^{T} m_{\rho}^{2} \left( p_{\mu} r_{\nu} - p_{\nu} r_{\mu} \right) \left( e^{(T)T} r \right) \int_{0}^{u} dv \int_{0}^{v} d\eta \left( h_{||}^{(t)}(\eta) - \frac{1}{2} \phi_{\perp}(\eta) - \frac{1}{2} h_{\perp}(\eta) \right). \] (4.16)

## 5 Helicity amplitudes

Now we are in a position to combine all results and calculate them the helicity amplitudes with the help of eq.(2.4).

We introduce short-hand notation

\[ C = i\alpha^{2} \frac{N^{2} - 1}{N^{2}} eQ_{q} \]

The contributions of the chiral-even configurations to various helicity amplitudes have the forms

\[ M_{\text{even}}^{+0} = -iC \int \frac{d^{2}k}{k^{2}(k - q)^{2}} \frac{d^{2}r \ du}{4\pi} (1 - 2u) \]

\[ \frac{(re^{(+))}}{r^{2}} \int_{0}^{u} dv \int_{0}^{v} d\eta \left( \phi_{||}(v) + \frac{1}{u} \phi_{||}(v) \right) \] (5.1)

\[ M_{++}^{\text{even}} = C \int \frac{d^{2}k}{k^{2}(k - q)^{2}} \frac{d^{2}r \ du}{4\pi} \int_{0}^{u} dv \int_{0}^{v} d\eta \left( \phi_{||}(v) + \frac{1}{u} \phi_{||}(v) \right) \]

\[ \frac{u\bar{u}}{2} \int_{0}^{u} dv \int_{0}^{v} d\eta \left( \phi_{||}(v) + \frac{1}{u} \phi_{||}(v) \right) \] (5.2)

\[ M_{\text{even}}^{+\text{even}} = C \int \frac{d^{2}k}{k^{2}(k - q)^{2}} \frac{d^{2}r \ du}{4\pi} \int_{0}^{u} dv \int_{0}^{v} d\eta \left( \phi_{||}(v) + \frac{1}{u} \phi_{||}(v) \right) \]

Next we calculate the two-dimensional integrals over \( r, k \) and the integral over \( u \). The result for the single spin-flip amplitude can be written as
\[ M_{+0}^{\text{even}} = C \frac{1}{2} \int \frac{d^2k}{k^2(k-q)^2} \, du \, f_\rho (1-2u) \]

\[ e^{(+) \left[ \frac{q}{q^2 u} + \frac{k-qu}{(k-qu)^2} - (u \leftrightarrow \bar{u}) \right] } \]

\[ = C f_\rho \frac{2\pi}{|q|^4} \left( \frac{qe^{(+)}}{|q|^4} \right) \int_0^1 \frac{du}{u \bar{u}} \left( 1 - 2u \right) \phi_{||}(u) \ln \frac{\bar{u}}{u} \]  
(5.4)

This formula was derived first in [5].

For the asymptotic form of \( \phi_{||}(u) = 6u \bar{u} \) we get

\[ M_{+0}^{\text{even}} = i s \alpha_s^2 \frac{N^2 - 1}{N^2} eQ_f \rho \frac{12\pi}{|q|^4} \frac{(qe^{(+)})}{|q|^4} . \]  
(5.5)

For the non spin-flip amplitude the integration over \( r \) and \( k \) leads to the expression

\[ M_{++}^{\text{even}} = -C f_\rho m_\rho \frac{2\pi}{|q|^4} \int_0^1 \frac{du}{u \bar{u}} g_{\perp}^{(e)WW}(u) . \]  
(5.6)

For the asymptotic form of the meson wave function (see eq. (4.5)) the integration over \( u \) gives

\[ M_{++}^{\text{even}} = -i s \alpha_s^2 \frac{N^2 - 1}{N^2} eQ_f \rho m_\rho \frac{6\pi}{|q|^4} \left( \ln \frac{1 - u_{\text{min}}}{u_{\text{min}}} - 1 + 2u_{\text{min}} \right) , \]  
(5.7)

where we restricted the integration region to \([1 - u_{\text{min}}, u_{\text{min}}]\), \( u_{\text{min}} = m_\rho^2/q^2 \). The integration over this interval gives the contribution to \( M_{++}^{\text{even}} \) of the region where the interquark distances \( r^2 \sim \frac{1}{q^2 u \bar{u}} < \frac{1}{m_\rho^2} \) are small, and where we can use pQCD.

For the double spin-flip amplitude the two-dimensional integrations over \( r \) and \( k \) give

\[ M_{+-}^{\text{even}} = -C f_\rho m_\rho \frac{4\pi}{|q|^4} \int_0^1 \frac{du}{u^2 \bar{u}^2} \left( \frac{1}{4} g_{\perp}^{(e)WW}(u) - u \bar{u} g_{\perp}^{(e)WW}(u) \right) \left[ \frac{1}{2} + (1 - 2u) \ln \frac{\bar{u}}{u} \right] \]  
(5.8)

For the asymptotic form of wave function (see eqs. (4.3), (4.10)) this reads

\[ M_{+-}^{\text{even}} = -i s \alpha_s^2 \frac{N^2 - 1}{N^2} eQ_f \rho m_\rho \frac{18\pi}{|q|^4} . \]  
(5.9)
The calculations of the chiral-odd contributions to helicity amplitudes are not so straightforward as in the chiral-even case.

Let us look again at eq. (2.4). This formula gives the amplitude as a convolution of the hard scattering amplitude $A$ and the vector meson light-cone wave function $\Psi$. The hard scattering amplitude is given by eqs. (3.1), (3.5), (3.3). It is well known [13] that at high energies such hard scattering amplitudes can be further factorized into a product of a dipole scattering amplitude and a perturbative photon wave function given by an $e\bar{u}\gamma_{\mu}u$ vertex. Therefore our scattering amplitude (2.4) can be rewritten as a product of the photon wave function, the dipole scattering amplitude and the vector meson wave function

$$M_{\text{odd}} = i\alpha_s^2 \frac{N^2 - 1}{N^2} \frac{1}{4\pi} \int \frac{d^2k}{k^2(k-q)^2} \frac{d^2r}{4\pi} \frac{du}{f_{\text{dipole}}} \Psi^\gamma(r, u) f_{\text{dipole}} \Psi^\rho(r, u). \quad (5.10)$$

This property is often called 'diffractive factorization'. It is valid even in the non-perturbative region where the photon splits into a dipole of large size. There are lots of phenomenological models in the literature which describe the photon wave function in this non-perturbative region.

Fortunately in our process high $t$ quark-dipole scattering chooses small interquark separations $r$. Therefore we need only the photon wave functions on the light-cone which are very similar to the light-cone wave functions of a vector meson [14].

To simplify the calculation we use the following trick which permits us to proceed in a similar way as in the chiral-even case. First we calculate the chiral-odd amplitude for the perturbative photon wave function with finite quark mass. For that we have to insert into eq. (2.4) the result of the trace of $\sigma_{\mu \nu}$ structure with the hard scattering amplitude (see the last eq. (5.12)) and the chiral-odd vector meson wave function (eq.(4.15)). For the no spin-flip case the result has the form

$$M_{++}^m = C \int \frac{d^2k}{k^2(k-q)^2} \frac{d^2r}{4\pi} \frac{du}{f_{\text{dipole}}} K_0(m r) f^T_{\rho} m \langle \langle e^{(+)}, e^{(+)\ast} \rangle \rangle \phi_\perp(u), \quad (5.11)$$

whereas for the single spin-flip transition it reads

$$M_{+0}^m = (-i)C \int \frac{d^2k}{k^2(k-q)^2} \frac{d^2r}{4\pi} \frac{du}{f_{\text{dipole}}} mK_0(m r) f^T_{\rho} m \langle \langle u\bar{u} \rangle \rangle \left( \int_u^1 \frac{dv}{v} \phi_\perp(v) - \int_0^u \frac{dv}{v} \phi_\perp(v) \right). \quad (5.12)$$

In these two cases only the quark spin configuration in which the spins of quark and antiquark are parallel to the that of the incoming photon contributes.
A comparison of (5.11) and (5.10) leads to the identification of the relevant chiral-odd meson and photon wave functions

\[ \Psi^\rho_{\text{odd}}(u) = \frac{2\pi}{\sqrt{2}} \frac{\delta^{ij}}{N} f^T_\rho \phi_\perp(u) , \]

(5.13)

\[ \Psi^\gamma_{\text{pert}} = eQ_q \sqrt{2} \frac{\delta^{ij}}{2\pi} m K_0(mr) , \]

(5.14)

with definite colours \(i\) and \(j\) of quark and antiquark respectively (note that the perturbative chiral-odd photon wave function is well known, see e.g. [17]). The non-perturbative light-cone wave function of the photon has to have a similar form as the wave function of vector mesons (5.13)

\[ \Psi^\gamma_{\text{odd}}(u) = \frac{2\pi}{\sqrt{2}} eQ_q \frac{\delta^{ij}}{N} f_\gamma \phi_\perp(u) . \]

(5.15)

Except the evident factor \(eQ_q\) the only difference between the meson and the photon wave function are the different dimensional coupling constants, \(f_\rho\) and \(f_\gamma^T\). The photon coupling constant \(f_\gamma\) is a product of the quark condensate \(\langle \bar{q}q \rangle\) and its magnetic susceptibility [15], [16]

\[ f_\gamma = \langle \bar{q}q \rangle \chi \approx 70\text{MeV} , \]

(5.16)

at the normalization point \(\mu = 1\text{GeV}\). Both parameters \(\langle \bar{q}q \rangle\) and \(\chi\) have been tested in various QCD sum rule applications. Let us emphasize in view of our final results that the value of \(\chi\) is large. The rough sum rule estimate which takes into account only the lowest lying hadronic state, namely the \(\rho\) meson is \(\chi = -2/m^2_\rho = -3.3\text{ GeV}^{-2}\). A more accurate analysis gives \(\chi = -4.4\text{ GeV}^{-2} \pm 0.4\) [16] (the value of \(f_\gamma\) in eq.(5.16) corresponds roughly to this value for \(\langle \bar{q}q \rangle = -0.017\text{ GeV}^3\)). Note that the quantity \(\chi\) being a parameter describing the QCD vacuum plays an important role in the known sum rules for the electromagnetic form factor of baryons.

Now in order to obtain the amplitude of the interest we have only to substitute

\[ \Psi^\gamma_{\text{pert}} \rightarrow \Psi^\gamma_{\text{non-pert}}(u) \]

(5.17)

in eqs. (5.11) and (5.12) and after taking into account the trace over colour indices obtain

\[
M^\text{odd}_{++} = \frac{C}{N} \int \frac{d^2k}{k^2(k - q)^2} \frac{d^2r}{4\pi} du f_{\text{dipole}} \\
2\pi^2 f_\gamma \phi_\perp^\gamma(u) f^T_\rho (e^{(+)e^{(+)}}) \phi_\perp(u) \\
= -\frac{C}{N} f^T_\rho f_\gamma \frac{4\pi^3}{|q|^2} \int_0^1 \frac{du}{u^2 u^2} \phi_\perp^\gamma(u) \phi_\perp(u) .
\]

(5.18)
For the asymptotic forms of the photon and the vector meson wave functions \( \phi_{\perp}(u) = \phi_{\perp}(u) = 6u\bar{u} \) this gives

\[
M^{\text{odd}}_{++} = -i\alpha_s^2 \frac{N^2 - 1}{N^3} eQ_q \frac{1}{|q|} f^T f_{\gamma} .
\]  

Let us compare this result with the chiral-even contribution to the non-flip amplitude (5.7). The origin of the large relative coefficient \( \sim (2\pi)^2 \) in the chiral-odd contribution can be related to the factors of \( 2\pi \) in Eqs. (5.13), (5.15) which in turn are related to QCD sum rules calculations for which the appearance of such factors is actually typical. When a quark loop is "broken up" by inserting a quark condensate \( < \bar{q}q > \) there is no integration over the full loop momentum range or \( \bar{q}q \) phase space left as the virtuality is restricted by some hadronic scale. The numerical factor is just the (two dimensional) minimal phase space volume and the actual expansion parameter is \( \sim (2\pi)^2 < \bar{q}q > \).

Due to the above mentioned large numerical factor and large value of \( \chi \) the chiral-odd contributions to helicity amplitudes are large. Note that the direct source of chiral symmetry breaking related to a nonzero quark mass \( m \) leads to negligible contributions to the helicity amplitudes since light quarks have very small current masses and the \( \gamma \to q\bar{q} \) chiral-odd vertex is proportional to this mass. If one would instead consider a model where the point-like form of the \( \gamma \to q\bar{q} \) chiral-odd vertex is used and the quark masses are of order of constituent quark masses, \( m \sim 200\text{MeV} \), then still the chiral-odd amplitudes would be much smaller since the value of constituent quark mass is much smaller than the parameter \( \sim (2\pi)^2 f_{\gamma} \).

For the single spin flip case we obtain

\[
M^{\text{odd}}_{++} = C \frac{\sqrt{2} \pi^2}{4\pi} \int \frac{d^2k}{k^2(k - q)^2} d^2r \int_0^1 du \phi_{\perp}(u) f^T m_{\rho} 
\]

\[
(r_x + i r_y) f_{\text{dipole}} u\bar{u} \left( \int \frac{dv}{v} \phi_{\perp}(v) - \int_0^u \frac{dv}{v} \phi_{\perp}(v) \right) 
\]

\[
= -C \frac{\sqrt{2} \pi (2\pi)^2}{|q|^5} \int_0^1 dv f_{\gamma} \phi_{\perp}^T m_{\rho} \frac{(1 - 2u)}{u^3\bar{u}^3} 
\]

\[
\left( \int \frac{dv}{v} \phi_{\perp}(v) - \int_0^u \frac{dv}{v} \phi_{\perp}(v) \right) .
\]

(5.20)

Using the asymptotic forms for \( \phi_{\perp}^T(u) \) and \( \phi_{\perp}(u) \) this expression takes the form

\[
M^{\text{odd}}_{++} = C \frac{f_{\gamma} f^T m_{\rho}}{|q|^5} \int_0^1 dv \frac{(1 - 2u)^2}{u\bar{u}} .
\]

(5.21)
Performing the remaining integral over $u$ we obtain

$$M_{+0}^{\text{odd}} = i\alpha^2 s \frac{N^2 - 1}{N^3} e Q f_\gamma f_\rho \, m_\rho \frac{144 \sqrt{2} \pi^3}{|q|^5} \left( \ln \frac{1 - u_{\text{min}}}{u_{\text{min}}} - 2(1 - 2 u_{\text{min}}) \right). \quad (5.22)$$

where we have again introduced suitable integration limit.

Finally we calculate the chiral-odd part of the double spin-flip amplitude. This case differs from the cases with no spin-flip and single spin-flip in one important aspect. In the last two cases only one spin configuration in the quark loop contributes, when the sum of quark helicities is equal to the helicity of incoming photon. For double spin-flip both chiral-odd spin configurations ($\lambda_q = +1/2$, $\lambda_{q\bar{q}} = +1/2$) (case (a)) and ($\lambda_q = -1/2$, $\lambda_{q\bar{q}} = -1/2$) (case (b)) contribute. For (a) the helicity of the initial photon $\lambda_\gamma = +1$ is carried by the helicity of quarks $S_z = \lambda_q + \lambda_{q\bar{q}} = +1$. After interaction the dipole acquires $L_z = -2$ which results in the meson helicity $\lambda_\rho = S_z + L_z = -1$. In the case (b) the photon with $\lambda_\gamma = +1$ splits into $q\bar{q}$ state with ($\lambda_q = -1/2$, $\lambda_{\bar{q}} = -1/2$) $S_z = -1$ and $L_z = +2$. A flip by 2 units of $L_z$ happens and the dipole has $S_z = -1$ and $L_z = 0$, which gives again the meson helicity $\lambda_\rho = S_z + L_z = -1$.

The contribution related to case (a) $M_{+}^{\text{odd a}}$ can be calculated in a similar way. Here only the second term in eq. (4.11) contributes and after introducing the chiral-odd non-perturbative photon wave function (5.15) and using the asymptotic forms of the functions $h_{||}^{(i)}(u) = 3(2u-1)^2$, $\phi_\perp(u)$ and $h_3(u) = 1 - C_2^{1/2}(2u-1)$ ($C_2^{1/2}$ is the Gegenbauer polynom) we obtain

$$M_{+}^{\text{odd a}} = -\frac{C}{N} \frac{3\pi}{4} f_\gamma f_\rho \, m_\rho^2 \left( \int \frac{d^2k}{k^2(k - q)^2} d^2r \, du \, \phi_\gamma^3(u) \left[ u^2 \bar{u}^2 f_{\text{dipole}}(e^{(+) r})(e^{(-)* r}) \right] \right). \quad (5.23)$$

After performing the relevant integrations the final result takes the form

$$M_{+}^{\text{odd a}} = i\alpha^2 s \frac{N^2 - 1}{N^3} e Q f_\gamma f_\rho \, m_\rho \frac{144 \pi^3}{|q|^6} \left( 2 \ln \frac{1 - u_{\text{min}}}{u_{\text{min}}} - 3(1 - 2 u_{\text{min}}) \right). \quad (5.24)$$

For case (b) we need the photon wave function with $S_z = -1$ and $L_z = +2$. For a vector meson the corresponding wave function is described by the second term in the square bracket of eq. (4.11). Unfortunately, we do not know a comprehensive QCD analysis of the photon wave function beyond twist-2. In this situation we make the assumption that the photon wave function with $S_z = -1$, $L_z = +2$ differs from the corresponding meson wave function only by the replacement $f_\rho^T \rightarrow e Q f_\gamma$. The

$$M_{+}^{\text{odd a}} = M_{+}^{\text{odd b}} \quad (5.25)$$
and

\[ M_{+}^{\text{odd}} = 2 M_{+}^{\text{odd} \, a}, \]  

(5.26)

where \( M_{+}^{\text{odd} \, a} \) is given by eq. (5.24).

### 6 Discussion

We have explicitly derived helicity amplitudes of the process \( \gamma q \rightarrow V q \). They are the sums of the chiral-even and the chiral-odd contributions

\[ M_{\lambda_{1} \lambda_{2}} = M_{\lambda_{1} \lambda_{2}}^{\text{even}} + M_{\lambda_{1} \lambda_{2}}^{\text{odd}}. \]  

(6.1)

The formulae for the amplitudes are given by eqs. (5.5), (5.7), (5.9), (5.19), (5.22), (5.26), (5.24), they are the main results of the present work.

At asymptotically high momentum transfer the dominant helicity amplitude is \( M_{+}^{0} \). Its chiral-even part \( M_{+}^{\text{even} \, 0} \) has the minimal, \( \sim 1/|q|^3 \), suppression. To discuss the onset of this asymptotic regime we consider the ratios

\[ \frac{M_{+}^{0}}{M_{+}^{\text{even} \, 0}} = 1 - \frac{24\pi^{2} f_{\gamma} f_{T} T_{\rho} m_{\rho}}{N f_{\rho} |q|^{2}} \left( \ln \frac{1 - u_{\min}}{u_{\min}} - 2(1 - 2u_{\min}) \right) \]  

(6.2)

\[ \frac{M_{+}^{+}}{M_{+}^{\text{even} \, 0}} = \frac{m_{\rho}}{|q| \sqrt{2}} \left( \ln \frac{1 - u_{\min}}{u_{\min}} - 1 + 2u_{\min} \right) + \frac{24\pi^{2} f_{\gamma} f_{T} T_{\rho}}{N f_{\rho} |q| \sqrt{2}} \]  

(6.3)

\[ \frac{M_{-}^{+}}{M_{+}^{\text{even} \, 0}} = \frac{3 m_{\rho}}{\sqrt{2} |q|} \left( \ln \frac{1 - u_{\min}}{u_{\min}} - 3(1 - 2u_{\min}) \right) \]  

(6.4)

The chiral-odd parts of the helicity amplitudes are proportional to dimension-full coupling constants \( f_{\gamma}, f_{T}^{T} \). The scale dependence for three active flavours is, see [10],

\[ \frac{f_{\gamma}^{T}(Q^{2})}{f_{\gamma}^{T}(\mu^{2})} = L^{1/27}, L = \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(\mu^{2})}. \]

The factorization scale in the case of our process is \( Q^{2} \sim -t \).

The counting of relative factors of \( 1/|q| \) for the chiral-even and the chiral-odd contributions to the helicity amplitudes in eqs. (6.2), (6.3), (6.4) can be easily understood. These contributions can be represented as convolutions of the corresponding photon and vector meson wave functions with the dipole scattering amplitude (3.10). The dipole amplitude does not depend on the helicity state of the quarks. (Helicity states do not change during the dipole interaction.) The projection of \( q\bar{q} \) angular momentum on the axises of dipole motion \( L_{z} \) can change due to the interaction, \( L_{z} \rightarrow L'_{z} \). The important observation is that in the case of high \( t \) scattering such a change (\( L_{z} \neq L'_{z} \)) is not suppressed by a factor \( 1/|q| \). Therefore the power counting of various contributions to helicity amplitudes is
determined entirely by the behaviour at small $r$ of the corresponding photon and vector meson wave functions.

The wave function describing the chiral-even $q\bar{q}$ fluctuation of the photon is given by first order perturbation theory. In the impact parameter representation it is simply a Fourier transform of the quark propagator, see eqs. (3.8) and (3.11). Note that it is power-like divergent at small interquark separations, $\sim r/r^2$. The total helicity of the quarks in the chiral even configuration is zero, $S_z = 0$, therefore the helicity of the photon is carried by the orbital angular momentum of the quarks, $\lambda_\gamma = S_z + L_z = L_z = \pm 1$.

In contrast in the non-perturbative chiral-odd $q\bar{q}$ configuration the helicity of the photon is carried by the total helicity of quarks $\lambda_\gamma = S_z$, $L_z = 0$ (the other possibility $\lambda_\gamma = +1$, $S_z = -1$, $L_z = +2$ is relevant only for $M_{+-}^{odd}$). The small $r$ asymptotics of this chiral-odd wave function is constant, it is described by the dimensionful non-perturbative parameter $f_\gamma$, see eq. (5.15). This asymptotics should be compared with the asymptotics of the chiral-even photon wave function $\sim 1/r \sim |q|$. Various vector meson wave functions have different small $r$ behaviour. The wave functions of (scaling) twist two describe $q\bar{q}$ pair with $L_z = 0$, they are constants at small $r$, see eqs. (4.2) and (4.15) for chiral-even and chiral-odd ones respectively. The configurations with $L_z = 0$ are described by the wave functions of higher twists. In the case $L_z = \pm 1$ they behave as $\sim r \sim 1/|q|$, see eqs. (4.4), (4.9) for chiral-even and (4.12) for chiral-odd wave functions. The state with $L_z = \pm 2$ is given by the the wave function $\sim r^2 \sim 1/q^2$, see eq. (4.16).

Let us discuss the helicity non flip amplitude (6.3). The chiral-even part of this amplitude is given by the configuration with $L_z = +1$, therefore the product of the corresponding photon and meson wave functions (4.4, 4.9) is $\sim \frac{1}{r} \cdot m_\rho f_\rho r = const$. The chiral-odd part in this amplitude is given by the configuration with $L_z = 0$ and in this case the product of chiral-odd photon wave function (5.15) and vector meson wave function (4.15) $\sim f_\gamma \cdot f_T = const$. This is the explanation why both parts of (6.3) have similar $1/|q|$ suppression.

In the chiral-odd case the leading twist $L_z = 0$ wave function of the meson enters the amplitude, in the chiral-even part it is the wave function of higher twist $L_z = \pm 1$. This difference is compensated by the difference in the small $r$ behaviour of the point-like chiral-even and the non-perturbative chiral-odd wave functions of the real photon. The same arguments apply to the spin-flip amplitudes (6.2) and (6.4).

We find that the chiral-odd contributions to the amplitudes are accompanied with astonishingly large numerical coefficients. In the case of $M_{+-}$ the chiral-even and the chiral-odd parts of (6.1) add with the same signs. In contrast, the chiral-even and the chiral-odd parts of $M_{+0}$ or $M_{+-}$ enters with opposite sign which leads to an effective reduction of these amplitudes for intermediate $|t|$. Even in the case of $M_{+0}$ (or $M_{+-}$) where the chiral-odd part is formally $\sim 1/q^2$ suppressed in comparison with the corresponding chiral-even part, the chiral-odd
part cannot be neglected up to very large momentum transfers. For instance in the \(|t|\) interval 3 GeV$^2 \div 8$ GeV$^2$ the ratio (6.2) takes values from 0.56 to 0.62, and the ratio (6.4) varies from −0.05 to 0.08. Though the chiral-even and the chiral-odd contributions to $M_{++}$ are of the same order with respect to $1/|q|$ counting the chiral-odd one can be dominant up to very large \(|t|\). According to (6.3) for the \(t\) range $3 \div 8$ GeV$^2$ the chiral-even part constitutes only $10 \div 20\%$ of the $M_{++}$ amplitude and for \(|t| \approx 100$ GeV$^2$ $M_{++}^{\text{even}} \approx 0.72 M_{++}^{\text{odd}}$.

Let us discuss next the relative magnitude of the various helicity amplitudes. On the one hand at asymptotically large \(|t|\) $M_{+0} \to M_{+0}^{\text{even}}$ will dominate. On the other hand in the intermediately large \(t\) region there is a large compensation between chiral-even and chiral-odd parts of $M_{+0}$, the similar compensation takes place for $M_{+-}$ amplitude. Therefore in this region the non spin-flip $M_{++}$ amplitude dominates strongly. According to eqs. (6.2), (6.3), $M_{+0}$ will exceed $M_{++}$ only at \(|t| > 40$ (GeV)$^2$. For the \(|t|\) interval 3 GeV$^2 \div 8$ GeV$^2$

\[
\frac{M_{+0}}{M_{++}} \sim 0.25 \div 0.35 \tag{6.5}
\]

\[
\frac{M_{+-}}{M_{++}} \sim -0.02 \div 0.04 \tag{6.6}
\]

Both chiral-even and chiral–odd parts of the amplitudes were calculated expecting \(|t|\) to be large, or in the leading order of $1/|q|$ expansion. As usual in the QCD approach to any exclusive reaction, the question about the region of applicability of these results is open until the power corrections have not studied. In our case the situation is more difficult because the factorization of the amplitude into hard and soft parts is violated for the chiral-even part of $M_{++}$ and the chiral-odd parts of $M_{+0}$ and $M_{+-}$. In these cases the corresponding integrals over the quark longitudinal momentum \(u\) contain the end point logarithmic singularities. This means that in these cases the contribution of the soft regions, where the transverse separation between quark and antiquark is large, is not power suppressed. The account of higher orders would result in an appearance of Sudakov like form factor which describes the suppression due to the change of the colour direction of motion without radiation of gluons. In the hard region, where $q\bar{q}$ pair scatters as a colourless dipole of the small size, this Sudakov suppression doesn’t work. It will come into a game for the scattering of dipoles of large sizes and therefore will lead to an effective suppression of the soft region.

At present we simply restrict the corresponding \(u\) integrals to the interval $[1 - u_{\text{min}}, u_{\text{min}}]$, $u_{\text{min}} = m^2_\rho / q^2$, which corresponds to the contribution of the hard region only. It is not known at the moment how to calculate in a model independent way the soft contributions to the chiral-even part of $M_{++}$ and the soft contributions to the chiral-odd parts of $M_{+0}$ and $M_{+-}$. The good news is, however, that the chiral-odd part $M_{++}^{\text{odd}}$ of the dominant in the intermediatly large \(|t|\) region helicity amplitude $M_{++}$ is free from this end-point singularity.
This chiral-odd part is numerically considerably larger than the hard contribution to its chiral-even counterpart $M^{even}_{++}$. Therefore we can expect that the relative uncertainty related with the uncalculated soft contributions to $M^{even}_{++}$ is small for $|t| \sim 3 \text{ GeV}^2 \div 8 \text{ GeV}^2$. For $M_{+0}$ and $M_{+-}$ the uncertainties related with the corresponding nonfactorizable parts of $M^{odd}_{+0}$ and $M^{odd}_{+-}$ can be larger. Despite that we believe that in the intermediately large $t$ region our predictions (6.2) and (6.4) for the ratios of the helicity amplitudes will be true on a qualitative level.

A recently reported ZEUS analysis [4] of the angular distribution of $\rho^0$ meson decay products from the process $\gamma p \rightarrow \rho^0 p$ at $-t \sim 1 \div 2 \text{ GeV}^2$ have actually demonstrated the dominance of the non spin-flip amplitude in this $t$ range. Unfortunately, the precision of these data is low and the only conclusions which can be drawn are that the spin-flip amplitudes are small, the relative sign between $M_{++}$ and $M_{+0}$ tends to be positive and the sign between $M_{++}$ and $M_{+-}$ tends to be negative. Note that this is in agreement with our predictions (6.2), (6.3), (6.4).

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