Deformation responses of reinforced concrete beams under pure torsion

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Abstract. The experimental programme that is the focus of this report dealt with twelve beams cast in three groups: Normal Concrete (NC), High Strength Concrete (HSC), and Self Compacting Concrete (SCC). The applied loads, resulting torsional moments, angles of twist and longitudinal strains are listed in Tables and presented in graphs, to aid discussion. The results of the practical work revealed that increasing the compressive strength of a section increases the stiffness of a beam, thus decreasing the angle of twist markedly. Increasing the steel fibre content in a section similarly decreases the value of the angle of twist of the beam, as well as resulting in an increase in stiffness. Increasing the compressive strength of a section also increases the stiffness of a beam, thus decreasing the longitudinal strain. While increasing the steel fibre content in a section decreases the value of longitudinal strain on all beams, resulting in an increase in stiffness, the highest improvement (100%) was seen in the HSC group.

1. Introduction

There are many civil engineering structures where torsional loading could be a significant loading condition, and the behaviour of an RC member must be modelled differently before and after cracking. Before first cracking, concrete behaves as an elastic material, and reinforcement can be ignored [1]. In members with square cross sections, the shear stress that develops in the beam flows around the member and is at a maximum at the midpoint of the outside surface. When the principle tensile stress reaches the concrete’s tensile strength, however, cracking occurs. In RC members under pure torsion, the stiffness of the uncracked member can be predicted by several theoretical approaches, and after cracking, the member behaves as a composite member and all properties of the concrete, its reinforcement, and their interactions must be considered to accurately predict the response to torsion. Many buildings and bridge elements are subjected to significant torsional moments that affect the design and may require strengthening. Clearly, research was required to address this gap in knowledge, and previous torsional strengthening investigations have focused on different strip layouts for ordinary RC beams [2, 3, 4, and 5].

Self-Compacting Concrete (SCC), has numerous advantages over conventional concrete. It is a type of concrete that does not require compaction, because it becomes levelled and compacted under its self-weight. SCC can thus spread and fill every corner of a formwork purely by means of its self-weight, eliminating the need for vibration or any type of compacting effort [6&7]. Fibre reinforced concrete (FRC) is a composite material in which steel fibres or other tension materials are incorporated to improve the tensile strength and other properties of the concrete. The quantity of fibres added to a concrete mix is measured as a percentage of the total volume of the composite (concrete and fibres); this may be termed Vf. Typically, the amount of steel fibre varies between 0.0 and 2.0 percent, by volume [8]. The advantages of adding fibres into a matrix include enhancements in compressive strength, tensile strength, flexural toughness, shear strength, durability and resistance to impact [9].

2. Study Plan

This research aims to study the effect of three variables on the torsion of beams; these variables are

- Type of concrete (NC, HSC, and SCC),
- Changes in compressive strength, and
- The amount of steel fibre included.
Twelve beam specimens of 100x200x2,000mm dimensions were made. These were reinforced with flexure and shear reinforcements ($\rho= 0.94\%$ with spacing between closed stirrups of 80mm c/c), as shown in Figure (1); four were of normal strength concrete (NSC), four were of high strength concrete (HSC), and four were of self-compacting concrete (SCC), all cast and tested under pure torsion up to failure for this purpose.

Three types of concrete mixes in varying strengths were used: NSC of 20 and 40MPa compressive strength, HSC of 60 and 75MPa and SCC of 40 and 60MPa. Two volume fraction ratios of 0.6 and 1.2 percent steel fibres were used in the three concrete types. For the SCC mix, the slump flow, L-box, U-box, and V-funnel tests were performed in the fresh state. In the hardened state, after the concrete had cured, the destructive tests were carried out; these included compressive strength, splitting tensile strength, and static modulus of elasticity, as listed in Table 1.

![Figure 1. Details of the tested beams](image)

| Mix Notation | Beam Symbol | Nominal Compressive Strength (MPa) | Steel Fibres ($V_f$) | Compressive Strength ($f'_c$) (MPa) | Modulus of Elasticity ($E_c$) (GPa) | Modulus of Rupture ($f_r$) (MPa) |
|--------------|-------------|------------------------------------|---------------------|------------------------------------|----------------------------------|-----------------------------|
| NSC          | NCA         | 20                                 | -                   | 21.00                              | 23.20                            | 3.53                        |
|              | NCB         | 40                                 | -                   | 39.34                              | 24.33                            | 4.73                        |
|              | NCA$_{0.6}$ | 20                                 | 0.6%                | 22.04                              | 23.64                            | 5.76                        |
|              | NCA$_{1.2}$ | 20                                 | 1.2%                | 20.22                              | 24.86                            | 7.65                        |
| HSC          | HSCC        | 60                                 | -                   | 61.23                              | 33.62                            | 6.84                        |
|              | HSCD        | 75                                 | -                   | 73.31                              | 42.53                            | 8.41                        |
|              | HSCC$_{0.6}$| 60                                 | 0.6%                | 65.36                              | 37.87                            | 8.44                        |
|              | HSCC$_{1.2}$| 60                                 | 1.2%                | 68.81                              | 38.82                            | 9.83                        |
| SCC          | SCCB        | 40                                 | -                   | 40.7                               | 24.05                            | 4.59                        |
|              | SCCC        | 60                                 | -                   | 68.32                              | 39.85                            | 7.76                        |
|              | SCCB$_{0.6}$| 40                                 | 0.6%                | 42.96                              | 24.44                            | 7.69                        |
|              | SCCB$_{1.2}$| 40                                 | 1.2%                | 44.15                              | 24.45                            | 8.08                        |
3. Results and Discussion

The beams were tested under torsional moments from eccentricity, with the applied load 750mm from the centre of the beam, as shown in Figure 2. The testing continued up to failure, and this failure mode led to excessive twisting angles and cracking, as is clear in Figure 3. Two dial gauges were positioned at the points of maximum torsional action to measure the angle of twist, and a further two dial gauges were positioned at the two ends of the tested beam to measure the longitudinal elongation of the beam due to the formation and propagation of torsional cracks. After that point, the longitudinal elongation is transformed to longitudinal strain, and the average of angles of twist and the total longitudinal strain were calculated for each tested beam, as shown in Table 2.

![Figure 2. Details of the tested beams](image1)

![Figure 3. Deformation of the Tested Beams](image2)
Table 2. Overall results from tests

| Mix Notation | Beam Symbol | Nominal Compressive Strength (MPa) | Steel Fibres (V_f) | Ultimate Load (P_u) (kN) | Torsional Moment (T) (kN/m) | Angle of Twist (Ø) (rad.) | Total Strain (ε) (%) |
|--------------|-------------|------------------------------------|-------------------|-------------------------|-----------------------------|--------------------------|---------------------|
| NSC          | NCA         | 20                                 | -                 | 25                      | 18.75                       | 0.01165                 | 0.0555              |
|              | NCA_0.6     | 20                                 | 0.6%              | 31                      | 23.25                       | 0.01165                 | 0.0471              |
|              | NCA_1.2     | 20                                 | 1.2%              | 34                      | 25.50                       | 0.01200                 | 0.0471              |
| HSC          | HSCC        | 60                                 | -                 | 57                      | 42.75                       | 0.01570                 | 0.0630              |
|              | HSCD        | 75                                 | -                 | 66.5                    | 49.88                       | 0.02120                 | 0.0770              |
|              | HSCC_0.6    | 60                                 | 0.6%              | 62                      | 46.50                       | 0.03896                 | 0.0590              |
|              | HSCC_1.2    | 60                                 | 1.2%              | 66                      | 49.50                       | 0.03320                 | 0.0550              |
| SCC          | SCCB        | 40                                 | -                 | 40                      | 30.00                       | 0.01085                 | 0.0395              |
|              | SCCC        | 60                                 | -                 | 55                      | 41.25                       | 0.02170                 | 0.0630              |
|              | SCCB_0.6    | 40                                 | 0.6%              | 47                      | 35.25                       | 0.02220                 | 0.0415              |
|              | SCCB_1.2    | 40                                 | 1.2%              | 51                      | 38.25                       | 0.01220                 | 0.0405              |

4. Angle of Twist (Ø)
The angle of twist is the three-dimensional deformation in the direction of torsional moment action. The average of two angles of twist in each tested beam was graphed against the torsional moment, as in Figure (4). The comparisons of the angle of twist for all tested beams were made at 25 kN, which is the failure load of NCA.

Figure 4. Torsional moment vs. angle of twist for all tested beams

4.1 Effect of Concrete Type on Angle of Twist
Comparing the three concrete types, NSC, HSC, and SCC, it can be noted that, the HSC group of beams is the group with the most stiffness according to the angle of twist, followed by the SCC group; see
figures 5, 6 and 7. On the other hand, a comparison between the NCB, which is a normal strength concrete beam, and the SCCB, which is self-compacting concrete beam, shows that the two beams have the same nominal strength of 40MPa, but that at a load of 25kN, the angle of twist of the SCCB is 0.0051 rad, which is smaller than that of the NCB, which was 0.0066 rad. In a similar manner, comparing the HSCC, which is a high strength concrete beam, and the SCCC, which is a self-compacting concrete beam, the two beams both have 60MPa nominal compressive strength, but the angle of twist of HSCC at a load of 25kN is 0.00235 rad, smaller than that of SCCC, which measures 0.00295 rad.
4.2 Effect of Compressive Strength on Angle of Twist
In the NSC group, a comparison between NCA, with 20MPa nominal compressive strength, and NCB, with 40MPa nominal compressive strength, showed that the angle of twist of NCB was smaller than that of NCA, at 0.0066 rad and 0.01165 rad, respectively.
In the HSC group, a similar comparison between HSCC, with 60MPa nominal compressive strength, and HSCD, with 75MPa nominal compressive strength, showed that the angle of twist of HSCD was smaller than that of HSCC, at 0.00216 rad and 0.00235 rad, respectively.
In the SCC group, the comparison between SCCB, with 40MPa nominal compressive strength, and SCCC, with 60MPa nominal compressive strength, showed that angle of twist of SCCC was smaller than that of SCCB, at 0.00235 rad and 0.0051 rad, respectively.
These results revealed that increasing the compressive strength of the section increases the stiffness of beam, thus decreasing the angle of twist markedly.

4.3 Effect of Steel Fibres on Angle of Twist
The results revealed that increasing the steel fibre content in the section decreased the value of angle of twist of the beam clearly, resulting in increases in stiffness.

4.4 Improvements in Angle of Twist
In the case of the angle of twist, an improvement is represented by a decrease in value, which reflects a reduction in the deformation in the member. Decreases in angle of twist were compared in terms of the angle of twist of the reference beam for each group of concrete types. The decrease in angle of twist of any beam was then compared with the angle of twist of the weakest beam in this work, which was the NCA at an applied load of 25kN, as seen in Table (3). From Table (3), it is clear that the angle of twist can be improved by increasing the compressive strength of the section; the highest improvement (100%) is seen in the HSC group. An increase in compressive strength is better than the use of steel fibres in terms of improving the angle of twist, as seen in the NCB and NCA0.6 in Table (3) (43.3 and 16.9%, respectively). While increasing the steel fibre content does decrease the angle of twist, this is not as significant as increasing the compressive strength of the beam; Figure (6) graphs the results in full.

Figure 5. Torsional moment vs. angle of twist for each type of concrete
Table 3. Improvement of angle of twist

| Mix Notation | Beam Symbol | Nominal Compressive Strength (MPa) | Steel Fibres (Vf) | Angle of Twist (Ø) (rad.) | Decreasing According to the Reference Beam (%) | Decreasing According to NCA Beam (%) |
|--------------|-------------|-----------------------------------|-------------------|---------------------------|---------------------------------------------|-------------------------------------|
| NSC * NCA    | 20          | -                                 | 0.01165           | -                         | -                                           | -                                   |
| NCB          | 40          | -                                 | 0.00660           | 43.3                      | 43.3                                        | 43.3                                |
| NCA0.6       | 20          | 0.6%                              | 0.00968           | 16.9                      | 16.9                                        | 16.9                                |
| NCA1.2       | 40          | 1.2%                              | 0.00900           | 22.7                      | 22.7                                        | 22.7                                |
| HSC * HSCC   | 60          | -                                 | 0.00235           | -                         | 79.8                                        |                                    |
| HSCD         | 75          | -                                 | 0                 | 100                       | 100                                         | 100                                 |
| HSCC0.6      | 60          | 0.6%                              | 0                 | 100                       | 100                                         | 100                                 |
| HSCC1.2      | 60          | 1.2%                              | 0                 | 100                       | 100                                         | 100                                 |
| SCC * SCCB   | 40          | -                                 | 0.00510           | -                         | 56.2                                        |                                    |
| SCCC         | 60          | -                                 | 0.00295           | 42.2                      | 74.7                                        |                                    |
| SCCB0.6      | 40          | 0.6%                              | 0.00400           | 21.6                      | 65.7                                        |                                    |
| SCCB1.2      | 40          | 1.2%                              | 0.00240           | 52.9                      | 79.4                                        |                                    |

Figure 6. Improving of angle of twist for types of beams

Further, Table.3 emphasises that, when a comparison adopted based on specific compressive strength, by continuing to increase the compressive strength of the section, the angle of twist will continue to improve, such that the HSC beams achieved the highest values (100%). Figure.7 graphs these effects.
5. **Total Longitudinal Strain (ε)**

The longitudinal strain is a unit less deformation in the direction of the beam’s longitudinal axis that represents the sum of the propagated torsional cracks’ width. The total of two longitudinal strains in each tested beam is graphed against the torsional moment in Figure (8). The comparisons of longitudinal strain for all tested beams were taken at 25 kN, which is the failure load of NCA.

![Figure 8. Torsional moment vs. longitudinal strain for each type of concrete](image-url)
5.1 Effect of Concrete Type on Longitudinal Strain
Making a comparison among the three concrete types, NSC, HSC, and SCC, it can be noted that the HSC group of beams is the group according with the most stiffness based on longitudinal strain, followed by the SCC group. By adopting a comparison between the NCB beam, which is a normal strength concrete beam, and the SCCB, which is a self-compacting concrete beam, both with the same nominal strength of 40MPa, it can be further found that at a load of 25kN, the longitudinal strain of SCCB is 0.0075, smaller than that of NCB, which is 0.008. Similarly, a comparison between HSCC, which is a high strength concrete beam, and SCCC, which is self-compacting concrete beam, both with 60MPa nominal compressive strength, shows that the longitudinal strain of HSCC at a load of 25kN is 0.001, smaller than that of SCCC, which is 0.009.

5.2 Effect of Compressive Strength on Longitudinal Strain
In the NSC group, a comparison between NCA, with 20MPa nominal compressive strength, and NCB, with 40MPa nominal compressive strength, shows that the angle of twist of NCB is smaller than that of NCA, at 0.008 and 0.0555, respectively.
In the HSC group, a comparison between HSCC, with 60MPa nominal compressive strength, and HSCD, with 75MPa nominal compressive strength, shows that the longitudinal strain of HSCD is smaller than that of HSCC, at zero and 0.001, respectively.
In the SCC group, a similar comparison between SCCB at 40MPa nominal compressive strength and SCCC at 60MPa nominal compressive strength shows that the longitudinal strain of SCCC is smaller than that of SCCB at 0.00045 and 0.0075, respectively.
The results show that an increase in compressive strength of a section will increase the stiffness of the beam, thus decreasing the longitudinal strain.

5.3 Effect of Steel Fibre on Longitudinal Strain
In the NSC group, comparison among NCA, NCA_{0.6} and NCA_{1.2} with the same nominal compressive strength (20MPa) but with different steel fibre contents showed that using steel fibres at 0.6 volume fraction and 1.2 volume fraction decreased the longitudinal strain of NCA_{0.6} and NCA_{1.2} to 0.033 and 0.005, respectively, both being less than that of NCA (0.0555).
In the HSC group, the comparison among HSCC, HSCC_{0.6} and HSCC_{1.2}, which also had the same nominal compressive strength (60MPa) but different steel fibre contents revealed that the longitudinal strain of both HSCC_{0.6} and HSCC_{1.2} decreased to be zero in comparison with that of HSCC (0.001).
In the SCC group, the comparison among SCCB, SCCB_{0.6} and SCCB_{1.2}, which also had the same nominal compressive strength (40MPa) but different steel fibre contents revealed that the longitudinal strain of SCCB_{0.6} and SCCB_{1.2} decreased to 0.0045 and 0.001, respectively, less than that of SCCB (0.0075).
These results suggest that increasing the steel fibre content in a section will decrease the value of longitudinal strain on the beam, resulting in an increase of stiffness.

5.4 Improvement of Longitudinal Strain
As in the case of angle of twist, the improvement of longitudinal strain is shown by a decrease in value which reflects a reduction in the deformation in the member. The decrease in longitudinal strain was thus compared to that of the reference beam for each group and then compared with the angle of twist of the weakest beam in this work, which it is NCA at an applied load of 25kN, as in Table 4.
Table 4. Improvement of longitudinal strain

| Mix Notation | Beam Symbol | Nominal Compressive Strength (MPa) | Steel Fibres ($V_f$) | Total Strain ($\varepsilon$) (%) | Decreasing According to the Reference Beam (%) | Decreasing According to NCA Beam (%) |
|--------------|-------------|-----------------------------------|---------------------|-----------------------------------|---------------------------------------------|-------------------------------------|
| NSC          | * NCA       | 20                                | -                   | 0.0555                            | -                                           | -                                  |
| NCB          | 40          | -                                 | 0.0080              | 85.6                              | 85.6                                        |
| NCA<sub>0.6</sub> | 20         | 0.6%                              | 0.0030              | 94.6                              | 94.6                                        |
| NCA<sub>1.2</sub> | 40         | 1.2%                              | 0.0050              | 90.9                              | 90.9                                        |
| HSC          | * HSCC      | 60                                | -                   | 0.0010                            | -                                           | 98.2                               |
| HSCD         | 75          | -                                 | 0                   | 100                               | 100                                         |
| HSCC<sub>0.6</sub> | 60         | 0.6%                              | 0                   | 100                               | 100                                         |
| HSCC<sub>1.2</sub> | 60        | 1.2%                              | 0                   | 100                               | 100                                         |
| SCC          | * SCCB      | 40                                | -                   | 0.0075                            | -                                           | 86.5                               |
| SCCC         | 60          | -                                 | 0.0005              | 94                                | 99.2                                        |
| SCCB<sub>0.6</sub> | 40         | 0.6%                              | 0.0045              | 40                                | 91.9                                        |
| SCCB<sub>1.2</sub> | 40        | 1.2%                              | 0.0010              | 86.7                              | 98.2                                        |

From Table 4, it is clear that the longitudinal strain can be improved by increasing the compressive strength of the section; the highest improvement (100%) is in the HSC group. On the other hand, an increase in compressive strength is better than the use of steel fibres for improving the angle of twist, with results of 94% and 40% in SCCC and SCCB<sub>0.6</sub>, as seen in Table 4 above. By increasing the steel fibre content, the angle of twist can be decreased, but this is not as effective as increasing the compressive strength of the beam, as seen in SCCC and SCCB<sub>1.2</sub> based on the results 94 and 86.7%. Figure 9 graphs these results.

![Graph of Table 4](image)

**Figure 9.** Improving of longitudinal strain in each type of beams

Table 4 also emphasises that, when a comparison is adopted according to specific compressive strength, by continuing to increase the compressive strength of the section, the angle of twist will improve; thus, the HSC beams achieved the highest values (100%). Additionally, increasing compressive strength is
better than improving the steel fibre fraction in terms of improving the angle of twist. Figure.10 graphs these effects.

![Figure 10. Improving longitudinal strain compared to NCA beam](image)

6. Conclusions

Based on the evidence from the experimental results reported above, the following conclusions can be drawn. It is emphasised that these conclusions must be limited to the variables studied:

1. The HSC group of beams is the strongest group, according to their ultimate carrying load, and have the most stiffness, followed by the SCC group.

2. Increasing the compressive strength of a section will increase the stiffness of the beam, thus clearly decreasing the angle of twist.

3. Increasing the steel fibre content in a section will decrease the value of angle of twist of the beam, resulting in an increase of stiffness.

4. Increasing the compressive strength of the section will increase the stiffness of the beam, decreasing the longitudinal strain.

5. Increasing the steel fibre content in a section will decrease the value of longitudinal strain in the beam, resulting in an increase in stiffness.

7. References

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