Numerical simulation of the interaction of the disturbed boundary layer with an incident shock

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Abstract. Interaction of the disturbed supersonic boundary layer with an incident oblique shock wave is studied numerically with eddy-resolving numerical simulations. Eigenmodes of the linear stability theory are used to generate the inflow boundary layer disturbances. The evolution of unstable boundary-layer disturbances, effects of the incident shock on the disturbances, effects of the disturbances on the boundary layer separation, flow dynamics in the separation zone, and laminar-turbulent transition are studied.

1. Introduction

Shock wave boundary layer interaction (SWBLI) is an important issue for the design of transonic and supersonic aircraft. SWBLI is often attended by the boundary layer separation which causes flow unsteadiness and additional stresses on aircraft structure. Comprehensive recent reviews of the SWBLI and associated phenomena are given in [1–3]. Most SWBLI studies are made for the turbulent boundary layer interacting with a shock wave. The development of more efficient commercial aircraft requires extending the laminar flow regime on most of aircraft surfaces to reduce viscous friction and thus provide better fuel efficiency. It is therefore desirable to study in detail the SWBLI in the case of a laminar boundary-layer with evolving unstable disturbances that can lead to laminar-turbulent transition farther downstream. Recently, the SWBLI in such transitional boundary layers has been reviewed [4] for flows at rather high supersonic Mach numbers. In this paper we perform the numerical simulation of the interaction of the disturbed transitional supersonic boundary layer on a flat plate with an oblique incident shock wave impinging on the plate. The simulations are performed at low supersonic Mach number which is more relevant for civil aircraft. Main topics of this study are the evolution of unstable boundary-layer disturbances, effects of the incident shock on the disturbances, effects of the disturbances on the boundary layer separation, flow dynamics in the separation zone, and laminar-turbulent transition.

2. Problem formulation and numerical techniques

The development of disturbances and the laminar-turbulent transition in the boundary layer during its interaction with an incident oblique shock wave are investigated on the basis of the unsteady Navier–Stokes numerical simulations with the resolution of the large-eddy structure of the flow. Supersonic boundary layer on a flat plate interacts with an incident shock wave generated by a wedge mounted at some distance above the plate. The flow parameters correspond to the experiments performed at the Institute of Theoretical and Applied Mechanics (ITAM) at low supersonic flow Mach number
M = 1.43, stagnation pressure $P_0 = 0.55 \cdot 10^5$ Pa, and stagnation temperature $T_0 = 293$ K. The plate temperature $T_w$ is fixed and equal to the temperature of a thermally insulated plate at laminar boundary layer conditions: $T_w/T_0 = 1.346$. The angle of the wedge generating the incident shock is $3^\circ$, which corresponds to the shock angle of incidence $48.2^\circ$. The flow parameters correspond to the conditions of our previous numerical study [5] conducted with both RANS and the eddy-resolving simulations. The left boundary of the computational domain is located at the distance $x = x_0$ downstream from the plate leading edge. The Blasius boundary layer thickness $\delta_0 = (\nu x_0/U)^{1/2}$ at the inflow is used as the length scale. Here $\nu$ is the kinematic viscosity, $U$ is the free-stream velocity. The computations are performed at $x_0$ corresponding to the Reynolds number $Re = U \delta_0 / \nu = 700$. The corresponding Reynolds number based on the distance from the leading edge is $Re_x = U x_0 / \nu = Re^2 = 0.49 \cdot 10^6$ m$^{-1}$. The three-dimensional instability waves of the linear stability theory (LST) are forced at the inflow boundary. Typically, two symmetrical fundamental instability waves with the non-dimensional circular frequency $\omega = 2\pi f U / \delta_0$ and the angles of the wave vector $\pm \chi$ are used for inflow forcing. Similar approach was used in our previous SWBLI computations [5, 6] and was also used in the simulations of the laminar-turbulent transition in the flat-plate boundary layer at $M = 2$ [7]. The LST calculations have been performed with the linear stability code VBLS3D developed at ITAM. The fundamental instability wave having the maximum growth rate at the inflow parameters corresponds to $\omega = 0.032$ and $\chi = 46^\circ$. The dimensional frequency of the disturbance computed at the experimental conditions is $f = 25.136$ kHz. The periodic boundary conditions along the $z$ coordinate are used in the computations. Initial amplitude of the disturbances $A_0$ at the inflow cross-section was varied in different computations. The dimensional Blasius thickness computed at the experimental conditions is $\delta_0 = 8.375 \cdot 10^{-4}$ m. The incident shock is introduced by setting the Rankine–Hugoniot jump conditions on part of the upper boundary starting from $x = x_0$.

Numerical simulations on the basis of the unsteady three-dimensional Navier–Stokes equations are performed with CFS3D and HyCFS CFD codes developed at ITAM. These numerical codes use modern high-order WENO schemes for spatial discretization of the convective fluxes and are suitable for simulations of the boundary layer with instability waves and its interaction with the incident shock wave. The CFS3D code is used to perform parallel computations on MPI clusters. HyCFS code is used to perform simulations on hybrid supercomputers with both CPUs and GPUs using three levels of parallelization: CUDA, OpenMP and MPI.

3. Results and discussion

To study the possible influence of the computational domain geometry and the resolution of the computational grid on the development of boundary layer disturbances and the SWBLI, simulations were performed for two different geometries of the computational domain and two different computational grids. In the first case, we used a region in the range of longitudinal coordinates $58 \, \text{mm} < x < 226 \, \text{mm}$, with the dimensions along three coordinate axes $L_x = 168 \, \text{mm}$, $L_y = 10 \, \text{mm}$, $L_z = 7.1 \, \text{mm}$. The grid in the respective coordinate directions has $N_x = 1152$, $N_y = 300$, $N_z = 96$ cells, the spatial resolution along the longitudinal coordinate $\Delta x = 0.146 \, \text{mm}$. In the second case, the longitudinal size of the region was reduced, and the size of the domain in the direction normal to the plate was increased: $58 \, \text{mm} < x < 184 \, \text{mm}$, $L_x = 126 \, \text{mm}$, $L_y = 17.5 \, \text{mm}$, $L_z = 7.1 \, \text{mm}$. In this case, the grid with $N_x = 1728$, $N_y = 400$, $N_z = 96$ was used, which made it possible to improve the spatial resolution along the longitudinal coordinate: $\Delta x = 0.073 \, \text{mm}$. The minimum cell size of the grid along the normal coordinate in the two calculated cases was the same $\Delta y_{\text{min}} = 3.7 \times 10^3 \, \text{mm}$.

The flow visualization of the disturbed boundary layer interacting with the incident shock is given in figure 1. The SWBLI causes adverse pressure gradient in the boundary layer and the formation of a separation zone, which leads to an increase in the boundary layer fluctuations, their transition to the nonlinear three-dimensional interaction regime and rapid laminar-turbulent transition directly downstream of the SWBLI zone. The distributions of the pressure and the skin friction coefficient $C_f$, averaged over time and the $z$ coordinate, given in figure 2, show that the size and position of the separation zone for the two calculated domains and grid resolutions varies slightly. The positions of
the separation and reattachment points measured by the zero values of the averaged $C_f$ coefficient are, respectively, $x_{sep} \approx 96$ mm and $x_{reat} \approx 114$ mm. The intended position of the shock reflection point on the plate was $x = 111$ mm in this case. The effect of increasing the height of the computational domain from $L_y = 10$ to $L_y = 17.5$ mm is manifested in the shift of the undesirable pressure disturbance coming from the upper boundary of the domain downstream from $x \approx 128$ mm to $x \approx 140$ mm.

Figure 1. SWBLI in the boundary layer disturbed with the fundamental instability wave with the amplitude $A_0 = 0.001$. Isosurface of Q criterion and color levels of Mach number.

Figure 2. Distributions of the averaged pressure (a) and skin friction coefficient (b) along the plate for two different computational domains and grids.

The distributions of the time-averaged coefficient $C_f$ over the plate surface given in figure 3 also show that the flow structure in the separation zone is in good agreement in the computations on the two computational grids. The observed non-uniformity of the $C_f$ distributions over the transverse
coordinate \( z \) is related to the presence of quasi-stationary vortex structures in the boundary layer, which are formed due to the nonlinear development and interaction of three-dimensional instability waves.

Figure 3. Surface distributions of averaged skin friction coefficient for two different domains and grids \( \Delta x = 0.146 \text{ mm} \) (а) and \( \Delta x = 0.073 \text{ mm} \) (b).

The interaction of the disturbed boundary layer with the incident shock is accompanied by a significant increase in pressure and mass-flow-rate fluctuations, see figure 4. Pressure fluctuations have a prominent peak downstream of the reattachment point at \( x \approx 122 \text{ mm} \), after which they decrease to an equilibrium level of \( \sim 1.2 \% \). The mass-flow-rate fluctuations increase rapidly in the interaction region and reach a level of \( \sim 10–15 \% \) at approximately the same values of the longitudinal coordinate. The pulsation characteristics of the disturbed boundary layer obtained in the computations performed on two grids with different resolution are qualitatively and quantitatively consistent with each other.

Figure 4. R.m.s. fluctuations of pressure (a) and mass flow rate (b) along the centerline \( z = L_z / 2 \) at height \( y = 0.14 \text{ mm} \).

The influence of the initial amplitude of disturbances has also been studied. In figure 5 the distributions of the averaged and pulsation characteristics for two different initial disturbance amplitudes at the inflow section are presented: \( A_0 = 0.001 \) and \( A_0 = 1 \cdot 10^{-4} \). The obtained data show that the change of the initial disturbance amplitude has a significant effect on the flow: the size of the separation region at \( A_0 = 1 \cdot 10^{-4} \) is significantly larger than at \( A_0 = 0.001 \). The positions of the separation and reattachment points obtained from the zero values of the averaged skin friction coefficient \( C_f \) are, respectively, \( x_{\text{sep}} \approx 91 \text{ mm} \), \( x_{\text{reat}} \approx 117 \text{ mm} \) at \( A_0 = 1 \cdot 10^{-4} \) and \( x_{\text{sep}} \approx 96 \text{ mm} \), \( x_{\text{reat}} \approx 114 \text{ mm} \) at \( A_0 = 0.001 \). Such a change in the geometric characteristics of the separation zone has a
significant effect on the pulsation characteristics of the flow. In the case of a small amplitude $A_0 = 1 \cdot 10^{-4}$, small local maximums of pressure pulsations at $x = 87$ mm and mass flow rate pulsations at $x = 89$ mm are noteworthy. The presence of these maximums is explained by the upstream shift of the separation point, which is observed in this case. At the same time, the beginning of the positive mean pressure gradient zone moves upstream from $x = 80$ mm at $A_0 = 0.001$ to $x = 75$ mm at $A_0 = 1 \cdot 10^{-4}$, which leads to a local increase in pulsations in this region at $A_0 = 1 \cdot 10^{-4}$. Farther downstream, within the SWBLI zone, the rise of the fluctuations at $A_0 = 1 \cdot 10^{-4}$ shifts downstream. Downstream of the interaction zone, after the transition to turbulence, the intensity of the fluctuations reaches approximately the same level in both cases.

![Figure 5. Surface distributions of averaged pressure (a) and skin friction coefficient (b); r.m.s. fluctuations of pressure (c) and mass flow rate (d) at height $y = 0.14$ mm for two different initial amplitudes $A_0$.](image)

**Conclusions**

The interaction of the disturbed supersonic boundary layer on the flat plate with the incident oblique shock wave has been investigated with eddy-resolving numerical simulations. The results of computations of the boundary layer forced by the fundamental instability wave show the non-uniformity of the distribution of the friction coefficient along the transverse coordinate, which is associated with the presence of quasi-stationary vortex structures in the boundary layer. It is shown that the interaction of the disturbance boundary layer with the incident shock is accompanied by a
significant increase in the pressure and mass-flow-rate fluctuations and quickly leads to laminar-
turbulent transition. The study of the influence of the initial disturbance amplitude on SWBLI zone
shows that the change in the initial disturbance amplitude has a significant effect on the size and
position of the separation zone and the pulsation characteristics of the flow.

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