Branes, Charge and Intersections

Donald Marolf

February 1, 2022

Physics Department, Syracuse University, Syracuse, New York 13244

Abstract

This is a brief summary of lectures given at the Fourth Mexican School on Gravitation and Mathematical Physics. The lectures gave an introduction to branes in eleven-dimensional supergravity and in type IIA supergravities in ten-dimensions. Charge conservation and the role of the so-called ‘Chern-Simons terms’ were emphasized. Known exact solutions were discussed and used to provide insight into the question ‘Why don’t fundamental strings fall off of D-branes,’ which is often asked by relativists. The following is a brief overview of the lectures with an associated guide to the literature.

1 Preface

This course was intended to be similar to the set of lectures I gave introducing branes in supergravity and string theory at the Third Mexican School on Gravitation and Mathematical Physics in 1998. A full write up of this previous set of lectures can be found in [1]. The presentation in [1] is in fact more similar to the lectures given at the Fourth School in 2000 than to the original 1998 lectures and I would still recommend [1] for an introduction to the subject. As with the 1998 lecture series, the style was intended for an audience with a relativity background as opposed to other introductions aimed more at those with a particle physics background. I apologize for the fact that several typographic errors remain in [1], though I believe that the equations are now correct. Readers of [1] are encouraged to e-mail me at
marolf@physics.syr.edu to point out such errors and make suggestions for future versions that will eventually be produced.

The more recent lecture series went beyond the material in [1] by including discussions of the so-called ‘Chern-Simons terms.’ These are terms in the supergravity actions that are responsible for the interesting features that arise in more complicated cases such as ‘intersections of branes.’ In particular, the goal was to include some more recent results from [2, 3, 4]. Unfortunately, this required that the basic introduction be shortened in the 2000 lecture series which likely made these lectures more difficult for the uninitiated. The reader interested in learning this additional material will surely benefit from taking the time to digest [1] in full before studying [2, 3, 4]. These latter papers were written as research papers and not as pedagogical introductions or reviews. Though I believe that [1] does contain sufficient introduction to allow the reader to follow [2, 3, 4], the reader is advised to read these latter papers somewhat more slowly than [1]. Note, by the way, that I do not mean to say that [1] will be a quick read for those who are new to the subject.

Below, I give a brief summary of the material covered in the 2000 lecture series. I am afraid that this summary is little more than a list of topics covered together with a somewhat more comprehensive guide to the literature. Nevertheless, I hope that it will be of use.

2 Summary

The 2000 lecture series began with an introduction to eleven-dimensional supergravity. Given certain generally accepted caveats, there is in fact a unique supergravity theory in eleven-dimensions and it was first obtained in [7]. With the same caveats, eleven is in fact the highest number of dimensions in which a supergravity theory exists. For reasons related to this observation, it turns out the eleven-dimensional supergravity is the simplest point for a physicist trained in General Relativity to begin to learn about string theory and supergravity. The point is that, to a first approximation, the dynamics of eleven-dimensional supergravity are essentially those of an Einstein-Maxwell theory (together with some Fermions). In particular, eleven-dimensional supergravity contains no ‘dilaton’ field as do other relevant supergravity theories. Theories with a dilaton are not minimally coupled in the sense of the strong equivalence principle and, as a result, hold a few extra surprises for relativists.
The idea of the lectures was to start by thinking of supergravity as being much like Einstein-Maxwell theory and then to slowly add back the features that distinguish it. The first such property is that while the familiar Maxwell field has a rank two field strength tensor $F$, the supergravity gauge fields have field strengths which are $p$-forms (i.e., rank $p$ covariant anti-symmetric tensors), each with a different value of $p$. It is the feature $p > 2$ which leads to the introduction of ‘branes.’ ‘Brane’ is a word for an extended object and is derived from the word membrane. In modern terminology, a membrane is known as a ‘2-brane’ because of it is extended in two spacelike directions; that is, it is 2+1 manifold. Similarly, strings are known as 1-branes and particles as 0-branes. Higher dimensional branes also arise in string theory and supergravity.

It turns out that the fundamental electric charges of rank $p$ gauge fields for $p > 2$ are necessarily such extended objects and that particles are necessarily neutral under such gauge fields. The details (as well as a similar discussion for magnetic charges) can be found in [1]. Indeed, a large part of [1] is devoted to this point.

One may either consider these gauge fields alone or one may include their couplings to gravity. In this latter case, one finds an associated set of so-called charged black $q$-brane solutions. These solutions are analogs of Reissner-Nordström [6] black holes but with horizons that are extended in $q$ directions instead of being compactly generated. For simplicity, these solutions are often discussed only in the extremal limit and this was the case both in the lectures and in [1]. In eleven-dimensional supergravity these black branes have smooth horizons even in the extremal limit just as in Einstein-Maxwell theory, though this property fails to hold in most other supergravity contexts. Readers interested in the non-extremal solutions should consult other reviews of black branes in string theory such as [7, 8, 9, 10].

Another complication of eleven-dimensional supergravity is of course the Fermions needed for supersymmetry. While I did not address the Fermions in the 2000 lecture series, one may find a brief introduction to their properties in [1]. For more details, the reader may wish to consult [5, 11, 12, 13].

The final complication arises from the so-called Chern-Simons term. This term has certain features in common with the distinctive term in 2+1 Chern-Simons theory, which is of course a topological field theory having no local degrees of freedom. However, the effects of this term discussed in the lectures have little to do with topological quantum field theory.

To understand just what these effects are, it turns out to be convenient
to postpone a discussion of the Chern-Simons term until after discussing how ten-dimensional supergravity arises from Kaluza-Klein reduction of the eleven-dimensional theory. Thus, Kaluza-Klein reduction was discussed next in the lectures and is the next topic in [1]. It is important to have some understanding of Kaluza-Klein reduction before moving on to the new material not included in [1]. In the lectures and [1] I consider only Kaluza-Klein reduction on circles, but [14] is a standard reference for more general reductions.

The reason that it is easier to first discuss Kaluza-Klein reduction and to only later address Chern-Simons terms is that the reduction process in fact creates additional Chern-Simons terms. This may seem like an additional complication, but on turning the picture around it yields insight into generic Chern-Simons terms. The point is that one may find a geometric picture of why the Chern-Simons terms arise in the dimensional reduction and this geometric picture clarifies properties of generic Chern-Simons terms. In particular, the effect of these ‘geometric’ Chern-Simons terms on intersections of branes is more easily seen as the effect of the twisting of space in the eleventh dimension on a simple configuration of branes in eleven dimensions. This point is not addressed in [1], though it forms the main theme of [3].

A particular example is considered in detail in [4], including a lower dimensional example that is much more easily visualized. The discussion of [4] is more elementary and may make a better starting point, though [3] addresses additional issues such as charge quantization. See also [18] for an earlier and rather general discussion of Chern-Simons terms and brane intersections.

The discussion above covers the main general topics addressed in the 2000 lecture series. In addition to describing this general structure, a final goal of the course was to describe how these general properties could be used to extract information about non-perturbative physics in string theory. The example given in the lectures considered fundamental strings attached to a particular type of D-brane, the famous branes on which fundamental strings are allowed to end. For an introduction to these branes the reader should consult the review [17] or the text [12].

The focus of this final discussion concerns the sensible question “what prevents fundamental strings from falling off of (i.e., separating from) D-branes and drifting off on their own?” I have often heard relativists ask this question of string theorists. A common answer to this question invokes charge conservation, though this answer can be confusing to relativists because the charge considered is not one that is seen in supergravity. Indeed, strictly
speaking this common answer turns out to be true only in the setting of perturbative string theory.

Nevertheless, a related answer can be obtained by studying the conservation of a supergravity charge. One finds that the fundamental string \textit{can} in fact separate from the D-brane, but only by transforming itself into a higher dimensional brane which, at least in the perturbative string limit, is in fact much more massive. In other words, there are energetic reasons for the fundamental string to remain bound to the D-brane and in the perturbative string limit this binding is very tight indeed. On the other hand, the binding is rather weak at large values of the string coupling so that at the non-perturbative level there can be significant fluctuations away from the D-brane. While this picture follows from general reasoning, the solutions constructed in \cite{2} and the related constructions of \cite{15,16} give a closely connected concrete example. The reader interested in more details should consult the second section of \cite{4}, in particular in the material associated with figure 1 of that reference. The treatment there is somewhat brief as the point is not central to that paper, but unfortunately I know of no other discussions of this point.

The reader who succeeds in absorbing the information outlined above will have gone a long way toward being able to understand discussions of branes in supergravity and even toward beginning research projects of their own. Although I have described it only briefly here, there is in fact a sizable amount of information to learn. I should state that the review \cite{1} based on my 1998 lectures addresses a number of additional topics that I was not able to include in the 2000 lecture series and that I have not mentioned in this summary. The reader pursuing a broad understanding of branes in supergravity should certainly digest such additional material as well, though those desiring only a mild introduction will be sufficiently occupied with the topics listed here.

New reviews of branes and supergravity appear on a regular basis, and I encourage the reader to scan the archives for further resources. I expect that a fully revised and expanded version of \cite{1} will be available before too many more years pass, but I hope that the present summary and guide to the literature can be of some use until this does in fact occur.

References
[1] D. Marolf, “String/M-branes for relativists,” gr-qc/9908045.
[2] A. Gomberoff and D. Marolf, “Brane transmutation in supergravity,” JHEP 0002, 021 (2000) [hep-th/9912184].
[3] D. Marolf, “Chern-Simons terms and the three notions of charge,” hep-th/0006117.
[4] D. Marolf, “T-duality and the case of the disappearing brane,” hep-th/0103098.
[5] E. Cremmer, B. Julia, and J. Scherk, “Supergravity Theory in Eleven-Dimensions,” Phys. Lett. B76 409 (1978).
[6] D. Brill and T. Dray, “Spell it Nordström,” General Relat. and Grav. 25 435-436 (1993).
[7] D. Youm, “Black Holes and solitons in string theory,” Phys. Rept. 316 1 (1999) hep-th/9710046.
[8] K. S. Stelle, “BPS branes in supergravity,” hep-th/9803116.
[9] M. J. Duff, “TASI lectures on branes, black holes and anti-de Sitter space,” hep-th/9912163.
[10] A. W. Peet, “TASI lectures on black holes in string theory,” hep-th/0008241.
[11] M. Green, J. Schwarz, and E. Witten, Superstring theory, (Cambridge U. Press, NY, 1987).
[12] J. Polchinski, String Theory (Cambridge U. Press, Cambridge, 1998).
[13] G. W. Gibbons and C. M. Hull, “A Bogomolny Bound for General Relativity and Solitons in N=2 Supergravity,” Phys. Lett. 109B 190 (1982).
[14] J. Maharana and J. H. Schwarz, “Noncompact symmetries in string theory,” Nucl. Phys. B 390, 3 (1993) hep-th/9207010.
[15] C. G. Callan, A. Guijosa and K. G. Savvidy, “Baryons and string creation from the fivebrane worldvolume action,” Nucl. Phys. B 547, 127 (1999) hep-th/9810092.
[16] C. G. Callan, A. Guijosa, K. G. Savvidy and O. Tafjord, “Baryons and flux tubes in confining gauge theories from brane actions,” Nucl. Phys. B 555, 183 (1999) [hep-th/9902197].

[17] C. V. Johnson, “D-brane primer,” [hep-th/0007170].

[18] Townsend, P.K. “Brane Surgery,” Nucl. Phys. Proc. Suppl. 58 163 (1997).