Asking Neutrons where they have been

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Abstract. Based on the results of Vaidman’s photonic three-path interferometer experiment with weak path marking [A. Danan, D. Farfurnik, S. Bar-Ad, and L. Vaidman, Phys. Rev. Lett. 111, 240402 (2013)] the authors claim that the photons have discontinuous particle trajectories. Here, we present a neutron optical version of the experiment by Danan et al., where in various beam paths of the interferometer the energy of the neutrons is slightly shifted. This is achieved by using resonance-frequency spin-rotators (SR) operating at different frequencies. The which-way information is derived from the time-dependent intensity, which is considered to result from the interfering cross terms between the stationary main component and the energy-shifted which-way signals. Our statements are based on a simple theoretical model, following the time evolution of the wave function of the neutrons, which clarifies the observation in the framework of standard quantum mechanics and reveals the multifold presence of the neutron’s wave function in the interferometer.

1. Introduction

Quantum theory provides the foundation for much of the technology of modern society and is probably the most successful theory until today. Nevertheless, some aspects remain unclear. One of these is the history of a quantum particle. When we measure a particle in our detector, we learn nothing about the trajectory it took to get there from the source - quantum theory simply has no answer to this question. What was the past of a photon which went through an interferometer? This question is the central point in the work proposed by Lev Vaidman [1] and an experimental realization with his co-workers [2, 3]. In this paper, entitled ”Asking photons where they have been”, Danan et al. use a Mach-Zehnder interferometer with absorbing mirrors in the same direction. This nested (inner) interferometer is then placed in one arm of a larger (outer) Mach-Zehnder interferometer, which is schematically illustrated in Figure 1. The authors probe the photons where they were in two situations: i) interference measurement and ii) when there is a which-way measurement using absorbers. The authors suggest in their paper discontinuous trajectories of the photons and argue that this can be intuitively understood: one should consider not just the possible paths of the photons traveling forwards in time through the system but also the paths that a photon traveling backwards in time from the detector to the source, according to the two-state vector formalism [4], a time-symmetric interpretation of quantum theory. Only at mirrors where both, forward and backwards evolving state, are present (A, B and C) they interfere,
Figure 1. Photonic setup and measured power spectrum. The two-state vector description of the photon inside the interferometer includes the standard forward evolving quantum state (yellow) and the backward evolving quantum state (green) of the photon detected by the photodetector.

leaving a faint trace. These faint trace are then found in the power spectrum (see Figure 1 top), indicating where the photon was inside the interferometer. From that the authors conclude that the particle can have trajectories that are not continuous. This particular statement triggered a heated debate [5, 6, 7, 8] (see [9], and references therein, for a detailed review of the debate). However, their experiment was carried out with a classical laser beam and their results can be fully explained within the framework of Maxwell theory. In [10] Zhou et al. present a single-photons version of the experiment performed by Danan et al., using electro-optic phase modulators (EOM) for which-way marking. However, their detection method is unable to extract all the available path information. To test the statement of Danan et al. an experiment of pure quantum nature and high detection efficiency is called for, which we have performed in a neutron interferometric experiment [11].

Since its first experimental verification [12] in 1974, neutron interferometry has been used to tests fundamental phenomena in quantum mechanics [13, 14]. More recently, entanglement between the neutron’s degrees of freedom [15, 16, 17] was applied for investigations of quantum contextuality with massive particles [18, 19, 20]. In addition, using the concept of weak measurements [21] in neutron optics [22] allowed to study quantum paradox such as the quantum Cheshire-Cat [23] or the quantum pigeon-hole principle [24].

2. Neutron optical Which-Way Experiment
Here, we present a single neutron which-way experiment, performed at the instrument S18 at the Institut Laue Langevin (ILL) in Grenoble, France. A schematical illustration of the neutron interferometric setup is given in Figure 2. The which-way marking is achieved by minimal-perturbing simultaneous which-way measurements using resonance-frequency spin-rotators (SR) [25, 26, 15], operating at different frequencies $\omega_I/2\pi = 74$ kHz, $\omega_{II}/2\pi = 77$ kHz, $\omega_{I+II}/2\pi = 80$ kHz, and $\omega_R/2\pi = 71$ kHz. A monochromatic neutron beam of mean wavelength
Figure 2. Setup of the neutron which-way measurement using a three-beam interferometer. In the lower panel the energy shifts, due to the interaction with the time dependent magnetic fields of the SRs of the individual beam path is illustrated. Letters A,B,C and F indicate the corresponding components of the photonic experiment [2].

The incoming neutrons are spin-polarized in positive z-direction by a magnetically birefringent prism splitting the unpolarized beam into two beams (one with the neutron spin aligned parallel to the positive z-direction and one aligned antiparallel). Even though the angular separation is just 4 seconds of arc only the beam with the spin-up component, denoted as $s_+$, fulfills Bragg’s condition at the first plate of the IFM. The wave functions in each path in the IFM consist of a spin and path wave function (with a respective phase shift), written as $\Psi_i = e^{i\chi_i} s_+ \otimes \psi_i$ ($i = I, II, R, I + II$). Between the second and the third plate of the IFM, the neutron’s paths are marked by slightly shifting the energy of the neutrons. This is achieved by

3. Pure Quantum Mechanical Model

In this Section, we want to introduce a simple model, which is completely inside the framework of standard quantum mechanic without the need for the two-state vector formalism. This model correctly predict the observed frequencies in the power spectrum, as well as their magnitudes.
the SRs, which their amplitudes tuned such that the spin is rotated only slightly by $\alpha_i = \pi/9$ (the wave functions before and after these SRs are still overlapping by 0.98). The amount of the energy shift $\Delta E = \hbar \omega$ is adjusted by the frequency $\omega$ of the oscillating magnetic-field of the corresponding SR. The rotation by a SRs is represented by the unitary transformation $\hat{U}_{SR}(\omega, \alpha)$ [27]. When applied on a wave function $\Psi_I$ calculated as $\Psi_i = \hat{U}_{SR}(\omega_i, \alpha_i) \Psi_i$

$$\Psi_i' = e^{i\chi_i} \left( \cos \frac{\alpha_i}{2} s_+ - i e^{\omega_i t} \sin \frac{\alpha_i}{2} s_- \right) \psi_i$$

$$= e^{i\chi_i} \left( s_+ - i e^{\omega t} \sin \frac{\alpha_i}{2} s_- \right) \psi_i + O(\alpha_i^2),$$

(1)

for small $\alpha_i$, with $i = I, II, R$. Here, $I, II, R$ corresponds to mirrors B, A and C of the photonic experiment [2] and form the nested (inner) IFM.

The path $I + II$ coming from the front loop is marked by SR$_{I+II}$ between the third and the fourth plate of the IFM, which corresponds to mirror F in the photonic experiment [2] (note that a SR corresponding to mirror E is omitted here). The wave function $\Psi_{I+II}'$ behind SR$_{I+II}$ is thus denoted as $\Psi_{I+II}' = \hat{U}_{SR}(\omega_{I+II}, \alpha) \frac{1}{\sqrt{2}} (\Psi_I' + \Psi_{II}')$. This beam is later recombined with the reference beam $\Psi_{R}'$ at the fourth plate of the IFM, resulting in the wave function in path $O$, which is expressed as $\Psi_O' = \frac{1}{\sqrt{2}} \Psi_{I+II}' + \frac{1}{2} \Psi_R'$.

Behind the IFM an energy compensation of $\Delta E_{EC} = \hbar \omega_{EC}$ is carried out by SR$_{EC}$ in path $O$, in order to reduce the overall energy shift of the which-way (WW) marking. This is realized by applying a spin-rotation of $\alpha_{EC}^2 = \pm \pi/2$ at frequency $\omega_{EC} = 68$ kHz. The resulting wave function behind SR$_{EC}$ is expressed as $\Psi_{EC}' = \hat{U}_{SR}(\omega_{EC}, \alpha_{EC}) \Psi_D'$. Before the beam reaches the O-detector a super-mirror spin-analyzer filters out the down-spin component, by spin-dependent reflection on a multilayer structure. The wave function behind the super-mirror $\Psi_{EC}^\pm_{SM} = \Pi_{s^\pm} \Psi_{EC}'$ is given by $\Psi_{EC}^\pm_{SM} = \Psi_{E_0} + \sum_i \Psi_{i}^\pm + O(\alpha_{ww}^2)$, where $\Pi_{s^\pm}$ denotes the projector onto the $s_\pm$ state. The wave-function $\Psi_{EC}^\pm_{SM}$ consists of the energy-unshifted main component $\Psi_{E_0} = \sum_i \frac{e^{i\chi_i}}{\sqrt{2}} \Psi_i$, ($i = I, II, R$) and the energy-shifted components $\Psi_i^\pm$, that is, the WW-signal given by $\Psi_i^\pm = \mp C_i e^{i\chi_i} \sin \left( \frac{\alpha_{ww}}{2} \right) e^{-i\Delta \omega t} s_\pm \otimes \psi_i$, for $i = I, II, R, I + II$ ($C_i = \frac{1}{\sqrt{2} \sqrt{2}}$ and $\frac{1}{2}$ for $i = I, II, R$ and $I + II$). The intensity at the O-detector can be calculated, up to the first order of $\alpha_{ww}$, by summing up the (stationary) intensity from the energy-unchanged main component $\Psi_{E_0}$ and the time-modulating one from the individual cross terms between the main component $\Psi_{E_0}$ and the marking components $\Psi_{i}^\pm$, as given in

$$I^\pm = |\Psi_{SM}^\pm|^2 = |\Psi_{E_0}|^2 + 2 \sum_i \text{Re} \left( \Psi_{E_0}^* \Psi_{i}^\pm \right) + O(\alpha_{ww}^2),$$

(2)

for $i = I, II, R, I + II$.

In our experiment, as in the photonic case [2], two cases are considered, namely (i) $\chi_{II} = 0$ and (ii) $\chi_{II} = \pi$ (both with $\chi_{I} = \chi_{R} = 0$). The respective intensities at the O-detector are calculated as

$$I^\pm_{(\chi_{II}=0, \chi_{R}=0)} = \frac{1}{32} \left[ 9 \pm 6 \sin \left( \frac{\alpha_{ww}}{2} \right) \cos(\Delta \omega_I t) + \cos(\Delta \omega_{II} t) + \cos(\Delta \omega_{II} t) + 2 \cos(\Delta \omega_{I+II} t) \right]$$

(3)

$$I^\pm_{(\chi_{II}=\pi, \chi_{R}=0)} = \frac{1}{32} \left[ 3 \pm 2 \sin \left( \frac{\alpha_{ww}}{2} \right) \cos(\Delta \omega_I t) - \cos(\Delta \omega_{II} t) + \cos(\Delta \omega_{II} t) \right]$$

(4)

(here, $\omega_I, \omega_{II}, \omega_R$ and $\omega_{I+II}$ corresponds to the frequencies $f_B, f_A, f_C$ and $f_f$ of the photonic experiment [2]).
Figure 3. Top: ideal simulation of the time-depended which-way signal $\Delta I$ for phase settings $\chi_{II} = 0$ (left) and for $\chi_{II} = \pi$ (right). Bottom: respective power spectra of simulated signals.

Since only relative phases between corresponding beams are relevant, only two phase shifters PS$_{II}$ and PS$_R$ are used in the actual experiment (the phase of path $I$ is set to $\chi_I = 0$). The phase shifter PS$_{II}$ tunes the phase $\chi_{II}$ of path $II$ relative to path $I$ and therefore controls the front loop, monitored by the H1-detector. The phase shifter PS$_R$ tunes the phase $\chi_R$ of the reference beam, which is monitored using the H2-detector. In our experiment, the phase shifter PS$_R$ is set to give $\chi_R = 0$ for all measurements. All three detectors are $^3$He counter tubes with a very high efficiency. A beam-blocker can be inserted into the IFM at two positions BB$_{I+II}$ and BB$_R$, in beam path $I + II$ and path $R$.

The paths, taken by neutrons, are marked and all rotation angles for the WW-marking SRs are set to $\alpha_i = \alpha_{ww} = \pi/9$. When the intensity difference $\Delta I = I^+ - I^-$ is determined, the stationary part cancel out and the time-modulating signal emerges explicitly.

In order to compare the measured intensity difference $\Delta I$ with the theoretical predictions from Equation 2 a simulation is required. The ideal simulation is done by calculating the intensities as given in Equation 4, which is plotted Figure 3 (upper panels) for $\chi_{II} = 0$ (left) and for $\chi_{II} = \pi$ (right), respectively. The time-dependent which-way signal $\Delta I$ is Fourier-analyzed in the next step, yielding the power spectra in Figure 3 (lower panel). For $\chi_{II} = 0$ all peaks are found at the expected frequency differences, $\Delta\omega_R = 3$ kHz, $\Delta\omega_I = 6$ kHz, $\Delta\omega_{II} = 9$ kHz, and $\Delta\omega_{I+II} = 12$ kHz. However, for $\chi_{II} = \pi$ two remarkable features occur: (i) The signal of SR$_{I+II}$ is zero and no peak is visible at frequency $\Delta\omega_{I+II}$, since SR$_{I+II}$ marks the wave function $(\psi_I + e^{i\pi}\psi_{II})/\sqrt{2} = 0$. (ii) The peaks at frequencies $\Delta\omega_R$, $\Delta\omega_I$ and $\Delta\omega_{II}$ drop to one third, since the amplitude of $\Psi_{E_0}$ becomes 1/3.

Next a contrast-corrected (c.c.) simulation is carried out by adding contrast parameters $C_{i,j}$, which denote actual (reduced) contrasts of interferograms. The c.c. intensity is given by

$$I_{\pm, c.c.} = \frac{1}{8} \sum_{i,j} C_{i,j} \Psi_i^* \Psi_j + 2 \sum_{i,j} \text{Re} \left( C_{i,j} \Psi_i^* \Psi_j^\pm \right),$$

(5)
with \( C_{ij} = 1 \). The measured contrast parameters are set for each pair of paths as \( C_{I,II} = 0.55, C_{I,R} = C_{R,I} = 0.60, \) and \( C_{II,R} = C_{R,II} = 0.50. \)

4. Results and Discussion

The intensity differences \( \Delta I \) of experimental data, which is plotted in Figure 4 (left) for \( \chi_{II} = 0 \) and (right) for \( \chi_{II} = \pi \), is in a next step Fourier transformed in the same manner as the both simulations. Note that for \( \chi_{II} = 0 \) the intensity in the H-beam (reflected) does not become zero, this is due to the uneven number of reflections and transmissions of the interfering beams. In O-direction (transmitted) the intensity may approach zero \( (\chi_{II} = \pi) \), since the numbers of transmissions and reflections are the same.

![Figure 4](image-url)

**Figure 4.** Measured time-depended which-way signal \( \Delta I \) obtained for phase settings \( \chi_{II} = 0 \) (left) and for \( \chi_{II} = \pi \) (right), together with phase settings indicated in IFM.

The final results of the interference measurements are presented in Figure 5. The structure of the plot is the following: on the left side simulations under ideal circumstances \( (C_{ii} = C_{ij} = 1) \) and with corrected contrast (c.c.) are shown by dashed blue lines and solid orange lines,

![Figure 5](image-url)

**Figure 5.** Simulations of power spectra for perfect contrast (dashed blue line) and contrast corrected (solid orange line) are plotted on the left. Power spectra of the interference measurements (solid red line) are plotted on the right. Aside are the respective phase settings of the IFM.
Figure 6. Simulations of power spectra for perfect contrast (dashed blue line) and contrast corrected (solid orange line) are plotted on the left. Power spectra of the absorber measurements (solid red line) are plotted on the right. Aside are the respective phase settings of the IFM.

respectively. The measured power spectra are plotted on the right panel. In the upper row of Figure 5, power spectra of simulations and the measurement are depicted for the phase setting $\chi_{II} = 0$. All peaks are found at the expected frequency differences, namely $\Delta \omega_R = 3$ kHz, $\Delta \omega_I = 6$ kHz, $\Delta \omega_{II} = 9$ kHz, and $\Delta \omega_{I+II} = 12$ kHz. The ideal simulation, the c.c. simulation, and the interference measurement have the same peak heights for the respective frequencies. The peaks at frequencies $\Delta \omega_R$, $\Delta \omega_I$, and $\Delta \omega_{II}$ are of the same height, while $\Delta \omega_{I+II}$ is twice as high as the others. This is because the WW-signal from SR$_{I+II}$ has to pass one beam splitter less than the signals from SR$_I$, SR$_{II}$, and SR$_R$ on the way to the detector. Consequently its amplitude is larger by a factor of $\sqrt{2}$. In addition, since SR$_{I+II}$ marks the superposed wave functions $\Psi'_I + \Psi'_II$/$\sqrt{2}$, the amplitude of the signal from SR$_{I+II}$ gains another factor of $\sqrt{2}$. Hence, the final amplitude of the signal of SR$_{I+II}$ is twice the amplitude of the signals from the other SRs, resulting in a doubling of the peak in the power-spectrum.

The power spectrum with the phase setting $\chi_{II} = \pi$ is plotted in the lower row of Figure 5. From Equation 3 and Equation 4, one expects two peculiarities for the ideal simulation (dashed blue line). In ideal circumstances, the beam in the path $I$ + $II$ is expected to have (absolutely) zero intensity due to destructive interference and the other peaks to decrease to one third. However, due to imperfect (destructive) interference capacity of our IFM ($C_{ij} < 1$) a leakage of neutrons from the front loop to the beam path $I$ + $II$ occurs, which is responsible for the non-zero peak at $\Delta \omega_{I+II}$ in the power-spectrum.

As a last step a beam-blocker (BB), made of 1 mm thick Cadmium, is inserted at position BB$_R$ or BB$_{I+II}$. The respective power spectra of simulations and the measurement are shown in Figure 6. When the beam-blocker in the position BB$_R$ (upper row), the beam in path $R$ no longer contributes and the ideal simulation shows no peaks at all. However, due to leakage from the front loop the peaks at frequencies $\Delta \omega_I$, $\Delta \omega_{II}$, and $\Delta \omega_{I+II}$ emerge. When the beam-blocker is put in the position BB$_{I+II}$ the WW-signals of SR$_I$, SR$_{II}$, and SR$_{I+II}$ are blocked and the respective peaks are invisible in the power spectrum (lower row). The height of the peak at frequency $\Delta \omega_R$ is unchanged. The c.c. simulation well reproduces the results of the measurements with absorbers inserted at position BB$_R$ or BB$_{I+II}$. 

5. Conclusions

Finally, we want to emphasize that the presented simple model, that is completely within the framework of standard quantum mechanics, gives both qualitative and quantitative correct predictions, without the need for the two-state vector formalism. The discontinues trajectories of the photons described in [2] stem from a misinterpretation of the observed faint traces. A faint trace is only a sufficient but not a necessary condition for the presence of the particle. Our experimental results witness the multifold presence of the neutron’s wave function in the interferometer.

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