Temperature dependence of nonlinear susceptibilities in an infinite range interaction model

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Abstract
We present a model to probe metamagnetic properties in systems with a variable number of interacting spins. Thermodynamic properties such as the magnetization per particle \(m(B, T, N)\), linear susceptibility \(\chi_1(T)\), and nonlinear susceptibilities \(\chi_3(T)\) and \(\chi_5(T)\) were calculated. The model produces a different magnetic response for \(N\) particles when comparing to \(N-1\) particles for small \(N \sim 1\). For an even number of particles, the susceptibilities show maxima in their temperature dependence. An odd number produces an additional free spin response that dominates at low temperatures. This free spin response also produces a step in the field-dependent magnetization per particle at \(B = 0\) for odd \(N\). The magnetization shows \(N/2\) steps at \(\gamma_B/J = n\) with integer \(n\) for even \(N\) and \((N-1)/2\) additional steps at \(\gamma_B/J = n + 1/2\) with integer \(n\) for odd \(N\). Small clusters respond with metamagnetism in an otherwise isotropic spin space, while the large clusters show no metamagnetism.

Keywords: metamagnetism, molecular magnetism, theory

(Some figures may appear in colour only in the online journal)

1. Introduction
Metamagnetism, a discontinuous rise in magnetization at zero temperature for a critical magnetic field, has long been a topic of interest [1–5]. Typically this happens as a discontinuity at low temperatures which smooths and broadens as temperature increases. In an insulating antiferromagnet the transition is described as a first order spin flop transition [1]. When the magnetic field is applied along some unpreferred direction, the low field response is weak because the spins are locked along the preferred direction. The spins line up along the magnetic field when the field exceeds the anisotropy energy. The anisotropy derives from crystalline electric fields and depends on the field orientation with respect to the lattice structure.

As opposed to the above, the strongly correlated metals (such as the heavy fermions) indicate a development of the length of the spin vector. This suggests that for strongly correlated metals like the heavy fermions, it is likely that the roots of metamagnetism lie in the exchange interaction instead of the anisotropy energy. In this case, the phenomena is not a balance between Zeeman and anisotropy energies, rather it appears to be a break of low field antiferromagnetic bond between the spins. This work proposes that a discrete energy level structure whose crossing represents the metamagnetism should be describable in terms of a spin Hamiltonian. Molecular metamagnetism belongs to problems of this class. There is macroscopic response coming from an ensemble of molecules with a finite number of spins and negligible intermolecular interaction.

We study an infinite range antiferromagnetic Heisenberg exchange interaction model [6] to complement the work done by Shivaram et al [7]. They developed an experimental framework involving nonlinear susceptibilities and their temperature dependence. The measured susceptibilities, both linear and nonlinear, have peaks that scale with the critical field, indicating a very small number of energy scales in the low lying many body energy levels.

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With a cluster of spins situated on lattice sites, the model is applicable to a case where all spins are equidistant and inter-cluster interactions are negligible. In passing, we note that the maximum number of equidistant spins in a cluster in $d$ dimensions is $d + 1$, making $N = d$ the largest cluster size which this model can exactly calculate. We have, however, proceeded to discuss the case where $N$ can be large, representing a cluster with near equally spaced spins.

This paper presents an overview of a model describing a cluster of $N$ fermionic (spin $s = 1/2$) spins interacting through a uniform interaction $J$. These interacting spins are influenced by an external magnetic field $B$ in the direction of the spin quantization axis $z$. Using the dimensionless variables $\tau = k_B T / J$ and $b = \gamma B / J$, the principal results are summarized here:

1. The responses for even and odd number of particles are qualitatively different. There are $(N + n_0)/2$ steps in the magnetization with $n_0 = N \mod 2$ describing the ‘oddness’ of $N$ (for even $N, n_0 = 0$ and for odd $N, n_0 = 1$). The steps occur at critical field values

$$B_c = \left( \frac{2n + n_0}{2} \right) \frac{J}{\gamma} n = 1, 2, ..., \frac{N - n_0}{2}.$$  

The magnetization per particle changes by $1/N$ at each $B_c$. When $N$ is odd, there is an additional step at $B_c = 0$ due to a free spin response which changes the magnetization per particle by $1/2N$. The fully saturated magnetization per particle is $m = 1/2$ for all $N$. This type of even–odd effect is common in quantum magnetism.

2. For odd $N$, the ground state is a Kramer’s doublet leading to a free spin response; a Curie law contribution to the total susceptibility.

3. The nonlinear susceptibilities are defined as

$$M - \chi_1(T)B = \chi_3(T)B^3 + \chi_5(T)B^5.$$  

The third and fifth order susceptibilities are negative at high temperatures. The susceptibilities rise at low temperatures and show a maximum at a characteristic temperature. For even $N$, they all go to zero at $T = 0$. For odd $N$, the free spin response dominates at low temperatures. The analysis of Shivaram et al. [7] is based on $H = \Delta S_z^2 - \gamma BS_z$ which shows similar results.

4. The specific heat as a function of temperature at $b = 0$ is Schottky-like. It rises exponentially at low temperatures and decays as $\tau^{-2}$ at high temperatures. As a function of magnetic field at low temperatures, the specific heat has a minimum at the critical fields buttressed by peaks both below and above the critical field. As the temperature increases, the minima stay fixed but the peaks move out.

The rest of the paper is organized as follows: section 2 focuses on the details of the infinite range Heisenberg exchange interaction model. The partition function and the observables are discussed for a generalized particle number $N$, commenting on the difference in the even $N$ and odd $N$ cases. Section 3 presents the results in the small $N$ limit ($N = 2, 3, 5$), and section 4 discusses the large $N$ limit ($N = 23, 24$) and the thermodynamic limit ($N \rightarrow \infty$). Section 5 contains a summary of the results and a discussion of the limitations of the model.

2. Model

In the model considered here, each spin interacts with all other spins. The Hamiltonian is

$$H = J \sum_{i < j} s_i \cdot s_j - \gamma B \cdot S.$$  

(1)

Here $J$ is an antiferromagnetic exchange interaction of infinite range, $\gamma$ is the gyromagnetic ratio, $B$ is the external field, and $S = \sum_i s_i$ is the total spin of the system. The eigenenergies of the system are calculated using the identity:

$$2 \sum_{i < j} s_i \cdot s_j = \left( \sum_i s_i \right)^2 - \sum_i s_i^2 = S^2 - \frac{3N}{4}.$$  

(2)

This produces the eigenenergies

$$\lambda = \frac{J}{2} S(S + 1) - \frac{3N}{8} J - \gamma B \mu,$$  

(3)

where $\mu$ is the $z$ component of spin and $S$ is the quantized spin eigenvalue. The range of values for $\mu$ and $S$ depend on whether there are an even or odd number of particles. The rewriting of the of the sum in equation (2) is a common trick in quantum magnetism. For generalization, it is useful to define the variable $n_0 = N \mod 2$, same as in principal result 2. The range of values for $\mu$ and $S$ can then be written $-S \leq \mu \leq S$ and $\frac{1}{2} n_0 \leq S \leq N/2$, with all values spaced by an integer. The partition function can then be written from these eigenenergies

$$Z = e^{N/2} \sum_{S = \frac{1}{2} n_0}^{N/2} e^{-S(S+1)/2} \sum_{\mu = -S}^{S} e^{\mu b / \tau}.$$  

(4)

Here $\tau = k_B T / J$ and $b = \gamma B / J$ are dimensionless variables describing the temperature and field. The prefactor $\exp(3N/2 \beta J)$ is a constant multiple that will cancel when calculating thermal properties, and will thus be dropped from here on. Finally the partition function leads to the Gibbs’ free energy such that $F = -k_B T \ln Z$, from which comes the thermodynamic properties: magnetization $M = -\partial F / \partial B$, magnetic susceptibilities $\chi_\mu = (-\partial^2 M / \partial B^2)_{B=0}$, specific heat $C = -T(\partial^2 F / \partial T^2)$, and pressure $P = -\partial F / \partial V$. For the pressure there is no explicit volume dependence. However the magnetic Gruneisen constant [8] $\gamma_B = \partial \ln J / \partial \ln V$ affords a simple expression of dimensionless pressure $PV/\gamma_B J = \partial F / \partial J$. The susceptibilities $\chi_n$ are nonzero only for odd $n$.

To more easily evaluate the thermodynamic properties, the partition function can be rewritten by changing the order of the summations. To simplify the expression, the field-independent function $A_\mu(N) = \sum_{\Delta S_z} e^{-S(S+1)/2\tau}$ is used. The even and odd partition functions are then,
\[
Z(N = 2n) = A_0(N) + 2 \sum_{\mu=1/2}^{N/2} A_\mu(N) \cosh \left( \frac{\mu b}{\tau} \right),
\]
\[
Z(N = 2n + 1) = 2 \sum_{\mu=1/2}^{N/2} A_\mu(N) \cosh \left( \frac{\mu b}{\tau} \right). \tag{5}
\]

From here the magnetization \( M \) for each becomes,
\[
M(N = 2n) = 2 \sum_{\mu=1/2}^{N/2} \mu A_\mu(N) \sinh(\mu b/\tau) 
\]
\[
2 \sum_{\mu=1/2}^{N/2} \mu A_\mu(N) \sinh(\mu b/\tau) \tag{6}
\]

The nonlinear susceptibilities follow from these expressions. Likewise the other thermodynamic observables such as specific heat or pressure (the field dependent part, using the implicit volume dependence of \( J \)) can be obtained using the other thermodynamic derivatives. It is possible to rewrite the magnetization for odd particle number \( M(N = 2n + 1) \) in order to highlight the free spin term. This calculation gives a term similar to what is seen for \( M(N = 2n) \) in equation (6).
\[
M(N = 2n + 1) = \frac{1}{2} \tanh \left( \frac{b}{2\tau} \right)
\]
\[
\frac{2 \sum_{\mu=1/2}^{N/2} \mu A_\mu(N) \sinh((\mu - 1/2)b/\tau)}{\sum_{\mu=1/2}^{N/2} A_\mu(N) \cosh((\mu - 1/2)b/\tau)}, \tag{7}
\]

where \( B_\mu(N) \) is defined from the functions \( A_\mu(N) \),
\[
B_\mu(N) = 2 \sum_{m=\mu}^{N/2} (-1)^{m-\mu} A_m(N). \tag{8}
\]

By reindexing the sums in equation (7) using \( \mu' = \mu - 1/2 \) and replacing \( B_{\mu'+1/2}(N) \) with \( A_{\mu'}(N - 1) \) the second term gives back the magnetization for even particle number. Deriving equation (7) can be done in a few steps:

1. Rewrite the partition function:
\[
Z(N = 2n + 1) = \cosh(\mu b/2\tau) F(b/\tau).
\]
2. Show the closed form for \( F(b/\tau) \):
\[
F(b/\tau) = 2 \sum_{\mu=1/2}^{N/2} A_\mu(N) f_\mu(b/\tau).
\]
3. Show the two recursive forms for \( f_\mu(b/\tau) \):
\[
f_\mu(b/\tau) = f_{\mu-1}(b/\tau) \cosh(b/\tau)
\]
\[
+ 2 \sinh((\mu - 1)b/\tau) \sinh(\mu b/2\tau)
\]
\[
= 2 \cosh((\mu - 1/2)b/\tau) - f_{\mu-1}(b/\tau),
\]

(3) Show the two recursive forms for \( f_\mu(b/\tau) \):
\[
f_\mu(b/\tau) = 2 \sum_{m=0}^{\mu-1/2} (-1)^m \cosh((\mu - 1/2 - m)b/\tau)
\]
\[
- (-1)^{\mu-1/2},
\]

where \( f_{1/2}(b/\tau) = 1 \). The first recursion comes from writing out steps (1) and (2). Writing out the first few terms of \( f_\mu \) gives evidence of the second recursion, which naturally leads to the closed form. By plugging the closed form from the second recursion into the first recursion, the equivalence between the two can be shown.

4) Find \( M \) by taking the necessary derivative of \( Z \) as written in 1).

The critical field values arise from the change in the ground state [9]4. In the low temperature limit, the transitions at the critical field values act as fermi functions. A low temperature analysis of the magnetization can show these critical field values. It is also possible to find the critical fields by calculating the magnetic field at the crossing of the ground state eigenenergies given in equation (3). In both cases the critical field values depend on whether there are even or odd number of particles. Using the ground state crossing method for even number of particles, it is trivial to show that a transition from the current ground state to the next ground state occurs at \( b_c = \gamma B_c/J = S_c + 1 \), where \( S_c \) is the spin of the current ground state, and the ground state at \( b = 0 \) is \( S_c = 0 \). This means that there is a transition for \( b_c = 1,2,...,N/2 \). Performing the same calculation for the odd particle number case, there will be a transition at \( b_c = 0 \), after which there is a transition at \( b_c = 3/2, 5/2,..., N/2 \).

3. Small \( N \)

At this stage it is important to look into expressions for specific values of \( N \), both to get a better, more comprehensive understanding and also to check the validity of some results in the space of hyperbolic functions. Some of these results can be derived by direct calculation of the partition function and used as a check.

3.1 \( N = 2 \)

Two spin half particles are described by a partition function \( Z = 1 + e^{-1/\tau}(1 + 2 \cosh b/\tau) \). The magnetization \( M \) is given by
\[
M = \gamma \frac{\sinh(b/\tau)}{C_2 + \cosh(b/\tau)} , \quad C_2 = \frac{1}{2} A_0(2) = \frac{1}{2} (1 + e^{1/\tau}) \tag{9}
\]

which leads to

4 For a similar model for molecular magnets with only short range interactions see [9].
\[ \chi_1(T) = \frac{\gamma^2}{2k_BT} \left( \frac{1}{1 + C_2} \right), \quad (10a) \]

\[ \chi_3(T) = \frac{\gamma}{3!} \left( \frac{\gamma}{k_BT} \right)^3 \frac{C_2 - 2}{(1 + C_2)^2}, \quad (10b) \]

\[ \chi_5(T) = \frac{\gamma}{5!} \left( \frac{\gamma}{k_BT} \right)^5 \frac{C_2^2 - 13C_2 + 16}{(1 + C_2)^3}. \quad (10c) \]

These are the principal nonlinear susceptibilities. The results for the zero temperature magnetization \([10]\) and linear susceptibility \([11]\) are well known. The results are shown in figures 1(a) and 2(a), respectively. The temperature dependence of the susceptibilities is qualitatively similar to the anisotropy based models \([7]\) (replacing \(C_2 \rightarrow \frac{2}{3} e^{1/\tau}\)). The linear susceptibility \(\chi_1\) has a maximum at \(T_1 = 0.624J\) and vanishes at both low and high temperatures. At high temperatures, it has an effective Curie–Weiss temperature \(\theta = J/4\). The third order susceptibility \(\chi_3(T)\) is negative at high temperatures but has a positive maximum at \(T_3 = 0.265J\) and vanishes at \(T = 0\). The next order nonlinear susceptibility, \(\chi_5(T)\) is qualitatively similar with a peak at \(T_5 = 0.176J\).

The specific heat \(N = 2\) is shown versus magnetic field in figure 1(d). The specific heat with respect to the field has a response with \(M\)-shaped peaks centered at the critical field value \(b = 1\). As the temperature increases, the two peaks move away from each other, but at the critical value \(b = 1\) the specific heat stays zero. As a function of temperature, the specific heat has the usual Schottky features, e.g. those of the specific heat of a two level system. They include an exponential rise at low \(T\) and an inverse power law decay at high temperatures.

The magnetic field dependence of the pressure is shown in figure 1(g). It consists of a threshold proportional to temperature at \(b = 1\), with flat regions for \(b \neq 1\).

### 3.2. \(N = 3, 5\)

Here the ground state is a Kramer’s doublet. This leads to several interesting effects. The partition function for \(N = 3\) is given by:

\[ Z = 2e^{-3/8\tau} \cosh \left( \frac{b}{2\tau} \right) \left[ 1 + 2e^{3/2\tau} \cosh \left( \frac{b}{\tau} \right) \right] \quad (11) \]

and the magnetization \(M\) is given by

\[ M = \frac{\gamma}{2} \tanh \left( \frac{b}{2\tau} \right) + \frac{\gamma}{C_3} \frac{\sinh(b/\tau)}{\cosh(b/\tau)}, \quad (12) \]

\[ C_3 = \frac{1}{2} B_{1/2}(3) = \frac{1}{2} e^{3/2\tau}. \]

The first term is the free particle \(S = 1/2\) response. With the odd number of particles this is the dominant contribution at low temperatures. The linear and nonlinear susceptibilities are given by

\[ \chi_1 = \frac{\gamma^2}{4k_BT} \left[ 1 + \frac{4}{1 + C_3} \right], \quad (13a) \]

\[ \chi_3 = \frac{\gamma}{3} \left( \frac{\gamma}{2k_BT} \right)^3 \left[ 1 + 2\frac{C_3 - 2}{(1 + C_3)^2} \right], \quad (13b) \]

\[ \chi_5 = \frac{2\gamma^5}{15} \left( \frac{\gamma}{2k_BT} \right)^5 \left[ 1 + 2^5 \frac{5C_3^2 - 13C_3 + 16}{(1 + C_3)^3} \right]. \quad (13c) \]

The first term in each of the above equations is the free particle \(s = 1/2\) response. The second term is (and the following terms are) the usual linear and nonlinear susceptibility albeit with an \(N\) dependent \(C_3\). There is a step at \(b = 0\) leading to a magnetization per particle \(m = 1/6\). This is followed by a step at \(b = 3/2\) with \(m = 1/2\). For odd number of particles without a magnetic field the ground state is doubly degenerate and the magnetization vanishes. However at the smallest field there is a separation in the energy levels for the \(m = \pm 1/6\) leading to a nonzero magnetization at low \(T\) and a step in \(m(b)\) at \(b = 0\).

The general features for an odd \(N\) are seen in figures 1 and 2 for both \(N = 3\) and \(N = 5\). The \(N = 5\) plots more clearly show these features. Figures 1(b) and (c) show the magnetization steps at the critical values of the magnetic field. The step at \(b = 0\) is followed by one at \(b = 3/2\) (for \(N = 3\)) at \(5/2\) (for \(N = 5\)). The linear (figure 2(a)) and nonlinear (figures 2(b) and (c)) susceptibilities with the low temperature limit are shown with the free spin response contribution removed. When included, all \(\chi_i\) diverge for the \(\tau \to 0\) limit. The features are similar to the even \(N\) response characteristic of the model; negative at high \(T\). As in the even \(N\) case, the specific heat (figures 1(e) and (f)) dips to zero at the critical field values. The pressure as a function of magnetic field (figures 1(h) and (i)) show the phase transition at the critical field values.

### 4. Large \(N\)

The partition function in equation (4) has a large \(N\) limit (by replacing \(J \to J/N\)) which can be studied in two alternative ways. Analytically, an infinite \(N\) limit can be studied by replacing the sum by an integral. The magnetization turns into a Gaussian integral of the form:

\[ M = \lim_{N \to \infty} \int_{m_0/2}^{m_0} dS e^{-S(S+1)/2N} f(S), \]

\[ f(S) = \frac{\sinh \left( \frac{(2S+1)b}{2\tau} \right)}{\sinh \left( \frac{b}{2\tau} \right)}, \]

which can be evaluated in terms of error functions. The results can be made more transparent by evaluating the sums directly over a large number of particles, such as \(N = 23\) or \(24\) (odd and even cases separately), and interpolating \(N \to \infty\). As discussed previously, this model is exact only for \(N \leq 4\), however setting the particle number significantly larger can give insight to what can be expected when the model breaks.

Figure 3 shows the full extension of the model to large particle number. Figure 3(a) plots the magnetization per particle \(m\) as a function of magnetic field at low temperatures for \(N = 23\). There is the expected step at \(b = 0\) followed by
11 more steps at $b = 3/2N, 5/2N, ..., 1/2$, which can be seen in both the magnetization per particle and the pressure in figure 3(c). By replacing $J \rightarrow J/N$, the critical field values are shifted to the field range $0 < b < 0.5$ due to the division by $N$. The M shape in the specific heat (figure 3(b)) at small temperatures can be seen for each of the critical field values. The even $N$ case for the magnetization per particle, specific heat, and pressure using $N = 24$ are seen in figures 3(d)–(f). Features similar to the $N = 23$ case are seen for $N = 24$.

The results here show that there will be steps in the magnetization per particle and pressure which quickly blur with increased temperature. This implies that with an increase in cluster size, the number of steps increases linearly with $N$. Since the change in the magnetization is the same in all cases, this means that the change in the magnetization per particle is always $1/N$, except for the initial $b = 0$ change for odd $N$, which is $1/2N$. This would result in a disappearance of the metamagnetism in the $N \rightarrow \infty$ case. The number of particles discussed in this section, however, are well outside of any physical system to which this model can appropriately be applied. This implies that since the metamagnetism is still present here, it will be present for all smaller number of particles where this model is physically relevant.

5. Discussion and conclusions

The infinite range interaction model, which is solvable through transparent intermediate steps, contains many of the properties found in metamagnetism. The model contains only one energy scale, $J$, apart from the thermodynamic control variables, the external magnetic field $B$ and the temperature $T$. We report here results for a finite, albeit wide ranging number of spins. The notable features of this model are:

(a) There are features due to the oddness of the total number of particles $N$. For odd $N$, there is a free spin contribution to the susceptibility that diverges as $T^{-1}$.

(b) There are quantum transitions at $T = 0$ which lose their singular transition properties at any non-zero temperature. The total magnetic susceptibility $\chi(B,T = 0)$ is singular at a series of critical fields $B = nB_c$ with $n = 1, 2, ...$ for an even number of spins. For an odd number of spins, the susceptibility diverges at $B = (n + 1/2)B_c$.

(c) The nonlinear susceptibilities, defined by

$$M/B = \chi_1(T) + \chi_3(T)B^2 + \chi_5(T)B^4,$$

have been measured for UPt$_3$ [7], and are negative at high temperatures. In metamagnetism, they become positive.
at low temperatures and go through a maximum. These Shivaram peaks [7] scale together as evidence of a minimal number of energy scales. Their relative magnitudes are in accord with the model here.

(d) Finally there are results for the field dependence of the specific heat and pressure (from which the magnetic field dependence of the compressibility and therefore the sound velocity can be derived). Principally, the specific heat vanishes at the critical fields and the pressure shows steps at $T = 0$.

Metamagnetism in a strongly correlated metal derives its properties from complicated interactions. It is usually described by a lattice of magnetic moments interacting with conduction electrons and often through the Anderson

Figure 2. Linear (a) and first two nonlinear (b) and (c) susceptibilities for $N = 2, 3, 5$. The susceptibilities for odd $N$ should diverge as $\tau \to 0$ due to the free spin response. To compare the $N = 3, 5$ susceptibilities to the $N = 2$ susceptibilities, the free spin response has been subtracted out. All properties are described below with increased temperature. (a) The linear susceptibilities show a single maximum and a tail asymptotically approaching zero. (b) The first nonlinear susceptibility $\chi_3 = (d^3M/db^3)_{b=0}$ peaks, decreases to a negative valued minimum, and then increases asymptotically towards zero. (c) The second nonlinear susceptibility $\chi_5 = (d^5M/db^5)_{b=0}$ shows similar, yet more exaggerated, features of $\chi_3$.

Figure 3. Thermodynamic quantities for $N = 23, 24$. (a)–(c) Are for $N = 23$ and (d)–(f) are for $N = 24$. The first column ((a) and (d)) are the magnetization per particle $m$ versus field $b$ for three temperatures $\tau = 0.0001, 0.1, 0.5$. The second column ((b) and (e)) shows the M shape in the specific heat $C$ versus field $b$ for $\tau = 0.05$. The third column ((c) and (f)) shows the pressure $P$ versus field $b$ for three temperatures $\tau = 0.0001, 0.05, 0.5$. 
model, causing the heavy mass of fermions as an outcome of a Kondo-like effect. The ground state corresponds to a singlet resonance between the local moment and the conduction electrons. Metamagnetism might happen when the magnetic field breaks the singlet resonance [12, 13]. Several experimental correlations [5, 14] appear reasonable within this formalism. One outcome that remains to be explored experimentally is that between the critical field and the effective mass.

In order to calculate a more complex observable, such as nonlinear susceptibilities, a truly microscopic calculation is a non-starter. An intermediate framework needs to be established which both incorporates multiple microscopic parameters and has the facility to proceed with more complex calculations. Described here is a model that seems reasonable from a microscopic point of view but is simple enough to yield nonlinear susceptibilities.

Metamagnetism in anisotropy based models describes a transition in the spin orientation. It is a specific component of the magnetization that changes discontinuously at some critical magnetic field. In contrast, the transition in the Wohlfarth–Rhoades type models is in the length of the spin vector, i.e. at the transition the magnitude of the macroscopic magnetization changes sharply. A simple view here is provided by an antiferromagnetic interaction. The macroscopic magnetization is zero, but the magnetic field changes the ground state so that the sample magnetization becomes finite at some critical magnetic field. In this paper we analyze the latter as independent of any anisotropy effects. Clearly when the macroscopic magnetization appears, it must experience structural orientation effects as well. In formal terms, the inertial term $J$ should be the gap in the spin wave frequency (depending on both the exchange and the anisotropy energies) and is governed by the spin dynamics including the anisotropy terms.

The model here is applicable to molecular magnets [9, 15, 16]$^4$, notwithstanding the (hopefully weak) intermolecular interactions. It is also more suitable for compact molecules where a uniform interaction is a better description, as opposed to a stretched molecule with a distribution of exchange magnitudes. In small molecules, there are effects depending on even–odd number of spins.

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