Generation of frequency multiplexed entangled single photons assisted by the entanglement

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We present a scheme to generate the frequency multiplexed entangled (FME) single photons based on the entanglement between two species atomic mixture ensemble. The write and read fields driven according to a certain timing sequence, the generation of FME single photons can be repeated until success is achieved. The source might have significant applications in wavelength division multiplexing quantum key distribution.

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In classical communication, wavelength division multiplexing (WDM) is a successful method for increasing the transmission capacity of optical fiber systems. In quantum communication, photon is the basic information carrier. To increase the amount of information, we may use the similar technique, but different channels may not be independent to each other. A source for the wavelength division multiplexing quantum key distribution (WDM-QKD) system must satisfy two conditions: (1) it must be a true single-photon source (to avoid eavesdropping); (2) it must contain photons with different wavelengths. A frequency multiplexed entangled (FME) single photon state has the form of (two frequency components)

$$|\psi(0)\rangle = c_1|\omega_1, 0\rangle + c_2|0, \omega_2\rangle,$$

which can be prepared in our scheme and might be an excellent candidate for WDM-QKD.

The generation of two- and multi-color light fields has been widely studied in three-level and double (or multi-) Λ configuration atomic systems. In these systems rich spectral features have been manifested, due to constructive or destructive quantum interference between different channels. For three-level atomic system, the studies of multi-color field generation have been reported theoretically and experimentally. Extending the analysis to a double (or multi-) Λ system, it is possible to simultaneously generate two- and multi-color light fields in the medium based on a dark state polariton (DSP) consisting of low-lying atomic excitations and photon states of two frequencies. The existence of the DSP in the Λ and double (or multi-) Λ atomic system required that the fields obey certain conditions for frequency, amplitude, and phase matchings. If the phase of the two (or multi-) color reading light fields is mismatched, then one of the pulses will be absorbed and lost. Here, we present a scheme that the generation of frequency entangled single photons is from two species atomic mixture ensemble without phase-matching requirement, and it is also a natural extension of existing work. Another potential approach to prepare the FME photon state based on coherent conversion was presented very recently.

In this brief report, we propose a scheme to generate the FME single photons based on the entanglement between two species atomic mixture ensemble. Here we first use the single-photon interference to realize the entanglement between two species atomic mixture ensemble, as the method is used to realize the entanglement between two identically distant atomic ensemble, but it is extension to two different species atoms. Based on the generated entanglement, then we can deterministically obtain the FME single photon.

The outline of our scheme is as follows. We consider an isotope mixture medium, containing N₁ and N₂ (I, II denote two different isotope) Λ-type two species atoms shown in Fig. 1. Here we consider a Rb⁺-Rb mixed atomic ensemble, and Fig. 2 shows their energy level diagrams. To generate the FME single photons, the system firstly is driven by the pumping fields and takes place the Raman scattering processes, then is driven by two read fields. There are four steps for this process. Step 1: optical pumping prepares all mixed atoms in the respective ground hyperfine levels |s⟩ and |s′⟩, and the whole state of the atomic ensemble and the Stokes fields can now be written as

$$|\psi(0)\rangle = |e(0)\rangle_1|e(0)\rangle_II|s_1\rangle|s_0\rangle,$$

where $|e(0)\rangle_1 \equiv |s_1, ..., s_{N_1}\rangle$ and $|e(0)\rangle_II \equiv |s'_1, ..., s'_{N_II}\rangle$ are...
collective atomic ground states, and $|0\rangle S_I$ and $|0\rangle S_{II}$ are the vacuum states of the Stokes fields $E_{S_I}$ and $E_{S_{II}}$, respectively. Step 2: sending in two off-resonant write fields $\Omega_{W1}$ and $\Omega_{W_{II}}$ interacting with the transition $|s\rangle \rightarrow |e\rangle$ and the transition $|s'\rangle \rightarrow |e'\rangle$, respectively, and the Stokes fields $E_{S_I}$ and $E_{S_{II}}$ with respective frequencies $\omega_{S_I} = \omega_{W1} + \omega_{sg}$ and $\omega_{S_{II}} = \omega_{W_{II}} + \omega_{sg'}$, are produced by spontaneous Raman process from each of the isotopes as illustrated in Fig. 2. We assume that the light-atom interaction time $\tau$ is short or the intensities of the write fields are weak, so both the mean photon numbers in the two forward-scattered Stokes pulse $E_{S_I}$ and $E_{S_{II}}$ are much smaller than 1. To erase the distinguishability of the photons from the Stokes fields $E_{S_I}$ and $E_{S_{II}}$, we choose the condition that the Stokes fields $E_{S_I}$ and $E_{S_{II}}$ have the same frequencies, i.e., $\omega_{S_I} = \omega_{S_{II}} (= \omega_0)$. Step 3: when only one photon is detected by the detector, a single spin wave excitation in each one of the two atomic species in the same region of space is generated. Because the frequencies of the two Stokes fields from the isotope are the same, we can not decide which isotope emit the photon, and the final state of the mixed ensemble is a superposition of two probabilities, i.e. an entangled state. Note that the generations of the entangled state from our scheme and from the DLCZ scheme [17] are based on the single-photon interference. But the mechanism are completely different. The former is based on the inner interference due to two Stokes fields use a common vacuum state. The latter is external interference based on a beam splitter where two Stokes fields are generated from different vacuum state. Step 4: after generation of the entangled state of the mixed ensemble, another two read fields $\Omega_{R_I}$ and $\Omega_{R_{II}}$ are driven on the isotope mixture system, and the spin wave is coherently converted into the photon with frequency $\omega_1$ or $\omega_{II}$ shown in Fig. 3. After driven by two read fields $\Omega_{R_I}$ and $\Omega_{R_{II}}$, the two-atomic-isotope ensemble radiates photons and both transitions lead to the same final atomic state of the whole ensemble $|e(0)\rangle_1|e(0)\rangle_{II}$, then the FME single photon will appear. The generation efficiency could be improved with the use of a recycled pulsed sequence shown in Fig. 4 which will be described in detailed later.

In order to illustrate the main idea, we give a detailed analysis of the generation process. Consider two species atomic mixture ensemble as shown in Fig. 1: the write fields $\Omega_{W1}$ and $\Omega_{W_{II}}$ inject the atomic ensemble, and the quantized Stokes fields are generated (see Fig. 2). Because the single photons $|1_{W1}\rangle S_I$ and $|1_{W_{II}}\rangle S_{II}$ are indistinguishable, i.e., $|1_{W1}\rangle S_I = |1_{W_{II}}\rangle S_{II} \equiv |1_{W}\rangle S$ then in this case the Stokes fields can be described by a common operator. For conceptual simplicity we assume the quantized field corresponds to a single mode of a running-wave cavity that is initially in a vacuum state. Note that this cavity field consideration is valid in the limit of unity finesse, i.e., in free space configuration. Then the Hamiltonian for two species atomic mixture ensemble in the rotating frame can be split into two parts $\hat{H} = \hat{H}_1 + \hat{H}_{II}$ according to different isotopes

$$\hat{H}_1 = -\hbar \Delta \hat{S}_{ee'} + [\hbar \Omega_{W1} \hat{S}_{es} + \hbar g_1 \hat{a} \hat{S}_{eg} + \text{H.c.}], \quad (3)$$

$$\hat{H}_{II} = \hbar \Delta \hat{S}_{e'e''} + [\hbar \Omega_{W_{II}} \hat{S}_{e's'} + \hbar g_{II} \hat{a} \hat{S}_{eg'} + \text{H.c.}], \quad (4)$$

where $\hat{S}_{\mu\nu} = \sum_j |\mu\rangle \langle \nu| \mu, \nu$ are collective atomic operators corresponding to transitions between atomic states $|\mu\rangle, |\nu\rangle$, and $\Delta$ is the detuning. $g_1$ and $g_{II}$ are the atomic-cavity field coupling constants of two isotope, respectively. $\Omega_{W1}$ and $\Omega_{W_{II}}$ are the respective Rabi frequencies of two write fields. The evolution of atomic operators is then described by Heisenberg-Langevin equations

$$\dot{\hat{S}}_{\mu\nu} = -\gamma_{\mu\nu} \hat{S}_{\mu\nu} + \frac{i}{\hbar} [\hat{H}, \hat{S}_{\mu\nu}] + \hat{F}_{\mu\nu}, \quad (5)$$

where $\gamma_{\mu\nu}$ is a decay rate of coherence $|\mu\rangle \rightarrow |\nu\rangle$, and $\hat{F}_{\mu\nu}$ are associated noise operators which have zero average and are correlated with $\delta$ associated diffusion coefficients. We can make the adiabatic approximation according to large single-photon detuning $\Delta \gg \gamma_1$ (let $\gamma_{eg} = \gamma_{es} = \gamma_1$, $\gamma_2$ (let $\gamma_{eg'} = \gamma_{e's'} = \gamma_2$), and first order in $\hat{a}$ $(\hat{S}_{es} \sim N_I \hat{S}_{e's'} \sim N_{II})$, then we obtain the equations of motion for the cavity mode and the ground state coherences $\hat{S}_I = (\hat{S}_{sg}/\sqrt{N_I})$ and $\hat{S}_{II} = (\hat{S}_{sg'}/\sqrt{N_{II}})$

$$\dot{\hat{a}} = -\kappa \hat{a} - \chi \hat{S}_{\dagger} - \chi \hat{S}_{\dagger} + \hat{F}_a, \quad (6)$$

$$\dot{\hat{S}}_I = -\gamma_{gs} \hat{S}_I + i\delta_{L_I} \hat{S}_I + i\chi_1 \hat{a} + \hat{F}_{S_I}, \quad (7)$$

$$\dot{\hat{S}}_{II} = -\gamma_{gs'} \hat{S}_{II} - i\delta_{L_{II}} \hat{S}_{II} + i\chi_1 \hat{a} + \hat{F}_{S_{II}}, \quad (8)$$

where $\kappa$ is the decay rate of the cavity mode, and $\chi_j = g_j \sqrt{N_j} \Omega_{W_j}/\Delta$ ($j = I, II$) is the coupling rate between the collective spin excitation $\hat{S}_j$ and the quantized field $\hat{a}$, and $\gamma_{L_j} = \gamma_j N_j \Omega_{W_j}/\Delta^2$ ($j = I, II$) is an optical pumping rate, and $\delta_{L_j} = g_j \Omega_{W_j}/\Delta$ (j = I, II) is the ac Stark shift. In this case the evolution of the entire system is described by an effective Hamiltonian

$$\hat{H}_{eff} = \hbar (\chi_1 \hat{S}_{\dagger} - \chi_{II} \hat{S}_{II \dagger}) \hat{a} + \text{H.c.} \quad (9)$$

FIG. 2: (Color online) Energy level diagrams of two species atoms $^{85}$Rb and $^{87}$Rb, respectively. The Stokes fields $E_{S_I}(\omega_0)$ and $E_{S_{II}}(\omega_0)$ are generated by the write fields $\Omega_{W1}$ and $\Omega_{W_{II}}$, respectively. $\Delta$ is the detuning.
The time evolution of an initial state $|\psi(0)\rangle$ is described by

$$|\psi(t)\rangle = e^{-i[(\chi_1 \hat{S}_1^x - \chi_2 \hat{S}_1^y)]t + H.c.}|\psi(0)\rangle.$$  \hspace{1cm} (10)

After a short interaction time $\tau$, the coefficient $P_j = \chi_j \tau$ ($P_j \ll 1, j = I, II$) is very weak. The final output state of the atomic collective mode and the forward-scattering Stokes mode can be approximated as

$$|\psi(\tau)\rangle = |e^{(0)}_I|c^{(0)}_{11}\rangle|0\rangle - i[P_1|c^{(1)}_I|c^{(0)}_{11}\rangle|1\rangle_{\omega_1}\rangle + P_{11}|c^{(0)}_I|c^{(1)}_{11}\rangle|1\rangle_{\omega_1}\rangle_s,$$  \hspace{1cm} (11)

where

$$|c^{(1)}_I\rangle = \frac{1}{\sqrt{N_1}} \sum_{l=1}^{N_1} e^{i(k_1^{(1)} - \omega_1/c)z_l}|g_1, \ldots, g_l, \ldots, s_{N_1}\rangle,$$

$$|c^{(1)}_{11}\rangle = \frac{1}{\sqrt{N_{11}}} \sum_{l=1}^{N_{11}} e^{i(k_{11}^{(1)} - \omega_{11}/c)z_{l'}}|s_{1}', \ldots, g_{l}', \ldots, s'_{N_{11}}\rangle,$$  \hspace{1cm} (12)

and $k_j^{(2)}$ ($j = I, II$) is the wave vector of the write fields. Here the terms with high order interactions are ignored, and the probability amplitudes $P_I$ and $P_{II}$ are dependent on the atom numbers $N_I$ and $N_{II}$, respectively. A click on the single-photon detector will project the atomic ensemble in the entangled state

$$|\phi\rangle = \frac{1}{\sqrt{P_I^2 + P_{II}^2}} \left[P_I|c^{(1)}_I|c^{(0)}_{11}\rangle - P_{II}|c^{(0)}_I|c^{(1)}_{11}\rangle\right].$$  \hspace{1cm} (13)

If $P_I = P_{II}$, the state is maximally entangled. After generation of the entangled state of the two species atomic ensemble, we can realize the FME single photons based on the generated entanglement of Eq. (13).

After generation of the entangled state of two species atomic ensemble, two read fields $\Omega_{R_I}$ and $\Omega_{R_{II}}$ nearly resonant with respective transition $|g\rangle \rightarrow |e\rangle$ and $|g'\rangle \rightarrow |e'\rangle$ are driven on the atomic ensemble, and the spin wave is coherently converted into the anti-Stokes photon with frequency $\omega_1$ or $\omega_{11}$ (see Fig. 3). Since both transitions lead to the same final atomic state $|e^{(0)}_I|c^{(0)}_{11}\rangle$, one cannot determine path. A frequency multiplexed entangled single-photon state will be generated, which has the form of (two frequency components)

$$|1_{\omega_1, \omega_{11}}\rangle = c_1|1\rangle_{\omega_1}|0\rangle_{\omega_{11}} + c_2|0\rangle_{\omega_1}|1\rangle_{\omega_{11}},$$  \hspace{1cm} (14)

where $c_1 = P_I/\sqrt{P_I^2 + P_{II}^2}$, and $c_2 = P_{II}/\sqrt{P_I^2 + P_{II}^2}$. The maximally entangled single-photon state $|1_{\omega_1, \omega_{11}}\rangle = (|1\rangle_{\omega_1}|0\rangle_{\omega_{11}} + |0\rangle_{\omega_1}|1\rangle_{\omega_{11}})/\sqrt{2}$ can be obtained by adjusting the intensities of the write fields and the mixing ratio of two species atoms. The emitted photon, which is, as before, only a single one, has imprinted on it the structure of the atomic excitation: its frequency spectrum consists of the separated lines of finite width. The scheme also provides an alternative method for measuring level splittings of different atoms.

Note that in the read process, the atomic excitation is mapped onto a FME single photon by application of two read fields. The physics behind this read process is identical to that utilized in electromagnetically induced transparency (EIT) \cite{19} “light storage” experiments \cite{20}. Due to the suppression of resonant absorption associated with EIT, the generated entangled single-photon is not absorbed by the large atomic population in state $|s\rangle$. The read laser converts the atomic spin wave into a DSP

$$\hat{\Psi}_j(z, t) = \cos \theta_j(t) \hat{E}_S' - \sin \theta_j(t) \hat{S}_j, \hspace{0.5cm} (j = I, II),$$  \hspace{1cm} (15)

where $\tan^2 \theta_j(t) = (g_j')^2 N_j/|\Omega_{R_j}|^2$ ($j = I, II$), $g_j'$ is the atom-field coupling constants. Under the adiabatic condition, the equation of motion for $\hat{\Psi}_j$ takes a simple form:

$$(\partial_t + v_y \partial_z)\hat{\Psi}_j(z, t) = 0.$$  \hspace{1cm} (16)

The DSP propagates out of the medium with group velocity $v_y$ and emerges as a single photon. For a single species three-level atomic system, the retrieval of a spin wave had been realized in generation of nonclassical photon pairs \cite{21, 22}. The direction, bandwidth, and central frequency of the entangled single-photon pulse are determined by the direction, intensity, and frequency of the retrieve laser \cite{23}.

Now, we consider the generation efficiency. First, the dark counts of the single-photon detectors are necessarily considered. The dark count gives a detector click,
but without real photon generation. Commercial actively quenched single-photon avalanche photodiodes (SPAD) modules that exhibit dark count rate from 400 Hz to 50 Hz \cite{24}. Extremely low dark count rate of 5 Hz is reported \cite{25}. Second, we must consider the influence of the finite detection efficiency. In our scheme, according to four steps operation, the success of generating the frequency multiplexed entangled single photon $|1_{2}, \omega_{2}\rangle$ depends on the success of generating the desired entangled state $|\phi\rangle$, which in turn depends on the detection result in step 3. When no photon is detected, there is no way to determine converted into a FME single photon by applying two read fields $\Omega_{R_{1}}$ and $\Omega_{R_{11}}$, which is quasideterministic in ideal case.

In fact, we can improve the generation efficiency by simplifying operation procedure-the write and read fields are driven on the ensemble according to the time sequence shown in Fig. \ref{fig:entanglement_data}. If the single photon generated by the write field occurs, then the frequency multiplexed entangled single photon can be generated by the read fields. If not, the read fields are used as the optical pumping for next round. The frequency multiplexed entangled single photons can be obtained by repeating the write and read fields according to the time sequence successively.

As a specific example for realization of our scheme proposed here, we consider a $^{85}$Rb/$^{87}$Rb mixed atomic ensemble. The energy level diagrams of the two species atoms is shown in Figs. \ref{fig:entanglement_data} and \ref{fig:entanglement_data2}. For the $D_{1}$ line of the Rb-85 species, the $\{|s\rangle, |g\rangle\}$ correspond to the levels $F = \{3, 2\}$ of $5S_{1/2}$, and $|e\rangle$ corresponds the level $F = 3$ of $5P_{3/2}$, respectively. For the $D_{1}$ line of the Rb-87 species, the $\{|s\rangle, |g\rangle\}$ correspond to the levels $F = \{2, 1\}$ of $5S_{1/2}$, and $|e\rangle$ corresponds the level $F = 1$ of $5P_{3/2}$, respectively. For a mode match, the write fields $\Omega_{W_{1}} (\omega_{W_{1}})$ and $\Omega_{W_{11}} (\omega_{W_{11}})$ are generated using phase modulation of a single-frequency laser pulse of $\Omega_{W_{3}} (\omega_{W_{3}} = (\omega_{ex} + \omega_{ex'})/2)$, where phase modulation is accomplished by an electro-optical phase modulator, which produces sidebands with frequencies $\omega_{\pm} = \omega_{W_{3}} + \delta \omega_{W}$ and $\omega_{W_{11}} = \omega_{W_{3}} - \delta \omega_{W}$. Also the the read fields $\Omega_{R_{1}} (\omega_{R_{1}})$ and $\Omega_{R_{11}} (\omega_{R_{11}})$ are obtained by another read field of $\Omega_{R_{3}} (\omega_{R_{3}} = (\omega_{ex} + \omega_{ex'})/2)$, which produces sidebands with frequencies $\omega_{R_{1}} = \omega_{R_{3}} - \delta \omega_{R}$ and $\omega_{R_{11}} = \omega_{R_{3}} + \delta \omega_{R}$. For the $^{85}$Rb/$^{87}$Rb isotope mixture system, the relating parameters are shown in Figs. \ref{fig:entanglement_data} and \ref{fig:entanglement_data2} then $\Delta = (\omega_{ex'} - \omega_{ex})/2 = 1.368$ GHz, $\delta \omega_{W} = (\omega_{ex} - \omega_{ex'})/2 = 1899.5$ MHz and $\delta \omega_{R} = \Delta = 1.368$ GHz.

In conclusion, we present a scheme to realize the entanglement between the two different species atoms, and then realize the FME single photon based on the entanglement. Injection the write and read fields according to a certain timing sequence, the generation of FME single photons can be repeated until success is achieved. Our scheme is feasible under the realistically experimental conditions.

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