CONVERSION OF DISLOCATION OSCILLATION WAVES TO SPIN ONES IN THE VICINITY OF OPT TEMPERATURES

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ABSTRACT. Dislocation waves in magnetic crystals in the vicinity of orientation phase transition (OPT) temperatures are considered in a frame of the field theory of the defects. The singularities of the dislocation flows, elastic deformations and magnetization occur if the magnetic subsystem is inhomogeneous and the dispersion of the media is not taken into account. Media dispersion causes a regularity of these parameters and a conversion of the spin wave to the dislocation wave.

1. INTRODUCTION

Real crystals contain defects of different nature, which influence its wave properties. The dislocations influence of phonon spectra of real crystals is especially notoriously. Ferromagnetic structure is sensitive to the different influences in the vicinity of the temperatures of Orientation Phase Transitions (OPT). The low stability of the magnetic state of the ferromagnetic near the temperature of OPT relieves an observation of different non-linear effects. Therefore an interaction between dislocations and magnetic structure of magnetics will be revealed at these temperatures.

Phenomenological description of the dislocation ensembles is based on the statistical effects. These effects are not a simple sum of the properties of a number of dislocations. Statistical properties of the dislocation ensembles reveal its wave characteristics. One of these wave effects is a screening of elastic deformations and correct description needs taking into account dislocation cores.

Elastic fields of separated dislocations are screened therefore the behavior of the dislocation core ensemble determinate the interaction with the crystalline structure. Dislocation ensembles move toward the direction of the influence in the case of directed external influence. Crystal glides of a separate dislocation of the ensemble realize a moving of the ensemble in non-crystallographic directions [1].

2. BASIC EQUATIONS

Quantitative description of the above model of the dislocation ensemble is presented by the equations, which are analogous to the Maxwell’s equations [1, 2]:

\[ \nabla \times \hat{j}(\mathbf{r}, t) = \partial_t \hat{\alpha}(\mathbf{r}, t), \]

\[ S \nabla \times \hat{\alpha}(\mathbf{r}, t) = -B \partial_t \hat{j}(\mathbf{r}, t) - \hat{\sigma}(\mathbf{r}, t) \]

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Here $\hat{j}$ is the tensor of the density of the defect flow; $\hat{\alpha}$ is the tensor of the defect density; $\hat{\sigma}$ is the tensor of the elastic deformations; $S$ is the constant which describes the potential energy of the defect over the unity of the length; $B$ is the constant, which describes the inertial properties of the defect. In addition we take into account the equation of the motion of the continuous media in the form:

\begin{equation}
\rho \partial_t^2 \hat{\sigma}(r,t) = \int \hat{C}(r,t;r_1,t_1) \left( \nabla \otimes \nabla \hat{\sigma}(r,t) + \rho \frac{\partial \hat{j}(r_1,t_1)}{\partial t_1} \right) d\mathbf{r}_1 dt_1,
\end{equation}

where $\rho$ is the density; $\hat{C}(r,t;r_1,t_1)$ is the elastic module tensor. We shall take into account the interaction of the dislocation oscillation waves and the magnetization oscillations putting the tensor $\hat{C}(r,t;r_1,t_1)$ as redefined one by magnetic-elastic interaction. When the materials is characterized by strong and intensive magnetic elastic interaction of the dislocation oscillation waves and the magnetic moments, a situation can results in the effects, which occur in the case when the electric-magnetic waves spread in such materials. We point out that the redefined part of $\hat{C}(r,t;r_1,t_1)$ is proportional to dynamic magnetic susceptibility tensor. Eq.(3) is integer-differential equation due to the time and spatial dispersions. Inhomogeneity of the considered material is determinate by the inhomogeneity of the magnetic sub-system. It may be easily achieved near the OPT temperatures by the temperature gradient, or the external magnetic field. Below we shall consider the cases when the sizes of the inhomogeneity along the $0x$ axis are much more than the wavelengths of the considered wave processes. In this case the inhomogeneity will be take into consideration by the dependencies of the ferrimagnetic resonance frequency on the coordinate $x$. We shall get in the accordance with the Eq.(3):

\begin{equation}
\sigma = -\int \hat{G}(r,t;r_1,t_1) \hat{C}(r_1,t_1;r_2,t_2) \frac{\partial \hat{j}(r_2,t_2)}{\partial t_2} d\mathbf{r}_1 dt_1 d\mathbf{r}_2 dt_2,
\end{equation}

Here $\hat{G}(r,t;r_1,t_1)$ is the 4-th rank tensor of the Green’s function of the Eq.(3) in the case $\hat{j}(r,t) = 0$.

## 3. Model simplification

To simplify the model, let us consider screw component oscillations plane wave, which is spread in the $z0x$ plane, $\alpha_{yz} = \alpha$; ($0y$- axis is the high-symmetry axis). In this case the Eqs.(1), (2) take the form:

\begin{equation}
\partial_t \alpha = \partial_z j_{xy} - \partial_x j_{zy},
\end{equation}

\begin{equation}
B \partial_t j_{xy} = S \partial_z \alpha - \sigma_{xy},
\end{equation}

\begin{equation}
B \partial_t j_{zy} = -S \partial_z \alpha - \sigma_{zy},
\end{equation}

In the absence of the magnetic-elastic interaction harmonic dislocation wave is described by the dispersion equation:

\begin{equation}
\omega^2 = \frac{1}{2B \rho} \rho C_0 + (B C_0 + \rho S) k^2 \pm \sqrt{k^4(\rho S - B C_0)^2 + 2\rho C_0 (B C_0 + \rho S) k^2 + \rho^2 C_0^2}
\end{equation}
Here the sign (−) corresponds to the acoustic wave, and the sign (+) corresponds to the dislocation one. Thus the spectrum of the oscillations of the dislocations has a gap $\omega_0$:

$$\omega_0 = \sqrt{\frac{C_0}{B}}.$$

Here $C_0 = C^0_{xyxy}$ is the elasticity module. Measured gap value and the value of Eq. 3 in the point $k = 0$ enable us to determine the constants $S$ and $B$.

4. Dislocation–magnetization interaction

Let us consider the interaction of the time-harmonic dislocation wave with the magnetic subsystem. Neglecting of the spatial dispersion and taking into account Eq. 4 and $G_{xyxy} = G_{zyzy} = 1/\rho \omega^2$, give us for long dislocation waves:

$$\sigma_{xy} = -i \left( \frac{C_0}{\omega} - \frac{4g \lambda^2 \Omega}{M \omega (\Omega^2 - \omega^2)} \right) j_{xy},$$

$$\sigma_{zy} = -i \left( \frac{C_0}{\omega} - \frac{4g \lambda^2 \Omega}{M \omega (\Omega^2 - \omega^2)} \right) j_{zy}.$$

Here $\omega$ is the dislocation wave frequency; $\lambda = \lambda_{xyxy} = \lambda_{zyzy}$ is the magnetic-elastic constant; $M$ is the sample magnetization, the easy axis is directed along $0y$ axis; $\Omega$ is the ferromagnetic resonance frequency, which depends on the coordinate $x$; $g$ is the gyro-magnet ratio. A substitution of the Eqs. 9, 10 to the Eqs. 6, 7 gives us:

$$S \partial_z \alpha = i \varepsilon(x) j_{xy},$$

$$-S \partial_x \alpha = i \varepsilon(x) j_{zy},$$

where

$$\varepsilon(x) = \omega B - \frac{1}{\omega} \left( C_0 - \frac{4g \lambda^2 \Omega}{M (\Omega^2 - \omega^2)} \right).$$

Then from the Eq. 3 we get:

$$\frac{S}{\varepsilon(x)} \partial^2_x \alpha + S \partial_x \left( \frac{1}{\varepsilon(x)} \partial_x \alpha \right) + \omega \alpha = 0.$$

Due to the dependence of the frequency $\Omega$ (Eq. 14) on the coordinate $x$ we can find a point $x_0$, in which $\varepsilon(x) = 0$. In this point

$$\Omega = \Omega_0 = -\frac{2g \lambda^2 \omega_0}{(\omega^2 - \omega_0^2)C_0 M} + \sqrt{\frac{4g^2 \lambda^4 \omega_0^2}{(\omega^2 - \omega_0^2)^2 C_0^2 M^2}} + \omega^2.$$

This point $x_0$ could be achieved in materials with $\lambda \sim 10^8 \text{erg/cm}^2$, $\Omega \sim 10^9 \text{s}^{-1}$ and $\omega \sim \omega_0 \sim (\omega - \omega_0) \sim \Omega$.

Let us put $x_0 = 0$. In the vicinity of this point we can write:

$$\varepsilon(x) = -\omega b Bx, \quad b > 0.$$
If a dependence of the ferromagnetic resonance frequency on the coordinate \( x \) caused by the temperature gradient in the inhomogeneous media, we have

\[
b = -\frac{4g\lambda^2(\Omega_0^2 + \omega^2)}{MB\omega^2(\Omega_0^2 - \omega^2)^2} \frac{d\Omega_0}{dT} \frac{dT}{dx}.
\]

Usually

\[
\frac{d\Omega_0}{dT} < 0,
\]

therefore to get \( b > 0 \), we have put

\[
\frac{dT}{dx} > 0.
\]

A substitution of the solution

\[
\alpha = \alpha_0(x)e^{i(\omega t - \kappa z)},
\]

to the Eq.(14) and taking into account the Eq.(15) gives us:

\[
\partial_x\left( \frac{1}{x} \partial_x \alpha_0(x) \right) - \left( \frac{\kappa}{x}^2 + \frac{bB\omega^2\nu}{S} \right) \alpha_0(x) = 0.
\]

Exact solution of this equation is complicated enough \([6]\). We are interested only in the part, which is connected with the singularity \([7]\):

\[
\alpha_0(x) = \alpha_0(1 + \frac{1}{2}\kappa^2\ln x).
\]

Therefore we have from this equation and the Eqs.(11), (12):

\[
\begin{align*}
\frac{j_{xy}}{\kappa B} &= \frac{\kappa \alpha_0 S}{b B \omega x} e^{i(\omega t - \kappa z)}, \\
\frac{j_{zy}}{\kappa B} &= \frac{i\frac{\kappa}{x}^2 \alpha_0 S \ln x}{b B \omega} e^{i(\omega t - \kappa z)}.
\end{align*}
\]

Hence we got the singularities of the components of the dislocation flow tensor and of the components of elastic stresses tensor, as it follows from Eqs.(9), (10).

The singularities vanish if we shall take into account magnetization oscillations attenuation:

\[
\Omega \Rightarrow \Omega - i\delta \frac{\omega}{gM},
\]

where \( \delta \) is small attenuation in magnetic subsystem \([9]\). In the Eqs.(11), (12) we have substitute \( \varepsilon(x) \) (Eq.(15)) by:

\[
\varepsilon(x) = -bB\omega x + iB\omega \nu.
\]

Here

\[
\nu = \frac{4\delta \lambda^2(\omega^2 + \Omega_0^2)}{\omega B \left( (\Omega_0^2 - \omega^2)^2 + \frac{(2\omega\Omega_0\delta gM)^2}{M^2} \right)^2}.
\]

Let us calculate averaged magnetic-elastic energy which is absorbed in the vicinity of the point \( x = 0 \):

\[
Q = \frac{i\omega}{2} \int \sigma_{xy} \varepsilon_{xy} dx.
\]

Here averaging is accomplished on the oscillation period; \( \varepsilon_{xy} \) is the component of the elastic deformation tensor.

So we get from the Eq.(9):

\[
Q = -\frac{1}{2\omega} \left( \frac{S\kappa \alpha_0}{B} \right)^2 \int \frac{\nu dx}{(bx)^2 + \nu^2}.
\]
If the value of $\nu$ is small, we can write:

$$\frac{\nu}{(bx)^2 + \nu^2} \Rightarrow \delta(bx).$$

Hence energy absorption is resonance-like, realized in the narrow area in the vicinity of the point $x = 0$, where the dislocation wave frequency coincides with the gap which is redefined by the magnetic-elastic interaction in the dislocation oscillations spectrum. Finally we got:

$$Q = -\frac{1}{2b\omega} \left( \frac{S\kappa\alpha_0}{B} \right)^2.$$

The velocity of the absorption of the dislocation oscillation energy depends not on the value of $\nu$ because the effective width of the region of intensive energy absorption is proportional to $\nu$.

A conversion of the dislocation waves to the spin ones may be realized due to the interaction of these waves with the magnetization oscillations. The space dispersion influence on this process is essential. Let us consider this effect. The value of the magnetic field, which arise due to the elastic deformation is given by

$$H_x = -\frac{\lambda}{M} \omega \Omega_0 = \frac{2i}{M} G(\omega) f_{xy},$$

in the absence of the dispersion. The magnetization we can write as:

$$M_x = \chi_{xx} H_x = ig \frac{S\lambda\kappa\Omega_0}{b\omega^2 (\Omega_0^2 - \omega^2)} e^{i(\omega t - \kappa z)},$$

$$M_z = \chi_{zz} H_x = -g \frac{S\lambda\kappa\Omega_0}{b\omega^2 (\Omega_0^2 - \omega^2)} e^{i(\omega t - \kappa z)}.$$

Here $\chi_{xx}$ and $\chi_{zz}$ are the components of the tensor of magnetic susceptibility.

One can see that in the point $x = 0$ magnetisation has a singularity. To take into account spin wave dispersion we have substitute in the Eqs. (11), (12)

$$k^2 \Rightarrow -\partial_x^2,$$

where $k$ is the $x$ component of the wave vector. Then we get for the spin waves

$$\beta \partial_x^2 M_x - b x M_x = i g S\alpha_0 \frac{\lambda\kappa\Omega_0}{B(\Omega_0^2 - \omega^2)} e^{i(\omega t - \kappa z)},$$

$$\beta \partial_x^2 M_z - b x M_z = -g S\alpha_0 \frac{\lambda\kappa}{B(\Omega_0^2 - \omega^2)} e^{i(\omega t - \kappa z)}.$$

The values of $\chi_{xx}$ and $\chi_{zz}$ are taken in the point $x = 0$ in these equations,

$$\beta = 2gMC_0^\gamma \frac{\Omega_0(\omega^2 - \omega_0^2)}{B(\omega^2 - \Omega_0^2)\omega_0^2} \omega_0^2 > 0,$$

where $\gamma$ is the inhomogeneous exchange parameter.
5. RESULTS AND DISCUSSION

The solution of the Eqs. (17), (18) describes spin wave which is created by dislocation one. This spin wave moves from the point \( x = 0 \):

\[
M_x = -\frac{gS\alpha_0\lambda\Omega_0}{B(\Omega_0^2 - \omega^2)\omega^2b^{2/3}\beta^{1/3}} e^{i(\omega t - \kappa z)} \int_0^\infty d\nu \exp \left\{ i \left[ x\nu \left( \frac{b}{\beta} \right)^{1/3} + \left( \frac{\nu}{3} \right)^3 \right] \right\},
\]

\[
M_z = -i\frac{gS\alpha_0\lambda\kappa}{B(\Omega_0^2 - \omega^2)\omega b^{2/3}\beta^{1/3}} e^{i(\omega t - \kappa z)} \int_0^\infty d\nu \exp \left\{ i \left[ x\nu \left( \frac{b}{\beta} \right)^{1/3} + \left( \frac{\nu}{3} \right)^3 \right] \right\}.
\]

In the region \( |x| >> \lambda, x < 0 \) we can estimate the values of \( M_x \) and \( M_z \) as:

\[
M_x = -\frac{i\Omega_0}{\omega} M_z = \frac{gS\alpha_0\lambda\kappa\Omega_0\sqrt{\pi}}{B(\Omega_0^2 - \omega^2)\omega b^{2/3}\beta^{1/3}} \exp \left\{ i \left( \omega t - \kappa z - \frac{2}{3} \sqrt{\frac{|b|x|}{\beta - \frac{\pi}{4}}} \right) \right\}.
\]

This effect takes place in the case when the dislocation wave is directed at some angle \( \theta \) to the \( 0x \) axis. We shall estimate the value of \( \theta \) when the considered conversion is maximal. In the approximation of the geometrical optic a wavelength of dislocation wave is much less than the size of the inhomogeneity

\[
\sqrt{(\omega^2 - \omega_0^2)\frac{B}{S}} >> b
\]

to the left-hand direction of the reflection point \( x_0 \) which is determined by:

\[
S\kappa^2 + Bb\omega^2 x_0 = 0,
\]

and the amplitude \( \alpha \) of the oscillations decreases exponentially. The point \( x = 0 \) is situated in the right-hand direction from the point \( x_0 \). The observation of this effect is possible in the case when the points \( x = 0 \) and \( x_0 \) are in proximity one to another:

\[
|k x_0| \approx 1,
\]

where \( k \) is the \( x \)-component of the wave vector.

At the large distance from the reflection point a magnetic-static influence is negligible, and the dispersion equation is:

\[
k^2 + \kappa^2 = \frac{B}{S}(\omega^2 - \omega_0^2).
\]

Hence we have

\[
\kappa^2 = \frac{B}{S}(\omega^2 - \omega_0^2)\sin^2\theta.
\]

It is easy to get in the vicinity of the point \( x = 0 \)

\[
k^2 + \kappa^2 = -\frac{Bb\omega^2}{S} x,
\]

and

\[
|k| \approx \kappa = \sqrt{\frac{B}{S}(\omega^2 - \omega_0^2)\sin^2\theta}.
\]

We substitute this equation to the equation for \( x_0 \), which follows from Eq. (20), then to the Eq. (21) and take into account Eq. (19) to get

\[
\sin^3\theta \approx \sqrt{\frac{S}{B}(\omega^2 - \omega_0^2)^{3/2}b} << 1.
\]
Here we can see that the angle $\theta$ is small but is not equal to zero.

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