Implementing PCAC in Nonperturbative Models of Pion Production

B. Blankleider, A. N. Kvinikhidze *
Department of Physics, Flinders University of South Australia, Bedford Park, SA 5042, Australia

Abstract

Traditional few-body descriptions of pion production use integral equations to sum the strong interactions nonperturbatively. Although much physics is thereby included, there has not been a practical way of incorporating the constraints of chiral symmetry into such approaches. Thus the traditional few-body descriptions fail to reflect the underlying theory of strong interactions, QCD, which is largely chirally symmetric. In addition, the lack of chiral symmetry in the few-body approaches means that their predictions of pion production are in principle not consistent with the partial conservation of axial current (PCAC), a fact that has especially large consequences at low energies. We discuss how the recent introduction of the “gauging of equations method” can be used to include PCAC into traditional few-body descriptions and thereby solve this long standing problem.

I. INTRODUCTION

Approximate chiral symmetry is an important property of quantum chromodynamics (QCD) and should therefore be an attribute of any effective description of strong interaction processes. Yet most nonperturbative effective descriptions of pion production in few-body processes ($NN \rightarrow \pi NN$, $\pi N \rightarrow \pi \pi N$, $\gamma N \rightarrow \pi N$, etc.) have been based on traditional few-body approaches which are not consistent with chiral symmetry – even if they are based on chiral Lagrangians (i.e. the input functions are chirally invariant). The essential idea of a traditional few-body approach is to take as input two-body $t$ matrices and pion production vertices, and then use integral equations to sum the multiple-scattering series nonperturbatively. A feature of such an approach is that the input can be constructed phenomenologically without the need to specify an explicit Lagrangian for the strong interactions. Despite the large amount of physics taken into account (e.g. all possible pair-like interactions are included), the lack of approximate chiral symmetry in this approach results in a pion production amplitude that does not obey PCAC. Although this is a particularly serious problem at low energies where low-energy theorems apply, the absence of PCAC is undesirable at any energy because of its inconsistency with QCD. In this contribution we

*On leave from the Mathematical Institute of Georgian Academy of Sciences, Tbilisi, Georgia
show how the gauging of equations method \cite{1,2} can be used to construct a nonperturbative few-body approach that is consistent with chiral symmetry and whose pion production amplitude obeys PCAC.

II. TRADITIONAL FEW-BODY MODEL OF NN $\rightarrow$ $\pi$NN WITHOUT PCAC

We shall base our discussion on the example of the relativistic $\pi$NN system for which a traditional few-body approach has recently been developed \cite{3,4}. For simplicity of presentation we shall treat the nucleons as distinguishable. The few-body approach to the $\pi$NN system provides a nonperturbative simultaneous description of the processes $NN \rightarrow \pi NN$, $NN \rightarrow NN$, and $\pi NN \rightarrow \pi NN$ within the context of relativistic quantum field theory. In this case the integral equations are four-dimensional and can be expressed symbolically as

$$T = V + VG_t T$$  \hspace{1cm} (1)$$

where, for distinguishable nucleons, $T$, $V$, and $G_t$ are $4 \times 4$ matrices written as

$$T = \begin{pmatrix} T_{NN} & T_N \\ T_N & T \end{pmatrix} ; \quad V = \begin{pmatrix} V_{NN} & F \\ F & G_0^{-1} I \end{pmatrix} ; \quad G_t = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 w^0 G_0 \end{pmatrix}$$.  \hspace{1cm} (2)$$

The elements of matrix $T$ are defined as follows. $T$ is a $3 \times 3$ matrix whose elements $T_{\lambda\mu}$ are Alt-Grassberger-Sandhas (AGS) amplitudes (generalised to four dimensions) describing the process $\pi NN \rightarrow \pi NN$. Note that the following “subsystem-spectator” labelling convention is used: $\lambda = 1$ or $2$ labels the channel where nucleon $\lambda$ forms a subsystem with the pion, the other nucleon being a spectator, while $\lambda = 3$ labels the channel where the two nucleons form the subsystem with the pion being the spectator. In a similar way, $T_{NN}$ is the amplitude for $NN$ scattering while $T_N$ and $\bar{T}_N$ are $3 \times 1$ and $1 \times 3$ matrices whose elements $T_{\lambda N}$ and $\bar{T}_{N\mu}$ describe $NN \rightarrow \pi NN$ and $\pi NN \rightarrow NN$, respectively. For simplicity of presentation we shall neglect connected diagrams that are simultaneously NN- and $\pi NN$- irreducible. Then the elements making up the kernel matrix $V$ specified in Eq. (2) take the following form:

$$V_{NN} = V_{NN}^{OPE} - \Delta$$  \hspace{1cm} (3)$$

where $V_{NN}^{OPE}$ is the nucleon-nucleon one pion exchange potential and $\Delta$ is a subtraction term that eliminates overcounting. $F$ is a $3 \times 1$ matrix with

$$F_\lambda = \sum_{i=1}^{2} \overline{\delta}_{\lambda i} F_i - B$$  \hspace{1cm} (4)$$

where $F_i = f_i d_j^{-1}$ consists of $f_i$, the vertex for $N_i \rightarrow \pi N_i$, and $d_j$, the Feynman propagator of nucleon $j \neq i$. The subtraction term $B$ in Eq. (4) likewise eliminates overcounting. $F$

\footnote{In this paper it should be understood that all amplitudes and currents with external nucleon legs are actually operators in Dirac space and need to be sandwiched between appropriate Dirac spinors $\bar{u}$ and $u$ to obtain the corresponding physical quantities.}
is the $1 \times 3$ matrix that is the time reversed version of $F$, $G_0$ is the $\pi NN$ propagator, and $\mathcal{I}$ is the $3 \times 3$ matrix whose $(\lambda, \mu)$’th element is $\tilde{\delta}_{\lambda, \mu}$. Finally the propagator term $\mathcal{G}_t$ is a diagonal matrix consisting of the $NN$ propagator $D_0$, and the $3 \times 3$ diagonal matrix $w^0$ whose diagonal elements are $t_1 d_2^{-1}$, $t_2 d_1^{-1}$, and $t_3 d_3^{-1}$, with $t_\lambda$ being the two-body $t$ matrix for the subsystem particles in channel $\lambda$ (for $\lambda = 1$ or 2, $t_\lambda$ is defined to be the $\pi N$ $t$ matrix with the nucleon pole term removed). The subtraction terms $\Delta$ and $B$ are defined with the help of Fig. 1 as follows:

$$\Delta = W_{\pi\pi} + W'_{\pi N} + W_{NN} + X + Y' - \tilde{B}'G_0B'$$  \hspace{1cm} (5)

where $W'_{\pi N} = W_{\pi N} + PW_{\pi N}P$, $Y' = Y + PYP$, and $B' = B + PBP$, $P$ being the nucleon exchange operator.

Despite the rich amount of physics incorporated into the $NN \rightarrow \pi NN$ amplitude within this model, it becomes evident from the following discussion that this amplitude cannot satisfy PCAC.

III. NEW FEW-BODY MODEL OF $NN \rightarrow \pi NN$ WITH PCAC

A. Choosing appropriate degrees of freedom

Although we do not specify the exact Lagrangian behind our new few-body approach, we do assume that this underlying Lagrangian is chirally invariant (up to a small explicit chiral symmetry breaking term). In turn, the chiral invariance of the Lagrangian puts a strong constraint on the nature of the fields that can be used to construct a practical few-body model. For example, if one would like to follow the $\pi NN$ model above and have only pions and nucleons as the degrees of freedom, then the underlying chiral Lagrangian would necessarily involve an isovector pion field $\vec{\xi}$ that transforms into functions of itself under a chiral transformation. Unfortunately, this can happen only in a nonlinear way [5]:

$$\vec{\xi} \rightarrow \vec{\xi} + \frac{1}{2} \vec{\theta}(1 - \xi^2) + \vec{\xi}\vec{\theta}$$ \hspace{1cm} (6)

where $\vec{\theta}$ is the vector of three (infinitesimally small) rotation angles. Thus one pion transforms into two pions, two pions transform into four pions, etc., a situation that would make it difficult to formulate a chirally invariant few-body description (i.e. a description whose
exposed states have a restricted number of pions – in our case 0 or 1). Alternatively, we can follow the example of the linear sigma model and choose an underlying Lagrangian that involves, in addition to pions and nucleons, the isoscalar field $\sigma$. In this case the $\sigma$, together with the pion field $\vec{\phi}$, transform under the chiral transformation in a linear way:

$$\vec{\phi} \to \vec{\phi} + \vec{\theta}\sigma, \quad \sigma \to \sigma - \vec{\theta} \cdot \vec{\phi}.$$  \hspace{1cm} (7)

Thus the four-component field $\phi \equiv (\sigma, \vec{\phi})$ transforms into itself - a situation that is ideal for a few-body approach. We shall therefore adopt the latter approach and treat pions and sigma particles on an equal footing. For the few-body $\pi NN$ model of Sec. 2, this means mostly a formal change where the usual (isospin) three-component pion is replaced by a four-component one. There is, however, one new aspect in that terms with a three-meson vertex ($\pi \pi \sigma$) now need to be included. For example, one will need the new subtraction term illustrated in Fig. 2. Nevertheless, the equations of Sec. 2 retain their structure, the only essential change being an increase in the size of the matrices like $T$, $T_N$ and $\bar{T}_N$ to take into account the introduction of a $\sigma NN$ channel. With these modifications we obtain few-body $\pi NN$ equations that are consistent with an underlying chiral Lagrangian.

FIG. 2. New subtraction term in the few-body $\pi NN$ model where pions and sigmas are treated on an equal footing.

B. Coupling the axial vector field everywhere

Now that we have achieved consistency with an underlying strong interaction chiral Lagrangian, our next goal is to construct a few-body $\pi NN$ model whose axial current is partially conserved. The problem at hand is analogous to the one of constructing a conserved electromagnetic current for a few-body system whose strong interactions are consistent with an underlying Lagrangian that conserves charge. In this case it is well known that exact current conservation is obtained by coupling an external electromagnetic field to all possible places in the strong interaction model. In a similar way, one would expect to obtain a partially conserved axial current by coupling an external axial vector field to all possible places in the strong interaction model. Until recently, however, what has not been known is the way to achieve this complete coupling for a few-body system whose strong interactions are described nonperturbatively by integral equations. Fortunately, the solution to this problem presented recently in the context of electromagnetic interactions \[1,2\], is based on a topological argument and therefore applies equally well to the present case of an axial vector field.

The basic idea is to add a vector index (indicating an external axial vector field) to all possible terms in the integral equations describing the strong interaction model. Thus, in the case of the $\pi NN$ system where the structure of the integral equations is given as in Eq. (4), coupling an external axial isovector field gives
\[ T^\mu = \mathcal{V}^\mu + \mathcal{V}^\mu \mathcal{G}_i T + \mathcal{V} \mathcal{G}_i^\mu T + \mathcal{V} \mathcal{G}_i T^\mu \] (8)

which can easily be solved to give a closed expression for \( T^\mu \):

\[ T^\mu = (1 + \mathcal{T} \mathcal{G}_i) \mathcal{V}^\mu (1 + \mathcal{G}_i T + \mathcal{T} \mathcal{G}_i^\mu T). \] (9)

\( T^\mu \) is a matrix of transition amplitudes \( T^\mu_{NN}, T^\mu_{N\Delta}, T^\mu_{Nd}, \) etc. (for the moment, we suppress the isovector index in these amplitudes). Here we are particularly interested in the transition amplitude \( T^\mu_{NN} \) as it is closely related to the pion production amplitude we seek.

It is easy to see that the transition amplitude \( T^\mu_{NN} \) given by Eq. (8) has the axial vector field being attached everywhere in \( T^\mu_{NN} \) except on the external nucleon legs (9). Including these external leg contributions then gives the complete axial vector transition current for \( NN \to NN \):

\[ j^\mu_{NN} = (\Gamma^\mu_i d_1 + \Gamma^\mu_2 d_2)T_{NN} + T_{NN}(d_1 \Gamma^\mu_1 + d_2 \Gamma^\mu_2) + T^\mu_{NN} \] (10)

where \( \Gamma^\mu_i \) is the axial vertex function of nucleon \( i \). The input to the \( \pi NN \) equations consists of the two-body \( t \) matrices \( t_i \), the pion production vertices \( f_i \), and the single particle propagators \( d_i \). Thus the input to Eq. (9) also includes the axial transition currents \( t^\mu_i, f^\mu_i \), and axial vertex function \( \Gamma^\mu_i \). In order for these input quantities to be consistent with an external axial vector field being attached everywhere, they must be constructed to satisfy the Axial Ward-Takahashi (AWT) identities (9). This is easily achieved by restricting the form of the corresponding bare quantities (9). Thus, for example, \( \Gamma^\mu_i \) needs to satisfy the AWT identity

\[ q_i \Gamma^\mu_i (k, p) = i \left[ d^{-1}_i(k)\gamma_5 + \gamma_5 d^{-1}_i(p) \right] t^a - if_\pi m^2 \pi f^a_i(k, p)d_\pi(q) \] (11)

where \( p \) and \( k \) are the initial and final momenta of the nucleon, \( q = k - p \), \( f^a_i(k, p) \) is the \( \pi NN \) vertex for a pion of isospin component \( a \), \( t^a \) is an isospin 1/2 matrix, \( m_\pi \) is the mass of the pion, and \( f_\pi \) is the pion decay constant. With the other input quantities constructed to satisfy similar AWT identities, it can be shown that the two-nucleon axial current given by Eq. (10) satisfies the AWT corresponding to exact PCAC:

\[ q_i j^\mu_{NN}(k_1 k_2, p_1 p_2) = i \left[ (\gamma_5 t^a)_{1} T_{NN}(k_1 - q, k_2; p_1 p_2) + T_{NN}(k_1 k_2; p_1 + q, p_2) (\gamma_5 t^a)_{1} \right] \\
+ i \left[ (\gamma_5 t^a)_{2} T_{NN}(k_1, k_2 - q; p_1 p_2) + T_{NN}(k_1 k_2; p_1, p_2 + q) (\gamma_5 t^a)_{2} \right] \\
- if_\pi m^2 \pi T_{N0}^a(k_1 k_2, p_1 p_2)d_\pi(q) \] (12)

where \( T_{N0}^a \) is the amplitude for \( \pi NN \to NN \) and in its time reversed form, is just the the pion production amplitude that we are seeking. Note that the axial current \( j^\mu_{NN} \) contains a pion pole (10):

\[ j^\mu_{NN}(k_1 k_2, p_1 p_2) = \tilde{j}_{NN}^\mu(k_1 k_2, p_1 p_2) + d_\pi(q)F^\mu_{\pi}(q)T_{N0}^a(k_1 k_2, p_1 p_2), \] (13)

where \( \tilde{j}_{NN}^\mu \) has no pion pole and \( F^\mu_{\pi} \) is the pion decay vertex function. Thus an alternative way of obtaining the pion production amplitude \( T_{N0}^a \) is to take the residue of Eq. (12) at the pion pole.
C. Physical content of the new model

Not only does the pion production amplitude $T_{N0}$ obey exact PCAC, but it contains a very rich amount of physics that goes beyond what is included in the traditional few-body approach of Sec. 2. A few of the infinite number of new contributions are illustrated in Fig. 3.

Finally we would like to stress the practicality of our approach: PCAC and inclusion of an infinite number of new pion production mechanisms has been achieved through a simple closed expression, Eq. (9), involving scattering amplitudes $T$ obtained from a traditional few-body model, Eq. (1). Our method also does not depend on the model used for kernel $V$—rather than Eq. (2), we could equally have used the simpler case of an $NN$ one-pion-exchange potential.

Acknowledgement. This work was supported by a grant from the Flinders University Research Committee.
REFERENCES

[1] A. Kvinikhidze and B. Blankleider, Phys. Rev. C 60, 044003 (1999).
[2] A. Kvinikhidze and B. Blankleider, Phys. Rev. C 60, 044004 (1999).
[3] A. N. Kvinikhidze and B. Blankleider, Nucl. Phys. A574, 788 (1994).
[4] D. R. Phillips and I. R. Afnan, Ann. Phys. (N.Y.) 247, 19 (1996).
[5] S. Weinberg, The Quantum Theory of Fields II (Cambridge University Press, Cambridge, 1966).
[6] W. Bentz, Nucl. Phys. A446, 678 (1985).