Harmonic electron-cyclotron maser emissions driven by energetic electrons of the horseshoe distribution with application to solar radio spikes

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ABSTRACT

Context. Electron-cyclotron maser emission (ECME) is the favored mechanism for solar radio spikes and has been investigated extensively since the 1980s. Most studies relevant to solar spikes employ a loss-cone-type distribution of energetic electrons, generating waves mainly in the fundamental \(X/O\) mode (\(X1/O1\)), with a ratio of plasma oscillation frequency to electron gyrofrequency (\(\omega_{pe}/\Omega_{ce}\)) lower than 1. Despite the great progress made in this theory, one major problem is how the fundamental emissions pass through the second-harmonic absorption layer in the corona and escape. This is generally known as the escaping difficulty of the theory.

Aims. We study the harmonic emissions generated by ECME driven by energetic electrons with the horseshoe distribution to solve the escaping difficulty of ECME for solar spikes.

Methods. We performed a fully kinetic electromagnetic particle-in-cell simulation with \(\omega_{pe}/\Omega_{ce} = 0.1\), corresponding to the strongly magnetized plasma conditions in the flare region, with energetic electrons characterized by the horseshoe distribution. We also varied the density ratio of energetic electrons to total electrons (\(n_{ce}/n_{0}\)) in the simulation. To analyze the simulation result, we performed a fast Fourier transform analysis on the fields data.

Results. We obtain efficient amplification of waves in the \(X\) and \(X2\) modes, with a relatively weak growth of \(O1\) and \(X3\). With a higher-density ratio, the \(X2\) emission becomes more intense, and the rate of energy conversion from energetic electrons into \(X2\) modes can reach \(\sim 0.06\%\) and \(0.17\%\), with \(n_{ce}/n_{0} = 5\%\) and \(10\%\), respectively.

Conclusions. We find that the horseshoe-driven ECME can lead to an efficient excitation of \(X2\) and \(X3\) with a low value of \(\omega_{pe}/\Omega_{ce}\), providing novel means for resolving the escaping difficulty of ECME when applied to solar radio spikes. The simultaneous growth of \(X2\) and \(X3\) can be used to explain some harmonic structures observed in solar spikes.

Key words. Sun: radio radiation – Sun: corona – radiation mechanisms: non-thermal – masers – waves – methods: numerical

1. Introduction

Electron-cyclotron maser emission (ECME) represents an important coherent radiation mechanism in plasmas in terms of a direct amplification of escaping electromagnetic radiation at the electron gyrofrequency (\(\Omega_{ce}\)) or its harmonics (in either extraordinary or ordinary modes, namely \(X/O\) modes) by gyroresonant wave-particle interaction. The radiations are believed to be driven by positive gradients of the velocity distribution in the perpendicular direction (i.e., \(\partial f/\partial v_{\perp} > 0\)) in the region in which the plasma oscillation frequency (\(\omega_{pe}\)) is lower than \(\Omega_{ce}\). The basic idea was originally suggested by Twiss (1958). Recognizing the significance of the relativistic correction in resonance condition, Wu & Lee (1979) achieved a major breakthrough in this theory, leading to extensive follow-up investigations (see also Wu 1985). They employed energetic electrons of the loss-cone distribution to explain the phenomenon of auroral kilometric radiation (AKR). Since then, the ECME model has been applied to radio emissions from various astrophysical objects, including decametric radiations from Jupiter, solar radio spikes, radio emissions from flare stars, pulsars, and blazar jets (see reviews by Treumann 2006; Melrose 2017).

Solar radio spikes are short-duration decimetric radio bursts with a narrow bandwidth, which are strongly correlated with hard X-ray (HXR) bursts (Guedel et al. 1991; White et al. 2011). They can be observed with high circular polarization (as high as 100%; Tarnstrom & Philip 1972; Slottje 1978). During a solar flare, up to ten thousand spikes can be observed (Benz 1985), and each individual spike was considered as an elementary burst of solar flares. As the particular type of solar radio bursts with the shortest characteristic time scale (<100 ms), highest brightness temperature (\(T_{B} \sim 10^{20} - 10^{21}\) K), very narrow relative bandwidth (~1–3\%), strong correlation with HXR bursts, as well as its significance in solar flare studies, the phenomenon has gained much attention since its discovery (see, e.g., Droge 1977; Benz 1986; Guangs Li 1987; Aschwanden et al. 1990; Fleishman & Mel'nikov 1998; Huang & Nakajima 2005; Chernov 2011). See Feng et al. (2018) and Feng (2019) for the latest reports.

Solar spikes are attributed to coherent radiations in magnetized solar plasmas. In the flare region, the magnetic fields are strong, therefore the ratio of characteristic frequencies (\(\omega_{pe}/\Omega_{ce}\)) can be lower than 1 (e.g., Régnier 2015). Hence, ECME has been accepted as one favorable mechanism of solar spikes (e.g., Holman et al. 1980; Melrose & Dulk 1982; Sharma et al. 1982; Sharma & Vlahos 1984; Winglee & Dulk 1986a). Most earlier
studies have assumed a version of the ECME that is driven by energetic electrons of the loss-cone distribution, typical of particles trapped within magnetic structures. Relevant investigations suggest that the fundamental X-mode (X1) emission is the dominant mode excited by ECME in plasmas with $\omega_{pe}/\Omega_{ce} < 0.3$, while harmonic excitations can be dominant only if $\omega_{pe}/\Omega_{ce}$ is close to or higher than 1 (Aschwanden 1990). Studies on ECME driven by ring-beam electrons (e.g., Wu & Freund 1984; Vlahos 1987) presented similar results.

The escaping difficulty of emissions at $\Omega_{ce}$ is one major problem of the ECME model for solar radio bursts. In the solar corona, the magnetic field strength decreases with height. When propagating outward from the source, it is in general difficult for fundamental emissions to pass through the second-harmonic layer, where strong gyromagnetic absorption applies (Melrose & Dulk 1982). To solve the problem, researchers have considered various possibilities such as that radiation can escape (1) through a process of partial re-emission above 2$\Omega_{ce}$ by the absorbing plasmas (McKean et al. 1989); (2) by propagating almost in parallel ($\delta\theta_g \approx 0$) in a low-temperature plasma with a weak effect of absorption (Robinson 1989); (3) or by propagating within a low-density wave-ducting tunnel (e.g., Wu et al. 2014; Melrose & Wheatland 2016). To address the problem, we explore the possibility of direct and efficient amplifications of second- and higher-harmonic emissions.

With a similar magnetic configuration and physical process in the source region, solar spires have been regarded as an analogy of the AKR, which currently is the only astrophysical radio emission whose source can be measured in situ by satellites (Melrose & Wheatland 2016). The horseshoe distribution, detected in its source region, has been identified as the main driver of ECME accounting for AKR (e.g., Ergun et al. 2000; Pritchett et al. 2002). Thus it is reasonable to assume that the horseshoe-driven ECME may also apply to solar spires. Melrose & Wheatland (2016) argued that this distribution can indeed form in flare loops when beam electrons, accelerated by the flare reconnection, fly toward a lower atmosphere with a stronger magnetic field with their magnetic momentum conserved. This distribution may lead to the excitation of the Z mode, which can convert into escaping modes in the density cavity according to the assumptions of Melrose & Wheatland (2016). However, a detailed study of the ECME driven by energetic electrons with the horseshoe distribution applicable to solar spires has not been reported to the best of our knowledge.

The linear kinetic cyclotron maser instability driven by the horseshoe distribution has been investigated by Yoon et al. (2016), and it was found that the growth rate of the second-harmonic X-mode (X2) can be very high if $\omega_{pe}/\Omega_{ce}$ is close to 1. In a recent study, Yousefzadeh et al. (2021) studied the velocity distribution of electrons propagating within a coronal loop and obtained an electron distribution with strip-like features that contains a significant positive gradient of velocity distribution. Further simulation suggested that the X2 emissions can be amplified through ECME driven by electrons associated with the strip feature, which seems to be similar to the horseshoe distribution in morphology. Thus, it is timely to clarify the applicability of horseshoe-driven ECME to solar spikes using fully kinetic electromagnetic particle-in-cell (PIC) simulations. This is the main motivation of the present study. We focus on the direct excitation of harmonic emissions and investigate the effect of the abundance of energetic electrons. In Sect. 2 we introduce the PIC code, parameter setup, and the electron distribution function. The main results are displayed in Sect. 3. In the last section, conclusions and a discussion are presented.

2. PIC code, parameter setup, and horseshoe distribution function

The numerical simulation was performed using the vector-PIC (VPIC) code developed and released by the Los Alamos National Laboratories. VPIC employs a second-order, explicit, leapfrog algorithm to update charged-particle positions and velocities in order to solve the relativistic kinetic equation for each species, along with a full Maxwell description for electric and magnetic fields evolved with a second-order finite-difference time-domain solver (Bowers et al. 2008, 2009).

With this code, we performed simulations in two spatial dimensions (2D) with three vector components (3V). The background magnetic field was set to be $B_0 = B_0 \hat{e}_z$, and the wave vector $(k)$ was in the xOz plane, so that $E_y$ represents the pure transverse component of the wave electric field. Periodic boundary conditions were used. We set $\omega_{pe}/\Omega_{ce}$ to be 0.1 to represent the strongly magnetized plasma condition of solar flares. The domain of the simulation was set to be $L_x = L_z = 1024 \Delta$, where $\Delta = 1.36 \Delta_0$ is the grid spacing, and $\Delta_0$ is the Debye length of background electrons. The simulation time step was $\Delta t = 0.012 \omega_{pe}^{-1}$. The unit of length is the electron inertial length $(d_e = c/\omega_{pe})$, and the unit of time is the plasma response time $(\omega_{pe}^{-1})$. The wave number and frequency range that can be resolved is $[-12, 12] \Omega_{ce}/c$ and $[0, 3.2] \Omega_{ce}$, respectively. Charge neutrality was maintained. A realistic proton-to-electron mass ratio of 1836 was used. We included 2000 macroparticles for each species in each cell. In addition, the VPIC code applies Marder passes (Marder 1987) periodically (every 100 time steps) to clean the accumulative errors of the divergence of electric and magnetic fields.

The background electrons and protons are Maxwellian with the same temperature, described by

$$f_0 = \frac{1}{(2\pi)^{3}/2\nu_0^3} \exp \left( -\frac{\nu^2}{2\nu_0^2} \right),$$

where $\nu$ is the momentum per mass of particle, $\nu_0 = 0.018c$ ($\sim 2$ MK), for electrons, and $c$ is the speed of light.

In relevant studies of the emission mechanism of AKR, the horseshoe distribution is mostly defined as an incomplete shell distribution with one-sided loss-cone feature. Because energetic electrons trapped within a coronal loop may be reflected a few times, leading to a distribution with a two-sided loss cone, we employed a horseshoe distribution consisting of a shell distribution and a two-sided loss cone as follows:

$$f_s(u, \mu) = A \frac{G(\mu)}{\sqrt{2\pi^2}} \exp \left( -\frac{(u - u_\mu)^2}{2\sigma^2} \right), \quad G(\mu) = 1 - \tanh \left( \frac{\mu - \mu_0}{\delta} \right),$$

where $u_\mu = 0.3c$ is the radius of the shell in momentum space, $\sigma = 0.01$ determines the width of the shell, and $A$ is the normalization factor. The cosine of the pitch angle is written $\mu = u_\parallel/u$, and $\mu_0 (=0.87)$ represents the cosine of the loss-cone angle ($\sim 30^\circ$), and $\delta (=0.1)$ defines the smoothness of the loss-cone boundary.

The abundance of energetic electrons varies from event to event and directly affects the value of the growth rate according to linear theory. In our simulations, we set the density ratio $n_e/n_0$ (where $n_e$ is the number density of energetic electrons, and $n_0$ refers to the number density of total electrons) to be 10% for the reference case, which was analyzed in detail. The initial velocity distribution is displayed in Fig. 1a. Then, we varied $n_e/n_0$ to
study its effect on the wave amplification. The simulations lasted 300 $\omega_{pe}^{-1}$ except in the cases with $n_e/n_0 = 1\%$ and $2.5\%$, in which the wave growth is relatively slow and we applied a longer simulation time (~500 $\omega_{pe}^{-1}$).

3. Simulation results

In this section we first present the reference solution with $n_e/n_0 = 10\%$, showing the detailed evolution of electron distribution and energies of the wave fields, as well as the dispersion relation analysis. Simulation results with varying $n_e/n_0$ are presented subsequently to study its effect on the amplification of harmonic emissions. Then we compare three cases with $n_e/n_0 = 5\%, 10\%$, and $50\%$ to analyze the detailed wave amplification and particle diffusion process. Finally, we examine the resonance conditions of the amplified wave modes to understand their excitation.

3.1. Harmonic ECME driven by horseshoe electrons (with $n_e/n_0 = 10\%$)

The evolution of the electron distribution for the reference solution is displayed in Fig. 1 and the accompanying animation. Initially, energetic electrons are distributed along the semicircle in velocity space, with a strong positive gradient below the radius. In the middle of the simulation ($t = 150 \omega_{pe}^{-1}$), significant diffusion of the horseshoe electrons can be seen. This leads to a smoother gradient and fills up the cavity at lower velocities. At the end of the simulation ($t = 300 \omega_{pe}^{-1}$), the gradient of the energetic electrons becomes even smoother. Background electrons are diffused slightly toward higher velocities along the perpendicular direction, indicating perpendicular heating or acceleration.

The temporal energy profiles of various wave field components and the decline in kinetic energy of the total electrons ($\Delta E_k$) are plotted in Fig. 2a, normalized to the initial kinetic energy of energetic electrons ($E_{\text{ke0}}$). At about 50 $\omega_{pe}^{-1}$, $E_y$ and $B_z$ start to increase sharply and reach maximum ($\sim 2 \times 10^{-2} E_{\text{ke0}}$) about 120 $\omega_{pe}^{-1}$, with five orders of magnitude above the corresponding noise level. Their temporal evolutions are consistent with each other. The energies of $E_x$, $E_z$, and $B_y$ later rise to a magnitude of $10^{-4} E_{\text{ke0}}$, while $B_z$ is the weakest component in energy ($\sim 10^{-5} E_{\text{ke0}}$). The profile of $\Delta E_k$ closely follows the increase of the dominant transverse components ($E_y$ and $B_z$). After 150 $\omega_{pe}^{-1}$, almost all field components are saturated.

Figure 3 presents the distribution of the maximum wave energy in the wave vector $k (k_x, k_z)$ space over the whole simulation (0–300 $\omega_{pe}^{-1}$) for $E_x$, $E_y$, and $E_z$. The waves are clearly amplified mainly along the perpendicular direction at $|k| \sim \Omega_{ce}/c$, $2 \Omega_{ce}/c$, and $3 \Omega_{ce}/c$, respectively. The modes carried by $E_y$ are much stronger than the modes carried by $E_x$ and $E_z$. This is consistent with the energy profiles plotted in Fig. 2a.

In Fig. 4, we present the $\omega$-$k$ dispersion diagrams of the perpendicular field components for the mode identification. According to the magnetotonic theory, for perpendicular propagation ($k = k_x \hat{e}_z$), the $X$ (and $Z$) modes are carried by $E_x$ and $E_z$, and the $O$ mode is carried by $E_y$. The dispersion curves of the modes are overplotted to facilitate mode identification. The wave modes carried by $E_x$ and $E_z$ are also carried by $B_z$ and $B_y$, respectively, according to the Faraday law ($k \times E = -\omega B$). According to the $\omega$-$k$ dispersion, in $E_x$ and $B_z$, the $X$ and $Z$ modes grow significantly at frequencies of 0.96 $\Omega_{ce}$ and 1.92 $\Omega_{ce}$, respectively, and the $X$ and $Z$ modes with $\omega \sim 2.9 \Omega_{ce}$ manifests a relatively weak growth. The excitation of all the wave modes can be found in a finite frequency range, with a relative bandwidth of $\sim 5\%$. For example, the amplification of the $Z$ mode mainly lies in a frequency range of 0.93–0.98 $\Omega_{ce}$. Fundamental and harmonic $O$ modes ($O1$ and $O2$) are also amplified, but with a much weaker intensity, according to the diagrams of $E_y$ and $B_x$. According to the online animation, all modes are amplified within an angular width of $10^\circ$–$20^\circ$ centered on the perpendicular direction, as is also indicated by Fig. 3. We note that at $90^\circ$, none of the amplified wave modes can be seen in the dispersion diagram of $B_z$, consistent with the divergence-free condition of $B$. The growth in energy of $B_z$ comes from the waves propagating at oblique directions (as is shown in the online animation of Fig. 4). The vertical features in panels a, b, and f are artifacts caused by the spectral leakage of the discrete Fourier transform (DFT) analysis.

The mode intensities were evaluated by integrating the energies of the corresponding field components within the respective ranges in $k$ space (marked in Fig. 3 with squares) for each time step, based on the Parseval theorem. As illustrated above, $Z$ and $X$ modes are mainly carried by $E_x$, $E_y$, and $B_z$, and $O1$ is carried by $E_z$ and $B_y$. To avoid the contamination of $Z$ mode in $E_x$, only the $E_y$ and $B_z$ components were considered for the evaluation of the $X$ intensity. We note that the $X$ energy in $E_z$ is relatively minor. The obtained temporal energy profiles are plotted in Fig. 2b. The energy of the $Z$ mode grows linearly from the beginning of the simulation until its saturation at 110 $\omega_{pe}^{-1}$. The fitted growth rate is about $0.016 \Omega_{ce}$, while the energy conversion rate.
that is, the ratio of the mode energy to the initial kinetic energy of energetic electrons, is about $4.1 \times 10^{-2}$. Both $X2$ and $X3$ start to grow later at $80 \omega_{pe}^{-1}$, with growth rates of 0.027 and 0.032 $\Omega_{ce}^{-1}$, respectively, and both end about $110 \omega_{pe}^{-1}$. The energy conversion rates of $X2$ and $X3$ are $1.7 \times 10^{-3}$ and $5 \times 10^{-5}$, respectively. The $O1$ growth rate is similar to that of the $Z$ mode, and its energy conversion rate is about $5 \times 10^{-4}$. The corresponding energies of $O2$ and $O3$ in the $E_z$ and $B_y$ components are lower than those of the $X2$ and $X3$ modes by about three orders of magnitude (not shown).

In summary, the horseshoe electrons with $n_e/n_0 = 10\%$ can drive significant cyclotron maser instability, generating waves mainly in $Z$ and $X2$ modes at frequencies slightly lower than $\Omega_{ce}$ and $2 \Omega_{ce}$, respectively, in the perpendicular direction. The growths of $O1$ and $X3$ modes are relatively weaker. We conclude that the horseshoe-driven maser emission is mainly in the $X2$ mode for the present case with $n_e/n_0 = 10\%$.

3.2. Effect of $n_e/n_0$ on wave excitations

During solar flares, a bulk energization of electrons may take place, producing an extremely large number of energetic electrons (Aschwanden 2002). The abundance of the energetic electrons ($n_e/n_0$) varies from event to event and from stage to stage of a single event. To explore its effect on wave excitations, we varied $n_e/n_0$ from 1% to 50% while the other parameters were fixed.

We start with detailed comparisons of the cases with $n_e/n_0 = 5\%$, 10\%, and 50\%. As displayed in Figs. 1 and 5 as well as in the accompanying animations, the electron diffusion becomes more efficient with more abundant energetic electrons. With $n_e/n_0 = 5\%$, the diffusion is relatively slow with a large gap even after saturation; with $n_e/n_0 = 10\%$, the gap is mostly filled up at the end of the simulation; and with $n_e/n_0 = 50\%$, the hole is entirely filled up before $100 \omega_{pe}^{-1}$, and the overall shape of distribution becomes similar to the Dory-Guest-Harri distribution.
Analytical dispersion relations of the $(a)$ $300 \omega_{pe}^{-1}$, $(b)$ $150 \omega_{pe}^{-1}$, and $(c)$ $50 \omega_{pe}^{-1}$ cases with $n_e/n_0 = 10\%$, over times $0–300 \omega_{pe}^{-1}$ in the direction of $90^\circ$.

Wave dispersion diagrams of the electric and magnetic components with $n_e/n_0 = 10\%$, over times $0–300 \omega_{pe}^{-1}$ in the direction of $90^\circ$. Analytical dispersion relations of the $X/Z$ mode and $O$ mode are overplotted as dotted white lines in panels $b$ and $c$, respectively. Amplified wave modes are marked. An animation of this figure is available online. The video begins at $\theta_{kB} = 70^\circ$ and advances $1^\circ$ at a time until it ends at $\theta_{kB} = 110^\circ$.

Snapshots of the velocity distribution of the simulations with $n_e/n_0 = 5\%$–$(a)$–$(c)$ and $50\%$–$(d)$–$(f)$. The distributions are obtained at $t = 0 \omega_{pe}^{-1}$ (left column), $150 \omega_{pe}^{-1}$ (middle column), and $300 \omega_{pe}^{-1}$ (right column). An animation of this figure is available online, starting at $t = 0 \omega_{pe}^{-1}$ and ending at $300 \omega_{pe}^{-1}$.

In this case, background electrons are accelerated considerably in the perpendicular direction, with a significant diffusion toward higher $v_z$ in the velocity space.

The linear stages for the three cases are $(50–150 \omega_{pe}^{-1})$, $(40–110 \omega_{pe}^{-1})$, and $(20–60 \omega_{pe}^{-1})$, indicating that more energetic electrons correspond to a faster evolution (Figs. 2 and 6). For all cases with different $n_e/n_0$, $2–5\%$ of $E_{(b0)}$ can be converted into the wave energy, with $E_z$ and $B_z$ being dominant with a similar evolutionary trend. The energies of $E_x$, $B_x$, and $B_y$ increase with increasing $n_e/n_0$. In the case of $n_e/n_0 = 50\%$, the energy evolution of $E_x$ shows an obvious decline after the saturation that is likely due to the damping of the corresponding $Z$ mode.
The intensity maps in the $k$ space are similar for the three cases, with significant perpendicular growth at $|k| \sim \Omega_{ce}/c$, $2\Omega_{ce}/c$, and $3\Omega_{ce}/c$ (see Figs. 3 and 7). With a higher density ratio, all the amplified modes become more intense in a broader range of $k$. Wave growths in $E_x$ and $E_z$ are negligible for $n_e/n_0 = 5\%$ and $10\%$, while the $E_z$ intensity at $|k| \sim \Omega_{ce}$ becomes very high with $n_e/n_0 = 50\%$, indicating a significant amplification of $O1$-mode waves. In Fig. 8 we display the $\omega$-$k$ dispersion analyses in $E_y$ (panels c, e, and h) with $\theta_{RB} = 90^\circ$, highlighting the amplified wave modes of $Z$, $X2$, and $X3$. We would like to point out that with $n_e/n_0 = 50\%$, the wave growth in $E_y$ occurs in a wide range of $k$ at frequencies close to the $Z$ mode, known as the relativistic mode (marked "R"), according to the earlier linear analysis by Pritchett (1984).

Energy profiles of various wave modes are shown in Figs. 2b and 6c–d. They are quite similar to those of the field components. When $n_e/n_0$ increases from 5% to 50%, the energy conversion rate of $Z$ mode decreases from $-5 \times 10^{-2}$ to $-4 \times 10^{-3}$, while the amplification of $X2$ becomes more efficient: its energy conversion rate increases from $-6 \times 10^{-4}$ to $-2 \times 10^{-3}$. The energy conversion rates of $O1$ and $X3$ are lower than those of the $Z$ and $X2$ modes, which also increase with $n_e/n_0$. With $n_e/n_0 = 50\%$, $X2$ becomes the most intense mode at the saturation stage, while the $Z$-mode energy declines with time, likely due to its damping with time, as mentioned above.

Fourier and energy analyses of various modes for cases with $n_e/n_0 = 1\%$, 2.5\%, 5\%, 7.5\%, 10\%, 25\%, 40\%, and 50\% are plotted in Fig. 8 for the $E_y$ $\omega$-$k$ dispersion and in Fig. 9 for profiles of mode energies and normalized growth rates. The $Z$, $X2$, and $X3$-mode waves become more intense with higher $n_e/n_0$ and increasing kinetic energies of energetic electrons. In addition, the $R$ mode appears with $n_e/n_0$ higher than 10\%.

The fitted growth rates (in unit of $\Omega_{ce}$) with varying density ratios are plotted in Fig. 9a. With increasing $n_e/n_0$ from 1\% to 50\%, all relevant modes ($Z$, $X2$, $X3$, and $O1$) present a faster evolution with higher growth rates. However, the growth rates are not proportional to the density ratio, as expected, while the increase in growth rate becomes slower with higher density ratio.

The energy conversion rate of the $Z$ mode decreases gradually, while those of the other modes ($X2$, $X3$, and $O1$) first increase to maximum values of $4 \times 10^{-2}$, $2 \times 10^{-4}$, and $3 \times 10^{-3} E_{kin}$, respectively, and then decline in general (Fig. 9b). It should be noted that the energy conversion rates shown here are the ratios of the mode energy to the kinetic energy of energetic electrons, representing the efficiency of the wave amplification for each case, rather than the obtained wave intensity.

As shown in Fig. 9a and b, the energy conversion rates are not directly correlated with the growth rates for different modes. For example, with $n_e/n_0 = 10\%$, $X3$ has the highest growth rate but the lowest energy conversion rate, while the $Z$ mode has the highest energy conversion rate with a low value of the growth rate. Similar results have been obtained by Yoon & Ziebell (1995) with a quasi-linear analysis of the cyclotron maser instability of loss-cone electrons. In their results, all the modes grow
with an initial energy of $1.0 \times 10^{-4} E_{\parallel 0}$ and the Z mode eventually dominates with a growth rate lower than X1. They explained that the growth rate of X1 is more likely to be affected by the evolution of the electron distribution and can only grow for a short duration, while the growth of the Z mode lasts longer. In our simulation, it is more complicated because the initial energy of the different modes varies greatly. The initial energy of the X3 mode is lower than that of the Z mode by two orders of magnitude, while the durations of growth for the two modes are also different. Additionally, the Z mode can be damped after the saturation

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Fig. 7. Maximum intensity of $(E_x, E_y, E_z)$ in the $\omega$ domain in the $k_\perp-k_\parallel$ space as shown by the color map of $20 \log_{10}((E_x, E_y, E_z)/(cB_0))$ for the whole simulation (0–300 $\omega_{pe}^{-1}$), for $n_e/n_0 = 5\%$ (panel a) and 50\% (panel b).

Fig. 8. $\omega$-$k$ dispersion diagrams of the pure transverse electric component ($E_y$) in the perpendicular direction. Each panel represents the result of the simulation with a certain $n_e/n_0$. The analyses are performed for the whole stage of each simulation case, which is 0–500 $\omega_{pe}^{-1}$ with $n_e/n_0 = 1\%$ and 2.5\%, and 0–300 $\omega_{pe}^{-1}$ for the others. “R” stands for the relativistic mode branch.

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stage in specific cases. Thus, the eventual energy conversion rate of different modes cannot be directly predicted with the growth rate.

In Fig. 9c we plot the variation in energy ratios of $O1$–$Z$, $O2$–$X2$, and $O3$–$X3$. The energy ratios of $O2$–$X2$ and $O3$–$X3$ remain lower than $10^{-2}$. This means that the obtained second- and third-harmonic emissions are almost in the sense of 100% $X$-mode polarization. The energy ratio of the $O1$–$Z$ mode is $\sim 10^{-4}$ with $n_c/n_0 < 5\%$, increasing to $\sim 0.15$ with $n_c/n_0 \geq 25\%$, indicating a more efficient excitation of $O1$ with larger $n_c/n_0$, consistent with the above analysis.

3.3. Resonance analysis

For a complete understanding of the results, we analyzed the resonance conditions of the amplified wave modes ($Z$, $X2$, $X3$, and $O1$). The cyclotron resonance condition can be written as

$$\omega - k_0v_0 - n\Omega_{ce}/\gamma = 0.$$  

(3)

Considering perpendicular wave propagation with $k_0 = 0$, the condition is simplified as

$$\omega = n\Omega_{ce}\sqrt{1 - \frac{v^2}{c^2}},$$  

(4)

where $n$ is the harmonic number. According to this equation, only waves at frequencies lower than $n\Omega_{ce}$ can be amplified perpendicularly. Therefore the resonance condition cannot be satisfied for the fundamental X mode, whose cutoff frequency is $\omega_X = 1/2\left(\Omega_{ce}^2 + 4\omega_{pe}^2 + \Omega_{ce}\right) > \Omega_{ce}$. The cutoff frequency and resonance frequency of the Z mode are $\omega_Z = 1/2\left(\Omega_{ce}^2 + 4\omega_{pe}^2 - \Omega_{ce}\right)$ and $\omega_{RZH} = \sqrt{\omega_{pe}^2 + \Omega_{ce}^2}$, respectively. Therefore the Z mode and harmonics of the X mode (e.g., $X2$ and $X3$) can be amplified. The resonance curves of the three modes are plotted in Fig. 1a with $\omega = 0.96\Omega_{ce}$ ($n = 1$), $\omega = 1.92\Omega_{ce}$ ($n = 2$), and $\omega = 2.88\Omega_{ce}$ ($n = 3$). The corresponding resonance curves overlap with each other and lie in a region with a significant positive gradient of the horseshoe distribution. The wave excitations of these modes are therefore attributed to the shell component of the horseshoe distribution, rather than to the loss-cone component, which occupies only a rather limited region. The frequency of the amplified $O1$-mode waves are the same as that of the $Z$ mode, corresponding to the same resonance curve.

4. Conclusion and discussion

We investigated the ECME driven by energetic electrons of the horseshoe distribution, rather than to the loss-cone component of the energetic gradient of the horseshoe distribution. The wave excitations overlap with each other and lie in a region with a significant positive gradient of the horseshoe distribution, rather than to the loss-cone component, which occupies only a rather limited region. The frequency of the amplified $O1$-mode waves are the same as that of the $Z$ mode, corresponding to the same resonance curve.

![Fig. 9. Variation in panel a: fitted linear growth rate (in unit of $\Omega_{ce}$), panel b: final energy conversion rate of the $Z$, $X2$, $X3$, and $O1$ modes with $n_c/n_0$. Panel c: ratio of the energy of the $O$ modes and $Z/X$ modes at certain harmonics ($E_{01}/E_Z$, $E_{O2}/E_{X2}$, and $E_{O3}/E_{X3}$) vs. $n_c/n_0$.](image)
duration, a narrow bandwidth, and 100% X-mode polarization. These characteristics are consistent with the observed features of solar spikes.

With the same assumption, we estimated the brightness temperature of X2 emissions to be $10^{11}$ and $10^{13}$ K with $n_e/n_0 = 5\%$ and 50\%, respectively. These estimates are in line with the observations of solar radio spikes. We would like to point out that bulk acceleration could occur during solar flares, as indicated by HXR observations (Raymond et al. 2012; Benz 2017). This makes it demanding to study wave amplifications in plasmas with $n_e/n_0 \geq 25\%$, although energetic electrons with a high proportion could cause an extremely quick response of the ambient plasmas and may not last steadily for seconds. Individual solar radio spikes occur within an extremely short time, however; the durations of each spike are as short as several to some dozen milliseconds.

For solar spikes, harmonic structures are reported in many events. Staehli & Magun (1986) observed the harmonic structure of solar spikes at frequencies of 3.47 and 5.2 GHz, with a frequency ratio of 2:3. Guedel (1990) reported nine events with harmonic structures, with frequency ratios being 2:3, 2:3:4, 3:4, etc. In the most recent study, Feng et al. (2018) reported an event at metric wavelength, with harmonic structures of 2:3:4. Excitations of various harmonics are necessary to interpret the observed multifrequency structures of solar spikes. In our simulation, simultaneous excitations of X2 and X3 are obtained. The energy of X3 is lower than that of X2 by two to three orders of magnitude, according to our simulation, but the X2 mode may undergo stronger absorption during its escape from the source. Emissions at higher harmonics could not be resolved in our solutions due to the limited grid resolution. Their intensity, if properly resolved, should be even weaker than the third-harmonic emission, according to the linear kinetic theory.

According to the simulation result with $n_e/n_0 = 50\%$, a significant decline in the energy of the Z mode is present after saturation (Fig. 6d). In this case, the Z mode mainly propagates perpendicularly to the background magnetic field, and the background electrons are heated considerably in the perpendicular direction, as was pointed out in Sect. 3.2. This indicates that the Z mode is damped by the cyclotron-resonant absorption of thermal electrons. The damping could contribute to the heating process of plasmas during solar flares. It is intriguing to further investigate the role of the Z mode in flare heating of coronal plasmas, which can be excited quite efficiently through electron-cyclotron maser instability (see, e.g., White et al. 1986).

It should be noted that our simulations were performed with a uniform-field assumption, driven by the analytically prescribed distribution function of the horseshoe type. In reality, solar radio bursts are a consequence of the multiscale process of solar eruption, involving large-scale eruption, magnetic reconnection, electron acceleration, and further transport. As mentioned in the introduction, Yousefzadeh et al. (2021) performed simulations of energetic electrons traveling along a large-scale coronal loop with the guiding-center method. They fed the obtained velocity distribution function into the PIC system and found efficient excitation of the X2 mode with $\omega_{pe}/\Omega_{ce} = 0.25$. Further studies along this line of thought with the electron distribution determined in a self-consistent manner should be pursued. In the present study, simulations were performed with $\omega_{pe}/\Omega_{ce} = 0.1$, representing the plasma condition in the flare regions with very strong magnetic field. Régnier (2015) estimated the $\omega_{pe}/\Omega_{ce}$ ratio in the solar corona by combining the force-free field extrapolation and hydrostatic models, and found that similarly low values of $\omega_{pe}/\Omega_{ce}$ exist in the corona for active regions with a complex magnetic topology. Nevertheless, future studies should consider the effect of $\omega_{pe}/\Omega_{ce}$. Other features such as the energy of horseshoe electrons and types of velocity distributions should also be evaluated.

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