Breaking of axial symmetry in excited heavy nuclei as identified in experimental data

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A phenomenological prediction for radioactive neutron capture is presented and compared to recent compilations of Maxwellian average cross sections and average radiative widths. The basic parameters for it, photon strength functions and nuclear level densities near the neutron separation energy are extracted from data without an ad-hoc assumption about axial symmetry - at variance to common usage. A satisfactory description is reached with an astonishingly small number of global fit parameters when theoretical predictions on triaxiality are inserted into conventional calculations of radioactive neutron capture. These predictions (constrained HFB calculations with the Gogny D1S interaction) were tabulated recently for a large number of nuclei. For the photon strength a parameterization of GDR shapes by the sum of three Lorentzians (TLO) is extrapolated to low energies. Level densities are influenced strongly by the significant collective enhancement based on the breaking of shape symmetry. In the predictions for both the replacement of axial symmetry by the less stringent requirement of invariance against rotation by $\pi(R_\pi)$ leads to a global set of parameters, which allows to cover the range in nuclear mass number $A$ from 50 to 250. The impact of non-GDR modes adding to the low energy slope of photon strength is also discussed including recent data on photon scattering and other radiative processes. These are shown to be of minor importance for a comparison with experimental cross sections for neutron capture by even target nuclei. Here the triple Lorentzian (TLO) fit method for a parameterization of giant dipole resonances is normalized in accordance to the dipole sum rule and the droplet model with surface dissipation accounts well for positions and widths without additional parameters. Thus a reliable prediction for compound nuclear reactions also outside the valley of stability is expected.

I. INTRODUCTION

The ongoing discussion about triaxial shapes in heavy nuclei, recently often studied far off stability, may provoke the question, how well the widely used assumption about axial symmetry of most less exotic nuclei is founded on sufficiently sensitive experimental data. In the sense of formal logics conclusions from masses and level energies seem to be an unsufficient condition. The splitting of the Isovector Giant Dipole Resonance (IVGDR) is used as an indicator of axial deformation; a respective proof needs an adjustment of the apparent width locally for the five isotopes regarded, and a more rigorous approach appears helpful. In the present work this feature will be reviewed for 23 nuclei (as examples) in a wide range of mass number $A$ under the assumption of broken axial symmetry. It enables a restriction to only one global fit parameter, which is valid for all nuclei regarded, to parametrize the IVGDR-width. As the energy integrated absorption cross section is fixed by a sum rule, no fit of it is needed. A similar reduction of the number of free parameters is found for the other topic of this paper, the accordance of capture resonance spacings to a Fermi gas prediction. Again the assumption of broken axial shape symmetry strongly modifies the analysis of the experimental information as obtained from neutron capture by 140 spin-0 target nuclei. As an introduction to the question of nuclear shape symmetry, information from low energy nuclear structure studies in support of triaxiality will be presented first. Detailed Coulomb excitation studies have indicated the importance of the breaking of nuclear shape symmetries in accord to predictions of electromagnetic nuclear properties as clearly influenced by this breaking. These studies go beyond the observation of energy spectra, which have been interpreted in the past preferentially assuming at least axial symmetry of the nuclear shape. And they are superior to the electromagnetic excitation of only the lowest $2^+$-level as a sufficient measure of axial deformation. Interesting insight is gained by a comparison of various experimental data to recent theoretical work allowing for the replacement of an ‘ad hoc’ requirement of axiality by the less stringent assumption of invariance against rotation by an angle $\pi, R_\pi$.

The paper will first review the various connections between nuclear shapes and electric quadrupole moments and transition rates; in the past these have significantly contributed our knowledge about fundamental nuclear properties. After a short discussion of electromagnetic sum rules for the nuclear dipole strength, a parameterization of the IVGDR as seen in photon reactions with heavy nuclei will be presented. In contrast to previous work the electric dipole strength is derived from IVGDR data without an assumption about spherical or axial symmetry as usually made. The results will be regarded with respect to their impact on the radiative capture process,
for which the low energy behavior of this strength is of major importance. As was noted recently for nuclei with mass number $A > 70$, the low energy tail of the electromagnetic dipole strength function depends sensitively on the proper description of the shape of the IVGDR, including not only its position, but in particular its width and eventual split. On the basis of calculated deformation and triaxiality very few global parameters suffice for a parameterization of the dipole strength of heavy nuclei. The theoretical shape parameters will be used to evaluate the electromagnetic strength as derived from photonuclear data in the IVGDR region. Photon emission and absorption data are important here as they also deliver information on the possible influence of other multipolarities. The breaking of nuclear shape symmetry on the level density will also be regarded, as it strongly influences the phase space for radiative neutron capture and hence the size of the cross section for this important process. The radiative capture of fast neutrons by heavier nuclei plays an important role in considerations for advanced nuclear systems and devices aiming for the transmutation of radioactive nuclear waste. This process is of interest also for the cosmic nucleosynthesis, especially for scenarios with such high fluxes of neutrons, that their capture becomes the main production process to reach heavier nuclides beyond Fe. As the experimental studies which are the basis for respective predictions can mainly be performed for nuclei in or close to the valley of beta-stability, a small number of global parameters increases the reliability of any extrapolation.

II. NUCLEAR QUADRUPOLE MOMENTS AND DEFORMATION

The electromagnetic response of nuclei has played an important role for the exploration of the size of nuclei and the deformation of their shape from spherical symmetry was first indicated by a splitting of atomic transitions due to the nuclear electromagnetic field. Much improved and accurate hyperfine structure measurements, partly using laser techniques, determined the ‘spectroscopic’ electric quadrupole ($\lambda = 2$) moment $Q_s$ of the ground state in nearly 800 odd nuclei. In addition, $Q_s$-values from the reorientation effect in Coulomb excitation as well as from muonic X-ray data became available for many even and odd nuclei. In even nuclei, Coulomb excitation-reorientation data can also yield ‘spectroscopic’ quadrupole moments for excited $2^+$-states $Q_s(2^+)$, as compiled for nearly 200 isotopes. In many cases the sizes and signs – positive and negative values are observed – of these quadrupole moments were not in agreement to predictions for single particle or hole configurations and a seemingly obvious picture interprets this as the result of the rotation of a nonspherical body with an ‘intrinsic’ quadrupole moment $Q_0 = -7/2Q_s(2^+)$. If the sign of $Q_s(2^+)$ was determined, it mostly was negative indicating an oblate charge distribution in the laboratory system and an intrinsic prolate deformation. For most even nuclei away from closed shells the large $Q_0$ values observed were interpreted as a strong indication for a cigar like deformation, especially for lanthanide ($A \approx 170$) and actinide nuclei ($A \approx 240$). Mainly for isotopes in the near magic Os-Pt-Hg region positive $Q_s(2^+)$ were observed, indicating oblate intrinsic shapes, i.e. $Q_0 < 0$. Assuming axial symmetry and a homogeneous distribution of the charge within the nuclear volume, a rotational model was formulated, in which the intrinsic electric quadrupole moment $Q_0$ of even nuclei is related to the radius difference $\Delta R$ between the long and the two short axes of the shape by

$$Q_0 \equiv \frac{\sqrt{\frac{3}{5\pi}} Z R^2 \beta (1 + b \beta)}{Q_s(2^+)};$$

$$\beta \equiv \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R} \approx 1.057 \frac{\Delta R}{R}$$

(1)

The rotational model relation between deformation, $Q_0$, and $\Delta R$ is widely applied when electromagnetic data are related to calculated nuclear (mass) deformations usually characterized by $\beta$. For years $b \approx 0.16$ was used, but reference is also often made to a compilation of electric quadrupole transition width, which proposed $b = 0$ as approximation. In the same paper $b = 0.36$ was applied in a more detailed comparison to calculations, performed assuming zero hexadecapole moments. Besides this ambiguity in $b$ several definitions proposed as deformation parameters in the literature differ from the ‘standard’ definition as given here. Not all of them – even the ones in widespread use – conserve the volume when departing from spherical symmetry, especially for large $\beta$ or $Q_0$. This may lead to unwanted and often neglected contributions from compressional energy in respective calculations.

Intense experimental investigations concerning the energies and angular momenta of the levels in heavy nuclei have been regarded in view of single particle as well as collective excitations like rotations or vibrations of the nuclear body as a whole. In the data analysis and in theoretical studies the spherical or axial symmetry was usually assumed, albeit a sensitive experimental verification was missing. Here observed level energies and differences between them, eminent in gamma transition energies, have played an important role. But eventual strong energy shifts due to configuration mixing may confuse the arguments making the enhancement of electromagnetic strength a more reliable indicator of nuclear collective motion. Subsequent to a more formal discussion of the influence of nuclear shapes on photon interactions data will be presented indicating how breaking axial symmetry influences electromagnetic strength and photonuclear reactions. Finally it will be shown how resulting new degrees of freedom enhance level densities and consequently radiative capture cross sections.
III. QUADRUPOLE TRANSITION PROBABILITIES AND TRIAXIALITY

The enhancement seen in experimental data on electric quadrupole (E2) transitions from the ground state [12] over predictions for a transition to a configuration formed by exciting only one or a few single particle orbitals is linked [6] to the breaking of spherical symmetry in heavy nuclei away from magic shells. The model of a rotating axially symmetric liquid drop with a quadrupole moment, representing an even nucleus, predicts one 2+ state with a “collective” i.e. enhanced E2-transition width. In this model the intrinsic structure for the ground state 0+ of the ground state and the excited level. The reduced matrix element (in e fm²) of the electric quadrupole transition $E_2$ (in MeV) of this ‘rotational’ state $r$ to ground is related [6, 8] to $Q_0$ (in fm²) by:

$$| \langle r | E_2 | 0 \rangle |^2 = B(E_2, 0 \rightarrow r) = \frac{5}{16\pi} Q_0^2$$

(2)

The E2 ground state decay width $\Gamma_{00}(E_\gamma)$ (in MeV) is obtained from the relation (valid in general):

$$\Gamma_{00}(E_\gamma; E_2) = \frac{4\pi}{\hbar c e} \frac{\alpha_e E_2^2}{g J_0} | \langle r | E_2 | 0 \rangle |^2$$

(3)

where $\alpha_e$, $\hbar$ and $c$ are the fine structure constant, Planck’s constant and the velocity of light; $J_0$ and $J_r$ are the spins of the ground state and the excited level. The reduced transition probability $B(E_2)$ used in [2] is often expressed in e²fm⁴, but in this work we prefer to divide the $E_2$ operator directly by the elementary charge $e$, and do the equivalent for the $B(E\lambda)$. The $Q_0$ and $B(E_2)$ are observables, and we will use them to express an experimentally determined deviation from sphericity, whenever possible; the deformation $\beta$ is a model parameter, and in view of ambiguities in the definition of $\beta$ it will mainly be used in relation to theoretical calculations and as defined there.

To describe the rotation related quadrupole transition between the ground and the lowest 2+-state one often assumes ad hoc a uniform axial charge distribution; the same assumption has been made in the past [6] to relate axis ratios derived from IVGDR shapes to $Q_0$ or $\beta$. The axis ratios used in this study are taken from the calculations [1] regarded as reliable source for the amount of triaxiality.

One serious shortcoming of the axial rigid rotor model is the fact, that it only predicts one ‘collective’ 2+-state. Experimentally at least two 2+ levels with enhanced transitions to the ground state are observed in nearly all even nuclei. This has led to the assumption [6, 22] that the coupling of the collective rotation to a collective quadrupolar vibration around a deformed basis state has to be invoked; it can either be along the symmetry axis ($\beta$-vibration) or perpendicular to it ($\gamma$-vibration) [6]. If for a respective Hamiltonian a solution is found, which describes the level scheme simultaneously to the $B(E2)$-values reasonably well, one may consider this a determination of the mean values of $\beta$ and $\gamma$, but for their dynamic uncertainty additional considerations are needed. Reference is often made to more microscopic calculations in a self-consistent scheme and various approximations [1, 29, 32] have been proposed recently. The first of these calculations is available for practically all heavy even nuclei between the neutron and proton drip lines. In Fig. 1 the correlation between $\gamma$ and $Q_0$ is depicted for nuclei in the valley of stability as it results from this CHFB-calculation. Assuming only $R_\pi$-invariance they find non-zero triaxiality ($\gamma$) $\neq 0$ for many nuclei, and in some cases the predicted standard deviation does not include $\gamma = 0$, i.e. $\cos(3\gamma) = 1$. In Fig. 1 the density of symbols is of significance, as all the nuclei from a small band near beta-stability are depicted. The clear clustering at $Q_0 \approx 200 \text{ fm}^2$ and $\cos(3\gamma) \approx 0.2$ play an important role for the discussion of IVGDR data in these numerous nuclei, previously often regarded transitional between axial and spherical in shape.
erator coordinate method, which is important at low angular momentum. Information on triaxiality obtained for the intrinsic system in Hartree-Fock-Bogolyubov (HFB) calculations is subject to significant change when the order of variation and projection on angular momentum in the observer’s frame are interchanged [1, 33]. Long ago it was pointed out [34], that the quantum mechanically proper variation after projection may shift the γ oscillation centered at axiality to γ ≠ 0. Another way to circumvent this problem is to parameterize the quadrupole degrees of freedom as seen by the observer in the laboratory, as is done implicitly by a interacting boson approximation (IBA) [32, 37]. A rather convincing agreement to E2-transition data as well as level energies is achieved by this group theoretical ansatz when adjusting parameters for a given region of the nuclide chart and then comparing experiments to the predictions for other nuclei in that region. A global description of all heavy nuclei was not reached yet, but by a distinction between neutron and proton ‘Bosonic’ modes an account for triaxiality was made (IBA-2) [38]; an inclusion of Giant Resonance modes would be of great interest. Another way to circumvent the problems related to the necessary projection into the observer’s frame is the construction of rotation invariants from experimentally observed transition rates [8]. Data from of heavy ion induced multiple Coulomb excitation of low lying levels in heavy nuclei were analyzed on the basis of such invariants [7, 39] and for many heavy nuclei the breaking of axial symmetry was indicated by these experimental data. This analysis excels an older, comparatively simple, model of a rigid triaxial rotor [40] which directly delivers two collective 2+-levels. Generalizing Eq. (2) $Q_0$ is replaced by $Q_i$ defined by the sum of several (in the case of a rigid triaxial rotor 2) squared E2-matrix elements:

$$Q_i^2 = \frac{16\pi}{3} \sum_{r=1,s} |\langle r|E2|0\rangle|^2$$  

(4)

Experimental data [41] show that a limitation to one term in the sum (s = 1) leads to an error of less than 10% in $Q_i$ and thus for the figures it suffices to use Eq. (2) instead of Eq. (4) to depict $Q$ derived from B(E2)-values. In the rotation invariant ansatz [8] the deviation from axial symmetry is described by the parameter $\cos(3\gamma)$, which is in principle directly related to transition rates observable in multiple Coulomb excitation [4, 7, 42, 44]. In quantal systems like nuclei only expectation values are accessible to measurements; $Q_0$ and $\cos(3\gamma)$ in Figs. (1, 2) are to be understood as symbols for the actually determined quantities; explicitly these are $(Q_0^2)$ and $(Q_0^2 \cos(3\gamma))$. In various nuclei the inclusion of triaxiality produced transition rates close to data [44, 47], especially when surface vibrations are included as a perturbation. But in most heavy even nuclei more than 2 collective 2+-levels are observed and this scheme has to be generalized further. Employing tensor algebra and assuming reflection symmetry as well as identical distributions of protons and neutrons Eq(4) was shown [8, 44] to be valid also for heavy nuclei in general, as long as the sum includes all relevant 2+-strength, especially all collective 2+-levels. Then Eq(1) with $s > 2$ represents an un-weighted sum rule which is rotation invariant and thus valid in the laboratory as well as in the intrinsic frame.

In Fig. (2) the correlation of the parameters $Q_0$ and $\cos(3\gamma)$ is displayed for more than 150 nuclei, for which relevant data have been published [4, 7, 36, 39, 42, 44, 47–49]. The rather complex experimental investigations on triaxiality were performed for a limited number of nuclei only, but a comparison to the prediction [1] shown in Fig. (1) indicates a clear trend to triaxiality with decreasing $Q_0$ is obvious. Here $\cos(3\gamma) \rightarrow 0$, whereas most well deformed nuclei and especially the actinides show smaller deviation from axiality. The small number of nuclei which are oblate already at low $E_x$ ($\cos(3\gamma) < 0$) does not allow similar conclusions, and for very small $Q_0$ a definition of triaxiality is not appropriate.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2}
\caption{(Color online) Correlation between $\cos(3\gamma)$ and $Q_0$ in ≈ 150 even nuclei with $A > 60$, for which respective experimental data are available. The bars correspond to experimental uncertainties and the dashed blue curve serves as eye guide to compare the trend of the data to the one of the calculations as presented in Fig. (1) where it is depicted as well.}
\end{figure}

If not determined directly, the sign of the latter was assumed to coincide with the one of $\cos(3\gamma)$; unambiguously opposite signs for these two quantities were not reported yet. Data for odd nuclei [50] are omitted in the Figure, we only state that they support the findings presented here. Also not given in the Figure are variances that can be derived in principle from an extension of the above formulae [1, 7, 8, 14], but the experimental uncertainties of respective data do allow a rough extraction of such information only. The experimental data show a very similar trend of the axiality increasing with $Q_0$ as seen in the calculation. The trend as indicated as blue dashed curve in figs. (1, 2) suggests a speculation of nuclear shapes represented by one parameter $Q_i$ only, with the axiality depending on it.
The split of the giant dipole resonances \[1\] is related to the axis lengths \(R_{1,2,3}\). Assuming \(R_{2}\)-invariance only, the combination of Eq. (13) to Eqs. (19-21) of ref. 8 results in relations for the three axes \(R_{i}\) of the ellipsoid in the body fixed system as derived from its shape parameters:

\[
\begin{align*}
5Q_{1}\cos(\gamma) &= Z(2R_{2}^{2} - R_{1}^{2} - R_{3}^{2}) \\
5Q_{1}\sin(\gamma) &= \sqrt{3}Z(R_{1}^{2} - R_{2}^{2}) \\
R_{0}^{2} &= R_{1}R_{2}R_{3}
\end{align*}
\]

The last line uses the concept of the conservation of the nuclear density in an equivalent ellipsoid, which has the same volume \(V = 4/3\pi R_{0}^{3}\), and the three harmonic oscillator constants \(\omega_{k}\) are inversely proportional to \(R_{k}\). Assuming the same charge \(Ze\) for the equivalent ellipsoid as for the non-spherical nucleus, the Eqs. \[\text{CHFB}\] can be related directly to the information about \(R_{0}\), the quadrupole moment \(Q_{1}\) and the triaxiality \(\gamma\) of that nucleus. The similarity between observations and the CHFB-calculations \[\text{CHFB}\], as shown in Figs. \[\text{CHFB}\] and \[\text{CHFB}\] suggests to use these as a reference. The equivalent sphere radius \(R_{0} = \sqrt{(5/3)R_{c}}\) with the charge radius \(R_{c}\), tabulated as well \[\text{CHFB}\], will also enter into the predictions discussed below. In ref. \[\text{CHFB}\] the tabulated \(\beta\) and \(\gamma\) values are related to the oscillator parameter ratios \(P_{\text{CHFB}}\) and \(Q_{\text{CHFB}}\), the direct outcome of these calculations (with the latter not describing a quadrupole moment). These are used in the following Eq. \(7\) as axis ratios to obtain the radii \(R_{i}\) from \(R_{0}\):

\[
\begin{align*}
R_{1} &= R_{0}(P_{\text{CHFB}}Q_{\text{CHFB}})^{-1/3} \\
R_{2} &= P_{\text{CHFB}}R_{1} \\
R_{3} &= Q_{\text{CHFB}}R_{1}
\end{align*}
\]

with

\[
\begin{align*}
x &= \frac{\beta}{2\beta + 1} \\
P_{\text{CHFB}} &= \exp\left(-\frac{x}{2}\sin\gamma\right) \\
Q_{\text{CHFB}} &= \exp\left(x\left[\frac{3}{2}\cos\gamma - \frac{\sqrt{3}}{2}\sin\gamma\right]\right)
\end{align*}
\]

In ref. \[\text{CHFB}\] supplemental material is given for 1712 even-even nuclei including their deformation parameters, from which the corresponding ‘triaxial oscillator parameters’ can be derived, which are inversely proportional to the axis lengths. The extraction of these axis ratios differs formally from Eq. \(6\) as well as from the Hill-Wheeler formula used by us before \[\text{CHFB}\]. It can easily been shown numerically, that for the small \(Q_{1}\) in the range of interest the differences are below 25% and hence not significant. For the calculation of fission barrier heights more exact prescriptions may be needed, but, as was pointed out already \[\text{CHFB}\] and \[\text{CHFB}\], the consideration of triaxiality is even more important in that case.

IV. PHOTON ABSORPTION SUM RULES AND GIANT RESONANCES

The non-resonant interaction between photons and objects of charge \(Ze\) and mass \(M\) is quantified by the Thomson scattering cross section \(8\pi(Z^{2}\alpha_{e}hc^{2})^{2}/3(Mc^{2})^{2}\) with \(\alpha_{e}\) denoting the fine structure constant, \(c\) the velocity of light and \(h\) the Planck constant divided by \(2\pi\); for \(^{208}\text{Pb}\) it amounts to 0.02 fm\(^{2}\) only. In addition to this direct process a photon of sufficiently high energy \(E_{\gamma} = E\) excites nuclei from the ground state resonantly; this is described by a Lorentzian centered at the resonance at \(E_{r}\) with total width \(\Gamma_{r}\):

\[
\sigma_{\gamma} \equiv I_{r0} \frac{2}{\pi} \frac{E^{2}\Gamma_{r}}{(E^{2} - E_{r}^{2})^{2} + E^{2}\Gamma_{r}^{2}}
\]

The integral of the absorption cross section \(\sigma_{\gamma}\) over the resonance, which has spin \(J_{r}\) is denoted by \(I_{r0}\):

\[
I_{r0} = \int \sigma_{\gamma}(E)dE = \frac{g(\pi \alpha_{e}^{2}\Gamma_{r})}{E_{r}^{2}}; \quad g = \frac{2J_{r} + 1}{2J_{0} + 1}
\]

The width \(\Gamma_{r}\) is the partial width of the transition between the resonant level \(\{E_{r}, J_{r}\}\) and the nuclear ground state \((0, J_{0})\). As described by Eq. \(5\) for multipole order \(\lambda = 2\), it is directly proportional to the electromagnetic transition matrix element; a respective relation exists for \(\lambda = 1\):

\[
\Gamma_{r0}(E_{\gamma}; E_{1}) = \frac{16\pi}{9} \frac{\alpha_{e}E_{1}^{3}}{g(\pi \alpha_{e}^{2})^{2}} |\langle r|E_{1}\rangle|^{2}
\]

As derived from very general conditions like causality, analyticity and dispersion relations the interaction of short wavelength photons with nuclei of mass number \(A = Z + N\) can be ‘integrated up to the meson threshold’ analytically, leading to the energy-weighted sum rule of Gall-Mann, Goldberger and Thirring (GGT) \[\text{GGT}\]:

\[
I_{E1} = \int_{0}^{E_{1}} \sigma_{\gamma}(E)dE \\
\cong 2\pi^{2} \alpha_{e}^{2} \frac{ZN}{m_{n}} \left[ \frac{Z}{A} + \frac{A}{10} \right] \\
= 5.97 \left[ \frac{Z}{A} + \frac{A}{10} \right] \text{MeVfm}^{2}
\]

Here \(m_{n}\) and \(m_{\pi}\) stand for the mass of nucleon and pion, respectively. The first term in the sum is the “classical” (TRK) sum rule” for electric dipole radiation \[\text{TRK}\] and the second “contains all of the mesonic effects” and is assumed \[\text{GGT}\] to be accurate within 30%. It was approximated by assuming “that a photon of extremely large energy interacts with the nucleus as a system of free nucleons”, and a correlation to hadronic shadowing was investigated to be weak \[\text{GGT}\]. Eq. \(10\) includes all multipole modes of photon absorption and no arguments \[\text{GGT}\] about the nuclear absorption of photons with energies above \(m_{\pi}c^{2}\) are needed. Absorption by the nuclei does not
contribute below $E_\gamma = m_e c^2$, but nucleon pairs and especially p-n-pairs are strongly dissociated by photons with $20 < E_\gamma < 200$ MeV. The respective "quasi-deuteron effect" has been derived from the expression valid for the free deuteron by correcting for Pauli blocking [57].

Photo-neutron data are available [58] for $^{208}$Pb up to energies above $m_e c^2$; they are shown in Fig. 3 and compared on an absolute scale to respective theoretical expressions:

1. a Lorentzian like in Eq. (7) with parameters from Eq. (12) (width $\Gamma_\gamma = 3.26$ MeV and pole energy $E_\gamma = E_\gamma^0 = 13.6$ MeV). $I_\gamma$ is normalized such that its integral agrees to the first term in Eq. (10), and

2. the expression for the absorption corresponding to the quasi-deuteron mode [57].

Apparently the cross section above 40 MeV is well reproduced on absolute scale and it arises that its integral is close to the second term in Eq. (10). It is worth mentioning that a similar situation was published [57] for other nuclei. In contrast to elastic electron scattering for other nuclei. In contrast to elastic electron scattering

\[ \sigma_{\text{abs}} (E_\gamma) = \frac{dI_{E1}}{dE_\gamma} (E_\gamma) \equiv \sigma_{\text{abs}}^{E1,IV} (E_\gamma) \]

\[ \approx 5.97 \frac{ZN}{A} \frac{2}{k\pi} \sum_{i=1,2} \frac{E_i^2 \Gamma_i}{(E_i^2 - E_\gamma^2)^2 + E_i^2 \Gamma_i^2} \text{fm}^2 \quad (11) \]

It will be shown in the next section, that the TRK sum rule is especially well fulfilled, when accounting for the breaking of axial symmetry. Especially the lanthanide and actinide nuclei appear to be quite close to axiality, as obvious in experimental data [66, 67]. It will be shown in sections V to VII that the extraction of total dipole strength from data is not very sensitive details in the region of the maximum, as long as the sum rule is respected, and the same is true for the height of the low energy tail. In Fig. 3, here approximating $^{208}$Pb as spherical even above 10 MeV, $k = 1$ is used. For heavy nuclei only very few parameters for pole positions and resonance widths are required in a global fit: The two historic theoretical treatments of the IVGDR predict its energy rather well for medium mass nuclei [65], respectively for the very heavy ones [63]. By using concepts of the droplet model these two approaches were unified [70] and hence IVGDR centroid energies $E_0 (Z, A)$ are well predicted in the range $50 < A < 254$. The nuclear radius used here is derived from the mean charge radius $\langle R_c \rangle$ taken from the CHFB calculations [1] as $R_0 = \sqrt{\langle f^2 \rangle / \langle f \rangle}$; symmetry energy $J = 32.7$ MeV and surface stiffness $Q = 29.2$ MeV are taken from the finite range droplet model [71]. Only one additional parameter, an effective nucleon mass has to be adjusted [13] to $m_{eff} = 800 \text{MeV}/c^2$ in an overall fit to the IVGDR data; due to the use of radii, deformation

![FIG. 3. (Color online)Cross section of photo-neutron production data [58] on $^{208}$Pb in comparison to a Lorentzian for the isovector IVGDR (full line, magenta) and the quasi-deuteron effect (blue dotted line). In $^{208}$Pb a deformation induced widening can be neglected as will become obvious in Fig. 4.](image-url)
and triaxiality from the CHFB calculations [1], it differs from our earlier work [13], where it was taken as 874 MeV. By the use of Eq. (7) the energies of the three resonance poles are derived from the spherical centroid \( E_0 \) and the well known proportionality between \( E_i \) and \( 1/R_i \). As previously [13, 72] we thus use (with all energies expressed in MeV):

\[
E_0 = \frac{hc}{R_0} \sqrt{\frac{8J}{m^*}} \frac{A^2}{4NZ} \left[ 1 + u - \varepsilon \cdot \frac{1 + \varepsilon + 3u}{1 + \varepsilon + u} \right]^{-1/2}
\]

\[
\varepsilon = 0.0768, \ u = (1 - \varepsilon) \cdot A^{-1/3} \cdot \frac{3J}{\gamma^2}
\]

\[
\Gamma_i = c_w E_i^{1.6}
\]

With one parameter for the energies and one for the widths, both adjusted to be equal for all heavy nuclei with \( A > 50 \) [13, 72] one gets a good agreement to measured resonance shapes, as will be shown in Figs. 6 to 25. In principle, the nature of the IVGDR does not directly allow for the determination of its width in analogy to Eq. (6), and the widening due to deformation has to be accounted for by using \( k > 1 \) in Eq. (11). A strict distinction between damping or spreading and the deformation induced splitting – to be discussed below – allows to neglect a photon energy dependence of the widths extracted from IVGDR data. Hydro-dynamical considerations [74] predict the dependence of the damping width \( \Gamma_i \) of an IVGDR on its pole energy \( E_i \) to be proportional to \( E_i^{1.6} \); this exponent lies between theoretical values [72] for one- and two-body dissipation. Of course, the proportionality constant \( c_w \) has an uncertainty related to the selection of nuclei, which are included in the fit. With the axis ratios available from the CHFB calculations an inclusion of all nuclides for which respective data have been tabulated yields \( c_w = 0.045(3) \). The width parameter is varying with \( E_i \) and not with \( E_i \) like in previous work [13]. As the slope of a Lorentzian sufficiently far away from \( E_0 \) is quasi proportional to \( \Gamma_i \), its uncertainty enters in the radiative width nearly linearly. In our earlier work, a value of \( c_w = 0.05 \) had been used [13, 72]; there single Lorentzians were adjusted to IVGDR data for \(^{88}\text{Sr}\) and \(^{208}\text{Pb}\), assumed to have one pole only. The new values for \( E_0 \) and \( c_w \) given here are based on the calculations [1, 30] and also account for the influence of the GQR’s, to be regarded below.

Individual local fits [67, 77] to the apparent widths seen in photo-neutron data (using \( k = 1 \) or 2 in Eq. (11), selected \( a d h o c \) to optimize the fit, result in a surprisingly large excess above the TRK sum rule. The result of such local fits as performed within the Reference Input Parameter Library (RIPL) project [31, 77] is depicted in the top part of Fig. 4. It differs somewhat from earlier data analysis [68] which yielded an average of 105(8)% for \( 100 < A < 200 \) and a finding for six Sn-isotopes, where an overshoot of only 10(7)% was found [78]. The large unsystematic scatter reported from both local fits yield strong arguments against their use for nuclear astrophysics, albeit proposed [79] in that field.

![FIG. 4. (Color online) Panel (a) shows the energies (top) and widths (bottom, both vs. mass number) resulting from \( \chi^2 \)-fits to the IVGDR in heavy nuclei, as compiled recently [77]. The fits are based on one or two Lorentzians and two points per nucleus are shown, if a 2-pole fit led to a smaller \( \chi^2 \). Our calculations with 3 poles as described in section IV and V (TLO) are depicted as drawn curves (in blue and magenta). In panel (b) the resulting GDR-integrals as obtained by the Lorentzian fits from ref. [31] are depicted in comparison to the TRK sum rule (dotted line), which is surpassed considerably in most cases, whereas TLO obeys it by definition.](image)
the level-density is rather small up to the IVGDR peak (and far below a Fermi-gas prediction; cf. section IX) such that fluctuations are observed as long as the experimental averaging is comparatively small. Obviously, several components are responsible for the apparent width of the IVGDR in heavy nuclei:

(a) Spreading into underlying complex configurations,
(b) Nuclear shape induced splitting,
(c) Fragmentation and
(d) Particle escape.

From calculations for heavy nuclei using the Rossendorf continuum shell model [80, 81] the escape width (d) in the IVGDR region was shown to be clearly smaller than the spreading width (a) derived by the global TLO-fit; a good agreement to data shows that $\Gamma$ in Eq. (12) depends on the pole energies only and one global fit parameter. For the concept of fragmentation (c) of the configurations belonging to e.g. the IVGDR a quantification of these configurations is needed. A detailed shell model calculation [82, 83] for the nucleus $^{208}$Pb is depicted in Fig. 5 which is based on a large number of configurations. From it one may conclude, that the parameterization of this IVGDR by a Lorentzian is justified as well as the width predicted as given in Eq. (12). It should be noted here, that this prediction, like TLO, is normalized to the "classical" sum rule, the main component in Eq. (10). The shape induced splitting (b) will be regarded in the next section.

![Graph](image)

FIG. 5. (Color online) Dipole absorption from a shell model calculation for $^{208}$Pb with configuration mixing [83] (black histogram for E1). For comparison the Lorentzian from Fig. 3 is superimposed as dotted curve (magenta); it corresponds to Eq. (12) reduced to one pole.

The agreement especially in the high energy slope as seen in Fig. 5 and Fig. 6 supports our concept of admitting no photon-energy-dependence of the width. Such a decrease was postulated by the KMF-model prediction for $f_{E1}$ allegedly derived from the theory for Fermi liquids [84]. A detailed relation to the work of Landau or Migdal is not given in that work, which explicitly states below its Eq. (29), that a "direct comparison" between the "spreading width" in a Lorentzian for the GDR and the one of the theory of Fermi liquids is "difficult...and not clear" [84], and they further assume ad hoc without any additional arguments, that they coincide. In this context it should be mentioned, that fundamental theoretical arguments have been used to show, that "Landau damping is not the appropriate process for describing the damping of the low-multipole giant resonances" [73].

V. ISOVECTOR GIANT DIPOLE RESONANCES IN HEAVY NUCLEI AND TRIAXIALITY

The coupling of dipole and quadrupole degrees of freedom in heavy nuclei has been discussed thoroughly [6, 27, 55, 57] and calculations within the interacting Boson model (IBA) have obtained reasonable fits to experimental data for two chains of even isotopes [88] and some other nuclei [89, 90]. In the dynamic collective model (DCM) [86, 91] calculations on the basis of hydrodynamical considerations were successfully compared to selected data [92, 93]. The importance of the breaking of axial symmetry for the GDR shapes was not discussed in the literature cited here and not in most of related work. In theoretical papers on a few nuclei triaxiality was mentioned with respect to the IVGDR [74, 94], but a full coverage of a wide range in the nuclide chart is missing. The parameterization presented earlier [13] and specified in this section is much less ambiguous concerning the mode coupling, but it has one advantage as it incorporates nuclear triaxiality explicitly by setting $k = 3$ in Eq. (11). For the resulting ‘triple’ Lorentzian (TLO) description the resonance energy $E_0$ is modulated by the ratios of the axis lengths $R_i$ in the spirit of Eqs. (6) and (7) (setting $E_i/E_0 = R_0/R_i$). This direct incorporation of triaxiality makes TLO differ from previous attempts to obtain Lorentzian fits to photo-neutron data for a large number of heavy nuclei [4, 15, 31, 66, 67, 77, 95]. In many nuclei, especially those of intermediate $Q_0$, the local fits presented there may lead to a seemingly better agreement, but often they require quite unreasonably large values for the width of the IVGDR and for the integrated strength in comparison to sum rule predictions. If triaxiality is accounted for in addition to the quadrupole deformation a good description of IVGDR shapes can be obtained [13] without treating the strength and width as free fit parameters. The deformation induced shift of the three axis lengths $R_i$ versus the equivalent radius $R_0$ is obtained from Eq. (7), which is based on a CHFB calculation [1, 30]. As outlined in section III the last-mentioned work lists values for the quadrupole deformation $\beta$ as well as the corresponding $\gamma$ for a large number of even nuclei. Hence, a global prescription to predict
electric dipole strength can be derived from it, not limited to using local information for single nuclei.

When a parameterization of the electric dipole strength in nuclei with non-zero \(Q_i\) is aimed for, the contribution (b) of the list in the previous section has to be treated sufficiently well. As shown in the top part of Fig. 6, the TRK sum rule, Eq. (10), disagrees in many nuclei to the Lorentzian fits \([67, 77]\) performed for the data of each nucleus independently without account for the possibility of broken axial symmetry. In these fits the width parameter was adjusted for each isotope separately to fit the peak region and for \(A\) between 90 and 150 an especially large discrepancy is observed as well as wide fluctuation with \(Z\) and \(A\) of this apparent breadth indicating a non-systematic variation which is difficult to conceive within the spreading concept. A similarly erratic dependence of the integrated IVGDR strength on \(Z\) and \(A\) was reported \([77]\) to result from this approach of fitting the photo-absorption data locally. In some cases the integrated cross section overshoots the classical sum rule given by Eq. (10) first term) by up to 100%. Apparently the two problems named are closely related, as the resonance integral is proportional to the product of height and width. As proposed previously \([12, 68, 97]\), a solution for this problem is found by allowing axial symmetry to be broken; this point will now be examined in further detail. As shown in section IIII accurate nuclear spectroscopic data suited to determine both deformation parameters are available only for a limited number of nuclei. The recent CHFB calculation \([1]\) delivers prior information for Eq. (7) above, inserted to obtain the resonance energies in the sum of Lorentzian functions in Eq. (11). This procedure leads to a significant splitting into three equally strong IVGDR components which increases with deformation. As seen in Fig. 1 for many nuclei the splitting between the three components is comparable in energy to their widths and thus not directly obvious from the data alone, especially in nuclides with \(Q_0 \approx 200 - 300 \text{fm}^2\). These are not rare as the clustering depicted in Fig. 1 shows, but the calculated triaxiality demands that all three axes are accounted for explicitly. This quite simple consideration explains the significant rise in apparent breadth as seen in Fig. 4 for the even Sm-isotopes with \(N = 86\) to \(N = 92\); \(^{148}\text{Sm}\) was not included because of the uncertain cross section for the \((\gamma, p)\)-reaction, and in \(^{148}\text{Sm}\), previously often regarded as spherical, the deformation induced split leads to a low tail, of importance for the envisaged extrapolation to even smaller energies. Special care is needed for nuclei near closed shells: The CHFB calculations do not fully account for the very deep mean field potential in such nuclei and thus they produce too much of collectivity \([30]\). Following what is said there, a reduction for nuclei only \(\delta\) nucleons away from a shell a factor for the \(\beta\)-deformation \([1]\) of \(0.4 + \delta/20\) is applied for \(\delta \leq 10\). This expression is used for protons as well as neutrons and the larger of the two correction factors is taken; it results in a reduction of the predicted \([1]\) \(\beta\)-values by 40, 30, 20 and 10% for the isotopes \(^{148}\text{Sm}, \text{ }^{150}\text{Sm}, \text{ }^{152}\text{Sm and }^{154}\text{Sm}\) and the corresponding agreement to the IVGDR data is shown in Fig. 5. It is worth mentioning here, that the increase of width \(\Gamma_i\) with \(E_i\) as predicted from Eq. (12) reduces the maximum in the upper of the two peaks, such that it is not two times higher than the lower, as expected from the near axial prolateness of \(^{154}\text{Sm}\).

The supplemental material based on the CHFB-calculations \([1]\) does not only list the mean values for the deformation parameters as obtained via the GCM, it also gives their variances as resulting from quantum mechanical zero point oscillation. The respective Gaussian distributions obtained thus allow an instantaneous shape sampling (ISS) as shown earlier \([90]\) for isotope chains Mo \([77]\) and Nd \([99]\). There the impact of ISS on the shape sampling (ISS) as shown earlier \([90]\) for isotope chains Mo \([77]\) and Nd \([99]\). There the impact of ISS on the height of the low energy tail and thus on radiative capture was demonstrated to be negligible. Results for the Sm chain as shown in Fig. 6 indicate a minor influence as well: the drawn black curves correspond to the TLO-prediction and the dashed purple curves stem from the analysis with TLO + ISS. The impact of this effect for other nuclei needs further study; we expect it to be small. The self-consistent extension to non-spherical nuclei introduces no extra free parameters in addition to the global ones of the Gogny-force \([1]\) and an attempt of a fit to the IVGDR energies and widths succeeds with only the four parameters introduced in connection to Eqs. (12).
and \[13\], two of which are known from LDM mass fits. In the evaluations presented in the next section the only local parameters for individual nuclei are the axis ratios calculated by CHFB \[1\], and no dependence of the widths \(\Gamma_i\) (cf. Eq. \[12\]) on the photon energy \(E_\gamma\) was allowed. Invoking widths varying only with the pole energies \(E_i\) (and not with \(A\) and \(Z\)) opens the possibility for a global prediction of nuclear photon strength also for heavy exotic nuclei and an extension to energies below the neutron emission threshold \(S_n\). This has the potential of consistent predictions for radiative capture processes, where full satisfaction is not reached with presently available methods \[101\]. But, as will also discussed below, other modes than the IVGDR contribute to photon absorption and their influence will be quantified in sections \[VII\] and \[X\].

VI. ELECTRIC DIPOLE STRENGTH IN THE IVGDR AND BELOW

The interaction of photons with heavy nuclei at energies above the neutron separation energy \(S_n\) is mostly resulting in reactions of type \((\gamma, xn)\), but for a determination of the complete absorption cross section the emission of \(p\), \(\alpha\)’s or fission might also be important. As will become obvious the photon absorption in nuclei with low level density below the IVGDR pole may be fragmented into many small peaks, overlapping at higher excitation energy only. The radiative neutron capture discussed later may proceed via this domain and it is thus indicated to use average photon strengths \(f_\lambda\) for the description of electromagnetic decay by multipolarity \(\lambda\). This average quantity, the strength function \(f_\lambda(E_\gamma)\), was introduced \[15\] for electromagnetic processes. The relation to the average photon absorption cross section and the integral \(f_{\text{abs}}^\lambda\) over a given energy interval \(\Delta E\) around \(E_\gamma\) is:

\[
f_\lambda(E_\gamma) = \frac{\langle \sigma^\lambda_{\text{abs}}(E_\gamma) \rangle}{(\pi\hbar c)^2 g_{\text{eff}} E_\gamma^2} = \frac{I_{\text{abs}}^\lambda}{(\pi\hbar c)^2 g_{\text{eff}} E_\gamma^2} \Delta E \tag{13}
\]

To use the strength functions \(f_\lambda(E_\gamma)\) for excitation as well as decay processes and thus connect photon and nucleonuclear processes one has to suppose them to be direction independent (Axel-Brink hypothesis) \[15\], Eqs. \[8\] \[9\] and \[13\] may then be used to directly relate \(f_\lambda\) to the electromagnetic decay widths of the resonant levels \(r\) in the integration interval \(\Delta E\) :

\[
f_\lambda(E_\gamma) = \frac{1}{\Delta E} \sum_{\Delta E} \frac{\Gamma_r}{E_r^{2\lambda+1}} = \frac{\langle \Gamma_r(E_\gamma) \rangle}{D_r E_r^2} \tag{14}
\]

The integration in Eq. \[13\] runs from \(E_\gamma - \Delta E/2\) to \(E_\gamma + \Delta E/2\) and for the sum in Eq. \[14\] all levels within this interval are included; the quantum-mechanical weight factor \(g_{\text{eff}}\) will be discussed in section \[VII\]. The average level spacing \(D_r\) at the upper of the two levels connected by \(E_\gamma = E_r - E_f\) is the inverse of the level density \(\rho_r = 1/D_r\). Increasing \(\langle \Gamma_r(E_\gamma) \rangle\) and the reduced matrix elements for the transition to the ground state results in an increase of \(f_\lambda(E_\gamma)\). But decays not to ground have to be corrected for \[14\], as they reduce the observed gamma-intensities and hence one approximates collective electromagnetic transition strengths of energy \(E_\gamma\) by \(f_\lambda(E_\gamma)\) to be independent of the energies \(E_r\) and \(E_f\); together with the notion of \(f_\lambda\) being valid for excitation and decay, this assumption is also part of the Axel-Brink hypothesis \[102\] \[103\]. In the present ansatz an eventual dependence of decays into a region of elevated nuclear temperature is not accounted for by a modified strength function, but rather by averaging over that region using the relevant level density (which may be characterized by an apparent temperature (cf. section \[IX\]).

Several facts have to be regarded when using experimental photo-neutron or photon absorption data to test the TLO parameterization as strength-function prediction:

1. Considerable discrepancies were reported for experiments performed e.g. at different laboratories \[104\] \[106\]; in some cases energy calibrations may differ somewhat.

2. Photo-neutron data were often obtained by using quasi-monochromatic photon beams with a rather wide energy distribution, which is incorporated by folding the calculations with a Gaussian of width \(\sigma = 0.3\) MeV, a value not as large as some recently made guesses \[107\] \[108\]. An apparent widening near the IVGDR peak is partly due to ISS as discussed above. Also in the case of a bremsstrahlung distribution used as photon source quite some uncertainty may arise \[109\].

3. In a number of cases the \((\gamma,p)\)-channel exhausts a significant portion of the photo-absorption cross section \[67\] \[77\] \[97\]. It was shown in an earlier paper \[97\], how the eventual influence of all open channels on the extraction of the absorption cross section from the existing data is tested by Hauser-Feshbach calculations.

4. At higher energy the competition by the \((\gamma,2n)\) channel becomes important and that requires involved subtraction procedures \[92\], which may be questioned \[107\] \[108\].

5. Most of the targets used contain unwanted isotopes, and when some of them have a lower \(S_n\) (like many odd isotones) the low energy yield has to be corrected \[97\].
6. Below and above the pole of the IVGDR contributions from the giant quadrupole (GQR) modes cause effects of some significance.

7. In heavy nuclei various excited states may exist aside from giant resonances, which cause observable photon strength also below the neutron threshold. It adds to the IVGDR tail not always smoothly, as Porter-Thomas fluctuations may randomly create strong peaks in spectra from a quasi-continuum of weakly populated levels.

In the IVGDR region averaged experimental photo-dissociation data as obtained with quasi-monochromatic photons from positron annihilation have been compiled and are available [66, 67, 70]. Such data do not exist for all stable isotopes and for some nuclei photo-absorption has been studied by an absorption technique, which may serve for a consistency check in spite of its systematic error due to the need to subtract the strong atomic absorption. As indicated repeatedly [104, 108, 110–113], cross sections obtained at Saclay should be reduced. The necessary reduction is probably related to difficulties in the analysis of multi-hit events in the neutron detector array [108, 112]. In accordance to a precision study [104], confirmed by results [97, 111] from the radiation source ELBE, the photo-neutron data of that origin are hence multiplied in this work by 0.9, considered as suitable for various A.

Items 2 to 4 influence the representation of the IVGDR peak region, but their effect on the tail a few widths $\Gamma$ below the pole is insignificant—in contrast to items 5 and 7. Photon strength data for energies below $S_n$ are even rarer [13]. Similar as in many photo-neutron studies experiments with quasi-monochromatic beams are performed to gain information on absorption from photon scattering: In early experiments the tagging system at Urbana was used [14, 115] and an increasing number of measurements are done at the laser-photon backscattering facilities HiS ($\gamma$-Ray Source) at Duke University [116, 117] and AIST (National Institute of Advanced Industrial Science and Technology) in Japan [106]. The photon intensities from these are not well known and bremsstrahlung data eventually serve for normalization. Results from scattering have to be corrected for branching to other than the ground state and in various cases the bremsstrahlung continua also feed higher excited levels and their decay yield has to be subtracted. In the case of less complex spectra use can be made of data from the decay to well isolated levels [118, 119]. More sophisticated schemes as developed e.g. at ELBE [83, 97, 120, 121] are based on statistical considerations already formulated some time ago [14, 15]. Due to the fact, that the electromagnetic strength is responsible for the absorption as well as the emission of photons, an iterative procedure can lead to a self-consistent solution [83, 122, 123]. Here, level spacings enter which often were taken from predictions and extrapolations eventually questioned; in view of new results [124] on level densities as outlined in section 11, a reduction of 30% in the $f_\lambda$ as compared to published values was applied in the case of ELBE-data and mentioned in the respective figure captions. In principle, also photon yields observed after nuclear collisions can deliver strength-information, when it can be normalized via an "external" fix-point. In the case of resonant neutron capture this is realized by "using the absolute gamma-ray intensities due to captured thermal and resonance neutrons" [125]. A similar normalization via neutron capture, now with respect to level densities, was attempted for data taken at the Oslo cyclotron [120, 128] with $^3$He and $^2$H projectiles. The method of relying on neutron capture resonance spacings requires the knowledge of the energy and spin dependence of level distances; here theoretical assumptions have to be used.

The subsequent compilation presents in figs. 7 to 25 photon strength function data and a comparison to the TLO parameterization with three IVGDR pole energies induced by the deformation. Here parameters are taken from the available CHFB-calculations [1] and globally determined fit values for $c_w$ in Eq. (12) and an effective mass quantifying the centroid $E_0$. The (black) dashed lines represent the prediction of $f_{EL}$ thus derived with the resonance integral from Eq. (11) (in accordance to the TRK sum rule) equally divided among the three poles of TLO. In all these figures the three are indicated as black bars at the energy axis. It is worth mentioning that the absolute heights in the low energy slope are nearly unchanged by the splits, albeit the apparent peak-height depends on it. As shown previously [83] and as will be further detailed in section 11, the effect of the photon strength on radiative neutron capture is strongest in the tail region below the neutron binding energy $S_n$. Here up to eight additional "minor" strength components may be of importance; they will be detailed in section 8. For a specification of their energy, strength and breadth we have used published data as listed with the figures. But the available information is by far not detailed enough to derive a systematic parameterization to evaluate their eventual influence on neutron capture with high precision. This is demonstrated by the experimental data inserted in the figures; the result of our adjustment of the relevant parameters to derive an approximate agreement is depicted by a full (blue) line. Like for TLO only globally fitted quantities enter the calculations for these plots; they are compiled in Table I. The plots start at 3 MeV as below Thomson scattering by the nuclear charge surmounts the IVGDR tail and the zero at $E_c = 0$ in Eq. (11). It should be stressed here again, that the presented ansatz does not aim for a full theoretical understanding of the coupling between the IVGDR to quadrupole modes or other excitations, but only for phenomenological prediction of photon strengths, which is global and hence extendable to many nuclides. In view of the already previously observed [15] "difficulty of accounting for the bump with the aid of a smoothly varying strength function", we clearly distinguish between the Lorentzian
tail and "minor" strength. From the figures this appears to describe existing data at least as well as previous attempts like the ones based on the KMF model [84, 93]. But it implies that for certain energy regions the strength published was observed only because it was significantly stronger than the one outside of them, which we assume to have been buried in the experimental background (e.g. due to Compton scattering in the photon detector).

The nuclei discussed in the following compilation were selected such that photo-neutron data and results for energies below $S_n$ are available; still different $A$, $Z$ and $Q_0$ are well represented. We deliver here a comprehensive collection of the data of interest, most of them were taken from an electronically accessible data base [76], for which they had been extracted from original work. To improve the visibility of the data points, some of the excitation functions have been re-binned to around 0.6 MeV/bin, and points of no significance were suppressed, e.g. when their uncertainty is comparable to the value. The subsequent figures will demonstrate what features of photon strength information can be derived experimentally, and how well systematic trends become visible. For nuclides as light as $^{54}$Fe channels competing to $(\gamma,n)$ cause difficulties for the extraction of the absorption cross section and hence the photon strength. In view of an increase of this problem with falling $A$ the reason for not extending the study to lower $A$ becomes obvious. Photo-dissociation $(\gamma,n)$ channels are added up. The $f_\lambda$ derived [130] at lower $E_\gamma$ from $\gamma$-decay in $^{56}$Fe seem to agree reasonably well with the sum of TLO and considerable extra 'pigmy' strength near 7 - 8 MeV (cf. Table I). But it should be mentioned, that between 6 and 10 MeV two published data sets lie clearly below [131] and above [128], and this is a typical example for problems arising for the kind of survey study as presented here. The observed rise below 4 MeV does not correspond to any prediction considered in the present study, but it may eventually be similar to M1 strength recently predicted for some heavier nuclei [132]. For 11–13 MeV, where the spin-flip M1 strength is expected, no data are available. In $^{78}$Se a significant

![Photo strength function for $^{78}$Se](image)

increase over the extrapolated tail is observed from photo-scattering investigated at the radiation source ELBE for $7 < E_\gamma < 10$ MeV, although these data [83] were reduced by a factor of 0.7. This is a correction for the disagreement of the level density ansatz used in the data analysis [83] with the one published recently [124], which will also be discussed in section IX. The IVGDR data were derived from photo-neutron experiments performed at Saclay [133].

Photon strength functions have been published previously for $^{88}$Sr [13, 124], as well as for $^{92–100}$Mo [97]; a width independent of $E_\gamma$ was used, a good agreement to the TLO prediction is observed and the integrated cross section obeys the TRK sum rule. The similarity between the low energy slopes in the experimental data of all Mo isotopes led to the suggestion [13, 97] of an extrapolation to lower $E_\gamma$ from $\gamma$-decay in $^{98}$Mo seem to agree reasonably well with the sum of TLO and considerable extra 'pigmy' strength near 7 - 8 MeV (cf. Table I). But it should be mentioned, that between 6 and 10 MeV two published data sets lie clearly below [131] and above [128], and this is a typical example for problems arising for the kind of survey study as presented here. The observed rise below 4 MeV does not correspond to any prediction considered in the present study, but it may eventually be similar to M1 strength recently predicted for some heavier nuclei [132]. For 11–13 MeV, where the spin-flip M1 strength is expected, no data are available. In $^{78}$Se a significant

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The data below 9 MeV are from elastic photon scattering sum of three Lorentzians (TLO) as described for Fig. 8. The neutron data [134] (black circles) in comparison to the recently revised data [106, 136] agree around 7 MeV from the uncertainty in level density mentioned above. The various isotopes in the target may cause a widening as compared to the data as shown in Fig. 9 and this may indicate, that the so-called Oslo method may suffer from the uncertainty in level density mentioned above. The recently revised data [106, 136] agree around 7 MeV with the photon scattering results and thus also with the TLO prediction with minor strength added. These new data overshoot at higher $E_\gamma$ and are lower by $\approx 40\%$ near 3 MeV; similar to Fe an increase of strength was observed at low energy and shown to be in accord to a shell model calculation for M1-strength in the nuclides $^{94-96}\text{Mo}$ [132]. New photo-neutron data below 11 MeV [106] with smaller error bars and smooth dependence on $E_\gamma$ replace the older ones [111].

Quasi-elastic photon scattering from natural Sn has been studied long ago [14] at the tagging set up installed at Urbana and "intermediate structure" in addition to the IVGDR tail has been identified. The absorption cross sections were derived from scattering data [14]: their branching correction by inserting constant average resonance widths may overestimate $\sigma_{abs}$ by at most 20% [14]. To improve the overlap with the ($\gamma,\text{n}$)-data a reduction by 0.8 was applied in Fig. 10 and the resulting values are shown together with the cross section for $^{118}\text{Sn}$ ($\gamma,\text{xn}$) [133], obtained with positron annihilation in flight; the surprisingly large strength near 10 MeV may be related to a target admixture of odd isotopes, similar to what was worked out for Mo targets [97]. Recent experiments with laser backscattered photons [100] support such an assumption. From a high resolution photon scattering experiment [119] with correction for branching losses a strength enhancement near 6.5 and 8 MeV was reported for $^{116}\text{Sn}$ and $^{124}\text{Sn}$, similar to what is shown here for $^{118}\text{Sn}$. A recent study of $^{112}\text{Sn}$ and $^{120}\text{Sn}$ [138] uses statistical corrections for inelastic scattering as proposed earlier [120] and again finds similar dipole strength as reported earlier. In addition the last-mentioned investigation has detected by a fluctuation analysis the need to increase the final photon strength by nearly a factor of two with respect to the sum of peaks observed in the spectra and resolved with $\approx 2$ keV (FWHM) energy resolution. Whereas data on $f_\gamma$ derived at Oslo for various Sn isotopes and other nuclides with $^3\text{He}$-induced reactions show significant differences from the photon scattering data shown here, the recent reanalysis for $^{118}\text{Sn}$ [139] does not - as depicted in Fig. 10. The overshoot seen at the higher energies can be explained by the influence of the giant quadrupole resonance and is of no importance for radiative neutron capture.

In Fig. 11 results from photo-neutron emission from natTe, multiplied by 0.9 - as done in general for data from Saclay - agree well to TLO above the IVGDR peak. The various isotopes in the target may cause a widening of it and below the isoscalar component of the GQR is expected from the systematics for this mode; its influence on the photoneutron cross section is not completely clear. The low energy data support the finding of extra strength near $0.42 E_\gamma$, also seen in Figs. 10 and 12 and to some extent in the data below 10 MeV for all nuclei presented here. This "intermediate structure" strength was observed since long [15, 115] by photon scattering in many heavy nuclei - and labeled pigmy strength. It may
partially overlap M1-strength [17] seen in various nuclei. Such peaked extra photon contributes little to the dipole sum in Eq. (10) at variance to a decrease of $\Gamma_{IVGDR}$ with decreasing $E_{\gamma}$, as favoured for long time [77, 84, 123]. Whereas this KMF-ansatz often disagrees to the dipole sum rule, the TLO-approach presented here always obeys it and still agrees to measurements. For $^{138}$Ba scattering data have been taken with quasi mono-energetic photons at the laser backscattering beam at HI\γS [117], whereas a bremsstrahlung experiment at ELBE was performed with $^{136}$Ba [122]. As one can assume that the two data sets result in very similar absorption cross sections, they are shown together in Fig. 12. A possible increase below 3 MeV [122] may be of similar origin as observations in Fe and for $N \approx 50$. Scattering experiments [141, 142] with isoscalar $\alpha$-projectiles have observed enhanced strength at $E_{\gamma} \approx 0.42E_{IVGDR}$ and its gamma-decay directly to the ground state indicating an isoscalar mode [19] which we label as low energy pigny in Table I. For a globally applicable quantification the yield not seen as individual spectral lines in a Ge-detector (looking like quasi-background) has to be taken into account [123].

For most of the nuclides to be discussed in the following ($^{146}$Nd to $^{238}$U) photon strength information for $E_{\gamma} < S_n$ was obtained from individually known branching ratios of gamma transitions following neutron capture via resonances near $S_n$ [15] or by analyzing primary gamma spectra following average resonance capture (ARC) [125] to reach the nucleus in question. The absolute photon widths can be derived from $\gamma/n$ ratios and the resonance widths determined by neutron time of flight (ntof). By an inspection of the gamma-ray angular distributions $\lambda = 1$ is assured, if polarized neutrons are used the decay multipolarity (E1 or M1) is known experimentally [25].

Inserting these widths and the average level spacings into Eq. (14, right) results in $f_\lambda(E_{\gamma})$, but this relies on a known level density $1/D_r$, and according to Eq. (14) this has lead to an increase by 30% [143] in the respective table of RIPL-3, which is used in Fig. 10 for the comparison to predictions on the basis of TLO. The dipole strength from such sources appears in the Figures as an addition to the IVGDR tail, suggesting that strong yield above it was not observable at other energies.

In Fig. 13 the case of $^{146}$Nd is shown and the regard of triaxiality in TLO leads to a reasonable description of data in the region of the IVGDR as well as below. In accord to Eq. (11) and (12) comparatively small $\Gamma_\gamma$ of 2.82, 3.33 and 3.76 MeV are used without a decrease with $E_{\gamma}$. In previous work [31, 66, 77] a single Lorentzian (SLO, $k = 1$) was proposed for $^{146}$Nd together with $\Gamma_{IVGDR} = 5.74$ MeV, also shown in Fig. 13 as red curve. It indicates that such a fit leads to a large resonance width $\Gamma$, as it emphasizes the pole region. Only with a decrease of the IVGDR width with photon energy, as was assumed [31, 95, 144, 145] in KMF-type analyses [31], agreement to the low energy data near 5 MeV can be reached in spite of the large value for $\Gamma$. At even lower energy a component is added to the Lorentzian for $\gamma$-decay calculation which violates the Axel-Brink hypothesis. In $^{144}$Nd and $^{146}$Nd the KMF-model was shown [95] to produce a similarly good agreement to data like TLO as proposed here (cf. Fig. 13), but the latter does not need any modification of the Lorentzian shape. And it works well also for deformed nuclei, what is not the case for KMF in $^{163}$Dy, as found [144] from an analysis of 2-step gamma-cascades after neutron capture. The data from $^{143}$Nd(n, $\gamma\alpha$) [144] [147] with their continuous
strength observed \[146\] below 1 MeV are often quoted as experimental support of the KMF theory – neglecting the fact, that there MI radiation may be favored \[148\], similar to what was indicated for \[149\] the fact, that there M1 radiation may be favored \[148\], as experimental support of the KMF theory – neglecting the tail down considerably, causing a significant effect especially in the region below 9 MeV (see Figs. 10 and 23 and Figs. 1 and 3 of \[95\]). Within TLO this is avoided by having \(\Gamma\) depend on \(E\) only and Eq. (12) causes the two upper poles (see figs. 14 to 17, 20 and 21 (and their sum) to have reduced height albeit all components have equal strength. Full conformance with the TRK-sum – is indicated near closed shells as well as for nuclei with large \(|Q_0|\). In Fig. 15 results of two different experiments on \(^{168}\)Er for the IVGDR range are depicted. The data with the larger error bars were obtained \[150\] by photon absorption with subsequent subtraction of the strongly dominating absorption by the atomic shell, which has to be determined by a precise calculation. As the other data \[151\] stem from a photo-neutron experiment performed with a natural target, deviations might not be related to \(^{168}\)Er. But still the agreement of both to TLO confirms our parameterization, which does not yet include ISS discussed with Fig. 14. The photon strength as observed in photon scattering (figs. 10 to 12, 14, 17 and 18) shows intermediate “pigmy” structure at \(E_\gamma \approx 0.43\) and/or 0.55\(E_{IVGDR}\), if these energies were covered experimentally. Also the \(\gamma\)-decay observed after \(n\)-capture is reasonably well accounted for in TLO (figs. 14 to 17), if an enhancement in the pigmy energy range is accounted for as minor strength, as parameterized in Table I. Its contribution to the summed strength in Eq. (10) is weaker by at least one order of magnitude as compared to the IVGDR sum. Already many years ago, experiments with tagged photons \[113\] have identified a resonance-like structure in the cross section of photon scattering on targets in the vicinity of \(^{208}\)Pb, and the case of Hg is especially significant. In \(^{208}\)Pb the energy of such minor modes lies in a region of small level density and hence
very large spacing between $1^-$-levels and it is intriguing to relate this structure to the strong $1^-$-level at 5.51 MeV \cite{83,150} and eventually weaker ones seen nearby in this region of low level density $\rho(E_x)$. This rather small $\rho(E_x)$ reaching up to the IVGDR range is related to the large shell correction in near-magic nuclei as will be discussed in section \ref{sec:shell_correction}. Thus Porter-Thomas fluctuations have a strong influence on the data not leading to smooth strength function curves as assumed for the approximate prediction used here. For $^{208}$Pb several investigations \cite{87,155,157,158} have explicitly looked at fine-structure at $E_x < 12$ MeV and some fluctuations in excess of the smooth IVGDR slope are seen; these are partly smeared out in the figures. Indications of contributions from the GQR are also observed: unfortunately related electron scattering data \cite{59,159,160} do not allow a fully consistent transfer of information. Inelastic proton scattering \cite{161} indicates a peak at 21.5 MeV which could be either IVGQR or IVGMR, whereas the ISGQR is indicated in figs. \ref{fig:13} to \ref{fig:18}, to partly overlap one or two of the IVGDR components. This excess over TLO contributes less than 5% to Eq. (10) similar to the pigmy addition.
contributing not more. Corrections of photon scattering yields have been performed using inelastic photon scattering data taken at the Urbana microtron laboratory [115], where also the neutron emission directly to low levels of $^{207}$Pb was observed [158]; it is of minor importance for the arguments discussed here. In Figs. 20 and

![Graph](image1)

**FIG. 19.** (Color online) Photon strength for $^{208}$Pb derived from photon scattering data using a quasi-monochromatic beam [115] (blue diamonds), from bremsstrahlung [83] (magenta star symbols) and from two cross section measurements for photoneutron production [153] (black circles), [157] (black x). TLO for the IVGDR is depicted as described for Fig. 8.

21 three sets of experimental data for $^{232}$Th and $^{238}$U, respectively, in the range of the IVGDR are displayed together: The data with the large error bars were obtained by photon absorption with subsequent subtraction of the dominating atomic absorption, which had to be determined by a precise calculation. They agree within uncertainty to data stemming from a photo-neutron experiment performed at Saclay, which were reduced here by 10%, as explained before. The agreement is not perfect, but indicates the reliability of both in the IVGDR regime. The agreement between two data sets is important with respect to the disagreeing data obtained at Livermore [162]. These cross sections for $^{232}$Th and $^{238}$U are exceptional large in the sense, that an analysis on the basis of Eq. (10) indicates an overshoot of $\approx 30\%$ as compared to the TRK sum. One or even two pigmy modes seem to be present in photon scattering by all nuclei with $A > 70$ regarded here. For lower energies the scissors M1 mode was predicted [17] to also be a general feature in all heavy nuclei, but in photon scattering it is difficult to separate from the E1 strength originating from quadrupole-octupole coupling. Results for a strong presumably magnetic mode presented recently [165] suffer from the missing parity assignment of the strength observed as well as from a questionable choice for the IVGDR tail. An identification as M1 is well possible by comparing e- and $\gamma$-scattering [166] and respective results indicate some difference to data from gamma decay after inelastic deuteron scattering [165] suggesting a need for further study. For all nuclei presented here a quite good agreement between observations and the TLO prediction for E1 (plus minor) strength is found. TLO relies on the
classical sum rule (TRK), the IVGDR mode energies $E_i$ are derived from the droplet model and their width $\Gamma_i$ is varying with it and hence slowly with $A$ and $Z$. There is no dependence of it on $E_i$, but on the three pole energies $E_n$, which are derived from the deformation dependent axis parameters determined by CHFB calculations \[1\]. There is one deficiency apparent in these: In nuclei with or near closed shells the calculations predict more deformation \[53\] as deduced from $B(E2)$-values and, as found within the study presented here, in discord to the deformation induced splitting of the IVGDR. For closed shell nuclei a small deformation (reduced by a factor 0.4 to 1 as discussed together with Fig. 6) results in a better description of the IVGDR peak shape; this reduction has no significant influence on the strength in the tail region, which will be shown to be essential for radiative capture. It is worth repeating here that the experimental data below $Sn$ for isotopes used as an argument for parameterizations other than TLO, if the extra ‘minor’ strength is added. Also an account for the variance of the deformation parameters by instantaneous shape sampling \[90\] only leads to small by TLO, if the extra 'minor' strength is added. Also an account for the variance of the deformation parameters by instantaneous shape sampling \[90\] only leads to small changes of the calculated strength in the IVGDR for deformed nuclei: The resonances are widened near the peak region, but the low energy tail remains unchanged. In closed shell nuclei and their neighbors the low level density induces narrow variations in the cross section data up to energies near $Sn$. These are eventually identified as one type of pigny resonances \[168\]; their contribution to the summed cross section stays below a few per cent. In general, the following conclusions can be drawn from the comparison of experimental data for 20 nuclei in the mass range from 50 to 250 to Eqs. \[11\] and \[13\]:

(a) The centroid IVGDR energies derived from droplet model fits to masses \[71, 160\] are in accord to the data, when an effective mass $m_eff c^2 = 800$ MeV and the neutron radii as predicted from the CHFB calculations \[1\] are used.

(b) There is no indication of a strong departure from the classical dipole sum rule \[53, 54\] if the yield at energies above the IVGDR is assigned to the quasi-deuteron strength. Previously reported excess may be due to overestimated resonance widths resulting from an ad hoc limitation to spherical or axial symmetry.

(c) The resonance widths vary only smoothly with $A$ and $Z$ and depend on the resonance energies via a power law (cf. Eq. \[12\]), predicted by hydro-dynamical considerations \[74, 85, 87\] and supported by experiment \[73\].

(d) The IVGDR data together with data obtained with different methods for lower energies do not allow for a strong decrease of the width with photon energy, as previously postulated \[84\]. Higher width values \[60, 67, 77\] compensate such a decrease and cause large strength at low energy.

(e) Only by allowing broken axial symmetry, three rather narrow resonance parts add up to a wide structure with a satisfactory large breadth of the IVGDR; this indicates the importance of calculations \[1\] or other input \[13\].

(f) Experimental data may suffer from missing exact information on detection efficiency as well as on beam resolution, eventually causing discrepancies between measurements at different laboratories– as was observed for $^{232}$Th and $^{238}$U.

(g) The full recognition of the Axel-Brink hypothesis and the TRK sum rule together with the independence of the IVGDR width from the photon energy are fundamental for our TLO-ansatz.

The comparisons of a very similar approach to data for a number of isotopes as published previously \[13, 72, 81, 83, 96, 97, 99, 114\] can be considered part of the present work, such that a quasi complete sample of IVGDR data in heavy nuclei are shown to be well described by TLO. A description of the IVGDR in deformed nuclei \[170\] not using the TRK sum rule and not based on a fully self-consistent calculation of the shape parameters was by far less successful in its predictions. The global TLO ansatz differs from locally adjusted Lorentzians presented previously \[14, 66, 106\] and thus leads to other conclusions concerning eventual excess of strength in the tail of the IVGDR.

### VII. SPECIAL CONSIDERATIONS FOR ODD NUCLEI

Concerning neutron s-wave capture on $J = 0$ targets it has been shown \[124\] that the TLO ansatz also works for the odd nucleus $^{89}$Y. When an average over the axis lengths for even neighbor nuclei is used to obtain $f_{E1}$, a good description for the IVGDR region is obtained. It was assumed, that for $J_0 \neq 0$ photon absorption into a mode $\Lambda$ populates $m$ members of a multiplet with $m=\min(2\lambda+1, 2J_0+1)$ and the decay widths to the ground state $\Gamma_0$ are equal for each member of the multiplet; the conditions for the validity of Eqs. \[13\] and \[14\] are thus fulfilled. The strength observed corresponds to the cross section summed over the multiplet and this can be described by an effective $g$, which according to Eq. \[15\] is:

$$g_{eff} = \sum_{r=1,m} \frac{2J_r + 1}{2\lambda_0 + 1} = 2\lambda + 1$$

This ansatz is valid in heavy nuclei \[13\] as it relates to the condition of weak coupling between the odd particle and the mode $\Lambda$. The TLO-calculations for odd-Å nuclei as shown in figs. \[22\] to \[24\] were performed on the basis of Eqs. \[11\] and \[13\] with $k = 3$. No extra spin dependent factors are needed and agreement to the experimental
data is found to be similar as in the case for even nuclei, also in the tail region below $S_n$. In Fig. 22 data for $^{133}$Cs are shown to be close to those for neighbouring even nuclei depicted in Figs. 12 and 13. The agreement to TLO is obvious and also "minor" strength as pigmy and the IVGQR are seen. For the nucleus $^{197}$Au not only the missing strength as compared to a single Lorentzian used there, the agreement is reasonable for TLO, when the discontinuity near 19 MeV is related to the known incorrect separation of the 2n-channel. The inclusion of triaxiality in TLO leads to a reduction of $\Gamma_i$ and thus of $\sigma_{abs}$ for sufficiently large $(E_i^2 - E_\gamma^2)^2$ in Eq. (11). The widths $\Gamma_i$ used previously are 2.9 and 4.0 MeV and an additional factor of 1.22 was obtained as compared to the TRK-sum rule. This factor is 1.0 for TLO and the values for $\Gamma_i$ are 2.7, 3.0 and 3.5 MeV. When $^{197,198}$Au is considered spherical $\Gamma \approx 4.5$ MeV results from a SLO-fit and this effects largely the strength predicted in the tail region. Also for $^{197}$Au the use of the KMF-model was proposed and the more satisfying agreement of TLO as presented in Fig. 24 again favours our ansatz over this model, as already observed for $^{146}$Nd, Fig. 13. In the bombardment with photons $^{239}$Pu mainly undergoes fission and the weaker neutron emission channel has to be added to obtain $\sigma_{abs}$. In Fig. 25 the result of this sum of Livermore data is compared to direct absorption data and a reasonable agreement is seen, as well as a good agreement to the TLO-prediction. This is remarkable in view of the disagreements depicted in Figs. 20 and 21 for the near neighbors $^{232}$Th and $^{238}$U and doubts about these older data seem justified.

Together with the agreement between $^{88}$Sr and $^{89}$Y reported recently the examples presented in this section support the TLO ansatz for the derivation of photon strength in odd nuclei. Our assumption, that the damping widths $\Gamma_i$ do only depend on $E_i$ and not on $E_\gamma$ clearly is at variance to previous proposals made for the electric dipole strength function and recently made modifications of this work. But we will show, that our new
FIG. 25. (Color online) Photon strength for $^{239}$Pu (black dashed curve: TLO; full blue curve: minor strength added to TLO). Data are derived from summing cross sections of fission and neutron emission induced by quasi-mono-energetic photons (black circles) and by discrete $\gamma$-rays from neutron capture (magenta stars on the low energy slope). The absorption data from [163] (black x) were obtained with bremsstrahlung.

ansatz allows to calculate average radiative strengths and neutron capture cross sections which compare well to observations.

VIII. PHOTON STRENGTH OF OTHER CHARACTER THAN ISOVECTOR ELECTRIC DIPOLE

The height of the low energy tail is nearly proportional to the IVGDR width and it depends weakly on its deformation induced splitting. A consequence hereof is a nearly full independence of the strength on any free parameter. But literature on photon scattering in this energy range has pointed out the possible importance of strength of other character than isovector electric dipole, as recently reviewed [179]. In the following also work published more recently is included and the discussion will be generalized to electromagnetic strength of different character. Even if it has minor importance for photon absorption and the sum rules, it may influence the decay of the compound nucleus and consequently also the reaction cross sections. Strength eventually adding to the IVGDR tail is characterized by approximate expressions listed in Table I and its eventual importance will be investigated. A direct relation exists between ground state transition widths, summed within an interval $\Delta E$, and the $f_{E\lambda}$ (in GeV$^{-(2\lambda+1)}$) as derived from Eqs. [13].

Strength information can hence be obtained by summing spectroscopic width data (in MeV) over a given energy range $\Delta E$ (also in MeV):

$$f_{\lambda}(E_\gamma) = \frac{1}{\Delta E} \sum \frac{\Gamma_{E\gamma}}{E_\gamma^{2\lambda+1}} = \frac{I_{\alpha\beta}}{(\pi hc)^2 g_{\gamma ff} E_\gamma^{2\lambda+1} \Delta E} \tag{16}$$

The second part of Eq. (16) allows to relate the strength function to integrated absorption cross sections $I_{\alpha \beta}$; these can be directly related to sum rules.

Electric dipole strength below the IVGDR

A low energy dipole strength was predicted to be formed by E3 strength coupled to low energy quadrupole modes [11]. In many even-even nuclei rather strong photon absorption into 1$^+$-levels with 1MeV $< E_x < 4$MeV has been observed [174] and a correlation of $E_x$ to the sum of the excitation energies of the low collective 2$^+$ and 3$^-$ modes was established. A systematic dependence of this dipole strength on $A$ and $Z$ has not been registered, probably due to possibly destructive interference between different contributions [175]. A functional dependence proposed for the 2$^+ \times 3^-$-mode from theoretical arguments was further specified [18, 176] and the B(E1)-values used here are in the range of observations in nuclei of various degrees of deformation. Photon scattering studies show similar strength for odd and even nuclei [174], but fragmentation away from closed shells and especially in odd nuclei cannot be excluded; it may cause a missing of small strength components. To estimate the influence of this low energy dipole mode on the photon emission we approximate it by a Gaussian distribution with parameters listed in Table I. The centroid energy $E_{q0}$ is taken as the sum of the energy of the 1st 2$^+$-level [12] and the corresponding value for the octupole (3$^-$) excitation [177]; theoretical approximations for exotic nuclei are available [178].

In principle Eq. (16) allows the inclusion of numerous recently published results for strong peaks not far below $S_{q0}$ near 7 MeV. In case an assignment to E1 is made, this strength is often denoted as PDR (‘Pygmy Dipole Resonance’) [19, 179], apparently in analogy to Bartholomew’s “pigmy resonance” in strength functions [13]. Such bumps were assigned to the “vibration of excess neutrons against a proton–neutron core” [180], or also as “an integral part of a toroidal E1 mode representing an example of vortex collective motion in nuclei” [179]. “Intermediate structure” in photon scattering was discussed in detail [80] and the concept of photon strength functions $f_{\lambda}(E_\gamma)$ was then introduced. At energies approaching $S_{q0}$ from below an extraction of $f_{\lambda}(E_\gamma)$ from photon scattering data becomes questionable as competing channels suppress the emission of elastic photons and often unobserved decay branches (inelastic photon scattering) are not easily accounted for. This may fake a fall-off which, together with the rise when approaching the IVGDR, appears as a peaking distribution. The strength in that energy range identified in the isotopes $^{112–124}$Sn...
by high resolution $\gamma$-spectroscopy was used to estimate the full dipole strength in the “PDR” by accounting for additional quasi-continuum contributions and the loss due to branching. Its peak energy increases weakly with decreasing $A$, but a clear change of the integrated strength is not observed even when combining the observations with data for $^{138}$Ba, $^{140}$Ce and recent work for Xe-isotopes $^{122}$. The analysis of observations on seemingly isolated peaks in gamma spectra deserves extra care: Porter-Thomas fluctuations may shan structures in data taken with a resolution larger than the level distance and thus not sufficient to resolve all levels. \textit{Sampling will give many small numbers and a few large ones and this is ideally suited for providing the pronounced peak structure that makes for the happiness of every dyed-in-the-wool spectroscopist.}

In a few earlier papers this problem was addressed experimentally with special care in the photon detection and strength data were published for structure seen in the region between 5 and 6 MeV with photon scattering by Zr and Sn and as well as for $A \approx 200$. As mentioned there, the contribution of the quasi-continuum below the lines is significant even after the real background due to unwanted radiative processes in the detector and the near-by environment has been quantified and subtracted. In the limit of $E_{\gamma} \to 0$ the strength $f_{E1}$ shold vanish as suggested by the well-known fact, that nuclei do not have an electric dipole moment. Unfortunately Thomson scattering dominates at long wavelengths and obscures resonant scattering processes. Fragmented parts of the electric dipole strength outside of the IVGDR may have isovector or isoscalar character and a distinction by experiments with isoscalar beams was proposed. This approach is not unambiguous, as only in collective models based on one incompressible fluid isoscalar electric dipole strength is isospin forbidden as it would correspond to a spurious motion of the centre of mass. Hints for the isoscalar character of electric dipole strength may indicate non-uniform proton-neutron distributions or compressional modes. Strength between $E_x \approx 5.5$ MeV and the neutron separation energy $S_n$ was shown to be of isoscalar nature in $^{40}$Ca, $^{58}$Ni, $^{90}$Zr and $^{208}$Pb by the coincident observation of inelastically scattered $\alpha$-particles and de-excitation $\gamma$-rays.

In the present study the various results reported for this energy range are considered an addition to the TLO-extrapolation and tentatively separated into two components named low and high energy pigmy mode (PM) in Table II. The parameters given there are selected such that the total strength included may be understood as an upper limit. In figs. 7 to 25 the low energy PM (a seemingly resonant strength near $0.43 \times E_{IVGDR}$) appears to be of similar magnitude for many different $A$, when compared to the IVGDR. Hence this intermediate structure resembles a prediction made for $^{208}$Pb. In a three-fluid hydrodynamical model of heavy nuclei, an oscillation of the excess neutrons against a core with $N = Z$ was shown to result in a peak of energy $E_x \leq 0.45E_0$, with $E_0$ valid for the full vibration of the neutrons against the protons, the IVGDR and $^{70}$. Experimental “evidence for a 5.5-MeV radiation bump”, an intermediate structure then named “pigmy” suggests our nomenclature high energy PM. Independent of an interpretation of these PM's valid for all heavy nuclei a Gaussian seems justified for both and the parameters used for them are given in Table II. These values were selected to clearly overpredict most available data after the TLO-integral over the energy interval is subtracted. They are purely phenomenological and independent of isospin and of models like the ones proposed to eventually explain extra strength below $S_n$.

\textbf{Magnetic Dipole Strength}

For both dipole modes (E1 and M1) Eq. (16) can be used together with the generalization of Eq. (9) to gather strength information from spectroscopic data. Here, the squared reduced matrix elements are inserted, which are direction independent in contrast to the still often used $B(E1, M \lambda)$ values. They are, expressed in $\text{fm}^2$, obtained from:

$$\Gamma_r(E_\gamma; E1, M1) = \frac{16\pi}{9} \frac{\alpha c E_\gamma^3}{g(\hbar c)^2} \langle r|E1, M1||0 \rangle^2$$  \hspace{1cm} (17)

The photon strength between nuclear ground states and the IVGDR region is predominantly of electric dipole (E1) character and also in the tail region around 5 MeV M1 decays are weaker as compared to E1. Magnetic (M1) spin flip strength occurs at higher energy than collective orbital magnetic strength (scissors mode), which is strong in nuclei with large quadrupole moment and possibly contributes to radiative capture. Both are built either on ground states or on excited levels and then contribute to the strength on top of them in the sense of the Axel-Brink hypothesis. High resolution photon scattering data as e.g. published by show spectral details indicating low level density, and these are especially significant in near magic nuclei. Again weak components at low energy are likely to be hidden in a quasi-continuum partly due to experimental background. Combined measurements of transition rates and polarization in the gamma decay of compound nuclei have been performed e.g. for $A = 79$ and the strength of low energy M1 transitions has been determined to be below that for E1. But measurements are rare for statistically significant information on E1 and M1 transitions in one nucleus. A possible way out is to average over many nuclei and a suitable compilation is available and updated regularly. The sorting of nearly 3000 transitions from compiled data for $80 < A < 180$ has resulted in the plot presented in Fig. 26. The M1-strengths $f_{M1}(E_\gamma)$ are apparently considerably larger than the E1-strengths $f_{E1}(E_\gamma)$ for $E_\gamma$ below 1 MeV whereas above the E1 strength becomes stronger up to the region of the
IVGDR. A direct measurement of the transition strength contributing to the decay width in the high level density region near $S_n$ was possible by using experimental information gained in $^{144}$Nd(n,$\gamma$)$\alpha$ experiments [146, 147]. In view of Fig. 20 this “low energy” strength is likely to be mainly M1 as pointed out before [148], but E1 was occasionally assumed to be the main component [95]. In any case the summed dipole strength function $f_1$ in this mode was shown [132, 140] to be small and concentrated at very small $E_\gamma$. Although such a low energy M1-component has been used [95, 184] eventually, we consider it unimpor-
tant for the prediction of radiative n-capture: The compilation of experimental data shown in Fig. 20 de-
sects a significant decrease for $E_\gamma \geq 1$MeV and for lower energy the cross section will be shown to be suppressed by the level density in the final nucleus. Recently very low energy M1 transitions resulting from orbital rearrange-
ment have been predicted by shell model calculations for Mo-isotopes [132] to eventually compete to E1 below 3 MeV.

In a recent review [17] the properties of three components of magnetic dipole strength have been described in detail: For energies between 2 and 4 MeV orbital M1 strength has been observed, and it has been shown that with increasing ground state deformation $\beta$ this orbital scissors mode reaches an integrated strength proportional to $Z^2\beta^2$ at an excitation energy $0.21E_{IVGDR}$, in agreement with data reviewed recently [187]. The enhancement of gamma decay between high lying levels as ob-
served in two step cascades [128, 145] was attributed to orbital scissors mode of about the strength seen by photon scattering. In addition to it a double humped structure was seen at higher energy (around $41A^{-1/3}$ MeV) and identified [17] as isoscalar and isovector spin-flip M1 modes. This is in agreement to the strength observed in heavy nuclei with polarized neutrons [95]. Including both regions at $\sim 34A^{-1/3}$ MeV and $\sim 42A^{-1/3}$ MeV [17] the energy weighted sum can be calculated within the energy range $\Delta E$ with reasonable accuracy; it reaches $\leq 2$ MeV fm$^2$, as the $A$-dependence of peak energy and peak integral approximately compensate each other [17]. The resulting estimate is smaller by at least 2 orders of magnitude as compared to the main component of the GGT sum rule (eq. 5.6 of [53]). This prediction was derived from very general arguments for the total photon strength in heavy nuclei, i.e. including all multipolarities. In Table I the estimates used by us are given; the scissors mode is considered as well as the two spin-flip M1 components, which are becoming important at higher photon energy. Polarization data are necessary to identify magnetic strength unambiguously and obser-
vations made with polarized photons on $^{88}$Sr [121] and $^{90}$Zr [188] were performed at two different laboratories and with two different techniques: off axis continuous bremsstrahlung and tagged photons. A surprisingly high magnetic strength was recently used in calculations for Zr-isotopes [105], although it superseded the systematics [17] by a factor of $\approx 3$. Combined to a rather low E1-strength as derived from a HFB+QRPA calculations [184] a suitable agreement to capture data results; in view of the M1-enhancement needed the underlying E1 model may have to be questioned.

Electric quadrupole modes

Low energy E2- transitions can be considerably enhanced and have played an important role in the spec-
troscopy of heavy nuclei – as discussed in sections II and III. The photon absorption cross sections can be de-

erved from Eq. 8; they have been found to be small as compared to E1 absorption [116] in the energy re-

dion near $S_n$. In figs. 7 to 25 $f_{E2}$ was multiplied with $E_{\gamma}^2$, the additional energy dependent phase space factor for absorption and decay, to allow a visual comparison to $f_1$; but for the calculation of the integral $I_{E2} = I_c$ (as listed in Table I) the correct decay width is used in Eq. 8. Extending previous work [12, 51, 116, 190] the influence of quadrupole giant resonance (QDR) contribu-
tions to photon absorption was investigated; information for quadrupole strength comes from sum rules and theo-

erical predictions [10, 116, 191] adjusted to electron scattering data [53, 60, 192]. The isoscalar ISGQR lies not far from the pole of the IVGDR such that it adds to the strength observed there. Strength modifications due to the IVGQR appear to be important, as seen in figs. 10 to 23 , and result in a difference from TLO at higher energy. The influence of quadrupole modes at even lower energy is investigated by assuming 5% of the ISGQR integral $\int \sigma_{abs}dE$ to be downshifted by a factor of three in energy. The approximations used in Table I for the integrals of the E1, E2 and M1 components were
TABLE I. Parameters of an upper limit for three minor electric and three magnetic dipole modes to the dipole strength function to be calculated with a Gaussian (Eq. 19). A very similar shape near the peak is reached for a Gaussian with a rms breadth $\sigma$, which is larger by a factor 2.5 as compared to the width $\Gamma$ of a Lorentzian. Three GQR modes are also listed, which are seen in photon absorption as well; they are proportional to the square of the nucleus’ charge radius $R_c$ (rms).

| Component                     | multipolarity | $E_c$ (MeV) | $I_c$ (fm$^2$MeV) | $\sigma_c$ (MeV) |
|-------------------------------|---------------|-------------|-------------------|-----------------|
| low $E_x$ pigmy mode          | E1            | 0.43$E_0$   | $7 \frac{2(\frac{1}{2}-1)}{Z}$ | 0.6             |
| high $E_x$ pigmy mode         | E1            | 0.55$E_0$   | $13 \frac{2(\frac{1}{2}-1)}{Z}$ | 0.5             |
| $0^+ \leftrightarrow (2^+ \times 3^-)_-$ | E1            | $\frac{160}{3} (1+\frac{40}{27})$ | 0.006$ZA\beta$ | 0.6             |
| orbital (scissors) mode       | M1            | 0.21$E_0$   | 0.03$ZA\beta$     | 0.4             |
| isoscalar spin-flip           | M1            | 42$A^{-1/3}$ | 17                | 0.8             |
| isovector spin-flip           | M1            | 47$A^{-1/3}$ | 27                | 1.3             |
| low $E_x$ quadrupole          | E2            | 19$A^{-1/3}$ | 0.1 $\frac{\alpha_s(\pi R_c E_x)^2}{[\frac{3A}{4\pi}]^{1/2}}$ | 1.0             |
| ISGQR                         | E2            | 63$A^{-1/3}$ | 2 $\frac{\alpha_s(\pi R_c E_x)^2}{[\frac{3A}{4\pi}]^{1/2}}$ | 1.0             |
| IVGQR                         | E2            | 48$A^{-1/6}$ | $\frac{\alpha_s(\pi R_c E_x)^2}{[\frac{3A}{4\pi}]^{1/2}}$ | 1.8             |

The values listed in the Table are upper limits deduced from experimental information $[17, 121, 188]$; this mode with its elevated energy has a reduced influence on radiative neutron capture. In contrast, the electric dipole strength due to the phonon coupling of the type $(2^+ \times 3^-)_-$ is of more importance; but it is difficult to quantify it in a scheme which covers the full range of nuclides in question, and thus our predictions clearly have some uncertainty. It also is mentioned, that for transitions between two high lying levels E1, M1 and E2 transitions may be important that are not accessible by studies with photon or electron beams.

IX. LEVEL DENSITIES

Intrinsic state density

Nuclear level densities $\rho(E_x, J^x)$ determine the final phase space for predictions of compound nuclear cross sections and decay rates. Within the RIPL project 31 various attempts were reviewed aiming for a simultaneous fit to data for s-wave neutron capture mean spacings and low energy discrete levels. It was shown that a parameterization based on Fermi gas formulae (modified to finite systems) leads to unsatisfactory predictions, but no mention of broken axial symmetry as possible cause can be found. In calculations based on fundamental principles, of interest especially if they need few free parameters only, a clear distinction has to be made between the quasi-particle state density $\omega_{qp}(E_x)$ and the level density $\rho(E_x, J^x)$ in the observer’s system. For the calculation of $\omega_{qp}$ on an absolute scale we use an analytical approach. It avoids as far as possible parameter adjustments and strongly relies on statistical laws for a Fermi gas - a system of independent spin 1/2-particles with weak mean attraction. As shown long ago $[194]$ it is characterized by a gap $\Delta(t)$ falling with rising temperature parameter $t$ down to 0 at a ‘critical’ temperature $t_{ct}$.

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[Link to the full document]
second order \(195-197\), governed by pairing. In infinite nuclear matter (nm) the backshift energy \(E_{bs}\), which describes a shift between the Fermi gas zero and the nuclear matter ground state, is equal \(196\) to the pairing condensation energy \(E_{con}\), defined below in Eq. (23). An application of text book statistical physics \(195\) to nuclear matter allows to evaluate the general features of this phase transition, and all effects appearing in infinite nuclei can be treated micro-canonical \(196\). Following earlier work \(198\) and a recent data analysis \(193-197\), the energy dependence of the state density is assumed to be exponential in the pairing dominated phase below the phase transition point. This region is hence approximated by the assumption of quasi constant temperature \(T_{ct}\), to be distinguished from the Fermi gas temperature \(t\). It will be discussed below, how these phenomenological parameters and \(\omega_{qp}(0)\) are found for the nuclei of interest.

If only quasiparticle excitations are considered the total state density (in the intrinsic frame) \(\omega_{qp}(E_x)\) at excitation energy \(E_x\) is approximated by:

\[
\omega_{qp}(E_x) = \omega_{qp}(0) \exp\left(\frac{E_x}{T_{ct}}\right) \quad \text{for} \quad E_x < E_{pt} \tag{19}
\]

\[
\omega_{qp}(E_x) = \frac{\sqrt{\pi} \exp(2\sqrt{\alpha(E_x - E_{bs}))}}{12\alpha^{1/4}(E_x - E_{bs})^{3/4}} \quad \text{for} \quad E_x \geq E_{pt} \tag{20}
\]

At the phase transition energy – corresponding to \(t_{pt}\) and given by \(E_{pt} = a_{nm} \cdot t_{pt}^2 + E_{bs} - \) a transition from a Fermi gas like behavior above to a pairing dominated regime below the transition point occurs. For the Fermi gas phase of infinite nuclear matter (nm) the “level density parameter” \(a_{nm}\) is required to be inversely proportional to the Fermi energy \(E_F\). It also determines the energy \(E_{con}\) of the pairing induced condensation \(31, 186, 196, 193, 200\). For \((E \geq E_{pt})\) in finite nuclei, the backshift \(E_{bs}\) stands for the energy between the Fermi gas zero and the nuclear ground state as in nuclear matter, but it has to be corrected for shell effects. In addition a surface term \(\delta a\) has to be added to \(a_{nm}\) to obtain a level density parameter \(\tilde{a}\) for a given \(A\). Using \(\alpha\) in Eq. (21) to quantify it, a comparison to capture resonance spacings finds \(\alpha\) to be as small as \(\alpha = 0.06\), when treated as a parameter, globally valid for all \(A\) under regard – actually the only one in the present fit to neutron capture data:

\[
a_{nm} = \frac{\pi^2 A}{4E_F} \approx \frac{A}{15} \tilde{a} = a_{nm} + \delta a; \quad \delta a = \alpha A^{2/3} \tag{21}
\]

\[
E_{con} = \frac{3}{\pi^2} a_{nm} \Delta_0^2 \quad E_{bs} = E_{con} - \delta E(Z, A) \quad E_{pt} = \tilde{a}_{pt}^2 + E_{bs} \tag{22}
\]

Various approximations are required \(124\); they are listed here albeit not all of them are of significant influence for the conclusions made later:

1. The pairing parameter \(\Delta((E_x = 0))\) is approximated by \(\Delta_0 = 12 \cdot A^{-1/2}\), independent of angular momentum.
2. \(\Delta_0\) is used for neutrons and protons and thus independent of neutron excess \(N - Z\).
3. Quasi-particle states are evenly spaced (on average) at the Fermi energy, not varying with \(N - Z\).
4. Fermi energy \(E_F = 37\) MeV and nuclear radius \(R = 1.16 \cdot A^{1/3}\) are independent of \(N - Z\); they control \(a_{nm}\) for nuclear matter and the phase transition \(193, 196\) at excitation energy \(E_{pt}\).
5. A direct dependence of equilibrium deformation on excitation energy \(E_x\) and angular momentum \(J\) is neglected beyond its implicit influence via \(\delta E(Z, A)\) in Eq. (23).
6. A factor \(1/4\) accounts for \(R_{\pi}\)-invariance \(201\).
7. The influence of shell effects is controlled by \(\delta E(Z, A)\), extracted by subtraction of the experimental mass from liquid drop values, following ref. \(203\).
8. At variance to previous work \(31, 186, 199, 200, 202\) the shell correction term is directly applied to the backshift energy \(E_{bs} \quad 198, 204, 203\). No dependence on deformation is included.

The expression given by Eq. (20) for \(\omega_{qp}\) in the Fermi gas regime – initially derived neglecting pairing \(6, 198\) – is in infinite Fermionic systems a good approximation for the formalism derived with a thorough (micro-canonical) \(196\) inclusion of pairing, if \(E_x\) is back-shifted by the condensation energy \(E_{con}\). In finite nuclei the back-shift \(E_{bs}\) as given in Eq. (23) is the sum of the pairing term \(E_{con}\) approximately independent of \(A\), and the effective shell correction \(\delta E(Z, A)\) which includes the odd-even mass difference. The intrinsic (quasi-particle) state density \(\omega_{qp}(E_x)\) for the Fermionic region as well as for \(E_{pt}\) are given by Eqs. (20) to (23) Below \(E_{pt}\) an interpolation of \(\omega_{qp}(E_x)\) to the ground state is found by Eq. (19); the simple approach of a logarithmic interpolation in analogy with an exponential increase of \(\omega_{con}((E_x))\) is in agreement to observations \(31, 187, 193, 197, 198\).

We fix \(E_{pt}\) using \(\tilde{a}, t_{pt}\) and \(E_{bs}\) and the requirement of a continuous transition in \(\omega_{qp}(E_x)\) at \(E_{pt}\) to determine \(T_{ct}\). In principle \(\omega_{qp}(0)\) can be fixed locally by regarding known spectral data at low \(E_x\), similar to what has been done previously \(183, 122, 190\). In a less stringent way we used as a first global approximation just above the ground state \(\omega_{qp}(0) = 0.1/\Delta_0\). The corresponding effect is demonstrated below with the predictions for average radiative widths. Eq. (20) shows, that a temperature parameter \(t = \sqrt{(E_x - E_{bs})/\tilde{a}}\) for a Fermi-gas approximated \(3, 196, 198\) at the saddle point of the Laplace transform, differs from an apparent nuclear temperature \(T_{app} = \frac{1}{R_{\pi}}\) as well as from \(T_{ct}\), which turns out to be
smaller than \( T_{\text{app}} \) by up to 35\%. The resulting sudden change in the slope of \( \omega_{qp}(E_x) \) at \( E_{pt} \) (cf. Fig. 28) is acceptable for a second order phase transition. Close to magic nuclei the large (negative) shell correction results in a then large energy \( E_{pt} \) for such a break. It is remarkable that in weakly bound nuclei (e.g. near the edge of stability) the Fermi gas phase starts already below \( E_x = 3 \text{ MeV} \). It can be seen in Fig. 27 that for more than 100 of the 146 nuclei investigated here \( E_{pt} \) is smaller than \( S_n \) and thus the neutron capture resonances fall into the Fermi gas regime, but the subsequent gamma decay mostly ends below \( E_{pt} \) and the level density there is important for the prediction of radiative capture cross sections. In the Figure it also becomes obvious, that in the Fermi gas regime \( \delta E(Z,A) \) is closely correlated to \( E_{pt} \) and thus also to \( \omega_{qp}(E_x) \); it is hence a quantity of large importance for a level density prediction and \( S_n \) plays a minor role, at variance to a previous assumption \cite{207}.

The quantities to be compared to observed level spacings have to be derived from \( \omega_{qp}(E_x) \) by a projection on angular momentum \( J \) in the observer system. The proposal was made \cite{6, 198, 206, 207} to consider the m-substate distribution of \( \omega_{qp}(E_x) \) as a Gaussian with width \( \sigma \) around \( m = 0 \) and to differentiate at \( m = J+1/2 \) with respect to \( m \). This leads to a spin dependent level density \cite{6, 31, 167, 186, 196, 199, 201}:

\[
\rho_{\text{ph}}(E_x, J^\pi) \approx \frac{2J+1}{2\sqrt{8\pi}\sigma^3} \exp \left( -\frac{(J+1/2)^2}{2\sigma^2} \right) \omega_{qp}(E_x)
\]

\[
\rho_{\text{small}}(E_x, J^\pi) \approx \frac{2J+1}{2\sqrt{8\pi}\sigma^3} \omega_{qp}(E_x) \text{ with } \sigma = \sqrt{\frac{3J}{h^2}} \quad (23)
\]

The spin dispersion \( \sigma \) can be related to the nucleus’ moment of inertia \( \mathfrak{I} \), often assumed to be the rigid rotor value \cite{6}. This redistribution of the quasi-particle states into levels of distinct spin implicitly assumes \cite{207} the nucleus to be exactly spherical symmetric even at \( E_x \approx S_n \). This neglects strongly mixed modes which, due to their collectivity, are pulled from their original quasi-particle energy down into the low excitation region. Without assuring spherical symmetry for the nuclei studied, Eq. (23) has found a widespread use \cite{31, 196, 198, 201, 203, 204}. In some work the rotational collectivity present in an axially symmetric nucleus was included at this stage as extra term \cite{6, 201, 205–207}, yielding a level density enhancement by a factor \( \sigma^2 \) \(( \approx A/5 \) as compared to Eq. (23). But still an agreement with observations was not reached without a significant enlargement of \( \sigma \) as compared to \( a_{nm} \) \cite{6, 31, 122, 206}; often it was adjusted in a fit.

**Collective enhancement**

It is obvious \cite{194, 201} that the absolute scale of any prediction for the level density is strongly influenced by breaking spherical symmetry in the Fermi gas regime and at \( E_{pt} \). In previous work \cite{31} deficits in the comparison to resonance spacings observed at the neutron binding energy \( S_n \) have been compensated not only by an increase of the level density parameter \( \tilde{a} \) \cite{31}, but also by an excitation energy dependence of it. Both measures are not supported by a ”standard theory” of a Fermi gas, which we accept as proper description of the statistics in highly excited nuclei. We will show by a comparison to experimental data that major modifications of \( \tilde{a} \) are not needed when giving up the scheme valid for the case of spherical or axial symmetry. An inclusion of the collective enhancement of \( \rho \) by breaking axial symmetry follows a proposal made long ago \cite{201} and also presented in textbook of Bohr and Mottelson \cite{6}, cf. Eq. (4-65b). One obtains from this an astonishingly simple expression for small \( J \), which to our knowledge was not taken up in any subsequent comparison to data; a factor 1/8 for \( \mathfrak{I} \)-invariance and parity conservation leads to:

\[
\rho(E_x, J^\pi) \xrightarrow{\text{small } J} \frac{2J+1}{8}\omega_{qp}(E_x) \quad (24)
\]

In view of the triple split observed in the IVGDR of nearly all nuclei with \( A > 60 \) – as described above – it is quite obvious to account for triaxiality in the enhancement of level densities. It results from the build-up of a rotational band on each intrinsic quasi-particle state: The total level spectrum, for a given angular momentum, is therefore obtained by summing over a set of intrinsic states rather than by a decomposition of the level spectrum, as for a spherical system \cite{201}. For an equilibrium shape that possesses all the rotational degrees of freedom of a three-dimensional body (and hence violates spherical and axial symmetry) such a rotational band on top of every intrinsic state involves \((2J+1)\) levels with total angular momentum \( J \). Each of these levels is itself \((2J+1)\)-fold degenerate, corresponding to the different

---

**FIG. 27.** (Color online) Phase transition energy \( E_{pt} \) for nuclei in the valley of stability vs. \( A \) (full line in black) in comparison to values for \( S_n \) (dashed in magenta) and the shell correction energy \( \delta E \) (blue dotted line), all in MeV.
m-substates \[201\]. The rotation induced collective enhancement may fade out at higher energy, but this will happen only far above the energy range around \(S_n\), of interest here \[34, 99, 203\]. As usual \[6, 194, 198\], it is assumed in Eq. \[21\] that the excitation energy \(E_x\) is large as compared to the rotational energy \(\hbar^2 J(J+1)/2\) and that the spin projections \(m\) are normally distributed around 0. Obviously the exponential spin cut off factor can be neglected in the limit for small \(J\). The reduction by a factor 4 is related to the invariance with respect to rotations by 180° about each of the three principal axes, valid for the most general quadrupole deformation \[6, 201\]. If the nuclear body is completely symmetric about one axis, i.e., axially deformed, a decrease of the level density by \(\sqrt{\pi/2}\sigma \approx 8\) is expected in the limit of small \(J\), and a reduction by a factor up to \(\approx 300\) – from \(\sqrt{\pi/2}\sigma^3\) – arises in the comparison between Eq. \(24\) and Eq. \(23\), valid for the level density of completely spherical nuclei \[6, 191, 201\]. The size of these factors indicates that the dependence of the absolute level density on the symmetry of the nuclei is appreciable, whereas the amount of deformation enters only via the spin cut off \(\sigma\) and this can be neglected for spins below \(I=1\). As the deformation parameters \(\beta\) and \(\gamma\) are subject to fluctuations around their mean values \[1\] a shape sampling equivalent to the one described for the axis lengths may be applied to the moments of inertia. Such sampling does not affect the symmetry class and its effect on \(\rho(E,J^\pi)\) was tested to be negligible by approximate calculations. Low energy oscillations in heavy nuclei have been assumed to lead to vibrational bands and to cause an additional \[201, 205\] collective enhancement of \(\rho(E,J^\pi)\). In our scheme extra level density due to quadrupolar modes is accounted for by the breaking of axial symmetry. When the effect of low energy quadrupole vibrations was investigated as proposed \[201\] with strengths derived from the isoscalar quadrupole giant resonance (ISGQR) using \(\hbar\omega_{\text{ISGQR}} = 4\) \(E_{\text{ISGQR}}(2^+)\) and \(I_{\text{ISGQR}} = \frac{1}{4} I_{\text{ISGQR}}(2^+)\). In analogy to the expression for the strength functions as compiled in Table \[1\], an enhancement resulted to be far below the effect of triaxiality.

Comparison to data

Average level distances \(D(E,J^\pi) = 1/\rho(E,J^\pi)\) are available from ensembles of discrete levels with known spin populated in nuclear reactions at low \(E_x\) and at the neutron separation energy \(S_n\) from compound resonances in the cross section for neutron capture in the eV and keV range. To demonstrate the energy dependence of the level density formalism presented here, results for \(^{81}\)Sr, \(^{113}\)Cd and \(^{255}\)U are given in Fig. \[28\]. For the plots odd-\(n\) nuclei with a satisfactory number of levels were selected from the data compiled within RIPL-3 \[31\]. The experimental information on \(\rho(Z,N,E-x,J^\pi)\) stems from counting bound levels up to an \(E_x\), above which completeness is no longer assured, as well as resonances just above \(S_n\) \[31, 186\]. As their spins are known, only two obvious assumptions are needed to obtain \(\omega_{\text{gr}}(E_x)\) from the data by using Eq. \[24\]:

1. Parities are equally distributed in the discrete spectrum;
2. \(\sigma_x\) in the exponential spin cut off correction is taken from systematics \[200\].

This procedure of applying the spin dependent factors to the data allows different spins to be shown in the same plot. As will be shown in section \[X\] the slope below \(S_n\) is of special interest for capture cross sections. It is reasonably well predicted even for \(^{81}\)Se, although some data are missed in absolute height. Extrapolations into domains away from stability need global parameterizations valid for an extended region of nuclei. For 132 nuclei with \(A > 70\) the average distance of s-wave neutron capture resonances is available \[143\]. As these all have spin \(1/2^+\), it is interesting to compare these data to Eq. \[20\], which represents the most general, triaxial case. It is worthwhile noting that for spin \(1/2\) the small \(J\) limit is correct within a few per cent only. Fig. \[29\] depicts the results from using Eqs. \[13\] to \[23\] and \[24\] in a comparison to measurements; obviously many of them lie close to the prediction. The shell correction plays an important role for the resulting agreement; this is improved by including a damping of it as proposed \[204\] on the basis of an extra entropy change with temperature. Agreement over many orders of magnitude is reached with the only fit parameter \(\alpha = 0.06\) and it is remarkable that even near \(^{208}\)Pb our "triaxial" ansatz does quite well. Some alternatives proposed earlier may eventually help to obtain an even better global fit: The reduction of the nucleon mass to an effective in-medium value was suggested \[167\], some of the expected deviations of nuclear radii from \(1.2\ A^{1/3}\) fm are included as calculated \[1\], model calculations of moments of inertia show considerable differences to rigid rotor values \[41\], and ambiguities may be seen in the choice of temperature \((T\ vs.t)\). These topics may add some to the collective enhancement given by Eq. \[24\], but an important influence on the intrinsic state density \(\omega_{\text{gr}}(E_x)\) as well as on \(\rho(E_x,J)\) was found to emerge from the choice made for \(\delta E\), by which the Fermi gas zero is fixed with respect to the nucleus’ ground state: Replacing the liquid drop model masses from ref. \[203\] by the one from \[202\] increases the level density for actinide nuclei by nearly an order of magnitude, whereas the use of a new LDM fit including a curvature term \[169\] has the opposite effect. The recent liquid drop model fit to masses \[202\] used in this study is based on fitting a volume and a surface term independently and the resulting agreement to resonance spacings are shown in Fig. \[29\]. A nearly equally good agreement is found \[124\] for a choice of LDM parameters \[208\] made many years ago. Unfortunately these LDM fits are made neglecting the breaking of axial symmetry, as its influence on ground state masses was calculated to be very small for most heavy nuclei.
FIG. 28. (Color online) For the state density $\omega_{qp}(E_x)$ in the nuclei $^{81}$Sr (a), $^{113}$Cd (b) and $^{235}$U (c) the prediction is shown together with respective data from RIPL-3 [31] obtained using discrete levels (black squares) [31, 143] as well as neutron resonance spacings (below vertical arrow); both are converted into state density $\omega$ by inverting Eq. (24). A change in slope at the phase transition energy $E_{pt}$ is clearly seen (diagonal arrow). The dotted blue lines depict results from Eq. (19 and 20) using parameters as given in Eqs. (21 and 23).

FIG. 29. (Color online) Average resonance spacings $D(S_n1/2^+)$ in nuclei with $51 < A < 253$ as observed in neutron capture by spin 0 target nuclei. Data (black data symbols) compiled by the RIPL collaboration [31, 143] are compared to the prediction with an effective shell correction [202] as presented in section IX (dotted curve in magenta). The drawn blue line depicts the case when it is damped with $E_x$, as proposed previously [204].

Hence more studies on the $A$-dependence of $\delta E$ seem indicated; a broad mass range should be regarded, as a large effect of the LDM fits to $\delta E$ is seen for $A \approx 60$ and $A \geq 230$, but only small differences appear between various LDM for $A \approx 80$ to 140. The agreement to experimental level densities from using this LDM fit [202] may well be regarded as an additional indicator for its quality, and it especially supports our parameterization.

X. RADIATIVE NEUTRON CAPTURE

General remarks

The radiative capture of fast neutrons by heavier nuclei plays an important role in considerations for advanced nuclear systems and it is of interest also for the cosmic nucleosynthesis. To test the combination of the present ansatz on the photon strength to the one for the level density - both allowing for a breaking of axial symmetry - a comparison on absolute scale of predicted to measured average radiative widths is demonstrated first. A sum over the decay channels to all bound states $J_b$ which can be reached from the capture resonances $J_r$ by photons of energy $E_\gamma = E_r - E_b$, multiplied by their density $\rho(E_b, J_b)$ leads to an effective averaging. The dependence of $\rho(E_b, J_b)$ on $J_b$ (cf. Eq. (24)) has the consequence that the $\gamma$-decay for dipole-transitions from $J_r = 1/2$ to $J_b = 3/2$ is favored as compared to final levels with $J_b = 1/2$ and a difference between $E1$ and $M1$ arises, if there is a parity dependence of the level density. In the ansatz discussed here, $\rho(E_x, J)$ above $E_{pt}$ is determined by the Fermi gas prescription and independent of parity by principle. One may account for differences in the low energy regime caused by a dominant parity through a respective estimate of $\omega(0) \equiv \omega(E_x < \Delta_0)$. If
respective information is available this may improve the predictions. The mean radiative width of capture resonances with \( J^c_r = 1/2^+ \) is the basis for the description of radiative capture by \( 0^+ \)-target nuclei with \( l_n = 0 \) as discussed in the following.

**Average radiative widths**

It was pointed out previously [92] that strength information can be extracted from capture data directly by regarding average photon widths \( \bar{\Gamma}_\gamma \). These are proportional to the ratio between the level densities at the capturing resonances \( r \) and at the final states \( b \) below \( S_n \) reached by \( E_r = E_r - E_b \). Eq. (13) shows that the first mentioned \( \rho(E_r, J_r) \) are included in \( f_1(E_r) \). Of course, the widths \( \bar{\Gamma}_\gamma \) depend in addition on the photon strength, i.e. the low energy IVGDR tail extrapolated from above. It is known that \( \bar{\Gamma}_\gamma \) does not vary with \( E_r \) [12, 209] and hence it can be approximated for \( J_r = 1/2^+ \) by setting \( \lambda = 1 \) in Eq. (13) and summing over all final bound levels \( b \in \Delta_b \), i.e. over \( \Delta_b = [0, S_n + E_r] \):

\[
\bar{\Gamma}_\gamma = \sum_{b \in \Delta_b} \Gamma_\gamma (r \rightarrow b)
\]

\[
\simeq \int_{\Delta_b} \rho(E_b, J_b) (\Gamma_\gamma (r \rightarrow b)) dE_\gamma \tag{25}
\]

\[
\simeq \int_{\Delta_b} \rho(E_b, J_b) f_1(E_r) dE_\gamma
\]

Average radiative widths were derived by a resonance analysis of neutron data taken just above \( S_n \) and tabulated [143] for 115 even-odd nuclei with \( 51 \leq A \leq 253 \). These \( \Gamma_\gamma \) allow for a combined test of predictions for photon strength and level density slopes, and respective data are shown in Fig. 30. Radiative capture into spin-0 targets through the s-channel only \((l_n = 0)\) is considered, such that the spin dependence of the level densities \( \rho(E_r, J_r) \), under study recently [200] can be neglected and the spin cut off in Eq. (24) is neglected here as the collective energy to be added to the intrinsic excitation is small as compared to \( E_r \). Magnetic (M1) strength was treated separately and then the number of levels with \( J_n = 1/2^+ \) and \( 3/2^+ \) at low energy enters explicitly. They play a minor role for the energy integrated photon interaction, as resulted from inserting tabulated [12, 31] values for the matrix elements and recent experimental data [13, 19, 123]. It appears that, similar to non-nuclear systems, electric dipole modes dominate radiative processes.

In Fig. 30 the agreement on absolute scale between prediction and data appears as satisfactory and this can be attributed to the combination of TLO to a LDM based shell correction. There are some discrepancies probably related to the neglect of shell effects (in addition to \( \delta E(Z, A) \)). In Fig. 30 the photon strength as discussed in sections VI to VIII and the level density obtained from the parameterization described in section IX are used. Previous work, which only covered limited numbers of nuclides, used parameters for \( \rho(E_r) \) locally adjusted to low lying levels and neutron resonances [93, 105, 106, 136, 144]. The good overall accordance to experimental data reached by us tests the various approximations applied: A decrease of \( \omega_{wp}(0) \) by a factor of 3 modifies \( \bar{\Gamma}_\gamma \) by 20 to 50% when regarding nuclei with \( A \approx 70 \) resp. \( A \approx 240 \); this shows that the agreement may be improved by an introduction of local spectroscopic information in addition to our parameterization, explicitly based on global properties only. The inclusion of all the various components of minor strength as listed in Table II increases \( \bar{\Gamma}_\gamma \) by \( \approx 55% \) for an average over \( A \). The largest effect (about 30%) was found for the coupled mode \((2^+ \times 3^-)_{1-}\), studied theoretically [11] since long. For its strength only data scattered in \( A \) and \( Z \) are available [18] and they were used as guide here. The second largest increase of \( \bar{\Gamma}_\gamma \) (by \( \approx 10% \)) and more for nuclei with large \( Q_0 \) is due to magnetic dipole strength (of the scissors mode). A low-energy E2 component of strength as discussed in the context of Table II was found to increase the decay of the \( 1/2^+ \)-levels just above \( S_n \), as populated via s-capture, by less than 7% albeit the opening of E2-decay to \( 5/2^+ \) levels. The inclusion of pigmy strength seems to have a small effect when an average over \( A \) is regarded: it stays below 10% for each of the two components.
Averaged cross sections for radiative capture

Above \( \approx 1 \) keV the neutron widths are that large, that \( \langle \Gamma_n \rangle \gg \Gamma_\gamma \) and the average width ratio in Eq. (20) can be replaced by \( \Gamma_\gamma \) with an accuracy of a few percent.

\[
\langle \sigma(n, \gamma) \rangle_r \equiv 2\pi^2 \chi_n^2 \sum_l (2l + 1) \frac{\Gamma_n \Gamma_\gamma}{\Gamma_n + \Gamma_\gamma} \cdot \rho(E_r, J_r) \quad \text{with} \quad \frac{\Gamma_n \Gamma_\gamma}{\Gamma_n + \Gamma_\gamma} \approx \Gamma_\gamma
\]  

As \( \Gamma_\gamma \) is contained implicitly in \( f_1(E_\gamma) \) one gets from Eq. (26) \( \sigma_{\text{capt}}(E_n) \) for \( E_n \approx E_r - S_n \):

\[
\sigma_{\text{capt}}(E_n) \equiv \langle \sigma(n, \gamma) \rangle_r \equiv 2\pi^2 \chi_n^2 \sum_{l, J_\gamma} (2l + 1) \int_0^{E_r} f_1(E_\gamma) \rho(E_b, J_b) dE_\gamma
\]  

This is known from measured neutron strengths as listed for RIPL-3 \cite{31, 143}. Approximating transmission coefficients (from optical potentials) for the compound nucleus formation and a full statistical treatment of its decay by a schematic ansatz \cite{140}, one finds:

In this cross section formula for \( \sigma_{\text{capt}}(E_n) \) the neutron energy enters mainly via \( \chi_n \) and the neutron angular momentum \( l \). Following a schematic compound nucleus model \cite{209}, one arrives at Eq. (23) for the Maxwellian averaged radiative capture cross section (MACS), averaged over many resonances. The case of non-zero target spins in radiative capture as well as the inclusion of \( l > 0 \), direct capture and inelastic scattering are not treated here; respective numerical tests indicate small changes only to our conclusions. A prediction of MACS based on the photon strength presented in sections VI to VIII is then possible and calculated by:

\[
 \langle \sigma(n, \gamma) \rangle_{kT} \approx \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sigma_{\text{capt}}(E_n) E_n e^{-E_n/kT} dE_n}{\int_0^\infty E_n e^{-E_n/kT} dE_n}
\]  

Here cross sections given by Eqs. (26) and (27) are folded with a Maxwellian distribution of neutron energies. As pointed out \cite{210}, the same can be done with experimental data. In view of the fact that \( D \gg \Gamma_r \geq \Gamma_\gamma \) the Maxwellian averages around 30 keV are formed incoherently with neglect of Porter-Thomas fluctuations. By only regarding the radiative capture by spin-zero targets effects related to ambiguities of spin cut off parameters and angular momentum coupling are suppressed. It is obvious from Fig. 31 that the TLO-parameterization with the proposed ansatz for \( \rho(A, J^*, E_x) \) work well: For the discussed range of \( A \) the overall agreement on absolute scale covering more than three decades is remarkable. The discrepancy observed in the region of \( A > 230 \) and \( A < 70 \) may be related to a false prediction of minor strength, or of shell correction influencing level densities, and details of the shell structure in these nuclei may be important for both. Interesting aspects become obvious from the regard of Figs. 30 and 31 together. Near closed shells like at \(^{208}\text{Pb} \) \( \Gamma_\gamma \) is large whereas a minimum appears for \( \langle \sigma(n, \gamma) \rangle_{kT} \). The average radiative widths directly rise with \( f_1(E_\gamma) \) and thus \( E_n - S_n \) and vary only with the slope of \( \rho(E_x, J) \) in the range from \( E_b \) to \( E_r \). In contrast, capture cross sections directly depend on the level density in the final nucleus \( \rho(E_b, J_b) \), and this results in a deep minimum at shell closure \( \text{cf.} \, \text{26-27} \). From this simplified study the main topic of our approach – the importance of triaxiality in heavy nuclei – is obvious and not influenced by the rather small discrepancies seen in Figs. 30 and 31.

FIG. 31. (Color online) Maxwellian averages of experimental cross sections \cite{211, 212} for radiative neutron capture into even nuclei with \( J = 0 \) and \( 50 < A < 250 \) shown as black data symbols for \( kT_{\text{AGB}} = 30 \) keV. They are plotted vs. \( A_{\text{CN}} \) in comparison to calculations based on Eq. (25) with TLO (dotted line in magenta) and including the minor components listed in Table II (full blue line). The level density prediction is described in section IX as a shell effect damping is included.

It was worked out earlier \cite{83} as well as in section VII that the impact of photon strength on radiative neutron capture cross sections is peaking in the region below \( E_\gamma \approx 6 \) MeV. Below, the factor \( E_\gamma^{2\Lambda+1} \) reduces the transition rates and above the density of levels to be reached becomes small. Thus the scissors mode has to be regarded for M1-transitions in deformed nuclei, and the...
low energy E1-strength in the $^{(2+ \times 3^-)}_{1^-}$ and the lower
pigmy modes have a strong effect in all nuclei. As de-
picted in Fig. 31 by the dashed curve, the cross section
is dominated by the IVGDR extension to low energy, es-
pecially for $A < 150$. Recent work [32, 73, 139] on the
influence of minor strength components has indicated a
more significant impact in comparison to the IVGDR tail,
which was quantified in a different way; it was shown in
section VII that single or double Lorentzians result in
erroneous estimates of the corresponding E1-strength in
the tail. This is especially so if a dependence of $\Gamma_j$ on
$E_i$ is imposed [34, 95] to seemingly improve the fit of the
IVGDR. Namely the gamma-energy dependence of the
resonance widths imposed often plays an important role.
In the TLO approach the sole variation of $\Gamma_1$ with the
pole energies $E_i$, its global adjustment and the strict im-
plementation of the TRK sum rule avoid irregularities in the
A-dependence of the dipole strength. We have shown that
this is appropriate in the valley of stability, but for
very neutron rich nuclei with their small $S_n$, experimental
tests may became possible eventually in newly available
radioactive beam facilities.

**Fast neutron induced transmutation**

Neutron capture processes play an important role in the
study of cosmic nucleosynthesis, especially for sce-
narios with such high neutron flux, that neutron capture
processes can be the cause for a production of nu-
clides beyond Fe [23, 24]. The radiative capture of fast
neutrons by heavier nuclei is of interest also for consid-
erations for advanced nuclear systems [20, 21] and
devices aiming for the transmutation of radioactive nuclear
waste. One – for many even the strongest – of the ar-
guments against a long-lasting commitment to nuclear
power as energy supply is the difficult permanent dis-
posal of the long lived radioactive waste produced in nu-
clear reactors. Significant efforts are thus being made
worldwide directed towards the minimization, manage-
ment, and disposal of highly radioactive nuclear waste.
Thus the partitioning of nuclear waste and transmuta-
tion of long-lived isotopes to nuclides with shorter life-
time are being investigated worldwide. Different schemes
have been proposed that may reduce the radioactivity and
radio-toxicity of the spent fuel after burn up. The
studies towards the choice of the best options make ex-
tensive use of simulation methods in order to predict the
system behaviour in a great variety of possible configura-
tions and running conditions [22]. A fundamental prereq-
usite for these computations is the availability of reliable
cross section data. These are needed for processes and
operating parameters that are significantly different from
those of current systems mostly operating with thermal
neutrons. A good understanding of the radiative capture
of fast neutrons is an important part for any theoreti-
cal attempt in this direction, and the agreement shown in
Fig. 31 appears promising. But a much more severe
problem with nuclear reactor waste is posed by the pres-
ence of nuclides beyond Uranium produced in the high
flux of slow neutrons captured by the $^{238}U$ usually form-
ing a large fraction of the fuel. By multiple neutron cap-
ture and subsequent $\beta$-decay an appreciable amount of
long lived nuclides are formed. As several of these “mi-
nor actinides” are $\alpha$-emitters the risk of their incorpo-
ration increases their high radiotoxicity. Attempts to ex-
and the TLO-based calculations of $(n,\gamma)$ cross sections
in actinide nuclei are hampered by serious discrepancies
between IVGDR data observed in different laboratories,
some of which are obvious from figures shown here. Neu-
tron capture by actinide nuclei is of great importance for
the transmutation of nuclear waste and a dedicated in-
vestigation of neutron capture cross sections for Th, U
and heavier nuclei, for which data were compiled [212], is
very neutron rich nuclei with their small $S_n$, experimental
useful. The fact, that TLO results in a good description
of the data for $^{239}Pu$ (cf. Fig. 25) and the good match
observed in Fig. 31 leads to consider the TLO approach
as encouraging to serve as guideline for predictions con-
cerning the radiative capture of fast neutrons also for
actinide nuclei. The approximations made to arrive at Eqs. 26 and 27 work well in the range of $E_n \approx 30$keV
(see Fig. 31), but the coupling to other channels like in-
elastic scattering has to be included at higher energies.
The effect of further contributions was shown recently in
detail for $^{238}U$ [62] and for other actinides [213], where
also the importance of the scissors mode was pointed out.
In view of the average overall agreement for more than
150 nuclides an inclusion of the TLO dipole strength and
the “triaxial” level density as an option in TALYS 180
or EMPIRE 214 appear highly desirable. These two
codes now use the “old” Lorentzian descriptions 51, 62
of the IVGDR shapes with only one or two poles and
locally adjusted IVGDR parameters.

The extension of our ansatz to targets with nonzero
spin needs additional caution as the transition to the ob-
server’s system in Eq. (24) assumes the rotational energy
to increase with spin. Data for resonance spacings near
$S_n$ as observed with neutron capture by odd nuclei in-
volve an elevated spin of usually single particle nature.
It is thus disputable to simply apply Eq. (24) to higher
spin, even if one of the several propositions 200 for the
spin cut off parameter $\sigma$ is inserted. A related ambiguity
emerges, when one of these is inserted into a factoriza-
tion for the spin dependence: $\rho(E_x, J) = S\rho(J)p(E_x)$.
The ansatz presented here for $J=1/2$ avoids this prob-
lem and further study is needed to extend it to higher
spin.

**XI. CONCLUSIONS**

Various spectroscopic information presented over the
years [1, 7, 8, 36, 42, 47] indicated triaxiality for a num-
ber of heavy nuclei. Admission of the breaking of axial
symmetry, albeit often weak, clearly improves a global
description of Giant Dipole Resonance (IVGDR) shapes
by a triple Lorentzian (TLO), introduced recently \[13, 72\] and discussed in detail by this paper. The three parts add up to the TRK sum rule \[53, 54\], when theoretical predictions for the $A$-dependence of pole energies from droplet model \[70\] and spreading widths based on one-body dissipation \[74\] are used. Two additional effects – hitherto not emphasized as such – indicate a breaking of axial symmetry in nearly all of the heavy nuclei: With one global parameter the scheme proposed by us reproduces observations for level densities in nuclei with $A > 50$ and $J = 1/2$, when (a) the Fermi gas prescription is only used above a phase transition at the critical temperature, (b) a pairing condensation energy is included in the backshift and (c) the collective enhancement due to symmetry breaking by triaxiality is included. Only a modest modification of the level density parameter $\tilde{\alpha}$ from its nuclear matter value is needed to fit resonance spacing data; this is a surprisingly small surface term –, the only free parameter of our level density prescription. Again only one global free parameter suffices to fit to the shape of the IVGDR peak by a triple Lorentzian photon strength (TLO), somewhat improved as compared to the original proposal \[13, 72\], well in accord to the TRK sum rule. It also predicts its low energy tail – without other modification than the addition of minor strength components – to match respective strength data. This consideration of broken spherical and axial symmetry – for low excitation in accord to CHFB calculations \[1\] and even more so for increasing excitation energy – allows for a combination of both parameterizations. Exact deformation parameters are unimportant for the tail of the E1-resonance as well as for the density of low spin states occurring in neutron capture by even targets, where neither spin cut off nor moments of inertia are involved. Thus a combination of the ansatz for photon strength to the novel approximation for level densities leads to a prediction on absolute scale for neutron capture in the range of unresolved resonances – including Maxwellian average cross sections compiled recently \[211\] for $\langle E_n \rangle = 30$keV.

Four points are stressed:

1. Ad hoc assumptions on shapes and collective enhancement in previous calculations \[31\] of compound nuclear rates are replaced by a direct account for broken symmetries.

2. Doing so, the prediction for level densities is improved such, that only one small parameter suffices to obtain agreement to data on absolute scale.

3. The low energy tail derived from the global triple Lorentzian (TLO) fit to IVGDRs is not modified by an extra energy dependence.

4. The predictions for average radiative widths and Maxwellian cross sections are sensitive to one ingredient to the calculations not controlled globally: The state density $\omega_{qp}(0)$ near the ground state of the final nucleus.

5. A similar statement has to be made concerning the low energy electric dipole strength induced by coupling nuclear quadrupole and octupole modes. For these a global prediction appears impossible and for Figures \[7\] to \[31\] values were used, which were derived as upper limits to experimentally determined data.

6. The possible increase of n-capture yields by some amount was discussed \[19\] to be caused by additional photon strength, predicted \[180\] to appear below $\approx 0.4 E_{IVGR}$. Such pigmy strength may eventually be related to a vibration of excess neutrons against a core. Another excess above the Lorentzian tail near $S_n$ seems to be related to a vortical proton motion \[179\]. Our study finds a rather weak influence of such strength components in comparison to the one mentioned under 5.

In conclusion: For more than 100 spin-0 target nuclei with $A > 50$ resonance spacing data and average capture cross sections are well described by a global fit with only a small number of adjusted quantities, which turn out to be $A$-independent. This successful ansatz can be used as basis for detailed studies in specific regions of the nuclide chart and this promises reliable predictions for other compound nuclear reactions besides radiative capture. The breaking of axial symmetry in excited heavy nuclei is demonstrated here on the basis of experimental data, most of which were published previously. Further studies on broken axial symmetry in addition to existing theoretical hints to the importance of triaxiality \[1, 3, 8, 27, 28, 33, 34, 38, 40, 71, 74, 215\] seem necessary, as nuclear theory studies often still prefer to assume axial symmetry ad hoc. We actually question that an experimental proof for such symmetry exists, and probably all heavy nuclei do not strictly obey it, in apparent resemblance to what was found for crystalline matter by Jahn and Teller in 1937 \[218\]. Such an analogy is suggested by the data analysis presented here, showing that the global ansatz derived on the basis of broken axial symmetry combined to the dipole sum rule allows remarkably good predictions for radiative neutron capture in general - with very few free fit parameters.

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