A Note on Positive Energy Theorem for Spaces with Asymptotic SUSY Compactification

Xianzhe Dai

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Abstract

We extend the positive mass theorem in [D] to the Lorentzian setting. This includes the original higher dimensional Positive Energy Theorem whose spinor proof is given in [Wi1] and [PT] for dimension 4 and in [Z1] for dimension 5.

1 Introduction and statement of the result

In this note, we formulate and prove the Lorentzian version of the positive mass theorem in [D]. There we prove a positive mass theorem for spaces which asymptotically approach the product of a flat Euclidean space with a compact manifold which admits a nonzero parallel spinor (such as a Calabi-Yau manifold or any special holonomy manifold except the quaternionic Kähler). This is motivated by string theory, especially the recent work [HHM]. The application of the positive mass theorem of [D] to the study of stability of Ricci flat manifolds is discussed in [DWW].

In general relativity, a spacetime is modeled by a Lorentzian 4-manifold $(\mathcal{N}, g)$ together with an energy-momentum tensor $T$ satisfying Einstein equation

\[ R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}. \]  

The positive energy theorem [SY1], [Wi1] says that an isolated gravitational system with nonnegative local matter density must have nonnegative total energy, measured at spatial infinity. More precisely, one considers a complete oriented spacelike hypersurface $M$ of $\mathcal{N}$ satisfying the following two conditions:

a). $M$ is asymptotically flat, that is, there is a compact set $K$ in $M$ such that $M - K$ is the disjoint union of a finite number of subsets $M_1, \ldots, M_k$ and each $M_l$ is diffeomorphic to $(\mathbb{R}^3 - B_R(0))$. Moreover, under this diffeomorphism, the metric of $M_l$ is of the form

\[ g_{ij} = \delta_{ij} + O(r^{-\tau}), \quad \partial_k g_{ij} = O(r^{-\tau-1}), \quad \partial_k \partial_l g_{ij} = O(r^{-\tau-2}). \]  

Furthermore, the second fundamental form $h_{ij}$ of $M$ in $\mathcal{N}$ satisfies

\[ h_{ij} = O(r^{-\tau-1}), \quad \partial_k h_{ij} = O(r^{-\tau-2}). \]

Here $\tau > 0$ is the asymptotic order and $r$ is the Euclidean distance to a base point.
b). \(M\) has nonnegative local mass density: for each point \(p \in M\) and for each timelike vector \(e_0\) at \(p\), \(T(e_0, e_0) \geq 0\) and \(T(e_0, \cdot)\) is a nonspacelike co-vector. This implies the dominant energy condition

\[
T^{00} \geq |T^{\alpha\beta}|, \quad T^{00} \geq (-T_{0i}T^{0i})^{\frac{1}{2}}.
\]  

The total energy (the ADM mass) and the total (linear) momentum of \(M\) can then be defined as follows [ADM], [PT] (for simplicity we suppress the dependence here on \(l\) (the end \(M_l\)))

\[
E = \lim_{R \to \infty} \frac{1}{4\omega_n} \int_{S_R} (\partial_i g_{ij} - \partial_j g_{ii}) * dx_j,
\]

\[
P_k = \lim_{R \to \infty} \frac{1}{4\omega_n} \int_{S_R} 2(h_{jk} - \delta_{jk} h_{ii}) * dx_j
\]  

(1.5)

Here \(\omega_n\) denotes the volume of the \(n-1\) sphere and \(S_R\) the Euclidean sphere with radius \(R\) centered at the base point.

**Theorem 1.1 (Schoen-Yau, Witten)** With the assumptions as above and assuming that \(M\) is spin, one has

\[
E - |P| \geq 0
\]

on each end \(M_l\). Moreover, if \(E = 0\) for some end \(M_l\), then \(M\) has only one end and \(N\) is flat along \(M\).

Now, according to string theory [CHSW], our universe is really ten dimensional, modelled on \(\mathbb{R}^{3,1} \times X\) where \(X\) is a Calabi-Yau 3-fold. This is the so called Calabi-Yau compactification, which motivates the spaces we now consider.

Thus, we consider a Lorentzian manifold \(N\) (with signature \((-+,\ldots,+)\)) of \(\dim N = n + 1\), with a energy-momentum tensor satisfying the Einstein equation. Then let \(M\) be a complete oriented spacelike hypersurface in \(N\). Furthermore the Riemannian manifold \((M^n, g)\) with \(g\) induced from the Lorentzian metric decomposes \(M = M_0 \cup M_\infty\), where \(M_0\) is compact as before but now \(M_\infty \simeq (\mathbb{R}^k - B_R(0)) \times X\) for some radius \(R > 0\) and \(X\) a compact simply connected spin manifold which admits a nonzero parallel spinor. Moreover the metric on \(M_\infty\) satisfies

\[
g = \bar{g} + u, \quad \bar{g} = g_{\mathbb{R}^k} + g_X, \quad u = O(r^{-\tau}), \quad \bar{g} u = O(r^{-\tau - 1}), \quad \bar{g}\bar{g} u = O(r^{-\tau - 2}),
\]  

(1.6)

and the second fundamental form \(h\) of \(M\) in \(N\) satisfies

\[
h = O(r^{-\tau - 1}), \quad \bar{g} h = O(r^{-\tau - 2}).
\]  

(1.7)

Here \(\bar{\nabla}\) is the Levi-Civita connection of \(\bar{g}\) (extended to act on all tensor fields), \(\tau > 0\) is the asymptotical order.

The total energy and total momentum for such a space can then be defined by

\[
E = \lim_{R \to \infty} \frac{1}{4\omega k \text{vol}(X)} \int_{S_R \times X} (\partial_i g_{ij} - \partial_j g_{aa}) * dx_j d\text{vol}(X),
\]

\[
P_k = \lim_{R \to \infty} \frac{1}{4\omega k \text{vol}(X)} \int_{S_R \times X} 2(h_{jk} - \delta_{jk} h_{ii}) * dx_j d\text{vol}(X).
\]  

(1.8)
Here the $*$ operator is the one on the Euclidean factor, the index $i,j$ run over the Euclidean factor while the index $a$ runs over the full index of the manifold.

Then we have

**Theorem 1.2** Assuming that $M$ is spin, one has

$$E - |P| \geq 0$$
on each end $M_l$. Moreover, if $E = 0$ for some end $M_l$, then $M$ has only one end. In this case, when $k = n$, $N$ is flat along $M$.

In particular, this result includes the original higher dimensional Positive Energy Theorem whose spinor proof is given in [Wi1] and [PT] for dimension 4 and in [Z1] for dimension 5.

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## 2 The hypersurface Dirac operator

We will adapt Witten’s spinor method [Wi1], as given in [PT], to our situation. The crucial ingredient here is the hypersurface Dirac operator on $M$, acting on the (restriction of the) spinor bundle of $N$. Let $S$ be the spinor bundle of $N$ and still denote by the same notation its restriction on (or rather, pullback to) $M$. Denote by $\nabla$ the connection on $S$ induced by the Lorentzian metric on $N$. The Lorentzian metric on $N$ also induces a Riemannian metric on $M$, whose Levi-Civita connection gives rise to another connection, $\bar{\nabla}$ on $S$. The two, of course, differ by a term involving the second fundamental form.

There are two choices of metrics on $S$, which is another subtlety here. Since part of the treatment in [PT] is special to dimension 4, we will give a somewhat detailed account here.

Let $SO(n,1)$ denote the identity component of the groups of orientation preserving isometries of the Minkowski space $\mathbb{R}^{n,1}$. A choice of a unit timelike covector $e^0$ gives rise to injective homomorphisms $\alpha$, $\hat{\alpha}$, and a commutative diagram

$$\begin{align*}
\alpha : \quad SO(n) & \to SO(n,1) \\
\hat{\alpha} : \quad Spin(n) & \to Spin(n,1).
\end{align*} \tag{2.9}
$$

We now fix a choice of unit timelike normal covector $e^0$ of $M$ in $N$. Let $F(N)$ denote the $SO(n,1)$ frame bundle of $N$ and $F(M)$ the $SO(n)$ frame bundle of $M$. Then $i^*F(N) = F(M) \times_\alpha SO(n,1)$, where $i : M \hookrightarrow N$ is the inclusion. If $N$ is spin, then we have a principal $Spin(n,1)$ bundle $P_{Spin(n,1)}$ on $N$, whose restriction on $M$ is then $i^*P_{Spin(n,1)} = P_{Spin(n)} \times_\hat{\alpha} Spin(n,1)$, where $P_{Spin(n)}$ is the principal $Spin(n)$ bundle of $M$. Thus, even if $N$ is not spin, $i^*P_{Spin(n,1)}$ is still well-defined as long as $M$ is spin.

Similarly, when $N$ is spin, the spinor bundle $S$ on $N$ is the associated bundle $P_{Spin(n,1)} \times_{\rho_{n,1}} \Delta$, where $\Delta = \mathbb{C}^{2n+1}$ is the complex vector space of spinors and $\rho_{n,1} : Spin(n,1) \to GL(\Delta)$ \tag{2.10}
Lemma 2.1 There is a positive definite hermitian inner product $\langle \cdot, \cdot \rangle$ on $\Delta$ which is $\text{Spin}(n)$-invariant. Moreover, $(s, s') = \langle e^0 \cdot s, s' \rangle$ defines a hermitian inner product which is also $\text{Spin}(n)$-invariant but not positive definite. In fact

$$(v \cdot s, s') = (s, v \cdot s')$$

for all $v \in \mathbb{R}^{n,1}$.

**Proof.** Detailed study via $\Gamma$ matrices [CBDM, p10-11] shows that there is a positive definite hermitian inner product $\langle \cdot, \cdot \rangle$ on $\Delta$ with respect to which $e^i$ is skew-hermitian while $e^0$ is hermitian. It follows then that $\langle \cdot, \cdot \rangle$ is $\text{Spin}(n)$-invariant. We now show that $(s, s') = \langle e^0 \cdot s, s' \rangle$ defines a $\text{Spin}(n)$-invariant hermitian inner product. Since $e^0$ is hermitian with respect to $\langle \cdot, \cdot \rangle$, $(\cdot, \cdot)$ is clearly hermitian. To show that $(\cdot, \cdot)$ is $\text{Spin}(n)$-invariant, we take a unit vector $v$ in the Minkowski space: $v = a_0 e^0 + a_i e^i$, $a_0, a_i \in \mathbb{R}$ and $-a_0^2 + \sum_{i=1}^n a_i^2 = 1$. Then

$$(v s, v s') = \langle e^0 v s, v s' \rangle$$

$$= a_0^2 \langle e^0 e^0 s, e^0 s' \rangle + a_i a_0 \langle e^0 e^i s, e^0 s' \rangle + a_0 a_i \langle e^0 e^0 s, e^i s' \rangle + a_i a_j \langle e^0 e^i s, e^j s' \rangle$$

$$= a_0^2 \langle s, e^0 s' \rangle - a_i a_j \langle e^i e^j s, s' \rangle$$

$$= a_0^2 \langle e^0 s, s' \rangle + a_i a_j \langle e^0 e^j s, s' \rangle$$

$$= a_0^2 \langle e^0 s, s' \rangle - a_i^2 \langle e^0 s, s' \rangle$$

$$= -(s, s')$$

Consequently, $(\cdot, \cdot)$ is $\text{Spin}(n)$-invariant. The above computation also implies that $v \cdot$ acts as hermitian operator on $\Delta$ with respect to $(\cdot, \cdot)$. $\blacksquare$

Thus the spinor bundle $S$ restricted to $M$ inherits an hermitian metric $(\cdot, \cdot)$ and a positive definite metric $(\cdot, \cdot)$. They are related by the equation

$$(s, s') = \langle e^0 \cdot s, s' \rangle.$$  \hspace{1cm} (2.12)

Now the hypersurface Dirac operator is defined by the composition

$$\mathcal{D} : \Gamma(M, S) \xrightarrow{\nabla} \Gamma(M, T^* M \otimes S) \xrightarrow{c} \Gamma(M, S),$$ \hspace{1cm} (2.13)

where $c$ denotes the Clifford multiplication. In terms of a local orthonormal basis $e_1, e_2, \cdots, e_n$ of $TM$,

$$\mathcal{D} \psi = e^i \cdot \nabla_{e_i} \psi,$$

where $e^i$ denotes the dual basis.

The two most important properties of hypersurface Dirac operator are the self-adjointness with respect to the metric $(\cdot, \cdot)$ and the Bochner-Lichnerowicz-Weitzenbock formula [Wil, [PT].
**Lemma 2.2** Define a $n-1$ form on $M$ by $\omega = \langle \phi, e^i \cdot \psi \rangle \text{int}(e_i) \text{dvol}$, where $\text{dvol}$ is the volume form of the Riemannian metric $g$. We have

$$[(\phi, \nabla \psi) - \langle \nabla \phi, \psi \rangle] \text{dvol} = d\omega.$$ 

Thus $\mathcal{D}$ is formally self adjoint with respect to the $L^2$ metric defined by $\langle \cdot, \cdot \rangle$ (and $\text{dvol}$).

**Proof.** Since $\omega$ is independent of the choice of the orthonormal basis, we do our computation locally using a preferred basis. For any given point $p \in M$, choose a local orthonormal frame $e_i$ of $TM$ near $p$ such that $\nabla e_i = 0$ at $p$. Extend $e_0, e_i$ to a neighborhood of $p$ in $N$ by parallel translating along $e_0$ direction. Then, at $p$, $\nabla e_i e^j = -h_{ij} e^0$ and $\nabla e_i e^0 = -h_{ij} e^j$. Therefore (again at $p$),

$$d\omega = \nabla_{e_i} \langle \phi, e^i \cdot \psi \rangle \text{dvol}$$

$$= [((\nabla e_i e^0) \cdot \phi, e^i \cdot \psi) + (e^0 \cdot \nabla e_i \phi, e^i \cdot \psi) + (e^0 \cdot \phi, (\nabla e_i e^i) \cdot \psi) + (e^0 \cdot \phi, e^i \cdot \nabla e_i \psi)] \text{dvol}$$

$$= [-h_{ij}(e^j \cdot \phi, e^i \cdot \psi) + (e^i \cdot e^0 \cdot \nabla e_i \phi, \psi) - h_{ii}(e^0 \cdot \phi, e^0 \cdot \psi) + \langle \phi, \nabla \psi \rangle] \text{dvol}$$

$$= [-\langle \nabla \phi, \psi \rangle + \langle \phi, \nabla \psi \rangle] \text{dvol}$$

Now the Bochner-Lichnerowicz-Weitzenbock formula.

**Lemma 2.3** One has

$$\mathcal{D}^2 = \nabla^* \nabla + \mathcal{R},$$

$$\mathcal{R} = \frac{1}{4} (R + 2R_{00} + 2R_{0i} e^0 \cdot e^i) \in \text{End}(S).$$

Here the adjoint $\nabla^*$ is with respect to the metric $\langle \cdot, \cdot \rangle$.

**Proof.** We again do the computation in the frame as in the proof of Lemma 2.2. Then

$$\mathcal{D}^2 = e^i \cdot e^j \cdot \nabla_{e_i} \nabla_{e_j} + e^i \cdot \nabla_{e_i} e^j \cdot \nabla_{e_j}$$

$$= -\nabla_{e_i} \nabla_{e_i} + \frac{1}{4} (R + 2 R_{00} + 2 R_{0i} e^0 \cdot e^i) - h_{ij} e^i \cdot e^0 \cdot \nabla_{e_j}.$$ 

Now

$$d[\langle \phi, \psi \rangle \text{int}(e_i) \text{dvol}] = e_i \langle \phi, \psi \rangle \text{dvol}$$

$$= (\nabla_{e_i} e^0 \cdot \phi, \psi) + (\nabla_{e_i} \phi, \psi) + (\phi, \nabla_{e_i} \psi)$$

$$= -h_{ij} (e^j \cdot \phi, \psi) + (\nabla_{e_i} \phi, \psi) + (\phi, \nabla_{e_i} \psi)$$

$$= -h_{ij} (e^0 \cdot e^j \cdot \phi, \psi) + (\nabla_{e_i} \phi, \psi) + (\phi, \nabla_{e_i} \psi)$$

This shows that $\nabla^*_{e_i} = -\nabla_{e_i} - h_{ij} e^j \cdot e^0$. The desired formula follows.

\[\mathcal{D}\]
3 Proof of the Theorem

By the Einstein equation,
\[ R = 4\pi(T_{00} + T_{0i}e^0 \cdot e^i). \]

It follows then from the dominant energy condition \[ (1.4) \] that
\[ R \geq 0. \] (3.15)

Now, for \( \phi \in \Gamma(M, S) \) and a compact domain \( \Omega \subset M \) with smooth boundary, the Bochner-Lichnerowicz-Weitzenbock formula yields
\[
\int_{\Omega}[|\nabla\phi|^2 + \langle \phi, R\phi \rangle - |D\phi|^2] \, dvol(g) = \int_{\partial\Omega} \sum \langle (\nabla_{e_a} + e_a \cdot D)\phi, \phi \rangle \, int(e_a) \, dvol(g) \tag{3.16}
\]
\[
= \int_{\partial\Omega} \sum \langle (\nabla_{\nu} + \nu \cdot D)\phi, \phi \rangle \, dvol(g|_{\partial\Omega}), \tag{3.17}
\]
where \( e_a \) is an orthonormal basis of \( g \) and \( \nu \) is the unit outer normal of \( \partial\Omega \). Also, here \( int(e_a) \) is the interior multiplication by \( e_a \).

Now let the manifold \( M = M_0 \cup M_\infty \) with \( M_0 \) compact and \( M_\infty \cong (\mathbb{R}^k - B_R(0)) \times X \), and \( (X, g_X) \) a compact Riemannian manifold with nonzero parallel spinors. Moreover, the metric \( g \) on \( M \) satisfies \[ (1.6) \]. Let \( e_a^0 \) be the orthonormal basis of \( \tilde{g} \) which consists of \( \frac{\partial}{\partial x_i} \), followed by an orthonormal basis \( f_{\alpha} \) of \( g_X \). Orthonormalizing \( e_a^0 \) with respect to \( g \) gives rise an orthonormal basis \( e_a \) of \( g \). Moreover,
\[
e_a = e_a^0 - \frac{1}{2} u_{ab} e_b^0 + O(r^{-2\tau}). \] (3.18)

This gives rise to a gauge transformation
\[
A : SO(\tilde{g}) \ni e_a^0 \to e_a \in SO(g)
\]
which identifies the corresponding spin groups and spinor bundles.

We now pick a unit norm parallel spinor \( \psi_0 \) of \( (\mathbb{R}^k, g_{2k}) \) and a unit norm parallel spinor \( \psi_1 \) of \( (X, g_X) \). Then \( \phi_0 = A(\psi_0 \otimes \psi_1) \) defines a spinor of \( M_\infty \). We extend \( \phi_0 \) smoothly inside. Then \( \nabla^0 \phi_0 = 0 \) outside the compact set.

**Lemma 3.1** If a spinor \( \phi \) is asymptotic to \( \phi_0 \): \( \phi = \phi_0 + O(r^{-\tau}) \), then we have
\[
\lim_{R \to \infty} \Re \int_{S_R \times X} \sum \langle (\nabla_{e_a} + e_a \cdot D)\phi, \phi \rangle \, int(e_a) \, dvol(g) = \omega_k vol(X) \langle \phi_0, E\phi_0 + P_k dx^0 \cdot dx^k \cdot \phi_0 \rangle,
\]
where \( \Re \) means taking the real part.

**Proof.** Recall that \( \nabla \) denote the connection on \( S \) induced from the Levi-Civita connection on \( M \). We have
\[
\nabla_{e_a} \psi = \nabla_{e_a} \psi - \frac{1}{2} h_{ab} e^0 \cdot e^b \cdot \psi. \tag{3.19}
\]
By the Clifford relation,

\[ \langle (\nabla e_a + e_a \cdot D) \phi, \phi \rangle = -\frac{1}{2} \langle [e^a, e^b] \nabla e_b \phi, \phi \rangle. \]

Hence

\[ \int_{S \times X} \sum \langle (\nabla e_a + e_a \cdot D) \phi, \phi \rangle \text{int}(e_a) \, d\text{vol}(g) = \]

\[ -\frac{1}{2} \int_{S \times X} \langle [e^a, e^b] \nabla e_b \phi, \phi \rangle \text{int}(e_a) \, d\text{vol}(g) + \frac{1}{4} \int_{S \times X} \langle [e^a, e^b] h_{bc} e^0 \cdot e^c \cdot \phi, \phi \rangle \text{int}(e_a) \, d\text{vol}(g). \]

Using (3.18) and the asymptotic conditions (1.7), the second term in the right hand side can be easily seen to give us

\[ \lim_{R \to \infty} \frac{1}{4} \int_{S \times X} 2(h_{ac} - \delta_{ac} h_{bb}) e^0 \cdot e^c \cdot \phi \text{int}(e_a) \, d\text{vol}(g) = \omega_k \text{vol}(X) \langle \phi_0, P_k dx^0 \cdot dx^k \cdot \phi_0 \rangle. \]

The first term is computed in [D] to limit to

\[ \omega_k \text{vol}(X) \langle \phi_0, E\phi_0 \rangle. \]

The following lemma is standard [PT], [Wi1].

**Lemma 3.2** If

\[ \langle \phi_0, E\phi_0 + P_k dx^0 \cdot dx^k \cdot \phi_0 \rangle \geq 0 \]

for all constant spinors $\phi_0$, then

\[ E - |P| \geq 0. \]

As usual, the trick to get the positivity now is to find a harmonic spinor $\phi$ asymptotic to $\phi_0$. Then the left hand side of (3.16) will be nonnegative since $R \geq 0$. Passing to the right hand side will give us the desired result.

**Lemma 3.3** There exists a harmonic spinor $\phi$ on $(M, g)$ which is asymptotic to the parallel spinor $\phi_0$ at infinity:

\[ D\phi = 0, \quad \phi = \phi_0 + O(r^{-\tau}). \]

**Proof.** The proof is essentially the same as in [D]. We use the Fredholm property of $D$ on a weighted Sobolev space and $R \geq 0$ to show that it is an isomorphism. The harmonic spinor $\phi$ can then be obtained by setting $\phi = \phi_0 + \xi$ and solving $\xi \in O(r^{-\tau})$ from the equation $D\xi = -D\phi_0$.

The rest of the Theorem follows as in [PT].
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