Cosmology with kSZ: breaking the optical depth degeneracy with Fast Radio Bursts

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The small-scale cosmic microwave background (CMB) is dominated by anisotropies from the kinematic Sunyaev-Zeldovich (kSZ) effect, and upcoming experiments will measure it very precisely, but the optical depth degeneracy limits the cosmological information that can be extracted. At the same time, fast radio bursts (FRBs) are an exciting new frontier for astrophysics, but their usefulness as cosmological probes is currently unclear. We show that FRBs are uniquely suited for breaking the kSZ optical depth degeneracy. This opens up new possibilities for constraining cosmology with the kSZ effect, and new cosmological applications for FRBs.

I. INTRODUCTION

As photons from the cosmic microwave background (CMB) travel through the Universe, a small fraction interact with free electrons. The kinematic Sunyaev-Zeldovich (kSZ) effect [1] is the result of CMB photons Compton scattering off free electrons that have non-zero peculiar velocities with respect to the CMB rest frame, which lead to additional anisotropies in the observed CMB radiation. As a result, we observe a small shift in the CMB temperature in the direction of those free electrons. This shift is proportional to the integrated momentum along the line-of-sight. Thus, kSZ measurements are potentially powerful observational probes of the peculiar velocities of systems of ionized gas that trace the total distribution of matter [e.g., 2–5]. Since the small-scale CMB is dominated by kSZ fluctuations and upcoming CMB surveys will measure it very precisely [6, 7], mapping the peculiar velocity distribution of the Universe with kSZ will provide competitive constraints on primordial non-Gaussianity [8]. Velocities probe the cosmological growth rate, which can allow further constraints on modified gravity models, the dark energy equation of state, and the sum of neutrino masses [e.g., 4, 6, 8–14].

The cosmological growth rate measured through the kSZ effect is however perfectly degenerate with the optical depth of galaxies or clusters [e.g., 15, 16] leading to an overall uncertainty in the inferred amplitude of the growth rate. This degeneracy with the optical depth is the limiting systematic uncertainty for measurements of the cosmological growth rate from kSZ tomography [e.g., 7, 12, 14].

Recent detections of multiple Fast Radio Burst Sources (FRBs1) along with theoretical models strongly suggest that there exist transient radio events originating (possibly) from energetic events at cosmological redshifts that are detectable with a rate greater than one per day [e.g., 17–19]. With future upgrades and outrigger stations, instruments like HIRAX [20] should localize of order 10 FRBs per day with sub-arcsecond accuracy which will enable one to acquire redshifts [21]. Plasma along the line of sight delays the FRBs in a frequency-dependent manner, with the delay in seconds approximately equal to $4.15 \times 10^{-3} \text{DM}/\nu_{\text{GHz}}^2$, where DM is the dispersion measure, in pc/cm$^3$, and is equivalent to the optical depth $\tau$ due to Compton scattering: $\text{DM} = (4.87 \times 10^5 \text{ pc cm}^{-3}) \tau$. Radio telescopes measure the DMs associated with these events quite precisely (typical measurement accuracies are 0.1%), which receive contributions from the host galaxy, the Milky Way, and any intervening free electrons [e.g., 22]. The third of these contributions is of great interest to the extragalactic and cosmological communities. With the promise of thousands of FRBs in the future, theoretical ideas and forecasts have been published regarding measuring the baryon content in the Intergalactic Medium [IGM, e.g., 23] and the Circumgalactic Medium [CGM, e.g., 24], regarding constraining the reionization epoch [25], and regarding measuring 3D clustering of free electrons [26] to name a few.

In this work, we propose to directly measure the galaxy optical depth through the contribution to FRB DMs from scattering of intervening free electrons, using the cross-correlation between the galaxy sample used in the kSZ measurement and a map of FRB dispersion measures. This cross-correlation can be directly interpreted as the galaxy optical depth as it is measuring the galaxy-electron power spectrum $P_g(k)$, thus breaking the optical depth degeneracy and allowing for sub-percent constraints on the growth rate. We focus on the information on the cosmological growth rate that we can extract with thousands or more of localized FRB measurements in combination with kSZ measurements made by upcoming CMB and galaxy surveys. We note that a recent paper [27] investigated the possibility of using FRBs for

1 Hereafter, FRBs refers to the sources, not the bursts.
cosmological tests, but found no interesting applications other than constraining the ionized gas distribution. We show in this work that constraining ionized gas (specifically, the galaxy-electron correlation) with FRBs enables cosmological applications of the kSZ effect.

II. THE GALAXY-ELECTRON SPECTRUM MEASURED WITH FRBS

The dispersion measure (DM) along a line of sight should be correlated with the density of foreground galaxies in that direction, since some of the electron fluctuations contributing to the DM originate from those galaxies. We are thus interested in cross correlating foreground galaxies with a map of DMs (not spatial locations) from FRBs. Note that this does not require FRBs to be clustered or for them to have redshift overlap with the galaxies. They instead act as a backlight for the free electrons in these galaxies, like quasars act for neutral hydrogen. The FRBs need to be localized with redshift information sufficient to inform whether or not the FRB in any given FRB-galaxy pair is behind the galaxy.

Because the DM is an integrated quantity along the line of sight, it is convenient to do the forecast using 2-d fields (not 3-d). For this preliminary investigation into the feasibility of using FRBs for cosmology, we work with a simplified geometry. We consider a thin shell of foreground galaxies, specifically a sample with a mean redshift of 0.75, redshift shell width of 0.3 and number density of $\sim 1.7 \times 10^{-4}$ Mpc$^{-3}$ expected to be provided by surveys like the Dark Energy Spectroscopic Instrument (DESI) [28]. All the FRBs are assumed to lie in a thin background shell centered at $z = 1$. In this thin-shell geometry, we can treat all fields in sight as 2-d fields.

Let \((\chi_g - \Delta \chi_g/2, \chi_g + \Delta \chi_g/2)\) be the comoving distance interval spanned by the foreground galaxies, and let \((\chi_f - \Delta \chi_f/2, \chi_f + \Delta \chi_f/2)\) be the comoving distance interval spanned by the background FRBs. We will use the notation \(\langle \cdot \rangle_g\) to mean “evaluated at the redshift of the galaxies”, e.g. \(z_g\) is the galaxy redshift.

We assume that the separation between the foreground and background shells is large enough that there are no spatial correlations between foreground galaxies and the spatial locations (or the host DMs) of background FRBs. Thus any galaxy-DM correlations can be attributed to correlations between the galaxies and the electrons along the line of sight in those galaxies.

The line-of-sight integral for the dispersion measure is (see e.g. [29]):

$$D(\mathbf{n}) = n_c z_0 \int_0^{\chi_f} d\chi (1 + z)(1 + \delta_e(\mathbf{n}, z)),$$

where \(\mathbf{n}\) is the line of sight direction, \(n_c z_0\) is the mean number density of free electrons at \(z = 0\), and \(\chi\) is the comoving distance. In the Limber approximation (equivalent to a small-angle approximation which is valid for the scales we consider), the cross-correlation between the 2-d DM field, \(D\), and the 2-d galaxy overdensity field, \(\delta_g\), is:

$$C_l^{Dg} = n_c z_0 \frac{1 + z_g}{\chi_g^2} P_{ge}(k, z_g)_{k = l/\chi_g},$$

where the \(C_l\) notation denotes angular power spectra at angular wavenumber or multipole \(l\), and \(P_{ge}\) is the 3D galaxy-electron cross power spectrum as a function of the magnitude \(k\) of the 3D Fourier wavenumber \(k\). Our proposed observable \(C_l^{Dg}\) thus measures the power spectrum \(P_{ge}\), which is an important quantity that captures how the free electron overdensity \(\delta_e\) is correlated with the galaxy overdensity \(\delta_g\). As explained in [7], the very same power spectrum \(P_{ge}\) is also measured by kSZ tomography. However, for cosmological applications of kSZ, \(P_{ge}\) appears in a nuisance parameter that multiplies the cosmologically informative cross power spectrum of galaxies and the cosmic velocity field \(P_{gv}\) [e.g., 7]. This motivates our external measurement of \(P_{ge}\) from FRB DMs.

To complete our forecast for the signal-to-noise-ratio (SNR) of \(C_l^{Dg}\), we also need the associated auto power spectra (again making the Limber approximation):

$$C_l^{DD} = n_c^2 \int_0^{\chi_f} d\chi \frac{(1 + z)^2}{\chi^2} P_{ee}(k, z)_{k = l/\chi},$$

$$C_l^{gg} = \frac{1}{\chi_g^2 (\Delta \chi_g)} P_{gg}(k, z_g)_{k = l/\chi_g},$$

where \(P_{ee}\) and \(P_{gg}\) are the electron and galaxy auto power spectra, respectively.
The small-scale power spectra above are calculated in the halo model following [7], with contributions from clustering of electrons and galaxies (the 2-halo term) and from the shape of the profiles of the electron and galaxy distributions (the 1-halo term). The calculated 2-d power spectra are shown in Fig. 1. When we “observe” the 2-d DM field, $D$, with a discretely sampled catalog of FRBs, there is an associated noise power spectrum $N_{DD}^D$ given by:

$$ N_{DD}^D = \frac{\sigma_D^2}{n_f^2} \quad (5) $$

Here, $n_f^2$ is the number density (per steradian) of FRBs, and $\sigma_D^2$ is the total variance of the DMs. The latter is the sum of three contributions: intrinsic scatter in the FRB host’s DM, residual uncertainty in the DM of our galaxy, and a term $\int d^2l/(2\pi)^2 C_{l}^{DD}$ from electron fluctuations along the line of sight that not associated with galaxies we are cross correlating with, the cosmological variance.

We will not worry about keeping track of these contributions separately, since the host contribution is a free parameter anyway. Since the RMS scatter of the DMs $\sigma_D$ is currently uncertain, we show forecasts for various plausible values given current detections of FRBs. We chose the range to be from 100 pc/cm$^3$ to 1000 pc/cm$^3$. This range is motivated by empirical measurements of the intrinsic DM of the host of the repeating FRB [30], which has DM of $\lesssim 324$ pc/cm$^3$ [21]. The cosmological DM RMS scatter is of order 100-1000 pc/cm$^3$ from our halo model calculations and from simulations [22]. The DM of our galaxy varies dramatically depending on sky location, however models exist [e.g., 31] to remove this contribution with only a small, uncorrelated residual left to contribute to the variance.

In terms of the above definitions, the total $S/N$ of the DM-galaxy cross power is given by:

$$ S/N^2 = \Omega \int \frac{d^2l}{(2\pi)^2} \left( \frac{C_{l}^{Dg}}{N_{l}^{Dg}} \right)^2, \quad (6) $$

where

$$ (N_{l}^{Dg})^2 = (C_{l}^{gg} + 1/n_f^2)(C_{l}^{DD} + \sigma_D^2/n_f^2), \quad (7) $$

$n_f^2$ is the number density of galaxies in the galaxy survey (per steradian), and $\Omega$ is the angular size of the survey in steradians which accounts for the partial sky coverage fraction $f_{sky}$ of the survey overlap through $\Omega = 4\pi f_{sky}$.

Using Eq. 2 and Eq. 6, we can also obtain the uncertainty on the bandpowers of the galaxy-electron power inferred from the DM-galaxy cross correlation (see Appendix A for details),

$$ \Delta P_{ge} = \frac{\chi_g}{n_{g0}(1 + z_g)} \left( \Omega \int_{k_{\min}}^{k_{\max}} \frac{k dk}{2\pi} \left( \frac{1}{(N_{l}^{Dg})^2} \right) \right)^{-1/2} \quad (8) $$

In Fig. 2, we show the galaxy-electron power spectrum along with the uncertainties on its bandpowers from a measurement made using the DESI galaxy sample cross-correlated with $10^4$ FRB DMs, assuming the simplified geometry described above and a DM RMS scatter of 300 pc/cm$^3$. For comparison, we also show the uncertainties on the $P_{ge}$ from a kSZ tomography [7] measurement using the proposed CMB-S4 experiment [32] and DESI, where we have assumed that the factor that multiplies $P_{ge}$ and depends on the cosmologically informative power spectrum $P_{g\gamma}$ has been fixed to a fiducial cosmology. We see
that FRB DMs measure $P_{ge}$ over a broad range of scales, while as noted in [7], kSZ tomography measures it very well only in a small range of scales in the 1-halo regime.

III. THE COSMOLOGICAL CONNECTION

The FRB-determined measurement of the small-scale cross-power-spectrum of galaxies and electrons $P_{ge}(k)$ detailed in the previous section can be used to break a degeneracy that limits the cosmological utility of kSZ tomography. Since the kSZ effect arises from the Doppler shifting of CMB photons that Compton scatter off free electrons with bulk radial velocities, the large scale cosmological velocity field modulates the cross-power-spectrum of the CMB temperature and galaxy overdensity field. This idea allows one to infer the large-scale cosmic velocity field from a combination of the CMB temperature anisotropies as measured by a CMB survey and the positions of galaxies as measured by a galaxy survey [7, 33] on small scales. However, this velocity field can only be inferred up to an overall constant $b_v$, since the kSZ effect is proportional to both the bulk radial velocity and the overdensity of free electrons. This unknown constant $b_v$ is in fact an integral over precisely the small-scale galaxy-electron power spectrum $P_{ge}(k)$ that can be measured with FRB DMs.

On large scales where linear theory is valid, the velocity reconstruction from kSZ tomography is directly proportional to the cosmic growth rate $f(a) \approx \frac{D_2(a)}{D_1(a)}$, where $D(a)$ is the growth factor for the matter spectrum that evolves as $P_{mm}(a) = D^2(a)P_{mm}(a = 1)$ and $a$ is the expansion scale factor. Since the velocity reconstruction is uncertain up to the amplitude $b_v$, in order to convert a kSZ tomography measurement to cosmological information on massive neutrinos, dark energy perturbations, modified gravity and other physics that can affect the growth rate, one needs an external measurement of $b_v$, or equivalently of $P_{ge}(k)$. This is the so-called “optical depth degeneracy”. To summarize, the program of constraining cosmology using linear theory with large-scale ($k << 0.1$ Mpc$^{-1}$) velocities from kSZ requires knowledge of an integral of the galaxy-electron power spectrum over extremely non-linear small scales ($0.1$ Mpc$^{-1} < k < 10$ Mpc$^{-1}$).

We have seen in the previous section that FRB DMs can provide this external measurement of small-scale $P_{ge}(k)$. At the back-of-the-envelope level, a 1% constraint on $P_{ge}(k)$ from FRBs (or equivalently $S/N = 100 \sigma$ on $C^D_{g}$) could translate to a 1% constraint on the velocity bias $b_v$. However, in practice, the velocity bias information in the FRB measurement is somewhat lower, because FRB DMs measure $P_{ge}(k)$ over a broad range of scales while (due to the squeezed bispectrum origin of the kSZ effect) the optical depth degeneracy is sourced primarily by small scales in the “1-halo” regime. In Appendix B, we obtain the 1-sigma constraint $\sigma(b_v)$ from FRB DMs properly accounting for this.

In Fig. 3 we show both the raw SNR for the measurement of $C^D_{g}$ (or $P_{ge}(k)$) using FRB DMs and the DESI galaxy survey (from Eq. 6), and the closely related relative uncertainty on the velocity bias $b_v$. As expected the SNR on the velocity bias is slightly lower. The SNR saturates at high FRB number density when it becomes limited by the sample variance $C_{g}^D$ in the contribution to DMs from intervening free electrons.

We can now obtain constraints on the cosmic growth rate that incorporate prior information on $b_v$ from FRBs. The large scale velocity field in linear theory inferred from kSZ tomography is now,

\begin{equation}
\hat{v}_{\text{rec}}(k) = b_v \frac{f a H}{k} \delta_m(k),
\end{equation}

where $\mathbf{k}$ is the 3-d wavevector, $\mu = k_r/k$ for radial component of the wavevector $k_r$ (along the line of sight), $H$ is the Hubble constant at the redshift of the galaxy sample and $\delta_m$ is the matter overdensity. The velocity reconstruction is performed over small-wavelength modes $k_S$ in the high-resolution CMB survey and the galaxy sur-

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2 Hereafter, we will refer to the unknown quantity (whose priors we obtain from FRBs) as the ‘velocity bias’, which can be loosely interchanged with ‘optical depth’.

3 Note however that scale-dependent effects, e.g. scale-dependent galaxy bias from primordial non-Gaussianity, can be constrained extremely well [34].
vey. The modes $k_S$ are limited to $0.1 \text{ Mpc}^{-1} < k < 10 \text{ Mpc}^{-1}$.

We marginalize over $b_g$ for a fiducial value of $b_v = 1$ but with the Gaussian prior determined earlier that depends on the number of FRBs, $N_{\text{FRB}}$.

![FIG. 4](image)

FIG. 4: The uncertainty on the combination of cosmic growth rate and amplitude of matter fluctuations $f\sigma_8$ from kSZ tomography with CMB-S4 and DESI as a function of the number of FRBs, $N_{\text{FRB}}$, available to break the ‘cluster optical depth degeneracy’ through cross-correlation of FRB DMs with the same DESI galaxy sample. The blue lines show the constraint from kSZ tomography with various shades corresponding to choices of the uncertain RMS scatter of FRB DMs $\sigma_D$. If RSD information is used in conjunction with kSZ (red dashed lines), the degeneracy is already broken to some degree but further improvement is possible with FRBs. The grey dot-dashed line shows the constraint from DESI RSD alone.

We consider survey combinations comprising of DESI and CMB-S4 and leave $N_{\text{FRB}}$, which are localized with redshifts as a free parameter. The assumed configurations of DESI and CMB-S4 can be found in [7]. DESI and CMB-S4 are used to obtain the above velocity reconstruction. The reconstruction $v_{\text{rec}}$ can then be combined with the galaxy overdensity field $\delta_g$ from DESI. The noise on the velocity reconstruction is given by [7],

$$N_{vv}(k) = \mu^{-2} \frac{\chi_\mu^2}{K_\mu^2}$$

$$= \left[ \frac{k_S d k_S}{2\pi} \left( \frac{P_{gg}(k_S)}{P_{\text{tot}}(k_S)C_{TT,\text{tot}}^{TT,\text{tot}}(k_S)} \right)_{l=kS\chi_S} \right]^{-1},$$

where $K_\mu$ is the kSZ radial weight function (defined in [7]) at the galaxy shell redshift, $\chi_\mu$ is the comoving distance to the galaxy shell redshift, and $C_{TT,\text{tot}}^{TT,\text{tot}}$ is the total angular power spectrum of CMB temperature anisotropies, including the late-time and reionization kSZ and foreground residuals after multi-frequency cleaning.

This combination gives us the following power spectra

$$P_{gg}(k, \mu) = (b_g + f(z)\mu^2)^2 P_{mm}(k),$$

$$P_{gg}(k, \mu) = b_v \left( \frac{f(z) aH(z)}{k} \right) (b_g + f(z)\mu^2) P_{mm}(k),$$

$$P_{vv}(k, \mu) = b_v^2 \left( \frac{f(z) aH(z)}{k} \right)^2 P_{mm}(k),$$

where $b_g$ is the linear galaxy bias, $P_{gg}$ is the galaxy-velocity cross power spectrum, $P_{gg}$ is the galaxy auto power spectrum, and $P_{vv}$ is the velocity auto power spectrum. We only include the redshift-space distortion (RSD) term $f\mu^2$ [35] in Eqs. 11 and 12 if explicitly mentioned from here on. As mentioned in [7], the velocity reconstruction formalism explicitly shows how the ‘octopolar pair sum’ estimator of [36] that utilizes higher moments of the galaxy-velocity correlation in redshift space can break the optical depth degeneracy. However, DMs from FRBs can be used as an independent way of breaking the optical depth degeneracy that is not affected by potential systematics in RSD measurements [37]. We thus do not include the $f\mu^2$ term in our baseline forecasts.

We can now forecast for cosmological parameters by constructing the Fisher matrix for the modes of the galaxy overdensity field and the reconstructed velocities

$$F_{ab} = \frac{V}{2} \int \frac{2\pi dk}{(2\pi)^3} \int_{-1}^1 d\mu \text{Tr} \left[ C_a C^{-1} C_b C^{-1} \right]$$

with covariance matrix,

$$C = \begin{bmatrix} P_{gg} + N_{gg} & P_{gg} & P_{gg} \\ P_{gg} & P_{vv} + N_{vv} & P_{vv} \end{bmatrix}$$

where $V$ is the total volume of the overlapping survey in Mpc$^3$, $N_{gg} = 1/n_{gal}$ is the shot noise contribution to the large scale galaxy power spectrum, with $n_{gal} = 1.7 \times 10^{-4}$ Mpc$^{-3}$ assumed for DESI. We consider a cosmological model parameterized by the scale-independent growth rate $f$ at $z = 0$ and the amplitude of matter fluctuations $\sigma_8$ at $z = 0$. We perform a Fisher analysis for the parameterization $\{b_g \sigma_8, f \sigma_8, b_v\}$ around fiducial parameters $\{b_g = 1.51, f = 0.53, \sigma_8 = 0.83, b_v = 1\}$ and use priors on $b_v$ obtained using the results in Appendix B. We then obtain the marginalized constraint on $f\sigma_8$ shown in Fig. 4.

**IV. RESULTS AND DISCUSSION**

We have shown that when the dispersion measures of FRBs are cross-correlated with a galaxy survey, we can reconstruct the galaxy-electron power spectrum $P_{ge}$,
which is precisely the observable that breaks the kSZ optical depth degeneracy, thus enabling cosmological applications of the kSZ effect. We find that the cross-correlation of DMs from FRBs with a galaxy survey like DESI is detectable, if around 100-1000 FRBs can be localized with sufficient redshift information to place them behind the DESI sample (Fig. 3). Such measurements translate into constraints on the optical depth of DESI galaxies at the 1% level for 100,000 localized FRBs if $\sigma_D \sim 300$ pc/cm$^3$. In Fig. 4, we show how such optical depth priors from FRB-galaxy cross-correlations translate to cosmological growth rate measurements from kSZ tomography with CMB-S4 and DESI. We show that $< 1\%$ level constraints can be obtained with $N_{\text{FRB}} > 10^5$ and $\sigma_D \sim 300$ pc/cm$^3$ independent of RSD measurements. Additionally, we show improvements of up to 50% if $\sigma$ DESI galaxies at the 1% level for 100,000 localized FRBs can also be made when combined with RSD for very large samples. Beyond breaking the optical depth degeneracy, the cross-correlation of DMs from FRBs with galaxy surveys provides constraints on the baryon distribution in galaxies and clusters. On small scales, the shape of $P_{\nu c}(k)$ is a measurement of the 1-halo electron free electron profile. As previous theoretical works have shown for real space (not Fourier space) [e.g., 23, 24, 40, 41], this provides valuable information on baryon density profiles of galaxies, groups, and clusters. Additionally, the profiles inferred from FRB DMs are unbiased. Obtaining such profiles provides information on the impact of baryons on the matter power spectrum [e.g., 42], which is currently unconstrained by empirical measurements, but is extremely important for future cosmological measurements that aim to probe the matter power spectrum on small scales [e.g., 43–45].

We have chosen the growth rate $f \sigma_8$ in these forecasts as it is a model-independent parameterization for the physics probed by cosmic velocities. The growth rate however can be affected by massive neutrinos, dark energy perturbations and modifications of General Relativity. We thus expect that the breaking of the optical depth degeneracy achieved using FRBs put forward in this work can yield significant constraints on extensions of the standard model of cosmology. This will also require going beyond the simplistic cosmological parameterization that we have considered here (for example, incorporating marginalization over the Hubble constant and matter density, while imposing priors from primary CMB measurements). These explorations are left for future work.

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The statistical error on the bandpower is then given by:

\[
\text{SNR}^2_{\text{bin}} = \frac{\Omega}{2\pi N_i^{\text{Dg}}/2} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{\mathrm{d}l}{\Delta l} \left(\frac{C_l^{\text{Dg}}}{N_i^{\text{Dg}}}\right)^2 \tag{A1}
\]

The statistical error on the bandpower is then given by:

\[
\Delta C_l^{\text{Dg}} = \frac{C_l^{\text{Dg}}}{\text{SNR}_{\text{bin}}} = \left(\frac{\Omega}{2\pi N_i^{\text{Dg}}/2} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{\mathrm{d}l}{\Delta l} \frac{1}{\left(\frac{N_i^{\text{Dg}}}{2}\right)^{1/2}} \right)^{-1/2} \tag{A2}
\]

In the thin-shell approximation, \(C_l^{\text{Dg}}\) is related to \(P_{ge}(k)\) by Eq. (2). Therefore, we can recast the preceding result as the statistical error on a \(P_{ge}\) bandpower over \(k\)-range \([k_{\text{min}}, k_{\text{max}}]\) to obtain Eq. 8.

**Appendix B: Velocity bias prior**

At back-of-the-envelope level, the constraint on \(b_v\) is 

\[\sigma(b_v) = 1/\text{SNR}, \text{ where the SNR of the FRB-galaxy cross-}\]
correlation was given in Eq. (6). However, this estimate is optimistic, since the SNR is obtained by summing all k-bins, whereas the kSZ velocity bias only depends on \( P_{ge} \) in a specific k-range.

To derive a better estimate for \( \sigma(b_v) \) which we use in the rest of this work, we recall that the kSZ velocity-bias \( b_v \) is defined by:

\[
b_v = \frac{\int dk_S F(k_S) P_{true}^{ge}(k_S)}{\int dk_S F(k_S) P_{fid}^{ge}(k_S)}
\]

(B1)

where

\[
F(k_S) = k_S \frac{P_{fid}^{ge}(k_S)}{P_{tot}^{gg}(k_S)} \left( \frac{1}{C_l^{TT,tot}} \right)_{l=k_S \chi_g}
\]

(B2)

and the integration range is over small-scale wavenumbers \( 0.1 \ Mpc^{-1} < k < 10 \ Mpc^{-1} \). We can obtain an estimate for the uncertainty \( \sigma(b_v) \) on \( b_v \) by relating it to the uncertainty \( \Delta P_{ge} \) on the bandpowers of \( P_{ge} \) through a quadrature sum of uncertainties, as is appropriate for uncorrelated bins that are normally distributed. To do this, we define a large number of \( k_S \)-bins, with width \( \Delta k_S \). Replacing the integral in the numerator of Eq. (B1) by a sum, the statistical error on \( b_v \) is:

\[
\sigma(b_v)^2 = \frac{\sum F(k_S)^2 \Delta P_{ge}(k_S)^2 (\Delta k_S)^2}{(\int dk_S F(k_S) P_{fid}^{ge}(k_S))^2}
\]

(B3)

where the sum in the numerator runs over \( k_S \)-bins. For notational compactness, we rewrite Eq. (8) in the form:

\[
(\Delta P_{ge}(k_S))^2 = G(k_S) (\Delta k_S)
\]

(B4)

where we have defined:

\[
G(k_S) = \left( \frac{\chi_g}{n_{e0}(1+z_g)} \right)^{-2} \left( \frac{k_S \Omega}{2\pi} \right) \left( \frac{1}{(C_l^{gg} + 1/n_2^{2d})(C_l^{DD} + \sigma^2_D/n_f^{2d})} \right)_{l=k_S \chi_g}
\]

(B5)

Plugging Eq. (B4) into Eq. (B3), we get our final expression for \( \sigma(b_v) \):

\[
\sigma(b_v)^2 = \frac{\sum F(k_S)^2 G(k_S)^{-1} (\Delta k_S)}{(\int dk_S F(k_S) P_{fid}^{ge}(k_S))^2}
\]

(B6)

where we have converted the sum back to an integral in the second line. It is possible to prove using this expression that \( \sigma(b_v) \geq 1/\text{SNR} \), so our “refined” estimate for \( \sigma(b_v) \) is more pessimistic than the back-of-the-envelope estimate \( \sigma(b_v) \approx 1/\text{SNR} \), as anticipated. In this work, wherever a prior on \( b_v \) is assumed, the “refined” estimate derived here is used.