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Abstract: Cosmic microwave background (CMB) radiation is characterized by well-established scales, the 2.7 K temperature of the Planckian spectrum and the 10^-5 amplitude of the temperature anisotropy. These features were instrumental in indicating the hot and equilibrium phases of the early history of the Universe and its large-scale isotropy, respectively. We now reveal one more intrinsic scale in CMB properties. We introduce a method developed originally by Kolmogorov, which quantifies a degree of randomness (chaos) in a set of numbers, such as measurements of the CMB temperature in a given region. Considering CMB as a composition of random and regular signals, we solve the inverse problem of recovering of their mutual fractions from the temperature sky maps. Deriving the empirical Kolmogorov’s function in the Wilkinson Microwave Anisotropy Probe’s maps, we obtain the fraction of the random signal to be about 20 per cent; i.e., the cosmological sky is a weakly random one. The paper is dedicated to the memory of Vladimir Arnold (1937-2010).

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A weakly random Universe?

V.G. Gurzadyan\textsuperscript{1}, A.E. Allahverdyan\textsuperscript{1}, T. G. Gharabamanyan\textsuperscript{1},
A.L. Kashin\textsuperscript{1}, H.G. Khachatryan\textsuperscript{1}, A.A. Kocharyan\textsuperscript{1,2},
S. Mirzoyan\textsuperscript{1}, E. Poghosian\textsuperscript{1}, D. Vetrugno\textsuperscript{3}, G. Yegorian\textsuperscript{1}

\textsuperscript{1} Yerevan Physics Institute and Yerevan State University, Yerevan, Armenia
\textsuperscript{2} School of Mathematical Sciences, Monash University, Clayton, Australia
\textsuperscript{3} Salento University and INFN, Sezione di Lecce, Lecce, Italy

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\textbf{ABSTRACT}

The cosmic microwave background (CMB) radiation is characterized by well-established scales, the 2.7 K temperature of the Planckian spectrum and the $10^{-5}$ amplitude of the temperature anisotropy. These features were instrumental in indicating the hot and equilibrium phases of the early history of the Universe and its large scale isotropy, respectively. We now reveal one more intrinsic scale in CMB properties. We introduce a method developed originally by Kolmogorov, that quantifies a degree of randomness (chaos) in a set of numbers, such as measurements of the CMB temperature in some region. Considering CMB as a composition of random and regular signals, we solve the inverse problem of recovering of their mutual fractions from the temperature sky maps. Deriving the empirical Kolmogorov’s function in the Wilkinson Microwave Anisotropy Probe’s maps, we obtain the fraction of the random signal to be about 20 per cent, i.e. the cosmological sky is a weakly random one. The paper is dedicated to the memory of Vladimir Arnold (1937-2010).

\textbf{Key words.} cosmology, cosmic background radiation

The high accuracy Planckian spectrum, the quadrupole anisotropy, the acoustic peaks of the power spectrum (Mather 1994, Smoot et al 1992, de Bernardis et al 2000) of the cosmic microwave background (CMB) appear to be carriers of a valuable information on the early Universe. Among other peculiarities of the CMB signal is that it is well described by Gaussian distribution of initial fluctuations. This fact is remarkable at least by two reasons: Gaussian fluctuations follow from the most simple single scalar field version of inflation. The Gaussian is also a limiting distribution, in accordance to the central limit theorem, at the sum of large number of independent random variables of finite variance. CMB fitting the Gaussian, certainly, does not reveal its randomness, as it might arise of random, as well as regular distributions.

Is it possible to reveal the fractions of random and regular signals in the CMB?

One can deal with this problem using a rigorous tool, the Kolmogorov distribution and the stochasticity parameter as a measure of randomness (Kolmogorov 1933, Arnold 2008a, Arnold 2008b has called it, concerning the asymptotic behavior of the distribution of $\lambda$ for any continuous $F$

\[ \lim_{n \to \infty} P\{\lambda_n \leq \lambda\} = \Phi(\lambda), \]

where the limiting distribution for any real $\lambda > 0$ is

\[ \Phi(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2\lambda^2}, \Phi(0) = 0, \]

the convergence is uniform, and $\Phi$, the Kolmogorov’s distribution, is independent on the distribution $F$ of the initial
random variable $X$. The interval of probable values of $\lambda$ yields $0.3 \leq \lambda_n \leq 2.4$ (Kolmogorov 1933). Such universality of Kolmogorov’s distribution marks it as a measure of stochasticity degree of datasets (Arnold 2008abc,2009ab).

This objective measure enables to consider composition of signals of various randomness, e.g. the behavior of $\lambda_n$ and $\Phi$ was studied at numerical simulations of sequences in (Ghahramanyan et al 2009) [Mirzoyan & Poghosian 2009].

A remarkable system this approach to be applied is the CMB dataset, just because its cumulative distribution function happen to be known and it is close to Gaussian! Indeed, then for Gaussian distribution function $F$ and via a properly developed strategy (for arithmetical and numerical details see (Gurzadyan et al 2009) [Gurzadyan & Kocharyan 2008] one can obtain the distribution of $\Phi$ for CMB, the Kolmogorov map for WMAP’s temperature data, as shown in Fig.1. Note, that not only the non-cosmological signal, the Galactic disk, is clearly outlined, which was not at all trivial a priori, but also such non-Gaussian feature as the Cold Spot appears to be also outlined by specific behavior of the function $\Phi$ compatible with its void nature (Gurzadyan and Kocharyan 2008,2009; Gurzadyan et al 2005,2009). The Gaussian nature of CMB is characterized by certain correlations studied by traditional methods, i.e. the harmonic analysis, correlations functions, power spectrum, which were particularly informative due to the structures of the acoustic peaks. The application of Kolmogorov’s approach will complement those studies quantifying the cumulative degree of correlations in the CMB signal. Kolmogorov’s parameter is also efficient in detecting point sources, radio and Fermi/LAT gamma-ray, and other anomalies in CMB maps (Gurzadyan et al 2010) [Gurzadyan et al 2008], i.e. this method enables the separation of the cosmological signal from non-cosmological ones.

In the present paper, for the first time, we undertake a novel analysis of the cosmological CMB signal revealed by the $\Phi$ distribution. First, we obtain the Kolmogorov distribution $\Phi(\lambda)$ for different regions of sky and WMAP’s 7-year W-band data (Jarosik et al 2010) in HEALPix representation (Gorski et al 2005). The procedure was as follows: circles of radii $1^\circ, 2^\circ$ have been considered, namely, in order to have enough number of pixels for the estimation of the mean $\Phi(\lambda)$, given that the $\lambda$ itself is a random parameter; the variance of the Gaussian then has been estimated for each circle region containing in average around 540 pixels. Then the resulting frequency distribution vs $\Phi$ has been obtained which appears to be decaying, corresponding to 10000 2D-balls randomly distributed over the sky with the consequent elimination of belts of coordinates $|b| < 20^\circ$, $|b| < 30^\circ$, $|b| < 40^\circ$, in order to evaluate the role of the contamination of the Galactic disk: Fig. 2 shows the plot for $|b| < 30^\circ$. The 2D-balls obviously cover also the cold spot regions studied in (Jarosik et al 2010) [Gurzadyan et al 2009]; the point sources revealed in CMB maps (Gurzadyan et al 2010) are also covered, however they cannot contribute to the distribution of $\Phi$ not only because of their small quantity but mainly because of their point-like feature, i.e. small number pixelization.

The first conclusion drawn from Figure 2 is an expected one: the CMB signal is certainly not random Gaussian, otherwise the stochasticity parameter and hence, the function $\Phi$ would have a uniform distribution, i.e. constant, as follows from Kolmogorov theorem.

![Fig. 1. The Kolmogorov map of cosmic microwave background radiation, i.e. the distribution of Kolmogorov’s function $\Phi$ over the sky for Wilkinson Microwave Anisotropy Probe’s 7-year W-band temperature dataset.](image1)

Now, once Kolmogorov theorem enables to estimate the randomness $\Phi$ for a given sequence, having the empirical distribution of $\Phi$ of Figure 2, one can inquire into the inverse problem: to reconstruct the initial signal which would result in that distribution.

To solve this inverse problem we consider a regular sequence represented in a form (cf. (Arnold 2008b))

$$c_n = (a + bn) \mod(b - a),$$

i.e. as an arithmetical progression repeated within an interval $[a, b]$, $h$ is an arbitrary constant.

Since the sequence (3) corresponds to a uniform distribution, one can use von Neumann’s method (von Neumann 1951) to extract from it a Gaussian non-random sequence of standard deviation $\sigma$ and mean value $m$. The latter we consider within the adopted interval $a = m - 10\sigma$ and $b = m + 10\sigma$. 

![Fig. 2. The empirical frequency distribution of Kolmogorov’s function $\Phi$ in the CMB map with extracted Galactic disk region $|b| < 30^\circ$ and the generated signal $c_n$ for the fraction of the random sequence $\alpha = 0.19$.](image2)
and mention with mean $\in$ with their varying proportion defined by the parameter $\alpha$. Thus, we have a sum of random and regular sequences i.e. the one derived by means of von Neumann method.

By generating many regular sequences with Gaussian distribution, i.e. when $\Phi$ is a Gaussian; if the original sequence $a_n$ is a random one, then one gets a random Gaussian sequence, and a non-random one otherwise.

By generating regular sequences with Gaussian distribution of mean and standard deviation corresponding to the WMAP’s CMB signal and calculating the Kolmogorov parameter for each of them, we were able to solve the inverse problem and retrieve the Kolmogorov theorem that we used at the analysis of real datasets, the WMAP’s 7-year CMB maps. Namely, we showed that the random component is a minor perturbation and properties of those processes.

The resulting $\alpha$ distributions are shown in Figure 2 along with WMAP’s frequency plot for the CMB map with extracted Galactic disk of $|b| < 20^\circ$. If the small difference, of few per cent, between the values of $\alpha$ of the CMB maps with extracted $|b| < 20^\circ$ and $|b| < 30^\circ$ regions is due to the Galactic disk contamination, then one can state that the latter contributes to a slight regularization of the CMB signal.

The methodology of Kolmogorov’s approach is based on the study of statistical properties, namely, of the degree of randomness of the signal via the stochasticity parameter and the distribution $\Phi$. As appears, those characteristics are enough informative to distinguish one signal from the other, for example, the cosmological CMB and the Galactic synchrotron one. So, although the photons of such different origin can have the same temperature, they are different by their degree of randomness and hence by their correlation properties, e.g. the synchrotron is characterized by a power-law spectrum, while CMB by a Planckian one. This difference is reflected in the statistical analysis. However, here we went further and quantified the contribution of the correlations and of chaotic components. The correlations in the CMB signal are carriers of information on various processes, from primordial and of the last scattering and reionization epochs up to the integrated Sachs-Wolfe and Sunyaev-Zeldovich effects. The revealing of the degree of correlations in CMB signal, particularly, reflects the cumulative contribution and properties of those processes.

Thus, considering CMB as composition of signals, we showed that the random component is a minor perturbation to a mostly regular signal. The model independent character of this result is due to the universality of the Kolmogorov theorem that we used at the analysis of real datasets, the WMAP’s 7-year CMB maps. Namely, we solved by numerical methods the inverse problem and recovered the mutual fractions of random and nonrandom signals which would lead to the empirical Kolmogorov’s distribution.

The fact that the CMB sky is a weakly random one and the derived 0.2 fraction of the random signal in CMB by their intrinsic statistical content may be compared with other scales of CMB, the 2.7K temperature of the Planckian spectrum and the $10^{-5}$ amplitude of the quadrupole anisotropy, indicating the hot equilibrium history of the Universe and the isotropy of the Universe, respectively. If the Planckian spectrum and the anisotropy amplitude do reflect and do scale certain correlations in CMB signal, the obtained fraction scales the correlations in the cosmological Gaussian perturbations.

What insights to the early Universe will open the new scaling?

We dedicate this paper to the memory of Vladimir Arnold (1937-2010), recalling the memorable discussions (VG,AAK) with him.
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