Primordial Black Holes With Variable Gravity

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ABSTRACT

We have studied the evolution of primordial black holes (PBHs) in a universe with a variable gravitational constant and bulk viscosity. We have found that the strength of gravity has changed appreciably in the early universe. The gravitational constant attained its greatest value at $t = 10^{-23}$ sec after the Big Bang. PBHs formed at the GUT and electroweak epochs would have masses about $1.85 \times 10^9$ g and $1.8 \times 10^{-7}$ g respectively. Their temperatures when they explode are $6 \times 10^8$ K and 6K respectively. PBHs formed during nuclear epoch are hard to detect at the present time. The gravitational constant ($G$) is found to increase as $G \propto t^2$ in the radiation epoch. The gamma rays bursts (GRBs) may have their origin in the evaporation of the PBHs formed during GUT time.

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1. INTRODUCTION

As well as being formed in the course of natural stellar or galactic evolution, black holes may also have been produced primordially, i.e., at the very earliest epochs of cosmological time. The process of a primordial black hole (PBH) formation in a cosmological model with variable gravitational constant $G$ creates an interesting problem. If one considers a Schwartzschild’s black hole formed in the very early Universe at time $t_f$ when $G$ had the value $G_f$, which may be different from the present gravitational constant $G_0$. The horizon size of a black hole is given by

$$R_f = \frac{2G_fM}{c^2} \sim ct_f,$$

where $M$ is the mass of the black hole. The horizon area ($A$) and the entropy ($S$) of the black hole are given by

$$A \propto M^2G^2 \quad \text{and} \quad S \propto GM^2.$$  \hspace{1cm} \text{(1)}

Thus as long as $G$ is increasing the entropy of the black hole increases. However, in the case of a decreasing $G$ one needs a further remedy to account for an increased entropy. Barrow [1] has conjectured the idea of a gravitational memory of the black hole (i.e., a black hole remembers the value of $G$ at the time of its formation). This will have a dramatic implications for the Hawking evaporation of PBHs as the temperature and the life time of the evaporating black holes are determined by the value of $G_f$ and not by $G_0$. These are given by [2]

$$\tau \sim G_f^2M^3 \quad \text{and} \quad T \sim G_f^{-1}M^{-1}.$$  \hspace{1cm} \text{(2)}

Black holes which explode today are those whose Hawking’s life time is equal to $t_0$ (the present age of the universe). Consequently one obtains

$$M_{ex} \simeq 4 \times 10^{14} \left(\frac{G_0}{G_f}\right)^{\frac{2}{3}} \text{ g},$$

\hspace{1cm} \text{(3)}

and their temperature when they explode is therefore given by,

$$T_{ex} \simeq 24 \times \left(\frac{G_0}{G_f}\right)^{\frac{1}{3}} \text{ MeV},$$

\hspace{1cm} \text{(4)}

where the suffix “ex” stands for explode.

3. EVAPORATION OF PRIMORDIAL BLACK HOLES

The radiated particles from a PBH have temperature $T$ given by [2]

$$kT = \frac{\hbar c^3}{8\pi GM} \sim 10^{26} M_g^{-1}$$

\hspace{1cm} \text{(5)}

and the entire mass is radiated away in a time given by

$$\tau \sim 3 \times 10^{-27} M_g^3 \text{ sec},$$

\hspace{1cm} \text{(6)}
where $M_g$ is the mass in grams. It is clear that as $M$ decreases $T$ increases and the mass loss increases until finally reaches a catastrophic limit (explosion or evaporation). Note that the stellar black hole ($M_g > 10^{23}$) is unlikely to explode in the life time of the Universe [3]. Carr [4] has investigated the PBH formation and evaporation in order to see whether the presently observed nucleon density as well as the microwave background radiation (MBR) can be explained in terms of emission of baryons, leptons, photons and so on, by a low mass black hole. PBHs formed from inhomogeneities at time $t$ must have an initial mass ($M_i$) of the order of the particle horizon mass ($M_H$) [5]:

$$M_i \approx M_H = c^3 G^{-1} t = 10^5 (t/s) M_\odot$$  \hspace{1cm} (7)

where $M_\odot$ is the solar mass. PBHs forming at Planck time ($10^{-43}$ sec) would have the Planck's mass ($10^{-5}$g), whereas those formed at $10^{-23}$ sec would have a mass $10^{15}$g required for PBHs which evaporate at the present epoch. The size of the PBH at any given time is limited by the size of the particle horizon. PBHs can radiate either elementary particles, e.g., quarks, gluons, which later emit particles such as baryon, meson and leptons; or composite particles directly (baryon, mesons, leptons). A PBH does not matter whether the emitted particle is a particle or an anti-particle. So the baryon number is not necessarily conserved. This possibility can be used to account for the observed baryon-to-photon ratio.

2. MACH'S PRINCIPLE AND THE VARIABLE GRAVITY

The inertial forces observed locally in an accelerated laboratory may be interpreted as gravitational effects having their origin in distant matter accelerated relative to the laboratory [6,7]. Einstein has tried to incorporate this principle in the formulation of his theory of general relativity (GR). Brans and Dicke have developed a theory which incorporates Mach’s principle. A model incorporating the elements of Mach’s principle was given by Sciama [8]. He, from dimensional argument, concluded that the gravitational constant $G$ is related to the mass distribution in a uniform expanding Universe through

$$\frac{GM}{Rc^2} \sim 1$$  \hspace{1cm} (8)

where $R$ and $M$ are the radius and the mass of the visible Universe, respectively. This relation suggests that either the ratio $M$ to $R$ should be fixed or the gravitational constant $G$ observed locally should be variable and determined by the mass distribution. Only mass ratio can be compared at different points, but not masses. It should be stated that the strong equivalence principle upon which GR stands is incompatible with variable $G$. In 1937 Dirac postulated the existence of very large numbers and constructed a cosmological model in which $G$ decreases with time as $G \propto t^{-1}$ in order not to change the atomic physics [12]. Unfortunately, his model could not resist the observational data.

We have recently presented a cosmological model with variable $G$ and bulk viscosity. The
gravitational constant $G$ is found to vary with time as \(9\),

\[ G \propto t^{\frac{2n-1}{1-n}}, \tag{9} \]

where \(n\) is the viscosity ‘index’, \(0 \leq n \leq 1\), and \(G \propto \exp(Bt)\), where \(B = \text{const.}\), during inflation \((n = 1)\).

3. STRONG GRAVITY

Salam [10,11] has considered the gravitational interaction mediated via heavy mesons and found that the gravitational forces are very strong. He remarked that the nuclear physics should better be called strong gravity. Sivaram and Sinha [11] identified the strong $f$-gravity metric with Dirac’s atomic metric and the large value of the coupling constant (that is $G_f = 10^{40}G_0$) provided the physical basis for the Large Number Hypothesis (LNH). So if one considers the earliest era when the Universe consisted of an extremely hot compact gas of hadrons, the epoch $10^{-23}$ sec, then if $G$ varied according to LNH right down to the present epoch, it would have a very large value $G = 10^{40}G_0$, at the beginning of the hadron era. This value is precisely the value $G_f$ found in considering the short range $f$-gravity mediated by massive $2^+ - f$ mesons. Thus in a region of strong curvature, nuclear physics is analogous to gravity.

It is evident from eq.(8) that if one considers the Planck epoch, the nuclear epoch and the present epoch we will get

\[
\frac{G_N M_N}{G_{Pl} M_{Pl}} = \frac{R_N}{R_{Pl}} \tag{10}
\]

and

\[
\frac{G_0 M_0}{G_{Pl} M_{Pl}} = \frac{R_0}{R_{Pl}} \tag{11}
\]

where the $G_N$($G_{Pl}$), $M_N$($M_{Pl}$), $R_N$($R_{Pl}$) are the values of the gravitational constant, mass and the radius of the Universe at the nuclear (Planck) epoch, respectively. It has been shown that throughout all epochs in the early universe, the relation [7]

\[ Gm^2 = \hbar c = \text{const.} \tag{12} \]

was valid, i.e., we had

\[ G_N m_N^2 = G_{Pl} m_{Pl}^2 = G_W m_W^2 = G_{GUT} m_{GUT}^2 = \hbar c, \tag{13} \]

where $G_N$, $G_W$, $G_{GUT}$ refer to the strong (nuclear) gravitational, weak and GUT coupling constants respectively and $m_p$, $m_{Pl}$, $m_W$ and $m_{GUT}$ refer to the nucleon mass, the Planck’s mass, the intermediate boson and GUT unification mass respectively. Inserting the numerical values: $R_0 = 10^{28}\text{cm}$, $R_N = 10^{-13}\text{cm}$, $R_{Pl} = 10^{-33}\text{cm}$, $M_0 = 10^{56}\text{g}$, $m_{Pl} = 10^{-5}\text{g}$, $m_W = 10^3\text{ GeV}$ and $m_{GUT} = 10^{15}\text{GeV}$: eqs.(10), (11) and (13) yield

\[ G_{GUT} = 10^8 G_0, \ G_W = 10^{32} G_0, \ G_N = 10^{40} G_0, \ \text{and} \ G_{Pl} = G_0. \tag{14} \]
This behavior of $G$ can not be interpreted by Dirac or Brans-Dicke model [12]. Barrow and Carr [5] have considered the evolution of the PBHs in the context of scalar-tensor theories and in particular to Brans-Dicke theory. The cosmological considerations investigated by them restrict the value of $\omega$ (the coupling constant) to the value $-4/3 > \omega > -3/2$. The case $\omega = -4/3$ was, however, excluded. Comparing these results with our model [9], we obtain the same constraint for $\omega$ in addition to the physical significance of the case $\omega = -4/3$. This case corresponds, in our model, to the inflationary solution. In fact, our temporal behavior of $G$ and the scale factor $R$ is defined for all values of $n$ (the viscosity index). The constraint made by Barrow and Carr on $\omega$ would imply an increasing gravitational constant.

From eq.(9), it can be seen that $G$ increases for $n > 1/2$, decreases for $n < 1/2$, remains constant for $n = 1/2$, and increases exponentially during inflation, i.e., when $n = 1$ [9]. Our model predicts that in the radiation epoch, for $n = 2/3$, $R \propto t, T \propto R^{-1}, G \propto t^2$ and $\rho \propto t^{-4}$. A similar variation is found by Abdel Rahman [13] in the radiation epoch. Thus for Planck, GUT, nuclear and electroweak epochs, one has

$$G_N/G_{Pl} = (t_N/t_P)^2 = 10^{40}, \quad G_N/G_W = (t_N/t_W)^2 = 10^8, \quad G_N/G_{GUT} = (t_N/t_{GUT})^2 = 10^{32}$$

and

$$R_N/R_{Pl} = t_N/t_P, \quad R_N/R_W = t_N/t_W, \quad R_N/R_{GUT} = t_N/t_{GUT},$$

or

$$R_N = 10^{-13} \text{ cm}, R_W = 10^{-17} \text{ cm}, R_{GUT} = 10^{-29} \text{ cm}, \quad R_{Pl} = 10^{-33} \text{ cm},$$

where $t_N = 10^{-23}\text{ sec}, t_P = 10^{-43}\text{ sec}, t_W = 10^{-27}\text{ sec}$ and $t_{GUT} = 10^{-39}\text{ sec}$, are the nuclear time, Planck’s time, electroweak time and GUT time. Thus both Abdel Rahman’s model and the present model predict the behavior of $G$ quoted in eq.(14). The relation $G \propto t^2$ is equivalent to eqs.(8) and (12). Note that in the Standard Model $R \propto t^{1/2}, T \propto R^{-1}, G = \text{const}$. This relation can’t account for the above relations. Thus it is suggestive to use this equation to predict a fundamental mass at any epoch in the early universe. If one combines this law with eqs.(8) and (12), one finds

$$m \propto t^{-1},$$

a relation that was valid in the early universe. It has been suggested by several authors that the PBH formed at the nuclear epoch ($10^{-23}\text{ sec}$) would have a mass of $10^{15}\text{g}$. However, if one considers the ‘correct’ value of $G$ we would obtain a value of $10^{-24}\text{g}$. This is, in fact, equal to the mass of the proton. PBHs formed in the early universe would have temperatures and masses, when exploded, given by (eqs.(3), (4) and (14))

$$T_{ex}^N = 1.29 \times 10^{-2} \text{ K and } M_{ex}^N = 8.6 \times 10^{-13} \text{ g},$$

$$T_{ex}^{GUT} = 6 \times 10^8 \text{ K and } M_{ex}^{GUT} = 1.85 \times 10^9 \text{g},$$

and

$$T_{ex}^W = 6 \text{ K and } M_{ex}^W = 1.8 \times 10^{-7} \text{g}.$$
One would therefore expect to observe PBHs emitting x-rays. These PBHs were formed during the GUT phase transition. The remnant of PBHs that formed during the nuclear epoch would be difficult to detect since they have a temperature below the cosmic background radiation. It is interesting to note that PBHs forming during Planck’s epoch are not affected, since \( G_{Pl} = G_0 \). Recently, Cline [14] has considered the PBH evaporation during the quark-gluon phase transition. He has shown that short gamma rays burst (GRB) occurs when the mass of the PBH is either \( 10^{14} \) or \( 10^9 \) g. Thus eq.(20) may indicate the emission of short GRBs. Hence, the spectra of these PBHs are different from those with constant \( G \). We would, therefore, expect to observe PBHs at a lower temperature. Thus a possible variation of \( G \) would alter the picture of the PBHs previously known.

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