Searching for a New Source(s) of T-Violation in Spin Dependent Total Cross Section Measurements

Guanghua Xu *

Department of Physics, University of Houston, 4800 Calhoun Rd, Houston, TX USA 77204

Abstract

We first re-prove with a more complete method that the minimum standard model, with the inclusion of the CKM-matrix, requires the T-odd/P-odd total cross section of two spin-1/2 particles to vanish in all orders [1]. Then we study the contribution to T-odd/P-odd total scattering cross sections from various channels within the Higgs sector, and optimize conditions for possible experimental measurements of these effects. These studies show that such contributions can appear at tree level, and that the spin dependent cross section asymmetry is measurable if the lightest Higgs particle is not too massive, e.g. $m_H \sim 200 \text{GeV}$, and if suitable reaction channels and beam energies and luminosities are chosen.

Key words: time reversal violation, spin dependent cross section, spontaneous CP violation

PACS: 11.30.Er, 12.15.Ji, 13.88.+e

1 Introduction

The minimal standard model [2] with the Cabibbo-Kobayashi-Maskawa mixing matrix [3], MSMCKM, explains CP-violations in heavy quark decays. However, it is natural to wonder if other CP-violations are possible. Various attempts to explain the baryon asymmetry of the universe require much larger CP-violation [4] than suggested by the MSMCKM, which may indicate additional CP or T violating mechanisms. Indeed, the possibility of CP (or T) violation

* Corresponding Author

Email address: gxu@uh.edu (Guanghua Xu).
due to the Higgs sector has been independently studied by the authors in refs. [5] and [6], and models proposed that introduce CP-violation through both neutral and charged Higgs-boson exchange[7].

Recently, it was shown[1] that while the MSMCKM gives a null result to all orders for an experimental test of time-reversal symmetry ($1/2 + 1/2 \rightarrow 1/2 + 1/2$) as suggested by ref. [8], this is not necessarily true if the Higgs sector also contributes to T (or CP) violation. Thus, ref. [1] indicates that such a measurement is a null test of extensions to the MSMCKM which could include CP (or T)-violation contributed by the Higgs sector. However, from an experimental viewpoint, one must know whether a test is feasible and what precision would be required. In what follows, we study several possible experimental tests. We propose 1) appropriate reaction channels for an experimental measurement, and 2) appropriate reaction energies.

In section 2, we re-prove the theorem that the T-odd/P-odd total cross section in the MSMCKM vanishes to all orders [1] by a different, and more complete, method. In section 3, the T-odd/P-odd total cross sections for various possible channels in a model extended to include contributions of the Higgs sector to T (or CP) violation, are studied as a function of the beam energy, in order to optimize the conditions for an experimental test. Finally, conclusions are given in section 4.

2 Proof of Null T-odd/P-odd $A_{x,y}$ within the MSMCKM

This theorem was originally proven in ref. [1]. Now consider the reaction of two spin-1/2 particles ($1/2 + 1/2 \rightarrow 1/2 + 1/2$). The forward-scattering matrix element is written as,[8]

$$M(0) = a_{0,0} + a_{0,z}\sigma_0\sigma_z + a_{z,0}\sigma_0\sigma_z + a_{x,x}(\sigma_x\sigma_x + \sigma_y\sigma_y) + a_{z,z}\sigma_z\sigma_z + a_{x,y}(\sigma_x\sigma_y - \sigma_y\sigma_x),$$  \hspace{1cm} (1)

where $\sigma_x \equiv \sigma \cdot x$, etc., and we have chosen a coordinate system by defining the unit vectors,

$$e_z = k_z/k_z$$
$$e_y = k \times k'/|k \times k'|$$
$$e_x = e_y \times e_z.$$  \hspace{1cm} (2)

Here $k$ and $k'$ are the incident and scattered momenta of the particles, respectively. The conditions for parity-non-conserving (PNC) and time-reversal-volated (TRV) amplitudes have the properties:
PNC (TRV) if \( n_x + n_z \) \((n_x)\) is odd. 

(3) In this equation \( n_x(z) \) is the number of \( x(z) \) subscripts. Thus, the only TRV amplitude in eq. (1) is \( a_{xy} \), which is also PNC, i.e. T-odd/P-odd.

Using the optical theorem, which relates the total cross section to the imaginary part of the forward-scattering amplitude, the total spin-correlation coefficient \( A_{x,y} \) for \( p_1 = p_x \) and \( p_2 = \pm p_y \) is given by,

\[
A_{x,y} = \frac{\text{Im}a_{x,y}}{\text{Im}a_{0,0}} .
\]

Therefore, this spin-correlation coefficient \( A_{x,y} \) is both time (T) and parity (P) odd. A non-zero \( A_{x,y} \) indicates not only that the reaction violates T and P but also that the T-odd/P-odd total cross section is not zero. This is a null test, and provides a framework to precisely investigate TRV processes.

The above derivation is obtained from the two-component spinor description, but the conclusions obtained using eq. (1) to eq. (4) can be applied in a four-component relativistic description, provided the center-of-mass frame (CMS) is used. This follows from 1) the four-component relativistic scattering matrix can be reduced to the two-component formalism in the center-of-mass frame (CMS); 2) the reduced Pauli scattering matrix has the same transformation properties as the non-relativistic scattering matrix under spatial reflections and time reversal; and 3) the spin vector in the non-relativistic treatment can be equated to the relativistic spin vector when the latter is measured in the particle rest-frame related to the CMS by a Lorentz transformation.

Ref. [1] proved that \( A_{x,y} \) of polarized scattering \( 1/2 + 1/2 \rightarrow 1/2 + 1/2 \) in the MSMCKM is identically zero. For a more complete proof, we note that in the MSMCKM, CP(T)-violation is caused by mixing among the three generations of quarks. A complex phase, \( \delta \), in the CKM-matrix provides a natural mechanism for the small, but non-zero violation, of CP conservation. The T (or CP) violating components of the Lagrangian are contained in the expression,

\[
\mathcal{L} = \frac{g}{\sqrt{2}} (J_\mu^+ W^\mu + J_\mu^- W^-\mu) ,
\]

where \( g \) is the coupling constant, \( W^\mu \) are the charged vector bosons, and \( J_\mu^\pm \) are the \( SU(2) \) fermionic currents. These have the values,

\[
g = e \sin \theta_W , \quad W^\pm_\mu = (A^1_\mu \mp iA^2_\mu)/\sqrt{2} , \quad J_\mu^+ = J_\mu^1 + iJ_\mu^2 = \frac{1}{2} U\gamma^\mu(1 - \gamma^5)VD , \quad J_\mu^- = J_\mu^{+\dagger} .
\]
Here $V$ is the CKM mixing matrix, $\theta_W$ is the Weinberg angle, and $U$ and $D$ are quark triplets ($u, c, t$) and ($d, s, b$), respectively. Based on eqs. (5) and (6), only the scatterings of quarks can possibly introduce T (or CP) violation components.

At tree level, the Feynman Diagrams of the forward scattering amplitude which could possibly contribute to T-odd total cross section are,

\begin{align*}
\text{(i)} & \quad \begin{array}{c}
\includegraphics[width=0.5\textwidth]{diagram1}\end{array} \\
\text{(ii)} & \quad \begin{array}{c}
\includegraphics[width=0.5\textwidth]{diagram2}\end{array}
\end{align*}

Fig. 1: The forward scattering amplitudes $\mathcal{M}(ab \rightarrow ab)$ at tree level that could possibly contribute to T-odd total cross section.

The forward scattering amplitude for Fig. 1(i) ($\mathcal{M}_{1i}$) and Fig. 1(ii) ($\mathcal{M}_{1ii}$) in Feynman-'t Hooft gauge are given by,

\begin{align*}
\mathcal{M}_{1i} &= -\frac{g^2}{8} \frac{1}{k_i^2 - M_W^2 + i\epsilon} V_{ab} V_{ab}^* \cdot [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] ; \\
\mathcal{M}_{1ii} &= -\frac{g^2}{8} \frac{1}{k_{ii}^2 - M_W^2 + i\epsilon} V_{ba} V_{ba}^* \cdot [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] ;
\end{align*}

with $k_i^2 = (p_a - p_b)^2$ and $k_{ii}^2 = (p_a + p_b)^2$.

From the T-even condition, $\mathcal{M}^T = \mathcal{M}^\dagger$, one can obtain the T-even and T-odd amplitudes as,

\begin{align*}
\mathcal{M}_{1i}^\text{even} &= -\frac{g^2}{8} \frac{1}{k_i^2 - M_W^2 + i\epsilon} Re(V_{ab} V_{ab}^*) [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] ; \\
\mathcal{M}_{1ii}^\text{even} &= -\frac{g^2}{8} \frac{1}{k_{ii}^2 - M_W^2 + i\epsilon} Re(V_{ba} V_{ba}^*) [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] ; \\
\mathcal{M}_{1i}^\text{odd} &= -\frac{g^2}{8} \frac{1}{k_i^2 - M_W^2 + i\epsilon} iIm(V_{ab} V_{ab}^*) [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] ; \\
\mathcal{M}_{1ii}^\text{odd} &= -\frac{g^2}{8} \frac{1}{k_{ii}^2 - M_W^2 + i\epsilon} iIm(V_{ba} V_{ba}^*) [\not{u}_b \gamma^\mu (1 - \gamma^5) u_a][\not{u}_a \gamma_\mu (1 - \gamma^5) u_b] .
\end{align*}

(8)
Since $Im(V_{ab} V_{ab}^*) = 0$ and $Im(V_{ba}^* V_{ba}) = 0$, the T-odd amplitudes are zero at tree level.

At one-loop level, the Feynman diagrams of the forward scattering amplitudes which could possibly contribute to T-odd total cross section are the following.

Since

$$[\bar{u}_c \gamma^\mu (1 - \gamma^5) u_a \bar{u}_d \gamma^\mu (1 - \gamma^5) u_b]^T = [\bar{u}_c \gamma^\mu (1 - \gamma^5) u_a \bar{u}_d \gamma^\mu (1 - \gamma^5) u_b]^\dagger, \quad (9)$$

the only factor that determines T-odd or T-even is the CKM-matrix elements in the amplitudes. The possible T-odd factors in Fig. 2(ii, iii) are the same as the factors in Fig. 1 and give zero T-odd amplitudes, i.e. the vertices that do not include CKM-matrix elements do not introduce T-odd factors. For Fig. 2(i), the multiplication of the matrix elements is given by,

$$V_{xa} V_{xb}^* V_{yb}^* V_{by} = |V_{xa} V_{xb}^* V_{yb}^* V_{by}|^2 = \text{real and therefore} \quad Im(V_{xa} V_{xb}^* V_{yb}^* V_{by}) = 0. \quad (10)$$

For Fig. 2(iv), the multiplication of the matrix elements is given by

$$V_{xa} V_{xb}^* V_{yb}^* V_{by} = |V_{xa} V_{xb}^* V_{yb}^* V_{by}|^2 = \text{real and therefore} \quad Im(V_{xa} V_{xb}^* V_{yb}^* V_{by}) = 0. \quad (11)$$

Therefore, at one-loop level, T-odd amplitude is zero.

For arbitrary $n$-th order, the forward scattering $a + b \rightarrow a + b$ would go
through combinations of the following processes as shown in Figs. 3 and Fig. 4, depending on the particles $a$ and $b$. \[10\]

\begin{align*}
\text{Fig. 3: The forward elastic scattering amplitudes } & \mathcal{M}(ab \rightarrow ab) \text{ at } n\text{-th order} \\
& \text{that could possibly contribute to T-odd total cross section.}
\end{align*}

\begin{align*}
\text{Fig. 4: Quark self-mass diagram.}
\end{align*}

If $a$ and $b$ belong to the $U(D)$ and $D(U)$ sectors respectively, the forward scattering goes through the processes shown in Figs. 3(a) and (b), but if $a$ and $b$ belong to $U(D)\bar{D}$ and $D(U)\bar{D}$ sectors respectively or $\bar{U}(D)$ and $D(\bar{U})$ sectors respectively, the forward scattering goes through the processes shown in Figs. 3(c).

We prove in the following that the multiplication of CKM matrix elements in each diagram of Figs. 3 and 4 is real; and since an arbitrary T-odd amplitude must be a combination of Figs. 3(i, ii, iii) and 4, the T-odd forward scattering amplitude for an arbitrary order is zero.

Fig. 4 is T-even which is obvious since the CKM-matrix contribution $V_{xa}V_{xa}^*$ is real. Without losing generality, let $a$ be in the $D$ sector and $b$ be in the $\bar{U}$ sector. The $n$-th order forward scattering amplitude could go through Figs. 3(i) if $n$ is an odd number or Figs. 3(ii) if $n$ is an even number.

The amplitude of Fig. 3(i) has the form,
The amplitude of Fig. 3(ii) has the form,

\[
\mathcal{M}_{3i,2}(0) = \sum_{x_i} \left( V_{y_i}^{-1} \bar{u}_{y_i} \gamma^{i} \mu_1 (1 - \gamma^5) u_{b_i} W^{+\dagger}_{\mu_1} \right),
\]

where the repeated indices \(x_i\) and \(y_i\) should be summed over the particles in the corresponding quark sectors and the corresponding momenta of the particles should be integrated based on conservation of momenta.

Based on the MSMCKM Lagrangian, \(x_i\) and \(y_{n-i+1}\) should be in the same quark sector.\(^{[10]}\) It is obvious that

\[
\mathcal{M}_{3i,1}(0) = \mathcal{M}_{3i,2}^\dagger(0) = \mathcal{M}_{3i,2}(0)^* \quad \text{and} \quad \mathcal{M}_{3i,1}(0) \mathcal{M}_{3i,2}(0) = \text{real}. \quad (13)
\]

Therefore, the T-odd amplitude from Fig. 3(i) is zero.

The amplitude of Fig. 3(ii) has the form,

\[
\mathcal{M}_{3i,2}(0) \propto \mathcal{M}_{3i,1}(0) \mathcal{M}_{3i,2}(0) \quad \text{with}
\]

\[
\mathcal{M}_{3i,1}(0) \propto \left( V_{x_i}^{-1} \bar{u}_{x_i} \gamma^{i} \mu_1 (1 - \gamma^5) u_{b_i} W^{+\dagger}_{\mu_1} \right) \left( V_{x_i}^{-1} \bar{u}_{x_i} \gamma^{i} \mu_1 (1 - \gamma^5) u_{b_i} W^{+\dagger}_{\mu_1} \right)
\]

\[
\mathcal{M}_{3i,2}(0) \propto \left( V_{y_i}^{-1} \bar{u}_{y_i} \gamma^{i} \mu_1 (1 - \gamma^5) u_{b_i} W^{+\dagger}_{\mu_1} \right) \left( V_{y_i}^{-1} \bar{u}_{y_i} \gamma^{i} \mu_1 (1 - \gamma^5) u_{b_i} W^{+\dagger}_{\mu_1} \right),
\]

where the repeated indices \(x_i\) and \(y_i\) should be summed over the particles in the corresponding quark sectors and the corresponding momenta of the particles should be integrated based on conservation of momenta.

Also, based on the MSMCKM Lagrangian, \(x_i(y_i)\) and \(x_{n-i+1}(y_{n-i+1})\) should be in the same quark sector\(^{[10]}\) and one should have,
\[ \mathcal{M}_{3ii,l}(0) = \mathcal{M}^\dagger_{3ii,l}(0) = \mathcal{M}^*_{3ii,l}(0) = \text{real}, \quad l = 1, 2. \]  

Therefore, the T-odd amplitude from Fig. 3(ii) is zero.

In Fig. 3(iii), a \( W^+ \) propagator placed after the annihilation of incoming quarks \( a \) and \( b \) could create a quark pair in \( U (D) \) and \( \bar{D} (\bar{U}) \) sectors, introducing a possible T-odd contribution. The created \( U (D) \) and \( \bar{D} (\bar{U}) \) pair will go through the processes of either Fig. 3(i) or Fig. 3(ii) before the final quark pair in the process is annihilated. This again creates a \( W^+ \) propagator. Therefore, Fig. 3(iii) can be broken down to a combination of Fig. 3(i) or Fig. 3(ii), a smaller part of the form of Fig. 3(iii), and Fig. 4. The smaller part of Fig. 3(iii) can be continuously divided into a combination of the smaller parts of Fig. 3(i) or Fig. 3(ii), and an even smaller part of the form of Fig. 3(iii), and Fig. 4. If this division is continued, one can eventually break Fig. 3(iii) into a combination of several components of Fig. 3(i) or Fig. 3(ii), Fig. 2(iv), and Fig. 4. As shown above, all these contributions from Fig. 3(i), Fig. 3(ii), Fig. 2(iv), and Fig. 4 do not contribute to T-odd amplitudes. Therefore, the T-odd amplitude from Fig. 3(iii) is zero.

Since an arbitrary possible T-odd forward scattering amplitude can be obtained from combinations of Fig. 3(i) or Fig. 3(ii), Fig. 3(iii) and Fig. 4, we conclude that T-odd forward scattering amplitude of a polarized reaction \( 1/2 + 1/2 \rightarrow 1/2 + 1/2 \) within the MSMCKM is identically zero to all orders. This implies that the T-odd total cross section of a polarized reaction \( 1/2 + 1/2 \rightarrow 1/2 + 1/2 \) is zero to all orders, by the optical theorem.

Because both T-odd/P-even and T-odd/P-odd amplitudes have the same CKM-matrix factors, both T-odd/P-even and T-odd/P-odd amplitudes should vanish to all order. We also note that T-odd/P-even amplitude should be zero based on eq. 2, and the zero T-odd/P-odd amplitude is due to the fact that the source of T-odd amplitude in the MSMCKM is introduced by the phase in the CKM matrix. The proof is therefore completed.

Two points need to be re-stated.

1) The above conclusion shows that a non-zero T-odd total cross section of a polarized reaction \( 1/2 + 1/2 \rightarrow 1/2 + 1/2 \) indicates the existence of additional \( T \) (or CP) violation source(s) besides the phase in the CKM matrix. Furthermore, it is a null test in which a high experimental accuracy can be achieved.

2) A zero T-odd total cross section does not indicate \( T \) (or CP) conservation in the physical process, i.e. a \( T \) (or CP) violation in a physical process is a necessary but not sufficient condition for an existence of a T-odd total cross section.
Ref. [1] showed that the T-odd/P-odd total cross section of a $\frac{1}{2} + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$ reaction could be non-zero if the Higgs sector contributes to T (or CP) violation. Thus, a measurement of a T-odd/P-odd total cross section, proportionate to the forward scattering amplitude, would indicate an additional mechanism(s) of T (or CP) violation. Based on the importance of T-odd total cross section measurement, possibilities to carry out such measurements will be estimated in the following. These estimates are based on very limited knowledge of the Higgs sector, so that the study is restricted to two Higgs doublets in which T (or CP) symmetry is broken by neutral Higgs boson exchange. If there are three Higgs doublets, both neutral and charged Higgs boson exchange could break T (or CP) symmetry. This would enhance the signals obtained from two Higgs doublets.

3 Several T-Odd/P-Odd Processes

The neutral scalar-quark interactive Lagrangian is given by,

$$\mathcal{L}_\phi = -\frac{1}{\sqrt{2} |\lambda_1|} \overline{Dm_D D\Phi_1} + \frac{i |\lambda_2|}{\sqrt{2} |\lambda_1| \sqrt{|\lambda_1|^2 + |\lambda_2|^2}} \overline{Dm_D} \gamma^5 D\Phi_3$$

$$-\frac{1}{\sqrt{2} |\lambda_2|} \overline{U m_U U\Phi_2} + \frac{i |\lambda_1|}{\sqrt{2} |\lambda_2| \sqrt{|\lambda_1|^2 + |\lambda_2|^2}} \overline{U m_U} \gamma^5 U\Phi_3 + h.c. \quad (16)$$

At tree level, there could be three possible T-odd/P-odd forward scattering amplitudes for $a(\frac{1}{2}) + b(\frac{1}{2}) \rightarrow a(\frac{1}{2}) + b(\frac{1}{2})$, shown in Fig. 5.

Fig. 5: The Higgs contributions to forward scattering amplitudes of $a + b \rightarrow a + b$

Here Fig. 5(i) is for the forward scattering of two arbitrary spin-1/2 particles, Fig. 5(ii) is the forward scattering for a spin-1/2 particle and its anti-particle, and Fig. 5(iii) is for the forward scattering of two identical, spin-1/2 particles. Based on the optical theorem, the T-odd/P-odd total cross sections of the processes shown in Fig. 5 are given by,
\[ \sigma_{T,P}^{5i} = 0, \]
\[ \sigma_{T,P}^{5ii} = \frac{-1}{v_{rel}} \frac{m_a^2 |\lambda_i| v_j v_3}{\sqrt{|\lambda_i|^2 + |\lambda_2|^2}} \frac{p_{az}}{p_{a0}} \frac{m_H \Gamma}{(4p_{a0}^2 - m_H^2)^2 + (m_H \Gamma)^2} s_{ax} s_{ay}, \]
\[ \sigma_{T,P}^{5iii} = \frac{-1}{v_{rel}} \frac{m_a^2 |\lambda_i| v_j v_3}{\sqrt{|\lambda_j|^2 + |\lambda_2|^2}} \frac{p_{az}}{p_{a0}} \frac{m_H \Gamma}{(a|\vec{p}_a|^2 + m_H^2)^2 + (m_H \Gamma)^2} s_{ax} s_{ay}, \]  

where \( v_{rel} \) is the relative velocity of two incoming particles, the momenta are measured in CMS, and \( i = 1(2), j = 2(1) \) if the particle \( a \) is in the \( U(D) \) sector. The effect of scalar exchange is assumed to be dominated by the lightest neutral-scalar particle of mass \( m_H \), i.e.,

\[ < \Phi_i \Phi_j >_k \sim \frac{v_i v_j}{k^2 - m_H^2 + i\epsilon}. \]  

Note from eq. (17) that \( \sigma_{T,P}^{5ii} \) reaches a maximum at \( p_{a0} = 0.5m_H \), which corresponds to the resonance region of the scattering, and that \( \sigma_{T,P}^{5iii} \) reaches a maximum when \( p_{a0} \) is slightly larger than 0. \[12\] Since there is no resonance in Fig. 3(iii), the maximum of \( \sigma_{T,P}^{5iii} \) can be orders of magnitude smaller than the maximum of \( \sigma_{T,P}^{5ii} \). Obviously, it is experimentally more favorable to choose scattering channels and incoming particle momenta which produce maximum T-odd/P-odd total cross sections. Further investigation of the magnitude of the T-odd/P-odd total cross sections will require knowledge of the Higgs sector and the masses of the quarks. The following assumptions are adopted.

1) There is no preferred coupling of the Higgs to quarks in the \( U \) and \( D \) sectors. This leads to,

\[ |\lambda_1| \sim |\lambda_2| \sim (\sqrt{2}G_F)^{-1/2} = 246 GeV. \]  

2) The order of magnitude of \( v_a v_b \) in eqs. (17) and (18) is approximately 1.

3) The \( u \) and \( d \) quark masses are approximately 5 MeV.

Due to the small quark masses and large vacuum expectation values, the couplings between quarks and the Higgs particle are very small, and small T-odd/P-odd total cross sections are expected. Therefore, one should attempt to find the largest open channels. We consider several processes in the following.

3.1 \( p\bar{p} \) scattering

We consider several factors in the T-odd/P-odd total cross section for \( p\bar{p} \) \[13\].
1) Since the valence quark composition of a proton is $uud$ and the valence quark composition of an anti-proton is $\bar{u}\bar{u}\bar{d}$, the dominant contributions to T-odd/P-odd total cross section are from $\sigma_{T,P}^{TP}$ in eq. (17).

2) Valence quarks in a proton only contribute about 30% of the proton spin. Thus, it is assumed that each valence quark contributes about 10% of the total spin.

3) Both valence and sea quarks in a proton contribute only about 50% of the proton total momentum. The most probable momentum for a valence quark in a proton is about at $x = 0.15$.

Then the maximum total cross section of $p\bar{p}$ is roughly given by,

$$
\sigma_{pp}^{TP} \simeq (\frac{2}{10})\sigma_{T,P}^{TP}(u\bar{u} \rightarrow u\bar{u}) + (\frac{1}{10})\sigma_{T,P}^{TP}(d\bar{d} \rightarrow d\bar{d})
$$

$$
= (\frac{2}{10}) - \frac{1}{v_{rel}} \frac{m_{u}^{2} |\lambda_{1}|^{2} |\lambda_{2}|^{2}}{v_{1}^{2} |\lambda_{1}|^{2} + |\lambda_{2}|^{2} p_{u0} m_{H}^{2}} \frac{1}{m_{H}^{2}} s_{ux} s_{uy}
$$

$$
+ (\frac{1}{10}) - \frac{1}{v_{rel}} \frac{m_{d}^{2} |\lambda_{1}|^{2} |\lambda_{2}|^{2}}{v_{1}^{2} |\lambda_{1}|^{2} + |\lambda_{2}|^{2} p_{d0} m_{H}^{2}} \frac{1}{m_{H}^{2}} s_{dx} s_{dy}.
$$

(20)

Using $m_{H}$ and $\Gamma$ given in ref. [14], and assuming $\sigma_{T,P}^{TP} \simeq 50 \text{ mb}$, an estimate of $\sigma_{pp}^{TP}$ and $A_{xy}$ is given in Table 1.

| $m_{H}$ (GeV) | $\Gamma_{H}$ (GeV) | $\sigma_{pp}^{TP}$ (mb) | $A_{xy}$ | beam energy in CMS (GeV) |
|--------------|------------------|------------------------|---------|-------------------------|
| 200          | 2                | $8 \times 10^{-14}$    | $2 \times 10^{-15}$ | 667          |
| 400          | 25               | $3 \times 10^{-15}$    | $7 \times 10^{-17}$ | 1333         |
| 600          | 100              | $6 \times 10^{-16}$    | $1 \times 10^{-17}$ | 2000         |
| 1000         | 450              | $8 \times 10^{-17}$    | $2 \times 10^{-18}$ | 3333         |

One can see from Table 1 that the larger the Higgs mass, the higher sensitivity is required to undertake a measurement. In any event, it requires high accuracy and sensitivity to measure such small cross sections.

Modern superconducting technology can measure current changes as low as $10^{-8} \sim 10^{-9} \text{ A}$. [15] If the lightest Higgs mass is not too large, e.g. $m_{H} \sim 200 \text{ GeV}$, and the luminosities of the beams are reasonably large, some of the small cross sections in Table 1 should be measurable.
3.2 \( pp \) scattering

To estimate the T-odd/P-odd total cross section of \( pp \), a few factors will be considered.\[13\]

1) The sea quarks in a proton are mainly found in the small \( x \) region. Thus, the major contributions to T-odd/P-odd forward scattering would most likely occur in valence quark collisions, i.e. \( \sigma_{T,P_{\text{iii}}} \), for beam energies below or around \( m_H/2 \). However, if the beam energies were much beyond \( m_H/2 \), contributions from sea quarks, i.e. \( \sigma_{T,P_{\text{ii}}} \), could also be important. Only beam energies below or around \( m_H/2 \) are considered. For beam energies much beyond \( m_H/2 \), one should refer to section 3.1.

2) The parton model is assumed to be valid at these beam energies.

3) As we only consider beam energies below or around \( m_H/2 \), points 2) and 3) in section 3.1 remain valid.

The total cross section of \( pp \) is roughly given by,

\[
\sigma_{pp}^{TP} \simeq \left( \frac{2}{10} \right) \sigma_{T,\text{iii}}^{TP}(uu \rightarrow uu) + \left( \frac{1}{10} \right) \sigma_{T,\text{ii}}^{TP}(dd \rightarrow dd)
\]

\[
\simeq -2 \frac{m_u^2 |\lambda_1| |v_2 v_3|}{10 v_{\text{rel}} |\lambda_2|^2 \sqrt{|\lambda_1|^2 + |\lambda_2|^2}} \frac{p_{uz}}{p_{u0}} \frac{m_H \Gamma}{(4 |\vec{p}_u|^2 + m_H^2)^2 + (m_H \Gamma)^2} s_{ux} s_{uy} + \\
-1 \frac{m_d^2 |\lambda_2| |v_1 v_3|}{10 v_{\text{rel}} |\lambda_1|^2 \sqrt{|\lambda_1|^2 + |\lambda_2|^2}} \frac{p_{dz}}{p_{d0}} \frac{m_H \Gamma}{(4 |\vec{p}_d|^2 + m_H^2)^2 + (m_H \Gamma)^2} s_{dx} s_{dy} \quad \text{(21)}
\]

Considering \( m_H = 200 \text{ GeV} \) and \( \Gamma = 2 \text{ GeV} \) and assuming \( \sigma_{pp} \simeq 50 \text{ mb} \), the estimated values of \( \sigma_{pp}^{TP} \) and \( A_{xy} \) are given in Table 2.

Table 2
\( \sigma_{pp}^{TP} \) vs beam energy in CMS for \( m_H = 200 \text{ GeV} \)

| beam energy in CMS (GeV) | \( \sigma_{pp}^{TP} \) (mb) | \( A_{xy} \) |
|-------------------------|-----------------|---|
| 5                       | \( 8 \times 10^{-18} \) | \( 2 \times 10^{-19} \) |
| 10                      | \( 8 \times 10^{-18} \) | \( 2 \times 10^{-19} \) |
| 50                      | \( 5 \times 10^{-18} \) | \( 1 \times 10^{-19} \) |
| 100                     | \( 2 \times 10^{-18} \) | \( 4 \times 10^{-20} \) |

As one can see from the results in Table 2, the T-odd/P-odd total cross sections for \( pp \) are much smaller than for \( pp \). This is understandable as there are no resonances in this channel.
One could also notice that the variations of $\sigma_{pp}^{TP}$ and $A_{xy}$ versus beam energies are small. This is due to the large mass of the Higgs particle, as compared to the incoming particle energies.

In general, both $\sigma_{pp}^{TP}$ and $\sigma_{pp}^{PP}$ are very small. However, as the above estimates are based on a neutral scalar boson, other possible source(s) of T (or CP) violation could be larger than these estimates. A careful comparisons between $pp$ and $p\bar{p}$ T-odd/P-odd total cross sections should provide more information of T (or CP) violation mechanisms.

3.3 $\bar{l}l$ and $ll$ scatterings

If the coupling between the lepton sectors and the Higgs sector is similar to the coupling between quark sectors and the Higgs sector, polarized $\bar{l}l$ and $ll$ scattering can also have T-odd/P-odd total cross sections. We consider the following points.

1) Leptons are elementary particles and one does not need to consider the unpolarized and polarized structure functions. Therefore, the result in eq. (17) can be directly used for the $\bar{l}l$ and $ll$ T-odd/P-odd total cross sections.

2) The total cross sections for $\bar{l}l$ and $ll$ should be significantly smaller than the $p\bar{p}$ and $pp$ cross sections since only electro-weak interactions are involved. For beam energies which are not in the $Z$ resonance region, $\sigma_{\bar{l}l} \sim \sigma_{ll} \sim 10 \mu b$ is assumed for simplicity[16].

The estimated $\sigma_{\bar{l}l}(\sigma_{ll})$ and $A_{xy}$ are given in Tables 3-6.

| $m_H$ (GeV) | $\Gamma_H$ (GeV) | $\sigma_{\bar{e}e}^{TP}$ (mb) | $A_{xy}$ | beam energy in CMS (GeV) |
|------------|-----------------|-------------------------------|----------|--------------------------|
| 200        | 2               | $3 \times 10^{-15}$           | $6 \times 10^{-13}$ | 100                      |
| 400        | 25              | $1 \times 10^{-16}$           | $2 \times 10^{-14}$ | 200                      |
| 600        | 100             | $2 \times 10^{-17}$           | $4 \times 10^{-15}$ | 300                      |
| 1000       | 450             | $2 \times 10^{-18}$           | $5 \times 10^{-16}$ | 500                      |

Similar to the results of baryon collisions, $\bar{l}l$ collisions have larger T-odd/P-odd total cross sections if the Higgs sector is one of the T-violation sources and if the coupling between the lepton sectors and the Higgs sector is similar to the one between the quark sectors and the Higgs sector. Especially for the $\mu\bar{\mu}$ channel, its T-odd/P-odd total cross section is orders of magnitude larger than the corresponding $p\bar{p}$ channel due to the larger masses of $\mu$ and $\bar{\mu}$. If there were high energy polarized $\mu$ and $\bar{\mu}$ beams available, the measurements...
Table 4
\(\sigma_{\mu\mu}^{TP}\) vs beam energy in CMS for \(m_H = 200\,\text{GeV}\)

| beam energy in CMS (GeV) | \(\sigma_{\mu\mu}^{TP} (mb)\) | \(A_{xy}\) |
|-------------------------|-----------------------------|-----------|
| 0.5                     | \(3 \times 10^{-19}\)       | \(6 \times 10^{-17}\) |
| 1                       | \(3 \times 10^{-19}\)       | \(6 \times 10^{-17}\) |
| 10                      | \(3 \times 10^{-19}\)       | \(6 \times 10^{-17}\) |
| 100                     | \(7 \times 10^{-20}\)       | \(1 \times 10^{-17}\) |

Table 5
Maximum \(\sigma_{\mu\bar{\mu}}^{TP}\) for various \(m_H\)

| \(m_H\) (GeV) | \(\Gamma_H\) (GeV) | \(\sigma_{\mu\bar{\mu}}^{TP} (mb)\) | \(A_{xy}\) | beam energy in CMS (GeV) |
|--------------|---------------------|-------------------------------|-----------|-------------------------|
| 200          | 2                   | \(1 \times 10^{-10}\)         | \(2 \times 10^{-8}\) | 100                     |
| 400          | 25                  | \(5 \times 10^{-12}\)         | \(1 \times 10^{-9}\) | 200                     |
| 600          | 100                 | \(8 \times 10^{-13}\)         | \(2 \times 10^{-10}\) | 300                     |
| 1000         | 450                 | \(1 \times 10^{-13}\)         | \(2 \times 10^{-11}\) | 500                     |

Table 6
\(\sigma_{\mu\mu}^{TP}\) vs beam energy in CMS for \(m_H = 200\,\text{GeV}\)

| beam energy in CMS (GeV) | \(\sigma_{\mu\mu}^{TP} (mb)\) | \(A_{xy}\) |
|-------------------------|-----------------------------|-----------|
| 0.5                     | \(1 \times 10^{-14}\)       | \(2 \times 10^{-12}\) |
| 1                       | \(1 \times 10^{-14}\)       | \(2 \times 10^{-12}\) |
| 10                      | \(1 \times 10^{-14}\)       | \(2 \times 10^{-12}\) |
| 100                     | \(3 \times 10^{-15}\)       | \(6 \times 10^{-13}\) |

of T-odd/P-odd total cross sections of \(\mu\) and \(\bar{\mu}\) collisions should provide a sensitive test.

On the other hand, the T-odd/P-odd total cross section of \(ll\) collisions is an order(s) of magnitude smaller than the corresponding \(l\bar{l}\) total cross section. These estimates are based on the assumption that the neutral Higgs particle contributes to T (or CP) violation. If a small T-odd/P-odd cross section of \(ll\) collisions is measured, a careful comparison between \(ll\) and the corresponding \(l\bar{l}\) T-odd/P-odd total cross sections would provide additional information about T (or CP) violation.
4 Summary and Future Prospect

This note addresses the possibilities of searching for additional sources of $T$ (or CP) violation through $T$-odd/$P$-odd total cross section measurements. It shows the following.

1. A non-zero $T$-odd/$P$-odd total cross section in the null test $1/2 + 1/2 \rightarrow 1/2 + 1/2$ will indicate that there is(are) additional source(s) of $T$ (or CP) violation besides the phase in CKM matrix.

2. The contributions to $T$-odd/$P$-odd total cross section from the Higgs sector can appear at tree level if the Higgs sector contribute to $T$ (or CP) violation, and the channels with resonance can be measurable with modern technology if the lightest Higgs mass is not too large and beam luminosities are reasonably large.

3. If the Higgs coupling to the leptons is similar to the coupling to quarks, $A_{xy}$ in both $l\bar{l}$ and $ll$ are larger than for $pp$ or $p\bar{p}$ due to the smaller $l\bar{l}$ and $ll$ total cross sections, and $\mu\bar{\mu}$ provides the most sensitive channel due to the larger muon mass.

4. The present study only considers the coupling of neutral Higgs particle as the additional source of $T$ (or CP) violation. Actual measurements could be larger if other mechanisms of $T$ (or CP) violation occur. A careful comparison between channels with and without resonance could reveal mechanisms of $T$ (or CP) violation beyond current ideas.

5. Measurement of $A(x, y)$ in the total cross section is a null test and as such has the possibility to give very accurate results.

6. The proposed measurements can provide information on possible extensions of MSMCKM and the properties of vacuum.

Acknowledgments

We would like to thank Professor Kwang Lau of Physics Department at University of Houston for helpful discussions. We are specially gratitude to Professor Ed V. Hungerford of Physics Department at University of Houston for detailed review of the manuscript and valuable contributions.

References

[1] Xu, Guanghua, Hungerford, Ed V., Nucl. Phys. B 649 (2003) 327-348.

[2] Weinberg, S. Phys. Rev.Lett. 19 1264-1266 (1967); Salam, A. (1968). In Elementary particle physics (Nobel Symp No. 8). (ed. N. Svartholm). Almqvist and Wilsell, Stockholm.

[3] Kobayashi, M. and Maskawa, M., Prog. theor. Phys. 49 (1973) 652.
[4] Dolgov, A. D., Phys. Rep. 222 (1992) 309 and references therein.

[5] Lee, T. D., Phys. Rep. 9C (1974) 143.

[6] Weinberg, S., Phys. Rev. Lett. 37 (1976) 657.

[7] Weinberg, S., Phys. Rev. Lett. 63 (1989) 2333; Phys. Rev. D 42 (1990) 860.

[8] Conzett, H. E., Phys. Rev. C 48 (1993) 423.

[9] Stapp, Henry P., Phys. Rev. 103 (1956) 425; Chou, Kuang-Chao, and Shirokov, M. I., Soviet Phys. JETP, 7 (1958) 851; Csonka, P. L., Moravesik, M. J., and Scadron, M. D., Ann. Phys. 40 (1966) 100.

[10] Here the $Z(\gamma)$ exchanges are excluded for they do not introduce additional $T$-odd factors.

[11] The only difference here from Fig. 3(i, ii) is that the initial and final quarks need to be summed over all components in the corresponding quark sector, but this difference does not change the conclusions of eqs. (13) and (15).

[12] The exact solution of $|\vec{p}_a|$ to have maximum $\sigma^{TP}_{Sii}$ can be determined from this cubic equation

$$64|\vec{p}_a|^3 + 16(m^2_H + 3m^2_a)|\vec{p}_a|^2 + 8m^2_a m^2_H |\vec{p}_a| + m^2_a m^2_H (m^2_H + \Gamma^2) = 0.$$ 

[13] This is just a rough estimate. A more accurate estimate needs calculations based on up-to-date quark structure functions and polarized quark structure functions. Sea quark contributions should also be considered.

[14] Vernon D. Barger, Roger J. N. Phillips, Collider Physics (Addison-Wesley Publishing Company, 1987).

[15] J. Clarke, Phil. May. 13 (1986) 115;

[16] For $\bar{l}l$ scattering, the maximum $\sigma^{TP}_{T}$ is at the Higgs resonance region, but the total cross section is not expected to be dominant in the Higgs resonance region due to the small coupling constants.