Shell-model description of $^{16}$C with the Idaho N$^{3}$LO NN potential

S Fujii$^{1}$, T Mizusaki$^{1,2}$, T Otsuka$^{1,3,4}$, T Sebe$^{5}$, A Arima$^{6}$

1 Center for Nuclear Study (CNS), University of Tokyo, Wako Campus of RIKEN, Wako 351-0198, Japan
2 Institute of Natural Sciences, Senshu University, Tokyo 101-8425, Japan
3 Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
4 RIKEN, Wako 351-0198, Japan
5 Faculty of Engineering, Hosei University, Koganei 184-8584, Japan
6 Science Museum, Japan Science Foundation, Tokyo 102-0091, Japan

E-mail: sfuji@cns.s.u-tokyo.ac.jp

Abstract. The structure of the neutron-rich carbon nucleus $^{16}$C is described within a new microscopic shell-model framework. The model space is composed of the 0$s$, 0$p$, 1$s$ and 0$d$ shells, and thus a sort of no-core type. The effective interaction is microscopically derived from the Idaho N$^{3}$LO NN potential and the Coulomb force through a unitary transformation theory. Calculated low-lying energy levels of $^{16}$C agree well with the experimental values. Furthermore, the anomalously hindered $B(E2; 2^+_1 \rightarrow 0^+_1)$ value for $^{16}$C, measured recently, is well described by the calculation with the bare charges.

1. Introduction

It is well-known that the $E2$ transition strength between the first $2^+$ and the ground $0^+$ states is rather strong in most of even-even nuclei. A recent experiment at RIKEN, however, has revealed that the $E2$ transition strength is extremely small in $^{16}$C, and its $B(E2; 2^+_1 \rightarrow 0^+_1)$ value was found to be $0.63 \pm 0.11$ (stat) $\pm 0.16$ (syst) $e^2$fm$^4$ [1]. The dominance of neutron excitation is also suggested for the same transition [2]. This means that the valence two neutrons in $^{16}$Ca are decoupled from the core part.

In a simple shell-model picture of the structure of the neutron-rich carbon isotopes with the mass number $A \geq 15$, neutrons begin to fill the 1$s_{1/2}$ and 0$d_{5/2}$ orbits, while the protons occupy mainly the orbits up to 0$p_{3/2}$. The 0$p_{1/2}$ orbit of the proton lies about 6 MeV higher than the 0$p_{3/2}$ orbit. Thus, it is expected that low-lying energy levels of such systems can be well described by a conventional shell model. In fact, 0$p_{1/2}0$d shell-model calculations succeed in reproducing those levels [3, 4, 5, 6]. However, the shell-model calculations give rather large $B(E2; 2^+_1 \rightarrow 0^+_1)$ values compared to the experimental ones. Especially, the discrepancy in $^{16}$C is noticeable. One of the reasons of the discrepancies may be the use of the effective charges which are determined so as to reproduce many experimental data in and near stable nuclei.

In the anti-symmetrized molecular dynamics (AMD) calculation with the bare charges which does not assume any arbitrary structure, the results of $B(E2; 2^+_1 \rightarrow 0^+_1)$ give smaller values than the shell-model results and are closer to the experimental data [7]. However, it seems that the results depend considerably on the parameters of the effective interaction, and as for the
energy level, the $0p1s0d$ shell model works better than the AMD. The discrepancies between the calculations and the experiments may be due to the fact that those models do not include certain exotic features of unstable nuclei because of their bases with some assumptions and adjustments in and near stable nuclei.

Given this situation, we have recently proposed a new shell-model framework to calculate for neutron-rich carbon isotopes including $^{16}$C without any information which has been accumulated in the study of stable nuclei [8]. In the following, we shall outline the method of the new shell model.

2. A new microscopic shell model
In the new shell-model framework, we start with a high-precision NN interaction and the Coulomb force and then derive a two-body effective interaction in a large model space through a unitary-transformation theory to take into account the short-range correlation of the original NN interaction. The three-or-more-body effective interactions are neglected for simplicity though those are generated from the original NN interaction. The unitary transformation $U$ is also employed in the unitary-model-operator approach (UMOA) [9, 10] and the ab initio no-core shell-model (NCSM) [11, 12], and its expression is given by

$$U = (1 + \omega - \omega^\dagger)(1 + \omega \omega^\dagger + \omega^\dagger \omega)^{-1/2}$$

where the operator $\omega$ satisfies $\omega = Q\omega P$ with the usual projection operators $P$ and $Q$ concerning the model space and its complement, respectively, and thus $\omega^2 = \omega^\dagger 2 = 0$ [13]. The large model space is specified by a boundary number $\rho_1$ which is given with the sets of the harmonic-oscillator (h.o.) quantum numbers \{\(n_a, l_a\)\} and \{\(n_b, l_b\)\} of two-body states as $\rho_1 = 2n_a + l_a + 2n_b + l_b$. If we diagonalize the Hamiltonian with the effective interaction in the large model space with a huge number of shell-model many-body basis states, it leads to the standard NCSM. In the NCSM, the value of $\rho_1$ is taken as large as possible so that the calculated result does not depend on this value. In the present study, however, since we intend to describe the structure of neutron-rich carbon isotopes which have the mass number $A \geq 14$, it is difficult to obtain a converging result in a sufficiently large model space in the usual NCSM manner due to a huge dimension of the matrix to be diagonalized. Therefore, we proceed to the next step for the effective interaction in a smaller model space in which the shell-model diagonalization can be performed.

In the present study, the small model space is set so as to contain the orbits from the $0s$ up to the $1p0f$ shell. The effective interaction in the small model space is derived again through the unitary transformation by using the effective interaction in the large model space as the original interaction. It is noted that the two-body effective interaction is self-consistently determined by introducing the one-body potential for the unperturbed ground-state wave function of $^{14}$C by means of iteration. This means that we derive the two-body effective interaction which can describe $^{14}$C as a good closed-shell nucleus. Although the self-consistent procedure is not necessarily needed in determining the effective interaction, such an effective interaction may be optimized for a restricted model space so as to obtain a good unperturbed energy. However, in some cases, this self-consistent calculation does not converges due to an incomplete small model space which is not completely separated from the complement in energy but in the orbits. Several matrix elements of the effective interaction of a step in the iteration are significantly different from those of another step. Therefore, in order to obtain a converged effective interaction, we restrict the value of $\rho_1$ as $\rho_1 = 6$ in the present study.

In addition, as for the particle-hole excitation, we need truncations in the actual shell-model calculation. The nucleon excitations from the hole states of $^{14}$C are restricted to up to two nucleons, and the excitations to the $1p0f$ shell are also up to two nucleons. Even in these restrictions, the dimension becomes very huge if we calculate for the neutron-rich carbon isotopes
with the mass number $A \simeq 20$ though we are planning to describe such systems by the present shell model.

Furthermore, we make a minimal refinement of neutron one-body energies for the $1s_{1/2}$, $0d_{5/2}$ and $0d_{3/2}$ orbits on top of $^{14}$C to realistically describe the structure of neutron-rich carbon isotopes. In the present shell-model calculation without any adjustable parameters for $^{15}$C, the ground $1/2^+$ state appears above the first excited $5/2^+$ state, contrary to the experiment. The refinement is introduced so that the results of the present shell-model diagonalization reproduce the UMOA results of the single-particle energies for the ground $1/2^+$ and the first excited $5/2^+$ states in $^{15}$C. The single-particle energy can be calculated in a fully microscopic way in a sufficiently large model space within the UMOA framework. In this way, we treat complex correlations coupled to a single-particle-like state in a large model space within the present shell model for a smaller model space. Thus, the present approach is a hybrid method combining a no-core type of shell model with single-particle information by the UMOA. Since the single-particle information plays an important role in the shell model, we expect that the present shell-model result with the refinement for neutron-rich carbon isotopes becomes close to the exact NCSM result which might be obtained in the near future. A similar approach may be useful, in particular, to some exotic systems for which single-particle levels are unknown experimentally. A more detailed explanation for the present shell model may be found in Ref. [8].

3. Results and discussion

Before showing shell-model results, in Fig. 1, first we illustrate calculated single-particle energies of $^{15}$C obtained by the UMOA using the N$^3$LO potential [14] based upon chiral perturbation theory, together with the experimental values. The result for “unpt.” denotes the unperturbed single-particle energy which is defined as the h.o. kinetic energy plus the self-consistent one-body potential, while the result for “2p1h” includes the two-particle one-hole excitation effect in a sufficiently large model space. In the UMOA calculation, we search for an optimal value of $\hbar\Omega$ for each state in nuclei which gives the energy minimum point. In the case of $^{15}$C for the N$^3$LO potential, the optimal values of $\hbar\Omega$ for the ground $1/2^+$ and the first excited $5/2^+$ states are 13.6 and 14.2 MeV, respectively. Note that these values are for the “$2p1h$” case. For the unperturbed single-particle energies, the energy minimum points do not appear at the same $\hbar\Omega$ values. For comparison, we show the unperturbed single-particle energies for the same $\hbar\Omega$ values as those for the “$2p1h$” case. In $^{15}$C, the ordering of the experimental single-particle $1/2^+$ and $5/2^+$ states are opposite to the case of $^{17}$O. Our result for “unpt.” does not reproduce this tendency, and the single-particle levels are rather repulsive to the experimental values. However, if we see the results for “$2p1h$”, the results become more attractive and the two levels are reversed, and then a good agreement with the experimental values is obtained.

As has been explained before, in the present shell model, we make a minimal refinement of neutron one-body energies for the $1s_{1/2}$, $0d_{5/2}$ and $0d_{3/2}$ orbits so that the results of the shell-model diagonalization reproduce the UMOA results, namely, the results for “$2p1h$” in Fig. 1. Since the refinement is introduced in a simple way, we use a common value of $\hbar\Omega$ in all the calculations for neutron-rich carbon isotopes. In the present study using the N$^3$LO potential, we employ the value of $\hbar\Omega = 14$ MeV. As for the magnitude of the refinement, denoting the one-body energy for a single-particle orbit $j$ of the neutron as $e^n(j)$, we need the variations as $\Delta e^n(1s_{1/2}) = -1.55$ and $\Delta e^n(0d_{5/2}) = +0.21$ MeV which are added to the original $e^n(j)$’s, namely, the h.o. kinetic energies. We also change the one-body energy for the $0d_{3/2}$ orbit of the neutron by the same amount as $\Delta e^n(0d_{5/2})$ for simplicity. We use these dressed one-body energies not only in the calculation for $^{15}$C but also for other neutron-rich carbon isotopes. Hereafter, we refer to the calculated results with the minimal refinement as “dressed”.

The results of low-lying energy levels of $^{16}$C with the above-mentioned refinement are shown in Fig. 2 with “dressed”, while the results with “original” represent those obtained without
any adjustable parameters. The calculated results become better by introducing the dressed one-body energies. Namely, the correct ordering of the low-lying energy levels is obtained.

In Table 1, the calculated and experimental values of $B(E2; 2^+_1 \rightarrow 0^+_1)$ for $^{16}$C are tabulated, together with the values for $^{14}$C to investigate the isotope dependence. Since the present model space is of no-core type and is rather large as compared to that of the conventional shell model, we do not employ the effective charges but use the bare charges in calculating $B(E2)$. Although, in principle, a sort of effective operator is needed for the calculation of electric transition strengths [15], we use the bare operator and investigate the validity of this treatment for the neutron-rich carbon isotopes.

For $^{16}$C, the calculated $B(E2)$ value becomes closer to the anomalously hindered experimental value by introducing the dressed one-body energies. Note that the value for “original” is still considerably smaller than that by the $0p1s0d$ shell-model results using the effective charges [5, 6]. For $^{14}$C, the results for “original” and “dressed” are identical, and show a slightly larger values than the experiment, but the discrepancies are still within the error bars. We note here that the calculated energy spacings between the $0^+_1$ and $2^+_1$ states for the cases “original” and “dressed” are 4.69 and 4.68 MeV, respectively, whereas the experimental value is 7.01 MeV. Thus, the calculated energy spacings are somewhat smaller than the experiment.

In Table 2, we show calculated occupation numbers for the $0^+_1$ and $2^+_1$ states in $^{14}$C and $^{16}$C for the case “dressed”. The occupation numbers of the proton for $^{16}$C are hardly changed between the $0^+_1$ and $2^+_1$ states. As for the neutron, only the occupation numbers for the $1s_{1/2}$ and $0d_{5/2}$ orbits are slightly different between the $0^+_1$ and $2^+_1$ states. This result supports the decoupling of the valence two neutrons from the core part [2, 5]. In the case of $^{14}$C, the proton occupation numbers for the $0p_{3/2}$ and $0p_{1/2}$ orbits are rather different between the $0^+_1$ and $2^+_1$ states in contrast to the case of $^{16}$C. These results indicate that the excitation mechanism is very different between $^{14}$C and $^{16}$C.

Here we comment on the ratio of the neutron and proton transition matrix elements for $^{16}$C. A large ratio has been found through the experiment as $M_n/M_p = 7.6 \pm 1.7$ [2]. In our calculation for the case “dressed”, the ratio is 4.82 and somewhat smaller than the experiment. Our small model space may be large enough for $M_p$ because of the inclusion of the $2\Omega$ excited $1p0f$ shell
and neutron (method can successfully describe the low-lying structure of NN potential and the Coulomb force through a unitary-transformation theory. The present probability most dominant configuration for the case "original" is \[0^+\] shell from the 0\(s\)/\(p\) shell, while it may not be sufficient for \(M_n\) due to the missing 2\(h\)\(\Omega\) excited 2\(s\)1d0\(g\) shell from the 1\(s\)0\(d\) shell. Thus, \(M_n\) can be enlarged to a certain extent in the calculation with a larger model space.

Finally, we investigate dominant configurations forming the ground 0\(^{+}\) state of 16\(^{C}\). The most dominant configuration for the case "original" is \([0s^2_1/2p^4_{3/2}][0s^2_1/2p^2_{1/2}p^2_{3/2}1s^2_{1/2}]\nu\) with the probability 8.4\%. The second one is \([0s^2_{1/2}p^4_{3/2}][0s^2_{1/2}p^2_{1/2}p^2_{3/2}d^2_{5/2}]\nu\) with the probability 28.1\%. In this case, the two neutrons in the 1\(s\)0\(d\) shell tend to occupy the 0\(d\)_5/2 state more than the 1\(s\)_1/2 state. On the other hand, these two configurations for "dressed" have the probabilities as 24.4 and 16.2\%, respectively. Thus, for the "dressed" case, both the two configurations are important for the ground 0\(^{+}\) state of 16\(^{C}\).

In summary, we have investigated low-lying energy levels and \(B(E2; 2^+_1 \rightarrow 0^+_1)\) of 16\(^{C}\) by introducing a new shell-model method of no-core type with the model space up to the 1\(p\)0\(f\) shell. The two-body effective interaction is microscopically derived from the Idaho N\(^3\)LO NN potential and the Coulomb force through a unitary-transformation theory. The present method can successfully describe the low-lying structure of 16\(^{C}\), and its anomalously hindered \(B(E2; 2^+_1 \rightarrow 0^+_1)\) value is well reproduced by the calculation with the bare charges. More detailed calculations for neutron-rich carbon isotopes are in progress, and its results will be reported elsewhere in the near future.

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### Table 1

| Isotopes | original  | dressed | Expt.         |
|----------|-----------|---------|---------------|
| 14\(^{C}\) | 4.11      | 4.11    | 3.74 \(\pm\) 0.50 |
| 16\(^{C}\) | 1.38      | 0.91    | 0.63 \(\pm\) 0.11 (stat) |

### Table 2

| Isotopes | \(J^\pi\) | Nucleon | 0\(p\)_3/2 | 0\(p\)_1/2 | 1\(s\)_1/2 | 0\(d\)_5/2 |
|----------|-----------|---------|------------|------------|------------|------------|
| 14\(^{C}\) | \(0^+_1\) | \(p\)    | 3.58       | 0.29       | 0.08       | 0.02       |
|          |           | \(n\)    | 3.89       | 1.95       | 0.06       | 0.02       |
|          | \(2^+_1\) | \(p\)    | 2.84       | 1.04       | 0.07       | 0.02       |
|          |           | \(n\)    | 3.89       | 1.95       | 0.06       | 0.03       |
| 16\(^{C}\) | \(0^+_1\) | \(p\)    | 3.41       | 0.44       | 0.11       | 0.02       |
|          |           | \(n\)    | 3.89       | 1.95       | 1.06       | 0.93       |
|          | \(2^+_1\) | \(p\)    | 3.38       | 0.47       | 0.10       | 0.02       |
|          |           | \(n\)    | 3.89       | 1.95       | 0.84       | 1.13       |
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