Parameter estimation for high dimensional classification model on colon cancer microarray dataset

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Abstract. In classification problems, logistic regression is among powerful techniques for discrimination. It provides directive probabilities of sample classification and interpretable coefficients. When it comes to model high dimensional dataset, however, logistic regression with its Newton-Raphson method of parameter estimation is no longer applicable, especially on low sample size and extremely high dimension. By applying cross-entropy algorithm on regularized logistic regression, it was able to well performing parameter estimation and highly accurate classification result.

1. Introduction
The advanced technologies in molecular biology and genetics such as DNA microarrays lead to the whole genome analysis instead of the study of genes individually. In a single experiment using the DNA microarray technology, the levels of thousands of gene expression can be measured. Biomedical research is one of the fields that take benefits from this technology. There are so many cancer classification based datasets resulted from microarray gene expression technology. With these datasets it allows us to compare the levels of gene expression on different instances in samples (tissues).

The problem of small sample size and high dimensionality in microarray dataset is still a challenging issue in cancer classification. The dataset is typically to have number of features (genes) more than the number of samples (tissues). Usually, the number of samples is about a hundred or less, while the number features is more than one thousand. Of these thousands features, most of them are irrelevant and only a small number of important genes are really responsible to cancer classification. Due to these characteristics of the datasets, the development of techniques for the purpose of feature selection remains a challenging task. A good selection of features could improve the classification accuracy and could also be providing a much simpler interpretation of the result [1, 2, 3].

Many authors have been proposing their methods for classification and feature selection almost any field of applications [2, 3]. A well known classical and powerful method for classification is logistic regression. This technique provides predictive probability of classification and easy interpretation of coefficients. However, when it comes to model high dimensional dataset which is also to have low sample size, the Newton-Raphson method of parameter estimation for logistic regression model is no
longer applicable. To overcome this situation, we proposed cross-entropy algorithm for parameter estimation on regularized logistic regression.

2. Regularized Logistic Regression

Logistic regression is a regression model to be used for binary classification [10]. Usually, the response variable has two values either 1 or 0. In this model the linear combination of predictor variables has a nonlinear relation with the binary responses. Suppose that we have \( n \) samples and \( p \) variables. Let the response variable denoted by \( y_i \in [0, 1] \) and \( \mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ip}) \) be the vector of predictor variables. The relation between response and predictor variables is formulated by

\[
\pi_i = p(y_i = 1 | \mathbf{x}_i) = \frac{\exp(x_i^T \beta)}{1 + \exp(x_i^T \beta)}, \quad i = 1, 2, \ldots, n
\]  

(1)

where \( \beta = (\beta_0, \beta_1, \ldots, \beta_p)^T \) is a vector of \( p \) unknown coefficients of predictor variables. For simplicity in estimation, we take the log-likelihood function (logit transformation) of Equation (1) as

\[
l(\beta) = \sum_{i=1}^{n} [y_i \log(\pi_i) + (1 - y_i) \log(1 - \pi_i)].
\]  

(2)

To control the size of variable coefficients, we can add a regularization term to the log-likelihood function. Tibshirani [4] proposed \( l_1 \)-norm regularization to simultaneously conduct variable selection and estimation. This regularization is also known as the least absolute shrinkage and selection operator (LASSO). Note that the log-likelihood function, \( l(\beta) \), is negative, while the regularization term is nonnegative. Consequently, estimation is conducted by adding the constraint to the negative log-likelihood function. As a result, the regularized logistic regression using LASSO is defined as

\[
\hat{\beta}_{LASSO} = \arg\min_{\beta} \left[ -\sum_{i=1}^{n} y_i \log(\pi_i) + (1 - y_i) \log(1 - \pi_i) + \lambda \sum_{j=1}^{p} |\beta_j| \right]
\]  

(3)

The constant \( \lambda \) is a tuning parameter. When \( \lambda = 0 \), it reduces to maximum likelihood estimator, while \( \lambda \to \infty \), it forces all coefficients of variables to be zeros [4].

3. The Cross-Entropy (CE) Algorithm

The CE algorithm provides a simple and efficient method to solve optimization problems. Since most of estimations are in relation to optimization (maximization or minimization) the CE can deal with this problem. The reputation of CE has been proven in almost any field of application, such as reliability analysis, performance analysis of telecommunication problem, and operation research [6, 7, 8, 9].

The CE involves two processes iteratively, random sample generation and parameter update of random mechanism in order to produce a better sample in the next iteration. In general, the CE algorithm for estimation problem is outlined as follows [7, 8].

- Generate random samples based on a mechanism (for example using normal distribution) as the candidates of parameter to estimate
- Fit these random samples into the objective function
- Sort the samples based on fitting values of objective function
- Take some elite samples (the samples with best fitting) and calculate the mean and the variance
- Repeat step 1 to 4 until the stopping criteria satisfied.

4. The Proposed Method and Dataset

In this chapter we are going to explain the procedure of our method. The algorithm of our method is organized as follows.

- Applying leave-one-out cross-validation (LOOCV) in this research, we separated the samples into training and testing. One sample was for testing and the rests were for training.
- After separating samples, we fixed the training samples into LASSO logistic regression model and estimated the parameters (coefficients) using the cross-entropy algorithm. These
coefficients were, then, sorted in relation to their absolute values from the largest to the smallest.

- Based on these coefficients, we conducted support vector machine (SVM) classification starting from variable which has the largest coefficient, and calculated the predicted accuracy. By adding one by one the second variable of the second largest coefficient until adding variable of the smallest coefficient, the SVM classification took place and the predicted accuracies were calculated.
- We plotted the accuracy trajectory of variables ordered by their absolute coefficients. We selected combination of variables that produced the highest accuracy.
- Using selected variables, we repeated step 2 to 4 until the maximum accuracy be reached.

The dataset used in this paper is colon cancer dataset. It is freely available from the web as it also published by [5]. This dataset consists of 62 samples and 2000 variables (genes). Of the 62 samples, 40 samples are cancerous and 22 others are noncancerous.

5. Result and Discussion
The experiment was conducted in 6 iterations. In the first iteration we selected 148 of 2000 variables. The trajectory plot can be seen in Figure 1.

![Figure 1](image-url)

**Figure 1.** Accuracy trajectory plot in the first iteration with vertical axis for accuracy and horizontal axis for number of selected variables

| Iteration | 1     | 2     | 3     | 4     | 5     | 6     |
|-----------|-------|-------|-------|-------|-------|-------|
|           | Number of variables before parameter estimation | 2000 | 148   | 76    | 51    | 28    | 26    |
| Accuracy using all variables after parameter estimation | 0.8065 | 0.9194 | 0.9355 | 0.9677 | **0.9839** | 0.9677 |
| Number of selected variables | 148 | 76    | 51    | 28    | 26    | 26    |
| Accuracy after sorting and selecting variables | 0.9839 | 1.0000 | 1.0000 | 1.0000 | **1.0000** | 0.9839 |

From Table 1, we can see that the best performance was reached in iteration 5 with the number of selected variables is 26 and the accuracy is 1.0000.
As seen in Figure 2, the best accuracy was achieved in iteration 5. Accordingly, the selection of variables should be based on the result in iteration 5. The coefficient value of each variable can be seen in Table 2. Note that the accuracy in Table 2 is a cumulative accuracy of the top best variables. In other words, Table 2 also shows the accuracy, when a model is using variable of first rank, is 0.807, and then by adding one variable of the next rank, it is 0.871, and so on. The perfect accuracy, 1.0, is achieved when using up to 26 variables.

Table 2. List of variables with their ranks, coefficient values and cumulative accuracies

| Rank | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Variable No. | 765 | 377 | 1772 | 1241 | 353 | 14 | 1976 | 493 | 792 | 1769 | 1482 | 554 | 1644 |
| Coefficient  | -1.52 | -1.14 | 0.74 | 0.64 | 0.55 | -0.45 | -0.41 | -0.35 | -0.32 | 0.29 | -0.26 | -0.25 | -0.25 |
| Accuracy     | 0.807 | 0.871 | 0.887 | 0.919 | 0.919 | 0.903 | 0.936 | 0.936 | 0.919 | 0.952 | 0.93 | 0.952 | 0.96 |

Table 2b. Rank 14 to 26

| Rank | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Variable No. | 1423 | 974 | 70  | 1325 | 783 | 590 | 1740 | 1560 | 164 | 1641 | 1346 | 1058 | 995 |
| Coefficient  | -0.21 | 0.21 | -0.21 | 0.20 | 0.20 | 0.17 | 0.17 | -0.17 | 0.15 | 0.12 | 0.11 | -0.07 | 0.04 |
| Accuracy     | 0.968 | 0.968 | 0.968 | 0.968 | 0.984 | 0.984 | 0.984 | 0.984 | 0.984 | 0.984 | 1   | 0.984 | 1   |

6. Conclusion

We have shown that by applying the cross-entropy algorithm on regularized logistic regression high performance accuracy can be achieved. Our proposed method can be applied on dataset that has high dimensionality and low sample size.

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