Higgs Bosons at Muon Colliders

M. S. Berger

Physics Department, Indiana University, Bloomington, IN 47405

(Dated: March 25, 2022)

We review the role of a muon collider in the study of Higgs bosons via production in the s-channel. Very precise measurements of a Standard Model-like Higgs boson mass and total width can be performed, and may lead to a discrimination between a Standard Model Higgs boson and the light Higgs boson of the minimal supersymmetric theory. The heavier Higgs bosons from a supersymmetric theory or an exotic Higgs sector can be studied in the s-channel. A muon collider may play a crucial role in separating the overlapping signals for two heavy nearly degenerate Higgs bosons, and may play an important role in precision tests of radiative corrections in the Higgs sector. The measurements at a muon collider will be complementary to the Higgs studies at the Large Hadron Collider and at an electron-positron Linear Collider.

I. INTRODUCTION

Interest has grown rapidly in muon colliders in the last several years as it became clear that the technological challenges might not be insurmountable[2]. Muon colliders are of interest to particle physics exploration for a number of reasons: a) the absence of significant bremsstrahlung allows one to contemplate circular accelerators of much higher energy than is possible with $e^+e^-$ machines, b) the coupling of Higgs bosons is proportional to particle mass (see Fig. (1)), and hence there is the possibility that Higgs bosons can be produced in reasonable numbers in the s-channel[3, 4], c) there are regions of parameter space for which it will be impossible for either the Large Hadron Collider (LHC) or a Linear Collider (LC) to discover the heavier Higgs bosons of supersymmetry or, in the case of a general two-Higgs-doublet or more extended model, Higgs bosons of any mass with small or zero $V V$ coupling, d) the neutrinos from the decays of muons can be used as a source for a neutrino factory[5, 6].

The large mass of the muon in comparison to that of the electron results in a number of advantageous features of a muon collider. The beam energy spreads of a muon collider can be very small, making them useful for studying narrow resonances like the SM Higgs boson. In addition, there is little bremsstrahlung, and the beam energy can be tuned to one part in a million through in situ spin-rotation measurements[7].

High rates of Higgs production at $e^+e^-$ colliders rely on substantial $V V$ Higgs coupling for the Higgs-strahlung process $Z+\text{Higgs}$ or for the $WW$ fusion process $WW\rightarrow\text{Higgs}$ ($WW$ fusion). In contrast, a $\mu^+\mu^-$ collider can provide a factory for producing a Higgs boson with little or no $V V$ coupling so long as it has SM-like (or enhanced) $\mu^+\mu^-$ couplings. Important examples of this last form of Higgs boson are the heavy neutral Higgs bosons $H^0$ and $A^0$ of the Minimal Supersymmetric Standard Model (MSSM).

If the a light ($<$ 130 GeV) Higgs boson exists, then both $e^+e^-$ and $\mu^+\mu^-$ colliders will be valuable; the Higgs boson would have been discovered at a previous higher energy collider (possibly a muon collider running at high energy), and then the Higgs factory would be built with a center-of-mass energy precisely tuned to the Higgs boson mass. The most likely scenario is that the Higgs boson is discovered at the LHC via gluon fusion ($gg\rightarrow H$) or perhaps earlier at the Tevatron via associated production ($q\bar{q}\rightarrow WH, t\bar{t}H$), and its mass is determined to an accuracy of about 100 MeV. If a linear collider has also observed the Higgs via the Higgs-strahlung process ($e^+e^-\rightarrow ZH$), one might know the Higgs boson mass to better than 50 MeV with an integrated luminosity of 500 fb$^{-1}$. The muon collider would be optimized to run at $\sqrt{s} \approx m_H$, and this center-of-mass energy would be varied over a narrow range so as to scan over the Higgs resonance.
II. SM-LIKE HIGGS BOSONS

The production of a Higgs boson (generically denoted $h$) in the $s$-channel with interesting rates is a unique feature of a muon collider \[3, 4\]. The resonance cross section is

$$
\sigma_h(\sqrt{s}) = \frac{4\pi \Gamma(h \to \mu\bar{\mu}) \Gamma(h \to X)}{(s - m_h^2)^2 + m_h^2 (\Gamma_h^{\text{tot}})^2}.
$$

In practice, however, there is a Gaussian spread ($\sigma_\sqrt{s}$) to the center-of-mass energy and one must compute the effective $s$-channel Higgs cross section after convolution assuming some given central value of $\sqrt{s}$:

$$
\sigma_h(\sqrt{s}) = \frac{1}{\sqrt{2\pi} \sigma_\sqrt{s}} \int \sigma_h(\sqrt{s}) \exp \left[-\frac{(\sqrt{s} - \sqrt{s_0})^2}{2\sigma_\sqrt{s}^2} \right] d\sqrt{s} \approx m_h \frac{4\pi}{m_h} \frac{\text{BF}(h \to \mu\bar{\mu}) \text{BF}(h \to X)}{\Gamma_h^{\text{tot}}} \left[1 + \frac{8}{\pi} \left(\frac{\sigma_\sqrt{s}}{\Gamma_h^{\text{tot}}}\right)^2 \right]^{1/2}.
$$

It is convenient to express $\sigma_\sqrt{s}$ in terms of the root-mean-square (rms) Gaussian spread of the energy of an individual beam, $R$:

$$
\sigma_\sqrt{s} = (2 \text{ MeV}) \left(\frac{R}{0.003}\right) \left(\frac{\sqrt{s}}{100 \text{ GeV}}\right).
$$

It is clear from Eq. (3) that a resolution $\sigma_\sqrt{s} \lesssim \Gamma_h^{\text{tot}}$ is needed to be sensitive to the Higgs width. Furthermore, Eq. (2) indicates that $\text{BF}(h \to \mu\bar{\mu})$ must not be extremely suppressed for there to be large event rates for Higgs production. The width of a light SM-like Higgs is very small (e.g. a few MeV for $m_{h_{SM}} \sim 110$ GeV), implying the need for $R$ values as small as $\sim 0.003\%$ for studying a light SM-like $h$. In addition to the very small beam energy spread, one must also be able to determine very accurately the beam energy to perform a scan over such a narrow resonance. This can be accomplished utilizing the spin precession of the muon noted above. A sample scan is illustrated in Fig. 2 for $m_{h_{SM}} = 110$ GeV SM Higgs boson.

The SM Higgs cross sections and backgrounds as well as the integrated luminosity required for a $5\sigma$ signal are shown in Fig. 3 for $R = 0.003\%$ and $m_{h_{SM}}$ values such that the dominant decay mode is $b\bar{b}$. However, the most recent experimental results from LEP have pushed the SM Higgs mass bound well above 91 GeV. For a Higgs mass $>\sim 130$ GeV, the Higgs width $\Gamma_h^{\text{tot}}$ becomes much larger as the $WW^*$ decay channel opens up.

The Higgs bosons in supersymmetric models are in general detectable at muon colliders. If the masses of the supersymmetric particles are large, the Higgs sector typically exhibits decoupling behavior in which the lightest supersymmetric Higgs boson $h^0$ will be very similar to the $h_{SM}$ when the other Higgs bosons are heavy, and the $h^0$ rates will be very similar to $h_{SM}$ rates. On the other hand, the heavier Higgs bosons in a typical supersymmetric model decouple from pairs of gauge bosons $VV$ at large mass and remain reasonably narrow.

---

**FIG. 1:** Feynman diagram for $s$-channel production of a Higgs boson.
FIG. 2: Number of events and statistical errors in the $b \bar{b}$ final state as a function of $\sqrt{s}$ in the vicinity of $m_{h_{\text{SM}}} = 110$ GeV, assuming $R = 0.003\%$, and $\epsilon L = 0.00125$ fb$^{-1}$ at each data point.

(<1 GeV unless the $t \bar{t}$ decay mode is open). As a result, their $s$-channel production rates remain large, and a muon collider can avoid the production channels that depend on a sizable coupling to gauge bosons.

What can a muon collider add to the LHC and LC? The LHC and quite likely a linear collider will be available already, and the Higgs boson will be detected and some of its properties determined before a muon collider will become operational. Current expectations for the luminosity at an LC are 500 fb$^{-1}$ over 1-2 years. This yields a SM Higgs boson production rate of greater than $10^4$ per year in the process $e^+e^- \rightarrow Zh$. Therefore the latest estimates of the luminosity at a linear collider yield numbers of Higgs bosons that are comparable to what will be available at a muon collider/Higgs factory with its more modest integrated luminosity (expected with the current machine parameters) of the order of one inverse femtobarn. A linear collider with such high luminosity can certainly perform quite accurate measurements of certain Higgs parameters such as the Higgs mass, couplings to gauge bosons, couplings to heavy quarks, etc.\[8\].

The $s$-channel production process allows one to determine the mass, total width, and the cross sections $\sigma_h(\mu^+\mu^- \rightarrow h \rightarrow X)$ for several final states $X$ to very high precision. The Higgs mass, total width and the cross sections can be used to constrain the parameters of the Higgs sector. For example, in the MSSM their precise values will constrain the Higgs sector parameters $m_{A^0}$ and $\tan \beta$ (where $\tan \beta$ is the ratio of the two vacuum expectation values (vevs) of the two Higgs doublets of the MSSM). The main question is whether these constraints will be a valuable addition to LHC and LC constraints.

Precise measurements of the couplings of the Higgs boson to the Standard Model particles are important tests of the mass generation mechanism. In the Standard Model with one Higgs doublet, this coupling is proportional to the particle mass. In the more general case there can be mixing angles present in the couplings. Precision measurements of the couplings can distinguish the Standard Model Higgs boson from the SM-like Higgs boson typically present in a more general model. If deviations are found, their magnitude can be extremely crucial for constraining the parameters of the more general Higgs sector. In particular, it might be possible to estimate the masses of the other Higgs bosons of the extended Higgs sector, thereby allowing a more focused search for them.

The precision possible at a muon collider for measuring $m_h$ and $\Gamma^\text{tot}_h$ of a SM-like $h$ with $m_h \sim 110$ GeV are $1 - 3 \times 10^{-6}$ and 0.2 respectively. To achieve these accuracies, one first determines the Higgs mass to about 1 MeV by the preliminary scan illustrated in Fig. 3. Then, a dedicated three-point fine scan near the
FIG. 3: The SM Higgs cross sections and backgrounds in $b\bar{b}$, $WW^*$ and $ZZ^*$. Also shown is the luminosity needed for a 5 standard deviation detection in $b\bar{b}$. From Ref. [3]. For a SM-like $h$, at $\sqrt{s} = m_h \approx 115$ GeV, the $b\bar{b}$ final state rates are $\approx 10^4$ events $\times L(fb^{-1})$ for both the signal and the background.

resonance peak using $L \sim 0.2$ fb$^{-1}$ of integrated luminosity (corresponding to a few years of operation) would be performed. For a SM Higgs boson with a mass sufficiently below the $WW^*$ threshold, the Higgs total width is very small (of order several MeV), and the only process where it can be measured directly is in the $s$-channel at a muon collider. An accurate measurement of $\Gamma_h^{tot}$ would be a very valuable input for precision tests of the Higgs sector. In particular, since all the couplings of the Standard Model $h_{SM}$ are known, $\Gamma_{h_{SM}}^{tot}$ is precisely predicted. Therefore, the precise determination of $\Gamma_h^{tot}$ obtained by this scan would be an important test of the Standard Model, and any deviation would be evidence for a nonstandard Higgs sector (or other new physics).

Other interesting measurements of Higgs boson properties can be performed at a muon collider in the case where at least a hundred inverse femtobarns of luminosity is available. Then the mass, width and spin of a SM-like Higgs boson can also be determined by operating either a muon collider or a linear collider at the $Zh$ production threshold where the rate is sensitive to the Higgs mass. With 100 fb$^{-1}$ of integrated luminosity, an error of less than 100 MeV can be achieved for $m_h < 150$ GeV. The shape of the $\ell^+\ell^- \rightarrow Zh$ threshold cross section can also be used to determine the spin and to check the CP nature of the Higgs.

III. HEAVY HIGGS Bosons

In supersymmetric models there are multiple physical Higgs bosons. Often the Higgs spectrum includes a SM-like Higgs boson with mass close to the $Z$ boson mass and some heavier Higgs bosons whose couplings are very much different than a SM particle of the same mass. For example, in the MSSM there is a light, neutral $h^0$ and two heavier neutral Higgs bosons, $H^0$ and $A^0$. As one adjusts the parameters of the theory to make the $H^0$ and $A^0$ heavier, the light Higgs boson $h^0$ becomes more and more like the SM Higgs boson. It may very well be the case that after the initial discovery of this SM-like Higgs boson the primary question will involve detecting deviations from the SM Higgs sector by a) measuring very precisely the SM-like Higgs boson properties, and/or b) directly discovering additional Higgs bosons.

In the context of the MSSM, It is highly likely that the process $e^+e^- \rightarrow Zh$ used to find and study the light Higgs state at a first generation LC will not be suitable for the heavier Higgs bosons, because in the decoupling
limit the coupling of the Higgs to gauge bosons is greatly suppressed (this is a corollary to the statement that the light Higgs boson in Standard Model-like). There is a $250 - 500$ GeV range of heavy Higgs boson masses for which discovery is not possible via $H^0 A^0$ pair production at a $\sqrt{s} = 500$ GeV LC. Further, the $A^0$ and $H^0$ cannot be detected in this mass range at either the LHC or LC for a wedge of moderate $\tan \beta$ values. (For large enough values of $\tan \beta$ the heavy Higgs bosons are expected to be observable in $b\bar{b} A^0, b\bar{b} H^0$ production at the LHC via their $\tau^+ \tau^-$ decays and also at the LC.) A linear collider operating in the $\gamma \gamma$ mode can produce Higgs bosons in the $s$-channel, and there have been a number of studies of such processes\cite{11, 12, 13, 14, 15, 16, 17}. This requires that such an option exists, and the energy of the $\gamma \gamma$ system is not as sharply peaked at the center-of-mass energy as it is for the muon collider.

A muon collider can fill some, perhaps all of this moderate $\tan \beta$ wedge. If $\tan \beta$ is large, the $\mu^+ \mu^- H^0$ and $\mu^+ \mu^- A^0$ couplings (proportional to $\tan \beta$ times a SM-like value) are enhanced, thereby leading to enhanced production rates in $\mu^+ \mu^-$ collisions. These bosons can be discovered via the radiative return mechanism\cite{3}, and once a peak is found the machine energy can be set to $m_{A^0}$ or $m_{H^0}$ and the muon collider becomes a Higgs factory for the heavier Higgs bosons. The resolution requirements for studying the heavy Higgs bosons in the $s$-channel are not as stringent as those for the light Higgs boson because the heavier Higgs boson widths are generally much larger. Since $R = 0.1\%$ is sufficient, much higher luminosity ($L \sim 2 - 10$ fb$^{-1}$/yr) would be possible as compared to that for $R = 0.01\% - 0.003\%$ as required for studying the $h^0$.

In the MSSM, the heavy Higgs bosons are largely degenerate, especially in the decoupling limit where they are heavy. In that case, a muon collider with sufficient energy resolution might be the only possible means for separating out these states. Examples showing the $H^0$ and $A^0$ resonances for $\tan \beta = 5$ and 10 are shown in Fig. 4. For the larger value of $\tan \beta$ the resonances are clearly overlapping. For the better energy resolution of $R = 0.01\%$, the two distinct resonance peaks are still visible, but they are smeared out and merge into one broad peak for $R = 0.06\%$.

Muon colliders excel at making precise measurements of Higgs boson masses since they can exploit the $s$-channel production process. This is reminiscent of the very accurate determination of the $Z$ boson mass to just $2.2$ MeV from the LEP measurements\cite{18}. Precise measurements of supersymmetric Higgs boson masses could provide a powerful window on radiative corrections\cite{19}. Supersymmetry together with gauge invariance in the MSSM implies the mass-squared sum rule

$$m_{h^0}^2 + m_{H^0}^2 = m_{A^0}^2 + m_Z^2 + \Delta, \quad (4)$$
where $\Delta$ is a calculable radiative correction (the tree-level sum rule results from setting $\Delta = 0$). This formula involves observables (masses) that can be precisely measured in the $s$-channel processes. Solving for the mass difference

$$m_{A^0} - m_{H^0} = \frac{m_{H^0}^2 - m_Z^2 - \Delta}{m_{A^0} + m_{H^0}},$$

and one obtains a form that indicates in the decoupling limit, $m_{A^0} \to \infty$, the mass difference between the heavy Higgs bosons becomes small. As discussed in the previous section, the light Higgs mass $m_{h^0}$ can be measured to less than an MeV in the $s$-channel. The masses of and the mass difference between the heavy Higgs states $H^0$ and $A^0$ can also be measured precisely by $s$-channel production. The ultimate precision that can be obtained on the masses of the $H^0$ and $A^0$ depends strongly on the masses themselves and $\tan \beta$. But a reasonable expectation is that a scan through the resonances should be able to determine the masses and the mass-difference to some tens of MeV with just $0.1 \text{fb}^{-1}$ of integrated luminosity. Altogether these mass measurements yield a value for the radiative correction $\Delta$ to a precision of order $10 \text{GeV}^2$. Since the typical size of $\Delta$ is of order $10^3 \text{GeV}^2$, this constitutes a measurement of roughly one part in $10^3$. The quantity $\Delta$ is calculable in terms of the self-energy diagrams of the Higgs bosons, and a comparison between the measured value and the theoretical prediction yields a test of radiative corrections in the MSSM. Further progress in the theoretical calculation of $\Delta$ would be needed to fully exploit the expected precision of the experimental measurements.

IV. CONCLUDING REMARKS

Recent experimental results hint that a muon collider may play a crucial role in studying the next generation of physics signals. There is the evidence from LEP for a Higgs boson near $m_H \simeq 115 \text{GeV}$. This $\geq 2\sigma$ signal is not definitive, but it has been taken very seriously since it is consistent with the current precision electroweak data and fits well with a supersymmetric interpretation. A Higgs boson with such a mass is in the optimal range for study at a Higgs factory. Such a Higgs boson sits comfortably above the $Z$-pole where there is a large background from $Z$ decay to $b\bar{b}$, and a 115 GeV mass is sufficiently below the $WW^*$ threshold that the decay width remains small and the ability of the muon collider to achieve a very narrow beam energy spread can be exploited.

In the MSSM such a Higgs boson mass of 115 GeV is near the theoretical upper limit of $m_{H^0} < 130 \text{GeV}$, and would indicate a value of the supersymmetry parameter $\tan \beta$ substantially above 1 (assuming stop masses $\lesssim 1 \text{TeV}$). This is consistent with recent evidence for non-SM contributions to the anomalous magnetic moment of the muon, which also can be explained in the MSSM with a moderately large value of $\tan \beta$. If these early indications prevail, and we are left with a supersymmetric Higgs sector with large $\tan \beta$, then it is likely that the heavy Higgs $H^0$ and $A^0$ will not be observable at the LHC or a LC. The detection of these Higgs bosons could be accomplished in the $s$-channel at a muon collider, and some precision tests involving the Higgs boson masses can be performed to check radiative corrections in the Higgs sector. More generally, the muon collider has the potential to find and study Higgs bosons that exist in more general models than the MSSM with extended Higgs sectors. In this more general context, the muon collider offers the possibility of studying the CP nature of the Higgs bosons that are found.

Finally the muon collider program encompasses much more than physics of Higgs factories described here. Interesting physics can be envisioned at all stages of the development of muon colliders: from neutrino factories to Higgs factories to even higher energies.

Acknowledgments

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91ER40661.

[1] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Physics of higgs factories (2001), arXiv:hep-ph/0110340.
[2] C. M. Ankenbrandt et al., Phys. Rev. ST Accel. Beams 2, 081001 (1999), arXiv:physics/9901022.
[3] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Phys. Rept. 286, 1 (1997), arXiv:hep-ph/9602415.
[4] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Phys. Rev. Lett. 75, 1462 (1995), arXiv:hep-ph/9504330.
[5] D. Ayres et al. (Neutrino Factory and Muon Collider), Expression of interest for r $\ell d$ towards a neutrino factory based on a storage ring and a muon collider (1999), arXiv:physics/9911009.
[6] N. Holtkamp et al., *A feasibility study of a neutrino source based on a muon storage ring*, slac-reprint-2000-054.
[7] R. Raja and A. Tollestrup, Phys. Rev. **D58**, 013005 (1998), arXiv:hep-ex/9801004.
[8] M. Battaglia and K. Desch (2000), arXiv:hep-ph/0101165.
[9] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Phys. Rev. Lett. **78**, 3991 (1997), arXiv:hep-ph/9612279.
[10] D. J. Miller, S. Y. Choi, B. Eberle, M. M. Muhlleitner, and P. M. Zerwas, Phys. Lett. **B505**, 149 (2001), arXiv:hep-ph/0102023.
[11] G. Jikia, Nucl. Phys. **B405**, 24 (1993).
[12] M. S. Berger, Phys. Rev. **D48**, 5121 (1993), arXiv:hep-ph/9307259.
[13] D. A. Dicus and C. Kao, Phys. Rev. **D49**, 1265 (1994), arXiv:hep-ph/9308330.
[14] G. J. Gounaris, P. I. Porfyriadis, and F. M. Renard, Eur. Phys. J. **C19**, 57 (2001), arXiv:hep-ph/0010006.
[15] M. S. Berger (1992), arXiv:hep-ph/9207275.
[16] M. M. Muhlleitner, M. Kramer, M. Spira, and P. M. Zerwas, Phys. Lett. **B508**, 311 (2001), hep-ph/0101083.
[17] D. M. Asner, J. B. Gronberg, and J. F. Gunion (2001), arXiv:hep-ph/0110320.
[18] D. E. Groom et al. (Particle Data Group), Eur. Phys. J. **C15**, 1 (2000).
[19] M. S. Berger, Phys. Rev. Lett. **87**, 131801 (2001), arXiv:hep-ph/0105128.
[20] M. S. Berger, Phys. Rev. **D41**, 225 (1990).
[21] R. Barate et al. (ALEPH), Phys. Lett. **B495**, 1 (2000), arXiv:hep-ex/0011045.
[22] P. Abreu et al. (DELPHI), Phys. Lett. **B499**, 23 (2001), arXiv:hep-ex/0102036.
[23] M. Acciarri et al. (L3), Phys. Lett. **B495**, 18 (2000), arXiv:hep-ex/0011043.
[24] G. Abbiendi et al. (OPAL), Phys. Lett. **B499**, 38 (2001), arXiv:hep-ex/0101014.
[25] A. N. Okpara (2001), arXiv:hep-ph/0105151.
[26] H. N. Brown et al. (Muon g-2), Phys. Rev. Lett. **86**, 2227 (2001), arXiv:hep-ex/0102017.