Analysis of Mathematical Ability of High School Students based on Item Identification of National Examination Set

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Abstract—This research aimed to compare the mathematical abilities of high school students which include mathematical connection ability, mathematical reasoning ability, mathematical communication ability in Indonesia based on gender and school status in 2016 National Examination. The research sample was 1870 high school students of the Natural Science Departments and 2000 high school students of the Social Science Departments taken with random sampling. Research data was in the form of scores of Jakarta high school students' works in 2016 National Examination. The results showed (1) mathematical connection ability and mathematical reasoning ability of Natural Science department students from private high schools are higher than public high schools, (2) mathematical connection ability and mathematical reasoning ability of Social Science department students from private high school are higher than public high schools.

Keywords—mathematical connection ability, mathematical reasoning ability, mathematical communication ability, High School, 2016 National Examination

I. INTRODUCTION

Mathematics is a science that cannot be separated from other sciences because mathematics is used to help solve other scientific problems or problems in everyday life. For example, to determine the growth of occupation of an area or the growth of bacteria, the calculation requires the concept of differential integral or geometrical series concept.

The concepts, principles, and procedures of mathematics conveyed in mathematical learning aim to develop five mathematical abilities, which are mathematical connection ability, mathematical communication ability, reasoning ability, mathematical representation ability and problem-solving ability [1]. Mathematical connection ability is students' ability to connect the knowledge they have had before with knowledge will be learned. Mathematical connection ability includes the relationship between one topic and another in mathematics, the relationship between mathematical topics and topics in other disciplines, as well as the relationship between mathematics and real life.

Mathematical communication is one of important aspect of mathematics learning. The process of mathematical communication can help in building students' understanding through the representation of mathematical ideas in verbal and written way so that students unwittingly have built an understanding of the mathematical concepts they have, and students are able to express and develop their ideas [1], [2], [3]. Indicators of mathematical communication ability are: 1) representing mathematical problems using pictures, tables and graphs, 2) communicating mathematical thinking in verbal and written way, 3) using mathematical language to represent ideas, 4) interpreting and evaluating mathematical ideas in verbal and written way, and 5) using terms, symbols, and structures to model mathematical situations or problems [1], [2], [4], [5].

The mathematical reasoning ability is the ability to process a logical conclusion based on relevant facts and sources; transformation is given in a certain order to draw conclusions [6]. Mathematical reasoning is a dynamic conceptualization of students’ mathematical power and dynamic activities that involve a variety of modes of thinking [1]. Reasoning ability is a major component of thinking that involves forming generalizations and describing valid conclusions about ideas and how those ideas are connected [7]. Indicators of mathematical reasoning ability are (1) making observations and identification processes using mathematical relationships, (2) drawing conclusions based on data trends, (3) developing evidence from various mathematical arguments [8], [9], [10], [11], [12].

Almost every year the Education Assessment Center for the Ministry of Education and Culture conducts research related to the results of the National Examination (UN). The results of their research are always surprising: (1) Indonesian students have not mastered the mathematical material in the form of Higher Order Thinking Skills (HOTS) which includes application, analysis, synthesis and evaluative abilities. The research only discusses the results of the UN and TIMMS in the implementation of data collection in that year. The results of 2016 UN analysis were only analyzed based on cognitive domains according to Bloom’s taxonomy include comprehension, application, analysis and synthesis; those have not been analyzed based on the mathematical connection ability, mathematical communication ability and mathematical reasoning ability and have not been analyzed based on gender and school status. Males and females have unique and different thinking abilities. Male students tend to use language skills in the left brain or use the right brain for spatial and mathematical skills, while females use both [13].
Gender is seen as one of the aspects that can influence a person's cognitive level. This is caused by psych biosocial factors that have an influence on gender differences [14]. These biological factors can make a difference in spatial ability, higher order thinking, and other cognitive aspects, thus giving rise to differences in the achievement of mathematical learning [15]. Males are seen as having a tendency to spatial aspects while females have a tendency to verbal abilities [15].

Some experimental research, development research, and classroom action research relating to the mathematical connections, mathematical communication and mathematical reasoning and HOTS abilities have also been widely practiced in Indonesia, including Kramarski & Mevarech [16] who conducted research developing mathematical reasoning ability and mathematical connections of high school students through problem-based learning; Naufal, et al [17] conducted problem-based learning research to improve mathematical reasoning abilities of junior high school students; Qohar [18] conducted research related to the development of mathematical communication instruments; Djamilah and Widjayanti [19] conducted qualitative research on mathematical problem solving ability of prospective mathematics teacher students: what and how to develop them. These researches only measure one ability on one subject.

Mathematical learning continues to be carried out by every teacher who has attended training to improve the quality of learning. Therefore, from the learning outcomes carried out by the teacher, problems arise on how the mathematical connection ability, mathematical communication ability and mathematical reasoning ability of Indonesian high school students are based on the identification of items in 2016 National Examination.

II. LITERATURE REVIEW

Mathematical connection ability is the ability of students in connecting ideas in learning mathematics with other contexts as well as everyday life so that students can better understand the learning received. Mathematical connection ability is developed with several indicators, such as (1) connecting concepts/rules of mathematics, (2) connecting mathematical concepts/rules with other fields of study, (3) connecting mathematical concepts/rules with applications in real life, and (4) looking for relationships between various concept representations and procedures [4], [20], [21], [22].

Mathematical communication ability is the process standards of communication and representation in the principles and standards for school mathematics to help the students develop mathematical literacy [23]. The mathematical communication ability related to various elements; critical thinking, systems thinking, problem solving, analysis, and judgment [24]. The communication in mathematics and its teaching and learning and the continuous professional development of mathematics teacher. The effective communication occurs in the classroom if it has the real critical aspects in student learning as its starting point. [25].

Mathematical reasoning ability is one of mathematical abilities which is done to make a conclusion or solution to mathematical problems using logical process and unlimited proof [11]. The process of mathematical reasoning divided into three stages, they are classifying material into different classes, finding order in each class, dan finding out relationship of two or more classes [26]. It is also explained that to succeed in studying mathematics, students need to master those three stages of reasoning ability process. In other words, reasoning ability for students is a very important skill for students.

III. METHOD

The independent variables are school status and gender, and the dependent variables are mathematical abilities including mathematical communication ability, mathematical connection ability, and mathematical reasoning ability. The samples of the research were 1870 high school students from Natural Science Department consisting of 863 male students and 1007 female students and 2000 high School students from Social Science Department consisting of 1064 male students and 936 female students selected with random sampling. Data on mathematical abilities include mathematical communication ability, mathematical connection ability, and mathematical reasoning ability. The connection ability, and mathematical reasoning ability are in the form of scores of Jakarta high school students' works in 2016 National Exam. The results of the research scores used were the students 'work from the students' responses to the National Examination of High School Natural Science Department. and to the to the National Examination of High School Social Science Department on Mathematics subject in 2016.

Indicators of mathematical communication ability are: 1) representing mathematical problems using pictures, tables and graphs, 2) communicating mathematical thinking in verbal and written way, 3) using mathematical language to represent ideas, 4) interpreting and evaluating mathematical ideas in verbal and written way, and 5) using terms, symbols, and structures to model mathematical situations or problems. Indicators of mathematical reasoning ability are (1) making observations and identification processes using mathematical relationships, (2) drawing conclusions based on data trends, (3) developing evidence from various mathematical arguments [8], [9], [10], [11], [12]. Indicators of mathematical connection ability are (1) connecting concepts/rules of mathematics, (2) connecting mathematical concepts/rules with other fields of study, (3) connecting mathematical concepts/rules with applications in real life, and (4) looking for relationships between various concept representations and procedures [4], [20], [21], [22].

The procedures of this research were (1) the 2016 National Examination questions are grouped into indicators of mathematical connection ability, mathematical communication ability and mathematical reasoning ability, (2) the mathematical ability of students was calculated by Rasch Model, (3) the significance based on school status factors i.e., public and private schools and gender was tested.
IV. RESULT AND DISCUSSION

A. Mathematical Abilities of High School Students from Natural Science Department

The average comparison of the mathematical connection ability of students in natural science department from public high schools and private high schools obtained significance value of 0.00000, so it is known that the mathematical connection ability of natural science students from private high schools is higher than public high school. The average comparison of the mathematical connection ability of male and female students in natural science department obtained a significance value of 0.59400, then the mathematical connection ability is no different between male and female student.

The average comparison of the mathematical communication ability of students in natural science department from public high schools and private high schools obtained significance value of 0.98900, so it is concluded that the mathematical communication ability in private high schools and public high schools is no different. The average comparison of the mathematical connection ability of male and female students in natural science department obtained a significance value of 0.05700, so it is concluded that the mathematical communication ability of male students and female students of natural science department is no different.

The average comparison of the mathematical reasoning ability of natural science department students from public and private high schools obtained a significance value of 0.74400 so it is concluded that the mathematical reasoning ability of male and female students is no different.

The results showed the mathematical communication ability in students of natural science department from public high schools and private high schools are no different, while the mathematical connection ability and mathematical reasoning ability in students of natural science department from private high schools are higher than that from public high schools.

In algebraic material, the difference in the average of number of private high school and public high school students who have mathematical connection ability is only 6%, in geometry material it is 10%, in calculus material it is 9.5% and in trigonometric material it is 7.5%. Descriptively, the difference in the average of students who have mathematical connection ability between private and public high school students is a maximum of 10%. This difference is very small, and this supports the inferential statistical test results showing no significant differences. The difference in the average of the number of students in natural science department from Private High School and Public High School measured mathematical connection ability is found the least in the algebraic material and the most in geometric material. Overall, only a small percentage of high school students from natural science department have mathematical connection ability.

In the statistical material, the average ability of the number of private high school students who correctly answered the questions that measured mathematical communication ability with the ability of representing mathematical problems using pictures, tables, and graphs is 58%, while that of public high school students is only 28%. In calculus material, the average ability of the number of high school students who correctly answered is 28%, while that of private high school students is 28%. The difference in the average number of private and public high school students in natural science department measured mathematical communication ability is found the least in calculus and the most in statistical material. Overall, students of public high school or private high school in natural science department have mathematical connection ability on the average below 50%.

The 2016 National Examination items that measured mathematical reasoning ability in this research only take algebraic material. The average ability of the number of private high school students who answered correctly the questions that measured reasoning ability is 50.5% and 44.5% are from public high schools. The difference in the average number of private and public high school students in natural science department measured reasoning ability is quite small, which is 6% so it descriptively supports the testing of inferential statistics which stated the average ability of the number of students who correctly answered the questions that measured mathematical reasoning ability of private high school students was higher than that of public high school students.

The average ability of the number of male students who answered correctly the questions that measured reasoning ability is 47% and female students get 48%. The difference in the number of female and male students who have mathematical reasoning ability is very small, that is only 1% so this result supports inferential statistics which stated that in answering questions that measured mathematical reasoning ability, female students’ mathematical reasoning ability are higher than high school students from natural science department. The average of male and female students has mathematical reasoning ability based on the national exam is less than 50%.

| Mathematical Ability | School Gender | Public High School and Private High School (Mean Difference) | Males and Females (Mean Difference) |
|----------------------|---------------|---------------------------------------------------------------|------------------------------------|
| Mathematical Connection Ability | \(0.00000\) | (0.46872) | 0.59400 (0.08595) |
| Mathematical Communication Ability | 0.98900 | (0.16063) | 0.05700 (0.14727) |
| Mathematical Reasoning Ability | \(0.00000\) | (-0.59285) | 0.74400 (0.09911) |

\(a\) Significance
The results of the research on the mathematical abilities of high school students in Natural Science department are shown in algebra, geometry, and calculus material. The average ability of the number of female and male students who answered correctly the questions that measured mathematical connection ability was not different; it was only 5.5% different in trigonometric materials. Descriptively the difference in the number of female and male students with maximum mathematical connection ability is 5.5%. Female and male students who have the most mathematical connection abilities are found in algebraic material and the least is in geometrical material. Overall female students or male high school students in natural science department who have mathematical connection ability on average are less than 50%.

In statistical material, 44% of male students answered correctly the questions that measured reasoning ability than female students. The difference is 44%. While in calculus material, 21% of female students correctly answered the questions that measured reasoning ability. Male students have more ability to represent mathematical problems by using pictures, tables, and graphs, while female students have more ability to communicate mathematical thinking in writing. Overall female or male students who mastered mathematical communication abilities are on average less than 50%.

C. Mathematical Abilities of High School Students from Social Science Department

The average comparison of the mathematical connection ability in students of social science department from public high schools and private high schools received a significance value of 0.00000, so it is concluded that the mathematical connection ability in private high school is higher than public high school. The average comparison of the mathematical connection ability for male and female students from the social science department obtained a significance value of 0.00000, so it is concluded that the mathematical connection ability of female is higher than male.

The average comparison of the mathematical communication ability in students from social science department from public high schools and private high schools obtained a significance value of 0.86800, so it is concluded that the mathematical communication ability in private high school is higher than public high school. The average comparison of the mathematical communication ability for male and female students from the social science department obtained a significance value of 0.00000, so it is concluded that the mathematical communication ability of female is higher than male.

The average comparison of the mathematical reasoning ability in students from social science department from public high schools and private high schools obtained a significance value of 0.00700, so it is concluded that the mathematical reasoning ability of female is higher than male.

| Mathematical Ability          | School                  | Gender               |
|------------------------------|-------------------------|----------------------|
|                              | Public High School      | Private High School  |
|                              | (Mean Difference)       | (Mean Difference)    |
| Mathematical Connection Ability | 0.00000 s (-0.69125)    | 0.00000 s (-0.26958) |
| Mathematical Communication Ability | 0.86800 (0.13811)       | 0.78300 (-0.02646)   |
| Mathematical Reasoning Ability | 0.00000 s (-0.56245)    | 0.00700 s (-0.16116) |

D. Mathematical Abilities of High School Students from Social Science Department in 2016 National Examination

The results showed that mathematical connection ability and mathematical reasoning ability of private high school students are higher than public high schools students in the social science department.

Algebraic material that measures mathematical connection ability includes sequence, function, and matrix. The probability material includes probability and multiplication rules. Calculus material includes derivatives of algebraic function, limit of algebraic functions, integration of algebraic functions. In algebraic material, the difference in the number of private high school and public high school students who have mathematical connection ability is only 22%, in geometry material the difference is 16% different, in calculus material it is 9% and in statistical material it is 10%. Private high school students who have mathematical connection ability are 12% more than public high school in all algebra, geometry, calculus and statistics materials. Overall only a small percentage of private high school or public high school students in social science department have students with mathematical connection ability, which is below 50%.

In statistical material, 11% of private high school students in social science department correctly answered questions about statistics that measured communication ability which is more than public high schools students. Overall only a small percentage of high school students in social science department have students with mathematical connection ability, which is below 50%.

The 2016 National Examination items from social science department that measured mathematical reasoning ability in this research only take algebraic and calculus material. In algebraic material, 11% of private high school students correctly answered questions that measured mathematical reasoning ability which is more than public high school students. Overall high school students in social science department with mathematical reasoning ability are below 55%.

The comparison results of the mathematical abilities based on gender showed that the mathematical connection ability and mathematical reasoning ability of female students is greater than male students. Meanwhile, mathematical communication ability of male and female students are no different. In algebraic material, the average ability of the
number of male students who have mathematical connection ability is 38%, in calculus material the difference is 41%, in probability material it is 3% and in statistical material it is 4%. In algebraic material, the average ability of the number of female students who have mathematical connection ability is 44%, in calculus material the difference is 44%, in probability material it is 44% and in the statistical material it is 36%. Female students who have more mathematical connection ability are 12% more than male students in all algebraic materials, calculus, probability and statistics.

The difference in the average ability of the number of male and female students who master mathematical communication ability is very small, which is 2%. The difference in the average ability of the number of male and female students who master mathematical reasoning ability is 5%. In algebraic material, the average ability of the number of female students who correctly answered the questions that measured mathematical reasoning ability is 6% more than the average ability of the number of male students. In calculus material, the average ability of the number of female students who correctly answered the questions that measured mathematical reasoning ability is 2.5% more than the average ability of the number of male students. Overall female or male students in social science department who master mathematical communication ability and mathematical reasoning ability are on the average of below 50%.

V. CONCLUSION

The conclusions of this research are (1) the mathematical connection ability and mathematical reasoning ability of students in natural science department from private high school are higher than public high school, while the mathematical communication ability of students in natural science department from private high schools and public high schools has no difference; (2) the mathematical connection ability and mathematical reasoning ability of students in social science department from private high school are higher than public high school, while the mathematical communication ability of students in social science department from private high schools and public high schools has no difference.

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