\( e \)-continuous and Somewhat \( e \)-continuity in \( N_{nc} \)-Topological Spaces

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Abstract. A new class of mapping functions called \( N_{nc}e \)-continuous map, somewhat \( N_{nc}e \)-continuous map, somewhat \( N_{nc}e \)-open maps has been established and defined by making use of \( N_{nc}e \)-open sets. Some Characterizations and properties of \( N_{nc}e \)-continuous mappings and somewhat \( N_{nc}e \)-continuous mapping functions are presented.

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1. Introduction

Smarandache’s neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc. [1, 2, 3, 4, 24, 25]. Topology is a classical subject, as a generalization topological spaces many types of topological spaces introduced over the year. Smarandache [16] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces \( nts \)’s introduced by Salama and Alblowi [13]. Lellis Thivagar et.al. [10] was given the geometric existence of \( N \) topology, which is a non-empty set equipped with \( N \) arbitrary topologies. Lellis Thivagar et al. [11] introduced the notion of \( N_n \)-open (closed) sets and \( N_n \) continuous in \( N \) neutrosophic crisp topological spaces. Al-Hamido [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into \( N \)-neutrosophic crisp topological spaces and investigate some of their basic properties. The importance of continuity and generalized continuity is significant in various areas of mathematics and related sciences. Recent progress in the study of characterizations and generalizations of continuity has been done by means of several generalized closed sets. The first step of generalizing closed set was done by Levine in 1970 [12]. As a generalization of closed sets, \( e \)-closed sets were introduced and studied by Ekici [6, 7, 8]. In 2020, Vadiel and John Sundar [21] the concept of \( N \)-neutrosophic crisp \( \delta \)-open, \( N \)-neutrosophic crisp \( \delta \)-semiopen and \( N \)-neutrosophic crisp \( \delta \)-preopen sets, \( N \)-neutrosophic crisp \( \alpha \)-continuous, \( N \)-neutrosophic crisp semi continuous and \( N \)-neutrosophic crisp pre continuous are introduced. In this paper, \( N_{nc}e \)-continuous, somewhat \( N_{nc}e \)-continuous and somewhat \( N_{nc}e \)-open are discussed by using \( N_{nc}eo \) sets and get some results on somewhat \( N_{nc}Cts \) functions.
2. Preliminaries

Salama and Smarandache [15] introduced the concept of a neutrosophic crisp set in a set Y and discussed some basic definition and properties in this section.

**Definition 2.2** [15] Let Y be a non-empty set. Then H is called a neutrosophic crisp set (in short, *ncs*) in Y if H has the form H = (H₁, H₂, H₃) where H₁, H₂, & H₃ are subsets of Y.

The neutrosophic crisp empty (resp., whole) set, denoted by φ₁ (resp., Yₙ) is an *ncs* in Y defined by φ₁ = (φ₁, φ₁, Y) (resp. Yₙ = (Y, Y, φ₁)). We will denote the set of all *ncs*’s in Y as ncS(Y).

**Definition 2.3** [9] Let H = (H₁, H₂, H₃), I = (I₁, I₂, I₃) ∈ ncS(Y). Then

(i) H ⊆ I if H₁ ⊆ I₁, H₂ ⊆ I₂ & H₃ ⊆ I₃ (contained),
(ii) H = I if H ⊆ I & I ⊆ H (equal),
(iii) Hᶜ in Y is Hᶜ = (H₃, H₂ᶜ, H₁) (complement),
(iv) H ∩ M in Y is H ∩ I = (H₁ ∩ I₁, H₂ ∩ I₂, H₃ ∩ I₃) (intersection),
(v) H ∪ M in Y is H ∪ I = (H₁ ∪ I₁, H₂ ∪ I₂, H₃ ∪ I₃) (union),
(vi) ∩ H_j in Y is ∩ H_j = (∩ H_j₁, ∩ H_j₂, ∩ H_j₃) (family of intersection),
(vii) ∪ H_j in Y is ∪ H_j = (∪ H_j₁, ∪ H_j₂, ∪ H_j₃) (family of union).

The following are the immediate results of Definition 2.3.

**Proposition 2.1** [9] Let H, I, J ∈ ncS(Y). Then

(i) φₙ ⊆ H ⊆ Yₙ,
(ii) if H ⊆ I & I ⊆ J, then H ⊆ J,
(iii) H ∩ I ⊆ H & H ∩ I ⊆ I,
(iv) H ⊆ H ∪ I & I ⊆ H ∪ I,
(v) H ⊆ I iff H ∩ I = H,
(vi) H ⊆ I iff H ∪ I = I.

**Proposition 2.2** [9] Let H, I, J ∈ ncS(Y). Then

(i) (Idempotent laws) : H ∪ H = H, H ∩ H = H,
(ii) (Commutative laws) : H ∪ I = I ∪ H, H ∩ I = I ∩ H,
(iii) (Associative laws) : H ∪ (I ∪ J) = (H ∪ I) ∪ J, H ∩ (I ∩ J) = (H ∩ I) ∩ J,
(iv) (Distributive laws) : H ∪ (I ∩ J) = (H ∪ I) ∩ (H ∪ J), H ∩ (I ∪ J) = (H ∩ I) ∪ (H ∩ J),
(v) (Absorption laws) : H ∪ (H ∩ I) = H, H ∩ (H ∪ I) = H,
(vi) (DeMorgan’s laws) : (H ∪ I)ᶜ = Hᶜ ∩ Iᶜ, (H ∩ I)ᶜ = Hᶜ ∪ Iᶜ,
(vii) (Hᶜ)ᶜ = H,
(viii) (a) H ∪ φₙ = H, H ∩ φₙ = φₙ.
(b) $H \cup Y_n = Y_n$, $H \cap Y_n = H$,
(c) $Y_n^c = \phi$, $\phi^c_n = Y_n$,
(d) in general, $H \cup H^c \neq Y_n$, $H \cap H^c \neq \phi_n$.

**Definition 2.4** [14] A neutrosophic crisp topology (briefly, nc) on a non-empty set $Y$ is a family $\Gamma$ of nc subsets of $Y$ satisfies

(i) $\phi_n, Y_n \in \Gamma$.
(ii) $H_1 \cap H_2 \in \Gamma \forall H_1 \& H_2 \in \Gamma$.

Then $(Y, \Gamma)$ is a neutrosophic crisp topological space (briefly, ncts) in $Y$. The $\Gamma$ elements are called neutrosophic crisp open sets (briefly, ncos) in $Y$. A ncs $C$ is called a neutrosophic crisp closed set (briefly, ncbs) if its complement $C^c$ is ncos.

**Definition 2.5** [5] Let $Y$ be a non-empty set. Then $n_c\Gamma_1, n_c\Gamma_2, \cdots, n_c\Gamma_N$ are $N$-arbitrary crisp topologies defined on $Y$ and the collection $N_{nc}\Gamma = \{A \subseteq X : A = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j), H_j, L_j \in n_c\Gamma_j\}$ is called $N$ neutrosophic crisp (briefly, $N_{nc}$)-topology on $Y$ if the axioms are satisfied:

(i) $\phi_n, Y_n \in N_{nc}\Gamma$.
(ii) $\bigcup_{j=1}^{\infty} A_j \in N_{nc}\Gamma \forall \{A_j\}_{j=1}^{\infty} \in N_{nc}\Gamma$.

Then $(Y, N_{nc}\Gamma)$ is called a $N_{nc}$-topological space (briefly, $N_{nc}$ts) on $Y$. The $N_{nc}\Gamma$ elements are called $N_{nc}$-open sets (briefly, $N_{nc}$os) on $Y$ and its complement is called $N_{nc}$-closed sets (briefly, $N_{nc}$cs) on $Y$. The elements of $Y$ are known as $N_{nc}$-sets (briefly, $N_{nc}$s) on $Y$.

**Definition 2.6** [5] Let $(Y, N_{nc}\Gamma)$ be $N_{nc}$ts on $Y & H$ be an $N_{nc}$s on $Y$, then the $N_{nc}$ interior of $H$ (briefly, $N_{nc}$int($H$)) and $N_{nc}$ closure of $H$ (briefly, $N_{nc}$cl($H$)) are defined as

$$N_{nc}$int($H$) = \bigcup \{B : B \subseteq H \& B \text{ is a } N_{nc}$os in $Y\}$$

$$N_{nc}$cl($H$) = \bigcap \{C : H \subseteq C \& C \text{ is a } N_{nc}$cs in $Y\}$$

**Definition 2.7** [5] Let $(Y, N_{nc}\Gamma)$ be any $N_{nc}$ts. Let $H$ be an $N_{nc}$s in $(X, N_{nc}\Gamma)$. Then $H$ is said to be a

(i) $N_{nc}$-regular open set [17] (briefly, $N_{nc}$ros) if $H = N_{nc}$int($N_{nc}$cl($H$)).
(ii) $N_{nc}$-semi open set (briefly, $N_{nc}$Sos) if $H \subseteq N_{nc}$cl($N_{nc}$int($H$)).
(iii) $N_{nc}$-pre open set (briefly, $N_{nc}$Pos) if $H \subseteq N_{nc}$int($N_{nc}$cl($H$)).
(iv) $N_{nc}$-\(\alpha\)-open set (briefly, $N_{nc}$aos) if $H \subseteq N_{nc}$int($N_{nc}$cl($N_{nc}$int($H$)))).
(v) $N_{nc}$-\(\gamma\)-open set[17] (briefly, $N_{nc}$gos) if $H \subseteq N_{nc}$cl($N_{nc}$int($H$)) $\cup$ $N_{nc}$int($N_{nc}$cl($H$)).
(vi) $N_{nc}$-\(\beta\)-open set [18] (briefly, $N_{nc}$bos) if $H \subseteq N_{nc}$cl($N_{nc}$int($N_{nc}$cl($H$)))).

The complement of an $N_{nc}$ros (resp. $N_{nc}$Sos, $N_{nc}$Pos, $N_{nc}$aos, $N_{nc}$gos & $N_{nc}$bos) is called an $N_{nc}$-regular (resp. $N_{nc}$-semi, $N_{nc}$-pre, $N_{nc}$-\(\alpha\), $N_{nc}$-\(\gamma\) & $N_{nc}$-\(\beta\)) closed set (briefly, $N_{nc}$cs (resp. $N_{nc}$Scs, $N_{nc}$Pos, $N_{nc}$aos, $N_{nc}$gos, $N_{nc}$gos, $N_{nc}$apos, $N_{nc}$apos, $N_{nc}$apos & $N_{nc}$apos) in $Y$.

The family of all $N_{nc}$ros (resp. $N_{nc}$cs, $N_{nc}$Sos, $N_{nc}$Sos, $N_{nc}$Pos, $N_{nc}$Pos, $N_{nc}$aos, $N_{nc}$gos, $N_{nc}$gos, $N_{nc}$apos, $N_{nc}$apos & $N_{nc}$apos) of $Y$ is denoted by $N_{nc}$ROS($Y$) (resp.$N_{nc}$CS($Y$), $N_{nc}$SOS($Y$), $N_{nc}$SCS($Y$), $N_{nc}$POS($Y$), $N_{nc}$PCS($Y$), $N_{nc}$OS($Y$), $N_{nc}$CS($Y$), $N_{nc}$BOS($Y$) & $N_{nc}$BCS($Y$)).
**Definition 2.8** [21] A set $H$ is said to be a

(i) $N_{nc}\delta$ interior of $H$ (briefly, $N_{nc}\delta\text{int}(H)$) if $N_{nc}\delta\text{int}(H) = \cup\{A : A \subseteq H & A$ is a $N_{nc}\text{ros}\}$.

(ii) $N_{nc}\delta$ closure of $H$ (briefly, $N_{nc}\delta\text{cl}(H)$) if $N_{nc}\delta\text{cl}(H) = \cup\{y \in Y : N_{nc}\text{int}(N_{nc}\text{cl}(L)) \cap H \neq \phi, y \in L & L$ is a $N_{nc}\text{mos}\}$.

**Definition 2.9** [21] A set $H$ is said to be a

(i) $N_{nc}\delta$- open set (briefly, $N_{nc}\delta\text{os}$) if $H = N_{nc}\delta\text{int}(H)$.

(ii) $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta\text{Pos}$) if $H \subseteq N_{nc}\text{int}(N_{nc}\delta\text{cl}(H))$.

(iii) $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta\text{Sos}$) if $H \subseteq N_{nc}\text{cl}(N_{nc}\delta\text{int}(H))$.

(iv) $N_{nc}\delta$- semi open set [22] (briefly, $N_{nc}\delta\text{os}$) if $H \subseteq N_{nc}\text{cl}(N_{nc}\delta\text{int}(H)) \cup N_{nc}\text{int}(N_{nc}\delta\text{cl}(H))$.

The complement of an $N_{nc}\delta\text{os}$ (resp. $N_{nc}\delta\text{Pos}$, $N_{nc}\delta\text{Sos}$ & $N_{nc}\delta\text{os}$) is called an $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi & $N_{nc}\delta$) closed set (briefly, $N_{nc}\delta\text{cs}$ (resp. $N_{nc}\delta\text{cs}$, $N_{nc}\delta\text{Scs}$ & $N_{nc}\delta\text{cs}$)) in $X$.

The family of all $N_{nc}\delta\text{Pos}$ (resp. $N_{nc}\delta\text{cs}$, $N_{nc}\delta\text{Sos}$, $N_{nc}\delta\text{Scs}$, $N_{nc}\delta\text{os}$ & $N_{nc}\delta\text{os}$) of $X$ is denoted by $N_{nc}\delta\text{POs}(Y)$ (resp. $N_{nc}\delta\text{PCS}(Y)$, $N_{nc}\delta\text{SOS}(Y)$, $N_{nc}\delta\text{SCS}(Y)$, $N_{nc}\delta\text{OS}(Y)$ & $N_{nc}\delta\text{SOS}(Y)$).

**Definition 2.10** [21] Let $(Y, N_{nc}\Gamma)$ be a $N_{nc}\text{ts}$ on $Y$ and $H$ be an $N_{nc}\text{os}$ on $Y$. Then

(i) $N_{nc}\delta\text{Pint}(H) = \cup\{B : B \subseteq H & B$ is a $N_{nc}\delta\text{Pos}$ set in $Y\}$.

(ii) $N_{nc}\delta\text{Pcl}(H) = \cap\{C : H \subseteq C & C$ is a $N_{nc}\delta\text{Pcs}$ set in $Y\}$.

**Definition 2.11** Let $(Y, N_{nc}\Gamma) & (Z, N_{nc}\Psi)$ be any two $N_{nc}\text{ts}$’s. A map $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ is said to be

(i) $N_{nc}$ [11] (resp. $N_{nc}\alpha$, $N_{nc}$ semi, $N_{nc}$ pre, $N_{nc}\gamma$ & $N_{nc}\beta$ [20])-continuous [19] (briefly, $N_{nc}\text{Cts}$ (resp. $N_{nc}\alpha\text{Cts}$, $N_{nc}\text{SCts}$, $N_{nc}\text{Pcts}$, $N_{nc}\gamma\text{Cts}$ & $N_{nc}\beta\text{Cts}$) if the inverse image of every $N_{nc}\text{os}$ in $(Z, N_{nc}\Psi)$ is a $N_{nc}\text{os}$ (resp. $N_{nc}\alpha\text{os}$, $N_{nc}\text{Sos}$, $N_{nc}\text{Pos}$, $N_{nc}\gamma\text{os}$ & $N_{nc}\beta\text{os}$) in $(Y, N_{nc}\Gamma)$.

(ii) $N_{nc}\delta$ (resp. $N_{nc}\delta$ semi & $N_{nc}\delta$ pre )-continuous [21] (briefly, $N_{nc}\delta\text{Cts}$ (resp. $N_{nc}\delta\text{Scts} & N_{nc}\delta\text{Pcts}$) if the inverse image of every $N_{nc}\text{os}$ in $(Z, N_{nc}\Psi)$ is a $N_{nc}\delta\text{os}$ (resp. $N_{nc}\delta\text{Sos}$ & $N_{nc}\delta\text{Pos}$) in $(Y, N_{nc}\Gamma)$.

3. $N_{nc}\delta$-continuous function

Throughout this section, Let $(Y, N_{nc}\Gamma) & (Z, N_{nc}\Psi)$ be any two $N_{nc}\text{ts}$’s.

**Definition 3.1** A map $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ is said to be $N_{nc}\delta$-continuous [23] (briefly, $N_{nc}\delta\text{Cts}$) if the inverse image of every $N_{nc}\text{os}$ in $(Z, N_{nc}\Psi)$ is a $N_{nc}\delta\text{os}$ in $(Y, N_{nc}\Gamma)$.

**Theorem 3.1** Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a function. Then

(i) Every $N_{nc}\text{Cts}$ is a $N_{nc}\alpha\text{Cts}$.

(ii) Every $N_{nc}\alpha\text{Cts}$ is a $N_{nc}\text{Pcts}$.

(iii) Every $N_{nc}\text{Pcts}$ is a $N_{nc}\gamma\text{Cts}$.

(iv) Every $N_{nc}\gamma\text{Cts}$ is a $N_{nc}\beta\text{Cts}$.

(v) Every $N_{nc}\beta\text{Cts}$ is a $N_{nc}\delta\text{Cts}$.

(vi) Every $N_{nc}\delta\text{Cts}$ is a $N_{nc}\text{SCts}$.

(vii) Every $N_{nc}\delta\text{SCts}$ is a $N_{nc}\delta\text{Cts}$.

(viii) Every $N_{nc}\delta\text{Pcts}$ is a $N_{nc}\delta\text{Pcts}$. 
(ix) Every $N_{nc}\delta P Cts$ is a $N_{nc}eCts$.

(x) Every $N_{nc}eCts$ is a $N_{nc}\beta Cts$.

**Proof.** Proof of (i) to (iii), (iv) and (v) to (vi) are proved in [19], [20] and [21]. We prove only (vii) to (x).

(vii) Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a $N_{nc}\delta SCts$ & $U_1$ is a $N_{nc}os$ in $Z$. Then $f^{-1}(U_1)$ is $N_{nc}\delta Sos$ in $Y$. By Proposition 3.1 in [22], every $N_{nc}\delta Soa$ is $N_{nc}eo$, $f^{-1}(U_1)$ is $N_{nc}eo$ in $Y$. Therefore $f$ is $N_{nc}\delta P Cts$.

(viii) Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a $N_{nc}P Cts$ & $U_1$ is a $N_{nc}os$ in $Z$. Then $f^{-1}(U_1)$ is $N_{nc}P o$ in $Y$. By Proposition 3.1 in [22], every $N_{nc}P o$ is $N_{nc}\delta P o$, $f^{-1}(U_1)$ is $N_{nc}\delta P os$ in $Y$. Therefore $f$ is $N_{nc}\delta P Cts$.

(ix) Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a $N_{nc}\delta P Cts$ & $U_1$ is a $N_{nc}os$ in $Z$. Then $f^{-1}(U_1)$ is $N_{nc}\delta P os$ in $Y$. By Proposition 3.1 in [22], every $N_{nc}\delta P o$ is $N_{nc}eo$, $f^{-1}(U_1)$ is $N_{nc}eo$ in $Y$. Therefore $f$ is $N_{nc}eCts$.

(x) It is similar to (ix).

**Remark 3.1** The diagram shows $N_{nc}eCts$ function of $N_{nc}$ts.

\[ \begin{array}{ccc}
N_{nc}Cts & \xrightarrow{nc\alpha Cts} & N_{nc}P Cts \\
\uparrow & & \downarrow \\
N_{nc}\delta Cts & \rightarrow & N_{nc}\delta P Cts
\end{array} \]

**Example 3.1** Let $Y = \{a_2, b_2, c_2, d_2, e_2\} = Z$, $nc\tau_1 = \{\phi_N, X_N, A, B, C\}$, $nc\tau_2 = \{\phi_N, X_N\}$. $A = \{\{c_2\}, \{\phi\}, \{a_2, b_2, c_2, d_2, e_2\}\}$, $B = \{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}$, $C = \{\{a_2, b_2, c_2\}, \{\phi\}, \{d_2, e_2\}\}$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C\}$. Define $f : (Y, 2_{nc}\tau) \rightarrow (Z, 2_{nc}\Psi)$ as

(i) $f(a_2) = c_2$, $f(b_2) = d_2$, $f(c_2) = a_2$, $f(d_2) = b_2$ & $f(e_2) = e_2$, then $f$ is $2_{nc}eCts$ but not $N_{nc}\delta P Cts$, the set $f^{-1}(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}) = \{\{c_2, a_2\}, \{\phi\}, \{a_2, b_2, e_2\}\}$ is a $2_{nc}eo$ but not $2_{nc}\delta P os$.

(ii) $f(a_2) = a_2$, $f(b_2) = c_2$, $f(c_2) = b_2$, $f(d_2) = d_2$ & $f(e_2) = e_2$, then $f$ is $2_{nc}eCts$ but not $N_{nc}\delta SCts$, the set $f^{-1}(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}) = \{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\}$ is a $2_{nc}eo$ but not $2_{nc}\delta Sos$.

(iii) $f(a_2) = a_2$, $f(b_2) = d_2$, $f(c_2) = c_2$, $f(d_2) = b_2$ & $f(e_2) = e_2$, then $2_{nc}\beta Cts$ but not $2_{nc}eCts$, the set $f^{-1}(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}) = \{\{a_2, d_2\}, \{\phi\}, \{b_2, c_2, e_2\}\}$ is a $2_{nc}\beta os$ but not $2_{nc}eo$.

**Theorem 3.2** Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a function. Then the conditions

(i) $f$ is $N_{nc}eCts$.

(ii) The inverse $f^{-1}(U_1)$ of each $N_{nc}es$ $U_1$ in $Z$ is $N_{nc}eo$ in $Y$ are equivalent.

**Proof.** The proof is obvious, since $f^{-1}(U_1) = f^{-1}(U_1)$ for each $N_{nc}es$ $U_1$ of $Z$.

**Theorem 3.3** Let $f : (Y, N_{nc}\Gamma) \rightarrow (Z, N_{nc}\Psi)$ be a function. Then the conditions

(i) $f(N_{nc}ecl(U_1)) \subseteq N_{nc}ecl(f(U_1)), \forall N_{nc} U_1$ in $Y$.

(ii) $N_{nc}ecl(f^{-1}(H_1)) \subseteq f^{-1}(N_{nc}ecl(H_1)), \forall N_{nc} H$ in $Z$
are equivalent.

**Proof.** (i) Since \(N_{nc\text{c}l}(f(U_1))\) is a \(N_{nc}\text{c}l\) in \(Z\) & \(f\) is \(N_{nc}\text{c}l\text{ts}\), then \(f^{-1}(N_{nc\text{c}l}(f(U_1)))\) is \(N_{nc}\text{c}c\) in \(Y\). Now, since \(U_1 \subseteq f^{-1}(N_{nc\text{c}l}(f(U_1)))\), \(N_{nc\text{c}l}(U_1) \subseteq f^{-1}(N_{nc\text{c}l}(f(U_1)))\). Therefore, \(f(N_{nc\text{c}l}(U_1)) \subseteq N_{nc\text{c}l}(f(U_1))\).

(ii) By replacing \(U_1\) with \(V_1\) in (i), we obtain \(f(N_{nc\text{c}l}(f^{-1}(H_1))) \subseteq N_{nc\text{c}l}(f(f^{-1}(H_1))) \subseteq N_{nc\text{c}l}(H_1)\). Hence, \(N_{nc\text{c}l}(f^{-1}(H_1)) \subseteq f^{-1}(N_{nc\text{c}l}(H_1))\).

**Remark 3.2** If \(f : (Y, N_{nc\Gamma}) \to (Z, N_{nc\Psi})\) is \(N_{nc\text{c}l}\text{ts}\), then

(i) \(f(N_{nc\text{c}l}(U_1))\) is not necessarily equal to \(N_{nc\text{c}l}(f(U_1))\) where \(U_1 \subseteq Y\).

(ii) \(N_{nc\text{c}l}(f^{-1}(H_1))\) is not necessarily equal to \(f^{-1}(N_{nc\text{c}l}(H_1))\) where \(H_1 \subseteq Z\).

**Example 3.2** In Example 3.1, \(f : (Y, 2_{nc\Gamma}) \to (Y, 2_{nc\Gamma})\) be an identity function. Then \(f\) is a \(2_{nc\text{c}l}\text{ts}\).

(i) Let \(U_1 = \{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\} \subseteq Y\). Then \(f(N_{nc\text{c}l}(U_1)) = f(2_{nc\text{c}l}(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}) = \{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}. But \(2_{nc\text{c}l}(f(U_1)) = 2_{nc\text{c}l}(f(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\})) = 2_{nc\text{c}l}(\{\{a_2, b_2\}, \{\phi\}, \{c_2, d_2, e_2\}\}) = \{\{a_2, b_2, d_2, e_2\}, \{\phi\}, \{c_2\}\}. Thus \(f(2_{nc\text{c}l}(U_1)) \neq 2_{nc\text{c}l}(f(U_1))\).

(ii) Let \(V_1 = \{\{a_2\}, \{\phi\}, \{b_2, c_2, d_2, e_2\}\} \subseteq Y\). Then \(2_{nc\text{c}l}(f^{-1}(V_1)) \subseteq 2_{nc\text{c}l}(f^{-1}(\{\{a_2\}, \{\phi\}, \{b_2, c_2, d_2, e_2\}\})) = 2_{nc\text{c}l}(\{\{a_2\}, \{\phi\}, \{b_2, c_2, d_2, e_2\}\}) = \{\{a_2\}, \{\phi\}, \{b_2, c_2, d_2, e_2\}\}. But \(f^{-1}(2_{nc\text{c}l}(V_1)) = f^{-1}(2_{nc\text{c}l}(\{\{a_2\}, \{\phi\}, \{b_2, c_2, d_2, e_2\}\})) = f^{-1}(Y) = Y\). Thus \(2_{nc\text{c}l}(f^{-1}(V_1)) \neq f^{-1}(2_{nc\text{c}l}(V_1))\).

**Theorem 3.4** Let \(f : (Y, N_{nc\Gamma}) \to (Z, N_{nc\Psi})\) be a function, then \(f^{-1}(N_{nc\text{int}(H_1)}) \subseteq N_{nc\text{c}l}(f^{-1}(H_1)), \) for all \(N_{nc}\text{c}l\) \(H_1 \) in \(Z\).

**Proof.** If \(f\) is \(N_{nc\text{c}l}\text{ts} \& \(H_1 \subseteq Z\). Then \(N_{nc\text{c}l}(H_1)\) is \(N_{nc}\text{c}l\) in \(Z\) and hence, \(f^{-1}(N_{nc\text{c}l}(H_1))\) is \(N_{nc}\text{c}l\) in \(Y\). Therefore \(N_{nc\text{c}l}(f^{-1}(N_{nc\text{c}l}(H_1))) = f^{-1}(N_{nc\text{c}l}(H_1))\). Also, \(N_{nc\text{c}l}(H_1) \subseteq U_1\), implies that \(f^{-1}(N_{nc\text{c}l}(H_1)) \subseteq f^{-1}(H_1)\). Therefore \(N_{nc\text{c}l}(f^{-1}(N_{nc\text{c}l}(H_1))) \subseteq N_{nc\text{c}l}(f^{-1}(H_1))\). That is \(f^{-1}(N_{nc\text{c}l}(H_1)) \subseteq f^{-1}(N_{nc\text{c}l}(H_1))\).

Conversely, let \(f^{-1}(N_{nc\text{c}l}(H_1)) \subseteq N_{nc\text{c}l}(f^{-1}(H_1))\) for every subset \(H_1\) of \(Z\). If \(H_1\) is \(N_{nc}\text{c}l\) in \(Y_1\), then \(N_{nc\text{c}l}(H_1) = H_1\). By assumption, \(f^{-1}(N_{nc\text{c}l}(H_1)) \subseteq N_{nc\text{c}l}(f^{-1}(H_1))\). Thus \(f^{-1}(H_1) \subseteq N_{nc\text{c}l}(f^{-1}(H_1))\). But \(N_{nc\text{c}l}(f^{-1}(H_1)) \subseteq f^{-1}(H_1)\). Therefore \(N_{nc\text{c}l}(f^{-1}(H_1)) = f^{-1}(H_1)\). That is, \(f^{-1}(H_1) = N_{nc}\text{c}l\text{ }H_1\) in \(Y\), \(\forall N_{nc}H_1\) in \(Z\). Therefore \(f\) is \(N_{nc}\text{c}l\text{ts} \) on \(Y\).

**Remark 3.3** If \((Y, N_{nc\Gamma}) \to (Z, N_{nc\Psi})\) is \(N_{nc}\text{c}l\text{ts}, then \(N_{nc\text{c}l}(f^{-1}(H_1))\) is not necessarily equal to \(f^{-1}(N_{nc\text{c}l}(H_1))\) where \(H_1 \subseteq Z\).

**Example 3.3** In Example 3.1, \(f\) is a \(2_{nc}\text{c}l\text{ts}\). Let \(V_1 = \{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\} \subseteq Z\). Then \(2_{nc\text{c}l}(f^{-1}(V_1)) \subseteq 2_{nc\text{c}l}(f^{-1}(\{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\})) = 2_{nc\text{c}l}(\{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\}) = \{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\}. But \(f^{-1}(2_{nc\text{c}l}(V_1)) = f^{-1}(2_{nc\text{c}l}(\{\{a_2, c_2\}, \{\phi\}, \{b_2, d_2, e_2\}\})) = f^{-1}(\{\{c_2\}, \{\phi\}, \{a_2, b_2, d_2, e_2\}\}) = \{\{c_2\}, \{\phi\}, \{a_2, b_2, d_2, e_2\}\}. Thus \(2_{nc\text{c}l}(f^{-1}(V_1)) \neq f^{-1}(2_{nc\text{c}l}(V_1))\).

**Theorem 3.5** Let \(f : (Y, N_{nc\Gamma}) \to (Z, N_{nc\Psi})\) be a function. Then the statements

(i) \(f\) is a \(N_{nc}\text{c}l\text{ts} \) function.

(ii) For every \(N_{nc}P\) \(p_{(p_1,p_2,p_3)} \in Y\) and each \(ncs\) \(U_1\) of \(f(p_{(p_1,p_2,p_3)})\), \(\exists\) an \(N_{nc}\text{c}l\text{ }H_1 \ni p_{(p_1,p_2,p_3)} \in H_1 \subseteq f^{-1}(U_1)\).

(iii) For every \(N_{nc}\) point \(p_{(p_1,p_2,p_3)} \in Y\) and each \(ncs\) \(U_1\) of \(f(p_{(p_1,p_2,p_3)})\), \(\exists\) an \(N_{nc}\text{c}l\text{ }H_1 \ni p_{(p_1,p_2,p_3)} \in H_1 \& f(H_1) \subseteq U_1\).
are equivalent.

Proof. (i) ⇒ (ii): If \( p_{(p_1,p_2,p_3)} \) is an \( N_{nc} \) set in \( Y \) and if \( U_1 \) is an \( N_{nc} \) function of \( p_{(p_1,p_2,p_3)} \), then \( \exists \) an \( N_{nc} \os W_1 \) in \( Z \ni f(p_{(p_1,p_2,p_3)}) \) in \( W_1 \subset U_1 \). Thus, \( f \) is a \( N_{nc}\text{Cts} \), \( H_1 = f^{-1}(W_1) \) is an \( N_{nc} \os \) & \( p_{(p_1,p_2,p_3)} \in f^{-1}(f(p_{(p_1,p_2,p_3)})) \) \( \subseteq f^{-1}(W_1) = H_1 \subseteq f^{-1}(U_1) \). Hence, (ii).

(ii) ⇒ (iii): Let \( p_{(p_1,p_2,p_3)} \) be an \( N_{nc} \) set in \( Y \) and let \( U_1 \) be an \( N_{nc} \) set of \( p_{(p_1,p_2,p_3)} \). Then \( \exists \) an \( N_{nc} \os U_1 \ni p_{(p_1,p_2,p_3)} \in H_1 \subseteq f^{-1}(U_1) \) by (ii). Thus, we have \( p_{(p_1,p_2,p_3)} \in H_1 \) & \( f(H_1) \subseteq f(f^{-1}(U_1)) \subseteq U_1 \). Hence, (iii).

(iii) ⇒ (i): Let \( H_1 \) be an \( N_{nc} \) set in \( Z \) & let \( p_{(p_1,p_2,p_3)} \in f^{-1}(H_1) \). Then, \( f(p_{(p_1,p_2,p_3)}) \in f(f^{-1}(H_1)) \subseteq H_1 \). Since \( H_1 \) is an \( N_{nc} \os \), it follows that \( H_1 \) is an \( N_{nc} \) set of \( f(p_{(p_1,p_2,p_3)}) \). \:. from (iii), \( \exists \) an \( N_{nc} \os U_1 \ni p_{(p_1,p_2,p_3)} \& f(U_1) \subseteq H_1. \Rightarrow p_{(p_1,p_2,p_3)} \in U_1 \subseteq f^{-1}(f(U_1)) \subseteq f^{-1}(H_1). \) Therefore, w.k.t \( f^{-1}(H_1) \) is an \( N_{nc} \os \) in \( Y \). Thus, \( f \) is a \( N_{nc}\text{Cts} \) function.

4. Somewhat \( N_{nc} \os \)-continuous functions

Throughout this section, Let \( (Y,N_{nc}\Gamma) \) \& \( (Z,N_{nc}\Psi) \) be any two \( N_{nc}\text{ts}'s \).

Definition 4.1 A map \( f : (Y,N_{nc}\Gamma) \to (Z,N_{nc}\Psi) \) is said to be somewhat \( N_{nc} \os \) (resp. \( N_{nc} \os \)-continuous (briefly, \( sw \) \( N_{nc}\text{Cts} \) (resp. \( sw \) \( N_{nc}\os\text{Cts} \))) if for \( B_1 \in N_{nc}\Psi \& f^{-1}(B_1) \neq \phi, \exists \) an \( N_{nc}\os \) (resp. \( N_{nc}\os \)) set \( A_1 \) of \( Y \ni A_1 \neq \phi \& A_1 \subseteq f^{-1}(B_1) \).

It is clear that every \( N_{nc}\text{Cts} \) function is \( sw \) \( N_{nc}\text{Cts} \) and every \( sw \) \( N_{nc}\os\text{Cts} \) is \( sw \) \( N_{nc}\os\text{Cts} \). But not converse.

Example 4.1 In Example 3.1,

(i) \( n_{nc}\Psi_1 = \{\phi_n,Y,N,A\}, n_{nc}\Psi_2 = \{\phi_n,X_n\} \). \( A = \langle \{a_2,e_2\}, \{\phi\}, \{b_2,d_2,e_2\} \rangle \), then we have \( 2_{nc}\Psi = \{\phi_n,Y,N,A\} \). Define \( f : (Y,2_{nc}\Gamma) \to (Z,2_{nc}\Psi) \) be an identity function. Then \( f \) is \( sw \) \( 2_{nc}\text{Cts} \) but not \( 2_{nc}\text{ts} \) function.

(ii) \( n_{nc}\Psi_1 = \{\phi_n,Y,N,A\}, n_{nc}\Psi_2 = \{\phi_n,X_n\} \). \( A = \langle \{e_2\}, \{\phi\}, \{a_2,b_2,d_2,e_2\} \rangle \), then we have \( 2_{nc}\Psi = \{\phi_n,Y,N,A\} \). Define \( f : (Y,2_{nc}\Gamma) \to (Z,2_{nc}\Psi) \) as \( f(a_2) = a_2, f(b_2) = c_2, f(e_2) = e_2 \). Then \( f \) is \( sw \) \( 2_{nc}\text{Cts} \) but not \( sw \) \( 2_{nc}\text{ts} \).

Definition 4.2 Let \( H_1 \) be an \( N_{nc}s \) in \( (Y,N_{nc}\Gamma) \). Then \( H_1 \) is said to be a \( N_{nc} \os \)-dense (resp. \( N_{nc} \os \) dense) (briefly, \( N_{nc} \os \ d \) (resp. \( N_{nc} \ os \)) if \( N_{nc} \os(H_1) = Y \) (resp. \( N_{nc} \os(H_1) = Y \)), equivalently if there is no proper \( N_{nc} \os \) (resp. \( N_{nc} \os \)) set \( C_1 \) in \( Y \ni H_1 \subseteq C_1 \subseteq Y \).

Theorem 4.1 For a surjective function \( f : (Y,N_{nc}\Gamma) \to (Z,N_{nc}\Psi) \), then the statements

(i) \( f \) is \( sw \) \( N_{nc}\os\text{Cts} \);

(ii) if \( C_1 \) is a \( N_{nc} \) set of \( Z \ni f^{-1}(C_1) \neq Y \), then there is a proper \( N_{nc}\os \) set \( F_1 \) of \( Y \ni f^{-1}(C_1) \subseteq F_1 \);

(iii) if \( E_1 \) is an \( N_{nc} \os \) set of \( Y \), then \( f(E_1) \) is a dense set of \( Z \)

are equivalent.

Proof. (i) ⇒ (ii): Let \( C_1 \) be a \( N_{nc} \) set of \( Z \ni f^{-1}(C_1) \neq Y \). Then \( Z \setminus C_1 \) is an \( N_{nc} \os \) set in \( Z \ni f^{-1}(Z \setminus C_1) = Y \setminus f^{-1}(C_1) \neq \phi \). By (i), \( \exists \) an \( N_{nc} \os \) set \( U_1 \) in \( Y \ni U_1 \neq \phi \& U_1 \subseteq f^{-1}(Z \setminus C_1) = Y \setminus f^{-1}(C_1) \). This means that \( f^{-1}(C_1) \subseteq Y \setminus U_1 \& Y \setminus U_1 = F_1 \) is a proper \( N_{nc} \os \) set in \( Y \).

(ii) ⇒ (i): Let \( V_1 \in \Psi \& f^{-1}(V_1) \neq \phi \). Then \( Z \setminus V_1 \) is \( N_{nc} \os \& f^{-1}(Z \setminus V_1) = Y \setminus f^{-1}(V_1) \neq Y \). By (ii), there exists a proper \( N_{nc} \os \) set \( F_1 \) of \( Y \ni f^{-1}(Z \setminus V_1) \subseteq F_1 \). \( \Rightarrow Y \setminus F_1 \subseteq f^{-1}(V_1) \& Y \setminus F_1 \in N_{nc} \os(Y) \) with \( Y \setminus F_1 \neq \phi \).
(ii)⇒(iii): Let \( E_1 \) be an \( N_{nc} e \) set in \( Y \). Suppose that \( f(E_1) \) is not \( N_{nc} d \) in \( Z \). Then there exists a proper \( N_{nc} e \) set \( C_1 \) in \( Z \ni f(E_1) \subseteq C_1 \subseteq Z \). Clearly \( f^{-1}(C_1) \neq Y \). By (ii), there exist a proper \( N_{nc} e \) subset \( F_1 \ni E_1 \subseteq f^{-1}(C_1) \subseteq F_1 \subseteq Y \). This is contradiction that \( E_1 \) is \( N_{nc} e \) in \( Y \).

(iii)⇒(ii): Suppose (ii) is not true. \( \exists \) a \( N_{nc} e \) set \( C_1 \) in \( Z \ni f^{-1}(C_1) \neq Y \) but there is not proper \( N_{nc} e \) set \( F_1 \) in \( Y \ni f^{-1}(C_1) \subseteq F_1 \). Then \( f^{-1}(C_1) \) is \( N_{nc} e \) in \( Y \). But by (iii), \( f(f^{-1}(C_1)) = C_1 \) must be \( N_{nc} d \) in \( Z \), which is a contradiction to \( C_1 \).

Definition 4.3 If \( Y \) is a set and \( N_{nc} \Gamma \) & \( N_{nc} \Gamma^* \) are \( N_{nc} \) topologies on \( Y \), then \( N_{nc} \Gamma \) is said to be \( N_{nc} e \) equivalent (resp. \( N_{nc} e \) equivalent) (briefly, \( N_{nc} e \) equ (resp. \( N_{nc} e \) equ)) to \( N_{nc} \Gamma^* \) provided if \( A_1 \ni N_{nc} \Gamma \& A_1 \neq \phi \) then there is an \( N_{nc} e \o \) (resp. \( N_{nc} o \)) set \( B_1 \) in \( (Y, N_{nc} \Gamma^*) \) such that \( B_1 \neq \phi \) and \( B_1 \subseteq A_1 \) and if \( A_1 \ni N_{nc} \Gamma^* \) & \( A_1 \neq \phi \) then there is an \( N_{nc} e \o \) (resp. \( N_{nc} o \)) set \( B_1 \) in \( (Y, N_{nc} \Gamma) \) such that \( B_1 \neq \phi \) and \( B_1 \subseteq A_1 \).

Now consider the identity function \( f : (Y, N_{nc} \Gamma) \to (Y, N_{nc} \Gamma) \) and assume that \( N_{nc} \Gamma \) & \( N_{nc} \Gamma^* \) are \( N_{nc} e \) equ. Then \( f : (Y, N_{nc} \Gamma) \to (Y, N_{nc} \Gamma^*) \) & \( f^{-1} : (Y, N_{nc} \Gamma^*) \to (Y, N_{nc} \Gamma) \) are \( N_{nc} e \) Cts. Conversely, if the identity function \( f : (Y, N_{nc} \Gamma) \to (Y, N_{nc} \Gamma^*) \) is \( N_{nc} e \) Cts in both directions, then \( N_{nc} \Gamma \) & \( N_{nc} \Gamma^* \) are \( N_{nc} e \) equ.

Theorem 4.2 If \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is a surjection \( N_{nc} e \) Cts \& \( N_{nc} \Gamma^* \) is a \( N_{nc} \) topology for \( Y \), which is equivalent to \( N_{nc} \Psi \), then \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is \( N_{nc} e \) Cts.

Proof. Let \( B_1 \) be an \( N_{nc} o \) set of \( Z \ni f^{-1}(B_1) \neq \phi \). Since \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is \( N_{nc} e \) Cts, \( \exists \) a nonempty \( N_{nc} e \o \) set \( A_1 \in (Y, N_{nc} \Gamma) \ni A_1 \subseteq f^{-1}(B_1) \). But by hypothesis \( N_{nc} \Gamma^* \) is \( N_{nc} e \) equ to \( N_{nc} \Gamma \). Therefore, \( \exists \) an \( N_{nc} e \o \) set \( A_1^* \in (Y, N_{nc} \Gamma^*) \ni A_1^* \subseteq A_1 \). But \( A_1 \ni f^{-1}(B_1) \). Then \( A_1^* \subseteq f^{-1}(B_1) \); hence \( f : (Y, N_{nc} \Gamma^*) \to (Z, N_{nc} \Psi) \) is \( N_{nc} e \) Cts.

Theorem 4.3 Let \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) be a \( N_{nc} e \) Cts surjection \& \( N_{nc} \Psi^* \) be a \( N_{nc} \) topology for \( Z \), which is equivalent to \( N_{nc} \Psi \). Then \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi^*) \) is \( N_{nc} e \) Cts.

Proof. Let \( B_1^* \) be an \( N_{nc} o \) set of \( (Z, \Psi^*) \ni f^{-1}(B_1^*) \neq \phi \). Since \( N_{nc} \Psi^* \) is equivalent to \( N_{nc} \Psi \), \( \exists \) a nonempty \( N_{nc} \o \) set \( A_1 \in (Z, N_{nc} \Psi) \ni A_1 \subseteq f^{-1}(B_1^*) \). Now \( f^{-1}(B_1) \subseteq f^{-1}(B_1^*) \). Since \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is \( N_{nc} e \) Cts, \( \exists \) a nonempty \( N_{nc} \o \) set \( A_1 \in (Y, N_{nc} \Gamma) \ni A_1 \subseteq f^{-1}(B_1^*) \). Then \( A_1 \subseteq f^{-1}(B_1^*) \); hence \( f : (Y, N_{nc} \Gamma^*) \to (Z, N_{nc} \Psi^*) \) is \( N_{nc} e \) Cts.

Theorem 4.4 A function \( f : (X_1, N_{nc} \Gamma) \to (X_2, N_{nc} \Psi) \) is \( N_{nc} e \) Cts & \( g : (X_2, N_{nc} \Psi) \to (X_3, N_{nc} \eta) \) is \( N_{nc} \) Cts, then \( f \circ g : (X_1, N_{nc} \Gamma) \to (X_3, N_{nc} \eta) \) is \( N_{nc} e \) Cts.

Definition 4.4 A function \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is said to be \( N_{nc} e \o \) (briefly, \( N_{nc} e \o \)) provided that if \( A_1 \ni N_{nc} \Gamma \& A_1 \neq \phi \), then \( \exists \) an \( N_{nc} e \o \) set \( B_1 \) in \( Z \ni B_1 \neq \phi \) & \( B_1 \subseteq f(A_1) \).

Theorem 4.5 A function \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \) is \( N_{nc} e \o \) iff for any \( A_1 \subseteq Y \), \( N_{nc} \text{int}(A_1) \neq \phi \) \( \iff \) \( N_{nc} \text{ext}(f(A_1)) \neq \phi \).

Theorem 4.6 For a function \( f : (Y, N_{nc} \Gamma) \to (Z, N_{nc} \Psi) \), the statements

(i) \( f \) is \( N_{nc} e \o \),

(ii) If \( D_1 \) is an \( N_{nc} e \) \( d \) subset of \( Z \), then \( f^{-1}(D_1) \) is an \( N_{nc} d \) subset of \( Y \) are equivalent.
Proof. (i) ⇒ (ii): Suppose $D_1$ is an $N_{nc}e$ d set in $Y$. We want to show that $f^{-1}(D_1)$ is a $N_{nc} d$ subset of $Y$. Suppose that $f^{-1}(D_1)$ is not $N_{nc} d$ in $Y$. Then $\exists$ a $N_{nc}e$ set $B_1$ in $Y \ni f^{-1}(D_1) \subseteq B_1 \subseteq Y$. By (i) and since that $Y \backslash B_1$ is $N_{nc}o$, $\exists$ a nonempty $N_{nc}eo$ set $E_1$ in $Z \ni E_1 \subseteq f(Y \backslash B_1)$. Therefore $E_1 \subseteq f(Y \backslash B_1) \subseteq f(f^{-1}(Z \backslash D_1)) \subseteq Z \backslash D_1$. That is, $D_1 \subseteq Z \backslash E_1 \subseteq Z$. Now, $Z \backslash E_1$ is a $N_{nc}ec$ set & $D_1 \subseteq Z \backslash E_1 \subseteq Z$. This implies that $D_1$ is not an $N_{nc}e d$ set in $Z$, which is a contradiction that $D_1$ is $N_{nc}e d$ in $Z$. Therefore, $f^{-1}(D_1)$ is a $N_{nc} d$ subset of $Y$.

(ii) ⇒ (i): Suppose $D_1$ is a nonempty $N_{nc}o$ set of $Y$. We show that $N_{nc}e int(f(D_1)) \neq \phi$. Suppose that $N_{nc}e int(f(D_1)) = \phi$. Then $N_{nc}e cl(f(D_1)) = Z$. Therefore by (ii) $f^{-1}(Z \backslash f(D_1))$ is $N_{nc} d$ in $Y$. But $f^{-1}(Z \backslash f(D_1))$ is $N_{nc} d$ in $Y$. But $f^{-1}(Z \backslash f(D_1)) \subseteq Y \backslash D_1$. Now $Y \backslash D_1$ is $N_{nc}c$. Therefore $f^{-1}(Z \backslash f(D_1)) \subseteq Y \backslash D_1$ gives $Y = N_{nc}e cl(f^{-1}(Z \backslash f(D_1))) \subseteq Y \backslash D_1$. This implies that $D_1 = \phi$ which is contradiction to $D_1 \neq \phi$. Therefore $N_{nc}e int(f(D_1)) \neq \phi$. Thus $f$ is $sw$ $N_{nc}e O$.

Theorem 4.7 For a bijective function $f : (Y, N_{nc} \Gamma) \rightarrow (Z, N_{nc} \Psi)$, the statements

(i) $f$ is $sw$ $N_{nc}e O$,

(ii) If $C_1$ is a $N_{nc}ec$ set of $Y \ni f(C_1) \neq Z$, then there is an $N_{nc}ec$ subset $F_1$ of $Z \ni F_1 \neq Z$ & $f(C_1) \subseteq F_1$

are equivalent.

Proof. (i) ⇒ (ii): Let $C_1$ be any $N_{nc}ec$ set of $Y \ni f(C_1) \neq Z$. Then $Y \backslash C_1$ is an $N_{nc}o$ set in $Y$ & $Y \backslash C_1 \neq \phi$. Since $f$ is $sw$ $N_{nc}eo$ $\exists$ an $N_{nc}eo$ set $B_1$ in $Z \ni B_1 \neq \phi$ & $B_1 \subseteq f(Y \backslash C_1)$. Put $F_1 = Z \backslash B_1$. Clearly $F_1$ is $N_{nc}ec$ in $Z$ and we claim $F_1 \neq Z$. If $F_1 = Z$, then $B_1 = \phi$ which is a contradiction. Since $B_1 \subseteq f(Y \backslash C_1)$, $f(C_1) = (Z \backslash f(Y \backslash C_1)) \subseteq Z \backslash B_1 = F_1$.

(ii) ⇒ (i): Let $A_1$ be any nonempty $N_{nc}o$ set of $Y$. Then $C_1 = Y \backslash A_1$ is $N_{nc}c$ set in $Y$ & $f(Y \backslash A_1) = f(C_1) = Z \backslash f(A_1) \implies f(C_1) \neq Z$. Therefore, by (ii), there is an $N_{nc}ec$ set $F_1$ of $Z \ni F_1 \neq Z$ & $f(C_1) \subseteq F_1$. Clearly $B_1 = Z \backslash F_1 \in N_{nc}e cl(Z, \Psi) \neq \phi$. Also $B_1 = Z \backslash F_1 \subseteq Z \backslash f(C_1) = Z \backslash f(Y \backslash A_1) = f(A_1)$.

5. $N_{nc}e$-resolvable spaces and $N_{nc}e$-irresolvable spaces

Definition 5.1 A $N_{nc}ets (Y, N_{nc} \Gamma)$ is said to be $N_{nc}e$-resolvable (resp. $N_{nc}$ resolvable) (briefly, $N_{nc}e rs$ (resp. $N_{nc} rs$) if $\exists$ an $N_{nc}e d$ (resp. $N_{nc} d$) set $A_1$ in $(Y, N_{nc} \Gamma) \ni Y \backslash A_1$ is also $N_{nc}e d$ (resp. $N_{nc} d$) in $(Y, N_{nc} \Gamma)$. A space $(Y, N_{nc} \Gamma)$ is called $N_{nc}e$-irresolvable (resp. $N_{nc}$ irresolvable) (briefly, $N_{nc}e irs$ (resp. $N_{nc} irs$)) if it is not $N_{nc}e rs$ (resp. $N_{nc} rs$).

Theorem 5.1 For a $N_{nc}ets (Y, N_{nc} \Gamma)$, the statements

(i) $(Y, N_{nc} \Gamma)$ is $N_{nc}e rs$,

(ii) $(Y, N_{nc} \Gamma)$ has a pair of $N_{nc}e d$ sets $A_1 \ni B_1 \ni A_1 \subseteq (Y \backslash B_1)$

are equivalent.

Proof. (i) ⇒ (ii): Suppose that $(Y, N_{nc} \Gamma)$ is $N_{nc}e rs$. $\exists$ an $N_{nc}e d$ set $A_1$ in $(Y, N_{nc} \Gamma) \ni Y \backslash A_1$ is $N_{nc}e d$ in $(Y, N_{nc} \Gamma)$. Set $B_1 = Y \backslash A_1$, then we have $A_1 = Y \backslash B_1$.

(ii) ⇒ (i): Suppose that (ii) holds. Let $(Y, N_{nc} \Gamma)$ be $N_{nc}e irs$. Then $Y \backslash B_1$ is not $N_{nc}e d$ & $N_{nc}e cl(A_1) \subseteq N_{nc}e cl(Y \backslash B_1) \neq Y$. Hence $A_1$ is not $N_{nc}e d$. This contradicts the assumption.

Theorem 5.2 For a $N_{nc}ets (Y, N_{nc} \Gamma)$, the statements

(i) $(Y, N_{nc} \Gamma)$ is $N_{nc}e irs$ (resp. $N_{nc} irs$),

(ii) For any $N_{nc}e d$ (resp. $N_{nc} d$) sets $A_1$ in $Y$, $N_{nc}e int(A_1) \neq \phi$ (resp. $N_{nc} int(A_1) \neq \phi$)

are equivalent.
In this work, we have introduced some new notions of $N_{nc}e$ functions and also a contra field in $N_{nc}e$. Theorem 4.6

\[ \text{Proof.} \]
(i) ⇒ (ii): Let $A_1$ be any $N_{nc}e$ set of $Y$. Then we have $N_{nc}e cl(Y \setminus A_1) \neq Y$, hence $N_{nc}e cl(A_1) \neq \phi$.

(ii) ⇒ (i): Suppose that $(Y, N_{nc}e \Gamma)$ is an $N_{nc}e$ rs space. \exists an $N_{nc}e$ set $A_1$ in $(Y, N_{nc}e \Gamma) \ni Y \setminus A_1$ is also $N_{nc}e$ in $(Y, N_{nc}e \Gamma)$. It follows that $N_{nc}e in(A_1) = \phi$, which is a contradiction; hence $(Y, N_{nc}e \Gamma)$ is $N_{nc}e$ irs.

**Definition 5.2** A $N_{nc}ets$ $(Y, N_{nc}e \Gamma)$ is said to be strongly $N_{nc}e$- irresolvable (briefly, $s$ $N_{nc}e$ irs) if for a nonempty set $A_1$ in $Y$, $N_{nc}e in(A_1) = \phi \implies N_{nc}e in(N_{nc}e cl(A_1)) = \phi$.

**Theorem 5.3** If $(Y, N_{nc}e \Gamma)$ is an $s$ $N_{nc}e$ irs space & $N_{nc}e cl(A_1) = Y$ for a nonempty subset $A_1$ of $Y$, then $N_{nc}e cl(N_{nc}e in(A_1)) = Y$.

**Theorem 5.4** If $(Y, N_{nc}e \Gamma)$ is an $s$ $N_{nc}e$ irs space & $N_{nc}e in(N_{nc}e cl(A_1)) \neq \phi$ for a nonempty subset $A_1$ of $Y$, then $N_{nc}e in(A_1) \neq \phi$.

**Theorem 5.5** Every $s$ $N_{nc}e$ irs is $N_{nc}e$ irr.

\[ \text{Proof.} \]
This is follows from Theorems 5.2 & 5.3.

**Theorem 5.6** If $f : (Y, N_{nc}e \Gamma) \rightarrow (Z, N_{nc}e \Psi)$ is a sw $N_{nc}e O$ function & $N_{nc}e in(B_1) = \phi$ for a nonempty subset $B_1$ of $Z$, then $N_{nc}e in(f^{-1}(B_1)) = \phi$.

**Proof.** Let $B_1$ be a nonempty set in $Z \ni N_{nc}e in(B_1) = \phi$. Then $N_{nc}e cl(Z \setminus B_1) = Z$. Since $f$ is sw $N_{nc}e O$ & $Z \setminus B_1$ is $N_{nc}e$ in $Z$, by Theorem 4.6 $f^{-1}(Z \setminus B_1)$ is $N_{nc}e$ in $Y$. Then $N_{nc}e cl(Y \setminus f^{-1}(B_1)) = Y$. Hence $N_{nc}e in(f^{-1}(B_1)) = \phi$.

**Theorem 5.7** If $f : (Y, N_{nc}e \Gamma) \rightarrow (Z, N_{nc}e \Psi)$ is a sw $N_{nc}e O$ function. If $Y$ is $N_{nc}$ irs, then $Z$ is $N_{nc}e$ irs.

**Proof.** Let $B_1$ be a nonempty set in $Z \ni N_{nc}e in(B_1) = Z$. We show that $N_{nc}e in(B_1) \neq \phi$. Suppose not, then $N_{nc}e cl(Z \setminus B_1) = Z$. Since $f$ is sw $N_{nc}e O$ & $Z \setminus B_1$ is $N_{nc}e$ in $Z$, we have by Theorem 4.6 $f^{-1}(Z \setminus B_1)$ is $N_{nc}e$ in $Y$. Then $N_{nc}e in(f^{-1}(B_1)) = \phi$. Now, since $B_1$ is $N_{nc}e$ in $Z$ and using Theorem 4.6 $f^{-1}(B_1)$ is $N_{nc}e$ in $Y$. Therefore, $N_{nc}$ d set $f^{-1}(B_1)$ that $N_{nc}e in(f^{-1}(B_1)) = \phi$, which is a contradiction to Theorem 5.2. Hence we must have $N_{nc}e in(B_1) \neq \phi$ for all $N_{nc}e$ d sets $B_1$ in $Z$. Hence by Theorem 5.2, $Z$ is $N_{nc}e$ irs.

6. Conclusion
In this work, we have introduced some new notions of $N_{nc}e Cts$, $sw$ $N_{nc}e Cts$ and $sw$ $N_{nc}e O$ functions and get results in $N_{nc}ets$. This can be extended to $N_{nc}e$- irresolute function, $N_{nc}e$- homeomorphism functions and also a contra field in $N_{nc}ets$.

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