Amplification of surface plasmons in graphene-black phosphorus injection laser heterostructures

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We propose and evaluate the heterostructure based on the graphene-layer (GL) with the lateral electron injection from the side contacts and the hole vertical injection via the black phosphorus layer (PL) \((p⁺PL-PL-GL\) heterostructure). Due to a relatively small energy of the holes injected from the PL into the GL (about 100 meV, smaller than the energy of optical phonons in the GL which is about 200 meV), the hole injection can effectively cool down the two-dimensional electron-hole plasma in the GL. This simplifies the realization of the interband population inversion and the achievement of the negative dynamic conductivity in the terahertz (THz) frequency range enabling the amplification of the surface plasmon modes. The later can lead to the plasmon lasing. The conversion of the plasmons into the output radiation can be used for a new types of the THz sources.

I. INTRODUCTION

The gapless energy spectrum of graphene layers (GLs) \([1, 2]\) enables their use in the interband photodetectors \([3, 4]\) (see also the review articles \([7, 11]\) and the references therein) and sources (for example, \([11, 17, 19-28]\) operating in the terahertz (THz) a far-infrared (FIR) spectral ranges. In particular, the optical \([12, 14, 17-21]\) and lateral injection pumping of the GLs from the side n- and p-contacts (i.e., from the chemically- or electrically-doped regions) \([13, 16, 22, 24-27]\) can lead to the interband population inversion and negative dynamic conductivity. This can enable the THz lasing experimentally demonstrated. The GL-based heterostructure with lateral current injection and the grating providing the distributed feedback exhibits a single-mode lasing at 5.2 THz and a broadband (1 - 8 THz) amplified spontaneous emission both at 100 K \([24, 26]\). To increase the operating temperature and further enhance the THz gain and lasing radiation intensity, the injection efficiency should be elevated.

The advantage of the carrier lateral double injection pumping from the side n- and p-contact regions in the GL-structures \([12, 16]\), in comparison with the optical pumping is associated with relatively low energies of the injected carriers. While the energy of the injected carriers is about \(\varepsilon_i \approx T_0\) \([30, 31]\), the initial energy of the photogenerated carries is equal to \(\varepsilon_{Opt} = h\Omega/2\) \([12, 28, 32]\). Here \(T_0\) is the lattice temperature, \(h\Omega\) is the energy of photons in the incident (pumping) radiation. In practical devices with the optical pumping using \(A_3B_5\) semiconductor interband lasers integrated with the GL-structure, \(h\Omega \sim 1\) eV. In the case of optical pumping by mid-IR quantum-cascade lasers, \(h\Omega\) can be markedly smaller, but the integration of the pumping source with the GL can be challenging due to the radiation polarization problems. The relatively high values of \(\varepsilon_{opt}\) determine rather high effective temperature \(T\) of the photogenerated two-dimensional electron-hole plasma (2DEHP) in the GL complicating the achievement of the strong interband population inversion and lasing \([32]\).

The efficiency of the lateral injection can be impaired by a decrease in the carrier density in the GL-heterostructure center caused by recombination (the sag of the carrier spatial lateral distribution \([30]\), which weakens the population inversion and decreases the net THz...
gain. This limits the lateral size of the device (spacing between the side n$^+$- and p$^+$-contacts to the GL by the carrier lateral ambipolar diffusion length. Shortening of the active part of the GL increases the leakage currents (electrons and holes reaching the p-contact and n-contact, respectively).

A compromise can be reached using of the lateral injection of one type of carriers (the electron injection from the side n$^+$-contacts) and the vertical injection of the other type (the hole injection via the bulk p-layer). A proper band alignment of the GL and the bulk material layer serving as the vertical injector could minimize or even avoid the 2DEHP heating by the injection of holes. This implies that the material for the hole injector should have the energy spacing, $\Delta_V$, between the Dirac point in the GL and the valence band top of the injector material as small as possible. One of such candidates for the injector material is the black phosphorus [33–51]. This material is now considered to be very promising for different electronic and optoelectronic devices applications (see, for example, [33–51]). The quantity $\Delta_V$ in the black phosphorous layers (PLs) comprising several atomic sheets is estimated as $\Delta_V \approx 100$ meV with the energy band $\Delta_C = \Delta_V + \Delta_C \approx 300$ meV ($\Delta_C$ is the GL-PL electron affinity). Since the energy of the holes injected into the GL from the PL’s is smaller than the energy of optical phonons in the GL (about of 200 meV), the hole injection can even cool in a substantial cooling of the 2DEHP under the vertical injection of holes from the side n$^+$ contacts and the vertical injection of holes from the bulk p$^+$-PL-GL -structure. We calculate the dependences of the carrier effective temperature, their quasi-Fermi energies, the 2DEHP frequency-dependent dynamic conductivity, and the coefficient of the surface plasmonic modes amplification as functions of the injected current for different structural parameters. Using these data, we find the conditions at which this conductivity is negative, and the coefficient of the surface plasmons amplification is positive. The plasmonic modes self-excitation in the latter case can lead to the plasmonic lasing followed by the conversion of these modes into the output THz radiation.

The cooling of the 2DEHP under the vertical injection might lead a substantial softening of the population inversion conditions and the conditions of the amplification and self-excitation of the photonic and plasmonic modes. Therefore, the proposed heterostructure can serve as an active part of the THz and FIR lasers with the photonic and plasmonic wave guides.

II. DEVICE STRUCTURE

Figure 1 shows the schematic view of this heterostructure with a relatively narrow-gap injector p-type black PL, GL, on a wide-gap substrate and its energy diagram at the operating bias voltage $U$ ($|U| > V_{bi} \approx \Delta_V/e$, where $V_{bi}$ the built-in voltage). As for the substrate, several relatively wide-gap materials can be used, in particular, hexagonal Boron Nitride (hBN) because the GLs on the hBN substrate exhibit exceptionally high mobility values. A wide gap in the hBN substrate provides high energy barrier for the electrons and holes in the GL and blocks their leakage to the substrate. At the applied bias voltage, the electron can freely fill in the GL conduction band, while the holes pass vertically from the heavily-

![Diagram of device structure](image)

**FIG. 1:** Schematic view of (a) the p$^+$-P-PL-GL heterostructure and (b) its band diagram at a voltage $U$. 
III. ENERGY AND DENSITY BALANCES IN THE 2DEHP

In each act of the interband and intraband emission/absorption of the GL optical phonons (with the energy \( \hbar \omega_S \approx 200 \text{ meV} \) and the interface optical phonons (with the energy \( \hbar \omega_{S_{\text{Opt}}} \approx 100 \text{ meV} \)) the energy of the 2DEHP decreases/increases by the quantity \( \hbar \omega_0 \). The resulting the energy balance equation is

\[
\frac{T}{T_0} = 1 + \frac{\Delta}{jG} \left( \frac{\Delta}{\hbar \omega_0} - \frac{1}{a} \right).
\]

The equation governing the electron and hole balance is given by:

\[
\exp\left(\frac{jG}{T}\right) \exp\left[\hbar \omega_0 \left(\frac{1}{T_0} - \frac{1}{T}\right)\right] - 1
+ s \left\{ \exp\left(\frac{jG}{T}\right) \exp\left[\hbar \omega_S \left(\frac{1}{T_0} - \frac{1}{T}\right)\right] - 1 \right\}
= \frac{j}{jG} \left(\frac{\Delta}{\hbar \omega_0}\right).
\]

IV. EFFECTIVE TEMPERATURE AND QUASI-FERMI ENERGIES AS FUNCTIONS OF THE INJECTED CURRENT

In the limit of small \( s \), which could correspond to the device with the substrate (instead of the hBN substrate) exhibiting very weak interaction of its phonon system with the carriers in the GL from Eqs. (1) and (2) we obtain

\[
\frac{T}{T_0} = 1 + \frac{\Delta}{jG} \left( \frac{\Delta}{\hbar \omega_0} - \frac{1}{a} \right),
\]

\[
\frac{\mu_e + \mu_h}{T} = \ln \left[ 1 + \frac{1}{jG} \left( \frac{\Delta}{\hbar \omega_0} - \frac{1}{a} \right) \right].
\]

From Eq. (3) one can see that \( T > T_0 \) if \( \Delta_i > \hbar \omega_0 \approx 200 \text{ meV} \) (heating of the 2DEHP by the injection current) and \( T < T_0 \) (cooling of this plasma) if \( \Delta_i < \hbar \omega_0 \). Simultaneously, fromEq. (4) we find that \( \mu_e + \mu_h < 0 \) and \( \mu_e + \mu_h > 0 \) when \( \Delta_i/\hbar \omega_0 > 1 + a \) and \( \Delta_i/\hbar \omega_0 < 1 + a \), respectively. In the case \( 1 < \Delta_i/\hbar \omega_i < 1 + a \), both \( T > T_0 \) and \( \mu_e + \mu_h \) are positive.
If $\Delta_i > \hbar \omega_0$, an increase in the injected current density $j$ results in a monotonic rise of the effective temperature. In this case, Eq. (3) yields the $T - j$ dependence, which diverges at a fairly large value $j = j_\infty$, where

$$j_\infty = j_G \frac{a [\exp(h \omega_0/T_0) - 1]}{(\Delta_i/h \omega_0)} = j_G \frac{a [\exp(h \omega_0/T_0)]}{(\Delta_i/h \omega_0)}. \quad (5)$$

Such a divergence means that at such a pumping the interaction of the carriers with optical phonons in the GL is not able to transfer the energy brought to the GL by the injected carriers to the optical phonon system. In reality, a sharp increase in the effective temperature might be limited by additional energy relaxation mechanisms engaging at very large temperatures.

When $j$ tends to $j_\infty$, from Eq. (4) we obtain

$$\frac{\mu_e + \mu_h}{T} \simeq \ln \left( \frac{a}{(1 - \Delta_i/h \omega_0)} \right). \quad (6)$$

The latter quantity can be both positive and negative (i.e., degenerate and non-degenerate 2DEHP, respectively).

In the most interesting case $\Delta_i < h \omega_0$, $j$ tends to the saturation current density

$$j_{sat} = j_G \frac{a}{(1 - \Delta_i/h \omega_0)}. \quad (7)$$

and the effective temperature $T$ steeply drops tending to zero. Apart from this, at $j \simeq j_{sat}$, the ratio $(\mu_e + \mu_h)/T$ tends to infinity, while $(\mu_e + \mu_h)$ tends to $h \omega_0$. In such a case, the hole quasi-Fermi energy can become close to $\Delta_i$. The latter, accompanied with a strong decrease in the effective temperature (and, hence, a strong carrier system degeneration), leads to a dramatic suppression of the hole capture into the GL because the GL valence band becomes overfilled up to the top of the barrier $(\mu_h \simeq h \omega_0/2 \sim \Delta_i)$. As a result, the injected current density can not markedly exceed $j_{sat}$ (the injected current saturation).

At $T = 300K$, setting $\frac{\Sigma_0}{\epsilon P_{Opt}} \simeq 10^{21}$ cm$^2$ s$^{-1}$, we obtain $j_G = 0.5 \times 10^2$ A/cm$^2$. The quantity $j_G$ can be of the same order of magnitude as $j_\infty$.

Equation (2) yields the sum of the electron and hole quasi-Fermi energies $\mu_e + \mu_h$ versus the injected (recombination) current $j$. An additional relationship between $\mu_e$ and $\mu_h$ on the one hand and $j$ on the other can be obtained considering the difference in the electron and hole densities, $\Sigma_e$ and $\Sigma_h$, in the GL determined by the electric field $E_{PG}$ at the PL and GL interface. Using Eq. (A6), we obtain

$$\Sigma_e - \Sigma_h = \frac{\kappa V}{4 \pi ed} = \frac{\kappa}{4 \pi \epsilon \epsilon_P N_a} j. \quad (8)$$

where $\kappa = (\varepsilon_P + \varepsilon_{hBN})/2$ is the effective dielectric constant determined by the dielectric constants of the layers ($\varepsilon_P$ and $\varepsilon_{hBN}$ are the dielectric constants of the BL and hBN, respectively) sandwiching the GL and $p_P$ is the hole mobility in the direction perpendicular to the heterostructure plane. Considering that the electron and hole densities in the GL are related to the quasi-Fermi energies (of the degenerate electron and hole components, $\mu_e, \mu_h > T$) as $\Sigma_e \simeq \mu_e^2/\pi \hbar^2 v_W^2$ and $\Sigma_h \simeq \mu_h^2/\pi \hbar^2 v_W^2$, where $v_W \simeq 10^8$ cm/s is the characteristic carrier velocity in the GLs, from Eq. (8) we arrive at (see also Appendix B)

$$\mu_e - \mu_h = \mu_e + \mu_h = \frac{\kappa h \omega_0^2}{4 e z P N_a j} T_0^2 D - \frac{j}{j_G}, \quad (9)$$

where

$$D = \frac{\kappa h \omega_0^2 \Sigma_0}{4 e b_P N_a \epsilon_{Opt} T_0} = \frac{\kappa h \omega_0^2 m \Sigma_0}{4 e^2 N_a \epsilon_{Opt} \epsilon_{Opt} T_0^2}. \quad (10)$$

For $\kappa \simeq 6, b_p = (250 - 500)$ cm$^2$/V s and $N_a = 5 \times 10^15$ cm$^{-3}$, Eq. (7) yields $D \simeq 0.049 \sim 0.038$.

Figure 2 shows the dependences of the carrier effective temperature $T$ in the GL, their net quasi-Fermi energy $(\mu_e + \mu_h)$, and the ratio $(\mu_e + \mu_h)/T$ on the normalized injection current density $j/j_G$ calculated using Eqs. (3) and (4), i.e., neglecting the contribution of the surface optical phonons ($s = 0$), for different values $\Delta_i$.

The plots in Figure 2 confirm the above qualitative analysis of the effective temperature and the quasi-Fermi energies behavior as functions of the injected current density. In particular, Fig. 2 demonstrates the possibility of a fairly strong cooling and degeneration of the 2DEHP in the GL with increasing injection current density providing that $\Delta_i < h \omega_0$ (curves “1” and “2”). But at $\Delta_i < h \omega_0$, Fig. 2 (curves “3” and “4”) demonstrates a moderate 2DEHP heating, which, nevertheless, is accompanied with the 2DEHP degeneration, although the latter is also moderate.

The inclusion an extra intraband and interband relaxation mechanism, like that associated with the carrier interaction with surface optical phonons ($s \neq 0$) with $\omega_s < \Delta_i < h \omega_0$, removes the tendency to the 2DEHP overcooling, so that the effective temperature decreases smoothly. This because when the effective temperature $T$ becomes sufficiently low due to the cooling effect of the high energy optical phonons, further decrease in this temperature is blocked by the energy absorption from the low energy optical phonons (i.e., the surface optical phonons). Although their number $N_s = \exp((h \omega_s/T_0) - 1)^{-1}$ is small, it, nevertheless, exceeds the number of the GL optical phonons $N_0 = \exp((h \omega_0/T_0) - 1)^{-1} \approx \exp((h \omega_s/T_0)$.

Figure 3 shows the same dependences as in Fig. 2 but calculated numerically for more general situations when both the GL optical phonons ($h \omega_0 = 200$ meV) and the surface optical phonons ($h \omega_0 = 100$ meV) contribute to the relaxation processes. As seen from Fig. 3, at the moderate injection current densities ($j \lesssim j_G$) assumed in
FIG. 2: The dependences of (a) carrier effective temperature $T$, (b) the net quasi-Fermi energy ($\mu_e + \mu_h$), and (c) the ratio $(\mu_e + \mu_h)/T$ on the normalized injection current density $j/j_G$ for different $\Delta V$: 1 - $\Delta V = 100$ meV, 2 - $\Delta V = 150$ meV, 3 - $\Delta V = 175$ meV, 1 - $\Delta V = 200$ meV.

FIG. 3: The same as in Fig. 2 but for values of the parameter $s$ characterizing the relative strength of the carrier interaction with the surface phonons: $\Delta V = 100$ meV, 1 - $s = 0$; 2 - $s = 0.001$; 3 - $s = 0.01$; 4 - $s = 0.1$, and 5 - $s = 1.0$.

V. DC CURRENT-VOLTAGE CHARACTERISTICS.

Disregarding the nonuniformity of the potential along the GL in the $x$-direction, (i.e., disregarding the current-crowding considered below in Sec. VIII), the device current-voltage characteristic can be found deriving $V$ as a function of the applied voltage $U$ (see Fig. 1(b)). Due to a smallness of the factor $D$, one can find from Eq. (6) that in reality $(\mu_e - \mu_h) \ll (\mu_e + \mu_h)$. Hence $\mu_e \approx (\mu_e + \mu_h)/2$. Considering, in particular, the case $s \ll 1$ in which Eqs. (3) and (4) are valid, we find

$$
\mu_e \approx \frac{T_0}{2} \cdot \frac{1}{1 - \frac{T_0}{\hbar \omega_0} \ln \left[ 1 + \frac{j}{j_G} \left( \frac{\Delta_i}{\hbar \omega_0} - 1 \right) \frac{1}{a} \right]} \times \ln \left[ 1 + \frac{j}{j_G} \left( \frac{\Delta_i}{\hbar \omega_0} - 1 \right) \frac{1}{a} \right].
$$

(11)
Considering Eq. (11), one can present the current-voltage characteristic \( U \) versus \( j/j_G \) in the following (in-explicit) form:

\[
U - \frac{\Delta V}{e} \approx V_0 j \frac{T_0}{j_G} + \frac{1}{1 - \frac{T_0}{\hbar \omega_0} \ln \left[ 1 + \frac{j}{j_G} \left( \frac{\Delta_1}{\hbar \omega_0} - 1 \right) \frac{1}{a} \right]} \times \ln \left[ 1 + \frac{j}{j_G} \left( \frac{\Delta_1}{\hbar \omega_0} - 1 \right) \frac{1}{a} \right]
\]

(12)

Here \( V_0 = d \Sigma_0 / N_0 b p \tau_{\text{Opt}} \). For the parameters used in above estimate, \( V_0 \approx 40 \text{ mV} \).

When \( \Delta_1 = \Delta_V + 3 T_0/2 < \hbar \omega_0 \), Eq. (12) describes a monotonically rising current-voltage characteristics tending to the saturation \( (j \approx j_\infty) \) at very high voltages.

If \( \Delta_1 < \hbar \omega_0 \), Eq. (12) yields the following expression for the voltage corresponding to the current saturation:

\[
U_{\text{sat}} = \frac{\Delta_V + \hbar \omega_0}{2e} + \frac{V_0 a}{(\Delta_V + 3 T_0/2)/\hbar \omega_0 - 1} \]

(13)

When the effect of the surface optical phonons is tangible, the current-voltage characteristics becomes a sub-linear.

VI. DYNAMIC CONDUCTIVITY

The contributions of the direct interband optical transitions and the intraband radiative transitions assisted with the carrier scattering (leading to the Drude absorption) to the pertinent components of the GL conductivity \( \sigma_{\text{inter}} = \sigma_{\text{inter}}^\text{Drude} + \sigma_{\text{inter}}^\text{optical} \) and \( \sigma_{\text{intra}} = \sigma_{\text{intra}}^\text{Drude} + \sigma_{\text{intra}}^\text{optical} \) constitute the GL net dynamic conductivity. In particular, \( \sigma_{\text{inter}} \) can be found as in references \[12, 57, 58\]:

\[
\text{Re} \sigma_{\text{inter}} = \frac{\hbar e^2}{4 \hbar} \sinh \left( \frac{\hbar \omega - (\mu_e + \mu_h)}{2T} \right) \cosh \left( \frac{\hbar \omega - (\mu_e + \mu_h)}{2T} \right) + \cosh \left( \frac{\mu_e - \mu_h}{2T} \right)
\]

\[
\approx \frac{\hbar e^2}{4 \hbar} \sinh \left( \frac{\hbar \omega - (\mu_e + \mu_h)}{2T} \right) \cosh \left( \frac{\mu_e - \mu_h}{2T} \right)
\]

(14)

Up to fairly large values of \( j/j_G \), the argument of the first \( \cosh \)-function in the denominator of the expression in the right-hand side of Eq. (14) is much larger than that in the second \( \cosh \)-function. Taking this into account, Eq. (14) can be reduced to the standard form \[12\]:

\[
\text{Re} \sigma_{\text{inter}} \approx \frac{e^2}{4 \hbar} \tanh \left( \frac{\hbar \omega - \mu_e - \mu_h}{4T} \right).
\]

(15)

The quantity \( \text{Im} \sigma_{\text{inter}} \) can be presented as \[57\]

\[
\text{Im} \sigma_{\text{inter}} = \frac{i e^2}{4 \hbar} \frac{4 \hbar \omega}{\pi} \int_0^\infty \frac{G(\varepsilon) - G(\hbar \omega/2)}{(\hbar \omega)^2 - 4 \varepsilon^2} d\varepsilon,
\]

(16)

where \( G(\varepsilon) = \tanh[2 \varepsilon - (\mu_e + \mu_h)/4T] \).

The intraband contributions \( \text{Re} \sigma_{\text{intra}}^\text{optical} + \text{Im} \sigma_{\text{intra}}^\text{optical} \) depend on the carrier momentum relaxation mechanisms in the GL, particularly, on the range of the effective carrier-carrier interactions and on disorder \[59\] (see also \[50\]). At fairly high carrier densities, expected under the injection conditions under consideration, the electron-hole interactions are the main mechanism of the momentum relaxation \[60, 62\]. Due to special features of the mutual scattering of the carriers with the linear dispersion law \[59, 61\], such scattering is a short range scattering.
The mutual carrier scattering is similar to the scattering on uncharged and screened charged impurities, as well as the acoustic phonons and defects. In this case, the momentum relaxation time as a function of the electron or hole momenta can be presented as \( \tau_p = \tau_0 p_0 / p ^{\frac{3}{2}} \) [43, 51], where \( p_0 = T_0 / v_W \) and \( \tau_0 \) is the characteristic carrier momentum relaxation time. If the dominant scattering mechanism is associated with the carrier interactions with weakly screened charged impurities or their clusters, i.e., with the long-range scatterers, one can set \( \tau_p = \tau_0 p_0 \). When the interaction with both the short- and long-range scatterers is important, the approximation \( \tau_p = \tau_0 = \text{const} \) could be used [12, 17, 52, 63]. Considering this, one can arrive at

\[
\text{Re} \sigma^{\text{intra}}_\omega + \text{Im} \sigma^{\text{intra}}_\omega = \left( \frac{e^2}{4\hbar} \right) \frac{8 \langle \epsilon_p \rangle \tau_0}{\pi \hbar (1 - i \omega \tau_p)}, \tag{17}
\]

where \( \langle \epsilon_p \rangle = T_0, \langle \tau_p \rangle = (2 \tau_0 T_0) / (\mu_e + \mu_h) \) at \( \tau_p \propto p^{-1} \) and \( \langle \epsilon_p \rangle = (\mu_e + \mu_h) / 2, \langle \tau_p \rangle = \tau_0 \) at \( \tau_p = \tau_0 = \text{const} \) (valid when \( \mu_e + \mu_h > T \)).

At \( \hbar \omega < \mu_e + \mu_h \), Eqs. (14) and (15) yields \( \text{Re} \sigma^{\text{inter}}_\omega < 0 \).

If the dominant scattering mechanism of the electrons and holes in the GL is their mutual interaction, the quantity \( \tau_0 \) calculated for \( T_0 = 25 \) meV and \( \kappa = 6 \) (for a GL sandwiched between the PL and hBN) is about of \( \tau_0 = 3.6 \) ps [62]. Accounting for other scattering mechanisms (impurities, acoustic phonons, and so on), one can set \( \tau_0 = 1 \) ps. Assuming \( 1.0 - 3.6 \) ps, the net real part of the dynamic conductivity is negative in the frequency range \( \omega / 2 \pi \geq (3.44 - 6.50) \) THz.

Figure 4 shows the spectral dependences of the real part of the net dynamic conductivity in the GL (\( \text{Re} \sigma^{\text{intra}}_\omega + \text{Re} \sigma^{\text{inter}}_\omega \)) calculated for the cases \( \tau_p \propto \tau_0 p^{-1} \) (solid lines) and \( \tau_p = \tau_0 = \text{const} \) (dashed lines) using Eqs. (15) and (17) with Eqs. (3) and (4) for \( T \) and \( (\mu_e + \mu_h) / T \) for different characteristic momentum relaxation \( \tau_0 \) and different values of the normalized injection current density \( j / j_G \). Other parameters used are \( T_0 = 300 \) K, \( \hbar \omega_0 = 200 \) meV, \( \Delta_V = 100 \) meV, (for \( \kappa \simeq 6 \)), \( a = 0.25 \), and \( s \ll 1 \).

As seen from Fig. 4, the real part of the dynamic conductivity of the 2DEHP can be negative at sufficiently strong injection pumping in a certain range of \( \hbar \omega \) (compare the curves for \( j / j_G = 0.52 \) and \( j / j_G = 0.78 \). An increase in the injection current density leads to the reinforcement of the negative dynamic conductivity and widening of the range where this conductivity is negative. This is mainly due to the rise of \( \text{Re} \sigma^{\text{inter}}_\omega \) when the net quasi-Fermi energy \( (\mu_e + \mu_h) \) increases [see Eq. (15)]. The comparison of the solid and dashed lines (corresponding to different momentum dependences of the momentum relaxation time) shows that they are rather close, although the character of the carrier scattering plays some role. The fact that the hBN substrate is virtually free of charged impurities (providing the long-range carrier scattering), is in favor of the dependence \( \tau_p \propto \tau_0 p^{-1} \).

![Figure 4: Spectral dependences of the real part of the net dynamic conductivity in the GL.](image)

Therefore, calculating plots in the consequent figures, we set \( \tau \propto \tau_0 p^{-1} \).

Figure 5 shows the spectral dependences of the real part of the 2DEHP dynamic conductivity similar to those in Fig. 4, but obtained for a higher value of the surface optical phonon parameter \( s \), namely for \( s = 0.1 \). Comparing the plots of Figs. 4 and 5, one can see that an increase in the parameter \( s \) results in a weakening of the negative dynamic conductivity effect. Enhancing the carrier mobility in the GL, i.e., and increase in \( \tau_0 \) can markedly reinforce the negative dynamic conductivity, and due to weakening of the intraband absorption. As follows from Fig. 3(c), the quantity \( \mu_e + \mu_h / T \) can markedly exceed unity even \( s \approx 1 \), but at relatively high injection current densities \( (j / j_G \sim 3 - 4) \). This implies that the effect of the negative dynamic conductivity can pronounced in the case of relatively strong carrier interaction with the surface optical phonons as well.

**VII. SURFACE PLASMOS AMPLIFICATION COEFFICIENT**

Using the equations for the GL dynamic conductivity under the injection pumping given in the Sec. VI, invoking the Maxwell equations, considering the structure geometry, and following the method applied previ-
ously \cite{17, 18, 22}, one can derive the dispersion equation for the surface plasmons with the frequency \( \omega \), in which the ac electric and magnetic fields components are proportional to \( \exp \left( i \rho \frac{\omega}{c} y - i \omega t \right) \) propagating in the direction parallel to the side contacts (along the axis \( y \)). Assuming (see Sec. VIII) that the plasmon absorption in the PL is due to the interaction with the holes (Drude absorption), one can arrive to the following dispersion equation:

\[
\varepsilon_{hBN} \sqrt{\varepsilon_z - \rho^2} + \varepsilon_z \sqrt{\varepsilon_{hBN} - \rho^2} + \frac{4\pi}{c} \sigma_\omega \sqrt{\varepsilon_z - \rho^2} \sqrt{\varepsilon_{hBN} - \rho^2} = 0
\]

with

\[
\varepsilon_z = \varepsilon_p \left( 1 - \frac{\omega_p^2}{\omega^2 + i \gamma_p \omega} \right).
\]

Here \( \sigma_\omega = \sigma_{\text{inter}} + \sigma_{\text{intra}} \) is the GL net dynamic conductivity, the low-frequency dielectric constants of the hBN \( \varepsilon_{hBN} \) is taken from \cite{64, 65}, \( \omega_p = \sqrt{4\pi e^2 N_a/m_e} \) is the plasma frequency of holes in the PL, \( \gamma_p = e/m_e \) is the plasma oscillation damping constant associated with the Drude absorption in the PL, and \( c \) is the speed of light in vacuum. The quantities \( \text{Re}(\rho) \) and \( 2\omega \text{Im}(\rho)/c \), obtained from the solution of Eq. (18), are the plasmon propagation index and the plasmon absorption or amplification coefficient (depending on the sign), respectively. Deriving the dispersion equation for the surface plasmons, we have accounted for the interaction of the electromagnetic radiation with phonons in PLs resulting in the single-phonon absorption if and only if the radiation is polarized along the axis \( z \). The pertinent absorption coefficient is two order of magnitude smaller than that in the standard polar semiconductors, although there is a narrow peak at 14 THz with the absorption coefficient about 500 cm\(^{-1}\). The two-phonon absorption is relatively week (about 15 cm\(^{-1}\) in the range 7.5 - 14 THz \cite{63}). Therefore, the Drude mechanism plays the main role in the plasmon absorption in the PL as was assumed above.

Figure 6 shows the spectral dependences of the plasmon amplification coefficient \( \alpha_\omega = -2\omega \text{Im}(\rho)/c \). We assumed that the acceptor density in the BL and the thickness of this layer are equal to \( N_a = 5 \times 10^{15} \) cm\(^{-3}\) and \( d = 10^{-4} \) cm, respectively. The injection current densities and other parameters are the same as for Fig. 5. As seen from Fig. 5, in the frequency range where the 2DEHP dynamic conductivity is negative, the amplification coefficient can be fairly large, of the order of \( \alpha_\omega \approx (1.5 - 2.0) \times 10^4 \) cm\(^{-1}\). The large amplification coefficient of the plasmonic mode in comparison with the photonic modes is attributed to a small plasmon propagation velocity compared to the speed of light.

As seen from Fig. 3, the reinforcement of the surface optical phonon scattering (increase in \( s \)) gives rise to pronounced variations of \( T \) and \( \mu_e + \mu_h \) and, hence, \( \alpha_\omega \). An increase in \( s \) corresponds to a drop of \( \alpha_\omega \). As seen, at \( j/j_G = 0.78 \) and \( s \geq 0.60 \), \( \alpha_\omega \) becomes negative. However, for a larger \( j/j_G \), \( \alpha_\omega \) can be positive at a larger \( s \).

The obtained values of the amplification coefficient are close to those in the GL-based structures with the side double injection. This is because the Drude absorption in the BL is relatively weak, at least, at \( N_a \leq 5 \times 10^{15} \) cm\(^{-3}\). At a higher doping of the PL, this absorption can decrease \( \alpha_\omega \) even leading to the transition from the amplification to the damping of the plasmonic modes as shown in Fig. 9. A weak Drude absorption is partially associated with strong localization of the y- and z-components of the plasmon electric field around the GL. The latter is demonstrated in Fig. 7. A strong localization of the plasmon electric field far from the contact p\(^+\)-PL (at the distance about 1 \( \mu \)m) prevents the plasmon damping due to the absorption in this layer.

![Figure 6: Spectral characteristics of the plasmon amplification](image)
The equation governing the electron and hole balance is given by:

\[
\frac{\mu_e + \mu_h}{T} + \frac{(1 + a)\hbar \omega_0}{1 + \frac{1}{T}} = \frac{j}{j_G} \frac{\Delta_i}{\hbar \omega_0} \tag{20}
\]

The variation of this energy associated with the Auger processes can be estimated as \(\varepsilon_{\text{Auger}} \sim T, \mu_e, \mu_h < \hbar \omega_0\), hence the contribution of the Auger processes to the 2DEHP energy balance can be disregarded. Considering this and using the linearized Eqs. (1) and (2), we arrive at

\[
\frac{\mu_e + \mu_h}{T} + (1 + a)\hbar \omega_0 \left( \frac{1}{T_0} - \frac{1}{T} \right) = \frac{j}{j_G} \frac{\Delta_i}{\hbar \omega_0} \tag{22}
\]

At relatively weak Auger processes \((a_A \ll 1)\), Eqs. (22) and (23) lead to the same dependences \((T - T_0)\) and \((\mu_e + \mu_h)\) on the injection current density \(j\) as obtained in Sec. III (for the relaxation on the GL optical phonons at small \(j/j_G\)).

Generally speaking, Eqs. (22) and (23) show that the Auger processes result in slowing down the cooling (which can occur at \(\Delta_i < \hbar \omega_0\)) of the 2DEHP with increasing injection current. If the Auger parameter \(a_A\) is sufficiently large \((a_A = \hbar \omega_0/\Delta_i - 1)\), the cooling gives way to the heating. At both cooling and heating of the 2DEHP, the splitting of the quasi-Fermi energies, i.e., the quantity \((\mu_e + \mu_h)\), increase when \(j\) increases providing that \(\Delta_i < \hbar \omega_0 < 1 + a\).

### B. Heating of optical phonons

The recombination and the intraband energy relaxation lead to the generation of nonequilibrium (hot) optical phonons. The generated hot optical phonons cool...
down through anharmonic decay to acoustic phonons which are subsequently absorbed into the substrate [54, 66, 68]. Direct cooling of the charge carriers also occurs via emission of the surface phonons of the underlying polar substrate.

As demonstrated experimentally, the optical phonon decay time in the GL-hBN heterostructures is about [54] $\tau_{\text{Opt}}^{\text{decay}} \sim 0.200 - 0.375 \text{ ps}$, i.e., is relatively short. At such short decay times, the deviation of the optical phonon system from equilibrium is insignificant, i.e., this system temperature $T_{\text{Opt}} \simeq T_0$. This justifies the omission of this effect in the model used above. An example of the inclusion of the optical phonon heating into a similar model could be found in [16, 32]. Due to the large specific heat capacity of hBN, the rise of the lattice temperature even under relatively strong pumping is small ($\sim 1 \text{ K}$) [54].

C. Current crowding in the GL

The finiteness of the GL conductivity can lead to a nonuniformity of the potential distribution $\varphi = \varphi(x)$ along the conductivity plane and, consequently, to a nonuniformity of the injection current $j = j(x)$, where axis $x$ is in the direction connecting the $n^+$-contacts (see Fig. 1). This effect is akin to the current-crowding effect in the bipolar transistors and light-emitting diodes, dominating at high current densities [69, 70]. The current crowding slows down the $j$ versus $U$ dependence. The general consideration of the current crowding requires a rather complex mathematical model with nonlinear differential equations describing the potential and current density distributions. This is beyond the scope of the present paper. Here we limit ourselves to the case when the current crowding is not too strong and find the pertinent conditions.

Since the resistance of the side contacts to the GL appears to be not a challenging issue [71, 72], we disregard the contribution of the contact resistance to the net potential drop, $U$, between the $p^+$-contact and the $n^+$-side contacts. The lateral variation of the injection current density in the in-plane direction $x$ (see Fig. 1) can be approximately found from the continuity equation:

$$\frac{d^2j}{dx^2} = K^2j$$

with the boundary conditions $j = j_0|_{x=\pm l}$ given at the side contact edges ($x = \pm l$). Here $2l$ is the spacing between the side-contacts to the GL, $j_0$ is given by Eq. (11), and $K \simeq \sqrt{(b_G/b_C)(N_a/\Sigma_Gd)}$, $b_C$ and $\Sigma_G$ are the mobility and density of the carriers in the GL, respectively.

Solving Eq. (26), we find

$$j = j_0 \frac{\cosh(Kx)}{\cosh(2Kl)}.$$  

The value of the injection current density sag $\delta j = [1 - \cosh^{-1}(Kl)] \simeq (Kl/2)^2$ is relatively small if $2l \ll L = 4K^{-1} = 4\sqrt{(b_G/b_C)(\Sigma_Gd/N_a)}$. This inequality implies that the lateral resistance of the GL is much smaller than the vertical resistance of the PL. Assuming $N_a = 5 \times 10^{15} \text{ cm}^{-3}$, $\Sigma_G = 10^{-12} \text{ cm}^{-2}$, $d = 10^{-4} \text{ cm}$, $b_G = 10,000 \text{ cm}^2/\text{V} \cdot \text{s}$, for $b_C = (250 - 500) \text{ cm}^2/\text{V} \cdot \text{s}$, we obtain that the current density nonuniformity can be disregarded if $2l \ll L = (25 - 30) \times \mu \text{m}$. Larger values of $2l$ correspond to the smaller contact leakage currents [30]. The latter inequality corresponds to the real device sizes.

On the contrary, in the GL-heterostructures with the lateral electron and hole double injection from the side contacts [30], the lateral nonuniformity of the carrier densities is determined by the diffusion length $L_D$. The latter is about a few micrometers. Since $L \gg L_D$, the GL-PL heterostructures with the combined injection can provide the negative dynamic conductivity in much larger area than the heterostructures with the lateral injection. This implies that the THz sources based on the GL-PL heterostructures can demonstrate markedly higher output power.

Conclusion

We proposed the $p^+$-PL-PL-GL heterostructures with the lateral electron and vertical hole injection as the active elements of the plasmonic lasers. Using the developed device model, we calculated the effective temperature of the carriers, their quasi-Fermi energies, and the dynamic conductivity of the 2DEHP in the GL. Under sufficiently strong injection current densities, the dynamic conductivity can be negative in a certain range of the plasmon energies providing positive and a fairly large amplification coefficient of the plasmonic mode. Due to a relatively small energy of the holes injected from the PL injecting contact in comparison with the optical phonon energy in the GL, the carrier effective temperature can be lower than the ambient temperature. This, together with the possibility of the negative dynamic conductivity realization in fairly large GL areas, promotes a more efficient THz lasing. Similar GL-based heterostructures can include the black arsenic injecting layers and other injecting layer materials with a proper band alignment to the GLs [73, 74]. Using the substrates providing weaker energy and momentum carrier relaxation in the GL (instead of hBN considered above, one can achieve a stronger negative dynamic conductivity and higher amplification amplification of the plasmonic modes at a weaker injection. The plasmonic lasing can be enabled by the plasmon reflection from the end faces and by the realization of the distributed feedback using the highly conducting saw-tooth (serrated) side contacts [26].

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The injected current coincides with the current across the p-PL in the hole injector. At low bias voltages, the injected current is associated with the hole diffusion across the BL. When \( V = V_{bi} \approx \Delta V \), i.e., when \( U = 0 \), its density can be estimated as \( j_0 \approx eD_p N_a / d = eD_p T_0 N_a / d \). Here \( D_p \) and \( b_p \) are the hole diffusion coefficient and mobility in the PL perpendicular to its plane (perpendicular to the atomic sheets forming the PL layers structure) and \( N_a \) the acceptor density in this layer.

At larger values of \( |U| \), when the voltage drop across the PL \( V > |U| - V_{bi} - \mu_e / e > 0 \) [see Fig. 1(b)], i.e., in the operation regime, the injected current is determined by the PL resistance. Taking into account that the holes in the p-PL should not be heated too strongly, we assume that the average electric field in this layer \( E = V/d \) is moderate, where \( d \) is the thickness of the PL. The acceptor density in the PL can be set \( N_a \sim (2 - 5) \times 10^{15} \text{cm}^{-3} \) [34, 35]. In such a situation, the hole density in the PL at voltages \( p \approx N_a \), and the current density across the PL \( J \) (which coincides with the density of the recombination current in the GL) is given by

\[
j = \frac{V}{d \rho_p}, \quad \rho_p = \frac{1}{eN_a b_p}.
\] (A1)

Here \( \rho \) is the PL resistivity.

Setting the acceptor density in the PL \( N_a \sim (2 - 5) \times 10^{15} \text{cm}^{-3} \) [34, 35], \( b_B = (250 - 500) \text{cm}^2 / V \text{s} \), \( d = 10^{-4} \text{cm} \), we obtain \( j_0 \approx 8 - 20 \text{A/cm}^2 \). If \( V = (0.1 - 1.0) \text{V} \), we obtain \( j = 2 \times 10^2 - 4 \times 10^3 \text{A/cm}^2 \).

Since at the normal device operation \( j_0 \ll j \), we can neglect \( j_0 \).

The hole effective temperature in the PL \( T_B \) can be estimated using the following equation:

\[
N_a \frac{(T_P - T_0)}{\tau_p} = j \frac{V}{d},
\] (A2)

so that

\[
T_P = T_0 + \frac{\tau_p}{e b_p N_a} j^2 = T_0 + \frac{m}{e^2 N_a^2 \tau_p^2} j^2.
\] (A3)

Here \( \tau_p \) and \( \tau_p^2 \) are the hole energy and momentum relaxation times in the PL. Considering Eq. (A3), one can find that

\[
\Delta_i = \Delta V + \frac{3T_0}{2} \left[ 1 + \Theta \left( \frac{j}{J_G} \right)^2 \right],
\] (A4)

where

\[
\Theta = \frac{m}{2N_a^2 T_0} \left( \frac{\Sigma_0}{\tau_{inter}^{\text{opt}}} \right)^2.
\]

Deriving the hole momentum relaxation time \( \tau_p^2 \) from the value of the hole mobility \( bP \) \( (\tau_p^2 \approx (0.4 - 0.8) \times 10^{-13} \text{s}) \) with \( m = 2.5 \times 10^{-28} \text{kg} \), setting \( \tau_p \approx 10 \tau_p^2 \) and \( \Sigma_0 / \tau_{inter}^{\text{opt}} = 10^{21} \text{cm}^{-2} \text{s}^{-1} \), for \( N_a = 5 \times 10^{15} \text{cm}^{-3} \), one obtains \( \Theta \approx 2.4 \times 10^{-3} \). The latter estimate implies that in the range of realistic current densities one can put \( \Delta_i = \Delta V + 3T_0/2 \approx 137 \text{meV} \).

**Appendix B. Nondegenerate electron-hole system**

When \( |\mu_e|, |\mu_h| < T \), the electron-hole system in the GL is non-degenerate, so that

\[
\Sigma_e \approx \frac{2T^2}{\pi \hbar^2 v_e^W} \exp \left( \frac{\mu_e}{T} \right), \quad \Sigma_h \approx \frac{2T^2}{\pi \hbar^2 v_h^W} \exp \left( \frac{\mu_h}{T} \right).
\] (D1)

As a result, taking into account Eq. (8), instead of Eq. (9) we obtain

\[
\mu_e - \mu_h \approx T_0 \frac{D}{2J_G} j,
\] (D2)

\[
\mu_e + \mu_h \approx T_0 \left[ 1 - \left( \frac{\Delta_i}{\hbar \omega_0} - 1 \right) \frac{1}{a} j \right],
\] (D3)

At \( V = 0.1 \text{V} \), \( N_a = 5 \times 10^{15} \text{cm}^{-3} \), \( d = 1.0 \mu\text{m} \), \( \kappa = 6 \) one obtains \( j \approx 1.6 \times (10^3 - 10^4) \text{A/cm}^2 \). This yields, \( (\mu_e + \mu_h) / T \approx 2.3 - 4.6 \) and \( (\mu_e - \mu_h) / T \ll 1 \).

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