The elementary interactions in the extraordinary optical transmission phenomenon

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Abstract. The electromagnetic interaction between nano-objects on the metal surface is driven by two different waves: the SPP mode and a quasi-cylindrical wave that persists on the surface over a few wavelength propagation distance. We discuss the impact of these two waves in the extraordinary transmission.

1. Introduction
The extraordinary light transmission (EOT) through metallic films perforated by nanohole arrays¹ has been first observed at optical frequencies. This observation has enabled many experimental and theoretical works on the transmission of electromagnetic waves through nano-apertures to understand the physical mechanisms responsible for the transmission.

When Thomas Ebbesen and his coworkers first reported on the EOT [1] they immediately suggested the involvement of SPPs. However, this explanation was not enough supported to be fully convincing. For instance similar EOTs have been reported in the long wavelength regime, where the involvement of SPPs is difficult to admit. As a consequence, several authors have challenged this interpretation [2,3].

Until now, the most advanced theoretical models [4-6] on the EOT fundamentally rely on classical grating theories and use modal expansions above the grating (Rayleigh plane waves) and inside the grating (hole modes). This well-accepted approach that have been used since the 60’s for metallic inductive grids for instance provides a macroscopic description of the phenomenon: its main prediction is the existence of a collective electromagnetic oscillation (often called the SPP Bloch-mode of the dressed interface) of the metal interface perforated by a 2D array of infinitely deep holes.

2. SPP-model
In this work, we rather provide a microscopic description. For that purpose, we abandon the classical mode-expansion approach, and we consider the elementary scattering events occurring among the individual 1D hole chain of the 2D array. These events are shown in Figs. 1a-1c for non-conical diffraction geometries (the \( y \)-component \( k_y \) of the in-plane wavevector momentum is null). Upon
interaction with the chain, the SPP-modes are partly excited, transmitted, reflected or scattered into the chain and into a continuum of outgoing plane waves. The interaction defines six elementary scattering coefficients that we have calculated by using a fully-vectorial method [7]. Two of the six coefficients, namely the SPP modal reflectance and transmittance coefficients $\rho$ and $\tau$, play a central role in the analysis. Their spectra are shown in Fig. 1e for near-infrared frequencies. For the computation, we have considered a real lossy metal, gold with a frequency-dependent permittivity $\varepsilon$ given by the tabulated data in the Palik’s book. It is worth emphasizing that all the six elementary scattering coefficients exhibit a smooth spectral behavior, only their associations through multiple scattering may result in anomalies or resonances.

Figure 1. Elementary processes involved in the EOT. They are all associated to the scattering of an electromagnetic field by a 1D hole chain under illumination by a the SPP mode, b the fundamental Bloch mode of the hole chain, and c an incident TM-polarized plane wave impinging with an oblique incidence defined by its in-plane wave-vector component $k_x$. The red and green arrows refer to the incident and scattered modes, respectively. The processes in a, b and c define six independent elementary scattering coefficients, $\rho$ (reflectance coefficient of the SPP mode), $\tau$ (transmittance coefficient of the SPP mode), $\alpha$ (scattering coefficient from the SPP mode to the fundamental Bloch mode), $\beta(k_x)$ (scattering coefficient from the SPP mode to the outgoing plane wave with an in-plane wave-vector component $k_x$), $t(k_x)$ (scattering coefficient from the fundamental Bloch mode to the plane wave) and $\rho_u$ (reflectance coefficient of the fundamental Bloch mode). d The transmittance and reflectance coefficients, $t_u(k_x)$ and $\rho_u(k_x)$, of the fundamental Bloch mode supported by the 2D gold membrane. e Spectra for the elastic scattering coefficients, $\rho$ and $\tau$, calculated at optical frequencies for a period $a=940$ nm and for square holes with a side length of $D=266$ nm.

Through a SPP coupled-mode model that coherently gathers these elementary processes, closed-form expressions for all the transmission spectrum characteristics, like the resonance wavelength, the peak transmission and the anti-resonance, can be derived. For normal incidence ($k_x=0$), the transmittance and reflectance coefficients, $t_u$ and $\rho_u$, of the fundamental Bloch mode supported by the 2D gold membrane (see Fig. 1d) can be written [8]

$$t_u(k_x = 0) = t + \frac{2\alpha \beta}{u^4 - (\rho + \tau)}$$  \hspace{1cm} (1)

$$\rho_u(k_x = 0) = r + \frac{2\alpha^2}{u^4 - (\rho + \tau)}$$  \hspace{1cm} (2)
where \( t \) and \( \beta \) denote \( t(k_x=0) \) and \( \beta(k_x=0) \), respectively. In the above equations, \( u = \exp(ik_{SP}a) \) is the phase delay accumulated by the SPP over a grating period, \( k_{SP} = k_0[\epsilon_g/(\epsilon_g+1)]^{1/2} \) being the complex propagation constant of a SPP on a flat gold-air interface. Equations (1) and (2) are very simple and their physical meaning will be discussed later on. From \( t_A(k_x) \) and \( r_A(k_x) \), the zeroth-order transmittance coefficient \( t_F(k_x) \) of the membrane is straightforwardly obtained by the well-known Fabry-Perot formula

\[
t_F(k_x) = \frac{t_A^2(k_x) \exp(ik_0nd)}{1 - r_A^2(k_x) \exp(i2k_0nd)}
\]

where \( n \) is the complex effective index of the fundamental Bloch mode of the 2D hole array, and \( d \) is the film thickness. The set of Eqs. (1)-(3), which solely involves SPP-modes as the mediator for the interaction, provides closed-form expressions for the EOT.

### 3. SPP-model predictions

In order to evaluate the role of SPP in the EOT, it is indeed interesting to compare the SPP-model predictions with fully-vectorial computational data. This is performed in Fig. 2. The later summarizes the main results of the inter-comparison obtained for a 2D hole array in a self-suspended gold membrane in air under illumination by a plane wave at normal incidence \( (k_x=0) \) at near-infrared frequencies. The fully-vectorial computational results are obtained with a grating solver that relies on the rigorous-coupled-wave-analysis, RCWA [9]. In a log scale, Fig. 2a shows the \((0,0)\)-order transmittance spectrum \( T(\lambda) = |t_F|^2 \) for an incident angle of \( \theta = 5^\circ \). The same quantity is shown in Fig. 2b in a linear scale for normal incidence.

\[\text{Figure 2. Comparison between the SPP-model predictions and fully-vectorial (RCWA) computation data obtained for the EOT of a gold hole-array membrane in air. a (0,0)-order transmittance spectrum } T \text{ for } \theta = 5^\circ \text{ in a log scale to evidence the deep minima. b Transmittance spectrum at normal incidence for } \theta = 0^\circ. \text{ All the results are obtained for a periodicity } a = 0.94 \, \mu m, \text{ a hole size } D = 266 \, nm \text{ (hole filling fraction of 8%) and a membrane thickness } d = 0.2 \, \mu m. \text{ The red and green curves correspond to RCWA data and to SPP-model predictions, respectively.}\]
The SPP model quantitatively captures all the salient features of the EOT, like the existence of two transmission peaks at oblique incidence (Fig. 2a) and of a single one at normal incidence (Fig. 2b), the Fano-like shape of the resonance with very deep minima located on the high energy side of each resonance (Fig. 2a) and with a broadband lineshape of the EOT (Figs. 2a and 2b).

4. Quasi-cylindrical waves
Although it well predicts the main features of the EOT at near-infrared frequencies, the multiple-scattering SPP-model does not provide a complete microscopic picture. For instance, it accounts for only one half of the total transmitted energy at peak transmittance (Fig. 2b). To analyze this in more details, we have calculated the field scattered by a single hole chain on the metal interface under illumination by a normally incident plane wave. The real part of the $y$-component $H_y$ of the total magnetic field-vector is shown in Fig. 3a as a function of the in-plane $x$- and $y$-coordinates. We have extracted the SPP-mode contribution $\beta \exp(ik_{SPP}x)$ (Fig. 3b), and by difference from the total field, the remaining contribution (Fig. 3c). As expected [10, 11] the total nearby field scattered on the interface by the nano-hole chain is not a pure SPP-mode, but additionally incorporates a quasi-cylindrical wave (CW) that persists along the interface over several wavelength distances, with an amplitude damping approximately scaling as $(1/\lambda)^{1/2}$.

Figure 3. Surface waves generated on a gold surface by a single hole chain, illuminated by a normally-incident plane wave polarized along the $x$-axis with a unitary magnetic field at the gold surface. a 2D map of the real part of the total $H_x$-component field scattered in the $x$-$y$ plane. A single period in the $y$-direction is shown. For clarity, the incident and specular-reflected plane waves have been removed. b and c Extracted SPP-mode and CW contributions. The computational results are obtained for an illumination at $\lambda=974$ nm.

The EOT is then understood as resulting from a surface-wave dynamical scattering process on the film interfaces. In agreement with recent theoretical [10] and experimental [11] works, the dynamics is shown to involve two very different fields, the classical (expected) SPP-mode of the flat interface and another field with a quasi-cylindrical behavior. The later creeps along the surface (like the SPP modes) and is absolutely necessary to fully understand the EOT: it is responsible for the fact that the pure SPP-model predicts only one half of the peak transmission in Fig. 2b, the other half being due to the CW.
5. Conclusion
At near-infrared frequencies, $\lambda \approx 1 \mu m$, it is found that the CW-field decay-rate is much faster than the SPP one, but at subwavelength hole distances like those encountered in 2D nano-hole arrays, the two waves almost equally contribute to build up the interaction between adjacent holes. It is therefore necessary to refer to a dual-wave picture to understand the EOT at those frequencies. At thermal infrared frequencies, the metal conductivity is higher and the SPP is only weakly excited by the holes. The multi-scattering process is then dominantly driven by the CW, which is equally excited at all energies. The physics is very similar to that recently discussed for slits [10].

We expect that the dual-wave picture will be useful for understanding other plasmonic phenomena.

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