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Some examples of global Poisson structures on $S^4$.

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The goal of this paper is to construct some concrete examples of global Poisson structures on the smooth manifold $S^4$, where a global Poisson structure is a bivector $w \in \Gamma(\wedge^2 TM)$ which vanishes under the Schouten bracket $[w, w] = 0$. This paper employs the twistor method to identify a subset of real Poisson structures on $S^4$ with the holomorphic Poisson structures on the complex manifold $\mathbb{C}P^3$. The holomorphic Poisson structures on $\mathbb{C}P^3$ has been completely classified [D. Cerveau and A. Lins Neto, Ann. Math. (2) 143, No. 3, 577–612 (1996, Zbl 0855.32015); F. Loray et al., Math. Nachr. 286, No. 8–9, 921–940 (2013, Zbl 1301.37032)], thus this allows for new Poisson structures on $S^4$ to be constructed.

The main technical result of this paper follows in two steps. First, the $g$-vectors on $\mathbb{C}P^3$ are characterized as pushforwards of $g$-vectors on $\mathbb{C}^4 \setminus \{0\}$ and the space of holomorphic Poisson structures are given an explicit description as a complex space with a real structure. In the second step, $S^4$ is identified with $\mathbb{H}P^1$ (where $\mathbb{H}$ is Hamilton’s quaternions), and a subspace of real Poisson structure on $\mathbb{H}P^1$ is identified with the real part of the space of holomorphic Poisson structures on $\mathbb{C}P^3$.

In Section 5, these results are generalized to the higher-dimensional cases of $\mathbb{C}P^n$ and $\mathbb{H}P^m$. In Section 6, a Poisson structures on $S^4$ is induced by a foliation of codimension-1 of degree 2 on $\mathbb{C}P^3$ for each of the six disconnected components of the space of such foliations.

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37F35 Conformal densities and Hausdorff dimension for holomorphic dynamical systems

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