Statistical CSI-Based Transmission Design for Reconfigurable Intelligent Surface-aided Massive MIMO Systems with Hardware Impairments

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Abstract—We consider a reconfigurable intelligent surface (RIS)-aided massive multi-user multiple-input multiple-output (MIMO) communication system with transceiver hardware impairments (HWIs) and RIS phase noise. Different from the existing contributions, the phase shifts of the RIS are designed based on the long-term angle informations. Firstly, an approximate analytical expression of the uplink achievable rate is derived. Then, we use genetic algorithm (GA) to maximize the sum rate and the minimum date rate. Finally, we show that it is crucial to take HWIs into account when designing the phase shift of RIS.

Index Terms—Reconfigurable intelligent surface (RIS), hardware impairments (HWIs), statistical CSI, intelligent reflection surface (IRS).

I. INTRODUCTION

With the rapid development of wireless communication technology, reconfigurable intelligent surface (RIS) has been recognized as a revolutionary technology for future wireless communication systems [1]. An RIS consists of passively reflecting elements and each element can independently induce certain phase shift changes to the incoming signal [2], [3]. RIS-aided communication systems have been extensively studied in [4], [5]. The authors in [4] investigated an RIS-assisted massive multiple-input multiple-output (MIMO) system, derived a closed-form expression of the rate. In [5], the performance of RIS-aided MIMO systems with direct links was studied based on statistical channel state information (CSI).

Recently, the transceiver hardware impairments (HWIs) were considered in [6]–[9]. Specifically, in [6], the authors focused on an RIS-assisted multi-antenna communication system with transceiver HWIs, and designed the transmit and reflecting beamforming. In [7], an RIS-aided communication system was studied based on the imperfect hardware, where the authors derived the spectral efficiency by considering imperfect CSI and presented a general methodology for the RIS’s reflecting beamforming (RB) optimization. In [8], the authors derived the optimal receive combining and transmit beamforming vectors, and provided the analytical upper and lower bounds on the maximal energy efficiency. In [9], the authors derived the closed-form expression of the average achievable rate with HWIs.

However, the HWIs under the scenario of RIS-aided massive MIMO have not been investigated. Against this background, we consider the uplink transmission of an RIS-aided massive MIMO system based on the statistical CSI, where transceiver has additive HWIs and RIS has phase noise. We consider the availability of statistical CSI because it changes more slowly than instantaneous CSI and it can also significantly relax the necessity of frequently reconfiguring the RISs [10], [11]. Besides, by increasing the element spacing of RIS, we can avoid spatial correlation. Specifically, an approximate analytical expression of the rate is derived. Then, to maximize the rate, GA is used to optimize the phase shifts. Finally, we show that it is crucial to take HWIs into account when designing the phase shift of RIS.

Notations: \( || \cdot || \) stands for the \( l_2 \) norm of a vector. \( \mathbf{x} \in \mathcal{C} \mathcal{N}(\mathbf{a}, \mathbf{\Sigma}) \) denotes that \( \mathbf{x} \) is a complex Gaussian random vector with mean \( \mathbf{a} \) and covariance matrix \( \mathbf{\Sigma} \). \( \text{diag}\{\cdot\} \) denotes a diagonal matrix whose diagonal elements are the same as the original matrix.

II. SYSTEM MODEL

![Fig. 1. A typical uplink RIS-aided MIMO system with direct links](image)

As depicted in Fig. 1, a typical uplink RIS-aided multi-user (MU) massive MIMO communication system with direct links is considered, where a BS is equipped with \( M \) antennas and the RIS is composed of \( N \) reflecting elements. The uniform square planar array (USPA) is adopted at both the BS and the RIS. \( K \) single-antenna users communicate with BS via the RIS, and the phase shift matrix of RIS is given by

\[
\Theta = \text{diag}\{e^{j\theta_1}, \ldots, e^{j\theta_K}, \ldots, e^{j\theta_N}\},
\]
where \( \theta_n \in [0, 2\pi) \) represents the \( n \)-th reflecting element of RIS.

Considering the imperfection of RIS, we assume that the phase noise at RIS can be written as \( \theta_i \in \mathcal{U}[-k_r \pi, k_r \pi] \) [12], where \( \mathcal{U} \) denotes the uniform distribution and \( k_r \) measures the severity of the residual impairments at the RIS. Therefore, the phase shift matrix of the RIS with phase noise can be expressed as

\[
\Theta = \text{diag} \left\{ e^{j(\theta_1 + \vartheta_1)}, \ldots, e^{j(\theta_n + \vartheta_n)}, \ldots, e^{j(\theta_N + \vartheta_N)} \right\}. \tag{2}
\]

We consider the Rician fading model for the RIS-related channel links. Specifically, the channel between the RIS and the BS is denoted by \( \mathbf{H}_{rb} \in \mathbb{C}^{M \times N} \), the channel between user \( k \) and the RIS is denoted by \( \mathbf{h}_k \in \mathbb{C}^{N \times 1} \), and the expressions of \( \mathbf{H}_{rb} \) and \( \mathbf{h}_k \) are respectively given by

\[
\mathbf{H}_{rb} = \sqrt{\mu} \left( \sqrt{\frac{1}{\rho + 1}} \mathbf{p}_{r} \mathbf{a}_{M}^{H} + \sqrt{\frac{1}{\rho + 1}} \mathbf{h}_{rb} \right), \tag{3}
\]

\[
\mathbf{h}_k = \sqrt{\mu_k} \left( \sqrt{\frac{\epsilon_k}{1 + \epsilon_k}} \mathbf{r}_{k} \mathbf{a}_{N} + \sqrt{\frac{1}{1 + \epsilon_k}} \mathbf{h}_k \right), \tag{4}
\]

where \( \mu \) and \( \mu_k \) denote the large-scale fading coefficients, \( \rho \) and \( \epsilon_k \) are Rician factors. \( \mathbf{H}_{rb} \) and \( \mathbf{h}_k \) are respectively given by (12), (13), (14) and (15). The transmit distortion is denoted by \( \mathbf{z} \), where \( \mathbf{z} \) models the joint effects of the non-linearities in power amplifier and digital-to-analog converters, the power amplifier noise and oscillator phase noise [6].

Based on the above definitions, the received signal can be expressed as

\[
\mathbf{y} = \mathbf{G}(\mathbf{P} \mathbf{x} + \mathbf{z}) + \mathbf{z}_r + \mathbf{n}, \tag{9}
\]

where \( \mathbf{P} = \text{diag}(\sqrt{\mu_1}, \ldots, \sqrt{\mu_K}) \) represents transmit power of the corresponding users, \( \mathbf{x} = [x_1, \ldots, x_K]^{T} \) represents the signal vector of users, and \( \mathbb{E}\{ |x_k|^2 \} = 1 \). \( \mathbf{n} \in \mathcal{C}\mathcal{N}(0, \sigma^2 \mathbf{I}_N) \) is the Additive white Gaussian noise.

### III. Analysis Of Uplink Achievable Rate

To reduce the computational and implementation complexity, the low-complexity maximal ratio combining (MRC) technique with receiver matrix \( \mathbf{G}^{H} \) is employed. Therefore, the received signal of the \( k \)-th user at the BS can be expressed as

\[
r_k = g_k^{H} \mathbf{y} = g_k^{H} \left( \sum_{i=1}^{K} g_i (\sqrt{\mu_i} x_i + z_{i,k}) + z_r + \mathbf{n} \right)
\]

\[
= g_k^{H} g_k \sqrt{\mu_k} x_k + \sum_{i=1, i \neq k}^{K} g_k^{H} g_i \sqrt{\mu_i} x_i + \sum_{i=1}^{K} g_k^{H} z_{i,k} + g_k^{H} z_r + g_k^{H} \mathbf{n}.
\]

By using [13, Lemma1], the uplink ergodic rate of user \( k \) can be approximated as

\[
R_k = \log_2 \left( 1 + \frac{p_k \mathbb{E}_{\text{signal}}(\mathbf{z})}{\sum_{i=1, i \neq k}^{K} p_i \mathbb{E}_{\text{interf}}(\mathbf{z}) + 2\sigma^2 \mathbb{E}_{\text{noise}}(\mathbf{z})} \right), \tag{11}
\]

where \( \mathbb{E}_{\text{signal}}(\mathbf{z}) = \mathbb{E}\{ |g_k|^4 \} \), \( \mathbb{E}_{\text{noise}}(\mathbf{z}) = \mathbb{E}\{ |g_k|^2 \} \), \( \mathbb{E}_{\text{interf}}(\mathbf{z}) = \mathbb{E}\{ |g_k g_i|^2 \} \), \( \mathbb{E}_{\text{noise}}(\mathbf{z}) \) and \( \mathbb{E}_{\text{interf}}(\mathbf{z}) \) are respectively given by (12), (13), (14) and (15). The derivations of the first three terms can be found in [5] and the fourth term is proved in Appendix A. Besides, \( a_k \triangleq \frac{\mu_k}{(\rho+1)\epsilon_k + 1} g_k \) is the \( m \)-th element of \( \mathbf{g}_k \), \( f_k(X) \triangleq a_k^{H}(\phi_{rb}^{(m)}, \phi_{rb}^{(r)}) X \).

We assume that the transmit power is scaled with the number of antennas according to \( p_k = p/M \), \( M \rightarrow \infty \), such that \( p \) is a fixed value. For simplicity, we set \( \rho = \epsilon_k = 0 \), i.e., only NLoS paths exist in the environment, then we have

\[
R_k \rightarrow \log_2 \left( 1 + \frac{p_1 A_1}{\sum_{i=1, i \neq k}^{K} A_2 + \sum_{i=1}^{K} A_3 + A_3 \sigma^2} \right), \tag{17}
\]

where \( A_1 = \nu_{u_k} N (\nu_{u_k} N + \nu_{u_k} + 2\xi_k) + \xi_k \), \( A_2 = \nu_{u_k} \mu_k N \), \( A_3 = \mu_k N + 2\xi_k \).

From equation (17), we can find that users in RIS-aided systems with imperfect hardware can scale down their transmit power by a factor of \( 1/M \) while the data rate will converge to a non-zero value as \( M \rightarrow \infty \).
\[
\mathbb{E}_{\text{signal}}(\Theta) = M^2a_k^2\rho^2\epsilon_c^2c^2 + 2a_kM\rho\epsilon_kc_k(\xi_k(M+1) + a_k(2M + 2MN\rho + MN + MN\epsilon_k + N + N\epsilon_k + 2)) + M^2\rho^2\epsilon_c^2 + 2\rho\epsilon_k + 2\rho\epsilon_k + 2\rho + 2\epsilon_k + 1) + a_k^2MN^2(\epsilon_c^2 + 2\rho\epsilon_k + 2\rho + 2\epsilon_k + 1)
\]
\[
\mathbb{E}_{\text{inter}}(\Theta) = M^2a_k\rho\epsilon_c\epsilon_kc_k + M\rho\epsilon_kc_k(a_k(\rho MN + N\epsilon_k + N + 2M) + \xi) + M\rho\epsilon_kc_k(a_k(\rho MN + N\epsilon_k + N + 2M) + \xi) + M^2a_k\rho\epsilon_k(\rho\epsilon_k + \epsilon_k + 2) + (\epsilon_k + 1) + 2M^2\rho\epsilon_k\epsilon_k\epsilon_k(\sin^2(k_\pi)R)\Re\{f^H_k(\Theta) f_l(\Theta) f^H_k h^H_l\} + (1 - \sin^2(k_\pi))N) + M(a_\epsilon\xi_kN(\rho + \epsilon_k + 1) + a_\epsilon\xi_kN(\rho + \epsilon_k + 1) + \xi_\epsilon_k).
\]
\[
\mathbb{E}_{\text{noise}}(\Theta) = M(a_\epsilon\rho\epsilon_kc_k + a_\epsilon\rho\epsilon_kN(\rho + \epsilon_k + 1) + \xi_\epsilon_k) + a_\epsilon\rho\epsilon_kN(\rho + \epsilon_k + 1) + \xi_\epsilon_k).
\]\
where \(c_k \triangleq (\sin^2(k_\pi) | f_k(\Theta)|^2 + (1 - \sin^2(k_\pi)|f_l(\Theta)|^2)^2 + (1 - \sin^2(k_\pi)N).
\]
\[
\mathbb{E}_{\text{wri}}(\Theta) = k_\lambda \left( \sum_{i=1,i\neq k}^{K} \mathbb{E}_{\text{inter}}(\Theta) + p_k\mathbb{E}_{\text{signal}}(\Theta) \right) + (1 + k_\lambda) k_\lambda \left( \sum_{i=1}^{K} p_i \sum_{m=1}^{M} |g_{i,m}|^2 |g_{k,m}|^2 \right),
\]
where \(|g_{i,m}|^2 |g_{k,m}|^2\) can be expressed as
\[
|g_{i,m}|^2 |g_{k,m}|^2 = (\sin^2(k_\pi)|f_k(\Theta)|^2 + (1 - \sin^2(k_\pi)|f_l(\Theta)|^2)^2 + (1 - \sin^2(k_\pi)N).
\]

\[\max_{\Theta} \sum_{k=1}^{K} R_k, \quad \text{s.t.} \quad \theta_n \in [0, 2\pi), \forall n.\]
\[\max_{\Theta} \min_{k} R_k, \quad \text{s.t.} \quad \theta_n \in [0, 2\pi), \forall n.\]

where \(R_k\) is given in (11).

Due to the complicated data rate expression, conventional optimization techniques are not applicable. To address this problem, we adopt GA. We need to discretize the angle, take the phase as the chromosome and design the objective function as the fitness function. The detailed steps of which are given in Algorithm 1. Specifically, we evaluate the fitness of individuals in each generation. Those with high fitness are retained as elites to the next generation, those with low fitness experience mutation operation to generate offspring, and those with medium fitness are used to generate parents, and then cross-parents to generate offspring. The complexity of the algorithm is proportional to \(qSMN^2\), where \(q\) is the number of iterations, \(S\) is the population size, \(M\) is the number of BS antennas, \(N\) is the number of reflecting elements [14].

IV. PHASE SHIFT OPTIMIZATION

In this section, we aim to optimize the phase shifts of RIS to maximize the sum rate and minimum user rate. Mathematically, the optimization problems can be formulated as follows

Algorithm 1 GA

1: Initialization: generate a population of \(S = S_c + S_m + S_p\) individuals and the \(i\)-th individual has a randomly generated chromosome \(\Theta\); the iteration number \(q = 1\);
2: \textbf{while} \(q \leq N \ast 100\) \textbf{do}
3: Fitness evaluation: Calculate the fitness of each individual by using the objective function in (18) or (20) and sort them in a descending order;
4: Selection: Based on the descending order, select the top \(S_c\) individuals as elites;
5: Mutation: Create \(S_m\) offspring from the last \(S_m\) individuals by using uniform mutation [4];
6: Crossover: Use stochastic universal sampling [4] to generate \(2S_p\) parents from the remaining \(S_p\) individuals. Then use two-points crossover [4] to create \(S_p\) offspring from \(2S_p\) parents;
7: Combine \(S_c\) elites, \(S_m + S_p\) offspring to form the next generation population; \(q = q + 1\);
8: \textbf{end while}
9: Output the chromosome of the most fit individual in the current population.

V. SIMULATIONS RESULTS

In this section, extensive simulation results are provided to validate the accuracy of our analytical expression. Unless otherwise stated, the simulation parameters are set as follows [10]:

the number of antennas of \(M = 50\), the number of reflecting elements of \(N = 25\), HWI coefficients \(k_c = k_u = k_b = 0.08\), \(\sigma^2 = -104\ \text{dBm}, p_k = 30\ \text{dBm}, \epsilon_k = 1, \forall k, \rho = 10, K = 4\), RIS-BS distance is denoted by \(l_{rb} = 1000\ \text{m}\). We assume that users are distributed on a semicircle with RIS as the center and a radius of 20 m. Therefore, user-RIS distance is denoted by \(l_{ur} = 20\ \text{m}\), the distance between user \(k\) and BS is denoted by \(l_{ub}^k\) and \(l_{ub}^k = 988\ \text{m}\), \(l_{ub}^k = l_{ub}^k = 980\ \text{m}\). The large-scale fading coefficients are \(\mu_k = 10^{-3}s^{-1/2}, v = 10^{-3}t^{-2.5}, \epsilon_k = 10^{-3}(l_{ub}^k)^{-4}, \forall k\). The AoA and the AoD are
Fig. 2. Rate versus $N$.

Fig. 3. Rate versus $M$.

Fig. 4. Rate versus the HWI coefficient.

all random values within $[0, 2\pi]$.

Fig. 2 depicts the max-sum rate and max-minimum user rate. The simulation shows that the Monte-Carlo (MC) simulation results are consistent with the derived results, which verify the correctness of the derived expression.

In Fig. 3, we compare the system performance in different scenarios. Obviously, with the increase of $M$, the performance gain of optimal phase shifts will become more and more prominent, which demonstrates the superiority of using GA to optimize the phase shifts in massive MIMO systems.

Fig. 4 shows the significance of investigating HWIs. In Fig. 4, one of the curves considers HWIs, while the other neglects HWIs to optimize the phase shift and substitute the obtained phase shift solution into the actual system with HWIs. $k_r, k_u, k_b = k_{hw}$). The simulation shows that with the increase of HWI coefficients, the performance gap between the two schemes becomes larger.

VI. CONCLUSION

We investigated an RIS-aided MU massive MIMO system with HWIs. The approximate analytical expression of uplink achievable rate has been derived based on Rician fading channel and MRC technique. Through MC simulation, we verified the accuracy of analytical expression. In addition, we showed that it is crucial to take HWIs into account when designing the phase shift of RIS. We will consider the spatial correlation at the RIS and the transmit power allocation in our future work.

APPENDIX A

DERIVATIONS OF $\mathbb{E}_{\text{HWI}}^k(\Theta)$

\[
\mathbb{E}\left\{|\mathbf{g}_{k}^H \mathbf{z}|^2\right\} = \alpha k_0(1 + k_u) \sum_{i=1}^{K} \sum_{m=1}^{M} \mathbb{E}\left\{|g_{im}|^2|g_{km}|^2\right\},
\]

where $(\alpha)$ is obtained by removing the zero terms.

Then, we have

\[
\sum_{i=1}^{K} \sum_{m=1}^{M} \mathbb{E}\left\{|g_{im}|^2|g_{km}|^2\right\} = p_k \sum_{m=1}^{M} \mathbb{E}\left\{|g_{km}|^4\right\} + \sum_{i=1}^{M} \sum_{m=1}^{M} \mathbb{E}\left\{|g_{im}|^2|g_{km}|^2\right\}.
\]

Let $\mathbf{v}_k = \mathbf{H}_k \mathbf{\Theta}_k$, we can rewrite $\mathbf{v}_{km}$ in the form of (24) at the top of the next page, where $\mathbf{v}_{km}$ and $\mathbf{a}_m(\psi_{rb}^{\text{opt}}, \psi_{rb}^{\text{opt}})$ are the $m$-th element of respective vectors, $[\mathbf{H}_{rb}]_{mn}$ denotes the $(m, n)$-th entry of matrix $\mathbf{H}_{rb}$, $h_{km}$ represents the $m$-th element of $\mathbf{h}_k$.

A. Derivations of $\mathbb{E}\{ |g_{km}|^4 \}$

We have

\[
\mathbb{E}\{ |g_{km}|^4 \} = 2\xi_k^2 + \mathbb{E}\{ |v_{km}|^4 \} + 2\xi_k \mathbb{E}\{ |v_{km}|^2 \} + 2\xi_k \mathbb{E}\{ (v_{km})(d_{km}^*) \}^2 + 4\mathbb{E}\{ (\mathbb{Re}\{ (v_{km})(d_{km}^*) \})^2 \},
\]

where $\mathbb{E}\{ |v_{km}|^2 \} = a_k(\rho\epsilon_k c_k + \rho N + \epsilon_k N + N)$ can be easily obtained, $\mathbb{E}\{ |v_{km}|^4 \}$ was derived in [4] and $\mathbb{E}\{ (\mathbb{Re}\{ (v_{km})(d_{km}^*) \})^2 \}$ can be expressed as

\[
\mathbb{E}\{ (\mathbb{Re}\{ (v_{km})(d_{km}^*) \})^2 \} = \alpha_k \xi_k \left[ \mathbb{E}\{ (\mathbb{Re}\{ (v_{km}^1)(\tilde{d}_{km}^*) \})^2 \} + \mathbb{E}\{ (\mathbb{Re}\{ (v_{km}^2)(\tilde{d}_{km}^*) \})^2 \} ight].
\]

(27)

Assume $v_{km}^1 = s + j \tilde{d}_{km} = p + jq$, where $p \in \mathcal{CN}(0, \frac{1}{2})$, $q \in \mathcal{CN}(0, \frac{1}{2})$. Thus, we can derive

\[
\mathbb{E}\{ (\mathbb{Re}\{ (v_{km}^1)(\tilde{d}_{km}^*) \})^2 \} = \frac{1}{2} \rho \epsilon_k c_k.
\]

(28)

\[
\mathbb{E}\{ (\mathbb{Re}\{ (v_{km}^2)(\tilde{d}_{km}^*) \})^2 \} = \frac{1}{2} \rho N.
\]

(29)

Substituting (27) into (26), we complete the derivations of $\mathbb{E}\{ |g_{km}|^4 \}$.

B. Derivations of $\mathbb{E}\{ |g_{im}|^2|g_{km}|^2 \}$, $\forall i \neq k$

\[
\mathbb{E}\{ |g_{im}|^2|g_{km}|^2 \} = \mathbb{E}\{ |d_{im}|^2|d_{km}|^2 \} + \mathbb{E}\{ |d_{im}|^2|v_{km}|^2 \} + \mathbb{E}\{ |d_{km}|^2|v_{im}|^2 \} + \mathbb{E}\{ |v_{km}|^2|v_{im}|^2 \},
\]

where $(\beta)$ is obtained by removing the zero terms.

We can readily obtain

\[
\mathbb{E}\{ |d_{im}|^2|d_{km}|^2 \} = \xi_i \xi_k \mathbb{E}\{ |v_{km}|^2 \},
\]

\[
\mathbb{E}\{ |d_{im}|^2|v_{km}|^2 \} = \xi_i \mathbb{E}\{ |v_{im}|^2 \},
\]

\[
\mathbb{E}\{ |d_{km}|^2|v_{im}|^2 \} = \xi_k \mathbb{E}\{ |v_{im}|^2 \}.
\]
\[ v_{km} = \sqrt{\frac{P_{\text{bk}}}{(\rho + 1)(\epsilon_k + 1)}} \left( \sqrt{\rho \epsilon_k a_{Mm} (\psi^a_{rb}, \psi^e_{rb}) f_k(\Theta)} + \sqrt{\rho a_{Mm} (\psi^a_{rb}, \psi^e_{rb})} \sum_{n=1}^{N} a^*_{Nn} \left( \phi^a_{rb} \phi^e_{rb} \right) e^{j(\theta_n + \theta_n)} \tilde{h}_{kn} \right) \]
\[ + \sqrt{\epsilon_k} \sum_{n=1}^{N} \left( \tilde{H}_{rb} \right)_{mn} e^{j(\theta_n + \theta_n)} a_N \left( \psi^a_{kr} \psi^e_{kr} \right) + \sum_{n=1}^{N} \left( \tilde{H}_{rb} \right)_{mn} e^{j(\theta_n + \theta_n)} \tilde{h}_{kn} \] 
\[ v_{km}^2 = \sum_{n=1}^{N} \sum_{m=1}^{4} \left( \tilde{H}_{rb} \right)_{mn} e^{j(\theta_n + \theta_n)} \tilde{h}_{kn} \] 

\[ \mathbb{E} \left[ |v_{im}|^2 |v_{km}|^2 \right] = 4 \rho \epsilon_k \sum_{\omega=1}^{4} \mathbb{E} \left[ \text{Re} \left( v_{km}^4 \right) \text{Re} \left( v_{im}^4 \right) \right] + \mathbb{E} \left[ \text{Re} \left( v_{km}^2 \right) \text{Re} \left( v_{im}^2 \right) \right] \] 
\[ + \mathbb{E} \left[ \text{Re} \left( v_{km}^2 \right) \text{Re} \left( v_{im}^4 \right) \right] + \mathbb{E} \left[ \text{Re} \left( v_{km}^2 \right) \text{Re} \left( v_{im}^2 \right) \right] \]

where \((\gamma)\) is obtained by removing the zero terms.

The expression of \( \mathbb{E} \left[ |v_{im}|^2 |v_{km}|^2 \right] \) is given by (30) at the top of this page and we will calculate the terms in (30).

The first one is
\[ \mathbb{E} \left[ \sum_{\omega=1}^{4} \left| v_{km}^\omega \right|^2 \sum_{\omega=1}^{4} \left| v_{im}^{\omega} \right|^2 \right] = \rho \epsilon_k \rho_c \epsilon_c (\rho_c \epsilon_c + \rho + \epsilon_i N + \epsilon_i N + N + \rho N (\rho_c \epsilon_c + \rho N + \epsilon_i (N + 1) + N + N (\rho_c \epsilon_c + \rho N + \epsilon_i (N + 1) + N + 1)). \]

Assume that
\[ a_{Mm} \left( \psi^a_{rb}, \psi^e_{rb} \right) f_k(\Theta) e^{j(\theta_n + \theta_n)} a^*_N \left( \psi^a_{ir}, \psi^e_{ir} \right) = \sigma^k_{mn} + j\sigma^s_{mn}, \]
\[ a_{Mm} \left( \psi^a_{rb}, \psi^e_{rb} \right) f_k(\Theta) e^{j(\theta_n + \theta_n)} a^*_N \left( \psi^a_{ir}, \psi^e_{ir} \right) = \sigma^c_{mn} + j\sigma^d_{mn}, \]
\[ \mathbb{E} \left[ \text{Re} \left( v_{km}^4 \right) \text{Re} \left( v_{im}^4 \right) \right] = \rho \epsilon_k \rho_c \epsilon_c \sum_{n=1}^{N} \left( \sigma^k_{mn} \sigma^k_{mn} s^2_{mn} + \sigma^s_{mn} \sigma^s_{mn} s^2_{mn} \right). \]

Thus, the second one is
\[ \mathbb{E} \left[ \text{Re} \left( v_{km}^4 \right) \text{Re} \left( v_{im}^4 \right) \right] = \frac{\rho \epsilon_k \rho_c }{2} \left( \sin^2(k_r \pi) \text{Re} \left( f_k(\Theta) \tilde{h}_{m} \tilde{h}_{i} f_k^*(\Theta) \right) \right) \]

Likewise, we have
\[ \mathbb{E} \left[ \text{Re} \left( v_{km}^{2} \right) \text{Re} \left( v_{im}^{2} \right) \right] = \frac{\rho \epsilon_k }{2} c_k, \]
\[ \mathbb{E} \left[ \text{Re} \left( v_{km}^{2} \right) \text{Re} \left( v_{im}^{2} \right) \right] = \frac{\rho \epsilon_i }{2} c_i, \]
\[ \mathbb{E} \left[ \text{Re} \left( v_{km}^{2} \right) \text{Re} \left( v_{im}^{2} \right) \right] = \frac{\rho N }{2}. \]

Substituting (31), (33), (34), (35) and (36) into (30), we complete the derivations of \( \mathbb{E} \left[ |g_{km}|^2 |g_{km}|^2 \right] \) and \( \mathbb{E} \left[ |g_{km}|^4 \right] \) into (16), we complete the proof of \( \mathbb{E}_{\tilde{h}_{km}}(\Theta) \).