Electric Vehicle Battery Sharing Game for Mobile Energy Storage Provision in Power Networks

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Abstract—Electric vehicles (EVs) equipped with a bidirectional charger can provide valuable grid services as mobile energy storage, under the ambit of vehicle to grid (V2G) service provision. However, proper financial incentives need to be in place to enlist EV drivers to provide services to the grid. In this paper, we consider two types of EV drivers who may be willing to provide mobile storage service using their EVs: commuters taking a fixed route, and on-demand EV drivers who receive incentives from a transportation network company (TNC) and are willing to take any route. We model the behavior of each type of driver using game theoretic methods, and characterize the Nash equilibrium (NE) of an EV battery sharing game where each EV driver withdraws power from the grid to charge its EV battery at the origin of a route, travels from the origin to the destination, and then discharges power back to the grid at the destination of the route. The driver earns a payoff that depends on the participation of other drivers and power network conditions. We characterize the NE in three situations: when there are only commuters, when there are only on-demand TNC drivers, and when the two groups of drivers co-exist. In particular, we show that the equilibrium outcome supports the social welfare in each of these three cases.

I. INTRODUCTION

Renewable generation is fast becoming the cheapest generation alternative, and scaling up variable renewable generation will require investment in electricity storage which can accommodate the uncertain and uncontrollable output. This trend is accompanied by a series of mandates in states like California and Massachusetts, which has spurred investment in utility scale battery projects [1]. At the same time, there is a push to ‘electrify everything’, i.e., move consumption from non-electric energy to electricity, which can then be supplied from clean, carbon-free resources like solar and wind.

Transportation causes 29% of global CO₂ emissions [2], and electrifying transport is an important step in any climate change mitigation plan. While transport sectors like long-haul trucking, shipping and aviation are hard to electrify, the passenger vehicle sector has seen rapid electrification in recent years. EVs are equipped with batteries which can also be used to function as mobile energy storage in the power network with the help of bidirectional chargers. EVs can move energy across both space and time by charging at the origin of a route and discharging at the destination. Such mobile energy storage in the form of EV batteries can help avoid time-consuming and expensive transmission line upgrades, and also serve as energy storage on the grid [3].

However, unlike utility scale battery projects that can be dispatched by the power system operator, these EVs are owned and operated by individual drivers. EV drivers will make independent decisions on whether to provide mobile storage service based on their individual costs and the value they create, which will be determined by the operation of the power network and on how much mobile storage capacity is available in the grid. They may have different motivations: some EV drivers may function as commuters, i.e. travel along fixed routes, and some EV drivers could be available on-demand to travel along specific routes to provide mobile storage service. A fundamental question arises: will the market equilibrium lead to a socially desirable level of mobile storage capacity?

This paper examines this question in three contexts: a) when there are a number of commuter EVs traveling along fixed routes in the power network which can provide mobile storage service along those routes, b) when there are a number of on-demand EV drivers which can provide mobile storage service along any route, and c) when there is a mix of commuter EVs and on-demand EVs in the network. The mobile storage service is provided in a wholesale market, and a transmission-constrained two-period economic dispatch problem is solved to determine the operation of the grid and mobile storage. This also determines the locational marginal prices, which are used to compensate mobile storage service providers. We make the following contributions to the literature: a) we develop novel game theoretic models in the context of sharing EV batteries as mobile energy storage, which incorporate both operation constraints of the power network and incentives for the EV drivers; b) we explicitly characterize the Nash Equilibrium (NE) of the proposed EV battery sharing games together with several benchmarks, and establish that the NE supports social welfare for all our settings. Proofs are presented in the extended version [4].

Our work is built upon two lines of recent research. The first is game theoretic analysis of storage sharing. Among many papers in this area, the closest related works include: [5], which studies the storage sharing and investment decisions of a collection of firms without considering network constraints, and [6], which analyzes the distributed storage investment game in power networks. [7] formulates a cooperative game for sharing energy storage within a residential microgrid, [8] formulates a Stackelberg game to model the sharing of cloud energy storage, and [9] studies a two-stage problem of a central storage owner sharing virtualized sections of the storage capacity with multiple users. The second line of research is the growing literature...
on utilizing EVs as mobile energy storage to provide grid services. See [10] for a cost-benefit analysis of the business model of sharing EVs to help commercial and industrial electricity users reduce their demand charges, [3] for the joint optimization of a power network and a fleet of mobile storage units, and [11] for a simulation study of the value of truck based mobile storage units in the California power grid.

II. MODEL

Consider a setting where EVs can provide mobile storage service in the power network using bidirectional chargers.

A. Power network

Consider a power network with \( n \) buses, indexed by \( i \in \mathcal{N} := \{1, \ldots, n\} \). For simplicity, we consider a daily operation setting and divide the day into two time periods: an off-peak period followed by a peak period, denoted by \( t \in \mathcal{T} := \{1, 2\} \). For each time period, we denote the generation and load over different buses in the power network by \( g_i(t), d_i(t) \in \mathbb{R}^n \) respectively. The convex generation cost function for the generator at bus \( i \) and time \( t \) is given by \( C_{i,t}(\cdot) \), and the total generation cost at time \( t \) is given by

\[
C_t(g_t) = \sum_{i=1}^{n} C_{i,t}(g_{i,t}), \quad t \in \mathcal{T}.
\]

The value of serving load at bus \( i \) and time \( t \) is given by the concave function \( B_{i,t}(\cdot) \), which quantifies the utility derived by the consumers who use the supplied electric energy. Let the total consumer utility be

\[
B_t(d_t) = \sum_{i=1}^{n} B_{i,t}(d_{i,t}), \quad t \in \mathcal{T}.
\]

The buses in the power network are connected by transmission lines, and the power flow along each line should not exceed the line capacity. Additionally, the total power injection into the network at any time should be zero. We model these linearized AC power flow constraints by

\[
P^t = 0, \quad \mathbb{H}P^t \leq \bar{f}, \quad t \in \mathcal{T},
\]

where \( P_t \in \mathbb{R}^n \) denotes the vector of power injections at each bus at time \( t \). Here \( \mathbb{H} \in \mathbb{R}^{m \times n} \) is the shift-factor matrix, where \( m \) is the number of transmission constraints and \( \bar{f} \) denotes the line capacities.

B. EVs as mobile storage in the power network

An EV can function as a mobile storage unit in the power network by charging at one bus during the off-peak period, moving to another bus and then discharging there during the peak period. In a power network with \( n \) buses, there are a total of \( n^2 \) routes that the EV can take, which include the “route” where the EV stays at the same location.

Consider an aggregate mobile storage capacity \( S_{i,j} \) moving from bus \( i \) at time \( t = 1 \) to bus \( j \) at time \( t = 2 \), which comprises of all of the EVs moving along that route and providing mobile energy storage service. The charging/discharging operation vector along route \( i \rightarrow j \) is given by \( u_{i,j} \in \mathbb{R}^2 \), where positive values of \( u_{i,j}(t) \) indicate charging and negative values indicate discharging. We assume that each EV starts with an empty battery at \( t = 1 \), which means that the aggregate state-of-charge at the end of time period \( t \in \mathcal{T} \) is given by \( \sum_{t=1}^{t} u_{i,j}(t) \), which is the sum of charging/discharging operations until that time. The state of charge must satisfy the capacity constraint of the aggregate storage capacity, i.e.,

\[
0 \leq \sum_{t=1}^{t} u_{i,j}(t) \leq S_{i,j}, \quad t \in \mathcal{T} \implies 0 \leq \mathbb{L}u_{i,j} \leq S_{i,j}1,
\]

where \( \mathbb{L} \in \mathbb{R}^{2 \times 2} \) is a lower triangular matrix with \( L_{t,t'} = 1 \) for all \( t \geq t' \). The vector of mobile storage capacities on all routes in the network is given by \( \mathbb{S} \in \mathbb{R}^n \), and has an element \( S_{i,j} \) corresponding to each route in the network. The power injection at each bus is the sum of generation and aggregate storage operation minus the demand, i.e.,

\[
p_i = g_i - \sum_{j=1}^{n} u_{i,j}(1), \quad p_j = g_j - \sum_{i=1}^{n} u_{i,j}(2),
\]

The storage operation \( u_{i,j}(1) \) at \( t = 1 \) occurs at the route origin (bus \( i \)), while the operation \( u_{i,j}(2) \) at \( t = 2 \) occurs at the route destination (bus \( j \)), which explains the asymmetrical definition of power injection.

C. Economic dispatch with mobile storage

In a centralized optimization setting, the system operator dispatches generation and storage to supply the flexible load at each bus. Given the aggregate mobile storage capacity moving along each route, the system operator solves the two-period economic dispatch problem to determine the operation of the grid and aggregate storage capacities:

\[
J(\mathbb{S}) = \min_{\mathbb{P},\mathbb{G},\mathbb{D},\mathbb{S}} \sum_{t \in \mathcal{T}} C_t(g_t) - B_t(d_t)
\]

s.t. \( p_i^{(1)} = g_i - d_i(1) - \sum_{j=1}^{n} u_{i,j}^{(1)}, \quad p_j^{(2)} = g_j - d_j(2) - \sum_{i=1}^{n} u_{i,j}^{(2)}, \quad \mathbb{1}^\top \mathbb{P} = 0, \quad t \in \mathcal{T}, \quad \mathbb{H}P^t \leq \bar{f}, \quad t \in \mathcal{T}, \quad 0 \leq \mathbb{L}u_{i,j} \leq S_{i,j}1, \quad i,j \in \mathcal{N}.
\]

Denote the optimal dual variables associated with constraints (1a) and (1b) by \( \lambda^{(1)}, \lambda^{(2)} \in \mathbb{R}^n \). The locational marginal price (LMP) at bus \( i \) at time \( t \) is the marginal cost of generating an additional unit of energy, or the marginal value of supplying an additional unit of load. The dual variable \( \lambda^{(1)}_i \) is the LMP at bus \( i \) at time \( t \). The LMP at a load bus determines the payment made by loads, and the LMP at a generation bus is the price at which generators are compensated. Mobile storage must pay for the electricity it consumes through charging at the LMP, and is also compensated at the LMP for discharging. The LMP depends on the mobile storage capacity available, i.e., it is a function of \( \mathbb{S} \) which is the vector of mobile storage capacities along each route in the network. We denote the LMPs by \( \lambda^{(t)}(\mathbb{S}) \) to emphasize this dependence, but omit it in places for notational convenience.
D. Commuter EV drivers with fixed routes

Consider an EV driven by a commuter who regularly travels along one of the $n^2$ possible routes in the network. The route choice for individual drivers in this case is exogenous. Each EV constitutes an infinitesimally small amount of storage traveling along a route, and we model the individual EVs as a continuum indexed by $k \in K_{i,j} = [0,1]$. An EV driver has the choice to use the EV battery as mobile storage, and in providing this service the EV driver buys electric energy at the LMP at the origin in the off-peak period, and sells that energy at the destination LMP in the peak period, thus capitalizing on the spatial-temporal LMP difference along the route. The value gained by an EV driver moving along the route $i \rightarrow j$ through providing mobile storage service per unit of storage capacity is

$$\lambda_j^{(2)}(S) - \lambda_i^{(1)}(S),$$

where $\lambda_j^{(2)}(S), \lambda_i^{(1)}(S)$ are the LMPs at the destination at time 2 and at the origin at time 1. The value depends on the amount of mobile storage capacity available in the network.

In order to provide this service, the driver has to cycle through the EV battery capacity, thus causing some battery degradation. We model this battery degradation as a cost $C$ of providing the service, which is incurred by the EV driver and is uniform across EV drivers. Further, the driver may have to park at a specialized charging station or wait longer than originally planned, and undergo some amount of inconvenience. For a driver $k \in K_{i,j}$ traveling along the route $i \rightarrow j$, we model this inconvenience as a cost $\theta_k$. The collection of drivers $K_{i,j}$ have a range of inconvenience costs, which can be modeled as a continuous range of $\theta_k$ values. Since the commuter EV moves along the route in any case, the travel cost does not factor into the decision to provide mobile storage service.

Each EV driver decides whether to provide the mobile storage service by comparing the value (the LMP difference in (2)) with the sum of battery degradation and inconvenience costs. We model this decision with a binary variable $s_k$, which is 1 when the driver $k$ provides mobile storage service, and 0 when she does not. The payoff for driver $k \in K_{i,j}$ is

$$\pi_k(s_k, S) = \left[ \lambda_j^{(2)}(S) - \lambda_i^{(1)}(S) - \theta_k - \kappa \right] s_k$$

per unit of storage capacity. The only difference in payoffs for drivers on the same route $i \rightarrow j$ is the inconvenience cost, which is different for each driver. Thus we can denote the decision to provide mobile storage service for each driver as a route-specific function $\sigma_{i,j} : \mathbb{R} \rightarrow \{0,1\}$ of the inconvenience cost, i.e.,

$$s_k = \sigma_{i,j}(\theta_k), \quad k \in K_{i,j}, \quad i,j \in \mathcal{N}.$$ 

The proportion of EVs that provide service is given by

$$S_{i,j} = \mathbb{E}[\sigma_{i,j}(\theta_k)] = \int_{k \in K_{i,j}} \sigma_{i,j}(\theta_k) dF_{i,j}(\theta_k) \leq 1,$$

where $F_{i,j}(\cdot)$ is the cumulative distribution of inconvenience costs of the drivers on route $i \rightarrow j$.

Remark 1: Here we implicitly assume that the maximum storage capacity on each route is 1 unit. We can scale the actual storage capacity, generation capacity and load across the network so that the available mobile storage capacity on each route $\int_{k \in K_{i,j}} dF(\theta_k)$ is scaled to be 1.

We denote the net arbitrage value of providing mobile storage service on route $i \rightarrow j$ as $\Delta_{i,j}^{(2)}(S)$, and define

$$\Delta_{i,j}^{(2)}(S) = \lambda_j^{(2)}(S) - \lambda_i^{(1)}(S) - \kappa.$$

E. On-demand EV drivers with flexible routes

Consider an EV which is signed up with a transportation network company (TNC) and can be requisitioned to provide mobile storage service along any of the $n^2$ possible routes in the network. Each EV driver has the choice to use the EV battery as mobile energy storage along any of the possible routes, or to not provide the service at all. In a large fleet of EVs, each EV is an infinitesimal small amount of storage and we model the individual EVs as a continuum indexed by $\ell \in \mathcal{L} = [0,1]$.

The value for the EV driver moving along $i \rightarrow j$ is the same as that for an EV driver with a fixed route, given in (2). However, the EV driver has to travel along $i \rightarrow j$ to provide this service, which she would not have otherwise since the sole purpose of the trip is providing mobile storage service. By providing this service, the EV battery will undergo some amount of degradation as well. The travel and battery degradation costs are modeled as a non-negative route-specific cost $\kappa_{i,j}$, and are the same for each EV traveling on this route. The EV driver has to spend time and effort in traveling and providing mobile storage service, and this inconvenience can be modeled as a non-negative cost $\theta_{i,j}$ which is specific to driver $\ell$ but is route-independent.

An EV driver signed up with a TNC has $n^2 + 1$ possible choices: providing the service at any of the $n^2$ routes, or not providing the service at all. We denote the decision to provide service on route $i \rightarrow j$ with $s_{\ell; i,j} \in \{0,1\}$, and note that $\sum_{i,j} s_{\ell; i,j} \leq 1$. The payoff for the EV driver $\ell \in \mathcal{L}$ is

$$\pi_{\ell}(s_{\ell}, S) = \sum_{i,j} \left( \lambda_j^{(2)}(S) - \lambda_i^{(1)}(S) - \theta_{\ell} - \kappa_{i,j} \right) s_{\ell; i,j}$$

per unit of storage capacity, where $s_{\ell} = \{s_{\ell; i,j}\}_{i,j \in \mathcal{N}} \in \mathbb{R}^{n^2}$. The only difference in payoffs for different EVs is their inconvenience cost, which in turn determines their route choice. We can then denote the optimal service provision choice by $s_{\ell} = \delta(\theta_{\ell}), \ell \in \mathcal{L}$, where $\delta : \mathbb{R} \rightarrow \{0,1\}^{n^2}$. We also define $s_{\ell} = 1 \sum_{i,j} s_{\ell; i,j} \in \{0,1\}$ which denotes whether the EV provides service along any route in the network. If $s_{\ell} = 0$, EV $\ell$ does not provide service along any route. We define $S_{i,j}$ as the amount of mobile storage available on $i \rightarrow j$. We have

$$S_{i,j} = \int_{\ell \in \mathcal{L}} \delta(\theta_{\ell}) s_{\ell; i,j} dF(\theta_{\ell}) \leq 1,$$

where $\delta(\theta_{\ell}) s_{\ell; i,j}$ is the decision to provide service on route $i \rightarrow j$, and $F(\cdot)$ is the cumulative distribution of
the inconvenience costs of the on-demand EVs. Note that
\[ \sum_{i,j} S_{i,j} \leq 1 \] as well.
We define the net arbitrage value of a route for commuter
EVs providing mobile storage service as
\[ \Delta_{i,j}^{\text{flex}}(S) = \lambda_{i,j}^{(2)}(S) - \lambda_{i,j}^{(1)}(S) - \kappa_{i,j}. \]

F. Solution concepts

Both commuter and on-demand EVs can be operated by
centralized operators and decentralized agents with different
objectives. We consider two benchmark solution concepts: 1) Myopic EV drivers: EVs act in a decentralized manner and
maximize their own individual payoffs without considering the effect of mobile storage service on the
LMPs, i.e., under the assumption that \( S = \mathbf{0} \).
2) Social welfare maximizing operator: A central operator
optimizes the mobile storage service provision of all the
EVs to maximize social welfare, i.e., minimize social
cost. The social cost is taken to be the sum of generation
cost, inconvenience and battery degradation costs for the
EVs (i.e., the surplus received by EV operators), minus the value of supplying electricity to loads.

The precise mathematical description of these benchmarks
differs depending on the type of EV drivers under considera-
tion (commuters or on-demand drivers), and will be provided in subsequent sections. We then consider the operation of EVs which operate in a decentralized manner to optimize their
individual payoffs, thus participating in an EV battery
sharing game. We define three game settings: 1) Commuter EVs only: The set of players is \( \cup_{i,j} \mathcal{K}_{i,j} \),
where each player can decide to provide mobile storage
service or not, with payoff defined in (3).
2) On-demand EVs only: The set of players is \( \mathcal{L} \), where
each player chooses from \( n \) routes to provide the
mobile storage service, or decides to not provide the
service at all, with payoff defined in (4).
3) Both commuter and on-demand EVs: Both types of players coexist, with their decisions and payoffs defined as before.

The game is an aggregate game, i.e., each player’s action
only impacts others’ payoffs via the aggregate storage capacities. We utilize Nash Equilibrium (NE) as the solution concept, under which no player has an incentive to unilat-
erally change their decision. We refer to the aggregate storage capacities induced by an NE as NE storage capacities. For each setting, we will compare the NE to the benchmarks discussed previously.

III. COMMUTER EVS: FIXED ROUTES

In this section, we consider the setting where there are only commuter EVs providing mobile storage service to the
grid. Each route in the power network has a population of
EV drivers \( k \in \mathcal{K}_{i,j} \) characterized by their inconvenience
cost \( \theta_k \), which are otherwise interchangeable. We partition
the population of EVs on route \( i \to j \) into \( \mathcal{K}_{i,j}^+ \) and \( \mathcal{K}_{i,j}^- \),
where EVs in \( \mathcal{K}_{i,j}^+ \) provide mobile storage service, and EVs in
\( \mathcal{K}_{i,j}^- \) do not. We posit that the EVs in \( \mathcal{K}_{i,j}^+ \) necessarily
have a lower inconvenience cost than the EVs in \( \mathcal{K}_{i,j}^- \) for
each solution concept discussed in section II-F (which will be mathematically defined subsequently), i.e.,

**Proposition 1:** For each solution concept discussed in this paper, there exists a threshold \( \bar{\theta}_{i,j} \) such that
\[ \mathcal{K}_{i,j}^+ = \{ k \in \mathcal{K}_{i,j} : \theta_k < \bar{\theta}_{i,j} \}, \quad i,j \in \mathcal{N}, \]
\[ \mathcal{K}_{i,j}^- = \{ k \in \mathcal{K}_{i,j} : \theta_k \geq \bar{\theta}_{i,j} \}, \quad i,j \in \mathcal{N}. \]

A. Benchmarks

1) Myopic EV drivers: Each myopic EV owner traveling along \( i \to j \) maximizes \( \pi_k(s_k,0) \). The optimal decision is
\[ s_{i,j}^{\text{myop}} = \begin{cases} 1, & \text{if } \Delta_{i,j}^{\text{myop}}(0) - \theta_k > 0, \\ 0, & \text{otherwise,} \end{cases} \]
which gives us threshold inconvenience cost for each route
\( \theta_{i,j}^{\text{myop}} = \Delta_{i,j}^{\text{myop}}(0) \), and the mobile storage proportion \( s_{i,j}^{\text{myop}} = F_{i,j}(\theta_{i,j}^{\text{myop}}). \)
2) Social welfare maximizing operator: A social welfare maximizing operator solves the following problem:
\[
\min_{\mathbf{S}, \mathbf{\theta}} J(\mathbf{S}) + \sum_{i,j} \int_{\theta < \theta_{i,j}} (\theta_k + \kappa) dF(\theta_k), \\
\text{s.t. } S_{i,j} = F_{i,j}(\theta_{i,j}), \quad i,j \in \mathcal{N},
\]
where \( J(\mathbf{S}) \) is the optimal solution of the economic dispatch problem in (1). We have

**Lemma 1:** The inconvenience cost threshold \( \theta_{i,j}^{\text{sw}} \) and the corresponding aggregate storage capacity \( S_{i,j}^{\text{sw}} \), \( i,j \in \mathcal{N} \), for
the socially optimal operation are given by the solution of
\[ \theta_{i,j}^{\text{sw}} = \Delta_{i,j}^{\text{sw}}(S_{i,j}^{\text{sw}}), \quad S_{i,j}^{\text{sw}} = F_{i,j}(\theta_{i,j}^{\text{sw}}); \quad i,j \in \mathcal{N}. \]

B. Nash equilibrium

Consider a situation where all the EVs are owned and
operated by distributed entities, e.g., the case where they are
all personal vehicles used for transport, and each EV driver participates in an EV battery sharing game. At the equilib-
rium there will be no EV which will be better off switching from \( \mathcal{K}_{i,j}^+ \) to \( \mathcal{K}_{i,j}^- \), or vice versa. Given the aggregate storage capacities, each EV maximizes its payoff, i.e.,
\[ \max_{s_k} \pi_k(s_k, S). \]
If each EV has a small storage capacity, then the operational
decision of one EV does not impact the LMPs, and
\[ s_{k}^{\text{NE}} = \sigma^{\text{NE}}_{k}(\theta_k) = \begin{cases} 1, & \text{if } \pi_k(1, S^{\text{NE}}) > 0, \\ 0, & \text{otherwise,} \end{cases} \]
for \( k \in \mathcal{K}_{i,j} \). In other words, any EV which can obtain a non-
negative payoff decides to provide mobile storage service.

**Lemma 2:** The Nash equilibrium inconvenience cost
threshold \( \theta_{i,j}^{\text{NE}} \) and the corresponding aggregate storage capacity
\( S_{i,j}^{\text{NE}} \), \( i,j \in \mathcal{N} \), are given by the solution of
\[ \theta_{i,j}^{\text{NE}} = \Delta_{i,j}^{\text{NE}}(S_{i,j}^{\text{NE}}), \quad S_{i,j}^{\text{NE}} = F_{i,j}(\theta_{i,j}^{\text{NE}}); \quad i,j \in \mathcal{N}. \]

**Theorem 1:** Any aggregate storage capacity corresponding
to the NE for commuter EVs supports the social welfare.

**Proof:** Proofs are given in the extended version [4].

**Remark 2:** This shows us that the equilibrium behavior of
commuter EVs providing mobile storage service will result in a socially optimal outcome.
C. Example

To gain explicit analytical insight, we consider a simple example with two periods... EVs is the solution of:

\[ \theta_{sw} = \Delta_{\text{flex}}^{i^*,j^*}(S_{sw}), \quad 1^T S_{sw} = F(\theta_{sw}), \quad (11) \]

where \( i^*, j^* \) are defined as in (9).

2) Social welfare maximizing operator: A central operator that maximizes social welfare solves the following problem:

\[
\min_{S, \theta} J(S) + \sum_{i,j} \kappa_{i,j} S_{i,j} + \int_{\theta_t} \theta_t \, dF(\theta_t) \quad (8)
\]

s.t. \( S \geq 0, \quad 1^T S = F(\bar{\theta}) \),

where \( J(S) \) is the optimal cost of the economic dispatch problem in (1). We can define the storage capacity on each route by \( S_{i,j}^\text{myop} \), and we know that \( \sum_{i,j} S_{i,j} = F(\bar{\theta}) \), where \( \bar{\theta} \) is the network-wide threshold of inconvenience costs determined by solving (8). From [3] we know that the value of increasing mobile storage capacity is given by \( -\nabla S_{i,j} J(S) = (\lambda^{(2)}(S) - \lambda^{(1)}(S))_+ \), which is the LMP increase along the route. However, increasing storage capacity along a route also increases the travel and inconvenience costs that need to be paid. The operator will add mobile storage capacity which maximizes the increase in social welfare \( (\lambda^{(2)}(S) - \lambda^{(1)}(S))_+ - \theta_t - \kappa_{i,j} \). We can ignore the inner positive part operator in this equation when we formulate our storage operation decision, since the LMP difference into their decision.

IV. ON-DEMAND EVs: FLEXIBLE ROUTES

In this section, we consider the setting where there are only on-demand EVs providing mobile storage service to the grid. The network has a population of on-demand EVs \( \ell \in \mathcal{L} \), which are characterized by their inconvenience cost \( \theta_\ell \), and are otherwise interchangeable. In order to define the optimal storage service, we partition the network-wide population of EVs into \( \mathcal{L}^+ \) and \( \mathcal{L}^- \) for each of the solution concepts, where EVs in \( \mathcal{L}^+ \) provide mobile storage service on any one route in the power network and EVs in \( \mathcal{L}^- \) do not provide mobile storage on any route. We can extend Proposition 1 and define a network-wide inconvenience cost threshold \( \bar{\theta} \) for each solution concept, such that

\[ \mathcal{L}^+ = \{ \ell \in \mathcal{L} : \theta_\ell < \bar{\theta} \}, \quad \mathcal{L}^- = \{ \ell \in \mathcal{L} : \theta_\ell \geq \bar{\theta} \}. \]

A. Benchmarks

1) Myopic EV drivers: The myopic EV driver indexed by \( \ell \in \mathcal{L} \) chooses \( s_\ell \) to maximize \( \pi_\ell(s_\ell, 0) \). Since the only difference in payoffs for EVs is the inconvenience cost \( \theta_\ell \), the route with the maximum potential payoff will attract all the myopic EV drivers. Let this route be \( i^*, j^* \), where

\[ (i^*, j^*) = \arg\max_{i,j} \Delta_{\text{flex}}^{i,j}(S_{sw}) \quad (9) \]

Then the decision of driver \( \ell \in \mathcal{L} \) to provide service is

\[ s_{\ell,i^*,j^*}^{\text{myop}} = \begin{cases} 1, & \text{if } \Delta_{\text{flex}}^{i^*,j^*}(S_{sw}) > \theta_\ell, \\ 0, & \text{otherwise}, \end{cases} \]

and with \( s_{\ell,i,j}^{\text{myop}} = 0 \) for \( (i,j) \neq (i^*, j^*) \). This gives us the inconvenience cost threshold \( \bar{\theta}^{\text{myop}} = \Delta_{i^*,j^*}(0) \), where \( i^*, j^* \) are defined as in (7). The total storage capacity for route \( (i^*, j^*) \) is given by \( S_{i,j}^{\text{myop}} = F(\bar{\theta}^{\text{myop}}) \), and \( S_{i,j}^{\text{myop}} = 0 \) for all other routes.

2) Social welfare maximizing operator: A central operator that maximizes social welfare solves the following problem:

\[
\min_{S, \theta} J(S) + \sum_{i,j} \kappa_{i,j} S_{i,j} + \int_{\theta_t} \theta_t \, dF(\theta_t) \quad (8)
\]

s.t. \( S \geq 0, \quad 1^T S = F(\bar{\theta}) \),

where \( J(S) \) is the optimal cost of the economic dispatch problem in (1). We can define the storage capacity on each route by \( S_{i,j}^{\text{myop}} \), and we know that \( \sum_{i,j} S_{i,j} = F(\bar{\theta}) \), where \( \bar{\theta} \) is the network-wide threshold of inconvenience costs determined by solving (8). From [3] we know that the value of increasing mobile storage capacity is given by \( -\nabla S_{i,j} J(S) = (\lambda^{(2)}(S) - \lambda^{(1)}(S))_+ \), which is the LMP increase along the route. However, increasing storage capacity along a route also increases the travel and inconvenience costs that need to be paid. The operator will add mobile storage capacity which maximizes the increase in social welfare \( (\lambda^{(2)}(S) - \lambda^{(1)}(S))_+ - \theta_\ell - \kappa_{i,j} \). We can ignore the inner positive part operator in this equation when we formulate our storage operation decision, since the LMP difference into their decision.

Lemma 3: The network-wide inconvenience cost threshold for socially optimal operation of on-demand EVs is the solution of

\[ \bar{\theta}^{\text{sw}} = \Delta_{i^*,j^*}^{\text{flex}}(S_{sw}), \quad 1^T S_{sw} = F(\bar{\theta}^{\text{sw}}), \quad (11) \]

where \( i^*, j^* \) are defined as in (9).
B. Nash equilibrium

Consider a situation where all EVs are owned and operated by distributed agents, e.g., when they are owned by individuals who sign up on TNC platforms to earn money for providing mobile storage service. Each EV driver participates in an EV battery sharing game, and makes an independent decision on whether to provide mobile storage service and which route to provide it on based on the payoff \( \pi_i(s_i, S) \). At the equilibrium, EVs will provide service in a manner such that no EV has an incentive to deviate from its chosen route and operation decision.

At the NE, given the aggregate storage capacities, each EV maximizes its own payoff, i.e., chooses a route and operational decision according to

\[
\max_{s_i} \pi_i(s_i, S) \quad \text{s.t.} \quad 1^T s_i \leq 1,
\]

where \( s_i \) is the decision vector of \( s_{\ell,i,j} \) for all routes on the network for driver \( \ell \). If each individual EV has a small storage capacity, then its operation decision will not impact LMPs and we can denote the (non-unique) optimal route choice of the marginal EV by

\[
(i^*, j^*) = \arg\max_{i,j} \Delta_{i,j}^{\text{fix}}(S^{\text{NE}}) \quad (12)
\]

and the mobile storage service provision by

\[
\begin{align*}
\delta_{i^*,j^*} & = \begin{cases} 1, & \text{if } \Delta_{i^*,j^*}^{\text{fix}}(S^{\text{NE}}) > 0, \\
0, & \text{otherwise,}
\end{cases} \\
\end{align*}
\]

For the equilibrium mobile storage \( S^{\text{NE}} \), there are two dimensions: each EV which provides mobile storage service on route \( i \to j \) at equilibrium will be no better off if (a) it decides to stop providing the service, or (b) it switches to a different route. Additionally, each EV which does not provide mobile storage service will be no better off if it does so on any route in the network.

\[ i^*, j^* \] are defined as in (12).

A) First, consider the case where \( \mathcal{L}^- \) is empty, i.e. all the EVs available provide mobile storage service on one route or the other. Then we have

\[
\Delta^{\text{fix}}_{i^*,j^*}(S^{\text{NE}}) = \Delta^{\text{fix}}_{i^*,j^*}(S^{\text{NE}}) \geq \Delta^{\text{fix}}_{i^*,j^*}(S^{\text{NE}})
\]

for all \( i^*, j^* \in \mathcal{N} \) where

\[
S^{\text{NE}}_{i^*,j^*} = 0,
\]

i.e., the marginal payoff for routes with non-zero mobile storage capacity is the same throughout the network, and is higher than the marginal payoff for routes with zero mobile storage capacity at equilibrium. We can relate the equilibrium mobile storage service to the socially optimal solution as:

\[ \text{Theorem 2: Any inconvenience cost threshold } \theta \text{ corresponding to the NE for on-demand EVs supports the social welfare.} \]

V. HYBRID: COMMUTER AND ON-DEMAND EVS

In this section, we consider the setting where there are both commuter and on-demand EVs providing mobile storage service to the grid. There is a mix of EVs in the power network; a population of commuter EVs on each route characterized by their inconvenience costs \( \theta_i, k \in K_{i,j} \), and a network-wide population of on-demand EVs characterized by their inconvenience costs \( \theta_{\ell}, \ell \in \mathcal{L} \). We denote the mobile storage capacity provided by commuter EVs with fixed routes by \( S_{\text{fix}} \), and the capacity provided by on-demand EVs with flexible routes by \( S_{\text{flex}} \). The total mobile storage capacity on all routes is represented by \( S \), and includes storage capacity from commuter and on-demand EVs. The two types of EVs provide the same service and are interchangeable in terms of value generated, but they have different inconvenience, travel and battery degradation costs. In order to define the optimal storage service for each of the solution concepts, we partition the population of commuter EVs on each route into two sets: those which provide service \( (K_{i,j}^+) \) and those which don’t \( (K_{i,j}^-) \). These sets are determined by a route specific inconvenience cost threshold \( \theta_{i,j} \). We also partition the network wide population of on-demand EVs into those which provide service on any route \( (\mathcal{L}^+) \) and those which don’t \( (\mathcal{L}^-) \). These sets are determined by a network-wide inconvenience cost threshold \( \theta \).

A. Benchmark: social welfare maximizing operator

To maximize the social welfare, a central operator solves the following problem:

\[
\min_{S} \sum_{i,j} \int_{\theta_k < \theta_{i,j}} (\theta_k + \kappa) dF \big( \theta_k \big) + \int_{\theta_k > \theta_{i,j}} \theta_k dF \big( \theta_k \big) + \sum_{i,j} \kappa_{i,j} S_{i,j}^{\text{flex}}
\]

s.t. \( S_{i,j}^{\text{fix}} = 0, 1^T S_{i,j}^{\text{flex}} = F(\theta_{i,j}^{\text{flex}}), \)

where \( \mathcal{S} = S_{i,j}^{\text{fix}} + S_{i,j}^{\text{flex}} \). From [3], we know that the marginal value of adding mobile storage on a route is

\[
-\nabla_{S_{i,j}^{\text{fix}}} J(S) = -\nabla_{S_{i,j}^{\text{fix}}} J(S) = (\lambda^2_{i,j}(S) - \lambda^1_{i,j}(S))_+,
\]
which is the same for both commuter and on-demand EVs. The operator will add mobile storage capacity to maximize social welfare, i.e., will increase mobile storage capacity on a route as long as the marginal value is greater than or equal to the cost for either type of EV,
\[ \Delta_{i,j}^{\text{fix}}(S) \geq \theta_{i,j}, \quad \Delta_{i,j}^{\text{flex}}(S) \geq \theta_{i,j}. \]

The operator will prioritize dispatching on-demand EVs to the route with the greatest marginal increase in social welfare. We can ignore the positive part operator in this expression, since the sum of costs on the right hand side is non-negative by definition.

**Lemma 5:** The inconvenience cost thresholds for the socially optimal storage operation of commuter and on-demand EVs are given by the joint solution of (5) and (11), where \( i^*, j^* \) are defined as in (9) and \( S^{\text{sw}} = S^{\text{fix, sw}} + S^{\text{flex, sw}} \).

**B. Joint Nash equilibrium**

Consider the situation where all of the EVs are operated independently irrespective of their type. Each EV driver participates in an EV battery sharing game, and makes an independent decision to provide mobile storage service and choose a route (for on-demand EVs) in order to maximize \( \pi_k(s_k, S) \) or \( \pi_\ell(s_\ell, S) \) as appropriate. At the equilibrium, there will be a combination of commuter and on-demand mobile storage on each route. No commuter EV should be better off if it switches from \( K^+_{i,j} \) to \( K^-_{i,j} \) or vice versa (characterized by \( \theta_{i,j}^{\text{fix, NE}} \)). Similarly, no on-demand EV should be better off switching from \( \ell^+ \) to \( \ell^- \) or vice versa (characterized by \( \theta_{i,j}^{\text{flex, NE}} \)), or by switching routes. The storage capacities at equilibrium are given by
\[ F^{\text{flex}}(\theta^{\text{flex, NE}}_{i,j}) = \begin{cases} 1 & \text{if } S^{\text{flex}}(\theta^{\text{flex, NE}}_{i,j}) = S^{\text{flex}}_{i,j} \quad i,j \in N, \\ 0 & \text{otherwise}. \end{cases} \]

where \( F^{\text{flex}}(\cdot) \), \( F^{\text{fix}}(\cdot) \) are the cumulative distributions of inconvenience costs of on-demand and commuter EVs on that route respectively. The equilibrium decision by a commuter EV is given by
\[ S^{\text{fix, NE}}_{k,i,j} = \begin{cases} 1 & \text{if } \Delta_{i,j}^{\text{fix}}(S^{\text{NE}}) > \theta_k \\ 0 & \text{otherwise}. \end{cases} \]

The equilibrium decision by an on-demand EV is given by the route choice \( (i^*, j^*) = \arg\max_{i,j} \Delta_{i,j}^{\text{flex}}(S^{\text{NE}}) \), and the mobile storage service provision by
\[ S^{\text{flex, NE}}_{\ell,i,j} = \begin{cases} 1 & \text{if } \Delta_{i,j}^{\text{flex}}(S^{\text{NE}}) > \theta_\ell \\ 0 & \text{otherwise}, \end{cases} \]

We can characterize the NE with these exhaustive cases:

A) For a route \( i \rightarrow j \), consider \( K^-_{i,j} \) and \( \ell^- \) are not empty, i.e. there are some commuter and on-demand EVs not providing mobile storage service. Then the marginal payoff for both type of EV is non-positive.

1) \( S^{\text{fix}}_{i,j} = 0 \) and \( S^{\text{flex}}_{i,j} = 0 \): then the marginal payoff for both type of EVs on that route should be zero, i.e.,
\[ \theta_{i,j}^{\text{NE}} = \Delta_{i,j}^{\text{flex}}(S^{\text{NE}}), \theta_{i,j}^{\text{flex, NE}} = \Delta_{i,j}^{\text{fix}}(S^{\text{NE}}). \]

2) \( S^{\text{fix}}_{i,j} = 0 \), which means \( \Delta_{i,j}^{\text{fix}}(S^{\text{NE}}) \geq \theta_{i,j}^{\text{flex, NE}}. \)

3) \( S^{\text{flex}}_{i,j} = 0 \), which means \( \Delta_{i,j}^{\text{flex}}(S^{\text{NE}}) \geq \theta_{i,j}^{\text{flex, NE}}. \)

B) For a route \( i \rightarrow j \), consider \( K^-_{i,j} \) is empty, i.e. all commuter EVs provide mobile storage service. Then \( S^{\text{fix}}_{i,j} = 1 \), and \( \Delta_{i,j}^{\text{flex}}(S^{\text{NE}}) \geq \theta_{i,j}^{\text{flex, NE}} \).

C) Consider \( \ell^- \) is empty, i.e. all on-demand EVs provide mobile storage service. Then \( \sum_{i,j} S^{\text{flex}}_{i,j} = 1 \), and \( \Delta_{i,j}^{\text{fix}}(S^{\text{NE}}) \geq \theta_{i,j}^{\text{flex, NE}} \) for at least one route \( i \rightarrow j \).

Except for the first situation, we cannot explicitly relate the equilibrium service by commuter and on-demand EVs.

**Lemma 6:** The inconvenience cost thresholds for the equilibrium operation of commuter and on-demand EVs are given by the joint solution of (6) and (13), where \( i^*, j^* \) are defined as in (12) and \( S^{\text{NE}} = S^{\text{flex, NE}} + S^{\text{fix, NE}} \).

**Theorem 3:** Any inconvenience cost thresholds for commuter and on-demand EVs corresponding to a joint Nash equilibrium also support the social welfare.

**VI. CONCLUSION**

We formulate and analyze a network EV battery sharing game, where distributed EVs provide mobile energy storage service to the grid. We model two different EV behaviors: commuter EVs which travel on fixed routes, and on-demand EVs which can travel on any route in the power network. We explicitly characterize the Nash equilibrium, and show that the socially optimal mobile storage operation is an NE for three network configurations: with commuter EVs only, with on-demand EVs only, and with a combination of commuter and on-demand EVs.

**REFERENCES**

[1] Wood Mackenzie, “US energy storage market shatters records in Q3 2020,” 2020, https://www.woodmac.com/press-releases/us-energy-storage-market-shatters-records-in-q3-2020/

[2] United States Environmental Protection Agency, “Inventory of U.S. Greenhouse Gas Emissions and Sinks,” Tech. Rep.

[3] U. Agwan, J. Qin, K. Poolla, and P. Varaiya, “Marginal value of mobile energy storage in power network,” in 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021, pp. 4936–4943.

[4] D. Zhao, H. Wang, J. Huang, and X. Lin, “Virtual energy storage in power network,” in 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021, pp. 4936–4943.

[5] D. Kalathil, C. Wu, K. Poolla, and P. Varaiya, “The sharing economy for the electricity storage,” IEEE Transactions on Smart Grid, 2017.

[6] J. Qin, S. Li, K. Poolla, and P. Varaiya, “Distributed storage investment in power networks,” in 2019 American Control Conference (ACC). IEEE, 2019, pp. 1579–1586.

[7] R. Carli, M. Dotoli, and V. Palmisano, “A distributed control approach based on game theory for the optimal energy scheduling of a residential microgrid with shared generation and storage,” in 2019 IEEE 15th International Conference on Automation Science and Engineering (CASE). IEEE, 2019, pp. 960–965.

[8] Y. Yang, U. Agwan, G. Hu, and C. J. Spanos, “Selling renewable utilization service to consumers via cloud energy storage,” arXiv preprint arXiv:2209.06461. [Online]. Available: https://arxiv.org/abs/2209.06461

[9] D. Kalathil, C. Wu, K. Poolla, and P. Varaiya, “The sharing economy for the electricity storage,” IEEE Transactions on Smart Grid, 2017.

[10] J. Qin, S. Li, K. Poolla, and P. Varaiya, “Distributed storage investment in power networks,” in 2019 American Control Conference (ACC). IEEE, 2019, pp. 1579–1586.

[11] R. Carli, M. Dotoli, and V. Palmisano, “A distributed control approach based on game theory for the optimal energy scheduling of a residential microgrid with shared generation and storage,” in 2019 IEEE 15th International Conference on Automation Science and Engineering (CASE). IEEE, 2019, pp. 960–965.

[12] Y. Yang, U. Agwan, G. Hu, and C. J. Spanos, “Selling renewable utilization service to consumers via cloud energy storage,” arXiv preprint arXiv:2209.06461. [Online]. Available: https://arxiv.org/abs/2209.06461

[13] D. Zhao, H. Wang, J. Huang, and X. Lin, “Virtual energy storage sharing and capacity allocation,” IEEE transactions on smart grid, vol. 11, no. 2, pp. 1112–1123, 2019.

[14] J. Qin, K. Poolla, and P. Varaiya, “Mobile storage for demand charge reduction,” IEEE Transactions on Intelligent Transportation Systems, 2021.

[15] G. He, J. Michalek, S. Kar, Q. Chen, D. Zhang, and J. F. Whitacre, “Utility-scale portable energy storage systems,” Joule, vol. 5, no. 2, pp. 379–392, 2021.