Mathematical modeling of the Stirling engine in terms of applying the composition of the power complex containing non-conventional and renewable energy

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Abstract. The opportunity of application of Stirling engine with non-conventional and renewable sources of energy. The advantage of such use. The resulting expression for the thermal efficiency of the Stirling engine. It is shown that the work per cycle is proportional to the quantity of matter, and hence the pressure of the working fluid, the temperature difference and, to a lesser extent, depends on the expansion coefficient; efficiency of ideal Stirling cycle coincides with the efficiency of an ideal engine working on the Carnot cycle, which distinguishes a Stirling cycle from the cycles of Otto and Diesel underlying engine. It has been established that the four input parameters, the only parameter which can be easily changed during operation, and which effectively affects the operation of the engine is the phase difference. Dependence of work per cycle of the phase difference, called the phase characteristic, visually illustrates mode of operation of Stirling engine. The mathematical model of the cycle of Schmidt and the analysis of operation of Stirling engine in the approach of Schmidt with the aid of numerical analysis. To conduct numerical experiments designed program feature in the language MathLab. The results of numerical experiments are illustrated by graphical charts.

1. Overview of possible applications and advantages of the Stirling engine
The objects of exploration, development of oil and gas wells, oil and gas extraction are often located in remote, inaccessible places far from electrical networks and highways. Autonomous power supply of these objects is currently carried out by diesel power plants, which is associated with a necessary supply of fuel and the associated large overhead. In this regard, of particular relevance is the use of non-renewable sources of energy (NRSE).
As such is known: wind energy, geothermal heat, solar radiation absorbed by either using solar panels in the form of electricity, or converted into thermal energy through solar collectors. This group should also include local energy sources such as energy of associated gas, peat, firewood, coal, coal shale, etc. Because of its specificity to ensure reliable energy, these sources must be integrated in the local energy sector, which should provide the necessary heat for heating dwellings and places of residence of staff and electricity needed for appliances and installations. Previous research of the authors [1-4] have shown the importance of optimal composition of the energy complex on the basis of RES. As
part of the local energy mix should include a Converter of thermal energy into electrical energy, has a number of properties: versatility of consumed energy, great lifespan, no need regular maintenance, reliability, low noise, high power density. The only, currently known, such transducer is the generator of a Stirling is a heat engine in which the generator is combined with a Stirling engine.

Stirling engine (DS) – refers to internal combustion engines, more specifically an external supply of heat. For work DS, you only need to provide its heat exchangers the temperature difference. The heater can use the heat of the burner, steam or water heated in the geothermal or solar collectors. It is possible to sum directly the solar energy from the solar concentrator. For these cases, provided the special design of the DS with a quartz head, through which concentrated solar radiation enters directly into the expansion chamber. For these engines you can even use geothermal water pumped directly from the well, or associated oil gas, which can have high corrosion aggressiveness. As refrigerator you can use the environment, chilled water, ice or snow.

2. The principle of operation of the Stirling engine. The classic Stirling cycle. Cycle Schmidt

Stirling engine [5-9] works in a closed cycle in which the working fluid expands at higher temperatures and contracts at much lower, which ensures the positive work over the cycle.

A prototype of the real cycle is the ideal Stirling cycle consists of two isotherms and two isochor. A Stirling machine can operate in the forward loop (in this case it is the engine) and selling (in this case, it is a refrigeration machine). The expansion of the working fluid occurs at a higher temperature of the heater T1 and contraction at refrigerator temperature T2 isothermal processes. The transition of the working fluid between these temperatures is carried out during the two isochoric.

We obtain an expression for cycle efficiency. The working fluid receives heat during the isochoric heating and the isothermal expansion:

$$ Q_n = q_1 + Q_1 = \nu \cdot C_V \cdot (T_1 - T_2) + \nu \cdot R \cdot T_1 \cdot \ln \frac{V_2}{V_1} \quad (1) $$

and gives during the isochoric cooling and isothermal compression:

$$ Q_s = q_2 + Q_2 = \nu \cdot C_V \cdot (T_1 - T_2) + \nu \cdot R \cdot T_2 \cdot \ln \frac{V_2}{V_1}. \quad (2) $$

Here:
- Q_1 – the amount of heat received working fluid with isothermal expansion;
- q_1 – the amount of heat received working fluid during isochoric heating;
- Q_2 – the amount of heat received by the worker body in an isothermal compression;
- q_2 – the amount of heat received working fluid during isochoric cooling;
- \( \nu \) is the number of moles of a substance, \( C_V \) is the molar heat capacity at constant volume.

Work done per cycle is equal to

$$ A = Q_1 - Q_2 = \nu \cdot R \cdot \ln \frac{V_2}{V_1} \cdot (T_1 - T_2) \quad (3) $$

Thermal efficiency is

$$ \eta = \frac{A}{Q_u} = \frac{\nu \cdot R \cdot \ln \frac{V_2}{V_1} \cdot (T_1 - T_2)}{\nu \cdot R \cdot T_2 \cdot \ln \frac{V_2}{V_1} \cdot T_1 + q_1} \quad (4) $$

Stirling has offered warmth q_1 not from the heater, and regenerator in which the heat enters into the heat exchange during the isochoric cooling of the gas. In this case, q_1 in the denominator disappears and the formula (4) has the form:

$$ \eta = \frac{T_1 - T_2}{T_1}, \quad (5) $$
i.e., coincides with the efficiency of a perfect engine. From this we can draw the following important conclusions:

The work per cycle is proportional to the quantity of matter, and hence the pressure of the working fluid at ambient temperature, the temperature difference and, to a lesser extent, depends on the expansion coefficient $V_2/V_1$.

Efficiency of the ideal Stirling cycle coincides with the efficiency of an ideal engine working on the Carnot cycle, which distinguishes a Stirling cycle from the cycles of Otto and Diesel the underlying engine.

Schmidt, using the representation of harmonic motion of the pistons and rest of the nodes, has left unchanged the other assumptions [10]: the equality of the instantaneous pressure at any point in the volume, ideal regeneration, isterichnost expansion and contraction, the instantaneous change of the gas temperature entering the corresponding cell (perfect heat transfer), the ideal gas as the working fluid.

Thus, the scheme of Schmidt, although it is close, certainly more realistic than the ideal Stirling cycle.

Let us turn to the derivation of the mathematical model of the cycle of Schmidt.

We assume that the working fluid is a perfect gas, obeying the equation

$$P \cdot V = n \cdot R \cdot T$$

how do we obtain an expression for the number of moles of the substance

$$n = \frac{P \cdot V}{R \cdot T},$$

Given that all the gas is distributed in three volumes: volume $V_E$ being at a temperature $T_1$, volume compression $V_c$, at a temperature of $T_2$ and dead volume $V_D$ at a temperature of $T_D = \frac{T_1 + T_2}{2}$, express the total amount of substance:

$$n = n_1 + n_2 + n_3 = \frac{P}{R} \left( \frac{V_E}{T_1} + \frac{V_c}{T_2} + \frac{V_D}{T_D} \right),$$ (6)

from which we obtain the expression for the pressure:

$$P = \frac{V \cdot R}{T_1 + \frac{V_c}{T_2} + \frac{V_D}{T_D}}.$$ (7)

In line with Schmidt's assumption of harmonic motion, consider the dependence of the volume expansion and contraction of the angle of rotation of the shaft, which is counted relative to the top dead center of the expansion piston:

$$V_E = \frac{V_0}{2} \left(1 + \cos \varphi \right); \quad V_c = \frac{k \cdot V_0}{2} \cdot \left(1 + \cos (\varphi - \delta) \right),$$

where $V_0$ – the amplitude of volume $V_E$, 
$\varphi$ – the rotation angle of the crankshaft (phase process),
$\delta$ – the gap phase of the piston compression from the piston extension.

Also: $T = \frac{T_1}{T_2}$; $V_D = X \cdot V_0$; $T_D = \frac{1}{2} \left( T_1 + T_2 \right)$. The expression for the pressure takes the following form:

$$P = \frac{V \cdot R \cdot T_2}{V_0} \cdot \left( \frac{2}{1} \cdot \cos \varphi + k \cdot \cos (\varphi - \delta) \right) + \frac{2x}{(1 + \tau)}.$$ (8)

The work in gas expansion is equal to

$$A_E = \int P_E \cdot d \cdot V_E = -n \cdot R \cdot T_2 \int \frac{1}{1} \cdot \cos \varphi + k \cdot \cos (\varphi - \delta) + \frac{2x}{(1 + \tau)}.$$ (9)

Work under the pressure
\[ A_c = \oint P_c \cdot d \cdot V_c = -\nu \cdot R \cdot T_2 \oint \frac{k \cdot \sin \varphi \cdot d \cdot \varphi}{\tau \cdot \cos \varphi + k \cdot \cos(\varphi - \delta)} + \frac{2x}{(1 + \tau)}. \] (10)

Here the symbol \( \oint \) indicates integration cycle:

\[ \oint = \int_{0}^{2\pi} \varphi = 0 \]

Total work per cycle is equal to

\[ A = A_E + A_c \] (11)

Performing the numerical integration in the above formulas, you can make a feature that allows numerically, according to the set of input parameters, the value of work per cycle. Such function was created on the MATHLAB language and used to analyze the operation of the Stirling engine in the approach of Schmidt.

3. Numerical analysis of cycle DS in the approach of Schmidt

The purpose of the analysis is to identify the dependence of the dimensionless parameters that characterize the performance of a loop \( \frac{P}{P_0}, \frac{V_1}{V_0}, \frac{V_2}{V_0}, \frac{A}{\nu R T_0}, \frac{Q_1}{\nu R T_0}, \frac{Q_2}{\nu R T_0} \) from the dimensionless parameters characterizing the design features of the process, its mode, or as \( \tau, k, X, \delta \). For the numerical analysis, a program is a function of the language MATHLAB. Using the program, you can analyze the work of the DS to determine the effect of various design parameters on its performance and determine the appropriate combination of parameters that optimizes the performance of the engine over the cycle, as will be shown below.

The Schmidt cycle

![The Schmidt cycle](image)

Figure 1 – Form cycle and the value of the dimensionless work for certain values of the parameters

The calculations showed that in the perfect work of the regenerator efficiency of the cycle coincides with the Schmidt DS or efficiency of the Carnot cycle.

Dimensionless work per cycle \( \frac{A}{\nu \cdot R \cdot T_2} \) depends on four dimensionless parameters: \( \delta, \tau, k, X \).

Especially noticeable influence of the phase difference \( \delta \), so that its value can be controlled by the output of a heat engine: if \( 0 < \delta < \pi \), \( A > 0 \), that is, the system operates as an engine, with \( \pi < \delta < 2\pi \), \( A < 0 \) as a refrigerating machine. Let us call the dependence of the dimensionless work from the phase difference between the phase characteristics. The maximum value of work the value of \( \delta \), which begins high, as does the kind of phase characteristics depend on the values of the three remaining parameters. Therefore, to characterize the influence of these parameters, it is convenient to represent in the graph the family of phase curves depending on any one of the three parameters at constant values of the other two. The following graph shows the family of phase characteristics at various values of the parameter \( k \).

Dependence of work on the phase difference
Figure 2 − Dependence of the phase characteristic from the relationship of the volume $k = V_C/V_E$ when $k \leq 1$. The arrow shows the sequence change of the curve in the collection at the specified change to the parameter $k$

The graph shows the influence of the phase difference $\delta$ to the value of the work per cycle. Given that the value of $\delta$ is technically easy to change in the process engine, the parameter can be used for power control of Stirling engines.

Many designs of Stirling engines vary the ratio of volume expansion and contraction $k = V_0/V$. 

Optimization of work per cycle on the parameter $k$

Figure 3 − Optimization of Stirling engine at the volume factor $k$. for different values of $\tau$ with constant value of $X$. Curves are shown $A_m(k)$ (in dimensionless form) and $\delta_m(k)$. Round markers indicate $A_{opt}(k_{opt})$ и $\delta_{opt}(k_{opt})$

From the previous picture you can see that the value of that $\delta = \delta_m$ maximizes value $A$ is different for different values of $k$. This value, as well as the value $A_{opt}$, depend on two other parameters: $\tau = \frac{T_1}{T_2}$

and $X = \frac{V_D}{V_0}$.

Given that the value of $\delta$ can be adjusted in the process of operation, while, as $k$ and $X$ are rigidly connected with the design of the engine, and the value of $\tau$ associated with the temperature regime of work, can also be considered constant, the optimization of $k$ can be realized as follows: by setting a specific value of $\tau$ and $X$, optimize And for each value of $k$ from a fairly wide range in the
neighborhood of the optimum, identifying $A_m(k)$ and $\delta_m(k)$, and then, from the obtained values to determine the optimum $k$.

The following is the result of the program MATHLAB, which uses the previously described function to perform such optimization and output in a graphic window graphics $A_m(k)$, $\delta_m(k)$ values $A_{opt}$, $\delta_{opt}$ and the corresponding annotations.

It follows from the figure that the optimum value of $k$ in advance should be selected based on the expected temperature range of the engine. So for $\tau=3$, which corresponds to $T_1=900$ K (627 °C), $T_2=300$ K (27 °C), $k_{opt}=1,63$, while for $\tau=1,2$, which corresponds to the temperature of the heater $T_1=360$ K (87 °C) at the same temperature of the refrigerator. The last case corresponds to using low-grade heat of hydrothermal reservoirs.

Each Stirling engine there is a dead volume $V_D$, which applies to the volume of the heater, regenerator, cooler ballast and other areas where the working fluid is not involved in the processes of compression-expansion. When designing a Stirling engine, it is important to have an idea about the impact of the proportion of dead volume to the engine. The following two drawings, made on the basis of numerical calculations explain this effect.

![Figure 4](image4.png)

**Figure 4** - Dependence of the phase characteristic of the proportion of dead volume

The figure shows that with increasing dead volume, the optimal phase shift varies almost from $0$ to $\frac{\pi}{2}$, and at zero volume, the maximum point is unstable. With a small spontaneous decrease in $\delta$ the process can go from positive to negative (the engine mode switches to the mode of refrigerating machine). From this circumstance we conclude that, although the growing proportion of the dead volume decreases maximum work, yet some of it is necessary for the stable operation of the DS.

Figure 5 shows in more detail the dependence of the optimized $\delta$ in the work and optimizes the phase difference of the proportion of dead volume $X$. 

![Figure 5](image5.png)
From figure 5 it is seen that the increase in the share of the dead volume significantly reduces engine performance. When using DS with various sources of thermal energy, it is important to know the effect of the temperature difference between the heater and the refrigerator on engine performance. In dimensionless form, the temperature difference relative to ambient temperature (refrigerator) is expressed by the parameter $\tau$. To obtain the dependence from the temperature difference, an approach was used similar to that used to obtain the graphs of Fig. 5: for each $\tau$ of a certain range this value was optimized on $\delta$ and the points obtained $(A_{opt}, \tau)$ and $(\delta_{opt}, \tau)$ displayed as curves on the chart. The dependence of the efficiency of the engine from the temperature difference in the dimensionless form shown in the following image.

Figure 6– the dependence of the work per cycle from the temperature difference

From figure 6 it is seen that for given $k$ and $X$, the work grows almost proportional to the difference of temperatures at an almost constant optimal $\delta$.

**Conclusions:**

As part of the local energy mix should include a Converter of thermal energy into electrical energy - generator Stirling – compact generator, combined with a Stirling engine (DS) have a number of
It is shown that with perfect regeneration and no heat losses, the efficiency of DS coincides with the efficiency of a Carnot cycle, which is the maximum possible.

It is shown that the work per cycle is proportional to the amount of substance of the working fluid, and thereby the pressure at ambient temperature.

A mathematical model of the cycle of Schmidt on the basis of which is composed of a program written in MATLAB that enables four input parameters to analyze the work of DS to determine a suitable combination of design parameters and their influence on the efficiency of the engine. The results of such research.

Found that of the four input parameters, the only parameter that can be easily changed during operation and which effectively affects the operation of the engine, is the phase difference \( \delta \). The dependence of the cycle work from \( \delta \), called the work phase characteristics, clearly illustrates the mode of operation of the DS. Given families of phase characteristics for different modes of operation.

The paper presents the results of optimization for different temperature parameters, corresponding to high and low potential sources of heat for various values of the ratio of the volume of the compression chamber to the expansion chamber (\( k \)). The optimal value of this parameter depends on the conditions under which the calculated engine.

The results of the corresponding calculations, from which it is concluded that the decrease in the proportion of dead volume (\( X \)) leads to a significant increase in efficiency, however, too small a dead volume can lead to loss of stability of engine operation.

It is shown that the maximum in \( \delta \), the work per cycle at constant values of \( k \), \( X \) is almost proportional to the temperature difference between the heater and the fridge.

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