The application of response surface method to optimization of precision ball end milling

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Abstract. This paper is focused on the multi criteria optimization of precision ball end milling process of hardened 55NiCrMoV6 steel. The proposed method enables the selection of optimal input parameters which affect the minimization of cutting forces and vibrations signals, as well as the maximization of process efficiency. The experiment includes the measurement of forces and vibrations during the milling tests with variable input parameters. Ultimately, the optimization of the ball end milling process with the application of response surface method is carried out.

1 Introduction

Ball end milling of molds and dies made of hardened alloy steels is very often conducted as a finish process, which consequently imposes restrictive requirements on machined surface’s quality and tool’s condition. The surface roughness of the machined surfaces can be affected by the kinematic-geometric parameters, frictional effects in the tool-work material inter-face [1], process stability [2], plastic-elastic deformations of work material induced by the ploughing mechanism [3, 4], as well as the thermal phenomena in the cutting zone [5-7]. However, one of the most significant factors affecting milled surface quality is tool-work piece relative displacement (vibrations) [8].

The vibrations are mainly induced by the cutting forces and milling process kinematics, and they are sensitive to the selected machining parameters and tool’s slenderness [9]. In addition, during ball end milling of curvilinear surfaces, the process dynamics is significantly affected by the surface inclination angle [10, 11]. Thus, the selection of cutting parameters and milling strategy which enable the reduction of forces and vibrations with the simultaneous maximization of process productivity is a significant task. In order to achieve it, the machining process optimization procedure can be applied.

The state of the art reveals that machining process’ optimization objectives include mainly surface roughness [12], cutting forces [13], tool life [14] and tool wear [15]. The most popular optimization approaches are based on the determination of Taguchi’s signal-to-noise ratio, grey relational analysis and response surface method. Durakbasa et al. [16] applied the Taguchi method for the optimization of process parameters and different
coating materials, combined with different tool radiiases to obtain the minimum surface roughness values during end milling process of AISI H13 hot work steel in dry cutting conditions. Zhou et al. [17] carried out the optimization of multi-axis ball-end milling of Ni-based superalloy Inconel 718 with the application of grey relational analysis. The purpose of the study was to simultaneously obtain minimum surface roughness and maximum compressive residual stress by optimizing the inclination angle, cutting speed, and feed. Masmiati et al. [18] applied the response surface method (RSM) to the optimization of surface integrity after ball end milling of S50C steel. Through the analysis, it was found that minimum residual stress and cutting force can be achieved during up milling, by adopting the MQL-SiO2 nanolubrication system.

Despite the comprehensive research regarding the optimization of milling, the minimization of forces and vibrations with the simultaneous maximization of process efficiency has not been reported in the literature. Therefore, this study is focused on the optimal selection of surface inclination angle $\alpha$ and feed per tooth $f_z$, in order to minimize the forces and vibrations generated during ball end milling and maximize the process efficiency. The proposed optimization procedure is carried out with the application of response surface method.

2 Experimental details

2.1 Work and tool materials

The experiments have been conducted on hardened alloy steel 55NiCrMoV6 sample with average hardness of 58 HRC. The selected tool was monolithic ball end mill made of fine-grained sintered tungsten carbide (WC) with diameter $D=10$ mm and number of teeth $z=2$. The tool’s geometry was as follows: $\gamma_o = -15^\circ$, $\lambda_s = 30^\circ$, $\alpha_o = 6^\circ$. Experiments were carried out on 5-axes CNC milling workstation (DMU 60monoBLOCK).

2.2 Research range and method

Experimental milling tests involved the measurement of cutting force components and acceleration of vibrations carried out in the range of variable surface inclination angles $\alpha$ and feed per tooth $f_z$. In order to obtain surface inclination during the milling process, tool’s rotational axis was inclined against the work piece in the range of various $\alpha$ angles (Fig. 1).

In all investigated cases tool’s effective diameter was lower than the value of pick feed – $D_{ef}<b_r$. The milling parameters applied in the research were depicted in table 1.
Table 1. Design of experiment.

| Cutting speed $v_c$ [m/min] | Depth of cut $a_p$ [mm] | Pick feed $b_r$ [mm] | Inclination angle $\alpha$ [°] | Feed per tooth $f_z$ [mm/tooth] | Overhang $l$ [mm] |
|----------------------------|------------------------|---------------------|-----------------------------|-----------------------------|------------------|
| 200                        | 0.3                    | 3.5–3.9             | 0 – 60                      | 0.05 – 0.2                  | 63               |
| 20 interval                |                        |                     | 20 interval                 | 0.05 interval               |                  |

The forces and accelerations of vibrations were measured in machine tool’s coordinates, in the three directions ($A_x$ [m/s²], $F_x$ [N] – feed normal direction, $A_y$ [m/s²], $F_y$ [N] – feed direction, $A_z$ [m/s²], $F_z$ [N] - thrust direction) with the application of the 3-directional piezoelectric Kistler 9257B force dynamometer and Brüel&Kjaer 4321 accelerometer. The piezoelectric sensors were connected through amplifiers and band-pass filters to a data acquisition computer. The dedicated software was applied for the calculation of signal’s peak values and filtration carried out with a low-pass filter.

2.3 Optimization based on response surface method

The multi-criteria optimization procedure was applied in the research. This optimization is based on the transformation of the approximated output factors’ values into the singular value of the resultant total utility (desirability) $U_{tot}$. This function is based on the idea that the "quality" of a process that has multiple quality characteristics (multi-criteria), with one of them outside of some "desired" limits, is completely unacceptable. The method finds operating conditions that provide the "most desirable" response values. Thus, the problem of simultaneous optimization of many output factors is simplified to the determination of one output parameter, which maximizes total response utility.

In the carried out research, the following optimization criteria were adopted:
- cutting force and acceleration of vibration components ($A_x$, $A_y$, $A_z$, $F_x$, $F_y$, $F_z$),
- material removal rate $Q_v$.

In the first part, the milling tests conducted on the basis of the central composite experimental design (with 16 repetitions and 1 block) were carried out. The material removal rate was calculated on the basis of equation:

$$Q_v = a_p \cdot b_r \cdot v_f$$  \hspace{1cm} (1)

The obtained milling tests’ results were the starting point to the definition of response utility profile, formulated for the investigated output factors ($F_i$, $A_i$, $Q_v$). During this optimization stage, all output factors’ values were assigned to the partial utilities, which values were contained in the range: <0, 1>. The equal weights of all responses have been selected. The response utility profiles for the responses were formulated in accordance with the recommendations presented in [19], on the basis of the following expressions:

$$Q_v \leq 3000 \text{ mm}^3/\text{min} \rightarrow U_{(Q_v)} = 0;$$  \hspace{1cm} (2)
$$3000 \text{ mm}^3/\text{min} < Q_v \leq 4000 \text{ mm}^3/\text{min} \rightarrow 0 < U_{(Q_v)} \leq 0.5;$$  \hspace{1cm} (3)
$$4000 \text{ mm}^3/\text{min} < Q_v < 5000 \text{ mm}^3/\text{min} \rightarrow 0.5 < U_{(Q_v)} < 1;$$  \hspace{1cm} (4)
$$Q_v \geq 5000 \text{ mm}^3/\text{min} \rightarrow U_{(Q_v)} = 1.$$  \hspace{1cm} (5)

$$A_{x,y,z} \leq 50 \text{ m/s}^2 \rightarrow U_{(A)} = 1;$$  \hspace{1cm} (6)
$$50 < A_{x,y,z} \leq 100 \text{ m/s}^2 \rightarrow 0.5 \leq U_{(A)} < 1;$$  \hspace{1cm} (7)
$$100 < A_{x,y,z} \leq 150 \text{ m/s}^2 \rightarrow 0 < U_{(A)} < 0.5;$$  \hspace{1cm} (8)
\[ A_{x,y,z} \geq 150 \text{ m/s}^2 \rightarrow U(A) = 0; \]  
\[ F_{x,y,z} \leq 100 \text{ N} \rightarrow U(F) = 1; \]  
\[ 100 \text{ N} < F_{x,y,z} \leq 150 \text{ N} \rightarrow 0.5 \leq U(F) < 1; \]  
\[ 150 < F_{x,y,z} < 200 \text{ N} \rightarrow 0 < U(F) < 0.5; \]  
\[ F_{x,y,z} \geq 200 \text{ N} \rightarrow U(F) = 0. \]

After the definition of response utility profiles, the optimal solution’s searching procedure was applied. This procedure included the mesh definition of the input variables \((f_z, \alpha)\). Subsequently, the output factors’ predictive values were calculated for the all mesh points and as a result, the partial utility values were received. In order to determine the total utility value, the mean geometric values of partial utilities were calculated in the each mesh point. The next step involved the formulation of the total utility model, with the application of spline function. After the generation of this model, the optimal solution searching algorithm was adopted. The optimal solution determines the values of input variables, corresponding to maximal value of the total utility. In order to achieve it, the simplex method was applied. Ultimately, the optimal input parameters’ \((f_{z,\text{opt}}, \alpha_{\text{opt}})\) values were received.

![Fig. 2](image_url)  
**Fig. 2.** Peak values of forces and accelerations of vibrations in function of \(\alpha\) and \(f_z\).

### 3 Results and discussion

The Figure 2 and Table 2 depict the measured peak values of forces \((F_x, F_y)\) and accelerations of vibrations \((A_x, A_y)\) in function of surface inclination angle \(\alpha\) and feed per tooth \(f_z\). Figures 2a,b reveal that increasing feed per tooth, \(f_z\), induces an increase in cutting
forces, which is a typical relation found during metal cutting. Furthermore, the most intense effect of \( f_z \) factor on forces is found for the lower investigated surface inclinations (\( \alpha \leq 20^\circ \)).

It can be also seen that the growth of the \( \alpha \) angle induces the decline in force values. According to [13], during slot milling (\( \alpha = 0 \)), cutting speed near ball end mill’s rotational axis is very low. Consequently, the large volume of the material which flows toward the cutter is not being transformed into the chip, but as a result of the large elastic-plastic deformations is pressed under the tool’s flank face. This phenomenon is followed by the generation of the relatively high ploughing forces, which consequently contribute to the growth of cutting force components. Figures 2c and 2d reveal also that the qualitative influence of the \( \alpha \) and \( f_z \) factors on the accelerations of vibrations is similar to that observed for the cutting forces. This observation is attributed to the proportionality of tool’s displacements to the value of force acting on the tool’s working part, according to the milling process’ dynamic vibration model [19].

The resultant total utility function for the surface inclination angle \( \alpha \) and feed per tooth \( f_z \) is presented in Figure 3. It can be concluded (Figure 3) that the optimum cutting parameters are for the higher ranges of inclination angle and the feed per tooth. The maximal value of utility function is achieved with feed per tooth \( f_{z, \text{opt}} = 0.163 \text{ mm/tooth} \) and surface inclination angle \( \alpha_{\text{opt}} = 45^\circ \). The application of these optimal cutting parameters gave cutting forces lower than 170 N, acceleration of vibrations lower than 55 m/s\(^2\) and material removal rate higher than 3820 mm\(^3\)/min.

**Table 2.** Experimental results.

| \( \alpha \), \(^\circ\) | \( f_z \), mm | \( F_x \), N  | \( F_y \), N  | \( F_z \), N  | \( A_x \), m/s\(^2\) | \( A_y \), m/s\(^2\) | \( A_z \), m/s\(^2\) |
|-----------------|--------|--------|--------|--------|----------------|----------------|----------------|
| 0               | 0.05   | 292    | 812    | 541    | 111            | 207            | 188            |
| 0               | 0.1    | 560    | 1276   | 970    | 162            | 318            | 252            |
| 0               | 0.15   | 771    | 1728   | 1337   | 200            | 424            | 335            |
| 0               | 0.2    | 881    | 2000   | 1656   | 237            | 517            | 383            |
| 20              | 0.05   | 168    | 220    | 81     | 24             | 50             | 59             |
| 20              | 0.1    | 230    | 274    | 149    | 46             | 91             | 99             |
| 20              | 0.15   | 307    | 403    | 266    | 76             | 142            | 142            |
| 20              | 0.2    | 363    | 663    | 451    | 107            | 186            | 182            |
| 40              | 0.05   | 91     | 120    | 62     | 20             | 20             | 28             |
| 40              | 0.1    | 153    | 186    | 65     | 30             | 30             | 37             |
| 40              | 0.15   | 211    | 244    | 87     | 45             | 39             | 65             |
| 40              | 0.2    | 277    | 292    | 120    | 57             | 53             | 88             |
| 60              | 0.05   | 65     | 119    | 93     | 15             | 14             | 23             |
| 60              | 0.1    | 94     | 143    | 89     | 19             | 15             | 27             |
| 60              | 0.15   | 130    | 159    | 92     | 33             | 26             | 47             |
| 60              | 0.2    | 156    | 213    | 98     | 47             | 44             | 48             |

**Fig. 3.** Total utility function for the \( \alpha \) and \( f_z \).

**4 Conclusions**

On the basis of the carried out investigations, the following conclusions are formulated:

• The \( \alpha \) and \( f_z \) factors have qualitatively similar effect on forces and vibrations’ values generated during precision ball end milling of hardened steel. Furthermore, the growth of the \( \alpha \) angle induced the decline in force and vibration values, whereas the increase of \( f_z \) caused the growth of the measured quantities.

• On the basis of the carried out optimization procedure, the following optimal milling parameters were obtained: \( f_{z, \text{opt}} = 0.163 \text{ mm/tooth} \), \( \alpha_{\text{opt}} = 45^\circ \).

• The selection of the optimized \( \alpha \) and \( f_z \) can result in the growth of productivity and reduction of dynamical load during milling, and thus improvement of machined surface quality and tool’s life.
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