Time Variations in the Scale of Grand Unification

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Abstract

We study the consequences of time variations in the scale of grand unification, $M_U$, when the Planck scale and the value of the unified coupling at the Planck scale are held fixed. We show that the relation between the variations of the low energy gauge couplings is highly model dependent. It is even possible, in principle, that the electromagnetic coupling $\alpha$ varies, but the strong coupling $\alpha_3$ does not (to leading approximation). We investigate whether the interpretation of recent observations of quasar absorption lines in terms of time variation in $\alpha$ can be accounted for by time variation in $M_U$. Our formalism can be applied to any scenario where a time variation in an intermediate scale induces, through threshold corrections, time variations in the effective low scale couplings.

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I. INTRODUCTION

Recent observations of quasar absorption lines have been interpreted as indicating variation in the fine structure constant on cosmological time scales \cite{1,2}:

\[
\frac{\delta \alpha}{\alpha} = (-0.57 \pm 0.10) \times 10^{-5} \text{ at } z \approx 0.2 - 3.7.
\]  

In the context of grand unified theories (GUTs), such a time variation in \( \alpha \) may be related to time variations in other gauge couplings and, in particular, to variations in the strong coupling constant \( \alpha_3 \) or, equivalently, in the QCD scale \( \Lambda_{\text{QCD}} \). Such related variations affect various observables in interesting ways \cite{3,4,5,6,7,8,9,10}.

Most previous works have assumed that the variation in \( \alpha \) is induced by a variation in the unified coupling at the Planck scale, \( \alpha_U(M_{\text{Pl}}) \) \cite{3,4,5,6,7,10}. The motivation for such studies comes mainly from string theories, where the value of \( \alpha_U(M_{\text{Pl}}) \) is determined by the value of the dilaton and other fields. We are here interested in another possibility, namely that the variation in \( \alpha \) is induced by a variation in the GUT scale \( M_U \), with \( M_{\text{Pl}} \) and \( \alpha_U(M_{\text{Pl}}) \) held fixed. In ref. \cite{11}, it is argued that if grand unification (in the sense of unification of the gauge interactions in a four-dimensional gauge group) arises within string theory, the GUT scale is likely to be a modulus.\(^1\) Such a situation can also arise in the framework of field theories, where symmetry breaking scales are determined by vacuum expectation values (VEVs) of scalar fields, but coupling constants are not. In either case, understanding why the modulus has the requisite properties is a difficult problem, which we will not address. We will show that the relation between \( \delta \alpha/\alpha \) and \( \delta \Lambda_{\text{QCD}}/\Lambda_{\text{QCD}} \) is highly model dependent. In particular, in contrast to the case that both are induced by \( \delta \alpha_U/\alpha_U \neq 0 \), it may happen that \( \delta \Lambda_{\text{QCD}}/\Lambda_{\text{QCD}} \ll \delta \alpha/\alpha \) (though this is not the generic case).

We are aware of two previous works that are closely related to ours. In ref. \cite{9}, the consequences of variations of \( M_U \) have been analyzed. It was assumed, however, that \( \alpha_U(M_U) \) remains fixed. It is then necessary that \( \alpha_U(M_{\text{Pl}}) \) also varies in a correlated way. We think that such a scenario is less plausible than the one that we investigate. In ref. \cite{8}, the consequences for the low energy couplings of variations in the mass of particles have been analyzed. Conceptually, our work takes a similar direction. In their discussion of grand unified theories, ref. \cite{8} examine a set of models in which only the masses of a subset of

\(^1\) Of course, in string theory, coupling unification does not necessarily require a unified gauge group.
GUT mass particles vary in time; in particular, only the masses of color-neutral, charged particles shift. In contrast, our basic assumption is that all heavy masses (of GUT gauge bosons, fermions and scalars) are proportional to a single breaking scale, $M_U$, and it is the variation of this scale that we study.\(^2\)

In this work we focus on the resulting relations between the low energy couplings. We do not concern ourselves here with the origin of the scale variation, nor with the connection to higher scale physics. Discussions of these topics can be found in [8, 12]. In addition, there are fundamental issues we will also not address. As pointed out in [13], it is difficult to understand, in the framework of local quantum field theory, how couplings could vary by so large an amount, without an enormous variation in the vacuum energy. This problem is not significantly different if it is changes in the unification scale which are responsible for this variation than if the coupling at the Planck or unification scale changes.

The plan of this paper is as follows. In section II we present the formalism for the variation of low energy couplings induced by variations in the scale of threshold corrections. In section III we obtain the relations between the variation of relevant cosmological observables and argue that, within our framework, they depend on a single parameter. In section IV we apply the formalism to the experimental data. We use it to constrain the parameter of section II. In section V we test specific models of grand unified theories with this constraint. We show that a variation in the GUT scale is unlikely to explain the claimed variation in $\delta \alpha/\alpha$. In section VI we explain how our formalism and results apply much more generically, to any model where there is a time variation in a scale where threshold corrections take place. We also mention some further subleading effects which might be non-negligible. We conclude in section VII.

II. FORMALISM

We assume that the variation in couplings is due to a variation in a single intermediate scale, $M_U$, where GUT threshold corrections take place. The gauge couplings $\alpha_i(Q)$ at a

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\(^2\) To understand the difference, one may think of the consequences of a variation in the VEV of the single Higgs doublet of the Standard Model, which would shift all masses, or a shift in the VEV of a hypothetical Higgs triplet, which would shift only neutrino masses.
scale $Q < M_U$ are given to one loop by the RGE solution
\[
\alpha_i^{-1}(Q) = \alpha_i^{-1}(M_U) + \frac{b_i}{2\pi} \ln \left( \frac{Q}{M_U} \right),
\]
where $\alpha_i(M_U)$ is the unified coupling at the scale $M_U$ and $b_i$ is the beta function coefficient (between $M_U$ and $Q$) for $\alpha_i$. The scale $M_U$ is related to the mass scale of heavy degrees of freedom that are integrated out, particularly the heavy GUT gauge bosons. Since the particle content of the theory below and above $M_U$ is different, the RGE coefficient $b_i$ changes at $M_U$. In particular, one obtains
\[
\alpha_i^{-1}(M_U) = \alpha_i^{-1}(M_{Pl}) + \frac{b_U}{2\pi} \ln \left( \frac{M_U}{M_{Pl}} \right) + \frac{b_i}{2\pi} \ln \left( \frac{Q}{M_U} \right),
\]
where $b_U$ is the beta function coefficient for the unified group. Combining (2) and (3) we write
\[
\alpha_i^{-1}(Q) = \alpha_i^{-1}(M_{Pl}) + \frac{b_U}{2\pi} \ln \left( \frac{M_U}{M_{Pl}} \right) + \frac{b_i}{2\pi} \ln \left( \frac{Q}{M_U} \right).
\]
Now we allow the GUT scale, $M_U$, to change by an amount of $\delta M_U$, while holding $\alpha_U(M_{Pl})$ and $M_{Pl}$ fixed. The resulting variation in the low scale couplings can be calculated from (4) (a similar derivation, for $\alpha_{EM}$, appears in [8]):
\[
\frac{\delta \alpha_i(Q)}{\alpha_i(Q)} = \alpha_i(Q) \frac{\Delta b_i \delta M_U}{2\pi M_U},
\]
where
\[
\Delta b_i \equiv b_U - b_i.
\]
Using (5), we can now relate the variation in low scale parameters to the variation $\delta M_U$. We focus on two parameters, the electromagnetic coupling $\alpha$ and the QCD scale $\Lambda_{QCD}$:
\[
\alpha^{-1} = \frac{5}{3} \alpha_1^{-1}(0) + \alpha_2^{-1}(0),
\]
\[
\Lambda_{QCD} = M_Z^{23/27} m_b^{2/27} m_c^{2/27} \exp \left( -\frac{2\pi}{9 \alpha_3(M_Z)} \right).
\]
Neglecting effects of threshold corrections below $M_U$ (which are discussed later), we obtain the relations:
\[
\frac{\delta \alpha}{\alpha} = \frac{\alpha}{2\pi} \left( \frac{5}{3} \Delta b_1 + \Delta b_2 \right) \frac{\delta M_U}{M_U},
\]
\[
\frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} = \frac{\Delta b_3}{9} \frac{\delta M_U}{M_U}.
\]
Finally, we use (9) and (10) to relate the variation of the two parameters:

\[
\frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} = \frac{2\pi}{9\alpha} \frac{\Delta b_3}{3\Delta b_1 + \Delta b_2} \frac{\delta \alpha}{\alpha} \\
\equiv R \frac{\delta \alpha}{\alpha} .
\]  

(11)

The factor \( R \approx \frac{0.75}{3\Delta b_1 + \Delta b_2} \) plays a crucial role in our analysis. As we show later, its value is highly GUT-model dependent.

III. OBSERVABLES

In this section, we use (11) to obtain expressions for the variation of several cosmological observables. The primordial \(^4\text{He} \) abundance is given by \([14, 15]\)

\[
Y_4 \equiv \frac{2(n_n/n_p)_{\text{NS}}}{1 + (n_n/n_p)_{\text{NS}}}, \tag{12}
\]

where the ratio of the number density of the neutron to that of the proton at nucleosynthesis is given by

\[
(n_n/n_p)_{\text{NS}} \simeq 0.8 \times e^{-Q/T_D} . \tag{13}
\]

Here \( T_D \simeq 0.8 \text{ MeV} \) is the decoupling temperature and \( Q \) is the mass difference between the neutron and the proton. Since \( T_D \) is a function of \( M_{\text{Pl}} \) and \( G_F \), it is independent of the Standard Model gauge couplings and we therefore neglect its variation. The mass difference \( Q \), on the other hand, depends on both \( \alpha \) and \( \Lambda_{\text{QCD}} \) \([16]\):

\[
Q \simeq 2.05 \text{ MeV} + C\alpha \Lambda_{\text{QCD}} , \tag{14}
\]

where \( C \) is a dimensionless order one parameter. Assuming variations in \( \alpha \) and \( \Lambda_{\text{QCD}} \), the value of \( Q \) is shifted [eq. (14)] from its present-day value of \( Q \simeq 1.29 \text{ MeV} \), and consequently \( (n_n/n_p)_{\text{NS}} \) would be shifted [eq. (13)] from its ‘standard’ value of \( (n_n/n_p)_{\text{NS}} \simeq \frac{1}{7} \). We thus obtain the change in \( Y_4 \) compared to its value calculated with the present values of the coupling constants:

\[
\frac{\delta Y_4}{Y_4} = \frac{1}{1 + (n_n/n_p)_{\text{NS}}} \frac{2.05 \text{ MeV} - Q}{T_D} \left( \frac{\delta \alpha}{\alpha} + \frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} \right) \approx 0.8 \left( 1 + R \right) \frac{\delta \alpha}{\alpha} . \tag{15}
\]
Two other relevant observables are $X$ and $Y$. The $X$ parameter gives the ratio between the hyperfine 21cm neutral hydrogen absorption transition to an optical resonance transition \[7\]:

$$
X \equiv \alpha^2 g_p \left( \frac{m_e}{m_p} \right),
$$

(16)

where $m_e$ is the electron mass, $m_p$ is the proton mass, and $g_p$ is the proton gyromagnetic moment. The $Y$ parameter determines the molecular hydrogen transition in the early universe \[8\]-\[19\],

$$
Y \equiv \frac{m_p}{m_e}.
$$

(17)

The proton mass is proportional, at first order, to $\Lambda_{QCD}$ \[4\]:

$$
\frac{\delta m_p}{m_p} \simeq \frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} = R \frac{\delta \alpha}{\alpha}.
$$

(18)

Assuming that the variations of $m_e$ and $g_p$ are negligible, we obtain:

$$
\frac{\delta X}{X} \simeq 2 \frac{\delta \alpha}{\alpha} - \frac{\delta m_p}{m_p} \simeq (2 - R) \frac{\delta \alpha}{\alpha},
$$

(19)

$$
\frac{\delta Y}{Y} \simeq R \frac{\delta \alpha}{\alpha}.
$$

(20)

IV. CONSTRAINING $R$ WITH EXPERIMENTAL DATA

Values for the observed variation of various observables were obtained through cosmological and nuclear studies. Table \[\text{I}\] summarizes these values (the details can be found in the quoted references). Using the formalism of the previous section, we relate the allowed range for each observable in Table \[\text{I}\] to an (in general, $R$-dependent) allowed variation in $\alpha$. The results are also given in Table \[\text{I}\].

For a given value of $R$, one obtains from Table \[\text{I}\] the implied variation of $\alpha$ at different $z$’s. The most interesting result, however, arises from requiring consistency of the three observables related to the intermediate redshift values. Note that $\delta Y/Y$ and $\delta X/X$ are consistent with each other for any value of $R$, because, at the $2\sigma$ level, they allow $\delta \alpha/\alpha = 0$, for which the value of $R$ becomes irrelevant. The constraints become nontrivial when one considers the claimed signal of $\delta \alpha/\alpha$ [eq. \[\text{I}\]]. The requirement of consistency between the observed $\delta Y/Y$ at $z \approx 2.3$–3 and the observed $\delta \alpha/\alpha$ at $z \approx 0.2$–3.7 favors a range in $R$ that is mostly negative in sign and not very large in magnitude, $R = O(-10)$. The
TABLE I: Cosmological and nuclear values for the variation of various observables and their implications for $\delta \alpha/\alpha$.

| Constraints | Redshift $z$ | Reference | Implied constraint on $\delta \alpha/\alpha$ |
|-------------|--------------|-----------|--------------------------------------------|
| $\frac{\delta Y}{Y} = (-6 \pm 7) \times 10^{-3}$ | $10^{10}$ | [14, 20] | $(-7 \pm 9) \times 10^{-3}/(1 + R)$ |
| $\frac{\delta \alpha}{\alpha} = (-0.57 \pm 0.10) \times 10^{-5}$ | $0.2$–$3.7$ | [1, 2] | $(-0.57 \pm 0.10) \times 10^{-5}$ |
| $\frac{\delta Y}{Y} = (5.7 \pm 3.8) \times 10^{-5}$ | $2.3$–$3$ | [18] | $(5.7 \pm 3.8) \times 10^{-5}/R$ |
| $\frac{\delta X}{X} = (0.7 \pm 1.1) \times 10^{-5}$ | $1.8$ | [17] | $(0.7 \pm 1.1) \times 10^{-5}/(2 - R)$ |
| $\frac{\delta \alpha}{\alpha} = (-0.36 \pm 1.44) \times 10^{-8}$ | $0.1$ | [21, 22] | $(-0.36 \pm 1.44) \times 10^{-8}$ |

requirement of consistency between the observed $\delta X/X$ at $z \approx 1.8$ and the observed $\delta \alpha/\alpha$ at $z \approx 0.2$–$3.7$ favors a range in $R$ that is mostly positive in sign and, again, not very large in magnitude, $R = O(+3)$. The most interesting results follows from fitting simultaneously all three observables. The result of a $\chi^2$ analysis is that the following range of $R$ is favored:

$$-1 \lesssim R \lesssim +6.$$  \hspace{1cm} (21)

Values of $R$ outside this range have a probability that is $\lesssim 5\%$ to yield the observed results. (Note that even the best fit point, with $R \sim +2$, has only a probability $\sim 15\%$ to yield the observed results.)

This result can be understood qualitatively: When $|R|$ is large, a value for $\delta \alpha/\alpha$ implies an even larger value for $\delta \Lambda_{\text{QCD}}/\Lambda_{\text{QCD}}$. This larger variation dominates the shift of nucleon masses. Since $Y$ is proportional to the proton mass while $X$ is inversely proportional to it, their relative variations must be of opposite signs, $\delta Y \delta X \leq 0$. The data, however, indicate that their variations are both biased to be positive. Consequently, only one of them can be consistent with a non-zero variation of $\alpha$. (Which one depends on the sign of $R$.) This situation is avoided only in case that the variation in $\Lambda_{\text{QCD}}$ does not dominate $\delta X/X$ or, in other words, in case of a small $R$.

To summarize, eq. (21) gives the consistent range for the parameter $R$ in theories where the variation of the low energy coupling constants is dominated by threshold effects at a single varying scale.
V. IMPLICATIONS FOR VARIOUS GUT MODELS

In this section we apply our results to specific models. We focus on the minimal supersymmetric standard model (MSSM) embedded in a grand unified theory (GUT), in which the varying scale is the scale of breaking of the unified group, $M_U$. In general, this is the mass scale of the GUT gauge supermultiplets, and we assume that it characterizes also the masses of all the heavy chiral supermultiplets. The one loop beta function coefficients below $M_U$ are those of the MSSM:

$$b_1 = \frac{33}{5}, \quad b_2 = 1, \quad b_3 = -3.$$  \hspace{1cm} (22)

Above the GUT scale, there is a single coefficient common to all gauge couplings, $b_U$. Thus, the ratio $R$ of eq. (11) is given by

$$R = \frac{2\pi}{9\alpha} \frac{b_U + 3}{\frac{3}{2}b_U - 12}.$$  \hspace{1cm} (23)

Before we discuss specific GUT models, let us make some general observations:

1. For asymptotically large (positive or negative) values of $b_U$, we obtain

$$|b_U| \gg 1 \implies R \to \frac{\pi}{12\alpha} \simeq +36.$$  \hspace{1cm} (24)

Large negative values of $b_U$ are not relevant, as demonstrated by the very conservative lower bounds set by examining the contributions from the gauge supermultiplet and from the minimal matter representations that are necessary to accommodate the MSSM fields:

$$b_U[SU(5)] > -8, \quad b_U[SO(10)] > -17.$$  \hspace{1cm} (25)

On the other hand, realistic models often have a large, positive $b_U$ and predict $R = \mathcal{O}(40)$.

2. Negative values of $R$ are achieved only for a very limited range of $b_U$:

$$R < 0 \text{ for } -3 < b_U < +\frac{9}{2}.$$  \hspace{1cm} (26)

3. For a hypothetical value of $b_U = \frac{9}{2}$, $\alpha$ does not vary (to the approximation that we use):

$$b_U = \frac{9}{2} \implies |R| \to \infty.$$  \hspace{1cm} (27)

This result explains the change of sign of $R$ between models with $b_U = 4$ and $b_U = 5$, and the very large magnitude of $R$ in these models, $R \simeq -502(+574)$ for $b_U = 4(5)$.
4. For $b_U = b_3 = -3$, $\Lambda_{QCD}$ does not vary (to the approximation that we use):

$$b_U = -3 \implies R = 0. \quad (28)$$

In general, the $O(100)$ factor in eq. (11), which is due to the exponential dependence of $\Lambda_{QCD}$ on $\alpha_3$, leads to a large $R$. In order to overcome this factor and obtain $|R| \sim O(1)$, the threshold correction of $\alpha_3$ needs to be highly suppressed, i.e., $\Delta b_3 \lesssim 0.01 \Delta b_{1,2}$. For MSSM GUT models with varying $M_{GUT}$, this would imply $-3.5 \lesssim b_U \lesssim -2.5$. In particular, when $b_U$ is shifted by one unit from the special value of $-3$, one already obtains rather large values of $|R|$. $R = +4.2(-5.5)$ for $b_U = -4(-2)$. We learn that, if $R$ is to be within the range of eq. (21), the only acceptable integer values for $b_U$ are $-3$ and $-4$.

We now move on to discuss several examples of specific GUT models. The $b_U$ coefficients and the resulting $R$-factors of various models are given in Table I. Each model is defined by its representation content. (Note that, for example, $n_5$ gives the number of SU(5) fundamentals plus the number of antifundamentals.) All SU(5) models have the quark and lepton fields in three generations of $10 + \bar{5}$ and the Higgs fields related to electroweak symmetry breaking in $5 + \bar{5}$. The minimal model has, in addition, a single $24$ to break $SU(5) \rightarrow G_{SM}$. This model is, however, excluded by combining constraints from coupling unification and from proton decay [23]. (It may still be viable with a special flavor structure of the supersymmetric mixing matrices [24]...) A viable, though fine-tuned, model can be constructed by adding another $5 + \bar{5}$ pair [23, 25]. To naturally induce doublet-triplet splitting, the $24$ has to be replaced by, at least, $50 + 50 + 75$ [26, 27, 28].

All SO(10) models have the quark and lepton fields in three generations of $16$. The minimal Higgs sector has $45 + 16 + 1\bar{6}$ to break $SO(10) \rightarrow G_{SM}$ and a single $10$ for electroweak symmetry breaking [29]. Note that we assume here that the breaking of $SO(10) \rightarrow G_{SM}$ is done in one step, and that the VEVs of all GUT-breaking Higgs fields vary together. The investigation of a two-step breaking, $SO(10) \rightarrow SU(5) \rightarrow G_{SM}$, with independent variation of each scale, will be presented in future work.\footnote{In case that the scale of $SO(10) \rightarrow SU(5)$ breaking varies but the scale of $SU(5) \rightarrow G_{SM}$ breaking does not, the result is effectively a variation in the unified coupling constant at the $SU(5)$ breaking scale. This result shows how our formalism can be applied also to models of time variation in $\alpha_U$: we should take $\Delta b_1 = \Delta b_2 = \Delta b_3$, which gives $R \approx 36$.}

Models where doublet-triplet splitting is achieved naturally require an additional $10$ and either an additional $16 + 1\bar{6}$ pair [30] or a
more complicated Higgs sector that includes several $45$ and $54$ multiplets \cite{31}. In each of these models, one could have a $126 + \overline{126}$ pair in place of a $16 + \overline{16}$ pair. Such a replacement increases $b_U$ by 66 and gives $R = \mathcal{O}(40)$.

| TABLE II: Various GUT models, their beta function coefficient $b_U$ and the resulting factor $R$. |
|-----------------------------------------------|
| SU(5)  | $n_5$ | $n_{10}$ | $n_{24}$ | $n_{50}$ | $n_{75}$ | $b_U$ | $R$   |
|--------|-------|---------|---------|---------|---------|-------|-------|
| 5      | 3     | 1       |         |         |         | $-3$  | 0     |
| 7      | 3     | 1       |         |         |         | $-2$  | $-5.5$|
| 5      | 3     |         | 2       | 1       |         | $+52$ | $+41.5$|
| SO(10) | $n_{10}$ | $n_{16}$ | $n_{45}$ | $n_{54}$ | | $b_U$ | $R$   |
| 1      | 5     | 1       |         |         |         | $-5$  | $+7.6$|
| 2      | 7     | 1       |         |         |         | 0     | $-23.9$|
| 2      | 5     |         | 3       | 2       |         | $+36$ | $+44.4$|

Other models of interest are reviewed in, for example, ref. \cite{32} and lead to similar results. We conclude that most GUT models give $|R| \gg 1$. A remarkable exception is given by models with $b_U = -3$. This possibility is realized in the minimal $SU(5)$ model which, without a very special flavor structure, is, however, phenomenologically excluded. Other models with $b_U = -3$ can be constructed within the framework of larger groups. There is, however, little motivation to consider such a scenario (which would require fine-tuning for the doublet-triplet splitting) for the sole purpose of deriving the desired $b_U$. We conclude that, within the framework of realistic GUT models that avoid fine-tuning, our mechanism predicts $|R| \gg 1$.

When this result is confronted with eq. (21), one is led to the conclusion that supersymmetric GUT models with a varying GUT scale are unlikely candidates to explain a variation in $\alpha$ of the size given in eq. (11).

VI. GENERALIZATION AND LIMITATIONS

Our formalism and many of our results have a much broader applicability than the $M_U$-variation in GUT models that we have focussed on. The formalism applies to any variation in a single intermediate scale where threshold corrections take place, $\mu_{\text{th}}$. Suppose that the
couplings are held fixed at some fixed high energy scale $\mu_0$. Define $b_i$ to be the one-loop beta function coefficients between $\mu_{\text{th}}$ and a low scale $Q$, while $b'_i$ are the corresponding coefficients between $\mu_0$ and $\mu_{\text{th}}$. We now define, as a generalization of eq. (6),

$$\Delta b_i \equiv b'_i - b_i. \quad (29)$$

With this definition, eq. (3) for $\delta \alpha/\alpha$, eq. (10) for $\delta \Lambda_{\text{QCD}}/\Lambda_{\text{QCD}}$, and eq. (11) for the ratio $R$, still apply.\(^4\) Similarly, the constraints on $\delta \alpha/\alpha$ in Table I and the allowed range for $R$ in eq. (21) hold in these more general circumstances.

An important comment is now in order: In principle, the variation $\delta \mu_{\text{th}}$ may induce a variation in all couplings at lower scales. In the most general case, this includes a variation in the masses of all the particles which are lighter than $\mu_{\text{th}}$. A variation of these masses could be induced by two sources:

1. The parameters that determine a particle mass, such as Yukawa couplings, may have threshold corrections at $\mu_{\text{th}}$. In this case, a variation analogous to (3) results. To calculate this effect, one should use the (linearized) beta function of these parameters below and above the scale $\mu_{\text{th}}$.

2. The variation in the gauge couplings, $\alpha_i$, below $\mu_{\text{th}}$, leads to variations in the low scale masses. To calculate this effect, one should use the RGE of the mass parameters below $\mu_{\text{th}}$.

When running the gauge couplings from $\mu_{\text{th}}$ to some low scale $Q$, all particles with masses $Q < m < \mu_{\text{th}}$ are integrated out. For example, in an MSSM GUT model, with $Q \sim m_b$ and $\mu_{\text{th}} = M_{\text{GUT}}$, we integrate out all supersymmetric particles at a scale $M_{\text{SUSY}}$ as well as heavy SM particles ($t, Z, W^\pm$). This integration out would usually introduce additional threshold corrections, which have small effects on the parameters at $m_b$.

We are mainly interested in small variations of $\alpha$ and $\Lambda_{\text{QCD}}$. As explained above, the variation $\delta \mu_{\text{th}}$ induces variations in all intermediate scales below $\mu_{\text{th}}$. But these variations induce, by threshold corrections at these scales, further variation in gauge couplings. The

\(^4\) One has to be careful about the normalization of $\alpha_1$ in non-GUT models. For example, if we take the conventional definition of $U(1)_Y$ in the Standard Model, the factors of $\frac{2}{3}$ in eqs. (7), (9) and (11) have to be omitted, while for eq. (22) one should use $b_1 = 11$.\(^{11}\)
whole process may be pictured as a ‘chain reaction’, with a final effect that could be significant, even if the individual variations at each intermediate scale are small. We postpone the detailed study of this effect to future work.

Here, we would only like to emphasize that when $R$ is large, as is the case in most of the GUT models that we have investigated, the variation of $\Lambda_{\text{QCD}}$ is the dominant effect at low energies, and it is well justified to ignore variations in couplings other than the gauge couplings. For small $R$, however, the modification due to threshold corrections of other parameters can become important. Thus, when we think of $R = 0$ models, one should not conclude that $\alpha$ would vary with $\Lambda_{\text{QCD}}$ remaining strictly fixed. Very likely, $\Lambda_{\text{QCD}}$ varies due to the effects of lower thresholds and of higher loops. But the effect is expected to be small [$R = \mathcal{O}(1)$] and our qualitative conclusions would not change.

VII. CONCLUSIONS

A variation in a physical scale in which threshold corrections take place leads to variations in low scale observables. Current experimental data constrain the relations between the threshold corrections of the three different gauge couplings. In particular, consistency between the claimed variation $\delta \alpha/\alpha$ [eq. (I)] and the allowed ranges for $\delta X/X$ and $\delta Y/Y$ [Table I] give

$$-1 \lesssim R \equiv \frac{2\pi}{9\alpha} \frac{\Delta b_3}{\frac{2}{3} \Delta b_1 + \Delta b_2} \lesssim +6,$$

(30)

where $\Delta b_i$ is the difference of the one-loop beta function coefficient above and below the varying scale. In other words, the difference between the beta function coefficients of the strong coupling $\alpha_3$ above and below the varying scale, should be suppressed by a factor of $\mathcal{O}(0.01)$ compared with those of the other two couplings, $\alpha_1$ and $\alpha_2$.

We focussed our investigation on the framework where the MSSM is embedded in GUT models and the varying scale is $M_{\text{GUT}}$. We demonstrated that the relation between the variation of $\Lambda_{\text{QCD}}$ and that of $\alpha$, that is, the ratio $R = \frac{\delta \Lambda_{\text{QCD}}/\Lambda_{\text{QCD}}}{\delta \alpha/\alpha}$, is highly model dependent. For example, we can construct SU(5) models with $R$ as high as $\mathcal{O}(+600)$ and as low as $\mathcal{O}(-500)$, and a model where $R = 0$. (The latter is the minimal SU(5) model, and the former have, respectively, eight and seven $\bf 5 + \bf \bar{5}$ pairs added to the minimal model.) We argued, however, that it is difficult to obtain consistency with the requirement (30) in realistic supersymmetric GUT models in our framework.
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