The spinon Fermi surface U(1) spin liquid in a spin-orbit-coupled triangular lattice Mott insulator YbMgGaO$_4$

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Motivated by the recent progress on the spin-orbit-coupled triangular lattice spin liquid candidate YbMgGaO$_4$, we carry out a systematic projective symmetry group analysis and mean-field study of candidate U(1) spin liquid ground states. Due to the spin-orbital entanglement of the Yb moments, the space group symmetry operation transforms both the position and the orientation of the local moments, and hence brings different features for the projective realization of the lattice symmetries from the cases with spin-only moments. Among the eight U(1) spin liquids that we find with the fermionic parton construction, only one spin liquid state, that was proposed and analyzed in Yao Shen, et. al., Nature 540, 559-562 (2016) and labeled as U1A00 in the present work, stands out and gives a large spinon Fermi surface and provides a consistent explanation for the spectroscopic results in YbMgGaO$_4$. Further connection of this spinon Fermi surface U(1) spin liquid with YbMgGaO$_4$ and the future directions are discussed. Finally, our results may apply to other spin-orbit-coupled triangular lattice spin liquid candidates, and more broadly, our general approach can be well extended to spin-orbit-coupled spin liquid candidate materials.

I. INTRODUCTION

The interplay between strong spin-orbit coupling (SOC) and strong electron correlation has attracted a significant attention in recent years$^1$. At the mean time, the abundance of strongly correlated materials with 5d and 4f electrons, such as iridates and rare-earth materials$^1,2$, brings a fertile arena to explore various emergent and exotic phases that arise from such an interplay$^3$–$^32$. The recently discovered quantum spin liquid (QSL) candidate YbMgGaO$_4$,$^{33}$ where the rare-earth Yb atoms form a perfect triangular lattice, is an ideal system that involves strong spin-orbital entanglement in the strong Mott insulating regime of the Yb electrons$^{34–41}$.

In YbMgGaO$_4$, the thirteen 4f electrons of the Yb$^{3+}$ ions are well localized and form a spin-orbit-entangled total moment $J$ with $J = 7/2$.$^{34,35}$ The eight-fold degeneracy of the $J = 7/2$ moment is further split by the $D_{3d}$ crystal electric fields. The resulting ground state Kramers doublet of the Yb$^{3+}$ ion, whose two-fold degeneracy is protected by the time-reversal symmetry, is well separated from the excited doublets and is responsible for the low-temperature magnetic properties of YbMgGaO$_4$. No signature of time-reversal symmetry breaking is observed for YbMgGaO$_4$ down to the lowest measured temperature$^{36–38}$. Applying the recent theoretical result on spin-orbit-coupled Mott insulators$^{37}$, two of us and collaborators have proposed YbMgGaO$_4$ to be the first QSL candidate in the spin-orbit-coupled Mott insulator with odd electron fillings$^{34–36,39}$. More broadly, YbMgGaO$_4$ represents a new class of rare-earth materials where the strong spin-orbit entanglement of the local moments meets with the geometrical frustration of the triangular lattice such that exotic quantum phases may be stabilized.

Apart from the absence of magnetic ordering, the heat capacity was found to be $C_v \propto T^{0.7}$ at low temperatures$^{33,34,37,43}$, and is close to the well-known $T^{2/3}$ heat capacity.$^{44–46}$ The latter was the one obtained within a random phase approximation for the spinon-gauge coupling in a spinon Fermi surface U(1) QSL.$^{44–46}$ More substantially, the broad continuum$^{36,37}$ of the magnetic excitation with a clear dispersion for the upper excitation edge agrees reasonably with the particle-hole continuum of the spinon Fermi surface.$^{46}$ However, due to the scattering with the phonon degrees of freedom, the thermal transport measurement in YbMgGaO$_4$ was unable to extract the intrinsic magnetic contribution to the thermal conductivity.$^{43}$ Partly motivated by the spin liquid behaviors in YbMgGaO$_4$ and more broadly by the families of rare-earth magnets with identical structures, in this paper, we carry out a systematic projective symmetry group (PSG) analysis for a triangular lattice Mott insulator with spin-orbital-entangled local moments. Unlike the cases for the spin-only moments in the pioneering work by X.-G. Wen$^{37}$, the space group symmetry opera-

![Figure 1](https://www.example.com/figure1.png)

**FIG. 1.** (a) The intralayer symmetries of the R̃3m space group for YbMgGaO$_4$.$^{35}$ (b) The same lattice symmetry group with a different complete set of elementary transformations. Here $S_0 \equiv C_3^{-1}I$. The bold arrow is the axis for the $C_2$ rotation (see Appendix).
tion, in particular, the rotation, transforms both the position and the orientation of the Yb local moments.\textsuperscript{35,39} We find that, among the eight U(1) QSL states, the spinon mean-field state that was introduced in Ref. 36 and labeled as the U1A00 state in our PSG classification, contains a large spinon Fermi surface and gives a large spinon scattering density of states that is consistent with the inelastic neutron scattering (INS) results.

The following part of the paper is organized as follows. In Sec. II, we describe the space group symmetry and the multiplication rules for the symmetry transformation. In Sec. III, we introduce the fermionic spinon construction and the fermionic spinon mean-field Hamiltonian. In Sec. IV, we explain the scheme for the projective symmetry group classification when the spin-orbit coupling is present. In Sec. V, we explain the relationship between the spinon band structure and the projective symmetry group of the spinon mean-field states. In Sec. VI, we focus on the U1A00 state and study the spectroscopic properties of this state. Finally in Sec. VII, we discuss the experimental relevance and remark on the thermal transport result and the competing scenarios and proposals. The details of the calculation are presented in the Appendices.

II. SPACE GROUP SYMMETRY

It was pointed out that the intralayer symmetries involve two translations, $T_1$ and $T_2$, one two-fold rotation, $C_2$, one three-fold rotation, $C_3$, and one spatial inversion $I$ (see Fig. 1(a))\textsuperscript{35,39}. Here we use a different complete set of elementary transformations for the space group symmetries that involve two translations, $T_1$ and $T_2$, one two-fold rotation, $C_2$, and one more operation, $S_6$ (see the definition in Fig. 1(b)). It is ready to confirm $I = S_6, C_3 = S_6^2$ with the definition $S_6 = C_3^{-1}T$. The multiplication rules of this symmetry group is given as

\begin{align*}
T_1^{-1}T_2T_1^{-1} &= T_1^{-1}T_2^{-1}T_1T_2 = 1, \\
C_2^{-1}T_1C_2T_2^{-1} &= C_2^{-1}T_2C_2T_1^{-1} = 1, \\
S_6^{-1}T_1S_6T_2 &= S_6^{-1}T_2S_6T_1^{-1} = 1, \\
(C_2)^2 &= (S_6)^6 = (S_6C_2)^2 = 1.
\end{align*}

Due to the presence of time reversal in YbMgGaO$_4$\textsuperscript{34,36–38}, we further supplement the symmetry group with the time reversal $\mathcal{T}$ such that $O^{-1}\mathcal{T}O = 1$ and $\mathcal{T}^2 = 1$, where $\mathcal{O}$ is a lattice symmetry operation.

III. FERMIONIC PARTICLE CONSTRUCTION

To describe the U(1) QSL that we propose for YbMgGaO$_4$, we introduce the fermionic spinon operator $f_{\alpha\sigma}(\alpha = \uparrow, \downarrow)$ that carries spin-1/2, and express the Yb local moment as

\begin{equation}
S_r = \frac{1}{2} \sum_{\alpha, \beta} f_{\alpha\sigma}^\dagger f_{\alpha\sigma} \sigma_{\alpha\beta} f_{\beta\sigma},
\end{equation}

where $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ is a vector of Pauli matrices. We further impose a constraint $\sum_{\alpha} f_{\alpha\sigma}^\dagger f_{\alpha\sigma} = 1$ on each site to project back to the physical Hilbert space of the spins. The choice of fermionic spinons allows a local SU(2) gauge freedom.\textsuperscript{47}

As a direct consequence of the spin-orbital entanglement, the spinon mean-field Hamiltonian for the U(1) QSL should generically involve both spin-preserving and spin-flipping hoppings, and has the following form

\begin{equation}
H_{\text{MF}} = - \sum_{(rr')} \sum_{\alpha \beta} \left[ t \sigma_{rr',\alpha\beta} f_{rr'}^\dagger f_{rr} + \text{h.c.} \right],
\end{equation}

where $t_{rr',\alpha\beta}$ is the spin-dependent hopping. The choice of the mean-field ansatz in Eq. (6) breaks the local SU(2) gauge freedom down to U(1). Here, to get a more compact form for Eq. (6), we follow Ref. 49 and introduce the extended Nambu spinor representation for the spinons such that $\Psi_r = (f_{r\uparrow}, f_{r\downarrow}, f_{r\uparrow}^\dagger, -f_{r\downarrow}^\dagger)^T$ and

\begin{equation}
H_{\text{MF}} = - \frac{1}{2} \sum_{(rr')} \left[ \Psi_{rr'}^\dagger \Psi_{rr'} + \text{h.c.} \right],
\end{equation}

where $u_{rr'}$ is a hopping matrix that is related to $t_{rr',\alpha\beta}$. With the extended Nambu spinor, the spin operator $S_r$ and the generator $G_r$ for the SU(2) gauge transformation are given by\textsuperscript{47,50–53}

\begin{align*}
S_r &= \frac{1}{4} \Psi_{r\uparrow}^\dagger (\sigma \otimes \mathbb{1}_{2\times2}) \Psi_{r\downarrow}, \\
G_r &= \frac{1}{4} \Psi_{r\uparrow}^\dagger (I_{2\times2} \otimes \sigma) \Psi_{r\downarrow},
\end{align*}

where $I_{2\times2}$ is a $2 \times 2$ identity matrix. Under the symmetry operation $O$, $\Psi_r$ transforms as

\begin{equation}
\Psi_r \rightarrow U_O G_O^\sigma \Psi_{O(r)},
\end{equation}

where $G_O^\sigma$ is the local gauge transformation that corresponds to the symmetry operation $O$, and we add a spin rotation $U_O$ because the spin components are transformed when $O$ involves a rotation. In Eq. (10), the gauge transformation and the spin rotation are commutative,\textsuperscript{54} simply because $[S_r^\sigma, G_O^\sigma] = 0$. Moreover, from Eq. (9), the gauge transformation $G_O^\sigma$ is block diagonal with $G_O^\sigma = I_{2\times2} \otimes W_O$, where $W_O$ is a $2 \times 2$ matrix (see Appendix).

| \(U(1)\) QSL | \(W_r^T\) | \(W_r^T\) | \(W_r^C\) | \(W_r^S\) |
|----------------|----------------|----------------|----------------|----------------|
| U1A00          | \(I_{2\times2}\) | \(I_{2\times2}\) | \(I_{2\times2}\) | \(I_{2\times2}\) |
| U1A10          | \(I_{2\times2}\) | \(I_{2\times2}\) | \(i\sigma^y\) | \(I_{2\times2}\) |
| U1A01          | \(I_{2\times2}\) | \(I_{2\times2}\) | \(i\sigma^y\) | \(I_{2\times2}\) |
| U1A11          | \(I_{2\times2}\) | \(I_{2\times2}\) | \(i\sigma^y\) | \(I_{2\times2}\) |

TABLE I. List of the gauge transformations for the four U1A PSGs. For the time reversal, all PSGs here have $W_r^T = I_{2\times2}$. The last two letters in the labels of the U(1) QSLs are extra quantum numbers in the PSG classification.\textsuperscript{48}
IV. PROJECTIVE SYMMETRY GROUP CLASSIFICATION

For the spinon mean-field Hamiltonian in Eq. (6), the lattice symmetries are realized projectively and form the projective symmetry group (PSG). To respect the lattice symmetries are realized projectively and form the symmetry turns into the following group relation for the QSL, \( \text{U(1)} \).

The ansatz itself is invariant under the so-called invariant gauge group (IGG) with \( u_{rr'} = G_{O(r)}^O u_{O(r)} G_{O(r')}^O \). The IGG can be regarded as a set of gauge transformations that correspond to the identity transformation. For an U(1) QSL, \( \text{IGG} = \text{U(1)} \).

A general group relation \( O_1 O_2 O_3 O_4 = 1 \) for the lattice symmetry turns into the following group relation for the PSG

\[
\mathcal{U}_{O_1} \mathcal{G}_{O_1}^O \mathcal{U}_{O_2} \mathcal{G}_{O_2}^O \mathcal{U}_{O_3} \mathcal{G}_{O_3}^O \mathcal{U}_{O_4} \mathcal{G}_{O_4}^O = \mathcal{I},
\]

where \( \mathcal{U}_{O_i} \) are the set of gauge transformations that commute with the spin rotation. As the series of rotations \( O_1 O_2 O_3 O_4 \) either rotate the spinons by 0 or \( 2\pi \),

\[
\mathcal{U}_{O_1} \mathcal{U}_{O_2} \mathcal{U}_{O_3} \mathcal{U}_{O_4} = \pm I_{4 \times 4},
\]

where \( I_{4 \times 4} \) is a 4 \( \times \) 4 identity matrix. Since \( \{I_{4 \times 4}\} \in \text{IGG}, \) then

\[
G_{O_1}^O G_{O_2}^O G_{O_3}^O G_{O_4}^O \in \text{IGG}.
\]

This immediately indicates that, to classify the PSGs for a spin-orbit-coupled Mott insulator, we only need to focus on the gauge part, first find the gauge transformation with the same procedures as those for the conventional Mott insulators with spin-only moments, and then account for the spin rotation.

For the mean-field ansatz in \( H_{\text{MF}} \), we choose the "canonical gauge" for the IGG with

\[
\text{IGG} = \{I_{2 \times 2} \otimes e^{i\phi \sigma_z} | \phi \in [0, 2\pi]\}.
\]

Under the canonical gauge, the gauge transformation associated with the symmetry operation \( O \) takes the form of

\[
G_{O}^O = I_{2 \times 2} \otimes W_{r}^O = I_{2 \times 2} \otimes [(i \sigma^z)^nO e^{i\phi_0[\sigma_1^\pm]r}],
\]

where \( n_O = 0, 1 \). For translations, one can always choose a gauge such that

\[
W_{r}^{T_1} = (i \sigma^z)^n_1,
\]

\[
W_{r}^{T_2} = (i \sigma^z)^n_2 e^{i\phi_2[x,y] \sigma^z}
\]

with \( n_1, n_2 = 0, 1 \) and \( \phi_2[0, y] = 0 \). The group relation in Eq. (3) further demands \( n_1 = n_2 = 0 \). Thus the group relation in Eq. (1) gives \( W_{r}^{T_1} = 1, W_{r}^{T_2} = e^{i(x,y) \sigma^z} \), where \( \phi_1 \) is the flux through each unit cell of the triangular lattice and takes the value of 0 or \( \pi \) (see Appendix). The PSGs with \( \phi_1 = 0 (\pi) \) are labeled by U1A (U1B).

Among the sixteen algebraic PSGs that we find, eight unphysical solutions have \( T^2 = 1 \) for the spinons and give vanishing spinon hoppings everywhere. In Tab. 1 and the Appendix, we list the remaining eight PSGs that have \( T^2 = -1 \) consistent with the fact that fermionic spinons are Kramers doublets (see Appendix).

V. MEAN-FIELD STATES

Here we obtain the spinon mean-field Hamiltonian from Tab. 1 and explain why the U1A00 state stands out as the candidate ground state for YbMgGaO4. We start with the U1A states. Among the four U1A states, the U1A10 state gives a vanishing mean-field Hamiltonian for the spinon hoppings between the first and the second neighbors, the remaining ones except the U1A00 state all have symmetry protected band touchings at the spinon Fermi level (see Fig. 2). To illustrate the idea, we consider the U1A01 state where the spinon Hamiltonian has the form

\[
H_{\text{MF}}^{U1A01} = \sum_k h_{\alpha\beta}(k) \gamma_1^T \gamma_1 \text{k}\beta \text{ in the momentum space and } h(k) = \sum_{\mu=1}^{3} d_{\mu}(k) \sigma^\mu.
\]

For this band structure there are nondegenerate band touchings at \( \Gamma, M \) and \( K \) points that are protected by the PSG of the U1A01 state. Under the operation \( S_0 \),

\[
\text{(a)} \quad \text{(b)} \quad \text{(c)} \quad \text{(d)}
\]

FIG. 2. (a,b,c) The mean-field spinon bands along the high-symmetry momentum lines (see (d)) of the U1A00, U1A01 and U1A11 states, where \( t_1, t_2 \) and \( t_3 \) are hoppings in their spinon mean-field Hamiltonians (see Appendix). The Dirac cones are highlighted in dashed circles. The dashed line refers to the Fermi level. (d) The Brilliouin zone of the triangular lattice.
the PSG demands that spinons to transform as
\[ f_{k\uparrow} \rightarrow -e^{-i\pi/3} f_{-k\downarrow}, \quad (21) \]
\[ f_{k\downarrow} \rightarrow e^{i\pi/3} f_{-k\uparrow}. \quad (22) \]
Applying \( S_0 \) three times and keeping \( H_{MF} \) invariant, we require
\[ h(k) = -[\sigma^y h(k) \sigma^y]^T \quad (23) \]
which forces \( d_0(k) = 0 \). The time reversal symmetry \((T = i\sigma^y \otimes I_{2 \times 2} K)\) further requires that \( d_\mu(k) = -d_\mu(-k) \). Thus we have symmetry protected band touchings with \( h(k) = 0 \) at the time reversal invariant momenta \( \Gamma \) and \( M \). The \( K \) points are invariant under \( C_2 \) and \( S_0 \) because the spinon partile-hole transformation is involved for \( S_0 \) (see Appendix). Using those two symmetries, we further establish the band touching at the \( K \) points. Likewise, for the \( U1A11 \) state, the PSG demands the band touchings at \( \Gamma \) and \( M \) points. Because there are only two spinon bands for the \( U1A \) states, these band touchings generically occur at the spinon Fermi level.

Due to the Dirac band touchings at the Fermi level, the low-energy dynamic spin structure factor, that measures the spinon particle-hole continuum, is concentrated at a few discrete momenta that correspond to the intra-Diraccone and the inter-Dirac-cone scatterings. Clearly, this is inconsistent with the recent INS result that observes a broad continuum covering a rather large portion of the Brillouin zone. For the \( U1B \) states, the spinons experience a \( \pi \) background field in each unit cell. The direct consequence of the \( \pi \) background field is that the \( U1B \) states support an enhanced periodicity of the dynamic spin structure in the Brillouin zone. Such an enhanced periodicity is absent in the INS result. In particular, unlike what one would expect for an enhanced periodicity, the spectral intensity at the \( \Gamma \) point is drastically different from the one at the \( M \) point in the existing experiments.

The above analysis leads to the conclusion that the \( U1A00 \) state is the most promising candidate \( U(1) \) QSL for \( YbMgGaO_4 \), and this conclusion is independent from any microscopic model. The spinon mean field Hamiltonian, allowed by the \( U1A00 \) PSG, is remarkably simple and is given as
\[ \tilde{H}_{MF}^{U1A00} = -t_1 \sum_{(rr'),\alpha} f_{r\alpha}^\dagger f_{r'\alpha} - t_2 \sum_{(rr'),\alpha} f_{r\alpha}^\dagger f_{r\alpha}, \quad (24) \]
where the spinon hopping is isotropic for the first and the second neighbors. This mean-field state only has a single band that is 1/2-filled, so it has a large spinon Fermi surface. From \( \tilde{H}_{MF}^{U1A00} \), we construct the mean-field ground state by filling the spinon Fermi sea,
\[ |\Psi_{MF}^{U1A00}\rangle = \prod_{\varepsilon_k < \varepsilon_F} f_{k\uparrow}^\dagger f_{k\downarrow} 0\rangle \quad (25) \]
where \( \varepsilon_k \) is the spinon dispersion and \( \varepsilon_F \) is the spinon Fermi energy. The mean-field variational energy is
\[ E_{\text{var}} = \langle \Psi_{MF}^{U1A00}| \hat{H}_{\text{spin}} |\Psi_{MF}^{U1A00}\rangle, \quad (26) \]
where
\[ H_{\text{spin}} = \sum_{\langle rr'\rangle} J_{zz} S_{r\uparrow}^z S_{r'\downarrow}^z + J_\pm (S_{r\uparrow}^+ S_{r'\downarrow}^- + S_{r\downarrow}^+ S_{r'\uparrow}^-) \]
\[ + J_{zz} (\gamma_{rr'} S_{r\uparrow}^z S_{r'\downarrow}^- + \gamma_{rr'} S_{r\downarrow}^z S_{r'\uparrow}^-) \]
\[ - \frac{i}{2} J_{zz} [\gamma_{rr'} S_{r\uparrow}^z S_{r'\downarrow}^- - \gamma_{rr'} S_{r\downarrow}^z S_{r'\uparrow}^-] \]
\[ + S_{r\uparrow}^z (\gamma_{rr'} S_{r'\downarrow}^- - \gamma_{rr'} S_{r'\uparrow}^-) \]
\[ (27) \]
is the microscopic spin model that was introduced in Refs. 34 and 35, and \( \gamma_{rr'} \) is a bond-dependent phase factor due to the spin-orbit-entangled nature of the Yb moments. The anisotropic nature of the spin interaction has been clearly supported by the recent polarized neutron scattering measurement. For the specific choice with \( J_{zz} = 0.915J_{zz} \), we find the minimum variational energy \( E_{\text{var}} = -0.39J_{zz} \) and occurs at \( t_2 = 0.2 t_1 \) (see Appendix). Here, the expectation values of the \( J_{zz} \) and \( J_{zz} \) interactions simply vanish, and this is an artifact of the free spinon mean-field theory with the isotropic hoppings in Eq. (24). We here establish that the \( U1A00 \) state is a spinon Fermi surface \( U(1) \) QSL.

**VI. SPECTROSCOPIC PROPERTIES**

For the \( U1A00 \) state, the dynamic spin structure essentially detects the spinon particle-hole excitation across the Fermi surface. The information about the Fermi surface is encoded in the profile of the dynamic spin structure factor. We evaluate the dynamic spin structure factor within the free spinon mean-field theory (see Appendix) (see Fig. 3(a)). Qualitatively similar to the mean-field theory with only first neighbor spinon hoppings, the improved free-spinon mean-field theory of \( H_{MF}^{U1A00} \) captures the crucial features of the INS results. The spinon particle-hole continuum covers a large portion of the Brillouin zone, and vanishes beyond the spinon bandwidth. More importantly, the “V-shape” upper excitation edge near the \( \Gamma \) point in Fig. 3(a) was clearly observed in the experiments, and the slope of the “V-shape” is the Fermi velocity.

Due to the isotropic spinon hoppings, \( H_{MF}^{U1A00} \) does not explicitly reflect the absence of spin-rotational symmetry that is brought by the \( J_{zz} \) and \( J_{zz} \) interactions. To incorporate the \( J_{zz} \) and \( J_{zz} \) interactions, we follow the phenomenological RPA treatment for the “t-\( J' \)” model in the context of cuprate superconductors and consider
\[ H = H_{MF}^{U1A00} + H_{\text{spin}}', \quad (28) \]
where \( H_{\text{spin}}' \) are the \( J_{zz} \) and \( J_{zz} \) interactions (see Appendix). While the free spinon results from \( H_{MF}^{U1A00} \) already capture the main features of the neutron scattering data, the anisotropic spin interaction \( H_{\text{spin}}' \) included by RPA, merely redistributes the spectral weight...
in the momentum space. We find in Fig. 3(b) that, the low-energy spectral weight at M is slightly enhanced, a feature observed in Refs. 36 and 37. From our choice of the parameters, it is plausible that this peak results from the proximity to a phase with a stripe-like magnetic order.35,36,39

VII. DISCUSSION

We have demonstrated that the spinon Fermi surface U(1) QSL gives a consistent explanation of the INS result in YbMgGaO4. Moreover, the anisotropic spin interaction, slightly enhances the spectral weight at the M points. The U(1) gauge fluctuation in the spinon Fermi surface U(1) QSL was suggested the cause for the sublinear temperature dependence of the heat capacity in YbMgGaO4.35,36,39,46

In YbMgGaO4, the coupling between the Yb moments is relatively weak.34 It is feasible to fully polarize the spin with experimentally accessible magnetic fields35,37,39,61 and to study the evolution of the magnetic properties under the magnetic field. Recently, two of us have predicted the spectral weight shift of the INS for YbMgGaO4 under a weak magnetic field, and the predicted spectral crossing at the Γ point and the dispersion of the spinon continuum have actually been confirmed in the recent INS measurement.62 Numerically, it is useful to perform numerical calculation with fixed Jz and Jzz that are close to the ones for YbMgGaO4, and obtain the phase diagram of our spin model by varying J± and Jzz.35,39,63 More care needs to be paid to the disordered region of the mean-field phase diagram35 where quantum fluctuation is found to be strong.35 The “2kF” oscillation in the spin correlation would be the strong indication of the spinon Fermi surface. Noteworthily recent DMRG works have actually provided some useful information about the ground states of the system, in particular, Ref. 65 suggested the scenario of exchange disorders. Certain amount of exchange disorder may be created by the crystal electric field disorder that stems from the Mg/Ga mixing in the non-magnetic layers, but recent polarized neutron scattering measurement did not find strong exchange disorder.59 Regardless of the possibilities of exchange disorders, the spin quantum number fractionalization, that is one of the key properties of the QSLs, could survive even with weak disorders. The approach and results in our present work are phenomenologically based and are independent of the microscopic mechanism for the possible QSL ground state in YbMgGaO4.

Ref. 43 claimed the absence of the magnetic thermal conductivity in YbMgGaO4 by extrapolating the low-temperature thermal conductivity data in the zero magnetic field. Here, we provide an alternative understanding for this thermal transport result. The hint lies in the field dependence of the thermal conductivity. It was found that, when strong magnetic fields are applied to YbMgGaO4, the thermal conductivity κzz/T at 0.2K is increased compared with the one at zero field.43 If one ignores the disorder effect and assumes the zero-field thermal conductivity is a simple addition of the magnetic contribution and the phonon contribution with

$$\kappa_{xx} = \kappa_{\text{spin,xx}} + \kappa_{\text{phonon,xx}}.$$  (29)

the strong magnetic field almost polarizes the spins completely and creates a spin gap for the magnon excitation, hence suppress the magnetic contribution. The high-field thermal conductivity would be purely given by the phonon contribution, and we would expect a decreasing of the thermal conductivity in the strong field compared to the zero field result. This is clearly inconsistent with the experimental result. Therefore, the zero-field thermal conductivity is not a simple addition of the magnetic contribution and the phonon contribution, i.e.,

$$\kappa_{xx} \neq \kappa_{\text{spin,xx}} + \kappa_{\text{phonon,xx}}.$$  (30)

This also strongly suggests the presence rather than the absence of magnetic excitations in the thermal conductivity result at zero magnetic field. If there is no magnetic excitation in the system at low temperatures, the low-temperature thermal conductivity at zero field should just be the phonon contribution, and we would expect the zero-field thermal conductivity to be the same as the one in the strong field limit, (although the intermediate field regime could be different). This is again inconsistent with the experiments. This means that the magnetic excitation certainly does not have a large gap and could just be gapless as we propose from the spinon Fermi surface state. In fact, the gapless nature of the magnetic excitation is consistent with the power-law heat capacity results in YbMgGaO4. What suppresses κzz could arise from the mutual scattering between the magnetic excitations and the phonons. In fact, similar field dependence of thermal conductivity κzz has been observed in other rare-earth systems such as Tb3Ti2O7 and Pr2Zr2O7.69 It was suggested there that the spin-phonon scattering is the cause. The Yb local moment, that is a spin-orbit-entangled object, involves the orbital mixing.
degree of freedom. The orbital degree of freedom is sensitive to the ion position, and thus couples to the phonon strongly. This is probably the microscopic origin for the strong coupling between the magnetic moments and the phonons in the rare-earth magnets. This is quite different from the organic spin liquid candidates and the herbertsmithite kagome system where the orbital degree of freedom does not seem to be involved.\textsuperscript{70–73}

If the ground state of YbMgGaO\textsubscript{4} is a QSL with the spinon Fermi surface, the field-driven transition from the QSL ground state to the fully polarized state is necessarily an unconventional transition beyond the traditional Landau’s paradigm and has not been studied in the previous spin liquid candidates\textsuperscript{34}. Since we propose YbMgGaO\textsubscript{4} to be a spinon Fermi surface U(1) QSL and gapless, the transition would be associated with the opening of the spin gap at the critical field. The continuous nature of the transition suggests the spin gap to open in a continuous manner. Moreover, the spin confinement would be concomitant with the spin gap that suppresses the spinon density of states and allows the instanton events of the U(1) gauge field to proliferate. Therefore, it might be interesting to identify the critical field and obtain the critical properties of the field-driven transition. Thermodynamic, spectroscopic, and thermal transport measurements with finer field variation would be helpful.

Finally, several families of rare-earth triangular lattice magnets have been discovered recently\textsuperscript{35,39,74–79}. Their properties have not been studied carefully. Our general classification results and the prediction of the spectroscopic properties would apply to the QSL candidates that may emerge in these families of materials. It is certainly exciting if one finds the new QSL candidates in these families behave like YbMgGaO\textsubscript{4}\textsuperscript{35}.

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\section{Appendix A: The coordinate System and space group symmetry}

Following our convention in Fig. 1 in the main text, we choose the coordinate system of the triangular lattice to be

\begin{align}
  a_1 &= (1, 0), \\
  a_2 &= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right). \tag{A1}
\end{align}

We label the triangular lattice sites by \( r = x a_1 + y a_2 \). Restricted to the triangular layer, the space group contains two translations \( T_1 \) along the \( a_1 \) direction, \( T_2 \) along the \( a_2 \) direction, a counterclockwise three-fold rotation \( C_3 \) around the lattice site, a two-fold rotation \( C_2 \) around \( a_1 + a_2 \), and the inversion \( I \) at the lattice site. Their actions on the lattice indices are

\begin{align}
  T_1 &: (x, y) \rightarrow (x + 1, y), \\
  T_2 &: (x, y) \rightarrow (x, y + 1), \\
  C_3 &: (x, y) \rightarrow (-y, x - y), \\
  C_2 &: (x, y) \rightarrow (y, x), \\
  I &: (x, y) \rightarrow (-x, -y). \tag{A7}
\end{align}

In the formulation introduced in the main text, we consider an equivalent set of generators, \( \{T_1, T_2, C_2, S_6\} \), where the operation \( S_6 \) is defined as \( S_6 \equiv C_3^{-1} I \) and acts on the lattice indices as

\begin{align}
  S_6 &: (x, y) \rightarrow (x - y, x). \tag{A8}
\end{align}

It is evident that these two sets of generators are equivalent, since we merely redefine the symmetry rather than introducing any new symmetry.

The multiplication rule of this symmetry group is given in the main text. For the convenience of the presentation below, we also list these rules here,

\begin{align}
  T_1^{-1} T_2 T_1^{-1} &= T_2^{-1} T_1 T_2 = 1, \tag{A9} \\
  C_2^{-1} T_1 C_2^{-1} &= C_2^{-1} T_2 C_2^{-1} = 1, \tag{A10} \\
  S_6^{-1} T_1 C_6 T_2 &= S_6^{-1} T_2 C_6 T_1^{-1} = 1, \tag{A11} \\
  (C_2)^2 &= (C_6)^6 = (S_6 C_2)^2 = 1. \tag{A12}
\end{align}

Including the time reversal symmetry, we further have

\begin{align}
  T_1^{-1} T T_1 T &= T_2^{-1} T T_2 T = 1, \tag{A13} \\
  C_2^{-1} T C_2 T &= S_6^{-1} T S_6 T = 1, \tag{A14} \\
  T^2 &= 1. \tag{A15}
\end{align}

\section{Appendix B: Projective symmetry group classification}

As we describe in the main text, we consider the U(1) QSL. The spinon mean-field Hamiltonian has the following form

\begin{equation}
  H_{\text{MF}} = - \sum_{\langle rr' \rangle} \sum_{\alpha \beta} \left[ t_{rr', \alpha \beta} f_{r \alpha}^\dagger f_{r' \beta}^\dagger + h.c. \right], \tag{B1}
\end{equation}
TABLE II. List of the gauge transformations for the symmetry operations of the eight U(1) PSGs, where \((x, y)\) is the coordinate in the oblique coordinate system. For time reversal symmetry, all PSGs have the same gauge transformation \(W'_{g} = I_{2 \times 2}\).

| \(U(1)\) QSL | \(W_{r}^{T_{1}}\) | \(W_{r}^{T_{2}}\) | \(W_{r}^{C_{2}}\) | \(W_{r}^{S_{6}}\) |
|----------------|----------------|----------------|----------------|----------------|
| U1A00 | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) |
| U1A10 | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(\sigma^{y}\) | \(I_{2 \times 2}\) |
| U1A01 | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(\sigma^{y}\) |
| U1A11 | \(I_{2 \times 2}\) | \(I_{2 \times 2}\) | \(\sigma^{y}\) | \(\sigma^{y}\) |
| U1B00 | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{x}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{x}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{x}I_{2 \times 2}^{-1}\) |
| U1B10 | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) |
| U1B11 | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) | \((I_{2 \times 2}^{-1})^{y}I_{2 \times 2}^{-1}\) |

where \(t_{r',r,\alpha\beta}\) is the spin-dependent hopping. With the extended Nambu spinor representation\(^{59}\) \(\Psi_{r} = (f_{r}^{+}, f_{r}^{-}, f_{r}^{-1}, f_{r}^{-1})^{T}\), \(H_{MF}\) has a more compact form

\[
H_{MF} = -\frac{1}{2} \sum_{(r', r)} [\Psi_{r'}^{T} u_{rr'} \Psi_{r'} + h.c.],
\]

where \(u_{rr'}\) is a hopping matrix that is related to \(t_{r',r,\alpha\beta}\),

\[
u_{rr'} = \begin{pmatrix} t_{r', r, \uparrow\uparrow} & 0 & t_{r', r, \uparrow\downarrow} & 0 \\ 0 & -t_{r', r, \downarrow\downarrow} & 0 & t_{r', r, \downarrow\uparrow} \\ 0 & 0 & t_{r', r, \downarrow\uparrow} & 0 \\ t_{r', r, \uparrow\uparrow} & 0 & 0 & -t_{r', r, \downarrow\downarrow} \end{pmatrix},
\]

1. Spatial symmetry

First of all, the gauge transformation and spin rotation are commutative. So in the PSG classification, we only need to focus on the gauge part of the PSG transformation. In the canonical gauge \(IGG = \{I_{2 \times 2} \otimes e^{i\phi_{r}} \mid \phi \in [0, 2\pi]\}\), the gauge transformation associated with a given symmetry operation \(O\) takes the form

\[
G_{r}^{O} = I_{2 \times 2} \otimes W_{r}^{O} \equiv I_{2 \times 2} \otimes \left\{ (i\sigma^{x})^{n_{O}} e^{i\phi_{O}} [r] \sigma^{x} \right\},
\]

where \(n_{O} = 0, 1\). For the symmetry multiplication rule \(O_{1}O_{2}O_{3}O_{4} = 1\) where \(O_{4}\) is an unitary transformation, the corresponding PSG relation becomes

\[
G_{r}^{O_{1}}G_{r}^{O_{2}}G_{r}^{O_{3}}(r)G_{r}^{O_{4}}(r)G_{r}^{O_{4}}(r) \in IGG
\]

or equivalently,

\[
W_{r}^{O_{1}}W_{r}^{O_{2}}W_{r}^{O_{3}}(r)W_{r}^{O_{4}}(r)W_{r}^{O_{4}}(r) \in \{e^{i\phi_{r}} \mid \phi \in [0, 2\pi]\}.
\]

We start with \(T_{1}\) and \(T_{2}\), where

\[
W_{r}^{T_{1}} = (i\sigma^{x})^{n_{T_{1}}}, \quad W_{r}^{T_{2}} = (i\sigma^{x})^{n_{T_{2}}} e^{i\phi_{T_{2}}[r] \sigma^{y}}.
\]

Through Eq. (A10) that connects \(T_{1}\) and \(T_{2}\), one immediately has \(n_{T_{1}} = n_{T_{2}}\). From Eq. (A11) where the total number of \(T_{1}\) and \(T_{2}\) is odd, one immediately has \(n_{T_{1}} = n_{T_{2}} = 0\). So we have

\[
W_{r}^{T_{1}} = 1, \quad W_{r}^{T_{2}} = e^{i\phi_{T_{2}}[r] \sigma^{y}}.
\]

Using Eq. (A9), we have

\[
[W_{r}^{T_{1}^{-1}}][W_{r}^{T_{2}T_{1}}][W_{r}^{T_{2}T_{1}}]^{-1} = T_{1}^{-1}(W_{r}^{T_{1}})^{-1}W_{r}^{T_{2}T_{1}}W_{r}^{T_{1}^{-1}}W_{r}^{T_{1}^{-1}},
\]

which leads to the result

\[
\phi_{T_{2}}[x + 1, y] - \phi_{T_{2}}[x, y] \equiv \phi_{1}\)

with \(\phi_{1}\) to be determined. Since it is always possible to choose a gauge such that \(\phi_{T_{2}}[0, y] = 0\), then we have \(\phi_{T_{2}}[x, y] = \phi_{1}x\).

Similarly, \(T_{1}^{-1}T_{2}^{-1}T_{1}T_{2} = 1\) leads to

\[
\phi_{T_{2}}[x + 1, y + 1] - \phi_{T_{2}}[x, y + 1] = \phi_{2}.
\]

It is ready to find \(\phi_{2} = \phi_{1}\).

We continue to find \(W_{r}^{S_{6}}\) and \(W_{r}^{C_{2}}\). For the operation \(S_{6}\) with \(W_{r}^{S_{6}} = (i\sigma^{x})^{n_{S_{6}}} e^{i\phi_{S_{6}}[x, y] \sigma^{x}}\), Eq. (A11) leads to

\[
-\phi_{S_{6}}[T_{1}(r)] + \phi_{S_{6}}[r] = -\phi_{1}y + \phi_{3},
\]

\[
-\phi_{S_{6}}[T_{2}(r)] + \phi_{S_{6}}[r] = \phi_{4} - \phi_{1}x + \phi_{1}y.
\]

for \(n_{S_{6}} = 0\), and

\[
-\phi_{S_{6}}[T_{1}(r)] + \phi_{S_{6}}[r] = -\phi_{1}y + \phi_{3},
\]

\[
-\phi_{S_{6}}[T_{2}(r)] + \phi_{S_{6}}[r] = \phi_{4} + \phi_{1}x + \phi_{1}y.
\]

for \(n_{S_{6}} = 1\). So we obtain

\[
\phi_{S_{6}}[r] = \frac{\phi_{1}xy - \phi_{3}x - \phi_{4}y - \phi_{1}y(y - 1)}{2},
\]

when \(n_{S_{6}} = 0\),

\[
\phi_{S_{6}}[r] = \frac{\phi_{1}xy - \phi_{3}x - \phi_{4}y - \phi_{1}y(y - 1)}{2}.
\]

For \(n_{S_{6}} = 1\), we further require \(\phi_{1} = 0, \pi\). \(S_{6}^{0} = 1\) is automatically satisfied with the above relations for both \(n_{S_{6}} = 0\) and \(n_{S_{6}} = 1\).

For \(W_{r}^{C_{2}} = (i\sigma^{x})^{n_{C_{2}}} e^{i\phi_{C_{2}}[x, y] \sigma^{x}}\), we need to consider two separate cases with \(n_{C_{2}} = 0, 1\), respectively.

If \(n_{C_{2}} = 0\), Eq. (A10) leads to

\[
-\phi_{C_{2}}[T_{2}^{-1}(r)] - \phi_{C_{2}}[T_{1}(r)] + \phi_{C_{2}}[r] = \phi_{5},
\]

\[
-\phi_{C_{2}}[T_{2}(r)] + \phi_{C_{2}}[T_{1}(r)] + \phi_{C_{2}}[r] = \phi_{6}.
\]
So we obtain $\phi_{C_2}[x, y] = -\phi_5 x - \phi_6 y - xy \phi_1$ and $\phi_1 = 0, \pi$ for $n_{C_2} = 0$. Similarly, for $n_{C_2} = 1$, we obtain $\phi_{C_2}[x, y] = -\phi_5 x - \phi_6 y - xy \phi_1$.

Using $C_2^2 = 1$, we further have $\phi_5 = -\phi_5$ for $n_{C_2} = 0$, and $\phi_6 = -\phi_6$ for $n_{C_2} = 1$. So we arrive at the result

$$n_{C_2} = 1, \quad \phi_{C_2}[x, y] = -\phi_5 x - \phi_6 y - xy \phi_1. \quad (B23)$$

Here, to simplify the above expression, we choose a pure gauge transformation $W_r^O = e^{i\sigma^z \phi}$. Under the pure gauge transformation, the gauge part of the PSG transforms as

$$W_r^O \rightarrow \tilde{W}_r^a W_r^O \tilde{W}_r^{a\dagger}_O^{-1}(r). \quad (B24)$$

Clearly $\tilde{W}_r^a$ only modifies $WT_1$ and $WT_2$ by an overall phase shift, but $W_r^{C_2}$ becomes

$$W_r^{C_2} = (i\sigma^x)^{n_{C_2}} e^{-iy \phi_1 \sigma^y} \quad (B25)$$

for both $n_{C_2} = 0, 1$, except that we require $\phi_1 = 0, \pi$ for $n_{C_2} = 0$.

For the relation $(S_0 C_2)^2 = 1$, we need to consider the four cases with $n_{S_0} = 0, 1$ and $n_{C_2} = 0, 1$.

For $n_{S_0} = n_{C_2} = 0$, we have $\phi_1 = \pi$, and $(S_0 C_2)^2 = 1$ gives $\phi_3 + 2\phi_4 = 0$. We then introduce a pure gauge transformation $W_r^b$,

$$\tilde{W}_r^b = e^{-i(x+y)\phi_4 \sigma^z}. \quad (B26)$$

After applying $\tilde{W}_r^b$, we have

$$\phi_{C_2} = -xy \phi_1, \quad (B27)$$

$$\phi_{S_0} = xy \phi_1 - \phi_3 \frac{y(y-1)}{2}. \quad (B28)$$

with $\phi_1 = 0, \pi$.

For $n_{S_0} = 0$ and $n_{C_2} = 1$, we obtain $\phi_3 = 0$. We introduce a pure gauge transformation $\tilde{W}_r^c$,

$$\tilde{W}_r^c = e^{-i(x-y)\phi_4 \sigma^z}. \quad (B29)$$

After applying $\tilde{W}_r^c$, we have

$$\phi_{C_2} = -xy \phi_1, \quad (B30)$$

$$\phi_{S_0} = xy \phi_1 - \phi_3 \frac{y(y-1)}{2}. \quad (B31)$$

For $n_{S_0} = 1$ and $n_{C_2} = 0$, we obtain $\phi_3 = 0$. We apply a pure gauge transformation $\tilde{W}_r^d$ and obtain

$$\phi_{C_2} = -xy \phi_1, \quad (B32)$$

$$\phi_{S_0} = xy \phi_1 - \phi_3 \frac{y(y-1)}{2}. \quad (B33)$$

For $n_{S_0} = 1$ and $n_{C_2} = 1$, we obtain $\phi_3 + 2\phi_4 = 0$. We apply a pure gauge transformation $\tilde{W}_r^e$ and obtain

$$\phi_{C_2} = -xy \phi_1, \quad (B34)$$

$$\phi_{S_0} = xy \phi_1 - \phi_3 \frac{y(y-1)}{2}. \quad (B35)$$

In summary, we have

$$W_r^{T_1} = 1, \quad W_r^{T_2} = e^{i\phi_1 x}. \quad (B36)$$

and

$$W_r^{C_2} = (i(\sigma^x)^n_{C_2} e^{-i\phi_1 x \sigma^y}, \quad (B37)$$

$$W_r^S = (i(\sigma^x)^n_{S_0} e^{-i\phi_1 [x-y - \frac{y(y-1)}{2}] \sigma^y}, \quad (B38)$$

where $\phi_1 = 0, \pi$ for $n_{C_2} = 0$ or $n_{S_0} = 1$.

2. Time reversal symmetry

Because time reversal is an antiunitary symmetry, the product $O^{-1} T^{-1} O T$ becomes

$$(W_r^O)^\dagger [\tilde{W}_r^T]^d W_r^O [\tilde{W}_r^O]^\dagger_{O^{-1}(r)}]^* \quad (B39)$$

for the PSGs, where $W_r^T$ is the gauge transformation associated with the time reversal. We here redefine

$$\tilde{W}_r^T = W_r^T (i\sigma^y), \quad (B40)$$

so that

$$O^{-1} T^{-1} O T \rightarrow (W_r^O)^\dagger (\tilde{W}_r^T)^\dagger W_r^O \tilde{W}_r^O_{O^{-1}(r)} \quad (B41)$$

$\tilde{W}_r^T$ has the general form $\tilde{W}_r^T = (i(\sigma^x)^n_T e^{i\phi_T \sigma^y} \sigma^x$.

We start with $n_T = 0$. The relation in Eq. (A13) leads to

$$\phi_T[x, y] - \phi_T[x - 1, y] = -\phi_7, \quad (B42)$$

$$\phi_T[x, y + 1] - \phi_T[x, y] = -\phi_8, \quad (B43)$$

so we have $\phi_T[x, y] = -\phi_T y - \phi_8 y$. Applying this result to Eq. (A14), we have

$$-\phi_{C_2} [x, y] - \phi_T [y, x] + \phi_{C_2} [y, x]$$

$$+ \phi_T [x, y] = \phi_9,$$

$$-\phi_{S_0} [x, y] - \phi_T [x, y] + \phi_{S_0} [y, x]$$

$$+ \phi_T [y, -x + y] = \phi_{10}. \quad (B44)$$

for $n_{C_2} = n_{S_0} = 0$. The above equations give $\phi_7 = \phi_8 = 0$, so we have $\tilde{W}_r^T = 1$. Other cases can be obtained likewise. We find that for both $n_T = 0$ and $n_T = 1$, there is $\phi_T[x, y] = 0$ and $\phi_1 = 0, \pi$. So we have

$$\tilde{W}_r^T = 1, \quad (B45)$$

where we have used a global and uniform rotation $e^{i\frac{\pi}{4} \sigma^y}$ to rotate $\sigma^y$ to the basis of $\sigma^y$.

Including the time reversal, there are 16 PSG solutions. But for $\tilde{W}_r^T = 1$, the mean-field ansatz is found to vanish everywhere. This makes sense as these PSGs have $T^2 = 1$ for the fermionic spinons that are expected to Kramers doublets. So only 8 of them with $T^2 = -1$ for the spinons survive. Replacing $e^{i\phi_1 \sigma^y}$ with $\pm 1$, we present the PSG solutions in the table of the main text.
Appendix C: Spinon band structures and mean-field Hamiltonians

As we establish in the previous section and the main text, there are four U1A PSGs and four U1B PSGs. In the main text, we have argued that the experimental results in YbMgGaO₄ is against the U1B states. So here we focus on the U1A states. From the U1A PSGs, it is straight to obtain the spinon transformations. We list the results in Tab. III.

1. Spinon band structures

Using Tab. III, we obtain the spinon mean-field Hamiltonian. In particular, the U1A10 state gives vanishing spinon hoppings on the first and second neighbors, and the U1A01 state gives an isotropic spinon hopping on both first and second neighbors. The U1A10 state, as we described in the main text, has symmetry protected band touchings at the Γ, M and K points. The U1A11 state has symmetry protected band touchings at the Γ and M points.

For the U1A10 state, the spinon mean-field Hamiltonian has the form

\[ H_{\text{MF}}^{\text{U1A10}} = \sum_{\mathbf{k}} h_{\alpha \beta}(\mathbf{k}) f_{\mathbf{k} \alpha}^\dagger f_{\mathbf{k} \beta}, \]  

where \( h_{\alpha \beta}(\mathbf{k}) \) is given by

\[ h(\mathbf{k}) = d_0(\mathbf{k}) I_{2 \times 2} + \sum_{\mu = 1}^3 d_\mu(\mathbf{k}) \sigma^\mu. \]  

In the main text, we have used \((S_6)^3\) and \(T\) to show \(d_0(\mathbf{k}) = 0\) and the band touchings at Γ and M. To account for the band touching at the K point, we need to use \(S_6\) and \(C_2\). Under \(S_6\),

\[ S_6 H S_6^{-1} = \sum_{\mathbf{k}} \left[ e^{i\pi/3} h(-S_6^{-1}(\mathbf{k})) f_{\mathbf{k} \uparrow}^\dagger f_{\mathbf{k} \downarrow} + h.c. \right] = \mathcal{H}, \]  

where \( h(\mathbf{k}) f_{\mathbf{k} \uparrow} = d_x(\mathbf{k}) - id_y(\mathbf{k}) \). Since \(K\) is invariant under \(S_6\),

\[ d_x(K) - id_y(K) = e^{i\pi/3} \left[ d_x(K) - id_y(K) \right], \]  

hence \(d_x(K) - id_y(K) = 0\).

The \(C_2\) symmetry constrains the \(d_z\) term, we have

\[ C_2 H C_2^{-1} = \sum_{\mathbf{k}} \left[ d_x(C_2^{-1}(\mathbf{k})) f_{\mathbf{k} \uparrow}^\dagger f_{\mathbf{k} \downarrow} - d_x(C_2^{-1}(\mathbf{k})) f_{\mathbf{k} \downarrow}^\dagger f_{\mathbf{k} \uparrow} \right] = \mathcal{H}. \]

Since \(K\) is also invariant under \(C_2\), we obtain \(d_x(K) = -d_x(K)\). Hence \(d_x(K) = 0\). We conclude that \(h(\mathbf{k}) = 0\) and there exists a band touching at \(K\).

For the U1A11 state, \(T\) and \(S_6\) are implemented in the same way as the U1A01 state, and we arrive at the same conclusion that there are band touchings at the Γ and M points. At the K point, however, the band structure is generally gapped due to a nonzero \(d_z\).

2. Spinon mean-field Hamiltonians

The U1A00 state has the isotropic spinon hoppings on first and second neighboring bonds, and the mean-field Hamiltonian \(H_{\text{MF}}^{\text{U1A00}}\) has already been given in the main text. This states gives a large spinon Fermi surface in the Broullin zone. The spinon mean-field states of the U1A01 state and the U1A11 state are given by

\[ H_{\text{MF}}^{\text{U1A01}} = \sum_{x,y} t_1 \left[ -i f_{(x+1,y),\uparrow}^\dagger f_{(x,y),\downarrow} - i f_{(x+1,y),\downarrow}^\dagger f_{(x,y),\uparrow} - e^{-i\pi/3} f_{(x,y+1),\uparrow}^\dagger f_{(x,y),\downarrow} \right. \]

\[ + e^{i\pi/3} f_{(x+1,y+1),\uparrow}^\dagger f_{(x,y),\downarrow} - e^{i\pi/3} f_{(x+1,y+1),\downarrow}^\dagger f_{(x,y),\uparrow} + e^{-i\pi/3} f_{(x+1,y+1),\downarrow}^\dagger f_{(x,y),\uparrow} + h.c. \]

\[ + t_2 \left[ e^{i\pi/3} f_{(x+1,y-1),\uparrow}^\dagger f_{(x,y),\downarrow} + e^{i\pi/3} f_{(x+1,y-1),\downarrow}^\dagger f_{(x,y),\uparrow} + f_{(x+1,y+2),\uparrow}^\dagger f_{(x,y),\downarrow} \right. \]

\[ - f_{(x+1,y+2),\downarrow}^\dagger f_{(x,y),\uparrow} + e^{i\pi/3} f_{(x+2,y+1),\uparrow}^\dagger f_{(x,y),\downarrow} + e^{-i\pi/3} f_{(x+2,y+1),\downarrow}^\dagger f_{(x,y),\uparrow} + h.c. \].
and

\[ R_{\text{MF}}^{U1A11} = \sum_{x,y} t_1 \left[ i f_{x,y}^\dagger f_{x+1,y} + i f_{x+1,y}^\dagger f_{x,y} + i f_{x,y}^\dagger f_{x,y+1} + i f_{x,y+1}^\dagger f_{x,y} + h.c. \right] + t'_1 \left[ \right. \right.

\[ + t_2 \left[ e^{i \pi \frac{y}{2}} f_{x,y-1}^\dagger f_{x,y} + e^{i \pi \frac{y}{2}} f_{x,y}^\dagger f_{x,y-1} - f_{x,y+1}^\dagger f_{x,y} + f_{x,y}^\dagger f_{x,y+1} + h.c. \right], \tag{C7} \]

where in both Hamiltonians \( t_1, t'_1 \) denote the first neighbor hoppings and \( t_2 \) denotes the second neighbor hopping.

The band structures for specific choices of the hopping parameters are plotted in the main text. Clearly, we observe the band touchings at the Γ, M and K points for the U1A01 state, and band touchings at the Γ and M points for the U1A11 state.

### Appendix D: The U1A00 state and the spectroscopic results

#### 1. Free spinon mean-field theory

The spinon mean-field Hamiltonian of the U1A00 state is

\[ H_{\text{MF}}^{U1A00} = -t_1 \sum_{(\mathbf{r},\mathbf{r}')} f_{\mathbf{r}'}^\dagger f_{\mathbf{r}} - t_2 \sum_{(\mathbf{r},\mathbf{r}')} f_{\mathbf{r}'}^\dagger f_{\mathbf{r}}, \tag{D1} \]

from which we compute the dynamic spin structure factor for different choices \( t_2/t_1 \). The dynamic spin structure factor is given by

\[ S(q,\omega) = \frac{1}{N} \sum_{\mathbf{r},\mathbf{r}'} e^{i \mathbf{q} \cdot \mathbf{r}'} \int dt e^{-i\omega t} \langle \Psi^{U1A00}_n(\mathbf{r}) | S_{\mathbf{r}'}(t) S_{\mathbf{r}'}^+(0) | \Psi^{U1A00}_n \rangle, \]

\[ = \sum_n \delta(\omega - \xi_{n\mathbf{q}}) \langle n | S_{\mathbf{q}}^+ | \Psi^{U1A00}_n \rangle |^2, \tag{D2} \]

where \( N \) is the total number of spins, the summation is over all mean-field states with the spinon particle-hole excitation. \( \xi_{n\mathbf{q}} \) is the energy of the \( n \)-th excited state with the momentum \( \mathbf{q} \). The results are depicted in Fig. 4(a-e) and are consistent with the inelastic neutron scattering results\( ^{36,37} \). All the results so far are independent from any microscopic spin interaction.
that was introduced in Refs. 34 and 35, the bandwidth \( B \) and \( \gamma \) seems to evolve with \( t_2/t_1 \). In all subfigures, the energy transfer is normalized against the corresponding bandwidth \( B \). The parameter \( \alpha \) is defined as \( J_{zz}/t_1 \).

2. Variational calculation and random phase approximation

Here we consider the microscopic spin Hamiltonian that was introduced in Refs. 34 and 35,

\[
H_{\text{spin}} = \sum_{\langle rr' \rangle} J_{zz} S_r^z S_{r'}^z + J_{\pm} (S_r^+ S_{r'}^- + S_r^- S_{r'}^+) \\
+ J_{\pm \pm} (\gamma_{rr'} S_r^z S_{r'}^z + \gamma_{rr'} S_r^z S_{r'}^z) \\
- \frac{i}{2} J_{\pm \pm} (\gamma_{rr'} S_r^z S_{r'}^z - \gamma_{rr'} S_r^z S_{r'}^z) \langle S_r^+ S_{r'}^- \rangle \\
+ \sum_{n,\pm} \langle \Psi_{\text{U1A00}}^n \rangle \langle S_r^z \rangle \langle S_{r'}^z \rangle, \tag{D3}
\]

where \( \gamma_{rr'} = 1, e^{i2\pi/3}, e^{-i2\pi/3} \) for \( rr' \) along the \( a_1, a_2 \) and \( a_3 \) bonds, respectively. Here, \( a_3 = -a_1 - a_2 \). It was suggested and demonstrated that the anisotropic \( J_{\pm \pm} \) and \( J_{\pm \pm} \) interactions compete with the XXZ part of the Hamiltonian and may lead to disordered states. Our calculation does show the enhancement of quantum fluctuation in certain regions of the phase diagram. Here we comment about the choices of the exchange couplings in the main text and in the following calculation. The \( J_{zz} \) and \( J_{\pm \pm} \) couplings can be determined by the mean-field variational energy calculation. For this spin Hamiltonian, the mean-field variational energy is given as

\[
E_{\text{var}} = \langle \Psi_{\text{U1A00}}^{\text{MF}} | H_{\text{spin}} | \Psi_{\text{U1A00}}^{\text{MF}} \rangle = \frac{1}{L^2} \sum_q \langle \Psi_{\text{U1A00}}^{\text{MF}} | J_{zz}(q) S_q^z S_{-q}^z + 2J_{\pm}(q) S_q^+ S_{-q}^- | \Psi_{\text{U1A00}}^{\text{MF}} \rangle \\
= \frac{1}{L^2} \sum_q \left[ J_{zz}(q) \sum_n \langle n | S_q^z | \Psi_{\text{U1A00}}^{\text{MF}} \rangle^2 + 2J_{\pm}(q) \sum_n \langle n | S_q^+ | \Psi_{\text{U1A00}}^{\text{MF}} \rangle^2 \right] \\
= \frac{1}{L^2} \sum_q \left[ J_{zz}(q) \frac{1}{4} \sum_{n,k} \langle n | f_{k+q+\uparrow} f_{k+\downarrow} - f_{k+q+\uparrow} f_{k+\downarrow} | \Psi_{\text{U1A00}}^{\text{MF}} \rangle^2 + 2J_{\pm}(q) \sum_n \langle n | f_{k+q+\uparrow} f_{k+\downarrow} | \Psi_{\text{U1A00}}^{\text{MF}} \rangle^2 \right], \tag{D4}
\]
where we have omitted \( J_{\pm \pm} \) and \( J_{\pm \pm} \) because they do not conserve spin, therefore their contribution to \( E_{\text{var}} \) is zero. This is an artifact of the free spinon theory of \( H^{\text{MF}}_{\text{var}} \) that only includes isotropic spinon hoppings for the first two neighbors.

Due to the isotropic spinon hoppings, \( H^{\text{MF}}_{\text{var}} \) does not explicitly reflect the absence of spin-rotational symmetry that is brought by the \( J_{\pm \pm} \) and \( J_{\pm \pm} \) interactions. To incorporate the \( J_{\pm \pm} \) and \( J_{\pm \pm} \) interactions, as we describe in the main text, we followed the phenomenological treatment for the “\( t-J \)” model in the context of cuprate superconductors\(^{60} \) and consider \( H = H^{\text{MF}}_{\text{var}} + H^{\prime}_{\text{spin}} \), where \( H^{\prime}_{\text{spin}} \) are the \( J_{\pm \pm} \) and \( J_{\pm \pm} \) interactions. In the parton construction, \( H^{\prime}_{\text{spin}} \) is treated as the spinon interactions and thus introduces a spin rotational symmetry breaking. With a random phase approximation for the interaction \( H^{\prime}_{\text{spin}} \), we obtain the dynamic spin susceptibility\(^{60} \)

\[
\chi^{\text{RPA}}(q, \omega) = [1 - \chi^0(q, \omega) \mathcal{J}(q)]^{-1} \chi^0(q, \omega),
\]

where \( \chi^0 \) is the free-spinon susceptibility, and \( \mathcal{J}(q) \) is the spin exchange matrix from \( H^{\prime}_{\text{spin}} \).

\[
\mathcal{J}(q) = 
\begin{pmatrix}
2(u_q - v_q)J_{\pm \pm} & -2\sqrt{3}w_qJ_{\pm \pm} & -\sqrt{3}w_qJ_{\pm \pm} \\
-2\sqrt{3}w_qJ_{\pm \pm} & 2(-u_q + v_q)J_{\pm \pm} & (u_q - v_q)J_{\pm \pm} \\
-\sqrt{3}w_qJ_{\pm \pm} & (u_q - v_q)J_{\pm \pm} & 0
\end{pmatrix}
\]

with \( u_q = \cos(q \cdot \mathbf{a}_1) \), \( v_q = \frac{1}{2} \left( \cos(q \cdot \mathbf{a}_2) + \cos(q \cdot \mathbf{a}_3) \right) \), and \( w_q = \frac{1}{2} \left( \cos(q \cdot \mathbf{a}_2) - \cos(q \cdot \mathbf{a}_3) \right) \). The renormalized \( \chi^{\text{RPA}}(q, \omega) \) can be read off from \( \chi^{\text{RPA}}(q, \omega) = -\frac{i}{2} \text{Im} \left[ \chi^{\text{RPA}}(q, \omega) \right]^{\pm} \) and is plotted in Fig. 3(b) in the main text.

The very precise values of \( J_{\pm \pm} \) and \( J_{\pm \pm} \) cannot be determined from the existing data-rich neutron scattering experiment in a strong field normal to the triangular plane. This is partly due to the experimental resolution, and is also due to the fact that the linear spin wave spectrum for the field normal to the plane is independent of \( J_{\pm \pm} \) and is not quite sensitive to \( J_{\pm \pm} \). In Fig. 3(b) of the main text, instead, we choose \( (J_{\pm \pm}, J_{\pm \pm}) \) to fall into the disordered region of the phase diagram in Ref. 35 where the quantum fluctuations are expected to be strong\(^{35} \).

**Appendix E: The U1B states**

In this section we use PSG to determine the free spinon mean-field Hamiltonian for the U1B states to the first and second spinon hoppings. In Fig. 5, we further present their spectroscopic features for comparison. Like the notation for U1As, the U1B states are also labeled by U1Bn_{C_2 \ n_{S_n}}.

1. **The U1B00 state**

For the \( \pi \)-flux states, the dynamic spin structure factor has an enhanced periodicity due to anticommutative lattice translations. One direct consequence of the periodicity is that \( \Gamma \) and \( M \) become equivalent, and the V-shaped upper excitation edge in Ref. 36 cannot be reproduced for the U1B states.

We choose the spinon basis in the momentum space \( \mathbf{j}_{k,l} = (\mathbf{j}_{A,k,\uparrow}, \mathbf{j}_{B,k,\uparrow}, \mathbf{j}_{A,k,\downarrow}, \mathbf{j}_{B,k,\downarrow})^T \), where \( A \) and \( B \) denote the two inequivalent sites in each unit cell due to the \( \pi \) flux.

The Hamiltonian is written in terms of the Dirac matrices \( \Gamma^a \) and their anticommutators

\[
\Gamma^{ab} = [\Gamma^a, \Gamma^b]/(2i).
\]

The representation is chosen to be \( \Gamma^{1,2,3,4,5} = (\sigma^x \otimes 1, \sigma^z \otimes 1, 1, \sigma^y \otimes \tau^z, \sigma^y \otimes \tau^z) \). \( \Gamma^a \) and \( \Gamma^{ab} \) is odd under time reversal except when \( a = 4 \) or \( b = 4 \). The Hamiltonian is thus

\[
h(k) = \sum_{a=1}^{5} d_a(k)\Gamma^a + \sum_{a < b = 1}^{5} d_{ab}(k)\Gamma^{ab}.
\]

For the U1B00 state, we have

\[
\begin{align*}
d_3(k) &= t'_1 \sin(k_x/2 - \sqrt{3}k_y/2), \\
d_4(k) &= t'_1 \cos(k_x/2 + \sqrt{3}k_y/2), \\
d_5(k) &= -2t'_1 \sin(k_x), \\
d_{13}(k) &= -2t_1 \sin(k_x/2 - \sqrt{3}k_y/2), \\
d_{14}(k) &= -2t_1 \cos(k_x/2 + \sqrt{3}k_y/2), \\
d_{15}(k) &= -2t_1 \sin(k_x), \\
d_{23}(k) &= \sqrt{3}t_1 \sin(k_x/2 - \sqrt{3}k_y/2), \\
d_{24}(k) &= \sqrt{3}t_1 \cos(k_x/2 + \sqrt{3}k_y/2), \\
d_{34}(k) &= 2t_2 \cos(\sqrt{3}k_y), \\
d_{35}(k) &= 2t_2 \sin(3k_x/2 - \sqrt{3}k_y/2), \\
d_{45}(k) &= 2t_2 \cos(3k_x/2 + \sqrt{3}k_y/2).
\end{align*}
\]

2. **The U1B01 state**

\[
\begin{align*}
d_3(k) &= t_2 \sin(3k_x/2 + \sqrt{3}k_y/2), \\
d_4(k) &= -t_2 \cos(3k_x/2 - \sqrt{3}k_y/2), \\
d_5(k) &= 2t_2 \sin(\sqrt{3}k_y), \\
d_{23}(k) &= -\sqrt{3}t_2 \sin(3k_x/2 + \sqrt{3}k_y/2), \\
d_{24}(k) &= -\sqrt{3}t_2 \cos(3k_x/2 - \sqrt{3}k_y/2).
\end{align*}
\]
FIG. 5. Dynamic spin structure factor for six free spinon mean-field states other than U1A00. Note the U1A10 Hamiltonian is identically zero for the first and second neighbor hoppings. None of them is consistent with the spinon Fermi surface picture. In all subfigures, the energy transfer is normalized against the corresponding bandwidth $B$.

3. The U1B10 state

\begin{align*}
    d_3(k) &= -\sqrt{3}t_1 \sin \left( (k_x - \sqrt{3}k_y)/2 \right), \\
    d_4(k) &= \sqrt{3}t_1 \cos \left( (k_x + \sqrt{3}k_y)/2 \right), \\
    d_{23}(k) &= -t_1 \sin \left( (k_x + \sqrt{3}k_y)/2 \right), \\
    d_{24}(k) &= -t_1 \cos \left( (k_x + \sqrt{3}k_y)/2 \right), \\
    d_{25}(k) &= 2t_1 \sin k_x. \\
\end{align*}

(E5)

4. The U1B11 state

\begin{align*}
    d_3(k) &= -\sqrt{3}t_2 \sin \left( (3k_x + \sqrt{3}k_y)/2 \right), \\
    d_4(k) &= -\sqrt{3}t_2 \cos \left( (3k_x - \sqrt{3}k_y)/2 \right), \\
    d_{23}(k) &= -t_2 \sin \left( (3k_x + \sqrt{3}k_y)/2 \right), \\
    d_{24}(k) &= t_2 \cos \left( (3k_x - \sqrt{3}k_y)/2 \right), \\
    d_{25}(k) &= -2t_2 \sin(\sqrt{3}k_y), \\
    d_{34}(k) &= 2t_1 \cos(k_x), \\
    d_{35}(k) &= -2t_1 \sin \left( (k_x + \sqrt{3}k_y)/2 \right), \\
    d_{45}(k) &= -2t_1 \cos \left( (k_x - \sqrt{3}k_y)/2 \right). \\
\end{align*}

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