Research on Precision Analysis of Five-Axis Machine Tool

Jinwei Fan, Haohao Tao and Peitong Wang
Beijing University of Technology, Beijing 100124, China

Abstract. Based on the theory of multi-body system and homogeneous coordinate transformation, this paper analyses the error of five-axis CNC machine tool and establishes the machining precision model firstly. Then taking the five-axis double wall CNC machine tools as an example and according to the structure of the machine and the assembly relationship of key parts, the geometric errors of the machine tools are analyzed. Meanwhile, the coordinate systems of all the key components and characteristic matrix among the bodies are established. Finally, the Machining Accuracy Model of Machine Tool was established. It lays the foundation for the accuracy of subsequent design work.

1. Introduction
CNC machine tools, as the most widely used and used high-tech products in the machinery manufacturing industry, are important basic equipment for precision and ultra-precision processing [1, 2]. In recent years, the demand for mechanical parts with complex geometry and high dimensional accuracy has increased dramatically in the manufacturing industry. Five-axis machine tools have been more and more widely used [3, 4]. Machining accuracy of machine tools is an important index to measure the working performance of machine tools, and the spatial error of machine tools coupled by geometric errors of key parts is the main reason that affects the machining accuracy [5]. Therefore, it is of great significance to study the accuracy model of CNC machine tools based on geometric errors of key components.

In precision analysis, a lot of research work has been done and many research results have been achieved. Many different mathematical models of spatial motion errors have been produced, such as geometric modeling, error matrix, quadratic relation model, mechanism modeling, rigid body kinematics and multi-body system theory. Denavit and Hartenberg [6] first established the motion model of multi-axis CNC machine tools by homogeneous coordinate transformation. Then Paul improved the model, which laid a foundation for the establishment of general error model. Cheng et al. [7] established the precision prediction model of precision vertical machining center by using the kinematics theory of multi-body system, and realized the precision prediction of machine tools by simulating typical samples, which provided a theoretical basis for accuracy allocation. In the same year, Zhu et al. [8] proposed a new method based on multi-body system theory to synthesize geometric error model, identification and error compensation. By identifying the geometric error of five-axis machining center, the effectiveness of this method in improving the accuracy of parts was verified. Based on the multi-body theory, this paper analyses the error of the whole machine tool, and uses the homogeneous coordinate transformation matrix to model the machining accuracy of the machine tool, which lays a theoretical foundation for the subsequent precision design work.
2. Structure analysis and description of NC machine tools
Take five-axis double-wall gantry CNC machine tool as an example. As shown in Figure 1, the machine consists of seven key parts: worktable, X guide rail, slide board, slider, C-axis, A-axis and cutter. The slider moves along the Y-axis, the pillow moves along the Z-axis, and the A-axis and the C-axis rotate around the X-axis and the Z-axis respectively.

![Fig. 1 Five-axis double-wall gantry CNC machine tool](image1)

Based on the theory of multi-reference, according to the assembly relationship between parts, the topological structure diagram shown in Fig. 2 is established.

![Fig. 2 Topological structure diagram](image2)

3. Establishment of machining accuracy model for NC machine tools
According to the seven key parts of CNC machine tools, it can be divided into two parts: workpiece branching and tool branching. Establish your own right-handed Cartesian coordinate system for each part of machine tool \( O_{j} - x_{j}, y_{j}, z_{j} \) (\( j = 0,1,2,\ldots \)). The workpiece coordinate system is \( O_{w} - x_{w}, y_{w}, z_{w} \). The tool coordinate system is \( O_{t} - x_{t}, y_{t}, z_{t} \). The machine tool coordinate system is \( O_{o} - x_{o}, y_{o}, z_{o} \). The tool forming point is \( P_{t} \), the tool posture is \( V_{t} \), the theoretical tool path of cutting tools is \( P_{w} \), the tool posture determined by machining technology is \( V_{w} \). From the above analysis, the positions of \( P_{t}, V_{t} \) and \( P_{w}, V_{w} \) in the workbench coordinate system \( O_{o} - x_{o}, y_{o}, z_{o} \) can be expressed as follows:
\[
\{P_i\}_0 = \prod_{i=1}^{n} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pp} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pe} \left[SE_j \left( j \right) E^{-1}(j) \right]_{et} \{r_i\} 
\]

(1)

\[
\{V_i\}_0 = \prod_{i=1}^{n} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pp} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pe} \left[SE_j \left( j \right) E^{-1}(j) \right]_{et} \{r_i\} 
\]

(2)

\[
\{P_s\}_0 = \prod_{i=1}^{n} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pp} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pe} \left[SE_j \left( j \right) E^{-1}(j) \right]_{et} \{r_s\} 
\]

(3)

\[
\{V_s\}_0 = \prod_{i=1}^{n} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pp} \left[SE_j \left( j \right) E^{-1}(j) \right]_{pe} \left[SE_j \left( j \right) E^{-1}(j) \right]_{et} \{r_s\} 
\]

(4)

where \{r_i\} and \{r_s\} are respectively the position matrix expressions of the tool forming point in the tool coordinate system \(O_x-y_z\) and the workpiece coordinate system \(O_x-y_z\). \{r_i\} and \{r_s\} are the matrix expressions of the tool's position and attitude in the tool coordinate system \(O_x-y_z\) and the workpiece coordinate system \(O_x-y_z\), respectively.

In the actual forming process of the machine tool, the actual trajectory of the tool forming point and the prop posture will inevitably deviate from the ideal trajectory and posture. Therefore, the machining accuracy model of machine tools can be expressed as:

\[
\{e_r\} = \left( e_{rx}, e_{ry}, e_{rz}, 0 \right)^T = \{P_w\} - \{P_t\} 
\]

(5)

\[
\{e_v\} = \left( e_{vx}, e_{vy}, e_{vz}, 0 \right)^T = \{V_w\} - \{V_t\} 
\]

(6)

4. Geometric error analysis and modeling of five-axis NC machine tool

4.1. Geometric error analysis

The main error sources affecting the machining accuracy of NC machine tools are geometric error, thermal error, load error, and servo system error. Among them, geometric errors are less affected by environmental factors and easy to measure. Whatever the errors, their final manifestations can be expressed by the geometric error analysis and motion modeling methods described above. Therefore, the emphasis of this paper is to take geometric error as the basic error source. Five-axis double-wall gantry CNC machine tool has seven key moving parts. The error of each key part can be divided into two parts: static error and motion error.

According to the six-freedom assumption theory of rigid body, six errors (trinomial displacement error and trinomial angular displacement error) will inevitably occur when rigid body moves in three-dimensional space, totaling 30 motion errors. In addition, there are three vertical errors between two of the three translational axes, and there are four errors between A-axis and Y-axis, Z-axis and C-axis and X-axis and Y-axis. Therefore, there are 37 geometric error parameters of five-axis double-wall gantry CNC machine tools, as shown in Table 1.
Table 1. Geometric errors of five-axis CNC machine tools

| Geometric Errors | εx(x) | εy(x) | εz(x) | δx(x) | δy(x) | δz(x) |
|------------------|-------|-------|-------|-------|-------|-------|
| Along X-axis     |       |       |       |       |       |       |
| Along Y-axis     |       |       |       |       |       |       |
| Along Z-axis     |       |       |       |       |       |       |
| Around C-axis    |       |       |       |       |       |       |
| Around A-axis    |       |       |       |       |       |       |
| Perpendicularity | εxy,  | εxz,  | εyz,  | εcx,  | εcy,  | εaz,  |

4.2. Establishment of machining accuracy model of Five-axis machine tool

Under the initial condition, the position of the actual tool center in the tool coordinate system is \( \{r_t\} = (0, 0, -L, 1)^T \), L is the tool length, and the expression of the actual tool pose in the tool coordinate system is \( \{r_t\} = (0, 0, -1, 0)^T \); in the workpiece coordinate system, the theoretical tool path is \( \{r_w\} = (x_w, y_w, z_w, 1)^T \). Thus, the theoretical tool pose \( \{r_w\} = (x_w, y_w, z_w, 0)^T \) can be obtained. In the worktable coordinate system, the position of tool forming point is:

\[
P_{0} = \prod_{t} \left( [SL'(j)L^{-1}(j)]_{p} SL'(j)L^{-1}(j)]_{w} SL'(j)L^{-1}(j)]_{w}(r_t) \right)
\]

(7)

The expression of the actual position and pose of the cutter is as follows:

\[
V_{0} = \prod_{t} \left( [SL'(j)L^{-1}(j)]_{p} SL'(j)L^{-1}(j)]_{w} SL'(j)L^{-1}(j)]_{w}(r_t) \right)
\]

(8)

The position of the theoretical tool path in the workbench coordinate system is as follows:

\[
P_{w0} = \prod_{t} \left( [SL'(j)L^{-1}(j)]_{p} SL'(j)L^{-1}(j)]_{w} SL'(j)L^{-1}(j)]_{w}(r_w) \right)
\]

(9)

The position of the theoretical tool path in the workbench coordinate system is as follows:

\[
V_{w0} = \prod_{t} \left( [SL'(j)L^{-1}(j)]_{p} SL'(j)L^{-1}(j)]_{w} SL'(j)L^{-1}(j)]_{w}(r_w) \right)
\]

(10)

Therefore, the machining accuracy model of machine tools is as follows:

\[
\begin{align*}
    e_{\chi} &= \delta_{\chi}(x) + \delta_{\chi}(y) - q_{\chi z} + x + \delta_{\chi}(c) \cos(c) + \delta_{\chi}(c) \sin(c) - e_{\chi z} \times y + q_{\chi z} \cos(c) \\
    e_{\gamma} &= \delta_{\gamma}(x) + \delta_{\gamma}(y) - q_{\gamma z} - y + \delta_{\gamma}(c) \sin(c) + \delta_{\gamma}(c) \cos(c) - q_{\gamma z} \cos(c) \\
    e_{\sigma} &= \delta_{\sigma}(x) + \delta_{\sigma}(y) - q_{\sigma z} + z + \cos(d) - \sin(d) \times e_{\sigma x} + \delta_{\sigma}(d)
\end{align*}
\]

(11)
\[
\begin{align*}
\{e_x\} &= \cos(\alpha) \times [-e_z \cos(\gamma) - e_y \sin(\gamma) + e_z \sin(\gamma) + e_y \cos(\gamma)] + e_x \times \sin(\gamma) - e_z \times \cos(\gamma) - e_y \times (z) \\
\{e_y\} &= -e_x \times \sin(\alpha) - e_z \times \sin(\gamma) - e_y \times (y) \times \sin(\gamma) - e_x \times (z) - e_y \times \cos(\gamma) + \cos(\alpha) \times [-e_x \times (x) + e_y \times (y)] \times \cos(\gamma) \\
\{e_z\} &= -e_x \times \cos(\gamma) - e_z \times \cos(\gamma) \times [-e_x \times (x) - e_y \times (y)] - e_y \times \cos(\alpha) + \sin(\alpha) \times \{e_z \times (x) + e_y \times (y) + e_x \times (z) - e_y \times (z)\} \times \cos(\gamma) + e_x \times \sin(\gamma)
\end{align*}
\]

(12)

5. Conclusion

(1) By analyzing the structure of the five-axis double-wall gantry CNC machine tool and describing the structure of the machine tool by using the topological structure, the machine tool is simplified to seven key parts, and the sub-coordinate system and the motion reference coordinate system are established for each moving part.

(2) The geometric errors of five-axis double-wall gantry CNC machine tools are analyzed, and 37 geometric errors are obtained.

(3) Based on the kinematics theory of multi-body system, the transformation matrix of relative motion coordinates among the moving parts of machine tool is derived, and the machining accuracy model of machine tool is further derived, which lays a foundation for the prediction of machine tool machining accuracy in the future.

Acknowledgments

This work is financially supported by the National Natural Science Foundation of China (No. 51275014), Science and Technology Major Projects of High-end CNC Machine Tools and Basic Manufacturing Equipment of China (No. 2016ZX04003001).

References

[1] WANG B. The study on thermal errors compensation of NC machine tools [D]. Beijing: College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, 2010.

[2] FAN J, TAO H, WU C, et al. Kinematic errors prediction for multi-axis machine tools’ guideways based on tolerance. Int J Adv Manuf Technol. 2018, 98 (5-8): 1131-1144.

[3] Y. Lin, Y. Shen. 2003 Modeling of five-axis machine tool metrology models using the matrix summation approach [J]. The International Journal of Advanced Manufacturing Technology. 21 (4): 243-248.

[4] S. W. Zhu, G. F. Ding, S. F. Qin, et al. 2012 Integrated geometric error modeling identification and compensation of CNC machine tool [J]. International Journal of Machine Tools and Manufacture. 52: 24-29.

[5] TAO H, FAN J, WU C, et al. An optimized single point offset method for reducing theoretical error of S-shaped test piece. Int J Adv Manuf Technol. 2019, 104 (1-4): 617-629.

[6] Denavit, J, Hartenberg, R. S. A kinematic notation for lower-pair mechanisms based on matrices [J]. Trans.of the Asme.journal of Applied Mechanics, 1955, 22: 215-221.

[7] CHENG Q, LIU G B, LIU Z F, et al. An identification approach for key geometric error sources of machine tool based on sensitivity analysis [J]. Journal of Mechanical Engineering, 2012, 48 (4): 92-100.

[8] Zhu S, Ding G, Qin S, et al. Integrated geometric error modeling, identification and compensation of CNC machine tools [J]. International Journal of Machine Tools & Manufacture, 2012, 52 (1): 24-29.