Statistical Mechanics of Nonlinear On-line Learning for Ensemble Teachers

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We analyze the generalization performance of a student in a model composed of nonlinear perceptrons: a true teacher, ensemble teachers, and the student. We calculate the generalization error of the student analytically or numerically using statistical mechanics in the framework of on-line learning. We treat two well-known learning rules: Hebbian learning and perceptron learning. As a result, it is proven that the nonlinear model shows qualitatively different behaviors from the linear model. Moreover, it is clarified that Hebbian learning and perceptron learning show qualitatively different behaviors from each other. In Hebbian learning, we can analytically obtain the solutions. In this case, the generalization error monotonically decreases. The steady value of the generalization error is independent of the learning rate. The larger the number of teachers is and the more variety the ensemble teachers have, the smaller the generalization error is. In perceptron learning, we have to numerically obtain the solutions. In this case, the dynamical behaviors of the generalization error are non-monotonic. The smaller the learning rate is, the larger the number of teachers is; and the more variety the ensemble teachers have, the smaller the minimum value of the generalization error is.

KEYWORDS: on-line learning, generalization error, ensemble teachers, true teacher

1. Introduction

Learning is to infer the underlying rules that dominate data generation using observed data. Observed data are input-output pairs from a teacher and are called examples. Learning can be roughly classified into batch learning and on-line learning.\textsuperscript{1} In batch learning, given examples are used more than once. In this paradigm, a student becomes to give correct answers after training if the student has had adequate freedom. However, it is necessary to have a long time and a large memory in which to store many examples. On the contrary, in on-line learning, examples once used are discarded. In this case, a student cannot give correct answers after training if the student has not had adequate freedom. However, it is possible to give correct answers if the student has had adequate freedom. In this case, a student can give correct answers after training if the student has had adequate freedom.
answers for all examples used in training. However, there are merits. For example, a large memory for storing many examples isn’t necessary, and it is possible to follow a time-variant teacher.

Recently, we analyzed the generalization performance of ensemble learning in a framework of on-line learning using a statistical mechanical method. Using the same method, we also analyzed the generalization performance of a student supervised by a moving teacher that goes around a true teacher. As a result, it was proven that the generalization error of a student can be smaller than that of a moving teacher, even if the student only uses examples from the moving teacher. In an actual human society, a teacher observed by a student does not always present the correct answer. In many cases, the teacher is learning and continues to change. Therefore, the analysis of such a model is interesting for considering the analogies between statistical learning theories and an actual human society.

On the other hand, in most cases in an actual human society, a student can observe examples from two or more teachers who differ from each other. Therefore, we analyze the generalization performance of such a model and discuss the use of imperfect teachers in this paper. That is, we consider a true teacher and teachers called ensemble teachers who exist around the true teacher. A student uses input-output pairs from ensemble teachers in turn or randomly.

A model in which the true teacher, the ensemble teachers and the student are all linear perceptrons with noise has already been solved analytically. In that case, it was proven that when the student’s learning rate satisfies $\eta < 1$, the larger the number $K$ of ensemble teachers is and the more variety the ensemble teachers have, the smaller the student’s generalization error is. On the other hand, when $\eta > 1$, the properties are completely reversed. If the variety of ensemble teachers is rich enough, the direction cosine between the true teacher and the student becomes unity in the limit of $\eta \to 0$ and $K \to \infty$.

However, linear perceptrons are somewhat special as neural networks or learning machines. Nonlinear perceptrons are more common than linear ones. Therefore, we analyze the generalization performance of a student in a model composed of nonlinear perceptrons, a true teacher, ensemble teachers, and the student. We obtain order parameters and the generalization errors analytically or numerically in the framework of on-line learning using a statistical mechanical method. We treat two well-known learning rules: Hebbian learning and perceptron learning. As a result, it is proven that the nonlinear model shows qualitatively different behaviors from the linear model. Moreover, it is clarified that Hebbian learning and perceptron learning show qualitatively different behaviors from each other. In Hebbian learning, we can analytically obtain the solutions. In this case, the generalization error monotonically decreases. The steady value of the generalization error is independent of the learning rate $\eta$. The larger the number $K$ of teachers is and the more variety the ensemble teachers have, the smaller the general-
generalization error is. In perceptron learning, we have to numerically obtain the solutions. In this case, the dynamical behaviors of the generalization error are non-monotonic. The smaller the learning rate $\eta$ is, the larger the number $K$ of teachers is; and the more variety the ensemble teachers have, the smaller the minimum value of the generalization error is.

2. Model

In this paper, we consider a true teacher, $K$ ensemble teachers and a student. They are all nonlinear perceptrons with connection weights $A$, $B_k$ and $J$, respectively. Here, $k = 1, \ldots, K$. For simplicity, the connection weights of the true teacher, the ensemble teachers and the student are simply called the true teacher, the ensemble teachers and the student, respectively. True teacher $A = (A_1, \ldots, A_N)$, ensemble teachers $B_k = (B_{k1}, \ldots, B_{kN})$, student $J = (J_1, \ldots, J_N)$ and input $x = (x_1, \ldots, x_N)$ are $N$-dimensional vectors. Each component $A_i$ of $A$ is drawn from $N(0,1)$ independently and fixed, where $N(0,1)$ denotes Gaussian distribution with a mean of zero and a variance of unity. Some components $B_{ki}$ are equal to $A_i$ multiplied by $-1$, and the others are equal to $A_i$. Which component $B_{ki}$ is equal to $-A_i$ is independent of the value of $A_i$. Hence, $B_{ki}$ also obeys $N(0,1)$. $B_{ki}$ is also fixed. The direction cosine between $B_k$ and $A$ is $R_{Bk}$ and that between $B_k$ and $B_{k'}$ is $q_{kk'}$. Each of the components $J_0^i$ of the initial value $J^0$ of $J$ is drawn from $N(0,1)$ independently. The direction cosine between $J$ and $A$ is $R_J$ and that between $J$ and $B_k$ is $R_{BkJ}$. Each component $x_i$ of $x$ is drawn from $N(0,1/N)$ independently. Thus,

$$\langle A_i \rangle = 0, \quad \langle (A_i)^2 \rangle = 1,$$

$$\langle B_{ki} \rangle = 0, \quad \langle (B_{ki})^2 \rangle = 1,$$

$$\langle J_0^i \rangle = 0, \quad \langle (J_0^i)^2 \rangle = 1,$$

$$\langle x_i \rangle = 0, \quad \langle (x_i)^2 \rangle = \frac{1}{N},$$

$$R_{Bk} = \frac{A \cdot B_k}{\|A\| \|B_k\|}, \quad q_{kk'} = \frac{B_k \cdot B_{k'}}{||B_k|| \|B_{k'}\|},$$

$$R_J = \frac{A \cdot J}{\|A\| \|J\|}, \quad R_{BkJ} = \frac{B_k \cdot J}{\|B_k\| \|J\|},$$

where $\langle \cdot \rangle$ denotes a mean. Figure 1 illustrates the relationship among true teacher $A$, ensemble teachers $B_k$, student $J$ and direction cosines $q_{kk'}, R_{Bk}, R_J$ and $R_{BkJ}$.

In this paper, the thermodynamic limit $N \to \infty$ is also treated. Therefore,

$$\|A\| = \sqrt{N}, \quad \|B_k\| = \sqrt{N}, \quad \|J^0\| = \sqrt{N}, \quad \|x\| = 1.$$  

Generally, a norm $\|J\|$ of the student changes as the time step proceeds. Therefore, ratios $l^m$ of the norm to $\sqrt{N}$ are introduced and called the length of the student. That is, $\|J^m\| = l^m \sqrt{N}$, where $m$ denotes the time step.
The internal potentials $y^m$ of the true teacher, $v^m_k$ of the ensemble teachers, and $u^m_l$ of the student are

$$y^m = A \cdot x^m,$$

$$v^m_k = B_k \cdot x^m,$$

$$u^m_l = J \cdot x^m,$$

respectively. Here, $y^m$, $v^m_k$ and $u^m$ obey the Gaussian distributions with means of zero and the covariance matrix $\Sigma$:

$$\Sigma = \begin{pmatrix} 1 & R_{Bk} & R_J \\ R_{Bk} & 1 & R_{BkJ} \\ R_J & R_{BkJ} & 1 \end{pmatrix}.$$

The outputs of the true teacher, the ensemble teachers, and the student are $\text{sgn}(y^m)$, $\text{sgn}(v^m_k)$ and $\text{sgn}(u^m_l)$, respectively. Here, $\text{sgn}(\cdot)$ is a sign function defined as

$$\text{sgn}(z) = \begin{cases} +1, & z \geq 0, \\ -1, & z < 0. \end{cases}$$

In the model treated in this paper, the student $J$ is updated using an input $x$ and the outputs of ensemble teachers $B_k$ for the input. That is,

$$J^{m+1} = J^m + f^m x^m,$$

where $f^m$ denotes a function that represents the update amount and is determined by the learning rule. In the well-known learning rules for nonlinear perceptrons, Hebbian learning and perceptron learning, $f^m$ are

$$f^m = \eta \text{sgn}(v^m),$$
respectively. Here, $\eta$ is the learning rate of the student and is constant. $\Theta(\cdot)$ is a step function defined as

$$
\Theta(z) = \begin{cases} 
+1, & z \geq 0, \\
0, & z < 0.
\end{cases}
$$

(16)

3. Theory

3.1 Generalization error

A goal of statical learning theory is to theoretically obtain generalization errors. We use

$$
e^m = \Theta(-y^m u^m)
$$

(17)
as the error of the student. The superscripts $m$, which represent the time step, are omitted for simplicity unless stated otherwise. Since the generalization error is the mean of errors for the true teacher over the distribution of new input, generalization error $\epsilon_g$ of student $J$ is calculated as follows:

$$
\epsilon_g = \int d\mathbf{x} P(\mathbf{x}) \epsilon
$$

(18)

$$
= \int dy du P(y, u) \epsilon(y, u)
$$

(19)

$$
= \frac{1}{\pi} \tan^{-1} \sqrt{1 - R_J^2}.
$$

(20)

Here, integration has been executed using the following: $y$ and $u$ obey $\mathcal{N}(0,1)$. The covariance between $y$ and $u$ is $R_J$.

3.2 Differential equations for order parameters

To simplify the analysis, the following auxiliary order parameters are introduced:

$$
r_J \equiv R_J l,
$$

(21)

$$
r_{BkJ} \equiv R_{BkJ} l.
$$

(22)

Simultaneous differential equations in deterministic forms,\textsuperscript{10} which describe the dynamical behaviors of order parameters, have been obtained based on self-averaging in the thermodynamic limits as follows:

$$
\frac{dr_{BkJ}}{dt} = \frac{1}{K} \sum_{k'=1}^{K} \langle f_{k'} v_k \rangle;
$$

(23)

$$
\frac{dr_J}{dt} = \frac{1}{K} \sum_{k=1}^{K} \langle f_k y \rangle;
$$

(24)

$$
\frac{dl}{dt} = \frac{1}{K} \sum_{k=1}^{K} \left( \langle f_k u \rangle + \frac{1}{2l} \langle f_k^2 \rangle \right).
$$

(25)
Here, dimension $N$ has been treated to be sufficiently greater than the number $K$ of ensemble teachers. Time is defined by $t = m/N$, that is, time step $m$ normalized by dimension $N$. Note that the above differential equations are identical whether the $K$ ensemble teachers are used in turn or randomly.

### 3.3 Hebbian learning

Since $y, v$ and $u$ obey the triple Gaussian distribution with means of zero and the covariance matrix of eq. (11), the four sample averages that appear in eqs. (23)–(25) in Hebbian learning can be calculated using eq.(14) as follows:

$$\langle f_k v_k \rangle = \eta^2 q_{kk}' \sqrt{2/\pi}, \quad \langle f_k y \rangle = \eta^2 R_{Bk} \sqrt{2/\pi},$$

$$\langle f_k u \rangle = \eta^2 R_{BkJ} \sqrt{2/\pi}, \quad \langle f_k^2 \rangle = \eta^2.$$

Since all components $A_i, J_0^i$ of true teacher $A$, and the initial student $J^0$ are drawn from $\mathcal{N}(0,1)$ independently and because the thermodynamic limit $N \to \infty$ is also treated, they are orthogonal to each other in the initial state. That is,

$$R_J = 0 \text{ when } t = 0. \quad \text{(30)}$$

In addition,

$$l = 1 \text{ when } t = 0. \quad \text{(31)}$$

Using eqs. (26)–(31), the simultaneous differential equations (23)–(25) can be solved analytically as follows:

$$r_{BkJ} = \frac{\eta}{K} \sum_{k'=1}^{K} \frac{2q_{kk'}}{\sqrt{2\pi}} t, \quad \text{(32)}$$

$$r_J = \frac{\eta}{K} \sum_{k=1}^{K} \frac{2R_{BkJ}}{\sqrt{2\pi}} t, \quad \text{(33)}$$

$$l^2 = \frac{\eta^2}{K} \sum_{k=1}^{K} \left( \frac{2}{K\pi} \sum_{k'=1}^{K} q_{kk'} t^2 + t \right) + 1. \quad \text{(34)}$$

### 3.4 Perceptron learning

Since $y, v$ and $u$ obey the triple Gaussian distribution with means of zero and the covariance matrix of eq. (11), the four sample averages that appear in eqs. (23)–(25) in perceptron learning can be calculated using eq.(14) as follows:
learning can be calculated using eq. (15) as follows:

$$\langle f_k v_k \rangle = \eta q_{kk} - R_{BkJ} \sqrt{2 \pi},$$  

(35)

$$\langle f_k y \rangle = \eta R_{Bk} - R_J \sqrt{2 \pi},$$  

(36)

$$\langle f_k u \rangle = \eta R_{BkJ} - 1 \sqrt{2 \pi},$$  

(37)

$$\langle f_k^2 \rangle = \frac{\eta^2}{\pi} \tan^{-1} \frac{1 - R_{BkJ}^2}{R_{BkJ}}.$$  

(38)

Since the simultaneous differential equations cannot be solved analytically in this case, we solve these equations numerically.

4. Results and Discussion

In this section, we treat the case where the direction cosines $R_{Bk}$ between the ensemble teachers and the true teacher, and the direction cosines $q_{kk'}$ among the ensemble teachers are uniform. That is,

$$R_{Bk} = R_B, \quad k = 1, \ldots, K,$$  

(39)

$$q_{kk'} = \begin{cases} q, & k \neq k', \\ 1, & k = k'. \end{cases}$$  

(40)

In Hebbian learning, since order parameters are analytically obtained, we can understand the dynamical behaviors clearly and deeply. Considering eqs. (21), (33), (34), (39) and (40), $R_J$ is obtained as follows:

$$R_J = \frac{R_B}{\sqrt{(K-1)q + 1} + \frac{q}{K} \left( \frac{1}{\eta^2} + \frac{1}{1} \right)}.$$  

(41)

Equation (41) shows the following: the dynamical behaviors of $R_J$ are monotonically increasing. The larger the learning rate $\eta$ is, the larger the direction cosine $R_J$ is. $R_J$ in the limit of $t \to \infty$ is obtained as follows:

$$R_J \to \frac{R_B}{\sqrt{\frac{1}{K} + (1 - \frac{1}{K})q}} = \frac{R_B}{\sqrt{q + \frac{1-q}{K}}}.$$  

(42)

This equation shows that the steady state value of $R_J$ is independent of the learning rate $\eta$. The larger the number $K$ of ensemble teachers is and the smaller the direction cosine $q$ among ensemble teachers is, the larger the steady state value of $R_J$ is.

Considering that the generalization error $\epsilon_g$ calculated by eq. (20) monotonically decreases as $R_J$ increases, $\epsilon_g$ in the case of Hebbian learning monotonically decreases. The larger $\eta$ is, the smaller $\epsilon_g$ is in the transient phase. The steady state value of $\epsilon_g$ is independent of $\eta$. However, the larger the number $K$ is and the smaller $q$ is, the smaller the steady state value of $\epsilon_g$ is. Therefore, the larger the number of teachers is and the more variety the ensemble
teachers have, the more clever the student can become.

![Fig. 2. Dynamical behaviors of generalization error $\epsilon_g$. Hebbian learning. Theory and computer simulations. Conditions other than $\eta$ are $K = 10$, $q = 0.49$ and $R_B = 0.7$.](image)

![Fig. 3. Steady state value of generalization error $\epsilon_g$. Hebbian learning. Theory and computer simulations. $R_B = 0.7$. When $q = R_B^2$ and $K = \infty$, the steady state value of $\epsilon_g$ is zero.](image)

Equation (42) shows $R_J \rightarrow R_B/\sqrt{q}$ in the limit of $K \rightarrow \infty$. On the other hand, when $S$ and $T$ are generated independently under conditions where the direction cosine between $S$ and $P$ and between $T$ and $P$ are both $R_0$, where $S$, $T$ and $P$ are high dimensional vectors, the direction cosine between $S$ and $T$ is $q_0 = R_0^2$, as shown in the appendix. Therefore, if ensemble teachers have enough variety that they have been generated independently under the condition that all direction cosines between ensemble teachers and the true teacher are
$R_B, R_B/\sqrt{q} = 1$, then the direction cosine $R_J$ between the student and the true teacher approaches unity in the limit of $K \to \infty$. Then, the generalization error approaches zero.

The dynamical behaviors of generalization error $\epsilon_g$ have been analytically obtained by eqs.(20) and (41) in Hebbian learning. Figures 2 and 3 show the analytical results of $\epsilon_g$ and the steady state value of $\epsilon_g$ with corresponding simulation results. In computer simulations, the dimension $N = 2000$ and $K$ ensemble teachers are used in turn. The generalization error $\epsilon_g$ was obtained by test for $10^4$ random inputs at each time step. In these figures, the curves represent theoretical results. The symbols represent simulation results. In Fig. 2, conditions other than $\eta$ are common: $K = 10, q = 0.49$ and $R_B = 0.7$. In Fig. 3, only $R_B$ is common: $R_B = 0.7$. The former discussions are confirmed in these figures.

On the other hand, in perceptron learning, we cannot solve eqs.(23)–(25) analytically. Therefore, we obtain the solutions numerically. The dynamical behaviors of generalization errors $\epsilon_g$ are shown in Figs. 4–6.

In Fig.4, conditions other than $\eta$ are $K = 10, q = 0.49$ and $R_B = 0.7$. In Fig.5, conditions other than $K$ are $\eta = 0.2, q = 0.49$ and $R_B = 0.7$. In Fig.6, conditions other than $q$ are $K = 10, \eta = 0.2$ and $R_B = 0.7$. Figure 4 shows that the dynamical behaviors of $\epsilon_g$ have non-monotonic properties when the learning rate $\eta$ is relatively small. However, Figs.5 and 6 show that the steady state value of the generalization error is independent of $K$ and $q$. These are remarkable differences from the properties of Hebbian learning.

![Fig. 4. Dynamical behaviors of generalization error $\epsilon_g$. Perceptron learning. Theory and computer simulations. Conditions other than $\eta$ are $K = 10, q = 0.49, R_B = 0.7$.](image)

When the learning rate $\eta$ is relatively small, the minimum value $\epsilon_g(min)$ of the generalization error exists and the smaller $\eta$ is, the smaller $\epsilon_g(min)$ is. The relationships between $K$ and $\epsilon_g(min)$, and $q$ and $\epsilon_g(min)$ are shown in Figs. 7 and 8, respectively. In Fig.7, conditions other than $\eta$ are $q = 0.49$ and $R_B = 0.7$. In Fig.8, conditions other than $\eta$ are $K = 10$ and
Fig. 5. Dynamical behaviors of generalization error $\epsilon_g$. Perceptron learning. Theory and computer simulations. Conditions other than $K$ are $\eta = 0.2$, $q = 0.49$ and $R_B = 0.7$.

Fig. 6. Dynamical behaviors of generalization error $\epsilon_g$. Perceptron learning. Theory and computer simulations. Conditions other than $q$ are $K = 10$, $\eta = 0.2$ and $R_B = 0.7$.

$R_B = 0.7$. These figures show that the larger the number $K$ is and the smaller the direction cosine $q$ is, the smaller the minimum value of generalization errors is. In other words, the larger the number of teachers is and the more variety the ensemble teachers have, the more clever the student can become.

In the case of the linear model, the properties were able to be summarized as follows: The smaller $\eta$ is, the smaller the steady state value of $\epsilon_g$ is. When the learning rate satisfies $\eta < 1$, the larger $K$ is and the smaller $q$ is, the smaller the steady state value of $\epsilon_e$ is. On the contrary, when $\eta > 1$, the properties are completely reversed. Comparing the linear model and the nonlinear model treated in this paper, there are qualitatively different properties.
Fig. 7. Relationship between $K$ and minimum values $\epsilon_g(\text{min})$ of generalization error. Perceptron learning. Theory. $q = 0.49, R_B = 0.7$.

Fig. 8. Relationship between $q$ and minimum values $\epsilon_g(\text{min})$ of generalization error. Perceptron learning. Theory. $K = 10, R_B = 0.7$.

5. Conclusion

We have analyzed the generalization performance of a student in a model composed of nonlinear perceptrons: a true teacher, ensemble teachers, and the student. We have calculated the generalization error of the student analytically or numerically using statistical mechanics in the framework of on-line learning. We have treated two well-known learning rules: Hebbian learning and perceptron learning. As a result, it has been proven that the nonlinear model shows qualitatively different behaviors from the linear model. Moreover, it has been clarified that Hebbian learning and perceptron learning show qualitatively different behaviors from each other. In Hebbian learning, we have analytically obtained the solutions. In this case, the generalization error monotonically decreases. The steady value of the generalization error is
independent of the learning rate. The larger the number of teachers is and the more variety the 
ensemble teachers have, the smaller the generalization error is. In perceptron learning, we have 
obtained the solutions numerically. In this case, the dynamical behaviors of the generalization 
error are non-monotonic. The smaller the learning rate is, the larger the number of teachers 
is, and the more variety the ensemble teachers have, the smaller the minimum value of the 
generalization error is.

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Appendix: Direction cosine $q$ among ensemble teachers

Let us consider the case where $S$ and $T$ are generated independently, satisfying the con-
dition that direction cosines between $S$ and $P$ and between $T$ and $P$ are both $R_0$, as shown 
in Fig. A-1, where $S$, $T$ and $P$ are $N$ dimensional vectors. In this figure, the inner product 
of $s$ and $t$ is

$$s \cdot t = \left( S - R_0 \frac{\|S\|}{\|P\|} P \right) \cdot \left( T - R_0 \frac{\|T\|}{\|P\|} P \right) \quad (A\cdot1)$$

$$= \|S\| \|T\| \left( q_0 - R_0^2 \right), \quad (A\cdot2)$$

where $s$ and $t$ are projections from $S$ to the orthogonal complement $C$ of $X$ and from $T$ to 
$C$, respectively. $q_0$ denotes the direction cosine between $S$ and $T$.

Incidentally, if dimension $N$ is large and $S$ and $T$ have been generated independently, $s$ 
and $t$ should be orthogonal to each other. Therefore, $q_0 = R_0^2$.

Fig. A-1. Direction cosine among ensemble teachers.
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