Outage Probability Analysis of AF Relaying Systems with Heterogeneous Ad Hoc Networks

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ABSTRACT

In this paper, we analyze the outage probability of AF relaying systems with Heterogeneous Ad Hoc Networks with respect to power allocation. The communication system can be decomposed into a two-tier heterogeneous ad hoc network and a single-input multiple-output (SIMO) system, where the information ad hoc network and the energy ad hoc network coexist and share the same spectrum resource. We firstly establish the end-to-end signal-to-noise ratio (SNR) expression considering that the energy/information signals are transmitted with different power levels, to maximize the SNR at the terminal node, we then investigate a power allocation scheme at the relay node. Closed-form expressions for the outage probability are analytically derived based on special functions, moreover, we propose the outage probability can be tightly lower bounded by a simple expression for the case when the SNR is sufficiently high. Numerical and simulation results are also reported to verify our theory. This paper provides a beneficial insight into performance optimization for wireless communication networks.

Keywords: Index Terms—heterogeneous ad hoc networks; AF relaying systems; outage probability

INTRODUCTION

Aiming at providing widespread coverage, high reliable transmission has been the main challenge in wireless communication systems. Recently, AF (amplifier-and-forward) technology has drawn substantial interest in Massive MIMO Systems [1][2][3], for it has been revealed it has obvious advantages on improving coverage and communication performance. On the other hand, AF relaying antenna can be deployed anywhere and provides power gain as well as signal reshaping. In [4][5], it has been shown conjugating AF-relay and MIMO strategies guarantees high data transmission rates. In addition, simultaneous wireless information and power transfer (SWIPT) and joint wireless information and energy transfer (JWIET) have been introduced into RF networks, R. Zhang et.al [6] has reported their effectiveness in providing perpetual energy supplies to wireless networks. The information receiver and the power receiver are separated and interference in conventional communication networks serves as a source for the energy harvesting (EH) in ad hoc networks.

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To improve information transmission quality in wireless networks, performance analysis and power allocation has been studied in various scenarios. In [7], the authors proposed an optimal power allocation (OPA) scheme based on the source and opportunistic relay. In [8], Quek et.al maximized SNR by utilizing the conic optimization theory under perfect channel state information (CSI) and relay power constraints. In [9], the researchers considered single-user MIMO systems and deduced ergodic achievable rates in details. In [10][11], Quality of Service (QoS) analysis was conducted for Heterogeneous Wireless Network and wireless mesh network, respectively. In [12], the authors analyzed the outage probability to study the response of a system for direct link/relay link in Cooperative Cognitive Radio Networks. However, to our best knowledge, few work has focused on the power allocation and outage probability in AF relaying systems equipped with Heterogeneous Information/Energy Ad Hoc Networks, which motivates our work.

This paper is organized as follows. In section 2, we establish the dual-hop communication system discussed in our paper, which includes a two-tier heterogeneous ad hoc network and a SIMO system. The two tiers in the ad hoc network are the information network and the energy network, respectively, which share the same spectrum resource, namely, the information receiver and the energy receiver see the identical MIMO channel from the transmitter. Further, we assume that the information access point (IAP) and the energy access point (EAP) are both independently distributed and can both be modeled as a homogeneous two-dimensional Possion point process (PPP). The SIMO system treats the typical information decoding (ID) (or EH) receiver as the transmitter and a uniform linear array (ULA) is adopted as terminal nodes. We then derive the received signal at terminal nodes and propose the OPA approach in this model. In section 3, we theoretically derive the closed solution for information outage probability, in addition, we present an approximate expression for the high SNR scene, which provides a lower bound and avoids complicated special functions. Numerical results and simulation are reported in section. Conclusions for this paper are drawn in section.

**Notation:** The mathematical symbols are defined as follows. We use $||x||_F$, $\Gamma(\cdot)$, $B(x,y)$, $I_0(\cdot)$ to denote Frobenius two-norm, Gamma-function, Beta-function, modified Bessel function of the first kind, respectively.

**PROBLEM FORMULATION AND POWER ALLOCATION**

The two hop AF relaying system we adopt is described in Fig.1, where the whole system can be decomposed into an Ad Hoc Network and a SIMO system. The channel characteristic between source nodes and the single relay node can be modeled as an Ad Hoc Network. An Ad Hoc Network concludes IAP, EAP and a receiving node (either ID or EH, which also serves as the relaying node in this scheme). Assume that IAP and EAP are equipped with $M_I$ and $M_E$ antennas, respectively. The location of IAPs and EAPs is assumed to obey homogeneous PPP. We depicts the homogeneous PPP of IAP and EAP as $\varphi(\lambda_I) = \{(X_{Ii}, h_{I0i}), i \in \mathbb{N}^+\}$ and $\varphi(\lambda_E) = \{(X_{Ej}, h_{E0j}), j \in \mathbb{N}^+\}$, respectively. $\lambda_I$ and $\lambda_E$ are utilized to represent the density of IAPs and EAPs, $X_{Ii}$ and $X_{Ej}$ are the positions of the $i$th IAP and the $j$th EAP, respectively. $h_{I0i} \in \mathbb{C}^{M_I\times1}$, $h_{E0j} \in \mathbb{C}^{M_E\times1}$ denotes the wireless channel vector from $i$th IAP ($j$th EAP ) to the relaying node, in addition, $h_{00} \in \mathbb{C}^{2M\times1}$ is the typical channel vector and $d_0$ is physical distance between IAP (EAP) and the relaying node. $\alpha$ is the fading coefficient. Then the received signal measured at the relay node can be expressed as:
\[ y_0 = d_0^{-\alpha/2} h_{00}^H x_0 + \sum_{i \in \phi(\lambda_i)} |X_i|^\alpha/2 h_{i0}^H x_{i} + \sum_{j \in \phi(\lambda_E)} |X_E|^\alpha/2 h_{E0}^H x_{Ej} + n_1 \]

where \( M_I + M_E = 2M \), \( x_{ii}, x_{Ej} \) is the information beamforming vector and energy beamforming vector, respectively, with \( \mathbb{E}\{x_{ii}^H x_{ii}\}/M_I = P_I \) and \( \mathbb{E}\{x_{Ej}^H x_{Ej}\}/M_E = P_E \) to satisfy power constraint. \( n_1 \) indicates the complex additive Gaussian White Noise (AWGN), \( n_1 \sim \mathcal{CN}(0, \sigma_1^2) \). In the second hop, a power amplifier is applied to provide a scaled version of \( y_0 \), i.e., the overall received signal at the ULA can be expressed as:

\[ y_{\text{final}} = G y_0 h_2 + n_2 \]

where: \( G, h_2 \in \mathbb{C}^{M \times 1}, n_2 \in \mathbb{C}^{M \times 1} \) is the amplify factor, channel vector between relay node and ULA, AWGN vector, respectively. When maximum ratio combining is applied at the destination node, the received signal is firstly transformed into

\[
\begin{align*}
    z &= \frac{h_2}{||h_2||_F} y_{\text{final}} \\
    &= (d_0^{-\alpha/2} h_{00}^H x_0 + \sum_{i \in \phi(\lambda_i)} |X_i|^\alpha/2 h_{i0}^H x_{i} + \sum_{j \in \phi(\lambda_E)} |X_E|^\alpha/2 h_{E0}^H x_{Ej}) \\
    \times \frac{y^*}{G||h_2||_F + G||h_2||_F n_1 + \frac{h_2^H n_2}{||h_2||_F}} \\
    \end{align*}
\]

Figure 1. System Configuration of Adopted Two-Hop Af Relay System Concluding a Two-Tier Heterogeneous Ad Hoc Network.
Note that (3) can be normalized with respect to the variance of the noise term, i.e., $z$ is divided by $\text{var}\{N_{\text{total}}\}$:

$$z' = \frac{G||h_2||_Fy^*}{\sqrt{G^2||h_2||_F^2n_1 + n_2}} + n'$$

(4)

where $n' \sim \mathcal{CN}(0,1)$. Assume $\mathbb{E}\{x_0^Hx_0\}/M = P_s$, the end-to-end SNR at the ULA is given by:

$$\text{var}\{z\} = \frac{P_sG^2||h_2||_F^2|y^*|^2}{G^2||h_2||_F^2N_1 + N_2}$$

(5)

At the relay node, the scalar gain is chosen to normalize $y_0$, i.e., (for space restriction, Eq (6) is shown at the top of next page), where the received signal is transmitted to the destination antenna array with power $P_r$:

Substitute (6) into (5) and we finally obtain (see the top of next page for Eq.7)

To simplify the expression in (7), we define:

$$s \triangleq d_0^{-\alpha}||h_{00}^Hx_0||^2_F$$

(8)

$$I_{\phi_I} \triangleq \sum_{i \in \varphi(\lambda_i)} |X_{IIi}||^{-\alpha}||h_{IIi}^Hx_{IIi}||^2_F$$

(9)

$$I_{\phi_E} \triangleq \sum_{j \in \varphi(\lambda_E)} |X_{IEj}||^{-\alpha}||h_{IEj}^Hx_{IEj}||^2_F$$

(10)

Then the total received SNR at the destination antenna can be rewritten as:

$$\text{SNR} = \frac{P_sP_rXY}{P_rX + Y + 1}$$

(11)

where:

$$X \triangleq ||h_2||^2_F/N_2$$

(12)

$$Y \triangleq (s + I_{\phi_I} + I_{\phi_E})/N_1$$

(13)
I. INFORMATION OUTAGE PROBABILITY

A. Obtain the Statistic Characteristics of X and Y

\[
G = \sqrt{\mathbb{E}\{y_0 y_0^*\}}
\]

\[
= \sqrt{d_0^{-\alpha} ||\mathbf{h}^H_{00} \mathbf{x}_0||_F^2 + \sum_{i \in \varphi(\lambda_I)} |X_{d_i}||^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_{d_i}||_F^2 + \sum_{j \in \varphi(\lambda_E)} |X_{E_j}||^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_{E_j}||_F^2 + N_1}
\]

(6)

\[
\text{SNR} = \frac{P_r \mathbb{E}_r}{P_s P_r ||\mathbf{h}_2||_F^2 (d_0^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_0||_F^2 + \sum_{i \in \varphi(\lambda_I)} |X_{d_i}||^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_{d_i}||_F^2 + \sum_{j \in \varphi(\lambda_E)} |X_{E_j}||^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_{E_j}||_F^2)}
\]

(7)

From (11), (12), and (13), it is reasonable to deduce the probability density of \( X \) and \( Y \). To simplify the derivation, we start by utilizing the remark in [13], which reveals \( I_{\lambda_I} \) and \( I_{\lambda_E} \), can be treated as a single homogeneous PPP by introducing a weighted node density \( \lambda_w = \lambda_I + \frac{P_E}{P_I} \lambda_E \), for a special case when the transmitted power \( P_I = P_E \), we have \( \lambda_w = \lambda_I + \lambda_E \). Then it can be verified

\[
T = I_{\phi_I} + I_{\phi_E} = \sum_{i \in \varphi(\lambda_I + \frac{P_E}{P_I} \lambda_E)} |X_{d_i}||^{-\alpha} ||\mathbf{h}_{00}^H \mathbf{x}_{d_i}||_F^2
\]

(14)

With the optimization strategies discussed in [14], to maximize the SNR, both IAP and EAP takes the maximum ratio transmission (MRT), namely, we have \( \text{rank}(\mathbf{x}_{d_i}^H \mathbf{x}_{d_i}) = 1 \) and \( \text{rank}(\mathbf{x}_{0}^H \mathbf{x}_{0}) = M \), which indicates

\[
||\mathbf{h}_{00}^H \mathbf{x}_0||_F^2 \sim \chi^2_{2M}, \quad ||\mathbf{h}_{00}^H \mathbf{x}_{d_i}||_F^2 \sim \chi^2_{2}
\]

(15)

**Lemma 1**: The probability density function (PDF) of \( T \) can be expressed as

\[
\mathcal{P}_T(x) = \frac{1}{2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\frac{N}{2})}{(\Gamma(k+1))^{\frac{N}{2}}} \frac{x^{\frac{N}{2}-1}}{x^{k+1}}
\]

Proof: We start by using the characteristic function (CF) of \( T \), which shows:
\[
CF_T(t) = \prod_{k=1}^{N} \mathbb{E} \left\{ \exp(jt|X_{il}|^{-\alpha}||h_{il}^H x_{il}||^2) \right\}
\]

where \(|\cdot|\) is defined as the number of elements in a set. Simply exploit the property of \(CF(t)\), we express \(p_T(x)\) as:

\[
p_T(x) = \frac{1}{2\pi} \int_0^{+\infty} C F_T(t) \exp(jtx) dt
\]

Perform Taylor’s Series of \(CF_T(t)\) with respect to \(jt\), i.e.,

\[
CF_T(t) = \sum_{k=0}^{+\infty} \frac{\partial^k CF_T(t)}{\partial (jt)^k} \bigg|_{t=0} \frac{(jt)^k}{k!}
\]

With the well-known Newton-Leibniz formula, \(\frac{\partial^k CF_T(t)}{\partial (jt)^k}\) can be calculated as;

\[
\frac{\partial^k CF_T(t)}{\partial (jt)^k} = \sum_{k_1+k_2+\cdots+k_N=k} \prod_{l=1}^{N} \frac{\partial^{k_l} (1 - 2j|X_{il}|^{-\alpha}t)^{-1}}{\partial^{k_l} t} \cdot \sum_{k_1+k_2+\cdots+k_N=k} C_{k}^{k_1} C_{k-k_1}^{k_2} \cdots C_{k-k_1-k_2-\cdots-k_{N-1}}^{k_N} \\
\times \prod_{l=1}^{N} \left( \frac{2|X_{il}|^{-\alpha}}{1 - 2j|X_{il}|^{-\alpha}t} \right)^{k_l} \frac{1}{1 - 2j|X_{il}|^{-\alpha}t}
\]

\[
= k! \sum_{k_1+k_2+\cdots+k_N=k} \prod_{l=1}^{N} \frac{1}{k_1! k_2! \cdots k_N!} \left( \frac{2|X_{il}|^{-\alpha}}{1 - 2j|X_{il}|^{-\alpha}t} \right)^{k_l} \frac{1}{1 - 2j|X_{il}|^{-\alpha}t}
\]

where \(|\varphi(\lambda_I + \frac{p_{E}}{P_I})| = N\). Substitute (19) into (18) and apply the property of Fourier Transform, it can be verified:
\[ p_T(x) = \sum_{k=0}^{+\infty} \frac{1}{2\pi} \sum_{k_1 + k_2 + \cdots + k_N = k} \prod_{i=1}^{N} \frac{(2|X_i|^{-\alpha})^{k_i}}{k_1!k_2!\cdots k_N!} \frac{(-1)^k}{(jx)^{k+1}} \]

To obtain a succinct form of (20), we simply define \( f_i(x) = u_i e^{c_i x} \), where \( 1 \leq i \leq N \). Note that

\[ S = \sum_{k_1 + k_2 + \cdots + k_N = k} \frac{c_1^{k_1}c_2^{k_2}\cdots c_N^{k_N}}{k_1!k_2!\cdots k_N!} u_1 u_2 \cdots u_N \]

is exactly the coefficient of \( x^k \) term of \( \prod_{i=1}^{N} f_i(x) = \prod_{i=1}^{N} u_i e^{\sum_{i=1}^{N} c_i x} \), that is, \( S = \prod_{i=1}^{N} u_i \frac{(\sum_{i=1}^{N} c_i)^k}{k!} \). By letting \( u_1 = u_2 = \cdots = u_N = 1 \) and \( c_i = 2|X_i|^{-\alpha} \), we can rewritten \( p_T(x) \) as:

\[ p_T(x) = \frac{1}{2\pi} \sum_{k=0}^{+\infty} \frac{(-1)^k}{(jx)^{k+1}} \frac{(\sum_{i=1}^{N} 2|X_i|^{-\alpha})^k}{(2\pi)^k} \]

which completes the proof.

We then focus on deriving the cumulative distribution function (CDF) of \( Y \), which can be written as:

\[
CDF_Y(y) = \Pr(Y < y) = \Pr(T + I < yN_1) = \int_{0}^{+\infty} \Pr(T + I < ty) f_{N_1}(t) dt = \frac{1}{d_0^{-\alpha}} \int_{0}^{+\infty} \int_{0}^{+\infty} \int_{0}^{ty-x} \frac{(r/d_0^{-\alpha})^{M-1} e^{-(r/2d_0^{-\alpha})}}{2^M \Gamma(M)} p_T(x) e^{-t} dr dx dt = \frac{1}{\Gamma(M)} \int_{0}^{+\infty} \int_{0}^{+\infty} \gamma(M, \frac{ty-x}{2d_0^{-\alpha}}) p_T(x) e^{-t} dx dt
\]

where \( \gamma(\alpha, x) \triangleq \int_{0}^{x} e^{-t} t^{\alpha-1} dt \). To solve (23), we start by replacing integration variable \( t, x \) with \( u, v \), which satisfies \( u = ty - x, v = x \). Then the Jacobian Determinant can be calculated by \( \partial(u, v)/\partial(x, t) = -y \). We can express (23) as:
\[
\text{CDF}_Y(y) = -y \left( \int_D y(M, u/2d_0^{-\alpha})p_T(v)e^{\frac{u-v}{y}} dudv - \int_D y(M, u/2d_0^{-\alpha})p_T(v)e^{\frac{u-v}{y}} dudv \right)
\]

\[
= -y \left( \int_0^{+\infty} ye^{-v/y}p_T(v)\Gamma(M)(1 + \frac{2d_0^{-\alpha}}{y})^{-M}dv + \int_0^{+\infty} ye^{v/y}p_T(v)\Gamma(M)(1 - \frac{2d_0^{-\alpha}}{y})^{-M}dv \right)
\]

\[
= (y^2(1 - \frac{2d_0^{-\alpha}}{y})^{-M} - y^2(1 + \frac{2d_0^{-\alpha}}{y})^{-M}) \times \mathbb{E}\{\exp(-T/y)\} \tag{24}
\]

\[
= (y^2(1 - \frac{2d_0^{-\alpha}}{y})^{-M} - y^2(1 + \frac{2d_0^{-\alpha}}{y})^{-M}) \times \prod_{i=1}^{N} \left(1 + \frac{|X_{1i}|^{-\alpha}}{y}\right)^{-1} \tag{25}
\]

where equation (a) comes from [15]: \( \int_0^{+\infty} e^{-\alpha x} \gamma(\beta, x)dx = \frac{1}{\alpha} \Gamma(\beta)(1 + \alpha)^{-\beta} \) and integration region \( D \) is \( D = \{(u, v) \in (0, +\infty) \times (0, +\infty)\} \). Equation (b) can be derived by applying the result of lemma 1.

B. Deduce the Closed Expression of Outage Probability

To provide the closed expression of outage probability, we then deduce the p.d.f of the random variable (RV): \( Q = \frac{axY}{bx+Y+c} \), where \( X, Y \) is any given RV, which is a general form of the SNR expression proposed in (11). Let \( \gamma_{th} \) denote the acceptable SNR threshold, the outage probability can be given by:

\[
P_{\text{out}} = \Pr(Q < \gamma_{th})
\]

\[
= \int_0^{+\infty} \Pr \left( \frac{axY}{bx+Y+c} < \gamma_{th} \right) f_X(x) dy
\]

\[
= \int_0^{+\infty} \Pr \left( (ax - \gamma_{th})Y < \gamma_{th}(bx + c) \right) f_X(x) dx
\]

\[
= \int_{\frac{\gamma_{th}}{a}}^{+\infty} \left( 1 - \text{CDF}_Y \left( \frac{bx + c}{ax - \gamma_{th}} \right) \right) f_X(x) dx + \int_0^{\frac{\gamma_{th}}{a}} f_X(x) dx
\]

\[
= 1 - \int_{\frac{\gamma_{th}}{a}}^{+\infty} C \text{DF}_Y \left( \frac{bx + c}{ax - \gamma_{th}} \right) f_X(x) dx
\]

\[
= 1 - \frac{1}{a} \int_0^{+\infty} C \text{DF}_Y \left( \frac{bu + by_{th} + ac}{au} \right) f_X \left( \frac{u + \gamma_{th}}{a} \right) du \tag{26}
\]

Note that the outage probability discussed in this paper can be calculated by letting \( a = P_sP_r \), \( b = P_r \) and
Exploiting the conclusion in [13], the p.d.f of RV $|\mathbf{h}_2||F/N_2$ is given as follows:

$$f_X(x) = \frac{\lambda^M}{(M-1)!} e^{-\lambda x} x^{M-1} \quad \lambda = \frac{M}{\sigma_1^2}$$

(27)

Theoretically, closed expression of $P_{out}$ can be calculated by combining (25) (26) and (27). However, we observe that it’s complicated to integrate over $y$ from 0 to $\infty$ due to the existence of continued product. Thus we take some manipulation to (27) by applying lemma2

**Lemma 2**: $\prod_{i=1}^{N} (1 + P_i x)^{-1} = \sum_{i=1}^{N} A_N(i) (1 + P_i x)^{-1}$, where we define the symbol $A_N(i) \triangleq \prod_{j\neq i,1 \leq j \leq N} \frac{P_i}{P_i - P_j}$

Proof: Firstly, we define a polynomial function with degree $N - 1$

$$h(x) = \sum_{i=1}^{N} P_i \prod_{j \neq i} (x - P_j) \prod_{j \neq i} (P_i - P_j)$$

(28)

Then it is easy to verify $h(0) = (-1)^{N-1} (\prod_{i=1}^{N} P_i) \sum_{i=1}^{N} A_N(i)$. Substitute $x = P_i, 1 \leq i \leq N$ into (28) and we may get $f(P_i) = P_i^N$, which implicates $P_1, P_2, \cdots, P_N$ construct the root set of $h(x)$, i.e., we may rewrite (28) as

$$h(x) = x^N + k(x - P_1)(x - P_2) \cdots (x - P_N)$$

(29)

(29) shows $h(0) = k(-1)^{N-1} \prod_{i=1}^{N} P_i$. $k$ can be determined by noticing that $\deg(h(x)) = N - 1$ if and only if $k = -1$. With the help of Fubini’s Principle, we find an important property of $A_N(i): \sum_{i=1}^{N} A_N(i) = 1$, which will be utilized later. Let $g_N(x) = \sum_{i=1}^{N} A_N(i) (1 + P_i x)^{-1}$, we may transform $g_N(x)$ into:

$$g_N(x) = \sum_{i=1}^{N-1} A_N(i)(1 + P_i x)^{-1} + A_N(N)(1 + P_N x)^{-1}$$

$$= \sum_{i=1}^{N-1} A_N(i)(1 + P_i x)^{-1} + (1 - \sum_{i=1}^{N-1} A_N(i))(1 + P_N x)^{-1}$$

$$= (1 + P_N x)^{-1} - \sum_{i=1}^{N-1} A_N(i) \frac{P_i x}{(1 + P_i x)(1 + P_N x)}$$

$$= (1 + P_N x)^{-1} \left[ 1 - \sum_{i=1}^{N-1} A_N(i) \frac{P_i x}{1 + P_i x} \right]$$

$$= (1 + P_N x)^{-1} \sum_{i=1}^{N-1} A_N(i) \frac{1}{1 + P_i x}$$

(30)

$$= (1 + P_N x)^{-1} g_{N-1}(x)$$

(31)
From the recurrence relation presented in (31), we finally obtain $g_N(x) = \prod_{i=2}^{N}(1 + P_i x)^{-1} g_1(x) = \prod_{i=1}^{N}(1 + P_i x)^{-1}$, which completes the proof.

With the finit sum representation given in lemma 2, it is obtained that the first term in Eq (25) is equivalent to:

$$y^2 (1 - 2d_0^{-\alpha} y)^{-M} \prod_{i=1}^{N} (1 + |X_{ii}|^{-\alpha} y)^{-1}$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{M+1} (-2d_0^{-\alpha})^{-M} |X_{ii}|^{-\alpha} a_N(i) a_{M+1}(j) \frac{y^{M+3}}{y + \delta_{ij}}$$

(32)

where $\delta_{ij}$ is defined as:

$$\delta_{ij} = \begin{cases} |X_{ii}|^{-\alpha} & j = 1 \\ -\frac{1}{2} d_0^{-\alpha} & 2 \leq j \leq M + 1 \end{cases}$$

(33)

Combine (25), (26), and (33), after some manipulation we may express the end-to-end outage probability in two-hop AF relaying system equipped with as:

$$P_{out} = P_{out_1}(\gamma_{th}) - P_{out_2}(\gamma_{th})$$

(34)

where the derivation of $P_{out_1}(\gamma_{th})$ are presented at the top of next page ( which is almost the same for $P_{out_2}(\gamma_{th})$), and Eq (a) comes from the effective integration result given in [15]:

$$\int_{u}^{+\infty} (x + \beta)^{2v-1}(x - u)^{2\rho-1} e^{-\mu x} dx$$

$$= \frac{(u + \beta)^{v+\rho-1}}{\mu^{v+\rho}} \exp \left[ (\beta - u) \frac{\mu}{2} \right] \Gamma(2\rho) W_{v-\rho, v+\rho-\frac{1}{2}}(u \mu + \beta \mu)$$

[u > 0, |arg(\beta + u)| < \pi, Re\mu > 0, Re\rho > 0]

(35)

where we assume $u = 0, \rho = \frac{r_1 + r_2 - 1}{2}, \beta = \frac{b \gamma_{th} + a c}{b + a \delta_{ij}}, \mu = \frac{\lambda}{a}, \text{ and } v = \frac{1 - r_1}{2}$, respectively. $W_{\lambda, \mu}(z)$ is the Whittaker function.

**Corollary 1**: In the following, we provide some discussion on the analytical result given in (38). It is easy to observe that the end-to-end outage probability implicitly depends on $\lambda_I, \lambda_E$ as well as $P_s, P_r$. For the case when the transmitted power at the source node and the relay node is greater than $\sigma_1^2$, namely, $P_s \gg \sigma_1^2$ and $P_r \gg \sigma_1^2$, we may deduce an approximate expression by using the following inequality:
\[
\frac{P_s P_r X Y}{P_r X + Y + 1} \leq P_s P_r \cdot \min\left\{ \frac{Y}{P_r}, \frac{X}{1 + 1/Y} \right\}
\]  

(39)

which provides a finite upper bound for RV Q. With (39) it is obtained \( P_{out}(\gamma_{th}) \) is approximated by:

\[
P_{out}^{\text{low}}(\gamma_{th}) \approx \Pr\left\{ \min\left\{ \frac{Y}{P_r}, \frac{X}{1 + 1/Y} \right\} \leq \frac{\gamma_{th}}{P_s P_r} \right\}
\]

\[
= 1 - (1 - \Pr(X \leq \frac{\gamma_{th}}{P_s}))(1 - \Pr(\frac{X}{1 + 1/Y} \leq \frac{\gamma_{th}}{P_s P_r}))
\]

(40)

It has been explained above that \( \frac{\gamma_{th}}{P_s P_r} (1 + 1/Y) \) is dominated by \( \frac{\gamma_{th}}{P_s P_r} \) while SNR is higher, we may then rewrite (40) as:

\[
P_{out}^{\text{low}}(\gamma_{th}) \approx 1 - (1 - \text{CDF}_{Y}(\frac{\gamma_{th}}{P_s}))
\]

\[
\times (1 - \frac{\text{CDF}_{Y}(\gamma_{th}/P_s)}{(M - 1)!})
\]

(41)

\[
= 1 - \left( 1 - \text{CDF}_{Y}(\frac{\gamma_{th}}{P_s}) \right)
\]

\[
\times \left[ e^{-\gamma_{th}/P_s P_r} \sum_{m=0}^{M-1} \frac{\left( \gamma_{th}/P_s P_r \right)^m}{m!} \right]
\]

(42)

which is our desired result.

**SIMULATION AND NUMERICAL RESULTS**

In this section, we perform computer simulation on derived outage probability. Analytic expressions are verified through the comparison with Monte-Carlo simulations. Simulation parameters are listed as follows. The noise variance at the relay node and the terminal node are assumed to be \( \sigma_1^2 = 10(\text{mW}) \) and and \( \mathbb{E}\{n^2\} = \sigma_2^2 \mathbf{I}_M (\sigma_2^2 = 10(\text{mW}) \) , respectively. As a typical value, the physical distance between the the IAP (or EAP) and the relay node are set as \( d_0 = 4m \) and the path-loss exponent is set as \( \alpha = 4 \).

In Fig.1, the probability density function when \( \lambda_I, \lambda_E, P_E \) and \( P_I \) varies are presented, where one can see the SNR at the relay node appears to be positively correlated to the equivalent node density \( \lambda_w = \lambda_I + \frac{P_E}{P_I} \). In Fig.2, we respectively give the outage probability for \( P_s \in \{50\text{mW, 100mW}\} \) and \( P_r \in \{10\text{mW, 50mW, 100mW}\} \). It can be obversed that though both \( P_s \) and \( P_r \) influences the outage probability, improving the power factor \( P_r \) brings more advantages to the effectiveness of transmission when the scale of ULA is large enough. In addition, the comparison between simulatiion and analytic results also proves the rationality of the closed expression we deduce in (38). Fig.3 plots \( P(\gamma_{th})|_{\gamma_{th}=100\text{mW}} \) with respect to the number of antennas equipped in the ULA, where \( P_s = 10\text{mW} \) and \( P_r \in \{10\text{mW, 25mW, 50mW}\} \). It is
reflected in Fig.3 that when $P_s$ and $P_r$ is fixed, the outage probability of the AF relay wireless network depicted in Fig.1 decreases with the increasing of $M$, but finally converges to a constant. Note in Fig.2 as well as Fig.3, $\gamma_{th}$ is energy threshold, which implicates the corresponding SNR is 10dB when $\gamma_{th} = 100mW$.

Figure 2. The p.d.f Of Rv Y for Different Two-Dimensional Ppp Density $\lambda_I$ and $\lambda_E$.

Figure 3. Outage Probability when $M = 10$, $\lambda_I = 0.2\text{nodes/m}^2$ And $\lambda_I = 0.2\text{nodes/m}^2$. 
\[ P_{\text{out}}(Y_{th}) = \lambda^M e^{-\frac{\lambda Y_{th}}{a}} \frac{(M-1)!}{\sum_{j=1}^{N+1} \sum_{i=1}^{M+3} (-2d_0^{-a})^{-M} |X_H|^a A_N(i) A_{M+1}(j) \binom{M + 3}{r} \frac{-\delta_{ij}}{b/a + \delta_{ij} + \frac{bY_{th} + ac}{a}} } \]

\[ \times \int_0^{+\infty} \sum_{t=0}^{M-1} \left( \frac{M-1}{t} \left( \frac{u}{a} \right)^t Y_{th}^{M-1-t} \right) du \]

\[ = \lambda^M e^{-\frac{\lambda Y_{th}}{a}} \sum_{i=1}^{N+1} \sum_{j=1}^{M+1} \sum_{r_1=0}^{M-3} \sum_{r_2=0}^{M-1} (-2d_0^{-a})^{-M} |X_H|^a A_N(i) A_{M+1}(j) \binom{M + 3}{r_1} \binom{M-1}{r_2} \frac{-\delta_{ij}a}{b + a\delta_{ij}} y_{th}^{M-1} \frac{\gamma_{th}^{M-1}}{a^{M-1}} \]

\[ \times \frac{r_1 + r_2 - 1}{2} \frac{\gamma_{th}}{a} \exp \left( \frac{\gamma_{th} + ac}{b + a\delta_{ij}} \right) \Gamma(r_1 + r_2 - 1) W_{1-r_1-r_2}^{1-r_1-r_2-1} \frac{(\gamma_{th} + ac)}{b + a\delta_{ij}} \]

Figure 4. Pr(Snr < 10db) with respect to the number of antennas M.

CONCLUSION

In this paper, we investigate the outage probability in an AF-relay system with a two-tier heterogeneous ad hoc network, where the distribution of IAP and EAP are both two-dimensional PPP. We firstly design a optimal power allocation scheme to maximum the SNR at the terminal node. We utilize the equivalent node density to simplify the problem and obtain the closed expression of outage probability in the form of Whittaker function. As a corollary, we also deduce that the outage probability can be tightly lower bounded
by a more simple expression. Simulation results verifies the analytic results coincide with the numerical results obtained by Monte-Carlo simulations.

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