Research on Edge Wave Deformation of High Strength Automobile Plate

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Abstract. Edge waves are the shape defects often encountered in the rolling process of automobile plates. The previous wave shape modal function cannot well reflect the situation that the on-site edge wave only occurs in a local area of the edge. In response to this problem, this article uses high-order trigonometric functions to describe the edge waves, and the obtained wave shape modes are consistent with the actual edge waves on site. On this basis, the energy method is used to calculate the critical buckling conditions of the edge waves. And using the perturbation method, the post-buckling path calculation model of the edge wave is established, and the post-buckling path of the edge wave is obtained. In order to verify the accuracy of the analytical calculation model, a finite element method is used to establish a simulation calculation model of the edge wave, and the buckling mode consistent with the scene was obtained. At the same time, the critical conditions of buckling are basically consistent with the analytical calculation results, reflecting the correctness of the analytical calculation model.

1. Introduction
Edge waves are the shape defects often encountered in the rolling process of automobile plates. In response to this problem, a large number of scholars have done a lot of work. At the end of the 1960s, Wistreich[1] proposed that the shape defects of the strip can be attributed to the stability of the elastic thin plate. The critical load at the time of instability is the result of the residual stress in the plate. Literature [2] calculated the rolling elongation difference and the corresponding residual stress according to the basic parameters given in the rolling process, and obtained the longitudinal wave form and corresponding parameters of the strip after rolling; Literature [3] gives the corresponding buckling wave forms under various tensile stress distributions; Literature [4-5] believes that the buckling mechanism of longitudinal waves is due to the uneven distribution of elongation in the length direction of the strip along the width. The result is that the middle part of the strip extends larger than the side part, which causes the middle part to be compressed and the side part is under tension; Literature [6] uses the finite strip method to numerically solve the buckling deformation of the rectangular strip under the residual stress; Literature[7] A general wave deflection function was constructed by using cubic spline function, and the longitudinal buckling of the strip after rolling was studied using the theory of large deflection and the principle of minimum capacity; Literature [8-9] used the finite element method to study the longitudinal buckling of the strip after rolling. The overall and local buckling and post-buckling paths of
the strip are studied by numerical simulation. In those papers, the wave-shaped defect described is full board wide, but it can be seen from Figure 1 that the edge waves that occurred at the scene actually did not cover the entire board width. It happens on the local width. Therefore, the previous literature describes the overall edge wave method is not accurate. In this paper, in order to more accurately describe the edge waves that occur on the local width, the high-order trigonometric function is used to describe the edge wave, deals with the experiment of buckling and the analytical of the post-buckling, and the critical buckling value and post-buckling path of the plate shape are solved.

2. The critical buckling conditions

2.1. Deflection function

According to the actual edge waves on site, which is periodic in the longitudinal direction and local in the width. In order to describe this board type defect, when the buckling region is $\Omega^{-b_1,-a,a}$, where $b$ is determined by the width of the edge wave, $a$ is determined by the half-wavelength of the edge wave, the deflection function can be expressed as follows:

$$W(x, y) = A(1 - \cos \frac{\pi x}{2b})^n \cos \frac{\pi y}{2a}$$

(1)

Where:

- $A$ —— Amplitude of the deflection;
- $n$ —— The power index, is determined by the wave form of the field measurement.

At this time, its obvious wave width area is ($-b_1$, $b_1$) & ($b_1$, $b$), When $x$ is in this interval, the following expression is satisfied:

$$(1 - \cos \frac{\pi x}{2b})^n \leq 0.05$$

(2)

For example, when $n=5$, the edge wave deflection function simulation diagram is obtained according to formula (1) as shown in the figure 2.

![Integral high-order edg wave simulation diagram of High-strength automotive plate](image)

Fig2 Integral high-order edg wave simulation diagram of High-strength automotive plate

2.2. Boundary conditions

According to the actual situation on site, the deflection should meet the following conditions:
\[ W(0,y) = 0, \quad \frac{\partial W}{\partial x} \bigg|_{x = \pm b} = 0 \]  
\( (3) \)

Both sides of the buckling region should meet:
\[
\begin{align*}
M_x(b,y) &= -D \frac{\partial^2 W}{\partial x^2} = 0 \\
M_x(-b,y) &= -D \frac{\partial^2 W}{\partial x^2} = 0
\end{align*}
\]  
\( (4) \)

2.3. Boundary load

The horizontal distribution of the stress function of the plate shape is consistent with the deflection function, and meets the stress self-balance condition, this paper defines the boundary load function of the edge wave as follows:
\[
\sigma_y = \sigma_0 \cdot f(x) = \sigma_0 \left\{ \frac{1}{2} (1 - \cos \frac{\pi x}{b})^n - d_N \right\}
\]  
\( (5) \)

Where, \( \sigma_0 \) is the peak value of the boundary uneven distribution stress.

To consider the stress should meet the balance condition, so:
\[
\int_{-b}^{b} \left\{ \frac{1}{2} (1 - \cos \frac{\pi x}{b})^n - d_N \right\} dx = 0
\]  
\( (6) \)

Then the \( d_N \) can be solved. The edge wave plate stress diagram is given in fig3.

![Plate-shaped stress diagram of integral high-order plate-shaped longitudinal buckling](image)

2.4. Critical stress

According to the small deflection theory and the energy principle, High-strength automotive sheet in a slightly bent state, small bending does not cause stretching of the midplane, only bending force produces strain energy. So just consider the work done by the bending energy and the force acting in the midplane. If the work done by these forces is less than the bending strain energy for each possible out-of-plane buckling, the balance of the plate is stable; otherwise, buckling deformation occurs. Therefore, the critical buckling stress obtained by the following formula:
\[
T - U = 0
\]  
\( (7) \)

The bending deformation energy of the rectangular plate and strip in the slightly bent state is:
\[
U = \frac{1}{2} \int_{a}^{b} \int_{-h/2}^{h/2} \left[ (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy}) \right] dx dy dz
\]
\[
= \frac{D}{2} \int_{\Omega} \left[ \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + \left( \frac{\partial^2 W}{\partial y^2} \right)^2 + 2\mu \left( \frac{\partial^2 W}{\partial x \partial y} \right)^2 + 2(1-\mu) \left( \frac{\partial^2 W}{\partial x \partial y} \right)^2 \right] dx dy
\]  
\( (8) \)

Where, \( D = \frac{E h^3}{12(1-\mu^2)} \), stands for the stiffness.
The work done by the mid-plane internal force acting in the mid-plane is:

\[
T = \frac{1}{2} \int_\Omega \left[ N_x \left( \frac{\partial W}{\partial x} \right)^2 + N_y \left( \frac{\partial W}{\partial y} \right)^2 + 2N_{xy} \frac{\partial W}{\partial x} \frac{\partial W}{\partial y} \right] \, dx \, dy
\]

(9)

Equations (7), (8) and (9) constitute the mathematical formulation for solving the critical stress. So, the \( \sigma \) can be expressed as:

\[
\sigma = -\frac{D}{h} \int_\Omega \left[ \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + \left( \frac{\partial^2 W}{\partial y^2} \right)^2 + 2\mu \frac{\partial^2 W}{\partial x \partial y} \frac{\partial^2 W}{\partial y \partial x} + 2(1-\mu) \left( \frac{\partial W}{\partial x \partial y} \right)^2 \right] \, dx \, dy
\]

\[
\int_\Omega f(x) \cdot \left( \frac{\partial W}{\partial y} \right)^2 \, dx \, dy
\]

(10)

Substituting deflection function (1) and plate stress function (5) into the Equations (9), the critical stress load can be obtained.

By seeking the minimum critical stress, that:

\[
\frac{\partial \sigma}{\partial a} = 0
\]

(11)

Through solving the Equations (10), the critical buckling half-wavelength can be obtained, and then the critical buckling stress can be solved.

\[
a = f(b, h)
\]

(12)

When the half board width \( b = 500mm \), thickness \( h = 0.5mm \), Elastic Modulus \( E = 210Gpa \), Poisson's ratio \( \mu = 0.3 \), the table 1 gives the critical stress of typical example of edge waves.

| Number | Power exponent n | Width of obvious wave area \( b_n/mm \) | Wave half wavelength \( a_n/mm \) | Critical load \( \sigma_{cr}/Mpa \) |
|-------|-----------------|------------------|-----------------|-----------------|
| 1     | 1               | 500              | 410.317         | 0.73546         |
| 2     | 2               | 400              | 244.139         | 1.46804         |
| 3     | 3               | 300              | 165.458         | 2.69448         |
| 4     | 4               | 200              | 124.605         | 4.33337         |
| 5     | 6               | 100              | 55.538          | 18.5483         |

It can be seen from Table 1 that as the obvious wave buckling area decreases, the critical wavelength of strip steel also decreases, while its critical load has an obvious increasing trend, and with the obvious wave buckling area As it becomes smaller, the critical load value increases more obviously.

3. Solving post-buckling path

In the production of cold-rolled thin strip steel, the strip shape quality after rolling has always been one of the main indicators to measure the level of rolled steel and product quality. Analytical research on the path of buckling generation after cold rolling of thin strip can effectively obtain the relationship between the strip shape stress and wave shape steepness (wave height/wavelength×100%) after rolling, and provide theoretical guidance for shape control. Therefore, the study of strip post-buckling is very meaningful for the control of strip shape quality.
Reference [10] [11], this paper uses perturbation method to solve the post-buckling path. In order to simplify the calculation process, this article does not consider the change of buckling area, assuming that the buckling regional of post-buckling process is the same as the critical buckling.

In the case of large deflection, the deformation coordination equation for edge wave calculation is:

\[
\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2}
\]

The balanced differential equation is:

\[
\begin{cases}
\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \\
\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \\
D \left(\frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) = N_x \frac{\partial^2 W}{\partial x^2} + N_y \frac{\partial^2 W}{\partial y^2} + 2N_w \frac{\partial^2 W}{\partial x \partial y}
\end{cases}
\]

Introduce a stress function \(G(x,y)\), the Karman equation of large deflection can be obtained:

\[
\begin{cases}
\frac{\partial^4 G}{\partial x^4} + 2 \frac{\partial^4 G}{\partial x^2 \partial y^2} + \frac{\partial^4 G}{\partial y^4} = \frac{\partial^2 G \partial^2 W}{\partial x^2} + \frac{\partial^2 G \partial^2 W}{\partial y^2} - 2 \frac{\partial^2 G \partial^2 W}{\partial x \partial y}
\end{cases}
\]

In order to simplify the calculation, this article does not consider the change of its buckling area, and assumes that the buckling area in the post-buckling process is the same as the buckling area at the critical buckling moment. Using the dimensionless perturbation method, the following type into dimensionless form:

\[
\begin{cases}
\theta = \frac{a}{b}, \quad \xi = \frac{x}{b}, \quad \eta = \frac{y}{a}, \quad w = \frac{W}{h} \\
f = \frac{12(1-\mu^2)F}{Eh^2}, \quad \frac{N_h}{h} = \frac{12(1-\mu^2)b^2}{Eh^2}, \quad N_h
\end{cases}
\]

Where \(N_h = \sigma \cdot h\), unit N/mm².

The Equations (16) substituted into the Karman large deflection equations (15), then:

\[
\begin{cases}
\theta^2 \frac{\partial^4 w}{\partial \xi^4} + 2 \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{1}{\theta^2} \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} = \frac{\partial^2 f \partial^2 w}{\partial \xi^2 \partial \eta^2} \\
\theta^2 \frac{\partial^4 f}{\partial \xi^4} = 12(1-\mu^2) \left[\left(\frac{\partial^2 w}{\partial \xi^2 \partial \eta}ight)^2 - \frac{\partial^2 w}{\partial \xi^2 \partial \eta} \frac{\partial^2 w}{\partial \xi^2 \partial \eta}\right]
\end{cases}
\]

Selected the center deflection \(w_0 = w(0,0) = \frac{W}{h}\) as perturbation parameters, get the perturbation equations.

\[
\begin{align*}
N_0 &= r_0 + r_2 w_0^2 + r_4 w_0^4 + r_6 w_0^6 + \cdots \\
w &= w_0^0(\xi,\eta)w_0^1 + w_0^1(\xi,\eta)w_0^2 + w_0^2(\xi,\eta)w_0^3 + \cdots
\end{align*}
\]

Where, the coefficients \(W\) of the expansion must satisfy the conditions:

\[
w_0(0,0) = 1, \quad w_0(0,0) = w_0(0,0) = w_0(0,0) = \cdots = 0
\]

The first order perturbation equation:
The second order perturbation equation:
\[ \frac{1}{\lambda^2} \frac{\partial^4 w_i}{\partial \xi^4} + 2 \frac{\partial^4 w_i}{\partial \xi^2 \partial \eta^2} + \lambda^2 \frac{\partial^4 w_i}{\partial \eta^4} = 0 \]  
(20)

The third order perturbation equation:
\[ \theta^2 \frac{\partial^4 g(r_{ij})}{\partial \xi^4} = 12(1 - \mu^2) \left[ \left( \frac{\partial^2 w_i}{\partial \xi \partial \eta} \right)^2 - \frac{\partial^2 w_i}{\partial \xi \partial \eta} \frac{\partial^2 w_i}{\partial \xi^2} \frac{\partial^2 w_i}{\partial \eta^2} \right] \]  
(21)

The fig. 4 gives the post-buckling mode and the fig. 5 gives post-buckling path of the quarter wave. The mode of post-buckling was in good accordance with the field quarter wave.

4. Finite Element Simulation Research

In order to verify the accuracy of the analytical calculation results, a finite element simulation calculation model of the edge wave was established and analyzed.

4.1. Edge wave simulation calculation model

Because SHELL63 element has two abilities of bending and thin film, it can bear in-plane load, including stress strengthening and large deformation ability, so SHELL63 element is adopted.

Select the material performance parameters as follows:

(1) The modulus of elasticity is \( 2.1 \times 10^5 \) MPa;
(2) Poisson's ratio is 0.3;
(3) The coefficient of thermal expansion of the strip in all directions is \( 12 \times 10^{-6} \degree \text{C}^{-1} \);
(4) The thickness is 0.001m.
The load form gives the internal stress of the strip by applying a temperature stress field, and the temperature stress distribution function is consistent with the load function in the analytical solution process.

When applying rigid body displacement constraints, constrain the X and Y displacements of the center point, and constrain the Z-direction displacement on the longitudinal centerline, leaving the longitudinal sides free.

4.2. Simulation results
Before buckling analysis, general static analysis is performed. After the static solution is obtained, eigenvalue buckling analysis is performed on the model to obtain eigenvalues. In the eigenvalue buckling analysis, the load applied in the static analysis is used. The obtained buckling mode is shown in the figure 6.

It can be seen from Table 2 that the errors of the finite element and analytical method calculation results are basically about 5%, that is, the finite element results basically verify the accuracy of the analytical method calculation results.

![Fig.6 Buckling mode of edge waves](image)

| Critical load/Mpa | Critical wavelength/mm |
|-------------------|------------------------|
| Finite element    | Analytical method      | Relative deviation |
| 0.7056            | 0.735416               | -4.05%             |
| 405               | 410.317                | 1.30%              |

5. Conclusions
In this paper, based on the buckling theory of plates, the edge waves of high-strength automotive panels was investigated in this paper. The following results are obtained:

(1) High-order trigonometric functions are used to describe edge wave patterns, it expresses the fact that the edge wave pattern does not cover the whole width.

(2) Using the stress distribution conditions and S.Timoshenko principle of least work, a new mechanical model of longitudinal buckling was established, and the mathematic model of critical buckling was achieved by Galerkin’s principle of virtual displacement, the path generation of post-buckling was found with perturbation-variational solution, and the mode of post-bucking was achieved.

(3) The finite element method is used to simulate the buckling critical conditions of the edge waves. The calculation results are in good agreement with the results obtained by the analytical method, which verifies the accuracy of the analytical calculation results.
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References
[1] Wistreich J G. Control of strip shape during cold rolling[J]. Journal of the Iron and Steel Institute, 1968, 206(12):1203~1206.
[2] Poplawski J V, Seccombe D A, Jr. Bethlehem's Contribution to the Mathematical Modeling of Cold Rolling in Tandem Mills [J]. Iron Steel Eng., 1980, 57(9):47~58.
[3] Roberts, W.L., Cold rolling of steel[M]. Dekker, New York, NY, 1978
[4] Yang Quan, Study on the cold rolled strip buckling and the target shape in the automatic flatness control[D]. Dissertation: University of Science and Technology Beijing, 1992.5.
[5] Yang Quan, Chen Xianlin. The deforming route of buckled waves of rolled Strip [J]. Journal of University of Science and Technology Beijing [J]. 1994, 16(1): 53-57.
[6] Lin Zhenbo, Finite strip method analysis on shape discrimination model in cold strip rolling mill [D]. Dissertation: Yanshan University, 1993.9.
[7] Bian Yuhong, Liu Hongmin. Universal method analysing the large deflection buckling deformation of rolled strip [J]. Chinese Journal of Mechanical Engineering, 1994, 30(Supp): 21~27.
[8] Qing Weijie, Yang Quan. Study on cold rolled strip global and local buckling, post-buckling using the finite element method[J]. Journal of University of Science and Technology Beijing, 2000, 22(4): 377~380.
[9] Qing Weijie, Study on the target shape in the automatic flatness control of WISGCO1700mm five-stand cold tandem rolling mill [D]. Dissertation: University of Science and Technology Beijing, 2000.3.
[10] Chang Tiezhu, Zhang Qingdong, Huang Shiqing. Analysis of transverse buckling for thin strip[J]. China Mechanical Engineering, 2009, 20(18): 2255~2259.
[11] Pan Liyu, Wang Shu. A perturbation-variational solution of the large deflection of rectangular plates under uniform load [J]. Applied Mathematics and Mechanics, 1986, 7(8): 675~688.