The Spallagenic Production Rates of Lithium, Beryllium and Boron

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Abstract

We calculate the production rates of $^6\text{Li}$, $^7\text{Li}$, $^9\text{Be}$, $^{10}\text{B}$ and $^{11}\text{B}$ via spallation of Carbon, Nitrogen and Oxygen nuclei by protons and $\alpha$-particles and by $\alpha-\alpha$ fusion reactions. We include recent measurements of the cross sections of $\alpha-\alpha$ fusion reactions and find that the computations yield rates of $^6\text{Li}$ and $^7\text{Li}$ production that are nearly a factor of two smaller than previously calculated. We begin by using the ‘straight ahead’ approximation for the fragment energy and the ‘leaky-box’ model for product capture in the Galaxy. In addition we test the straight ahead approximation by recalculating the production rates using an empirical description of the fragment energy distribution and find that the results closely match. We have also calculated the rates for various cosmic ray spectra and find that the hardest spectra tested decrease the rates with CR CNO by approximately an order of magnitude relative to our chosen standard. Finally we have computed the Population I elemental ratios and the Population II scaling relations for our standard and find that our computations predict an abundance of Lithium for a given abundance of Beryllium that is 1/4 smaller than previously derived.
I. INTRODUCTION

The spallation production rates of the isotopes of Lithium, Beryllium and Boron (LiBeB) are a necessary component in any calculation of the evolution of these nuclei in the Galaxy. In particular, $^6$Li, $^9$Be and $^{10}$B are thought to be produced solely by spallation and thus give clues to the contribution of spallagenic $^7$Li and $^{11}$B to the total abundance of these nuclei. For example it is the comparison of the observed solar $^{11}$B/$^{10}$B isotopic ratio of 4.05 [1] to the spallagenic prediction of $\sim 2.5$ [2] that has led many authors to include the production of $^{11}$B by supernova into their models of Galactic chemical evolution [3]. LiBeB have also been observed in many metal poor halo stars [4–10] and the data indicates that the abundances of these light nuclei are linearly (or almost linearly) proportional to the abundance of Iron in these stars and contrary to the standard model prediction of a quadratic proportion that would be expected if the light nuclei were a secondary product [11]. This would seem to indicate either a primary origin by some other mechanism or a production rate dominated by spallation of CRs enriched in CNO on interstellar p and $\alpha$. It is the purpose of this paper to compute the spallagenic rates of production of these light nuclei with inclusion of new $\alpha - \alpha$ cross section data and to examine the details of these calculations: in particular, to test the ‘straight ahead’ approximation to the fragment energy distribution. In the last section we use our new production rates and rederive the elemental ratios for the Population I and the scaling relations for the Population II environments previously derived in Steigman & Walker [21].

II. THE STRAIGHT AHEAD CALCULATION

The production rates $J_{PTF}$ for the reaction $P + T \rightarrow F + ...$ are formally calculated from the equation

$$J_{PTF} = \int dE_P \frac{d\phi_P(E_P)}{dE_P} \sigma_{P+T\rightarrow F}(E_P) \left\{ \int dE_F \rho(E_F[E_P]) S_F(E_F) \right\} ,$$

(1)
where P labels the incident CR projectile, T the ISM target and F the fragment produced. $d\phi_P(E_P)/dE_P$ is the interstellar cosmic ray spectrum, $\sigma^{P+T\rightarrow F}(E_P)$ the relevant cross section for the process, $\rho(E_F[E_P])$ is the fragment energy distribution function and $S_F(E_F)$ is a factor that accounts for the successful trapping of the fragment F in the Galaxy. Here, and in the remainder of this paper, all energies/momenta are in per nucleon units. At present there is no accepted standard form for the CR spectrum since the interstellar cosmic ray spectrum below $\sim 5 \text{ GeV/nucleon}$ cannot be measured directly \cite{12}. The cosmic ray spectrum as measured by balloon borne experiments at the top of the atmosphere \cite{13}, by satellites or instruments carried into orbit \cite{14} or by interplanetary spacecraft \cite{15} has been modulated by the geomagnetic effects of the sun. Theoretical descriptions of this modulation use these observations as tests for the models and all require the interstellar cosmic ray spectrum as input \cite{12,16}. Many studies of chemical evolution \cite{17,18} employ a technique where the interstellar cosmic ray spectrum is formed from propagating a source spectrum through the ISM. For the present we choose a CR spectrum of the form

$$
\frac{d\phi_P(E_P)}{dE_P} = \frac{1.6\,E_0^{1.6}}{(E_P + E_0)^{2.6}} \text{/(MeV/nucleon)}/(\text{cm}^2/\text{s}) \tag{2}
$$

which was presented by Gloeckler & Jokipii \cite{19} and used by Walker, Matthews & Viola \cite{20}, Steigman & Walker \cite{21} and Urch & Gleeson \cite{22}. The parameter $E_0$ is the nucleon mass. We shall consider modifications of this form in section \textsection{IV}.

If the Galaxy trapped every fragment produced by the reaction $P + T \rightarrow F + ...$ then $S_F(E_F) = 1$ and the integral over the fragment energy $E_F$ is trivial. This is true in the cases of p and $\alpha$ as projectiles, however this is not the case when the projectile is CNO. The fragment energy distribution $\rho(E_F[E_P])$ and S-factor are usually taken to be

$$
\rho(E_F[E_P]) = \delta(E_P - E_F), \tag{3}
$$

and

$$
S_F(E_F) = \begin{cases} 
1 & P \in \{\text{H,He}\} \\
\exp\left[-R_F(E_F)/\Lambda\right] & P \in \{\text{C,N,O}\}
\end{cases}, \tag{4}
$$
where $R_F(E_F)$ is the range of F and $\Lambda$ is the stopping power of the ISM. The $\delta$ function form for $\rho(E_F[E_P])$ is known as the ‘straight ahead’ approximation and the form of $S_F(E_F)$ is often called the ‘leaky-box’ model [23]. The range $R_F(E_F)$ is tabulated up to 12 MeV/nucleon for each fragment by Northcliffe & Schilling [24]. Above this energy we use the range for protons as tabulated by Janni [25] and rescale in exactly the same manner as Fields, Olive & Schramm [17]. We assume that $\Lambda$ is independent of energy although Fields, Olive & Schramm [17] introduced an energy dependent form while Tsao, Silberberg, Barghouty & Sihver [26] use a rigidity dependent $\Lambda$ introduced by Gupta & Webber [27].

The cross sections needed to create LiBeB were tabulated by Read & Viola [28] and we have included the recent measurements of the $\alpha - \alpha$ fusion reaction cross sections by Mercer et al [29] and the reanalysis by Mercer, Glagola and Austin [30]. The new measurements show that both these cross sections fall approximately exponentially with projectile energy and have reduced the uncertainty in these rates due to the necessity of extrapolation to higher energies.

There are some complications in these computations that are frequently left unstated in the literature: the cross sections in Read & Viola [28] are for the isobars, in the case of $A = 7$ this is a $\sim 50:50$ mixture of $^7\text{Li}$ and $^7\text{Be}$ and likewise for $A = 11$ which contains both $^{11}\text{B}$ and $^{11}\text{C}$. These other nuclei will eventually decay to $^7\text{Li}$ and $^{11}\text{B}$ respectively and even though the half-lives are normally short at 53.28 days and 20.3 minutes respectively [31] this cannot occur until they have been trapped by the Galaxy since the decay mode is inner orbital electron capture. Thus, for $R_{^7\text{Li}}$ and $R_{^{11}\text{B}}$ we average the trapping factors of these products. In a similar manner the $A = 6$ isobaric cross section includes the production of $^6\text{He}$ but in this case the decay mode is $\beta$ decay with a half-life of 0.807 s [31] and so any $^6\text{He}$ produced will quickly decay to $^6\text{Li}$ and no averaging is required.

In table I: columns a) and b) we present the rates as calculated by equation (1) using the forms for the various terms as listed in equations (2) through (4) using two different values of $\Lambda$. An initial examination of the results shows that fragment loss for the reactions with $P \in \{\text{C, N, O}\}$ reduces the production rate by a factor $\lesssim 10$ relative to the rate when
the projectile and target are interchanged. While this may appear to render these reactions unimportant it must be remembered that, at the present time, the abundances of CNO in the CR are enhanced relative to their ISM values by approximately equivalent factors. We also find that the new $\alpha - \alpha$ rates are smaller than those calculated by Walker, Matthews & Viola [20]: the rate for $\alpha + \alpha \rightarrow ^6\text{Li}$ decreases by a factor of 1.52 while the rate for production of $^7\text{Li}$ decreased by a factor of 1.70. Thus for fixed projectile fluxes and target densities, the production of Li (that includes an $\alpha-\alpha$ contribution) relative to that of Be and B (that does not include $\alpha-\alpha$) decreases.

### III. TESTING THE ‘STRAIGHT AHEAD’ APPROXIMATION

In section [1] we used the ‘straight ahead’ approximation to calculate $J^{PTF}$ and it is this that we now wish to test in the same manner as Tsao, Silberberg, Barghouty & Silver [20]. The momentum distributions of the isotopes produced by fragmentation of various projectile and target nuclei are discussed by Morrissey [32] and Hufner [33]. They are found to be Gaussian distributed in the rest frame of the spalled nucleus with narrow dispersions and a small mean momentum in the direction parallel to the incident projectile. In particular the momentum distributions of the fragments from $^{12}\text{C}$ and $^{16}\text{O}$ projectiles upon various targets ranging in mass from Be to Pb were measured by Greiner et al [34] who found that their results had no significant correlation with target mass or beam energy. Goldhaber [35] and more recently Bauer [36] explain these results in terms of a nuclear model with minimal correlations between the nucleon momenta and predict the dependence of the momentum per nucleon distribution $\rho(P_F)$ on fragment mass $A_F$ as

$$\rho(P_F) = \frac{1}{(2\pi\alpha_F)^{3/2}} \exp \left( \frac{-(P_F - <P_F>)^2}{2\alpha_F^2} \right),$$  \hspace{1cm} (5)$$

$$\alpha_F^2 = \alpha_0^2 \frac{A_{\text{CNO}} - A_F}{A_F(A_{\text{CNO}} - 1)},$$  \hspace{1cm} (6)$$

and
\[
\langle P_F \rangle = 8 \frac{(A_{\text{CNO}} - A_F)}{A_{\text{CNO}}} \frac{\gamma + 1}{\beta \gamma} \text{ MeV/nucleon,}
\]

where \( \alpha_0 \sim 100 \text{ MeV} \), \( A_{\text{CNO}} \) is the mass of the nucleus to be spalled while \( \beta = v/c \) and \( \gamma = (1 - \beta^2)^{-1/2} \) are those of the projectile. Equation (5) assumes that the transverse momentum dispersion is equal to the longitudinal and equation (7) is a semi-empirical relation by Morrissey [32]. We also ignore the change in \( \alpha \) that occurs for values of \( E_P \lesssim 100 \text{ MeV/nucleon} \) indicated by Stokstad [37]: at such small energies the value of \( S_F \) is very close to unity and every fragment isotope is captured by the Galaxy rendering the decrease in \( \alpha \) irrelevant. Thus we can rewrite equation (1) for reactions involving CNO that includes this better approximation to the product fragmentation energy distribution:

\[
J_{PTF} = \int dE_P \frac{1.6E_0^{1.6}}{(E_P + E_0)^{2.6}} \sigma^{P+T\to F}(E_P) \left\{ \int \frac{d^3P_F'}{(2\pi\alpha_F^2)^{3/2}} \exp \left( -\frac{(P_F' - \langle P_F' \rangle)^2}{2\alpha_F^2} \right) \exp \left( -\frac{R_F(E_F)}{\Lambda} \right) \right\}.
\] (8)

The dependence of \( E_P \) upon \( E_F \) is more subtle than before and enters into the integral over momentum per nucleon \( P_F' \) since this is evaluated in the CNO rest frame.

In table I: columns c) and d) we present the rates as calculated by equation (8) again for \( \Lambda = 5 \text{ g/cm}^2 \) and 10 g/cm\(^2\). A glance at the results shows that indeed the ‘straight ahead’ approximation accurately (within numerical error) predicts the production rate of those reactions with \( P \in \{p, \alpha\} \). This result was to be expected: the spread in fragment momenta \( \alpha_F \) in equation (8) is only of order \( \sim 100 \text{ MeV} \) and the central value corresponds to a stopping range much less than \( \Lambda \) so all fragments are trapped. For the reactions with \( P \in \{C, N, O\} \) there appears to be a slight increase in the production rates relative to the ‘straight ahead’ calculations for the smaller value of \( \Lambda \) and a slight decrease for the larger value of \( \Lambda \) but the differences are small and do not warrant rejection of the approximation. Again, the approximation’s success has a simple explanation: the fragments that escape must have large momenta (\( \gtrsim 500 \text{ MeV} \)) and the small values of \( \alpha_F \) mean that \( P_F \sim P, E_F \sim E_P \).
IV. CHANGING THE CR SPECTRUM

The CR spectrum (2) used in sections II and III is by no means the only one that has been used. As previously stated, solar modulation significantly depresses the low energy ($\lesssim 5$ GeV) region of the CR spectrum and the extent of this alteration can be mitigated by changing the unmodulated spectrum to yield similar observations. Yet it is exactly this region that dominates the integrands in equation (8) since it corresponds to where $S_F(E_F)$ is $\approx 1$. To address this uncertainty we also calculate the production rates $J^{PTF}$ with interstellar spectra that differ from equation (2). More specifically, we rewrite equation (2) as

$$\frac{d\phi_P(E_P)}{dE^\mu} = \frac{E_0^{\nu-\mu-1}}{B(\mu+1, \nu-\mu-1)} \frac{E_P^\mu}{(E_P + E_0)^\nu} / \text{(MeV/nucleon)} / \text{(cm}^2/\text{s}),$$

where $B(\mu+1, \nu-\mu-1)$ is the Beta function. This functional form is sufficiently flexible to mimic virtually any spectrum desired but, to avoid a plethora of results, we restrict ourselves to considering values of $\mu$ and $\nu$ such that $\nu - \mu = 2.6$, $0 \leq \mu \leq 1$ and keep $E_0$ at it’s previous value. This family of spectra asymptotically approaches the form of equation (4) at high energies but ‘push’ some of their low energy component to higher values relative to the $\mu = 0, \nu = 2.6$ standard. Consequently it should be expected that the rates for CR CNO on ISM H,He would be reduced due to a decrease in the fraction of captured fragments. The rates with $P \in \{p, \alpha\}$ will be affected but only to a slight degree since all fragments are still captured but the cross sections are not constants. The effect of the reduction of the spectrum at lower energies on the $\alpha + \alpha$ fusion production rates of $^6$Li and $^7$Li should be more pronounced because the cross sections for both of these processes are predominant at lower energies. The results of using equation (3) are listed in table II for the values $\mu \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ and $\Lambda$ fixed at 5 g/cm$^2$. As predicted the rates for $P \in \{p, \alpha\}$ are almost unaltered but the rates for the reactions with $P \in \{C, N, O\}$ and $\alpha + \alpha \rightarrow ^6$Li, $^7$Li are significantly affected by the change in the CR spectra, indeed the rates for $\mu = 1, \nu = 3.6$ are approximately an order of magnitude smaller than the rates for $\mu = 0, \nu = 2.6$. If these hard spectra are indeed indicative of the true interstellar cosmic ray spectrum then even
the present CR CNO enrichment mentioned previously cannot bring the ‘inverse’ reactions into competition with their ‘forward’ brethren and the production of the $^9$Be and $^{10}$B in the Galaxy by this process has always been dominated by reactions with $p$ and $\alpha$ as the projectiles.

V. POPULATION I ELEMENTAL RATIOS AND POPULATION II SCALING RELATIONS

Finally we have recomputed the scaling relations and elemental ratios previously presented in Steigman&Walker [21] for our standard case of $\Lambda = 5$ g/cm$^2$, $\mu = 0, \nu = 2.6$. The approximate nucleosynthetic yields can be written as

$$y_F \approx \sum_p \sum_T \alpha_p \ y_T \ J^{PTF} \Delta t = 1.18 \times 10^{-12} \ R_F \ \Delta t_{\text{Gyr}}.$$  \hspace{1cm} (10)

The uncertainty in the time dependence of the CR spectrum, the S-factor and the abundances of the targets and CRs can be largely removed by considering the ratios of the elements rather than individual yields. The Population I ISM is taken to be

\begin{align*}
y_H &= 1, & \alpha_p &= 8.05, \\
y_{He} &= 0.1, & \alpha_\alpha &= \alpha_p/10, \\
y_C &= 4.2 \times 10^{-4}, & \alpha_C &= \alpha_p/310, \\
y_N &= 8.7 \times 10^{-5}, & \alpha_N &= \alpha_p/5120, \\
y_O &= 6.9 \times 10^{-4}, & \alpha_O &= \alpha_p/240,
\end{align*}

and we compute

\begin{align*}
R_7/R_6 &= 1.49, \hspace{1cm} (11) \\
R_7/R_9 &= 6.22, \quad R_{6+7}/R_9 &= 10.39, \hspace{1cm} (12) \\
R_{6+7}/R_{10+11} &= 0.62, \hspace{1cm} (13) \\
R_{11}/R_{10} &= 2.48, \hspace{1cm} (14)
\end{align*}
We find that in comparison with S&W that the our new calculations for the production rates have increased the $7/6$ ratio slightly and reduced all the remainder except for the $11/10$ ratio which has remained unaltered. The inclusion of the ‘reverse’ reactions and the lower $^6\text{Li}$ and $^7\text{Li}$ yields have both decreased the denominators and increased the numerators.

In the early Galaxy we adopt the same scaling relations as S&W, namely

\[
\begin{align*}
y^{II}_{H} &= 1, \\
y^{II}_{He} &= 0.08, \\
y^{II}_{C} &= y^I_C \times 10^{[\text{Fe}/\text{H}]}, \\
y^{II}_{N} &= y^I_N \times 10^{[\text{Fe}/\text{H}]}, \\
y^{II}_{O} &= y^I_O \times 10^{[\text{Fe}/\text{H}] + 1/2}, \\
\alpha^{II}_p &= \alpha^I_p, \\
\alpha^{II}_\alpha &= \alpha^I_p \times 0.08, \\
\alpha^{II}_C &= \alpha^I_C \times 10^{[\text{Fe}/\text{H}]}, \\
\alpha^{II}_N &= \alpha^I_N \times 10^{[\text{Fe}/\text{H}]}, \\
\alpha^{II}_O &= \alpha^I_O \times 10^{[\text{Fe}/\text{H}] + 1/2},
\end{align*}
\]

and find

\[
\begin{align*}
R_6 &= 1.27(1 + 16.94 \times 10^{[\text{Fe}/\text{H}]}) \\
R_7 &= 2.09(1 + 13.95 \times 10^{[\text{Fe}/\text{H}]}) \\
R_9 &= 5.97 \times 10^{[\text{Fe}/\text{H}]} \\
R_{10} &= 25.98 \times 10^{[\text{Fe}/\text{H}]} \\
R_{11} &= 60.74 \times 10^{[\text{Fe}/\text{H}]}
\end{align*}
\]

(15) (16) (17)

Once again, in comparison with S&W: the contribution of the $\alpha - \alpha$ reactions to the production of $^6\text{Li}$ and $^7\text{Li}$ have almost halved the constant in $R_6$ and $R_7$ and the CNO terms have doubled. The $^9\text{Be}$, $^{10}\text{B}$ and $^{11}\text{B}$ yields are approximately doubled too. While the new yields do not alter the basic conclusions of S&W they do make it weaker and the amount of $^6\text{Li}$ and $^7\text{Li}$ that would be inferred by observation of the amount of $^9\text{Be}$ in the oldest stars would be reduced. For example: at a metalicity of $[\text{Fe}/\text{H}] = -3$ the ratio of Lithium to Beryllium is now only $\sim 550$ compared to a value of $\sim 2200$ found in Steigman & Walker, a factor of 4 difference. We also obtain a better agreement with the $^6\text{Li}$ to $^9\text{Be}$ ratios observed in metal-poor halo stars: for the star HD84937 Hobbs, Thorburn & Rebull find the ratio to be $73 \pm 18$ whereas the prediction is $37.3$: for BD+26$^\circ$3578 the observed ratio is $22 \pm 13$ and the prediction is $57.0$. 
VI. CONCLUSIONS

We have calculated the production rates of $^6$Li, $^7$Li, $^9$Be, $^{10}$B and $^{11}$B via spallation of Carbon, Nitrogen and Oxygen nuclei by protons and $\alpha$-particles and the production of $^6$Li and $^7$Li by $\alpha-\alpha$ fusion reactions. We have found that the new $\alpha-\alpha$ fusion cross section data produces smaller production rates than previously computed by factors of $\sim 1.5$ and 1.7 for $^6$Li and $^7$Li respectively. By employing a better description of the fragment energy as a function of the projectile’s energy we relaxed the ‘straight ahead’ approximation and found that the rates were only slightly affected. We also computed the production rates with increasingly harder spectra and found that they decreased by up to an order of magnitude compared to our reference values. We conclude that, if these spectra represent the true interstellar spectrum, the $^9$Be and $^{10}$B production in the Galaxy is always dominated by the production from the ‘forward’ reactions of CR p/$\alpha$ upon CNO in the ISM.
| rate (P + T → F) | a)     | b)     | c)     | d)     |
|------------------|--------|--------|--------|--------|
| H + C → $^6$Li   | 1.21E-26 | 1.21E-26 | 1.21E-26 | 1.21E-26 |
| C + H → $^6$Li   | 1.57E-27 | 2.37E-27 | 1.68E-27 | 2.36E-27 |
| H + C → $^7$Li   | 2.15E-26 | 2.15E-26 | 2.15E-26 | 2.14E-26 |
| C + H → $^7$Li   | 3.80E-27 | 5.35E-27 | 3.98E-27 | 5.30E-27 |
| H + C → $^9$Be   | 4.09E-27 | 4.09E-27 | 4.10E-27 | 4.10E-27 |
| C + H → $^9$Be   | 3.97E-28 | 6.19E-28 | 4.10E-28 | 6.22E-28 |
| H + C → $^{10}$B | 2.28E-26 | 2.28E-26 | 2.28E-26 | 2.28E-26 |
| C + H → $^{10}$B | 4.22E-27 | 6.05E-27 | 4.29E-27 | 6.07E-27 |
| H + C → $^{11}$B | 5.73E-26 | 5.73E-26 | 5.73E-26 | 5.73E-26 |
| C + H → $^{11}$B | 1.43E-26 | 1.90E-26 | 1.44E-26 | 1.90E-26 |
| H + N → $^6$Li   | 1.82E-26 | 1.82E-26 | 1.82E-26 | 1.82E-26 |
| N + H → $^6$Li   | 2.61E-27 | 3.79E-27 | 2.81E-27 | 3.74E-27 |
| H + N → $^7$Li   | 1.05E-26 | 1.05E-26 | 1.05E-26 | 1.05E-26 |
| N + H → $^7$Li   | 2.24E-27 | 2.95E-27 | 2.34E-27 | 2.94E-27 |
| H + N → $^9$Be   | 4.67E-27 | 4.67E-27 | 4.66E-27 | 4.66E-27 |
| N + H → $^9$Be   | 8.24E-28 | 1.17E-27 | 8.57E-28 | 1.16E-27 |
| H + N → $^{10}$B | 1.16E-26 | 1.16E-26 | 1.16E-26 | 1.16E-26 |
| N + H → $^{10}$B | 3.58E-27 | 4.40E-27 | 3.65E-27 | 4.39E-27 |
| H + N → $^{11}$B | 2.41E-26 | 2.41E-26 | 2.41E-26 | 2.41E-26 |
| N + H → $^{11}$B | 6.93E-27 | 8.76E-27 | 7.02E-27 | 8.76E-27 |
| H + O → $^6$Li   | 1.29E-26 | 1.29E-26 | 1.29E-26 | 1.29E-26 |
| O + H → $^6$Li   | 1.99E-27 | 2.85E-27 | 2.12E-27 | 2.83E-27 |
| H + O → $^7$Li   | 2.06E-26 | 2.06E-26 | 2.06E-26 | 2.06E-26 |
| O + H → $^7$Li   | 3.50E-27 | 5.02E-27 | 3.73E-27 | 4.95E-27 |
| H + O → $^9$Be   | 4.12E-27 | 4.12E-27 | 4.12E-27 | 4.13E-27 |
| Reaction                        | E1    | E2    | E3    | E4    |
|--------------------------------|-------|-------|-------|-------|
| O + H → $^9$Be                 | 7.11E-28 | 1.03E-27 | 7.48E-28 | 1.01E-27 |
| H + O → $^{10}$B               | 1.49E-26 | 1.49E-26 | 1.49E-26 | 1.49E-26 |
| O + H → $^{10}$B               | 3.11E-27 | 4.31E-27 | 3.22E-27 | 4.26E-27 |
| H + O → $^{11}$B               | 2.79E-26 | 2.79E-26 | 2.79E-26 | 2.79E-26 |
| O + H → $^{11}$B               | 7.70E-27 | 1.00E-26 | 7.88E-27 | 1.00E-26 |
| He + C → $^6$Li                | 4.36E-26 | 4.36E-26 | 4.36E-26 | 4.36E-26 |
| C + He → $^6$Li                | 9.03E-27 | 1.21E-26 | 9.54E-27 | 1.19E-26 |
| He + C → $^7$Li                | 5.97E-26 | 5.97E-26 | 5.97E-26 | 5.98E-26 |
| C + He → $^7$Li                | 1.33E-26 | 1.77E-26 | 1.38E-26 | 1.76E-26 |
| He + C → $^9$Be                | 1.19E-26 | 1.19E-26 | 1.20E-26 | 1.19E-26 |
| C + He → $^9$Be                | 3.45E-27 | 4.36E-27 | 3.53E-27 | 4.38E-27 |
| He + C → $^{10}$B              | 4.94E-26 | 4.94E-26 | 4.94E-26 | 4.94E-26 |
| C + He → $^{10}$B              | 1.28E-26 | 1.67E-26 | 1.30E-26 | 1.66E-26 |
| He + C → $^{11}$B              | 9.32E-26 | 9.32E-26 | 9.33E-26 | 9.33E-26 |
| C + He → $^{11}$B              | 2.53E-26 | 3.28E-26 | 2.55E-26 | 3.28E-26 |
| He + N → $^6$Li                | 1.55E-26 | 1.55E-26 | 1.55E-26 | 1.55E-26 |
| N + He → $^6$Li                | 3.03E-27 | 3.98E-27 | 3.19E-27 | 3.95E-27 |
| He + N → $^7$Li                | 2.18E-26 | 2.18E-26 | 2.18E-26 | 2.18E-26 |
| N + He → $^7$Li                | 3.97E-27 | 5.49E-27 | 4.17E-27 | 5.44E-27 |
| He + N → $^9$Be                | 7.40E-27 | 7.40E-27 | 7.38E-27 | 7.40E-27 |
| N + He → $^9$Be                | 1.54E-27 | 2.06E-27 | 1.59E-27 | 2.04E-27 |
| He + N → $^{10}$B              | 3.57E-26 | 3.57E-26 | 3.57E-26 | 3.57E-26 |
| N + He → $^{10}$B              | 8.43E-27 | 1.11E-26 | 8.62E-27 | 1.11E-26 |
| He + N → $^{11}$B              | 7.08E-26 | 7.08E-26 | 7.09E-26 | 7.09E-26 |
| N + He → $^{11}$B              | 1.60E-26 | 2.18E-26 | 1.63E-26 | 2.18E-26 |
| He + O → $^6$Li                | 1.28E-26 | 1.28E-26 | 1.28E-26 | 1.28E-26 |
| O + He → $^6$Li                | 2.17E-27 | 2.98E-27 | 2.32E-27 | 2.93E-27 |
| Reaction                  | Case 1 | Case 2 | Case 3 | Case 4 |
|---------------------------|--------|--------|--------|--------|
| He + O → $^7$Li           | 1.79E-26 | 1.79E-26 | 1.79E-26 | 1.78E-26 |
| O + He → $^7$Li           | 3.21E-27 | 4.46E-27 | 3.39E-27 | 4.43E-27 |
| He + O → $^9$Be           | 6.88E-27 | 6.88E-27 | 6.87E-27 | 6.86E-27 |
| O + He → $^9$Be           | 1.27E-27 | 1.76E-27 | 1.32E-27 | 1.75E-27 |
| He + O → $^{10}$B         | 2.16E-26 | 2.16E-26 | 2.16E-26 | 2.16E-26 |
| O + He → $^{10}$B         | 4.54E-27 | 6.22E-27 | 4.69E-27 | 6.20E-27 |
| He + O → $^{11}$B         | 3.84E-26 | 3.84E-26 | 3.84E-26 | 3.84E-26 |
| O + He → $^{11}$B         | 8.95E-27 | 1.21E-26 | 9.16E-27 | 1.20E-26 |
| He + He → $^6$Li          | 8.58E-28 | 8.58E-28 |
| He + He → $^7$Li          | 1.41E-27 | 1.41E-27 |

TABLE I. The reaction rates $J^{PTE}$ in units of $/s$. There are 4 cases listed:

a) $\Lambda = 5$ g/cm$^2$: straight-ahead approximation

b) $\Lambda = 10$ g/cm$^2$: straight-ahead approximation

c) $\Lambda = 5$ g/cm$^2$: momentum integral equation

d) $\Lambda = 10$ g/cm$^2$: momentum integral equation
| Rate \((P + T \rightarrow F)\) | \(a)\) | \(b)\) | \(c)\) | \(d)\) | \(e)\) | \(f)\) |
|--------------------------|-------|-------|-------|-------|-------|-------|
| \(H + C \rightarrow ^6\text{Li}\) | 1.21E-26 | 1.21E-26 | 1.22E-26 | 1.22E-26 | 1.22E-26 | 1.22E-26 |
| \(C + H \rightarrow ^6\text{Li}\) | 1.57E-27 | 1.21E-27 | 9.20E-28 | 6.86E-28 | 5.05E-28 | 3.70E-28 |
| \(H + C \rightarrow ^7\text{Li}\) | 2.15E-26 | 2.12E-26 | 2.12E-26 | 2.11E-26 | 2.11E-26 | 2.10E-26 |
| \(C + H \rightarrow ^7\text{Li}\) | 3.80E-27 | 2.93E-27 | 2.21E-27 | 1.65E-27 | 1.22E-27 | 8.94E-28 |
| \(H + C \rightarrow ^9\text{Be}\) | 4.09E-27 | 4.25E-27 | 4.45E-27 | 4.61E-27 | 4.75E-27 | 4.86E-27 |
| \(C + H \rightarrow ^9\text{Be}\) | 3.97E-28 | 3.19E-28 | 2.50E-28 | 1.93E-28 | 1.48E-28 | 1.12E-28 |
| \(H + C \rightarrow ^{10}\text{B}\) | 1.21E-26 | 1.21E-26 | 1.22E-26 | 1.22E-26 | 1.22E-26 | 1.22E-26 |
| \(C + H \rightarrow ^{10}\text{B}\) | 1.57E-27 | 1.21E-27 | 9.20E-28 | 6.86E-28 | 5.05E-28 | 3.70E-28 |
| \(H + C \rightarrow ^{11}\text{B}\) | 2.15E-26 | 2.12E-26 | 2.12E-26 | 2.11E-26 | 2.11E-26 | 2.10E-26 |
| \(C + H \rightarrow ^{11}\text{B}\) | 3.80E-27 | 2.93E-27 | 2.21E-27 | 1.65E-27 | 1.22E-27 | 8.94E-28 |
| \(H + N \rightarrow ^6\text{Li}\) | 1.82E-26 | 1.80E-26 | 1.81E-26 | 1.81E-26 | 1.81E-26 | 1.81E-26 |
| \(N + H \rightarrow ^6\text{Li}\) | 2.61E-27 | 1.97E-27 | 1.46E-27 | 1.07E-27 | 7.77E-28 | 5.62E-28 |
| \(H + N \rightarrow ^7\text{Li}\) | 1.05E-26 | 1.01E-26 | 1.01E-26 | 9.97E-27 | 9.92E-27 | 9.88E-27 |
| \(N + H \rightarrow ^7\text{Li}\) | 2.24E-27 | 1.61E-27 | 1.15E-27 | 8.19E-28 | 5.84E-28 | 4.18E-28 |
| \(H + N \rightarrow ^{9}\text{Be}\) | 4.67E-27 | 4.62E-27 | 4.65E-27 | 4.64E-27 | 4.64E-27 | 4.63E-27 |
| \(N + H \rightarrow ^{9}\text{Be}\) | 8.24E-28 | 6.35E-28 | 4.80E-28 | 3.59E-28 | 2.66E-28 | 1.97E-28 |
| \(H + N \rightarrow ^{10}\text{B}\) | 1.16E-26 | 1.08E-26 | 1.05E-26 | 1.02E-26 | 1.00E-26 | 9.97E-27 |
| \(N + H \rightarrow ^{10}\text{B}\) | 3.58E-27 | 2.55E-27 | 1.80E-27 | 1.27E-27 | 9.00E-28 | 6.42E-28 |
| \(H + N \rightarrow ^{11}\text{B}\) | 2.41E-26 | 2.28E-26 | 2.23E-26 | 2.20E-26 | 2.18E-26 | 2.17E-26 |
| \(N + H \rightarrow ^{11}\text{B}\) | 6.93E-27 | 4.88E-27 | 3.48E-27 | 2.53E-27 | 1.86E-27 | 1.38E-27 |
| \(H + O \rightarrow ^6\text{Li}\) | 1.29E-26 | 1.27E-26 | 1.28E-26 | 1.27E-26 | 1.26E-26 | 1.26E-26 |
| \(O + H \rightarrow ^6\text{Li}\) | 1.99E-27 | 1.51E-27 | 1.14E-27 | 8.46E-28 | 6.19E-28 | 4.49E-28 |
| \(H + O \rightarrow ^7\text{Li}\) | 2.06E-26 | 2.04E-26 | 2.05E-26 | 2.05E-26 | 2.04E-26 | 2.03E-26 |
| \(O + H \rightarrow ^7\text{Li}\) | 3.50E-27 | 2.75E-27 | 2.10E-27 | 1.59E-27 | 1.18E-27 | 8.73E-28 |
| \(H + O \rightarrow ^{9}\text{Be}\) | 4.12E-27 | 4.09E-27 | 4.11E-27 | 4.10E-27 | 4.09E-27 | 4.08E-27 |
| Reaction                | 7.11E-28 | 5.65E-28 | 4.38E-28 | 3.34E-28 | 2.52E-28 | 1.88E-28 |
|------------------------|----------|----------|----------|----------|----------|----------|
| O + H → 9Be            | 1.49E-26 | 1.47E-26 | 1.47E-26 | 1.46E-26 | 1.46E-26 | 1.45E-26 |
| H + O → 10B            | 3.11E-27 | 2.47E-27 | 1.92E-27 | 1.47E-27 | 1.12E-27 | 8.43E-28 |
| O + H → 10B            | 2.79E-26 | 2.70E-26 | 2.66E-26 | 2.62E-26 | 2.59E-26 | 2.56E-26 |
| O + H → 11B            | 7.70E-27 | 6.04E-27 | 4.64E-27 | 3.51E-27 | 2.64E-27 | 1.97E-27 |
| He + C → 6Li           | 4.36E-26 | 4.18E-26 | 4.11E-26 | 4.04E-26 | 3.98E-26 | 3.94E-26 |
| C + He → 6Li           | 9.03E-27 | 6.61E-27 | 4.75E-27 | 3.38E-27 | 2.40E-27 | 1.69E-27 |
| He + C → 7Li           | 5.97E-26 | 5.76E-26 | 5.67E-26 | 5.59E-26 | 5.54E-26 | 5.49E-26 |
| C + He → 7Li           | 1.33E-26 | 9.86E-27 | 7.20E-27 | 5.22E-27 | 3.77E-27 | 2.71E-27 |
| He + C → 9Be           | 1.19E-26 | 1.12E-26 | 1.08E-26 | 1.04E-26 | 1.02E-26 | 1.00E-26 |
| C + He → 9Be           | 3.45E-27 | 2.52E-27 | 1.82E-27 | 1.30E-27 | 9.28E-28 | 6.62E-28 |
| He + C → 10B           | 4.94E-26 | 4.74E-26 | 4.67E-26 | 4.60E-26 | 4.55E-26 | 4.51E-26 |
| C + He → 10B           | 1.28E-26 | 9.62E-27 | 7.12E-27 | 5.26E-27 | 3.88E-27 | 2.86E-27 |
| He + C → 11B           | 9.32E-26 | 8.97E-26 | 8.84E-26 | 8.74E-26 | 8.66E-26 | 8.60E-26 |
| C + He → 11B           | 2.53E-26 | 1.90E-26 | 1.42E-26 | 1.07E-26 | 7.98E-27 | 5.98E-27 |
| He + N → 6Li           | 1.55E-26 | 1.49E-26 | 1.47E-26 | 1.46E-26 | 1.45E-26 | 1.45E-26 |
| N + He → 6Li           | 3.03E-27 | 2.07E-27 | 1.41E-27 | 9.77E-28 | 6.80E-28 | 4.76E-28 |
| He + N → 7Li           | 2.18E-26 | 2.13E-26 | 2.14E-26 | 2.13E-26 | 2.12E-26 | 2.12E-26 |
| N + He → 7Li           | 3.97E-27 | 2.94E-27 | 2.16E-27 | 1.58E-27 | 1.15E-27 | 8.42E-28 |
| He + N → 9Be           | 7.40E-27 | 7.18E-27 | 7.14E-27 | 7.10E-27 | 7.07E-27 | 7.05E-27 |
| N + He → 9Be           | 1.54E-27 | 1.11E-27 | 8.05E-28 | 5.81E-28 | 4.21E-28 | 3.06E-28 |
| He + N → 10B           | 3.57E-26 | 3.45E-26 | 3.42E-26 | 3.40E-26 | 3.39E-26 | 3.38E-26 |
| N + He → 10B           | 8.43E-27 | 6.13E-27 | 4.47E-27 | 3.29E-27 | 2.43E-27 | 1.81E-27 |
| He + N → 11B           | 7.08E-26 | 6.97E-26 | 6.98E-26 | 6.97E-26 | 6.96E-26 | 6.95E-26 |
| N + He → 11B           | 1.60E-26 | 1.24E-26 | 9.52E-27 | 7.28E-27 | 5.55E-27 | 4.23E-27 |
| He + O → 6Li           | 1.28E-26 | 1.25E-26 | 1.25E-26 | 1.24E-26 | 1.24E-26 | 1.24E-26 |
| O + He → 6Li           | 2.17E-27 | 1.56E-27 | 1.11E-27 | 7.89E-28 | 5.60E-28 | 3.98E-28 |
|reaction | $J^{P\Gamma}$ | $T$ | $F$ |
|---------|---------------|-----|-----|
|He + O → $^7$Li | 1.79E-26 | 1.76E-26 | 1.76E-26 | 1.75E-26 | 1.75E-26 | 1.75E-26 |
|O + He → $^7$Li | 3.21E-27 | 2.39E-27 | 1.76E-27 | 1.29E-27 | 9.47E-28 | 6.93E-28 |
|He + O → $^9$Be | 6.88E-27 | 6.77E-27 | 6.78E-27 | 6.77E-27 | 6.77E-27 | 6.76E-27 |
|O + He → $^9$Be | 1.27E-27 | 9.54E-28 | 7.09E-28 | 5.25E-28 | 3.87E-28 | 2.85E-28 |
|He + O → $^{10}$B | 2.16E-26 | 2.12E-26 | 2.13E-26 | 2.12E-26 | 2.12E-26 | 2.11E-26 |
|O + He → $^{10}$B | 4.54E-27 | 3.47E-27 | 2.62E-27 | 1.98E-27 | 1.49E-27 | 1.12E-27 |
|He + O → $^{11}$B | 3.84E-26 | 3.76E-26 | 3.75E-26 | 3.74E-26 | 3.73E-26 | 3.73E-26 |
|O + He → $^{11}$B | 8.95E-27 | 6.86E-27 | 5.22E-27 | 3.96E-27 | 3.01E-27 | 2.29E-27 |
|He + He → $^6$Li | 8.58E-28 | 5.32E-28 | 3.26E-28 | 1.99E-28 | 1.23E-28 | 7.77E-29 |
|He + He → $^7$Li | 1.41E-27 | 7.99E-28 | 4.43E-28 | 2.42E-28 | 1.32E-28 | 7.28E-29 |

**TABLE II.** The reaction rates $J^{P\Gamma}$ in units of /s. The 6 cases listed:

a) $\nu = 0.0; \mu = 2.6$ (standard)

b) $\nu = 0.2; \mu = 2.8$

c) $\nu = 0.4; \mu = 3.0$

d) $\nu = 0.6; \mu = 3.2$

e) $\nu = 0.8; \mu = 3.4$

f) $\nu = 1.0; \mu = 3.6$
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