Highly conducting correlated state of crossed electronic bands

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Introduction. — In 2018, an unconventional superconducting state was detected in twisted bilayer graphene (tBG) [1]. Further observations revealed a whole family of strongly correlated electronic states also including insulating [2, 3] and magnetic [4, 5] ones. These observations have been followed by several attempts to probe the mechanism responsible for such a diversity [6–12] and by an explosive growth of theoretical publications trying to explain the measurements [13–30]. Even wider diversity of electronic correlated states have been found in twisted graphene multilayers [31–35], which demonstrate abrupt drops in the resistivity as the temperature is lowered [31–35]. In this Letter, we propose a minimal model able to explain a non-superconducting resistivity drop in strongly correlated electron systems at low temperatures.

The main model component is the band-crossing point around which the electrons are paired, Fig. 1(a). The well-known massless Dirac fermions (MDFs) could be seen as an example of such a band structure. The electron-electron correlations of MDFs are mostly studied around which the electrons are paired, Fig. 1(a). The wave vector \( \mathbf{k} \equiv -\mathbf{k} \). The strongly correlated electron systems at low temperatures may occur in twisted multilayer graphene structures, which are experimentally available now.

Even though our pairing formally involves two bands, it relies solely on electron-electron BCS-like coupling and does not require a previous excitation creating the hole states, as it would do in the case of excitonic coupling. In this Letter, we show that our pairing does not lead to superconductivity but instead “freezes” the upper layer of the Fermi sea, see Fig. 1(b). The resulting correlated ground state can be written as

\[
|G\rangle_Q = \prod_{k > k_0} \left( \frac{1}{\sqrt{2}} \left(1 + c_{\mathbf{k} \uparrow} c_{-\mathbf{k} \downarrow}^\dagger \right) \right)
\times \prod_{k \leq k_0} \left( \frac{1}{\sqrt{2}} \left(1 + c_{\mathbf{k} \uparrow} c_{-\mathbf{k} \downarrow}^\dagger \right) \right)
\times \left( \frac{1}{\sqrt{2}} \left(1 + c_{\mathbf{k} \downarrow} c_{-\mathbf{k} \uparrow}^\dagger \right) \right) |0\rangle,
\]

where \(|0\rangle\) denotes the electron vacuum, \(c_{\mathbf{k} \sigma}^\dagger\) is the electronic creation operator for a given band index \(\kappa = \{\pm\}\), spin \(\sigma = \{\uparrow, \downarrow\}\), and wave vector \(\mathbf{k} (|k| \equiv k)\) with \(k \equiv -k\). The wave vector \(k_0 = \Delta_0/(\hbar v)\) (with the order parameter \(\Delta_0\), Dirac velocity \(v\), and Planck constant \(\hbar\)) separates the “frozen” and normal electrons in \(k\)-space and plays the same role as the inverse coherence length in the BCS state [42]. In contrast to BCS, however, the order parameter \(\Delta_0\) is not equivalent to the gap, which is zero in our case. Moreover, a certain minimal interaction strength is needed to “freeze” electrons. Most importantly, the “frozen” electrons turn out to be protected.
FIG. 1. (a) The normal ground state, $|G\rangle$. Unconventional pairing occurs between time-reversed states having antiparallel momenta and spins (yellow arrows). The energy window $\hbar\omega_c$ is determined by the pairing mechanism (phonon- or plasmon-assisted). The blue gradient illustrates electron population at finite temperature. The crossed bands are assumed to have different orbital characters (red arrows) that precludes conventional (same-energy) pairing. (b) The correlated ground state, $|G\rangle_Q$, splits into two sectors with $k \leq k_0$ and $k > k_0$. The former has a BCS-like structure, whereas the latter represents normal electrons, and the two sectors together can be seen as a “frozen” Fermi sea with “Fermi ice” floating on Fermi liquid. The thickness of the “frozen” layer is $2\Delta_0$ with $\Delta_0$ being the self-consistent order parameter. (c) The correlated ground state and excitations in terms of the quasiparticle operators. The $\gamma_{1k}$-excitations are gapped by $\Delta_0$ even though the overall spectrum is gapless.

against any elastic scattering by the symmetry of the Bogoliubov transformation. The resulting correlated state may appear as highly conducting but not superconducting at low enough temperatures when electron-phonon scattering is suppressed and the electron transport is normally limited by disorder. Unlike a superconductor, we expect the “Fermi ice” will have no unusual signatures in spectroscopy measurements.

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The model Hamiltonian. — From the mean-field perspective the particular choice of the pairing mechanism is not of principle importance (so the pairing potential could be generated by either electron-phonon coupling with the Debye frequency defining the interaction energy window [28] or electron-electron interactions with the plasmon frequency playing the cut-off role [43]). Our unconventional pairing assumes that the kinetic energy of an upper-band electron increases with momentum, whereas it decreases for the lower one — the case of X-shaped crossing, see Fig. 1(a). Including this feature into the model BCS-like Hamiltonian we have $H = H_{\uparrow\downarrow} \oplus H_{\downarrow\uparrow}$, where $H_{\sigma\sigma'}$ is given by [44]

$$H_{\sigma\sigma'} = \sum_k \left( \epsilon_k c_{k\sigma}^\dagger c_{-k\sigma} - \epsilon_k c_{-k\sigma'}^\dagger c_{k\sigma'} \right) + \sum_{kk'} V_{kk'} c_{k\sigma}^\dagger c_{k'\sigma'} c_{-k'\sigma'} + \epsilon_{-k'\sigma}.$$  \hspace{1cm} (2)

Here, $\epsilon_k = \hbar v k \geq 0$ is the linear dispersion, and $V_{kk'}$ is the Fourier transform of the pairing potential. Introducing the two terms $H_{\uparrow\downarrow}$ and $H_{\downarrow\uparrow}$ is a formal way to avoid unintentional spin polarization of the bands. The two Hamiltonians $H_{\uparrow\downarrow}$ and $H_{\downarrow\uparrow}$ have the same structure, and all the properties deduced for $H_{\uparrow\downarrow}$ automatically apply to $H_{\downarrow\uparrow}$.

The Hamiltonian (2) does not involve the dimensionality of the wave vector $k$ explicitly and can be used regardless. However, the interaction term is reduced [44], as it includes only pair interactions between collinear electrons moving in opposite directions making pairing one-dimensional in that regard [43]. Since the right and left moving electrons do not interact within a given band, they may be belong to the different valleys or have different orbital characters. The nature of $V_{kk'}$ is not specified at this point, but we later show that the pairing must be plasmon-assisted to overcome the critical interaction threshold.

The mean-field Hamiltonian. — The mean-field approximation with the order parameter

$$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$  \hspace{1cm} (3)
results in the following Hamiltonian

\[ H_{1\uparrow}^{MF} = \sum_{k} \left( c_{k+\uparrow}^\dagger c_{\uparrow} - \epsilon_k \right) \left( c_{-k-\downarrow}^\dagger c_{\downarrow} - \Delta_k \right) + \sum_{k} \left( \Delta_k \left( c_{-k-\downarrow} c_{k+\uparrow} - \epsilon_k \right) \right), \]  

and \[ H_{1\downarrow}^{MF} \] can be obtained by swapping the spin indexes in equation (4). Using the Cooper’s assumption (12), \[ V_{kk'} = -V_0 \epsilon_k - \epsilon_{k'} \leq \hbar\omega_c/2 \] and \[ V_{kk'} = 0 \] otherwise, we obtain the order parameter given by

\[ \Delta_0 e^{i\delta_0} = V_0 \sum_{\epsilon_k \leq \omega_c/2} \langle c_{-k-\downarrow} c_{k+\uparrow} \rangle, \]  

Here, \( V_0, \Delta_0, \) and \( \delta_0 \) are the real and positive numbers independent of \( k \). Diagonalizing and transforming \[ \gamma \] into the canonical form we obtain

\[ H_{1\uparrow}^{MF} = \sum_{k} \Delta_0 \left( 1 + e^{-i\delta_0} c_{-k-\downarrow} c_{k+\uparrow} \right) + \sum_{k} \left[ (\epsilon_k - \Delta_0) \gamma_{0k}^\dagger \gamma_{0k} + (\epsilon_k - \Delta_0) \gamma_{1k}^\dagger \gamma_{1k} \right]. \]  

Here, \( \gamma_{0,1k} \) and \( \gamma_{0,1k}^\dagger \) are the quasiparticle creation and annihilation operators related to the original electron operators via Bogolubov transformation given by

\[ \begin{pmatrix} c_{k+\uparrow}^\dagger \\ c_{-k-\downarrow}^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\delta_0} \\ 1 & -e^{-i\delta_0} \end{pmatrix} \begin{pmatrix} \gamma_{0k}^\dagger \\ \gamma_{1k}^\dagger \end{pmatrix}, \]  

and

\[ \langle c_{-k-\downarrow} c_{k+\uparrow} \rangle = \frac{1}{2} e^{i\delta_0} \left( \langle \gamma_{0k}^\dagger \gamma_{0k} \rangle + \langle \gamma_{1k}^\dagger \gamma_{1k} \rangle - 1 \right) \]  

is the ground-state average with \( \langle \gamma_{0k}^\dagger \gamma_{0k} \rangle = \langle \gamma_{1k}^\dagger \gamma_{1k} \rangle = 0 \), \( \langle \gamma_{0,1k}^\dagger \gamma_{0,1k} \rangle = n_F(\epsilon_k - \Delta_0) \), where \( n_F(\epsilon_k) \) is the Fermi-Dirac distribution. The order parameter can be found from equations (5) and (8) that results in

\[ \Delta_0 = V_0 \sum_{\epsilon_k \leq \omega_c/2} \left[ n_F(\epsilon_k - \Delta_0) - n_F(\epsilon_k + \Delta_0) \right] \]  

and gives the order parameter function \( \Delta_0(T) \). As the phase \( \delta_0 = 0 \) does not enter any physical quantity we set it to zero in our further analysis. The Hamiltonian \( H_{1\uparrow}^{MF} \) is transformed in the same way by introducing a set of similar quasiparticle operators \( \beta_{0,1k}^\dagger \) and \( \beta_{0,1k} \).

**Basic properties.** — One can prove directly that |\( G \rangle_Q \) given by Eq. (1) is the ground state of \( H_{1\uparrow}^{MF} \). Acting with either of \( \gamma_{0k}^\dagger \) \( (k > k_0) \), \( \gamma_{0k} \) \( (k \leq k_0) \), \( \gamma_{1k} \) on \( |G\rangle_Q \) creates an excited state with the positive excitation energies indicated in Fig. 1(c). Doing the same with either of \( \gamma_{0k} \) \( (k > k_0) \), \( \gamma_{0k}^\dagger \) \( (k \leq k_0) \), \( \gamma_{1k}^\dagger \) on \( |G\rangle_Q \) annhilates the ground state. The same is true for \( H_{1\downarrow}^{MF} \) with \( \beta_{0,1k} \) and \( \beta_{0,1k}^\dagger \).

Note that the Bogolubov transformation (7) depends on neither \( \epsilon_k \) nor \( \Delta_0 \). The spectrum remains gapless and the quasiparticle density of states is the same as in the non-interacting case, regardless the order parameter value. This is in strong contrast to the conventional superconducting pairing (12).

Solving Eq. (9) explicitly involves a density of states (DOS) that, in turn, depends on dimensionality of the problem. In a two-dimensional case, Eq. (9) takes the following form

\[ \frac{2\Delta_0}{\hbar \omega_c} = \frac{V_0^2}{8\pi^2 v^2} \int_0^1 dx \left[ n_F \left( x \hbar \omega_c - \Delta_0 \right) - n_F \left( x \hbar \omega_c \right) \right]. \]  

At \( T = 0 \), we have \( \Delta_0 = V_0^2/(32\pi v^2) \), if \( \Delta_0 > \hbar \omega_c/2 \), and \( \Delta_0 = 8\pi^2 v^2 / V_0 \), if \( \Delta_0 \leq \hbar \omega_c/2 \). Most importantly, there is no solution at all as long as \( V_0 < 16\pi v^2 / \hbar \omega_c \). The critical \( V_0 \) also exists in the case of conventional BCS pairing near the band-crossing (13). Such a conventional \( V_0 \) occurs because DOS of MDF’s vanishes at the band-crossing point in a two-dimensional case. We emphasize that the criticality is intrinsic to our model and exists even if DOS is a constant. To show that we consider a one-dimensional case with Eq. (9) explicitly written as

\[ \frac{2\Delta_0}{\hbar \omega_c} = \frac{r_s}{\pi} \int_0^1 dx \left[ n_F \left( x \hbar \omega_c - \Delta_0 \right) - n_F \left( x \hbar \omega_c + \Delta_0 \right) \right] \]  

with \( r_s = V_0/(\hbar v) \). Inset of Figure 2 shows its solution for different \( r_s \). The critical temperature, \( T_c \), can be estimated by taking the limit \( \Delta_0 \rightarrow 0 \), and the equation reads \( r_s = \pi \tan\left(\hbar \omega_c / (4T_c)\right) \). Obviously, \( \tan \) cannot be larger than 1, hence, \( T_c \) does not exist as long
as \( r_s < \pi \). The order parameter at \( T = 0 \) is given by \( \Delta_0 = \hbar c_0 r_s / (2\pi) \) and is always larger than \( \hbar \omega_c / 2 \). Hence, our correlated state can exist if and only if the interaction parameter is larger than a certain critical value. Using the same parametrization in the case of conventional pairing one can show that the correlated (superconducting) state exists at any \( r_s \) as long as \( T \) is low enough [40].

We compare the thermodynamic potential of our correlated state, \( \Omega_Q \), with that of a normal state, \( \Omega_N \), to make sure that the former has lower energy than the latter, and the electrons condense into that special state. We start from the partition function for a given spin channel \( Z = \text{Tr} [e^{-\mathcal{H}_F^{MF}/T}] \), which in our case explicitly reads

\[
Z = \prod_k \left[ \left( 1 + e^{-\frac{\epsilon_k - \Delta_0}{\gamma}} \right) \left( 1 + e^{-\frac{\epsilon_k + \Delta_0}{\gamma}} \right) \right]
\times e^{-\frac{\gamma}{\hbar c_0}} \sum_k \left[ n_F(\epsilon_k - \Delta_0) - n_F(\epsilon_k + \Delta_0) + 2 \right].
\]

We calculate \( \Delta_0 = -T \ln Z \), and \( \Delta_N \) is given by the same formula with \( \Delta_0 = 0 \). The difference \( \Delta_0 = \Omega_Q - \Omega_N \) is shown in Fig. 2 and becomes negative below a critical temperature. In the limit of \( T = 0 \), \( \Delta_0 = \hbar c_0^2 (1 - r_s / \pi) / (4\pi \gamma) \).

In two-dimensional version of the model, Eq. (10) suggests two critical temperatures, \( T_c \) and \( T'_c \), but only the former has a physical meaning [46]. The higher critical temperature \( T_c \) indeed describes transition into a correlated state with lower energy. This is unlike the \( T'_c \) where no actual transition occurs [46].

Construction of the correlated state. — The proposed correlated state \( \mid \Omega \rangle \) is constructed in two steps. First, we create an empty state, \( \mid 0 \rangle_Q \). The new empty and ground states are not equivalent, \( \mid 0 \rangle_Q \neq \mid G \rangle_Q \). The situation is similar to the normal state, when \( \mid 0 \rangle \neq \mid G \rangle \). The empty state can be formally obtained by means of the conventional BCS transformation [42] by setting the mean-field order parameter to infinity and shifting \( \delta_0 \) by \( \pi \). The coherences then become equal \( \langle u_k = v_k = 1/\sqrt{2} \rangle [42] \), and our empty state can be written as

\[
\mid 0 \rangle_Q = \prod_k \frac{1}{\sqrt{2}} \left( 1 - c^\dagger_{+k} c^\dagger_{-k} \right) \frac{1}{\sqrt{2}} \left( 1 - c^\dagger_{+k} c^\dagger_{-k} \right) \mid 0 \rangle.
\]

Second, we fill the empty state \( \mid 0 \rangle_Q \) with quasiparticles up to the Fermi level (the cut-off momentum \( k_0 = \Delta_0 / (\hbar \nu) \)), see Fig. 1. Using our Bogolubov transformation [7] the action of \( \gamma^\dagger_{0k} \) on \( \mid 0 \rangle_Q \) can formally be written as

\[
\gamma^\dagger_{0k} \frac{1}{\sqrt{2}} \left( 1 - c^\dagger_{+k} c^\dagger_{-k} \right) \mid 0 \rangle = c^\dagger_{+k} \mid 0 \rangle,
\]

i.e. we break down one pair and create one free electron. The operator \( \gamma_{1k} \) acts in a similar way but creates an electron in the opposite band having the opposite momentum and spin \( (c^\dagger_{-k}, \text{instead of } c^\dagger_{+k}) \). The quasiparticle creation operators \( \gamma_{0,1k} \) do the same with the pairs having swapped spins. It is however instructive to fill the quasiparticle states also in pairs, i.e. by creating \( \gamma_{1k} \gamma^\dagger_{0k} \mid 0 \rangle_Q \) at once. Indeed, we find that

\[
\gamma^\dagger_{1k} \gamma^\dagger_{0k} \frac{1}{\sqrt{2}} \left( 1 - c^\dagger_{+k} c^\dagger_{-k} \right) \mid 0 \rangle = \frac{1}{\sqrt{2}} \left( 1 + c^\dagger_{+k} c^\dagger_{-k} \right) \mid 0 \rangle,
\]

i.e. creating a quasiparticle pair \( \gamma^\dagger_{1k} \gamma^\dagger_{0k} \mid 0 \rangle_Q \) changes the sign in front of \( c^\dagger_{+k} c^\dagger_{-k} \) but the structure of the state holds. Note that the BCS state does not have such a property because the operator \( \gamma^\dagger_{1k} \) swaps the coherence factors \( u_k \) and \( v_k \) that leads to gradual depairing upon further filling. In our case, the Bogolubov transformation does not depend on energy, which makes it possible to fill the states at any energy without destroying the paired structure. We have to make sure, however, that any quasiparticle created in the lower band \( \gamma_{1k} \) is accompanied by another one created in the upper band \( \gamma_{0k} \). Those quasiparticles having no companion in the upper band form a conventional unpaired electron sea. The paired quasiparticles form the “frozen” layer.

Manifestations. — In the conventional superconducting BCS model, the operators \( \gamma_{0,1k} \) acting on the ground state change the average particle number by a factor \( u_k^2 - v_k^2 \), which can be seen as a signature of the BCS

![FIG. 3.](image-url) (a) Calculated band structure of tDBG with the twist angle \( \theta = 1.33^\circ \) and without interlayer potential difference. The black (solid) and red (dashed) bands have different orbital characters. (b) The X-shaped band-crossing along the \( M_1 - \Gamma \) direction with the Fermi level corresponding to the filling factor \( \nu = 1.75 \).

This particular crossing point dominates as having the highest DOS. (c) The electron velocity in units of \( 10^7 \text{ m/s} \) extracted from panel (b) and interaction parameter \( r_s \) with \( \bar{\varepsilon} = 6.85 \) relevant for h-BN [47, 48]. (d) The phase diagram indicating the regions of correlated and normal electronic states. The critical temperature at \( \theta = 1.33^\circ \) is about \( \hbar \omega_c / 2 \). Assuming \( \hbar \omega_c = 1.75 \text{ meV} \) we arrive at \( T_c = 10 \text{ K} \), which is in good agreement with the temperature reported in Ref. [34] at which the drop in resistivity occurs.
superconducting state \[^{[43]}\]. The factor obviously vanishes in our case \((u_k = u_k' = 1/\sqrt{2})\) indicating that the state \(|\Psi\rangle\) is not a conventional BCS state. The resulting quasiparticle spectrum is gapless, and we are not able to distinguish between our normal and correlated states spectroscopically.

Nevertheless, electron pairing may influence electron scattering and, as a consequence, electrical conductivity. Let us consider the non-magnetic disorder Hamiltonian written in terms of the quasiparticle operators according to the Bogolubov transformation as

\[
H_{\text{dis}} = \sum_{k, k', \gamma \kappa} U_{kk'} \left( c_{\gamma k' \uparrow}^\dagger c_{\kappa k \uparrow} + c_{\gamma k' \downarrow}^\dagger c_{\kappa k \downarrow} \right)
\]

\[
= \sum_{k, k'} U_{kk'} \left( \gamma_{0k' \uparrow}^\dagger \gamma_{1k \uparrow} + \gamma_{1k' \uparrow}^\dagger \gamma_{0k \uparrow} + \beta_{0k' \downarrow}^\dagger \beta_{1k \downarrow} + \beta_{1k' \downarrow}^\dagger \beta_{0k \downarrow}
+ \beta_{0k' \uparrow}^\dagger \beta_{1k \uparrow} + \beta_{1k' \uparrow}^\dagger \beta_{0k \uparrow} + \gamma_{1k' \downarrow}^\dagger \gamma_{0k \downarrow} + \gamma_{0k' \downarrow}^\dagger \gamma_{1k \downarrow} \right). \tag{13}
\]

Here, \(U_{kk'}\) is the matrix element of a smooth potential unable to induce intervalley scattering. The low-energy excitations are created by quasiparticle operators of “0” type, see Fig. 1(c). However, all terms in Eq. 13 contain one quasiparticle operator of type “1”. Hence, calculating the low-energy scattering matrix elements we obtain an expectation value of a quartic product in which three quasiparticle operators are always of “0” type and one operator of type “1”. The expectation value must necessarily vanish, since the numbers of “0”-type and “1”-type operators are not balanced. The scattering becomes only possible when it is accompanied by substantial electron energy change larger than \(\Delta_{0} \[^{[10]}\]. In fact, the low-energy quasiparticle states are protected from scattering by the symmetry of the Bogolubov transformation having equal coherences. The symmetry can be broken by shifting the Fermi level away from the band-crossing but the effect is negligible as long as the shift is much less than \(\hbar \omega_{c} \[^{[10]}\].

Our estimates of the pairing potential suggest that the electron-phonon coupling would be too weak to form a “frozen” state. Indeed, the typical value for the product between electronic DOS and \(V_{0}\) lies between 0.2 and 0.4 in conventional superconductors \[^{[42]}\], whereas in our case the critical \(r_{s}\) is one order of magnitude higher. It is however possible to satisfy the critical condition using a plasmon-assisted pairing with \(V_{0} \sim e^{2}/\bar{\epsilon}\) with \(\bar{\epsilon}\) being the effective dielectric constant. The coupling constant \(r_{s} = e^{2}/(\hbar \omega_{p})\) should then be seen as a rough characteristic of the Coulomb interaction strength responsible for the plasmon-assisted pairing. The true plasmon-mediated pairing is described by the frequency-dependent Eliashberg gap equations, as shown in our previous paper \[^{[43]}\].

The electron transport measurements \[^{[34]}\] clearly demonstrate a resistivity drop of twisted double bilayer graphene (tDBG) with twist angle \(\theta = 1.3^{\circ}\) and filling factor \(\nu = 1.7\) as temperature is lowered below 10 K. Figure 3(a,b) shows the relevant band structure with an X-shaped band crossing along the \(M_{1} \rightarrow \Gamma\) direction. The scattering bands have opposite orbital characters, which supposedly preclude conventional pairing. The band-crossing intersects with the Fermi level at \(\theta = 1.33^{\circ}\) and \(\nu = 1.75 \[^{[51]}\]. \[^{[50]}\] The filling factor \(\nu = 4\) corresponds to a fully occupied band. There is no on-site energy difference between the layers \[^{[46]}\]. Figure 3(c,d) shows \(r_{s}\) as a function of twist angle, and the phase diagram with normal and correlated states. If the temperature drops below \(T_{c}\) and the electrons transition to our correlated state, then the upper layer of the Fermi sea turns into “ice”, and all the elastic scattering processes get turned off including charged impurities and other smooth defects. One is only left with magnetic impurities, if any, and inelastic scatterers, like phonons, which are much weaker at low temperatures. This is what might take place in tDBG at \(T < T_{c}\) and manifest itself in the resistivity measurements \[^{[34]}\].

Conclusions. — The only two crucial ingredients in this theory are (i) X-shaped crossing electronic bands and (ii) strong electron-electron pairing across the band intersection. The flatter the crossing bands are, the lower the critical interaction strength is. As discussed in \[^{[46]}\], we have found that this correlated state is stable against perturbations including: (i) shift in Fermi energy level within the energy window \(\hbar \omega_{p}\); (ii) dimensionality of the problem; (iii) change in the specific dispersion at the band crossing. As flat-band touching is generic in twisted graphene systems, the connection between the highly conductive states observed in Refs. \[^{[34]}\] and our model is suggestive.

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[46] See following Supplemental Material will be available upon journal publication: explicit expressions describing the Bogolubov transformation, excited quasiparticle states, quasiparticle scattering matrix elements, electron scattering matrix elements, as well as equations describing a two-dimensional version of the model, shifted Fermi level, conventional pairing, and the continuum model of twisted double bilayer graphene.

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