Theoretical analysis of averaging methods for intermodal fiber interferometer

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Abstract. Theoretical expressions describing the signal of an intermodal fiber interferometer are presented. Two methods of signal averaging are investigated theoretically: the ensemble averaging and the averaging over a “long realization”. The methods are examined by conducting numerical experiments. The averaging efficiency is analyzed depending on number of realizations and the fiber length increment.

1. Introduction

Fiber-optic sensing based on intermodal fiber interferometers (IFI) is a relevant research topic. A variety of sensing solutions in such areas as vibration [1], security [2, 3], biomedical sensing [4, 5] and others [6-8] are proposed in recent times. The basic principle of IFI’s operation is associated with the analysis of an interferometric signal, formed as a speckle pattern at the output multimode fiber (MMF) facet [9]. A speckle-pattern is sensitive to external fiber perturbations, as they cause changes in phase differences of propagating modes.

IFI is an attractive sensing scheme due to its simplicity and low cost, however its practical application is complicated by statistical nature of IFI signals [9]. To obtain stable IFI transfer function and response to external perturbations, a various of signal processing methods, mostly related to correlation [10, 11] or averaging approaches [12, 13], are usually applied. The latter one is a convenient method for IFI transfer function calculation, which is referred to as the method of averaged Amplitude Characteristics [14]. Considering IFI signal averaging, a several methods can be utilized: ensemble averaging, averaging over “long realization” and averaging over lateral coordinates [15].

In this paper, we theoretically consider ensemble averaging and the averaging over “long realization” methods. Theoretical expressions describing the intermodal fiber interferometer signals averaged by these methods are analyzed depending on realizations’ number for ensemble averaging and on signal length for “long realization” averaging method. The investigation is based on the method of averaged Amplitude Characteristics, proposed in [14].

2. Theoretical model

The schematic of an IFI is presented in figure 1. The laser excites a certain distribution of fiber modes, which propagate through a multimode fiber forming an interference speckle pattern at the fiber output. Propagating modes obtain different phase shifts caused by an external perturbation of the fiber in accordance with different propagation constants that leads to the transformation of interference spots in the speckle pattern. An array of photodetectors records speckle pattern evolution over time. Each
photodetector records a spatially filtered interferometric signal, which contains the information about external fiber perturbation. Herewith, the overall light intensity at the fiber output remains constant.

Figure 1. Schematic of an intermodal fiber interferometer.

The light intensity at a certain point of the fiber facet can be described as [14]:

\[ I(r, \phi, M, L) = \frac{1}{M} \sum_{k=1}^{M} A_i^2 (E_i(r, \phi))^2 + \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1}^{M} A_i A_k E_i(r, \phi) E_k(r, \phi) \cos[(\beta_i - \beta_k)L + (\phi_i - \phi_k)] \]  

(1)

where \( \beta_i \) is the propagating constant of the \( i \)-th mode; \( \phi_i \) is the random phase shift of the \( i \)-th mode, caused by stationary longitudinal fiber inhomogeneities and mode coupling as a result of them; \( A_i \) and \( E_i(r, \phi) \) are mode amplitude and mode function of the \( i \)-th mode respectively, \( M \) is the number of modes excited in the fiber; \( L = L_0 + \delta L \), \( L_0 \) is fiber length, and \( \delta L \) is the fiber length increment (deterministic value) as a result of external impact. The first term in equation (1) is a constant component and the second term is an interferometric component. Division by \( M \) enables overall light intensity to remain constant under variation of the number of propagating modes provided that excited modes have equal intensity. Modes can be replaced by mode groups in the case of parabolic-index MMF due to modes’ propagating constant approximate equality within a mode group. Examples of the IFI signal caused by fiber elongation, calculated using equation (1) for different number of mode groups in parabolic-index MMF are showed in figure 2. Calculated plots illustrate the common property of IFI: signal parameters significantly depend on the operating point, determined by random phase shifts \( \phi_i \) and fiber length \( L \) in equation (1).

Figure 2. Simulated IFI signal for different number of mode groups: \( M=2, 8, 16 \).

A convenient way to analyze IFI properties depending on its parameters is demonstrated [14], and referred to as the method of averaged amplitude characteristics. The calculation of the Amplitude Characteristic requires the computation of standard deviation of the interferometric signal for a range of deterministic fiber elongations \( \delta L \) (\( \delta L_{\text{max}} \approx 1.1 \text{ mm} \) as a beat length of adjacent mode groups in a parabolic MMF), and then its normalization to the maximum value [14]:

\[ ACh = \frac{\sigma(\delta L)}{\sigma(\delta L_{\text{max}})} \]  

(2)

The standard deviation of the IFI signal for a given perturbation \( \delta L \) can be expressed as

\[ \sigma(r, \phi, M, L, \delta L) = \sqrt{D(r, \phi, M, L, \delta L)} \]  

(3)

where

\[ D(r, \phi, M, L, \delta L) = I^2(r, \phi, M, L, \delta L) - \overline{I(r, \phi, M, L, \delta L)^2} \]  

(4)
\[
I(r, \phi, M, L, \delta L) = \frac{1}{\delta L} \int_{-L_o}^{L_o} I(r, \phi, M, L) dL
\]

\[
I^2(r, \phi, M, L, \delta L) = \frac{1}{\delta L} \int_{-L_o}^{L_o} I^2(r, \phi, M, L) dL
\]  \hspace{1cm} (5)

Using Eq. (1), (4), (5), dispersion \( D(r, \phi, M, L, \delta L) \) can be expressed as

\[
D(r, \phi, M, L_0, \delta L) = \sum_{i=1}^{N} A_{ik}^2 E_i^2(r, \phi) \left( \frac{1 + \frac{1}{2} \cos[\Delta \beta_i (\delta L)]}{M^2 - M} \right)
\]

\[
\times \left( \frac{\sin(\Delta \beta_i \delta L)}{\Delta \beta_i \delta L} \right) - \left( \cos^2 \frac{0.5 \Delta \beta_i (\delta L)}{0.5 \Delta \beta_i \delta L} \right)
\]

\[
\times \left( \frac{\sin^2 \left(0.5 \Delta \beta_i (\delta L)\right)}{\left(0.5 \Delta \beta_i \delta L\right)^2} \right)
\]  \hspace{1cm} (6)

where \( \Delta \beta_i = \beta_i - \beta_i \), \( \Delta \phi_i = \phi_i - \phi_i \),

\[
\beta_i = \frac{2 \pi n}{\lambda} \left[ 1 - 2 \Delta \left( \frac{k}{M_{\text{max}}} \right)^{2 a} \right]^{\frac{1}{2}},
\]  \hspace{1cm} (7)

where \( n \) is the core refractive index, \( \lambda \) is a wavelength, \( \Delta \) is relative core-cladding refractive index difference, \( a \) is profile parameter, \( M_{\text{max}} \) is the maximum number of mode groups possible to propagate in the fiber. \( M_{\text{max}} \) can be calculated as [16]

\[
M_{\text{max}} = \frac{V^2 \cdot \alpha}{\sqrt{2(\alpha + 2)}},
\]  \hspace{1cm} (8)

where

\[
v = \frac{\pi \cdot d \cdot n \sqrt{2 \Delta}}{\lambda},
\]  \hspace{1cm} (9)

\( d \) is a core diameter.

Equation (3) and (6) enables to obtain non-averaged Amplitude Characteristic based on equation (2). With this, the arguments of cosines in equation (6) determine the operating point in the absence of external fiber perturbations. Over different measurements, random \( \Delta \phi_i \) results in Amplitude Characteristic’s and IFI’s response to external perturbation instability.

Considering equation (6), two methods of signal averaging are obvious, and related to the argument of cosine: averaging over a random \( \Delta \phi_i \) at fixed \( L_0 \) and averaging over a fiber length \( L_0 \) at fixed \( \Delta \phi_i \). The first method is, in fact, ensemble averaging. In practice, it can be achieved by multiple measurement of IFI’s response to the same external perturbation, randomly shifting some fiber segment from one stable position to another for each measurement.

In the second averaging approach, the fiber position is fixed, saving \( \Delta \phi_i \) unchanged, but \( L_0 \) is randomly varied for every measurement. A specific modification of this method can be attractive for the Amplitude Characteristic measurement. Here, the IFI signal is recording when \( L_0 \) is linearly changing from initial value \( L_0 \) to some value \( L_0 + \Delta L_0 \) in the absence of any additional fiber perturbations. In such a case, segments of lengths \( \delta L < \Delta L_0 \) at the recorded signal can be considered as IFI signals, caused by fiber elongations \( \delta L \), and can be averaged over their different locations at the entire recorded signal, corresponding to \( \Delta L_0 \). Thus, performing such an averaging procedure for signal segments, corresponding to a complete set of \( \delta L \), Amplitude Characteristic can be calculated. Herein, fiber length increment \( \Delta L_0 \) should be large enough for providing effective averaging for all values \( \delta L \) of the Amplitude Characteristic. In [15], we refer to the described method as averaging over a “long realization”. In fact, it can be considered as some analogue of well-known time averaging. In contrast to ensemble averaging, this technique does not require multiple records, what can be convenient in practice.

In calculations, we used the following fiber parameters: fiber length \( L_0 = 500 \) m, core diameter \( d=62.5 \) μm, profile parameter \( a=2 \), relative core-cladding refractive index difference \( \Delta =0.017 \), core refractive index \( n=1.48 \), and wavelength \( \lambda=1.3 \) μm. For these parameters, maximum number of
propagating mode groups was calculated to be 20 mode groups. For simplicity, we assumed that excited mode groups have equal amplitudes ($A_1=A_2=\ldots=A_M$), and we utilized equations for Hermite-Gauss modes as mode functions of mode groups [17].

3. Ensemble averaging
Performing averaging over random initial phases $\phi_i$ (or their differences ($\phi_i-\phi_k$)), uniformly distributed over the interval $[-\pi, \pi]$, equation (6) can be converted to the following form:

$$D(M, r, \phi, \delta L) = \frac{1}{2} \sum_{i=1}^{M} \sum_{l=1}^{2} a_i^l a_i^l E_i^2(r, \phi) E_i^2(r, \phi) \left[ 1 - \frac{\sin^2(0.5\Delta \beta_i \delta L)}{(0.5\Delta \beta_i \delta L)^2} \right]$$

(10)

Examples of non-averaged (random) normalized Amplitude Characteristics and corresponding averaged Amplitude Characteristics for three cases of excited mode groups are presented in figure 3. Calculated plots demonstrate the possibility to obtain typical averaged IFI Amplitude Characteristics containing linear and saturation parts. The slope of the linear part is determined by the number of excited mode groups $M$.

**Figure 3.** Averaged over ensemble Amplitude Characteristics for 3 cases of excited mode groups and examples of corresponding not averaged Amplitude Characteristics.

An important practical question is related to the dependence of measured signal deviation from the true value on the number of realizations in the ensemble. To investigate this, we repeatedly calculated the value of averaged Amplitude Characteristic for a certain fiber perturbation ($\delta L=10 \mu m$) corresponding to initial linear part of the Amplitude Characteristic for different number of ensemble realizations. The value of averaged Amplitude Characteristic, calculated by the equation (10) for a fiber perturbation $\delta L=10 \mu m$, was taken as a true value $x_{true}$. Then, we calculated the normalized standard deviation of averaged Amplitude Characteristic’s values $x_n$ with respect to the true value $x_{true}$ according to the formula

$$\sigma_n = \frac{1}{x_{true}} \sqrt{\frac{\sum_{j=1}^{m} (x_n - x_{true})^2}{m-1}}$$

(11)

where $m$ is the number of experiments for the measurement of the Amplitude Characteristic’s value $x_n$, averaged over $n$ ensemble realizations (we used $m=100$ in calculations). Figure 4 demonstrates the dependency of standard deviation $\sigma_n$ on the number of realizations in the ensemble $n$. The dependency has a shape of $1/\sqrt{n}$ that agrees with the theory of random variables averaging [18]. The results also show that the $\sigma_n$ value decreases with an increase in the number of excited modes in an optical fiber.
4. Averaging over long realization

The “long realization” averaging process can be described as averaging the dispersion $D$ (equation (6)) over increment $\Delta L_0$ of fiber length $L_0$:

$$D_{LR}(M, r, \phi, L_0, \Delta L_0, \partial L) = \frac{1}{\Delta L_0} \int D(M, r, \phi, L_0, \partial L) dL$$

(12)

Substituting equation (6) to equation (11) we obtained the following expression for the averaged over “long realization” $D_{LR}$:

$$D_{LR}(M, r, \phi, L_0, \Delta L_0, \partial L) = \sum_{x=1}^{M} \sum_{i} a_i^e E_i^j(r, \phi) E_i^j(r, \phi) \left[ \frac{1}{2} \cos(\Delta \beta_i (2L_0 + \Delta L_0 + \partial L)) \sin(\Delta \beta_i (\Delta L_0)) - \frac{1}{2} \cos(\Delta \beta_i (2L_0 + \Delta L_0 + \partial L) + 2\Delta \phi_i) \sin(\Delta \beta_i (\Delta L_0)) \right]$$

(13)

The latter expression was utilized in the numeral experiments for the “long realization” averaging. Amplitude characteristics, averaged over “long realization” ($\Delta L_0=1\text{mm}$), for different number of mode groups are presented in figure 5. Calculated dependences confirm the efficiency of the “long realization” averaging method.

![Figure 5](image-url) **Figure 5** Averaged over “long realization” Amplitude Characteristics for 3 cases of excited mode groups.

![Figure 6](image-url) **Figure 6.** Dependence of averaged normalized measurement error ($\sigma_{AL}$ in equation (14)) on the number of ensemble realization.

Similarly to ensemble averaging, we analyzed averaging efficiency depending on the value of $\Delta L_0$. As the true value $x_{true}$, we utilized the value of the Amplitude Characteristic, averaged over $\Delta L_0=10\text{mm}$ ($\delta L=10 \mu m$). Equation (11) in this case can be represented as

$$\sigma_{AL} = \frac{1}{x_{true}} \sum_{j=1}^{m} (x_{AL, j} - x_{true})^2$$

(14)

We used the same value $m=100$ in calculations, as in the ensemble averaging case. Obtained results, however, significantly differ from those obtained for the ensemble averaging (figure 6). The dependences has a shape of decreasing curve with the presence of zero values. The shape of $1/\sqrt{n}$ is not observed. Similar to the case of ensemble averaging, the results depend on the number of excited mode groups: a greater number of excited mode groups improves the averaging efficiency. It should be emphasized, that calculations demonstrate the measurement error striving for zero significantly earlier than expected (we expected sufficient $\Delta L_0$ to be around 1 mm, as a beat length of adjacent mode groups), especially for large number of excited mode groups.
5. Conclusion
In this paper, we theoretically considered two methods for IFI signals averaging: the ensemble averaging and the averaging over “long realization”. Both methods were confirmed to be effective methods enabling to obtain averaged Amplitude Characteristic. The methods demonstrated some features. Ensemble averaging demonstrated the decrease of measurement error as a function of square root of the ensemble realizations number. The dependency of measurement error on the number of realizations (for ensemble averaging) and on fiber length increment (for the “long realization”) depend on the number of excited mode groups, and for the long realization averaging method this dependency has more complicated shape in the contrary to ensemble averaging.

The results confirm that both averaging methods can be used in practice. Calculated error dependences on the number of realizations and fiber length increment can be useful for planning of experiments with the intermodal fiber interferometers.

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