Maximum Entropy, the Collective Welfare Principle and the Globalization Process

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Although both systems analyzed are described through two theories apparently different (quantum mechanics and game theory) it is shown that both are analogous and thus exactly equivalents. The quantum analogue of the replicator dynamics is the von Neumann equation. A population can be represented by a quantum system in which each subpopulation playing strategy $s_i$ is represented by a pure ensemble in state $|\Psi_k(t)\rangle$ and with probability $p_k$. The probability $x_i$ of playing strategy $s_i$ or the relative frequency of the individuals using strategy $s_i$ in that population can be represented as the probability $\rho_{ii}$ of finding each pure ensemble in the state $|i\rangle$.

Quantum mechanics could be used to explain more correctly biological and economical processes. It could even encloses theories like games and evolutionary dynamics. We can take some concepts and definitions from quantum mechanics and physics for the best understanding of the behavior of economics and biology. Also, we could maybe understand nature like a game in where its players compete for a common welfare and the equilibrium of the system that they are members. All the members of our system will play a game in which its maximum payoff is the equilibrium of the system. They act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare. A system where its members are in Nash Equilibrium (or ESS) is exactly equivalent to a system in a maximum entropy state. A system is stable only if it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable an eventually collapses. The results of this work shows that the “globalization” process has a behavior exactly equivalent to a system that is tending to a maximum entropy state and predicts the apparition of big common markets and strong common currencies that will find its “equilibrium” by decreasing its number until they get a state characterized by only one common currency and only one common market around the world.

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I. INTRODUCTION

With the purpose of beginning with our analysis we will start from the fact that a physical system is modeled through quantum mechanics and a socioeconomical system through evolutionary game theory. The relationships between these two systems are analyzed and it is shown how although both systems are described through two theories apparently different both are analogous and thus exactly equivalents.

The quantum analogue of the replicator dynamics is the von Neumann equation. The natural trend of these systems is to a maximum entropy state which is their state of equilibrium. A system in where all its members are in Nash equilibrium is equivalent to a system in a maximum entropy state. The “globalization” process has a behavior exactly equivalent to a system that is tending to a maximum entropy state and it is predicted the apparition of big common markets and strong common currencies that will reach the “equilibrium” by decreasing its number until they get a state characterized by only one common currency and one common market around the world.

II. THE REPLICATOR DYNAMICS & EVOLUTIONARY GAME THEORY

We could understand nature like a game in where its players compete for a common welfare and the equilibrium of the system that they are members. All the members of our system will play a game in which its maximum payoff is the equilibrium of the system. They act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare. The stability of the system is given by the maximization of the welfare of the collective. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable an eventually collapses.
The central equilibrium concept in game theory is the Nash Equilibrium. A Nash equilibrium (NE) is a set of strategies, one for each player, such that no player has an incentive to unilaterally change his action. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if he remained with his current strategy. A Nash equilibrium satisfies the following condition

\[ E(p, p) \geq E(r, p), \]

where \( E(s_i, s_j) \) is a real number that represents the payoff obtained by a player who plays the strategy \( s_i \) against an opponent who plays the strategy \( s_j \). A player can not increase his payoff if he decides to play the strategy \( r \) instead of \( p \).

Evolutionary game theory \cite{2,6,9,10} does not rely on rational assumptions but on the idea that the Darwinian process of natural selection \cite{7} drives organisms towards the optimization of reproductive success \cite{8}. Instead of working out the optimal strategy, the different phenotypes in a population are associated with the basic strategies that are shaped by trial and error by a process of natural selection or learning. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics \cite{2,6,9,10,11} whose stable fixed points are Nash equilibria \cite{2}. The central equilibrium concept of evolutionary game theory is the notion of Evolutionary Stable Strategy (ESS) introduced by J. Smith and G. Price \cite{4,11}. An ESS is described as a strategy which has the property that if all the members of a population adopt it, no mutant strategy could invade the population under the influence of natural selection. ESS are interpreted as stable results of processes of natural selection.

Consider a large population in which a two person game \( G = (S, E) \) is played by randomly matched pairs of animals generation after generation. Let \( p \) be the strategy played by the vast majority of the population, and let \( r \) be the strategy of a mutant present in small frequency. Both \( p \) and \( r \) can be pure or mixed. An evolutionary stable strategy (ESS) \( p \) of a symmetric two-person game \( G = (S, E) \) is a pure or mixed strategy for \( G \) which satisfies the following two conditions

\[ E(p, p) > E(r, p), \]

\[ E(p, p) = E(r, p) \geq E(r, r). \]

Since the stability condition only concerns to alternative best replies, \( p \) is always evolutionarily stable if \( (p, p) \) is a strict equilibrium point. An ESS is also a Nash equilibrium since is the best reply to itself and the game is symmetric. The set of all the strategies that are ESS is a subset of the NE of the game. A population which plays an ESS can withstand an invasion by a small group of mutants playing a different strategy. It means that if a few individuals which play a different strategy are introduced into a population in an ESS, the evolutionarily selection process would eventually eliminate the invaders.

The model used in EGT is the following: Each agent in a \( n \)-player game where the \( i^{th} \) player has as strategy space \( S_i \) is modeled by a population of players which have to be partitioned into groups. Individuals in the same group would all play the same strategy. Randomly we make play the members of the subpopulations against each other. The subpopulations that perform the best will grow and those that do not will shrink and eventually will vanish. The process of natural selection assures survival of the best players at the expense of the others. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics

\[ \frac{dx_i}{dt} = [f_i(x) - \langle f(x) \rangle] x_i. \]

The probability of playing certain strategy or the relative frequency of individuals using that strategy is denoted by frequency \( x_i \). The fitness function \( f_i = \sum_{j=1}^{n} a_{ij} x_j \) specifies how successful each subpopulation is, \( \langle f(x) \rangle = \sum_{k,l=1}^{n} a_{kl} x_k x_l \) is the average fitness of the population, and \( a_{ij} \) are the elements of the payoff matrix \( A \).

\[ \frac{dx_i}{dt} = \left[ \sum_{j=1}^{n} a_{ij} x_j - \sum_{k,l=1}^{n} a_{kl} x_k x_l \right] x_i. \]

The replicator dynamics rewards strategies that outperform the average by increasing their frequency, and penalizes poorly performing strategies by decreasing their frequency. The stable fixed points of the replicator dynamics are Nash equilibria, it means that if a population reaches a state which is a Nash equilibrium, it will remain there.

Quantum games have proposed a new point of view for the solution of the classical problems and dilemmas in game theory. Quantum games are more efficient than classical games and provide a saturated upper bound for this efficiency \cite{12,13,14,15,16,17}.

### III. THE VON NEUMANN EQUATION & QUANTUM STATISTICAL MECHANICS

An ensemble is a collection of identically prepared physical systems. When each member of the ensemble is characterized by the same state vector \( |\Psi(t)\rangle \) it is called pure ensemble. If each member has a probability \( p_i \) of being in the state \( |\Psi_i(t)\rangle \) we have a mixed ensemble. Each member of a mixed ensemble is a pure state and its evolution is given by Schrödinger equation. To describe correctly a statistical mixture of states it is necessary the
introduction of the density operator
\[ \rho(t) = \sum_{i=1}^{n} p_i |\Psi_i(t)\rangle \langle \Psi_i(t)| \] (5)

which contains all the physically significant information we can obtain about the ensemble in question. Any two ensembles that produce the same density operator are physically indistinguishable. The diagonal elements \( \rho_{nn} \) of the density operator \( \rho(t) \) represents the average probability of finding the system in the state \( |n\rangle \) and its sum is equal to 1. The non-diagonal elements \( \rho_{np} \) expresses the interference effects between the states \( |n\rangle \) and \( |p\rangle \) which can appear when the state \( |\Psi_i\rangle \) is a coherent linear superposition of these states. The time evolution of the density operator is given by the von Neumann equation
\[ i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho] \] (6)

which is only a generalization of the Schrödinger equation and the quantum analogue of Liouville’s theorem.

**IV. RELATIONSHIPS BETWEEN QUANTUM MECHANICS & GAME THEORY**

In table 1 we compare some characteristic aspects of quantum mechanics and game theory \cite{18, 19, 20}.

| Quantum Mechanics       | Game Theory          |
|-------------------------|----------------------|
| n system members        | n players            |
| Quantum States          | Strategies           |
| Density Operator        | Relative Frequencies Vector |
| Von Neumann Equation    | Replicator Dynamics  |
| Von Neumann Entropy     | Shannon Entropy      |
| System Equilibrium      | Payoff               |
| Maximum Entropy         | Maximum Payoff       |

Table 1

It is easy to realize the clear resemblances and apparent differences between both theories and between the properties both enjoy. This was a motivation to try to find an actual relationship between both systems.

It is important to note that the replicator dynamics is a vectorial differential equation while von Neumann equation can be represented in matrix form. If we would like to compare both systems the first we would have to do is to try to compare their evolution equations by trying to find a matrix representation of the replicator dynamics and this is \cite{21}
\[ \frac{dX}{dt} = G + G^T, \] (7)

where the relative frequencies matrix \( X \) has as elements \( x_{ij} = (x_i x_j)^{1/2} \) and
\[ (G + G^T)_{ij} = \frac{1}{2} \sum_{k=1}^{n} a_{ik} x_k x_{ij} \]
\[ \quad + \frac{1}{2} \sum_{k=1}^{n} a_{jk} x_k x_{ji} \]
\[ \quad - \sum_{k,l=1}^{n} a_{kl} x_k x_{il} x_{ij} \] (8)

are the elements of the matrix \( (G + G^T) \). From this matrix representation we can find a Lax representation of the replicator dynamics \cite{21}
\[ \frac{dX}{dt} = [Q, X] \] (9)

and with \( \Lambda = [Q, X] \)
\[ \frac{dX}{dt} = [\Lambda, X]. \] (10)

The matrix \( \Lambda \) is equal to
\[ (\Lambda)_{ij} = \frac{1}{2} \left[ \left( \sum_{k=1}^{n} a_{ik} x_k \right) x_{ij} - x_{ji} \left( \sum_{k=1}^{n} a_{jk} x_k \right) \right] \] (11)

and \( Q \) is a diagonal matrix which has as elements
\[ q_{ii} = \frac{1}{2} \sum_{k=1}^{n} a_{ik} x_k. \] (12)

It is easy to realize that the matrix commutative form of the replicator dynamics \cite{10} follows the same dynamic than the von Neumann equation \cite{6} and the properties of their correspondent elements (matrixes) are similar, being the properties corresponding to our quantum system more general than the classical system.

Although a physical system is modeled and described mathematically through quantum mechanics while a socioeconomical is modeled through game theory both systems seem to have a similar behavior. Both are composed by \( n \) members (particles, subsystems, players, states, etc.). Each member of our systems is described by a state or a strategy which has assigned a determined probability. The quantum mechanical system is described by a density operator \( \rho \) whose elements represent the system average probability of being in a determined state. The socioeconomical system is described through a relative frequencies matrix \( X \) whose elements represent the frequency of players playing a determined strategy. The evolution equation of the relative frequencies matrix \( X \) (which describes our socioeconomical system) is given by a Lax form of the replicator dynamics which was shown that follows the same dynamic than the evolution equation of the density operator (the von Neumann equation).
Some specific resemblances between quantum statistical mechanics and evolutionary game theory are summarized in the next table.

| Quantum Statistical Mechanics | Evolutionary Game Theory |
|------------------------------|--------------------------|
| $n$ system members           | $n$ population members   |
| Each member in the state $| \Psi_k \rangle$ with $p_k \rightarrow \rho_{ij}$ & Each member plays strategy $s_i$ $s_i \rightarrow x_i$ |
| $\rho$, $\sum_{ij} \rho_{ij} = 1$ & $X$, $\sum_{i} x_i = 1$ |
| $i\hbar \frac{\partial}{\partial t} = \hat{H}, \rho$ & $\frac{dX}{dt} = [\Lambda, X]$ |
| $S = -Tr\{\rho \ln \rho\}$ & $H = -\sum_{i} x_i \ln x_i$ |

In table 3 we compare the properties of the matrixes $\rho$ and $X$.

| Density Operator | Relative Freq. | Matrix |
|------------------|----------------|--------|
| $\rho$ is Hermitian | $X$ is Hermitian |
| $Tr\rho(t) = 1$ | $TrX = 1$ |
| $\rho^2(t) \leq \rho(t)$ | $X^2 = X$ |
| $Tr\rho^2(t) \leq 1$ | $TrX^2(t) = 1$ |

We can also propose the next “quantization relationships”

\[
    x_i \rightarrow \sum_{k=1}^{n} \langle i | \Psi_k \rangle p_k \langle \Psi_k | i \rangle = \rho_{ii},
\]

\[
    (x_i x_j)^{1/2} \rightarrow \sum_{k=1}^{n} \langle i | \Psi_k \rangle p_k \langle \Psi_k | j \rangle = \rho_{ij}. \tag{13}
\]

A population will be represented by a quantum system in which each subpopulation playing strategy $s_i$ will be represented by a pure ensemble in the state $|\Psi_k(t)\rangle$ and with probability $p_k$. The probability $x_i$ of playing strategy $s_i$ or the relative frequency of the individuals using strategy $s_i$ in that population will be represented as the probability $\rho_{ii}$ of finding each pure ensemble in the state $|i\rangle$ \[21\]. Through these quantization relationships the replicator dynamics (in matrix commutative form) takes the form of the equation of evolution of mixed states i.e. the von Neumann equation is the quantum analogue of the replicator dynamics. And also $X \rightarrow \rho$, $\Lambda \rightarrow -\frac{i}{\hbar} \hat{H}$, (where $\hat{H}$ is the Hamiltonian of the physical system) and $H(x) \rightarrow S(\rho)$ \[22, 23\].

**VI. THE COLLECTIVE WELFARE PRINCIPLE & THE QUANTUM UNDERSTANDING OF CLASSICAL SYSTEMS**

If our systems are analogous and thus exactly equivalents, our physical equilibrium (maximum entropy) should be also exactly equivalent to our socioeconomical equilibrium (NE or ESS). And if the natural trend of a physical system is to a maximum entropy state, should not a socioeconomical system trend be also to a maximum entropy state which would have to be its state of equilibrium? Has a socioeconomical system something like a “natural trend”? This can be seen like a microscopical cooperation between quantum objects to improve their states with the purpose of reaching or maintaining the equilibrium of the system. All the members of our quantum system will play a game in which its maximum payoff is the equilibrium of the system. The members of the system act as a whole besides individuals like they obey a rule in where they prefer the welfare of the collective over the welfare of the individual. This equilibrium is represented in the maximum system entropy in where the system resources are fairly distributed over its members. This system is stable only if it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable and eventually it collapses (Collective Welfare Principle) \[18, 19, 20, 21, 22, 23\].

A. **Globalization**

Globalization represents the inexorable integration of markets, nation-states, technologies \[24\] and the intensification of consciousness of the world as a whole \[25\].
This refers to increasing global connectivity, integration and interdependence in the economic, social, technological, cultural, political, and ecological spheres.  

Globalization has various aspects which affect the world in several different ways such as the emergence of worldwide production markets and broader access to a range of goods for consumers and companies (industrial), the emergence of worldwide financial markets and better access to external financing for corporate, national and subnational borrowers (financial), the realization of a global common market, based on the freedom of exchange of goods and capital (economical), the creation of a world government which regulates the relationships among nations and guarantees the rights arising from social and economic globalization (political), the increase in information flows between geographically remote locations (informational), the growth of cross-cultural contacts (cultural), the advent of global environmental challenges that can not be solved without international cooperation, such as climate change, cross-boundary water and air pollution, over-fishing of the ocean, and the spread of invasive species (ecological) and the achievement of free circulation by people of all nations (social).  

In economics, globalization is the convergence of prices, products, wages, rates of interest and profits towards developed country norms. Globalization of the economy depends on the role of human migration, international trade, movement of capital, and integration of financial markets. Economic globalization can be measured around the four main economic flows that characterize globalization such as goods and services (e.g. exports plus imports as a proportion of national income or per capita of population), labor/people (e.g. net migration rates; inward or outward migration flows, weighted by population), capital (e.g. inward or outward direct investment as a proportion of national income or per head of population), and technology. To what extent a nation-state or culture is globalized in a particular year has until most recently been measured employing simple proxies like flows of trade, migration, or foreign direct investment, as described above. A multivariate approach to measuring globalization is the recent index calculated by the Swiss Think tank KOF. The index measures the three main dimensions of globalization: economic, social, and political.  

Maybe the firsts of these so called states-nations, communities, “unions”, common markets, etc. were the Unites States of America and the USSR (now the Russian Federation). Both consists in a set or group of different nations or states under the same basic laws or principles (constitution), policies, objectives and an economy characterized by a same currency. Although each state or nation is a part of a big community each of them can take its own decisions and choose its own way of government, policies, laws and punishments (e.g. death penalty) but subject to a constitution (which is no more than a “common agreement”) and also subject to the decisions of the “congress” of the community which regulates the whole and the individual decisions of the parts. The United States of America consists in 50 states and a federal district. It also has many dependent territories located in the Antilles and Oceania. The currency in The United States is the Dollar. The Russian Federation consists in a big number of political subdivisions (88 components). There are 21 republics inside the federation with a big degree of autonomy over most of the aspects. The rest of territory consists in 48 provinces known as oblast and six regions (kray), between which there are 10 autonomic districts and an autonomic oblast and 2 federal cities (Moscow and San Petersburg). Recently, seven federal districts have been added. The currency in Russia is the Rublo.  

The European Union stands as an example that the world should emulate by its sharing rights, responsibilities, and values, including the obligation to help the less fortunate. The most fundamental of these values is democracy, understood to entail not merely periodic elections, but also active and meaningful participation in decision making, which requires an engaged civil society, strong freedom of information norms, and a vibrant and diversified media that are not controlled by the state or a few oligarchs. The second value is social justice. An economic and political system is to be judged by the extent to which individuals are able to flourish and realize their potential. As individuals, they are part of an ever-widening circle of communities, and they can realize their potential only if they live in harmony with each other. This, in turn, requires a sense of responsibility and solidarity.  

The meeting of 16 national leaders at the second East Asia Summit (EAS) on the Philippine island of Cebu in January 2007 offered the promise of the politically fractional but economically powerful Asian mega-region. The summit was coalescing into a single meaningful unit.  

Seth Kaplan has offered the innovative idea of a West African Union to help solve West Africa’s deep-rooted problems. The 15 West African countries stretching from Senegal to Nigeria face some of the worst problems of the developing world: Pervasive inter-group conflict; borders that fail to reflect the cultural landscape; weak national cohesion; corrupt officials and impotent institutions; a dearth of skilled workers exacerbated by brain drain; poor investment climates; and AIDS. Britain’s Department for International Development considers ten of the fifteen countries fragile. Seventy-five percent of the area’s people live under governments that cannot deliver many of the most basic services—excluding, in many cases, security. More than 25,000 peacekeepers are needed to maintain a fragile peace in the region’s war zones. Conflicts spill easily across borders, as do refugees, arms, and instability.
In South America has been proposed the creation of a Latin-American Community which is an offer for the integration, the struggle against the poverty and the social exclusion of the countries of Latin-America. It is based on the creation of mechanisms to create cooperative advantages between countries that let balance the asymmetries between the countries of the hemisphere and the cooperation of funds to correct the inequalities of the weak countries against the powerful nations. The economy ministers of Paraguay, Brazil, Argentina, Ecuador, Venezuela and Bolivia agreed in the “Declaración de Asunción” to create the Bank of the South and invite the rest of countries to add to this project. The Brazilian economy minister Mantega stand out that the new bank is going to consolidate the economic, social and politic block that is appearing in South America and now they have to point to the creation of a common currency. Recently, Uruguay has also accepted the offer of the creation of the bank and the common currency and is expected that more countries add to this offer [33].

B. The Equilibrium Process

After analyzing our systems we conclude that a socioeconomical system has a behavior exactly equivalent that a physical system. Both systems evolve in analogous ways and to analogous points. A system where its members are in Nash Equilibrium (or ESS) is exactly equivalent to a system in a maximum entropy state. The stability of the system is based on the maximization of the welfare of the collective above the welfare of the individual. The natural trend of a physical system is to a maximum entropy state, should not a socioeconomical system tend be also to a maximum entropy state which would have to be its state of equilibrium? Has a socioeconomical system something like a “natural trend”? Socioeconomically and based on our analysis, our world could be understood as a statistical mixture of “ensembles” (countries for example). Each of these ensembles are characterized by a determined state and a determined probability. But more important, each “country” is characterized by a specific “temperature” which is a measure of the socioeconomical activity of that ensemble. That temperature is related with the activity or with the interactions between the members of the ensemble. The system will evolve naturally to a maximum entropy state. Each pure ensemble of this statistical mixture will vary and accommodate its state until get the “thermal equilibrium” first with its nearest neighbors creating new big ensembles characterized each of them by a same temperature. Then with the time, these new big ensembles will seek its “thermal equilibrium” with its nearest neighbors creating new bigger ensembles but less in number. The system will continue evolving naturally in time until the whole get a state characterized by a same “temperature”.

This behavior is very similar to what have been called globalization and actually the process of equilibrium that is absolutely equivalent to a system that is tending to a maximum entropy state is the actual globalization. This analysis predicts the apparition of big common “markets” or (economical, political, social, etc.) communities of countries (European Union, Asian Union, Latin-American Community, African Union, Mideast Community, Russia and USA) and strong common currencies (dollar, euro, yen, inti, etc.). The little and poor economies eventually will finish unavoidably absorbed by these “markets” and these currencies. If this process continues these markets or communities will find its “equilib-
rimum” by decreasing its number until there will be a moment in where there exists a big common community or market and a same common currency around the world.

VII. CONCLUSIONS

Although both systems analyzed are described through two theories apparently different (quantum mechanics and game theory) both are analogous and thus exactly equivalents. A socioeconomical system has a behavior exactly equivalent that a physical system. Both systems evolve in analogous ways and to analogous points. The quantum analogue of the replicator dynamics is the von Neumann equation. A system where its members are in Nash Equilibrium (or ESS) is exactly equivalent to a system in a maximum entropy state. The stability of the system is based on the maximization of the welfare of the collective above the welfare of the individual. The natural trend of both systems is to its maximum entropy state which is its state of equilibrium. The results of this work show that the “globalization” process has a behavior exactly equivalent to a system that is tending to a maximum entropy state and predicts the apparition of big common markets and strong common currencies that will find its “equilibrium” by decreasing its number until they get a state characterized by a big common community or market and a same common currency around the world.

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