Correlation functions in lattice formulations of quantum gravity

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We compare different models of a quantum theory of four-dimensional lattice gravity based on Regge’s original proposal. From Monte Carlo simulations we calculate two-point functions between geometrical quantities and estimate the masses of the corresponding interaction particles.

1. INTRODUCTION

There are two related schools attempting at a lattice theory of quantum gravity, dynamical triangulation and Regge quantum gravity. We concentrate on the latter approach and refer to [1,2] for excellent reviews of the former framework. In particular we investigate different formulations, all of them based on the original work by Regge [3], namely conventional Regge gravity [4,5], a group theoretical approach [6], $Z_2$-link Regge-theory [7], and Regge gravity coupled to SU(2)-gauge theory [8]. Each model exposes phase transitions at some critical gravitational couplings separating small and large curvature phases and allows to look for a continuum limit [9,10]. A candidate realistic quantum theory of gravity should reproduce the expected long range interaction behavior observed in nature. Thus we calculate two-point functions to probe the existence of massless quanta of the gravitational field.

2. LATTICE QUANTUM GRAVITY

Any smooth $d$-manifold can be approximated by appropriately glueing together pieces of flat space, called $d$-simplices, ending up with a simplicial lattice. We take the edge lengths as degrees of freedom and leave the triangulation of the lattice fixed. Adopting the Euclidean path integral we can write down the partition function

\[ Z = \int D[q, U] e^{-I(q, U)} \]

with an action $I$ that covers all of the models introduced in more detail below. The functional integration extends over the squared edge lengths $q$ and (if switched on) over the non-Abelian gauge fields $U$.

One of the problems with (1) is the ambiguity in performing the link-lengths integration. Commonly the measure is written as

\[ D[q] = \prod_l dq_l q_l^{-\sigma} F(q_l), \]

with $F$ a function of the squared edge lengths $q$ being equal to one if the Euclidean triangle inequalities are fulfilled and zero otherwise. The question remains whether such a local measure is sufficient and how the power $\sigma$ might be chosen. A recent calculation for two-dimensional pure gravity pointed out that simple measures like (3) do not respect the infinite volume of the diffeomorphism group. Only by appropriate gauge fixing and including the corresponding Faddeev-Popov term the correct continuum limit can be obtained [11]. However, the generalization of this procedure to higher dimensions and its numerical implementation are technically demanding.

Working in Euclidean space, i.e. with positive definite metric, the conformal mode renders the continuum Einstein-Hilbert action unbounded from below. This unpleasant feature persists in the discretized Regge-Einstein action but need not necessarily lead to an ill-defined path integral [11]. Indeed, numerical simulations reveal the existence of a well-defined phase with finite expectation values within a certain range of the bare Planck mass, $(m_P^+)^2 \leq m_P^2 \leq (m_P^-)^2$ [11].
On a triangulated lattice an action is given by

\[ I(q, U) = -2m_P^2 \sum_t R_t(A_t, \delta_t) + \lambda \sum_s V_s + \frac{\beta}{2} \sum_t W_t \text{Re}[\text{Tr}(1 - U_t)] \, . \]  

(3)

The first sum runs over all triangle areas \( A_t(q) \) and corresponding deficit angles \( \delta_t(q) \) yielding the curvature \( R_t \). A cosmological term consisting of the cosmological constant \( \lambda \) times the sum over the volumes \( V_s(q) \) of all four-simplices \( s \) follows. Finally, an additional non-Abelian gauge action composed of the inverse gauge coupling \( \beta \), the weight factors \( W_t(q) \) and the ordered product \( U_t \) of SU(2) matrices around the triangle \( t \) is appended. The weights

\[ W_t = \text{const} \times \frac{V_t}{A_t^2} \, , \]  

(4)

with a four-volume \( V_t \) assigned to every triangle, describe the coupling of gravity to the gauge fields.

Now we are ready to define the four different models for subsequent numerical treatment. Monte Carlo simulations have been performed on regularly triangulated hypercubic lattices with toroidal topology and \( 4^3 \times 8 \) vertices. The gravitational couplings \( m_P^2 \) were chosen close to \( (m_r^-)^2 \) where there is a certain chance for a continuous phase transition \cite{[6]}.

2.1. Conventional Regge gravity

Here we employ the Regge-Einstein action, putting \( R_t = A_t \delta_t \) with the gravitational couplings \( m_P^2 = \{ -0.0775, -0.0785, -0.0795 \} \), and include a cosmological term with \( \lambda = 1 \). No additional gauge fields are present in the action, \( \beta = 0 \), and the gravitational measure is chosen to be uniform, \( \sigma = 1 \).

2.2. Group theoretical approach

Constructing the dual of a simplicial lattice, Poincaré transformations can be assigned to its links to yield an action in which the \( \sin \) of the deficit angle enters, \( R_t = A_t \sin \delta_t \). Again we use a cosmological constant, \( \lambda = 1 \), vanishing inverse gauge coupling, \( \beta = 0 \), the uniform measure, \( \sigma = 1 \), and vary \( m_P^2 = \{ -0.055, -0.0555, -0.056 \} \).

2.3. \( Z_2 \)-link Regge-gravity

This model is defined by restricting the squared link lengths to take on only two possible values \( q_t \sim 1 + c \sigma_t \, , \quad \sigma_t \in \mathbb{Z}_2 \, , \quad c \leq c_{\text{max}} \in \mathbb{R}^+ \) . \( (5) \)

Then the quantum-gravity path-integral can be rewritten as the partition function of a spin system with somewhat complicated, yet local spin interactions \cite{[7]}.

2.4. Gauge fields coupled to Regge gravity

In the system of SU(2) gauge fields coupled to quantum gravity we set \( R_t = A_t \delta_t \), \( \lambda = 0 \) and use a scale invariant measure, \( \sigma = 0 \) \cite{[8,10]} . The following pairs of gravitational and gauge coupling, respectively, are considered: \( (m_P^2, \beta) = \{ (-0.0025, 1.6), (-0.0055, 1.0) \} \).

3. TWO-POINT FUNCTIONS

In order to examine the physical relevance of the well-defined phase mentioned above, we compute correlation functions of certain geometrical quantities. The gravitational weak-field propagator can be cast into a spin-two and a spin-zero contribution \cite{[12]}. The volume correlations are sensitive to the scalar part

\[ G_V(d) = \langle \sum_{s \leq v_0} V_s \sum_{s' \geq v_d} V_{s'} \rangle_c \]  

(6)

and the curvature correlations are due to the presence of spin-2 particles

\[ G_R(d) = \langle \sum_{t \leq v_0} R_t \sum_{t' \geq v_d} R_{t'} \rangle_c \, . \]  

(7)

The local operators in \( (6) \) and \( (7) \) should be measured at two vertices \( v_0 \) and \( v_d \) separated by the geodesic distance. We take the distance \( d \) to be equal to the index distance along the main axes of the skeleton. This seems a reasonable approximation in the well-defined phase with its small average curvature. In general one expects for \( (6) \) and \( (7) \) at large distances the functional form

\[ G \sim \frac{e^{-md}}{d^s} \, . \]  

(8)

A power law with \( a = 2 \) and a vanishing effective mass \( m = 0 \) would hint at Newtonian gravity with massless gravitons.
(a) Conventional Regge gravity: \( \Diamond \, \ldots m_P^2 = -0.0775, \, \Box \, \ldots m_P^2 = -0.0785, \, \Delta \, \ldots m_P^2 = -0.0795 \)

(b) Group theoretical approach: \( \Diamond \, \ldots m_P^2 = -0.055, \, \Box \, \ldots m_P^2 = -0.0555, \, \Delta \, \ldots m_P^2 = -0.056 \)

(c) Gauge fields coupled to gravity: \( \Diamond \, \ldots (m_P^2 = -0.0025, \beta = 1.6), \, \Box \, \ldots (m_P^2 = -0.005, \beta = 1.0) \)

Figure 1. Volume (left plots) and curvature (middle with magnification in the right plots) correlation functions for (a) conventional Regge gravity, (b) the group theoretical approach, and (c) the system of non-Abelian gauge fields coupled to Regge gravity. Error bars not explicitly drawn are in the size of the symbols. The curves correspond to fits with the function (8).
Table 1

|                      | (a) Conventional Regge gravity | (b) Group theoretical approach | (c) Gauge fields |
|----------------------|--------------------------------|--------------------------------|------------------|
| $m^2_P$              | −0.0775                | −0.0785              | −0.0795         |
| $m$                  | 2.9                  | 3.1                  | 3.0              |

4. RESULTS

Fig. 1 displays our Monte Carlo data of the two-point functions $\langle 6 \rangle$ and $\langle 7 \rangle$. The volume correlations have already been studied around the transition at positive coupling where fits to an exponential decay have been obtained [13]. Such a fit procedure seems to be more difficult at negative couplings.

The curvature correlations are more suitable for a fit with $\langle 8 \rangle$. In order to test whether they obey a power law we fixed $a = 2$ and fitted the effective masses $m$. We took only distances $d \geq 2$ into account. $G_R(d = 1)$ is presumably plagued by lattice artifacts due to contact terms [14]. We are anyhow interested in the large distance behavior.

Table 1 contains the obtained mass parameters $m$. For (a) conventional Regge gravity $m$ stays rather constant towards the critical coupling whereas in (b) the group theoretical approach the mass decreases for $m^2_P \to (m^-)^2$. In the case of SU(2) fields on the fluctuating lattice we get only one reasonable fit again indicating a nonzero mass. For all fits the uncertainties in the mass parameters are large. The $Z_2$-link approximation of Regge gravity seems to behave differently. For this model the curvature correlations are compatible with a power law. The results of a more extended study of the spin approach are reported in [14].

5. SUMMARY

We computed two-point functions close to the critical bare Planck mass in the negative gravitational coupling regime. Altogether they exhibit a very similar behavior for the considered models. Except for $Z_2$-link Regge-gravity we found no convincing evidence for long-range correlations corresponding to massless spin-zero or spin-two excitations. One reason might be, that contrary to the models described here, the $Z_2$-link theory is well-defined for all couplings $m^2_P$ and computationally much less demanding, which allows to perform simulations exactly at the critical point.

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