Study of $B_c^- \rightarrow X(3872) \pi^- (K^-)$ decays in the covariant light-front approach

Wei Wang, Yue-Long Shen, Cai-Dian Lü

Institute of High Energy Physics, Chinese Academy of Sciences, P.O.Box 918, Beijing 100049, P.R. China

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Abstract. In the covariant light-front quark model, we calculate the form factors of $B_c^- \rightarrow J/\psi$ and $B_c^- \rightarrow X(3872)$. Since the factorization of the exclusive processes $B_c^- \rightarrow J/\psi \pi^- (K^-)$ and $B_c^- \rightarrow X(3872) \pi^- (K^-)$ can be proved in the soft-collinear effective theory, we can easily get the branching ratios for these decays from the form factors. Taking the uncertainties into account, our results for the branching ratio of $B_c^- \rightarrow J/\psi \pi^- (K^-)$ are consistent with previous studies. By identifying $X(3872)$ as a $1^{++}$ charmonium state, we obtain $\text{BR}(B_c^- \rightarrow X(3872) \pi^-) = (1.7^{+0.7+0.1+0.4}_{-0.6-0.2-0.4}) \times 10^{-4}$ and $\text{BR}(B_c^- \rightarrow X(3872) K^-) = (1.3^{+0.5+0.3}_{-0.2-0.3}) \times 10^{-5}$. Assuming $X(3872)$ to be a $1^{--}$ state, the branching ratios will be one order of magnitude larger than those of the $1^{++}$ state. These results can easily be used to test the charmonium description for this mysterious meson $X(3872)$ at the LHCb experiment.

1 Introduction

$X(3872)$ was first observed by Belle in the exclusive decay $B^\pm \rightarrow K^\pm X \rightarrow K^{\pm} \pi^{\mp} \pi^{\mp} J/\psi$ [1], and subsequently confirmed by the CDF, DØ and BaBar collaborations in various decay and production channels [2–4]. At present a definite answer to the question of its internal properties is not well established, but the current experimental data strongly support a $1^{++}$ state [5]. Enormous interest in the study of $\bar{c}c$ resonance spectroscopy followed this discovery and there exist many interpretations of this meson. The first and most natural assignment of this state is the first radial excitation of the $1P$ charmonium state $\chi_{c1}$ [6]. However, this interpretation has encountered two difficulties: its decay width ($< 2.3$ MeV, 95% C.L.) is tiny compared with other charmonia; and there is a gap of about $100$ MeV between the measured mass and the quark model prediction [7]. Motivated by these two difficulties, many non-charmonium explanations were proposed, such as it being a $\bar{c}\bar{c}g$ hybrid meson [8], a glueball [9], a diquark cluster [10, 11], and a molecular state [12–15]. But in fact, there are few experimental data that could provide a clear discrimination among these descriptions, and this makes the situation more obscure. Recently the CLQCD collaboration studied the mass for the first excited states of $1^{++}$ charmonium and found that it is consistent with the measured mass of $X(3872)$ [16]. Consistence indicates that $X(3872)$ can be the first radial excited state of $\chi_{c1}$, and it seems that the mass difficulty trails off. Now, in order to investigate the structure of this meson more clearly, a large amount of experimental data and theoretical studies on the productions and decays of $X(3872)$ are strongly needed.

In $B_{u,d,s}^- \rightarrow \bar{c}c$ decays involving the charmonium final states, the emitted meson is a heavy charmonium. The non-factorizable contribution should be large to induce large uncertainties [17]. As the energy release is limited, these decays may also be polluted by the final state interactions, which are non-perturbative in nature. But fortunately the production of charmonia in $B_c$ decays could provide unique insight in these mesons. Since the emitted meson here is a light meson ($\pi$ or $K$), the factorization of $B_c \rightarrow (\bar{c}c) M (M$ is a light meson) could be proved in the framework of the soft-collinear effective theory (SCET) to all orders of the strong coupling constant in the heavy quark limit, which is similar to $B^0 \rightarrow D^+ \pi^-$ and $B^- \rightarrow D^0 \pi^-$ [18]. The decay matrix element can be decomposed into a $B_c \rightarrow (\bar{c}c)$ form factor and a convolution of a short distance coefficient with the light-cone wave function of the emitted light meson.

Although SCET provides a powerful framework to study the factorization of the exclusive modes, the non-perturbative form factors could not be directly studied. We can only extract them via the experimental data or rely on some non-perturbative method. In the present paper, we will use the light-front quark model to calculate these $B_c \rightarrow M(\bar{c}c)$ form factors. As pointed out in [19], the light-front QCD approach has some unique features that are particularly suitable for use in a description of a hadronic bound state. The light-front quark model [20–23] can provide a relativistic treatment of the movement of the hadron and also give a full treatment of the hadron spin by using the so-called Melosh rotation. Light-front
wave functions, which describe the hadron in terms of their fundamental quark and gluon degrees of freedom, are independent of the hadron momentum and thus are explicitly Lorentz invariant. Furthermore, in the covariant light-front approach [24], the spurious contribution, which is dependent on the orientation of the light front, is eliminated by including the zero-mode contributions properly. This covariant model has been successfully extended to the study of the decay constants and form factors of the ground state s-wave and the low-lying p-wave mesons [25–28].

The paper is organized as follows. The formalism for the form factor calculations, taking $B_c^- \to J/\psi$ as an example, is presented in the next section. The numerical results for form factors and decay rates of $B_c^- \to J/\psi\pi^-(K^-)$, $B_c^- \to J/\psi\rho^-(K^-)$, $B_c^- \to X(3872)\pi^-(K^-)$ and $B_c^- \to X(3872)\rho^-(K^-)$ are given in Sect. 3. The conclusion is given in Sect. 4.

2 Calculation of the form factors and the branching ratios

In the following, we use $X$ to denote $X(3872)$ for simplicity. Different from the $B_{c,d,s}$ mesons, the $B_c^-$ system consists of two heavy quarks, $b$ and $c$, which can decay individually. Here we will consider $b$ decays, while $c$ acts as a spectator. At the quark level, $B_c^- \to J/\psi\pi^-$ and $B_c^- \to X(3872)\pi^-$ decays are characterized by the $b \to (c\bar{d}u)$ transition and the corresponding effective Hamiltonian is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)) + \text{h.c.},$$

(1)

where $V_{ij}$ are the corresponding CKM matrix elements. The local four-quark operators $O_{1,2}$ are defined by

$$O_1(\mu) = (\bar{s}_a \gamma_\mu (\mu-1-\gamma_5) b_\nu) \gamma_\mu (\mu-1-\gamma_5) b_\nu ,$$

$$O_2(\mu) = (\bar{s}_a \gamma_\mu (\mu-1-\gamma_5) b_\nu) \gamma_\mu (\mu-1-\gamma_5) b_\nu ,$$

(2)

where $\alpha$ and $\beta$ are the color indices. Since the four quarks in the operators are different from each other, there is no penguin contribution, and thus there is no CP violation. The left handed current is defined as $(\bar{q}_a \gamma_\mu V_{-A}) = \bar{q}_a \gamma_\mu (1-\gamma_5) q_A$. For the $b \to (c\bar{d}u)$ transition, $V_{ss}^*$ is replaced by $V_{cs}^*$, while the $d$ quark field in the four-quark operator is replaced by $s$. With the effective Hamiltonian given above, the matrix element for the $B_c^- \to J/\psi\pi^-$ transition can be expressed as

$$M = \langle J/\psi(P'',\varepsilon'')|H_{\text{eff}}|B_c^-(P') \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* a_1(\mu) \langle J/\psi(P'',\varepsilon'')|O_2(\mu)|B_c^-(P') \rangle,$$

(3)

with $P^{(\prime)}$ being the incoming (outgoing) momentum, $\varepsilon''$ the polarization vector of $J/\psi$ and $a_1 = C_2 + C_1/3$ the Wilson coefficient.

In the effective Hamiltonian, the degrees of freedom heavier than the $b$ quark mass $m_b$ are included in the Wilson coefficients, which can be calculated using perturbation theory. Then the task that is left is to calculate the operators’ matrix elements between the $B_c^-$ meson state and the final states, which suffer large uncertainties. Nevertheless, the problem becomes tractable if factorization becomes applicable. Thanks to the development of SCET, the proof of the factorization can be accomplished in an elegant way [30, 31]. In SCET, the heavy meson is described by the heavy quarks $h_u$ and soft gluons $A_s$ in its rest frame; the final state light meson moves very fast, and it is described by the collinear quarks $\xi_c$ and the collinear gluons $A_c$. In [18], it has been shown that the collinear gluons do not connect to the particle in the heavy meson, while the soft gluons do not connect to those in the light meson to all orders in $\alpha_s$ and leading power in $A_{QCD}/m_{B_c}$. In phenomenological language, the non-factorizable diagrams cancel each other because of color transparency. Furthermore, there is no annihilation contribution as the quarks in the final state meson are different from each other. Thus the decay amplitude can be expressed as the product of the $B_c \to J/\psi$ form factor and a convolution of a short distance Wilson coefficient with the non-perturbative light-cone distribution amplitude of the light meson. Without the higher order QCD corrections, the convolution is reduced to the decay constant of the light meson.

The form factors for the $B_c \to J/\psi$ and $B_c \to X(3872)$ transitions induced by the vector and axial-vector currents are defined by

$$\langle J/\psi(P'',\varepsilon'')|V_{12}|B_c^-(P') \rangle = \frac{1}{m_{B_c} + m_{J/\psi}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\mu\nu} P^\alpha q_3 V^{PV}(q^2),$$

(4)

$$\langle J/\psi(P'',\varepsilon'')|A_{12}|B_c^-(P') \rangle = i \left( m_{B_c} + m_{J/\psi} \right) \varepsilon^{\mu\nu\alpha}_\mu A_1^{PV}(q^2) - \frac{\varepsilon^{\mu\nu\alpha}_\mu \cdot P}{m_{B_c} + m_{J/\psi}} A_2^{PV}(q^2) - 2m_{J/\psi} \frac{\varepsilon^{\mu\nu\alpha}_\mu \cdot P}{q^2} q_\mu [A_3^{PV}(q^2) - A_0^{PV}(q^2)],$$

(5)

$$\langle X(P'',\varepsilon'')|V_{12}|B_c^-(P') \rangle = \left( m_{B_c} - m_x \right) \varepsilon^{\mu\nu\alpha}_\mu V_1^{PA}(q^2) - \frac{\varepsilon^{\mu\nu\alpha}_\mu \cdot P'}{m_{B_c} - m_X} P^\mu V_2^{PA}(q^2) - 2m_x \frac{\varepsilon^{\mu\nu\alpha}_\mu \cdot P'}{q^2} q_\mu [V_3^{PA}(q^2) - V_0^{PA}(q^2)],$$

(6)

$$\langle X(P'',\varepsilon'')|A_{12}|B_c^-(P') \rangle = -i \frac{1}{m_{B_c} - m_X} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\mu\nu} P^\rho q^\sigma A^{PA}(q^2),$$

(7)

where $P = P' + P''$, $q = P' - P''$, and the convention $\epsilon_{0123} = 1$ is adopted. To cancel the poles at $q^2 = 0$, we must have $A_1^{PV}(0) = A_0^{PV}(0)$, $V_3^{PA}(0) = V_0^{PA}(0)$. The form fac-

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1 For a review, see [29].