INCENTIVE CONTRACT DESIGN FOR SUPPLIER SWITCHING WITH CONSIDERING LEARNING EFFECT

QIAN WEI
School of Management, Tianjin University of Technology
Tianjin, 300384, China

JIANXIONG ZHANG AND XIAOJIE SUN∗
College of Management and Economics, Tianjin University
Tianjin, 300072, China

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ABSTRACT. For minimizing purchase cost, a buying firm would switch to suppliers with providing more favorable prices. This paper investigates the optimal switching decision of a buyer that may switch to an entrant supplier with production learning ability (which is regarded as a private information) under a principal-agent framework. The results obtained show that the switching cost and the learning effect have significant impacts on the buyer’s switching decision. Only when the fixed component of the switching cost is relatively low, the buyer can be better off from a partial switching strategy; otherwise, the buyer should take an all-or-nothing switching strategy or no switching strategy. As the learning ability of the entrant supplier increases, the buyer prefers to make more switching. Finally, a benefit-sharing contract is proposed to evaluate the performance of the principal-agent contract, and we demonstrate that the principal-agent contract almost completely dominates the benefit-sharing contract.

1. Introduction. In the face of increasing market competition, more and more buying firms have recognized the importance of multi-sourcing to reduce purchase cost. Many buyers tend to divide their purchase plans into two or more suppliers, instead of maintaining a close business relationship with an exclusive supplier. With the extensive application of new materials and advanced technology, alternative suppliers acting as new entrants to the market usually have a natural advantage on the initial production cost. This induces buyers to switch partly or completely to these entrant suppliers for purchasing with lower prices. In addition, entrant suppliers may reduce their production costs through learning-by-doing, coming from the fact that workers get more familiar with their jobs and have a better knowledge on how to improve the production process with accumulated experience. The learning effect plays a critical role in reducing production cost. Therefore, buyers will

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∗Corresponding author: Xiaojie Sun.
potentially obtain more benefits if cooperating with entrant suppliers with higher learning ability.

The supplier switching problem with assuming the production cost of an entrant supplier as an asymmetric information, has been examined in some studies (e.g., [31] and [37]). Note that the initial production cost of the entrant supplier can be estimated by applying various techniques and methodologies developed over the recent years. Hence, it is more reasonable that the initial production cost of the entrant supplier is considered as a common information. However, the production learning ability is often regarded as an inherent competitive advantage of the entrant supplier, and thus is difficult to be observed by its buyer. In general, the buyer have only limited information about the entrant supplier’s learning ability and faces the risk of choosing an undesirable supplier. To deal with this problem, the buyer has to provide incentives for the entrant supplier to guarantee the revelation of the hidden learning ability.

While the buyer would benefit from switching to the entrant supplier via a lower price, it will also incur a corresponding switching cost [36]. Therefore, the buyer needs to make a trade-off in obtaining a lower price and undertaking a corresponding switching cost when making the switching decisions. In this paper, we attempt to answer the following questions:

(1) What is the optimal switching decision for the buyer under a principal-agent contract?
(2) How do the switching cost and the learning effect affect the switching decision of the buyer?
(3) How effective is the principal-agent contract compared with a benefit-sharing contract in dealing with supplier switching under asymmetric information?

To address these research questions, we establish a supplier switching model with considering learning effect under a principal-agent framework, and propose a benefit-sharing contract for comparison. For the principal-agent contract, the optimal switching decision is obtained by solving an equivalent optimization problem. Furthermore, the impacts of the switching cost and the learning effect on the switching decision of the buyer are investigated. We show that only when the fixed component of the switching cost is relatively low, adopting a partial switching strategy is better off for the buyer. Otherwise, the buyer should take an all-or-nothing switching strategy or no switching strategy. As the learning ability of the entrant supplier increases, the buyer tends to make more switching. As for the benefit-sharing contract, applying backward induction method, we get the optimal sharing proportion numerically. Moreover, the detailed difference between the two contracts is described. Our analysis reveals that the principal-agent contract almost completely dominates the benefit-sharing contract.

2. Literature review. The literature related to our work mainly belongs to two streams: learning effect and supplier switching.

The theory of learning curve was first proposed by [32] when investigating the impact of human behavior on production efficiency in the aerospace industry. Since then, a great deal of references have emerged to study learning-by-doing. Economists found that the learning effect strongly reveals economic and market aspects, e.g., economic growth [2], competitive behavior [28], capital investment [11], market structure, and trade policies [7]. [34] gave a comprehensive review for the applications of learning curve. [1] explored some of the behavioral processes that give
rise to the learning curve, which highlights the complex relationship between first-order and second-order learning. [16] surveyed the work that deals with the effect of learning on the lot-size problem. [35] constructed a new theoretical framework for learning and making improvements based upon learning cycles. [3] developed a continuous review inventory model in which the learning effect in the production process is included in the processing time component of the lead-time. [5] investigated the learning effect of the unit production time on optimal lot size for the imperfect production system with allowable shortages. [15] incorporated the learning effect of setup costs into an inventory replenishment system. [25] examined how the learning curve theory could inform better management of new technology implementation projects. [33] studied the impacts of the supply-side cost learning effect on dynamic pricing strategies and the channel efficiency in a decentralized supply chain. [22] investigated the learning effect in various scheduling problems. [38] focused on the green supply chain performance, in which production cost is jointly affected by cost learning and operational inefficiency effects. [13] addressed a closed-loop supply chain with considering learning and forgetting in production. [9] proposed an inventory model to determine optimal lot-sizing and pricing strategies with trade credit and learning effect. Apart from those mentioned above, [19], [27], [26], [6], [21], etc. also devoted to the research of learning effect.

Although learning effect has attracted lots of attention from researchers due to its significant impacts on cost reduction, it hasn’t been considered in supplier switching problems. With the consideration of suppliers’ production costs, buyers should take learning effect into account when making switching decisions. In other words, the learning ability as the inherently competitive advantage of an entrant supplier has significant impacts on its buyer’s switching decision. Therefore, it is of great importance to explore the problem of switching decision incorporating learning effect.

Supplier switching enabling buyers to reduce their purchase costs has attracted much attention from scholars. [8] examined how source switching is optimally structured. [14] showed that individual antecedents have different effects on switching behavior. Using a real-options (contingent claims) approach, [17] analyzed and evaluated supply contracts in a setting characterized by exchange rate uncertainty, supplier-switching options, order-quantity flexibility, profit sharing, and supplier reaction options. [18] highlighted the role of a buying firm’s switching inertia in the supplier-selection process, and demonstrated the usefulness of their framework for the industrial automation industry. The pioneering work on supplier switching under a principal-agent framework was conducted by [31]. They developed and analyzed supplier switching model theoretically with the consideration of asymmetric information and switching cost. [12] investigated consumers’ motives for negative attitudes towards switching in three deregulated markets. Assuming that the supplier learns the production costs over time, [24] extended the basic framework in [31] to a dynamic one. [20] examined how asymmetric information alters key variables of a firm’s supplier switching process, such as the timing of contracting (hurried versus delayed contracting), transfer payments, set-up, switching, and abandonment decisions. [10] studied a firm’s cost-based sourcing decision of whether to invest in an incumbent supplier or switch to an alternative supplier in order to realize lower purchase costs. [36] constructed a supplier switching model based on the principal-agent theory to minimize the buyer’s purchase cost. They found that the forms of the optimal supplier switching strategies are directly related to the concavity
and convexity of the switching cost functions. [37] extended the model in [31] by considering a volume-dependent switching cost and a general price function of the incumbent supplier. [29] studied an infinite-horizon optimal switching problems for underlying processes that exhibiting “fast” mean-reverting stochastic volatility. [4] employed a practical perspective to study and conceptualize supplier-switching processes in business relationships. [30] analyzed how global supplier switching decisions (reshore and relocate) are influenced by buyers’ cost-focus and competitive strategies. Different from these works, we give a detailed characterization for the production cost of an entrant supplier, and focus on the impacts of the production cost structure of the entrant supplier and the volume-dependent switching cost on a buyer’s switching decision under asymmetric information. Moreover, the performance of different contracts on supplier switching is studied in this paper, generating more managerial insights and implications.

The remainder of this paper is organized as follows. In Section 3, we formulate the switching model with learning effect under a principal-agent framework. A benefit-sharing contract is proposed in Section 4 for comparison. In Section 5, we conduct numerical studies to illustrate the effectiveness of the methods. Finally, Section 6 concludes this paper and provides some suggestions for the future work.

3. Principal-agent contract. Consider a buyer (B, she) purchases a product from an incumbent supplier (S_1) at a unit price \( p \). Without loss of generality, the total purchase quantity of the buyer is normalized to be 1. When an alternative supplier comes to the market, especially, when the new supplier may reduce production cost by learning-by-doing, the buyer has incentives to switch partly or completely to the entrant supplier (S_2, he) to purchase the product with a lower price. The buyer decides the switching ratio \( r \in [0, 1] \). Specifically, \( r = 0 \) means no switching, \( r = 1 \) means a complete switching, and values between 0 and 1 mean a partial switching.

We assume that the entrant supplier S_2 provides a completely substitutable product, that is, the products provided by the two suppliers are same. Different from the incumbent supplier S_1, the production cost of S_2 can be reduced through learning-by-doing. Specifically, the production cost of S_2 is given by

\[
c = c_0 - Xr,
\]

where \( c_0 \) is the initial production cost of S_2, and \( X \) denotes S_2’s learning rate as his private information. The more the buyer B switches to S_2, the lower the production cost of S_2 will be.

In practice, the buyer often cannot exactly acquire S_2’s learning rate \( X \), and knows only the distribution function \( F(x) \) and the density function \( f(x) \) of the learning rate within the support \([0, \gamma]\). The failure rate of the learning rate distribution \( X \) is defined as \( h(x) = \frac{f(x)}{1-F(x)} \). The Mills ratio is defined as \( m(x) = \frac{1}{h(x)} \) that is commonly used in the economics literature. We assume that the Mills ratio \( m(x) \) satisfies \( m'(x) \leq 1 \), which is more general than the requirement of increasing failure rate as discussed in [39].

When the buyer decides to purchase from S_2, she will incur a switching cost. As argued in [30], the switching cost \( s \) is modeled as switching volume-dependent, namely,

\[
s(r) = kr + Isgn(r), \quad r \in [0, 1],
\]
where $I \geq 0$ is the fixed switching cost, $k > 0$ is the unit variable switching cost, and

$$\text{sgn}(r) = \begin{cases} 
0, & r = 0, \\
1, & r > 0.
\end{cases}$$

Since the buyer cannot observe the entrant supplier’s true learning ability, she has to design a mechanism to motivate the entrant supplier to report truly his private information for minimizing her purchase cost. It should be mentioned that waiting for a price offered by the entrant supplier is not a good strategy for the buyer. This is because of the fact that an entrant supplier knowing the switching cost of the buyer and the purchase price of the incumbent supplier would choose a price just below the level where the buyer would stay with the incumbent supplier. As studied by [31], this ultimately leads to a situation, where the entrant supplier obtains all profits from the relationship with the buyer, and the buyer only marginally benefits from switching. Based on the discussion, we can know that the buyer won’t benefit from the entrant supplier’s lower cost resulting from the learning effect. In the following, we show that inducing the entrant supplier to reveal his true learning ability will be a better strategy for the buyer.

Under the principal-agent contract, the buyer would design a mechanism under which the entrant supplier would report his true learning rate, and the buyer pays to the access of this information, according to the revelation principle [23]. Specifically, the buyer requires $S_2$ to render a learning rate $y$, and she minimizes her total purchase cost by deciding the switching strategy $r(y)$ and transfer payment $t(y)$ according to the reported $y$. Denote $\pi(x, y)$ as $S_2$’s profit when the true learning rate is $x$ but he reports $y$, and then $\pi(x, y) = t(y) - r(y)(c_0 - xr(y))$. On one hand, the incentive-compatibility constraint (IC) is proposed to guarantee that $S_2$ can optimize his profit when he truly reports the learning rate, namely,

$$\pi(x, x) = t(x) - r(x)(c_0 - xr(x)) \geq \pi(x, y), \ \forall x, y \in [0, \gamma]. \quad (\text{IC})$$

On the other hand, the participation constraint (PC) should be addressed. Specifically, $S_2$ should get the reservation profit at least for reporting the true learning rate, which is set to zero for simplicity, namely,

$$\pi(x, x) = t(x) - r(x)(c_0 - xr(x)) \geq 0, \ \forall x \in [0, \gamma]. \quad (\text{PC})$$

Denote the buyer’s total purchase cost as $C$, which is composed of three parts: (1) the transfer payment $t(x)$ to the entrant supplier $S_2$; (2) the transfer payment $(1 - r(x))p$ to the incumbent supplier $S_1$; (3) the switching cost $s(r(x))$.

The expectation of $C$ can be described as

$$E(C) = \int_0^{\gamma} \left[ t(x) + (1 - r(x))p + s(r(x)) \right] f(x)dx. \quad (2)$$

With the objective of minimizing $E(C)$ by deciding $r(\cdot)$ and $t(\cdot)$, one can get the optimization problem under the incentive-compatibility constraint, participation constraint, and decision constraint as follows:

$$\begin{align*}
\text{min} & \quad E(C) \\
\text{subject to:} & \\
(\text{IC}) & \quad \pi(x, x) \geq \pi(x, y), \ \forall x, y \in [0, \gamma], \\
(\text{PC}) & \quad \pi(x, x) \geq 0, \ \forall x \in [0, \gamma], \\
& \quad 0 \leq r(x) \leq 1, \ \forall x \in [0, \gamma].
\end{align*} \quad (3)$$
The following proposition proposes an equivalent solvable optimization problem of the original problem (3). For clarity of the paper, all the proofs are presented in Appendix.

**Proposition 1.** The transfer payment \( t(x) = c_0r(x) - xr^2(x) + \int_0^x r^2(z)dz \), the original problem (3) is equivalent to the following dynamic optimization problem:

\[
\begin{align*}
\min_{r(\cdot)} & \int_0^\gamma \left[ 1 - F(x) - xf(x) \right] r^2(x) + (c_0 - p + k)r(x) + p + Isgn(r(x)) \right] f(x)dx \\
\text{subject to:} & \\
r'(x) & \geq 0, \\
0 & \leq r(x) \leq 1.
\end{align*}
\]

(4)

As can be seen from Proposition 1, the number of control variables in the original optimization problem (3) is reduced to one. Before proceeding to solve the optimization problem (4), we first analyze the coefficient of \( r^2(x) \) in the objective function.

Denoting \( g(x) = 1 - F(x) - xf(x) \), we can get the following lemma with the assumption of \( m'(x) \leq 1 \).

**Lemma 1.** When \( m'(x) \leq 1 \), for \( x \in [0, \bar{x}) \), \( g(x) > 0 \); for \( x \in (\bar{x}, \gamma] \), \( g(x) < 0 \), where \( \bar{x} \) is the unique solution of the equation \( g(x) = 0 \).

Lemma 1 shows that there exists a unique solution for \( g(x) = 0 \). Equipped with this conclusion, we obtain the optimal switching strategies as illustrated in the following theorem.

**Theorem 1.** The optimal switching strategies are listed in Tables 1 and 2.

With the optimal switching ratio \( r^*(x) \) obtained in Theorem 1, the optimal transfer payments \( t^*(x) \) can be calculated out as presented in Tables 3 and 4. Substituting \( r^*(x) \) and \( t^*(x) \) into the objective function (2), we can get the buyer’s minimal expected total purchase cost.

| \( \Psi = c_0 - \gamma + k + I - p \) | \( \Psi \geq 0 \) | \(-m(0) - \gamma \leq \Psi < 0 \) | \( \Psi < -m(0) - \gamma \) |
|---|---|---|---|
| \( r^*(x) \) | \( r^*(x) = 0 \), \( x \in [0, \bar{x}] \) | \( r^*(x) = \begin{cases} 0, & x \in [0, x_1^*), \\ 1, & x \in [x_1^*, \gamma]. \end{cases} \) | \( r^*(x) = 1 \), \( x \in [0, \gamma] \). |

As illustrated in Theorem 1, the switching cost and the learning rate have significant impacts on the switching decision. Only when the fixed switching cost is relatively low, the buyer would adopt a partial switching strategy; otherwise, the buyer would take an all-or-nothing switching or no switching strategy.

We first consider the case when the fixed component of the switching cost is relatively high, i.e., \( I \geq m(0) \). Denote \( \Psi = c_0 - \gamma + k + I - p \), which represents the difference of the total purchase cost between \( r = 1 \) and \( r = 0 \). If \( \Psi \geq 0 \), that is, the minimal production cost of \( S_2 \) plus the maximal switching cost \( s(1) = k + I \) is higher than the price of \( S_1 \), the exorbitant fixed switching cost prevents the buyer
When the difference $\Psi$ becomes smaller, i.e., $\Psi = c_0 - \gamma + k + I - p$ from switching. Thus both the optimal switching ratio and transfer payment are 0. When the difference $\Psi$ becomes smaller, i.e., $-m(0) - \gamma \leq \Psi < 0$, the optimal

**Table 2. Optimal switching strategies with $I < m(0)$**

| $\Psi = c_0 - \gamma + k + I - p$ | $\Psi \geq 0$ | $-I - \gamma \leq \Psi < 0$ |
|----------------------------------|--------------|-----------------------------|
| $r^*(x)$                         | $r^*(x) = 0$, $x \in [0, \gamma]$. | $r^*(x) = \left\{ \begin{array}{ll} 0, & x \in [0, x_1^*], \\ 1, & x \in [x_1^*, \gamma]. \end{array} \right.$ |
| $I - 2\sqrt{Im(0)} - \gamma < \Psi < -I - \gamma$ | $I - 2m(0) - \gamma \leq \Psi \leq I - 2\sqrt{Im(0)} - \gamma$ | $\Psi \leq I - 2m(0) - \gamma$ |
| $r^*(x) = \left\{ \begin{array}{ll} 0, & x \in [0, x_1^*], \\ \frac{-c_0 - p + k}{2[m(x) - x_1^*]}, & x \in [x_1^*, x_2^*], \\ 1, & x \in [x_2^*, \gamma]. \end{array} \right.$ | $r^*(x) = \left\{ \begin{array}{ll} -\frac{c_0 - p + k}{2[m(x) - x_1^*]}, & x \in [0, x_2^*], \\ 0, & x \in [x_2^*, \gamma]. \end{array} \right.$ | $r^*(x) = 1$, $x \in [0, \gamma]$. |

**Table 3. Optimal transfer payments with $I \geq m(0)$**

| $\Psi = c_0 - \gamma + k + I - p$ | $\Psi \geq 0$ | $-m(0) - \gamma \leq \Psi < 0$ | $\Psi < -m(0) - \gamma$ |
|----------------------------------|--------------|-----------------------------|-----------------------------|
| $t^*(x)$                         | $t^*(x) = 0$, $x \in [0, \gamma]$. | $t^*(x) = \left\{ \begin{array}{ll} 0, & x \in [0, x_1^*], \\ c_0 - x_1^*, & x \in [x_1^*, \gamma]. \end{array} \right.$ | $t^*(x) = c_0 - x_1^*$, $x \in [0, \gamma]$. |

**Table 4. Optimal transfer payments with $I < m(0)$**

| $\Psi = c_0 - \gamma + k + I - p$ | $\Psi \geq 0$ | $-I - \gamma \leq \Psi < 0$ | $I - 2\sqrt{Im(0)} - \gamma < \Psi < -I - \gamma$ |
|----------------------------------|--------------|-----------------------------|-----------------------------|
| $t^*(x) = \left\{ \begin{array}{ll} 0, & x \in [0, x_1^*], \\ c_0 - x_1^*, & x \in [x_1^*, \gamma]. \end{array} \right.$ | $t^*(x) = \left\{ \begin{array}{ll} 0, & x \in [0, x_1^*], \\ c_0 - x_1^*, & x \in [x_1^*, \gamma], \\ c_0 - x_2^* + \int_{x_1^*}^{x_2^*} r^2(v)dv, & x \in [x_1^*, x_2^*], \\ c_0 - x_2^* + \int_{x_2^*}^{x_1^*} r^2(v)dv, & x \in [x_2^*, \gamma]. \end{array} \right.$ | $t^*(x) = c_0 - x_1^*$, $x \in [0, \gamma]$. |
| $I - 2m(0) - \gamma \leq \Psi \leq I - 2\sqrt{Im(0)} - \gamma$ | $\Psi \leq I - 2m(0) - \gamma$ | $\Psi \leq I - 2m(0) - \gamma$ |
| $t^*(x) = \left\{ \begin{array}{ll} c_0 - x_1^*, & x \in [0, x_1^*], \\ c_0 - x_2^* + \int_{x_1^*}^{x_2^*} r^2(v)dv, & x \in [x_1^*, x_2^*], \\ x \in [x_2^*, \gamma]. \end{array} \right.$ | $t^*(x) = c_0 - x_1^*$, $x \in [0, \gamma]$. |
switching is the all-or-nothing policy. Specifically, if the learning rate is less than $x_1^*$, the sum of the transfer payment and the switching cost is larger than the purchase cost from $S_1$, thus no switching is optimal and no transfer payment is paid. In the region where $S_2$'s learning rate is larger than the threshold $x_1^*$, the buyer takes a complete switching strategy due to the high learning ability of $S_2$. When the difference $\Psi$ is smaller than $-m(0) - \gamma$, the buyer completely switches to $S_2$ so as to minimize the total purchase cost.

Table 2 shows the optimal switching strategies when the fixed switching cost is relatively low, i.e., $I < m(0)$. When the difference $\Psi$ is in an intermediate region, i.e., $I - 2m(0) - \gamma < \Psi < -I - \gamma$, partial switching is optimal for the buyer. Specifically, for $I - 2\sqrt{Im(0)} - \gamma < \Psi < -I - \gamma$, the contract menu is separated into three regions by two threshold $x_3^*$ and $x_2^*$. The region with $x \in [0, x_3^*)$ corresponds to a lower learning ability of $S_2$, thus no switching is the buyer’s optimal strategy. The region with $x \in [x_3^*, x_2^*)$ corresponds to an intermediate learning rate of $S_2$, and it is a partial switching strategy that makes the buyer pay a minimal cost in this region. The region with $x \in (x_2^*, \infty)$ corresponds to a high learning rate of $S_2$, thus the buyer completely switches to $S_2$ to take full advantage of the learning effect from $S_2$. For $I - 2m(0) - \gamma < \Psi \leq -2\sqrt{Im(0)} - \gamma$, the optimal switching falls into two regions. The buyer will make a partial switching or complete switching strategy, depending on the learning rate of $S_2$. For $\Psi \leq I - 2m(0) - \gamma$, complete switching to $S_2$ is the optimal strategy for the buyer.

4. Benefit-sharing contract. To evaluate the performance of the principal-agent contract, we propose a commonly used contract, i.e., benefit-sharing contract (which often is in the form of revenue-sharing or profit-sharing contract) for comparison. This contract usually serves as an efficient tool for coordinating a supply chain and is widely adopted in literature and practice. In this paper, the benefit-sharing contract may be easily implemented by sharing the total purchase cost reduction with the new supplier, since the supplier switching decision will help the buyer to reduce the purchase cost. Intuitively, the buyer will benefit from switching to the new supplier once his production cost is significantly low, and the total purchase cost will decrease with the switching action. Here, we consider the total purchase cost reduction as the benefit. Thus the buyer has incentives to share the benefit from switching with the entrant supplier. Particularly, the buyer would share a proportion of the total purchase cost reduction with the entrant supplier to guide his participation. Note that with the benefit-sharing contract, the revelation principle for inducing the entrant supplier to truly report learning rate is not involved, that is, the incentive-compatibility constraint vanishes.

We assume that the buyer provides a contract $(\phi, \hat{\phi}, \hat{r})$ to $S_2$ with $\hat{r} = \frac{y}{\gamma}$, where $\hat{r} \in [0, 1]$ is the switching ratio, $y$ is the reported learning rate of $S_2$, and $\phi$ is the sharing proportion of the benefit (the total amount of cost reduction) to $S_2$, which is the buyer’s decision variable. This is a stylized Stackelberg game between the buyer (as the leader) and $S_2$ (as the follower). The sequence of events is as follows: first, the buyer announces the sharing proportion $\phi$; then, $S_2$ reports learning rate $y$ based on $\phi$ and his true learning rate $x$.

When $S_2$ reports the learning rate $y$ and chooses the contract $(\phi, \hat{\phi}, \hat{r})$, one can get the total cost reduction denoted by $\pi_B$ as follows:

$$\pi_B = p - (1 - \hat{r})p - \hat{r}(c_0 - y\hat{r}) - s(\hat{r}).$$  \hspace{1cm} (5)
For the case $c_0 - p + k + I \geq \gamma$, it can be verified that the total cost reduction $\pi_B \leq 0$ for any $y \in [0, \gamma]$, thus no switching is the optimal choice under the benefit-sharing contract. Recall that, under the principal-agent contract, the optimal switching is $y^* = 0$ for any $\phi \in \Omega$. Therefore, when the condition $c_0 - p + k + I \geq \gamma$ holds, no switching occurs under both contracts. Hereafter, we only focus on the case $c_0 - p + k + I < \gamma$.

By backward induction, we begin with solving the decision-making problem of $S_2$ to identify its response function. The profit of $S_2$ with the true learning rate $x$ denoted by $\hat{\pi}$ can be described as follows:

$$\hat{\pi}(\phi, y, x) = \hat{r}(c_0 - y\hat{r} - (c_0 - x\hat{r})) + \phi \pi_B$$

$$= \hat{r}^2(x - y) + \phi \pi_B. \quad (6)$$

It can be easily checked that there exists a threshold $\bar{y}$ so that when $y \in [0, \bar{y}]$, $\pi_B \leq 0$. Therefore, once $S_2$ reports a learning rate $y \leq \bar{y}$, the buyer will not switch to $S_2$, i.e., $\hat{r} = 0$. Here, $\bar{y}$ solves the equation $\pi_B = 0$. Accordingly, the optimization problem of $S_2$ is formulated as follows:

$$\max_{y \in [0, \gamma]} \hat{\pi}(\phi, y, x). \quad (7)$$

By solving the optimization problem (7), we obtain the optimal response function $y^*(\phi, x)$ of $S_2$ as described in the following proposition.

**Proposition 2.** The optimal response function $y^*(\phi, x)$ of $S_2$ is obtained as follows:

$$y^*(\phi, x) = \begin{cases} 
\gamma, & A_1 \geq 0, A_2 \geq 0, \\
\frac{x + \sqrt{\Theta}}{3(1 - \phi)}, & A_1 < 0, A_3 \geq 0, A_4 \geq 0, \\
\bar{y}, & \text{otherwise},
\end{cases} \quad (8)$$

where

$$\Theta = x^2 + 3\phi(1 - \phi)(p - c_0 - k)\gamma,$$

$$A_1 = -3(1 - \phi)\gamma + 2x + \phi(p - c_0 - k),$$

$$A_2 = -(1 - \phi)\gamma + x + \phi(p - c_0 - k) - \phi I,$$

$$A_3 = (2x - \sqrt{\Theta})(x + \sqrt{\Theta})^2 + 9\phi(1 - \phi)(p - c_0 - k)\gamma(x + \sqrt{\Theta}) - 27\phi(1 - \phi)^2 \gamma^2 I,$$

$$A_4 = (x + \sqrt{\Theta})^3 + 9(1 - \phi)^2(p - c_0 - k)\gamma(x + \sqrt{\Theta}) - 27(1 - \phi)^3 \gamma^2 I.$$

The expected total cost reduction of the buyer, denoted as $\hat{R}(\phi)$, is described as

$$\hat{R}(\phi) = (1 - \phi) \int_0^\gamma (p - (1 - \hat{r})p - \hat{r}(c_0 - y\hat{r}) - s(\hat{r})) f(x) dx. \quad (9)$$

Then, substituting $y^*(\phi, x)$ into (9), we can get the analytical expression of $\hat{R}(\phi)$ with respect to $\phi$ in the following proposition.

**Proposition 3.** The expected total cost reduction $\hat{R}(\phi)$ with respect to $\phi$ is obtained as follows:
i. When $p - c_0 - k \geq \frac{3}{2} I$, 

\[
\hat{R}(\phi) = \begin{cases} 
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx, & 0 < \phi < \frac{\gamma}{\gamma + p - c_0 - k}, \\
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx & (\gamma + p - c_0 - k - I), \\
(1 - \phi)(\gamma + p - c_0 - k - I), & \frac{\gamma}{\gamma + p - c_0 - k} \leq \phi < 1.
\end{cases}
\]

ii. When $I \leq p - c_0 - k < \frac{3}{2} I$, 

\[
\hat{R}(\phi) = \begin{cases} 
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx, & 0 < \phi < \frac{\gamma}{\gamma + p - c_0 - k}, \\
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx & (\gamma + p - c_0 - k - I), \\
(1 - \phi)(\gamma + p - c_0 - k - I), & \frac{\gamma}{\gamma + p - c_0 - k} \leq \phi < 1.
\end{cases}
\]

iii. When $0 \leq p - c_0 - k < I$, 

\[
\hat{R}(\phi) = \begin{cases} 
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx, & 0 < \phi < \frac{\gamma}{\gamma + p - c_0 - k}, \\
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx & (\gamma + p - c_0 - k - I), \\
(1 - \phi)(\gamma + p - c_0 - k - I), & \frac{\gamma}{\gamma + p - c_0 - k} \leq \phi < 1.
\end{cases}
\]

iv. When $p - c_0 - k < 0$, 

\[
\hat{R}(\phi) = \begin{cases} 
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx, & 0 < \phi < \frac{\gamma}{\gamma + p - c_0 - k}, \\
(1 - \phi) \left[ \frac{\hat{\gamma}}{27\gamma^3(1-\phi)} + \frac{\gamma}{3\gamma(1-\phi)} \right] \int f(x) \, dx & (\gamma + p - c_0 - k - I), \\
(1 - \phi)(\gamma + p - c_0 - k - I), & \frac{\gamma}{\gamma + p - c_0 - k} \leq \phi < 1.
\end{cases}
\]

Here, $\hat{x} = \max\{\hat{x}_3, \hat{x}_4, 0\}$, $\hat{x}_3$ is obtained by solving $A_3 = 0$, and $\hat{x}_4$ is obtained by solving $A_4 = 0$. 
It should be mentioned that the concavity of $\hat{R}(\phi)$ with respect to $\phi$ is hardly verified by an analytic approach. We have done a large number of numerical studies and found the unimodality of $\hat{R}(\phi)$ with respect to $\phi$. The optimal sharing proportion $\phi^* = \arg\max\{\hat{R}(\phi)\}$ can be obtained numerically.

5. Numerical studies. In this section, we focus on the comparison between the principal-agent contract and the benefit-sharing contract. Note that the IC constraint disappears in the benefit-sharing contract. Therefore, the discussion on when should the buyer develop the IC constraint is of great importance. First, we examine the switching decisions and expected total costs under both contracts.

We use a specific distribution function to illustrate the main results. Specifically, the learning rate of $S_2$ is uniformly distributed on the interval $[0, \gamma]$, i.e., $X \sim U[0, \gamma]$. Then, the density function is $f(x) = \frac{1}{\gamma}, 0 \leq x \leq \gamma$, and the cumulative distribution function is $F(x) = \frac{x}{\gamma}, 0 \leq x \leq \gamma$. Thus, the Mills ratio $m(x)$ is calculated as $m(x) = \frac{1-F(x)}{f(x)} = \gamma - x, x \in [0, \gamma]$.

Consider the following parameter setting in the subsequent analysis: $c_0 = 1.6, p = 3, k = 0.2, I = 0.5, \gamma = 1$. According to Theorem 1, under the principal-agent contract, the optimal switching ratio is

$$r^*(x) = \begin{cases} 
0, & 0 \leq x < 0.14, \\
\frac{3}{\gamma(1-2x)}, & 0.14 \leq x < 0.2, \\
1, & 0.2 \leq x \leq 1. 
\end{cases}$$

The minimal total cost is achieved as $E(C) = 2.2772$. Figure 1 shows the optimal switching ratio $r^*(x)$ and transfer payment $t^*(x)$.
By virtue of the benefit-sharing contract presented in Section 4, we can get

\[
\hat{R}(\phi) = \begin{cases} 
1 - \phi & \text{if } \phi < 0.238 \\
\int_{\hat{x}}^{1} \frac{(x + \sqrt{x^2 + 3.6 \phi (1 - \phi)}^3 - 0.5) \, dx}{5(1 - \phi)} & \text{if } 0.238 \leq \phi < 0.714 \\
1.7(1 - \phi) & \text{if } \phi \geq 0.714,
\end{cases}
\]

where \(\hat{x}\) is calculated as \(\hat{x} = 0.01\).

As shown in Figure 2, \(\hat{R}(\phi)\) is unimodal with respect to \(\phi\), and the equilibrium benefit-sharing proportion \(\phi^* = 0.494\) with \(\hat{R}(\phi^*) = 0.6974\). The corresponding equilibrium strategies are rendered as

\[
y^*(x) = r^*(x) = \begin{cases} 
0.6588(x + \sqrt{x^2 + 0.8998}) & \text{if } x < 0.4626, \\
1 & \text{if } x \geq 0.4626.
\end{cases}
\]

The total cost in expectation under the benefit-sharing contract is obtained as \(\hat{E}(C) = 2.3026\), which is larger than that under the principal-agent contract. Therefore, it is concluded that the principal-agent contract outperforms the benefit-sharing contract.

To gain more managerial insights, we carry out sensitivity analysis of the expected total cost with respect to the key parameters. When one parameter varies, the other parameters keep unchanged. Moreover, to explore the value of proposing the IC constraint, we compare the expected total costs under the two contracts. Denote the difference of the expected total cost between the principal-agent contract and the benefit-sharing contract as \(\Delta E\), i.e., \(\Delta E = \hat{E}(C) - E(C)\).

As shown in Table 5, when switching occurs, the principal-agent contract always dominates the benefit-sharing contract. In other words, the buyer will always obtain a lower expected total cost when employing the principal-agent contract. The advantage of using the principal-agent contract is highlighted with a relatively small initial production cost of \(S_2\). As \(c_0\) increases to 2.88, the buyer will stay with \(S_1\).
Table 5. Sensitivity analysis with respect to system parameters $c_0, p, k, I$.

|       | -80%  | -60%  | -40%  | -20%  | 0     | 20%   | 40%   | 60%   | 80%   |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $E(C)$ | 1.0200 | 1.3400 | 1.6982 | 2.2772 | 2.5727 | 2.6831 | 2.8631 | 2.9559 |       |
| $\hat{E}(C)$ | 1.3609 | 1.6088 | 1.8492 | 2.0805 | 2.3026 | 2.5406 | 2.7265 | 2.8656 | 2.9559 |
| $\Delta E$ | 0.3409 | 0.2688 | 0.1510 | 0.1030 | 0.0254 | 0.0167 | 0.0074 | 0.0025 | 0     |

and there is no difference between the two contracts. Moreover, the total cost of the buyer will increase when the initial cost of $S_2$ increases under both the two contracts. Since $c_0$, $p$, and $k$ always appear as a whole in both the two contracts, the impacts of $p$ and $k$ can be analyzed analogously. In addition, as the fixed component of the switching cost increases, the total cost of the buyer will increase under both the two contracts. From the above discussion, we can draw the conclusion that the principal-agent contract will always be a better choice for the buyer from the ex-ante perspective.

Under the principal-agent contract, the total cost reduction denoted as $T_p(x)$ and can be written as

$$T_p(x) = p - (1 - r^*_x)p - r^*_x(c_0 - x r^*_x) - s(r^*_x).$$

Under the benefit-sharing contract, the total cost reduction denoted as $T_b(x)$ and can be written as

$$T_b(x) = p - (1 - \hat{r}^*_x)p - \hat{r}^*_x(c_0 - x \hat{r}^*_x) - s(\hat{r}^*_x).$$
Thus the difference of total cost reduction between the two contracts is given by
\[
\Delta T(x) = T_p(x) - T_b(x).
\]

The equilibrium profit \( \pi^*(x, x) \), \( \hat{\pi}^*(x) \) of \( S_2 \) and the difference of total cost reduction \( \Delta T(x) \) between the two contracts are presented in Figure 3. Note that there may exist strategic reporting of \( S_2 \) under the benefit-sharing contract. When \( x \in [0, 0.35] \), it can be seen from Figure 3 that \( \hat{\pi}^*(x) > \pi^*(x, x) \), which implies that \( S_2 \) benefits from the benefit-sharing contract for a relatively small learning rate. The equilibrium learning rate \( y \) submitted is determined by the true learning rate of \( S_2 \) and the buyer’s benefit-sharing proportion. Note that the advantage of \( S_2 \) under the benefit-sharing contract achieves the peak at \( x_1 = 0.14 \). For the learning rate greater than 0.14, this advantage is weakened as the learning rate increases and there is only a tiny difference between the two contracts for \( x \in [0.35, 1] \).

![Figure 3. S2’s profit and the total cost reduction difference under the two contracts via x.](image)

From an ex-post perspective, for the case \( x \in [0, 0.14] \) (in which there is no switching under the principal-agent contract), the buyer prefers the benefit-sharing contract which is confirmed by the fact \( \hat{\pi}^*(x) - \pi^*(x, x) < -\Delta T(x) \) shown in Figure 3. For the case \( x \in [0.14, 0.46] \), \( \Delta T(x) > 0 \) indicates that the system consisting of the buyer and \( S_2 \) benefits more from the principal-agent mechanism compared to the benefit-sharing contract. With the fact that the increased payoff \( \hat{\pi}^*(x) - \pi^*(x, x) \) to \( S_2 \) is less than \( \Delta T(x) \), it is obvious that the buyer is better off under the principal-agent contract. When the true leaning rate \( x \) of \( S_2 \) falls into the interval \( x \in [0.46, 1] \), the buyer takes a completely switching strategy under both contracts. The buyer is slightly better off when applying the benefit-sharing contract from an ex-post perspective, since the total cost reductions are equal under the two contracts while \( \hat{\pi}^*(x) < \pi^*(x, x) \). For instance, with learning rate at point \( x = 0.5 \), \( \hat{\pi}^*(x) = 0.34 \) and \( \pi^*(x, x) = 0.35 \).

To sum up, our results reveal that the principal-agent contract brings a lower expected total cost than the benefit-sharing contract. From an ex-post perspective,
the switching strategy on the basis of the principal-agent theory dominates the one based on the benefit-sharing contract when the learning rate is in an intermediate region, while the result is opposite for a relatively large or small learning rate.

6. **Concluding remarks.** In this paper, a supplier switching model with considering learning effect as asymmetric information is studied. To minimize the total purchase cost of the buyer, it is of great importance to motivate the entrant supplier to report his true learning ability. The introduction of the IC constraint guarantees the revelation of true information. We first formulate the incentive model under a principal-agent framework, and then propose a benefit-sharing contract for comparison. The optimal switching decisions under both contracts are obtained. Moreover, the impacts of the switching cost and the learning ability on the switching decision are investigated. Furthermore, we compare the two contracts both at the beginning of the planning stage and from an ex-post perspective.

The results in this paper have the following theoretical contributions and managerial implications: (1) With characterizing the production cost of the entrant supplier in a detailed way, we establish the switching model with considering learning effect and volume-dependent switching cost in a principal-agent framework, and obtain the optimal switching decision. (2) We investigate the impacts of the switching cost and the learning effect on the switching decision of the buyer. It is shown that only when the fixed component of the switching cost is relatively low, adopting a partial switching strategy may be better off for the buyer. Otherwise, the buyer should take an all-or-nothing switching strategy or no switching strategy. Moreover, the buyer prefers to make more switching as the learning ability of the entrant supplier increases. (3) The performance of different contracts on supplier switching is studied in this paper. Specifically, we propose a benefit-sharing contract to compare with the principal-agent contract both at the beginning of the planning stage and from an ex-post perspective. We find that the principal-agent contract almost completely brings a lower expected total cost than the benefit-sharing contract. From an ex-post perspective, the switching strategy on the basis of the principal-agent theory dominates the one based on the benefit-sharing contract when the learning rate is in an intermediate region, while the result is opposite for a relatively large or small learning rate.

There are several potential extensions to our research. The reaction of the incumbent supplier can be incorporated, since the incumbent supplier may decrease his price to avoid the loss of the order quantity from the buyer. In addition, the production cost of the entrant supplier with learning ability can be described in a multi-period setting which is of great interest for future study. Moreover, other forms of the switching cost function may also be considered in future work.

**Appendix.**

**Proof of Lemma 1.**

*Proof.* Note that \( g(x) = \frac{1 - F(x) - xf(x)}{f(x)} = m(x) - x \), since \( g(0) = \frac{1}{f(0)} > 0, g(\gamma) = -\gamma < 0 \), there exists at least one \( x \) denoted by \( \bar{x} \) that solves the equation \( g(x) = 0 \).

Taking the derivative of \( g(x) \), we can get \( g'(x) = m'(x) - 1 \leq 0 \). Therefore, for \( x \in [0, \bar{x}), g(x) > 0 \), for \( x \in (\bar{x}, \gamma], g(x) < 0 \), and the equation \( g(x) = 0 \) has a unique solution. \( \square \)
Proof of Proposition 1.

Proof. Note that the incentive-compatibility constraint in (3) indicates \( \frac{\partial \pi(x,y)}{\partial y} \bigg|_{y=x} = 0 \) according to the first-order condition. Since \( \frac{\partial \pi(x,y)}{\partial y} = t'(y) - c_0 r'(y) + 2x r(y) r'(y) \), one has
\[
t'(x) - c_0 r'(x) + 2x r(x) r'(x) = 0. \tag{11}
\]
Then, differentiating (11) with respect to \( x \), one can get
\[
t''(x) = c_0 r''(x) - 2x r(x) r''(x) - 2r(x) r'(x) - 2x r'(x))^2. \tag{12}
\]
Note that the function \( \pi(x,y) \) in \( y \) peaks at \( y = x \), one can obtain \( \frac{\partial^2 \pi(x,y)}{\partial y^2} \bigg|_{y=x} \leq 0 \), that is \( t''(x) - c_0 r''(x) + 2x r'(x))^2 + 2x r(x) r''(x) \leq 0 \). With (12), we can get
\[
r'(x) \geq 0. \tag{13}
\]
Additionally, combining (11) and (13), we can get
\[
t(y) - t(x) = \int_x^y t'(r)dr = \int_x^y r'(z)(c_0 - 2z r(z))dz \\
= c_0(r(y) - r(x)) - \int_x^y 2z r(z) r'(z)dz \\
\leq c_0(r(y) - r(x)) - \int_x^y 2x r(x) r'(z)dz \\
= c_0(r(y) - r(x)) - x[r^2(y) - r^2(x)],
\]
which implies
\[
t(x) - r(x)(c_0 - x r(x)) = \pi(x,x) \geq t(y) - r(y)(c_0 - x r(y)) = \pi(x,y).
\]
Therefore, the incentive-compatibility constraint is equivalent to equations (11) and (13). Then, with \( \frac{\partial \pi(x,y)}{\partial y} \bigg|_{y=x} = 0 \), we can obtain
\[
\frac{d\pi(x,x)}{dx} = \left( \frac{\partial \pi(x,y)}{\partial y} + \frac{\partial \pi(x,y)}{\partial x} \right) \bigg|_{y=x} = r^2(x) \geq 0, \tag{14}
\]
which means \( \pi(x,x) \) is a non-decreasing function with respect to \( x \). Thus, for any \( x \in [0,\gamma] \), \( \pi(x,x) \geq 0 \) if the condition \( \pi(0,0) \geq 0 \) holds, i.e., the participation constraint PC is guaranteed by \( \pi(0,0) \geq 0 \).

It can be easily checked that the buyer’s objective function (2) increases in \( t(x) \). Thus it is optimal for the buyer to set a binding PC, that is \( t(0) = r(0)c_0 \).

Integrating (11) from 0 to \( x \), one can get
\[
\int_0^x t'(r)dr = \int_0^x r'(z)(c_0 - 2z r(z))dz \\
= c_0 r(x) - c_0 r(0) - z r^2(z)\bigg|_0^x + \int_0^x r^2(z)dz \\
= c_0 r(x) - c_0 r(0) - x r^2(x) + \int_0^x r^2(z)dz,
\]
which by virtue of \( t(0) = r(0)c_0 \) renders
\[
t(x) = c_0 r(x) - x r^2(x) + \int_0^x r^2(z)dz. \tag{15}
\]
Then, combining (15) with \( E(C) \) in (2) and taking integration by parts, one can get

\[
E(C) = \int_0^\gamma \left[ h(x) + (1 - r(x))p + s(r(x)) \right] f(x) dx
\]

\[
= \int_0^\gamma \left[ |cr(x) - x^2r^2(x) + \int_0^x r^2(z) dz + (1 - r(x))p + s(r(x)) \right] f(x) dx
\]

\[
= \int_0^\gamma \left[ \left( c_0r(x) - x^2r^2(x) + (1 - r(x))p + s(r(x)) \right) - \frac{F(x)}{f(x)} \right] f(x) + r^2(x) \right] dx
\]

\[
= \int_0^\gamma \left[ \left( 1 - F(x) - x^2f(x) \right) r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x)) \right] f(x) dx
\]

Therefore, the original problem (3) turns to

\[
\begin{align*}
\min_{r(x)} & \int_0^\gamma \left[ \frac{1 - F(x) - x^2f(x)}{f(x)} r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x)) \right] f(x) dx \\
\text{subject to:} & \\
& r'(x) \geq 0, \\
& 0 \leq r(x) \leq 1.
\end{align*}
\]

(16)

**Proof of Theorem 1.**

Proof. To solve the optimization problem (4), we first omit the constraints. It should be mentioned that the necessary condition for the functional

\[
\int_0^\gamma \left[ \frac{1 - F(x) - x^2f(x)}{f(x)} r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x)) \right] f(x) dx
\]

achieving its minimum is that \( r(x) \) satisfies the Euler-Lagrange equation. That is,

\[
\frac{\partial}{\partial r} \left[ \frac{1 - F(x) - x^2f(x)}{f(x)} r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x)) \right] f(x) = 0.
\]

Note that \( f(x) \) is independent with \( r(x) \). Therefore, we minimize the term

\[
\frac{1 - F(x) - x^2f(x)}{f(x)} r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x))
\]

in the following analysis to find the optimal \( r^*(x) \).

Denote \( l(r(x)) = g(x) r^2(x) + (c_0 - p + k) r(x) + p + I\operatorname{sgn}(r(x)) \). Using the first-order condition, we can get \( \bar{r}(x) = -\frac{c_0 - p + k}{2g(x)} \). From Proposition 2, there exists a unique \( \bar{x} \) that solves the equation \( g(x) = 0 \), and for \( x \in [0, \bar{x}] \), \( g(x) > 0 \), for \( x \in (\bar{x}, \gamma] \), \( g(x) < 0 \). It is shown that \( l(r(x)) \) is convex with respect to \( r \), for \( r \in [0, \bar{x}] \), and is concave with respect to \( r \), for \( r \in (\bar{x}, \gamma] \). Therefore, for \( x \in [0, \bar{x}] \), the minimum total cost can be obtained at \( r = 0 \), \( r = 1 \), or \( r = \bar{r} \). For \( x \in [\bar{x}, \gamma] \), the minimum value can be achieved at the boundary point \( r = 0 \) or \( r = 1 \). It can be verified that for \( x \leq x^*_1 \), \( l(0) \leq l(1) \), for \( x > x^*_1 \), \( l(1) < l(0) \), here \( x^*_1 \) is the unique solution of the equation \( m(x) - x + c_0 - p + k + I = 0 \). For \( x \leq x^*_2, \bar{r} \leq 1, \) for \( x > x^*_2, \bar{r} > 1, \) here \( x^*_2 \) is the unique solution of the equation \( m(x) - x + \frac{c_0 - p + k}{2} = 0 \). For \( x \leq x^*_3, \)
\( l(0) \leq l(r) \), for \( x > x_3^* \), \( l(0) > l(r) \), here \( x_3^* \) is the unique solution of the equation\n\[ m(x) - x - \frac{(c_0 - p + k)^2}{4I} = 0. \]

Consider the case \( I < m(0) \), a similar procedure can be applied to obtain the solution in the case \( I \geq m(0) \).

i) For the case \( \Psi \geq 0 \), \( l(0) \leq l(1) \).

When \( c_0 - p + k \geq 0 \), for \( x \in [0, \bar{x}] \), \( \bar{r}(x) = -\frac{c_0 - p + k}{2[m(x) - x]} \leq 0 \), thus, the minimum cost is obtained at \( r = 0 \).

When \( c_0 - p + k < 0 \), it can be verified that \( -\frac{c_0 - p + k}{2} > \frac{(c_0 - p + k)^2}{4I} > 0 > -(c_0 - p + k + I) \), thus, we can get \( x_2^* < x_3^* < \bar{x} < x_1^* \).

Therefore, the optimal switching is given by\n\[ r^*(x) = 0, \quad 0 \leq x \leq \gamma. \]

ii) For the case \(-I - \gamma \leq \Psi < 0\).

When \( c_0 - p + k \geq 0 \), for \( x \in [0, \bar{x}] \), \( \bar{r}(x) = -\frac{c_0 - p + k}{2[m(x) - x]} \leq 0 \), thus, the minimum cost is obtained at \( r = 0 \). The optimal switching is given by\n\[ r^*(x) = \begin{cases} 0, & 0 \leq x < x_1^*, \\
1, & x_1^* \leq x \leq \gamma. \end{cases} \]

When \( c_0 - p + k < 0 \), it can be verified that \( -\frac{c_0 - p + k}{2} > \frac{(c_0 - p + k)^2}{4I} > -(c_0 - p + k + I) \), thus, \( x_2^* < x_3^* < x_1^* \). The optimal switching is given by\n\[ r^*(x) = \begin{cases} 0, & 0 \leq x < x_1^*, \\
1, & x_1^* \leq x \leq \gamma. \end{cases} \]

Next, for the case \( \Psi < -I - \gamma \), it can be verified that \( \frac{(c_0 - p + k)^2}{4I} > -(c_0 - p + k + I) > -\frac{c_0 - p + k}{2} > 0 \), then, we can get \( x_3^* < x_1^* < x_2^* \).

iii) If \( c_0 - p + k > -2\sqrt{Im(0)} \), then \( x_3^* > 0 \), therefore, the optimal switching is given by\n\begin{equation}
\begin{aligned}
r^*(x) = \begin{cases} 0, & 0 \leq x < x_3^*, \\
-\frac{c_0 - p + k}{2[m(x) - x]}, & x_3^* \leq x < x_2^*, \\
1, & x_2^* \leq x \leq \gamma. \end{cases}
\end{aligned}
\end{equation}

iv) When \( -2m(0) < c_0 - p + k \leq -2\sqrt{Im(0)} \), \( x_3^* \leq 0 < x_2^* \), and the optimal switching is given by\n\begin{equation}
\begin{aligned}
r^*(x) = \begin{cases} -\frac{c_0 - p + k}{2[m(x) - x]}, & 0 \leq x < x_2^*, \\
1, & x_2^* \leq x \leq \gamma. \end{cases}
\end{aligned}
\end{equation}

v) For \( c_0 - p + k < -2m(0) \), \( x_2^* < 0 \). The optimal switching is given by\n\[ r^*(x) = 1, \quad 0 \leq x \leq \gamma. \]

It can be verified that the optimal switching obtained above satisfies the constraints. \( \square \)

**Proof of Proposition 2.**

**Proof.** The optimization problem of the entrant supplier can be written as follows:
\[ \max_{y \in [y, \gamma]} \hat{\pi}(\phi, y, x) = -\frac{(1 - \phi)}{\gamma^2} y^3 + \frac{x}{\gamma^2} y^2 + \frac{\phi}{\gamma} (p - c_0 - k)y - \phi I \text{sgn}(y - \bar{y}). \]
Then, taking derivative of $\hat{\pi}(\phi, y, x)$ with respect to $y$, we can get

$$\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y} = - \frac{3(1 - \phi)}{\gamma^2} y^2 + \frac{2x}{\gamma^2} y + \frac{\phi}{\gamma} (p - c_0 - k).$$

We first consider the case $\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y} \bigg|_{y=\gamma} \geq 0$ and $\hat{\pi}(\phi, \gamma, x) \geq 0$. Note that $\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y} \bigg|_{y=\gamma} \geq 0$ and $\hat{\pi}(\phi, \gamma, x) \geq 0$ are equivalent to the conditions $A_1 = -3(1 - \phi)\gamma + 2x + \phi(p - c_0 - k) \geq 0$ and $A_2 = -(1 - \phi)\gamma + x + \phi(p - c_0 - k) - \phi I \geq 0$. In this case, if $p - c_0 - k \geq 0$, it can be easily checked that for $y \in [\hat{y}, \gamma]$, $\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y} \geq 0$.

Therefore, $\hat{\pi}(\phi, y, x)$ is increasing in $y$, the maximum is obtained at the point $y = \gamma$. If $p - c_0 - k \geq 0$, it can be verified that $\hat{\pi}(\phi, y, x)$ firstly decreases and then increases in $y$. It should be mentioned that $\hat{\pi}(\phi, y, x) < 0$ on the decreasing interval. Thus, the maximum is achieved at the point $y = \gamma$.

Next, we consider the case $\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y} \bigg|_{y=\gamma} < 0$, i.e., $A_1 < 0$. Note that one root of $\frac{\partial \hat{\pi}(\phi, y, x)}{\partial y}$ is negative and another is $y = \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}$, here, $\Theta = x^2 + 3\phi(1 - \phi)(p - c_0 - k)\gamma$. If $p - c_0 - k \geq 0$, when $\hat{y} \leq \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}$ holds, it can be easily verified that $\hat{\pi}(\phi, y, x)$ is increasing in $y$ for $y \in [\hat{y}, \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}]$ and decreasing in $y$ for $y \in [\frac{x + \sqrt{3\Theta}}{3(1 - \phi)}, \gamma]$. Note that $\hat{y} \leq \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}$ and $\hat{\pi}(\phi, \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}, x)$ are equivalent to the conditions $A_4 \geq 0$ and $A_3 \geq 0$. Therefore, when $A_3 \geq 0$ and $A_4 \geq 0$ hold, the maximum is obtained at the point $y = \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}$. Otherwise, $y^*(\phi, x) = \hat{y}$. Similar analysis can be done for $p - c_0 - k < 0$.

According to the above analysis, the optimal response function $y^*(\phi, x)$ of $S_2$ can be obtained as follows:

$$y^*(\phi, x) = \begin{cases} \gamma, & A_1 \geq 0, A_2 \geq 0, \\ \frac{x + \sqrt{3\Theta}}{3(1 - \phi)}, & A_1 < 0, A_3 \geq 0, A_4 \geq 0, \\ \hat{y}, & \text{otherwise}, \end{cases}$$

where $\Theta = x^2 + 3\phi(1 - \phi)(p - c_0 - k)\gamma$, $A_1 = -3(1 - \phi)\gamma + 2x + \phi(p - c_0 - k)$, $A_2 = -(1 - \phi)\gamma + x + \phi(p - c_0 - k) - \phi I$, $A_3 = (2x - \sqrt{3\Theta})(x + \sqrt{3\Theta})^2 + 9\phi(1 - \phi)(p - c_0 - k)\gamma(x + \sqrt{3\Theta}) - 27\phi(1 - \phi)I^2$, $A_4 = (x + \sqrt{3\Theta})^3 + 9(1 - \phi^2)(p - c_0 - k)\gamma(x + \sqrt{3\Theta}) - 27(1 - \phi^3)I^2$. 

Proof of Proposition 3.

**Proof.** The expected total cost reduction of the buyer $\hat{R}(\phi)$ can be written as follows:

$$\hat{R}(\phi) = (1 - \phi) \int_0^\gamma (p - (1 - \hat{r})p - \hat{r}(c_0 - y\hat{r}) - s(\hat{r}))f(x)dx. \quad (19)$$

We divide the range of $p - c_0 - k$ into four regions, i.e., i: $p - c_0 - k \geq \frac{3}{2}I$, ii: $I \leq p - c_0 - k < \frac{3}{2}I$, iii: $0 \leq p - c_0 - k < I$ and iv: $p - c_0 - k \leq 0$. Denote
\( \hat{x} = \max\{\bar{x}_3, \bar{x}_4, 0\}, \) \( \bar{x}_3 \) is obtained by solving \( A_3 = 0 \), and \( \bar{x}_4 \) is obtained by solving \( A_4 = 0 \).

We first consider the case \( p - c_0 - k \geq \frac{3}{2}I \). It can be easily verified that for \( 0 < \phi < \frac{\gamma}{3\gamma + p - c_0 - k} \), \( A_1 < 0 \). Thus, for \( x \in [\hat{x}, \gamma] \), \( y^*(\phi, x) = \frac{x + \sqrt{\Theta}}{3(1 - \phi)} \), for \( x \in [0, \hat{x}] \), \( y^*(\phi, x) = \bar{y} \), no switching occurs. Substituting \( y^*(\phi, x) = \frac{x + \sqrt{\Theta}}{3(1 - \phi)} \) into (19), we can get for \( 0 < \phi < \frac{\gamma}{3\gamma + p - c_0 - k} \), \( \hat{R}(\phi) = (1 - \phi) \int_{\gamma}^{\hat{x}} \left[ \frac{(x + \sqrt{\Theta})^3}{27\gamma^2(1 - \phi)^3} + \frac{(x + \sqrt{\Theta})(p - c_0 - k)}{3\gamma(1 - \phi)} - I \right] f(x)dx \).

For \( \frac{\gamma}{3\gamma + p - c_0 - k} \leq \phi < \frac{3\gamma}{3\gamma + p - c_0 - k} \), the optimal response function \( y^*(\phi, x) \) can be obtained as follows:

\[
y^*(\phi, x) = \begin{cases} 
\gamma, & x \geq \frac{3\gamma - \phi(3\gamma + p - c_0 - k)}{2} \\
\bar{y}, & x \in [\hat{x}, \frac{3\gamma - \phi(3\gamma + p - c_0 - k)}{2}], \\
\frac{\gamma}{3(1 - \phi)}, & \text{otherwise},
\end{cases}
\]

Then, substituting (20) into (19), we can get the expression of \( \hat{R}(\phi) \). Specifically, for \( \frac{\gamma}{3\gamma + p - c_0 - k} \leq \phi < \frac{3\gamma}{3\gamma + p - c_0 - k} \),

\[
\hat{R}(\phi) = (1 - \phi) \int_{\gamma}^{\hat{x}} \left[ \frac{(x + \sqrt{\Theta})^3}{27\gamma^2(1 - \phi)^3} + \frac{(x + \sqrt{\Theta})(p - c_0 - k)}{3\gamma(1 - \phi)} - I \right] f(x)dx + (1 - \phi) \int_{\gamma}^{\hat{x}} \left( \gamma + p - c_0 - k - I \right) f(x)dx
\]

Using integration by parts on the second term of \( \hat{R}(\phi) \), we can get

\[
\hat{R}(\phi) = (1 - \phi) \int_{\gamma}^{\hat{x}} \left[ \frac{(x + \sqrt{\Theta})^3}{27\gamma^2(1 - \phi)^3} + \frac{(x + \sqrt{\Theta})(p - c_0 - k)}{3\gamma(1 - \phi)} - I \right] f(x)dx + (1 - \phi) \left[ - F(\frac{3\gamma - \phi(3\gamma + p - c_0 - k)}{2}) \right] (\gamma + p - c_0 - k - I).
\]

For \( \frac{3\gamma}{3\gamma + p - c_0 - k} \leq \phi < 1 \), \( A_1 \geq 0 \). In addition, it can be verified that with the condition \( p - c_0 - k \geq \frac{3}{2}I \), \( A_2 \geq 0 \) always holds. Therefore, for \( \frac{3\gamma}{3\gamma + p - c_0 - k} \leq \phi < 1 \), \( y^*(\phi, x) = \gamma \). Then substituting the optimal response function \( y^*(\phi, x) = \gamma \) into (19), we can get

\[
\hat{R}(\phi) = (1 - \phi) \int_{\gamma}^{\hat{x}} (\gamma + p - c_0 - k - I) f(x)dx = (1 - \phi) (\gamma + p - c_0 - k - I).
\]

From the above analysis, we can get the expression of \( \hat{R}(\phi) \) for the case \( p - c_0 - k \geq \frac{3}{2}I \) as follows:

\[
\hat{R}(\phi) = \begin{cases} 
(1 - \phi) \int_{\gamma}^{\hat{x}} \left[ \frac{(x + \sqrt{\Theta})^3}{27\gamma^2(1 - \phi)^3} + \frac{(x + \sqrt{\Theta})(p - c_0 - k)}{3\gamma(1 - \phi)} - I \right] f(x)dx, & 0 < \phi < \frac{\gamma}{3\gamma + p - c_0 - k}, \\
(1 - \phi) \int_{\gamma}^{\hat{x}} \left[ \frac{(x + \sqrt{\Theta})^3}{27\gamma^2(1 - \phi)^3} + \frac{(x + \sqrt{\Theta})(p - c_0 - k)}{3\gamma(1 - \phi)} - I \right] f(x)dx + (1 - \phi) \left[ - F(\frac{3\gamma - \phi(3\gamma + p - c_0 - k)}{2}) \right] (\gamma + p - c_0 - k - I), & \frac{\gamma}{3\gamma + p - c_0 - k} \leq \phi < \frac{3\gamma}{3\gamma + p - c_0 - k}, \\
(1 - \phi) (\gamma + p - c_0 - k - I), & \frac{3\gamma}{3\gamma + p - c_0 - k} \leq \phi < 1.
\end{cases}
\]

The similar method can be used in other three cases, we omit detailed computation process here. \( \square \)
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REFERENCES

[1] P. S. Adler and K. B. Clark, *Behind the learning curve: A sketch of the learning process*, *Management Science*, 37 (1991), 267–281.
[2] K. J. Arrow, *The economic implications of learning by doing*, *Review of Economic Studies*, 29 (1962), 155–173.
[3] M. Ben-daya and M. Hariga, *Lead-time reduction in a stochastic inventory system with learning consideration*, *International Journal of Production Research*, 41 (2003), 571–579.
[4] L. E. Bygballe, *Toward a conceptualization of supplier-switching processes in business relationships*, *Journal of Purchasing and Supply Management*, 23 (2017), 40–53.
[5] C. K. Chen, C. C. Lo and Y. X. Liao, *Optimal lot size with learning consideration on an imperfect production system with allowable shortages*, *International Journal of Production Economics*, 113 (2008), 459–469.
[6] M. Cheng, S. Xiao and G. Liu, *Single-machine rescheduling problems with learning effect under disruptions*, *Journal of Industrial & Management Optimization*, 14 (2018), 967–980.
[7] P. Dasgupta and J. Stiglitz, *Learning-by-doing, market structure and industrial and trade policies*, *Oxford Economic Papers*, 40 (1988), 246–268.
[8] J. S. Demski, D. E. M. Sappington and P. T. Spiller, *Managing Supplier Switching*, *The RAND Journal of Economics*, 18 (1987), 77–97.
[9] L. Feng and Y. L. Chan, *Joint pricing and production decisions for new products with learning curve effects under upstream and downstream trade credits*, *European Journal of Operational Research*, 272 (2019), 905–913.
[10] G. Friedl and S. M. Wagner, *Supplier development or supplier switching?* *International Journal of Production Research*, 50 (2012), 3066–3079.
[11] D. Fudenberg and J. Tirole, *Capital as a commitment: Strategic investment to deter mobility*, *Journal of Economic Theory*, 31 (1983), 227–250.
[12] A. Gamble, E. A. Juliussson and T. Gärling, *Consumer attitudes towards switching supplier in three deregulated markets*, *The Journal of Socio-Economics*, 38 (2009), 814–819.
[13] B. C. Giri and C. H. Glock, *A closed-loop supply chain with stochastic product returns and worker experience under learning and forgetting*, *International Journal of Production Research*, 55 (2017), 6760–6778.
[14] J. B. Heide and A. M. Weiss, *Vendor consideration and switching behavior for buyers in high-technology markets*, *Journal of Marketing*, 59 (1995), 30–43.
[15] T. W. Hung and P. T. Chen, *On the optimal replenishment in a finite planning horizon with learning effect of setup costs*, *Journal of Industrial & Management Optimization*, 6 (2010), 425–433.
[16] M. Y. Jaber and M. Bonney, *The economic manufacture/order quantity (EMQ/EOQ) and the learning curve: Past, present, and future*, *International Journal of Production Economics*, 59 (1999), 93–102.
[17] B. Kamrad and A. Siddique, *Supply contracts, profit sharing, switching, reaction options*, *Management Science*, 50 (2004), 64–82.
[18] S. L. Li, A. Madhok, G. Flaschka and R. Verma, *Supplier-switching inertia and competitive asymmetry: A demand-side perspective*, *Decision Sciences*, 37 (2006), 547–576.
[19] T. Li, S. P. Sethi and X. L. He, *Dynamic pricing, production, and channel coordination with stochastic learning*, *Production and Operations Management*, 24 (2015), 857–882.
[20] C. Löfler, T. Pfeiffer and G. Schneider, *Controlling for supplier switching in the presence of real options and asymmetric information*, *European Journal of Operational Research*, 223 (2012), 690–700.
[21] M. March, S. Zanoni, and M. Y. Jaber, *Economic production quantity model with learning in production, quality, reliability and energy efficiency*, *Computers & Industrial Engineering*, 129 (2019), 502–511.
[22] G. Mosheiov, *Scheduling problems with a learning effect*, *European Journal of Operational Research*, 132 (2002), 687–693.
[23] R. B. Myerson, *Incentive compatibility and the bargaining problem*, *Econometrica: Journal of the Econometric Society*, 47 (1979), 61–73.
[24] T. Pfeiffer, A dynamic model of supplier switching, *European Journal of Operational Research*, 207 (2010), 697–710.

[25] M. Plaza, O. K. Ngwenyama and K. Rohlf, A comparative analysis of learning curves: Implications for new technology implementation management, *European Journal of Operational Research*, 200 (2010), 518–528.

[26] S. Shum, S. L. Tong and T. T. Xiao, On the impact of uncertain cost reduction when selling to strategic consumers, *Management Science*, 63 (2016), 843–860.

[27] L. Silbermayer and S. Minner, Dual sourcing under disruption risk and cost improvement through learning, *European Journal of Operational Research*, 250 (2016), 226–238.

[28] A. M. Spence, The learning curve and competition, *The Bell Journal of Economics*, 12 (1981), 49–70.

[29] A. T. Tsekrekos and A. N. Yannacopoulos, Optimal switching decisions under stochastic volatility with fast mean reversion, *European Journal of Operational Research*, 251 (2016), 148–157.

[30] M. Uluskan, A. B. Godfrey and J. A. Joines, Impact of competitive strategy and cost-focus on global supplier switching (reshore and relocation) decisions, *The Journal of The Textile Institute*, 108 (2017), 1308–1318.

[31] S. M. Wagner and G. Friedl, Supplier switching decisions, *European Journal of Operational Research*, 183 (2007), 700–717.

[32] T. P. Wright, Factors affecting the cost of airplanes, *Journal of Aeronautical Sciences*, 3 (1936), 122–128.

[33] K. Xu, W. Y. Chiang and L. Liang, Dynamic pricing and channel efficiency in the presence of the cost learning effect, *International Transactions in Operational Research*, 18 (2011), 579–604.

[34] L. E. Yelle, The learning curve: Historical review and comprehensive survey, *Decision Sciences*, 10 (1979), 302–328.

[35] W. I. Zangwill and P. B. Kantor, The learning curve: A new perspective, *International Transactions in Operational Research*, 7 (2000), 595–607.

[36] J. X. Zhang, W. S. Tang and M. M. Hu, Optimal supplier switching with volume-dependent switching costs, *International Journal of Production Economics*, 161 (2015), 96–164.

[37] J. X. Zhang, Q. Wei, G. W. Liu and W. S. Tang, A supplier switching model with the competitive reactions and economies of scale effects, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47 (2017), 2831–2843.

[38] Q. Zhang, W. S. Tang and J. X. Zhang, Green supply chain performance with cost learning and operational inefficiency effects, *Journal of Cleaner Production*, 112 (2016), 3267–3284.

[39] X. Zhao, Z. Pang and K. E. Stecke, When does a retailer’s advance selling capability benefit manufacturer, retailer, or both?, *Production and Operations Management*, 25 (2016), 1073–1087.

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E-mail address: qwei@tju.edu.cn
E-mail address: jxzhang@tju.edu.cn
E-mail address: xjsun@tju.edu.cn