Non - Darcian and Non - Uniform Salinity Gradients on Triple Diffusive Convection in Composite Layers

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Abstract: The effect of uniform and non-uniform salinity gradients on the onset of triple diffusive convection in a system of composite layers enclosing an incompressible, three component, electrically conducting fluid which lies above a saturated porous layer of the identical fluid is studied analytically. The upper boundary of the fluid layer and the lower boundary of the porous layer are static and both the boundaries are insulating to heat and mass. At the interface, the velocity, shear stress, normal stress, heat, heat flux, mass and mass flux are presumed to be continuous, intended for Darcy-Brinkman model. An Eigenvalue problem is attained and the same is solved by the regular perturbation approach. The critical Rayleigh number which is the guiding principle for the invariability of the system is accomplished for every salinity profile individually. The effects of various physical parameters on the onset of Triple diffusive convection are considered for all the profiles graphically.

Keywords: Triple diffusion, non-uniform Salinity gradients, Regular perturbation method, Darcy-Brinkman model.

1. INTRODUCTION

In standard Benard problem, density difference was the only destabilizing source due to which the system was unstable. This unsteadiness is due to the difference in temperature between the two surface boundaries of the fluid. This situation where the temperature is the only diffusing component is referred to as single component diffusion. If the fluid has additional salt dissolved in it then there are two destabilizing sources for the density difference i.e. temperature field and salt field, which is known as double diffusion. Along with the temperature, if there are two more agencies (salts) present dissolved in the fluid the convection is referred to as triple diffusive convection. The effect of a third diffusive agent is receiving much attention in present day research field as there are numerous physical systems with two dissolved salts diffusing independently along with temperature field.

Griffiths [4], Turner [19] recognized that there are many situations where more than two dissolved salts are present along with the temperature field. For instance: solidification of molten alloys, geothermally heated lakes, oceanography, high-quality crystal production, oceanography, production of pure medication, groundwater flow and many more.

Griffiths [4], Pearlstein et al [8] and Lopez [5] investigated theoretically the onset of convection in an infinite horizontal layer of triple diffusive fluid. Shivakumara I S and Kumar [16] investigated the bifurcation analysis of a triply diffusive coupled stress fluid in terms of a simplified model consisting of seven nonlinear ordinary differential equations. Shivakumara I S and Kumar [17] have studied the linear and weakly nonlinear triple diffusive convection in couple stress fluid layer. K.R. Raghunathanaad I.S Shivakumara [9] have investigated the triple diffusive convection in an Oldroyd-B fluid-saturated porous layer by performing linear and weakly nonlinear stability analyses. Sameena Tarannum and S. Pranesh [14] have studied a nonlinear triple diffusive convection in a rotating couple stress liquid to study the effect of heat and mass transfer by deriving Ginzburg-Landau equation. Chand S [1] studied theoretically the triple-diffusive convection in a micropolar ferrofluid layer heated and soluted below with transverse uniform magnetic field along with uniform vertical rotation. Rana G.C et al [11] have studied the onset of triple-diffusive convection in a horizontal layer of nano fluid heated from below and salted from above and below both analytically and numerically. Rionero [12] studied a triply convective diffusive fluid mixture saturating a porous horizontal layer, heated from below and salted from above. Rionero [13] also investigated the multicomponent diffusive convection in the porous layer for the more general case when heated from below and salted by m salts partly from above and partly from below. Zhao, Wang and Zhang [22] investigated the problem of triply diffusive convection in a Maxwell fluid saturated porous layer. K.R. Raghunath et al [10] investigated the weakly nonlinear stability of...
the triple diffusive convection in a Maxwell fluid saturated porous layer. Mukesh Kumar Awasthi et al. [6] have performed a linear stability analysis for the onset of triple-diffusive convection in the presence of internal heat source in a Maxwell fluid saturated porous layer.

All the above literature are confined to the single layer of fluid or porous layer but in many physical systems, the occurrence of composite layer and salinity gradients is natural which motivated us to study the onset of triple diffusive convection in fluid - porous composite layer for uniform and non-uniform salinity gradients.

For Fluid layer,
\[
\nabla \cdot \mathbf{q} = 0
\]
\[
\rho_0 \left[ \frac{\partial q}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla P + \mu \nabla^2 \mathbf{q} - \rho g \mathbf{k}
\]
\[
\frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa \nabla^2 T
\]
\[
\frac{\partial C_1}{\partial t} + (\mathbf{q} \cdot \nabla) C_1 = \kappa_1 \nabla^2 C_1
\]
\[
\frac{\partial C_2}{\partial t} + (\mathbf{q} \cdot \nabla) C_2 = \kappa_2 \nabla^2 C_2
\]

where
\[
\rho = \rho_0 \left[ 1 - \alpha (T - T_0) + \alpha_{s1} (C_1 - C_0) + \alpha_{s2} (C_2 - C_0) \right]
\]

and for the porous layer,
\[
\nabla_m \cdot \mathbf{q}_m = 0
\]
\[
\rho_0 \left[ \frac{1}{\varepsilon} \frac{\partial q_m}{\partial t} + \frac{1}{\varepsilon^2} (\mathbf{q}_m \cdot \nabla) \mathbf{q}_m \right] = -\nabla_m P_m + \mu \nabla^2 \mathbf{q}_m - \frac{\mu}{K} \mathbf{q}_m - \rho_m g \mathbf{k}
\]
\[
A \frac{\partial T_m}{\partial t} + (\mathbf{q}_m \cdot \nabla_m) T_m = \kappa_m \nabla^2 T_m
\]
\[
\phi \frac{\partial C_{m1}}{\partial t} + (\mathbf{q}_m \cdot \nabla_m) C_{m1} = \kappa_{m1} \nabla^2 C_{m1}
\]
\[
\phi \frac{\partial C_{m2}}{\partial t} + (\mathbf{q}_m \cdot \nabla_m) C_{m2} = \kappa_{m2} \nabla^2 C_{m2}
\]

where
\[
\rho_m = \rho_0 \left[ 1 - \alpha_{m1} (T_m - T_0) + \alpha_{m1} (C_{m1} - C_0) + \alpha_{m2} (C_{m2} - C_0) \right]
\]

and the symbols in the above equations have the following meaning
\[
\mathbf{q} = (u, v, w)
\]
is the velocity vector, \( t \) is the time, \( \mu \) is the fluid viscosity, \( P \) is the total pressure, \( \rho_0 \) is the fluid density, \( g \) is the acceleration due to the gravity, \( A \) is the ratio of heat capacities, \( C_p \) is the specific heat, \( K \) is the permeability of the porous medium, \( T \) is the temperature, \( \kappa \) is the thermal diffusivity of the fluid.
are the concentrations or the salinity fields, \( \kappa_m \) is the solute diffusivity of the fluid,

\[
\alpha_t = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{P,T} \quad \alpha_{s1} = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial C} \right)_{P,C} \quad \alpha_{s2} = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial C} \right)_{P,C}
\]

\( \phi \) is the porosity and the subscripts \( m \) and \( f \) refer to the porous medium and the fluid respectively.

The basic steady state is assumed to be quiescent and we consider the solution of the form, in the fluid layer,

\[
[u, v, w, P, T, C_1, C_2] = \left[ 0, 0, 0, P_i(z), T_b(z), C_{b1}(z), C_{b2}(z) \right]
\]  
(13)

and in the porous layer

\[
[u_m, v_m, w_m, P_m, T_m, C_{m1}, C_{m2}] = \left[ 0, 0, 0, P_{mb}(z_m), T_{mb}(z_m), C_{mb1}(z_m), C_{mb2}(z_m) \right]
\]  
(14)

where the subscript \( \text{b} \) denotes the basic state.

The temperature distributions \( T_b(z) \) and \( T_{mb}(z_m) \) are found to be

\[
T_b(z) = T_0 + \frac{(T_u - T_0)}{d} z \quad \text{in} \quad 0 \leq z \leq d
\]  
(15)

\[
T_{mb}(z_m) = T_0 - \frac{(T_t - T_0)}{d_m} z_m \quad \text{in} \quad 0 \leq z_m \leq d_m
\]  
(16)

\[ T_0 = \frac{\kappa d_m T_u + \kappa_m d T_t}{\kappa d_m + \kappa_m d} \]

is the interface temperature.

The concentration distributions \( C_{b1}(z) \), \( C_{mb1}(z_m) \), \( C_{b2}(z) \) and \( C_{mb2}(z_m) \) are found to be

\[
-\frac{\partial C_{b1}}{\partial z} = \frac{C_{io} - C_{in}}{d} h(z) \quad \text{in} \quad 0 \leq z \leq d
\]  
(17)

\[
-\frac{\partial C_{mb1}}{\partial z_m} = \frac{C_{10} - C_{1n}}{d_m} h_m(z_m) \quad \text{in} \quad 0 \leq z_m \leq d_m
\]  
(18)

\[
C_{b2}(z) = C_{20} + \frac{(C_{2u} - C_{20})}{d} z \quad \text{in} \quad 0 \leq z \leq d
\]  
(19)

\[
C_{mb2}(z_m) = C_{20} - \frac{(C_{2t} - C_{20})}{d_m} z_m \quad \text{in} \quad 0 \leq z_m \leq d_m
\]  
(20)

where

\[ h(z), \ h_m(z_m) \]

are salinity gradients in fluid and porous layers respectively At the interface \( h(z) = h_m(z_m) \) and

\[ C_0 = \frac{\kappa d_m C_u + \kappa_m d C_t}{\kappa d_m + \kappa_m d} \]

is concentration at the interface.

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form,

\[
[q_i, P, T, C_1, C_2] = \left[ 0, P_i(z), T_b(z), C_{b1}(z), C_{b2}(z) \right] + \left[ q'_i, P', \theta, S_1, S_2 \right]
\]  
(21)

and

\[
[q_m, P_m, T_m, C_{m1}, C_{m2}] = \left[ 0, P_{mb}(z_m), T_{mb}(z_m), C_{mb1}(z_m), C_{mb2}(z_m) \right] + \left[ q'_m, P'_m, \theta_m, S_{m1}, S_{m2} \right]
\]  
(22)
The primed quantities in the above equations are the perturbed ones over their equilibrium counterparts. Eqs. (21) and (22) are substituted into the Eqs. (1) to (12) and are linearized in the usual manner, the pressure term is eliminated from (2) and (8) by taking curl twice on

\[ \left( x, y, z \right) = d \left( x', y', z' \right) \] and \[ \left( x_m, y_m, z_m \right) = d_m \left( x_m', y_m', z_m' - 1 \right) \]

In this manner the detailed flow fields in both the fluid and porous layers can be clearly obtained for all the depth ratios \[ \hat{d} = \frac{d_m}{d} \]. The non-dimensionalised basic equations are subjected to normal mode expansion and we seek solutions for the dependent variables in the fluid and porous layers (following Venkatachalappa M et al [20]). Assuming that the principle of exchange of instabilities holds for the superposed layers (following Chen and Chen [2], D.A Nield [7]), so that each layer is of unit depth with these two equations and only the vertical component is retained. The separate length scales are chosen for the two layers (following Chen and Chen [2], D.A Nield [7]), and denoting the differential operator \[ \frac{\partial}{\partial z} \] by \[ D \] and \[ D_m \] respectively, an Eigen value problem consisting of the following ordinary differential equations is obtained for the first concentration distribution is obtained as below,

In \([0 \leq z \leq 1]\)

\[ (D^2 - a^2)^2 W = Ra^2 \Theta - R_{sl} a^2 \Sigma_1 - R_{s_2} a^2 \Sigma_2 \] 
\[ (D^2 - a^2) \Theta + W = 0 \] 
\[ \tau_1 \left( D^2 - a^2 \right) \Sigma_1 + Wh(z) = 0 \] 
\[ \tau_2 \left( D^2 - a^2 \right) \Sigma_2 + W = 0 \] 

For the fluid layer, \[ R = \frac{g \alpha_1 (T_0 - T_u) d^3}{\nu K} \] is the Rayleigh number, \[ R_{s_1} = \frac{g \alpha_1 (C_{10} - C_{lu}) d^3}{\nu K} \] are the Solute Rayleigh numbers, \[ \tau_1 = \frac{K_{s_1}}{K}, \tau_2 = \frac{K_{s_2}}{K} \] are the diffusivity ratios. For the porous layer, \[ \beta^2 = \frac{K}{d_m^2} = Da \] is the Darcy number, \[ \hat{\mu} = \frac{v_m}{v} \] is the viscosity ratio, \[ R_m = \frac{g \alpha_1 (T_0 - T_u) d_m K}{\nu K_m} = RDa \] \[ R_{s_m} = \frac{g \alpha_1 (C_{10} - C_{lu}) d_m K}{\nu K_m} = R_{s_m} Da \] are the Solute Rayleigh – Darcy number in porous medium, \[ \tau_{pm1} = \frac{K_{s_m1}}{K_m}, \tau_{pm2} = \frac{K_{s_m2}}{K_m} \] are the diffusivity ratios, \[ A \] and \[ a_m \] are the non-dimensional horizontal wave numbers, \[ \Theta \] and \[ \Theta_m \] are the temperature in fluid and porous layers, \[ S \] and \[ S_m \] are the concentration in fluid and porous layers and

\[ \int_0^1 h(z)dz = \int_0^1 h_m(z_m)dz_m = 1. \]

Eqs. (23) to (30) are twentieth order ordinary differential equation which are to be solved using the below mentioned boundary conditions.
III. BOUNDARY CONDITIONS

The boundary conditions after non-dimensionalisation and Normal mode expansion are

\[ W(1) = 0, \quad DW(1) = 0, \quad D\Theta(1) = 0, \quad DS_1(1) = 0, \quad DS_2(1) = 0, \quad D_m^2 S_{m1} (0) = 0, \quad D_m^2 S_{m2} (0) = 0 \]

\[ \hat{T}W(0) = W_m(1), \quad \hat{T}\hat{d}DW(0) = D^2_m W_m (1), \quad \hat{T}\hat{d}^2 (D^2 + \alpha^2)W(0) = \hat{\mu} (D^2_m + \alpha^2_m)W_m (1) \]

\[ \Theta(0) = \hat{T}\Theta_m (1), \quad D\Theta(0) = D_m\Theta_m (1), \quad S(0) = S_{m1} (1), \quad DS(0) = D_m S_{m1} (1) \]

\[ S_1(0) = S_{m2} (1), \quad DS_2(0) = D_m S_{m2} (1), \quad W_m (0) = 0, \quad D_m W_m (0) = 0, \quad D_m \Theta_m (0) = 0, \]

\[ \hat{T}\hat{d}^2 \beta \left(D^2 W(0) - 3\alpha^2 DW(0) \right) = -D_m W_m (1) + \hat{\mu} \hat{\beta} \left(D^2_m W_m (1) - 3\alpha^2_m D_m W_m (1) \right) \]

where

\[ \hat{T} = (T - T_0) / (T_0 - T_0), \quad \hat{\kappa} = \kappa_m / \kappa, \quad \hat{\alpha} = \alpha / \hat{\alpha}, \quad \hat{\beta} = \beta / \hat{\beta}, \quad \hat{\Theta} = \Theta / \hat{\Theta}, \quad \hat{S} = (C_d - C_{i0}) / (C_{i0} - C_{in}) \]

for \( \hat{\iota} = 1, 2 \)

\[ \hat{\kappa} = \kappa_m / \kappa = \hat{\alpha} / \hat{T}, \quad \hat{\kappa}_1 = \kappa_{m1} / \kappa, \quad \hat{\alpha}_1 = \hat{\alpha} / \hat{T}, \quad \hat{\kappa}_2 = \kappa_{m2} / \kappa, \quad \hat{\alpha}_2 = \hat{\alpha} / \hat{T}, \quad \hat{\kappa}_1, \quad \hat{\kappa}_2 \]

and \( \hat{\kappa}_1 \) are the thermal diffusivity and the solutal diffusivity ratios respectively. The Energy Equations are solved using respective boundary conditions from (29) (following Shivakumara I.S et al [15]).

IV. SOLUTION BY REGULAR PERTURBATION TECHNIQUE

For the constant heat and mass flux boundaries convection sets in at small values of horizontal wavenumber ‘\( a \)’, accordingly, we expand

\[ \begin{bmatrix} W \\ \Theta \\ \Sigma_1 \end{bmatrix} = \sum_{j=0}^{\infty} a^{2j} \begin{bmatrix} W_j \\ \Theta_j \\ \Sigma_{j1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} W_m \\ \Theta_m \\ \Sigma_{m1} \\ \Sigma_{m2} \end{bmatrix} = \sum_{j=0}^{\infty} a^{2j} \begin{bmatrix} W_{mj} \\ \Theta_{mj} \\ \Sigma_{mj1} \\ \Sigma_{mj2} \end{bmatrix} \]

With an arbitrary factor, the solutions for zero order equations are:

\[ W_0 (z) = 0, \quad \Theta_0 (z) = \hat{T}, \quad \Sigma_{10} (z) = \hat{S}_1, \quad \Sigma_{20} (z) = \hat{S}_2 \]

\[ W_{m0} (z_m) = 0, \quad \Theta_{m0} (z_m) = 1, \quad \Sigma_{m10} (z_m) = 1, \quad \Sigma_{m20} (z_m) = 1 \]

The equations at first order in \( a^2 \) are,

For fluid layer,

\[ D^4 W_1 - R T_1 + R_{11} \hat{S}_1 + R_{12} \hat{S}_2 = 0 \quad (31) \]

\[ D^2 \Theta_1 - \hat{T}_1 + W_1 = 0 \quad (32) \]

\[ \tau_1 D^2 \Sigma_{11} - \tau_1 \hat{S}_1 + W_1 h(z) = 0 \quad (33) \]

\[ \tau_2 D^2 \Sigma_{21} - \tau_2 \hat{S}_2 + W_1 = 0 \quad (34) \]

For porous layer,

\[ \hat{\mu} \hat{\beta}^2 D^4 m_{m1} W_{m1} - D^2 m_{m1} W_{m1} - R_{m1} + R_{m11} + R_{m12} = 0 \quad (35) \]

\[ D^2 m_{m1} - 1 + W_{m1} = 0 \quad (36) \]

\[ \tau_{m1} D^2 m_{m1} - \tau_{m1} + W_{m1} h_m (z_m) = 0 \quad (37) \]

\[ \tau_{m2} D^2 m_{m2} - \tau_{m2} + W_{m1} h_m (z_m) = 0 \quad (38) \]

The corresponding boundary conditions are,
The solutions of the Eqs.(32) and (36) give \( W_1 \) and \( W_{ml} \) respectively are important in obtaining the Eigen values and are found to be,

\[
W_1(z) = C_1 + C_2 z + C_3 z^2 + C_4 z^3 + \left( R_\beta T - R_\gamma S_1 - R_\delta T S_2 \right) \frac{z^4}{24}
\]

\[
W_{ml}(z_m) = C_5 + C_6 z_m + C_7 e^{R\nu_m} + C_8 e^{-R\nu_m} - \left( R_m - R_{ml1} - R_{ml2} \right) \frac{z_m^2}{2}
\]

Where \( p = \sqrt{\frac{1}{\lambda B^2}} \) and \( C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8 \) are constants which are determined using the velocity boundary conditions and are as follows

\[
C_1 = \Delta_1 C_7 + \Delta_8 C_8 = \frac{d^2 B}{2T}, \quad C_2 = \Delta_2 C_7 + \Delta_6 C_8 = \frac{d^2 B}{2T}, \quad C_3 = \Delta_3 C_7 + \Delta_4 C_8 = \frac{\mu B}{2T},
\]

\[
C_4 = \Delta_4 C_7 + \Delta_2 C_8 = \frac{B}{64 T^2 \beta^2}, \quad C_5 = -C_7 + C_8, \quad C_6 = p C_8 - p C_7, \quad C_7 = A \Delta_{17} + B \Delta_{18},
\]

\[
C_8 = A \Delta_{14} + B \Delta_{15}, \quad A = R_\beta T - R_\gamma S_1 - R_\delta T S_2, \quad B = R_m - R_{ml1} - R_{ml2},
\]

\[
\Delta_1 = \frac{\mu p^2 e^p - p - p e^p + p}{64 T^2 \beta^2}, \quad \Delta_2 = \frac{p e^{-p} - p - \mu \beta^2 p^3 e^{-p}}{64 T^2 \beta^2}
\]

The solutions of the Eqs.(32) and (36) give \( W_1 \) and \( W_{ml} \) respectively are important in obtaining the Eigen values and are found to be,
4.1 Solvability condition

The differential equations and boundary conditions corresponding to temperature and concentrations yield the compatibility condition

\[
\Delta_{20} = \Delta_7 \Delta_{18} + \Delta_{16} \Delta_8 - \frac{d^2}{2T}, \quad \Delta_{21} = \Delta_3 \Delta_{17} + \Delta_{15} \Delta_6, \quad \Delta_{22} = \Delta_5 \Delta_{18} + \Delta_{16} \Delta_6 - \frac{d}{T}, \quad \Delta_{28} = -\Delta_{18} - \Delta_{16}.
\]

\[
\Delta_{23} = \Delta_3 \Delta_{17} + \Delta_{14} \Delta_4, \quad \Delta_{24} = \Delta_3 \Delta_{18} + \Delta_{16} \Delta_4 - \frac{\mu}{2T}, \quad \Delta_{25} = \Delta_4 \Delta_{17} + \Delta_{15} \Delta_2, \quad \Delta_{30} = p \left( \Delta_{16} - \Delta_{18} \right).
\]

\[
\Delta_{26} = \Delta_4 \Delta_{18} + \Delta_{16} \Delta_2 + \frac{1}{6Td^2}, \quad \Delta_{27} = -\Delta_{17} - \Delta_{15}, \quad \Delta_{29} = p \left( \Delta_{15} - \Delta_{17} \right).
\]

4.2 Linear Salinity Profile:

In this profile

\[
\begin{align*}
\hat{h}(z) &= h_m(z_m) = 1 \quad (42)
\end{align*}
\]

The critical Rayleigh number for this model is obtained by substituting (42) in (41) and is found to be

\[
R_{c1} = \frac{\delta_1 + \left( R_{11} \hat{S}_1 + R_{12} \hat{S}_2 \right) \delta_2 + \left( R_{m1} + R_{m2} \right) \delta_6}{T \left( \delta_3 + \frac{d^3 \beta^2 \delta_6}{\kappa} \right)}
\]

where

\[
\begin{align*}
\delta_1 &= \Delta_{19} + \frac{\Delta_{13}}{2} + \frac{\Delta_{15}}{3} + \frac{\Delta_{17}}{4} + \frac{1}{120}, \quad \delta_2 = \Delta_{20} + \frac{\Delta_{12}}{2} + \frac{\Delta_{24}}{3} + \frac{\Delta_{26}}{4}, \\
\delta_3 &= \Delta_{27} + \frac{\Delta_{30}}{2} + \Delta_{17} \left( \frac{e^p - 1}{p} \right) + \Delta_{15} \left( \frac{1 - e^{-p}}{p} \right), \\
\delta_4 &= \Delta_{28} + \frac{\Delta_{32}}{2} + \Delta_{18} \left( \frac{e^p - 1}{p} \right) + \Delta_{16} \left( \frac{1 - e^{-p}}{p} \right) - \frac{1}{6}, \quad \delta_5 = \left( 1 + \tau_{m1} + \tau_{m2} \right) \delta_4 + \hat{d}^2 \left( 1 + \tau_1 + \tau_2 \right) \delta_6, \\
\delta_6 &= \left( 1 + \tau_{m1} + \tau_{m2} \right) \delta_6 + \hat{d}^2 \left( 1 + \tau_1 + \tau_2 \right) \delta_7, \quad \delta_7 = \hat{T} + \hat{d}^2 + \tau_1 \tau_{m1} \left( \hat{S} + \hat{d}^2 \right) + \tau_2 \tau_{m2} \left( \hat{S} + \hat{d}^2 \right).
\end{align*}
\]

\[
\Delta_i \] remains same as earlier.

4.3 Parabolic salinity profile:

Following Sparrow et al [18], \( h(z) = 2z, \ h_m(z_m) = 2z_m \) \quad (43)

The critical Rayleigh number for this model is obtained by substituting (43) in (41) and is found to be

\[
R_{c2} = \frac{\delta_1 + \left( R_{11} \hat{S}_1 + R_{12} \hat{S}_2 \right) A_2 + \left( R_{m1} + R_{m2} \right) A_3}{T \left( A_2 + \hat{d}^3 \beta^2 A_3 \right)}
\]

where
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\[
A_2 = \Delta_{19} \delta_2 + \Delta_{21} \delta_3 + \Delta_{23} \delta_4 + \Delta_{25} \delta_5 + \delta_6 + \Delta_{27} \delta_7 + \Delta_{29} \delta_8 + \Delta_{17} \delta_9 + \Delta_{15} \delta_{10}
\]
\[
A_3 = \Delta_{25} \delta_2 + \Delta_{22} \delta_3 + \Delta_{24} \delta_4 + \Delta_{26} \delta_5 + \Delta_{28} \delta_7 + \Delta_{30} \delta_8 + \Delta_{19} \delta_9 - \delta_{11}
\]

\[
\delta_1 = \hat{T} + \hat{d}^2 + \tau_1 \tau_{pm1} \left( \hat{S} + \hat{d}^2 \right) + \tau_2 \tau_{pm2} \left( \hat{S}_2 + \hat{d}^2 \right)
\]
\[
\delta_2 = 1 + \tau_{pm1} + \tau_{pm2}, \quad \delta_3 = \frac{2 \tau_{pm1}}{3} + \frac{1 + \tau_{pm2}}{2}, \quad \delta_4 = \frac{\tau_{pm1}}{2} + \frac{1 + \tau_{pm2}}{3}, \quad \delta_5 = \frac{2 \tau_{pm1}}{5} + \frac{1 + \tau_{pm2}}{4},
\]
\[
\delta_6 = \frac{\tau_{pm1}}{72} + \frac{1 + \tau_{pm2}}{120}, \quad \delta_7 = \hat{d}^2 \left( 1 + \tau_1 + \tau_2 \right), \quad \delta_8 = \hat{d}^2 \left( \frac{2 \tau_1}{3} + \frac{1 + \tau_2}{2} \right),
\]
\[
\delta_9 = \hat{d}^2 \left( 2 \tau_1 \left( e^p - \frac{e^p - 1}{p} \right) + \frac{1 + \tau_2}{2} \right),
\]
\[
\delta_{10} = \hat{d}^2 \left( 2 \tau_1 \left( e^p - \frac{e^p - 1}{p} \right) + \frac{1 + \tau_2}{2} \right), \quad \delta_{11} = \hat{d}^2 \left( \frac{\tau_1}{4} + \frac{1 + \tau_2}{6} \right)
\]

\[\Delta'_{i}\] remains same as earlier.

4.4 Inverted Parabolic salinity profile:

For this case \( h(z) = 2(1 - z), \quad h_m(z_m) = 2(1 - z_m) \) (44)

The critical Rayleigh number for this model is obtained by substituting (44) in (41) and is found to be

\[
R_{c3} = \frac{\delta_1 + (R_{s1} \hat{S}_1 + R_{s2} \hat{S}_2) A_2 + (R_{pm1} + R_{pm2}) A_1}{\hat{T}} \left( A_2 + \frac{\hat{d}^2 \beta^2 A_1}{\kappa} \right)
\]

Where

\[
A_2 = \Delta_{19} \delta_2 + \Delta_{21} \delta_3 + \Delta_{23} \delta_4 + \Delta_{25} \delta_5 + \delta_6 + \Delta_{27} \delta_7 + \Delta_{29} \delta_8 + \Delta_{17} \delta_9 + \Delta_{15} \delta_{10}
\]
\[
A_3 = \Delta_{25} \delta_2 + \Delta_{22} \delta_3 + \Delta_{24} \delta_4 + \Delta_{26} \delta_5 + \Delta_{28} \delta_7 + \Delta_{30} \delta_8 + \Delta_{19} \delta_9 - \delta_{11}
\]

\[
\delta_1 = \hat{T} + \hat{d}^2 + \tau_1 \tau_{pm1} \left( \hat{S} + \hat{d}^2 \right) + \tau_2 \tau_{pm2} \left( \hat{S}_2 + \hat{d}^2 \right)
\]
\[
\delta_2 = 1 + \tau_{pm1} + \tau_{pm2}, \quad \delta_3 = \frac{\tau_{pm1}}{3} + \frac{1 + \tau_{pm2}}{2}, \quad \delta_4 = \frac{\tau_{pm1}}{2} + \frac{1 + \tau_{pm2}}{3}, \quad \delta_5 = \frac{\tau_{pm1}}{6} + \frac{1 + \tau_{pm2}}{4},
\]
\[
\delta_6 = \frac{\tau_{pm1}}{120} + \frac{1 + \tau_{pm2}}{144}, \quad \delta_7 = \hat{d}^2 \left( 1 + \tau_1 + \tau_2 \right), \quad \delta_8 = \hat{d}^2 \left( \frac{\tau_1}{3} + \frac{1 + \tau_2}{2} \right),
\]
\[
\delta_9 = \hat{d}^2 \left( 2 \tau_1 \left( e^p - \frac{e^p - 1}{p} \right) + \frac{1 + \tau_2}{2} \right),
\]
\[
\delta_{10} = \hat{d}^2 \left( 2 \tau_1 \left( e^p - \frac{e^p - 1}{p} \right) + \frac{1 + \tau_2}{2} \right), \quad \delta_{11} = \hat{d}^2 \left( \frac{\tau_1}{12} + \frac{1 + \tau_2}{6} \right)
\]

\[\Delta'_{i}\] remains same as earlier.
4.5 Piecewise linear Salting below Salinity profile:

For this case following Currie [3],
\[
h(z) = \begin{cases} 
  \varepsilon^{-1}, & 0 \leq z \leq \varepsilon \\
  0, & \varepsilon \leq z \leq 1
\end{cases}, \quad h_m(z_m) = \begin{cases} 
  \varepsilon^{-1}, & 0 \leq z_m \leq \varepsilon_m \\
  0, & \varepsilon_m \leq z \leq 1
\end{cases}
\] (45)

The critical Rayleigh number for this model is obtained by substituting (45) in (41) and is found to be
\[
R_{c5} = a_4 + \left( R_{1}, \hat{S}_1 + R_{2}, \hat{S}_2 \right) A_2 + \left( R_{m1} + R_{m2} \right) A_3
\]
\[
\hat{T} \left( A_2 + \frac{d^3 \beta^2 A_m}{\kappa} \right)
\]

where
\[
A_1 = \Delta_1 \hat{S} + \Delta_2 \hat{S}_2 + \Delta_3 \hat{S}_2 + \Delta_4 \hat{S}_2 + \Delta_5 \hat{S}_2 + \Delta_6 \hat{S}_2 + \Delta_7 \hat{S}_2 + \Delta_8 \hat{S}_2 + \Delta_9 \hat{S}_2 + \Delta_10 \hat{S}_2 - \Delta_11,
\]
\[
\delta_1 = \hat{T} + d^2 + \delta_1 \tau_{pm} \left( \hat{S} + d^2 \right) + \delta_2 \tau_{pm} \left( \hat{S}_2 + d^2 \right), \quad \delta_2 = 1 + \tau_{pm} \tau_{pm}, \quad \delta_3 = \frac{1}{2} \left( \epsilon \tau_{pm} + \tau_{pm} \right),
\]
\[
\delta_3 = \frac{1}{3} \left( \epsilon^3 \tau_{pm} + 1 + \tau_{pm} \right), \quad \delta_4 = \frac{1}{4} \left( \epsilon^3 \tau_{pm} + 1 + \tau_{pm} \right), \quad \delta_5 = \frac{1}{120} \left( \epsilon^4 \tau_{pm} + 1 + \tau_{pm} \right),
\]
\[
\delta_10 = \frac{d^2}{\epsilon_m} \left( 1 - e^{-\epsilon_m} \right) + (1 + \tau_2) \left( 1 - e^{-\epsilon} \right), \quad \delta_{11} = \frac{d^2}{2} \left( \tau_2 \epsilon_m^2 + \left( 1 + \tau_2 \right) \frac{6}{6} \right).
\]
\[\Delta_i\] are defined earlier.

4.6 Piecewise linear Salinity profile Desalting above:

For this case following Vidal and Acrivos [21],
\[
h(z) = \begin{cases} 
  0, & 0 \leq z \leq (1 - \varepsilon) \\
  \varepsilon^{-1}, & (1 - \varepsilon) \leq z \leq 1
\end{cases}, \quad h_m(z_m) = \begin{cases} 
  0, & 0 \leq z_m \leq (1 - \varepsilon_m) \\
  \varepsilon_m^{-1}, & (1 - \varepsilon_m) \leq z \leq 1
\end{cases}
\] (46)

The critical Rayleigh number for this model is obtained by substituting (46) in (41) and is found to be
\[
R_{c5} = a_5 + \left( R_{1}, \hat{S}_1 + R_{2}, \hat{S}_2 \right) A_2 + \left( R_{m1} + R_{m2} \right) A_3
\]
\[
\hat{T} \left( A_2 + \frac{d^3 \beta^2 A_m}{\kappa} \right)
\]

where
\[
A_1 = \Delta_1 \hat{S} + \Delta_2 \hat{S}_2 + \Delta_3 \hat{S}_2 + \Delta_4 \hat{S}_2 + \Delta_5 \hat{S}_2 + \Delta_6 \hat{S}_2 + \Delta_7 \hat{S}_2 + \Delta_8 \hat{S}_2 + \Delta_9 \hat{S}_2 + \Delta_10 \hat{S}_2 - \Delta_11,
\]
\[
A_1 = \Delta_1 \hat{S} + \Delta_2 \hat{S}_2 + \Delta_3 \hat{S}_2 + \Delta_4 \hat{S}_2 + \Delta_5 \hat{S}_2 + \Delta_6 \hat{S}_2 + \Delta_7 \hat{S}_2 + \Delta_8 \hat{S}_2 + \Delta_9 \hat{S}_2 - \Delta_{11}.
\]
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\[ \delta_i = \hat{T} + \hat{d}^2 + \tau_{pml} (\hat{S} + \hat{d}^2) + \tau_{pm2} (\hat{S}_2 + \hat{d}^2), \quad \delta_2 = 1 + \tau_{pm1} + \tau_{pm2}, \]

\[ \delta_4 = \frac{1}{2} \left( \frac{\tau_{pm1}}{\epsilon} \left( 1 - (1 - \epsilon)^2 \right) + (1 + \tau_{pm2}) \right), \quad \delta_3 = \frac{1}{3} \left( \frac{\tau_{pm1}}{\epsilon} \left( 1 - (1 - \epsilon)^3 \right) + (1 + \tau_{pm2}) \right), \]

\[ \delta_5 = \frac{1}{4} \left( \frac{\tau_{pm1}}{\epsilon} \left( 1 - (1 - \epsilon)^2 \right) + (1 + \tau_{pm2}) \right), \quad \delta_6 = \frac{1}{120} \left( \frac{\tau_{pm1}}{\epsilon} \left( 1 - (1 - \epsilon)^3 \right) + (1 + \tau_{pm2}) \right), \]

\[ \delta_7 = \frac{\hat{d}^2}{2} (1 + \tau_1 + \tau_2), \quad \delta_8 = \frac{\hat{d}^2}{2} \left( \frac{\tau_1}{\epsilon} \left( 1 - (1 - \epsilon_m)^2 \right) + (1 + \tau_2) \right), \]

\[ \delta_9 = \frac{\hat{d}^2}{p} \left( \frac{\tau_1}{\epsilon_m} (e^p - e^{p(1 - \epsilon_m)}) + (1 + \tau_1) (e^p - 1) \right), \quad \delta_{10} = \frac{\hat{d}^2}{p} \left( \frac{\tau_1}{\epsilon_m} (-e^{-p} + e^{-p(1 - \epsilon_m)}) + (1 + \tau_2) (1 - e^{-p}) \right), \]

\[ \delta_{11} = \frac{\hat{d}^2}{6} \left( \frac{\tau_1}{\epsilon_m} \left( (1 - \epsilon_m)^3 - 1 \right) + (1 + \tau_2) \right). \]

are defined earlier.

4.7 Step function salinity profile:
In this profile the basic concentration/solute/salt drops suddenly by an amount \( \Delta S \) at \( z = \epsilon \) and \( \Delta S_m \) at \( z_m = \epsilon_m \) otherwise uniform. Accordingly

\[ h(z) = \delta(z - \epsilon), \quad h_m(z_m) = \delta(z_m - \epsilon_m) \]

(47)

where \( \epsilon \) is the solutal depth in the fluid layer and \( \epsilon_m \) is the solutal depth in the porous layer.

The critical Rayleigh number for this model is obtained by substituting (47) in (41) and is found to be

\[ R_{c6} = \frac{\delta_i + (R_{11} \hat{S}_1 + R_{21} \hat{S}_2 + R_{31}) A_2 + (R_{pm1} + R_{pm2}) A_3}{\hat{T}(A_2 + \hat{d}^2 A_3 / \kappa)} \]

where

\[ A_1 = \Delta_{19} \delta_2 + \Delta_{21} \delta_3 + \Delta_{23} \delta_4 + \Delta_{25} \delta_5 + \Delta_6 + \Delta_{27} \delta_7 + \Delta_{29} \delta_8 + \Delta_{17} \delta_9 + \Delta_{15} \delta_{10} \]

\[ A_2 = \Delta_{20} \delta_2 + \Delta_{22} \delta_3 + \Delta_{24} \delta_4 + \Delta_{26} \delta_5 + \Delta_{28} \delta_6 + \Delta_{31} \delta_7 + \Delta_{34} \delta_8 + \Delta_{19} \delta_9 - \delta_{11} \]

\[ \delta_i = \hat{T} + \hat{d}^2 + \tau_{pml} (\hat{S} + \hat{d}^2) + \tau_{pm2} (\hat{S}_2 + \hat{d}^2), \quad \delta_2 = 1 + \tau_{pm1} + \tau_{pm2}, \quad \delta_3 = \epsilon \tau_{pm1} + \frac{1 + \tau_{pm2}}{2}, \]

\[ \delta_4 = \epsilon^2 \tau_{pm1} + \frac{1 + \tau_{pm2}}{3}, \quad \delta_5 = \epsilon^2 \tau_{pm1} + \frac{1 + \tau_{pm2}}{4}, \quad \delta_6 = \frac{\epsilon^2 \tau_{pm1}}{24} + \frac{1 + \tau_{pm2}}{120}, \quad \delta_7 = \hat{d}^2 (1 + \tau_1 + \tau_2), \]

\[ \delta_8 = \hat{d}^2 \left( \frac{\epsilon_m \tau_1 + 1 + \tau_2}{2} \right), \quad \delta_9 = \hat{d}^2 \left( \frac{\tau_1 e^{p \epsilon_m} + (1 + \tau_2) \left( e^p - 1 \right)}{p} \right), \]

\[ \delta_{10} = \hat{d}^2 \left( \frac{\tau_1 e^{p \epsilon_m} + (1 + \tau_2) \left( 1 - e^{-p} \right)}{p} \right), \quad \delta_{11} = \hat{d}^2 \left( \frac{\tau_1 e_m^2 + (1 + \tau_2)}{6} \right). \]

\( \Delta_i \) are defined earlier.
V. RESULTS AND DISCUSSIONS

For Linear, Parabolic and Inverted Parabolic Salinity Profiles:

![Graph showing the variation of critical thermal Rayleigh number for different profiles with respect to the depth ratio.](image_url)

**Fig. 1.** The variation of critical thermal Rayleigh number $R_c$ for Linear, Parabolic and Inverted parabolic salinity profiles with respect to the depth ratio $\hat{d} = \frac{d_m}{d}$.

Figure 1 shows the variation of critical Rayleigh number $R_c$ for different profiles with respect to the depth ratio for fixed values of $Da = 0.1$, $\kappa = 1$, $\mu = 2$, $\tau_1 = \tau_2 = 0.25$, $\tau_{pm1} = \tau_{pm2} = 0.75$, $\hat{S}_1 = \hat{S}_2 = 1$, $R_{\xi_1} = R_{\xi_2} = 5$ and $\hat{T} = 1$. Graphically it is evident that the parabolic salinity profile is the most stable. Inverted parabolic profile is unstable for $0 \leq \hat{d} \leq 0.65$, and linear profile is unstable for $0.65 \leq \hat{d} \leq 1$. At $\hat{d} = 0.65$, linear and inverted parabolic profiles have same effect on $R_c$.

![Graph showing the effect of $\hat{\mu}$ on critical Rayleigh number $R_c$ for different profiles.](image_url)

**Fig. 2.** The effect of $\hat{\mu}$ on critical Rayleigh number $R_c$ for Linear, Parabolic and Inverted parabolic profiles with respect to the depth ratio $\hat{d} = \frac{d_m}{d}$.

Figure 2 shows the variation of critical Rayleigh number $R_c$ for different profiles with respect to the depth ratio for fixed values of $Da = 0.1$, $\kappa = 1$, $\tau_1 = \tau_2 = 0.25$, $\tau_{pm1} = \tau_{pm2} = 0.75$, $R_{\xi_1} = R_{\xi_2} = 5$, $\hat{S}_1 = \hat{S}_2 = 1$, and $\hat{T} = 1$. The effects of the viscosity ratio $\hat{\mu} = \frac{\mu_{pm}}{\mu}$ which is the ratio of the effective viscosity of the porous matrix to that of the fluid viscosity is displayed in the above graphs. For fixed values of depth ratio, the increase in the value of $\hat{\mu}$ increases the value of critical Rayleigh number $R_c$, i.e., the system is stabilized. Thus the onset of triple diffusive convection is delayed.
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Fig. 3. The effect of first solute Rayleigh number $R_{s1}$ on critical Rayleigh number $R_c$ for Linear, Parabolic and Inverted parabolic profiles with respect to the depth ratio $\frac{d}{d_m}$.

$Da = 0.1, \kappa = 1, \mu = 0.5, \tau_1 = \tau_2 = 0.25, \tau_{m1} = \tau_{m2} = 0.75, R_{s1} = 50, \tilde{S}_1 = \tilde{S}_2 = 1$, and $\hat{T} = 1$. The effect of solute Rayleigh number $R_{s1}$ is large for small change in the value of depth ratio. From the curves it is evident that for fixed values of depth ratio, the increase in the value of solute Rayleigh number $R_{s1}$ increases the value of critical Rayleigh number $R_c$. The increasing values of solute Rayleigh number $R_{s1}$ will affect the onset of convection only for larger values of the depth ratio $\frac{d}{d_m}$ that is, in porous layer dominant composite systems the convection is delayed.

Fig. 4. The effect of $\tau_1$ on critical Rayleigh number $R_c$ for Linear, Parabolic and Inverted parabolic profiles with respect to the depth ratio $\frac{d}{d_m}$.

Figure 4 shows the effects of the diffusivity ratio $\tau_1 = \frac{\kappa_1}{\kappa}$, which is the ratio of first saline diffusivity to thermal diffusivity of the fluid on critical Rayleigh number $R_c$ for different profiles with respect to the depth ratio for fixed values of $Da = 0.1, \kappa = 1, \mu = 0.5, \tau_2 = 0.25, \tau_{m1} = \tau_{m2} = 0.75, R_{s1} = 50, \tilde{S}_1 = \tilde{S}_2 = 1$, and $\hat{T} = 1$. It is clear from the graphs that for larger values of the depth ratio $\frac{d}{d_m}$ there is no effect of any variation in the values of $\tau_1$. The effect of $\tau_1$ is prominent for fluid layer dominant composite systems. For a fixed value of depth ratio, the increase in the value of $\tau_1$ increases the value of the critical thermal Rayleigh number. Thus increasing values of $\tau_1$ makes the system stable and hence delay the convection.
For Salting below, Desalting above and Step function Salinity Profiles:

Fig.5: The variation of critical Rayleigh number \( R_c \) for Step function, Desalting above and Salting below profiles with respect to the saline depth \( E \).

Figure 5 shows the variation of critical Rayleigh number \( R_c \) for different profiles with respect to the saline depth \( E \) for fixed values of \( Da = 0.1, \dot{\mu} = 0.5, \dot{d} = 1, \varepsilon_m = 1, \kappa = 1, \dot{S}_1 = \dot{S}_2 = 1, \hat{T} = 1, R_{s1} = R_{s2} = 5, \tau_1 = \tau_2 = 0.25, \tau_{pm1} = \tau_{pm2} = 0.75 \).

Graphically it is evident that the step function salinity profile is the unstable profile. Salting below salinity profile is the stable profile for the depth ratio \( 0 \leq \dot{d} \leq 0.45 \) and Desalting Above salinity profile is the stable profile for the depth ratio \( 0.45 \leq \dot{d} \leq 1 \).

At \( \dot{d} = 0.45 \) both salting below and desalting above profiles have same effect on \( R_c \).

Fig.6: The effect of \( \dot{\mu} \) on critical Rayleigh number \( R_c \) for Step function, Desalting above and Salting below profiles with respect to the saline depth \( E \).

Figure 6 shows the effect of the viscosity ratio \( \dot{\mu} = \frac{\mu_m}{\mu} = 1.5, 2, 2.5 \) which is the ratio of the effective viscosity of the porous matrix to that of the fluid layer on critical Rayleigh number \( R_c \). For fixed value of \( Da = 0.1, \dot{d} = 1, \varepsilon_m = 1, \kappa = 1, \dot{S}_1 = \dot{S}_2 = 1, \hat{T} = 1, R_{s1} = R_{s2} = 5, \tau_1 = \tau_2 = 0.25, \tau_{pm1} = \tau_{pm2} = 0.75 \).

With the increase in the value of \( \dot{\mu} \) increases the critical thermal Rayleigh \( R_c \) which stabilizes the system, so the onset of triple diffusive convection is delayed. In other words, when the effective viscosity of the porous medium \( \mu_m \) is made larger than the fluid viscosity \( \mu \), the onset of the convection in the fluid layer can be delayed.
From the above graphs it is evident that for fixed values of saline depth \( \varepsilon \), the increase in the value of solute Rayleigh number \( R_{s1} \) increases the value of critical Rayleigh number \( R_c \), i.e., the system is stabilized. Thus the onset of triple diffusive convection is delayed.

From the graph it is clear that critical Rayleigh number \( R_c \) decreases as \( \tau_{pm} \) increases in step function and desalting above profiles and \( R_c \) increases with increase in \( \tau_{pm} \) in Salting below profile. Thus the system is destabilized for step function and desalting above profile and stabilized for Salting below profile.
VI. CONCLUSION:

6.1. For Linear, Parabolic and Inverted Parabolic salinity Profile:

i) The curves of solute Rayleigh number of first solute \( R_{s1} \) are diverging, indicating that, in porous layer dominant composite systems the convection is delayed by increasing solute Rayleigh number \( R_{s1} \).

ii) The curves of diffusivity ratio \( \mu \) are converging, indicating that, in porous layer dominant composite systems the convection can be made fast by increasing the concentration of first salt.

6.2. For Salting below, Desalting above and Step function Profile:

i) By increasing the parameters \( \mu \) and \( R_{s1} \), triple diffusive convection for the above profiles is delayed.

ii) By increasing the thermal diffusivity ratio \( \tau_1 \), the triple diffusive convection in the above profiles is quick.

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