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# Sparse Signal Reconstruction with Multiple Side Information using Adaptive Weights for Multiview Sources

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## ABSTRACT

This work considers reconstructing a target signal in a context of distributed sparse sources. We propose an efficient reconstruction algorithm with the aid of other given sources as multiple side information (SI). The proposed algorithm takes advantage of compressive sensing (CS) with SI and adaptive weights by solving a proposed weighted $n$-$\ell_1$ minimization. The proposed algorithm adaptively exploits the multiple weights in two levels, first each individual intra-SI and then inter-SI weights are iteratively updated at every reconstructed iteration. This two-level optimization leads to exploiting the multiple SIs with different qualities. We experimentally perform our algorithm on generated sparse signals and also correlated feature histograms as multiview sparse sources from a multiview image database. The results show that RAMSIA significantly outperforms both classical CS and CS with single SI, and RAMSIA with higher number of SIs gained more than the one with smaller number of SIs.

**Index Terms**— Side information, compressive sensing, sparse signal, $n$-$\ell_1$ minimization, adaptive weights.

## 1. INTRODUCTION

Recent emerging applications such as visual sensor surveillance and mobile augmented reality are challenging in a distributed sensing scenario of smart tiny devices within heterogeneous networks in real-time. By cooperatively using multiple sensors, we can further enhance gathering and processing information under time-resource constraints for such devices. In particular, multiview object tracking and recognition [1, 2] are challenging how to utilize correlations among the multiview images without communications among them. The distributed sensing challenge may be addressed by representing it as a distributed sparse representation of multiple sources [3]. This raises the need for representing and reconstructing the sparse sources along with exploiting the correlated information among them. One of the key problems is how to robustly reconstruct a compressed source given supported observations, i.e., other available sources as SI. Attractive CS techniques [4–12] may enable these distributed sources to deal with such reconstruction problems.

Recently, some CS reconstructions with prior information [7–12] have been investigated in terms of theoretical as well as application aspects. It was shown that if the prior information is good enough, a $\ell_1$-$\ell_1$ minimization [7, 8], which integrates SI into classical CS, improves the reconstruction dramatically. The work in [9–12] shows efficient reconstructions with SI for sparse signals from a limited number of measurements in background subtraction and MRI imaging applications. These schemes are restricted to considering only one SI in conditions of good enough qualities. However, we are interested in the multiview heterogeneous sources with quality changing in time, i.e., unknown prior correlations, correspondences, and SI qualities among sources. Therefore, we are aiming at improving the reconstruction given arbitrary SIs, whose possible poor qualities and the multiple SIs are taken into account.

In this work, we propose an efficient reconstruction algorithm with multiple SIs using adaptive weights by solving a weighted $n$-$\ell_1$ problem. The algorithm adaptively reconstructs a sparse source from a small number of measurements, which are obtained by a random projection, given other sources. Our previous work [13] proposed a reconstruction algorithm with multiple SI, where the weights are iteratively updated for each given individual index cross all SIs under only its local constraint. To improve performance, the algorithm we propose here contributes to solving the $n$-$\ell_1$ problem that computes the adaptive weights in two levels. Intra-SI weights are first computed in each individual SI and then global inter-SI weights are updated at every iteration of the reconstruction. By this way, the proposed algorithm can robustly take advantage of exploiting both intra-source and inter-source correlations to adapt to on-the-fly correlation changes in time, e.g., multiview images with different SI qualities.

The rest of this paper is organized as follows. Section 2 reviews related works including fundamental problems of signal recovery and CS with SI. Our problem statement and proposed algorithm are presented in Sec. 3. We demonstrate performances of the proposed algorithm on distributed sparse sources in Sec. 4.

## 2. RELATED WORK

In this section, we review a fundamental problem of signal recovery from low-dimensional signals [4–6] and CS with SI...
Signal Recovery. Low-dimensional signal recovery arises in a wide range of applications in signal processing. Most signals in such applications have sparse representations in some domain. Let \( x \in \mathbb{R}^n \) denote a sparse source, which is indeed compressible. The source \( x \) can be reduced by sampling via a projection \([5]\). We denote a random measurement matrix for \( x \) by \( \Phi \in \mathbb{R}^{m \times n} \) \((m < n)\), whose elements are sampled from an i.i.d. Gaussian distribution. Thus, we get a compressed vector \( y = \Phi x \), also called measurement, consisting of \( m \) elements. The source \( x \) can be recovered \([4, 5]\) by solving:

\[
\min_{x} ||x||_1 \text{ subject to } y = \Phi x, \tag{1}
\]

where \( ||x||_p := (\sum_{i=1}^{n} |x_i|^p)^{1/p} \) is \( \ell_p \) norm of \( x \) wherein \( x_i \) is an element of \( x \).

Problem (1) becomes finding a solution to

\[
\min_{x} \{ H(x) = f(x) + g(x) \}, \tag{2}
\]

where \( f : \mathbb{R}^n \rightarrow \mathbb{R} \) is a smooth convex function with Lipschitz constant \( L \) \([6]\) of gradient \( \nabla f \) and \( g : \mathbb{R}^n \rightarrow \mathbb{R} \) is a continuous convex function possibly non-smooth. Problem (1) is obviously a special case of (2) with \( g(x) = \lambda ||x||_1 \), where \( \lambda > 0 \) is a regularization parameter, and \( f(x) = \frac{1}{2} ||\Phi x - y||_2^2 \). The results of using proximal gradient methods \([6]\) give that \( x^{(k)} \) at iteration \( k \) can be iteratively computed by:

\[
x^{(k)} = \Gamma_{\frac{1}{\lambda}}(x^{(k-1)} - \frac{1}{L} \nabla f(x^{(k-1)})), \tag{3}
\]

where \( L \geq L_{\nabla f} \) and \( \Gamma_{\frac{1}{\lambda}}(x) \) is a proximal operator that is defined by

\[
\Gamma_{\frac{1}{\lambda}}(x) = \arg \min_{v \in \mathbb{R}^n} \left\{ \frac{1}{L} g(v) + \frac{1}{2} ||v - x||_2^2 \right\}. \tag{4}
\]

CS with SI. CS with prior information or SI via \( \ell_1-\ell_1 \) minimization is improved dramatically if SI has good enough quality \([7, 8, 10]\). The \( \ell_1-\ell_1 \) minimization considers reconstructing \( x \) given a SI, \( z \in \mathbb{R}^n \) by solving the problem (2) with \( g(x) = \lambda (||x||_1 + ||x - z||_1) \), i.e., solving:

\[
\min_{x} \left\{ \frac{1}{2} ||\Phi x - y||_2^2 + \lambda (||x||_1 + ||x - z||_1) \right\}. \tag{5}
\]

The \( \ell_1-\ell_1 \) minimization (5) has an expression for the bound \([7, 8, 10]\) on the number of measurements required to successfully reconstruct \( x \) that is a function \([7, 8, 10]\) of the quality of SI \( z \).

3. RECONSTRUCTION WITH MULTIPLE SI USING ADAPTIVE WEIGHTS

3.1. Problem Statement

We consider a problem statement regarding how to efficiently reconstruct a sparse source given multiple SIs. The reconstruction for the distributed sensing of sparse sources is motivated by an idea of how multiple heterogeneous-dynamic lightweight cameras can perform robustly on the desired applications under resource-time constraints, where sensing and processing are expensive. CS \([4–6]\) is emerged as an elegant technique to deal with these challenges for such real-time sensing and processing. Despite prohibiting the communication between tiny cameras, we can take advantage of their correlations at the decoder through supported SIs. The known SIs can be some other reconstructed sources existing at the decoder, where we can exploit inter-source redundancies for a robust reconstruction from the reduced sparse source. Therefore, we need an advanced reconstruction flexibly adapting to on-the-fly changes according to the heterogeneous sparse sources.

Let us consider the example scenario of the distributed compressed sensing of tiny cameras for multiview object recognition in Fig. 1, where a corresponding feature histogram acquired from a given camera is considered as a sparse source \( x \) (Fig. 1(b)). Figure 1 shows three-view images, View 1 (Fig. 1(a)), View 2 (Fig. 1(c)), View 3 (Fig. 1(e)), of the given object 60 in the COIL-100 database \([14]\) with corresponding SIFT-feature \([15]\) points to create feature histograms as sparse sources. The feature histograms are created by extracting all SIFT \([15]\) features from the image then propagating down a hierarchical vocabulary tree based on a hierarchical \( k \)-means algorithm \([16]\). In reality, we may need very high-dimensional histograms, it is essential to reduce the source dimension by CS before further processing. The idea of CS is reducing the source without prior knowledge of the source distribution. Thus a reduced \( y \), which is obtained by compressing 1000 dimensions (D) \( x \) (Fig. 1(b)), is to be conveyed to the joint decoder. We use the available compressed \( y \) to reconstruct \( x \) with given known SIs, 1000-D \( z_1 \) (Fig. 1(d)) and 1000-D \( z_2 \) (Fig. 1(f)), which are feature histograms of neighbor views. Obviously, \( x, z_1, z_2 \) are naturally correlated in some degree.

In order to reconstruct \( x \) from \( y \) given \( z_1, z_2 \), we may go straight to the solution (5) \( \ell_1-\ell_1 \) minimization with only one input SI either \( z_1 \) or \( z_2 \). The measurements of \( y \) required to successfully reconstruct \( x \) is a function of the quality of \( z_1, z_2 \). There may be a chance that the \( \ell_1-\ell_1 \) reconstruction performs worse than the \( \ell_1 \) reconstruction, incurred by a not good enough SI, e.g., \( z_2 \), exposing the drawback of the \( \ell_1-\ell_1 \) reconstruction. Therefore, we come up with a new Reconstruction Algorithm with Multiple Side Information using Adaptive weights (RAMSIA) uses the measurement \( y \), which is obtained by \( y = \Phi x \), and \( J \) known SIs, \( z_1, z_2, ..., z_J \in \mathbb{R}^n \) as inputs. The final goal is to reconstruct output \( \hat{x} \) as best as we can.

RAMSIA is built up using the advances of the \( \ell_1-\ell_1 \) minimization (5) and additionally adaptive weights to robustly work on multiple SIs. The objective function of RAMSIA shall be created as an \( n-\ell_1 \) minimization based on the prob-
lem in (2) with

\[ g(x) = \lambda \left( \beta_0 \|W_0 x\|_1 + \sum_{j=1}^{J} \beta_j \|W_j (x - z_j)\|_1 \right), \]  

where \( \beta_j > 0 \) are weights among SIs and \( W_j \) are a diagonal weight matrix for each SI, \( W_j = \text{diag}(w_{j1}, w_{j2}, ..., w_{jn}) \), wherein \( w_{ji} > 0 \) is the weight in \( W_j \) at index \( i \) for a given SI \( z_j \). To conform the first term \( \beta_0 \|W_0 x\|_1 \) to the remaining terms, we rewrite it as \( \beta_0 \|W_0 (x - z_0)\|_1 \) where \( z_0 = 0 \), thus \( g(x) = \lambda \sum_{j=0}^{J} \beta_j \|W_j (x - z_j)\|_1 \) from (6). We may emphasize as a difference compared with our previous work [13] that we here compute weights in two levels. First \( w_{ji} \) for intra-SI weights and then inter-SI weights \( \beta_j \) among SIs as global weights are computed to optimize the weights. We end up formulating the objective function of the \( n-\ell_1 \) problem by:

\[ \min_x \left\{ H(x) = \frac{1}{2} \|\Phi x - y\|_2^2 + \lambda \sum_{j=0}^{J} \beta_j \|W_j (x - z_j)\|_1 \right\}. \]  

It can be noted that this (7) would become the problem (5) if \( \beta_j = 1 \), \( W_0 = W_1 = I \), and \( W_j = 0 \) \((j \geq 2)\), where \( I \) is a unit diagonal matrix (size of \( n \times n \)), \( I = \text{diag}(1, 1, ..., 1) \).

3.2. The proposed RAMSIA

Before solving the \( n-\ell_1 \) problem in (7), we need to address an arisen question how to determine the weight values to improve the reconstruction as well as avoid declination by the negative SIs to the reconstruction. We should distribute relevant weights not only in the intra-SI but also inter SIs. To optimize the objective function in (7), we impose constraints

Fig. 1. SIFT-feature histograms of object 60 in the COIL-100 [14].
on both all $\beta_j$ and $W_j$, by this way, we will be able to assign weights on multiple SIs according to their qualities during the iterative process. We propose to solve the $n \times \ell_i$ problem in (7) based on the proximal gradient method [6], i.e., at every iteration $k$ we need to update, on the one hand, the weights $\beta_j$, $W_j$ and on the other hand compute $x$. We may have different strategies to update the weights depending on our constraints.

To solve the minimization problem in (7), we propose SI constraints by $\sum_{j=1}^{n} w_{ji} = n$ at the intra SI level and $\sum_{j=1}^{J} \beta_j = 1$ at the inter SI level. We minimize the objective function $H(x)$ in (7) in two steps, one for $w_{ji}$ and the remain for $\beta_j$ during the iterative reconstruction. Referring to our previous work [13], a different weighting strategy was used, where $\sum_{j=0}^{n} w_{ji} = 1$ at a given index $i$ cross all SIs and all $\beta_j = 1$.

Firstly, by considering $x$ and $\beta_j$ fixed, for each SI $z_j$, we compute $W_j$ by optimizing (7), i.e., $\argmin_{W_j} \{H(x)\}$ is equal to

$$\argmin_{W_j} \left\{ \sum_{j=0}^{J} \beta_j ||W_j(x-z_j)||_1 \right\} = \argmin_{W_j} \left\{ \lambda \sum_{j=0}^{n} w_{ji} ||x_i-z_{ji}||_1 \right\},$$

(8)

where $z_{ji}$ is an element of $z_j$ at index $i$. We can achieve the minimization of (8) (following Cauchy inequality) when all $w_{ji}$ are equal to a positive parameter $\eta$, i.e., $w_{ji} = \eta/\left(||x_i-z_{ji}||_1+\epsilon\right)$ with a small added $\epsilon$. Combining with the intra SI constraint $\sum_{i=1}^{n} w_{ji} = n$, we get intra SI weights $w_{ji}$ by:

$$w_{ji} = n/\left(1+\left(||x_i-z_{ji}||_1+\epsilon\right)\right).$$

(9)

Secondly, given $x$ and $W_j$, we compute $\beta_j$ at the inter-SI level by optimizing the problem in (7):

$$\argmin_{\beta_j} \{H(x)\} = \argmin_{\beta_j} \left\{ \lambda \sum_{j=0}^{J} \beta_j ||W_j(x-z_j)||_1 \right\}.$$  

(10)

Similar to (8), from (10) we obtain $\beta_j$ as this formula:

$$\beta_j = \eta/\left(||W_j(x-z_j)||_1+\epsilon\right).$$

(11)

Combining (11) with the inter SI constraint $\sum_{j=0}^{J} \beta_j = 1$, we get the inter SI weights $\beta_j$ by:

$$\beta_j = 1/\left(1+\left(||W_j(x-z_j)||_1+\epsilon\right)\right) \left(\sum_{i=0,\neq j}^{J} \left(||W_i(x-z_i)||_1+\epsilon\right)^{-1}\right).$$

(12)

Given $W_j$ and $\beta_j$, we compute $x^{(k)}$ at iteration $k$ to minimize the problem in (7). Based on the proximal gradient method [6], $x^{(k)}$ is obtained from (3), where $g(x) = \lambda \sum_{j=0}^{J} \beta_j ||W_j(x-z_j)||_1$. The remaining key question is

Algorithm 1: The proposed RAMSIA algorithm.

Input: $y, \Phi, z_1, z_2, ..., z_J$;
Output: $\tilde{x}$;

// Initialization.
$W_0^{(1)} = I; \beta_0^{(1)} = 1; W_1^{(1)} = 0; \beta_1^{(1)} = 0 \ (1 \leq j \leq J);$
$u^{(1)} = \tilde{x}^{(0)} = 0; L = L_{\gamma}^z; \lambda, \epsilon > 0; t_1 = 1; k = 0;$
while Stopping criterion is false do

$k = k + 1;$
/ Solving given the weights.
$\n^{(k)}(k) = \Phi^T(\Phi u^{(k)} - y);$ 
$x^{(k)} = \sum_{l=1}^{L} \Gamma_{\lambda,\epsilon}(u^{(k)} - \frac{1}{L} \nabla f(u^{(k)})); \Gamma_{\lambda,\epsilon}(.)$ is given by (13); 
/ Computing the updated weights.
$w_{ji}^{(k+1)} = \frac{1}{1+\left(||W_j^{(k+1)}(x^{(k)}-z_i)||_1+\epsilon\right)} \left(\sum_{j=0}^{J} ||W_j^{(k+1)}(x^{(k)}-z_j)||_1+\epsilon\right)^{-1};$
/ Updating new values.
$t_{k+1} = (1+\sqrt{1+4t^2})/2; u^{(k+1)} = x^{(k)} + \frac{1}{t_{k+1}} (x^{(k)} - x^{(k-1)});$
end
return $x^{(k)}$;

how to compute the proximal operator $\Gamma_{\lambda,\epsilon}(x)$. From (26) (derived in Appendix), $\Gamma_{\lambda,\epsilon}(x)$ is obtained by:

$$\Gamma_{\lambda,\epsilon}(x_i) = \begin{cases} x_i - \eta \sum_{j=0}^{J} \beta_j w_{ji} (-1)^{b_i} & \text{if } (19); \\ 0 & \text{if } (20); \end{cases}$$

Finally, we sum up the proposed RAMSIA in Algorithm 1 based on a fast iterative FISTA algorithm [6], where the main difference from FISTA is the iterative computation of the updated weights. RAMSIA iteratively updates two weight levels in turn $w_{ji}$ in (9) and $\beta_j$ in (12). It can be noted that the Stopping criteria in Algorithm 1 can be either a maximum iteration number $k_{max}$, a relative variation of the objective function $H(x)$ (7), or a change of the number of non-zero components of the estimate $x^{(k)}$. In this work, the relative variation of $H(x)$ (7) is chosen.

4. EXPERIMENTAL RESULTS

First we consider the reconstruction of multiview feature histograms as sparse sources, which could be used in multiview object recognition. Given an image, its feature histogram is formed as in Sec. 3.1, e.g., $x$. The size of the vocabulary tree [16] depends on the value of $k$ and number of hierarchies, e.g., if $k = 10$ and 3 hierarchies, $n = 1000$ vocabularies as 1000-D, which may be a very big number in reality. Because of the small number of features in a single image, the histogram
Fig. 2. Success probabilities of reconstructing the original 1000-D \( x \) vs. number of dimensions of the compressed \( y \) using 1,2,3 SIs.

vector \( x \) is sparse. Therefore, \( x \) is first projected into the compressed vector \( y \) that is to be sent to the decoder. At the joint decoder, we suppose \( x \) is to be reconstructed given some already decoded histograms of neighbor views, e.g., \( z_1, z_2, z_3 \). In this work, we use the COIL-100 [14] containing multiview images of 100 small objects with different angle degrees. For the sake of ensuring our experimental setup a realistic circumstance of multiple distributed sources without given correspondences, we randomly select the 4 neighbor views (3 neighbors as SIs) of representative objects 16 (Fig. 2(a)) and 60 (Fig. 2(b)) over 72 views captured through 360 degrees in the COIL-100 [14] multiview database. In addition, we consider the reconstruction of a generated sparse source \( x \) given three known SIs (Fig. 2(c)). We generate \( x \) with \( n = 1000 \) with 100 supports (number of non-zeros), which is generated from the standard i.i.d. Gaussian distribution. To illustrate a
scenario of the less correlated SIs, we generate SIs with all supports of subtractions, \( x - z_1, x - z_1, x - z_1 \) are equal to 300.

We will evaluate and compare the reconstruction accuracy of the proposed RAMSIA exploiting different subsets of SIs against the existing \( \ell_1 \) reconstructions. Let RAMSIA-J-\( \ell_1 \) denote the RAMSIA reconstruction, where \( J \) indicates number of SIs, e.g., RAMSIA-1-\( \ell_1 \), RAMSIA-2-\( \ell_1 \), RAMSIA-3-\( \ell_1 \) are RAMSIA using 1 \((z_1)\), 2 \((z_1,z_2)\), and 3 \((z_1,z_2,z_3)\) SIs, respectively. Similarly, RAMSI-1-\( \ell_1 \), RAMSI-2-\( \ell_1 \), RAMSI-3-\( \ell_1 \) are three versions in our previous work [13]. Let FISTA-\( \ell_1 \), FISTA-\( \ell_2 \), FISTA-\( \ell_3 \) denote the \( \ell_1, \ell_2, \ell_3 \) CS reconstruction with one SI (\( z_1 \)) [7].

The existing FISTA [6] and FISTA-\( \ell_1 \) reconstructions [7] are used for comparison. The original source 1000-D \( x \) is compressed into different lower-dimensions \( y \). We assess the accuracy of a reconstructed \( \hat{x} \) versus the \( x \) via the success probability, denoted as \( \Pr(\text{success}) \), versus the number of dimensions. For a fixed dimension, the \( \Pr(\text{success}) \) is the number of times, in which the source \( x \) is recovered as \( \hat{x} \) with an error \(||\hat{x} - x||_2/||x||_2| \leq 10^{-3}\), divided by the total 100 trials (each trial considered randomly selecting 4 neighbor views for objects 16,60 (Figs. 2(a)+2(b)) and different generated \( x, z_1,z_2,z_3, \Phi \) for generated signals (Fig. 2(c))). RAMSIA (Algorithm 1) thus has \( n = 1000, m < n, J = 1, 2, 3 \) and we set parameters \( \epsilon = 0.1, \lambda = 10^{-5} \).

Figure 2 presents the performances of the proposed RAMSIA as well as RAMSI [13] with 1,2,3 SIs, FISTA [6], and FISTA-\( \ell_1 \) [7] in terms of the success probabilities versus dimensions. It shows clearly that RAMSIA significantly improves the reconstruction accuracy. Furthermore, the results using 3 SIs, RAMSIA-3-\( \ell_1 \) gives highest accuracy and the performance of RAMSIA-2-\( \ell_1 \) is higher than that of RAMSIA-1-\( \ell_1 \). For instance, RAMSIA requires 300, 320, 340 measurements corresponding to using 3, 2, 1 SIs to successfully reconstruct Object 16 (Fig. 2(a)). Despite exploiting only the same one SI, RAMSIA-1-\( \ell_1 \) is absolutely better than FISTA-\( \ell_1 \),\( \ell_1 \). For the signal in Fig. 2(c), the accuracies of FISTA-\( \ell_1 \),\( \ell_1 \) are worse than those of FISTA, i.e., SI does not help, however, RAMSIA-\( \ell_1 \) still significantly outperforms FISTA. These results may highlight the drawback of \( \ell_1 \) minimization when SI qualities are not good enough. Therefore, if we exploit the weighted \( \ell_1 \) minimization as RAMSIA, our reconstruction can deal with the negative instances by adaptive weights, e.g., less weights on such negative cases of SI \( z_1 \), to achieve the improvements for all scenarios.

\section{5. CONCLUSION}

This paper proposed an sparse signal reconstruction with multiple SIs for the distributed sources by solving the general \( \ell_1 \) minimization problem. The proposed RAMSIA scheme iteratively updated the adaptive weights in two levels not only in each intra-SI but among SIs to adaptively optimize the reconstruction. RAMSIA took advantage of exploiting both intra-source and inter-source redundancies to adapt to the changes of the heterogeneous sparse sources to deal with the different SI qualities. We experimentally tested RAMSIA on the multiple feature histograms as multiview sparse sources captured from distributed cameras and also the generated signals. The results showed that RAMSIA robustly outperformed the conventional CS and recent \( \ell_1 \) minimization methods. Moreover, RAMSIA with higher number of SIs gained more improvements than the one with smaller number of SIs.

\section{6. APPENDIX}

We shall compute the proximal operator \( \Gamma_{\beta y}(x) \) (4) with \( g(x) = \lambda \sum_{j=0}^{J} |\mathbf{W}_j(x - z_j)|_1 \). From (4), \( \Gamma_{\beta y}(x) \) is expressed by:

\[
\Gamma_{\beta y}(x) = \arg\min_{x \in \mathbb{R}} \left\{ \lambda \sum_{j=0}^{J} \beta_j |\mathbf{W}_j(x - z_j)|_1 + \frac{1}{2} ||x - \beta_{\beta}||_2^2 \right\}. 
\]  

(14)

We may note that both terms in (14) are separable in \( v \) and thus we can minimize each element \( v_i \) of \( v \) individually as

\[
\Gamma_{\beta y}(x) = \arg\min_{v_i \in \mathbb{R}} \left\{ \lambda \sum_{j=0}^{J} \beta_j w_{ji} |v_i - z_{ji}|_1 + \frac{1}{2} ||v_i - x_i||_2^2 \right\}.
\]  

(15)

We consider the \( \partial h(v_i)/\partial v_i \). Without loss of generality, we assume \(-\infty \leq z_0 \leq z_1 \leq \cdots \leq z_J \leq \infty \). For convenience, let us denote \( z_{-1} = -\infty \) and \( z_{J+1} = \infty \). When \( v_i \) is located in one of the intervals, we suppose \( v_i \in (z_{li}, z_{li+1}) \) with \(-1 \leq l \leq J \), where \( \partial h(v_i) \) exists. Taking the derivative of \( h(v_i) \) in \((z_{li}, z_{li+1})\) delivers

\[
\frac{\partial h(v_i)}{\partial v_i} = \lambda \sum_{j=0}^{J} \beta_j w_{ji} \text{sign}(v_i - z_{ji}) + (v_i - x_i),
\]  

(16)

where \( \text{sign}(\cdot) \) is a sign function. In addition, let \( b(\cdot) \) denote a boolean function, i.e., \( b(l < j) = 1 \) if \( l < j \), otherwise \( b(l < j) = 0 \). Consequently, \( sign(v_i - z_{ji}) = (-1)^{b(l < j)} \) and from (16), we rewrite:

\[
\frac{\partial h(v_i)}{\partial v_i} = \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (-1)^{b(l < j)} + (v_i - x_i).
\]  

(17)

When setting \( \partial h(v_i)/\partial v_i = 0 \) to minimize \( h(v_i) \), we derive:

\[
v_i = x_i - \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (-1)^{b(l < j)}.
\]  

(18)

This \( v_i \) (18) is only valid in \((z_{li}, z_{li+1}), \) i.e.,

\[
z_{li} + \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (-1)^{b(l < j)} < x_i < z_{li+1} + \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (-1)^{b(l < j)}.
\]  

(19)
We will prove that \( h \) in the following Lemma 6.1.

### Lemma 6.1

Given \( x_i \) belonging to the intervals represented in (20), \( h(v_i) \) in (15) is minimum when \( v_i = z_{ti} \).

**Proof.** We re-express \( h(v_i) \) (15) by:

\[
h(v_i) = \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (v_i - z_{ti}) - (z_{ji} - z_{ti}) + \frac{1}{2} ((v_i - z_{ti}) - (x_i - z_{ti}))^2.
\]

(21)

Applying a simple inequality \( |a - b| \geq |a| - |b| \), where \( a, b \in \mathbb{R} \), to the first term and expanding the second term in (21), we obtain:

\[
h(v_i) \geq \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} |v_i - z_{ti}| - \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} |z_{ji} - z_{ti}| + \frac{1}{2} (v_i - z_{ti})^2 - (v_i - z_{ti}) (x_i - z_{ti}) + \frac{1}{2} (x_i - z_{ti})^2.
\]

(22)

It can be noted that \( -(v_i - z_{ti}) (x_i - z_{ti}) \geq -|v_i - z_{ti}| |x_i - z_{ti}| \). Thus the inequality in (22) is equivalent to:

\[
h(v_i) \geq |v_i - z_{ti}| \left( \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} |x_i - z_{ti}| \right) + \frac{1}{2} |v_i - z_{ti}|^2 + \frac{1}{2} (x_i - z_{ti})^2.
\]

(23)

Without difficulty, from the expression in (20), we get:

\[-\frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} \leq x_i - z_{ti} \leq \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} \iff |x_i - z_{ti}| \leq \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji}.\]

(24)

Eventually, we observe that the part including \( v_i \) in the right hand side of the inequality \( h(v_i) \) in (23) is:

\[
|v_i - z_{ti}| \left( \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} |x_i - z_{ti}| \right) + \frac{1}{2} |v_i - z_{ti}|^2.
\]

(25)

With (24), the expression in (25) is minimum when \( v_i = z_{ti} \). Therefore, we deduce that \( h(v_i) \) (15) is minimum when \( v_i = z_{ti} \).

In summary, from (18) with conditions in (19), (20) and Lemma 6.1, we obtain:

\[
\Gamma_{\beta, L}(x_i) = \begin{cases} 
  x_i - \frac{\lambda}{L} \sum_{j=0}^{J} \beta_j w_{ji} (-1)^b l < j) & \text{if (19)}; \\
  z_{ti} & \text{if (20)};
\end{cases}
\]

(26)

### 7. REFERENCES

[1] M. Taj and A. Cavallaro, “Distributed and decentralized multicamera tracking,” *IEEE Signal Process. Mag.*, vol. 28, no. 3, pp. 46–58, 2011.

[2] A. Sankaranarayanan, A. Veeraraghavan, and R. Chellappa, “Object detection, tracking and recognition for multiple smart cameras,” *Proc. of IEEE*, vol. 96, no. 10, pp. 1606–1624, 2008.

[3] A. Y. Yang, M. Gastpar, R. Bajcsy, and S. Sastry, “Distributed sensor perception via sparse representation,” *Proc. of IEEE*, vol. 98, no. 6, pp. 1077–1088, 2010.

[4] D. Donoho, “For most large underdetermined systems of linear equations the minimal \( \ell_1 \)-norm solution is also the sparsest solution,” *Communications on Pure and Applied Math.*, vol. 59, no. 6, pp. 797–829, 2006.

[5] E. Candès and T. Tao, “Near-optimal signal recovery from random projections: Universal encoding strategies?” *IEEE Trans. Inf. Theory*, vol. 52, no. 12, pp. 5406–5425, Apr. 2006.

[6] A. Beck and M. Teboulle, “Fast iterative shrinkage-thresholding algorithm for linear inverse problems,” *SIAM Journal on Imaging Sciences*, vol. 2(1), pp. 183–202, 2009.

[7] J. F. Mota, N. Deligianni, and M. R. Rodrigues, “Compressed sensing with side information: Geometrical interpretation and performance bounds,” in *IEEE Global Conf. on Signal and Information Processing*, Austin, Texas, USA, Dec. 2014.

[8] ——, “Compressed sensing with prior information: Optimal strategies, geometry, and bounds,” ArXiv e-prints, Aug. 2014.

[9] J. Scarlett, J. Evans, and S. Dey, “Compressed sensing with prior information: Information-theoretic limits and practical decoders,” *IEEE Trans. Signal Process.*, vol. 61, no. 2, pp. 427–439, Jan. 2013.

[10] J. F. Mota, N. Deligiannis, A. Sankaranarayanan, V. Cevher, and M. R. Rodrigues, “Dynamic sparse state estimation using \( \ell_1-\ell_1 \) minimization: Adaptive-rate measurement bounds, algorithms and applications,” in *IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Brisbane, Australia, Apr. 2015.

[11] L. Weizman, Y. C. Eldar, and D. B. Bashat, “Compressed sensing for longitudinal MRI: An adaptive-weighted approach,” *Medical Physics*, vol. 42, no. 9, pp. 5195–5207, 2015.
[12] G. Warnell, S. Bhattacharya, R. Chellappa, and T. Basar, “Adaptive-rate compressive sensing using side information,” *IEEE Trans. Image Process.*, vol. 24, no. 11, pp. 3846–3857, 2015.

[13] H. V. Luong, J. Seiler, A. Kaup, and S. Forchhammer, “A reconstruction algorithm with multiple side information for distributed compression of sparse sources,” in *Data Compression Conference*, Snowbird, Utah, USA, Apr. 2016.

[14] S. A. Nene, S. K. Nayar, and H. Murase, “Columbia object image library (coil-100),” Technical Report CUCS-006-96, Feb. 1996.

[15] D. G. Lowe, “Object recognition from local scale-invariant features,” in *IEEE Int. Conf. on Computer Vision*, Kerkyra, Greece, Sep. 1999.

[16] D. Nistér and H. Stewénius, “Scalable recognition with a vocabulary tree,” in *IEEE Int. Conf. on Computer Vision and Pattern Recognition*, New York, USA, Jun. 2006.