Disaggregating a Regional Extent Digital Soil Map using Bayesian Area-to-Point Regression Kriging for Farm-Scale Soil Carbon Assessment

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Abstract
Most soil management activities are implemented at farm scale, yet digital soil maps are commonly available at regional or national scale. Disaggregating these regional/national maps is to be applicable for farm scale tasks particularly in data poor or limited situations. Although disaggregation is a frequently discussed topic in recent literature, the uncertainty of the disaggregation process is not often discussed. Underestimation of inferential or predictive uncertainty in statistical modelling leads to inaccurate statistical summaries and overconfident decisions. The use of Bayesian inference allows for quantifying the uncertainty associated with the disaggregation process. In this study, a framework of Bayesian Area-to-Point Regression Kriging (ATPRK) is proposed for downscaling soil attributes, in particular, maps of soil organic carbon. Estimation of point support variogram from block supported data, was carried out using the Monte Carlo integration via the Metropolis-Hasting Algorithm. A regional soil carbon map with a resolution of 100 m (block support) was disaggregated to 10 m (point support) information for a farm in the northern NSW, Australia. The derived point support variogram has a higher partial sill and nugget while the range and parameters do not deviate much from the block support data. The disaggregated fine-scale map (point support with a grid spacing of 10m) using Bayesian ATPRK had an 87% concordance correlation with the original coarse scale map. The uncertainty estimates of the disaggregation process were given by a 95% confidence interval (CI) limits. Narrow CI limits indicate, the disaggregation process gives a fair approximation of mean SOC content of the study site. The Bayesian ATPRK approach was compared with dissever; a regression-based disaggregation algorithm. The disaggregated maps generated by dissever had 96% concordance correlation with the coarse-scale map. dissever achieves this higher concordance correlation through an iteration process while Bayesian ATPRK is a one-step process. The two disaggregated products were validated with 127 independent topsoil carbon observations. The validation concordance correlation coefficient for Bayesian ATPRK disaggregation was 23% while downscaled maps generated from dissever had 18% CCC. The advantages and limitations of both disaggregation algorithms are discussed.
1 Introduction

The proliferation of digital soil mapping (DSM) and modelling techniques during the past decade has made available vast amounts of digital soil information (Lagacherie, 2008; Grunwald, 2009; Minasny and McBratney, 2016). For example, the GlobalSoilMap project (Arrouays et al., 2014) aspired towards delivering a global coverage of soil attributes at 100 m resolution for all 

regression techniques. Area to point kriging (ATPK) (Kyriakidis, 2004) and its variants such as area to point regression kriging (ATPRK) are the most common geostatistical techniques applied to disaggregate soil resource information. Incorporating both point and areal data in the kriging system is such a variant of ATPK presented by Goovaerts (2011). That study demonstrated ATPK to map the variability within soil units and secondly introduced a general formulation for incorporating area-to-area, area-to-point, and point-to-point covariance in the kriging system. Replacing the areal mean with summary statistics in the kriging system is another improved ATPK technique suggested by Orton et al. (2014). Román Dobarco et al. (2016) extended this approach using summary statistics in the ATPRK system. Further, a Bayesian regression kriging approach was presented by Ramos et al. (2013). The model used novel image fusion technique for panchromatic sharpening of low-resolution satellite images using images from unmanned aerial vehicle flown over agricultural fields to demonstrate their model.

Regression approaches are also a popular approach to downscale soil attributes. For example, Malone et al. (2012), formalised a statistical downscaling algorithm; dissever based on the Liu and Pu (2008) algorithm for downscaling continuous earth resources with a case study of downscaling soil organic carbon content. This method was refined by Roudier et al. (2017), allowing the user to choose between a range of regression models apart from the originally contained Generalised additive models (GAM). “DSMART” (Disaggregation and Harmonisations of Soil Map Units through Resampled Classification Trees) is another popular regression-based approach, proposed by Ogders et al. (2014) to downscale soil map units to soil series information. However it is different to the discussed method since there is no fine gridding involved.

Disaggregation is a change of support problem (Cressie, 1991; Gotway and Young, 2002). Spatial support is the geometrical size, shape and orientation spatial unit of an observation or a prediction. In the disaggregation process, when the support changes the associated statistical and spatial properties of the variable also change, adding a major source of uncertainty (Truong et al., 2014). ATPK is a theoretically-based method that uses the classical kriging principles to predict at point support using block support information (Kyriakidis, 2004). Incorporating fine scale covariate data in the kriging system (ATPRK) is superior to ATPK as it allows...
for within class variability, spatial autocorrelation and also ancillary covariates representing “scorpan” factors (McBratney et al., 2003) represented by ancillary covariates (Kerry et al., 2012).

In disaggregating soil information, quantifying the uncertainties of this process is not often discussed. Underestimation of inferential or predictive uncertainty in statistical modelling can lead to inaccurate statistical summaries and overconfident decisions (Draper, 1995). Therefore, it is important to account for the underlying parameter uncertainty associated with the disaggregation process. The use of Bayesian inference in ATPRK system enables quantifying the uncertainty associated with inferring point support parameters from the block supported system, in addition to estimating the product uncertainty (Diggle et al., 1998).

To the best of our knowledge, Bayesian ATPRK has not been used for downscaling soil information. In this paper, we present a Bayesian ATPRK approach for downscaling soil attributes. Specifically, the study focuses on downscaling regional soil organic carbon (SOC) maps to high-resolution farm scale maps. Soil-organic carbon SOC is arguably a key soil property due to its conferring benefits to the soil physical and chemical properties and potential to store atmospheric carbon. Accurate estimations of SOC stocks at farm scale are important for precision agricultural needs. In addition, the fine-scale maps can be used for designing sampling strategies for quantification of SOC stocks (de Gruijter et al., 2016) required by carbon sequestration auditing process such as carbon farming initiatives in Australia (Malone et al., 2017).

A regional soil carbon map with a resolution of 100 m (block support) was disaggregated into 10 m (point support) information for a farm in the Spring Ridge region in the northern NSW. Application of the Bayesian ATPRK approach is presented and demonstrated within the study. In addition, a comparative analysis with the dissever algorithm, a regression-based downscaling method, is performed to assess the performance of Bayesian ATPRK in the context of spatial downscaling of digital soil carbon mapping.

### 2 Theoretical background

Assuming we are observing a continuous Gaussian process, we denote the process as \( S(p) \) for locations \( p \in D \), where \( D \) is the region of interest. For block support (B) data, where \( B \subset D \), we presume that the observations are as block averages:

\[
S(B) = |B|^{-1} \int_B S(p) \, dp
\]

where \( |B| \) denotes the area of \( B \).

The underlying Gaussian process \( S \) can be expressed as

\[
S = \sum \alpha_m \beta + \epsilon
\]

where \( \mathbf{m} \) is the design matrix of the trend function and \( \beta \) vector of coefficients, and \( \epsilon \) is the stochastic residuals with mean of zero and covariance function \( C(\mathbf{0}) \), where \( \mathbf{0} \) represent the parameters of the covariance function.

Since the block supported observations; \( S_B \) and point support observations; \( S_p \) are jointly Gaussian, the prediction problem can be introduced as:

\[
\begin{pmatrix}
S_B \\
S_p
\end{pmatrix} = N
\begin{pmatrix}
\mathbf{m} \\
\beta_p \\
\beta_m
\end{pmatrix}
\begin{pmatrix}
C_{pp} & C_{pm} & C_{mm}
\end{pmatrix}
\]
where $C_{pp}$ is the variance-covariance matrix of $S_p$, $C_{nn}$ the variance-covariance matrix of $S_n$, $C_{pn}$ and $C_{np}$ are the variance-covariance matrices between $S_p$ and $S_n$ and vice versa. $m$ is the design matrix of for fine-scale covariates of $S_p$.

Since the joint distribution is Gaussian the optimal predictions at unsampled point locations can be given as,

$$
\hat{S}_p = m^T \beta + C_{pn} C_{nn}^{-1} (S_n - M \beta) \quad (4)
$$

where $\hat{S}_p$ is the vector of predicted soil carbon at $N$ point locations. And the variance-covariance matrix for the prediction error is,

$$
\text{var}(S_p|S_n) = C_{pp} - C_{pn} C_{nn}^{-1} C_{np} \quad (5)
$$

This shows that the inference of $S_p$ from $S_n$ is straightforward and resembles a simple kriging formulation (Cressie and Wikle, 2011). However, the estimation of point support variogram from the block support data is not a trivial task. Usually, the estimation is done via deregulation or deconvolution of the empirical variogram. Goovaerts (2008); Gotway and Young (2007); Pardo-Igúzquiza and Atkinson (2007) proposed an iterative numerical procedure for the deconvolution of the block support variogram for irregular block support. Wang et al. (2015) suggested an integration approach, while Nagle et al. (2011) used a maximum likelihood approach. Gelfand et al. (2001) used Markov Chain Monte Carlo (MCMC) integration to estimate point support variogram from block support data where the parameters of point support variogram were given as non-informative priors. Truong et al. (2014) used a similar approach, but the parameters of the point support variogram were given as informative priors. In this paper, the variogram parameters were inferred directly from the data via Bayesian estimation with an objective of inferring the uncertainty of the disaggregation process.

### 2.1 Bayesian inference

The likelihood function of Gaussian process $S$ can be given as

$$
f(S|\beta, \theta) = N(m \beta, C(\theta)) \quad (6)
$$

where $S/S_n \beta, \theta$ is distributed as

$$
N \left( M \beta + C_{pp}(\theta) C_{np}(\theta) (S_n - m \beta) \right) \quad (7)
$$

Each entry of the distribution can be estimated through a Monte Carlo integration. Then we can replace Eq. 3 with

$$
\tilde{f} \left( (S_p, S_n) \mid \beta, \theta \right), \text{where ‘hat’ denotes a Monte Carlo integration. Then it is apparent that,}
$$

$$
\tilde{f} \left( (S_p, S_n) \mid \beta, \theta \right) = f \left( (S_p, S_n) \right) \mid \beta, \theta \quad (8)
$$
To predict at \( \mathbf{S}_p \), we require \( f(\mathbf{S}_p, \mathbf{S}_B) \), which is given by Eq. 3. Using \( f(\mathbf{S}_p, \mathbf{S}_B) \mid \beta, \theta \), and using Eq. 8 we can obtain \( f(\mathbf{S}_p, \mathbf{S}_B) \mid \beta, \theta \) to sample \( \mathbf{S}_p \).

Then given the priors \( [\beta, \theta] \), the Bayesian model is specified, and a Monte Carlo fitting procedure can be carried out to maximise following log likelihood function. A complete description of deducing the Bayesian area to point inference can be found in Gelfand et al. (2001).

\[
L(\beta, \theta; \mathbf{S}_B) = \frac{n}{2} \log(2\pi) + \frac{1}{2} \text{log} |\mathbf{C}_{\text{BB}}| - \frac{1}{2} (\mathbf{S}_n - \mathbf{M}\beta)^T \mathbf{C}_{\text{BB}}^{-1} (\mathbf{S}_n - \mathbf{M}\beta) \tag{9}
\]

Where \( \mathbf{C}_{\text{BB}} \) is block to block covariance and can be approximately calculated as

\[
\mathbf{C}_{\text{BB}}(i,j) = \begin{cases} 
\frac{1}{K} \left( 2 \left( \frac{2}{K(K-1)} \sum_{k=1}^{K} \sum_{l=1}^{K} (s_k + c_0 - \gamma_p(s_k - s_l)) \right) \right), & i \neq j \\
\frac{1}{K} (c_1 + c_0 - \gamma_p(s_k - s_l)), & i = j 
\end{cases} 
\tag{10}
\]

Where \( i, j \) index are block supported observations, \( k, l \) index discretised points within a block and \( K \) is the number of discretisation points per block (Truong et al., 2014). \( c_1, c_0 \) are the partial sill and the nugget component of the semivariogram; \( \gamma_p \).

### 3 Case study

#### 3.1 Study area and coarse scale soil carbon map

We illustrate the methodology by disaggregating a regional topsoil organic carbon map of the University of Sydney E. J. Holtsbaum Agricultural Research Station (Fig. 1). The landholding, also known as “Nowley”, is located in the Spring Ridge district of the central/north west slopes region of NSW, Australia. Its area is approximately 2300ha and is managed as a mixed farming enterprise of cattle grazing throughout the year, and wheat, barley, and canola in winter, and sorghum and sunflower in the summer. Cropping is performed on soils derived from basalt parent materials. The average annual rainfall is 600mm.

The available map is a New South Wales (NSW) topsoil carbon map, which was generated using a local regression kriging technique (Somarathna et al., 2016). These maps are analogues to the nationally available soil carbon maps of Australia which are available in separate layers corresponding to the depth intervals definition in the GlobalSoilMap product specifications (Arrouays et al., 2014). The standard soil depth layers are 0-5cm, 5-15cm, 15-30cm, 30-60cm, and 60-100cm.

This NSW soil carbon map has grid spacing 100m x 100m, with block support of the same size. Hence, map spatial resolution and the support are equal, which in turn mean that the pixel values that make up the map represent a predicted mean for the area of each pixel. Commonly the pixel values of a digital soil map are on point support, with the pixel value representing the value at a point usually at the centre of each pixel (Malone et al., 2013). Therefore, block kriging was used convert the maps to point supported maps before the analysis.
The topsoil of the study area was represented by the depth interval of 0-7.5 cm, as this was the depth interval used in previous work conducted at this site for soil carbon auditing (de Gruijter et al., 2016). The respective coarse (NSW) map for the 0-7.5 cm depth interval was predicted by using mass-preserving splines (Malone et al., 2009) of the 0-5 cm and 5-15 cm standardised maps. This map will be hereafter referred to as the coarse scale map. It is this map that subsequent descriptions of spatial downscaling will refer to. The experimental variogram of the block supported coarse scale map is given in Fig. 2.

3.2 Environmental covariates

Elevation, topographic wetness index (TWI), Gamma radiometric Thorium, gamma radiometric potassium, Landsat band 4 and Landsat band 7 were used as environmental covariates in the spatial downscaling model. Landsat data has a resolution of 30 m. Since the coarse map was disaggregated into 10 m resolution, the Landsat data were pan-sharpened using the panchromatic band (15 m), and then resampled to 10 m.

The elevation data had been collected during a ground survey using an all-terrain vehicle at with 20 m swath. Then the data were processed using a local block kriging approach to produce an elevation data at a 10 m resolution. Based on the developed digital elevation model (DEM), the Topographic Wetness Index (TWI) was derived. The gamma radiometric data was derived from aerial survey (Minty et al., 2009) which were resampled to 10 m. A more detailed description of data collection and post-processing can be found in Malone et al. (2017).

3.3 Model parameter inferencing using the MCMC simulation

Suppose our observed soil carbon data (S) follows a second-order stationary Gaussian process which is characterised by the mean (trend) and the variogram where \( S_\mathbf{p} \sim N(\mathbf{S}_\mu, \gamma_\mathbf{h}) \) where \( \gamma_\mathbf{h} = (h, \theta) \) : is a function of Euclidian distance \( h \) and \( \theta \) vector of parameters of Matérn variogram (Eq. 14). The block averages \( S_\mathbf{p} \) have the same spatial mean as \( S_\mu \) but a different spatial structure (Gotway and Young, 2002). The block support covariance \( C_{

\begin{align*}
C_{ij} &= c_0 \delta_{ij} + c_1 \left[ \frac{1}{2\pi r} \Gamma(v) K_v \left( \frac{h}{r} \right) \right] 
\end{align*}

where \( C_{ij} \) is the covariance between observation i and j, h represents the separation distance between i and j, \( \delta_{ij} \) denotes the Kronecker delta (\( \delta_{ij} = 1 \) if i=j and \( \delta_{ij} = 0 \) when \( i \neq j \)), \( c_0 + c_1 \) signifies the sill variance, \( K_v \) is the modified Bessel function of the second kind of order \( v \), \( \Gamma \) is the gamma function, \( r \) denotes the distance or ‘range’ parameter and \( v \) is the spatial ‘smoothness’. Calculating \( C_{BB} \) requires discretisation and it was done using a regular grid pattern, with equal 10 m spacing between discretising points within a block. Then \( C_{BB} \) was calculated according to Eq. 11.

The priors were given as informative priors and considered as normally distributed. The coarse scale soil carbon map was regressed against the selected covariates to estimate the approximate prior values for the coefficient of the linear trend (\( \beta \)). Gaussian priors were assigned for both spatial trend (\( \beta \)) and covariance
component (θ) of the spatial model. MCMC integration was carried out using the Metropolis Hasting algorithm. Metropolis Hasting is a computationally efficient algorithm that allows sampling from conditional distributions (Diggle et al., 1998). Calibration of the model was done using two parallel chains of MCMC with 100,000 iterations.

3.4 Spatial prediction
The mean, 95% upper and lower CI limits of spatial model parameters were calculated using the values of the last 10,000 MCMC iterations. These calibrated parameters were used to predict SOC content at the fine scale. The predictions were made on the 10m x 10m points based on Eq. 4.

3.5 Disaggregation using dissever
The proposed Bayesian model also compared with the dissever regression–based disaggregation approach. dissever is a regression-based downscaling approach proposed by Malone et al. (2012). dissever operates in two steps; initialisation and iteration. At initialisation step, the coarse map is resampled using nearest neighbour resampling technique to match the resolution of available fine scale covariates. Then the resampled map is regressed against the fine scale covariates. Next, the regressed fine map is upscaled through averaging, and then the mass balance deviation (deviation factor; DF) from the coarse map is calculated. The regressed fine map is corrected using the DF, and then the algorithm progresses to the iteration step. In the iteration step, regression and upsampling continue until the mass balance deviation between the upscaled map and the original map is less than a set minimum (threshold). The following flow diagram (Fig. 3) is an illustrative description of the dissever algorithm. This differs from Bayesian ATPRK since it is not focused to address the change of support problem and it is a fine gridding exercise. A detailed and comprehensive account of dissever can be found in Malone et al. (2012) and Malone et al. (2017).

Following the procedure mentioned above, the coarse scale carbon map of the area was disaggregated into 10m using dissever. The environmental covariates used in the Bayesian approach were similarly made available for dissever.
3.6 Independent validation

3.6.1 Validation data

The accuracy of the model was tested using an independent dataset which consists of 127 soil samples from 0 - 7.5 cm soil cores (cross-section area of 41 cm$^2$ and height of 7.5 cm). The samples are considered to have point support. This data was collected over two separate soil surveys during 2014 and 2015 using stratified random sampling. Stratification was based on the SOC prediction fields generated from digital soil mapping. A detailed description about the stratified random sampling process can be found in de Gruijter et al. (2016). The carbon content of collected soil cores was measured using dry combustion by Elementar Vario Max CNS macro elemental analyser (Elementar Analyses System GmbH, Hanau, Germany). The measured carbon contents are in carbon concentrations (g of C per 100 g of soil).

3.6.2 Validation statistics

The model accuracy was compared using Mean squared error (MSE) and Lin’s concordance correlation coefficient (CCC) (Lin, 1989) between the disaggregated values and observed independent values at sampling points.

CCC is given by

$$\text{ccc} = \frac{2\rho \sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2 + (\mu_x - \mu_y)^2}$$  \hspace{1cm} (12)

Where $\rho$ is the correlation coefficient, $\sigma_x, \sigma_y$ are the variances of observed (x) and predicted (y) values and $\mu_x, \mu_y$ are the respective means. CCC is scaled between -1 and 1, the latter implying perfect agreement and the former implying perfect reverse agreement.

MSE is given by

$$\text{MSE} = \frac{1}{n'} \sum_{i=1}^{n'} (S_i - \hat{S}_i)^2$$  \hspace{1cm} (13)

where $S_i$ is the observed values of SOC at independent sampling points, $\hat{S}_i$ predicted SOC values at the fine scale and $n'$ number of independent SOC samples at a fine scale. MSE is an indication of the accuracy of predictions, while CCC indicates both accuracy and precision of the disaggregation exercise.

4 Results and Discussion

The use of LMM (linear mixed model) as the spatial prediction model allows integration of fine-scale covariate information and spatial correlation in a single model. Unlike regression kriging in which regression and residual kriging are used in two phases of modelling, LMM is a one-step process. We used Bayesian inference techniques to estimate parameters of LMM to deliver uncertainty estimates of model parameters. This section...
provides a description of posterior parameters of the spatial model followed by a comparison of spatial disaggregated maps using Bayesian ATPRK and Dissever.

### 4.1 Bayesian parameter estimation of the spatial model

Fig. 4 illustrates the posterior statistics of the linear trend parameters. The upper diagonal panel displays correlation coefficients among the covariates, the lower diagonal depicts the scatter plots, while the diagonal shows the posterior distributions. The diagonal histograms are well defined which indicates that all parameters of the linear trend are relatively well-defined. The correlations among covariates are minimal, which indicates that there is no multicollinearity between the covariates.

Fig. 5 displays the posterior distribution of Matérn variogram parameters and their correlations. Posterior values of the range parameter are more or less similar to the prior values. However, the partial sill and nugget parameters are different from their priors. The posterior partial sill is higher than the prior sill, which agrees with the fact that the coarse scale variability of soil carbon is lower compared to the finer scale (Fig. 5b).

Table 1, 2 provides the estimates of the parameters of the spatial model along with the 95% confidence limits which is an indication of the uncertainty of the parameters. The estimated posterior parameters have narrow CI limits indicating the estimated mean provides a good approximation of the soil carbon content of the study area.

### 4.2 Spatial Disaggregation using Bayesian ATPRK

Fig. 6 displays the Bayesian ATPRK disaggregated soil carbon content of Nowley at 10m along with the 95% confidence interval limits. Unlike other methods which only give a single estimate of mean soil carbon content, the Bayesian technique provides a range/interval of the most probable value with a 95% confidence. These estimates are more reliable as they are generated using numerous repeated samplings. Accuracy and confidence in estimates are important parameters for planning, designing, and implementation of soil management. Therefore, Bayesian ATPRK provides a comprehensive output of soil carbon estimates at the farm scale.
4.3 Comparison of Bayesian ATPRK and Dissever disaggregation methods

The disaggregated maps from both techniques were upscaled using a spatial overlay to assess the accuracy of disaggregation. CCC values were calculated for both techniques. Fig. 7 shows the agreement between the cell values of coarse map and the upscaled disaggregated maps.

Concordance correlation between original coarse values and upscaled product were 87% and 96% for Bayesian ATPRK and dissever, respectively. Dissever iterates until the agreement between upscaled and original coarse cell reaches a set minimum, while Bayesian ATPRK output is a result of one simulation considering both the correlation with the covariates and covariance structure of the residuals. This explains the stronger concordance associated with dissever.

Fig. 8 illustrates the comparison between dissever and Bayesian ATPRK disaggregated product with the coarse scale source map. A closer examination reveals that Bayesian ATPRK product resembles the spatial pattern of SOC more closely than the dissever product while dissever seemingly over predicts the SOC content. Conversely, dissever is computationally more efficient compared to Bayesian ATPRK. Calculating the block to point and block to block distance matrices is a computational expensive procedure. The whole study area cannot be computed in a single run using a desktop computer. Therefore, the study area has to be split into tiles, and predictions were carried out for each of the tiles. This resulted in abrupt changes between two tiles in the Bayesian ATPRK disaggregated map.

The use of parallel computing, high performing computing can improve the processing time.

4.4 Validation of Disaggregated maps

Independent validation results between disaggregated maps using dissever and Bayesian ATPRK indicate that Bayesian ATPRK technique is slightly more accurate compared to dissever produced output in general. CCC value of Bayesian ATPRK was 23% while that of dissever was 18%. Greater CCC indicates more accurate and precise predictions. Mean squared error (MSE) of Bayesian ATPRK product was 0.45 while MSE for dissever was 0.49, which indicates that Bayesian ATPRK disaggregation is slightly more accurate than dissever disaggregation.

Although the validation CCC is low, it still is a sufficient approximation in terms of SOC as the SOC prediction accuracy generally remains low. The coarse map is a product of regression for the whole region, which also has a lower concordance correlation with the sampled data. Through the disaggregation process, the errors tend to propagate widening the gap between the actual and predicted SOC content.

Overall, we have demonstrated the use Bayesian ATPRK for downscaling coarse scale rasters to more informative fine scale rasters using available fine scale covariates. The study proves that Bayesian ATPRK provides more accurate results with uncertainty estimates of the disaggregation process. There are few limitations...
of this study. Firstly, we did not deal with the uncertainty from all source of measurement errors. The coarse scale map always carries a certain degree of error/uncertainty. Also, there is an uncertainty associated with the splining of the maps. Both of these uncertainties can be treated as measurement errors and filtered by adding a noise variance along the diagonal of the covariance matrix $C_{bb}$. Another way of correcting the bias is to use the measured data in the spatial model to reflect the actual ground conditions. Malone et al. 2017 demonstrated that the inclusion of the ground observations improves the accuracy of the disaggregated maps. A further limitation of our method is that the disaggregated map produced from Bayesian ATPRK has a block effect due to the final map is a collection of tiles produced separately due to high computing demands of the model. This tilting effect can be eliminated by using high performing and parallel computing solutions to generate the fine scale image at one go. Also, low rank- representations (Wikle, 2010) of the covariance can also be used as possible solutions to computational burden of the process.

Furthermore, although we have used the proposed Bayesian ATPRK in DSM domain, this method can also be used to downscale coarse scale satellite data. Even though there is a wide range of fine resolution satellite images available in present context, there is still sparsity of fine resolution data for certain parts of the world. Spatial downscaling is a solution to this data sparsity. There is a range of disaggregation/pan sharpening techniques available in the literature. However, very few techniques can tackle the uncertainty of source images and the uncertainty of the disaggregation process. Unlike other methods, Bayesian ATPRK can explicitly help to tackle observation errors of the satellite images.

5 Conclusions and Recommendations

The study demonstrates that Bayesian techniques can be effectively used to address the change of support problem. Monte Carlo integration provides the estimates of point supported covariance structure using block supported data along with uncertainty estimates of parameter inference. This study used informative priors of covariance parameters as the soil carbon data were available for the study site. In the absence of prior information, priors can be given as non-informative priors.

Bayesian ATPRK provides an effective way of disaggregating coarse scale SOC maps. The output of this study facilitates policy making and soil management activities by providing the maps with confidence limits which lead to wider understanding of the SOC content of the area. However, the method is computationally expensive and the use of high performing computing techniques is recommended for faster and smoother predictions.

Further, the noise of coarse scale observation can be filtered through incorporating the noise variance in the diagonal of $C_{bb}$. The use of point data (if available) in the spatial model can improve the accuracy of the disaggregation. Finally, the use of both of these techniques is recommended for lower uncertainty and improved accuracy of the disaggregation process.

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Figure 1: The study area (left) (Sources: Esri, DigitalGlobe, GeoEye, i-cubed, USDA FSA, USGS, AEX, Geotab, Aerogrid, IGN, IGP, swisstopo, and the GIS User Community) and the coarse scale topsoil soil organic carbon map (right).
Figure 2: Block support experimental variogram of the coarse scale (100 m resolution) soil carbon map.

Figure 3: Schematic representation of dissever disaggregation technique.
Figure 4: Correlations and posterior distribution of linear trend parameters of the spatial model.
Figure 5: (a) Correlations and posterior distribution of variogram parameters (b) and the point and block support variograms.
Figure 6: Bayesian ATPRK disaggregated map with the 95% confidence limits.
Figure 7: Correlation plots between upscaled disaggregated values from Bayesian ATPRK, dissever and original coarse scale block values.
Figure 8: Comparing the disaggregated map with original coarse scale SOC map.
Table 1. Posterior parameters of the linear trend model.

|                | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ |
|----------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Lower CI limit | 1.360     | -0.003    | -0.291    | -0.045    | 0.002     | -0.019    | -0.030    |
| Mean           | 1.416     | 0.002     | -0.281    | -0.040    | 0.007     | -0.014    | -0.024    |
| Upper CI limit | 1.471     | 0.007     | -0.270    | -0.034    | 0.012     | -0.009    | -0.018    |

$\beta_0$, $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, and $\beta_5$ are the coefficients of the linear trend for intercept, elevation, TWL gamma radiometric thorium, gamma radiometric potassium, Landsat band 4 and Landsat band 7 parameters respectively.

Table 2. Posterior parameters of the point support Matern variogram model.

| Method         | Nugget | sill     | range   | smoothness |
|----------------|--------|----------|---------|------------|
| Lower CI limit | -0.05  | -0.02    | 759.10  | 0.45       |
| Mean           | 0.005  | 0.034    | 788.24  | 0.50       |
| Upper CI limit | 0.06   | 0.09     | 817.37  | 0.57       |
Response to Reviewer’s comments

We would like to thank the reviewers for providing constructive comments and efforts towards improving this manuscript. We highlight the reviewers’ comments in red and our responses are given in black.

Reviewer 1 comments and Responses

General comments: This manuscript proposed the use of Bayesian Area-to-Point Regression Kriging (ATPRK) for spatial disaggregation of regional scale digital soil map to farm scale. This method is able to provide the uncertainty estimate associated with the disaggregation process. Throughout it had lower concordance correlation with the coarse map than dissever algorithm, the independent data showed that Bayesian ATPRK had a higher concordance correlation than dissever. Generally, the proposed method is interesting as it can provide the uncertainty related to the disaggregation process. However, from my point of view, the major uncertainty of disaggregated map comes from the original map, thus it is more important to incorporate the uncertainty of the original map into disaggregation. Is the proposed method able to integrate this? Another limitation is the Results and Discussion section, where authors focused mostly on the results and rarely provided a more general discussion. I look forward to seeing the feedback from the authors on these two issues.

We have identified the uncertainty of the original should also be considered in the disaggregation process. We have mentioned this in our conclusions section as a recommendation. Our main focus of this paper was to estimate the uncertainty of the disaggregation process which is overly neglected in the current literature. However, the proposed Bayesian method is capable of handling the uncertainty of the original map by incorporating the uncertainty as a noise variance in the covariance matrix, $C_{bb}$. This enables filtering the noise. Also, we have added more detailed discussion about filtering measurement errors. (lines 341 -348)

Another limitation is the Results and Discussion section, where authors focused mostly on the results and rarely provided a more general discussion.

We agree with the reviewer that the discussion section should be little more generalised. Therefore, a more generalised discussion was added at the of the results and discussion section (lines 341 -361).

Specific comments: Line 21: Why only mentioned underestimation? How about overestimation?

Here we are referring to the fact that in DSM literature the uncertainty estimates of the disaggregation process are often neglected.

Line 35: What is the difference in computing time? I know Bayesian ATPRK is a one-step process, and it saves computing time? Because sometimes an iteration process can also be efficient.

The difference in computing time is approximately 3 hrs

Line 45: It is not “five standard soil depths” but “six standard depth intervals”.

The typo was corrected

Lines 71-72: I do not agree with the statement here, DSMART can also produce maps in a fine grid.

Here, we are referring to the fine riding process where we disaggregated a coarse grid cell (100m) into many (10 x 10) fine grid cells. While the rasters created by DSMART based on the most probable class assigned after generating realisations from classification trees. The classification trees are built using the values taken from random samples from each soil class polygon from the legacy maps.
What do you mean “at N point”? Since you listed many methods for deconvolution of the empirical variogram, it is better to show here why you chose Bayesian estimation.

We have included the objective of choosing Bayesian techniques to infer the point support variogram. (line 137-138)

Lines 187-189: Do you think the proposed framework can integrate the uncertainty from the fitted mass-preserving splines as well as from the uncertainty estimates associated to these two SOC maps (0-5, 5-15 cm)?

Yes, the uncertainty from all sources can be accounted using the proposed Bayesian techniques. This is also included in the added new section to the results and discussion section.

I understand that the Bayesian ATPRK can provide the uncertainty in the step of disaggregation, however, I think the major uncertainty comes from the original map, which should be not ignored in disaggregation.

We have identified this as a limitation of the study, and it was further elaborated in this revised version (line 341-361.)

Lines 196-197: Why you choose 10 m as the final resolution? The finest covariate used here is in 10 m (Elevation data), I think you should mention it in the very beginning.

There were two reasons behind choosing 10m as the fine resolution. One is the resolution of available data at fine scale and secondly, we believe that 10m maps are informative enough for supporting decision making process at fine scale. In another word, 10m maps would capture most of the spatial variability of SOC and therefore we selected 10m as our fine scale’s resolution.

Line 268: I suggest adding PICP to assess the uncertainty.

We present CI estimates of model parameters and maps. CI values provide an insight to the uncertainty estimates. Therefore, we decided not to include PICPs.

Lines 341-342: reduce the processing time? However, parallel computing can still not solve abrupt changes between two tiles. Maybe you need a laptop with larger RAM?

Yes, it needs a bigger RAM to disaggregate the whole study area at once.

Line 579: Figure 6 still showed an obvious block effect. In order to compare the original SOC map (Figure 1), please use the same legend and then we can tell the difference between them.

Figure 1 was changed to the same legend.

Line 595: The map from Disserver looks smoother.

Disserver disaggregate the whole study area at once resulting a smoother map. Therefore, the maps are smoother.

Technical corrections Line 21: DSM has not been defined in the previous texts. Line 48: DSM has been defined before.

Respective corrections were made.

Line 91: SOC has not been defined yet.

SOC defined

Line 200: C2 SOILD Interactive comment Printer-friendly version Discussion paper DEM has not been defined yet and Topographic Wetness Index can be replaced by TWI here.

DEM was defined and the Topographic Wetness Index was replaced by TWI.

Line 209: and should not be in italic. Please have a careful check of all the parameters which should be in italic.

Line 260: In “cm2”, 2 should be in superscript.
Respective corrections were made

Line 280: predicted SOC values.

Typo was corrected

Line 285: LMM has not been defined.

LMM was defined

Line 290: In other texts, dissever is used. Please make it consistent.

Line 539: Please use either Figure or Fig. in the text. Currently, it is a mix.

The term “Figure” has been used as the title of figures while “Fig.” was used in the text when the figures are described

Line 565: Better to put (a) and (b) in the upper right position.

Changed the positions of a and b

Reviewer 2 comments and responses

The manuscript is well-written and follows up from earlier work of Malone et al. 2017 on disaggregation of coarse scale SOC maps. This is particular important for facilitating management at the field/farm scale. Although I am not a geostatistical expert, the data analysis is sound and the results are convincing, although not much different from the ones obtained by the ‘dissever’ approach.

Lines 19-20 Is not there a verb missing in the sentence starting with ‘Disaggregating’?

The typo was corrected

Line 21 Define ‘DSM’ at first time use.

DSM was defined

Line 52 I appreciate that you try to use the original meaning of scale ‘large’ and ‘small’, but the word ‘or’ might create confusion. How about replacing ‘or’ by ‘i.e.’?

Line 53 …into two categories. . .

Typo was corrected

Line 62 Please rephrase this sentence and break it up. e.g. They demonstrated that a Gaussian process model can be used as a novel image fusion technique. They used images from unmanned aerial vehicle flown over agricultural fields for panchromatic sharpening of low-resolution satellite images.

The sentence was rephrased.

All the following typos were corrected.

Line 65 Avoid using ‘approach’ twice in one sentence.

Please rephrase line 74 are not there some commas missing?

Line 79 …and also ancillary covariates such as the ‘scorpan factors’ (McBratney et al., 2003; Kerry et al., 2012).

Line 83 …underlying…

Line 89 …soil organic carbon (SOC) maps… SOC is arguably…

Line 236 …approach…

Line 264 Specify the manufacturer and the country for the Variomax. Did you check the samples for inorganic carbon (carbonates)?

Line 285 Has ‘LMM’ been defined before? Avoid using ‘model’ twice in one sentence.

Line 351 The MSE has units.
