Coherence of Currents in Mesoscopic Cylinders

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Abstract

The persistent currents driven by the pure Aharonov-Bohm type magnetic field in mesoscopic normal metal or semiconducting cylinders are studied. A two-dimensional (2D) Fermi surfaces are characterized by four parameters. Several conditions for the coherence and enhancement of currents are discussed. These results are then generalized to a three-dimensional (3D) thin-walled cylinder to show that under certain geometric conditions on the Fermi surface, a novel effect - the appearance of spontaneous currents is predicted.

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1 Introduction

Quantum coherence in mesoscopic systems arose a great interest in the last years because it leads to many interesting phenomena such as persistent currents, conductance oscillations etc. In a series of papers [2, 3] we investigated persistent currents in a collection of isolated mesoscopic rings stacked along one axis. The current $I(\phi)$ induced by the magnetic flux $\phi$, due to the Aharonov-Bohm effect, was therein calculated under the assumption, that the electrons from different rings interact via the magnetostatic (current-current) interaction and the interaction was considered in the mean field approximation. It was shown that due to the selfconsistent equation for
the current, in some cases, a novel effect - the spontaneous self-sustaining currents at the zero external flux - can be predicted. A similar effect, due to the Aharonov-Casher effect was also recently proposed [4].

In the present paper we investigate persistent currents in 2D and 3D cylinders made of a clean normal metal or a semiconductor (ballistic regime). It is known that the magnitude of the currents in 2D systems depends strongly on the correlations of currents from different channels [5]. To study this correlation we calculate the currents in systems with different shapes of the Fermi surface (FS). We also search for the possibility of obtaining spontaneous currents for the most favorable FS shapes.

2 Persistent Currents in a Single Channel

Let us consider a 2D normal metal or semiconductor system of a cylindrical geometry. The circumference and height of the cylinder are denoted by $L_x$ and $L_y$ respectively. We neglect the width of its wall, assuming it is of the order of $\lambda_F$ - the Fermi wavelength. We also assume that the magnetic flux $\phi$ threads the cylinder axially and is confined to its center, so that the electrons move in a field-free space. Here we use the cylindrical coordinate system, in which the height coordinate is $y$ and the azimuthal angle is $\Theta \in [0; 2\pi)$. It is convenient to replace $\Theta$ by $x = \frac{L_x}{2\pi} \Theta$ so that $x$ varies between 0 and $L_x$.

For the wave function of an electron in the cylinder we apply periodic boundary conditions in the azimuthal direction and infinite potential barrier along the $y$ axis. In presence of the magnetic field $\phi$ the wave function obeys the following boundary conditions [6]

$$\psi(L_x) = \exp\left(\frac{2\pi i \phi}{\phi_0}\right) \psi(0),$$

$$\left.\frac{d\psi}{dx}\right|_{x=L_x} = \exp\left(\frac{2\pi i \phi}{\phi_0}\right) \left.\frac{d\psi}{dx}\right|_{x=0},$$

where $\phi_0 = \frac{\hbar}{e}$ denotes the flux quantum.

Solving the Schrödinger equation in the free electron approximation we
get quantized energy levels

\[ E_{qr}(\phi) = \frac{\hbar^2}{2m} [k_x^2(q, \phi) + k_y^2(r)], \]  

(2)

where \( k_x(q, \phi) \) and \( k_y(r) \) are components of the wave vector \( k \) in the azimuthal and \( y \) direction, respectively,

\[ k_x(q, \phi) = \frac{2\pi}{L_x} \left( q + \frac{\phi}{\phi_0} \right), \quad q = 0, \pm 1, \pm 2, \ldots, \]  

(3)

\[ k_y(r) = \frac{r\pi}{L_y}, \quad r = 0, 1, 2, \ldots. \]  

(4)

In our 2D system we are interested in the net current along the azimuthal direction. The Fermi surface (FS) is in this case 2D. Apart from momentum in the \( k_x \) direction the electrons can move in the \( y \) direction. The collection of them, having the same \( k_y \) or \( r \) quantum number will be called a channel. Each channel is a semi one-dimensional (1D) system with a characteristic radius \( K_x \), which is the maximal possible value of the \( x \) component of the wave vector \( |k_x| \leq K_x \). This radius as well as the number of channels \( M \) depend on the shape of the 2D FS.

The current carried by a single electron of the energy \( E_\alpha \) is given by

\[ I_\alpha(\phi) = -\frac{\partial E_\alpha(\phi)}{\partial \phi}, \]  

(5)

where \( \alpha \) is an arbitrary set of indices. In case of the energy (2) it is a linear function of \( \phi \). The total current in the 2D system is the sum over the contributions \( I_\alpha \) of all states, where the occupation probability is the Fermi-Dirac (FD) function.

In the present and in the the next chapter we assume that the temperature is zero. We first recall the current \( I_r(\phi) \) corresponding to a single channel \( r \) with \( N \) electrons. It is a sum over \( q \) of currents \( I_{qr}(\phi) \) corresponding to the first \( N \) lowest lying states \( E_{qr}(\phi) \). In the following we use the Fourier series expansion formula [5, 7] for this current

\[ I_r(\phi) = \sum_{l=1}^{\infty} \frac{2l_1}{\pi l} \cos(lK_xL_x) \sin \left( \frac{2\pi l\phi}{\phi_0} \right), \]  

(6)
where

\[ I_1 = \frac{e v_F}{L_x} = \frac{e h K_x}{m L_x} \]  

(7)

is the amplitude of the current in the 1D system and \( K_x = \frac{\pi N}{L_x} \) is the 1D radius of the FS corresponding to the channel \( r \) (\( v_F \) is the velocity of electrons at the FS). The total current for the single channel \( r \) has a sawtooth shape with the period 1 in \( \phi \) and the amplitude inversely proportional to the circumference of the ring. The current for a system with an even number of electrons has a discontinuous jump up at \( \phi = 0 \) whereas in the case of an odd number of electrons at \( \frac{\phi}{\phi_0} = 0.5 \). More generally if the number of electrons in the system varies between even and odd as a function of \( \phi \), the current assumes one of the above values corresponding to the number of electrons at the particular value of the flux \([5, 8]\). In the next chapter we show that this property to some extent "survives" the transition to a 2D system.

3 The Currents Depending on the Shape of the Fermi Surface

The total current in the 2D case depends on the strength of the interchannel correlations. It has been already shown \([3]\) that if the FS is spherical then, for short cylinders \( L_x \geq L_y \) the channel currents add without phase correlation, whereas for long cylinders there exist some phase correlation and the total current is bigger. A large phase correlation among channel currents means that the increase of the flux \( \phi \) results in an almost simultaneous cross of the FS by the large number of channels. The most favorable situation takes place if the separation between the last occupied level and the FS from channel to channel is nearly the same. There exists then a perfect correlation among the channel currents because the \( M \) levels cross the FS simultaneously while the flux is changed by one fluxoid - we get then the largest amplitude of the total current.

The phase correlation and the value of the total current depend on the shape of the FS. In case of the free electron approximation considered in chapter 2 the FS is circular.
\[ K_F^2 = K_x^2 + K_y^2. \]  

(8)

It is characterized by a single parameter \( K_F \), specifying its volume. The radius \( K_x \) of the FS in the given channel \( r \) can be easily calculated from (8).

However for the interacting electrons moving in the crystal lattice the dispersion relation \( E(k) \) is in general a complicated function of the wave vector \( k \), leading to different shapes of the FS. For example, in the tight binding approximation, for 3D systems we can get, depending on the crystal symmetry and on the filling factor, the FS being the sphere, the cube, the cube with rounded corners, octahedron etc. [9]. The FS can be in addition differently oriented with respect to \( k_\alpha (\alpha = x, y, z) \) axes in the reciprocal lattice. For 2D systems or for 3D systems with 2D conduction (an example of such systems are high \( T_c \) superconductors in a normal state) the FS can change from a circle to a square [10, 11] via different other shapes.

In the following we study the influence of the shape of the FS on persistent currents in multichannel systems changing its shape in the following way

\[ K_F^n = \left( \frac{K_x}{\alpha} \right)^n + \left( \frac{K_y}{\beta} \right)^n. \]  

(9)

The shape of this Fermi surface depends on four positive parameters: \( K_F \), \( n \), \( \alpha \) and \( \beta \). Generally speaking the parameter \( n \) controls the convexity of the FS, whereas \( \alpha \) and \( \beta \) measure its curvature at the places where it crosses \( k_x \) or \( k_y \) axes. To understand how the above parameters influence FS and the corresponding current in this chapter we consider three cases in which some of the parameters are kept constant while the other are varied.

(i) \( K_F = \text{const.}, n = 2 \) and \( \alpha, \beta \) are varied:

\[ K_F^2 = \frac{K_x^2}{\alpha^2} + \frac{K_y^2}{\beta^2}. \]  

(10)

In this case the FS is elliptical with its major axis along \( k_x \) or \( k_y \) depending on whether \( \alpha > \beta \) or \( \beta > \alpha \), respectively. The curvature of the FS at the point, where it crosses the \( k_x \) axis is equal to \( \frac{\beta}{\alpha} K_F \). We assume in addition that \( \alpha \beta = 1 \) because it ensures the independence of the volume under the FS on \( \alpha \) and \( \beta \). Under this assumption the
number of states under the FS maintains independent on $\alpha$ (and $\beta$) in the limit of $K_F \to \infty$ whereas for finite $K_F$ it depends only slightly due to the effects on the edge of the FS.

Replacing $K_y$ by its actual value at the $r$-th channel (4) the Eq. (10) yields for the 1D radius $K_x$ of the FS in this channel

$$K_x = \alpha K_F \sqrt{1 - \left(\frac{k_y(r)}{\beta K_F}\right)^2}.$$  \hspace{1cm} (11)

The current in the whole 2D system is the sum of $M + 1$ single channel currents (3) taken with the appropriate $K_x$

$$I_M(\phi) = \sum_{r=0}^{M} \sum_{l=0}^{\infty} \frac{2I_0}{\pi l} \alpha \sqrt{1 - \left(\frac{k_y(r)}{\beta K_F}\right)^2} \cos \left[ lK_F L_x \alpha \sqrt{1 - \left(\frac{k_y(r)}{\beta K_F}\right)^2} \sin \left(\frac{2\pi l \phi}{\phi_0}\right)\right],$$

$$\text{where } I_0 = \frac{e h K_F}{m L_x}.$$  \hspace{1cm} (12)

For the circular Fermi surface ($\alpha = \beta = 1$) in a 44-channel system for $L_x = 1000\text{Å}$, $L_y = 100\text{Å}$ and assuming $E_F = \frac{\hbar^2 K_F^2}{2m} = 7eV$ (typical for Cu) the $I-\phi$ characteristic is shown in Fig.1 (solid line). It is a integer Fig.1 ragged function, built of intervals of the linear decrease and discontinuous jumps up of the current. Each discontinuous increase of the current value in Fig.1 corresponds to a single electron leaving or coming under the Fermi surface. This structure of the current function is characteristic for $T = 0$ and remains the same in other cases in this chapter.

If the parameter $\alpha$ increases then the circular FS turns into an ellipse with the major axis along the $k_x$-axis. An increase of $\alpha$ up to 10 (Fig.1 - dashed line) results in the decrease of the number of channels ($M \sim \beta$) and the current value. In this case we don’t observe any correlation between currents from different channels. An increase of the parameter $\beta$ is equivalent to turning the major axis of the ellipse along $k_y$-axis. If $\beta$ increases up to 10, then a substantial enhancement of the current is observed (Fig.1 - dotted line). This is because the perpendicular to the
The $k_x$ axis part of the FS is very flat (its radius of curvature is proportional to $\frac{\beta}{\alpha}$) causing an enhancement of the current correlation.

It is worth noticing that the current correlation change similarly for the circular FS ($\alpha = \beta = 1$) when the circumference and the height of the cylinder are changed in such a way that $L_x L_y = \text{const.}$. In this case an increase of the cylinder height produces the same enhancement of current correlation as the increase of $\beta$. The total current however is even more enhanced because, at the same time, the circumference of the cylinder decreases causing the increase of $I_0$ (cf (12)).

(ii) $K_F = \text{const.}, \alpha = \beta = 1$, $n$ is varied:

$$K_F^n = |K_x|^n + |K_y|^n.$$  

(13)

Here we consider $n$ to be an arbitrary positive real number. For $n = 1$ the Fermi surface is of a triangular shape (Fig.2). With increasing $n$ it becomes convex and changes from triangular through circular ($n = 2$) to rectangular for $n \rightarrow \infty$. On the other hand, decreasing $n$ below $n = 1$ yields concave Fermi surfaces shown in Fig.3. Such FS are frequently observed in HTSC materials. In our numerical calculations we adjust $K_F$ to $n$ in such a way that the volume under the FS remains constant.

In the present case $K_x$ is

$$K_x = K_F \sqrt{1 - \left(\frac{k_y(r)}{K_F}\right)^n}. $$  

(14)

Inserting this into (3) and (7) yields

$$I_M(\phi) = \sum_{r=0}^{M} \sum_{l=0}^{\infty} \frac{2I_0}{\pi l} \sqrt{1 - \left(\frac{k_y(r)}{K_F}\right)^n} \cos \left[lK_FL_x \sqrt{1 - \left(\frac{k_y(r)}{K_F}\right)^n} \sin \left(\frac{2\pi l\phi}{\phi_0}\right)\right]. $$  

(15)

The case with $n = 0.4$ corresponds to the FS from Fig.3. It is the case where the number of channels is greater compared to the circular
FS \( (n = 2) \). Taking again \( L_x = 1000\,\text{Å}, L_y = 100\,\text{Å} \) and \( E_F = 7\,\text{eV} \) the \( I-\phi \) characteristic for \( n = 0.4 \) is shown in Fig.4 (dotted line). We observe here a moderate increase of the current value compared to the circular FS (solid line). For \( n = 1 \) the FS is triangular and in the \( I-\phi \) characteristic (Fig.4 - dashed line) there is a substantial increase of correlation of phases from different channels and the greater current value. It is because all the states at the FS leave (or get in) the FS simultaneously. More generally, in case of \( n = 1 \), there is a full correlation and enhancement of the current provided the following geometrical condition

\[
\frac{1}{2} \alpha L_x = \mu \beta L_y, \tag{16}
\]

where \( \mu \) is an arbitrary positive integer, is obeyed. In case of the dashed line of Fig.4, \( \mu = 5 \).

However, if the relation (16) do not hold the I(\( \phi \)) current can be much smaller. Therefore the geometrical amplification of the current in case of \( n = 1 \) is a matter of a careful adjustment of the cylinder dimensions \( L_x \) and \( L_y \) in such a way that, for a given \( \alpha \) and \( \beta \), the condition (16) is fulfilled. Such a dependence on a cylinder dimensions does not take place if we consider persistent currents for the systems with \( n > 2 \). For \( n = 5 \) we have a case in between circular and rectangular FS. In the \( I-\phi \) characteristic (Fig.5 dotted line) some correlation among the channel currents exists what results in an increase of the current amplitude compared to the circular FS (solid line). The case with \( n \to \infty \), i.e. rectangular FS, is the most suitable to obtain a big current amplitude. The \( I-\phi \) characteristic is shown in Fig.5 (dashed line). We observe here a perfect coherence (all the terminal states simultaneously leave or get into the Fermi surface) and great increase of the current value. The amplitude of the current is equal to the number of channels \( M \) in \( I_0 \)-units. Thus in general, with the departure of \( n \) from 2, we observe the increasing amplitude of the M-channel current.

(iii) \( \alpha \ll \beta, n = 2, K_F \) is varied:

In the previous two cases we have changed the shape of the FS leav-
ing the area under it invariant. As a result of that some currents were enhanced and some were suppressed compared to the spherical FS. Changing parameters $\alpha$, $\beta$ and $n$ we could not, however, predict whether the current is positive or negative in a given interval. This important property of the current is governed by the area under the FS. In Fig. 6 the currents are shown for an elliptical FS with $\beta \gg \alpha$. Different current characteristic in this figure correspond to $K_F$ increased up to 2% from its initial value. The plot shows that in this way the paramagnetic currents can be converted into diamagnetic and vice versa. The $I-\phi$ characteristic is therefore an almost periodic function of the volume under the FS. This important property is reminiscent of the current behavior in a single channel case, when the number of electrons in the system changes by two [7, 8]. Here we show that this property of a single channel survives the transition to the 2D case.

So far we considered a perfect cylinder. However it has been shown [1, 3, 12] that the phenomenon will survive modest scattering, both elastic and inelastic. Let us assume that our mesoscopic system contains a small number of impurities (ballistic regime). The average current, where the average is taken over impurity configurations, has been calculated in [12]. The formula for the total average current for 2D cylinder is

$$\bar{I}(\phi) = I(\phi) \exp \left( -\frac{L_x}{2\lambda} \right),$$

(17)

where $\lambda$ is the mean free path. For a sufficiently clean material $\lambda$ is of the order of a few microns and in the mesoscopic regime the impurities do not decreases the current significantly.

## 4 Spontaneous Currents in the Mesoscopic Cylinder

In this chapter we generalize our considerations to 3D systems and to nonzero temperatures. The formula (3) for a current induced by the magnetic
field in the 1D system of the mesoscopic size in $T > 0$ is replaced by

$$I(\phi) = \sum_{l=1}^{\infty} \frac{4I_0 T}{\pi T^*} \exp \left( -\frac{lt}{T^*} \right) \frac{\exp \left( -\frac{2lt}{T^*} \right)}{1 - \exp \left( -\frac{2lt}{T^*} \right)} \cos(lK_x L_x) \sin \left( \frac{2\pi l \phi}{\phi_0} \right),$$

where $T^*$ is the characteristic temperature, defined by the energy gap between energy levels at the Fermi surface

$$T^* = \frac{\hbar^2 N}{m L_x^2}.$$  

(19)

The formula (18) is valid assuming the grand canonical ensemble [7]. It is applicable in our case because single channels can exchange electrons according to $K_x$, which determines their chemical potential. Temperature influences the current in such a way, that all discontinuities in the $I-\phi$ characteristic (all transitions in the current value) are smoothed, and the maximum of the amplitude decreases.

Consider now a 3D system of the cylinder geometry made of a clean non-superconducting material. The circumference, height and the width of the wall are denoted by $L_x, L_y$ and $L_z$ respectively. We assume that the width of the cylinder is small compared to the other dimensions $L_z \ll L_x, L_z \ll L_y$ and therefore we can assume with a good approximation that the vector potential $A$ does not depend on $z$.

Generalizing equation (9) to a 3D system the current can be obtained by replacing $K_x$ in (18) by

$$K_x = \alpha K_F \sqrt{1 - \left( \frac{k_y(r)}{\beta K_F} \right)^n - \left( \frac{k_z(s)}{\gamma K_F} \right)^n},$$

(20)

where $\alpha \beta \gamma = 1$ and

$$k_z(s) = \frac{s\pi}{L_z}, \quad s = 0, 1, 2, \ldots$$

(21)

Therefore we obtain

$$I(\phi) = \sum_{r=0}^{M} \sum_{s=0}^{P} \sum_{l=0}^{\infty} \frac{4I_0 T}{\pi T^*} \left[ \frac{\exp \left( -\frac{lt}{T^*} \right)}{1 - \exp \left( -\frac{2lt}{T^*} \right)} \right] \cos(lK_x L_x) \sin \left( \frac{2\pi l \phi}{\phi_0} \right).$$

(18)
\[ \times \cos \left[ l K_F L_z \alpha \sqrt{1 - \left( \frac{k_y(r)}{\beta K_F} \right)^n - \left( \frac{k_z(s)}{\gamma K_F} \right)^n} \right] \sin \left( \frac{2 \pi l \phi}{\phi_0} \right). \] (22)

Changing the Fermi surface in the same way as in 2D case yields a very similar effect. In cases where the current coherence was high the third dimension substantially increases the number of channels \((P \sim L_z)\) and enhances the current.

It has been recently shown in a series of papers [2, 3] that a metallic or a semiconducting system made of a set of mesoscopic quasi 1D rings stacked along certain axis can exhibit a transition to a low temperature state with a spontaneous orbital current.

In this chapter we concentrate on searching for a possibility of obtaining such spontaneous currents in a clean metallic or semiconducting 3D cylinder. We consider here the case when the magnetic field \(\phi\) enclosed by the cylinder, which drives the persistent current \(I(\phi)\) around it, is the sum of the externally applied flux \(\phi_e\) and of the flux \(\phi_I\) from the persistent current itself [2]

\[ \phi = \phi_e + \mathcal{L} I, \] (23)

where \(\mathcal{L}\) is the self-inductance of the cylinder (for cylinder with height \(L_y\), width \(L_z\) and a radius \(R = \frac{L_x}{2\pi}\), \(\mathcal{L} = \frac{\mu_0 \pi R^2}{L_y} (\sqrt{\frac{L_x^2}{L_y^2} - R^2} - R)\)). Most of theoretical discussions [3, 4] neglect the second term in (23). Relations (23) and (22) represent the system of two selfconsistent equations to calculate the current \(I(\phi)\) and the magnetic flux \(\phi\). The question of existence of the spontaneous persistent currents reduces into a problem whether these equations have stable, non-vanishing solutions at \(\phi_e = 0\)

\[ \frac{\phi}{\mathcal{L}} = I(\phi). \] (24)

If there is any nonzero solution of (24) then we have the spontaneous current, corresponding to the point of intersection the straight line (LHS of (24)) and the nonlinear curve (RHS of (24)) which is given by (22). The nonzero solution is stable provided the intersection is at the place where the slope of the curve (22) is negative.

We have made model calculations for a cylinder with the following parameters \(L_x = 25000\pi \text{Å}, L_y = 10000\text{Å}, L_z = 100\text{Å}, E_F = 7eV\) with \(\alpha = \beta = \)
\( \gamma = 1 \) and \( n = 4, 5 \) (Figs 7, 8). We see that for \( n \geq 4 \) there is always a nonzero stable solution at sufficiently low temperatures (circles at Figs 7 and 8). The temperature at which the spontaneous current occurs increases with increasing \( n \). One can show also that the condition \( \alpha = \beta = \gamma = 1 \) is not necessary and the spontaneous currents occur even if the above parameters are different. It means that for all the Fermi surfaces similar to rectangular the spontaneous currents can be obtained.

Similar transition to a spontaneous currents occur for a generalized triangular FS i.e. for \( n = 1 \). In this case the FS is of a tetrahedral shape and the geometrical condition necessary to obtain spontaneous currents is

\[
\frac{1}{2} \alpha L_x = \mu \beta L_y = \nu \gamma L_z, \tag{25}
\]

where \( \mu \) and \( \nu \) are positive integers.

For 2D systems we do not observe spontaneous currents neither for high value of \( n \) nor for reasonable values of \( \alpha \) and \( \beta \) coefficients.

5 Conclusions

In this paper we presented same model considerations of persistent currents in clean multichannel systems of a cylindrical geometry (ballistic regime).

We discussed persistent current behavior due to the pure Aharonov-Bohm effect — we neglected attenuation of the current due to the field penetrating through the system. We also ignored the spin-orbit coupling. One can estimate that such assumptions pose no serious problems for the cylinders under study. We also neglected here the Coulomb interaction. Recently it has been shown that the Coulomb interaction does not influence the persistent current behavior in clean systems \cite{13}, whereas it enhances the current in the diffusive regime \cite{14}. There is also an indication that the Coulomb interaction of the Hubbard type enhances the typical value of the persistent current in disordered rings in any dimensions \cite{15}. The decrease of currents with disorder was taken into account (see eq. \cite{17}) following \cite{12}.

The current in multichannel system depends strongly on the phase correlations between currents of different channels. We discussed persistent currents as a function of a shape of the Fermi surface by comparing them to
the standard circular FS. In case of the elliptical FS the current-flux characteristics show substantial increase of the current magnitude with increasing and attenuation with decreasing curvature of the Fermi surface at the point where it crosses the $k_x$ axis. The triangular FS enhances the current provided its parameters obey the geometrical condition (16). The departure into concave FS only slightly increases the current. In the opposite case of the convex FS the current amplitude increases from its value in case of the circular FS to the maximal value in case of the rectangular FS ($n \rightarrow \infty$). We found also that the current-flux characteristic is an almost periodic function of an area under the FS.

We also investigated the possibility of spontaneous currents in 2D and 3D cylinders. We found that for 3D cylinders spontaneous currents can be obtained for $n \geq 4$ and for $n = 1$ provided the geometrical dimensions of the cylinder are properly adjusted. This can be a hint for a possible experimental verification of the currents. The characteristic temperatures $T_c$ increase with increasing $n \geq 4$. The most favorable situation takes place for $n \rightarrow \infty$ what is equivalent to perfect correlation of currents from different channels. Such Fermi surfaces can be obtained e.g. for bcc crystals in tight binding approximation for nearly half filled or half filled band. Preferable materials would be metals or semiconductors with a long phase coherence length and with a small number of impurities.

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7 Figure captions

Figure 1. Persistent current vs flux at T=0, for elliptical Fermi surfaces (n=2), with β = 0.1 (dashed line), β = 1 (solid line) and β = 10 (dotted line). In the inserted circle the magnified curve (α = β = 1) show the details of the current behavior.

Figure 2. Convex Fermi surfaces according to Eq.(9) for α = β = 1 and n = 1, 2, 5 and ∞.

Figure 3. Concave Fermi surfaces according to Eq.(9) for α = β = 1 and n = 0.8, 0.6 and 0.4.

Figure 4. Persistent current vs flux at T=0 and α = β = 1, for spherical, n = 2 (solid line), triangular, n = 1 (dashed line) and concave n = 0.4 (dotted line) Fermi surfaces.

Figure 5. Persistent current vs flux at T=0 and α = β = 1, for convex Fermi surfaces, n = 2 (solid line), n = 5 (dotted line) and rectangular FS, n → ∞ (dashed line).

Figure 6. Persistent current vs flux at T=0 and β ≫ α and n = 2, for the elliptical Fermi surface and different volumes $K_{F_1} < K_{F_2} < \ldots < K_{F_5}$. The characteristic is an almost periodic function of $K_F$ (only half of the period in $K_F$ is shown).

Figure 7. The graphical solution of a set of selfconsistent equations (22) and (24) for different temperatures and the convex Fermi surface n = 4, α = β = 1. The nonzero crossings of the straight line (24) with the current-flux characteristic (22) denoted by circles correspond to spontaneous currents in the 3D cylinder.

Figure 8. The graphical solution of a set of selfconsistent equations (22) and (24) for different temperatures and the convex Fermi surface n = 5, α = β = 1. The nonzero crossings of the straight line (24) with the current-flux characteristic (22) denoted by circles correspond to spontaneous currents.
in the 3D cylinder.
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