Research on Simulation System of Rotor-Dynamics of Ball-Bearing and Turbo-Pump Rotor

Xiong Zhang*, Aimei Tian
School of Astronautics, Beihang University, Beijing, China
*Corresponding author e-mail: axiongzhang1993@163.com

Abstract. In this paper, considering the effect of inertia ball, the static calculation model of angular contact ball bearings is established, the ball and the inner and outer raceway contact series parallel connection stiffness, deduced the formula of stiffness of the bearings, the use of FORTRAN language rotor-dynamics of the bearing calculation program, through the existing literature examples verify the correctness of the program. Combined with the actual turbo-pump rotor system of turbo-pump radial ball bearing radial bearing radial load stiffness is calculated, considering the additional influence of sealing fluid induced stiffness, at variable speed under the condition of calculation of bearing seal rotor system whirling frequency, the results accord with the actual engineering. In this paper, a multi factor simulation system for rotor-dynamics is developed, which provides a reference for design phase analysis of turbo-pump rotor-dynamics.

1. Introduction
Most of the bearings on the liquid rocket engine turbo-pumps are angular contact ball bearings. Since the turbo-pump rotors often work at high speeds, they are mainly subjected to radial loads and also bear certain axial loads, so radial ball bearings and centripetal bearings are often used, contact angle $0^\circ \leq \alpha \leq 45^\circ$. It is very necessary to accurately calculate the dynamic characteristics of high-speed ball bearings. This facilitates the analysis of the dynamic characteristics and the stability analysis of the turbine pump rotor under variable speed conditions.

When the ball bearing is running at a high speed, the centrifugal force and the gyro moment of the ball rolling body will change significantly. With this in mind, based on the Hertz contact theory, Harris [1] proposed a pseudo-static calculation method for a high-speed ball bearing, which transforms the dynamic equation of the bearing into a set of nonlinear algebraic equations. Jiang Xingqi [2] performed detailed calculation and analysis of the dynamic performance of angular contact ball bearings with pure axial loading. Feng Jilu [3] analyzed the influence of structural parameters on the radial stiffness of the bearing. Zhang Jinlong [4] calculated and analyzed the dynamic stiffness of the high-speed ball bearing of the electric spindle, and was applied in the analysis of the dynamic characteristics of the rotor system. In this paper, for the high-speed ball bearings used in turbo-pumps, considering the influence of centrifugal force of the ball and the change of gyro torque with the rotation speed, a pseudo-static calculation model of the ball bearings was established. The numerical simulation program for solving the dynamic performance of the bearings was compiled using the FORTRAN language. The example verifies the correctness of the program. Based on the calculation
of the radial stiffness of the radially loaded turbo pump radial ball bearing, the dynamic characteristics of the turbo pump rotor with the rotation speed are analyzed. The purpose is to design the turbo pump rotor at the beginning, provide some reference when estimating dynamic characteristics.

2. Ball Bearing Pseudo-static Calculation Model

2.1. Bearing Inner Ring Force Analysis
The load acting on the inner ring of the bearing and the force acting on the inner ring of the ball remain balanced. Therefore, the equilibrium equation set of the entire bearing is established according to the combined force of the inner ring of the bearing:

$$F_a = \sum_{j=1}^{z} Q_j \sin \alpha_j \sin \varphi_j$$  \hspace{1cm} (1)

$$F_t = \sum_{j=1}^{z} Q_j \cos \alpha_j \cos \varphi_j$$  \hspace{1cm} (2)

$$M = \sum_{j=1}^{z} Q_j \sin \alpha_j \cos \varphi_j$$  \hspace{1cm} (3)

2.2. Bearing Dynamic Calculation
The radial stiffness, axial stiffness, and angular stiffness of the bearing are obtained using the series-parallel relationship of the contact stiffness:

$$K_p = \sum_{j=1}^{z} \frac{K_{ij} \cos \alpha_j \times K_{ij} \cos \alpha_j \cos \varphi_j}{K_{ij} \cos \alpha_j + K_{ij} \cos \alpha_j} \times \frac{2\pi}{z} (j-1)$$  \hspace{1cm} (4)

$$K_a = \sum_{j=1}^{z} \frac{K_{ij} \sin \alpha_j \sin \alpha_j}{K_{ij} \sin \alpha_j \sin \alpha_j}$$  \hspace{1cm} (5)

$$K_\theta = d_a R \sum_{j=1}^{z} \frac{K_{ij} \sin \alpha_j \sin \alpha_j \times K_{ij} \sin \alpha_j \sin \alpha_j \cos \varphi_j}{K_{ij} \sin \alpha_j \sin \alpha_j + K_{ij} \sin \alpha_j \sin \alpha_j} \times \frac{2\pi}{z} (j-1)$$  \hspace{1cm} (6)

3. Bearing Basic Equations Solution
In order to further verify the reliability of the model and program, the internal dynamics of the B7005C/P4 ball bearing given in paper [2] was simulated. The number of rolling balls was 15, the ball diameter was 5.5mm, the initial contact angle was 15°, the radius radii of the inner and outer raceway grooves were 0.54 and 0.57, respectively, and the axial load was 50N. The contact angle and contact load were firstly related to the rotation speed.
Figure 1. Radial stiffness of bearing.

Figure 2. Axial stiffness of bearing.

Figure 3. Angular stiffness of bearing.
With the increase of the speed, the bearing diameter. The stiffness increases while the axial stiffness and angular stiffness decrease rapidly. This is due to an increase in the rotational speed, the increase in centrifugal force of the ball makes the outer ring contact load increases, so the outer ring normal contact stiffness increases, but the inner ring contact load is almost constant, so the inner ring contact stiffness is basically the same, the inner and outer ring contact stiffness. The series results increase the radial stiffness, while the reduction of the contact angle of the outer ring results in a significant reduction of the axial stiffness and the angular stiffness. It is basically consistent with the calculation result in paper [2], which verifies the correctness of the calculation program of the bearing in this paper.

4. Analysis of Dynamic Characteristics of Turbo Pump Rotor at Variable Speed

Based on the analysis of the dynamic stiffness of the bearing and considering the influence of the additional stiffness induced by the sealing fluid, the rotor system of the turbo-pump is analyzed under variable speed conditions.

The parameters of the two bearings at the pump end and the turbine end are shown in Table 1. The effect of axial force is ignored here. Both bearings only consider the effect of radial loads. Using the calculation program of the bearing dynamic characteristics compiled in this paper, the stiffness of the two bearing supports changes with the rotation speed as shown in Fig4. Under larger radial loads, due to the influence of the centrifugal force of the ball and the gyro moment, the radial stiffness slightly decreases with the increase of the rotation speed.

A labyrinth seal was placed before and after the centrifugal impeller. The flow field was analyzed with Fluent software to obtain the dynamic characteristics of the sealed fluid. The main stiffness, cross stiffness, main damping and cross damping were related to the rotation speed, the main stiffness of the pump, the cross stiffness is shown in Fig5.

| Position position | Pump end | Turbine side |
|-------------------|----------|--------------|
| Rolling balls     | 10       | 12           |
| Contact angle     | 0°       | 18°          |
| Ball diameter     | 23.812mm | 30.162mm     |
| Radial load       | 5KN      | 6KN          |

![Figure 4. Relationship between radial stiffness and rotational speed of bearing.](image)
Combining the dynamic stiffness of the bearing and the additional dynamic stiffness of the seal with the turbo-pump rotor dynamics simulation system based on the Riccati transfer matrix method, the turbine pump rotor is relatedly calculated. The eddy frequency changes with the speed as shown in Fig6, the first-order critical speed is 20493.095r/min, and the second-order critical speed is 22133.968r/min.

5. Conclusion
The internal motion of angular contact ball bearings is very complex. It is more in line with engineering practice to use numerical methods to solve the dynamic stiffness of dynamic angular contact bearings.
When calculating the dynamic characteristics of a rotor under the influence of multiple factors such as bearings and seals, it is necessary to give the relationship between the influencing factors and the rotational speed, and to calculate the bearing-seal-rotor whirl frequency more fully and accurately.

In this paper, the rotor dynamics simulation system with multiple influencing factors developed by the self-programming program provides the corresponding calculation basis for the analysis of the dynamic characteristics of the turbo pump rotor in the design stage.

References
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