A Model With Dynamical R-parity Breaking
and Unstable Gravitino Dark Matter

Xiangdong Ji,$^{1,2}$ Rabindra N. Mohapatra,$^1$ Shmuel Nussinov,$^3$ and Yue Zhang$^{2,1}$

$^1$Maryland Center for Fundamental Physics and Department of Physics,
University of Maryland, College Park, Maryland 20742, USA

$^2$Center for High-Energy Physics and Institute of Theoretical Physics,
Peking University, Beijing 100871, China

$^3$Tel Aviv University, Israel

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Abstract

An unstable gravitino with lifetime longer than $10^{26}$ sec or so has been proposed as a possible dark matter candidate in supergravity models with R-parity breaking. We find a natural realization of this idea in the minimal supersymmetric left-right models where left-right symmetry breaking scale in the few TeV range. It is known that in these models, R-parity must break in order to have parity breaking as required by low energy weak interactions. The sub-eV neutrino masses imply that R-parity breaking effects in this model must be highly suppressed. This in turn makes the gravitino LSP long lived enough, so that it becomes the dark matter of the Universe. It also allows detectable displaced vertices at the LHC from NLSP decays. We present a detailed analysis of the model and some aspects of its rich phenomenology.
I. INTRODUCTION

It is now widely believed that about 25% of the energy density in the Universe is in the form of a cold dark matter. The nature of the elementary particle which constitutes this dark component is however not known. The scenario with an unstable gravitino which is the lightest supersymmetric particle (LSP) with a very long life time as a dark matter is particularly attractive [1, 2], having interesting consequences for astrophysics and cosmology, as well as colliders.

The LSP gravitino in an R-parity conserving supersymmetric (SUSY) theory is absolutely stable and could be a dark matter candidate [3]. An unstable gravitino, in a theory with broken R-parity symmetry, needs to be sufficiently long lived to be a dark matter. The strengths of the R-parity violating (RPV) couplings (usually denoted by $\lambda, \lambda'$) needed to achieve required longevity must be highly suppressed i.e. $\lambda, \lambda' \leq 10^{-6}$, and one would then like to understand the origin of such small couplings. It is therefore interesting to explore models where such small couplings may arise naturally. We find that TeV scale supersymmetric left-right (SUSYLR) models which have been discussed in connection with neutrino masses and strong CP problems provides one such framework, with two interesting features: (a) R-parity breaking is dynamically induced in the global minimum of the potential in order for the theory to break parity; (b) smallness of neutrino masses guarantees that the resulting R-parity violating interactions are highly suppressed.

The left-right symmetric theories based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$[4] were originally introduced to explain parity violation in the standard model and were found to have a number of interesting properties. These models provide a natural framework for understanding the small neutrino masses via the seesaw mechanism[5] and became specially interesting after the discovery of neutrino masses. The right handed neutrino which is essential for seesaw mechanism is automatically contained and furthermore the B-L gauge symmetry[6] whose breaking provides the heavy Majorana mass to the right handed neutrinos relates the small neutrino masses to the parity breaking scale. This implementation of the seesaw mechanism is different from those in the SM context where the right handed neutrino mass is a free parameter.

The seesaw scale (or the parity breaking scale) however, still remains undetermined. It can take values anywhere from near a TeV if the Dirac Yukawa couplings $Y_\nu \sim 10^{-6} \approx Y_e$ or it can be much higher if the $Y_\nu$'s are larger. It is important to note that a TeV seesaw scale is perfectly natural and does not require any higher fine tuning than that present in SM. A TeV seesaw scale is clearly of great interest for the LHC. It is worth noting that the current searches by the CDF and
D0 collaborations have yielded limits on the parity breaking scale in the 750 GeV range [7].

In this paper we consider the minimal left-right (SUSYLR) extension of the MSSM since it combines the advantages of supersymmetry while making MSSM realistic by providing a way to understand neutrino masses. It also cures certain problems of the MSSM such as making R-parity a good symmetry if one uses B-L=2 Higgs triplets to break parity so that proton is stable. Furthermore, it provides a solution to the SUSY and strong CP problems. The minimal version of this model (without any gauge singlets) has two striking features: (i) the parity and SUSY breaking scales are related, thereby predicting that the seesaw scale is necessarily in the TeV range and (ii) while R-parity conservation is automatic above the parity breaking scale, the ground state of the theory can break parity only if R-parity is spontaneously broken by a vacuum expectation value (VEV) of the right-handed sneutrino fields [8]. Thus the model breaks R-parity dynamically. Furthermore in this case there is an upper limit on the $W_R$ scale in the range of a few TeV’s [9], which is to be expected since parity breaking and susy breaking are intimately linked. This makes it possible to test the theory using LHC data expected in coming years.

An interesting feature of the resulting dynamical R-parity breaking is that it only breaks lepton number and keeps baryon number intact and therefore the proton is absolutely stable in this model. It is also worth noting that while the effective R-parity breaking below the $W_R$ scale has some properties similar to MSSM with bilinear R-parity breaking [12], it has many properties which are characteristic of the SUSYLR theory that can provide distinguishing tests.

The immediate question that then arises is whether this SUSY model which starts out promising a stable dark matter, does indeed have a dark matter after parity and R-parity breaking. We address this question in this paper. We find that despite R-parity breaking, the unstable LSP gravitino in our model can be the dark matter of the universe [1]. The reason for this is that requiring sub-eV neutrino masses suppresses the strengths of the R-parity breaking interactions responsible for gravitino decay to such a level that the gravitino becomes sufficiently long lived and dark matter. Secondly, we also find that despite the neutrino-Higgsino mixing induced by spontaneous R-parity breaking, the seesaw results for neutrino masses remain essentially intact. Another consequence of this small strength of R-parity breaking is that the next-to-lightest SUSY particle (NLSP) which can be a neutralino or stau or sneutrino produced at LHC has a lifetime such that it can give rise to displaced vertices in the LHC detector [13].

The paper is organized as follows: Section II presents the basic contents of the model, and reviews the result of Ref. [8] that R-parity indeed must break in the model if ordinary parity has
to break. In Section III, we consider implications of RPV on the neutrino mass in this model. In Section IV, we discuss contribution to various RPV couplings due to bilinear terms in the superpotential. In Section V, we derive cosmological implications for gravitino as the dark matter. In Section VI, we discuss collider signatures of the model and in Section VII, we briefly discuss some other consequences of the model. In the appendix, we display the minimization of the potential and obtain bounds on the $W_{R}$-boson mass and the right-handed sneutrino VEV.

II. BASIC FEATURES OF SUSYLR MODEL AND SPONTANEOUS R-PARITY VIOLATION

The gauge group in the minimal SUSYLR model is $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$. The chiral left and right handed quark superfields are $Q \equiv (u, d)(2, 1, \frac{1}{3}, 3)$ and $Q^c \equiv (u^c, d^c)(1, 2, -\frac{1}{3}, 3^*)$ respectively, and similarly the lepton superfields are given by $L \equiv (\nu, e)(2, 1, -1, 1)$ and $L^c \equiv (\nu^c, e^c)(2, 1, +1, 1)$, where flavor indices have been implicit. The symmetry breaking is achieved by the following bi-fundamental and $B-L$ triplet Higgs superfields: $\phi_a(2, 1, 0, 1) (a = 1, 2)$, $\Delta(3, 1, +2, 1)$, $\bar{\Delta}(3, 1, -2, 1)$, $\Delta_c(3, 1, -2, 1)$, $\bar{\Delta}_c(3, 1, +2, 1)$.

The superpotential of the model is:

\[
W = Y_u Q^T \tau_2 \Phi_1 \tau_2 Q^c + Y_d Q^T \tau_2 \Phi_2 \tau_2 Q^c \\
+ Y_\nu L^T \tau_2 \Phi_1 \tau_2 L^c + Y_\nu L^T \tau_2 \Phi_2 \tau_2 L^c \\
+ if \left( L^T \tau_2 \Delta L + L^c T \tau_2 \Delta^c L^c \right) + \mu_{ab} \text{Tr} \left( \Phi_a^T \tau_2 \Phi_b \tau_2 \right) \\
+ \mu_\Delta \text{Tr} \left( \Delta \bar{\Delta} + \Delta^c \bar{\Delta}^c \right),
\]

where $Y$'s are Yukawa couplings, $f$ is the Majorana coupling and $\mu_\Delta$ is the $\mu$-term for triplets. Note that we do not have any gauge singlet fields in the model.

First point to note is that if $M_R \gg M_{SU SY}$, the SUSY breaking scale, the right handed gauge symmetry remains unbroken since we must have $\langle \Delta^c \rangle = \langle \bar{\Delta}^c \rangle = 0$ to preserve supersymmetry. From this it follows that these vevs i.e. $\langle \Delta^c \rangle = v_R$ and $\langle \bar{\Delta}^c \rangle = \bar{v}_R$ and $\mu$-term must have TeV scale vevs, i.e., the theory is necessarily one with TeV scale parity breaking. From now on we will assume that all the mass parameters in the theory are of order of a TeV.

To do a detailed analysis of the ground state of the theory, we write down the scalar potential including the soft SUSY-breaking terms:

\[
V = V_F + V_D + V_S,
\]
We first give $V_D$:

$$V_D = \frac{g_R^2}{8} \sum_m |\tilde{\nu}^c \delta_m|^2 + 2 \text{Tr} \left( \Delta^c \tau_m \Delta^c + \bar{\Delta}^c \tau_m \bar{\Delta}^c \right) + \text{Tr}(\Phi \tau^T \Phi^\dagger)$$

$$+ \frac{g^2}{8} \sum_m 2 \text{Tr} \left( \Delta^\dagger \tau_m \Delta + \bar{\Delta}^\dagger \tau_m \bar{\Delta} \right) + \text{Tr}(\Phi^\dagger \tau_m \Phi)$$

$$+ \frac{g^2_{BL}}{8} |\tilde{\nu}^c|^2 + 2 \text{Tr} \left( \Delta^\dagger \bar{\Delta} - \Delta^c \bar{\Delta}^c + \Delta^\dagger \bar{\Delta}^c - \bar{\Delta}^c \bar{\Delta}^c \right) \right|^2. \quad (3)$$

Below we give some of the terms from $V_F = \sum_a |\frac{\partial W}{\partial \phi_a}|^2$, with $\phi_a$ going over all the fields in the model and the soft susy breaking term $V_S$ [8, 14] which are relevant for our discussion:

$$V_F + V_S = A_L \tilde{L}^T \tau_2 \Phi_a \tau_2 \tilde{L}^c$$

$$+ i A \left( \tilde{L}^T \tau_2 \Delta \tilde{L} + \tilde{L}^T \tau_2 \Delta^c \tilde{L}^c \right)$$

$$+ b_{ab} \text{Tr} \left( \Phi^T \tau_2 \Phi_b \tau_2 \right) + B \text{Tr} \left( \Delta \bar{\Delta} + \Delta^c \bar{\Delta}^c \right) + \text{h.c.}$$

$$+ M_{ab}^2 \text{Tr} \left( \Phi^\dagger \Phi_b \right) + m_0^2 \left( \tilde{L}^\dagger \tilde{L} + \tilde{L}^c \tilde{L}^c \right)$$

$$+ M_{\Delta}^2 \text{Tr} \left( \Delta^\dagger \Delta + \Delta^c \Delta^c \right) + M_{\Delta}^2 \text{Tr} \left( \bar{\Delta}^\dagger \bar{\Delta} + \bar{\Delta}^c \bar{\Delta}^c \right), \quad (4)$$

where in addition to the terms dependent on Higgs fields, we have also kept slepton terms since sneutrino is electrically neutral and can in principle have vev. We have omitted all terms involving the squarks. All soft parameters with mass dimensions are assumed to have TeV SUSY-breaking scale.

It was shown in [8] that if the ground state of this tree level potential has to break parity, it must break R-parity by giving a vev to the $\tilde{\nu}^c$ field. We review this argument in the appendix.

The global minimum of the model is then characterized by the following vev pattern of the fields

$$\langle \tilde{L}_i^c \rangle = \begin{pmatrix} \langle \tilde{\nu}_i^c \rangle \\ 0 \end{pmatrix}, \langle \Delta^c \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \langle \bar{\Delta}^c \rangle = \begin{pmatrix} 0 & \bar{v}_R \\ 0 & 0 \end{pmatrix}. \quad (5)$$

In this case, there is an upper bound on the $v_R$ scale as discussed in [9], given roughly by

$$\frac{m_0}{f} < v_R < \frac{(A + f \mu_\Delta)}{2f}. \quad (6)$$

where the parameters $A$ and $m_0$ are susy breaking parameters in the theory, of the order of a few hundred GeVs and $f$ is a typical Majorana coupling of the $\Delta$ fields. Using these, we get few TeV upper limit on the right-handed scale if $f \sim 1/10$. A detailed derivation of the upper bound is given in the appendix.
It has recently been shown that in the case of the model with an additional singlet, once one takes one loop corrections into account, in the domain of parameters where $m_{\tilde{c}}^2 \leq 0$, R-parity conserving and electric charge conserving minimum indeed becomes the global minimum[10]. In this case one generically needs a higher parity breaking scale to get correct mass spectrum for sleptons. Our discussions in the subsequent sections will remain valid for the complementary domain i.e. $m_{\tilde{c}}^2 \geq 0$ in the presence of loop effects. R-parity conserving ground state can also be a global minimum if one includes higher dimensional operators provided the parity breaking scale is above $10^{10}$ GeV[11]. Since we are interested in TeV scale $W_R$, these new operators do not affect our considerations.

In the rest of the paper, we will assume that the component of the right handed sneutrinos that acquires a vev must align along the electron flavor direction so that the neutrino masses will remain in the sub-eV range. It appears that it may be possible to ensure this by choosing the $A_{ee}$ term associated with $\nu_\ell^c \nu_\ell^c \Delta^c$ coupling to be sufficiently negative while keeping this for other flavors to be positive. We do not pursue the details of this calculation here.

III. NEUTRINO MASS

In this section, we address the question of how to understand small neutrino masses in this model. As is well known, neutrinos acquire Dirac masses after electroweak symmetry is broken by the $\Phi_{1, 2}$ vevs. We assume that only $< H_u^0 >$ and $< H_d^0 >$ fields acquire vevs $\kappa_1$ and $\kappa_2$ respectively. In this model, B-L breaking gives large Majorana masses to the right handed neutrinos which combined with the the Dirac masses leads to the usual type I seesaw mechanism. In the absence of the sneutrino vev, the type I seesaw mechanism yields sub-eV left-handed neutrino masses from TeV-scale right handed-neutrino masses provided the neutrino Dirac Yukawa couplings $Y_\nu$ are of the same order as that of the electron in the standard model. An upper bound on the $v_R$ scale dictated by the dynamics of the model, implies that all elements of the neutrino Yukawa coupling matrix in our model must have an upper bound of order $10^{-6}$. There are no type II contributions in the renormalizable supersymmetric left-right model in the supersymmetric limit. After SUSY breaking, a finite but small type II contribution is induced which can be comparable to the type I contribution. This does not affect the analysis of neutrino masses done below and we ignore it here. As we show later, the small Dirac Yukawa couplings needed for understanding small neutrino masses have the important implication, that the gravitino lifetime which is inversely
proportional to $Y_\nu^2$ is long enough that it can become the dark matter of the Universe.

The presence of right handed sneutrino condensate complicates the analysis of neutrino masses since it introduces a mixing term of the form $LH_u$ in the superpotential making the neutrino-Higgsino-gaugino mass matrix to be a $15 \times 15$ mass matrix. As we show below, if we assume that the right handed sneutrino that acquires a vev is aligned along the electron flavor direction, one can still use successive seesaw approximation so that neutrino masses remain in the sub-eV range as in the simple type I seesaw models prior to neutrino Higgsino mixing.

To proceed with the neutrino masses, note that the terms in the superpotential are induced by the sneutrino vev and expand the seesaw matrix are given by

$$\Delta W = (Y_\nu)_{ij} x_j L_i^T (i\tau_2) L H_u + (Y_l)_{ij} x_j L_i^T (i\tau_2) L H'_u , \quad (7)$$

where $H_u$'s are defined through $\Phi_1 = (H'_d, H_u)$ and $\Phi_2 = (H_d, H'_u)$, and $x_i = \langle \tilde{\nu}^c_i \rangle$. These terms mix lepton superfields with the Higgs superfields and therefore break R-parity. They lead to the mixing of lefthanded neutrinos with the neutralinos. These mixing terms enlarge the $6 \times 6$ normal seesaw neutrino mass matrix to $15 \times 15$. However, we can employ successive decoupling to simplify the analysis of this matrix to get neutrino masses. For $v_R$ in the multi-TeV range, the $v_R$-scale fermions decouple first, leaving a $9 \times 9$ mass matrix. On the further application of a second stage seesaw through integrating out the electroweak scale fields, small neutrino masses follow.

For convenience of discussion, we choose a basis in which the $Y_l$ is diagonal and all neutrino mixings arise from $Y_\nu$. We will also choose all Majorana couplings of right handed neutrinos, $f$, to be diagonal.

As noted before we will assume the RH-sneutrino vev to align along the electron flavor i.e. $\langle \tilde{\nu}^c_{\mu,\tau} \rangle = 0$ so that the neutrino-Higgsino mixing is in the MeV range. The symmetric neutrino-neutralino mass matrix can now be written in the basis of
\{ \nu_L^i, \tilde{H}_u^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{H}_d^0, (-i\lambda_L^3), (-i\lambda_R^3), (-i\lambda_{BL}), \nu^c_i, \tilde{\Delta}^0_c, \tilde{\Delta}^{0c} \} \text{ as}
\[
\begin{pmatrix}
0 & (Y_\nu)_{ik} x_k & 0 & (Y_\nu)_{ij} \kappa_1 & 0 & 0 \\
-\mu_{12} & 0 & -\mu_{11} - \frac{g_L}{\sqrt{2}} \kappa_1 & -\frac{g_B}{\sqrt{2}} \kappa_1 & 0 & 0 \\
-\mu_{12} & 0 & -\mu_{22} & \frac{g_L}{\sqrt{2}} \kappa_2 & \frac{g_B}{\sqrt{2}} \kappa_2 & 0 \\
0 & -\mu_{12} & 0 & 0 & 0 & 0 \\
-\mu_{11} & 0 & -\mu_{12} & 0 & 0 & 0 \\
-\mu_{12} & 0 & \ldots & M_L & 0 & 0 \\
-\mu_{11} & 0 & \ldots & M_R & 0 & \sqrt{2} g_R v_R - \sqrt{2} g_R \bar{v}_R \\
-\mu_{12} & 0 & \ldots & M_{BL} & -\sqrt{2} g_{BL} x_j & \sqrt{2} g_{BL} \bar{v}_R \\
-\mu_{12} & 0 & \ldots & \bar{f}_{ij} v_R & \bar{f}_{ki} x_k & 0 \\
-\mu_{11} & 0 & \ldots & 0 & -\mu_R & 0 \\
-\mu_{12} & 0 & \ldots & -\mu_R & 0 & 0
\end{pmatrix},
\]

where we have assumed the vev’s of doublet fields have the following pattern
\[
\Phi_1 = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \kappa_2 & 0 \\ 0 & 1 \end{pmatrix}.
\] (8)

To simplify this matrix to obtain the neutrino masses, we proceed in two steps as mentioned: first we integrate out the TeV scale fields $\nu^c_i, \tilde{\Delta}^0_c, \tilde{\Delta}^{0c}$ and a combination of $\lambda_L^3, \lambda_{BL}$. The remaining combination $\tilde{B} = \frac{g_{BL}}{\sqrt{g_R^2 + g_{BL}^2}} (-i \lambda_{BL}) + \frac{g_B}{\sqrt{g_R^2 + g_{BL}^2}} (-i \lambda_{BL})$ stays at the electroweak scale. The mass matrix for the electroweak scale active fields is
\[
\nu_i \begin{pmatrix}
-\frac{\xi_i}{v_R} (Y_\nu f^{-1} Y_\nu^T)_{ij} (Y_\nu)_{ik} x_k & 0 & (Y_\nu)_{ij} \kappa_1 & 0 & 0 & 0 \\
-\mu_{12} & 0 & -\mu_{11} - \frac{g_L}{\sqrt{2}} \kappa_1 & \frac{g_B}{\sqrt{2}} \kappa_1 & 0 & 0 \\
-\mu_{12} & 0 & \frac{g_L}{\sqrt{2}} \kappa_2 & \frac{g_B}{\sqrt{2}} \kappa_2 & 0 & 0 \\
0 & -\mu_{12} & 0 & 0 & 0 & 0 \\
-\mu_{11} & 0 & -\mu_{12} & 0 & 0 & 0 \\
0 & -\mu_{12} & \ldots & M_L & 0 & \sqrt{2} g_R v_R - \sqrt{2} g_R \bar{v}_R \\
\end{pmatrix},
\] (9)

where $M_B = \frac{g_{BL}^2}{g_{BL}^2 + g_R^2} M_R + \frac{g_B^2}{g_{BL}^2 + g_R^2} M_{BL}$ and $g'$ satisfies $g'^{-1} = g_R^2 + g_{BL}^2$. 

8
When $Y_\nu$ and $Y_l$ are small, we can still use the seesaw formula to estimate the neutrino mass contributions from R-parity violating terms.

$$\delta M_\nu = -[(Y_\nu)_{ik} x_k, 0, (Y_l)_{ik} x_k, 0, 0] \times \begin{bmatrix}
0 & -\mu_{12} & 0 & -\mu_{11} - \frac{g_L^2 g^2}{\sqrt{2}} \kappa_1 - \frac{g_R^2}{\sqrt{2}} \kappa_1 \\
-\mu_{12} & 0 & -\mu_{22} & 0 \frac{g_L^2 g^2}{\sqrt{2}} \kappa_2 + \frac{g_R^2}{\sqrt{2}} \kappa_2 \\
0 & -\mu_{22} & 0 & -\mu_{12} 0 0 \\
-\mu_{11} & 0 & -\mu_{12} & 0 0 0 \\
-\frac{g_L^2}{\sqrt{2}} \kappa_1 & \frac{g_L^2 g^2}{\sqrt{2}} \kappa_2 & 0 & 0 M_L 0 \\
-\frac{g_R^2}{\sqrt{2}} \kappa_1 & \frac{g_R^2}{\sqrt{2}} \kappa_2 & 0 & 0 0 M_{R\bar{R}}
\end{bmatrix}^{-1} \begin{bmatrix}
(Y_\nu^T)_{kj} x_k \\
0 \\
(Y_l^T)_{kj} x_k
\end{bmatrix}. \quad (10)
$$

The sneutrino condensates picks up 11, 13, 31 and 33 elements of the inverse matrix. To apply the seesaw formula, we concentrate on the corresponding cofactors of these elements.

$$\text{cof}_{11} = \frac{1}{2} \mu_{12}^2 \kappa_2^2 \left( g_L^2 M_{R\bar{R}} + g^2 M_L \right) = \frac{g_L^2 g^2}{2} \mu_{12}^2 \kappa_2^2 \left( \frac{M_L}{g_L^2} + \frac{M_R}{g_R^2} + \frac{M_{BL}}{g_{BL}^2} \right),$$

$$\text{cof}_{13} = \text{cof}_{31} = -\frac{g_L^2 g^2}{2} \mu_{12} \mu_{11} \kappa_2^2 \left( \frac{M_L}{g_L^2} + \frac{M_R}{g_R^2} + \frac{M_{BL}}{g_{BL}^2} \right),$$

$$\text{cof}_{33} = \frac{g_L^2 g^2}{2} \mu_{11}^2 \kappa_2^2 \left( \frac{M_L}{g_L^2} + \frac{M_R}{g_R^2} + \frac{M_{BL}}{g_{BL}^2} \right). \quad (11)$$

Therefore, the neutrino mass can be written as

$$\begin{bmatrix}
(M_\nu)_{ij} = -\frac{\kappa_1}{\mu_{R}} (Y_\nu f^{-1} Y_\nu^T)_{ij} - \frac{g_L^2 g^2 \kappa_2}{2 M_L M_{R\bar{R}} (\mu_{11} \mu_{22} - \mu_{12}^2)} \left( \frac{M_L}{g_L^2} + \frac{M_R}{g_R^2} + \frac{M_{BL}}{g_{BL}^2} \right) \times \left[ \left( \frac{\mu_{12}}{\mu_{11}} \right) (Y_\nu)_{ik} (Y_\nu)_{jk} + x_1^2 (Y_\nu)_{i1} y_e \delta_{j1} + x_1^2 (Y_\nu)_{1j} y_e \delta_{i1} - \left( \frac{\mu_{11}}{\mu_{12}} \right) g^2 \delta_{i1} \delta_{j1} \right] \quad (12)
\end{bmatrix}$$

where the first term is the usual see-saw formula for neutrino; we have worked in the basis that the charged lepton Yukawa coupling is diagonal and assumed only $x_1 \neq 0$. In the following sections, we will see that this constraint helps us to avoid too large RPV terms, which not only makes gravitino life-time sufficiently long but also has important collider implications.

An interesting aspect of the neutrino mass formula is the presence of the term $\left( \frac{M_L}{g_L^2} + \frac{M_R}{g_R^2} + \frac{M_{BL}}{g_{BL}^2} \right)$. In the framework of mSUGRA and gauge mediated SUSY breaking, one typically gets

$$\frac{|M_L|}{g_L^2} = \frac{|M_R|}{g_R^2} = \frac{|M_{BL}|}{g_{BL}^2}. \quad (13)$$

This relation is preserved by renormalization group evolution. In SUSYLR case, we have $M_L = M_R$ can be complex, while $M_{BL}$ must be real. In a special case where $\arg(M_L) = \pm 2\pi/3$, the
second term in Eq. (12) vanishes, i.e. $\delta M_\nu = 0$. In this case, we can also allow $x_2, x_3 \neq 0$ without affecting the neutrino mass discussion above. However no such assumption is necessary if we only choose $x_1 \neq 0$.

Note that in the context of MSSM with R-parity breaking bilinear terms, getting small neutrino masses generically requires to fine-tune the neutrino-Higgsino coupling terms [22, 23]. In our case, the smallness of these terms is now guaranteed by the smallness of the Dirac Yukawa couplings together with the alignment of $\langle \tilde{\nu}^c \rangle$ along the electron direction.

IV. SOURCES OF R-PARITY VIOLATION IN THE MODEL

Our model is different from all previous models of R-parity breaking [21, 22] because R-parity breaking is forced by the tree level dynamics of the theory to enable parity breaking [8]: We do not have the freedom of choosing the tree level parameters leading to an alternative vacuum with R-parity conservation. Here R-parity is broken by the right-handed sneutrino vev which leads to bilinear R-parity violating coupling at low energies. As noted recently, in models with singlets, once one loop corrections are included [10], in a parameter subdomain, one can indeed have an R-parity conserving vacuum as a global minimum without breaking electric charge. If the same discussion were to apply to the minimal case under discussion here, the considerations of this paper will still remain valid in the complementary parameter domain.

The dominant R-parity violating interaction in the effective theory below the TeV scale, after the right-handed sneutrino and the heavy $\Delta$ fields are integrated out, is given by

$$\Delta W = \epsilon_i L_i^T (i\tau_2) H_u + \epsilon'_1 L_1^T (i\tau_2) H'_u.$$ (14)

There are analogous terms coming from the soft-breaking trilinear terms given by

$$\mathcal{L}_1 = B_i \tilde{L}_i^T (i\tau_2) H_u + B'_1 \tilde{L}_1^T (i\tau_2) H'_u,$$ (15)

where $\epsilon_i = (Y_\nu)_{i1} x_1$ and $\epsilon'_1 = Y_e x_1 \delta_{i1}$. The terms in the second equation above arise when we implement the soft SUSY breaking on the effective R-parity breaking Lagrangian (say via usual mSUGRA). Since in general the coefficients are arbitrary and we ignore them in what follows.

In order to understand neutrino masses, we have assumed that only the right-handed electron sneutrino gets a VEV. The main consequence of this is that since $Y_e \sim 10^{-5.5}$ and $Y_{\nu, ij} \leq 10^{-6}$, the strengths of RPV interactions are all very small. In the presence of these terms, the gravitino is
unstable; however, we will see later that the small magnitude of these coupling strengths required to understand neutrino masses, allows the gravitino to live long enough to be the dark matter of the Universe. Thus the gravitino dark matter is connected to the smallness of the neutrino mass, and the viability of the scenario does not require tuning of a separate parameter.

\[ \mathbf{L} \mathbf{H} \mathbf{d} \mathbf{c} \sim \mathbf{H} \mathbf{u} \sim \mathbf{c} \mathbf{Z} \mathbf{(W)} \]

FIG. 1: “Flavor-violating” gauge couplings due to bilinear R-parity violation. The shaded blob is an R-Parity breaking vertex.

\[ \mathbf{L} \mathbf{H} \mathbf{u} \mathbf{c} \mathbf{L} \mathbf{H} \mathbf{d} \mathbf{c} \mathbf{f} \mathbf{j} \mathbf{f} \mathbf{i} \mathbf{L} \mathbf{H} \mathbf{d} \mathbf{c} \mathbf{f} \mathbf{j} \mathbf{f} \mathbf{i} \]

FIG. 2: The induced tri-linear RPV couplings. The shaded blobs are R-Parity breaking vertices.

The bilinear R-parity breaking terms in the superpotential lead to mixings between neutralinos, \( \tilde{H}_u \) and neutrinos \( \nu \) as discussed in the previous section and similarly between charginos and charged leptons. These terms can also generate mixing between sleptons and \( H_d \) boson through the F-term of \( H_u \) and \( H'_u \)

\[
F_{H_u} = \mu_{11} H'_d + \mu_{12} H_d + \epsilon_1 L_i ,
\]
\[
F_{H'_u} = \mu_{21} H'_d + \mu_{22} H_d + \epsilon'_1 L_1 .
\]
To calculate R-parity violating decays, a general way is to diagonalize all the mass matrices and then to rotate all fields into physical states. In the present work, since we only estimate the order of magnitude for the R-parity violating effects, we adopt an approximate but more convenient approach. We take the bilinear RPV terms as mass perturbations which are to be inserted into all the amplitudes. It is these mass insertions that act as sources of R-parity violation. In Fig. 1, we illustrate how to get the “flavor-violating” gauge coupling induced by bilinear RPV term \[^{[27]}\] which plays an important role in neutralino decay. The tri-linear couplings of the conventional R-parity breaking \( \lambda LLE^c \) and \( \lambda' QLD^c \) terms can also be generated from the bilinear terms. The diagram on the left-panel in Fig. 2 is generated from neutrino-neutralino mixing in Eq. (9), and that on the right is generated through slepton-Higgs mixing due to Eq. (16). As expected, the right-handed (s)leptons can be coupled to leptons, whereas the left-handed (s)leptons couple to both hadrons and leptons.

V. GRAVITINO DARK MATTER

In generic SUGRA theories, the gravitino is a very weakly coupled particle with mass ranging from eV to many TeV’s. It can be produced in early universe plasma and remains in equilibrium with rest of the cosmic soup at very high temperatures i.e. \( (T \sim M_{pl}) \). Slightly once the universe cools below the Planck temperature, the gravitinos decouple. Since their annihilation or decay rate are very slow, their number density dilutes only due to entropy dumped into the cosmic bath at different annihilation thresholds of other particles. This dilution is not a large effect. Therefore in the absence of inflation, if gravitino is the LSP and R-parity is conserved, its mass must not exceed 1 keV in order not to over-close the universe.

In the inflationary scenario however, any initial gravitino abundance will be diluted to very tiny values. However, secondary production of gravitinos in the reheating process can be appreciable and proportional to the reheating temperature. This has been estimated in various papers to be \[^{[15]}\]  

\[
\Omega_{3/2} h^2 \approx 0.27 \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_\tilde{g}}{1 \text{ TeV}} \right)^2 ,
\]

\( (17) \)

where \( T_R \) is the reheating temperature, and \( m_{3/2} \) and \( m_\tilde{g} \) are the gravitino and gluino masses, respectively. While this would permit gravitinos in the 100 GeV mass range, one must be mindful of other constraints when R-parity is conserved: the NLSP (often neutralino or stau, etc.) decays late, after freeze-out and often after big-bang nucleosynthesis (BBN) \( (T \lesssim 1 \text{ MeV}) \), and produces
a large amount of entropy mainly as photons into the universe, thereby drastically changing the ratio of $\frac{n_B}{n_\gamma}$. Also the decay products can destroy the produced elements making it hard to understand the successes of BBN \[16\]. Therefore, in a consistent picture of the universe described by supergravity theories, the NLSP must decay quickly ($< 10^2$s). If R-parity is conserved, typical NLSP lifetime however is anywhere from a few days to years as can be inferred from the formula

$$\tau_{NLSP} \approx 9 \text{ days} \left( \frac{m_{3/2}}{100\text{GeV}} \right)^2 \left( \frac{100\text{GeV}}{m_{NLSP}} \right)^5. \quad (18)$$

Here we need $m_{3/2} \ll 1\text{ GeV}$ for $m_{NLSP}$ mass around 100 GeV. So whereas the overclosure constraints by gravitinos can be reconciled with multi-GeV mass gravitinos by adjusting the reheat temperature, the NLSP lifetime constraints cannot be accommodated without extreme fine tuning.

This late-decay problem can be solved if R-parity is violated by a small amount \[1\]. [For an R-parity conserving theory, there are also ways to avoid this; see \[17\].] In this case, the neutralino decays very quickly into RPV channels and there is no longer an upper-bound from BBN considerations. Our model leads naturally to this scenario since the gravitino can now decay to several channels, as shown in Fig. 3. To see if it lives long enough to become a viable dark matter, we need to estimate its lifetime.

The dominant decay-modes of gravitino are $\gamma + \nu$, $W^\pm + l^\mp$, $Z^0 + \nu$ and $h^0 + \nu$ \[18\]. From the following analysis of cosmological constraints, we find the gravitino should be generally lighter than $W$- or $Z$-boson. In this case, gravitino mainly decays into photon and neutrino. The strength of the $\gamma + \nu$ decay that occurs through the diagram depends on the value of the photino-neutrino

\[FIG. 3: \text{Left-panel: Dominant PRV two-body gravitino decay mode } (\tilde{G} \rightarrow \gamma \nu_L) \text{ in our model. Right-panel: a typical three-body gravitino decay mode.}\]
mixing. In our model this mixing occurs via the one-loop diagram in Fig. 4 and the corresponding mixing parameter $U_{\tilde{\gamma}-\nu}$ can be estimated to be

$$U_{\tilde{\gamma}-\nu} = \frac{<\tilde{\nu}^c> \nu wk \sin \beta \mu e h_e}{16\pi^2 M_{\text{susy}}^2 M_{\tilde{\gamma}}} ,$$

(19)

and is of order $10^{-6} - 10^{-7}$ for $\mu \sim M_{\text{SUSY}} \sim M_{\tilde{\gamma}} \sim <\tilde{\nu}^c> \sim 100$ GeV and $h_e \sim 10^{-5}$. The decay rate is then given by

$$\Gamma(\tilde{\gamma} \rightarrow \nu + \gamma) \approx \frac{|U_{\nu\tilde{\gamma}}|^2 m_{3/2}^3}{32\pi M_{\text{pl}}^2} ,$$

(20)

where $|U_{\nu\tilde{\gamma}}|^2$ is the percentage of photino in neutrino mass eigenstate in the presence of R-parity violation $\nu \approx \nu_0 + U_{\nu\tilde{\gamma}}\tilde{\gamma} + \cdots$ as in Fig. 4.

The transverse and longitudinal components of a decaying gravitino give comparable couplings to final state photon and neutrino. As a rough estimate, taking $m_{3/2} \sim 2$ GeV and $|U_{\nu\tilde{\gamma}}| \approx 10^{-6} - 10^{-7}$ (as given in our model), one gets $\Gamma(\tilde{G} \rightarrow \nu + \gamma)^{-1} \approx 10^{49} - 10^{51}$ GeV$^{-1} \approx 10^{25} - 10^{27}$ sec. Thus we see that even though the gravitino is unstable via R-parity violating interactions, it can become a viable dark matter of the Universe. The gravitino also has three-body decay modes with $\Gamma_{3\text{-body}}^{-1} = \left(\frac{h_v h_y (\tilde{\nu}^c) m_{3/2}^3}{192\pi^2 M_{\text{pl}} M_{\text{susy}}^3}\right)^{-1}$ but this rate is small compared to the two body decay rate.

The gravitino decay also produces energetic extra-galactic diffuse $\gamma$-rays in the universe. From the above decay rate, one can derive the photon energy-flux per unit solid angle assuming gravitino
has homogeneous distribution throughout the universe \[2\]

\[
E^2 \left| \frac{dJ(E(t_0))}{dE} \right| = \frac{\rho_c \Omega_{3/2} \Gamma(\tilde{G} \rightarrow \nu + \gamma)}{8\pi H_0 \Omega^{1/2}_M} \left( \frac{2E}{m_{3/2}} \right)^{5/2} \left[ 1 + \frac{\Omega_A}{\Omega_M} \left( \frac{2E}{m_{3/2}} \right)^3 \right]^{-1/2}
\]

where the last step follows if \( \tilde{G} \rightarrow \nu + \gamma \) is the dominant decay channel of gravitino. A \( \theta \)-function \( \theta \left( 1 - \frac{2E}{m_{3/2}} \right) \) is implicit because the photon frequency is cut off at \( m_{3/2} / 2 \), where the flux is peaked. Experimentally, EGRET observes the extragalactic cosmic gamma-ray flux has an excess over the power law spectrum, which can be as large as \( 2.23 \times 10^{-6} \) (cm\(^2\) str s\(^{-1}\) GeV for photon energy between 2 and 20 GeV \[24\]). Attributing part of this excess to the decay of gravitino dark matter in the universe, one has an upper bound on the mass of gravitino for a given neutrino-photino mixing \( |U_{\nu\gamma}| \). Taking the mixing parameter around \( 10^{-6} \) we find that \( m_{3/2} \lesssim 2 \) GeV for it to be consistent with cosmology.

VI. NLSP AND VERTICED DISPLACED AT LHC

The above R-parity breaking scenario can be tested at LHC. Here we discuss the possible signatures following from various decays of various possible NLSP’s.

A. Neutralino as NLSP

R-parity conserving decays The next-to-lightest superparticle can be the neutralino, i.e. a linear combination of neutral gauginos and higgsinos. It has both RPC and PRV decay channels. The R-parity conserving channels are \( \tilde{\chi}^0 \rightarrow \tilde{G} + \gamma, \tilde{G} + Z^0, \tilde{G} + h^0 \), with gravitino dominantly in the longitudinal component. These decay rates are calculated in the same way to gravitino decaying into neutrino and photon \([19]\), yielding

\[
\Gamma(\tilde{\chi}^0 \rightarrow \tilde{G} \gamma) = \frac{|U_{\tilde{\chi}^0 \tilde{\gamma}}|^2 k^2 m_{\chi^0}^5}{16\pi F^2},
\]

\[
\Gamma(\tilde{\chi}^0 \rightarrow \tilde{G} Z^0) = \frac{|U_{\tilde{\chi}^0 \tilde{Z}}|^2 k^2 m_{\chi^0}^5}{16\pi F^2} \left( 1 - \frac{M_{Z^0}^2}{m_{\chi^0}^2} \right)^4,
\]

\[
\Gamma(\tilde{\chi}^0 \rightarrow \tilde{G} h^0) = \frac{|U_{\tilde{\chi}^0 \tilde{h}}|^2 k^2 m_{\chi^0}^5}{16\pi F^2} \left( 1 - \frac{M_{h^0}^2}{m_{\chi^0}^2} \right)^4,
\]

(22)
$|U_{\tilde{\chi}^0_i}|^2$ is the percentage of the i-th species (photino, zino, higgsino) in the neutralino NLSP and $\frac{k_i}{\lambda}$ is the coupling constant of goldstino with matter fields. As will be seen, such decays are much slower than R-parity violating modes.

**R-parity violating two-body decays** The light neutral gaugino (wino or bino) does not directly couple to right-handed sneutrino (standard-model charge-free), so the higgsino component in NLSP controls its R-parity violating decays. If the neutralino NLSP is heavier than the $W$ or $Z$-boson, it can decay into $W^\pm + l^\mp$ or $Z^0 + \nu$. The corresponding Feynman-diagrams are shown in Fig. 5 and the decay rates are on the order of

$$
\Gamma(\tilde{\chi}^0 \rightarrow W^\pm l^\mp) \approx \frac{G_F m_N^3}{8\sqrt{2}\pi} |U_{\tilde{\chi}^0 \nu}|^2 \left(1 + \frac{2M_W^2}{m_{\tilde{\chi}^0}^2}\right) \left(1 - \frac{M_W^2}{m_{\tilde{\chi}^0}^2}\right)^2,
$$

$$
\Gamma(\tilde{\chi}^0 \rightarrow Z^0 \nu_L) \approx \frac{G_F m_N^3}{32\sqrt{2}\pi} |U_{\tilde{\chi}^0 \nu}|^2 \left(1 + \frac{2M_Z^2}{m_{\tilde{\chi}^0}^2}\right) \left(1 - \frac{M_Z^2}{m_{\tilde{\chi}^0}^2}\right)^2. \quad (23)
$$

![Feynman diagrams](image_url)

**FIG. 5**: A typical two-body (left) and three-body (right) PRV decay diagram of neutralino NLSP.

To make a rough estimate, we take $|U_{\tilde{\chi}^0 \nu}| = 10^{-6}$ and $m_{\tilde{\chi}^0} = 100 - 200$ GeV. We then find that $\Gamma_{2\text{-body}} \approx 10^{-13} - 10^{-12}$ GeV which corresponds to a neutralino lifetime of $\sim 10^{-11} - 10^{-12}$ sec. This will lead to a vertex displacement in the detector of about $0.1 - 1$ mm.

This decay rate is fast enough not to ruin the success of BBN, and slow enough to produce collider signatures such as vertex displacement at LHC. The displacement of secondary vertex where NLSP decays is around $c\tau \approx 1$ mm, which is observable within the detector.

**R-parity violating three-body decay** If the neutralino is lighter than $W^\pm + l^\mp$ or $Z^0 + \nu$ mass, the two-body RPV decay channels are forbidden. It can only decay into a three-body final state...
FIG. 6: Decay rates as a function of neutralino mass. Red curve is for $\tilde{\chi}^0 \to W^\pm l^\mp$, blue is for $\tilde{\chi}^0 \to Z^0 \nu$, and green is for $\tilde{\chi}^0 \to \nu_L l_L \bar{b}_R$, where we choose $|U_{\chi\nu}| \approx 10^{-6}$ and $|U_{\chi Z}| \sim O(1)$. We also choose $\tan \beta = 3$ and $m_{\tilde{\chi}} = 300$ GeV.

through a virtual $W$- or $Z$-boson or sparticles, which couples to two SM particles. The dominant three-body decay mode is going to $b + \bar{b} + \nu$ [25], as shown in Fig. 5. The three-body decay rates have been calculated in Ref. [26]. In Fig. 6, we plot the two- and three-body decay rates as a function of the neutralino NLSP mass. We see that the three-body decay has a longer lifetime and can hardly produce an observable vertex displacement inside the detector near the vertex.

B. Stau as NLSP

The stau can also serve as the NLSP in the model. Generally, there is a small mixing between left- and right-handed staus. The lighter stau $\tilde{\tau}_1$ is a linear combination of them

$$\tilde{\tau}_1 = \alpha \tilde{\tau}_L + \beta \tilde{\tau}_R ,$$

where $|\alpha|^2 + |\beta|^2 = 1$. From the discussions in Section IV, we know that at the R-parity violating vertices, the right-handed sleptons mainly couple to lepton final states, while left-handed sleptons can decay either leptonically or hadronically. Typical diagrams for stau decay are shown in Fig. 7.
The decay rates are
\[
\begin{align*}
\Gamma(\tilde{\tau}_L \rightarrow t\bar{b}) & \approx \frac{3G_FM^2m_{\tilde{\tau}_1}^2}{4\sqrt{2}\pi}|U_{\tilde{\tau}_L H^-}|^2 \left(1 - \frac{m_t^2}{m_{\tilde{\tau}_1}^2}\right)^2, \\
\Gamma(\tilde{\tau}_R \rightarrow \tau\bar{\nu}) & \approx \frac{G_FM^2m_{\tilde{\tau}_1}^2}{4\sqrt{2}\pi}|U_{\nu\bar{H}_0}|^2 \left(1 - \frac{m_{\tau}^2}{m_{\tilde{\tau}_1}^2}\right)^2,
\end{align*}
\]

where the mixing due to RPV mass-insertion can be estimated to be $|U_{\tilde{\tau}_L H^-}|, |U_{\nu\bar{H}_0}| \approx \frac{\alpha Y_{\nu}(l)}{\mu} \approx 10^{-6}$.

In Fig. 8, we plot the stau decay rate as a function of its mass. There is a kink at $m_{\tilde{\tau}_1} = m_t$ where a significant hadronic final-state decay channel opens. For stau mass less than 300 GeV, the decay rate is of the order of $10^{-13}$ GeV, which corresponds to a vertex displacement 1 mm.

Actually, the NLSP can also be left-handed sneutrino. The decay diagrams are similar to those of stau NLSP, and one the sneutrino has a similar decay rate to sleptons.
VII. ADDITIONAL COMMENTS

In this section, we discuss some other aspects of the model.

**Strong and SUSY CP** It has been pointed out\[29\] that the constraint of left-right symmetry restricts the mass matrices and phases in the model in such a way that it provides a solution to both the strong CP and SUSY CP problem. Key to solving the strong CP problem is the hermiticity of the quark mass matrices in the model. Once R-parity is broken, it is not clear that this property will hold. However we find that if the right handed sneutrino vev is aligned along the electron flavor, the induced phases in the Det $M_q$ are of order $(\frac{Y_e \langle \nu_e^c \rangle}{\mu})^2 \sim 10^{-10}$ which is below the bound on the $\theta$ provided by electric dipole moment of the neutron. SUSY CP problems are not affected by this phase.

**Decays of the doubly-charged Higgsino** One of the distinguishing features of the B-L=2 triplet Higgses is the presence of doubly charged Higgs fields. These are accompanied by their fermionic superpartners ($\tilde{\Delta}^{++}$). These particles can have masses in the sub-TeV to TeV range and may therefore be accessible to LHC. In the presence of R-parity breaking, these fermions will decay to $\tilde{\Delta}^{++} \rightarrow \tau^c t \bar{b}$ (as shown in Fig. 9). There are no standard model background for such decay modes.

![FIG. 9: A Feynman diagram for doubly-charged Higgsino decay.](image)

**R-parity breaking processes in the early universe** We now make a few more comments on the impact of R-parity violating in our model in the early universe. Note that due to the smallness of the strength of the $\epsilon_i LH_u$ term in the superpotential i.e. $\epsilon_i \sim 10^{-4}$ GeV, the effective $\lambda (LL\bar{c})$ and $\lambda' (QL\bar{d}^c)$ interactions are of order or less than equal to $10^{-8}$. It was noted in ref.\[28\], for such small strengths, the lepton number violating interactions involving sparticles are out of equi-
librium. As a result one could perhaps contemplate generating lepton asymmetry by the decays of the NLSP particle e.g. neutralino slightly above the electroweak scale and have them converted to baryons via the sphaleron effects. This question is under consideration.

VIII. CONCLUSION

In conclusion, we have discussed the implications of the minimal renormalizable supersymmetric left-right seesaw model as an extension of MSSM to include neutrino masses and show that the model leads to dynamical R-parity violation by its ground state in order to break parity symmetry. We have analyzed a particular realization of the model where spontaneous R-parity breaking occurs along the $\tilde{\nu}_e$ direction. We first show that small neutrino masses in this model can be understood only by the usual low scale seesaw condition that the Dirac Yukawa couplings are tuned to the value of $Y_\nu \lesssim 10^{-6}$ and without any further tuning. We then show that if gravitino is the LSP, then its decays are automatically suppressed by the same condition that guarantees the smallness of neutrino masses, making the gravitino long lived enough to be an unstable dark matter of the Universe. We also point out that the NLSP decays in this model may lead to displaced vertices which can provide a clear LHC signal. These models have many other collider implications such as doubly charged Higgs and Higgsino fields [30] that have been discussed extensively in the literature as well as low energy lepton number violating signals such as muonium-anti-muonium oscillations [31].

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Appendix

In this appendix, we discuss the minimization of the Higgs potential to obtain an upper bound on the right-handed scale $v_R$.

First we will show that in the SUSYLR model, if R-parity is preserved by the vacuum, parity symmetry cannot be broken i.e. $v_R = \bar{v}_R = 0$ as shown in [8]. A more precise statement perhaps
is that, if we express the potential, $V$ as functions of the vevs of the neutral Higgs fields ($v_R, \bar{v}_R$ and $x$ and look for a minimum of $V$ along the direction $x = 0$ (i.e. R-parity conserving), the minimum occurs at $v_R = \bar{v}_R = 0$. We then show that once we include R-parity breaking effect by the vacuum i.e. $\langle \tilde{\nu} \rangle \equiv x \neq 0$ i.e. along the direction $x \neq 0$, there appear global minima that break parity i.e. $v_R, \bar{v}_R \neq 0$ and also that it occurs only below a certain value for $v_R$ and $\bar{v}_R$ i.e. there is an upper limit on the parity breaking scale. This proves our assertion that R-parity breaking in this theory is a dynamical phenomenon.

To show this let us start with the potential in Eq. (2) which consists of the field vevs $v_R$ and $\bar{v}_R$ and $x \equiv \langle \tilde{\nu} \rangle$ and look for its minimum:

$$V = \left[ M_\Delta^2 v_R^2 + M_\Delta^2 \bar{v}_R^2 - 2B|v_R\bar{v}_R| \right] + \left[ f^2 x^4 - (2A_{v_R} + 2f \mu_\Delta \bar{v}_R - m_0^2 - 4f^2 v_R^2)x^2 \right] + \left[ \frac{g_{L}^2 + g_{BL}^2}{8} (x^2 - 2v_R^2 + 2\bar{v}_R^2)^2 \right],$$

(26)

We have set the $\Phi$ and $\Delta, \bar{\Delta}$ vevs to zero. Note that we have kept the $\langle \tilde{\nu} \rangle$ in the potential. For simplicity, we have set $f = f_1$.

The first point to note is that to ensure a lower bound on the potential, we must satisfy the conditions on the parameters:

$$M_\Delta^2 + M_{\bar{\Delta}}^2 \geq 2B,$$

$$M_\Delta M_{\bar{\Delta}} > B.$$  

(27)

The first constraint comes from looking at the direction $v_R = \bar{v}_R$ and $x = 0$ demanding that the potential is bounded from below. The second comes from looking along the QED breaking vacuum so that the D-terms vanish and setting $x = 0$ and again demanding positivity. Once these two conditions are imposed, for $x = 0$, the minimum of the potential corresponds to $v_R = \bar{v}_R = 0$ and hence no parity violation.

It is worth pointing out that the form of the potential for $x = 0$ is same as in the case of MSSM, where of course we know that symmetry breaking occurs. The difference in the case of SUSYLR is the observation [8] that there exist QED breaking directions along which for arbitrary $v_R$ and $\bar{v}_R$, the D-term vanishes so that one has the second condition in Eq. (27). In the case of MSSM, the second condition does not exist and in fact to break the gauge symmetry in SUSYLR case, one needs the opposite of the second condition i.e. $M_\Delta M_{\bar{\Delta}} < B$. 

21
In order to show that along the nonzero $x$ directions, one can indeed have a minimum that breaks the gauge symmetry, it is convenient to rewrite the potential in Eq. (26) as follows:

$$V = (f^2 + \frac{g^2}{8})[x^2 - C]^2 + D(v_R, \bar{v}_R),$$

(28)

where

$$D(v_R, \bar{v}_R) = \left[ M^2 v^2_R + M^2 \Delta v^2_R - 2|B|v_R\bar{v}_R + \frac{g^2}{2}(v^2_R - \bar{v}^2_R) \right] - \left( f^2 + \frac{g^2}{8} \right) C^2,$$

$$C = \frac{[2Av_R + 2f\mu\Delta\bar{v}_R - m_0^2 - 4f^2 v^2_R + \frac{1}{2}g^2 (v^2_R - \bar{v}^2_R)]}{2(f^2 + \frac{g^2}{8})},$$

(29)

where we define $\tilde{g}^2 = (g^2_{R} + g'^2)$. Advantage of rewriting this way is that we can now minimize with respect to $x$ very easily and get

$$x^2 = C.$$  

(30)

Since $x$ is a real number, the above equation implies that $C \geq 0$. For $v_R$ and $\bar{v}_R$ close to each other, the $C \geq 0$ condition turns into an upper limit on the $v_R$ scale of

$$\frac{(A + f\mu\Delta) - \sqrt{(A + f\mu\Delta)^2 - 4f^2m_0^2}}{4f^2} \leq v_R \leq \frac{(A + f\mu\Delta) + \sqrt{(A + f\mu\Delta)^2 - 4f^2m_0^2}}{4f^2}.$$  

(31)

This is clear from the expression for $C$ since for large $v_R$, the $-f^2v^2_R$ term in $C$ dominates making $C < 0$ and hence driving the vev of $x$ to zero in which case, the minimum corresponds to $v_R = \bar{v}_R = 0$ as noted and hence no parity violation. This is in accord with the observation of ref. [9] that the tree level potential for the minimal SUSYLR model requires R-parity violation if parity has to break, as required to get the standard model and the parity breaking scale has an upper limit in the TeV range, making the theory experimentally testable at LHC.

We have done a numerical analysis of the potential and find that the full potential indeed has a negative minimum value only when the $x \neq 0$ and $v_R \neq 0$ and $\bar{v}_R \neq 0$ giving the desired parity violating ground state. We also find from this numerical analysis that once we set $C \leq 0$ or equivalently $x = 0$, the ground state corresponds to $v_R = \bar{v}_R = 0$. In the table below, we give some numerical examples of solutions for the desired vacua for specific TeV scale parameters in the potential for the case when $x \neq 0$. We check that the potential has a minimum for each case with a value of the potential at the minimum which is negative so that it is indeed a global minimum.
TABLE I: The other parameters are chosen to be $M_\Delta = 1$ TeV, $M_\bar{\Delta} = 1.1$ TeV and $B = 1.099$ TeV$^2$.

Incidentally, along the $x = 0$ direction in the potential, there is also no electroweak symmetry breaking by similar arguments as above [8] i.e. $\langle \phi \rangle = 0$; however along the above $x \neq 0$ direction, one immediately gets $\langle \phi \rangle \neq 0$ also along with breaking of $SU(2)_R \times U(1)_{B-L}$ symmetry.

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